A remarkable experimental achievement of the 1980s and 1990s was the coherent coupling of a single atom existing in one of two states to a single photon trapped in a cavity. These results showed that it is possible to create a quantum superposition of light and matter, and that the spin degree of freedom has a coherence time that can potentially exceed that of superconducting qubits, and cavity photons can serve to effectively overcome the limitation of short-range interaction inherent to spin qubits. Here, we review recent advances in hybrid ‘super–semi’ quantum systems, which coherently couple superconducting cavities to semiconductor quantum dots. We first present an overview of the physics governing the behaviour of superconducting cavities, semiconductor quantum dots and their modes of interaction. We then survey experimental progress in the field, focusing on recent demonstrations of cavity quantum electrodynamics in the strong-coupling regime with a single charge and a single spin. Finally, we broadly discuss promising avenues of future research, including the use of super–semi systems to investigate phenomena in condensed-matter physics.
hybrid quantum systems integrate the most desirable properties of semiconductor spin qubits and superconducting quantum devices.

- Single electron charges can be coherently coupled to single microwave-frequency photons.
- Using spin–orbit interactions, a single electron spin can be coherently coupled to a single photon.
- Coherent charge–photon and spin–photon coupling may enable long-range qubit interactions that are mediated by microwave-frequency photons.
- Hybrid quantum devices are also finding utility as sensitive probes of Kondo and valley physics, and perhaps of Majorana fermions.

spins confined in semiconductor QDs. We begin by laying the theoretical groundwork underlying the experiments, with a description of the superconducting cavity, the ‘artificial atom’, which, in most experiments, consists of a semiconductor double quantum dot (DQD), their modes of interaction and the figures of merit that describe the quantum coherence of the system. We then survey experiments involving the charge degree of freedom, which interacts with the cavity electric field through the electric dipole interaction. A combination of electric dipole coupling and spin–orbit coupling enables coherent spin–photon interactions, which we discuss next. Lastly, we give examples of how semiconductor-circuit QED could impact fundamental science and engineering in areas that range from topological physics to surface microscopy and quantum technology.

Cavity QED with DQDs

At a basic level, a typical cavity QED system (Fig. 1a) consists of just two components: a photonic cavity and a two-level quantum system with closely matched resonant frequencies. The first demonstrations of cavity QED in atomic physics used microwave transitions between Rydberg states of single caesium atoms. These results were eventually extended to the visible spectral range.

Cavity QED can also be implemented using a wide variety of solid-state systems (Fig. 1b). Colour centres in diamond, such as nitrogen and silicon vacancy centres, have spin–full ground states and narrow, spin-selective microwave and optical transitions, which allows the realization of cavity QED with integrated photonic structures. Using nanofabrication techniques, it is possible to build mesoscopic semiconductor and superconducting devices that are quantum coherent. Semiconductor DQDs can be used to isolate single electrons; in these systems, the charge degree of freedom can be controlled with electric fields and the spin degree of freedom with magnetic fields and the exchange interaction through changes to the confining potential.

Superconducting circuits combine a capacitance and a Josephson inductance to create a quantum system with an anharmonic energy-level spectrum. Cavity QED experiments involving superconducting quantum devices are reviewed in refs. 1-4. In this Review, we focus on experiments involving semiconductor DQDs, whose electrical tunability opens the door to cavity QED using long-lived spin states.

Charge–photon interaction. A single electron trapped in a DQD forms a fully tunable, two-level system. Electric dipole interactions couple the electron trapped in the DQD to the cavity photon with a strength described by the charge–photon coupling rate \( g \). The interdot spacing is typically on the order of \( d = 100 \text{ nm} \), which leads to a large electric dipole moment \( \vec{d} \). The coupling rate is given by the product of this dipole moment with the root mean square of the vacuum electric field \( E_\text{vac} \) of the cavity. Near zero detuning, the charge states are strongly hybridized, leading to the maximum in the charge–photon coupling rate. Away from zero detuning, the charge states are only weakly admixed.

In atomic physics, Fabry–Pérot cavities are typically employed to trap optical photons. For the much larger quantum-dot devices, typical energy scales are on the order of 20–40 µeV, so it becomes convenient to use superconducting resonators to trap microwave-frequency photons (1 GHz ~ 4.2 µeV). Cavities are never perfect and can have internal losses, described by a decay rate \( \kappa_\text{int} \) and losses through the cavity ports, \( \kappa_L \) and \( \kappa_R \). Cavity QED systems can be conveniently probed by measuring the transmission through (or reflection from) the cavity. For example, in Fig. 1a, port 1 is driven by a weak input field \( \alpha_\text{in} \) and the signal \( \alpha_\text{out} \) exiting port 2 is measured.

