Solving 3D magnetohydrostatics with RBF-FD: applications to the solar corona. (English) J. Comput. Phys. 462, Article ID 111214, 15 p. (2022)

Summary: We present a novel magnetohydrostatic numerical model that solves directly for the force-balanced magnetic field in the solar corona. This model is constructed with Radial Basis Function Finite Differences (RBF-FD), specifically 3D polyharmonic splines plus polynomials, as the core discretization. This set of PDEs is particularly difficult to solve since in the limit of the forcing going to zero it becomes ill-posed with a multitude of solutions. For the forcing equal to zero there are no numerically tractable solutions. For finite forcing, the ability to converge onto a physically viable solution is delicate as will be demonstrated. The static force-balance equations are of a hyperbolic nature, in that information of the magnetic field travels along characteristic surfaces, yet they require an elliptic type solver approach for a sparse overdetermined ill-conditioned system. As an example, we reconstruct a highly nonlinear analytic model designed to represent long-lived magnetic structures observed in the solar corona.

MSC:

65Dxx Numerical approximation and computational geometry (primarily algorithms)
65Nxx Numerical methods for partial differential equations, boundary value problems
41Axx Approximations and expansions

Keywords: magnetohydrostatics; radial basis functions finite differences; RBF-FD; solar corona; static constrained PDEs

Full Text: DOI

References:
[1] Bayona, V., An insight into RBF-FD approximations augmented with polynomials, Comput. Math. Appl., 77, 9, 2337-2353 (2019) · Zbl 1442.41007
[2] Bayona, V.; Flyer, N.; Fornberg, B., On the role of polynomials in RBF-FD approximations: III. Behavior near domain boundaries, J. Comput. Phys., 380, 378-399 (2019) · Zbl 1451.65012
[3] Dalmasse, K.; Saveliev, A.; Gibson, S. E.; Fan, Y.; Nychka, D. W.; Flyer, N.; Mathews, N.; DeLuca, E. E., Data-optimized coronal field model. I. Proof of concept, Astrophys. J., 877, 2, 111 (June 2019)
[4] Dove, J. B.; Gibson, S. E.; Rachmeler, L. A.; Tomczyk, A.; Judge, P., A ring of polarized light: evidence for twisted coronal magnetism in cavities, Astrophys. J. Lett., 731, 1, Article L1 pp. (Apr. 2011)
[5] Flyer, N.; Barnett, G.; Wicker, L., Enhancing finite differences with radial basis functions: experiments on the Navier-Stokes equations, J. Comput. Phys., 316, 39-62 (2016) · Zbl 1349.76460
[6] Flyer, N.; Fornberg, B.; Bayona, V.; Barnette, G., On the role of polynomials in RBF-FD approximations: I. Interpolation and accuracy, J. Comput. Phys., 321, 21-38 (2016) · Zbl 1349.65642
[7] Fornberg, B.; Elcrat, A. R., Some observations regarding steady laminar flows past bluff bodies, Philos. Trans. R. Soc. A, 372, Article 20130353 pp. (2014) · Zbl 1353.76042
[8] Fornberg, B.; Flyer, N., A Primer on Radial Basis Functions with Applications to the Geosciences (2015), SIAM Press: SIAM Press Philadelphia, PA · Zbl 1358.86001
[9] Fasshauer, G. F., Meshfree Approximation Methods with MATLAB, vol. 6 (2007), World Scientific Publishers Singapore · Zbl 1123.65001
[10] Gibson, S. E.; Fan, Y., Partially ejected flux ropes: implications for interplanetary coronal mass ejections, J. Geophys. Res. Space Phys., 113, A9 (2008)
[11] Gibson, S. E.; Low, B. C., A time-dependent three-dimensional magnetohydrodynamic model of the coronal mass ejection, Astrophys. J., 493, 460 (1998)
[12] Gibson, S. E.; Low, B. C., Three-dimensional and twisted: an MHD interpretation of on-disk characteristics of coronal mass ejections, J. Geophys. Res., 105, A8, 18187-18202 (2000)
[13] Low, B. C.; Flyer, N., The topological nature of boundary value problems for force-free magnetic fields, Astrophys. J., 668, 1, 557-570 (2007)
[14] Lugaz, N.; Farruglia, C. J.; Manchester, W. B.; Schwadron, N., The interaction of two coronal mass ejections: influence of relative orientation, Astrophys. J., 778, 1, 20 (2013)

