Generalized Conformal Symmetry and Recovery of $SO(8)$ in Multiple M2 and D2 Branes

Yoshinori Honma\textsuperscript{a}, Satoshi Iso\textsuperscript{a}, Yoske Sumitomo\textsuperscript{a}, Hiroshi Umetsu\textsuperscript{b} and Sen Zhang\textsuperscript{a}

\textsuperscript{a}Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK) and Department of Particles and Nuclear Physics, The Graduate University for Advanced Studies (SOKENDAI), Oho 1-1, Tsukuba, Ibaraki 305-0801, Japan

\textsuperscript{b}Department of General Education, Kushiro National College of Technology Otanoshike-Nishi 2-32-1, Kushiro 084-0916, Japan

Abstract

We investigate conformal symmetries of the Aharony - Bergman - Jafferis - Maldacena (ABJM) theory for multiple M2 branes and the Lorentzian Bagger - Lambert - Gustavsson (L-BLG) theory which can be obtained by taking a scaling limit $k (\gg N) \to \infty$ of the ABJM theory. The conformal symmetry is maintained in the L-BLG by considering general space-time varying solutions to the constraint equations. The dual geometry is reduced to $d = 10 \text{AdS}_4 \times \text{CP}^3$ in the scaling limit and has the same conformal symmetry. The curvature radius $R$ satisfies $l_p^{(11)} \ll l_p^{(10)} \ll R \ll l_s$ ($l_p^{(d)}$ and $l_s$ are the $d$-dimensional Planck lengths and the string scale), and the theory is in a region where an $\alpha'$ expansion is not valid. We also study how the $SO(8)$ covariance is recovered in the $\text{AdS}_4 \times \text{CP}^3$ geometry by taking the scaling limit.

\textsuperscript{*}yhonma@post.kek.jp
\textsuperscript{†}satoshi.iso@kek.jp
\textsuperscript{‡}sumitomo@post.kek.jp
\textsuperscript{§}umetsu@kushiro-ct.ac.jp
\textsuperscript{¶}zhangsen@post.kek.jp
1 Introduction

Bagger and Lambert, and Gustavsson discovered the $\mathcal{N} = 8$ superconformal $(2 + 1)$-dimensional field theories with a $SO(8)$ global symmetries by exploiting the 3-algebraic structures $[1, 2]$ and an effective theory of multiple M2 branes was proposed, which was based on the 3-algebra with a Lorentzian signature $[3, 4, 5]$ (the L-BLG theory). Another very interesting proposal for multiple M2 branes was also made recently by Aharony, Bergman, Jafferis and Maldacena (the ABJM theory) $[6]$.

An earlier proposal of L-BLG $[3, 4, 5]$ has the desired symmetries, $SO(8)$ and superconformal symmetries but there are several problems. First, because of the Lorentzian signature, the model contains fields $X^I_0$ and $X^I_{-1}$ which may endanger the unitarity of the theory. The fields $X^I_{-1}$ (and the fermionic partner $\Psi_{-1}$) are, however, contained in the action only linearly and can be integrated out to give the following constraints on the ‘conjugate’ fields $X^I_0$ and $\Psi_0$:

$$\partial^2 X^I_0 = 0, \quad \Gamma^\mu \partial_\mu \Psi_0 = 0.$$ (1.1)

Hence the would-be ghost modes can be removed from the propagating degrees of freedom. If we take a constant solution $X^I_0 = v^I$ to the constraint equation, the L-BLG theories are reduced to the action of the $N$ D2 branes in flat space $[9, 5, 10, 11]$. The specific choice of the solutions breaks the conformal invariance and the $SO(8)$ to $SO(7)$ and it has been suspected that the L-BLG theory is nothing more than a theory of D2 branes in $d = 10$ spacetime. This is another problem of the L-BLG theory.

In this paper we would like to emphasize that the constraint equations (1.1) should be more carefully treated as we did in $[10, 12]$ and show that the interpretation of the L-BLG as the ordinary D2 branes is not appropriate (see also a recent work $[13]$). In $[10]$, we revisited the constraint equation and considered general spacetime dependent solutions and the theory around it $[1]$. By considering such space-time dependent solutions, the theory is shown to have a generalized conformal symmetry as well as the manifest $SO(8)$ invariance $[12]$. The purpose of the present paper is to investigate these symmetries extensively both in the field theories and in the gravity duals.

Another very interesting proposal for multiple M2 branes was made by Aharony et.al. $[6]$. They generalized the superconformal Chern-Simons matter theories $[17, 18]$ to the $\mathcal{N} = 6$ superconformal $U(N) \times U(N)$ theories. The levels of the Chern-Simons gauge fields

$^\ast$There are attempts to kill the ghost fields by gauging a shift symmetry $[7, 8]$.

$^\dagger$In $[10]$, we have also studied the constraint equations of the mass-deformed theory $[14, 15]$. In this case, the constraint equation is modified to $(\partial^2 + \mu^2)X^I_0 = 0$ and there are no constant solutions. We studied the theory around a background of the spacetime dependent solution $X^I_0 = \exp(\mu x) \delta^I_0$ and called such field theories Janus field theories. It was also shown in $[16]$ that the spacetime dependent coupling in the massless theory can be reinterpreted as a coordinate-dependent mass term with a constant coupling.
are $(k, -k)$ and the theory is conjectured to describe the low energy limit of $N$ M2-branes probing $\mathbb{C}^4/\mathbb{Z}_k$. Hence at large $N$, it is dual to the M-theory on $\text{AdS}_4 \times S^7/\mathbb{Z}_k$. In the formulation the 3-algebraic structure does not seem to play any role, but recently Bagger and Lambert showed that if the condition of the antisymmetry of the structure constant is relaxed, the ABJM theory can be written in terms of the new 3-algebra [19]. There are also interesting works of the ABJM theory for the relation to a M5-brane through the Basu-Harvey equations [20, 21, 22], $\mathcal{N} = 6$ Chern-Simons theory with the other gauge groups [23, 24] and the $\mathcal{N} \leq 8$ superconformal theories from the 3-dimensional gauged supergravity models [25]. For other related works of the ABJM theory, see [26].

Now we have two different proposals for the formulation of multiple M2 branes, L-BLG and ABJM, and it is important to understand the relation between them, especially how the conformal symmetries and $SO(8)$ invariance are realized in these theories. In [12], we have explicitly shown that the L-BLG theory can be obtained by taking the following scaling limit of the ABJM theory (fermions are omitted here):

$$
B_\mu \to \lambda B_\mu,
$$

$$
X^I_0 \to \lambda^{-1} X^I_0,
$$

$$
k \to \lambda^{-1} k,
$$

(1.2)

where $B_\mu$ is an axial combination of the two gauge fields $B_\mu = (A^{(L)}_\mu - A^{(R)}_\mu)/2$ and $X^I_0$ are trace components of the bifundamental matter fields. The other bosonic fields are kept fixed and then take the limit:

$$
\lambda \to 0.
$$

(1.3)

The gauge group $SU(N) \times SU(N)$ of the ABJM theory is reduced to the semi-direct sum of $SU(N)$ and translations a la Inonü-Wigner contraction [27]. Taking the scaling limit, the action of the ABJM theory is reduced to the action of the L-BLG theory. Furthermore, the same constraint equations (1.1) can be obtained by requiring finiteness of the action in the $\lambda \to 0$ limit. We emphasize here that the scaling limit is taken before taking the large $N$ limit. Hence the 't Hooft coupling $N/k$ vanishes in the scaling limit from ABJM to L-BLG.

The M2 branes described by the ABJM theory has conformal symmetry. The scaling limit mentioned above corresponds to locating M2 branes far from the origin of the $\mathbb{Z}_k$ orbifold as well as taking $k(\gg N) \to \infty$. Since the coupling of the scaled theory is promoted to an $SO(8)$ vector $X^I_0(x)$, we showed in [12] that the scaled theory of L-BLG has an enhanced symmetries, i.e. generalized conformal symmetry and $SO(8)$ invariance. These symmetries are not expected to exist in the effective theory of the ordinary D2 branes. This generalized conformal symmetry is essentially the same as that proposed by Jevicki, Kazama and Yoneya [28] 10 years ago for general Dp-branes.
In this paper we further investigate the conformal symmetry and recovery of $SO(8)$ invariance in the ABJM and L-BLG theories. In section 2, we first analyze the conformal invariance of the ABJM theory, in particular the invariance under the special conformal transformations. Since the scaling limit is compatible with the conformal invariance, the L-BLG theory also has the same conformal invariance. We also show the constraint equations (1.1) are compatible with the conformal symmetries. It should be emphasized that the conformal invariance can be preserved only when we consider a set of spacetime dependent solutions to the constraint equations, and a specific choice to the equations generally breaks the conformal invariance.

In section 3, we discuss the conformal symmetry and the recovery of $SO(8)$ in the gravity dual. In [6], the dual geometry of the ABJM theory is conjectured to be $AdS_4 \times S^7/\mathbb{Z}_k$ where the $S^7$ is considered as $U(1)$ Hopf fibration on $\mathbb{CP}^3$ and the $U(1)$ direction is orbifolded. In the scaling limit of ABJM to L-BLG, $k$ is taken to infinity and the $\mathbb{Z}_k$ identification looks like a circle identification of the $d = 11$ theory. In this reduction to $d = 10$, the dilaton field takes a constant value and the reduced $d = 10$ geometry is given by $AdS_4 \times \mathbb{CP}^3$. In the original discussion of ABJM, the ’t Hooft coupling $N/k$ is kept fixed and hence the radius of curvature of $AdS_4$ is finite in the string units. However in our scaling limit to L-BLG, $k$ is taken to infinity before taking the large $N$ and the radius becomes much smaller than the string scale:

$$\left( \frac{R}{l_s} \right)^2 = R_{str}^2 \sim \sqrt{\frac{N}{k}} \to 0.$$  \hspace{1cm} (1.4)

On the other hand, comparing $R$ with the $d = 10$ Planck length, the ratio is given by

$$\left( \frac{R}{l_{(10)}^p} \right)^2 \sim k^{1/8} N^{3/8} \to \infty$$ \hspace{1cm} (1.5)

and the type IIA supergravity approximation itself is good. Hence the reduced geometry of ABJM in the scaling limit $k \to \infty$ to L-BLG can be described by $AdS_4 \times \mathbb{CP}^3$, but it cannot be considered as a low energy approximation of type IIA superstring. The scaled theory (L-BLG) may be more appropriately interpreted as M2 branes in $d = 10$ that is dimensionally reduced from the $d = 11$ supergravity.

