Towards observational constraints on a negative \((1 + z)^4\) type contribution in the Friedmann equation

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Abstract

We discuss certain issues related to the limitations of density parameters for a “radiation”-like contribution to the Friedmann equation using kinematical or geometrical measurements. We analyse the observational constraint of a negative \((1 + z)^4\)-type contribution in cosmological models. We argue that it is not possible to determine the energy densities of individual components of matter scaling like radiation from astronomical observations. We find three different interpretations of the presence of the radiation term: 1) the FRW universe filled with a massless scalar field in a quantum regime (the Casimir effect), 2) the Friedmann-Robertson-Walker (FRW) model in the Randall Sundrum scenario with dark radiation, 3) the cosmological model with global rotation. From supernovae type Ia (SNIa) data, Fanaroff-Riley type IIb (FRIIb) radio galaxy (RG) data, baryon oscillation peak and cosmic microwave background radiation (CMBR) observations we obtain bounds for the negative radiation-like term. A small negative contribution of dark radiation can reconcile the tension in nucleosynthesis and remove also the disagreement between \(H_0\) values obtained from both SNIa and the Wilkinson Microwave Anisotropy Probe (WMAP) satellite data.

1 Introduction

In constraining the negative energy contribution to the Friedmann equation, a variety of astronomical observations were used, such as SNIa data \([1,2]\), FRIIb RG data \([3]\), baryon oscillation peak and CMBR observations. Although we can obtain the bounds on the sum of density parameters for all fluids scaling like radiation, it is impossible to determine the contribution coming from each component. In particular it is impossible to determine the negative energy...
density arising from the Casimir effect or dark radiation and positive energy density arising from radiation because they scale with respect to redshift in the same way in the Friedmann equation. We note that the CMBR and big-bang nucleosynthesis (BBN) offer stringent conditions on this term, which can be regarded as an established upper limit on any individual components of negative energy density, and therefore on the Casimir effect, global rotation, discreteness of space following loop quantum gravity or brane dark radiation. We argue that even if the precise value of the density parameter for fictitious fluid is known from observations, it is still not possible to determine the nature (or, say, the origin) of the global rotation of the Universe using measurements basing on geometrical optics only, i.e. null geodesics $ds^2 = 0$. For example, if we consider the origin of this negative radiation-like term from global rotation effects, then it is not possible to come up with different forms of contribution leading to the same expression of $-\Omega_{r,0}(1+z)^4$ type. There are, in principle, three different interpretations of the presence of the negative radiation term in the $H^2(z)$ relation, as follows.

The Casimir effect is a simple observational consequence of the existence of quantum fluctuation [4]. The Casimir force between conducting plates leads to a repulsive force, like the positive cosmological constant. Moreover, different laboratory experiments were designed to measure the Casimir effect with increased precision and thus strengthen the constraints on corrections to Newtonian gravitational law [5]. For a survey of recently obtained results in Casimir-energy studies see [6]. For our purpose, it is important that the effect of Casimir energy, which scales like radiation, can contribute into the $H^2(z)$ relation — crucial for any kinematical test. It is also interesting that the same type of contribution to the effective energy density can be produced by loop quantum theory effects in semi-classical quantum cosmology [7,8]. These effects give rise to an evolutionary scenario in which the initial singularity is replaced by a bounce. It is worth mentioning that the Casimir-type contributions arising from tachyon condensation are possible [9].

In the brane world scenario, our universe is a submanifold which is embedded in a higher-dimensional spacetime, called ‘bulk space’. In the Randall and Sundrum scenario [10], the Einstein equations restricted to the brane reduce to a generalization of the FRW equation with additional terms. One of these terms, called dark radiation, is of considerable interest because it scales like radiation [11]. This term arises from the non-vanishing electric part of the five dimensional Weyl tensor. Dark radiation strongly effects both BBN and CMBR. Ichiki et al. [12] gave limits on the possible contribution as $-1.23 < \rho_{dr}/\rho_\gamma \leq 0.11$ from BBN and $-0.41 < \rho_{dr}/\rho_\gamma \leq 0.105$ at the 95% confidence level from CMBR measurements. Let us note that a small negative contribution of dark radiation can also reconcile the tension between the observed $^4$He and $D$ abundances [13].
Let us consider Newtonian cosmology following Senovilla et al. [14]. Then we can define, following the authors, a homogeneous Newtonian cosmology with $\rho$ and $p$ having no spatial dependence, i.e. $\rho = \rho(t)$ and $p = p(t)$, and we require that the velocity vector fields depends linearly on the spatial coordinates. Then the equation, which represents shear-free Newtonian cosmologies with expansion and rotation, well known as the Heckmann-Schücking model [15] takes the following form

