MATHMATICAL MODELING OF THE SURFACE ROUGHNESS OF THE GRINDING WHEEL DURING STRAIGHTENING

Purpose. Research on the mechanism of influence of the straightening conditions of the grinding wheel, including the relative oscillations of the wheel and a multipoint diamond dresser, on the roughness of the ground surface and other machining results.

Methodology. Straightening a grinding wheel with a multipoint diamond dresser is a process of high-speed destruction of a hard, abrasive material and its bond under the instantaneous forces, abrasive grains with a hard surface of a diamond crystal. During the grinding wheel straightening, the total component of normal forces causes correspondingly less elastic deformations in the "wheel – straightening tool" system, which increases the accuracy of the geometric shape of the grinding wheel working surface.

Findings. The research results make it possible to determine the parameters of the surface roughness of a workpiece and to find ways to control it to increase the efficiency of the grinding process.

Originality. The regularities of the influence of the grinding wheel straightening conditions on the state of its working surface have been established. The paper shows that the initial arrangement of grains along the normal to the surface of the wheel is determined by its characteristics. When the abrasive grains hit the surface of the straightening tool, some of the vertices are chipped off, as a result of which the density of the grain vertices on the outer surface of the wheel increases. The straightening process was further developed in the direction of the non-uniform character of the location of the vertices of abrasive grains. The distribution of the grinding wheel position at the wheel bond depends on the straightening conditions. Since the removal of the allowance in the process of grinding is carried out by the most protruding grain vertices, then, consequently, the result of grinding will depend on their location and the conditions for the wheel straightening.

Practical value. Application of the research results obtained in the work, namely, mathematical modeling of the surface roughness of the grinding wheel during straightening, makes it possible to calculate the roughness parameter of the ground surface. The work also shows that the level of chipping of the grain vertices depends on the grinding wheel straightening conditions, in particular, on the value of the axial feed of the straightening tool. In this case, lower stresses arise in the grains and the bond, and the tool works as a harder one. Straightening conditions affect the stability of the grinding wheel and its self-sharpening process in the machining zone. This determines the significant role of straightening in the results of the grinding process.

Keywords: profile deviation, straightening conditions, grinding wheel

Introduction. An integral part of the grinding process is the grinding wheel straightening, which largely determines the machining results. The wheel straightening is necessary not only to give the working surface of the grinding wheel a given geometric shape. It is known that the straightening process conditions depend on the consumption of the abrasive tool, the state of its working surface, its cutting properties, and, consequently, the roughness of the surface of the workpiece being ground, the quality of its surface layer, its productivity, and the cost of machining. So, only by changing the longitudinal feed during straightening, the roughness of the ground surface can vary from $R_2 \approx 2.5$ to $R_{\alpha} \approx 0.05 $ μm [1]. The wheel straightening process has a significant effect on the grinding accuracy and the physical and mechanical state of the surface layer of the workpiece [2]. Grinding wheel consumption during straightening ranges from 50 to 95 % of its net volume [3]. Depending on the various types and conditions of grinding, the straightening time ranges from 10 to 80 % of the technological time. As a result, 5–70 % of the cost of the grinding operation is the cost of the grinding wheel straightening [4]. Thus, increasing the efficiency of the grinding process is largely associated with the research and improvement of the straightening processes for grinding wheels, with the rational use of modern straightening tools.

The most common type of straightening tool is a multipoint diamond dresser. The widespread use of the multipoint diamond dresser in production is facilitated by its versatility, ease of use, relatively low cost, and high wear resistance. Apart from geometric factors, the results of the grinding process during straightening with the multipoint diamond dresser are likely to be influenced by many other factors. The simplified representation of the surface of the grinding wheel in the form of a helical line [5], in particular, cannot explain the fact that under the same straightening and grinding conditions, the roughness of the ground surface is different. Even in different sections of the same surface, the height of microroughnesses can differ several times.

During the straightening process, the vibrations of the wheel and the straightening tool always take place, which can have both positive and negative effects on the machining results. The fact that these vibrations have an impact on the results of grinding is evidenced by, at least, the experience of using ultrasonic straightening of the wheel [6], which allows reducing the roughness of the ground surface and the temperature in the cutting zone. However, the mechanism of the influence of the relative oscillations of the grinding wheel and the straightening tool in the straightening process on the grinding results remains unclear, and this makes it difficult to find rational ways to control these oscillations and to increase the efficiency of the grinding process.

This work is devoted to the analysis of progressive processes of the grinding wheel straightening. Particular attention is paid to studying the mechanism of the influence of the multipoint diamond dresser vibrations during straightening on the state of the working surface of the wheel and the grinding results. New progressive methods for straightening the grinding wheel have been considered, the optimal conditions for their implementation have been determined.

Literature review. Currently, the most common type of straightening tool is a diamond tool.

In the straightening process, simultaneously with the destruction of the surface of the grinding wheel, the diamond wear occurs. The reason for diamond wear is the numerous impacts of abrasive grains of the grinding wheel on its working surface, as well as the friction of the diamond on the filled wheel surface [7].