In Appendix [A] we discuss the effect of $U(1)$ gauge field in the scaling limit of the $U(N) \times U(N)$ ABJM theories. In Appendix [B] the recovery of $SO(8)$ in $\mathbb{C}^4/U(1)$ is discussed. In Appendix [C] we review the ordinary reduction from $d = 11$ M2 branes to $d = 10$ D2 branes.
2 Conformal Symmetry of ABJM and L-BLG

2.1 Conformal invariance of ABJM

The ABJM theory of $d = 3, \mathcal{N} = 6$ superconformal theory is proposed as a dual field theory of M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$. As shown in [29], the ABJM theory is invariant under the superconformal transformations. Here we study the invariance of the ABJM theory under the conformal transformations, in particular the special conformal transformations.

The action of the ABJM theory is given by

$$S = \int d^3x \, \text{tr}[-(D_\mu Y^A)^\dagger D^\mu Y^A + i\psi^A_{\bar{\mu}} \Gamma^\mu D_\mu \psi^A] + S_{CS} - S_{V_f} - S_{V_b} \tag{2.1}$$

where

$$S_{CS} = \frac{1}{2} d^3x \, 2K \epsilon^{\mu \nu \lambda} \text{tr}[A^{(L)}_\mu \partial_\nu A^{(L)}_\lambda + \frac{2i}{3} A^{(L)}_{\nu A} A^{(L)}_{\lambda} - A^{(R)}_\mu \partial_\nu A^{(R)}_\lambda - \frac{2i}{3} A^{(R)}_{\nu A} A^{(R)}_{\lambda}] \tag{2.2}$$

$$S_{V_b} = -\frac{1}{48K^2} \int d^3x \, \text{tr}[Y^A Y^B Y^C Y^D Y^B Y^C Y^D + 4Y^A Y^B Y^C Y^D Y^B Y^C - 6Y^A Y^B Y^C Y^D Y^B Y^C Y^D],$$

$$S_{V_f} = \frac{i}{4K} \int d^3x \, \text{tr}[Y^A Y^B Y^C Y^D \psi^B \psi_B - Y^A Y^B \psi^A \psi_B^B + 2Y^A Y^B \psi^A \psi_B^B - 2Y^A Y^B \psi^A \psi_B^B + \epsilon^{ABCD} Y^A Y^B \psi^C \psi^D - \epsilon^{ABCD} Y^A Y^B \psi^C \psi^D]$$

and $A = 1, 2, 3, 4$. We used the notation of [30] and $K = k/8\pi$.

It is a $U(N) \times U(N)$ or $SU(N) \times SU(N)$ gauge theory. The other choices of gauge groups are possible but here we consider these two types. The actions of the gauge fields are given by the Chern-Simons action with coefficients $k$ and $-k$. Matter fields $Y^A$ and $\psi^A$ are in the bifundamental representation and the covariant derivative is defined by

$$D_\mu Y = \partial_\mu Y + iA^{(L)}_\mu Y - iY A^{(R)}_\mu. \tag{2.2}$$

The action is invariant under $\mathcal{N} = 6$ superconformal transformations. In the following we check the explicit invariance under the conformal transformations.

First it is obvious that the action is invariant under the dilatation. Dilatation is defined by $x \rightarrow e^e x$ and simultaneously we transform each field by multiplying $e^{-n\epsilon}$ where $n$ is the conformal weight. The scalars $Y^A$, fermions $\psi^A$ and the gauge fields $A_\mu$ have weights 1/2, 1, 1 respectively.

A little more nontrivial transformation is a special conformal transformation. It is given by

$$\delta x_\mu = 2\epsilon \cdot xx_\mu - \epsilon_\mu x^2. \tag{2.3}$$
If we write the infinitesimal transformation for each field \( Y(x) \) as \( \delta Y(x) = Y'(x') - Y(x) \), they are given by

\[
\begin{align*}
\delta Y^A(x) &= -\epsilon \cdot x Y^A(x), \\
\delta A^{(L,R)}_\mu(x) &= -2\epsilon \cdot x A^{(L,R)}_\mu(x) - 2(x \cdot A^{(L,R)})\epsilon_\mu - \epsilon \cdot A^{(L,R)} x_\mu, \\
\delta \psi^A(x) &= -2\epsilon \cdot x \psi^A(x) - \epsilon_{\mu\lambda}\epsilon^\nu x^\lambda \Gamma^{\mu}_\lambda \psi^A(x).
\end{align*}
\]

These transformations can be understood as follows. They look like the general coordinate transformations, but are different since the theory is restricted to live in the flat space-time with a fixed metric and the change of the metric under the general coordinate transformations must be compensated by the transformations of the fields. The first terms in each transformation reflect the conformal weight of each field. The second term in the transformation of the fermion is the local Lorentz transformation which pulls back terms in each transformation.

The action is invariant under the above special conformal transformations. In order to see it, the following transformation rules are useful:

\[
\begin{align*}
d^3x &\to e^{6\epsilon \cdot x}d^3x, \\
\partial_\mu &\to e^{-2\epsilon \cdot x}[\partial_\mu - 2(\epsilon_\mu x_\nu \partial_\nu - x_\mu \epsilon^\nu \partial_\nu)], \\
D_\mu Y &\to e^{-3\epsilon \cdot x}[D_\mu Y - \{Y + 2x^\nu \partial_\nu Y + 2i(x \cdot A^{(L,R)}Y - Y \cdot A^{(R)})\} \epsilon_\mu \\
&\quad + \{2\epsilon^\nu \partial_\nu Y + 2i(\epsilon \cdot A^{(L,R)}Y - Y \epsilon \cdot A^{(R)})\} x_\mu], \\
F_{\mu\nu} &\to e^{-2\epsilon \cdot x}[F_{\mu\nu} - 2(\epsilon_\nu x^\rho F_{\mu\rho} - \epsilon_\mu x^\rho F_{\nu\rho}) + 2(x_\nu \epsilon^\rho F_{\mu\rho} - x_\mu \epsilon^\rho F_{\nu\rho})].
\end{align*}
\]

Though \( \epsilon \) is an infinitesimal parameter, we write the overall factors as \( e^{-2n\epsilon \cdot x} \) for convenience. They are cancelled in the action because \( n \) is the conformal weight of each field and coordinates.

Here let us check the invariance of the Chern-Simons term as an example. First the derivative part transforms as

\[
\epsilon^{\mu\nu\lambda}\text{tr} F_{\mu\nu} A_\lambda \\
\to \epsilon^{\mu\nu\lambda} e^{-6\epsilon \cdot x} \text{tr}[F_{\mu\nu} A_\lambda + 4(\epsilon_\mu x^\rho - x_\mu \epsilon^\rho) A_\lambda F_{\nu\rho} - 2F_{\mu\nu}(x \cdot A\epsilon_\lambda - \epsilon \cdot A x_\lambda)].
\]

The pre-factor \( e^{-6\epsilon \cdot x} \) is cancelled with the transformation of \( d^3x \) in (2.5). The rest vanishes because

\[
\epsilon^{\mu\nu\lambda}\text{tr}[2(\epsilon_\mu x^\rho - x_\mu \epsilon^\rho) A_\lambda F_{\nu\rho} - F_{\mu\nu}(x \cdot A\epsilon_\lambda - \epsilon \cdot A x_\lambda)] \\
= \epsilon^{\mu\nu\lambda}\text{tr}[2\epsilon_\mu \rho\alpha f_\alpha F_{\nu\rho} A_\lambda - \epsilon_\lambda \rho\alpha f_\alpha F_{\mu\rho} A_\lambda] = 0.
\]

5
In the second line we have defined $f^\alpha = \epsilon^{\mu\nu\alpha} x_\mu \epsilon_\nu$. Similarly the invariance of the term $\epsilon^{\mu\nu\lambda} A_\mu A_\nu A_\lambda$ can be shown by noting that the gauge field transforms as

$$A_\mu \rightarrow e^{-2\epsilon \cdot x} (A_\mu + 2\epsilon_{\mu\alpha\beta} f^\alpha A^\beta).$$

(2.8)

Hence the Chern-Simons terms are invariant under the special conformal transformation. Though we have checked it explicitly, the invariance can be naturally understood because the Chern-Simons term is independent of the metric if it is defined in a curved background space-time.

The other terms in the action are also straightforwardly shown to be invariant under the special conformal transformations.

### 2.2 ABJM to L-BLG

As shown in [12], the L-BLG theory is obtained by taking a scaling limit of the ABJM theory with a gauge group $SU(N) \times SU(N)$. In the gauge theory with $U(N) \times U(N)$ there is a subtlety in the scaling of the $U(1)$ part. We will discuss the issue in the Appendix A and here restrict the discussions to the $SU(N) \times SU(N)$ case.

