REVISITING THE DISTRIBUTIONS OF JUPITER’S IRREGULAR MOONS: II. ORBITAL CHARACTERISTICS

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Abstract. From the statistical point of view, this paper mainly emphasizes the orbital distribution laws of Jupiter’s irregular moons, most of which are located in Ananke group, Carme group and Pasiphae group. By comparing 19 known continuous distributions, it is verified that there are suitable distribution functions to describe the distribution of these natural satellites. For each distribution type, interval estimation is used to estimate the corresponding parameter values. At a given significant level, one-sample Kolmogorov-Smirnov nonparametric test is applied to verify the specified distribution, and we often select the one with the largest $p$-value. The results show that all the semi-major axis, mean inclination and the orbital period of the moons in Ananke group and Carme group obey the Stable distribution. In addition, according to Kepler’s third planetary motion law, and by comparing the theoretically calculated best-fit cumulative distribution function (CDF) with the observed CDF, we demonstrate that the theoretical distribution is in good agreement with the empirical distribution. Therefore, these characteristics of Jupiter’s irregular moons are indeed very likely to follow some specific distribution laws, and it will be possible to use these laws to help study certain features of poorly investigated moons or even predict undiscovered ones.

1. Introduction

The giant Jupiter system is often referred to a miniature solar system \cite{1}. Jupiter’s gravity is strong enough to keep objects in orbit over 18.6 million miles away. This means that there is a particularly large space around Jupiter for researchers to examine, possibly hiding natural satellites that have not yet been discovered. Since the composition of Jupiter is similar to the Sun, the exploration of Jupiter can help to gain a deeper understanding of the solar system. The moons of Jupiter are divided into regular ones and irregular ones. Irregular moons are characterized by a high degree of eccentricity and inclination, which are distinct from the near-circular, uninclined orbits of regular moons. These distant retrograde moons are grouped into at least three main different orbital groupings and are considered to be the remnants of three once larger parent bodies that are broken apart during collision with asteroids, comets or other natural satellites \cite{2}. These three groups are Ananke, Carme and Pasiphae, respectively. The specific classification of Jupiter’s irregular moons can be found from Table 7 in Appendix A. The ‘current’ in Table 7 corresponds to the current discovery that Jupiter has 79 moons and its latest classification, while ‘previous’ refers to Jupiter’s 69 moons and their previous classifications before July 2018.

Carruba et al. \cite{3} integrated orbits of a variety of hypothetical Jovian moons on a long timescale and found that the Lidov-Kozai effect due to the solar perturbations plays the most prominent role in the secular orbital evolution. Recently, Aschwanden \cite{4} interpreted the observed quasi-regular geometric patterns of planet or moon distances in terms of a self-organizing system. Then researchers must be curious as to whether there are specific rules for the irregular moons of Jupiter. When the total number of Jupiter’s moons was still 69, based on the classification of Sheppard and Jewitt \cite{5}, Gao et al. \cite{6} investigated the distributions of orbital and

\textit{Key words and phrases.} Jupiter’s irregular moons; Distribution law; Orbital characteristics; Kolmogorov-Smirnov test.
physical characteristics of Jupiter’s moons by using one-sample Kolmogorov-Smirnov (K-S) nonparametric test (please see [7] for details). Several features of Jupiter’s moons have been found to obey the logistic distribution and the $t$ location-scale distribution. In addition, they verified that the distribution results were helpful in predicting some characteristics of the moons that have not been well studied. Moreover, they also believed that if future observations will allow an increase in the number of Jupiter’s moons, the distribution laws may be slightly different, but it will not change significantly over a long period of time.

In recent years, the number of known irregular moons has been greatly increased with the powerful observation [8]. It is worth mentioning that the Carnegie Institution for Science announced the discovery of 12 new Jupiter’s moons in July 2018 [9]. In addition to the temporary designation of 2 moons in June 2017, 10 of these newly discovered moons are part of the outer group moons that orbit the Jupiter in retrograde or opposite directions. This exciting and important discovery has increased the total number of Jupiter satellites to 79 [9, 10]. Considering that the newly discovered Jupiter’s moons are small and a few moons have been regrouped (please see Table 7 in Appendix A for details), do their orbital characteristics still follow certain potential laws?

In this paper, according to the updated moons’ data of Jupiter, we continue to study the distribution laws for Jupiter moons’ orbital characteristics, including semi-major axis, inclination, eccentricity, argument of periapsis, longitude of the ascending node, mean anomaly and period. These features, except for the period, constitute the six so-called orbital elements, they are the parameters that are often used to specify an orbit. Semi-major axis and eccentricity determine the shape and size of the orbit. Longitude of ascending node and inclination define the orientation of the orbital plane in which the ellipse is embedded, the argument of periapsis is the angle from the ascending node to periapsis, measured in the direction of motion. Mean anomaly is only an angle in mathematical sense that varies linearly with time. It can be converted to true anomaly, which is often used to indicate the position of a point on the orbit [11].

By using the one-sample KS test method in statistics, we verify the dozens of commonly used distributions one by one and calculate the $p$-values corresponding to these distributions. For the same orbital feature, the distribution corresponding to the largest $p$-values is theoretically the one we are looking for, and the closer the $p$-value is to 1, the more likely we will find that the distribution is correct. In order to be able to describe these distributions analytically, we also calculate the values of the parameters corresponding to these distributions and the confidence intervals corresponding to these parameters from the perspective of statistical data inference. Moreover, whether the theoretical results of the statistical prediction are valid can also be verified. Because some of the orbital characteristics are not independent, but are coupled to each other. For example, the nonlinear relationship between the semi-major axis ($sma$) and the orbital period ($T$) can be expressed in accordance with the third Kepler’s laws of planetary motion. If we infer from the observation data that the $sma$ and $T$ obey a certain distribution $d_1$ and another distribution $d_2$, respectively. However, the distribution of the $sma$ can also be analytically calculated according to the third law of Kepler and the distribution $d_2$. The distribution of $sma$ obtained by analytical calculation here is recorded as $d_3$. Therefore, for the orbital characteristics of the $sma$, the rationality of data inference can be verified by comparing the distribution $d_1$ inferred by KS test method and the distribution $d_3$ obtained by analytical calculation.
2. Distribution inference based on different moons’ groups

2.1. Ananke group. The Ananke group had only 11 moons a few months ago, but currently admits 19 moons, including the latest three moons S/2017 J7, S/2017 J3 and S/2017 J9, and the five moons Euporie, Orthosie, Helike, S2003 J18 and S/2016 J1 formerly in the Pasiphae group are now reclassified to the Ananke group (see Appendix A for a comparison of Jupiter’s moons regrouping).

Based on the one-sample KS test method, we obtain the best-fit distribution of these orbital features in Table 1. In the fourth column of the table, the confidence interval shows that the true value of these parameters falls close to the measurement with a certain probability. Although the distribution parameter values can be calculated from the observed data, whether the null hypothesis be accepted is greatly influenced by the significance level (usually set to 0.05 or 0.01), so we study the distribution of orbital features by using p-values that can avoid pre-determined significance levels. If the p-value is greater than 0.05, we accept the null hypothesis, otherwise reject it. However, if the p-values of several distributions corresponding to the same orbital characteristics are greater than 0.05, we should try to choose the distribution with the largest p-value, which is a bit one-sided, but it is reasonable, because an event often occurs with a greater probability.

It can be seen from Table 1 that the semi-major axis, the mean inclination and the period all obey Stable distributions (See [12] [13] for more details), and the p-values are all greater than 0.9, which is much larger than the commonly used 0.05 and 0.01. Since most of the probability density functions (PDF) of the Stable distribution have no closed form expressions except for a few special cases, they are conveniently represented by a characteristic function (CF). If a random variable admits a PDF, then the CF is a Fourier transform of PDF. Therefore, it provides the basis for an alternative path to analysis results as compared to directly using a PDF. The relationship between CF and PDF can be expressed by the following formula

\[
f_X(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_X(t) e^{-itx} dt,
\]

where \( f_X \) and \( \phi_X \) are the PDF and CF of the random variable \( X \), respectively.

The random variable \( X \) is called Stable ([12] [13]) if its CF can be written as

\[
\phi_X(t; \alpha, \beta, c, \mu) = \exp \left( it\mu - |ct|^\alpha (1 - i\beta \text{sgn}(t) \Phi) \right), \quad t \in \mathbb{R},
\]

where \( \alpha \in (0,2] \) is the characteristic exponent responsible for the shape of the distribution, \( \beta \in [-1,1] \) is called the skewness of the distribution and used to measure asymmetry (\( \beta=0 \) means symmetry), \( c \in (0, +\infty) \) is the scale parameter, which narrows or extends the distribution around, \( \mu \in \mathbb{R} \) is the location parameter that shifts the distribution to the left or the right, \( \text{sgn}(t) \) is the usual sign function and

\[
\Phi = \begin{cases} 
\tan \left( \frac{\pi \alpha}{2} \right), & \alpha \neq 1, \\
-\frac{2}{\pi} \log |t|, & \alpha = 1.
\end{cases}
\]

If \( \alpha = 0.5 \) and \( \beta = 1 \), the corresponding special case is called Levy distribution. And if \( \alpha = 1 \) or \( \alpha = 2 \), it is defined as Cauchy or Gaussian distribution, respectively.

