Fractal, Diffraction-encoded Space-Division Multiplexing for Free Space Optical Communication

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ABSTRACT
With free space optical (FSO) systems, information is generally transmitted to minimize diffraction. Here, we demonstrate an alternate paradigm in which multiple spatial bit streams are diffraction-encoded, an approach that enables a wider cone for reception and is especially robust to noise. We leverage the fact that diffracted fractal patterns or diffractals redundantly encode information over large areas as they propagate to the far field. This scheme enables a roving receiver to capture multiple spatial bits simultaneously when sampling a portion of the far-field beam. Our numerical FSO studies with and without atmospheric turbulence are a basis for new design considerations governing roaming area, beam divergence, and proper diffraction encoding. Concepts related to the sparse complexity of fractal, diffracted patterns show promise for diffractal space-division multiplexing and may also be applied to channel marking, sensing, imaging, and other FSO systems.

1. INTRODUCTION

Although fractals are characterized by high visual complexity, their information content is low: they can be easily generated via simple, recursive algorithms. B. Mandelbrot elaborated on the fractal self-similar geometry and their mathematical notation in his books. M. Berry reported that the diffracted waves from fractal structures exhibit spatiotemporal intensity spiking in their linear propagation dynamics. To emphasize their uniqueness, he referred to the diffracted waves from fractals as ‘diffractals’. Fractal geometries and diffractal scattering have attracted widespread attention in many branches of science with applications in engineering such as digital image processing, especially image compression and antenna design. Such applications exploit a high level of information redundancy, which is organized in strongly-corrugated spatial patterns.

The implications of sparsity and redundancy in diffractals for communications systems are underexplored and there is strong potential for their application in the area of wireless communications given the drive to increase data rates. In the past, communication networks have embraced other advanced modulation and multiplexing schemes. Commonly used multiplexing techniques in optical fiber communication today include space-division multiplexing (SDM) or spatial multiplexing, wavelength-division multiplexing (WDM) using disjoint frequency bins, orthogonal frequency division multiplexing (OFDM) or coherent WDM (CoWDM) using spectrally overlapping yet orthogonal subcarriers, and polarization-division multiplexing (PDM) using both orthogonal polarizations supported by a single-mode fiber for independent bit streams. Among these approaches, spatial multiplexing has recently drawn significant interest, as the technology is still under development, particularly with FSO systems.

One potential approach for FSO spatial multiplexing uses beams with orbital angular momentum (OAM). Since OAM states are mutually orthogonal, they are simultaneously transmitted or multiplexed along the same beam axis and demultiplexed at the receiver. For the same carrier frequency, the system’s aggregate capacity is equal to the number of system state modes. OAM-multiplexed systems have achieved Tbit/s-scale transmission rates over free space. Additionally, J. Wang et al. have experimentally demonstrated a free-space data link with an aggregate transmission capacity of 1.036 Pbit/s and a high spectral efficiency of 112.6 bit/s/Hz using 26 OAM modes simultaneously with other multiplexing technologies. However, since multiple OAM states are multiplexed along the same beam axis, coaxial propagation and reception are required, which means that coherent, OAM-multiplexed links are sensitive to misalignment compared to non-OAM, single-beam communication links. This is an important challenge for FSO and will become worse in the presence of atmospheric turbulence. Propagating through atmospheric turbulence, the intensity profile of Gaussian and OAM beams can be significantly corrupted, making it harder to align and track using their intensity gradient, and greater efforts are necessary to evaluate and attenuate the receiver error.

Another approach to FSO that may be used
in tandem with OAM for SDM is multiple-input multiple-output (MIMO), where multiple independent bit streams are transmitted simultaneously and multiple aperture elements are employed at the transmitter/receiver. Zhao et al. claim that conventional, line-of-sight (LOS) MIMO SDM systems outperform OAM.28–30 As a well-established technique in radio wireless systems,31, 32 MIMO approach could provide capacity gains relative to single aperture systems and increase link robustness for FSO communications.33 However in practice, MIMO is prone to interference between the transmitted and received beams at different aperture elements; this interference arises when these apertures are not sufficiently spatially separated.29, 34, 35

