Certified Adversarial Defenses Meet Out-of-Distribution Corruptions: Benchmarking Robustness and Simple Baselines

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Abstract

Certified robustness guarantee gauges a model’s robustness to test-time attacks and can assess the model’s readiness for deployment in the real world. In this work, we critically examine how the adversarial robustness guarantees from randomized smoothing-based certification methods change when state-of-the-art certifiably robust models encounter out-of-distribution (OOD) data. Our analysis demonstrates a previously unknown vulnerability of these models to low-frequency OOD data such as weather-related corruptions, rendering these models unfit for deployment in the wild. To alleviate this issue, we propose a novel data augmentation scheme, FourierMix, that produces augmentations to improve the spectral coverage of the training data. Furthermore, we propose a new regularizer that encourages consistent predictions on noise perturbations of the augmented data to improve the quality of the smoothed models. We find that FourierMix augmentations help eliminate the spectral bias of certifiably robust models enabling them to achieve significantly better robustness guarantees on a range of OOD benchmarks. Our evaluation also uncovers the inability of current OOD benchmarks at highlighting the spectral biases of the models. To this end, we propose a comprehensive benchmarking suite that contains corruptions from different regions in the spectral domain. Evaluation of models trained with popular augmentation methods on the proposed suite highlights their spectral biases and establishes the superiority of FourierMix trained models at achieving better-certified robustness guarantees under OOD shifts over the entire frequency spectrum.1

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1Pre-print under review. Codes and benchmark datasets will be open-sourced upon paper acceptance.

1. Introduction

Developing machine learning (ML) systems that are robust to adversarial variations in the test data is critical for applied domains that require ML safety [20], such as autonomous driving and cyber-security. Unfortunately, a large body of work in this direction has fallen into the cycle where new empirical defense techniques are proposed, followed by new adaptive attacks breaking these defenses [2, 50]. Therefore, significant efforts have been dedicated to developing methods that are certifiably robust [16, 42, 52] which provide provable robustness guarantees. Most promising among these certified defenses are randomized smoothing (RS) based certified defenses [8, 31, 32] which are scalable to deep neural networks (DNNs) and high-dimensional datasets. Specifically, the RS-based certification procedure relies on a smoothed version of the original classifier, which outputs the class most likely returned by the original classifier under random noise perturbations of the input. Prediction from the RS procedure at the test time is accompanied by a radius in which the predictions of the smoothed classifier are guaranteed to remain constant, thereby making them resilient to adversarial attacks within the neighborhood. Training methods such as [8, 46, 58] have been proposed to maximize the average certified radius (ACR), and models trained using these procedures achieve state-of-the-art (SOTA) adversarial robustness guarantees, all while assuming that the test data is identically distributed to the training data. In this work, we take a critical look at the current status of certifiably robust ML and consider whether these certifiably robust models are ready for deployment in the real world.

Our work takes the first steps towards answering this question by evaluating RS-based provably robust ML models on out-of-distribution (OOD) shifts, as mismatches between the training and deployment distributions are ubiqui-
tious in the real world. Our analysis shows that OOD data pose a serious threat to certifiably robust models. We therefore highlight a previously unrecognized threat to certifiably robust models and thereby show that these models are not yet ready for deployment in the real world. Specifically, we found state-of-the-art certifiably robust models to be surprisingly brittle to low-frequency perturbations, such as weather-related corruptions (e.g., fog and frost). Vulnerability to such corruptions could lead to a detrimental performance of ML models on safety-critical applications. For example, 35%–75% performance drop is observed on low-frequency corruptions rendering RS-based robustness guarantees useless (Figure 1).

Motivated from our analysis, which shows RS-based smoothed classifiers to be brittle to low-frequency corruptions, we propose a novel data augmentation method that uses spectrally diverse yet semantically consistent augmentations of the training data. Specifically, our proposed data augmentation method FourierMix generates augmented data samples by using Fourier-based transformations on the input data to increase the spectral coverage of the training data. FourierMix randomly perturbs the amplitude and phase of the images in the training data and then combines them with the affine transformation of the data, producing spectrally diverse augmentations. To encourage the model to produce consistent predictions on different augmentations of the data, we propose a hierarchical consistency regularizer (HCR). The use of HCR as the regularizer leads to semantic consistency of representations across random noise perturbations as well as FourierMix generated augmentations of the same input image. The proposed scheme consistently achieves significantly better certified robustness guarantees as compared to existing state-of-the-art data augmentation schemes extended to build a smoothed classifier, across a range of OOD benchmarks. We further analyze these smooth models using Fourier sensitivity analysis in the spectral domain. In comparison to other methods, models trained on FourierMix augmentations coupled with hierarchical consistency regularization are significantly more resilient to perturbations across the entire frequency spectrum.

Our empirical evaluation of certifiably robust models on various OOD benchmark datasets uncovers another peculiar phenomenon—popular benchmark datasets may be biased towards certain frequency regions. Due to the complexity of real-world data, it is extremely challenging and tedious to uncover the spectral biases of the models and to identify their failure modes. Because of this, improvements in the performance of the models on these benchmark datasets may not generalize to other OOD scenarios. Thus, we should be cautious and avoid over-reliance on a specific leaderboard, especially to judge the robustness of models on OOD data. To enable the designers to understand the spec-

![Certified Robustness Is Fragile under Distribution Shifts](image)

Figure 1. Robustness guarantees of certified models [8] degrade significantly on out-of-distribution data, suggesting that these models are not yet ready for deployment in the real world.

tral biases of their models and obtain a more comprehensive view of the model robustness to OOD data, we propose a complementary new benchmark that includes a collection of OOD test sets, each focusing on specific frequency ranges while collectively covering the entire frequency spectrum. Evaluation of the certified robustness of different models on the proposed dataset shows that the smooth models obtained after training with existing data augmentation schemes are indeed biased towards certain frequencies regions. This justifies the observed performance (and ranking) variations across different benchmarks. On the other hand, models trained with our FourierMix based data augmentations perform significantly better than the competitors across the entire frequency spectrum. This further demonstrates that our data augmentation can produce spectrally diverse data which help alleviate the biases of the models to different frequencies. Our main contributions are as follows:

- We highlight a previously unknown vulnerability of certified adversarial defenses to OOD data and show their failure on mid/low-frequency perturbations of the data.
- We present a novel data augmentation scheme FourierMix that produces spectrally diverse augmentations of the training data combined with a new consistency regularization term. Our evaluation shows that they significantly enhance the robustness of certifiably robust models to OOD data (e.g., 18.3%/26.3% average improvements on CIFAR-10/100-based OOD benchmarks compared to other baselines).
- We propose a novel OOD benchmark suite providing broad spectral coverage of the corruptions that may be encountered at test time. The dataset can help probe the spectral biases of the models, which may not be unveiled by evaluation on existing OOD benchmark datasets.

2. Related Work

Deep neural networks (DNNs) trained using standard gradient descent optimizers [44] have been shown vulnerable to adversarial examples [48]. A number of white-
and black-box attacks have been proposed [5–7, 23, 53] to construct adversarial examples with small \( \ell_p \) distances to the original data that mislead these DNN models. Besides adversarial attacks, recent studies have devoted efforts to characterizing model performance under out-of-distribution (OOD) shifts [3,21], where natural corruptions lead to a significant impact on the accuracy of SOTA ML models. Thus, it has become imperative to study how ML models can be made robust to test data coming from different distributions when the models are deployed in the real world.

**Certified Robustness and Defenses.** The authors in [48] have discovered the adversarial examples in DNN models, after which many defenses have been presented to mitigate such vulnerability [2]. However, many of the proposed countermeasures have been shown to rely on gradient obfuscation, limiting malicious agents from accessing the accurate gradients. Such defenses are vulnerable to adaptive attacks, which give a false sense of security [2] of the models. Certified defenses are thus highly desirable. Along with a prediction on the test point, these defenses output a certified radius \( r \) such that for any \( \| \delta \|_2 < r \), the model continues to have the same prediction. Such techniques include convex polytope [52], recursive propagation [16], and linear relaxation [42, 59]. These methods provide a lower bound on the perturbation required to change the model’s prediction on a target point. However, such methods can merely be applied to shallow models, which limits their practicality. Recently, [8,31,32,39] have proposed randomized smoothing (RS)-based certified defenses that produce better lower bounds and are scalable to large networks. In this paper, we study the OOD robustness of such certified defenses. Unlike a recent work [36], which uses data poisoning attacks to hurt the robustness guarantees of the RS-based models, our work demonstrates the failure of these models on test-time corruptions, which might be encountered by the model deployed in the real world.

