Higher derivative corrections to the entropic force from holography

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The entropic force has been recently argued to be responsible for dissociation of heavy quarkonia. In this paper, we analyze \( R^2 \) corrections and \( R^4 \) corrections to the entropic force, respectively. It is shown that for \( R^2 \) corrections, increasing \( \lambda_{GB} \) (Gauss-Bonnet factor) leads to increasing the entropic force. While for \( R^4 \) corrections, increasing \( \lambda \) (‘t Hooft coupling) leads to decreasing the entropic force. Also, we discuss how the entropic force changes with the shear viscosity to entropy density ratio, \( \eta/s \), at strong coupling.

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I. INTRODUCTION

The experimental programs at LHC and RHIC have produced a new state of matter so-called "strongly coupled quark-gluon plasma (sQGP)" \[1–3\]. One of the main experimental signatures for sQGP formation is dissociation of quarkonia \[4\]. It was suggested earlier that the color screening is the main mechanism responsible for this suppression \[3\]. Subsequently, some authors argued that the imaginary part of the heavy quark potential may be a more important reason than screening \[5, 6\]. Recently, it was proposed by D. E. Kharzeev \[9\] that the entropic force would be responsible for melting the quarkonia as well.

The entropic force is related to the increase of the entropy with the separation between the constituents of the bound state. It is an emergent force and does not describe other fundamental interactions. Based on the second law of thermodynamics, it stems from multiple interactions that drive the system toward the state with a larger entropy. The entropic force was developed in \[10\] to explain the elasticity of polymer strands in rubber. Subsequently, Verlinde argued \[11\] that it would be responsible for gravity, but this interesting idea may be controversial (see for \[12\]) and will not be discussed here. Recently, it was argued \[13\] that the entropic force can drive the dissociation process if one considers the process of deconfinement as an entropic self-destruction. This argument is based upon the Lattice results that show that there is a peak in the heavy quark entropy around the crossover region of the sQGP \[13, 14\]. However, it should be noticed that the entropic force cannot be taken as a fundamental property of the system, but it allows us to understand the behavior of complicated microscopic systems not amenable to microscopic treatment. In this work, we will restrict ourselves to its application in dissociation of quarkonia in the sQGP.

AdS/CFT, the duality between a string theory in AdS space and a conformal field theory in the physical space-time, has yielded many important insights for studying different aspects of the sQGP. In this approach, K. Hashimoto et al have carried out the entropic force associated with a heavy quark pair for \( \mathcal{N} = 4 \) SYM theory in their seminal work \[19\]. There, it is found that the peak of the entropy near the transition point is related to the nature of deconfinement and the growth of the entropy with the distance can yield the entropic force. Soon after \[19\], investigations of the entropic force with respect to a moving quarkonium appeared in \[20\]. It is shown that the velocity has the effect of increasing the entropic force thus enhancing the quarkonia dissociation. Recently, we have studied the effect of chemical potential on the entropic force and observed that the chemical potential increases the entropic force implying that the quarkonia dissociation is enhanced at finite density \[21\].

In general, string theory contains higher derivatives corrections due to the presence of stringy effects. Although very little is known about the forms of higher derivative corrections in string theory, given the vastness of the string landscape one may expect that generic corrections do occur \[22\]. As a concrete example, type IIB string theory on \( AdS_5 \times S^5 \) is dual to \( \mathcal{N} = 4 \) SYM theory. Using the relation \( \sqrt{\lambda} = \frac{L \alpha'}{\alpha'} \) (\( L \) is the radius of \( AdS_5 \) and \( \alpha' \) the reciprocal of the string tension), the \( \mathcal{O}(\alpha') \) expansion in type IIB string theory becomes the \( \frac{1}{\sqrt{\lambda}} \) expansion in SYM theory. The leading order corrections in \( 1/\sqrt{\lambda} \) (\( R^4 \) corrections) come from stringy corrections to the type IIB tree level effective

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action of the form $\alpha'^3 R^4$. It was argued $[23, 24]$ that $R^4$ corrections to $\eta/s$ are positive, consistent with the viscosity bound $[24, 26]$. On the other hand, curvature squared interactions (corresponding to $R^2$ corrections) can be induced in the gravity sector in $AdS_5$ by including the world-volume action of D7 branes $[27, 29]$. It was shown $[30, 32]$ that in the five dimensional gravity theories with $R^2$ corrections $\eta/s$ can be lower than $1/(4\pi)$. Also, there are other observables or quantities that have been studied in theories with higher derivative corrections, see e.g. $[33–36]$.

