Abstract. The differential cross section for $p(e, e'\omega)p$ has been studied at $Q^2 \sim 5.5 \text{(GeV/c)}^2$. Here $Q^2$ represents the four momentum squared of the virtual photon in the excitation of baryonic resonances by an electron projectile. In order to extract the $\omega$-meson differential cross section from the JLAB data, the data was compared to a full Monte Carlo simulation of the detector based on events generated for omega production in a way that the production cross section was varied to achieve a match to the data. The bin selected for this procedure takes into account the measure of robustness of the stripping of the $\omega$ peak from the multi-pion background as well as the statistics in the measured data and the Monte Carlo simulation of the signal and background physics. An error estimation technique for the cross section was based on determining the dependence of the extracted cross section parameters on the experimental set-up (including parameters for the spectrometer, target beam geometries and performance). We compare our results with a Regge-based model for hadronic content in the $t$-channel exchange of a photon in $Q^2$ region of overlap. There is an extension of this data into a completely new region, which is the highest yet measured.

1. Introduction
The extraction of the measured exclusive $\omega$ differential cross section at the highest achievable $Q^2$ values in the valence quark region has been performed. The importance of our study is that this average $Q^2$ of 5.5 (GeV/c)$^2$ falls in the region where the transition from low $Q^2$, low $t$ physics, where soft, non-perturbative QCD processes characterised by constituent quarks dominate, to the high $Q^2$, high $t$ regime where hard processes characterised by current quark correlations are expected to play an increasingly important role [1]. This paper describes the experiment, the extraction of the cross section from the data, the bin selection and the error estimation. Finally the data are discussed.

2. Experimental setup
The E01-002 experiment in Hall C at the Thomas Jefferson National Accelerator Facility (JLab) consists of an electron beam energy of 5.75 GeV incident on a hydrogen target, two spectrometers, associated electronics and software for reconstruction of events. The scattered electrons are detected using the Short Orbit Spectrometer (SOS) [2, 3], which is a resistive $QD\bar{D}$ (quadrupole, dispersive dipole, anti-dispersive dipole) spectrometer. The recoil protons are detected using the High Momentum Spectrometer (HMS), which is a $QQQ\bar{D}$ spectrometer. The $\omega$ particles were identified by the missing mass method which involves using event reconstruction and conservation laws. The instrumentation layout of the experiment can be found in Ref. [1].
3. Selection of data

The electron spectrometer was fixed in both angle and momentum therefore defining a central three-momentum transfer $q$. The direction of a boosted decay cone of protons is determined in turn by the vector $q$. In order to capture as much of this decay cone possible, the proton spectrometer was stepped in angle and momentum, with kinematics chosen such that adjacent settings overlapped in angle and momentum [4]. With such an approach, the systematic uncertainties associated with the imperfect knowledge of the spectrometer acceptance is reduced.

The kinematic settings for the experiment is shown in Table 1.

| Electron Arm | Electron Arm | Proton Arm | Proton Arm |
|--------------|--------------|------------|------------|
| $P_{\text{SOS}}$ [GeV/c] | $\theta_{\text{SOS}}$ [degrees] | $P_{\text{HMS}}$ [GeV/c] | $\theta_{\text{HMS}}$ [degrees] |
| 1.74 | 47.5 | 2.13 | 13.5, 16.5, 19.5, 22.5 |
| 1.74 | 47.5 | 2.23 | 18.0, 12.0, 15.0, 21.0 |
| 1.74 | 47.5 | 2.57 | 11.2, 13.5, 16.5, 19.5, 22.5 |
| 1.74 | 47.5 | 2.69 | 12.0, 15.0, 18.0, 21.0, 24.0 |
| 1.74 | 47.5 | 3.10 | 11.2, 13.5, 16.5, 19.5, 22.5 |
| 1.74 | 47.5 | 3.24 | 12.0, 15.0, 18.0, 21.0, 24.0 |
| 1.74 | 47.5 | 3.73 | 11.2, 13.5, 16.5, 19.5, 22.5 |
| 1.74 | 47.5 | 3.90 | 12.0, 15.0, 18.0, 21.0 |
| 1.74 | 47.5 | 4.50 | 11.2, 13.5, 16.5, 19.5 |
| 1.74 | 47.5 | 4.70 | 15.0, 18.0 |

Table 1. Spectrometer settings for the $Q^2 \simeq 5.36 \text{ GeV}^2/c^2$, $(E = 5.5 \text{ GeV})$ data

The SOS was used to separate the electrons from the negatively charged pions. This was done by using both a threshold gas Čerenkov detector and a lead-glass calorimeter. In the HMS, protons were separated from positively charged pions using a combination of coincidence time (the difference between the trigger times of two the spectrometers) and particle velocity, or equivalently the time of flight $\beta_{\text{tof}}$.

