Strange and nonstrange baryon spectra in the relativistic interacting quark-diquark model with a Gürsey and Radicati-inspired exchange interaction

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The relativistic interacting quark-diquark model, constructed in the framework of point form dynamics, is extended to strange baryons. The strange and non-strange baryon spectra are calculated and compared with the experimental data. The mass of \( \Lambda^{0}(1405) \), which is a long-standing problem of three quarks constituent quark models, is well reproduced in our quark-diquark picture of baryons.

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I. INTRODUCTION

The three quark constituent quark models (QMs) are quite successful in describing many baryon observables, like the magnetic moments, the open-flavor decays and the electromagnetic form factors of the nucleon. These models show some differences, for example concerning the particular form of Hamiltonian they are based on, but share the main features: 1) they are built upon the effective degrees of freedom of three constituent quarks; 2) their mass operator contains a confining potential which, in general, is linear or quadratic in the quark relative coordinate. In the 80’s, the predictive power of QMs has been extended with the unquenched Fock model (UQM) formalism, which introduced the higher Fock \( qqq-q\bar{q} \) components in baryon wave functions, arising from the coupling to the meson-baryon continuum. This formalism makes it possible to access to a number of problems which cannot be treated in the naive three quark QMs, such as the calculation of the flavor asymmetry of the nucleon sea or the strange content of the nucleon electromagnetic form factors.

One of the main difficulties of three quarks QMs is that they predict a number of states much larger than that of the experimentally observed baryons. This is the well-known problem of the missing resonances. One may try to look for these resonances in channels such as \( N\eta, N\eta', N\omega \) and \( K^+\Lambda \), where the final state meson is different from the pion. Indeed, it is well-known that the majority of baryon resonances is seen in reactions in which the pion is present either in the incoming (e.g. \( N\pi \to N\pi \)) or outgoing (e.g. \( N\gamma \to N\pi \)) channel. Thus, it would not be surprising if some baryon resonances, very weakly coupled to the single pion, were missing from the experimental results. The other possibility is that the problem of missing resonances has to do with the choice of the effective degrees of freedom. Thus, in quark-diquark models, the effective degree of freedom of diquark is introduced to describe baryons as bound states of a constituent quark and diquark. Since the degrees of freedom of two quarks are frozen in the diquark, the state space will be greatly reduced.

The diquark concept dates back to 1964, when its possibility was hypothesized by Gell-Mann in his original paper on quarks. Many articles have been written on this subject since its introduction (for a review, see Ref. and more recently the diquark concept has been used in several studies, ranging from one-gluon exchange to lattice QCD calculations. Important phenomenological indications for diquark-like correlations have also been provided. This makes plausibly enough to make diquarks a part of baryon wave functions.

In this paper, we provide a mass formula for the strange and nonstrange baryon resonances within the interacting quark-diquark model, and then compute the strange and nonstrange baryon spectra. The relativistic interacting quark-diquark model is constructed with the point form formalism, which was already used to develop point form three quark QMs for baryons such as those of Refs. and more recently the diquark concept has been introduced to describe baryons as bound states. The relative motion between the two constituents and the Hamiltonian of the model are functions of the relative coordinate \( \vec{r} \) and its conjugate momentum \( \vec{p} \). The Hamiltonian contains a direct (Coulomb + linear confining) interaction and an exchange one, depending on the spins and isospins of the quark and the diquark. The extension of the model to strange and heavy (e.g. charmed and bottomed) baryons only needs some small changes in the spin-flavor exchange potential of the model. Specifically, it requires the substitution of the previous spin and isospin-dependent exchange interaction with a more general Gürsey-Radicati inspired one.

In the end, we compare our theoretical results to the experimental data and show that the strange baryon spectrum is reproduced reasonably well, with a quality.
comparable to that of other three quark QMs [2, 18, 52].

\[ M_{(n,n)} - M_{(n,s)} - M_{(s,n)} - M_{(s,s)} - M_{(n,n)} - M_{(n,s)} - M_{(s,n)} - M_{(s,s)} \]

| Source |
|----------------|
| 28 |
| 29 |
| 38 |
| 39 |
| 53, 54 |
| 55 |
| 56 |
| 57 |
| 58 |
| 59 |
| 60 |
| 61 |
| 62 |
| 63 |

TABLE I: Mass difference (in GeV) between scalar and axial-vector diquarks according to some previous studies.

