Modeling and simulation of basic components and transmission line based on Time Scale Frame Transformation

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Abstract. With the application of HVDC and flexible AC transmission in the power system, the rapid transient process of power electronic switches makes it impossible for electromechanical transient model based on the ‘quasi-steady state’ assumption unable to accurately reflect its changing process. At the same time, the increasing scale of the power system and the large number of higher harmonics brought by power electronic switches make the simulation results of the traditional electromagnetic transient model, which considers small step size, not fast enough. Based on the analysis and summary of the existing frequency reduction principles, this paper proposes time scale transformation for electromagnetic transient simulation with large step size, and uses it to establish the time scale frame phasor models of basic AC components and transmission lines. Finally, an example is given to show that the method is correct and effective.

1. Introduction
With the increasing scale of power system, the access of various new energy sources, the large-scale use of HVDC(high voltage direct current) and FACTS (flexible AC transmission systems), the harmonic components in the system have greatly increased, and the traditional electromechanical transient simulation and the traditional small-step electromagnetic transient simulation are no longer applicable[1-3].

[4]-[9] presents a method of shifted frequency analysis (SFA). Its principle is to construct the complex form of electrical signal, and then multiply it by a complex number to make frequency offset, thus reducing the change speed of signals and achieving the purpose of large step simulation. [6] deduces the mathematical models of synchronous motor, transmission line and transformer by using SFA, but its signal frequency only considers the vicinity of fundamental wave. [10] proposes the application of dynamic phasor(DP) to the simulation of power electronic circuits. The dynamic phasor method obtains the k-th Fourier coefficient by Fourier decomposition of electrical signals and then averaging. In order to reduce the calculation amount, only the fundamental frequency or 2~3 harmonic dynamic phasor model is established in the literatures, which has large harmonic truncation error. [11]-[16] apply dynamic phasor method to AC/DC system, in which [11] establishes the mathematical model of a single lossless line in detail by using dynamic phasor method, and [12] deduces the mathematical models of three-phase lossless line and three-phase lossy line in detail. The models of transmission lines in these two documents only consider the fundamental frequency or the lower harmonics around the fundamental frequency.

In this paper, the author first proposed the idea of time scale frame transformation(TSFT) in literature [17]. The essence of this idea is to construct complex analytic signal of single-phase real
signal, and then to reduce the frequency of original signal by taking advantage of the rotation characteristics of analytic signal, so as to simulate with large step size. Based on this method, the corresponding time scale frame phasor(TSFP) models of basic AC components and transmission lines are established. Finally, the feasibility and effectiveness of the proposed method are verified by an example.

2. Time scale frame transformation

2.1. Analysis of rotation characteristics of analytic signal

2.1.1. Analysis of mathematical meaning of $e^{jx}$. Mathematically, the natural base $e$ can be defined as:

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = \lim_{n \to \infty} \left(1 + \frac{100\%}{n}\right)^n$$  \hspace{1cm} (1)

Equation (2) shows that for $e^x$, $x$ can be divided into $n$ step, and the growth rate of each step is $x/n$ of the previous step. In fact, $x$ can be either real or imaginary. When the $x$ is real number, the growth is in the direction of the initial value. When $x$ is an imaginary number, the direction of growth is perpendicular to the direction of the initial value and maintains a vertical relationship with the growth process. It is shown in figure 2.

According to equation (2), when $x=j\theta$, $e^x=e^{j\theta}$ can be expressed as:

$$e^{j\theta} = \lim_{n \to \infty} \left(1 + \frac{j\theta}{n}\right)^n = \lim_{n \to \infty} \left(1 + \frac{100\% j\theta}{n}\right)^n$$  \hspace{1cm} (3)

The growth process of $e^{j\theta}$ can be understood as the growth of the complex number 1+j0: the growth rate of each step is $j\theta/n$, and the growth direction of each step is perpendicular to the direction before the growth. When $n$ is infinite, the modulus of the complex number remains unchanged. At this point, the growth process can be seen as the complex number 1+j0 doing counterclockwise rotation, the rotation angle is $\theta$, as shown in figure 3.
The growth direction follows a circle.