In this paper, we study $R^2$ corrections and $R^4$ corrections to the entropic force. More specially, we would like to see how these corrections affect the entropic force as well as the quarkonia dissociation. On the other hand, $\eta/s$ is different than $1/(4\pi)$ in the theories with higher derivative corrections, so the connection between $\eta/s$ and the entropic force in these theories may be an interesting fact that comes for free in holography. These are the main motivations of the present work.

The organization of the paper is as follows. In section 2, we analyze $R^2$ corrections to the entropic force and explore how these corrections affect the quarkonia dissociation. Also, we discuss how the entropic force changes with $\eta/s$ in this case. In section 3, we investigate $R^4$ corrections to the entropic force as well. Finally, we provide a concluding discussion in section 4.

II. $R^2$ CORRECTIONS

In string theory, the $R^2$ interactions is argued to arise from the world-volume action of D7 branes $[27, 29]$. Restricting to the gravity sector in $AdS_5$, the effective gravity action to leading order can be written as $[30, 31]$

$$I = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[ R + \frac{12}{L^2} + L^2(c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho_1\rho_2} R^{\mu\nu\rho_1\rho_2}) \right],$$

where $G_5$ is the 5 dimensional Newton constant, $R_{\mu\nu\rho_1\rho_2}$ is the Riemann tensor, $R$ is the Ricci scalar, $R_{\mu\nu}$ is the Ricci tensor, $L$ is the radius of $AdS_5$ at leading order in $c_i$ with $\lim_{\lambda \to \infty} c_i = 0$. Other terms with additional derivatives or factors of $R$ are suppressed by higher powers of $\frac{1}{L^4}$. However, at this order only $c_3$ is unambiguous while $c_1$ and $c_2$ can be arbitrarily altered by a field redefinition $[30, 32]$. To avoid this issue, one applies the Gauss-Bonnet (GB) gravity, a special case of the action (1), in which $c_i$ are fixed in terms of a single parameter $\lambda_{GB}$. The GB gravity gives the following action $[37]$

$$I = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[ R + \frac{12}{L^2} + \frac{\lambda_{GB}}{2} L^2(R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho_1\rho_2} R^{\mu\nu\rho_1\rho_2}) \right],$$

where $\lambda_{GB}$ is constrained in

$$-\frac{7}{36} < \lambda_{GB} \leq \frac{9}{100},$$

where the lower bound originates from requiring the boundary energy density to be positive-definite $[38]$ and the upper bound comes from avoiding causality violation in the boundary $[31]$. The black brane solution of GB gravity can be written as $[39]$

$$ds^2 = -a^2 \frac{r^2}{L^2} f(r) dt^2 + \frac{r^2}{L^2} d\vec{x}^2 + \frac{L^2}{r^2} \frac{dr^2}{f(r)},$$

with

$$f(r) = \frac{1}{2\lambda_{GB}} \left[ 1 - \sqrt{1 - 4\lambda_{GB}(1 - \frac{r_h^4}{r^4})} \right],$$

and

$$a^2 = \frac{1}{2}(1 + \sqrt{1 - 4\lambda_{GB}}),$$

where $\vec{x} = x_1, x_2, x_3$ represent the boundary coordinates and $r$ denotes the coordinate of the 5th dimension. The boundary is located at $r = \infty$. The horizon is located at $r = r_h$. Moreover, the temperature is

$$T = \frac{ar_h}{\pi L^2}.$$
It was argued \[30–32\] that
\[
\eta/s = \frac{1}{4\pi}(1 - 4\lambda_{GB}), \tag{8}
\]
one can see that \(\eta/s \geq \frac{1}{4\pi}\) can be violated for \(\lambda_{GB} > 0\). And, by increasing \(\lambda_{GB}\), \(\eta/s\) decreases.

