ON THE INCONSISTENCY BETWEEN COSMIC STELLAR MASS DENSITY AND STAR FORMATION RATE UP TO $z \sim 8$

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Received 2015 October 12; accepted 2016 February 3; published 2016 March 29

ABSTRACT

In this paper, we test the discrepancy between the stellar mass density (SMD) and instantaneous star formation rate in the redshift range $0 < z < 8$ using a large observational data sample. We first compile the measurements of SMDs up to $z \sim 8$. Comparing the observed SMDs with the time-integral of instantaneous star formation history (SFH), we find that the observed SMDs are lower than that implied from the SFH at $z < 4$. We also use the Markov chain Monte Carlo (MCMC) method to derive the best-fitting SFH from the observed SMD data. At $0.5 < z < 6$, the observed star formation rate densities are larger than the best-fitting one, especially at $z \sim 2$ where they are larger by a factor of about two. However, at lower ($z < 0.5$) and higher redshifts ($z > 6$), the derived SFH is consistent with the observations. This is the first time that the discrepancy between the observed SMD and instantaneous star formation rate has been tested up to very high redshift $z \sim 8$ using the MCMC method and a varying recycling factor. Several possible reasons for this discrepancy are discussed, such as underestimation of SMD, initial mass function, and evolution of cosmic metallicity.

Key words: cosmology: observations – Galaxy: stellar content

1. INTRODUCTION

Thanks to the development of telescopes, more and more important and accurate data are being obtained in almost all fields of astronomy. Multi-wavelength observations with Hubble and Spitzer Space Telescopes and large ground-based telescopes give us much information on high-redshift galaxies. This gives us a great chance to measure the stellar mass density (SMD) from galaxy surveys (Labbé et al. 2013; Grazian et al. 2015, hereafter G15; Santini et al. 2015) and instantaneous star formation rate density (SFRD) (Bouwens et al. 2012a, 2012b; Schenker et al. 2013) up to about $z = 8$. Because of the physical connection between the star formation history (SFH) and SMD, we expect that these two measured quantities should be consistent with each other.

By integrating the instantaneous SFH over redshift and making some correction for mass loss during stellar evolution through stellar winds and explosive processes (Renzini & Voli 1981; Woosley & Weaver 1995), we can get the predicted stellar mass-density history (SMH). Meanwhile, after the same correction for stellar evolution, the SFH can be obtained from the derivative of SMH with respect to redshift. As mentioned above, both the SFH and SMH can be measured independently. So one can compare them to check whether the instantaneous SFH and SMH are consistent with each other. However, in order to make a reasonable comparison, the SFH and SMH should be derived under same assumptions, such as initial mass function (IMF), metallicity, and dust correction. Furthermore, to get the unbiased SFH and SMH, we also need to obtain the correct calibration for the SFRD and SMD, respectively, because all of these factors can affect the final comparison result (G15; Hopkins & Beacom 2006; Wilkins et al. 2008; Madau & Dickinson 2014).

Numerous studies have been done to compare those two quantities, but their results are not consistent with each other. For example, some authors have found that there is a good agreement between the measured SFH and SMH (Madau et al. 1998; Fontana et al. 2004; Arnouts et al. 2007; Reddy 2011; Behroozi et al. 2013). Reddy (2011) found that the SFH and SMH were in general agreement if considering some systematic effects, such as the evolution of the UV luminosity function of a galaxy, stellar mass in UV-faint galaxies, and the variation of dust attenuation with luminosity. Behroozi et al. (2013) constrained the SFH and SMH based on the relation between stellar mass and halo mass and found that both SMH and SFH are consistent with observations. In contrast, other authors have found that there is puzzling disagreement between them (G15; Hopkins & Beacom 2006; Wilkins et al. 2008; Santini et al. 2012; Madau & Dickinson 2014; Leja et al. 2015; Tomczak et al. 2016). Hopkins & Beacom (2006, hereafter HB06) found that the SMH inferred from the best-fitting SFH is larger than the observed one for redshifts $z < 3$, and that the peak of the larger factor at redshift $z \sim 2$ is about four. Wilkins et al. (2008, hereafter W08) compared the SMD and SFRD for redshifts $z \leq 4$. They found that the SFH derived from the observed SMH is lower than the observed SFH and the discrepancy peaks at $z \sim 3$ by 0.6 dex. Madau & Dickinson (2014, hereafter MD14) have also found this discrepancy, although it is not so significant. They found that the inferred SMD history was larger than the observed one by only a factor of about 60%. However, these works used different stellar IMF assumptions. HB06 used the IMFs, noted as BG IMF and SalA IMF in HB06, given in Baldry & Glazebrook (2003). W08 defined a new IMF with low-mass slope of $-1.0$ ($0.1 M_\odot < M < 0.5 M_\odot$), and used this IMF in their work, while MD14 used a traditional Salpeter IMF with slope of $-2.35$. The choices of IMF can lead to some deviations in the comparison, which we will show in Section 5. Several explanations are proposed for this discrepancy, including underestimation of the stellar mass (Maraston et al. 2010; Bernardi et al. 2013; Courteau et al. 2014). Instead, some authors claimed that the observed SFRD is overestimated in UV and IR bands (Utomo et al. 2014) or FUV and U bands (Boquien et al. 2014). Alternatively, a possible evolution of stellar IMF will also affect the estimations of observed SFH.
and SMH (HB06; W08). Because this discrepancy is still under debate, we use a large quantity of observed SFRD and SMD data, including the latest observations, to reinvestigate this problem by using the Markov chain Monte Carlo (MCMC) method and considering the effect of stellar evolution in a more detailed way.

