Hořava-Lifshitz Dark Energy

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Abstract: We formulate Hořava-Lifshitz cosmology with an additional scalar field that leads to an effective dark energy sector. We find that, due to the inherited features from the gravitational background, Hořava-Lifshitz dark energy naturally presents very interesting behaviors, possessing a varying equation-of-state parameter, exhibiting phantom behavior and allowing for a realization of the phantom divide crossing. In addition, Hořava-Lifshitz dark energy guarantees for a bounce at small scale factors and it may trigger the turnaround at large scale factors, leading naturally to cyclic cosmology.
1. Introduction

Recently, a power-counting renormalizable, ultra-violet (UV) complete theory of gravity was proposed by Hořava in [1, 2, 3, 4]. Although presenting an infrared (IR) fixed point, namely General Relativity, in the UV the theory possesses a fixed point with an anisotropic, Lifshitz scaling between time and space of the form \( x \rightarrow \ell x, t \rightarrow \ell^z t \), where \( \ell, z, x \) and \( t \) are the scaling factor, dynamical critical exponent, spatial coordinates and temporal coordinate, respectively.

Due to these novel features, there has been a large amount of effort in examining and extending the properties of the theory itself [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. Additionally, application of Hořava-Lifshitz gravity as a cosmological framework gives rise to Hořava-Lifshitz cosmology, which proves to lead to interesting behaviors [32, 33]. In particular, one can examine specific solution subclasses [34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46], the phase-space behavior [47, 48, 49, 50, 51], the gravitational wave production [52, 53, 54, 55, 56], the perturbation spectrum [57, 58, 59, 60, 61, 62, 63, 64, 65, 66], the matter bounce [67, 68, 69, 70, 71, 72], the black hole properties [73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87], the dark energy phenomenology [88, 89, 90, 91, 92], the astrophysical phenomenology [93, 94, 95, 96, 97], the thermodynamic properties [98, 99, 100] etc.
In the present form of Hořava-Lifshitz cosmology, one combines the aforementioned modified gravitational background with a scalar field that reproduces (dark) matter. Doing so he obtains a dark-matter universe, with the appearance of a cosmological constant and an effective “dark radiation” term. Although these terms are interesting cosmological artifacts of the novel features of Hořava-Lifshitz gravitational background, they still restrict the possible scenarios of Hořava-Lifshitz cosmology.

In the present work we are interested in formulating Hořava-Lifshitz cosmology in a way that an effective dark energy, with a varying equation-of-state parameter, will emerge. Thus, we add a second scalar field, which dynamics will be combined with the cosmological constant and dark radiation terms. Although such a scalar field could put into question the renormalizability of the theory, it is still interesting to investigate what would be its effects on the cosmological behavior of the universe. Indeed, it proves that the effective Hořava-Lifshitz dark energy that arises, can have very interesting cosmological implications at both early and late times, such as to trigger a bounce and a turnabout. Its equation-of-state parameter can present quintessence behavior, or surprisingly enough, it can quite generally give rise to phantom behavior and to the crossing of the phantom divide. Although we construct it under the detailed-balance condition, the basic features of the model at hand are independent of that and are expected to be present even if we relax this constraint.

This paper is organized as follows. In section 2 we formulate Hořava-Lifshitz cosmology with both dark matter and dark energy fields. In section 3 we examine the cosmological behavior of the model and we discuss its implications. Finally, section 4 is devoted to the summary of the obtained results.

2. Hořava-Lifshitz cosmology with dark matter and dark energy fields

We begin with a brief review of Hořava-Lifshitz gravity. The dynamical variables are the lapse and shift functions, $N$ and $N_i$ respectively, and the spatial metric $g_{ij}$ (roman letters indicate spatial indices). In terms of these fields the full metric is

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt),$$

where indices are raised and lowered using $g_{ij}$. The scaling transformation of the coordinates reads ($z=3$):

$$t \rightarrow l^3 t \quad \text{and} \quad x^i \rightarrow lx^i.$$  \hspace{1cm} (2.2)