The intensity of diamond wear is influenced by the characteristics of the grinding wheel, straightening modes, the shape of the diamond, and its position relative to the wheel.

© Strelchuk R. M., Trokhimchuk S. M., 2021
The area formed in the process of diamond wear has a great influence on straightening quality. However, diamond wear is hundreds of times less than that of monolithic carbide discs.

The high wear resistance of the diamond straightening tool and low forces during straightening, the rigidity and simplicity of the design ensured high precision of the polished products and a wide spread of diamond straightening. For abrasive wheel straightening, multipoint diamond dressers, straightening diamonds, diamond needles, plates, combs, rollers, and others are used. The best raw materials from the entire range of synthetic polycrystalline diamonds are currently SV (CB) and ASPK (ACITK) materials. Their performance is at the level of natural diamonds. A variety of tools made of cubic boron nitride have become widespread.

High machining accuracy, according to [8], is obtained with wheels straightened with diamond rollers. At the same time, the average deviation from the cylindricality of the polished surface of the workpiece ranges from 5 to 10 μm, which is 3–6 times lower than during straightening with carbide rollers.

The roughness of the surface to be ground largely depends on the straightening scheme and the grinding method. During plunge straightening, the microprofile of the roller is copied onto the abrasive wheel, therefore, during plunge grinding, the size of diamonds and their concentration in the roller significantly affect the roughness of the machined surface [9].

During straightening on the pass, the roughness of the machined surface does not significantly depend on the size of the diamonds, which makes it expedient to equip the rollers with coarse grains to increase their wear resistance without a noticeable decrease in roughness. In this case, with different grinding schemes, the roughness of the processed surface is within the same parameter.

The value of the traverse and the longitudinal feed of the diamond roller has no significant effect on the surface roughness of the part and, as follows from [10], is in the range of \( R_a \approx 0.4–0.8 \mu m \). According to some reports, during the straightening of the wheel with a diamond roller with blunt diamond grains, it is possible to achieve a roughness of the ground surface \( R_a = 0.25 \mu m \).

Straightening of grinding wheels with diamond rollers is carried out with abundant cooling at a liquid pressure of 2.1–6.3 Pa. Their service life is influenced by the longitudinal feed and the traverse, straightening speed, grain size, and other parameters. In most cases, the service life of diamond rollers ranges from 10 000 to 100 000 workpieces.

The use of diamond rollers, the profile of which is a shaped surface corresponding to the workpiece, allows the straightening of grinding wheels by the plunge-in method, which makes it possible to significantly increase the productivity of the grinding process and simplify the design of the machine. The disadvantage of this method of wheel straightening is the high cost of the diamond roller. Therefore, at present, it is economically feasible only in conditions of mass production.

Efficient tools for profile straightening of grinding wheels are diamond combs, bars, plates, and cutters, the working part of which consists of small diamonds firmly interconnected by a special alloy. The roughness of the machined surface depends on the characteristics of the diamonds used in the tool and on the straightening modes and reaches \( R_a = 0.63 \mu m \). The maximum profile accuracy is within the range of 0.015–0.02 mm. The widespread use of the above tool is limited by its high cost.

The multipoint diamond dresser is a universal and thus the most widespread type of straightening tool. Its advantage is the ease of use, high wear resistance, and relatively low cost. Therefore, in recent years, much attention has been paid to improving this type of straightening tool and studying the influence of the straightening process with this tool on the grinding results.

One of the ways to improve the performance of diamond straightening dressers is the preliminary metallization of rough diamonds. The use of diamonds with a coating that forms a continuous film connected to the surface of the crystal contributes to a stronger fixation of diamond grains in the bond, reduces the destruction and chipping of diamonds in the work process. As a result, the wear resistance of such a tool increases 1.2–1.4 times as compared to a similar tool made of ordinary diamonds.

The straightening conditions of the multipoint diamond dresser have a great influence on the grinding results.

The wear of the grinding wheel is mainly influenced by the temperature in the contact zone, the amount of pressure per grain per unit time. The largest load is taken by the most protruding grains and the grains that are at a depth hardly participate in cutting. Therefore, the coarser the straightening mode is, the fewer grains are on the wheel surface, the more loads act on them and the more significant the wear of the grinding wheel occurs. This is well confirmed by the graph in Fig. 1, showing the dependence of the wheel wear on the size of the removed allowance and the straightening mode [11].

In the work [12], it is shown that the ratio of forces during straightening and grinding has a significant effect on the accuracy of the ground surfaces. Based on the obtained theoretical dependence and experimental work, the conclusion is drawn: with an increase in the difference in forces during grinding and straightening, the influence of the imbalance of the grinding wheel on the waviness of the ground surface increases. The smallest surface undulation occurs during straightening with a force equal to the grinding force.

In this work, we will consider the process of straightening grinding wheels as the process of forming a helical groove on the working surface of the wheel, the profile of which coincides with the profile of the diamond. In the process of grinding, the helical surface of the grinding wheel is copied to the ground surface, causing the formation of regular roughnesses on it.

