Mathematical model of a micropolar lubricating stuff

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Abstract. The article is devoted to the mathematical analysis model of the radial bearing of infinite length, lubricated with lubricating stuff and molten low-melting metal coating on the surface of the bearing bush, having the general rheological properties in the laminar condition of micro-polar lubrication properties, taking into account the dependence of viscosity rheological properties of micro-polar lubricating stuff, and the permeability of the porous coating from the pressure. The authors on the basis of the motion equations of a viscous incompressible fluid having micro-polar properties, and for “lamina”, of the continuity equation, Darcy equation and determining from the expression for the dissipation rate of mechanical energy, the profile of molten contour of the bearing bush, while taking account of the dependence of the overall rheological properties of the lubricating stuff and melt the low-melting metal coating having the micro-polar properties and the permeability of the porous coating of the pressure found the asymptotic and exact self-similar solution of the differential equation system on the parameter, due to the melt and the dissipation rate of mechanical energy to zero (without melt) and first (including melt) approximation.

1. Purpose of study
Investigation of the field performance of a plain journal bearing based on the development of refined calculation models based on the classical Navier-Stokes, Darcy, continuity equations with additional consideration for the dissipation rate of mechanical energy, implementation of numerical analysis to create algorithms and software for solving practical tasks.

2. Scientific novelty
The proposed solution and refine design models is to assess the impact of the molten low-melting metal coating the surface of the bearing bush, a porous surface coating of the axle journal, as well as the dependence of viscosity of general rheological properties of the lubricating stuff and melt, with the micro-polar properties of the pressure.

3. Problem statement
Consider the motion of micro-polar lubricating stuff in the working gap of the radial bearing surface of the bearing bush is covered with fusible metal alloy, and the shaft neck – porous layer. The bearing bush made of a material with a low melting point is stationary, and the shaft covered with a porous layer rotates at a speed of $\Omega$. All the heat during rotation is used to melt the material surface of the bearing bush, which is coated with a low-melting metal alloy. The rheological properties of the lubricating stuff used and the
permeability of the porous coating depend on the pressure and are given as:

\[
\mu' = \mu_0 e^{\nu \theta}, \quad \kappa' = \kappa_0 e^{\nu \theta}, \quad \gamma' = \gamma_0 e^{\nu \theta}, \quad \tilde{K}' = \tilde{K}_0 e^{\nu \theta}.
\]

(1)

where \(\mu'\) is the coefficient of dynamic viscosity of the lubricating stuff, \(\kappa', \gamma'\) is the viscosity coefficients of the micro-polar lubricating stuff; \(\mu_0\) - characteristic viscosity of Newtonian lubricating stuff; \(p'\) - hydrodynamic pressure in the lubricant layer, \(\tilde{a}_n\) - experimental constant, \(\kappa'\) - permeability of the porous coating, \(\kappa'\) - characteristic permeability.

In the polar coordinate system (Fig.1) \(C_i: r' = r - \tilde{H}; C_o: r' = r; C_c: r' = r_1(1+H') + \lambda f(\theta), (2)\)

where \(H = \varepsilon \cos \theta - \frac{1}{2} \varepsilon^2 \sin^2 \theta + ...\), \(\varepsilon = \frac{e}{r_0}\), \(r_0\) - the axle radius, \(r_1\) - the radius of the bearing bush, covered with fusible metal alloy, \(e\) – eccentricity, \(\varepsilon\) - the eccentricity ratio, \(\tilde{H}\) – the thickness of the porous layer, \(\lambda f(\theta)\) - the function to be determined.

![Fig.1. Working scheme](image)

The initial equation is a system of dimensionless equations of motion of micro-polar lubricating stuff, Darcy’s law, the continuity equation, and the equation that determines, taking into account the expression for the dissipation rate of mechanical energy, the profile of the molten contour of a bearing bush coated with a low – melting metal alloy with an accuracy of up to terms \(O(\xi^2)\) - is written as:

\[
\begin{align*}
\frac{\partial^2 u}{\partial r^2} + N^2 \frac{\partial u}{\partial r} + \frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial \theta^2} &= N_1 + 1 \frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial r} + \frac{\partial v}{\partial r} = 0, \\
\frac{\partial^2 P}{\partial r^2} + \frac{1}{r^2} \frac{\partial P}{\partial r} + \frac{1}{r^2} \frac{\partial^2 P}{\partial \theta^2} &= 0, \\
\frac{d\Phi(\theta)}{d\theta} &= K \int_{e_0}^{1} \left( \frac{\partial M}{\partial r} \right)^2 dr;
\end{align*}
\]

(3)

where \(K = \frac{2\mu \Omega_{\beta}}{L^2\delta}, \quad \eta = \frac{e}{\delta}, \quad \eta_1 = \frac{\lambda}{\delta}, \quad \Phi(\theta) = \eta_1 f(\theta)\).

