Shakeup spectrum in a two-dimensional electron gas in a strong magnetic field.

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Abstract

The shakeup emission spectrum in a two-dimensional electron gas in a strong magnetic field is calculated analytically. The case of a localized photocreated hole is studied and the calculations are performed with a Nozières-De Dominicis-like Hamiltonian. The hole potential is assumed to be small compared to the cyclotron energy and is therefore treated as a perturbation. Two competing many-body effects, the shakeup of the electron gas in the optical transition, and the excitonic effect, contribute to the shakeup satellite intensities. It is shown, that the range of the hole potential essentially influences the shakeup spectrum. For a short range interaction the above mentioned competition is more important and results in the shakeup emission quenching when electrons occupy only the lowest Landau level. When more than one Landau level is filled, the intensities of the shakeup satellites change with magnetic field nonmonotonically. If the interaction is long range, the Fermi sea shakeup processes dominate. Then, the satellite intensities smoothly decrease when the magnetic field increases and there is no suppression of the shakeup spectrum when the only lowest Landau level is filled. It is shown also that a strong hole localization is not a necessary condition for the SU spectrum to be observed. If the hole localization length is not small com-
pared to the magnetic length, the SU spectrum still exists. Only the number of contributions to the SU spectrum reduces and the shakeup processes are always dominant.

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I. Introduction

It is known that many body processes can have a dramatic effect on the optical spectra of a degenerate electron gas. One of the most notable examples is optical absorption and emission processes in metals where a localized hole state is involved, the so-called X-ray Fermi edge singularity (FES) phenomena [1]- [3]. This effect is dominated by two competing contributions, the Anderson orthogonality catastrophe [4] due to the Fermi sea shakeup and the Mahan “exciton” [5], due to the electron-hole interaction. The first effect leads to a strong reduction of the absorption and the emission intensity close to the X-ray absorption edge, while the second one is manifested by the power law divergence at the threshold energy [6]. The shakeup effect that occurs in optical transitions in which excitations of the electron gas are created, also leads to a low-energy tail in the emission spectrum [7]. The same type of processes were studied in semiconductor quantum well structures with a degenerate two-dimensional electron gas (2DEG). The FES effect in the luminescence spectra of modulation-doped InGaAs-InP quantum wells was observed in Refs. [8], [9]. The effect was attributed to the hole localization due to the alloy fluctuations.

The experiments in 2DEG structures were also carried out in the quantum Hall regime. The case of a 2DEG in a semiconductor in a strong perpendicular magnetic field essentially differs from that of metals, since in the 2D case the conduction-band states are quantized into discrete Landau levels (LLs). As a result the photoluminescence spectrum turns into a series of equally spaced magnetoluminescence (ML) lines with the spacing given by the electron cyclotron energy $\hbar \omega_B = eB/m_e c$. In addition, the low-energy photoluminescence tail is also transformed into a series of satellite lines, so-called shakeup (SU) satellites, which are below the main ML spectrum. The reason is that in the presence of a magnetic field the shakeup process generates discrete inter-LL excitations, “magnetoplasmons” (MPs), in which an electron is promoted from one LL to a higher one, across the Fermi energy.

For the first time, the FES effect (a strong enhancement of the intensities of the ML spectral lines towards the Fermi energy) and the SU satellites were reported in Ref. [8] and
Ref. [10], respectively. Further experimental investigations of the SU spectra in 2D structures were carried out in Refs. [11]-[19]. In early Refs. [9], [11] was found that the strength of the shakeup is controlled by the localization of the photocreated hole. In addition, a strong suppression of shakeup processes at high magnetic fields was observed and qualitatively discussed in Refs. [9], [11]. The discussion is based on the theoretical results for the hole Green’s function at zero magnetic field which are obtained in Ref. [20]. In recent Refs. [13], [15] periodic changes of the shakeup line intensities in the filling factor were reported. In Ref. [16] a role of the hole localization on the main and SU magnetoluminescence spectra was studied. In Ref. [14] 2D structures with delocalized photocreated holes were investigated and a strong suppression of shakeup processes was observed when electrons occupy only the lowest LL. The shakeup processes due to the creation of the spin-wave excitations were studied in Refs. [17], [18].

A theory of optical and magneto-optical phenomena in a 2DEG was given in Refs. [21]-[23], where numerical calculations of the optical spectra are presented. In Ref. [22] the influence of the hole mass, temperature and the electron-hole interaction strength on the main magneto-optical spectra is discussed. It is noted also that the SU spectrum is more pronounced when the interaction strength increases. Recently the FES problem in a 2DEG was solved analytically for the case of a weak magnetic field when a large integral number of Landau levels are filled [24]. In Ref. [14] the theoretical model of the SU spectrum was developed for the case of an unlocalized photocreated hole. Observations of a strong suppression of the shakeup effects at a filling factor close to \( \nu = 1 \) are explained by the competition between the interband and intraband scattering processes.