When a DQD is placed inside a superconducting microwave resonator, the electric field \( E_\text{res} \) inside the resonator tilts the energy landscape and the difference \( \epsilon \) between the energy levels of the left and right dots becomes \( \epsilon + \epsilon E_\text{res}/d \). Because \( d \) is much smaller than the wavelength of the electromagnetic waves inside the resonator, we can apply the electric dipole approximation and consider \( E_\text{res} \) constant within the entire volume of the DQD. The quantized electric-field operator can be expressed in terms of creation and annihilation operators \( a \) and \( a^\dagger \) of the electromagnetic field mode inside the resonator (these are equivalent to the ladder operators of the quantum harmonic oscillator), so that \( E_\text{res} = E_\text{vac}(a + a^\dagger) \). The coupling of the single electron trapped in the DQD (which forms a charge qubit, BOX 1) to the resonator mode is described by the Hamiltonian

\[
H = H_\text{c} + H_\text{int} + H_\text{res} \quad \text{with} \quad H_\text{int} = g(a + a^\dagger)\tau_z
\]

in units in which \( h = 1 \), with the charge-cavity coupling \( g = eE_\text{res}/d \) and the quantum operator \( \tau_z \) defined via \( \tau_z |0,0\rangle = |0,0\rangle \) and \( \tau_z |1,0\rangle = -|1,0\rangle \). The electric dipole of the DQD with one electron, \( \epsilon \tau_z \), can be probed via the microwave transmission through the cavity. Theoretically, this means that the DQD and cavity need to be treated as an open quantum system. The transmission can be efficiently calculated using input–output theory (BOX 2).

Typical experiments operate in a regime in which the charge–cavity coupling \( g \) is a weak perturbation compared with the bare energy scale of the qubit \( \Omega \). In this operating regime, it is convenient to first diagonalize \( H_\text{c} \) and then transform \( H_\text{int} \) into the eigenbasis of \( H_\text{c} \). Transforming into a frame rotating with the probe field frequency and neglecting fast oscillating terms within the rotating-wave approximation, one finds

\[
\frac{\Delta}{\tau} = f_\text{c} - f_\text{g}, \quad \langle f_\text{c} \rangle \text{ is the probe frequency; often } f_\text{c} = f_\text{g}, \text{ and, thus, } \Delta = 0. \quad \tau_\text{c} \text{ and } \tau_\text{g} \text{ are the raising and lowering operators.}
\]
Cavity quantum electrodynamics

\[ \chi = \frac{g_c}{-\Omega + i\gamma/2}. \]

For simplicity, we can consider a symmetric cavity without intrinsic losses (\(\kappa_{\text{int}} = 0\)), such that \(\kappa_1 = \kappa_2 = \kappa/2\). The cavity is also often probed on resonance (\(\Delta = 0\)). In the absence of a DQD, \(\chi = 0\) and microwaves are transmitted unhindered through the cavity (\(\chi = 1\)). Charge dynamics within the DQD results in an effective microwave admittance that loads the superconducting cavity, changing the cavity amplitude and phase response\(^{66-72}\). The electric susceptibility \(\chi\) is greatest (and, thus, \(\chi, A, \kappa, d\) smallest) for a symmetric DQD (\(\varepsilon = 0\)), because, in this configuration, the electron is most easily transferred between dots and the dipole moment is maximized.

**Quantum-coherent charge–photon coupling.** The scale of the susceptibility, \(\varepsilon \ll \kappa\), and transmission, \(A \gg 1/g^2\) (assuming \(\kappa \ll g\) and \(\varepsilon = 0\) in the case of a DQD), are both determined by the electric dipole and vacuum cavity electric field via the electron–dipole coupling strength \((g = E/\ell_d)\). For a Rydberg atom in an optical cavity, \(E_0 \approx 1 \text{ mV m}^{-1}\) and \(d \approx 100 \mu\text{m}\), leading to couplings \(g\) roughly in the 10–100 kHz range\(^{73}\). This coupling can, in principle, be strengthened in two ways: by increasing the electric dipole through an increase in the DQD size or by increasing the vacuum electric field \(E_0 = \sqrt{\langle n \rangle} / 2 \varepsilon_0 V\), where \(V\) is the cavity mode volume. Although superconducting circuit microwave resonators typically have a slightly lower resonance frequency than 3D cavities for cavity QED based on Rydberg atoms, their mode volume can be thousands of times smaller than that of 3D cavities\(^{65-67}\). The vacuum electric field can, therefore, be several orders of magnitude stronger in superconducting resonators, which allows for qubit–resonator couplings of \(\frac{g}{\ell_d} \approx 1 - 10 \text{ MHz for QDs} (d \approx 100 \text{ nm})\) and \(\frac{g}{\ell_d} \approx 10 - 100 \text{ MHz for superconducting qubits} (d \approx 1 \mu\text{m})\). Crucially, such large values of \(g\), can easily exceed both the cavity linewidth \(\kappa\) and qubit decoherence rate \(\gamma\). The limit in which \(g \gg (\gamma, \kappa)\) is called the strong-coupling regime of cavity QED\(^{74}\). Achieving strong coupling is significant because the qubit and photon degrees of freedom become directly entangled with each other\(^1\). In addition to being of fundamental interest, this entanglement can be exploited for applications in quantum information science\(^75\).

