Universal interferometric signatures of a black hole’s photon ring

Michael D. Johnson1,2,*, Alexandru Lupsasca2,3,4,*, Andrew Strominger2,3,*, George N. Wong5,6, Shahar Hadar2,3, Daniel Kapeč7, Ramesh Narayan1,2, Andrew Chael1,2,8,9, Charles F. Gammie5,10, Peter Galison2,3,11, Daniel C. M. Palumbo1,2, Sheperd S. Doeleman1,2, Lindy Blackburn1,2,12, Joseph R. Farah1,2,12, James M. Moran1,2

The Event Horizon Telescope image of the supermassive black hole in the galaxy M87 is dominated by a bright, unresolved ring. General relativity predicts that embedded within this image lies a thin “photon ring,” which is composed of an infinite sequence of self-similar subrings that are indexed by the number of photon orbits around the black hole. The subrings approach the edge of the black hole “shadow,” becoming exponentially narrower but weaker with increasing orbit number, with seemingly negligible contributions from high-order subrings. Here, we show that these subrings produce strong and universal signatures on long interferometric baselines. These signatures offer the possibility of precise measurements of black hole mass and spin, as well as tests of general relativity, using only a sparse interferometric array.

INTRODUCTION

The Event Horizon Telescope (EHT) Collaboration has recently published images of the supermassive black hole in M87 using very long baseline interferometry (VLBI) at 1.3-mm wavelength (1–6). These images reveal a bright ring of emission with a diameter of approximately 40 μas. However, while the diameter of this ring is resolved by the EHT, its thickness and detailed substructure are not. Here, we show that general relativity predicts an intricate substructure in the ring that presents distinctive signatures for interferometric measurements. These signatures offer a promising approach for precisely determining the mass and spin of black holes and for testing general relativity using sparse interferometers, such as an extension of the EHT to space.

Neglecting opacity, a telescope with perfect resolution directed at a black hole observes an infinite number of nested images of the universe. These images arise from photons that differ by the number \( n \) of half-orbits that they complete around the black hole on the way from their source to the detector. Each such image is thus an increasingly delayed and demagnified snapshot of the universe as seen from the black hole. In an astrophysical setting, this self-similar sequence of relativistic images is dominated by the luminous matter surrounding the black hole and produces in its image a feature known as the “photon ring” of the black hole (7–10). The leading \((n = 1)\) subring appears as a sharp, bright feature in ray-traced images from many general relativistic magnetohydrodynamic (GRMHD) simulations (see Fig. 1). Successive subrings have exponentially sharper profiles and asymptotically approach the boundary of the black hole “shadow.” For large \( n \), these profiles mirror the leading subring in a manner that universally depends on the spacetime geometry, with the ratio of successive subring flux densities determined by Lyapunov exponents that characterize the instability of bound photon orbits. Hence, measuring the size, shape, and thickness of the subrings would provide new and powerful probes of a black hole spacetime.

Both GRMHD simulations and analytic estimates suggest that the photon ring should provide only ~10% of the total image flux density. This dimness may appear to preclude observations of the photon ring and its substructure, which is dimmer still. However, interferometric measurements are sensitive to more than just overall flux: They also natively filter images by their spatial wave numbers and therefore naturally isolate contributions from individual photon subrings. Sufficiently long baselines also resolve out diffuse flux in an image and are thus dominated by power from the photon ring. Hence, although sharp elements of the photon ring produce a negligible contribution to the total flux in an image, they can still provide a pronounced, dominant signal on long baselines.

In this Research Article, we explore the photon ring’s theoretical underpinnings and show that, unexpectedly, precise measurements of the photon ring and even its subrings are feasible using interferometry. We describe the shell of bound photon orbits of a Kerr black hole and its relation to the photon ring. We also present a decomposition of the photon ring into subrings indexed by half-orbit number and derive their self-similar structure, which is universally governed by Lyapunov exponents that characterize orbital instability.

