A three-body calculation of incoherent $\pi^0$ photoproduction on a deuteron

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Abstract

Incoherent $\pi^0$ photoproduction on a deuteron in the $\Delta(1232)$ region is treated in a three-body scattering approach using separable two-body interactions. Results are presented for total and differential cross sections. It turns out that the role of higher order terms beyond the first order in the multiple scattering series is insignificant, and their inclusion cannot explain the existing discrepancy between theory and experiment.

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I. INTRODUCTION

The role of the final state interaction (FSI) in incoherent $\pi^0$ photoproduction on a deuteron

$$\gamma + d \rightarrow \pi^0 + n + p$$

has been studied by various groups [1–6]. In general, the theoretical treatment of this reaction was based on the multiple scattering picture, in which, however, only the first order terms with respect to the final $NN$ and $\pi N$ interactions are taken into account. According to these studies, the main FSI effect arises from $NN$ rescattering, whereas the contribution from $\pi N$ rescattering is rather small.

It is well known that in the reaction in Eq. (1) the first-order inclusion of the $NN$ interaction has a particularly strong effect compared to the impulse approximation (IA). The reason for this feature is the fact, that in contrast to processes with charged pions, $\gamma d \rightarrow \pi^+nn/\pi^-pp$, for incoherent $\pi^0$ production (1) the impulse approximation contains a spurious contribution of the coherent reaction ($\gamma d \rightarrow \pi^0d$) since the final plane wave is not orthogonal to the deuteron ground state [3, 7, 8]. Indeed, projecting out the ground state from the final plane wave, the so-called modified IA, comprises already the dominant part of the correction of the inclusion of FSI in first order [3]. The remaining FSI effect is of the same order as for charged pion production. Further incorporation of $\pi N$ rescattering gives an additional (however much less significant) decrease. Thus the total first-order FSI effect in the $\Delta$-resonance region is a decrease of the total cross section by about 30 % compared to the one predicted by the pure spectator model (IA).

On the other hand, the calculation including only the first order rescattering terms still overestimates the experimental total cross section [9, 10] by about 15 % at the $\Delta(1232)$ peak. It appears reasonable to assume that the remaining difference could be assigned to the neglect of the higher order terms in the multiple scattering series. At least for two reasons these terms can be important: (i) multiple scattering can cause an additional broadening of the $\Delta$ resonance and, consequently, can lead to a lowering of the $\Delta$ peak in the cross section, and (ii) inclusion of the intermediate $NN$ states in which the pion is absent allows one to take into account the mechanism of two-nucleon absorption of pions, which may also decrease the cross section.

Therefore it is the aim of the present work to study the role of higher orders of the
multiple scattering series in the reaction (1). To this end, we calculate the reaction amplitude using three-body scattering theory. In the next section we briefly outline the formalism. Our approach is based on a separable representation of the driving two-body $\pi N$ and $NN$ interactions. As is well known, in this case the original three-body equations simplify to a set of equations of Lippman-Schwinger type for a system of coupled quasi-two-body channels. To reduce this set into an easily solvable one-dimensional form, we apply an expansion into partial waves. In Sect. III we present our results and compare them with existing experimental data. We also discuss the importance of the multiple scattering corrections. Conclusions are given in the final Section IV.

II. FORMALISM

In the present approach we use for the description of the final $\pi^0 np$ three-body state three coupled two-body channels, each consisting of a quasiparticle formed by two of the three particles and the remaining one as a spectator. Thus each quasiparticle is an interacting two-body system. The three channels are in detail:

1. Channel named “$d$” consists of a deuteron as a quasiparticle with two interacting nucleons and a pion as spectator,

2. Channel named “$\Delta$” consists of an interacting nucleon-pion system forming a $\Delta$ as quasiparticle and a spectator nucleon,

3. Channel named “$N$” consists of a nucleon as a nucleon-pion quasiparticle and a spectator nucleon.

In the following we use $\alpha, \beta, \cdots \in \{d, \Delta, N\}$ to label the channels and the corresponding quasiparticles, while $a, b, \ldots$ are used for the corresponding spectators. In this notation the channel $\alpha$ consists of a spectator $a$ and two interacting particles ($bc$) forming the quasiparticle $\alpha$.

