About islands of stability and limiting mass of the atomic nuclei

V S Okunev

Bauman Moscow State Technical University, Moscow, Russia.
E-mail: okunevvs@bmstu.ru

Abstract. The maximum magic number of neutrons for existing nuclei can be 260, protons - 126. The maximum deformation of existing super-heavy nuclei corresponds to the number of protons is about 120, the number of neutrons is about 126, 155 and 222. The islands (or shallows) of stability can be formed not only around the $^{298}$Fl, but also around the $^{320}$120 and $^{342}$126. The islands are extended toward larger numbers of neutrons. An increase in the half-life should be observed for the $^{348}$126 and $^{386}$126. The half-life of $^{298}$Fl is $\sim 10^7...10^8$ years. This is an understated estimate. The half-life of the longest-lived isotope of the 126th element with a mass number of 342 is $\sim 10^6...10^7$ s (or 0.03...0.33 years).

1. Introduction
1.1. A task
One of the fundamental tasks of modern nuclear physics is to determine the possibility of the existence of a second island of stability for super-heavy nuclei. The problem is closely related to the determination of the upper bound of the masses of atomic nuclei. Its solution is possible on the basis of predicting the properties of unknown nuclides.

1.2. Background. Islands of stability
The drip model predicts the instability of heavy nuclei with the divisibility parameter $Z^2/A \approx 45...49$ ($Z$ — nuclear charge, $A$ — mass number). Such nuclei do not have a fission threshold. The shell model assumes the existence of a stabilizing effect of nuclear shells. The microscopic model suggests that filled proton and neutron shells increase the height of the fission threshold and slow down the decays of super-heavy nuclei.

A significant increase in the average lifetime of a super-heavy nucleus with a magic number of protons ($Z = 114$) and neutrons ($N = 184$) was predicted by V.M. Strutinsky in 1966 on the basis of the microscopic theory of the atomic nucleus [1, 2]. Simultaneously, W.D. Myers and W. Swiatecki hypothesized a possible increase in stability (with respect to spontaneous fission) of nuclei with magical $Z = 114$ and 126 and $N = 184$ [3]. Myers and Swiatecki developed a "semi-empirical theory of nuclear masses and deformations", which formed the basis of a microscopic model [4]. Thus, in 1966, the existence of stability islands in the region of trans-actinides ($Z > 103$). There can be several islands. Their centers are determined by the magic numbers $N$ and $Z$. The contours of the first island
(Z = 114, N = 184) from the side of neutron-deficient nuclei are already known: in the laboratories of the world, such nuclei have been obtained.

1.3. Known theoretical and experimental facts

Theory.
(a) There is a hypothesis of 1966 about the possibility of the existence of two islands of stability formed around doubly magical super-heavy nuclei (298Fl and 310Cd).
(b) A theoretical transition from super-heavy nuclei to neutron stars is possible. To do this, it is sufficient to include the gravitational component in the Weizsacker formula. It is possible to extrapolate the optimal N/Z ratio taking into account the gravitational interaction
(c) The existence of bubble nuclei is predicted (Z = 400, N = 900).
Experiments and modern realities.
(a) Super-heavy elements (up to Z = 118) have been synthesized in various laboratories of the world.
(b) Traces of super-heavy elements (up to Z = 130) in the meteorites have been found.

2. Methods of research

2.1. Traditional methods
Traditionally, the methods for predicting the properties of yet undiscovered atomic nuclei are based on the adaptation of nuclear models to the region of unknown nuclides. However, it is difficult to confirm the applicability of these models in the region of unknown nuclei. The correct extrapolation of models to the region of super-actinides (122 ≤ Z ≤ 157) is laborious and may prove to be unjustified [5]. The use of known models for super-heavy nuclei is controversial [6].

2.2. Forecasting based on analysis of properties of known nuclides

The author uses an approach based on an analysis of the periodicity of the change in the properties of known atomic nuclei, and extrapolation of known semi-empirical relationships to the region of unknown nuclei. This requires the analysis of large amounts of information. The effectiveness of the analysis is determined by the successful choice of the way data is presented. Some simple considerations, based on well-known facts and the existing vast base of nuclear data, allow us to draw new interesting conclusions [7].

Based on the use of nuclear models, it was assumed that Z = 108 and N = 162 are magic numbers [8], but the simplest analysis showed that the increased stability of the nuclei of this region is related to the maximum deformation of the nuclei [9, 10]. The existence of the first island of stability around the nucleus of 298Fl is beyond doubt. The analysis shows that the role of completely filled shells in the stabilization of nuclei is exaggerated. It is noticeable only near the optimal N/Z, which determines the equilibrium of the forces of nuclear (manifestation of strong), electromagnetic and weak interactions [8, 9]:

\[ N/Z = 0.98 + 0.015 A^{2/3}. \]  

(1)

The relation (1) holds up to parity of N and Z in the nucleus. It is also characteristic of the longest-lived heavy and super-heavy nuclei [11]. There is reason to believe that not only at high energies, but also at small distances between interacting objects, all interactions behave as a single interaction. (This was discussed by V.L. Ginzburg [12].) The analysis allows doubting the existence of a doubly magic nuclide Z = 126 and N = 184 [9-11].

