The effect of subgrade reaction modulus on the nonlinear behavior of reinforced concrete plates

Hameed Stephan Tozy* and Dr. Emad Noel Naaom

1 College of Engineering, University of Musol, Musol, Iraq
*Email: enghameedtozy@gmail.com

Abstract. The research investigates the nonlinear behavior of reinforced concrete plates based on soil by using the finite elements method. This investigation has done by using computer program (DARC3) that mean Dynamic Analysis of Reinforced Concrete in 3D which is written in (FORTRAN) language and it is modified in this study to include the effect of elastic subgrade reaction soil using Winkler's method under effect of different static loads. 20-node isoparametric brick elements are used to represent the nonlinear behavior of reinforced concrete plate, the reinforcement idealized by smeared layer within brick elements, assuming perfect bond between the steel and the concrete. The finite elements results are obtained the load-deflection response. The effects of boundary conditions, plate thickness, type of load and the modulus of vertical subgrade reaction of the soil layer are studied, it is shown that in case of increasing the subgrade reaction values and plate thickness lead to decreasing in the deflection and vice versa. Also it is shown that the fixed boundaries give less deflection than that in case of simply supported boundaries. The results obtained from this study are compared with the previous study, and good agreements results are obtained with percentage (1.52%).

Keywords: subgrade reaction, Winkler theory, finite elements method, nonlinear analysis, R.C. plate.

1. Introduction
The bending properties of the plates depend greatly on its thickness in comparison with other dimensions. Plates can be classified according to its thickness into three types, thin plates with small deflection, thin plates with large deflection and thick plates [1], it is also can be classified according to the type of material that are made, woody plates, metal plates and concrete plates (slabs), the latter type is considered in this study, the supports of the slab has many types like supported into beam, carried by columns, carried on walls, and supported on the soil.

The concrete plates that are based on the soil, which are the focus of this study, can be defined as those that are supported by the soil, and the main purpose is to transfer the loads directly to the soil. A Winkler model was adopted to represent the soil layer upon which the concrete plate is based. Winkler assumes that the soil layer is composed of converging linear springs, in which the displacement occurs directly under the bearing region, and outside this region the displacement is zero [2], thus this model ignores any interference between adjacent springs. In recent study, the effect of subgrade reaction, plate thickness and boundary conditions are considered to show their effects on the nonlinear behavior of the plate under the static loads.

2 Literature review
In 1983 Baumann and Weisgerber [3] developed a yield line method for analyzing reinforced concrete plates supported on the soil to determine the collapse loads. Many cases were discussed with different load position.

For each case, it was taken into consideration many factors that include the properties of the material and the geometric shape. The load-deflection curve was provided to determine the collapse load and compared it with the results of previous research. It was concluded that the modified yield line theory gave a good results compared with other researches for different dimensions of the plates and different load positions.

In 1998 Daloglu and Vallabhan [4] were used non-dimensional finite elements to analyze a supported slab on multi-layered soils, a method was developed to evaluate the coefficient equivalent to soil layer subgrade reaction (k) which used in Winkler’s model. It was used a constant value for Poisson's ratio of soil of (0.25). Graphs were provided by which the value of (k) can be calculated when the complete engineering and general system characteristics are known. It was used a numerical example to show comparisons of results for (Winkler k) equivalent values with those values suggested by other modified studies. It was concluded that if a fixed value of the subgrade reaction of the soil layer is used in the case of applying a uniformly distributed load, the values of the deflections are uniform and there are no bending moments or shear forces in the slab. To obtain realistic results, higher (k) values should be used near the edges of the slab. Also, the value of (k) depends on the depth of the soil layer.

In 2010 Al-Obaidie and Al-Azzawi [5] studied the effect of non-linear analysis of supported reinforced concrete beams on flexible foundations using specific 3D elements with steel reinforcing bars as smeared layers inside the concrete. The non-linear behavior of the materials used includes cracks in concrete, plastic flow, crushing in concrete as well as access to the stress of submission to steel reinforcement. The flexible basis was modeled to act linearly using the Winkler model.

Several examples of a concrete beam with simple support under the influence of different conditions of loads were analyzed, the effect of the width of the beam and the subgrade reaction of the soil on the beam deflection were studied. It was concluded that the maximum deflection decreases in the case of increasing the width of the beam, and the maximum stress of the reinforcement decreases with the increase in the width and depth of the beam. The maximum deflection decreases with increasing the vertical and horizontal subgrade reactions of the soil layer.

