Calculation of radiation heat transfer in staggered drop-shaped tubes bundle

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Abstract. Tube bundles can be used as a separation heat exchanger in the “organic” Rankine cycle power plants (ORC), while the hot gas passes over the outer surface, and the working substance in ORC flows inside the tubes. Numerical study of the hydrodynamics and heat transfer of a staggered bundle of drop-shaped tubes at different angle of attack ($\theta$) of the incoming gas flow is carried out using the ANSYS package. The distribution of velocity, pressure, temperature in the studied bundle are obtained and the relation for calculating the average Nusselt number for the stress-strain state of the tubes bundle is proposed. The numerical method of calculation of the elementary, local and average radiation view factors for staggered bundle of drop-shaped tubes is developed using the “crossed-strings method”. The radiation net fluxes in the systems of drop-shaped and circular tubes with the same equivalent diameter are determined.

1. Introduction

In single-circuit recycling power units operating on the "organic" Rankine cycle (ORC), one of the main heat exchangers is the separating heat exchanger for "exhaust gases – the working substance of the ORC" (figure 1). The paper presents a numerical study of heat transfer and hydrodynamics of such a heat exchanger, which is a 6-row staggered bundle of drop-shaped tubes with longitudinal and transverse pitch of 37 mm, with accounting for the effect of deformation caused by pressure drop inside and outside the tubes.

Figure 1. Schematic view of the ORC recycling unit.
1-Waste heat; 2- Separating heat exchanger with drop-shaped tubes bundle; 3- Turbine; 4- Condenser; 5- Feed pump.
2. Convection heat transfer calculation

The geometric characteristics of the cross-section of the drop-shaped tube are shown in figure 2a. The angle of attack $\theta$ could be adjusted by simultaneously rotating all the tubes in the bundle by 0, 45, 135, 180 degrees clockwise (figure 2b). Mathematical and numerical model for the calculation of heat transfer and hydrodynamics of a bundle of drop-shaped tubes, considering the stress-strain state using the ANSYS package, is developed. Simulation of the stress-strain state is carried out in ANSYS Static Structural. The deformations caused by the pressure difference inside (14 bar) and outside the tubes (1 bar) are determined. Figure 2c shows the cross-section of the tube after deformation.

![Figure 2. a) A drop-shaped tube cross-section dimensions; b) The adjusting of flow attack angle; c) Stress-strain state.](image)

The distribution of pressure, velocity, temperature, and heat transfer coefficient in the bundle with a change of the velocity of the incoming flow in the range of 1.33÷7 m/s is numerically investigated. Figure 3 presents the calculation results of the air flow through the bundle at a speed of 1.33 and 7 m/s.

![Figure 3. Distribution of pressure (a), velocity (b) and dimensionless temperature (c) at $\theta= 0^\circ$.](image)

In [1], experimentally and numerically studied heat transfer in a 3-row staggered bundle of drop-shaped tubes (without deformation), the data in [1] are used to verify the developed numerical model. Figure 4 shows the bundle’s averaged Nusselt numbers corresponding to the $Re_a$ numbers calculated as well as in [1] by the incoming flow velocity and the angles of attack $\theta = 0, 45, 135, 180$ degrees. The obtained good agreement allows us to use the developed numerical model to calculate the states of such systems.

The results of the computational experiment (3-row bundle of drop-shaped tubes with deformation and without heat transfer by radiation) are processed as the following similarity equation for the heat transfer coefficient averaged over the surface area of the bundle:

$$
\bar{Nu} = a \cdot Re^b \cdot Pr^{\frac{1}{2}} \cdot \left(1 + \frac{\theta^\circ}{90^\circ}\right)^c
$$

(1)
in which thermophysical properties are calculated at the average temperature of the incoming flow; as the characteristic dimension in the similarity numbers the equivalent diameter \( D_{eq} \) is used; calculation of the Reynolds number \( Re = \frac{U_{av}D_{eq}}{\nu} \) was performed by the average flow velocity \( (U_{av}) \) in the narrow cross-section of the bundle.