While $\theta$ changes over time, such as $\theta=\omega t$; for equation (3), $e^{j\theta}=e^{j\omega t}$. As can be seen from figure 3, at this time, $e^{j\theta}$ can be seen as doing circular motion with $\omega$ as angular velocity. And when $\omega$ is a constant, it is doing uniform circular motion.

2.1.2. Analysis of mathematical meaning of $e^{j\theta}$. From the analysis in the above section, $e^{j\theta}$ moves in a circle with the change of time, indicating that it has rotation characteristics. When $\omega>0$, it rotates counterclockwise, and it rotates clockwise when $\omega<0$.

The relationship between $e^{j\omega t}$ and analytic signals is established through Euler's formula, and the real signal $u(\omega, t)$ is set as follows:

$$u(\omega, t) = A(t)\cos(\omega t + \theta)$$  \hspace{1cm} (4)

Where, $\omega$ is the angular frequency of the signal, $\theta$ is the initial phase. And $A(t)$ is the amplitude envelope of the signal, and its change frequency is much lower than the signal frequency.

In signal theory, the Hilbert Transform (HT) of $u(\omega, t)$ is $v(\omega, t)$: 

$$v(\omega, t) = H\left[v(\omega, t)\right] = A(t)\sin(\omega t + \theta)$$  \hspace{1cm} (5)

The analytic signal $Z_{uv}(\omega, t)$ can be defined as:

$$Z_{uv}(\omega, t) = u(\omega, t) + jv(\omega, t)$$  \hspace{1cm} (6)

The Euler formula is known as:

$$e^{j\theta} = \cos \theta + j\sin \theta$$  \hspace{1cm} (7)

According to the orthogonality of Hilbert transformation, the analytic signal $Z_{uv}(\omega, t)$ can also be expressed by Euler's formula:

$$Z_{uv}(\omega, t) = u(\omega, t) + jv(\omega, t)$$

$$= A(t)\cos(\omega t + \theta) + jA(t)\sin(\omega t + \theta)$$

$$= A(t)e^{j\theta}e^{j\omega t}$$  \hspace{1cm} (8)

It can be seen from the analysis of 2.1.1: $e^{j\omega t}$ has rotation characteristics. Therefore, $Z_{uv}(\omega, t)$ can be regarded as the complex number $A(t)\cos \theta + jA(t)\sin \theta$, and it rotates with time changing. Its movement trajectory in the two-dimensional plane is shown in figure 4. The figure shows that in the complex domain, the analytic signal is a rotating vector. Its rotation characteristics are derived from $e^{j\omega t}$ in equation (8), and the initial position is $\theta$, the amplitude is $A(t)$, and the rotation velocity is $\omega$. Therefore, the mapping of real signal $u(\omega, t)$ in complex space can be obtained by constructing the analytic signal $Z_{uv}(\omega, t)$, and its mapping in complex domain has the rotation property with the change of time.
2.2. Analysis of frequency reduction principle of existing methods

Dynamic phasor, frequency shift and other methods can reduce the signal frequency, although the application scene is quite different. This section analysis the frequency reduction principle of these methods and studies whether there are common mathematical principles.

2.2.1. Dynamic phasor. In essence, dynamic phasor can be regarded as the Fourier transform of real signal in time domain with rectangular window. As the ‘window’ moves with time, the original time domain signal is fourier decomposed in the window, and each order fourier series is the dynamic phasor of each order. Each order of dynamic phasor changes slowly with time, so it can be simulated by taking large step. The dynamic phasor is used to decompose the signal frequency and retain the frequency order with larger amplitude.

