FD Cell-Free mMIMO: Analysis and Optimization

Soumyadeep Datta*, Ekant Sharma†, Dheeraj N. Amudala*, Rohit Budhiraja* and Shivendra S. Panwar‡

*Indian Institute of Technology Kanpur †NYU Tandon School of Engineering

{soumyadeep, ekant, dheeraja, rohitbr}@iitk.ac.in, {sd3927, sp1832}@nyu.edu

Abstract—We consider a full-duplex cell-free massive multiple-input-multiple-output system with limited capacity fronthaul links. We derive its downlink/uplink closed-form spectral efficiency (SE) lower bounds with maximum-ratio transmission (MRT)/maximal ratio combining (MRC) and optimal uniform quantization. To reduce carbon footprint, this paper maximizes the non-convex weighted sum energy efficiency (WSEE) via downlink and uplink power control, and successive convex approximation framework. We show that with low fronthaul capacity, the system requires a higher number of fronthaul quantization bits to achieve high SE and WSEE. For high fronthaul capacity, higher number of bits, however, achieves high SE but a reduced WSEE.

Index Terms—Energy efficiency, full-duplex (FD), fronthaul.

I. INTRODUCTION

Cell-free (CF) massive multiple-input-multiple-output (mMIMO) technology envisions communication without cell boundaries, where all access points (APs), connected via fronthaul links to a central processing unit (CPU), communicate with all the user equipments (UEs). It promises substantial gains in spectral efficiency (SE) and fairness over conventional small-cell deployments [1], [2]. Full-duplex (FD) CF mMIMO is a relatively recent area of interest [3]–[5], where APs simultaneously serve downlink and uplink UEs on the same spectral resource. The existing FD CF mMIMO literature assumes high-capacity fronthaul links [3]–[5]. These links, however, have limited capacity, and the information needs to be quantized and sent over them. This aspect has only been investigated for HD CF mMIMO [6], [7].

Energy efficiency (EE) metric is now being commonly used to design modern wireless systems [8]. The global EE (GEE) metric, defined as the ratio of the network throughput and its total energy consumption, has been used to design CF mMIMO systems [5], [7], [9]. A UE with limited energy availability will accord a much higher importance to its EE than an another UE with a sufficient energy supply. The network-centric GEE metric cannot accommodate such heterogeneous EE requirements [8]. The weighted sum energy efficiency (WSEE) metric, defined as the weighted sum of individual EEs [8], is better suited [8], [10]. The current work, unlike the existing CF mMIMO literature [5], [7], [9], optimizes WSEE.

We next list our main contributions in this context:

1) We consider a FD CF mMIMO system with maximal ratio combining/maximal ratio transmission (MRC/MRT) at the APs, and a limited fronthaul with optimal uniform quantization. This is unlike the existing works on FD CF mMIMO [3]–[5], which consider perfect high-capacity fronthaul. We derive achievable SE expressions for uplink and downlink UEs considering multiple antennas at each AP.

2) We maximize the non-convex WSEE using successive convex approximation (SCA)-based iterative approach. We first equivalently recast the WSEE objective and then locally approximate it as a generalized convex program (GCP).

II. SYSTEM MODEL

We consider, as shown in Fig. 1, a FD CF mMIMO system where M APs serve K single-antenna HD UEs on the same spectral resource, comprising $K_u$ UEs on the uplink and $K_d$ UEs on the downlink, with $K = (K_u + K_d)$. Each AP has $N_t$ transmit antennas and $N_r$ receive antennas, and is connected to the CPU using a limited-capacity fronthaul link, which carries quantized uplink/downlink information to/from the CPU. We see from Fig. 1 that due to the FD model, • uplink receive signal of each AP is interfered by its own downlink transmit signal (intra-AP) and other APs (inter-AP).

Additionally, the UEs experience multi-UE interference (MUI) as the APs serve them on the same spectral resource.

![Fig. 1: System Model for FD CF mMIMO communications](image)

Channel description: The vector $g_{mk}^d \in \mathbb{C}^{N_t \times 1}$ is the channel from the $k$th downlink UE to the transmit antennas of the $m$th AP, and $g_{ml}^u \in \mathbb{C}^{N_r \times 1}$ is the channel from the $l$th uplink UE to the receive antennas of the $m$th AP [1]. We model them as $g_{mk}^d = (\beta_{mk}^d)\frac{1}{2}\tilde{g}_{mk}^d$ and $g_{ml}^u = (\beta_{ml}^u)\frac{1}{2}\tilde{g}_{ml}^u$. Here $\beta_{mk}^d$ and $\beta_{ml}^u \in \mathbb{R}$ are corresponding large scale fading coefficients, which are same for all antennas at the $m$th AP [1]. The vectors $\tilde{g}_{mk}^d$ and $\tilde{g}_{ml}^u$ denote small scale fading with independent and identically distributed (i.i.d.) $\mathcal{CN}(0,1)$ entries. The UDI channel between the $k$th downlink UE and $l$th uplink UE is modeled similar to [3]. as $h_{kl} = (\beta_{kl})\frac{1}{2}\tilde{h}_{kl}$, where $\beta_{kl}$ is the large scale fading coefficient and $\tilde{h}_{kl} \sim \mathcal{CN}(0,1)$ is the small scale fading. The inter- and intra-AP channels from the transmit antennas of the $i$th AP to the receive antennas

1We, henceforth, consider $k = 1$ to $K_d$, $l = 1$ to $K_u$ and $m = 1$ to $M$, to avoid repetition, unless mentioned otherwise.
of the $m$th AP are denoted as $H_{mi} \in \mathbb{C}^{N_t \times N_i}$, $i = 1$ to $M$.