For the sake of simplicity, the distribution types of other orbital features are briefly introduced (see Tables 1-3 for details). The mean eccentricity obeys Extreme Value distribution with location parameter \( \mu \) and scale parameter \( \sigma \). As the value of \( \sigma \) increases, the density function curve disperses gradually. The mean and variance
Table 1. Inference of the distribution of each orbital characteristics in the Ananke group

| Characteristic       | Distribution      | Parameters | Confidence Intervals | p-value  |
|----------------------|-------------------|------------|----------------------|----------|
| Semi-major axis(km)  | Stable            | $\alpha = 1.32899$, $\beta = -1$, $c = 211134$, $\mu = 2.10713 \times 10^7$ | $\alpha \in [0.2]$, $\beta \in [-1.1]$, $c \in [0, \infty]$, $\mu \in [-\infty, \infty]$ | 0.911782098004567 |
| Mean inclination(deg)| Stable            | $\alpha = 1.420563$, $\beta = -0.205584$, $c = 1.62162$, $\mu = 148.744$ | $\alpha \in [0.74677, 2]$, $\beta \in [-1, 0.999048]$, $c \in [0.8027, 2.44053]$, $\mu \in [147.487, 150.002]$ | 0.910877985227074 |
| Mean eccentricity    | Extreme Value     | $\mu = 0.235049$, $\sigma = 0.0444536$ | $\mu \in (0.213908, 0.256189]$, $\sigma \in (0.0318622, 0.0620209]$ | 0.494461231786516 |
| Argument of periapsis| Loglogistic       | $\alpha = 5.10234$, $\beta = 0.374292$ | $\alpha \in [4.791185, 5.41282]$, $\beta \in [0.258487, 0.541979]$ | 0.654341556407226 |
| Longitude of the ascending node | Generalized Extreme Value | $k = -0.784386$, $\sigma = 127.426$, $\mu = 188.251$ | $k \in [-1.1982, -0.370579]$, $\sigma \in [79.4528, 204.364]$, $\mu \in [124.222, 252.28]$ | 0.708753279867213 |
| Mean anomaly         | Generalized Pareto| $\mu = -1.07975$, $\xi = 0$ | $\mu \in [-1.07975, -1.07975]$, $\xi \in [0, \infty]$ | 0.918361274248861 |
| Period(days)         | Stable            | $\alpha = 1.20629$, $\beta = -1$, $c = 8.28652$, $\mu = 624.981$ | $\alpha \in [0.2]$, $\beta \in [-1, 1]$, $c \in [0, \infty]$, $\mu \in [-\infty, \infty]$ | 0.987823575757576 |

Table 2. Inference of the distribution of each orbital characteristics in the Carme group

| Characteristic       | Distribution      | Parameters | Confidence Intervals | p-value  |
|----------------------|-------------------|------------|----------------------|----------|
| Semi-major axis(km)  | Stable            | $\alpha = 0.987755$, $\beta = 0.0533906$, $c = 112441$, $\mu = 2.32477 \times 10^7$ | $\alpha \in [0.2]$, $\beta \in [-1.1]$, $c \in [0, \infty]$, $\mu \in [-\infty, \infty]$ | 0.953386728226813 |
| Mean inclination(deg)| Stable            | $\alpha = 0.835022$, $\beta = -0.329864$, $c = 0.242389$, $\mu = 165.084$ | $\alpha \in [0.424668, 1.24538]$, $\beta \in [-0.948243, 0.288516]$, $c \in [0.147881, 0.336988]$, $\mu \in [164.949, 165.219]$ | 0.97493148191565 |
| Mean eccentricity    | Stable            | $\alpha = 1.27463$, $\beta = 0.000479687$, $c = 0.0119451$, $\mu = 0.256685$ | $\alpha \in [0.60643, 1.88884]$, $\beta \in [-1.1]$, $c \in [0.00554199, 0.0183482]$, $\mu \in [0.248013, 0.265357]$ | 0.999628336728153 |
| Argument of periapsis| Generalized Pareto| $k = -0.359188$, $\sigma = 119.284$, $\mu = 149.7$ | $k \in [-0.987774, 0.093978]$, $\sigma \in [76.1928, 186.747]$, $\mu \in [86.0337, 213.366]$ | 0.805760848421139 |
| Longitude of the ascending node | Loglogistic       | $\alpha = 0.60721$, $\beta = 0.477994$ | $\alpha \in [4.62636, 5.38805]$, $\beta \in [0.33058, 0.691149]$ | 0.775558039736224 |
| Mean anomaly         | Generalized Pareto| $\mu = -1.21365$, $\xi = 0$ | $\mu \in [-1.21365, 0]$, $\xi \in [0, \infty]$ | 0.920490638431330 |
| Period(days)         | Stable            | $\alpha = 0.940971$, $\beta = 0.163918$, $c = 6.45023$, $\mu = 724.273$ | $\alpha \in [0.2]$, $\beta \in [-1, 1]$, $c \in [0, \infty]$, $\mu \in [-\infty, \infty]$ | 0.993588758405022 |

of Extreme Value distribution are $\mu + \nu \sigma$ and $\frac{\pi^2}{6}$, respectively, and here $\nu$ is the Euler constant. The argument of Pariaphis obeys Loglogistic distribution, of which $\alpha$ is scale parameter and it is also the median of the distribution. The parameter $\beta > 0$ is a shape parameter. The distribution is unimodal when $\beta > 1$ and its dispersion decreases as $\beta$ increases. The longitude of the ascending node obeys Generalized Extreme Value distribution with shape parameter $k$, scale parameter $\sigma$, and location parameter $\mu$. Mean anomaly obeys Generalized Pareto distribution with the mean $\mu + \sigma/(1 - \xi)$ ($\xi < 1$), and variance $\sigma^2/[(1-\xi)^2(1-2\xi)]$ ($\xi < 1/2$).
### Table 3. Inference of the distribution of each orbital characteristics in the Pasiphae group

| Characteristic | Distribution        | Parameters                  | Confidence Intervals                  | $p$-value     |
|---------------|---------------------|-----------------------------|---------------------------------------|---------------|
| Semi-major axis(km) | Extreme Value       | $\mu = 2.38737 \times 10^7$, $\sigma = 493901$ | $\mu [2.36096e \times 10^7, 2.41378e \times 10^7]$, $\sigma [334031, 730286]$ | 0.995638769628093 |
| Mean inclination(deg) | Normal              | $\mu = 151.213$, $\sigma = 4.33280$ | $\mu [148.814, 153.612]$, $\sigma [3.17163, 6.83213]$ | 0.998947065611077 |
| Mean eccentricity | Birnbaum-Saunders   | $\beta = 0.33437$, $\gamma = 0.24734$ | $\beta [0.292838, 0.375902]$, $\gamma [0.158832, 0.335847]$ | 0.72797549729721 |
| Argument of periapsis | Generalized Extreme Value | $k = 0.829357$, $\sigma = 50.6921$, $\mu = 87.4714$ | $k \in [1.017129, 1.83044]$, $\sigma [23.1042, 111.222]$, $\mu [51.4714, 123.471]$ | 0.574187377012528 |
| Longitude of the ascending node | Stable              | $\alpha = 0.759928$, $\beta = -1$, $c = 18.0216$, $\mu = 309.064$ | $\alpha [0, 2]$, $\beta [-1, 1]$, $c \in [0, \text{Inf}]$, $\mu \in [\text{Inf}, \text{Inf}]$ | 0.979888663536014 |
| Mean anomaly     | Stable              | $\alpha = 2$, $\beta = 0.0313276$, $c = 69.6119$, $\mu = 205.542$ | $\alpha [0, \text{Inf}]$, $\beta [-1, 1]$, $c \in [0, \text{Inf}]$, $\mu \in [\text{Inf}, \text{Inf}]$ | 0.945043051567604 |
| Period(days)     | Generalized Extreme Value | $k = -0.460585$, $\sigma = 26.9631$, $\mu = 729.292$ | $k \in [-0.825924, -0.095246]$, $\sigma [17.6755, 41.1308]$, $\mu \in [714.26, 744.325]$ | 0.999746193251320 |

### Table 4. Distribution inference summary

| Characteristic | Ananke group     | Carme group      | Pasiphae group   |
|---------------|------------------|------------------|-----------------|
| Semi-major axis(km) | Stable           | Stable           | Extreme Value   |
| Mean inclination(deg) | Stable          | Stable           | Normal          |
| Mean eccentricity  | Extreme Value    | Stable           | Birnbaum-Saunders |
| Argument of periapsis | Logistic        | Generalized Extreme Value | Generalized Extreme Value |
| Longitude of the ascending node | Generalized Extreme Value | Logistic | Stable |
| Mean anomaly     | Generalized Pareto | Generalized Pareto | Stable          |
| Period(days)     | Stable           | Stable           | Generalized Extreme Value |

2.2. **Carme group.** There are 20 moons in Carme group, of which S/2017 J2, S/2017 J5, S/2017 J8 are the latest discovered moons. S/2003 J19 and S/2011 J1 were not classified before, but are now classified as Carme group. The semi-major axis, the mean inclination, the eccentricity, and the period are subject to Stable distribution. The argument of periapsis obeys Generalized Extreme Value distribution. The longitude of the ascending node follows Logistic distribution. The mean anomaly obeys Generalized Pareto distribution (please see Table 2 for details).