In 2012 Vanam et al. [6] studied the behavior of symmetric rectangular plates under different static loads and supporting conditions. It was used a finite elements method to represent the plates by rectangular shape element, depended on the geometry of the entire plate with four nodes.

During the analysis process, it was taken several values of the plate thickness started from (0.01) to (0.18) m, under different loading conditions, included a uniformly distributed load of (500 N/m²) and a concentrated load at the center of (5000 N) with simply and fixed supported edges. The numerical analysis was performed through the development of programming in the mathematics program (MATLAB), and it was also performed the numerical analysis using the finite element analysis program (ANSYS), the results obtained gave a good agreement with the results of exact solutions that were calculated through Kirchhoff’s theory of plates. From the above mentioned numerical results, it was concluded when the plate thickness increased the deflection was decreased.

In 2014 Overli [7] performed a laboratory study of a reinforced concrete slab that was fixed supported and rested on the ground under the influence of a concentrated load located in the different positions of the slab. The nonlinear behavior of the slab was represented by performing a nonlinear finite element analysis. Soil was considered as a layer that did not withstand tensile strength, the smeared cracks approach of the cracked concrete was used. A finite element model was used to assess the effect of soil layer hardness and shrinkage.

The study focused on how cracks form on the upper surface, which are often of concern in constructed buildings.
The study compared the experimental results with the theoretical results, and it was matched well. The results indicated that drying shrinkage can cause severe cracking of the slabs supported on the soil.

In 2016 Walker and Holland [8] suggested the subgrade reaction value of the soil layer by deriving an equation, which is a function of several variables, and how to choose this value. It was discussed which type of foundation may not be suitable for like raft foundation, large and heavy equipment foundations, tank foundations, continuous beams.

In 2019 Abdel-Rahman et al. [9] analyzed reinforced concrete slabs of industrial buildings supported on soil. The nonlinear finite element method was applied to study the structural response of an industrial slab supported on soil to bear truck lift by loading of single wheel axle. Many parameters were studied, the slab dimension ratios, the reinforcement content and arrangement for the load position in relation to the edges of the slabs and the subgrade reaction of the soil layer. The soil layer was represented by boundary spring elements without the tensile strength of the springs to simulate the bearing properties of the soil. The numerical results were compared with the estimated response using linear finite element. It was concluded that the thickness of the slab has a dominant effect on the bearing capacity results of the load. As for the reinforcement, it has a negligible effect on the structural response. Linear finite element analysis yields acceptable load-carrying capacities as compared to the results of NLF EA.

3. Finite element method

3.1 Finite element method for representing concrete

In the current study, the structure is considered a three-dimensional problem, where the concrete is represented by equal dimensional brick elements consisting of (20) nodes. The effect of normal reinforcement was also incorporated using smeared layers as shown in ‘figure 1’.

![Figure 1](image)

Figure 1. The brick element with isometric (20) nodes [10]

Each nodal point contains three degrees of freedom represented by (u, v, w) along the Cartesian coordinates (x, y, z) respectively. In deriving finite element models for global coordinates, (u, v, w) are displacements at a point with local coordinates (ξ, η, ζ), (ui, vi, wi) are the displacement values in the node (i), (xi, yi, zi) are the global coordinates of the point and (xi, yi, zi) are the node (i) coordinates. The element stiffness matrix can be calculated using the following equation [11]:

\[
[K]_c = \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} [B(\xi, \eta, \zeta)]^T [D[B(\xi, \eta, \zeta)]] [B(\xi, \eta, \zeta)] d\xi d\eta d\zeta
\]  

(1)
Where: \([B]\) represents the strain displacement matrix, \([D]\) represents the material matrix, \(|J|\) represents the determinant of the Jacobian matrix of the integral points of the brick elements.

### 3.2 Representation of the reinforcement smeared layer

The analysis of reinforced concrete structures with the finite element method requires a simple but accurate method of representing reinforcement.

