Reduced sensitivity of the \((d,p)\) cross sections to the deuteron model beyond adiabatic approximation

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It has recently been reported [Phys. Rev. Lett. 117, 162502 (2016)] that \((d,p)\) cross sections can be very sensitive to the \(n-p\) interactions used in the adiabatic treatment of deuteron breakup with nonlocal nucleon-target optical potentials. To understand to what extent this sensitivity could originate in the inaccuracy of the adiabatic approximation we have developed a leading-order local-equivalent continuum-discretized coupled-channel model that accounts for non-adiabatic effects in the presence of nonlocality of nucleon optical potentials. We have applied our model to the astrophysically relevant reaction \(^{26}\text{Al}(d,p)^{27}\text{Al}\) using two different \(n-p\) potentials associated with the lowest and the highest \(n-p\) kinetic energy in the short-range region of their interaction, respectively. Our calculations reveal a significant reduction of the sensitivity to the high \(n-p\) momenta thus confirming that it is mostly associated with theoretical uncertainties of the adiabatic approximation itself. The non-adiabatic effects in the presence of nonlocality were found to be stronger than those in the case of the local optical potentials. These results argue for extending the analysis of the \((d,p)\) reactions, measured for spectroscopic studies, beyond the adiabatic approximation.

Introduction. One nucleon transfer in \((d,p)\) reactions is an important source of information about the single-particle strength in atomic nuclei, quantified by spectroscopic factors and asymptotic normalization coefficients. They are obtained from a comparison of experimental and theoretical cross sections calculated using direct transfer reaction theory and, therefore, are influenced by its uncertainties. The uncertainties arising due to the input optical potentials and the shape of the mean field that binds the transferred neutron has been known for a very long time. Recently, new theoretical uncertainties have been identified in Ref. [1], associated with the \(n-p\) interaction used in adiabatic treatment of deuteron breakup with nonlocal nucleon optical potentials. This work studied the \(^{26}\text{Al}(d,p)^{27}\text{Al}\) reaction, measured in [2] to pin down the \(^{26}\text{Al}\) destruction by the \((p,\gamma)\) reactions in novae explosions, and used several deuteron models: Hulthén model [3], AV18 [4], Reid soft core [5], CD-Bonn [6] and the chiral effective field theory at N4LO with five different regulators [7]. All these models produce exactly the same deuteron wave functions \(\phi_d\) and the vertex functions \(V_{np}\phi_d\), where \(V_{np}\) is the \(n-p\) potential, at the \(n-p\) separations \(r\) larger than than 2 fm. However, the model predictions for these quantities at \(0 < r < 2\) fm are very different. This sensitivity to the short-range \(n-p\) wave functions (and the corresponding sensitivity to the high \(n-p\) momenta) seems puzzling given the relatively low deuteron incoming energies, about 10 MeV, for which the \((d,p)\) calculations have been done in [1]. Such sensitivity may indicate that other important effects, associated with \((d,p)\) reaction mechanisms, are missing in these calculations.

In this paper, we show that most of the sensitivity of the \(A(d,p)B\) cross sections to the high \(n-p\) momenta goes away when deuteron breakup is treated beyond the adiabatic distorted-wave approximation (ADWA). The latter is based on the dominant term in the Weinberg state expansion of the \(A+n+p\) wave function, calculated neglecting the couplings to all the other Weinberg components [8]. In ADWA with local \(n-A\) and \(p-A\) optical potentials, the adiabatic potential \(U_{dA}(R)\), given by the sum \(U_{nA}(R) + U_{pA}(R)\) [9], does not depend on deuteron model. However, the nonlocal adiabatic potential explicitly depends on the average \(n-p\) kinetic energy over the (short) range of their interaction, given by the matrix element \(\langle T_{np}\rangle_V \equiv \langle \phi_d|V_{np}T_{np}|\phi_d\rangle/\langle \phi_d|V_{np}|\phi_d\rangle\) [10, 11]. This matrix element is very sensitive to high \(n-p\) momenta, which is reflected in the ADWA cross sections.

