Neutron moderation spectrum considering inelastic scattering

V.D. Rusov\textsuperscript{a,*}, V.A. Tarasov\textsuperscript{a}, S.A. Chernezhenko\textsuperscript{a}, A.A. Kakaev\textsuperscript{a}, V.P. Smolyar\textsuperscript{a}, V.V. Urbanovich\textsuperscript{a}, V.M. Vashchenko\textsuperscript{b}

\textsuperscript{a}Odessa National Polytechnic University, Shevchenko av., 1, 65044, Odessa, Ukraine. \\
\textsuperscript{b}State Ecological Academy of Post-Graduate Education and Management, V. Lypkivskogo str., 35, bldg.2, 03035, Kyiv, Ukraine

Abstract

For the first time an analytic expression for inelastic neutron scattering law for isotropic neutron source considering moderating medium temperature as a parameter was obtained in the present paper within gas model. The obtained scattering law is based on solution of kinematic problem of inelastic scattering of a neutron at a nucleus in laboratory coordinate system (L-system) in general case. I.e. in case not only a neutron but also a nucleus have arbitrary velocity vector in L-system. Analytic expression for neutron flux density and neutron moderation spectrum in reactor fission medium are found for elastic scattering law obtained by the authors before and for new law of inelastic scattering. They consider elastic and inelastic scattering and depend on medium temperature. The obtained expressions for neutron moderation spectra enable reinterpretation of physical nature of processes that determine the shape of neutron spectrum in thermal neutron energy range.

Keywords: nuclear, nucleus, neutron, moderation, inelastic, elastic, scattering, temperature, kinematics, spectrum, interaction, cross-section, flux density.

1 Introduction

Neutron moderation theory is an important issue in nuclear reactor physics [1–9]. Traditional to nuclear reactor physics theory of neutron moderation does not consider temperature of the moderating medium, because it is based on neutron scattering law not considering medium nuclei thermal motion. Within this theory the neutron moderation spectrum is described by an analytic expression known as Fermi moderation spectrum [1–9]. Fermi moderation spectrum also does not depend on moderating medium temperature and therefore it cannot accurately describe experimental neutrons moderation spectrum correctly in low-energy range, e.g. neutron spectra of thermal nuclear power reactors. Due to the fact that experimental neutron spectra for thermal nuclear reactors are similar to Maxwell distribution [1,3,7,10,11], an assumption has been made that neutron moderation spectrum can be described by this distribution in low-energy range. This assumption enabled development of widely used contemporary semi-empiric calculation algorithm for neutron moderation spectra calculation. According to this algorithm the neutron moderation spectrum is a combination of high-energy range as Fermi moderation spectrum and low-energy range as Maxwell’s distribution. Hence it contains a formula to determine neutron gas temperature based on moderating

\textsuperscript{*}Corresponding author: siis@te.net.ua
medium temperature which determines low-energy spectrum range [1–9]. And the formula itself was obtained by numerical approximation of experimental spectra of several thermal nuclear reactors of different kinds, available at that moment in the past [1], and it is still widely used in nuclear reactors physics e.g. see [5–9, 12–16]. Therefore in the strict sense there is no neutron moderation theory available, and creating one is an essential task.

Let’s also note, that neutron moderation spectra in different media can be modeled by Monte-Carlo approach. This method is the basis for neutron moderation spectra calculation implemented in different reactor computer algorithms. E.g. available software with corresponding interfaces designed to model transfer of neutrons, photons, etc in different media by Monte-Carlo method. includes MCNP4, GEANT4 [17,18] and others. They enable determination of neutron moderation spectrum by Monte-Carlo method, i.e. by numerical solution of kinematics problem of neutron scattering considering possible neutron absorption for moderating medium that absorbs neutrons. This computation is repeated multiple times for random neutron and nucleus initial velocities to obtain resulting neutron moderation spectrum by averaging of the accumulated results across all samples. But this is rather a modeling approach than a theory.

Lack of neutron moderation theory first of all hinders investigation of emergency modes of nuclear reactors and development of new generation of reactors physics, e.g. soliton fission reactors [19,20], impulse reactors, neutron breeders (boosters), subcritical assemblies [21–23] and natural-occurring nuclear reactors, e.g. georeactor [24].

This paper proposes a further development of our recently published work [25] which provided a base theory for neutron moderation considering temperature of fission environment. The base theory model of neutron moderation does not fully consider inelastic scattering reactions and therefore it is applicable to moderating media consisting of nuclei with negligibly small inelastic scattering cross-section. The current paper develops a neutron moderation theory for neutrons emitted by an isotropic neutron source considering inelastic scattering. Account for inelastic scattering reactions is especially important for moderating media consisting of heavy nuclei. E.g. according to [11] inelastic neutron scattering by heavy nuclei will manifest at neutron energy greater than several hundreds kiloelectronvolts. In contrast to light nuclei for which inelastic scattering will manifest for energy higher than one or several megaelectronvolts.

The Authors’ scientific interest is physics of fission neutron-multiplying media (reactor fuel media) and therefore the neutron moderation theory was developed first of all to be applicable to such media. The assumption of neutron source isotropy corresponds to neutron source density in fission neutron-multiplying media.

Any fission medium (nuclear fuel during the reactor operation, but in general case any medium wherein a chain reaction takes place) is thermodynamically unstable due to fission processes followed by release of high amounts of energy and emission of neutrons and other particles. Moreover, its nuclide compound, thermal conductivity, radiation defects dynamics change, sometimes leading to changing of the environment shape and sometimes even rupturing it, etc. This way, the fission medium of the reactor where fission processes take place is an open physical system in non-equilibrium thermodynamical state. Such system may be described within the margins of non-linear, non-equilibrium thermodynamics of open physical systems. Non-equilibrium stationary states, satisfying Prigogine criterion [26–28]: minimum entropy production – may exist in such systems. From non-linear non-stationary thermodynamics it is known that occurrence and kind of such stationary mode depend not only on system internal parameters (internal entropy) but also on boundary conditions (entropy flow at boundaries). For example, implementation of a stationary mode in a non-equilibrium system (so called non-equilibrium stationary state) demands constancy of boundary conditions, e.g. [27].
In this paper the following simplifications are used for building the neutron moderation process model in a fission medium. In a fission medium two thermodynamic sub-systems are distinguished: moderating neutrons sub-system and moderating medium nuclei sub-system. Each of these is an open physical system interacting with one another. According to the previous paragraph, in reality both systems are in non-equilibrium state. However, in our model we use a simplification that the moderating medium nuclei sub-system is in state close to equilibrium due to its inertness relative to perturbations. Therefore by neglecting influence of neutrons sub-system at nuclei sub-system we assume that moderating medium nuclei sub-system is in thermodynamic equilibrium. This simplification enables us to introduce moderating medium temperature into the model and to describe the nuclei kinetic motion energy distribution by Maxwell’s distribution. We consider the moderating neutrons sub-system in non-equilibrium state and we introduce no temperature for it in our model.

Let’s emphasize, that as noted above, in traditional approach to deriving neutron moderation spectrum a neutron gas sub-system temperature is introduced. This implies a simplified assumption that neutrons sub-system is also in thermodynamic equilibrium state which we avoided. Moreover, as noted above, neutron gas temperature formula is includes moderating medium temperature through a ratio empirically fit to experimental values.

For the first time an analytic expression for inelastic neutron scattering law for isotropic neutron source considering moderating medium temperature as a parameter was obtained in the present paper within gas model. The obtained scattering law is based on solution of kinematic problem of inelastic scattering of a neutron at a nucleus in laboratory coordinate system (L-system) in general case. I.e. in case not only a neutron but also a nucleus have arbitrary velocity vector in L-system. Analytic expression for neutron flux density and neutron moderation spectrum in reactor fission medium are found for elastic scattering law obtained by the authors before and for new law of inelastic scattering. They consider elastic and inelastic scattering and depend on medium temperature. The obtained expressions for neutron moderation spectra enable reinterpretation of physical nature of processes that determine the shape of neutron spectrum in thermal neutron energy range.