Decomposing the gravitational action into a kinetic and a potential part as $S_g = \int dt d^3x \sqrt{g} N (\mathcal{L}_K + \mathcal{L}_V)$, and under the assumption of detailed balance [3], which apart form reducing the possible terms in the Lagrangian it allows for a quantum inheritance principle [1] (the $D+1$ dimensional theory acquires the renormalization
properties of the D-dimensional one), the full action of Hořava-Lifshitz gravity is given by

\[
S_g = \int dtd^3x \sqrt{gN} \left\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) - \frac{\kappa^2}{2w^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2w^2} \frac{e^{ijk}}{\sqrt{g}} R_{il} \nabla_j R^l_k - \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(1 - 3\lambda)} \left[ \frac{1 - 4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right] \right\},
\]

where

\[
K_{ij} = \frac{1}{2N} (g_{ij} - \nabla_i N_j - \nabla_j N_i),
\]

is the extrinsic curvature and

\[
C^{ij} = \frac{\epsilon^{ijk}}{\sqrt{g}} \nabla_k (R_i^j - \frac{1}{4} R \delta_i^j)
\]

the Cotton tensor, and the covariant derivatives are defined with respect to the spatial metric \(g_{ij}\). \(\epsilon^{ijk}\) is the totally antisymmetric unit tensor, \(\lambda\) is a dimensionless constant and \(\Lambda\) is a negative constant which is related to the cosmological constant in the IR limit. Finally, the variables \(\kappa\), \(w\) and \(\mu\) are constants with mass dimensions \(-1\), \(0\) and \(1\), respectively.

Inserting a scalar field in the construction and imposing the corresponding symmetries consistently, one results to the following action \[32, 33, 62\]:

\[
S_\phi = \int dtd^3x \sqrt{gN} \left[ \frac{3\lambda - 1}{4} \frac{\phi^2}{N^2} + m_1 m_2 \phi \nabla^2 \phi - \frac{1}{2} m_2 \phi \nabla^4 \phi + \frac{1}{2} m_3 \phi \nabla^6 \phi - V_\phi(\phi) \right],
\]

where \(V_\phi(\phi)\) acts as a potential term and \(m_i\) are constants (note that for simplicity we have absorbed the possible term \(-\frac{1}{2} m_1^2 \phi^2\) inside \(V_\phi(\phi)\)). Clearly, this is a simplified consideration which cannot cope with the current knowledge of dark matter properties, but it allows for a first investigation on the subject. Finally, we mention that one could add the matter sector through a hydrodynamical approach, adding a cosmological stress-energy tensor to the gravitational field equations and demanding to recover the usual general relativity formulation in the low-energy limit \[17, 47\].

Since the scalar-field approach to matter seems to have a better theoretical justification, in this work we follow it. However, our results are independent of the specific way that matter is incorporated, and one could equivalently follow both the above matter-formulations.

In principle one could include additional scalars in the theory. The role of scalar fields in cosmology has become crucial the last decades, mainly in inflation \[101\] or in dark energy phenomenology \[102\], as well as in many other cases. However, in the end of the day one should provide an explanation for their appearance, and the
usual approach is that the scalars arise from some fundamental (probably higher-dimensional) theory of nature, unknown up to now. Thus, although the additional scalar fields have not a robust theoretical justification, nor it is clear how they behave at the quantum level, it is still interesting to study their effects on cosmology.

In this work we will allow for an additional scalar field, in which we attribute the dark energy sector. Clearly, such an extra field could put into question the renormalizability of the theory, which must be examined in detail before the present model can be considered as a realistic cosmology. However, the interesting cosmological implications of the scenario at hand motivate us to perform such a cosmological analysis, even with the renormalizability subject, as well as the possible conceptual and theoretical problems of Hořava-Lifshitz gravity itself \[11, 12, 17, 22, 103, 104\], open for the moment. Thus, we add a second scalar \( \sigma \), with action

\[
S_\sigma = \int dtd^3x \sqrt{g}N \left[ \frac{3\lambda - 1}{4} \dot{\sigma}^2 + h_1 h_2 \sigma \nabla^2 \sigma - \frac{1}{2} h_2^2 \sigma \nabla^4 \sigma + \frac{1}{2} h_3^2 \sigma \nabla^6 \sigma - V_\sigma(\sigma) \right],
\]

where \( V_\sigma(\sigma) \) accounts for the potential term of the \( \sigma \)-field and \( h_i \) are constants.