**Robustness against Common Corruptions – Benchmarks and Defenses.** Pioneering studies have identified vulnerabilities of deep learning models to common corruptions. Dodge et al. find that standard trained DNNs are vulnerable to blur and Gaussian noise [13]. Hendrycks et al. [21] present CIFAR-10/100-C and ImageNet-C, consisting of fifteen different common corruptions with five severity levels to facilitate robustness evaluations of CIFAR [30] and ImageNet [11] models. Recently, Mintum et al. further propose CIFAR-10/100-C and ImageNet-C to provide new corruptions [37]. There are two popular lines of work on improving the robustness against common corruptions: **test-time adaptation** [47] and **data augmentation** [10, 22]. The authors in [45] propose a method to update the batch normalization (BN) statistics for improving domain adaptation. Another recent method, TENT [51] updates both the affine transformation and statistics of BN by using self-entropy minimization. On the other hand, methods such as AutoAugment [10] leverages reinforcement learning to learn an augmentation policy that produces a diverse set of augmentations to help make the models robust to OOD data. Another popular method, AugMix [22] achieves impressive performance improvement on corrupted data using augmentations generated by mixing up images obtained from applying randomly sampled operations along with using a Jenson-Shannon based consistency loss during training. Unlike existing data augmentation schemes which intend to improve the empirical robust accuracy of the models, the data augmentation schemes of interest to this paper aim to improve the adversarial robustness guarantees on OOD data.

**Certified Semantic Robustness.** Recent work [14,34,40] have also focused on developing techniques to provide performance guarantees to seen (or known) common corruption types (such as rotation or brightness changes). However, in this work, we are interested in more realistic scenarios with unseen (or unknown) test-time corruptions. It is worth noting that the susceptibility analysis and defense techniques developed in this work can be extended to SOTA semantic robustness techniques.

3. Are Certifiably Robust Models Ready for Deployment in the Wild?

Predictions of certifiably robust ML models are guaranteed to stay constant in a neighborhood of a test point, making them provably resilient to adversaries at the test time. This feature of certified defenses makes them an attractive candidate for real world safety-critical applications. However, progress in this area has been assessed by evaluating these models in idealistic scenarios (i.e., the in-distribution setup), which is not representative of real world data distributions. To better understand the performance of certified defenses in the real world, in this section, we evaluate SOTA certified defenses against OOD shifts.

3.1. Background on SOTA Certified Defenses

We focus our attention on the SOTA certification technique based on randomized smoothing (RS) which is efficient and scalable. Let us consider a base classifier \( M \) trained on samples \( x \in \mathcal{X} \subset \mathbb{R}^d \times \mathbb{R}^3 \) and their corresponding labels \( y \in \mathcal{Y} \subset \mathbb{R}^+ \), obtained from an underlying data distribution \( D \).

**Certification.** The RS-based certification uses a base classifier \( M \) and provides certified robustness guarantees for its smoothed version defined as \( \hat{M}(x) = \arg \max_{c \in \mathcal{Y}} P(M(x + \delta) = c) \) where \( \delta \sim \mathcal{N}(0, \sigma^2 I) \). Intuitively, \( \hat{M} \) returns the most probable class returned by \( M \) on Gaussian perturbations of the input \( x \). The certification guarantees that the prediction of the smoothed classifier \( \hat{M} \) is consistent in the \( \ell_2 \) radius [8] of \( CR(M, \sigma, x; y) = \).
\[ \mathbb{P}(\Phi^{-1}(p_A) - \Phi^{-1}(p_B)) \],
where \( \Phi^{-1} \) is the inverse CDF of the standard Gaussian distribution, \( p_A = \mathbb{P}(\mathcal{M}(x + \delta) = c_A) \) is probability of the top class \( (c_A) \) and \( p_B = \max_{c \neq c_A} \mathbb{P}(\mathcal{M}(x + \delta) = c_A) \) is the probability of the runner-up class. Monte Carlo-based sampling [17] is utilized to approximate \( p_A \leq p_B \) and \( p_B = 1 - p_A \geq p_B \). The certified radius can still be computed using the same formula by replacing \( p_A \) and \( p_B \) with \( \overline{p_A} \) and \( \overline{p_B} \).

Improved Training. It has been observed empirically [8] that models trained using the standard training procedure do not provide reasonable certified robustness. Therefore, there is an increasing interest in developing improved training techniques to maximize the certified robustness. Several works [33] have made significant advances on the training techniques to maximize the certified radius, respectively. Specifically, new training methods such as Gaussian augmentation [8], SmoothAdv [46] and MACER [58] have been proposed. Intuitively, Cohen et al. [8] propose to leverage Gaussian augmentation with variance \( \sigma^2 \) to train the base classifier. SmoothAdv [46] and MACER [58] both use Gaussian augmentation and further improve Cohen et al.’s baseline method by adversarial training and introducing an auxiliary objective to maximize the certified radius, respectively. However, the effect of OOD data on the robustness guarantees of these models has been unexplored in the literature.

Evaluation Metrics. Similar to previous works [36, 46, 58], we use the average certified radius (ACR) as our metric to evaluate the robustness of the models on in-distribution (or clean) test data. Specifically, ACR := \( \frac{1}{|D_{test}|} \sum_{(x,y) \in D_{test}} \text{CR}(\mathcal{M}(\sigma,x) ; y) \times 1_{\mathcal{M}(x,\sigma)=y} \). which is also equivalent to the area under the certified radius-accuracy curve. We assign \( \text{CR}(\cdot) = 0 \) for incorrect prediction of \( \mathcal{M} \). For OOD performance, we measure the mean ACR (mACR) as an overall metric, mACR := \( \frac{1}{c} \sum_{c=1}^{c} \text{ACR}_c \), where \( c \) is the number of corruptions leveraged in a specific test set. For example, \( c = 15 \) and 10 in CIFAR-10/100-C and -C datasets, respectively. We also report the ACR for each corruption type. Unlike previous studies on empirical defenses, we do not use the empirical clean and robust accuracy [8, 46, 58] as a metric in this work since we focus on the certified robustness.

3.2. Analyzing Certified Defenses in OOD Settings

Real-world test data oftentimes do not follow the training data distribution \( D \), although tangible improvements have been made on certifying robustness of in-distribution data. Therefore, evaluating the performance of \( \mathcal{M} \) on out-of-distribution (OOD) data \( \{\hat{x}, y\}_1, \ldots, \{\hat{x}, y\}_n \sim D \) becomes a major concern. We consider the impact of OOD data on models trained using SOTA robust training methods [8, 46, 58] and RS-based certified defenses.

3.2.1. Degradation of Certified Robustness Guarantees on Common Corruptions

To measure the performance of certified defenses on OOD data, we use the popular common corruptions dataset CIFAR-10-C [21]. CIFAR-10-C contains 15 different corruptions from four categories (with 5 severity levels): noise, blur, weather, and digital corruptions. We arrange the corruption dataset into three groups and evaluate the ACR by increasing the severity level of the corruptions. Grouping is performed based on the visual similarity of the amplitude spectrum of corrupted images (see Appendix A). Group-H corruptions (roughly categorized as high-frequency corruption type) consist of \{Gaussian noise, impulse noise, shot noise, pixelate, JPEG\}; Group-M corruptions (roughly categorized as mid-frequency corruption type) consist of \{defocus blur, frosted glass blur, motion blur, zoom blur, elastic\}; and Group-L corruptions (roughly categorized as low-frequency corruption type) consist of \{brightness, fog, frost, snow, contrast\}.

The performance of SOTA certified defenses on these groups of corruptions is presented in Figure 2. SmoothAdv and MACER both achieve tangible enhancements in ACR on in-distribution CIFAR-10 data compared to the Gaussian augmentation baseline. However, all methods show a sharp performance drop in ACR as we move from Group-H (high-frequency) to Group-L (low-frequency). We see that these methods are surprisingly brittle in low-frequency corruption regime, e.g., we see up to 54% drop in ACR when moving from severity 0 (i.e., in-distribution) to severity 5. We
emphasize that this performance drop points to a methodological shortcoming and is not due to the corruptions in Group-L being too difficult since the empirical robust accuracy (Figure 8 in Appendix B) remains consistently high on all the groups and severity levels for empirically robust models [22, 29, 43]. Even though the performance of any ML model is expected to suffer on test data that lies far away from the data used during training, the drastic performance degradation of RS-based certifiably robust models on low-frequency corruptions is particularly concerning.

