Searching for tetraquarks on the lattice

S. Prelovsek¹, T. Draper², C.B. Lang³, M. Limmer³, K.-F. Liu², N. Mathur⁴ and D. Mohler⁵

¹ Department of Physics, University of Ljubljana and Jozef Stefan Institute, Slovenia.
² Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506, USA.
³ Institut für Physik, FB Theoretische Physik, Universität Graz, A-8010 Graz, Austria.
⁴ Department of Theoretical Physics, Tata Institute of Fundamental Research, Mumbai, India.
⁵ 4004 Wesbrook Mall Vancouver, BC V6T 2A3 Canada, Canada

DOI: will be assigned

We address the question whether the lightest scalar mesons $\sigma$ and $\kappa$ are tetraquarks. We present a search for possible light tetraquark states with $J^{PC} = 0^{++}$ and $I = 0, 1/2, 3/2, 2$ in the dynamical and the quenched lattice simulations using tetraquark interpolators. In all the channels, we unavoidably find lowest scattering states $\pi(k)\pi(-k)$ or $K(k)\pi(-k)$ with back-to-back momentum $k = 0, 2\pi/L, \cdots$. However, we find an additional light state in the $I = 0$ and $I = 1/2$ channels, which may be related to the observed resonances $\sigma$ and $\kappa$ with a strong tetraquark component. In the exotic repulsive channels $I = 2$ and $I = 3/2$, where no resonance is observed, we find no light state in addition to the scattering states.

It is still not known whether the lightest observed nonet of scalar mesons $\sigma$, $\kappa$, $a_0(980)$ and $f_0(980)$ [2] are conventional $qq$ states or exotic tetraquark $qqqq$ states. Tetraquark interpretation was proposed by Jaffe back in 1977 [1] and it is supported by many phenomenological studies, for example [2, 3]. The tetraquarks, composed of a scalar diquark and anti-diquark, form a flavor nonet and are expected to be light. The observed ordering $m_{\kappa} < m_{a_0(980)}$ favors tetraquark interpretation since the $I = 1$ state [$\bar{s}d$][us] with additional valence pair $\bar{s}s$ is naturally heavier than the $I = 1/2$ state [$\bar{s}d$][du].

It is important to determine whether QCD predicts any scalar tetraquark states below 1 GeV from a first principle lattice QCD calculation. Previous lattice simulations [4, 5] have not given the final answer yet. The strongest claim for $\sigma$ as tetraquark was obtained using the sequential Bayes method to extract the spectrum [4] and needs confirmation using a different method. Our new results, given in this proceeding, are presented with more details in [6, 7].

We calculate the energy spectrum of scalar tetraquark states with $I = 0, 2, 1/2, 3/2$ in dynamical and quenched lattice simulations. Our dynamical simulation ($a \approx 0.15$ fm, $V = 16^3 \times 32$) uses dynamical Chirally Improved $u/d$ quarks [8] and it is the first dynamical simulation intended to study tetraquarks. The quenched simulation ($a \approx 0.20$ fm, $V = 16^3 \times 28$) uses overlap fermions, which have exact chiral symmetry even at finite $a$.

The energies of the lowest three physical states are extracted from the correlation functions

$$C_{ij}(t) = \langle 0 | O_i(t) O^\dagger_j(0) | 0 \rangle_{\beta = 0} = \sum_n Z_n^i Z_n^* e^{-E_n t}$$

with tetraquark interpolators $O \sim \bar{q}qqq$, where $Z_n^i \equiv \langle 0 | O_i | n \rangle$. In all the channels we use three different interpolators that are products of two color-singlet currents [6]. In addition, we use two types of diquark anti-diquark interpolators in $I = 0, 1/2$ channels [6].

When calculating the $I = 0, 1/2$ correlation matrix, we neglect the so-called single and double disconnected quark contractions [5], as in all previous tetraquark studies. The resulting

508 LP09
states have only a $\bar{q}q\bar{q}q$ Fock component in this approximation, while they would contain also a $\bar{q}q$ component if single disconnected contractions were taken into account [5]. Since we are searching for “pure” tetraquark states in this pioneering study, our approximation is physically motivated.

All physical states $n$ with given $J^{PC} = 0^{++}$ and $I$ propagate between the source and the sink in the correlation functions. Besides possible tetraquark states, there are unavoidable contributions from scattering states $\pi(k)\pi(-k)$ for $I = 0, 2$ and scattering states $\pi(k)K(-k)$ for $I = 1/2, 3/2$. Scattering states have discrete momenta $\vec{k} = \frac{2\pi}{L}\vec{j}$ on the lattice of size $L$ and energy $(m_{\pi}^2 + \vec{k}^2)^{1/2} + (m_{\pi,K}^2 + \vec{k}^2)^{1/2}$ in the non-interacting approximation. Our main question is whether we find some light state in addition the scattering states in $I = 0, 1/2$ channels. If such a state is found, it could be related to the resonances $\sigma$ or $\kappa$ with a strong tetraquark component.

The energies $E_n$ are extracted from the correlation functions $C_{ij}(t)$ via the eigenvalues $\lambda^\alpha(t) \propto e^{-E_{\alpha}(t-t_0)}$ of the generalized eigenvalue problem $C(t)\vec{u}^\alpha(t) = \lambda^\alpha(t,t_0)C(t_0)\vec{u}^\alpha$ at some reference time $t_0$ [9].

