Students’ activities for understanding function shifting by using GeoGebra

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Abstract. The needs of worker for STEM-related jobs caused STEM education becomes an important focus. Designing activities by combining Science, Technology, Engineering and Mathematics itself (STEM) is possible to do in mathematics learning. This paper aimed to describe students’ understanding on function shifting rules in calculus course through activities using GeoGebra as a technology application. A contextual problem was given to students and required them to solve it by using GeoGebra and using their prior knowledge in mathematics and physics. Teaching experiment was conducted to first-year mathematics education students. The data were obtained from the students’ worksheet and depth interview. The results showed that the students were interested with the learning approach and they could understand the rules of function shifting.

1. Introduction

The needs of worker for STEM-related jobs these days caused STEM education becomes an important focus [1]. However, the fact revealed that the interest of Students in learning STEM subjects declined this last decade [2]. STEM education concerned on improving science and mathematics with little integration and attention given to technology or engineering several decades recently [2]. Integrated STEM education defined as approaches that explore teaching and learning between/among any two or more of the STEM subject areas, and/or between a STEM subject and one or more other school subjects [3]. The outcomes for learning at least one of the other STEM subjects should be purposely designed in a course—such as a math or science learning outcome in a technology or engineering class. Integrated STEM education is also described as an effort to combine some or all of the four disciplines of STEM subjects into one class, unit, or lesson that is based on connections between the subjects and real-world problems [4]. In summary, STEM Education is an approach that combining some or all of the four STEM subjects into one lesson that is designed intentionally.

STEM education programs of high quality should include (a) integration of technology and engineering into science and math curriculum at a minimum; (b) promote scientific inquiry and engineering design, include rigorous mathematics and science instruction; (c) collaborative approaches to learning, connect students and educators with STEM fields and professionals; (d) provide global and multiperspective viewpoints; (e) incorporate strategies such as project-based learning, provide formal and informal learning experiences; and (f) incorporate appropriate technologies to enhance learning[5]. Based on those characteristics of STEM education program, students’ activity using GeoGebra for understanding the rules of function shifting, a subtopic in Calculus Differential course was designed and implemented in a class of first-year mathematics students.
Educators are as the center of the learning activity in most of higher mathematics and science learning processes [6]. Recent study by Căprioară (2013) showed that there are some didactic practices of mathematics teacher that negatively impact to the learning process. Mathematics teachers were more subject oriented rather than to the learning process itself. In other words, most of mathematics teachers still involved the students into the learning process at a minimum level of participation. Considering those research results, mathematics learning should involve the students actively and should be students oriented. Context in science and engineering are possible to be engaged in mathematics. Technology is also provided in vary to support mathematics learning. GeoGebra is one of them. GeoGebra has helped the students to enhance their performance in learning mathematics, one of them is in coordinate system topic [8]. Combining real world problem and the use of technology have potential to increase students’ engagement, interest and sense of ownership and autonomy over their learning process [9]. Function shifting rules usually only memorized by some students. When they failed to memorize the rules, they could not determine the graph of the new function when it was shifted or they could not justify what rules has applied so that we can obtain the new graph from the old one. Function shifting is flexible to be learnt by using both of real life context and involving technology application. Accordingly, to help the students understand the concept of function shifting, the activity involving real life context and integrated with GeoGebra was design and implemented.

2. Research Method

This study was qualitative research. The method started with content analysis. Function shifting as subtopic of Graph Transformation in calculus course was selected to be developed because it is possible to use GeoGebra actively and effectively in the learning process. This topic is also can be generated by using real life context to be modelized mathematically. In this research, some activities were designed in a form of students’ worksheet. The learning resources designed including the worksheet was reviewed by an expert to state the validity. The next step was implementing the activities designed in an experiment class and observing the teaching and learning process in the classroom in order to look for students’ response toward the tasks being given. Finally, interviews were undertaken to find information about students’ perception and response toward the learning approach as well as their understanding to the topic.

The learning activity including some tasks was implemented to a class mathematics education program as the experiment class. During the implementation, video recorder and field note was employed to record the teaching and learning process including students’ activities when doing the tasks given or when specific related responses of the teachers appear. Data from the video record and the field note were coded and analyzed after the data reduction.