II. NONRELATIVISTIC QUARK-DIQUARK STATES

In a quark-diquark model, baryons are assumed to be composed of a constituent quark, \( q \) and a constituent diquark, \( Q \). In the energy range we are interested into, i.e. up to 2 GeV, the diquark can be described as two correlated quarks with no internal spatial excitations, thus in S-wave [38, 39]. Then, its color-spin-flavor wave functions must be antisymmetric. Rainbow-ladder DSE calculations confirmed that the first spatially excited diquark, the vector diquark, has a mass much larger than those of the scalar and axial-vector diquark, i.e. the ground state diquarks [53, 55]. Moreover, as we take only light baryons into account, composed of \( u, d, s \) quarks, the internal group is restricted to SUsf(6). Using the conventional notation of denoting spin by its value and flavor and color by the dimension of the representation, the quark has spin \( s_q = \frac{1}{2} \), flavor \( F_q = 3 \), and color \( C_q = 3 \). Since the hadron must be colorless, the diquark must transform as \( \bar{3} \) under SUc(3) and therefore one can have only the symmetric SUsf(6) representation \( 21_{\text{sf}}(S) \), containing \( s_2 = 0, F_1 = \bar{3}, \) and \( s_1 = 1, F_1 = 6 \), i.e., the scalar and axial-vector diquarks, respectively.

In the following, we will indicate the possible diquark states by their constituent quarks (denoted by \( s \) if strange or \( n \) otherwise) in square (scalar diquarks) or brace brackets (axial-vector diquarks). The possible scalar diquark configurations are thus \( \{n,n\} \) and \( \{n,s\} \), while the possible axial-vector diquark configurations are \( \{n,n\} \), \( \{n,s\} \) and \( \{s,s\} \) [29]. For quark-diquark states, we use the notation

\[ \langle q,q| \langle F_1, F_2; (t_1, t_2)T; (s_1, s_2)S \rangle \]

or

\[ \langle q,q| \langle F_1, F_2; (t_1, t_2)T; (s_1, s_2)S \rangle \]

where the SUf(3) representations of the diquark, \( F_1 = \bar{3} \) or 6, and the quark, \( F_2 = 3 \), are coupled to the SUf(3) representation of the baryon, \( F \). Similarly, the spins (isospins) of the diquark, \( s_1 (t_1) \), and of the quark, \( s_2 (t_2) \), are coupled to the total spin (isospin) of the baryon, \( S (T) \). Finally, the quark-diquark basis states for \( N, \Delta, \Lambda, \Sigma, \Xi \) and \( \Omega \)-type baryons, written in the notation of Eq. (1), are given in App. [53]. See also Table I where we report some estimations of the masses of axial-vector and scalar diquarks according to some previous studies [28, 29, 38, 45, 53, 56].

III. THE MASS OPERATOR

We consider a quark-diquark system, where \( \vec{r} \) and \( \vec{q} \) are the relative coordinate between the two constituents and its conjugate momentum, respectively. The baryon rest frame mass operator we consider is

\[ M = E_0 + \sqrt{\vec{q}^2 + m_1^2 + \vec{q}^2 + m_2^2 + M_{\text{dir}}(r) + M_{\text{ex}}(r)} \]

where \( E_0 \) is a constant, \( M_{\text{dir}}(r) \) and \( M_{\text{ex}}(r) \) respectively the direct and the exchange diquark-quark interaction, \( m_1 \) and \( m_2 \) stand for diquark and quark masses, where \( m_1 \) is either \( m_{[q,q]} \) or \( m_{[q,q]} \) according if the mass operator acts on a scalar or axial-vector diquark [28, 29, 38, 50, 70].
The direct term we consider,
\[ M_{\text{dir}}(r) = -\frac{\mathbf{r}}{r} \left( 1 - e^{-\mu r} \right) + \beta r , \]
is the sum of a Coulomb-like interaction with a cut off and a linear confinement term.

