Quantum dots with even number of electrons: Kondo effect in a finite magnetic field

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We study a small spin-degenerate quantum dot with even number of electrons, weakly connected by point contacts to the metallic electrodes, and subject to an external magnetic field. If the Zeeman energy $B$ is equal to the single-particle level spacing $\Delta$ in the dot, the system exhibits Kondo effect, despite the fact that $B$ exceeds by far the Kondo temperature $T_K$. A possible realization of this in tunneling experiments is discussed.

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Zero-bias anomaly of tunneling conductance, discovered in the early sixties, has been explained in terms of scattering by magnetic impurities located in the insulating layer of the tunneling junction \cite{1}, in close analogy with the Kondo explanation of the resistivity minimum in metals \cite{2}. Recently, this problem gained renewed attention following theoretical predictions that very similar effects should be detectable in tunneling of electrons through small semiconductor quantum dots \cite{3}, \cite{4}. It was indeed observed in quantum dots formed in GaAs/AlGaAs heterostructures by the gate-depletion technique \cite{5}-\cite{8}. These peaks are observable even at zero temperature, and the value of the conductance at the zero-bias peak splits into two peaks $\approx T_K$ \cite{9}. These peaks are never reaches the unitary limit \cite{10}, \cite{11}, \cite{12}.

For $N = \text{even}$ this consideration is inapplicable, since in the ground state of the spin-degenerate quantum dot all single-particle energy levels are occupied by pairs of electrons with opposite spins, and the total spin is zero. Therefore, the Kondo physics is not expected to emerge in this case. Not surprisingly, investigations of dots with $N = \text{odd}$ appear more attractive. Yet, as we demonstrate below, quantum dots with $N = \text{even}$ may exhibit a generic Kondo effect at certain value of the Zeeman energy $B \gg T_K$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{（a）The spinless ground state of the dot with $N = \text{even}$ electrons. （b）Excited state which has $S^z = 1$. States (a) and (b) differ by adding a spin-down or spin-up electron accordingly to the state $|\Omega\rangle$ of $N − 1$ electrons in the dot, shown at (c). The states (a) and (b) are denoted as $|\downarrow\rangle$ and $|\uparrow\rangle$ in (c).}
\end{figure}

In quantum dots, charge and spin excitations are controlled by two energy scales, $E_c$ and $\Delta$ respectively, which typically differ by the order of magnitude \cite{12}. This separation of the energy scales allows to change the spin state of the dot, without changing its charge. Indeed, if $N = \text{even}$, the ground state of the dot has spin $S = 0$. The lowest excited state with non-zero spin $S = 1$, has energy $\Delta$ (see Fig.1). These two states, however, are...
affected very differently by the magnetic field, and for $B = \Delta$, they become degenerate (see Fig. 2). Since they differ by flipping the spin of a single electron in the dot, this system is a natural candidate for realizing the Kondo effect. Moreover, this Kondo effect is unique in a sense that the presence of a large magnetic field $B = \Delta \gg T_K$ is a necessary condition for its very occurrence.

To check this idea, we consider the following model Hamiltonian for a quantum dot, such as studied in \cite{3-8}, attached to a single metallic electrode:

$$\mathcal{H} = H_0 + H_d + H_T. \quad (1)$$

Here the Hamiltonian of the lead electrons is

$$H_0 = \sum_{k\sigma} \epsilon_k \psi_{k\sigma}^\dagger \psi_{k\sigma}, \quad (2)$$

where $\epsilon_k$ is the energy, measured from the Fermi level $\epsilon_F$. An in-plane magnetic field has no influence on the two-dimensional electron gas in the lead, provided that $B$ is small compared to the Fermi energy (which is certainly the case for $B \sim \Delta$) \cite{3-8}. As for the dot Hamiltonian $H_d$ we consider just the two single-particle energy levels, closest to $\epsilon_F$ (Note that the Anderson model description of the $N = \text{odd} \ \text{case} \ [3, 8, 5]$ is based on a similar approximation, which is valid, provided that the conductances of the tunneling junctions are small \cite{3-8}). Hence, $H_d = \sum_{p=\pm 1} \frac{1}{2} \left( p \Delta - \sigma B \right) d_{p\sigma}^\dagger d_{p\sigma} + E_c (N - 2)^2. \quad (3)$