2.3. **Pasiphae group.** Up to now, 15 moons have been classified as Pasiphae group, including S/2017 J6, which has also been newly discovered. The orbital characteristics in this group obey a distribution that is significantly different from the other two groups in the Tables 1 and 2. Specifically, the semi-major axis follows an Extreme Value distribution, while the corresponding best-fit distribution in the Ananke group and the Carme group is the Stable distribution. The mean inclination is subject to a Normal distribution. The mean eccentricity obeys Birnbaum-Saunders distribution. Both the argument of periapsis and period are subject to Generalized Extreme Value distribution. The longitude of the ascending node and mean anomaly follow Stable distribution.

### 3. Comparison of Previous and Current Orbital Properties Distributions

As can be seen from Table 5 that the mean inclination obeys the Stable distribution with a $p$-value of 0.974931481491565. However, the previous best-fit distribution is the $T$ location-scale distribution with a
Table 5. Distributions of current and previous orbital characteristics in the Carme group

| Characteristic          | Distribution | Parameters                           | p-value | Distribution | Parameters                           | p-value |
|------------------------|--------------|--------------------------------------|---------|--------------|--------------------------------------|---------|
| Semi-major axis (km)   | Stable       | $\alpha = 0.987755$, $\beta = 0.0533906$, $c = 112441$, $\mu = 2.32477 \times 10^7$ | 0.953386728226811 | Logistic | $\mu = 2.3326 \times 10^7$, $\sigma = 65133.9$ | 0.998691608208985 |
| Mean inclination (deg) | Stable       | $\alpha = 0.835022$, $\beta = -0.329864$, $c = 0.242389$, $\mu = 165.084$ | 0.907558909358380 | T location-scale | $\mu = 165.117$, $\sigma = 0.17015$, $\nu = 0.875108$ | 0.666238390424803 |
| Mean eccentricity      | Stable       | $\alpha = 1.27463$, $\beta = 0.006476987$, $c = 0.0119451$, $\mu = 0.256685$ | 0.999628336728153 | Birnbaum-Saunders | $\beta = 0.254254$, $\gamma = 0.030888$ | 0.624411997733947 |

Table 6. Distributions of current and previous orbital characteristics in the Pasiphae group

| Characteristic          | Distribution | Parameters                           | p-value | Distribution | Parameters                           | p-value |
|------------------------|--------------|--------------------------------------|---------|--------------|--------------------------------------|---------|
| Semi-major axis (km)   | Extreme Value | $\mu = 2.39737 \times 10^7$, $\sigma = 89901$ | 0.995638769628093 | T location-scale | $\mu = 3.39242 \times 10^7$, $\sigma = 273627$, $\nu = 0.730397$ | 0.372989759556375 |
| Mean inclination (deg) | Normal       | $\mu = 151.214$, $\sigma = 4.33206$ | 0.998947065610777 | Logistic | $\mu = 151.357$, $\sigma = 65333.9$ | 0.898736358327201 |
| Mean eccentricity      | Birnbaum-Saunders | $\beta = 0.33437$, $\gamma = 0.24734$ | 0.727975497229721 | Logistic | $\mu = 0.296251$, $\sigma = 0.0599373$ | 0.955007875761940 |

p-value of only 0.666238390424803. Similarly, the optimal distribution of the mean eccentricity is a Stable distribution with a p-value of 0.999628336728153. All optimal distributions of these three orbital elements are the Stable distribution, which may indicate that the moons in the Carme group are likely to have the same origin, that is, they may be born from the split of the same parent asteroid.

In the Pasiphae group, the distributions of orbital features are different from the other two groups. As shown in Table 6, the best-fit distribution of the semi-major axis is the Extreme Value distribution, and its p-value is 0.995638769628093, which is much larger than the value in the literature [6]. The cause of this phenomenon may be the change in the classification of moons, and more distributions have been tested in this paper.

To more intuitively observe the difference between the current distribution and the previous distribution, the observed cumulative distribution function (CDF) and best-fit CDF were plotted respectively. Based on the previous data and the new data, the orbital properties of moons’ data can be compared more specifically and conveniently, and then we get the following CDF figures (Figures 1 and 2).

4. Verification of the rationality of theoretical results

In this section, the reasonability that the best-fit distribution of the semi-major axis and the orbital period is demonstrated analytically based on Kepler’s third law of planetary motion.

4.1. Carme group. As can be seen from Table 2, whether the semi-major axis or the period, the distribution with the largest p value, is Stable distribution. However, as mentioned in the Section 2, the probability density function (PDF) of Stable distribution is given by the characteristic function. Three simple cases including Gaussian distribution, Cauchy distribution and Levy distribution have not occurred, and it is difficult
Figure 1. (a), (b) and (c) are the best-fit CDF and observed CDF of current distributions and previous distributions of orbital characteristics in the Carme group.
Figure 2. (a), (b) and (c) are the best-fit CDF and observed CDF of current distributions and previous distributions of orbital characteristics in the Pasiphae group.
to calculate its probability density function and draw its pdf image. Therefore, we discuss the second largest
$p$-value $T$ location-scale distribution. When the semi-major axis obeys the $T$ location-scale distribution, the $p$
value is $0.900021606617932$, and the parameters are $(2.32508, 0.01141172, 1.39731)$. When the period obeys
the $T$ location-scale distribution, the $p$ value is $0.979168426854978$ and the parameters are $(724.436, 6.75988,$
$1.01233)$. The pdf of the $T$ Location-Scale distribution is given

It can be seen from Table 2 that both the semi-major axis and the period obey the Stable distribution, and
according to Kepler’s third law, there is a relationship $T = \sqrt{\frac{4\pi^2 a^3}{GM}}$ (GM is mass parameter) between
them. In theory, if the distribution type of the semi-major axis or the period is known, then the other one can
be derived analytically. If the analysis results are consistent with the statistical inference results, at least very
close, it means that the result of statistical inference is effective. But now there is a problem, as described in
Section 2, the Stable distribution of the PDF can be given by the CF. In addition to its three special cases
of distribution, including Gaussian distribution, Cauchy distribution and Levy distribution are currently well
studied, but in other cases the Stability distribution, its PDF is still poorly studied. Therefore, we have to
discuss the distribution that is very close to its $p$-value, i.e. $T$ location-scale distribution. When the semi-major
axis obeys the $T$ location-scale distribution with a $p$-value of $0.900021606617932$, the corresponding parameters
are $(2.32508, 0.01141172, 1.397731)$. When the period obeys the $T$-location-scale distribution of the
$p$-value of $0.979168426854978$, the corresponding parameters are $(724.436, 6.75988, 1.01233)$.

Note that the PDF of the $T$ location-scale distribution can be defined as

$$f = \frac{\Gamma(\frac{\nu+1}{2})}{\sigma \sqrt{\nu \pi} \Gamma(\frac{\nu}{2})} \left[ \frac{\nu + (\frac{x-\mu}{\sigma})^2}{\nu} \right]^{-\frac{\nu+1}{2}},$$

where $\Gamma(\cdot)$ is the gamma function.

So the corresponding predicted PDFs can be written as

$$f_{\text{pre, sma}}(a; \mu, \sigma, \nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\sigma \sqrt{\nu \pi} \Gamma(\frac{\nu}{2})} \left[ \frac{\nu + (\frac{x-\mu}{\sigma})^2}{\nu} \right]^{-\frac{\nu+1}{2}},$$

$$= \frac{23.88443557}{(1 + 0.715660805(70.83557646a - 164.6983821)^2)^{1.109665}},$$

and

$$f_{\text{pre, peri}}(T; \mu, \sigma, \nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\sigma \sqrt{\nu \pi} \Gamma(\frac{\nu}{2})} \left[ \frac{\nu + (\frac{T-\mu}{\sigma})^2}{\nu} \right]^{-\frac{\nu+1}{2}},$$

$$= \frac{0.04720551091}{(1 + 0.9878201772(0.1479513181a - 107.1812611)^2)^{1.006165}}.$$ Then, based on the Kepler’s third law, the PDF of the semi-major axis can be derived as follows

$$f_{\text{ana, sma}}(T; \mu, \sigma, \nu) = \frac{3\pi}{GM} \sqrt{\frac{a}{GM}} f_{\text{pre, peri}}\left(\sqrt{\frac{4\pi^2 a^3}{GM}}; 724.436, 6.75989, 1.01233\right),$$

$$= \frac{14.4750061 \sqrt{a}}{(1 + 0.9878201772(30.24500303\sqrt{a}^3 - 107.1812611)^2)^{1.006165}}.$$ From Figure 3 (a), we can find that $f_{\text{ana, sma}}(T; \mu, \sigma, \nu)$ (PDF represented by the red curve) obtained by analytical method is very consistent with $f_{\text{pre, sma}}(a; \mu, \sigma, \nu)$ (PDF represented by blue curve) obtained by statistical inference.
Figure 3. (a), (b) are the comparison of PDF curves of semi-major axis in the Carme group and the Pasiphae group respectively.