We then derive generically expected interferometric signatures of the photon ring. We show that its subrings produce a cascade of damped oscillations on progressively longer baselines, with the visibility of each subring conveying precise information about its diameter, width, and angular profile. Last, we discuss observational prospects for detecting these signatures with extensions of the EHT. In particular, we highlight the possibility of detecting the leading \( n = 1 \) subring using a station in low Earth orbit, the \( n = 2 \) subring using a station on the Moon, and the \( n = 3 \) subring using a station in the Sun-Earth L2 orbit.
and angular momentum $M$ of a black hole of mass $M$ using Boyer-Lindquist coordinates, in which the metric of a Kerr black hole fattens to a three-dimensional spherical shell. It is best described neither escape to infinity nor fall across the event horizon. For spacetime containing bound null geodesics or "bound orbits" that the orbits reside entirely in the equatorial plane. At generic points, on the other hand, they oscillate in the $\theta$ direction between polar angles $\theta_{\pm}$ determined by equatorial annuli. Their intersection with the equatorial plane serves as the photon shell boundary. These coordinates have the special property that each bound orbit lies at some fixed value of $r$ in the range $r_{\pm} \leq r \leq r_{\pm}^1$.

$$r_{\pm} = 2M \left[ 1 + \cos \left( \frac{2}{3} \arccos \left( \pm \frac{a}{M} \right) \right) \right]$$

(2B)

Every point in the equatorial annulus $r_{\pm}^1 \leq r \leq r_{\pm}$, $\theta = \pi/2$, has a unique bound orbit passing through it. On the boundaries $r = r_{\pm}$, the orbits reside entirely in the equatorial plane. At generic points, on the other hand, they oscillate in the $\theta$ direction between polar angles $\theta_{\pm} = \arccos \left( \mp \sqrt{u_{\pm}} \right)$.

$$\theta_{\pm} = \arccos \left( \mp \sqrt{u_{\pm}} \right)$$

(3)

where

$$u_{\pm} = a^2 (r - M)^2 \left[ -r^3 + 3M^2 r - 2a^2 M \pm 2 \sqrt{M^2 \left( 2r^3 - 3Mr^3 + a^2 M \right) - \Delta} \right]$$

(4)

We will refer to one such complete oscillation (e. g., from $\theta_{-}$ back to itself) as one orbit, since the photon typically returns to a point near, but not identical to (since the azimuthal angle $\phi$ also shifts), its initial position.
To summarize, the photon shell is the spacetime region
\[ r^2 \leq r \leq r^2_{\gamma}, \quad 0 \leq \theta \leq \theta_{\gamma}, \quad 0 \leq \phi < 2\pi \] (5)
depicted in Fig. 2) for all times \(-\infty < t \leq \infty\).

The bound orbit at radius \( r \) has the energy-rescaled angular momentum
\[ \ell = \frac{M(r^2 - a^2) - r\Delta}{a(r - M)} \] (6)

The inner circular equatorial orbit at \( r^2_{\gamma} \) is prograde, while the outer one at \( r^2_{\gamma} \) is retrograde: \( \ell(r^2_{\gamma}) \approx 0 \). The overall direction of the orbits reverses at the intermediate value \( r^2_0 \) for which \( \ell \) vanishes. At that radius, \( \{\theta_{\gamma}, \theta_{\gamma}\} \) equals \([0, \pi],\) and the orbits can pass over the poles.

The bound geodesics are unstable in the sense that, if perturbed slightly, they either fall into the black hole or escape to infinity where they can reach a telescope. The observed photon ring image arises from photons traveling on such “nearly bound” geodesics. Consider two geodesics, one of which is bound, with the other initially differing only by an infinitesimal radial separation \( \delta r \). The equation of geodesic deviation shows that, after \( n \) half-orbits between \( \theta_{\gamma} \), their separation grows to
\[ \delta r_n = e^{\gamma n} \delta r_0 \] (7)