**Experimental demonstrations**

Hybrid quantum devices comprising gate-defined QDs coupled via their electric dipole moment to microwave cavities have been successfully demonstrated using multiple material systems, including GaAs/AlGaAs heterostructures\(^{11,26-28}\), InAs nanowires\(^{77}\), graphene\(^{13,14}\), carbon nanotubes\(^{15,14,84}\), and Si/SiGe heterostructures\(^{16,14,43,53}\). The microwave cavity is often realized as a superconducting coplanar waveguide resonator\(^{21,11,14,22}\) (left panel of FIG. 2a). To maximize the quality factor of the cavity and the chance of reaching the strong-coupling regime, each gate line leading to the DQD is sometimes filtered by an...
Double quantum dots

A quantum dot (QD) is a nanoscale object that confines an electron in all three spatial dimensions. Single quantum dots are described by the electrostatic charging energy $E_q = e^2/2C$, which is the energy cost associated with adding or removing an electron from the system. $e$ is the elementary charge of an electron and $C = \pi \epsilon_0 / \lambda^2$ denotes the capacitance of the QD, with $\epsilon_0$ the permittivity of free space and $\lambda$ the radius of the QD. The orbital particle-in-a-box energy scale is governed by $E_{\text{orb}} = h^2 / 2m^* \lambda^2$, where $h$ is the reduced Planck’s constant and $m^*$ the effective mass of the electron. Both energy scales are set by the physical dimensions of the dot ($a_0$) and its materials parameters ($m^*$ and $\epsilon_0$), and are, therefore, difficult to change in situ. Fortunately, it is possible to create an artificial molecule by placing two QDs next to each other and forming a double quantum dot (DQD). In semiconductor DQDs, a single electron is confined to two boxes that are capacitively coupled via a small capacitance $C$.

The energy-level separation $\epsilon$ and the interdot tunnelling rate $t$ can be electrically tuned. A DQD containing a single electron can be viewed as a charge qubit, a voltage-tunable double-well potential containing a single charge (part a of the figure). The interdot spacing is typically on the order of $d = 100 \text{ nm}$, which leads to a substantial electric dipole moment. The charge physics of a DQD is described by the Hamiltonian

$$H_0 = \frac{\epsilon}{2} a^2 + t c_s a c,$$

which is written in the basis $|L\rangle = |(1, 0)\rangle$, $|R\rangle = |(0, 1)\rangle$, where $|N_L, N_R\rangle$ denotes a DQD charge state with $N_L$ and $N_R$ electrons occupying the left and right dot, respectively. In other words, $|L\rangle$ and $|R\rangle$ describe two charge states in which the electron is in the left or right dot, respectively. If the right and left dot energy levels are aligned ($\epsilon = 0$), then the $|L\rangle$ and $|R\rangle$ states hybridize to form molecular bonding and antibonding states $|\pm\rangle \propto |L\rangle \pm |R\rangle$, whereas at large detuning ($|\epsilon| \gg t$) the states $|L\rangle$ and $|R\rangle$ are essentially unperturbed by tunnelling. The energy of the two levels is plotted as a function of $\epsilon$ in part c of the figure. The states $|\pm\rangle$ allow for electric dipole transitions around $\epsilon = 0$ and play the role of the atomic levels for cavity quantum electrodynamics. As the detuning parameter is increased from zero, the effective dipole moment is reduced by a factor of $2 \epsilon / 4 \epsilon + 4 t^2$, and becomes negligible for large detunings. To take account of the fact that an electron is endowed with a spin-1/2 degree of freedom, the basis can be extended to $|\pm \uparrow/\downarrow\rangle = |(1, 0), (1, 0\uparrow, 0\downarrow), (0, 1\uparrow, 0\downarrow)\rangle$, where the arrow indicates the spin state of the electron.

A simplified schematic of the hybrid device is depicted in the middle panel of Fig. 2a. At the fundamental resonance frequency $f_c$, the vacuum fluctuation of the cavity generates a high transconductance $g_\text{c}$, which commonly fall between a few hundred MHz and several GHz. The charge–photon coupling regime for DQD charge qubits is generally described by a linear array of superconducting quantum interference devices (SQUIDs) made from Al Josephson junctions, leading to $f_c \approx 1.5 \text{ kHz}$ by virtue of the large Josephson inductance of each SQUID. Another approach, discussed in the next section, utilizes the large kinetic inductance of a nanowire made from NbTiN.

Strong charge–photon coupling. Achieving the strong–photon coupling regime for DQD charge qubits is generally challenging owing to their large decoherence rates $\gamma$, which commonly fall between a few hundred MHz and several GHz. These values often exceed the superconducting transition frequency $f_c$ by one or more orders of magnitude. Therefore, a significant reduction in $\gamma$ is needed to access the strong–photon coupling regime $g_\text{c} > (\gamma, \kappa, \kappa)$. Both approaches have recently been successful in two experiments that we discuss here.