Treating the electromagnetic interaction perturbatively in lowest order one obtains for the reaction $T$-matrix

$$ T = \sum_{\alpha \in \{d, \Delta, N\}} X^\alpha \tau^\alpha g_\alpha , \quad (2) $$
where \(X^\alpha\) denotes a channel amplitude, \(\tau^\alpha\) the channel propagator and \(g_\alpha\) the quasiparticle vertex for \((bc) \rightarrow b + c\). A graphical representation of the \(T\)-matrix in terms of the channel amplitudes is shown in Fig. 1.

\[
\gamma \Xrightarrow{\pi} T^dN = X^d \circ \tau_d \circ g_d + X^\Delta \circ \tau_\Delta \circ g_\Delta + X^N \circ \tau_N \circ g_N
\]

FIG. 1: Diagrammatic representation of the \(T\)-matrix in Eq. (2).

The amplitudes \(X^\alpha\) obey a set of coupled equations, which can be derived from the Faddeev three-body formalism under the assumption of separable two-body interactions. In operator form they read

\[
X^\alpha = Z^{\gamma d,\alpha} + \sum_{\beta \in \{d, \Delta, N\}} X^\beta \tau^\beta Z^{\beta,\alpha}, \quad \alpha \in \{d, \Delta, N\}.
\]  (3)

These equations are shown in a graphical representation in Fig. 2.

\[
\gamma \Xrightarrow{\pi} X^d = 2 \left( X^\Delta \circ Z^{\Delta d+} + X^N \circ Z^{Nd} \right)
\]

\[
\gamma \Xrightarrow{\Delta} X^\Delta = Z^{\gamma d,\Delta} + X^d \circ Z^{d\Delta} + X^\Delta \circ Z^{\Delta d+} + X^N \ circ Z^{\Delta N} + X^N \ circ Z^{NN}
\]

FIG. 2: Diagrammatic representation of the system of three-body equations of Eq. (3). The factor 2 in the first equation arises from the symmetrization of the two nucleons.

The driving terms \(Z^{\alpha,\beta}\) describe the exchange of a particle \(c\) between the quasiparticles \(\alpha\) and \(\beta\). The terms \(Z^{\gamma d,\alpha}\), forming the inhomogeneous part of the set (3), contain the electromagnetic vertices \(\gamma N \rightarrow \Delta (Z^{\gamma d,\Delta})\) and \(\gamma N \rightarrow N (Z^{\gamma d, N})\). Obviously one has \(Z^{\gamma d, d} = 0\).

In momentum space the potentials \(Z^{\alpha,\beta}\) have the following form

\[
Z^{\alpha,\beta}(\vec{p}_a, \vec{p}_b; W) = \frac{g_\alpha(q_a)g_\beta(q_b)}{W - E_a(p_a) - E_b(p_b) - E_c(|\vec{p}_a + \vec{p}_b|) + i\epsilon}.
\]  (4)
Here $W$ denotes the total energy in the center-of-mass (c.m.) frame, $\vec{p}_a$ and $\vec{p}_b$ the c.m.-momenta of the spectator particles of the channels $\alpha$ and $\beta$, respectively. The relative momenta $\vec{q}_{\alpha/\beta}$ between the spectators of the channels $\beta/\alpha$, respectively, and the exchanged particle in the arguments of the vertices $g_{\alpha/\beta}(q_{\alpha/\beta})$ are treated nonrelativistically, e.g.

$$\vec{q}_\alpha = \vec{p}_b + \frac{M_b}{M_b + M_c} \vec{p}_a,$$

with $M_c$ denoting the mass of the exchanged particle, whereas for the particle energies we use the relativistic relation $E_a(p) = \sqrt{p^2 + M_a^2}$.

