Provenance for Regular Path Queries

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1 Introduction

It has been recognized that the result of a database query should be annotated with \textit{provenance}, i.e. information about how, why, where, with what level of certainty or security clearance, etc a particular fact of the query was derived. The seminal paper by Green, Tannen and Karvounarakis\textsuperscript{4} convincingly showed that all major forms of provenance can be uniformly captured within the algebraic framework of \textit{semirings}. Green et al. show that a suitably semiring-annotated positive (negation-free) relational algebra and datalog can capture the provenance of query results. Furthermore, the various database semirings form a partial order where coarser ("smaller") semirings can be obtained as homomorphic images of semirings with a finer grain of information. Green et al. also show that the annotated positive relational algebra and datalog form congruences within their semiring hierarchy.

\textit{Regular path queries} (RPQs) is the ubiquitous mechanism for querying graph databases\textsuperscript{1}. RPQs are in essence regular expressions over the edge symbols. The answer to an RPQ on a given graph database is the set of pairs of objects \((a, b)\), which are connected by paths spelling words in the language of the regular path query. An annotated pair in the answer would naturally contain the set of words that spell paths between \(a\) and \(b\). However, a finer grain of provenance can be obtained by annotating the words with the intermediate vertices of each path spelling the word.

Since graph databases have their roots in automata theory, and automata have their roots in the algebraic theory of semiring-automata\textsuperscript{8}, an investigation into how the provenance algebra of Green et al. can be paired with the algebra of semiring-automata is called for. The paper at hand represents a first step in this direction.

2 Databases and Regular Path Queries

We consider a database to be an edge-labeled graph. Intuitively, the nodes of the database graph represent objects and the edges represent relationships between the objects. The edge labels are drawn from a finite alphabet \(\Delta\). Elements of \(\Delta\) will be denoted \(r, s, \ldots\). As usual, \(\Delta^*\) denotes the set of all finite words over \(\Delta\). Words will be denoted by \(u, w, \ldots\). We also assume that we have a universe of objects, and objects will be denoted \(a, b, c, \ldots\).
Associated with each edge is a weight expressing the “strength” of the edge. Such a “strength” can be multiplicity, cost, distance, etc, and is expressed by an element of some semiring \( \mathcal{R} = (R, \oplus, \otimes, 0, 1) \) where

1. \((R, \oplus, 0)\) is a commutative monoid with 0 as the identity element for \( \oplus \).
2. \((R, \otimes, 1)\) is a monoid with 1 as the identity element for \( \otimes \).
3. \(\otimes\) distributes over \(\oplus\): for all \(x, y, z \in R\),
   \[
   (x \oplus y) \otimes z = (x \otimes z) \oplus (y \otimes z),
   \]
   \[
   z \otimes (x \oplus y) = (z \otimes x) \oplus (z \otimes y).
   \]
4. 0 is an anihilator for \(\otimes\): \(\forall x \in R, x \otimes 0 = 0 \otimes x = 0\).

For simplicity we will blur the distinction between \(\mathcal{R}\) and \(R\), and will only use \(\mathcal{R}\) in our development.

In this paper, we will in addition require for semirings to have a total order \(\preceq\). If \(x \preceq y\), we say that \(x\) is better than \(y\). “Better” will have a clear meaning depending on the context.

Now, a database \(D\) is formally a graph \((V,E)\), where \(V\) is a finite set of objects and \(E \subseteq V \times \Delta \times \mathcal{R} \times V\) is a set of directed edges labeled with symbols from \(\Delta\) and weighted by elements of \(\mathcal{R}\).

We concatenate the labels along the edges of a path into words in \(\Delta^*\). Also, we aggregate the weights along the edges of a path by using the multiplication operator \(\otimes\). Formally, let \(\pi = (a_1, r_1, x_1, a_2), \ldots, (a_n, r_n, x_n, a_{n+1})\) be a path in \(D\). We define the start, the end, the label, and the weight of \(\pi\) to be

\[
\alpha(\pi) = a_1, \quad \beta(\pi) = a_{n+1}, \quad \lambda(\pi) = r_1 \cdots r_n \in \Delta^*, \quad \kappa(\pi) = x_1 \otimes \cdots \otimes r_n \in \mathcal{R}
\]

respectively.