An analytical description of the SU spectrum for the case of a localized hole is absent to-day, to our knowledge. At that reason, some theoretical problems are open up to now. For example, the role of the excitonic effect and hole localization on the SU spectrum is not clear. Usually, it is assumed that the SU spectrum is due to the Anderson orthogonality catastrophe and the contribution of the Mahan effect is ignored (see, e.g., Refs. [9], [11]). On the other hand, in Ref. [24] is shown that in the case of the main absorption and emission
spectra a balance between the Mahan effect and the orthogonality catastrophe is important, similarly to the case of the X-ray problem in metals. In the early experimental works [9], [11] was supposed also that both the FES effect and SU spectrum are very sensitive to the localization degree of the photocreated hole, as noted above. However, this assumption is in contradiction to the numerical calculations in Ref. [22] and the experimental results in Refs. [14], [16]. In addition, in experiments some peculiarities of the SU spectrum noted above (a strong suppression of shakeup processes at high magnetic fields [9], [11] and the periodic changes of the shakeup line intensities in the filling factor [13], [15]) are observed. These properties of the SU spectrum can not be explain by a simple discussion given in Refs. [9], [11] and require an analytical treatment.

In this paper we present an analytical study of the SU emission spectrum due to transitions between a localized level and a 2DEG, subjected to a strong magnetic field. Calculations of the shakeup spectrum are performed with a Nozières - De Dominicis-like Hamiltonian [21], [24], in which we treat the interaction part as a perturbation. The electron-hole interaction is assumed to be much smaller than the cyclotron energy. In this case the SU spectrum can be calculated perturbatively, as the SU satellites are due to the creation (by the optical transition) of the electron-"hole" pairs at different Landau levels. We show that the SU spectrum is determined by both the Anderson orthogonality and the Mahan effect. In the general case the contribution due to the Fermi sea shakeup is dominant. However, for the special case of a short range hole potential, the excitonic effect competes with the shakeup effect. This competition results to a strong supression of the SU spectrum if electrons occupy only the lowest Landau level. We show also that a strong hole localization is not a necessary condition for the SU spectrum to be observed. In addition, we discuss the filling factor dependence of the SU lines, as well as the influence of the hole potential range.
II. PROBLEM FORMULATION

We consider here a model that is similar to the one used in the study of core level optical spectra in metals [5], [6], [25], since most of the experiments on magneto-optical spectra in a 2DEG were carried out in samples with a disorder due to composition fluctuations. In such structures some amount of holes are localized due to alloy fluctuations and behave like the core holes in the X-ray problem. Note that a very small hole localization length of 10Å − 30Å was reported in Ref. [11] for an InGaAs/InP quantum well. For comparison, in GaAs crystals the magnetic length $l_B \geq 50\text{Å}$ at a magnetic field of $B \leq 25T$.

Thus, we assume that the photocreated hole state is nondegenerate, and strongly localized, with a localization length $a$ much smaller than the magnetic length $l_B$. The electron-electron interaction in the conduction band is ignored and the Hamiltonian of the problem consists of three contributions [21], [24], [25].

$$H = H_0 + H_h + H_{int}. \quad (1)$$

The first term describes noninteracting electrons, taken here as spinless, and the operator $c_n^\dagger$ creates an electron in the Landau state $|n\rangle$ with an energy of $\varepsilon_n$,

$$H_0 = \sum_n \varepsilon_n c_n^\dagger c_n. \quad (2)$$

The second term describes the hole, and the operator $d^\dagger$ creates a hole in a nondegenerate state $|h\rangle$, with a negative energy $E_0$, measured from the bottom of the conduction band at $B = 0$,

$$H_h = E_0 d^\dagger d. \quad (3)$$

The third term describes the coupling between the conduction electrons and the hole which creates a potential $V(\vec{r})$,

$$H_{int} = d^\dagger d V, \quad V = \sum_{n,n'} V_{nn'} c_n^\dagger c_{n'}. \quad (4)$$
The emission spectrum is given by the real part of the Fourier transform of the response function

\[ I(\omega) = N \times \text{Re} \int_0^\infty dt \, e^{-i\omega t} F^e(t), \quad F^e(t - t') = \sum_{nn'} M_n M_{n'}^* F_{nn'}^e(t - t'), \tag{5} \]

where \( N \) is a normalization constant. The optical matrix element \( M_n \) describes the electron-photon coupling, \( M_n = P_{vc} \langle n|h \rangle \), where the interband momentum matrix element \( P_{vc} \) incorporates the effect of the lattice periodicity and \( \langle n|h \rangle \) is the overlap between the electron and hole envelope functions. The two-particle Green’s function is given by

\[ F_{nn'}^e(t - t') = \langle \hat{i}_h^e | T \{ c_n^\dagger(t) d(t') c_{n'}(t') \} | \hat{i}_h^e \rangle, \tag{6} \]

where \( T \) is time-ordering operator, the operators \( c_n(t) = e^{iHt} c_n(0) e^{-iHt} \), and \( d(t) = e^{iHt} d(0) e^{-iHt} \), are taken in the Heisenberg representation. The state \( | \hat{i}_h^e \rangle \) is the initial state of the electron system in the presence of a localized hole. This state is a product of the hole state \( | h \rangle \), and the Slater determinant \( | \hat{i}^e \rangle \) of single-particle electron states, which differ from the Landau electron states \( | n \rangle \) due to the hole potential, and are the eigenstates of the Hamiltonian \( H^e = H_0 + V \).

