Sigma meson and lowest possible glueball candidate in an extended linear \(\sigma\) model

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Abstract. We formulate an extended linear \(\sigma\) model of a quarkonia nonet and a tetraquark nonet as well as a complex iso-singlet (glueball) field to study the low-lying scalar meson. Chiral symmetry and \(U_A(1)\) symmetry and their breaking play important role to shape the scalar meson spectrum in our work. Based on our study we will comment on what may be the mass of the lowest possible scalar and pseudoscalar glueball states. We will also discuss on what may be the nature of the sigma or \(f_0(600)\) meson.

Keywords: Linear Sigma model, light scalar meson, quarkonium, tetraquark, glueball

PACS: 12.39.Fe,13.75.Lb,14.40.Be

INTRODUCTION

Theoretical understanding of the so-called Higgs Boson of QCD namely the \(\sigma\) or \(f_0(600)\) meson is important as they play an important role in chiral symmetry breaking and can be used as a probe for QCD vacuum. While the confirmation of its existence from \(\pi\pi\) scattering process puts an end to the decades-long controversy [1, 2], the agreement on the nature of \(\sigma/f_0(600)\) however has not yet been achieved. The tetraquark model [3] supports \(\sigma/f_0(600)\) to be a tetraquark state. Whereas, recent data from \(\pi\pi\) and \(\gamma\gamma\) scattering [4, 5] suggest it has a sizable fraction of glueball and another study from K-matrix analysis [6] suggested that \(f_0(600)/\sigma\) should be a glueball dominant state.

On the other hand the agreement on the lowest possible iso-scalar and pseudoscalar glueball states has neither been achieved. The result from Lattice simulations suggests \(f_0(1500)\) or \(f_0(1700)\) could be a glueball rich iso-scalar, while \(\eta(1489)\) can be a glueball rich pseudoscalar [7].

The great success of chiral symmetry in understanding the nature of lightest pseudoscalars motivates us to use an extended version of linear \(\sigma\) models [8, 9] to study the nature of these low-lying isoscalars and pseudoscalars. The motivation for constructing an extended linear sigma model consisting of effective quarkonia, tetraquark and glueball fields comes from the physical considerations that scalar condensates are allowed by the QCD vacuum. So in principle apart from quarkonia and tetraquark condensates, scalar glueball condensate should also be present in the QCD vacuum and need to be considered while studying the vacuum excitation. The mixing pattern followed in our framework comes from the consideration that mesons having identical external quan-
tum numbers can mix even if they have different internal flavour structures. Based on these premises we attempt to study the nature of $\sigma/f_0(600)$ meson as well as lowest isoscalar and pseudoscalar glueball candidate. In the following section we briefly review the model Lagrangian and the symmetry breaking pattern. At last we will present our results and conclude.

**MODEL**

The extended linear $\sigma$ model can be systematically formulated under the symmetry group $SU_R(3) \times SU_L(3) \times U_A(1)$. Three types of chiral fields are included: a $3 \times 3$ matrix field $\Phi$ which denotes the quarkonia states, a $3 \times 3$ matrix field $\Phi'$ which denotes the tetraquark state, and a complex chiral singlet field $Y$ which denotes the pure glueball states.

Up to the mass dimension $O(p^4)$ (we assume that they are the most important operators to determine the nature of light scalars of ground states), the Lagrangian of our model can include two parts: the symmetry invariant part $\mathcal{L}_S$ and the symmetry breaking one $\mathcal{L}_{SB}$. The symmetry invariant part includes those terms which respect $SU(3)_L \times SU(3)_R$ symmetry as well as $U_A(1)$ symmetry:

$$\mathcal{L}_S = Tr(\partial_\mu \Phi \partial^\mu \Phi^\dagger) + Tr(\partial_\mu \Phi' \partial^\mu \Phi'^\dagger) + \partial_\mu Y \partial^\mu Y^* - m_\Phi^2 Tr(\Phi^\dagger \Phi) - m_{\Phi'}^2 Tr(\Phi'^\dagger \Phi') - \mu Y^2 - \lambda_1 Tr(\Phi^\dagger \Phi \Phi^\dagger \Phi) - \lambda_2 Tr(\Phi'^\dagger \Phi \Phi'^\dagger \Phi') - \lambda_3 (Y Y^*)^2 - \lambda_4 \epsilon_{abc} e^{def} \Phi_d a \Phi_f b + h.c. + [k Y Det(\Phi) + h.c.],$$

