Can wavelets produce a clearer picture of weak-form market efficiency in Bitcoin?

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Abstract

This study proposes a new approach for testing for random walk behavior in daily Bitcoin returns (19/07/2010–03/03/2022) by contextualizing the Dickey-Fuller test in time-frequency space using continuous complex wavelet transforms. By splitting our full sample into smaller sub-sample periods segregated by Bitcoin halving dates, we find that Bitcoin returns are most predictable or least market efficient (i) at higher frequency or short-run cycles of between 2 and 16 days, (ii) between November-February months, (iii) during ‘bubble’ periods, (iv) across the consecutive halving dates, (v) during the ‘Black Swan event’ caused by financial market turmoil arising from the COVID-19 pandemic, and (vi) subsequent to the announcements of new COVID-19 variants. Altogether, our findings have important policy implications for different stakeholders in Bitcoin markets.

Keywords Bitcoin · Efficient market hypothesis (EMH) · Adaptive market hypothesis (AMH) · Random walks · Wavelet coherence

1 Introduction

The pseudonymous person(s) Satoshi Nakamoto developed Bitcoin as an open source software code that implements a decentralized peer-to-peer system for electronic transactions and its design is “…based on cryptographic instead of proof, allowing two willing parties to transact directly with each other without the need for a trusted third party…” (Nakamoto, 2009). Naturally, earlier academic work on Bitcoin, and the development of cryptocurrency markets, was mainly centered around research in fields of computer sciences (Merediz-Sola & Bariviera, 2019) but quickly attracted
interest amongst financial economists and regulatory authorities following the exponential growth of Bitcoin prices, trade volume and market capitalization experienced over the last decade or so (Smith & Kumar, 2018; Corbert et al., 2019).

The growing consensus within the economics paradigm is that Bitcoin bears certain characteristics of traditional financial assets. For instance, Baur et al., (2018) find that Bitcoin has financial features of a speculative asset as opposed to a medium of exchange and performs poorly as a unit of account or store of value because of its high volatility. Other authors find that Bitcoin can be used for hedging purposes against equity assets (Garcia-Jorcano & Benito, 2020), currencies (Urquhart & Hanxiong, 2019; Bedi & Nashier, 2020) and commodities such as gold (Pal & Mitra, 2019) and oil (Symitsi & Chalvatzis, 2019). Moreover, in similarity to other traditional financial assets, Bitcoin is affected by policy announcements (Pyo & Lee, 2020), social media (Shen et al., 2019; Kraaijeveld and De Smedt, 2020; Geugan & Renault, 2021), google trends/investor sentiments (Urquhart, 2018; Shen et al., 2019; Bouri & Gupta, 2021) and has been punctuated by episodes of bubble-like behavior (Chaim & Laurini, 2019; Xiong et al. 2020; Hafner 2020; Kyriazis et al., 2020; Moosa, 2020; Enoksen et al., 2020).

Due to its similarity to financial assets, Bitcoin has inherited some of the classical academic debates on assets markets, chief among these is the efficient market hypothesis (EMH), which questions whether financial markets are informationally efficient or not. Fama (1970) identifies three forms of information (i.e. private, public and historic) which rational participants can use to influence market efficiency. Markets are considered to be informational efficient if new information is immediately absorbed into asset prices and cannot be used for predicting purposes. From the Bitcoin literature, most studies focus on weak form market efficiency, that is, determining whether market participants can use historic information from the Bitcoin time series to predict future returns and thus ‘beat the market’. These studies employ a wide range of statistical tools to determine whether Bitcoin returns evolve as a random walk/martingale process (efficient) or stationary/long-memory process (inefficient) (see Merediz-Sola & Bariviera, 2019; and Corbert et al. 2019, for indepth reviews).

