Scaling properties of cavity-enhanced atom cooling

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I. INTRODUCTION

For a large variety of atoms, laser cooling has proven to be an extremely successful and widely applicable method to generate fairly large ensembles of ultra-cold neutral atoms. Nevertheless, some intrinsic effects, like re-absorption of the spontaneously scattered light, optical pumping to unwanted states or dipole heating, limit its applicability and the final phase space density that can be achieved. It has been suggested that these problems could be partly cured by the help of high-finesse optical resonators to enhance the cooling process and make it more selective in frequency space as well as in position space. For single atoms moving in the field of only one or a few photons in high-Q microscopic resonators these results could largely be demonstrated experimentally. Although this nicely confirms the theoretical concepts and model approximations, the scaling towards larger particle and photon numbers as well as large interaction volumes is not entirely clear. Some initial results on the scaling properties of this cooling scheme were obtained for ensembles of well trapped particles, where a harmonic approximation for the trapping potential can be assumed. Neglecting cavity induced momentum diffusion, collective equations for the atomic motion could be derived from which the scaling properties with atom number and volume could be read off. Similarly in the very far detuned weak field case, where spontaneous emission and field induced atom-atom interaction can be neglected, estimates of the scaling properties could be derived from geometrical considerations.

The range of the validity of both of these approaches is, however, not entirely clear and does not cover a large parameter regime. In many practical cases neither the tight binding situation allowing a collective description of the properties of a whole ensemble, nor the weak field assumption treating each atom separately will be realized. In that case one cannot expect reliable answers from these models. Only a more general description bridging the gap from the single atom to the collective description and from a single photon to a coherent field with many photons should provide adequate answers on the efficiency and realizability of this cooling scheme.

In this work we use a quite general semiclassical approach which only relies on the assumptions of a weak saturation and a not too low temperature $T \gg T_{\text{recoil}}$, which should be true for most practical setups. The model as defined in section II consistently includes spontaneous emission, cavity decay, atom-atom interaction via the field, and realistic mode functions. For a single atom in a standing wave field simple analytic expressions for the final temperature and the cooling time are derived in section III. In a rescaled version these are then applied to the $N$-atom case and tested against numerical simulations in section IV. In section V we compare the results to standard Doppler cooling and discuss the possibility of cooling without spontaneous emission in section VI. Finally, in section VII we briefly discuss the scaling properties for the case of several degenerate cavity modes.

II. SEMICLASSICAL MODEL

We consider a dilute gas of $N$ identical two-level atoms of mass $m$, transition frequency $\omega_0$, and spontaneous emission rate $\Gamma$ interacting with $M$ modes of a high-finesse optical cavity with resonance frequencies $\omega_k$, cavity decay rates $\kappa_k$, and mode functions $f_k(x)$, $k = 1..M$, respectively. The mode functions are mutually orthogonal and fulfill the normalization condition $\int |f_k|^2 \, dx = V$, where $V$ is the mode volume. The spatially averaged single photon Rabi frequency is denoted by $g$. The cavity is driven by a pump laser of frequency $\omega_p$, the spatial overlap of the pump with the mode functions at the driven cavity mirror yields pump strengths $\eta_k$.

Generalizing previous work, we derive a semiclassical model of the system described by a set of coupled stochastic differential equations (SDEs) for the atomic positions $x_n$, momenta $p_n$, and mode amplitudes $\alpha_k$.
\[ dx_n = \frac{P_n}{m} \, dt, \quad (1a) \]
\[ dp_n = -U_0 \left[ \mathcal{E}(x_n) \nabla_n \mathcal{E}^* (x_n) + \mathcal{E}^* (x_n) \nabla_n \mathcal{E}(x_n) \right] \, dt \\
+ i \gamma \left[ \mathcal{E}(x_n) \nabla_n \mathcal{E}^* (x_n) - \mathcal{E}^* (x_n) \nabla_n \mathcal{E}(x_n) \right] \, dt \\
+ dP_n, \quad (1b) \]
\[ da_k = -\eta_0^* \, dt + i \left[ \Delta_k \alpha_k - U_0 \sum_n \mathcal{E}(x_n) f_k^* (x_n) \right] \, dt \\
- \left[ \kappa \Delta_k \alpha_k + \gamma \sum_n \mathcal{E}(x_n) f_k^* (x_n) \right] \, dt + dA_k, \quad (1c) \]