4.2. Pasiphae group. The corresponding predicted PDFs can be denoted as

\[ f_{\text{pre, sma}}(a; \mu, \sigma) = \frac{e^{-\frac{a - \mu}{\sigma}}}{\sigma} e^{-e^{-\frac{a - \mu}{\sigma}}} \]

and

\[ f_{\text{pre, peri}}(T; k, \mu, \sigma) = e^{-\frac{1+(T-\mu)/\sigma}{k}} (1 + k((T - \mu)/\sigma)^{-1/k}) \]

Similarly, by using Kepler’s third law, the PDF of semi-major axis can also be derived analytically as follows

\[ f_{\text{ana, sma}}(T; \mu, \sigma) = 3\pi \sqrt{\frac{a}{GM}} f_{\text{pre, peri}}(\sqrt{\frac{4\pi^2a^3}{GM}}; -0.460585, 26.9631, 729.292) \]

From Figure 3 (b), we can also find that \( f_{\text{ana, sma}}(T; \mu, \sigma) \) (PDF represented by the blue curve) obtained by analytical method is in good agreement with \( f_{\text{pre, peri}}(T; k, \mu, \sigma) \) (PDF represented by red curve) obtained by statistical inference.

5. Conclusions

Based on the reference [6], we continue to use the K-S test method to study the distribution of the six orbital elements and orbital periods of the latest Jupiter irregular moons in this paper. It is found that these orbital features mainly obey Stable distribution, Extreme Value distribution, Loglogistic distribution, Generalized Extreme Value distribution, Generalized Pareto distribution, Normal distribution and Birnbaum-Saunders distribution.

Moreover, we also made some comparisons on the semi-major axis, the mean eccentricity as well as the mean inclination. From the comparison results, the best-fit distribution of the three features in this paper has larger \( p \)-value. From the figures of best-fit CDF and the CDF based on the observational data, both the current best-fit
distribution and the previous one are well matched. There are two possible reasons for this result. First, the number of tested distribution functions are greater than in [6]. Second, the classification of Jupiter’s moons has changed and 12 newly discovered moons have been added. Furthermore, based on Kepler’s third law, the PDF obtained by the analytical method is very close to the PDF obtained by statistical inference, so it is reasonable to say that the best fit distribution of these orbital features is reasonable.

In addition, Table 5 shows that the orbital elements of some moons have the same Stable distribution. This interesting result may indicate that they have the same origin, which may have originated from the same parent asteroid. We will continue to pay attention to whether they are really ‘brothers’.

APPENDIX

A. Classification of Irregular Moons

see Table 7

B. Distribution Inference Results

See Tables 8-19
**Table 7. Classification of Irregular Satellites**

| Number | Name      | Designation  | Number | Name      | Designation  |
|--------|-----------|--------------|--------|-----------|--------------|
|        |           | Ananke group (Current) |        |           | Ananke group (Previous) |
| XXXIV  | Euporie   | S/2001 J10   | LIV    |          | S/2003 J18   |
| LII    |           | S/2010 J2    | LII    |          | S/2010 J2    |
| LIV    | S/2016 J1 | S/2017 J3    |        |           |              |
| XXXIII | Euanthe   | S/2001 J7    | L V    |          | S/2003 J18   |
| XXXV   | Orthosie  | S/2001 J9    |        |           |              |
| XXIX   | Thyone    | S/2001 J2    | XX IX  |          | S/2001 J2    |
| XL     | Mneme     | S/2003 J21   | XL     | Mneme    | S/2003 J21   |
| XXII   | Harpalyke | S/2000 J5    | XX II  |          | S/2000 J5    |
| XXX    | Hermippe  | S/2001 J3    | XXX    |          | S/2001 J3    |
| XXVII  | Praxidike | S/2000 J7    | XXVII  |          | S/2000 J7    |
| XLI    | Thelxinoe | S/2003 J22   | XLI    | Thelxinoe| S/2003 J22   |
| L X    | S/2003 J3 | S/2003 J3    |        | L         | S/2003 J3    |
| XLI    | Helike    | S/2003 J6    |        | Ananke   | S/2003 J6    |
| XXIV   | Iocaste   | S/2000 J3    | XXIV   | Iocaste  | S/2000 J3    |
| XII    | Ananke    | S/2017 J9    |        |           |              |

| Carme group (Current) | Carme group (Previous) |
|-----------------------|------------------------|
| S/2003 J19            |                        |
| XXIII Arche           | S/2002 J1              |
| XXXVIII Pasithee       | S/2001 J6              |
| L Herse               | S/2003 J17             |
| XXI Chaldene          | S/2000 J10             |
| XXXVII Kale            | S/2001 J8              |
| XXVI Isonoe           | S/2000 J6              |
| XXXI Aitne            | S/2001 J11             |
| XXV Erinome           | S/2000 J4              |
| L I                   | S/2010 J1              |
| XX Tayget             | S/2000 J9              |
| XI Carme              | S/2017 J2              |
| XXIII Kalyke          | S/2000 J2              |
| XLVII Eukelade        | S/2003 J1              |
| LVII                  | S/2003 J5              |
| XLV Kallichore        | S/2003 J11             |
|                      | S/2011 J1              |

| Pasiphae group (Current) | Pasiphae group (Previous) |
|--------------------------|---------------------------|
| S/2017 J6                |                          |
| LVIII                    | S/2003 J15               |
| XXXII Eurydome           | S/2001 J4                |
| XXVIII Autonoe           | S/2001 J1                |
| LVI                      | S/2011 J2                |
| XXXVI Sponde             | S/2001 J5                |
| LIX                      | S/2017 J1                |
| VIII Pasiphae            | VIII Pasiphae            |
| X IX Megacleite          | XIX Megacleite           |
| IX Sinope                | IX Sinope                |
| XXXIX Hegemone           | S/2003 J8                |
| XL I Aoede               | S/2003 J7                |
| XVII Callirrhoe          | S/1999 J1                |
| XLVIII Cyllene           | S/2003 J13               |
| XLIX Kore                | S/2003 J14               |
| XXXIV Euporie            | S/2001 J10               |
| LV                      | S/2003 J18               |
| LIV                     | S/2016 J1                |
| XXXV Orthosie           | S/2001 J9                |
| XLI Helike               | S/2003 J6                |
| Table 8. Ananke group |
|-----------------------|
| Beta | Birnbaum-Saunders | Exponential | Extreme Value | Gamma | Generalized Exponential Value | Generalized Pareto | Half Normal | Logistic | Loglogic |
|-------|------------------|-------------|---------------|-------|-------------------------------|-------------------|------------|---------|---------|
| h     | 0                | 0.4579329  | 0.4558849     | 0.0000003 | 0.0000000000                 | 0.0000000000     | 0.000000   | 0.6609235 | 0.664948  |
| p     | 0.379263        | 90505560   | 51387251      | 0.173291  | 0.07025703                    | 0.001015       | 0.122054   | 4390458   | 49863483  |
| Semi-major axis (km) | parameters | a=0.391716, b=5249.25 | β=0.024846 | γ=0.020486 | μ=0.106985e+07, σ=326466 | μ=0.0000000000 | 0.000000   | a=0.168567 | b=0.012476 |
| confidence interval | α=[76.8666, 25.3625, 9796.65] | [1.0164924, 0.0316948] | [2.06328e+07, 2.10839e+07, 4.6567e+07] | [0.342491, 765706, 472498] | [0.09114, 0.0316948] | [0.106985e+07, 3.26466] | [0.0000000000, 0.0000000000] | [0.0168567, 0.0012476] |
| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rican | Stable | T location-Scale | Weibull |
| h     | 0.4856020       | 45314695   | 0.4583649     | 0.4874359 | 0.0000014 | 0.0000017 | 0.441103 | 0.917820 | 0.7030019 | 0.6530247 |
| p     | 0.82948522      | 16297056   | 0.842680     | 0.733619  | 0.000014    | 0.000017   | 0.441103 | 0.917820 | 0.7030019 | 0.6530247 |
| Semi-major axis (km) | parameters | μ=0.168533, σ=0.0247039 | μ=2.086e+07, σ=501144 | λ=0.086446e+07 | B=0.17557e+07 | 0.483786 | 0.711134 | 0.2107e+07 | 0.637906 |
| confidence interval | α=[235.346, 893.149] | [1.0164924, 0.0316948] | [2.062097e+09, 2.11079e+07] | [0.342491, 765706, 472498] | [0.09114, 0.0316948] | [0.086446e+07, 0.483786] | a=0.0000000000 | 0.0000000000 | 0.0000000000 |
| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rican | Stable | T location-Scale | Weibull |
| h     | 0.5067797       | 88636145   | 0.4922221     | 0.503083 | 0.503083   | 0.6695392 | 6.5792133 | 5.201161 | 0.7766837 | 0.5893362 |
| p     | 0.89175056      | 16297056   | 0.842680     | 0.733619  | 0.000014    | 0.000017   | 0.441103 | 0.917820 | 0.7030019 | 0.6530247 |
| Mean inclination (deg) | parameter | a=0.18024, b=10343.3 | β=0.48374, γ=0.2018171 | μ=0.148999 | γ=0.292228 | a=0.212135 | 2177.212 | 0.139052 | 0.513902 | 0.7766837 |
| confidence interval | α=[1045.98, 3104.61] | [153.273, 246.441] | [0.0177789, 0.0379391] | [0.11886, 0.398942] | [0.037819, 0.132669] | [0.014899, 0.481319] | 0.112863 | 2177.212 | 0.139052 | 0.513902 |
| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rican | Stable | T location-Scale | Weibull |
| h     | 0.48206014      | 75913992   | 0.5137527     | 0.513822 | 0.1042826 | 0.86555178 | 5.247669 | 0.9108779 | 0.6031086 |
| p     | 0.8758951       | 80756951   | 0.5137527     | 0.513822 | 0.1042826 | 0.86555178 | 5.247669 | 0.9108779 | 0.6031086 |
| Mean inclination (deg) | parameter | μ=4.9995, σ=0.0224131 | μ=0.14847, σ=3.00301 | λ=0.3487, a=0.104144 | 0.104144 | 0.104144 | 0.104144 | 0.104144 | 0.104144 |
| confidence interval | α=[4.98869, 5.0103] | [149.964, 153.851] | [2.015994, 2245.1] | [0.14683, 1.9784] | [0.234784, 0.04539] | 0.14683 | 153.851 | 153.851 | 0.14683 |
| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rican | Stable | T location-Scale | Weibull |
| h     | 0.48206014      | 75913992   | 0.5137527     | 0.513822 | 0.1042826 | 0.86555178 | 5.247669 | 0.9108779 | 0.6031086 |
| p     | 0.8758951       | 80756951   | 0.5137527     | 0.513822 | 0.1042826 | 0.86555178 | 5.247669 | 0.9108779 | 0.6031086 |
| Mean inclination (deg) | parameter | μ=4.9995, σ=0.0224131 | μ=0.14847, σ=3.00301 | λ=0.3487, a=0.104144 | 0.104144 | 0.104144 | 0.104144 | 0.104144 | 0.104144 |
| confidence interval | α=[4.98869, 5.0103] | [149.964, 153.851] | [2.015994, 2245.1] | [0.14683, 1.9784] | [0.234784, 0.04539] | 0.14683 | 153.851 | 153.851 | 0.14683 |
### Table 9. Continued

