Determining the structure of $X(3872)$ in heavy ion collisions

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Abstract.

We study the time evolution of the $X(3872)$ abundance in the hot hadron gas produced in the late stage of heavy ion collisions. We use effective field Lagrangians to obtain the production and dissociation cross sections of $X(3872)$. In this evaluation we include diagrams involving the anomalous couplings $\pi D^* \bar{D}$ and $X \bar{D} D^*$ and also the couplings of the $X(3872)$ with charged $D$ and $D^*$ mesons. With these new terms the $X(3872)$ interaction cross sections are much larger than those found in previous works. Using these cross sections as input in rate equations, we conclude that during the expansion and cooling of the hadronic gas, the number of $X(3872)$, originally produced at the end of the mixed QGP/hadron gas phase, is reduced by a factor of 4.

1. Introduction

Several new charmonium states have been observed [1, 2, 3, 4] during the last decade. Among these new states, the most well known is the $X(3872)$. The structure of this new state has been subject of an intense debate. Calculations using constituent quark models give masses for possible charmonium states with $J^{PC} = 1^{++}$ quantum numbers, which are too large: $2^3 P_1(3990)$ and $3^3 P_1(4290)$ [5]. These results motivated the conjecture that these objects are multiquark states. The vicinity of the $X$ mass to the $\bar{D}D^*$ threshold inspired the proposal that the $X(3872)$ could be a molecular $\bar{D}D^*$ bound state with a small binding energy [6, 7]. Another interpretation of the $X(3872)$ is that it can be a tetraquark state resulting from the binding of a diquark and an antidiquark [8, 9]. There are other proposals as well [2, 3, 4, 10, 11].

Hadronic collisions seem to be a new testing ground for ideas about the structure of the new states. It has been shown [3, 12] that it is extremely difficult to produce meson molecules in $p p$ collisions. In the molecular approach the estimated cross section for $X(3872)$ production is two orders of magnitude smaller than the measured one. On the other hand, in Ref. [13] a simple model was proposed to compute the $X$ production cross section in $p p$ collisions in the tetraquark approach.

As pointed out in Refs. [14, 15], high energy heavy ion collisions offer an interesting scenario to study the production of multiquark states. In these processes, a quite large number of heavy
quarks is expected to be produced, reaching as much as $20 \, c\bar{c}$ pairs per unit rapidity in Pb + Pb collisions at the LHC. If the production mechanism is coalescence, then the production yield of these exotic hadrons at the moment of their formation strongly reflects their internal structure. In particular, it was shown that in the coalescence model the production yield of the $X(3872)$, at RHIC or LHC energies, is almost 20 times bigger for a molecular structure than for a tetraquark configuration.

In the last phase of heavy ion collisions, the $X(3872)$ interacts with other hadrons during the expansion of the hadronic matter. It can be destroyed in collisions with the comoving light mesons and it can also be produced through the inverse reactions.

The interactions of the $X$ in a hadronic medium have been studied in the framework of $SU(4)$ effective Lagrangians in Ref. [16], where it was found that the absorption cross section is two orders of magnitude larger than the production one. In particular, it was found that $<\sigma v>_{X\to D^+D^*} \simeq 30 <\sigma v>_{D^+D^*\to X}$. In spite of this difference, the authors of Ref. [16] arrived at the intriguing conclusion that the number of $X$’s stays approximately constant during the hadronic phase. In Ref. [16] the coupling of the $X(3872)$ with charged charm mesons (such as $D^-D^{*+}$) was neglected and only neutral mesons were considered (such as $D^0\bar{D}^{0*}$). Moreover, the terms with anomalous couplings were not included in the calculations. In Ref. [17] we showed that the inclusion of the couplings of the $X(3872)$ to charged $D$’s and $D^*$’s and those of the anomalous vertices, $\pi D^0 D^*$ and $X D^* D^*$, increases the cross sections by more than one order of magnitude. Similar results were also observed in the case of $J/\psi \pi$ cross section [18]. These anomalous vertices also give rise to new reaction channels, namely, $\bar{D} + D^* \to \pi + X$ and $\pi + X \to \bar{D} + D^*$. Thus it is important to evaluate the changes that the above mentioned contributions can produce in the $X$ abundance (and in its time evolution) in reactions as those considered in Ref. [16]. This was done in [19].

The formalism used in Refs. [16] and [17] was originally developed to study the interaction of the $J/\psi$ with light mesons in a hot hadron gas many years ago [18, 20].