A regular path query (RPQ) is a regular language over \(\Delta\). For the ease of notation, we will blur the distinction between regular languages and regular expressions that represent them. Let \(Q\) be an RPQ and \(D = (V,E)\) a database. Now let \(a\) and \(b\) be two objects in \(D\), and \(w \in \Delta^*\). We define

\[
\Pi_{w,D}(a,b) = \{ \pi \text{ in } D : \alpha(\pi) = a, \beta(\pi) = b, \lambda(\pi) = w \},
\]

\[
\Pi_{Q,D}(a,b) = \bigcup_{w \in Q} \Pi_{w,D}(a,b).
\]

Then, the answer to \(Q\) on \(D\) is defined as

\[
\text{Ans}(Q,D) = \{ [(a,b), x] \in (V \times V) \times \mathcal{R} : \Pi_{Q,D}(a,b) \neq \emptyset \text{ and } x = \oplus \{ \kappa(\pi) : \pi \in \Pi_{Q,D}(a,b) \} \}.
\]
If \([(a, b), x] \in \text{Ans}(Q, D)\), we say that \((a, b)\) is an answer of \(Q\) on \(D\) with weight \(x\).

Let \(w \in Q\). Suppose \(\Pi_w D(a, b) \neq \emptyset\). Clearly, \((a, b)\) is an answer of \(Q\) on \(D\) with some weight \(x\). We say \(w\) is the \(basis\) of a “reason” for \((a, b)\) to be such an answer. This basis has obviously a weight (or strength) coming with it, namely \(y = \oplus \Pi_w D(a, b)\). We say that \((w, y)\) is a \(reason\) for \((a, b)\) to be an answer of \(Q\) on \(D\). In general, there can be many such reasons. We denote by

\[ \Xi_{Q, D}(a, b) \]

the set of reasons for \((a, b)\) to be an answer of \(Q\) on \(D\). It can be seen that \(x = \oplus \{y : (w, y) \in \Xi_{Q, D}(a, b)\}\).

In the rest of the paper we will be interested in determining whether a pair \((a, b)\) has “the same or stronger” reasons than another pair \((c, d)\) to be in the answer of \(Q\) on \(D\).

Evidently, \(\Xi_{Q, D}(a, b) \subseteq \Delta^* \times \mathcal{R}\), but we have a stronger property for \(\Xi_{Q, D}(a, b)\). It is a partial function from \(\Delta^*\) to \(\mathcal{R}\). We complete \(\Xi_{Q, D}(a, b)\) to be a function by adding \(0\)-weighted reasons.

An \(\mathcal{R}\)-annotated language (AL) \(L\) over \(\Delta\) is a function

\[ L : \Delta^* \to \mathcal{R}. \]

Frequently, we will write \((w, x) \in L\) instead of \(L(w) = x\). From the above discussion, \(\Xi_{Q, D}(a, b)\) is such an AL.

Given two \(\mathcal{R}\)-ALs \(L_1\) and \(L_2\), we say that \(L_1\) is \(contained\) in \(L_2\) iff \((w, x) \in L_1\) implies \((w, y) \in L_2\) and \(x \preceq y\).

Now, we say that \(\Xi_{Q, D}(a, b)\) is \(the same or stronger\) than \(\Xi_{Q, D}(c, d)\), iff,

\[ \Xi_{Q, D}(a, b) \preceq \Xi_{Q, D}(c, d). \]

It might seem strange to use \(\preceq\) to say “stronger”, but we are motivated by the notion of distance in real life. The shorter this distance, the stronger the relationship between two objects (or subjects) is.

