Deimos Encounter Trajectory Design for Piggyback Spacecraft Launched for Martian Surface Reconnaissance*

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This paper discusses a trajectory design for imaging both Mars and Deimos, which meets the requirements of the next Chinese mission to Mars and Deimos. Compared to Viking-1, being weak in systematic schedules in its original and extended missions, which resulted in the use of more fuel and reducing encounter opportunities, a multipurpose design is addressed in this paper for imaging both Mars and Deimos at Pre-Phase A, not after launch. A frozen and repeating orbit is employed to provide as many as 280 periodic encounters within 100 km of Deimos, in contrast to the fly-around mode used by Phobos-1/2 and Phobos-Grunt, requiring much fuel to guide the spacecraft to the vicinity of Phobos from the highly elliptical captured orbit. To enable encounters with Deimos under all of the perturbations and orbital control and determination errors, the station-keeping strategies for the arguments of periareon and latitude are implemented by some corrections to the semi-major axis and inclination, respectively. A numerical simulation is used to verify the encounter opportunities with the help of the station-keeping strategies. Detailed investigations on imaging, lighting and access conditions show that a working orbit is beneficial for imaging both the Martian surface and Deimos. Therefore, it is concluded that designing the trajectory of the piggyback spacecraft to be carried by the main orbiter in the next Chinese Mars mission during Pre-Phase A is practical from the engineering perspective.

Key Words: Mission Design and Analysis, Martian Surface Reconnaissance, Piggyback Spacecraft, Deimos Encounter

Nomenclature

- $n$: angle rate of the working orbit
- $n_{\text{Deimos}}$: orbital angular rate of Deimos orbiting Mars
- $\alpha$: periareon argument
- $\Omega$: right ascension of ascending node
- $P_{\text{s/c}}$: period of spacecraft orbiting Mars
- $a$: semi-major axis
- $e$: eccentricity
- $p$: semilatus rectum
- $u$: argument of latitude
- $r$: distance from Mars to spacecraft
- $J_n$: coefficient of zonal harmonics terms, $n = 2, 3, ..., 7$
- $R_e$: Martian equatorial radius
- $G$: constant of universal gravitation
- $M_{\text{Mars}}$: mass of Deimos
- $\mu$: gravitational constant of Mars, $\mu = \sqrt{GM_{\text{Mars}}}$
- $M_{\text{Deimos}}$: mass of Deimos
- $M_{\text{Sun}}$: mass of the Sun
- $e_x = e \cdot \cos \omega$
- $e_y = e \cdot \sin \omega$
- $p_{\text{SR}}$: solar radiation constant near Mars
- $m$: mass of spacecraft
- $A$: effective projection area of spacecraft
- $a_{\text{SRP}}$: characteristic acceleration due to solar radiation pressure
- $r_{\text{Sun}}$: distance from Sun to Mars

- $\beta$: gravity constant of Sun,
  \[ \beta = \frac{GM_{\text{Sun}}}{r_{\text{Sun}}^3} \]
- $\Delta V$: velocity increment supplied by the maneuver
- $\Delta \Omega$: average drifting rate of the right ascension of ascending node
- $\Delta t$: encounter period
- $C$: solar radiation pressure parameter,
  \[ C = \frac{3}{2} a_{\text{SRP}} \frac{a^2}{q^2} \]
- $W$: oblateness parameter,
  \[ W = \frac{3}{2} \frac{R_e^2}{a^2} \]
- $c_R$: reflectivity coefficient
- $\delta_{\text{Sun}}$: true longitude of the Sun
- $r$: position vector of spacecraft in the Mars-central inertial frame
- $r_{\text{Sun}}$: position vector from Sun to Mars
- $a_{\text{Sun}}$: perturbing acceleration resulting from the Sun’s gravity
- $i_{\text{Sun}}$: inclination of Sun respective to the Martian equator
- $r_{\text{encounter}}$: radius at the encounter
- $R$: disturbing function up to the $J_n$ perturbation
- $\delta_i$: difference between the actual and nominal inclination
- $r_{\text{dn}}$: radius at the descending node
- $I_{\text{sp}}$: specific impulse of hydrazine
- $g$: gravitational acceleration
- $L$: distance between the spacecraft and the Mars at the time of encounter

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Δω: drift of the argument of latitude

\[ V_0: \text{orbital velocity at the apoareon} \]

\[ \delta: \text{altitude of sub-satellite point on the Martian surface} \]

\[ \mathbf{r}_{\text{Deimos}}: \text{position vector of spacecraft in the Mars-central inertial frame} \]

\[ \Delta V_f: \text{main accumulative effect by Deimos’ gravity} \]

1. Introduction

Mars has been long the subject of human fascination.\(^1\) – \(^3\) As a terrestrial planet inside the solar system, Mars is similar to Earth in its size and atmosphere. There have been several completed or planned missions to explore Mars and its moons since the 1960s.\(^2\),\(^3\) Engineering interplanetary journeys is very complicated, so the exploration of Mars has experienced a high failure rate, especially in earlier attempts. Roughly two-thirds of all spacecraft destined for Mars failed to complete their missions, and there are some that failed even before their observations could begin. As the first mission to successfully land and operate on Mars, the famous Viking program consisted of a pair of American space probes sent to Mars, Viking-1 and 2.\(^4\) – \(^6\) Based on the earlier Mariner 9 spacecraft, each spacecraft was designed to photograph the surface of Mars from orbit with a fully fueled mass of 3,527 kg. In order to use two vidicon cameras for imaging, a working orbit of 320 km × 56,000 km × 39.3° (i.e., altitude of periareon and apoareon, and inclination) was allocated. Nozomi, known before launch as Planet-B, was the first Mars-flyby spacecraft developed by Japan. It was intended to have a highly eccentric Mars orbit 150 km × 50,925 km × 170°.\(^7\) The total mass of Nozomi at launch was 540 kg, including 282 kg of propellant, an imaging camera, and other devices. However, due to electrical failure, the spacecraft flew by Mars on December 14, 2003 and went into a roughly 2-year heliocentric orbit. Another interesting mission was the Mars Global Surveyor, developed by NASA and launched in 1996.\(^8\) It was sent on a global mapping mission that examined the entire planet using the Mars Orbiter Camera,\(^9\) and also performed atmospheric occultation monitoring with the help of sister orbiters from an early mission. Then, after aerobraking and thrusters slowly converting the elliptical inserting orbit of 171.4 km × 17,836 km × 93° into a nearly circular Sun-synchronous orbit of 378 km × 378 km × 93°, which allowed all images of the same surface features that were taken by the spacecraft on different dates to be taken under identical lighting conditions. The spacecraft weighed 1,060 kg with full propellant at the time of launch. The similar Sun-synchronous orbit was used in a later Mars-orbiting mission. The Mars Odyssey was launched in 2001 and equipped with a thermal emission imaging system.\(^10\) Moreover, a lower altitude of 339 km was adopted to improve image resolution. The spacecraft had a launch mass of 725 kg, including 348.7 kg of fuel. The Mars Reconnaissance Orbiter used a Sun-synchronous orbit to conduct reconnaissance of the Martian surface as well.\(^11\) With the help of aerobraking and thrusters, it dropped from the capture orbit of 426 km × 44,500 km × 93° to an imaging orbit of 250 km × 316 km × 93°. A high-resolution imaging science experiment camera was carried to produce a resolution of 0.3 m from an altitude of 300 km. The launch mass was 2,180 kg including 1,149 kg of propellant and pressurant. Yinghao-1, the first Chinese orbiter to explore Mars, was planned to separate from Russia’s Phobos-Grunt spacecraft after the orbit inserting maneuver provided by the main engine of the Phobos-Grunt spacecraft.\(^12\),\(^13\) Its working orbit was assigned by the main spacecraft at 80 km × 80,000 km × 5° because no orbital maneuver was allowed by Yinghao-1. However, this mission ended in failure because of a launch vehicle problem.\(^14\) Besides, several Mars-orbiting probes are planned by aerospace organizations, such as MAVEN (NASA)\(^15\) and MOM (India),\(^16\) which are on the way to Mars. The next Chinese Mars mission is expected to be launched in 2016.\(^16\) In conclusion, imaging the Martian surface is a basic task assigned to most of Mars missions. Generally, it is required to perform imaging at the altitude of less than 600 km with good lighting conditions.