2. Thermal cross sections

In this section we calculate the cross sections averaged over the thermal distributions for the processes $D\bar{D} \to \pi X$, $D^* D \to \pi X$ and $D^{**} D^* \to \pi X$, and for the inverse reactions. In Fig. 1 we show the different diagrams contributing to each process. In Ref. [16] it was shown that the contribution from the reactions involving the $\rho$ meson is very small compared to the reactions with pions and thus we neglect the former in what follows. The cross sections for the processes shown in Fig. 1 were obtained in Ref. [17]. For more details about the calculations, we refer the reader to Ref. [17]. As explained in that paper, we use monopole form factors to obtain the cross sections of the processes in Fig. 1. The thermally averaged cross section for a process $ab \to cd$ can be calculated using the expression

$$
\langle\sigma_{ab\to cd} v_{ab}\rangle = \frac{\int d^3p_a d^3p_b f_a(p_a) f_b(p_b) \sigma_{ab\to cd} v_{ab}}{\int d^3p_a d^3p_b f_a(p_a) f_b(p_b)}
$$

where $f_a$ and $f_b$ are Bose-Einstein distributions, $\sigma_{ab\to cd}$ are the cross sections computed in [17], $v_{ab}$ represents the relative velocity of the two interacting particles ($a$ and $b$).

In Fig. 2 we show the total thermally averaged cross sections for the processes involving the production of the $X(3872)$ state, i.e., $D\bar{D} \to \pi X$, $D^* D \to \pi X$ and $D^{**} D^* \to \pi X$ reactions, while in Fig. 3 we show the inverse processes, i.e., the dissociation of $X(3872)$ through the reactions $\pi X \to \bar{D} D$, $\pi X \to D^* D$, $\pi X \to D^{**} D^*$, respectively. For the latter cases, we use the principle of detailed balance to determine the corresponding cross sections. Figs. 2 and 3 should be compared with the Fig. 3 of Ref. [16]. Our cross sections are a factor 100 larger in
Figure 1. Diagrams contributing to the processes $\bar{D}D \rightarrow \pi X$ [(a) and (b)], $\bar{D}^*D \rightarrow \pi X$ [(c) and (d)] and $\bar{D}^*D^* \rightarrow \pi X$ [(e), (f), (g) and (h)]. The filled box in the diagrams (d), (g) and (h) represents the anomalous vertex $XD^*\bar{D}^*$, which was evaluated in Ref. [17].

reactions with $D^*\bar{D}^*$ in the initial or final state. This can be attributed mostly to the inclusion of the anomalous terms. Moreover, our cross sections are a factor 10 larger in the case of $DD$ mesons in the initial or final state. The difference comes from the inclusion of the coupling of the $X$ to charged charged $D$'s and $D^*$'s, which was not considered in Ref. [16].

3. Time evolution of the $X(3872)$ abundance

In order to compute the time evolution of the $X$ abundance, we need to know how the temperature changes with time and this is highly model dependent. Fortunately, as one can see in Figs. 2 and 3 the dependence of $<\sigma v>$ on the temperature is relatively weak.

We study the yield of $X(3872)$ in central Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV at RHIC. By using the thermally averaged cross sections obtained in the previous section, we can now analyze the time evolution of the $X(3872)$ abundance in hadronic matter, which depends on the densities and abundances of the particles involved in the processes of Fig. 1, as well as the cross sections associated with these reactions (and the corresponding inverse reactions), Figs. 2 and 3. The momentum-integrated evolution equation has the form [16, 21, 22, 23]

$$\frac{dN_X(\tau)}{d\tau} = R_{QGP}(\tau) + \sum_{c,c'} \left[ \langle \sigma_{cc' \rightarrow \pi_Xv_{cc'}} \rangle n_c(\tau) N_{c'}(\tau) - \langle \sigma_{\pi_X \rightarrow cc'v_{\pi X}} \rangle n_{\pi}(\tau) N_X(\tau) \right],$$

(2)

where $N_X(\tau), N_c(\tau), n_c(\tau)$ and $n_{\pi}(\tau)$ are the abundances of $X(3872)$, of charmed mesons of type $c'$, of charmed mesons of type $c$ and of pions at proper time $\tau$, respectively. The term
Figure 2. Thermally averaged cross sections.
a) \( \bar{D}D \to \pi X(3872) \) (dashed line), \( \bar{D}^*D \to \pi X(3872) \) (dark-shaded region) and \( \bar{D}^*D^* \to \pi X(3872) \) (light-shaded region).

Figure 3. Thermally averaged cross sections.
\( \pi X(3872) \to \bar{D}D \) (dashed line), \( \pi X(3872) \to \bar{D}^*D \) (dark-shaded region) and \( \pi X(3872) \to \bar{D}^*D^* \) (light-shaded region).