The Hall C event builder code called ENGINE [1] was used for the offline replay of the collected raw signals. ENGINE reads each of the events, reconstructs trajectories, and generates kinematical and particle identification information for each event. In essence, the replay ENGINE converts raw data into calibrated physical quantities on an event-by-event basis. The analysis codes operates on the ENGINE output files (HBOOK) and converts them to ROOT files. From this point on the data is analysed within the ROOT framework using C++ code.

In the process $p(e,e'p)\omega$, the kinematics of the undetected particle is reconstructed by calculating the missing mass squared $m^2_x$ from the conservation of the four-momentum. The extraction of the $\omega$ was accomplished by applying a cut on the $m^2_x$ spectrum around the omega peak and subtracting the background. To ensure the quality of the measurement, events of interest are selected by applying the set of ‘standard’ cuts to the data as listed in Table 2. Cuts such as the particle momentum deviation at both the SOS and HMS ($\delta_p$ and $\delta_{\theta}$) are applied to ensure that only particles within the well understood region of the spectrometer momentum acceptance are used. Also used, but not listed in Table 2, are cuts on the collimators at both spectrometers [5]. These are to ensure that the path of the reconstructed track of a detected particle can be traced back through acceptable regions of the collimator slits.

For the purpose of extraction of the cross section as indicated in Table 3, the data are binned in $W$, $\cos \theta_\omega$, $\phi_{cm}$ and $m^2_x$, where $W$ is the invariant mass of the hadronic system, $\theta_\omega$ is the polar angle between the direction of the $\omega$ and the three-momentum transfer vector $\vec{q}$ in the centre-of-mass of the resonance, $\phi_{cm}$ is the azimuthal angle of the $\omega$ with respect to the electron.
**Table 2.** Set of ‘standard’ cuts applied to the data and to the simulation where applicable. † The Particle Identification cuts were not applied to the simulation.

| Quantity | Cut | Purpose |
|----------|-----|---------|
| Coincidence time | $|t_{\text{coin}} - t_{\text{cent}}| \leq 1.5$ | Selecting proton |
| HMS particle momentum deviation, | $|\delta_h| \leq 9\%$ | HMS acceptance |
| SOS particle momentum deviation, | $-15\% \leq \delta_s \leq +20\%$ | SOS acceptance |
| SOS x position focal plane, | $-20 \text{ cm} \leq X_{\text{SOS,f.p}} \leq +22 \text{ cm}$ | SOS acceptance |
| $^{1}\text{SOS shower counter sum, } E_{\text{norm}}$ | $E_{\text{norm}} \geq 0.7$ | selecting electron |
| $^{1}\text{SOS Cerenkov number of photons, } N_{p.e.}$ | $N_{p.e.} \geq 0.5$ | selecting electron |

$^{1}\text{ Missing mass squared}$

$$m_\omega^2 \quad 0.56 \text{ GeV}^2 \leq m_\omega^2 \leq 0.66 \text{ GeV}^2$$

selecting $\omega$ particle

scattering plane, and $m_\omega^2$ is the square of the missing mass for $p(e, e'p)\omega$.

| Variable | Range | Bins |
|----------|-------|------|
| $W$ (GeV) | $1.72 \leq W \leq 1.92$ | 10 |
| $\cos \theta_\omega$ | $-1.0 \leq \cos \theta_\omega \leq 1.0$ | 5 |
| $\phi_\omega$ (rad) | $0 \leq \phi_\omega \leq 2\pi$ | 3 |

**Table 3.** $\omega$ analysis binning showing the ranges of $W$, $\cos \theta_\omega$ and $\phi_\omega$.

For the $\omega$ analysis, the Monte Carlo ratio method is used to extract the cross section. The Hall C Monte Carlo simulation package (SIMC) included simulations of variety of effect as well as models for electroproduction cross section. In practice, the experimentally measured differential cross section is distorted by several systematic and random effects such as radiative processes, multiple scattering and other nuclear reactions, spectrometer acceptance and efficiency issues, to mention a few. These effects can not be reliably deconvoluted from the data. Instead, events generated from a model of the theoretical differential cross section are subjected to a Monte Carlo procedure which simulates the same systematic and random effects of the actual experiment (spectrometers, target, beam). The method used is outlined in four basic steps. Firstly, we obtain the data yield. Secondly, we generate the synthetic the Monte Carlo data. Thirdly, the ratio of the data yield and the Monte Carlo is calculated. Fourthly, the measured experimental cross section is scaled from the ratio applied to the theoretical cross section which had input to the Monte Carlo.