| Beta | Birnbaum-Saunders | Exponential | Extreme Value | Gamma | Generalized Extreme Value | Generalized Pareto | Half Normal | Logistic | Loglogistic |
|------|-------------------|-------------|---------------|-------|---------------------------|-------------------|-------------|----------|-------------|
| h    | 0                 | 0           | 1             | 0     | 0                         | 0                 | 1           | 1        | 1           |
| P    | 0.147725          | 0.082776    | 0.008918      | 0.494462 | 0.125407                  | 0.355089          | 0.0145267  | 0.0009874 | 0.433606    |
| parameter | a=12.9547     | β=0.204059  | µ=0.211263    | σ=0.444536 | k=0.15873                 | σ=0.0532617       | σ=0.196666 | σ=0.21506  | α=1.55125   |
|         | b=48.4209        | γ=0.265447  |               |        |                           |                   |             | σ=0.216997 | β=0.14478   |
|        |                   |             |               |        |                           |                   |             |           |             |

**Mean eccentricity**

| Confidence interval | α=[0.45226, 26.0102] | β=[0.15919, 0.2282] | γ=[0.181049, 0.349846] |
|---------|-----------------------|---------------------|------------------------|
|          | [0.141101, 0.350897]  | [0.213908, 0.62029]  | [0.062029, 0.252217]   |
|          | [0.170663, 0.271116]  | [0.213908, 0.62029]  | [0.062029, 0.252217]   |

| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rician | Stable | T location-Scale | Weibull |
|-----------|----------|--------|---------|----------|--------|--------|-----------------|--------|
| h         | 0        | 0.092487 | 0.1682309 | 0.2422713 | 4.7085970 | 0.0160108 | 0.236867 | 0.312338 | 0.243124 |
| P         | 0.032525 | 0.032525 | 0.1682309 | 0.2422713 | 4.7085970 | 0.0160108 | 0.236867 | 0.312338 | 0.243124 |
| parameter | µ=1.58648 | σ=0.270232 | µ=1.211263 | σ=0.0509137 | λ=0.211263 | B=0.15344 | σ=0.020496 | σ=0.050382 | σ=0.20302 |
|          |          |          |          |          |          |          |          |          |          |
|          |          |          |          |          |          |          |          |          |          |

**Mean eccentricity**

| Confidence interval | µ=[-1.71672, -1.45023] | σ=[0.20419, 0.39962] |
|---------|------------------------|---------------------|
|          | [0.186724, 0.25580]   | [0.540098, 0.759224] |
|          | [0.181539, 0.28293]   | [0.6362405, 0.700438] |

### Argument of periasis

| Beta | Birnbaum-Saunders | Exponential | Extreme Value | Gamma | Generalized Extreme Value | Generalized Pareto | Half Normal | Logistic | Loglogistic |
|------|-------------------|-------------|---------------|-------|---------------------------|-------------------|-------------|----------|-------------|
| h    | 0                 | 0           | 1             | 0     | 0                         | 0                 | 1           | 1        | 1           |
| P    | 0.6118055         | 0.4918258  | 0.00092739    | 0.4851798 | 0.4778767               | 0.4537667         | 0.4356800  | 0.485620  |
| Argument of periasis | parameter | µ=0.978907 | σ=0.635986 | µ=189.636 | σ=103.79 | λ=189.636 | B=151.881 | σ=130.046 | δ=189.637 |
|        | 0.461358          |             |               |        |               |       |             |          |             |
|          | [0.138271, 0.41958] | [0.170241, 0.349846] |
|          | [0.015738, 0.252217] | [0.062029, 0.252217] |

| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rician | Stable | T location-Scale | Weibull |
|-----------|----------|--------|---------|----------|--------|--------|-----------------|--------|
| h         | 0        | 0.6118055 | 0.4918258 | 0.00092739 | 0.4851798 | 0.4778767 | 0.4537667 | 0.4356800 | 0.485620  |
| P         | 0.6118055 | 0.4918258 | 0.00092739 | 0.4851798 | 0.4778767 | 0.4537667 | 0.4356800 | 0.485620  | 0.485620  |
| parameter | µ=0.978907 | σ=0.635986 | µ=189.636 | σ=103.79 | λ=189.636 | B=151.881 | σ=130.046 | δ=189.637 | A=214.793 |
| Argument of periasis |          |          |          |          |          |          |          |          |          |
|          |          |          |          |          |          |          |          |          |          |
|          |          |          |          |          |          |          |          |          |          |
### Table 10. Continued

| Beta          | Birnbaum-Saunders | Exponential | Gamma  | Generalized Gamma | Generalized Pareto | Half Normal | Logistic | Loglogistic |
|---------------|-------------------|-------------|--------|-------------------|-------------------|------------|----------|-------------|
| Mean anomaly  |                   |             |        |                   |                   |            |          |             |
| h             | 0.1047716         | 796.05681   | 0.0077817 | 0.0480541         | 0.5641869         | 0.0804612 | 0.7087532 | 0.382847   |
| P             |                   | 970.04894   | 7037.128 | 50439605          | 99481093          | 7980721    | 9828547   | 9.2484194  |
| Longitude of the ascending node | parameter | a=1.42533 | b=5.95015 | 0.102405 | 1.27373 | 0.194.95 | 0.251.496 | 0.15648   |
| Confidence interval | beta | 0.349886 | 0.766834 | 0.00077410 | 1.07405 | 0.2820135 | 0.4212443 | 0.4712900 |
| Mean anomaly  |                   |             |        |                   |                   |            |          |             |
| h             | 0.0575628         | 560.24808   | 0.108860 | 0.4757158         | 0.000074610       | 0.2820135 | 0.4212443 | 0.4819786  |
| P             |                   | 303.5053    | 9368.7063 | 3230844198 | 8872529       | 0.452286   | 0.4819786 | 1.0826936  |
| Longitude of the ascending node | parameter | a=4.94319 | b=1.10303 | 0.628956 | 0.99456 | 0.19945 | 0.14944 | 0.1223 |
| Confidence interval | beta | 0.51257 | 0.144476 | 0.000074610 | 0.19945 | 0.14944 | 0.1223 |
| Normal        | Lognormal         | Nakagami    | Normal  | Poisson           | Rayleigh          | Rician     | Stable    | T location-Scale | Weibull |
| (h, P)        |                   |             |         |                   |                   |            |          |             |
| (0, 0)        |                   |             |         |                   |                   |            |          |             |

