IBM Small-Scale Superconducting Quantum Computer Not Gate Error Test

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Abstract—Quantum NOT gates play an important role in the process of quantum information conversion. However, when the X-gate operation is executed on a real quantum computer, there is a large deviation between the actual operation result and the theory, which will lead to inaccurate results when the quantum algorithm containing the X-gate operation is executed. In order to facilitate users to understand the error fluctuations of the X gate in time before executing the quantum algorithm containing the X gate operation on the IBM quantum cloud platform, this paper proposes a method to measure the X gate error. By measuring the X-gate error of four small-scale superconducting quantum computers on the back end of the IBM quantum cloud platform, analyze the degree of fluctuation of the actual measurement value of the quantum device; at the same time, under different execution times shots, the influence of the X-gate test is analyzed. The test results show that the measured value of each qubit of different quantum devices fluctuates to different degrees; the execution times of shots are different, and the test results of X-gate error will be affected to different degrees. This test method helps users select the optimal performance of the qubits before executing quantum algorithms with X gate on IBM’s small-scale quantum computers.

1. Introduction
In recent years, quantum technology has developed rapidly, using the inherent parallelism of quantum to surpass supercomputers in terms of computing power [1]. However, compared with traditional computers, current quantum computers cannot be owned by every user like a classic computer due to the difficult manufacturing process, high cost, and extremely easy to be affected by the external environment. To this end, IBM opened the 5-qubit cloud platform in 2016, its name is IBM Quantum Experience [2]. Today, the IBM quantum computing cloud platform has provided users with a total of six 5-qubit superconducting quantum computers for free: ibmq_belem, ibmq_bogota, ibmq_lima, ibmq_manila, ibmq_quito, and ibmq_santiago [3]; at the same time, eight chip indicators [4] are provided to users: connectivity graph, energy relaxation time T1 [5], phase relaxation time T2 [6], device frequency, readout error, single qubit gate error, CNOT gate error and quantum volume [7]. It is designed to facilitate users to run quantum algorithms and test the performance indicators of quantum computers on real quantum computers.

Although the IBM quantum cloud platform provides great convenience for users to compile algorithms. However, as a precision instrument, a real quantum computer is easily affected by the inherent errors of the instrument and the noise of the external environment. When running the algorithm on the IBM quantum cloud platform, since the quantum gates that make up the quantum algorithm can be divided into single-qubit gates, double-qubit gates and multi-qubit gates according to
the number of bits, there will be varying degrees of error in the implementation of different quantum
gates. This leads to deviations between the actual operating results of the overall quantum algorithm
and the theoretical results. Therefore, before executing the quantum algorithm, it is necessary to obtain
the quantum gate operation error and the final state measurement error in the IBM quantum cloud
platform in time.

For the test of quantum gates, Ref [8,9] proposed quantum process tomography, which can obtain
the magnitude and type of the quantum system noise, but the number of measurements increases
exponentially with the increase in the number of gate operations. The Ref [10-13] proposed a random
test benchmark, which is mainly used to calculate the error probability of each gate operation.
However, when the quantum computer system is large in scale, it is very difficult to characterize
quantum noise. In order to solve the problem of the increase in the scale of the quantum system and
the increase in the number of gate operations, the Ref [14] used the small-scale superconducting
quantum computer supported by the back-end of the IBM quantum cloud platform to test only the
CNOT gate error and the readout error of the quantum device. This test intuitively reflects the
fluctuation of the CNOT gate error.

In order to facilitate users to obtain the error information in time before executing the single
quantum gate algorithm, based on the Ref [14], this paper provides four small-scale superconducting
quantum computers ibmq_belem, ibmq_lima, ibmq_manila and ibmq_quito ( In alphabetical order)
the single-qubit operation gate -- X gate's error floating degree is tested and analyzed. Simultaneously,
verify whether there is an impact on the X-gate error test under different execution times shots.

