Research on Prediction of Metro Surface Deformation Based on Ensemble Kalman Filter

Zengxin Li*, Yuanzhong Luan, Yaodong Liang and Zhaolei Ji
Shandong University of Science and Technology, Qingdao, China

*Corresponding author: zengxinli@sdust.edu.cn

Abstract. In order to improve the prediction ability of the statistical model of the subway surface deformation, the equal-dimensional innovation grey system theory model and Kalman filter model are used to predict the surface deformation of the subway. The ensemble Kalman filter (EnKF) algorithm is used to assimilate the two groups of prediction data to improve the accuracy of the prediction data. The absolute difference and the root mean square error between the EnKF assimilation prediction data, the Kalman filter prediction data, and the equal-dimensional and innovation grey model prediction data and the measured data are compared and analysed. The results show that the accuracy of EnKF assimilation prediction data is greatly improved compared with the prediction data of grey system theory model and Kalman filter model, and the short-term prediction effect is better.

Keywords: Subway tunnel surface deformation, grey system theoretical model, Kalman filter model, ensemble Kalman filter, accuracy analysis.

1. Introduction
With the rapid development of society, subway has become one of the indispensable modes of transportation in urban development and progress. With the excavation of urban tunnels, the movement and deformation of overlying strata and ground surface will inevitably be caused, which will affect the life and safety of urban residents. Therefore, it is of great significance to establish a high-precision surface settlement prediction model by using the ground observation data of subway [1]. At present, the settlement data processing methods mainly include: regression analysis, time series model, grey system theory (GM) model, Kalman filter model, generalized regression neural network, etc [2-4]. However, a single prediction model will have certain limitations. Through data assimilation, we can combine two or more kinds of data to make full use of the advantages of each model, and then obtain high-quality prediction data. Ensemble Kalman filter (EnKF) is a sequential data assimilation algorithm proposed by Evensen [5-6] in 1994 based on Epstein's stochastic dynamic forecast theory. After nearly ten years of research and development, the ensemble Kalman filter algorithm has gradually developed and matured, and it has gradually received the attention of data assimilation researchers. Different authors have conducted extensive discussions on the theory and application of the ensemble Kalman filter algorithm [7-13].

In this paper, based on the measured data of point 6 on the monitoring section of J17 of Zhengzhou Metro Line 1, the optimized grey system theory model and Kalman filter model are used for prediction,
and the ensemble Kalman filter assimilation algorithm is used to fuse the two groups of prediction data. Through the accuracy comparison and analysis with the measured data, it is confirmed that the prediction accuracy of the ensemble Kalman filter assimilation data is higher, and the short-term prediction effect is better.

2. Basic principles

2.1. Grey theory model of equal dimension and new information

Obtain the original sequence from known information:

\[ X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\} \quad x^{(0)}(i) \geq 0, \ i = 1,2,\ldots,n \]  

(1)

Perform square root processing on the original data \(X^{(0)}\) to get the deformed sequence \(X^{(0)}_{1}\):

\[ X^{(0)}_{1}(i) = \sqrt{X^{(0)}(i)}, \ i = 1,2,\ldots,n \]  

(2)

Perform an accumulation 1-AGO (Accumulated Generating Operator) sequence \(X^{(1)}\):

\[ X^{(1)}(i) \sum_{k=1}^{i} X^{(0)}(k), \ i = 1,2,\ldots,n \]  

(3)

New background value construction formula \(Z^{(1)}\):

\[ z^{(1)}(k) = \frac{x^{(1)}(k-x^{(1)}(k-1))}{\ln[x^{(1)}(k-x^{(1)}(k-1))]-\ln[x^{(1)}(k-1)]} \]  

(4)

Establish grey theory model:

\[ X^{(0)}_{1}(i) + aZ^{(1)}(i) = b, \ i = 1,2,\ldots,n \]  

(5)

In the formula, \(X^{(0)}(i)\) is the grey derivative; \(Z^{(1)}(i)\) is the whitening background value; \(a\) is the development coefficient, \(b\) is the grey effect, and \(a\) and \(b\) are parameters to be estimated.