$$a^2 = \frac{\rho(t_0)}{3a} - \frac{2\omega^2}{3a^2} + C \quad (1)$$

where $\omega$ is the rotation scalar and $C$ is an integration constant. We interpret it in terms of curvature constant although in the Newtonian spacetime the curvature is zero. For our purpose, it is important that the effect of rotation produces a negative term in the Newtonian analogue of the Friedmann equation.

In the Newtonian cosmology in contrast to general relativity rotation can appear in homogenous and isotropic space. In general relativity the effect of rotation are strictly related to non-vanishing shear. The homogeneous universe with non-vanishing shear may expand and rotate relative to local gyroscopes. The relation between the rotation of the universe and the origin of the rotation of galaxies was also investigated [16,17,18,19]. Additionally, the role of rotation of objects in the Universe, their significance and astronomical measurements was recently addressed by [20,21]).

2 Observational constraints on the FRW model parameter with a negative radiation-like term

Usually the fundamental test of a cosmological model is based on the luminosity distance $d_L$ as a function of redshift [22]. For the distant SNIa, one can directly observe their apparent magnitude $m$ and redshift $z$. Because the absolute magnitude $M$ of the supernovae is related to its absolute luminosity $L$, the relation between distance modulus $\mu$, the luminosity distance $d_L$, the observed magnitude $m$ and the absolute magnitude $M$ has the following form

$$\mu \equiv m - M = 5 \log_{10} d_L + 25 = 5 \log_{10} D_L + M \quad (2)$$

where $D_L = H_0 d_L$ and $M = -5 \log_{10} H_0 + 25$. The luminosity distance of a supernova can be obtain as the function of redshift:

$$d_L(z) = (1 + z) \frac{c}{H_0} \frac{1}{\sqrt{|\Omega_{k,0}|}} \mathcal{F} \left( H_0 \sqrt{|\Omega_{k,0}|} \int_{0}^{z} \frac{dz'}{H(z')} \right) \quad (3)$$
\[
\left( \frac{H}{H_0} \right)^2 = \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^4 + \Omega_{r,0}(1+z)^4 + \Omega_{\text{dr},0}(1+z)^4 + \Omega_{\Lambda,0}, \quad (4)
\]

where \( \Omega_{k,0} = -\frac{k}{H_0^2} \) and \( \mathcal{F}(x) \equiv (\sinh(x), x, \sin(x)) \) for \( k < 0, k = 0, k > 0 \), respectively. We assumed \( \Omega_{r,0} = \Omega_{\gamma,0} + \Omega_{\nu,0} = 2.48 h^{-2} \times 10^{-5} + 1.7 h^{-2} \times 10^{-5} \simeq 0.0001 \) [11].

Daly and Djorgovski [23] (see also [24, 25]) proposed the inclusion of radio galaxies in the analysis. To accomplish this, it is useful to use the coordinate distance defined as:

\[
y(z) = \frac{H_0 d_L(z)}{c(1+z)}. \quad (5)
\]

Daly and Djorgovski [3] have compiled a sample comprising the data on \( y(z) \) for 157 SNIa in the Riess et al. [1] Gold dataset and 20 FRIIb radio galaxies. In our data sets we also include 115 SNIa compiled by Astier et al. [2].

The coordinate distance \( y(z) \) does not depend on the value of \( H_0 \) from definition. However, when we compute the coordinate distance from the luminosity distance (or the distance modulus \( \mu \)) the knowledge of the value of \( H_0 \) is required. For each sample we choose the values of \( H_0 \) which were used in original papers. We used the distance modulus presented in Ref. [1, 2] for the calculation of the coordinate distance. For each sample we choose the values of \( H_0 \) appropriate to the data sets. For Riess et al.’s Gold sample we fit the value of \( h = 0.646 \) as the best fitted value and this value is used for calculation of coordinate distance for SNIa belonging to this sample. In turn the value \( h = 0.70 \) was assumed in the calculations of the coordinate distance for SNIa belonging to Astier et al.’s sample, because the distance moduli \( \mu \) presented in Ref. [2, Tab. 8] was calculated with such an arbitrary value of \( h = 0.70 \).

