Bi-maximal Mixing and Bilinear R Violation

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Abstract

We perform a general analytic study of feasibility of obtaining a combined explanation for the deficits in the solar and the atmospheric neutrino fluxes with two large mixing angles in supersymmetric model with bilinear $R$ parity violations. The required hierarchy among the solar and atmospheric neutrino mass scales follows in this framework in the presence of an approximate Higgs - slepton universality at the weak scale. The solar mixing angle is shown to be related to non-universality in slepton mass terms specifically to differences in soft parameters of the first two leptonic generations. It is shown that this flavour universality violation should be as strong as the Higgs-slepton universality violation if solar neutrino mixing angle is to be large. The standard supergravity models with universal boundary conditions at a high scale lead to the required Higgs-slepton universality violations but the predicted violation of flavour universality among the first two generations is much smaller than required. This model therefore cannot provide an explanation of large solar neutrino mixing angle unless some universality violations in soft supersymmetry breaking parameters are introduced at a high scale itself.

1 Introduction

Experimental observations of deficits in the solar $\odot$ and atmospheric $\odot$ neutrino fluxes have provided concrete ground to believe in neutrino oscillations. These experimental results are consistent with a simple picture of three active neutrinos mixing with each other. Within this picture, two independent (mass)$^2$
differences ($\Delta_{\odot}, \Delta_{atm}$) among three neutrinos govern the oscillations of the solar and atmospheric neutrinos respectively. One needs $\Delta_{\odot}/\Delta_{atm} \leq 10^{-2}$. Two of the mixing angles determining amplitudes of these oscillations are required to be large. The third mixing angle measured by the survival probability of the electron neutrinos in laboratory experiments such as CHOOZ is found to be much smaller $\leq 0.1$.

Different theoretical possibilities have been suggested for obtaining the above neutrino spectrum with two large mixing angles. One potentially interesting possibility in this regard is supersymmetric standard model containing bilinear $R$ parity and lepton number violation. The following features of the model make it an ideal candidate for the description of neutrino masses. (1) The lepton number violations and hence neutrino masses and mixing are described in this model in terms of three parameters. Ratios of these parameters control neutrino mixing which can be naturally large. (2) The mechanism for suppression of neutrino masses compared to other fermion masses is automatically built-in for two of the most popular supersymmetry breaking scenario namely the minimal supergravity model (mSUGRA) and models with gauge mediated supersymmetry breaking (GMSB). Extensive studies of these models have been carried out in the literature. Our aim in this paper is to discuss under what conditions the bilinear model can lead to two large mixing angle among neutrinos. We discuss this issue analytically and in the process show that the two scenarios mentioned above cannot lead to two large mixing angles although small angle mixing solution to the solar neutrino problem is possible.

The suppression in neutrino masses in mSUGRA and GMSB arises due to equality at a high scale ($\equiv M_X$) of soft parameters of one of the Higgs fields ($\equiv H'_1$) with the corresponding parameters of the leptonic doublets having the same quantum numbers as $H'_1$. Small differences arise in these equal parameters at the weak scale due to RG scaling. For example, one finds in case of mSUGRA

$$\Delta m^2_i \equiv (m^2_{\nu i} - m^2_{H'_i}) \approx \frac{3h^2}{4\pi^2} \ln \frac{M_X}{M_Z} m^2_{susy} \approx 2 \cdot 10^{-3} m^2_{susy}$$

(1)

Feasibility of only small mixing angle solution was pointed out also in [14]. Our analysis considerably differs from theirs.
where \( m^2_{\nu_i} (i = 1, 2, 3), m^2_{H'_1} \) respectively denote the weak scale values of the soft SUSY breaking masses of the sneutrino and \( H'_1 \) respectively and \( m_{SUSY} \) is the typical SUSY breaking scale \( \sim \mathcal{O}(100 \text{ GeV}) \). The \( h_b \) in the above equation refers to the \( b \)-quark Yukawa coupling. The neutrino masses in this model involve the above and similar differences among \( B \) parameters. The suppression in these differences leads to suppression in neutrino masses. Thus the smallness of neutrino masses is linked to near universality of the \( Higgs (H'_1) \) and \( sneutrino \) soft parameters. As we will discuss in this paper, the solar neutrino mixing angle is directly linked to \emph{flavour} universality violation, \emph{i.e.} to differences in sneutrino mass parameters themselves. More specifically, the solar neutrino mixing angle involves the parameter

\[
\delta = \frac{m^2_{\nu_2} - m^2_{\nu_1}}{\Delta m^2_1 + \Delta m^2_2},
\]

which is required to be \( \mathcal{O}(1) \) implying that the weak scale universality violation among different flavours are required to be as strong as the corresponding Higgs-slepton universality violations. This is in sharp contrast with the expectations based on mSUGRA and GMSB where the former violations are mainly controlled by the muon Yukawa coupling while the latter by the \( b \) or \( \tau \) Yukawa couplings. Thus \( \delta \) in eq.\((2)\) is of \( \mathcal{O}(10^{-4}) \) instead of being one.

Link between universality violation and large mixing was brought out in the numerical study of [13]. In contrast to their work, our analytical study allows us to determine specific pattern of universality violation and also allows us to quantify the amount of violation needed to obtain the LMA solution for the solar neutrino problem.

We present our results in the following manner. The next section outlines general formalism we adopt and our assumptions. It also contains analytic discussion of neutrino mixing and masses in this scheme. The close link between large angle solar neutrino solution and flavour violation is emphasised in section (3) which also contains results based on numerical analysis. The last section contains a summary. Some of the technical aspects relevant to discussions in the text are elaborated in the appendices.
2 Sources of neutrino masses

In this section, we derive analytical conditions on the low energy susy parameters in order to have a phenomenologically consistent neutrino spectrum. To this extent we do not assume any specific structure of the soft masses. Further assumptions regarding various contributions to neutrino mass spectrum within mSUGRA inspired scenarios are discussed as and where required. The superpotential takes the following form in our case:

\[
W = h_{ij}^u Q_i u_j^c H_2 + h_{ij}^d Q_i d_j^c H'_1 + h_{ij}^e L_i^c e_j^c H'_1 + \mu' H'_1 H_2 + \epsilon_i L_i^c H_2. \tag{3}
\]

Without loss of generality, we have chosen above the basis in which the down quarks and charged lepton masses are diagonal. The \(\epsilon_i\) characterise lepton number violation in this basis.

