Extragalactic sources in Cosmic Microwave Background maps

G. De Zotti\textsuperscript{a,b,1} G. Castex,\textsuperscript{a} J. González-Nuevo,\textsuperscript{c} M. Lopez-Caniego,\textsuperscript{d} M. Negrello,\textsuperscript{b} Z.-Y. Cai,\textsuperscript{e} M. Clemens,\textsuperscript{b} J. Delabrouille,\textsuperscript{f} D. Herranz,\textsuperscript{g} L. Bonavera,\textsuperscript{g} J.-B. Melin,\textsuperscript{h} M. Tucci,\textsuperscript{i} S. Serjeant,\textsuperscript{l} M. Bilicki,\textsuperscript{m,n} P. Andreani,\textsuperscript{o} D.L. Clements,\textsuperscript{p} L. Toffolatti\textsuperscript{c,q} and B.F. Roukema\textsuperscript{r}

\textsuperscript{a}SISSA, via Bonomea 265, 34136 Trieste, Italy
\textsuperscript{b}INAF-Osservatorio Astronomico di Padova, vicolo dell’Osservatorio 5, I-35122 Padova, Italy
\textsuperscript{c}Departamento de Física, Universidad de Oviedo, C. Calvo Sotelo s/n, 33007 Oviedo, Spain
\textsuperscript{d}European Space Agency, ESAC, Planck Science Office, Camino bajo del Castillo, s/n, Urbanización Villafranca del Castillo, Villanueva de la Cañada, Madrid, Spain
\textsuperscript{e}CAS Key Laboratory for Research in Galaxies and Cosmology, Department of Astronomy, Astronomy and Space Science, University of Science and Technology of China, Hefei, Anhui 230026, China
\textsuperscript{f}APC, 10, rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13, France
\textsuperscript{g}Instituto de Física de Cantabria (CSIC-UC), avda. los Castros s/n, 39005 Santander, Spain
\textsuperscript{h}DSM/Irfu/SPP, CEA-Saclay, F-91191 Gif-sur-Yvette Cedex, France
\textsuperscript{i}Département de Physique Théorique and Center for Astroparticle Physics, Université de Genève, 24 quai Ansermet, CH-1211 Genève 4, Switzerland
\textsuperscript{j}Department of Physical Sciences, The Open University, Walton Hall, Milton Keynes MK7 6AA, U.K.
\textsuperscript{k}Astrophysics, Cosmology and Gravity Centre, Department of Astronomy, University of Cape Town, Private Bag X3, Rondebosch, South Africa
\textsuperscript{l}Kepler Institute of Astronomy, University of Zielona Góra, ul. prof. Z. Szafrana 2, 65-246 Zielona Góra, Poland
\textsuperscript{m}European Southern Observatory, Karl-Schwarzschild-Straße 2, D-85748, Garching, Germany

\textsuperscript{1}Corresponding author.

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Abstract. We discuss the potential of a next generation space-borne CMB experiment for studies of extragalactic sources with reference to COrE+, a project submitted to ESA in response to the call for a Medium-size mission (M4). We consider three possible options for the telescope size: 1 m, 1.5 m and 2 m (although the last option is probably impractical, given the M4 boundary conditions). The proposed instrument will be far more sensitive than Planck and will have a diffraction-limited angular resolution. These properties imply that even the 1 m telescope option will perform substantially better than Planck for studies of extragalactic sources. The source detection limits as a function of frequency have been estimated by means of realistic simulations taking into account all the relevant foregrounds. Predictions for the various classes of extragalactic sources are based on up-to-date models. The most significant improvements over Planck results are presented for each option. COrE+ will provide much larger samples of truly local star-forming galaxies (by about a factor of 8 for the 1 m telescope, of 17 for 1.5 m, of 30 for 2 m), making possible analyses of the properties of galaxies (luminosity functions, dust mass functions, star formation rate functions, dust temperature distributions, etc.) across the Hubble sequence. Even more interestingly, COrE+ will detect, at $|b| > 30^\circ$, thousands of strongly gravitationally lensed galaxies (about 2,000, 6,000 and 13,000 for the 1 m, 1.5 m and 2 m options, respectively). Such large samples are of extraordinary astrophysical and cosmological value in many fields. Moreover, COrE+ high frequency maps will be optimally suited to pick up proto-clusters of dusty galaxies, i.e. to investigate the evolution of large scale structure at larger redshifts than can be reached by other means. Thanks to its high sensitivity COrE+ will also yield a spectacular advance in the blind detection of extragalactic sources in polarization: we expect that it will detect up to a factor of 40 (1 m option) or of 160 (1.5 m option) more radio sources than can be detected by Planck and, for the first time, from several tens (1 m option) to a few hundreds (1.5 m option) of star forming galaxies. This will open a new window for studies of the global properties of magnetic fields in star forming galaxies and of their relationships with star formation rates.

Keywords: CMBR experiments, galaxy evolution, active galactic nuclei, galaxy surveys

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1 Introduction

We investigate the impact on studies of extragalactic sources of the planned fourth generation Cosmic Microwave Background (CMB) mission named Cosmic Origins Explorer plus (COrE+). Various options are being considered. The COrE+ “light” concept envisages an effective telescope size of \( \simeq 1 \) m and a 60–600 GHz frequency range with a total of 2040 detectors. More ambitious options, requiring substantive contributions from international partners (“COrE+ extended”), contemplate telescope sizes from \( \simeq 1.5 \) m (baseline) to 2 m (although the last option is unlikely to fit within the M4 constraints) and a frequency range from 60 to 800 GHz (but we consider also the effect of an extension to 1200 GHz) with a total of 5800 detectors. The instrument comprises more than two times as many frequency channels as Planck, to make possible an efficient separation of foreground emissions. The very large number of state-of-the-art detectors ensures a much better sensitivity per channel.

The plan of the paper is the following. In section 2 we discuss the source detection in CMB maps. In section 3 we briefly describe the populations of extragalactic sources in the COrE+ frequency range. Section 4 is about the extraction of the rich information content on sources below the detection limit. In section 5 we deal with counts in polarization. Finally, in section 6 we summarize our main results.

Throughout this paper we adopt the fiducial ΛCDM cosmology with best-fit Planck + WP + high-resolution CMB data of the Atacama Cosmology Telescope (ACT) and South Pole Telescope (SPT) experiments as provided by [1].
Figure 1. Left panel: power spectra of the astrophysical components present in COrE+ maps at 70 GHz at high Galactic latitudes ($|b| \geq 30^\circ$), computed from the Planck Sky Model [2], masking the region at $|b| < 30^\circ$ with a $10^\circ$ apodization for $30^\circ < |b| < 40^\circ$. The three dot-dashed straight green lines show the power spectra of radio sources, that are the dominant extragalactic population at this frequency, computed after having masked sources brighter than the detection limits for the three options for the COrE+ telescope, shown in figure 4. The contributions to the power spectrum of Galactic emission are detailed in the adjacent half-panel. Right panel: power spectra of the astrophysical components present in COrE+ maps at 150 and 353 GHz. The lines have the same meaning as in the left panel. However at these frequencies the only relevant Galactic emission is the one of thermal dust. Also, fluctuations from dusty galaxies, which have a large contribution from clustering, are increasingly important with increasing frequency.

2 Detection limits for CMB maps

2.1 Basics

The development of algorithms for point source detection has a long history in astronomy. Images produced by CMB experiments have, from this point of view, important peculiarities [3] that make inadequate the algorithms consecrated by decades of successful usage in many wavebands, from radio to X-rays, such as CLEAN [4], DAOFIND [5] and SExtractor [6]. The peculiarities mostly follow from the limited angular resolution of CMB experiments.

