Influence of a Non-Dipole Magnetic Field on the Peak Energies of Cyclotron Absorption Lines

Osamu NISHIMURA
Department of Electronics and Computer Science, Nagano National College of Technology;
716 Tokuma, Nagano, Nagano 381-8550
nishi@ei.nagano-nct.ac.jp

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Abstract

We calculate cyclotron lines in a neutron star slab assuming a non-dipole surface magnetic field. We consider the influences of a non-dipole surface field on the properties of the cyclotron resonant scattering lines. When the magnetic field strength decreases with height in the line-forming region, the ratios of the higher harmonics to the fundamental at the peak energies of the cyclotron lines become less than classical harmonic ratios, $1 : 2 : 3 \ldots$. On the other hand, when the magnetic field strength increases with height in the line-forming region, the ratios at the peak energies of the cyclotron lines become more than the classical harmonic ratios. The deviation from the harmonic ratios, which has actually been observed in some accretion-powered X-ray pulsars, is more significant than that expected from a relativistic effect in the cyclotron resonant energy. This may suggest the line-forming region is threaded by a strong non-dipole magnetic field in some accreting X-ray pulsars. In observations, the ratios are more or less than the classical harmonic ratios. This could imply that the magnetic field strength in the line-forming region decreases or increases with the altitude or the horizontal distance from an emission region.

Key words: stars: atmospheres — stars: magnetic fields — stars: neutron

1. Introduction

Cyclotron lines in the spectra of neutron stars provide a powerful tool to directly measure the magnetic field strength of neutron stars. They have been detected in the spectra of more than 10 accretion-powered X-ray pulsars, indicating commonly broad and shallow features (Coburn et al. 2002). Makishima et al. (1999) pointed out that the observed line widths are somewhat larger than those expected by thermal Doppler effects of electrons. This probably suggests that other factors, for example magnetic field variations and so on, contribute to the line widths. In addition, more than two cyclotron absorption features were detected in some accreting X-ray pulsars. Harmonic ratios of the line energies seem to be larger than or less than classical ones, $1 : 2 : 3 \ldots$. In observations of 4U 1907+09 (Cusumano et al. 1998) and Vela X-1 (Kreykenbohm et al. 1999), the harmonic ratios are more than the classical ones, while the harmonic ratios are less than the classical ones in the observations of 4U 0115+63 (Heindl et al. 1999; Santangelo et al. 1999).

Bulik et al. (1992, 1995) compared the pulse phase-dependent spectra of 4U 1538−−52 and Vela X-1 obtained from Ginga to a series of static model slabs of curved accreting polar caps. Bulik et al. (1992) performed fits to the spectra with the magnetic fields corresponding to cyclotron energies in the 16–26 keV range, consisting of a combination of six spectra with the cyclotron ground energy increasing in steps of 2 keV. As a result, they found that even maximal pole-on Doppler broadening is unable to fit the line profiles with a single field strength or a dipole distribution at a constant radius. Bulik et al. (1995) also fitted to the spectra with two bins corresponding to the components as follows. One component is characterized by the usual magnetic field and is responsible for the formation of the cyclotron line. The other component is characterized by a smaller magnetic field and is responsible for filling in the line produced by the previous component. The field strength difference between the high and low components is a factor of 5 or larger, i.e., much larger than that expected from a simple dipole field variation along the stellar surface inside the polar cap dimensions. This could be due to the presence of random or chaotic small-scale variations of the surface field from small-scale convective motions that became frozen at the time of the crust crystallized. Higher order multipoles could also be present due to thermomagnetic field evolution effects in the presence of nonuniform heat flux, as in the scenario of Blandford et al. (1983), or due to crustal platelet migrations, as pointed out in Ruderman (1991a, 1991b, 1991c). In fact, there is evidence for a complex magnetic field structure in some white dwarfs (e.g. Ferrario, Wickramasinghe 1989).

In accreting neutron stars, there is a possibility of the presence of departures from a simple dipole structure. An alternative possibility for the variation of the field could be due to a height distribution of the line-forming region. For a dipole field the variation would be $\Delta B/B \sim 3\Delta R/R$, so the required height range would be $\geq 0.3R$. This scattering could occur in an accretion mound, as in the model of Blandford et al. (1991), or in a suspended scattering atmosphere, as in the model of Dermer and Sturmer (1991) and Sturmer and Dermer (1994). Thus, there is a possibility that the observed spectrum could be the product of a combination of scattering near the surface and scattering in the magnetosphere. Thus, these models require a high field gradient in the line-forming region.

