SU(N) Yang-Mills on a circle, loop variables, Mandelstam identities and quantization ambiguities

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Abstract

We consider SU(N) Yang-Mills on a circle (cylindrical space-time) and quantize the eigenvalues of the holonomy. In this way the Mandelstam identities associated with the holonomy are trivially solved. Furthermore we indicate that there are exactly two physically inequivalent representations of the algebra of gauge-invariant operators, resulting in different spectra.

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1 Introduction

Wilson loop variables have been used for a long time in the study of Yang-Mills theories. In recent years it has also been used in the context of general relativity [1]. When using the Wilson loop variable, which is the trace of the holonomy around a closed loop, there always appear a number of non-linear constraints, the Mandelstam identities, that encode that the Wilson loop variable is really the trace of an $N \times N$-matrix. These identities are discussed in [2, 3, 4]. While Yang-Mills theory in 3 + 1-dimensions is a difficult subject, Yang-Mills theory in 1 + 1-dimensions is much simpler. In fact it is almost completely trivial, having only a finite number of global degrees of freedom on a circle. However, certain aspects of the higher dimensional theories remain, in particular the essence of the Mandelstam identities. Thus Yang-Mills theory on a circle is a useful toy model. It can be solved exactly. In [5] and [6] this is accomplished using other methods than ours. However, their results differ which is discussed recently in [7]. We will also discuss this difference. In fact, as we will indicate, these are the only inequivalent ways of quantizing. In our formalism these two inequivalent ways arise as different representations of the algebra of gauge-invariant operators.

2 Classical theory

Our starting point is the gauge-invariant part of the $SU(N)$ Yang-Mills Hamiltonian,

$$H = \frac{1}{2} \int_0^L dx \, \text{tr}(E^2(x)),$$

(1)

where $L$ is the length of the circle. The basic Poisson bracket is,

$$\{A^a(x), E^b(y)\} = \delta^{ab} \delta(x - y),$$

(2)

and $A(x) = A^a(x) t^a$, $E(x) = E^a(x) t^a$ where $t^a$ are the $N \times N$-matrix generators of the group ($a = 1, \ldots, \dim(SU(N))$). Here we have chosen

$$\text{tr}(t^a t^b) = \delta^{ab}$$

which implies the identity,

$$(t^a)^{ij} (t^a)^{kl} = \delta_{il} \delta_{jk} - \frac{1}{N} \delta_{ij} \delta_{kl},$$

(3)

where $i, j, k, l$ denotes matrix indices. We assume that the connection and electric field are periodic fields on the circle i.e. $A(x) = A(x + L)$, $E(x) = E(x + L)$. We also have the first class constraint (Gauss' law),

$$D_x E(x) = \partial_x E(x) + ig[A(x), E(x)] \approx 0,$$

(4)
where $g$ is the coupling constant. Let us define parallel transport by

$$U(x, y) = \mathcal{P} \exp(ig \int_x^y dx' A(x')),$$  \hfill (5)

where $\mathcal{P}$ denotes path ordering, i.e. $U(x, y)$ is the solution to the integral equation

$$U(x, y) = 1 + ig \int_x^y dx' U(x, x')A(x').$$

$U(x, y)$ is an element of the group $SU(N)$. The holonomy $h(x)$ is defined by

$$h(x) = U(x, x + L),$$

i.e. parallel transport once around the whole circle. Note also that $U(x, x + nL) = h^n(x)$ where $n$ is an integer which follows from the basic sewing property of path ordered exponentials,

$$U(x, x')U(x', y) = U(x, y).$$

Let $\Lambda(x)$ be a (finite) $SU(N)$ gauge-transformation (generated by (4)). Then

$$A'(x) = \Lambda(x)A(x)\Lambda^{-1}(x) + \frac{1}{ig}\Lambda(x)\partial_x\Lambda^{-1}(x) \quad \hfill (6)$$

$$E'(x) = \Lambda(x)E(x)\Lambda^{-1}(x). \quad \hfill (7)$$

This implies that $U(x, y)$ transforms homogeneously i.e.

