Remarks on the Upper Bounds on Higgs Boson Mass from Triviality

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ABSTRACT

We study the effects of the one-loop matching conditions on Higgs boson and top quark masses on the triviality bounds on the Higgs boson mass using $\beta_\lambda$ with corrected two-loop coefficients. We obtain quite higher results than previous ones and observe that the triviality bounds are not nearly influenced by varying top quark mass over the range measured at CDF and D0. The effects of typo errors in $\beta_\lambda^{(2)}$ and the one-loop matching condition on the top quark mass are negligible. We estimate the size of effects on the triviality bounds from the one-loop matching condition on the Higgs boson mass.

PACS numbers : 14.80.Bn

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The tree level Higgs potential in the standard model (SM) is given by

\[ V = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{24} \phi^4. \quad (1) \]

As well known, a pure scalar \( \lambda \phi^4 \) theory allows only \( \lambda_R = 0 \) where \( \lambda_R \) is the renormalization coupling. If \( \lambda_R \) is not zero there is a singularity in the evolution of the running coupling constant. This is the triviality problem of the pure \( \lambda \phi^4 \) theory.

But one can expect that the interactions of the standard Higgs with other particles in the SM can make \( \lambda_R \) to have a non-zero value. Beg et al. [1] studied this possibility with a consideration of the fact that the \( U(1) \) coupling \( g_1 \) always produces a Landau singularity at a very high scale \( \Lambda_{LS} \approx 10^{42} \) GeV. They obtained the upper bound of the Higgs boson mass as a function of the top quark mass from a condition of \( \Lambda_{\lambda} > \Lambda_{LS} \) where \( \Lambda_{\lambda} \) is a scale of the singularity of the running coupling \( \lambda \). (actually they used \( \Lambda_B = 4 \times 10^{37} \) GeV instead of \( \Lambda_{LS} \))

But many people think that the SM is a low energy effective theory which is embedded in some more fundamental theory at a scale \( \Lambda \). Usually this scale \( \Lambda \) is less than \( \Lambda_{LS} \). For example, the Planck scale \( \Lambda_{PL} = 10^{19} \) GeV where something new must appear is much less than \( \Lambda_{LS} \). Therefore it was required to calculate the triviality bounds on the Higgs boson mass in the case that the new physics scale \( \Lambda \) is less than \( \Lambda_{LS} \). Lindner et al. implemented this calculation at one- and two-loop level. [2,3]

Grzadkowski and Lindner implemented, at two-loop level, analyses of the triviality Higgs boson mass bounds in the SM using tree level matching conditions on Higgs boson and top quark masses. [3] And they expect the effects of the one-loop matching condition on Higgs boson mass to be small for \( M_H < 500 \) GeV.

It was noted that there were two errors in the \( \beta \) function of \( \lambda \), \( \beta_\lambda \). [4] These errors are the electroweak contributions to the two-loop coefficients of the \( \beta_\lambda \) function. [5] We denote the two-loop part of the \( \beta_\lambda \) function as \( \beta_\lambda^{(2)} \).

Considering only the interactions of gauge particles and top quark with the Higgs particle, the one-loop matching conditions are

\[ \lambda(\mu_0) = 3 \frac{M_H^2}{v^2} [1 + \delta_\lambda(\mu_0)], \]

\[ h_t(\mu_0) = \sqrt{2} \frac{M_t}{v} [1 + \delta_t(\mu_0)], \quad (2) \]
where $h_t$ is top-Yukawa coupling, $\mu_0$ is the renormalization scale and $v = (\sqrt{2}G_F)^{-1/2} = 246$ GeV. $M_H$ and $M_t$ denote Higgs boson and top quark mass respectively. Explicit form of $\delta_\lambda(\mu_0)$ and $\delta_t(\mu_0)$ can be found in Ref. [6].

Recently the existence of the top quark is clear. [7]

$$M_t = 180 \pm 12 \text{GeV} \quad \text{(CDF + D0)}.$$ (3)

For this range of $M_t$ value and $M_H < 1$ TeV, $\delta_t(\mu_0)$ is $\sim 5\%$ or less for $\mu_0 = M_Z$ and $M_t$. So this effect is negligible in the study of the upper bounds on the Higgs boson mass. But $\delta_\lambda(\mu_0)$ is quite large and heavily depends on the choice of $\mu_0$.

In this paper we study the effects of matching condition on the Higgs boson mass on the triviality bounds on the Higgs boson mass using $\beta_\lambda$ with corrected two-loop coefficients.