Mars has two moons similar to a C-type asteroid (i.e., Phobos and Deimos).\(^18\) Both have irregular shapes, and move along the Martian equatorial plane. Studying the surface of the soil of Phobos and Deimos helps to understand the origin and evolutionary information of stars and planets in the solar system.\(^4\) The masses of Phobos and Deimos are so small that the region of influence lies completely below the surface. Thus, Keplerian-type orbits around the small bodies, similar to what is possible around larger bodies, are consequently impossible. Two flight modes have potential applications for imaging or sample-return missions. One is to fly around Phobos or Deimos in a special kind of orbit designated as a quasi-synchronous orbits (QSO).\(^19\) In the rotating synodic reference frame, a coplanar QSO relative to the orbital plane can be described as an elliptical epicycle drifting back and forth in the direction of the orbital velocity of the body. An advantage of this mode is keeping the spacecraft in the vicinity of the moons for a long time, which is more beneficial for imaging. The other is to fly-by Phobos or Deimos at the encounter intersection between the trajectories of the spacecraft and the body. Actually, both of the modes are driving the spacecraft to orbit Mars. The orbital elements of the fly-by mode can be quite different from those of Phobos or Deimos; however, the fly-around mode allows similar orbital elements to the orbits. The QSO was almost coplanar with the moon orbits around Mars. Both the moons orbit nearly directly over the Mars’ equator, so the fly-round spacecraft should have a slight inclination, which is not suitable for global Martian surface reconnaissance. Besides, more orbital maneuvers are required to guide the spacecraft to the vicinity of the moon from the highly elliptical captured orbit.\(^20\)

Except for some planned missions like OSIRIS-REx,\(^21\) Phobos Surveyor and Gulliver,\(^22\) two actual projects with QSO of one of the moons as the destination are the USSR Phobos-1 and 2 missions (1988) and the Russian Phobos-Grunt mission, although they failed to finish their tasks. Both Phobos-1 and 2 weighed 6,200 kg, including a 3,600 kg pro-

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pulsion system, affording the most fuel-expensive maneuvers for Mars orbit insertion and the corrections used to come closer to Phobos.\textsuperscript{23} Russia carried out the Phobos-Grunt Sample Return Mission, which was planned to orbit Phobos to explore it, obtain a sample and then return with the sample.\textsuperscript{23} However, the Phobos-Grunt mission ended in failure because of a launch vehicle problem, along with the Ying-huo-1.\textsuperscript{14} Apart from these above, observations of Phobos and Deimos have been made by close encounters using spacecraft in different contexts, such as the close fly-bys of Mariner-9 in 1971, Viking-1 in 1977, Mars Global Surveyor in 1998, Mars Reconnaissance Orbiter in 2007 and 2008, and so on. Due to the higher periareon altitude than Phobos, the Viking-1 orbiter had more opportunities to have a glance at Phobos than other orbiters in Sun-synchronous orbits. It was trimmed to a highly elliptical orbit of 1,513 km × 33,000 km × 39°. Obviously, the working orbit designed for its primary mission was only to photograph Mars, without any consideration of the extended mission to obtain images of Phobos. Thus, only 17 encounters within 300 km in February 1977 were achieved by several maneuvers mainly to prevent the drift of periareon argument.\textsuperscript{23} The total velocity increment $\Delta V$ requirements for both encounters was 47 m/s. An opportunity for a series of Deimos encounters occurred in October 1977. A 50 m/s impulse was required to ensure five encounters within 100 km. Thus, the limited encounters is a disadvantage of the Viking-1 mission. In conclusion, compared with the long-stay, fly-around mode, the fly-by mode is an economical option for fuel to image the moon. However, regular encounters during the mission life are required in the new strategy to improve the lack of encounters.

Furthermore, Viking-1 was not proficient in terms of systematic scheduling when imaging both Mars and Phobos. The original mission was planned to photograph the Martian surface without an consideration of Phobos, and later imaging Phobos was added as an extended mission after launch, which resulted in using more fuel to prevent the drift of periareon argument. Even so, there were few encounter opportunities with Phobos. Due to the expensive cost of launch vehicles for Earth-to-Mars transfer, a multipurpose mission is preferable. Such a mission may consists of orbiter and lander, and even micro-spacecraft, such as Viking-1 and 2, Phobos-1 and 2, Mars 96, Mars Pathfinder, Phobos-Grunt and Ying-huo-1. Thus, it is quite practical to take a piggyback micro-spacecraft to Mars with the main orbiter. Some extra reconnaissance and investigation of the Martian surface, atmosphere and moons can be achieved using the micro-spacecraft. Besides, considering the high failure rate of Mars exploration, a fractionated architecture containing several functional probes is more competitive.

This paper discusses a multipurpose design for imaging both Mars and Deimos in the Pre-Phase A. It meets the requirements of the next Chinese mission to Mars and Deimos (i.e., Ying-huo-2). A frozen and repeating orbit is employed to provide as many as 280 periodic encounters within 100 km of Deimos. Compared with the other fly-around mode used by Phobos-1/2 or Phobos-Grunt missions, which required much fuel to guide the spacecraft to the vicinity of Phobos from the highly elliptical captured orbit, such as the 3,600 kg of fuel carried by the 6,200 kg Phobos-1/2,\textsuperscript{23} only 50.07 kg hydrazine fuel will be needed using the orbit inserting strategy proposed in this paper. To perform encounters with Deimos under all of the perturbations, orbital control and determination errors, the station-keeping strategies on the arguments of periareon and latitude are implemented using some corrections to the semi-major axis and inclination, respectively. A numerical simulation is used to verify the encounter opportunities with the help of the station-keeping strategies. Some detailed investigations on imaging, lighting and accessing conditions show that designing the working orbit is beneficial for photographing both the Martian surface and Deimos. Therefore, it is concluded that the trajectory in the Pre-Phase A for the piggyback spacecraft to be carried by the main orbiter of the next Chinese Mars mission is practical from an engineering perspective.