3.2.2. Validating the Brittle ness of Smoothed Models Through a Spectral Lens

To highlight that the vulnerability to low-frequency corruptions is a limitation of provably robust ML models, in this section, we perform a more systematic analysis that corroborates that our finding is not limited to a specific benchmark and holds more broadly. To achieve this, we perform a spectral domain analysis of SOTA smoothed models by utilizing the Fourier sensitivity analysis [57], which we briefly summarize next.

A Fourier basis image in the pixel space is a real-valued matrix $U_{i,j} \in \mathbb{R}^{d \times d}$ where its $||U_{i,j}||_2 = 1$, and $\text{FFT}(U_{i,j})$ only has two non-zero elements at $(i, j)$ and $(-i, -j)$ in the coordinate that views the image center as the origin. Given a test set and a smoothed model, we evaluate the CR$(\epsilon)$ of $x_{orig} = x + rU_{i,j}$ for each $x$ in the test set and compute their ACR, where $r$ is randomly sampled in $\{-1, 1\}$, $\epsilon$ is the perturbation in $\ell_2$ norm, and we treat the RGB channels independently. Each of the evaluated ACR corresponds to a data point in the heat map located at $(i, j)$. Figure 3 shows the heatmaps of models trained with Gaussian augmentation [8], SmoothAdv [46], and MACER [58] using $\epsilon = 4$ [57]. The center and edges of the heatmap contain evaluation on the lowest and highest frequency perturbations, respectively. The results in Figure 3 show that the certifiably robust classifiers achieve small ACR on OOD data belonging to the low-frequency region (around the center of the image) whereas they achieve a high ACR in the high-frequency region (near the edges). In particular, the ACRs are always less than 0.3 for all three methods in the mid-to-low frequency range, while they perform well in high-frequency regime. We emphasize that the Fourier sensitivity analysis in Figure 3 is general and is not specific to corruptions appearing in CIFAR10-C. Based on our analysis, we find that certifiably robust models are biased towards high-frequency noises and perform surprisingly poor on low-frequency OOD data.

Following this insight, we develop a data augmentation method capable of producing spectrally diverse augmentations to make certifiably robust models perform well on OOD data across the entire frequency spectrum in § 4.

### Algorithm 1: FourierMix Pseudocode

| Line | Description |
|------|-------------|
| 1    | $x_{aug} = 0$ |
| 2    | Sample mixing weights $(w_1, ..., w_k) \sim \text{Dirichlet}(\alpha, ..., \alpha)$ |
| 3    | for $i = 1, 2, ..., k$ do |
| 4    | Sample random affine transformation $T_i$ |
| 5    | $x_{fourier} = \text{FFT}(x_{aug})$ |
| 6    | Sample severity level of operations $s_A, s_P$ |
| 7    | $x_{fourier} = (A_{s_A} \circ P_{s_P})(x_{fourier})$ |
| 8    | $x_f = \text{IFFT}(x_{fourier})$ |
| 9    | Sample weight $t \sim \text{Beta}(\alpha, \alpha)$ |
| 10   | $x_{aug} = w_i \cdot (t x_f + (1 - t) T_i^T \cdot x_{aug})$ |
| 11   | end |
| 12   | Sample weight $m \sim \text{Beta}(\alpha, \alpha)$ |
| 13   | $x_{aug} = m x_{aug} + (1 - m) x_{aug}$ |

4. FourierMix: Data Augmentation with Broad Spectral Coverage

To improve the certified robustness of RS-based methods on OOD data, it is intuitively desirable to make the base classifier $M$ robust against different types of corruptions and their Gaussian perturbations. Motivated by our Fourier sensitivity analysis (§ 3), we propose a novel data augmentation method, FourierMix, to boost the certified robustness performance on OOD data. To improve the spectral coverage, we introduce Fourier-based operations that manipulate the image in the frequency domain. We also leverage randomly sampled affine transformations to enrich the augmentations in FourierMix. We adopt the high-level framework of AugMix [22] for chaining and mixing different augmented images. Figure 4 shows the overall pipeline and Algorithm 1 presents the pseudocode of FourierMix.

**Fourier Operations.** Two dimensional images can be converted into the frequency domain by applying the Fourier transform and vice versa. Fourier transform has the duality property, which provides a unique but equivalent perspective for image analysis. We use fast Fourier transform (FFT) and inverse FFT (IFFT) for the transformation between the pixel and frequency domains. $\text{FFT}(x)$ is complex in general, i.e., $\text{FFT}(x) = \text{FFT}_{\text{real}}(x) + i\text{FFT}_{\text{imag}}(x)$, with $A = \text{FFT}_{\text{real}}(x)$ as its amplitude and $P = \arctan(\text{FFT}_{\text{imag}}(x)/\text{FFT}_{\text{real}}(x))$ as its phase. The amplitude spectrum of natural images generally follows a power-law distribution, i.e., $\frac{1}{f^\eta}$, where $f$ is the azimuthal frequency and $\eta \approx 2$ [4, 49], resulting in extremely small power in the high-frequency areas. However, the amplitude spectrum of the IID Gaussian noise is a uniform distribu-
FourierMix

Affine Transformation

Fourier Perturbation

Amplitude

Phase

Additive Gaussian Noise

Generate Augmented Sets Using FourierMix

Data

Generate Augmented Sets Using FourierMix

FourierMix

FourierMix Data

Model Training

Training Loss on Gaussian Augmentations

Hierarchical Consistency Regularization (HCR)

FourierMix

Out-of-Distribution Test Data

Randomized Smoothing Certification

Improved Certified Robustness

Figure 4. Overview of our FourierMix pipeline for generating spectrally diverse data augmentations and training of certifiably robust models with the proposed hierarchical consistency regularization (HCR). The bottom-right figure shows the significant improvement achieved by our method (in terms of ACR) in comparison to Cohen et al. [8] on severity 5 corruptions from different frequencies in CIFAR-10-C.

So, Gaussian augmentation biases the models towards the high-frequency regime relative to original images. In order to have broad and unbiased spectral coverage, the core of FourierMix is to allocate similar proportions of augmentations across all frequencies. We use two spectral perturbation methods in FourierMix to achieve this goal:

\[ A(u, v) = A^{\text{orig}}_{u,v} \cdot U(1 - s_A, 1 + s_A) \]  
\[ P(u, v) = P^{\text{orig}}_{u,v} + \Lambda^{\text{truncated}}_{u,v}(0, \sigma^2 I) \]

where \((u, v)\) is the coordinate of the 2D frequency in the spectrum, and \(s_A\) and \(s_P\) control the severity levels of two perturbations. Formally, the PDF of \(A^{\text{truncated}}_{u,v} = \frac{\phi(x/\sigma)}{\sigma \cdot \Phi(x/\sigma)}\), where \(\phi(\cdot)\) and \(\Phi(\cdot)\) denote the PDF and CDF functions of a standard normal distribution, respectively. On one hand, we apply multiplicative factors sampled from a uniform distribution \(U(\cdot)\) to all frequencies in the amplitude spectrum. Therefore, \(A(u, v)\) ensures that the proportions of augmentation are similar across all frequencies relative to the original spectrum. On the other hand, since the magnitude of a phase spectrum is not correlated with the 2D frequency [35], additive noises are able to assign similar proportions of augmentations across 2D frequencies. As it is widely acknowledged that the phase component retains most of the high-level semantics [28, 54, 56], we leverage additive truncated Gaussian to constrain \(P(u, v)\) so that it will not destroy the semantics of the training images. Some sample images generated using FourierMix are provided in Appendix D.

