Didactical design for distance concept in solid geometry to develop mathematical representation ability in vocational high school

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Abstract. This is qualitative research about the didactical design of distance concept in solid geometry, aiming to develop mathematical representation ability. This research employs a qualitative method with didactical design research (DDR) consisting of three stages of the didactical situation: didactical and pedagogical anticipation, metapedadidactical analysis, and retrospective analysis. Hypothetical didactical design was arranged by learning obstacle and hypothetical learning trajectory with the theory of didactical situations. The research subjects were three students of State Vocational High School in Garut as representatives of 30 students who had experience learning Distance Concepts in Solid Geometry and had been given an identification test for learning obstacles. The study identified that learning obstacles are the didactic and epistemological obstacle, which covered five inaccuracies in (1) Students' ability in presenting concepts in various mathematical representation; (2) Understanding of the concept orthogonal projection between points on a line in the geometry; (3) construct geometric shapes of space to clarify the problem; (4) Understanding students' visual representations related to the ability to determine the position of a point or a line perpendicular to a line; (5) Understanding of calculation procedures using the Pythagorean theorem and algebraic concepts.

1. Introduction

Vocational high school is a unique school different from senior high school [1]. The objective of mathematics learning in vocational high school must be integrated with the objectives of the expertise competency program or related to other subjects. Thus, it supports the achievement of the objectives of the expertise competency program of each department, which will be in line with the development of the world of work [1,2,3].

In the 2013 curriculum, one of the objectives of learning mathematics is that students can solve problems and mathematical representations [4]. Gagne & Mayer argued that several studies show that students who have good representation skills will find successful solutions in problem-solving[5]. The representation can make problems easier and simpler; even though, at first, it looks difficult and complicated. Any problems that arise can be solved more easily [6]. Representation defines revealing ideas using some ways, such as spoken language, written, symbols, pictures, diagrams, models,
graphics, or using physical objects [7]. In line with what was stated by Sabirin, representation is a form of interpretation of one's thoughts on a problem situation [8].

Through representation, students' mathematical ideas can be made more concrete [9,10]. Moreover, representation is seen as an important part of mathematical activities and means capturing mathematical concepts; student success in solving problems can't be separated from the role of representation [5,11,12,13]. Students' understanding of learning mathematics has a huge effect on representation abilities [14]. In the process of learning mathematics, every teacher should be able to develop students' representational abilities.

A branch of science that plays an important role in learning mathematics is geometry [15]. Learning geometry uses many symbols and algebraic calculations; thus, students are required to use reasoning and representation skills [16]. Although geometry is indispensable in the context of real-life, many students do not master this concept. This is illustrated by the data on the results of 2015, 2016, 2017, 2018, and 2019 national exams, in which the absorption of national exam scores of Grade 12 students of the vocational school tended to decline throughout Indonesia, especially in the geometry and trigonometry concept presented in Table 1 [17].

Table 1. The Absorption/understanding of Geometry and Trigonometry Concept from 2015 to 2019

| Year | Understanding geometry concept |
|------|--------------------------------|
| 2015 | 44.84                          |
| 2016 | 34.18                          |
| 2017 | 41.10                          |
| 2018 | 31.92                          |
| 2019 | 28.39                          |

Table 1 above shows that the research assumes that the average achievement of students' mathematics skills, especially in geometry and trigonometry, is low. This is proved by the percentage of student achievement in answering the 2019 national exam questions on geometry material, which tends to below.

Teaching school mathematics, for example, geometry and arithmetic, create many learning obstacles in mathematics communication [18]. The representation of geometric objects is a major problem in geometry [19]. One of the reasons for students' low representation ability in solid geometry is that they cannot build their representations.

The arrangement of teaching materials that can activate students in constructing geometric concepts and building their understanding is an effort that can be made to overcome student difficulties in learning geometry [20]. While the didactic design development plays an important role in the mathematics learning process and affects how students conduct learning [21,22].