[15] MacTaggart, D.; Elsheikh, A.; McLaughlin, J. A.; Simitev, R. D., Non-symmetric magnetohydrostatic equilibria: a multigrid approach, Astron. Astrophys., 556, A40 (2013)

[16] Malamushenko, A.; Flyer, N.; Gibson, S., Convolutional neural networks for predicting the strength of near-Earth magnetic field caused by interplanetary coronal mass ejections, Front. Astron. Space Sci., 7, 62 (2020)

[17] Manchester, W. B.; Gombosi, T. I.; Roussev, I.; Ridley, A.; De Zeeuw, D. L.; Sokolov, I. V.; Powell, K. G.; Toth, G., Modeling a space weather event from the Sun to the Earth: CME generation and interplanetary propagation, J. Geophys. Res. Space Phys., 109, A2 (2004)

[18] Manchester, W. B.; Vourlidas, A.; Toth, G.; Lugaz, I. I.; Roussev, N.; Sokolov, I. V.; Gombosi, T. I.; de Zeeuw, D. L.; Opher, M., Three-dimensional MHD simulation of the 2003 October 28 coronal mass ejection: comparison with LASCO coronograph observations, Astrophys. J., 684, 2, 1448-1460 (2003)

[19] Mathews, N. H.; Flyer, N.; Gibson, S. E., Reconstructing the coronal magnetic field: the role of cross-field currents in solution uniqueness, Astrophys. J., 898, 70 (2020)

[20] Nindos, A.; Patsourakos, S.; Vourlidas, A.; Cheng, X.; Zhang, J., When do solar erupting hot magnetic flux ropes form?, Astron. Astrophys., 642, A109 (2020)

[21] Paige, C. C.; Saunders, M. A., LSQR: an algorithm for sparse linear equations and sparse least squares, ACM Trans. Math. Softw., 8, 1, 43-71 (1982) · Zbl 0478.65016

[22] Shu, C.; Ding, H.; Yeo, K. S., Local radial basis function-based differential quadrature method and its application to solve two-dimensional incompressible Navier-Stokes equations, Comput. Methods Appl. Mech. Eng., 192, 7-8, 941-954 (Feb. 2003)

[23] Tolstykh, A. I.; Shirobokov, D. A., On using radial basis functions in a “finite difference mode” with applications to elasticity problems, Comput. Mech., 33, 1, 68-79 (Dec. 2003)

[24] Tominec, I.; Larsson, E.; Heryudono, A., A least squares radial basis function finite difference method with improved stability properties, SIAM J. Sci. Comput., 43, 2, A1441-A1471 (2021) · Zbl 1472.65153

[25] Török, T.; Kliem, B., Confined and ejective eruptions of kink-unstable flux ropes, Astrophys. J., 630, 1, L97-L100 (Aug. 2005)

[26] Wiegelmann, T.; Neukirch, T., An optimization principle for the computation of MHD equilibria in the solar corona, Astron. Astrophys., 457, 3, 1053-1058 (2006)

[27] Wright, G.; Fornberg, B., Scattered node compact finite difference-type formulas generated from radial basis functions, J. Comput. Phys., 212, 99-123 (2006) · Zbl 1089.65020

[28] Zhu, X.; Wiegelmann, T., On the extrapolation of magnetohydrostatic equilibria on the Sun, Astrophys. J., 866, 130 (2018)

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