Searching for and exploring double-parton scattering effects
in four-jet production at the LHC

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Abstract

We discuss four-jet production at the LHC. We calculate cross section for both single-parton scattering (SPS) using the ALPGEN code as well as for double-parton scattering (DPS) in leading-order collinear approach. Our results are compared with experimental data obtained recently by the CMS collaboration. We show that the ALPGEN code relatively well describes distributions in transverse momenta and rapidity of each of the four jets ordered by their transverse momenta (leading, subleading etc.). The SPS mechanism does not explain the distributions at large rapidity for the leading jet. The DPS mechanism considerably improves the agreement with the experimental data in this corner of the phase space. In order to enhance the relative DPS contribution we propose to impose different cuts. The relative DPS contribution increases when decreasing the lower cut on the jet transverse momenta as well as when a lower cut on the rapidity distance between the most remote jets is imposed. We predict very flat distribution in azimuthal angle between the most remote jets with low lower cuts on jets transverse momentum. We identify phase-space corners where the DPS content is enhanced relatively to the SPS one.

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I. INTRODUCTION

The physics of multiparton interactions (MPI), in general, or double parton scattering (DPS) for hard processes gained recently a new impulse related to the experiments at Large Hadron Collider (LHC) [1–4].

The DPS contributions were discussed for several reactions to mention double Drell-Yan [5, 6], creation of \( b\bar{b}b\bar{b} \) [7], etc. It was predicted [8] and recently found [9, 10] that the double parton scattering leads to very abundant production of (at least) two \( c\bar{c} \) pairs at the LHC. Another process where DPS seems to be seen is double \( J/\psi \) production in the region of large rapidity separation between the two \( J/\psi \)'s [11, 12].

We will not discuss in this Letter evident progress in theoretical understanding of the DPS or MPI interaction in general, which took place in recent years. In contrast to very quantitative and precise studies of many SPS perturbative partonic processes the field of MPI and DPS lacks similar precision. So far most of the experimental studies have concentrated on the extraction of \( \sigma_{\text{eff}} \) in so-called factorized ansatz\(^1\). In our opinion, at present some efforts should be made in order to make the studies of DPS processes really quantitative and differential in kinematical variables. In order to make more detailed phenomenological studies one needs also more clear cases where the DPS effects could be identified without any doubts.

Four-jet production was traditionally discussed in the context of double parton scattering. Actually it was a first process where the DPS was claimed to be observed experimentally [14]. However, in most of the past as well as current analyses the DPS contribution to four-jet production is relatively small and single parton scattering (SPS) driven by the \( 2 \to 4 \) partonic processes dominates. To pin down the DPS contribution one has to make rather complicated studies comparing Monte Carlo templates and experimental data. In such a case the result is not very transparent and one may worry whether the final result is not an artifact of an inadequate Monte Carlo generator. A more evident result for DPS in four-jet production would clearly provide a new impulse for the MPI community.

On the theoretical side the DPS effects in four-jet production were discussed in Refs. [15–20]. A first theoretical estimate of SPS four-jet production, including only some

\(^1\) In some cases even experimental extraction of \( \sigma_{\text{eff}} \) may be subtle [13].
partonic subprocesses, and its comparison to DPS contribution was presented in Ref. [17] for Tevatron. Some new kinematical variables useful for identification of DPS were proposed in Ref. [19]. A model dependence on collision energy and minimal transverse energy of $\sigma_{eff}$ was studied in Ref. [18]. Presence of perturbative parton splitting mechanism in the context of four-jet production was discussed in Ref. [20].

In our recent studies we have shown how big can be the contribution of DPS for (two) jets widely separated in rapidity [21]. Understanding of this contribution is important in the context of searching for BFKL effects or in general QCD higher-order effects [22]. We have found that with the present cuts used in the CMS analysis [23] the DPS contribution can be of the order of 10-20%. It could be still somewhat enhanced imposing further cuts on transverse momentum of the dijets or azimuthal angle between jets.

In the present letter we wish to explore exclusive four-jet sample where the situation in the context of searching for DPS should be even better. We shall discuss how to maximize the DPS contribution by selecting relevant kinematical cuts.