We have the following soft supersymmetry breaking terms at the weak scale:

\[
V_{\text{soft}} = \frac{1}{2} |H_1^0|^2 + \frac{1}{2} |H_2^0|^2 + \frac{1}{2} |\tilde{\nu}_i^*|^2 - B \mu' H'_1 H_2 + c.c. - B \epsilon_i \left( \tilde{\nu}_i^* H_2 + c.c. \right) + \ldots \tag{4}
\]

Note that the above equation refers to soft terms at the weak scale. For simplicity we have displayed only the terms involving neutral fields in the above equation. The following comments are needed in connection with eq.(4):

(i) Although we have allowed for arbitrary diagonal sneutrino masses, we have not included off-diagonal sneutrino masses in this primed basis since such off-diagonal masses are severely constrained by flavour violating processes, e.g. \(\mu \rightarrow e\gamma\). (ii) \(V_{\text{soft}}\) does not contain sneutrino-Higgs mixing terms of the form \(m_{\tilde{\nu}_i H_1^0}\tilde{\nu}_i^{*} H_1^0\) although they are allowed by the gauge symmetry. Such terms are not present in the minimal supergravity theory at high scale. The renormalization group (RG) equations for \(m_{\tilde{\nu}_i H_1^0}\) given in the appendix, eq.(41) show that these terms cannot get generated even at the weak scale if they are not present at high scale. Thus it is meaningful to omit these terms. We should emphasise that this statement is very specific to the particular basis in which bilinear terms are not rotated away from the superpotential until the weak scale and neglect of such terms would not be justified in any other basis. In our case, the \(\tilde{\nu}_i^* H_1^0\)
term would make its appearance when we go to the basis with no bilinear \( R \) violating terms in the superpotential at the weak scale.

The neutrino masses arise from several sources in this model. Discussion of these sources becomes transparent if we re-express eq.(3) in the new basis in which bilinear terms are rotated away from \( W \):

\[
H_1 = \frac{\mu' H'_1 + \sum_i \epsilon_i L'_i}{\mu'}, \\
L_i = \frac{\mu' L'_i - \epsilon_i H'_1}{\mu'}.
\]

This basis are simple but are orthonormal only up to \( \mathcal{O}(\epsilon^2) \). This approximation is sufficient for most of our discussions since \( \epsilon_i \) are required to be much smaller than the typical SUSY scale \( \mu' \) in order to reproduce the scale of neutrino masses correctly. Generalisation of eq.(5) valid to higher order in \( \epsilon_i \) and its consequences are discussed in the appendix B. Eq.(3) takes the following form in the unprimed basis:

\[
W = h_{ij}^u Q_i u^c_j H_2 + h_{ij}^d Q_i d^c_j H_1 + h_{ij}^e Q_i e^c_j H_1 - \lambda'_{ijk} L_i Q_j L_k + \lambda'_{ijk} L_i d^c_j - \lambda^e_{ijk} L_i e^c_j + \mu H_1 H_2.
\]

where

\[
\lambda^e_{ijk} = \frac{\epsilon_i \mu}{\mu'} h^e_{ij} \delta_{jk}, \quad \lambda'_{ijk} = (\delta_{ik} h^e_{ij} \epsilon_j - \delta_{jk} h^e_{ij} \epsilon_i).
\]

Similarly, after rotating primed terms in eq.(3) and adding the contribution of the supersymmetric part, we get the following expression for the full scalar potential in the unprimed basis:

\[
V_{\text{scalar}} = (m_{H_1}^2 + \mu^2) |H_1|^2 + (m_{H_2}^2 + \mu^2) |H_2|^2 + m_{\bar{\nu}_i}^2 |\bar{\nu}_i|^2 + \Delta m^2_{\nu} \epsilon_i \epsilon_j \left( \bar{\nu}_i H_1^0 + \text{c.c} \right) + \frac{1}{8} \left( g^2 + g'^2 \right) \left( |H_1|^2 + |\bar{\nu}_i|^2 - |H_2|^2 \right)^2,
\]

\(^2\)Note that this definition of a new basis is same as that of Ref.\cite{16}. However in the present work, this rotation is done only at the weak scale in contrast to \cite{16} where it is scale-dependent.
where
\[ \Delta m_i^2 \equiv m_{\tilde{\nu}_i}^2 - m_{H^0_i}^2 \quad \Delta B_i \equiv B_i - B_\mu. \] (9)

Two major sources of neutrino masses arise from eqs. (7,8). Minimization of eq.(8) generates sneutrino vev:
\[ < \nu_i > \equiv \epsilon_i k_i \] (10)

where
\[ k_i \approx v_1 \frac{(-\Delta m_i^2 + \tan \beta \mu \Delta B_i)}{\mu (m_{\tilde{\nu}_i}^2 + 1/2M_Z^2 \cos 2\beta)} \] (11)

\( v_1 = < H_1^0 > \) and \( M_Z \) represents the Z boson mass. Sneutrino vevs lead to neutrino masses through their mixing with neutral-gaugino s:
\[ M_{\text{tree}} \equiv A_0 < \tilde{\nu}_i > < \tilde{\nu}_j > = A_0 \epsilon_i \epsilon_j k_i k_j. \] (12)

\( A_0 \) is obtained by diagonalizing the 7 \times 7 neutrino-neutralino mass matrix in the standard way \[ [17] \] :
\[ A_0 = \frac{\mu (g^2 + g'^2)}{2 (-c\mu M_2 + M^2_W \sin 2\beta (c + \tan^2 \theta_W))}, \] (13)

where \( \theta_W \) represents the Weinberg angle and \( M_W \) represents the W-boson mass. \( c \) is given by \( 5g^2/3g'^2 \sim 0.5 \) with \( M_2 \) representing the standard gaugino mass parameter.

