Nonvanishing pion masses for vanishing bare quark masses

Aiichi Iwazaki
Nisho-kakusha University,
6-16 Sanbancho Chiyoda-ku Tokyo 102-8336, Japan.
(Dated: Jan. 10, 2019)

It is generally thought that pion masses vanish when bare light quark masses $m_q$ vanish. This is because QCD is believed to be chiral symmetric when the bare quark masses vanish. We discuss a possibility that even when $m_q = 0$, the chiral symmetry is explicitly broken in strong coupled QCD, while it is not in weakly coupled QCD. Assuming that QCD monopoles are dynamical degrees of freedom in the strong coupled QCD, we show that the chiralities of quarks are not conserved in monopole quark scatterings. The chiral non symmetric interactions are very weak relative to the other chiral symmetric strong interactions of quarks and gluons. This relatively weak interactions produce small pion masses. Analyzing vacuum energies of quarks interacting with monopoles, we find that chiral condensate $\langle \bar{q}q \rangle \neq 0$ is caused by monopole condensate.

PACS numbers: 12.38.-t,12.38.Aw,11.30.Rd,14.80.Hv

I. INTRODUCTION

Pions are Nambu-Goldstone bosons associated with chiral symmetry SU$_A$(2) and their nonzero masses are produced by the bare light quark masses. Namely, the chiral symmetry is spontaneously broken and simultaneously chiral condensate $\langle \bar{q}q \rangle$ arises. The condensate is the order parameter of the chiral symmetry. But, it is assumed in the arguments that we can define the chiral symmetric strong coupled QCD when the bare light quark masses $m_q$ vanish. Because chiral symmetric weakly coupled QCD can be defined perturbatively, it has been expected that the chiral symmetry holds even in the strong coupled QCD.

In this paper we discuss that the chiral symmetry does not hold in the strong coupled QCD where QCD monopoles are supposed to be quasi-particles. We show that the monopole quark interactions explicitly break the chiral symmetry. We also discuss that the monopole condensation induces the chiral condensate. We cannot define chiral symmetric strong coupled QCD even if the bare quark masses vanish. On the other hand, weakly coupled QCD is defined such that it is chiral symmetric.

It has recently been pointed out that QCD monopoles are present in strong coupled QCD and play important roles for strong coupled QGP as well as the quark confinement. It has been discussed that they are dominant components of the QGP near the transition temperature $\sim 160$MeV, while they are absent in perturbative QCD. Such monopoles have been also discussed to play a role for chiral symmetry breaking. That is, it has been shown that the monopole pair production of massless quarks takes place in the monopole condensed vacuum when a classical color charge is put in the vacuum. Furthermore, their roles for the chiral symmetry breaking have been pointed out in lattice gauge theory. In particular, the association with zero modes of Dirac operators has been examined: Addition of a pair of monopole and anti-monopole to gauge field configuration in Dirac operators increases the number of the zero modes. It suggests that the monopoles play a role for the generation of the chiral condensate.

In this paper we assume that the monopoles are present in strong coupled QCD and show that the chiralities of the monopoles are not conserved in the monopole quark scatterings. It has been discussed that in the scattering, we need to impose a chiral non symmetric boundary condition for quarks at the location of the monopoles. We may effectively describe such an interaction that $\bar{q}q\Phi^\dagger \Phi$ where $q$ denotes up and down quarks, and $\Phi$ denotes the monopole. The interaction is similar to quark mass terms. Using the interaction, we calculate the vacuum energies of the quarks and find that the chiral condensate $\langle \bar{q}q \rangle \neq 0$ takes place only when the monopole condensation $\langle \Phi \rangle \neq 0$ is generated.