The structure of this paper is organized as follows. We will give an introduction to the SMD and SFH in Sections 2 and 3 respectively. In Section 4, we will introduce our method and give our result. Some potential causes for the discrepancy between observed SFH and SMH are discussed in Section 5. Finally, a summary will be given in Section 6. For simplicity, we assume the Salpeter IMF with index of $-2.35$ (Salpeter 1955) and solar metallicity as a universal metallicity in our work. The flat $\Lambda$CDM cosmology with $h = 0.7$ and $\Omega_m = 0.3$ is adopted.

2. THE SMD

The SMD $\rho_s(z)$ is the stellar mass in a unit co-moving cosmic volume at redshift $z$. It can be obtained by integrating the galaxy stellar mass function (GSMF) $\Psi(M)$ at a certain redshift,

$$\rho_s(z) = \int_0^\infty M \Psi(M) \, dM,$$

where $\Psi(M) \, dM$ represents the number of galaxies with mass between $M$ and $M + dM$ in a unit co-moving cosmic volume. In practice, we integrate the GSMF from $M_{\text{min}}$ to $M_{\text{max}}$ instead of $0$ to $\infty$, where $M_{\text{min}}$ and $M_{\text{max}}$ are the lower and upper limits of the stellar mass of galaxies. Generally, $M_{\text{min}}$ and $M_{\text{max}}$ are taken as $10^8 M_\odot$ and $10^{13} M_\odot$, respectively. In this work, all the SMD data from G15 are obtained by integrating their GSMF over the range $10^8 M_\odot < M < 10^{13} M_\odot$. However MD14 adopted a range of about $10^{9.5} M_\odot < M < 10^{13} M_\odot$ (see MD14 for more detail). Fortunately, this little difference in the lower mass limit does not make much difference to the observed SMD (MD14). Carefully, we also compare those observed SMD data in G15 and MD14 and find that the difference is less than 1% and can be neglected.

Fundamentally, the method of estimating the SMDs is to fit the observed galaxy spectral energy distributions (SEDs) with a library of template SEDs. Then we can obtain the optimal mass-to-light ratio $M/L$ of the galaxies. It should be noted that the IMF plays a very important role in estimating the stellar mass of a galaxy. It represents the number ratio of stars with a certain mass among a stellar population that includes all the stars formed at the same time. Usually, bright massive stars emit almost all the light of a galaxy while faint low-mass stars dominate its stellar mass, so the low-mass slope of the IMF affects the estimation of SMD. Meanwhile, there is a remarkable difference in the evolution of stars with different masses. Massive stars evolve faster and lose more mass than low-mass stars. Therefore, assumptions on IMF will affect the mass-to-light ratio $M/L$ of a galaxy as well as the recycling fraction of stellar mass, which means the mass fraction of each generation of stars returned to the interstellar medium through stellar wind, explosion or some other processes. All of these will bias the estimation of the stellar mass of a galaxy. For simplicity, a simple power-law IMF of Salpeter (1955) in the mass range $0.1 M_\odot$–$100 M_\odot$ is adopted in our work, although it is challenged by some observations. There are also some other IMFs used in previous works (Bastian et al. 2010), such as the Chabrier IMF (Chabrier 2003) and Modified Salpeter IMF (Baldry & Glazebrook 2003). The converting factor of SMD from one IMF to another can be obtained using a population synthesis code like PEGASE (Fioc & Rocca-Volmerange 1997) or FSPS (Conroy & Gunn 2010).

Metallicity is another important factor to affect the estimation of SMDs. A low-metallicity star evolves faster while a high-metallicity star will lose more mass through a strong stellar wind. Therefore, different metallicities will give different recycling fractions of stellar mass. Moreover since the average cosmic metallicity evolves with redshift or the age of the universe, we should, in principle, use different metallicities at different redshifts. But this is so complex that we just adopt the solar metallicity $Z_\odot = 0.02$ as the universal metallicity in our work, which is consistent with that of MD14. We will leave discussion of the systematic bias analysis of the metallicity assumption to Section 5.

Thanks to large galaxy surveys such as SDSS and 6dFGRS, and also some large telescopes such as HST, Spitzer, and VLT, SMD data can be accurately measured to $z \sim 8$. We choose 124 observed SMD data from previous literature over the redshift range $0 < z \leq 8$. Since the SMD data from different groups might be estimated in different IMFs, we should rescale them to the Salpeter IMF. Luckily, these SMD data scaled by the Salpeter IMF can be found in MD14 and G15. All of these data and their references are listed in Table 2.