The scaling is given as follows:

$$B_\mu \rightarrow \lambda B_\mu,$$

$$X_0^I \rightarrow \lambda^{-1} X_0^I,$$

$$\psi_{A0} \rightarrow \lambda^{-1} \psi_{A0},$$

$$k \rightarrow \lambda^{-1} k$$

(2.9)

where

$$Y^A = X_0^{2A-1} + iX_0^{2A} - \hat{X}^{2A} + i\hat{X}^{2A-1}, \quad B_\mu = \frac{1}{2} (A_{\mu}^{(L)} - A_{\mu}^{(R)})$$

(2.10)

and $X_0^I$ and $\psi_{0A}$ are trace components of the bifundamental matter fields, and $I = 1, \cdots , 8$. When we take $\lambda \rightarrow 0$ limit and keep the other fields fixed, the action of the ABJM theory is reduced to the action of the L-BLG theory. Since the $k \rightarrow \infty$ limit is taken before taking the large $N$, our scaling corresponds to a vanishing ’t Hooft coupling $N/k \rightarrow 0$. Besides the action, the same constraint equations as those in the L-BLG theory can be obtained from the ABJM theory:

$$\partial^2 X_0^I = 0, \quad \Gamma^\mu \partial_\mu \psi_0 = 0,$$

(2.11)

by requiring finiteness of the action in the $\lambda \rightarrow 0$ limit.
In the above scaling limit we arrive at the L-BLG theory:

\[ L_0 = \text{Tr} \left[ -\frac{1}{2} (\hat{D}_\mu \hat{X}^I - B_\mu X_0^I)^2 + \frac{1}{4} (X^K_0)^2 (\{ \hat{X}^I, \hat{X}^J \})^2 - \frac{1}{2} (X_0^I [\hat{X}^I, \hat{X}^J])^2 \right. \\
\left. + \frac{i}{2} \bar{\Psi} \Gamma^\mu \hat{D}_\mu \hat{\Psi} + i \bar{\Psi}_0 \Gamma^\mu B_\mu \hat{\Psi} - \frac{1}{2} \bar{\Psi}_0 \hat{X}^I [\hat{X}^J, \Gamma_{IJ} \hat{\Psi}] + \frac{1}{2} \bar{\Psi} X_0^I [\hat{X}^J, \Gamma_{IJ} \hat{\Psi}] \right. \\
\left. + \frac{1}{2} \epsilon^{\mu\nu\lambda} \hat{F}_{\mu\nu} B_\lambda - \partial_\mu X_0^I B_\mu \hat{X}^I \right]. \quad (2.12) \]

In the original formulation of the L-BLG theory, the constraint equations (2.11) are derived by integrating the auxiliary fields \( X_{-1} \) and \( \Psi_{-1} \):

\[ L_{gh} = (\partial_\mu X_0^I)(\partial^\mu X_{-1}^I) - i \bar{\Psi}_{-1} \Gamma^\mu \partial_\mu \Psi_0. \quad (2.13) \]

Since the above scaling is compatible with the conformal transformations discussed in the previous section, the action (2.12) is invariant under the conformal transformations (see also [31]). The action for the auxiliary fields (2.13) is also invariant if we define the transformations for them as

\[ \delta X_{-1}^I(x) = -\epsilon \cdot x X_{-1}^I(x), \]
\[ \delta \Psi_{-1}(x) = -2\epsilon \cdot x \Psi_{-1}(x) - \epsilon_{\mu\nu\lambda} \epsilon^\nu x^\lambda \Gamma^\mu \Psi_{-1}(x). \quad (2.14) \]

### 2.3 Generalized conformal symmetry in D2 branes

Now integrate the \( B_\mu \) gauge field. It has been discussed that if we pick up a specific solution to the constraint equation (2.11), especially a constant solution

\[ X_0^I = v \delta^I 8, \quad \Psi_0 = 0, \quad \delta^I 8 \]

the L-BLG theory is reduced to the action of the ordinary D2 branes whose Yang-Mills coupling constant is given by \( g_{YM} = v \):

\[ L = \text{Tr} \left[ -\frac{1}{4v^2} \hat{F}_{\mu\nu}^2 - \frac{1}{2} (\hat{D}_\mu \hat{X}^A)^2 + \frac{1}{4} v^2 [\hat{X}^A, \hat{X}^B] + \frac{i}{2} \bar{\Psi} \Gamma^\mu \hat{D}_\mu \hat{\Psi} + \frac{1}{2} v \bar{\Psi} [\hat{X}^A, \Gamma_{8A} \hat{\Psi}] \right]. \quad (2.16) \]

where \( A, B = 1, \cdots, 7 \). Then \( SO(8) \) is spontaneously broken to \( SO(7) \) because we have specialized the 8-th direction. The conformal invariance is also broken. Though the action is the same as that of the D2 branes, we see later that the interpretation of the L-BLG theory as an effective theory of the ordinary D2 branes is not appropriate since the radius of curvature is much smaller than the string scale in the gravity dual.

The constraint equations (2.11) have more general solutions than (2.15) which depend on the spacetime coordinates. Then the resulting action becomes a Yang-Mills theory with
a spacetime dependent coupling \[^{10}\]. As we have shown \[^{12}\], the action with the spacetime dependent coupling is invariant under the conformal transformations if we consider a set of spacetime dependent solutions. The conformal invariance is discussed in more details in the next section.

We here consider the simplest spacetime dependent solutions:

\[ X_0^I = v(x) \delta^{I,8}, \quad \Psi_0 = 0, \quad \partial^2 v(x) = 0. \]  

(2.17)

Then the L-BLG theory is reduced to the same action as that of the D2 branes but with a spacetime varying coupling:

\[
\mathcal{L} = \text{Tr} \left[ -\frac{1}{4v(x)^2} \hat{F}_{\mu\nu}^2 - \frac{1}{2} (\hat{D}_\mu \hat{X}^A)^2 + \frac{1}{4} v(x)^2 [\hat{X}^A, \hat{X}^B]^2 \\
+ \frac{i}{2} \bar{\Psi} \Gamma^\mu \hat{D}_\mu \hat{\Psi} + \frac{1}{2} v(x) \bar{\Psi} [\hat{X}^A, \Gamma_{8,A} \hat{\Psi}] \right].
\]  

(2.18)

\(SO(8)\) symmetry is spontaneously broken to \(SO(7)\) as well, but the action with a varying \(v(x)\) has a generalized conformal symmetry if the coupling transforms as

\[ \delta v(x) = -(\epsilon \cdot x) v(x). \]  

(2.19)

This transformation is originated in the special conformal transformation of the scalar field \(^{24}\). The generalized conformal transformation for Dp branes were first proposed by Jevicki, Kazama and Yoneya \[^{28}\]. In the present case, the transformation \((2.19)\) is naturally derived since the coupling constant of the Yang-Mills action is determined by the center of mass coordinates \(X_0^I(x)\) of the M2 branes.

It is worth noting that the generalized conformal transformation \((2.19)\) is compatible with the constraint equations \((2.11)\) only when \(p = 2\). We will discuss it in the next section.

### 2.4 Conformal symmetry and \(SO(8)\) invariance of L-BLG

The space-time dependent coupling \(v(x)\) can be promoted to an \(SO(8)\) vector \(X_0^I(x)\) by considering general solutions to the constraint equations \((2.11)\) as shown in \[^{10}\]. Then the resultant action after integrating the \(B_\mu\) gauge field becomes D2 branes effective action with space-time dependent couplings in a vector representation of the \(SO(8)\). In \[^{12}\] we showed that if we consider space-time dependent solutions the theory has the \textit{generalized conformal symmetry} as well as the manifest \(SO(8)\) invariance.

In this section we study more details of the generalized conformal symmetry of the L-BLG theory. Especially we show that the conformal transformations are closed under the constraint equations \((2.11)\).
By integrating the $B_\mu$ gauge field, we get the action $S = \int d^3x (\mathcal{L}_0 + \mathcal{L}')$:

\[
\mathcal{L}_0 = \text{Tr} \left[ -\frac{1}{2} (\hat{D}_\mu P^I)^2 + \frac{1}{4} X_0^2 [P^I, P^J]^2 + \frac{i}{2} \tilde{\Psi} \Gamma^\mu \hat{D}_\mu \hat{\Psi} + \frac{1}{2} \tilde{\Psi} [P^I, (X_0^J \Gamma_J) \Gamma_I \hat{\Psi}] 
+ \frac{1}{2 (X_0^2)^2} \left( \frac{1}{2} e^{\mu \lambda \nu} \hat{F}_{\nu \lambda} + i \tilde{\Psi}_0 \Gamma^\mu \hat{\Psi} - 2 P_I \partial^\mu X_0^I \right)^2 \right] - \frac{1}{2} \tilde{\Psi}_0 \Gamma_{IJ} \hat{\Psi} [P^I, P^J],
\]

\[
\mathcal{L}' = \frac{1}{X_0^2} \text{Tr} \left[ \left( -\tilde{\Psi}_0 \Gamma_I (X_0^J \Gamma_J) [P^I, \hat{\Psi}] - i \tilde{\Psi}_0 \Gamma_\mu \hat{D}_\mu \hat{\Psi} \right) (X_0^K \hat{X}^K) \right].
\]

(2.20)

where we have defined a new scalar field $P_I$ with 7 degrees of freedom by using the projection operator

\[
P_I(x) = \left( \delta_{IJ} - \frac{X_0^0 X_0^J}{X_0^2} \right) X^J.
\]

(2.21)

The $X^I_0(x)$ field is constrained to satisfy $\partial^2 X^I_0 = 0$. This is a generalization of (2.18).

We called this theory a Janus field theory of (M)2-branes since the coupling constant is varying with the space-time coordinates.

The action of the gauge field is no longer the Chern-Simons action but we can again show that it is invariant under the conformal transformations. Under the dilatation $x^\mu \rightarrow e^{n} x^\mu$, each field is multiplied by $e^{-n\epsilon}$ where $n$ is the conformal weight. The weights of $P, X_0, A_\mu, \Psi, \Psi_0$ are 1/2, 1/2, 1, 1, 1 respectively. The action is evidently invariant.

Special conformal transformation is similarly given by

\[
\delta x^\mu = 2 \epsilon \cdot x x^\mu - \epsilon^\mu x^2
\]

(2.22)

and the fields transform as

\[
\begin{align*}
\delta P^I(x) &= -\epsilon \cdot x P^I(x), \\
\delta X^I_0(x) &= -\epsilon \cdot x X^I_0(x), \\
\delta A_\mu(x) &= -2 \epsilon \cdot x A_\mu(x) - 2 (x \cdot A \epsilon_\mu - \epsilon \cdot A x_\mu), \\
\delta \tilde{\Psi}(x) &= -2 \epsilon \cdot x \tilde{\Psi}(x) - \epsilon_{\mu \lambda} \epsilon^\nu x^\lambda \Gamma^\mu \tilde{\Psi}(x), \\
\delta \Psi_0(x) &= -2 \epsilon \cdot x \Psi_0(x) - \epsilon_{\mu \lambda} \epsilon^\nu x^\lambda \Gamma^\mu \Psi_0(x).
\end{align*}
\]

(2.23)

It is now straightforward to show the invariance of the action. The Lagrangian is not invariant but changes by total derivatives.