If \(\Xi_{Q, D}(a, b) \preceq \Xi_{Q, D}(c, d)\) and \(\Xi_{Q, D}(c, d) \preceq \Xi_{Q, D}(a, b)\), we say that \((a, b)\) and \((c, d)\) are in the answer of \(Q\) on \(D\) for \(exactly\) the same reasons and write

\[ \Xi_{Q, D}(a, b) = \Xi_{Q, D}(c, d). \]

### 3 Computing Reason Languages

An \(annotated\) \(automaton\) \(A\) is a quintuple \((P, \Delta, \mathcal{R}, \tau, p_0, F)\), where \(\tau\) is a subset of \(P \times \Delta \times \mathcal{R} \times P\). Each annotated automaton \(A\) defines an AL, denoted by 
\([A]\) and defined by

\[ [A] = \{(w, x) \in \Delta^* \times R : w = r_1 r_2 \ldots r_n, x = \oplus \{ \otimes_{i=1}^n x_i : (p_{i-1}, r_i, x_i, p_i) \in \tau, p_n \in F\}\}. \]
An AL $L$ is a regular annotated language (RAL), if $L = [A]$, for some semiring automaton $A$.

Given an RPQ $Q$, a database $D$, and a pair $(a, b)$ of objects in $D$, it turns out that the reason language $\Xi_{Q, D}(a, b)$ is RAL.

An annotated automaton for $\Xi_{Q, D}(a, b)$ is constructed by computing a “lazy” Cartesian product of a (classical) automaton $Q$ for $Q$ with database $D$. For this we proceed by creating state-object pairs from the query automaton and the database. Starting from object $a$ in $D$, we first create the pair $(p_0, a)$, where $p_0$ is the initial state in $Q$. We then create all the pairs $(p, b)$ such that there exist a transition $t$ from $p_0$ to $p$ in $Q$, and an edge $e$ from $a$ to $b$ in $D$, and the labels of $t$ and $e$ match. The weight of this edge is set to be the weight of edge $e$ in $D$.

In the same way, we continue to create new pairs from existing ones, until we are not anymore able to do so. In essence, what is happening is a lazy construction of a Cartesian product graph of $Q$ and $D$. Of course, only a small (hopefully) part of the Cartesian product is really constructed depending on the selectivity of the query. The implicit assumption is that this part of the Cartesian product fits in main memory and each object is not accessed more than once in secondary storage.

Let us denote by $C_{Q, D}(a, b)$ the above Cartesian product. We can consider $C_{Q, D}(a, b)$ to be a weighted automaton with initial state $(a, p_0)$ and set of final states $\{(p, b) : p \in F_Q\}$, where $F_Q$ is the set of final states of $Q$. It is easy to see that $\Xi_{Q, D}(a, b) = [C_{Q, D}(a, b)]$.

4 Some Useful Semirings

We will consider the following semirings in this paper.

- **boolean** $\mathcal{B} = (\{T, F\}, \lor, \land, F, T)$
- **tropical** $\mathcal{T} = (\mathbb{N} \cup \{\infty\}, \min, +, \infty, 0)$
- **fuzzy** $\mathcal{F} = (\mathbb{N} \cup \{\infty\}, \min, \max, \infty, 0)$
- **multiplicity** $\mathcal{M} = (\mathbb{N}, +, \cdot, 0, 1)$

$T$ and $F$ stand for “true” and “false” respectively, and $\lor, \land$ are the usual “and” and “or” Boolean operators. On the other hand, $\min, \max, +, \cdot$ are the usual operators for integers.

It is easy to see that a Boolean annotated automaton $A = (P, \Delta, \mathcal{B}, \tau, p_0, F)$ is indeed an “ordinary” finite state automaton $(P, \Delta, \tau, p_0, F)$, and a RAL over $\mathcal{B}$ is a an “ordinary” regular language over $\Delta$. In this case it can be seen that

$$\Xi_{Q, D}(a, b) \preceq \Xi_{Q, D}(c, d) \iff \Xi_{Q, D}(a, b) \supseteq \Xi_{Q, D}(c, d).$$

Since the containment of regular languages is decidable, we have that the provenance problem is decidable in the case of semiring $\mathcal{B}$.

For semiring $\mathcal{F}$ we show later that the problem is decidable.

On the other hand, for semirings $\mathcal{T}$ and $\mathcal{M}$ the problem is unfortunately undecidable. For these results we refer to [7] and [2], respectively. [7] shows an
even stronger result that the problem of RAL equivalence, which is \( L_1 \preceq L_2 \) and \( L_2 \preceq L_1 \) at the same time, is undecidable. On the other, it is interesting to note that for the case of \( \mathcal{A} \), only the containment problem is undecidable, whereas the equivalence is in fact decidable in polynomial time via a reduction to a linear algebra problem [2].

