On the Utility of Directional Information for Repositioning Errant Probes in Central Force Optimization

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Abstract. Central Force Optimization is a global search and optimization algorithm that searches a decision space by flying “probes” whose trajectories are deterministically computed using two equations of motion. Because it is possible for a probe to fly outside the domain of feasible solutions, a simple errant probe retrieval method has been used previously that does not include the directional information contained in a probe’s acceleration vector. This note investigates the effect of adding directionality to the “repositioning factor” approach. As a general proposition, it appears that doing so does not improve convergence speed or accuracy. In fact, adding directionality to the original errant probe retrieval scheme appears to be highly inadvisable. Nevertheless, there may be alternative probe retrieval schemes that do benefit from directional information, and the results reported here may assist in or encourage their development.

30 May 2010
Brewster, Massachusetts

Ver. 2 (Fig. 1 improved for clarity; minor typos corrected.)

6 June 2010
Brewster, Massachusetts

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1. Introduction

This note makes some observations about the utility of including directional information in the errant probe repositioning scheme used in Central Force Optimization [1-14]. CFO is a deterministic Nature-inspired global search and optimization metaheuristic that has been successfully applied to practical problems such as linear and circular array design [1,5], microstrip patch antenna design [15], and matching network optimization [1]. CFO also has performed well on recognized benchmark functions compared to other state-of-the-art algorithms [11-13].

CFO searches a decision space $\Omega$ by flying “probes” whose trajectories are based on a metaphor drawn from gravitational kinematics. Because CFO’s probes may fly outside $\Omega$, a methodology is needed to deal with such errant probes. In previous implementations they were placed inside $\Omega$ using a simple deterministic scheme based on the “repositioning factor,” $F_{rep}$, that does not include directional information. If any probe coordinate fell outside the decision space, then that coordinate was changed independently of the others, thereby losing the directional information contained in the acceleration that caused the probe to fly outside $\Omega$ in the first place.

It seems reasonable to speculate that retaining directionality would improve CFO’s convergence by only truncating the errant probe’s trajectory instead of changing its directional as well. This note takes a preliminary look at this question. It describes an errant probe methodology that combines the repositioning factor with acceleration directional information in order to investigate whether or not including directionality is beneficial. Perhaps somewhat surprisingly, not only does including directional information fail to improve convergence, in most cases it is an impediment.

2. The CFO Algorithm

CFO locates the maxima of an $N_d$-dimensional objective function $f(\bar{x})$ defined on a decision space of feasible solutions $\Omega: \{ \bar{x} | x_i^{\text{min}} \leq x_i \leq x_i^{\text{max}}, 1 \leq i \leq N_d \}$, $x_i \in \mathbb{R}$, where $\bar{x} = (x_1, x_2, ..., x_N)$. $\Omega$ is bounded by $2N_d$ planes $P_i: \{ \bar{x} | \bar{x} = (x_1, ..., x_{i-1}, x_i, x_{i+1}, ..., x_N) \}$, where

$$X_{ik} = \begin{cases} x_i^{\text{min}}, & k = 1 \\ x_i^{\text{max}}, & k = 2 \end{cases}$$

(note that $i = 1, ..., N_d$; $k = 1, 2$ throughout).

CFO samples $\Omega$ by flying “probes” through it at a series of “time” steps (iterations). $f(\bar{x})$’s value, its fitness, is computed step-by-step at each probe’s location which is
specified by its position vector. At step \( j-1 \) probe \( p \) is located at \[ \vec{R}_j^p = \sum_{i=1}^{N_j} x_i^{p,j-1} \hat{e}_i \]
where \( \hat{e}_i \) is the unit vector along the \( i^{th} \) coordinate axis, \( 0 \leq j \leq N_i \) the iteration index, and \( N_i \) the total number of steps (note that the first step is \#0). \( 1 \leq p \leq N_p \) is the probe number and \( N_p \) the total number of probes. Probe \( p \) moves from the point \( \vec{R}_j^p \) at step \( j-1 \) to \[ \vec{R}_j^p = \sum_{i=1}^{N_j} x_i^{p,j} \hat{e}_i \]
at step \( j \) under the influence of the (constant) acceleration \[ \vec{a}_{j-1}^p = \sum_{i=1}^{N_j} a_i^{p,j-1} \hat{e}_i \]
produced by the CFO “masses” discovered by the probe distribution at step \( j-1 \).

Probe \( p \) ’s motion in “CFO space” is computed from two deterministic “equations of motion” for the probe’s trajectory and acceleration, respectively, as follows:

\[ \vec{R}_j^p = \vec{R}_{j-1}^p + \vec{a}_{j-1}^p \]  \hspace{1cm} (1)
\[ \vec{a}_{j-1}^p = \sum_{n=1}^{N_p} U(M_{j-1}^n - M_j^p) \cdot (M_{j-1}^n - M_j^p) \times \frac{(\vec{R}_{j-1}^p - \vec{R}_j^p)}{||\vec{R}_{j-1}^p - \vec{R}_j^p||} \]  \hspace{1cm} (2).

\( M_{j-1}^n = f(x_1^{n,j-1}, x_2^{n,j-1}, \ldots, x_{N_p}^{n,j-1}) \) is the objective function’s fitness at probe \( p \) ’s location at time step \( j-1 \). Each of the other probes at that step (iteration) has associated with it fitness \( M_{j-1}^n, n = 1, \ldots, p-1, p+1, \ldots, N_p \). \( U(\cdot) \) is the Unit Step function defined as \[ U(z) = \begin{cases} 1, & z \geq 0 \\ 0, & \text{otherwise} \end{cases} \]. Note that \( \vec{a}_{j-1}^p = 0 \) for \( \vec{R}_{j-1}^p = \vec{R}_j^p, n \neq p \), because probe \( n \) then has coalesced with probe \( p \) and cannot exert any gravitational force on \( p \). In this case the acceleration expression is indeterminate because \( M_{j-1}^n = M_{j-1}^p \); and accordingly it is set to zero. Note that these equations have been simplified as described in [12,13].