This approach made it possible to conclude that the ways to reduce the roughness of the ground surface are based on a decrease in the height and roughness width vs. straightening. The main influence on the roughness of the ground surfaces is no longer the grain size of the wheel, which is determined by the grain size, but the straightening mode. A small straightening width makes a coarse-grained wheel appear to be a fine-grained one, which significantly expands the range of application of medium-grain wheels.

As a result of the study, a so-called “fine straightening zone” of the grinding wheel carried out with the longitudinal feed \( S_{x} = 0.05 \text{ mm/rev} \) or less has been found. Using the indicated feed rates on the final passes of the diamond, it is possible to obtain a surface roughness \( R_a = 0.125–0.032 \mu m \) and geometric shape accuracy within 1–2 \( \mu m \) with wheels of 40 grit on a ceramic bond.

**Fig. 1. Wear of the grinding wheel depending on the straightening mode**
The necessary conditions for finishing grinding are no vibrations, good condition of the spindle, careful balancing of the wheel, the ability to carry out small longitudinal feeds with uniform and smooth movement.

Thus, the most significant influence from the grinding wheel straightening conditions on the roughness of the ground surface is exerted by the value of the axial feed of the diamond dresser and the degree of dullness of the diamond straightening vertex. However, the use of small values of axial feeds of diamond during straightening is limited by a sharp decrease in the productivity of straightening and grinding processes, the danger of defects on the surface of the workpiece, the need to use high-precision equipment, as well as to ensure the absence of vibrations in the technological system.

Due to the above reasons, the method of “fine straightening” of the wheel has not received a wide practical application. Attempts have been made to reduce the roughness of the ground surface by changing the direction of rotation of the grinding wheel in the process of straightening and the speed of movement of the multipoint diamond dresser along the grinding wheel generatrix. Such measures provide some reduction in the roughness of the polished surface. However, this significantly complicates the kinematics of the machine.

Research results on the influence of the relative vibrations of the grinding wheel and the straightening tool during straightening are very contradictory. Most researchers believe that the vibration of the technological system during the straightening process negatively affects the grinding results, increasing the roughness of the ground surface, its waviness.

In [13], as a result of probabilistic studies, it was found that with insufficient rigidity of the straightening tool, the diamond dresser can receive significant vibrations from the impact of grains on it, as a consequence of which the density of the abrasive grains is reduced, the roughness of the ground surface increases. However, this work assumes a random arrangement of abrasive grains on the wheel surface, while during straightening of the grinding wheel with the multipoint diamond dresser, there is a tendency toward their ordered arrangement.

As a result, the above studies do not explain the fact that, in some cases, fluctuations of the multipoint diamond dresser during the straightening process can have a positive effect on the machining results of the workpiece. Thus, when applying ultrasonic vibrations to the multipoint diamond dresser during straightening of the grinding wheel, the roughness of the ground surface decreases, and the grinding performance increases.

A continuation of work in this direction was the development of a device for ultrasonic straightening using a piezoelectric transducer and generator. Consequently, with practically the same output parameters of the grinding process, it was possible to significantly reduce the dimensions and power consumption of the generator using the ultrasonic method of the wheel straightening, to avoid cooling the concentrator. The oscillation frequency of the master oscillator was adjusted by some modernization of the electrical circuit to achieve the maximum oscillation amplitude of the diamond. The disadvantages of ultrasonic straightening of grinding wheels are the complexity and cumbersomeness of the construction of the straightening tool, high power consumption, the complexity of setting up equipment, and its high cost. Therefore, it has not found wide application in the industry.

However, studies on the ultrasonic wheel straightening method have shown that applying certain vibrations to the straightening tool can have a positive effect on the grinding results. But the mechanism of this influence remains unclear, which makes it difficult to find rational ways to control wheel vibrations and improve the straightening tool. A possible way of straightening and to increase the efficiency of the grinding process is associated with many technological difficulties. In the process of the abrasive wheel straightening, a helical groove is cut on its working surface, which in the process of grinding is kinematically copied on the ground surface. As a result, it is necessary to sharply underestimate the straightening mode to obtain the minimum surface roughness of the workpiece. This, on the one hand, leads to a decrease in the productivity of straightening and the grinding process as a whole, and on the other hand, it causes the risk of burning on the workpiece surface.

**Purpose.** Research on the mechanism of influence of the straightening conditions of the grinding wheel, including the relative oscillations of the wheel and a multipoint diamond dresser, on the roughness of the ground surface and other machining results.

**Methods.** A grinding wheel straightening with a multipoint diamond dresser is a process of high-speed destruction of a hard-abrasive material and its bond under the instantaneous forces, arising from the contact of fast-moving abrasive grains with a hard surface of a diamond crystal. During straightening of a 25 grit of the grinding wheel with a peripheral speed of 25 m/s, the contact time of the abrasive grain with the diamond tool is only 0.00001 s, which is significantly shorter than the contact time during straightening by the rolling method or the grinding method with non-diamond tools. Given this, the total component of normal forces causes correspondingly smaller elastic deformations in the wheel — the straightening tool system, which increases the accuracy of the geometric shape of the working surface of the grinding wheel.

