Open string fields as matrices

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We show that the action expanded around Erler–Maccaferri’s N D-brane solution describes the N + 1 D-brane system where one D-brane disappears due to tachyon condensation. String fields on multi-branes can be regarded as block matrices of a string field on a single D-brane in the same way as matrix theories.

Subject Index B23, B26, B28

1. Introduction

Open string field theory has the possibility of revealing non-perturbative aspects of string theory. Recently, Erler and Maccaferri have proposed a method to construct classical solutions, which are expected to describe any open string background [1]. This indeed implies that open string field theory is able to give a unified description of various D-branes regarded as non-perturbative objects of string theory.

Multi-brane solutions in Ref. [1] provide a correct vacuum energy and gauge-invariant observables. Accordingly, in order to prove whether the theory describes a multi-brane system, it is necessary to clarify open and closed string spectra in the background of the solution. However, it is difficult to give a definite answer to this problem, because there are some subtleties concerning BRST cohomology in the background [1].

There is another question related to the degree of freedom of string fields in the background. We have one string field in the theory on a single D-brane. However, in the case that the multi-brane solution provides the background of the N D-branes, the number of string fields increases to N² around the solution. Intuitively, N² fluctuation fields in the multi-brane background seem to be introduced as redundant degrees of freedom. Here, it is natural to ask how to generate N² string fields or Chan–Paton factors from one string field.

On the other hand, it is well known that matrix theories are able to describe various D-branes [2,3]. In matrix theories, D-branes are created by classical solutions as block diagonal matrices. After expanding a matrix around the solution, block matrices can be understood as representing open strings connecting each D-brane. Here, it should be noted that there are similarities between the matrix and the open string field: the matrix is deeply tied to the open string degree of freedom and an open string field is interpreted as a matrix in which the left and right indices correspond to the left and right half-strings [4]. Then, it seems plausible that N² string fields on N D-branes are embedded like block matrices in a string field on a D-brane.
The purpose of this paper is to clarify the origin of the $N^2$ string fields in the background of an $N$ D-brane solution. We will show that the theory expanded around the solution is regarded as an open string field theory on $N + 1$ D-branes, but in which a D-brane vanishes as a result of tachyon condensation. Then, the $N^2$ string fields will be given as block matrices in a string field as an infinite-dimensional matrix. Consequently, we can expect that the $N$ D-brane solution correctly reproduces the open and closed string spectra in the $N$ D-brane background.

The paper is organized as follows. In Sect. 2, after a brief explanation of multi-brane solutions by Erler–Maccaferri [1], we will introduce projection operators acting on a space of string fields. Then, we will analyze a string field theory expanded around the $N$ D-brane solution in terms of the projectors. In Sect. 3, we will give concluding remarks.

2. Open string field theory around multi-brane solutions

2.1. Erler–Maccaferri’s solution for $N$ D-branes

The action of bosonic cubic open string field theory is
\[
S[\Psi; Q_B] = -\frac{1}{g^2} \int \left( \frac{1}{2} \Psi Q_B \Psi + \frac{1}{3} \Psi^3 \right).
\] (2.1)

From the action, the equation of motion is given by
\[
Q_B \Psi + \Psi^2 = 0.
\] (2.2)

To construct multi-brane solutions for $N$ D-branes, Erler and Maccaferri introduced $N$ pairs of regularized boundary-condition-changing operators, $\Sigma_a$ and $\bar{\Sigma}_a$ ($a = 1, \ldots, N$) [1]. These operators satisfy
\[
\bar{\Sigma}_a \Sigma_b = \delta_{ab},
\] (2.4)
and
\[
Q_T \Sigma_a = Q_T \bar{\Sigma}_a = 0,
\] (2.5)
where $Q_T$ is a modified BRST operator on the tachyon vacuum. From (2.5), we find that
\[
Q_B \Sigma_a = \Sigma_a \psi_T - \psi_T \Sigma_a, \quad Q_B \bar{\Sigma}_a = \bar{\Sigma}_a \psi_T - \psi_T \bar{\Sigma}_a.
\] (2.6)

where $\psi_T$ denotes the tachyon vacuum solution of (2.2). Here, we only assume that $\Sigma_a$ and $\bar{\Sigma}_a$ satisfy Eqs. (2.4) and (2.5) (or equivalently (2.6)) for a tachyon vacuum solution $\psi_T$, regardless of wedge-based [5,6] or identity-based [7,8] solutions.

