Possibility to determine the radius of accretion disk by gravitational waves

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Abstract. We investigate gravitational waves from a dust disk around a Schwarzschild black hole to focus on whether we can extract any of its physical properties from a direct detection of gravitational waves. We adopt a black hole perturbation approach in a time domain, which is a satisfactory approximation to illustrate a dust disk in a supermassive black hole. We find that we can determine the radius of the disk by using the power spectrum of gravitational waves and that our method to extract the radius works for a disk of arbitrary density distribution. Therefore we believe a possibility exists for determining the radius of the disk from a direct observation of gravitational waves detected by the Laser Interferometer Space Antenna.

1. Introduction

Gravitational wave projects have been launched in many countries to explore a new world in gravitational physics, in astrophysics, and in cosmology. Nowadays, several ground-based laser interferometric gravitational wave detectors such as TAMA300, LIGO, GEO600, and VIRGO have been seeking for a direct detection of gravitational waves (See Ref. [1] for the current status of the worldwide network of gravitational wave detectors). The sensitive frequency for the ground-based interferometers is within the range of $10 - 1000 \text{ Hz}$. The representative sources of gravitational waves are the inspiral of binary systems composed of neutron stars and black holes, spinning neutron stars, and low-mass X-ray binaries. There will be also a gravitational wave detector in space, Laser Interferometer Space Antenna (LISA). The sensitive frequency band in LISA is in the range of $10^{-4} - 10^{-1} \text{ Hz}$, and one of the representative gravitational wave sources this detector is studying is supermassive black holes whose masses are $M \sim 10^6 - 10^9 M_\odot$. There is increasing evidence that a supermassive black hole exists at the center of every galaxy.

There is abundant observational evidence for dense gas in galactic nuclei [2]. For example, our galactic nuclei contains a $4 \times 10^6 M_\odot$ supermassive black hole surrounded by an $\sim 10^4 M_\odot$ molecular gas torus. Also the disruption of a compact star by a supermassive black hole spreads the gas in black hole spacetime [3]. From a theoretical point of view, the tidal disruption of a
star by a supermassive black hole has been studied in Newtonian gravity (e.g. [4]) and in post-
Newtonian gravity [5], where the black hole is regarded as a point mass. There are also several
studies of tidal disruption in a Schwarzschild black hole [6] and in a Kerr black hole (e.g. [7]).
In all cases, the fragments of the disrupted star are either swallowed by the supermassive black
hole or following a highly eccentric orbits. There is also an indication, based on an observation,
that a disk can form around a supermassive black hole [8].

Black hole perturbation approach is one of the satisfactory tools to illustrate gravitational
waves from a compact star around a supermassive black hole. Many works have been published
in post-Newtonian expansion [9], in numerical analysis [10] to compute gravitational waves
from a test particle in a Kerr black hole. However all of the above works have been investigated in a
Fourier domain, which means that the orbital time of the particle is infinite. The basic equations
of gravitational waves are ordinary differential ones, but we have to treat “infinite” orbital time
by restricting our orbit to a special one. There is another way of computing linearized Einstein
equations: a time domain approach, which directly solves the basic equations in partially
differential ones. The main advantages of this time domain approach are that we can impose
an initial condition as an appropriate distribution of the compact object and that we can treat
an arbitrary orbit of the particle in general. There are many studies in the field of time domain
computation of the head-on collision of two black holes perturbatively (e.g. [11]), of black
hole perturbation in Schwarzschild spacetime (e.g. [12]) and in Kerr spacetime (e.g. [13]), of
perturbation in spherical stars (e.g. [14]), of perturbation in nonrotating objects [15] and of
perturbation in slowly rotating stars (e.g. [16]).

Our purpose in this paper is to investigate whether we can extract any physical property
of the disk around a supermassive black hole from gravitational waves. Several studies have
investigated extracting physical properties of a tidally disrupted star in supermassive black
holes (e.g. [17, 18]). When the disk is formed from tidal disruption of a star by a supermassive
black hole, test particle approximation to illustrate the fragments is a satisfactory tool since
the pressure gradient is no longer dominant [18]. Saijo and Nakamura [17] investigated a tidally
disrupted star falling into a black hole and found that we can extract the size of the star from
the energy spectrum of gravitational waves. Although they investigated gravitational radiation
in a Fourier domain, inappropriate description of the pre-disruption of the star, their Fourier
domain approach does almost illustrate the picture of post-disruption appropriately since most
gravitational waves are radiated at a quasi-normal ringing phase. Here we mainly focus on
gravitational waves from a disrupted star around a supermassive black hole in a time domain
to learn whether we can extract any physical property of a dust disk. We have to deal with two
types of test particle orbit in our dust disk calculation due to the angular momentum depletion.
Particles both plunging into and orbiting a Schwarzschild black in the same calculation. To
perform the above situation from the computational point of view, time domain approach is
quite easy to handle rather than Fourier domain one.