Teaching is considered effective if the teacher chooses their methods, forms, and means (teaching strategies) by taking into account students' various characteristics [23]. Thus, in implementing a lesson, a teacher should have compiled a design that contains the teaching materials to be used. This design is called a didactic design. Every child can learn mathematics, but some children learn and make connections faster than others [24]. Mathematical abilities in a child are often inactive and are not attended to by the child or the teacher. This potential can be lost forever if it is not found and supported at the right time.

Based on the case, this research offers an effort to improve learning by developing a didactic design that includes activities for students' mathematical representations that can be applied in the classroom in order to improve students' understanding of the concept of distance in spatial geometry. Hypothetic didactic designed to reduce students' difficulties with the material related to students’ mathematical representation ability. Thus, This study aims to 1) Identify and analyze the characteristics of learning obstacles toward students related to distance in solid geometry learning; 2)
Formulate a hypothetical learning trajectory (HLT) that can be developed in the distance in solid geometry.

2. Research Method
Before the research began, the researchers conducted a preliminary study to review students' representation abilities at the research location. Based on the preliminary study that has been conducted, the researcher assumes that the mathematical representation questions have not been mastered by students (respondents).

This research used a didactical research design that is a design focusing on developing teaching concepts to overcome or reduce the barriers that arise in students. Three stages in didactic design research carried out to compile a didactic design of mathematics learning on distance in the geometry of space, as follow:

1) Didactic situation analysis, the didactic situation will generate ideas about predictions and anticipation of student responses before learning is carried out.

2) Metapedadidactical analysis is the ability of teachers before learning, during learning, and after testing teaching materials. The metapedadidactical analysis stage in this research did not reach the didactic design implementation stage. It was hampered by the COVID-19 pandemic situation where schools throughout Indonesia changed their learning systems using the Distance Learning system (PJJ).

3) Retrospective analysis is the activity of linking the didactic situation analysis results before learning with the results of the metapedadidactical analysis obtained in the learning process. This research focuses on the obstacles related to the representation ability of distance in the Solid Geometry concept.

This research will be conducted at one of the vocational high school in Garut, Indonesia. The subjects were selected by purposive sampling. The learning obstacle test was given to 30 Grade 12 students. Based on the average score of the test results, students were classified into three groups: the upper group, the medium group, and the low group. From each of these groups, one person was taken randomly to be the research subjects.

The researcher is the main instrument in this research, which has a role in determining the focus of the research during data collection, data analysis, and concluding. Meanwhile, the supporting instruments are in the form of test instruments and non-test instruments. The test instrument is in the form of description questions used to identify learning obstacles related to the representation ability of distance concept in solid geometry. Likewise, the non-test instrument in the form of an interview guide for teachers and students is used to determine why students give answers to the tests given and understand the flow of students' thinking. The steps for developing a test instrument in this study are making a question grid, composing questions, and conducting instrument assessments in terms of language accuracy, content suitability, and construct aspects by people who are seen as experts, namely two mathematics lecturers and one partner teacher.

3. Result and discussion
This section reveals results found in this research about the didactic design of distance in the concept in solid geometry to develop Grade 12 students' representation abilities. Based on the findings obtained in the preliminary study, to develop students' mathematical representation abilities, it is necessary to pay attention to the achievement indicators of increasing mathematical representations. Indicators of mathematical representation in this study were adapted from Gagatsis and Elia [25], presented in Table 2 below.
Table 2. Indicators of mathematics representation ability.

| No | Aspect | Indicator |
|----|--------|-----------|
| 1  | Visual | a) Representing data or information from a representation in the form of diagrams, graphs, or tables.  
|    |        | b) Using visual representations to solve problems |
| 2  | Image  | a) Drawing geometric patterns  
|    |        | b) Drawing geometric shapes to clarify problems and facilitate resolution |
| 3  | Symbolic (Mathematical Equation or Expression) | a) Making mathematical equations or models from given representations  
|    |        | b) Solving the problem by involving mathematical expressions |
| 4  | Verbal (Word or written text) | a) Making problem situations based on the data or representation provided  
|    |        | b) Writing an interpretation of a representation  
|    |        | c) Writing down solutions to problems through written sentences  
|    |        | d) Using math solving steps with words |

By taking into account learning obstacles, a solid geometry concept learning design was designed to develop the mathematical representation abilities in Grade 12 students. The representation indicators developed were not much different from the mathematical representation indicators during the preliminary research. The teaching topics developed in this research were in the form of Student Worksheets. In addition to making teaching materials, researchers made predictions of anticipatory responses. The following are the analysis results of the learning obstacle in the form of an epistemological obstacle indicated on each item.