2. Mission Analysis

The idea of this article comes from a candidate proposal for the next Chinese Mars mission, Ying-huo-2, the successor project following the unsuccessful Ying-huo-1.\textsuperscript{13} According to the Roadmap 2050 for the Chinese Space Programme,\textsuperscript{17} another Mars exploration probe is not expected to be launched until 2016 at the earliest, even though the failure of Ying-huo-1 strongly affected Chinese Mars research. A competitive scheme for this mission includes a main orbiter, a micro-spacecraft and a demonstration lander, the wet masses of which are 2,600 kg, 300 kg and 300 kg, respectively.\textsuperscript{25–27} Due to the constraints of their launch vehicle (i.e., Rocket CZ-3B), only a total mass of 300 kg is allocated for the micro-spacecraft discussed in this paper. The orbiter is proposed to implement a global mapping of Martian surface, which requires a CCD visible light camera and a polar orbit. The targeting site of the tiny lander is the southern fringe of the southern pole. The micro-spacecraft is expected to assist the orbiter to photograph the Martian surface, and also to have a glance at one of the Martian moons, like Deimos. It is also planned to investigate the Martian ionosphere using the satellite-to-satellite occultation observation between the micro-spacecraft and main orbiter.

Considering the effect of photographing the Martian surface, a Sun-synchronous orbit is proposed, with its local time at the descending node of 10:30 AM and an inclination of 92.7°.\textsuperscript{26} To balance the aerodynamic drag, both the periareon and apoareon altitudes are chosen as 500 km, which is used to indicate this orbit as 500 × 500 km.\textsuperscript{28} Furthermore, according to the Earth-to-Mars transfer trajectory, the orbiter and piggyback spacecrafts will arrive at Mars on September 21, 2016.\textsuperscript{28} After the braking maneuver $\Delta V = 1,176$ m/s, the targetted orbit is 500 × 76,450 km with the captured inclination of 92.7°. Then, several deceleration maneuvers in the orbital plane will be implemented at the periareon to drop the apoareon altitude from 76,450 km to 500 km step by step,
as shown in Fig. 1. From the viewpoint of configuration of the combination, the micro-spacecraft is attached behind the orbiter for the convenience of separation, and the lander is locating between the orbiter and the spacecraft.

Essentially, the micro-spacecraft is an enhanced version of Yinghuo-1 equipped with an occultation receiver, optical imager, fluxgate magnetometer and so on. Compared to the unavailable orbital maneuvering of Yinghuo-1, the micro-spacecraft has a significant improvement in terms of the capability to perform orbital maneuvers. Inheriting from Yinghuo-1 are the size (0.75 × 0.75 × 0.60 m$^3$), a solar array length of 5.6 m, and mean and peak powers of 90 W and 180 W respectively. Due to the constraints of the rocket and orbiter, the total mass of this spacecraft comes to 300 kg. According to the conceptual system design in Pre-Phase A, a preliminary analysis is implemented to balance the masses of all the functional subsystems. It shows that a fuel mass of no more than 55 kg is acceptable, which should cover both consumption periods during insertion and station-keeping the nominal orbit.

In conclusion, what is required for the trajectory design are the scientific objectives, requirements from the cameras, the end of the Earth-to-Mars transfer trajectory, the orbiter’s maneuver strategy to drop the apoaereon altitude, and the total fuel utilized by the micro-spacecraft, which are listed as follows. Firstly, the cameras carried by the spacecraft require acceptable lighting conditions for the solar elevating angles and the velocity-height ratios when photographing the Martian surface and Deimos. The spacecraft will periodically encounter Deimos at a distance of less than 100 km with a tolerable Mars shadow during the mission life. Secondly, according to the preliminary design for this mission, the three combined probes arrive at the periareon with a semi-major axis of 3,771.99 km, eccentricity of 2.0331 and inclination of 92.7°. Thirdly, after braking the hyperbolical trajectory to a 500 × 76,450 km elliptical orbit, several deceleration maneuvers are required by the main orbiter to drop the apoaereon altitude to 500 km step by step, which may help the micro-spacecraft to enter its targeted orbit economically. Finally, the fuel carried by the micro-spacecraft can provide a total velocity increment of ΔV = 425.3 m/s according to the specific impulse of hydrazine used by the orbital maneuver engine.

### 3. Design Methodologies and Control Strategies for Working Orbit

#### 3.1. Design methodologies for working orbit

The main task of following orbiter is photographing Deimos many times in its life-span, and helping the head orbiter to photograph the Martian surface and explore its environment in the meanwhile. To have a glance at Deimos, two alternative orbits can be potentially applied to perform regular encounters with this Martian moon. One is a quasi-synchronous orbit (QSO) keeping around Deimos, which is considered to be the best approach for photographing Deimos all the time. However, the Keplerian-type way cannot be used to orbit Deimos because the mass of Deimos is too small in relation to its close distance to Mars; especially considering the fact that the Lagrange points are located inside the surface of Deimos. Quite similar to the formation flying theory, both the spacecraft and Deimos are dominated by the gravitational attraction of Mars. Thus, the semi-major axis, eccentricity and inclination expected by QSO are close to those of Deimos, i.e., 23,460 km, 0.0002 and 1.793°, which is very far from both the captured Earth-to-Mars trajectory and the main orbiter’s working orbit. However, the fuel carried by the micro-spacecraft can provide a total velocity increment of ΔV = 425.3 m/s. Thus, this candidate nominal orbit is beyond the capability of orbital maneuvering, although it would benefit photographing Deimos greatly.

The other choice is a frozen and repeating orbit to keep regular relative positions between the spacecraft and Deimos. This can avoid the situation of the two orbits having no intersection point due to arch movement; simultaneously, and also avoid the frequency of encounters decreasing caused by phase deviation in the two orbits. According to Section 3.3, the most economical way for this spacecraft is to use the main orbiter’s engine to brake the hyperbolical trajectory to the elliptical orbit before it separates from the main orbiter. Thus, a large amount of fuel is spent on changing the inserting inclination to the working inclination. According to the preliminary analysis in Pre-Phase A, the insertion inclination into Mars is designed as 92.7°. Although 63.43° and 116.57° can fix the argument of periareon, the latter is closer to 92.7°, and it will cost less fuel to change to the targeted value. Thus, only the frozen and repeating orbit with a critical inclination of 116.57° is chosen as the nominal orbit in this paper.

Moreover, the repeating condition between the orbital angular rate $n$ and the drifting rate of longitude ascending node $Ω$ is satisfied as follows:

$$\begin{align*}
\n_{\text{Deimos}} \cdot \Delta t + \Omega \cdot \Delta t &= m_1 \cdot 360°^\circ \\
n \cdot \Delta t &= m_2 \cdot 360°^\circ \text{ or } \Delta t = m_2 \cdot P_{s/c}
\end{align*}$$

where $\n_{\text{Deimos}}$ is the orbital angular rate at which Deimos circles Mars, $\Omega$ is the average drifting rate of the ascending node longitude, $\Delta t$ is the encounter period, both $m_1$ and $m_2$ are co-prime integers, and $P_{s/c}$ is the orbital period of the nominal elliptical orbit. Although the angular velocity varies momentarily due to the elliptical orbit, the angular velocity
average can be obtained from the orbital period (i.e., \( n = 360^\circ / P_{s/c} \)).