We also need an exchange interaction, since this is the crucial ingredient of a quark-diquark description of the SUf(3) Gell-Mann matrices. In the non-strange sector, we also have a contact interaction
\[ M_{\text{cont}} = \left( \frac{m_{[n,n]} \theta_{[n,n]}}{e} \right)^{1/2} \left( \frac{D}{2} \right) e^{-\eta r^2} \delta_{L,0} \delta_{s_1,1} , \]
which was introduced in the mass operator of Ref. 39 to reproduce the Δ − Ν mass splitting. It is worthwhile comparing the exchange interactions of Eq. (4) and that of Ref. 39,
\[ M_{\text{ex}}(r) = (-1)^{L+1} e^{-\sigma r} \left[ A_S \mathbf{s}_1 \cdot \mathbf{s}_2 + A_F \lambda^I_1 \cdot \lambda^I_2 + A_I \mathbf{t}_1 \cdot \mathbf{t}_2 \right] , \]
one can notice that the spin-isospin \( (\mathbf{s}_1 \cdot \mathbf{s}_2)(\mathbf{t}_1 \cdot \mathbf{t}_2) \) term of Eq. (6) has here been substituted with a flavor-dependent one. The isospin dependence is still necessary in Eq. (4), because there are resonances which have the same quantum numbers except the isospins. These baryons, belonging to the same SUf(3) representation, have different isospins that result from different combinations of the isospins of the quark and the diquark, like Λ(1600) and Σ(1193) (see Tables V and VII). Thus, without the introduction of an isospin dependence into the exchange interaction, the previous states, Λ(1600) and Σ(1193), would become degenerate and lie at the same energy.

Finally, it has to be noted that in the present work all the calculations are performed without any perturbative approximation.

The eigenfunctions of the mass operator of Eq. (2) can be seen as eigenstates of the mass operator with interaction in a Bakerian-Thomas construction. Point form means that the Lorentz generators and with the non-interacting Lorenz generators and with the non-interacting four velocity [73].

The dynamics is given by a point form Bakamjian-Thomas construction. Point form means that the Lorentz group is kinematic. Furthermore, since we are doing a point form Bakamjian-Thomas construction, \( P = MV_0 \) where \( V_0 \) is the noninteracting four-velocity (with eigenvalue \( v \)).

The general quark-diquark state, defined on the product space \( H_1 \otimes H_2 \) of the one-particle spin \( s_1 \) (0 or 1) and spin \( s_2 \) (1/2) positive energy representations \( H_1 = L^2(R^3) \otimes S^1_0 \) or \( H_1 = L^2(R^3) \otimes S^1_0 \) and \( H_2 = L^2(R^3) \otimes S^2_0 \) of the Lorentz group, is given by [39]
\[ |p_1, p_2, \lambda_1, \lambda_2 \rangle , \]
where \( p_1 \) and \( p_2 \) are the four-momenta of the diquark and the quark, respectively, while \( \lambda_1 \) and \( \lambda_2 \) are, respectively, the z-projections of their spins.

The velocity states are introduced as [39, 43, 44]
\[ |v, \vec{k}_1, \lambda_1, \vec{k}_2, \lambda_2 \rangle = U_{B(v)} |k_1, s_1, \lambda_1, k_2, s_2, \lambda_2 \rangle_0 , \]
where the suffix 0 means that the diquark and the quark three-momenta \( \vec{k}_1 \) and \( \vec{k}_2 \) satisfy the condition:
\[ \vec{k}_1 + \vec{k}_2 = 0 . \]

Following the standard rules of the point form approach, the boost operator \( U_{B(v)} \) is taken as a canonical one,
obtaining that the transformed four-momenta are given by $p_{1,2} = B(v)k_{1,2}$ and satisfy
\[ p_i^\mu + p_{i-}^\mu = \frac{P_N^\mu}{M_N} \left( \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2} \right), \]
where $P_N^\mu$ is the observed nucleon four-momentum and $M_N$ is its mass. The important point is that Eq. 6 redefines the single particle spins. Since canonical boosts are applied, the conditions for a point form approach are satisfied. Thus, the spins on the left hand state of Eq. 6 perform the same Wigner rotations as $\vec{k}_1$ and $\vec{k}_2$, allowing to couple the spin and the orbital angular momentum as in the non relativistic case, while the spins in the ket on the right hand of Eq. 6 undergo the single particle Wigner rotations.

