Hard Pomeron in exclusive meson production at ILC

R. ENBERG\textsuperscript{a,b}, B. PIRE\textsuperscript{a}, L. SZYMANOWSKI\textsuperscript{c,d} S. WALLON\textsuperscript{e}

\textsuperscript{a}CPHT, École Polytechnique, F-91128 Palaiseau, France\textsuperscript{2}
\textsuperscript{b}Lawrence Berkeley Laboratory, Berkeley, Ca, USA\textsuperscript{3}
\textsuperscript{c}Soltan Institute for Nuclear Studies, Hoża 69, 00-681 Warsaw, Poland
\textsuperscript{d}Université de Liège, B4000 Liège, Belgium
\textsuperscript{e}LPT, Université Paris-Sud, F-91405 Orsay, France\textsuperscript{4}

We calculate the exclusive process $\gamma^*_L(Q_1^2)\gamma^*_L(Q_2^2) \rightarrow \rho^0_L\rho^0_L$, at high energy. The Born order estimate and the leading (LLA) and next to leading order (NLLA) BFKL resummation effects show the feasibility of experimental detection in a quite large range of $Q^2$ values at future high energy $e^+e^-$ linear colliders.

\textsuperscript{1}Presented at Photon2005, International Conference on the Structure and Interactions of the Photon, Warsaw 31.08-04.09.2005, by Samuel Wallon.
\textsuperscript{2}Unité mixte 7644 du CNRS.
\textsuperscript{3}present address
\textsuperscript{4}Unité mixte 8627 du CNRS.
1 Introduction

Future $e^+e^-$ colliders will offer the possibility of clean testing of QCD dynamics through the scattering of two virtual photons. By selecting events in which two vector mesons are produced with a large rapidity gap and no accompanying particles, one is getting access to the kinematical regime in which the perturbative approach to exclusive scattering is justified. When the photon virtualities are comparable, the perturbative Regge dynamics of QCD should dominate and allow the use of resummation techniques of the BFKL type. We have studied [1] these effects for the reaction

$$\gamma_1^*(q_1) \gamma_2^*(q_2) \to \rho_L^0(k_1) \rho_L^0(k_2),$$

where the virtualities $Q_i^2 = -q_i^2$ of the scattered photons play the role of the hard scales. Up to now, the available experimental data are restricted at rather small values of the energy [2] and are analysed in terms of generalized distribution amplitudes [3]. Recently the same process in the forward limit was studied in the full NLLA BFKL approach in Ref. [4].

2 Born order study

The Born approximation study of the scattering amplitude of the process (1) proves the feasibility of a dedicated experiment. The impact representation of the scattering amplitude has the form, in terms of transverse momenta,

$$\mathcal{M} = is \int \frac{d^2k}{(2\pi)^4k^2(r-k)^2} \mathcal{J}^\gamma_{L}(q_1) \to \rho_L^0(k_1)(r-k) \mathcal{J}^\gamma_{L}(q_2) \to \rho_L^0(k_2)(-r,-k+k),$$

where $\mathcal{J}^\gamma_{L}(q_i) \to \rho_L^0(k_i)$ are the impact factors corresponding to the transition of $\gamma_i^*(q_i)$ via the $t$–channel exchange of two gluons, with $r = 1/2(k_1 - k_2)$ and $\vec{r}^2 = t_{\text{min}} - t$. In this approximation, the amplitude (2) depends linearly on $s$ and can be expressed as

$$\mathcal{M} = is 2\pi \frac{N_c^2 - 1}{N_c^2} \alpha_s^2 \alpha_{em} f_2^2 Q_1 Q_2 \int dz_1 dz_2 z_1 \bar{z}_1 \phi(z_1) z_2 \bar{z}_2 \phi(z_2) M(z_1, z_2),$$

with (denoting $\mu_i^2 = Q_i^2 z_i$)

$$M(z_1, z_2) = \int \frac{d^2k}{k^2(r-k)^2} \left[ \frac{1}{z_1^2 L^2 + \mu_1^2} + \frac{1}{z_2^2 L^2 + \mu_2^2} - \frac{1}{(z_1 L - k)^2 + \mu_1^2} \right] (1 \leftrightarrow 2).$$