$$U'(x, y) = \Lambda(x)U(x, y)\Lambda^{-1}(y), \quad \hfill (8)$$

and in particular,

$$h'(x) = \Lambda(x)h(x)\Lambda^{-1}(x), \quad \hfill (9)$$

if $\Lambda(x)$ is periodic. However, this is not the most general situation. We might allow for non-periodic gauge-transformations still keeping $A$ periodic. These satisfy,

$$\Lambda(x + L) = \mathbb{Z}_N \Lambda(x),$$  \hfill (10)

where $\mathbb{Z}_N$ is any element in the center of the group i.e. an $N$:th root of unity $\xi$, $\xi^N = 1$. We will call such non-periodic gauge-transformations $\mathbb{Z}_N$ transformations. Under such a transformation, the holonomy transforms as,

$$h'(x) = \xi \Lambda(x)h(x)\Lambda^{-1}(x). \quad \hfill (11)$$

The $\mathbb{Z}_N$ symmetry is really of secondary importance since it only holds for pure Yang-Mills. As soon as we couple fermions this symmetry is lost.
2.1 Loop variables

Following [1] we introduce the following functions on phase space, the loop variables,

\[ T^0(n) = \text{tr}(h^n(x)) \]  \hspace{1cm} (12)
\[ T^1(x; n) = \text{tr}(E(x)h^n(x)) \]  \hspace{1cm} (13)
\[ T^2(x; n, y; m) = \text{tr}(E(x)U(x, y + nL)E(y)U(y, x + mL)) \]  \hspace{1cm} (14)

They are easily seen to be gauge-invariant. Furthermore, \( T^0(nN) \) etc, is \( \mathbb{Z}_N \) invariant.

Note also that \( T^0(n) \) is independent of \( x \), motivating the notation. In fact, on the constraint surface \( T^1(x; n) \) is also independent of \( x \) since

\[ \partial_x T^1(x; n) = \text{tr}((D_x E(x))h^n(x)) \approx 0. \]  \hspace{1cm} (15)

Similarly, \( T^2(x; n, y; m) \) is independent of \( x \) and \( y \). Using the identity,

\[ T^2(x + n'L; n, y; m) = T^2(x; n - n', y; m + n') \]

it also follows that \( T^2(x; n, y; m) \) is independent of \( n - m \) on the constraint surface, i.e. \( T^2(x; n, y; m) = T^2(n + m) \). Analogously, one may consider loop variables of higher order in \( E \). To calculate Poisson brackets we need,

\[ \frac{\delta U(x, y)}{\delta A^a(x')} = ig\theta(x, y, x')U(x, x')t^aU(x', y), \]  \hspace{1cm} (16)

where

\[ \theta(x, y, x') = \int_x^y dx'' \delta(x'' - x'). \]

In particular,

\[ \frac{\delta h(x)}{\delta A^a(x')} = igU(x, x')t^aU(x', x)h(x). \]  \hspace{1cm} (17)

In what follows, all brackets will be evaluated on the constraint surface, where they simplify. Using (17) and (8) we obtain,

\[
\begin{align*}
\{ T^0(n), T^0(m) \} & = 0 \\
\{ T^1(n), T^1(m) \} & = ig(n - m)T^1(n + m) - \frac{ig}{N}(nT^1(n)T^0(m) - mT^1(m)T^0(n)) \\
\{ T^2(n), T^0(m) \} & = -2igmT^1(n + m) - \frac{1}{N}T^1(n)T^0(m) \\
\{ T^2(n), T^1(m) \} & = ig(n - 2m)T^2(n + m) + \frac{ig}{N}(2mT^1(n)T^1(m) - nT^2(n)T^0(m)) \\
\{ H, T^0(n) \} & = -ignLT^1(n) \\
\{ H, T^1(n) \} & = -ignLT^2(n).
\end{align*}
\]  \hspace{1cm} (18)
The last two identities follow since $H = \frac{L^2}{2}T^2(0)$. Brackets not considered here lead to higher order loop variables. We also have the reality conditions,

\[
(T^0(n))^* = T^0(-n) \\
(T^1(n))^* = T^1(-n) \\
(T^2(n))^* = T^2(-n)
\]

\[H^* = H, \]

where $*$ denotes complex conjugation.

