Temperature characteristics modeling of Preisach theory

Hao Chen¹, Qifeng Xu¹*, Yukai Xiang¹ and Yifan Huang¹

¹College of Electrical Engineering and Automation, Fuzhou University, Fuzhou, China

Abstract. This paper proposes a modeling method of the temperature characteristics of Preisach theory. On the basis of the classical Preisach hysteresis model, the Curie temperature, the critical exponent and the ambient temperature are introduced after which the effect of temperature on the magnetic properties of ferromagnetic materials can be accurately reflected. A simulation analysis and a temperature characteristic experiment with silicon steel was carried out. The results are basically the same which proves the validity and the accuracy of the method.

1 Introduction

It is important to accurately grasp the magnetic properties of ferromagnetic materials for modeling and simulation of power transformers, voltage transformers and current transformers. However, as the hysteresis loop will be affected by temperature (in actual applications there is widespread temperature fluctuation in these transformers), it is necessary to analyze the influence of temperature on the hysteresis loop. The native Preisach operator is static and does not incorporate time or temperature dependent effects. Therefore, a modeling method of the temperature characteristics of Preisach theory is proposed in this paper in which the Curie temperature, the critical exponent, and the ambient temperature of magnetic material are introduced into the classical Preisach hysteresis model. This method can be used to simulate the magnetic properties of ferromagnetic materials at different temperatures to make up the defect that the classical Preisach theory can not simulate the temperature characteristics of ferromagnetic materials.

2 Preisach Hysteresis Model

According to the Preisach theory, magnetic material is composed of a large number of magnetic dipoles with the positive reversal threshold value \( \alpha \) and the negative reversal threshold value \( \beta \) of dipoles being statistically distributed [1-3]. The distribution density of the magnetic dipole is expressed by the non-negative two-variables function \( \mu(\alpha, \beta) \), which is called Preisach function as shown below:

\[
\mu(\alpha, \beta) = \begin{cases} 
0 & (\alpha > H_{sat} \text{ or } \beta < -H_{sat} \text{ or } \alpha < \beta) \\
\mu(\alpha, \beta) & (\alpha > -\beta, -\alpha)
\end{cases}
\]

(1)

The dipole magnetization characteristics described by the Preisach theory is shown in Fig.1. The region \( S^+ \) indicates that the dipole is in the +B state, and the region \( S^- \) indicates that the dipole is in the -B state.

![Figure 1. Magnetization characteristics of the dipoles of Preisach Model](image)

In the case where the magnetic material is not saturated, it is required to use the equation (2) to double integral calculation. Assuming the Preisach function variables can be separated [4], so:

\[
B_r \mu(\alpha, \beta) = \mu_\alpha(\alpha)\mu_\beta(\beta)
\]

(3)

The hysteresis loop equation can be derived under different conditions [5, 6]. If the ferromagnetic material is magnetized by the complete demagnetization state, it will run in the initial magnetization curve:

\[
F(H) = \int_0^H \mu(\alpha) \, d\alpha
\]

(5)

Then

\[
\int_{-H_{sat}}^H \mu(-\beta) \, d\beta = -F(H)
\]

(6)

If \((H_0, B_0)\) is the extreme point of the magnetization locus, the ascending and descending branches starting from the extreme point are:

\[
B'_{H_0, B_0}(H) = B_r(H) - B_r(-H_0) + B_0 + 2F(H_0)F(H), \quad H > H_0
\]

(7)

\[
B''_{H_0, B_0}(H) = B_r(H) + B_r(-H_0) + B_0 - 2F(-H_0)F(-H), \quad H < H_0
\]

(8)
Where:
\[ B_x(H) = -B_x(-H) \] (9)
\[ F(H) = \frac{B_x(H) - B_x(-H)}{2|B_x(H)|} \] \( H \geq 0 \) (10)
\[ F(H) = \sqrt{B_x(-H)} \] \( H < 0 \) (11)

Each branch is related to the function \( F(H) \), and \( F(H) \) is related to the function of descending branch \( B_d(H) \). Therefore, as long as the data are obtained, the hysteresis loops under various circumstances can be described by the equations (4)-(11).

3 Temperature Correction Method for Preisach Model

3.1 Analysis of temperature characteristics of ferromagnetic materials

The \( M_S-T \) curves of saturation magnetization of ferromagnetic materials varies with temperature as shown in Fig. 2. It can be found that the saturation magnetization \( M_S \) decreases continuously with the increase of temperature. When the temperature increases to the Curie temperature the saturation magnetization \( M_S \) decreases to zero and the ferromagnetic material changes from ferromagnetic to paramagnetic [7].