**Uplink channel estimation:** We assume a coherence interval of duration $T_c$ (in s) with $\tau_c$ samples, which is divided into: a) channel estimation phase of $\tau_1$ samples, and b) downlink and uplink data transmission of $(\tau_c - \tau_1)$ samples. We consider $\tau_1^d$ and $\tau_1^u$ pilots for the downlink and uplink UEs, respectively, with $\tau_1 = \tau_1^d + \tau_1^u$. All the downlink (resp. uplink) UEs, on the same spectral resource, simultaneously transmit $\tau_1^d$ (resp. $\tau_1^u$)-length uplink pilots to the APs, which estimate the channel to perform MRT (resp. MRC). The $k$th downlink UE (resp. $l$th uplink UE) transmits pilot signals $\sqrt{\tau_1^d} \varphi_k^d \in \mathbb{C}^{\tau_1^d \times 1}$ (resp. $\sqrt{\tau_1^u} \varphi_l^u \in \mathbb{C}^{\tau_1^u \times 1}$). We assume that the pilots i) have unit norm i.e., $\|\varphi_k^d\| = \|\varphi_l^u\| = 1$; ii) have equal normalized transmit signal-to-noise ratio (SNR), $\rho_i$; and iii) are intra-set orthonormal i.e. $\langle \varphi_k^d, \varphi_l^u \rangle_H = 0 \forall l \neq l'$ and $\langle \varphi_k^d, \varphi_l^u \rangle_H = 0 \forall l \neq k'$. Therefore, $\tau_1^d \geq K_d$ and $\tau_1^u \geq K_u$.

The pilots transmitted by the downlink (resp. uplink) UEs are received by the transmit (resp. receive) antennas of the APs. The transmit antennas operate in the receive mode while estimating channel $[3]$. The APs then project the received signal on the pilot signals to compute the linear minimum-mean-squared-error (MMSE) channel estimates $[3]$, denoted as $g_m^d$ and $g_m^u$, respectively. The estimation error vectors are defined as $e_m^u = g_m^d - \tilde{g}_m^d$ and $e_m^d = g_m^u - \tilde{g}_m^u$. With MMSE estimation, $g_m^d$, $e_m^u$, and $g_m^u$, $e_m^d$ are mutually independent and their individual terms are i.i.d. with pdf $\mathcal{CN}(0, \gamma_m^d)$, $\mathcal{CN}(0, \beta_m^u)$, $\mathcal{CN}(0, \gamma_m^u)$, $\mathcal{CN}(0, \beta_m^d)$, respectively, with $\gamma_m^d = \tau_1^d \rho_i \beta_m^u + \frac{\sigma_m^d}{\tau_1^u \rho_i \beta_m^d}$ and $\gamma_m^u = \tau_1^u \rho_i \beta_m^u + \frac{\sigma_m^u}{\tau_1^d \rho_i \beta_m^d}$ $[3]$.  

**Downlink data transmission:** The CPU chooses message symbol with pdf $\mathcal{CN}(0, 1)$ for the $k$th downlink UE, which is denoted as $s_k^d$. To send this symbol to the $m$th AP via the limited-capacity fronthaul, the CPU first multiplies $s_k^d$ with a power-control coefficient $\eta_{mk}$, and then quantizes it. The $m$th AP uses the MMSE channel estimates to perform MRT on the quantized signal, and generates its transmit signal as following:

$$x_m^d = \sqrt{\rho_d} \sum_{k \in \kappa_{dm}} (\tilde{g}_m^d)_{k^*} (\tilde{a} \sqrt{\eta_{mk}} s_k^d + \tilde{c}_m^d).$$  

(1)

Here $\rho_d$ is the normalized maximum transmit SNR at each AP. The $m$th AP, due to limited fronthaul capacity, serves only the UEs in the subset $\kappa_{dm} \subset \{1, \ldots, K_d\}$, as discussed later in Section III. The CPU sends downlink symbols for UEs in the set $\kappa_{dm}$. The quantization operation is modeled as a multiplicative attenuation, $\tilde{a}$, and an additive distortion, $\tilde{c}_m^d$, for the $l$th uplink UE in the fronthaul link between the $m$th AP and the CPU, with $E\{c_m^d\} = (b - \tilde{a}) \mathcal{CN}(0, 1)$. Satisfying the average transmit SNR constraint, $E\{x_m^d)^2\} \leq \rho_d$, we get

$$\rho_d \tilde{a} \sum_{k \in \kappa_{dm}} \eta_{mk} \mathbb{E}\{|\tilde{g}_m^d|_k^2\} \leq \rho_d \Rightarrow \tilde{a} \sum_{k \in \kappa_{dm}} \gamma_m^d \eta_{mk} \leq \frac{1}{N_t}$$  

