Tracing the Gauge Origin of Yukawa and Higgs Parameters Beyond the Standard Model

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Abstract

We discuss possible realizations of the hypothesis that all the fundamental interactions of the elementary particles should be of gauge type, including the Yukawa and Higgs ones. In the minimal SUSY extension of the standard model, where the quartic Higgs couplings are “gauged” through the D-terms, it is also possible to generate radiatively the Yukawa matrices for the light generations, thus expressing them as functions of gauge couplings. The program can also be applied to the SUSY LR model, where the possibility to induce radiatively the mixing angles, can help to make viable the parity solution to the strong CP problem. The superpotential of the model still includes some non-gauge couplings, namely, the Yukawa constants for the third generation and the trilinear terms $\lambda \chi_L \Phi \chi_R$ and $\lambda' \chi_L^c \Phi \chi_R^c$ involving the Higgs bi-doublet ($\Phi$) and two pairs of doublets ($\chi_L, \chi_R$ and their conjugates). Additional progress to relate these parameters to gauge couplings, can be made by embedding the LR model within a SUSY model $SU(4)_W \times U(1)_{B-L}$ in five dimensions, where the Higgs bi-doublet is identified as the extra component of the 5D gauge field.
1 Introduction

The Standard Model (SM) of the strong and electroweak interactions has met with extraordinary success; it has been tested already at the level of quantum corrections [1]. These corrections give some hints about the nature of the Higgs sector, pointing towards the existence of a relatively light Higgs boson, with a mass of the order of the electroweak scale, $m_{\phi_{SM}} \approx v$ [2]. However, the SM is plagued with aesthetic and naturalness problems, which have motivated the search for theories beyond the SM, where those problems could be solved. In particular, one would like to have an understanding of the SM parameters. These parameters could be classified as follows:

1. Dimensionless gauge parameters, i.e. those associated with the gauge symmetries ($g_1, g_2, g_3$ and $\theta_{QCD}$).

2. The dimensionfull parameter of the Higgs potential $\mu^2$, which fixes the scale of electroweak symmetry breaking (EWSB).

3. Non-gauge dimensionless parameters, i.e. the quartic Higgs coupling ($\lambda$) and the Yukawa matrices ($Y_f, f = u, d, l, \nu$), which are not associated with a known symmetry.

In the light of this classification, we could ask how many “forces” are included in the SM. One may think that it only contains three (gauge) forces. However, from the point of view of quantum field theory, the parameters $\lambda$ and $Y_f$ describe interactions too, i.e. they induce the Higgs self-coupling and the Higgs-fermion vertices. However, these “forces” are not associated with a gauge symmetry, and one may wonder why does nature allows the presence of such arbitrary non-gauge interactions. If the gauge principle has been so successful in describing the electroweak and strong interactions, one could expect that those non-gauge parameters should appear related to the gauge interactions in the theory that will replace the SM.

In this paper, we attempt to explore possible models that could realize this radical form of the gauge principle, where the only allowed parameters should be the gauge coupling constants and possibly gauge-invariant mass terms. Then, all the other interactions that appear to be of non-gauge type, should be derived from the gauge ones. Some of them could still be fundamental and will appear related to gauge forces because of the presence of extra symmetries, others may not be fundamental and would be induced as higher order effects of the gauge fields.

In fact, one of the simplest attempts to solve the problem of quadratic divergences in the SM, through an accidental cancellation [3], implies a relationship between the quartic Higgs coupling and the Yukawa and gauge constants, namely:

$$\lambda = y_t^2 - \frac{1}{8}[3g^2 + g'^2]$$

Unfortunately, this relation implies a Higgs mass $m_\phi = 316$ GeV, that seems already excluded. Nevertheless, this relation makes us suspect that $\lambda$ and $y_t$ could be of fundamental type.