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$^1$ $\Sigma_a$ and $\bar{\Sigma}_a$ are constructed by boundary-condition-changing (bcc) operators, $\sigma_a$ and $\bar{\sigma}_a$, satisfying the operator product expansion: $\bar{\sigma}_a(z') \sigma_b(z) \rightarrow \delta_{ab}$ ($z' \rightarrow z$). In the Minkowski background, a zero momentum condition for the bcc operators is not necessarily required. So, the simplest bcc operators are given as
\[
\sigma_a(z) = e^{i k_{a} \cdot X(z)}, \quad \bar{\sigma}_a(z) = e^{-i k_{a} \cdot X(z)},
\] (2.3)

where $k_{a}^{\mu}$ satisfy $k_{a}^{2} = 0$ and $k_{a} \cdot k_{b} < 0$ ($a \neq b$). For example, we can take $k_{a}^{\mu} = (a, 1, \sqrt{a^2 - 1}, 0, \ldots, 0)$.
Using $\psi_T$, $\Sigma_a$, and $\Sigma_{\bar{a}}$, Erler–Maccaferri provided a multi-brane solution as [1]

$$
\Psi_0 = \psi_T - \sum_{a=1}^{N} \Sigma_a \psi_T \Sigma_{\bar{a}}.
$$

We can calculate the action for $\Psi_0$ with the help of (2.4) and (2.5):

$$
S[\Psi_0; Q_B] = -(N - 1) S[\psi_T; Q_B].
$$

Then, the solution $\Psi_0$ provides a correct vacuum energy for $N$ D-branes. Expanding the string field around the solution as $\psi = \Psi_0 + \psi$, we can obtain the action for the fluctuation $\psi$:

$$
S[\psi; Q_B] = S[\Psi_0; Q_B] + S[\psi; Q_{\Psi_0}],
$$

where the operator $Q_{\Psi_0}$ denotes the shifted BRST operator by the solution $\Psi_0$.

### 2.2. Projectors

To clarify the physical interpretation of $S[\psi; Q_{\Psi_0}]$, we introduce $N$ projection states as follows:

$$
P_a = \Sigma_a \Sigma_{\bar{a}} \quad (a = 1, \ldots, N),
$$

where the same indices $a$ are not summed. Here we have to notice that, as pointed out in Ref. [1], $\Sigma_{\bar{a}}$ should be multiplied to $\Sigma_a$ from the left and so these projectors should be dealt with carefully. More precisely, we define the projections for arbitrary string fields $A$ and $B$ as follows:

$$
AP_a B = (A \Sigma_a)(\Sigma_{\bar{a}} B).
$$

From (2.4) and (2.11), we can easily find that

$$
P_a P_b A = \Sigma_a (\Sigma_{\bar{a}} P_b A) = \Sigma_a ((\Sigma_{\bar{a}} \Sigma_b)(\Sigma_{\bar{b}} A)) = \delta_{ab} P_a A.
$$

This is a sufficient definition of the projectors for later calculation. But it suggests that we need to insert some infinitesimal worldsheet to separate $\Sigma_a$ and $\Sigma_{\bar{a}}$. We will discuss this point further in the last section.

