Reorganization of a dense granular assembly: the ‘unjamming response function’

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We investigate the mechanical properties of a static dense granular assembly in response to a local forcing. To this end, a small cyclic displacement is applied on a grain in the bulk of a 2D disordered packing under gravity and the displacement fields are monitored. We evidence a dominant long range radial response in the upper half part above the solicitation and after a large number of cycles the response is ‘quasi-reversible’ with a remanent dissipation field exhibiting long range streams and vortex-like symmetry.

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It is always striking that apparently different cellular materials like foams, emulsions or granular systems share in common many rheological properties [1]. All these systems can flow like fluids when a sufficiently high external stress is applied but jam into an amorphous rigid state below a critical yield stress. This jamming transition is associated with a slowdown of the dynamics which led Liu et al. [1] to propose an analogy between the process of jamming and the glass transition for glass-forming liquids. Although the nature of this jamming transition is still unclear experimentally [2, 3], several theoretical attempts were made to adapt the concepts of equilibrium thermodynamics to athermal systems out of equilibrium [4, 5, 6]. For packing made of grains with a size larger than few microns, thermal fluctuations are too small to allow a free exploration of the phase space and grains are trapped into metastable configurations. The system can not evolve until an external mechanical perturbation like shear or vibration is applied, which allows grains to overcome energy barriers and triggers structural rearrangements. In this case, the free volume and configurations accessible to each grain are capital notions that were used to define the new concept of ‘effective temperature’ [4]. Recently it was proposed that this notion could account for the transports properties in the vicinity of a jammed state [6]. However besides this large number of theoretical and numerical works there are only few experiments connecting the motion at the grain level to the macroscopic mechanical behavior [8]. It appears that there is a crucial need for understanding the connection between the local geometrical properties and the possible motion of a grain since structural rearrangements, and therefore displacement fields, are the key for understanding the rheology of dense systems.

On the other hand, there is still a debate to understand the elasto-plastic behavior of amorphous materials [9, 10, 11, 12]. Recent experiments on the response of a granular pile to a small force perturbation revealed an elastic-like behavior, which is very sensitive to the preparation, i.e. to the microscopic texture [13]. Similar conclusions were drawn in the context of sound propagation [14]. However, at this stage it is not entirely clear which features of the texture (such as coordination number, contact distribution, various fabric tensors) are useful to build relevant macroscopic constitutive relations [15]. When the external drive increases an irreversible yield occurs that is usually described by a plasticity theory. Yet the study of the early stages of plasticity is of crucial interest for a better understanding of yield properties and the elucidation of strain localization (shear bands). Recent theoretical attempts were made to explain global plastic deformations from a modelling of local structural rearrangements named ‘shear transformation zone’. This approach was firstly introduced to account for the onset of plasticity in amorphous solids [11] but was extended to granular materials [17].

In this paper, we present a conceptually simple experiment which aim is to study the response of a dense disordered granular media to a small perturbation induced by the displacement of a grain-scale intruder. The forces applied to the intruder grain are large enough to unjam this initially static packing but the driving is slow enough to stay in the quasi-static regime. Deformations induced by the local perturbation are small (less than 10^-3) and so we are rather far from a fully developed plasticity regime that would be induced, for example, by a moving rod [17]. We propose a new path of study for the jammed state by monitoring the displacement fields in response to a localized cyclic perturbation that brings the system above the jamming transition. We call it the ‘unjamming response’ function which should be a characteristic feature of the packing configurations and of its reorganization properties. Note that, very recently, similar displacement response experiments have been performed by Moukarzel et al. [18].