Let the time-domain continuous signal \( x(t) \) with period \( T \) be decomposed into Fourier Series in time interval \( \tau \in (t-T, t) \):

\[
 x(\tau) = \sum_{k=-\infty}^{\infty} X_k(t) e^{jk\omega t} \tag{9}
\]

Where: \( \omega = 2\pi/T \) is the angular frequency of fundamental wave; \( X_k(t) \) is the Kth Fourier series, which is a complex number.

\( X_k(t) \) is defined as the Kth ‘dynamic phasor’. According to equation (9), at any time \( t \), we can get:

\[
 X_k(t) = \frac{1}{T} \int_{t-T}^{t} x(\tau) e^{-jk\omega \tau} d\tau \tag{10}
\]

According to equation (10), when \( k=1 \) and \( X_1(t) \) amplitude is constant, the dynamic phasor \( X_1(t) \) is the traditional phasor.

The following formula is defined as:

\[
 F_k(t) = X_k(t) e^{jk\omega t} \tag{11a}
\]

We can get:

\[
 X_k(t) = F_k(t) e^{-jk\omega t} \tag{11b}
\]

Since \( X_1(t) \) is complex, \( F_1(t) \) is also complex. Each order of dynamic phasor \( X_k(t) \) has the function of frequency reduction, which can be mathematically understood as \( F_1(t) \) being reversed rotation transformation by using \( e^{jk\omega t} \).

2.2.2. Frequency shift. In a communication system, the input signal is usually modulated to a high frequency carrier for information transmission. After the signal is sent, the high-frequency carrier signal is removed when the signal is received at the receiving end, and the original input signal is extracted. The frequency shift method is one similar to this communication process.

In signal theory, a narrow-band signal \( u(t) \) can be defined as:

\[
 u(t) = u'(t) \cos \omega t - u'^Q(t) \sin \omega t \tag{12}
\]

Its Hilbert transform \( v(t) \) is:

\[
 v(t) = H[u(t)] = u'(t) \sin \omega t - u'^Q(t) \cos \omega t \tag{13}
\]

The analytic signal \( F_{an}(t) \) is composed of \( u(t) \) and \( v(t) \).

\[
 F_{an}(t) = u(t) + jv(t) = u'(t) \cos \omega t + ju'^Q(t) \sin \omega t = A_m e^{j\theta} e^{j\omega t} \tag{14}
\]

Where:

\[
 A_m = \sqrt{\left(u'(t)\right)^2 + \left(u'^Q(t)\right)^2} \tag{15}
\]

\[
 \theta = \arctan\left(\frac{u'^Q(t)}{u'(t)}\right)
\]
The frequency shift of $F_{uv}(t)$ can be obtained:

$$Z_{dq}(t) = F_{uv}(t)e^{-j\omega t} = A_{m}e^{j\theta}e^{j\omega t}e^{-j\omega t} = A_{m}e^{j\theta}$$

(16)

Compared with formula (16) and formula (11), it can be found that frequency shift can also be understood as the reverse rotation transformation applied by $e^{-j\omega t}$ to the analytic signal $F_{uv}(t)$, and $Z_{dq}(t)$ is the projection of the analytic signal $F_{uv}(t)$ in the d-q rotational coordinate system.

By comparing the principle of frequency reduction of dynamic phasor and frequency shift with the rotation characteristics of analytic signal, it can be found that the principle of frequency reduction in the above method can be unified as the reverse rotation transformation of analytic signal with rotation characteristics, which is the projection of analytic signal in the same direction of rotation coordinate system with the same rotation speed and angular frequency of signal.