\( R_{QGP}(\tau) \) in Eq. (2) represents the \( X \) yield from the quark-gluon plasma in the mixed phase, since the hadronization of the QGP takes a finite time, and it is given by \([16, 23]\):

\[
R_{QGP}(\tau) = \begin{cases} 
N_X^0 & \text{if } \tau_C < \tau < \tau_H \\
0 & \text{otherwise} 
\end{cases}
\]

(3)

where the times \( \tau_C = 5.0 \text{ fm/c} \) and \( \tau_H = 7.5 \text{ fm/c} \) determine the beginning and the end of the mixed phase respectively. The constant \( N_X^0 \) corresponds to the total number of \( X(3872) \) produced from quark-gluon plasma. To solve Eq. (2) we assume that the pions and charmed mesons in the reactions contributing to the abundance of \( X \) are in equilibrium. Therefore \( N_{c'}(\tau) \), \( n_c(\tau) \) and \( n_\pi(\tau) \) can be written as \([16, 21, 22, 23]\)

\[
N_{c'}(\tau) \approx \frac{1}{2\pi^2} \gamma_c g_D m_{D(*)}^2 T(\tau) V(\tau) K_2 \left( \frac{m_{D(*)}}{T(\tau)} \right),
\]

\[
n_c(\tau) \approx \frac{1}{2\pi^2} \gamma_c g_D m_{D(*)}^2 T(\tau) K_2 \left( \frac{m_{D(*)}}{T(\tau)} \right),
\]

\[
n_\pi(\tau) \approx \frac{1}{2\pi^2} \gamma_\pi g_\pi m_\pi^2 T(\tau) K_2 \left( \frac{m_\pi}{T(\tau)} \right),
\]

(4)

where \( \gamma_i \) and \( g_i \) are the fugacity factor and the spin degeneracy of particle \( i \) respectively. As can be seen in Eq. (4), the time dependence in Eq. (2) enters through the parametrization of the temperature \( T(\tau) \) and volume \( V(\tau) \) profiles suitable to describe the dynamics of the hot hadron gas after the end of the quark-gluon plasma phase.

Following Refs. \([16, 22, 23]\), we assume the \( \tau \) dependence of \( T \) and \( V \) to be given by \([16, 22, 23]\)

\[
T(\tau) = T_C - (T_H - T_F) \left( \frac{\tau - \tau_H}{\tau_F - \tau_H} \right) \frac{4}{5},
\]

\[
V(\tau) = \pi \left[ R_C + v_C (\tau - \tau_C) + \frac{\alpha_C}{2} (\tau - \tau_C)^2 \right]^2 \tau_C.
\]

(5)
In the above equation \( R_C = 8.0 \) fm denotes the final size of the quark-gluon plasma, while \( v_C = 0.4 c \) and \( a_C = 0.02 c^2/\text{fm} \) are its transverse flow velocity and transverse acceleration at \( \tau_C \) respectively. The critical temperature of the (quark gluon plasma to hadronic matter) transition is \( T_C = 175 \) MeV; \( T_H = T_C = 175 \) MeV is the temperature of the hadronic matter at the end of the mixed phase. The freeze-out takes place at the freeze-out time \( \tau_F = 17.3 \) fm/c, when the temperature drops to \( T_F = 125 \) MeV.

To solve Eq. (2), we assume that the total number of charm quarks in charm hadrons is conserved during the production and dissociation reactions, and that the total number of charm quark pairs produced at the initial stage of the collisions at RHIC is 3, yielding the charm quark fugacity factor \( \gamma_C \approx 6.4 \) in Eq. (4) \[16, 15\]. In the case of pions, we follow Ref. \[23\] and work with the assumption that their total number at freeze-out is 926, which fixes the value of \( \gamma_\pi \) appearing in Eq. (4) to be \( \approx 1.725 \).

Assuming that the state in question is a tetraquark with \( J^{PC} = 1^{++} \), the number of \( X(3872) \) produced at the end of the mixed phase is \[15\]:

\[
N_{X(4q)}^0 = N_{X(4q)}(\tau_H) \approx 4.0 \times 10^{-5}.
\] (6)

In order to determine the time evolution of the \( X(3872) \) abundance we solve Eq. (2) starting at the end of the mixed phase, i.e. at \( \tau_H = 7.5 \) fm/c, and assuming that the \( X(3872) \) is a tetraquark. The initial condition is given by Eq. (6). We use this initial abundance to integrate Eq. (2) and we show the result in Fig. 4. In the figure the solid line represents the result obtained using the same approximations as those made in Ref. \[16\]. Our curve is slightly different from that of Ref. \[16\] because we did not include the contribution of the \( \rho \) mesons, as discussed earlier. The dashed line shows the result when we include the couplings of the \( X(3872) \) to charged \( D \)'s and \( D^* \)'s. The light-shaded band shows the results obtained with the further inclusion of the diagrams containing the anomalous vertices. The band reflects the uncertainty in the \( XD^{*}D^{*} \) coupling constant, which is \( g_{XDD^{*}D^{*}} = 12.5 \pm 1.0 \) \[17\].