We evaluated $M(z_1, z_2)$ analytically and checked that the production amplitude dramatically decreases with $t$ so that its magnitude at $t = t_{\text{min}}$ dictates the rate of the reaction. In this simpler case, the integral over $\vec{k}$ can easily be performed and gives

$$M(z_1, z_2) = \frac{4\pi}{z_1 \bar{z}_1 z_2 \bar{z}_2 Q_1^2 Q_2^2 (z_1 \bar{z}_1 Q_1^2 - z_2 \bar{z}_2 Q_2^2)} \ln \frac{z_1 \bar{z}_1 Q_1^2}{z_2 \bar{z}_2 Q_2^2}.$$
The amplitude $\mathcal{M}$ can then be analytically computed through $z_1, z_2$ integration. In the special case where $Q = Q_1 = Q_2$, it simplifies to

$$
\mathcal{M}_{t_{\text{min}}} (Q_1 = Q_2) = -i s \frac{N_c^2 - 1}{N_c^2} \alpha_s^2 \alpha_{em} f_{\rho}^2 \frac{9\pi^2}{2Q^4} (24 - 28 \zeta(3)) . \tag{6}
$$

The peculiar limits $R = Q_1/Q_2 \gg 1$ and $R \ll 1$ correspond to the kinematics typical for deep inelastic scattering on a photon target described through collinear approximation. In the limit $R \gg 1$ the amplitude simplifies into

$$
\mathcal{M}_{t_{\text{min}}} \sim i s \frac{N_c^2 - 1}{N_c^2} \alpha_s^2 \alpha_{em} \alpha (k_1) \beta (k_2) f_{\rho}^2 \frac{96\pi^2}{Q_1^2 Q_2^2} \left( \ln \frac{R}{R - 1} - \frac{1}{6R} \right) . \tag{7}
$$

We show on Fig.1L the differential cross section $d\sigma/dt = |\mathcal{M}|^2 / (16\pi s^2)$ at the threshold $t = t_{\text{min}}$ as a function of $Q_2^2/Q_1^2$. Its rather large order of magnitude when $Q_1^2 \approx Q_2^2 \approx 1–10$ GeV$^2$ implies the feasibility of its experimental measure at the International Linear Collider.

Figure 1: Left: Differential cross-section for the process $\gamma_L^* \gamma_L^* \rightarrow \rho_L^0 \rho_L^0$ at Born order, at the threshold $t = t_{\text{min}}$, as a function of $Q_2^2/Q_1^2$. The asymptotical curves, given by Eq. (7), are valid for large $Q_2^2/Q_1^2$. The dots correspond to the special case $Q_1 = Q_2$ where the formula (6) can be applied. Right: Cross section for LLA BFKL (dotted curves) and for the NLLA corrected kernel (full curves), for the three cases $Q = Q_1 = Q_2 = 2$ GeV, 3 GeV and 4 GeV (from top to bottom in the plot).

### 3 BFKL effects

We studied BFKL effects in the leading order approximation at $t = t_{\text{min}} \sim -Q_1^2 Q_2^2 / s$. In the BFKL framework, the amplitude of the process can be expressed through the inverse Mellin transform with respect to $s$ as

$$
A(s, t = t_{\text{min}}) = i s \int \frac{d\omega}{2\pi i} e^{\omega Y} f_{\omega} (\omega^2 = 0) , \tag{8}
$$
where $Y = \ln(s/s_0)$ is the rapidity. The BFKL Green’s function reads