First, we would like to show that the chiralities are not conserved in the monopole quark scattering. When we consider monopole quark interaction in QCD, we notice that the monopoles arise only in the low energy region of QCD. In the region the coupling strength is so large as for quarks and gluons themselves to be not appropriate dynamical variables. It has been discussed that the assumption of Abelian dominance may hold in the low energy region. The assumption has been examined in lattice gauge theories using Maximal Abelian gauge. According to the assumption, the dynamical degrees of freedom relevant to the low energy physics are massless color triplet quarks, maximal Abelian (diagonal) gauge fields and monopoles. Off diagonal components of the gauge fields are massive so that they are not relevant. Although the monopoles are massive, they are nearly massless at a critical temperature.
where they begin to condense in vacuum. After their condensation, the masses of excited monopoles are much less than those of off diagonal gluons. Thus, relevant excitations to the low energy QCD are massless quarks, Abelian (diagonal) gluons and the monopoles. In the present paper we assume the Abelian dominance in order to treat explicitly the monopole excitations in strong coupled QCD.

Under the assumption we discuss monopole quark scatterings. In order to do so, we briefly explain monopole excitations in $SU(3)$ gauge theory. In $SU(3)$ gauge theory, we have three types of monopoles, which are characterized by root vectors of $SU(3)$, $\tilde{c}_1 = (1,0), \tilde{c}_2 = (-1/2,-\sqrt{3}/2)$ and $\tilde{c}_3 = (-1/2,\sqrt{3}/2)$. They describe the couplings with the maximal Abelian gauge fields, $A_a^{\mu,8}$ such as $\tilde{c}_1 A_a^{\mu}$. For example a monopole with $\tilde{c}_1$ couples only with $A_a^{\mu} = \tilde{c}_1 A_a^{\mu}$. Thus, the quarks coupled with the monopole are a doublet $q = (q^+, q^-, 0)$ of the color triplet. Here the index $\pm$ of $q^\pm$ denotes a positive or negative charged component associated with the gauge field $A_2^\mu \lambda^3/2; \lambda^a$ are Gell-Mann matrices. Similarly the other monopoles couple with the quark doublets, $q = (0, q^+, q^-)$.

Thus, we consider scattering of a monopole and a massless quark doublet $(q^+, q^-)$.

$$\gamma_\mu (i \partial^\mu + \frac{g}{2} A^\mu) q^\pm = 0,$$

where the gauge potentials $A^\mu$ denotes a Dirac monopole given by

$$A_\phi = g_m (1 - \cos(\theta)), \quad A_0 = A_\tau = A_\theta = 0$$

where $\bar{A} \cdot d\vec{x} = A_\phi dx + A_\theta d\theta + A_\phi d\phi$ with polar coordinates $r, \theta$ and $\phi = \arctan(y/x)$. $g_m$ denotes a magnetic charge with which magnetic field is given by $B = g_m r^2/\sqrt{\pi}$. The magnetic charge satisfies the Dirac quantization condition $g_m g = n/2$ with integer $n$ where $g$ denotes the gauge coupling of $SU(3)$ gauge theory. Hereafter, we assume the monopoles with the magnetic charge $g_m = 1/2g$.

The monopole quark (in general, fermion) dynamics has been extensively explored, in particular in the situation of monopole catalysis of baryon decay (so called Rubakov effect). The point is that conserved angular momentum has an additional component. That is, it is given by

$$\vec{J} = \vec{L} + \vec{S} + g g_m \vec{r}/r$$

where $\vec{L}$ ($\vec{S}$) denotes orbital (spin) angular momentum of quark. The additional term $\pm gg_m \vec{r}/r$ play an important role of chiral symmetry breaking. Owing to the term we can show that either the charge or the chirality is not conserved in the monopole quark scattering. When the chirality (or helicity $\sim \vec{p} \cdot \vec{S}/|\vec{p}| |\vec{S}|$) is conserved, the spin must flip $\vec{S} \rightarrow -\vec{S}$ after the scattering because the momentum flips after the scattering; $\vec{p} \rightarrow -\vec{p}$. Then, the charge must flip $g \rightarrow -g$ because of the conservation of $\vec{J} \cdot \vec{r}$, i.e. $\Delta(\vec{J} \cdot \vec{r}) = \Delta(\vec{S} \cdot \vec{r}) + \Delta(g g_m r) = 0$. ($\Delta(Q)$ denotes the change of the value $Q$ after the scattering.) On the other hand, when the charge is conserved (it leads to $0 = \Delta(\vec{J} \cdot \vec{r}) = \Delta(\vec{S} \cdot \vec{r})$), the chirality $\vec{p} \cdot \vec{S}/|\vec{p}| |\vec{S}|$ must flip because the spin does not flip $\vec{S} \rightarrow -\vec{S}$. Thus we find that either the charge or the chirality conservation is lost in the scattering. (In the discussion we assume that the mass of the monopole is sufficiently large so that the collision does not change the state of the monopole. That is, the energy of the quark is much less than the mass of the monopole.)

