Group Delay Control in Longitudinal Offset Coupled Resonator Optical Waveguides

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We study the differential group delay that can be obtained as the result of the phase imbalance in coupled micro-resonator waveguide arrays with longitudinal offset couplings. Various devices based on this effect are proposed and analyzed using a finite-differences time-domain method.

I. INTRODUCTION

The recent advances in fabrication technologies have promoted the microring resonator devices to the status of preferred platform for many photonic signal processing functionalities, such as optical filtering, dispersion compensation, sensing, or group delay control [1–8]. Although microring structures can be realized using different materials, most proposals of passive structures have focused on silicon-on-insulator (SOI) devices to date, due to obvious reasons of convenience and the available enabling technology.

In designing photonic structures at the top-level, a waveguide coupler is most conveniently treated as an ideal, lumped component characterized by the numerical value of its coupling constant. However, in the standard fabrication process, the coupling constant is determined mainly by the transversal offset, i.e. by the separation between the coupler waveguides. To meet the accuracy typically demanded for the value of the coupling constant, such separation must be set with an accuracy of around a few nanometers, which is near the current technological limit.

To overcome the difficulty described above, an alternative approach was proposed in [9,10]. This technique consists in varying the coupling length rather than the inter-guide separation, which remains constant. Such variation is in turn obtained by changing the longitudinal offset between the two parallel coupled straight waveguides of fixed length. Photolithographic production is then feasible with this technique as the resolution requirements are strongly relaxed.

As it stands, the longitudinal offset technique unavoidably imposes a certain type of phase unbalance between the fields at the two outputs of any coupling section, even if the physical lengths of both optical paths are the same. Thus, relative phase-shifts appear and accumulate throughout the whole structure. Whether they are irrelevant or not for the operation of the device depends on the specific architecture considered and the functionality sought. In this work, we study the control of the relative group delay that can be attained and its use in various photonic devices. A proof of principle of the operation of these designs is obtained by the simulation of their performance using a finite-difference time-domain (FDTD) solver for 2D Maxwell equations.

![FIG. 1: Geometry of a coupled resonator optical waveguide implementing the longitudinal offset technique.](image-url)
II. MODEL EQUATIONS

The geometry of a CROW implementing the longitudinal offset technique is shown in Figure 2. With reference to Figure 2, \( L_c \) is the length of the straight waveguide sections and corresponds to the coupler length for zero lateral displacement, \( L_R = \pi R \), is the length of each curved waveguide section of radius \( R \). The half ring length is \( L = L_R + L_c \) and will be assumed constant. The longitudinal offset \( L_{\text{off}} \) is defined in the figure and will be regarded as positive whenever the displacement is along the direction of the propagation of the field in that section and negative otherwise.

We can trace the effect of the coupling and the propagation phase in Figure 2 to write the field relations

\[
E^+_{i+1} = -j\beta E^+_{i} \exp(-j\beta(L + L_{\text{off},i})) + r E^-_{i+1} \exp(-j\beta L) \\
E^-_{i} = r E^+_{i+1} \exp(-j\beta L) - j\beta E^-_{i+1} \exp(-j\beta(L - L_{\text{off},i}))
\]

(1)

Rewriting the above expressions as

\[
E^+_{i} = \frac{1}{jt} (-E^+_{i+1} \exp(j\beta L + L_{\text{off},i}) + r E^-_{i+1} \exp(j\beta L_{\text{off},i})) \\
E^-_{i} = \frac{1}{jt} (-r E^+_{i+1} \exp(j\beta L_{\text{off},i} + L_{\text{off},i}) + E^-_{i+1} \exp(-j\beta(L - L_{\text{off},i})))
\]

(2)

permits to obtain the one-cell matrix relation

\[
\begin{pmatrix} E^+_{i} \\ E^-_{i} \end{pmatrix} = \frac{1}{jt} \exp(j\beta L_{\text{off},i}) \begin{pmatrix} -\exp(j\beta L) & r_l \\ -r_l & \exp(-j\beta L) \end{pmatrix} \begin{pmatrix} E^+_{i+1} \\ E^-_{i+1} \end{pmatrix}.
\]