3. THE SFH

Determining the cosmic SFH is a key problem in many fields of astronomy, such as the formation of galaxies and the evolution of cosmic metallicity. Many works have been done to measure the cosmic SFH using different methods. HB06 used UV and IR luminosities as tracers to measure the SFRD up to $z \approx 6$. There are also other tracers of the SFRD such as the $\text{H}\alpha$ line, radio, and X-ray emissions (for a review, see MD14). SFRs are usually measured from the typical information on very massive stars, since they have a very short life compared with the typical timescale for star formation. The UV emission of a newly formed stellar population is dominated by those massive stars, so it can be an instantaneous indicator of the SFRs (Kennicutt 1998; Salim et al. 2007; Haardt & Madau 2012; Schenker et al. 2013). Besides, since interstellar dust can absorb the UV emission from those massive stars and re-radiate at MIR and FIR wavelengths, IR observations can be another important indicator of the SFRs (Magnelli et al. 2011, 2013; Gruppioni et al. 2013). Generally, dust extinction is negligible in the FIR band while it is much more significant in the UV band, especially for those star-forming regions surrounded by dense clouds. Therefore, the correction of dust extinction in order to use the UV luminosity as SFRD indicator is very important. For the IR-band radiation, since the dust can also be heated by old low-mass stars or active galactic nuclei, it is not good enough to use IR luminosity to estimate the SFRD when the cosmic SFR is very small or for those galaxies with low SFR such as our Milky Way (Lonsdale Persson & Helou 1987). The IR luminosity becomes a robust indicator at $1 < z < 4$, where the larger SFR makes the newborn massive stars dominate the dust heating. However, IR detectors are not sensitive enough to measure the IR luminosity of high-redshift galaxies, while the UV emission can be measured more easily at $z > 1$ as it is redshifted to the optical
band. Therefore, combining UV and IR observations can give us a better estimation of the cosmic SFRD over the whole redshift range. What is more, since the life of massive stars is short, their death events can also be used as tools to measure the SFRs: for example, core-collapse supernovae (Dahlen et al. 2004; Li et al. 2011; Horiuchi et al. 2013) and long-duration gamma-ray bursts (Kistler et al. 2009; Wang & Dai 2009; Wanderman & Piran 2010).

Being the same as SMD, the estimation of SFRD also depends on the choice of IMF since those indicators can only trace the formation rates of massive stars. We need to factor the total SFR over the entire mass range based on an IMF assumption. MD14 chose the SFR data estimated from FUV and IR data based on the Salpeter IMF and gave the SFH up to $z \sim 8$ by fitting the observed data. It has the form

$$
\psi(z) = a \frac{(1+z)^b}{1 + [(1+z)/c]^d} M_\odot \text{yr}^{-1} \text{Mpc}^{-3}
$$

with the optimal parameters $(a, b, c, d) = (0.0166, 0.1848, 1.9474, 2.6316)$ considering dust extinction. The data without correction for absorption yield $(a, b, c, d) = (0.0, 0.0798, 1.658, 3.105)$. HB06 used a modified Salpeter IMF, denoted as SalA IMF, and gave the optimal parameters as $(a, b, c, d) = (0.0170, 0.13, 3.3, 5.3)$. In the following analysis, we will use both these two forms of SFH to remove the possible effect of the form of SFH. For the form in MD14, we simply adopt their optimal parameters. For the Cole form, we adopt the optimal parameters given in HB06. Because of the different choices of IMF, we use the factor 0.77 suggested in HB06 to convert the SFH to the Salpeter one.

4. METHODS AND RESULTS

The cosmic SMD at a certain redshift is the cumulative mass of all the stars formed at higher redshifts. Therefore the SMD $\rho_s(z)$ can be expressed by the integration of the SFH $\psi(z)$ as

$$
\rho_s(z) = (1 - R) \int_0^{z(z)} \psi(t') dt',
$$

where $H(z) = H_0 \sqrt{\Omega_m (1 + z)^3 + (1 - \Omega_m)}$ is the Hubble parameter in a flat $\Lambda$CDM cosmology, and the recycling fraction factor $R$ represents the mass fraction of each stellar population returned to the interstellar medium. This fraction factor can be obtained by using stellar population synthesis code. In previous work, MD14 used a constant fraction factor $R = 0.27$ while G15 used a value of 0.28.

The above equation is just an approximation of the actual mass recycling process because it is based on the assumption that the recycling process happens instantaneously. However, a new stellar generation would have returned only little mass while a generation formed at an early time would have returned more mass to the interstellar medium. Therefore, a more accurate equation should be expressed as (W08)

$$
\rho_s(z) = \int_0^{z(z)} \psi(t') (1 - f_r (t - t')) dt',
$$

where $f_r (t - t')$ is the mass fraction of the stellar generation formed at time $t'$ that has been returned to the interstellar medium at time $t$. If $f_r (t - t')$ is a constant, Equation (5) reduces to Equation (4).