The error of the coordinate distance can be computed as

\[
\sigma^2(y_i) = \left( \frac{10^\frac{\mu}{5}}{c(1+z)10^5} \right)^2 \left( \sigma^2(H_0) + \left( \frac{H_0 \ln 10}{5} \right)^2 \sigma^2(\mu_i) \right) \quad (6)
\]

where \( \sigma_i(\mu_i) \) denotes the statistical error of distance modulus determination (note that for Astier et al.’s sample the intrinsic dispersion was also included) and \( \sigma(H_0) = 0.8 \text{ km/s Mpc} \) denotes error in \( H_0 \) measurements.

Recently Eisenstein et al. have analysed the baryon oscillation peaks (BOP) detected in the Sloan Digital Sky Survey (SDSS) Luminosity Red Galaxies [27]. They found that value of \( A \)

\[
A \equiv \frac{\sqrt{\Omega_{m,0}}}{E(z_1)} \left( \frac{1}{z_1 \sqrt{|\Omega_{k,0}|}} \mathcal{F} \left( \sqrt{|\Omega_{k,0}|} \int_0^{z_1} \frac{dz}{E(z)} \right) \right)^\frac{2}{3} \quad (7)
\]
(where \( E(z) \equiv H(z)/H_0 \) and \( z_1 = 0.35 \) is equal \( A = 0.469 \pm 0.017 \). The quoted uncertainty corresponds to one standard deviation, where a Gaussian probability distribution has been assumed.

It is also possible to include in our analysis the so-called the (CMBR) “shift parameter”

\[
R \equiv \sqrt{\Omega_{m,0}} y(z_{lss}) = \sqrt{\frac{\Omega_{m,0}}{|\Omega_{k,0}|}} \mathcal{F} \left( \sqrt{\frac{\Omega_{k,0}}{|\Omega_{m,0}|}} \int_0^{z_{lss}} \frac{dz}{E(z)} \right)
\]

where \( R_0 = 1.716 \pm 0.062 \) [28].

In our combined analysis, we can obtain a best fit model by minimizing the pseudo-\( \chi^2 \) merit function [29]:

\[
\chi^2 = \chi^2_{SN+RG} + \chi^2_{SDSS} + \chi^2_{CMBR} = 
\sum_i \left( \frac{y_i^{\text{obs}} - y_i^{\text{th}}}{\sigma_i(y_i)} \right)^2 + \left( \frac{A^{\text{mod}} - 0.469}{0.017} \right)^2 + \left( \frac{R^{\text{mod}} - 1.716}{0.062} \right)^2,
\]

where \( A^{\text{mod}} \) and \( R^{\text{mod}} \) denotes value of \( A \) and \( R \) obtained for particular set of the model parameter. For Astier et al. SNIa [2] sample additional error in \( z \) measurements were taken into account. Here \( \sigma_i(y_i) \) denotes the statistical error (including error in \( z \) measurements) of the coordinate distance determination.

In order to constrain the cosmological parameters, one can minimize the following likelihood function \( \mathcal{L} \propto \exp(-\chi^2/2) \). However, to constrain a particular model parameter, the likelihood function marginalized over the remaining parameters of the model should be considered [29]. Our results are presented in Table 1, Table 2 and Fig. 1. Table 1 refers to the minimum \( \chi^2 \) method, whereas Table 2 shows the results from the marginalized likelihood analysis.

We obtain as the best fit a flat universe with \( \Omega_{m,0} = 0.3, \Omega_{\Lambda,0} = 0 \) and \( \Omega_{dr,0} = 0.7 \). For the dark radiation term, we obtain the stringent bound \( \Omega_{dr,0} > -0.00035 \) at the 95% confidence level (\( -0.00026 \) at the 68.3% confidence level).

Please note that if \( \Omega_{r,0} + \Omega_{dr,0} < 0 \), then we obtain a bouncing scenario [30,31,26] instead of a big bang. For \( \Omega_{m,0} = 0.3, \Omega_{dr,0} = -0.00035 \) and \( h = 0.65 \) bounces (\( H^2 = 0 \)) appear for \( z \simeq 1200 \). In this case, the BBN epoch never occurs and all BBN predictions would be lost.