2. Related Concepts

2.1. Superconducting quantum computer
At present, the more popular physical implementations of quantum computers mainly include ion traps,
semiconductors, and superconductors. The ion trap quantum computer is a quantum computer that
uses electric field trapping in a vacuum cavity and laser cooling of ions [15]. However, the computer
integration of this physical realization method is difficult. The idea of semiconductor quantum
computer [16] is derived from the semiconductor technology of classical computers, which can be
integrated on a large scale, but there is charge noise between qubits.

Compared with the first two design schemes, the superconducting quantum computer is a computer
scheme designed based on the principle of superconducting circuits. The core component is the
superconducting Josephson junction [17], the middle is a thin layer of insulating material, and the two
sides are superconductors, is a special LC resonator.

Superconducting quantum computers have the advantages of high integration, strong scalability,
and mature processing technology. They are the focus of research by top teams at home and abroad.
For example, in June 2021, the University of Science and Technology of China research team
developed a 66-qubit quantum processor "Zuchongzhi" [18], which is a superconducting system; the
four quantum computers tested in this article are also superconducting physics implementations.

2.2. Quantum not gate
Among single-qubit gates, quantum NOT gates (X gates) are widely used in algorithms such as
Deutsch-Jozsa (D-J) [19], Quantum Phase Estimation (QPE) [20], and factorization Shor [21] . Is a
common quantum operation gate, a kind of Pauli gate, sometimes it can also be called Pauli-X gate
[22]. On the Bloch sphere, the X gate rotates by π units around the x axis. Its function is similar to that
of the classical NOT gate. It can convert the ground state |0> to the excited state |1> and the excited
state |1> to the ground state |0>.

2.3. Qubit Connected Graph
The connectivity of a quantum computer is similar to that of graph theory, which refers to the
maximum value of the qubit dimension in a quantum topology graph. The four 5-qubit
superconducting quantum computers in the IBM quantum cloud platform used in this experiment contain two connected graphs. Here, the ibmq_manila quantum computer connection diagram is shown in Figure 1; the ibmq_belem, ibmq_lima, and ibmq_quito quantum computer connection diagrams are the same, and the qubit Q1 is coupled with Q0, Q2, and Q3 respectively, as shown in Figure 2.

Each circle in the quantum computer connection diagram represents a qubit, and the connecting line between the two circles indicates that the double qubit gate can directly act on the two qubits. If the double-qubit gate acts on two qubits that do not have a connecting line, the quantum compiler needs to add a series of switch gates (SWAP gate [23]) to realize the interaction between non-connected qubits. However, increasing the SWAP gate will introduce additional errors and reduce the depth of the quantum circuit. Therefore, the researchers proposed an all-to-all quantum connected graph, in which each qubit of the connected graph is connected to other qubits, that is, there is a coupling relationship. The higher the connectivity of the qubit, the stronger the hardware programmability and operability of the quantum computer.

2.4. Fidelity
In quantum computing, fidelity refers to the degree of fit between the real experimental results of a quantum computer and the ideal results. The mathematical expression is [24]

\[ F = \nu \left( \sqrt{\rho \sigma \rho} \right) \]  

(1)

Here, \( F \) is the fidelity, and \( \rho \) is the experimental realization of ideal \( \sigma \). In quantum circuits, fidelity can be divided into initial state preparation fidelity, gate operation fidelity and readout fidelity.

2.4.1. Fidelity of initial state preparation
Before executing the algorithm, the qubit needs to be initialized to reduce the energy level of the qubit to the ground state. However, due to the combined influence of the circuit noise inside the chip and the environmental noise outside the chip, the qubit cannot be completely initialized to the ground state. Therefore, before compiling the quantum algorithm, the initialization error needs to be measured. The fidelity of the initial state preparation is an index that measures the degree of fit between the actual ground state and the ideal ground state.

2.4.2. Qubit gate operation fidelity
The fault-tolerant law of quantum computers states that quantum computers can only work normally when the gate error rate is very low and the crosstalk is weak enough. Therefore, the engineering realization of quantum computers must reduce gate operation errors. Quantum gate fidelity can be used as an indicator of quantum gate operation to measure the degree of deviation between the gate operation performed in the experimental state and the ideal gate operation.