Let’s note that anisotropy of elastic and inelastic scattering reactions was not considered and this task is postponed for further theory development together with problem of accounting for moderating medium nuclei interaction.

Let’s also note that background of the issue is given in detail in [25].

2 Kinematics of inelastic neutron scattering at moderating medium nucleus

If the neutron inelastic scattering reaction is characteristic to neutron moderating medium (e.g. for heavy nuclei, such as Uranium-238), its kinematics presented in our paper [25] should be adjusted.

According to [5,6] neutron inelastic scattering reaction at a nucleus progresses through an excited compound nuclei state, i.e. at the first phase of the reaction the neutron moderating medium nucleus captures a neutron:

\[(A,Z) + 1_0 n \rightarrow (A + 1, Z)^*,\]  \hspace{1cm} (1)

where \((A,Z)\) is neutron moderating medium nucleus with mass number \(A\) and charge \(Z\); \(1_0 n\) – neutron \((0 – zero charge, 1 – neutron mass number); \((A + 1, Z)^*\) – a compound nucleus with mass number \(A + 1\) and charge \(Z\); \(^*\) – denotes nucleus excited state. During the next phase of the reaction the compound nucleus decomposes producing a nucleus of the neutron
moderating medium in excited state and emitting a neutron. The nucleus is de-excited by emitting a $\gamma$-quant:

$$(A + 1, Z)^* \rightarrow (A, Z)^* + ^{1}_0 n \rightarrow (A, Z) + ^{1}_0 n + \gamma. \quad (2)$$

Here we should note that the compound nucleus decomposition is a two-particle nuclear reaction, and kinematics problem solution is univalent, e.g. see [29].

Let’s consider neutron inelastic scattering at a moderating medium nucleus. Neutron moderating medium (following [25]) is described within framework of gas model. I.e. it is assumed that the medium nuclei do not interact with one another, but possess kinetic energy due to thermal motion. Following [25] let’s introduce two laboratory coordinates systems (fig. 1):

- a stationary laboratory coordinate system or $L$-system
- a non-stationary laboratory coordinate system or $L'$-system that moves relative to $L$-system with a constant velocity equal to velocity of thermal motion of the moderating medium nucleus at which the neutron is scattered.

Let’s note, that here we consider a special case when orientation of $L$ and $L'$ laboratory coordinate systems axes is equal and radius vector of $L'$-system origin in $L$-system equals to moderating medium nucleus radius vector in $L$-system, i.e. the moderating medium nucleus initially stands still in $L'$-system.

Here we use the same notation as in [25]:

- $m_1 = m_{\text{neutron}}$ - neutron mass;
- $m_2 = m_{\text{nucleus}}$ - nucleus mass;
- $\vec{r}_1^{(L)}$ - neutron radius vector in $L$-system;
- $\vec{r}_2^{(L)}$ - nucleus radius vector in $L$-system;
- $\vec{r}_C^{(L)}$ - inertia center radius vector in $L$-system;
- $\vec{r}_1^{(L')}$ - neutron radius vector in $L'$-system;
- $\vec{r}_2^{(L')}$ - nucleus radius vector in $L'$-system;
- $\vec{r}_C^{(L')}$ - radius vector of inertia center in $L'$-system
- $\vec{V}_{10}^{(L)}$ - neutron velocity before impact with the nucleus in $L$-system;
- $\vec{V}_{1}^{(L)}$ - neutron velocity after impact with the nucleus in $L$-system;
- $\vec{V}_{20}^{(L)}$ - nucleus velocity before impact with the neutron in $L$-system;
- $\vec{V}_{2}^{(L)}$ - nucleus velocity after impact with the neutron in $L$-system;
- $\vec{V}_{10}^{(L')}$ - neutron velocity before impact with the nucleus in $L'$-system;
- $\vec{V}_{1}^{(L')}$ - neutron velocity after impact with the nucleus in $L'$-system.
\[ V_{20}^{(L')} \] – nucleus velocity before impact with the neutron in \( L' \)-system;
\[ V_{12}^{(L')} \] – nucleus velocity after impact with the neutron in \( L' \)-system;
\[ \vec{V}_C^{(L')} \] – inertia center velocity in \( L' \)-system.

Interdependence of radius vectors of a point in \( L \) and \( L' \) systems is given by:
\[ \vec{r}^{(L)} = \vec{r}_O^{(L')} + \vec{r}^{(L')} \],
(3)

where \( \vec{r}_O^{(L')} \) is radius vector of \( L' \)-system origin of coordinates in \( L \)-system (see fig. 1).

Therefore, according to the problem definition:
\[ \vec{V}_{10}^{(L)} = \frac{d\vec{r}_1^{(L)}}{dt} \neq 0 \quad \text{and} \quad \vec{V}_{20}^{(L)} = \frac{d\vec{r}_2^{(L)}}{dt} \neq 0. \]
(4)

In \( L' \)-system before the impact the nucleus is standstill, i.e.:
\[ \vec{V}_{20}^{(L')} = 0. \]
(5)

The interrelation between coordinates of \( m_1 \) and \( m_2 \) in \( L \) and \( L' \)-systems is given by (3). And interrelation between velocities (inertial systems velocity addition law, based on Galilean relativity principle):
\[ \vec{V}_{1}^{(L')} = \vec{V}_{1}^{(L)} - \vec{V}_{20}^{(L)} \quad \text{and} \quad \vec{V}_{2}^{(L')} = \vec{V}_{2}^{(L)} - \vec{V}_{20}^{(L)}. \]
(6)

Due to the fact that the moderating medium nucleus stands still in \( L' \)-system, the solution of kinematic problem of inelastic scattering of the neutron at the nucleus in \( L' \)-system may be expressed analogous to solution of two-particle nuclear reaction kinematics with non-zero reaction thermal effect, e.g. given in [29]. Indeed, according to the inelastic neutron scattering reaction schema given by (1) and (2), the thermal effect of such reaction equals to emitted \( \gamma \)-quant energy, further denoted by \( E_\gamma \). Next, let’s consider our problem analogous to [29], but within our notation.

The problem of two particles impact is convenient to solve in inertia center coordinate system, i.e. \( C \)-system.

For inelastic neutron scattering reaction in \( C \)-system the total isolated system momentum conservation law is valid. However, the total kinetic energy conservation law in this isolated system doesn’t work.

From momentum conservation law for two impacting particles in \( C \)-system we obtain:
\[ \vec{P}_{10}^{(C)} + \vec{P}_{20}^{(C)} = \vec{P}_{1}^{(C)} + \vec{P}_{2}^{(C)} = 0, \]
(7)

where:
\[ \vec{P}_{10}^{(C)} \] – neutron momentum before impact with the nucleus in \( C \)-system;
\[ \vec{P}_{1}^{(C)} \] – neutron momentum after impact with the nucleus in \( C \)-system;
\[ \vec{P}_{20}^{(C)} \] – nucleus momentum before impact with the neutron in \( C \)-system;
\[ \vec{P}_{2}^{(C)} \] – nucleus momentum after impact with the neutron in \( C \)-system.

