Simple Progression Law in Predicting the Damage Onset and Propagation in Composite Notched Laminates

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Abstract

The aim of this article is to simulate the damage initiation and progression in unidirectional (UD) laminates. A three-dimensional (3D) failure criteria of Puck incorporated with degradation scheme is developed. Two types of degradation law known as sudden degradation and progressive degradation are used to predict the damage progression in UD laminates. The establishment of constitutive law in progressive damage model (PDM) is achieved through implementation of user subroutines in Abaqus. The failure analysis is applied to various composite stacking sequences and geometries, as well as different fiber reinforced polymer (FRP) composite materials. The comparative studies revealed that the predicted ultimate failure load agree well with those available in the literature.

Keywords: Progressive damage model, composite laminate, open-hole tension.

1. Introduction

Damage initiation and evolution in composite structures is very crucial to be analyzed which can occur in several failure mechanisms such as fibre breakage, fibre buckling, matrix cracking, material debonding and also delamination. The failure can occur either individually or several failure modes are combined. Due to increasingly computational capability to handle such complicated behaviors, many attempts are made to predict the failure of composite laminates. Although research concerning failure theories and prediction of strength of composite laminates on fibre-reinforced polymer composite has existed for many decades still, no such a robust and reliable method can precisely predict the failure characteristics of the laminates [1]. In general, not all cases fit the experimental results.

Failure theories are initially developed for unidirectional composite materials and tailored based on the static regime. Two categories of methods are distinguished: non-interactive and interactive criteria. Non-interactive criteria assume all failure modes are not connected to each other. Failure will occur if stresses/strains in the principal material orientation exceed the respective strengths/strains. Maximum stress/strain criteria imply that technique to identify the maximum failure load. Apart from that, interactive criteria assume interaction between failure mechanisms which focusing on failure surface or envelope. Among the famous interaction criteria that are still used until now are Tsai-Arzi [2], Tsai-Wu [3] and Hoffmann [4]. Based on the above criterion, the laminates are treated as orthotropic materials [5]. Only a single equation is required to relate the interaction between different stress components in the material frame. Hence, still many scientific works adopt it due to their simple forms and acceptability of the accuracy of the strength prediction. A major drawback of those theories is that the approach used not reflects the level of complexity in composite structures. Therefore, the classical failure theories fail to distinguish the mode of failure in composite laminates. Failure mode based theories are needed to overcome that situation and also will make the analysis more meaningful. Hashin and Rotem [6] proposed a two-dimensional (2D) interaction failure initiation theory based on fibre and matrix failure modes, later extended to 3D forms by Hashin [7] itself. Yamada and Sun [8] proposed a prediction method of composite laminate using in-situ shear strength evaluated in the form of cross-ply laminate that suited for the fibre controlled laminates. Motivated by initial work of Yamada and Sun, Chang and Chang [9] developed an extension model by introducing shear non-linearity failure mode. Puck and Schürmann [10] improved the initiation failure theory of Hashin by introducing an angle of fracture plane. Determination of angle of fracture on action plane is governed by shear stresses acting on it. In the case of plane stress, three different modes are distinguished for inter-fiber failure (IFF) modes. Further development of Puck’s theories is carried out by Davila et al. [11] which included evaluation of fracture planes for matrix cracking and utilization of fracture mechanics approach to determine the in-situ strength. This criterion is meant for 2D failure analysis and later extended to 3D problems including shear non-linearity developed by Pinho et al. [12].