For equation (3), the boundary conditions have the following form:

\[
u(0) = 0, \quad u(r_0) = M \frac{\partial P}{\partial r} \Big|_{r_0}, \quad \nu(0) = 1, \quad p = P \quad \text{при} \quad r' = \frac{r_0}{\tilde{H}}, \quad \frac{\partial P}{\partial r} \Big|_{r_0} = 0, \quad p(0) = p(2\pi\tilde{H}) \frac{P_0}{p}
\]

(4)

where \(M = \frac{k^3}{\beta \delta^2}\)

the relationship between non-dimensional and dimensional values is given as:

\[
r' = \tilde{r} + \delta r, \quad \tilde{\delta} = \delta - \Omega t_0, \quad n' = \Omega \tilde{r} v, \quad p' = p P, \quad \rho' = \frac{(2\mu_0 + \kappa_0) \Omega^2_{\beta} \delta^2}{2\delta^2}, \quad \nu' = \nu, \quad \mu' = \mu_0 \mu, \quad \kappa' = \kappa_0 \kappa, \quad \gamma' = \gamma_0 \gamma'.
\]
\[ N^2 = \frac{\kappa_0}{2\mu_0 + \kappa_0}, \quad N_1 = \frac{2\mu_0}{\delta^2\kappa_0}, \quad \ell^2 = \frac{\gamma_0}{4\mu_0} \]

(5)

Similarly, in a porous layer: \[ P' = p'P, \quad r' = \hat{H}r, \quad \hat{k}' = \hat{k}\hat{k} \]

(6)

Taking into account the smallness of the gap, as well as equality \( \nu = 0 \) on moving and stationary surfaces, we average the second equation of the system (3) over the thickness of the lubricant layer, we get:

(7)

The equation solution (7) will be found in the form:

\[ \nu = A(\theta) r^2 + A_1(\theta) r + A_2(\theta) \]

(8)

Taking into account the boundary conditions (4), we obtain:

\[ \nu = A(\theta)(r^2 - (h - \Phi)r - \Phi h) \]

(9)

Substituting (9) in (7) up to the terms \( O\left(\frac{\Phi}{N_1}\right), \quad O\left(\frac{\Phi^2}{N_1^2}\right) \) we get:

\[ \nu = \frac{1}{2N_h} (r^2 - rh), \quad \frac{\partial \nu}{\partial r} = -\frac{1}{2N_h} (2r - h), \quad A_1 = -\frac{1}{2N_h} \]

(10)

Taking into account (10), the equation system (3) in the approximation we have taken has the form:

\[ \frac{\partial P}{\partial r} = 0, \quad \frac{\partial^2 u}{\partial r^2} + \frac{N^2}{2N_h} (2r - h) = e^{\alpha \omega} \frac{dP}{d\theta}, \quad \nu = \frac{1}{2N_h} (r^2 - rh) \]

\[ \frac{\partial v}{\partial r} + \frac{\partial v}{\partial \theta} = 0, \quad \frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \nu}{\partial \theta^2} = 0, \quad \frac{d\Phi(\theta)}{d\theta} = K \int_0^{\Phi(\theta) \frac{\partial u}{\partial r}} \frac{\partial u^2}{\partial r} \, dr \]

(11)

We will enter the designation \( Z = e^{\alpha \omega} \). Differentiating both parts of the equation, we get:

\[ \frac{dZ}{d\theta} = -\alpha e^{\alpha \omega} \frac{dP}{d\theta} \quad \text{or} \quad e^{\alpha \omega} \frac{dP}{d\theta} = -\frac{1}{\alpha} \frac{dZ}{d\theta} \]