The following is an overview of the error in the process of working with representative students from the upper, middle, and lower groups in solving the concept of the distance between two points in solid geometry presented in the figures below.

Figure 1. The answer to question 1 by students in the upper group.
Figure 2. The answer to question 1 by students in the middle group.

Figure 3. The answer to question 1 by students in the lower group.

Overall, the identified obstacles from question number 1 were students' mistakes in constructing the distance on the cube and students' lack of understanding of the concepts used to determine the point-to-point distance in spatial geometry. This shows that students still have difficulty using visual representations to solve problems. It depicts errors in the processing of student representatives from the upper, middle, and lower groups in solving the concept of the distance between two points in solid geometry.
Figure 4 shows that students have understood the concept of calculating the distance of points and lines, such as the distance of point P to the QR line, yet there is an algebraic calculation error conducted by students. The algebraic calculation error of the student lies in determining the result of $5^2$. After being interviewed, the student did not know the result of $5^2$ but the inaccuracy factor.

The depiction of the error in working with representative students from the middle group in solving the concept of point and line distance problems in solid geometry is presented in Figure 5. Figure 5 shows that students have understood the concept of calculating the distance of points and lines, such as the distance of point P to the QR line, but there are algebraic calculation errors. The error in calculating algebra lies in using the Pythagorean Theorem when calculating the length of the OP that is the distance between point P and the QR line.

Figure 5. The answer to question 2 by students in the middle group.

An overview of the error in working with representative students from the lower group in solving the concept of point and line distance problems in spatial geometry is presented in Figure 6 below.
Figure 6 shows the answer to question number 2 by students in the lower group. The students simply can't calculate the distance of the point P to the QR line, which is based on the error of visualizing the plane of a triangle that shouldn't be right to be a right triangle. Overall, the identified obstacles from question number 2 were students' mistakes in constructing the distance from point to line in spatial geometry, students' lack of understanding of how to project a point on a line, and students' lack of understanding of the concepts used to determine the point to point distances in the geometry of space. This shows that students still have difficulty in making a geometric shape to clarify problems and facilitate their solutions. They still have difficulty writing solutions to problems through written sentences.

Question number 3: A lidless box ABCD EFGH is made of a cardboard 40cm long and 40cm wide. How to make the box is by cutting the four corners of the cardboard to form a square with a size of x cm.

a. Draw the problem above!

b. If the surface area of the box is 700 cm$^2$, determine the length, width, and height of the box! Explain why!

c. Make a question from the situation above related to the concept of distance in shapes, then solve it! Below is an error/mistake in the process of working with representative students from lower groups in solving the concept of the distance between two points in the geometry of space.

Figure 7. The answer to question 3 by students in the lower group.
Figure 7. It is known that students begin to understand the context of the questions. Thane to understand the process of the shape of the box from a square sheet of zinc. This is shown that the students have accurately described the block nets that are formed, but they have not been able to construct them into a box or box without a lid.

Overall, the identified obstacles from question number 3 indicate that students can't use information from a representation in the form of images. Students cannot create mathematical equations or mathematical models of the problems given, and students can't create problem situations based on data or representations given.

Question number 4: Andi will hold his 17th birthday party by decorating the living room in his house in the form of a block. The decoration is connected the ribbon from a lamp located in the middle of the living room ceiling with a corner point on the floor of the living room. The living room ceiling is 4 x 2 meters, and the height is 3 meters.

a. Sketch Andi's problem in a picture!

b. Identify the problem faced by Andi and write down the solution to the solution using words!

c. What is the minimum length of the ribbon that Andi must provide?

The figure below is a mistake in the processing of student representatives from the upper groups in solving the concept of the distance between two points in the geometry of space.

