TOPOLOGICAL INDICES OF K-TH SUBDIVISION AND
SEMI TOTAL POINT GRAPHS

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Abstract. Graph theory has provided a very useful tool, called topological
indices which are a number obtained from the graph $G$ with the property
that every graph $H$ isomorphic to $G$, value of a topological index must be
same for both $G$ and $H$. In this article, we present exact expressions for
some topological indices of $k$-th subdivision graph and semi total point graphs
respectively, which are a generalization of ordinary subdivision and semi total
graph for $k \geq 1$.

1. Introduction

Let, $G = (V, E)$ be a connected, undirected simple graph with vertex set $V = V(G)$ and edge set $E = E(G)$. The degree of a vertex $v$ in $G$ is defined as the
number of edges incident to $v$ and denoted by $d_G(v)$. Let $n$ and $m$ denote the order
and size of the graph $G$. In this paper we consider only connected graphs without
any self loop and parallel edges. In chemical graph theory, a topological index is a
number obtained from a graph which is structurally invariant, that is, every graph
$H$ isomorphic to $G$, value of a topological index must be same for both $G$ and $H$.
Among different topological indices the vertex degree based topological indices has
shown there applicability in chemistry, biochemistry, nanotechnology.

The most popular Zagreb indices were introduced more than forty years ago \[1\] by Gutman and Trinajstić where they have examined the dependence of the total
$\pi$-electron energy on molecular structure. These indices are denoted by $M_1(G)$ and
$M_2(G)$ and respectively defined as

$$M_1(G) = \sum_{v \in V(G)} d_G(v)^2 = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]$$

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operations.
and

\[ M_2(G) = \sum_{uv \in E(G)} d_G(u)d_G(v). \]

These indices have been studied intensively by researchers in recent past (see [2, 3, 4, 5, 6]).

The “forgotten topological index” or F-index of a graph \( G \) is denoted by \( F(G) \) and was introduced in the same paper where first and second Zagreb indices were introduced [1] and is defined as

\[ F(G) = \sum_{v \in V(G)} d_G(v)^3 = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2]. \]

We encourage the interested readers to consult the papers [7, 8, 9, 10, 11] for some recent study of this index.

The multiplicative variants of additive graph invariants was introduced by Todeschini et al. [12, 13], which applied to the Zagreb indices would lead to the first and second Multiplicative Zagreb Indices. Thus the multiplicative Zagreb indices are defined as

\[ \Pi_1(G) = \prod_{v \in V(G)} d_G(v)^2 \]

and

\[ \Pi_2(G) = \prod_{uv \in E(G)} d_G(u)d_G(v). \]

For different studies on these two indices see [14, 15, 16, 17, 18, 19].

One modified version of Zagreb index, named as hyper Zagreb index, was introduced by Shirredel et al. in [20] and is defined as

\[ HM(G) = \sum_{uv \in E(G)} |d_G(u) + d_G(v)|^2. \]

We refer the reader to [21, 22], for some recent study and application of hyper Zagreb index.

The Symmetric division deg index of a graph \( G \) is defined as [23]

\[ SSD(G) = \sum_{uv \in E(G)} \frac{\max\{d_G(u), d_G(v)\}}{\min\{d_G(u), d_G(v)\}} + \frac{\min\{d_G(u), d_G(v)\}}{\max\{d_G(u), d_G(v)\}}. \]

For different recent study of this index see [24, 25].

There are various studies of different derived graphs in recent literature. The subdivision related derived graphs were introduced by Sampathkumar in [26, 27, 28] and named as semitotal-point graph and semitotal-line graph. First of all the subdivision graph \( S(G) \) of a graph \( G \) is defined as follows:

**Definition 1.1.** The subdivision graph \( S(G) \) is obtained from \( G \) by adding a new vertex corresponding to every edge of \( G \), that is, each edge of \( G \) is replaced by a path of length two.
Hande et al. in [29], defined a generalization of $S(G)$, called the k-th subdivision graph, denoted by $S_k(G)$ and is defined as follows:

**Definition 1.2.** The k-th subdivision graph of $G$, denoted by $S_k(G)$, is the graph obtained by inserting k number of new vertices to each edge of $G$. Thus the graph of $S_k(G)$ consists of total $(n + km)$ number of vertices and has $(1 + k)m$ number of edges.