Here, the so-called Lyapunov exponent \( \gamma \) is a function on the space of bound orbits given by (see the Supplementary Materials)
\[ \gamma = \frac{4}{a} \sqrt{\frac{2M\Delta}{(r - M)^2}} \int_0^1 \frac{dr}{(1 - \lambda^2)(u_+r^2 - u_-)} \] (8)

A closely related formula appears in (14). Hence, the nearby bound geodesic will typically cross the equatorial plane a number of times of order
\[ n \approx \frac{1}{\gamma} \ln \frac{\delta r_n}{\delta r_0} \] (9)

until \( \delta r_n \gg \delta r_0 \), when the geodesic is well separated from the bound orbit and it shoots off to infinity (or crosses the event horizon if \( \delta r_0 < 0 \)). These Lyapunov exponents are central and potentially observable quantities that characterize the geometry of the Kerr photon shell.

**Photon ring and subrings**

The photon ring is the image on the observer screen produced by photons on nearly bound geodesics (7). In the limit in which the photons become fully bound, it may be shown that their images approach a closed curve \( C_{\gamma} \) given by
\[ \rho = D^{-1} \sqrt{a^2 \cos^2 \theta_{\obs} - u_+ u_- + \ell^2} \] (10A)
\[ \varphi_{\rho} = \arccos \left( \frac{\ell}{\rho D \sin \theta_{\obs}} \right) \] (10B)

where \((\rho, \varphi_{\rho})\) are dimensionless polar coordinates on the observer screen, while \((D, \theta_{\obs})\) denote the observer’s distance and inclination from the Kerr spin axis, respectively. We can view \( C_{\gamma} \) as parameterized by the shell radius \( r^2_{\gamma} \leq r \leq r^2_{\gamma} \), from which the photon originated. For each value of \( r \), Eq. 10B has two solutions for \( \varphi_{\rho} \) in the range \( 0 \leq \varphi_{\rho} \leq 2\pi \), so each radius in the photon shell appears at two positions on \( C_{\gamma} \). A notable consequence of Eqs. 10A and 10B is that for \( \theta_{\obs} \neq 0 \), both \( \ell \) and \( \rho \), and hence \( \varphi_{\rho} \), are functions only of \( r, \theta_{\obs}, \) and \( D \). Hence, a measurement at a specific angle \( \varphi_{\rho} \) along the ring probes a specific radius \( r \) of the Kerr geometry and not, as might have been expected, a specific angle around the black hole!

Astrophysically observed photon intensities \( I_{\text{ring}}(\rho, \varphi_{\rho}) \) at the screen can be computed by backward ray tracing. One follows the null geodesics from the observer screen back into the Kerr spacetime, integrating the Doppler-shifted strength \( J \) of matter sources along the geodesic, with attenuation factors accounting for the optical depth. Scattering effects are negligible because the expected plasma frequency and electron gyroradius are in the megahertz range, several orders of magnitude below the observing frequencies that we consider. For the images in this paper, we used ipole (15). A light ray aimed exactly at the curve \( C_{\gamma} \) is captured by the photon shell and (unstably) orbits the black hole forever. Those aimed inside \( C_{\gamma} \) fall into the black hole, while those aimed outside escape to infinity. Therefore, \( C_{\gamma} \) is the edge of the black hole shadow.

If we shoot a light ray very near, a distance \( \delta \rho \) from the shadow edge at \( \rho_0 \), it will circle many times through the emission region before falling into the black hole or escaping to infinity. The affine length of the ray and its number of half-orbits accordingly diverge as \( \delta \rho \to 0 \)
\[ n \approx -\frac{1}{\gamma} \ln \frac{\delta \rho}{\delta \rho_0} \] (11)

This follows from Eq. 9 together with a computed relation between \( \delta \rho \) and \( \delta \rho_0 \). For optically thin matter distributions, Eq. 11 implies a mild divergence in the observed ring intensity \( I_{\text{ring}} \sim n \) as the shadow edge is approached, since a light ray that completes \( n \) half-orbits through the emission region can collect \( n \sim \text{times more photons along its path.} \) The photon ring is then the bump in the photon intensity containing this logarithmic divergence at the shadow edge. Although the divergence is cut off by a finite optical depth, this notable feature remains visually prominent in many ray-traced images of GRMHD simulations, as in Fig. 1.