As follows from relations (7):
\[ m_1 \cdot \vec{V}_{10}^{(C)} = -m_2 \cdot \vec{V}_{20}^{(C)} \quad \text{and} \quad m_1 \cdot \vec{V}_{1}^{(C)} = -m_2 \cdot \vec{V}_{2}^{(C)}. \]
(8)

Velocity modulus can be obtained from (8):
\[ \left| \vec{V}_{10}^{(C)} \right| = V_{10}^{(C)} = \frac{m_2}{m_1} \left| \vec{V}_{20}^{(C)} \right| = \frac{m_2}{m_1} V_{20}^{(C)} \]
(9)
and
\[ |\vec{V}_1(C)| = V_1(C) = \frac{m_2}{m_1} |\vec{V}_2(C)| = \frac{m_2}{m_1} V_2(C). \]  

(10)

By introducing neutron \( A_{\text{neutron}} = 1 \) and nucleus \( A_{\text{nucleus}} = A \) mass numbers and assuming \( m_1 = m_{\text{neutron}} \approx A_{\text{neutron}} = 1 \) and \( m_2 = m_{\text{nucleus}} \approx A_{\text{nucleus}} = A \) we obtain the following expressions for (8) and (9)-(10):

\[ \vec{V}_1(C)_{10} = -A \cdot \vec{V}_2(C)_{20} \quad \text{and} \quad \vec{V}_1(C)_{1} = -A \cdot \vec{V}_2(C)_{2}, \]

(11)

and

\[ V_{10}(C) = +A \cdot V_{20}(C) \quad \text{and} \quad V_1(C) = +A \cdot V_2(C). \]

(12)

The moderating medium nucleus and neutron inertia center coordinates may be expressed in the following way:

\[ \vec{r}(L')_{C} = \left( 1 \cdot \vec{r}(L')_{1} + A \cdot \vec{r}(L')_{2} \right) \cdot \frac{1}{A+1}. \]

(13)

Considering the fact, that in \( L' \)-system the initial nucleus velocity prior to impact is \( \vec{V}_{20}(L') = 0 \), the inertia center velocity for an isolated system consisting of two particles (i.e. the neutron and the nucleus) in \( L' \)-system is the following:

\[ \vec{V}_C(L') = \frac{1}{A+1} \cdot \vec{V}_{10}(L'). \]

(14)

Due to total momentum conservation law the inertia center velocity in \( L' \)-system doesn’t change after the impact. Therefore we omit corresponding indexes denoting values prior and after the impact for the inertia center velocity.

Due to the fact that inertia center coordinate system \( (C\text{-system}) \) moves relative to laboratory system with velocity of inertia center in \( L' \)-system, the neutron velocity prior to impact in \( C \)-system is:

\[ \vec{V}_1(C)_{10} = \vec{V}_1(L')_{10} - \vec{V}_C(L'). \]

(15)

Substituting it into (14) we obtain:

\[ \vec{V}_1(C)_{10} = \vec{V}_1(L')_{10} - \frac{1}{A+1} \cdot \vec{V}_{10}(L') = \frac{A}{A+1} \cdot \vec{V}_{10}(L'). \]

(16)

Using (8) and considering (14), we find the nucleus velocity in \( C \)-system prior to impact:

\[ \vec{V}_{20}(C) = -\frac{1}{A+1} \cdot \vec{V}_{10}(L'). \]

(17)

The total kinetic energy of the system (two-particle system consisting of \((A,Z)^*\) and \( _0^1 n \)) after the reaction \( T(C) \) equals to:

\[ T(C) = T_0(C) + E_\gamma, \]

(18)

where \( T_0(C) \) – total kinetic energy of the system before the reaction.

Using (16) and (17), we can express the total kinetic energy of the system prior to reaction:

\[ T_0(C) = \frac{\left| \vec{P}_{10}(C) \right|^2}{2\mu} = \frac{\mu \left( V_{10}(L')^2 \right)}{2}, \]

(19)

where \( \mu = A/(A+1) \) – reduced mass of the particles prior to reaction.
Then kinetic energy of the system after the reaction according to (1) and (2) equals:

\[ T(C) = \frac{1}{2\mu} |\vec{p}_{1}^{(C)}|^{2} = \frac{1}{2\mu} |\vec{p}_{10}^{(C)}|^{2} - E_{\gamma} = \frac{\mu \left(V_{10}^{(L')}\right)^{2}}{2} - E_{\gamma}. \]  

(20)

The neutron momentum modulus after the reaction can be expressed based on (20):

\[ |\vec{p}_{1}^{(C)}| = \sqrt{2\mu' T(C)} = \sqrt{2\mu' \left(T_{0}^{(C)} - E_{\gamma}\right)} = \sqrt{2\mu' \left[\frac{\mu \left(V_{10}^{(L')}\right)^{2}}{2} - E_{\gamma}\right]}, \]  

(21)

where \( \mu' = \left(A + E_{\gamma}/c^{2}\right)/(A + 1) \) – reduced mass of the particles after the reaction, where \( c \) is the speed of light in vacuum.

Denoting a unit vector along the neutron velocity in \( C \)-system after the reaction as \( \vec{e}_{1} \), according to (21) we obtain:

\[ \vec{V}_{1}^{(C)} = \sqrt{2\mu'} \left[\frac{\mu \left(V_{10}^{(L')}\right)^{2}}{2} - E_{\gamma}\right] \cdot \vec{e}_{1}. \]  

(22)

Let’s note that angular distribution of inelastic neutron scattering may be almost always considered spherically-symmetric [5, 7] just as for elastic scattering.

The neutron velocity vector after the reaction in \( L' \)-system may be obtained by adding neutron velocity vector after impact in \( C \)-system (22) and velocity vector in \( C \)-system (14):

\[ \vec{V}_{1}^{(L')} = \vec{V}_{1}^{(C)} + \frac{1}{A+1} \vec{V}_{10}^{(L')} \]  

(23)

Based on velocity parallelogram shown in fig. 2, square of neutron velocity modulus in \( L' \)-system after the impact is:

\[ \left(\vec{V}_{1}^{(L')}\right)^{2} = \left(\vec{V}_{1}^{(C)}\right)^{2} + \left(V_{10}^{(L')} \cdot \frac{1}{A+1}\right)^{2} + 2 \cdot \left(\vec{V}_{1}^{(C)} \cdot \frac{1}{A+1}\right) \cdot \left(V_{10}^{(L')} \cdot \frac{V_{10}^{(L')}}{A+1}\right) \cdot \cos \theta, \]  

(24)

where \( \theta \) is neutron exit angle in \( C \)-system (fig. 2).

Emitted \( \gamma \)-quant energy \( E_{\gamma} \) is confined within an interval:

\[ 0 \leq E_{\gamma} \leq \frac{A}{A+1} \cdot \left(\frac{\left(V_{10}^{(L')}\right)^{2}}{2} \right) < E_{f}, \]  

(25)
where $E_f$ – is the margin neutron energy, that causes fission of the compound nucleus $(A + 1, Z)^*$. 

As follows from neutron and nucleus (standstill in $L'$-system) isolated system total energy conservation law, the compound nucleus excitation energy equals to (e.g. see [5,6]):

$$E^* = \varepsilon_n + \frac{A}{A+1} T_n^{(L')}$$

(26)

where $\varepsilon_n$ – neutron bond energy in the compound nucleus, $T_n^{(L')}$ – neutron kinetic energy prior to scattering in $L'$-system.