We now follow the calculations of \([19]\) to analyze the entropic force for the background metric \([4]\). The entropic force is defined as \([9]\)
\[
F = T \frac{\partial S}{\partial x}, \tag{9}
\]
where \(T\) is the temperature of the plasma, \(S\) represents the entropy, \(x\) denotes the inter-quark distance.

The Nambu-Goto action is
\[
S_{NG} = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma L = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-\det g_{\alpha\beta}}, \tag{10}
\]
where \(g_{\alpha\beta}\) is the induced metric and parameterized by \((\tau, \sigma)\) on the string world-sheet. \(g_{\mu\nu}\) is the metric, \(X^\mu\) is the target space coordinate.

For our purpose, we take the static gauge
\[
t = \tau, \quad x_1 = \sigma, \tag{12}
\]
and assume that \(r\) depends only on \(\sigma\),
\[
r = r(\sigma). \tag{13}
\]

Under this assumption, the lagrangian density is found to be
\[
L = a \sqrt{\frac{f(r) r^4}{4} + \dot{r}^2}, \tag{14}
\]
with \(\dot{r} = \frac{dr}{d\sigma}\).

Note that \(L\) does not depend on \(\sigma\) explicitly, so the corresponding Hamiltonian is a constant,
\[
\mathcal{L} = \frac{\partial L}{\partial \dot{r}} \dot{r} = \text{constant}. \tag{15}
\]

Imposing the boundary condition at \(\sigma = 0\),
\[
\dot{r} = 0, \quad r = r_c \quad (r_h < r_c), \tag{16}
\]
where \(r = r_c\) is the deepest point of the U-shaped string.

One finds
\[
\frac{f(r) r^4}{\sqrt{f(r) r^4 + L^4 \dot{r}^2}} = \sqrt{f(r_c) r_c^4}, \tag{17}
\]
with
\[
f(r_c) = \frac{1}{2\lambda_{GB}} \left[1 - \sqrt{1 - 4\lambda_{GB}(1 - \frac{r_h^4}{r_c^4})}\right], \tag{18}
\]
results in
\[
\dot{r} = \frac{dr}{d\sigma} = \sqrt{\frac{r^4 f(r) [r^4 f(r) - r_c^4 f(r_c)]}{L^4 r_c^4 f(r_c)}}. \tag{19}
\]
FIG. 1: Left: $xT$ versus $\epsilon$ for different $\lambda_{GB}$. From top to bottom $\lambda_{GB} = -0.1, 0.01, 0.06$ respectively. Right: $S^{(2)}/\sqrt{\lambda}$ versus $xT$ for different $\lambda_{GB}$. From right to left $\lambda_{GB} = -0.1, 0.01, 0.06$ respectively.

By integrating (19) the inter-quark distance of $Q\bar{Q}$ is obtained

$$x = 2\int_{r_c}^{\infty} dr \sqrt{\frac{L^4 r_c^4 f(r_c)}{r^4 f(r) [r^4 f(r) - r_c^4 f(r_c)]}}.$$  \hfill (20)

To study the effect of $R^2$ corrections on the inter-distance, we plot $xT$ versus $\epsilon$ with $\epsilon \equiv r_h/r_c$ for different $\lambda_{GB}$ in the left panel of Fig.1. In the plots from top to bottom $\lambda_{GB} = -0.1, 0.01, 0.06$ respectively. We can see that for each plot there exists a maximum value of $xT$, and that $xT$ is an increasing function of $\epsilon$ for $xT < xT_{\text{max}}$ but a decreasing one for $xT > xT_{\text{max}}$. In fact, for the later case, one needs to consider some new configurations [40] which are not solutions of the Nambu-Goto action. Here we are interested mostly in the region of $xT < xT_{\text{max}}$. For convenience, we write $c \equiv xT_{\text{max}}$. From the left panel of Fig.1, one also finds that increasing $\lambda_{GB}$ leads to decreasing $c$. Therefore, one concludes that with increasing $\lambda_{GB}$ the inter-distance decreases, similarly to what occurred in [35].