To calculate $f_r (t - t')$, we consider the mass evolution of an instantaneously formed stellar population using the FSPS code (Conroy & Gunn 2010). This code can give the evolution of the current remaining stellar mass fraction, which means $1 - f_r$, of a simple stellar population after setting some necessary parameters. We choose the Salpeter IMF and solar metallicity while leaving other parameters as their default values in the code.3 Figure 1 shows the evolution of the current mass fraction $1 - f_r (t)$ of a simple stellar population. We can see that it has almost no mass loss within 1 Myr, then the fraction is up to about 0.27 at 14 Gyr. If we just adopt a constant recycling fraction factor $R = 0.27$, we will overestimate the effect of mass loss in stellar evolution. The choice of IMF and metallicity will affect the evolution of $f_r$, which will be discussed in detail in Section 5.

Given the form of SFH $\psi(z)$, we can predict the SMH with Equation (4) or Equation (5). Both MD14 and G15 used Equation (4), but their recycling fraction factors are slightly different, being 0.27 and 0.28 respectively. In our work, we use an evolving $f_r$ instead of a constant one (hereafter, $f_r$ represents the evolving recycling factor while $R$ represents the constant one). Since many previous works have been done to determine the evolution of cosmic SFH, we can use their results to predict the SMH with our $f_r$.

\footnote{For the parameters in the code, we use verbose = 0, which means that the Padova isochrones model is used. The parameter of the dust absorption model is dust_type = 0, which corresponds to the power-law attenuation model. For the dust emission model, add_dust_emission = 1 means the Draine & Li (2007) model. We consider the nebular continuum component, i.e., add_neb continuum = 1. More detailed information can be found in the manual for FSPS 2.5 code, which can be downloaded from https://github.com/c Congley/2016 April 1}
In the top panel of Figure 2, the green and blue solid circles with 1σ errors are the observed SMD data given in MD14 and G15. The black solid line represents the SMH predicted from observed SFHs with an evolving factor $f_i$ and the black dotted-dashed line represents the SMH predicted from observed SFHs with a constant factor $R$. The magenta line and the gray region are the inferred SMH from our best-fitting SFH from the observed SMD data and the 95% confidence region obtained with the MCMC method. Middle panel: the ratio of the predicted SMHs from observed SFHs with 1σ errors are the observed SMD data given in MD14 and G15. The black solid line represents the SMH predicted from observed SFHs with $f_i$ and constant $R$ at different redshifts; it is up to about 1.2 at $z = 8$. Bottom panel: the ratio between SMHs from observed SFHs in MD14 and our best-fitting SFHs from observed SMD data with evolving $f_i$; it peaks at $z \sim 1.5$ with a factor of about 2.

In Cole et al. (2001) and obtain the optimal parameters $(a, b, c, d) = (0.030, 0.058, 2.361, 2.707) \pm (0.007, 0.012, 0.256, 0.134)$, which are different from those of HB06. Cole et al. (2001) obtained the optimal parameters as $(a, b, c, d) = (0.0166, 0.1848, 1.9474, 2.6316)$ after considering dust extinction. The SFH is much lower if there is no dust absorption and the optimal parameters are $(a, b, c, d) = (0.0, 0.0798, 1.658, 3.105)$. The difference in these two cases is about a factor of three at $z \approx 2$. Our best-fitting SFH from the observed SMD data lies between the SFHs of those two cases. Comparing with the predicted SMHs from the best-fitting SFHs from observed SMD data, we find that the observed SFHs in both MD14 and HB06 overpredict the SMH. From these figures, we find that both instantaneous observed SFHs overpredict the SMH, and the SFH of MD14 gives a better prediction. Moreover, we also find that $f_i$ gives different predictions from a constant recycling factor, especially at high redshifts. From the middle panels of these two figures, we can find that the SMHs predicted by observed SFHs with $f_i$ are a little higher than those with $R$, and the larger factors are up to about 20% at $z \sim 8$ in both cases.

We also use the MCMC method to derive the best-fitting SFH from the observed SMD data. We choose the SFH form as Equation (2) used in MD14. And then we use Equation (5) and the MCMC method to fit the observed SMD data and obtain the optimal parameters. Our result is $(a, b, c, d) = (0.023, 1.66, 2.81, 3.67) \pm (0.003, 0.34, 0.33, 0.17)$, which is quite different from the best-fitting parameters $(a, b, c, d) = (0.015, 2.7, 2.9, 5.6)$ obtained from observed SFR data given in MD14. In Figure 2, the magenta solid line represents the SMH predicted from our best-fitting SFH from observed SMD data with the SFH form in MD14 and the gray region shows the 95% confidence region. We also use the SFH form in Cole et al. (2001) and find that both instantaneous observed SFHs overpredict the SMH, and the SFH of MD14 gives a better prediction. Moreover, we also find that $f_i$ gives different predictions from a constant recycling factor, especially at high redshifts. From the middle panels of these two figures, we can find that the SMHs predicted by observed SFHs with $f_i$ are a little higher than those with $R$, and the larger factors are up to about 20% at $z \sim 8$ in both cases.