During decomposition of the compound nucleus into a moderating medium nucleus in a non-exicted state, neutron and $\gamma$-quant (see reaction schema (2)) a fraction of the excited nucleus energy $\varepsilon_n$ deontes work against strong interaction forces during neutron emission, and the remaining excitation energy fraction equals to sum of system kinetic energy (nucleus and neutron) in $C$-system after scattering and $\gamma$-quant energy:

$$\frac{A}{A+1} T_n^{(L')} = \left(\vec{P}_1^{(C)}\right)^2 + E_\gamma.$$  

(27)

A stochastic multitude of reactions of neutrons inelastic scattering at moderating medium nuclei contribute to total neutron moderation spectrum, herewith the $\gamma$-quant kinetic energy (and also square of the neutron momentum after scattering in $C$-system univocally related to it through (27)) varies randomly within the energy range determined by (25). There are no reasons to suggest that some value of $\gamma$-quant kinetic energy is somehow emitted more frequently than others. Therefore it may be assumed that a random $\gamma$-quants energy value within the inelastic scattering of a multitude of neutrons at nuclei of the moderating medium will be given by equally probable distribution and the probability density is determined by the following expression:

$$\rho(E_\gamma) = \rho\left(\vec{P}_1^{(C)}\right) = \frac{1}{\left(\frac{A}{A+1}\right)\left(\frac{V_{10}^{(L')}}{2}\right)^2}.$$  

(28)

The average value of square of neutron momentum and average value of square of neutron velocity after scattering in $C$-system (which are equal because the neutron has $A_{neutron} = 1$) in case of equiprobable random distribution can be easily expressed as a sum of its maximal (at $E_\gamma = 0$) and minimal (at $E_\gamma = \frac{A}{A+1} T_n^{(L')})$ values divided by 2:

$$\left(\vec{V}_1^{(C)}\right)^2 = \frac{1}{2} \left[\left(\frac{A}{A+1}\right)^2 \left(\frac{V_{10}^{(L')}}{2}\right)^2 + 0\right]^2 = \frac{1}{2} \left(\frac{A}{A+1}\right)^2 \left(\frac{V_{10}^{(L')}}{2}\right)^2 = \frac{A^2 B^2}{(A+1)^2} \left(\frac{V_{10}^{(L')}}{2}\right)^2,$$

(29)

$$B^2 = \frac{1}{2}.$$  

(30)

Knowing average value of neutron velocity modulus after scattering in $C$-system, we can analogous to expression (22) multiply it by unit vector of neutron exit direction $\vec{e}_1$ in $C$-system and obtain an expression for average neutron velocity vector after scattering:

$$\vec{V}_1^{(C)} = \frac{AB}{A+1} V_{10}^{(L')} \cdot \vec{e}_1.$$  

(31)
Therefore, the average value of square of neutron velocity after scattering in $L'$-system can be obtained analogous to (31), i.e.:

$$
\left( \vec{V}_{1}^{(L')} \right)^2 = \left( \vec{V}_{1}^{(C)} \right)^2 + \left( V_{10}^{(L')} \cdot \frac{1}{A+1} \right)^2 + 2 \cdot \left( \vec{V}_{1}^{(C)} \cdot \frac{V_{10}^{(L')}}{A+1} \right) \cdot \cos \theta. \tag{32}
$$

Substituting $\vec{V}_{1}^{(C)}$ from (31) into (32) we obtain the following expression:

$$
\left( \vec{V}_{1}^{(L')} \right)^2 = \left( V_{10}^{(L')} \right)^2 \cdot \frac{A^2 B^2 + 2AB \cos \theta + 1}{(A+1)^2}. \tag{33}
$$

The ratio of square of average neutron velocity after inelastic scattering $\left( \vec{V}_{1}^{(L')} \right)^2$ to square of neutron velocity prior to scattering $\left( V_{10}^{(L')} \right)^2$ in $L'$-system can be derived from (33) considering inelastic scattering of multitude of neutrons that have equal velocity module prior to scattering. This ratio is also equal to ratio of average neutron kinetic energy after scattering $E_{2}^{(L')}$ to neutron kinetic energy prior to scattering $E_{1}^{(L')}$:

$$
\frac{\left( \vec{V}_{1}^{(L')} \right)^2}{\left( V_{10}^{(L')} \right)^2} = \frac{E_{2}^{(L')}}{E_{1}^{(L')}} = \frac{A^2 B^2 + 2AB \cos \theta + 1}{(A+1)^2}. \tag{34}
$$

By introducing inelastic scattering coefficients in the following way:

$$
\tilde{\alpha}_{1} = \left( \frac{AB - 1}{AB + 1} \right)^2 \quad \text{and} \quad \tilde{\alpha}_{2} = \left( \frac{AB + 1}{A+1} \right)^2, \tag{35}
$$

expression (34) transforms into:

$$
\frac{E_{2}^{(L')}}{E_{1}^{(L')}} = \frac{\tilde{\alpha}_{2}}{2} \left[ (1 + \tilde{\alpha}_{1}) + (1 - \tilde{\alpha}_{1}) \cos \theta \right]. \tag{36}
$$

Now, expressing the vectors of average neutron velocity after scattering in $C$-system through analogous vector of average neutron velocity after scattering in $L$-system, we can rewrite (36) in the following form:

$$
\frac{\left( \vec{V}_{1}^{(L)} - \vec{V}_{20}^{(L)} \right)^2}{\left( \vec{V}_{10}^{(L)} - \vec{V}_{20}^{(L)} \right)^2} = \frac{E_{2}^{(L')}}{E_{1}^{(L')}} = \frac{\tilde{\alpha}_{2}}{2} \left[ (1 + \tilde{\alpha}_{1}) + (1 - \tilde{\alpha}_{1}) \cos \theta \right]. \tag{37}
$$

From expression (37) for inelastic scattering we obtain an expression analogous to elastic
scattering expression obtained in [25]:

\[
\frac{\left(\vec{V}_1^{(L)}\right)^2}{\left(\vec{V}_{10}^{(L)}\right)^2} = \frac{E_1^{(L)}}{E_{10}^{(L)}} = \\
\frac{\tilde{\alpha}_2^2}{2} \left[ (1 + \tilde{\alpha}_1) + (1 - \tilde{\alpha}_1) \cos \theta \right] \left[ 1 - 2 \frac{V_{10}^{(L)} V_{20}^{(L)} \cos \beta}{V_{10}^{(L)}^2} + \frac{V_{20}^{(L)}}{V_{10}^{(L)}^2} \right] + \\
\frac{\tilde{\alpha}_2^2}{2} \left[ (1 + \tilde{\alpha}_1) + (1 - \tilde{\alpha}_1) \cos \theta \right] - \frac{\tilde{\alpha}_2^2}{2} \left[ (1 + \tilde{\alpha}_1) + (1 - \tilde{\alpha}_1) \cos \theta \right] \frac{V_{10}^{(L)} V_{20}^{(L)} \cos \beta}{V_{10}^{(L)}^2} + \\
+ 2 \frac{\vec{V}_1^{(L)} \cdot \vec{V}_{20}^{(L)}}{V_{10}^{(L)}^2} - \left\{ 1 - \frac{\tilde{\alpha}_2^2}{2} \left[ (1 + \tilde{\alpha}_1) + (1 - \tilde{\alpha}_1) \cos \theta \right] \right\} \frac{1}{A} \frac{E_{20}^{(L)}}{E_{10}^{(L)}},
\]

(38)

where \( E_1^{(L)} = \frac{1}{2} \left( \vec{V}_1^{(L)} \right)^2 \); \( \cos \beta \) – cosine of the angle between \( \vec{V}_{10}^{(L)} \) and \( \vec{V}_{20}^{(L)} \), expressed through scalar product \( \vec{V}_{10}^{(L)} \cdot \vec{V}_{20}^{(L)} \) in the following way:

\[
\cos \beta = \frac{\vec{V}_{10}^{(L)} \cdot \vec{V}_{20}^{(L)}}{V_{10}^{(L)} V_{20}^{(L)}} = \frac{V_{10}^{(L)} V_{20}^{(L)}}{V_{10}^{(L)} V_{20}^{(L)}},
\]

(39)

\( \cos \gamma \) – cosine of the angle between \( \vec{V}_1^{(L)} \) and \( \vec{V}_{20}^{(L)} \):

\[
\cos \gamma = \frac{\vec{V}_1^{(L)} \cdot \vec{V}_{20}^{(L)}}{V_1^{(L)} V_{20}^{(L)}} = \frac{\vec{V}_1^{(L)} \cdot \vec{V}_{20}^{(L)}}{V_1^{(L)} V_{20}^{(L)}}.
\]

(40)

