Propagation of microwave radiation through an inhomogeneous plasma layer in a magnetic field

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Abstract. The problem of reliable microwave communication through a plasma sheath has its origin from the beginning of space flights. During reentry of spacecraft, the plasma layer can interrupt the communication. At sufficiently high plasma density, the plasma layer either reflects or attenuates radio wave communications to and from the vehicle. In this work, we present a simple analytical one-dimensional algorithm to study the propagation of electromagnetic (EM) waves through a nonuniform plasma layer in a static nonuniform magnetic field. The experimental study of the EM wave transmission and reflection through plasma layer was carried out on the (i) microwave set and (ii) on the unit using a high-voltage pulsed discharge.

1. Introduction

When a hypersonic vehicle travels through the atmosphere, a high-density plasma sheath forms around it. The density of the plasma layer is about $10^{15}$ to $10^{20}$ m$^{-3}$, and this density is high enough to prevent radio signals from passing through the layer causing so-called radio blackout. One well known possible way to diminish the radio signal attenuation is to use a static magnetic field which strongly effects on plasma properties [1]. The calculation and experiments were performed with model plasma regions also demonstrate the possibility of increasing the passage of microwave (MW) through a plasma layer placed in a magnetic field.

In this work, we present a simple analytical one-dimensional (1D) algorithm for simulation of the propagation of electromagnetic (EM) waves through a nonuniform plasma layer in a static nonuniform magnetic field. On the basis of this algorithm one can estimate the degree of the influence of the magnetic field on the reflection, transmission and attenuation of microwave energy. As an example of application, we considered the passage of linearly polarized microwaves through the shock layer around a cone-shaped body moving in the atmosphere of the Earth with a velocity of 7500 m/s at a 50-km height [2]. The realization of experiments in a well respective to the calculated conditions is very difficult and requires expensive equipment. Therefore, at the initial stage of the research, we found possible to use quite simple and effective ways to perform our operations. The use of pulsed discharges with parameters similar to the calculated allowed...
us to fix the influence of magnetic field on the propagation of EM waves through a plasma layer.
We hope that the modeling of plasma layers with the required configurations and parameters can be realized with the use of the magnet-plasma compressor and of the pulsed plasma jet [3].

2. Theory

The theoretical study is carried out in the wavelength range

$$\lambda = 1-10 \text{ cm.}$$

(1)

Under the condition (1) within the period of the electric field oscillation $T$ the ions are immobile. In addition, we neglect the thermal motion of electrons. We consider the problem within the scope of the 1D model. Such approach is justified when the curvature radius of the plasma sheath considerably exceeds the wavelength.

A linearly polarized EM wave with $\text{Re} \{E_W \exp[i (\omega t - kz)]\}$, where $k = \omega / c$, of amplitude $A_W(|E_W| = A_W)$ interacts with an infinite and uniform in the $x, y$ directions plasma layer. The layer with parameters $N_p(z), \nu(z)$, where $N_p$ is a plasma density, $\nu$ is an electron–neutral collision frequency is localized on the interval $0 \leq z \leq \Delta$. It is assumed that the magnetic induction vector $B$ is in parallel to the wavevector $k$. In the general case, the magnetic induction varies in the $z$ direction, $B(z)$.

Outside the plasma sheath the solution of Maxwell curl equations for the complex amplitudes of the electric field is

$$E(z) = \begin{cases} E_W \exp(-ikz) + E_R \exp(ikz), & \text{if } z \leq 0, \\ E_T \exp(-ikz), & \text{if } z \geq \Delta, \end{cases}$$

(2)

where $E_R, E_T$ are the amplitudes of reflected and transmitted waves, respectively.

In order to solve the wave equation within the plasma sheath, the interval $[0, \Delta]$ was divided into $M \gg 1$ equal parts, $\delta = \Delta/M$. Assuming that inside the elementary layer $z_m - \delta \leq z \leq z_m$ ($z_m = m\delta, m \leq M$) the plasma density, $N_p$, the collision frequency, $\nu_m$, and the magnetic field, $B_m$, are constant the exact solution for the complex amplitude of the electric field can be written in the form

$$E_m = \sum_{j=1}^{2} E_{jm}^{(+)} \exp(-i k \sqrt{\varepsilon_{jm}}) + E_{jm}^{(-)} \exp(i k \sqrt{\varepsilon_{jm}}),$$