Hierarchical Consistency Regularization (HCR). Motivated from [24] that enforces consistency on in-distribution data, we propose hierarchical consistency regularization (HCR) to further boost the performance of FourierMix in terms of the ACR on OOD test sets:

\[ \mathcal{L}_G = \frac{1}{s} \sum_{i=0}^{s} \text{KL}(\mathcal{M}(x_j + \delta_i) || \overline{\mathcal{M}}(x_j, \delta)) \]  
\[ \mathcal{L}_{HCR} = \frac{1}{k+1} \sum_{j=0}^{k} \left[ \lambda \text{KL}(\mathcal{M}(x_j, \delta)||\overline{\mathcal{M}}(x, \delta)) + \eta \mathcal{L}_G \right] \]

where \(\overline{\mathcal{M}}(x, \delta) = \mathbb{E}_{j \in \{0, 1, \ldots, k\}}[\mathcal{M}(x_j, \delta)]\), \(\mathcal{M}(x_j, \delta) = \mathbb{E}_{s \in \{1, 2, \ldots, s\}}[\mathcal{M}(x_j + \delta_s)]\), \(x_0\) is the original training image, and \(\text{KL}(\cdot||\cdot)\) denotes the Kullback–Leibler divergence (KLD) [27]. We use \(k = 2\) and \(s = 2\) for the FourierMix and Gaussian augmentation with \(\delta_s = \mathcal{N}(0, \sigma^2 I)\), respectively. Since Jensen–Shannon divergence (JSD) [15] uses the KLD to calculate a normalized score that is symmetrical, HCR essentially stacks two levels of JSD while training the base classifier to enforce the consistent representations over both augmentations. The first level of consistency \(\mathcal{L}_G\) is applied to the Gaussian augmentation, rendering the Gaussian perturbed neighbors of \(x_{0,1,2}\) have similar outputs, and the second level of consistency is on the whole \((k+1)s\) set to enforce FourierMix augmented images with consistent outputs. We utilize \(\lambda\) and \(\eta\) as hyper-parameters to tune the weights of two levels of consistency. The overall training loss is:

\[ \mathcal{L} = \frac{1}{s} \sum_{i=1}^{s} \mathcal{L}(x_0 + \delta_i, y) + \mathcal{L}_{HCR} \]

Comparison with AugMix. The key difference between FourierMix and AugMix is the base augmentation operations used in the pipeline. AugMix leverages the operations from AutoAugment [10] that do not overlap with ImageNet-C. In contrast, the augmentations in FourierMix utilize a simpler set of generic augmentations. We compare the performance (i.e., ACR) of FourierMix with AugMix on multiple OOD datasets in our evaluation (§5 and §6).

5. Experiments on Popular OOD Benchmarks

Experimental Setup. As introduced in §3.1, we use ACR and mACR as the main evaluation metrics. We utilize the official implementation from [8] to compute the certified radius CR(·). We use the same base architectures leveraged in prior arts [8, 24, 46, 58], i.e., ResNet-110 and ResNet-50 as the backbones, for experiments on CIFAR-10/100 and ImageNet [18], respectively. We use Gaussian augmentation.
Table 1. Models trained with FourierMix and HCR achieve significant improvements in the certified robustness (ACR and mACR) guarantees on all popular OOD datasets. **Bold** and **underline** denote the best and runner-up results, respectively.

| Augmentation | CIFAR-10 | CIFAR-10-C | CIFAR-10-C | CIFAR-100 | CIFAR-100-C | ImageNet | ImageNet-C | ImageNet-C |
|--------------|----------|------------|------------|----------|-----------|----------|-----------|-----------|
|              | ACR      | mACR-Low-Mid-High | ACR      | mACR-Low-Mid-High | ACR      | mACR-Low-Mid-High | ACR      | mACR-Low-Mid-High |
| Gaussian     | 0.461    | 0.363 0.301 0.353 0.435 | 0.314    | 0.149 0.131 0.182 0.208 | 0.130    | 0.076 0.195 0.228 0.385 | 0.266    | 0.246 0.258 0.385 0.435 |
| +JSD         | 0.535    | 0.439 0.346 0.451 0.520 | 0.393    | 0.281 0.232 0.167 0.248 0.200 | 0.196    | 0.175 0.239 0.202 0.382 0.594 | 0.395    | 0.395 0.395 0.395 0.395 |
| +HCR         | 0.727    | 0.550 0.444 0.372 0.312 | 0.520    | 0.374 0.232 0.167 0.240 0.200 | 0.314    | 0.281 0.239 0.202 0.382 0.594 | 0.444    | 0.444 0.444 0.444 0.444 |

**Baseline.** We evaluate the certified robustness of models trained with following augmentation schemes on OOD data: Gaussian [8], AutoAugment [10], and AugMix [22]. We also compare HCR with the baseline JSD regularization [24]. We follow Cohen et al. [8] and Jeong et al. [24] to train the Gaussian and Gaussian+JSD baseline models, respectively. For other augmentation methods, we apply Gaussian noise $N(0, \sigma^2 I)$ to half of the training samples in the mini-batch to ensure good certification performance using RS, and we follow Hendrycks et al. to apply JSD to these augmentation methods [22].

**Datasets.** For the in-distribution evaluation, we use CIFAR-10/100 [30] and ImageNet [11] datasets. CIFAR-10/100 consist of small $32 \times 32$ images belonging to 10/100 classes and ImageNet consists of 1.2 millions images with 1,000 classes. We crop images in ImageNet into the same size of $224 \times 224 \times 3$ pixels. For OOD data, we use the common corruptions datasets [21] (CIFAR-100-C, ImageNet-C) and a recently proposed dataset [37] (CIFAR-100-C ImageNet-C) which contains human interpretable and perceptually different corruptions as compared to those contained in CIFAR-C/ImageNet-C.

5.1 Results on CIFAR-Based OOD Benchmarks

The results in Tables 1 show the overall mACR of the models trained on CIFAR-10 using different augmentation and regularization methods when evaluated on CIFAR-10-C and CIFAR-10-C, respectively. The results show that FourierMix consistently achieves the highest mACR across different corruption types. FourierMix+HCR significantly improves upon the baseline of Gaussian augmented training by 26.7% and 33.4% in terms of the overall mACR on CIFAR-10-C and CIFAR-10-C, respectively and also improves upon the stronger baseline, AugMix+HCR, by 5.3% and 6.6% on the two datasets. We find consistency regularization to be helpful for certified robustness on OOD data. Specially, on mid- and high frequency corruptions adding JSD to Gaussian augmentations significantly improves the robustness on OOD data. We see that combining HCR with FourierMix achieves SOTA ACRs on all corruptions types providing significant gains even on low-frequency corruptions. This success is attributed to the spectrally diverse corruptions produced by FourierMix. Interestingly, we find AutoAugment overfits to corruptions in CIFAR-10-C since it suffers a major performance degradation on corruptions in CIFAR-10-C. We believe the large overlap between the leveraged augmentations and corruptions in CIFAR-10-C and limited spectral diversity are the primary reasons for this performance degradation of AutoAugment. Detailed results for each corruption type in CIFAR-10-C/C are shown in Tables 2 and 3 in Appendix C.1.

Next, we present the mACR (Table 1) of the models trained with CIFAR-100 when evaluated on OOD data (CIFAR-100-C and CIFAR-100-C). Similar to the performance of models trained with CIFAR-10, FourierMix achieves the highest overall mACR among all augmentation methods on both OOD datasets. Specifically, FourierMix+HCR outperforms the Gaussian baseline by 54.4% and 74.6% on two datasets, respectively. Compared to AugMix+HCR, FourierMix+HCR improves the performance by 4.8% and 7.6% on the two datasets, respectively. Detailed results for each corruption type in CIFAR-100-C/C are shown in Tables 4 and 5 in Appendix C.2.

To further corroborate our findings on OOD benchmarks, we carry out the Fourier sensitivity analysis of models trained on CIFAR-10/100 in Figure 5. Adding a consistency loss (Gaussian+JSD) improves the ACR of the model in the high-frequency region but is still worse than the ACR achieved by the addition of consistency loss (JSD and HCR) with FourierMix augmentations in low-to-mid frequency regions. Similar to our quantitative results, AutoAugment does not improve much over the baseline of Gaussian augmentation which suggests that models trained with AutoAugment may be biased towards high frequency regions.