Then the equation system (11) will be written as:

\[ \frac{\partial Z}{\partial \theta} = 0, \quad \frac{\partial^2 u}{\partial r^2} + \frac{N^2}{2N_h} (2r - h) = -\frac{1}{\alpha} \frac{dZ}{d\theta}, \quad \nu = \frac{1}{2N_h} (r^2 - rh) \]

\[ \frac{\partial u}{\partial r} + \frac{\partial v}{\partial \theta} = 0, \quad \frac{\partial^2 P}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0, \quad Z \frac{d\Phi(\theta)}{d\theta} = K \int_0^{\Phi(\theta) \frac{\partial v}{\partial r}} \frac{\partial v^2}{\partial r} \, dr \]

(12)

With appropriate boundary conditions:

\[ u = 0, \quad \nu = 0 \quad \text{at} \quad r = 1 + \eta \cos \theta + \Phi(\theta), \quad u \big|_{r=1} = \bar{M} \frac{\partial P}{\partial r} \bigg|_{r=1}, \quad \nu(0) = 1 \]

(13)

\[ p = P \quad \text{at} \quad r' = \frac{r_0}{R}, \quad \frac{\partial P}{\partial r'} \bigg|_{r'=\frac{r_0}{R}} = 0, \quad Z(0) = Z(2\pi) = e^{\alpha \omega} \]

The boundary conditions for non-dimensional velocity components \( u \) and \( \nu \) on the contour \( r = -\Phi(\theta) \) can be written as:

\[ \nu(1 + \eta \cos \theta + H(x)) = \nu(1 + \eta \cos \theta) - \left( \frac{\partial \nu}{\partial \theta} \right)_{r=1+\eta \cos \theta} \cdot H(\theta) - \left( \frac{\partial^2 \nu}{\partial \theta^2} \right)_{r=1+\eta \cos \theta} \cdot H^2(\theta) + \ldots = 0 \]

(14)

\[ u(1 + \eta \cos \theta H(x)) = u(1 + \eta \cos \theta) - \left( \frac{\partial u}{\partial \theta} \right)_{r=1+\eta \cos \theta} \cdot H(\theta) - \left( \frac{\partial^2 u}{\partial \theta^2} \right)_{r=1+\eta \cos \theta} \cdot H^2(\theta) + \ldots = 0 \]
The asymptotic solution of the differential equation system (12) taking into account the boundary conditions (13) and (14) is found in the form:

\[ v(r, \theta) = v_0(r, \theta) + K\Phi_0(r, \theta) + K^2 v_1(r, \theta) + \ldots \]
\[ u(r, \theta) = u_0(r, \theta) + Ku_0(r, \theta) + K^2 u_1(r, \theta) + \ldots \]
\[ \Phi(\theta) = -K\Phi_0(\theta) - K^2\Phi_1(\theta) - K^3\Phi_2(\theta) - \ldots \]
\[ Z(\theta) = Z_0(\theta) - KZ_1(\theta) - K^2Z_2(\theta) - K^3Z_3(\theta) - \ldots \]

(15)

By substituting (15) into the differential equation system (12), taking into account the boundary conditions (13) and (14), we obtain the following equations:

For the zeroth-order approximation:

\[ \frac{\partial^2 u_0}{\partial r^2} + \frac{N_1^2}{2N_1 h} (2r - h) = -\frac{1}{\alpha} \frac{dZ_0}{d\theta}, \quad \frac{\partial u_0}{\partial r} + \frac{\partial v_0}{\partial \theta} = 0, \quad \frac{\partial^2 P_0}{\partial r^2} + \frac{1}{r} \frac{\partial P_0}{\partial r} + \frac{1}{r^2} \frac{\partial P_0}{\partial \theta^2} = 0. \]

(16)

With boundary conditions:

\[ u_0 = 0, v_0 = 1 \quad \text{at} \quad r_0 = 0, \quad u_0 = 0, \quad v_0 = 0 \quad \text{at} \quad r_0 = 1 + \eta \cos \theta, \]

\[ u(0) = \bar{M} \frac{\partial P_0}{\partial r}, \quad p_0 = P_0 \quad \text{at} \quad r = r_0 = \frac{r_0}{H}, \quad \frac{\partial P_0}{\partial r} \bigg|_{r = r_0 = \frac{r_0}{H}} = 0, \quad Z_0(0) = Z_0(2\pi) = e^{-\frac{\rho}{r}}. \]