The results generated from the Monte Carlo where adjusted to be reasonably close to the data such that the same computational treatment could be used to analyze both the results from the Monte carlo and the data. For each run the data and the Monte Carlo results were binned.
in \((W, \cos{\theta}_\omega, \phi_\omega)\). A missing mass squared \((m^2_x)\) cut which contains the \(\omega\) particle peak is applied to the data and the Monte Carlo results, and the results integrated over the \(m^2_x\). For the individual bins, the experimental differential cross-section is determined by the calculation of the ratio of the number of data events to the number of simulated Monte Carlo events, then this ratio is used to scale the model cross-section. \(\sigma^{\text{data}} = \sigma^{\text{mod}} \frac{N_{\text{data}}}{N_{\text{MC}}}\), where \(N_{\text{data}}\) is the number of \(\omega\) particles in the data obtained by multiplying the data yield by the measured experimental charge; while \(N_{\text{MC}}\) is the number of \(\omega\) particle generated from SIMC using a model input cross section \(\sigma^{\text{mod}}\).

The background and signal contributions are estimated with a two-parameter fit in each \((W, \cos{\theta}_\omega, \phi_\omega)\) bin. Assuming the shape given by the model is correct, the parameters are the normalization of the signal and background. The fit minimized the \(\chi^2\) difference between the total Monte Carlo (signal + background) and the data. However, in some cases (out-of-plane \(\phi\) bins), the multipion background and the \(m^2_x\) of the \(\omega\) spectra are too similar to have a reliable result from the fit. This is due to the decrease in acceptance, and in these cases it is assumed that the background normalization parameter is constant with respect to \(\phi\). This constraint was applied to the fit.

The main source of background is from events with more than one undetected particle, mainly multiple pions. The treatment of the background was done by simulating the \(m^2_x\) spectra of the background using SIMC with a model of the largest contributing reactions (mainly pions). These yielded our approximation to the shape of the multipion background to a normalisation factor.

For local analysis, we retain bins in \((W, \cos{\theta}_\omega, \phi_{\text{cm}})\) if they passed the following three criteria. For the first criterion, considering the number of entries in the bins in the region of the \(\omega\) missing mass squared peak, at least two of these must have at least 10 events. Secondly, a threshold for non-overlap of signal \((S)\) and background \((B)\) contributions of at least 10%. This ensures that the fit applied to the data is reliable from the physics point of view. This second criterion also includes a rejection of the case when there is no background or no signal. Equation 1 indicates the metric used for the separation of the \(S\) and the \(B\).

\[
\frac{1}{2} \int \frac{(S - B)^2}{(S + B)} \, dx
\]  

The third criterion is to ensure that there is reliable amount of statistics for the fit. This is necessary as there is lack of statistics from background simulation to give good results at the highest \(W\) bins of our measurement. The efficiency of the three criteria is 52%. There is an overlap of 26% between the first criterion and second criterion which together contribute 94% of the efficiency. A typical missing mass squared plot showing a bin selected for passing through the three criteria and subsequently used for the extraction of the cross-section is displayed in Fig. 1.

4. Data Corrections

In order to extract reliable results from our data, corrections were made on a run-by-run basis, on the track reconstruction inefficiencies, dead times and also on offsets. All the corrections applied to the data to acquire physical quantities are shown in Table 4. The entire set of kinematic offsets that were used during the replay of our present data is listed in Table 5.

A 4% correction due to proton absorption is done for the trigger efficiency. The condition for a proton to cause a trigger in the HMS is that it had to deposit enough energy to create above-threshold signals in at least three out of four scintillator planes in the detector stack. A trigger inefficiency for proton detection in the HMS is produced by protons which are not
Figure 1. (Colour online) Plot representing one of the bins from which the cross-section is extracted as it is selected after passing through the three criteria. The (blue) points are the data while the solid line (red) is the sum of the simulations. The multipion background component is the green filled area and the $\omega$ production simulation component is within the area of the red line. The dashed blue lines show the region within which the background fit is done while the solid dark lines show the region within which the $\omega$ model cross-section is extracted.

