Non-singlet QCD analysis in the NNLO approximation

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\textbf{Abstract.} A non-singlet QCD analysis of the structure function up to NNLO is performed based on the orthogonal polynomials approach. The results of valence quark distributions up to NNLO are in good agreement with the available theoretical models.

1. Introduction
The global parton analysis of deep inelastic scattering (DIS) and the related hard scattering data are generally performed at next-to leading order (NLO). Presently the next-to leading order is the standard approximation for most of the important processes in QCD. Analyzing DIS at next-to-next-to-leading order (NNLO) is important as we may be able to investigate the hierarchy LO \(\rightarrow\) NLO \(\rightarrow\) NNLO for the processes using the most precise available data.

The NNLO corrections should be included in order to arrive at quantitatively reliable predictions for hard processes occurring at present and future high-energy colliders.

Recently much effort has been invested in computing NNLO QCD corrections to a wide variety of partonic processes and therefore it is needed to generate parton distributions also at NNLO, so that the theory can be applied in a consistent manner.

During the recent years the interest to use CCFR data \cite{1} for \(x F_3\) structure function in the higher orders, based on the orthogonal polynomial expansion method has increased \cite{2, 3, 4, 5}.

In this paper we determine the flavor non-singlet parton distribution functions, \(x u_v(x, Q^2)\) and \(x d_v(x, Q^2)\), using the Jacobi polynomial approach up to the NNLO level. This calculation is possible now, as the non-singlet anomalous dimension coefficients in \(N\)-Moment space in three loops has already been introduced \cite{6}.

2. QCD Formalism
In this work we choose the following parameterization for the valence quark densities

\[ x v_i(x, Q^2) = A_i x^{\alpha_i} (1-x)^{\beta_i} (1 + \rho_i \sqrt{x} + \gamma_i x) , \]

with \(v_i = u_v, d_v, i = u, d\). Three valence–projections are considered.
The non–singlet (NS) parts of the structure functions \( F_{2}^{p,d}(x, Q^{2}) \) for \( x > 0.3 \) are related to valence quarks combinations. Since in this region valence quark is dominate, the structure functions \( F_{2}^{p,d}(x, Q^{2}) \) at LO given by

\[
\begin{align*}
F_{2}^{p}(x, Q^{2}) &= \frac{5}{18} [x(u - \bar{u})(x, Q^{2}) + x(d - \bar{d})(x, Q^{2})] + \frac{1}{6} [x(u - \bar{u})(x, Q^{2}) - x(d - \bar{d})(x, Q^{2})] \\
&= \frac{4}{9} xu_{v}(x, Q^{2}) + \frac{4}{9} xd_{v}(x, Q^{2}) ,
\end{align*}
\]

\[
\begin{align*}
F_{2}^{d}(x, Q^{2}) &= \frac{5}{18} [x(u - \bar{u})(x, Q^{2}) + x(d - \bar{d})(x, Q^{2})] \\
&= \frac{5}{18} x(u_{v} + d_{v})(x, Q^{2}) .
\end{align*}
\]

In the region \( x \leq 0.3 \) for the difference of the proton and deuteron data we use

\[
F_{2}^{NS}(x, Q^{2}) = 2[F_{2}^{p}(x, Q^{2}) - F_{2}^{d}(x, Q^{2})] = \frac{1}{3} [x(u + \bar{u})(x, Q^{2}) - x(d + \bar{d})(x, Q^{2})] = \frac{1}{3} x(u_{v} - d_{v})(x, Q^{2}) + \frac{2}{3} x(\bar{u} - \bar{d})(x, Q^{2}) ,
\]

here sea quarks cannot be neglected for \( x \) smaller than about 0.3. We use the \( x(\bar{d} - \bar{u}) \) distribution at \( Q_{0}^{2} = 4 \text{ GeV}^{2} \) as applied in Ref. [7, 8], but plays a marginal role in our analysis.