In addition to $P_a$, we define the 0th projection as a complementary projector:

$$
P_0 = 1 - \sum_{a=1}^{N} P_a,
$$

where 1 denotes the identity string field. By definition, these $N + 1$ projections satisfy

$$
\sum_{a=0}^{N} P_a = 1,
$$

where the Greek indices are used for values $0, 1, \ldots, N$. From (2.5), it follows that $Q_T P_a = 0$ and then we have

$$
Q_B P_a = P_a \psi_T - \psi_T P_a.
$$

Moreover, we can find some relations among $P_a$, $\Sigma_a$, and $\Sigma_{\bar{a}}$:

$$
P_a \Sigma_b = \Sigma_a \delta_{ab}, \quad \Sigma_{\bar{a}} P_b = \Sigma_{\bar{a}} \delta_{ab}, \quad P_b \Sigma_{\bar{a}} = 0, \quad \Sigma_a P_0 = 0.
$$
With the help of these projectors, the string field $\Psi$ can be partitioned into $(N + 1) \times (N + 1)$ blocks:

$$\Psi = \sum_{\alpha=0}^{N} \sum_{\beta=0}^{N} P_{\alpha} \Psi P_{\beta} = \begin{pmatrix}
\Psi_{00} & \Psi_{01} & \cdots & \Psi_{0N} \\
\Psi_{10} & \Psi_{11} & \cdots & \Psi_{1N} \\
\vdots & \vdots & \ddots & \vdots \\
\Psi_{N0} & \Psi_{N1} & \cdots & \Psi_{NN}
\end{pmatrix}, \quad (2.17)$$

where $\Psi_{\alpha\beta}$ is defined as the $(\alpha, \beta)$ sector of $\Psi$, i.e., $\Psi_{\alpha\beta} \equiv P_{\alpha} \Psi P_{\beta}$.

According to Ref. [1], the second term in (2.7) is a solution to the equation of motion at the tachyon vacuum. From (2.16), the second term is represented as

$$- \sum_{a=1}^{N} \Sigma_a \Psi T \Sigma_a = \begin{pmatrix}
0 & 0 & 0 & \cdots & 0 \\
0 & -\Sigma_1 \Psi T \Sigma_1 & 0 & \cdots & 0 \\
0 & 0 & -\Sigma_2 \Psi T \Sigma_2 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & -\Sigma_N \Psi T \Sigma_N
\end{pmatrix}. \quad (2.18)$$

Accordingly, it turns out that the $N$ D-brane solution at the tachyon vacuum is given as a block diagonal matrix. This is a similar result to the case of matrix theories [2,3].

### 2.3. Background described by the solution

Now, we consider the fluctuation $\psi$ around the $N$ D-brane solution. According to the previous subsection, $\psi$ can be written by matrix representation:

$$\psi = \sum_{\alpha=0}^{N} \sum_{\beta=0}^{N} \tilde{\phi}_{\alpha\beta}, \quad (2.19)$$

where $\tilde{\phi}_{\alpha\beta} = P_{\alpha} \psi P_{\beta}$. $\tilde{\phi}_{\alpha\beta}$ represents a block matrix of $\psi$ with infinite dimension.

Here, we consider change of variables of $\tilde{\phi}_{\alpha\beta}$. $\phi_{ab}$ can be rewritten as

$$\tilde{\phi}_{ab} = P_{a} \phi_{ab} P_{b} = \Sigma_a (\Sigma_a \phi_{ab} \Sigma_b) \Sigma_b. \quad (2.20)$$

So, we can change the variables from $\tilde{\phi}_{ab}$ to $\phi_{ab} = \Sigma_a \tilde{\phi}_{ab} \Sigma_b$. Similarly, writing $\tilde{\phi}_{0a} = \chi_a \Sigma_a, \tilde{\phi}_{a0} = \Sigma_a \tilde{\chi}_a$, the fluctuation $\psi$ is represented as

$$\psi = \chi + \sum_{a=1}^{N} \chi_a \tilde{\chi}_a + \sum_{a=1}^{N} \Sigma_a \tilde{\chi}_a + \sum_{a=1}^{N} \sum_{b=1}^{N} \Sigma_a \phi_{ab} \Sigma_b$$

$$= \begin{pmatrix}
\chi \\
\Sigma_a \tilde{\chi}_a \\
\Sigma_a \phi_{ab} \Sigma_b
\end{pmatrix}, \quad (2.21)$$

where we rewrite $\tilde{\phi}_{00}$ as $\chi$. 