Finally we need to check that the transformation is closed within the constraint equations (2.11). Namely if the field $X^I_0(x)$ satisfies $\partial_\epsilon^2 X^I_0(x) = 0$, the transformed field $X^I_0'(x')$ must also satisfy $\partial_\epsilon^2 X^I_0'(x') = 0$. For an infinitesimal special conformal transformation, this is equivalent to show $\partial^2 \delta X^I_0(x) = 0$ where $\delta X^I_0(x)$ is the transformation at the numerically same point defined by

\[
\begin{align*}
\delta X^I_0(x) &= X^I_0'(x) - X^I_0(x) = \delta X^I_0(x) - \delta x^\mu \partial_\mu X^I_0(x), \\
\delta \Psi_0(x) &= \Psi_0'(x) - \Psi_0(x) = \delta \Psi_0(x) - \delta x^\mu \partial_\mu \Psi_0(x).
\end{align*}
\]

(2.24)
In the following, in order to see the specialty for M2 (or D2)-branes, we generalize the special conformal transformation to Dp-branes [28]:

\[ \tilde{\delta}X^I_0(x) = -(3 - p)\epsilon \cdot x X^I_0 - (2\epsilon \cdot xx^\mu - \epsilon x^2)\partial_\mu X^I_0 \]  

(2.25)

It is easy to show

\[ \partial_2^2(\tilde{\delta}X^I_0(x)) = 2(p - 2)\epsilon^\mu \partial_\mu X^I_0 \]  

(2.26)

where we have used the constraint equation \( \partial_2^2X^I_0 = 0 \). This vanishes at \( p = 2 \) only.

Similarly, \( \tilde{\delta}\Psi_0 \) is given by

\[ \tilde{\delta}\Psi_0(x) = -2(3 - p)\epsilon \cdot x \Psi_0 - \epsilon_{\mu\nu\lambda}^\gamma x^\lambda \Gamma^\mu\nu\lambda\Psi_0 - (2\epsilon \cdot xx^\mu - \epsilon x^2)\partial_\mu \Psi_0 \]  

(2.27)

and satisfies

\[ \Gamma^\alpha\partial_\alpha(\tilde{\delta}\Psi_0(x)) = 2(p - 2)\Gamma^\alpha\epsilon_\alpha\Psi_0 \]  

(2.28)

where we used the constraint equation \( \Gamma^\alpha\partial_\alpha \Psi_0 = 0 \). Again \( \Gamma^\alpha\partial_\alpha(\tilde{\delta}\Psi_0(x)) = 0 \) vanishes at \( p = 2 \) only. Both of the constraints are compatible with the generalized conformal transformations at \( p = 2 \). It shows a specialty of M2 (or D2) branes.

We have shown that the constraint equations are compatible with the generalized conformal transformations. If the solutions are restricted to constant ones as in (2.15), we no longer have the generalized conformal symmetry. It can be maintained only when we consider a set of space-time dependent solutions to the constraint equations.

Recently H. Verlinde [13] also considered space-time dependent solutions to the constraint equations and discussed the conformal symmetry of the L-BLG theory. In his study the constraint equation is imposed everywhere except at \( z_i \) where a local operator \( \mathcal{O}_i(z_i) \) is inserted,

\[ X^I_0(x) = \sum \frac{q^I_i}{|x - z_i|}. \]  

(2.29)

This is an inhomogeneous solution to the equation

\[ \partial_2^2X^I_0 = -4\pi \sum q^I_i \delta^3(x - z_i). \]  

(2.30)

We can add the homogeneous solutions to the above. If \( q^I \) and \( z \) (omitting the index \( i \)) transform as

\[ \delta q^I = \epsilon \cdot z q^I \]
\[ \delta z_\mu = 2(\epsilon \cdot z) z_\mu - \epsilon_\mu z^2, \]  

(2.31)

the transformation of \( X^I_0 \)

\[ \delta X^I_0(x) = -(\epsilon \cdot x)X^I_0(x) \]  

(2.32)
is reproduced and the L-BLG action is invariant under the conformal transformations. Note that \( q^I \) cannot be a constant. If \( q^I \) is kept fixed, the set of solutions is not closed under the conformal transformations. In order to recover the conformal invariance, \( q^I \) should be a position \( z \)-dependent charge.

We have shown that the L-BLG theory has both of the \( SO(8) \) invariance and the conformal symmetry. In the next section we discuss the symmetry properties of the gravity dual of the ABJM theory.

3 \( SO(8) \) and Conformal Symmetry in Dual Geometry

3.1 Large \( k \) limit of ABJM geometry

In the paper \cite{6}, it was pointed out that the \( U(N) \times U(N) \) ABJM theory is dual to the M-theory on \( AdS_4 \times S^7/\mathbb{Z}_k \), which is a \( d = 11 \) supergravity solution of M2 branes probing the orbifold \( C^4/\mathbb{Z}_k \). We first review the solution of supersymmetric M2 branes in \( d = 11 \) supergravity.

The \( d = 11 \) metric of the multiple M2-branes is given by

\[
ds^2 = H^{-\frac{n}{4}} \left( \sum_{\mu,\nu=0}^{2} \eta_{\mu\nu} dx^\mu dx^\nu \right) + H^{\frac{n}{4}} \left( dr^2 + r^2 d\Omega_7^2 \right),
\]

\[
H(r) \equiv 1 + \frac{R^6}{r^6},
\]

where \( R^6 = 32\pi^2 N' l_p^6 \) and \( d\Omega_7^2 \) is the metric of a unit 7-sphere. \( N' \) is the number of the M2 branes and identified with \( N' = kN \). The three form field is also given as

\[
C^{(3)} = H^{-1} dx^0 \wedge dx^1 \wedge dx^2
\]

and the 4-form flux normalized by the world volume is proportional to \( N' \).

By focusing on the near horizon region of the M2-brane, the geometry becomes \( AdS_4 \times S^7 \) geometry. In the near horizon limit \( R \gg r \), \( H(r) \) is replaced by \( H(r) = (R/r)^6 \) and the metric becomes

\[
ds^2 = \left( \frac{r}{R} \right)^4 \left( \sum_{\mu,\nu=0}^{2} \eta_{\mu\nu} dx^\mu dx^\nu \right) + \left( \frac{R}{r} \right)^2 dr^2 + R^2 d\Omega_7^2
\]

\[
= R^2 \left[ \frac{1}{4} ds_{AdS}^2 + d\Omega_7^2 \right]
\]

where we have rescaled the M2 brane world volume coordinates by a factor \( 2/R^3 \). Hence the near horizon geometry of the supersymmetric M2 branes is given by \( AdS_4 \times S^7 \) with a radius \( R \). In the large \( N' = kN \) limit, the radius becomes much larger than the \( d = 11 \) Planck length and the \( d = 11 \) supergravity approximation is valid.
The ABJM theory describes M2 branes on $\mathbb{C}^4/\mathbb{Z}_k$ orbifold. The dual geometry can be obtained by first specifying the polarization (choice of the complex coordinates) in $\mathbb{R}^8$ and then dividing $\mathbb{C}^4$ by $\mathbb{Z}_k$.

Since $S^7$, parameterized by $z^A$ ($A = 1, \cdots, 4$) with $|z^A|^2 = 1$, is a $U(1)$-fibration on $\mathbb{C}\mathbb{P}^3$, the metric of $S^7$ is written as

$$d\Omega_7^2 = (d\varphi' + \omega)^2 + ds_{\mathbb{C}\mathbb{P}^3}^2$$

where $\varphi'$ is the overall phase of $z^A$. The details of the definition of coordinates are written in Appendix B.

We now perform the $\mathbb{Z}_k$ quotient by dividing the overall phase of each $z^A$, namely the $\varphi'$ direction. By rewriting $\varphi' = \varphi/k$ with $\varphi \sim \varphi + 2\pi$, the metric of $S^7/\mathbb{Z}_k$ becomes

$$ds_{S^7/\mathbb{Z}_k}^2 = \frac{1}{k^2} (d\varphi + k\omega)^2 + ds_{\mathbb{C}\mathbb{P}^3}^2. \tag{3.5}$$

Before performing the $\mathbb{Z}_k$ quotient, the metric has the conformal symmetry associated with the $AdS_4$ geometry and $SO(8)$ symmetry of $S^7$. The orbifolding breaks the $SO(8)$ symmetry to $SU(4) \times U(1)$ but the conformal invariance still exists. This is the bosonic symmetry of the ABJM theory.

The L-BLG action can be derived by taking the scaling limit (2.9) of the ABJM theory. In the gravity side, this scaling corresponds to locating the probe M2 branes far from the orbifold singularity and taking the large $k$ limit. As we show in the next section, the former process recovers the $SO(8)$ if the positions of the M2 branes are considered to be dynamical variables. The latter makes the radius of the $\varphi'$ circle small and $d = 11$ geometry is reduced to $d = 10$.