Thompson et al. (2002) also suggested the presence of higher multipoles in SGRs from more complicated pulse
profiles. The pulse profile of the 1998 August 27 giant flare provides direct evidence for the presence of higher magnetic multipoles in SGR 1900+14: four subpulses of a large amplitude appeared during the intermediate portion of the burst, which repeated coherently with a 5.16 s spin period (Feroci et al. 2001; Thompson, Duncan 2001).

In 4U 0115+63, Cen X-3, and Vela X-1 (perhaps also GX 301−2), the dipole magnetic field inferred from $P$ and $\dot{P}$ is lower than those inferred from the cyclotron line energies. In Vela X-1, this feature becomes remarkable. These results may imply that the strength of the magnetic field in the line-forming region is enhanced by a crust-anchored dipole or multipole field. The cyclotron first harmonic lines in the spectra in Vela X-1 could also be difficult to detect. The line at the first harmonic commonly has a highly complex shape due to multiple resonant scattering or “photon spawning” from resonant Raman scattering at the higher harmonics. However, most of the first harmonic lines in accretion-powered X-ray pulsars are shallower and broader, even when the power-law index is large. This suggests that other effects than photon spawning would probably play an important role on smearing the lines.

The crust gives rise to small-scale anomalies that can be modeled by a number of crust-anchored dipoles oriented in different directions (Blandford et al. 1983; Arons 1993). Gil et al. (2002) considered the scenario where the magnetic field on the surface of a neutron star is non-dipolar, assuming the presence of both the fossil field in the core and the crustal field structures. They modeled the actual surface magnetic field by superposition of the star-centered global dipole, $d$, and the crust-anchored local dipole moment, $m$.

The resultant surface magnetic field is

$$B_s = B_d + B_m,$$  

where

$$B_d = \left(\frac{2d \cos \theta}{r^3}, \frac{d \sin \theta}{r^3}, 0\right)$$

and

$$B_m = \frac{3(r-r_s)|m \cdot (r-r_s)| - m|r-r_s|^2}{|r-r_s|^3}.$$  

Here, $r_s$ is a vector pointing to the location of a crust-anchored dipole moment when considering the neutron star center as the origin. There are small-scale deviations of the surface magnetic field from the global dipole.

Gil et al. (2002) mostly considered an axially symmetric case in which both $d$ and $m$ are directed along the $z$-axis (parallel or antiparallel). Since we consider the magnetic field in one dimension, the global dipole and the crust anchored dipole magnetic field is respectively given by

$$B'_d = \frac{2d}{r^3}$$

and

$$B'_m = \frac{2m}{(r-r_s)^3}.$$  

As a result, the resultant surface magnetic field is described by

$$B = B'_d + B'_m = \frac{2d}{r^3} + \frac{2m}{(r-r_s)^3}.$$  

In the present work, we adopt a magnetic field distribution, $B(z)$, that varies linearly with the height of the slab as a more general approach than that given by equation (6) derived from the model proposed by Gil et al. (2002). The field distribution $B(z)$ is given by

$$B(z) = B_0 \pm \frac{z}{z_{\text{max}}} \Delta B.$$  

Here, $z$ is the height from the bottom of the slab, $z_{\text{max}}$ is the height of the slab, $B_0$ is the magnetic field strength at the bottom of the slab and $\Delta B$ is the total variation in the magnetic field strength. In our works, the width of the cyclotron energies, $\hbar \omega_{\text{R}}$, is taken to be 10 keV, which varies linearly between 20 and 30 keV. This model has an advantage that we can easily specify the range of the cyclotron resonant energies, even though this is not a real field. Moreover, the resulting spectra are very similar to those calculated by using equation (6) derived from a model proposed by Gil et al. (2002), while a linear dependence of the field on the height is only a first approximation. Thus, we adopt this model for our present calculations. We calculate the X-ray spectra by solving the radiative transfer through the slab threaded by the magnetic field given by equation (7). We refer to a magnetic field that decreases with height as a “decreasing magnetic field”, and one that increases with height as an “increasing magnetic field.”

2. Calculation Method

2.1. Surface Magnetic Field

According to Urpin et al. (1986), the magnetic field near the surface of neutron stars has a non-dipolar component. The field configuration likely consists of a number of magnetic spots with a typical size $\sim 100$ m. Moreover, Gil and Mitra (2001) have demonstrated that only the presence of a strong non-dipolar surface magnetic field can favor such vacuum-gap formation. Gil et al. (2002) modeled the actual surface magnetic field by superposition of the star-centered global magnetic dipole, $d$, and the crust-anchored local dipole moment, $m$.