3. Results

The results of this study were divided into two main parts. The first result the implementation of the learning activity and the students understanding toward the topic, and the second is the students respond towards the learning activity.

3.1. The implementation of the learning activity and the students understanding toward function shifting

There is a contextual problem formulated in several tasks that were given in the worksheets. The problem is about finding a basketball trajectory equation when it is thrown upward. The first task of the problem is asking the students to sketch the graph of the function based on given data of the height of the ball at some specific times and then confirm their work by using GeoGebra, a dynamics and interactive mathematics software for learning and teaching mathematics in science.

| t (time) | f(t) (height) |
|---------|---------------|

Table 1. Data of the ball height at some instantaneous times
The second task of the problem is collecting similar data of the height of the ball when it is thrown by the player jumping upright 0.5 m above ground level. Based on the data obtained, students were asked to draw the graph of the function by using GeoGebra. The final task is asking the students to conclude from the two graphs how to obtain the second graph from the initial graph and give the reason. As an exercise, students were asked to do ball throwing 0.5 m lower by following similar lines as the previous case.

The class was divided into small groups that consist of 3 students. Each group was supplied with worksheets which started with finding the formula of basketball trajectory. Students did some analysis to obtain the rules of shifting by following the guidance in the worksheet and utilizing GeoGebra to do the experiment. When technological tools are available, students can focus on higher level of thinking such as decision making, reflection, reasoning, and problem solving [8].

In this research, there were some tasks that should be done by the students separated into two worksheets. The first worksheet consist of activities that require the students to: 1) modelling the problems into mathematical function; 2) determining the domain and the range of the function formed; 3) sketching the graph manually; 4) sketching the graph using GeoGebra and comparing with the graph that was sketched manually. The second worksheet was required students to: 1) do manipulation to the function that was obtained in the activity 1 by adding or subtracting several arbitrary constants; 2) comparing the graph function that was obtained by using GeoGebra before and after manipulation; 3) draw the conclusion of shifting rule; 4) solve the problems in the figure 1. Table 2 below described the implementation of the activities and divided into three parts i.e. the activities being implemented, the problems occurred, and the intervention of the educator to solve the problems.

| Worksheet | Activity | Problems | Intervention |
|-----------|----------|----------|--------------|
|           | Modelling the problems into mathematical function | Students do not know how to start solving the problems. | Questioning the students. The educator tried to remind the students by asking, “did you remember the shape of trajectory of a ball throwing? All of you had already learned it in physics, hadn’t you?” |
| 1         | Determining the domain and the range of the function formed | Give the incorrect domain and the range. The domain and the range are \((-\infty, \infty)\) and \((-\infty, \frac{D}{4a})\), respectively because they forgot that time which corresponds to the domain and height of the ball which corresponds to the range cannot be negative. | Remind the context of the problem by questioning. |
|           | Sketching the graph manually | Initially only drew the points of the given data without connecting them and without continuing it for the possible | Remind the context of the problem questioning. |

### Table 2. Summary of The Learning Activity
We can see from the Table 2 above that each step of activity students had their own difficulties and to help the students, the educator gave interventions such as questioning, giving a hint, or asking other students to share their opinion.

There are few problems experienced by the students in doing the activities. One of them was determining the data for the shifted graph. For example, students confused when determining the sample points for graphing the function \( y = f(t + c) \), where \( c \) is an arbitrary real constant. This was because they wrongly paired each value \( t \) taken from the domain with its corresponding range. For example, for \( f(t) = -5t^2 + 9t + 2 \) and taking \( c = 1 \), they inputted \( \text{Polynomial}[{(1,6),(1.5,4.25),(2,0)}] \). As a result, they obtained exactly the same function and the same graph as the initial one as seen in figure 1. The educator asked the students to check the data table they constructed and to inspect each ordered pair. He reminded that the students should not put the sum \( t + 1 \) as the abscissa of the points. Instead, they should put each value of \( t \) which were taken as the abscissa and pairing it with the map of \( t + 1 \) under the function. So, they would have inputted the following command: \( \text{Polynomial}[{(0,6),(0.5,4.25),(1,0)}] \) to obtain a new graph that was actually obtained by shifting the initial graph to the left 1 unit as far as seen in figure 2.
The other problem that students met was drawing the conclusion for \( f(x \pm c) \). They got confused to decide which one is move to the right and which is move to the left. It was because they were still influenced by the first conclusion for \( f(x) \pm c \). The conclusion of \( f(x) \pm c \) was easier to understand. The movement of \( f(x) \) is depended to the value of \( c \). If \( c \) is positive, \( f(x) \) move upward as far as \( c \) units and vice versa.