From Figure 4, we find that the derived SFH from observed SMD data is much different from that given in MD14. Compared with the observed SFH of MD14, our best-fitting SFH is consistent with it at $z < 0.5$ and $z > 6$. But in the range $0.5 < z < 6$, our best-fitting SFH is lower. The lower factor reaches a peak of about two at about $z = 2$. For the SFH form of Cole et al. (2001), Figure 5 shows a similar result as Figure 4. Wilkins et al. (2008) found that the observed SFH is consistent with the SFH inferred from SMD at $z < 1$ but about four times larger at $z \approx 3$, which is different from ours. This may be caused by the different assumptions of IMF. Besides, Wilkins et al. (2008) only considered the SFH and SMD at $z < 4$. We consider a large redshift range up to $z \sim 8$ and find that the SFH inferred from observed SMD data is consistent with observed SFH at $z > 6$. 

Figure 2. The redshift evolution of SMH. We use the SFH form of MD14 and their best-fitting parameters in this case. Top panel: the green and blue circles with 1σ errors are the observed SMD data given in MD14 and G15, respectively. The black solid line represents the SMH predicted from observed SFHs with an evolving factor $f_i$ and the black dotted-dashed line represents the SMH predicted from observed SFHs with a constant factor $R$. The magenta line and the gray region are the inferred SMH from our best-fitting SFH from the observed SMD data and the 95% confidence region obtained with the MCMC method. Middle panel: the ratio of the predicted SMHs from observed SFHs between evolving $f_i$ and constant $R$ at different redshifts; it is up to about 1.2 at $z = 8$. Bottom panel: the ratio between SMHs from observed SFHs in MD14 and our best-fitting SFHs from observed SMD data with evolving $f_i$; it peaks at $z \sim 1.5$ with a factor of about 2.

Figure 3. Same as Figure 2 but using the SFH form of Cole et al. (2001) and the observed SFH given in HB06. In the bottom panel, the ratio peaks at $z \sim 2$ with a factor of about 4.
observed SMD data and the best-fitting SFH from observed SMD data and the 95% confidence region obtained with the MCMC method. The form of SFH used for MCMC fitting is given by MD14. Bottom panel: the ratio between our best-fitting SFH derived from observed SMD data and the best-fitting SFH in MD14.

Figure 5. Same as Figure 4 but the SFH form of Cole et al. (2001) is used for MCMC fitting. It shows a similar result to Figure 4.

5. DISCUSSION

In this work, we use a large observational data sample to test the discrepancy between the SMH and instantaneous SFH over the redshift range $0 < z < 8$. We find that there is a discrepancy between observed SMD and instantaneous SFH data. As mentioned above, we choose a single power-law Salpeter IMF and solar metallicity in our analysis for simplicity. However, the estimations of SMD and SFRD depend on the assumptions of IMF and metallicity. Therefore we discuss the possible effect of the choice of IMF and metallicity on our result and some other potential causes of this discrepancy in this section.

Generally, the high-mass slope of the IMF affects the estimation of SFRs since young massive stars in a newborn stellar population dominate the emission, especially the UV emission. The low-mass slope of the IMF affects the estimation of SMDs since old low-mass stars dominate the stellar mass of a galaxy. Therefore, a more top-heavy IMF will generate more massive stars, which emit more radiation, leading to a high ratio of luminosity to SFR. Then we will obtain a low SFH for a certain observed luminosity. Figure 6 shows the conversion factors of SFRD and SMD from the traditional Salpeter IMF to different IMFs. The top two panels, which are obtained from simple stellar populations, show the conversion factors of SFRDs in UV and IR luminosity respectively. The bottom two panels, which are obtained from complex stellar populations with a constant SFR, are for the SMDs. For the conversion factors of SFRD, we choose the value in the first 1 Gyr, while for factors of SMD we use the value after 1 Gyr since the ratios are roughly unchanged in those time periods. The rough conversion factors are listed in Table 1. These conversion factors are consistent with those given in previous literature (HB06, W08, MD14).

What is more, the choice of IMF can also affect the evolution of the recycling factor of a stellar population, since stars with different initial masses have very different evolutionary processes and mass-loss rates. The top panel in Figure 7 shows the dependence of the recycling factor on different IMFs. We find that, compared with the traditional Salpeter IMF, other IMFs with a heavy massive end or light low-mass end give larger recycling factors, which can lead to a lower predicted SMH from the observed SFH. The rough recycling factors of different IMFs are listed in Table 1. Considering the effects of the choice of different IMFs, the discrepancy between the observed SFRD and SMD will be relieved if we use the conversion factors obtained with UV luminosity. For example, if we consider the observed SFH given in MD14 and the conversion factor of UV luminosity, the IMFs of Chabrier (2003) and Kroupa (2001) will relieve the discrepancy at about the 40% level, while the IMF of Baldry & Glazebrook (2003) will relieve it even at about the 90% level. The IMF of Baldry & Glazebrook (2003) can decrease the discrepancy to about the 30% level even using the conversion factors obtained from IR luminosity.