2.4.3. Read fidelity
In the quantum circuit, after the quantum gate operation is completed, the result of the operation needs to be measured. However, due to the inherent error of the readout instrument and the error caused by the external environment, the quantum state measurement value is not accurate. For this reason, it is necessary to read the index of fidelity. The readout fidelity, that is, the fidelity of the final state
measurement, measures the degree of fit between the actual final state measurement value of the quantum computer and the ideal measurement value.

3. Test Method Design

The design steps of the quantum X-gate error test method in this paper include three aspects:

- **Readout error test.** Add measurement operations on the qubits q₀ - q₄ respectively, and measure the readout fidelity (measurement fidelity) of each qubit, according to the formula

  \[ E_r = 1 - F_r \]  

  (2)

  Obtain the read error \( E_r \). Among them, \( F_r \) is the read fidelity. As shown in Figure 3, it is the readout fidelity test circuit diagram of qubit q₀, and its measurement error pseudo code is shown in Table 1.

  **Table 1. Read error pseudo code**
  
  **Algorithm: Readout error**

  | Input: Qubit measurement circuit | Output: Qubit measurement error |
|----------------------------------|--------------------------------|
| 1: import the toolkit            | 2: provider ← Visit Quantum Cloud Platform |
| 3: Quantum Machine, qubit, shots ← Define the name of the IBM quantum computer, the number of qubits and the number of executions | 4: backend ← Visit the Quantum Machine |
| 5: qreg_q ← Define quantum register | 6: creg_c ← Define classic registers |
| 7: circuit ← Define quantum circuits | 8: i ← 0 |
| 9: while i < qubit do | 10: cq ← Measure qreg_q, creg_c |
| 11: job ← Execute quantum circuit | 12: R_error_results[i] ← Execution circuit result |
| 13: print R_error_results        | |

  The readout error measured by this circuit includes the initial state preparation error.

- **Add X gate error test.** Using the method of adding X gates to qubits q₀ - q₄ and measuring, the probability of |00001>, |00010>, |00100>, |01000>, and |10000> is the total of qubits q₀ - q₄. Fidelity, according to the formula

  \[ E_z = 1 - F_z \]  

  (3)

  Find the total error \( E_z \). Here, \( F_z \) is the overall fidelity. As shown in Figure 4, it is the circuit diagram of the fidelity test of adding X gate of qubit q₀. The pseudo code of the measurement error is shown in Table 2.

  **Table 2. Add X gate error pseudo code**
  
  **Algorithm: Add X gate error**

  | Input: Qubit plus X gate measurement circuit | Output: Qubit plus X gate measurement error |
|---------------------------------------------|---------------------------------------------|
| 1: import the toolkit                        | 2: provider ← Visit Quantum Cloud Platform |
| 3: Quantum Machine, qubit, shots ← Define the name of the IBM quantum computer, the number of qubits and the number of executions | 4: backend ← Visit the Quantum Machine |
| 5: qreg_q ← Define quantum register          | 6: creg_c ← Define classic registers |
| 7: circuit ← Define quantum circuits         | |
8: \( i \leftarrow 0 \)
9: while \( i < \text{qubit} \) do
10: Circuit plus X gate operation
11: \( c_q \leftarrow \text{Measure} \ qreg_q, \ creg_c \)
12: \( \text{job} \leftarrow \text{Execute quantum circuit} \)
13: \( \text{Add}_X\_\text{results}[i] \leftarrow \text{Execution circuit result} \)
14: print \( \text{Add}_X\_\text{results} \)

The error measured by this circuit includes three aspects: initial state preparation error, X gate error and measurement error (reading error).