The next step is to calculate the entropy $S$, given by

$$S = -\frac{\partial F}{\partial T},$$  \hfill (21)

where $F$ is the free energy of $Q\bar{Q}$. This quantity has been studied from the AdS/CFT correspondence, see e.g. [41–43]. There are two cases for the free energy.

1. If $x > \frac{c}{T}$, the fundamental string breaks in two disconnected strings implying the quarks are completely screened. In this case, the free energy is

$$F^{(1)} = \frac{a}{4\pi\alpha'}\int_{r_h}^{\infty} dr.$$  \hfill (22)

In terms of (21), one gets

$$S^{(1)} = a\sqrt{\lambda}\theta(L - \frac{c}{T}).$$  \hfill (23)

2. If $x < \frac{c}{T}$, the fundamental string is connected. In this case, the free energy is actually the total energy of the quark pair which can be derived from the on-shell action of the fundamental string in the dual geometry. Plugging (19) into (10), one finds

$$F^{(2)} = \frac{1}{4\pi\alpha'}\int_{r_c}^{\infty} dr \sqrt{\frac{a^2 r_c^4 f(r)}{r^4 f(r) - r_c^4 f(r_c)}}.$$  \hfill (24)

As $r_h$ is related to $T$, one can rewrite (21) as

$$S = \frac{\partial F}{\partial T} = \frac{\partial F}{\partial r_h} \frac{\partial r_h}{\partial T} = -\frac{\pi L^2}{a} \frac{\partial F}{\partial r_h}.$$  \hfill (25)
By virtue of \[25\], one gets
\[
S^{(2)} = - \frac{1}{2a} \int_{r_c}^{\infty} dr \frac{[a'(r)b(r) + a(r)b'(r)][a(r) - a(r_c)] - a(r)b(r)[a'(r) - a'(r_c)]}{\sqrt{a(r)b(r)[a(r) - a(r_c)]}},
\] (26)
with
\[
a(r) = \frac{a^2 f(r)r^4}{L^4}, \quad a(r_c) = \frac{a^2 f(r_c)r^4}{L^4}, \quad b(r) = a^2,
\] (27)
where we have used the relation \( a' = \frac{L^2}{\sqrt{\lambda}} \). Also, the derivatives in the above equation are with respect to \( r_h \).

To proceed further we have to resort to numerical methods. In the right panel of Fig 1, we plot \( S^{(2)}/\sqrt{\lambda} \) versus \( xT \) for different \( \lambda_{GB} \). In the plots from right to left \( \lambda_{GB} = -0.1, 0.01, 0.06 \), respectively. From the figures, one can see that increasing \( \lambda_{GB} \) leads to larger entropy at small distances. On the other hand, from \( \lambda \) one finds that the entropic force is related to the growth of the entropy with the distance. As a result, increasing \( \lambda_{GB} \) the quarkonia dissociation is enhanced. Interestingly, it was argued \[23\] that increasing \( \lambda_{GB} \) leads to increasing the imaginary potential thus making the quarkonia melt easier, consistent with the findings here.

Also, it follows from \[25\] that increasing \( \lambda_{GB} \) leads to decreasing \( \eta/s \). While increasing \( \lambda_{GB} \) leads to enhancing the quarkonia dissociation. Thus, one concludes that in the case of \( R^4 \) corrections the quarkonia dissociation is enhanced as \( \eta/s \) decreases.

### III. \( R^4 \) Corrections

In this section we study \( R^4 \) corrections to the entropic force. These corrections are related to \( \alpha' \) corrections on the string theory side \[44, 45\] and correspond to leading order correction in \( 1/\lambda \) on the gauge theory side. The \( \alpha' \)-corrected metric is given by \[44, 45\]
\[
ds^2 = G_{tt} dt^2 + G_{xx} dx^2 + G_{rr} dr^2,
\] (28)
with
\[
G_{tt} = -r^2(1 - w^{-4})T(w), \quad G_{xx} = r^2 X(w), \quad G_{rr} = r^{-2}(1 - w^{-4})^{-1}U(w),
\] (29)
where
\[
T(w) = 1 - k(75w^{-4} + \frac{1225}{16}w^{-8} - \frac{695}{16}w^{-12}) + ..., \quad X(w) = 1 - \frac{25k}{16}w^{-8}(1 + w^{-4}) + ..., \quad U(w) = 1 + k(75w^{-4} + \frac{1175}{16}w^{-8} - \frac{4585}{16}w^{-12}) + ...
\] (30)
with \( w = \frac{r}{r_h} \).