In what follows we shall discuss how to formulate a successful program along these ideas. We shall show that the construction of an extension of the SM that fulfills this program, only requires assembling several theoretical ideas that have appeared in the past,
and will argue that the models discussed in this paper that achieve some success, seem to point towards the existence of both supersymmetry (SUSY) and Extra-dimensions (XD) at scales beyond $O(1)$ TeV. We shall assume that the quartic Higgs couplings and the Yukawa couplings of the third generation are fundamental, and they are directly related to gauge couplings at tree-level, because of supersymmetry and extra-dimensions. On the other hand, the Yukawa matrices for the light generations will be derived from the fundamental couplings, and expressed as functions of the gauge couplings and other parameters of the theory (i.e. ratios of gauge invariant masses).

The first realization of this program, to be discussed in the next section, appears in the minimal SUSY extension of the SM (MSSM). We shall elaborate on how the presence of D-terms allows to relate the quartic Higgs coupling to the SM gauge coupling constants. It will be shown that in this model it is also possible to express the Yukawa couplings for the light generations in terms of gauge couplings, using a radiative mechanism, i.e. from loop diagrams that involve the fermion-fermion-gaugino couplings. This mechanism also allows to express the quark mixing angles in terms of gauge interactions, which then helps to make viable the so called parity solution to the strong CP problem, as given by some left-right SUSY models, which is discussed in section 3. Finally, we shall consider embedding the model in extra-dimensions, where additional progress can be made to express the couplings of chiral superfields that appear in the superpotential, as functions of gauge couplings. This could be accomplished, for instance, within the context of a five dimensional SUSY model $SU(4)_W \times U(1)$, where the Higgs bi-doublet is identified as the extra component of the 5D gauge field.

## 2 Yukawa and Higgs parameters in the MSSM

The minimal implementation of SUSY in fundamental particle physics (MSSM) has met with mixed success. On the positive side one could count: the stabilization of the Higgs mass and Radiative EWSB, the unification of the gauge coupling constants, and the prediction of a Dark matter candidate. Whereas the non-observation (yet) of the superpartners, and the corresponding mechanisms of SUSY breaking and transmission, needed to make them heavy enough, are among its unpleasant aspects.

However, by its own virtues SUSY also solves the problem of the Higgs self coupling (through the D-terms), a success that in our opinion should be counted at the same level as the gauge coupling unification. Furthermore, SUSY also offers some new avenues to discuss the problem of the Yukawa couplings.

### 2.1 The quartic Higgs coupling in the MSSM

In the MSSM, the gauge and Higgs particles are placed in gauge and chiral supermultiplets, respectively. However, because the SUSY formalism requires to have equal bosonic and fermionic degrees of freedom in each supermultiplet, one needs to introduce the auxiliary fields, which can be eliminated by the solution to the equations of motion. For the non-abelian case, the vector multiplet is written as,

$$V^a = (\lambda^a, \psi^a_\mu, D^a) = (\lambda^a, \psi^a_\mu, \sum g\phi^T a \phi)$$

(2)
where $\lambda^a$ denotes the gaugino, the superpartner of the gauge field $v^a_\mu$, $D^a$ is the (gauge) auxiliary field and $T^a$ denote the group generators. The sum runs over all the scalar fields of the model, which opens the window to incorporate Higgs bilinears into the vector multiplet. Then, the quartic Higgs couplings are naturally related to the gauge couplings ($g, g'$); the quartic terms in the Higgs potential will appear as follows:

$$V_4 = \frac{g^2}{4} ((H^*_u r^i H_a)^2 + (H^*_d r^i H_d)^2) + \frac{g'^2}{4} [(H^*_u H_a)^2 - (H^*_d H_d)^2]$$  \hfill (3)

However, the resulting value for the mass of the light Higgs boson predicted by the model, $m_h \simeq m_Z$, is getting into conflict with current Higgs mass bounds ($m_h \geq 115$ GeV), and something should come to the rescue. In first place, large Radiative corrections can make $m_h \simeq 130$ GeV [4], while new gauge contributions could induce an even heavier Higgs mass [5].

The possibility to express the scalar quartic couplings as gauge constants, is valid not only for the Higgs boson, but also for all the scalar superpartners (squarks and sleptons). Furthermore, this property of SUSY is independent of the SUSY soft-breaking, and it survives even in models where the superpartners are very heavy, such as in the more minimal MSSM [6] where they could be of $O(10)$ TeV, or even of $O(M_{pl})$, as in the recently proposed split SUSY models [7].