### Table 11. Continued

| Beta          | Birnbaum-Saunders | Exponential | Gamma  | Generalized Gamma | Generalized Pareto | Half Normal | Logistic | Loglogistic |
|---------------|-------------------|-------------|--------|-------------------|-------------------|------------|----------|-------------|
| Mean anomaly  |                   |             |        |                   |                   |            |          |             |
| h             | 0.732828          | 949.38695   | 0.2170562 | 0.3105308         | 0.7780143         | 0.6828752 | 0.8395948 | 0.9183612  |
| P             |                   | 90312764    | 4245996 | 47634998          | 5144426          | 13649792   | 74248861  | 64735279   |
| Longitude of the ascending node | parameter | a=1.61475 | b=7.78967 | 0.165152 | 0.74763 | 0.22544 | 0.179831 | 0.1235 |
| Confidence interval | beta | 0.695799 | 0.980299 | 0.141553 | 0.7780143 | 0.6828752 | 0.8395948 | 0.9183612 |
| Normal        | Lognormal         | Nakagami    | Normal  | Poisson           | Rayleigh          | Rician     | Stable    | T location-Scale | Weibull |
| (h, P)        |                   |             |         |                   |                   |            |          |             |
| (0, 0)        |                   |             |         |                   |                   |            |          |             |
| Period (days) | Beta | Birnbaum-Saunders | Exponential | Extreme Value | Gamma | Generalized Extreme Value | Generalized Pareto | Half Normal | Logistic | Loglogistic |
|--------------|------|-------------------|-------------|--------------|-------|----------------------------|---------------------|------------|----------|------------|
| h 0          | 0.2464753 | 0.2241271 | 0.6216052 | 0.2296189 | 0.9351105 | 5.983399856 | 1.88811080321486E-07 | 0.5352926 | 0.5154073 | 0.5154073 |
| P 19505.533 | 26465747  | 40367347 | 10574908 | 64142756 | 60949E-07 | 321486E-07 | 88931487 | 49513219 | 49513219 |
| Parameter a = 335.873 | β = 614.532 | γ = 0.0451084 | μ = 614.911 | σ = 13.7502 | α = 0.224237 | 7.20524658270882E-07 | 0.6216052 | 0.2296189 | 0.2296189 |
| | | | | | | | | | | |
| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rician | Stable | T location-scale | Weibull |
| h 0         | 0.2356051 | 0.2345935 | 0.2515045 | 0.2589681 | 0.96323135 | 0.2403968 | 0.9878233 | 0.8519166 | 0.5765628 |
| P 66173821  | 48910143  | 93585989 | 26771509 | 104714E-06 | 92121989 | 575757E-06 | 2741636 | 6290288 | 6290288 |
| Parameter μ = 6.42087 | σ = 0.0360603 | μ = 614.911 | σ = 21.565 | λ = 641.911 | B = 435.061 | μ = 620.502 | σ = 13.4808 | ν = 2.87E-08 | 6290288 |
| | | | | | | | | | |
| confidence interval | 0.176.163, 649.37 | 0.105.692, 418.675 | 0.1021.34 | 0.1562.3 | 0.1547.03 | 0.1547.03 | 0.1547.03 | 0.1547.03 | 0.1547.03 |
| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rician | Stable | T location-scale | Weibull |
| h 0         | 0.2345935 | 0.2515045 | 0.2589681 | 0.2598681 | 0.56323135 | 0.2403968 | 0.9878233 | 0.8519166 | 0.5765628 |
| P 66173821  | 48910143  | 93585989 | 26771509 | 104714E-06 | 92121989 | 575757E-06 | 2741636 | 6290288 | 6290288 |
| Parameter μ = 6.42087 | σ = 0.0360603 | μ = 614.911 | σ = 21.565 | λ = 641.911 | B = 435.061 | μ = 620.502 | σ = 13.4808 | ν = 2.87E-08 | 6290288 |
| | | | | | | | | | |
| confidence interval | 0.176.163, 649.37 | 0.105.692, 418.675 | 0.1021.34 | 0.1562.3 | 0.1547.03 | 0.1547.03 | 0.1547.03 | 0.1547.03 | 0.1547.03 |
Table 12. Carne group

| Parameter | Beta | Birnbaum-Saunders | Exponential | Extreme Value | Gamma | Generalized Extreme Value | Generalized Pareto | Half Normal | Logistic | Loglogistic |
|-----------|------|-------------------|-------------|---------------|-------|--------------------------|-------------------|------------|----------|-------------|
| h         | 0    | 0.6026355        | 1           | 0             | 0     | 0                        | 1                 | 1          | 0        | 0           |
| p         | 0.6706755 | 0.0026355 | 1.44454998 | 0.3563632 | 0.6677067 | 0.6140590 | 0.05850879 | 1.0158714 | 0.834737 | 0.8296567 |
|            | 70789827 | 592919E-07 | 592919E-07 | 592919E-07 | 592919E-07 | 592919E-07 | 592919E-07 | 592919E-07 | 592919E-07 | 592919E-07 |
| Semi-major axis (km) | a=3773.99 | b=12446.7 | | | | | | | | |
|              | $\beta=2.32642e+07$ | $\mu=2.32656e+07$ | $\sigma=2.3432e+07$ | | | | | | | |
| confidence interval | $\mu=[1.56831e+07, 3.89093e+07]$ | $\sigma=2.34097e+07$ | $\sigma=2.32687e+07$ | | | | | | | |
|              | $\nu=16.956, 16.968$ | $\nu=16.956, 16.968$ | $\nu=16.956, 16.968$ | | | | | | | |
| Semi-major axis (km) | $\alpha=2258.13, 6307.43$ | $\omega=744.41, 20810.2$ | $\nu=0.018686$ | | | | | | | |
|              | $\beta=2.32642e+07$ | $\mu=2.32656e+07$ | $\sigma=2.3432e+07$ | | | | | | | |
| confidence interval | $\mu=[1.56831e+07, 3.89093e+07]$ | $\sigma=2.34097e+07$ | $\sigma=2.32687e+07$ | | | | | | | |
|              | $\nu=16.956, 16.968$ | $\nu=16.956, 16.968$ | $\nu=16.956, 16.968$ | | | | | | | |

| Parameter | Beta | Birnbaum-Saunders | Exponential | Extreme Value | Gamma | Generalized Extreme Value | Generalized Pareto | Half Normal | Logistic | Loglogistic |
|-----------|------|-------------------|-------------|---------------|-------|--------------------------|-------------------|------------|----------|-------------|
| h         | 0    | 0.6026355        | 1           | 0             | 0     | 0                        | 1                 | 1          | 0        | 0           |
| p         | 0.6706755 | 0.0026355 | 1.44454998 | 0.3563632 | 0.6677067 | 0.6140590 | 0.05850879 | 1.0158714 | 0.834737 | 0.8296567 |
|            | 70789827 | 592919E-07 | 592919E-07 | 592919E-07 | 592919E-07 | 592919E-07 | 592919E-07 | 592919E-07 | 592919E-07 | 592919E-07 |
| Semi-major axis (km) | a=3773.99 | b=12446.7 | | | | | | | | |
|              | $\beta=2.32642e+07$ | $\mu=2.32656e+07$ | $\sigma=2.3432e+07$ | | | | | | | |
| confidence interval | $\mu=[1.56831e+07, 3.89093e+07]$ | $\sigma=2.34097e+07$ | $\sigma=2.32687e+07$ | | | | | | | |
|              | $\nu=16.956, 16.968$ | $\nu=16.956, 16.968$ | $\nu=16.956, 16.968$ | | | | | | | |
| Semi-major axis (km) | $\alpha=2258.13, 6307.43$ | $\omega=744.41, 20810.2$ | $\nu=0.018686$ | | | | | | | |
|              | $\beta=2.32642e+07$ | $\mu=2.32656e+07$ | $\sigma=2.3432e+07$ | | | | | | | |
| confidence interval | $\mu=[1.56831e+07, 3.89093e+07]$ | $\sigma=2.34097e+07$ | $\sigma=2.32687e+07$ | | | | | | | |
|              | $\nu=16.956, 16.968$ | $\nu=16.956, 16.968$ | $\nu=16.956, 16.968$ | | | | | | | |
Table 3. Continued

| Beta          | Birnbaum-Saunders | Exponential | Gamma | Generalized Extreme Value | Generalized Pareto | Half Normal | Logistic | Loglogistic |
|---------------|-------------------|-------------|-------|----------------------------|--------------------|-------------|----------|-------------|
| h             | 0                 | 0           | 1     | 0                           | 0                  | 1           | 1        | 0           |
| P             | 0.6211999         | 0.6945566   | 5.68904522 | 0.2061099                   | 0.6541297         | 0.6325293   | 0.00012879 | 2.01745155  |
| Parameter     | a=69.849         | b=200.824   | 0.10323     | 0.271899                    | a=94.4514         | 0.2500657   | k=-1.57434  |
|               | σ                | 0.28899     | 0.00273209  | 0.247682                    | 0.247682         | 0.247682    | 0.247682   | 0.247682    |
| Mean eccentricity | 0.245667  | 0.267448   | 0.0205325    | 0.260176                    | 0.260176         | 0.260176    | 0.260176   | 0.260176    |
| Confidence interval | [0.198473, 0.0205325] | [0.0186206, 0.0385901] | [0.012186, 0.00085052] | [0.019212] | [0.019212] |
| Lognormal     | Normal            | Poisson     | Rayleigh    | Rician                      | Stable            | T location-Scale | Weibull   |
| h             | 0                 | 0           | 1         | 0                           | 0                 | 0           | 0        | 0           |
| P             | 0.6668948         | 0.60860154  | 0.5410066  | 1                           | 1                 | 1           | 1        | 1           |
| Parameter     | μ=-1.3599         | σ=0.010574  | 0.258085   | 0.258085                    | 0.258085         | 0.258085    | 0.258085  | 0.258085    |
| Mean eccentricity | 0.245667  | 0.267448   | 0.0205325    | 0.260176                    | 0.260176         | 0.260176    | 0.260176   | 0.260176    |
| Confidence interval | [0.128292, -1.31042] | [0.0615105, -0.0399221] | [0.0856963, 0.298418] | [0.0195618, 0.348285] |
| Logistic      | Nakagami          | Normal      | Poisson    | Rayleigh                    | Rician            | Stable      | T location-Scale | Weibull   |
| h             | 0                 | 0           | 1         | 0                           | 0                 | 1           | 1        | 0           |
| P             | 0.6714204         | 0.6049809   | 0.5410066  | 1                           | 1                 | 1           | 1        | 1           |
| Parameter     | μ=-1.38628        | σ=0.13664   | 0.179526   | 0.179526                    | 0.179526         | 0.179526    | 0.179526  | 0.179526    |
| Mean eccentricity | 0.245667  | 0.267448   | 0.0205325    | 0.260176                    | 0.260176         | 0.260176    | 0.260176   | 0.260176    |
| Confidence interval | [0.056325, 0.29806] | [0.0615105, 0.348285] | [0.0856963, 0.298418] | [0.0195618, 0.348285] |
| Logistic      | Nakagami          | Normal      | Poisson    | Rayleigh                    | Rician            | Stable      | T location-Scale | Weibull   |
| h             | 0                 | 0           | 1         | 0                           | 0                 | 1           | 1        | 0           |
| P             | 0.6714204         | 0.6049809   | 0.5410066  | 1                           | 1                 | 1           | 1        | 1           |
| Parameter     | μ=-1.38628        | σ=0.13664   | 0.179526   | 0.179526                    | 0.179526         | 0.179526    | 0.179526  | 0.179526    |
| Mean eccentricity | 0.245667  | 0.267448   | 0.0205325    | 0.260176                    | 0.260176         | 0.260176    | 0.260176   | 0.260176    |
| Confidence interval | [0.056325, 0.29806] | [0.0615105, 0.348285] | [0.0856963, 0.298418] | [0.0195618, 0.348285] |