The parameter \( k \) is related to \( \lambda \) by
\[
k = \frac{\zeta(3)}{8} \lambda^{-3/2} \sim 0.15\lambda^{-3/2}.
\] (31)

The horizon is \( r = r_h \) and the temperature is
\[
T = \frac{r_h}{\pi L^2 (1 - k)}.
\] (32)

In addition, it was argued \[23, 24\] that
\[
\eta/s = \frac{1}{4\pi}(1 + \frac{135}{8}\zeta(3)\lambda^{-3/2}),
\] (33)
one can see that \( \eta/s \geq 1/4\pi \) remains valid in theories with \( R^4 \) corrections. Also, decreasing \( \lambda \) leads to increasing \( \eta/s \).
The next analysis is almost parallel to the previous section, so we present the final results here. One finds

$$x = 2 \int_{r_c}^{\infty} dr \frac{a(r_c)b(r)}{a(r)^3 - a(r)a(r_c)}, \quad (34)$$

with

$$a(r) = r^4(1 - w^{-4})T(w)X(w), \quad a(r_c) = r_c^4(1 - w_1^{-4})T(w_1)X(w_1), \quad b(r) = T(w)U(w), \quad (35)$$

where \( w_1 = \frac{\lambda}{r_h} \), \( T(w_1) = T(w)|_{w=w_1} \), \( X(w_1) = X(w)|_{w=w_1} \).

Likewise, to analyze \( R^4 \) corrections to the inter-distance, we plot \( xT \) versus \( \varepsilon \) with different \( \lambda \) in the left panel of Fig.2. Note that the behavior of \( R^4 \) corrections is not the same as the \( R^2 \) corrections. Here one can see that the value of \( xT_{\max} \) increases as \( \lambda \) increases, in agreement with the findings of [34].

On the other hand, the free energy \( F^{(2)} \) is found to be

$$F^{(2)} = \frac{1}{\pi \alpha'} \int_{r_c}^{\infty} dr \frac{a(r)b(r)}{a(r)^3 - a(r)a(r_c)}. \quad (36)$$

After some manipulations, one finds

$$\frac{S^{(2)}}{\sqrt{\lambda}} = \frac{k - 1}{2} \int_{r_c}^{\infty} dr \frac{[a(r)b(r) + a(r)b'(r)][a(r) - a(r_c)] - a(r)b(r)[a'(r) - a'(r_c)]}{a(r)b(r)[a(r) - a(r_c)]^2}, \quad (37)$$

with

$$a'(r) = r^4(T'(w)X(w) + T(w)X'(w)) + 4w^{-5}w'T(w)X(w) - w^{-4}T'(w)X(w) - w^{-4}T(w)X'(w),$$

$$a'(r_c) = r_c^4(T'(w_1)X(w_1) + T(w_1)X'(w_1)) + 4w_1^{-5}w_1'T(w_1)X(w_1) - w_1^{-4}T'(w_1)X(w_1) - w_1^{-4}T(w_1)X'(w_1),$$

$$b'(r) = T'(w)U(w) + T(w)U'(w). \quad (38)$$

and

$$T'(w) = 300kw^{-5}w' + \frac{1225}{2}kw^{-9}w' - \frac{2085}{4}kw^{-13}w',$$

$$T'(w_1) = 300kw_1^{-5}w_1' + \frac{1225}{2}kw_1^{-9}w_1' - \frac{2085}{4}kw_1^{-13}w_1',$$

$$X'(w) = \frac{25}{4}w^{-9}w' + \frac{75}{4}w^{-13}w',$$

$$X'(w_1) = \frac{25}{4}w_1^{-9}w_1' + \frac{75}{4}w_1^{-13}w_1',$$

$$U'(w) = -300kw^{-5}w' - \frac{1175}{2}kw^{-9}w' + \frac{13755}{4}kw^{-13}w', \quad (39)$$

where the derivatives are with respect to \( r_c \).