To obtain stronger constraints on the model parameters, it is useful to use the CMBR observations. The hotter and colder spots in the CMBR can be interpreted as acoustic oscillations in the primeval plasma during the last scattering. It is very interesting that the locations of these peaks are very sensitive to variations in the model parameters. Therefore, they can be used
as another way of constraining the parameters of cosmological models. The acoustic scale $\ell_A$ which gives the locations of the peaks is defined as

$$\ell_A = \pi \frac{\int_0^{z_{\text{dec}}} \frac{dz'}{H(z')}}{\int_{z_{\text{dec}}}^{\infty} c_s \frac{dz'}{H(z')}}$$

where, for the flat model, equation (4) reduces to

$$H(z) = H_0 \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{\Lambda,0}}.$$  

where $c_s$ is the speed of sound in plasma. Knowing the acoustic scale we can determine the location of the $m$-th peak $\ell_m \sim \ell_A (m - \phi_m)$ where $\phi_m$ is the phase shift caused by the plasma driving effect. The CMBR temperature angular power spectrum provides the locations of the first two peaks $\ell_1 \sim 220.1^{+0.8}_{-0.8}$, $\ell_2 \sim 546^{+10}_{-10}$ [32]. Using three years of WMAP data, Spergel et al. obtained that the Hubble constant $H_0 = 73$ km/s Mpc, the baryonic matter density $\Omega_{b,0} = 0.0222h^{-2}$, and the matter density $\Omega_{m,0} = 0.128h^{-2}$ [33], which are in good agreement with the observation of position of the first peak (see Fig. 2) but lead (assuming the $\Lambda$CDM model) to a value $\Omega_{m,0} = 0.24$. There is also disagreement between $H_0$ values obtained from SNIa and WMAP. We compute the location of the first peak as a function of $\Omega_{\text{dr,0}}$ assuming $H_0 = 65$ km/s Mpc ($\Omega_{m,0} = 0.3$). Now we obtain agreement with the observation of the location of the first peak for non-zero values of the parameter $\Omega_{\text{dr,0}}$ (Fig. 2). We obtain $-1.05 \times 10^{-5} < \Omega_{\text{dr,0}} < -0.5 \times 10^{-5}$ at the 95% confidence level. Please note that our limit is stronger than that obtained by Ichiki et al. [12], which provides bounds of $-7.22 \times 10^{-5} < \Omega_{\text{dr,0}} \leq 0.65 \times 10^{-5}$ (in the case of the BBN) and $-2.41 \times 10^{-5} < \Omega_{\text{dr,0}} \leq 0.62 \times 10^{-5}$ (in the case of the CMBR).

All these values of $\Omega_{\text{dr,0}}$ are in agreement with the result obtained from the combined analysis because the $2\sigma$ confidence interval for this parameter obtained from the SNIa data contains the area allowed from the CMBR. While the combined analysis gives the possibility that $\Omega_{\text{dr,0}}$ is equal to zero, the CMBR location of the first peak seems to exclude this case for $h = 0.65$.

## 3 Conclusion

In our paper we analysed the observational constraints on the negative $(1+z)^4$-type contribution in the Friedmann equation. We discussed different proposals for the presence of such a dark radiation term. Although it is not possible, with present kinematic observations, to determine the energy densities of individual components which scales like radiation, we show that some stringent bounds on the value of this total contribution can be given. Our detailed conclusions are the following.

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Fig. 1. The 68.3% and 95.4% confidence levels (obtained from the combined analysis of SN+RG+SDSS+CMBR) on the plane ($\Omega_{m,0}, \Omega_{\Lambda,0}$).

Table 1
Results of the statistical analysis of the model with the negative radiation term obtained from $\chi^2$ best fit. The upper section of the table represents the constraint $\Omega_{k,0} = 0$ (flat model).