3. The Errant Probe Problem

The acceleration computed from eq. (2) may large enough that trajectory eq. (1) flies probe \( p \) outside the domain of feasible solutions, in which case it somehow must be returned to \( \Omega \) or extinguished. While many return schemes are possible, the one that consistently has been used in previous CFO papers relies on either a fixed or variable repositioning factor \( 0 < F_{rep}^\min \leq F_{rep} \leq 1 \). Coordinate-by-coordinate, if the coordinate lies outside \( \Omega \) then it is brought inside \( \Omega \) as follows:

If \( \vec{R}_j^p \cdot \hat{e}_i < x_i^{\min} \) \hspace{1cm} : \hspace{1cm} \vec{R}_j^p \cdot \hat{e}_i = \max\{ x_i^{\min} + F_{rep} (\vec{R}_{j-1}^p \cdot \hat{e}_i - x_i^{\min}), x_i^{\min} \} \hspace{1cm} (3a)

If \( \vec{R}_j^p \cdot \hat{e}_i > x_i^{\max} \) \hspace{1cm} : \hspace{1cm} \vec{R}_j^p \cdot \hat{e}_i = \min\{ x_i^{\max} - F_{rep} (x_i^{\max} - \vec{R}_{j-1}^p \cdot \hat{e}_i), x_i^{\max} \} \hspace{1cm} (3b)
where the dot denotes vector scalar (inner) product. The out-of-bounds coordinate is set to a fraction \( F_{\text{rep}} \) of the difference between its starting value and its boundary value. \( F_{\text{rep}} \) starts at an arbitrary initial value \( F_{\text{rep}}^{\text{init}} \) which then is incremented at each iteration by an arbitrary amount \( \Delta F_{\text{rep}} \). If \( F_{\text{rep}} > 1 \) it is set to \( F_{\text{rep}} = F_{\text{rep}}^{\min} \), and the process continued. This procedure improves \( \Omega \)'s sampling by distributing errant probes throughout the decision space in an arbitrary but precise manner (see [10] for a more detailed discussion). At every step each probe’s position is known exactly, so that every CFO run with the same setup parameters returns exactly the same results, thus preserving CFO’s inherent determinism. What this scheme loses, however, is the directional information contained in the acceleration vector because a repositioned errant probe moves in a direction that generally is different from its initial trajectory.

4. Repositioning with Directional Information

The straight line defined by \( \bar{S}(\eta) = \bar{R}_{j-1}^p + \eta(\bar{R}_j^p - \bar{R}_{j-1}^p) \), \(-\infty \leq \eta \leq \infty\), passing through probe \( p \)'s successive positions \( \bar{R}_{j-1}^p \) and \( \bar{R}_j^p \) (note \( \bar{S}(0) = \bar{R}_{j-1}^p \), \( \bar{S}(1) = \bar{R}_j^p \)) intersects each of the boundary planes \( P_{ik} : X_{ik} \) at \( \bar{S}_{ik} = \bar{S}(\eta_{ik}) = \bar{R}_{j-1}^p + \eta_{ik}(\bar{R}_j^p - \bar{R}_{j-1}^p) \). Fig. 1 illustrates this geometry in two dimensions (2D). Planes \( P_{11}, P_{12}, P_{21} \), and \( P_{22} \) bound \( \Omega \). Point \( \bar{R}_{j-1}^p \) lies inside \( \Omega \), while \( \bar{R}_j^p \) is outside. The acceleration \( \bar{a}_{j-1}^p \) has flown probe \( p \) outside \( \Omega \), so that it must be repositioned somewhere inside the decision space. If the probe’s trajectory is constrained to lie along the direction of the acceleration, then its maximum displacement is \( d_{\text{max}} \), that is, the “distance” between the probe’s starting point inside \( \Omega \) and the closest boundary plane in the direction of the acceleration as shown in the figure.

The coefficients \( \eta_{ik} \) at the points where \( \bar{S}(\eta) \) and \( P_{ik} \) intersect are determined by the requirement \( \hat{e}_j \cdot \bar{S}_{ik} = X_{ik} = \hat{e}_j \cdot \left\{ \bar{R}_{j-1}^p + \eta_{ik}(\bar{R}_j^p - \bar{R}_{j-1}^p) \right\} = x_{i,j-1}^p + \eta_{ik}(x_{i,j}^p - x_{i,j-1}^p) \), so that \( \eta_{ik} = \frac{X_{ik} - x_{i,j-1}^p}{x_{i,j}^p - x_{i,j-1}^p} \). The nearest intersection point in the direction of \( p \)'s motion is \( \bar{S}^* = \bar{S}(\eta^*) = \bar{R}_{j-1}^p + \eta^*(\bar{R}_j^p - \bar{R}_{j-1}^p) \) where \( \eta^* = \text{MIN}_{\eta > 0}(\eta_{ik}) \), \( 0 \leq \eta^* \leq 1 \). Thus, probe \( p \)'s maximum displacement, which corresponds to placing it on the decision space boundary, is \( d_{\text{max}} = \left\| \bar{S}^* - \bar{R}_{j-1}^p \right\| = \eta^* \left\| \bar{R}_j^p - \bar{R}_{j-1}^p \right\| \).

The new repositioning scheme used in this note consequently still utilizes \( F_{\text{rep}} \), but includes directionality follows:

\[
\text{If } \bar{R}_{j-1}^p \in \Omega \text{ and } \bar{R}_j^p \notin \Omega : \quad \bar{R}_j^p = \bar{R}_{j-1}^p + F_{\text{rep}} d_{\text{max}} \hat{a}_{j-1}^p \quad (4),
\]
where \( \hat{\mathbf{a}}_{j-1}^{p} = \frac{\mathbf{a}_{j-1}^{p}}{\|\mathbf{a}_{j-1}^{p}\|} \) is the unit vector in the direction of probe \( p \)'s trajectory between steps \( j-1 \) and \( j \). Because \( 0 < F_{rep}^{\text{min}} \leq F_{rep} \leq 1 \), the probe cannot be displaced more than \( d_{\text{max}} \), thus constraining it to lie inside \( \Omega \) or on the boundary; and the unit vector \( \hat{\mathbf{a}}_{j-1}^{p} \) insures that \( p \) moves only in the direction of its initial acceleration. The value of \( F_{rep} \) is set arbitrarily in the same way as before.