![Illustration](image)

Figure 8. The answer to question number 4 by students in the upper group.

Figure 8. shows that students can construct the problem into an image. However, they have not been able to identify the problem and write down the solution verbally, and students mistakenly determine the minimum length of tape needed by Andi by using the concept of the area of a triangle.

Overall, the identified obstacles from question number 4 were students' errors in restating data or information from a representation in the form of images, writing problem solutions through written sentences, and solving problems through mathematical expressions or mathematical equations.

Based on the results of the analysis of students' mistakes in working on the four questions regarding the concept of distance in the geometry of space above, it was revealed that several errors commonly experienced by students were as follow:

1. Students' ability to represent some types of mathematical representation. In this case, students' understanding of the concept of point and line distance was simply by constructing a line segment that connected the two objects without paying attention to the perpendicular position of the line segment. This mistake is categorized as a didactical obstacle caused by the learning process, including the teaching materials.

2. The mistake is related to the material orthogonal projection between points on a line in space geometry. Students stated that the projection of the point toward the line would always be in the middle of the line. This mistake is an epistemological obstacle caused by the limited
context of students' knowledge to link the concept of orthogonal projection to the problem of
distance in the geometry of space.

3. Students' visual representation mistakes are related to the student's ability to determine the
position of a point or line in a geometric spatial shape. While students still had difficulty
determining the position of a point or line that was perpendicular to other lines or not
perpendicular at all. This error is an epistemological and ontogenic obstacle. Epistemological
obstacles occurred because they only understood the context of triangles in two dimensions.
Yet, while looking at the context of triangles in three dimensions or the space geometry,
students had obstacles.

4. Mistakes related to calculation procedures using the Pythagorean theorem and algebraic
concepts. This mistake is an ontogenic obstacle caused by the mental development of students.
The student made a mistake in transforming from a right triangle shape into the Pythagorean
theorem rule.

The arrangement of didactic design for distance in the space geometry concept is based on a
hypothetical learning trajectory. It is the alleged trajectory of students' thinking that will provide
direction to the teacher to take actions that are tailored to the teacher's perceptions and students'
understanding [18]. Learning trajectories are compiled based on the analysis results from textbooks
related to distance material in spatial geometry and the results of learning obstacle analysis. The
researcher made a chapter design divided into two hypothetic lesson designs, consisting of two
didactic design meetings. It should be noted that before students begin learning distance in the
geometry of space, an initial meeting was held to discuss the prerequisite material designed by the
researcher (image design) outside the didactic design implementation lesson hours. This is conducted
based on the learning obstacle found by researchers, including students experiencing obstacles in
finding a relationship or connection between the concept of distance in the geometry of space and the
prerequisite material of Didactic Design Concept Situation 1 (distance between two points).

4. Conclusion
Based on the research results some conclusions are drawn. First, the learning obstacle identified is the
didactical obstacle, and epistemology obstacle. The learning obstacles found related to mathematical
representation abilities consist of: (1) the ability of students to present concepts in various forms of
mathematical representations, (2) understanding orthogonal projection material between points on
lines in spatial geometry, (3) constructing spatial geometric figures to clarify the problem (4)
understanding the procedure for using the Pythagorean formula and algebraic concepts. Also, from the
HLT perspective found in this research, before students are given distance in solid geometry, the
teacher should ensure in advance the students' understanding of the prerequisite material needed.

Furthermore, the ability of representation in distance solid geometry is inseparable from
prerequisite concepts such as drawing proportional geometric shapes, orthogonal projections of points,
lines, and planes so that in developing a didactic design, the concept of distance is available in
situations that specifically build students' understanding of the prerequisite concept.

It is hoped that the hypothetical didactic designed will minimize the learning obstacle related to the
representation ability occurred in the preliminary study. The didactic design obtained can be used as
an alternative that can be used as a reference in carrying out the solid geometry learning, especially in
building the concept of distance in solid geometry. Besides, the hypothetic didactic design the concept
of distance in space geometry is recommended to be implemented through the three stages of
Didactical Research Design (DDR) to obtain a didactic design that minimizes the shortcomings,
including learning obstacles related to representation skills.

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