where \( \mathcal{E}(x) = \sum_k f_k (x) \alpha_k \) is the field amplitude at position \( x \), \( U_0 = \Delta_0 g^2 / (\Delta_0^2 + \Gamma^2) \) the light shift per photon \( (\Delta_0 = \omega_p - \omega_a) \), \( \gamma = \Gamma g^2 / (\Delta_0^2 + \Gamma^2) \) the photon scattering rate, and \( \Delta_k = \omega_p - \omega_k \) the detuning of the \( k \)th mode from the pump laser.

The interpretation of the various terms in eqs. (1) is rather simple. The two terms in eq. (1a) correspond to the dipole force and the radiation pressure force, respectively. Equation (1b) describes the pumping of the cavity modes, the frequency detuning from the pump laser shifted according to the coupling to the atoms, and the mode damping due to cavity decay and photon scattering by the atoms. \( dP_n \) and \( dA_k \) are white noise increments which can be characterized by a diffusion matrix \( D_{ij} = \langle dF_i dF_j \rangle \), where \( F_i = \{ P_n, A_k \} \). The diffusion matrix exhibits nontrivial cross correlations between momentum diffusion and cavity amplitude and phase fluctuations.

We may decompose the noise terms by writing
\[ dP_n = dP_n^{spont} + dP_n^{ind}, \quad (2a) \]
\[ dA_k^r = dA_k^{r, spont} + \sum_n dA_{k,n}^{r, ind}, \quad (2b) \]
\[ dA_k^i = dA_k^{i, spont} + \sum_n dA_{k,n}^{i, ind}. \quad (2c) \]

The first terms correspond to momentum and mode amplitude fluctuations due to spontaneous emission of photons into vacuum modes. The second terms describe the correlated momentum and field fluctuations, which occur when the \( n \)th atom scatters a photon among different cavity modes. The precise form of all noise terms is given in Appendix A.

The set of SDEs (1) is well suited for a numerical implementation. Note that the number of equations grows linearly with the number of atoms and modes, in sharp contrast to full quantum descriptions, which become exceedingly large for all but the simplest situations. The inclusion of the quantum noise terms, on the other hand, allows to obtain reliable results, for example, for steady-state temperatures and trapping times, which cannot be derived from a simple classical model.

Let us now specify the parameter regime we are concentrating on in this paper. We assume an optical cavity of sufficiently high finesse with large separation between the various eigenfrequencies. Then we can neglect all modes, which are not close to resonance with the pump field frequency. For a standing wave cavity the parameters can be chosen such, that only a single mode will fulfill this condition. For a ring cavity setup one should include at least the two degenerate counterpropagating running waves, while in a quasiconfocal or quasishperical setup many transverse modes can be involved in the dynamics. We will discuss these situations later in section VII. For the following, we will specialize the model to a one-dimensional standing wave setup, neglecting transverse atomic motion. Therefore, only a single mode function \( f(x) = \cos(kx) \) is considered and we will drop the subindex \( k \) in the notation for the field amplitude, detuning, and pump strength. Since the derivation of the SDE is based on an adiabatic elimination of the internal atomic degrees of freedom, we must ensure a small atomic saturation parameter, that is, \( s = |a|^2 g^2 / (\Delta_0^2 + \Gamma^2) \lesssim 1 \).

To this end we assume a large atomic detuning \( \Delta_0 \gg \Gamma \). In order to observe a significant effect of the atomic motion onto the cavity dynamics, we require \( \Delta \approx -\kappa \) and \( N |U_0| \lesssim \kappa \). Hence, for large atom numbers \( N \), we assume \( |U_0| \ll \kappa \). Although these parameters are known to give rise to cooling independent of the sign of the atomic detuning (and hence the sign of \( U_0 \)), we will in the following use negative values of \( U_0 \) since, in a full model, this additionally gives rise to three dimensional trapping.