**Results.** During straightening of the grinding wheel with the multipoint diamond dresser, a helical surface is formed on its working surface, which, during machining, is copied onto the workpiece surface to be machined, mainly determining its roughness. Fig. 1 shows a geometric diagram of the formation of a ground surface during straightening of a grinding wheel with a single diamond. After the wheel straightening with a diamond with a radial feed \( S_{a,1} \) and an axial feed \( S_{a,2} \), a helical groove appears on the surface of the wheel with a width \( T = S_{a,1} \) and a depth \( h_{a} \). During contact with a given section of the workpiece, the abrasive wheel will rotate at a certain angle relative to its axis, as a result of which its helical surface will be displaced by some value \( h_{k} \). In Fig. 2, the solid line represents the part of the radial section of the wheel that is in contact with the given section of the part, and the dotted line represents the same part of the wheel, but already coming out of contact with this section when it rotates. In the case of plunge grinding, during the movement of this section of the workpiece in the contact zone with the wheel, a part of the metal from the lateral surface of the section of the workpiece will be cut off, and the height of the protrusion will decrease and become equal to \( h_{p} \). If during the time of contact with a given workpiece cross-section the grinding wheel had time to make a complete revolution, then the side surface of the helical groove would cut off the entire protruding part on the workpiece.

When grinding with the longitudinal feed, the height and the roughness profile of the workpiece will also be influenced.

![Fig. 2. Geometric diagram of the polished surface shaping during the abrasive wheel straightening with a single diamond in the absence of vibrations](image-url)
by the magnitude and the longitudinal feed direction $S_l$. If during the rotation of the grinding wheel to the contact arc with the given section the workpiece moves in the longitudinal direction by the amount of the helical line displacement, and the velocity vectors of these movements coincide, then a full profile groove will be cut on the ground surface.

In this case, the height of the microroughness of the workpiece will be equal to $h_a = h_h$. If the direction and the longitudinal feed value of the wheel do not coincide with the direction and magnitude of the helical line displacement during its rotation, then the roughness height on the workpiece surface $h_a$ will be less than the depth of the helical groove $h_h$. The geometric shape of the helical groove profile on the wheel working surface, and, consequently, the workpiece roughness shape, is largely determined by the geometric shape of the working surface of the diamond straightening. Various researchers in different ways describe the geometric shape of the profile of the working part of the straightening diamond. Some of them believe that a flat area is formed on the diamond working surface during the straightening tool operation, and the size of this area characterizes the wear of the tool. Others, when determining the wear of the diamond straightening surface, find the presence of a bulge on that surface. Many researchers a priori describe the shape of a diamond as a geometric figure [14].

Since various researchers did not come to a common opinion regarding the shape of the diamond profile, special experimental studies have been carried out to solve this issue. For this purpose, BMI-Ts (БМ-Ц) toolmaker’s microscope was used. Using a microscope, the width $b$ of the diamond protruding profile was determined at various distances from its vertex. The maximum value $h$ was 50 µm since in most cases, it is this part of the diamond grain that takes part in the grinding wheel straightening.

Based on these measurements, we will further take the parabola $b = 2\sqrt{\rho_a}a$ as the geometric shape of the profile of the diamond vertex of the straightening dresser, and the degree of dullness of the diamond will be characterized by the radius of the vertex corner $r_p = K/8$.

The considered geometric scheme of the formation of the workpiece ground surface by the screw surface of the grinding wheel is valid in the absence of vibration during the straightening process. In fact, in addition to longitudinal movement along the grinding wheel profile, the straightening tool vertex, under the influence of vibrations in the technological system, will also perform relative movements in the radial direction of the wheel, creating waviness along the helical line.

The location of the vertices of abrasive grains on the wheel surface can be represented as the superposition of a random component in the form of a sinusoidal surface on a deterministic periodic basis as a helical groove, which is formed as a result of relative oscillations of the wheel and the diamond under the dynamic sources.

The random component is generated by the imbalance of rapidly rotating parts, the vibration of rolling bearings, uneven movement of hydraulic drive elements, and so on.

One of the most important sources of these vibrations is the impact of abrasive grains on the working surface of the diamond during straightening. On impact with abrasive grit, the multipoint diamond dresser is displaced radially and oscillates around its midpoint. At a random time, the multipoint diamond dresser will encounter the next abrasive grain, and, as a result of this impact, the vibrations of the multipoint diamond dresser can be increased or decreased.

Such vibrations from the impact of the grains on the straightening tool surface can also be obtained by the grinding wheel. However, due to its greater mass compared to the mass of the straightening tool, these fluctuations will be less significant.

If there are relative oscillations of the grinding wheel and the multipoint diamond dresser during straightening, the vertices of abrasive grains will no longer be located on the wheel helical surface, as in the absence of these oscillations, but will be displaced relative to this surface. In the process of grinding, the vertices of abrasive grains protruding beyond the wheel helical surface will leave random scratches on the workpiece surface to be ground. The greater the diamond vibration amplitude is, the deeper the scratches are left on the workpiece surface by individual abrasive grains, and the greater the effect on the resulting surface roughness of the workpiece is exerted by its random component.

Mathematical model. Let us consider the formation of a microprofile of the workpiece ground, taking into account the lateral displacement of the helical surface of the wheel relative to the workpiece surface during grinding.