As noted by Grzadkowski and Lindner, the evolution behaviors of $\lambda$ is drastically changed at two-loop level. [3] At two-loop level there is no more singularity, there exists a new UV fixed-point. As scale increases, two-loop part of $\beta_\lambda$, $\beta_\lambda^{(2)}$, starts to dominate and exactly cancels the one-loop contribution at a very high scale. This is a signal of a breakdown of a perturbation expansion. So the existence of this UV fixed-point doesn’t mean that one can remove the singularity without embedding since the point takes place behind a perturbation area. They study the dependence of $\lambda(\mu = 0)$ as a function of $\lambda(\mu = \Lambda)$ and find a plateau in the plane $(\lambda(0), \lambda(\Lambda))$. This plateau originates from the existence of the UV fixed-point. So the existence of the plateau is a signal of a breakdown of a perturbation expansion and corresponds to the pole. From their automatic fixing procedure designed to find the end of the plateau, they obtained the value of $\bar{\lambda}(\Lambda)$ which is about $60^2$ for $M_t < 210$ GeV and $\Lambda < 10^{15}$ GeV.

For numerical calculations we give boundary conditions for the gauge couplings at $M_Z$ as follows

$$g_1(M_Z) = 0.3578,$$
$$g_2(M_Z) = 0.6502,$$

Note that the coefficient of the quartic term of the scalar potential which we are using is not $\lambda$ but $\frac{\lambda}{24}$. We are using the form of the potential given in Ref. [4]. The Higgs potential used by authors of Ref. [2,3] corresponds to $V = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{24}\phi^4$. Therefore the value of our $\lambda$ is three times larger than that in Ref. [2,3]
\[ \alpha_s(M_Z) = \frac{g_2^2(M_Z)}{4\pi} = 0.123. \]

We use \( \beta \)-functions found in Ref. [4].

In Fig. 1 we show the running of \( \lambda \) for \( M_t = 180 \text{ GeV} \) and \( M_H = 200, 300, 400, 500 \text{ GeV} \). We use tree level matching condition with \( \mu_0 = M_Z \). For each value of \( M_H \), diverging line is the result from one-loop calculations and the other is that from two-loop ones. We observe that the effects of the corrected coefficients of \( \beta^{(2)}_\lambda \) on the running of \( \lambda \) is negligible for \( M_H > 200 \text{ GeV} \). So the effects of wrong coefficients of \( \beta^{(2)}_\lambda \) on bounds on the Higgs boson mass are negligible and the results of Ref. [3] are not influenced by these corrections. From Fig. 1 we can observe the existence of the UV fixed-point where \( \lambda \) is about 72. The scale where one-loop calculation diverges and the other scale where two-loop calculation starts to be stationary are close to each other. And we can see the effects of the two-loop corrections diminishes the slope of \( \lambda(\mu) \). This results from the fact that \( \beta^{(2)}_\lambda \) is negative. The corresponding pole position at two-loop level is expected to be smaller than that at one-loop level. Therefore we can expect that triviality bounds computed at two-loop level are higher than those computed at one-loop level. When we calculate the corresponding pole position at two-loop level we require the calculation is consistent with this expectation.

We define \( \Lambda^{(1)}_{\mu} \) as the pole position of \( \lambda \) computed at one-loop level using tree level matching condition with \( \mu_0 = M_Z \). In Fig. 2 we plot \( \Lambda^{(1)}_{\mu} \) as a function of \( M_H \) for three values of \( M_t = 160, 180, 200 \text{ GeV} \). This is the very same framework as that of Ref. [2]. A requirement \( \Lambda^{(1)}_{\mu} > \Lambda \) gives the upper bound on \( M_H \) as a function of \( \Lambda \). [2] This requirement means that the triviality is removed by adding fields which couple to the Higgs particle in the new physics. Numerically, we identify the pole position as the scale at which \( \lambda \) starts to be bigger than 1000. If new physics appears at the Planck scale, \( M_H < \sim 220 \text{ GeV} \) to satisfy the requirement \( \Lambda^{(1)}_{\mu} > \Lambda_{\text{PL}} \). This result is different from that of Ref. [2]. Our result is somewhat higher than that of Ref. [2]. From Fig. 2 we observe that the effects of varying \( M_t \) from 160 to 200 GeV on the Higgs boson mass bound is less than about 10 GeV when \( \Lambda < 10^{20} \text{ GeV} \). Contrary to the results of Ref. [2] we can not observe the large effects of varying \( M_t \). It seems like that the origin of these discrepancies come from the differences of numerical treatments of running of
λ and the singularity. But the origin is unclear. We observe that the smaller Λ, the less dependence on \( M_t \).