5.2. Results on ImageNet-Based OOD Benchmarks

Table 1 presents the mACR of the models trained on ImageNet when evaluated on ImageNet-C and ImageNet-C. Due to poor performance of some of the methods on CIFAR-10/100, we chose not to pursue them for ImageNet scale experiments (denoted by "-" in Table 1). We observe...
that OOD shifts lead to a drastic decline in the certified robustness on ImageNet. The drop between the ACR of clean data and the mACR of OOD data is \( \sim 57\% \), whereas it was \( \sim 30\% \) on CIFAR-10/100. Encouragingly, FourierMix continues to achieve the highest mACR compared to other baselines. FourierMix outperforms the baseline of Gaussian augmented training and AugMix+JSD by 55.9% and 2.1% in terms of the overall mACR, respectively. FourierMix also realizes consistent good performance across the spectrum, whereas Gaussian+JSD and AugMix+JSD are biased to high-frequency and low-frequency corruptions, respectively. Despite HCR not making a significant difference over JSD regularization, it is worth noting that substantial improvements can still be gained by FourierMix (over other baseline augmentations) on ImageNet due to its broad spectral coverage. Detailed results for ImageNet-C/C can be found in Tables 6 and 7 in Appendix C.3.

Overall Insights. Our results in this section not only highlight the vulnerability of SOTA certified defenses to OOD data but also uncovers spectral biases in the benchmark datasets that are used to measure OOD robustness. In particular, methods that perform well on one corrupted dataset may not work well on another dataset due to differences in the spectral signatures of the corruptions. For example, AutoAugment performs well on CIFAR-10-C corruptions but is significantly worse on CIFAR10-C and CIFAR100-C/C. Moreover, we find that such ranking discrepancies not only exist when measuring ACR but are also evident when measuring empirical robust accuracy. Figure 9 in Appendix B shows that there is no single SOTA model from RobustBench leaderboard [9] which performs the best on all benchmark datasets or corruption types. This makes it incredibly important to obtain a comprehensive view of the model robustness to avoid issues such as leaderboard bias [38] and model overfitting to a specific benchmark [37]. To enable researchers achieve this objective, next we propose a new benchmark which has a collection of spectrally diverse OOD datasets.

6. A Spectral OOD Benchmarking Suite

Here we discuss the creation and evaluation of models on the proposed OOD benchmarking suite. The goal of this new OOD suite is to complement (and not replace) the existing benchmark datasets and enable researchers to uncover and resolve spectral biases of their models.

6.1. Protocol for Dataset Generation

The proposed benchmark is a collection of datasets (CIFAR-10/100-F) each focusing on a specific frequency range while collectively covering the entire frequency spectrum. Different from the Fourier sensitivity analysis that only perturb a single frequency using the Fourier basis, CIFAR-10/100-F leverages power law-based noise [1] to generate complex perturbations in the spectral domain [25]. Note that power spectrum of several natural data distributions (e.g., natural images) follow power-law distributions.

The proposed benchmark is a collection of datasets (CIFAR-10/100-F) each focusing on a specific frequency range while collectively covering the entire frequency spectrum. Different from the Fourier sensitivity analysis that only perturb a single frequency using the Fourier basis, CIFAR-10/100-F leverages power law-based noise [1] to generate complex perturbations in the spectral domain [25]. Note that power spectrum of several natural data distributions (e.g., natural images) follow power-law distributions.
Figure 6. ACRs on the proposed CIFAR-10/100-F dataset averaged over 3 severity levels show that FourierMix based models perform consistently better than other baselines across entire spectrum. Increasing $\alpha$ (from left to right), decreases the spread of the frequencies. Sharp dips in ACRs in mid-frequency regions (e.g., (c) and (d)) demonstrate the vulnerability of models to low-to-mid frequency corruptions.

Figure 7. Sample images from CIFAR-10/100-F when $\epsilon = 12$. Corruption is formulated as $\delta_{\text{Fourier}}(f) = U(0, 2\pi)$ to simulate real world noises. Given each pair $(x^i, y^i)$ in the original validation set, we synthesize CIFAR-10/100-F images as

$$x^i_f = x^i + \gamma \cdot \text{IFFT}(\delta_{\text{Fourier}}),$$

where $\gamma = \frac{\text{IFFT}(\delta_{\text{Fourier}})}{\epsilon}$. Normalizes the spreading effect of the power-law distribution and, thus, controls the severity level of CIFAR-10/100-F. We create both CIFAR-10/100-F with 3 severity levels with $\epsilon \in \{8, 10, 12\}$. As the images in CIFAR-10/100 are of size $32 \times 32$, their FFT spectrums have discrete azimuthal frequencies from 0 to 16. Since zero-frequency noise is a constant in the pixel space, we set the center frequency $f_c \in \{1, 2, ..., 16\}$. We leverage $\alpha \in \{0.5, 1, 2, 3\}$ because power law noises with $0 < \alpha \leq 3$ arise in both natural signals and in man-made processes [1]. In total, our CIFAR-10/100-F datasets consist of $3 \times 4 \times 16 = 192$ test sets from different regions of the frequency spectrum thereby increasing the spectral coverage of the dataset.

Visual Effect of Varying $\alpha$ and $f_c$. To better understand our proposed benchmark, we explain the effect of $\alpha$ and $f_c$ with some sample images. As shown in Figure 7, $\alpha$ controls the frequency dispersion of the corruption at $f_c$. With a smaller $\alpha$, e.g., $\alpha = 0.5$, the spreading effect of the power law distribution is more significant. The corrupted images thus contain noises across all azimuthal frequencies. In contrast, for larger $\alpha$, the corruptions will be focused more on a single frequency e.g., $\alpha = 3$. As evident from Figure 7 higher $f_c$ leads to a higher corruption frequency. More images from CIFAR-10/100-F from different classes and severity levels are shown in Appendix F.

6.2. Results on CIFAR-10/100-F

Figure 6 reports the performance of models used in § 5.1 on CIFAR-10-F benchmark. Our results show that both AutoAugment [10] and AugMix [22] based smoothed models are relatively biased towards low-frequency corruptions. The effect of high frequency corruptions is more pronounced on models trained with AutoAugment which behave similar to the simple baseline of Gaussian augmentation (Figures 6(c) and 6(d)). The intersection of the curves of AugMix+JSD and Gaussian+JSD in the mid frequency region in CIFAR-10-F (Figure 6), illustrates the different spectral biases introduced by different augmentation methods. Unlike CIFAR10-F, we find that Gaussian and Gaussian+JSD perform relatively worse on CIFAR-100-F compared to other augmentation methods. In comparison to other methods, we find that models trained with FourierMix and HCR do not show significant spectral biases and serve as a strong baseline. Specifically, models trained with FourierMix+HCR, on average, outperforms AugMix+HCR, by 11.8% and 16.0% on CIFAR-10/100-F, respectively. We emphasize that models trained with FourierMix do not over-
fit to CIFAR-10/100-F datasets since they have different formulations and even visual patterns (see Appendix D and F). Moreover, FourierMix models provide consistently better performance on other OOD benchmarks as well, demonstrating its generality.

7. Discussion and Conclusion

Our work showed that certified defenses are surprisingly brittle to OOD shifts such as low-frequency corruptions. To alleviate this issue, we proposed FourierMix augmentation to increase the spectral coverage of the training data. We also presented a benchmarking suite to gain a comprehensive understanding of the model’s OOD robustness. Some of our findings are consistent with past results in that we also show that the model evaluation in OOD settings is a challenging problem, and one should not rely on a single benchmark [19,37]. However, as opposed to the existing works that focus on empirical robustness, we show that these issues persist and may even be more prominent on the problem of certified adversarial defense. Even though evaluation against all possible types of OOD data is infeasible, our results highlighted that eliminating spectral biases of the models improves the certified robustness on OOD data.