II. THEORETICAL FORMALISM

Partonic cross sections used to calculate DPS are only in leading order. The cross section for dijet production can be then written as:

$$\frac{d\sigma(ij \rightarrow kl)}{dy_1 dy_2 d^2 p_t} = \frac{1}{16\pi^2 s^2} \sum_{i,j} x_1 f_i(x_1, \mu^2) x_2 f_j(x_2, \mu^2) |M_{ij \rightarrow kl}|^2.$$  \hspace{1cm} (2.1)

In our calculations we include all leading-order $ij \rightarrow kl$ partonic subprocesses. The $K$-factor for dijet production is rather small, of the order of $1.1 - 1.3$ (see e.g. [24, 25]), but can be easily incorporated in our calculations. It was shown that already the leading-order approach gives results in sufficiently reasonable agreement with recent ATLAS and CMS inclusive jet data [21].

This simplified leading-order approach can be however used easily in calculating DPS differential cross sections. The multi-dimensional differential cross section can be written as:

$$\frac{d\sigma^{DPS}(pp \rightarrow 4\text{jets} \ X)}{dy_1 dy_2 d^2 p_{1t} dy_3 dy_4 d^2 p_{2t}} = \sum_{i_1,j_1,k_1,l_1,i_2,j_2,k_2,l_2} \frac{C}{\sigma_{eff}} \frac{d\sigma(i_1j_1 \rightarrow k_1l_1)}{dy_1 dy_2 d^2 p_{1t}} \frac{d\sigma(i_2j_2 \rightarrow k_2l_2)}{dy_3 dy_4 d^2 p_{2t}},$$  \hspace{1cm} (2.2)
where $C = \begin{cases} \frac{1}{2} & \text{if } i_1j_1 = i_2j_2 \land k_1l_1 = k_2l_2 \\ 1 & \text{if } i_1j_1 \neq i_2j_2 \lor k_1l_1 \neq k_2l_2 \end{cases}$ and partons $j, k, m = g, u, d, s, \bar{u}, \bar{d}, \bar{s}$. The combinatorial factors include identity of the two subprocesses. Each step of DPS is calculated in the leading-order approach (see Eq.(2.1)).

Experimental data from Tevatron \cite{27, 28} and LHC \cite{29–31} provide an estimate of $\sigma_{\text{eff}}$ in the denominator of formula (2.2). In the calculations we have taken in most cases $\sigma_{\text{eff}} = 15 \text{ mb}$. Phenomenological studies of $\sigma_{\text{eff}}$ are summarized e.g. in \cite{32} with the average value $\sigma_{\text{eff}} \approx 15 \text{ mb}$.

Now we proceed to the SPS production mechanisms of four-jet production. The elementary cross section for the SPS mechanism has the following generic form:

$$d\hat{\sigma}_{ij \to klmn} = \frac{1}{2^{\delta}} |M_{ij \to klmn}|^2 d^4PS,$$  \hspace{1cm} (2.3)

where the invariant phase space reads:

$$d^4PS = \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \frac{d^3p_3}{2E_3(2\pi)^3} \frac{d^3p_4}{2E_4(2\pi)^3} (2\pi)^4 \delta^4(p_1 + p_2 + p_3 + p_4).$$  \hspace{1cm} (2.4)

Above $p_1, p_2, p_3, p_4$ are four-momenta of outgoing partons (jets). Many possible subprocesses $ij \to klmn$ are possible in general. In some corners of the phase space only some processes are really important and others may be safely neglected.

The hadronic cross section is then given by the integral

$$d\sigma = \int dx_1 dx_2 \sum_{ijklmn} f_i(x_1, \mu_F^2) f_j(x_2, \mu_F^2) d\sigma_{ij \to klmn}.$$  \hspace{1cm} (2.5)

Above $f_i$ and $f_j$ are parton (gluon, quark, antiquark) distribution function.

Instead of explicitly using the formulae above we shall use its Monte Carlo version as implemented in the event generator ALPGEN \cite{33}. Weighted events from the generator will be used to construct distributions presented in the next section. Only light quarks/antiquarks are included in our calculation.