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Similar to the equation \( Q \psi_0(\Sigma_a A \bar{\Sigma}_b) = \Sigma_a (Q_B A) \bar{\Sigma}_b \) given in Ref. [1], by using (2.6) and (2.15), we have

\[
Q \psi_0(P_0 A P_0) = P_0(Q_T A) P_0, \tag{2.22}
\]
\[
Q \psi_0(P_0 A \bar{\Sigma}_a) = P_0(Q_{T0} A) \bar{\Sigma}_a, \tag{2.23}
\]
\[
Q \psi_0(\Sigma_a A P_0) = \Sigma_a(Q_{OT} A) P_0, \tag{2.24}
\]

where the operator \( Q_{\psi_1 \psi_2} \) is defined as \( Q_{\psi_1 \psi_2} A = Q_B A + \psi_1 A - (-1)^{|A|} A \psi_2 \) for two classical solutions \( \psi_1 \) and \( \psi_2 \) [1], and then \( Q_{T0} = Q_{\psi_T 0} \) and \( Q_{0T} = Q_0 \psi_T \). Using these relations, we can obtain a matrix representation of \( Q \psi_0 \psi \):

\[
Q \psi_0 \psi = \left( \begin{array}{cc}
P_0(Q_T \chi) P_0 & P_0(Q_{T0} \chi_b) \bar{\Sigma}_b \\ 
\Sigma_a(Q_{OT} \bar{\chi}_a) P_0 & \Sigma_a(Q_B \phi_{ab}) \bar{\Sigma}_b \end{array} \right). \tag{2.25}
\]

Consequently, the action expanded around \( \Psi_0 \) can be rewritten as

\[
S[\psi; Q \psi_0] = S[\phi_{ab}; Q_B] + S'[\chi, \chi_a, \bar{\chi}_a, \phi_{ab}], \tag{2.26}
\]

where each action is given by

\[
S[\phi_{ab}; Q_B] = -\frac{1}{g^2} \int \left( \frac{1}{2} \sum_{a=1}^{N} \sum_{b=1}^{N} \phi_{ba} Q_B \phi_{ab} + \frac{1}{3} \sum_{a=1}^{N} \sum_{b=1}^{N} \sum_{c=1}^{N} \phi_{ab} \phi_{bc} \phi_{ca} \right)
= -\frac{1}{g^2} \int \text{tr} \left( \frac{1}{2} \phi Q_B \phi + \frac{1}{3} \phi^3 \right), \tag{2.27}
\]

and

\[
S'[\chi, \chi_a, \bar{\chi}_a, \phi_{ab}] = -\frac{1}{g^2} \int \left( \frac{1}{2} \chi Q_T \chi + \sum_{a=1}^{N} \bar{\chi}_a Q_{T0} \chi_a + \frac{1}{3} \chi^3 \right. \\
\left. + \sum_{a=1}^{N} \bar{\chi}_a \chi_a + \sum_{a=1}^{N} \sum_{b=1}^{N} \chi_a \phi_{ab} \bar{\chi}_b \right). \tag{2.28}
\]

In (2.27), \( \phi \) represents a matrix \( (\phi_{ab}) \) and the trace denotes the sum of the diagonal elements with indices \( a, b \). Obviously, (2.27) represents the action for \( N \) D-branes; namely, \( \phi_{ab} \) is a string field of an open string attached on the \( a \)th and \( b \)th D-branes. Moreover, in the action (2.28), \( \chi \) is a string field on a D-brane with tachyon condensation, and \( \chi_a \) and \( \bar{\chi}_a \) represent string fields of an open string attaching on a D-brane with tachyon condensation and on one of the \( N \) D-branes, on which \( \phi_{ab} \) also attach. Accordingly, the actions (2.27) and (2.28) describe the theory for \( N + 1 \) D-branes in which a D-brane vanishes due to tachyon condensation. This system should be physically equivalent to the \( N \) D-brane system because \( Q_T \) and \( Q_{T0} \) have trivial cohomology\(^2\) and therefore this result is consistent with the expectation that the solution (2.7) is regarded as an \( N \) D-brane solution.