In Point form dynamics Eq. 2 corresponds to a good mass operator as it commutes with the Lorentz generators and with the four velocity. We diagonalize the Hilbert space spanned by the velocity states. Instead of the internal momenta $\vec{k}_1$ and $\vec{k}_2$, one can also use the relative momentum $\vec{q}$, conjugate to the relative coordinate $\vec{r} = \vec{r}_1 - \vec{r}_2$, thus considering the following velocity basis states:
\[ |v, \vec{q}, \lambda_1, \lambda_2\rangle = U_B(v)|k_1, s_1, \lambda_1, k_2, s_2, \lambda_2\rangle_0 . \]

IV. RESULTS AND DISCUSSION

In this section, we show our results for the strange and non-strange baryon spectra. Because this paper in mainly focused on the extension of the interacting quark-diquark model to strange baryons, here we present the results of two fits to the experimental data [17]. In the first, ”Fit 1”, we fit the model mass formula to the strange and non-strange baryon spectra, while in the second, ”Fit 2”, we focus our attention on the strange sector only. Obviously, in this second case we expect to get a better reproduction of the experimental data in the strange baryon sector and, perhaps, to increase the predictive power of our model for still unobserved strange baryon resonances.

Tables III and IV show the comparison between the experimental data and the results of our quark-diquark model calculation, obtained with the set of parameters of Table I (“Fit 1”). Figures I and Tables IV show our quark-diquark model results, obtained with the set of parameters of Table II (“Fit 2”). In this second case, the overall quality in the reproduction of the experimental data is reasonably good and comparable to that of other three quark QMs [2, 4, 18, 52].

A long standing problem of three quarks QMs in the strange sector is that of $\Lambda^\ast(1405)$, since its experimental mass is not reproduced with a reasonable accuracy within this kind of models. Here, the mass of this resonance is well reproduced in terms of a quark-diquark picture of baryons. It is also interesting to note that in our model $\Lambda(1116)$ and $\Lambda^\ast(1520)$ are described as bound states of a scalar diquark $|n, n\rangle$ and a quark $s$, where the quark-diquark system is in $S$ or $P$-wave, respectively. This is in accordance with the observations of Refs. [29, 30] on $\Lambda^\ast$’s fragmentation functions, that the two resonances can be described as $|n, n\rangle - s$ systems. See Table VII.