The general consensus derived from literature is that Bitcoin is only market (in) efficient over certain time periods (i.e. time-varying efficiency), as insinuated by the adaptive market hypothesis (AMH) of Lo (2004), although these studies identify different time periods or structural break points when the Bitcoin market switches from being inefficient to market efficient or vice versa (Urquhart, 2016; Nadarajah & Chu, 2017; Bariviera et al., 2017; Bariviera, 2017; Tiwari et al., 2018; Jiang et al., 2018, Alvarez-Ramirez et al., 2018; Zhang et al., 2018; Vidal-Tomas et al., 2018; Khuntia & Pattanayak 2018; Aggarwal, 2019; Chu et al., 2019; Sensoy, 2019; Tran & Leirvik, 2020; Vidal-Tomas, 2020; Wu & Chen, 2020; Lopez-Martin et al., 2021; Manahov & Urquhart, 2021; Yaya et al., 2021). There also exists a smaller and more recent strand of studies (Corbet & Paraskei, 2020; Naeem et al., 2021) which identify a different mechanism for asymmetric behavior in the Bitcoin returns in which market efficiency switches between bear and bulls markets. These later studies insinuate asymmetric cyclical behavior in market efficiency for Bitcoin returns which differ from the time-varying dynamics identified in other literature.
Our study proposes a novel method of simultaneously examining time-varying and cyclical-varying dynamics in the random walk model of Bitcoin prices as a means of testing for market efficiency. Using continuous wavelet transforms, we develop a random walk testing procedure for Bitcoin returns in a time frequency domain which are consistent with the assumptions underlying the traditional Dickey-Fuller test for (non)stationarity. The mathematical precision wavelets present in capturing the temporal and spectral dynamics in the co-movement between a pair of time series allows us to gain new insights to random walk behavior of Bitcoin time series as we are able to capture different cycles of weak-form market efficiency/inefficiency across different time periods. Notably, similar wavelet coherence techniques have been widely applied in the cryptocurrency literature to examine the time-frequency co-movement between Bitcoin and gold (Kang et al., 2019), Bitcoin returns and volatility (Qiao et al., 2020), Bitcoin and COVID-19 health statistics (Goodell & Goutte, 2021) and Bitcoin and COVID-19 fear index (Rubbanly et al., 2021).

The main contribution of our study is that it harmonizes ‘bits-and-pieces’ of seemingly contradictory empirical evidences found in the previous literature which have investigated the efficiency of Bitcoin markets. Firstly, our study demonstrates that the Bitcoin market has been alternating between efficiency and in(efficient) more frequently yet irregularly than previously studies suggested. Secondly, we find that Bitcoin returns are more predictable or market inefficient during November to February periods every year which reflect calendar effects anomalies recent confirmed by Kaiser (2019), Baur et al., (2019), Kinateder & Papavassiliou (2021) and Qadan et al., (2021). Thirdly, we find that Bitcoin markets are most inefficient during periods in which Bitcoin markets experience bubble build-ups and crashes. Fourthly, our study is consistent with literature which suggest that predictability in Bitcoin markets is more prominent over the short-run or at high-frequency oscillations (Kakinaka & Umeno, 2021). Fifthly, consistent with more recent literature, we find that COVID-19 pandemic has an adverse impact on market efficiency of Bitcoin markets (Manif et al., 2020; Kakinaka & Umeno 2021; Naeem et al., 2021).

We also present novel evidence suggesting that overall market efficiency in Bitcoin markets has been progressively diminishing across the Bitcoin halving dates in which the supply of the cryptocurrency is consecutively halved in four-year cycles. Moreover, our study further finds that during the post-2021 period, whom Rouatbi et al., (2021) dub the ‘immunization period of the COVID-19 pandemic’, Bitcoin has been become more frequently market inefficient as more variants of the diseases have been announced by the health community. Altogether, this study not only adds new knowledge to the scientific literature but also has important implications for different stakeholders in Bitcoin markets.