The predicted value of the solution of the whitening equation is:

\[ \hat{x}(1)(t) = \left(X^{(0)}(1) - \frac{b}{a}\right)e^{-a(t)} + \frac{b}{a} \]  

(6)

By accumulating reduction, you can get the reduction value of the original sequence:

\[ \hat{x}(0)(t) = (1 - e^{a}) \left(X^{(0)}(1) - \frac{b}{a}\right)e^{-a(t)} \]  

(7)

2.2. Kalman filter model

The Kalman filter method is based on probability theory and minimum variance estimation. According to the current monitoring data and the optimal estimation value of the previous moment, the optimal estimation value of the current moment is calculated, and the monitoring value of the next moment can be reasonably predicted \cite{14}. The state equation and observation equation of Kalman filter are:

\[ Y_{k+1} = \varphi_{(k+1,k)}Y_{k} + \Omega_{k} \]  

(8)

\[ y_{k+1} = B_{k+1}Y_{k+1} + \Delta_{k+1} \]  

(9)
Where $Y_{k+1}$ and $Y_k$ are the state vectors at time $t_{k+1}$ and $t_k$ respectively; $\varphi(k_{k+1}, k)$ is the state transition matrix from time $t_k$ to time $t_{k+1}$; $\Omega_k$ is the dynamic noise at time $t_k$; $y_{k+1}$ and $\Delta_{k+1}$ are the measured displacement value and observation noise at time $t_k$ respectively; $B_{k+1}$ is the observation matrix at time $t_{k+1}$.

Assuming that the noise, the state vector and the observation noise are independent of each other, in the discrete linear system:

$$
\begin{align*}
E(\Omega_k) &= 0, E(\Delta_k) = 0 \\
\text{Cov}(\Omega_k, \Omega_j) &= D_{\Omega}(k)\delta_{kj}, \text{Cov}(\Delta_k, \Delta_j) = D_{\Delta}(k)\delta_{kj} \\
E(Y_0) &= Y(0/0), Var(Y_0) = D_Y(0)
\end{align*}
$$

In the formula, $E$ is the mean value; $D$ is the variance; $k$ and $j$ are time values, when $j = k, \delta_{kj} = 1$; when $j \neq k, \delta_{kj} = 0$.

$$
Y(k/(k-1)) = \varphi(k_{k-1}, k)Y((k-1)/(k-1))
$$

$$
D_Y(k/(k-1)) = \varphi(k_{k-1}, k)D_Y((k-1)/(k-1))\varphi(k_{k-1}, k)^T + D_\Omega(k-1)
$$

$$
J_k = D_Y(k/(k-1)) \cdot B^T[B_kD_Y(k/(k-1))B_k^T + D_\Delta(k)]^{-1}
$$

$$
Y(k/k) = Y(k/(k-1)) + J_k[Y_k - B_kY(k/(k-1))]
$$

$$
D_Y(k/k) = (I - J_kB_k)D_Y(k/(k-1))
$$

In the formula, $Y((k-1)/k-1)$ is the state vector at $t_{k-1}$; $Y(k/(k-1))$ is the transition value of the state vector; $J$ is the filter gain vector; $Y(k/k)$ is the updated value at $t_k$; $D$ is the mean square error matrix of the state vector transition.

2.3. Ensemble Kalman filter

The basic idea of EnKF is: according to the background field and observation field error statistics, the ensemble disturbances of a limited sample are randomly selected and added to the background field and the observation field to generate the background field set and the observation field set, and the short-term forecast of the background field set is used to estimate the forecast The error covariance matrix is then assimilated with the observations at the new time to obtain a set of analysis fields, which are used for short-term forecasts and then assimilated at the next observation time, and so on[15].