Fig. 1. Errant probe in a 2D decision space.
5. CFO Implementation

Pseudocode for this implementation is in Fig.2 [Appendix has core routine source code, and a complete electronic listing is available on request, rf2@ieee.org]. Note that the user inputs only the decision space parameters and the objective function to be maximized.

Procedure CFO \([f(\tilde{x}), N_d, \Omega] \]

Internals: \(N_t, F^\text{init}, \Delta F^\text{rep}, F^\text{min}, \left(\frac{N_p}{N_d}\right)_{\text{MAX}}\), \(\gamma_{\text{start}}, \gamma_{\text{stop}}, \Delta \gamma\).

Initialize \(f^{\text{global}}(\tilde{x}) = \text{very large negative number, say, } -10^{4200}\).

For \(N_p/N_d = 2\) to \(\left(\frac{N_p}{N_d}\right)_{\text{MAX}}\) by 2:

1. Total number of probes: \(N_p = N_d \cdot \left(\frac{N_p}{N_d}\right)_{\text{MAX}}\)

For \(\gamma = \gamma_{\text{start}}\) to \(\gamma_{\text{stop}}\) by \(\Delta \gamma\):

1. Re-initialize data structures for position/acceleration vectors & fitness matrix.
2. Compute IPD [see [6,11-13]].
3. Compute initial fitness matrix, \(M^0_p, 1 \leq p \leq N_p\).
4. Initialize \(F^\text{rep} = F^\text{init}\).

For \(j = 0\) to \(N_t\) [or earlier termination – see text]:

1. Compute position vectors, \(\tilde{R}^p_j, 1 \leq p \leq N_p\) [eq.(1)].
2. Retrieve errant probes \((1 \leq p \leq N_p)\):
   a. Without directional information:
      i. \(\tilde{R}^p_j \cdot \hat{e}_i < x_i^{\text{min}}\) : \(\tilde{R}^p_j \cdot \hat{e}_i = \max\{x_i^{\text{min}} + F^\text{rep}(\tilde{R}^p_{j-1} \cdot \hat{e}_i - x_i^{\text{min}}), x_i^{\text{min}}\}\)
      ii. \(\tilde{R}^p_j \cdot \hat{e}_i > x_i^{\text{max}}\) : \(\tilde{R}^p_j \cdot \hat{e}_i = \min\{x_i^{\text{max}} - F^\text{rep}(x_i^{\text{max}} - \tilde{R}^p_{j-1} \cdot \hat{e}_i), x_i^{\text{max}}\}\)
3. With directional information:
   i. If \(\tilde{R}^p_j \in \Omega\) and \(\tilde{R}^p_{j-1} \notin \Omega\) : \(\tilde{R}^p_j = \tilde{R}^p_{j-1} + F^\text{rep} d_{\text{max}} \hat{a}^p_{j-1}\)
   ii. Compute fitness matrix for current probe distribution, \(M^p_j, 1 \leq p \leq N_p\).
   iii. Compute accelerations using current probe distribution and fitnesses [eq. (2)].
   iv. Increment \(F^\text{rep} : F^\text{rep} = F^\text{rep} + \Delta F^\text{rep}\); If \(F^\text{rep} > 1\) : \(F^\text{rep} = F^\text{min}\).
   v. If \(j \geq 20\) and \(j \ MOD 10 = 0\) :
      i. Shrink \(\Omega\) around \(\tilde{R}_{\text{best}}\) [see [11-13] for details].
      ii. Retrieve errant probes [procedure Step (c)].
   vi. Reset \(\Omega\) boundaries [values before shrinking].
   vii. If \(f^\text{max}(\tilde{x}) \geq f^{\text{global}}(\tilde{x})\) : \(f^{\text{global}}(\tilde{x}) = f^\text{max}(\tilde{x})\).

Next \(j\)

Next \(\gamma\)

Next \(N_p/N_d\)

Fig. 2. CFO Pseudocode.
6. Results

Table 1 shows results for CFO with and without directionality in the probe repositioning for a suite of 23 benchmark functions. It compares CFO directly to the Group Search Optimizer (GSO) algorithm [16,17] and indirectly to Particle Swarm (PSO) and Genetic Algorithm (GA) algorithms, which were used in [16] for comparison with GSO. Details of the algorithms, the benchmark suite, and the experimental setup appear there.

The first CFO column shows results with the usual $F_{rep}$ repositioning scheme (eq. (3), no directional information, data reproduced from [13]). The next two columns contain results for directional information using the scheme in eq. (4) applied in two ways: (i) at every step, and (ii) at every other step, respectively. The best fitnesses are highlighted in red, but no highlighting is applied if CFO and the best other algorithm (GSO, PSO, or GA) return essentially the same fitnesses. Note that performance of the other algorithms is described statistically because they are all inherently stochastic, whereas CFO’s results are based on a single run because CFO is inherently deterministic. $N_{eval}$ is the total number of CFO function evaluations.

### Table 1. CFO with/without directional information applied to GSO benchmarks.

| $F_{rep}$ | $N_{d}$ | $F_{max}$ | Best Fitness / Other Algorithm | Reposition without Directional Info | Reposition with Directional Info | Reposition with Mixed Directional Info |
|----------|--------|----------|-------------------------------|-----------------------------------|----------------------------------|--------------------------------------|
|          |        |          |                               | Best Fitness                      | $N_{eval}$ Best Fitness $N_{eval}$ Best Fitness $N_{eval}$ |                          |
| (1)      | (1)    | (2)      |                               | Best Fitness                      | $N_{eval}$ Best Fitness $N_{eval}$ Best Fitness $N_{eval}$ |                          |

Unimodal (average of 1000 runs)

Results for a single run because CFO is deterministic.

