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Coarse grid corrections in Krylov subspace evaluations of the matrix exponential. (English)

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Summary: A coarse grid correction (CGC) approach is proposed to enhance the efficiency of the matrix exponential and \( \varphi \) matrix function evaluations. The approach is intended for iterative methods computing the matrix-vector products with these functions. It is based on splitting the vector by which the matrix function is multiplied into a smooth part and a remaining part. The smooth part is then handled on a coarser grid, whereas the computations on the original grid are carried out with a relaxed stopping criterion tolerance. Estimates on the error are derived for the two-grid and multigrid variants of the proposed CGC algorithm. Numerical experiments demonstrate the efficiency of the algorithm when it is employed in combination with Krylov subspace and Chebyshev polynomial expansion methods.

MSC:

- 65F60 Numerical computation of matrix exponential and similar matrix functions
- 65M20 Method of lines for initial value and initial-boundary value problems involving PDEs
- 65M55 Multigrid methods; domain decomposition for initial value and initial-boundary value problems involving PDEs

Keywords:

- matrix exponential
- phi matrix function
- multigrid
- Krylov subspace methods
- exponential residual
- exponential time integration

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