Although we have taken some first steps to address this challenging problem, there are still many questions that remain to be answered. First, bridging the gap between robustness guarantees in high-frequency and low-frequency corruption regimes is still an open problem. A deeper theoretical understanding of this phenomenon will likely motivate systematic approaches to overcome this issue. Next, we encourage future research to pursue test-time adaptation ideas [12,41] in the context of robustness certification. We expect that designing data-efficient and unsupervised adaptation methods that improve robustness guarantees under OOD shifts can be a worthwhile direction. Finally, the adaptation methods that improve robustness guarantees un-

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References

[1] 1/f noise. http://www.scholarpedia.org/article/1/f_noise, 2021. 8, 9
[2] Anish Athalye, Nicholas Carlini, and David Wagner. Obfuscated gradients give a false sense of security: Circumventing defenses to adversarial examples. In International conference on machine learning, pages 274–283. PMLR, 2018. 1, 3
[3] Saikiran Bulusu, Bhavya Kailkhura, Bo Li, Pramod K Varshney, and Dawn Song. Anomalous example detection in deep learning: A survey. IEEE Access, 8:132330–132347, 2020. 3
[4] Geoffrey J Burton and Ian R Moorhead. Color and spatial structure in natural scenes. Applied optics, 26(1):157–170, 1987. 5
[5] Nicholas Carlini and David Wagner. Towards evaluating the robustness of neural networks. In 2017 IEEE Symposium on Security and Privacy (SP), pages 39–57, 2017. 3
[6] Pin-Yu Chen, Yash Sharma, Huan Zhang, Jinfeng Yi, and Cho-Jui Hsieh. EAD: elastic-net attacks to deep neural networks via adversarial examples. In Proceedings of the AAAI Conference on Artificial Intelligence, pages 10–17, 2018. 3
[7] Pin-Yu Chen, Huan Zhang, Yash Sharma, Jinfeng Yi, and Cho-Jui Hsieh. ZOO: Zeroth order optimization based black-box attacks to deep neural networks without training substitute models. In ACM Workshop on Artificial Intelligence and Security, pages 15–26, 2017. 3
[8] Jeremy Cohen, Elan Rosenfeld, and Zico Kolter. Certified adversarial robustness via randomized smoothing. In International Conference on Machine Learning, pages 1310–1320. PMLR, 2019. 1, 2, 3, 4, 5, 6, 7, 13, 14, 15
[9] Francesco Croce, Maksym Andriushchenko, Vikash Sehwag, Eduordo Debenedetti, Nicolas Flammarion, Mung Chiang, Prateek Mittal, and Matthias Hein. Robustbench: a standardized adversarial robustness benchmark. arXiv preprint arXiv:2010.09670, 2020. 8, 13
[10] Ekin D Cubuk, Barret Zoph, Dandelion Mane, Vijay Vasudevan, and Quoc V Le. Autoaugment: Learning augmentation strategies from data. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 113–123, 2019. 3, 6, 7, 9
[11] Jia Deng, Wei Dong, Richard Socher, Li-Jia Li, Kai Li, and Li Fei-Fei. Imagenet: A large-scale hierarchical image database. In 2009 IEEE conference on computer vision and pattern recognition, pages 248–255. Ieee, 2009. 3, 7
[12] James Diffenderfer, Brian R Bartoldson, Shreya Chaganti, Jize Zhang, and Bhavya Kailkhura. A winning hand: Compressing deep networks can improve out-of-distribution robustness. arXiv preprint arXiv:2106.09129, 2021. 10
[13] Samuel Dodge and Lina Karam. Understanding how image quality affects deep neural networks. In 2016 eighth international conference on quality of multimedia experience (QoMEX), pages 1–6. IEEE, 2016. 3
[14] Marc Fischer, Maximilian Baader, and Martin Vechev. Certified defense to image transformations via randomized smoothing. arXiv preprint arXiv:2002.12463, 2020. 3
...
[46] Hadi Salman, Greg Yang, Jerry Li, Pengchuan Zhang, Huan Zhang, Ilya Razenshteyn, and Sebastien Bubeck. Provably robust deep learning via adversarially trained smoothed classifiers. *arXiv preprint arXiv:1906.04584*, 2019. 1, 4, 5, 6, 14

[47] Steffen Schneider, Evgenia Rusak, Luisa Eck, Oliver Bringmann, Wieland Brendel, and Matthias Bethge. Improving robustness against common corruptions by covariate shift adaptation. *Advances in Neural Information Processing Systems*, 33, 2020. 3

[48] Christian Szegedy, Wojciech Zaremba, Ilya Sutskever, Joan Bruna, Dumitru Erhan, Ian Goodfellow, and Rob Fergus. Intriguing properties of neural networks. *arXiv preprint arXiv:1312.6199*, 2013. 2, 3

[49] DJ Tolhurst, Y. Tadmor, and Tang Chao. Amplitude spectra of natural images. *Ophthalmic and Physiological Optics*, 12(2):229–232, 1992. 5

[50] Florian Tramer, Nicholas Carlini, Wieland Brendel, and Aleksander Madry. On adaptive attacks to adversarial example defenses. *arXiv preprint arXiv:2002.08347*, 2020. 1

[51] Dequan Wang, Evan Shelhamer, Shaoteng Liu, Bruno Olshausen, and Trevor Darrell. Tent: Fully test-time adaptation by entropy minimization. In *International Conference on Learning Representations*, 2021. 3, 15

[52] Eric Wong and Zico Kolter. Provable defenses against adversarial examples via the convex outer adversarial polytope. In *International Conference on Machine Learning*, pages 5286–5295. PMLR, 2018. 1, 3

[53] Chaowei Xiao, Bo Li, Jun-Yan Zhu, Warren He, Mingyan Liu, and Dawn Song. Generating adversarial examples with adversarial networks. *arXiv preprint arXiv:1801.02610*, 2018. 3

[54] Qinwei Xu, Ruipeng Zhang, Ya Zhang, Yanfeng Wang, and Qi Tian. A fourier-based framework for domain generalization. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 14383–14392, 2021. 6

[55] Greg Yang, Tony Duan, J Edward Hu, Hadi Salman, Ilya Razenshteyn, and Jerry Li. Randomized smoothing of all shapes and sizes. In *International Conference on Machine Learning*, pages 10693–10705. PMLR, 2020. 10

[56] Yanchao Yang, Dong Lao, Ganesh Sundaramoorthi, and Stefano Soatto. Phase consistent ecological domain adaptation. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 9011–9020, 2020. 6

[57] Dong Yin, Raphael Gontijo Lopes, Jonathon Shlens, Ekin D Cubuk, and Justin Gilmer. A fourier perspective on model robustness in computer vision. *arXiv preprint arXiv:1906.08988*, 2019. 5, 13

[58] Runtian Zhai, Chen Dan, Di He, Huan Zhang, Boqing Gong, Pradeep Ravikumar, Cho-Jui Hsieh, and Liwei Wang. Macer: Attack-free and scalable robust training via maximizing certified radius. In *International Conference on Learning Representations*, 2020. 1, 4, 5, 6, 14

[59] Huan Zhang, Tsui-Wei Weng, Pin-Yu Chen, Cho-Jui Hsieh, and Luca Daniel. Efficient neural network robustness certification with general activation functions. In *Advances in Neural Information Processing Systems*, pages 4944–4953, 2018. 3
Appendix

A. Amplitude Spectrum of CIFAR-10-C/¯C

As introduced in § 3, we arrange the amplitude spectrum of corruptions on CIFAR-10-C into three groups, roughly categorized as high/mid/low-frequency corruptions. Specifically, we compute the the $\mathbb{E}[\text{FFT}(x)]$ and $\mathbb{E}[\text{FFT}(C(x) = x)]$ by averaging over all the validation images [57] for CIFAR-10 and each corruption in CIFAR-10-C, respectively, where $C(\cdot)$ denotes the corruption function. As Figure 10 shows, CIFAR-10 (clean) images follow a distribution of $\frac{1}{\sqrt{2\pi}}$, where $f = \sqrt{u^2 + v^2}$ is the azimuthal frequency and $\alpha \approx 2$. Therefore, clean images have extremely low power in the high-frequency regions (the edges and corner). Due to this, all the noise perturbations corresponding to JPEG and pixelate can be considered as high-frequency corruptions, relative to the clean images’ distribution. On the other hand, weather-related and contrast corruptions are all centered in the low-frequency region. We categorize remaining perturbations as mid-frequency corruptions.

We also visualize the amplitude spectrum of corruptions from CIFAR-10-C in Figure 11. We find that most of the corruptions from CIFAR-10-C are centered in the low/mid-frequency ranges, explaining why FourierMix achieves larger improvements on CIFAR-10-C than CIFAR-10-C compared to spectrally-biased baselines.