A first step towards the determination of the charge–photon coupling rate is the detection of the charge states within the DQD. This is traditionally accomplished by measuring the conductance of a proximal quantum point contact that is sensitive to the charge distribution within the QDs. Charge-state detection may also be performed by measuring the transmission properties of the cavity, which are sensitive to the tunnel-rate-dependent complex admittance of the QDs. An example is shown in Fig. 2c, where the cavity transmission amplitude $A/A_0$ ($A_0$ is a normalization constant) is fixed at a constant frequency $f = f_c$, which is measured as a function of gate voltages $V_N_1$ and $V_N_2$ (see Si/SiGe in Fig. 2b), which control the chemical potentials of the QDs. A charge–photon coupling rate $g_\text{c} > (\gamma, \kappa, \kappa)$ is needed to access the strong–photon coupling regime $g_\text{c} > (\gamma, \kappa, \kappa)$. Both approaches have recently been successful in two experiments that we discuss here.

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on-chip, low-pass LC-filter to suppress photon leakage from the cavity.

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**Input–output theory primer**

A system formed by a cavity and a double quantum dot can be accurately described using techniques developed to describe open quantum systems\(^{6,10,47}\). In this formulation, we break up the total Hamiltonian \( H = H_0 + H_R + V_c \) into a system Hamiltonian \( H_0 \) for the double quantum dot, its surrounding environment and a single mode of the cavity and bosonic operator \( a \), a reservoir Hamiltonian \( H_R \) describing a bath of electromagnetic modes \( b(f) \) for each port (‘mirror’) coupled to the cavity mode and an interaction Hamiltonian \( V_c \) that couples the cavity mode to the reservoir.

\[
V_c = \hbar \sum_n \int d\lambda_n(f) [b_n(f)a^\dagger + a b_n^\dagger(f)]
\]

Under the condition that the coupling constants \( \lambda_n(f) \) are approximately independent of the frequency \( f \) over the frequency range of interest, we can treat the reservoir as a Markovian bath. Formally integrating the Heisenberg equation of motion for \( b_n(f,t) \) starting from an initial time \( t_0 < t \), we arrive at closed Heisenberg equations of motion for \( a(t) \):

\[
a(t) = \frac{i}{\hbar} [H_0, a^\dagger] - \frac{\kappa}{2} a(t) + \sum_n \sqrt{\kappa_n} a_{n,\text{in}}(t),
\]

\[
a_{n,\text{in}}(t) = \sqrt{2\pi} \int df b_n(f) e^{-2\pi f(t - t_0)},
\]

where \( \kappa = \kappa_{\text{int}} + \sum_n \kappa_n \) is the total cavity decay rate including intrinsic loss \( \kappa_{\text{int}} \), \( \kappa_n = 2\pi |\lambda_n(f)|^2 \) the decay rate through port \( n \) of the cavity and \( a_{n,\text{in}}(t) \) the ‘input’ field incident on port \( n \) of the cavity. Applying a boundary condition on \( b_n(f,t) \) at \( t > t_0 \) gives rise to a similar equation for \( a(t) \) in terms of ‘output’ fields \( a_{n,\text{out}}(t) \). The input and output fields have the simple relation

\[
a_{n,\text{out}}(t) = \sqrt{\kappa_n} a(t) - a_{n,\text{in}}(t),
\]

which allows for a complete description of the cavity response. In the main text, we consider a two-port system with an input field on port 1 and \( (a_{1,\text{in}}(t)) = 0 \) (FIG. 1a). The measured transmission coefficient \( A \) is then given by the ratio of the port 2 output field \( (a_{2,\text{out}}(t)) = \sqrt{\kappa_2} (a(t)) \) to the port 1 input field \( (a_{1,\text{in}}(t)) \).

**Spin–photon coupling**

The quantum coherence of the spin-\( \frac{1}{2} \) of individual electrons in QDs or defects in silicon typically lasts between tens of microseconds and several millisecond\(^{6,9,12} \) and can, in some cases, even approach a second\(^{13} \), whereas nuclear spin coherence times approach 1 minute\(^{113} \). Spin is, therefore, the primary choice as a qubit for quantum information processing in semiconductors\(^{17,56} \). Because the exchange interaction is short ranged\(^{54} \), this naturally leads to the question of how to couple two electron spins that are separated by a large distance using spin–electric coupling to a common cavity mode. This might seem extremely challenging because the spin of an electron does not directly couple to the electric field of the cavity. However, there are several techniques to hybridize the spin and charge degrees of freedom (qubits) of an electron. All these methods endow the spin with an effective electric dipole that enables its interaction with the electric field of the cavity. For multi-electron spin qubits, the Pauli exclusion principle provides a way to couple orbital and spin degrees of freedom\(^{12,14,115} \) (a recent experiment using this method has attained strong spin–qubit–photon coupling\(^{17} \)). One mechanism that works for single
electron spins is the natural built-in spin–orbit coupling due to relativistic effects, which is sizeable in a number of semiconductor materials\(^{3,16,17}\). The intrinsic spin–orbit coupling may work particularly well for holes in the valence band of some semiconductors\(^ {18}\). Without relying on such intrinsic effects, one can engineer a spin–electric interaction using controlled magnetic fields, either time-dependent fields that induce electron spin resonance\(^{11,19–21}\) or static but spatially varying fields produced by an on-chip microscale ferromagnet\(^ {42,43,111,122–127}\).