The semi total point graph $R(G)$ of a graph $G$ is defined as follows:

**Definition 1.3.** The semi total point graph $R(G)$ is obtained from $G$ by adding a new vertex corresponding to every edge of $G$, then joining each new vertex to the end vertices of the corresponding edge that is, each edge of $G$ is replaced by a triangle.

Jog et al. in [30], introduced a generalization of $R(G)$, denoted by $R_k(G)$, called the k-th semi total point graph of $G$. The k-th semi total point graph of $G$ is defined as follows:

**Definition 1.4.** The k-th semi total point graph $R_k(G)$ is the graph obtained from $G$ by adding $k$ vertices corresponding to each edge and connecting them to the end points of the edge considered. Clearly the graph of $R_k(G)$ is of order $(n+mk)$ and has $(1+2k)m$ number of edges.

In [31], Mehranian obtained formulas for the Laplacian polynomial and Kirchhoff index of the k-th semi total point graphs. For different study of different derived graphs see the recent papers [32, 33, 34]. In this article, we determine the different topological indices such as first and second Zagreb indices, multiplicative Zagreb indices, F-index, hyper-Zagreb index and Symmetric division deg index of k-th subdivision and semi total point graphs respectively.

### 2. Main Results

In this section, first we derive exact expressions of certain topological indices of k-th subdivision graph and then proceeding similarly we obtain different topological indices of k-th semi total point graph respectively.

#### 2.1. K-th subdivision graphs.**

The k-th subdivision graph of $G$ is the graph obtained by inserting $k$ number of new vertices of degree 2 to each edge of $G$. Thus in $S_k(G)$ there are $km$ number vertices of degree 2 and the remaining vertices are of same as $G$. The degree of the vertices of $S_k(G)$ are given as follows

\[
d_{S_k(G)}(v) = \begin{cases} 
  d_G(v), & \text{if } v \in V(G) \\
  2, & \text{if } v \in V(S_k(G)) \setminus V(G).
\end{cases}
\]

From definition, it is clear that $S_0(G) = G$ and $S_1(G) = S(G)$. In the following theorems, we now calculate different topological indices k-th subdivision graph $S_k(G)$ respectively.
Theorem 2.1. Let $G$ be a connected graph. Then
\[ M_1(S_k(G)) = M_1(G) + 4km. \]

Proof. Since $S_k(G)$ has $(n + mk)$ number of vertices and $(1 + k)m$ number of edges, from definition of first Zagreb index, we have
\[
M_1(S_k(G)) = \sum_{v \in V(S_k(G))} d_{S_k(G)}(v)^2
\]
\[
= \sum_{v \in V(S_k(G)) \cap V(G)} d_{S_k(G)}(v)^2 + \sum_{v \in V(S_k(G)) \setminus V(G)} d_{S_k(G)}(v)^2
\]
Also, since for $S_k(G)$, there are $n$ number of vertices are of degree $d_G(v)$ and $mk$ number of vertices are of degree 2, we have
\[
M_1(S_k(G)) = \sum_{v \in V(G)} d_G(v)^2 + \sum_{v \in V(S_k(G)) \setminus V(G)} 2^2
\]
\[
= M_1(G) + 4km.
\]
Hence the desired result follows. \(\Box\)

Theorem 2.2. Let $G$ be a connected graph. Then
\[ M_2(S_k(G)) = 2M_1(G) + 4(k - 1)m. \]

Proof. We know that, in $S_k(G)$, there are $n$ number of vertices are of degree $d_G(v)$ and $mk$ number of vertices are of degree 2. Also a vertex $v$ in $G$ is adjacent with $d_G(v)$ number vertices of degree 2 in $S_k(G)$ and for every edge in $G$ there must be $(k-1)$ edges in $S_k(G)$ with both the end vertices are of degree 2. So from definition of second Zagreb index of a graph we have
\[
M_2(S_k(G)) = \sum_{uv \in E(S_k(G))} d_{S_k(G)}(u)d_{S_k(G)}(v)
\]
\[
= \sum_{uv \in E(S_k(G)) \cap E(G)} d_{S_k(G)}(u)d_{S_k(G)}(v)
\]
\[
+ \sum_{uv \in E(S_k(G)) \setminus E(G)} d_{S_k(G)}(u)d_{S_k(G)}(v)
\]
\[
= \sum_{uv \in E(S_k(G)) \cap E(G)} 2d_G(u) + \sum_{uv \in E(S_k(G)) \setminus E(G)} 2d_G(u)
\]
\[
= 2M_1(G) + 4(k - 1)m.
\]
Which is the desired result. \(\Box\)
Theorem 2.3. Let $G$ be a connected graphs. Then