![Figure 4](image)

**Figure 4.** Comparison of heat transfer from the staggered drop-shaped tubes in present work and in [1].

The coefficients \( a, b \) and \( c \) in equation (1) were computed in MathCad package using the least square technique (Table 1). The obtained equation is applicable for \( Re_a = 1.8 \times 10^3 \sim 9.4 \times 10^3 \) and for Prandtl number \( (Pr = 0.7) \).

| \( \theta \) | \( a \) | \( b \) | \( c \) |
|---------|--------|--------|--------|
| \( 0^\circ \leq \theta \leq 45^\circ \) | 0.318 | 0.574 | -0.797 |
| \( \theta = 135^\circ \) | 0.318 | 0.574 | -0.336 |
| \( \theta = 180^\circ \) | 0.318 | 0.574 | -0.027 |

### 3. Radiation heat transfer calculation

Radiant heat fluxes should be accounted for in the heat recovery schemes of the exhausting gases of gas turbine and steam-gas units. Analysis of the results presented in figure 3c shows that the non-uniformity of the temperature field in the flow along the perimeter of the tubes may be significant. This fact dictates the need to include the inhomogeneity of radiation in the tubes bundle in the methodology. This can be done only by a detailed calculation of the elementary view factors of radiation at each node of the computational domain. The complexity of determining the radiation fluxes in drop-shaped tube systems is due to their shape (figure 5a). To test the calculation results, the system of circular tubes was also considered (figure 5b).

![Figure 5](image)

**Figure 5.** The design scheme of the tubes system a) drop-shaped, b) circular shape.
In the considered systems (figures 5 a, b) the control area ABCD is allocated. To determine the fluxes of inhomogeneous radiation leaving the surfaces "1"."4" and the radiation fluxes incident on these surfaces, each surface of the system is divided into elementary surfaces (figure 6) having such a minimum required size that the radiation fluxes from them (on them) can be considered uniform. The two surfaces are selected as elementary sites, and the other two surfaces close the system (figure 7).

Figure 6. Discretization of the computational domain.

Figure 7. Examples of lengths of "crossed-strings" in an elementary system.

Thus, the problem is reduced to the known method for determining homogeneous radiation fluxes for all elementary surfaces of a new system (in computational terms). In this paper the Polyak’s method [2] was used for homogeneous flows applied to elementary systems. As a result, for homogeneous radiation on one row of drop-shaped tubes (figure 8a) the following relations can be used:

Non-crossing strings length (GC+CB+BF):

\[ F_{12} = \frac{\pi - \arccos\left(\frac{d_1 - d_2}{2b}\right)}{2} \cdot d_1 + b \sqrt{1 - \left(\frac{d_1 - d_2}{2b}\right)^2} + \arccos\left(\frac{d_1 - d_2}{2b}\right) \cdot d_2 \]  \hspace{1cm} (2)

When calculating the length of crossing strings (figure 8b):

The length of the segment HP is equal to (HT+TP):

\[ HP = \frac{\sqrt{\left(\frac{d_1}{d_2 + d_1}\right)^2 S^2 + b^2 - d_1^2}}{4} + \frac{\sqrt{\left(\frac{d_2}{d_1 + d_2}\right)^2 S^2 + b^2 - d_2^2}}{4} \]  \hspace{1cm} (3)

The arc length GH is:

\[ GH = \frac{\pi}{2} - \left[\arccos\left(\frac{d_2 + d_1}{2\sqrt{S^2 + b^2}}\right) - \arccos\left(\frac{S}{\sqrt{S^2 + b^2}}\right)\right] \cdot d_1 \]  \hspace{1cm} (4)

The angle of PO_{2}Q is equal to the angle of HO_{1}G, so the arc length PQ is:

\[ PQ = \frac{\pi}{2} - \left[\arccos\left(\frac{d_2 + d_1}{2\sqrt{S^2 + b^2}}\right) - \arccos\left(\frac{S}{\sqrt{S^2 + b^2}}\right)\right] \cdot d_2 \]  \hspace{1cm} (5)
Figure 8. The calculation of the lengths of: a) non-crossing strings: (the length of the arc GC + straight line CB + length of arc BF); b) the crossing strings: (the length of the arc GH + straight line HP + the length of the arc PQ).