It was shown in Ref. [6] that the calculation of the optical spectra reduces to a “one-body” problem when the conduction electrons are scattered by a transient potential due to the hole, switched on/off suddenly at the time of the transition. As a result the two-particle Green’s function in the time representation is \( \text{exactly} \) a product of the hole Green’s function, which accounts for the shakeup effect, and the so-called transient electron Green’s function which accounts for the exciton effect,

\[ F_{nn'}^e(t - t') = -g_{nn'}^e(t', t | t, t') D^e(t - t'). \tag{7} \]

In Eq. (7) the hole Green’s function is but the average of the evolution operator,

\[ D^e(t - t') = e^{-iE_0(t - t')} \langle \hat{i}_h^e | \hat{S}(t, t') | \hat{i}_h^e \rangle, \quad \hat{S}(t, t') = T \, \exp \{ i \int_{t'}^t d\tau \tilde{V}(\tau) \}, \tag{8} \]

where \( \tilde{V}(\tau) \) is the interaction \( V \) from Eq. (4), in the Heisenberg representation with the Hamiltonian \( H^e \). The transient electron Green’s function is given by
and it satisfies the Dyson equation, that is known in the X-ray problem as the Nozières-De Dominicis equation [6]:

\[ g_{nn'}^e(\tau, \tau' | t, t') = G_{nn'}^e(\tau - \tau') + i \int_{t'}^t d\tau'' \sum_{ss'} G_{ns}^e(\tau - \tau'') V_{ss'} g_{ns'}^e(\tau'', \tau' | t, t'). \]

The conduction electron Green’s function \( G_{nn'}^e(t) \) entering Eq. (10) is defined by

\[ G_{nn'}^e(t - t') = \langle i^e | T \{ \tilde{c}_n(t) \tilde{c}_n^\dagger(t') \} | i^e \rangle, \]

and it obeys the Dyson equation,

\[ G_{nn'}^e(t) = G_{n}^{(0)}(t) \delta_{nn'} - i \sum_{n'n} \int_{-\infty}^{+\infty} d\tau \ G_{n}^{(0)}(\tau) V_{nn''} G_{n''n'}^e(t - \tau), \]

where the bare electron propagator

\[ G_{n}^{(0)}(t) = e^{-i\varepsilon nt} \left[ \theta(t) - n(\varepsilon_n) \right], \]

and \( n(\varepsilon_n) \) are the electron occupation numbers.

III. GENERAL RESULTS

As was noted above, the low-energy satellites of the emission spectrum are supposedly due to the shakeup effect, when the optical transition creates “magnetoplasmons” [9], [11]. (In what follows the MPs are the free electron-“hole” pairs, since the electron-electron interaction in the conduction band is neglected.) The energies which are needed to create the MPs are determined by the cyclotron energy \( \hbar \omega_B \). Therefore, one can expect that in the case of a strong magnetic field the interaction Hamiltonian \( H_{int} \) in Eq. (1) can be treated as a perturbation if the hole potential is much smaller than \( \hbar \omega_B \). Following this approximation, we calculate the hole Green’s function (8) and the transient electron Green’s function (9) up to the second-order in the interaction potential \( H_{int} \), Eq. (4), by iterations of Eqs. (12) and (10), respectively. Then, we collect the same order terms in the two-particle Green’s
function (7) and perform its Fourier transform. The hole level shift due to the electron-hole interaction is ignored. Finally, we find that the corrections to the main ML spectrum appear already in the first order,

$$I_{ML}(\omega) = N \sum_n \delta(h\omega - h\omega_n) n(\varepsilon_n) \left\{ |M_n|^2 + 2 M_n \sum_s \frac{V_{ns}}{\varepsilon_n - \varepsilon_s} M^*_s [1 - n(\varepsilon_s)] \right\}. \quad (14)$$

The first term in Eq. (14) is a well known result for the emission spectrum in the one-electron description of the optical process. All the occupied LLs with $M_n \neq 0$ contribute to this term and the energy $h\omega_n = |E_0| + \varepsilon_n$ determines the spectral position of the $n$th ML line. The second term is due to the lowest-order many body corrections to the main ML spectrum. This term has its origin in the transient electron Green’s function and determines the change of the ML line intensity due to the hole potential. At zero temperature quantum numbers $n$ and $s$ refer to different LLs, $n$ refers to occupied states, while $s$ to empty ones. The closer the $n$th ML line is to the Fermi energy, the larger is its intensity change, due to the corresponding energy differences in the denominators.

Unlike the main spectrum Eq. (14), the shakeup emission spectrum is absent in a single-particle approximation, since it is due to many-body effects. We find that the SU spectrum appears in the second order of the perturbation and is given by

$$I_{SU}(\omega) = N \sum_{n,n',s} \delta(h\omega - h\omega_n + (\varepsilon_s - \varepsilon_{n'})) \left\{ |M_n|^2 \frac{|V_{sn'}|^2}{(\varepsilon_s - \varepsilon_{n'})^2} ight\}$

$$- M_n \frac{V_{ns}V_{sn'}}{(\varepsilon_s - \varepsilon_n)(\varepsilon_s - \varepsilon_{n'})} M^*_n \{n(\varepsilon_n) n(\varepsilon_{n'}) [1 - n(\varepsilon_s)] \}. \quad (15)$$

As in Eq. (14), for zero temperature the quantum numbers $s$ refer to empty states, while $n$ and $n'$ refer to occupied ones. Therefore, the $\delta$–functions in Eq. (15) ensure that the shakeup satellites appear below the main ML peaks. The first term in the brackets arises from the hole Green’s function (8), and describes the Fermi sea shakeup effect in the emission process. The second term has its origin in the electron transient propagator (9), and is due to the interaction of the hole with the extra electron that is created in the optically active Landau state with $M_n \neq 0$, in the process of absorption. Thus, the SU spectrum is a
result of a balance between the contribution of the orthogonality catastrophe and that of the excitonic effect.

