Symmetries of Sasakian Generalized Sasakian-Space-Form Admitting Generalized Tanaka-Webster Connection

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Abstract. The object of this paper is to study certain symmetric properties of Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection. We studied semi-symmetry and Ricci semi-symmetry of Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection. Further we obtain results for Ricci pseudosymmetric and Ricci-generalized pseudosymmetric Sasakian generalized Sasakian-space-form.

1 Introduction

In a Riemannian manifold, a curvature tensor given by $K(X, Y) = R(X, Y, Y, X)$ for an orthonormal pair of vectors $(X, Y)$, is known as the sectional curvature. A Riemannian manifold with constant sectional curvature $c$ is called a real-space-form, and its curvature tensor $R$ satisfies

$$R(X, Y)Z = c\{g(Y, Z)X - g(X, Z)Y\}.$$ |

A Sasakian manifold with constant $\phi$-sectional curvature $c$ is called a Sasakian-space-form and its curvature tensor $R$ is given by

$$R(X, Y)Z = \frac{c + 3}{4}[g(Y, Z)X - g(X, Z)Y] + \frac{c - 1}{4}[g(X, \phi Z)\phi Y - g(Y, \phi Z)\phi X + 2g(X, \phi Y)\phi Z] + \frac{c - 1}{4}[\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X] + g(X, Z)\eta(Y)\xi - g(Y, Z)\eta(X)\xi.$$ (1.1)
In 2004, Alegre et al. [2] generalized the Sasakian-space-form by replacing the constant quantities $\frac{c+3}{4}$ and $\frac{c-1}{4}$ with differentiable functions. Such space is called generalized Sasakian-space-form.

The generalized Sasakian-space-form have been studied by many authors such as Sarkar and De ([17, 10, 11]), Singh ([18, 19]), De and Majhi ([8, 9, 12]), Kishor et al. [15], Alegre and Carriazo [3, 4], Akbar and Sarkar [1], Sular and Ozgur [20, 21] and many others.

In 2008, Alegre and Carriazo studied structures on generalized Sasakian-space-form [4] and studied generalized Sasakian-space-form admitting trans-Sasakian structure. In this paper we studied generalized Sasakian-space-form admitting Sasakian structure and we called such manifold as Sasakian generalized Sasakian-space-form.

In 1989, Tanno [23] defined the generalized Tanaka-Webster connection for contact metric manifolds, which generalized the connection given by Tanaka [22] and Webster [24]. The generalized Tanaka-Webster connection have been studied by De [7], de Dios Pérez [16] and others.

A manifold is said to be semi-symmetric and Ricci semi-symmetric [26, 27] if the Riemannian curvature tensor $R$ and Ricci tensor $S$ satisfies $R.R = 0$ and $R.S = 0$ respectively. That is

$$R(X, Y).R(U, V)W = 0$$  \hspace{1cm} (1.2)

and

$$R(X, Y).S(U, V) = 0$$  \hspace{1cm} (1.3)

for all $X, Y, U, V, W \in \chi(M)$.

There are two notions of pseudosymmetric manifolds which are defined by Chaki in 1987 [6] and Deszcz in 1992 [13]. Throughout the paper we consider pseudosymmetric manifolds defined by Deszcz. An $n$-dimensional Riemannian manifold $M, n > 2$, is called pseudosymmetric manifolds if $R.R$ and $Q(g, R)$ are are linearly dependent, i.e.,

$$R.R = FQ(g, R),$$  \hspace{1cm} (1.4)

holds on the set $U_R = \{x \in M : Q(g, R) \neq 0 \text{ at } x\}$, where $F$ is some function on $U_R$.

And the manifold is called Ricci pseudosymmetric and Ricci-generalized pseudosymmetric manifold if

$$R.S = f'Q(g, S)$$  \hspace{1cm} (1.5)

and

$$R.R = fQ(S, R)$$  \hspace{1cm} (1.6)
holds on the set \( U_S = \{ x \in M : Q(g, S) \neq 0 \text{ at } x \} \) and \( U_R = \{ x \in M : Q(g, R) \neq 0 \text{ at } x \} \) respectively, where \( f' \) and \( f \) are some function on \( U_S \) and \( U_R \).