(2)

**Uplink data transmission:** The $K_u$ uplink UEs also transmit simultaneously to all the $M$ APs on the same spectral resource. The $l$th uplink UE transmits its signal $x_l^u = \sqrt{\rho_u} \varphi_l^u s_l^u$ with $s_l^u \sim \mathcal{CN}(0, 1)$ being its message symbol, $\rho_u$ being the maximum uplink transmit SNR and $\theta_l$ being the power control coefficient. To satisfy the average transmit SNR constraint

$$E\{|x_l^u|^2\} \leq \rho_u \Leftrightarrow 0 \leq \theta_l \leq 1.$$  

(3)

The FD APs not only receive uplink UE signals but also their own downlink transmit signals and that of other APs, referred to as intra-AP and inter-AP interference. The intra- and inter-AP interference channels vary extremely slowly and the FD APs can thus estimate them with very low pilot overhead. We assume that these channel estimates are imperfect. The receive antenna array of each AP, similar to [3], can therefore only partially mitigate the intra- and inter-AP interference. The residual intra-/inter-AP interference (RI) channel $H_{mi} \in \mathbb{C}^{N_t \times N_i}$, $i = 1$ to $M$, similar to [3], is modeled as a Rayleigh faded channel with entries i.i.d. $\sim \mathcal{CN}(0, \gamma_{\text{RI},mi})$. Here $\gamma_{\text{RI},mi} \triangleq \beta_{\text{RI},mi} \gamma_{\text{RI}}$ with $\beta_{\text{RI},mi}$ being the large scale fading coefficient from the $i$th AP to the $m$th AP, and $\gamma_{\text{RI}}$ being the RI power after interference cancellation. Using (1), received uplink signal at the $m$th AP:

$$y_m^u = \sum_{l=1}^{K_u} g_m^u x_l^u + \sum_{i=1}^{M} H_{mi} x_i^d + w_m^u = \sqrt{\rho_u} \sum_{l=1}^{K_u} g_m^u \tilde{a} \sqrt{\eta_{mk}} s_l^u + w_m^u.$$  

(4)

Here $w_m^u \in \mathbb{C}^{N_i \times 1}$ is the additive receiver noise at the $m$th AP with i.i.d. entries $\sim \mathcal{CN}(0, 1)$.

**Decoding of received signal on the downlink and uplink:** The $k$th downlink UE receives its desired message signal from a subset of all APs, denoted as $M_k^d \subset \{1, \ldots, M\}$. The $m$th AP serves the $k$th downlink UE iff $k \in \kappa_{dm} \Leftrightarrow m \in M_k^d$.

The $m$th AP performs MRC on the received signal, $y_m^u$, for the $l$th uplink UE using $\left\{g_m^u\right\}^H$. It quantizes the combined signal before sending it to the CPU, modeled using a constant attenuation $\tilde{a}$, and additive distortion, $\tilde{c}_m^d$, for the $l$th uplink UE in the fronthaul link between the $m$th AP and the CPU, with $E\{c_m^d\} = (b - \tilde{a}) \mathcal{CN}(0, 1)$. The CPU receives signals for the $l$th uplink UE from the APs in the subset $M_l^u \subset \{1, \ldots, M\}$, due to limited fronthaul capacity, as discussed in detail later in Section III. We denote the subset of uplink UEs served by the $m$th AP as $\kappa_{um} \subset \{1, \ldots, K_u\}$. The $m$th AP serves the $l$th uplink UE iff $l \in \kappa_{um} \Rightarrow m \in M_l^u$.

Using (1)-(4), the signal received by the $k$th downlink UE and by CPU for the $l$th uplink UE are $[5]$ and $[4]$, respectively.

**Quantization, limited fronthaul and AP selection:** The fronthaul link between the $m$th AP and the CPU uses $\nu_m$ bits to quantize the real and imaginary parts of transmit signal of the $k$th downlink UE, i.e. $\sqrt{\eta_{mk}} s_k^d$, and the uplink receive signal after MRC, i.e. $\left\{g_m^u\right\}^H y_m^u$. Due to the limited-capacity fronthaul, the $m$th AP serves only $K_{\text{um}}(\pm |\kappa_{um}|)$ and $K_{\text{dm}}(\pm |\kappa_{dm}|)$ UEs on the uplink and downlink, respectively $[5]$. For each UE, we recall that there are $(\tau_c - \tau_1)$ data samples in each coherence interval of duration $T_c$.