5 Spheres and Stripes

Let \( L \) be an annotated language over a semiring \( \mathcal{R} \). We have

**Definition 1.** Let \( x \in \mathcal{R} \).

1. The \( x \)-inner sphere of \( L \) is
   \[
   L^x = \{(w, y) \in \Delta^* \times \mathcal{R} : (w, y) \in L \text{ and } y \preceq x\}.
   \]

2. The \( x \)-outer sphere of \( L \) is
   \[
   L^\bar{x} = \{(w, y) \in \Delta^* \times \mathcal{R} : (w, y) \in L \text{ and } x \preceq y\}.
   \]

3. The \( x \)-stripe of \( L \) is
   \[
   L^\dot{x} = \{(w, y) \in \Delta^* \times \mathcal{R} : (w, y) \in L \text{ and } y = x\}.
   \]

We now give the following characterization theorem [3].

**Theorem 1.** Let \( L_1 \) and \( L_2 \) be two annotated languages over a discrete semiring \( \mathcal{R} \). Then, \( L_1 \preceq L_2 \), if and only if,

1. \([L_1]\subseteq [L_2]\),
2. \([L_2^\dot{x}] \cap [L_1] \subseteq [L_1^\dot{x}]\), for each element \( x \) of \( \mathcal{R} \).

**Proof.** If. Let \((w, x) \in L_1 \). By condition (1), \( w \in [L_2]\), and thus, there exists \( y \) in \( \mathcal{R} \), such that \((w, y) \in L_2 \). Now, we want to show that \( y \preceq x \). For this, observe that \( w \in [L_2^\dot{y}] \) and since also \( w \in [L_1] \), we have \( w \in [L_2^\dot{y}] \cap [L_1] \).

By condition (2), \([L_2^\dot{y}] \cap [L_1] \subseteq [L_1^\dot{y}]\), i.e. \( w \in [L_1^\dot{y}] \). The latter means that \((w, x) \in L_1^\dot{y}\), i.e. \( y \preceq x \).

Only. If \( L_1 \subseteq \preceq L_2 \), then, clearly, condition (1) directly follows. Now, let \( w \in [L_2^\dot{y}] \cap [L_1] \), for some \( y \) in \( \mathcal{R} \). From this, we have that \((w, y) \in L_2 \) and \((w, x) \in L_1 \) for some \( x \) in \( \mathcal{R} \). By the fact that \( L_1 \preceq L_2 \), \( y \preceq x \). Thus, \((w, x) \in L_2^\dot{y}\), which in turn means that \( w \in [L_1^\dot{y}] \). Since \( y \) was arbitrary, we have that condition (2) is satisfied as well. \( \square \)

Observe that that conditions (1) and (2) of Theorem 1 are about containment checks of pure languages that we obtain if we ignore the weight of the words in the corresponding annotated languages. These containments are decidable when \( L_1 \) and \( L_2 \) are RALs.
We have the following useful equalities

\[
\lfloor L^x \rfloor = ([L] \setminus \lfloor L^x \rfloor) \cup \lfloor L^\dot{x} \rfloor
\]

(1)

\[
\lceil L^x \rceil = ([L] \setminus \lceil L^x \rceil) \cup \lceil L^\dot{x} \rceil.
\]

(2)

We also define here the notion of “discrete” semirings.

**Definition 2.** A semiring \( \mathcal{R} = (R, \oplus, \otimes, 0, 1) \) is said to be discrete iff for each \( x \neq 0 \) in \( R \) there exists \( y \) in \( R \), such that

1. \( x \prec y \), and
2. there does not exist \( z \) in \( R \), such that \( x \prec z \prec y \).

\( y \) is called the next element after \( x \), whereas \( x \) is called the previous element before \( y \).

Observe that all the semirings we list in Section 4 are discrete.