In this paper we studied symmetries of Sasakian generalized Sasakian-space-form admitting generalized Tanaka-Webster connection. After introduction in preliminaries section, we showed some known relation in Sasakian manifold and generalized Sasakian-space-form. In the third section, we have given the expression for curvature tensor with respect to genaralized Tanaka-Webster connection in generalized Sasakian-space-form. The next section is dedicated for the study of semi-symmetry and Ricci semi-symmetry. In the last two sections we studied Ricci pseudosymmetric and Ricci-generalized pseudosymmetric manifolds.

2 Preliminaries

An \( n \)-dimensional smooth manifold \( M \) is said to be an almost contact metric manifold if it admits an almost contact metric structure \( (\phi, \xi, \eta, g) \) consisting of a tensor field \( \phi \) of type \( (1, 1) \), a vector field \( \xi \), a 1-form \( \eta \) and a Riemannian metric \( g \) satisfying [5]

\[
\phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \phi \xi = 0, \quad \eta \circ \phi = 0,
\]

and

\[
g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y). \tag{2.1}\]

On such a manifold, the fundamental \( \Phi \) of \( M \) is defined as

\[
\Phi(X, Y) = g(\phi X, Y), \quad X, Y \in \Gamma(TM).
\]

An almost contact metric manifold is called a Sasakian manifold if and only if [25]

\[
(\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X, \quad \nabla_X \xi = -\phi X. \tag{2.2}\]

On a Sasakian manifold \( M \), the following relations are held [25]

\[
R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.3}
\]

\[
R(X, \xi)Y = \eta(Y)X - g(X, Y)\xi, \tag{2.4}
\]

\[
\eta(R(X, Y)Z) = \eta(X)g(Y, Z) - \eta(Y)g(X, Z), \tag{2.5}
\]

\[
\eta(R(X, Y)\xi) = 0. \tag{2.6}
\]
\[ S(X, \xi) = (n - 1)\eta(X), \]  
\[ Q\xi = (n - 1)\xi, \]  
\[ (\nabla_X \eta) Y = g(X, \phi Y). \]  

In a generalized Sasakian-space-form the following properties holds \[2\]

\[ R(X, Y)Z = f_1 [g(Y, Z)X - g(X, Z)Y] \]
\[ + f_2 [g(X, \phi Z)\phi Y - g(Y, \phi Z)\phi X + 2g(X, \phi Y)\phi Z] \]
\[ + f_3 [\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X, Z)\eta(Y)\xi] \]
\[ - g(Y, Z)\eta(X)\xi, \]  
\[ S(X, Y) = [(n - 1)f_1 + 3f_2 - f_3]g(X, Y) \]
\[ - [3f_2 + (n - 2)f_3]\eta(X)\eta(Y), \]  
\[ QX = [(n - 1)f_1 + 3f_2 - f_3]X - [3f_2 + (n - 2)f_3]\eta(X)\xi, \]  
\[ S(X, \xi) = (n - 1)(f_1 - f_3)\eta(X), \]  
\[ Q\xi = (n - 1)(f_1 - f_3)\xi, \]  
\[ R(X, Y)\xi = (f_1 - f_3)\{\eta(Y)X - \eta(X)Y\}, \]  
\[ R(\xi, Y)Z = (f_1 - f_3)\{g(Y, Z)\xi - \eta(Z)Y\}, \]  
\[ R(\xi, Y)\xi = (f_1 - f_3)\{\eta(Y)\xi - Y\}. \]  
\[ r = n(n - 1)f_1 + 3(n - 1)f_2 - 2(n - 1)f_3, \]  
where \[ r = \sum_{i=1}^{n} S(e_i, e_i) \] is the scalar curvature.
3 Generalized Tanaka-Webster connection

Tanno [23], defined the generalized Tanaka-Webster connection \( \tilde{\nabla} \) for contact metric manifolds by

\[
\tilde{\nabla}_X Y = \nabla_X Y + (\nabla_X \eta)(Y)\xi - \eta(Y)\nabla_X \xi - \eta(X)\phi(Y)
\]  

(3.1)

for all \( X, Y \in \chi M \), and \( \nabla \) is the Riemannian connection.