To explicitly define the scatterings, it has been discussed that we need to imposed a boundary condition for the quarks at the location of the monopoles. It is either of charged conserved but chirality non conserved boundary condition or chirality conserved but charge non conserved one. The charge is strictly conserved because the charge conservation is guaranteed by the gauge symmetry. When the quark flips its charge, monopoles are charged or heavy charged off diagonal gluons must be produced to preserve the charge conservation. But the processes cannot arise in the low energy scattering. Charged monopoles are dyons and they are heavy. Thus, inevitably the chirality is not conserved. The right handed quark $q_R$ is transformed to the left handed quark $q_L$ in the scattering. That is, we need to impose the boundary conditions $q^\pm_R (r = 0) = q^\mp_L (r = 0)$ at the location of the monopole, which breaks the chiral symmetry. The detailed analyses have shown that the chiral symmetry breaking is caused by chiral anomaly in QCD. (Even if we impose the chirality conserved but charge non conserved boundary condition $q^\pm_R (r = 0) = q^\mp_L (r = 0)$ at the location of the monopole, we can show that the charge is conserved, but chirality is not conserved. The chirality non conservation arises from chiral condensate $\langle q \bar{q} \rangle \propto 1/r^3$ locally present around each monopole at $r = 0$. The condensate is formed by the chiral anomaly when we take into account quantum effects of gauge fields $A_\mu = \delta A_\mu^{\text{quantum}} + A_\mu^{\text{monopole}}$. Eventually, the chirality non conserved boundary condition is realized in physical processes. These results are by-products in the analyses of the Rubakov effect.)
Furthermore, quarks may change their flavors in the scattering. For instance, $u$ quark is transformed into $d$ quark. Then, the monopole must have a SU(2) flavor after the scattering. But it is impossible because there are no such monopoles with SU(2) flavors in QCD. Therefore, quarks cannot change their flavors in the scattering with the monopole. Quarks change only their chiralities. It results in SU$_A(2)$ chiral symmetry breaking in the scattering.

We should mention that the monopole quark interaction is weak compared with hadronic interactions. We note that the interaction does not depend on the strong coupling $\alpha_s = g^2/4\pi$ of QCD in the tree level. This is because the quarks have color charges $g$ and the monopoles have magnetic charges $g_m = 1/2g$. Therefore, the monopole quark interaction ($\propto g \times g_m$) does not involve $\alpha_s$ in the tree level. On the other hands, the quark gluon interactions depend on $\alpha_s$ even in the tree level. More explicitly, the monopole quark interaction is proportional to $gg_m = 1/2$, while quark gluon or quark quark interactions are proportional to $g^2 = 4\pi\alpha_s$. Because $\alpha_s(Q) \sim 0.5$ even for the energy scale $Q \sim 1$GeV, the monopole quark interaction $gg_m = 1/2$ is much smaller than the other hadronic interactions $g^2 \sim 2\pi$.