2.3. Principle and method of TSFT

The rotation transformation $e^{j\omega t}$ is been assumed in this paper, and $\omega$ is the rotation speed. Multiply both sides of the analytic signal $Z_{uv}(\omega, t)$ of equation (8) by $e^{j\omega t}$ to get:

$$Z_{uv}(\omega, t)e^{j\omega t} = A(t)e^{j\theta}e^{j\omega t}e^{j\omega t}$$

$$= A(t)e^{j\theta}e^{j(\omega+\omega)t}$$

$$= A(t)e^{j(\omega+\omega)t}$$

$$= X_{dq}(\Delta\omega, t)$$

(17)

Where, $\Delta\omega = \omega + \omega$

$$X_{dq}(\Delta\omega, t) = A(t)e^{j\theta}e^{j\Delta\omega t}$$

$$= x_{d}(\Delta\omega, t) + jx_{q}(\Delta\omega, t)$$

$$\begin{cases} x_{d}(\Delta\omega, t) = A(t)\cos(\Delta\omega t + \theta) \\ x_{q}(\Delta\omega, t) = A(t)\sin(\Delta\omega t + \theta) \end{cases}$$

(18)

(19)

In signal theory, the frequency of signal corresponds to its time scale. In this paper, the above rotation transformation is referred to as the Time Scale Frame Transformation (TSFT). When $\omega<0$, time scale decreases and signal frequency decreases, which is called time scale forward transformation. When $\omega>0$, the time scale increases and the signal frequency increases, which is called inverse transformation of time scale. It can be seen that, through the flexible value of $\omega$, time scale transformation unifies the principle of frequency reduction of dynamic phasor and frequency shift. And the relative rotation transformation of $e^{j\omega t}$ has been giving a clear explanation of frequency reduction physically. In addition, TSFT expands the range of frequency selection, and makes frequency shift and other methods not limited by the condition of power frequency fundamental wave. The result of TSFT of signal is called TSFP.

In addition, the original real signal can be obtained by equation (20).

$$u(\omega, t) = \text{Re}\left\{z_{uv}(\omega, t)\right\} = \text{Re}\left\{X_{dq}(\Delta\omega, t)e^{j\omega t}\right\}$$

(20)

3. Modeling of AC basic components and transmission lines

The basic AC components and transmission lines are both linear components. Their TSFP models are established below. And three-phase coupling is considered in the modeling.
3.1. Basic components models

3.1.1. Three-phase resistance model. For a three-phase resistance, as shown in figure 5, the relationship between the time domain voltage and current is shown in equation (21).

\[
\begin{bmatrix}
  u_a \\
  u_b \\
  u_c
\end{bmatrix} =
\begin{bmatrix}
  R_a & R_a & R_a \\
  R_b & R_b & R_b \\
  R_c & R_c & R_c
\end{bmatrix}
\begin{bmatrix}
  i_a \\
  i_b \\
  i_c
\end{bmatrix}
\]  
(21)

Using TSFT, the above formula can be rewritten as:

\[
\begin{bmatrix}
  u_{dq,a} (t) \\
  u_{dq,b} (t) \\
  u_{dq,c} (t)
\end{bmatrix} =
\begin{bmatrix}
  R_a & R_a & R_a \\
  R_b & R_b & R_b \\
  R_c & R_c & R_c
\end{bmatrix}
\begin{bmatrix}
  i_{dq,a} (t) \\
  i_{dq,b} (t) \\
  i_{dq,c} (t)
\end{bmatrix}
\]  
(22)

The TSFP difference model of the three-phase resistance is shown in equation (23), and its equivalent circuit is shown in figure 6.

\[
\begin{bmatrix}
  i_{dq,a} (t) \\
  i_{dq,b} (t) \\
  i_{dq,c} (t)
\end{bmatrix} = R_{abc}^{-1}
\begin{bmatrix}
  u_{dq,a} (t) \\
  u_{dq,b} (t) \\
  u_{dq,c} (t)
\end{bmatrix}
\]  
(23)

3.1.2. Three-phase coupling inductance model. For a three-phase coupling inductance branch, as shown in figure 7, the relationship between the time domain voltage and current is shown in equation (24).