Now, in order to focus on cosmological frameworks, we impose an FRW metric,

\[
N = 1, \quad g_{ij} = a^2(t) \gamma_{ij}, \quad N^i = 0,
\]

with

\[
\gamma_{ij} dx^i dx^j = \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2,
\]

where \( k = -1, 0, 1 \) correspond to open, flat, and closed universe respectively. In addition, we assume that the scalar fields are homogenous, i.e \( \phi \equiv \phi(t) \) and \( \sigma \equiv \sigma(t) \).

By varying \( N \) and \( g_{ij} \), we obtain the equations of motion:

\[
H^2 = \frac{\kappa^2}{6(3\lambda - 1)} \left[ \frac{3\lambda - 1}{4} \dot{\phi}^2 + V_\phi(\phi) \right] + \frac{\kappa^2}{6(3\lambda - 1)} \left[ \frac{3\lambda - 1}{4} \dot{\sigma}^2 + V_\sigma(\sigma) - \frac{3\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^2} - \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \right] + \frac{\kappa^4 \mu^2 \Lambda k}{8(3\lambda - 1)^2 a^2},
\]

\[
\dot{H} + \frac{3}{2} H^2 = -\frac{\kappa^2}{4(3\lambda - 1)} \left[ \frac{3\lambda - 1}{4} \dot{\phi}^2 - V_\phi(\phi) \right] - \frac{\kappa^2}{4(3\lambda - 1)} \left[ \frac{3\lambda - 1}{4} \dot{\sigma}^2 - V_\sigma(\sigma) - \frac{\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \right] + \frac{\kappa^4 \mu^2 \Lambda k}{16(3\lambda - 1)^2 a^2},
\]
where we have defined the Hubble parameter as \( H \equiv \frac{2}{a} \). Finally, the equations of motion for the scalar fields read:

\[
\ddot{\phi} + 3H\dot{\phi} + \frac{2}{3\lambda - 1} \frac{dV_\phi(\phi)}{d\phi} = 0 \tag{2.12}
\]

\[
\ddot{\sigma} + 3H\dot{\sigma} + \frac{2}{3\lambda - 1} \frac{dV_\sigma(\sigma)}{d\sigma} = 0. \tag{2.13}
\]

At this stage we can define the energy density and pressure for the scalar fields. Concerning \( \phi \), the corresponding relations are

\[
\rho_\phi = \frac{3\lambda - 1}{4} \dot{\phi}^2 + V_\phi(\phi) \equiv \rho_M
\]

\[
p_\phi = \frac{3\lambda - 1}{4} \dot{\phi}^2 - V_\phi(\phi) \equiv p_M, \tag{2.14}
\]

and as we have mentioned they constitute the (dark) matter content of the Ho\v{r}ava-Lifshitz universe. Concerning the dark energy sector, we can define

\[
\rho_{DE} \equiv \frac{3\lambda - 1}{4} \dot{\sigma}^2 + V_\sigma(\sigma) - \frac{3\kappa^2 \mu^2 k^2}{8(3\lambda - 1) a^4} - \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \tag{2.15}
\]

\[
p_{DE} \equiv \frac{3\lambda - 1}{4} \dot{\sigma}^2 - V_\sigma(\sigma) - \frac{\kappa^2 \mu^2 k^2}{8(3\lambda - 1) a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)}. \tag{2.16}
\]