| $F_{rep}$ | $N_{d}$ | $F_{max}$ | Other Algorithm | Reposition without Directional Info | Reposition with Directional Info | Reposition with Mixed Directional Info |
|-----------|--------|----------|-----------------|-----------------------------------|----------------------------------|--------------------------------------|
| 1         | 30     | 0        | -3.6927x10^{-7} / PSO | 0                                | 222,960 0                        | 564,020                             |
| 2         | 30     | 0        | -2.9168x10^{-5} / PSO | 0                                | 237,540 0                        | 757,920                             |
| 3         | 30     | 0        | -1.1979x10^{-5} / PSO | -6.1861x10^{-8} 397,320 0         | 773,400 -1.5468x10^{-7} 1,292,820 |
| 4         | 30     | 0        | -0.1078 / GSO         | 0                                | 484,260 0                        | 317,220                             |
| 5         | 30     | 0        | -3.7582 / PSO         | -4.6823x10^{-6} 436,680 0         | 1,649,580 -7.8625x10^{-5} 1,058,700|
| 6         | 30     | 0        | -1.600x10^{-5} / GSO  | 0                                | 176,380 0                        | 336,420                             |
| 7         | 30     | 0        | -9.9104x10^{-6} / PSO | -1.2919x10^{-4} 399,960 0         | 399,960 -8.483x10^{-4} 397,620    |

Multimodal, Many Local Max. (avg 1000 runs)

Results for a single run because CFO is deterministic.

| $F_{rep}$ | $N_{d}$ | $F_{max}$ | Other Algorithm | Reposition without Directional Info | Reposition with Directional Info | Reposition with Mixed Directional Info |
|-----------|--------|----------|-----------------|-----------------------------------|----------------------------------|--------------------------------------|
| 1         | 30     | 12,569.5 | 12,569.4882 / GSO | 12,569.4865                      | 415,500 12,569.4551 633,420 12,569.4376 626,220 |
| 2         | 30     | 0        | -0.6509 / GA     | 0                                | 397,080 0                        | 347,880                             |
| 3         | 30     | 0        | -2.6548x10^{-4} / GSO | 4.7705x10^{-8} 518,820 4.7705x10^{-8} 627,600 4.7705x10^{-8} 730,020 |
| 4         | 30     | 0        | -3.0792x10^{-7} / GSO | -1.7057x10^{-10} 235,800 -9.337x10^{-7} 185,520 -2.1875x10^{-2} 192,360 |
| 5         | 30     | 0        | -2.7648x10^{-11} / GSO | -2.1541x10^{-3} 292,080 -1.4003x10^{-3} 729,720 -1.5237x10^{-1} 437,220 |
| 6         | 30     | 0        | -4.6948x10^{-7} / GSO | -1.8293x10^{-4} 300,000 -3.9782x10^{-4} 945,420 -8.8243x10^{-4} 676,020 |

Multimodal, Few Local Maxima (avg 50 runs)

Results for a single run because CFO is deterministic.

| $F_{rep}$ | $N_{d}$ | $F_{max}$ | Other Algorithm | Reposition without Directional Info | Reposition with Directional Info | Reposition with Mixed Directional Info |
|-----------|--------|----------|-----------------|-----------------------------------|----------------------------------|--------------------------------------|
| 1         | 2      | -1       | -0.9980 / GSO   | -0.9980 78,176 0                 | -0.9980 83,812 -0.9980 79,716     |
| 2         | 4      | -3.075x10^{-3} / GSO | -3.7713x10^{-6} 143,152 -3.4513x10^{-5} 121,408 -4.8177x10^{-2} 426,264 |
| 3         | 2      | 1.0316285 | 1.0316285 / GSO | 1.031588 87,240 1.031627 88,560 1.031628 86,504 |
| 4         | 2      | -0.398   | -0.3979 / GSO   | -0.3979 82,096 -0.3979 80,240 -0.3979 94,316 |
| 5         | 2      | -3       | -3 / GSO        | -3 100,996 -3 105,544 -3 100,472 |
| 6         | 3      | 3.86     | 3.8628 / GSO    | 3.8628 160,338 3.8628 155,010 3.8628 174,846 |
| 7         | 6      | 3.32     | 3.2697 / GSO    | 3.3219 457,836 3.3219 439,656 3.3219 491,076 |
| 8         | 4      | 10       | 7.5349 / PSO    | 10.1532 251,648 10.1532 243,364 10.1532 216,864 |
| 9         | 4      | 10       | 8.3553 / PSO    | 10.4029 316,096 10.4029 303,664 10.4029 238,344 |
| 10        | 4      | 10       | 8.9394 / PSO    | 10.5364 304,312 10.5364 326,224 10.5364 271,816 |

(1) Negative of the functions in [16] are computed by CFO because CFO searches for maxima instead of minima.
(2) Data reproduced from [16].
(3) Data reproduced from [13].
(4) Directional information used on every step.
(5) Directional information used on alternate steps.
The (perhaps) surprising conclusion from these data is that, generally, adding directionality to the original $F_{rep}$ repositioning scheme does not improve convergence, either in terms of speed or accuracy. Figs. 3 and 4 plot convergence speed across the benchmark suite for the two ways in which directional information is included. The charts show the fractional change in the number of function evaluations, $1 - \frac{N'}{N} = \frac{N - N'}{N}$, where $N$ is the number of evaluations without directional information and $N'$ the number with it. A value of zero corresponds to no change in convergence speed, while positive values reflect improvement and negative ones degradation.

For the group of low dimensionality multimodal functions with few local maxima ($F_{14}$-$F_{23}$), adding directionality at every step (Fig. 3) makes only a slight difference in convergence, with improvement on 6 of the 10 functions. A similar behavior occurs when directional information is applied on alternate steps (Fig. 4), but the variability is greater, and in one case ($F_{15}$) the degradation is substantial.

![Convergence Speed (directional info every step)](image)

Fig. 3. CFO Convergence speed with directionality on every step (better > 0, worse < 0).