### 2.2 Conjugacy classes

As seen by (9), the holonomy transforms under gauge-transformations by conjugation in $SU(N)$. Gauge-invariant functions of the holonomy are therefore class functions $f$,

\[f(h) = f(ghg^{-1}), \quad \forall g \in SU(N).\]

A particular example of a class function is $T^0(n)$. Let us note some properties of the conjugacy classes of $SU(N)$, the classic source of information being [8]. Any $SU(N)$ matrix is conjugate to a diagonal matrix $D$. Two diagonal matrices are conjugate if and only if their eigenvalues are related by permutation. Let $D = \text{diag}(\lambda_1, \ldots, \lambda_N)$. Since $\det D = 1$ we have,

\[\lambda_N = \lambda_1^{-1} \cdots \lambda_{N-1}^{-1}.\]  \hfill (20)

Furthermore, since $D$ is unitary the eigenvalues all have modulus 1 i.e. $\lambda_i = e^{i\varphi_i}$, ($\varphi_i$ real $i = 1, \ldots, N - 1$). Any class function $f$ is therefore a function of $N - 1$ eigenvalues, symmetric under permutations

\[\lambda_i \leftrightarrow \lambda_j, \quad i, j = 1, \ldots, N\]

where $\lambda_N$ is given by (20), e.g. for $N = 2$, $f(\lambda_1) = f(\lambda_1^{-1})$. From now on, permutations will always mean permutations of all $N$ eigenvalues, $\lambda_N$ being given by (20). We can express $T^0(n)$ in terms of the eigenvalues of $h(x)$ (which are independent of $x$),

\[T^0(n) = \lambda_1^n + \ldots + \lambda_{N-1}^n + \lambda_1^{-n} \cdots \lambda_{N-1}^{-n}.\]  \hfill (21)

### 3 Quantization

In the Dirac quantization approach, the quantized constraint operators should annihilate physical states. Therefore, by (13),

\[\partial_x T^1(x; n) \Psi_{\text{phys}} = 0,\]
i.e. $\hat{T}^1(x; n) = \hat{T}^1(n)$ and similarly for $\hat{T}^2$ (or rather, $\Psi_{\text{phys}}$ has support only on $\hat{T}^1(x; n)$s that are independent of $x$). Hence quantize the Poisson bracket algebra (18) as,

$$[\hat{T}^0(n), \hat{T}^0(m)] = 0$$  \hspace{1cm} (22)

$$[\hat{T}^1(n), \hat{T}^0(m)] = gh(m \hat{T}^0(n + m) - \frac{1}{N} \hat{T}^0(n) \hat{T}^0(m))$$  \hspace{1cm} (23)

$$[\hat{T}^1(n), \hat{T}^1(m)] = gh(m - n) \hat{T}^1(n + m) + \frac{gh}{N}(n \hat{T}^0(m) \hat{T}^1(n) - m \hat{T}^0(n) \hat{T}^1(m))$$ (24)

$$[\hat{\mathcal{H}}, \hat{T}^0(n)] = gh \ln \hat{T}^1(n)$$ \hspace{1cm} (25)

$$[\hat{\mathcal{H}}, \hat{T}^1(n)] = gh \ln \hat{T}^2(n).$$  \hspace{1cm} (26)

We have refrained from quantizing the algebra involving $\hat{T}^2$ since it is ordering dependent. Instead we define $\hat{T}^2(n)$ by (26). Assuming $\hat{T}^2(n)$ to be continuous in $n$ we obtain,

$$\lim_{n \to 0} \frac{1}{2ghn}[\hat{\mathcal{H}}, \hat{T}^1(n)] = \hat{\mathcal{H}}.$$  \hspace{1cm} (27)

Note that the last two terms in (24) look ordering dependent, but in fact they are not due to (23) as long as one orders both $\hat{T}^1$'s either to the left or to the right of the $\hat{T}^0$'s. Now let these operators act on wavefunctions that are class functions, i.e. symmetric functions of $N - 1$ eigenvalues of the holonomy $h(x)$. The eigenvalue operators themselves (which are not gauge-invariant), act simply by multiplication and hence $\hat{T}^0(n)$ acts as,

$$\hat{T}^0(n) \Psi(\lambda_1, \ldots, \lambda_{N-1}) = (\lambda_1^n + \ldots + \lambda_{N-1}^n + \lambda_1^{-n} \cdots \lambda_{N-1}^{-n}) \Psi(\lambda_1, \ldots, \lambda_{N-1}).$$  \hspace{1cm} (28)