Quarks change their chirality without the change of their flavors in the monopole quark scattering; the chiralities change at the location of the monopole. Effectively, we can describe such an interaction by using monopole field $\Phi$ that $-g'q\bar{q}|\Phi|^2(\bar{u}_L u_R + \bar{u}_R u_L + \bar{d}_L d_R + \bar{d}_R d_L) = -g'|\Phi|^2(\bar{u}u + \bar{d}d)$ with $g' > 0$. The parameter $g'$ is roughly given by the inverse of the monopole mass $M$ times the ratio of the coupling strength $gg_m$ to $g^2$, i.e. $g' \sim M^{-1}gg_m/g^2 = M^{-1}/4\pi$. The mass $M$ is taken to be of the order of $\Lambda_{QCD}$. (More precisely, the monopole quark interaction is described by using three types of the monopole fields $\Phi_i$ ($i = 1 \sim 3$) such that $-g'|\Phi|^2(\bar{q}_1 q_1 + \bar{q}_2 q_2 + \bar{q}_3 q_3) + |\Phi|^2(\bar{q}_2 q_2 + \bar{q}_3 q_3)$ where indices $i$ of $q_i$ denotes color component of quark $q = u$ or $d$. The interaction preserves Weyl symmetry in QCD.)

The chiral symmetry SU$_A(2)$ is explicitly broken by the monopole quark interaction even if the bare light quark masses $m_q$ vanish. The symmetry breaking effects of the interaction are bigger than those of the bare mass terms. Indeed, when the monopole condenses, the term $g'q\bar{q}|\Phi|^2$ gives rise to a quark mass $g'|\Phi|^2$. We suppose that both scales of the monopole mass $M$ and the monopole condensate $\langle \Phi \rangle$ are of the order of $\Lambda_{QCD} \sim 250$MeV. Then, the mass $g'|\Phi|^2 \sim 20$MeV ($g' \sim M^{-1}gg_m = M^{-1}/2$) is bigger than those of the bare quark masses ($m_q \sim 5$MeV). Although the estimation is very rough, it seems natural that pion masses ($\sim 140$MeV) are mainly generated by the monopole quark interaction. (We notice that pion masses are unnaturally bigger than the bare quark masses.) The pions are Nambu-Goldstone bosons in the absence of both the monopole quark interaction and the bare light quark masses. Their physical masses are mainly given by the non vanishing monopole quark interaction, not the bare quark masses.

Now, we would like to show that the chiral condensate $\langle \bar{q}q \rangle \neq 0$ arises only when the monopoles condense, $\langle \Phi \rangle \neq 0$. That is, the chiral condensate arises simultaneously when the quark confinement is realized. The coincidence in both condensations is caused by the monopole quark interaction. First, we rewrite the potential terms of the quarks and the monopoles, using auxiliary fields in the following,

$$-g'\bar{q}q|\Phi|^2 - V(\Phi) \rightarrow -\sigma|\Phi|^2 - \lambda(\sigma - g'\bar{q}q) - V(\Phi),$$

where $V(\Phi)$ denotes monopole potential. When we integrate with respect to auxiliary fields $\lambda$ and then $\sigma$, we recover the potential $-g'\bar{q}q|\Phi|^2 - V(\Phi)$. Obviously, $\sigma$ represents $g'\bar{q}q$ and $\lambda g'$ does quark mass.

It is easy to evaluate the vacuum energy of the quarks $q$,

$$V_q(\lambda g') = -\sum_{\bar{q},q} \sqrt{p^2 + (\lambda g')^2} = -\frac{N A^4}{8\pi^2 x^4}(x(x^2 + 1)^{3/2} - \frac{1}{2}x(x^2 + 1)^{1/2} - \frac{1}{2}\log(x + (x^2 + 1)^{1/2}))$$

with $x \equiv |\Lambda/\lambda g'| \geq 0$, where $N$ denotes quark’s internal degrees of freedom, that is, $N = 2 \times 2 \times 3 = 12$ (spin×flavour×color). We take a cut off parameter $\Lambda$ in the integral over the range of $|\bar{p}|$. The parameter should be in a range (200 ~ 500)MeV. Beyond the value, the assumption of Abelian dominance does not hold, or the monopole excitations are absent.