As shown below, the expression (38) would suffice for our goals and it’s not necessary to present further transformations of (38) and (39) and associated to them (40) that provide a final set of expressions giving an exact solution of the considered kinematic problem of neutron inelastic scattering at a nucleus, considering thermal motion of the nucleus, because it leads only to a set of lengthy expressions because intermediate solution of the problem (38) includes cosines of the angles between neutron and nucleus vectors (39) and (40). Therefore these cosines values must be transformed into \( L \)-system, i.e. it requires several additional relations of transforming unit vectors that determine neutron and nucleus velocity after scattering into \( L \)-system. Therefore reducing the solution to a single analytic expression in this case and its consideration in this paper is pointless.
3 Neutron inelastic scattering law considering moderating medium nuclei thermal motion

According to neutron inelastic scattering at a moderating medium nucleus kinematics given in section 2 by (38), the probability that a neutron with kinetic energy \( E_{10} \) prior to scattering at a nucleus in \( L \)-system, after the scattering will have kinetic energy withing an interval from \( E_1 \) to \( E_1 + dE_1 \) may be written in a following way:

\[
P \left( E_1 \right) dE_1 = P \left( \theta, \beta, \gamma, E_1 \right) d\theta d\beta d\gamma dE_1 = P(\theta) d\theta \cdot P(\beta) d\beta \cdot P(\gamma) d\gamma \cdot P \left( E_1 \right) dE_1.
\]  

(41)

As known in nuclear physics, as long as the compound nucleus decomposition does not depend on history of its formation, the inelastic scattering of neutrons in inertia center coordinate system is spherically symmetric (isotropic). Therefore for \( P(\theta) d\theta \) we obtain:

\[
P(\theta) d\theta = \frac{2\pi}{4\pi r^2} \int_0^{2\pi} \sin \theta d\phi d\theta = \frac{1}{2} \sin \theta d\theta,
\]  

(42)

where \( \phi \) denotes azimuth angle of spherical coordinates \( r, \theta, \phi \), introduced in inertia center coordinates system.

Due to the fact that thermal motion of the moderating medium nuclei is chaotic and the neutron source is isotropic (neutron source emits a neutron group with given energy and isotropic spatial distribution of their velocity vectors directions), the distribution of velocity vectors directions in space for neutrons after inelastic scattering is given by equiprobable random distribution law along \( \beta \) and \( \gamma \) angles as a part of (38). I.e. it is also spherically symmetric (isotropic). Therefore analogous to the previous case we obtain:

\[
P(\beta) d\beta = \frac{1}{2} \sin \beta d\beta,
\]  

(43)

\[
P(\gamma) d\gamma = \frac{1}{2} \sin \gamma d\gamma.
\]  

(44)

Let’s average kinetic energy of the neutron after inelastic scattering at a nucleus (given by (38)) across spherically-symmetric distribution of thermal motion velocities of the moderating medium nuclei the and across isotropic neutron source (across isotropic spatial distribution of neutron velocity vectors with given energy, emitted by the neutron source). Sequentially we obtain the following expression:

\[
\overline{E}_1 = \int_0^\pi \int_0^\pi E_1 P(\beta) d\beta P(\gamma) d\gamma = \overline{E}_1 \left\{ \frac{\tilde{\alpha}_1^2}{2} \left| (1 + \tilde{\alpha}_1) + (1 - \tilde{\alpha}_1) \cos \theta \right| - \left[ 1 - \frac{\tilde{\alpha}_1^2}{2} \left| (1 + \tilde{\alpha}_1) + (1 - \tilde{\alpha}_1) \cos \theta \right| \frac{E_N}{A \cdot E_1} \right] \right\}.
\]  

(45)

Here \( \overline{E}_1 \) - neutron energy for isotropic neutron source averaged across neutron momentum directions, which is equal to energy of neutrons emitted by the source \( E_{10} \), i.e. \( \overline{E}_1 = E_{10} \). And \( E_N \) is nucleus kinetic energy, determined by Maxwell’s distribution for
neutron moderating medium nuclei [31], which depends on temperature of the moderating medium as a parameter and has the following form:

\[
P(E_N^{(L)}) \, dE_N^{(L)} = \frac{2}{\sqrt{\pi(kT)^3}} \exp \left( -\frac{E_N^{(L)}}{kT} \right) \sqrt{\frac{E_N^{(L)}}{kT}} \, dE_N^{(L)}. \tag{46}
\]

Let’s average the expression (45) across Maxwell’s distribution of the neutron moderating medium nuclei thermal motion (46), considering \( \overline{E}_N^{(L)} = E_{10}^{(L)} \) and using known expression [31]

\[
\overline{E}_N^{(L)} = \int_0^\infty E_N^{(L)} P\left( E_N^{(L)} \right) \, dE_N^{(L)} = \frac{3}{2} kT,
\tag{47}
\]

we obtain the following:

\[
\overline{E}_1^{(L)} = E_{10}^{(L)} \left\{ \frac{\bar{\alpha}_2^2}{2} \left[ (1 + \bar{\alpha}_1) + (1 - \bar{\alpha}_1) \cos \theta \right] - \left[ 1 - \frac{\bar{\alpha}_2^2}{2} \left[ (1 + \bar{\alpha}_1) + (1 - \bar{\alpha}_1) \cos \theta \right] \frac{3}{4} kT \right. \alpha \cdot E_{10}^{(L)} \right. \right\}.
\tag{48}
\]

Therefore, due to the fact that a functional relationship between \( \overline{E}_1^{(L)} \) and \( \theta \) is univalent as follows from (48), the probability that a neutron with kinetic energy \( E_{10}^{(L)} \) prior to scattering at a nucleus in \( L \)-system, after scattering at a chaotically moving moderating medium nucleus will have kinetic energy averaged by nuclei thermal motion \( \overline{E}_1^{(L)} \) within \( \overline{E}_1^{(L)} \) ... \( \overline{E}_1^{(L)} + dE_1^{(L)} \) interval, \( P\left( \frac{\overline{E}_1^{(L)}}{E_1^{(L)}} \right) \, dE_1^{(L)} \) is determined by \( P(\theta)d\theta \) (42) distribution and therefore we obtain the following relation (to simplify notation here we drop averaging signs and reference to \( L \)-system, i.e. denoting \( P\left( \frac{\overline{E}_1^{(L)}}{E_1^{(L)}} \right) \, dE_1^{(L)} = P(E_1) \, dE_1 \):

\[
P(E_1) \, dE_1 = P(\theta) \, d\theta = P(\theta) \left. \frac{d\theta}{dE_1} \right|_{dE_1 =} = \frac{1}{2} \sin \theta \left. \frac{1}{E_{10}^{(L)} \left[ \frac{\bar{\alpha}_2^2}{2} (1 - \bar{\alpha}_1) \sin \theta + \frac{\bar{\alpha}_2^2}{2} (1 - \bar{\alpha}_1) \sin \theta - \frac{3}{4} kT \right. \alpha \cdot E_{10}^{(L)} \right. \right|_{dE_1} \]

\[
= \left. \frac{1}{E_{10}^{(L)} + \frac{1}{4} \cdot \frac{3}{2} kT} \bar{\alpha}_2^2 (1 - \bar{\alpha}_1) \right. \tag{49}
\]

Therefore the obtained neutron inelastic scattering law considering thermal motion of moderating medium nuclei is the following:

\[
\left\{ \begin{array}{ll}
P(E_1) \, dE_1 = \frac{dE_1}{E_{10}^{(L)} + \frac{1}{4} \cdot \frac{3}{2} kT} \bar{\alpha}_2^2 (1 - \bar{\alpha}_1), & \text{in case } \bar{\alpha}_2^2 \bar{\alpha}_1 \left( E_{10}^{(L)} + \frac{1}{4} \cdot \frac{3}{2} kT \right) \leq E_1 \leq \left( E_{10}^{(L)} + \frac{1}{4} \cdot \frac{3}{2} kT \right); \\
P(E_1) = 0, & \text{in case } E_1 < \bar{\alpha}_2 \bar{\alpha}_1 \left( E_{10}^{(L)} + \frac{1}{4} \cdot \frac{3}{2} kT \right) \text{ or } E_1 > \left( E_{10}^{(L)} + \frac{1}{4} \cdot \frac{3}{2} kT \right). \tag{50}
\end{array} \right.
\]
In the conclusion of the section let’s stress that the law of inelastic scattering (50) is given for averaged neutron energy after scattering $E_1$. The neutron energy averaging is made across thermal (chaotic) motion of the neutron moderating medium nuclei and across isotropic neutron source. As follows from the scattering law (50) all the neutrons emitted by the isotropic neutron source and having energy $E_{10}$ prior to scattering at the moderating medium nuclei, after inelastic scattering at moderating medium nuclei will have energy $E_1$ averaged across thermal (chaotic) motion of the moderating medium nuclei and across isotropic neutron source with probability given by (50).

