Determining phase-space properties of the IHEP RFQ output beam using the RMS beam widths from wire-scanners

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Abstract: A beam line is built after the IHEP RFQ for halo study. To determine transverse emittance and ellipse parameters of the RFQ output beam, beam size data obtained from the first two of 14 wire scanners are employed. By using the transfer matrix method and the least square method, a set of linear equations were set up and solved. The solutions were then applied as initial beam parameters in multi-particle simulations to check the method of calculation. It is shown that difference between the simulated RMS beam size and the measured one at the measurement location is less than 7\%, which is acceptable in our experiments.

Key words: high intensity, proton, wire scanner, ellipse parameters, transverse matching

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1 Introduction

A beam line is installed after the IHEP RFQ, of which the output energy is 3.5MeV and the operation frequency is 352MHz. The purpose of this beam line is to study beam halo formation and to do comparison between measurements and multi-particle simulations. The layout of the beam line is shown in Fig.1 [1]. It consists of 28 quadrupoles. The first four quadrupoles can be independently adjusted to do matching or mismatching. The other 24 quadrupoles are used to form FD lattice. Several types of beam diagnostic devices are installed. The key diagnostic component is a group of 14 wire scanners, which locate in the middle of drift spaces between quadrupoles. A pair of wire scanners which locate after the 5\textsuperscript{th} and 6\textsuperscript{th} quadrupoles are used to determine horizontal and vertical ellipse parameters of the RFQ output beam respectively. These parameters are essential for the upcoming halo experiment. The wire scanners after quadrupole 17 to 22 are used to measure beam profiles in X direction and Y direction alternately. Simultaneously, the wire scanners after quadrupole 23 to 28 are used to measure beam profiles in X direction only.

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2 Least-squares analysis

For measured beam profile, we generally fitted it with a Gaussian function and then obtained RMS beam size $X_{RMS}$. For a beam transporting through a lattice, the beam ellipse transformation can be expressed in matrix form as [2]

$$\sigma_2 = R\sigma_1 R^T$$

(1)

Where $R$ is the transfer matrix between two positions and $R^T$ is the transpose of the matrix $R$. The $\sigma$-matrix is defined as

$$\sigma = \begin{pmatrix} \beta \varepsilon & -\alpha \varepsilon \\ -\alpha \varepsilon & \gamma \varepsilon \end{pmatrix}$$

(2)

Where $\alpha, \beta, \gamma$ are the Twiss parameters and $\varepsilon$ is the un-normalized beam emittance. By carrying out matrix multiplication of Eq. (1), we can get

$$\beta_2 \varepsilon_2 = R^2_1 \beta_1 \varepsilon_1 - 2R^2_1 R^2_1 \alpha_1 \varepsilon_1 + R^2_1 \gamma_1 \varepsilon_1$$

(3)

Here, we use $\alpha_1, \beta_1, \gamma_1, \varepsilon_1$ to denote the initial ellipse parameters at the exit of RFQ, and $\alpha_2, \beta_2, \gamma_2, \varepsilon_2$ to denote the ellipse parameters at the measurement location. Since the Twiss parameters satisfy the relationship of $\beta \gamma - \alpha^2 = 1$ and the RMS beam size is equal to $\sqrt{\beta_2 \varepsilon_{2,RMS}^2}$, then Eq. (3) can be expressed as

$$X_{RMS}^2 = R^2_1 \alpha_1 + 2R^2_1 R^2_1 \beta_1 + R^2_1 \gamma_1$$

(4)

where $a = \beta \varepsilon_1$, $b = -\alpha \varepsilon_1$, $c = (1 + \alpha^2)\varepsilon_1 / \beta_1$.

According to Eq. (4), if the gradients of quadrupoles between two points are changed, the RMS beam size at the wire scanner will also change. And hence three unknown elements in Eq. (4) can be deduced from a set of equations obtained from three different quadrupole settings.