The resultant surface magnetic field is

$$B_s = B_d + B_m,$$  

where

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2.2. Geometry

We consider two cases of the 1–0 geometry and the 1–1 geometry as the line-forming regions (Isenberg et al. 1998), as shown in Figure 1. The 1–0 geometry is a slab in which the column depth between the source plane and the bottom of the slab is zero, i.e., a slab illuminated from below. The 1–1 geometry is a slab with the source plane embedded in the middle of the slab. In the present work, \( N_e \) is defined as the column depth between the top surface and the bottom of the slab.

![Diagram of a slab threaded by a non-dipole magnetic field oriented parallel to the slab normal. The 1–0 geometry is a slab with the source plane located at the bottom of the slab. The 1–1 geometry is a slab with the source plane embedded in the middle of the slab.](https://example.com/diagram)

For the 1–0 geometry, we use the boundary conditions of slabs injected with the power-law photon distribution, \( I(\nu) \), from below. The inner boundary condition is free, and the outer one assumes no incoming radiation from outside. Assuming no radiation from above at \( \tau = 0 \),

\[
\mu \left( \frac{\partial u_{\mu\nu}}{\partial \tau_{\mu\nu}} \right)_0 = u_{\mu\nu}(0). \tag{8}
\]

The radiation is assumed to be the intensity, \( I(\nu) = E^{-\alpha} \) from below at \( \tau = \tau_{\max} \); then,

\[
\mu \left( \frac{\partial u_{\mu\nu}}{\partial \tau_{\mu\nu}} \right)_{\tau_{\max}} = I(\nu) - u_{\mu\nu}(\tau_{\max}). \tag{9}
\]

Here, the variable \( u \equiv (1/2)[I(\mu) + I(-\mu)] \), which has a mean-intensity-like character (Mihalas 1978). This boundary condition may correspond to a line-forming region in the magnetosphere of a neutron star (Dermer, Sturner 1991; Sturner, Dermer 1994).

For the 1–1 geometry, we used the boundary conditions with the power-law photon distribution, \( I(\nu) \), emitted in the middle of the slab. The inner boundary condition is free, and the outer one assumes no incoming radiation from outside. Assuming no radiation from above at \( \tau = 0 \),

\[
\mu \left( \frac{\partial u_{\mu\nu}}{\partial \tau_{\mu\nu}} \right)_0 = u_{\mu\nu}(0). \tag{10}
\]

The radiation is assumed to be the intensity, \( I(\nu) = E^{-\alpha} \) emitted in the middle of the slab. At \( \tau = \tau_{\max} \),

\[
\mu \left( \frac{\partial u_{\mu\nu}}{\partial \tau_{\mu\nu}} \right)_{\tau_{\max}} = 0. \tag{11}
\]

This boundary condition may correspond to a line-forming region in a semi-infinite slab at the stellar surface (Slater et al. 1982; Wang et al. 1988, 1989).

3. Results

In Gil et al. (2002)’s model, one should expect that both cases \( m \cdot d > 0 \) and \( m \cdot d < 0 \) will occur with approximately equal probability. We therefore investigate the properties of the cyclotron resonant scattering lines for both cases of a “decreasing magnetic field” and an “increasing magnetic field” in the 1–0 and the 1–1 geometries. In this section, the height of the slab is taken to be 5 m.

3.1. 1–0 Geometry

The 1–0 geometry means that the photon source illuminates the line-forming region from below, which may correspond to a line-forming region in the magnetosphere of a neutron star.

3.1.1. The magnetic field strength decreases with height

Figure 2 shows the cyclotron lines formed through a slab threaded by a magnetic field that decreases linearly with the height. The properties of the cyclotron lines are very similar to those which are formed in the line-forming region threaded by a dipole magnetic field (Nishimura 2003). The ratios of the peak energies of the higher harmonic lines to the fundamental are more than the classical harmonic ratios primarily due to the difference in the physical process of line formation between each harmonic. As discussed in Nishimura (2003), the absorption line in the first harmonic becomes deeper at the redward, since the peak of the first harmonic absorption line forms at the top of the slab due to the effect of scattering in cyclotron resonance. In addition, the optical depth increases with a decrease of the magnetic field strength. In the second harmonic, the process of cyclotron resonance can be mostly considered as pure absorption due to resonant Raman scattering and the optical depth is independent of the magnetic field strength. In the third harmonic, the features of the absorption line are similar to those in the second harmonic, but the absorption line becomes deeper at the blueward since the peak of the third harmonic absorption line forms at the bottom of the slab due to the effect of scattering in cyclotron resonance. In addition, the optical depth increases with a decrease of the magnetic field strength.

