Day-ahead stochastic electricity market clearing considering renewable power uncertainty

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Abstract. The electricity market development in China faces a new background of increasing proportion of renewable power. Different from conventional power plants, the output of renewable power plants usually has strong uncertainty, which is a challenge for electricity market clearing strategy. This paper proposes a day-ahead electricity market clearing method that considers the uncertainty of renewable power sources, which is modelled based on the truncated versatile distribution. The scheduled power of conventional power plants and renewable power plants is determined by minimizing the social cost. Case studies show that the proposed method can greatly reduce the total social cost.

1. Introduction
At present, China's electricity market reform is gradually being carried out. The focus of the new round of power system reform is to build a unified, open and competitive electricity market system, giving full play to the decisive role of the market in resource allocation.

Different from the electricity market formation in countries such as Europe and the United States, China's electricity market construction is accompanied by a new background in the development of renewable power. China's research institutes suggest that China will achieve 60% of electricity from renewable power in 2050 [1].

Electricity market has received extensive attention and there are many research studies. Studies in [2] focus on the dynamics in forward and spot electricity markets. Electricity and natural gas interdependency in the market model are studied in [3]. Day-ahead electricity markets in Europe are formulated in [4]. Studies in [5] analyze market power and trading strategies on the electricity market. A novel model for the clearing of the European day-ahead electricity market incorporating “Prezzo Unico Nazionale” orders is presented in [6].

However, as generation from renewable sources of energy reaches grid parity, renewable power producers are asked to participate in electricity markets under the same rules as conventional power plants [7]. Renewable power such as wind power and solar photovoltaic power has strong uncertainty compared with traditional power sources such as thermal power. The above studies [2] have not considered the renewable power uncertainty in the design of the electricity market clearing.

Stochastic market clearing model with probabilistic participation of wind and electric vehicles cost is proposed in [8], in which the expected cost of reserve activation due to probabilistic EV aggregator and wind units is considered by scenarios. However, limited number of scenarios could not represent the uncertainty accurately.

Renewable power producers participating in day-ahead electricity markets must forecast their output before market close time [9]. If the forecast renewable power is scheduled in the electricity
markets without considering the uncertainty, on the one hand, power shortages may occur if the actual renewable power is lower than the forecast one. On the other hand, if actual renewable power is larger than the forecast one, renewable power would be wasted. To this end, there would be more social cost for ancillary services.

This paper proposes a day-ahead electricity market clearing method that considers the uncertainty of renewable power sources, which determine the scheduled power of conventional power plants and renewable power plants. The renewable power uncertainty is modelled based on the truncated versatile distribution. The renewable power plants cost model is proposed to consider the renewable power uncertainty.

2. Modelling renewable power uncertainty based on truncated versatile distribution

The uncertainty of renewable power can be modelled based on the recent proposed truncated versatile distribution [10], which has the probability density function (PDF) as follows.

\[
    f(x) = \begin{cases} 
        \frac{\alpha \beta e^{-\alpha(x-\gamma)}}{M} \quad & x \leq x \leq \bar{x} \\
        0 \quad & x \leq \bar{x}, x \geq \bar{x}
    \end{cases}
\]  

(1)

where \( x, \bar{x} \) is the lower and upper boundaries of the domain, which is PDF value strictly non-zero domain interval of the truncated versatile distribution. In this paper, the output of renewable power plants domain is \([0, 1]\), i.e., \( x=0, \bar{x}=1 \) when we use the truncated versatile distribution (1) to model the renewable power uncertainty. \( \alpha, \beta, \) and \( \gamma \) are the vertical parameter, skewness parameter and horizontal displacement, respectively. \( \alpha>0, \beta>0 \) and \( -\infty<\gamma<+\infty \). \( M \) is the normalization constant and can be calculated by

\[
    M = (1 + e^{-\alpha(\bar{x}-\gamma)})^\beta - (1 + e^{-\alpha(x-\gamma)})^\beta
\]  

(2)

The cumulative distribution function (CDF) of the truncated versatile distribution is defined as:

\[
    F(x) = \begin{cases} 
        0 \quad & x < \bar{x} \\
        \frac{1}{M} \left[(1 + e^{-\alpha(x-\gamma)})^\beta - (1 + e^{-\alpha(\bar{x}-\gamma)})^\beta\right] \quad & x \leq x \leq \bar{x} \\
        1 \quad & x > \bar{x}
    \end{cases}
\]  

(3)

As shown in Figure 1, the truncated versatile distribution has flexible curve form and limited domain interval to represent the renewable power distribution. Furthermore, the CDF of truncated versatile distribution has analytical forms as shown in (3), which will greatly reduce the computational efficiency of the market clearing algorithm [10].