We choose the continuum-discretized coupled-channel (CDCC) approach [13, 14] to treat deuteron breakup in \(A(d,p)B\) reactions beyond the adiabatic approximation. The CDCC, developed and used for local nucleon-target optical potentials only, in some cases predicts significantly different cross sections than the ADWA does [12]. Extending the CDCC to the case of nonlocal \(n-A\) and \(p-A\) potentials, in principle, could be done on the basis of the exact nonlocal ADWA formalism of Ref. [12]. However, it would involve time-consuming calculations of nonlocal kernels when the \(d\)-wave component in deuteron is included, making the whole task difficult. For this reason, based on ideas of [10, 11] we have developed a leading-order local-equivalent CDCC approximation to have a quick assessment of the role of the high \(n-p\) momenta in \((d,p)\) reactions. In the ADWA, the leading order solution deviates from the exact one by about 10% but the sensitivity to the deuteron model is present in both of them in the same proportions [12], which justifies using of the leading order local-equivalent CDCC for our purposes.

Nonlocal CDCC model. In the CDCC, the wave function \(\Phi(R, r)\) of the \(A+n+p\) system includes expansion over the \(n-p\) continuum bins \(\phi_i(r)\). To begin with, we
assume that the bins represent only the $s$-wave motion and that all spins are neglected. In this case,

$$\Phi(R, r) = \sum_{i=0}^{n-1} \chi_i(R) \phi_i(r)$$  \hspace{1cm} (1)

and channel function $\chi_i$ are found from the three-body nonlocal Schrödinger equation given by Eq. (9) of Ref. [11]. In Eq. (1) and everywhere below we assume that $\phi_0$ is the deuteron bound state wave function $\phi_d$. We assume that nonlocal potentials $U_{NA}$ and $U_{pA}$ have the Perey-Buck form [18],

$$U_{NA}(r, r') = H(r - r')U_{NA}[(r + r')/2],$$  \hspace{1cm} (2)

where $H(x)$ is the coupling potential and its total angular momentum $s$ and all spins are neglected. In this case, $H(x)$ is the kinetic energy operator, $U_C$ is the Coulomb potential energy, and $E_d$ is the center-of-mass energy of the $d - A$ system.

$$\mathcal{V}_{\nu'}(s, R) = \sum_N \int dx \phi_i^*(x + \alpha_1 s) U_{NA} \left( \frac{x}{2} - R \right) \phi_i(x)$$  \hspace{1cm} (5)

where $n$ is an integer greater than $n$, $\alpha_1 = A/(A + 1)$ and $\alpha_2 = (A + 2)/(A + 1)$. Because of the short range of $H(s)$ the wave function $\chi_{\nu'}$ can be represented by the leading-order expansion that retains only spherical components in $s$ [11].

$$\chi_{\nu'} \left( \frac{\alpha_2 s}{2} + R \right) \approx \sum_{n=0}^{n_{\text{max}}} \frac{s^{2n}}{\beta_{2n}} \gamma_n T_R \chi_{\nu'}(R),$$  \hspace{1cm} (6)

in which

$$\gamma_n = \frac{(-)^n}{n!(2n + 1)!} \left( \frac{\mu_d \alpha_2^2 \beta^2}{4\hbar^2} \right)^n,$$  \hspace{1cm} (7)

where $\mu_d$ is the reduced mass of $A + d$. Then Eqs. (11) become

$$(T_R + U_C(R) - E_d) \chi_i(R) = - \sum_{n=0}^{n_{\text{max}}} \gamma_n \sum_{\nu'} U_{\nu'}^{(n)}(R) T_R \chi_{\nu'}(R),$$  \hspace{1cm} (8)

with the coupling potentials

$$U_{\nu'}^{(n)}(R) = \int dx \left[ \tilde{\phi}_i^{(n)}(x) \right]^{\dagger} \left[ \sum_N U_{NA} \left( \frac{x}{2} - R \right) \right] \phi_i(x)$$  \hspace{1cm} (9)

that contain the modified-by-nonlocality functions

$$\tilde{\phi}_i^{(n)}(x) = \int ds \mathcal{H}(s) \left( \frac{s}{\beta} \right)^{2n} \phi_i(x + \alpha_1 s).$$  \hspace{1cm} (10)