The quartic couplings among squarks and Higgs bosons contribute to sfermion masses, and its effect could be tested by measuring the mass-difference among scalars that only differ by their gauge quantum numbers, for instance:

$$m^2_{\tilde u_L} - m^2_{\tilde d_L} = \cos 2\beta m^2_{\tilde W} = m^2_{\tilde \nu} - m^2_{\tilde l}$$  \hfill (4)

These mass relations could be tested at the NLC [8], which will be able to verify the generality of the SUSY solution to the problem of relating the quartic scalar couplings to the gauge constants.

### 2.2 Radiative Yukawa couplings

SUSY also has the elements that may allow to express the Yukawa parameters as functions of gauge couplings. Namely, there are certain types of SUSY interactions that involve two fermions and one scalar, the fermion-sfermion-gaugino vertices, which are given in terms of gauge couplings and formally are similar to a Yukawa coupling, in the sense that both involve two fermions and one scalar. Within the MSSM, these couplings can be closed into a loop and generate the Yukawa parameters. Radiative corrections to fermion masses by SUSY loops, were discussed some time ago, both to generate the full fermion masses [9], and to correct the GUT mass relations [10, 11]. More realistic models have been discussed recently [12]. Henceforth, we shall assume that the third generation masses appear at tree-level, while the light masses and mixing angles are the ones that could be generated radiatively.

To describe our method to evaluate the generation of fermion masses through SUSY loop, we start by writing the diagonal masses for d-type quarks in powers of the Cabibbo angle $\lambda = 0.22$, namely:

$$\bar{M}_d = \text{diag}(d\lambda^4, s\lambda^2, 1) \cdot m_b$$  \hfill (5)
where $d, s$ are $O(1)$ coefficients needed to factor out the b-quark mass $m_b$. Then, assuming that the quark mass matrices are orthogonal, i.e. the diagonalizing matrices satisfy $V^T d = V^T L$, the non-diagonal mass matrix for d-type quarks can be expressed as: $M_d = V_{CKM} M_d V_{CKM}^T$. Then, using Wolfenstein parametrization for the CKM matrix one gets:

$$M_d = \begin{pmatrix}
(d + s)\lambda^4 & s\lambda^3 & A\rho\lambda^3 \\
s\lambda^3 & s\lambda^2 & A\lambda^2 \\
A\rho\lambda^3 & A\lambda^2 & 1
\end{pmatrix} m_b,$$

(6)

One can then determine whether the entries in $M_d$ can be induced as a SUSY loop effect. For the quark sector it is enough to consider the gluino-squark loop, which gives:

$$M_d^{\text{rad}} = \frac{2\alpha_s}{3\pi} \left( m_{\tilde{g}} M_{LR}^2 I(x) \right) m_{\tilde{q}}^2,$$

(7)

where the loop integral $I(x)$ depends on the ratio of squark and gluino masses, $m_{\tilde{q}}^2/m_{\tilde{g}}^2$, and is typically of order 0.5. For d-type squarks and sleptons:

$$M_{2 LR}^2 = v [A_f \cos \beta - \mu Y_f \sin \beta]$$

(8)

Given a typical size for SUSY parameters, namely $m_{\tilde{q}} = m_{\tilde{g}} = A_f = O(500)$ GeV, one gets a natural size for the induced quark mass of the order $v \times 10^{-2} \simeq 1$ GeV or less, while for leptons the corresponding value is about 0.1 GeV, which seems suitable to generate masses for the first and second families.

