Analysis of Thin-Walled Bars Stress State with an Open Section

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Abstract. The analysis of the thin-walled structural problem is still a topic of interest and research by many scientists with many different solutions and theories. This paper presents the occurrence of stress components on the thin-walled bars with open sections. Vlasov's theory has been applied to analyze the stress state on thin rods. At the same time, using the Midas civil structure calculation program based on finite element principle for structural analysis with taking into account sectional warping. From the results obtained by the finite element method, it is necessary to select and design carefully for thin-walled bars.

1. Introduction

Thin-walled bars with open-section has wide application in building structures because which have many advantages such as light and easy to transport. However, the special properties of the thin section can significantly affect the bearing capacity of the structure [1-3]. Therefore, thin-walled structures before use should be subjected to a thorough analysis of the carrying capacity. Theory calculation based on the hypothesis of flat sections is not applicable to thin-walled members due to their small thickness. For thin-walled with an open section, the process is affected by the load, the results obtained are far from Saint Venant’s findings [4]. When a twisted deformation is formed, the part twists around its center but does not maintain the plane but, at the same time, does not remain plane and deformed out-of-plane distortion (sectional warping distortion). So, when the thin-walled bars are affected by moment torque. Torsion of thin-walled structures with open-section when warping of the section to additional stress, which make a significant contribution to the total stresses [5]. This stress intensity cannot be neglected for thin-walled, and applying Saint Venant's theory can lead to errors.

Two geometric hypotheses of Vlasov's theory are applied: the section of the thin-walled bars is considered rigid, and the shape of the section is undeformable; the shearing strains on the average cross-section are assumed not to exist [6].

In this paper with the development of the finite element method to analyze of thin-walled bars stress state with an open section. The result is a reference for the design of thin-walled bars steel structures with open sections.

2. Methods

2.1. The differential equations of Vlasov's theory for thin-walled open-section bars
In this study, based on Reissner’s transformation principle and the relationship from Timoshenko beam theory, Vlasov has given the displacement differential equations corresponding to the seven degrees of freedom of the thin section of the open section as follows [7].

\[
\begin{align*}
    E A \ddot{\xi} = q_x \\
    -E J_y \dddot{\xi} + q_z = 0 \\
    -E J_x \ddot{\eta} + q_y = 0 \\
    -E J_0 \dddot{\theta} + G J_d \dot{\theta} + m_x - m_{b1} = 0
\end{align*}
\]  

Figure 1. C-Section of thin-walled bars

Where: A - cross-sectional area; E - Young’s modulus; J_x, J_y - profile area moments of inertia about x and y-axes; J_0 - warping torsion constant for a profile; J_d - the torsion constant; q_x, q_y, q_z - the distributed load according to the x, y and z-axes, respectively; m_x, m_y - the bending moment is distributed according to the z and y-axes, respectively.

Consider the general displacement functions as follows: \( \xi = \xi(x), \eta = \eta(x), \zeta = \zeta(x) \) in the x, y and z-axes, respectively. The fourth equation of the system of equations (1) is rewritten as follows [8]:

\[
\theta'' - k^2 \theta = \frac{m_z - m_{b1}}{EJ_0}
\]  

Where: k - coefficient of flexural–torsional of a cross-section and is determined by the formula (3):

\[
k = \sqrt{\frac{GJ_d}{EJ_0}}
\]  

The solution of equation (2) we obtain the following function (4):

\[
\theta(x) = f_o(x) + f(x)
\]  

Where: \( f_o(x) \) - the common integral of the homogeneous differential equation corresponding to (2) is:

\[
f_o(x) = A \sinh(kx) + B \cosh(kx) + Cx + D
\]  

\( f(x) \) - The partial integral of equation (2), depending on the nature of the loading of the bars and arbitrary integration constants A, B, C, D depending on the boundary conditions.

