Prospects of detecting the QCD critical point

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Abstract

We investigate the possibility to observe the QCD critical point in $A + A$ collisions at the SPS. Guided by the QCD phase diagram expressed in experimentally accessible variables we suggest that the process $C + C$ at $158 \text{ GeV}/n$ freezes out very close to the critical point. We perform an analysis of the available preliminary experimental data for a variety of SPS processes. The basic tool in our efforts is the reconstruction of the critical isoscalar sector which is formed at the critical point. Our results strongly support our proposition regarding the $C + C$ system.

1 Critical properties of QCD

The study of the QCD phase diagram in the baryonic chemical potential-temperature plane is a subject of rapidly increasing interest in the last decade. Recent investigations suggest that in the real world where the $u$ and $d$ quarks have a small current mass ($O(10 \text{ MeV})$) and the strange quark is much heavier ($O(100 \text{ MeV})$) there is a second order critical point as endpoint of a first order transition line. This critical endpoint is located at low baryonic density (compared to the baryonic density in the nuclear matter) and high temperature ($O(100 \text{ MeV})$) values. The order parameter characterizing the critical behaviour has isoscalar quantum numbers and the underlying symmetry which breaks spontaneously at the critical point is the $Z(2)$ symmetry classifying the QCD critical point in the $3 − D$ Ising universality class. However this symmetry does not represent an obvious symmetry of the original QCD Langrangian but it is rather an invariance of the effective thermal QCD action.

The fluctuations of the condensate formed at the critical point correspond to isoscalar particles which are distributed in phase space producing a characteristic self-similar pattern with fractal geometry determined by the
The isothermal critical exponent of the 3 - D Ising universality class \[4\]. The properties of the isoscalar condensate \(\sigma(\vec{x})\) are strongly affected by the baryonic environment:

\[ \sigma_\rho \approx \lambda \left( \frac{\rho - \rho_c}{\rho_c} \right) \sigma_o \]  

(1)

where \(\rho\) is the baryonic density in the critical region, \(\rho_c\) is the critical baryonic density and \(\lambda\) is a dimensionless parameter of order one. Eq. (1) relates the isoscalar condensate at zero baryonic density \((\sigma_o)\) with its value at baryonic density \(\rho\). The form of eq. (1) suggests that the difference \(\rho - \rho_c\) can be considered as an alternative order parameter (besides the isoscalar condensate \(\sigma\)) characterizing the QCD critical point. Projecting the baryonic density onto the rapidity space and using the scaling properties of the critical baryonic fluid formed in a \(A + A\)-collision process, one obtains the relation \[5\]:

\[ A_{\perp}^{-2/3} n_b = \Psi(z_c, \frac{\mu}{\mu_c}, \rho_c) \]  

(2)

where \(A_{\perp}\) is the total number of nucleons of the \(A + A\) system in the plane transverse to the beam, \(n_b\) is the net baryon density at midrapidity and \(\Psi\) is a scaling function. The variable \(z_c\) is defined as: \(z_c = A_{\perp}^{-2/3} A_t L^{-1}\) with \(A_t\) the total number of participating nucleons in the \(A + A\) collision and \(L\) the size of the system in rapidity space. The scaling function \(\Psi\) depends also on the ratio of the chemical potentials \(\frac{\mu}{\mu_c}\) and for \(\mu = \mu_c\) simplifies to:

\[ \Psi(z_c, 1, \rho_c) = \begin{cases} 
\rho_c + \frac{2}{\pi} (z_c - \rho_c) + C (z_c - \rho_c)^4 & ; \quad z_c > \rho_c \\
\rho_c & ; \quad z_c \leq \rho_c 
\end{cases} \]

In fact the scaling relation (2) represents an alternative description of the QCD phase diagram in terms of measurable quantities [5]. In Fig. 1 we present a plot of eq. (2) in the \((z_c, \xi)\) plane (we use the notation \(\xi = A_{\perp}^{-2/3} n_b\)). In the same plot we show also the coordinate pairs \((z_{c,i}, \xi_i)\) for a variety of \(A + A\) processes in running (NA49, RHIC), passed (NA35) and future experiments (LHC). We also mark the QCD critical point in this graph. One can easily see that the \(C + C\) system at the SPS energies (158 GeV/n) is very close to the critical point.