(17)

For the initial approximation:

\[ \frac{\partial^2 u_i}{\partial r^2} - \frac{1}{\alpha} \frac{dZ_i}{d\theta}, \quad \frac{\partial u_i}{\partial r} + \frac{\partial v_i}{\partial \theta} = 0, \quad \frac{\partial^2 P_i}{\partial r^2} + \frac{1}{r} \frac{\partial P_i}{\partial r} + \frac{1}{r^2} \frac{\partial P_i}{\partial \theta^2} = 0. \]

(18)

With boundary conditions:

\[ u_i = 0, v_i = \left( \frac{\partial u_i}{\partial r} \right)_{r=0}, \Phi_i(\theta), \quad u_i = 0, v_i = 0, u_i = 0 \quad \text{at} \quad r = 1 + \eta \cos \theta, \]

\[ u_i(0) = \bar{M} \frac{\partial P_i}{\partial r}, \quad p_i = P_i \quad \text{at} \quad r = r_i = \frac{r_i}{H}, \quad \frac{\partial P_i}{\partial r} \bigg|_{r = r_i = \frac{r_i}{H}} = 0, \quad Z_i(0) = Z_i(2\pi) = 0, \]

\[ K\Phi_i(0) = \bar{K}, \Phi(0) = \Phi(2\pi) = \bar{a}. \]

For the zeroth-order approximation, we look for the exact solution in the form:

\[ U_0 = \frac{\partial \psi_0}{\partial r} + V_0(r, \theta), \quad V_0 = -\frac{\partial \psi_0}{\partial r} + U_0(r, \theta), \quad \psi_0(r, \theta) = \bar{\psi}_0(\xi), \quad \xi = \frac{r}{h(\theta)}, \]

\[ V_0(r, \theta) = \bar{v}(\xi), \quad U_0(r, \theta) = -\bar{u}_0(\xi) \cdot h'(\theta). \]

(20)

Substituting (20) in (16)-(17), we have:

\[ \bar{\psi}_0 = \bar{C}_2, \quad \bar{u}_0 = \bar{C}_1 - \frac{N_1^2}{2N_1} (2\xi - 1), \quad \bar{u}_0 + \xi \bar{\psi}_0 = 0, \quad dZ_0 = -\alpha \left( \frac{\bar{C}_1}{h'(\theta)} + \frac{\bar{C}_2}{h'(\theta)} \right). \]

(21)

And boundary conditions:

\[ u(0) = 0, \quad \bar{v}(0) = 0, \quad \bar{u}_0(1) = 0, \quad \frac{\partial P_0}{\partial r} \bigg|_{r = 0} = \bar{M} \frac{\partial P_0}{\partial r}, \quad p_0 = P_0 \quad \text{at} \quad r = r_i = \frac{r_i}{H}, \]

\[ u(1) = 0, \quad \bar{v}(0) = 0, \quad \bar{u}_0(1) = 1, \quad \int_0^1 \bar{u}_0(\xi) d\xi = 0, \quad Z_0(0) = Z_0(2\pi) = e^{-\frac{\rho}{r}}, \]

(22)

By immediate integration we get:

\[ \bar{\psi}_0(\xi) = \bar{C}_1 (\xi - \frac{\xi}{3} - \frac{1}{3}), \quad \bar{u}_0(\xi) = \bar{C}_1 (\xi - \frac{1}{3} - \frac{1}{3}) - \bar{C}_2 \left( \frac{N_1^2}{2N_1} \bar{C}_1 + 1 \right) \xi + 1, \]

\[ Z_0 = -\alpha \left[ \bar{C}_1 (\theta - 2\eta \sin \theta) + \bar{C}_2 (\theta - 3\eta \sin \theta) \right] + e^{-\frac{\rho}{r}}. \]

(23)
\[
Z_0(0) = Z_0(2\pi) = \frac{e^{-\alpha r_0^2}}{r_0^2}
\]

We get the following expression from the condition:
\[
\tilde{C}_2 = -\tilde{C}_1
\]  
(24)

Hydrodynamic pressure determination.