III. FIRST RESULTS

To start our analysis we wish to check how reliable our SPS four jet calculation is. In Fig. \[\text{Fig. 1}\] we confront the results of calculation with the leading-order code ALPGEN \cite{33} with recent CMS experimental data \cite{22}. In this analysis the CMS collaboration imposed...
different transverse momentum cuts on the leading, subleading, 3rd and 4th jets (see the figure caption). In this calculation we have used an extra K-factor to effectively include higher-order effects. How big is the K-factor was discussed recently in [34]. It was found that the next-to-leading order contributions are important and lead to $K < 1$. In our calculation here we will use the K-factor found there. We get relatively good description of both transverse momentum and pseudorapidity distributions of each of the four (ordered in transverse momentum) jets. Therefore we conclude that the calculation with the ALPGEN generator can be a reliable SPS reference point for the DPS effects.

![Graphs showing transverse momentum and rapidity distributions](image)

FIG. 1: Transverse momentum (left panel) and rapidity (right panel) distributions of each of the four-jets (ordered in transverse momentum) in the four-jet sample together with the CMS experimental data [22]. The calculations were performed with the ALPGEN code [33]. Here kinematical cuts relevant for the experiment were applied to allow for a comparison.

In Fig.2 we show in addition the calculated contributions of DPS for different values of the $\sigma_{\text{eff}}$ parameter. The $\sigma_{\text{eff}}$ parameter effectively includes both traditional (independent emissions) DPS and effects related to parton splitting [35, 36]. In general, the relative contribution of both mechanisms may depend on different kinematical variables. Phenomenological exploration of the dependences may help in understanding the interplay of the different contributions and shed more light on the underlying mechanism. We observe that the DPS contribution is rather small (independent of $\sigma_{\text{eff}}$ parameter) at larger transverse momenta and in midrapidities of jets. Their contribution increases when going to small transverse momenta and large rapidities. We observe a clear improvements at large pseudorapidities ($|\eta| > 3$) when the DPS contribution is added to the SPS con-
The presented CMS data were not optimized for search for DPS effects and now we wish to explore how to improve the situation, i.e., to enhance sample of the DPS events.

**FIG. 2**: Transverse momentum (left column) and rapidity (right column) distributions for the leading jet of the four-jet sample. The solid line represents the SPS four-jet contribution, whereas the dashed line corresponds to the DPS one. Different values of $\sigma_{eff}$ (given in the figure) were used in different rows.
Having shown that our approach is consistent with existing LHC four-jet data we can focus on finding optimal conditions for “observing” the DPS effects. As shown in our previous paper on dijets widely separated in rapidity \[21\] the distribution in rapidity separation of such jets seems a very good observable for observing the onset of the DPS enhancement or even its dominance. In Fig. 3 we show some examples of such distributions for different cuts on the jet transverse momenta for two collision energies \(\sqrt{s} = 7\) TeV and \(\sqrt{s} = 14\) TeV obtained with the condition of the four-jet observation. We focus only on the distance between the most remote jets; the other two jets are then in between. The higher collision energy or the smaller the lower transverse momentum cut the bigger the relative DPS contribution. In such cases one can therefore expect a considerable deficit when only SPS four jets are included. Such cases would be very useful to “extract” the \(\sigma_{eff}\) parameter which for the selected sample does not to be the same as for other cases discussed in the literature. Any deviation from the “canonical” value of 15 mb would therefore shed some new light on the underlying dynamics. For example, a two-component model discussed in Refs. \[35, 36\] strongly suggests such dependences.

Another observable discussed in Ref. \[21\] is the distribution in transverse momentum of the most remote jets. It was argued there that the jets coming from different partonic collisions are not correlated and therefore large transverse momenta of such uncorrelated jet pairs are possible in general. Here we wish to explore the situation in this respect quantitatively for the four-jet sample. In Fig. 4 we show an example for one energy and one cut on jet transverse momenta. In these calculations we have not included extra cut on jet rapidity separation. The DPS and SPS distributions look rather similar for the four-jet sample. This shows that an extra cut on the transverse momentum of the pair of the most remote jets would almost not help to enhance the DPS contribution.