The typical displacements induced by the perturbation are small at the scale of a grain: we observe a range of displacements between 1/2000 and 1/3 of a grain size d but these are still very large compared to the local displacements induced by the granular contact deformations. A
simple order of magnitude calculation shows that grains can be considered as rigid, since for the metallic grains we use and the load experienced by the packing under gravity, elastic displacements at contacts are as small as $10^{-8} d$. This huge separation of scales shows that we are probing the response of the granular assembly solely due to grain reorganizations. It corresponds either to contact opening/closing or to a change of contact direction (rolling contact). Under gravity these processes may be partially reversible and depend in a very sensible way on the packing geometrical properties (texture, density) and how far we are from the jamming transition. This sensitivity can be seen as the hallmark of the ‘fragile’ character of granular assemblies and the questions raised to understand this relation are important to obtain a fully consistent picture of condensed jammed phases as described for example by O’Hern et al.\[19\].

A sketch of the experimental set-up is shown in figure inset. We prepare disordered 2D packings of brass hollow cylinders by mixing small (diameter $d_1 = 4$ mm) and big ones ($d_2 = 5$ mm) in order to avoid macroscopic crystallization. We chose to have the total mass of small cylinders to be equal to that of the big ones, so that their numbers are respectively in proportion of 7 to 4. This geometry allows a precise monitoring of the displacements for each grain in the visualization field. All cylinders have a 3 mm height and lay on a low frictional glass plane. The lateral and bottom walls are made of plexiglass and delimits a rectangular frame of $L = 26.8$ cm $\simeq 54 d_2$ width and an adjustable height of typically $H = 34.4$ cm $\simeq 70 d_2$. The bottom plane can be tilted at an angle $\varphi = 33^\circ$ such as to control the confinement pressure inside the granular material by an effective gravity field $g \sin \varphi$. The angle $\varphi$ value is larger than the static Coulomb angle of friction between the grains and the glass plate. The intruder is a big grain of diameter $d_2$ located in the median part of the container at a 21.2 cm (i.e. $\simeq 42 d_2$) depth from the upper free surface. It is attached to a rigid arm in plexiglass moved by a translation stage and a stepping motor driven by a computer, so that the intruder motion is characterized by cycles of displacement along the median axis $Y$ of the container. In this report the intruder is moving up then down in a quasi-static way at a velocity of 156 $\mu m/s$ separated by rest periods of 9 s. The intruder displacement value $U_0$ is only a fraction of a grain diameter ($U_0 = 1.25$ mm). A high resolution CCD camera ($1280 \times 1024$ pixels) is fixed above the experimental setup with its plane parallel to the tilted plane which is homogeneously illuminated from behind. The image frame is centered slightly above the intruder and covers a zone of area $39 d_2 \times 31 d_2$ – see figure\[1\]. The camera is triggered by a signal coming from the motor which allows the capture of an image in the rest phase one second before each intruder displacement. In the following, we use the notation $i$ for the index corresponding to the $i^{th}$ image just before the $i^{th}$ displacement (upwards or downwards) and $n$ for the cycle number with $n = int \left( \frac{i+1}{2} \right)$. The center of each grain is determined with precision using the computation of the correlation between an image of the packing and two reference images corresponding to both grain types ($d_1$ or $d_2$). We obtain, for each image, the locations of more than a thousand grains with a resolution down to 0.05 pixels. The displacements of each single grain in response to the intruder motion is determined with a precision of less than 10 $\mu m$.

To compute the averaged displacement fields we first coarse grain individual grain displacements in little cells of typical size $1.2 d_2 \times 1.2 d_2$ regularly located in the cartesian coordinates ($O, X, Y$) reference frame – see figure\[1\]. We then use an ensemble average. To this end, 16 equivalent experiments were performed by fixing the ini-
FIG. 3: a) and b) Mean rescaled response functions in polar coordinates $ru_r$ and $ru_\theta$ for the upward motion $i = 17$ ($n = 9$) as a function of $\theta$. This data collapse has been obtained with radial distances ranging from 7.4 to 21.8$d_2$, represented by different symbols – e.g. ▼ for $r = 15.8d_2$. c) and d) Vertical and horizontal components of the irreversible displacement field $\vec{v}$ for cycle $n = 9$ in cartesian coordinates as a function of $X$. The horizontal span corresponds to the whole cell width. Here the data correspond to $Y$ between 14.5 and 26.7$d_2$ – e.g. $Y = 24.2d_2$ is ▱. Rescaling factors are indicated on the axis legends. The quantities $b_n$ and $\epsilon_n$ are the maxima of the mean profiles a) and c) respectively.