Let us consider an on-shell closed string coupling to an open string field. In the complex plane, a closed string vertex operator is given by \( \mathcal{V}(z, \bar{z}) = c(z)c(\bar{z})V_{\text{matt}}(z, \bar{z}) \), where \( V_{\text{matt}} \) is a vertex

\(^2\) In Ref. [9], it is shown that a homotopy operator exists for \( Q_T, Q_{T0}, \) and \( Q_{0T} \) if a homotopy state is given for \( Q_T \). For the identity-based tachyon vacuum solution [10], \( Q_{T0} \) and \( Q_{0T} \) also have vanishing cohomology, as does \( Q_T \) [8], since a homotopy state can be constructed for the solution.
operator with the conformal dimension \((1,1)\) in the matter sector. We can give a BRST invariant state using \(\mathcal{V}\) as
\[
V = \mathcal{V}(i, -i) I,
\]  
(2.29)
where the point \(z = i\) corresponds to the midpoint of an open string. Since the vertex is inserted at the midpoint, the state \(V\) commutes with any string field \(A\): \(VA = AV\). For the open string field \(\Psi\), an interaction term with the closed string vertex is given as a gauge-invariant overlap [11]:
\[
O_V(\Psi) = \int V \Psi.
\]  
(2.30)
In the background of the \(N\) D-brane solution, using (2.4) and (2.16), we can easily find couplings of the fluctuation fields to the closed string as
\[
O_V(\psi) = O_V(\chi) + \sum_{a=1}^{N} O_V(\phi_{aa}).
\]  
(2.31)
This correctly provides a closed string interaction to open strings on the \(N + 1\) D-branes.

Next, we consider the correspondence between gauge symmetries in the original action (2.1) and the expanded action (2.26). The original gauge transformation is given by
\[
\delta_A \Psi = Q_B \Lambda + \Psi \Lambda - \Lambda \Psi.
\]  
(2.32)
Since \(\Psi = \Psi_0 + \psi\), the gauge transformation for \(\psi\) is given by
\[
\delta_A \psi = Q_{\Psi_0} \Lambda + \psi \Lambda - \Lambda \psi,
\]  
(2.33)
where we note that \(\Lambda\) is the same parameter as in (2.32). Here, we decompose \(\Lambda\) into \(\tilde{\Lambda}_{a\beta} = P_a \Lambda P_\beta\) by the projectors. Then, changing variables as
\[
\tilde{\Lambda}_{ab} = \Sigma_a \Lambda_{ab} \Sigma_b, \quad \tilde{\Lambda}_{0a} = \lambda_a \Sigma_a, \quad \tilde{\Lambda}_{a0} = \Sigma_a \tilde{\lambda}_a,
\]  
(2.34)
and writing \(\tilde{\lambda}_{00} = \lambda\), we find that
\[
\delta_A \psi = \sum_{a=1}^{N} \sum_{b=1}^{N} \Sigma_a (\delta_A \phi_{ab}) \tilde{\Sigma}_b + \sum_{a=1}^{N} P_0 (\delta_A \chi_a) \tilde{\Sigma}_a + \sum_{a=1}^{N} \Sigma_a (\delta_A \tilde{\chi}_a) P_0 + P_0 (\delta_A \chi) P_0,
\]  
(2.35)
where the gauge transformations for the components are given as
\[
\delta_A \phi_{ab} = Q_B \Lambda_{ab} + \phi_{ac} \Lambda_{cb} - \Lambda_{ac} \phi_{cb} + \tilde{\chi}_a \lambda_b - \tilde{\lambda}_a \chi_b
\]
\[
\delta_A \chi_a = P_0 (Q_{\Sigma} \lambda_a) + \chi \lambda_a + \chi_b \Lambda_{ba} - \lambda \chi_a - \lambda_b \phi_{ba},
\]
\[
\delta_A \tilde{\chi}_a = (Q_{\Sigma} \tilde{\lambda}_a) P_0 + \tilde{\chi}_a \lambda + \phi_{ab} \tilde{\lambda}_b - \tilde{\lambda}_a \chi - \Lambda_{ab} \tilde{\chi}_b,
\]
\[
\delta_A \chi = P_0 (Q_{\Sigma} \lambda) P_0 + \chi \lambda + \chi_a \tilde{\lambda}_a - \lambda \chi - \lambda_a \tilde{\chi}_a.
\]  
(2.36)