The rest of the study is structured as follows. The next section presents the empirical data and study outlines the empirical approach of the study. The third section presents the empirical results whilst section four concludes the study.
2 Data and preliminary analysis

Our study uses daily data of Bitcoin closing prices in US$ ($p_t$) collected from 19th July 2010 to 3rd March 2022, sourced from COINDESK and we compute the returns ($r_t$) on Bitcoin as $r_t = \log(p_t) - \log (p_{t-1})$. As a preliminary exercise, we compute the summary statistics and perform the ADF and DF-GLS unit root tests on Bitcoin prices and returns. Moreover, for comparative purposes we split our full sample into four smaller samples which are segregated by the Bitcoin halving dates of 28th October 2012, 8th July 2016 and 11th May 2020. We consider these dates as relevant break points as they describe the ‘halving mechanism’ of issuing new bitcoins in four cycles until its diminish in 2040 and these dates represent an important technical aspect in the functioning of Bitcoin (Meynkhard, 2019; Eksi & Schreiti, 2022). To validate these break points, we perform Chow test at each of the three halving dates and report statistics of 321.35 (3.38), 1559.25 (1.96) and 9362.07 (0.28) for Bitcoin prices (returns), respectively. We therefore conclude on observing statistically significant breaks for all halving dates for the case of Bitcoin prices whereas those for Bitcoin returns are only significant at the first halving date.

Table 1 reports the descriptive statistics and unit root tests for Bitcoin prices and returns series and based on the results we summarize four important stylized facts which may be difficult to observe from the time series plots presented in Figs. 1 and 2, respectively. Firstly, we note that the averages and volatilities of Bitcoin price (returns) has been increasing (decreasing) across consecutive halving sub-sample periods with the sole exception of the case of Bitcoin returns between 3rd and 4th sub-samples which experience increases in average values. Secondly, Bitcoin prices have been progressively decreasing from a positive skew in the first three sub-samples to a negative skew in the 4th sub-sample whilst Bitcoin returns sharply changed from positive to negative skew between the 1st and 2nd subsamples and thereafter has progressively moved towards being positively skewed in the last sub-sample period. Thirdly, both Bitcoin prices and returns are characterized by relatively larger fat tails in all subsamples although kurtosis values progressively become smaller across the sub-sample periods. Lastly, we find that whilst both ADF and DF-GLS tests confirm unit root process (stationary) in Bitcoin prices (returns) across all sub-samples except for the DF-GLS test on the full sample and 1st subsample for Bitcoin returns which confirms a unit root process in the series.

Overall, we treat our findings from Table 1 as providing preliminary evidence of time-varying weak-form market efficiency as advocated by the AMH, particularly for the DF-GLS tests which indicate that Bitcoin returns have evolved from being efficient for periods prior to the first halving period and yet turns inefficient in subsequent subsamples. In the next section, we outline our main empirical framework which uses continuous wavelet tools to model a Dickey-Fuller type test for random walk behavior in Bitcoin returns across a time-frequency plane.
3 Empirical approach

Our empirical approach is centered on the random walk test of Dickey and Fuller (1979) who proposed testing for random walk behavior in a bi-variate regression of a time series \( y_t \) regressed on it’s own first lag \( y_{t-1} \) i.e.

\[
y_t = \beta y_{t-1} u_t
\]

And evaluating whether the coefficient on \( y_{t-1} \) is significantly different from unity i.e. \( H_0: \beta = 1 \), implying that the returns series evolve as a random walk, whereas the series

Table 1 Descriptive statistics of Bitcoin price and returns

|                  | Full sample | 1st halving period | 2nd halving period | 3rd halving period | 4th halving period |
|------------------|-------------|--------------------|--------------------|--------------------|--------------------|
| **Mean**         | 7379.05     | 5.41               | 340.96             | 5744.25            | 34473.40           |
| **Median**       | 630.34      | 4.93               | 318.56             | 6285.86            | 37341.59           |
| **Maximum**      | 67549.74    | (11/09/2021)       | 1119.79            | 19166.98           | 67549.74           |
| **Minimum**      | 0.05        | (26/07/2010)       | 12.28              | 555.93             | 8608.13            |
| **Std. dev.**    | 14038.98    | 4.91               | 216.14             | 3693.36            | 18047.52           |
| **Skewness**     | 2.49        | 0.95               | 0.59               | 0.34               | -0.13              |
| **Kurtosis**     | 8.31        | 3.87               | 3.02               | 2.73               | 1.63               |
| **J-B (p-value)**| 0.00        | 0.00               | 0.00               | 0.00               | 0.00               |
| **ADF**          | -0.37[]     | -1.55[ ]          | -1.86[19]          | -1.88[0]           | -1.46[0]           |
| **DF-GLS**       | 0.15[ ]     | -0.53[ ]          | -0.33[19]          | -0.58[0]           | -0.15[0]           |
| **Chow test**    | N/A         | 321.35***          | 1559.25***         | 9362.07***         | N/A                |
| **Obs.**         | 4246        | 846               | 1319               | 1402               | 661                |