- Calculate the X gate error \( E_x \). Using the difference between the X gate test error and the read error test error, the mathematical expression is

\[
E_x = E_i - E_r
\]

(4)

Calculate the X gate error \( E_i \), the error pseudo code is shown in Table 3.

| Table 3. X gate error pseudo code |
|----------------------------------|
| **Algorithm:** X gate error      |
| **Input:** Qubit measurement error, qubit plus X gate measurement error |
| **Output:** X gate error result  |
| 1: \( i \leftarrow 0 \)          |
| 2: While \( i < \text{qubit} \) do |
| 3: \( \text{X}_\text{error}_\text{results}[i] \leftarrow \text{Add}_X\_\text{results}[i] - \text{R}_\text{error}_\text{results}[i] \) |
| 4: print \( \text{X}_\text{error}_\text{results} \) |

In order to prevent deviations in the indicator test results of real quantum computers in different time periods, and to ensure the accuracy of the X-gate error test results, this experiment parallelizes steps “Readout error test” and “Add X gate error test”.

![Figure 3. Readout fidelity test circuit.](image1)

![Figure 4. The fidelity test of adding X gate circuit.](image2)

**4. Test Results**

In this experiment, four X-gate error tests were performed on four 5-qubit superconducting quantum computers, *ibmq_belem*, *ibmq_lima*, *ibmq_manila* and *ibmq_quito*. The results show that in the four floating degree test experiments, the qubits contained in each quantum computer have different degrees of floating; In the shots comparison experiment, the shots were 8192, 4096, and 2048 respectively. By testing the X gate error of the bits in the quantum machine, it is proved that the test results of each quantum computer are affected to different degrees.
4.1. X gate error floating degree test

This experiment provides four X gate tests of quantum devices, analyzes the average, variance, maximum, minimum, and difference of the X gate error of each qubit, and observes the degree of fluctuation. Here, the average value, maximum value and minimum value of X-gate error analysis data of four quantum computers are shown in Figure 5-8, and the number of executions shots = 8192.

Here, the upper end of the error line is the maximum value of the qubit X gate error, and the lower end of the error line is the minimum value. At the same time, the variance and difference of the test data are shown in Table 4-7.

- **ibmq_belem device (Figure 5 and Table 4):** The average value of qubits Q0-Q4 is 0.06195; the X gate error test value of Q1 is smaller than other qubits; the floating degree of Q3 is smaller than other qubits.

  | Q0   | Q1   | Q2   | Q3   | Q4   |
  |------|------|------|------|------|
  | Variance | 3.50E-4 | 1.10E-4 | 1.88E-4 | 8.07E-5 | 1.49E-3 |
  | Difference | 0.042   | 0.024   | 0.030   | 0.019   | 0.084   |

- **ibmq_lima device (Figure 6 and Table 5):** The average value of qubits Q0-Q4 is 0.04990; the X gate error test value of Q0 is smaller than other qubits; at the same time, the degree of fluctuation of Q0 is also smaller than other qubits.
Table 5. *ibmq_lima* machine X-gate error floating

|       | Q0      | Q1      | Q2      | Q3      | Q4      |
|-------|---------|---------|---------|---------|---------|
| Variance | 4.67E-6 | 1.67E-6 | 1.37E-5 | 2.87E-5 | 4.60E-5 |
| Difference | 0.005   | 0.009   | 0.008   | 0.011   | 0.015   |

- *ibmq_manila* device (Figure 7 and Table 6): The average value of qubits Q0-Q4 is 0.02975; the X gate error test value of Q3 is smaller than other qubits; the degree of floating of Q4 is smaller than other qubits.

Table 6. *ibmq_manila* machine X-gate error floating

|       | Q0      | Q1      | Q2      | Q3      | Q4      |
|-------|---------|---------|---------|---------|---------|
| Variance | 1.98E-4 | 1.35E-4 | 3.56E-4 | 1.00E-5 | 4.67E-6 |
| Difference | 0.030   | 0.027   | 0.041   | 0.007   | 0.005   |

- *ibmq_quito* device (Figure 8 and Table 7): The average value of qubits Q0-Q4 is 0.05550; the X gate error test value of Q4 is smaller than other qubits; at the same time, the floating condition of Q4 is also smaller than other qubits.