Another approach to predict the initial failure of composite structures is to use the continuum damage mechanics (CDM) method at ply level. This method is originally developed by Kachanov [13] and is widely used in analyzing the failure of composite laminates. This approach has great ability to predict fibre fracture, matrix cracking and delamination failure mechanism. Ladeveze and Dantec [14] developed an in-plane failure theory for elementary ply using damage mechanics approach. This theory is designed to predict fiber/matrix debonding and matrix micro-cracking in UD composite. Vaziri et al. [15] implemented CDM failure theory on CFRP composite laminates under plane stress condition. They adopted constitutive equations proposed by Matzenmiller et al.
At early development of this theory, maximum stress/strain criteria are used to identify the fibre failure (FF). Later, Puck realized that those criteria are not really physically correct [21], and new analytical equations for unidirectional (UD) composite laminate are written in the following equations:

\[
FF = \left\{ \begin{array}{ll}
\frac{1}{X_1} \left[ \sigma_1 - \left( v_{12} - v_{12f} \right) m_y E_{11} \right] \left( \sigma_2 + \sigma_3 \right) & \text{for } \ldots 
\leq 0 \\
\frac{1}{X_2} \left[ \sigma_2 - \left( v_{12} - v_{12f} \right) m_y E_{11} \right] \left( \sigma_1 + \sigma_3 \right) & \text{for } \ldots < 0 
\end{array} \right.
\]

(1)

Where \(X_1\) and \(X_2\) are the tensile and compressive strengths of a UD layer in the longitudinal direction; \(v_{12}\) and \(v_{12f}\) are the major Poisson’s ratio for UD lamina and fibre, respectively. The mean stress magnification factor, \(m_y\) is assumed to be 1.3 for glass fibre and 1.1 for carbon fibre [23].

For the inter-fibre failure, the concept of fracture plane is introduced since the stresses acting on this plane is assumed to create the fracture. The fracture plane is predicted with respect to the material plane between the angle \(\theta\) of \(-90^\circ\) and \(+90^\circ\) (180° due to symmetry plane). General tensor transformations are used in order to obtain the normal and shear stresses acting on the action-plane.

\[
\sigma_n(\theta) = \sigma_z \cos^2(\theta) + \sigma_y \sin^2(\theta) + 2\tau_{yz} \sin \theta \cos \theta \\
\tau_{nn}(\theta) = (\sigma_1 - \sigma_2) \sin \theta \cos \theta + \tau_{12}(\cos^2 \theta - \sin^2 \theta) \\
\tau_{nt}(\theta) = \tau_{12} \sin \theta + \tau_{12} \cos \theta 
\]

In this analysis, \(\sigma_n(\theta)\) is the stress normal to fracture plane, \(\tau_{nn}(\theta)\) and \(\tau_{nt}(\theta)\) are the shear stresses parallel and perpendicular to fibre direction in fracture plane, whereby \(\theta\) is the inclination angle (arbitrary) on fracture plane (see Figure 1). The equation for tensile and compressive inter fibre failure (IFF) are written as:

\[
IFF(\theta) = \left\{ \begin{array}{ll}
\left( \frac{1}{R_{11}^+} \frac{F^i}{R_{11}^{m+}} \sigma_1(\theta) \right)^2 + \left( \frac{\tau_{nn}(\theta)}{R_{11}^+} \right)^2 & \text{for } \sigma_n \geq 0 \\
\left( \frac{\tau_{nn}(\theta)}{R_{11}^+} \right)^2 + \left( \frac{\tau_{nt}(\theta)}{R_{11}^{m+}} \sigma_2(\theta) \right)^2 & \text{for } \sigma_n < 0 
\end{array} \right. \]

(3)

The parameter \(\psi\) denotes the shear angle in action plane, \(R_{11}^+\) is failure resistance normal to fibers direction, and \(R_{11}^{m+}, R_{11}^+\) and \(R_{11}^{m+}\) are the fracture resistances of the action plane due to the shear stressing. The additional information on Puck’s parameters used here is shown in Table 1. Other related equation can be referred in publications [22, 20, 23].

| Type            | Inclination parameter (°) |
|-----------------|--------------------------|
| Glass fibre     | \(P'_1\) \(P'_2\) \(P'_3\) \(P'_4\) |
| Carbon fibre    | \(0.3\) \(0.25\) \(0.25\) \(0.25\) |

Table 1: Recommended Puck’s parameters [23]
2.2. Damage Evolution

The use of failure criteria independently is not adequate to represent total failure behavior of laminated composite structures. The suitable degradation technique is required to degrade properly the material properties or stiffness matrix itself. The most famous and easy way is to apply ply-discount method whenever damage is identified. This approach is implemented using subroutines and realized using FORTRAN language.