In this paper, we demonstrate the novel approach: diffractal space-division multiplexing (DSDM), which is illustrated in Fig. 1. The unique properties of diffractal redundancy enable the simultaneous transmission of multiple independent bit streams;36, 37 in the far field, arbitrary parts of a diffractal contain sufficient information to recreate the entire original (sparse) signal.38, 39 Transmitted beams with higher fractal orders achieve higher reconstruction accuracy [see right column of Fig. 1]; in prior work, this result has been demonstrated experimentally with a 4-F system.36 Since DSDM does not rely on wavelength or polarization, it could be used with WDM and PDM techniques to further improve system capacity. Additionally, DSDM may be used to improve data transmission capacity in adverse environments in a manner analogous to other FSO techniques in which different parts of a signal are referenced to reduce receiver error.40

One reason why DSDM may be underexplored is due to diffraction issues (i.e., diffractals generate a wide cone of high spatial frequencies as they propagate, which is counter to many paradigms for FSO). Additionally, the strong diffraction from irregularly corrugated beams is a challenge to simulate reliably. Nevertheless, DSDM presents several important advantages over other FSO multiplexing technologies:

- Robust to misalignment: with DSDM, receivers may sample arbitrary beam parts, entirely off-axis.
- Wide reception cone: DSDM enables a roaming area for the non-coaxial transmitter and receiver.
- Simple design: compared to MIMO, DSDM uses a single transmitter/receiver aperture pair.
- Robust to turbulence: diffractals provide redundant encoding to capture multiple bits per frame.
- Swift decoding: DSDM uses optical processing for demultiplexing and a simple soft thresholding for reconstruction.
- Simple receiver requirements: the same optics may be used to demultiplex all data channels.
- High detection sensitivity: focusing lens enables capture of low intensity optical signals.
- Scalability: aggregate capacity is only limited by number of pixels available at the transmitter.
More generally, DSDM may be relevant to other applications where the alignment between transmitter and receiver is not fixed, when a receiver “roaming area” is needed, or when an object or data needs to be encrypted, marked, or tracked. Spatial kernel patterns may be used as channel codes or to enable additional channel coding for error correction. In order to advance ideas on redundant spatial diffraction encoding, here we establish basic parameters for fractal propagation or diffraction encoding to the “far-field”: we measure the reconstruction accuracy and robustness of DSDM. This article is organized to elaborate on novel opportunities for DSDM in FSO communication systems. In Sec. 2, we show a simple transmitter and receiver design and illustrate the experimental implementation of DSDM using a spatial light modulator(SLM). We illustrate diffractal propagation characteristics, which determine the roaming area described in later analyses. In Sec. 3 we illustrate design considerations tied to receiver size and the propagation distance between transmitter and receiver, which influences the communication channel’s performance. In Sec. 4, we show that, even in the presence of turbulence, with the implementation of higher fractal order beams, accuracy of DSDM system remains high. In fact, with only 81x81 pixel transmitters, we achieve a 10\(^{-3}\) bit error rate (BER) with 5 dB signal-noise-ratio (SNR) (the receiver size collects only 30% of the off-axis roaming area at 2.5 km propagation distances without any error-correcting schemes). This result indicates outstanding possibilities for robust FSO with fractal-based, diffraction-encoded signals.

2. DIFFRACTAL SPACE-DIVISION MULTIPLEXING

The DSDM system involves: multiplexing (where the transmitted data is a fractal from kernel data), diffraction encoding (when the beam propagates to the receiver), and demultiplexing (which is composed of coherent optical and electronic processing). The DSDM deploy the intensity modulation/direct detection (IM/DD) scheme, which is common for FSO communication systems\(^{41}\). In Fig. 2(a), we show an experimental setup with a transmissive spatial light modulator (SLM, Holoeye LC2012). The SLM amplitude-modulates with on/off keying (OOK), where the ‘1’s are in-phase optical pulses that occupy the bit duration and ‘0’s are bits in the absence of optical pulses. For fractal orders (FO) of 3, 4, and 5, the beams transmitted by the SLM are 1, 3, and 9 mm in diameter.