According to the mission requirements listed above, the altitude of the periareon should be the same as the one for the orbiter (i.e., 500 km). Thus, several pairs of \((m_1, m_2)\), like \((1, 1)\), \((1, 2)\), \((1, 3)\), \((2, 3)\), \((1, 4)\), \((3, 4)\) and so on, are used ergodically to substitute into Eq. (1) to solve the nominal orbit from the repeating condition. The solutions show that only the repeating condition of

\[
m_2 = \frac{3}{2} \frac{m_1}{n_1} \tag{2}
\]

is practical because the apoaareon altitude solved from this pair is 28,511.2 km, which is between the braking and working orbit of the main orbiter. Therefore, it can be assumed that the semi-major axis is 17,902.6 km, and the orbital period is 1,212.1 min, and the eccentricity is 0.7823. From the geometric viewpoint, the four periareon arguments can keep the micro-spacecraft’s trajectory intersect with that of Deimos (i.e., 25.87\(^{3}\)). Actually, the Mars-insertion periareon argument for the three combined probes is designed to be 334.13\(^{3}\) according to the preliminary analysis in Pre-Phase A, so the nominal periareon argument is 334.13\(^{3}\).

According to Eq. (1), it is obtained that the interval between two successive encounters is 2.53 days. Thus, the micro-spacecraft can encounter Deimos by fly-by as many as 280 times during the mission life of two years, as shown in Fig. 2.

Obviously, it is critical to the success of photographing Deimos whether or not the periareon argument can be frozen. Therefore, it is necessary to evaluate the efforts of all the perturbations including the solar radiation pressure, Sun’s gravity, Deimos’ gravity, Martian geopotential model, and orbital control and determination errors.

To check the drift of the periareon argument during the mission life, analytical and numerical approaches are employed to evaluate the efforts of the solar radiation pressure (SRP), Sun’s gravity, Deimos’ gravity, Martian geopotential model, and orbital control and determination errors. According to the orbital geometry demonstrated in the following figure, the drift of \( \omega \) will cause normal and radial variations in the encounter distance, denoted as \( \delta r_1 \) and \( \delta r_2 \), respectively. The normal variation is caused by the unfrozen periareon argument as \( \delta r_1 = r \cdot \delta \omega \) and the radial variation can be derived from the differentiation of the Kepler radius equation of

\[
r = \frac{p}{1 + e \cos \omega}
\]

as

\[
\delta r_2 = \frac{pe \sin \omega}{(1 + e \cos \omega)^2} \delta \omega,
\]

as shown in Fig. 3. Thus, the variations in encounter distance are formulized as

\[
\delta r = \sqrt{\delta r_1^2 + \delta r_2^2}. \tag{3}
\]

Take \( \omega = 334.13^3 \) for example, a drift of 0.01\(^3\) in \( \omega \) will cause a variation of 6.248 km in encounter distance.

The analytical and numerical approaches are used to evaluate the efforts of the solar radiation pressure (abbr. SRP), Sun’s gravity, Deimos’ gravity, Martian geopotential model, and orbital control and determination errors.

The SRP has some contribution to the eccentricity and periareon argument, which is formulized as following\(^{20}\):

\[
\begin{align*}
\frac{\mathrm{d}e_x}{\mathrm{d}t_{\text{Sun}}} &= e_x \left(1 - \frac{W}{1 - e_x^2 - e_y^2}\right) \\
\frac{\mathrm{d}e_y}{\mathrm{d}t_{\text{Sun}}} &= -C \sqrt{1 - e_x^2 - e_y^2 - e_z} \left(1 - \frac{W}{1 - e_x^2 - e_y^2}\right)
\end{align*}
\]

where \( e_x = e \cos \omega \) and \( e_y = e \sin \omega \), \( t_{\text{Sun}} \) are the true longitude of the Sun, and \( W \) and \( C \) are the SRP and oblateness parameters respectively, listed as

\[
C = \frac{3}{2} a_{\text{SRP}} \frac{a^2}{G/M_{\text{Mars}}} \quad \text{and} \quad W = \frac{3}{2} \frac{I_2}{a^5}.
\]

For an average spacecraft, the reflectivity coefficient \( c_R \) is considered to be 1.3. The total mass of the spacecraft picked-up is \( m = 300 \text{ kg} \), and the solar array length and width inherited from Yinghuo-1 are 5.6 m and 0.75 m (i.e., the effective projection area \( A \) is considered as 4.2 m\(^2\)). Moreover, the SRP constant is \( p_{SR} = 1.955 \times 10^{-6} \text{ N/m}^2 \) near Mars. Thus, the characteristic acceleration due to SRP is \( a_{\text{SRP}} = p_{SR} c_R A / m = 6.7 \times 10^{-8} \text{ m/s}^2 \). According to the theory developed by Colombo et al.\(^{20}\) the pair (\( e_x, e_y \)) will evolve around a fixed equilibrium point (i.e., frozen eccentricity). However, the drift of \( \omega \) during the mission life is just 0.0002\(^3\), which is equivalent to 0.121 km according to Eq (3).

Fig. 2. Working orbit of micro-spacecraft encountering Deimos.

Fig. 3. Normal and radial variations in encounter distance caused by the drift of periareon argument.
The perturbing acceleration resulting from the Sun’s gravity is expressed as
\[
a_{\text{Sun}} = \beta \left( \frac{3}{r_{\text{Sun}}^3} (r_{\text{Sun}} \cdot r) r_{\text{Sun}} - r \right)
\]  
(5)

where the gravity constant of the Sun
\[
\beta = \frac{GM_{\text{Sun}}}{r_{\text{Sun}}^3}
\]

\(G\) is the gravitational constant, \(M_{\text{Sun}}\) is the mass of the Sun, \(r_{\text{Sun}}\) and \(r_{\text{Sun}}\) are the position vector and the distance from Sun to Mars, respectively, and \(r\) is the position vector of the spacecraft in the Mars-central inertial frame. Thus, the characteristic acceleration for this picked-up spacecraft is 5.2 \(\times 10^{-7}\) m/s\(^2\). The variation of \(\omega\) due to the Sun is expressed, following Chao,\(^{30}\) as
\[
\dot{\omega} = \frac{3}{4} \beta \cdot a^4 n \left( 1 - \frac{3}{2} \sin^2 i_{\text{Sun}} \right) \left( \frac{3}{2} \sqrt{1 - e^2} \right)
\]  
(6)

where \(i_{\text{Sun}}\) is the inclination of the Sun respective to the Mars’ equator. Therefore, the drift of the periareon argument during the mission life is just 0.0008°, which is equivalent to 0.502 km according to Eq. (3).