Assuming the time subscript is $t$, the ensemble forecast of the variable:

$$
C^b = (c_1^b, \cdots, c_N^b)
$$

The background error covariance, the set average is:

$$
\overline{c^b} = \frac{1}{N} \sum_{i=1}^{N} c_i^b
$$

Therefore

$$
B^b = \frac{(c^b - \overline{c^b})(c^b - \overline{c^b})^T}{N-1}
$$

Calculate the gain matrix:
3. Data assimilation process

3.1. Prediction of equal-dimensional new-information grey theory model and Kalman filter model

First of all, according to the measured deformation data of period 35-55 on the J17 monitoring section of the second phase of Zhengzhou Metro Line 1, the parameters of the grey theoretical model of equal dimension and new information are obtained as: \([a, b] = [-0.0341, -909219]\)^T. The gray theoretical model of equal dimension and new information is:

\[
\hat{x}(t + 1) = (1 - e^{-0.0341})(-7.89 - 9.9219/0.0341)e^{-0.0341t}
\]

Still using the 35-55 period data as the modeling data, a grey theoretical model of equal dimensions and new information with a dimension of 20 is established, and the 56-60 period data is predicted, and the comparison and analysis with the traditional grey theoretical model are performed, as shown in Table 1.

| Number of periods | Measured value /mm | Grey theory model of equal dimension and new information | Traditional grey theory model |
|-------------------|-------------------|-----------------------------------------------------|-----------------------------|
|                   | Predictive value /mm | Absolute difference /mm | Predictive value /mm | Absolute difference /mm |
| 56                | -16.48             | -16.21                      | 0.27                      | -16.95                      | 0.47                     |
| 57                | -17.05             | -17.39                      | 0.34                      | -17.54                      | 0.59                     |
| 58                | -17.66             | -17.99                      | 0.33                      | -18.12                      | 0.46                     |
| 59                | -18.21             | -18.68                      | 0.47                      | -18.72                      | 0.51                     |
| 60                | -19.63             | -20.15                      | 0.52                      | -20.27                      | 0.64                     |

Average relative error 2.2% 3.3%

It can be seen from Table 1 that the average relative error of the prediction data of the equal-dimensional and new-information grey theoretical model is 2.2%, while the average relative error of the traditional grey theoretical model is 3.3%, indicating that the prediction accuracy of the equal-dimensional and new-information grey theoretical model is higher than that of the traditional grey theoretical model.

The Kalman filter model is used to filter the measured data in periods 35-55, the mean value of the measured data in periods 35 and 36 is taken as \(Y_0\), and the corresponding variance matrix is used as the initial variance matrix. The state vector can be preliminarily predicted by formula (11). Formula (12) predicts the variance, formula (13) calculates the filter gain according to the predicted variance, formula (14) combines the state vector predicted value and the filter gain to obtain the optimal filter value of the state vector, and formula (15) updates the variance to the next round. The filter calculation is prepared and looped in turn to obtain Kalman filter forecast data, forecast 56-60 period data, and compare and analyze with the equal-dimensional new-information grey theory model, as shown in Figure 1. It can be seen from Figure 1 that the prediction accuracy of the Kalman filter model is higher than that of the
equal-dimensional and new-information grey theory model, which is a preparation for EnKF assimilation data.

![Figure 1. Comparison between the predicted values of the Kalman filter model and the equal dimension new information grey theoretical model](image)

**Figure 1.** Comparison between the predicted values of the Kalman filter model and the equal dimension new information grey theoretical model

3.2. **EnKF data assimilation process**

The process of data assimilation refers to the fusion of data obtained by different monitoring methods to become similar or the same; let A represent equal-dimensional and new-information gray model prediction data, and B represent Kalman filter model prediction data, then there are three ways of assimilation[16-17]: First, A data remains unchanged, and B data is assimilated to A data; second, A data remains unchanged, and B data is assimilated to A data; third, A and B data are assimilated in a way closer to each other. In order to make full use of the advantages of the two types of data, this article chooses the third assimilation method. Considering that the prediction data of the Kalman filter model is higher than the prediction data of the equal-dimensional and new-information gray model, the Kalman filter model prediction data is mainly used (Observation field) assimilation method, the specific EnKF assimilation process is shown in Figure 2.
4. EnKF assimilation data analysis

In order to investigate the prediction effect of the fusion data, based on the settlement data of point 6 on the J17 monitoring section of Zhengzhou Metro Line 1, the prediction of equal dimension innovation grey model and Kalman filter model, as well as EnKF data fusion processing, are carried out by using MATLAB language.