To solve the coupled equations (8) we use the local energy-approximation. In the case of a single channel, this approximation means $T_R \approx E - U_C - U_{\nu'}^{loc}(R)$ with $U_{\nu'}^{loc}$ obtained from a transcendental equation [18]. The multichannel CDCC case we introduce a generalization of the local-energy approximation,

$$(T_R + U_C(R) - E_d) \chi_i(R) = - \sum_{\nu'} U_{\nu'}^{loc}(R) \chi_{\nu'}(R),$$  \hspace{1cm} (11)

where $n_{\text{max}}$ times to the r.h.s. of Eq. (8) neglecting commutators between $T_R$ and $U_{\nu'}^{loc}$. For one-channel case, the corrections beyond this assumption, determined by $\beta^4$, are very small [11]. Imposing the requirement that the local-equivalent coupling potentials $U_{\nu'}^{loc} \chi_{\nu'}(R)$ satisfy

$$(T_R + U_C(R) - E_d) \chi_i(R) = - \sum_{\nu'} U_{\nu'}^{loc}(R) \chi_{\nu'}(R),$$  \hspace{1cm} (12)

we obtain a system of the transcendental matrix equations

$$f_{ij}^{(n)}(0) - (E_j - U_C) \delta_{ij} + \sum_k (j^{(1)}_{\nu'} + \delta_{ik}) X_{kl} + \sum_k j^{(2)}_{\nu'} X_{kl} + \ldots = 0,$$  \hspace{1cm} (13)

for

$$X_{ij} = (E_{\nu'} - U_C) \delta_{ij} - U_{\nu'}^{loc},$$  \hspace{1cm} (14)

in which $j^{(n)}_{ij} = \gamma_n U_{\nu'}^{(n)}$. We solve equations (13) using Newton method and then read $U_{\nu'}^{loc}$ into the CDCC reaction code, which in our case was FRESCO [19].

The scheme described above remains unchanged when all spins are included. We will assume in the following that the target has spin 0, although it can be proved that a non-zero target spin simply introduces an overall factor in the coupling potentials. In the coupling scheme, consistent with FRESCO ($l + s_n = j_n$, $j_n + s_p = I$ and $L + I = J$), the bin functions $\tilde{\phi}_i$ are labeled by a set of quantum numbers $\alpha = \{i, l, j_n\}$, where $i$ includes both the bin energy and its total angular momentum $I$, $l$ is the $n$-$p$ orbital momentum and $j_n$ is the total momentum of the neutron. The channel functions $\chi_{LJ}$ depend on the $d - A$ relative orbital momentum $L$ and total momentum $J$. We require that the local-equivalent coupling potentials $U_{\nu'}^{loc}$ satisfy

$$(T_R + U_C(R) - E_d) \chi_{LJ}(R) = - \sum_{\nu'} C_{\nu'}^{LJ} \chi_{\nu'}^{loc}(R) \chi_{\nu'}^{L'J'}(R),$$  \hspace{1cm} (15)

where the (un)primed quantities correspond to the (initial) final state,

$$C_{\nu'}^{LJ} = (-)^{J+L+J} \left\{ \begin{array}{ccc} L & I & J \\ L' & I' & J' \end{array} \right\} L'(L'0\lambda0|L0),$$  \hspace{1cm} (16)
and \( a = \sqrt{2a + 1} \). The \( U^{\text{loc}}_{ij\lambda} \) are also found from a system of transcendental matrix equations

\[
g^{(0)}_{ii'\lambda} - (E_{i'} - U_C) \hat{I} \delta_{ii'} \delta_{\lambda 0} + \sum_{k_1i_1j_1, j_1i_1} \left( g^{(1)}_{k_1i_1} + \hat{I} \delta_{k_1i_1} \delta_{l_1i_0} \right) \frac{\lambda_{j_1l_1}}{2} C_{j_1l_1i_1}^{ij} \lambda_{k_1i_1}^i X_{k_1i_1}^{(l_2)} + \sum_{k_1k_2j_2, j_2i_2} \left( g^{(2)}_{k_1j_2} \frac{\lambda_{k_2j_2}}{2} C_{k_2j_2l_2}^{ij} \lambda_{j_2l_2}^j \right) X_{k_2j_2}^{(l_2)} X_{i_2j_2}^{(l_3)} X_{i_2j_2}^{(l_3)} \lambda_{i_2j_2}^{i_2} + \ldots = 0, \tag{17}
\]

with \( D_{ij\lambda\tilde{ij}\lambda'}^{LII} = (-)^\lambda' \tilde{I} L \tilde{I}^{-1} C_{L\tilde{L}\lambda\lambda'}^{LII} \) and \( j_i \) being the spin of state \( k_i \), written for

\[
X^{(\lambda)}_{ii'} = (E_{i'} - U_C) \hat{I} \delta_{ii'} \delta_{\lambda 0} - U^{\text{loc}}_{ii'\lambda}.
\tag{18}
\]