On figure 3, we show the ensemble average and coarse-grained displacement field obtained after the second upward motion of the intruder (cycle $n = 2$, displacement $i = 3$). We clearly notice that the granular motion is not localized in the vicinity of the intruder and that this small perturbation of only one third of a grain diameter produces a far field effect. The presence of two displacement rolls is observed near the intruder. They are located symmetrically on each side of the intruder but turn in opposite directions. Besides this near field effect, the main response principally occurs above the intruder with displacement vectors that tend to align along the radial directions from the intruder. On figures 3(a) and 3(b) we display a typical response to the $n = 9^{th}$ upward motion ($i = 17$) in polar coordinates $(0, r, \theta)$. By defining $\vec{u}_n^\uparrow = u_{n,r} \vec{e}_r + u_{n,\theta} \vec{e}_\theta$, we display the rescaled quantities: $ru_{n,r}$ and $ru_{n,\theta}$ as a function of the angle $\theta$. The collapse of the data onto the same curve (within experimental uncertainties) at distances far enough from the intruder ($r \geq 6d_2$), shows that we can extract a dominant term for the far field displacement of the type:

$$\vec{v}_n^\uparrow \simeq b_n U_0 d_2 \frac{f(\theta)}{r} \vec{e}_r,$$

(1)

The parameter $b_n$ is a dimensionless decreasing function of the cycle number $n$ (see further) and $f(\theta)$ is an even function of $\theta$ with $f(0) = 1$. Note that we could not observe any significant shape variation of the function $f(\theta)$ with $n$. The preceding formula holds for the zone above the intruder, i.e. $-90^\circ \leq \theta \leq 90^\circ$, as...
the decay of the response below the intruder is much faster than $1/r$. According to equation (1) the projection $u_{n,Y}(X,Y)$ of $\overrightarrow{a}_{n}$ along $\overrightarrow{Y}$ should be maximal in $X = 0$ for a given vertical distance $Y$ from the intruder, leading to $u_{n,Y,\text{max}}(Y) \simeq b_{n}U_{0}d_{2}/Y$. Simple integration of $u_{n,Y}$ along the $X$ axis shows that the quantity $b_{n}U_{0}d_{2}$ is proportional to the effective area displaced by the intruder on the horizontal line seen at a remote vertical distance $Y$. Thus, in this interpretation, parameter $b_{n}$ can be seen as an ‘effective transmission’ parameter or a ‘displacement impedance’ connecting the externally driven motion of the intruder to the granular reorganizations in the bulk. A good determination of $b_{n}$ can be inferred from average of the experimental quantity $\beta_{n} = (u_{n,Y,\text{max}}/(U_{0}d_{2}))$ over values of $Y$ in between $6d_{2}$ and $27d_{2}$.

Now we study the behavior of the irreversible field $\overrightarrow{a}_{n}$ for the transient part of the response. After few cycles its amplitude is relatively small: one tenth or less of the displacement field magnitude. On figures 4(c) and 4(d), we display the cartesian coordinates of $\overrightarrow{a}_{n} = a_{n,X} \overrightarrow{X} + a_{n,Y} \overrightarrow{Y}$, for a cycle number $n = 9$ and for different heights far from intruder level. We observe a striking feature: the dominant part of the field has an amplitude almost independent of the vertical distance and its influence spans the whole width of the container ($27d_{2}$ on each side). It can be described as a quasi vertical columnar flow with a linear decay of its amplitude out of the median axis, as to first order, the shape of the field is triangular with a maximum value $a_{n,\text{max}}$ in $X = 0$. The measurement error bars prevent a more precise determination of the field shape. We also notice that the horizontal part of the field $a_{n,X}$ is not exactly zero and could actually show a slight tendency to point outwards the median line but we are at the limit of the signal over noise ratio to go further in the analysis.