In what follows we present a more detailed study of the SU spectrum, Eq. (15). To proceed, we rewrite this equation using the axial gauge representation for the Landau states $|n\rangle = |Nm\rangle$ with a center at the hole position, where $N$ is the Landau number and $m$ is the angular momentum. Then we arrive at

$$I_{SU}(\omega) = N \sum_{k=1}^{\infty} \delta(\hbar \omega - \hbar \omega_0 + k \hbar \omega_B) \left[ A_k - M_k \right],$$

where

$$A_k = \sum_{N,m,N',m'} \sum_{m_s} |M_{Nm}|^2 \frac{|V_{(N+N'+k)m_s,N'm'}|^2}{(\varepsilon_{N+N'+k} - \varepsilon_{N'})^2} F_{NN'k},$$

$$M_k = \sum_{N,m,N',m'} \sum_{m_s} M_{Nm} V_{Nm(N+N'+k)m_s} V_{N(N'+N+k)m_s,N'm'} M^*_{N'm'} F_{NN'k},$$

with

$$F_{NN'k} = n(\varepsilon_N) n(\varepsilon_{N'}) \left[ 1 - n(\varepsilon_{N+N'+k}) \right].$$

It is clearly seen from Eq. (16) that the $k$th satellite ($SU_k$) of the shakeup spectrum appears at an energy $k \hbar \omega_B$ below the lowest-energy main ML line of energy $\hbar \omega_0 = |E_0| + \frac{1}{2} \hbar \omega_B$. In Eq. (17) the summation over the LL numbers $N$ and $N'$ refers to the occupied LLs. In the shakeup term $A_k$ the number $N$ indicates the LL which contributes to the emission process, while $N'$ indicates the LL which is involved in the MP creation. Note that the state $|N'm'\rangle$ is not necessarily optically active. In the exciton term $M_k$, however, the electron state that is involved in the MP creation is always optically active. Therefore, evidently, all channels contribute to the MP creations, except those which contribute to the optical transitions. Indeed, from Eq. (17) it follows that the terms with $N = N'$ and $m = m'$ in $A_k$ and $M_k$ cancel each other. The remaining terms in $M_k$ are nonzero if the LLs with $N \neq N'$ are optically active, i.e., $M_N \neq 0$ and $M_{N'} \neq 0$.

As it follows from Eq. (16), each SU satellite results from a superposition of replicas from different ML lines. As an illustration, the possible options for the first SU$_1$ satellite are shown.
schematically in Fig. 1. Electrons occupy the three lowest LLs which are assumed to be optically active. Thus, the ML spectrum, Eq. (14), contains three lines, their corresponding transitions are shown by the solid arrows in Fig. 1. Each of these transitions contributes to the SU1 satellite at $\hbar\omega_0 - \hbar\omega_B$, if it is accompanied by a MP excitation, shown by the dashed arrows. The ML line at $\hbar\omega_0$ contributes to the SU1 satellite if it is accompanied by an $\hbar\omega_B$ MP excitation, which can happen only from the uppermost filled LL, $N = 2$. The ML line at $\hbar\omega_0 + \hbar\omega_B$ contributes to the SU1 satellite if it is accompanied by a $2\hbar\omega_B$ MP excitation, which can occur from LLs $N = 2$ and $N = 1$. Similarly, the line at $\hbar\omega_0 + 2\hbar\omega_B$ contributes to the SU1 satellite when a $3\hbar\omega_B$ MP is excited from one of the occupied LLs $N = 2$, $N = 1$, or $N = 0$.

To proceed further, we assume that the hole wave function $\Psi_h(\vec{r})$ is spherically symmetric. Then only zero-angular momentum states contribute to the optical transitions, $M_{Nm} = M_{N0}\delta_{m0}$. The usual selection rule for interband magneto-optical transitions $N_{\text{electron}} = N_{\text{hole}}$, which is valid for a nonlocalized hole, is lifted. For a localized hole, on the contrary, all LLs can be optically active. Indeed, if the hole is localized on a scale smaller than $l_B$, the optical matrix element $M_{N0}$, according to its definition above, is given by

$$M_{N0} = P_{vc} \int d\vec{r} \Psi_{N0}^*(\vec{r}) \Psi_h(|\vec{r}|) = P_{vc} \Psi_{N0}^*(0) \int d\vec{r} \Psi_h(|\vec{r}|),$$

(19)

where $\Psi_{Nm}(\vec{r})$ is the electron wave function. Thus, the optical matrix elements entering Eq. (17) are defined by the electron wave function at the origin, $\Psi_{N0}(0) = 1/\sqrt{2\pi l_B}$, and the hole localization length $a \equiv \int d\vec{r} \Psi_h(|\vec{r}|)$, giving $M_{N0} = P_{vc} a/\sqrt{2\pi l_B} \equiv M$. As a result, the optical transitions from all the populated LLs to the hole level $E_0$ are allowed. We assume also that the electron-hole interaction occurs via a screened potential[17]. In what follows we choose the hole potential to be $V(|\vec{r}|) = V_0 \exp{-r^2/2L^2}$ with the potential amplitude $V_0$, and the potential range $L$. Note that the amplitude $V_0$ in this case is defined by two parameters, the hole localization length $a$ and the potential range $L$. Depending on the potential amplitude $V_0$, the localization length $a$ can be larger or smaller than the potential range $L$. In the first case the hole potential is short ranged, while in the second case it can
be long ranged. Since the interaction potential \( V(|\vec{r}|) \) is spherically symmetric, the electron scattering takes place only between Landau states with the same angular momentum, \( V_{Nm,N'm'} = V_{Nm,N'm} \delta_{mm'} \).