The first parts of these expressions, namely \( \frac{3\lambda - 1}{4} \dot{\sigma}^2 + V_\sigma(\sigma) \) and \( \frac{3\lambda - 1}{4} \dot{\sigma}^2 - V_\sigma(\sigma) \) correspond to the energy density and pressure of the \( \sigma \)-field, \( \rho_\sigma \) and \( p_\sigma \) respectively. The term proportional to \( a^{-4} \) is the usual “dark radiation term”, present in Ho\v{r}ava-Lifshitz cosmology [32, 33]. Note that it is present even if we take the IR limit, that is it is a cosmological artifact reflecting the novel features of Ho\v{r}ava-Lifshitz gravitational background. Finally, the constant term is just the explicit (negative) cosmological constant. Therefore, in expressions (2.15),(2.16) we have defined the energy density and pressure for the effective dark energy, which incorporates the aforementioned contributions. We mention that we could absorb the constant term inside the potential (equivalently define an effective potential as \( \tilde{V}_\sigma(\sigma) = V_\sigma(\sigma) - \frac{3\lambda^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \)), but we prefer to maintain it explicitly just to keep track of the origin of various terms, having in mind that \( V_\sigma(\sigma) \) must be sufficiently positive to assure for a positive \( \rho_{DE} \) as required by realistic cosmologies.

Using the above definitions, we can re-write the Friedmann equations (2.10),(2.11) in the standard form:

\[
H^2 = \frac{\kappa^2}{6(3\lambda - 1)} \left[ \rho_M + \rho_{DE} \right] + \frac{\beta k}{a^2} \tag{2.17}
\]

\[
\dot{H} + \frac{3}{2} H^2 = -\frac{\kappa^2}{4(3\lambda - 1)} \left[ p_M + p_{DE} \right] + \frac{\beta k}{2a^2}. \tag{2.18}
\]
In these relations we have defined $\beta \equiv \frac{\kappa^4 \mu^2 \Lambda}{8(3\lambda - 1)^2}$, which is the coefficient of the curvature term. Additionally, we could also define an effective Newton’s constant and an effective light speed [32, 33], but we prefer to keep $\frac{\kappa^2}{6(3\lambda - 1)}$ in the expressions, just to make clear the origin of these terms in Hořava-Lifshitz cosmology. Finally, note that using (2.12), (2.13) it is straightforward to see that the aforementioned dark matter and dark energy quantities verify the standard evolution equations:

$$\dot{\rho}_M + 3H(\rho_M + p_M) = 0 \quad (2.19)$$
$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = 0. \quad (2.20)$$

3. Cosmological implications and discussion

In the previous section we formulated Hořava-Lifshitz dark energy, that is we considered two scalar fields, one responsible for dark matter fluid and one contributing to the dark energy sector, in the framework of Hořava-Lifshitz gravity. In this section we examine the cosmological implications of this model and in particular the dark energy phenomenology.

As usual, a central observable quantity is the dark energy equation-of-state parameter, defined as:

$$w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} = \frac{3\lambda - 1}{4} \dot{\sigma}^2 - V_\sigma(\sigma) - \frac{\kappa^2 \mu^2 k^2}{8(3\lambda - 1)\alpha_\sigma} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)}. \quad (3.1)$$

In the following we explore this expression in some cosmological scenarios.

3.1 Absent $\sigma$-field

Let us first consider the simplified case of the complete absence of the $\sigma$-field. We remind that even in this scenario, one cannot avoid a constant potential term, in order to acquire a positive-defined $\rho_{DE}$. In a sense, in the present formulation of Hořava-Lifshitz dark energy, one cannot eliminate the $\sigma$-field presence completely, since it will always be (trivially but non neglectably) manifested itself through a constant potential term. However, even if this case seems special it leads to very interesting cosmological implications, acting as a valuable example.

We start considering a flat ($k = 0$) spacetime. In this case, as expected, relation (3.1) leads to $w_{DE} = -1$ that is we obtain the simple cosmological constant universe. It is interesting to see that the spacetime flatness sets to zero the “dark radiation” term that is present in (3.1) (which is a difference comparing to a similar term arising in braneworld models [105]), and thus with the addition of the disappearance of the $\sigma$-field terms we acquire the simple result $w_{DE} = -1$.