As a research method, extrapolation of known dependencies to unknown nuclei is used based on the analysis of existing files of estimated nuclear data. Such extrapolation may not have a physical meaning. Therefore, extrapolation results require additional analysis.

The simplest problem of extrapolation over an extreme node is to restore the values of the function f at the node \( x_{n+1} \) according to the known values of this function at the nodes \( x_1, x_2, \ldots, x_n \). If the node \( x_{n+1} \) is far apart from the nodes \( x_1, x_2, \ldots, x_n \), the known information about the behavior of the function f in the knots \( x_1, x_2, \ldots, x_n \) is weakly used [13]. If the nodes are located close, then the role of errors in the
initial information increases [13]. The choice of extrapolation nodes is not simple [13]. However, in this problem, the nodes \( x_1, x_2, \ldots, x_{n+1} \) are known in advance and they can’t be selected. When specifying a function by a graph, preference is given to approximation by polynomials. Often the approximating function is expediently sought in the form \( f(x) = g(x, a_1, a_2, \ldots, a_n) \) [13, 14]. The parameters \( a_1, a_2, \ldots, a_n \) are determined from the condition that \( f(x) \) coincides with the approximating function at the nodes \( x_1, x_2, \ldots, x_n \). The case of linear interpolation (extrapolation) is widespread. The approximation is sought in the form of a sum over \( i \) from 1 to \( n \):

\[
g(x, a_1, a_2, \ldots, a_n) = \sum a_i \varphi_i(x),
\]

where \( \varphi_i(x) \) - the fixed functions, the values of the coefficients \( a_i \) are determined from the condition of coincidence with the approximating function at the nodes \( x_1, x_2, \ldots, x_n \) [13, 14]. In this problem, such a method of extrapolation gives incorrect results. Linear extrapolation beyond the last node of the sequence of eight known magic numbers 2, 8, 20, 28, 50, 82, 114, 126 gives a value of 140 instead of 184. Extrapolation of the nine known magic numbers is 178, i.e. the monotonicity of a function drawn through given points is violated. Linear extrapolation of the number of neutrons (protons) on the shell for the ninth shell gives 30 instead of 58.

Nonlinear extrapolation is necessary. Interpolation (extrapolation) formulas for tables with constant step are known. Using the Bessel polynomial gives some advantages over using the second-degree polynomial [13]. In [15], an algorithm for choosing the most universal function ("universal pattern") is proposed for approximation based on a fairly simple cascade neural network model. However, much simpler models can be used to solve the problem.

If necessary, a plot of the known points is made using a pattern [14]. If the points are sparse, experienced engineers use a metal ruler placed on the edge, bending it so that the edge of the ruler passes through all points at once (flexible pattern) [14]. The flexible ruler is an elastic bar. The equation of its free equilibrium is \( \varphi_{IV}(x) = 0 \). Hence, in the interval between two neighboring nodes, the interpolation function is a polynomial of the third degree [14]. It is convenient to write it in the form:

\[
\varphi(x) = a_i + b_i (x - x_{i-1}) + c_i (x - x_{i-1})^2 + d_i (x - x_{i-1})^3, x_{i-1} \leq x \leq x_i.
\]

The method of solution is presented in [14]. It is applicable for solving the task.

To solve the problem, it is important to consider the following aspects, identified by the authors [14] for multidimensional interpolation.

(a) It is important that the function passing through the given nodes be monotonous. When extrapolating, it is desirable to preserve the monotonicity of the function.

(b) Not any number of nodes is advantageous, not every arrangement of nodes is permissible.

(c) Sequential interpolation (and hence extrapolation) has several advantages. Moreover, it is better to approximate the discrete dependence (function) by a continuous dependence, i.e. the tabulation of the original data is transferred to the graph. It is also necessary to use the smoothing function passing through the specified nodes. Equally important is the way data is presented. Ideally, it is good to reduce the function to a linear or quasilinear dependence.