2. Black hole perturbation approach in Schwarzschild spacetime

Here we describe the basic equations of gravitational waves in order to compute gravitational
waves from a test particle around a Schwarzschild black hole. First, we describe the motion
of a test particle with a rest mass $\mu$ around a Schwarzschild black hole with a gravitational
mass $M$. In this paper, we only consider the motion of a test particle in the equatorial plane.
The motion is specified by giving the two parameters $\tilde{E}_p$ and $\tilde{L}_p$ in units of $\mu$ as $\tilde{E}_p = E_p/\mu$
and $\tilde{L}_p = L_p/\mu$, where $E_p$ and $L_p$ are the energy and the total angular momentum of a
test particle observed at infinity, respectively. With these parameters, the equation of motion of a
test particle is described as

\[
\frac{d^2 r_s}{dt^2} = \frac{1}{E_p^2} \left( 1 - \frac{2M}{r} \right) \left[ -\frac{M}{r^2} + \frac{\tilde{L}_p^2}{r^3} \left( 1 - \frac{3M}{r} \right) \right],
\]

where \( r_s = r + 2M \ln(r/2M - 1) \) is the tortoise coordinate. We also introduce the effective potential \( \tilde{V}_p \) by integrating the radial motion of a test particle (1) as

\[
\left( \frac{dr_s}{dt} \right)^2 = 1 - \left( \frac{\tilde{V}_p(r)}{E_p} \right)^2, \quad \tilde{V}_p(r) = \sqrt{\left( 1 - \frac{2M}{r} \right) \left( 1 + \frac{\tilde{L}_p^2}{r^2} \right)}.
\]

Next, we describe the basic equations of gravitational waves. These equations are obtained by linearizing the Einstein equations in Schwarzschild spacetime called Regge-Wheeler equations for odd parity and Zerilli equations for even parity. The radial wave function of gravitational waves obeys Regge-Wheeler-Zerilli equations as

\[
\left[ \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_s^2} - V_l(r)^{(\text{odd/even})} \right] \Psi_{lm}^{(\text{odd/even})}(t, r) = S_{lm}^{(\text{odd/even})}(t, r), \tag{4}
\]

where \( \Psi_{lm}^{(\text{odd/even})}, S_{lm}^{(\text{odd/even})}, V_l^{(\text{odd/even})} \) is the radial wave function, source term, and effective potential of gravitational waves, respectively. With radial wave function \( \Psi_{lm}^{(\text{odd/even})} \), the energy flux \( (dE_{gw}/dt) \) of gravitational waves at the wave zone is given by [Eq. (B.26) of Ref. [12]]

\[
\frac{dE_{gw}}{dt} = \frac{1}{64\pi} \sum_{l,m} \frac{(l+2)!}{(l-2)!} \left[ \left| \frac{\partial}{\partial t} \Psi_{lm}^{(\text{even})} \right|^2 + 4 \left| \Psi_{lm}^{(\text{odd})} \right|^2 \right]. \tag{5}
\]

3. Gravitational waves from a test particle in Schwarzschild spacetime

We use a time domain approach to solve Eq. (4). In order to test our newly developed code, we compare our results in two cases with well-known semi-analytical results. One is gravitational waves from a particle in a circular orbit around a Schwarzschild black hole, which has been studied up to the 5.5 post-Newtonian order [19]. The other is gravitational waves from a particle in a circular orbit around a Schwarzschild black hole in a Fourier domain [20]. We set the initial condition of the radial wave function as

\[
\Psi_{lm}^{(\text{odd/even})} \bigg|_{t=0} = 0, \quad \frac{\partial \Psi_{lm}^{(\text{odd/even})}}{\partial t} \bigg|_{t=0} = 0, \tag{6}
\]

with the boundary condition that there is no incoming wave to our system. Namely, we impose the outgoing wave boundary condition for the radial wave function at horizon and at a spatial infinity as