The evolution equations are solved in Mellin–N space and the Mellin transforms of the above distributions are denoted by \( F_{2}^{NS}(N, Q^{2}), F_{2}^{p,d}(N, Q^{2}) \), respectively. The non–singlet structure functions are given by

\[
F_{2}^{k}(N, Q^{2}) = \left[ 1 + a_{s}(Q^{2})C_{1}(N) + a_{s}^{2}(Q^{2})C_{2}(N) \right] F_{2}^{k}(N, Q^{2}) ,
\]

for the three cases above. Here \( a_{s}(Q^{2}) = a_{s}(Q^{2})/(4\pi) \) denotes the strong coupling constant and \( C_{1}(N)(Q^{2}) \) are the non–singlet Wilson coefficients. The solution of the non–singlet evolution equation for the parton densities to 3–loop order reads

\[
\begin{align*}
F_{2}^{k}(N, Q^{2}) &= F_{2}^{k}(N, Q_{0}^{2}) \left( \frac{a}{a_{0}} \right)^{-\beta_{0}(N)/\beta_{0}} \left\{ 1 - \frac{1}{\beta_{0}} (a - a_{0}) \left[ \hat{P}_{1}^{+}(N) - \frac{\beta_{1}}{\beta_{0}} \hat{P}_{0}^{+} \right] \\
&- \frac{1}{2\beta_{0}} (a^{2} - a_{0}^{2}) \left[ \hat{P}_{2}^{+}(N) - \frac{\beta_{2}}{\beta_{0}} \hat{P}_{1}^{+} + \left( \frac{\beta_{1}^{2}}{\beta_{0}^{2}} - \frac{\beta_{2}}{\beta_{0}} \right) \hat{P}_{0}^{+}(N) \right] \\
&+ \frac{1}{2\beta_{0}^{2}} (a - a_{0})^{2} \left[ \hat{P}_{1}^{+}(N) - \frac{\beta_{1}}{\beta_{0}} \hat{P}_{0}^{+} \right] \right\} ,
\end{align*}
\]

Here, \( \hat{P}_{k} \) denote the Mellin transforms of the \( (k + 1)–\)loop splitting functions.

3. The method of the QCD analysis of SF

One of the simplest and fastest possibilities in the PSF reconstruction from the QCD predictions for its Mellin moments is Jacobi polynomials expansion. The Jacobi polynomials are especially suited for this purpose since they allow one to factor out an essential part of the \( x \)-dependence of the SF into the weight function [9]. According to Ref. [10] we have a relation between the structure function and its Mellin moments

\[
\begin{align*}
\mathcal{F}_{2}^{k}(x, Q^{2}) &= x^{\beta}(1 - x)^{\alpha} \sum_{n=0}^{N_{\text{max}}} \Theta_{n}^{\alpha,\beta}(x) \sum_{j=0}^{n} c_{j}^{(n)}(\alpha, \beta) \mathcal{F}_{2}^{k}(j + 2, Q^{2}) ,
\end{align*}
\]
where $F^{j+2}_{2}(J, Q^2)$ are the moments determined by Eqs. (5,6). $N_{\text{max}}$, $\alpha$ and $\beta$ have to be chosen so as to achieve the fastest convergence of the series on the R.H.S. of Eq. (7) and to reconstruct $F^{j+2}_{2}$ with the required accuracy. In our analysis we use $N_{\text{max}} = 9$, $\alpha = 3.0$ and $\beta = 0.5$. The same method has been applied to calculate the polarized structure function $xg_1$ from their moments [10]. Obviously the $Q^2$-dependence of the structure function is defined by the $Q^2$-dependence of the moments.

Now we can use the structure function data measured in charged lepton proton and deuteron deep inelastic scattering. We used all of the world data for $F^{p,d,NS}_{2}$ as discussed in Ref. [8] and references there in. Using the CERN subroutine MINUIT, we defined a global $\chi^2$ for all the experimental data points in the NNLO case. Fig.1 illustrates the evolution of the valence quark densities $xu_v(x, Q^2)$ and $xd_v(x, Q^2)$ for $Q^2 = 1 \text{ GeV}^2$ and $Q^2 = 10 \text{ GeV}^2$ at NNLO.

**Figure 1.** The parton distribution $xu_v$ and $xd_v$ at some different values of $Q^2$. The solid (dotted) line is our model for Jacobi approach (Bernstein approach [5]), the dashed line is the MRST model [11], dashed-dotted line is the A05 model [12], dashed-dotted-dotted line is the BBG model [8].

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