Eqs. (17) now include all necessary angular momentum couplings. They contain functions

\[
g^{(n)}_{ii'\lambda}(R) = \gamma_n \sum_{l', n_{l'}} (-)^{l + s_p + s_j + j'} \tilde{I} L \tilde{I}^{-1} \hat{I} \tilde{I}^\gamma_{l'n_{l'}} \frac{\lambda^2}{4\pi} \left( j_n, j_{l'}, l \right) \left( j_n, j_{l'}, l \right) \left( j_n, j_{l'}, l \right) \left( j_n, j_{l'}, l \right) U^{(n)}_{\alpha'\lambda}(R),
\tag{19}
\]

determined by the multipoles of the coupling potentials folded between the original \( \phi_i \) and modified \( \phi_i \) functions:

\[
U^{(n)}_{\alpha'\lambda}(R) = \int_0^\infty dx x^2 \left[ \frac{\phi_{\alpha'}(x)}{\gamma_n} \right]^n \sum_{N} U^{(\lambda)}_{N\alpha}(x, R) \phi_{\alpha}(x)
\]

\[
U^{(\lambda)}_{N\alpha}(x, R) = 2\pi \int_{-1}^1 du U_{N\alpha}(\frac{x}{2} - R) P_\lambda(u),
\tag{20}
\]

with \( u \) being the cosine between \( x \) and \( R \) and \( P_\lambda(u) \) the Legendre polynomial.

**Application to the \( 26^m Al(d, p)27^m Al \) reaction.** We apply the newly developed local-equivalent CDCC model to the \((d, p)\) reaction recently measured in inverse kinematics with isomeric \( 26^m Al \) beam \[21\]. Because of the \( 0^+ \) spin of this isomer transfers to the final \( 27^m Al \) states will involve only one orbital momentum, thus facilitating extraction of spectroscopic factors.

We have performed the CDCC calculations for three incident deuteron energies, 9.2, 25 and 50 MeV, typical for the TRIUMF, GANIL and RIKEN facilities. We used the Gianinni-Ricco systematics of energy-independent nonlocal nucleon optical potentials for \( N = Z \) targets \[21\] and two nucleon-nucleon (NN) potentials; Hulthen and RSC. In Ref. [1] the calculations with these potentials gave the lowest and the highest \( 26^g Al(d, p)27^m Al \) cross sections, respectively. Both \( s^- \) and \( d^- \) wave continuum bins were used in the calculations. For the reaction at 9.2 MeV three bins were taken for each component considered equispaced for proton-neutron energies from 0 to 6 MeV (closed channels start at 6.3 MeV). At 25 MeV, five bins were taken from 0 to 20 MeV (closed channels at 20.97 MeV) and at 50 MeV, four bins from 0 to 44 (closed channels at 44.18 MeV). Convergence with bin mesh was checked in calculations with local potentials at all energies and with nonlocal potentials at 9.2 MeV. We were also made aware that contributions from the closed channels at low \( E_d \) are negligible \[22\].

We have calculated the local-equivalent coupling potentials \( U^{\text{loc}}_{ij\lambda}(R) \) at each point \( R \) from 0 to 50 fm by solving Eq. (\ref{eq:17}) using the Newton method. The choice of \( n_{\text{max}} = 3 \) was sufficient for \( U^{\text{loc}}_{ij\lambda} \) to converge, similar to findings in the one-channel study \[11\]. For some \( d^- \)-wave channels, \( n_{\text{max}} = 2 \) was sufficient. The \( U^{\text{loc}}_{ij\lambda} \) have been read into FRESCO which calculated the channel functions \( \chi_i \) and then the finite range transfer cross sections using the same NN potentials in the transfer vertex. In the case of the RSC, both the \( s^- \) and \( d^- \) wave deuteron

![FIG. 1](image-url)