Then, to reproduce the mass hierarchy that appears in equation (6), we shall assume that the trilinear $A$ terms have precisely that hierarchy, and therefore the LR mass term too. Thus, one can write:

$$[M_{2 LR}^2]_{ij} = \gamma_{ij} \lambda^{n_{ij}} m_{\tilde{q}}^2$$

(9)

where $\gamma_{ij}$ are parameters of order one and the integers $n_{ij}$ are chosen to have the same pattern as in equation (6), e.g. $n_{11} = 4, n_{12} = 3$, etc. Then, we have:

$$(M_d^{\text{rad}})^{\text{ij}} = \frac{2\alpha_s I(x) m_{\tilde{g}}}{3\pi} \gamma_{ij} \lambda^{n_{ij}}$$

(10)

In order to get the correct values for a particular mass matrix entry, we have just to verify that the condition $M_d = M_d^{\text{rad}}$ can be satisfied. This will amount to find solutions to the system of equations for the parameters $d, s, A, \rho$ written in terms of the spectrum of SUSY particles and the parameters $\gamma_{ij}$. Indeed, one can see that to get a correct d-quark mass we need to satisfy: $\gamma_{11} m_{\tilde{g}} \approx 6\pi (s + d) m_b/\alpha_s$, which only requires $m_{\tilde{g}} = O(1)$ TeV for $s, d, \gamma_{11} \simeq 1$. For the strange quark one can also get a correct mass with reasonable values of parameters, and similar results hold for the u-, c-quarks and charged leptons [12]. However, one would need a very large gluino mass to generate the b-quark mass, which we did not assume possible anyway.

A complete program to generate all the light fermion masses and the mixing angles, requires that the elements of $M_{2 LR}^2$ and the SUSY parameters needed to generate the correct textures, should not be in conflict with current FCNC bounds [14]. For d- and s-quarks, since only the diagonal elements of $M_{2 LR}^2$ are needed, there are no problems with SUSY-induced FCNC. Furthermore, even when one generates the Cabibbo angle through the 12
element of $M_{LR}^2$, there are no problems neither, because the bounds are of the same order ($\lambda^3$) that we assumed for the 12 entry of $M_{LR}^2$. In addition, one also needs a SUSY breaking scheme that generates the correct pattern of soft-breaking terms. In fact, one would still need to find out how to relate the soft-breaking terms to gauge couplings, but to discuss this problem and its possible solutions we need to address physics at much higher scales, of $O(100)$ TeV in gauge mediated models or even higher, $O(M_{pl})$ in SUGRA models [13]. Recently, radiative fermion masses were studied within a model with $U(2)$ flavor symmetry [12], which also discussed how to get the soft SUSY breaking pattern. Other effects of these SUSY loops for top and Higgs decays are discussed in [15].

Finally, to have complete success one would also like to relate the third generation Yukawa couplings to gauge couplings. For the top quark, its Yukawa constant is so large that it seems difficult to conceive models where it could come from loop effects, since this will require $\gamma_{33}m_{\tilde{g}} = O(100)$ TeV. However, the fact that the top Yukawa is actually of the order of the SM gauge couplings, opens the window to other ideas, such as gauge-Higgs unification in extra dimensions, which will be explored in the final section, before that we shall dwell into the strong CP problem.

3 Radiative Yukawas and the parity solution to the strong CP problem

Despite the success of the MSSM, it says little about other open issues of the SM, such as the flavor and CP problems. The MSSM also bring other problems in its own, such as the mu problem or the lepton (L) and Baryon (B) number non-conservation. Moreover, the non-observation of the light Higgs boson, and the superpartners, have re-introduced fine-tuning problems in the MSSM [16], which have motivated the search for alternatives or extensions of the MSSM.

One of these major problems is associated with the CP-violating parameter $\bar{\theta}$ that characterize the non-trivial vacuum structure of the QCD lagrangian. The term $\bar{\theta}G_{\mu\nu}G^{\mu\nu}$ contributes to the neutron electric dipole moment ($d_n \simeq 5 \times 10^{-16} \bar{\theta}$ e-cm). In order to avoid conflict with current bounds $d_n \leq 6.3 \times 10^{-26}$, one has to assume that $\bar{\theta} \leq 10^{-10}$, which is considered another hierarchy problem [17].