The photon ring can be subdivided into subrings arising from photons that have completed \( n \) half-orbits between their source and the screen. This definition for the photon ring agrees with that in (16) but differs from the later usage in (9) and (10) by the inclusion of the \( n = 1 \) and \( 2 \) contributions. These low \( n \) contributions fully account for the thin ring image visible in Fig. 1. To orbit at least \( n/2 \) times around the black hole, the photon must be aimed within an exponentially narrowing window
\[ \frac{\delta \rho_\rho}{\delta \rho} \approx e^{-\gamma n} \] (12)
around the shadow edge. Hence, the subrings occupy a sequence of exponentially nested intervals centered around \( C_{\gamma} \).

Each subring consists of photons lensed toward the observer screen after having been collected by the photon shell from anywhere in the universe. Hence, in an idealized setting with no absorption, each subring contains a separate, exponentially demagnified image of the entire universe, with each subsequent subring capturing the visible universe at an earlier time. Together, the set of subrings are akin to the frames of a movie, capturing the history of the visible universe as seen from the black hole. In an astrophysical setting, these images are dominated by the luminous matter around the black hole. For a black hole surrounded by a uniform distribution extending over the poles, the contributions made by each subring to the total intensity profile cannot be told apart, and the individual subrings cannot be
distinguished on the image. However, for a realistic disk or jet with emission peaked in a conical region, the subrings are visibly distinct: the nth subring is approximately a smooth peak of width $e^{-\gamma}$. Summing these smooth peaks, like layers in a tiered wedding cake (see Fig. 3), reproduces the leading logarithmic divergence in the intensity (Eq. 11).

The photons comprising successive subrings for the same angle $\varphi_p$ traverse essentially the same orbits and hence encounter the same matter distribution around the black hole. Apart from source variations on the time scale of an orbit, intensities of the nth and $(n+1)$th subring differ only because they correspond to windows whose widths $\delta \varphi_n$ and $\delta \varphi_{n+1}$ differ by a factor of $e^{-\gamma}$. Hence, for large enough $n$, the intensities are related by

$$I_{n+1}^{\text{ring}}(\rho_c + \delta \rho, \varphi_p) \approx I_n^{\text{ring}}(\rho_c + e^{\gamma} \delta \rho, \varphi_p)$$

We therefore find the angle-dependent subring flux ratio

$$\frac{I_{n+1}^{\text{ring}}}{I_n^{\text{ring}}} \approx e^{-\gamma}$$

Equations 13 and 14 are matter-independent predictions for the photon ring structure that involve only general relativity. The prediction holds only for “large enough” $n$: At small $n$, there are non-universal matter-dependent effects from photons that do not traverse exactly the same region around the black hole. Insight into when $n$ is large enough might be obtained from GRMHD simulations.

Since the exponent $\gamma$ depends on $a$, $\theta_{\text{obs}}$, and $\varphi_p$ (see the Supplementary Materials), the flux ratio asymmetry in Eq. 14 provides a new method for determination of the spin. For Schwarzschild, $\gamma = \pi$ (8), corresponding to a demagnification factor of $e^{-\pi} \approx 4\%$. For a black hole of maximal spin $a = M = 1$ viewed from an inclination $\theta_{\text{obs}} = 17^\circ$ [as estimated for M87; (17)], the factor $e^{-\gamma}$ is as large as 13% on the part of the ring where the black hole spins toward the observer. Although Eq. 12 breaks down for $n = 0$, this suppression factor suggests that the leading $n = 1$ subring should provide $\sim 10\%$ of the total luminosity, in order-of-magnitude agreement with GRMHD simulations.

**Interferometric signatures of a photon ring**

This section explores the response of an interferometer to the photon ring through a series of increasingly refined geometrical models. We first review the interferometric signatures of an infinitesimally thin, uniform, and circular ring. We then extend this treatment to include rings with nonuniform brightness, nonzero thickness, and noncircular structure. We conclude this section by discussing specific features expected for the photon ring and its subrings.