At a distance of 10 m from the SLM, a fraction of the laser beam is captured off-axis by a camera placed in the focal plane of the lens. The beam diverges over 10-times in width and the light intensity at the receiver is too low to be detected without the focusing lens. Only a small portion of the far-field beam is captured. The detector sensor, placed in the focal plane of the lens, captures the demultiplexed data with FO = 3, 4, and 5. [Figs. 2(b1-b3)], which easily reproduces the transmitted “J” kernel. This diffractal transmitter/receiver scheme is explained further below.

2.1 Multiplexer: fractal, spatially-modulated transmission

The kernel OOK data is a binary \(s \times s\) array. The transmitted data is produced with a fractal mask or a screen pattern generated with the Kronecker product [see left column of Fig. 1]. The Kronecker product of the kernel with itself involves placement of the kernel at the location of each bit ‘1’ and the placement of an all-zeros matrix at each bit ‘0’. When the beam’s fractal order is equal to \(n\), the Kronecker product repeats \(n\) times. We define the smallest sub-square of the fractal as the pixel. Therefore, each transmitted pattern contains \(s^n \times s^n = s^{2n}\) pixels. Details of this kernel-dependent diffraction are provided in Sec 1.1 of the supplemental document.

2.2 Diffractal beam divergence

Diffractal beam divergence is an important design consideration: it defines the effective roaming area for receivers. From diffraction theory, a spatially-corrugated
beam such as a fractal diverges faster than a Gaussian-profiled beam. We numerically propagate diffractals with $s = 3$. The total transmitted power is unit normalized and the transmitted beam area and pixel power varies with fractal orders and kernel shapes. The pixel size ($W_{px} = 2$ mm) and wavelength ($\lambda = 1550$ nm) remains fixed and constant regardless of beam shape. Care is necessary to ensure that the power is either conserved in the simulations or repeatable with doubled boundary widths.

For the calculation of diffracted beam radius, the concept of beam mode field radius (MFR) is employed and given by the equation:

$$\text{MFR} = \sqrt{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |u(x,y)|^2 (x^2 + y^2) \, dx \, dy / \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |u(x,y)|^2 \, dx \, dy} \quad (1)$$

where $x$ and $y$ are the transverse spatial coordinates and $u(x,y)$ is the electric field of the FSO beam. The fraction of the total beam power within the maximum roaming area is less than or equal to $1 - 1/e^2$. Different kernel shapes have different degrees of diffraction. Moreover, the range of diffraction for different $3 \times 3$ kernels varies by a factor of 2: In Fig. 3(a), the kernel “R” diffracts at half the rate as the kernel “X”. The upper limit for the beam divergence speed is that for a single pixel. A diffractal with the kernel “X” spreads almost as much as a single pixel at the same propagation distance. We observe that a diffractal’s beam divergence scales approximately with the number of internal edges in the kernel shape or with the largest independent block length of the kernel. Different kernel shapes experience different degrees of diffraction and the extent to which this varies is also tied to the image complexity that characterizes all diffractals.

In fact, even the slowest diffractals diverge at a rate much greater than a Gaussian of the same initial width due to their highly corrugated structure. For example, the kernel ”R” diverges at a rate 26 times faster than a Gaussian beam with the same waist radius. In Fig. 3(b), the initial beam radius of the kernel ”R” is around 10 cm. This beam radius increases by a factor of $\sqrt{2}$ after a propagation distance of $z = 0.8$ km. Meanwhile, the Rayleigh length of a Gaussian beam with a 10 cm waist radius is 21 km, given by $z_R = \pi/\lambda(w_0)^2$, where $w_0$ is the beam waist. This extreme divergence of diffractals relative to the divergence of Gaussian beams is a result of the high degree of structure, intrinsic to fractal-modulated beams.

Diffractals uniquely exhibit non-Gaussian beam diffraction statistics, meaning that their propagation in the near field is populated with spatiotemporal spikes. However, many aspects of their propagation at long distances are similar to Gaussian beam propagation. For example, the diffractal beam radius MFR increases linearly at longer propagation distances. The MFR at longer distances ultimately scales in proportion with the size of a single pixel or with the largest independent block length of the kernel rather than the initial beam waist. In other words, with the same kernel shape and pixel size, beams with different fractal orders have different initial beam waists but similar far-field MFR. This convergence, which depends on the kernel shape but not on fractal order is illustrated in Fig. 3(c).