For the deformation of No. 6 point, EnKF is used to assimilate the forecast data of the next 20 periods. The results of the 60-66 period forecast data are shown in Table 2. The comparison chart of the measured data, EnKF assimilation data, Kalman filter prediction data and the prediction data of the equal dimension innovation grey model is shown in Figure 3.

As shown in Table 1, through the comparative analysis of absolute difference and RMSE (root mean square error), the RMSE value of EnKF assimilation prediction data is 0.19mm, the RMSE value of Kalman filter prediction data is 0.21mm, and the RMSE value of equidimensional innovation grey model prediction data is 0.46mm. It is found that the EnKF assimilation prediction data is closer to the measured data and the accuracy is improved than the Kalman filter prediction data and the equal-dimensional innovation grey model prediction data. At the same time, it can be seen that the accuracy of Kalman filter prediction data is higher than that of equal dimension innovation grey model. As shown in Figure 4, the 70-76 period forecast data is displayed. Based on the prediction data of Kalman filter and the prediction data of equal dimension innovation grey model, with the increase of observation period, the accuracy of prediction value obviously decreases, which proves that EnKF assimilation data has better short-term prediction effect, and it is also found that the equidimensional innovation grey model can only make short-term prediction for time series data.
Figure 3. Comparison of forecast data for periods 60-66

Figure 4. Comparison of forecast data for periods 70-76
Table 2. Comparison table of settlement prediction data and measured data

| Number of periods | Measured value /mm | EnKF assimilation prediction data | Kalman filter prediction data | Equal-dimensional and new-information grey model forecast data |
|------------------|--------------------|-----------------------------------|------------------------------|-------------------------------------------------------------|
|                  |                    | Predictive value /mm | Absolute difference /mm | Predictive value /mm | Absolute difference /mm | Predictive value /mm | Absolute difference /mm |
| 60               | -19.63             | -19.34               | 0.29                      | -19.31            | 0.32                      | -19.42            | 0.21                       |
| 61               | -20.47             | -20.23               | 0.24                      | -20.17            | 0.30                      | -20.42            | 0.05                       |
| 62               | -20.98             | -21.03               | 0.05                      | -20.81            | 0.17                      | -21.17            | 0.19                       |
| 63               | -21.52             | -21.73               | 0.21                      | -21.57            | 0.05                      | -22.23            | 0.29                       |
| 64               | -22.17             | -22.31               | 0.14                      | -22.05            | 0.12                      | -22.84            | 0.67                       |
| 65               | -22.89             | -23.11               | 0.22                      | -22.75            | 0.08                      | -23.52            | 0.63                       |
| 66               | -23.45             | -23.53               | 0.08                      | -23.25            | 0.20                      | -24.12            | 0.67                       |
| RMSE/mm          | 0.19               | 0.21                 | 0.46                      |                  |                          |                  |                            |

5. Conclusions

(1) According to the characteristics of the subway system subject to external disturbances, this paper uses the Kalman filter prediction model and the equal-dimensional and new-information gray model to predict the settlement of the subway monitoring points. It is found that the two prediction models have good short-term prediction effects, however, as time goes by, its prediction accuracy will become lower and lower, and it is found that the Kalman filter model has a better prediction effect than the equal-dimensional and new-information gray model. When receiving the paper, we assume that the corresponding authors grant us the copyright to use the paper for the book or journal in question. Should authors use tables or figures from other Publications, they must ask the corresponding publishers to grant them the right to publish this material in their paper.

(2) Based on the prediction data of Kalman filter model and equal dimension innovation grey model, the prediction data of them are assimilated by EnKF, and compared with the measured data, Kalman filter prediction data and equal dimension innovation grey model prediction data. The results show that the prediction data based on EnKF assimilation is better and more accurate than Kalman filter prediction model and equal dimension innovation grey model, and the short-term prediction effect is good, which can be a more valuable reference for Metro deformation monitoring.

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