The fronthaul data rate between the $m$th AP and the CPU:

$$R_{th,m} = \frac{2\nu_m(K_{\text{dm}} + K_{\text{um}})(\tau_c - \tau_1)}{T_c}.$$  

(7)
\[ r_k^d = \sum_{m=1}^{M} (g_{mk}^d)^T x_m^d + \sum_{l=1}^{K_u} h_{kl} x_l^u + u_k^d = \bar{a} \sqrt{\rho_d} \sum_{m \in \mathcal{M}_r^d} \sqrt{\eta_{mk}} (g_{mk}^d)^T (g_{mq}^d)^* s_q^d + \bar{a} \sqrt{\rho_d} \sum_{m=1}^{M} q \in \mathcal{K}_{dn} \setminus \{k\} \sqrt{\eta_{mq}} (g_{mk}^d)^T (g_{mq}^d)^* s_q^d \]

message signal

+ \sum_{l=1}^{K_u} \sqrt{\rho_u} \sqrt{\theta} h_{kl} s_{l1}^u + \sum_{m=1}^{M} \sum_{q=1, q \neq l}^{K_u} \sqrt{\rho_u} \sqrt{\theta} g_{ml}^u (g_{ml}^d)^* s_{mq}^u \]

uplink downlink interference, UDI

\[ r_l^u = \bar{a} \sum_{m \in \mathcal{M}_r^u} \sqrt{\rho_u} \sqrt{\theta} h_{kl} s_{l1}^u + \bar{a} \sum_{m=1}^{M} q \in \mathcal{K}_{dn} \setminus \{k\} \sqrt{\eta_{mq}} (g_{mk}^d)^T (g_{mq}^d)^* s_q^d + u_l^d \]

multi-UE interference, MI

\[ \sum_{m \in \mathcal{M}_r^u} \sqrt{\rho_u} \sqrt{\theta} g_{ml}^u (g_{ml}^d)^* s_{mq}^u \]

AWGN at receiver

\[ r_l^u = \bar{a} \sum_{m \in \mathcal{M}_r^u} \sqrt{\rho_u} \sqrt{\theta} g_{ml}^u (g_{ml}^d)^* s_{mq}^u + \sum_{m \in \mathcal{M}_r^u} \sqrt{\rho_u} \sqrt{\theta} g_{ml}^u \bar{H} s_{mq}^u + \bar{a} \sum_{m \in \mathcal{M}_r^u} \sqrt{\rho_u} \sqrt{\theta} g_{ml}^u H s_{mq}^u + \sum_{m \in \mathcal{M}_r^u} \sqrt{\rho_u} \sqrt{\theta} g_{ml}^u \bar{H} \tilde{u}_m^u + \sum_{m \in \mathcal{M}_r^u} \sqrt{\rho_u} \sqrt{\theta} g_{ml}^u H \tilde{u}_m^u \]

residual interference (inter-AP and intra-AP), RI

Algorithm 1: Fair AP selection for disconnected UEs

1. For \( k \leftarrow 1 \) to \( K_d \) do
2. If \( \mathcal{M}_r^d = \emptyset \) then
   Sort the APs in descending order of channel gains, \( \beta_{mk}^d \), and find the AP \( n \) with the largest channel gain.
   For the \( n \)th AP, sort downlink UEs in \( \kappa_{dn} \) in descending order of channel gains and find the \( q \)th downlink UE with minimum channel gain and at least one more connected AP.
   Remove the \( q \)th downlink UE from the set \( \kappa_{dn} \) and add the \( k \)th downlink UE to it.
3. Repeat the same procedure for all the uplink UEs \( l = 1 \) to \( K_u \).

lower bounds. We rewrite the received signal at the \( k \)th downlink UE in (5) and at the CPU for the \( l \)th uplink UE in (6), as follows:

\[ r_k^d = \bar{a} \sum_{m \in \mathcal{M}_r^d} \sqrt{\rho_d} \sqrt{\theta} h_{kl} (g_{ml}^d)^* s_{mq}^d + n_\phi^d, \]

desired signal, DS

where the first term, denoted as \( DS_\phi^d \), is the “true” desired signal received over the channel mean. The term \( n^d_\phi \) denotes the effective additive noise. It contains, besides the various interferences (MUI, RI, UDI), noise (N) and quantization distortion (TQD) terms as expressed in (5)-(6), an additional beamforming uncertainty term, denoted as \( BU_{\phi}^d \), which is a “signal leakage” term received over deviation of channel from mean. It is expressed as:

\[ BU_{\phi}^d = \bar{a} \rho_u \sum_{m \in \mathcal{M}_r^d} \sqrt{\rho_u} \sqrt{\theta} h_{kl} (g_{ml}^d)^* s_{mq}^d + n_\phi^d. \]

It is easy to see that \( n_\phi^d \) is uncorrelated with \( DS_\phi^d \). We treat them as worst-case additive Gaussian noise, which is guaranteed to be a tight approximation for massive MIMO.