In high resolution images, intensity peaks on the angular scale of the Full Width at Half-Maximum (FWHM) of the telescope are due to either point sources or to detector noise and the highest significance peaks are straightforwardly interpreted as point sources. On the contrary, in CMB images also structure in diffuse emissions (the CMB itself and Galactic radiations) shows up on the FWHM scale, complicating the detection of point sources.

Optimal methods for source detection in CMB maps need the knowledge of the probability density functions (PDFs) of each signal contributing to the “background” (for a review see [3]). In general, however, our knowledge of the PDFs is highly incomplete. A further complication comes from the non-stationarity of Galactic emissions, hampering the realization of detection methods valid throughout the sky. Still, methods able to efficiently filter out the “background” noise have been devised [7]. The widely used matched filter characterizes the “background” noise in terms of its power spectrum. Although this approach has known limitations [3] it constitutes a useful reference.

The power spectra at high Galactic latitude ($|b| \geq 30^\circ$) of the astrophysical components present in COrE+ maps at selected frequencies (70, 150 and 353 GHz) are illustrated in figure 1. They were computed using the most recent version of the Planck Sky Model [2] with an apodized Galactic mask (see the caption of figure 1). These figures illustrate the
relative importance of the various contributions to the fluctuation field and how they vary with frequency.

The root mean square (rms) fluctuations, \( \sigma \), are related to the power spectrum, \( C_\ell \), by

\[
\sigma^2 = \omega^2 \sum_\ell \frac{2\ell + 1}{4\pi} b^2_\ell C_\ell \simeq \frac{\omega^2}{2\pi} \int_0^\infty d\ell \, \ell b^2_\ell C_\ell ,
\]

(2.1)

where

\[
\omega = 2\pi \int_0^\infty d\theta \, \theta \exp \left[ -\frac{1}{2} \left( \theta/\theta_b \right)^2 \right] = 2\pi \theta_b^2
\]

(2.2)

is the solid angle of the instrument, \( b_\ell \) is the experimental beam function, which for a Gaussian beam with standard deviation \( \theta_b \) is well approximated by

\[
b_\ell = \exp \left[ -\frac{1}{2} \theta_b^2 \ell (\ell + 1) \right].
\]

(2.3)

The integral approximation to the sum holds if \( \theta_b \ll 1 \) rad (so that the relevant \( \ell \gg 1 \)).

Poisson fluctuations are uncorrelated. Hence the only non-zero term of the correlation function is the one at zero-lag and the power spectrum, which is the transform of the correlation function, is independent of \( \ell \):

\[
C_\ell = \int_0^{S_d} dN/S \, S^2 dS ,
\]

(2.4)

where \( dN(S)/dS \) are the differential number counts per steradian of sources weaker than the detection limit \( S_d \). Then

\[
\int_0^\infty d\ell \, \ell b^2_\ell C_\ell = \frac{\pi C_\ell}{2\theta_b^2}
\]

(2.5)

and

\[
\sigma^2 \simeq \frac{\omega^2}{2\pi} \frac{\pi C_\ell}{\omega} = \frac{\omega}{2} C_\ell = \omega_{\text{eff}} \int_0^{S_d} dN/S \, S^2 dS ,
\]

(2.6)

which is the classical expression for the variance of a Poisson distribution of sources weaker than \( S_d \), within a solid angle \( \omega_{\text{eff}} \) defined by

\[
\omega_{\text{eff}} = 2\pi \int_0^\infty d\theta \, \theta \, f^2(\theta, \phi).
\]

(2.7)

Adopting a Gaussian instrumental response function (the “beam” function), \( f(\theta, \phi) \), in polar coordinates:

\[
f(\theta, \phi) = \exp \left[-\frac{1}{2} \left( \frac{\theta}{\theta_b} \right)^2 \right],
\]

(2.8)

we get

\[
\omega_{\text{eff}} = \pi \theta_b^2 = \frac{\omega}{2} = \frac{\pi}{2 \ln(2)} \left( \frac{\text{FWHM}}{2} \right)^2,
\]

(2.9)

where we have used the relation \( \text{FWHM} = 2\sqrt{2 \ln 2} \theta_b \simeq 2.355 \theta_b \). Note that the solid angle to be used to compute Poisson fluctuations is half of that to be used in the general case (eqs. (2.1) and (2.2)).
Figure 2. Contributions of the various astrophysical components to the root mean square (rms) intensity fluctuations at high Galactic latitude ($|b| \geq 30^\circ$) at the angular resolutions of the three options for the CORe+ telescope, as a function of the observed frequency. Fluctuations are dominated by the CMB up to 271 GHz and by Galactic thermal dust at higher frequencies. The dominance of diffuse emissions highlights the importance of efficient component separation algorithms. The WMAP and Planck experiences have demonstrated that fluctuations due to diffuse emissions can be largely filtered out by source extraction algorithms. The contributions of extragalactic sources have been computed after masking sources brighter than the detection limits shown in figure 4.

Figure 3. Root mean square (rms) intensity fluctuations at the resolution of the measurements as a function of the FWHM of the telescope. The deviation of Poisson fluctuations from the direct proportionality, $\sigma \propto \text{FWHM}$, suggested by the combination of eqs. (2.6) and (2.9), is due to the decrease of the detection limit, $S_d$, in eq. (2.6) as the angular resolution improves. The decrease of the fluctuation amplitude with decreasing $S_d$ is stronger for flatter slopes of the source counts.

The dependence of rms fluctuations on frequency for diffraction limited observations with a 1 m, 1.5 m and a 2 m telescope is illustrated by figure 2. Figure 3 shows the trend of rms fluctuations with angular resolution at 3 frequencies. The relative importance of fluctuations due to unresolved point sources increases with frequency and with decreasing FWHM. Their dominant contribution in the FWHM range of interest here comes from AGN
Figure 4. PCCS 90% completeness limits (thick red bars; [17]) compared with 1σ fluctuations due to CMB and Galactic emissions. The latter have been estimated using the mask described in the caption of figure 1. The fact that the detection limits are generally not far from such rms fluctuations, while the threshold signal to noise ratios for source detection range from 4 to 4.9, illustrates the power of source extraction algorithms in filtering out diffuse emissions. The estimated 4σ detection limits for the 3 COrE+ options (dot-dashed green lines) are also shown for comparison. The dashed orange curve shows the 5σ noise levels for the Planck nominal mission (15 months).

powered radio sources\(^1\) up to about 150 GHz and from dusty galaxies at higher frequencies. Although the intrinsic clustering of high-z radio sources is strong (e.g., [13]), their observed spatial distribution is almost indistinguishable from a Poisson one [14, 15]. This is due to their very broad luminosity function, implying that at any flux density limit they span a wide redshift range that washes out much of the clustering signal [16]. On the contrary, in the case of dusty galaxies the contribution of clustering dominates except on the smallest angular scales.

The above discussion illustrates why, in spite of the very high sensitivity of modern CMB experiments, they only provide shallow surveys of extragalactic sources. For example, the 90% completeness limits of the Planck Catalogue of Compact Sources (PCCS; [17]) at 70, 100, 217 and 857 GHz are 776, 300, 180 and 680 mJy, respectively, while the 5σ instrumental noise limits at the same frequencies, for the 15 month survey, are \( \approx 235, 131, 143, \) and 220 mJy, respectively [18, 19]. This means that the surveys are limited by fluctuations of sky signals, not by instrumental noise: there is a lot of useful information in the fluctuation field! A comparison between the PCCS 90% completeness limits at all Planck frequencies and the corresponding 5σ instrumental noise levels is presented in figure 4.