Now we consider the $k \to \infty$ limit of the dual geometry of the ABJM theory. Following the prescription of ABJM, we shall interprete the coordinate $\varphi$ as the compact direction in reducing from M-theory to type IIA superstring. Using the reduction formula [32]

$$ds_{11}^2 = e^{-\frac{2}{3}\phi} ds_{10}^2 + e^{\frac{4}{3}\phi} (l_p)^2 (d\varphi + A)^2 \tag{3.6}$$

we get the $d = 10$ metric and the dilaton field in type IIA supergravity as

$$ds_{10}^2 = \frac{r}{k l_p} H^{-\frac{1}{2}} \left( \sum_{\mu,\nu=0}^2 \eta_{\mu\nu} dx^\mu dx^\nu \right) + \frac{r}{k l_p} H^{\frac{1}{4}} (dr^2 + r^2 ds_{\mathbb{C}\mathbb{P}^3}^2), \tag{3.7}$$

$$e^{2\phi} = \left( \frac{r}{k l_p} \right)^3 H^{\frac{1}{2}} = \left( \frac{R}{k l_p} \right)^3. \tag{3.8}$$

Hence in the $k \to \infty$ limit, the metric becomes $AdS_4 \times \mathbb{C}\mathbb{P}^3$:

$$ds_{10}^2 = \frac{R^3}{k} \left[ \frac{1}{4} ds_{AdS_4}^2 + ds_{\mathbb{C}\mathbb{P}^3}^2 \right] \tag{3.9}$$
where the radius of curvature in string units is
\[ R_{\text{str}}^2 = \left( \frac{R}{l_s} \right)^2 = \frac{R^3}{k l_p^3} = 2^{5/2} \pi \sqrt{\frac{N}{k}}. \] (3.10)

The dilaton is a constant and this is the reason why the \( d = 10 \) metric still has a conformal symmetry associated with the \( AdS_4 \) geometry. This is different from the ordinary reduction of the M2 branes to D2 branes by compactifying the 11th direction of the Cartesian coordinate (see Appendix C). Note that in the type IIA picture, in addition to the four-form RR flux \( F_4 \), there is a 2-form RR flux:
\[ F_4 = \frac{3 R^3}{8 l_p^3} \hat{\epsilon}_4, \]
\[ F_2 = dA = k d\omega \] (3.11)
where \( \hat{\epsilon}_4 \) is the volume form in a unit radius \( AdS_4 \) space. Hence the geometry is described by the \( AdS_4 \times \mathbb{CP}^3 \) compactification with \( N \) units of the four form flux on \( AdS_4 \) and \( k \) units of the two-form flux on the \( \mathbb{CP}^1 \) in \( \mathbb{CP}^3 \) space.

In the \( k \to \infty \) limit with \( N/k \) fixed, the compactification radius along the \( \varphi \)-direction \( R_{11} \) becomes very small compared to the \( d = 11 \) Planck length:
\[ \frac{R_{11}}{l_p} = \frac{R}{k l_p} \sim \frac{(Nk)^{1/6}}{k} \to 0. \] (3.12)

Thus the theory is reduced to a ten-dimensional type IIA superstring on \( AdS_4 \times \mathbb{CP}^3 \). However the scaling limit from ABJM to L-BLG is taking large \( k \) limit before taking the large \( N \) and the ’t Hooft coupling \( N/k \) becomes 0 in this limit. Since \( R_{11} = g_s^{2/3} l_p \), the string coupling constant \( g_s = e^{\phi} \) also becomes 0:
\[ g_s = e^{\phi} \sim k^{-\frac{2}{3}} N^{\frac{1}{2}} \to 0. \] (3.13)

Since \( d = 11 \) Planck length \( l_p \) and \( d = 10 \) Planck length \( l_p^{(10)} \) are related to the string length as \( l_p = g_s^{1/3} l_s \) and \( l_p^{(10)} = g_s^{1/4} l_s \), the ratios of the radius of the IIA geometry with \( l_s \) and \( l_p^{(10)} \) are given by
\[ \left( \frac{R}{l_s} \right)^2 \sim \sqrt{\frac{N}{k}} \to 0, \quad \left( \frac{R}{l_p^{(10)}} \right)^2 \sim k^{1/8} N^{3/8} \to \infty. \] (3.14)

Therefore the Type IIA supergravity approximation itself is good but the \( \alpha' \) expansion is not good and the theory cannot be considered as the low energy approximation of type IIA superstring. On the other hand, the radius \( R \) is much larger than the \( d = 11 \) Planck length and it may be more appropriately interpreted as a dimensional reduction of M2 branes in the \( d = 11 \) supergravity.
We summarize the various length scales in the scaling limit of the ABJM theory to the L-BLG theory:

\[ R_{11} \ll l_{p}^{(11)} \ll l_{p}^{(10)} \ll R_{AdS} \ll l_s. \tag{3.15} \]

The compactification radius \( R_{11} \) is much smaller than any other scales and the theory is reduced to \( d = 10 \). But the radius of the \( AdS_4 \times \mathbb{CP}^3 \) is smaller than the string length and larger than the \( d = 10 \) and \( d = 11 \) Planck scales.

In the ordinary case of the duality between type IIB superstrings on \( AdS_5 \times S^5 \) and \( \mathcal{N} = 4 \) SYM in \( d = 4 \), the radius of curvature \( R \) is given by

\[ \left( \frac{R}{l_s} \right)^4 \sim g_s N, \quad \left( \frac{R}{l_{p}^{(10)}} \right)^4 \sim N. \tag{3.16} \]

Thus it is usually assumed that both of \( g_s N \) and \( N \) are large so that the type IIB supergravity approximation and the \( \alpha' \)-expansion are valid. Unless \( g_s N \) is large, \( \alpha' \) corrections cannot be neglected and the supergravity description itself is not valid. In the weak coupling limit, the dual field theory is usually considered to be more appropriate. In our case, we can consider the \( d = 10 \) supergravity as a dimensional reduction of \( d = 11 \) supergravity. However membranes wrapping the \( \varphi \) direction become very light strings in the unit of the radius of curvature \( R \), and this may invalidate the supergravity approximation of the M-theory.

### 3.2 Recovery of \( SO(8) \) in dual geometry of L-BLG

In taking the scaling limit \( k(\gg N) \rightarrow \infty \) of the ABJM theory to the L-BLG theory, the eleven-dimensional geometry is reduced to the ten-dimensional \( AdS_4 \times \mathbb{CP}^3 \):

\[ ds^2 = H^{-\frac{2}{3}} \left( \sum \eta_{\mu\nu} dx^{\mu} dx^{\nu} \right) + H^\frac{1}{3} (dr^2 + r^2 ds_{\mathbb{CP}^3}^2) \]

\[ H(r) = \frac{R^6}{r^6}. \tag{3.17} \]

In this section we discuss how the \( SO(8) \) can be recovered in the scaling limit of the ABJM geometry to the L-BLG geometry. The L-BLG geometry is obtained by taking \( k \rightarrow \infty \) limit of \( AdS_4 \times S^7/\mathbb{Z}_k \) and simultaneously locating the probe M2 brane far from the origin of the orbifold. In the large \( k \) limit, the geometry becomes \( d = 10 \ AdS_4 \times \mathbb{CP}^3 \), and there are only 7 transverse directions to the M2 brane world volume, However the radial distance in (3.17) is given by the distance in \( d = 8 \):

\[ r^2 = \sum_{I=1}^{8} (X^I)^2. \tag{3.18} \]
It is invariant under the original $SO(8)$ rotation and the $Z_k$ quotient leaves $r$ invariant.

Now we consider a probe M2 brane in the above geometry. In the static gauge, the M2 brane world volume is identified with the coordinates $x^\mu$ ($\mu = 0, 1, 2$) and the position of the M2 brane is given by $X^I(x)$ where $I = 1, \cdots, 8$. There are only 7 independent propagating modes among 8, and the direction that is removed is the $\varphi$-direction. Remember that the $\varphi$ is the overall phase of the complex coordinate $z^i$ of the transverse $R^8$. Assuming that the probe M2 brane is located far from the source branes, we can separate the probe M2 brane coordinates into the classical background fields $X^I_0(x)$ and the quantum fluctuations $\hat{X}^I(x)$. Since the M2 brane is on $\mathbb{C}^4/U(1)$, all the points on the gauge orbit generated by the $\varphi$-rotation are identified. Here the position of the M2 brane is represented by the coordinates of $\mathbb{R}^8$; a point on the gauge orbit is singled out by fixing the gauge (see Appendix [B]).

If the probe M2 brane is located at

$$X^I_0 = v \delta^{I, 8}$$  \hspace{1cm} (3.19)

where $v$ is much larger than the scale of the fluctuations, the rotation along the $\varphi$-direction is approximated by

$$\delta X^7 = -\delta \varphi \ v, \hspace{1cm} \delta X^I = 0 \ , \ I \neq 7.$$  \hspace{1cm} (3.20)

This shows that in the large $v$ limit the $\varphi$ direction can be identified with the 7th direction $X^7$. Since the $Z_k$ orbifolding with large $k$ corresponds to gauging away the $\varphi$-direction, the fluctuation along the 7th direction is killed and the field $\hat{X}^I(x)$ can fluctuate only in the other 7 directions. This means that the $SO(7)$ rotation acts among the other 7 directions around the classical background of (3.19). If the classical background $X^I_0(x)$ takes different directions at different world volume points, the killed direction also changes locally on the world volume.

In order to get a manifest $SO(8)$ covariant formulation of this mechanism, it is convenient to separate the classical background field of the M2 brane and the fluctuations in the complex coordinates as

$$Z^A(x) = Z^A_0(x) + \hat{Z}^A(x).$$  \hspace{1cm} (3.21)

If the fluctuations are much smaller than the classical background field, the $\varphi$ rotation can be approximated as

$$\delta Z^A = i \delta \varphi Z^A_0.$$  \hspace{1cm} (3.22)

---

\footnote{\ref{3.19} has fixed a gauge of the $\varphi$ rotation and \ref{3.20} is nothing but the direction parallel to the gauge orbit. If we change a gauge, e.g. to $X^I_0 = v \delta^{I, 7}$, \ref{3.20} is also changed accordingly.}
If we write

\[ Z_0^A = X_0^{2A-1} + iX_0^A \]
\[ \hat{Z}^A = i\hat{X}^{2A-1} - \hat{X}^A, \]  

(3.23)

where \( A = 1 \cdots 4 \), the propagating degrees of freedom along the direction \( (3.22) \) are killed and the fluctuations are restricted to obey

\[ X_0^I \hat{X}^I = 0. \]  

(3.24)

Note that the decomposition of the complex fields into the real and the imaginary parts are different between the classical background \( Z_0^A \) and the fluctuations \( \hat{Z}^A \) in \( (3.23) \). With this definition, if \( X_0^I = v\delta^{I,8} \), the killed direction becomes the 8th direction of \( \hat{X}^I \). We can write the fluctuations perpendicular to \( \hat{X}^I \) in \( (3.24) \) as

\[ P^I = \left( \delta^{IJ} - \frac{X_0^I X_0^J}{(X_0^0)^2} \right) \hat{X}_J. \]  

(3.25)

This \( P^I \) automatically satisfies the condition \( (3.24) \) and 7 degrees of freedom are projected among the 8 degrees of freedom. Now everything is written in a manifestly \( SO(8) \) covariant way. The \( SO(8) \) covariance is recovered because we have assumed that the fluctuation is much smaller than the classical background fields of the probe M2 brane. This assumption is consistent with the scaling limit of the ABJM theory to the L-BLG theory.