Table 4. Corrections applied to the data. Indicated by the parentheses are the range of correction sizes applied on a run-by-run ($^\dagger$) or a bin-by-bin basis ($^\ddagger$).

| Effects                  | Correction in % |
|--------------------------|-----------------|
| Proton absorption        | +4 ± 1          |
| $^\dagger$Computer dead time | +(1.0 – 19.1)   |
| $^\dagger$HMS tracking    | +(2.3 – 14.3)   |
| $^\dagger$SOS tracking    | +(0.3 – 0.9)    |
| $^\dagger$Electronic dead time | +(0.0 – 2.4)   |
| $^\ddagger$Random coincidence | -(0.0 – 7.6)   |

Table 5. Kinematic offsets applied to the data during the replay phase.

| Quantity | HMS                     | SOS                     |
|----------|-------------------------|-------------------------|
| $\theta$ | 0.0 ± 0.5 mrad           | 0.0 ± 0.5 mrad           |
| $\phi$   | +1.1 ± 0.5 mrad          | +3.2 ± 0.5 mrad          |
| $p$      | -0.13 ± 0.05 %           | -1.36 ± 0.05 %           |
| $E_e$    | 0.00 ± 0.05 %            | 0.00 ± 0.05 %            |

detected in their interaction with the scintillator and by protons that do not pass through all the scintillators due to absorption.

The radiative corrections for our experiment were calculated within SIMC which is a Monte Carlo framework for applying radiative corrections in $(e,e'p)$ coincidence reactions at
GeV energies [1]. The size of the radiative corrections implemented by SIMC is done by running
the full simulation with and without including radiative effects. The uncertainty in the radiative
corrections was estimated to be 2%.

5. Systematic Error Analysis and Uncertainties Estimates

For a meaningful result, the statistical uncertainties must be correctly calculated and the
estimates for systematic uncertainties should be reasonable. The determination of the statistical
uncertainty on the cross-section is done by using the uncertainties in \( N_{\text{data}} \) and \( N_{MC} \). For the
\( N_{\text{data}} \), the uncertainty is given by the statistical uncertainty (Poisson) in the number of measured
real events and the contribution from accidental coincidnece events and events from the cell
walls. As the Monte Carlo is done on for high statistics, the relative statistical error \( R = \frac{N_{\text{data}}}{N_{MC}} \)
is dominated by the uncertainty in the number of real events.

The concept and procedure used in the systematic studies in this work corresponds to that
applied in Ref. [1] to the \( p(e, e'p)\eta \) data. Depending on their source, estimates of the systematic
uncertainties are accounted for in one of two ways. On one hand, we applied the assumed global
uncertainty to the overall data to error sources that are independent of the binned kinematic
variable \( (W, \cos\theta_\omega, \phi_\omega) \) of the extracted data.

On the other hand, for the systematic uncertainties that are expected to be kinematic-
variable dependent are treated on a bin-by-bin basis. The estimation of such uncertainties are
done by comparing for each bin the value of the cross section extracted with a nominal set
of variables used in the Monte Carlo simulation to that extracted from using adjusted set of
variables. The nominal set of variables are the best set of variables used for the ‘standard’
analysis from which the final \( \omega \) differential cross-section is calculated. One at a time, each
set of variables (variable) are altered and the analysis redone up to the stage the differential
cross-section is acquired.

Table 6 lists various sources of kinematically-dependent systematic errors considered in the
analysis. The values of these variables are either not known precisely or the determination
of the cross-section can be sensitive to their changes. As a fact, the target position in the
beam direction, \( z_{\text{targ}} \), is only known within 3 mm. The \( z_{\text{targ}} \) defines the entrance angles to the
spectrometers which, in other words, could affect the spectrometer quantities. For the standard
analysis, an offset of 1.5 mm from the nominal centre is used. The variaton chosen is 1.5 mm
from either side of the nominal value. The Short Orbit Spectrometer (SOS) is found to be out-
of-plane, but the exact amount is unknown [1]. Through a recent survey, a nominal value \( x'_{\text{sos}} = 2.62 \) mr is used in the extraction and offsets of 1.5 mr and 3.5 mr are used for the systematic
analysis. The dependence of the missing mass squared, \( m^2_{x} \), cut is checked by widening and
subsequently tightening the cuts over reasonable limits on the region of background fits and the
region of the extraction of the model cross-section.