It is assumed that the steel bars are distributed as a layer within the element, thus concrete and reinforcement are represented by one element [12]. In the present work it is assumed that there is a perfect bond between the reinforcement and the surrounding concrete. Each group of reinforcing bars is assumed to be a 2D membrane layer of thickness equivalent to steel bars. The layer is placed inside the solid element to match the surface corresponding to \((\xi, \eta, \text{constant}), (\xi, \text{constant}, \zeta)\) or \((\text{constant}, \eta, \zeta)\) as appropriate, the resulting element shows the second state in the ‘figure 2’. Reinforcement bars are supposed to resist axial stresses only toward the bar. A local cartesian coordinate system must be established at each integration point in the reinforcement layer with one of the axes directed along the rod direction. The stresses and the local stiffness matrix of the reinforcement are first evaluated in the local system, then transformed to the global coordinates. Finally, the stiffness contribution of each layer is added to the solid concrete component.

\[
[K]_s = \int_{-1}^{1} \int_{-1}^{1} [B]^T [T]^T [D_s] [T] [B] \cdot |J_s| \, d\xi \, d\eta
\]  

(2)

whereas:

- \([K]_s\) : is the stiffness matrix of the reinforcement layer,
- \([T]\) : is the transformation matrix from local to global system,
- \([D_s]\) : elasticity matrix for normal reinforcement,
- \(|J_s|\) : is the determinant of the Jacobian matrix at the integral points of the equivalent reinforcement layer.

### 3.3 Representation of the soil layer (foundation)

For the soil layer on which the reinforced concrete plate is based, a Winkler model was adopted to represent this layer for both the compressive and frictional constraints.
As for the subgrade reaction of soil, it is an imaginary relationship between soil pressure and deflection. It can be measured using the plate-load test. With this test, the load-deflection curve is adopted. The subgrade reaction of the subsoil layer (\(K_Z\)) can be calculated using the following equation:

\[
K_Z = \frac{P}{W}
\]  
(3)

Where
- \((K_Z)\) is the vertical subgrade reaction of the soil layer,
- \((P)\) is the pressure on the plate,
- \((W)\) is the deflection of the plate [14].

There are two methods of using the Winkler model, including linear and nonlinear. The Linear model of Winkler is used as a part of this study.

The approved stiffness matrix for soil layer is [15]:

\[
[K]_f = \begin{bmatrix}
[R_w] & 0 & 0 & 0 & 0 \\
0 & [R_w] & 0 & 0 & 0 \\
0 & 0 & [R_w] & 0 & 0 \\
0 & 0 & 0 & [R_w] & 0 \\
0 & 0 & 0 & 0 & [R_w]
\end{bmatrix}^{n*n}
\]  
(4)

\[
[R_w] = \begin{bmatrix}
Kf1 & 0 & 0 \\
0 & Kf2 & 0 \\
0 & 0 & Kf3
\end{bmatrix}
\]  
(5)

whereas:

\[
K_{fi} = \int_{-1}^{1} \int_{-1}^{1} Ni \cdot Kx \cdot |J| d\xi d\eta
\]  
(6)

\[
K_{gi} = \int_{-1}^{1} \int_{-1}^{1} Ni \cdot Ky \cdot |J| d\xi d\eta
\]  
(7)

\[
K_{gi} = \int_{-1}^{1} \int_{-1}^{1} Ni \cdot Kz \cdot |J| d\xi d\eta
\]  
(8)

Where
- \((Ni)\) is the shape function of the node (i),
- \((Kx)\), \((Ky)\) and \((Kz)\), which are the subgrade reaction of the soil layer in the directions (x), (y) and (z) respectively.

Thus, the final stiffness matrix is equal to the sum of the stiffness matrices resulting from concrete and normal reinforcement and the soil layer on which the plate is supported, which can be written as follows:

\[
[K] = [K]_c + [K]_s + [K]_f
\]  
(9)

Where \([K]\) is the final stiffness matrix which represents the sum of the stiffness matrices of concrete, normal reinforcement, and the layer of soil on which the plate is supported.

4. Materials

4.1 Modeling concrete in compression
The behavior of concrete in compression is simulated in the present work by an elasto-plastic work hardening model followed by a perfectly plastic, which is terminated at the onset of crushing. The yield criterion used in the present work has been successfully used by many researchers [15] and is given by:

\[ F_0(\sigma, \sigma_o) = c I_1 + \sqrt{(c^2 I_1^2 + 3\beta_p J_2)} - \sigma_o = 0 \]  

(10)

Where \((I_1)\) and \((J_2)\) are the first and second stress invariants. The constants \((c)\), \((\beta_p)\) are evaluated from experimental tests [16]. 

\((\sigma_o)\) is the equivalent effective stress at the onset of plastic deformation that can be determined from the uniaxial compression test as shown in figure 3.

Figure 3. Uniaxial stress-strain curve for concrete in compression [15]

4-2 Modeling of concrete during crushing
The isotropic expansion of the post-loading surface is terminated when the effective stress reaches the peak compressive stress, further than that ideal response is assumed to occur. The plastic ideal flow continues until the maximum deformation amplitude of the concrete is reached and the material ultimately appears crushing failure.