$$F(S_k(G)) = F(G) + 8km.$$ 

Proof. As $S_k(G)$ has $(n + mk)$ number of vertices and $(1 + k)m$ number of edges, from definition of $F$-index index, we have

$$F(S_k(G)) = \sum_{v \in V(S_k(G))} d_{S_k(G)}(v)^3 = \sum_{v \in V(S_k(G)) \cap V(G)} d_{S_k(G)}(v)^3 + \sum_{v \in V(S_k(G)) \setminus V(G)} d_{S_k(G)}(v)^3.$$ 

Again, we have for $S_k(G)$, there are $n$ number of vertices are of degree $d_G(v)$ and $mk$ number of vertices are of degree 2, we have similarly

$$F(S_k(G)) = \sum_{v \in V(G)} d_G(v)^3 + \sum_{v \in V(S_k(G)) \setminus V(G)} 2^3 = F(G) + 8km.$$ 

Hence we get the desired result. □

Theorem 2.4. Let $G$ be a connected graphs. Then

$$\Pi_1(S_k(G)) = 4km \Pi_1(G).$$ 

Proof. As $S_k(G)$ has $(n + mk)$ number of vertices and $(1 + k)m$ number of edges, from definition of first multiplicative Zagreb index index, we have

$$\Pi_1(S_k(G)) = \prod_{v \in V(S_k(G))} d_{S_k(G)}(v)^2 = \prod_{v \in V(S_k(G)) \cap V(G)} d_{S_k(G)}(v)^2 \prod_{v \in V(S_k(G)) \setminus V(G)} d_{S_k(G)}(v)^2.$$ 

Again, for the graph $S_k(G)$, there are $n$ number of vertices are of degree $d_G(v)$ and $mk$ number of vertices are of degree 2 and hence we can write

$$\Pi_1(S_k(G)) = \prod_{v \in V(G)} d_G(v)^2 \prod_{v \in V(S_k(G)) \setminus V(G)} 2^2 = 4km \Pi_1(G).$$ 

Which is the desired result. □

Theorem 2.5. Let $G$ be a connected graphs. Then

$$\Pi_2(S_k(G)) = 4km \Pi_2(G).$$ 

Proof. We know that, $S_k(G)$ has $(n + mk)$ number of vertices and $(1 + k)m$ number of edges. Thus, from definition of second multiplicative Zagreb index, we
Also, we have for $S_k(G)$, there are $n$ number of vertices are of degree $d_G(v)$ and $mk$ number of vertices are of degree 2. Then, we have

$$\Pi_2(S_k(G)) = \prod_{v \in V(S_k(G)) \setminus V(G)} d_{S_k(G)}(v)d_{S_k(G)}(v)$$

Also, we have

$$\Pi_2(S_k(G)) = \prod_{v \in V(S_k(G)) \setminus V(G)} d_{G}(v)^{d_{G}(v)} \prod_{v \in V(S_k(G)) \setminus V(G)} 2^2 = 4^{km}\Pi_2(G).$$