Substituting the value \(\tilde{C}_2\) in equation (23) for \(Z_0\), we get:
\[
Z_0 = -\alpha \tilde{C}_1 \eta \sin \theta + e^{-a \frac{r_0}{r_0^2}}
\]

(25)

Using the asymptotic decomposition to determine \(P\), we arrive at the following approximate equation:
\[
1 - ap + \frac{\alpha^2 p^2}{2} \left(1 - \alpha \frac{p_x}{p} + \frac{\alpha^2}{2} \left( \frac{p_x}{p} \right)^2 \right) + \alpha \tilde{C}_1 \eta \sin \theta = 0
\]  
(26)

To find the hydrodynamic pressure, we solve equations (26) up to \(O(\eta^2), O\left(\alpha^2 \left( \frac{p_x}{p} \right)^2 \right)\), we get
\[
P = \frac{p_x}{p} + \left[1 + \alpha \frac{p_x}{p} - \frac{\alpha^2}{2} \left( \frac{p_x}{p} \right)^2 \right] \tilde{C}_1 \eta \sin \theta
\]  
(27)

Given the expression (27), Darcy’s equation takes the form:
\[
P(r, \theta) = R(r^*) \tilde{C}_1 \eta \sin \theta + \left[1 + \alpha \frac{p_x}{p} - \frac{\alpha^2}{2} \left( \frac{p_x}{p} \right)^2 \right] \tilde{C}_1 \eta \sin \theta
\]

(28)

Substituting (28) in Darcy’s equation to determine the function \(R(r^*)\), we proceed to the following differential equation:
\[
R^*(r^*) + \frac{R'}{r} - \frac{R'}{r^2} = 0
\]

(29)

and boundary conditions:
\[
\frac{dR}{dr^2} |_{r=r^*} = 0, R(\frac{r_0}{H}) = 1
\]

(30)

Immediate integration of (29) with (30) for the function \(R(r^*)\) allows us to obtain the equation:
\[
R(r^*) = \frac{r_0^2 H r^2}{2r_0^2 - 2H r_0 + H^2} + \frac{r_0^2 \left( r_0^2 - 2H r_0 + H^2 \right)}{2r_0^2 - 2H r_0 + H^2} r^2
\]

(31)

With the expression
\[
\tilde{M} \frac{\partial P}{\partial r} |_{r=r^*, \eta} = \frac{1}{\tilde{a}(\xi)} d\xi
\]

(32)

For \(\tilde{C}_1\) we will have:
\[
\tilde{C}_1 = \frac{6 \tilde{a} \left( 2r_0^2 - 2H r_0 + H^2 \right) \left( 1 + a \frac{p_x}{p} - \frac{\alpha^2}{2} \left( \frac{p_x}{p} \right)^2 \right)}{12 \tilde{H} \tilde{M} \left( r_0 - \tilde{H} \right) \left( 1 + a \frac{p_x}{p} - \frac{\alpha^2}{2} \left( \frac{p_x}{p} \right)^2 \right)^2 + 6 \tilde{a} \left( 2r_0^2 - 2H r_0 + H^2 \right)}
\]

(33)

Then for \(p_0\) we get:
\[ p_0 = \frac{6\eta \sin \theta (2\alpha^2 - 2\hat{H}_g + \hat{H}^2) \left(1 + \frac{a P_r}{\rho} - \frac{a^2 (P_r)}{\rho^2} \right)}{12\hat{H}^2 \hat{M} (\eta - \hat{H}) \left(1 + \frac{a P_r}{\rho} - \frac{a^2 (P_r)}{2 \rho^2} \right)} + \frac{p_r}{\rho^2}. \]  

To determine \( F_i(\theta) \), taking into account equation (23), we come to the following expression:

\[ \frac{d\Phi_i(\theta)}{d\theta} = \frac{h(\theta)}{Z_0} \int \left( \frac{\hat{\psi}_i^*(\xi)}{h^2(\theta)} + \frac{\hat{\psi}_i(\xi)}{h(\theta)} \right)^2 d\xi. \]