A further cue comes from the consideration of the number of detections per resolution element. For example, the PCCS has 191, 629, 1409 and 6773 detections at 70, 100, 217 and 857 GHz, respectively, above the 90% completeness level in the extragalactic zone. This means that there are 1940, 1119, 1868 and 516 resolution elements per source, well above the values (30–40 resolution elements/source) usually corresponding to the confusion limit.

A key issue is the ability of source extraction algorithms to filter out fluctuations due to diffuse emissions, such as the CMB and Galactic radiations. Figure 4 shows that the component separation algorithms developed for the Planck project [7, 20] are quite efficient

\(^1\)Also star-forming galaxies are radio sources at some level (e.g., [9–12]), but their contribution to fluctuations in CMB maps is negligible.
at doing that: over a broad frequency range the PCCS 90% completeness limits are close to the 1 $\sigma$ fluctuations due to diffuse emissions. In other words most of their contributions to fluctuations are filtered out.

2.2 Estimate of COrE+ detection limits

To estimate the detection limits for the three COrE+ options considered in this paper we have carried out realistic simulations of sky maps including all the relevant foregrounds as well as the instrumental noise for a three year mission. Simulations with resolutions $\geq 4$ arcmin have been carried out using the latest version of the Planck Sky Model (PSM; [2]). The PSM includes, in addition to a model of the CMB, Galactic diffuse emissions (synchrotron, free-free, thermal and spinning dust, CO lines), Galactic HII regions, extragalactic radio sources, dusty galaxies, thermal and kinetic Sunyaev-Zeldovich signals from clusters of galaxies. Each component is simulated by means of educated interpolations/extrapolations of the available data, complemented by state-of-the-art models.

Distinctive features of the simulations are spatially varying spectral properties of synchrotron and dust; different spectral parameters for each point source; modelling of the clustering properties of extragalactic sources and of the power spectrum of fluctuations in the cosmic infrared background (CIB), in close agreement with the latest observational determinations.

On angular scales below 4 arcmin, reached at frequencies above $\approx 160$–320 GHz (depending on the telescope size), the PSM has not been well tested yet. So, at high frequencies we have performed simpler simulations, taking advantage of the fact that, on small scales, the fluctuation field at high Galactic latitudes is dominated by extragalactic sources making up the CIB and fluctuations due to the CMB and to diffuse Galactic emissions can safely be neglected. The method developed by [15] was used to distribute the sources on the sky consistent with the measured CIB power spectra at different sub-mm wavelengths [21–23], i.e. properly allowing for their clustering properties.

The source detection on simulated maps was performed using the IFCAMEX detection pipeline$^2$ and specifically its implementation of the second member of the Mexican Hat Wavelet family (MHW2). This wavelet is obtained applying the Laplacian operator two times on the two-dimensional Gaussian function. The MHW2 is used as a filter to reduce the contribution from the background, including both the small scale noise and the large scale diffuse emission from our own Galaxy, thus enhancing the detection efficiency. The analysis was carried out in projected flat $3.66^\circ \times 3.66^\circ$ square patches, corresponding to $128 \times 128$ pixels for HEALPix$^3$ (Hierarchical Equal Area isoLatitude Pixelation; [24]) maps with $N_{\text{side}} = 2048$.

For this purpose the map was divided into partially overlapping flat patches. Dividing the sky map into small patches we can optimize the filter taking into account the statistical properties of the map in the vicinity of each source. In practice, we did first a blind run and then a second non-blind run at the position of each source detected in the blind run. The second step allowed us to remove artifacts in the blind catalogue, mostly due to border effects in the filtered images.

Since the flat patches overlap, we can have multiple detections of the same source. The repetitions are removed, keeping for each source only the detection with the highest signal-to-noise ratio. Further details on the MHW2 and the IFCAMEX implementations can be

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$^2$http://max.ifca.unican.es/IFCAMEX.

$^3$http://healpix.sourceforge.net.
found in [25] and in [26]. The IFCAMEx detection pipeline was previously used to build the New Extragalactic WMAP Point Source (NEWPS) catalogue [27, 28] and the part of the PCCS at $\nu \leq 70$ GHz; it also being used to build the second Planck catalogue of compact sources (PCCS2) at the same frequencies. A different implementation of the same algorithm was used for the PCCS and PCSS2 catalogues at higher frequencies.

The $4\sigma$ detection limits, $S_d$, that, according to our simulations, correspond to $\gtrsim 90\%$ completeness and reliability in the “extragalactic zone” ($|b| > 30^\circ$), where Galactic emissions are quite low, are shown in figures 4 and 5. The values of $S_d$ obtained from the simulations are well approximated by the formula:

$$S_d = 4\left[\sigma^2_{\text{conf}} + \sigma^2_{\text{noise}} + (0.12 \sigma_{\text{CMB}})^2\right]^{1/2},$$

(2.10)

with

$$\sigma^2_{\text{conf}} = \sigma^2_{\text{P,radio}} + \sigma^2_{\text{P,dusty}} + \sigma^2_{\text{clust,dusty}} + \sigma^2_{\text{SZ}}.$$  (2.11)

In the above formulae the various contributions to the fluctuation field have been computed using the equations given in section 2.1. The CMB power spectrum corresponds to the Planck best-fit cosmological parameters [1]. The rms confusion fluctuations, $\sigma_{\text{conf}}$, include the contributions of radio sources, of dusty galaxies and of galaxy clusters (Sunyaev-Zeldovich, SZ, effect). The Poisson contribution from radio sources was computed via eq. (2.6) using the [29] model at frequencies of up to 100 GHz and the [30] model at higher frequencies. The detection limit, $S_d$, was computed iteratively. A check with the PSM has shown that, as expected, the clustering of radio sources adds a negligible contribution (we recall that the PSM contains, down to faint flux density levels, real radio sources, at their real positions in the sky). The contributions of dusty galaxies and of galaxy clusters were computed using, respectively, the model by [31], that fits accurately the measured CMB power spectra, and the power spectrum of the thermal SZ effect measured by [32]. These power spectra comprise both the Poisson and the clustering contributions.

3 Extragalactic sources in the COrE+ frequency range

By a lucky (for CMB studies) coincidence the CMB peak occurs at a frequency close to a deep minimum of the intensity of the extragalactic background light as well as of the Galactic emissions. The steep increase with frequency of the dust emission spectrum in the mm/sub-mm region (typically $S_\nu \propto \nu^{3.5}$), while the radio emissions (synchrotron and free-free) decline, makes the crossover frequency between radio and dust emission components only weakly dependent on their relative intensities. Moreover, dust temperatures tend to be higher for distant high luminosity sources, partially compensating for the effect of redshift. As a consequence there is an abrupt change in the populations of bright sources above and below $\sim 1$ mm: radio sources dominate at longer wavelengths, while in the sub-mm region dusty galaxies take over (figure 5).

The Planck Early Release Compact Source Catalogue (ERCSC; [40]) reported sources extracted from maps built using the data obtained from the scans of the sky between 2009 August 13 and 2010 June 6, comprising the first all-sky survey and about 60% of the second. It was exploited by [34] to derive counts of extragalactic radio sources in the range 30–217 GHz and by [36] to investigate the statistical properties of infrared and radio extragalactic sources between 100 and 857 GHz.

Subsequent releases of the Planck compact source catalogue benefited of additional data (15 months, “the nominal mission”, for the PCCS, full mission for the PCCS2) as well as of
Figure 5. Spectral energy distributions (SEDs) of a bright radio source ([HB89] 0234+285; dashed black line) and of a bright infrared galaxy (M61; solid black line) compared with the estimated 4σ detection limits for the three COrE+ telescope sizes (dot-dashed green lines). The radio emission of M61 was estimated exploiting the mean relationships between IR and radio emissions derived by [33]. The synchrotron and free-free contributions are shown by the dashed black lines at the bottom of the figure. Synchrotron dominates at low frequencies. Note that the frequency at which the radio spectrum of [HB89] 0234+285 crosses the dust emission of M61 is only 0.3 dex higher than the crossing frequency of its own radio emission, which is almost 2 orders of magnitude fainter.

improved data processing. As a result, the completeness limits of the PCCS are fainter than those of the ERCSC by average factors of ≃ 1.5 at LFI frequencies (30–70 GHz) and of ≃ 2.5 in the range 100–353 GHz; the improvement is larger at higher frequencies: a factor of 3.5 at 545 GHz and of 4.5 at 857 GHz. No information is available yet on the PCCS2. However major improvements are not expected.