![PDF curves of truncated versatile distribution.](image-url)
3. Electricity market clearing model considering the renewable energy uncertainty

Based on the truncated versatile distribution for renewable power uncertainty, an electricity market clearing model for determining the scheduled power of conventional power plants and renewable power plants is proposed in this section.

The day-ahead electricity market clearing horizon is denoted as the set of (market clearing) time intervals \( T = \{1, 2, \ldots, T\} \), i.e., \( t = 1, 2 \ldots T \) time intervals. The set of conventional power plants in the power system is denoted as \( I = \{1, 2, \ldots, I\} \), i.e., \( i = 1, 2 \ldots I \) conventional power plants. The set of renewable power plants in the power system is denoted as \( J = \{1, 2, \ldots, J\} \), i.e., \( j = 1, 2 \ldots J \) renewable power plants. The day-ahead electricity market clearing is realized by the market operator as in this section.

3.1. Renewable power plants cost model

The possible actual power of renewable power plant \( j \) at time \( t \) is denoted as \( w_j^t \) and the scheduled power in the day-ahead market clearing is denoted as \( p_{w,j}^t \). Note that \( w_j^t \) is a random variable with PDF in (1) and CDF in (3). \( p_{w,j}^t \) is the decision variable.

For the reason that the actual power of renewable power plant \( j \) at time \( t \) that can be supplied to system is a random variable \( w_j^t \), if the possible actual power \( w_j^t \) is larger than the scheduled power \( p_{w,j}^t \), renewable power producers could sell the excess renewable power to the electricity market or just discard it. If the possible actual power \( w_j^t \) is smaller than the scheduled power \( p_{w,j}^t \), renewable power producers need to purchase power shortages in the spot electricity market as ancillary services. As discussed above, the generation cost of renewable power plant \( j \) in the electricity market clearing horizon can be calculated by the following formulas.

\[
C_{w,j}^t(p_{w,j}^t) = b_j \cdot \int_{p_{w,j}^t}^{p_{w,j}^m} (p_{w,j}^t - w_j^t) f(w_j^t) dw_j^t - s_i \cdot \int_{p_{w,j}^t}^{p_{w,j}^m} (w_j^t - p_{w,j}^t) f(w_j^t) dw_j^t
\]

where \( b_j \) is the coefficient of power purchasing; \( s_i \) is the electricity price for sale to the spot electricity market.

3.2. Day-ahead electricity market clearing model

The day-ahead electricity market clearing model aims to minimize the generation cost of conventional power plants and renewable power plants as follows.

\[
\min f_c(p_i^t) + f_w(p_{w,j}^t)
\]

where \( f_c(p_i^t) \) and \( f_w(p_{w,j}^t) \) are the generation cost of conventional power plants and renewable power plants, respectively; \( p_i^t \) is the scheduled power of conventional power plant \( i \) at time \( t \) in the day-ahead electricity market clearing, \( i \in I = \{1, 2, \ldots, I\} \); \( f_c(p_i^t) \) can be calculated by the following formulas.

\[
f_c(p_i^t) = \sum_{t=1}^{T} \sum_{i=1}^{I} C_{i,j}^t(p_i^t) = \sum_{t=1}^{T} \sum_{i=1}^{I} \left[ \frac{1}{2} a_i (p_i^t)^2 + b_i p_i^t + c_i \right]
\]

The constraints of the day-ahead electricity market clearing model is as follows.

\[
p_i^{\text{min}} \leq p_i^t \leq p_i^{\text{max}}, \quad i \in I, \quad t \in T
\]
\[ -\Delta p_{i}^{\max} \leq p_{i}^{t} - p_{i}^{t-1} \leq \Delta p_{i}^{\max}, \quad i \in I, \quad t \in T \] (9)

\[ 0 \leq p_{w,j}^{t} \leq p_{w,j}^{\max}, \quad j \in J, \quad t \in T \] (10)

\[ \sum_{i=1}^{I} p_{i}^{t} + \sum_{j=1}^{J} p_{w,j}^{t} - \sum_{b \in B} L_{b}^{t} = 0, \quad t \in T \] (11)

\[ -\bar{F}_{l} \leq \sum_{i=1}^{I} K_{b} p_{i}^{t} + \sum_{j=1}^{J} K_{b} p_{w,j}^{t} - \sum_{b \in B} K_{b} L_{b}^{t} \leq \bar{F}_{l}, \quad l \in L \] (12)

where:

(8) is the conventional power plant scheduled power capacity constraint; \( p_{i}^{\max} \) and \( p_{i}^{\min} \) are the upper and lower generation limit of the \( i \)'th conventional power plant, respectively;

(9) is the conventional power plant ramp-rate constraint; \( \Delta p_{i}^{\max} \) is the maximum amount of upward and downward ramp rate of \( i \)'th conventional power plant within a specific time period (e.g., one hour);

(10) is the renewable power plant power capacity constraint; \( p_{w,j}^{\max} \) is the capacity of \( j \)'th renewable power plant;

(11) is the supply-demand balance constraint; \( L_{b}^{t} \) is the forecast power demand on the \( b \)'th bus at time \( t \); the set of buses in the power system is denoted as \( B = \{1,2,...,B\} \), i.e., \( b=1,2...B \) buses;

(12) is the transmission capacity constraint; \( \bar{F}_{l} \) is the transmission capacity limit on transmission line \( l \); \( K_{b} \), \( K_{b} \), and \( K_{b} \) are the shift factor of \( i \)'th convention power plant, \( j \)'th renewable power plant and \( b \)'th bus, respectively; the set of transmission lines in the power system is denoted as \( L = \{1,2,...,L\} \), i.e., \( l=1,2...L \) transmission lines.

The proposed day-ahead electricity market clearing model, i.e., (4)~(7), s.t.: (8)~(12) can be proved to be convex and can be solved by sequential linear programming algorithm.

4. Case study

The proposed model is solved based on the Matlab toolbox CVX. \( T=24 \) and each market clearing time interval is one hour. The day-ahead electricity market clearing model determines the scheduled power of conventional power plants and renewable power plants from 1h to 24h of the next day. The system topology is shown in Figure 2. Three conventional power plants (G1, G2 and G3) are connected on the first, second and sixth buses, respectively. A wind power plant with installation capacity of 200 MW is connected to the fifth bus [11]. The parameters of the conventional power plants are shown in Table 1. The loads of the third, fourth and fifth buses are 100, 200, and 100 MW, respectively. The cost coefficient of the renewable power plant's power shortage purchase \( b_{t} \) is 100$/MWh, and the excess renewable price sold to the spot market \( s_{t} \) is 0$/MWh.
Table 1. Cost coefficients of Conventional power plants.

| Generator | $p_i^{max}$ (MW) | $p_i^{min}$ (MW) | $\Delta p_i^{max}$ (MW/h) | $a_i$ ($/MW^2$) | $b_i$ ($/MW$) | $c_i$ ($) |
|-----------|------------------|------------------|---------------------------|-----------------|---------------|-----------|
| G1        | 220              | 100              | 55                        | 0.05            | 10            | 100       |
| G2        | 100              | 10               | 45                        | 0.01            | 22            | 162       |
| G3        | 100              | 10               | 45                        | 0.01            | 22            | 162       |

4.1. Schedule results
The scheduled power of three conventional power plants and wind power plant is shown in Figure 3. Figure 4 shows the forecast wind power (red line) and the scheduled wind power when $b_t=100$ MWh (green line). We can see that the forecast wind power and scheduled wind power have the same trend, i.e., increasing from 1h to 8h, decreasing from 8h to 19h and increasing from 19h to 24h. With the decrease of wind power, the scheduled power of conventional power plants increases to meet the load demand, as shown in Figure 3.

Table 2 compares the system cost of two day-ahead electricity market clearing models. The electricity market clearing model regard the forecast wind power as the scheduled wind power (not considering the uncertainty), resulting in a large renewable power shortage purchase. By contrast, the proposed electricity market clearing model could seek the best balance of conventional power plant output and renewable power uncertainty, which can reduce the total social cost.

Table 2. System cost of day-ahead electricity market clearing model.

|                     | The proposed model | Model without considering renewable power uncertainty |
|---------------------|--------------------|-------------------------------------------------------|
| Conventional power plant cost($) | 152976             | 146896                                               |
| Renewable power plant cost($)     | 5813               | 16811                                               |
| Total cost($)            | 158789             | 163702                                               |

Figure 4. Scheduled wind power under different renewable power shortage purchase coefficients.
4.2. Sensitivity analysis of renewable power shortage purchase coefficients

Figure 4 shows the scheduled wind power under different renewable power shortage purchase coefficients. We can see that because the renewable power producers need to purchase the power shortages when the possible actual power is smaller than the scheduled power, all scheduled wind power is lower than the forecast wind power. With the increase of renewable power shortage purchase coefficients, the scheduled power of renewable power plants decreases to avoid too much power shortages purchasing.

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