Integrating (33) we get:

\[ \frac{d\Phi_i(\theta)}{d\theta} = \frac{1}{Z_0} \left( \frac{\Delta_i \Delta_i h(\theta)}{h^3(\theta)} + \frac{\Delta_i h(\theta)}{h^2(\theta)} + \frac{\Delta_i \Delta_i h(\theta)}{h(\theta)} \right). \]

where \( \Delta_i = \int_0^1 (\hat{\psi}_i^*(\xi))^2 d\xi = \frac{C^2_i}{12}; \quad \Delta_2 = \int_0^1 2\psi^\star(\xi) \cdot \hat{\psi}(\xi) d\xi = \frac{1}{6} C_1 C_2; \quad \Delta_3 = \int_0^1 (\hat{\psi}(\xi))^2 d\xi = 4 + \frac{N^4}{720N_i^2} \)

Solving equations (36)-(37) taking into account \( CF_i(\theta) = \kappa \bar{a} \), we get:

\[ \Phi_i(\theta) = \frac{1}{68} \left[ (\theta - \eta \sin \theta) \left( -\tilde{C}^2_i + 4 + \frac{N^4}{720N^2_i} \right) + \bar{a} \right]. \]

Then for the initial approximation, we get:

\[ u_i = -\frac{\hat{\psi}_i}{\hat{\theta}} + U_i(r, \theta); \quad v_i = -\frac{\hat{\psi}_i}{\hat{r}} + V_i(r, \theta); \quad \psi_i(r, \theta) = \hat{\psi}_i(\xi); \quad \xi = \frac{r}{h(\theta)}; \quad V_i(r, \theta) = \nu(\xi); \quad U_i(r, \theta) = -u_i(\xi) \cdot h'(\theta); \]

\[ \frac{dZ_i}{d\theta} = -\alpha \left( \frac{\tilde{C}_1}{h^2(\theta)} + \frac{\tilde{C}_2}{h^2(\theta)} \right); \quad h(\theta) = 1 + \eta \cos \theta. \]

Substituting (39) into the differential equation system (18), taking into account the boundary conditions (19), we obtain the following differential equation system:

\[ \hat{\psi}'' = \tilde{C}_1, \quad \tilde{u}''(\xi) + \xi \tilde{v}''(\xi) = 0, \quad \frac{dZ_i}{d\theta} = -\alpha \left( \frac{\tilde{C}_1}{h^2(\theta)} + \frac{\tilde{C}_2}{h^2(\theta)} \right). \]

And boundary conditions:

\[ \hat{\psi}'(0) = 0, \quad \hat{\psi}'(1) = 0, \quad \tilde{u}(1) = 0, \quad \tilde{v}(1) = 0, \quad \nu_i(0) = 0, \quad \nu_i(1) = 0, \quad u_i(0) = M, \quad \nu_i(0) = 0, \quad \frac{\tilde{a}_i}{\xi} \frac{dZ_i}{d\theta} = 0. \]

By immediate integration we get:

\[ \hat{\psi}'(\xi) = \frac{\tilde{C}_1}{2} (\xi^2 - \xi), \quad \tilde{u}_i(\xi) = \tilde{C}_i \frac{\xi^2}{2} - \left( -\frac{\tilde{C}_1}{2} + M \right) \xi + M, \quad \tilde{C}_i = 6M \]

From the condition \( Z_i(0) = Z_i(2\pi) = 0 \) we get:
\[
\tilde{C}_i = -M \tilde{C}_i
\]  
\hspace{1cm} (43)

Where
\[
M = \sup_{\{0^2 \leq \varepsilon \}} \frac{\partial u_0}{\partial r} |_{r=0} \cdot \Phi_1(\theta) = \sup_{\{0^2 \leq \varepsilon \}} \left[ \left( -\frac{1}{\eta \cos \theta} - \tilde{C}_i \eta \cos \theta + \frac{N^2}{6N_1} - (1 - 2\eta \cos \theta) - \frac{N^2}{4N_1} (1 + \eta \cos \theta) \right) \times \right] \\
\times \frac{1}{68} \left[ (\theta - \eta \sin \theta) \left( \tilde{C}_i^2 + 4 + \frac{N^4}{720N_1^2} \right) + \tilde{a} \right]
\]

With (40) taken into account, \( Z_i \) we get
\[
Z_i = -\alpha \tilde{C}_i M \eta \sin \theta \]  
\hspace{1cm} (44)