Under the assumptions above we obtain for the SU spectrum

\[
\mathcal{A}_k = |M|^2 \sum_{N,N'} \sum_m \left| V_{(N+N'+k)m,N'm} \right|^2 \frac{\delta_{mm'}}{\varepsilon_{N+N'+k} - \varepsilon_{N'}} F_{NN'k},
\]

\[
\mathcal{M}_k = |M|^2 \sum_{N,N'} \frac{V_{N0,N'0} V_{N0,N'0}}{(\varepsilon_{N+N'+k} - \varepsilon_{N})} F_{NN'k}. \tag{20}
\]

All angular momentum channels contribute to the shakeup term \( \mathcal{A}_k \), as all states are shaken up by the disappearance of the hole in the optical transition. On the other hand, only the zero-angular momentum channel contributes to the exciton term \( \mathcal{M}_k \), since only this channel is involved in the optical process. Thus, in the general case, the SU spectrum is mostly due to the Fermi sea shakeup effect.

**IV. SHORT RANGE HOLE POTENTIAL**

When the hole potential \( V(|\vec{r}|) \) is short range compared to \( l_B \), \( L \ll l_B \), nonzero-angular momentum shakeup processes are strongly suppressed. Indeed, at \( L \ll l_B \), using \( \Psi_{Nm}(0) = \delta_{m0} / \sqrt{2\pi l_B} \) one obtains

\[
V_{Nm,N'm'} = \int d\vec{r} \Psi_{Nm}(\vec{r}) V(|\vec{r}|) \Psi_{N'm'}^{\ast}(\vec{r})
= \Psi_{Nm}(0) \Psi_{N'm'}^{\ast}(0) \int d\vec{r} V(|\vec{r}|) = V_0 \frac{L^2}{2\pi l_B^2} \delta_{m0}. \tag{21}
\]

As a result, only zero-angular momentum term survives in Eq. (20), and the intensity of the \( k \)th satellite reduces to the following simple form:

\[
I_{SU_k} = I_{ML}^0 \alpha^2 \sum_{N,N'} \left\{ \frac{1}{(N + k)^2} - \frac{1}{(N + k)(N' + k)} \right\} F_{NN'k}, \tag{22}
\]

where \( I_{ML}^0 = \mathcal{N}|M|^2 \) and the parameter \( \alpha = (V_0/2\pi \bar{h} \omega_B) (L/l_B)^2 = V_0 m_e L^2 / 2\pi \hbar^2 \) defines the interaction strength and is magnetic field independent.
Thus, for a short range hole potential, the shakeup contribution to the SU\(_k\) satellite is strongly suppressed by the excitonic contribution. One can see from Eq. (22) that only the processes in which optical transitions and MP excitations occur from different LLs, \(N \neq N'\), now survive. In this case, if the three lowest LLs are populated, there is no contribution to the SU\(_1\) satellite from the ML line at \(\hbar \omega_0 + \hbar \omega_B\) when a \(2\hbar \omega_B\) MP is excited from the \(N = 1\) LL (see Fig. 1). In addition, there is no contribution from the ML line at \(\hbar \omega_0 + 2\hbar \omega_B\) when a \(3\hbar \omega_B\) MP is excited from the \(N = 2\) LL. The most important consequence of the result above is that when only the lowest LL is occupied, i.e., the Landau numbers are \(N = N' = 0\) in Eq. (22), the shakeup spectrum disappears. This conclusion is, probably, not surprising. Under the conditions above, the problem reduces effectively to the one-electron problem, since there is only one electron in the state \(|N = 0, m = 0\rangle\) which is allowed for both optical and scattering transitions. As was noted above, a strong suppression of the SU spectrum in the case in which electrons occupy only the lowest LL, was observed in Ref. [14] for GaAs/AlGaAs quantum wells.

When more than one LL is occupied, the intensity of the \(k\)th satellite decreases with its number \(k\). When this number is larger than the number of the filled LLs \(N_\mu = \nu, k \gg \nu\), the SU\(_k\) peak intensity decreases as \(k^{-4}\),

\[
I_{SU_k} = I_{ML}^0 \alpha^2 \left(\frac{\nu - 1}{2}\right)^\nu \left(\frac{\nu + 1}{2}\right)^{\nu - 1} \frac{1}{k^4}.
\]  

In Eq. (23) the filling factor \(\nu\) is taken for the spinless case.

As it follows from Eq. (22), the intensity of the each SU\(_k\) satellite increases linearly with the magnetic field \(B\) due to the optical matrix element \(|M|^2 \sim B\). There is also an implicit dependence on \(B\) due to the occupation numbers entering Eq. (22) via the function \(F_{NN'k}\), Eq. (18). When the magnetic field \(B\) increases, the upper LL depopulates and its contribution to the SU spectrum changes. Therefore, the linear increase in the shakeup intensity can be violated for some regions of \(B\), and a nonmonotonic dependence on the magnetic field can be expected.