Note here that the \( SO(8) \) rotation changes the gauge choice of the \( \varphi \) rotation and \( SO(8) \) is mixed with the \( U(1) \) gauge transformation. Also note that because of the different assignments of \( X^I \) to \( Z^A \) for \( Z_0 \) and \( \hat{Z} \), the \( SO(8) \) is different from the original \( SO(8) \) before taking the orbifolding.

The analysis here and in the previous section shows why the L-BLG theory has both of the conformal symmetry and the invariance under \( SO(8) \). The compactification direction along the \( \varphi \) direction is different from the ordinary reduction to \( d = 10 \) by compactifying the 11th transverse direction. The dilaton becomes constant and the \( AdS_4 \) geometry is preserved. This is the reason why there is a conformal symmetry in the effective field theory of L-BLG.

The \( SO(8) \) invariance is more subtle. In the scaling limit of ABJM to L-BLG, we take \( k \to \infty \) limit and simultaneously locate the probe M2 brane far from the origin of the orbifold. Then the killed direction of the fluctuations by \( Z_k \) \( (k \to \infty) \) orbifolding is given by the \( SO(8) \) vector of the classical background fields \( X_0^I \) after specifying the gauge choice, and defining the projection operator by using \( X_0^I \) the manifest \( SO(8) \) covariance is obtained.
3.3 Actions of probe branes in $AdS_4 \times \mathbb{CP}^3$

In this section we study possible forms of the effective field theory of probe M2 branes in the background geometry (3.17). The analysis in the section follows the prescription of [33] and [34] that a classical scalar field in the radial direction is interpreted as the Yang-Mills coupling. We will study probe M2 branes in a curved background while flat 11-dimensional background is used there.

By using the metric of (3.17), the generally covariant kinetic term can be written as

$$S_0 = -\frac{1}{2} \int d^3 x \sqrt{-\text{det} g} g^{\mu\nu} g_{IJ} \text{tr}[D_\mu X^I D_\nu X^J],$$

(3.26)

where $\mu, \nu = 0, 1, 2$ are the world volume indices and $I, J = 1, \cdots, 8$ are the target space indices, and $D_\mu = \partial_\mu - iA_\mu$ is the covariant derivative to assure that $X^I$ lies on $\mathbb{C}^4/U(1)$ (see Appendix B).

Both of the world volume metric $g^{\mu\nu}$ and the target space metric $g_{IJ}$ are functions of the position of the M2 branes $X^I(x)$. A static gauge is taken and the world volume metric $g_{\mu\nu}$ is given by the induced metric in the curved space-time (3.17).

This kinetic term can be simplified as follows. The metric $g^{\mu\nu}$ and $g_{IJ}$ are functions of the M2 brane position through $r$. As we did in the previous section, we separate the 8 scalar fields $X^I(x)$ of the probe M2 branes into a classical background and quantum fluctuations. If the probe M2 branes are located far from the origin of the orbifold singularity, the position of the M2 branes is approximated by the value of the classical background fields $X^I_0(x)$ and $r \sim \sqrt{(X^I_0(x))^2}$. Inserting the explicit form of the metric, the kinetic term can be simplified (see Appendix [B]) as

$$S_0 = -\frac{1}{2} \int d^3 x \eta^{\mu\nu} \eta_{IJ} \text{tr}[\partial_\mu P^I \partial_\nu P^J],$$

(3.27)

where $P^I(x)$ is the projected fluctuating fields (3.23). In deriving this action, we have used that the classical background fields $X^I_0$ are slowly varying. Note that all the dependence of $H(r)$ vanishes and the kinetic term of the fluctuating fields does not have the explicit dependence on the position of M2 branes.

The position of the M2 branes $X^I_0$ must satisfy the classical equation of motion on the geometry (3.17). Because of the cancellation of $H(r)$, it looks like a free field equation of motion. But the fields $X^I_0$ are restricted to be on the geometry where the $\varphi$-direction is killed, and they are slightly different from the constraint equation (2.11) in the L-BLG theory, or that in the scaling limit of the $SU(N) \times SU(N)$ ABJM theory. This is related to the effect of the $U(1)$ gauge field of the ABJM theory. We discuss it in Appendix [A].

In the rest of this section, we dare to generalize the discussion of the kinetic term of the scalar field to the other possible terms in the effective action of the probe M2 branes in the geometry (3.17). First assume that a gauge field is induced on the effective
action of the probe M2 branes and its action is given by the ordinary Yang-Mills type. Then the general coordinate invariant YM action in the curved metric \((3.17)\) is given by

\[
-\frac{1}{4} \int d^3 x \sqrt{\det g} g^{\mu \rho} g^{\nu \sigma} \text{tr} [F_{\mu \nu} F_{\rho \sigma}] = -\frac{1}{4} \int d^3 x \left( \frac{R}{r} \right)^2 \eta^{\mu \rho} \eta^{\nu \sigma} \text{tr} [F_{\mu \nu} F_{\rho \sigma}].
\]

(Since we are considering the \(d = 11\) theory, there is no freedom to multiply a dilaton dependence in the action.) In this case, \(H(r)\) dependence remains and the effective Yang-Mills coupling is given by the following field dependent value:

\[
g_{YM}^2 (x) = \frac{r^2}{R^2} = \frac{(X^I_0(x))^2}{R^2}.
\]

Similarly if we assume that the scalar field acquires a quartic potential, the general coordinate and \(SO(8)\) invariance require its form to be

\[
\frac{1}{4} \int d^3 x \sqrt{\det g} g^{IK} g_{JL} \text{tr} [P^I, P^J] [P^K, P^L] = \int d^3 x \frac{1}{4} \frac{(X^I_0)^2}{R^2} \eta^{IK} \eta_{JL} \text{tr} [P^I, P^J] [P^K, P^L].
\]

Here \(P^I\) are projected scalar fields \((3.25)\).

Summing up these three terms, we have the following forms of the effective action:

\[
S = -\frac{1}{2} \int d^3 x \left( \text{tr} [\partial_\mu P^I \partial^\mu P^I] - \frac{1}{4} \frac{R^2}{(X^I_0)^2} \text{tr} [F_{\mu \nu} F^{\mu \nu}] + \frac{1}{4} \frac{1}{R^2} \text{tr} [P^I, P^J]^2 \right).
\]

Of course there is little justification of the above analysis but it is amusing to see that this is nothing but the bosonic part of \((2.20)\). The analysis might support an interpretation that the action of L-BLG is the effective action of the probe M2 branes in the geometry of \((3.17)\). The \(X^I_0\) dependence of the coefficients will be related to the conformal invariance of the M2 branes. It will be interesting to constrain possible forms of the effective action including fermions, higher derivative terms, or generic potential terms by the generalized conformal invariance.

## 4 Conclusions

In this paper, we investigated the conformal symmetries and the recovery of \(SO(8)\) invariance of the Aharony-Bergman-Jafferis-Maldacena (ABJM) theories and Lorentzian Bagger-Lambert-Gustavsson (L-BLG) theories. The conformal invariance, in particular, the invariance under the special conformal transformations does hold in the L-BLG theory only when we consider a set of spacetime dependent solutions to the constraint equations \(\partial^2 X^I_0 = 0\). The conformal symmetries in the field theories are consistent with the gravity
duals; $AdS_4 \times S^7/\mathbb{Z}_k$ geometry for the ABJM theory and $AdS_4 \times \mathbb{C}P^3$ geometry for the L-BLG.

It may sound strange that the L-BLG has a $SO(8)$ global symmetry while the dual geometry $AdS_4 \times \mathbb{C}P^3$ does not have it manifestly. In order to resolve the problem, we investigated the recovery of $SO(8)$ by considering a dual geometry around a classical background. We have shown how the $SO(8)$ covariance is recovered in the geometry probed by a slowly varying M2 brane located far from the orbifold singularity.

Although the radius of $AdS_4$ is larger than the $d = 10$ Planck length and the type IIA supergravity approximation is good, it is much smaller than the IIA string scale and the dual geometry of the scaled theory of L-BLG cannot be interpreted as the low energy effective theory of type IIA superstring. But the radius is larger than the $d = 11$ Planck length and it can be considered as a dimensional reduction of the $d = 11$ supergravity solution.

We have also studied the effective action of probe M2 branes in a curved geometry that is obtained by taking the scaling limit of $AdS_4 \times S^7/\mathbb{Z}_k$. It is amusing and also somewhat surprising that the position dependent coefficients of the coupling constant can be correctly reproduced; $g_{YM}^2$ is proportional to a square of the position of the M2 branes. In particular, if we assume that the scalar potential is quartic, the potential is shown to be multiplied by a square of the center of mass coordinates of the M2 branes. This is consistent with the sextic potential which is expected for the effective theory of M2 branes.

Finally we would like to comment on a subtlety related to the $U(1)$ factor when interpreting the L-BLG theory as a $k \to \infty$ scaling limit of ABJM theory. The L-BLG can be obtained by taking the scaling limit of the $SU(N) \times SU(N)$ ABJM theory. If the gauge group is $U(N) \times U(N)$, the classical background $X_0^I$ must obey a classical equation of motion restricted on $\mathbb{C}^4/U(1)$, not on the full $\mathbb{C}^4$. This constraint is consistent with the dual geometric picture of the $U(N) \times U(N)$ ABJM theory. Thus the original L-BLG theory will be necessary to be supplemented by an additional constraint in order to interpret it as the M2 branes probing $\mathbb{C}^4/U(1)$.