### 2.3 Diffractal propagation to the ”far field”

With DSDM, diffraction provides part of the signal spatial encoding; the accuracy in the reconstructed data depends on how far the beam travels between the trans-
mitter and receiver. We use the concept of a "Fraunhofer distance" to quantify the diffraction distance to the far-field,

$$z_{DFF} > 2s^2nL_{dfpx},$$

(2)

where $n$ is the fractal order and $L_{dfpx} = \pi/\lambda(W_{px}/2)^2$ is the confocal parameter for a single pixel of width $W_{px}$. We note that the strong spiking spatiotemporal behavior described by $^4$ is part of the diffraction-encoding for DSDM and occurs when $z < z_{DFF}$. Our simulations suggest that, in order to fully take advantage of the diffractal encoding, the receiver should be at a distance $z > z_{DFF}$ from the transmitter. At the same time, as we demonstrate in the following section, good reconstruction is still achieved at $z < z_{DFF}$ when the detector is sufficiently large.

The propagation distance influences the DSDM diffraction encoding as well as the roaming area. By increasing the FO, we may not significantly increase the roaming area [Fig. 3(c)]; however, we do increase $z_{DFF}$ and require longer distances for diffraction encoding. To ensure proper diffraction encoding at shorter distances $z$, smaller pixel size $W_{px}$ may be used; nevertheless, this correspondingly increases the MFR.

2.4 Detection and demultiplexing

Longer propagation distances result in larger roaming areas. At the receiver, a portion of this roaming area is captured. Fig. 4(a) shows the definitions of receiver detector width (DW) and roaming radius (R) with respect to the beam mode field radius (MFR). The green dotted circle represents the possible roaming area and the green solid circle is defined as the maximum roaming area with radius equal to $\sqrt{2}\text{MFR}$.

DSDM demultiplexing is performed both optically (with a convex lens and Fourier-plane camera) and soft thresholding (with a simple threshold algorithm). A vignetted lens focuses light onto a sensor. Diffractal propagation is unique because in the focal plane of an arbitrarily placed lens, a pattern similar to the initial data kernel is produced, even when the lens is off-axis and captures a fraction of the far-field beam. The receiver (lens and camera) may move freely within the maximum roaming area with radius equal to $\sqrt{2}\text{MFR}$.

![Figure 4](image.png)

Figure 4. (a) The far-field beam pattern for the kernel "J" and definitions of coverage area (CW), receiver width (DW), and roaming radius (R). Several randomly-positioned receivers are shown (white squares). (b) From left to right, optical deconvolution from an off-axis receiver DW = 0.75m to the sensor. The captured area represents 5.8% of total beam power and 16% of the coverage area. (c) 9 sub-blocks of the received image. During reconstruction, each sub-block is thresholded to ‘1’ or ‘0’ depending on the sub-block intensity.

3. DESIGN CONSIDERATIONS

The roaming radius, FO, and receiver size all play a critical role in DSDM. In the sections below, we consider these parameters where the receiver aperture is significantly smaller than the diffracted beam or maximum roaming area $R = \sqrt{2}\text{MFR}$. To draw statistics, we sample over 4000 random locations across the beam to calculate the kernel bit-error-rate (K-BER), which varies spatially. The K-BER is the accuracy calculated for a fixed kernel instead of a random pattern; this fixed kernel pattern is a measure of the accuracy if DSDM is applied for channel marking or tracking.

3.1 Roaming radius

Not surprisingly, the error probability increases as the receiver moves away from the far-field beam center axis, however decreases with larger receiver areas. The roaming radius of the receiver directly influences the K-BER performance.

Figure 5 illustrates the reconstruction and K-BER performance as a function of the roaming radius $R$ at a propagation distance of $z = 2.5$ km without turbulence, where the kernel is “J” and FO = 4. Fig. 5(b) shows the pattern observed in the Fourier-plane of the receiver lens for different sampling locations of the coverage area. Figure 5(c) shows the reconstructed data of corresponding images in 5(b). In general, as the receiver moves farther away from the center of the diffracted beam, the K-BER gradually increases; the
Figure 5. Illustrations of transmitted kernel “J” with fractal order (FO) of 4 at propagation distance $z = 2.5$ km. (a) K-BER vs roaming radius ($R$) for different receiver widths (DW). (b) Deconvolved and (c) reconstructed data over 25 equally-spaced patches across the coverage area. The dashed squares indicate a possible roaming area wherein the sampled, reconstructed 9-bit images are all correct.