3. Results

3.1. "Triangles of stability". Islands and shallows of stability

On the Z-N-diagram the triangles close in form to rectangular triangles can be distinguished (figure 1). We call them "triangles of stability" [9-11, 16]. One of the vertices of each of the triangles (at the right angle) is the double magic nucleus, distant from the equilibrium curve (1). On the legs are the magic nuclei. Hypotenuse is determined by the relation (1). Since the latter is true to the parity of \( N \) or \( Z \), the hypotenuse is a sawtooth line. On this line there are stable or relatively long-lived nuclei. An analysis of the physical properties of nuclei in such triangles is a universal tool for predicting the properties of unknown nuclides.
The nuclei maximally deformed in the ground energy state are characterized by the largest internal quadrupole electric moment $Q$ in absolute value. These nuclei are characterized by increased stability. As a result, these nuclei are similar to magical and double magic nuclei for some physical characteristics. Both have a maximum half-life $T_{1/2}$ in comparison with the neighboring nuclides of the $Z$-$N$ diagram, the minimum cross section for interaction with hadrons, and the maximum threshold for inelastic processes (with the exception of the neutron capture neutron capture reaction). For magic nuclei $Q(N) = 0$ or $Q(Z) = 0$, in doubly magic $Q(N) = Q(Z) = 0$. At the vertex at the right angle of the stability triangle, a nucleus with a maximum $|Q|$ [9, 10].

Figure 1. The arrangement of double magic nuclei and stability triangles: 1 - equilibrium curve (1), 2 – proton drip line.

In figure 2 schematically depicts the location of the island (indicated by the number 1) and the shallows (2, 3) of stability for super-heavy nuclides. Index 4 corresponds to the condition of optimal $N/Z$. 5 - triangle stability of trans-actinides. The shallow 2 is formed around the nucleus of the 120th element ($N = 200, A = 320$) with the maximum $|Q|$.

Figure 2. Diagram of the location of the triangle, the islands (shallows) stability of trans-actinides.

The mutual Coulomb repulsion of protons tends to break the nucleus. For this reason, atomic nuclei can withstand neutron overloading more easily than protons. The islands and shallows of stability should be stretched towards the large $N$ (see figure 2). The patterns of changes in the nuclear-physical properties of nuclides are the same for all stability triangles and are periodic in nature. It is possible to isolate stability triangles that contain not yet discovered nuclides (see figure 2), and predict the properties of these nuclides [9-11, 16]. So, the
double magic nucleus of the $^{298}$Fl ($N = 184, Z = 114$) can turn out to be relatively long-lived not only due to the filling of the nuclear shells, but also due to the optimal $N/Z$ ratio. The double magic nucleus of the 126th element ($A = 310$) is far from the curve (1) and near the proton stability boundary. It can be non-existent. With $Z = 126$ and $A = 342$, the ratio $N/Z \rightarrow \text{opt}$, which makes this nucleus the contender for the center of the shallow.

3.2. Extrapolation for the purpose of determining the numbers $N$ and $Z$ corresponding to the maximum stability of the nucleus in the region of super-heavy nuclides

3.2.1. General provisions. In the first approximation, especially if the distance between the shells is small, we can assume that the maximum deformation corresponds to half-filled shells, i.e. the arithmetic mean of adjacent magic numbers. The arithmetic average of the magic numbers 114 and 126 gives a value of 120; 126 and 184 gives 155. The exact value of $N$ and $Z$ corresponding to the maximum deformation of the nucleus can differ from 120 and 155 and is determined experimentally. It has been experimentally confirmed that the numbers 126 and 184 are magic for neutrons. It is not known whether the magic numbers of neutrons and protons are the same for $N$ or $Z \geq 126$. In accordance with the modern concept of nuclear shells, the magic numbers of protons and neutrons at large mass numbers must differ. It is also not known whether the values of magic numbers can change when going to bubble nuclei.

Suppose that super-heavy elements (up to $Z = 130$), found in trace amounts in meteorites, were formed as a result of nuclear reactions, for example, when cosmic objects collide at high velocities, and the double magic nucleus of the $^{298}$Fl is the heaviest of the relatively long-lived ones. At $Z > 114$, in the $N$-$Z$ coordinates there are only shallows (at $Z$ about 120 and 126) containing short-lived nuclei.

The following questions remain.

1. Are there islands at $Z \gg 126$, at the centers of which there are double magic nuclei with a half-life exceeding the half-life of $^{298}$Fl or even of the nuclei composing shallows at $Z$ about 120 or 126? Here we should distinguish two tasks for analysis.