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Fig. 2. Spectra for a slab thickness of $5 \times 10^2$ cm, an electron number density of $N_e = 3.14 \times 10^{21}$ electron cm$^{-2}$, the temperature $kT = 5$ keV, the 1–0 geometry and a “decreasing magnetic field”. The solid, dashed, dot-dashed, and thick-dotted curves represent spectra for $\mu = 0.1834$, 0.5255, 0.7967, and 0.9603, respectively. The dotted line denotes the power-law spectrum with $\alpha = 1$ emitted at the source plane. First, second, and third harmonic absorption lines are seen around 20, 40–50, and 80 keV, respectively. The properties of the cyclotron lines are very similar to those which are formed in the line-forming region threaded by a dipole magnetic field. The ratios of the peak energies of the higher harmonic lines to the fundamental are more than the classical harmonic ratios.

and photon spawning from resonant Raman scattering at higher harmonics. Consequently, a broad absorption feature in the first harmonic tends to turn into absorption and emission features at the blue wing, as shown in figure 2. We can therefore conclude that the properties of the cyclotron lines are very similar to those that are formed in the line-forming region threaded by a dipole magnetic field. However, cyclotron lines formed through the slab threaded by a non-dipole magnetic field reveal these features, even in the cyclotron line-forming region with a height of 5 m, while the height of the line-forming region is required to be more than about 500 m in a slab with a dipole magnetic field.

3.1.2. The magnetic field strength increases with height

When the magnetic field strength increases with height, the shapes of the cyclotron lines in the first harmonic become broad and shallow, as shown in figure 3. The shapes of the lines at the first harmonic are similar to those observed in the spectra of accreting X-ray pulsars. Moreover, the ratios of the centroid energies of the second harmonic lines to the fundamental are less than 2, which have actually been observed in 4U 1538–52 (Heindl et al. 1999; Santangelo et al. 1999). Consequently, the peak energy of the absorption line at the first harmonic corresponds to a strong magnetic field compared to those at the higher harmonics. Thus, the scattering events in the cyclotron first harmonic make the peak energy only of the first harmonic line noticeably deviate from the harmonic ratios in the peak energies of all harmonic cyclotron lines. The higher harmonic lines would therefore almost have a harmonic relationship with a spacing of half the peak energy of the second harmonic line. The way of the deviation in the ratios is in agreement with results observed in some accreting X-ray pulsars, such as 4U 0115+63. These properties of the ratios in the peak energies may therefore suggest that the strength of the magnetic field increases with altitude or horizontal distance from an emission region in the line-forming region.

When the shallow portion in the first harmonic absorption feature is formed at the energy side with a larger photon number in this power-law spectrum, a broad absorption feature holds, even if refilling it by photons spawning from resonant Raman scattering at higher harmonics. This is because the portion of the shallow absorption feature is not sufficiently buried by the cyclotron scattered photons in the first harmonic due to the inclination of the incident photon spectrum. Consequently, the portion of the shallow absorption
feature is not completely refilled by photon spawning from resonant Raman scattering at higher harmonics, so that the broad absorption feature can be formed. For \( \alpha < 0 \), the shape of the first harmonic line therefore becomes broader and shallower, as shown in figure 3, when the magnetic field strength increases with height. On the other hand, the decrease in the magnetic field strength with height tends to alter the broad absorption feature to absorption and emission features at the blue wing, as shown in figure 2. As a result, the shape of the first harmonic line is not very broad in spite of the presence of various strengths of the magnetic field.

3.2. 1–1 Geometry

The 1–1 geometry means that the photon source is embedded in the line-forming region, which may correspond to a line-forming region on, or near to, the surface of a neutron star in an accretion column.