The trilinear terms in eq.(7) lead to the second contribution to neutrino masses at 1-loop level. Since these couplings are proportional to the Yukawa couplings, the dominant contributions arise due to exchanges of the \( b \)-quark-squark and \( \tau \)-lepton-slepton in the loops. The loop induced mass matrix is of the form:
\[ (M_{\text{loop}})_{ij} = \epsilon_i \epsilon_j (A_b + A_\tau (1 - \delta_{i3})(1 - \delta_{j3})) \] (14)

where
\[ A_b = \frac{3}{16\pi^2} \frac{v_1}{\mu^2} \frac{M^2_{W_b}}{M^2_{H_b}} \sin \phi_b \cos \phi_b \ln \left( \frac{M^2_{H_b}}{M^2_{W_b}} \right), \] (15)
\[ A_\tau = \frac{1}{16\pi^2} \frac{v_1}{\mu^2} \frac{M^2_{W_\tau}}{M^2_{H_\tau}} \sin \phi_\tau \cos \phi_\tau \ln \left( \frac{M^2_{H_\tau}}{M^2_{W_\tau}} \right). \] (16)
Here $\phi_{b, \tau}$ denotes mixing between the left and the right handed squark (sneutrino) fields. These mixing angles are proportional to the $b$ and $\tau$ Yukawa couplings. Approximating them by $\frac{m_{b, \tau}}{m_{\text{susy}}}$ we get the following numerical values

\begin{align*}
A_0 & \approx 5 \cdot 10^{-3} \text{ GeV}^{-1}, \\
A_b & \approx 3 \cdot 10^{-10} \text{ GeV}^{-1}, \\
A_\tau & \approx 4 \cdot 10^{-12} \text{ GeV}^{-1}.
\end{align*}  

for $m_{\text{susy}} \sim 100 \text{ GeV}$. There are other loop contributions to neutrino masses and a complete discussion is given in [7, 13, 18]. We have retained here only those contributions which are known [18] to be dominant in case of mSUGRA and GMSB. The additional contributions not included in the text come from, (a) $R$ parity violating mixing of the charged sleptons with Higgs fields (b) sneutrino exchange diagrams through $R$-parity violating sneutrino-Higgs mixing and (c) loop contribution to the tree level neutrino neutralino mixing. These contributions are sub-dominant as long as the parameters $\Delta m^2_i, \Delta B_i$ are suppressed [18]. Such suppression is required purely from the phenomenological point as we argue below. It is then consistent to omit these sub-dominant terms for the analytical discussion that follows. We however discuss these additional contributions in the appendix B.

The total neutrino mass matrix is given by

\begin{equation}
(M_{\text{tot}})_{ij} = A_0 \epsilon_i \epsilon_j k_i k_j + \epsilon_i \epsilon_j (A_b + A_\tau (1 - \delta_{i3}) (1 - \delta_{j3})) .
\end{equation}  

The desired hierarchy among neutrino masses is automatically built in the above equations in view of typical numerical values of the parameters $A_{0,b,\tau}$. The tree contribution dominates over the rest (unless $k_i$ are enormously suppressed) but it leads to only one massive neutrino. Switching on the $b$-quark contribution gives mass to the other neutrino, one neutrino still remaining massless at this stage. The latter obtains its mass from somewhat less dominant contribution due to $A_\tau$. Note that hierarchy among the first two neutrino masses need not be very strong due to similar magnitudes of $A_{b,\tau}$. The above statements are made explicit below which also contains discussion on neutrino mixing.
2.1 Neutrino masses and mixing

The tree-level neutrino mass matrix can be easily diagonalized:

\[ U_0 M_{\text{tree}} U_0^T = \text{diag}\{0, 0, m_\nu^3\}, \]

(19)

where

\[ U_0^T = \begin{pmatrix} c_2 & s_2 c_3 & s_2 s_3 \\ -s_2 & c_2 c_3 & c_2 s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}, \]

(20)

with \( s_{2,3} = \sin \theta_{2,3} \) and

\[ \tan \theta_2 = \frac{\epsilon_1 k_1}{\epsilon_2 k_2} \quad \text{and} \quad \tan \theta_3 = \sqrt{\frac{\epsilon_1^2 k_1^2}{\epsilon_2^2 k_2^2 + \epsilon_3^2 k_3^2}}. \]

(21)

The total mass matrix eq. (18), assumes the following form in basis with diagonal tree mass matrix:

\[ U_0 M_{\text{tot}} U_0^T = \begin{pmatrix} a_1^2 (A_b + A_r) & a_1 (A_b a_2 + A_r b_2) & a_1 (A_b a_3 + A_r b_3) \\ a_1 (A_b a_2 + A_r b_2) & A_b a_2^2 + A_r b_2^2 & A_b a_2 a_3 + A_r b_2 b_3 \\ a_1 (A_b a_3 + A_r b_3) & A_b a_2 a_3 + A_r b_2 b_3 & A_b a_3^2 + A_r b_3^2 \end{pmatrix}, \]

(22)

where

\[ a_1 = \frac{\epsilon_1 \epsilon_2}{\omega_\perp} (k_1 - k_2), \]

\[ a_2 = \frac{\epsilon_3}{\omega_\perp \omega} (\epsilon_1^2 k_1 (k_1 - k_3) + \epsilon_2^2 k_2 (k_2 - k_3)), \]

\[ a_3 = \frac{1}{\omega} (\epsilon_1^2 k_1 + \epsilon_2^2 k_2 + \epsilon_3^2 k_3), \]

\[ b_2 = \frac{\epsilon_3 k_3}{\omega_\perp \omega_\perp}, \]

\[ b_3 = -\frac{1}{\omega} (\epsilon_1^2 k_1 + \epsilon_2^2 k_2), \]

(23)

with

\[ \omega = (\epsilon_1^2 k_1^2 + \epsilon_2^2 k_2^2 + \epsilon_3^2 k_3^2)^{1/2}, \]

\[ \omega_\perp = (\epsilon_1^2 k_1^2 + \epsilon_2^2 k_2^2)^{1/2}. \]

(24)

The subsequent diagonalization can be approximately done if we neglect terms \( O(\frac{A_b}{A_{0 b}}) \). Let

\[ U_1^T = \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \]