   (a) The first. Yu.T. Oganesyan suggested the possibility of a transition from traditional nuclides with a constant density of nuclear matter in the central part to exotic nuclei with a lower density of nuclear matter at the center ($Z \geq 120$) and the existence of bubble nuclei (at $Z = 400$ and $N = 900$). It should be noted that for known neutron-deficient nuclei the yield of the proton (protons) in the halo is accompanied by a significant (by orders of magnitude) increase of the half-life [9, 17]. This is due to the deceleration (stabilization) of radioactive decays by the electromagnetic interaction between the proton (protons) of the halo and the protons of the nucleus. For known nuclei with an optimal $N/Z$ ratio, the halo does not exist. However, if a part of the neutrons and one or two protons with a close to zero binding energy in the super-heavy nucleus with the optimal $N/Z$ evaporating will become part of the halo, and for some time they will be retained by nuclear forces, then the decay of the super-heavy nucleus electromagnetic interaction. Such nuclear structures are not known. The transition of all the nucleons of the nucleus to the halo formally corresponds to the nucleus-bubble.

   (b) The second. Are there magic numbers greater than 184?

2. What are the objects located on the equilibrium curve between super-heavy (possibly, exotic) nuclei and neutron stars? It is not excluded that these are groups of nuclei connected by forces of nuclear interaction. Let’s leave it for later analysis.

To study the first question, we use extrapolation. We have at our disposal a sequence of nine magic numbers (2, 8, 20, 28, 50, 82, 114, 126, 184), a sequence containing eight numbers corresponding to the distances between the nuclear shells, that is, between the magic numbers, equal to the number of nucleons of this species on the shell: 2, 6, 12, 8, 22, 32, 32, 12, 58. Sequences can be constructed for experimentally and theoretically determined maximally deformed nuclei, i.e., for $N$ and $Z$ for nuclei with maximal $Q$ (elongated nuclei), minimal $Q$ (oblate nuclei), and accordingly sequences containing $N$ and $Z$ located between maximally deformed nuclei.
In figure 3 (a) shows the dependencies of the values of $N$ or $Z$ on the ordinal number of the magic number. In figure 3 (b) shows the dependencies of intervals between magic numbers on the ordinal number of the interval. For the convenience of extrapolation, here and below, the points corresponding to the magic numbers are connected by straight lines. The discrete dependence is preliminarily approximated by a continuous dependence.

![Figure 3. The magic numbers (a) and the number of neutrons (protons) on the shell (b).](image)

Note that the magic number 28 was determined later than others. The magic number 114 was determined theoretically in the 1960s, but experimental confirmation was received only at the beginning of the 21st century. It was assumed that the numbers $Z = 108$ and 120 are also magical [8, 9]. The number 114 somewhat distorts the "perfection" of the curve in figure 3, and the number 28 distorts the curve in figure 3 (b). This makes extrapolation difficult. The role of different magic numbers in the stability of atomic nuclei is not the same. Analysis of ENDF-B/VII.1 data [18] shows that this role is less for $N$ and $Z$, equal to 28. Perhaps it is less for $N$ and $Z$, equal to 114. For medium atomic nuclei, the stabilization factor associated with an increase in the average specific binding energy of $\varepsilon$ (per nucleon) is noticeable.

3.2.2. Extrapolation of magic numbers and the number of neutrons (protons) on the shell. So, we exclude from consideration the magic numbers 28 and 114, thus passing to a monotonic function (figure 4). Let's leave the magic numbers 2, 8, 20, 50, 82, 126, 184 and the intervals between them 2, 6, 12, 30, 32, 44, 58. Extrapolation for the extreme node (184) allows us to get the next magic number - 260 and the interval between 184 and the next magic number (the number of nucleons on the shell) is 76 (see figure 4). This suggests the possibility of the existence of double magic nuclei at $Z = 126$, $N = 260$, $A = 386$ and $Z = 184$, $N = 260$, $A = 444$. The ratio of $N/Z$ in the first case is 2.06, in the second - 1.41.

Let us determine the physical basis for such extrapolation. Because of mutual repulsion of protons in the nucleus, the average specific binding energy of nucleons in a heavy nucleus decreases with increasing mass number or number of protons. Extrapolating the known dependence of $\varepsilon (A)$ in the region of super-heavy nuclei, we obtain that the specific binding energy is zero at $A \approx 500 \ldots 600$. On the other hand, according to the drop model, the nuclei with the divisibility parameter $Z/A \approx 45 \ldots 49$ and more should be divided spontaneously: the fission threshold is absent. The divisibility parameter of the $^{298}$Fl is 43.6, of the $^{342}$126 is 46.4. The divisibility parameter of the nucleus with $Z = 126$, $N = 260$, $A = 386$ is 48.64. This nucleus can exist because of magic $Z$ and $N$. The divisibility parameter of the nucleus with $Z = 184$, $N = 260$, $A = 444$ is 76.25. This nucleus is at a considerable distance from the equilibrium curve (1) of the forces of nuclear, electromagnetic, and weak interaction. It nuclei does not exist. At mass number $A = 386$, the optimum values are $Z = 139$, $N = 247$, or $N/Z = 1.78$. At $A = 444$, the optimal values are $Z = 156$, $N = 288$, or $N/Z = 1.85$. The divisibility parameters for these
two nuclei are 50.05 and 54.81, respectively. Such nuclei should not exist. Finally, the double magic nucleus at \( Z = 184, N = 260, A = 444 \) is characterized by the divisibility parameter of 76.25, which is characteristic for non-existent nuclei.