**Visibilities for a thin, uniform, and circular ring**

Each baseline joining two elements of an interferometer samples a complex visibility $V(u)$, which corresponds to a single Fourier component of the sky image $I(x)$ (18)

$$V(u) = \int I(x) e^{-2\pi i u \cdot x} d^2 x$$

Here, $u$ is the dimensionless vector baseline projected orthogonal to the line of sight and measured in units of the observation wavelength $\lambda$, while $x$ is a dimensionless image coordinate measured in radians.

In terms of polar coordinates $(\rho, \varphi_p)$ on the observer screen (Eqs. 10A and 10B), the image and corresponding visibility function of an infinitesimally thin, uniform, and circular ring are

$$I(\rho, \varphi_p) = \frac{1}{\pi d} \delta \left( \rho - \frac{d}{2} \right)$$

$$V(u, \varphi_u) = I_0(\pi d u)$$

where $d$ is the ring diameter in radians and the image is normalized to have a total flux density of unity, $V(0) = 1$. $I_m$ denotes the $m$th Bessel function of the first kind, which admits the asymptotic expansion

$$J_m(\pi d u) \approx \frac{1}{\pi} \sqrt{\frac{2}{\pi d u}} \cos \left( \pi \left( du - \frac{2m + 1}{4} \right) \right)$$

valid for $\pi d u \gg m^2$. Hence, $V(u)$ is a weakly damped pure frequency with period $\Delta u = 2/d$ inside an envelope that falls as $1/\sqrt{u}$.
Visibilities for a nonuniform ring

The image of a thin ring with nonuniform brightness in $\varphi_\rho$ decomposes into a sum over angular Fourier modes

$$I(\varphi_\rho) = \frac{1}{\pi d} \delta(\rho - \frac{d}{2}) \sum_{m=-\infty}^{\infty} \beta_m e^{im\varphi_\rho},$$

(18)

where $\beta_m = \beta^*_m$ since the image is real. The total image flux density is given by $\beta_0 > 0$.

The corresponding visibility function is

$$V(u, \varphi_u) = \sum_{m=-\infty}^{\infty} \beta_m J_m(\pi du) e^{im(\varphi_u - \pi/2)}$$

(19)

Using Eq. 17, for long baselines, we may approximate

$$V(u, \varphi_u) \approx \frac{\alpha_z(\varphi_u) \cos (\pi du) + \alpha_z(\varphi_u) \sin (\pi du)}{\sqrt{du}}$$

$$\alpha_z(\varphi_u) \equiv \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \beta_m e^{im[\varphi_u + \pi(m-1+1)]}$$

(20)

Thus, for sufficiently long baselines, the radial visibility function of a nonuniform thin ring is determined by a single pair of weakly damped, orthogonal modes $\alpha_z(\varphi_u)$. Their envelope still fall as $\sqrt{\varphi_u} \sim 1/\sqrt{u}$, and the modes have a common period of $\Delta u = 2/d$ in complex visibilities (or $\Delta u = 1/d$ in visibility amplitudes). The angular spectrum of the image $\{\beta_m\}$ is easily retrieved from the angular spectrum of the visibilities (see the Supplementary Materials).

Visibilities for a thick ring

Baselines of length $u \gtrsim 1/L$ are required to resolve image features of size $\lesssim L$. Hence, the visibility function of any ring with diameter $d$ and thickness $w \ll d$ has two asymptotic regimes

$$\text{(I)}: \frac{1}{d} \ll u \ll \frac{1}{w}, \quad \text{(II)}: \frac{1}{d} \ll \frac{1}{w} \ll u$$

(21)

Baselines in regime (I) resolve the diameter of the ring but not its thickness, while longer baselines in regime (II) resolve both. Hence, the visibility function in regime (I) behaves like that of a thin ring (a damped periodicity with envelope $\sqrt{\varphi_u} \sim 1/\sqrt{u}$), while the envelope of the visibility function in regime (II) is sensitive to the radial profile of the ring. In general, the visibility of any smooth ring decays exponentially in regime (II), although images with discontinuous derivatives, such as a uniform disk or annulus, can have slower, power-law falloffs.
th subring dominates the signal in the regime (I).