(3)

where $\varepsilon_{1,2m} = 1 + (\varepsilon_m - 1)/(1 \pm \Omega_m)$, $\varepsilon_m = 1 + N_m/(1 - i \nu_m)$, $N_m = N_p/N_{cr}$, $N_{cr}$ is the density at which the electron plasma frequency is equal to the microwaves frequency, $\nu_m = \nu_m/\omega$, $\Omega_m = \Omega_m/(\omega - i \nu_m)$, $\Omega_m = |e| B_m/m_e$, $m_e$ is the electron mass, $e$ is the electron charge. The complex amplitudes $E_R, E_{jm}^{(\pm)}$, $E_T$ are to be determined from continuity conditions for the $x, y$ components of the electric field $E$ and their derivatives $dE/dz$ at the points $z = 0, z_m, \Delta$:

$$E(z = -0) = E(z = +0),$$
$$E(z = z_m - 0) = E(z = z_m + 0),$$
$$E(z = \Delta - 0) = E(z = \Delta + 0),$$
$$\left. \frac{dE}{dz} \right|_{z=-0} = \left. \frac{dE}{dz} \right|_{z=+0},$$
$$\left. \frac{dE}{dz} \right|_{z=z_m-0} = \left. \frac{dE}{dz} \right|_{z=z_m+0},$$
$$\left. \frac{dE}{dz} \right|_{z=\Delta-0} = \left. \frac{dE}{dz} \right|_{z=\Delta+0}.$$
Finally, the solution for the complex amplitudes within the sheath at the points $z_m$ is given by

$$
\begin{align*}
\left( \frac{E_x}{E_y} \right)_m &= E_{W_x} \sum_{j=1}^{2} \left( \frac{1}{h_j} \right) e_{jm},
\end{align*}
$$

where $h_1 = i$, $h_2 = -i$ (circular polarization),

$$
\begin{align*}
e_{jm>1} &= e_{jm-1} \frac{(1 + Q_{jm}) \beta_jm}{1 + Q_{jm} \beta_jm^2}, \\
\beta_{jm} &= \exp \left( -ik\sqrt{\varepsilon_{jm}} \right),
\end{align*}
$$

$$
Q_{jM-1} = \frac{\sqrt{\varepsilon_{jM-1}} - \sqrt{\varepsilon_{jm} + Q_{jm} \beta_jm^2 (\sqrt{\varepsilon_{jM-1}} + \sqrt{\varepsilon_{jm}})}}{\sqrt{\varepsilon_{jM-1}} + \sqrt{\varepsilon_{jm} + Q_{jm} \beta_jm^2 (\sqrt{\varepsilon_{jM-1}} - \sqrt{\varepsilon_{jm}})}}.
$$

The expressions for the complex amplitudes of reflected and transmitted waves are

$$
\begin{align*}
\left( \frac{E_{R_x}}{E_{R_y}} \right) &= \frac{E_{W_x}}{2} \sum_{j=1}^{2} \left( \frac{1}{h_j} \right) \frac{1 - \sqrt{\varepsilon_{j1} + (\sqrt{\varepsilon_{j1}} + 1) Q_{j0}}}{1 + \sqrt{\varepsilon_{j1} - (\sqrt{\varepsilon_{j1} - 1) Q_{j0}}},} \\
\left( \frac{E_{T_x}}{E_{T_y}} \right) &= E_{W_e} \sum_{j=1}^{2} \left( \frac{1}{h_j} \right) e_{jM}.
\end{align*}
$$

Using the expressions (6), one readily obtains the following expressions for the coefficients of reflection, $R$, and transmission, $T$,

$$
\begin{align*}
R &= \sum_{j=1}^{2} R_j, \\
T &= \sum_{j=1}^{2} T_j,
\end{align*}
$$

where

$$
R_j = \frac{1}{2} \frac{1 - \sqrt{\varepsilon_{j1} + (\sqrt{\varepsilon_{j1}} + 1) Q_{j0}}}{1 + \sqrt{\varepsilon_{j1} - (\sqrt{\varepsilon_{j1} - 1) Q_{j0}}},}^2, \\
T_j = 2 |e_{jM}|^2.
$$

Since the exact values of the plasma density and electron–neutral collision frequency are not known, we have used the following approximation of the numerical simulation [2]

$$
N_p(z) = N_{p0} \exp (\Lambda z), \quad 0 \leq z \leq \Delta,
$$

where $\Lambda = \frac{1}{\Delta} \ln \frac{N_{p\text{max}}}{N_{p0}}$, $\Delta = 0.5$ cm, $N_{p\text{max}} = 5 \times 10^{19}$ m$^{-3}$, $N_{p0}$ is the ambient electron density, $N_{p0} \ll N_{cr}$. In this report, we assume that (i) within the layer the electron–neutral collision frequency is about $5 \times 10^9$ s$^{-1}$, (ii) the magnetic induction is independent of $z$.