(25)
where
\[
\tan 2\theta_1 = \frac{2a_1 (A_b a_2 + A_\tau b_2)}{A_b (a_2^2 - a_1^2) + A_\tau (b_2^2 - b_1^2)}.
\]
We then have
\[
U_1 U_0 M_{\text{tot}} U_0^T U_1^T = \begin{pmatrix}
m_{\nu_1} & 0 & 0 \\
0 & m_{\nu_2} & 0 \\
0 & 0 & m_{\nu_3}
\end{pmatrix} + \mathcal{O}(\frac{A_b, \tau}{A_0}).
\]
(27)
The eigenvalues are approximately given by
\[
m_{\nu_1} \approx A_\tau \frac{a_1^2(a_2 - b_2)^2}{(a_1^2 + a_2^2)},
\]
\[
m_{\nu_2} \approx A_b (a_1^2 + a_2^2),
\]
\[
m_{\nu_3} \approx A_0 \omega^2.
\]
(28)
The mixing among neutrinos is described by
\[
U \equiv U_0^T U_1^T = \begin{pmatrix}
c_2 s_1 c_3 & c_2 s_1 + c_1 s_2 c_3 & s_2 s_3 \\
-s_2 c_1 - s_1 c_2 c_3 & -s_2 s_1 + c_1 c_2 c_3 & c_2 s_3 \\
s_1 s_3 & -c_1 s_3 & c_3
\end{pmatrix}.
\]
(29)
Let us now discuss consequences of the above algebraic results.
(1) It follows from eqs. (17, 28) that the neutrino masses obey the desired hierarchy:
\[
m_{\nu_1} \lesssim m_{\nu_2} \ll m_{\nu_3}.
\]
(2) The neutrino masses relevant for the solar and atmospheric scales are respectively given by \(A_0 \epsilon^2 k^2\) and \(A_b \epsilon^2\) leading to
\[
\frac{\Delta_{\odot}}{\Delta_{\text{atm}}} \approx \left(\frac{A_b}{A_0}\right)^2 \frac{1}{k^4},
\]
where \(\epsilon, k\) represent typical values of \(\epsilon_i, k_i\). It follows that the ratio of the solar to atmospheric scales is independent of the \(R\) violating parameters \(\epsilon_i\) and depends upon the values of the soft parameters represented by \(k\). One typically needs
\[
\epsilon \sim 10^{-1} \text{ GeV} \quad ; \quad k \sim 10^{-3} - 10^{-4}
\]
in order to reproduce the scales correctly. This shows in particular that irrespective of details of the SUSY breaking the Higgs-slepton universality (corresponding to very small values of \(k\)) is unavoidable in this model if neutrino masses are to be correctly reproduced.
A particularly interesting limit would be the case where \( k \sim \mathcal{O}(1) \). This is what one expects in a truly non-universal regime of soft masses \(^\[19\]\). From the above, we see that in this limit, one can in fact choose \( \epsilon_i \sim \mathcal{O}(\text{MeV}) \) and generate neutrino masses of \( \mathcal{O}(\text{eV}) \). However only one neutrino would be massive in this scenario, with the other neutrinos being with extremely negligible masses and without any relevance for phenomenology.

If exact flavour universality were to hold between the first two generations then \( k_1 = k_2 \) (see eq.\((11)\)). In this case \( a_1 \) as defined in eq.\((23)\) would be zero leading to \( s_1 = 0 \) in eq.\((25)\). The \( s_1 \) is required to be large in order to obtain the large mixing angle solution and obtaining this solution would need very sizable departures from the flavour universality among the first two generations. We quantify these remarks in the next section.

### 3 Neutrino mixing and departure from flavour universality

We derived approximate expressions for the neutrino masses and mixing without any specific assumption on the soft symmetry breaking sector. The entire neutrino spectrum can be parameterized in terms of three \( \epsilon_i \) and three \( k_i \) of which \( k_i \) depend upon the soft SUSY breaking parameters. We now quantify the amount of flavour universality violations needed for obtaining the most preferred large angle solution to the solar neutrino problem. The following two parameters are introduced as a measure of universality violation:

\[
x = (k_1 - k_3)/(k_1 + k_3) \quad ; \quad y = (k_1 - k_2)/(k_1 + k_2)
\]

We regard \( x \) and \( y \) as independent parameters but restrict their variation to values between (-1,1) in the numerical analysis that follows.

The neutrino mixing is determined by the matrix \( U \) in eq.\((29)\). Due to hierarchical mass spectrum, the survival probabilities for the solar and atmospheric neutrinos approximately assume two generation form. The corresponding mixing angles \( \theta_\odot \) and \( \theta_{\text{atm}} \) are given in terms of elements of the mixing matrix \( U \) as follows:

\[
\sin^2 2\theta_{\text{atm}} \sim 4 \, U_{\mu\bar{\nu}_3}^2 (1 - U_{\mu\bar{\nu}_3})^2 \approx 0.8 - 1.0
\]
\[
\sin^2 2 \theta_{\odot} \sim 4 U_{e2}^2 U_{e1}^2 \approx 0.75 - 1.0 \\
\sin^2 \theta_{\text{CHOOZ}} \sim U_{e3}^2 \leq 0.01 ,
\] (32)

where numbers on the RHS correspond to the required values for these parameters based on two generation analysis of the experimental data \[3\].

We can convert the above restrictions on \(\theta_{\odot}, \theta_{\text{atm}}\) to restrictions on the mixing angles \(s_{1,2,3}\) entering the definition of \(U\). The CHOOZ result requires \(|s_2 s_3| \leq 0.1\) and the nearly maximal atmospheric mixing is obtained with \(|c_2 s_3| \approx \frac{1}{\sqrt{2}}\). This requires small \(s_2\) and large \(s_3\). The solar neutrino mixing angle defined in eq.(32) coincides with \(s_1\) in this limit. We thus need \(\sin^2 2 \theta_1 \sim 0.75 - 1\). Large value of \(s_1\) in turn needs sizable departure from flavour universality as will be argued in the last subsection.