where $N = \sum_{p\sigma} d_{p\sigma}^\dagger d_{p\sigma}$, $p = \pm 1$ refers to single-particle energy levels in the dot, $\sigma = \pm 1$ stands for up- and down-spin. In writing the interaction term in (3) we assumed that the system is tuned to the middle of the $N = \text{even} \ \text{valley of the Coulomb blockade}$. The coupling between the dot and the electron gas is described by the tunneling Hamiltonian

$$H_T = \sum_{p\sigma} t_p \psi_{p\sigma}^\dagger \psi_{p\sigma} + \text{H.c.}, \quad (4)$$

where $L$ is a normalization constant, and we have allowed an explicit dependence of the tunneling amplitudes on $p$. The two states of the dot which become degenerate at $B = \Delta$, are

$$|\uparrow\rangle = d_{+1\uparrow}^\dagger |\Omega\rangle, \quad |\downarrow\rangle = d_{-1\downarrow}^\dagger |\Omega\rangle, \quad (5)$$

with $|\Omega\rangle = d_{1\uparrow}^\dagger |0\rangle$, in which $|0\rangle$ is the ground state of the dot with $N - 2$ electrons. It is useful to define spin operators, built on the states (5):

$$S^z = \frac{1}{2} \left( |\uparrow\rangle \langle \uparrow\rangle - |\downarrow\rangle \langle \downarrow\rangle \right), \quad S^+ = S^x + i S^y = |\uparrow\rangle \langle \downarrow|.$$ 

These operators act on different spin states of the dot. Since $E_c \gg \Delta$, virtual charge excitations to states with $N \neq 2$ can be integrated out by means of a Schrieffer-Wolf transformation. The resulting effective Hamiltonian now reads,

$$H = H_0 + H_p + H_{ex}. \quad (6)$$

It contains a potential scattering part

$$H_p = U_c \rho + U_s \sigma^z, \quad \rho = \frac{1}{2} (\rho_\uparrow + \rho_\downarrow), \quad \sigma^z = \frac{1}{2} (\rho_\uparrow - \rho_\downarrow), \quad \rho_\sigma = \psi_\sigma^\dagger \psi_\sigma, \quad \sigma^\pm = \psi_\uparrow^\dagger \psi_\downarrow, \quad \sigma^- = \psi_\downarrow^\dagger \psi_\uparrow. \quad (9)$$

The operators appearing in (9) act on the conduction electrons at the site of the dot. They are defined as

$$J^z_c = 2 t_c, \quad J^\pm_c \sim \frac{J^z_c}{E_c} \frac{t^\pm_1}{t^\perp_1} \frac{t^\pm_2}{t^\perp_2}, \quad U_c = 2 t_c E_c. \quad \text{It should be noticed that the value of} \ U_c \ \text{depends strongly on the } J^\pm_c \ \text{of the same order to} \ U_c. \ \text{In addition to the contributions, listed in (5-8) there also appears a term, proportional to} \ S^z. \ \text{This term, evidently, represents a correction to} \ \text{the level spacing} \ \Delta, \ \text{and does not lift the degeneracy of the states (5), if} \ B \ \text{is properly adjusted (we assume that} \ \text{the tunneling alone does not change the symmetry of the ground state of the dot).}$$

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{fig2}
\caption{Low-energy states of a spin-degenerate quantum dot in magnetic field.}
\end{figure}
In the usual Kondo effect, the spin-charge separation ensures that potential scattering makes no contribution. In the Hamiltonian \[ H \] the charge degrees of freedom are coupled to the spin through the term \( J_z \rho S^z \); in addition, there appears a potential scattering in the spin channel, \( U_c \sigma^z \). The main effect of the potential scattering terms \( H_{sc} \) is to introduce small spin-dependent corrections to the density of states \([14]\). For our purpose it is sufficient to replace the weakly energy-dependent corrections to the density of states \([14]\). For simplicity, we add an additional index for the channel, \( \alpha \), drain electrodes. To describe this situation, we flow to the strong coupling fixed point for the dimensionless coupling constants \( J \), \( \nu \).