Within the general MSSM, there are a plethora of new phases associated with the gaugino and sfermion soft-breaking lagrangian. In particular, the phase of the gluino mass contributes to the $\bar{\theta}$ parameter, and it has to be highly suppressed too; this is called the SUSY CP problem. Thus, in the MSSM one needs to explain the size of the effective parameter $\bar{\theta}_{eff}$, which includes the following contributions:

$$\bar{\theta}_{eff} = \bar{\theta}_{QCD} + \bar{\theta}_{EW} + \bar{\theta}_{\tilde{g}}$$

(11)

$\bar{\theta}_{QCD}$ arises from QCD instanton effects, $\bar{\theta}_{EW} = arg Det[M_uM_d]$ is associated with chiral symmetry breaking in the electroweak (EW) sector and $\bar{\theta}_{\tilde{g}} = -3arg(m_{\tilde{g}})$ comes from SUSY breaking.

A beautiful solution to this problem can be found within the left-Right (LR) symmetric extension of the MSSM [18]. In this case the presence of the parity symmetry (P) makes the parameter $\bar{\theta}_{QCD} = 0$. In addition, the model requires the quark mass matrices to be hermitian, while the gluino mass is real, and these conditions are sufficient to have $\bar{\theta}_{eff} =$
0 at tree-level [19]. Furthermore, it was shown in ref. [19], that this solution survives at higher orders, since only a safely small $\theta_{\text{eff}}$ is generated at one-loop level.

The SUSY LR model has other virtues, such as making more natural the assumption of $R-$parity conservation in the MSSM, thus protecting the proton from decaying too fast. It also relates the hypercharges of quarks and leptons with the gauged $B - L$ number. The particle content of the model, including the Higgs sector, is shown in Table 1, which displays their quantum numbers under the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$. The breaking of the gauge symmetry can occur in several steps. For instance, one can break first: $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$, using the Higgs doublets $\chi_R, \chi_R^c$. Breaking of the EW symmetry: $SU(2)_L \times U(1)_Y \rightarrow U(1)_{QED}$, can be done with another pair of Higgs doublets $\chi_L$ and $\chi_L^c$. Furthermore, to generate masses for quarks and leptons one needs to include the Higgs bi-doublet ($\Phi$).

|         | $SU(2)_L$ | $SU(2)_R$ | $U(1)_{B-L}$ | $SU(3)_c$ |
|---------|-----------|-----------|-------------|------------|
| $Q_L$   | 2         | 1         | $\frac{2}{3}$ | 3          |
| $Q_R$   | 1         | 3         | $-\frac{1}{3}$ | 3          |
| $L_L$   | 2         | 1         | -1           | 1          |
| $L_R$   | 1         | 2         | 1            | 1          |
| $\Phi$  | 2         | 2         | 0            | 1          |
| $\chi_L$| 2         | 1         | 0            | 1          |
| $\chi_R$| 1         | 2         | 0            | 1          |

However, with a single Higgs bi-doublet the model is unable to generate quark mixing, while neutrinos remain massless. Several solution based on the inclusion of additional Higgs multiplets have been proposed in the literature; for instance adding another bi-doublet can allow the generation of quark mixing [19], while Higgs triplets are used to generate neutrino masses [21].

In this paper we want to keep the minimum number of Higgs multiplets, which should make easier our hope to relate their interactions to gauge couplings. Thus we shall stick to the minimal model, which only includes the Higgs doublets and a single Higgs bi-doublet. This choice assures that the conditions needed to keep the solution to the strong CP problem are respected [19]. The superpotential of the model is then:

$$ W = m_{\chi L} \chi_L^c + m'_{\chi R} \chi_R^c + \lambda_{\chi L} \Phi \chi_R + \lambda'_{\chi L} \Phi \chi_R^c + Y_q Q_L \Phi Q_R + Y_l L_L \Phi L_R $$ (12)

Although $W$ still allows for the presence of the diagonal Yukawa matrices, their number is substantially reduced because the model implies up-down unification ($Y_d = Y_u$). In fact we only need to assume the presence of the Yukawa constants for the third generation, as the light ones could be induced radiatively too. The couplings $\lambda, \lambda'$ are also arbitrary, i.e. of non-gauge type, however, in the next section we shall discuss how to extend the LR model in such a way that they are related to gauge couplings too.