The expression for the systematic uncertainty in the differential cross section which results
in a change in a given variable in the \( i \)th bin is given as : \( \delta_i^v = \frac{| x_i - y_i^v |}{2} \) where \( x_i \) is the
differential cross-section obtained from the nominal value of the variable in the main analysis,
and \( y_i^v \) is the differential cross section obtained from the analysis with the change \( v \) applied to
that variable.

The mean systematic error for all bins, weighted by the statistical error of the measurement
in each bin is : \( \langle \delta^v \rangle = \frac{\sum_i \delta_i^v / \sigma_i^2 \Sigma 1 / \sigma_i^2}{\Sigma 1 / \sigma_i^2} \) where \( \sigma_i \) is the statistical error of the differential cross-section
in bin \( i \). The calculation of the total systematic uncertainty for the \( i \)th bin, \( \delta_{\text{tot}}^i \), is done by
the addition in quadrature of the systematic uncertainty for each variation, \( \delta_i^v \), and the global
systematic uncertainties, \( \delta_{\text{glo}} : \delta_{\text{tot}} = \sqrt{\Sigma_v (\delta_i^v)^2 + \Sigma \delta_{\text{glo}}^2} \)
| Source                  | Nominal value | Systematic variation |
|-------------------------|---------------|----------------------|
| $z_{\text{tag}}$ offset | 1.5           | 0.0                  |
|                         | 1.5           | 3.0                  |
| $x_{\text{so}}$ offset  | 2.62          | 1.5                  |
|                         | 2.62          | 3.5                  |
| $m_{x_1}^2$, background-fit region (GeV$^2$) | 0.4 \leq m_{x_1}^2 \leq 0.8 | 0.3 \leq m_{x_1}^2 \leq 0.9 | 2.3% |
|                         | 0.5 \leq m_{x_1}^2 \leq 0.7 | 1.4% |
| $m_{x_1}^2$, cross-section region (GeV$^2$) | 0.56 \leq m_{x_1}^2 \leq 0.66 | 0.46 \leq m_{x_1}^2 \leq 0.76 | 0.3% |
|                         | 0.59 \leq m_{x_1}^2 \leq 0.63 | 0.1% |

Table 6. Sources of kinematic-dependent errors, values used for the main analysis (nominal values), their systematic variations and the weighted mean systematic error for all bins ($\langle \delta^v \rangle$).

6. Results and Conclusion

The centre-of-mass differential cross-sections for the $\omega$ are extracted at an average $Q^2$ of 5.36 GeV$^2$ at an invariant mass range of 1.72 GeV $< W < 1.92$ GeV at angle range $-1.0 < \cos \theta < 1.0$, with full $\phi_{cm}$ coverage. We compute the cross-sections in five $\cos \theta_{cm}$ bins and three $\phi_{cm}$ bins. Shown in Fig. 2 are the computed cross-sections against the invariant mass $W$. For lower $W$ bins where we have sufficient statistics, there is a strong correlation between our measured $\omega$ cross-sections (black dots) and the input model cross-sections (red triangle) [1]. The agreement between our extracted cross-sections and the input model cross-sections decreases for higher $W$ bins due to lack of statistics from our data.

We also compare our measured cross-section with Regge-based model [6, 7] for hadronic content in the t-channel exchange of a photon in a similar $Q^2$ region. As shown in Fig 3, in the region of invariant mass 1.72 GeV $< W < 1.92$ GeV and $Q^2 \sim 5$ GeV$^2$ where there is an overlap with our data, there is a good agreement between the model and our measurement.

Our present data do not have sufficient angular coverage at higher $W$ to comment comprehensively on the presence of resonance mass region. The use of our data which is characterized by its very backward angular coverage can further be used to test additional models if more statistics at high invariant mass and scattering angle are made available. This data extends into a completely new region, which is the highest yet measured.

Future work includes a multipion background simulation at higher invariant mass and scattering angles. This will concurrently include the $\rho$ meson which also has a similar mass but broader peak than the $\omega$ particle.
Figure 2. (Colour online) Extracted $ep \rightarrow ep\omega$ differential cross-sections against invariant mass $W$ and flat in $\phi$. The red triangles represent the input model cross-section and black dots represent the extracted differential cross-sections. The inner error bars are statistical and the outer error bars are the quadrature sum of the statistical and systematic errors.
Figure 3. (Colour online) Measured cross-section (green dots) and model (black triangle) used in Refs. [6, 7] as a function of the scattering angle.

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