2.2.1. Reducing Stiffness Matrix

To describe the elastic-brittle behavior of fibre-reinforced composites, a constitutive model suited for composite material is used, and a successive attempt is made by Lee et al. [5], and later this approach is called method 1. A 3D-damaged stiffness matrix is written as:

\[
C(d) = \begin{bmatrix}
    k_{1}C_{1} & k_{2}C_{12} & k_{3}C_{13} & 0 & 0 & 0 \\
    k_{2}C_{12} & k_{1}C_{2} & k_{2}C_{23} & 0 & 0 & 0 \\
    k_{3}C_{13} & k_{2}C_{23} & k_{3}C_{3} & 0 & 0 & 0 \\
    0 & 0 & 0 & k_{k}G_{12} & 0 & 0 \\
    0 & 0 & 0 & 0 & k_{k}G_{13} & 0 \\
    0 & 0 & 0 & 0 & 0 & k_{k}G_{23}
\end{bmatrix}
\]  

(4)

Where \( C_{k} \) is undamaged stiffness component, and \( G_{12}, G_{13} \) and \( G_{23} \) are the in-plane and out-of-plane shear modulus of composite material. The multiplication factors \( k_{1}, k_{2} \) and \( k_{3} \) are defined as following:

\[
k_{1} = 1 - d_{f} \\
k_{2} = (1 - d_{f})(1 - d_{c}) \\
k_{3} = (1 - S_{e}d_{w})(1 - S_{e}d_{w})
\]  

(5)

Where \( d_{f} \) and \( d_{c} \) are the global damage variables corresponding to fibre and inter-fibre failure, respectively. Individual damage variables based on failure mode are represented by \( d_{fb}, d_{cc}, d_{cm} \) and \( d_{cm} \) for fibre failure in tension and compression and inter-fibre failure in tension and compression, respectively. The relationship between global and local variables is defined as \( d_{f} = 1 - (1 - d_{fb})(1 - d_{fb}) \) and \( d_{c} = 1 - (1 - d_{cc})(1 - d_{cc}) \). The control parameters, \( S_{e} \) and \( S_{w} \) are 0.9 and 0.5, respectively as recommended in Abaqus manual.

Local damage variables are evaluated by using constant factor as shown in following equation:

\[
d_{i} \begin{cases} 
1 & \text{for } f_{i} > 1 \\ 
0 & \text{else} 
\end{cases}
\]  

(6)

The subscript \( i \) represents \( ft, fc, nt, \) and \( mc \). The constant factor successfully implemented by several researchers [5, 24, 25] beause of its simplicity of working principle. This approach also known as sudden degradation. The approach is implemented by User-material (UMAT) subroutine. Finally, at the damaged material points, the constitutive model expressed in terms of stress-strain relation can be updated as:

\[
\sigma = C(d) \varepsilon
\]  

(7)

2.2.2. Reducing Elastic Constants

The other approach for degrading the stiffness of material is to reduce the elastic constants accordingly based on failure modes. The progressive model using 3D Puck’s formulation is written using user-defined field (USDFLD) subroutine, and is achieved by reducing the elastic properties. This concept is stated as method 2 in further sections. Reduced elastic constants as mentioned below:

\[
E_{1} = 0.001E_{11}^{0}, G_{12} = 0.001G_{12}^{0}, G_{13} = 0.001G_{13}^{0}, \nu_{12} = \nu_{13} = 0
\]  

(8)

And for matrix failure in both loading directions,

\[
E_{2} = 0.001E_{22}^{0}, E_{33} = 0.001E_{33}^{0}, G_{23} = 0.001G_{23}^{0}, \nu_{23} = 0
\]  

(9)

If more than one failure modes occur, all elastic constants are reduced to 1% of its original magnitude, except for Poisson’s ratio which are truncated to zero value. The similar pattern of reduction [26] is successfully carried out using different reduction factor. The stresses from previous increment are called into USDFLD subroutine at the beginning of new increment and used to evaluate Puck’s criteria. Once criteria is satisfied, the elastic properties are degraded by multiplying the degradation factor with original stiffness values.

