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Unsharp measurements and the conceptual problems of Quantum Theory

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Abstract

The paper emphasis the role of unsharpness in the body of Quantum Theory and the relations to the conceptual problems of the Theory

Key words: quantum measurement, unsharpness, effect, positive operator-valued measure
In a letter to Heisenberg (1921) Pauli wrote:

"One may view the world with a p-eye (an eye for the momentum space) and one may view it with a q-eye (an eye for the position space), but if one opens both eyes simultaneously, one than gets crazy."

This challenging affirmation is characteristic for a whole ideology raised on the structure of quantum mechanics. However, nowadays, this became an old-fashioned ideology; thus, one may reformulate Pauli sentence as:

"One may imagine viewing the world with a p-eye and one may imagine viewing it with a q-eye, but if one imagine opening both eyes simultaneously, one than imagine craziness."

I. Quantum mechanics in the sharp measurement interpretation

Quantum mechanics needed another mathematical representation from the point of view of measurement formalism. The states are represented by positive trace-class operators $\rho$ on a separable Hilbert space $\mathcal{H}$, while properties seemed to be represented by projector operators $p$ on the closed subspaces of $\mathcal{H}$. Defining the counterproperty $\tilde{p}$ as the orthogonal complement $p^\perp$ and identifying logical conjunction of properties with the projector on the intersection of the corresponding subspaces, one may see that all these properties are sharp ones: $p \land \tilde{p} = O$ [2].

For any state there are many self-adjoint operators, that may define it completely. By spectral theorem, to these self-adjoint operators correspond spectral measures whose ranges are the collections of properties (projectors) which, by the upper definition, defines an observable. Two projectors (sharp properties) are coexistent if and only if they commute:

$$p_1 \cdot p_2 = p_2 \cdot p_1$$

Also, in quantum mechanics one has to define the system as an ensemble of identically prepared physical (micro-)objects. For any complete observable there exist an experimental procedure which assigns to every projector from the range of the spectral measure a filter, so that an individual object, interacting with it, will pass or not; consequently the corresponding sharp property would be true, or false. Note that for another individual object the
answer need not be the same; when the answer is always true or always false the property is said to be real for that state.

Some projector $P_1$ may not belong to the range of the spectral measure of any other complete observable where $P_2$ is present. So, there exist non-coexistent properties in quantum mechanics. The filters corresponding to such non-coexistent properties cannot be built up together. This is the case of properties related to non-commuting operators, known as complementary observables. Consequently, the experimental procedures corresponding to two sharp complementary observables cannot be built up together simultaneously. Heisenberg had been the first who investigated the result of two consecutive non-coexistent measurements, and obtained an euristic relation (related to a similar one from the wave theory), which sounds like a relation obtained by independent means from the mathematical formalism of the quantum theory. The first is known as Heisenberg relation, while the later as Robertson’s [3]:

$$\Delta A \cdot \Delta B \geq \frac{\hbar}{2}$$

This similarity was, at that time, the single reason to assign the significance of imprecision in measuring an observable to the second order symmetric moment of it, i.e. $\Delta A, \Delta B$. Nevertheless, this is the real significance, but the sharp version of the quantum formalism was not able to produce the proof, and this was the standpoint for a lot of conceptual confusions.

Briefly, if one can "see" the world only by sharp properties, reality has to be split in complementary sub-realities, each giving us full access unless we totally disregard the other sub-realti(-es). Nevertheless, this kind of knowledge has to be complete, because one can fully find the state of the system. This claiming has challenged A. Einstein scientific realism who proved [4] that (sharp) quantum theory cannot be simultaneously: real, causal, local and complete[5]. Many scientists interpreted Einstein’s criticism as a conscription for finding a classical-type (hidden-variable) theory, which was intended to fill the presumable incompleteness of quantum theory. Nevertheless a theorem due to J.S. Bell [6] followed by experimental verifications [7] confirmed that predictions of quantum theory for sharp observables are in perfect concordance with physical reality, contrary to hidden-variable-classical-type theories.
II. Quantum mechanics in the unsharp measurement interpretation

There were many attempts to solve these problems by giving up the reality, causality or locality conditions, contrary to Einstein views, and to common scientific sense, too. By the upper presentation, it is clear that one has another solution for Einstein incompleteness problem, i.e., giving up the sharpness condition. Indeed, this renunciation is a quite natural one: by the M. Born interpretation, the sole quantity subject to direct experimental verification is the probability:

\[ w = Tr(\rho \cdot p) \]

Of course is subject to the condition:

\[ 0 \leq w \leq 1 \] (1)

which corresponds in the set of the projectors to:

\[ 0 \leq p \leq I \] (2)