To study the effects of two-loop corrections to \( \beta \) functions and one-loop matching condition on the Higgs boson mass bounds, we consider following two cases. The first one is the case of tree level matching condition with \( \mu_0 = M_Z \) and two-loop \( \beta \) function. The second one is the case of one-loop matching condition with \( \mu_0 = M_Z, M_t, M_H \) and one-loop \( \beta \) function. The corresponding pole positions of the first and second cases are denoted by \( \Lambda_p^{(2)} \) and \( \Lambda_p^{(1),(\mu_0)} \) respectively.

In Fig. 3 we plot \( \Lambda_p^{(2)} \) as a function of \( M_H \) for \( M_t = 180 \) GeV where \( \Lambda_p^{(2)} \) is the scale of perturbative expansion breakdown at two-loop level using tree level matching condition. We also plot \( \Lambda_p^{(1)} \) for comparisons. \( \Lambda_p^{(2)} \) is defined as the scale at which \( \lambda \) starts to be bigger than some value \( \lambda_{\text{cut}} \) around the scale \( \Lambda_p^{(1)} \), or \( \lambda (\mu > \Lambda_p^{(2)}) > \lambda_{\text{cut}} \). We choose three values \( \lambda_{\text{cut}} = 40, 60, 70 \) to examine the effects of choice of the value of \( \lambda_{\text{cut}} \). We take \( \lambda_{\text{cut}} = 60 \) because this value is the one used in Ref. [3] in interested region of \( M_t \). We take \( \lambda_{\text{cut}} = 70 \) because this value is very near to the UV fixed-point. We observe that the case of \( \lambda_{\text{cut}} = 40 \) produces lower bound than one-loop case for \( \Lambda < 10^5 \) GeV. Although smaller value of \( \lambda_{\text{cut}} \) is more realistic in view of the demand that the two-loop effects make sense perturbatively, the choice of small \( \lambda_{\text{cut}} \) produces lower bound than that obtained by calculations at one-loop level. This is not consistent with the general expectation of the two-loop effects. We expect that two-loop effects on bound are positive since \( \beta^{(2)}_\lambda \) is negative. Therefore we estimate an uncertainty of the two-loop effects by varying \( \lambda_{\text{cut}} \) from 40 to 70. Taking 60 as a center value and for \( M_t = 180 \) GeV, we estimate the bounds on the Higgs boson mass as follows

\[
\begin{align*}
M_H &< 270 \pm 10 \text{ GeV} \quad \text{for} \quad \Lambda = 10^{15}\text{GeV} \\
M_H &< 340 \pm 25 \text{ GeV} \quad \text{for} \quad \Lambda = 10^{10}\text{GeV} \\
M_H &< 500 \pm 70 \text{ GeV} \quad \text{for} \quad \Lambda = 10^6\text{GeV}.
\end{align*}
\]

The triviality bound has a tendency to increase when we consider two-loop effects and these increments are larger for smaller \( \Lambda \). Since these theoretical errors are quite large, the effects of the matching condition are not important if the effects are small.

We denote the pole position computed at one-loop level using the one-loop matching condition as \( \Lambda^{(1)} \).
condition on the Higgs boson mass as $\Lambda^{(1),\mu_0}_p$. In Fig. 4 we plot $\Lambda^{(1),\mu_0}_p$ as a function of $M_H$ for three choices of the renormalization scale $\mu_0 = M_Z$, $M_t$, $M_H$ and $M_t = 180$ GeV. One can find a rather similar figure in Ref. [8] From this figure we can estimate the size of the effects of the one-loop matching condition on the Higgs boson mass. For large value of $M_H$ and $\mu_0 = M_Z$, $M_t$, $\delta_\lambda(\mu_0)$ can be less than -1. The fact $\delta_\lambda(\mu_0 = M_Z, M_t)$ can be less than -1 illustrates that the perturbation results are not reliable anymore and $\lambda$ at those $\mu_0$ is negative. So we compute $\Lambda^{(1),\mu_0}_p$ only for the values of $M_H$ satisfying $\delta_\lambda(\mu_0) > -1$. $\Lambda^{(1),\mu_0}_p$ for large $M_H$ with small $(1 + \delta_\lambda(\mu_0))$ is the same order of magnitude as that for small $M_H$ with small $\delta_\lambda(\mu_0)$. From Fig. 4 we observe that $\mu_0 = M_H$ choice gives nearly the same, but a litter bit lower, triviality bound. For the $M_H < \sim 500$ GeV where the perturbative results are reliable when $\mu_0 = M_Z$, $M_t$, the effects of taking into account matching condition on Higgs boson mass give the same order of magnitude as given by the differences between triviality bounds computed at one- or two-loop levels. In the case $\mu_0 = M_H$, the effects are negligible.