The presence of more diquark types, with respect to

| Resonance | Status | $M_{\text{Exp.}}$ (MeV) | $J^P$ | $L^P$ | $S$ | $n_r$ | $M_{\text{Calc.}}$ (MeV) |
|-----------|--------|--------------------------|------|------|----|------|--------------------------|
| $N(939)$  | $P_{11}$ | 939          | $1^-$ | $0^+$ | 0  | 0    | 939          |
| $N(1440)$ | $P_{11}$ | 1420 - 1470   | $1^-$ | $0^+$ | 0  | 1    | 1511         |
| $N(1520)$ | $D_{13}$ | 1515 - 1525   | $1^-$ | $0^+$ | 0  | 0    | 1537         |
| $N(1535)$ | $S_{11}$ | 1525 - 1545   | $1^-$ | $0^+$ | 0  | 0    | 1537         |
| $N(1650)$ | $S_{11}$ | 1645 - 1670   | $1^-$ | $0^+$ | 1  | 0    | 1625         |
| $N(1675)$ | $D_{15}$ | 1670 - 1680   | $1^-$ | $0^+$ | 1  | 1    | 1746         |
| $N(1680)$ | $F_{15}$ | 1680 - 1690   | $2^+$ | $0^+$ | 0  | 0    | 1799         |
| $N(1700)$ | $D_{13}$ | 1650 - 1750   | $1^-$ | $0^+$ | 1  | 0    | 1625         |
| $N(1710)$ | $P_{11}$ | 1680 - 1740   | $0^+$ | $1^+$ | 0  | 0    | 1776         |
| $N(1720)$ | $P_{13}$ | 1700 - 1750   | $0^+$ | $1^+$ | 1  | 0    | 1648         |
| $N(1875)$ | $D_{13}$ | 1820 - 1920   | $1^-$ | $0^+$ | 0  | 1    | 1888         |
| $N(1880)$ | $P_{11}$ | 1835 - 1905   | $0^+$ | $0^+$ | 0  | 2    | 1890         |
| $N(1895)$ | $S_{11}$ | 1880 - 1910   | $1^-$ | $0^+$ | 0  | 1    | 1888         |
| $N(1900)$ | $P_{13}$ | 1875 - 1935   | $0^+$ | $1^+$ | 1  | 1    | 1947         |
| $\Delta(1232)$ | $P_{33}$ | 1230 - 1234   | $0^+$ | $1^+$ | 1  | 0    | 1247         |
| $\Delta(1600)$ | $P_{33}$ | 1500 - 1700   | $0^+$ | $1^+$ | 1  | 1    | 1689         |
| $\Delta(1620)$ | $S_{11}$ | 1600 - 1660   | $1^-$ | $0^+$ | 1  | 0    | 1830         |
| $\Delta(1700)$ | $D_{33}$ | 1670 - 1750   | $1^-$ | $0^+$ | 1  | 0    | 1830         |
| $\Delta(1750)$ | $P_{31}$ | 1708 - 1875   | $0^+$ | $1^+$ | 0  | 1    | 1489         |
| $\Delta(1900)$ | $S_{31}$ | 1840 - 1920   | $1^-$ | $0^+$ | 1  | 0    | 1910         |
| $\Delta(1905)$ | $F_{31}$ | 1855 - 1910   | $2^+$ | $0^+$ | 1  | 0    | 2042         |
| $\Delta(1910)$ | $P_{31}$ | 1860 - 1920   | $2^+$ | $0^+$ | 1  | 1    | 1827         |
| $\Delta(1920)$ | $P_{33}$ | 1900 - 1970   | $2^+$ | $1^+$ | 1  | 0    | 2042         |
| $\Delta(1930)$ | $D_{35}$ | 1900 - 2000   | $1^-$ | $0^+$ | 1  | 0    | 1910         |
| $\Delta(1940)$ | $D_{33}$ | 1940 - 2060   | $1^-$ | $0^+$ | 1  | 0    | 1910         |
| $\Delta(1950)$ | $F_{31}$ | 1915 - 1950   | $2^+$ | $1^+$ | 1  | 0    | 2042         |
the non-strange case of Ref. [32], makes the reproduction of the experimental data below the energy of 2 GeV more difficult than before. In particular, one can notice that in the present case (see results from "Fit 2", Tables V, VII) there are 19 missing resonances below the energy of 2 GeV, while in the non strange sector [30] there were no missing states under 2 GeV. Indeed, in the strange sector one has two scalar diquarks, \([n,n]\) and \([n,s]\), and three axial-vector diquarks, \([n,n]\), \([n,s]\) and \([s,s]\), while in the non-strange sector one only has a scalar diquark, \([n,n]\), and an axial-vector diquark, \([n,n]\). Nevertheless, we think that the number of missing resonances of our model may decrease when new experimental data from more powerful experiments and more precise data analy-
The properties of heavy hadrons. The application of our
testing in light of the recent experimental effort to study
and/or bottomed baryons [78], which can be quite inter-
results converge very well.

A variational basis of 100 harmonic oscillator shells, the
based on harmonic oscillator trial wave functions. With
nalized by means of a numerical variational procedure,
in the table arises in the difference
ones. The main deviation from the evaluations reported
result for the mass difference between the axial-vector
model dependent, their difference is not. Comparing our
from PDG [17] (blue boxes).

A possible solution to the puzzle of missing reso-
non-strange sector. A
predicted [17]. For example, in the non-strange sector up
to an excitation energy of 2.41 GeV, on average about 45
N states are predicted, but only 12 have been established
(four- or three-star) and 7 are tentative (two- or one-star)
[17]. A possible solution to the puzzle of missing reso-
nances is the introduction of a new effective degree of
freedom: the diquark. This is what we tried to do in the
present paper and in Ref. 39 in the non-strange sector.

FIG. 1: (Color online) Comparison between the calculated
masses (black lines) of the 3\( \Lambda \) and 4\( \Lambda^* \) resonances
(up to 2 GeV; from "Fit 2") and the experimental masses
from PDG [17] (blue boxes).

While the absolute values of the diquark masses are
model dependent, their difference is not. Comparing our
result for the mass difference between the axial-vector
and scalar diquarks to those of Tab. 8 it is interesting to
note that our estimations are comparable with the other
ones. The main deviation from the evaluations reported
in the table arises in the difference \{n, s\} − \{n, s\}.

The whole mass operator of Eq. 2 has been dia-
goingalized by means of a numerical variational procedure,
based on harmonic oscillator trial wave functions. With
a variational basis of 100 harmonic oscillator shells, the
results converge very well.