Panel B: Bitcoin returns

|                  | Mean | Median | Maximum | Minimum | Std. dev. | Skewness | Kurtosis | J-B (p-value) | ADF          | DF-GLS      | Chow test | Obs. |
|------------------|------|--------|---------|---------|-----------|----------|----------|--------------|--------------|-------------|-----------|------|
| **Mean**         | 0.13 | 0.25   | 0.13    | 0.08    | 0.08      | 0.10     |          |              | -23.13***[5] | -1.29[20]  | N/A       | 4246     |
| **Median**       | 0.098| 0.11   | 0.09    | 0.09    | 0.09      | 0.12     |          |              | -19.71***[1] | -2.00[12]  | 3.38*     | 846      |
| **Maximum**      | 15.64| 15.65  | 9.81    | 9.32    | 6.54      |          |          |              | -13.71***[4] | -2.00[12]  | 3.38*     | 1319     |
| **Minimum**      | -21.89| -13.54 | -21.89  | -13.72  | -5.83     |          |          |              | -13.71***[4] | -2.00[12]  | 3.38*     | 1402     |
| **Std. dev.**    | 2.06 | 2.68   | 2.01    | 1.85    | 5.07      |          |          |              | -19.71***[1] | -2.00[12]  | 3.38*     | 1402     |
| **Skewness**     | -0.49| 0.24   | -1.76   | -0.57   | 0.09      |          |          |              | -13.71***[4] | -2.00[12]  | 3.38*     | 1402     |
| **Kurtosis**     | 15.47| 10.46  | 25.57   | 8.61    | 5.07      |          |          |              | -13.71***[4] | -2.00[12]  | 3.38*     | 1402     |
| **J-B (p-value)**| 0.00 | 0.00   | 0.00    | 0.00    | 0.00      |          |          |              | -19.71***[1] | -2.00[12]  | 3.38*     | 1402     |
| **ADF**          | -23.13***[5] | -19.71***[1] | -13.71***[4] | -38.67***[0] | -26.10***[0] |          |          |              | -19.71***[1] | -2.00[12]  | 3.38*     | 1402     |
| **DF-GLS**       | -1.29[20] | -2.00[12] | -12.53***[4] | -24.10***[1] | -5.99***[4] |          |          |              | -19.71***[1] | -2.00[12]  | 3.38*     | 1402     |
| **Chow test**    | N/A | 3.38*  | 1.96    | 0.28    |          |          |          |              | -19.71***[1] | -2.00[12]  | 3.38*     | N/A      |
| **Obs.**         | 4246 | 846    | 1319    | 1402    | 661       |          |          |              | -19.71***[1] | -2.00[12]  | 3.38*     | 661      |

Note: ‘***’,‘**’,‘*’ denote 1%, 5% and 10% significance levels, respectively for ADF, DF-GLS and Chow tests. Optimal lag lengths for ADF and DF-GLS tests as determined by modified AIC are reported in [ ]. Dates corresponding to minimum and maximum values are reported in ()
is stationary if \(0 < |\beta| < 1\) or a white noise if \(\beta = 0\). Econometrically, the null hypothesis of unit root behaviour can also be specified as:

\[
\beta = \frac{\text{cov}(y_t, y_{t-1})}{\text{var}(y_t)} = 1
\]  

(2)