Then, we can use the SUSY loops to generate quark mixing and neutrino masses. Although SUSY loops based on the soft-terms $M^2_{LL}$, have been employed previously in the literature for the SUSY LR model [22], we believe that our formulation, which is based on the soft terms $M^2_{LR}$, allows to write the conditions needed to find a successful model in a simpler and more concise form. In fact, the formulae presented in the previous section are quite general, such that one can get the conditions needed to generate quark mixing.
For instance, to generate the cabibbo angle, one needs to have $M_{d2}^d = s\lambda^3 = M_{rad}$, which then means that the SUSY parameters must satisfy: $\gamma_{12}m_{\tilde{g}} = 3\pi s m_b / \alpha_s$, and again this only requires $m_{\tilde{g}} = O(1)$ TeV. Further, having $M_{LR}^2 \simeq \lambda^3 \tilde{m}^2$, is not in conflict with current bounds on FCNC.

On the other hand, to discuss neutrino masses, one could use the radiative mechanism through the $W_L - W_R$ mixing [23], or alternatively, we can follow the discussion of refs. [24], where it was shown how to induce the Yukawa couplings for the left-handed (LH) neutrinos, using the sneutrino-neutralino loop. Assuming the presence of a trilinear term ($A_\nu$) that induces a mixing among the LH and RH sneutrinos, and a mixing angle ($\delta$) in the RH sneutrino sector, one obtains the following expression for the light neutrino masses,

$$m_\nu \simeq \frac{g^2}{384 \pi^2} \frac{A_\nu^2 v^2 \delta^2}{\tilde{m}_s}$$  \hspace{1cm} (13)

Taking $m_{\tilde{\nu}_L} \simeq m_{\tilde{n}} \simeq m_{\tilde{B}}$ allows to generate neutrino masses of order $0.1 - 1$ eV, which is in the correct range to explain atmospheric neutrino data; similar loop effects can also explain the solar neutrino data. A complete model of neutrino masses along these lines will be presented elsewhere [35].

## 4 Yukawa and Higgs self-couplings from extra-dimensions.

Although a significant reduction of couplings has been achieved in the LR SUSY model discussed in the previous section, we still need to worry about the interaction between the Higgs bi-doublet and the doublets that appear in the superpotential, equation (12), and the large Yukawa coupling for the third generation. We shall show in this section that by embedding the model in extra-dimensions, it is possible to achieve further progress to fulfill our program.

Theories with extra dimensions have received much attention recently, mainly because of the possibility they offer to address the problems of the SM from a new geometrical perspective. These range from a new approach to the hierarchy problem [25, 26] up to a possible explanation of flavor hierarchies in terms of field localization along the extra dimensions [27]. Model with extra dimensions have been applied to neutrino physics [28], GUT models [29] and Higgs phenomenology [30], among many others.

An interesting scenario within the extra-dimensional (XD) approach, consists in identifying the Higgs boson as a component of an XD gauge field [31]. Promising models could be constructed in five and six dimensions, with or without SUSY [32, 33]. Unification of Higgs and matter with the gauge multiplets has also been discussed [34].

To illustrate the idea of symmetry breaking through orbifolds, we shall consider a gauge theory in 5D with a gauge group $G$, which is compactified on a $S^1 / Z_2$ orbifold. The 5D gauge bosons are $A_M = T^A A^A_M$ ($M = (\mu, 5)$). The full gauge symmetry can be broken by the orbifold boundary conditions (O.B.C.):

$$A_\mu(x, y) \rightarrow A_\mu(x, -y) = +PA_\mu(x, y) P^{-1}, \quad (14)$$

$$A_5(x, y) \rightarrow A_5(x, -y) = -PA_5(x, y) P^{-1}, \quad (15)$$

$P$ acts on gauge space as an “inner automorphism”, such that the gauge symmetry is broken: $G \rightarrow H$. Thus, O.B.C. split the group generators into two sets, $T^A = \{ T^a, T^k \}$,
$T^A \epsilon G$, $T^a \epsilon H$. Since $A_\mu^a$ has even $Z_2$-parity, it has zero modes in the spectrum. On the other hand, $A_\mu^k$ has odd $Z_2$-parity, and does not have zero modes in the spectrum. Furthermore, $A_\mu^5$ (odd-odd) has zero modes, and its v.e.v. can break the symmetry further, namely $H \rightarrow H'$.