In order to find the minimum of the potential $V_q(\lambda g') + \sigma(\lambda + |\Phi|^2) + V(\Phi)$, we solve the equations,

$$\partial_\lambda V_q(\lambda g') + \sigma = 0, \quad \lambda + |\Phi|^2 = 0, \quad \text{and} \quad \Phi(\sigma + \partial_\Phi V(\Phi)) = 0.$$  

We note that the solution of $\sigma$ represents the chiral condensate; $\sigma = g'(\bar{q}q)$. Because $\sigma + \partial_\lambda V_q(\lambda g') = \sigma + x^2 g'/\Lambda \partial_\lambda V_q(\lambda g') = 0$, we obtain
\[ \sigma = g' \langle \bar{q}q \rangle = -\frac{Ng'\Lambda^3}{4\pi^2x^3} \left( x \sqrt{x^2 + 1} - \log(x + \sqrt{x^2 + 1}) \right). \] (7)

Thus, we can see that \( \langle \bar{q}q \rangle < 0 \) for \( \lambda \neq 0 \) (\( \Phi \neq 0 \)) and \( \langle \bar{q}q \rangle = 0 \) only when \( \lambda = 0 \) (\( \Phi = 0 \)). It implies that the chiral condensate is generated by the monopole condensation \( \langle \Phi \rangle \neq 0 \). The result holds even if we take any the monopole potential \( V(\Phi) \). The coincidence of the chiral and monopole condensations has been previously discussed\[13\] in lattice gauge theory.

The presence of the monopole condensation depends on the choice of the potential \( V(\Phi) \). In order to see whether or not the monopole condensation takes place, we need to specify the monopole potential \( V(\Phi) \). Here we simply adopt the potential in a dual superconducting model of monopoles; \( V(\Phi) = -\mu^2|\Phi|^2 + \lambda' |\Phi|^4 \) with \( \mu^2 > 0 \) and \( \lambda' > 0 \). The model describes quark confinement, which is realized by the monopole condensation.

\[ V(\Phi) = \Lambda^4 \left( -\frac{z}{x} + \frac{w}{x^2} \right) \] (8)

with \( z \equiv \mu^2/(g'\Lambda^3) > 0 \) and \( w \equiv \lambda'/(g^2\Lambda^2) > 0 \).

Without the monopole quark interaction, the monopoles condense, \( \langle \Phi \rangle = \sqrt{\mu^2/2\Lambda} \). But it is not obvious for the monopoles to condense when the monopole quark interaction is switched on. We can show that the potential \( V_q(\lambda g') + V(\Phi = \sqrt{-\lambda}) \) with \( N = 12 \) has the minimum at nonzero \( x > 0 \) (\( \lambda < 0 \)) because \( V_q + V \to \Lambda^4w/x^2 \) as \( x \to 0 \) (\( \Phi \to \infty \)) and \( V_q + V \to -3\Lambda^4/(2\pi^2) - \Lambda^4z/x \) as \( x \to +\infty \) (\( \Phi \to 0 \)). Because \( x = -\Lambda/g'\lambda \) and \( \lambda = -|\Phi|^2 \), we find that the monopole condensation takes place. Therefore, the monopole condensation \( \langle \Phi \rangle \neq 0 \) generates the chiral condensate \( \langle \bar{q}q \rangle < 0 \). This is caused by the monopole quark interaction \( g'\bar{q}q|\Phi|^2 \). As we explained, the interaction effectively describes the chirality non conservation in the monopole quark scattering. Quarks change their chiralities at the location of the monopoles owing to the chirality non conserved boundary condition for quarks.

The result is consistent with our previous result\[2\]. Namely, the chiral non symmetric pair production \( \langle dQ_5/dt \rangle \neq 0 \) takes place in the monopole condensed vacuum \( \langle \Phi \rangle \neq 0 \) when an external classical color charge is put in the vacuum. \( Q_5 \) denotes the difference of the number of the quarks with positive (negative) chirality; \( Q_5 = N_+ - N_- \). Obviously the chiral symmetric pair production \( \langle dQ_5/dt \rangle = 0 \) takes place in a vacuum without the monopoles when the classical charge is put in the vacuum. The classical charge generates electric field. It is the standard Schwinger mechanism.