In the case of a static magnetic field gradient \(\nabla B\) produced by a micromagnet, an applied electric field \(E_a\) shifts the electron position in a single QD by

\[
x_e = \frac{e E_a a_s^2}{E_{orb}}.
\]

where \(E_{orb}\) and \(a_s\) denote the energy-level spacing and size of the QD, respectively. For an oscillatory electric field, this means that the magnetic field seen by the electron also becomes oscillatory, \(B(x_e + x_s \sin \omega t)\), allowing for electric dipole spin resonance. For the quantized cavity-field, \(E_{\text{cav}} \sin \omega t\) is replaced by \(E_{\text{cav}} = E_a(a + a')\), resulting in a spin–photon coupling \(g_s \sim \frac{e^2 \hbar}{E_{\text{cav}}} a_s a_s^2\). The spin–phonon coupling \(g_p\) in a DQD can be much larger and more controllable than in a single QD\(^ {128,129}\).

To study the combined charge and spin dynamics of a single electron in a DQD, one can employ the 4 × 4 Hamiltonian in the basis \(\{|1,0\}, \{|1,1\}, \{|0,1\}, \{|0,0\}\}\):

\[
H_0 = \frac{1}{2} \begin{pmatrix}
\varepsilon + B_z & \Delta B_x & 2t_c & 0 \\
\Delta B_x & \varepsilon - B_z & 0 & 2t_c \\
2t_c & 0 & -\varepsilon + B_z & -\Delta B_x \\
0 & 2t_c & -\Delta B_x & -\varepsilon - B_z
\end{pmatrix}
\]

which includes the Zeeman coupling \(H_Z = \mathbf{S} \cdot \mathbf{B}(r)\) of the spin \(\mathbf{S}\) to an external magnetic field \(\mathbf{B}(r)\) (in energy units)\(^ {122}\). A magnetic field \(B_z\) pointing in the \(z\)-direction leads to an energy splitting between the spin-up and spin-down states.

### a) Cavity-coupled double quantum dots

The coplanar waveguide cavity is located in the middle of the sample and is coupled to measurement ports through finger capacitors located at each end of the cavity (upper inset). A gate-defined double quantum dot (DQD) is positioned at an antinode of the cavity electric field. Low-pass LC-filters with inductance \(L_c\) and capacitance \(C_c\), represented by the circuit diagram in the lower inset, allow for dc biasing of the DQD gate electrodes and reduce photon losses from the cavity. Middle: schematic representation of the device. A half-wavelength standing wave is formed in the coplanar waveguide cavity and couples to a single electron trapped in the DQD via the electric dipole interaction. In a typical experiment, port 1 of the cavity is driven by a coherent microwave field \(a_{\text{mic}}\), and the signal exiting port 2 of the cavity \(a_{\text{out}}\) is measured. \(\kappa_1\) and \(\kappa_2\) denote the coupling rate between the cavity and port 1 and port 2, respectively. Right: circuit representation of the device. Here, the microwave cavity is modelled as a parallel LC-oscillator with an effective inductance \(L_c\) and capacitance \(C_c\). The DQD is modelled as a pair of charge islands with a mutual capacitance \(C_m\), and dot 2 is coupled to the LC-oscillator through a capacitance \(C_c\). \(\kappa_1\) and \(\kappa_2\) are set by the port capacitance \(C_{g1}\) and \(C_{g2}\) respectively.