The present work can be expanded to include charmed
and/or bottomed baryons [78], which can be quite inter-
rests in light of the recent experimental effort to study
the properties of heavy hadrons. The application of our

model to the description of heavy baryons is straightfor-
ward and does not require a modification of the mass
operator.

### Appendix A: Quark-diquark basis

For \( N \)-type states, we have the following states:

\[
|n, n\rangle; \ (3 \otimes 3) 8; \ \left( \begin{array}{c} 0, \frac{1}{2} \\ \frac{1}{2} \end{array} \right); \ \left( \begin{array}{c} 0, \frac{1}{2} \\ \frac{1}{2} \end{array} \right); \quad (A1a)
\]

\[
|n, n\rangle; \ (6 \otimes 3) 8; \ \left( \begin{array}{c} 1, \frac{1}{2} \\ \frac{1}{2} \end{array} \right); \ \left( \begin{array}{c} 1, \frac{1}{2} \\ \frac{1}{2} \end{array} \right); \quad (A1b)
\]

\[
|n, n\rangle; \ (6 \otimes 3) 8; \ \left( \begin{array}{c} 1, \frac{1}{2} \\ \frac{1}{2} \end{array} \right); \ \left( \begin{array}{c} 1, \frac{1}{2} \\ \frac{3}{2} \end{array} \right); \quad (A1c)
\]

For \( \Delta \)-type states, one has:

\[
|n, n\rangle; \ (6 \otimes 3) 10; \ \left( \begin{array}{c} 1, \frac{1}{2} \\ \frac{3}{2} \end{array} \right); \ \left( \begin{array}{c} 1, \frac{1}{2} \\ \frac{1}{2} \end{array} \right); \quad (A2a)
\]
TABLE V: Comparison between the experimental values [17] of Σ and Σ*-type resonance masses (up to 2 GeV) and the numerical ones (all values are expressed in MeV), from "Fit 2". $J^P$ and $L^P$ are respectively the total angular momentum and the orbital angular momentum of the baryon, including the parity $P$; $S$ is the total spin, obtained by coupling the spin of the diquark $s_1$ and that of the quark; $Q^2q$ stands for the diquark-quark structure of the state; $F$ and $F_1$ are the dimensions of the SU(3) representations for the baryon and the diquark, respectively; $I$ and $t_1$ are the isospins of the baryon and the diquark, respectively; finally $n_r$ is the number of nodes in the radial wave function.