To solve equation (2), the common steps have been presented in the study [9]. When determining the functions \( \zeta(z), \xi(z), \eta(z) \) and \( \theta(z) \) from the system of equations (1), normal stresses, shear stresses and moment of pure torsion are obtained by the system of equations (6).
Where: $\hat{d}$ - profile thickness; $S_y$ and $S_z$ - static moments of cross section relative to the $y$ and $z$-axes; $S_\omega$ - analogous sectorial static moment; $H$ - a moment of pure torsion.

Figure 2 shows determine the geometrical characteristics of the section [10]. The sectorial coordinate (sectorial area) is a doubled area, described by the radius vector $PA$ when the point $A$ moves along the contour from the origin $O$ to some value of the arc $s$ ($OA = s$). Considering a differential length element $AB = ds$.

The sectorial coordinate $\omega$ ($\text{cm}^2$) is given by

$$\omega = \int_0^s d\omega = \int_0^s r ds$$ (7)

Where: $r$ - a radius measured from the shear center $P$ to the peripheral coordinates on the median line of the thin-wall; and $\omega$ represents twice the enclosed area of the cell.

The sectorial static moment $S_\omega$ ($\text{cm}^4$) and the sectorial moment of inertia $J_\omega$ ($\text{cm}^6$) is determined by the formula (8):

$$S_\omega = \int_\Lambda \omega dA; \quad J_\omega = \int_\Lambda \omega^2 dA$$ (8)

When the thin-walled bars subject to general load, the axial force $N$, bending moments $M_y$, $M_z$ about the $y$ and $z$-axes and bimoment $B$ are obtained over the cross-section are defined as (9):

$$N = EA\xi; \quad M_y = -EJ_y\xi; \quad M_z = -EJ_z\eta; \quad B_\omega = -EJ_\omega\theta$$ (9)

Based on the superposition principle, the stress is equal to the sum of four different cases of load action. It is possible to know the normal stress in equation (6) is rewritten as follows [11]:

$$\sigma = \frac{N}{A} + \frac{M_y}{J_y}z + \frac{M_z}{J_z}y + \frac{B_\omega}{J_\omega}\omega$$ (10)

Second equation and third of equations (6) is a component tangential stress distribution on the thickness of the section. Shear force and Bimoment can be calculated as:
The shearing stresses can be obtained by using (12) [12]:

\[ \tau = \pm \frac{H\delta}{J_d} \pm \frac{Q_x S_y}{J_x \delta} \pm \frac{Q_y S_x}{J_y \delta} \pm \frac{M_{\omega} S_{\omega}}{J_{\omega} \delta} \]  

(12)

Where: \( Q_x, Q_y, N \) - internal cross-axis and long-axis forces; \( M_{\omega} \) - warping torque.

2.2. Finite element method to determine stress for thin-walled open-section bars

When calculating for normal bars, the degrees of freedom is determined in the nodes of the elements includes three linear displacements and three rotational displacements. In the construction of thin-walled bars with open section be considered the seventh degree of freedom of assembly (warping of the open-section). Therefore, each element of the thin-walled bars has 14 degrees of freedom [5, 13]. Figure 3 shows the components movable nodes of a finite element.

![Thin-walled bars element with 14 degrees of freedom](image)

Each bar is connected to the local coordinate system \( O_1X_1Y_1Z_1 \) randomly compared within the total coordinates system \( OXYZ \). \( u_1, v_1, w_1 \) are displacements in \( X_1, Y_1, \) and \( Z_1 \) directions respectively. \( \alpha_1, \beta_1, \gamma_1 \) are the angle of rotation in \( X_1, Y_1, \) and \( Z_1 \) directions, respectively. \( \delta_1 \) is sectional warping. At each node, the corresponding displacement in the global coordinate system is \( u, v, w, \alpha, \beta, \gamma \) and \( \delta \). The symbols \( H \) and \( K \) are used to designate the parameters of displacement of the two nodes of the thin-walled element.

The relationship between displacement and load can be expressed as [5, 14].

\[ [K]_{7x7} \{U\}_{7x1} = \{P\}_{7x1} \]  

(13)

Where: \([K]_{7x7}\) - stiffness matrix in the global coordinate system; \([U]\) - displacement vector fields; \([P]\) - the load column matrix in the global coordinate system.