The story is quite different across the two high dimensionality function groups. Adding directionality on the unimodal functions ($F_1$-$F_7$) significantly degrades convergence on five of them with essentially no change on one ($F_7$) and improvement by about a factor of 2 on another ($F_4$). This behavior is evident in both Figs. 3 and 4. A similar trend is seen in the multimodal functions with many local maxima ($F_8$-$F_{13}$). Adding directional information, regardless of the method used, substantially degrades convergence on four functions ($F_8$, $F_{10}$, $F_{12}$, $F_{13}$), while a moderate improvement is seen on one ($F_{11}$). On $F_9$, directionality at every step results in significantly slower convergence, whereas adding it on alternate steps results in a modest improvement.
The fitness data in Table 1 show that, for the most part, adding directional information does not materially improve CFO’s accuracy. CFO’s fitnesses were quite similar in all cases except two. On function $F_3$, directionality at every step results in CFO’s locating the known maximum exactly, and including it on alternate steps improves the fitness by more than two orders of magnitude. In this case, adding directional information has improved the accuracy substantially. But in marked contrast, adding directionality on $F_5$ yields very poor results. On this function, whose known maximum is zero, CFO returns a best fitness of $-4.8623 \times 10^{-5}$ with no directional information. Adding directionality on alternate steps results in a fitness of $-7.8625 \times 10^{-4}$, which is worse by a factor of 16. And adding it at every step yields a fitness of $-3.1836$, which is worse yet, by a huge factor of 65,475. This example illustrates that including directionality in some cases can be a substantial impediment to CFO’s performance. The conflicting results for $F_3$ and $F_5$, which are quite disparate and no doubt a consequence of the functions’ unusual topologies, suggests that a very cautious approach should be taken to including directionality in CFO.

7. Conclusion

This note compares CFO’s performance using the original $F_{rep}$ errant probe retrieval scheme that contains no directional information and using a modified scheme that adds directionality in two ways: (i) at every step and (ii) at every other step. It was speculated that doing so would improve the algorithm’s performance, both in terms of convergence speed and accuracy. But the results are somewhat surprising and decidedly contrary to that expectation. For the most part, adding directionality worsens convergence speed. In a very limited number of cases it materially improves speed; but, by far, in most cases convergence is substantially slower. The effect on CFO’s accuracy is less pronounced,
with the best fitnesses being the same or very similar across the great majority of test functions whether or not directionality is included. In two cases, significant changes are observed, one with a marked increase in accuracy, but the other with a far worse decrease in accuracy.

It appears that there is little if any benefit from adding directionality to CFO’s original $F_{rep}$ repositioning scheme, and that doing so likely will slow convergence and, in some cases depending on the objective function’s topology, will result in much worse accuracy. Of course, other completely different errant probe repositioning methodologies might benefit from probe directional information, and such different approaches should be developed and investigated. The results reported here hopefully will encourage work in this area.

30 May 2010
Brewster, Massachusetts

Ver. 2 (Fig. 1 improved for clarity; minor typos corrected.)
6 June 2010
Brewster, Massachusetts

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Appendix: Source code listing for core CFO routines.

Note: see references [11,13] for complete listings that contain the procedures not included below.

'Program 'CFO_04-07-2010(MiniBenchmarks)_VER2.BAS' compiled with
'Power Basic/Windows Compiler 9.04.0122 (www.PowerBasic.com).

'LAST MOD 04-07-2010 - 2116 HRS EDT

=====================================================================================================#COMPILE EXE
#DIM ALL
#USEMACROS = 1
#INCLUDE "Win32API.inc"
DEFEXT A-Z

===================================================================================================='

'----- MAIN PROGRAM -----
FUNCTION PBMAIN () AS LONG
  ------ CFO Parameters ----- LOCAL Nd%, Np%, Nt& LOCAL G, DeltaT, Alpha, Beta, Frep AS EXT LOCAL PlaceInitialProbes$, InitialAcceleration$, RepositionFactor$ LOCAL R(), A(), M(), Rbest(), Mbest() AS EXT 'position, acceleration & fitness matrices LOCAL FunctionName$ 'name of objective function

  ------------------------- Miscellaneuous Setup Parameters ----------------------- LOCAL N%, i%, YN&, Neval&&, NevalTotal&&, BestNpNd%, NumTrajectories%, Max1DprobesPlotted%, LastStepBestRun&, Pass%
LOCAL A$, RunCFO$, NECfileError$
LOCAL BestGamma, BestFitnessThisRun, BestFitnessOverall, StartTime, StopTime AS EXT
LOCAL Best ProbeNum%, Best TimeStep&, Best Probe Number Overall%, Best Time Step Overall&, StatusWindowHandle???

  -------------------- Global Constants --------------------- REDIM Aij(1 TO 2, 1 TO 25) '(GLOBAL array for Shekel's Foxholes function) CALL FillArrayAij CALL MathematicalConstants 'NOTE: Calling order is important!! CALL AlphabetAndDigits CALL SpecialSymbols CALL EMconstants CALL ConversionFactors ': CALL ShowConstants 'to verify constants have been set

  --------------------------- General Setup ---------------------------- CFOversion$ = "CFO Ver. 04-06-2010"
RANDOMIZE TIMER 'seed random number generator with program start time DESKTOP GET SIZE TO ScreenWidth&, ScreenHeight& 'get screen size (global variables) IF DIR$("wgnuplot.exe") = "" THEN MSGBOX("WARNING!  'wgnuplot.exe' not found.  Run terminated.") : EXIT FUNCTION END IF

' ------------------------------------------------------ CFO RUN PARAMETERS ---------------------------------------------------------------

SELECT CASE Nd%
CASE 1 : CALL Plot1Dfunction(FunctionName$,R()) : REDIM R(1 TO Np%, 1 TO Nd%, 0 TO Nt&) 'erases coordinate data in R()used to plot function
CASE 2 : CALL Plot2Dfunction(FunctionName$,R()) : REDIM R(1 TO Np%, 1 TO Nd%, 0 TO Nt&) 'ditto

END IF

' CALL GetTestFunctionNumber(FunctionName$)' : exit function 'DEROG CALL GetFunctionRunParameters(FunctionName$,Nd%,Np%,Nt%,G,DeltaT,Alpha,Beta,Frep,R(),A(),M(),DiagLength,PlaceInitialProbes$,InitialAcceleration$,RepositionFactor$)' NOTE: Parameters returned but not used in this version!!