To reduce Eq. (3) to a numerically manageable form we exploit a partial wave expansion of the amplitudes $X^\alpha$ in terms of the total angular momentum $J$ and the isospin $T$. We use the $LS$ coupling scheme by coupling the total angular momentum $\vec{j}_\alpha$ of the quasiparticle with the spin $\vec{s}_a$ of the third particle to the total channel spin $\vec{S}_\alpha$. The orbital momentum $\vec{L}_\alpha$ is then coupled to $\vec{S}_\alpha$ to the total angular momentum $\vec{J}$. For the given values of photon polarization $\vec{\epsilon}_\lambda$, initial deuteron spin projection $M_d$, and the total spin $S_\alpha$ with projection $M_{S_\alpha}$ of the final quasi-two-body state $\alpha$ the partial wave expansion of the channel amplitudes $X^\alpha_{\lambda M_d S_\alpha M_{S_\alpha}}(\vec{k}, \vec{p}; W)$ reads

$$X^\alpha_{\lambda M_d S_\alpha M_{S_\alpha}}(\vec{k}, \vec{p}; W) = N_d \sum_{J^\pi, L, S} \sum_{L, S, M} X^\alpha_{J^\pi L S L_\alpha S_\alpha}(\vec{k}, \vec{p}; W) \sqrt{\frac{2L + 1}{4\pi}} Y^*_{L J S L_\alpha S_\alpha}(\hat{\vec{p}}) \times (1\lambda M_d | S M_S)(L_0 S M_S | J M_S)(L_\alpha M_{L_\alpha S_\alpha} M_{S_\alpha} | J M_S),$$

where the factor $N_d$ takes into account the deuteron normalization. In Eq. (6) the $z$ axis is chosen along the initial photon momentum $\vec{k}$.

With the help of this partial wave decomposition one obtains from Eq. (3) a set of one-dimensional coupled equations for each value of $J$, parity $\pi$ and isospin $T$ in the following form (for simplicity we drop the energy $W$ in the arguments)

$$X^\alpha_{J^\pi T L S L_\alpha S_\alpha}(\vec{k}, p) = Z^\gamma_{J^\pi T L S L_\alpha S_\alpha}(\vec{k}, p)$$

$$+ \sum_{\beta \in \{d, \Delta, N\}} \sum_{L', S'} \int \frac{p'^2 dp'}{(2\pi)^2} X^\beta_{J^\pi T L' S'}(\vec{k}, p') \tau^\beta(w_\beta(p')) Z_{L' S' L_\alpha S_\alpha}^\gamma(\vec{p}', p'),$$

where $\alpha \in \{d, \Delta, N\}$ and $k$ denotes the momentum of the incident photon. The argument $w_\beta$ of the propagator $\tau^\beta$ is the quasiparticle energy calculated on the assumption that the corresponding spectator $b$ is on-shell:

$$w_\beta^2(p') = W^2 - 2W E_b(p') - M_b^2,$$
with $E_h(p') = \sqrt{p'^2 + M_h^2}$. The spin $S$ of the initial $\gamma d$ state in Eq. (7) is a vector sum of the deuteron spin $\vec{s}_d$ and the photon circular polarization vector $\vec{e}_\gamma^\lambda$, with components $(\vec{e}_\gamma^\lambda)_\mu = -\delta_{\mu\lambda}$. The partial wave components of the driving terms $Z_{L'S',L\alpha}^{\beta,\alpha}(J^{ST})$ can be obtained using the formalism developed, e.g., in Ref. [11].