FIG. 1. The differential cross sections of \( 26^m Al(d, p)27^m Al \) at \( E^{lab}_d = 9.2 \) MeV for population of the \( 27^m Al(5/2^-) \) ground state \((a)\) and the excited 1/2\(^++\) states at \( E_x = 0.84, 6.8 \) and 10.2 MeV \((b, c, d)\), respectively. In the cases with experimental data \[21\] all calculations have been multiplied by the spectroscopic factor obtained for the RSC-CDCC calculation.
TABLE I. Various ratios of the $^{26_{\text{Al}}}(d, p)^{27_{\text{Al}}}$ cross sections calculated in the ADWA and CDCC with two different NN potentials, RSC and Hulthén, for $E_d^{\text{lab}} = 9.2$, 25 and 50 MeV and for four final states in $^{27_{\text{Al}}}$. The ratios were calculated at the maxima of cross sections. All energies are given in MeV.

| $E_x/E_d^{\text{lab}}$ | $\sigma_{\text{ADWA}}/\sigma_{\text{H}}$ | $\sigma_{\text{CDCC}}/\sigma_{\text{H}}$ | $\sigma_{\text{ADWA}}/\sigma_{\text{CDCC}}$ | $\sigma_{\text{RSC}}/\sigma_{\text{H}}$ | $\sigma_{\text{RSC}}/\sigma_{\text{CDCC}}$ |
|----------------------|-------------------------------|-------------------------------|---------------------------------|-------------------------------|---------------------------------|
| 0.00                 | 1.17                         | 1.38                         | 1.76                            | 1.04                         | 1.08                            | 1.05                            | 1.71                         | 1.80                            | 1.55                            | 1.81                         | 2.35                            | 2.60                            |
| 0.84                 | 1.03                         | 1.12                         | 2.24                            | 1.01                         | 0.97                            | 1.14                            | 1.64                         | 1.29                            | 1.78                            | 1.67                         | 1.50                            | 3.54                            |
| 6.8                  | 1.21                         | 0.86                         | 2.09                            | 1.00                         | 0.96                            | 1.07                            | 1.57                         | 1.28                            | 1.37                            | 1.89                         | 1.14                            | 2.68                            |
| 10.2                 | 1.24                         | 0.81                         | 1.69                            | 1.00                         | 0.96                            | 1.05                            | 1.52                         | 1.27                            | 1.10                            | 1.89                         | 1.08                            | 1.76                            |

The leading order nonlocal CDCC and ADWA calculations are shown in Fig. 1 and 2 for deuteron incident energies of 9.2 and 50 MeV, respectively, and for four final $^{27_{\text{Al}}}$ states: the ground $J^\pi = 5/2^+$ state and three astrophysically relevant excited $J^\pi = 1/2^+$ states. In all cases, the CDCC cross sections are significantly lower than the ADWA ones. Their ratio in the maximum, shown in Table I, in most cases is higher than an average value of 1.25 fm and diffuseness $a = 0.65$ fm.

The CDCC calculations show that the sensitivity to the $n-p$ model is significantly reduced. It is less than 4% for $E_d^{\text{lab}} = 9.2$ MeV but can understandably increase with the deuteron incident energy up to 14%.

Although our main aim is the comparison of ADWA and CDCC calculations, given the existence of experimental data for $^{27_{\text{Al}}}(1/2^+)$ [20], we deduced spectroscopic factors from these data using both ADWA and CDCC and both NN potentials. They are presented in Table II and compared to previous ADWA calculations with local optical potentials. Both CDCC calculations and the Hulthén-ADWA reproduce the shape of experimental data but RSC-ADWA overestimates the data at larger angles for the states at $E_x = 0.84$ and 6.8 MeV. The spectroscopic factors extracted with CDCC are larger than those determined in [20], but this difference decreases with the excitation energy.

Understanding reduced sensitivity. The strong sensitivity of the ADWA cross sections to the NN model comes from the coefficient $M_0$ in the transcendental equation for the local-equivalent adiabatic potential $U^{\text{loc}}$, $U^{\text{loc}} = M_0(U_{nA} + U_{pA}) \exp \left[-\gamma(E - U_C - U^{\text{loc}})\right]$, (21)

![FIG. 2. The same as in Fig. 1 but for $E_d^{\text{lab}} = 50$ MeV.](image-url)

TABLE II. Spectroscopic factors obtained from the $^{26_{\text{Al}}}(d, p)^{27_{\text{Al}}}(1/2^+)$ cross sections calculated with ADWA and CDCC with two different NN potentials, RSC and Hulthén for $E_d^{\text{lab}} = 9.2$ MeV. All energies are in MeV.