\[
\begin{bmatrix}
  u_a (t) \\
  u_b (t) \\
  u_c (t)
\end{bmatrix} =
\begin{bmatrix}
  L_a & L_m & L_m \\
  L_m & L_a & L_m \\
  L_m & L_m & L_a
\end{bmatrix}
\begin{bmatrix}
  i_a (t) \\
  i_b (t) \\
  i_c (t)
\end{bmatrix}
\]  
(24)

According to the differential characteristic of TSFT, the above formula can be rewritten as:

\[
\begin{bmatrix}
  u_{dq,a} (t) \\
  u_{dq,b} (t) \\
  u_{dq,c} (t)
\end{bmatrix} =
\begin{bmatrix}
  L_a & L_m & L_m \\
  L_m & L_a & L_m \\
  L_m & L_m & L_a
\end{bmatrix}
\begin{bmatrix}
  i_{dq,a} (t) \\
  i_{dq,b} (t) \\
  i_{dq,c} (t)
\end{bmatrix}
\] + \begin{bmatrix}
  L_a & L_m & L_m \\
  L_m & L_a & L_m \\
  L_m & L_m & L_a
\end{bmatrix}
\begin{bmatrix}
  i_{dq,a} (t) \\
  i_{dq,b} (t) \\
  i_{dq,c} (t)
\end{bmatrix}
\]  
(25)

Its TSFP difference mathematical model is shown in equation (26), and its equivalent circuit is shown in figure 8.

\[
\begin{bmatrix}
  i_{dq,a} (t) \\
  i_{dq,b} (t) \\
  i_{dq,c} (t)
\end{bmatrix} = Z_n^{-1}
\begin{bmatrix}
  u_{dq,a} (t) \\
  u_{dq,b} (t) \\
  u_{dq,c} (t)
\end{bmatrix} + \begin{bmatrix}
  i_{dq,a} (t) \\
  i_{dq,b} (t) \\
  i_{dq,c} (t)
\end{bmatrix}
\]  
(26)

Where.
\[
E = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, L = \begin{bmatrix} L_x & L_m & L_n \\ L_m & L_x & L_n \\ L_n & L_m & L_x \end{bmatrix}
\]

\[
Z_{n1}^{-1} = Z_{m1}^{-1}Y_{n4} \cdot Z_{n1} = E + \frac{\Delta t}{2} (j \omega E), Z_{n2} = E - \frac{\Delta t}{2} (j \omega E), Y_{n3} = \frac{\Delta t}{2} L^{-1}, Y_{n4} = \frac{\Delta t}{2} L^1
\]

\[
i_{dq_{line}} (t - \Delta t) = -Z_{n2}^{-1}Z_{n1}^{-1}i_{dq_{line}} (t - 2\Delta t) + \left( Z_{n2}^{-1}Z_{n1}^{-1}Z_{n1}^{-1}Y_{n4}\right) u_{dq_{abc}} (t - \Delta t)
\]

### 3.1.3. Three-phase coupling capacitance model

For a three-phase coupling capacitance branch, as shown in figure 9, the relationship between the time domain voltage and current is shown in equation (27).

![Figure 9. Branch of three-phase coupling capacitance](image)

![Figure 10. The equivalent circuit of three-phase Coupling Capacitance](image)

\[
\begin{bmatrix} i_a (t) \\ i_b (t) \end{bmatrix} = \begin{bmatrix} C_s & C_m & C_m \\ C_m & C_s & C_m \\ C_m & C_m & C_s \end{bmatrix} \begin{bmatrix} u_a (t) \\ u_b (t) \\ u_c (t) \end{bmatrix}
\]

\[
\begin{bmatrix} i_{dq_a} (t) \\ i_{dq_b} (t) \end{bmatrix} = \begin{bmatrix} C_s & C_m & C_m \\ C_m & C_s & C_m \\ C_m & C_m & C_s \end{bmatrix} \begin{bmatrix} u_{dq_a} (t) \\ u_{dq_b} (t) \\ u_{dq_c} (t) \end{bmatrix} + j \omega \begin{bmatrix} C_s & C_m & C_m \\ C_m & C_s & C_m \\ C_m & C_m & C_s \end{bmatrix} \begin{bmatrix} u_{dq_a} (t) \\ u_{dq_b} (t) \\ u_{dq_c} (t) \end{bmatrix}
\]