We define \( \gamma_{ml}^u \) as the effective additive noise. It contains, besides the various interferences (MUI, RI, UDI), noise (N) and quantization distortion (TQD) terms as expressed in (5)-(6), an additional beamforming uncertainty term, denoted as \( BU_{\phi}^d \), which is a “signal leakage” term received over deviation of channel from mean. It is expressed as:

\[ BU_{\phi}^d = \bar{a} \rho_u \sum_{m \in \mathcal{M}_r^d} \sqrt{\rho_u} \sqrt{\theta} h_{kl} (g_{ml}^d)^* s_{mq}^d + n_\phi^d. \]

III. ACHIEVABLE SPECTRAL EFFICIENCY

We now derive the achievable SE for the \( k \)th downlink UE and the \( l \)th uplink UE, denoted respectively as \( S_k^d \) and \( S_l^u \). We use \( \{d, u\} \) to denote downlink and uplink, respectively; \( \phi \in \{k, l\} \) to denote the \( k \)th downlink UE and the \( l \)th uplink UE, respectively; and \( \{m, n\} \) for \( \phi = k, \theta_l \) for \( \phi = l \). We employ use-and-then-forget (UatF) technique to derive SE
Theorem 1. An achievable lower bound to the SE for the \(k\)th downlink UE with MRT and \(l\)th uplink UE with MRC is
\[
S^d_k = \tau_f \log_2 \left( 1 + \frac{\sum_{m=1}^M A^d_{mk} \theta_{mk}}{\sum_{m=1}^M B^d_{mk} \theta_{mk} + \sum_{m=1}^M D^d_{mk} \theta_{mk} + P_{\text{fix},k} + P_{\text{th},m}} \right),
\]
\[
S^u_l = \tau_f \log_2 \left( 1 + \frac{A^u_l \theta_{l1}}{\sum_{q=1}^Q B^u_{lq} \theta_{lq} + \sum_{q=1}^Q D^u_{lq} \theta_{lq} + P_{\text{fix},l} + P_{\text{th},l}} \right).
\]

Here \(\eta^m \in \mathbb{C}^{M \times K_u}, \Theta^u \in \mathbb{C}^{K_u \times 1}\) and \(\nu^m \in \mathbb{R}^{M \times 1}\) are the variables on which the SE for each UE depends.

**Proof:** Refer to Appendix [A].

IV. WSEE MAXIMIZATION FOR FD CF MMIMO

We now maximize WSEE by calculating the optimal power control coefficients \(\{\eta^m, \Theta^u\}\). We denote \(\varepsilon \triangleq \{d, u\}\) for the downlink and uplink, respectively; \(\phi \triangleq \{k, l\}\) for the \(k\)th downlink UE and \(l\)th uplink UE, respectively; and define the individual EEs for each UE as \(\xi^\phi_{uk} = \frac{S^\phi_{uk}}{P^\phi_{uk}}\) [7], with \(B\) being the bandwidth and \(P^\phi_{uk}\) denoting the individual power consumption for each UE. The power consumed by the \(k\)th downlink UE and the \(l\)th uplink UE are obtained respectively as [7]
\[
p^d_k = P_{\text{fix},k} + N_t p_d N_0 \sum_{m=1}^M \frac{1}{\alpha^d_{mk}} |\eta^m_{mk}|^2 + P^d_{\text{th},k},
\]
\[
p^u_l = P_{\text{fix},l} + p_u \sum_{m=1}^M \frac{1}{\alpha^u_l} |\theta_{lq}|^2 + P^u_{\text{th},l}.
\]

Here \(\alpha^d_{mk}, \alpha^u_l\) are power amplifier efficiencies at the \(m\)th AP and the \(l\)th uplink UE respectively [3], \(N_0\) is noise power. The \(P_{\text{fix},k}\) is the fixed per-UE power consumed by the APs i.e.,
\[
P_{\text{fix},k} = \frac{1}{K} \sum_{m=1}^M \left( P_{0,m} + (N_t + N_r) P_{\text{th},m} + P_{\text{fix},m} \right).
\]

Here \(P_{\text{fix},k}\) is the power consumed by the \(k\)th AP that has a fixed component \(P_{0,m}\), and a traffic-dependent component, proportional to the data rate \(R_{\text{th},m}\), attaining a maximum of \(P_{\text{th},m}\) at capacity \(C_{\text{th},m}\).

The WSEE is defined as weighted sum of individual EEs [3]
\[
\text{WSEE} = \sum_{k=1}^{K_u} w^d_k \xi^d_{uk} + \sum_{l=1}^{K_u} w^u_l \xi^u_{ul} \triangleq B \left( \sum_{k=1}^{K_u} w^d_k S^d_k (\eta, \Theta, \nu) + \sum_{l=1}^{K_u} w^u_l S^u_l (\eta, \Theta, \nu) \right),
\]

where \(w^d_k\) are weights assigned to downlink and uplink EEs to account for heterogeneous EE requirements of UEs [10].