Notice that the remaining \(A + A\) processes at the SPS \((Si + Si, Pb + Pb)\) are not so close to the critical point although they still lie in the scaling region. It is therefore expected that nonstatistical fluctuations will be present in all these processes and become stronger as we approach the critical point. How to reveal these fluctuations, it will be discussed in the following sections.
2 Statistical description of the isoscalar condensate

The isoscalar condensate formed at the critical point can be described as a critical (Feynman-Wilson) fluid in local thermal equilibrium \[6\]. Universality class arguments determine the effective action for the dynamics of the condensate \( \sigma(\vec{x}) \) at energies \( \approx T_c \) in \( 3-D \) as:

\[
\Gamma_c[\sigma] = T_c^{-1} \int d^3\vec{x} \left[ \frac{1}{2} (\nabla \sigma)^2 + G T_c^4 (T_c^{-1} \sigma)^{\delta+1} \right]
\]  \( (3) \)

Eq. (3) leads to the correct equation of state:

\[
\frac{\delta \Gamma}{\delta \sigma} \sim G \sigma^\delta
\]

where \( \delta = 5 \) is the isothermal critical exponent of the \( 3-D \) Ising model. The coupling \( G \) has been calculated in \[7\] for the \( 3-D \) Ising model on the
lattice leading to $G \approx 2$. The field $\sigma(\vec{x})$ in eq. (3) is in fact macroscopic, i.e. the quantum fluctuations are integrated out to get the effective action (3) and therefore it possesses classical properties. Following ref. (5) we recall here that the experimentally accessible quantity is not the field $\langle \sigma \rangle$ itself but the quantity $\langle \sigma^2 \rangle$ which represents density fluctuations of the $\sigma$-particles created at $T = T_c$.

Based on the effective action (3) we can now proceed to determine the partition function of the condensates as a functional integral:

$$Z = \int \mathcal{D}[\sigma] e^{-\Gamma_c[\sigma]}$$

(4)

The path summation in the above equation is dominated by the saddle points of the corresponding action which have an instanton-like form [4]. Within this approximation we can determine the density-density correlation of the critical system both in configuration as well as in momentum space. Then using the calculated density-density correlation function we find the distribution of the corresponding $\sigma$-particles in phase space. We end up with a pattern formed through the overlap of several self-similar clusters with fractal mass dimension determined by the isothermal critical exponent $\delta$ [4]. Using the Fourier transform of the spatial density-density correlation function we obtain the corresponding quantity in momentum space. A similar pattern occurs also in momentum space. The number of clusters as well as the multiplicity within each cluster are the same in both spaces while the local fractal dimension differs. Another property determining the geometrical features of the critical system is the shape of its evolution. For a cylindrical evolution the number of clusters is in general greater than one while in the case of spherical evolution the system consists a single cluster. Also the corresponding fractal dimensions are influenced by the geometrical shape of the evolving system [6]. A less influenced property is the fractal dimension of the transverse momentum space which turns out to be $\approx 0.7$ for cylindrical systems and $\approx 1$ for spherical systems.

**The Critical Monte Carlo (CMC) event generator**

Using the results of the saddle point approximation to the partition function of the critical system one can develop a Monte-Carlo algorithm to simulate the production of the critical $\sigma$-particles in an $A + A$ collision. We restrict our interest to the the distribution of the sigmas in momentum space since the coordinates in this space are experimentally accessible. The momentum coordinates of the centers of the $\sigma$-clusters are treated as random variables distributed according to an exponential decay law with range determined by
the critical temperature. Within each cluster the particles are strongly correlated and possess a fractal geometry. The corresponding fractal dimension is given in terms of the exponent $\delta$ while the multiplicity within each cluster is determined through the transverse radius of the entire system, its size in rapidity, the critical coupling $G$ and the critical temperature $T_c$. The momenta of the sigma-particles within each cluster are generated using the tool of Lévy walks [8].