\[
\left( \frac{\partial}{\partial t} + \frac{\partial}{\partial r_s} \right) \Psi_{lm}^{(\text{odd/even})} = 0 \quad \text{at} \quad r_s = r_s^{\text{in}}, \tag{7}
\]

\[
\left( \frac{\partial}{\partial t} - \frac{\partial}{\partial r_s} \right) \Psi_{lm}^{(\text{odd/even})} = 0 \quad \text{at} \quad r_s = r_s^{\text{out}}, \tag{8}
\]
where \( r_{in} \) and \( r_{out} \) are the inner and outer edge of the computational grid.

Although the initial condition we choose (6) is somewhat artificial, the particle motion is unaffected by gravitational radiation in our test particle approximation, and hence, after a certain evolution time, the radial wave function should converge into an appropriate solution. Therefore, we simply ignore the early stage of the evolution throughout this paper. We should carefully deal with the delta function \( \delta(r - \hat{R}) \), which appears in the source term in Eq.(4). One treatment is to approximate the delta function to a step function that only covers one grid cell in the null coordinates [12]. The other is to adopt, as we do here, an approximate function to describe a delta function, which appears in the source term, as [14]

\[
\delta(r - \hat{R}) = \frac{1}{\sqrt{\pi \sigma}} \exp \left[ -\frac{(r - \hat{R})^2}{\sigma^2} \right],
\]

where \( \sigma \) is a standard deviation parameter. Note that Eq. (9) becomes exactly a delta function in the limit of \( \sigma \to 0 \). We solve Eq. (4) in second order accuracy in space and time with the scheme based on two times iterated Crank-Nicholson method. We set the step-size for space and time as \( \Delta r^* = 1.0 M \) and \( \Delta t = 0.5 \times \Delta r^* \), which satisfies the Courant condition. Although we vary the location of the inner boundary \( r^* \leq -1000 M \), the difference of gravitational wave amplitude is within the round-off error. We set the standard deviation parameter \( \sigma \) as \( \sigma = \Delta r^* \).

When \( \sigma \) is less than \( \Delta r^* \), it is not certain to specify the location of a test particle due to the less spatial grid resolution, and therefore we discard such region.

4. Gravitational wave from a dust disk

Here we explain our method for constructing a dust disk in Schwarzschild spacetime. We make five assumptions for constructing a dust disk as follows:

(i) A disk has no self-interaction with each component, i.e., a disk is composed of test particles.
(ii) A disk is thin so that the only components are located in the equatorial plane.
(iii) The density distribution for each radius of the disk is uniform.
(iv) Each radius of the ring has a Keplerian orbit, that is to say, a circular orbit.
(v) A disk is dynamically and radially stable.

From Assumption (v), our disk is only located outwards of the innermost stable circular orbit of a test particle in Schwarzschild spacetime. Note that the disk is composed of test particles from Assumption (i) in a circular orbit at each radius. Since we have a symmetry in the azimuthal direction of the particle motion, we can construct a ring from test particles using Assumption (iii). Then, we compose a disk from rings. We regard the mass distribution per unit width of the disk at radius \( r \) as \( m(r) \) given by

\[
m(r) = 2\pi C \mu r^{1-n},
\]

where \( C, \mu, \) and \( n \) represent the normalization constant that has a dimension of \( r^{n-2} \), the rest mass of a test particle, and the factor of the mass distribution, respectively. Note that \( n = 0 \) denotes the constant mass density distribution of the disk. The total mass of the disk \( \mu_{\text{disk}} \) is

\[
\mu_{\text{disk}} = \int_{6M}^{R} dr \ m(r) = \begin{cases} 
\frac{2^{-\frac{n}{2}} \pi C \mu [R^{2-n} - (6M)^{2-n}]}{2 \pi C \mu [\ln(R/M) - \ln 6]} & (n \neq 2), \\
\frac{2\pi C \mu [\ln(R/M) - \ln 6]}{2 \pi C \mu [\ln(R/M) - \ln 6]} & (n = 2).
\end{cases}
\]

4
In order to construct a disk numerically from a ring, we set each ring at the location which satisfies the energy
\[
\tilde{E}_p^j = \tilde{E}_p^1 + \frac{j - 1}{N_d - 1} (\tilde{E}_p^{N_d} - \tilde{E}_p^1) \quad (j = 1, \cdots, N_d),
\]
where \(\tilde{E}_p^1\) is the energy at \(r = 6M\), \(\tilde{E}_p^{N_d}\) is the energy at \(r = R\), \(R\) is the radius of the disk, and \(N_d\) is the number of the rings. In order to initiate the radial motion of a dust disk, we slightly deplete the angular momentum of each ring. Since we maintain the energy conserved during the depletion, we still follow the geodesic equation for a ring after the depletion.