The expressions for mixing angles and masses obtained in the last section can be used to approximately determine the allowed ranges of parameters \(k_i, \epsilon_i\) which explain the solar and atmospheric neutrino anomalies. We approximately need \(|s_2| \leq \sqrt{2} U_{e3}\) and \(|s_3| \approx \frac{1}{\sqrt{2}}\). This implies:

\[
\epsilon_2^2 k_2^2 \approx \epsilon_3^2 k_3^2 , \\
|\epsilon_1 k_1| \approx \sqrt{2} |U_{e3} \epsilon_2 k_2| .
\] (33)

The magnitude of \(\epsilon_3 k_3\) is then approximately fixed by the atmospheric mass scale:

\[
\epsilon_3^2 k_3^2 \approx \frac{m_{\nu_3}}{2A_0} \approx \frac{\Delta \text{atm}}{2A_0} ,
\] (34)

while the solar scale and mixing angle determines \(\epsilon_3^2\):

\[
\epsilon_3^2 \approx \left( \frac{\sqrt{\Delta \odot}}{2A_0 \cos^2 \theta_{\odot}} \right) \frac{(1 + x)^2(1 - y)^2}{(x - y)^2} .
\] (35)

Eqs.(33,34,35) allow us to express magnitudes of all \(\epsilon_i, k_i\) in terms of \(x, y\), approximately known \(A_{0,b}\) and the experimentally measurable quantities.

The solar neutrino mixing angle following from eq.(26) is given in the limit \(A_\tau \ll A_b\) by

\[
\tan^2 \theta_{\odot} \approx \tan^2 \theta_1 \approx \frac{4U_{e3}^2 y^2(1 - x)^2}{(x - y)^2} .
\] (36)

We have used eq.(33) in deriving the above relation. It is clear that large \(\theta_1\) require sizable departure from flavour universality, i.e. sizable \(y\). Moreover,
one typically needs \(|x - y| \approx 2|U_{e3}y(1 - x)|\) in order to obtain a sizable solar neutrino mixing angle.

We now numerically determine the region in the \(x, y\) plane needed to reproduce the required ranges in mixing angle and masses. We make use of eqs. (33-35) to determine the approximate input values of \(\epsilon_i, k_i\) in terms of the \(\Delta_\odot, \Delta_{atm}\) and \(x, y\). We allow input values to vary by varying \(\Delta_\odot, \Delta_{atm}\) over the experimentally allowed ranges. We also randomly choose \(x, y\) between -1 and 1. Through this procedure, we choose a set of \(1.5 \times 10^5\) different values for the input parameters \(\epsilon_i, k_i\). Then we numerically diagonalize the total neutrino mass matrix, eq.(18) for each of these values of \(\epsilon_i, k_i\) and determine a set of \(x, y\) values which correctly reproduces the allowed ranges of the solar and atmospheric neutrino parameters and lead to \(|U_{e3}| \leq 0.1\). We obtain about 2024 \(x, y\) values leading to the correct description of neutrino anomalies. These points in the \(x, y\) plane are displayed in Fig.(1). This figure, based on complete diagonalization clearly shows the features obtained through approximate formulas. All the allowed values of \(x\) and \(y\) are in the range \(-0.9 - -0.6\) and sizable departure from universality is clearly seen. Also most points satisfy approximate equality \(|x - y| \sim 2U_{e3}y\) needed to obtain large solar neutrino mixing angle. As an illustration, we give below a typical set of \(\epsilon_i, k_i\) which correctly reproduces all the parameters:

\[
\begin{align*}
\epsilon_3 & \sim 0.1 \text{ GeV} \quad ; \quad k_3 \sim 1.1 \cdot 10^{-3} \\
\epsilon_2 & \sim 0.031 \text{ GeV} \quad ; \quad k_2 \sim 3.5 \cdot 10^{-3} \\
\epsilon_1 & \sim 0.087 \text{ GeV} \quad ; \quad k_1 \sim 9.1 \cdot 10^{-4}
\end{align*}
\]  

Typically, one needs \(\epsilon_i \sim \mathcal{O}(10^{-1} \text{ GeV})\) and \(k_i \sim 10^{-3}\) as argued before.

Let us now compare above phenomenological restrictions with expectations based on specific framework like mSUGRA. In order to obtain correct neutrino masses one needs parameters \(k_i\) (11) to be suppressed, typically \(k \sim 10^{-3} - 10^{-4}\) as in eq.(30). The other constraint is that \(y\) should be \(\mathcal{O}(1)\). The \(k_i\) provide a measure of the Higgs-slepton universality violation. Typical value of \(k_i\) obtained in mSUGRA follows from eq.(1) and is in the range required from phenomenology. Thus mSUGRA provides a very good framework to understand neutrino mass hierarchy as has been demonstrated in number of papers through
detailed numerical calculations [7, 10, 13]. However mSUGRA would not be able to provide the required value of $y$. This can be seen as follows. Theoretically, $y$ can be approximately written using eq.(11) as follows:

$$y \approx \frac{\mu \tan \beta (B_1 - B_2) - (m_{\tilde{\nu}_1}^2 - m_{\tilde{\nu}_2}^2)}{\mu \tan \beta (\Delta B_1 + \Delta B_2) - (\Delta m_1^2 + \Delta m_2^2)}$$

(38)

where we have neglected terms of order $(\Delta m_i^2)^2$, $(\Delta B_i)^2$ etc. Within mSUGRA, $y$ is identically zero at the high scale as $B_1 = B_2$ and $m_{\tilde{\nu}_1}^2 = m_{\tilde{\nu}_2}^2$ due to the universal boundary conditions. At the weak scale, this universality condition is broken solely by RG evolution. In the limit of neglecting first two generation Yukawa couplings, $y$ is identically zero even at the weak scale. A rough estimate of parameters appearing in $y$ can be obtained by approximately integrating the RG equations, eqs.(42,43) given in appendix A. We see that

$$\frac{m_{\tilde{\nu}_1}^2 - m_{\tilde{\nu}_2}^2}{\Delta m_1^2 + \Delta m_2^2} \approx \frac{1}{6} \left(\frac{m_\mu}{m_b}\right)^2 \approx 10^{-4}$$

$$\frac{(B_1 - B_2)}{\Delta B_1 + \Delta B_2} \approx \frac{1}{6} \left(\frac{m_\mu}{m_b}\right)^2 \approx 10^{-4}$$

(39)

Together they would imply very small value for $y \sim 0$ instead of the required value of $O(1)$. Thus universal boundary conditions of mSUGRA cannot lead to a large mixing angle solution to the solar neutrino problem.