$$f_\omega(0) = \frac{1}{2(2\pi)^4} \int \frac{dk^2}{k^3} \frac{dk'^2}{k'^3} J_{\gamma^* L}^{\gamma^* L}(q_1 \rightarrow \rho_1^0 (k_1)) (k, -k) J_{\gamma^* L}^{\gamma^* L}(q_2 \rightarrow \rho_1^0 (k_2)) (-k', k')$$

$$\times \int_{-\infty}^{\infty} d\nu \frac{1}{\omega - \omega(\nu)} \left( \frac{k^2}{k'^2} \right)^{i\nu},$$

where the integration over angles has been performed. $\omega(\nu)$ is the BFKL characteristic function defined as $\omega(\nu) = \bar{\alpha}_s \chi(\nu)$, with $\bar{\alpha}_s \equiv \alpha_s N_c / \pi$ and

$$\chi(\nu) = 2 \psi(1) - \psi(1/2 + i\nu) - \psi(1/2 - i\nu), \quad \psi(x) = \Gamma'(x)/\Gamma(x).$$

At LLA accuracy, $\bar{\alpha}_s$ is a fixed parameter. We choose, however, let it depend on the given $Q_1$ and $Q_2$, which are external to the pomeron but provide a reasonable choice, through $\bar{\alpha}_s = N_c \frac{\pi}{\alpha_s} (Q_1 Q_2)$.

To estimate higher order effects, two improvements to the LLA BFKL amplitude have been made: BLM scale fixing (for the running of the coupling entering the impact factors) and renormalization group resummed BFKL kernel.

We show on Fig.1R the differential cross-section corresponding to the LLA BFKL and NLLA corrected kernel. The amplification factor due to LLA BFKL resummation is indeed large. It is larger than the enhancement predicted for the total cross-section [5], because the enhancement at the level of amplitude needs to be squared for exclusive processes. This effect is reduced when including NLL corrections.

4 Conclusion

Since we expect the ILC to cover a quite large region in rapidity, experiments will allow us to test the dynamics of Pomeron exchange. The Born approximation estimate of this reaction, and the increase of the amplitude due to BFKL resummation effects show that the process $\gamma^* \gamma^* \rightarrow \rho \rho$ should be measurable by dedicated experiments at the ILC, for virtualities of the photons up to a few GeV$^2$. We plan to pursue this line of research with Odderon exchange processes such as $\gamma^* \gamma^* \rightarrow \pi^0 \pi^0$ or, through interference effects in charge asymmetric observables in $\gamma^* \gamma^* \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ [6]

Work of L.Sz. is supported by the Polish Grant 1 P03B 028 28. He is a Visiting Fellow of the FNRS (Belgium).

References

[1] B. Pire, L. Szymanowski and S. Wallon, arXiv:hep-ph/0507038, arXiv:hep-ph/0501155 and arXiv:hep-ph/0410108. R. Enberg, B. Pire, L. Szymanowski and S. Wallon, arXiv:hep-ph/0508134

[2] P. Achard et al. [L3 Collaboration], Phys. Lett. B 568, 11 (2003); Phys. Lett. B 597, 26 (2004); Phys. Lett. B 615, 19 (2005).
[3] M. Diehl et al., Phys. Rev. Lett. 81, 1782 (1998) and Phys. Rev. D 62, 073014 (2000); I. V. Anikin et al., Phys. Rev. D 69, 014018 (2004) and arXiv:hep-ph/0506277.

[4] D. Y. Ivanov and A. Papa, arXiv:hep-ph/0508162.

[5] J. Bartels et al., Phys. Lett. B 389, 742 (1996); J. Kwiecinski and L. Motyka, Phys. Lett. B 462, 203 (1999) and Eur. Phys. J. C 18, 343 (2000); M. Boonekamp et al, Nucl. Phys. B 555, 540 (1999).

[6] P. Hägler et al., Phys. Lett. B 535, 117 (2002), Eur. Phys. J. C 26, 261 (2002).