For instance, if an image is stretched, thus far do not require an image with perfectly circular structure.

Although the photon ring is nearly circular for all black hole spins and inclinations, the primary interferometric signatures discussed so far do not require an image with perfectly circular structure. For instance, if an image is stretched, \( I(x, y) \rightarrow I'(x, y) = I(ax, by) \); then, its visibility function is correspondingly compressed, \( V(u, v) \rightarrow V'(u, v) = |ab|^{-1}V(au, bv) \). Thus, the visibility profiles of a stretched ring share the properties and asymptotic expansions derived for a circular ring (e.g., Eq. 20), except that the radial periodicities become a function of position angle. To leading order in the asymmetry \( a/b \), the diameter corresponding to a damped radial periodicity in the visibility domain matches that of the stretched ring along the baseline’s position angle. For the black hole in M87, the asymmetry is expected to be a few percent at most, even for a maximally rotating black hole (see fig. S2).

**Visibilities of the photon subrings**

As discussed earlier, the photon ring decomposes into subrings labeled by the photon half-orbit number \( n \). According to Eq. 14, the width of the radial intensity profile produced by the \( n \)th subring is

\[
\text{w}_n \sim \text{w}_0 \gamma^n,
\]

while the brightness remains approximately constant with \( n \) (until some \( n_{\text{max}} \) determined by the optical depth). Each subring thus contributes a periodically modulated visibility,

\[
V_n(u) = \text{w}_0 \gamma^n |u|^{1/2} |V(\text{w}_0 u)|^{-1/2},
\]

which falls more steeply for baselines \( u > 1/w_n \). Hence, the \( n \)th subring dominates the signal in the regime

\[
\frac{1}{w_{n-1}} < u < \frac{1}{w_n}
\]  

(22)

This implies that the totality of subring contributions has an envelop defined by this turnover behavior

\[
V(u) \approx \sum \frac{w_n}{\sqrt{u}} \sim \frac{1}{u^{3/2}}
\]  

(23)

Together, the subrings then form a cascade of damped oscillations on progressively longer baselines, each dominated by the image of a single subring and conveying precise information about its diameter, thickness, and angular profile. Figure 4 displays visibilities of the time-averaged GRMHD image in Fig. 1, which exhibit the expected damped periodicity, as well as clear contributions on long baselines from distinct subrings (see Fig. 5 for a schematic illustration of this cascade).

**DISCUSSION**

Detection of the photon ring’s universal interferometric signatures requires measurements on longer baselines, with finer angular resolution than those currently available to the EHT. This extension can be achieved either by observing at higher frequencies or on longer physical baselines via space VLBI. For reference, ALMA (Atacama Large Millimeter/submillimeter Array) currently observes up to 950 GHz (ALMA band 10) with higher frequencies (up to 1.53 THz) possible in the future (19, 20). Figure 5 shows baseline lengths for a variety of array configurations and observing frequencies.

For the EHT to observe the photon ring, it must also achieve sufficient sensitivity to detect its visibilities. For both Sgr A* and M87, the horizon-scale emission has a total flux density of \( F_{\text{tot}} \sim 1 \) Jy at \( \lambda \sim 1 \) mm (21, 22), with a fraction \( f_{\text{ring}} \approx 10\% \) expected to come from the photon ring. The expected amplitude of the photon ring on long interferometric baselines is thus

\[
|V(u)| \sim 30 \text{ mJy} \left( \frac{u}{10 \text{ G}\lambda} \right)^{-3/2} \left( \frac{d}{40 \text{ mas}} \right)^{-1/2} \left( \frac{f_{\text{ring}}}{0.1} \right) \left( \frac{F_{\text{tot}}}{1 \text{ Jy}} \right)
\]  