3.2.1. The magnetic field strength decreases with height

There are some differences in the shapes of the first harmonic lines between the 1–1 and the 1–0 geometry. The significant emission features in the blueward wing are formed in the 1–1 geometry, as shown in figure 4. The reason is as follows. One broad emission feature is formed in the source plane, since line photons cross the source plane a few times by multiple scattering. Moreover, this emission feature in the source plane is formed over a broad energy band corresponding to the different strengths of the magnetic field, which exist in the line-forming region, as shown in figure 5. The absorption feature is formed in the upper slab with a weaker magnetic field after the formation of the broad emission feature in the source plane, which is formed by photons scattered in the slab with different strengths of the magnetic field. Consequently, strong asymmetry shapes are formed in the first harmonic lines due to the effect of scattering in cyclotron resonance. In contrast, the properties of the lines in the higher harmonics are similar to those in the 1–0 geometry, since the resonant processes in the higher harmonics are nearly pure absorption due to resonant Raman scattering. However, the width of the absorption line is about half of that in the 1–0 geometry, because the higher harmonic lines are formed in the upper half of the total domain in the 1–1 geometry.

3.2.2. The magnetic field strength increases with height

In the 1–1 geometry, one emission and the absorption features of the first harmonic line are different from one broad absorption line formed in the 1–0 geometry, as can be seen in figures 3 and 6. The properties of the cyclotron lines in the 1–1 geometry with the non-dipole magnetic field employed in the present work are as follows. First, the first harmonic lines are broader than those formed through the slab threaded by a uniform magnetic field, but not as broad as those formed in the 1–0 geometry due to the formation of an emission feature at the red wing. Moreover, those consist of one broad emission and absorption features. These features are also different from two strong emission features at both wings of an absorption line in the slab with a uniform magnetic field (cf. figure 12). In addition, these can be shallow, as observed in accreting X-ray pulsars, which were not able to be reproduced only by a global dipole field (Nishimura 2003). Second, the ratios of the peak energies of the higher harmonic absorption lines with respect to the fundamental are less than the classical harmonic ratios. This is because the peak energy of the first harmonic absorption line corresponds to a stronger magnetic field than those in the higher harmonic lines, as the 1–0 geometry. This is in agreement with the ratios of the centroid energies observed in 4U 0115+63 (Heindl et al. 1999; Santangelo et al. 1999), as discussed in section 4.
Figure 7 shows the spectra of photons moving upward ($\mu > 0$) with the angle $\mu = 0.577$ at three locations of the slab in the 1–1 geometry. At the source plane, the complex emission feature forms with the peak energy at $\sim 24$ keV, the energy of which corresponds to the magnetic field strength at the source plane, since some line photons scattered in the vicinity of the source plane cross the source plane many times. Some photons are injected toward the bottom of the slab ($\mu < 0$), and then they are scattered upward and crossed the source plane. After other photons are injected toward the top of the slab ($\mu > 0$), and then scattered in the vicinity of the source plane ($\sim$ within Doppler width), they cross the source plane an even number of times while they are scattered multiply. Thus, at the source plane, a broad and complex emission feature is generated through multiple scatterings in the presence of various strengths of the magnetic field. The emission feature is more complex than that formed through a slab with a uniform magnetic field (Isenberg et al. 1998). The reason is as follows. The energy peak of the emission feature at the source plane indicates the cyclotron energy corresponding to the strength of the magnetic field at the source plane, as shown in figure 7. This is mainly because only the first harmonic line photons scattered in the vicinity of the source plane can cross it many times, since the resonance energy varies with the height. Subsequently, the broad emission feature formed at the source plane is deformed by scattering events in the upper slab with a stronger magnetic field than in the source plane. Consequently, the broad emission feature comes to emerge only at the redward wing. In addition, this emission feature contains photons spawning from resonant Raman scattering at the higher harmonics in the presence of various strengths of the magnetic field. Thus, the broad and complicated emission features at the redward wing are formed due to cyclotron resonant scattering in the presence of various strengths of the magnetic field.

4. Comparison with Observations

In this section, we compare the observations with numerical calculations, while supposing that the peak energies of the calculated cyclotron absorption lines correspond to the centroid energies of the observed cyclotron lines. This peak energy may be different from the fitted centroid of the observed cyclotron absorption line, but in most cases we can expect that the difference is very small.