### III. Analytical Results for a Single Atom

Before we discuss the results obtained from numerical simulations of the SDEs, we will briefly summarize the analytical results obtained earlier for a single atom in a single mode of a standing wave cavity and specialize these to the present parameters. As we will see later, this provides an excellent basis for the discussion of the multi-atom case.

The analytical results (2) were derived under the assumption of weak pumping \( \eta \) such that there is at most one excitation in the system. However, it can be shown that under the conditions required for the adiabatic elimination of the atomic excited state, these results also hold for higher photon numbers as long as \( s \ll 1 \). Thus, we may apply the formulas in the semiclassical limit discussed here. For atomic velocities \( v \) fulfilling \( kv < \kappa \) the force on a moving atom can be expanded to lowest order in \( kv/\kappa \) as
\[ F = F_0 + F_1 v, \quad (3) \]
where \( F_0 \) is the stationary dipole force from eq. (1) for a fixed atomic position, and \( F_1 \) is the linear friction coefficient,
\[ F_1(x) = -2k^2 \cos(kx)^2 \sin(kx)^2 \frac{\eta^2 U_0^2}{\kappa^4}. \quad (4) \]
Neglecting localization effects, the position-averaged friction force thus reads
\[ T_1 = -k^2 \eta^2 U_0^2 \frac{\eta}{4\kappa^4}. \]

Similarly, the averaged momentum diffusion coefficient is found to be
\[ \overline{\mathcal{P}} = k^2 \kappa \eta^2 U_0^2 \frac{\eta}{8\kappa^4} \]

and hence the steady-state temperature is
\[ k_B T = -\frac{\overline{\mathcal{P}}}{F_1} = \frac{\kappa}{2}. \]

Thus, we can get sub-Doppler cooling for \( \kappa < \Gamma \).

Finally, from the friction coefficient we obtain an estimate for the cooling time \( \tau_c \) defined from the exponential decrease of the kinetic energy \( E(t) \) of the form
\[ E(t) = \left[ E(0) - \frac{k_B T}{2} \right] \exp(-t/\tau_c) + \frac{k_B T}{2} \]

with
\[ \tau_c = \frac{m}{2 |\mathcal{P}|} = \frac{\kappa^4}{2 \eta^2 U_0^2 \omega_R^4}. \]

Here \( \omega_R = \hbar k^2/(2m) \) is the recoil frequency. Hence we get faster cooling for larger cavity field intensity and larger optical potentials per photon. Moreover, the cooling time is proportional to the atomic mass such that the scheme works best for relatively light atoms.

IV. COOLING OF MANY ATOMS

Let us now consider \( N \) atoms. As they all couple to the same mode, one might expect that in general they will perturb each other and interact via this mode [11–14]. In fact, for a few atoms and a few photons inside a high-finesse cavity such effects have recently been demonstrated experimentally [15]. In order to investigate the cooling properties of many atoms, we numerically integrate the SDEs (1) and average over many different realizations. Choosing a flat initial distribution of atoms in space and a large initial kinetic energy we calculate the time evolution of the kinetic energy. In the following we will concentrate on the steady-state temperature and, in particular, on the cooling time scale \( \tau_c \).

In Fig. 1 we plot the numerically obtained mean kinetic energy per atom obtained from numerical simulations of the SDEs averaged over 100 realizations (solid curves). The dashed curves are exponential fits. Lower curves: \( N = 1 \), upper curves: \( N = 10 \). The parameters are \( NU_0 = -0.6\kappa \), \( N^2 \gamma = 0.03\kappa \), \( \Delta = -0.6\kappa \), \( \eta = 3\sqrt{N}\kappa \), \( \kappa = 415\omega_R \).

FIG. 1. Time evolution of the mean kinetic energy per atom obtained from numerical simulations of the SDEs averaged over 100 realizations (solid curves). The dashed curves are exponential fits. Lower curves: \( N = 1 \), upper curves: \( N = 10 \). The parameters are \( NU_0 = -0.6\kappa \), \( N^2 \gamma = 0.03\kappa \), \( \Delta = -0.6\kappa \), \( \eta = 3\sqrt{N}\kappa \), \( \kappa = 415\omega_R \).