In the present calculation we included states with total angular momentum up to $J_{\text{max}} = 7$ of both parities (the maximum orbital momentum $L_{\text{max}} = 9$). Furthermore, since the dominating $\Delta$-resonance term only enters states with total isospin $T = 1$, we neglect contributions of the $T = 0$ part. Therefore, we omit in the subsequent equations the isospin notation. As a result, for each total spin and parity $J^{\pi}$ we have at most eight coupled one-dimensional integral equations, two equations for each of the channels $\alpha = d, N$ and four for $\alpha = \Delta$.

Our basic ingredient is a separable representation of the scattering amplitudes in the $\pi N$ and $NN$ two-body subsystems. For $\pi N \to \Delta \to \pi N$ we take

$$t_\Delta(\vec{q}, \vec{q}'; z) = -\frac{g_\Delta(\vec{q}) g_\Delta^\dagger(\vec{q}')}{z - M_0 - \Sigma_{\pi N}(z)},$$

where $\Sigma_{\pi N}$ is the $\Delta \to \pi N$ self-energy and $M_0$ a parameter. The vertex functions are taken in the standard form (the isospin part is omitted) with a monopole form factor

$$g_\Delta(\vec{q}) = \frac{f_{\pi N \Delta}(\vec{s}_{\Delta N} \cdot \vec{q})}{m_\pi} \frac{\beta_\Delta^2}{\beta_\Delta^2 + q^2},$$

where $m_\pi$ denotes the pion mass and $\vec{s}_{\Delta N}$ the spin transition operator. The parameters $M_0$, $f_{\pi N \Delta}$ and $\beta_\Delta$ were adjusted to the $\pi N$ phase shifts in the $P_{33}$ channel. The resulting fit, presented in panel (a) of Fig. 3, gives $M_0 = 1306$ MeV, $f_{\pi N \Delta}^2/4\pi = 0.8113$, and $\beta_\Delta = 295$ MeV.

Similarly, for $\pi N \to N \to \pi N$ we use

$$t_N(\vec{q}, \vec{q}'; z) = \frac{g_N(\vec{q}) g_N^\dagger(\vec{q}')}{{z - M_N}},$$

where $M_N$ stays for the nucleon mass, and the vertex function reads

$$g_N(\vec{q}) = \frac{f_{\pi NN}(\vec{s}_{NN} \cdot \vec{q})}{m_\pi} \frac{\beta_N^2}{\beta_N^2 + q^2},$$

with $f_{\pi NN}^2/4\pi = 0.0735$ and $\beta_N = 800$ MeV.

For the transition $\gamma N \to \Delta$ only the dominant $M1$ part is taken into account. The corresponding vertex function was parametrized in the form

$$g_\Delta^\lambda(z, \vec{k}) = e \frac{G_{M1}(z)}{2M_N} (\vec{s}_{\Delta N} \cdot (\vec{k} \times \vec{e}_\lambda)), $$

where $G_{M1}(z)$ is a function of the scaling variable $z = (M_0 - \Sigma_{\pi N}(z))/M_0$.
FIG. 3: Panel (a): Present fit to the $P_{33}$ $\pi N$ phase shifts using Eqs. (9)-(13). The data are taken from the compilation in Ref. [13]. Panel (b): The $M_{1+}^{(+)}$ multipole for $\gamma N \rightarrow \pi N$. Solid and dashed curves: our fit for real and imaginary parts, respectively. The full circles and squares show the energy independent multipole analysis from Refs. [14, 15].

with the magnetic transition moment

$$G^{M_1}(z) = \mu_\Delta(z) e^{i\Phi_\Delta(z)}, \quad (14)$$

and $e$ for the elementary charge.

The off shell-behavior of the vertex [13] is determined by the analytic continuation of the function $G^{M_1}(z)$ of Eq. (14) into the complex plane of $z$. Below the single-nucleon threshold we use

$$G^{M_1}(z) = G^{M_1}(m_\pi + M_N), \quad (15)$$

for $\Re z < m_\pi + M_N$. The approximation (15) obviously violates analyticity of the amplitude. However, as the direct calculation shows, the subthreshold region provides only a small fraction of the resulting cross section, at least in the energy region not very close to the threshold, so that this shortcoming of our model does not visibly affect the results.