(1)

While the symmetry breaking part includes the following terms

$$\mathcal{L}_{SB} = [Tr(B, \Phi) + h.c.] + [Tr(B', \Phi') + h.c.] + (D, Y + h.c.) - [\lambda_m Tr(\Phi \Phi'^\dagger) + h.c.].$$

(2)

To construct the symmetric terms in the Lagrangian we closely follow [10] where the choice of terms are limited by the number of internal quark plus antiquark lines at the effective vertex, which is set to 8. This condition is relaxed for two terms with coupling constants $\lambda_1'$ and $\lambda_2$ and the reason to include these two terms stems from the practical consideration of making sure our potential is bounded. For detailed discussion on the terms appearing in the Lagrangian please refer to [11]. As evident from the Lagrangian both chiral symmetry $SU(3)_L \times SU(3)_R$ and $U(1)_A$ symmetry are explicitly broken by the terms in $\mathcal{L}_{SB}$. The $3 \times 3$ matrices $B$ and $B'$ responsible for the breaking of the symmetry can be parametrized as: $B(B') = T_a b_a(T_a b_a)'$ ($T_a$ are the generators of $U(3)$ with $T_0 = \sqrt{\frac{1}{6} I}$).

Since the vacuum expectation values of the quarkonia and tetraquark fields can carry those quantum numbers which are allowed by the QCD vacuum, only $a = (0, 3, 8)$ fields are allowed. The choice of this external fields $b_a(b_a')$ control the nature and extent of the symmetry breaking. Out of various possible symmetry breaking scenarios, we consider the following case in our present study where:

- $b_0(b_0') \neq 0$, $b_3(b_3') = 0$ and $b_8(b_8') \neq 0$. In this case, $U(3)_A$ and $SU(3)_V$ both are explicitly broken and $SU(3)_V$ is broken to $SU(2)_V$. As a result $m_u = m_d \neq m_s$, 
TABLE 1. The categorization of scalar and pseudo-scalar states in term of isospin quantum number are demonstrated. States in the same category can mix with each other.

| Isospin | \( I = 1 \) | \( I = \frac{1}{2} \) | \( I = 0 \) |
|---------|-----------|----------------|-----------|
| PseudoScalars(P=-1) | \{ \pi, \pi' \} | \{ \eta_1, \eta_2, \eta_3, \eta_4, \eta_5 \} | \{ K, K' \}, \{ K^*, K^{*'} \} |
| Scalars(P=1)      | \{ a, a' \}, \{ \kappa, \kappa' \}, \{ \kappa^*, \kappa'^{\prime} \} | \{ f_1, f_2, f_3, f_4, f_5 \} |

where \( m_i \) is the quark mass of the \( i \)th flavour.

This is reasonable considering the up and down quark masses are nearly equal to each other and thereby indicating \( SU(2)_\Omega \) is a good (approximate) symmetry. The remnant \( SU(2)_\Omega \) isospin symmetry allows us to represent two condensates for quarkonia and tetraquark fields each as: \( v_0, v_8 \) and \( v'_0, v'_8 \) respectively. While the gluonic condensate in our theory is labeled as \( v_\gamma \).

RESULTS AND CONCLUSION

Due to the unbroken \( SU_\Omega(2) \) isospin symmetry, physical scalar and pseudo-scalar states can be categorized into three groups with isospin quantum numbers as \( I = 1 \) (triplet), \( \frac{1}{2} \) (doublet) and 0, respectively. Only bare quarkonia, tetraquark and glueball fields with the same isospin quantum number can mix with each other to form physical states. Moreover, there is no mixing between scalar and pseudoscalar fields. Thus the chiral singlet glueball field can only mix with the isosinglet singlets of quarkonia and tetraquark fields. Using these facts, the physical states below 2 GeV can be tabulated as given in Table 1, where the isodoublet \( \{ K, K' \} \) is connected with the isodoublet \( \{ K^*, K^{*'} \} \) by charge conjugation. And a similar relation holds for \( \{ \kappa, \kappa' \} \) and \( \{ \kappa^*, \kappa'^{\prime} \} \).