Hence the desired result follows. \hfill \Box

**Theorem 2.6.** Let $G$ be a connected graphs. Then

$$HM(S_k(G)) = F(G) + 4M_1(G) + 16km - 8m.$$  

**Proof.** Since for every edge in $G$ there are $(k - 1)$ edges in $S_k(G)$ with both the end vertices are of degree 2 and also a vertex $v$ in $G$ is adjacent with $d_G(v)$ number vertices of degree 2. So from definition of hyper Zagreb index of a graph, we have

$$HM(S_k(G)) = \sum_{u \in E(S_k(G))} [d_{S_k(G)}(u) + d_{S_k(G)}(v)]^2$$

$$= \sum_{u \in E(S_k(G))} [d_{S_k(G)}(u) + d_{S_k(G)}(v)]^2$$

$$+ \sum_{u \in E(S_k(G))} [d_{S_k(G)}(u) + d_{S_k(G)}(v)]^2$$

$$\sum_{u \in V(G)} [2 + d_G(u)]^2d_G(u) + \sum_{u \in V(S_k(G)) \setminus V(G)} [2 + 2]^2$$

$$= \sum_{u \in V(G)} [d_G(u)^3 + 4d_G(u)^2 + 4d_G(u)] + 16(k - 1)m$$

$$= F(G) + 4M_1(G) + 8m + 16km - 16m.$$  

Which is the required expression. \hfill \Box

**Theorem 2.7.** Let $G$ be a connected graphs. Then

$$SSD(S_k(G)) = \frac{1}{2}M_1(G) + 2(k - 1)m + 2n.$$  

**Proof.** Since for $S_k(G)$, there are $n$ number of vertices are of degree $d_G(v)$ and $mk$ number of vertices are of degree 2 and for every edge in $G$ there are $(k - 1)$ edges in $S_k(G)$ with both the end vertices are of degree 2. Also a vertex $v$ in $G$ is
adjacent with $d_G(v)$ number vertices of degree 2. So from definition of Symmetric division deg index of a graph we have

$$SSD(S_k(G)) = \sum_{u \in E(S_k(G))} \frac{d_{S_k(G)}(u)^2 + d_{S_k(G)}(v)^2}{d_{S_k(G)}(u)d_{S_k(G)}(v)}$$

$$= \sum_{u \in E(S_k(G))} u \in V(G), v \in V(S_k(G)) \setminus V(G)} \frac{d_{S_k(G)}(u)^2 + d_{S_k(G)}(v)^2}{d_{S_k(G)}(u)d_{S_k(G)}(v)}$$

$$+ \sum_{u \in E(S_k(G))} u \in V(S_k(G)) \setminus V(G)} \frac{d_{S_k(G)}(u)^2 + d_{S_k(G)}(v)^2}{d_{S_k(G)}(u)d_{S_k(G)}(v)}$$

$$= \sum_{u \in V(G)} \frac{2^2 + d_G(u)^2}{2d_G(u)}d_G(u) + \sum_{u \in E(S_k(G))} \frac{2^2 + 2^2}{2}$$

$$= \frac{1}{2} \sum_{u \in V(G)} [d_G(u)^2 + 4] + 2(k-1)m$$

$$= \frac{1}{2} M_1(G) + 2n + 2(k-1)m.$$ 

Thus the desired result follows. 

**Corollary 2.1.** From the above theorems the following results follows as a direct consequence.

(i) $M_1(S_k(G)) = M_1(S_{k-1}(G)) + 4m,$ 
(ii) $M_2(S_k(G)) = M_2(S_{k-1}(G)) + 4m,$ 
(iii) $F(S_k(G)) = F(S_{k-1}(G)) + 8m,$ 
(iv) $\Pi_1(S_k(G)) = 4^n\Pi_1(S_{k-1}(G)),$ 
(v) $\Pi_2(S_k(G)) = 4^n\Pi_2(S_{k-1}(G)),$ 
(vi) $HM(S_k(G)) = HM(S_{k-1}(G)) + 16m,$ 
(vii) $SSD(S_k(G)) = SSD(S_{k-1}(G)) + 2m.$ 

**Corollary 2.2.** Let $G$ be a $r$-regular graph with $n$ vertices and $m (\geq \frac{nr}{2})$ number of edges. Then using theorem 2.1-2.7, the following result follows after direct calculation:

(i) $M_1(S_k(G)) = nr^2 + 2nk,$ 
(ii) $M_2(S_k(G)) = 2nr(r + k - 1),$ 
(iii) $F(S_k(G)) = nr^3 + 4nr,$ 
(iv) $\Pi_1(S_k(G)) = 2^{enr}r^{2n},$ 
(v) $\Pi_2(S_k(G)) = 2^{nknr}r^n,$ 
(i) $HM(S_k(G)) = nr(r^2 + 4r + 8k - 4),$ 
(i) $SSD(S_k(G)) = \frac{1}{2} nr^2 + nr(k - 1) + 2n.$
Note that, from the results obtained in theorem 2.1-2.7, we can easily obtain the result for ordinary subdivision graph by putting $k = 1$. For instance, putting $k = 1$ in theorem 2.1-2.7 we get the same results of subdivision graph $S(G)$ for first and second Zagreb index, F-index and multiplicative Zagreb indices as in $[5, 6, 10, 19]$ respectively. Thus here we generalized the above results for some $k \geq 1$.

2.2. K-th semi total point graphs. The k-th semi total point graph of $G$, denoted by $R_k(G)$, is the graph obtained by adding $k$ number of vertices to each edge of $G$ and joining them to the end points of the respective edges. Clearly the graph of $R_k(G)$ is of order $(n + mk)$ and has $(1 + 2k)m$ number of edges. The degree of the vertices of $R_k(G)$ are given as follows

$$d_{R_k(G)}(v) = \begin{cases} 
(k + 1)d_G(v), & \text{if } v \in V(G) \\
2, & \text{if } v \in V(R_k(G)) \setminus V(G). 
\end{cases}$$

Also, it is clear that $R_0(G) = G$ and $R_1(G) = R(G)$. In the following theorems, we now calculate different topological indices k-th semi total graph $R_k(G)$ respectively.

**Theorem 2.8.** Let $G$ be a connected graphs. Then

$$M_1(R_k(G)) = (k + 1)^2M_1(G) + 4km.$$ 

**Proof.** Since, $R_k(G)$ has $(n + mk)$ number of vertices and $(1 + 2k)m$ number of edges, from definition of first Zagreb index, we have

$$M_1(R_k(G)) = \sum_{v \in V(R_k(G))} d_{R_k(G)}(v)^2 = \sum_{v \in V(R_k(G)) \setminus V(G)} d_{R_k(G)}(v)^2 + \sum_{v \in V(R_k(G)) \setminus V(G)} d_{R_k(G)}(v)^2.$$ 

Again, since in $R_k(G)$ there are $n$ number of vertices are of degree $(k + 1)d_G(v)$ and $mk$ number of vertices are of degree 2, we have

$$M_1(R_k(G)) = \sum_{v \in V(G)} (k + 1)^2d_G(v)^2 + \sum_{v \in V(R_k(G)) \setminus V(G)} 2^2 = (k + 1)^2M_1(G) + 4km,$$

which proves the desired result. \qed

**Theorem 2.9.** Let $G$ be a connected graphs. Then

$$M_2(R_k(G)) = 2k(k + 1)M_1(G) + (k + 1)^2M_2(G).$$
Proof. From definition of second Zagreb index, we have

\[ M_2(R_k(G)) = \sum_{uv \in E(R_k(G))} d_{R_k(G)}(u)d_{R_k(G)}(v) \]

\[ = \sum_{uv \in E(S_k(G)) \setminus V(G)} d_{R_k(G)}(u)d_{R_k(G)}(v) \]

\[ + \sum_{uv \in E(S_k(G))} d_{R_k(G)}(u)d_{R_k(G)}(v) \]

\[ = \sum_{uv \in E(R_k(G)) \setminus V(G)} 2(k+1)d_G(u) + \sum_{uv \in E(R_k(G)) \setminus V(G)} (k+1)^2d_G(u)d_G(v) \]

\[ = 2k(k+1)M_1(G) + (k+1)^2M_2(G). \]

Hence, we get the required result. □

Theorem 2.10. Let \( G \) be a connected graphs. Then

\[ F(R_k(G)) = (k+1)^3F(G) + 8km. \]

Proof. From definition of F-index and since \( R_k(G) \) has \((n + mk)\) number of vertices and \((1 + 2k)m\) number of edges, we have

\[ F(R_k(G)) = \sum_{v \in V(R_k(G))} d_{R_k(G)}(v)^3 \]

\[ = \sum_{v \in V(R_k(G)) \setminus V(G)} d_{R_k(G)}(v)^3 + \sum_{v \in V(R_k(G)) \setminus V(G)} d_{R_k(G)}(v)^3. \]