And from Eq. (2) it easy to see that the null hypothesis holds only if \(\text{cov}(y_t, y_{t-1}) = \text{var}(y_t)\) which implies that the \(\text{var}(y_t)\) and \(\text{var}(y_{t-1})\) are equal. Alternatively, the null hypothesis can be re-written in terms of the autocorrelation coefficient between \(y_t\) and \(y_{t-1}\) as:

\[
\beta = \frac{\text{cov}(y_t, y_{t-1})}{\text{var}(y_t)} = 1
\]  

(2)

\[
\beta = \frac{\text{cov}(y_t, y_{t-1})}{\text{var}(y_t)} = 1
\]  

(2)
\[ \rho (y_t, y_{t-1}) = \frac{\text{cov}(y_t, y_{t-1})}{\text{sd}(y_t) \text{sd}(y_{t-1})} = 1 \]  

(3)

Where Eq. (3) only holds if standard deviation (and consequentially variances) of \( y_t \) and \( y_{t-1} \) are equal. In our study we propose a random walk test of correlation between \( y_t \) and \( y_{t-1} \) over time and frequency domain using continuous wavelets to transform the time series, \( \Delta y_t \) and \( y_{t-1} \). These wavelets or ‘daughter wavelets’ we transform the time series into a two-dimensional time-frequency space originate from an analytical or mother wavelet by dilation and translation, i.e.

\[ \psi_s(t) = s^{-0.5} \left( \frac{t-s}{s} \right) \]  

(4)

Where \( \tau \) is the translation parameter controlling the width of the wavelet, \( s \) is the scaling parameter controlling the length of the wavelet and \( s^{-0.5} \) is a normalizing factor which ensures that the daughter wavelets and the mother wavelets keep the same ‘energy’. The continuous wavelet transforms (CWT) of the time series \( y_t \) and \( y_{t-1} \) with respect to \( \psi \), is obtained by comparing the time series with the family of wavelet daughters:

\[ W_{y(t)}(s, t) = \int_{-\infty}^{\infty} y(t) \frac{1}{\sqrt{s}} * \left( \frac{t-s}{s} \right) dt \]  

(5)

\[ W_{y(t-1)}(s, t) = \int_{-\infty}^{\infty} y(t-1) \frac{1}{\sqrt{s}} * \left( \frac{t-s}{s} \right) dt \]  

(6)

Where \( * \) denotes a complex conjugation. Whilst there exist many variations of ‘wavelet families’ to choose from, we make use to the ‘Morlet wavelet’ which has optimal joint time-frequency concentration and is defined as:

\[ (t) = -^{0.25} \exp (i_0 t) \exp (-\frac{1}{2} t^2) \]  

(7)

To ensure that the parameterization of the Morlet wavelet depicts an inverse relation between wavelet scales and the frequencies, \( t \approx s^{-1} \), the Morlet is set at \( 2\pi \) so that the wavelet scale, \( s \), is equal to the Fourier period. The wavelet power spectrum (WPS) of the time series, which measures the variance of the series across a time-frequency plane, can be defined as \( W_{y(t)y(t)} = \left| W_{y(t)} \right|^2 \) and \( W_{y(t-1)y(t-1)} = \left| W_{y(t-1)} \right|^2 \), respectively. From the WPS of the series, the Cross-Wavelet Power Spectrum (CWPS), which is analogous to the autocovariance between \( y(t) \) and \( y(t-1) \) in time-frequency domain, is given as \( (WPS)_{y(t-1)y(t)} = W_{y(t-1)y(t)} = \left| W_{y(t-1)y(t)} \right| \). Finally, wavelet coherence, which is analogous to the correlation between \( y(t-1) \) and \( y(t) \) across time and frequency, is
computed as the ratio of the cross spectrum to the product of the spectrums of the individual series i.e.

\[
R_n(s) = \frac{S(W_{xy})}{[(SW_x^2)(SW_y^2)]^{\frac{1}{2}}}
\]  

(8)

Where \( S \) is a smoothing operator in both time and scale. To distinguish between negative and positive correlation between a pair of time series as well as identifying lead-lag causal relationships between the variables, we make use of phase difference dynamics we are defined as:

\[
\phi_{x,y} = \tan^{-1}\left(\frac{I\{W_x\}}{\Re\{W_x\}}\right)
\]