Exactly at the critical temperature $T_c$ the mass of the sigma particles is zero (for an infinite system). As the system freezes out the sigma-mass increases and when it overcomes the two pion threshold the sigmas decay into pions which constitute the experimentally observable sector of the critical system. Unfortunately there is no theoretical description of this process based on first principles. A possible treatment of the $\sigma$-decay into pions is to introduce the probability density $P(m)$ for a sigma to have mass $m$ and then using pure kinematics to determine the momenta of the produced pions. The mass $m$ is assigned to the decaying sigmas randomly. A more detailed description of the whole algorithm can be found in [6].

3 SPS data analysis (preliminary)

If the mass of the decaying sigma is well above the two-pion threshold the momenta of the produced pions are very distorted with respect to the momentum of the initial sigma and the fractal geometry of the critical condensate is not transferred to the pionic sector. Therefore in order to reveal the critical fluctuations in an analysis of the final pions one has to isolate the part of phase space for which the mass of the decaying sigmas is very close to the two-pion threshold. In this case the fractal properties of the sigma-momenta are transferred to final pions. Our proposal is to perform an event by event analysis in an $A + A$-dataset forming for each event all the pairs of pions with opposite charge and filtering out those pairs with invariant mass within a narrow window just above the value $2m_\pi$ [6]:

$$2m_\pi \leq \sqrt{m^2_{\pi^+\pi^-}} \leq 2m_\pi + \epsilon \quad ; \quad m^2_{\pi^+\pi^-} = (p_{\pi^+} + p_{\pi^-})^2$$ (5)

In (5) $p_{\pi^\pm}$ are the four-momenta of the positive (negative) charged pions respectively. The parameter $\epsilon$ is assumed to be very small compared to the pion mass. We apply first our analysis to a large set of CMC generated events (100000). The CMC input parameters have been chosen to meet the properties of the $C + C$ system at the SPS:

- The size in rapidity $\Delta = 6$, corresponding to $\sqrt{s} = 158 \text{ GeV}/n$
• The transverse radius $R_{\perp} = 15 \text{ fm}$. With this choice we can fix the mean multiplicity of charged pions to be $\approx 50$ close to the corresponding value in the $C+C$-system at the SPS.

• The critical temperature $T_c \approx 140 - 170 \text{ MeV}$

• The self-coupling $G = 2$ and the isothermal critical exponent $\delta = 5$ determined by the universality class of the transition.

The essential parameters in our approach is the transverse radius $R_{\perp}$ which controls the mean pion multiplicity in the simulation of an $A+A$-process, the size in rapidity $\Delta$ which controls the total energy of the system and the isothermal exponent $\delta$ determining the fractal geometry of the isoscalar fluctuations.

Having produced the $10^5$ CMC events we calculate the factorial moments in transverse momentum space of the final produced charged pions. For the decay of the $\sigma$-s into pions we use a Gaussian $\sigma$-mass probability distribution with a large mean value (300 MeV) and large standard deviation (100 MeV). With such a deviation in mass we expect that the critical fluctuations present in the sigma-sector will be strongly suppressed in the charged pion-sector. This choice of $P(m_\sigma)$ may be quite conservative but it consists a good test for the efficiency of our data-analysis algorithm.