REDIM R(1 TO Np%, 1 TO Nd%, 0 TO Nt%), A(1 TO Np%, 1 TO Nd%, 0 TO Nt%), M(1 TO Np%, 0 TO Nt%) 'position, acceleration & fitness matrices REDIM Rbest(1 TO Np%, 1 TO Nd%, 0 TO Nt%), Abest(1 TO Np%, 1 TO Nd%, 0 TO Nt%), Mbest(1 TO Np%, 0 TO Nt%) 'overall best position & fitness matrices

' -------- PLOT 1D and 2D FUNCTIONS ON-SCREEN FOR VISUALIZATION --------

IF Neval& = 2 AND INSTR(FunctionName$,"PBM_") > 0 THEN CALL CheckNECFiles(NECfileError$)
IF NECfileError$ = "YES" THEN EXIT FUNCTION ELSE MSGBOX("[WARNING!  wgnuplot.exe] not found.  Run terminated.") : EXIT FUNCTION END IF

SELECT CASE Nd%
CASE 1 : CALL Plot1Dfunction(FunctionName$,R()) : REDIM R(1 TO Np%, 1 TO Nd%, 0 TO Nt%) 'erases coordinate data in R()used to plot function
CASE 2 : CALL Plot2Dfunction(FunctionName$,R()) : REDIM R(1 TO Np%, 1 TO Nd%, 0 TO Nt%) 'ditto

12 of 17
YN& = MSGBOX("RUN CFO ON FUNCTION " + FunctionName$ + "?" + Chr$(13) + Chr$(13) + "Get some coffee & sit back...", %MB_YESNO, "CONFIRM RUN") : IF YN& = %IDYES THEN RunCFO$ = "YES"

IF RunCFO$ = "YES" THEN

StartTime = TIMER

CALL CFO(FunctionName$, Nd%, Nt&, R(), A(), M(), DiagLength, BestFitnessOverall, BestNpNd%, BestGamma, Neval&&, Mbest(), BestProbeNumberOverall%, BestTimeStepOverall&, LastStepBestRun&, Alpha, Beta)

StopTime = TIMER

MSGBOX(FunctionName$ + Chr$(13) + "Total Function Evaluations = " + STR$(Neval&&) + Chr$(13) + "Runtime = " + STR$(ROUND((StopTime - StartTime)/3600##, 2)) + " hrs")

END IF

ExitPBMAIN:

END FUNCTION 'PBMAIN()
FOR p% = 1 TO Np%: FOR i% = 1 TO Nd%: A(p%,i%,0) = 0##: NEXT i%: NEXT p% 

'STEP (A4) -------------- Initialize Frep ----------------

Frep = 0.5##

'============================================================================ LOOP ON TIME STEPS STARTING AT STEP #1============================================================================

BestFitnessThisRun = -1E4200 'very large NEGATIVE number
FOR j& = 1 TO Nt&

'STEP (B) ----------- Compute Probe Position Vectors for this Time Step --------

FOR p% = 1 TO Np%: FOR i% = 1 TO Nd%: R(p%,i%,j&) = R(p%,i%,j&-1) + A(p%,i%,j&-1): NEXT i%: NEXT p% 'note: factor of 1/2 combined with G=2 to produce coefficient of one

'STEP (C) ----------- Retrieve Errant Probes ---------------

IF j& MOD ReposInterval% = 0 THEN 'use directional scheme 1/3 of the time
 CALL RetrieveErrantProbes2(Np%,Nd%,j&,A(),Frep) 'added 04-01-10
 ELSE
 CALL RetrieveErrantProbes(Np%,Nd%,j&,A(),Frep)
END IF

'STEP (D) ----------- Compute Fitness Matrix for Current Probe Distribution -----------

FOR p% = 1 TO Np%: FOR i% = 1 TO Nd%

IF k% <> p% THEN

dummy index

FOR L% = 1 TO Nd%  : SumSQ = SumSQ + (R(k%,L%,j&)-R(p%,L%,j&))^2 : NEXT L% 'dummy index

IF SumSQ <> 0## THEN 'to avoid zero denominator (added 03-20-10)
 Denom = SQR(SumSQ) : Numerator = UnitStep((M(k%,j&)-M(p%,j&)))*(M(k%,j&)-M(p%,j&))
 A(p%,i%,j&) = A(p%,i%,j&) + (R(k%,i%,j&)-R(p%,i%,j&))*Numerator^Alpha/Denom^Beta 'ORIGINAL VERSION WITH VARIABLE Alpha & Beta
END IF 'added 03-20-10

END IF 'dummy index

NEXT k% 'dummy index

NEXT i% 'coord (dimension) #

NEXT p% 'probe #

' --------- Get Best Fitness & Corresponding Probe # and Time Step ---------

CALL GetBestFitness(M(),R(),j&,BestProbeNumber,BestTimeStep)

IF BestFitness >= BestFitnessThisRun THEN

BestFitnessThisRun = BestFitness : BestProbeNumberThisRun = BestProbeNumber : BestTimeStepThisRun = BestTimeStep
END IF

' ----- Increment Frep ----- 

Frep = Frep + DeltaFrep

IF Frep > 1## THEN Frep = 0.05## 'keep Frep in range [0.05,1]

' --------- Starting at Step #20 Shrink Decision Space Around Best Probe Every 20th Step ---------

IF j& MOD 20 = 0 AND j& >= 20 THEN

FOR i% = 1 TO Nd% : XiMin(i%) = XiMin(i%)+(R(BestProbeNumber%,i%,BestTimeStep&)-XiMin(i%))/2## : XiMax(i%) = XiMax(i%)-(XiMax(i%)-R(BestProbeNumber%,i%,BestTimeStep&))/2## : NEXT i% 'shrink DS by 0.5

IF j& MOD ReposInterval% = 0 THEN
 CALL RetrieveErrantProbes2(Np%,Nd%,j&,R(),A(),Frep) 'added 04-01-10
 ELSE
 CALL RetrieveErrantProbes(Np%,Nd%,j&,R(),Frep) 'TO RETRIEVE PROBES LYING OUTSIDE SHRUNKEN DS 'ADDED 02-07-2010
END IF

' ----- If SlopeRatio Changes Abruptly, Shrink Decision Space Around Best Probe ------ 'SOMETIMES SEEMS TO IMPROVE PERFORMANCE, BUT NOT WITH VERY ERRATIC Fitness Evolution PLOTS...