As will be shown below, the obtained scattering law in form (50) enables obtaining expressions for neutron flux density and neutron energy spectra, moderating in different moderating media considering temperature of the moderating medium.

4 Neutron moderation in neutron moderating and absorbing media, containing several kinds of nuclei

According to [25] an analytic solution for stationary balance equation for moderated neutrons may be found as neutron flux density. In the case, considered in [25] the expression for the elastically scattered moderating neutron flux density is:

$$
\Phi_1(E) = \frac{\infty \Sigma_{el}(E')}{E \Sigma_{el}(E')} Q(E') \cdot \exp \left[ - \int_{E}^{\infty} \frac{\Sigma_a(E'') dE''}{\xi_{el} |\Sigma_t(E'')| E'' + \frac{1}{3} kT} \right] dE' + \frac{\Sigma_{el}(E)}{\Sigma_t(E)} Q(E), \tag{51}
$$

where $Q(E)$ is the quantity of neutrons generated with energy $E$ per unit volume per unit time, $\Sigma_{el}$ – total macroscopic cross section of elastic scattering of the moderating medium ($\Sigma_{el} = \sum_i \Sigma^i_{el}$, where $\Sigma^i_{el}$ – macroscopic cross section of the i-th nuclide in the moderator medium compound), $\Sigma_t = \Sigma_s + \Sigma_a$ – total macroscopic cross section, $\Sigma_s$ – total fission environment macroscopic cross section, $\Sigma_a$ – total neutron absorption macroscopic cross section, $|\xi_{el}|$ – modulus of average-logarithmic energy decrement for elastic scattering $\xi_{el}$, which is determined by analogy to standard neutron moderation theory (e.g., see [4–6, 25]), but through a new neutron elastic scattering law in [25]. Let us recall that $\Sigma_s = \Sigma_{el} + \Sigma_{in}$, where $\Sigma_{el} = \Sigma_p + \Sigma_{rs}$ – total macroscopic cross section of elastic scattering, $\Sigma_p$ – total macroscopic cross section of potential scattering, $\Sigma_{rs}$ – total macroscopic cross section of resonant scattering, $\Sigma_{in}$ – inelastic scattering macroscopic cross section, e.g. see [6].

The expression (51) contains probability function of resonant neutron non-absorption [4–6, 25, 32], now containing moderating medium temperature:

$$
\varphi(E) = \exp \left[ - \int_{E}^{\infty} \frac{\Sigma_a(E'') dE''}{\xi_{el} |\Sigma_t(E'')| E'' + \frac{1}{3} kT} \right]. \tag{52}
$$

Given the inelastic scattering law (50) and preforming calculations analogous to the one made for elastic scattering in [25], we obtain an expression for inelastically scattered moderating neutrons flux density in the following form:

$$
\Phi_2(E) = \frac{\infty \Sigma_{in}(E')}{E \Sigma_{in}(E')} Q(E') \cdot \exp \left[ - \int_{E}^{\infty} \frac{\Sigma_a(E'') dE''}{\xi_{in} |\Sigma_t(E'')| E'' + \frac{1}{3} kT} \right] dE' + \frac{\Sigma_{in}(E)}{\Sigma_t(E)} Q(E), \tag{53}
$$
where average-logarithmic energy decrement of inelastic scattering $\xi_{in}$ is introduced by analogy to standard neutron moderation theory (e.g., see [4–6,25]), but through inelastic neutron scattering law (50). After performing calculations analogous to calculations preformed for $\xi_{el}$ in (24), we obtain the following expression:

$$\xi_{in} = \frac{\tilde{\alpha}_1}{1 - \tilde{\alpha}_1} \ln \left( \tilde{\alpha}_2^2 \tilde{\alpha}_1 \right) + \frac{1 - \tilde{\alpha}_2^2 \tilde{\alpha}_1}{\tilde{\alpha}_2^2 (1 - \tilde{\alpha}_1)} + \ln \left( \frac{E}{E + \frac{A}{2} kT} \right). \quad (54)$$

Therefore, considering elastic and inelastic neutron scattering it is possible to obtain an expression for total moderating neutrons flux density:

$$\Phi(E) = \Phi_1(E) + \Phi_2(E). \quad (55)$$

And the neutron spectrum will be defined by a standard expression:

$$\rho(E) = \frac{n(E)}{\int_0^{\infty} n(E')dE'} = \frac{1}{\sqrt{2E}} \Phi(E). \quad (56)$$

Let’s note, that considering discreteness of energy levels of moderating medium nuclei at which neutrons are scattered inelastically, it is obvious that the inelastic scattering is possible not at any value of neutron kinetic energy, but with its kinetic energy above a threshold given by the following expression (see (26)):

$$E_{in}^{thold} = \frac{A + 1}{A} E_{min}^*, \quad (57)$$

where $E_{min}^*$ is the minimal energy of excitation of the scattering nucleus.

This is confirmed by experimental results, e.g. according to [11], neutron inelastic scattering at heavy nuclei is observed only at neutron energy higher than several hundreds kiloelectronvolts. And for light nuclei – at energies higher than one or several meaelectronvolts.

For fission reactor environments $Q(E)$ is determined by fission spectrum of the fission nuclide or fission nuclides combination, which according to [1, 7, 25, 33, 34] may be given by the following expression:

$$Q(E) = \tilde{Q} \cdot q \cdot \exp \left( -aE \right) \cdot \text{sh}(\sqrt{bE}), \quad (58)$$

where $a$, $b$ and $q$ – constants, given in table 1 below, $E$ – neutron energy nondimensionalized by 1 MeV, $\tilde{Q} = \int_0^{\infty} Q(E)dE$ – total neutrons quantity generated in a unit volume in a unit time.

| Constant | U$_{235}^\text{235}$ | Pu$_{239}^\text{239}$ | U$_{233}^\text{233}$ | Pu$_{241}^\text{241}$ |
|----------|------------------|------------------|------------------|------------------|
| $a$      | 1.036            | 1                | 1.05 ± 0.03      | 1.0 ± 0.05       |
| $b$      | 2.29             | 2                | 2.3 ± 0.10       | 2.2 ± 0.05       |
| $q$      | 0.4527           | $\sqrt{\frac{2}{\pi e}} = 0.48394$ | 0.46534          | 0.43892          |
Fig. 3 shows the energy spectrum of neutrons moderating in hydrogen medium calculated using the Eq. (51). The neutron source was determined by the Eq. (58) for uranium-235 (Table 1), the medium temperature was set 1200 K. The cross-sections of neutron reactions in hydrogen were taken from the ENDF/B-VII.1 database [35].

Fig. 4 shows the moderating neutrons spectrum in hydrogen obtained by Monte-Carlo simulation using GEANT4 package [36]. The neutron source was given by Eq. (58) for uranium-235, and the moderator temperature was taken about 1000 K.
The comparison of the two neutron spectra in hydrogen moderator, obtained using different methods and presented in figures 3 and 4 demonstrates a good agreement between them.