4.1. Decreasing Magnetic Field

Figures 8 and 9 show the spectra in the 1–0 and the 1–1 geometries for the case of a “decreasing magnetic field” with injected source photon spectra, $\gamma^{-1.3} \times \exp(-h\nu/kT)$, which was employed to fit the spectra in 4U 0115+63 observed with BeppoSAX (Santangelo et al. 1999), respectively. The continuum spectra tend to be deformed by the emission feature at the blue wing, while the three broad absorption features are seen in the emergent spectra. The peak energies of the second harmonic absorption features could be detected to be higher due to the steep slope of the continuum at the higher energies. Two cyclotron line harmonics were detected in the average spectrum of the high-mass X-ray binary 4U 1907+09 observed with the BeppoSAX satellite on 1997 September 27 and 28 (Cusumano et al. 1998). They reported that the second line appears deeper than the first line. The ratio of the centroid energy of the second line with respect to the first line is also more than 2, in which two absorption features were detected at 18.8 keV and 39.4 keV, respectively. This ratio may be
attributed to the distribution of the magnetic field strength. According to Kreykenbohm et al. (1999), the ratio of the centroid energies also seems to be more than 2 in the accreting X-ray pulsar Vela X-1. They reported that the second harmonic line seems not to be coupled by a factor of 2 in centroid energy to the fundamental, but by a factor of 2.3 to 2.5 instead. In these X-ray pulsars, we can explain these ratios by considering either a large-scale line-forming region (∼1 km) with a dipole magnetic field or a small-scale slab (∼1–10 m) with a non-dipole magnetic field that decreases with height.

4.2. Increasing Magnetic Field

An “increasing magnetic field” tends to render the features of the cyclotron lines in the first harmonic broad and shallow, especially for the 1–0 geometry. Figures 10 and 11 show the spectra in the 1–0 and the 1–1 geometries for the case of an “increasing magnetic field” with injected source photon spectra, \( \gamma^{-1.3} \exp(-h\nu/kT) \), which was employed to fit the spectra in 4U 0115+63 observed with BeppoSAX. Here, we assumed an electron number density of \( N_e = 1.5 \times 10^{21} \text{ electron cm}^{-2} \), \( z = 5 \times 10^2 \text{ cm} \).

most significant in the main and secondary pulses, respectively. Four absorption-like features have also been observed in the spectrum of the recurrent hard pulsating X-ray transient 4U 0115+63 with BeppoSAX (Santangelo et al. 1999). The ratios between the centroid energies of the lines with respect to the fundamental are 1 : (1.9) : (2.8) : (3.9). These ratios are appreciably different from the classical values, 1 : 2 : 3 : 4. They, however, pointed out that this result can be attributed entirely to the value of the centroid of the first harmonic, i.e., for a power-law plus cutoff model, the first harmonic would be at 12.79 ± 0.05 keV, while a fit to the other three harmonics gives a spacing of 12.02 ± 0.02 keV. The determination of its centroid energy could be responsible for a somewhat inadequate modeling of the continuum, since the first harmonic line is located close to the energy interval over which the slope of the X-ray spectrum steepens rapidly.

However, our model is able to explain these ratios by the slab with an “increasing magnetic field”, regarding the peak energies of the cyclotron absorption lines as the centroid energies of the observed cyclotron lines. The peak energy of the first harmonic line only deviates significantly from the classical harmonic ratios in the peak energies of the cyclotron lines due to the effect of scattering, while those of the higher harmonic lines do not show considerable deviations from the classical harmonic ratios due to almost absorption in the cyclotron resonance processes. As a result, the higher harmonic lines would almost have a harmonic relationship with the X-ray spectrum from the classical harmonic ratios in the peak energies of the observed cyclotron lines. The peak energy of the first harmonic line only deviates significantly from the classical harmonic ratios in the peak energies of the cyclotron lines due to the effect of scattering, while those of the higher harmonic lines do not show considerable deviations from the classical harmonic ratios due to almost absorption in the cyclotron resonance processes. As a result, the higher harmonic lines would almost have a harmonic relationship with the X-ray spectrum from the classical harmonic ratios in the peak energies of the observed cyclotron lines.
of the peak energies of the higher harmonic absorption lines with respect to the fundamental are $1 : 1.84 : 2.76$, where the peak energy of each harmonic absorption line indicates $29.0$ keV, $53.5$ keV, $80.0$ keV, respectively. The peak energy of the third harmonic absorption line, however, is about three-times half of that of the second harmonic. The ratio of the peak energies of the absorption lines at the second to the fundamental are clearly less than those in a slab threaded by a uniform magnetic field. The deviation from harmonic ratios expected by a relativistic effect therefore seems not to be sufficient to explain that observed in some accreting X-ray pulsars, such as 4U 0115+63. The variation in the magnetic field strength, such as the non-dipole component, should probably be effective in the formation of the cyclotron lines. This result may imply that the strength of the magnetic field increases with altitude, or horizontal distance, in the line-forming region of some accreting X-ray pulsars.