According to the standard, the roughness average is defined as the average value of the distances of the profile points to its centerline at the base length. The position of the centerline, with sufficient accuracy for practice, can be determined based on the equality of the areas on both sides of this line. Then we can write

$$R_a = \frac{2}{l_b}S_{ha}(l_a),$$

where $l_b$ is base profile length; $S_{ha}(l_a)$ is the total area of the roughness ridges of a given workpiece cross-section at the level of the profile centerline within the base length.

Let us draw the coordinate axes from the vertex of the protrusion formed by the helical surface of the wheel (Fig. 3): $OY$ axis — perpendicular to the ground surface; $OX$ axis — parallel to it. Let us calculate what the area of this protrusion at the centerline level is.

Fig. 3 shows that $S_{ha}(l_a) = 2S_{ECD}$

$$S_{ECD} = S_{OBD} - S_{ABC} - S_{OAC}.$$

It is known that the area of a parabola in the Cartesian coordinate system $X$ and $Y$ is equal to

$$S_{par} = \frac{2}{3}xy.$$  (3)

From the parabola equation $y = \sqrt{\rho_a x}$, using the extreme values of the variables $y = \frac{S_{ha}}{2}, x = h_a$, we define

$$S_{ha} = \frac{4}{3}\rho_a h_a.$$  (4)

Then the parabola equation will take the form

$$y = \frac{S_{ha}}{2\sqrt{\rho_a}}.$$  (5)

In this case, the expression (3) will be written as follows

$$S_{par} = \frac{1}{3}\sqrt[3]{\rho_a}x^{1.5}.$$  (6)

![Fig. 3 Diagram of the ground surface shaping during straightening of a grinding wheel with a single-point diamond](image)
Based on the expression (6), it is easy to determine the area of the protrusion of the parabolic workpiece surface outside the profile centerline

\[ S_{OPN} = \frac{S_{h0}}{3} h_0 \left( 1 - \frac{\lambda_m}{h_m} \right)^{1.5}. \]  

(7)

In the same way, we find from Fig. 1

\[ S_{ABC} = \frac{S_{h0}}{3} h_0 \left( 1 - \frac{h_m}{h_0} \right)^{1.5}, \]  

\[ S_{O,ACE} = \frac{S_{h0}}{2} (h_0 - h_m)^{1.5} (h_m - \lambda_m) = \frac{S_{h0}}{2} \left( \frac{h_m}{h_0} \right)^{0.5} \left( h_m - \lambda_m \right). \]

Then from the expression (2), the area of the protrusion at the level \( \lambda_m \) will be equal to

\[ S_{ECF} = \frac{S_{h0}}{3} h_0 \left( 1 - \frac{\lambda_m}{h_m} \right)^{1.5} - \frac{S_{h0}}{2} h_0 \left( 1 - \frac{h_m}{h_0} \right)^{0.5} (h_m - \lambda_m). \]

We have got half the area of one protrusion. Considering that the number of protrusions on the base length \( l_b \) is equal to \( h_0/S_{h0} \), from the previous expression, we obtain the area of the profile intersected by the centerline

\[ S_p(\lambda_m) = 2 \frac{l_b}{S_{h0}} \left( \frac{S_{h0}}{3} h_0 \left( 1 - \frac{\lambda_m}{h_m} \right)^{1.5} - \frac{S_{h0}}{2} h_0 \left( 1 - \frac{h_m}{h_0} \right)^{0.5} (h_m - \lambda_m) \right). \]

Substituting the found value \( S_p(\lambda_m) \) into the formula (1), we will obtain

\[ R_l = \frac{4}{3} h_0 \left( 1 - \frac{\lambda_m}{h_m} \right)^{1.5} - 4 h_0 \left( 1 - \frac{h_m}{h_0} \right)^{1.5} - 2 \left( 1 - \frac{h_m}{h_0} \right)^{0.5} (h_m - \lambda_m). \]

Let us determine the \( h_a \), \( h_b \), and \( \lambda_m \) values. From the expression (4)

\[ h_b = \frac{S_{h0}}{8 \rho_a}. \]  

(10)

Since in the process of plunge grinding when the wheel rotates, the helical groove is displaced in the axial direction, the maximum height of microroughnesses on the workpiece surface (Fig. 1) will be

\[ h_a = \frac{L}{2 \pi D}. \]  

(11)

where \( L \) is the value of the lateral displacement of the wheel helical surface relative to the workpiece.

For one revolution of the grinding wheel, the helical groove will move in the axial direction by the width size \( T = S_{h0} \). Therefore, when turning the wheel by the length of the arc \( l_k \), the displacement amount will be

\[ l_k = S_{h0} \frac{L}{\pi D}. \]  

(12)

where \( D \) is the grinding wheel diameter.

It is known that the total length of the arc when the wheel is in contact with a given workpiece cross-section is approximately determined by the equality

\[ L = 2 \sqrt{h_a A} \left( \frac{V_a}{V_d} \pm 1 \right). \]  

(13)

where \( A' = D - \) at surface grinding with the periphery of the wheel; \( A' = \frac{D d}{D+d} \) – at circular external grinding; \( A' = \frac{D d}{D-d} \) – at internal grinding; \( d \) is the workpiece diameter.