![Figure 4. The results of extrapolation (dashed lines) of magic numbers (a) and the number of neutrons (protons) on the shell (b).](image)

Stabilized can be maximally deformed super-heavy nuclei (with half-filled nuclear shells). If the magic number is 260, then the maximum strain may correspond to \( N \) or \( Z \), equal to 222. Optimal \( N/Z \) at \( Z = 126 \) corresponds to \( A = 342 \). At \( Z = 126, N = 222 \) the mass number is \( A = 348 \). This nucleus is overloaded with only six neutrons. Thus, the shallows can be formed around the 126th element with a mass number of 342 and 348.

So, the simplest analysis suggests that the last magic number of neutrons can be considered 184 and, possibly, 260. If \( Z = 184 \) turns out to be magical, then the nuclei with such a charge should not exist. The shallows formed around the nuclei of the 126th element with a mass number of 342 and 348 are formed by the heaviest of the relatively long-lived elements.

3.2.3. Extrapolation of the numbers of neutrons (protons) corresponding to the maximum deformation of the nucleus. The maxima of \( Q \) (indicated in parentheses) and the minima of \( Q \) corresponding to the maximum difference from the spherical shape of the nucleus are attained at \( N \) or \( Z \) values equal to 1, 3, 6, 9, 12, 16, 18, \( \approx20, 25, 28, 34, 44, 47, 52, 70, 83-84, 108, 128 \) \[9\]. In this case, the numbers 20 and 28 of this sequence are magical, i.e. the points corresponding to \( Q = 0 \) and \( |Q| \to \text{max} \) in this part of the curve \( Q(N) \) or \( Q(Z) \) are close.

In figure 5 the values of \( N \) (or \( Z \)) corresponding to the maximum elongated and maximally flattened nuclei are presented. Extrapolation suggests that the next largest mass of the most elongated nucleus corresponds to \( N \) (or \( Z \)), equal to 160, and maximally oblate - 184. But 184 is the magic number for neutrons. Hence, it corresponds to a stable or a long-lived nuclei.

If we consider the entire sequence \( N \) (or \( Z \)) for which \( |Q| \to \text{max} \), then the next number at which the deformation is maximal should be 152 (figure 6). Combining this sequence with magic numbers, we get the next number \( N \) (or \( Z \)) equal to 260, at which the nuclei is the most stable (see figure 6). However, as can be seen from figure 6 (b), extrapolation in this case is not informative, since its results are not unambiguous.
3.3. Prediction of physical properties of super-heavy nuclei

3.3.1. Initial data. Preliminary conclusions. The method of extrapolation can be selected on the basis of an analysis of known data, for example, the corresponding region of transition from stable magic atomic nuclei of lead and bismuth to short-lived polonium isotopes. Analysis of the nuclei located in different stability triangles allows us to draw preliminary conclusions.

The half-life of a doubly magic nucleus, far removed from the equilibrium condition (1), can be noticeably smaller than the neighboring once-magic nuclides of the stability triangle. So, \( T_{1/2}(^{132}_{50}\text{Sn}) < T_{1/2}(^{134}_{50}\text{Sn}) < T_{1/2}(^{132}_{50}\text{Sb}) < T_{1/2}(^{134}_{50}\text{Sb}) \) [9, 11]. A neutron-deficient nucleus with magic \( N \) can have a shorter half-life than a near-magic (\( N \) by 1 less), despite the odd number of neutrons in the near-magic nucleus. So, \( T_{1/2}(^{213}_{83}\text{Ac}) < T_{1/2}(^{214}_{83}\text{Ac}) < T_{1/2}(^{216}_{83}\text{Th}) < T_{1/2}(^{217}_{83}\text{Th}) \) [9, 11]. As follows from the analysis of the databases [18, 19], for magical and near-magical nuclei, located at some distance from the curve (1) in the \( Z-N \) coordinates, the maximum of half-life
(depending on $A$, $N$ or $Z$) is diffuse and covers neighboring nuclides. Moreover, if the nuclei are overloaded with protons ($^{215}$Ac, $^{216}$Th and other magic by $N$), then the isotope with a smaller $N$ or an isoton with a larger $Z$ will be the longest-lived one. This result is usually a much less significant reason for the stabilization of decays: a decrease in the mass number with the simultaneous transition of one of the protons of the nucleus into a neutron. The filling of nuclear shells and the parity of the number of protons and neutrons in the nucleus stabilize radioactive decays well only in the case of the optimal $N/Z$ in the nucleus [7]. In the general case, they are less significant stabilizing factors than the equilibrium of nuclear, Coulomb, and weak forces [7, 9].