The WSEE maximization problem is formulated as
\[
P_1 : \max_{\eta, \Theta, \nu} B \left( \sum_{k=1}^{K_u} w^d_k S^d_k (\eta, \Theta, \nu) + \sum_{l=1}^{K_u} w^u_l S^u_l (\eta, \Theta, \nu) \right)
\]
\[
\text{s.t. } S^d_k (\eta, \Theta, \nu) \geq S^d_{\text{ok}}, S^u_l (\eta, \Theta, \nu) \geq S^u_{\text{ol}}, \quad R_{\text{th},m} \leq C_{\text{th},m}, \quad (17a)
\]
\[
\lambda_k \geq 0, \quad \nu^m \geq 0, \quad \lambda^u \geq 0. \quad (17b)
\]

The quality-of-service (QoS) constraints in (17a) guarantee a minimum SE, \(S^d_{\text{ok}}\) and \(S^u_{\text{ol}}\), for each downlink and uplink UE respectively. We observe that the number of quantization bits, \(\nu\), makes \(P_1\) a difficult integer optimization problem [7]. We therefore fix \(\nu\) such that it satisfies the first constraint in (17b) [7] and numerically investigate \(\nu\) in Section V. We now linearize the non-convex objective in \(P_1\), after omitting the constant \(B\), using epigraph transformation [11]:
\[
P_2 : \max_{\eta, \Theta, \nu} \sum_{k=1}^{K_u} \frac{K_u}{u^d_k} f_k^d (\eta, \Theta, \nu) + \sum_{l=1}^{K_u} \frac{K_u}{u^u_l} f_l^u (\eta, \Theta, \nu)
\]
\[
\text{s.t. } f_k^d \leq \frac{S^d_k (\eta, \Theta)}{p^d_k (\eta)}, \quad f_l^u \leq \frac{S^u_l (\eta, \Theta)}{p^u_l (\Theta)}, \quad (18)
\]

Here, \(f^d_k \in [f_1^d, \ldots, f_{K_u}^d] \in \mathbb{C}^{K_u \times 1}\) are slack variables representing the individual EEs, \(\xi^o\), divided by bandwidth \(B\). We simplify non-convex constraints (17a) and (17b) as follows:

- We introduce slack variables \(\Psi^d \triangleq [\psi_1, \ldots, \psi_{K_u}], \zeta^u \triangleq [\zeta_1, \ldots, \zeta_{K_u}], \lambda^d \triangleq [\lambda_1, \ldots, \lambda_{K_u}] \in \mathbb{C}^{K_u \times 1}\), representing square roots of individual SEs, \(S^o\); individual signal-to-interference-and-noise-ratios (SINRs), and square roots of numerators of individual SINRs, respectively. We substitute \(C \triangleq \{c_{mk} \triangleq \frac{1}{\sqrt{\tau_{mk}}} \in \mathbb{C}^{M \times K_u}\}\), for concave variable \(\sqrt{\tau_{mk}}\).
- We now use (12) and (13) respectively, to expand the individual SEs, \(S^o\), and the power consumptions, \(P^o\), in terms of the optimization variables \(C, \Theta, f^d, \Psi^d, \zeta^u, \lambda^d\).
- To linearize the right-hand side of jointly convex constraints, at the \(n\)th SCA iteration, we substitute first-order Taylor approximations, \(f_k^d \approx 2 f_k^{(n)} (f_k^{(n)} / f_k^{(n)}) f_k \leq \Lambda (f_k^{(n)}) / f_k^{(n)}\), as global under-estimators [11], to recast \(P_2\) into a GCP as follows
The residue after the \( n \)th iteration, \( r_{SCA}^{(n)} \), has a magnitude
\[
\| r_{SCA}^{(n)} \| = \sqrt{\| C^{(n+1)} - C^{(n)} \|^2_F + \| (\Theta^{(n+1)} - \Theta^{(n)}) \|^2_F}.
\]
We provide an iterative method to solve P3 in Algorithm 2.

Algorithm 2: WSEE maximization algorithm

Input:
1. Initialize power control coefficients \( \{C, \Theta\} \) \((r)\) by allocating equal power to all downlink UEs being served and full power to all uplink UEs.
2. Initialize \((f^d, f^u, \Psi^d, \Psi^u, \zeta^d, \zeta^u, \lambda^d, \lambda^u, C, \Theta) (r)\) by replacing \((19), (20), (38), (39), (39)\) by equality.

Output: Globally optimal power control coefficients \( \{C, \Theta\} \) \((r)\).

while \( \| r_{SCA}^{(n)} \| \leq \varepsilon_{SCA} \) \((convergence threshold)\) do

1. Solve P3 for the \((n)\)th SCA iteration to obtain optimal variables, \((f^d, f^u, \Psi^d, \Psi^u, \zeta^d, \zeta^u, \lambda^d, \lambda^u, C, \Theta) (r)\) \((n)\).
2. Assign the SCA iterates for the \((n+1)\)th iteration, \((f^d, f^u, \Psi^d, \Psi^u, \zeta^d, \zeta^u, \lambda^d, \lambda^u, C, \Theta) (r)\) \((n+1)\).