In the expression (13), the plus sign is used for climb grinding, and the minus sign – for up grinding. Substituting the value \( L \) from equality (13) into the expression (12), we will get

\[ l_k = 2 S_{h0} \left( \frac{V_a}{V_d} \pm 1 \right) \sqrt{h_a A} \frac{1}{\pi D}. \]

The numerical value of the lateral displacement of the wheel helical surface along the workpiece axis \( l_k \) will also be influenced by the longitudinal feed value during grinding and its direction. It is easy to show that with workpiece longitudinal movement in the direction of displacement of the helix when turning the wheel, the value \( l_k \) will decrease by the value

\[ I_{r_k} = S_{h0} \frac{l_k}{\pi D}. \]

where \( I_{r_k} \) is an arc of continuous contact of the given workpiece cross-section with the grinding wheel.

With the workpiece longitudinal movement in the direction opposite to the displacement of the helix, the \( I_{r_k} \) value increases by the \( I_{r_k} \) value. The resulting relative displacement of the wheel helical surface \( I'_{r_k} \) will be the absolute sum of this displacement due to the rotation of the grinding wheel around its axis and the displacement of the helical surface due to its longitudinal feed, i.e.

\[ I_{r_k} = I_{r_k} + I_{r_k} = 2 S_{h0} \left( \frac{V_a}{V_d} \pm 1 \right) \sqrt{h_a A} \frac{1}{\pi D} \pm S_{h0} \frac{2}{\pi d} \sqrt{h_a A}. \]

Substituting the found value \( I_{r_k} \) in equality (11), we will obtain

\[ h_a = h_a - \frac{1}{8 \rho_a} \left[ 2 S_{h0} \left( \frac{V_a}{V_d} \pm 1 \right) \sqrt{h_a A} \frac{1}{\pi D} \pm S_{h0} \frac{2}{\pi d} \sqrt{h_a A} \right]. \]

Solving this equation for \( h_a \), we will find

\[ h_a = \frac{S_{h0}}{2 \rho_a} \left[ 1 + \frac{A'}{2 \rho_a} \left( \frac{S_{h0}}{D} \left( \frac{V_a}{V_d} \pm 1 \right) \pm \frac{S_{h0}}{d} \right)^2 \right]. \]  

(14)

where the value \( A' \) has the same meaning as in equality (13).

Substituting the values \( h_a \) from equality (10) into the expression (14), we will finally get

\[ h_a = \frac{S_{h0}^2}{8 \rho_a + 4 \pi A'} \left[ \frac{S_{h0}}{D} \left( \frac{V_a}{V_d} \pm 1 \right) \pm \frac{S_{h0}}{d} \right]^2. \]  

(15)

If we sum up the area of the protrusions above the centerline with the areas of the protrusions below it, we will get a rectangle filled with metal with \( l_k \) and \( \lambda_m \) sides. In this case, the area of the protrusions at the level of the depressions \( S_p \) is hence

\[ S_p = S_{h0} l_b, \]

\[ \lambda_m = \frac{1}{l_b} S_p. \]

The \( S_p \) value is easy to determine from the expression (8), taking \( \lambda_m = 0 \)
Thus, all the necessary values of the variables in the expression (9) to determine the roughness average value workpiece $R_a$ formed by the helical surface of the grinding wheel, taking into account its lateral displacement along the workpiece axis, have been found. In this case, the method for calculating the $R_a$ value is as follows. According to the values of the axial feed of the diamond during straightening $S_{ax}$ and its corner radius $\rho$, as well as from the given values of the diameter of the wheel $D$ and the workpiece $d$ and their movement speeds $V_d$ and $V_p$, we will find the values $h_y$ (10), $h_h$ (14), and $\lambda_m$ (16). Substituting these values into equation (9), we determine the value of $R_a$.

Here is an example of calculating the workpiece roughness average. Initial data: 3D642E (3Д642Е) machine, a straight workpiece, $S_{ax} = 40$ m/min, $V_p = 10$ m/min. The straightening mode: multipoint diamond dresser $\rho = 1$ mm, $S_{ax} = 0.16$ mm/r.

By the formula (10)

$$h_y = \frac{0.16^2}{8.1} \cdot \frac{0.0032}{0.0032} = 0.003.$$  

By the formula (15)

$$h_h = \frac{0.16^2}{8.1} \cdot \frac{0.0032}{0.0032} = 0.003.$$  

By the formula (16)

$$\lambda_m = \frac{2}{3} \left( \frac{0.0032}{0.0032} - \frac{0.003}{0.0032} \right)^{0.5} = 0.005.$$  

Substituting the found $h_y$, $h_h$, and $\lambda_m$ values into the formula (9), we will define

$$R_a = \frac{4}{3} \left( 1 \cdot \frac{0.0015}{0.0032} \right)^{1.5} - 4 \cdot \frac{0.0032}{0.0032} \left( 1 - \frac{0.003}{0.0032} \right)^{1.5} - 2 \cdot \frac{0.003}{0.0032} \left( 0.003 - 0.0015 \right) = 0.0017.$$  

It is easy to determine that without taking into account the lateral displacement of the wheel helical surface relative to the workpiece, $h_y = h_h$, $\lambda_m = \frac{2}{3} h_h$, $R_a = 0.257 h_h$. For the conditions of this example $R_a = 0.257 \cdot 0.0032 = 0.00082$.