B. Empirical Robust Accuracy of SOTA Models on Corrupted Data

The results in Figure 9 show empirical robust accuracy of state-of-the-art models on existing OOD benchmarks. We use the recently proposed RobustBench [9] benchmark and selected the top-performing models on CIFAR-10-C for this experiment [22, 29, 43]. As evident from the figure, the performance of the models varies across datasets and corruption types showing that a single model is not able to achieve the best performance on all types of OOD data. Evaluating the models on a single benchmark is not enough to obtain the true picture of the OOD robustness of a model. Thus to eliminate the biases present in OOD benchmarks, one should gauge the OOD robustness of a model by evaluating it on a variety of datasets. Our proposed CIFAR-10/100-F benchmark can be used by designers to probe the spectral biases of the models.

C. Detailed Evaluation Results

C.1. CIFAR-10-Based OOD Benchmarks

In this section, we present detailed results for our evaluation on CIFAR-10-C/¯C. We fix $\eta = 10$ and use $\lambda = 40$ for HCR (Equation 4) in our experiments on CIFAR-10. Tables 2 and 3 present the ACR on individual corruption types from CIFAR-10-C/¯C, respectively. FourierMix consistently achieves the highest ACR on most of the corruption types in both OOD datasets. Especially, we find FourierMix helps achieve larger improvements on weather-related corruptions, which have real-world implications (e.g., safety of autonomous driving).

C.2. CIFAR-100-Based OOD Benchmarks

In this section, we present detailed results for our evaluation on CIFAR-100-C/¯C and CIFAR-100-F. We fix $\eta = 10$ and use $\lambda = 20$ for HCR in our experiments on CIFAR-100. Tables 4 and 5 present the ACR on individual corruption types from CIFAR-100-C/¯C, respectively. CIFAR-100 is more difficult for RS-based certification compared to CIFAR-10. We find that FourierMix+HCR helps achieve the highest ACR on all corruption types in both datasets with significant enhancements compared to existing augmentation methods.

C.3. ImageNet-Based OOD Benchmarks

ImageNet appears to be the most challenging dataset for certified defenses, to which only RS-based techniques can be applied [8]. We select representative combinations of augmentations and regularization schemes that perform well on CIFAR-10/100 for our experiments on ImageNet. We exclude the input normalization layer, which trades off the ACR on clean data for the ACR on OOD data. We use $\eta = 5$ and $\lambda = 5$ for our experiments with HCR. Tables 6 and 7 present the detailed results on our evaluation on ImageNet-C/¯C. Note the the corruption types in ImageNet-C are different from the ones in CIFAR-10/100-C. We find that the spectral biases of other baselines become much more noticeable on ImageNet-based OOD benchmarks. Gaussian+JSD accomplishes the highest ACR on high-frequency corruptions, while AugMix+JSD performs the best on several low-frequency corruptions in ImageNet-C. As RS-based models generally suffer performance degradation on low-frequency corruptions, Gaussian+JSD beats AugMix+JSD in terms of overall mACR. However, FourierMix performs well across the spectrum, reaching the highest mACR on both datasets. Although tangible improvements have been realized by FourierMix on ImageNet-based OOD benchmarks, we want to highlight that there is still large room for future research to improve over our baselines. We hope this work will motivate more studies on certified defenses for ImageNet under OOD shifts, as discussed in § 7.

D. FourierMix Details

Hyper-parameter Settings. We detail the chosen hyper-parameters used in the experiments with FourierMix. As illustrated in Algorithm 1 and Equations 1 and 2, we leverage 5 different severity levels and truncated Gaus-
Sample Images from FourierMix. We visualize randomly sampled images from CIFAR-10/100 and ImageNet in Figure 13, 14, and 15, respectively.

E. Training and Evaluation Details

Training. We train CIFAR-10/100 and ImageNet models for 200 and 90 epochs for all methods with an SGD optimizer, respectively [44]. We exclude the input normalization layer as it will degrade the certification performance on OOD data. We use different σ to train CIFAR-10/100 and ImageNet models, as specified in § 5.

Evaluation. Recall from the theorem derived in § 3 of Cohen et al., CR(·) approaches ∞ when p_A approaches the value 1 [8]. However, this will also require the Gaussian perturbed samples n ∼ ∞. Consider that the base classifier M(· + δ) has observed n samples that all equal to c_A, p_A ≥ α(1/n) has a probability 1 − α [8]. To both constrain the computational complexity and achieve a tight bound, we use n = 100, 000, n_0 = 100, and α = 0.001 as the hyper-parameters to get high confidence of the computed radius, following prior arts [8,24,46,58]. Since we need to evaluate OOD datasets with 125× larger sizes than the original test sets, we certify 500 and 350 examples from each corruption and each severity level of the CIFAR-10/100 and ImageNet OOD datasets (i.e., -C). For the Fourier sensitivity analysis of CIFAR-10/100, each data point in the heat map is the corresponding ACR of 200 examples.

F. Sample Images from CIFAR-10/100-F

We visualize more sample images from our created datasets in Figure 12 using different classes. It is also
Table 3. Average Certified Radius (ACR) of Models Trained with Different Methods on CIFAR-10-C. Models trained with FourierMix and HCR achieve significant improvements in the certified robustness (ACR) guarantees on corruptions from the CIFAR-10-C dataset.

| Augmentation | Blue | Brown | Checkerboard | Circular | Inv. | Sparkle | Lines | Pinch | Ripple | Sparkles | Trans. | Chromatic |
|--------------|------|-------|--------------|----------|------|---------|-------|-------|--------|----------|--------|-----------|
| Gaussian     | 0.314| 0.351 | 0.255       | 0.310    | 0.386| 0.222   | 0.336 | 0.398 | 0.365  | 0.251    | 0.259  |
| +JSD         | 0.395| 0.458 | 0.303       | 0.395    | 0.452| 0.252   | 0.340 | 0.492 | 0.346  | 0.269    | 0.306  |
| +FourierMix  | 0.304| 0.351 | 0.265       | 0.312    | 0.393| 0.223   | 0.348 | 0.406 | 0.248  | 0.233    | 0.256  |
| +HCR         | 0.346| 0.354 | 0.297       | 0.335    | 0.445| 0.258   | 0.374 | 0.436 | 0.402  | 0.269    | 0.308  |
| +FourierMix  | 0.348| 0.389 | 0.269       | 0.334    | 0.439| 0.233   | 0.358 | 0.416 | 0.397  | 0.272    | 0.307  |
| +JSD         | 0.382| 0.429 | 0.303       | 0.372    | 0.483| 0.255   | 0.404 | 0.467 | 0.450  | 0.306    | 0.350  |
| +FourierMix  | 0.393| 0.442 | 0.309       | 0.384    | 0.486| 0.268   | 0.419 | 0.471 | 0.464  | 0.320    | 0.368  |
| +HCR         | 0.394| 0.391 | 0.269       | 0.351    | 0.441| 0.237   | 0.368 | 0.432 | 0.401  | 0.280    | 0.325  |
| +FourierMix  | 0.397| 0.445 | 0.307       | 0.395    | 0.482| 0.265   | 0.430 | 0.490 | 0.463  | 0.320    | 0.377  |

worth noting that FourierMix augmented images (Figure 13 and 14) have different patterns with CIFAR-10/100-F.