![Figure 2. Variation of spontaneous magnetization with temperature](image)

Generally, the Curie temperature of ferromagnetic material is very high. For example, the temperature of silicon steel can reach 700°C. However, under the general operating temperature of ferromagnetic material it is difficult to reach the Curie temperature. Therefore, it is usually only considered that operating temperature of ferromagnetic material is lower than the Curie temperature. In this case, the ferromagnetic material is in a ferromagnetic state.

3.2 Temperature characteristics modeling of Preisach theory

The temperature characteristics of ferromagnetic materials are studied according to the Weiss theory and the mean field theory [8, 9]. Below using the Curie temperature, the critical exponent \( \gamma \) and the Curie temperature \( T_C \) are introduced to obtain the approximate expression of \( M_S-T \) curve:

\[
M_S(T) = M_S(0) \left( \frac{T_C - T}{T_C} \right)^\gamma \quad T < T_C
\] (12)

Where \( M_S(0) \) expresses the saturation magnetization of the magnetic material at absolute zero temperature (K). The temperature \( T_0 \) is taken as a reference temperature \((T = T_0 = 298K)\) and it is introduced into the equation (12):

\[
M_S(T_0) = M_S(0) \left( \frac{T_C - T_0}{T_C} \right)^\gamma \quad T < T_C
\] (13)

Simultaneously using equation (12) and (13), then eliminate \( M_S(0) \):

\[
M_S(T) = M_S(T_0) \left( \frac{T_0 - T}{T_0 - T_C} \right)^\gamma \quad T < T_C
\] (14)

The relation between saturation magnetization \( M_S \) and magnetic induction intensity \( B_s(T) = \mu_0[H_m + M_S(T)] \) is plugged into equation (2):

\[
B(H, T) = \mathcal{B}(H, T) = \int \mu_0[B_s(T)\mu(H, T)]d\alpha d\beta - \int \mu_0[B_s(T)\mu(H, T)]d\alpha d\beta
\] (15)

For the separated Preisach function only its descending branch is required, so it is easy to get:

\[
B_d(H, T) = B_d(H, T_0) = \frac{M(T) + H}{M(T_0) + H}
\] (16)

The modified Preisach model with the temperature correction can be obtained through plugging equation (14) and (16) into equation (4)-(11). By measuring the...
hysteresis loop at normal atmospheric temperature, the hysteresis loop can be obtained at any temperature below the Curie temperature.

3.3 Temperature parameters acquisition method

It is found that the two temperature parameters of the critical exponent $\gamma$ and the Curie temperature $T_C$ are key to this Preisach temperature model. There are various methods to obtain the Curie temperature, such as the $M_S$-$T$ curve method, the induction method, the initial permeability curve method, the magnetoresistance effect method and so on [7, 10]. At present, the $M_S$-$T$ curve method using physical property measurement system (PPMS) to measure $T_C$ is most accurate. This paper uses the $M_S$-$T$ curve method with the curve shown in Fig.2. When the $M_S$ in the $M_S$-$T$ curve is close to zero, the corresponding temperature is the Curie temperature $T_C$ [11, 12].

As for the critical exponent $\gamma$, two $M$-$H$ curves are measured respectively at the normal atmospheric temperature $T_0$ and another temperature $T$ experimentally, then the saturation magnetization $M_S(T_0)$ and $M_S(T)$ can be obtained. Finally, the initial critical exponent $\gamma_0$ can be obtained by importing $M_S(T_0)$ and $M_S(T)$ into the equation (14). There will be an error in this initial critical exponent, so the initial critical exponent should be further optimized through a successive approximation method [13] as shown in Fig.4.

4 Simulation and Experimental Verification

4.1 Experimental principle and parameters

The experiments to obtain the hysteresis loop using the same coil at different temperatures has been carried out with Z110 (cold-rolled silicon steel sheet) manufactured by Nippon Steel. The thickness of the silicon steel sheet is 0.23mm, the inner radius of iron core coil is 30mm, the outer radius of iron core coil is 70mm and the width of iron core coil is 40mm, as shown in Fig.5.

The software flow chart of successive approximation method to get the critical exponent is shown in Fig.4.

(a) Take the initial critical exponent $\gamma_0$ into the Preisach temperature model and the $M$-$H$ simulation curve can be obtained at temperature $T$.

(b) Calculate the mean square error $E$ of the simulated $M$-$T$ curve and the $M$-$T$ curve obtained by experiment at temperature $T$.