Within the context of $N = 1$ SUSY models in 5 dimensions, the quartic Higgs couplings are given in terms of gauge constants through the D-terms, while the Yukawa interactions could become arise from the covariant derivative, and therefore can be expressed in terms of gauge constants too. In fact, it is worth mentioning that 5D $N = 1$ SUSY is equivalent to $N = 2$ SUSY, from the 4D perspective, and a pure $N = 2$ SUSY theory only has one gauge coupling constant, although its superpotential includes Yukawa-type interactions.

The SUSY LR model of the previous section could be embedded into a 5D models, One could consider, for instance, a SUSY model with bulk gauge symmetry $SU(4)_W \times U(1)_{B-L}$ [34]. Appropriate orbifold boundary conditions can break the symmetry to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, and the extra-components of the gauge fields include a Higgs bi-doublet $\Phi$, with the right quantum numbers to induce diagonal Yukawa couplings. The interaction of the Higgs bidoublet with matter is given by the 5D SUSY lagrangian, which includes terms like,

$$L_5 = \frac{1}{\sqrt{2}} \int d^2 \theta \Psi_L (\partial_5 - \Phi) \Psi_R$$  \hspace{1cm} (16)$$

where $\Psi_{L,R}$ denote the chiral matter supermultiplets (quarks and leptons).

Furthermore, in order to complete the Higgs sector of the SUSY LR model, we shall assume the presence of additional scalars, namely the extra pairs of Higgs doublets ($\chi_L$, $\chi_R$ and their conjugates) needed to break the LR and EW symmetries. However, the couplings of these fields to the Higgs bi-doublet will also arise from terms similar to those appearing in equation (16). Therefore the couplings will satisfy $\lambda = \lambda'$, and will only depend on the 5D gauge coupling constant ($g_5$) and the compactification radius ($R$). This result will have implications for the Higgs spectrum that will be explored in the future [35]. Thus, by combining SUSY and extra-dimensions we find further progress for our program to relate all the couplings in terms of gauge couplings.

5 Comments and conclusions

In this paper we have proposed a program to achieve a gauge description for all the interactions of the elementary particles, including the Yukawa and Higgs ones. In the minimal SUSY extension of the standard model, we have reviewed how the D-terms allow to express the quartic Higgs couplings in terms of gauge constants. Furthermore, we also argued that the Yukawa matrices for the light generations can be expressed in terms of gauge constants by generating them radiatively (through gaugino-sfermion loops). The SUSY radiative mechanism can in turn help to make viable the parity solution to the strong CP problem. This is realized for instance in the SUSY left-right model with a Higgs sector that includes only a single Higgs bi-doublet ($\Phi$) and two pairs of doublets ($\chi_L$, $\chi_R$ and their conjugates). The superpotential of the model includes two non-gauge couplings, for the trilinear terms $\chi_L \Phi \chi_R$ and $\chi_R^c \Phi \chi_L^c$, which can be “gauged” by embedding the model in extra-dimensions; for instance by working within the context of a SUSY model $SU(4)_W \times U(1)_{L-R}$ in five dimensions, where the Higgs bi-doublet is identified as the extra component of the 5D gauge field.
Thus, one needs both SUSY and extra-dimensions, in order to make further progress to express all the Yukawa and Higgs parameters in terms of gauge couplings. However, the origin of the SUSY soft breaking terms remains as an open problem. This issue may have to wait for some experimental input or further progress on the theoretical side. In this regard, having found some models that show at least some partial progress in expressing most of its couplings as functions of gauge couplings, can be considered as a possible footprint of String Theory at low energies. String Theory would fulfill completely this program, from a top-bottom approach, since it starts with a single coupling at the Planck scale and claims that all other couplings should be derived quantities.

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