4-3 Concrete modeling in tensile strength
In this study, a smeared crack model was adopted with fixed perpendicular cracks to represent concrete fracture. Under additional loading, secondary cracking may occur at the sampling point that was originally cracked in one direction. The model is described in terms of cracking criterion, post-fracture formulation and shear retention model.

4-4 Reinforcement modeling
The Reinforcement is merge into the concrete brick element assuming perfect bonding and has uniaxial properties towards the bars. An elastic-viscoplastic model was used in the present study. A stress-strain curve was assumed for normal stress as shown in ‘figure 4’.
5. Analysis of a reinforced concrete plate resting on soil

The computer program (DARC3) written in (FORTRAN) language was used and it is modified in this study to include the effect of elastic subgrade reaction soil using (Winkler’s) method. The modified program is used to analyze soil-based reinforced concrete plates under effect of static load.

To check the validity of the modified program, a symmetrical reinforced concrete plate resting on the soil, square in shape, dimensions (3500 * 3500 mm) and thickness (120 mm), was studied by (Øverli) [7]. The plate is based on the soil, the subgrade reaction value of the soil layer is (0.15 N/mm³) which represents the stiffness of a soil layer formed from fractured rock. while the reinforcement is represented as a smeared layer in both directions (x) and (y) with the perfect bond between the concrete and the reinforcement. All sides of the plate are considered fixed, the properties of the materials are shown below:

| Table 1: properties of the materials |
|-------------------------------------|
| E<sub>c</sub> | f<sub>c</sub> | f<sub>t</sub> | F<sub>y</sub> | K<sub>soil</sub> |
| 26727 N/ mm² | 32.1 N/ mm² | 5.1 N/mm² | 564 N/ mm² | 0.15 N/ mm³ |

The plate is loaded by many values of central concentrated load. The load-deflection results obtained from the numerical analysis in this research paper are compared with the reference [7] and as shown in ‘figure 5’.
Figure 5. The load-deflection curve of a R.C plate resting on the soil

The load-deflection curve of present study has a same behavior of that in Ref. [7]. Many case studies are done to show their effects on the plate behavior.

5-1 Effect of subgrade reaction of soil
To study the effect of the vertical subgrade reaction of the soil layer ($K_Z$) on the deflection of the concrete plate, several values of the vertical subgrade reaction of the soil were taken under a concentrated load in the middle of the plate. ‘Figure 6’ shows the relation between subgrade reaction values and midspan deflection. Through this relationship, it is found that when increasing the values of the vertical subgrade reaction of the soil from (0.0 to 0.2 N/mm$^3$), the amount of decreasing in deflection was about (3.4) % at ultimate load.

Figure 6. The effect of the vertical subgrade reaction of the soil on the load-deflection curve at midspan (plate thickness=120mm)

5-2 Effect of reinforced concrete plate thickness
In order to investigate the effect of the concrete plate thickness for constant vertical subgrade reaction value of the soil, several values of the plate thickness are considered. The thickness values ranged from (100 to 160 mm) under effect of a concentrated load in the center of the plate with the value of the vertical subgrade reaction of soil equal to (0.15 N/mm$^3$). As shown in ‘Figure 7’. It is noticed for deflection is decreasing when the thickness increased from (100 to 160 mm) by (81.7%) at ultimate load point. In this figure a semi log nondimensional relation between the factor of central deflection ($\delta/a$) vs. the applied load ($P/Er^2$).
Figure 7. Effect of plate thickness on central deflection of Reinforced Concrete Plate on Elastic Foundation (Kz = 0.15 N/mm³)

5.3 Effect of support type
The type of supporting boundaries of reinforced concrete plate are considered which has a great influence on the maximum deflection. To demonstrate this effect, two cases of supports, which are the fixed supported and simply supported from all sides of the reinforced concrete plate loaded by a many values of a central concentrated load and the vertical subgrade reaction value of soil equal to (0.15 N/mm³). It is noticed from ‘Figure 8’ that the amount of increasing in ultimate load in case of fixed supported equal to (77%) more than that in case of simply supported, that is mean the load-deflection curve for fixed supported case is more stiffer than simply supported case.