As can be seen, without the inclusion of the anomalous coupling terms, the abundance of \( X \) remains basically constant. This is because the magnitude of the thermally averaged cross sections for the \( X \) production and absorption reactions obtained within this approximation is so small that the second term in the right hand side of Eq. (2) is completely negligible compared to the first term. When including the coupling of the \( X \) to charged \( D \)'s and \( D^* \)'s we basically do not find any important change for the time evolution of the \( X \) abundance, since, as it was shown in \[19\], the thermally averaged cross sections do not change drastically in this case. On the other hand, the inclusion of the anomalous coupling terms, depicted in Figs. 1c, 1d, 1f, 1g and 1h, modifies the behavior of the \( X(3872) \) yield, producing a fast decrease of the \( X \) abundance with time. We emphasize that the \( X(3872) \) abundance is the one which comes from the QGP and is what could be observed if the \( X(3872) \) is a tetraquark state. However, if the \( X(3872) \) is a molecular state, it will be formed by hadron coalescence at the end of the hadronic phase. According to Ref. \[16\], at this time the average number of \( X \)'s, considering it as a \( DD^{*} \) molecule, is

\[
N_{X\text{(mol)}} \approx 7.8 \times 10^{-4},
\] (7)

which is about 80 times larger than the contribution for a tetraquark state at the end of the hadronic phase (see Fig. 4). We can then conclude that the QGP contribution for the \( X(3872) \) production (as a tetraquark state and from quark coalescence) after being suppressed by hadronic interactions, becomes insignificant at the end of the hadronic phase.

4. Conclusions

We have presented a calculation of the time evolution of the \( X(3872) \) abundance in heavy ion collisions. If the \( X(3872) \) is a tetraquark state it will be produced at the mixed phase by
Figure 4. Time evolution of the $X(3872)$ abundance as a function of the proper time $\tau$ in central Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The solid line, the dashed line and the light-shaded region represent the results obtained considering only the neutral $D$'s and $D^*$'s, adding the contribution from charged $D$'s and $D^*$'s and including contributions from the anomalous vertices respectively. The initial condition is the abundance of the $X(3872)$ considered as a tetraquark Eq. (6).

quark coalescence. It can be destroyed in collisions with the comoving light mesons, such as $X + \pi \rightarrow \bar{D} + D$, $X + \pi \rightarrow \bar{D}^* + D^*$ but it can also be produced through the inverse reactions, such as $D + \bar{D} \rightarrow X + \pi$, $\bar{D}^* + D^* \rightarrow \pi + X$. We have considered the contributions of anomalous vertices, $\pi D^* \bar{D}^*$ and $X \bar{D}^* D^*$, and the contributions from charged $D$ and $D^*$ mesons to the $X(3872)$ production and dissociation cross sections. These vertices, apart from enhancing the cross sections associated with the $\bar{D}^* D^*$ channel, give rise to additional production/absorption mechanisms of $X$, which are found to be relevant.

The cross sections, averaged over the thermal distribution, have been used to analyze the time evolution of the $X(3872)$ abundance in hadronic matter. We have found that the abundance of a tetraquark $X$ drops from $N_{X(4q)} \approx 4.0 \times 10^{-5}$ at the beginning of the hadronic phase [16] to $N_{X(4q)} \sim 1.0 \times 10^{-5}$ at the end of the hadronic phase.

On the other hand, if the $X(3872)$ is a molecular state it will be produced by hadron coalescence at the end of the hadronic phase. According to Ref. [16], at this time the average number of $X$'s, considering it as a $DD^*$ molecule, is $N_{X(mol)} \approx 7.8 \times 10^{-4}$, which is about 80 times larger than $N_{X(4q)}$.

As expected, the results show that the $X$ multiplicity in relativistic ion collisions depends on the structure of $X(3872)$. Our main conclusion is that the contribution from the anomalous vertices play an important role in determining the time evolution of the $X(3872)$ abundance and they lead to strong suppression of this state during the hadronic phase. Therefore, within the uncertainties of our calculation we can say that if the $X(3872)$ is observed in a heavy ion
collision it must have been produced at the end of the hadronic phase and, hence, it must be a molecular state.

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