Alternative techniques of population transfer in multilevel systems

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Abstract. We consider a five-level atomic system (M-system) driven by four near-resonant laser pulses and study the possibilities of population transfer via different combinations of Stimulated Raman Adiabatic Passage (STIRAP) and bright-STIRAP (b-STIRAP) processes. We examine these combinations analytically and numerically from the point of view of efficiency of population transfer. It is shown, in particular, that the combination of the processes above enhances the efficiency. Analytical expressions are obtained for the quasienergies of the system in two regimes differing by conditions imposed on resonance detunings. Interaction adiabaticity conditions are analyzed. We also study the necessary successions of turning on and off of pulses which ensure efficient transfer of atomic populations. The obtained analytical results agree well with direct numerical calculations.

1. Introduction
A widespread technique for population transfer in Λ-systems employs stimulated Raman process at adiabatic passage of laser pulses (STIRAP) [1]. This method is based on counterintuitive sequence of pulses when the Stokes pulse coupling the final state and the intermediate, excited, state precedes the pump pulse which couples the excited state with the initial one. In multilevel systems the population transfer is realized with use of successive STIRAP processes or STIRAP-chains [2].

Recently an alternative efficient method of population transfer where the intuitive sequence of pulses is used with a large one-photon detuning has been predicted and demonstrated experimentally. Unlike the STIRAP for which the dynamics adiabatically projects onto the dark state, i.e., the excited state component is absent, the intuitive sequence follows a bright state. This process has been called bright STIRAP (b-STIRAP) [3]. An essential distinction of b-STIRAP from STIRAP is the rate of population transfer in media which is superluminal [4].

The purpose of the present study was to find other ways of transfer of level populations in multilevel systems through combination of STIRAP and b-STIRAP (STIRAP-b-STIRAP, b-STIRAP-b-STIRAP, b-STIRAP-STIRAP), rather than purely STIRAP-chains. We restrict our study to five-level system which can be considered as two three-level systems with a common level, so it is supposed that STIRAP and b-STIRAP provide more possibilities being combined in different successions. The advantages of one or another combination will be analyzed depending on the configuration of five atomic levels.

2. Atomic system and basic equations
Consider a five-level atomic system with M-type configuration of levels with numbering of levels and Rabi frequencies of incident pulses shown in Fig.1 and the Hamiltonian

\[
H = \sum \sigma_i \delta_{i-1} + \left( \sum \sigma_{i+4} \Omega_i \right) + \text{h.c.}
\]

(1)

where the detunings are defined as

\[
\delta_1 = \omega_{21} - \omega_1, \quad \delta_2 = \omega_{31} + \omega_2 - \omega_1, \quad \delta_3 = \omega_{43} - \omega_3 + \delta_2, \quad \delta_4 = \omega_{54} + \omega_4 - \omega_3 + \omega_2 - \omega_1
\]

with \( \omega_{ij} \) being the frequency separations between the respective levels, \( \omega_i \) the frequencies of laser pulses, and Rabi frequencies are defined as usual.

The characteristic equation \( \det(H - \lambda E) = 0 \) has the following explicit form:

\[
\lambda(\lambda - \delta_1)(\lambda - \delta_2)(\lambda - \delta_3) - \Omega_1^2(\lambda - \delta_1)(\lambda - \delta_2) - \Omega_2^2(\lambda - \delta_2)(\lambda - \delta_3) + \lambda \Omega_3^2 \Omega_4^2 = 0,
\]

(2)

The conventional STIRAP-STIRAP chain is realized for the eigenstate corresponding to the eigenvalue \( \lambda = 0 \). As follows from Eq.(2), it has a zero root, if the conditions of exact two- and four-photon resonances are met:

\[
\delta_2 = \delta_4 = 0.
\]

(3)

By eliminating this root we obtain a fourth order equation whose roots can be written in not very cumbersome form if some additional conditions are fulfilled. So, under condition \( \delta_1 = \delta_3 \) the roots of characteristic equation are

\[
\lambda_0 = 0, \quad \lambda_{1,2} = \frac{1}{2} \left( \delta_1 \pm \sqrt{\delta_1^2 + 4x_1} \right), \quad \lambda_{3,4} = \frac{1}{2} \left( \delta_1 \pm \sqrt{\delta_1^2 + 4x_2} \right)
\]

(4)

where \( x_{1,2} = (1/2)(\Omega_1^2 \pm (\Omega_2^2 - 4V^2)^{1/2}) \), while the quantities \( \Omega \) (generalized Rabi frequency) and \( V \) are defined as follows: \( \Omega_1^2 = \Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2 \), \( \Omega_1^4 = \Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2 \). Adiabaticity of interaction will take place under the conditions:

\[
V^4/T/\delta_1 \gg 1, \quad \Omega^2/T/\delta_1 \gg 1, \quad \delta_1 T \gg 1,
\]

where \( T \) is the time of interaction between the system and fields of pulses.