Unlike in \([3]\), the coefficients in \((13)\) are spin-dependent currents in the conduction electrons: \( \nu_\sigma \). We are interested in the contribution to the tunneling conductance due to the Kondo effect. Therefore, we will ignore these terms (thereby neglecting small corrections to the densities of states, similar to \([11]\)). It is convenient to perform a canonical transformation \( a_\sigma = \alpha_\sigma \psi_{L,\sigma} + \beta_\sigma \psi_{R,\sigma}, \quad c_\sigma = \alpha_\sigma \psi_{L,\sigma} - \beta_\sigma \psi_{R,\sigma}, \quad (13) \) where

\[
\alpha_\uparrow = t_{L,\uparrow}/t_\uparrow, \quad \beta_\uparrow = t_{R,\uparrow}/t_\uparrow, \\
\alpha_\downarrow = t_{L,\downarrow}/t_\downarrow, \quad \beta_\downarrow = t_{R,\downarrow}/t_\downarrow,
\]

and

\[
t_\uparrow = \sqrt{t_{L,\uparrow}^2 + t_{R,\uparrow}^2}, \quad t_\downarrow = \sqrt{t_{L,\downarrow}^2 + t_{R,\downarrow}^2}, \quad (14)
\]

Unlike in \([3]\), the coefficients in \([13]\) are spin-dependent as a result of the asymmetry of the tunneling amplitudes. It turns out that only \( a_\sigma \) enter the interaction terms in the effective Hamiltonian which acquire the same form, as \([3]\) with

\[
J_z^c = \frac{2 \left( t_{\uparrow}^2 - t_{\downarrow}^2 \right)}{E_c}, \quad J_s^c = \frac{2 \left( t_{\uparrow}^2 + t_{\downarrow}^2 \right)}{E_c}, \quad J^\perp = \frac{4t_\uparrow t_\downarrow}{E_c}, \quad (15)
\]

and with definitions of the operators analogous to \([3]\) (with \( \psi_\sigma \) replaced by \( a_\sigma \)).

In the weak coupling regime \( T \gg T_K \) (the characteristic energy scale of the problem - the Kondo temperature \( T_K \) - is discussed below) the Kondo contribution \( G_K \) to the differential conductance can be calculated perturbatively from \([8]\), \([13]\). The resulting expression is lengthy, therefore we present it only for the symmetric case \( t_{qp} = t_q \), when \( t_\uparrow = t_\downarrow = t \) and \( J_s^c = J_\perp = J = 4t^2/E_c \). In this case the result can be written in a compact form

\[
G_K = \frac{e^2}{\pi \hbar} g_0 \left( \frac{3\pi^2/8}{\ln^2(T/T_K)} \right) \sum_\sigma \left( \frac{2t_L t_R}{t_\uparrow^2 + t_\downarrow^2} \right)^2, \quad (16)
\]

In the strong coupling regime \( T \ll T_K \), the spin-flip scattering is suppressed, and the system allows an effective Fermi-liquid description (see, for example, \([19]\)). The zero-bias conductance then follows immediately from the Landauer formula,

\[
G_K = \frac{e^2}{2\pi \hbar} \sum_\sigma \tau_\sigma, \quad \tau_\sigma = (2\alpha_\sigma \beta_\sigma)^2. \quad (17)
\]

In the symmetric case \([14]\) reduces to \( G_K = (e^2/\pi \hbar) g_0 \). By virtue of the universality of the Kondo model, the two independent parameters, \( g_0 \) and \( T_K \), are sufficient for the description of \( G_K \) in the whole temperature range \( T \ll \Delta \). Notice that, due to the asymmetry of the coefficients in \([13]\), the transmission probabilities \( \tau_\sigma \) retain the spin dependence even in the unitary limit, unlike in the \( N = \text{odd} \) Kondo effect \([3]\). This reveals itself in the spin current in response to the applied voltage, with the corresponding spin conductance given by

\[
G_K = (e^2/2\pi \hbar) \sum_\sigma \sigma \tau_\sigma \neq 0. \quad \text{However, this effect might be difficult to measure.}
\]