In Fig. 2 we plot the numerically obtained mean kinetic energy as a function of time for a single atom and for ten atoms. For the many atom case we rescale the parameters such that \( NU_0 \) and the optical potential depth (proportional \( U_0^2/\kappa^2 \)) are constant. We find that the curves are very well approximated by an exponential fit as in eq. 8 with cooling times \( \tau_c = 142\kappa^{-1} \) (\( N = 1 \)) and \( \tau_c = 1110\kappa^{-1} \) (\( N = 10 \)). For the given parameters, the analytical estimate given by eq. (9) yields \( \tau_c = 118.6\kappa^{-1} \) and \( 1186\kappa^{-1} \), respectively, which is surprisingly good. Therefore, the single atom cooling time appears to make sense for the many atom case. However, the operating conditions differ for varying atom numbers, according to the condition that \( NU_0 \) remains constant.

FIG. 2. Steady-state kinetic energies per atom (a) and cooling times \( \tau_c \) (b) for atom numbers from 1 to 100. Crosses show the numerical results for \( \Delta = -0.6\kappa \), circles for \( \Delta = -\kappa \). All other parameters are the same as in Fig. 1. The solid and dashed lines are linear fits to the numerical data.
The steady-state kinetic energies obtained from the exponential fits of Fig. 1 are $466\omega_R \,(N = 1)$ and $510\omega_R \,(N = 10)$. While these are approximately of the same size, they are significantly larger than the value of $\kappa/4 \approx 100\omega_R$ obtained from eq. (4). However, eq. (4) was derived assuming $U_0 \ll \kappa$ and a flat spatial distribution in the steady state. Both of these conditions are violated for the present parameters.

It should also be emphasized that an exponential fit as in Fig. 1 only works for sufficiently hot atoms where the friction force (4) can be replaced by its mean value and can therefore be treated as constant. For very cold and well localized atoms the position dependence of the friction must be taken into account. In fact, eq. (4) predicts that the friction force vanishes completely at the bottom of the potential wells. Hence, if in the course of the cooling process the localization increases, the time scale of the cooling increases and the kinetic energy no longer decreases according to an exponential law. For such parameters, a unique cooling time $\tau_c$ can only be deduced from the initial stages of the cooling while the atoms are still uniformly distributed in space.

For the same scaling of the parameters with $N$ as discussed above, we plot the steady-state kinetic energy and the cooling time for particle numbers from one to 100 in Fig. 3. We find an approximately constant final temperature and a cooling time proportional to $N$. Taking the scaling of $U_0$ with the particle number $N$ into account, these results agree with the predictions of the single-atom estimates of the previous section.

In Fig. 3, a different scaling of the system parameters is applied. We choose a small value of $U_0$ and keep all parameters constant while changing the atom number. For the steady-state temperature we obtain approximately constant values as long as $N|U_0| \lesssim \kappa/2 \,(N \lesssim 10)$. For $10 \lesssim N \lesssim 25$ the frequency shift induced by the atoms is of the order of the cavity detuning $\Delta = -\kappa$, which reduces the cooling efficiency and consequently increases the steady-state temperature. For even larger atom numbers the cavity is shifted into positive detuning where the friction force changes sign and the atoms are accelerated. In this limit no stationary state is achieved and no cooling time can be defined.

For the same parameters, Fig. 3(b) shows the cooling time for varying atom number. We see that $\tau_c$ slightly decreases with increasing $N$. Close inspection of the numerical data shows, however, that this decrease can be entirely explained by an increase of the cavity photon number according to the increase of $NU_0$, which shifts the cavity closer to resonance for larger atom numbers. Keeping the photon number constant, e.g., by adjusting the pump strength $\eta$, would thus give rise to a cooling time essentially independent of $N$.