Relation (2) is a necessary and sufficient condition for (1), but is not a necessary condition for \( p \) to be a projector. Indeed, there exist more general operators, called effects, which obey relation (2) and can stand for generalized properties (E. Davies and C. Helstrom were the first who introduced this notion. Afterwards, the topic was rigorously grounded by G. Ludwig, and developed by E. Prugovecki, S. Gudder, etc.). One may define the counterproperty:

\[ \bar{p} = I - p \] (3)

This is not, generally, an orthocomplement, because the conjunction need not necessarily exist, and if it exist, it is not necessarily \( O[2] \). The operator \( \frac{1}{2} \cdot I \) is called semitransparent effect. The effects which are neither less than the semitransparent effect nor greater than it, are called regular effects. These regular effects correspond to the most general notion of property. The complementation (4) is a kind of orthocomplementation in the set of regular effects, i.e., regular effects are sharp ones in this sense (of course they are not real sharp ones). Now, one may define a complete unsharp observable by a
collection of effects, which completely defines the state. Two unsharp properties are coexistent if there exist a complete observable, which contains both of them (of course, it has to be an unsharp one). In general, there does not exist any self-adjoint operator corresponding to an unsharp observable. Indeed, instead of the spectral (projectorial) measure of the sharp observable, here one has a positive-operator-valued measure (POVM), which cannot define, by itself, a self-adjoint operator, but only a maximal symmetric operator.

A POVM $F$ can be obtained by smearing a projectorial measure $E$:

$$F(X) = \int_{\Omega} w(X, \lambda) \cdot dE_{\lambda}$$

where $\Omega$ is the space of the measurement outcomes, $X$ is a Borel set on and is a measurable function on for fixed $X$ and a probability measure on the Borelians class for fixed $\lambda$ [2]. The smearing operation induces some degree of uncertainty, which is due to the absence of sharply defined criteria of ascribing numerical values to the equivalence classes produced by measurement process. Also, POVM may come out in the theory of open systems and, consequently, in the theory of quantum measurements.

A theorem due to Neimark ensures the extension of any closed maximal symmetric $A$ operator acting on $\mathcal{H}$ to a selfadjoint one acting on $\hat{\mathcal{H}} \supset \mathcal{H}$ and, consequently, of any POVM to a projectorial one. However, this extension is not unique, which means that one cannot state the sharp version of Quantum Theory as a fundamental one, able to produce, via smearing processes, particular unsharp cases. By the contrary, reality is itself fundamentally unsharp, while the sharp situations are available only for gedanken-experiments.

The conceptual problems of Quantum Theory are, most of all, related to such gedanken-experiments. The genuine role of such "experiments" was to clarify the concepts of the new theory vs. those of the classical mechanics. (The later have been constructed by similar procedures; e.g., the idea of a material point moving without constraints comes from a gedanken-experiment invented by Galilei). Unfortunately, as we shown upwards, the expected conceptual clarification did not come.

It was stated upwards that from the formal theory of (experimental) measurements one yields the statistical character of the concept of system. In Classical Physics this does not make any harm, because all definable properties are mutually coexistent, so one can fully assign them to any individual object described by the system. By the contrary, in Quantum Physics this
cannot be done, so its concepts seem to bear a kind of incompleteness. Unfortunately, there exists a great reluctance against the idea of incompleteness of the scientific knowledge: a good theory has to offer a complete knowledge, even at risk of changing the meaning of the word ”complete”. This reluctance is closely related to the deep origins of the modern (experimental) physics, which stays in the Middle Age occidental monasteries. Their new spiritual perspective, contrary to the Orthodox Christian one, proclaimed (implicitly), for the first time in the human history, a kind of ”God’s death”: God is far away, and the world is all ours! So, we can handle it as we want, because God let us; from here, one yields the inquisitorial method, which later became the experimental method: the subject (of knowledge) owns a kind of pre-knowledge (some information about some witchcraft vs. a theory to be tested), while the object (of knowledge) has to obey to rules of the subject, i.e. to answer his pre-conceived questions (what kind of witchery he had done vs. what definite property it posses). Unsharpness is a way of sparing the completeness taboo, i.e. a way which preserves the experimental method (restricting to regular effects is very important!). Nevertheless, one may conceive a Physics beyond the experimental method, which is David Bohm's approach to a hidden variable theory [10]. But, unfortunately, Bohm’s approach is undermined by the same sin as the experimentist approach [11]. The latter imposes a verifiable unsharp structure to reality, but the method itself is not (meta-)verifiable, while the former tries to change the method (by changing the immutable linguistic form of Quantum Theory as a universal theory), making it (meta-)verifiable, but plays, unavoidable, with imposed unverifiable sharp structures.

The Occam’s razor tells us that unsharp measurement interpretation of quantum theory is, for the moment, the most appropriate way to understand fundamental physics these days.

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