This ensures that (22) is satisfied. Furthermore, this representation of $\hat{T}^0$ automatically satisfies the Mandelstam identities, e.g. when $N = 2$,

$$\hat{T}^0(n) = \hat{T}^0(-n)$$

$$\hat{T}^0(n) \hat{T}^0(m) = \hat{T}^0(n + m) + \hat{T}^0(n - m).$$

Under a $Z_N$ transformation, $\Psi(\lambda_1, \ldots, \lambda_{N-1})$ transforms into $\Psi(\xi \lambda_1, \ldots, \xi \lambda_{N-1})$, where $\xi^N = 1$. Let’s try to find a representation of $\hat{T}^1$ satisfying (23) as a (pure) first order differential operator in the eigenvalues. The unique solution is, labeling this specific representation by $\hat{T}^1_0$,

$$\hat{T}^1_0(n) \Psi(\{\lambda\}) = gh \sum_{i=1}^{N-1} (\lambda_i^n - \frac{1}{N} \hat{T}^0(n)) \lambda_i \partial_{\lambda_i} \Psi(\{\lambda\}).$$  \hspace{1cm} (29)

where $\{\lambda\} = (\lambda_1, \ldots, \lambda_{N-1})$. A check shows that this representation of $\hat{T}^1$ is invariant under permutations (of the eigenvalues) and that it satisfies (24). However, we might add a zeroth order term to this representation, i.e. a class function, as long as (24) is satisfied ((23) is obviously still satisfied). We will investigate such choices in what follows. First however consider (25). As an ansatz assume that $\hat{\mathcal{H}}$ is a (pure) second order differential operator in angles $\varphi_i$, i.e. a (pure) second order polynomial in $\lambda_i \partial_{\lambda_i}$. Then compare only
the first order part of the left- and righthand sides of (25). The unique solution is given by,

\[ \hat{H}_0 \Psi(\{\lambda\}) = \left( g \bar{h} \right)^2 L \sum_{i=1}^{N-1} (\lambda_i \partial_{\lambda_i})^2 - \frac{1}{N} \sum_{i=1}^{N-1} (\lambda_i \partial_{\lambda_i})^2 ) \Psi(\{\lambda\}). \]  

(30)

Let us note some properties of \( \hat{H}_0 \). Introduce

\[ \Xi_{(n_1, \ldots, n_{N-1})}(\{\lambda\}) = \lambda_1^{n_1} \cdots \lambda_{N-1}^{n_{N-1}}, \]  

(31)

where \( n_1, \ldots, n_{N-1} \) are integers. \( \Xi \) is an eigenvector of \( \hat{H}_0 \), i.e.

\[ \hat{H}_0 \Xi_{(n_1, \ldots, n_{N-1})}(\{\lambda\}) = \left( g \bar{h} \right)^2 \frac{2}{N} L \sum_{i=1}^{N-1} n_i^2 - 2 \sum_{i>j=1}^{N-1} n_i n_j \]  

(32)

where

\[ P_N(\{n\}) = (N-1) \sum_{i=1}^{N-1} n_i^2 - 2 \sum_{i>j=1}^{N-1} n_i n_j. \]  

(33)

To \( \hat{H}_0 \) we might add a first order term. We will return to this point in a while.

### 3.1 Symmetric representation

Let us choose \( \hat{H} = \hat{H}_0 \). Checking, it is found that it satisfies (24). This is in fact, in our formalism, the Hamiltonian derived in [5]. \( \hat{T}_0^1 \) and \( \hat{T}_0^2 \) are determined by (24) and (26) respectively. The eigenstates are totally symmetric linear combinations of \( \Xi_{(\{n\})} \) (remember that physical states are class functions), i.e.

\[ \Psi_{(n_1, \ldots, n_{N-1})}(\{\lambda\}) = \sum_{\text{perms}} \Xi_{(n_1, \ldots, n_{N-1})}(\pi(\lambda_1), \ldots, \pi(\lambda_{N-1})), \]

where \( \pi \) permutes all \( \lambda_i \)'s including \( \lambda_N \). Evidently, not all indices \( (n_1, \ldots, n_{N-1}) \) correspond to different eigenstates. If we want these states to be \( \mathbb{Z}_N \) invariant we have to require \( \sum_{i=1}^{N-1} n_i \) to be a multiple of \( N \). The eigenenergies are given by (32). The action of the loop variables is very simple on the eigenstates, e.g.

\[ \hat{T}_0^n(\Psi_{(n_1, \ldots, n_{N-1})})(\{\lambda\}) = \sum_{i=1}^{N-1} \Psi_{(n_1, \ldots, n_i+n, \ldots, n_{N-1})}(\{\lambda\}) + \Psi_{(n_1, \ldots, n_{N-1}-n)}(\{\lambda\}). \]