As a next step we allow for a non-zero curvature. In this case the dark radiation term appears, leading to a more complex $w_{DE}$-behavior. In particular, setting
\[ V_\sigma(\sigma) = V_0 \text{ (the sufficiently positive constant discussed earlier), and defining} \]

\[ \tilde{V}_0 = \frac{8V_0}{\kappa^2\mu^2} - \frac{3\Lambda^2}{3\lambda - 1}. \] (3.2)

(3.1) leads to:

\[ w_{DE} = \frac{\tilde{V}_0 - \frac{1}{3(\lambda-1)a^4}}{V_0 - \frac{3}{3(\lambda-1)a^4}}. \] (3.3)

We mention that in principle, both \( V_0 \) and \( \Lambda \) can be arbitrary. However, since effectively \( \tilde{V}_0 \) will play the central role of dark energy in the current, observable universe, one has to fine-tune \( V_0 \) and \( \Lambda \) in order to quantitatively lead to a very small \( \tilde{V}_0 \), consistently with observations.

A first observation is that the “running” behavior of Hořava-Lifshitz background is reflected in the “running” behavior of \( w_{DE} \). Furthermore, surprisingly enough we observe that \( w_{DE} < -1 \) always, that is we result to an effective dark energy presenting a phantom behavior. This behavior is a pure effect of the dark radiation term, and enlightens the discussion about the novel implications of Hořava-Lifshitz dark energy. Progressively, as the scale factor increases, dark radiation dilutes and \( w_{DE} \) asymptotically goes to \(-1\) at very large times. Thus, although presenting a phantom behavior, this scenario is free of a Big-Rip \([106, 107, 108, 109]\).

The cosmological evolution is also very interesting in the other “direction”, that is going to small scale factors. Due to the presence of the dark radiation term, one can easily see from the Friedmann equations \((2.10), (2.11)\) that at some particular moment \( \dot{H} \) changes sign leading to a bounce \([67, 68]\). Therefore, \( \rho_{DE} \) will never become negative and the universe will never become singular.

### 3.2 Flat universe with \( \sigma \)-field

Let us now consider a flat universe, with the presence of both matter and \( \sigma \) fields. In this case we obtain:

\[ w_{DE} = \frac{\frac{3\lambda-1}{4} \dot{\sigma}^2 - V_\sigma(\sigma) + \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}}{\frac{3\lambda-1}{4} \dot{\sigma}^2 + V_\sigma(\sigma) - \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}}. \] (3.4)

As we see, \( w_{DE} \) presents the “running” behavior that is inherited from the Hořava-Lifshitz background. Clearly, in the IR fixed point \( \lambda = 1 \), and under the absorption of the constant term in the potential, we re-obtain the usual equation-of-state parameter of standard dark energy formalism \([110, 111, 112]\). Therefore, since dark energy observations are by far inside the IR, Hořava-Lifshitz artifacts of this scenario would not be observable and the model is not distinguishable from standard dark energy paradigms. Finally, concerning the cosmological behavior of \( w_{DE} \), we see that it lies always at the quintessence regime \((-1 < w_{DE})\) and the evolution is free of a Big Rip. The reason for these features is that the universe flatness kills the dark radiation term and thus it disappears the novel and unexpected phenomena.
3.3 Non-flat universe with $\sigma$-field

In this scenario the dark energy equation-of-state parameter is given by (3.1) in full generality. Similarly to the previous subsection, we observe that $w_{DE}$ depends explicitly on $\lambda$, with its value at $\lambda = 1$ corresponding to the IR limit. However, the Hořava-Lifshitz nature of the model is manifested in an unexpected cosmological implication which is independent of the energy scale. Namely, the effective dark energy can experience the crossing of the phantom divide $-1$, for a suitable potential, as can be easily observed from (3.1). Amazingly, this is achieved despite taking the IR limit of the theory. Thus, in this case, artifacts of Hořava-Lifshitz gravity could be detected through dark energy observations. However, one still cannot distinguish between this model and alternative models that allow for the realization of $w_{DE} < -1$ phase, such are modified gravity [113, 114, 115, 116, 117] or models with phantom [106, 118, 119, 120] or quintom fields [121, 122, 123, 124, 125].