Following Ref. [16] we fit the energy dependence of $\mu_\Delta(z)$ and $\Phi_\Delta(z)$ in such a way that the resulting $\gamma N \rightarrow \Delta \rightarrow \pi N$ amplitude

$$t_\Delta^\gamma (\vec{k}, \vec{q}; z) = g_\Delta^\gamma (z, \vec{k}) \tau(z) g_\Delta^\dagger (\vec{q}) \quad (16)$$
reproduces the isovector magnetic amplitude $M_{1+}^{(+)}$ in the energy region from threshold up to 450 MeV (panel (b) in Fig. 3). Thus, we do not treat the background terms (the crossed nucleon pole and the $\omega$-exchange) exactly, but their contribution is effectively included via adjustment of the ansatz in Eqs. (13)-(16) to the data of $M_{1+}^{(+)}$. The magnitude $\mu_\Delta(z)$ and the phase $\Phi_\Delta(z)$ in Eq. (14) are parametrized as

$$
\mu_\Delta(z) = \sum_{n=0}^{4} C_n \left( \frac{z}{M_\Delta} \right)^n, \quad \Phi_\Delta(z) = \sum_{n=0}^{4} D_n \left( \frac{z}{M_\Delta} \right)^n,
$$

with $M_\Delta = 1232$ MeV. The constants $C_n$ and $D_n$ resulting from the fit in Fig. 3 are collected in Table I.

For the $\gamma N \rightarrow N$ vertex we also use only the dominant magnetic part:

$$
g^{(v)}_N(\vec{k}) = e \frac{\mu_\Delta^{(v)}}{2M_N} (\vec{\sigma} \cdot (\vec{k} \times \vec{\epsilon})) ,
$$

where $\mu_\Delta^{(v)} = 2.353$ is the isovector part of the nucleon magnetic moment.

In the $NN$ sector only the $s$-wave states $^3S_1$ and $^1S_0$ are taken into account, neglecting the contribution of the tensor component $^3D_1$. For the $s$-wave interactions we use the rank-one separable parametrization of the Paris potential from Ref. [17]:

$$
v_d^{(s)}(\vec{q}, \vec{q}') = -g_d^{(s)}(q) g_d^{(s)}(q')
$$

with

$$
g_d^{(s)}(q) = \sqrt{2} \pi \sum_{n=1}^{6} \frac{C_n^{(s)}}{q^2 + \beta_n^{(s)}},
$$

where the index $s$ refers to the singlet or triplet state. The parameters $C_n^{(s)}$ and $\beta_n^{(s)}$ are listed in [17].

The coupled integral equations in Eq. (7) were solved using the matrix inversion method. To overcome the problem of singularities we used the well known procedure in which the integration contour is shifted from the real axes to the fourth quadrant of the complex $p'$
plane. This technique is quite well known (see, e.g., Ref. [18]), and there is no need to describe it here in detail. Some formal aspects related to the relativistic kinematics were considered in Ref. [19].

Here we would like to comment only on the details concerning the treatment of the singularities in the case when the singularities of the driving terms \( Z_{L'S',L\alpha S\alpha}^{\beta,\alpha} \) have the configuration shown in Fig. 4. It is known, that in order to find the \( X^\alpha \)-matrix at real momenta, one has to perform a continuation of the driving terms \( Z_{L'S',L\alpha S\alpha}^{\beta,\alpha} \) in Eq. (7) onto the second Riemann sheet (the part \( BCD \) of the integration contour in Fig. 4). However, in this case near \( p' = 0 \) the driving term \( Z_{L'S',L\alpha S\alpha}^{\beta,\alpha} \) behaves like \( (1/p')^{L+1} \). As a result, the values of the integrand in Eq. (7) strongly diverge near the origin \( C \), and, at the same time, their contributions from near this point coming from the intervals \( BC \) and \( CD \) essentially cancel. This leads to a significant loss of numerical accuracy via a small difference of two large numbers. This problem was also mentioned in Ref. [20], where the break-up reaction \( nd \to npn \) was studied. In Ref. [20] the three-body equations were solved only for low values of \( L \) (\( L \leq 3 \)), whereas for higher \( L \) only the first order approximation or the inhomogeneous term was considered.