For all the discrete semirings, we can compute \( L^\dot{x} \) by computing \( ([L] \setminus \lfloor L^x \rfloor) \cup \lfloor L^\dot{x} \rfloor \) or \( ([L] \setminus \lceil L^x \rceil) \cup \lceil L^\dot{x} \rceil \) where \( u \) is the previous element before \( x \). [Initially, \( \lfloor L^\dot{1} \rfloor = \lfloor L^1 \rfloor \).]

For such semirings then, in order to decide \( L_1 \preceq L_2 \) based on Theorem 1, we need to be able to compute either inner or outer spheres.

Nevertheless, Theorem 1 does not necessarily give a decision procedure for \( L_1 \preceq L_2 \) when semirings \( \mathcal{T} \) and \( \mathcal{N} \) are considered, even if \( L_1 \) and \( L_2 \) are RALs. This is because the number of inner (and outer) spheres might be infinite for these semantics.

Interestingly, Theorem 1 gives an effective procedure for deciding \( L_1 \preceq L_2 \) when the fuzzy semiring \( \mathcal{F} \) is considered, and \( L_1 \) and \( L_2 \) are RALs. This is true because for this semiring, the number of inner-spheres for each RAL \( L \) is finite; this number is bounded by the number of transitions in an annotated automaton for \( L \).

Regarding the \( \mathcal{T} \) and \( \mathcal{N} \) semirings, the number of spheres is finite, if and only if, the languages are bounded, that is, there is bound or limit on the weight each word can have. Fortunately, the boundedness for RALs over \( \mathcal{T} \) and \( \mathcal{N} \) is decidable.

For a RAL \( L \) over \( \mathcal{T} \), determining whether there exists a bound coincides with deciding the “limitedness” problem for “distance automata”. The later problem is widely known and positively solved in the literature (cf. for example [5,9,12,6]). The best algorithm is by [9], and it runs in exponential time in the size of an AL recognizing \( L \). If \( L \) is bounded, then the bound is \( 2^{4n^3+n\lg(n+2)+n} \), where \( n \) is the number of states in an AL recognizing \( L \).

For a RAL \( L \) over \( \mathcal{N} \), determining whether there exists a bound is again decidable [14]. This can be done in polynomial time. However, if \( L \) is bounded, the bound is \( 2^{n \lg n + 2.0566n} \), where \( n \) is the number of states in an AL recognizing \( L \).

6 Computing Spheres

6.1 Tropical Semiring

In this section we present an algorithm, which for any given number \( k \in \mathbb{N} \) constructs the \( k \)-th inner-sphere \( L^k \) of a RAL \( L \).
For this, we build a mask automaton $M_k$ on the alphabet $K = \{0, 1, \ldots, k\}$, which formally is as follows: $M_k = (P_k, K, \tau_k, p_0, F_k)$, where $P_k = F_k = \{p_0, p_1, \ldots, p_k\}$, and

$$\tau_k = \{(p_i, n, p_{i+n}) : 0 \leq i \leq k, \text{ and } 0 \leq n \leq k - i\}.$$ 

As an example, we give $M_3$ in Fig. 1. The automaton $M_k$ has a nice property. It captures all the possible paths (unlabeled with respect to $\Delta$) with weight equal to $k$.

It can be shown that

**Theorem 2.** $M_k$ contains all the possible paths $\pi$ with weight $|\pi| \leq k$, and it does not contain any path with weight greater than $k$.

It can be easily seen that the size of automaton $M_k$ is $O(k^2)$. Now by using $M_k$, we can extract from an annotated automaton $A$ for $L$ all the transition paths with a weight less or equal to $k$, giving so an effective procedure for computing the $k$-th sphere $L^k$.