The limited K-BER performance is largely influenced by compression noise or the reconstruction error that arises from using only a portion of the entire diffracted beam. As long as the receiver aperture is smaller than the diffracted beam, compression noise exists, regardless of whether there is additional noise or not. Compression noise decreases as the receiver size increases. For small roaming radius $R$, the K-BER drops sharply, and this drop occurs for smaller $R$ with larger receiver size. Compression noise decreases quickly when the receiver size is larger. The receiver aperture covers most of the high-intensity central area of the far-field beam. Not surprisingly, DSDM with smaller roaming areas and larger receivers have the best performance.

### 3.2 Influence of Fractal Order

DSDM performance is significantly improved when we increase the FO of the transmitted data kernel. As FO increases, the accuracy over the roaming area increases and smaller DWs are possible. The far field beam exhibits smaller similar speckle features and information is encoded at higher spatial frequencies. Therefore, when the FO is large, the detector image produced from an arbitrary subsection of the roaming area closely resembles the transmitted data.

Figure 6(a) shows the K-BER versus DW at a propagation distance of $z = 10$ km and compares the K-BER performance of FO = 2, 3, and 4 beams. The trend clearly shows that higher fractal orders achieve higher accuracy. The K-BER for FO = 4 is lower than the K-BER for FO = 2 and 3. In order to reach the same K-BER level of $10^{-3}$, the FO = 3 channel needs a receiver size that is about 1.6 times larger than that of the FO = 4 channel. To put the receiver sizes into perspective, at Fig. 6(a), when FO = 4, the K-BER of $10^{-3}$ is achieved with a receiver size less than 25% of the maximum roaming area (the receiver area = (DW)$^2$ = (2.5m)$^2$ = 6.25 m$^2$; maximum roaming area = 25 m$^2$). The Fourier-plane detector images carry more self-similar, iterated features with FO = 4 compared to FO = 2 [Fig. 6(b-e)]. The greater degree of redundancy in these features leads to smaller K-BER with higher FO.

Figure 6 illustrates that DSDM with larger FO is an effective way to improve system performance. However, larger FO beams require more transmitted pixels, which require longer propagation distances for encoding [Eq. 2]. As noted above, smaller pixels may be used to decrease the necessary propagation distance but this also increases the rate of beam divergence. Thus, careful design of beam divergence and receiver area is needed for higher-FO DSDM.

### 3.3 Influence of Receiver Size and Kernel Shape

One main advantage of DSDM is that the receiver aperture can be much smaller than the whole diffracted beam; however, this advantage varies with the diffraction encoding and kernel data. In the previous graph’s trend (for the kernel “J”) where the K-BER vs DW relationship is smooth [Fig. 6(a)], the propagation distance $z = 10$ km puts the receiver approximately in the far field or $z \approx z_{DFF}$. At a shorter distance $z = 2.5$ km, the K-BER vs DW for different FO = 4 kernels
shows more subtle features [Fig. 7(a)]. While this figure shows that a larger receiver size results in a lower error probability, at this shorter $z$, we observe features from partial diffraction encoding where the beam has not reached the "far field". The inflection points in the curves in Fig. 7(a) are one feature of partial diffraction encoding. A comparison with the smooth curve of Fig. 6(a) indicates two obvious turning points that separate the trend lines into three regions.

The K-BER performance in Region I is again limited by compression noise.\textsuperscript{43} By comparing Fig. 7(c1) and (c3), we see the influence of compression noise: a detector image with a larger receiver, where $DW = 0.4$m, contains more detailed information than with a smaller receiver, $DW = 0.2$m. The upper left corner of the receivers of different size are located at the same place [see white squares in Fig. 7(b)]. In Region III, the K-BER drops sharply as before with increasing receiver size. Results indicate that when $DW \leq 0.45$m (where receiver size is 30% of the maximum roaming area) K-BER is below the forward error correction limit $10^{-3}$.\textsuperscript{44} This indicates that the K-BER is reduced simply by increasing DW.