| Resonance | Status | $M^{exp.}$ (MeV) | $J^P$ | $L^P$ | $S$ | $s_1$ | $Q^2q$ | $F$ | $F_1$ | $I$ | $t_1$ | $n_r$ | $M^{calc.}$ (Fit 2) (MeV) |
|-----------|--------|-----------------|------|------|-----|------|-------|-----|------|-----|------|------|------------------------|
| Σ(1193) $P_{11}$ | **** | 1189 - 1197 $\frac{3}{2}^+$ | 0+ | $\frac{3}{2}$ | 0 | [n, s]n | 8 | 3 | 1 | $\frac{1}{2}$ | 0 | 1211 |
| Σ(1620) $S_{11}$ | ** | 1620 $\frac{1}{2}^-$ | 1- | $\frac{1}{2}$ | 1 | [n, n]s | 8 | 6 | 1 | 1 | 0 | 1753 |
| Σ(1660) $P_{11}$ | *** | 1630 - 1690 $\frac{1}{2}^+$ | 0+ | $\frac{1}{2}$ | 1 | [n, n]s | 8 | 6 | 1 | 1 | 0 | 1546 |
| Σ(1670) $D_{13}$ | **** | 1665 - 1685 $\frac{3}{2}^-$ | 1- | $\frac{3}{2}$ | 1 | [n, n]s | 8 | 6 | 1 | 1 | 0 | 1753 |
| Σ(1750) $S_{11}$ | *** | 1730 - 1800 $\frac{1}{2}^-$ | 0 | $\frac{1}{2}$ | 0 | [n, s]n | 8 | 3 | 1 | $\frac{1}{2}$ | 0 | 1868 |
| Σ(1770) $P_{11}$ | * | 1770 $\frac{3}{2}^+$ | 0+ | $\frac{3}{2}$ | 1 | [n, n]s | 8 | 6 | 1 | $\frac{1}{2}$ | 0 | 1668 |
| Σ(1775) $D_{13}$ | **** | 1770 - 1780 $\frac{1}{2}^-$ | 0 | $\frac{1}{2}$ | 0 | [n, n]s | 8 | 6 | 1 | 1 | 0 | 1753 |
| Σ(1880) $P_{11}$ | ** | 1880 $\frac{3}{2}^+$ | 0+ | $\frac{3}{2}$ | 0 | [n, s]n | 8 | 3 | 1 | $\frac{1}{2}$ | 1 | 1801 |
| Σ(1915) $F_{15}$ | **** | 1900 - 1935 $\frac{3}{2}^+$ | 2+ | $\frac{3}{2}$ | 0 | [n, s]n | 8 | 3 | 1 | 2 | 0 | 2061 |
| Σ(1940) $D_{13}$ | *** | 1900 - 1950 $\frac{1}{2}^-$ | 0 | $\frac{1}{2}$ | 0 | [n, s]n | 8 | 3 | 1 | $\frac{1}{2}$ | 0 | 1868 |
| missing | – | – | 0 | $\frac{1}{2}$ | 1 | [n, n]s | 8 | 6 | 1 | 1 | 0 | 1895 |
| Σ(2000) $S_{11}$ | * | 2000 $\frac{1}{2}^-$ | 1- | $\frac{1}{2}$ | 1 | [n, n]s | 8 | 6 | 1 | 1 | 0 | 1895 |
| Σ*(1385) $P_{13}$ | **** | 1382 - 1388 $\frac{3}{2}^+$ | 0+ | $\frac{3}{2}$ | 1 | {n, n}s | 10 | 6 | 1 | 1 | 0 | 1334 |
| Σ*(1840) $P_{13}$ | * | 1840 $\frac{3}{2}^+$ | 0+ | $\frac{3}{2}$ | 1 | {n, n}s | 10 | 6 | 1 | 2 | 0 | 1439 |
| Σ*(2080) $P_{13}$ | ** | 2080 $\frac{3}{2}^+$ | 0+ | $\frac{3}{2}$ | 1 | {n, n}s | 10 | 6 | 1 | 1 | 1 | 1924 |

TABLE VI: As table V but for Ξ, Ξ* and Ω-type resonances.

$\{n, n\}n$; $\{6 \otimes 3\} 10$: $\left(\frac{1}{2} \otimes \frac{3}{2}\right) \left(\frac{1}{2} \otimes \frac{3}{2}\right)$; (A2b) $\{n, s\}n$; $\{3 \otimes 3\} 8$: $\left(\frac{1}{2} \otimes \frac{1}{2}\right) 0$; $\left(0, \frac{1}{2}\right) \left(\frac{1}{2}\right)$, (A3b)

For Λ-type states one has:

$\{n, n\}s$; $\{3 \otimes 3\} 8$: $(0, 0) 0$; $\left(0, \frac{1}{2}\right) \left(\frac{1}{2}\right)$, (A3a) $\{n, s\}n$; $\{6 \otimes 3\} 8$: $\left(\frac{1}{2} \otimes \frac{1}{2}\right) 0$; $\left(1, \frac{1}{2}\right) \left(\frac{1}{2}\right)$, (A3c)
for $\Sigma$-type states one has:

\begin{align*}
\{n, s\}n; \ (6 \otimes 3) \ 8: \ & \left( \frac{1}{2}; \frac{1}{2} \right) 0; \left( 1, \frac{1}{2} \right) \frac{3}{2} \rangle ; \quad \text{(A3d)} \\
\{n, s\}n; \ (6 \otimes 3) \ 8: \ & \left( \frac{1}{2}; \frac{1}{2} \right) 1; \left( 1, \frac{1}{2} \right) \frac{1}{2} \rangle . \quad \text{(A5d)}
\end{align*}

for $\Lambda$-type states one has:

\begin{align*}
\{n, s\}n; \ (3 \otimes 3) \ 1: \ & \left( \frac{1}{2}; \frac{1}{2} \right) 0; \left( 0, \frac{1}{2} \right) \frac{3}{2} \rangle ; \quad \text{(A4a)} \\
\{n, s\}n; \ (6 \otimes 3) \ 8: \ & \left( \frac{1}{2}; \frac{1}{2} \right) 1; \left( 1, \frac{1}{2} \right) \frac{3}{2} \rangle ; \quad \text{(A5e)}
\end{align*}