The role of a finite bias \( eV \gg T_K \), and of the magnetic field’s departures from the degeneracy points \( B = \pm \Delta \) is similar to that for the case \( N = \text{odd} \) \([3], [14]\): At \( B = \pm \Delta \), the conductance exhibits peaks near zero bias, whose width saturates to \( T_K \) in the Kondo regime \( T \ll T_K \). When the degeneracy is lifted, each of these peaks splits to two. Therefore, the peaks at \((B, eV)\) plane are located at the points, which satisfy either an equation \( |B - \Delta| \approx eV \), or \( |B + \Delta| \approx eV \). For a fixed \( eV \neq 0 \), these equations have four solutions for \( B \).

The possibility of experimental realizations of the proposed Kondo effect depends crucially on the value of the Kondo temperature \( T_K \). Since for \( \nu_\sigma = \nu_0 \) the scaling invariant \([15]\)

\[
C^2 = (J_z^c)^2 - (J_\perp)^2 = \left( \frac{2\nu_0}{E_c} \right)^2 \left( t_\uparrow^2 - t_\downarrow^2 \right)^2 \geq
\]

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Since one finds where the width is of the order of $0$, the first of the scaling equations (11) can be written as
\[
\ln \frac{E_c}{D} = \frac{1}{2C} \ln \left( \frac{J^z - C}{J^z + C} \right) \left( \frac{J^z + C}{J^0 - C} \right).
\]
Here we have taken into account that the initial bandwidth is of the order of $D$. The condition $J^z (D = T_K) \sim 1$ gives the logarithmic estimate of the Kondo temperature. Since $C \ll 1$, one finds
\[
T_K \sim E_c \exp \left[ -\frac{A}{J_0^z} \right],
\]
where $A = \frac{1}{2\lambda} \ln \left( \frac{1+\lambda}{1-\lambda} \right)$, $\lambda = C/J_0^z$, and $0 \leq \lambda < 1$. In the isotropic limit $\lambda \to 0$ one has $A \to 1$ and (18) reduces to the usual expression $T_K \sim E_c \exp \left( -1/\nu_0 J^z_0 \right)$. For a given $J_0^z$ this value is significantly higher than that corresponding to the strongly anisotropic limit, when $\lambda \to 1$ and factor $A$ diverges as $\ln (1-\lambda)^{-1}$. The parameters $J_0^z$ and $C$, which control the Kondo temperature $T_K$, can be expressed in terms of the Kondo temperatures $T_K^{N \pm 1} \sim E_c \exp \left( -1/\nu_0 J_{N \pm 1} \right)$ for the nearby Coulomb blockade valleys with odd number of electrons $N \pm 1$, since the corresponding exchange constants are given by $J_{N-1} = 4t_1^2/E_c$ and $J_{N+1} = 4t_2^2/E_c$:
\[
J_0^z \approx \frac{1}{2} \left( \frac{1}{\ln E_c/T_{K^{-1}}} + \frac{1}{\ln E_c/T_{K^{+1}}} \right),
\]
\[
C \approx \frac{1}{2} \left( \frac{1}{\ln E_c/T_{K^{-1}}} - \frac{1}{\ln E_c/T_{K^{+1}}} \right).
\]
From these equations and from (18) follows, that
\[
\min T_K^{N \pm 1} \lesssim T_K \lesssim \max T_K^{N \pm 1}.
\]
That is, $T_K$ is intermediate between the corresponding scales for the neighboring Coulomb blockade valleys with odd number of electrons. It ensures the observability of the proposed effect in the systems, studied in [5]-[8].

In conclusion, we argue in this Letter that spin-degenerate quantum dots with even number of electrons exhibit Kondo effect in a finite magnetic field, when the Zeeman energy is equal to the single-particle level spacing in the dot, and, therefore, is much larger than the Kondo temperature. The effect appears due to a large difference between the characteristic energy scales for spin and charge excitations of quantum dots, and can not be realized with the usual magnetic impurities.

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