However, in the experiments, because of errors, the equation sets usually have no solution or the solutions differ greatly between each other. To avoid these problems, more than three independent linear equations are taken into account and the least square method is adopted.

For example, if several groups of beam sizes and quadrupole gradients are known, a set of linear equations can be obtained as:
\[
\begin{aligned}
\left\{ \begin{array}{l}
R_{1,1}^2a + 2R_{1,1}R_{1,12}b + R_{1,12}^2c = \chi_{1,\text{RMS}}^2 \\
\vdots \\
R_{n,1}^2a + 2R_{n,1}R_{n,12}b + R_{n,12}^2c = \chi_{n,\text{RMS}}^2
\end{array} \right.
\]

where \( n \geq 3 \).

The variance, or the sum of squared residuals, is calculated as

\[
\chi^2 = \sum_{i=1}^{n} (R_{i,1}^2a + 2R_{i,1}R_{i,12}b + R_{i,12}^2c - \chi_{i,\text{RMS}}^2)^2
\]  

According to the method of least squares, the best solution of Eq.(5) has the property that make \( \chi^2 \) minimum, which means, at this point, the differential of \( \chi^2 \) on \( a, b, c \) are equal to zero respectively, as

\[
\begin{aligned}
\frac{\partial}{\partial a} \sum_{i=1}^{n} (R_{i,1}^2a + 2R_{i,1}R_{i,12}b + R_{i,12}^2c - \chi_{i,\text{RMS}}^2)^2 &= 0 \\
\frac{\partial}{\partial b} \sum_{i=1}^{n} (R_{i,1}^2a + 2R_{i,1}R_{i,12}b + R_{i,12}^2c - \chi_{i,\text{RMS}}^2)^2 &= 0 \\
\frac{\partial}{\partial c} \sum_{i=1}^{n} (R_{i,1}^2a + 2R_{i,1}R_{i,12}b + R_{i,12}^2c - \chi_{i,\text{RMS}}^2)^2 &= 0
\end{aligned}
\]

Expanding these differentials, we get

\[
\begin{aligned}
\sum_{i=1}^{n} 2R_{i,1}^4a + 4R_{i,1}^3R_{i,12}b + 2R_{i,1}R_{i,12}^2c - 2\chi_{i,\text{RMS}}^2R_{i,12}^2 &= 0 \\
\sum_{i=1}^{n} 4R_{i,1}^3R_{i,12}a + 8R_{i,1}^2R_{i,12}^2b + 4R_{i,1}R_{i,12}^3c - 4\chi_{i,\text{RMS}}^2R_{i,12}R_{i,11} &= 0 \\
\sum_{i=1}^{n} 2R_{i,1}^2R_{i,12}^2a + 4R_{i,1}R_{i,12}^3b + 2R_{i,12}^4c - 2\chi_{i,\text{RMS}}^2R_{i,12} &= 0
\end{aligned}
\]

then \( a, b, c \) can be solved from Eq.(7), and the beam parameters will be deduced as

\[
\varepsilon = \sqrt{ac - b^2}, \quad \beta = \frac{a}{\sqrt{ac - b^2}}, \quad \alpha = -\frac{b}{\sqrt{ac - b^2}}.
\]

3 EXPERIMENT DATA ANALYSIS

As mentioned in the first section, the 1st and the 2nd wire scanner were used to calculate X direction and Y direction beam parameters respectively. For each direction, only one quadrupole between the RFQ and the wire scanner was adjusted to produce different beam size at the wire scanner, while the gradients of other quadrupoles keep constant.

We used a code TRACE-3D [3] to set a model for computing transfer matrices and tracking beam. Because transfer matrices had relations with initial ellipse parameters while beam current wasn’t equal to zero, an iteration procedure was used. Firstly, we assumed the beam current was zero and it was easy
to get transfer matrices and set up equations. Secondly, we used the zero-current solution as initial ellipse parameters in model to get new transfer matrices and set up new equations. The process was iterated again and again until the difference between two successive solutions was less than 10^{-3}. In order to get solutions efficiently and correctly, a code was written to solve the equation set, to edit TRACE-3D input file, to call TRACE-3D code and to read the transfer matrix element automatically.