| Sample          | $\Omega_{k,0}$ | $\Omega_{m,0}$ | $\Omega_{\text{dr},0}$ | $\Omega_{\Lambda,0}$ | $\chi^2$ |
|-----------------|----------------|----------------|-------------------------|------------------------|----------|
| SN              | -              | 0.31           | 0.000                   | 0.690                  | 297.5    |
| SN+RG           | -              | 0.32           | -0.001                  | 0.681                  | 320.4    |
| SN+RG+SDSS      | -              | 0.30           | 0.000                   | 0.700                  | 322.4    |
| SN+RG+SDSS+CMBR | -              | 0.30           | 0.000                   | 0.700                  | 322.5    |
| SN              | -0.28          | 0.43           | 0.000                   | 0.850                  | 296.0    |
| SN+RG           | -0.20          | 0.40           | -0.001                  | 0.801                  | 319.5    |
| SN+RG+SDSS      | 0.06           | 0.29           | 0.000                   | 0.650                  | 321.1    |
| SN+RG+SDSS+CMBR | 0.00           | 0.30           | 0.000                   | 0.700                  | 322.5    |
Fig. 2. The location of the first peak as a function of $-\Omega_{dr,0}$. Note that $l_1 \simeq 220$ for $h = 0.73$ favours $\Omega_{dr,0} \simeq 0$, while for $h = 0.65$ lead to $\Omega_{dr,0} \neq 0$.

Table 2
Results of the statistical analysis of the model with the negative radiation term. The values of the model parameters are obtained from the marginalized likelihood analysis. We present the maximum likelihood values with 68.3% confidence ranges. The upper section of the table represents the constraint $\Omega_{k,0} = 0$ (flat model).

| Sample                   | $\Omega_{k,0}$ | $\Omega_{m,0}$      | $\Omega_{dr,0}$     | $\Omega_{\Lambda,0}$ |
|--------------------------|-----------------|----------------------|----------------------|-----------------------|
| SN                       | -               | $0.34^{+0.10}_{-0.03}$ | $0.000^{−0.066}_{−0.00}$ | $0.66^{+0.02}_{−0.04}$ |
| SN+RG                    | -               | $0.35^{+0.10}_{−0.04}$ | $0.000^{−0.070}_{−0.00}$ | $0.65^{+0.02}_{−0.04}$ |
| SN+RG+SDSS               | -               | $0.31^{+0.02}_{−0.02}$ | $0.000^{−0.007}_{−0.00}$ | $0.69^{+0.01}_{−0.01}$ |
| SN+RG+SDSS+CMBR          | -               | $0.30^{+0.02}_{−0.01}$ | $0.00000^{−0.00022}_{−0.00}$ | $0.70^{+0.01}_{−0.02}$ |
| SN                       | $-0.44^{+0.24}_{−0.22}$ | $0.63^{+0.19}_{−0.15}$ | $0.000^{−0.130}_{−0.00}$ | $0.87^{+0.13}_{−0.13}$ |
| SN+RG                    | $-0.36^{+0.22}_{−0.22}$ | $0.59^{+0.20}_{−0.14}$ | $0.000^{−0.131}_{−0.00}$ | $0.82^{+0.12}_{−0.13}$ |
| SN+RG+SDSS               | $0.11^{+0.08}_{−0.07}$ | $0.29^{+0.02}_{−0.02}$ | $0.000^{−0.023}_{−0.00}$ | $0.61^{+0.05}_{−0.06}$ |
| SN+RG+SDSS+CMBR          | $0.00^{+0.02}_{−0.03}$ | $0.30^{+0.02}_{−0.01}$ | $0.00000^{−0.00026}_{−0.00}$ | $0.70^{+0.02}_{−0.02}$ |

(1) The combined analysis of SNIa data and FRIIb radio galaxies using baryon oscillation peaks and CMBR “shift parameter” give rise to the almost flat universe with $\Omega_{m,0} \simeq 0.3$.

(2) From the above-mentioned combined analysis, we obtain an upper bound $-\Omega_{dr,0} < 0.00035$ at the 95% confidence level. This is a stronger limit than obtained previously by us from SNIa data only [21].
(3) We find new stringent limits on a negative component scaling like radiation from the location of the peak in the CMBR power spectrum, $-1.05 \times 10^{-5} < \Omega_{\text{dr},0} < -0.5 \times 10^{-5}$ at the 95% confidence level. This bound is stronger than that obtained from the BBN and CMBR by Ichiki et al. [12].

(4) From the limit $-\Omega_{\text{dr},0} < 1.05 \times 10^{-5}$ we obtain that $\Omega_{\text{dr},0} + \Omega_{\text{r},0} > 0$. This implies that $H^2(z)$ is always greater than zero ($H^2(z) > 0$) and the bounce does not appear which means that the big bang scenario is strongly favoured over the bounce scenario.

(5) The discussed model with a small contribution of dark radiation type can also resolve the disagreement between values of $H_0$ obtained from SNIa and WMAP.

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