Also, in \( R_k(G) \) there are \( n \) number of vertices are of degree \((k+1)d_G(v)\) and the remaining \( mk \) number of vertices are of degree 2, so we have

\[ F(R_k(G)) = \sum_{v \in V(G)} (k+1)^3d_G(v)^3 + \sum_{v \in V(R_k(G)) \setminus V(G)} 2^3 \]

\[ = (k+1)^3F(G) + 8km. \]

Which is the desired expression. □

Theorem 2.11. Let \( G \) be a connected graphs. Then

\[ \Pi_1(R_k(G)) = 4km(k + 1)^2\Pi_1(G). \]
Proof. Since, $R_{k}(G)$ has $(n + mk)$ number of vertices and $(1 + 2k)m$ number of edges, from definition of first multiplicative Zagreb index, we have

$$
\Pi_1(R_{k}(G)) = \prod_{v \in V(R_{k}(G))} d_{R_{k}(G)}(v)^2
$$

$$
= \prod_{v \in V(R_{k}(G)) \cap V(G)} d_{R_{k}(G)}(v)^2 \prod_{v \in V(R_{k}(G)) \setminus V(G)} d_{R_{k}(G)}(v)^2.
$$

Again, similarly the graph $R_{k}(G)$ has $n$ number of vertices of degree $(k + 1)d_{G}(v)$ and $mk$ number of vertices are of degree 2, we have

$$
\Pi_1(R_{k}(G)) = \prod_{v \in V(G)} (k + 1)^2d_{G}(v)^2 \prod_{v \in V(R_{k}(G)) \setminus V(G)} 2^2
$$

$$
= 4^{km}(k + 1)^{2n}\Pi_1(G).
$$

Hence the desired result follows. \qed

Theorem 2.12. Let $G$ be a connected graphs. Then

$$
\Pi_2(R_{k}(G)) = 4^{km}\left\{ (k + 1)^{2m}\Pi_1(G) \right\}^{(k+1)}.
$$

Proof. We have from definition of second multiplicative Zagreb index

$$
\Pi_2(R_{k}(G)) = \prod_{u \in E(R_{k}(G))} d_{R_{k}(G)}(u)d_{R_{k}(G)}(v)
$$

$$
= \prod_{v \in V(R_{k}(G)) \cap V(G)} d_{R_{k}(G)}(v)d_{R_{k}(G)}(v) \prod_{v \in V(R_{k}(G)) \setminus V(G)} d_{R_{k}(G)}(v)d_{R_{k}(G)}(v).
$$

Again, since $R_{k}(G)$ has $(n + mk)$ number of vertices and out of which $n$ numbers of vertices are of degree $(k + 1)d_{G}(v)$ and $mk$ number of vertices are of degree 2, we have

$$
\Pi_2(R_{k}(G)) = \prod_{v \in V(G)} \left\{ (k + 1)d_{G}(v) \right\}^{(k+1)}d_{G}(v) \prod_{v \in V(R_{k}(G)) \setminus V(G)} 2^2
$$

$$
= (k + 1)^{2m(k+1)} \left\{ \prod_{v \in V(G)} d_{G}(v)d_{G}(v) \right\}^{(k+1)} \prod_{v \in V(R_{k}(G)) \setminus V(G)} 2^2
$$

$$
= 4^{km}\left\{ (k + 1)^{2m}\Pi_1(G) \right\}^{(k+1)}.
$$

Which proves the desired expression. \qed

Theorem 2.13. Let $G$ be a connected graphs. Then

$$
HM(R_{k}(G)) = (k + 1)^2HM(G) + k(k + 1)^2F(G) + 4k(k + 1)M_1(G) + 8km.
$$
Proof. From definition of hyper-Zagreb index, we have

\[
HM(R_k(G)) = \sum_{uv \in E(R_k(G))} [d_{R_k(G)}(u) + d_{R_k(G)}(v)]^2
\]

\[
= \sum_{uv \in E(R_k(G))} \left[ d_{R_k(G)}(u) + d_{R_k(G)}(v) \right]^2
+ \sum_{u \in V(R_k(G)) \setminus V(G)} \left( d_{R_k(G)}(u) + d_{R_k(G)}(v) \right)^2
\]