Differential counts at several frequencies of radio sources and of dusty galaxies are shown in figures 6 and 7, respectively. The Planck counts (yellow data points) were obtained from the ERCSC (the PCCS has not been exploited yet for this purpose). The 90% completeness limits of the PCCS and of the 3 COrE+ options are also shown. The scientific advances made possible by COrE+ are discussed in the following sub-sections.

3.1 Classical radio sources

COrE+ covers the frequency range where the information on the spectral energy distribution (SED) of radio-loud Active Galactic Nuclei (AGNs) is still scanty: even the SEDs of very bright sources like [HB89] 0234+285 (figure 5) are not measured at sub-mm wavelengths. Yet, in this spectral region important spectral features, carrying essential information on physical conditions of sources, show up.

At the high COrE+ radio frequencies the dominant radio source populations are the compact flat- and inverted-spectrum ones, and primarily blazars (BL Lac objects and flat-spectrum radio quasars). For these sources, observations at mm/sub-mm wavelengths often reveal the transition from optically thick to optically thin radio emission in the most compact regions, i.e. are informative on the maximum self-absorption frequency which roughly corresponds to the synchrotron peak frequency (in terms of νLν). A systematic survey in
Figure 6. Differential number counts of radio sources at 70, 150, 220 and 550 GHz. The Planck data points are from [34] at 70 GHz and from [36] at higher frequencies (counts based on the PCCS are not available yet; the 90% completeness limits of the latter catalogue in the “extragalactic zone” are indicated by the vertical solid lines). The South Pole Telescope (SPT) points are from [35] and the Herschel point at 550 GHz is from [37]. The curves are the model predictions by [29] at 70 GHz and by [30] at higher frequencies. The vertical dashed lines correspond to the 4σ detection limits for the 3 COrE+ telescope options.

Figure 7. Differential number counts of dusty galaxies at 150, 220, 550, 850 and 1200 GHz (2 mm, 1.4 mm, 545 µm, 353 µm and 250 µm). The Planck ERCSC data points are from [36] and [38]. The SPT points are from [35] and the Herschel points from [39]. The curves are the model predictions by [31]: the red lines show the contributions of unlensed (solid) and strongly lensed (dashed) protospheroidal galaxies, the cyan lines show the counts of late-type normal and starburst galaxies, the back lines are the total. The vertical dashed lines correspond to the 4σ detection limits for the 3 COrE+ telescope options while the vertical solid lines indicate the 90% completeness limits of the PCCS in the “extragalactic zone”.

the COrE+ range will, for example, allow us to see if there are systematic differences in the synchrotron turnover frequencies between BL Lacs and flat-spectrum radio quasars, as would be expected if their jets have different distributions of the angular separations from the line of sight, implying different amounts of Doppler boosting. Correlations between turnover frequency and luminosity, which is also boosted by relativistic beaming effects, would help testing current models.

Major high radio frequency flares have been observed in several compact radio sources [41–43], including the recent giant outburst of 3C 454.3 [44]. Several mechanisms can yield strong variability of relativistically beamed sources (blazars; e.g. [42] and references therein): shocks travelling along the jet, changes of the bulk Lorentz factor causing variations of the Doppler factor, changes of the viewing angle due, e.g., to precession of binary black
hole systems or to helical trajectories of plasma elements or to rotating helical jets. Thus variability studies provide clues into the physical properties of emitting regions. Variability due to geometrical effects is expected to affect weakly if at all the source spectra, while shocks induce strong spectral variations with intensity peaks that generally move downwards in frequency with shock age. The CORe+ surveys may catch the rise of the flare at the highest frequencies, missed by ground based observations.

Extended, steep-spectrum radio sources are minor contributors to millimeter-wave counts at bright flux densities. Nevertheless Planck and WMAP surveys have detected a few tens of these sources [17, 27, 28, 45, 46]. Although extended, with few exceptions, they are unresolved by the large beams of CMB experiments. In the case of the 2m options we expect the detection of about 400 of them at 60 GHz and $|b| > 20^\circ$. Their spectral break frequency, $\nu_b$, at which the synchrotron spectrum steepens due to electron energy losses, is related to the magnetic field, $B$, and to the synchrotron age, $t_s$ (in Myr), by $\nu_b \simeq 96(30 \mu G/B)^3 t_s^{-2} \text{GHz}$. The systematic multi-frequency study at the CORe+ frequencies will thus provide a statistical estimate of the radio source ages, primarily at relatively low redshifts. At high redshifts the dominant energy loss mechanism of relativistic electrons may be inverse Compton scattering off CMB photons because the CMB energy density, which grows as $(1+z)^4$, can exceed the magnetic energy density in radio lobes. As a consequence, the synchrotron emission of extended, steep-spectrum, sources is suppressed with increasing redshift down to lower and lower frequencies, while compact sources are almost unaffected [47]. This has obvious implications for the statistics of extended sources as a function of redshift and of the survey frequency. Detailed predictions depend on poorly known quantities, such as the distribution of magnetic field intensities, of source sizes and of injected relativistic electrons. A comparison of low-frequency ground based surveys with the high frequency surveys by CORe+ can provide key constraints on these quantities.

Excess far-IR/sub-mm emission due to dust mostly heated by young stars is often observed from radio galaxies, particularly at high $z$ [48]. CORe+’s broad frequency coverage will allow an extensive study of the spectral energy distribution from radio to sub-mm wavelengths and therefore of the presence of dust in host galaxies and of the relationships between the radio emission and the evolutionary status of the host galaxies.

According to the current notion, host galaxies of blazars are passive ellipticals [49]. This view has however already been challenged by Planck observations of catalogued blazars associated to galaxies with intense star formation activity (3 examples are shown in figure 8). Since dust emission from the host galaxy is detectable by Planck only for nearby galaxies (but blazars are rare locally) or for extreme IR luminosities, these objects are likely only the tip of the iceberg. The higher sensitivity of CORe+ will allow us to extend the study to much larger samples, shedding light on the possible relation of the blazar phenomenon with star-formation activity.

3.2 Special radio source populations

High radio frequency surveys are crucial to investigate special classes of radio sources, self-absorbed up to cm wavelengths. A particularly important class of sources in this category are extreme GHz Peaked Spectrum (GPS) sources or high frequency peakers (HFPs; [50]). GPS sources are powerful $[\log(L_{1.4\text{GHz}}/\text{erg s}^{-1}\text{Hz}^{-1}) \gtrsim 32]$, compact ($\text{sizes} \lesssim 1\text{kpc}$) radio sources with convex spectra peaking at GHz frequencies. Conclusive evidence that they correspond to the earliest stages of the evolution of powerful radio sources, when the radio-emitting region grows and expands within the interstellar medium of the host galaxy, came
Figure 8. SEDs of three blazars detected by Planck showing a sub-mm excess attributable to host galaxies endowed with active star formation. Since, with the exception of galaxies with extreme SFRs, Planck could only detect the far-IR emission from nearby galaxies, the sub-mm detection of these blazars implies that a substantial fraction of these sources are hosted by star-forming galaxies, contrary to the notion that their host galaxies are passive ellipticals. The Planck data are represented by the black points with error bars; the other data points were taken from the NASA/IPAC Extragalactic Database, NED. For clarity, the SEDs of GB6 1347+1217 and of NGC 1275 have been raised by $\Delta \log(\nu f_\nu) = 3$ and $= 6$, respectively.

from VLBI measurements of propagation velocities of up to $\simeq 0.4c$, implying dynamical ages of $\sim 10^3$ years (see [51] for a review and references). The identification of these sources is, therefore, a key element in the study of the birth of radio activity.