4.3. Trends of the Peak Energies of Cyclotron Lines in Each Harmonics

The peak energy of the first harmonic absorption line shifts to redward of the line center for a “decreasing magnetic field” and blueward of line center for an “increasing magnetic field”, principally due to the effect of scattering in the cyclotron resonance. The peak energy of the second harmonic tends not to shift appreciably, since the opacity at the second harmonic is independent of the strength of the magnetic field. The peak energy of the third harmonic tends to shift somewhat to blueward of the line center for both a “decreasing magnetic field” and an “increasing magnetic field”, because the opacity at the third harmonic increases with the strength of the magnetic field. Consequently, the peak energy of the third harmonic absorption line would tend to become somewhat higher than three-times half of the peak energy of the second harmonic. The trend at the third harmonic is detected in such observations as 4U 0115+63 (figure 9 of Heindl et al. 2004).

5. Conclusions

We investigated the influence of a strong change in the field strength, such as the presence of a non-dipole component in the line-forming region, on the properties of the cyclotron lines. For a “decreasing magnetic field”, the properties of the
cyclo-tron resonant scattering lines are similar to those formed through the slab with the dipole magnetic field (Nishimura 2003). The ratios of the cyclotron peak energies of the higher harmonics to the fundamental tend to be less than the classical harmonic ratios. In contrast, for an “increasing magnetic field”, the ratios of the cyclotron peak energies of the higher harmonics to the fundamental tend to be less than the classical harmonic ratios. Cyclotron absorption lines in the X-ray spectra of accreting X-ray pulsars sometime indicate non-harmonicity in the ratios of the centroid energies of the line at the higher harmonics to the fundamental. In observations, the ratios are more than or less than the classical harmonic ratios. This could imply that the magnetic field strength in the line-forming region decreases or increases with altitude or horizontal distance. On considering Gil et al. (2002)’s model, the centroid energy ratios will be expected to become more than or less than the classical harmonic ratios with the same probability. We, however, have only three observed examples: 4U 1907+09, Vela X-1 (more than the classical harmonic ratios), and 4U 0115+63 (less than the classical harmonic ratios). We will need more observational data to discuss these features in more detail. In the present paper, we adopt the magnetic field that varies linearly with height as a more general approach. The resulting spectra are very similar to those calculated by equation (6) derived from a model proposed by Gil et al. (2002). The variation in the magnetic field strength, such as the non-dipole component, should be considered in the line-forming region to explain the ratio of the centroid energies of the cyclotron lines of the higher harmonics to the fundamental in some accreting X-ray pulsars.

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Appendix. Radiative Transfer

We assume a static, plane-parallel slab threaded by a vertical magnetic field. We use the Feautrier method to solve the radiative transfer problem with cyclotron resonant scattering in the presence of a magnetized plasma. Computations were mainly carried out with 50 frequencies and 4 angles. The radiative transfer equation can be written as one second order equation carried out with 50 frequencies and 4 angles. The radiative transfer equation in the X-ray spectra of accreting X-ray pulsars sometime indicate non-harmonicity in the ratios of the centroid energies of the line at the higher harmonics to the fundamental. In observations, the ratios are more than or less than the classical harmonic ratios. This could imply that the magnetic field strength in the line-forming region decreases or increases with altitude or horizontal distance. On considering Gil et al. (2002)’s model, the centroid energy ratios will be expected to become more than or less than the classical harmonic ratios with the same probability. We, however, have only three observed examples: 4U 1907+09, Vela X-1 (more than the classical harmonic ratios), and 4U 0115+63 (less than the classical harmonic ratios). We will need more observational data to discuss these features in more detail. In the present paper, we adopt the magnetic field that varies linearly with height as a more general approach. The resulting spectra are very similar to those calculated by equation (6) derived from a model proposed by Gil et al. (2002). The variation in the magnetic field strength, such as the non-dipole component, should be considered in the line-forming region to explain the ratio of the centroid energies of the cyclotron lines of the higher harmonics to the fundamental in some accreting X-ray pulsars.

The photon propagation angle relative to the magnetic field, $hν$ is also the energy of a photon. The source function is given by

$$S_ν = \int_{ν_{max}}^{ν_{min}} \int_0^1 dν'dμ'R(ν, μ; x', μ')u(μ', ν'), \quad (A3)$$

where $R(ν, μ; x', μ')$ represents the redistribution function of the cyclotron resonant scattering. We use the complete redistribution, $φ(x, μ)φ'(x', μ')$, for the redistribution function. Here, $φ(x, μ)$ is the nonrelativistic polarization-averaged scattering profile (cf. Wasserman, Salpeter 1980). The absorption is also negligible, since the density is assumed to be sufficiently low.