eta = 0.7##

IF SlopeRatio(M(),R(),j&) >= 3## THEN

FOR i% = 1 TO Nd% : XiMin(i%) = XiMin(i%)+eta*(R(BestProbeNumber%,i%,BestTimeStep&)-XiMin(i%)): XiMax(i%) = XiMax(i%)-eta*(XiMax(i%)-R(BestProbeNumber%,i%,BestTimeStep&)): NEXT i% 

IF j& MOD ReposInterval% = 0 then
 CALL RetrieveErrantProbes2(Np%,Nd%,j&,A(),Frep) 'added 04-01-10
 ELSE
 CALL RetrieveErrantProbes(Np%,Nd%,j&,A(),Frep) 'TO RETRIEVE PROBES LYING OUTSIDE SHRUNKEN DS 'ADDED 02-07-2010
END IF

END IF

' --------- Early Run Termination ---------

IF HasFITNESSsaturated$(25,j&,Np%,Nd%,M(),R(),DiagLength) = "YES" THEN

LastTimeStep = j& : EXIT FOR 'exit TIME STEP Loop
END IF
NEXT j& 'END TIME STEP LOOP

'------------------------------------------------------------------------ Best Overall Fitness & Corresponding Run Parameters ----------------------------------------

IF BestFitnessThisRun >= BestFitnessOverall THEN

BestFitnessOverall = BestFitnessThisRun : BestProbeNumberOverall% = BestProbeNumberThisRun% : BestTimeStepOverall& = BestTimeStepThisRun&

CALL CopyBestMatrices(Np%,Nh%,M(),R(),Mbest(),Rbest())

END IF

'STEP (C) ----- Reset Decision Space Boundaries to Initial Values -----  
CALL ResetDecisionSpaceBoundaries(Nh%)
NEXT GammaNumber% 'END GAMMA LOOP
NEXT NumProbesPerDimension% 'END Np/Nd LOOP
END SUB 'CFO()

============================================================================================================================================= 

SUB IPD(Np%,Nd%,Nt&,R(),Gamma) 'Initial Probe Distribution [IPD] on "Probe Lines" Parallel to Coordinate Axes
LOCAL DeltaXi, DelX1, DelX2, Di AS EXT
LOCAL NumProbesPerDimension%, p%, i%, k%, NumX1points%, NumX2points%, x1pointNum%, x2pointNum%

IF Nd% > 1 THEN
NumProbesPerDimension% = Np%
ELSE
NumProbesPerDimension% = Np%
END IF

FOR i% = 1 TO Nd%
FOR p% = 1 TO Np%
R(p%,i%,0) = XiMin(i%) + Gamma*(XiMax(i%)-XiMin(i%))
NEXT Np%
NEXT i%

FOR i% = 1 TO Nd% 'place probes probe line-by-probe line (i% is dimension [coordinate] number)
DeltaXi = (XiMax(i%)-XiMin(i%))/(NumProbesPerDimension%-1)
FOR k% = 1 TO NumProbesPerDimension%
R(p%,i%,0) = XiMin(i%) + (k%-1)*DeltaXi
NEXT k%
NEXT i%
END SUB 'IPD()

'-------------
SUB GetBestFitness(M(),Np%,StepNumber&,BestFitness,BestProbeNumber%,BestTimeStep&)
LOCAL p%, jj&, A$
BestFitness = -1E4200 'very large negative number
FOR jj& = 0 TO StepNumber&
FOR p% = 1 TO Np%
IF M(p%,jj&) >= BestFitness THEN
BestFitness = M(p%,jj&) : BestProbeNumber% = p% : BestTimeStep& = jj&
END IF
NEXT p%
NEXT jj&
END SUB 'GetBestFitness()

FUNCTION HasFITNESSsaturated$(NavgSteps&,j&,Np%,Nd%,M(),R(),DiagLength)
LOCAL A$, B$
LOCAL k&, p%
LOCAL BestFitness, SumOfBestFitnesses, BestFitnessStepj, FitnessSatTOL AS EXT

AS = "NO" : B$ = "$j$:STEP(j)+CHS(13)
FitnessSatTOL = 0.000001### 'tolerance for FITNESS saturation
IF j% < NavgSteps& - 10 THEN GSTO EmitHasFITNESSsaturated 'execute at least 10 steps after averaging interval before performing this check
SumOfBestFitnesses = 0##
FOR k% = j%-NavgSteps&+1 TO j% 'GET BEST FITNESSES STEP-BY-STEP FOR NavgSteps& INCLUDING THIS STEP j% AND COMPUTE AVERAGE VALUE.
' BestFitness = M(k&,1) 'ORIG CODE 03-23-2010: THIS IS A MISTAKE!
BestFitness = -1E4200 'THIS LINE CORRECTED 03-23-2010 PER DISCUSSION WITH ROB GREEN.
'INITIALIZE BEST FITNESS AT k&-th TIME STEP TO AN EXTREMELY LARGE NEGATIVE NUMBER.

FOR p% = 1 TO Np% 'PROBE-BY-PROBE GET MAXIMUM FITNESS
IF M(p%,k&) >= BestFitness THEN BestFitness = M(p%,k&)
NEXT p%

IF k& = j& THEN BestFitnessStepJ = BestFitness 'IF AT THE END OF AVERAGING INTERVAL, SAVE BEST FITNESS FOR CURRENT TIME STEP j&

SumOfBestFitnesses = SumOfBestFitnesses + BestFitness

NEXT k&

IF ABS(SumOfBestFitnesses/NavgSteps&-BestFitnessStepJ) <= FitnessSatTOL THEN A$ = "YES" 'saturation if (avg value - last value) are within TOL

ExitHasFITNESSsaturated:
HasFITNESSsaturated$ = A$
END FUNCTION 'HasFITNESSsaturated$

'---------------------------------------------------------------
'
SUB RetrieveErrantprobes2(Np%,Nd%,j&,R(),A(),Frep)
LOCAL A$, ProbeInsideDSstepJ$, ProbeInsideDSstepJminus1$

LOCAL p%, i%, k%
LOCAL Xik, dMax, EtaIK(), EtaStar, SumSQ, MagRjRj1, MagAj1, Numerator, Denom AS EXT