Predictions for the counts of GPS sources have been worked out by [29, 52] and [53]. Observational estimates are complicated by the contamination of candidate GPS samples by flaring blazars that also exhibit spectral peaks at GHz frequencies [54]. Recent estimates differ by substantial factors, but the uncertainties are very large due to the poor statistics. [55] found that only 3 sources with $S_{20\text{GHz}} = 200\text{mJy}$ selected over an area of about 6.1 sr could be classified as genuine GPSs, with a possible incompleteness by a factor of 2. On the other hand, [56] have 2 confirmed GPSs with $S_{20\text{GHz}} > 200\text{mJy}$, both with $\nu_p > 80\text{GHz}$, over a much smaller area; this would correspond to 24–155 (68% confidence limits for a Poisson distribution) GPSs over 6.1 sr. Only all sky surveys can produce significant samples of these rare objects, but they need to go deeper than Planck. COrE+, especially with the largest telescope size, has the right properties.

### 3.3 Local dusty galaxies

The (sub-)mm surveys performed by all-sky CMB experiments are the ideal way to carry out an unbiased census of dusty galaxies in the local neighbourhood, down to volume densities beyond the reach of more sensitive surveys over small areas of the sky, such as those carried out by Herschel. The Planck ERCSC has offered the first opportunity to accurately determine the luminosity functions in the very local Universe at several (sub-)millimetre wavelengths, unaffected by cosmological evolution [38], and to investigate their properties [57]. These studies have not yet been extended to the PCCS.

In section 2 we have shown that, at sub-mm wavelengths, COrE+ will reach, at 500–600 GHz flux densities a factor of $\simeq 4$, 6.7 and 9.6, for the 1 m, 1.5 m and 2 m option,
respectively, fainter than the PCCS, i.e. to explore, respectively, a volume a factor of 8, 17 and 30 times larger. This means that COre+ can detect at 600 GHz, from \( \simeq 7,900 \) (1 m option) to \( \simeq 17,800 \) (1.5 m option) to \( \simeq 31,500 \) (2 m option) star forming galaxies out to \( z = 0.1 \) in the “extragalactic zone”.

This redshift range is within that covered by the Sloan Digital Sky Survey (SDSS) whose DR10 release extends over 14,555 square degrees \([58]\). Additional redshifts are provided by other, albeit shallower, large-area spectroscopic redshift surveys: the all-sky 2MASS Redshift Survey (2MRS; \([59]\)) and the IRAS PSC Redshift Survey (PSCz; \([60]\)); the hemispherical Six-Degree Field Galaxy Survey (6dFGS; \([61]\)), and others, such as the Two-Degree Field Galaxy Survey (2dFGRS; \([62]\)); see, e.g., \([63]\) for a compilation of redshift measurements, dubbed 2M++.

In the near future the low-redshift Southern sky will be comprehensively mapped spectroscopically by the currently starting TAIPAN survey aiming at a complete flux-limited sample down to \( r \lesssim 18 \) mag at \( \delta \lesssim 10^\circ \) \([64]\). On a somewhat longer timescale, the 4-metre Multi-Object Spectroscopic Telescope (4MOST; \([65]\)) will provide multifibre \( R = 5000 \) resolution spectroscopy of 10 million \( z < 1 \) emission line galaxies. Similar swaths of the sky are also expected to be observed in HI emission-line surveys such as WALLABY with ASKAP \([66]\); the Square Kilometre Array (SKA) will conduct the biggest spectroscopic galaxy survey ever, aimed at detecting the 21cm HI emission line from around a billion galaxies over 3/4 of the sky \([67]\).

Accurate photometric redshifts have been obtained for many more galaxies detected by wide-angle surveys \([68–70]\). The 2MASS Photometric Redshift (2MPZ) sample of \([68]\) should be particularly useful in this respect due to its almost full sky coverage, suitable depth and size (1 million galaxies with a median \( z = 0.07 \); for comparison, the median values of the low-\( z \) peaks of the distributions in figure 9 are \( \simeq 0.02–0.03 \)). In addition, COre+ local dusty galaxies missed by 2MASS should be present in the WISE catalogue \([71]\); a large fraction, if not all of them, will have reliable photometric redshifts available over most of the sky \([72]\, and Bilicki et al. in preparation].

Other wide-angle multi-band photometric surveys, being currently undertaken, are expected to provide photometric redshifts in large volumes of the relatively low-redshift universe, especially when these data-sets are combined with each other and/or with WISE. These include, but are not limited to: Pan-STARRS \([73]\), the VISTA Hemisphere Survey (VHS; \([74]\)), the Dark Energy Survey (DES \([75]\)) and Skymapper \([76]\).

The large number of galaxies for which spectroscopic or photometric redshifts will be available will make possible analyses of the properties of galaxies (luminosity functions, dust mass functions, star formation rate functions, dust temperature distributions, etc.) across the Hubble sequence. Of special interest will be the study of IR emission from IR-faint galaxies such as early-type or dwarf galaxies. Combining the COre+ with the available multi-wavelength data, especially with those from WISE, 2MASS, GALEX and Euclid we will get a complete view of the SEDs and will be able to investigate relationships between the dust content, the star formation rate (SFR), the stellar mass, the environment and more.

For dusty galaxies, that dominate the extragalactic source population at high frequencies, lower frequency measurements provide information on the relationship, if any, between star-formation and nuclear radio activity and on the radio emission (synchrotron and free-free) powered by star formation.
Figure 9. Predicted redshift distributions of dusty galaxies at 295, 390 and 600 GHz at the 4 \( \sigma \) detection limits of the 1 m, 1.5 m and 2 m options, from top to bottom. Such limits are 81.0, 44.1 and 29.1 mJy at 295 GHz; 104.5, 60.7 and 41.6 mJy at 390 GHz; 150.5, 90.4 and 63.5 mJy at 600 GHz. The distributions are strikingly bi-modal with two well separated peaks. The low-\( z \) peak is due to relatively local (\( z \lesssim 0.1 \)) star-forming galaxies while the high-\( z \) (\( z \gtrsim 1.5 \)) peak is dominated by strongly lensed galaxies.

3.4 High redshift dusty galaxies

Sub-mm surveys (see, e.g., [77] for a recent review) have clearly demonstrated the importance of the FIR/sub-mm data in reconstructing a complete picture of the history of galaxy formation and evolution. The high-redshift sources detected in these surveys are expected to be the progenitors of the present-day massive elliptical galaxies [78–80]. These galaxies can reach extreme IR luminosities (\( \geq 10^{13} L_{\odot} \); [81–84]), that can be further boosted by strong gravitational lensing [85–89] to the point that some were detected by Planck, in spite of the shallowness of its surveys [90–93].

Using the [31] model, that accurately fits a broad variety of infrared to millimeter-wave data on extragalactic sources\(^4\) (multi-frequency and multi-epoch luminosity functions of galaxies and AGNs, redshift distributions, number counts, total and per redshift bins) we find that the deeper COrE+ surveys, compared to Planck, will detect unlensed IR galaxies up

\(^4\)See figures in http://staff.ustc.edu.cn/~zcai/.
to redshifts $> 1$ (for telescope sizes $\geq 1.5 \text{ m}$) and many more strongly lensed (amplification $\mu > 2$) galaxies. In the “extragalactic zone” ($|b| > 30^\circ$) we expect, at 600 GHz, $\simeq 540$ unlensed galaxies at $z > 1$ for the 1.5 m option. For the 2 m option this number jumps to $\simeq 15,900$.