In order to provide an explicit but general example of the aforementioned crossing behavior we consider power-law solutions of the form $\sigma(t) = c_\sigma t^q$ and $a = c_\alpha t^p$, which satisfy the Friedmann and field equations for appropriately constructed potentials. In this case, the crossing is realized at

$$a_{cr} = c_\alpha \left[ \frac{\kappa \mu}{c_\sigma^2 c_\alpha^q q(3\lambda - 1)} \right]^{\frac{p}{q + 2p - 1}}. \quad (3.5)$$

We mention that according to the parameter values, the $-1$-crossing can be realized at an arbitrary scale factor, either at early or at late times.

3.4 Bounce, turnaround and cyclic behavior

Apart from interesting dark energy phenomenology, the general scenario of the presence of both scalar fields in a non-flat universe allows for interesting cosmological behavior at all epochs. In particular, at small scale factors the role the dark radiation term inside Hořava-Lifshitz dark energy is enhanced and one naturally obtains a bounce, thus avoiding the possible singular behavior of the universe. We mention that due to the presence of the $\sigma$-field the bounce is always obtained and it is not restricted to slowly-diluted matter contents [67, 68]. On the other hand, one can easily construct a large sub-class of $\sigma$-potentials which at late times allow the negative cosmological constant to dominate the evolution (the positivity of $\rho_{DE}$ is not affected) and thus to trigger a turnaround. In other words, the competition of the dark radiation and cosmological constant terms, can make Hořava-Lifshitz dark energy naturally addressing cyclic cosmology. A construction of such a model is left for future investigation.

4. Conclusions

In this work, using an additional canonical scalar field, we have formulated Hořava-
Lifshitz cosmology with an effective dark energy sector. Hořava-Lifshitz dark energy, due to the inherited features from the novel Hofava-Lifshitz gravitational background, proves to present very interesting behaviors, possessing a varying equation-of-state parameter, exhibiting phantom behavior and allowing for a realization of the phantom divide crossing. Although according to current observations one cannot distinguish it from well-studied scenarios such as quintessence, that is from General Relativity with an extra scalar field, its novelty is that it possesses an improved gravitational background with UV completeness. Moreover, Hořava-Lifshitz dark energy can always guarantee for a bounce at small scale factors, offering a simple way to avoid singular cosmological evolution. Furthermore, it can easily accept simple solution sub-classes where it can trigger both bounce and turnaround and thus lead naturally to cyclic cosmology. These features were the motivations of the present work.

The existence of a second scalar field offers larger possibilities to control the aforementioned behaviors, either at small or large scales factors, or even simultaneously for both cosmological regimes. Furthermore, such a construction could alleviate the undesirable phenomena that could arise in the gravitational sector \cite{1, 2}. Although for simplicity we have performed the analysis under the detailed-balance condition, the basic features of the construction are independent of that and are expected to be present even if we relax this constraint. On the contrary, a similar investigation without detailed-balance will make the construction more robust and free of possible problems that detailed-balance could bring in Hořava-Lifshitz gravity itself \cite{11, 33}. Lastly, note that the presence of the Cotton tensor prevents high-momentum pathological behavior \cite{33}. However, it would be interesting to use Renormalization Group methods to study the running of the quantities, leading to improved dynamics, similarly to the earlier works \cite{126, 127}. But such an investigation lies beyond the scope of the present work.

Finally, we mention that the present work is just a first approach on the subject of dark energy in Hořava-Lifshitz cosmology, definitely far from a solution to the dark energy problem. It is interesting to note that one faces the same two ways to acquire acceleration as in “conventional” cosmology. In particular, in the later it is widely known that one can either use an additional scalar field (quintessence \cite{111, 112, 113} or phantom one \cite{100, 118}), or modify the gravitational sector itself \cite{113, 114, 115, 116, 117}. Similarly, in the case of Hořava-Lifshitz cosmology, one can either use an additional scalar field, an approach followed in this work, or he can modify and generalize the gravitational action of Hořava-Lifshitz gravity \cite{13, 14, 92}. Each approach exhibits its own advantages and disadvantages.

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