\[
= \sum_{u \in V(G)} [2 + (k + 1)d_G(u)]^2 kd_G(u)
+ \sum_{u \in V(G)} [(k + 1)d_G(u) + (k + 1)d_G(v)]^2
\]

\[
= k \sum_{u \in V(G)} [(k + 1)^2 d_G(u)^3 + 4(k + 1)d_G(u)^2 + 4d_G(u)]
+ (k + 1)^2 HM(G)
\]

\[
= k(k + 1)^2 F(G) + 4k(k + 1)M_1(G) + 8km + (k + 1)^2 HM(G).
\]

Thus the required result follows. □

Theorem 2.14. Let \( G \) be a connected graphs. Then

\[
SSD(R_k(G)) = SSD(G) + \frac{1}{2}(k + 1)M_1(G) + \frac{2n}{(k + 1)}.
\]

Proof. From definition of Symmetric division deg index and k-th semi total graph, we have

\[
SSD(R_k(G)) = \sum_{uv \in E(R_k(G))} \frac{d_{R_k(G)}(u)^2 + d_{R_k(G)}(v)^2}{d_{R_k(G)}(u)d_{R_k(G)}(v)}
\]

\[
= \sum_{uv \in E(R_k(G)) \cup V(G), v \in V(R_k(G)) \setminus V(G)} \frac{d_{R_k(G)}(u)^2 + d_{R_k(G)}(v)^2}{d_{R_k(G)}(u)d_{R_k(G)}(v)}
\]
ber of edges. Then, using theorems 2.8-2.14, the following result follows directly:

\[ \text{SSD} \]

\[ \text{Π} \]

\[ \text{F} \]

\[ \text{Π} \]

\[ \text{HM} \]

Note that, from the results obtained in Theorem 2.8-2.14, we can also obtain the results for ordinary semi total point graph by putting \( k = 1 \) (see [5, 6, 10, 19]).

3. Conclusion

In this article, we determine the topological indices such as Zagreb indices, multiplicative Zagreb indices, F-index, hyper-Zagreb index and Symmetric division.
deg index of $k$-th subdivision and semi total point graphs respectively. In this paper, we generalized the results for subdivision and semi total point graphs for some $k \geq 1$.