### b) Double quantum dot materials systems

Fig. 2 | Constructing cavity-coupled double quantum dots. a) Left: optical image of a Nb coplanar waveguide cavity fabricated on top of a Si/SiGe heterostructure. The coplanar waveguide cavity is located in the middle of the sample and is coupled to measurement ports through finger capacitors located at each end of the cavity (upper inset). A gate-defined double quantum dot (DQD) is positioned at an antinode of the cavity electric field. Low-pass LC-filters with inductance \(L_c\) and capacitance \(C_c\), represented by the circuit diagram in the lower inset, allow for dc biasing of the DQD gate electrodes and reduce photon losses from the cavity. Middle: schematic representation of the device. A half-wavelength standing wave is formed in the coplanar waveguide cavity and couples to a single electron trapped in the DQD via the electric dipole interaction. In a typical experiment, port 1 of the cavity is driven by a coherent microwave field \(a_{\text{mic}}\), and the signal exiting port 2 of the cavity \(a_{\text{out}}\) is measured. \(\kappa_1\) and \(\kappa_2\) denote the coupling rate between the cavity and port 1 and port 2, respectively. Right: circuit representation of the device. Here, the microwave cavity is modelled as a parallel LC-oscillator with an effective inductance \(L_c\) and capacitance \(C_c\). The DQD is modelled as a pair of charge islands with a mutual capacitance \(C_m\), and dot 2 is coupled to the LC-oscillator through a capacitance \(C_c\). \(\kappa_1\) and \(\kappa_2\) are set by the port capacitance \(C_{g1}\) and \(C_{g2}\) respectively. b) Scanning electron micrographs of cavity-coupled DQDs fabricated from a variety of host materials, including GaAs, InAs, graphene, carbon nanotubes and Si/SiGe. The electrode connected to the microwave cavity is indicated for each material. Panel 1, GaAs, adapted with permission from Ref.\(^ {18}\), AAAS. Panel 2, GaAs, adapted with permission from Ref.\(^ {18}\), AP; InAs, adapted from Ref.\(^ {18}\), CC BY 4.0 (https://creativecommons.org/licenses/by/4.0/); graphene, adapted with permission from Ref.\(^ {18}\), ACS; carbon nanotube, adapted from Viennot et al. Stamping single wall nanotubes for circuit quantum electrodynamics. Appl. Phys. Lett. \(104\), 113108 (2019)\(^ {18}\), with the permission of AIP Publishing.
spin-down states. As long as the field has the same strength in both dots, that is, \( \mathbf{B} \) does not depend on the position \( \mathbf{r} \), the spin and charge qubits are completely separate. In this case, only the charge qubit interacts with the electric field of the cavity, whereas the spin qubit is decoupled from it. However, as soon as a magnetic field perpendicular to the homogeneous field component is applied, the charge and spin qubits are hybridized, allowing for a coupling of the spin qubit to the cavity electric field. The coupling to the cavity is again obtained by replacing \( \varepsilon \) with \( \varepsilon + e \mathbf{E}_\text{cav} \cdot \mathbf{r} \). In this way, we obtain the Hamiltonian \( H = H_0 + H_{\text{int}} \) as discussed above. The four relevant energy levels \( |n\rangle \) of the DQD are found by diagonalizing the matrix \( H_0 \), whereas the electric dipole transition matrix elements \( d_{nm} \) can be determined by transforming \( H_{\text{int}} \) into the eigenbasis of \( H_0 \):

\[
H_{\text{int}} = g_n (a + a^\dagger) \sum_{n,m} d_{nm} |n\rangle \langle m|.
\]

For the understanding of the most important mechanisms for the spin–photon interaction, it is sufficient to consider an effective two-level model. Introducing the rotating-wave approximation, one arrives at the Jaynes–Cummings model \( H = \frac{1}{2} \Delta \sigma_z + g_n (a + a^\dagger) \sigma_z \), where the Pauli operators \( \sigma_z \) act on the low-energy hybridized spin states, and \( \Delta = B_z -hf_z \) is the detuning of the spin Zeeman splitting \( B_z \) from the photon energy \( hf_z \). For a symmetric DQD with \( \varepsilon = 0 \), one finds a spin–photon coupling rate \( g_n \approx \frac{\Delta hf_z}{\hbar} \), where \( \delta = 2t_c - hf_z \) can be controlled.

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**Fig. 3 | Strong charge–photon coupling.** a | Top: the charge-stability diagram of a double quantum dot (DQD) can be extracted by measuring the cavity transmission amplitude \( A/A_0 \) as a function of the gate voltages \( V_{P1} \) and \( V_{P2} \). Dot–lead transitions correspond to the exchange of an electron between the source or drain reservoirs and the DQD. Interdot charge transitions correspond to the transfer of an electron between dots and are also visible in the data. \((N_1/N_2)\) denotes a charge state with \( N_1 \) electrons in dot 1 and \( N_2 \) electrons in dot 2. Bottom: \( A/A_0 \) measured in the vicinity of the \((1,0)\)–\((0,1)\) interdot transition (area highlighted by the rectangle in the top panel) after increasing the interdot tunnel coupling \( t_c \) and with \( 2t_c/h < f_s \). The white lines indicate the eigenenergies of the Jaynes–Cummings Hamiltonian describing the charge–photon system when \( g_n / 2\pi = 6.7 \) MHz (solid lines) or 0 MHz (dashed lines). Bottom left: \( A/A_0 \) as a function of \( f \) when \( \varepsilon = 6 \) μeV and \( \varepsilon = 0 \) μeV. The dashed lines are fits to cavity input–output theory. Bottom right: cavity reflectance spectrum \( |S_{11}| \) of a GaAs-based device taken with \( 2t_c/h < f_s \). Panel b is adapted with permission from [REF\( ^{11} \)] (top and bottom left), AAAS and [REF\( ^{11} \)] (bottom right), APS.
by adjusting the DQD tunnel coupling $t_c$. Although this two-level model explains the vacuum Rabi splitting that has been observed experimentally, there are more subtle effects such as the asymmetry of the Rabi peak heights that require a three-level model to be explained\textsuperscript{122}.