FIG. 4: Integration contours and configuration of the cuts in the complex \( p' \) plane. The dashed line shows the part of the contour on the second sheet.

In the present case, we use another integration contour (the polygonal curve \( AFG \) in Fig. 4). The driving terms are always calculated on the first Riemann sheet, and the integration does not pose any numerical problem. A certain disadvantage of this method lies in
the fact that the position of the momentum $p_0$, where the contour is squeezed between the logarithmic cuts (see Fig. 4), depends on the value of the on-shell momentum $p$ in Eq. (7). For this reason one has to solve the set of Eq. (7) separately for each value of $p$ of the chosen mesh.

After inversion of the system in Eq. (7) and the determination of the partial wave amplitudes $X_{LS,LaS_a}^{\alpha(J^\pi,T)}(k,p;W)$ one obtains the corresponding channel amplitudes $X_{\lambda M_d \lambda M_s}^{\alpha}(\vec{k},\vec{p};W)$ from Eq. (6), from which, finally, the reaction amplitude $T_{\lambda M_d m_1m_2}$ (see Eq. (2)) as function of the momenta of the final particles $\vec{q}_\pi$, $\vec{p}_1$, and $\vec{p}_2$ follows

$$T_{\lambda M_d m_1m_2}(k,\vec{p}_1,\vec{p}_2,\vec{q}_\pi) = \frac{1}{\sqrt{2}} X_{\lambda M_d \lambda M_s}^{d}(k,\vec{q}_\pi;W) \left( \frac{1}{2} m_1 \frac{1}{2} m_2 \right) T_d(w_d(q_\pi)) g_d(q_{NN})$$

$$+ \left[ \sum_{\alpha \in \{\Delta,N\}} \sum_{j_\alpha m_\alpha} \sum_{S_\alpha} \sqrt{\frac{2}{3}} X_{\lambda M_d \lambda M_s}^{\alpha}(k,\vec{p}_1;W) \left( \frac{1}{2} m_1 1 m_\alpha - m_1 \right) j_\alpha m_\alpha \right]$$

$$\times \left( \frac{1}{2} m_2 j_\alpha m_\alpha \right) S_\alpha M_s$$

$$\tau^\alpha(w_\alpha(p_1)) g_\alpha(q_{NN}) - (1 \leftrightarrow 2).$$  \hspace{1cm} (21)

Here $q_{\pi N_i}, i = 1, 2,$ denotes the relative momentum in the subsystem $\pi N_i$, and $q_{NN}$ the relative momentum of the two final nucleons. As is mentioned above, in the present calculation we took into account only configurations with total isospin $T = 1$, since those with $T = 0$ do not contain the dominant $N\Delta$-configuration. Therefore, in the first term on the rhs of Eq. (21) only the two-nucleon states with total spin $S_d = 1$ contribute.

Using the amplitude of Eq. (21) the fully exclusive differential cross section for the present reaction in the overall center-of-mass frame is given in terms of the $T$-matrix (Eq. (21))

$$\frac{d\sigma}{dq_\pi d\Omega_\pi d\Omega_{NN}^*} = \frac{1}{(2\pi)^5} \frac{E_d M_N^2}{W} \frac{q_{\pi N}^2 p_{NN}^*}{4\omega_\gamma \omega_\pi \omega_{NN}} \frac{1}{6} \sum_{\lambda,M_d,m_1,m_2} |T_{\lambda M_d m_1m_2}(k,\vec{p}_1,\vec{p}_2,\vec{q}_\pi)|^2,$$  \hspace{1cm} (22)

where $E_d$, $\omega_\gamma$, and $\omega_\pi$ denote the total energies of the corresponding particles, and $\omega_{NN}$ the invariant $NN$ energy. The nucleon momentum in the $np$ center-of-mass system is denoted by $p_{NN}^*$ and its spherical angle by $\Omega_{NN}^*$. 