Since we neglect the interaction between each component of the disk, gravitational waves emitted from the disk are obtained by superposing the waves emitted from each ring. For each gravitational wave emitted from each ring, we impose the same initial condition (6) and the same boundary condition (7) and (8) as we used for the case of a test particle. In order to avoid the wave reflection at the boundary after a long evolution, we take the spatial grid as \(-5000M \leq r_* \leq 10000M\). We also set the observer at \(R_{\text{obs}} = 5000M\). The radial wave function of gravitational waves \(\Psi_l^{\text{disk}}(t, r)\) is given by
\[
\Psi_l^{\text{disk}}(t, r) = \frac{1}{l_{\text{disk}}} \int_0^r \frac{m(r_0)}{r_0} \Psi_l^{\text{odd/even}}(t_0) \bigg|_{r(0)=r_0},
\]
where \(r_0\) represents the radius of the ring at \(t = 0\). Only the \(m = 0\) mode contributes to the gravitational waves from the ring due to its axisymmetric nature, where \(m\) is an index of spherical harmonics \(Y_{lm}\). Therefore the amplitude of the gravitational wave is zero until the angular momentum is depleted. We only take the \(l = 2\) mode into consideration because it is the dominant mode for the observer in the equatorial plane. In this paper we consider only radial mode due to \(m = 0\). However the radial mode has a fundamental physics and there is also in a realistic situation. Thus it is considered that we can extract some important evidence by this analysis. Additionally we define the power spectrum of the gravitational waveform as
\[
P(\omega) = \lim_{T \to \infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} [h_+(t) - i h_\times(t)] e^{-i\omega t} dt \right|^2,
\]
where \(h_+(t)\) and \(h_\times(t)\) are the perturbed metric in the wave zone, and \(\omega\) is a frequency. In our computation, we choose as large a \(T\) as possible, namely \(T = 4000M\), assuming that the phase difference apart from the periodic waves at the edge of the integration should be negligible. The valid frequency range of the power spectrum is several \(\times (2\pi/T) \lesssim \omega\), namely \(0.005 \lesssim M\omega\), in all our computations. Hereafter we calculate the gravitational waves with \(\Delta r_* = \sigma = 0.25M\) and \(\Delta t = 0.5 \times \Delta r_*\).

**Table 1.** Relationship between the initial radius of the ring and the characteristic frequency of gravitational waves from a ring.

| \(R_{\text{ring}} [M]\) | 8.0    | 10.0   | 14.0   | 20.0   | 30.0   |
|-------------------------|-------|--------|--------|--------|--------|
| \(M\omega_{gw}\)       | 0.0204| 0.0204 | 0.0141 | 0.00942| 0.00471|
5. Numerical results

First we assume that the rest mass density distribution of the disk is uniform, namely \( n = 0 \) in Eq. (10). We vary the outermost radius of the disk as \( R = 10M, 14M, 20M, \) and \( 30M \). We deplete 1% of the angular momentum of each ring at \( (t - r)/M = 0 \) to initiate the radial motion. As a matter of fact, the innermost radially stable fragments of the disk approach \( r \sim 7.85M \). Since a uniform ring with a circular orbit retains an axisymmetric nature in the system, the system does not emit gravitational waves before the depletion. We choose \( N_d = 50, 100, 175, \) and \( 300 \) for \( R = 10M, 14M, 20M, \) and \( 30M \), respectively, so that the step-size of the integration in Eq. (13) should be the same. As the distance between the closest two radii of the components at \( t = 0 \) is \( 0.035M \sim 0.26M \) depending on the radius, the components have sufficient numbers of particles to compose a disk. We find that since the waveforms (except for the disk with \( R = 10M \)) are very similar to each other, it is difficult to obtain information on the disk solely from the waveforms. However, we also find that there is a clear difference in the power spectrum of a gravitational waveform, varying the radius of the disk (Fig. 1). The value in Fig. 1 corresponds to the frequency \( M\omega \) of each peak. From these power spectra, the peak frequencies are almost the same, irrespective of the disk radius. However, for \( R = 14M, 20M, \) and \( 30M \) there is another peak frequency, which is lower than the frequency of the first peak in the case where \( R = 10M \).