FOR p% = 1 TO Np% 'check every probe, probe-by-probe

' ---------------------- Determine Probe Locations at Steps j& and j&-1 -------------------------------

ProbeInsideDSstepJ$ = "YES" 'presume probe is inside DS at step j&

FOR i% = 1 TO Nd% 'check to see if probe p lies outside DS (any coordinate beyond corresponding boundary coordinate)
IF (R(p%,i%,j&) > XiMax(i%) OR R(p%,i%,j&) < XiMin(i%)) THEN 'probe lies outside DS
ProbeInsideDSstepJ$ = "NO" : EXIT FOR 'need only one coordinate outside DS
END IF

NEXT i%

ProbeInsideDSstepJminus1$ = "YES" 'presume probe is inside DS at step j&-1

FOR i% = 1 TO Nd% 'check to see if probe p lies outside DS (any coordinate beyond corresponding boundary coordinate)
IF (R(p%,i%,j&-1) > XiMax(i%) OR R(p%,i%,j&-1) < XiMin(i%)) THEN 'probe lies outside DS
ProbeInsideDSstepJminus1$ = "NO" : EXIT FOR 'need only one coordinate outside DS
END IF

NEXT i%

' ------------------------------- If Probe is Outside at Both Steps, Use Old Scheme to Reposition ---------------------------------------

IF ProbeInsideDSstepJ$ = "NO" AND ProbeInsideDSstepJminus1$ = "NO" THEN 'probe p% is outside DS at both time steps => use old Frep scheme to reposition

FOR i% = 1 TO Nd%
IF R(p%,i%,j&) < XiMin(i%) THEN R(p%,i%,j&) = MAX(XiMin(i%) + Frep*(R(p%,i%,j&-1)-XiMin(i%)),XiMin(i%))
IF R(p%,i%,j&) > XiMax(i%) THEN R(p%,i%,j&) = MIN(XiMax(i%) - Frep*(XiMax(i%)-R(p%,i%,j&-1)),XiMax(i%))

NEXT i%

END IF 'ProbeInsideDSstepJ$ = "NO" AND ProbeInsideDSstepJminus1$ = "NO"

' -------------- If Probe is Outside at Step j& but Inside at Step j&-1 Then Use Reposition Using Directional Information ---------------

IF ProbeInsideDSstepJ$ = "NO" AND ProbeInsideDSstepJminus1$ = "NO" THEN 'probe p% is outside DS at step j& and inside at step j&-1 => use scheme that preserves directional information

REDIM EtaIK(1 TO Nd%, 1 TO 2) 'Eta(i%,k%)

FOR i% = 1 TO Nd% 'compute array of Eta values

FOR k% = 1 TO 2
SELECT CASE k%
CASE 1 : Xik = XiMin(i%)
CASE 2 : Xik = XiMax(i%)
END SELECT

Numerator = Xik-R(p%,i%,j&-1) : Denom = R(p%,i%,j&)-R(p%,i%,j&-1)

IF ABS(Denom) <= 1E-10 THEN
EtaIK(i%,k%) = 0## 'DO NOT REPOSITION
ELSE
EtaIK(i%,k%) = Numerator/Denom

END IF

NEXT k%

NEXT i%

' -------------------------------------------------------------
'
END SUB RetrieveErrantprobes2
EtaStar = 1E4200 'very large POSITIVE number
FOR i% = 1 TO Nd% 'get min Eta value >= 0
  FOR k% = 1 TO 2
    IF EtaIK(i%,k%) <= EtaStar AND EtaIK(i%,k%) >= 0## THEN EtaStar = EtaIK(i%,k%)
  NEXT k%
NEXT i%
IF EtaStar < 0## OR EtaStar > 1## THEN MSGBOX("WARNING! EtaStar="+STR$(EtaStar))
SumSQ = 0## : FOR i% = 1 TO Nd% : SumSQ = SumSQ + (R(p%,i%,j&) - R(p%,i%,j&-1))^2 : NEXT i% : MagRjRj1 = SQR(SumSQ) 'magnitude of 
[(Rp at step j) MINUS (Rp at step j-1)]
  dMax = EtaStar*MagRjRj1 'distance to nearest boundary plane from position of probe p at step j-1
  SumSQ = 0## : FOR i% = 1 TO Nd% : SumSQ = SumSQ + A(p%,i%,j&-1)^2 : NEXT i% : MagAj1 = SQR(SumSQ) 'magnitude of acceleration at 
step j-1
  IF i% = 1 OR i% = Nd% 'reposition probe p using acceleration directional information
    R(p%,i%,j&) = R(p%,i%,j&-1) + Frep*dMax*A(p%,i%,j&-1)/MagAj1 'unit vector in direction of acceleration preserves acceleration 
directional information
  END IF
NEXT i%
END IF 'ProbeInsideDSstepJ$ = "NO" AND ProbeInsideDSstepJminus1$ = "YES"
NEXT p% 'process next probe
END SUB 'RetrieveErrantProbes()

'-------------------------------
SUB RetrieveErrantProbes(Np%,Nd%,j&,R(),Frep) 'original version, does not include acceleration vector directional information
LOCAL p%, i%
FOR p% = 1 TO Np%
  FOR i% = 1 TO Nd%
    IF R(p%,i%,j&) < XiMin(i%) THEN R(p%,i%,j&) = MAX(XiMin(i%) + Frep*(R(p%,i%,j&-1)-XiMin(i%)),XiMin(i%)) 'CHANGED 02-07-10
    IF R(p%,i%,j&) > XiMax(i%) THEN R(p%,i%,j&) = MIN(XiMax(i%) - Frep*(XiMax(i%)-R(p%,i%,j&-1)),XiMax(i%))
  NEXT i%
NEXT p%
END SUB 'RetrieveErrantProbes()

'-----------------------------
SUB ResetDecisionSpaceBoundaries(Nd%)
LOCAL i%
FOR i% = 1 TO Nd% : XiMin(i%) = StartingXiMin(i%) : XiMax(i%) = StartingXiMax(i%) : NEXT i%
END SUB 'ResetDecisionSpaceBoundaries()

'--------------------------------------
END OF PARTIAL SOURCE CODE LISTING