(3)

and the input-output matrix relation for the whole chain

\[
\begin{pmatrix} E^+_0 \\ E^-_0 \end{pmatrix} = \exp \left( j\beta \sum_{i=0}^{N} L_{\text{off},i} \right) \left[ \prod_{i=0}^{N} \frac{1}{jt} \begin{pmatrix} -\exp(j\beta L) & r_l \\ -r_l & \exp(-j\beta L) \end{pmatrix} \right] \begin{pmatrix} E^+_N \\ E^-_N \end{pmatrix}.
\]

(4)

If the input signal is placed at \( E^+_0 \), taking into account that \( E^-_{N+1} = 0 \), we can obtain \( E^+_N = 1/m_{11} E^+_0 \), where \( m_{11} \) is the top diagonal element in the matrix relation (4).

Expression (4) evidences that, for the specific case of a CROW chain and within the scope of the ideal model (wavelength-independent coupling constants), the offset-induced phase unbalance is harmless as far as the amplitude response is concerned (this was misinterpreted in [11]), while a contribution to the group delay on the system appears that is the cumulative effect of the longitudinal offsets. This effect can be engineered to yield added functionalities to the structure, as described below.

Quite remarkably, if the other available set of input/output ports is used, there is a change in the transmission group delay since for the new device topology there is a sign flip for every value of \( L_{\text{off}} \) along the signal path. Therefore,
there exist two possible transfer functions implemented by the same physical system that are identical except for a
group delay term that takes two opposite values $\tau_B = -\tau_A = n/L_{\text{off}} \equiv \tau_{\text{off}}$.
This is illustrated in Figure 3 for a $N = 3$ ring CROW. The transmission path for the fast $A$ input-output ports
are marked with dashed lines and the slow path relating the $B$ input-output ports are marked with a solid line. The
transfer functions are

$$H_A(\omega) = \exp(j\tau_{\text{off}}\omega)H(\omega) \quad H_B(\omega) = \exp(-j\tau_{\text{off}}\omega)H(\omega),$$

where $H(\omega)$ is a common transfer function response term that would correspond to the same system implemented
without the longitudinal offsets.
FIG. 5: Time-domain multiplexer.

FIG. 6: Mach-Zehnder implementation using a longitudinal displacement structure.

III. APPLICATIONS

Using a Y-branch splitter, as shown in Figure 4, we can simultaneously obtain two outputs with the same amplitude spectral shaping and a relative delay. The shape used for the representation of the signals intends to reflect the existing relation in the filtering due to the common term $H(\omega)$ in (5). Cascading this type of structures would permit to obtain a series of relatively delayed signals obtained from the same input with a given filtering. Reversing the scheme, as shown in Figure 5, permits to multiplex two signals with a controlled relative time-delay and simultaneously apply the same filtering to both inputs.

The connection of the two inputs using a 3 dB splitter and the outputs with a 3 dB coupler, as shown in Fig. 6.
FIG. 7: Geometry of the coupler/splitter implemented in the simulations.

FIG. 8: Transmission spectrum (a) and group delays for the two outputs.

leads to the implementation of a Mach-Zehnder interferometer and an overall response

$$H_{MZ}(\omega) = \cos(\tau_{\text{off}}\omega)H(\omega).$$  (6)

This type of response can be exploited to double the FSR of an optical filter, by adequately tuning of the relative delay, or to realize notch filters.

IV. RESULTS AND DISCUSSION

The systems depicted in Figures 4, 5 and 6 for $N = 3$ ring CROWs have been analyzed using the FDTD method. In each case, independent simulations have been performed for the subsystems bounded by the dashed-dotted lines in the figures, and the computed transfer functions have been used to obtain the total response.

The computations are performed on an equivalent 2D reduced model obtained using the effective index method. All the waveguides in our devices are 350 nm wide and an effective index of $n_{\text{eff}} = 2.4$ has been assumed. The bent
waveguide sections have an inner radius of $r = 2 \, \mu m$ in all cases. The evanescent couplers are implemented with a waveguide separation of 200 nm. The geometry used for the 3dB Y-branch couplers and splitters is depicted in Fig. 7. The coupling and splitting responses obtained are very close to ideal 3 dB in all the cases.