As a summary, we plot the triviality bound on $M_H$ as a function of $M_t$ for several $\Lambda$ values computed at two-loop level with $\lambda^{\text{cut}} = 60$ using one-loop matching condition with $\mu_0 = M_H$. (see Fig. 5) The effects of the one-loop matching condition with $\mu_0 = M_H$ lower the bounds obtained by calculation at two-loop level using tree level matching condition. The bounds are lowered by 10, 15 and 40 GeV for $\Lambda = 10^{15}$, $10^{10}$ and $10^6$ GeV respectively. We summarize as follows

$$
M_H < 260 \pm 10 \pm 2 \text{ GeV} \quad \text{for} \quad \Lambda = 10^{15}\text{GeV}
$$

$$
M_H < 325 \pm 25 \pm 2 \text{ GeV} \quad \text{for} \quad \Lambda = 10^{10}\text{GeV}
$$

$$
M_H < 460 \pm 70 \pm 7 \text{ GeV} \quad \text{for} \quad \Lambda = 10^6\text{GeV}.
$$

The first and second errors are related with the choice of the value $\lambda^{\text{cut}}$ and varying top quark mass from 150 to 210 GeV respectively. We observe that the dependence on $M_t$ are small and obtain quite higher results than those of Ref. [2]. The theoretical errors of these bounds are larger for smaller $\Lambda$. The effects of typo errors in $\beta^{(2)}_\lambda$ and matching condition on the top quark mass on triviality bounds are negligible.
Acknowledgments

This work was supported in part by Korea Science and Engineering Foundation.
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Figure Captions

Fig. 1: Plots of \( \lambda(\mu) \) for \( M_t = 180 \) GeV and \( M_H = 200 \) (solid line), 300 (dashed line), 400 (dotted line) and 500 (dash-dotted line) GeV. For each value of \( M_H \), diverging line is the result from one-loop calculations and the other is that from two-loop ones.

Fig. 2: Plots of \( \Lambda_p^{(1)} \) as a function of \( M_H \) for three values of \( M_t = 160 \) (dashed line), 180 (solid line) and 200 (dotted line) GeV. \( \Lambda_p^{(1)} \) is the pole position of \( \lambda \) computed at one-loop level without using matching condition.

Fig. 3: Plots \( \Lambda_p^{(1), (2)} \) as a function of \( M_H \) for \( M_t = 180 \) GeV. \( \Lambda_p^{(2)} \) is defined as the scale at which \( \lambda \) starts to be bigger than some value \( \lambda_{\text{cut}} \). We denote \( \Lambda_p^{(1)} \) as a solid line, \( \Lambda_p^{(2)} \) with \( \lambda_{\text{cut}} = 60 \) as a dashed line, \( \Lambda_p^{(2)} \) with \( \lambda_{\text{cut}} = 40 \) as a dotted line and \( \Lambda_p^{(2)} \) with \( \lambda_{\text{cut}} = 70 \) as a dashed-dotted line.

Fig. 4: Plot of \( \Lambda_p^{(1)} \) (solid line) and plots of \( \Lambda_p^{(1), (\mu_0)} \) as a function of \( M_H \) for three choices of the renormalization scale \( \mu_0 = M_Z \) (dashed line), \( M_t \) (dotted line) and \( M_H \) (dash-dotted line) and \( M_t = 180 \) GeV. We denote the pole position computed at one-loop level using matching condition on the Higgs boson mass as \( \Lambda_p^{(1), (\mu_0)} \).

Fig. 5: Plots of the triviality bounds on \( M_H \) as a function of \( M_t \) for several values of \( \Lambda = 10^{19} \) (thick solid line), \( 10^{15} \) (solid line), \( 10^{10} \) (dashed line), \( 10^6 \) (dotted line) and \( 10^3 \) (dash-dotted line) GeV. We use two-loop \( \beta \) functions with \( \lambda_{\text{cut}} = 60 \) and one-loop matching condition with \( \mu_0 = M_H \).