Figure 8. The effect of the support condition of the reinforced concrete plate on the load-deflection curve at mid-span (plate thick.=120mm, Kz=0.15N/mm³)

5.4 Type of load
To illustrate the effect loading type, a comparison was made between a concentrated load in the center of the reinforced concrete plate and a uniformly distributed load on the upper surface of the reinforced concrete plate with several values of the subgrade reaction of the soil layer on which the plate is supported and ranges between (0.0 - 0.2 N/mm³). ‘Figure 6’ and ‘9’ show the load-deflection curve under many values of (Kz) for concentrated load and uniformly distributed load respectively.
Figure 9. The effect of the vertical subgrade reaction of the soil layer on the load-deflection curve at mid-span with Uniformly distributed load

6. Conclusion

The main conclusions obtained through this research work of the reinforced concrete plates supported on soil are as follows:

1. The results of the non-linear behavior for reinforced concrete plate based on soil obtained in this study are in good agreement comparing with previous studies.
2. In case of increasing the vertical subgrade reaction value of the soil layer the deflection at midspan decreased.
3. The increasing of R.C. plate thickness affect the nonlinear behavior of this plate by decreasing the midspan deflection.
4. The amount of increasing in ultimate load in case of fixed supported is more than that in case of simply supported, that is mean the load-deflection curve for fixed supported case is more stiffer than simply supported case.

References

[1] Timoshenko S. and Woinowsky S 1959 Theory of Plates & Shells. (2nd edition. McGraw Hill. New York, New York, United States)

[2] Kerr A D 1964 Elastic and viscoelastic foundation models J. Appl. Mech. Trans. ASME 31(3) 491–8

[3] Baumann R A, Weisgerber F E and Asce M 1983 Yield-Line Analysis of Slabs-on-Grade J. of Struct. Eng. 109 (7) 1553-68

[4] Daloglu A T and Girija Vallabhan C V 2000 Values of k for Slab on Winkler Foundation J. of Geotech. and Geoenvironmental Eng. 126 (5) 463-71

[5] Al-Azzawi A A and Al-Obaidie A M 2010 Nonlinear Three Dimensional Finite Element
Analysis of Reinforced Concrete Beams on Elastic Foundations Journal of the Serbian Society for Computational Mechanics 4 (1) 66-87

[6] Vanam B. C. L 2012 Static analysis of an isotropic rectangular plate using finite element analysis (FEA) Journal of Mechanical Engineering Research 4(4) 148-162

[7] Øverli J 2014 Experimental and numerical investigation of slabs on ground subjected to concentrated loads Central European Journal of Engineering Cent. Eur. J. Eng. 4 210–25

[8] Walker W W and Holland J A Modulus of Subgrade Reaction-Which One Should be Used? SSI Structural Services, Inc ENGINEERING BULLETIN http://www.ssiteam.com/publications

[9] Shaaban I G 2018 Analysis of Reinforced Concrete Slabs-on-Grade in Industrial Buildings In: 38th Cement and Concrete Science Conference. Coventry University, Coventry, UK. (Unpublished)

[10] Cervera M., Hinton E., Bonet J. and Bicanic N. 1988 Numerical methods and software for dynamic analysis of plates and shells (Pineridge Press Limited 54, Newton Road, Mumbels, Swansea, U.K.)

[11] Robert Cook D, David Malkus S. and Michael Plesha E. 1989 Concept and Application of Finite Element Analysis (third edition John Wiley and Sons, Hoboken, United States)

[12] Mohammed M S 2002 Time-Dependent Nonlinear Finite Element Analysis of Reinforced Concrete Deep Beams M.Sc.Thesis Al- Mustansiria University, Iraq

[13] Behnam A B 2002 Nonlinear Finite Element Analysis of Prestressed Reinforced Concrete Beams M.Sc.Thesis University of Mosul, Iraq

[14] Al-Azzawi A. A. and Hasanain M. Ghalban 2016 Analysis of Thick Rectangular Plates Resting on Nonlinear Winkler Foundation Journal of Engineering and Sustainable Development 20 (1) 118-134

[15] Al-Sharbaa IA, Al-Azzawi AA and Mahmood RG 2011 Nonlinear Finite Elment Analysis of Reinforced Cconcrete Plates on Elastic Foundations The Iraqi Journal For Mechanical And Material Engineering Special issue A 34-41

[16] Kupfer H, Hilsdorf KH and Rush H 1969 Behavior of concrete under biaxial stresses ACI Proceeding 66 656-666

[17] Barzegar F and Maddipude S 1997 Three-dimensional Modeling of Concrete Structurs. I: Plain Cconcrete Journal of Structural Engineering 123 (10) 1339-46

[18] Owen D. R. J. and Hinton E. 1980 Finite Elements in Plasticity (Pineridge Press Limited 91 West Cross Lane, West Cross, Swansea U. K)