3. Numerical Analyses

To demonstrate the effectiveness of the proposed damage model, numerical analyses are carried out on open-hole coupons of laminated composite subjected to in-plane tensile loading.

3.1. Finite Element Model

Finite element (FE) model developed as shown in Figure 3 are performed using Abaqus 6-13, while Figure 2 is used as general dimension for all cases performed in this publication. Even though some of stacking sequences are symmetry, full scale of laminated plies are modelled in order to increase the accuracy of results. The models are simply supported on one side and loaded in longitudinal direction by means of prescribed displacement.

Each plies are modelled using reduced integration brick element C3D8R. The plies are stacked using equivalent single layer (ESL) technique provided by Abaqus via lay-up editor. Thus, only one element through thickness is adequate for the simulation of laminated composite plate. To ease the process of extracting load-displacement data, a reference point (RF) is located at the end of the composite plate, which the prescribed displacement is applied. Nodes on free face (pulled face) are tied to the RF using equation constrains so that equal displacement of master node (RF) and slave nodes (pulled face) can be achieved. The global element size is 1 mm, whereas smaller element size is modelled at the vicinity of the hole.

3.2. User-Subroutines

The requirement of using more sophisticated failure theory is crucially needed to perform failure analysis of composite laminates.
Furthermore, the increase of computational capability has encouraged many researchers to implement complicated theory in order to make better prediction. Thus, in this research, a 3D Puck’s failure criteria coupled with damage evolution formulation is implemented and realised by the help of user-defined field (USDFLD) and user-material (UMAT) subroutines. The routines are coded using FORTRAN and linked to Abaqus via FORTRAN compiler. The written codes are called for each integration point during analysis of damage in composite structures.

4. Results and discussion

4.1. Computational Effort

The analysis is performed under windows 64-bit platform, Intel Core i7 CPU and 8 GB of random access memory (RAM). The pre- and post-processing are conducted using Abaqus version 6-13, while subroutines are linked using Intel FORTRAN compiler. There were no complicated algorithms which could affect the elapsed time for simple ply discount evolution law. The tangent stiffness operator or DDSDDE (notation in Abaqus subroutine) was equal to damaged stiffness matrix.

4.2. Mesh Dependency Analysis

To investigate the influence of mesh size on failure load of each test configurations, a simple composite plate with central hole under tension loading is developed for both method 1 and method 2. The geometry and mesh are illustrated in Figure 5 by modelling size of geometry since the calculation time was very fast. This simple laminate composed of a single layer 90° (perpendicular to loading direction) and thickness of 1mm. The material properties used was T300/1034-C as described in Table 2 in the next section.

4.3. Analysis of T300/1034-C Laminates

The central notch laminated composite plates are composed of two lay-up configurations referred as [(0/90)₆]s and [(0/±45/90)₃]s. The corresponding thickness for both set of laminates were 2.6162 and 3.175 mm, respectively. The corresponding length, width and diameter of hole are 203.2 mm, 25.4 mm and 6.35 mm respectively. The properties of the material in Table 2 are adopted from Lee et al. [5] and Zhao and Zhang [27]. From the evaluation of micromechanical formulation of composite, the out-of-plane shear modulus can be calculated using relation of $G_{23} = E_{22}/(1+v_{23})$. The
Christensen’s equation of $v_{yi} = v_{yi}(1-v_{yi}(E_{yi}/ E_{tt}))$/1-v_{yi} is used to calculate the Poisson’s ratio in direction 2-3. The experimental data for OHT specimens of T300/1034-C are taken from Lee at al. [5] and Zhang and Zhao [27], which are originated from work of Chang and Chang [9].