There are 15 parameters in our model. To solve these parameters we treat the tetraquark vacuum condensates as well as the mixing angles for isotriplet \( (\theta_\pi) \) and isodoublet \( (\theta_k) \) fields as input parameters. Then from the mass matrices of \( a, a', \kappa \) and \( \kappa' \), parameters related to isotriplet and isodoublet sectors are solved. The symmetry parameters \( \{ b_0, b_8, b'_0, b'_8, D \} \) are solved from the vacuum stability conditions. Whereas, to solve for the parameters related to glueball sector, we use following two conditions:

\[
\text{Tr}[M_{\eta^2}]_{\text{Model}} = \text{Tr}[M_{\eta^2}]_{\text{Exp}}, \quad \text{Det}[M_{\eta^2}]_{\text{Model}} = \text{Det}[M_{\eta^2}]_{\text{Exp}}. \tag{3}
\]

At the end we are left with one free parameter which is bare glueball mass \( m_Y \) and is used in our study as a scanning parameter.

For space constraint we refer to [11] for details on parameter fixing and method to choose the best fit solution. In choosing the best fit solution we vary the \( \pi' \) mass from \( 1.2 - 1.4 \) GeV and choose those solution which give the tree level decay width for \( f_0(600) \to \pi \pi \) between \( 0.35 - 0.9 \) GeV.

Our best fit parameter set is presented in Table (2). We would like to highlight a few features out from it. 1) In absence of the explicit symmetry breaking terms, it is the negative mass parameter \( m_q^2 \) that would trigger the spontaneous chiral symmetry breaking. 2) The sign of \( v_\gamma \) is correlated with the sign of \( k \), and the sign of \( k \) is determined
FIGURE 1. (Left figure) The dependence of $\lambda_Y$ upon $m_Y^2$ is demonstrated. A solid circle marker shows the point $\lambda_Y = 0$, which corresponds to $m_Y^2 = 3.452$ ($m_Y = 1.858$). (Right figure) The dependence of mass of $f_0$ upon $m_Y^2$ is demonstrated. A vertical line with $m_Y^2$ is drawn to read out the lowest mass $m_{f_0} = 1.86$.

TABLE 2. The values of parameters in our fit are shown where the best value of $m_{\pi'}$ is found to be $m_{\pi'} = 1.2$ GeV.

| Parameter | Value | Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|-----------|-------|
| $\theta_\pi$ (radian) | -0.604 | $\lambda_1'$ | 8.248 | $m_{\Phi^2}$ (GeV$^2$) | -0.025 |
| $\theta_K$ (radian) | -0.714 | $\lambda_2$ | 76.428 | $b_0^1$ (GeV$^3$) | 0.166 |
| $v_0$ (GeV) | 0.074 | $\lambda_3$ (GeV) | -0.738 | $m_{\Phi^2}$ (GeV$^2$) | 0.744 |
| $v_8$ (GeV) | -0.115 | $\lambda_Y$ | 38.327 | $b_5^1$ (GeV$^3$) | 0.18 |
| $v_0'$ (GeV) | 0.203 | $k$ | -78.15 | $\lambda_1$ | 35.465 |
| $v_8'$ (GeV) | 0.126 | $\lambda_m$ (GeV$^2$) | -1.044 | D (GeV$^3$) | -0.265 |
| $v_Y$ (GeV) | -0.109 | $b_0$ (GeV$^3$) | -0.085 | $m_Y^2$ (GeV$^2$) | 3.0 |
| $m_Y^2$ (GeV$^2$) | 3.0 | $b_8$ (GeV$^3$) | -0.161 |

from the mass spectra of pseudoscalar sector. 3) The couplings $\lambda_1$, $\lambda_1'$, $\lambda_Y$ are positive which guarantee the potential is bounded from below. 4) The values of $\lambda_1$, $\lambda_2$, and $\lambda_Y$ as well as $k$ are large, which demonstrate the non-perturbative nature of the model.