The counts of strongly lensed galaxies were computed using again the [31] model, but adopting a maximum amplification $\mu_{\text{max}} = 30$ to take into account the sources size (at variance with [31] who assumed point sources). As shown by figure 2 of [94] the adopted $\mu_{\text{max}}$ is consistent with the counts and the redshift distribution of strongly lensed galaxies detected by the South Pole Telescope (SPT; [35, 95]).

Even the 1 m option will detect thousands of strongly lensed galaxies at $|b| > 30^\circ$ (about 740 at 390 GHz and over 2,000 at 600 GHz). For the 1.5 m option these numbers increase to $\simeq 2,900$ and $\simeq 6,200$, respectively; for the 2 m option they are $\simeq 6,700$ and $\simeq 12,700$, respectively.

For all the options considered, the COrE+ counts will cover the gap between the bright (sub-)mm counts measured by Planck and those measured by the deep SPT and Herschel surveys (orange and green points, respectively, in figures 6 and 7). The transition from the bright counts, dominated by nearby galaxies, which have an Euclidean slope, to the fainter ones made much steeper by the combined effect of strong cosmological evolution and of the positive K-correction [96, 97] provides a strong test for galaxy evolution models. Many of them predict a smoother transition than suggested by the present data, but only the much better statistics that will be provided by COrE+ will make it possible to reach firm conclusions.

The selection of strongly lensed galaxies will be extremely easy at about 300–400 GHz. As illustrated by figure 9, essentially all high-$z$ galaxies detected at these frequencies will be strongly lensed. The other detected sources will be easily recognizable low-$z$ late-type galaxies, plus a small group of radio sources, also easily identifiable in low frequency radio catalogs. At 600 GHz the fraction of strongly lensed galaxies among $z > 1$ objects is still very close to 100% for the 1 m option and only slightly decreases (to 92%) for 1.5 m option. For the 2 m option the strongly lensed fraction among 600 GHz detections with $z > 1$ is 44%; but in this case we expect $\simeq 6,700$ strongly lensed galaxies in the “extragalactic zone” already at 390 GHz, where they are $\simeq 87\%$ of objects at $z > 1$. This means that COrE+ will allow us to find very easily thousands of strongly lensed galaxies with a close to 100% efficiency (see also [87]).

While other facilities will also be generating large reliable gravitational lens catalogues on a comparable timescale (e.g. Euclid Hα lenses [105], Gaia lensed quasars [e.g. 106]), the critical advantage of COrE+ will be in extending the sources and lenses to much higher redshifts. This will make it possible to probe the evolution of dark matter halo substructure and the stellar Initial Mass Function (IMF) to much higher redshifts (key tests of semi-
analytic models of galaxy evolution, e.g. \cite{107, 108}). This large, high-redshift lens catalogue will also be ideal to probe cosmological parameters using rare double lenses (see, e.g., \cite{109} for the use of dynamical constraints to remove model degeneracies, e.g. \cite{110}). The background source population will mostly be ultra-luminous star-forming systems that in themselves have always posed the strongest challenges to semi-analytic galaxy evolution models. Only in large lens samples can the rarest high-magnification events be found, and the angular magnification afforded by gravitational lensing in these systems will make them the ideal laboratories for determining the physical processes that dominate star formation and feedback in the early Universe.

4 Extracting information on extragalactic sources below the detection limit

4.1 Proto-clusters of dusty galaxies

As mentioned in section 2, thanks to the very low instrumental noise, the fluctuation field measured by COrE+ is signal dominated. This implies that it contains a lot of useful information. In regions with low dust content, several intensity peaks not associated to individual sources just below the detection limit were found to correspond to clumps of dusty galaxies likely to evolve into rich galaxy clusters \cite{93, 111}. This was predicted by \cite{112} on the basis of the argument summarized and slightly updated below.

Sub-mm surveys proved to be most efficient in detecting high-$z$ massive galaxies, interpreted as progenitors of present day giant ellipticals, caught during their star-formation phase. There is evidence of strong clustering of these sources \cite{21–23, 113–115}, consistent with them being tracers of strongly overdense regions. Sub-mm surveys are therefore optimally suited to look for proto-clusters at earlier redshifts than can be reached by optical/near-IR, X-ray, SZ surveys.

The analysis in section 2 has demonstrated that, in high Galactic latitude regions, intensity peaks below the point source detection limits at sub-mm wavelengths are dominated by fluctuations in the distribution of faint sources making up the CIB. Such fluctuation field is highly skewed. One example, computed using the PSM at the Planck resolution, is displayed in figure 10. The clustering contribution overcomes Poisson fluctuations on scales larger than several arcmin (see figure 3). This means that sub-mm maps filtered with resolutions of several arcmin are a powerful tool to detect candidate proto-clusters of dusty galaxies.

In other words, the relatively large beams used by CMB experiments collect photons from regions with Mpc physical sizes at $z = 1–3$ (at these redshifts the physical scale is $\sim 0.5\,\text{Mpc/arcmin}$) thus summing the contributions from tens of star-forming galaxies in overdense regions. As first pointed out by \cite{112} the galaxy clumps may then become detectable, even if individual galaxies are well below the detection limit. Unbiased searches of high-$z$ protoclusters thus become possible, overcoming the need of targeting possible signposts of high density peaks (high-$z$ radiogalaxies or powerful QSOs), as done so far.

It must be stressed, however, that over-densities are not necessarily proto-clusters. Some may be random alignments, projected on the plane of the sky, of large scale structures either unrelated or connected by a cosmic filament almost aligned with the line of sight. Quantitative predictions were worked out by \cite{112}. Briefly, within the standard gravitational clustering scenario, we expect that the distribution of intensity peaks due to source over-densities has a very large variance, resulting from three contributions, discussed below (see \cite{116}). The
mean number of objects inside a volume $V$ centered on a source is:

$$\langle N \rangle_p = n \int_V [1 + \xi(r)] dV,$$

(4.1)

where $n$ is the mean source number density. The variance around $\langle N \rangle_p$ is:

$$\langle (N - \langle N \rangle_p)^2 \rangle_p = \langle N \rangle_p + n^2 \int_V \int_V \left[ \zeta(r_1, r_2) + \xi(r_{12}) - \xi(r_1)\xi(r_2) \right] dV_1 dV_2$$

(4.2)

with

$$\zeta(r_1, r_2) = Q(z) \left[ \xi(r_1)\xi(r_2) + \xi(r_1)\xi(r_{21}) + \xi(r_{12})\xi(r_2) \right],$$

(4.3)

$Q(z)$ being the amplitude of the three-point angular correlation function.