end

V. Simulation results

We now numerically investigate the performance of a FD CF mMIMO system with limited capacity fronthaul links using the proposed optimization approach. We assume a realistic system model wherein \( M \) APs, \( K_d \) downlink UEs and \( K_u \) uplink UEs are scattered in a square of size \( D \times D \) km. To avoid the boundary effects, we wrap the APs and UEs around the edges. The large-scale fading coefficients for the channels between the APs and the downlink and uplink UEs, denoted as \( \beta_{md} \) and \( \beta_{um} \) respectively, are modeled as
\[
\beta_{md} = 10^{-\frac{d_{md}}{10}} \quad \text{and} \quad \beta_{um} = 10^{-\frac{d_{um}}{10}}. \quad (20)
\]

Here \( 10^{-\frac{d_{md}}{10}} \) and \( 10^{-\frac{d_{um}}{10}} \) are log-normal shadowing factors having a standard deviation \( \sigma_{d} \) in dB and \( z_{md} \) and \( z_{um} \) follow a two-components correlated model. The path losses, \( PL_{md} \) and \( PL_{um} \)= \((\text{in dB})\) follow a three-slope model. We, similar to [3], model the large-scale fading coefficients for the inter-AP RI channels, i.e., \( \beta_{rlm} \), \( \forall l \neq m \), as in (20), and assume that the large-scale fading for the intra-AP RI channels, which experience no shadowing, are modeled as \( \beta_{rlm} = 10^{-\frac{d_{rlm}}{10}} \). The inter-UE large scale fading coefficients, \( \beta_{rl} \), are also modeled similar to (20). We consider, for brevity, the same number of quantization bits \( \nu \), and the same capacity \( C_{fh} \) on all fronthaul links. We, henceforth, denote the transmit powers on the downlink and uplink as \( p_d (= p_d N_0) \) and \( p_u (= p_u N_0) \) respectively, and the pilot transmit power as \( p_p (= p_p N_0) \). We fix the system model and power consumption model parameters, similar to [1], [3], [7], as in Table 1.

Table 1: FD CF mMIMO system and power consumption parameters

| Parameter | Value |
|-----------|-------|
| \(D, r_c, T_c\) | 1 km, 200, 1 ms |
| \(\sigma_d, B\) | 2 dB, 20 MHz |
| Fronthaul parameters \(\nu, C_{fh}\) | 2, 100 Mbps |
| B, PL_{by} | \(-20, -81.84\) |
| P_{bh}, P_{b,m}, P_{c,m} = P_{c,b} = P_{c,1}, p_{t} (in W) | 10, 0.825, 0.2, 0.2 |
| N_0, \alpha_m = \alpha_g' | \(-121.4\) dB, 0.4 |

Validation of achievable SE expressions: We consider an FD CF mMIMO system with \( M = 32 \) APs, each having \( N_t = N_r = 2 \) transmit and receive antennas, and \( K_d = K_u = 10 \) uplink and downlink UEs and consider equal transmit power for uplink and downlink data transmission, i.e., \( p_d = p_u = p \). We verify in Fig. 2a the tightness of the derived SE lower bound in [(2) - (3)], labeled as LB, by comparing it with the numerically obtained ergodic SE labeled as upper-bound (UB) as it requires instantaneous CSI. The large-scale fading coefficients are set to unity, i.e., \( \beta_{md} = \beta_{um} = 1, \forall k \in K_d, l \in K_u \). We, similar to [1], allocate equal power to all downlink UEs and full power to all uplink UEs, i.e., \( \eta_{mk} = (b N_t (\sum_{k \in K_d} d_{mk})^{-1})^{-1}, \forall k \in K_d \) and \( \theta_1 = 1 \). We consider two cases: i) perfect high-capacity fronthaul with \( a = b = 1 \), and ii) limited fronthaul links, with \( \nu = 2 \) quantization bits and capacity \( C_{fh} = 10 \) Mbps. We see that the derived lower bound is tight for both perfect and limited fronthaul links. With limited fronthaul, the sum SE significantly drops, as quantization distorts the signal and limits the number of UEs that each AP can serve (see [9]).

WSEE maximization - optimal system parameters: We now study WSEE variation with system parameters. We consider \( M = 32 \) APs each having \( N_t = N_r = 2 \) antennas, \( K_d = K_u = 10 \) downlink and uplink UEs and QoS constraints \( S_{ok} = S_{ol} = 0.1 \) bits/s/Hz, unless mentioned otherwise.

We plot in Fig. 2b the WSEE versus transmit power \( p \), which is taken to be equal for downlink and uplink, i.e., \( p_d = p_u = p \). We consider optimal power allocation (OPA) in Algorithm 2 labeled as “OPA”, and compare it with three sub-optimal power allocation schemes: i) equal power allocation of type 1, labeled as “EPA 1”, where \( \eta_{mk} = (b N_t (\sum_{k \in K_d} d_{mk})^{-1})^{-1}, \forall k \in K_d \) and \( \theta_1 = 1 \) (7), (9), ii) equal power allocation of type 2, labeled as “EPA 2”, where \( \eta_{mk} = (b N_t (K_d d_{mk})^{-1})^{-1}, \forall k \in K_d \) and \( \theta_1 = 1 \) (9), and iii) random power allocation, labeled as “RPA”, where power control coefficients are chosen randomly from a uniform distribution between 0 and the “EPA 1” value. We note that existing literature has not yet optimized the WSEE metric for CF mMIMO systems, hence we can only compare with above sub-optimal schemes. We note that our proposed approach far outperforms the baseline schemes.