However, the trend lines in Region II in Fig. 7(a) are flattened or slightly raised, which appears in violation of the trend described above. However in this Region, the K-BER performance is dominated by partial diffraction encoding, or not having propagated far enough to reach the "far field". It is not easy to observe Region II at a longer propagation distance of $z = 10$ km, which is closer to $z_{DFF}$ or the "far field" [Fig. 6(a)]. Figure 7(b) shows different receiver sizes $DW = 0.2, 0.3$, and $0.4$m in the roaming area. Accurate reconstruction is achieved when $DW = 0.2$ and $0.4$m [Fig. 7(c1,c3)]. One important area of future work will be the reconstruction of beams with partial diffraction encoding such as those in region II. An illustration is shown in Fig. 7(c2) when a receiver $DW = 0.3$m is located in the top left of the far field. In this case, the bottom right corner of the receiver samples only a part of the high-intensity central area. This area remains localized and is as large as the original, transmitted beam. Since the intensity of the central portion is much higher than the other sampled parts, the upper-left corner of the deconvolved image is much brighter. With our on/off threshold reconstruction algorithm, only the brightest area is considered as '1', whereas the other dark areas are '0'. This sampling, in combination with the current threshold algorithm, results in a higher error probability with $DW = 0.3$m than $DW = 0.2$m.

We note that, at many instances in our study, the numerical reconstruction algorithm fails to identify the kernel pattern even though the detector images would easily be classified by human visual inspection. We tried other reconstruction algorithms besides the one used based on a threshold; kernel reconstruction algorithms based on intensity differentials and image boundaries do, in some cases, reduce the error probability compared to our simple intensity threshold approach. The simplest reconstruction algorithm, however, distills clearer understanding of the diffraction encoding, which is one scope of this article. In the future, we anticipate that more advanced reconstruction algorithms will significantly improve the accuracies beyond the results presented here.

4. ROBUSTNESS TO TURBULENCE

In the presence of atmospheric turbulence, DSDM has the advantage of redundant encoding. We simulate the transmitter-receiver propagation in the presence of...
weak and strong atmospheric turbulence with random phase screens. Two common parameters of atmospheric turbulence—the index of refraction structure $C_n^2$ and propagation distance $z$—are varied to simulate different turbulence strengths. Moreover, the Rytov variance is a fundamental scaling parameter that depicts the strength of the wave fluctuations, which is defined by $\sigma_I^2 = \frac{1}{2} C_n^2 k^7/6 L^{11/6}$, where $k = 2\pi/\lambda$ is the optical wavenumber and $\lambda$ is the wavelength. Another scaling parameter, the signal-to-noise ratio (SNR), is defined as:

$$SNR = 10 \log_{10} \left( \frac{\text{Signal}}{\text{Noise}} \right) = 10 \log_{10} \left( \frac{\sum_1^N \sum_1^N |u_{AT}|^2}{\sum_1^N \sum_1^N |u_{AT} - u_{vac}|^2} \right)$$

where $u_{AT}$ and $u_{vac}$ are the complex, electric-field profiles of the diffracted beams with and without atmospheric turbulence.

K-BER performance under different atmospheric turbulence strength is simulated and estimated in Fig. 8(a1,a2). The propagation distance is $z = 2.5$ km, and receiver width DW ranges from 5 to 60 cm. The K-BER vs DW trends are similar to Fig. 7(c), except that the turbulence phase screens are added during propagation. The K-BER is again calculated by averaging 4000 single-aperture receivers at random locations within the maximum roaming area. Figure 8(a1) shows the K-BER under weak turbulence, where $C_n^2 = 10^{-15} m^{-2/3}$, and scintillation index is $\sigma_I^2 = 0.11$, corresponding to an SNR of 5 dB. Figure 8(a2) shows K-BER under strong turbulence, where $C_n^2 = 10^{-14} m^{-2/3}$, and scintillation index is $\sigma_I^2 = 1.11$, corresponding to a SNR close to 0 dB. The fractal order is $FO = 4$ and the propagation distance is $z = 2.5$ km. (b) The received bit error at different locations within the maximum roaming area radius of 2.8 m with increasing receiver size (DW) in columns and fractal order in rows. Here, $z = 10$ km.