The Deimos’ gravity is so slight that the conlinear libration points in the Mars-Deimos restricted three-body problem (i.e., \(L_1\) and \(L_2\)) are located inside the body of Deimos,\(^{19}\) and even less than the \(J_2\) perturbing acceleration at the encounter points. At the encounter, the acceleration due to \(J_2\) is
\[
a_{J_2} = \frac{3}{2} J_2 R e^2 \frac{GM_{\text{Mars}}}{r_{\text{encounter}}^3} = 2.6 \times 10^{-3} \text{ m/s}^2,
\]

where \(R e\) is the equatorial radius of Mars, and \(r_{\text{encounter}}\) is the radius at the encounter. However, the acceleration due to Deimos’ gravity has a significant effect in the vicinity of the encounter, which is 6.0 \(\times 10^{-3}\) m/s\(^2\) and 1.5 \(\times 10^{-5}\) m/s\(^2\) at the encounter distances of 50 km and 100 km, respectively. In addition, the encounter velocity between the spacecraft and Deimos is as fast as 2 km/s. Thus, the main accumulative effect is to impose a radical impulse at the encounter, which is integrated as
\[
\Delta V_e = \int_0^t \frac{GM_{\text{Deimos}} (r - r_{\text{Deimos}})}{||r - r_{\text{Deimos}}||^3} \, dt \approx 0.00022 \text{ m/s},
\]

where \(r_{\text{Deimos}}\) is the position vector of the spacecraft in the Mars-central inertial frame. The total impulse during the mission life is \(\Delta V = n_{en} \cdot \Delta V_e = 0.064 \text{ m/s}\), where \(n_{en}\) is the number of encounter times with Deimos (i.e., 288). According to Lagrange’s perturbation equation, the drift of \(\omega\) during the mission life can be obtained from
\[
\Delta \omega = \int_0^t \dot{\omega} \, dt \approx \Delta V_e \cdot -\sqrt{1 - e^2} \cos \omega = 0.0017°,
\]

which is equivalent to 1.062 km according to Eq. (3).

Both the osculating and secular terms caused by the Martian geopotential model are discussed in this section. According to Brouwer’s theory,\(^{31}\) the osculating short-periodic terms of the eccentricity and periareon argument are mainly gained from \(J_2\) perturbation, which are expanded as following:
\[
\Delta e_\epsilon = k_6 \cos i + k_7 \sin i
\]

\[
\Delta e_\xi = k_8 \cos i + k_7 \sin i
\]

(7)

where
\[
k_6 = \frac{1}{4} J_2 \left( \frac{R e}{a} \right)^2 \left( 6 - \frac{21}{2} \sin^2 i \right), \quad k_7 = \frac{7}{8} J_2 \left( \frac{R e}{a} \right)^2 \sin^2 i
\]

and
\[
k_8 = \frac{1}{4} J_2 \left( \frac{R e}{a} \right)^2 \left( 6 - \frac{15}{2} \sin^2 i \right).
\]

At the encounter, the periodic osculating eccentricity and periareon argument are 0.0039°, which is equivalent to 2.428 km according to Eq. (3).

The critical inclination of 116.36° is considered to freeze the eccentricity and periareon argument under the \(J_2\) perturbation. However, it is necessary to check the efforts from the higher-order geopotential perturbations. Therefore, the averaging technology developed by Kozai\(^{32}\) is employed to remove the osculating short-periodic terms and implement the long-term orbit propagator. The averaging periareon argument can be integrated numerically using Lagrange’s equation, i.e.,
\[
\omega = -\frac{\cos i}{na^2 \sqrt{1 - e^2} \sin i} \frac{\partial R}{\partial t} + \frac{\sqrt{1 - e^2}}{na^2 e} \frac{\partial R}{\partial e}
\]

The details of the disturbing function \(R\) can be found in Chao\(^{30}\) during the mission life with an integration step of 86,400 s using the seventh and eighth orders of the Runge-Kutta method. Actually, the disturbing function is presented up to the \(J_4\) perturbation in Chao\(^{30}\) and Kozai.\(^{32}\) Thus, to promote the assessment, a disturbing function up to the \(J_7\) perturbation \(R\) is derived in this paper, as follows:

\[
R = A_2 \frac{a^3}{a^3} (1 - e^2)^{-3/2} \left( \frac{1}{3} - \frac{3}{2} \sin^2 i \right) + A_3 \frac{a^3}{a^3} (1 - e^2)^{-5/2} e \left( \frac{3}{2} + \frac{15}{8} \sin^2 i \right) \sin \omega \sin i
\]

\[
+ A_4 \frac{a^6}{a^6} (1 - e^2)^{-7/2} \left[ \left( 1 + \frac{3}{2} e^2 \right) \left( \frac{3}{35} - \frac{3}{7} \sin^2 i + \frac{3}{8} \sin^4 i \right) + \frac{3}{4} e^2 \sin^2 i \frac{3}{7} \frac{1}{2} \sin^2 i \right] \cos 2\omega
\]

\[
+ A_5 \frac{a^6}{a^6} (1 - e^2)^{-9/2} \left[ 2e + \frac{3}{2} e^3 \right] \left( \frac{5}{12} - \frac{5}{18} \sin^2 i + \frac{1}{8} \sin^4 i \right) \sin \omega + \frac{1}{2} e^2 \left( \frac{5}{18} \sin^2 i + \frac{3}{16} \sin^4 i \right) \sin 3\omega \right]
\]
+ \frac{\mu A_\mu}{a^3} (1 - e^2)^{-11/2} \left[ \left( 1 + 5e^2 + \frac{15}{8} e^4 \right) \left( -\frac{5}{231} + \frac{5}{22} \frac{\sin^2 i}{\sin^2 \mu} - \frac{45}{88} \frac{\sin^4 i}{\sin^4 \mu} - \frac{5}{16} \frac{\sin^6 i}{\sin^6 \mu} \right) + \frac{5}{2} \left( 1 + \frac{1}{2} e^2 \right) \left( -\frac{5}{22} \frac{\sin^2 i}{\sin^2 \mu} + \frac{15}{22} \frac{\sin^2 i}{\sin^2 \mu} - \frac{15}{32} \frac{\sin^4 i}{\sin^4 \mu} \right) \cos 2\omega + \frac{5}{16} e^4 \left( -\frac{5}{88} \frac{\sin^4 i}{\sin^4 \mu} + \frac{3}{16} \frac{\sin^6 i}{\sin^6 \mu} \right) \cos 4\omega \right]
+ \frac{\mu A_\mu}{a^3} (1 - e^2)^{-13/2} \left[ \left( 3e + \frac{15}{2} e^2 + \frac{15}{8} e^4 \right) \left( 105 \frac{3}{143} \frac{\sin^2 i}{\sin^2 \mu} - \frac{88}{143} \frac{\sin^4 i}{\sin^4 \mu} + \frac{15}{64} \frac{\sin^6 i}{\sin^6 \mu} \right) \sin \omega + \frac{3}{16} e^4 \left( -\frac{11}{143} \frac{\sin^4 i}{\sin^4 \mu} + \frac{5}{64} \frac{\sin^6 i}{\sin^6 \mu} \right) \sin 5\omega \right] \sin \mu

where

$$A_\mu = -J_2 \Re \frac{P_1}{P_0}$$

and

$$P_1 = 3, \quad P_3 = 1, \quad P_4 = \frac{35}{8}, \quad P_5 = \frac{63}{8}, \quad P_6 = \frac{231}{16}, \quad P_7 = 429, \quad P_8 = \frac{429}{16}.$$  

The history of averaging the periareon argument is achieved by integrating the variation equation of the averaging \(\omega\), and is shown as follows, where the periareon argument deviates 0.012\(^\circ\) from the nominal value, which is equivalent to 7.498 km according to Eq. (3).