According to the differential characteristic of TSFT, the above formula can be rewritten as:

\[
\begin{bmatrix} i_{dq_a} (t) \\ i_{dq_b} (t) \end{bmatrix} = \begin{bmatrix} C_s & C_m & C_m \\ C_m & C_s & C_m \\ C_m & C_m & C_s \end{bmatrix} \begin{bmatrix} u_{dq_a} (t) \\ u_{dq_b} (t) \\ u_{dq_c} (t) \end{bmatrix}
\]

\[
\begin{bmatrix} i_{dq_{line}} (t - \Delta t) \\ i_{dq_{line}} (t - \Delta t) \\ i_{dq_{line}} (t - \Delta t) \end{bmatrix} = -Z_{n2}^{-1}Z_{n1}^{-1}i_{dq_{line}} (t - 2\Delta t) + \left( Z_{n2}^{-1}Z_{n1}^{-1}Z_{n1}^{-1}Y_{n4}\right) u_{dq_{abc}} (t - \Delta t)
\]

### 3.2. Single lossless line model based on TSFT

For the long transmission line pq, as shown in figure 11, the differential equation of the line can be obtained under the premise of ignoring its loss.
In the formula, $L_0$ and $C_0$ are respectively inductance and capacitance per unit length of long transmission line. According to the differential characteristic of TSFT, the above formula can be rewritten as:

$$
\begin{cases}
-\frac{\partial u(x,t)}{\partial x} = L_0 \frac{\partial i(x,t)}{\partial t} \\
-\frac{\partial i(x,t)}{\partial x} = C_0 \frac{\partial u(x,t)}{\partial t}
\end{cases}
$$

(30)

According to the traveling wave theory, at the head end and the end of the line, i.e. $x$ takes 0 and $l$, a TSFP difference mathematical model is finally obtained as shown in equation (32). The equivalent circuit is shown in figure 12.

$$
\begin{cases}
i_{dq,p}(t) = \frac{1}{Z} u_{dq,p}(t) + i_{dq,hisp}(t - \Delta t) \\
i_{dq,q}(t) = \frac{1}{Z} u_{dq,q}(t) + i_{dq,hsq}(t - \Delta t)
\end{cases}
$$

(32)

Where,

$$
\Delta t = \frac{l}{v}, \quad \theta_n = \omega l/v, \quad Z = \sqrt{L_0/C_0}
$$

$$
\begin{cases}
i_{dq,hisp}(t - \Delta t) = -\frac{1}{Z} u_{dq,q}(t - \Delta t) e^{-j\theta_n} - i_{dq,q}(t - \Delta t) e^{-j\theta_n} \\
i_{dq,hsq}(t - \Delta t) = -\frac{1}{Z} u_{dq,p}(t - \Delta t) e^{-j\theta_n} - i_{dq,p}(t - \Delta t) e^{-j\theta_n}
\end{cases}
$$

3.3. Single lossy line model based on TSFT

The single lossy line model is based on Bergeron model and approximately considers the line resistance. It divides the transmission line into several lossless sections, which can be divided into two sections according to the actual application of the project. Then, the lumped resistor $R$ of the transmission line is divided into three parts: the first and last ends are connected in series with $R/4$, and the middle is connected in series with $R/2$. Finally, the Dommel model of a single lossy line can be obtained by using Bergeron equivalent to these two lossless lines and adding the resistances at the first and last ends and in the middle.
Therefore, as an improved Bergeron model, TSFT is also applicable to Dommel model. The modeling process is basically the same as Bergeron model based on TSFT, so detailed derivation is not required, and only the TSFP difference mathematical model is listed, as shown in equation (33). The equivalent circuit is shown in figure 14.