**Conclusions.** Thus, the paper shows the influence of the straightening conditions of the grinding wheel on the workpiece surface roughness. The level of shaping of the vertices of the grains depends on the values of the axial feed of the diamond during straightening $S_{ax}$, the corner radius $\rho$, the diameter of the wheel $D$ and the workpiece $d$, and their movement speeds $V_p$ and $V_d$. That is, the level of shaping corresponds to the distance from the bond level to the vertices of the grains chipped during straightening. With a decrease in the value of the axial feed of the diamond during straightening $S_{ax}$ and an increase in the number of passes of the straightening tool, the grain vertices will be closer to the bond. Under these conditions, lower stresses arise in the grains and the bond, the grinding wheel wakes up to work as a wheel with a more solid structure. Therefore, the straightening conditions depend on the resistance of the grinding wheel and the process of its self-sharpening in the machining zone. This determines the significant role of straightening in the grinding results.

**References.**

1. Kuzin, V. V., Fedorov, S. Y., & Grigor’ev, S. N. (2017). Correlation of Diamond Grinding Regime with Surface Condition of Ceramic Based on Zirconium Dioxide. *Refractories and Industrial Ceramics*, 57(6), 625-630. https://doi.org/10.1007/s11418-017-0033-8.

2. Tu, L., Li, J., & Shi, W. (2020). Investigation on experiment and simulation of the grinding process of cast iron. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 234(13), 2653-2661. https://doi.org/10.1177/0954406220907928.

3. Patel, A., Bauer, R. J., & Warkentin, A. (2019). Investigation of the effect of speed ratio on workpiece surface topography and grinding power in cylindrical plunge grinding using grooved and non-grooved grinding wheels. *The International Journal of Advanced Manufacturing Technology*, 105(7), 2977-2987. https://doi.org/10.1007/s00170-019-04406-6.

4. Hung, L. X., Hong, T. T., Ky, L. H., Tuan, N. Q., Tung, L. A., Long, B. T., & Pi, V. N. (2019). A Study On Calculation Of Optimum Exchanged Grinding Wheel Diameter When Internal Grinding. *Materials Today: Proceedings*, 18, 2840-2847. https://doi.org/10.1016/j.matpr.2019.07.151.

5. Shi, X., & Xiu, S. (2020). Study on the hardness model of grinding for structural steel. *The International Journal of Advanced Manufacturing Technology*, 106(7), 3563-3573. https://doi.org/10.1007/s00170-019-04787-8.

6. Suya Prem Anand, P., Arunachalam, N., & Vijayaraghavan, L. (2018). Effect of grinding on subsurface modifications of pre-sintered zirconia under different cooling and lubrication conditions. *Journal of the Mechanical Behavior of Biomedical Materials*, 86, 122-130. https://doi.org/10.1016/j.jmbbm.2018.06.026.

7. Strelchuk, R., Trokhymchuk, S., Sofronova, M., & Ospova, T. (2020). Revealing patterns in the wear of profile diamond wheels. *Eastern-European Journal of Enterprise Technologies*, 3(1(105)), 30-37. https://doi.org/10.15587/1729-4061.2020.203685.

8. He, Q., Xie, J., Lu, K., & Yang, H. (2020). Study on in-air electro-contact discharge (ECD) truncating of coarse diamond grinding wheel for the dry smooth grinding of hardened steel. *Journal of Materials Processing Technology*, 276, 116402. https://doi.org/10.1016/j.jmatprotec.2019.116402.

9. Denkova, B., Grove, T., & Sutharakumaran, V. (2020). New profiling approach with geometrically defined cutting edges for sintered metal bonded CBN grinding layers. *Journal of Materials Processing Technology*, 278, 116473. https://doi.org/10.1016/j.jmatprotec.2019.116473.

10. Zhou, K., Ding, H., Wang, K., Yang, J., Guo, J., Liu, Q., & Wang, W. (2020). Experimental investigation on material removal mechanism during rail grinding at different forward speeds. *Tribology International*, 143, 106040. https://doi.org/10.1016/j.triboint.2019.106040.

11. Guo, Y., Liu, M., & Li, C. (2020). Modeling and experimental investigation on grinding force for advanced ceramics with different removal modes. *The International Journal of Advanced Manufacturing Technology*, 106(11), 5483-5495. https://doi.org/10.1007/s00170-020-05013-6.

12. Kalchenko, V. V., Yeroshenko, A. M., Boyko, S. V., & Ignatenko, P. L. (2019). Determination of instantaneous temperature in the cutting zone during abrasive processing. *Naukovi Visnyk Natsionalnoho Hirnychoho Universytetu*, (5), 35-40. https://doi.org/10.29207/nvuo/2019/1/4.