*Note: All parameters are estimated using maximum likelihood estimation.*
| Longitude of the ascending node | Beta | Birnbaum-Saunders | Exponential | Gamma | Generalized Pareto | Generalized Extreme Value | Half Normal | Logistic | Loglogistic |
|-------------------------------|------|------------------|-------------|-------|-------------------|---------------------------|------------|----------|-------------|
| a=0.172632387856651 | 0.185689404370139 | 0.5448213 | 0.540237 | 0.347907 | 0.566073 | 0.1856894 | 0.5811526 | 0.5841180 | 0.4509683 | 0.7755500 |
| b=90.78288 | 0.6184814 | 0.347907 | 0.540237 | 0.5448213 | 0.566073 | 0.1856894 | 0.5811526 | 0.5841180 | 0.4509683 | 0.7755500 |
| µ=182.742 | 0.347907 | 0.540237 | 0.5448213 | 0.566073 | 0.1856894 | 0.5811526 | 0.5841180 | 0.4509683 | 0.7755500 |
| σ=98.5839 | 0.540237 | 0.5448213 | 0.566073 | 0.1856894 | 0.5811526 | 0.5841180 | 0.4509683 | 0.7755500 |

Table 14. Continued
| Period (days) | Beta | Birnbaum-Saunders | Exponential | Extreme Value | Gamma | Generalized Extreme Value | Generalized Pareto | Half Normal | Logistic | Loglogistic |
|--------------|------|-------------------|-------------|---------------|-------|--------------------------|--------------------|-------------|----------|------------|
| h | 0 | 0.0638585 | 9945.758 | 1.11154812754144E-06 | 38449682 | 0.038988838449682 | 1.89807879956283E-09 | 0.0309629547672015 | 0.36013169582307 | 0.36013169582307 |
| p | 1 | 0.0638585 | 0540822 | 1.11154812754144E-06 | 38449682 | 0.038988838449682 | 1.89807879956283E-09 | 0.0309629547672015 | 0.36013169582307 | 0.36013169582307 |
| a=125.437 | b=48.7261 | 0.0537534 | 0.720.405 | 0.720.405 | μ=720.405 | 1.89807879956283E-09 | 0.0309629547672015 | 0.36013169582307 | 0.36013169582307 |
| parameter | | | | | | | | | | |
| confidence interval | a=[80.8464, 194.621] | b=[28.282, 83.9487] | | | | | | | | |
| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rician | Stable | T location-Scale | Weibull |
| h | 0.033719 | 0.043534 | 0.0480125 | 0.00002207 | 0.0497154 | 0.00002207 | 0.0497154 | 0.00002207 | 0.0497154 | 0.00002207 |
| p | 281.83435 | 6373.1779 | 0.0480125 | 0.00002207 | 0.0497154 | 0.00002207 | 0.0497154 | 0.00002207 | 0.0497154 | 0.00002207 |
| μ=6.57845 | σ=0.0505649 | ω=5.20260 | μ=720.405 | 1.89807879956283E-09 | 0.0309629547672015 | 0.36013169582307 | 0.36013169582307 | 0.36013169582307 | 0.36013169582307 |
| Poisson | | | | | | | | | | |
| confidence interval | | | | | | | | | | |
| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rician | Stable | T location-Scale | Weibull |
| h | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 |
| p | 9945.758 | 9945.758 | 9945.758 | 9945.758 | 9945.758 | 9945.758 | 9945.758 | 9945.758 | 9945.758 | 9945.758 |
| μ=6.57845 | σ=0.0505649 | ω=5.20260 | μ=720.405 | 1.89807879956283E-09 | 0.0309629547672015 | 0.36013169582307 | 0.36013169582307 | 0.36013169582307 | 0.36013169582307 | 0.36013169582307 |
| Poisson | | | | | | | | | | |
| confidence interval | | | | | | | | | | |
| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rician | Stable | T location-Scale | Weibull |
| h | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 | 0.00138447 |
| p | 9945.758 | 9945.758 | 9945.758 | 9945.758 | 9945.758 | 9945.758 | 9945.758 | 9945.758 | 9945.758 | 9945.758 |
| μ=6.57845 | σ=0.0505649 | ω=5.20260 | μ=720.405 | 1.89807879956283E-09 | 0.0309629547672015 | 0.36013169582307 | 0.36013169582307 | 0.36013169582307 | 0.36013169582307 | 0.36013169582307 |
| Poisson | | | | | | | | | | |
| confidence interval | | | | | | | | | | |

Table 15. Continued
| Beta | Birnbaum-Saunders | Exponential | Generalized Pareto | Generalized Exponential Value | Half Normal | Logistic | Loglogistic |
|------|------------------|-------------|-------------------|-------------------------------|------------|----------|------------|
| h    | 0                | 0           | 1                 | 0                             | 1          | 0        | 0          |
| p    | 0.876069         | 0.862536    | 5.7961816         | 0.995637                      | 0.871049   | 0        | 0          |
|      | a=1287.8         | b=4169.49   | µ=-2.35979e+07    | μ=-2.38737e+07                | 1          | 1        | 1          |
|      | µ=2.19260756     | σ=0.49433   | σ=4.37823e+07     | 0                             | 1          | 1        | 1          |
|      | σ=5.6048224      | µ=0         | σ=2.36408e+07     | 0.956204                      | 0         | 0        | 0          |
|      | α=0.961544       | β=0.931140  | 0                   | 0                             | 0         | 0        | 0          |
|      | a=[502.373, 3301.21] | b=1617.45, 10749.5 | µ=2.39061e+07 | μ=2.34595e+07                  | 1          | 1        | 1          |
|      | 0.009403, 2.33302e+07 | 0.0217123 | 0                   | 0                             | 0         | 0        | 0          |
|      | 6857.03, 2.38036e+07 | 0.097164 | 0                   | 0                             | 0         | 0        | 0          |
|      | [2.31082e+07, 2.38108e+07] | 510534 | 0                   | 0                             | 0         | 0        | 0          |

**Lognormal**

| Beta | Nakagami | Normal | Poisson | Rayleigh | Rician | Stable | T location-Scale | Weibull |
|------|----------|-------|--------|----------|--------|--------|-----------------|--------|
| h    | 0        | 0     | 0      | 0        | 0      | 0      | 0               | 0      |
| p    | 0.847076 | 0.879144 | 0.872036 | 0.0001637 | 0.881600 | 0.8872265 | 0.8870214 | 0.994106 |
|      | 51791922 | 50289622 | 05231185 | 703716694 | 69518538 | 04169972 | 09454305 | 69251343 |
|      | µ=16.9764 | µ=422.098 | µ=2.35979e+07 | µ=0.183919 | µ=571145 | µ=2.35979e+07 | µ=2.38683e+07 | µ=48.1023 |
|      | σ=0.0252785 | σ=5.57188e+14 | σ=59671 | σ=59671 | σ=59671 | σ=59671 | σ=59671 | σ=59671 |
|      | 2.35979e+07 | B=1.66911e+07 | s=2.3591e+07 | s=2.3591e+07 | s=2.3591e+07 | s=2.3591e+07 | s=2.3591e+07 | s=2.3591e+07 |
|      | 2.35979e+07 | B=151.213 | A=2.35979e+07 | A=2.35979e+07 | A=2.35979e+07 | A=2.35979e+07 | A=2.35979e+07 | A=2.35979e+07 |

**Beta**

| Probability Distribution | Value |
|-------------------------|-------|
| Lognormal               | 0.9982467 |
| Nakagami               | 0.9982467 |
| Normal                 | 0.9982467 |
| Poisson                | 0.9982467 |
| Rayleigh               | 0.9982467 |
| Rician                | 0.9982467 |
| Stable               | 0.9982467 |
| T location-Scale        | 0.9982467 |
| Weibull               | 0.9982467 |