Due to the deviation in inclination from 116.57\(^\circ\) as result of the ODC error, there exists a secular drift on \(\Delta i\). The drift rate can be evaluated from the linear equation as

$$\dot{\omega} = \frac{3}{4} J_2 \Re e^2 \sqrt{\mu} \frac{5 \cos^2 i - 1}{a^{7/2}(1 - e^2)^2}. 5 \sin 2i \cdot \delta i,$$

where \(\delta i\) is the difference between the actual and nominal inclination. According to the ODC technologies developed during the Chinese lunar mission, CE-2,\(^{33}\) the insertion accuracy of \(\delta i\) can be improved to less than 0.01\(^\circ\) after a series of inserting maneuvers, so that the total drift of the periareon argument during the mission life can be estimated from the linear equation as 0.038\(^\circ\), which means the drift in the encounter distance will be 23.744 km according to Eq. (3).

In conclusion, all the factors, including SRP, Sun’s gravity, Deimos’ gravity, Martian geopotential model and ODC errors, contribute to the drift in the encounter distance of no more than 36.07 km during the mission life. Therefore, to further freeze the orbit, a station-keeping strategy is proposed in the following section.

### 3.2. Station-keeping strategies for working orbit

To perform a mission analysis during an encounter with Deimos, all of the perturbations from the SRP, Sun’s gravity, Deimos’ gravity and geopotential model are considered in this section. The encounter opportunities are achieved by two approaches: one is to keep the geometrical intersection between the spacecraft’s trajectory and Deimos’ orbit, and the other is to keep the spacecraft and Deimos synchronous in the argument of latitude (i.e., \(u\)).

The geometrical intersection can be guaranteed using a frozen periareon argument. Only the ignorable drift of \(\omega\) can hold the regular encounters between Deimos and the nominal working orbit. Fortunately, the perturbations have only a slight effect on the drift of \(\omega\), while the orbital control or determination error in inclination has the most influence. Even in the case without station-keeping control of inclination, the encounter distance \(\Delta L\) varies from 16.4–83.6 km during the mission life due to the drift of \(\omega\). Accordingly, an efficient inclination controller is implemented to hold the periareon argument during the mission life.

According to \(J_2\)’s contribution on the secular rate of periareon argument, i.e.,

$$\omega = \frac{3}{4} J_2 \Re e^2 \sqrt{\mu} \frac{5 \cos^2 i - 1}{a^{7/2}(1 - e^2)^2} 5 \sin 2i \cdot \delta i,$$

a slight change in inclination will cause a variation in the secular rate:

$$\dot{\omega} = \frac{3}{4} J_2 \Re e^2 \sqrt{\mu} \frac{5 \cos^2 i - 1}{a^{7/2}(1 - e^2)^2} 5 \sin 2i \cdot \Delta i$$

Thus, the drift rate of \(\omega\) will depend on whether or not the actual inclination is above or below 116.57\(^\circ\). Therefore, a simple but practical strategy is to employ a single burn to change the positive and negative sign of \(\delta i\). The controlled output \(\Delta i\) of every single burn should be greater than the ODC accuracy of \(\delta i\), such as \(\Delta i = \pm 0.02\(^\circ\)\). If the allowed boundary of the periareon argument is set as 0.01\(^\circ\) equivalent to 6.248 km, the interval between two successive maneuvers is about one year, so that only two maneuvers are required during the mission life, as demonstrated in Fig. 5. Both of the maneuvers are performed at the descending or ascending node. However, the descending node is chosen from the viewpoint of saving the velocity increment

$$\Delta V = \sqrt{\mu} \frac{\Delta i}{r_{dn}}$$
where the radius at the descending node $r_{\text{dn}}$ is much greater than the ascending node for $\omega = 334.13^\circ$. Thus, a total velocity increment of $2 \times 0.8 \text{ m/s}$ is provided by the micro-spacecraft engine. This fuel cost will be

\[ \Delta m = m \frac{\Delta V}{I_{sp} \cdot g} = 0.228 \text{ kg}, \]

where $I_{sp}$ is the specific impulse of hydrazine, and $g$ is the gravitational acceleration (i.e., effective exhaust velocity $I_{sp} \cdot g$ is 2,100 m/s).

On the other hand, the spacecraft and Deimos should arrive at the encounter point at the same time. This means that the spacecraft should reach the appropriate argument of latitude (i.e., $\omega$). Compared to the slow drift in $\omega$, the fast drift in $u$ is caused by the difference between the actual and nominal semi-major axis (i.e., $a$). However, due to the primary errors from orbital determination and control (abbr. ODC), it is difficult to hold the expected $a$ at the encounter. Taking the error of $\Delta a = 1 \text{ km}$ as an example, the encounter distance during two successive encounters can be increased to 30 km, which is considered as the main threat to the mission. Actually, the Sun’s gravity and geopotential model have no long-term effect on $a$, due to the perturbation theory of conservative force. Besides, very few Mars shadows (listed in Section 4.1) during the mission life make the SRP cause more than a slight contribution to $a$ as compared to $e_i$ and $e_\gamma$. According to the discussion in Section 3.1, the main accumulative effect of Deimos’ gravity is to impose a radical impulse at the encounter, which does not contribute to $a$.

The ODC errors in the semi-major axis make the micro-spacecraft go forward or backward in the direction of flight, which may lead to a collision between the spacecraft and Deimos. The spacecraft will fail to hold a safe distance in the flight direction because the ODC errors may drive the spacecraft towards Deimos gradually, pass by pass. Different from the long-term drift imposed by ODC errors in the flight direction, no long-term drift occurs in the radial direction, which is a technical merit of relative astrodynamics and has the potential application for avoiding collision. Thus, a safe distance $\Delta L$ in the radial direction is reserved to hold the spacecraft far from Deimos, even if ODC errors occur. Generally, to locate the nominal encounter point 50 km away from Deimos, on the Mars’ equator, is preferable according Viking-1 and other Phobos or Deimos fly-by missions, shown in Fig. 6.

Based on the analysis mentioned above regarding the drifts in flight and radial directions, the ODC error in a semi-major axis has a significant effect on the actual encounter distance between the micro-spacecraft and Deimos as compared to other elements. Thus, a correction strategy for the semi-major axis is required to hold the argument of latitude in the flight direction. All of the correction maneuvers are performed at the apoareon of the working orbit. According to the orbital determination technology developed during the Chinese lunar probe CE-2, the accuracy of the semi-major axis can be improved as 1 km. It is practical to set each correction maneuver larger than the accuracy, so an acceptable correction of $\Delta a$ is $\pm 2 \text{ km}$. Similar to the station-keeping strategy for the periareon argument, the argument of latitude is expected to fluctuate between the positive and negative boundaries allowed.