In order to understand the peak frequencies in the power spectra, we calculate the characteristic frequency of a particle around a Schwarzschild black hole. We summarize the characteristic frequencies of gravitational waves from a test particle with 1% depletion of the angular momentum from a circular orbit in Table 1. From Fig. 1 and Table 1, the peak frequency \( M\omega = 0.0204 \), which appears in all disk models, corresponds to the characteristic frequency of a ring \( \omega_{gw} \) of \( R_{\text{ring}} = 8M \) and 10M, and another peak frequency lower than \( M\omega = 0.0204 \) corresponds to the characteristic frequency of a particle at the outermost edge of the disk. Note that the peak frequencies above \( M\omega = 0.0204 \) correspond to the higher-order frequency of \( M\omega = 0.0204 \). In fact, the corresponding frequencies have the following rule: \( 0.0408 = 0.0204 \times 2, 0.0628 \approx 0.0204 \times 3, 0.0832 \approx 0.0204 \times 4 \). The peak of the frequency below \( M\omega = 0.0204 \) comes from the ring at the outer edge of the disk, and the peak at \( M\omega = 0.0204 \) comes from the ring at the inner edge of the disk. The reason for no peak at the frequency below \( M\omega = 0.0204 \) of a disk with \( R = 10M \) is that the frequency corresponding to \( R = 10M \) is the same as that of the peak at \( M\omega = 0.0204 \). The phase cancellation effect plays a role in the emission of gravitational waves from the rings between the two edges of the disk. Therefore, it is possible to determine the radius of a dust disk from the power spectrum if the...
disk radius is $\gtrsim 12M$. The characteristic frequency $\omega_{gw}$ coincides with the radial frequency $\omega_p$ of a particle at $r = R_{\text{ring}}$ due to the axisymmetric motion of the disk that corresponds to the $m = 0$ mode. In Fig. 2, we plot the relation between the radial frequencies $\omega_p$ of the particle at $R_{\text{ring}}$ and initial particle radius $R_{\text{ring}}$. Note that we deplete 1% of the angular momentum of the ring to initiate the radial motion. Since our frequency resolution is $M \Delta \omega \approx 0.005$ as we mentioned before, the frequency $\omega_p$ and $\omega_{gw}$ have a good correspondence. As mentioned above, because a peak corresponds to $\omega_{gw}$ at $R_{\text{ring}} = R$ in the power spectrum of gravitational waves, it is expected that we can determine the radius of a dust disk using the relation between $\omega_p$ and $R_{\text{ring}}$. The quasi-normal ringing in the waveform represents the character of black hole; we can determine the character of the central black hole from the ringing. In a dust disk system, however, we cannot find the quasi-normal ringing in the waveform because there is a continuous inflow of fragments into a Schwarzschild black hole. Therefore the ringing is canceled by the different phase of the wave generated by each inflow fragment.

![Figure 2](image_url)

**Figure 2.** Radial frequency $\omega_p$ of the particle as a function of initial location $R_{\text{ring}}$.

Next we consider the non-constant mass density distribution of a dust disk, which means that the rest mass density of the outermost edge of the disk is lower than that of the innermost edge, i.e., $n > 0$. In this case we found that it is also difficult to obtain information on the disk from the observational waveforms. However, in the power spectra, the frequency corresponds to the peak having the same feature as the one in the case where $n = 0$. Thus, the peak frequencies in the power spectrum correspond to those of gravitational waves from a ring at $R_{\text{ring}} = R$ and at the inner region of the disk. So, we conclude that we can determine the radius of the disk irrespective of the density distribution using the power spectrum of GWs.

6. Conclusions

We study gravitational waves from a dust disk around a Schwarzschild black hole using a black hole perturbation approach in a time domain. We especially focus on whether we can obtain the radius of the disk from gravitational waves. We find that it is difficult to obtain information on the dust disk only by the observational waveforms. However, it is possible to determine the radius of the disk using the power spectrum of GWs, irrespective of its density distribution. In addition, there is also a peak corresponding to a frequency of gravitational waves from a particle located at the inner edge of the disk. Since the standard deviation of the inner edge of the disk from $R = 6M$ is a consequence of the radiated angular momentum, we could estimate the amount of radiation from the radius standard deviating from the innermost circular orbit. Although the detail disruption process requires 3D general relativistic calculation.
with the whole energy transport process, our statement from the simple model is still helpful to understand the disruption process by observing the gravitational wave spectrum.