A similar pattern is observed for all nuclides. The regularities in the change in the properties of nuclei in the transition from $Z = 114$ to 115 and 116 should be similar to the transition from $Z = 82$ to 83 and 84. In figure 7 shows the dependences $T_{1/2}(N)$ and $T_{1/2}(Z)$. For the $^{209}$Bi $T_{1/2} = (1.9 \pm 0.2) \cdot 10^{19}$ years [20], the remaining data correspond to [18]. The curves in figure 7 the dependencies are characteristic for four cases: for magic by $N$ (126) for $N/Z \rightarrow$ opt, for magic by $N$ (114) neutron-deficient nuclei; for maximally deformed nuclei with a half-filled neutron shell ($N = 120$) and for nuclides maximally close to the proton drip line, whose properties are known ($N = 105$). For illustration, the half-lives of stable nuclides in figure 7 (b) are assumed to be equal to the experimentally determined $T_{1/2}$ for $^{209}$Bi.

The first case corresponds to the transition from the heaviest stable elements to the short-lived elements. In the second case, there is an increase of $T_{1/2}$ for the magic nucleus ($N = 114$) at $Z = 83$, following the magic number 82. The superfluous proton in the nucleus has the same stabilization effect as the almost filled shell for the deformed nuclei. In the third case, the picture corresponds to the conventional one, we can say classical: the most long-lived is the magic nuclide of lead, the effect of proton pairing is clearly noticeable, as $Z$ increases by 1, the half-life decreases to a greater extent than with decreasing. In the fourth case, the pairing effect is not manifested, with the filled ($Z = 82$) and almost filled ($Z = 81$) proton shell $T_{1/2}(N) \rightarrow$ max and decreases with increasing $Z$ (for $Z > 82$). Similar patterns are also characteristic of other magical and near-magical nuclei. The physical properties of the nuclides that determine their half-life in the stability triangles vary symmetrically with respect to the optimal $N/Z$ ratio. This only confirms the universality of the approach to predicting the properties of atomic nuclei, based on the analysis of the triangles of stability.
3.3.2. Features of maximally deformed nuclei. The experimentally obtained $T_{1/2}$ ($N$) dependences (with respect to spontaneous fission) for certain isotopes of U ($T_{1/2} \rightarrow \text{max at } N = 143$), Pu (146), Cm (146), Cf (150), Fm (152), No (152) in the vicinity of the optimal $N/Z$, as well as theoretical dependencies for neutron-deficient isotopes of the 106th (maximum at $N = 164$), 108th (162) and 110th (162) elements. These dependences support the assumption of nuclear deformation at $N = 152$ and 162.

Note that $N = 152$ corresponds to the maximum of $T_{1/2}$ only for the isotopes Fm and No, and $N = 162$ for the theoretical maximum for the isotopes of the 108th and 110th elements. For U, Pu, Cm and Cf, the maxima are fairly flat: three isotopes of U, four isotopes of Pu, five isotopes of Cm and two isotopes of Cf are characterized by a half-life close to the maximum. In figure 8 shows the dependence of $T_{1/2}$ on $Z$ ($a$) and $N$ ($b$) for some actinides with $N/Z \rightarrow \text{opt}$ and for their longest-lived isotopes (they correspond to maximally deformed nuclei). The minimum values of $T_{1/2}$ ($\alpha$-decay and spontaneous fission) are given. Only some nuclei with $N/Z \rightarrow \text{opt}$ ($^{231}\text{Pa}$, $^{237}\text{Np}$ and $^{243}\text{Am}$) are the longest-lived isotopes of this element.

![Figure 8](https://example.com/figure8.png)

**Figure 8.** Dependence of the half-life on the nuclear charge ($a$) and the number of neutrons in the nucleus ($b$) for nuclides with the optimal $N/Z$ (●) and for their longest-lived isotopes (○).

Most of the long-lived nuclei considered correspond to an odd number of neutrons $N = 153$, under which the deformation of the nucleus in the ground state is maximal. Near ($N = 150...152$) in the Z-N diagram are also relatively long-lived isotopes, i.e. $N = 152$ corresponds to the maximum of $Q$.

3.3.3. Estimates of the half-life of super-heavy nuclei. The patterns obtained for magical and maximally deformed heavy nuclei are characteristic of super-heavy nuclides. Their half-lives may be higher than the values of $T_{1/2}$ for deformed heavy nuclei (in the range from $Z = 90$ to 110), but lower than the values corresponding to the magic nuclei of the lead-196 and 208 in triangles of stability.