The length of the crossing strings is equal:

\[ F_5 = GH + HP + PQ \] (6)

Then:

\[ \bar{\Phi}_{13} = \frac{F_5 - F_2}{s} \quad \bar{\Phi}_{12} = \frac{1 - \bar{\Phi}_{13}}{2} \] (7)

The method to determine the elementary view factors of radiation and average view factors (2-7), by the “crossed-strings” method, is implemented in Mathcad. For the given sizes of drop-shaped tubes (figure 2a) the matrix of view factors is calculated (table 2 “a”):

Table 2. Calculated average view factors: a) drop-shaped tube \( d_1 = 0.0116 \text{ m}; d_2 = 0.0058 \text{ m}; b = 0.0215 \text{ m}; s = 0.037 \text{ m}; \) b) equivalent round tube \( b_{\text{test}} = 0.0000001 \).

|       | 1  | 2       | 3  | 4       |
|-------|----|---------|----|---------|
| a) \( \Phi \) (\( d_1, d_2, b, s \)) | 1  | 0       | 0.338 | 0.342 | 0.338 |
|       | 2  | 0.342   | 0.342 | 0.317  |
|       | 3  | 0.342   | 0.317 | 0.342  |
|       | 4  | 0.342   | 0.338 | 0.342  |

|       | 1  | 2       | 3  | 4       |
|-------|----|---------|----|---------|
| b) \( \Phi \) (\( d_1, d_1, b_{\text{test}}, s \)) | 1  | 0       | 0.221 | 0.557 | 0.221 |
|       | 2  | 0.45    | 0.45  |
|       | 3  | 0.557   | 0.221 | 0 |
|       | 4  | 0.45    | 0.101 | 0.45  |

The comparison with the known solution for a multi-row system of circular tubes [3] is made for homogeneous radiation on one row of circular tubes:

\[ \bar{\Phi}_{12} + \bar{\Phi}_{14} = 1 - \frac{d}{s} \left[ \sqrt{\left( \frac{s}{d} \right)^2 - 1 - \arccos \frac{d}{s}} \right] \] (8)

According to (8) the averaged \( \bar{\Phi}_{12} + \bar{\Phi}_{14} = 0.221 \), i.e. the solutions (see table 2 “b”) coincide. Similarly, the view factor for the 2\(^{\text{nd}}\), 3\(^{\text{rd}}\), etc. rows of the tubes bundle can be found, assuming approximately that the radiant flux passing through the tubes of the first row is uniformly distributed on the plane behind the tubes of the first row with a value of \( (1 - 2\bar{\Phi}_{12}) \) fraction of the incident radiation on the
tubes of the first row. Assuming the same conditions for each subsequent row, a generalized formula for k rows is written as follows:

$$\bar{\phi} = \frac{\phi_{12}}{\bar{\phi}_{12}} \cdot \sum_{n=1}^{n=k} (1 - 2 \phi_{12})^{n-1}$$

(9)

The equation (9) is applicable for the same values of S=s/d in each row of the tubes bundle and it can be used for both aligned and staggered tube bundles.

4. Conclusions

The mathematical numerical model has been developed to calculate the heat transfer coefficient of the staggered drop-shaped tubes bundle using the ANSYS package, taking into account the stress-strain state of the tubes. The distributions of flow velocity, pressure, temperature, mechanical stresses and heat transfer coefficient along the perimeter of drop-shaped tubes of 6-row bundle were obtained in computational experiments. The equation to calculate the 3-row staggered drop-shaped tubes bundle’s $\tilde{N}u$ depending on the average flow velocity in the minimum free cross section and the discrete flow angles of attack $\theta = 0^\circ, 45^\circ, 135^\circ$ and $180^\circ$ is proposed.

To determine the inhomogeneous radiation fluxes in a bundle of drop-shaped tubes, the Polyak’s method for homogeneous fluxes, applied to elementary systems, was used. Elementary view factors of radiation have been found by the "crossed-strings method". Moreover, quite a complex logic in calculating the shape and length of the crossing strings, consisting of straight and curved elements, requires using vector’s and Boolean’s algebra, and solving a set of vector equations in programming. The paper presents the mathematical relations implemented in the Mathcad environment and allows estimating the radiation heat transfer in the systems of circular and drop-shaped tubes and the radiation net fluxes on the basis of numerical methods for calculating the elementary, local, average radiation view factors.

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