| $E_x$     | ADWA Hulthén | ADWA RSC | CDCC Hulthén | CDCC RSC | Ref |
|-----------|-------------|--------|-------------|--------|----|
| 0.84      | 0.07        | –      | 0.13        | 0.11   | 0.08 |
| 6.8       | 0.14        | –      | 0.14        | 0.14   | 0.11 |
| 10.2      | 0.13        | 0.08   | 0.18        | 0.18   | 0.16 |
where γ is a constant. This coefficient is given by

$$M_0 = N \int ds dx \, H(s) \phi_d(x + \alpha_1 s) V_{np}(x) \phi_d(x)$$

with \(N = \langle \phi_d | V_{np} | \phi_d \rangle^{-1}\) (see Eq. [11] for the link between \(M_0\) and \((T_{np})_V\)). Because of the short range of \(V_{np}\), \(M_0\) is highly sensitive to the details of \(\phi_d\) at small \(x\). In the CDCC, the main channel corresponds to the folding model with the \(U_{loc}^{\prime}\) found from Eq. [21] and \(M_0\) generated by Eq. [22] with \(N = 1\) and without \(V_{np}\): \n
$$M_0 = \int dx \, \phi_d^*(x) \phi_d(x).$$

Because of the small deuteron binding energy this \(M_0\) is determined by the large values of \(x\), corresponding to small \(n-p\) momenta, where all the NN models agree. Also, because of the small range of nonlocality \(\beta\), in this range \(\bar{\phi}_d \approx \phi_d\) (see Fig. 3a) and, therefore, \(M_0 \approx 1\). The same statements are relevant for low-energy continuum bins which are affected by the nonlocality and differences in the NN potentials only at small \(x\) (Fig. 3b) thus explaining the reduced sensitivity to the deuteron model in the \((d,p)\) calculations with CDCC. The differences in the NN model affect high-energy bins (Fig. 3c) where modifications due to nonlocality are stronger. As a result, the sensitivity to the high \(n-p\) momenta is stronger for a large deuteron incident energy, as seen from Table I.

The ADWA could be corrected by including more Weigberg states in the expansion of \(\Phi(R, r)\) \[8, 23\]. This would involve calculations of nondiagonal local-equivalent coupling potentials \(U_{li}^{loc}\) that depend on the coefficients given by [22] but with Weigberg states \(\phi_i^W\) instead of \(\phi_d\). Such coefficients (and, therefore, the \(U_{li}^{loc}\) and the corresponding \((d,p)\) cross sections) would be determined by the model-dependent short-range behaviour of \(V_{np} \phi_i^W\). It was shown in [20] that continuum bins could be expanded over Weigberg states. Therefore, sufficient number of NN-dependent Weigberg states should recover the almost-NN-independent CDCC calculations. It is worth mentioning that for local optical potentials the non-adiabatic corrections explicitly depend on the same NN model-dependent matrix element \((T_{np})_V\) \[27\] that features in the nonlocal ADWA.

**Conclusions.** Based on our newly developed local-equivalent CDCC model with nonlocal optical potentials, we have shown that the previously reported strong sensitivity of the adiabatic \((d,p)\) cross sections, calculated with nonlocal nucleon optical potentials, is significantly reduced. For low deuteron incident energies it is now less than 4% but can increase up to 14% for higher energies.

We have also found that non-adiabatic effects are much stronger than those in the case of local optical potentials. To confirm this finding, the nonlocal CDCC should be extended beyond the leading order. Exact ADWA cross sections with nonlocal potentials are smaller than the leading-order cross sections [12] but this tendency may not necessarily be the same in the CDCC case. It is conceivable that the difference between exact nonlocal CDCC and nonlocal ADWA can be smaller than that obtained in this work.

The sensitivity to high \(n-p\) momenta due to uncertainties of the adiabatic approximation suggests that theoretical analysis of \((d,p)\) experiments should be extended beyond the adiabatic approximation when nonlocal optical potentials are used. This is an important message given the current interest of other groups in ADWA with nonlocal potentials, such as in [28, 29]. Full nonlocal CDCC calculations could help to refine the spectroscopic factors and asymptotic normalization coefficients obtained from \((d,p)\) reactions. We note that present results were obtained with energy-independent optical potentials. A proper treatment of energy-dependence within the three-body context is a challenge, in particular in the CDCC formalism, where the energy between nucleon and target is not well defined in the considered final states. Whether approximate prescriptions to take this dependence into account could result in additional NN-model dependence of \((d,p)\) cross sections remains to be investigated.

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