Assuming it is acceptable to photograph Deimos within 100 km using the camera carried by the micro-spacecraft, the station-keeping box of $\pm 86.6 \text{ km}$ can be obtained using the triangle illustrated in Fig. 7. The time interval between the two successive maneuvers is 18 orbital periods (i.e., 15
days). Although the allowed boundary is ±86.6 km, the actual imaging distance can vary from 50 to 100 km during an interval due to the safe distance reserved in the radial direction.

For each maneuver of $\Delta a = \pm 2$ km, the velocity increment

$$\Delta V = \frac{\Delta a}{2a} V_i$$

($V_i$ is the orbital velocity at the apoapereon) is provided by the micro-spacecraft engine (i.e., 0.21 m/s), which will cost fuel of 0.03 kg. Thus, a total fuel mass of 1.5 kg is required for a total of 50 station-keeping maneuvers during the mission life. Compared to Viking-1, which was weak in systematic schedules in its original and extended missions, thereby resulting in using more fuel but having fewer encounter opportunities, a suitable station-keeping strategy is addressed during Pre-Phase A, not after launch, in order to save fuel consumption significantly.

Furthermore, a numerical simulation including all of the perturbations is implemented to present the encounter opportunities during the mission life with the help of the control strategies, including the station-keeping control of $\omega$ and the station-keeping control of $u$. The history of encounter distance using the combined control strategies is shown in Fig. 8. During the mission life, 48 maneuvers for station-keeping $u$, labeled by the red pentagons, are implemented at the moment when the encounter distance tends to go beyond 100 km; while two maneuvers for $i$, labeled by the black circles, are implemented when $\omega$ arrives at its boundary. Due to the perturbations and ODC errors, the actual encounter distance can be held between 46.04 and 103.54 km, which is nearly equal to the expected values.

### 3.3. Inserting strategy for working orbit

According to the candidate proposal for the next Chinese Mars mission, it includes a 2,600 kg main orbiter, and the 300 kg micro-spacecraft discussed in this paper. There exist two alternatives to insert the nominal orbit. The main difference between the two strategies is whether or not the micro-spacecraft will share the braking maneuver in the hyperbolical trajectory to 500 × 76, 450 km elliptical orbit with the main orbiter. Moreover, the 55 kg fuel carried by the micro-spacecraft can only provide a total velocity increment of $\Delta V = 425.3$ m/s. Thus, the winning strategy should require less fuel consumption.

The first strategy is to separate the micro-spacecraft and main orbiter before they arrive at Mars. Generally, the Earth-to-Mars transfer trajectory is aiming at the desired orbit for the main orbiter, not the orbit for the piggyback spacecraft. Actually, the interplanetary trajectory ends at its periar-
eon altitude is designed to be 5.7736 km/s according to the preliminary analysis in Pre-Phase A, and the velocity of the desired orbit at the same periareon is 4.4472 km/s. Thus, the first strategy will cost a total velocity increment of \( \Delta V = 1.326.4 \text{ m/s} \) for both direct or step-by-step drops. However, compared with the second strategy with \( \Delta V = 425.3 \text{ m/s} \) just for changing the inclination, the first strategy requires more fuel and is beyond the maneuver capability of the micro-satellite, which accounts for why it is not chosen in this paper.

Apart from the allocated inclination, the argument of latitude is also required to achieve an encounter with Deimos at the descending node. Similar to the station-keeping strategy, a slight correction at the semi-major axis \( \Delta a \) can be used to initiate the expected drift of the argument of latitude \( \Delta u \). The effort of \( \Delta a \) during the \( j \) periods can be derived from the Keplerian equation as

\[
\Delta u = \left( \frac{\mu}{(a + \Delta a)^3} - \frac{\mu}{a^3} \right) \cdot P_{i/c} \cdot j = -3\pi \frac{\Delta a \cdot j}{a} \tag{9}
\]

where \( j \) indicates the adjustment duration cost of \( j \) periods. Therefore, a longer duration can be used to enhance the effort on \( \Delta u \).

The equation mentioned above indicates that the fuel cost varies inversely with the adjustment duration. Thus, an economical way to capture the destination of latitude is proposed as follows. If the required adjustment satisfies \( 0^\circ < \Delta u_0 < 180^\circ \), the spacecraft is controlled by, firstly, lowering the semi-major axis to drive the drift of \( \Delta u \) until it meets the required \( \Delta u_0 \), and then raising the semi-major axis to brake the drift. However, if \( 180^\circ < \Delta u_0 < 360^\circ \), the spacecraft is controlled by, firstly, raising and then lowering the semi-major axis to the required \( \Delta u_0 \). Taking the maximum \( \Delta u_0 = 180^\circ \) for instance, the fuels cost using the adjustment strategy is shown in Table 1. A tolerable adjustment duration is 25 days in practical engineering, which requires a fuel mass of no more than 2.61 kg.

To conclude, some economical control strategies are proposed in this paper to meet the requirements of the next Chinese mission to photograph both Mars and Deimos, including inserting maneuvers that adjust the inclination from 92.7° to 116.57°, two raising and lowering maneuvers to allocate the argument of latitude, two station-keeping maneuvers to prevent the drift of periareon argument, and several station-keeping maneuvers to hold the spacecraft inside the flight direction box. Therefore, a fuel mass of no more than 45.518 kg is required for both the inserting and station-keeping strategies, and 10% of the mass mentioned above is required by the attitude control system of the micro-spacecraft according to the experiential assessment of a Chinese satellite,\(^{31}\) which results in a total fuel mass of 50.07 kg being needed for the whole mission. Thus, the fuel mass of 55 kg carried by the micro-spacecraft can cover all of the maneuvers expected during mission life.

4. Analysis of Photographing, Lighting and Accessing Conditions

4.1. Photographing and lighting conditions

According to the preliminary analysis in Pre-Phase A, the micro-spacecraft is scheduled to photograph Mars near the periareon to perform a global survey for the Martian surface together with the main orbiter, and to photograph Deimos by itself during the fly-by.

Due to the imaging resolution on the Martian surface, only an altitude of less than 600 km is feasible for the camera. Therefore, the accessible interval of argument of latitude can be solved from the radius equation of \( r = p/[1 + e \cos(u - \omega)] \). Then the specified altitudes to photograph the Martian surface are obtained from \( \delta = \arcsin(\sin i \sin u) \), as \(-4^\circ \rightarrow 40^\circ \), demonstrated in Fig. 9. Only the ascending passes from \(-4^\circ \rightarrow 40^\circ \) are chosen with the flight duration of 608 s. This is because the mean altitude of the descending passes reaches 11,036 km, which prevents the acquisition of high-resolution images.

Another parameter for photographing the surface is the ratio between the relative velocity in a Martian fixed coordinate system and the altitude above the surface, shown in Fig. 10. The velocity-height ratio (VHR) of the ascending passes ranges from 0.006 to 0.008 s\(^{-1} \), reaching its maximum at the periareon (i.e., at the latitude of \(-23.2^\circ \)). Besides, the impractical ratio of descending passes varies from \(1.6 \times 10^{-5} \) to \(1.5 \times 10^{-4} \text{ s}^{-1} \), which accounts for not being chosen as well.