(c) Determine whether the mean square error is in the required error range $e_{\text{min}}$-$e_{\text{max}}$. If the error is too large, $\gamma_1=\gamma_0-\Delta \gamma$ will be substituted into step (1) for two iterations. If the error is too large, $\gamma_1=\gamma_0+\Delta \gamma$ will be substituted into the step (a) for the next iteration. Only when the mean square error is within error range, the critical exponent $\gamma$ which is suitable for the whole $M$-$H$ curve will be output.
By adjusting the transformer T to generate an input current to provide a magnetic field strength of $H$ the primary current $I$ can be measured by a current probe of oscilloscope (Tektronix A622), where the voltage $U_{R_s}$ on the resistor $R_s$ and the magnetic field strength $H$ have a positive proportional relation $H = N_1 \ast U_{R_s}/L_{Rs}$. The voltage probe of the oscilloscope (Tektronix TPP0100) is connected to two sides of the capacitor $C$ on the secondary side to measure the voltage $U_C$. $U_C$ and the magnetic induction intensity $B$ have a positive proportional relation of $B = (RCU_C)/(N_2A)$. In such a way the B-H curve can be obtained by oscilloscope [14, 15]. The temperature experiment only demands the iron core coil to be put into the temperature test box.

The temperature metrical method is pointed out in section 4.1 in which the $M_s$-$T$ curve of the silicon steel sheet is obtained by using PPMS-9T, and the Curie temperature $T_C$ is also determined. The initial critical exponent $\gamma_0$ is obtained by using the $M_s$-$T$ curve and the equation (14), then $\gamma_0$ would be further optimized through the method proposed in section 3.3 to get $\gamma$. The specific values of the relevant parameters identified by the material are shown in table 2.

| Parameters                          | Value  |
|-------------------------------------|--------|
| Reference temperature               | 298    |
| Curie temperature (K)               | 1104   |
| Critical exponent                   | 0.3536 |
| Mass density(kg/m$^3$)              | 7650   |
| Silicon content(%)                  | 3.2    |
| Saturation magnetization at room temperature(kA/m) | 1365   |

### 4.2 Experiment and simulation results

The working temperature of power equipment is generally much lower than the Curie temperature. Taking the normal atmospheric temperature of 25°C as a reference to the actual operating temperature of general power system equipment (The general operating temperature range of transformers is -15°C to 85°C), the simulation of the limit hysteresis loop is obtained by selecting the temperature points of -15°C, 0°C and 85°C, as shown in the Fig.7. In practical applications, the hysteresis loops in a limited temperature range (-15°C to 85°C) is considered to be approximately reversible when the temperature is close to the normal atmospheric one. The experimental and simulation studies are carried out on the basis of those conditions [16-18].

Figure 7. Hysteresis loop with different temperature

The simulation results are compared with the experimental ones as shown in Fig.8. Fig.8(a) shows the basic magnetization curve at normal atmospheric temperature. Fig.8(b) shows a comparison of experimental results with simulation results in the descending part of -15°C and 85°C. Fig.8(c) shows a comparison between the experiment and the simulation at 0°C. Fig.8(d) shows the magnetization curves of saturated section at -15°C, 0°C, 25°C and 85°C respectively.

![Figure 7: Hysteresis loop with different temperature](image)

![Figure 8: Experimental results](image)

Magnetization decreases with the increase of temperature under the same magnetic field strength. The changing regularity in the experimental curve is consistent with simulation results and the experimental data fit well with simulation results.

### 4.3 Error analysis

The standard errors $\theta$ of simulation and experimental data at -15°C and 85°C are calculated respectively.

$$\theta = \sqrt{\frac{1}{n} \sum (X_i - \bar{X})^2}$$  

The error in the experimental results with reference to the simulation data is 3.214% at -15°C and 3.269% at 85°C. This means that the model proposed in the paper can reflect the influence of temperature on the hysteresis loop of ferromagnetic material very accurately. The causes of error is that the Preisach temperature model is only an approximate fitting one which can not with strict accuracy express the effect of temperature on the hysteresis loop.

### 5 Conclusions

In this paper the modeling method of Preisach theory with temperature characteristics is proposed, and the hysteresis loop model with temperature characteristics is built up. The Preisach model can be employed to
simulate the hysteresis loop at different temperatures by introducing the critical exponent, the Curie temperature and the ambient temperature. The comparison between experimental data and simulation results shows that the model is effective and accurate. The model is suitable for the coupling simulation of the magnetic properties and the temperature characteristics of electromagnetic equipment such as electric transformers.