Both reversible and irreversible displacement fields evolve with the number of cycles. On the upper part of figure 4 we display the evolution of the values $b_{n}$ as a function of the displacement number $i$. The error bar on $b_{n}$ is a witness of the dispersion of the quantity $\beta_{n}$: it is larger for the two first cycles where there is a departure from the scaling proposed in (1). From figure 4, we observe a quasi exponential relaxation to a limiting value $b_{0} = 0.19 \pm 0.01$ which is 3 times smaller than the original one. This decrease of $b_{n}$ can be interpreted as a progressive screening of the perturbation and is due to a local arching effect that takes place at a distance of few grains around the intruder. These observations seem to validate, at least in the limit of experimental uncertainties, the interpretation of $b_{n}$ as a displacement impedance. The screening due to this arching effect progressively ceases to vary for a number of cycles larger than few tens as evidenced by the steady saturation of the $b_{n}$ values for $n \gtrsim 30$. In parallel (figure 4, bottom part), we monitor the mean maximal amplitude $a_{n,\text{max}}$ of the irreversible component $a_{n,Y}$. We observe its decay to zero for $i \gtrsim 60$, i.e. $n \gtrsim 30$ which corresponds to the onset of a quasi-reversible response observed from the curve $b_{n}(i)$.

In the quasi-reversible regime ranging from $n = 30$ up to $n = 262$ for the longest experiments we did, we could not observe any sensible variation of the ensemble average fields ($\overrightarrow{a}_{n}^{\uparrow}$ or $\overrightarrow{a}_{n}$) and compacity. On the other hand, a closer look at each individual experimental realization of the irreversible displacement field, shows a very striking
The irreversible displacement field has radically changed its symmetry: coherent streams and vortex-like structures appear in the whole packing. What is not clear yet is whether this structuring will lead to a further slower evolution of the packing or whether it corresponds to some steady-state feature i.e. a rem-

ant steady dissipation field. The long range coherent structures found in many different situations are in our opinion of a central importance since they could carry some universal features characterizing the collective dissipa-
tion modes of condensed jammed phases.

In conclusion, we propose the first experimental deter-
mination of the ‘unjamming response’ i.e. the reorganiza-
tion field due to a localized cyclic displacement experi-
enced by a packing of hard-grains under gravity. We use a small perturbation such as to investigate intermediate values of deformations which are large compared to the deformations at granular contacts but small compared with usual experiments where a plastic deformation field is fully developed. The response to an upward pertur-

bation can be separated into three distinct parts. Far from the intruder we observe a dominant radial displace-
ment field which amplitude scales as the inverse of the distance to the perturbation. Close to the perturbation we observe displacement rolls on both sides as well as a vault forming in the immediate surrounding of the in-
truder. In the part below the intruder the displacement decays rapidly to zero. A local arching effect progres-
sively screens the perturbation as seen far from the in-
truder and is accompanied by an irreversible field shaped as a quasi-vertical columnar flow. This irreversible down-
ward flow is too small to induce noticeable changes of compacity but is sufficient to modify the subsequent me-

chanical response. Thus the granular material can be seen as an auto-adaptive material screening the exerted perturbation. Once this flow ceases, the response is quasi-

reversible but on each realization, we still observe a rema-
nant irreversibility flow spanning the whole container and characterised by a different symmetry: vortex-like struc-
tures and coherent long range streams. These vortex-like structures remind strongly what was observed recently in simulations for the non-affine components of the elastic field in amorphous media \[12\] and also for the particle displacement fluctuations in a quasi statically sheared granular flow \[21\]. The question whether these features are linked to the 2D aspect of the studied systems, and that 3D granular assemblies may behave differently has to be investigated. In the future we plan to compare these experimental data with theoretical predictions, e.g. elastic calculations such as those computed in \[12\] or \[21\] in the context of 2D amorphous media. At last, it would be interesting to monitor the change of the unjamming re-

sponse function with respect to initial compacity and/or to texture parameters in association with force measure-
ments. This systematic study could bring some crucial informations on the nature of the jamming/unjamming transition.

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