As an illustration, the intensities for the first three satellites, SU\(_1\), SU\(_2\), and SU\(_3\), as
a function of the magnetic field $B$ are shown in Fig. 2. The magnetic field $B$ increases from the initial magnitude of $B = B_{4/5}$ which corresponds to the location of the Fermi energy $E_F$ halfway between LLs $N = 4$ and $N = 5$. The occupation numbers entering Eqs. (18) and (22) are modeled by the function $n(\varepsilon_N) \to \frac{1}{2} [1 + \Phi((E_F - \varepsilon_N)/\gamma)]$, where $\Phi(x)$ is the probability integral [26], and $\gamma \ll \hbar \omega_B$ is the small phenomenological width of the LL. The curves in Fig. 2 are calculated using the parameters $\gamma/\hbar \omega_{B_{4/5}} = 0.2$ and $\alpha = 0.1$.

It can be seen from Fig. 2 that the shakeup intensities show a pronounced nonmonotonic dependence on the magnetic field $B$. The changes are periodic with $B$ and the intensity extrema appear when the uppermost LL crosses the Fermi level. The oscillations are more pronounced for the first SU$_1$ satellite. As was noted above, periodic changes of the shakeup intensities as a function of the magnetic field are observed in Ref. [13] at low temperatures in a modulation-doped quantum well of InGaAs.

V. INFLUENCE OF THE POTENTIAL RANGE

It is evident that the condition above for the hole potential to be short range, $L \ll l_B$, can be violated at high $B$ due to the decrease in magnetic length, $l_B \sim B^{-1/2}$. Thus, the results above can be not valid, for example, when the SU spectrum is measured at high $B$. We will study here the influence of the potential range on the SU spectrum and consider first the case when only the lowest LL is occupied, in which case the potential range is crucial. Then, calculating the scattering matrix elements and performing the summation over the angular momentum $m$ in Eq. (20), with the model potential above, one obtains

$$I_{SU_k} = I^0_{ML} \frac{V^2}{(k\hbar \omega_B)^2} W(L/l_B) n^2(\varepsilon_0).$$

(24)

where the function

$$W(x) = x^4 \left[ \frac{1}{(2x^2 + 1)^2} - \frac{1}{(x^2 + 1)^4} \right], \quad W(x) = \begin{cases} 2x^8, & x \ll 1, \\ \frac{1}{4}(1 - 1/x^2), & x \gg 1. \end{cases}$$

(25)

For a short range potential when $L \ll l_B$, the function $W(L/l_B)$ is very small, see Eq. (25), and therefore, the SU spectrum is strongly suppressed, in agreement with the results.
above. This is not a case, however, when the potential is long range and \( L \geq l_B \). Then the SU\(_k\) satellite intensity can be not small. In this case it is determined mostly by the interaction strength \( V_0/\hbar \omega_B \), and its number \( k \).

When the magnetic field \( B \) decreases, the SU\(_k\) intensity taken in units of the ML intensity \( I_{ML}^0 n(\varepsilon_0) \), decreases as \( B^{-2} \) due to the cyclotron energy. The function \( W(L/l_B) \) decreases with increasing \( B \), and also there is an additional decrease \( \sim B^{-1} \) due to the occupation numbers \( n(\varepsilon_0) \). Introducing a phenomenological width for the \( N = 0 \) LL in the same manner as in Fig. 2, the occupation numbers \( n(\varepsilon_0) \) are replaced by the filling factor \( \nu \sim 1/B \). Note that the filling factor of the \( N = 0 \) LL tends to zero only at \( B \to \infty \), as the lowest LL cannot be empty for a given electron density. Fig. 3 illustrates the magnetic field dependence of the intensity of the SU\(_1\) satellite, Eq. (24), at different potential range parameters \( L/l_B \). The magnetic field \( B \) increases from the initial magnitude of \( B = B_0/1 \) which corresponds to the location of the Fermi level halfway between the \( N = 0 \) and \( N = 1 \) LLs. The curves in Fig. 3 are calculated using the following parameters: \( V/\hbar \omega_{B_0/1} = 0.3 \), and \( L/l_{B_0/1} = 0.6, 1, 1.4 \). From Fig. 3 one can conclude the following. When only one LL is occupied, the SU\(_1\) intensity smoothly decreases with increasing \( B \). At high magnetic fields it is almost independent of the potential range \( L \) and is very small. At low magnetic fields \( B \simeq B_0/1 \), the longer the potential range, the larger the intensity. In addition, the decay of the intensity is more pronounced at larger \( L \), and is faster than \( B^{-2} \).

If several LLs are filled, the shakeup intensities are given by expressions similar to Eq. (24). However, the dependence on the potential range parameter \( L/l_B \) is now more complicated, because of the increasing number of possible shakeup processes. In this case the satellite intensities are proportional to \( (L/l_B)^2 \) at \( L \ll l_B \), see Eq. (22), and are almost independent of \( L/l_B \) at \( L \gg l_B \). Therefore, for a short range potential, the satellite intensity Eq. (22) increases linearly with increasing \( B \) due to the optical matrix elements. When the uppermost LL depopulates, its contribution to the satellite intensity decreases with increasing \( B \) and overbalances the linear increase. As a result, a reduction of the satellite intensity occurs when ever the uppermost LL crosses the Fermi energy. On the contrary, when the
potential is long range, the satellite intensity decreases with increasing $B$ as $B^{-1}$, due to the optical matrix elements and the cyclotron energy, and it decreases also with $B$ due to the occupation numbers. Thus, one can expect that the oscillating dependence of the shakeup intensities on the magnetic field $B$, shown in Fig. 2, will be much less pronounced for the case of a long range potential. This is illustrated in Fig. 4.