It is found that the condition $\lambda_Y > 0$ can predict the lightest glueball scalar should be around 2.0 GeV or so, as can be read off from Fig. (1b), while the lightest glueball pseudo scalar should be $\eta_5$. The mass splitting between these two glueball states is controlled by parameters $v_Y$ and $\lambda_Y$ and is found to be around 0.15 GeV. When compared with the Lattice QCD prediction for the glueball bare mass reported in [12] where the mass is 1.611 GeV, our result $m_Y = 1.73$ GeV is slightly heavier than this prediction. When $m_Y = 1.611$ GeV is taken, then the predicted mass of the lightest glueball is $m_{f_0} = 2.29$ GeV. 3) The lightest scalar $f_0^0(600)$ is found to be 0.27 GeV or so and is a quarkonia dominant state.

In Figure 1, we demonstrate the dependence of $\lambda_Y$ and $f_0^0$ masses upon the free parameter $m_Y^2$ with the rest of parameters are given in Table (2). As shown in Fig. (1a), when $m_Y^2$ is larger than 3.4 GeV$^2$, the $\lambda_Y$ becomes negative. Then the potential of our model has to confront with the problem of unbounded vacuum from below. In the allowed values of $m_Y^2$, the masses of $f_i^0, i = 1, 2, 3, 4$ are almost independent of its value,
TABLE 3. Mass spectra and components for the pseudo-scalar and scalar mesons based on our fit are shown where the best value of \( m' \) is found to be \( m' = 1.2 \text{ GeV} \).

| Meson | Our Value (GeV) | quarkonia (%) | tetraquark (%) | glueball (%) | Experimental Value (GeV) |
|-------|----------------|--------------|---------------|--------------|-------------------------|
| \( \eta_5 \) | 1.858 | 0.037 | 0.001 | 99.962 | 1.756 ± 0.009 |
| \( \eta_4 \) | 1.380 | 75.803 | 24.167 | 0.03 | 1.476 ± 0.004 |
| \( \eta_3 \) | 1.291 | 26.700 | 73.294 | 0.006 | 1.294 ± 0.004 |
| \( \eta_2 \) | 0.907 | 15.852 | 84.145 | 0.003 | 0.958 ± 24 × 10^{-5} |
| \( \eta_1 \) | 0.595 | 81.607 | 18.393 | 0.0 | 0.548 ± 24 × 10^{-6} |
| \( f_5^0 \) | 2.09 | 0.01 | 0.0 | 99.99 | - |
| \( f_4^0 \) | 1.487 | 77.469 | 22.53 | 0.001 | 1.505 ± 0.006 |
| \( f_3^0 \) | 1.347 | 22.177 | 77.82 | 0.003 | 1.2 - 1.5 |
| \( f_2^0 \) | 1.124 | 21.561 | 78.439 | 0.0 | 0.980 ± 0.010 |
| \( f_1^0 \) | 0.274 | 78.784 | 21.211 | 0.005 | 0.4 - 1.2 |

as demonstrated in Fig. (1b). The upper bound of \( m_f^2 \) is determined from the condition \( \Gamma_{f_1^0} > 0.35 \text{ GeV} \).

To conclude, our model predicts that the isoscalar glueball should be heavier than 2.0 GeV when the pseudoscalar \( \eta(1726) \) is the best glueball candidates. The lowest isoscalar \( f_0(600) \) is found to be quarkonia dominant state with a considerable tetraquark component.

ACKNOWLEDGMENTS

This work is supported by the NSFC under Grant Nos. 11175251, 11250110058 and 11275213, DFG and NSFC (CRC 110), CAS key project KJCX2-EW-N01, CAS 2011Y2JB05, K.C.Wong Education Foundation, and Youth Innovation Promotion Association of CAS.

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