Let \( R \) and \( \tilde{R} \) denotes the Riemannian curvature tensors of Sasakian manifold with respect to \( \nabla \) and \( \tilde{\nabla} \) respectively. A relation between \( R \) and \( \tilde{R} \) is given by [7]

\[
\tilde{R}(X, Y)Z = R(X, Y)Z + \left[ g(X, Z)\eta(Y) - g(Y, Z)\eta(X) \right] \xi \\
+ g(Y, \phi Z)\phi X + g(X, \phi Z)\phi Y + 2g(Y, \phi X)\phi Z \\
- \eta(Y)\eta(Z)X + \eta(X)\eta(Z)Y.
\]  

(3.2)

Contracting (3.2) we obtain

\[
\tilde{S}(Y, Z) = S(Y, Z) - g(Y, Z) - (n - 3)\eta(X)\eta(Y).
\]  

(3.3)

Using (2.10) and (2.11) in the above equations we have

\[
\tilde{R}(X, Y)Z = (f_1 - 1)\left[ g(Y, Z)X - g(X, Z)Y \right] \\
+ f_2\left[ g(X, \phi Z)\phi Y - g(Y, \phi Z)\phi X + 2g(X, \phi Y)\phi Z \right] \\
+ f_3\left[ \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X, Z)\eta(Y)\xi \\
- g(Y, Z)\eta(X)\xi - g(Y, \phi Z)\phi X + g(X, \phi Z)\phi Y \\
+ 2g(Y, \phi X)\phi Z - \eta(Y)\eta(Z)X + \eta(X)\eta(Z)Y, \right.
\]  

(3.4)

and

\[
\tilde{S}(Y, Z) = \left[ (n - 1)f_1 + 3f_2 - f_3 \right] g(X, Y) \\
- \left[ 3f_2 + (n - 2)f_3 \right] \eta(X)\eta(Y) \\
- g(Y, Z) - (n - 3)\eta(X)\eta(Y).
\]  

(3.5)

Now we have

\[
\tilde{R}(X, Y)\xi = (f_1 - f_3 - 1)\left\{ \eta(Y)X - \eta(X)Y \right\},
\]  

(3.6)

\[
\tilde{R}(\xi, X)Y = (f_1 - f_3 - 1)\left\{ g(Y, Z)\xi - \eta(Z)Y \right\},
\]  

(3.7)

\[
\tilde{R}(\xi, X)\xi = (f_1 - f_3)\left\{ \eta(Y)\xi - Y \right\},
\]  

(3.8)

\[
\tilde{S}(X, \xi) = (n - 1)(f_1 - f_3 - 1)\eta(X),
\]  

(3.9)

\[
\tilde{S}(\xi, \xi) = (n - 1)(f_1 - f_3 - 1).
\]  

(3.10)


4 Semi-symmetric and Ricci semi-symmetric

Suppose that the Sasakian generalized Sasakian-space-form is semi-symmetric with respect to generalized Tanaka-Webster connection, then from (1.2) we get

$$\tilde{R}(X, Y) \tilde{R}(U, V)W = 0.$$  \hspace{1cm} (4.1)

It is well known that

$$\tilde{R}(X, Y) \tilde{R}(U, V)W = \tilde{R}(Y, V)W - \tilde{R}(Y, U)W - \tilde{R}(X, Y)U, V)W.$$  \hspace{1cm} (4.2)

Now setting $X = U = \xi$ in (4.1) and using (4.2) we get

$$(f_1 - f_3 - 1)^2 \{g(Y, W)V - g(V, W)Y\} + (f_1 - f_3 - 1)\tilde{R}(Y, V)W = 0,$$

which can be written as

$$\tilde{R}(Y, V)W = (f_1 - f_3 - 1)\{g(V, W)Y - g(Y, W)V\},$$  \hspace{1cm} (4.3)

provided $f_1 - f_3 - 1 \neq 0$.

Thus we have

**Theorem 4.1.** In a semi-symmetric Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection, we have

$$\tilde{R}(Y, V)W = (f_1 - f_3 - 1)\{g(V, W)Y - g(Y, W)V\},$$

provided $f_1 - f_3 - 1 \neq 0$.