**Strong spin–photon coupling.** Compared with charge–photon coupling, reaching the strong-coupling regime of spin–photon interaction faces a distinct challenge: the direct magnetic-dipole coupling rate $g_B$ between a single electron spin and a single photon is mostly limited to 10–500 Hz, which is too slow to overcome single-spin dephasing rates or cavity-loss rates\textsuperscript{121,135}. At the same time, a low level of charge noise is required of the device because spin–charge hybridization subjects the electron spin to charge-noise-induced spin dephasing. A cavity-coupled carbon nanotube DQD has been used to hybridize spin and charge via ferromagnetic leads to achieve $g_B/2\pi = 1.3$ MHz but remained in the weak-coupling regime, owing to a relatively large spin decoherence rate $\gamma_s/2\pi = 2.5$ MHz\textsuperscript{138}. More recently, two experiments using Si-based DQDs have successfully attained the strong-coupling regime between a single spin and a single photon\textsuperscript{125,143}. In this section, we review these experiments, as well as a separate experiment that coupled a three-electron spin state in a GaAs triple QD to a single photon\textsuperscript{44}.

It is remarkable that strong spin–photon coupling via spin–charge hybridization does not require strong charge–photon coupling. This follows from the different scaling of the spin–photon coupling $g_B$ and induced spin decoherence rate $\gamma_s$ on the degree of spin–charge hybridization controlled by the ratio of magnetic field gradient $\Delta B_z$ and the controllable energy detuning $\delta = 2\mu\nu - \hbar f_s$, whereas $\frac{\gamma_s}{2\pi} \propto \frac{\Delta B_z}{\delta}$ is linear in the spin–charge hybridization $\gamma_s \propto \frac{\Delta B_z}{\delta}$, thus allowing for $\frac{\gamma_s}{2\pi} \gg \frac{\Delta B_z}{\delta}$ at sufficiently small $\frac{\Delta B_z}{\delta}$.
The setup for the first experiment we discuss is illustrated in Fig. 4a. The device is a gate-defined DQD on top of a Si/SiGe heterostructure coupled to a coplanar waveguide cavity, similar to devices used in previous work on strong charge–photon coupling but including a micron-sized Co magnet on top of the DQD gate electrodes. The micromagnet creates a large magnetic field gradient, such that the quantization axis of the electron spin is dependent on its location, hybridizing the spin and charge degrees of freedom. A plot of the DQD energy levels including the spin degree of freedom is provided in Fig. 4b.

To search for spin–photon coupling, the frequency of the single-spin qubit $E_s/h = g_\mu_B B_{ext}/h$ is tuned into resonance with the cavity by changing the external magnetic field $B_{ext}$. Here, $E_s$ is the Zeeman energy, $g$ the $g$-factor of the electron and $\mu_B$ the Bohr magneton. $B_{ext}$ is the total magnetic field spatially averaged over the electron’s wave function, having contributions from both the externally applied field and the intrinsic field of the micromagnet. The cavity transmission amplitude as a function of $f$ and $B_{ext}$, taken at $\varepsilon = 0$, is shown in Fig. 4c. A clear avoided crossing is observed around $B_{ext} = 92.2$ mT, where the resonance condition $E_s/h = f_c$ is met. A vacuum Rabi splitting $2g/2\pi = 11.0$ MHz is obtained, indicating strong coupling between the single electron spin and a cavity photon. The spin–photon coupling rate $g/2\pi = 5.5$ MHz exceeds direct magnetic-dipole coupling rates by four to five orders of magnitude $\gamma^{16,15}$. A second experiment, also involving a DQD defined on a Si/SiGe heterostructure, used a high-impedance cavity design composed of a thin NbTiN nanowire with a large kinetic inductance $\gamma^{15}$. Strong spin–photon coupling was also achieved in this device, as shown by the avoided crossing in Fig. 4d.