Table 7. *ibmq_quito* machine X-gate error floating

|       | Q0      | Q1      | Q2      | Q3      | Q4      |
|-------|---------|---------|---------|---------|---------|
| Variance | 1.39E-4 | 7.87E-5 | 1.32E-4 | 1.50E-3 | 5.16E-5 |
| Difference | 0.025   | 0.020   | 0.027   | 0.076   | 0.016   |

In summary, according to Figure 5-8 and Table 4-7, it can be seen that

- In the same quantum device: The bit with the smallest X gate error test value may not necessarily have the smallest floating degree.

- For quantum computers with the same connectivity (*ibmq_belem*, *ibmq_lima*, and *ibmq_quito*): the Q1 bit X gate error average of each quantum device is less than the Q0-Q4 bit X gate error average. Simultaneously, after comparison, the stability of the *ibmq_lima* device is higher than that of the other two devices.

4.2. The shots test

This test compares the impact of the X-gate error test on 4 superconducting quantum computers when the number of execution shots is 2048, 4096, and 8192. The experimental results are shown in Figure 9-12.

![Figure 9. *ibmq_belem* device test data under different shots.](image-url)
According to Figure 9-12, when the number of execution shots is 2048, 4096, 8192, it can be seen from the average value of the X gate error test of each qubit of the quantum computer,
• ibmq_belem device (Figure 9): It has the least impact on the X gate error of qubit Q1; when shots = 2048, the X gate error value of Q1 is the smallest, which is 0.0315.
• ibmq_lima device (Figure 10): It has the least impact on the X gate error of qubit Q0; when shots = 2048, the X gate error value of Q0 is the smallest, which is 0.0280.
• ibmq_manila device (Figure 11): It has the least impact on the X gate error of qubit Q4; when shots = 2048, the X gate error value of Q3 is the smallest, which is 0.0153.
• ibmq_quito device (Figure 12): It has the least impact on the X gate error of qubit Q4; when shots = 2048, the X gate error value of Q2 is the smallest, which is 0.0283.

In order to analyze the data in Figure 9-12 more accurately, this experiment calculates the variance value of the X gate test of the above four quantum devices under different shots, as shown in Figure 13-16.

![Figure 13. ibmq_belem device test variance under different shots.](image1)

![Figure 14. ibmq_lima device test variance under different shots.](image2)
Based on Figure 9-12 and Figure 13-16, we can see that for each qubit of the different devices in this experiment, when the shots are 2048, 4096, and 8192, there are different degrees of influence. On the other hand, for the quantum devices ibmq_belem, ibmq_lima, and ibmq_quito, under different execution times shots, the ibmq_lima device qubit X gate test variance average is smaller than the other two devices. Therefore, the X gate error test value of the ibmq_lima device has the least impact.

5. Conclusion
This paper proposes a method for testing quantum X-gate error, and uses four small-scale quantum computers provided by the IBM cloud platform as experimental objects to measure the X-gate error and analyze the degree of floating of each quantum computer. The experimental results show that through four error tests, each quantum device has different degrees of fluctuation. Simultaneously, verify if the number of execution shots is 2048, 4096, 8192, whether there is any impact on the test data. The verification results show that, with different shots, the bits of each quantum device do have varying degrees of impact.

In the process of quantum information conversion, the quantum NOT gate plays an extremely important role. This article aims to test the quantum X-gate error in IBM's small-scale superconducting quantum computer, so that users can obtain the quantum computer's X-gate error
fluctuation degree in time when executing quantum algorithms with X-gates, so that it is better to choose the quantum computer with the best performance. At the same time, the test method has scalability and portability, and can be used in quantum computers of larger scale and different physical realization systems. Therefore, the X-gate test method in this article has important practical significance.

Here, using the X-gate error test method to evaluate domestic quantum computers, while comparing the bit performance of domestic and foreign quantum computers will be a future work.

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