Our current knowledge of the IMF remains remarkably poor (Kroupa 2002). Some observations suggest that the actual IMF may deviate from the Salpeter IMF (Davé 2008; van Dokkum 2003; Kroupa et al. 2013; Weidner et al. 2013). The redshift distribution of gamma-ray bursts can be explained by an evolving IMF (Wang & Dai 2011). There are also some alternative IMFs such as those of Kroupa (2002) and Chabrier (2003). However, some observations show that the IMF in the local universe was not drastically different from the Salpeter IMF over a wide range of environments (Weisz et al. 2015). So, more accurate and reliable observations and constraints on the IMF are needed for further study of the cosmic SFH and SMH.

Apart from the IMF, the cosmic metallicity is also an important factor for the estimation of SFH and SMH. The systematic effect of the metallicity assumption needs to be discussed. We use the FSPS code of Conroy & Gunn (2010) to calculate the evolution of $f_\text{c}$ and the UV and IR luminosities of stellar populations under different metallicities. The top panel in Figure 7 shows the dependence of $f_\text{c}$ on metallicity in the Salpeter IMF. We can see that the dependence is not significant, since $f_\text{c}$ changes by less than 3% while the metallicity changes by a factor of 100. The top two panels of Figure 8 present the UV and IR luminosities of different
metallicities under the Salpeter IMF for the model of simple stellar populations. The bottom two panels are for the model of complex stellar populations. From those figures, we can see that the dependence of UV and IR luminosities on metallicity is significant since it will change 1.5–4 times while the metallicity changes by a factor of 100. Considering that observations of the evolution of cosmic metallicity, such as those by Rafelski et al. (2012), suggested that the cosmic metallicity will less than 0.1 \(Z_\odot\) at \(z > 2\), this result suggests that it would be better to take the effect of the evolution of cosmic metallicity into account in further analysis.

From Figures 2 and 3 we can find that the observed SMD data are lower than the predicted value. Therefore, the SMD might be systematically underestimated. It is known that the mass of a galaxy is dominated mainly by its old low-mass stars but its luminosity is dominated by young massive stars, which will lead to the outshining problem. Maraston et al. (2010) studied the synthetic spectrum of a composite population formed with a constant SFR for 1 Gyr and found that the total spectrum was dominated by those stars formed in the latest 0.5 Gyr. The stellar mass of those galaxies with recent star formation will be systematically underestimated since the older stars are lost in the bright light of young stars. Therefore, we think that in the middle redshift range, such as 1 < \(z\) < 3, the metallicities under the Salpeter IMF for the model of simple stellar populations. The bottom two panels are for the model of complex stellar populations. From those figures, we can see that the dependence of UV and IR luminosities on metallicity is significant since it will change 1.5–4 times while the metallicity changes by a factor of 100. Considering that observations of the evolution of cosmic metallicity, such as those by Rafelski et al. (2012), suggested that the cosmic metallicity will less than 0.1 \(Z_\odot\) at \(z > 2\), this result suggests that it would be better to take the effect of the evolution of cosmic metallicity into account in further analysis.

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**Table 1**

| Power-law Slope (Unit of Mass: \(M_\odot\)) | Conversion Factor |
|------------------------------------------|------------------|
| IMF                                      | \(f_r\) | SFRD<sup>b</sup> | SMD |
| Salpeter                                  | 2.35 | 2.35 | 2.35 | 0.27 | 1 | 1 |
| Chabrier                                  | ... | ... | -2.3 | 0.44 | 0.57 | 0.61 |
| Kroupa                                    | 1.3 | 2.3 | 2.3 | 0.41 | 0.60 | 0.65 |
| Wilkins                                   | 1.0 | 2.35 | 2.35 | 0.41 | 0.65 | 0.65 |
| Baldry                                    | 1.5 | 2.15 | 2.15 | 0.47 | 0.42 | 0.58 |
| SIA                                       | 1.5 | 2.35 | 2.35 | 0.37 | 0.73 | 0.73 |
| SIB                                       | 1.5 | 1.5 | 2.35 | 0.46 | 0.59 | 0.59 |

Notes.

- \(f_r\) is measured at about 14 Gyr.
- For those IMFs with two conversion factors, the one in brackets is obtained by comparing the IR luminosity and the other is from UV luminosity. Where only one conversion factor is given, it means the factors from UV and IR luminosities are the same.
- The IMF of Chabrier (2003) is \(dn/dm \propto \exp\left(-\log m - \log 0.08\right)^2 / (2 \times 0.69^2) / m\) for \(m < 1 M_\odot\).
### Table 2