Conditions \( \delta_1 = - \delta_3 \) and \( \Omega_1^2 + \Omega_2^2 = \Omega_3^2 + \Omega_4^2 \) also lead to a simple appearance of the roots of Eq.(2)

\[
\lambda_0 = 0, \quad \lambda_{1,2,3,4} = \pm \left( \frac{1}{2} \left( (\delta_1^2 + \Omega_1^2) \mp \left( (\delta_1^2 + \Omega_2^2)^2 - 4V^2 \right)^{1/2} \right)^{1/2} \right)
\]

(5)

Knowledge of eigenvalues (4) and (5) enables us to construct corresponding eigenvectors. However, we will only consider several special cases where an efficient transfer of population from level 1 to level 5 is possible.
3. Combination of b-STIRAP and STIRAP

If $\Omega_1 = \Omega_4$, expressions (4) are simplified essentially:

$$
\lambda_{1,2} = \frac{1}{2} \left( \delta_1 \mp \sqrt{\delta_1^2 + 4\Omega_2^2} \right)
$$

$$
\lambda_{3,4} = \frac{1}{2} \left( \delta_1 \mp \sqrt{\delta_1^2 + 4\Omega_4^2} \right)
$$

(6)

where the generalized Rabi frequency $\Omega_s$ is defined by the relation $\Omega_s^2 = \Omega_1^2 + \Omega_2^2 + \Omega_3^2$. Let us assume the one-photon detuning to be positive ($\delta_1 > 0$) and consider the root $\lambda_1$ of Eq.(2). An interesting peculiarity of this root is that it is independent of fields $\Omega_2$ and $\Omega_3$. The state vector corresponding to this eigenvalue has the form

$$
|\Psi\rangle = \cos\Phi\cos\Theta|1\rangle - \sin\Phi\cos\Theta|2\rangle + \sin\Phi\sin\Theta|4\rangle + \cos\Phi\sin\Theta|5\rangle
$$

(7)

where the angles $\Phi$ and $\Theta$ are defined by relations $\tan\Phi = \lambda_1/\Omega_1$ and $\tan\Theta = \Omega_2/\Omega_3$. The state (7) does not contain the state $|3\rangle$ and the dipole moments of the transitions $1 \rightarrow 2$ and $4 \rightarrow 5$ are equal in modulus, but opposite in sign. Figure 2 shows the succession of turning on the pulses and results of direct numerical calculation demonstrating an efficient population transfer. It is seen that the field $\Omega_1$ is turned on prior to $\Omega_2$ (b-STIRAP) and the field $\Omega_3$ prior to $\Omega_4$ (STIRAP).

Fig.2. (Color online) Sequence of pulses and dynamics of level populations in case $\Omega_1 = \Omega_4$ and $\delta_1 > 0$.

The same chain may be realized via eigenstate corresponding to the root $\lambda_4$. In this case the state vector involves all the bare atomic states:

$$
|\Psi\rangle = \cos\Phi_1\cos\Theta_1\sin\Theta|1\rangle - \sin\Phi_1\sin\Theta|2\rangle - \sin\Phi_1\cos\Theta_1|3\rangle + \cos\Phi_1\sin\Theta_1|4\rangle + \cos\Phi_1\cos\Theta_1\cos\Theta_1|5\rangle.
$$

(8)

where $\Phi_1$ and $\Theta_1$ are defined similarly to (7): $\tan\Phi_1 = \lambda_2/\Omega_4$ and $\tan\Theta_1 = \Omega_2/\Omega_1$. For efficient population transfer through the state (8) pulses of different durations should be used, as shown in Fig.3.
4. Combination of STIRAP and b-STIRAP

This chain can be realized when the fields coupling transitions 1→2 and 4→5 are different, while the transitions 2→3 and 3→4 are driven by degenerate pumping, i.e., $\Omega_2 = \Omega_3$. The succession of pulse switching and the evolution of level populations are depicted in Fig.4. It is seen that this case also provides efficient transfer of population from state 1 to state 5.
Although the advantages of the above-considered schemes as compared with the conventional STIRAP-STIRAP chain are not seen for the studied single-atom problem, in case of atomic medium they may turn to be more efficient from the point of view of the rate of population transfer and from the point of view of increasing the medium length at which the full transfer is possible. Indeed, the main restriction on the medium length in STIRAP-chain transfer is the condition of pumping (field $\Omega_1$) depletion. In the schemes considered above different fields play the role of pumping, so a photon absorbed from the field in one transition will in another transition be emitted back into the same field. However, detailed investigation of the population transfer problem in media is out of scope of the present work and will be performed elsewhere.

5. Combination b-STIRAP-b-STIRAP

Such a sequence, like STIRAP-STIRAP, may be realized by two pulses with $\Omega_1=\Omega_3$ and $\Omega_2=\Omega_4$. The detailed analysis shows, however, that the population can in this case be transferred efficiently to only the level 3, as depicted in Fig.5.

![Fig.5. (Color online) b-STIRAP-b-STIRAP chain with two pulses.](image)

**Conclusion**

The coherent population transfer is at the heart of many quantum processes with applications for quantum information processing, the control of chemical reaction, nonlinear and atomic optics et al. We have studied a five-level M-type atomic system in the sense of possibilities of population transfer between the levels. We have demonstrated that adiabatic transfer of populations in this system may be realized not only via dark states in traditional STIRAP-chain, but also via other adiabatic states. We have proposed three novel alternative schemes of coherent population transfer between lower levels of an M-type atomic system. We found analytically and numerically the possible combinations of parameters and sequences of corresponding laser pulses providing complete transfer of population to a specific level by optimum combinations of STIRAP and b-STIRAP processes. Although the upper levels in considered schemes are during interaction populated provisionally, their populations can be made negligibly low by providing large single-photon detunings. Advantages and shortcomings of proposed schemes as compared with conventional STIRAP-chain are associated with those of the b-STIRAP technique (see [4,5]) and will be analyzed in a subsequent paper in connection with their realization in macroscopic volumes and use for designing logic gates.
Acknowledgment

Work was in part supported by the Ministry of Education and Science of Armenia (MESA), grant no. 11-1c124 of State Science Committee of MESA, IRMAS International Associated Laboratory, and the Volkswagen Stiftung I/84 953.

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