13. Li, C. L., Peng, Y., Chen, D. L., & Pan, T. L. (2018). Theoretical investigation of vertical elliptic vibration-assisted grinding (EVAG) technology. The International Journal of Advanced Manufacturing Technology, 94(5), 2315-2324. https://doi.org/10.1007/s00170-017-0989-3.

14. Kalchenko, V. V., Yeroshenko, A. M., & Boyko, S. V. (2017). Mathematical modeling of abrasive grinding working process, Naukovi Visnyk Natsionalnoho Hirnychoho Universytetu, (6), 76-82.

Математичне моделювання шорсткості поверхні шліфувального круга при правці

П. М. Стрельчук, С. М. Трохимчук
Українська інженерно-педагогічна академія, м. Харків, Україна, e-mail: r.m.strelchuk@gmail.com

Мета. Дослідження механізму впливу умов правки шліфувального круга, у тому числі відносів коливань круга та алмазно-металевого олівця на шорсткість шліфованої поверхні та інші результати обробки.

Методика. Правка шліфувального круга алмазно-металевим олівцем виявляє собою процес високошвидкісного руйнування твердого абразивного матеріалу та його зв’язки під дією миттєвих сил, абразивних зерен із твердою поверхнею кристала алмаза. При правці шліфувального кругу сумарна складова нормальних сил викликає, відповідно, менші пружні деформації в системі «круг — правлячий інструмент», що підвищує точність геометричної форми робочої поверхні шліфувального круга.

Результати. Результати дослідження дозволяють визначити параметри шорсткості поверхні деталі та знати способи управління ними з метою підвищення ефективності процесу шліфування.

Наукова новизна. Встановлені закономірності впливу умов правки шліфувального круга на стан його робочої поверхні. У роботі показано, що вихідне розташування зерен по формі та орієнтації має вплив на їхні властивості й шорсткість робочої поверхні. Порівняння результатів з роботами інших авторів підтвердило ефективність використання математичного моделювання.

Практична значимість. Применение результатов исследований, полученных в работе, а именно математическое моделирование шероховатости поверхности шлифовального круга при правке, предоставляет возможность рассчитать параметр шероховатости шлифованной поверхности. В работе показано также, что от условий правки шлифовального круга, в частности — от величины припуска и свойств правящего инструмента, зависит устойчивость шлифовального круга в процессе шлифования, а также результаты процесса шлифования самой детали. Получены прямые зависимости между параметрами шероховатости и упругими характеристиками системы «круг — правящий инструмент», что повышает точность геометрической формы рабочей поверхности шлифовального круга.

Результаты. Результаты исследования позволяют определить параметры шероховатости поверхности детали и найти способы управления ими с целью повышения эффективности процесса шлифования.

Научная новизна. Установлены закономерности влияния условий правки шлифовального круга на состояние его рабочей поверхности. Научное значение работы заключается в том, что проведенное исследование позволяет рассчитать параметры шероховатости поверхности детали и найти способы управления ими с целью повышения эффективности процесса шлифования.

Научная новизна. Установлены закономерности влияния условий правки шлифовального круга на состояние его рабочей поверхности. Научное значение работы заключается в том, что проведенное исследование позволяет рассчитать параметры шероховатости поверхности детали и найти способы управления ими с целью повышения эффективности процесса шлифования.

Цель. Исследование механизма влияния условий правки шлифовального круга, в том числе относительных колебаний круга и алмазно-металлического каната на шероховатость шлифованной поверхности и другие результаты обработки.

Методика. Правка шлифовального круга алмазно-металлическим карандашом представляет собой процесс высокоскоростного разрушения твердого абразивного материала и его связки под действием мгновенных сил, абразивных зерен с твердой поверхностью кристалла алмаза. При правке шлифовального круга суммарная соотношение нормальных сил вызывает соответственно менее упругие деформации в системе «круг — правящий инструмент», что повышает точность геометрической формы рабочей поверхности шлифовального круга.

Результаты. Результаты исследования позволяют определить параметры шероховатости поверхности детали и найти способы управления ею с целью повышения эффективности процесса шлифования.

Научная новизна. Установлены закономерности влияния условий правки шлифовального круга на состояние его рабочей поверхности. Научное значение работы заключается в том, что проведенное исследование позволяет рассчитать параметры шероховатости поверхности детали и найти способы управления ими с целью повышения эффективности процесса шлифования.

Практическая значимость. Применение результатов исследований, полученных в работе, а именно математическое моделирование шероховатости поверхности шлифовального круга при правке, предоставляет возможность рассчитать параметр шероховатости шлифованной поверхности. В работе показано также, что от условий правки шлифовального круга, в частности — от величины припуска и свойств правящего инструмента, зависит устойчивость шлифовального круга в процессе шлифования, а также результаты процесса шлифования самой детали. Получены прямые зависимости между параметрами шероховатости и упругими характеристиками системы «круг — правящий инструмент», что повышает точность геометрической формы рабочей поверхности шлифовального круга.

Ключевые слова: отклонение профиля, условия правки, шлифовальный круг

Recommended for publication by O. A. Permyakov, Doctor of Technical Sciences. The manuscript was submitted 22.08.20.