For this, let $A = (P_A, \Delta, \tau_A, q_0, F_A)$ be an annotated automaton for $L$. We construct a Cartesian product automaton

$$C_k = A \times M_k = (P_A \times P_k, \Delta, \tau, (q_0, p_0), F_A \times F_k),$$

where $\tau = \{((q, r), n, (q', p')) : (q, r, n, q') \in \tau_A \text{ and } (p, n, p') \in \tau_k\}$. It can be verified that

**Theorem 3.** $[C_k] = L^k$.

### 6.2 Fuzzy Semiring

In order to compute $L^k$, where $L$ is a RAL, and $k \in \mathbb{N}$, we simply build an annotated automaton $A$ for $L$, and then throw out all the transitions weighted by more than $k$. Let $A'$ be the annotated automaton thus obtained. It can be verified that

**Theorem 4.** $[A'] = L^k$. 
6.3 Multiplicity Semiring

One can indeed derive a method for computing inner or outer spheres for languages over \( \mathcal{A} \) using complex results spread out in several chapters of [2] and [11]. We will follow here instead a different, much simpler approach based on ordinary automata. This approach computes outer spheres.

Let \( A = (P_A, \Delta, \tau_A, p_{A,0}, F_A) \) be a weighted automaton for \( L \). From \( A \) we obtain an “ordinary automaton” \( B = (P_B, \Delta, \tau_B, p_{B,0}, F_B) \), where \( P_B = P_A \), \( p_{B,0} = p_{A,0} \), \( F_B = F_A \), and

\[
\tau_B = \{(p, r, p'), \ldots, (p, r, p') : (p, r, p', n) \in \tau_B \}
\]

We can show that

**Theorem 5.** \((w, k) \in [A] \) if and only if, \( B \) has \( k \) accepting transition paths that spell \( w \).

The importance of this theorem is that we now transformed the problem of computing inner or outer spheres of \( L \) into the problem of computing the sets of words spelled out in \( B \) by a number of accepting transition paths, which is greater or smaller than the sphere index. For simplicity, we will focus here on outer-spheres.

Interestingly, the set of all the words spelled out by at least \( k \) accepting transition paths in \( A \) is indeed computable. For this, we present a simple construction which was hidden as an auxiliary construction in [13].

The construction is as follows. Let \( \Psi_k \) be the set of \( k \times k \) Boolean matrices. We build a Cartesian product automaton

\[
B^k = (P^k_B, \Delta, \tau^k_B, p^k_{B,0}, F^k_B)
\]

where

\[
P^k_B = P_B \times \ldots \times P_B \times \Psi_k
\]

\[
p^k_{B,0} = (p_{B,0}, \ldots, p_{B,0}, \psi), \quad \text{where} \quad \psi[i, j] = 0 \text{ for } i \neq j, \quad \text{and} \quad \psi[i, j] = 1 \text{ for } i = j
\]

\[
\tau^k_B = \{(p_1, \ldots, p_k, \psi), r, (p'_1, \ldots, p'_k, \psi') : (p_i, r, p'_i) \in \tau_B, \text{ for } i \in [1, k], \text{ and} \quad \psi'[i, j] = 1 \text{ if } \psi[i, j] = 1 \text{ or } s_i \neq s_j \}
\]

\[
F^k_B = F_B \times \ldots \times F_B \times \{\psi^*\}, \quad \text{where} \quad \psi^*[i, j] = 1, \text{ for } i, j \in [1, k].
\]

Let \( w \) be a word spelled by \( k \) or more transition paths in \( B \). Let \( \rho_1, \ldots, \rho_k \) be \( k \) of these transition paths. In the Cartesian product automaton \( B^k \) we will have a transition path \( \rho \) that corresponds to the combination of \( \rho_1, \ldots, \rho_k \). It can be verified that the last state of \( \rho \) in \( B^k \) will have its matrix equal to \( \psi^* \). This is because since \( \rho_i \) is different from \( \rho_j \), for each \( i, j \in [1, k] \), at some point the matrix of some state in \( \rho \) will have 1 for its \( i, j \) entry. Then for each subsequent
state in \( \rho \), the corresponding matrix will retain 1 in its \( i, j \) entry. Therefore, if \( \rho_1, \ldots, \rho_k \) are accepting transition paths, then \( B^k \) will accept \( w \). Considering \( B^k \) From all the above we have

**Theorem 6.** A word \( w \) is accepted by \( B^k \), if and only if, there are at least \( k \) accepting paths spelling \( w \) in \( B \).

From this theorem and Theorem 11 we then have

**Theorem 7.** \( [L^k] = L(B^k) \).

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