Table 2: Material properties of T300/1034-C, IM7/977-3 and AS4/PEEK

| Elastic constants | Material       | T300/1034-C | IM7/977-3 | AS4/PEEK |
|-------------------|----------------|------------|-----------|----------|
| $E_{tt}$ (MPa)    |                | 146860     | 164000    | 127600   |
| $E_{22}$ (MPa)    |                | 11376      | 8980      | 10300    |
| $E_{12}$ (MPa)    |                | 11376      | 8980      | 10300    |
| $G_{12}$ (MPa)    |                | 6185       | 5020      | 6000     |
| $G_{23}$ (MPa)    |                | 6185       | 5020      | 6000     |
| $G_{13}$ (MPa)    |                | 4006       | 3000      | 3700     |
| $v_{12}$          |                | 0.3        | 0.32      | 0.32     |
| $v_{23}$          |                | 0.3        | 0.32      | 0.32     |
| $X_{1}$ (MPa)     |                | 1379       | 1680      | 1234     |
| $X_{2}$ (MPa)     |                | 86.5       | 100       | 92.7     |
| $Y_{1}$ (MPa)     |                | 268.2      | 247       | 176      |
| $S_{12}$ (MPa)    |                | 134.0      | 80        | 82.6     |

Table 3 shows the comparison of strength between experiments and simulations for two different stacking sequences as well as thickness of T300/1034C composite rectangular plate. Both method 1 and 2 are discussed in terms of accuracy of predicting the ultimate failure load. The most accurate estimation is achieved via the laminate C for method 1 and 2 that give only -3.245% and -5.147% differences, respectively. In general, the approaches implemented here work quite well with test data for error differences ranging from -11.19 % to 6.182 %.

For the sake of comparison, the simulated load-displacement curve for specimens C and D using both methods are shown in Figure 7 in which the quality of predictions are not so close. This is due to different philosophy used for both methods as described in section 2. The effect of giving discount to the stiffness matrix yielded to the premature failure load and displacement, while reducing the elastic properties resulted in over-predicted ultimate load in many cases. However, the selection of degradation factors plays an important role to change the failure estimation in method 2. In the same figure, it is clearly observed that decreasing the thickness of laminate resulted in lower ultimate load and slope of the elastic region.

The patterns of each internal damage variables in each ply can be obtained from the simulation. For the specimen A and B, the damage patterns in Figure 8 indicated that the damage path establishes in perpendicular direction with respect to loading direction. Combination of matrix cracking and fibre breakage appears in a 0 layer, while matrix failure prominently contributed to failure in a 90 plies. The similar patterns are shown in different sub laminate, thus is not necessary displayed and discussed here.

The progression of damage represented fiber and matrix damage for the 0, 45, -45, and 90 layer of the first sub laminate (from top) with thickness of 2.616 mm using method 1 is exhibited in Figure 9, while the other sub laminates are not shown here since the patterns are identical to sublamine 1. The damage patterns only visualized at the peak load and end of analysis. The failure initiated at the vicinity of hole, in direction perpendicular to loading path. The trends basically followed the nature of damage, where the weakest sections of layers will be the initiation as well as progression of damage path [28]. At 0, 45 and -45 plies, similar damage patterns can be observed for matrix and fibre failure, however, matrix cracking is more prominent at 90 layer.

4.4. Analysis of IM7/977-3 Laminates

The rectangular laminates are made of IM7/977-3 composite and had the following dimension: length $L = 138$ mm, width $W = 38.1$ mm, and diameter $D = 6.35$ mm. The thickness of each plies was 0.127 mm. Three different stacking sequences are considered in this analysis, namely, (0/45/90/-45)s, (60/0/-60)s, and (30/60/90/-60/30)s. A set of material properties are adopted from references [29-31]. The test data are extracted from Wang et al. [30], which were tested in the Air Force Research laboratory, Dayton.