As it was noted in our recent paper [25], the analytic expression obtained within the new theory of neutron moderation, describes the entire spectrum of moderating neutrons – i.e. in the entire band of possible energies depending on the moderator composition and temperature. It may contain two maxima (high-energy and low-energy) or a single maximum (high-energy for fast neutrons or low-energy for thermal neutrons, or an intermediate one). Such transformation of the theoretical spectrum is demonstrated below by the series of images depicting the spectra of neutrons moderating in uranium-carbon media of different composition.

Figs. 5 - 11 show the energy spectra of neutrons moderating in homogeneous uranium-carbon medium. The theoretical curves were calculated using Eq. (56), and the Monte-Carlo simulated ones were obtained using GEANT4 software. The neutron source was determined by Eq. 58) for uranium-235 in all cases. The neutron reactions cross-sections for uranium-238 (Fig. 13) and carbon (Fig. 12) were taken from ENDF/B-VII.1 [35].

From comparison of the Figs. 5 - 11 it is clear that the theoretically calculated spectra are in good agreement with the Monte-Carlo simulated ones.

The series of images demonstrates the gradual transformation of the neutron spectrum from fast to thermal with increase of the carbon percentage in the moderating mixture.

Let us mention the sharp peak in the thermal part of the theoretical spectrum (seen in Figs. 8 - 11). It is associated with the logarithmic energy decrement $\xi$ crossing zero, since the neutron flux density formally tends to infinity in this case (see (51) or (53)). As it was noted in our recent paper [25], for the new scattering law, the energy decrement is not constant, in contrast to the standard moderation theory. Instead, it depends on the neutron energy and

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Energy spectrum of the moderating neutrons in uranium medium (100\% $^{238}\text{U}$) calculated using Eq. (56) (Theory) and using GEANT4 software. The initial spectrum was that of fission (58). The medium temperature is 600 K.}
\end{figure}
Figure 6: Energy spectrum of the moderating neutrons in uranium-carbon medium (50% $^{238}\text{U} + 50\% \text{C}$) calculated using Eq. (56) (Theory) and using GEANT4 software. The initial spectrum was that of fission (58). The medium temperature is 600 K.

Figure 7: Energy spectrum of the moderating neutrons in uranium-carbon medium (20% $^{238}\text{U} + 80\% \text{C}$) calculated using Eq. (56) (Theory) and using GEANT4 software. The initial spectrum was that of fission (58). The medium temperature is 600 K.
Figure 8: Energy spectrum of the moderating neutrons in uranium-carbon medium (10% $^{238}\text{U} + 90\% \text{C}$) calculated using Eq. (56) (Theory) and using GEANT4 software. The initial spectrum was that of fission (58). The medium temperature is 600 K.

Figure 9: Energy spectrum of the moderating neutrons in uranium-carbon medium (5% $^{238}\text{U} + 95\% \text{C}$) calculated using Eq. (56) (Theory) and using GEANT4 software. The initial spectrum was that of fission (58). The medium temperature is 600 K.
Figure 10: Energy spectrum of the moderating neutrons in uranium-carbon medium (1% $^{238}$U + 99% C) calculated using Eq. (56) (Theory) and using GEANT4 software. The initial spectrum was that of fission (58). The medium temperature is 600 K.

Figure 11: Energy spectrum of the moderating neutrons in carbon medium (100% C) calculated using Eq. (56) (Theory) and using GEANT4 software. The initial spectrum was that of fission (58). The medium temperature is 600 K.
Figure 12: The microscopic cross-sections for the neutron reactions in carbon at 600 K.

Figure 13: The microscopic cross-sections for the neutron reactions in uranium-238 at 600 K.
moderator temperature (see e.g. (54)). In the energy range below the thermal energies the logarithmic energy decrement $\xi$ becomes negative. This seems reasonable, since such neutrons would obviously gain some extra energy when interacting with the thermally moving nuclei. So the neutrons are "pushed" away from the zero energy, and our new theory of neutron moderation "captures" this feature.

Let us consider the point which corresponds to the zero value of the logarithmic energy decrement. The theoretical expression for the neutron flux is divergent at this point. According to the definition of the logarithmic energy decrement (e.g. [4–6, 25]), the energy of such neutrons after the interaction remains the same as before. So the neutrons with such energy tend not to change it in the course of interaction with moderator nuclei. Therefore such neutrons would accumulate.

However if we look at the kinematics of two-particle elastic scattering, the case when the light incident particle after collision has the same energy as before, corresponds to the only point of the momentum diagram – when both particles after collision move along the same line in the L-system. I.e. $P_1^{(L)} \parallel P_2^{(L)}$, where $P_1^{(L)}$ and $P_2^{(L)}$ are the momenta of the incident and scattering particles in L-system after collision. This point corresponds to the solution $P_1^{(L)} = P_1^{(L)}$ and $P_2^{(L)} = 0$, $P_1^{(L)}$ is the incident particle momentum in L-system after collision. This solution is traditionally considered as valid, but it is obviously not physical, since the situation when the classical particle is scattered on another one, while preserving its momentum vector, just cannot happen. So the point at which the logarithmic energy decrement is zero, should be excluded from consideration as non-physical.

It is interesting though that when considering the inelastic scattering through the stage of compound nucleus, this point cannot be excluded. The neutron can be ejected from the compound nucleus with the same momentum vector it had before. This problem is smoothed over by the fact that the inelastic scattering is a threshold reaction (with the threshold about $10^5$ keV), and its cross-section is zero for the neutron energy corresponding to the point of $\xi = 0$.

5 Conclusions

For the first time a general analytic expression for inelastic neutron scattering law considering temperature of the moderating medium as a parameter is obtained for isotropic neutron source. Also analytic expressions for neutron flux density and neutron moderation spectrum are obtained for isotropic neutron source in neutron moderating and absorbing media (e.g. different reactor media), containing nuclei of different kinds and also depending on medium temperature.

The obtained expressions for moderating neutrons spectra open a new way to interpret physical nature of processes that determine the shape of neutron spectrum in low-energy range. The cross-sections of elastic and inelastic neutron scattering at moderating medium nuclei behavior and logarithmic energy decrement influence at neutron moderation spectrum maximum shape in low-energy spectrum range was discovered. The nature of this maximum is coupled with non-stationary neutron system moderation by scattering at thermalized moderation medium nuclei system. Therefore its peculiarities cannot be explained within classic approach considering neutrons system in thermodynamic equilibrium and obeying Maxwell’s distribution.

The energy spectra of neutrons moderating in homogeneous hydrogen and uranium-carbon media are presented. The graphs were obtained using the analytical expression 56 and the GEANT4 Monte-Carlo code [36]. The comparative analysis of the neutron spectra obtained using both methods reveals a good agreement.
Finally, a significantly different behavior of elastic and inelastic neutron scattering at different moderating reactor media (e.g. see ENDF/B-VII.0 or [7]) provides opportunities for experimental investigation of elastic and inelastic scattering of neutrons at moderating medium nuclei cross-section behavior influence at neutron moderation spectrum maximum shape in low-energy spectrum range and experimental test of the obtained analytic expressions.

References

[1] A.M. Weinberg, and E.P. Wigner. *The Physical Theory of Neutron Chain Reactors*. The University of Chicago Press, 1958.

[2] A.I. Akhiezer, and I.Ya. Pomeranchuk. *Introduction into the theory of neutron multiplication systems (reactors)*. IzdAT, Moscow, 2002, 367 p. [in Russian]

[3] A.D. Galanin. *Thermal reactor theory*. Pergamon Press, New York, 1960.

[4] S.M. Feinberg, S.B. Shikhov, and V.B. Troyanskii. *The theory of nuclear reactors, Volume 1*. Atomizdat, Moscow, 1978, 400 p. [in Russian]

[5] G.G. Bartolomey, G.A. Bat, V.D. Baybakov, and M.S. Alkhutov. *Basic theory and methods of nuclear power installation calculations*. Energoatomizdat, Moscow, 1989, 512 p. [in Russian]

[6] S.V. Shirokov. *The nuclear reactor physics*. Naukova Dumka, Kyiv, 1998, 288 p. [in Russian]

[7] W.M. Stacey. *Nuclear Reactor Physics*. Wiley-VCH, New York, 2001, 707 p.