Figure 1 shows the coefficients of reflection, $R$, transmission, $T$, and (for completeness) attenuation degree, $D$ ($D = -10 \log T$), of microwave energy versus the wavelength in the case $B = 0$. It can be seen from the figure that (i) in the limit $\lambda < 0.5$ cm, the plasma sheath (8) is practically transparent ($R \to 0, T \approx 1$), (ii) in the wavelength range $\lambda > 1$ cm, almost 70% of microwave energy is reflected from the layer and this value is independent upon the wavelength, (iii) the function $D(\lambda)$ most rapidly increases with increasing wavelength near the value $\lambda \approx 0.5$ cm, (iv) in the range $4 \leq \lambda \leq 10$ cm $D(\lambda) \approx 40$.

Figure 2 shows the coefficients of reflection and transmission of microwave energy versus the magnetic induction for three values of the wavelength. It can be seen from the figure 2 that (i) the coefficient of reflection depends weakly upon the magnetic induction and changes within the range $R \approx 0.6 \pm 0.2$, (ii) the coefficient of transmission noticeably increases with increasing magnetic induction when $B > B_{\text{cr}}$, where $B_{\text{cr}} \approx 0.6$, 0.3 and 0.2 T for $\lambda = 2$, 6 and 10 cm, respectively.
Figure 1. The coefficients of reflection $R$ (curve 1), transmission $T$ (curve 2) and attenuation degree $D$ (curve 3) of microwave energy as the functions of the wavelength in the case $B = 0$, $\nu = 5 \times 10^9$ s$^{-1}$.

Figure 2. The coefficients of reflection (a) and transmission (b) of microwave energy as a function of the magnetic induction in the case $\nu = 5 \times 10^9$ s$^{-1}$. The curves are for the wavelengths $\lambda = 2$ (dots), 6 (dashed lines) and 10 cm (solid lines).

3. Experiment
In this stage investigation, the main purpose of our experiments was to determine the degree of static magnetic field influence on the transmission of the MW radiation interacting with some different plasma configurations [3–5]. Here we used the configurations obtained on the MW installation and on the high-voltage pulsed unit. The diagnostic wavelengths were 2.3 and 1 cm. The photos of a typical pulsed discharge at different pressures are shown in figure 3.

The distance between the electrodes is 40 mm, the breakdown voltage is in the range 400–1500 V. The main channel and a faint glow around it are well seen in figure 3. The transverse dimension of the glow is about 3 cm. Thus, the diagnostic beam with the wavelength 1 cm is completely overlapped by the discharge plasma ($N_e > 10^{19}$ m$^{-3}$). Magnetic field changes the configuration, the dimensions and a color of the channel. Signals from the photoelectron
Figure 3. Photos of a pulsed discharge in the cases $B = 0$ (a, c), $B = 0.2$ T (b, d), $P \approx 15$ Torr (a, b), $P \approx 5$ Torr (c, d).

Figure 4. The typical signals from PMT (top signal) and from the receiving horn (bottom signal) in the cases $B = 0$ (a), $B = 0.2$ T (b) at the pressure 15 Torr and $\lambda = 1$ cm.

multiplier (PEM) and from the receiving horn with the diode section displayed on the 4-beam oscilloscope TDS 2014 in the cases $B = 0$ and 0.2 T are presented in figure 4. The time evolution of the signals reflects the dynamics of the scattered power.

Without the magnet the amplitude of the passed signal in front of the glow falls almost to zero and remains close to it. A clear trend of increase in the passage of microwaves through the plasma in a magnetic field is fixed. Signal amplitude may increase up to 30%. Analysis of the results of numerous experiments with the discharge of the dipole type ($N_e > 10^{20}$ m$^{-3}$ [5]) or combined with the surface discharge (at air pressure 70 Torr) also demonstrates the increase of the passed signal in a magnetic field. The waveforms showing a typical behavior of the signals passed through the plasma in magnetic field are presented at figure 5.

It is well seen how the signal from the microwave sensor is changed when the plasma is in a magnetic field. Signal amplitude under these conditions increase at the range of 10–30% and
depends on the relative location of the discharge and magnet. It should emphasize that the value of the magnetic field in our experiments was lower than predicted in calculations.

4. Conclusions
In this report, we present a simple analytical 1D algorithm to study the propagation of an EM waves through a nonuniform plasma layer in a static nonuniform magnetic field. We considered the passage of linearly polarized microwaves through the shock layer around a cone-shaped body moving in the atmosphere of the Earth with a velocity of 7500 m/s at a 50-km height. In this condition, the transmission noticeably increases with increasing magnetic induction when $B > B_{cr}$, where $B_{cr} \approx 0.6, 0.3$ and 0.2 T for $\lambda = 2, 6$ and 10 cm, respectively. The results of the experiments with pulsed discharges also show the possibility of increasing the amplitude of the transmitted wave through the plasma in a quite weak magnetic field up to tens of percent.

Acknowledgments
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