Figure 8. (a) K-BER performance under (a1) weak turbulence conditions, $C_n^2 = 10^{-15} m^{-2/3}$, and scintillation index is $\sigma_I^2 = 0.11$, corresponding to an SNR of 5 dB. (a2) strong turbulence conditions, where $C_n^2 = 10^{-14} m^{-2/3}$ and scintillation index is $\sigma_I^2 = 1.11$ corresponding to a SNR of 0 dB. The fractal order is $FO = 4$ and the propagation distance is $z = 2.5$ km. (b) The received bit error at different locations within the maximum roaming area radius of 2.8 m with increasing receiver size (DW) in columns and fractal order in rows. Here, $z = 10$ km.

we show the bit error from 40x40 single receivers shifted in position and at evenly distributed locations over the roaming area. In Fig. 8(b), different colors represent 0 to 9 received error bit values (there are 9 bits in each kernel).

By column, receiver widths from 0.26 to 2.32 m are tested. For the same FO, larger receiver widths correspond with fewer error bits, consistent with the declining curves in Fig. 8(a1,a2). By row, we show FO from 2 to 6. For the same receiver size, when the FO increases, the bit error decreases. Larger FO’s generally improve DSDM robustness to atmospheric turbulence; however, a larger FO increases the distance needed for diffraction
encoding $z_{DFF}$ [Eq. 2]. This issue of diffraction encoding is highlighted with $FO = 4, 5, 6$ at $DW = 0.77$ m in Fig. 8(b). Higher FO's have a smaller K-BER up to $FO = 5$, but when the FO increases to 6, the corresponding bit error value increases instead of decreases. This increase appears to break the trend where smaller error accompanies higher FO. In fact, the distance $z = 10$ km is significantly less than the minimum diffraction-encoded distance and is not far enough for $FO = 6$ to reach the “far-field”. Again, our results indicate that DSDM is still promising when the propagation distance is less than the Fraunhofer diffraction length $z < z_{DFF}$ [Eq. 2].

5. DISCUSSION AND CONCLUSION

DSDM leverages the fact that fractal patterns of kernel data are redundantly encoded over large areas as they propagate to the far field. As a result, a small portion of the far-field carries information to reproduce the original kernel data. Provided that the receiver detector has sufficient sensitivity, the best reconstruction accuracy is achieved from beams that have propagated to the far field. However, the far field—the point beyond which the radiation pattern scales but does not change shape with propagation—is not yet explicitly defined for diffractals. Additionally, in this article, we have shown that diffractional beam divergence and propagation to the far field depend strongly on kernel shape. Our results are relevant to computational sensing, imaging, and communication systems.

Although diffractals exhibit considerably more stable propagation in the far field, we are able to implement DSDM with larger DW in the near field ($z < z_{DFF}$). As the beam propagates to the far field, the intensity patterns exhibit spatiotemporal spiking as part of the process of diffraction encoding. We provide an analysis of the dependence on the receiver size and influence of fractal order in DSDM. In many cases with incomplete diffraction encoding, the detector images are inaccurately classified using our linear threshold algorithm but easily classified by visual inspection. Judging from, the incorporation of a neural network or optimization scheme could improve the BER by several orders over already-promising results.

In conclusion, we show enormous potential for DSDM in FSO communication and channel marking, where only a few percent of the off-axis diffracted beam power is needed to reconstruct spatially encoded kernel data. DSDM may be used in practical free-space propagation systems to achieve high-transmission capacity in combination with other degrees of freedom, such as polarization and wavelength multiplexing. With DSDM, information is redundantly encoded spatially so that, with a sufficiently large receiver, DSDM communication is robust to atmospheric turbulence. With 81x81 transmitted pixels, we achieve BER of $10^{-3}$ under weak turbulent conditions (5 dB SNR) when the receiver sizes are 30% of the roaming area over propagation distances of 2.5 km. These simulation results would be improved further with the use of higher-FO beams. Higher-FO beams are technologically feasible now but beyond our current capability with simulations. To implement DSDM experimentally over similar distances with higher FO, smaller pixels ensure proper diffraction encoding. The effect of spatially-modulating beams with smaller pixels is a larger reception cone area, which, far from being disadvantageous, may be valuable for FSO systems where the transmitter and receiver are roaming or not coaxial.

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