For simplicity, the density of the slab is assumed to be uniform. We employ the equilibrium temperature, $T_e$ ~ (1/4)$ω_B$, for the temperature of the cyclotron line-forming region. This equilibrium temperature was calculated by Lamb et al. (1990) as a function of the field strength and column depth through the slab from a balance of the cooling and heating that arise from cyclotron resonant and nonresonant scattering. Here, $ω_B$ is taken to be the lowest cyclotron frequency of the range (= 20–30 keV) considered in our calculations. The temperature of the slab can be influenced by a rapid variation in the magnetic field strength. It, however, would not significantly affect the peak energies at each absorption line, i.e., our conclusions.

We also used the relativistic cross sections summed over the final spin states and averaged over the photon polarizations (Harding, Daugherty 1991) up to the third harmonic, which are the same one as Nishimura (2002). Wang et al. (1988) argue that the polarization-averaged cross sections are suitable for first harmonic scattering in optically thick media when the vacuum contribution to the dielectric tensor dominates the plasma contribution, i.e.,

$$\frac{w}{\delta} = 3 \times 10^{-6} \left( \frac{N_e}{10^{23} \text{ cm}^{-3}} \right) \left( \frac{B}{B_c} \right)^{-4} \ll 1, \quad (A4)$$

where $w = (ω_p/ω_B)^2$ is the plasma frequency parameter, $ω_p$ the plasma frequency, $ω_B$ the cyclotron frequency, $δ$ the magnetic vacuum polarization parameter (Adler 1971), $N_e$ the electron number density. The present calculations are performed under these conditions. The cyclotron energy is given by

$$\hbar ω_B = mc^2[(1 + 2nB′ sin^2 θ)/2 - 1]/sin^2 θ. \quad (A5)$$

Here, $n$ represents the quantum number in the Landau levels. For a slab column density of $N_e$, the polarization- and frequency-averaged optical depth in the cyclotron first harmonic in the limit $n(B/B_c) \ll 1$ is

$$τ_1 \propto N_e B_{12}^{-1} \left( \frac{T}{\text{keV}} \right)^{-1/2} \left( \frac{1 + μ^2}{|μ|} \right), \quad (A6)$$

where $B = B_{12} \times 10^{12}$ G and $T$ is the electron temperature in keV (cf. Wang et al. 1993). The optical depth at the second and third cyclotron harmonics in the limit $n(B/B_c) \ll 1$ are

$$τ_2 \propto N_e \left( \frac{T}{\text{keV}} \right)^{-1/2} \left( \frac{1 + μ^2 sin^2 θ}{|μ|} \right), \quad (A7)$$

and
\[ \tau_3 \propto N_e B_{12} \left( \frac{T}{\text{keV}} \right)^{-1/2} \frac{(1 + \mu^2) \sin^4 \theta}{|\mu|}. \] (A8)

We assume the cold-plasma approximation \((kT_e \ll \hbar \omega_B)\) in which the Landau level spacing is much larger than the typical electron energy, so that the Landau levels are not collisionally populated. In addition, the photon densities are assumed to be sufficiently low, so that the levels are not radiatively populated. We can therefore consider that the initial and final electron state is the Landau ground state \((n = 0)\) in each scattering. We take the distribution of the electron momenta along the field, \(f(p)\), to be a one-dimensional nonrelativistic Maxwellian, \(f(p)dp = N_e [\exp(-u^2)/\sqrt{\pi}] du\), where \(u = p/\sqrt{2m_e T}\). The effect of the relativistic scattering profile is ignored. The shape of the scattering profile is deformed by a relativistic effect (Lamb et al. 1989). However, it would not strongly affect the ratio of the energies of the line center in each harmonics from the results of Wang et al. (1993). This approximation, therefore, would not affect the conclusions in the present paper.

We employed relativistic transition rates that were the same as those of Nishimura (2002). When the cyclotron energy, for instance, is 30 keV, most electrons (\(\sim 92\%\)) excited to the second Landau state by absorbing a photon at the cyclotron resonant scattering decay to the first excited state, and then to the ground state. In the case of excitation to the third Landau state, the electron decay to the second excited state with a transition probability of about 84\%. Thus, the cyclotron resonant scattering at the second and third harmonics can be almost considered as true absorption. In addition, variations with height in the transition rates are only by about 2\% relative to the transition rate at the middle of slab. We use therefore the transition rate calculated by using the magnetic field strength in the middle of slab.

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