For a survey with a Gaussian angular response function, $f(\theta)$ (eq. (2.8)), and $\xi(r) = (r_0/r)^{1.8}$ we have

$$nJ_2 = n \int_V \xi(r) f(\theta) dV \simeq 25.9 n r_0^{1.8} [DA(z)\theta_b]^{1.2}$$

(4.4)

where $DA$ is the angular diameter distance and

$$\xi(r, z) = b^2(M_{\text{eff}}, z) \xi_{\text{DM}}(r, z)$$

(4.5)

$b(M_{\text{eff}}, z)$ being the bias factor, $M_{\text{eff}}$ the effective halo mass, $\xi_{\text{DM}}(r, z)$ the spatial correlation function of dark matter halos. The mean luminosity of the “clump” around the central source with luminosity $L_m$ is

$$\bar{L}_{cl}(z) = L_m(z) + \int_L dL' L' \Phi(L', z) \int_V [1 + \xi(r, z)] f(\theta) dV$$

(4.6)
with variance:

\[
\sigma_{L_{cl}}^2 = \sigma_L^2 + \int_L dL' L'^2 \Phi(L', z) \int_V [1 + \xi(r, z)] f(\theta) dV \\
+ \left[ \int dL' L' \Phi(L', z) \right]^2 \int \left[ \zeta(r_1, r_2) + \xi(r_{12}) - \xi(r_1)\xi(r_2) \right] f(\theta_1) f(\theta_2) dV_1 dV_2. 
\]

The first contribution to the variance refers to the luminosity of the most luminous source, acting as a beacon signalling the presence of the proto-cluster. The second refers to fluctuations in the total luminosity of the surrounding sources, that can be seen as random samples of the luminosity function, assuming that their number is equal to the mean. The third takes into account that “clumps” are random samples of the galaxy spatial distribution. In other words, the third term accounts for fluctuations in the effective number of galaxies within the beam, independently of their luminosity; the second term accounts for the fluctuations in their total luminosity at fixed number of galaxies.

All-sky surveys, such as that by COre+, are optimally suited to pick up the rare cases in which all contributions conspire to yield an exceptionally high luminosity of the clump. Particularly favourable situations may occur when the main source is strongly lensed or the clump is within a filament almost aligned with the line of sight, so that the contribution of neighbouring sources (seen in projection) to the observed signal can be very large.

Under the assumption that the statistics of the matter density distribution can be described by a log-normal function \[112\], the probability distribution function of \(L_{cl}\) is:

\[
p(L_{cl}, z) = \frac{\exp \left[ -\frac{1}{2} \ln(L_{cl}) - \mu_g(z))^2/\sigma^2_g(z) \right]}{\sqrt{2\pi}\sigma^2_g L_{cl}} \]

where

\[
\mu_g(z) = \ln \left[ \frac{L_{cl}(z)}{\sqrt{\sigma^2_{L_{cl}}(z) + L_{cl}(z)^2}} \right], \\
\sigma^2_g(z) = \ln \left[ \frac{\sigma^2_{L_{cl}}(z)}{L_{cl}(z)^2 + 1} \right].
\]

The clump luminosity function \(\Psi(L_{cl}, z)\) obeys the normalization condition:

\[
\int \Psi(L_{cl}, z) L_{cl} dL_{cl} = \int \Phi(L, z) L dL. 
\]

A search for such clumps has been carried out by \[111\] by investigating the nature of *Planck* ERCSC sources that lie within an area of \(\sim 90\) deg\(^2\) observed as part of the *Herschel* Multitiered Extragalactic Survey (HerMES). Four sources were found to be associated with overdensities of *Herschel* sources with sub-mm colours suggesting redshifts \(\simeq 1–2\). The estimated surface density of these candidate high-\(z\) clumps was found to be consistent with the predictions by \[112\], although with large uncertainties.

However new calculations taking into account the more recent determinations of the redshift-dependent sub-millimeter luminosity functions (Negrello et al., in preparation) yield lower clump surface densities than estimated by \[112\], and fall short of accounting for the \[111\]
result. A very similar conclusion was reached by [117] using a completely independent approach, coupling hydro-dynamical zoom-in simulations with the recently developed radiative transfer code GRASIL3D. The origin of the discrepancy is unclear. On one side it may suggest that models and simulations underestimate the IR luminosities of the clumps. Alternatively, the overdensities discovered by [111] might be not individual proto-clusters but positive fluctuations in the number of proto-clusters within the \textit{Planck} beam. In both cases the study of these overdensities provides important clues on the early evolution of large scale structure.

4.2 Power spectrum of the Cosmic Infrared Background (CIB)

The better angular resolution, at high frequencies, of all COre+ options, compared to \textit{Planck}, will allow us to measure, in a uniform way, the CIB power spectrum over an unprecedented range of frequencies and of angular scales (from \(\sim 1\) arcmin to tens of degrees), thus breaking the degeneracy between the Poisson contribution and that of non-linear effects (one-halo term), that complicates the interpretation of \textit{Planck} measurements, without resorting to external (\textit{Herschel}) data. Although there is good agreement between the latest determinations of the power spectra by \textit{Planck} and \textit{Herschel} [22, 23], at \(545\) GHz the \textit{Herschel} data tend to be slightly but systematically higher, although compatible within the error bars.

Accurate determinations of the CIB power spectrum at different frequencies provide on one side constraints on the evolution of the cosmic star formation density and, on the other side, on halo masses associated to sources of the CIB. Except on the smallest angular scales, the CIB power spectrum is determined by galaxy clustering. The currently standard approach to describe it starts from the consideration that galaxies are hosted by dark matter halos. The power spectrum of the galaxy distribution is parameterized as the sum of the one-halo term that dominates on small scales and depends on the distribution of galaxies within the same halo, and the two-halo term that dominates on large scales and is related to correlations among different halos.

The halo occupation distribution, which is a statistical description of how dark matter halos are populated with galaxies, is modelled using a central-satellite formalism (see, e.g., [118]). The two-halo term is essentially proportional to the square of the effective bias factor times the linear theory power spectrum of dark matter halos. Since the bias factor is a steep function of the halo mass and redshift, the two-halo term is a sensitive probe of the effective halo mass of CIB sources (i.e. of galaxies that contribute most to the cosmic star formation) and of its evolution with redshift.

The large number of COre+ channels will make it possible to investigate in detail the decorrelation of power spectra measured in different frequency bands. A decorrelation is expected because the redshift distribution of sources of the CIB shifts to higher and higher redshifts with decreasing frequency [31, 84, 119, 120], and was observed [22, 23]. Accurate measurements of the CIB cross spectra for different frequency channels sets strong constraints on the frequency dependence of redshift distributions, hence on the evolution of the cosmic SFR.

5 Polarization of extragalactic sources

Our calculations yield 90\% completeness limits in polarized flux density that are much lower than the corresponding limits in total intensity, shown in figure 4, and not far from the 5\(\sigma\) noise levels. This is because, in the COre+ frequency range, the sky is much less complex in
polarization than in temperature: the free-free, the spinning dust, the SZ and the CIB emissions are either unpolarized or very weakly polarized; the CMB is also weakly polarized and, moreover, has a different polarization pattern (dominated by the E-mode) than foreground emissions, which have comparable E- and B-mode polarization. The source confusion level is also low since, roughly, the rms polarization fluctuations, $\sigma_p$, are related to the fluctuations in total intensity, $\sigma_i$, by $\sigma_p \simeq \Pi \sigma_i$, where $\Pi$ is the mean polarization degree \cite{121}. For flat-spectrum radio sources, that dominate the counts at few cm to mm wavelengths, \cite{122} found $\Pi \simeq 0.026$ at 18 GHz; the mean polarization degree of dusty galaxies is expected to be substantially lower (see below).

The estimated COrE+ 90% completeness limits in polarization for the 1 m option decrease from $\simeq 25$ mJy to $\simeq 6$ mJy as the frequency increases from 60 to 200 GHz and increase at higher frequencies up to $\simeq 23$ mJy at 350 GHz. For the 1.5 m option the 5 $\sigma$ noise levels in polarization vary from $\simeq 11$ mJy at 60 GHz to $\simeq 3$ mJy at 200 GHz, to $\simeq 8$ mJy at 350 GHz.