Therefore, the numerical simulations show that for $NU_0 < \kappa$ the cooling time for an ensemble of $N$ atoms is of the same order of magnitude as the cooling time for a single atom. This suggests that the individual atoms in the cloud are cooled independently from each other, although they are all coupled to the same cavity mode.

![Fig. 3. Steady-state kinetic energies per atom (a) and cooling times $\tau_c$ (b) for atom numbers from 1 to 10. Parameters are $U_0 = -0.05\kappa$, $\gamma = 2.5 \times 10^{-4}\kappa$, $\Delta = -\kappa$, $\eta = 10\kappa$, $\kappa = 415\omega_R$.](image)

For the case of $NU_0 \ll \kappa$ (no effect of the atoms on the cavity mode) this has also been predicted by Vuletic and Chu [3]. The basic argument there is that the cavity enhances backscattering of photons by moving atoms according to the Doppler shift, similar to free-space Doppler cooling. We now see that the cooling of atoms is nearly independent of the atom number also in cases where the cavity photon number is significantly changed by the presence of the atoms ($NU_0 \lesssim \kappa$). We attribute this to the fact that, considering the motion of an individual atom out of the whole ensemble, only the change in the cavity field induced by that atom is correlated with the atomic motion, while the influence of all other atoms is uncorrelated and hence on average cancels. Note that this argument is based on the assumption of no correlation between the motion of different atoms. Thus, for deep (harmonic) optical potentials in the classical limit where all particles oscillate with the same frequency a completely different scaling with the atom number has been found [5]. For the parameters considered here, the anharmonicity of the sinusoidal potential implies a different oscillation frequency for different atoms. Together with the independent part of the momentum diffusion this ensures fast dephasing between the individual atomic
oscillations. Despite the fact that they commonly interact with the same field these mechanisms guarantee that the atoms can be considered as independent. Such decorrelation (phase space mixing) has also been shown to be important for stochastic cooling of atoms in optical traps [6,7], which has been suggested as another alternative to cool atoms without spontaneous emission.

V. COMPARISON WITH FREE-SPACE DOPPLER COOLING

Since we have seen that the single atom formulas of Sec. III provide a good approximation for the many atom case, too, we will investigate these in the following a bit further.

Let us first introduce the spatially averaged atomic saturation $s$ as

$$ s = |a|^2 \frac{g \cos(k_x)}{\Delta_a} \approx \frac{\eta^2 U_0^2}{4 \kappa^2 g^2}. \quad (10) $$

For the validity of the SDEs (9) we required $s \ll 1$. We can then rewrite the averaged friction force (9) in the form

$$ F_1 = -sk^2 \left( \frac{g}{\kappa} \right)^2 \quad (11) $$

and the cooling time as

$$ \tau_c = \frac{\kappa^2}{4sg^2 \omega_R^{-1}}. \quad (12) $$

Hence, for a given atom and a given saturation the maximum friction force and minimum cooling time $\tau_c$ is limited by the ratio of the atom-cavity coupling $g$ and the cavity decay rate $\kappa$. Cooling is most efficient if this ratio is large, that is, for high-finesse cavities.

For standard Fabry-Perot resonators [3] with mirror curvature larger than the cavity length $l$, the mode waist scales as $l^{1/4}$ and thus the mode volume $V$ as $l^{3/2}$. Since $g \propto 1/\sqrt{V}$ and $\kappa \propto 1/l$ we find $(g/\kappa)^2 \propto \sqrt{l}$. Thus, cavity enhanced cooling seems more favorable for longer cavities, which also would allow to include more atoms. However, this route is limited by the capture range of the cooling given by $kv < \kappa$. Thus, for a small cavity decay rate only already very cold atoms are subject to the cavity cooling force.