Note that (37) is complicated and one needs to resort to numerical methods. In the right panel of Fig.2, we plot \( S^{(2)}/\sqrt{\lambda} \) versus \( xT \) for different \( \lambda \). From the figures, one can see that increasing \( \lambda \) leads to smaller entropy at small distances, which means the entropic force decreases as \( \lambda \) increases. In other words, decreasing \( \lambda \) enhances quarkonia dissociation. Interestingly, it was argued that [34] decreasing \( \lambda \) leads to larger imaginary potential or smaller dissociation length, consistent with the findings here. Moreover, it follows from (33) that decreasing \( \lambda \) leads to increasing \( \eta/s \). Thus, one concludes that in the case of \( R^4 \) corrections the quarkonia dissociation is enhanced as \( \eta/s \) increases.

However, it should be emphasized that without computing at least the next-to-leading order corrections in the ’t Hooft coupling one can not assure that it is physically meaningful to go all the way down from infinity coupling to \( \lambda \sim 5.5 \) by just considering \( R^4 \) corrections.

IV. CONCLUSION

The entropic force may represent a mechanism for melting the heavy quarkonia. In this paper, we studied the effects of higher derivative corrections to the entropic force and discussed how the entropic force changes with \( \eta/s \) at strong
coupling. It is shown that for $R^2$ corrections, increasing $\lambda_{GB}$ leads to increasing the entropic force thus enhancing the quarkonia dissociation. While for $R^4$ corrections, increasing $\lambda$ leads to decreasing the entropic force thus suppressing the quarkonia dissociation. It is found that for $R^2$ corrections the entropic force is enhanced as $\eta/s$ decreases, while for $R^4$ corrections the entropic force is enhanced as $\eta/s$ increases. Namely, $R^2$ corrections affect the entropic force in the opposite way of $R^4$ corrections. This is conceivable, because $R^2$ corrections are of different nature than $R^4$ corrections (for the origin of the two corrections, see [25, 27]). In fact, a similar problem has been explained in the study of $\eta/s$ [27]. Therein, it was argued that in certain regimes of the parameter space, i.e., $\lambda = 6\pi, N_c = 3$, it is not unreasonable to include both $R^2$ corrections and $R^4$ corrections as making independent and comparable contributions to the CFT properties.

Certainly, one may doubt why the entropic force is the correct (and useful) approach to understand dissociation of quarkonia. Although we cannot provide a clear interpretation at present, we believe that the entropic self destruction is an intriguing idea and worth studying. Actually, in some sense one can check the effectiveness of this idea by comparing the same effect on the entropic force and with that on the imaginary potential. To our knowledge, the velocity effect [20, 46, 47], the chemical potential effect [21], $R^2$ corrections [35] and $R^4$ corrections [34] on the two quantities give consistent results regarding the quarkonia dissociation. These agreements support that if the imaginary potential is right the entropic force may be also effective.

Finally, it should be noticed that the background considered here is a purely gravitational background with no matter fields in the bulk and no dynamical breaking of the conformal symmetry. It is of great interest to pursue in the investigations performed on top of phenomenologically realistic gauge/gravity backgrounds. We hope to report our progress in this regard in the future.