An inner product is determined by requiring \( (\hat{T}_0^n(\{\varphi\})) = \hat{T}_0^n(-n) \) and \( \hat{H}^\dagger = \hat{H} \). Then (23) and (26) implies that all the classical reality conditions, (19), are quantized exactly. Hence, (up to an overall factor),

\[ < \Phi, \Psi > = \int d\varphi_1 \cdots d\varphi_{N-1} \Phi^\dagger(\{\varphi\}) \Psi(\{\varphi\}). \]  

(34)

Here all integrals are taken from \(-\pi\) to \(\pi\) in the angles. Alternatively we can integrate over the eigenvalues,

\[ \frac{d\lambda_i}{i\lambda_i} = d\varphi_i. \]

Different eigenstates are orthogonal using this inner product. The groundstate is \( \Psi_{(0, \ldots, 0)} \) and it has zero energy.
3.2 Antisymmetric representation

Now choose \( \hat{H} = \Delta^{-1} \hat{H}_0 \Delta \) where \( \Delta \) is,

\[
\Delta = \prod_{j>i=1}^N (\lambda_i - \lambda_j).
\]

We note that it is a well defined choice, being invariant under permutations of eigenvalues. Pulling \( \hat{H}_0 \) through to the right, one sees that it corresponds to having added a certain first order term to \( \hat{H}_0 \). Furthermore, it satisfies (27). This is the Hamiltonian considered in [3]. It is (up to a constant) the radial part of the Laplacian on \( SU(N) \), see [3]. \( \Delta \) is totally antisymmetric under permutations of eigenvalues. Hence eigenstates of \( \hat{H} \) are given as,

\[
\Psi_{(n_1,\ldots,n_{N-1})}(\{\lambda\}) = \Delta^{-1} \sum_{\text{perms}} \text{sgn}(\pi) \Xi_{(n_1,\ldots,n_{N-1})}(\pi(\lambda_1),\ldots,\pi(\lambda_{N-1})).
\]

These are the characters of \( SU(N) \). Eigenenergies are still given by (32). The groundstate is \( \Psi_{(1,\ldots,N-1)} \) with energy

\[
(gh)^2 L \frac{N}{24} (N^2 - 1).
\]

The spectrum of \( \hat{H} \) is a proper subset of that of \( \hat{H}_0 \). Hence these Hamiltonians are clearly physically inequivalent. The action of loop variables on eigenstates is the same as for the symmetric representation. The inner product is,

\[
\langle \Phi, \Psi \rangle = \int d\phi_1 \cdots d\phi_{N-1} \Delta \Delta^* \Phi^*(\{\phi\}) \Psi(\{\phi\}).
\]

The measure density \( \Delta \Delta^* \) is the measure density induced by the Haar-measure on the group. Note how utterly sensible it is from the point of view of the group, e.g. the conjugacy class \( \lambda_1 = \ldots = \lambda_{N-1} = 1 \) consists of a single group element, the unit matrix, in contrast to a generic conjugacy class having all eigenvalues distinct which consists of a set of group elements forming a submanifold of the group with non-zero dimension. Thinking about the group it is natural to give a larger weight to this generic conjugacy class than the unit element class. \( \Delta \) does just this as it vanishes on the unit element class. In general, the so called singular set which is the set of conjugacy classes having not all eigenvalues distinct, has Haar-measure zero (\( \Delta \) is zero on this set).

3.3 Generalities

Returning to the issue of adding first order terms to \( \hat{H}_0 \) (of which the antisymmetric representation is a particular example), consider for simplicity \( N = 2 \). In this case the most general representation also satisfying (27) is,

\[
\hat{H} \Psi(\varphi_1) = -(gh)^2 L \frac{\partial^2}{\partial \varphi_1^2} + 2f(\varphi_1) \partial \varphi_1 + f(\varphi_1)^2 + f'(\varphi_1))\Psi(\varphi_1),
\]