Possibilities to circumvent the requirement for a strong magnetic field gradient generated by an on-chip micromagnet include the use of materials with strong spin–orbit coupling, such as holes in germanium $\gamma^{14,16}$, or the use of the Pauli exclusion principle in multi-spin qubits, such as the resonant-exchange qubit $\gamma^{15}$. The resonant-exchange qubit, like other exchange-only qubits, uses a degenerate spin subspace ( decoherence-free subspace) of three or more electrons in which all-electrical control is possible $\gamma^{14,15,14}$. The idea is based on the availability of electron spin states $\gamma^{67}$. The charge susceptibility of a two-electron double quantum dot (DQD) depends on the electron spin configuration, leading quite naturally to cavity-based readout of electron spin states $\gamma^{7}$. Cavity quantum electrodynamics (QED) has been adapted with permission from Ref. $\gamma^{114}$, APS. Panel $\gamma$ is adapted from Shim et al. Induced quantum dot probe for material characterization. Appl. Phys. Lett. $\gamma^{114}$, 152105 (2019) $\gamma^{166}$, with the permission of AIP Publishing. Panel $\delta$ is adapted from Hartke et al. Microwave detection of electron-phonon interactions in a cavity-coupled double quantum dot. Phys. Rev. Lett. $\gamma^{112}$, 097701 (2018) $\gamma^{114}$.
of two qubit states with identical spin quantum numbers (for example, $s = 1/2$ and $s_0 = +1/2$), between which spin-conserving, electrically controlled exchange interactions can operate. An experiment with a resonant-exchange qubit formed in a triple QD in GaAs filled with three electrons and embedded into a NBTiN high-impedance cavity demonstrated a coupling between the resonant-exchange qubit and cavity photons of $g/2\pi = 31$ MHz. With a qubit decoherence rate of $\gamma/2\pi = 20$ MHz and a cavity decay rate of $\kappa/2\pi = 47$ MHz, this experiment achieved the strong-coupling regime, resolving the two vacuum Rabi peaks, because $2g > \kappa/2 + \gamma$. The resonant-exchange qubit offers an extended tunability of $g$, and $\gamma$ via two electrostatic control parameters that control the dipolar (linear tilt) and quadrupolar (centre dot energy) component of the triple-dot potential. By tuning these two parameters, the resonant-exchange qubit can be operated at a sweet spot where the qubit energy splitting becomes insensitive to small electrostatic fluctuations due to external charge noise.

To apply spin–photon cavity QED devices to quantum-information-processing tasks, such as the implementation of long-range two-qubit gates, it is necessary to rapidly switch on and off the spin–photon coupling rate $g$. This allows the spin qubit to be manipulated in an isolated state ($g = 0$), in which it is protected from cavity-induced Purcell decay and read out via the cavity when the coupling is back on. One way to tune $g$, is by tilting the DQD potential (Fig. 4c). As $\varepsilon$ is increased from zero, a strong decrease of spin–photon coupling from $g/2\pi = 5.5$ MHz ($\varepsilon = 0$) to $g/2\pi \ll 1$ MHz ($\varepsilon = 40\mu$V) is observed. This change is due to the fact that, with $|e| \gg \hbar$, the electron wave function becomes strongly localized within one dot (Fig. 4b), and interdot tunnelling is largely suppressed. Here, the displacement of the electron wave function by the cavity photon is limited to about 3 pm. The effective magnetic field generated by a cavity photon is, therefore, very small, effectively turning off the spin–photon coupling.

**Conclusions**

The strong and controllable coupling between individual spin qubits embedded in a superconducting microwave resonator allows for long-distance spin–spin coupling mediated by microwave photons. Resonant interactions between two spins separated by 4 mm have recently been achieved using circuit QED. With device improvements, this coupling could be employed to perform two-qubit gates between spins separated by several millimetres. One should keep in mind that 1 mm is a very long distance compared with the 80-nm separation of nearest-neighbour spin qubits in Si. Their small footprint on a semiconductor chip is one characteristic feature of semiconductor spin qubits, which makes them strong contenders for a scalable quantum-information-processing platform. In addition to enabling the entanglement of distant spin qubits, non-local two-qubit gates may facilitate quantum error correction in the framework of a fault-tolerant quantum-computing architecture. The possibility of creating a network of spin qubits with engineered coupling may be very useful for realizing a surface code.

Moreover, the possibility of creating a network of spin qubits with coupling geometries ranging from local to ‘all-to-all’ opens interesting perspectives for the quantum simulation of interacting quantum many-body systems. Spin–photon coupling has important implications beyond the generation of long-range quantum entanglement (Fig. 5a). The coupling of the electron spin to an electromagnetic cavity also allows for the dispersive readout of the quantum state of a spin qubit and lays the foundation for the development of quantum non-demolition and single-shot readout methods. Because the spin–photon coupling rate $g$, depends strongly on the detuning $\varepsilon$, electrically switching on the cavity coupling of each spin qubit may allow for selective readout in large arrays of spin qubits (Fig. 5b).

Moreover, the superconducting qubit community has adopted the use of frequency multiplexing for quantum-state readout of multi-qubit devices. A similar approach could be adopted for spins.

Looking beyond spin qubits, the nascent field of hybrid-circuit QED could have a major impact on condensed-matter physics as a whole. Cavity measurements have been proposed to investigate Majorana modes and provide an alternative to the somewhat ambiguous measurements of zero-bias conductance peaks. It has even been suggested that microwave cavities could be used to detect the braiding of Majorana fermions and for quantum-state readout of multi-qubit devices. A similar approach could be adopted for spins.

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G.B.M. and X.M.J.R. researched data for the article and discussed the content. All authors contributed to the writing and editing of the paper.

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