**Acknowledgements**

We thank Y. Hikida, A. Ishibashi and S. Moriyama for useful discussions. The work of S. I. was partly supported by Grant-in-Aid for Scientific Research.
A  

**U(1) part in ABJM theory**

In scaling the ABJM theory to the L-BLG theory, we have mainly concerned with the $SU(N) \times SU(N)$ gauge theory. In this appendix we consider the scaling limit of the $U(N) \times U(N)$ ABJM theory, especially the effect of the $U(1)$ part. For simplicity we consider the bosonic terms only. In the presence of the $U(1)$ gauge field, the covariant derivative is modified to

$$D_\mu Y = \hat{D}_\mu \hat{Y} + 2i B_{0\mu} \hat{Y} + i \{ \hat{B}_\mu, \hat{Y} \} + \partial_\mu Y_0 + 2i \hat{B}_\mu Y_0 + 2i B_{0\mu} Y_0,$$

where $B_{0\mu}$ is the axial combination of the $U(1) \times U(1)$ gauge field

$$B_{0\mu} = \frac{1}{2}(A^{(L)}_\mu - A^{(R)}_\mu).$$

The gauge field $B_{0\mu}$ is associated with the gauge transformation of the complex field $Y^A \to e^{i\phi Y^A}$. Hence if the dual geometry is described by $C^4/U(1)$, we need the gauge symmetry even after the scaling to L-BLG. Therefore, we do not scale the $B_{0\mu}$ field unlike $B_\mu$. The scaling is given by

$$\hat{B}_\mu \to \lambda \hat{B}_\mu, \quad Y_0 \to \lambda^{-1} Y_0, \quad B_{0\mu} \to B_{0\mu}$$

and take the limit $\lambda \to 0$. The kinetic term of the scalar fields becomes

$$-\frac{1}{2} \text{tr} |D_\mu Y_A|^2 = \text{tr} \left[ -\frac{1}{2} (\hat{D}_\mu \hat{Y}_A + 2i \hat{B}_\mu Y_{0A} + 2i B_{0\mu} \hat{Y}_A) \hat{D}^\mu \hat{Y}^A + 2i \hat{B}^\mu Y_0^A + 2i B_{0\mu} \hat{Y}^A \right] - \frac{1}{2} \lambda^2$$

$$-i (\partial_\mu Y_{0A} + 2i B_{0\mu} Y_{0A}) \hat{B}^\mu \hat{Y}^A + i (\partial_\mu Y_0^A + 2i B_{0\mu} Y_0^A) \hat{B}^\mu \hat{Y}^A \right].$$

The difference from the $SU(N) \times SU(N)$ case is that all the derivative is replaced by the covariant derivative with respect to $B_{0\mu}$. Requiring finiteness of the action, one can obtain the modified constraint

$$D^2_{U(1)} Y_0^A \equiv (\partial_\mu + 2i B_{0\mu})(\partial^\mu + 2i B_{0\mu}) Y_0^A = 0.$$  

The gauge field $B_{0\mu}$ does not have a kinetic term and it is nothing but the auxiliary gauge field $A_\mu$ introduced in the $C^4/U(1)$ gauged model discussed in Appendix B.

In the presence of the vector-like $U(1)$ gauge field

$$A_{0\mu} = \frac{1}{2}(A^{(L)}_\mu + A^{(R)}_\mu),$$

there is a coupling of $B_{0\mu}$ to $A_{0\mu}$ through the Chern-Simons term. If we do not scale the $A_{0\mu}$ either, it is given by

$$4 \lambda^{-1} K \epsilon^{\mu\nu\rho} \text{tr} B_{0\mu} F_{0\nu\rho},$$
where $F_{0\mu\nu} = \partial_\mu A_{0\nu} - \partial_\nu A_{0\mu}$. Then because of the $\lambda^{-1}$ coefficient this must vanish too.

If we instead scale the $A_{0\mu}$ gauge field with $\lambda$, the coefficient becomes of the order $\lambda^0$, and an integration over $B_{0\mu}$ solves it as

$$2B_{0\mu}^{(0)} = -\frac{i}{2|Y_0|^2}(Y_0^A \partial_\mu \bar{Y}^A - \bar{Y}_0^A \partial_\mu \hat{Y}^A) - 2K\epsilon_{\mu\nu\rho}F_0^{\nu\rho}. \quad (A.8)$$

**B  SO(8) recovery in C⁴/U(1) model**

In Section 3.2 we showed the recovery of $SO(8)$ invariance in the scaling limit of $AdS_4 \times \mathbb{CP}^3$. In this appendix, we study a $C^4/U(1)$ sigma model and see the recovery of $SO(8)$. This is a generalization of the equivalence of a gauged model on $\mathbb{CP}^1$ and an $O(3)$ nonlinear $\sigma$ model to a higher dimensional target space.

$C^4$ is parameterized by the following angular variables:

$$
\begin{align*}
    z^1 &= \rho e^{i(\phi_1 + \phi')} \cos \theta, \\
    z^2 &= \rho e^{i(\phi_2 + \phi')} \sin \theta \cos \psi, \\
    z^3 &= \rho e^{i(\phi_3 + \phi')} \sin \theta \sin \psi \cos \chi, \\
    z^4 &= \rho e^{i\phi'} \sin \theta \sin \psi \sin \chi, \\
    0 &\leq \phi' \leq 2\pi, \quad 0 \leq \theta, \psi, \chi, \phi_1, \phi_2, \phi_3 \leq \pi.
\end{align*}
$$

(B.1)

We then consider a scalar field on $C^4/U(1)$ by identifying

$$z_i \sim e^{i\phi'} z_i. \quad (B.2)$$

The Lagrangian of the scalar field $Z_i(x)$ on $C^4/U(1)$ must be invariant under the local gauge transformation

$$Z_i(x) \rightarrow e^{i\phi'} Z_i(x) \quad (B.3)$$

and the action can be written by introducing an auxiliary gauge field $A_\mu$ as

$$S = \int d^3x |(\partial_\mu - iA_\mu)Z_A|^2. \quad (B.4)$$

In the ABJM theory, the gauge field comes from the $U(1)$ part of the axial combination of the two $U(N)$ gauge fields $B_{0\mu}$ (see Appendix A). The gauge field does not have a kinetic term and and it can be eliminated by solving the equation of motion as

$$A_\mu = \frac{i}{2|Z^A|^2}(Z^A \partial_\mu \bar{Z}^A - \bar{Z}^A \partial_\mu Z^A). \quad (B.5)$$
By substituting the solution to the action, we obtain a nonlinear action which depends on the $Z^A$ fields only. The action (B.4) becomes

$$S = \int d^3x (|\partial Z^A|^2 - A_\mu^2 |Z^A|^2).$$  \hspace{1cm} (B.6)

In the case of CP$^1$ model, it is well known that the model is nothing but the nonlinear $\sigma$-model on $S^2$. In our case, it is a nonlinear model on $\mathbb{C}^4/U(1)$.

Now we expand the field around a classical background and expand the field as

$$Z^A(x) = Z_0^A + \hat{Z}^A.$$  \hspace{1cm} (B.7)

The classical background satisfies the equation of motion. Assume that the classical background is very slowly varying and much larger than the fluctuation $\hat{Z}^A$:

$$|Z_0^A| \gg |\hat{Z}^A|, \ |dZ_0^A|.$$  \hspace{1cm} (B.8)

Under the assumption (B.8), the quadratic terms of the fluctuations in the action (B.6) become

$$S \sim \int d^3x (|\partial \hat{Z}^A|^2 - A_\mu^{(0)2} |Z_0^A|^2)$$  \hspace{1cm} (B.9)

where

$$A_\mu^{(0)} = \frac{i}{2|Z_0^A|^2} (Z_0^A \partial_\mu \hat{Z}^A - \hat{Z}_0^A \partial_\mu Z^A).$$  \hspace{1cm} (B.10)

If we decompose the complex fields into real components as

$$Z_0^A = X_0^{2A-1} + i X_0^{2A}$$
$$\hat{Z}^A = i \hat{X}^{2A-1} - \hat{X}^{2A},$$  \hspace{1cm} (B.11)

the gauge field can be written as

$$A_\mu^{(0)} = \frac{1}{(X_0^I)^2} X_0^I \partial_\mu \hat{X}^I.$$  \hspace{1cm} (B.12)

Thus the action can be written as a manifestly $SO(8)$ covariant expression:

$$S = \int d^3x \{ (\partial \hat{X}^I)^2 - \frac{4}{X_0^2} (X_0^I \partial \hat{X}^I)^2 \}.$$  \hspace{1cm} (B.13)

In terms of the projected scalar field

$$P^I = \hat{X}^I - \frac{X_0^I X_0^J \hat{X}^J}{(X_0^I)^2},$$  \hspace{1cm} (B.14)

the action is written (under the assumption (B.8))

$$S = \int d^3x (\partial_\mu P^I)^2.$$  \hspace{1cm} (B.15)

It is manifestly invariant under the $SO(8)$ transformations. But note that the $SO(8)$ transformation is different from the $SO(8)$ acting on the original $\mathbb{R}^8$ because of the different decompositions of the complex fields into the real components in (B.11).
In this appendix, we remind the reader of the ordinary reduction of M2 branes in $d = 11$ supergravity to D2 branes in $d = 10$ type IIA supergravity to clarify the difference from the reduction adopted in the ABJM theory. By compactifying $x_{11}$ direction and identifying $x_{11} \sim x_{11} + 2\pi R_{11}$ the M2 brane solution is given by replacing the metric (3.1) with a smeared harmonic function \[ H(r) = \sum_{n=-\infty}^{\infty} \frac{R^6}{(r^2 + (x_{11} + 2\pi n R_{11})^2)^3}. \] (C.16)

where $r$ is the radial distance in the 7 non-compact transverse directions. The string coupling constant is given by $R_{11} = g_s l_s$. Then we can get the solution of the multiple D2-branes in the string frame by using the reduction rule and the Poisson resummation at distance much larger than $R_{11}$:

\[
\begin{align*}
    ds_{D2} &= H^{-\frac{1}{2}} \left( \sum_{\mu, \nu = 0}^{2} \eta_{\mu\nu} dx^\mu dx^\nu \right) + H^{\frac{1}{2}} \left( dr^2 + d\Omega_6^2 \right), \\
    e^\phi &= H^{\frac{1}{4}}, \\
    H(r) &= \frac{6\pi^2 g_s N l_5^5}{r^5}.
\end{align*}
\] (C.17)

It is quite different from (3.9). Especially the dilaton is not a constant and the conformal symmetry of the M2 brane geometry is broken; it is no longer $AdS_4$. The transverse direction is given by the radial direction and $S^6$, and therefore it has the $SO(7)$ invariance.