\[
\begin{align*}
\begin{cases}
    i_{dp,p}(t) & = \frac{1}{Z'} u_{dp,p}(t) + i_{dp,hq}(t - \Delta t) \\
i_{dq,q}(t) & = \frac{1}{Z'} u_{dq,q}(t) + i_{dq,hq}(t - \Delta t)
\end{cases}
\end{align*}
\]

(33)

Where,

\[
Z' = Z + R/4 = \sqrt{L_0/C_0} + R/4, \quad k = (Z - R/4)/(Z + R/4)
\]

\[
\begin{align*}
\begin{cases}
i_{dphq}(t - \Delta t) & = -\frac{1 + k}{2} \left[ \frac{u_{dphq}(t - \Delta t)}{Z'} + ki_{dphq}(t - \Delta t) \right] e^{-j\omega \Delta t} - \frac{1 - k}{2} \left[ \frac{u_{dphq}(t - \Delta t)}{Z'} + ki_{dphq}(t - \Delta t) \right] e^{j\omega \Delta t} \\
i_{dqhq}(t - \Delta t) & = -\frac{1 + k}{2} \left[ \frac{u_{dqhq}(t - \Delta t)}{Z'} + ki_{dqhq}(t - \Delta t) \right] e^{-j\omega \Delta t} - \frac{1 - k}{2} \left[ \frac{u_{dqhq}(t - \Delta t)}{Z'} + ki_{dqhq}(t - \Delta t) \right] e^{j\omega \Delta t}
\end{cases}
\end{align*}
\]

3.4. Three-phase lossless distributed parameter line model based on TSFT

The time domain wave equation of the three-phase lossless line is:

\[
\begin{align*}
& \frac{\partial u_{abc}(x,t)}{\partial x} = L_{abc} \frac{\partial i_{abc}(x,t)}{\partial t} \\
& \frac{\partial i_{abc}(x,t)}{\partial x} = C_{abc} \frac{\partial u_{abc}(x,t)}{\partial t}
\end{align*}
\]

(34)

Where,

\[
u_{abc}(x,t) = \left[u_a(x,t), u_b(x,t), u_c(x,t)\right]^T, i_{abc}(x,t) = \left[i_a(x,t), i_b(x,t), i_c(x,t)\right]^T.
\]

\[
L_{abc} = \begin{bmatrix} L_a & L_m & L_m \\ L_m & L_a & L_m \\ L_m & L_m & L_a \end{bmatrix}, \quad C_{abc} = \begin{bmatrix} C_e & C_m & C_m \\ C_m & C_e & C_m \\ C_m & C_m & C_e \end{bmatrix}
\]

According to the differential characteristic of TSFT, the above formula can be rewritten as:

\[
\begin{align*}
& \frac{\partial u_{abc}(x,t)}{\partial x} = L_{abc} \frac{\partial i_{abc}(x,t)}{\partial t} + j\omega L_{abc} i_{abc}(x,t) \\
& \frac{\partial i_{abc}(x,t)}{\partial x} = C_{abc} \frac{\partial u_{abc}(x,t)}{\partial t} + j\omega C_{abc} u_{abc}(x,t)
\end{align*}
\]

(35)

For a three-phase lossless distributed parameter circuit, in order to obtain its mathematical model, a reversible matrix \( T \) is first used to diagonalize the inductance and capacitance matrices so as to decouple the three-phase line. Then the decoupled three-phase line can use the same method as the above TSFP model for single-phase line to solve the model of three-phase lossless line. Here, the
modeling method will not be deduced in detail, only the final TSFP difference mathematical model will be given, as shown in equation (36). The equivalent circuit diagram is shown in figure 15.