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[1] J. Adams et al. [STAR Collaboration], Nucl. Phys. A 757, 102 (2005).
[2] K. Adcox et al. [PHENIX Collaboration], Nucl. Phys. A 757, 184 (2005).
[3] E. V. Shuryak, Nucl. Phys. A 750, 64 (2005).
[4] Karen M. Burke, et al, [JET Collaboration], Phys. Rev. C 90, 014909 (2014).
[5] T. Matsui, H. Satz, Phys. Lett. B 178, 416 (1986).
[6] M. Laine, O.Philipsen, P. Romatschke and M. Tassler, JHEP 03 (2007) 054
[7] A. Beraudo, J.-P. Blaizot, and C. Ratti, Nucl. Phys. A 806, 312 (2008).
[8] N. Brambilla, J. Ghiglieri, A. Vairo, and P. Petreczky, Phys. Rev. D 78, 014017 (2008).
[9] D. E. Kharzeev, Phys. Rev. D 90, 074007 (2014).
[10] K. H. Meyer, G. Susich, and E. Valk, Kolloid Z. 59, 208 (1932).
[11] E. P. Verlinde, JHEP 04 (2011) 029.
[12] D.-C. Dai and D. Stojkovic, JHEP 11 (2017) 007.
[13] O. Kaczmarek, F. Karsch, P. Petreczky and F. Zantow, Phys. Lett. B, 543, 41 (2002).
[14] O. Kaczmarek and F. Zantow, [hep-lat/0506019].
[15] P. Petreczky and K. Petrov, Phys. Rev. D 70, 054503 (2004).
[16] J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998).
[17] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. B 428, 105 (1998).
[18] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. 323, 183 (2000).
[19] K. Hashimoto and D. E. Kharzeev, Phys. Rev. D 90, 125012 (2014).
[20] K. B. Fadafan, S. K. Tabatabaei, Phys. Rev. D 94, 026007 (2016).
[21] Z. Zhang, D. Hou, and G. Chen, Phys. Lett. B 768, 180 (2017).
[22] M.R. Douglas and S. Kachru, Rev. Mod. Phys. 79 (2007) 733.
[23] A. Buchel, J. T. Liu, and A. O. Starinets, Nucl. Phys. B 707, 56 (2005).
[24] P. Benincasa and A. Buchel, JHEP 01 (2006) 103.
[25] A. Buchel, R. C. Myers, M. F. Paulos, and A. Sinha, Phys. Lett. B 669, 364 (2008).
[26] R. C. Myers, M. F. Paulos, and A. Sinha, Phys. Rev. D 79, 041901 (2009).
[27] A. Buchel, R. C. Myers and Aninda Sinha, JHEP 03 (2009) 084.
[28] O. Aharony, J. Pawelczyk, S. Theisen, and S. Yankielowicz, Phys. Rev. D 60, 066001 (1999).
[29] O. Aharony and Y. Tachikawa, JHEP 01 (2008) 037.
[30] M. Brigante, H. Liu, R.C. Myers, S. Shenker, S.Yaida, Phys.Rev.D 77, 126006 (2008).
[31] M. Brigante, H. Liu, R.C. Myers, S. Shenker, and S. Yaida, Phys. Rev. Lett. 100, 191601 (2008).
[32] Y. Kats and P. Petrov, JHEP 01 (2009) 044.
[33] J. Noronha and Adrian Dumitru, Phys. Rev. D 80 014007 (2009).
[34] K.B. Fadafan and S.K. Tabatabaei, Eur. Phys. J. C 74 2842 (2014).
[35] S.I. Finazzo and J. Noronha, JHEP 11 (2013) 042.
[36] Z. Zhang, D. Hou, Y. Wu and G. Chen, Adv. High Energy Phys, 2016, 9503491(2016).
[37] B. Zwiebach, Phys. Lett. B 156, 315 (1985).
[38] D.M. Hofman, J. Maldacena, JHEP 05 (2008) 012.
[39] R.G. Cai, Phys. Rev. D 65, 084014 (2002).
[40] D. Bak, A. Karch, L. G. Yaffe, JHEP 08 (2007) 049.
[41] J.M. Maldacena, Phys. Rev. Lett. 80, 4859 (1998).
[42] A. Brandhuber, N. Izhaki, J. Sonnenschein and S. Yankielowicz, Phys. Lett. B 434, 36 (1998).
[43] S.J. Rey, S. Theisen and J.T. Yee, Nucl. Phys. B 527, 171 (1998).
[44] J. Pawelczyk, S. Theisen, JHEP 09 (1998) 010 .
[45] S. S. Gubser, I. R. Klebanov, A. A. Tseytlin, Nucl. Phys. B 534, 202 (1998).
[46] S. I. Finazzo, J. Noronha, JHEP 01 (2015) 051.
[47] M. A. Akbari, D. Giataganas and Z. Rezaei, Phys. Rev. D 90, 086001 (2014).