In Fig. 4 the SU$_1$ satellite intensity is shown as a function of the magnetic field $B$, for different potential ranges. The Fermi energy is halfway between the $N = 1$ and $N = 2$ LLs at the magnetic field $B = B_{1/2}$. The curves in Fig. 4 are calculated using the parameter $\gamma/\hbar\omega_{B_{1/2}} = 0.2$, $V/\hbar\omega_{B_{1/2}} = 0.3$, and $L/l_{B_{1/2}} = 1, 1.4, 3.2$ (curves 1, 2, and 3, respectively). The occupation numbers $n(\varepsilon_1)$ and $n(\varepsilon_0)$ are modeled in the same manner as in Fig. 2 and Fig. 3. From Fig. 4 it can be seen, that the shorter the potential range, the more pronounced the nonmonotonicity of the SU$_1$ intensity as a function of $B$, as expected. Thus, when the potential range $L \simeq l_B$, the SU$_1$ intensity has a wide peak at those magnetic fields in which the uppermost LL depopulates, see curves 1, and 2. However, at larger $L \simeq 3l_B$ it decreases almost smoothly with increasing $B$, as is shown in Fig. 4 by curve 3.

VI. DISCUSSION

The results above are obtained for a hole wave function and a hole potential that are spherically symmetric. They are valid also for a nonsymmetric case, when the hole potential is short range. Indeed, at $L \ll l_B$, the scattering matrix elements are defined only by the properties of the electron wave function at $\vec{r} = 0$, but not by the potential symmetry. It refers also to the optical matrix elements as the localization length is assumed to be small, $a \ll l_B$. In the opposite case, that of a long range potential, there are, however, changes. Unlike the previous case of a symmetric potential, now electron scattering occurs also between different angular momentum states. Therefore, additional shakeup channels appear and the results must change quantitatively. However, the shakeup processes in this case will dominate, as before. The reason is the same as above, namely, all the angular
momentum states are shaken up, but the excitonic contribution involves optically active states with only zero angular momentum.

The other restriction is that the hole localization length is much smaller than the magnetic length, $a \ll l_B$. When this condition is violated, the optical matrix elements $M_{Nm}$ entering Eq. (17) are changed. They depend now on the electron quantum number $N$ and the hole localization length $a$. Choosing the hole wave function to be $\Psi(|\vec{r}|) = (2a)^{-1} \exp \{-r^2/4a^2\}$, one obtains

$$M_{Nm} = \frac{P_{vc}}{\sqrt{2\pi a}} \frac{l_B}{l_B^2 - 1} N \frac{l_B^2}{a^2} (l_B^2 + 1)^{-N-1} \delta_{m0}. \quad (26)$$

In addition, the scattering matrix elements Eq. (21) have to be substituted by

$$V_{NmSm} = \int d\vec{r}_1 d\vec{r}_2 \Psi_{Nm}(\vec{r}_1) \Psi_{Sm}^*(\vec{r}_1) V(\vec{r}_1 - \vec{r}_2) |\Psi_h(\vec{r}_2)|^2. \quad (27)$$

The matrix elements (27) describe the Auger-like electron-hole scattering processes. When $a$ is not small compared to $l_B$, the scattering matrix elements (27) are finite for all angular momentum, even for a short range hole potential. Thus, the shakeup processes are now always dominant and the SU spectrum exists in this case also at filling factors $\nu \leq 1$. To demonstrate this consider, for example, the case of $a \simeq l_B$. In this case the optical transitions from the lowest LL $N = 0$, are the most intensive, $M_{Nm} \simeq M_0 \delta_{N0} \delta_{m0}$, where $M_0 = P_{vc}/2\sqrt{2\pi}$. There is only one line in the main ML spectrum with energy $\hbar \omega_0$ and, therefore, the number of contributions to the SU spectrum reduces. In Fig. 1, for example, only one contribution to the SU$_1$ survives, when the ML line at $\hbar \omega_0$ is accompanied by an $\hbar \omega_B$ MP excitation from the $N = 2$ LL. The intensity of the SU$_k$ satellite at $a \simeq l_B$ is given by

$$I_{SU_k} = N |M_0|^2 \frac{1}{(k\hbar \omega_B)^2} \left\{ \sum_{N,m} |V_{Nm(N+k)m}|^2 F_{0Nk} - |V_{000}|^2 F_{00k} \right\}, \quad (28)$$

where $N$ numbers the occupied LLs. The shakeup contribution is dominant and, moreover, the excitonic term does not contribute to each SU$_k$ satellite. Indeed, the function $F_{00k}$ differs from zero only if the $k$th LL is empty, see Eq. (18). Therefore, the excitonic term contributes
only to the satellites, with a number \( k \) equal or larger than the number of occupied LLs. For example, when the three lowest LLs \( N = 0, N = 1, \) and \( N = 2 \) are occupied, as in Fig. 1, this term do not contribute to the first SU_1 and second SU_2 satellites, only to the third and higher satellites. Thus, a strong hole localization is not a necessary request for the SU spectrum to be realized. It is not a case, however, for the FES effect. As it follows from Eq. (14), in the case of \( a \approx l_B \) the many-body corrections to the main spectrum vanish, as only the lowest Landau state is optically active and the optical transitions from the higher LL \( N \neq 0 \) are much less intensive, \( M_N \approx 0 \). The result above for the SU spectrum is in contradiction to the statement in the early experimental works (see, e.g., Refs. [9], [11]) that both the FES effect and SU spectrum are very sensitive to a degree of the hole localization. Our conclusions are, however, in agreement with the results from Refs. [22] and [16]. The numerical calculations in Ref. [22] (see Fig.1(b) in Ref. [22]) and recent experimental observations in Ref. [16] show that the SU spectrum, contrary to the FES effect, do not depend strongly on the hole localization.