3. Concluding remarks

We have shown that the theory expanded around the \(N\) D-brane solution given by Erler–Maccaferri describes an \(N + 1\) D-brane system with a vanishing D-brane due to the tachyon condensation. By projectors made of regularized bcc operators, an open string field in the original theory is divided into multi-string fields with matrix indices. Then, these indices can be regarded as Chan–Paton factors in the \(N\) D-brane background. We have found that \(N^2\) string fields on \(N\) D-branes are embedded in
a string field as block matrices. Similarly, gauge transformation parameters in the expanded theory are represented as block elements of a gauge parameter string field in the original theory.

From the matrix representation (2.19), the string fields $\tilde{\phi}_{\alpha\beta}$ are mutually independent variables and then the degrees of freedom of $\tilde{\phi}_{\alpha\beta}$ are equivalent to those of the string field $\psi$. Then, it is natural to expect that the path integral measure of the fluctuation $\psi$ is given by the product of measures of $\tilde{\phi}_{\alpha\beta}$. As seen in the previous section, we can rewrite $\tilde{\phi}_{\alpha\beta}$ as $\phi_{ab}$, $\chi$, $\chi_{a}$, and $\bar{\chi}_{a}$ by linear transformations. Therefore, the measure of $\psi$ is expressed by the measures of the string fields on the $N + 1$ D-branes:

$$D\psi = \prod_{a=1}^{N} \prod_{b=1}^{N} D\phi_{ab} D\chi \prod_{a=1}^{N} D\chi_{a} \prod_{a=1}^{N} D\bar{\chi}_{a}.$$  
(3.1)

Hence, the matrix interpretation of open string fields ensures that the quantum measure for the $N$ D-brane system is correctly derived from the classical solution in the string field theory.

Finally, we should comment on the multiplicative ordering of $\Sigma_{a}$ and $\bar{\Sigma}_{a}$ in the projectors. As in (2.11), we have defined the projectors such that $\Sigma_{a}$ does not operate on $\bar{\Sigma}_{a}$, because bcc operators break associativity, as discussed in Ref. [1]. To get a more definite result, we should separate these states by some worldsheet. This is a similar approach to that adopted in Ref. [12] to remedy the problem due to another nonassociativity. Accordingly, we need to regularize $P_{a}$ by inserting some worldsheet between $\Sigma_{a}$ and $\bar{\Sigma}_{a}$. In the case that $\psi_{T}$ is given by the Erler–Schnabl solution, one possible choice for regularization is

$$P_{a} = \Sigma_{a} Q_{T} \left( \frac{B}{1 + K} e^{-\epsilon K} \right) \bar{\Sigma}_{a},$$  
(3.2)

where $\epsilon$ is a positive infinitesimal parameter. It is noted that $B/(1 + K)$ is a homotopy operator for $Q_{T}$ and this construction is parallel to that of the regularized bcc operators from $\sigma$ and $\bar{\sigma}$ [1]. It can easily be seen that $P_{a}P_{b} = \delta_{ab}P_{a}$ and $Q_{T}P_{a} = 0$.

In this regularization, the limit $\epsilon \to 0$ should be taken after calculating the correlation functions related to trace (or integration) of string fields. It should never be done in string fields; e.g., the state $P_{a}A$ keeps the parameter $\epsilon$ until correlation functions are calculated. Evidently, the state with the regularization parameter is regarded as a kind of distribution as in Ref. [13] and indeed it is outside the usual Fock space like the phantom term in Schnabl’s tachyon vacuum solution [5]. We hope that, in terms of the projectors, it will be possible to obtain a deeper understanding of a space of string fields, in particular, the topology in the space beyond the single Fock space in string field theories [14].

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