It was easier because the movement did not counter the \( y \)-axis. But the movement of \( f(x \pm c) \) countered the \( x \)-axis. When the value of \( c \) is positive, the graph of the function moved to left. At this stage, educator asked a group to explain and demonstrate the problem using GeoGebra in front of the class for several arbitrary \( c \). Then the educator and students made the conclusion. Many groups could draw a correct conclusion although not in a complete sentence as the following.

"The second graph was obtained by shifting the first graph 0.5 unit upward”.

This is interesting since they understood the idea but could not explain using a clear and complete sentence. At this stage, the educator checked each group by asking them to repeat the conclusion for the purpose to assure that they draw a correct conclusion.

3.2. Students response toward the learning process
The other interesting result is the information about students’ response toward the teaching approach. From the interview, these data was obtained. Some students confessed that the approach was interesting (Vignette 1, line 16 - 18). Although, they were confused at the beginning, they found that the approach helped them to understand the topic better because they experienced themselves how the formulas come from (Vignette 1, line 3 – 5). They felt that from the learning process, they received some advantages (Vignette 1, line 9-12). The transcript of vignette 1 below reveals the facts.

**Vignette 1.** Transcript of students response toward the learning process

*Interviewer*: “What do you think about the teaching and learning approach?”

*Student*: “At first, it was confusing because we don’t know where we are up to. But, after the educator brought the results of each group classically, we realized our mistakes. This made us understood better.”
“We could learn to make reflection of our work. Firstly, we work separately in groups and then when the educator discuss each group work in class we realized our mistakes and value other group’s work.”

“What do you prefer? Using this learning approach or the educator explains straight away?”

“I prefer this approach because, for me, it is really well-suited. The learning process was more interesting because I experienced myself constructing my understanding together with my group’s member about the concept. If it is explained by the educator straight away usually I will easily forget what I understood from the educator’s explanation.”

The transcript in the vignette 1 shows positive response from the students toward the approach being implemented. They enjoyed the learning process which challenge and help them at the same time to understand the concept of function shifting.

4. Conclusion

The learning activities using GeoGebra had been implemented in this study could facilitate the students to derive and use the formula of shifting function. In the learning process, the educator could implement approaches of teaching in which the educator must be perceptive toward all students’ responses. The students found few problems in the learning activities such as give the incorrect domain and the range, found difficulties in sketching the graph, and drawing a conclusion. The educator may questioning, give a hint, invite others’ opinion or else such that students will think deeper, explore, or share opinion.

The implementation of the activity using GeoGebra supported students’ to have a good understanding about the topic. GeoGebra helped the students in to do trial and error until the find the hypothesis of the general form of function shifting. Students are not only able to answer the questions related to the shifting rules but also able to explain their answers by providing examples to support their answers. In addition, being experienced with the learning activity, the students were more comfortable in learning mathematics. They enjoyed the learning process such as: they could discuss with teammates, could reflect their works with other teams’ result, and on the other hand, they felt that the educator’s role really helped them when they stuck or had a deadlock in the discussion. Students satisfied with the learning process where they could construct their own concept of function shifting.

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Acknowledgments
This research is funded by DP2M Dirjen Dikti, The Ministry of Research, Technology, and Higher Education (Kemenristekdikti), Republic of Indonesia Under the scheme of Penelitian Unggulan Perguruan Tinggi, year 2016, DIPA Universitas Negeri Surabaya, contract number: 004.12/UN38.11-P/LT/2016. This research also got full support by the Department of Mathematics, Universitas Negeri Surabaya, Indonesia.