We next quantify in Fig. 2c the joint variation of WSEE and the sum SE, with the number of quantization bits \( \nu \), in the fronthaul links. We note that the WSEE is calculated using Algorithm 2. We consider transmit power \( p_d = p_u = p = 30 \) dBm and take two different cases: i) high fronthaul capacity, \( C_{fh} = 100 \) Mbps, which is sufficiently high to support all the UEs, and ii) limited fronthaul capacity, \( C_{fh} = 10 \) Mbps, which limits the number of UEs a single AP can serve. We observe that for \( C_{fh} = 100 \) Mbps, the WSEE falls with increase in \( \nu \), even though the corresponding sum SE increases. For \( C_{fh} = 10 \) Mbps, the sum SE and the WSEE simultaneously increase with increase in \( \nu \). To explain this behaviour, we note that increasing \( \nu \) improves the sum SE for both the cases due to a reduction in the quantization attenuation and distortion. For \( C_{fh} = 100 \) Mbps, the APs serve all the UEs, i.e., \( K_d = K_u \), and \( K_{um} = K_{ud} \), so increasing \( \nu \) linearly increases the fronthaul data rate, \( R_{fh} \) (see [7]). This, as seen from (16), increases the traffic-dependent fronthaul power consumption. Using lower number (1-2) of quantization bits is therefore more energy-efficient, as it provides sufficiently good SE with
a low energy consumption. However, for $C_{fh} = 10$ Mbps, $K_{um}$ and $K_{dm}$ have an upper limit, given by (9), which is inversely related to $\nu$. The product, $\nu(K_{um} + K_{dm})$, remains nearly constant for all values of $\nu$. Thus, $R_{fh}$, (see (7)) doesn’t increase with increase in $\nu$ and remains close to the capacity, $C_{fh}$. The traffic-dependent fronthaul power consumption, given in (16), hence, remains close to $P_h$. A higher number (3-4) of quantization bits therefore provides a higher sum SE and hence, also maximizes the WSEE.

VI. CONCLUSION

We derived the achievable SE expressions for a FD CF mMIMO wireless system with fronthaul quantization. We optimized the non-convex WSEE using SCA framework which in each iteration solves a GCP. We numerically investigated the WSEE dependence on various system parameters which gives important insights to design energy-efficient systems.

APPENDIX A

We now derive the achievable SE for the $k$th downlink UE in (12) and the $l$th uplink UE in (13). We use $\varepsilon \triangleq \{d, u\}$ to denote downlink and uplink, respectively; $\xi \triangleq \{t, r\}$ to denote transmit and receive, respectively; $\phi \triangleq \{k, l\}$ to denote the $k$th downlink UE and the $l$th uplink UE, respectively; and $v_{m,\phi} \triangleq \{\{m, k\} \forall \phi = k, \{m, l\} \forall \phi = l\}$. From Section III we know that $g_{m,\phi} = \tilde{g}_{m,\phi} + e_{m,\phi}$, where $\tilde{g}_{m,\phi}$ and $e_{m,\phi}$ are independent and $E\{\|\tilde{g}_{m,\phi}\|^2\} = N_\xi \gamma_{m,\phi}$. We express the desired signal for the $k$th downlink UE and the $l$th uplink UE as

$$E\{|DS_k|^2\} = a^2 \rho_s E\left\{ \sum_{m \in M_\phi} \sqrt{v_{m,\phi}} E\{ (\tilde{g}_{m,\phi})^T (\tilde{g}_{m,\phi})^* s_m^2 \} \right\}$$

$$= a^2 N_\xi \rho_s \left( \sum_{m \in M_\phi} \sqrt{v_{m,\phi}} \|\tilde{g}_{m,\phi}\|^2 \right).$$

(21)

We now calculate the beamforming uncertainty for the $k$th downlink UE and the $l$th uplink UE as $E\{|BU_k|^2\}$

$$= a^2 \rho_s \sum_{m \in M_\phi} E\left\{ \sqrt{v_{m,\phi}} ((\tilde{g}_{m,\phi})^T (\tilde{g}_{m,\phi})^*) - E\{ (\tilde{g}_{m,\phi})^T (\tilde{g}_{m,\phi})^* \} \right\}^2$$

$$= a^2 \rho_s \sum_{m \in M_\phi} \sqrt{v_{m,\phi}} (N_\xi (\gamma_{m,\phi} + 1) (\gamma_{m,\phi})^2 + N_\xi (\gamma_{m,\phi} (\beta m,\phi - \gamma_{m,\phi} - N_\xi (\gamma_{m,\phi} (\beta m,\phi - \gamma_{m,\phi}))^2$$

$$= a^2 N_\xi \rho_s \sum_{m \in M_\phi} \sqrt{v_{m,\phi}} \beta m,\phi \gamma_{m,\phi}.$$