In Table 1, the calculated beam parameter is shown. We got two groups of beam parameters for different beam current. It is shown that the beam parameters change slightly when the beam current is increased. In the Y direction, the calculated beam parameters are more close to the PARMTEQM prediction than that in the X direction. And in both directions, there are about 50% difference between the simulated emittance and the calculated ones.

Table 1 Beam parameters at the exit of RFQ

| RFQ exit beam | α_x (mm/mrad) | β_x (mm/mrad) | α_y (mm/mrad) | β_y (mm/mrad) | ε_x (π.mm.mrad) | ε_y (π.mm.mrad) |
|---------------|--------------|---------------|--------------|--------------|----------------|----------------|
| PARMTEQM(I=20mA) | -0.126 | 0.079 | -0.709 | 0.218 | 0.205 | 0.21 |
| Experiment-1(I=22mA) | 3.129 | 0.385 | -0.546 | 0.112 | 0.344 | 0.327 |
| Experiment-2(I=26mA) | 3.753 | 0.461 | -0.611 | 0.113 | 0.333 | 0.33 |

Using these calculated beam parameters as initial beam parameters, we simulated proton beam transporting through the beam line and got simulated RMS beam sizes at the measurement location. These simulated beam sizes were compared with those measured ones to check the method of calculation. Besides Trace-3D code, another multi-particle code PARMILA [4] was also used since it can simulate nonlinear space-charge force. In PARMILA simulations, the initial beam distribution was 6D Waterbag distribution, and 50000 macro-particles per bunch were used. Space charge effect was calculated via the 2-dimensional particle in cell (PIC) method. The simulated RMS beam sizes from two codes are shown in Fig. 2 and 3. In Fig. 2, the difference between the measured data and the simulated data is less than 3% in X direction and less than 7% in Y direction. In Fig.3, the difference between the measured data and the simulated data is less than 7% in X direction and less than 3% in Y direction.

Fig.2. I=22mA, RMS beam size at the measurement location. (The diamonds show the measured data, the circles show the simulated data by using Trace-3D and the triangles show the simulated data by using PARMILA)
4 Transverse Matching

Once the initial Twiss parameters were obtained, we used the Trace-3D code to find a proper quad setting for beam matching. However, we found that the matching was not good as proposed. As shown in Fig. 4, we use black dots to represent the measured RMS beam size while quadrupoles are set at the Trace-3D value. As can be seen, the RMS beam sizes measured at the same point of FD periods were not agreed with each other. On the other hand, the measured beam sizes differed from the simulated ones, which were obtained from PARMILA code (The model setting is same with the actual setting). The reasons led to this problem were the measurement uncertainties, the simplified space charge effects in Trace-3D code and the difference between model and real machine. To get a better matching, quadrupole scans were done from 0.8 to 1.2 times of the Trace-3D value. The figure of merit was to obtain equal RMS beam sizes along the beam line. Finally, we found a better matching, of which the gradient of Q4 was 10% higher than the Trace-3D solution. As shown in Fig. 4, the blue circle represents the measured value after quadrupole scans. It is also shown that the measured beam sizes are almost the same with the simulated ones at wire scanners.

5 Conclusions

In this paper, we described how to calculate ellipse parameters of the IHEP RFQ output beam using only the RMS beam sizes from wire scanners. However, this method highly depends on the beam
quality. For getting accurate parameters, we need to collect a lot of data and abandon ones deviating a lot from others. Once the beam parameters obtained, we can use them to do matching. Due to some unavoidable errors, the matching may be not perfect as proposed. However, it can be refined by adjusting quadrupoles a little based on the Trace-3D solution.

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