Table 3: Simulated failure load of the T300/1034-C central notched laminated coupon

| Label | Thickness (mm) | Failure load (kN) | Difference (%) |
|-------|----------------|-------------------|---------------|
| Test  | Method 1       | Method 2          | Method 1      | Method 2      |
| A     | 3.175          | 28.97             | 25.78         | 27.73         | -10.79        | -4.28 |
| B     | 2.616          | 23.92             | 21.243        | 23.021        | -11.19        | -3.75 |
| C     | 3.175          | 22.80             | 22.06         | 23.985        | -3.245        | 5.197 |
| D     | 2.616          | 20.98             | 19.73         | 22.277        | -5.97         | 6.182 |

A = [0(90)3]s, C = D = [0(±45/90)3]s

Fig. 7: Comparison of failure load of OHT coupon T300/1034-C with different thickness using both methods.

Fig. 8: Failure patterns of specimen A in sublaminate 1 using method 1.

Fig. 9: Damage path at 0°

Fig. 10: Damage path at 90°

Matrix failure | Fibre failure | Ultimate load | End of analysis | Ultimate load | End of analysis |
|---------------|--------------|---------------|-----------------|---------------|----------------|

Damage path at 0°

Damage path at 90°

Damage path at 0°

Damage path at 45°
Table 4 and Figure 10 present the prediction results of failure load and error analysis regarding to three different layup of IM7/977-3 composite specimen. Both prediction methods performed reasonably well in all cases considering the simplicity of progressive damage model applied, although method 1 showed an error of higher than 10% (-11.42%). Most predicted results are under prediction when compared to experimental data, except for method 2 in specimen G. Overall, method 2 performed better from method 1 in terms of accuracy of the prediction.

Table 4: Simulated failure load of the IM7/977-3 central notched laminated coupon

| Label | Diameter | Failure load (kN) | Difference (%) |
|-------|----------|------------------|----------------|
| Test  | Method 1 | Method 2         | Method 1       | Method 2 |
| E     | 42.89    | 38.93            | 42.58          | -9.23    | -0.72 |
| F     | 47.29    | 41.89            | 41.89          | -11.419  | -3.244 |
| G     | 39.58    | 36.69            | 41.353         | -7.355   | 4.48  |

Table 5 analyses the accuracy of proposed models to predict the ultimate failure by comparing with experimental results for AS4/PEEK laminates. Method 1 produced slightly lower value of failure load when compared to test results, but still within acceptable range. The error about ±5% proved that method 2 performed excellently to predict the peak load for this laminate. Besides, the increase size of diameter from 2 to 10 mm gradually changed the ultimate load as can be seen in Figure 13. Depending on model used, the failure load decreased about 1.5 to 5 kN for different diameter applied in the simulations.

Table 5: Simulated failure load of the AS4/PEEK notched laminates

| Label | Diameter | Failure load (kN) | Difference (%) |
|-------|----------|------------------|----------------|
5. Conclusion

In the present study, the initiation and growth of cracks in fibre reinforced polymer (FRP) composite laminate are satisfactorily estimated using finite element simulations that employ 3D Puck's failure criteria together with ply discounting method as the evolution law. In addition, the proposed models are able to predict the effect of specimen sizes including thickness and diameter of the laminate, as well as different stacking sequences. The performance of method 2 are better than method 1 due to smaller reduction of stiffness matrix. Furthermore, the degradation of stiffness is dependent on the selection of degradation factor (arbitrary), which in this case following the rules proposed from literature. In future, the effect of delamination need to be considered in order to predict out of plane failure and deformation. Although some discrepancy between test data and simulations, the proposed PDM’s could be considered as a practical design tool for detecting the onset of failure and damage progression in FRP composite laminated structures.

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