References

[1] J. Bagger and N. Lambert, “Gauge Symmetry and Supersymmetry of Multiple M2-Branes,” Phys. Rev. D \textbf{77}, 065008 (2008) arXiv:0711.0955 [hep-th]. “Modeling multiple M2’s,” Phys. Rev. D \textbf{75}, 045020 (2007) arXiv:hep-th/0611108. “Comments On Multiple M2-branes,” JHEP \textbf{0802}, 105 (2008) arXiv:0712.3738 [hep-th].

[2] A. Gustavsson, “Algebraic structures on parallel M2-branes,” arXiv:0709.1260 [hep-th]. A. Gustavsson, “Selfdual strings and loop space Nahm equations,” JHEP \textbf{0804}, 083 (2008) arXiv:0802.3456 [hep-th].

[3] J. Gomis, G. Milanesi and J. G. Russo, “Bagger-Lambert Theory for General Lie Algebras,” arXiv:0805.1012 [hep-th].

[4] S. Benvenuti, D. Rodriguez-Gomez, E. Tonni and H. Verlinde, “N=8 superconformal gauge theories and M2 branes,” arXiv:0805.1087 [hep-th].
[5] P. M. Ho, Y. Imamura and Y. Matsuo, “M2 to D2 revisited,” arXiv:0805.1202 [hep-th].

[6] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, “N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals,” arXiv:0806.1218 [hep-th].

[7] M. A. Bandres, A. E. Lipstein and J. H. Schwarz, “Ghost-Free Superconformal Action for Multiple M2-Branes,” arXiv:0806.0054 [hep-th].

[8] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk and H. Verlinde, “The Superconformal Gauge Theory on M2-Branes,” arXiv:0806.0738 [hep-th].

[9] S. Mukhi and C. Papageorgakis, “M2 to D2,” JHEP 0805, 085 (2008) arXiv:0803.3218 [hep-th].

[10] Y. Honma, S. Iso, Y. Sumitomo and S. Zhang, “Janus field theories from multiple M2 branes,” arXiv:0805.1895 [hep-th].

[11] B. Ezhuthachan, S. Mukhi and C. Papageorgakis, “D2 to D2,” arXiv:0806.1639 [hep-th].

[12] Y. Honma, S. Iso, Y. Sumitomo and S. Zhang, “Scaling limit of N=6 superconformal Chern-Simons theories and Lorentzian Bagger-Lambert theories,” arXiv:0806.3498 [hep-th].

[13] H. Verlinde, “D2 or M2? A Note on Membrane Scattering,” arXiv:0807.2121 [hep-th].

[14] J. Gomis, A. J. Salim and F. Passerini, “Matrix Theory of Type IIB Plane Wave from Membranes,” arXiv:0804.2186 [hep-th].

[15] K. Hosomichi, K. M. Lee and S. Lee, “Mass-Deformed Bagger-Lambert Theory and its BPS Objects,” arXiv:0804.2519 [hep-th].

[16] M. Blau and M. O’Loughlin, “Multiple M2-Branes and Plane Waves,” arXiv:0806.3253 [hep-th].

[17] J. H. Schwarz, “Superconformal Chern-Simons theories,” JHEP 0411, 078 (2004) arXiv:hep-th/0411077.

[18] D. Gaiotto and X. Yin, “Notes on superconformal Chern-Simons-matter theories,” JHEP 0708, 056 (2007) arXiv:0704.3740 [hep-th].

[19] J. Bagger and N. Lambert, “Three-Algebras and N=6 Chern-Simons Gauge Theories,” arXiv:0807.0163 [hep-th].
[20] S. Terashima, “On M5-branes in N=6 Membrane Action,” arXiv:0807.0197 [hep-th].

[21] J. Gomis, D. Rodriguez-Gomez, M. Van Raamsdonk and H. Verlinde, “A Massive Study of M2-brane Proposals,” arXiv:0807.1074 [hep-th].

[22] K. Hanaki and H. Lin, “M2-M5 Systems in N=6 Chern-Simons Theory,” arXiv:0807.2074 [hep-th].

[23] K. Hosomichi, K. M. Lee, S. Lee, S. Lee and J. Park, “N=5,6 Superconformal Chern-Simons Theories and M2-branes on Orbifolds,” arXiv:0806.4977 [hep-th].

[24] M. Schnabl and Y. Tachikawa, “Classification of N=6 superconformal theories of ABJM type,” arXiv:0807.1102 [hep-th].

[25] E. A. Bergshoeff, M. de Roo, O. Hohm and D. Roest, “Multiple Membranes from Gauged Supergravity,” arXiv:0806.2584 [hep-th]. E. A. Bergshoeff, O. Hohm, D. Roest, H. Samtleben and E. Sezgin, “The Superconformal Gaugings in Three Dimensions,” arXiv:0807.2841 [hep-th].

[26] A. Mauri and A. C. Petkou, arXiv:0806.2270 [hep-th]. J. Bhattacharya and S. Minwalla, arXiv:0806.3251 [hep-th]. T. Nishioka and T. Takayanagi, arXiv:0806.3391 [hep-th]. Y. Imamura and K. Kimura, arXiv:0806.3727 [hep-th]. J. A. Minahan and K. Zarembo, arXiv:0806.3951 [hep-th]. A. Armoni and A. Naqvi, arXiv:0806.4068 [hep-th]. A. Hanany, N. Mekareeya and A. Zaffaroni, arXiv:0806.4212 [hep-th]. D. Gaiotto, S. Giombi and X. Yin, arXiv:0806.4589 [hep-th]. J. Ahn, arXiv:0806.4807 [hep-th]. J. Bedford and D. Berman, arXiv:0806.4900 [hep-th]. G. Arutyunov and S. Frolov, arXiv:0806.4940 [hep-th]. B. . j. Stefanski, arXiv:0806.4948 [hep-th]. G. Grignani, T. Harmark and M. Orselli, arXiv:0806.4959 [hep-th]. K. Hosomichi, K. M. Lee, S. Lee, S. Lee and J. Park, arXiv:0806.4977 [hep-th]. G. Grignani, T. Harmark, M. Orselli and G. W. Semenoff, arXiv:0807.0205 [hep-th]. S. Terashima and F. Yagi, arXiv:0807.0368 [hep-th]. N. Gromov and P. Vieira, arXiv:0807.0437 [hep-th]. C. Ahn and P. Bozhilov, arXiv:0807.0566 [hep-th]. N. Gromov and P. Vieira, arXiv:0807.0777 [hep-th]. C. S. Chu, P. M. Ho, Y. Matsuo and S. Shiba, arXiv:0807.0812 [hep-th]. S. Cherkis and C. Samann, arXiv:0807.0808 [hep-th]. B. Chen and J. B. Wu, arXiv:0807.0802 [hep-th]. Y. Zhou, arXiv:0807.0890 [hep-th]. T. Li, Y. Liu and D. Xie, arXiv:0807.1183 [hep-th]. N. Kim, arXiv:0807.1349 [hep-th]. Y. Pang and T. Wang, arXiv:0807.1444 [hep-th]. M. R. Garousi, A. Ghodsi and M. Khosravi, arXiv:0807.1478 [hep-th]. A. Hashimoto and P. Ouyang, arXiv:0807.1500 [hep-th]. D. Astolfi, V. G. M. Puletti, G. Grignani, T. Harmark and M. Orselli, arXiv:0807.1527 [hep-th]. C. Ahn and R. I. Nepomechie, arXiv:0807.1924 [hep-th]. D. Bak and S. J. Rey, arXiv:0807.2063 [hep-th]. Y. Imamura and K. Kimura,
[27] E. Inönü and E. P. Wigner, “On the Contraction of groups and their representations,” Proc. Nat. Acad. Sci. 39 (1953) 510.

[28] A. Jevicki, Y. Kazama and T. Yoneya, “Generalized conformal symmetry in D-brane matrix models,” Phys. Rev. D 59, 066001 (1999) [arXiv:hep-th/9810146].

[29] M. A. Bandres, A. E. Lipstein and J. H. Schwarz, “Studies of the ABJM Theory in a Formulation with Manifest SU(4) R-Symmetry,” arXiv:0807.0880 [hep-th].

[30] M. Benna, I. Klebanov, T. Klose and M. Smedback, “Superconformal Chern-Simons Theories and AdS$_4$/CFT$_3$ Correspondence,” arXiv:0806.1519 [hep-th].

[31] M. A. Bandres, A. E. Lipstein and J. H. Schwarz, “N = 8 Superconformal Chern–Simons Theories,” JHEP 0805, 025 (2008) [arXiv:0803.3242 [hep-th]].

[32] E. Witten, “String theory dynamics in various dimensions,” Nucl. Phys. B 443, 85 (1995) [arXiv:hep-th/9503124].

[33] S. Banerjee and A. Sen, “Interpreting the M2-brane Action,” arXiv:0805.3930 [hep-th].

[34] S. Cecotti and A. Sen, “Coulomb Branch of the Lorentzian Three Algebra Theory,” arXiv:0806.1990 [hep-th].

[35] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, “Large N field theories, string theory and gravity,” Phys. Rept. 323, 183 (2000) [arXiv:hep-th/9905111].