III. RESULTS AND DISCUSSION

We start the discussion by considering the role of different partial waves in the total cross section as is shown in Fig. 5 for the IA (panel (a)) and the full three-body calculation (panel (b)). Similar to the coherent photoproduction $\gamma d \rightarrow \pi^0 d$ [21] the largest contribution in the incoherent reaction comes from the $2^+$ wave, which predominantly is an $M1$ transition,
leading to the production of pions with angular momentum $l_\pi = 1$ with respect to the $np$ system. This partial wave alone contributes almost 45% to the total IA-cross section in the $\Delta$ region. The next important partial waves are $2^-$ and $3^-$ generating basically pions with $l_\pi = 2$. The other partial waves give much smaller contributions to the total cross section.

Inclusion of FSI (see panel (b) of Fig. 5) leads to a visible decrease of $\sigma(2^+)$ by a factor 2-3, whereas $\sigma(3^+)$ (predominantly $M2$) is considerably enhanced and becomes comparable to $\sigma(2^+)$. Next in importance is $\sigma(2^-)$, which is also increased by FSI by about 10%. The contributions of the higher partial waves is still quite insignificant.

As already mentioned, the corresponding partial wave series was cut-off at $J_{\text{max}} = 7$ and $L_{\text{max}} = 9$. In order to demonstrate the good convergence for this value of $J_{\text{max}}$ we show in Fig. 6 the semi-exclusive differential cross section with respect to the final pion as function of a few lower $J_{\text{max}}$-values. Similar to the results for the elastic pion-deuteron scattering of Ref. [22], this approximation provides a satisfactory convergence in the $\Delta$-resonance region as one can see: changing $J_{\text{max}}$ from 5 to 7 has already quite a small effect, so that the choice $J_{\text{max}} = 7$ appears to be acceptable.

In Fig. 7 we show in panel (a) the total cross section as well as in panel (b) the differential cross section at $E_\gamma = 330$ MeV, calculated in the present three-body model. These results...
can be compared with our previous calculation in Ref. [3]. In the latter case the single nucleon amplitude $t(\gamma N \to \pi N)$ was taken from the MAID analysis [24] and the inclusion of FSI was reduced to the first order contributions (i.e., to $np$ and $\pi N$ rescatterings in the final state). The present result, which is obtained in a somewhat oversimplified model for $\gamma N \to \pi N$ (pure resonance ansatz for the $M^{(+)}_{1+}$ multipole, and neglect of the tensor component of the deuteron wave function) agrees quite well with those of Ref. [3].

As the present calculation shows, the role of the $NN$ configurations turns out to be vanishingly small, less than 0.5% in the whole energy region considered. Even at very backward angles of the produced pions, where the short internucleon distances start to play a role their contribution can safely be neglected. It is also worth noting that electromagnetic transition to the $NN$ state represented by the potential $Z^{\gamma d,N}$ (the second inhomogeneous term in Fig. 2) contributes only 1-1.5% to the total cross section and can be disregarded as well. Fig. 7 clearly shows that the multiple scattering corrections do not visibly change the reaction dynamics. Inclusion of only the first order corrections, i.e. $np$ and $\pi N$ rescatterings in the final state, turns out to be sufficient.

In the same Fig. 7 we compare our results with experimental data from Ref. [9]. Whereas
FIG. 7: Total (panel (a)) and differential (panel (b)) cross sections of the reaction $\gamma d \rightarrow \pi^0 np$.