The further solution is the same as for the zeroth-order approximation, we get:
\[
\tilde{C}_1 = \frac{6Mr_i \left( 2r_i^2 - 2\tilde{H}r_i + \tilde{H}^2 \right) \left( 1 + \alpha \frac{p_g}{p} - \frac{\alpha^2}{2} \left( \frac{p_g}{p} \right)^2 \right)}{12\tilde{H}^2 \tilde{M} \left( r_i - \tilde{H} \right) \left( 1 + \alpha \frac{p_g}{p} - \frac{\alpha^2}{2} \left( \frac{p_g}{p} \right)^2 \right) + r_i \left( 2r_i^2 - 2\tilde{H}r_i + \tilde{H}^2 \right)}
\]  
\hspace{1cm} (45)

\[
p_i = \frac{6Mr_i \eta \sin \theta \left( 2r_i^2 - 2\tilde{H}r_i + \tilde{H}^2 \right) \left( 1 + \alpha \frac{p_g}{p} - \frac{\alpha^2}{2} \left( \frac{p_g}{p} \right)^2 \right)}{12\tilde{H}^2 \tilde{M} \left( r_i - \tilde{H} \right) \left( 1 + \alpha \frac{p_g}{p} - \frac{\alpha^2}{2} \left( \frac{p_g}{p} \right)^2 \right) + r_i \left( 2r_i^2 - 2\tilde{H}r_i + \tilde{H}^2 \right)}
\]  
\hspace{1cm} (46)

The results of the study:
\[
R_x = p^* r^2 \int_0^{2\pi} \left( p_0 - \frac{p_g}{p} + Kp_l \right) \cos \theta d \theta = 0.
\]
\[
R_y = p^* r^2 \int_0^{2\pi} \left( p_0 - \frac{p_g}{p} + Kp_l \right) \sin \theta d \theta = \frac{\mu \Omega p^*}{2} \tilde{C}_1 \tilde{C}_pi \left( 1 + \alpha \frac{p_g}{p} - \frac{\alpha^2}{2} \left( \frac{p_g}{p} \right)^2 \right) (1 + KM)
\]
\[
L_{np} = \mu \int_0^{2\pi} \left[ \tilde{C}_i \frac{\partial u_0}{\partial r} \bigg|_{r=0} + K \tilde{C}_i \frac{\partial u_0}{\partial r} \bigg|_{r=0} \right] d \theta = \mu \left( 1 - \alpha \eta \frac{2}{2} \right) \left( 1 - \frac{N^2}{6N_1} + \frac{N^2}{4N_1} \right) \left( -2\pi + \frac{K^2 \pi^2}{34} \tilde{C}_i^2 + 4 + \frac{N^4}{720N_1^2} \right)
\]  
\hspace{1cm} (47)
Figure 2. Dependency graph of vector component of the supporting force from the permeability parameters of the porous coating $K$ the and $\alpha$ pressure-viscosity ratio.

Figure 3. Dependency graph of the friction force on the permeability parameters of the porous coating $K$ and $\alpha$ of the viscosity versus pressure.

Results
As a result, the velocity and pressure fields in the lubricant and porous layers were determined, taking into account the dependence of the rheological properties of the lubricating stuff and the melt of a low-melting metal coating. Having micro-polar properties, as well as the permeability of the porous coating from pressure. Also, the function $F_1(\theta)$ due to the melt of the bearing bush surface and the main operating characteristics: load-bearing capacity and friction force are determined. But the authors also determined the influence of the parameters $\kappa$ due to the melt surface of the bearing bush, coated with low-melting metal alloy, $H$ the thickness of the porous coating, $\alpha$ characterizes the dependence of viscosity of the micro-polar lubricating stuff pressure, $K$ characterize the permeability of the porous layer from the pressure of the structurally-viscous parameters, $N_1$ and $N_2$ micro-polar lubricating stuff on the bearing capacity and friction force.