[8] V.I. Vladimirov. *Practical problems of nuclear reactor operation*. Energoatomizdat, Moscow, 1986, 304 p. [in Russian]

[9] G.P. Verkhivker, and V.P. Kravchenko. *Base for calculation and design of nuclear power reactors*. TEC Publishing, Odesa, 2008, 409 p. [in Russian]

[10] I.I. Gurievich, and L.V. Tarasov. *Physics of low energy neutrons*. Nauka, Moscow, 1965, 608 p. [in Russian]

[11] N.A. Vlasov. *Neutrons*. Nauka, Moscow, 1971, 551 p. [in Russian]

[12] V.D. Rusov, E.P. Linnik, V.A. Tarasov, T.N. Zelentsova, I.V. Sharph, V.N. Vaschenko, S.I. Kosenko, M.E. Beglaryan, S.A. Chernezhenko, P.A. Molchinikolov, S.I. Saulenko, and O.A. Byegunova. *Traveling wave reactor and condition of existence of nuclear burning soliton-like wave in neutron-multiplying media*. Energies (Special Issue ”Advances in Nuclear Energy”), 2011, Vol. 4, P. 1337-1361.

[13] V.D. Rusov, V.A. Tarasov, and S.A. Chernezhenko. *The modes with the sharpening in the uranium-plutonium fission environment of the technical nuclear reactors and georeactor*. Problems of Atomic Science and Technology, 2011, Vol. 2(97), P. 123-131. [in Russian]

[14] V.D. Rusov, V.A. Tarasov, V.M. Vaschenko, E.P. Linnik, T.N. Zelentsova, M.E. Beglaryan, S.A. Cherneghenko, S.I. Kosenko, P.A. Molchinikolov, V.P. Smolyar, and E.V. Grechan. *Fukushima plutonium effect and blow-up regimes in neutron-multiplying media*. World Journal of Nuclear Science and Technology, 2013, No. 3, P. 9-18. arXiv:1209.0648v1 [nucl-th].
[15] V.D. Rusov, D.A. Litvinov, E.P. Linnik, V.M. Vaschenko, T.N. Zelentsova, M.E. Beglayyan, V.A. Tarasov, S.A. Chernevengo, V.P. Smolyar, P.A. Molchinkolov, K.K. Merkotan, and P.E. Kavatskyy. Kamland-experiment and soliton-like nuclear georeactor. Part 1. Comparison of theory with experiment. Journal of Modern Physics, 2013, No. 4, P. 528-550.

[16] V.D. Rusov, V.A. Tarasov, S.A. Chernegenko, and T.L. Borikov. Blow-up modes in uranium-plutonium fissile medium. Proc. Int. Conf. ”Problems of physics of high energy densities. XII Khariton Topical Scientific Readings”, RFNC-VNIIEF Publishing, Sarov, 2010, P. 94-102. [in Russian]

[17] MCNP – A General Monte Carlo N-particle Transport Code. Version 4C / Ed. J.F. Briesmeister. Los Alamos National Laboratory, NM (USA). Report No. LA-13709-M. March 2000, 788 p.

[18] S. Agostinelli, J. Allison, K. Amak, et al. Geant4 – a simulation toolkit. Nuclear Instruments and Methods in Physics Research. Section A: Accelerators, Spectrometers, Detectors and Associated Equipment., 2003, Vol. 506, Iss. 3, P. 250-303.

[19] V.D. Rusov, V.A. Tarasov, I.V. Sharf, V.M. Vaschenko, E.P. Linnik, T.N. Zelentsova, M.E. Beglayyan, S.A. Chernevengo, S.I. Kosenko, P.A. Molchinkolov, V.P. Smolyar, and E.V. Grechan. On some fundamental peculiarities of the traveling wave reactor operation. Science and Technology of Nuclear Installations. 2015, Vol. 2015, P. 1-23. arXiv:1207.3695v1 [nucl-th].

[20] V.D. Rusov, V.A. Tarasov, M.V. Eingorn, S.A. Chernezhenko, A.A. Kakaev, and V.N. Vaschenko. Ultraslow wave nuclear burning of uranium-plutonium fissile medium on epithermal neutrons. Progress in Nuclear Energy. 2015, Vol. 83, P. 105-122. arXiv:1409.7343v2 [nucl-th].

[21] V.F. Kolesov. Aperiodic pulse reactors. Vol.1. RFNC-VNIIEF Publishing, Sarov, 2006, 553 p. [in Russian]

[22] A.V. Lukin. Physics of the Pulse Nuclear Reactors. RFNC-VNIITF Publishing, Sniezhinsk, 2006, 528 p. [in Russian]

[23] A.V. Arapov, A.A. Deviatkin, I.Yu Drozdov, and M.V. Mochkaiev. The results of the physical launch of the BR-1M reactor. Problems of the physics of high energy density. XII Kharitonov thematic scientific readings, RFNC-VNIIEF Publishing, Sarov, 2010, P. 22-27. [in Russian]

[24] V.D. Rusov, V.N. Pavlovich, V.N. Vaschenko, V.A. Tarasov, et al. Geoantineutrino spectrum and slow nuclear burning on the boundary of the liquid and solid phases of the Earth’s core. Journal of Geophysical Research, 2007, vol. 112, B09203, P. 1-16.

[25] V.D. Rusov, V.A. Tarasov, S.A. Chernezhenko, A.A. Kakaev, and V.P. Smolyar. Neutron moderation theory with thermal motion of the moderator nuclei. Eur. Phys. J. A (2017) 53: 179, arXiv: 1612.06838v1 [nucl-th].

[26] I. Prigogine. Introduction to Thermodynamics of Irreversible Processes. John Wiley & Sons, New York, 1968.

[27] I.F. Bakhareva. Nonlinear nonequilibrium thermodynamics. Saratov University Publishing, Saratov, 1976, 140 p. [in Russian]

[28] I.A. Kvasnikov. Thermodynamics and statistical physics. Vol.3. Theory of nonequilibrium systems. Editorial URSS, Moscow, 2003, 448 p. [in Russian]

[29] O.G. Sitenko. Scattering theory. Lybid, Kyiv, 1993, 335 p. [in Russian]
[30] B.G. Levich. *Theoretical physics: an advanced text*. Vol.2, Nauka, Moscow, 1971, 936 p. [in Russian]

[31] B.G. Levich. *Theoretical physics: an advanced text*. Vol.1, Nauka, Moscow, 1969, 910 p. [in Russian]

[32] V.D. Rusov, V.A. Tarasov, S.I. Kosenko, and S.A. Chernegenko. *The resonance absorption probability function for neutron and multiplicative integral*. Problems of Atomic Science and Technology, 2012, Iss.2(78), P. 68-72. [in Russian]. arXiv:1208.1019v1 [nucl-th].

[33] N.D. Fedorov. *A brief reference book for engineer-physicists*. State publishing of literature in the field of nuclear science and technology, Moscow, 1961, 507 p. [in Russian]

[34] Yu.M. Shirokov, and N.P. Yudin. *Nuclear Physics*. Nauka, Moscow, 1972, 671 p. [in Russian]

[35] Chadwick M.B. ENDF/B-VII.1 Nuclear Data for Science and Technology: Cross Sections, Covariances, Fission Product Yields and Decay Data. / M.B. Chadwick, M. Herman, P. Obloinski et al. // Nuclear Data Sheets. - 2011. - Vol. 112, Iss. 12. - P. 2887 - 2996.

[36] Agostinelli S. Geant4 - a simulation toolkit. / S. Agostinelli, et al. // Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment. 2003. Vol. 506. Issue 3. P. 250-303.