Then using the charged pion momenta of each event one can form $\pi^+ - \pi^-$ pairs with invariant mass very close to the two-pion threshold. In our actual calculations we have used as a window $\epsilon = 4 \text{ MeV}$ to filter out pion pairs with invariant mass in the range $2m_\pi \leq \sqrt{m_{\pi^+\pi^-}^2} \leq 2m_\pi + \epsilon$. Then we consider each charged pion pair as a sigma-particle. Performing the factorial moment analysis in the new momenta (of the reconstructed sigmas) we expect a partial restoration of the critical fluctuations. Indeed this characteristic behaviour is clearly shown in Fig. 2a where we present the results of the calculation of the second factorial moment in transverse momentum space for both the negative pions as well as the sigmas. The effect of the restoration of the critical fluctuations in the reconstructed sigma sector combined with the large suppression of the fluctuations in the negative pions, as predicted, is impressive. The theoretical expectation, for an infinite critical system, for the corresponding intermittency index is $s_{2,cr}^{(2D)} \approx 0.67$ while our analysis leads to $s_{2}^{(2D)} \approx 0.54$. We have applied the same analysis to data sets obtained from the NA49 experiment at the SPS. We have analysed 13731 events of the $C+C$ system at 158 GeV/n, 76065 events of the $Si+Si$ system at the same energy, 13420 events of the $Pb+Pb$ system at 40 GeV/n, 384 events of the same system at 80 GeV/n and finally 5584 events of the $Pb+Pb$ system at
158 GeV/n. It must be noted that all the data sets used in our analysis are only preliminary and there is a need for further investigations with improved data. In Fig. 2b we show the results for the second moment in transverse momentum space in the $C + C$ system. There is an impressive agreement between simulated and real data. In fact the slope $s^{(2D)}_{2}$ of the $C+C$ system turns out to be $\approx 0.58$.

In Fig. 3 we show the second factorial moment for all the available NA49 experimental data sets both for negative pions as well as sigmas. A gradual increment of the slope $s^{(2D)}_{2}$ as we approach the $C+C$ system - and according to Fig. 1 the critical point - is observed close to our theoretical expectations. For all the systems the effect of the reconstruction of the critical fluctuations in the $\sigma$-sector is clearly seen.

Figure 2: The reconstruction of $\sigma$-momenta in (a) $10^5$ CMC events and (b) 13731 preliminary $C + C$ data.
The analysis described so far concerns a finite kinematic window above the two pion threshold. It is interesting to extrapolate the properties of the various systems exactly at the two pion threshold. In this case no distortion due to the $\sigma$-decay into pions will be present and we expect to reproduce the theoretically expected results for the critical system. Therefore we have to take the limit $\epsilon \to 0$. In order to extract this information one has to calculate $s^{(2D)}_{2,\epsilon}$ for various values of the kinematical window $\epsilon$ and use an interpolating function to extrapolate to $\epsilon = 0$. The obtained value $s^{(2D)}_{2,0}$ can be directly compared with the theoretical expected value for $s^{(2D)}_{2,cr}$. To be able to perform this analysis one has to study a system with very large charged pion multiplicity per event and/or to use a very large dataset. For this reason we have applied our approach to two systems: (i) the 5584 $Pb + Pb$ events at 158 $GeV/n$ and (ii) the $10^5$ CMC generated critical events (simulating the $C + C$ system at 158 $GeV/n$). The results of our calculations are presented in Fig 4. The solid circles are the values of $s^{(2D)}_{2,\epsilon}$ for the $Pb + Pb$ system while the open triangles describe the CMC results for various values of $\epsilon$. The dashed lines present a corresponding exponential fit.

For the CMC events we find $s^{(2D)}_{2,0} = 0.69 \pm 0.03$ a value which is very close to the expected $s_{2,cr} = 0.67$, while for the $Pb + Pb$ at 158 $GeV/n$ system we get $s^{(2D)}_{2,0} = 0.34$. The last value corresponds to a strong effect, owing to the fact that the $Pb + Pb$ system lies in the scaling region around the critical point. However it is clearly smaller than the theoretical value at the endpoint, in accordance with the fact that this system freezes out in a distance from the critical point in terms of the variables in Fig. 1.

In summary we have introduced an algorithm to detect critical fluctuations related to the formation of an isoscalar condensate in $A + A$-collisions. First analysis, using preliminary SPS-NA49 data, indicates the proximity to the critical point of the freeze-out area in the collisions with nuclei of medium size ($C + C$ or $Si + Si$).

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reconstruction of $\sigma$ - SPS data

Figure 3: The second factorial moment in transverse momentum space for all the analysed SPS processes. Represented are only the results obtained after the reconstruction of the isoscalar sector.
Figure 4: The slope $s_2^{(2D)}$ for different values of the kinematic window $\epsilon$ both for the $10^5$ CMC events as well as for 5584 $Pb + Pb$ events at 158 GeV/n using preliminary SPS-NA49 data.