(24)

For comparison, a baseline from ALMA to a 4-m orbiter with 32 GHz of averaged bandwidth and a 10-min coherent integration period would have a thermal noise of \( \sigma_{\text{ALMA}} \approx 3 \) mJy at 950 GHz and \( \sigma_{\text{EHT}} \approx 1.3 \) mJy at 690 GHz. For baselines from ALMA to a 10-m orbiter, such as the proposed Millimetron mission for \( L_2 \) (23, 24), the thermal noise would be \( \sigma_{\text{ALMA}} \approx 1 \) mJy and \( \sigma_{\text{EHT}} \approx 0.5 \) mJy. Another possibility would be to place a VLBI station on or orbiting the Moon, which could sample many periods of the \( n = 2 \) regime of M87 at current EHT observing frequencies. A 10-m dish on the Moon could achieve \( \sigma \approx 0.1 \) mJy on baselines to ALMA with coherent integrations of 10 min and a bandwidth of 32 GHz.

These sensitivities could be substantially improved via simultaneous multifrequency observations. In addition to having more sensitive receivers and longer coherence times, lower frequencies give correspondingly shorter baselines and thus increased interferometric power from the photon ring. Phase calibration with lower frequencies could then allow substantially longer integration times at higher frequencies (see the Supplementary Materials for additional details and discussion). Interferometric signatures of the photon ring are most prominent when the image has little small-scale power outside the ring and when the ring has a smooth angular profile dominated by low mode numbers \( m \). Both of these conditions are met in time-averaged images of black hole accretion flows, such as in Fig. 1. Because visibilities of a time-averaged image are equal to time-averaged visibilities of a variable image, developing capabilities for long, coherent averaging could significantly improve the prospects for unambiguous detection and characterization of the photon ring.
In summary, precise measurements of the size, shape, thickness, and angular profile of the $n$th photon subring of M87 and Sgr A* may be feasible for $n = 1$ using a high-frequency ground array or low Earth orbits, for $n = 2$ with a station on the Moon and for $n = 3$ with a station in L2.

**SUPPLEMENTARY MATERIALS**

Supplementary material for this article is available at http://advances.sciencemag.org/cgi/content/full/6/12/eaaz1310.

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Acknowledgments: We thank A. Broderick, Z. Frankel, J. Goldfield, S. Gralla, E. Himwich, H.-Y. Pu, and Z. Younsi. Funding: This work was supported by the National Science Foundation (AST-1440254, AST-1716536, AST-1716327, OISE-1743747, and PHY-1205550), the Gordon and Betty Moore Foundation (GBMF-5278), the Department of Energy (DE-SC0009998), (DE-SC0007870 to A.S.), NASA (no. HST-HF2-51431.001-A), and the Black Hole Initiative at Harvard University through a grant (60477) from the John Templeton Foundation. Author contributions: M.D.J., A.L., A.S., S.H., R.N., A.C., P.G., and D.C.M.P. formulated the ideas in this paper in discussions on the theoretical aspects of the photon shell and ring. S.H., D.K., A.S. and D.C.M.P. provided significant text and contributions. All authors contributed to review and editing.

Competing interests: The authors declare that they have no competing interests. Data and materials availability: All data needed to evaluate the conclusions in the paper are present in the paper and/or the Supplementary Materials. Additional data related to this paper may be requested from the authors.

Submitted 14 August 2019
Accepted 19 December 2019
Published 18 March 2020
10.1126/sciadv.aaz1310

Citation: M. D. Johnson, A. Lupsasca, A. Strominger, G. N. Wong, S. Hadar, D. Kapec, R. Narayan, A. Chael, C. F. Gammie, P. Galison, D. C. M. Palumbo, S. S. Doeleman, L. Blackburn, M. Wielgus, D. W. Pesce, J. R. Farah, J. M. Moran, Universal interferometric signatures of a black hole’s photon ring. Sci. Adv. 6, eaaz1310 (2020).