Finally in Fig. 5 we show an example of azimuthal angle distribution between the most remote jets. While the distribution for SPS peaks at \(\phi = \pi\), the DPS contribution is very flat as the most remote jets come dominantly from different independent and uncorrelated (in azimuthal angle) partonic scatterings. Since the DPS dominates we predict very flat distribution in relative azimuthal angle. Limiting to small \(\phi_{jj}\) would further enhance the relative amount of DPS without considerable lowering statistics.

Table I illustrates and summarizes integrated contributions of SPS and DPS for selected transverse momentum cuts imposed on all four jets and on rapidity distance be-
FIG. 3: Distribution in rapidity distance of the most remote jets from the four-jet sample for $\sqrt{s} = 7$ TeV (left column) and $\sqrt{s} = 14$ TeV (right column) for different cuts on jet transverse momenta (identical for all four jets).

FIG. 4: Distribution in transverse momentum of the most remote jets for $\sqrt{s} = 14$ TeV.

tween the most remote jets. Already with the lower cut on transverse momentum of 35
FIG. 5: Azimuthal correlation between the most remote jets for SPS (dashed line) and DPS (solid line) contributions for $\sqrt{s} = 14$ TeV.

GeV (used already in some CMS analyses) the DPS contribution is about 40% for $\sqrt{s} = 7$ TeV and about 60% for $\sqrt{s} = 14$ TeV. Lowering the lower cut on jet transverse momentum to 20 GeV gives already 70% and 80% of DPS, respectively. Imposing in addition that the distance between the remote jets is bigger than $\Delta y > 7$ enhances the DPS contribution to almost 90%. Therefore imposing such cuts would help to extract fairly precisely the $\sigma_{\text{eff}}$ parameter from such experimental studies.

TABLE I: Integrated cross sections in nanobarns for two collision energies and different cuts on jet transverse momenta and rapidity distance between the most remote jets. Here, $\sigma_{\text{eff}} = 15$ mb has been used for calculating the DPS cross section.

| Kinematical cuts: | $\sqrt{s} = 7$ TeV | $\sqrt{s} = 14$ TeV |
|-------------------|---------------------|---------------------|
|                   | $\sigma_{\text{SPS}}$ | $\sigma_{\text{DPS}}$ | $\sigma_{\text{SPS}}$ | $\sigma_{\text{DPS}}$ |
| $|y| < 4.7$         | 40.55               | 29.92               | 197.74               | 275.23               |
| $35 < p_T < 100$ GeV | 1 047.37           | 2 443.77           | 4 194.11           | 16 652.39           |
| $20 < p_T < 100$ GeV | 18.56               | 113.26               | 151.70               | 1 194.28               |
| $\Delta y > 7.0$   |                     |                      |                     |                      |
| $20 < p_T < 100$ GeV | 291.68               | 1 221.88               | 1 157.15               | 8 326.19               |
| $0 < \phi_{jj} < \pi/2$ |                     |                      |                     |                      |
IV. CONCLUSIONS

In the present paper we have explored how to enhance the relative contribution of double-parton scattering for four-jet production. Compared to our previous studies, where we focussed on jets with large rapidity separation (important in searches for the BFKL dynamics), here we have studied the case of exclusive four-jet production. We have shown that already some present data for four-jet production obtained at the LHC by the CMS collaboration with relatively small cuts on transverse momenta indicate some evidence of DPS at large pseudorapidities of the leading jet.

We have shown that imposing a lower cut on transverse momenta and rapidity distance between the most remote jets improves the situation considerably, i.e. enhances the relative contribution of DPS. A dedicated analysis of the DPS effect is possible already with the existing data sample at $\sqrt{s} = 7$ TeV. The situation at larger energies, relevant for LHC Run 2 should be even better. We predict that azimuthal correlation between jets widely separated in rapidity should disappear in the considered kinematical domain. We have found that in some corners of the phase space the DPS contribution can go even above 80%, not necessarily at the expense of lowering the cross section (statistics of experimental data).

We have presented the detailed predictions. Once such cross sections are measured, one could try to extract the $\sigma_{eff}$ parameter and try to obtain its dependence on kinematical variables. Such dependence can be expected due to several reasons (parton-parton correlations, hot spots, perturbative parton splitting) and detailed experimental data would help us to understand the situation much better.

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