(9)

Where \( \phi_{x,y} \) is parametrized in radians, bound between \( \pi \) and \(-\pi\). If \( \phi_{x,y} \in (0, \pi) \) and \( \phi_{x,y} \in (0, -\pi) \), then the series are said to be in-phase (positive correlation) with \( y \) leading \( x \) in the former and \( x \) leading \( y \) in the latter. Conversely, If \( \phi_{x,y} \in (\pi, \pi) \) and \( \phi_{x,y} \in (\pi, -\pi) \), then the series are said to be in an anti-phase (negative correlation) with \( x \) leading \( y \) in the former and \( y \) leading \( x \) in the latter. A phase-difference of zero implies co-movement between the pair of series at the specified frequency.

4 Results

To facilitate the analysis of our empirical findings from our main wavelet analysis, we split our data into year timeframes corresponding to the Bitcoin halving dates which are presented in 4 interconnected wavelet coherence plots in Figs. 3, 4, 5 and 6. One notable advantage with wavelet coherence analysis, is that the results are not altered by narrowing or widening the time window in the analysis. This differs from conventional regression analysis where results for the full sample differ from those of the sub-samples and hence previous studies have exercised caution in selecting break-dates used to sample-split the data (Wu & Chen, 2020). Nevertheless, the reasoning for employing different sub-periods in our analysis is to ‘zoom in’ closer into the wavelet coherence plots and observe monthly effects which would be otherwise difficult to do using the entire sample period as a time window. Moreover, there are no previous studies which have used Bitcoin halving periods as structural break-points in the previous literature focused on examining bitcoins market efficiency.

The wavelet coherence plots describe the time frequency dynamics between the time series are visual presented using ‘heat maps’ and ‘arrow orientation’. On one hand, the heat maps show the strength of the co-movement of the time series at different frequencies across the time domain. The warm contours in the heat maps indicate strong correlation whilst cooler colors represent weaker co-movement between the series. On the other hand, the arrows in the heat map indicate the phase dynamics.
between the series being predictable if synchronizations are anti-phase (negative) denoted by arrow orientation ↓, ← and ↖. The faint white lines surrounding the color contours represent the 5% significance level.

From Figs. 3, 4, 5 and 6, we generally observe periodic significant anti-phase synchronizations between previous Bitcoin returns and current returns indicating the Bitcoin returns are stationary only during certain periods of the different years and we note certain similarities in cyclical oscillations in all four subsample periods. For instance, we observe that higher frequency synchronization of between 2 and 16 days are most prominent in all plots. This finding fits well with some recent literature which similarly finds more inefficiency over short-run cycles (Kakinaka & Umeno, 2021). We also observe that in all sub-samples, the most consistent co-movements are observed in the November to February periods at frequency oscillations of 2–16 days, with reflects ‘month-of-the-year’ effects similarly found by Kaiser (2019), Baur et al., (2019), Kinateder & Papavassiliou (2021) and Qadan et al., (2021). Moreover, in all sub-sample periods we observe additional cycles of predictability or market
inefficiency around periods of observed bubble ‘build-up and burst’ behavior in Bitcoin prices and during these periods we observe strong frequency synchronizations ranging up to oscillations as high as 32-day cycles. These include periods of the first Bitcoin bubble between April 2011 and July 2011; the second bubble between January 2013 and April 2013; the third bubble in Mt. Gox between November 2013 and February 2014; the fourth bubble between November 2017 and December 2018; and the more recent ‘Black Friday cryptocrash’ between October 2020 and January 2021 (Kyriazis et al., 2020; Enoksen et al., 2020).