According to the data of [21], the dependence $T_{1/2}$ ($N$) for Hs has a local ($N = 162$) and global ($N = 184$) maxima. For $^{118}\text{Fl}$, the local maximum corresponds to $N = 162$, the global $N = 182...184$. A similar phenomenon is characteristic of $\alpha$-decays. According to [21], the maximum $T_{1/2}$ with respect to $\alpha$-decay in the $T_{1/2}$ ($N$) dependence is observed at $N = 164$, the global one at $N = 184$ for Hs, and at $N = 164$ and $N = 180...184$ respectively for Fl. Obviously, the local maximum corresponds to the maximally deformed state of the nucleus ($Q \rightarrow \text{max}$), the global maximum corresponds to the magic number $N = 114$. $Q$ ($N=114$) = 0. According to estimates [21], obtained with the help of a microscopic model of nuclei, for double magic nuclides the half-life with respect to spontaneous fission can reach $\sim 10^7$ years. The lower limit of $T_{1/2}$ for super-heavy doubly magic nuclei $^{298}\text{Fl}$ can be estimated at $\sim 10^5...10^6$ years, the upper limit obtained on the basis of the theory of relativity of the limiting masses of atomic nuclei is $3 \times 10^8$ years [21].

In practice, interpolation and extrapolation are often used by second-degree polynomials or trigonometric polynomials. The discrete dependencies $T_{1/2}$ ($N$) and $T_{1/2}$ ($Z$) can easily be reduced to
continuous differentiable functions and a uniform grid of $N$ and $Z$ is used. Detailed tables of $T_{1/2} (N)$ and $T_{1/2} (Z)$ dependencies allow the use of the simplest extrapolation methods that are easily performed on paper [14]. In a number of cases it is advisable to use the analysis of $\log T_{1/2} (N)$ or $\log T_{1/2} (Z)$. The form of the function depends essentially on the purpose of extrapolation. A method of linear interpolation or extrapolation with indefinite coefficients is well used [13].

For correct extrapolation, nuclides with even and odd $N$ and $Z$ were considered separately. In figure 9 shows the $T_{1/2} (N)$ and $T_{1/2} (Z)$ dependencies for trans-actinides (according to data [21-23]).

The nature of the change in the properties of super-heavy maximum deformed or spherical nuclei as a function of $N$ and $Z$ is completely subject to the laws revealed for much lighter, well-studied nuclei. The simplest analysis of the triangles of stability of lead, thorium, and other actinides and their vicinities in $Z$-$N$ coordinates, analysis of the known $T_{1/2} (N)$ and $T_{1/2} (Z)$ dependencies allow predicting the properties of new unknown elements forming a triangle of trans- and super-actinide stability with vertices at the points $Z = 126$, $N = 184$ (double magic element); $Z = 114$, $N = 184$ ($^{298}$Fl; $N/Z \rightarrow \text{opt}$), $Z = 126$, $N = 216$ ($Z$-magic nuclide with optimal $N/Z$; $A = 342$).

The half-life of the nuclei varies considerably. For this reason, the most informative representation of the dependence $T_{1/2} (N)$ or $T_{1/2} (Z)$ on a semilogarithmic scale. Linear extrapolation using the logarithmic scale for $T_{1/2}$ turned out to be the most preferable from the point of view of the efficiency of predictions of the properties of strongly deformed nuclei. Such extrapolation for neutron-deficient nuclides in the region of optimal $N/Z$ under conditions of incompleteness of the initial information (there is no data on the properties of nuclei with $N/Z \rightarrow \text{opt}$) allowed us to estimate $T_{1/2}$ with good accuracy (table 1). The error is greatest if the longest-lived isotope of this element has a somewhat
excess \( N \) with respect to the optimal \( N/Z \) or the maximally deformed nucleus (in the ground state of energy) with respect to the spherical shape. In this case, there are two relatively long-lived nuclei.

### Table 1. Estimation and exact values (ENDF/B-VII.1 [18]) of \( T_{1/2} \) (s) for some actinides.

| Nucleus | Estimated value | Exact value |
|---------|-----------------|-------------|
| \(^{227}\text{Ac}\) | \(~10^9\) | 6.87·10^8 |
| \(^{232}\text{Th}\) | \(~10^{15}...10^{18}\) | 4.43·10^{17} |
| \(^{236}\text{U}\) | \(~10^{17}\) | 7.39·10^{14} |
| \(^{238}\text{U}\) | \(~10^{17}...10^{20}\) | 1.47·10^{17} |

| Nucleus | Estimated value | Exact value |
|---------|-----------------|-------------|
| \(^{244}\text{Pu}\) | \(~10^{16}...10^{19}\) | 2.52·10^{15} |
| \(^{247}\text{Cm}\) | \(~10^{14}...10^{16}\) | 5.05·10^{14} |
| \(^{249}\text{Cf}\) | \(~10^{10}\) | 1.11·10^{10} |
| \(^{251}\text{Cf}\) | \(~10^{10}\) | 2.83·10^{10} |