Adopting the slope of 1.3 for the integral counts of radio sources in terms of polarized flux density ($N(> S_p) \propto S_p^{-1.3}$) found by \cite{122} at 18 GHz (see also \cite{123}), we find that COrE+ will detect up to a factor of 40 (1 m option) or of 160 (1.5 m option) more polarized radio sources than can be detected by Planck. In other words, COrE+ will provide blindly selected samples of thousands of polarized radio sources. We caution that these estimates are only tentative because of the complex spectral behaviour of the polarized flux density \cite{122} that make extrapolations in frequency quite uncertain. On the other hand, this uncertainty adds interest to COrE+ measurements.

Little is known on the polarization degree of dusty galaxies, but it is likely to be low because the complex structure of galactic magnetic fields with reversals along the line of sight and the disordered alignment of dust grains reduce the global polarized flux when integrated over the whole galaxy. The measurements at 850 $\mu$m of M82 by \cite{124} gave a global net polarization degree of only 0.4%.

If the net polarization degree of M82 is typical of star forming galaxies, using again the \cite{31} evolutionary model, we expect the detection in polarization of $\simeq 7.5$ sr$^{-1}$ dusty galaxies for the 1 m option and of $\simeq 37$ sr$^{-1}$ for the 1.5 m option, corresponding to several tens or a few hundred galaxies, respectively, in the “extragalactic zone” ($|b| > 30^\circ$). This will provide valuable constraints on the global properties of magnetic fields in star forming galaxies and on their relationships with SFRs.

6 Discussion and conclusions

COrE+ will perform substantially better than Planck also in the case of a telescope of a similar or somewhat smaller (1 m) size. There are two main reasons for this. COrE+ will be confusion limited up to the highest frequencies, in contrast to Planck. For example, at 545 GHz (550 $\mu$m) the Planck beam has a FWHM = 3.8', while the diffraction limit is 1.5'. The better resolution implies substantially deeper point source detection limits because the survey depth is mostly limited by fluctuations of sky signals. Realistic simulations give, at 545 GHz, a 90% completeness level of 96 mJy for the COrE+ 1.5 m option and of 141 mJy for the 1 m option. For comparison the corresponding flux density limit for the PCCS is of 570 mJy.

At low frequencies, where Planck resolution is also at the diffraction limit, the better performance by COrE+ is due to its lower instrumental noise. Source detection algorithms efficiently filter out diffuse emissions (CMB and Galaxy), whose power spectra sink down
rapidly at the relevant resolutions, but suffer instrumental noise, which has a flat, roughly white noise, power spectrum. Our PSM-based simulations give, at 100 GHz, 90% completeness limits of 85 and 180 mJy for the 1.5 m and 1 m options, respectively, to be compared with the limit of 300 mJy for the PCCS.

At higher frequencies, the Planck ERCSC has already provided large samples of dusty galaxies mostly at distances < 100 Mpc, thus offering the first opportunity to accurately determine the local luminosity function unaffected by cosmological evolution and to investigate their properties. The PCCS reaches distances about a factor 1.55 larger, increasing the explored volume by about a factor of 3.7. COoE+ will reach still further, allowing us to explore a volume a factor of about 17 (1.5 m option) or of about 8 (1 m option) larger than the PCCS. Spectroscopic or photometric redshifts will be available for essentially all these galaxies thanks to the large area surveys from the ground and to the Euclid slitless spectroscopy. This will make analyses of the properties of galaxies (luminosity functions, dust mass functions, star formation rate functions, dust temperature distributions, etc.) possible across the Hubble sequence.

But the main improvement of COoE+ compared to Planck observations in total intensity will be in early galaxy evolution. The sub-mm surveys by COoE+ will cover the gap between the essentially Euclidean portion of the counts measured by Planck and the steep one measured by the deep SPT and Herschel surveys. The transition between the two portions provides a strong test for galaxy evolution models. Many of them predict it to be smoother than suggested by present data, but only the much better COoE+ statistics will make possible to reach firm conclusions.

Planck has already found high-redshift galaxies with extreme intrinsic IR luminosities, further boosted by strong gravitational lensing. Hyperluminous IR galaxies are very interesting in themselves since have always posed one of the strongest challenges to semi-analytic galaxy evolution models. Based on our simulations we expect that COoE+ will detect, at 600 GHz, from about 321 sr$^{-1}$ (1 m option) to about 992 sr$^{-1}$ (1.5 m option) strongly lensed galaxies, i.e. from a few to several thousands of such objects in the high Galactic latitude sky. Such large samples are of extraordinary astrophysical value in many fields because the magnification makes galaxies that are intrinsically too faint become observable, while the corresponding increase of the apparent size makes it possible to measure the internal structure to levels otherwise unattainable. Optical spectroscopy of galaxies acting as lenses can be exploited to measure the mass distribution of their dark matter halos as a function of redshift. This will allow a direct test on the evolution of large-scale structure. Samples of thousands of strongly lensed galaxies are essential for many other astrophysical and cosmological applications, e.g. to measure cosmological parameters. Essentially all high-$z$ dusty galaxies will be strongly lensed. The other sources detected at high frequencies will be easily recognizable low-$z$ late-type galaxies, plus a small group of radio sources, also easily identifiable in low frequency radio catalogs. The selection of strongly lensed galaxies will then be extremely easy.

At sub-mm wavelengths fluctuations on scales of several arcmin are dominated by clustering of high-$z$ massive galaxies, interpreted as progenitors of present day giant ellipticals. Sub-mm surveys are therefore optimally suited to look for proto-clusters at earlier redshifts than can be reached by optical/near-IR, X-ray and SZ surveys. Herschel observations of intensity peaks with red sub-mm colours, suggestive of high redshifts, in Planck high frequency maps have indeed shown that several of them represent the joint emission from multiple starbursts at high $z$, as expected for a young galaxy cluster.
Again, COrE+ can do much better than Planck. Its high sensitivity sub-mm maps, filtered at several arcmin resolution corresponding to proto-clusters scales at $z = 1–3$, will be a powerful tool to carry out unbiased searches of the long-sought galaxy proto-clusters whose intergalactic gas has not yet reached the virial temperature (i.e. is not detectable in X-rays or via the SZ effect) and whose member galaxies are dust obscured, i.e. optically faint. Only very large area surveys, such as those by COrE+, can provide enough statistics on these rare objects. Higher sensitivity and higher angular resolution follow-up (sub-)mm measurements will assess the nature of candidate proto-clusters. This would help to solve a central problem in cosmology: how did the large scale structure of galaxies form?

COrE+ will also measure, in a uniform way, the CIB power spectrum over an unprecedented range of frequencies and of angular scales (from $\sim 1'$ to tens of degrees), thus breaking the degeneracy between the Poisson contribution and that of non-linear effects (one-halo term), that complicated the interpretation of Planck measurements, without resorting to external (e.g. Herschel or SPT) data.

Moreover, COrE+ will extend the counts of radio sources compared to Planck, most notably at high frequencies, where very little is known. Above 217 GHz the counts will be determined for the first time over a substantial flux density range with good statistics. This will enable the first investigation of the (sub)-mm spectral energy distribution (SED) and of the evolutionary properties of radio sources at (sub-)mm wavelengths. The vast majority of these sources are expected to be blazars, and the accurate determination of their spectra will help to understand how physical processes occurring along relativistic jets shape the SED. We will also get numerous samples of ‘extreme’ radio sources allowing us to investigate the rich phenomenology of radio sources at (sub-)mm wavelengths.

A spectacular advance will be made possible by the high COrE+ sensitivity in the blind detection of extragalactic sources in polarization. We estimate that COrE+ will detect up to a factor of 40 (1 m option) or of 160 (1.5 m option) more polarized radio sources than can be detected by Planck. We also expect, for the first time, blind samples of several tens (1 m option) to a few hundred (1.5 m option) of star forming galaxies detected in polarization in the “extragalactic zone”. These samples will open a new window for studies of the global properties of magnetic fields in star forming galaxies and of their relationships with SFRs.

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