\[
\begin{align*}
\dot{i}_{dq, pabc} (t) &= Z_{abc}^{-1} u_{dq, pabc} (t) + \dot{i}_{dq, hispabc} (t - \Delta t) \\
\dot{i}_{dq, qabc} (t) &= Z_{abc}^{-1} u_{dq, qabc} (t) + \dot{i}_{dq, hisqabc} (t - \Delta t)
\end{align*}
\]

(36)

Where,

\[
Z_{abc} = \begin{bmatrix} Z_1 & \phantom{f} \phantom{f} \phantom{f} \\
Z_2 & Z_3 \end{bmatrix}, \quad e^{-j\theta_\omega t} = \begin{bmatrix} e^{-j\theta_\omega} \\
\phantom{f} e^{-j\theta_\omega} \end{bmatrix}
\]

\[
\begin{align*}
\dot{i}_{dq, hispabc} (t - \Delta t) &= -Z_{abc}^{-1} u_{dq, qabc} (t - \Delta t) e^{-j\theta_\omega} - \dot{i}_{dq, hisqabc} (t - \Delta t) e^{-j\theta_\omega} \\
\dot{i}_{dq, hisqabc} (t - \Delta t) &= -Z_{abc}^{-1} u_{dq, pabc} (t - \Delta t) e^{-j\theta_\omega} - \dot{i}_{dq, pabc} (t - \Delta t) e^{-j\theta_\omega}
\end{align*}
\]

4. Example verification

Using the example in reference [11], the system topology is shown in figure 16. In the circuit, \( e(t) = \cos(\omega t) \), \( R=10\Omega \), \( L=0.3\text{H} \). The length of the distributed parameters line between point 2 and 3 is 300km, and \( L_0=0.885\text{mH/km} \). When the switch is closed, the voltage is the maximum value, and the transient process of the line under no load can be observed at the same time.

\[
\begin{align*}
& R \\
& L \\
& 1 \\
& 2 \\
& 3
\end{align*}
\]

Figure 16. Topology diagram of study case

The simulation is carried out according to different cases in table 1, the simulation time is 0.3s. And the voltage of point 2 and point 3 can be observed.

| Case   | Simulation mode | Frequency | step |
|--------|-----------------|-----------|------|
| Case1  | EMT             | 50Hz      | 50μs |
| Case2  | TSFT            | 50Hz      | 500μs|

The circuit is simulated according to Case2, and the simulation results are inversely transformed back to the time domain to compare with the simulation results of Case1, as shown in figure 17. It can be seen that the larger error between Case2 and Case1 occurs at the beginning, and the error becomes smaller as time goes by. It should be noted that the numerical oscillation suppression technique is not
used in this example, and the accuracy will be further improved after the application. And Case2 uses only 0.022s, while Case1 uses 0.118s. It can be seen that the simulation speed of TSTF is much faster than EMT.

![Voltage waveform](image)

Figure 17. Voltage waveform

5. Conclusions
In this paper, TSFT is briefly introduced, and the TSFP models of the basic components and transmission lines are established. Finally, the feasibility and validity of TSFT are verified by an example. The conclusions are as follows:

1) By analyzing the mathematical meaning of $e^{j\omega t}$, this paper studies the rotation characteristics of analytical signals. Through the analysis of the characteristics of signal frequency reduction by dynamic phasor and frequency shift, the principle of frequency reduction of the above method is unified for the first time as the inverse rotation transformation $e^{-j\omega t}$ of the analytic signal. The principle and method of TSFT are established by extending $e^{-j\omega t}$ to the case that the rotation speed is different from the signal frequency.

2) Based on TSFT, three-phase RLC model, single lossless line, single lossless line and three-phase lossless line mathematical model are established respectively. It can be seen from the example that TSFP model not only has fast simulation speed, but also has acceptable errors compared with traditional electromagnetic transient model.

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