#### Stellar Mass Density Data

| Redshift Range | $\log (\rho/M_\odot \text{ Mpc}^{-3})$ | Reference |
|----------------|-------------------------------------|-----------|
| 0.05–0.22      | 8.78$\pm$0.07                     | Gallazzi et al. (2008) |
| 0.07           | 8.59$\pm$0.06                     | Li & White 2009 |
| 0.2–0.3        | 8.59$\pm$0.05                     | Moustakas et al. (2013) |
| 0.3–0.4        | 8.59$\pm$0.06                     |             |
| 0.4–0.5        | 8.55$\pm$0.08                     |             |
| 0.2–0.4        | 8.46$\pm$0.09                     | Bielby et al. (2012) |
| 0.4–0.6        | 8.33$\pm$0.03                     |             |
| 0.6–0.8        | 8.45$\pm$0.08                     |             |
| 0.8–1.0        | 8.42$\pm$0.06                     |             |
| 1.0–1.2        | 8.25$\pm$0.04                     |             |
| 1.2–1.5        | 8.14$\pm$0.06                     |             |
| 1.5–2.0        | 8.16$\pm$0.03                     |             |
| 0.0–0.2        | 8.75$\pm$0.12                     | Pérez-González et al. (2008) |
| 0.2–0.4        | 8.61$\pm$0.06                     |             |
| 0.4–0.6        | 8.57$\pm$0.04                     |             |
| 0.6–0.8        | 8.52$\pm$0.05                     |             |
| 0.8–1.0        | 8.44$\pm$0.05                     |             |
| 1.0–1.3        | 8.36$\pm$0.05                     |             |
| 1.3–1.6        | 8.18$\pm$0.07                     |             |
| 1.6–2.0        | 8.02$\pm$0.07                     |             |
| 2.0–2.5        | 7.87$\pm$0.09                     |             |
| 2.5–3.0        | 7.76$\pm$0.18                     |             |
| 3.0–3.5        | 7.65$\pm$0.14                     |             |
| 3.5–4.0        | 7.49$\pm$0.13                     |             |
| 0.2–0.5        | 8.55$\pm$0.08                     | Ilbert et al. (2013) |
| 0.5–0.8        | 8.47$\pm$0.08                     |             |
| 0.8–1.1        | 8.50$\pm$0.08                     |             |
| 1.1–1.5        | 8.34$\pm$0.07                     |             |
| 1.5–2.0        | 8.11$\pm$0.06                     |             |
| 2.0–2.5        | 7.87$\pm$0.09                     |             |
| 2.5–3.0        | 7.64$\pm$0.15                     |             |
| 3.0–4.0        | 7.24$\pm$0.20                     |             |
| 0.2–0.5        | 8.61$\pm$0.06                     | Muzzin et al. (2013) |
| 0.5–1.0        | 8.46$\pm$0.03                     |             |
| 1.0–1.5        | 8.22$\pm$0.03                     |             |
| 1.5–2.0        | 7.99$\pm$0.05                     |             |
| 2.0–2.5        | 7.62$\pm$0.11                     |             |
| 2.5–3.0        | 7.52$\pm$0.09                     |             |
| 3.0–4.0        | 6.84$\pm$0.20                     |             |
| 0.3            | 8.79$\pm$0.12                     | Arnouts et al. (2007) |
| 0.5            | 8.64$\pm$0.11                     |             |
| 0.7            | 8.62$\pm$0.08                     |             |
| 0.9            | 8.70$\pm$0.15                     |             |
| 1.1            | 8.51$\pm$0.08                     |             |
| 1.35           | 8.39$\pm$0.10                     |             |
| 1.75           | 8.13$\pm$0.13                     |             |
| 0.1–0.35       | 8.58                            | Pozzetti et al. (2010) |
| 0.35–0.55      | 8.49                            |             |
| 0.55–0.75      | 8.50                            |             |

### Table 2 (Continued)

| Redshift Range | $\log (\rho/M_\odot \text{ Mpc}^{-3})$ | Reference |
|----------------|-------------------------------------|-----------|
| 0.75–1.00      | 8.42                            |             |
| 0.5–1.0        | 8.63                            | Kajisawa et al. (2009) |
| 1.0–1.5        | 8.30                            |             |
| 1.5–2.5        | 8.04                            |             |
| 2.5–3.5        | 7.74                            |             |
| 1.3–2.0        | 8.11                            | Marchesini et al. (2009) |
| 2.0–3.0        | 7.75                            |             |
| 3.0–4.0        | 7.47                            |             |
| 1.9–2.7        | 8.10                            | Reddy et al. (2012) |
| 2.7–3.4        | 7.87                            |             |
| 3.0–3.5        | 7.32                            | Caputi et al. (2011) |
| 3.5–4.25       | 7.05                            |             |
| 4.25–5.0       | 6.37                            |             |
| 3.8            | 7.24                            | González et al. (2011) |
| 5.0            | 6.87                            |             |
| 5.9            | 6.79                            |             |
| 6.8            | 6.46                            |             |
| 3.7            | 7.30                            | Lee et al. (2012) |
| 5.0            | 6.75                            |             |
| 5.0            | 7.19                            | Yabe et al. (2009) |
| 8.0            | 5.78                            | Labbé et al. (2013) |