The solar elevating angle (SEA) is critical for photographing the Martian surface, which sets the lighting conditions for the sub-satellite points on the Martian surface. It can be obtained from the angle between the Mars-Sun vector and the tangent plane at the sub-satellite point.\(^{30}\) The SEAs during both the ascending and descending passes are illustrated on the latitudes of \(-4^\circ \) and \(-40^\circ \), respectively in Fig. 11. It is concluded from this figure that the SEAs on the ascending passes tend to decrease at the beginning of mission life,

Table 1. Fuel cost using the adjustment strategy in the case of \( \Delta u_0 = 180^\circ \).

| Adjustment duration [day] | Numbers of orbital periods \( j \) | \( \Delta u \) [kg] |
|--------------------------|---------------------------------|------------------|
| 8.4                      | 10                              | 2.92             |
| 16.8                     | 20                              | 3.96             |
| 25.2                     | 30                              | 2.61             |
| 33.6                     | 40                              | 1.98             |

Fig. 9. Track of sub-satellite point of the working orbit for the micro-spacecraft on the Martian surface.
but for the descending passes increase gradually from 0° until reaching a maximum. After the SEAs on the descending passes decrease from the maximum to 0°, those on the ascending passes have rise to 70° by the end of mission life. It is interesting that both the ascending and descending passes at the beginning possess an excellent SEA at a latitude of 40°. Therefore, it is preferred that the micro-spacecraft photographs the Martian surface on the ascending passes, where both velocity-height ratio and SEA are beneficial for photography.

On the other hand, it is reasonable to locate the micro-spacecraft between the Sun and Deimos for the purpose of photographing Deimos, which means the encounter points are designed for the descending nodes. Thus, good imaging and lighting conditions for photographing Deimos exist.

Considering the parallel light case, a numerical simulation is implemented to judge whether or not the micro-spacecraft is positioned behind the Mars and inside the cylinder centered in the Sun-Mars vector. Thus, the duration of Martian shadow demonstrated in Fig. 13 shows that there exists no shadow until the 316th day, and then a slight shadow of less than 32 min lasting until the end of mission life. However, the longest shadow, of approximately 202.7 min, will appear on the 977th day, which should be considered for the extended mission if the micro-spacecraft can survive until then.

### 4.2. Accessing condition for spacecraft-Earth and spacecraft-orbiter modes

Due to the long distance from Mars or Deimos to any specified earth-based station, it is necessary to investigate the accessing conditions between them. A numerical simulation is implemented to count the distance and accessing duration between the micro-spacecraft and some different ground stations, which are almost identical to each other. It was concluded, from Figs. 14 and 15, that there will be one or two links between the spacecraft and station every day, with a minimum accessing time of 424 min, the mean
being 668 min and the maximum being 836.2 min. The link distance reaches its maximum of \(40.025 \times 10^7\) km on the 300th day, and reaches its minimum of \(6.205 \times 10^7\) km at the end of the mission.

Based on a detailed analysis on photographing conditions, it was concluded that, at the beginning of the mission, it is suitable to transmit the measurement/control data and images at a low bit rate due to the radio signals being reduced up to 100 dB because of the long distance, so transmitting the signal at a low bit rate is suitable. However, a high bit rate is applicable at the end of the mission owing to the shorter distance.

The main orbiter is accessible relative to the micro-spacecraft on each pass, even though they fly at different orbit altitudes, making it possible to study the Martian atmosphere using occultation technologies. The occultation is investigated by statistically counting the opportunities when the spacecraft-orbiter vector is at a specified height (60 km for this paper) above the Martian surface. Figure 16 shows the latitude of occultation during the mission life. Generally, there are two times of occultation during every orbital period of the micro-spacecraft. These take place at the moments when the spacecraft is approaching and departing its periareon. As the periareon argument is frozen by the critical elliptical orbit, the latitude boundary is achieved from \(\delta_0 = \arcsin(\sin i \cdot \sin \omega) = -23^\circ\). Hence, the occultations occurring on the descending passes are above the latitude of \(-23^\circ\), while the occultations occurring on the ascending passes are below the latitude of \(-23^\circ\).

5. Conclusion

Exploration of the primary Martian moon (Deimos) has drawn much attention from astronomical and astronautical communities, for the potential merit of understanding Deimos’ origin and its genetic relation to Mars. It is challenging to spend only 55 kg of fuel to deliver a piggyback spacecraft to an appropriate orbit that has tolerable lighting conditions to photograph both the Martian surface and Deimos. As a comparison, Viking-1 was weak in systematic schedules for photographing both Mars and Phobos. The original mission was planned only for photographing Martian surface, without consideration of Phobos. Later it was decided to photograph Phobos after launch as an extended mission. This resulted in requiring more fuel to prevent the drift of \(\omega\). As a result, there were fewer encounter opportunities with Phobos. Thus, preparing a multipurpose design at the Pre-Phase A, not after launch, for photographing both Mars and Deimos is addressed in this paper.

Moreover, this paper proposes trajectory design for a piggyback micro-spacecraft carried by the main orbiter of the next Chinese Mars mission, named Yinghuo-2. Compared to the fly-around QSO mode used by Phobos-1/2 and Phobos-Grunt missions, requiring much fuel to guide the spacecraft to the vicinity of Phobos from the highly elliptical captured orbit, the first contribution is to employ a frozen and repeating orbit to meet the encounter requirements of having as many as 280 opportunities to photograph Deimos within 100 km.
Among some candidate insertion strategies, it is proposed in this paper that the micro-spacecraft share the braking maneuver in the hyperbolic trajectory to 500 × 76, 450 km elliptical orbit with the main orbiter. Then several deceleration maneuvers will be implemented by the main orbiter at the periapse to drop the apoapse altitude from 76,450 km to 500 km step by step. An intermediate orbit of 500 × 28,522.3 km will be used during deceleration, where the micro-spacecraft will separate from the orbiter. Only an inclination adjustment from 92.7° to 116.57° will be required to guide the micro-spacecraft into its nominal orbit. Furthermore, to perform encounters with Deimos regularly under all of the perturbations and orbital control and determination errors, the station-keeping strategies for the arguments of periapse and latitude will be implemented using some corrections to the semi-major axis and inclination, respectively. A numerical simulation is used to verify the encounter opportunities with the help of the station-keeping strategies.

Considering the camera working at an altitude of less than 600 km, the ascending passes from the latitude of −4° to 40° are scheduled for photographing the Martian surface, while the solar elevating angles vary from 15° to 80°, and the velocity-height ratios vary from 0.006 s⁻¹ to 0.008 s⁻¹. For photographing Deimos within 100 km, the angles can be 80° early in the mission, and the ratios vary from 0.02 s⁻¹ to 0.04 s⁻¹. In addition, all durations of Mars’ shadow are less than 32 min in the mission life of two years. Therefore, it is concluded that the trajectory design in Pre-Phase A for the piggyback spacecraft carried by the main orbiter of the next Chinese Mars mission is practical from an engineering perspective.

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