The effect of the electron spin can be accounted for if we assume that the spin of the conduction electron is conserved in the scattering event with the localized hole. In fact this approximation is implicit in the statement above that only one localized level is involved. Under such a restriction the exchange scattering processes in which the conduction electron and the localized hole would both reverse their spin, are ignored. In this case the only change is that the emission intensities Eqs. (14) and (15) are multiplied by a factor 2 as there are twice as many electrons involved.

The results above can not be extended to the case of delocalized holes. In this case, as well as in the case of a many state localized hole, the Nozières - De Dominicis formalism, i.e., the “one-body” representation for the two-particle Green’s function, is not valid and to obtain the emission spectrum one has to calculate the two-particle Green’s function.

The case of delocalized holes is studied in Ref. [14], where a strong suppression of the SU_1 satellite at \( \nu \leq 1 \) was observed for GaAs/AlGaAs quantum wells, in which the holes are assumed to be unlocalized. The amplitude for the shakeup optical transition was calculated
up to the second order in the perturbation, namely, up to the first order in the coupling with electromagnetic field, and up to the first order in the electron-hole or electron-electron interaction. The states of the electron system are given by Slater determinants. It was shown that when an electron from the lowest LL recombines with a valence hole, the contributions to the shakeup transition amplitude due to the electron-hole and electron-electron scattering processes cancel each other. Such a mechanism for the SU emission quenching differs from the one presented above. In our model the electrons in the conduction band are assumed to be noninteracting and the SU spectrum disappears as there are no shakeup channels when the hole potential is short range.

VII. CONCLUSION

The emission spectrum due to transitions between a localized level and a 2DEG in a strong magnetic field is calculated analytically. Calculations are performed with a Nozières-De Dominicis-like Hamiltonian where the interaction part is treated as a perturbation, since the hole potential is assumed to be small compared to the cyclotron energy, $V_0 \ll \hbar \omega_B$. The shakeup spectrum appears only when the emission is calculated up to the second-order in this perturbation and is a result of a balance between the orthogonality catastrophe and the excitonic effect contributions. In the general case the shakeup effect is dominant as all the angular momentum states are shaken up, while the excitonic contribution involves the optically active states with zero angular momentum only. For the special case of a short range hole potential, however, the excitonic effect competes with the shakeup effect as nonzero-angular momentum shakeup processes are strongly suppressed. If electrons occupy only the lowest LL, i.e., the filling factor $\nu \leq 1$, there are no channels to shake up the Fermi sea in the optical transition and the SU spectrum disappears. When electrons occupy two or more LLs, the satellite intensities oscillate as a function of the magnetic field. For the opposite case, that of a long range hole potential, the shakeup spectrum is determined mostly by the Fermi sea shakeup processes, and the excitonic contribution is much less
important. In this case the satellite intensities decrease smoothly when the magnetic field increases and there is no suppression of the SU spectrum when electrons occupy the lowest LL only. A strong hole localization, $a \ll l_B$, is not a crucial condition for the SU spectrum to be observed. If the hole localization length is not small compared to $l_B$, the SU spectrum still exists. Only the number of contributions to the SU spectrum reduces and the shakeup processes are always dominant, as compared with the case of a strong hole localization.

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FIGURES

FIG. 1. A schematical illustration of different contributions to the first SU$_1$ shakeup satellite. The solid lines show the occupied LLs which are also optically active. The dashed lines show the empty LLs. The solid and the dashed arrows show the optical transitions and the magnetoplasmon creations, respectively.

FIG. 2. The intensities of shakeup satellites as a function of the magnetic field $B$, for a short range potential. The intensities are given in units of $I_{ML}^0$ at $B = B_{4/5}$.

FIG. 3. The intensity of the first SU$_1$ shakeup satellite as a function of the magnetic field $B$ for different localization parameters. The parameter $L/l_{B_0/1} = 1.4, 1, 0.6$ for curves 1, 2, and 3, respectively. At $B = B_{0/1}$ electrons occupy the lowest LL $N = 0$. The intensity is given in units of $I_{ML_0}^0 n(\varepsilon_0)$.

FIG. 4. The intensity of the first SU$_1$ shakeup satellite as a function of the magnetic field $B$ for different localization parameters. The parameter $L/l_{B_0/1} = 1, 1.4, 3.2$ for curves 1, 2, and 3, respectively. At $B = B_{1/2}$ electrons occupy the two lowest LLs $N = 0$, and $N = 1$. The shakeup intensity is given in units of $I_{ML}^0$ at $B = B_{1/2}$.
"Shakeup spectrum in a two-dimensional electron gas..."
E. Tsitsishvili, Y. Levinson

FIG. 1
"Shakeup spectrum in a two-dimensional electron gas..." E. Tsitsishvili, Y. Levinson

FIG. 2

Graph showing the shakeup spectrum with three identified features labeled SU₁, SU₂, and SU₃.

B/B₄/₅
FIG. 3

"Shakeup spectrum in a two–dimensional electron gas..." E. Tsitsishvili, Y. Levinson
"Shakeup spectrum in a two-dimensional electron gas..." E. Tsitsishvili, Y. Levinson