Now using (3.2) in (4.3) we get

$$R(Y, V)W = (f_1 - f_3 - 1)\{g(V, W)Y - g(Y, W)V\}$$

$$- \left[ g(Y, W)\eta(Y) - g(V, W)\eta(Y) \right] \xi + g(V, \phi W)\phi Y$$

$$- g(Y, \phi W)\phi V - 2g(V, \phi Y)\phi W$$

$$+ \eta(V)\eta(W)Y - \eta(Y)\eta(W)V,$$  \hspace{1cm} (4.4)

provided $f_1 - f_3 - 1 \neq 0$. Thus we can state that:

**Theorem 4.2.** In a semi-symmetric Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection the Riemannian curvature tensor is given by (4.4), provided $f_1 - f_3 - 1 \neq 0$. 

Again suppose that the Sasakian generalized Sasakian-space-form is Ricci semi-symmetric with respect to generalized Tanaka-Webster connection, then from (1.3) we get

\[ \tilde{R}(X,Y)\tilde{S}(U,V) = 0. \]  

(4.5)

It implies

\[ \tilde{S}(\tilde{R}(X,Y).U,V) + \tilde{S}(U,\tilde{R}(X,Y)V) = 0. \]  

(4.6)

Setting \( X = U = \xi \) in (4.6) we get

\[ (f_1 - f_3 - 1)\{(n - 1)(f_1 - f_3 - 1)g(Y,V) - S(Y,V)\} = 0. \]

Which implies

\[ S(Y,V) = (n - 1)(f_1 - f_3 - 1)g(Y,V), \]  

(4.7)

provided \( f_1 - f_3 - 1 \neq 0 \).

We have

**Theorem 4.3.** A semi-symmetric Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection is Einstein manifold provided \( f_1 - f_3 - 1 \neq 0 \).

5 Ricci-generalized pseudosymmetric manifold

Suppose that the Sasakian generalized Sasakian-space-form is Ricci-generalized pseudosymmetric with respect to generalized Tanaka-Webster connection, then from (1.6)

\[ \tilde{R}(X,Y)\tilde{R}(U,V)W = fQ(\tilde{S},\tilde{R})(U,V;W;X,Y). \]

This is equivalent to

\[ \tilde{R}(X,Y)\tilde{R}(U,V)W = f\{(X \wedge \tilde{S} Y)\tilde{S}(U,V)\}, \]  

(5.1)

where \( ((X \wedge \tilde{S} Y))Z = \tilde{S}(Y,Z)X - \tilde{S}(X,Z)Y \) for all \( X, Y, Z \).

Thus we get

\[
\begin{align*}
\tilde{R}(X,Y)\tilde{R}(U,V)W & = \tilde{R}(\tilde{R}(U,V)X,Y)W - \tilde{R}(X,\tilde{R}(U,V)Y)W \\
-\tilde{R}(X,Y)\tilde{R}(U,V)W & = f\{(X \wedge \tilde{S} Y)\tilde{R}(U,V)W - \tilde{R}((X \wedge \tilde{S} Y)U,V)W \\
& \quad - \tilde{R}(U,(X \wedge \tilde{S} Y)V)W - \tilde{R}(U,V)(X \wedge \tilde{S} Y)W\}. 
\end{align*}
\]
or

\[
\begin{align*}
\bar{R}(X,Y)\bar{R}(U,V)W &= \bar{R}(\bar{R}(U,V)X,Y)W - \bar{R}(X,\bar{R}(U,V)Y)W \\
-\bar{R}(X,Y)\bar{R}(U,V)W &= f\{\bar{S}(Y,\bar{R}(U,V)W)X - \bar{S}(X,\bar{R}(U,V)W)Y \\
&- \bar{S}(Y,U)\bar{R}(X,V)W + \bar{S}(X,U)\bar{R}(Y,V)W \\
&- \bar{S}(Y,W)\bar{R}(U,V)X + \bar{S}(X,W)\bar{R}(U,V)Y\}.
\end{align*}
\]

(5.2)