References
[1] Levanov I, Doykin A, Zadorozhnaya E and Novikov R 2017 Investigation of Antiwear Properties of Lubricants with the Geo-Modifiers of Friction Tribology in Industry 39 (3) pp 302–306
[2] Mukchortov I V, Pochkaylo K A and Zadorozhnaya E A 2016 The Influence of Anti-Wear Additives on the Bearings Hydro-Mechanical Characteristics *Procedia Engineering* **150** pp 607–611

[3] Zadorozhnaya E 2013 The Research of Non-Newtonian Properties and Rheology of Thin Lubricant Layers in Hydrodynamic Journal Bearings *Society of Tribologists and Lubrication Engineers Annual Meeting and Exhibition* pp 95–97

[4] Vasilenko V V, Lagunova E O, Mukutadze M A and Prikhodko V M 2017 International Journal of Applied Engineering Research ISSN 0973-4562 Vol 12 **19** pp 9138–9148

[5] Mukutadze M A, Lagunova E O, Garmonina A N and Vasilenko V V 2018 Radial Slip Bearing with a Pliable Supporting Surface *Russian Engineering Research* Vol 38 **3** pp 166–171

[6] Mukutadze M A 2016 Radial Bearing with Porous Elements *Procedia Engineering* **150** pp 559–570

[7] Mukutadze M A, Lagunova E O and Vasilenko V V 2018 Development of the Design Model of a Hydrodynamic Lubricating Material Formed during Melting of the Axial Bearing, in the Presence of Forced Lubrication *Journal of Machinery Manufacture and Reliability* Vol 47 pp 271–277

[8] Akhverdiev K S, Lagunova E O and Mukutadze M A Calculated Model of Wedge-Shaped Sliding Supports in Turbulent Friction Regime *Advances in Engineering Research (AER)* Vol 157 *International Conference “Actual Issues of Mechanical Engineering”* (AIME 2018) pp 346–353

[9] Akhverdiev K S, Lagunova E O and Mukutadze M A 2018 Calculated Model of Wedge-Shaped Sliding Supports Taking into Account Rheological Properties of Viscoelastic Lubricant *Advances in Engineering Research, Volume 158, International Conference on Aviamechanical Engineering and Transport* (AviaENT 2018) pp 246–253

[10] Lagunova E O and Mukutadze M A 2018 Radial Friction Bearings Conditioned by Melt Cooling *Proceedings of the 4th International Conference on Industrial Engineering, Lecture Notes in Mechanical Engineering (ICIE 2018)* pp 897–910

[11] Lagunova E O and Mukutadze M A Calculation of a Radial Slider Bearing with a Fusible Coating *Journal of Friction and Wear* 2019, Vol **40** 1 pp 88–94

[12] Mukutadze M A, Mukutadze A M and Vasilenko V V 2019 Simulation Model of Thrust Bearing with a Free-Melting and Porous Coating of Guide and Slide Surfaces *IOP Conf. Series: Materials Science and Engineering* **560** DOI:10.1088/1757-899X/560/1/012031

[13] Opatskikh A N, Mukutadze M A and Mukutadze A M 2019 V-Shaped Journal bearings Using Microporous Lubricants Caused by a Melt Accounting for the Dependence of Lubricant Viscosity and Porous Layer Permeability on Pressure *Journal of Physics: Conference Series* **1353** 012025 DOI:10.1088/1742-6596/1353/1/012025

[14] Mukutadze M A, Vasilenko V V, Mukutadze A M and Opatskikh A N 2019 Mathematical Model of a Plain Bearer Lubricated with Molten Metal *IOP Conf. Series: Earth and Environmental Science* **378** 012021 DOI:10.1088/1755-1315/378/1/012021

[15] Akhverdiev K S, Alexandrova E V, Kruchinina E E and Mukutadze M A 2010 Stratified Course Of Two-Layer Lubrication In The Gap Of A Threshold Bearing, With An Increased Bearing Ability *Vestnik DGTU* Vol 10 **2** (45) pp 217–223

[16] Akhverdiev K S, Aleksandrova E E, Mukutadze M A and Kopotun B E Stratified Course Of Two-Layer Lubrication In The Radial Bearing Clearance With An Increased Bearing Capacity And Damping Properties *Vestnik RGURS* 2009 **4** (36) pp 133–139

[17] Akhverdiev K S, Aleksandrova E E and Mukutadze M A 2010 Stratified Two-Layer Lubrication Current In The Clearance Of A Complexly Loaded Radial Bearing Of A Final Length With An Increased Loading Capacity *Vestnik RGURS* 2010 **1** (37) pp 132–137