for $\Sigma^*$-type states one has:

\begin{align*}
\{n, s\}n; \ (3 \otimes 3) \ 8: \ & \left( \frac{1}{2}; \frac{1}{2} \right) 0; \left( 0, \frac{1}{2} \right) \frac{1}{2} \rangle ; \quad \text{(A4b)} \\
\{n, s\}n; \ (6 \otimes 3) \ 10: \ & \left( 0, \frac{1}{2} \right) \frac{3}{2} \rangle , \quad \text{(A6b)}
\end{align*}

for $\Sigma$-type states one has:

\begin{align*}
\{n, s\}n; \ (3 \otimes 3) \ 8: \ & \left( \frac{1}{2}; \frac{1}{2} \right) 1; \left( 0, \frac{1}{2} \right) \frac{1}{2} \rangle ; \quad \text{(A5a)} \\
\{n, s\}n; \ (6 \otimes 3) \ 10: \ & \left( 1, \frac{1}{2} \right) \frac{3}{2} \rangle , \quad \text{(A6b)}
\end{align*}

\begin{align*}
\{n, s\}n; \ (6 \otimes 3) \ 8: \ & \left( 1, \frac{1}{2} \right) \frac{1}{2} \rangle , \quad \text{(A5b)} \\
\{n, s\}n; \ (6 \otimes 3) \ 10: \ & \left( 1, \frac{1}{2} \right) \frac{3}{2} \rangle , \quad \text{(A6b)}
\end{align*}

\begin{align*}
\{n, s\}n; \ (6 \otimes 3) \ 8: \ & \left( 1, \frac{1}{2} \right) \frac{3}{2} \rangle , \quad \text{(A5c)} \\
\{n, s\}n; \ (6 \otimes 3) \ 10: \ & \left( 1, \frac{1}{2} \right) \frac{3}{2} \rangle , \quad \text{(A6d)}
\end{align*}
for Ξ-type states one has:

\[ |n, s\rangle; \ (3 \otimes 3) 8; \ \left( \frac{1}{2}, 0 \right) \frac{1}{2}; \ (0, \frac{1}{2}) \frac{1}{2} \], \quad (A7a)

\[ |n, s\rangle; \ (6 \otimes 3) 8; \ \left( \frac{1}{2}, 0 \right) \frac{1}{2}; \ (1, \frac{1}{2}) \frac{1}{2} \], \quad (A7b)

\[ |n, s\rangle; \ (6 \otimes 3) 8; \ \left( \frac{1}{2}, 0 \right) \frac{1}{2}; \ (1, \frac{1}{2}) \frac{3}{2} \], \quad (A7c)

\[ |s, s\rangle; \ (6 \otimes 3) 8; \ \left( 0, \frac{1}{2} \right) \frac{1}{2}; \ (1, \frac{1}{2}) \frac{1}{2} \], \quad (A7d)

\[ |s, s\rangle; \ (6 \otimes 3) 8; \ \left( 0, \frac{1}{2} \right) \frac{1}{2}; \ (1, \frac{1}{2}) \frac{3}{2} \], \quad (A7e)

for Ξ*-type states one has:

\[ |n, s\rangle; \ (6 \otimes 3) 10; \ \left( \frac{1}{2}, 0 \right) \frac{1}{2}; \ (1, \frac{1}{2}) \frac{3}{2} \], \quad (A8a)

\[ \{n, s\}; \ (6 \otimes 3) 10; \ \left( \frac{1}{2}, 0 \right) \frac{1}{2}; \ (1, \frac{1}{2}) \frac{3}{2} \], \quad (A8b)

\[ |s, s\rangle; \ (6 \otimes 3) 10; \ \left( 0, \frac{1}{2} \right) \frac{1}{2}; \ (1, \frac{1}{2}) \frac{3}{2} \], \quad (A8c)

\[ |s, s\rangle; \ (6 \otimes 3) 10; \ \left( 0, \frac{1}{2} \right) \frac{1}{2}; \ (1, \frac{1}{2}) \frac{3}{2} \]; \quad (A8d)

finally for Ω-type states the only possibility is:

\[ \{s, s\}; \ (6 \otimes 3) 10; \ (0, 0); \ \left( \frac{1}{2}, \frac{1}{2} \right) \frac{3}{2} \]. \quad (A9)

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