**Mean Value**

| Probability Distribution | Value |
|-------------------------|-------|
| Lognormal               | 14589926 |
| Nakagami               | 14589926 |
| Normal                 | 14589926 |
| Poisson                | 14589926 |
| Rayleigh               | 14589926 |
| Rician                | 14589926 |
| Stable               | 14589926 |
| T location-Scale        | 14589926 |
| Weibull               | 14589926 |
| Argument of periphasis | Beta | Birnbaum-Saunders | Exponential | Extreme Value | Gamma | Generalized Extreme Value | Generalized Pareto | Half Normal | Logistic | Loglogistic |
|------------------------|------|------------------|-------------|---------------|-------|-----------------------------|------------------|-------------|----------|------------|
|                        | h    |                  |             |               |       |                             |                  |             |          |            |
|                        | p    |                  |             |               |       |                             |                  |             |          |            |
| Mean eccentricity      |      |                  |             |               |       |                             |                  |             |          |            |
|                        | a=10.755 | β=0.3347 | μ=0.3446 | σ=0.0962249 | b=0.020839 |                     |                  |             |          |            |
|                        | a=21.7175 | β=0.3833 | μ=0.230055 | σ=0.1067426 | b=0.00992847 |                       |                  |             |          |            |
|                        | a=39.2136 | β=0.358847 | μ=0.13628 | σ=0.405685 | b=0.041985 |                       |                  |             |          |            |

Table 17. Continued.
### Table 18. Continued

| Beta | Birnbaum-Saunders | Exponential | Gamma | Generalized Exponential | Generalized Pareto | Half Normal | Logistic Logistic |
|------|-------------------|-------------|-------|--------------------------|------------------|-------------|------------------|
| **h** | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0.0474001 |
| **P** | 7.3359159 | 97556768 | 0.0119097 | 0.0154574 | 0.414957 | 0.0405711 | 0.904586 | 0.7589793 |
| **Longitude of the ascending node** | | | | | | | | |
| **parameter** | a=3.29271 | b=10.0629 | γ=0.10748 | μ=260.139 | 317.907 | 0.889461 | 0.984077 | 0.6970371 |
| **confidence interval** | b[2.876, 35.2991] | γ[0.47683, 1.08897] | σ[34.9735, 86.5525] | k[-1.21492, 252.872] | k[-1.23683, 252.872] | k[-1.21492, 252.872] | k[-1.21492, 252.872] | k[-1.21492, 252.872] |
| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rician | Stable | T location | Scale |
| **h** | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0.0519302 |
| **P** | 9923868 | 73202663 | 0.0403010 | 0.0118418 | 0.00000892 | 0.0273103 | 0.0975674 | 0.0736593 |
| **Longitude of the ascending node** | | | | | | | | |
| **parameter** | μ=5.38597 | σ=0.681073 | μ=13.8863 | σ=9.94678 | μ=250.967 | σ=93.6478 | λ=250.967 | B=188.69 |
| **confidence interval** | σ[10.6963, 304.949] | ω[46347.1, 104905] | λ[242.95, 258.984] | B[150.785, 252.217] | s[176.54, 284.218] | σ[64.058, 141.529] | μ[141.529, 284.218] |
| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rician | Stable | T location | Scale |
| **h** | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0.0258709 |
| **P** | 537644E-13 | 198459E-06 | 8.07683173 | 0.0436519 | 0.0697037 | 0.0810148 | 0.6894300 | 0.6927896 |
| **Mean anomaly** | | | | | | | | |
| **parameter** | a=1.21564 | b=0.01555 | μ=16.7356 | σ=3.42722 | μ=205.542 | σ=85.4244 | μ=187.22 | σ=442.782 |
| **confidence interval** | a[1.002652, 0.337] | b[1.15571, 16.1408] | μ[131.255, 367.241] | σ[7.01284, 163.838] | σ[57.5329, 126.838] | σ[250.777, 252.872] | σ[126.838, 252.872] |
| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rician | Stable | T location | Scale |
| **h** | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0.0417302 |
| **P** | 9561357 | 66577983 | 0.1192421 | 0.0253812 | 0.0017620 | 0.0594391 | 0.9216530 | 0.9450430 |
| **Mean anomaly** | | | | | | | | |
| **parameter** | μ=4.89152 | σ=1.66695 | μ=50.542 | σ=101.901 | μ=205.542 | σ=101.901 | μ=108.97 | σ=69.6119 |
| **confidence interval** | μ[139.111, 261.973] | σ[74.6046, 160.708] | μ[198.287, 252.872] | σ[72.6643, 160.708] | σ[72.6643, 160.708] | σ[72.6643, 160.708] | σ[72.6643, 160.708] |
| Lognormal | Nakagami | Normal | Poisson | Rayleigh | Rician | Stable | T location | Scale |
| **h** | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0.0369839 |
| **P** | 5.81465 | 1.05213 | μ=0.17605 | σ=101.901 | μ=205.542 | σ=101.901 | μ=108.97 | σ=69.6119 |
| **Mean anomaly** | | | | | | | | |
| **parameter** | μ=3.96839 | σ=1.22042 | μ=199.111 | σ=212.797 | μ=198.287 | σ=212.797 | μ=126.838 | σ=212.797 |
| **confidence interval** | μ[149.111, 261.973] | σ[74.6046, 160.708] | μ[198.287, 252.872] | σ[72.6643, 160.708] | σ[72.6643, 160.708] | σ[72.6643, 160.708] | σ[72.6643, 160.708] |

- **Beta, Birnbaum-Saunders, Exponential, Gamma** represent the parameters of different probability distributions.
- **Generalized Exponential, Generalized Pareto** indicate the generalized form of exponential and Pareto distributions.
- **Half Normal, Logistic, Logistic** refer to different distribution types.
| Period (days) | Beta Birnbaum-Saunders | Exponential | Extreme Value | Gamma | Generalized Extreme Value | Generalized Pareto | Half Normal | Logistic | Loglogistic |
|--------------|------------------------|-------------|---------------|-------|--------------------------|------------------|------------|---------|-------------|
| parameter    |                        |             |               |       |                          |                  |            |         |             |
| a=227.447    |                        |             |               |       |                          |                  |            |         |             |
| b=81.6014    |                        |             |               |       |                          |                  |            |         |             |
| h            | 0                      | 0           | 1             | 0     | 0                        | 0                | 1          | 1       | 0           |
| P            | 0.9995033              | 0.9983989   | 8.7583614     | 0.9976648 | 0.9986640 | 19325132      | 520109E-07 | 1.21915041 | 0.999458    |
|              | 81065451               | 58389214    | 553776E-06    | 87071864 | 85631321   | 19325132      | 861873E-06 | 0.999709   | 48029618   |
| h            | 0                      | 0.9999998   | 8.7583614     | 0.9876648 | 0.9986640 | 19325132      | 520109E-07 | 1.21915041 | 0.999458    |
| P            | 81065451               | 58389214    | 553776E-06    | 87071864 | 85631321   | 19325132      | 861873E-06 | 0.999709   | 48029618   |
| h            | 0                      | 0           | 1             | 0     | 0                        | 0                | 1          | 1       | 0           |
| P            | 0.9995033              | 0.9983989   | 8.7583614     | 0.9976648 | 0.9986640 | 19325132      | 520109E-07 | 1.21915041 | 0.999458    |
|              | 81065451               | 58389214    | 553776E-06    | 87071864 | 85631321   | 19325132      | 861873E-06 | 0.999709   | 48029618   |

**Table 19. Continued**

| Period (days) | Lognormal | Nakagami  | Normal       | Poisson | Rayleigh | Rician       | Stable        | T location-Scale | Weibull     |
|--------------|-----------|-----------|--------------|---------|----------|--------------|----------------|-----------------|-------------|
| parameter    |           |           |              |         |          |              |                |                 |             |
| a=6.60059    |           |           |              |         |          |              |                |                 |             |
| b=0.0356735  |           |           |              |         |          |              |                |                 |             |
| h            | 0         | 0         | 0            | 0       | 0        | 0            | 0              | 0               | 0           |
| P            | 0.9986122 | 0.9988946 | 0.999374     | 0.9961258 | 0.000029118 | 0.9990974    | 0.9990965      | 0.9990970       | 0.9933114     |
|              | 10308261  | 92961846  | 34153382     | 9306309 | 4398467647 | 91549839     | 47156696       | 93941786        | 33289137      |
| h            | 0         | 0         | 0            | 0       | 0        | 0            | 0              | 0               | 0           |
| P            | 0.9986122 | 0.9988946 | 0.999374     | 0.9961258 | 0.000029118 | 0.9990974    | 0.9990965      | 0.9990970       | 0.9933114     |
|              | 10308261  | 92961846  | 34153382     | 9306309 | 4398467647 | 91549839     | 47156696       | 93941786        | 33289137      |
| h            | 0         | 0         | 0            | 0       | 0        | 0            | 0              | 0               | 0           |
| P            | 0.9986122 | 0.9988946 | 0.999374     | 0.9961258 | 0.000029118 | 0.9990974    | 0.9990965      | 0.9990970       | 0.9933114     |
|              | 10308261  | 92961846  | 34153382     | 9306309 | 4398467647 | 91549839     | 47156696       | 93941786        | 33289137      |
Data Availability
The data used to support the findings of this study are available from the corresponding author upon request.

Author Contributions
Formal analysis, Fabao Gao; Software, Xia Liu; Writing—original draft, Xia Liu; Writing—review & editing, Fabao Gao.

Funding
This research was funded by the National Natural Science Foundation of China (NSFC) though grant No.11672259 and the China Scholarship Council through grant No.201908320086.

Conflicts of Interest
The authors declare that there is no competing interests.

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