Currently, based on the principle of finite element software such as Ansys, Abaqus, Midas civil, etc. have been applied to solve many structural problems in general and steel structure in particular. In this study, with the help of Midas civil software to solve the problem of thin-walled bars with 14 degrees of freedom on each element taking into account the influence of Bimomen due to the sectional warping under the effect of load.
3. Numerical example

3.1. Describe the example

As shown in Figure 4, a thin-walled bars has a C-shaped with the following dimensions: \( h = 24 \text{ cm}, \ b = 12 \text{ cm}, \ \delta = 1 \text{ cm}, \) the length of the bars \( L = 200 \text{ cm}. \) The elastic modulus \( E \) and shear modulus \( G \) are \( 200 \) and \( 80 \text{ kN/mm}^2 \), respectively. The bars are fixed one point; the free point is subject to force \( F = 15 \text{ KN}. \)

![Figure 4. Thin-walled bars (a) and cross section C (b)](image)

3.2. Results

Using the Midas civil structure calculation program based on finite element principle for structural analysis. The finite element of thin-walled bars model consists of 21 node points and 21 elements. The model of thin-walled bars is shown in Figure 5. We consider the stress distribution on the section of the fixed end at four points 1, 2, 3, 4 (Figure 6) with load combination such as self-weight, concentrated force (creating stress due to bending moment) and taking into account warping of the section (creating stress due to warping torque). The bimoment diagram under the effect of load \( F \) on thin-walled bars is shown in Figure 7. Figures 8-10 show that distribution of normal stress at fixed section at four points 1, 2, 3, 4.

![Figure 5. Finite element model for thin-walled bars](image)

![Figure 6. Investigate the stress in the position of a fixed end](image)
Figure 7. The distribution of the Bimoment $B_{\omega}$ ($B_{\omega,\text{max}} = 1569.60\,\text{N.m}$)

Figure 8. Normal stress due to bending ($\sigma_0 = 81.24\,\text{MPa}$)

Figure 10 shows the maximum normal stress due to Bimoment $B_{\omega}$ at the cross-section at the fixed at four points 1, 2, 3, 4.

Figure 9. Normal stresses ($\sigma_0$) due to Bimoment $B_{\omega}$

a – Point 1 ($\sigma_0^1 = 133.08\,\text{MPa}$)  
b – Point 2 ($\sigma_0^2 = -176.74\,\text{MPa}$)  
c – Point 3 ($\sigma_0^3 = -133.08\,\text{MPa}$)  
d – Point 4 ($\sigma_0^4 = +176.74\,\text{MPa}$)
Figure 10. Normal stress due to load combination ($\sigma$)

Figure 10 shows the maximum normal stress $\sigma_1$, $\sigma_2$, $\sigma_3$, $\sigma_4$ occurring in the cross-section at the fixed end are $\sigma_1 = +214.32$ MPa, $\sigma_2 = -95.50$ MPa, $\sigma_3 = -214.32$ MPa, $\sigma_4 = -95.50$ MPa.

For this example, the normal stresses of the fixed end at points 1, 2, 3 and 4 due to Bimomen ($B_\omega$) exceed the normal stress due to bending moment ($M_\mu$) of 1.35 times. Therefore, with the thin-walled bars, when analyzing stress state, it is necessary to consider warping torque.
The maximum rotation of thin-walled bars is at the free end and equal \( \theta = 0.51 \) rad/m, as shown in Figure 11.

4. Conclusions
The following statements should be presented in conclusions:

1. For steel structures such as beams, columns, thin-walled bars, etc. with open cross-section, restriction of torque formation is necessary. When the torque is present, it is necessary to take control measures in the dangerous sections.

2. Using the FEM method can quickly determine the stress-deformation state on the sections of the thin-walled bars with complex boundary conditions and high reliability.

3. The results show that the cross-section deformation (warping of the section) should be taken into account in the process of analyzing the stress state.

4. Future research: the authors continue to study the working structure in the plastic flow phase with complex conditions. Structural analysis works during the plastic flow phase to optimize the cross section in the steel structure design.

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