Setting \( X = U = \xi \) in (5.2) we get

\[
(f_1 - f_3 - 1)^2 \{ g(Y,W)V - g(V,W)Y \} + (f_1 - f_3 - 1)\bar{R}(Y,V)W \\
= f\left[ (n-1)(f_1 - f_3 - 1)\{ \bar{R}(Y,V)W - g(V,W)Y \} \\
+ g(Y,W)\eta(V)\xi + g(V,Y)\eta(W)\xi \right] \\
- \bar{S}(Y,V)\eta(W)\xi - \bar{S}(Y,W)\{ \eta(V)\xi - V \}.
\]

(5.3)

Again setting \( V = \xi \) in (5.3) we get

\[
f(f_1 - f_3 - 1)[g(Y,W)\xi - \eta(W)Y] \\
= f(f_1 - f_3 - 1)^2[g(Y,W)\xi - \eta(W)Y].
\]

(5.4)

We have either

\[
(f_1 - f_3 - 1) = 0, \tag{5.5}
\]

or

\[
(f_1 - f_3 - 1) = 1, \tag{5.6}
\]

provided \( f \neq 0 \).

Setting \( W = \xi \) in (5.3) and using (5.5) we get

\[
S(Y,V) = 0, \tag{5.7}
\]

for all \( Y, V \in \chi M \), provided \( f \neq 0 \) and \( f_1 - f_3 - 1 \neq 1 \).

Thus we have

**Theorem 5.1.** A Ricci-generalized pseudosymmetric Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection is Ricci flat provided \( f \neq 0 \) and \( f_1 - f_3 - 1 \neq 1 \).
Again setting $W = \xi$ in (5.3) and using (5.6) we get

$$S(Y, V) = (n - 1)g(V, Y),$$

(5.8)

for all $Y, V \in \chi M$, provided $f \neq 0$ and $f_1 - f_3 - 1 \neq 0$.

We have

**Theorem 5.2.** A Ricci-generalized pseudosymmetric Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection is Einstein manifold provided $f \neq 0$ and $f_1 - f_3 - 1 \neq 0$.

### 6 Ricci-pseudosymmetric manifold

Suppose that the Sasakian generalized Sasakian-space-form is Ricci-pseudosymmetric with respect to generalized Tanaka-Webster connection, then from (1.5)

$$\tilde{R}(X, Y).\tilde{S}(U, V) = f'Q(g, \tilde{R})(U, V; X, Y).$$

This is equivalent to

$$\tilde{R}(X, Y).\tilde{S}(U, V) = f'\{(X \wedge g Y).\tilde{S}(U, V)\},$$

(6.1)

where $(X \wedge g Y) Z = g(Y, Z) X - g(X, Z) Y$ for all $X, Y, Z$.

Thus we get

$$\tilde{S}(\tilde{R}(X, Y).U, V) + \tilde{S}(U, \tilde{R}(X, Y)V) = f'\{\tilde{S}((X \wedge g Y)U, V) + \tilde{S}(U, (X \wedge g Y)V)\}.$$  

or

$$\tilde{S}(\tilde{R}(X, Y).U, V) + \tilde{S}(U, \tilde{R}(X, Y)V) = f'\{g(Y, U)\tilde{S}(X, V) - g(X, U)\tilde{S}(Y, V) + g(Y, V)\tilde{S}(U, X) - g(X, V)\tilde{S}(U, Y)\}.$$  

(6.2)

Setting $X = U = \xi$ in (6.2) we get

$$(f_1 - f_3 - f' - 1)\{S(Y, V) - (n - 1)(f_1 - f_3 - 1)g(Y, V)\} = 0.$$  

Which implies

$$S(Y, V) = (n - 1)(f_1 - f_3 - 1)g(Y, V),$$

(6.3)

for all $Y, V \in \chi M$, provided $(f_1 - f_3 - f' - 1) \neq 0$.

We have
**Theorem 6.1.** A Ricci-pseudosymmetric Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection is Einstein manifold provided $f_1 - f_3 - f' - 1 \neq 0$.

Now using Theorem 4.2 of [4] and (6.3) we get the following corollary

**Corollary 6.2.** An $n$-dimensional connected Sasakian generalized Sasakian-space-form, ($n \geq 5$), which is Ricci-pseudosymmetric with respect to generalized Tanaka-Webster connection is Ricci flat provided $f' \neq 0$.

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