One need not consider a generic non-universal scenario to introduce universality violations. It is clear from the forgoing discussion that one only needs small Higgs-slepton universality violation as well as flavour violation of similar magnitude to obtain a large solar neutrino mixing angle. These violations can come from either non-universal slepton mass terms or from non-universal B-terms or both. It is possible that such a scenario can arise from a higher theory of flavour either based on string theory [20] or through abelian [21] or non-abelian [22] flavour symmetries. However, for a phenomenological understanding it is clear that the existence of Higgs-slepton and flavour universality violations at the high scale would lead to correct neutrino mass spectrum at the weak scale. Knowing the value of $x$ and $y$ required for a correct neutrino spectrum at the weak scale, it is possible to estimate the amount of non-universality
required at the high scale. For example, using $y$, we have in the limit of neglecting contributions from $\Delta B$ terms, the required slepton flavour universality violations to be of order:

$$m_{\tilde{\nu}_2}^2(0) - m_{\tilde{\nu}_1}^2(0) \approx y(m_{\tilde{\nu}_2}^2(0) + m_{\tilde{\nu}_1}^2(0)) + 2 y \frac{m_{\tilde{\nu}_2}^2(0) m_{\tilde{\nu}_1}^2(0)}{m_{H_1}^2(0) + \delta m_{H_1}^2}$$

where $\delta m_{H_1}^2$ represents the correction to the high scale Higgs mass due to RG scaling. From the above we see that for a large negative $y$, $m_{\tilde{\nu}_1}^2(0)$ should be at least a factor of 3 times larger than $m_{\tilde{\nu}_2}^2(0)$. Introducing such a non-universality at the high scale would lead to the correct neutrino spectrum at the weak scale.

A similar analysis can also be considered for the B-terms [9]. Moreover, such a pattern can be incorporated naturally in non-minimal models of GMSB [23]. In these models, identical gauge quantum numbers of all sneutrinos assure almost universal sneutrino masses at the weak scale as in the case of mSUGRA. In contrast, there is no natural reason within these models for the flavour universal $B$ parameters. In fact, the $B$ parameters are assumed to vanish in the minimal version of the scheme [24, 25]. Thus the universality of $B$ parameters at supersymmetry breaking scale holds by default. It is possible to choose non-universal and non-zero $B_{1,2}$ terms to start with in this model. This does not significantly influence the conventional phenomenology of the minimal version as long as the parameters $\epsilon_i$ are much smaller than the $\mu$-parameter in the superpotential. But it allows the LMA solution as has been demonstrated through a detailed numerical work [9].

From the above discussion we see that the phenomenological requirement of $k \sim O(10^{-3})$ and $x, y \sim O(1)$ leads to a specific class of non-universality at the high scale. A generic non-universal soft spectrum might lead to much larger class of solutions. Such an analysis is beyond the scope of present work.

4 Comments

Supersymmetric model with bilinear $R$ parity violations provides a potentially interesting framework to study neutrino masses and mixing. The dominant sources of neutrino masses can be parameterized in this scenario in terms of three dimensionful parameters $\epsilon_i$ and three dimensional parameters $k_i$. The $k_i$
depend on the structure of soft supersymmetry breaking terms at the weak scale. We have tried to obtain phenomenological restrictions on $\epsilon_i$ and $k_i$ without making specific assumptions on the values of the soft supersymmetry breaking parameters. While neutrino masses can be suppressed by lowering the overall scale $\epsilon_i$ of $R$ parity violation, phenomenologically preferred hierarchy in neutrino masses require that both $\epsilon_i$ and $k_i$ are suppressed, see eq.(30). $k_i$ provide a measure of the Higgs-slepton universality and suppression in their values indicate very small amount of this violation. Such violation of universality is already built in the mSUGRA and GMSB scenario.

A large solar neutrino mixing angle can be obtained consistently within these scenarios only if flavour universality violations in the soft parameters of the first two generations are almost as large as the violation of Higgs-slepton universality. This feature does not emerge in models where these universality violations are generated solely by RG scaling as in the case of mSUGRA. Thus mSUGRA seems more suitable to describe the less preferred small mixing angle solution to the solar neutrino problem.

We concentrated throughout on the most dominant sources of neutrino masses in this theory. This is a good assumption in case of small universality violation. The other sources of neutrino masses would become important in case of large universality violation. It is not unlikely that these contributions could also lead to a large solar neutrino mixing angle in such scenarios.

Another way to achieve the Large Mixing Angle solution is to consider some \[26, 27\] or all \[28\] of the dimensionless lepton number violating couplings ($\lambda$, $\lambda'$ ) to be present in the superpotential. Several features of the neutrino mass spectrum like hierarchy and large mixing are still preserved within these models making them phenomenologically viable. However, unlike the bilinear model considered in this work, these models are less constrained simply due to the large number of additional couplings present in the model. This can lead to the large solar neutrino mixing angle even without relaxing the universality constraints on the soft spectrum \[26, 28\].
5 Appendix A

The renormalization group equations for various parameters appearing in the soft scalar potential are basis dependent. We have chosen a specific basis in which bilinear terms in the potential are kept in the superpotential till the weak scale. These terms are rotated only after evolving to weak scale. We collect here RG equations for relevant parameters with this specific choice. They differ for example from the ones derived in [18] where relevant rotation is performed at each scale. The following equations follow in a straightforward manner from the formalism given by Falck[29]:

\[
\frac{d}{dt} m_{\nu_i H'_1}^2 = m_{\nu_i H'_1}^2 (-\frac{1}{2}Y_i^E - \frac{1}{2}Y_\tau - \frac{3}{2}Y_b), \tag{41}
\]

\[
\frac{d}{dt} (\Delta m_i^2) = 3Y_b (m_{Q3}^2 + m_{D3}^2 + m_{H'_1}^2 + \tilde{A}_b^2) - Y_i^E (m_{L_i}^2 + m_{E_i}^2 + m_{H'_1}^2 + \tilde{A}_i^E)^2, \tag{42}
\]

\[
\frac{d}{dt} (\Delta B_i) = \tilde{A}_i Y_\tau + 3Y_b \tilde{A}_b - 3Y_i^E \tilde{A}_i^E. \tag{43}
\]

In the above, we have used standard notation for all the soft parameters appearing in the equations with the exception of \( \tilde{A} \) which represents the soft trilinear couplings.