G. Discussion on Test-Time Adaptation

As discussed in Appendix 2, another widely acknowledged approach to counter OOD shifts is test-time adaptation. We thus perform a preliminary study on how test-time adaptation will affect the certified robustness. Specifically, we use BN [45] and TENT [51] as representative methods. Since the theorem derived by Cohen et al. [8] requires the base classifier $M$ to be deterministic, we cannot apply BN and TENT in an online manner. To deal with such a problem, while evaluating the ACR of OOD data from a specific corruption type, we randomly sample 500 (out of 10,000) images from the OOD test set for the adaptation. We follow other settings specified in [45, 51] for our experimentation. Table 9 presents the detailed results on CIFAR-10-C. We find that test-time adaptations fail to improve the ACR in the OOD setting. The reason is that one-shot adaptation relies upon a small amount of data which is not sufficient to correct the OOD shift. In contrast, it may cause the base classifier $M$ to become biased towards the small subset of test data used for adaptation. We highlight that certification of adaptive models is also a potential direction that can help with OOD certified robustness. More theoretical support is needed in this direction, and we leave it as a promising future work.
Table 5. Average Certified Radius (ACR) of Models Trained with Different Methods on CIFAR-100-\(\bar{C}\). Models trained with FourierMix and HCR achieve significant improvements in the certified robustness (ACR) guarantees on corruptions from the CIFAR-100-\(\bar{C}\) dataset.

| Augmentation | ImageNet  | -Low  | -Mid  | -High | Gaussian | Shot | Impulse | Defocus | Glass | Motion | Zoom | Snow | Frost | Fog | Bright | Contrast | Elastic | Pixel | JPEG |
|--------------|-----------|-------|-------|-------|----------|------|--------|--------|-------|--------|------|------|------|-----|--------|----------|--------|-------|------|
| Gaussian     | 0.660     | 0.395 | 0.239 | 0.564 | 0.567    | 0.542| 0.378  | 0.351  | 0.374 | 0.254  | 0.245| 0.013 | 0.551 | 0.519 | 0.640  | 0.702   |
| +JSD         |           |       |       |       |          |      |        |        |       |        |      |      |      |     |        |          |        |       |      |
| +AugMix + JSD| 0.714     | 0.391 | 0.288 | 0.550 | 0.496   | 0.489| 0.473  | 0.329  | 0.395 | 0.376  | 0.352 | 0.255  | 0.286 | 0.041 | 0.542  | 0.481  | 0.622  | 0.668 |
| +FourierMix + JSD | 0.751 | 0.399 | 0.242 | 0.564 | 0.515   | 0.493| 0.483  | 0.315  | 0.384 | 0.380  | 0.370 | 0.254  | 0.302 | 0.041 | 0.544  | 0.473  | 0.637  | 0.694 |
| +FourierMix + HCR | 0.414 | 0.562 | 0.544 | 0.479 | 0.370   | 0.366 | 0.366  | 0.367  | 0.210 | 0.408  | 0.215 | 0.415  | 0.541 | 0.541 | 0.472  | 0.483  | 0.550  |

Table 6. Average Certified Radius (ACR) of Models Trained with Different Methods on ImageNet-C. Spectrally diverse augmentations from FourierMix brings significant gains to certified robustness of the models trained on ImageNet against corruptions from ImageNet-C.

| Augmentation | mACR | Blue | Brown | Caustic | Checkboard | Cocolentric | Inv. Sparkle | Perlin | Plasma | Single Freq. | Sparkle |
|--------------|------|------|-------|---------|------------|-------------|--------------|--------|--------|------------|---------|
| Gaussian     | 0.266| 0.394| 0.284 | 0.325   | 0.250      | 0.235       | 0.152        | 0.274  | 0.065  | 0.284      | 0.400   |
| +JSD         | 0.395| 0.579| 0.395 | 0.512   | 0.370      | 0.374       | 0.224        | 0.404  | 0.113  | 0.408      | 0.567   |
| +AugMix + JSD| 0.579| 0.560| 0.381 | 0.461   | 0.363      | 0.342       | 0.212        | 0.413  | 0.121  | 0.397      | 0.538   |
| +FourierMix + JSD | 0.413 | 0.562 | 0.544 | 0.479   | 0.370      | 0.366       | 0.215        | 0.413  | 0.227  | 0.417      | 0.547   |
| +FourierMix + HCR | 0.411 | 0.565 | 0.535 | 0.481   | 0.365      | 0.367       | 0.210        | 0.408  | 0.215  | 0.415      | 0.550   |

Table 7. Average Certified Radius (ACR) of Models Trained with Different Methods on ImageNet-\(\bar{C}\). Spectrally diverse augmentations from FourierMix brings significant gains to certified robustness of the models trained on ImageNet against corruptions from ImageNet-C.

| Augmentation | mACR | Gauss. | Shot | Impulse | Defocus | Glass | Motion | Zoom | Snow | Frost | Fog | Bright | Contrast | Elastic | Pixel | JPEG |
|--------------|------|--------|------|--------|--------|-------|--------|------|------|-------|-----|--------|----------|--------|-------|------|
| Gaussian     | 0.363| 0.448  | 0.448 | 0.421  | 0.380  | 0.346 | 0.338  | 0.357 | 0.394 | 0.347 | 0.187| 0.439  | 0.137    | 0.342  | 0.420 | 0.440 |
| +JSD         | 0.356| 0.441  | 0.442 | 0.417  | 0.369  | 0.338 | 0.326  | 0.345 | 0.392 | 0.347 | 0.181| 0.436  | 0.133    | 0.332  | 0.411 | 0.432 |
| +HCR         | 0.357| 0.442  | 0.442 | 0.419  | 0.369  | 0.337 | 0.328  | 0.346 | 0.394 | 0.348 | 0.182| 0.436  | 0.133    | 0.330  | 0.412 | 0.434 |

Table 8. Average Certified Radius (ACR) of In-distribution (CIFAR10) and OOD (CIFAR10-C) Data with \(\sigma = 0.25\) Using SOTA Certified Defense Methods.

| Method       | clean mACR | Gauss. | Shot | Impulse | Defocus | Glass | Motion | Zoom | Snow | Frost | Fog | Bright | Contrast | Elastic | Pixel | JPEG |
|--------------|------------|--------|------|--------|--------|-------|--------|------|------|-------|-----|--------|----------|--------|-------|------|
| Gaussian     | 0.461      | 0.363  | 0.448 | 0.448  | 0.421  | 0.380 | 0.346  | 0.338 | 0.357 | 0.394 | 0.187| 0.439  | 0.137    | 0.342  | 0.420 | 0.440 |
| MACER        | 0.539      | 0.426  | 0.509 | 0.492  | 0.460  | 0.436 | 0.422  | 0.433 | 0.443 | 0.381 | 0.232| 0.477  | 0.185    | 0.428  | 0.490 | 0.503 |
| SmoothAdv    | 0.519      | 0.411  | 0.483 | 0.485  | 0.471  | 0.448 | 0.426  | 0.423 | 0.425 | 0.418 | 0.361| 0.222  | 0.451    | 0.175  | 0.415 | 0.472 |

Table 9. Failure of Existing Methods: Average Certified Radius (ACR) of CIFAR10-C with \(\sigma = 0.25\) Using Test-Time Adaptation.

| Adaptation   | mACR | Gauss. | Shot | Impulse | Defocus | Glass | Motion | Zoom | Snow | Frost | Fog | Bright | Contrast | Elastic | Pixel | JPEG |
|--------------|------|--------|------|--------|--------|-------|--------|------|------|-------|-----|--------|----------|--------|-------|------|
| Gaussian     | 0.363| 0.448  | 0.448 | 0.421  | 0.380  | 0.346 | 0.338  | 0.357 | 0.394 | 0.347 | 0.187| 0.439  | 0.137    | 0.342  | 0.420 | 0.440 |
| +BN          | 0.356| 0.441  | 0.442 | 0.417  | 0.369  | 0.338 | 0.326  | 0.345 | 0.392 | 0.347 | 0.181| 0.436  | 0.133    | 0.332  | 0.411 | 0.432 |
| +ENT         | 0.357| 0.442  | 0.442 | 0.419  | 0.369  | 0.337 | 0.328  | 0.346 | 0.394 | 0.348 | 0.182| 0.436  | 0.132    | 0.330  | 0.412 | 0.434 |
Figure 10. Amplitude Spectrum $A$ of Different Corruptions in CIFAR-10/100-C with severity 3.

Figure 11. Amplitude Spectrum $A$ of Different Corruptions in CIFAR-10/100-C with severity 3.
Figure 12. Sample Images from CIFAR-10/100-F with $\epsilon = 12$. From top down row-wise, the images are from $\alpha \in \{0.5, 1, 2, 3\}$ and from left to right column-wise, the images are from $f_c \in \{1, 2, ..., 16\}$.
Figure 13. Sample Images from FourierMix Data Augmentation on CIFAR-10. To better highlight the visual patterns of FourierMix, we utilize the highest severity level for $A(\cdot)$ and $P(\cdot)$ in this figure.
Figure 14. Sample Images from FourierMix Data Augmentation on CIFAR-100. To better highlight the visual patterns of FourierMix, we utilize the highest severity level for $A(\cdot)$ and $P(\cdot)$ in this figure.
Figure 15. Sample Images from FourierMix Data Augmentation on ImageNet. To better highlight the visual patterns of FourierMix, we utilize the highest severity level for $A(\cdot)$ and $P(\cdot)$ in this figure.