Figure 8 (a) shows the amplitude responses and Fig. 8 (b) the group delays for the $A$ (solid line) and $B$ (dashed line) input/output transfer functions of a $N = 3$ ring system with $L_c = 45 \, \mu m$ and constant $L_{off} = 33 \, \mu m$. As predicted by the expressions (5), the amplitude responses are nearly identical, and the group delay difference between the two outputs is very close to a constant value of approximately 2 psec. The variation of the coupling coefficient with the wavelength is apparent in the results.

Figure 9 shows the transmission amplitude spectrum for a single input/output (dashed line) and the response from the MZ configuration in solid line, showing the FSR doubling effect. In this case, $L_c = 32.833 \, \mu m$ and $off = 5.948 \, \mu m$.

V. CONCLUSION

In this paper we wave analyzed a CROW chain realized by means of the convenient longitudinal offset technique. An analytical expression has been presented which explicitly displays the contribution of the inherent accumulated phase unbalances and is valid for the most general case, including apodization or non-periodicity. The information thus gained has been used to propose several architectures for photonic processing based on a periodic, alternating chain. We have described the generation of two identically filtered, truly delayed optical signals, as well as their multiplexing with a controlled time-delay. A Mach-Zender structure which permits to double the FSR has also been presented. All the proposed devices have been proved numerically through 2D FDTD simulation, which accounts for non-ideal effects such as coupler wavelength-dependence and radiation losses, the agreement with the expected results being excellent.

[1] G. Lenz and C. K. Masden, “General optical all-pass filter structures for dispersion control in WDM systems,” J. Lightwave Technol., vol. 17, no. 7, pp. 1248–1254, Jul. 1999.
[2] P. Chamorro-Posada, F. J. Fraile-Pelaez and F. J. Diaz-Otero, ”Micro-ring chains with high-order resonances,” J. Lightw. Technol., vol. 29, no. 10, pp. 1514–1521, 2011.
[3] C. K. Madsen, G. Lenz, A. J. Bruce, M. A. Cappuzzo, L. T. Gomez, and R. E. Scotti, “Integrated all-pass filters for tunable dispersion and dispersion slope compensation,” IEEE Photon. Technol. Lett., 11, pp. 1623–1625, Dec. 1999.
[4] C. Y. Chao, W. Fung, and L. J. Guo, “Polymer microring resonators for biochemical sensing applications,” IEEE J. Sel. Top. Quantum Electron., vol. 12, no. 1, pp. 134–142, Jan./Feb. 2006.
[5] R. W. Boyd, D. J. Gauthier, and A. L. Gaeta, “Applications of Slow Light in Telecommunications,” Opt. Photonics News, vol. 17, no. 4, pp. 18–23, Apr. 2006.
[6] P. Chamorro-Posada, and F. J. Fraile-Pelaez, “Fast and slow light in zigzag microring resonator chains,” Opt. Lett., vol. 34, no. 5, pp. 626–628, Mar. 2009.
[7] J. E. Heebner and R. W. Boyd, “Slow and fast light in resonator-coupled waveguides,” J. Mod. Opt., vol. 49, no. 14/15, pp. 2629-2636, Nov./Dec. 2002.

[8] J. K. Poon, L. Zhu, G. A. DeRose, and A. Yariv, “Transmission and group delay of microring coupled-resonator optical waveguides,” Opt. Lett., vol. 31, no. 4, pp. 456–458, Feb. 2006.

[9] J. D. Doménech, P. Muñoz, and J. Capmany, “The longitudinal offset technique for apodization of coupled resonator optical waveguide devices: concept and fabrication tolerance analysis,” Opt. Express, vol. 17, no. 23, pp. 21050–21059, Nov. 2009.

[10] J.D. Doménech, P. Muñoz, and J. Capmany, “Transmission and group-delay characterization of coupled resonator optical waveguides apodized through the longitudinal offset technique,” Opt. Lett., vol. 36, no. 2, pp. 136–138, Jan. 2011.

[11] P. Chamorro-Posada and F.J. Fraile Peláez, “Phase asymmetry effect in longitudinal offset coupled resonator optical waveguides,” IEEE Photon. Technol. Lett., vol. 26, no. 15, Aug. 2014.