6 Appendix B

In this appendix we justify the neglect of additional contributions to neutrino masses not included in the main text. We also discuss flavour violating processes \( \mu \to e\gamma \) and show that the corresponding branching ratio is very small in the present context.

Detailed analysis of the additional 1-loop diagrams contributing to neutrino mass matrix has been done in [7, 13, 18]. While Refs. [7, 13] calculate all the 1-loop self-energy diagrams to the \( 7 \times 7 \) neutrino-neutralino mass matrix and re-diagonalise it, Ref.[18] follows the effective mixing matrix approach. In addition to the contributions considered in the text, large contributions are also expected from diagrams which are not Yukawa suppressed, thus involving only gauge vertices. These can be visualized as diagrams with two R-parity violating mass insertions proportional to \( \Delta m_{\tau_i}^2, \Delta B_i \) as given in eq.(8), with neutralino...
(chargino), sneutrino (charged slepton) and neutral Higgs (charged Higgs) in the loops [12, 18]. Typical magnitude of these diagrams is given by

\[ M_{ij}^\lambda \approx \frac{g^2}{16\pi^2} \epsilon_i \epsilon_j k'_i k'_j m_{\text{susy}}^{-1} \]  

with

\[ k'_i \approx c_1 \Delta m^2_i + c_2 \Delta B_i^2 \]

\[ \frac{m_{\nu_i}}{m^2_{\text{susy}}} \]

\[ m_{\text{susy}} \] is a typical supersymmetry breaking scale and \( c_{1,2} \) are coefficients of order one following from the scalar mass matrices of the model. \( k'_i \) are similar to parameters \( k_i \) defined in eq.(11). It is natural then to choose \( k'_i \sim \frac{\mu_1}{v_1} k_i \) for order of magnitude estimates. Comparing the 1-loop gaugino contribution with the \( b \)-quark contribution (eq.(14)) \( M^b \) we obtain

\[ \frac{M_{ij}^\lambda}{M_{ij}^b} \approx \frac{g^2}{16\pi^2 A_b} \left( \frac{\mu}{v_1} \right)^2 \frac{k_i k_j}{m_{\text{susy}}} \]  

The numerical value of \( A_b \) is given in eq.(15). As argued above, we typically need \( k_i \sim 10^{-3} \sim 10^{-4} \). It is seen that the \( b-\text{quark} \) contribution retained in the main text dominates over the gaugino contribution in this case and it is consistent to neglect the latter. The other contributions to neutrino masses are even less dominant than the gaugino contribution. They come from 1-loop diagrams with two Yukawa vertices. These can be seen as a) diagrams with \( \lambda \) and \( h_\tau \) vertices with a R-parity violating mass insertion in the internal line connecting charged slepton and charged Higgs, and b) diagrams with \( h_\tau \) couplings at both the vertices with two R-parity violating mass insertions proportional to the sneutrino vev. Both these sets of diagrams are suppressed by the \( \tau \)-Yukawa coupling. They have been analyzed in detail in Ref.[18] where it has been shown that they can become comparable in magnitude to \( A_\tau \) in large \( \tan \beta \) regions. However as we have seen earlier this contribution is always subdominant compared to the contribution from bottom Yukawa couplings, \( A_b \). Thus it is justified to neglect these contributions within the present analysis.

**Effects of Basis Rotation up to higher order in \( \epsilon \)**: We now generalize the basis [5] to higher order in \( \epsilon \) and discuss its consequences. Such generalization

\[ ^3\text{For a detailed discussion of the various diagrams in mass insertion approximation, see} \ [15] \]
becomes necessary for discussion of flavour violating transitions such as $\mu \to e\gamma$.

Eq. (3) can be re-rewritten as follows:

$$
\begin{pmatrix}
H_1 \\
L_1 \\
L_2 \\
L_3
\end{pmatrix}
= 
\begin{pmatrix}
1 - \frac{1}{2} \hat{e}^2 & \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\
-\hat{e}_1 & 1 - \frac{1}{2} \hat{e}_1^2 & -\frac{1}{2} \hat{e}_1 \hat{e}_2 & -\frac{1}{2} \hat{e}_1 \hat{e}_3 \\
-\hat{e}_2 & -\frac{1}{2} \hat{e}_1 \hat{e}_2 & 1 - \frac{1}{2} \hat{e}_2^2 & -\frac{1}{2} \hat{e}_2 \hat{e}_3 \\
-\hat{e}_3 & -\frac{1}{2} \hat{e}_1 \hat{e}_3 & -\frac{1}{2} \hat{e}_2 \hat{e}_3 & 1 - \frac{1}{2} \hat{e}_3^2
\end{pmatrix}
\begin{pmatrix}
H'_1 \\
L'_1 \\
L'_2 \\
L'_3
\end{pmatrix}, + \mathcal{O}(\hat{e}^3)
$$

(46)

where $\hat{e}_i = (\epsilon_i)/\mu$, $\hat{e}^2 = \hat{e}_1^2 + \hat{e}_2^2 + \hat{e}_3^2$. The $V_{soft}$ in eq. (4) assumes the form

$$
V_{soft} = m_{H_1}^2 |H_1^0|^2 + m_{H_2}^2 |H_2^0|^2 + m_{\tilde{\nu}_i}^2 |\tilde{\nu}_i|^2 + \Delta m_i^2 \hat{e}_i (\tilde{\nu}_i^* H_1^0 + c.c) + (-\mu B H_1^0 H_2^0 + c.c)
$$

$$
- \epsilon_i \Delta B \left( \tilde{\nu}_i H_2^0 + c.c \right) - \frac{1}{2} \sum_{i<j} \hat{e}_i \hat{e}_j (\Delta m_i^2 + \Delta m_j^2) (\tilde{\nu}_i^* \tilde{\nu}_j + c.c),
$$