(35)
where \( f(-\varphi_1) = -f(\varphi_1) \) to make \( \hat{H} \) gauge-invariant, i.e. invariant under permutations of eigenvalues. Defining \( \hat{T}^2 \) by \( \left[ \hat{T}^2, \hat{H} \right] \) implies that \( \hat{T}^2 \) satisfies its quantized bracket algebra with a particular ordering, modulo some quantum corrections in \( \left[ \hat{T}^2, \hat{T}^1 \right] \). These corrections are independent of \( f(\varphi_1) \). This is not surprising as we will see. In the language of \( [10] \) the Hamiltonian in (35) and \( \hat{H}_0 \) are related by a quantum canonical transformation. The measure density in the inner product determined by this Hamiltonian is found to be,

\[
\mu(\varphi_1) = k e^{2F(\varphi_1)} ,
\]

where \( k \) is a constant and \( F'(\varphi_1) = f(\varphi_1) \). From the measure density we can construct a quantum canonical transformation by letting,

\[
\mu(\varphi_1) = C^{-2}(\varphi_1) . \tag{36}
\]

A solution is \( C(\varphi_1) = e^{-F(\varphi_1)} \) having set \( k = 1 \). It is easily seen that \( \hat{H} = C \hat{H}_0 C^{-1} \) and hence \( \hat{H} \) and \( \hat{H}_0 \) are related by a canonical transformation. It would seem that \( \hat{H} \) and \( \hat{H}_0 \) are physically equivalent. There is however a subtlety. States are required to be invariant under permutations of eigenvalues, which in this case means even in \( \varphi_1 \). We have chosen a \( C(\varphi_1) \) which is even, hence eigenstates of \( \hat{H} \) are,

\[
\Psi(\varphi_1) = C(\varphi_1) \cos n\varphi_1 .
\]

Thus in this case, \( \hat{H} \) and \( \hat{H}_0 \) are completely equivalent. We might however have chosen a different square root in (36) such that \( C(\varphi_1) \) is odd. For such a \( C \), eigenstates of \( \hat{H} \) are,

\[
\Psi(\varphi_1) = C(\varphi_1) \sin n\varphi_1 .
\]

This is exactly what happens in the antisymmetric representation, having \( f(\varphi_1) = \cot \varphi_1 \). The measure is,

\[
\mu(\varphi_1) = e^{2 \log |\sin \varphi_1|} = \sin^2 \varphi_1 .
\]

A possible choice of \( C \) is obviously \( 1/|\sin \varphi_1| \) which is an odd function of \( \varphi_1 \). Hence eigenstates of the Hamiltonian \( \sin^{-1} \varphi_1 \hat{H}_0 \sin \varphi_1 \) has eigenstates \( \sin \varphi_1 \). It is thus clear that all choices of Hamiltonians are either equivalent to the symmetric or the antisymmetric representation and that this quantization ambiguity arises when choosing a particular square root in (36). This conclusion generalizes to higher \( N \). For any \( N \), we can write the most general Hamiltonian \( \hat{H} \) as,

\[
\hat{H} = \hat{H}_0 + \sum_{i=1}^{N-1} f_i(\{\lambda\}) \lambda_i \partial_{\lambda_i} + g(\{\lambda\}) ,
\]

where \( g(\{\lambda\}) \) is determined from (27). The functions \( f_i \) are not independent as the Hamiltonian is required to be invariant under permutations. This Hamiltonian determines a certain measure density \( \mu(\{\lambda\}) \) which is invariant. Setting \( \mu = C^{-2} \) determines \( C \) up to a sign (a sign which can vary from point to point). We can e.g. choose \( C \) either totally symmetric or totally antisymmetric. Then everything works fine. There are however other choices of \( C \) which are neither totally symmetric nor totally antisymmetric. But in these cases it seems impossible to construct eigenfunctions of the Hamiltonian which are totally symmetric. Thus we must exclude these choices i.e. in general \( \hat{H} = C \hat{H}_0 C^{-1} \) and this Hamiltonian is equivalent to either the symmetric or the antisymmetric representation.
4 Conclusion

We have seen that considering the eigenvalues of the holonomy is very fruitful when quantizing Yang-Mills theory. This feature is expected to generalize to higher dimensions. The quantization ambiguity is seen to arise as follows. Even if two different Hamiltonians are related by a quantum canonical transformation, their eigenstates might not be related by this canonical transformation since eigenstates are required to be invariant under permutations of eigenvalues. As far as pure Yang-Mills theory goes, there is no good reason to reason to prefer one representation to the other. However we can speculate that coupling (fundamental) fermions might change matters since then the whole group becomes important and not just the conjugacy classes.

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