Figure 1: The resulting spectrum $E_n$ for $I = 0, 2, 1/2, 3/2$ in the dynamical (left) and the quenched (right) simulations. Note that there are two states (black and red) close to each other in $I = 0$ and $I = 1/2$ cases. The lines at $I = 0, 2$ present the energies of non-interacting $\pi(k)\pi(-k)$ with $k = j\frac{2\pi}{L}$ and $j = 0, 1, \sqrt{2}$. Similarly, lines at $I = 1/2, 3/2$ present energies of $\pi(k)K(-k)$.
The resulting spectrum $E_n$ for all four isospins is shown in Fig. 1. The lines present the energies of the scattering states in the non-interacting approximation. Our dynamical and quenched results are in qualitative agreement.

In the repulsive channel $I = 2$, where no resonance is expected, we indeed find only the candidates for the scattering states $\pi(0)\pi(0)$ and $\pi(\pm \pi)L\pi(\mp \pi)L$ with no additional light state. The first excited state is higher than expected due to the smallness of $3 \times 3$ basis. Similar conclusion applies for the repulsive $I = 3/2$ channel with $\pi K$ scattering states.

In the attractive channel $I = 0$ we find two (orthogonal) states close to the threshold $2m_\pi$ and another state consistent with $\pi(\pm \pi)L\pi(\mp \pi)L$, so we do find an additional light state. This leads to a possible interpretation that one of the two light states is the scattering state $\pi(0)\pi(0)$ and the other one corresponds to $\sigma$ resonance with strong tetraquark component (see more general discussion in [10]). In the attractive $I = 1/2$ channel we similarly find the candidates for the lowest two $\pi(k)K(-k)$ scattering states and a candidate for a $\kappa$ resonance with a large tetraquark component. These results have to be confirmed by another independent lattice simulation before making firm conclusions.

We investigate two criteria for distinguishing the one-particle (tetraquark) and two-particle (scattering) states in [7]. The first criteria is related to the time dependence of $C_{ij}(t)$ and $\lambda^n(t)$ at finite temporal extent of the lattice. The second is related to the volume dependence of the couplings $\langle 0|O_i|n\rangle$.

The ultimate method to study $\sigma$ and $\kappa$ on the lattice would involve the study of the spectrum and couplings in presence of $\bar{q}qq \leftrightarrow \bar{q}q \leftrightarrow \text{vac} \leftrightarrow \text{glue}$ mixing, using interpolators that cover these Fock components. Such a study has to be done as a function of lattice size $L$ in order to extract the resonance mass and width using the Lüscher’s finite volume method [10, 11].

Acknowledgments

This work is supported by the Slovenian Research Agency, by the European RTN network FLAVIAnet (contract MRTN-CT-035482), by the Slovenian-Austrian bilateral project (contract BI-AT/09-10-012), by the USA DOE Grant DE-FG05-84ER40154, by the Austrian grant FWF DK W1203-N08 and by Natural Sciences and Engineering Research Council of Canada.

References

[1] R. L. Jaffe, Phys. Rev. D 15 (1977) 267 and 281; R. L. Jaffe, Exotica, hep-ph/0409065.
[2] Note on the scalar mesons, C. Amsler et al., Review of Particle Physics, Phys. Lett. B667 (2008) 1.
[3] L. Maiani et al., Phys. Rev. Lett. 93 (2004) 212002; G. ’t Hooft et al., Phys. Lett. B 662 (2008) 424; Hee-Jung Lee, N.I. Kochelev, Phys. Rev. D78 (2008) 076005.
[4] N. Mathur et al., χQCD collaboration, Phys. Rev. D76 (2007) 114505.
[5] S. Prelovsek and D. Mohler, Phys. Rev. D79 (2009) 014503; M. Alford and R. Jaffe, Nucl. Phys. B578 (2000) 367; H. Suganuma et al., Prog. Theor. Phys. Suppl. 168 (2007) 168; M. Loan, Z. Luo, Y. Y. Lam, Eur. Phys. J. C57 (2008) 579.
[6] S. Prelovsek, T. Draper, C.B. Lang, M. Limmer, K.-F. Liu, N. Mathur and D. Mohler, arXiv: 0909:5134, PoS LAT2009 (2009) 103.
[7] S. Prelovsek, T. Draper, C.B. Lang, M. Limmer, K.-F. Liu, N. Mathur and D. Mohler, to be published.
[8] C. Gattringer, C. Hagen, C.B. Lang, M. Limmer, D. Mohler and A. Schäfer, Phys. Rev. D79 (2009) 054501.
[9] M. Lüscher and U. Wolff, Nucl. Phys. B339 (1990) 222; B. Blossier et al., JHEP 0904 (2009) 094.
[10] S. Sasaki and T. Yamazaki, Phys. Rev. D74 (2006) 114507, PoS LAT2007 (2007) 131.
[11] M. Lüscher, Comm. Math. Phys. 104 (1986) 177; Nucl. Phys. B354 (1991) 531; Nucl. Phys. B364 (1991) 237; Zhi-Yuan Niu, Ming Gong, Chuan Liu, Yan Shen, Phys. Rev. D80 (2009) 114509.