References

1. Li Fuhua, Liu Dezhi, Chen Junquan, Wang Dong and Guo Yun. Identification of a Preisach Hysteresis Model With Lorentzian Function and Its Verification. Transactions of China Electrotechnical Society, 26, (2011), 1-7.
2. Zhao Guosheng and Li Langru. A Nonlinear Vector Preisach Model Considering Reversibility of the Hysteresis. Proceedings of the CSEE, 20, (2000), 4-10.
3. Li Zhen, Li Qingmin, Li Changyun, Sun Qiuqin and Lou Jie. Queries on the J-A Modeling Theory of the Magnetization Process in Ferromagnets and Proposed Correction Method. Proceedings of the CSEE, 31, (2011), 124-131.
4. S.R. Naidu. Simulation of the hysteresis phenomenon using preisach’s theory. IEE Proc-C, 137, (1990),73-79.
5. Zhang Xingang and Wang Zezhong. Research on CT Modeling Based on the Preisach Theory. Proceedings of the CSEE, 25, (2005), 68-72.
6. Zhao Yongfu. Research on Modeling of High-voltage Current Transformer and Effect of Its Saturation on Differential Protection. Master dissertation, North China Electric Power University, Baoding, (2009).
7. Xie Guangbin, Jin Xi, Lang Tong, Jiang Lei, Chen Xue-min and Cai Yongbin. The Research of Contact Feeling Temperature Indicator Based on Curie temperature for Transmission Line. Electrical Measurement and Instrumentation, 49, (2012), 59-63.
8. P. Andrei, A. Stancu, H. Hauser and P. Fulmek. Temperature, Stress and Rate Dependent Numerical Implementation of Magnetization Processes in Phenomenological Models. Journal of optoelectronics and advanced materials, 9, (2007), 1137-1139.
9. A. Arrott and J. Noakes. Approximate Equation of State for Nickel Near Its Critical Temperature. Phys. Rev. Lett., 19, (1967), 786-789.
10. Ji Song, Qian Kunning, Tan Suokui and Zhang Yansong. Development of Computer Aided Curie Temperature Metrical Instrument for Soft Magnetic Materials Applying Hopkinson Effect. Journal of Magnetic Materials and Devices, 36, (2005), 34-37.
11. B. R. Knight, J. A. Bain and T. E. Schlesinger. Hamr Adjacent Track Stability in the Presence of a Medium Curie Temperature Distribution. IEEE Transactions on Magnetics, 46, (2010), 2462-2465.
12. Wang Yiming and Zhu Jiangang. SNR Impact of Media Anisotropy Near Curie Temperature in Heat-Assisted Magnetic Recording. IEEE Transactions on Magnetics, 47, (2011), 2368-2370.
13. Li Xiaoping, Peng Qingshun, Li Jinbao, Wen Xishan and Ru Hailiang. Parameter Identification of Hysteresis Loop Model for Transformer Core. Power System Technology, 36, (2012), 200-205.
14. Ma Yuli and Dai Xinrui. Optimized design of apping method for dynamic hysteresis loop of ferromagnetic material. Physics and Engineering, 22, (2012), 32-34.
15. Zhao Xiuke. Switching Power Supply of Magnetic Components, Nanjing University of Aeronautics and Astronautics Institute of Automation: Nanjing, (2004), 48-49.
16. Aina He, Anding Wang, Shiqiang Yue, Chengliang Zhao and Chuntao Chang et al. Dynamic magnetic characteristics of Fe78Si13B9 amorphous alloy subjected to operating temperature. Journal of Magnetism and Magnetic Materials, 408, (2016), 159-163.
17. Yongfeng Liang, Junpin Lin, Feng Ye, Yanli Wang and Guoliang Chen. Effect of Heat Treatment on Microstructure and Properties of Heavily Cold Rolled Fe-6.5 wt %Si Alloy Sheet. Metallic Functional Materials, 17, (2010), 43-47.
18. Yufeng Du. Research on Coupling Simulation of Magnetic Field and Temperature Field Based on Temperature Characteristics Test of Ferromagnetic Materials, M.S. dissertation, Shenyang University of Technology, (2016).
19. He Yuan, Li Xin and Luo Jian. Distinguishing Magnetizing Inductance of Power Transformer Based on Hysteresis Loop. Power System Protection and Control, 41, (2013), 19-24.
20. P. R. Wilson, J. N. Ross, and A. D. Brown. Simulation of Magnetic Component Models in Electric Circuits Including Dynamic Thermal Effects. IEEE Transactions on Power Electronics, 17, (2002), 55-65.
21. Vittorio Basso, Cinzia Beatrice, Martino LoBue, Paola Tiberto, and Giorgio Bertotti. Connection between hysteresis and thermal relaxation in magnetic materials. PHYSICAL REVIEW B, 61, (2000), 1278-1285.
22. Alexander Sutor, Stefan J. Rupitsch, Shasha Bi, and Reinhard Lerch. A modified Preisach hysteresis operator for the modeling of temperature dependent magnetic material behavior. JOURNAL OF APPLIED PHYSICS, 109, (2011), 07D338-07D340.