Data listed below are used in Grazian et al. (2015) and represented by blue dots in Figures 2 and 3:

| Redshift Range | $\log (\rho/M_\odot \text{ Mpc}^{-3})$ | Reference |
|----------------|-------------------------------------|-----------|
| 3.5–4.5        | 7.36                            | Grazian et al. (2015) |
| 4.5–5.5        | 7.20                            |             |
| 5.5–6.5        | 6.94                            |             |
| 6.5–7.5        | 6.90                            |             |
| 4.0            | 7.58                            | Duncan et al. (2014) |
| 5.0            | 7.39                            |             |
| 6.0            | 6.98                            |             |
| 7.0            | 6.86                            |             |
| 0.625          | 8.42                            | Tomczak et al. (2014) |
| 0.875          | 8.36                            |             |
| 1.125          | 8.24                            |             |
| 1.375          | 8.09                            |             |
| 1.75           | 8.09                            |             |
| 2.25           | 7.94                            |             |
| 0.6–1.0        | 8.45                            | Santini et al. (2012) |
| 1.0–1.4        | 8.28                            |             |
| 1.4–1.8        | 8.16                            |             |
| 1.8–2.5        | 8.18                            |             |
| 2.5–3.5        | 8.10                            |             |
| 3.5–4.5        | 7.91                            |             |
| 6.95           | 6.57                            | Labbé et al. (2010) |
| 0.225          | 8.68                            | Pozzetti et al. (2007) |
| 0.55           | 8.57                            |             |
SFR is so large that it will lead to a systematic underestimation of the SMDs because of the outshining problem. From Figures 4 and 5 we can see that, although the observed SMD data are lower than the SMD inferred from SFH at z < 0.5, this does not mean that the best-fitting SFHs are also lower than the observed ones. In Figures 4 and 5, the observed SFRs are much larger than the derived SFRs in the middle redshift range 0.5 < z < 6. The observed SFR is about twice the derived one at z ~ 2. So the discrepancy between SMD and SFH can also be caused by an overestimation of SFRs, especially in the middle redshift range 0.5 < z < 6. Cole et al. (2001) found that the SFH would be much lower if there were no dust absorption. The difference in these two cases is about a factor of three at z ~ 2. Our best-fitting SFH from observed SMD data lies between the SFHs of those two cases. This hints that if we overestimate the dust absorption effect, we will overestimate the observed SFRDs.

### 6. SUMMARY

In this article, we use a large observational data sample to test the discrepancy between the SMH and instantaneous SFH in the redshift range 0 < z < 8. At first, we integrate the observed SFH over redshift to obtain the predicted SMH. We find that the inferred SMH is overpredicted at redshift z < 4, especially for the observed SFH of HB06. Although the SMH derived from the observed SFH of MD14 makes a much better comparison, it still has the problem of overprediction by a factor of about two at redshift z ~ 1.5. Compared with the results of the case using a constant recycling factor R, the evolving f_d will give 20% more SMH at z ~ 8. Second, with the form of SFH in MD14, we use the MCMC method to fit the observed SMD data and obtain the optimal parameters (a, b, c, d) = (0.028, 1.88, 2.40, 3.69) ± (0.005, 0.47, 0.36, 0.29), which are remarkably different from the result in MD14. This result is shown in Figure 2. Figure 4 shows the comparison between the SFH derived from observed SMD data and the observed SFRs. Comparing with the observed SFH of MD14, we can see that our best-fitting SFH is consistent with it at z < 0.5 and z > 6. But in the range 0.5 < z < 6, our best-fitting SFH is lower and even only half the observed one at about z = 2. In order to remove the possible effect of the form of SFH, we perform the same analysis with the form of SFH in Cole et al. (2001). For this SFH form, Figure 5 shows a similar result to Figure 4. Wilkins et al. (2008) found that the observed SFH is consistent with the...
SFH inferred from SMD at \( z < 1 \) but about four times larger at \( z \approx 3 \), which is different from ours. Besides, Wilkins et al. (2008) only considered the SFH and SMD at \( z < 4 \), but we consider a large redshift range up to \( z \sim 8 \) and find that the SFH inferred from observed SMD data is consistent with the observed SFH at \( z > 6 \).

We discuss some systematic effects of the assumptions of IMF and metallicity. A top-heavy or bottom-light IMF can relieve the discrepancy between observed SFH and SMH. For example, if we consider the observed SFH given in MD14, the IMFs of Chabrier (2003), Kroupa (2001), and Baldry & Glazebrook (2003) will relieve the discrepancy at about the 40% to 90% level, respectively. We find that metallicity does not affect the evolution of recycling factor \( f_r \) significantly, only by about 3% for a change in metallicity by a factor of 100. However, the metallicity affects the UV and IR luminosities of stellar populations. We also discuss the effect of possible underestimation of SMD and overestimation of SFRD because of the outshining problem and the possible overestimation of dust extinction.

In order to solve the discrepancy between the observed SMH and SFH, we still need more accurate and reliable observations of the evolution of cosmic IMF and metallicity. The next generation of 30 m ground-based telescopes or space telescopes is awaited.

We thank an anonymous referee for useful suggestions and comments. We also thank A. Grazian for providing some SMD data used in this work. This work is supported by the National Basic Research Program of China (973 Program, grant No. 2014CB845800), the National Natural Science Foundation of China (grants 11422325 and 11373022), the Excellent Youth Foundation of Jiangsu Province (BK20140016), and the Program for New Century Excellent Talents in University (grant No. NCET-13-0279).

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Figure 8. The dependence of UV and FIR luminosities on age and metallicity. The top two panels are obtained from simple stellar populations, and the bottom two panels from complex stellar populations with a constant SFR.