For \(^{208}\text{Pb}\) and \(^{209}\text{Bi}\) nuclei, too low estimates are obtained (by several orders of magnitude), which is associated with a sharp change in the function \( T_{1/2} \) and the derivative. For this reason, these nuclides are not represented in Table 1. If the \( T_{1/2} \) estimate is acceptable for deformed nuclei, then the estimate for the doubly magical and adjacent nuclides on the \( Z-N \) diagram is strongly underestimated and this extrapolation is not valid. It can be expected that the use of such a simple extrapolation to estimate the half-life of the double magic nuclei of the \(^{208}\text{Fl}\) will give a greatly underestimated estimate.

### 4. Findings

The \(^{298}\text{Fl}\) nucleus will be the longest-lived trans-actinide stability triangle. The nuclei with charge \( Z > 114 \) will have a shorter half-life. Comparing \( T_{1/2} \) (\( N \)) for the magic nuclei of lead and maximally deformed actinides, whose lifetime is determined by \( \alpha \)-decay (\( T_{1/2} \) with respect to spontaneous fission is large), the half-life of \(^{298}\text{Fl}\) (for which \( \alpha \)-activity also predominates) can be estimated in \(~10^7...10^8\) years. This is a crude and highly underestimated estimate (one should expect \( T_{1/2} \gg 10^7...10^8 \) years), which is in good agreement with the data [21].

The nucleus with \( Z = 126 \) and \( N = 216 \) should be relatively long-lived, but with a much smaller (approximately 10 orders of magnitude) half-life than \(^{298}\text{Fl}\).

Between \(^{298}\text{Fl}\) and \(^{342}\text{Lc}\) there should be an island (or shallow) for long-lived highly deformed nuclides with a charge of about 120 and optimal or close to the optimal \( N/Z \) ratio (around the nucleus \(^{320}\text{Lc}\)). This island (shallow) corresponds to the imposition of several triangles of stability (by analogy with actinides [10]).

An element 127 with an optimal \( N/Z \), up to parity of \( N \) and \( Z \), should have a half-life of less than \(^{342}\text{Lc}\). Next in \( Z \), even an even-even nuclide with the optimal \( N/Z \), may turn out to be short-lived. Extrapolation of \( T_{1/2} \) and the detected traces of the 130th element allow us to make an assumption about the possible existence of nuclei \(^{346}\text{Lc}\) and \(^{350}\text{Lc}\).

At \( N = 216 \), the deformation of the nucleus should be noticeable (the location of a very shallow maximum is not yet known), which is also a stabilizing factor.

The second island of stability (or rather, the shallows) can form around the \( Z \)-magic nuclide \(^{342}\text{Lc}\) (a rough estimate yields \( T_{1/2} \sim 10^5...10^7 \) s) with the optimal \( N/Z \) ratio.

All islands and shallows should be stretched towards a larger number of neutrons. The nuclei are easier to withstand neutron overload than protons. This is due to the zero electric charge of the neutron and the absence of additional Coulomb forces destabilizing the nucleus.

### 5. Conclusion

A comparative analysis of the physical properties of atomic nuclei, carried out with the help of stability triangles, and the simplest extrapolation make it possible to qualitatively estimate the half-lives of the still unknown super-heavy nuclides. These are very rough estimates, since different stability triangles contain a different number of atomic nuclei. It is possible that the shallow formed around the nucleus \(^{342}\text{Lc}\), stretched towards the larger \( N \), will contain relatively long-lived even-even...
nuclei of the 128th and 130th elements, as well as the 127th element with even N, and necessarily with a close to the optimal ratio N/Z.

It is possible that these are the heaviest nuclides from relatively long-lived ones. The longest-lived atomic nucleus of this shallow is its center - the nucleus $^{342}_{126}$. The heaviest existing nucleus may be $^{216}_{126}$. All the shallows will be stretched to the side of congestion by neutrons, since the nuclei can withstand neutron overloading more easily than protons.

Super-heavy elements with a charge $Z > 126$ can’t exist with the traditional form of the nucleus, when the density of nuclear matter is maximal and approximately constant in the central part of the nucleus. The extrapolation of nuclear properties considered can precede traditional methods of predicting the properties of as yet undiscovered nuclides associated with the extrapolation of known nuclear models to the region of unknown super-heavy nuclei.

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