Altogether our findings bind together ‘bit-and-pieces’ of previous empirical literature on Bitcoin market (in)efficiency in a harmonious way. For instance, our findings correspond to those of Urquhart (2016), Zhang et al., (2018) and Bariviviera (2018) who find evidence of Bitcoin being less inefficient between 2011–2014 period as well as those of Sensoy (2019) and Tran & Leirvik (2020) who find improved efficiency from 2016 onwards particularly in 2017–2019 period. Our findings can also concur with those of Tiwari et al., (2018), Kristoufek & Vosvrda (2019) and Alvarez-Ramirez & Rodriguez (2021) who find Bitcoin is inefficient during April-August 2013, August-November 2016, January 2015-June 2017, January 2016-March 2017, respectively, and also partially correspond to the findings of Wu & Chen (2020) who find efficiency between 2014–2016. Similarly, our findings are also in line with Yaya et al., (2021) recent findings of Bitcoin being more inefficient during the pre-crash period of August 2015 – October 2017 compared to the post-crash period of October 2017-November 2018. Finally, our findings are also in line with the more recent findings of Manif et al. (2020), Kakinaka & Umeno (2021), Naeem et al., (2021) and Alvarez-Ramirez & Rodriguez (2021), who similarly observe that the pandemic affected market efficiency, particularly during the initial global outbreak of the virus in March – April 2020.

Moreover, our empirical analysis contributes new knowledge to the existing literature on Bitcoin market efficiency in two main ways. Firstly, we find that market efficiency in Bitcoin has been affected by the halving periods of Bitcoin supply which have progressively diminished market efficiency, in the weak-form sense. This finding is novel since previous studies have not used examined Bitcoin market efficiency in context of it’s halving dates. Secondly, our study uses longer and more recent empirical data which has allowed us to examine market efficiency for periods corresponding to the more recent ‘Black Friday cryptocrash’ experienced between October 2020 and January 2021, the immunization period of post 2021 identified in Rouatbi et al., (2021); as well as the announcement of five different COVID-19 variants (alpha, beta, gamma, delta and omicron) between December 2020 and November 2021. Our study shows that despite the rollout of mass vaccinations in early 2021, Bitcoin has become more frequently market inefficient has more variants of the disease have been announced.

5 Conclusions

This paper presents a new approach for examining market efficiency in Bitcoin markets by contextualizing the conventional Dickey-Fuller tests for random walk behav-
ior in a time-frequency domain using continuous wavelet tools. Using Bitcoin daily returns computed over the period 19/07/2010 to 31/10/2021 and we find (i) Bitcoin is more predictable over short-frequency oscillations of 2–16 days, (ii) Bitcoin markets are affected by monthly calendar effects and are more predictable between December and February, (iii) Bitcoin is less efficient during months of bubble build up and bursts, (iv) Bitcoin has progressively become progressively market inefficient across the consecutive halving dates, and (v) Since the COVID-19 related ‘Black Swan’ event in March 2020, market efficiency in Bitcoin has progressively deteriorated despite the recent rollout of mass vaccination and overall improvement in COVID-19 global health status.

Altogether, our study has implications for investors, market regulators, health practitioners and academics who have different interests in Bitcoin. Firstly, our results are relevant towards investors, fund managers and other market participants in Bitcoin markets who rely on technical analysis as their main trading strategy. Our study shows that strategies based on historic information can be developed to beat the mar-
ket and obtain above-average returns. Secondly, our empirical evidence of increasing periodic informational inefficiency in Bitcoin implies that these markets may need formal regulation as there is currently no legal framework which can protect the safe investors from speculators whose behavior fuels overpricing in cryptomarkets and increases the risks associated with these assets. Thirdly, we show that whilst health outcomes are at least important for informational efficiency in Bitcoin markets, the recent mass vaccination of people around the world has not improved market efficiency in Bitcoin. Instead, our findings show that market inefficiency has been fueled by the announcement of new COVID-19 variants which, in turn, highlights the role that health practitioners can play in stabilizing the Bitcoin market by preventing future outbreaks of newer variants of the disease. Lastly, for academics, our study presents a novel empirical method for testing weak-form market efficiency in time-frequency space and has proven to be virtuous in knitting together ‘bits-and-pieces’ of seemingly contradictory empirical evidences by capturing the different periods of market (in)efficiency identified in previous studies in a harmonious manner. We propose that future research endeavors can focus on using similar wavelet coherence framework to test for time-varying and frequency varying market efficiency in other cryptocurrencies, financial assets, precious metals and commodity markets.

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