Our result is derived by using the anomaly equations; \( \partial_\mu J_5^\mu \propto \vec{E} \cdot \vec{B} \). Such a chirality production\[16,17\] has been discussed in the decay of glasma produced just after high energy heavy ion collisions.

Analyzing the monopole quark scattering, we have found that the chiralities of the quarks are not conserved. The chiral non conservation takes place at the location of the monopoles. That is, the quarks change their chiralities at the location of the monopoles. We have represented such a interaction as \( g'\bar{q}q\Phi^2 \). The parameter \( g' \) is roughly given by the inverse of the monopole mass. As we have explained, the interaction is relatively weak compared with the other quark gluon interactions. Therefore, the chiral symmetry is explicitly broken even if the bare light quark masses \( m_q \) vanish. It leads to the result that pion masses do not vanish even when \( m_q \) vanishes. The result could be examined in lattice gauge theory. The small pion masses are caused by the weak monopole quark interaction. Furthermore, we have shown that the chiral condensate is generated by the monopole condensation. The result is derived by analyzing the vacuum energy of the quarks interacting with the monopoles by the term \( g'\bar{q}q|\Phi|^2 \). Based on the present analysis, we have proposed a phenomenological model\[18\] of hadrons coupled with the QCD monopoles, in which only a color singlet monopole has been shown to be observable.

The author expresses thanks to Prof. O. Morimatsu, KEK theory center for useful comments and discussions.

\[ \[1\] J. Liao and E. Shuryak, Phys. Rev. C 75 (2007) 054907.
J. Xu, J. Liao and M. Gyulassy, Chin. Phys. Lett. 32 (2015) 092501.
A. Iwazaki, Phys. Rev. C93 (2016) no.5, 054912.
[2] Y. Nambu, Phys. Rev. D10 (1974) 4626.
G. 'tHooft, High Energy Physics, Proceedings of the EPS International Conference, Palermo, Italy edited by A. Zichichi (1975) Editrice Compositori, Bologna 1976.
S. Mandelstam, Phys. Rep. 23C (1976) 245.
[3] A. Iwazaki, Int.J.Mod.Phys. A32 (2017) no.23n24, 1750139.
[4] A. Di Giacomo and M. Hasegawa, Phys. Rev. D91 (2015) 054512.
A. Ramamurti and E. Shuryak, [arXiv:1801.06922]
[5] T. Banks, A. Casher, Nucl. Phys. B169 (1980) 103.
[6] Y. Kazama, C.N. Yang and A.S. Goldhaber, Phys. Rev. D15 (1977) 2287.
[7] Z.F. Ezawa and A. Iwazaki, Phys. Rev. D25 (1982) 2681; Phys. Rev. D26 (1982) 631.
[8] T. Suzuki and I. Yotsuyanagi, Phys. Rev. D42 (1990) 4257.
[9] S. Maedan and T. Suzuki, Prog. Theor. Phys. 81 (1989) 229.
[10] H. Suganuma, S. Susaki, and H. Toki, Nucl. Phys. B435 (1995) 207.
[11] M.N. Chernodub, Phys. Lett. B474 (2000) 73.
[12] V.A. Rubakov, Nucl. Phys. B203 (1982) 311; Rept. Prog. Phys. 51 (1988) 189.
[13] C. G. Callan Jr., Phys. Rev. D25 (1982) 2141.
[14] Z. F. Ezawa and A. Iwazaki, Z. Phys. C20 (1983) 335.
[15] O. Miyamura, Phys. Lett. B353 (1995) 91.
[16] A. Iwazaki, Phys. Rev. C80 (2009) 052202.
[17] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. Lett. 104 (2010) 212001.
[18] A. Iwazaki, arXiv:1810.07270.