References

[1] I. Gutman, N. Trinajstić, Graph theory and molecular orbitals. Total $\pi$-electron energy of alternant hydrocarbons, *Chem. Phys. Lett.*, 17(1972), 535-538.
[2] G.H. Fath-Tabar, Old and new Zagreb indices of graphs, *MATCH Commun. Math. Comput. Chem.*, 65(2011), 79-84.
[3] B. Zhou, I. Gutman, Further properties of Zagreb indices, *MATCH Commun. Math. Comput. Chem.*, 54(2005), 233-239.
[4] K.C. Das, K. Xu, J. Nam, On Zagreb indices of graphs, *Front. Math. China*, 10(2015), 567-582.
[5] I. Gutman, B. Furtula, Z. Kovijanic Vukicevic, G. Popivoda, Zagreb indices and coindices, *MATCH Commun. Math. Comput. Chem.*, 74(2015), 5-16.
[6] B. Basavanagoud, I. Gutman, C. S. Gali, On second Zagreb index and coindex of some derived graphs, *Kragujevac J. Sci.*, 37(2015), 113-121.
[7] B. Furtula, I. Gutman, A forgotten topological index, *J. Math. Chem.*, 53(2015), 1184-1190.
[8] N. De, F-index of graphs based on four operations related to the lexicographic product, arXiv:1706.00464v1.
[9] N. De, S.M.A. Nayeem and A. Pal, F-index of some graph operations, *Discrete Math. Algorithm. Appl.*, 8(2), 1650025 (2016).
[10] N. De, F-index and coindex of some derived graphs, *Bull. Int. Math. Virtual Inst.*, 8(2018), 81-88.
[11] N. De, F-index of total transformation graphs, *Discrete Math. Algorithm. Appl.*, 9(3), (2017), 17 pages.
[12] R. Todeschini, D. Ballabio and V. Consonni, Novel molecular descriptors based on functions of new vertex degrees, *Univ. Kragujevac*, 2010, 73-100.
[13] R. Todeschini and V. Consonni, New local vertex invariants and molecular descriptors based on functions of the vertex degrees, *MATCH Commun. Math. Comput. Chem.*, 64(2010), 359-372.
[14] M. Eliasi, A. Iranmanesh, I. Gutman, Multiplicative versions of first Zagreb index, *MATCH Commun. Math. Comput. Chem.*, 68(2012), 217-230.
[15] I. Gutman, Multiplicative Zagreb Indices of Trees, *Bull. Soc. Math. Banja Luka.*, 18(2011), 17-23.
[16] K. Xu, H. Hua, A unified approach to extremal multiplicative Zagreb indices for trees, unicyclic and bicyclic graphs, *MATCH Commun. Math. Comput. Chem.*, 68(2012), 241-256.
[17] J. Liu, Q. Zhang, Sharp Upper Bounds for Multiplicative Zagreb Indices, *MATCH Commun. Math. Comput. Chem.*, 68(2012), 231-240.
[18] T. Reti, I. Gutman, Relations between Ordinary and Multiplicative Zagreb Indices, *Bull. Internat. Math. Virt. Inst.*, 2(2012), 133-140.
[19] B. Basavanagoud, S. Patil, Multiplicative Zagreb indices and coindices of some derived graphs, *Opuscula Math.*, 36(2016), 287-299.
[20] G.H. Shirrdel, H. Rezapour and A.M. Sayadi, The hyper Zagreb index of graph operations, *Iran. J. Math. Chem.*, 4(2013), 213-220.
[21] N. De, Hyper Zagreb index of bridge and chain graphs, arXiv:1703.08325v1.
[22] W. Gao, M.K. Jamil and M.R. Farahani, The Hyper-Zagreb index and some graph operations, *J. Appl. Math. and Comput.*, 54(2017), 263-275.
[23] D. Vukicevic, Bond additive modeling 2 mathematical properties of max-min rodeg index, *Croatica Chemica. Acta.*, 83(2010), 261-273.
[24] V. Alexander, Upper and lower bounds of symmetric division deg index, *Iran. J. Math. Chem.*, 5,2(2014), 91-98.
[25] C. K. Gupta, V. Lokesha, Shwetha B. Shetty, On the Symmetric division deg index of graph, *South East Asian J. Math.*, **41**(1)(2016), 1-23.
[26] E. Sampathkumar and S. B. Chikkodimath, The semi-total graphs of a graph-I *J. Karnatak Univ. Sci.*, **18**(1973), 274-280.
[27] E. Sampathkumar, S. B. Chikkodimath, Semitotal graphs of a graph - II, *J. Karnatak Univ. Sci.*, **18**(1973), 281-284.
[28] E. Sampathkumar, S. B. Chikkodimath, Semitotal graphs of a graph - III, *J. Karnatak Univ. Sci.*, **18**(1973), 285-296.
[29] S. P. Hande, S. R. Jog, H. S. Ramane, P. R. Hampiholi, I. Gutman, and B. S. Durgi, Derived graphs of subdivision graphs, *Kragujevac J. Math.*, **37**(2)(2013), 319-323.
[30] S. R. Jog, S. P. Hande, I. Gutman and S. B. Bozkurt, Derived graphs of some graphs, *Kragujevac J. Math.*, **36** (2012), 309-314.
[31] Z. Mehranian, The Laplacian polynomial and Kirchhoff index of the k-th semi total point graphs, *Iran. J. Math. Chem.*, **5**, (2014), 7-15.
[32] N. De, Narumi-Katayama index of some derived graphs, *Bull. Int. Math. Virtual Inst.*, **7**(2017), 117-128.
[33] D. Sarala, S.K. Ayyaswamy and S. Balachandran, The Zagreb indices of graphs based on four new operations related to the lexicographic product, *Appl. Math. and Comput.*, **309**, (2017), 156-169.
[34] N. De, S.M.A. Nayeem, A. Pal, Total eccentricity index of the generalized hierarchical product of graphs, *Int. J. Appl. Comput. Math.*, **2**(2016), 411-420.

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