Let us now compare eqs. (11) and (12) with the corresponding results for free-space Doppler cooling [1]. This provides optimum cooling for $\Delta_a = -\Gamma$. The friction force is given by

$$ F_{1}\textit{Dopp} = -sk^2, \quad (13) $$

and the cooling time is therefore

$$ \tau_{c}\textit{Dopp} = \frac{1}{4s} \omega_R^{-1}. \quad (14) $$

We note that the only difference with the cavity-induced cooling is the absence of the factor $(g/\kappa)^2$. Hence, only in high-Q cavities with $g > \kappa$ the cooling scheme discussed here is faster than standard Doppler cooling. The achievable steady-state temperatures of the order of $\kappa/2$, on the other hand, can be significantly lower than the Doppler limit of $\Gamma/2$ even for cavities with $g < \kappa$ but at the expense of a smaller capture range as already discussed before.

VI. SPONTANEOUS ATOMIC DECAYS

Cavity-induced cooling has been suggested as a possible means to cool particles without closed optical transitions such as certain atomic species or molecules. The motivation behind this proposal is that the cooling scheme relies on induced internal transitions of the particle rather than on spontaneous emission of photons. On the other hand, equation (11) shows that for large atomic detuning and correspondingly small saturation, which is required for a small photon scattering rate $\gamma$, the cooling time also becomes very long. The figure of merit in this respect is the number of photons $N_{ph}$ which are spontaneously scattered per particle during one cooling time $\tau_c$. Only for $N_{ph} \approx 1$ effective cooling of, for example, a molecule becomes feasible.

Using the results of the previous section, we can easily obtain an estimate for this number from the definition $N_{ph} = 2s\tau_c$, yielding

$$ N_{ph} = \frac{\gamma \kappa^2}{2 \omega_R U_0^2} = \frac{1}{2} \frac{\Gamma}{\omega_R} \left( \frac{\kappa}{g} \right)^2. \quad (15) $$

Note that this result is independent of the pump strength $\eta$ and the atom detuning $\Delta_a$, and is entirely fixed by atom and cavity properties.

As in the previous section, it is interesting to compare this with the corresponding result for standard Doppler cooling given by

$$ N_{ph}\textit{Dopp} = \frac{1}{2} \frac{\Gamma}{\omega_R} \gg 1. \quad (16) $$

Again, the only difference is the missing factor of $(\kappa/g)^2$. Therefore, only for $g > \kappa$ the number of spontaneously emitted photons per cooling time is reduced with respect to free-space Doppler cooling. Despite the fact that cavity-induced cooling works at arbitrarily large atomic detuning, the increased cooling time counteracts this reduction of the scattering rate $\gamma$, such that the overall efficiency falls behind Doppler cooling for cavities of not sufficiently high finesse. For high-Q cavities, on the other hand, the reabsorption problem, which limits the achievable particle density in free-space laser cooling, is strongly reduced too, as a photon which has decayed through the cavity mirror cannot be reabsorbed [20].

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VII. MULTI-MODE CAVITIES

Let us now return to the more general situation where the optical cavity supports several (nearly) degenerate modes as described by the SDEs (1).

In a first step, we use the friction coefficient obtained for a single atom in a ring cavity [21] to derive analytical estimates analogously to those in sections III and VI for a single-mode standing-wave cavity. We find

\[ s = \frac{\eta^2 U_0^2}{4\kappa^2 g^2}, \quad k_B T = \frac{\kappa}{2}, \quad \tau_c = \frac{1}{2}\frac{\kappa^4}{\eta U_0^2\omega_R^2}, \quad N_{ph} = \frac{1}{4}\frac{\Gamma}{\omega_R^2} \left(\frac{\kappa}{g}\right)^2 \]

Hence, for the same mean atomic saturation we obtain the same steady-state temperature but only half the cooling time and only half the number of spontaneously scattered photons per atom compared to the single-mode cavity.

This suggests that using \( M \) degenerate optical modes, for instance in a near-confocal or near-planar setup, improves the efficiency of cavity-enhanced cooling by a factor of \( M \). In fact, numerical simulations of ten atoms and one to ten modes have confirmed this simple scaling. Therefore, a multi-mode cavity reduces the cooling time and the number of incoherently scattered photons per atom by a factor of \( M(g/\kappa)^2 \) compared to free-space Doppler cooling. Finally, it should be emphasized that in this case and using again \( \Delta \approx -\kappa \), cooling occurs if \( MN|U_0| \) (instead of \( N|U_0| \) for a single mode) is of the order of or smaller than \( \kappa \).