We also mention the target of the gravitational wave source and its detectability. The typical frequency and the strength in our dust disk model are

\[ f = \frac{\omega}{2\pi} = 6.59 \times 10^{-4} \left( \frac{10^6 M_\odot}{M} \right) \left( \frac{M \omega}{0.0204} \right), \]  
\[ h = 4.78 \times 10^{-23} \left( \frac{1 \text{Mpc}}{r} \right) \left( \frac{M}{10^6 M_\odot} \right) \left( \frac{\mu/M}{10^{-6}} \right) \left( \frac{r h_+ + \mu}{0.01} \right). \]  

Therefore it is possible to detect gravitational waves from a dust disk oscillating around a supermassive black hole by LISA.

We have only investigated the outermost radius of the disk up to \( \sim 30M \) due to a limitation on computational time. However, our finding can be extrapolated into a more general astrophysical situation, such as the black hole formation phase of a supermassive star collapse. In this phase, the collapse forms a supermassive black hole and a disk [21], and the fragments of the disk could fall into the supermassive black hole due to some dissipative mechanism or instabilities of the fluid that lead to a different configuration. The interaction between each material component of the disk is considerably small for soft equation of state such as a supermassive star, the radial oscillation of the disk may take a dominant role in exciting a peak in the power spectrum of gravitational waves. Therefore it could be one source generation scenario of gravitational waves useful for determining the size of the disk.

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References
[1] B. C. Barish, in Proceedings of the 17th International Conference on General Relativity and Gravitation, edited by P. Florides, B. Nolan and A. Ottewill (World Scientific, New Jersey, 2005), p. 24.
[2] F. Melia and H. Falcke, Annu. Rev. Astron. Astrophys. 39, 309 (2001).
[3] J. P. Halpern, S. Gezari, and S. Komossa, ApJ, 604, 572 (2004).
[4] J. A. Marek, A. Lioure, and S. Bonazzola, Astron. Astrophys. 306, 666 (1996).
[5] S. Ayal, M. Livio, and T. Piran, ApJ, 545, 772 (2000).
[6] P. Laguna, W. A. Miller, W. H. Zurek, and M. B. Davies, ApJ, 410, L83 (1993).
[7] P. Diener, V. P. Frolov, A. M. Khokhlov, I. D. Novikov, and C. J. Pechick, ApJ, 479, 164 (1997).
[8] L. G. Cannizzo, H. M. Lee, and J. Goodman, ApJ, 351, 38 (1990).
[9] M. Sasaki and H. Tagoshi, Living Rev. Relativity 6, 6 (2003).
[10] T. Nakamura, K. Oohara, and Y. Kojima, Prog. Theor. Phys. Suppl. 90, Parts II and III (1987).
[11] C. O. Lousto and R. H. Price, Phys. Rev. D 57, 1073 (1998).
[12] K. Martel, Phys. Rev. D 69, 044025 (2004).
[13] E. Pazo-Avalos and C. O. Lousto, Phys. Rev. D 72, 084022 (2005).
[14] J. Ruoff, P. Laguna, and J. Pullin, Phys. Rev. D 63, 064019 (2001).
[15] A. Nagar, G. Diaz, J. A. Pons and J. A. Font, Phys. Rev. D 69, 124028 (2004).
[16] J. Ruoff, A. Stavridis, and K. D. Kokkotas, Mon. Not. R. Astron. Soc. 332, 676 (2002).
[17] M. Saijo and T. Nakamura, Phys. Rev. D 63, 64004 (2001).
[18] S. Kobayashi, P. Laguna, E. S. Phinney, and P. Meszaros, ApJ, 615, 855 (2004).
[19] T. Tanaka, H. Tagoshi, and M. Sasaki, Prog. Theor. Phys. 96, 1087 (1996).
[20] M. Davis, R. Ruffini, W. H. Press, and R. H. Price, Phys. Rev. Lett. 27, 1466 (1971).
[21] M. Shibata and S. L. Shapiro, ApJ, 572, L39 (2002).