(47)

where

$$
m_{H_1}^2 = m_{H_1}^2 (1 - \hat{e}^2) + m_{\tilde{\nu}_i}^2 \epsilon_i^2,
$$

$$
m_{\tilde{\nu}_i}^2 = m_{H_1}^2 \epsilon_i^2 + m_{\tilde{\nu}_i}^2 (1 - \hat{e}_i^2),
$$

$$
B = B_\mu (1 - \hat{e}^2) + B_\epsilon \hat{e}_i^2.
$$

(48)

The rotation has generated off-diagonal flavour violating sneutrino mixing terms at $\mathcal{O}(\hat{e}^2)$. Since these terms conserve lepton number, they do not directly contribute to the neutrino masses but lead to flavour violating transitions such as $\mu \to e\gamma$.

The rotation in eq. (46) induces mixing among the charged leptons which were diagonal to start with. Define the charged lepton mass matrix as

$$
L_i M_l e^c,
$$

then

$$
M_l =
\begin{pmatrix}
h_{11} d_{1i} & \hat{e}_1 \hat{e}_2 h_{2i} & \hat{e}_1 \hat{e}_3 h_{3i} \\
\hat{e}_1 \hat{e}_2 h_{1i} & h_{22} d_{2i} & \hat{e}_1 \hat{e}_3 h_{3i} \\
\hat{e}_1 \hat{e}_3 h_{1i} & \hat{e}_2 \hat{e}_3 h_{2i} & h_{33} d_{3i}
\end{pmatrix},
$$

(49)

where

$$
d_i \equiv v_1 (1 + \frac{1}{2} (\hat{e}_i^2 - |\epsilon|^2) + \epsilon_i^2 k_i - \epsilon_i^2 k_l, \\
f_i \equiv \frac{1}{2} v_1 + k_i.
$$
The $k_i$ appearing in above are defined in eq.(11) and they signify sneutrino vev contribution to the charged lepton mass matrix. As argued in the text, $k_i$ are required to be small $\sim (10^{-3} - 10^{-4})$ in order to account for the correct neutrino masses. It then follows that sneutrino vev contribution to each element in $M_l$ is suppressed compared to the corresponding contribution of $v_1$. Thus this contribution can be neglected while diagonalizing $M_l$ in any realistic theory.

Even after neglecting it, the $\mathcal{O}(\hat{\epsilon}^2)$ contribution does produce additional mixing among charged leptons that is not Yukawa suppressed. This is easily seen in the simplified case of two generation. The $2 \times 2$ version of the charged lepton mass matrix is obtained from eq.(49) by setting $\epsilon_3 = 0$. The following rotation on the basis $(e_1, e_2)$ is needed to diagonalize the charged lepton masses:

$$
\begin{pmatrix}
  e \\
  \mu
\end{pmatrix}
= \begin{pmatrix}
  1 & \frac{1}{2} \hat{\epsilon}_1 \hat{\epsilon}_2 \\
  -\frac{1}{2} \hat{\epsilon}_1 \hat{\epsilon}_2 & 1
\end{pmatrix}
\begin{pmatrix}
  e_1 \\
  e_2
\end{pmatrix}.
$$

(50)

($e, \mu$) here refers to the flavour basis. This additional rotation affects the neutrino mixing terms in eq.(47) which can be re-written in the flavour basis as

$$
V_{\text{soft}} = m^2_{H_1} |H_1^0|^2 + m^2_{H_2} |H_2^0|^2 + m^2_{\tilde{v}_1} |\tilde{v}_e|^2 + m^2_{\tilde{v}_2} |\tilde{v}_\mu|^2
+ \left( \Delta m^2_{1e} \tilde{v}_e^* H_1^0 + \Delta m^2_{2e} \tilde{v}_e^* H_1^0 + \text{c.c} \right)
- \left( \mu B H_1^0 H_2^0 + \text{c.c} \right) - \left( \epsilon_1 \Delta B_1 \tilde{v}_e H_2^0 + \epsilon_2 \Delta B_2 \tilde{v}_e H_2^0 + \text{c.c} \right)
- \hat{\epsilon}_1 \hat{\epsilon}_2 \left( \Delta m^2_{1e} + \Delta m^2_{2e} - m^2_{\tilde{v}_1} + m^2_{\tilde{v}_2} \right)(\tilde{v}_e^* \tilde{v}_\mu + \text{c.c}).
$$

(51)

One sees that there are no additional lepton number violating mass terms other than present at $\mathcal{O}(\hat{\epsilon})$. Thus discussion on additional contribution to neutrino masses just given remains unchanged. However, eq.(51) contains lepton conserving but flavour violating contribution proportional to $\tilde{v}_e^* \tilde{v}_\mu$. This can lead to process such as $\mu \rightarrow e \gamma$. The branching ratio for this process is given by

$$
BR(\mu \rightarrow e \gamma) = \frac{12\pi^2}{G^2_F m^2_\mu} |B|^2.
$$

(52)

In the present case, the amplitude $B$ arises due to insertion of the flavour violating sneutrino mass term given in the last term in eq.(51). This is approximately given by

$$
|B| \sim \frac{e^3 m_\mu \frac{1}{2} \epsilon_1 \epsilon_2}{16\pi^2 \frac{1}{2} m^2_\mu \mu^2 k}.
$$

(53)
where $k$ is a typical magnitude of $k_i$ and $m_{\nu}^2$ is sneutrino (mass)$^2$. As already argued, we need $\epsilon_i \sim 0.1$ GeV and $k \sim 10^{-3}$. Given this, last equation is seen to give very small contribution to $BR(\mu \to e\gamma) \sim \mathcal{O}(10^{-13} |\epsilon|^4|k|^2)$ which makes it unobservable in both present and future experiments.

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Figure 1: Allowed values of $x$ and $y$ for which all the neutrino oscillation constraints are satisfied. The input values of parameters are chosen in a way described in the text.