VIII. CONCLUSIONS

In this paper we have investigated the possibility of cooling a cloud of neutral particles by a cavity-enhanced cooling scheme. Numerical simulations of the stochastic differential equations describing the situation of many two-level atoms confined in a single-mode standing-wave optical resonator show that the friction force and therefore the cooling time is of the same order of magnitude as for a single atom for the same system parameters. However, the operating regime which is ideal for one atom cannot be used for many atoms, so that the best achievable cooling time increases approximately linearly with the atom number. The analytical results obtained earlier for that situation can thus be specialized to the parameter regime relevant for many-particle cooling and have been shown to agree well with the numerical results. Comparison with the analogous expressions for free-space Doppler cooling indicate that cavity-induced cooling is mainly favorable for high-finesse optical cavities fulfilling \( g > \kappa \). Numerical simulations also suggest that the cooling efficiency is increased in cavities which support many degenerate optical modes.

Using the parameters of the high-Q resonators used in recent cavity-QED experiments [21] one obtains spontaneously scattered photon numbers per cooling time as low as 5-10. Replacing the heavy rubidium or cesium atoms by lithium, for example, therefore yields photon numbers below one which would allow efficient cooling without spontaneous emission. Similarly, only the lightest molecules form possible candidates for this cooling scheme, with the further difficulty of the reduced molecular dipole moment between any given ground and excited state. In any case, according to the small size of these resonators only very few particles can be trapped and cooled simultaneously.

With larger cavities the efficiency of cavity-induced cooling falls behind that of Doppler cooling. However, these systems are well suited to demonstrate the cooling effect on large ensembles by itself. For example, assuming sodium atoms, an atomic saturation of \( s = 0.1 \), a cavity decay rate \( \kappa \) of the order of one MHz and a coupling constant \( g \) of about 100 kHz [23], cooling times of the order of a ms should be achievable.

An alternative way to reach small mode volumes combined with large finesse is the use of evanescent wave fields generated by total internal reflection inside microoptical devices. Examples range from optical microspheres, thin high index surface layers or optical band gap guides on surfaces (integrated optics). Such devices look rather promising for the realization of dissipative walls or cooling schemes in microtraps.

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APPENDIX A: CORRELATED MOMENTUM AND CA VITY FLUCTUATIONS

In the following we give the full expressions for the correlated noise terms introduced in Sec. II. The fluctuations induced by spontaneous emissions read

\[ dP^{\text{spont}}_n = k_0 \sqrt{2\gamma|\mathcal{E}(x_n)|^2} dW_n, \]
\[ dA_{k}^{\text{spont}} = \sqrt{\kappa/2} dW_{k}, \quad (A2) \]
\[ dA_{i}^{\text{spont}} = \sqrt{\kappa/2} dW_{i}, \quad (A3) \]

where \( k_0 = \omega_p/c \). The noise terms according to rescattering of photons within the cavity modes are given by

\[
\begin{pmatrix}
    dP_{n}^{\text{ind}} \\
    dA_{k, n}^{\text{ind}} \\
    dA_{k, n}^{\text{ind}}
\end{pmatrix} = \sqrt{2} \gamma \left( \sqrt{2} \gamma \right) \{ v_n e^{i\phi_n/2} \} dW_{+}^{n} + \sqrt{2} \gamma \{ v_n e^{i\phi_n/2} \} dW_{-}^{n},
\]

(A4)

where \( v_n \) and \( \phi_n \) are defined by

\[
v_n = \begin{pmatrix}
    \nabla_n \mathcal{E}(x_n) \\
    -if_k(x_n)/2 \\
    f_k(x_n)/2
\end{pmatrix}, \quad e^{i\phi_n} = \frac{\nabla_n \mathcal{E}^{*}(x_n)^2}{|\nabla_n \mathcal{E}(x_n)|^2}.
\]

(A5)

Here \( dW_n \) is a three dimensional, all other \( dW_x \) are one-dimensional real Gaussian stochastic variables of mean zero and variance one.