Dotted curves: impulse approximation (IA) (the inhomogeneous $Z^{\gamma d,\alpha}$ terms in Fig. 2). Dash-dot and double-dot curves include first order $np$ and in addition $\pi^0 N$ rescattering contributions. Solid curves: full three-body calculation.

in the region below the $\Delta(1232)$ peak the agreement is satisfactory, in the region near the maximum ($E_\gamma \geq 300$ MeV) the theory is too high. This discrepancy was already discussed in Refs. [2–4]. In particular it was conjectured in [3] that the difference may come from the neglect of the $\Delta N$ interaction which might lead to a broadening of the $\Delta$ resonance due to additional inelasticity. The present calculation, which effectively takes into account the $\Delta N$ interaction, shows, that this effect is negligible in the incoherent reaction and cannot explain the discrepancy.

IV. CONCLUSION

In this paper we have presented a calculation of total and differential cross sections for the incoherent reaction $\gamma d \rightarrow \pi^0 np$ in the energy region from threshold up to the $\Delta$ resonance. The calculation is based on a three-body model for the inclusion of the final $\pi NN$ interaction. Although we use some simplifications (non-relativistic three-body equations, neglect of the deuteron $d$-wave), the most significant features of the process are preserved, including the
importance of the $M1$ multipole transition $\gamma N \rightarrow \Delta$ and the dominance of the $\Delta$ resonance in $\pi N$ scattering.

The results show that the corrections due to multiple scattering are quite insignificant in the major part of the kinematical region. As already mentioned in the introduction, the major importance of FSI in the $\pi^0 np$ channel is related to the orthogonality of the initial and the final $np$ wave functions. This effect is taken into account essentially already by the first order FSI contributions. This can be seen from the fact that inclusion of the $np$ rescattering (dashed curve on the right panel of Fig. 7) leads to a significant decrease of the differential cross section at very forward pion angles. This agrees with our expectation that the reduction due to orthogonality should be maximal in the region where the momentum transferred to the nucleon system is small.

According to our results, the full three-body calculation changes the cross section compared to the first-order rescatterings only by about $1 - 2\%$ in the $\Delta$ resonance region. Since the multiple scattering corrections are insignificant their inclusion cannot explain the existing deviation between the theoretical and experimental results. The theory still visibly overestimates the data, as is shown in panel (a) of Fig. 7. The problem concerning the difficulties in describing the photoproduction of $\pi^0$ on a deuteron in the first resonance region was also addressed in Ref. [10]. In this work the authors had analysed the inclusive cross section $\gamma d \rightarrow \pi^0 X$ with $X$ being either a deuteron or a neutron-proton scattering state.

As is shown in Ref. [23], using the closure approximation for the final two-nucleon state, the sum of both cross sections should be equal to the sum of the free-nucleon cross sections, folded with the nucleon momentum distribution in a deuteron. The latter is approximately equal to the cross section $\sigma_{IA}$ of $\sigma(\gamma d \rightarrow \pi^0 np)$ calculated in the spectator model. However, as the calculation in Ref. [10] shows, $\sigma_{IA}$ overestimates by about $15\%$ the experimental total cross section for $\gamma d \rightarrow \pi^0 X$. Since the free proton cross section is well known the natural conclusion would be that the real free neutron cross section is overestimated by the existing multipole analyses (in Ref. [10] the MAID [24] and SAID [25] analyses are considered). In order to bring the theory into agreement with the data of [9, 10] the theoretical neutron cross section has to be decreased by about $25\%$. Such a strong isospin dependence of the elementary amplitude can hardly be explained within the existing models for $\gamma N \rightarrow \pi N$. According to these models the reaction is strongly dominated by the $\Delta(1232)$ resonance so that the proton and neutron cross sections are nearly equal. Thus the question about the
source of the discrepancy between theoretical predictions and data in the $\Delta(1232)$ region remains open.

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