PyHHMM: A Python Library for Heterogeneous Hidden Markov Models

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Abstract

We introduce PyHHMM, an object-oriented open-source Python implementation of Heterogeneous-Hidden Markov Models (HHMMs). In addition to HMM’s basic core functionalities, such as different initialization algorithms and classical observations models, i.e., continuous and multinoulli, PyHHMM distinctively emphasizes features not supported in similar available frameworks: a heterogeneous observation model, missing data inference, different model order selection criterias, and semi-supervised training. These characteristics result in a feature-rich implementation for researchers working with sequential data. PyHHMM relies on the numpy, scipy, scikit-learn, and seaborn Python packages, and is distributed under the Apache-2.0 License. PyHHMM’s source code is publicly available on Github\(^1\) to facilitate adoptions and future contributions. A detailed documentation\(^2\), which covers examples of use and models' theoretical explanation, is available. The package can be installed through the Python Package Index (PyPI).

Keywords: Heterogeneous Hidden Markov Models, python, missing data inference, semi-supervised training

1. Introduction

Hidden Markov Models (HMMs), as defined by Rabiner (1989), are generative models where the modeled system is assumed to be a Markov process, in which an observation model explains the observed data through a hidden variable.

Different frameworks that implement these well-known models are publicly available. However, these implementations usually lack features required to use these models with real-world datasets: missing data inference, ability to manage heterogeneity, semi-supervised training support to increase hidden states’ interpretability, and synthetic data generation. Aiming to provide an HMM implementation that both academics and industry professionals can use, we present PyHHMM, an open-source Python toolbox that, in contrast to existing libraries, supports the previously enumerated features and constitutes a valuable alternative to work with sequential data.

\(^1\) https://github.com/fmorenopino/HeterogeneousHMM  
\(^2\) https://pyhhmm.readthedocs.io/en/latest

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2. Heterogeneous Hidden Markov Model

Hidden Markov Models’ objective is to learn the hidden states sequence, denoted $S = \{s_1, s_2, \ldots, s_T : s_t \in 1, \ldots, I\}$, with $t = \{1, 2, \ldots, T\} \in \mathbb{N}$, that better explain the observed data, $Y = \{y_1, y_2, \ldots, y_T : y_t \in \mathbb{R}^M\}$. To do so, they use an observation model $p(y_t|s_t)$. Therefore, each observation $y_t$ depends exclusively on its associated state $s_t$. Nevertheless, standard HMM implementations use Multinomial and Gaussian observation models, depending on the probability distribution chosen to model the emission probabilities $p(y_t|s_t)$. Heterogenous-HHMs (HHMMs) encompass these two variations of the HMM, combining Gaussian and discrete observations. Therefore, the HHMM can manage heterogeneous data by using different observation emission probability distributions to model the conditional dependencies of the hidden states on the observations.

HHMMs can be fully characterized via the hidden states sequence, $S$; the continuous observations sequence, $Y$; its associated continuous observations emission probabilities, $B = \{b_i : p_{b_i}(y_t) = p(y_t | s_t = i)\}$; the discrete sequence observations, $L = \{l_1, l_2, \ldots, l_T : l_t \in 1, \ldots, J\}$; its associated discrete observations emission probabilities, $D = \{d_{im} : d_{im} = P(l_t = m | s_t = i)\}$; the state transition probabilities, $A = \{a_{ij} : a_{ij} = P(s_{t+1} = j | s_t = i)\}$; and the initial state probability distribution, $\pi = \{\pi_i : \pi_i = P(s_1 = i)\}$. HHMM’s architecture is represented in Figure 1.

![Figure 1: Heterogeneous HMM architecture. Gray represents observed data.](image)

HHMMs, as classic HMMs, present three inference problems:

1. Given the observed data sequences $\{Y, L\}$ and the model’s parameters $\theta = \{A, B, D, \pi\}$, to estimate the probability of the observed sequence given the model, $p(Y, L|\theta)$.
2. Given $\{Y, L\}$ and $\theta$, to determine the optimal hidden states sequence $S$ that better explains the data. This can be achieved by using the Forward-Backward algorithm, Yu and Kobavashi (2003), calculating $p(s_t|y_t, l_t)$ each time-step, or through the Viterbi algorithm, Forney (1973), which maximizes the probability of the hidden states sequences by considering all time-steps $t = \{1, 2, \ldots, T\}$, i.e., calculating $p(S|y_t, l_t)$.
3. To determine the optimal parameters $\theta$ that maximize the conditional probability $p(Y, L|\theta)$, which can be achieved through the Baum-Welch algorithm, Frazzoli (2013). The joint distribution required for this third task, which may be modified to support any other type of observation emission probability’ distribution, is expressed in Equation 1.
For more information regarding these problems and how to solve them, we refer the readers to Rabiner (1989), Anikeev and Makarevich (2006), and Cappé et al. (2009).

\[
p(S, Y, L) = \prod_{n=1}^{N} \left( p(s_i^n) \prod_{t=2}^{T_n} p(s_i^n | s_{i-1}^n) \right) \left( \prod_{t=1}^{T_n} p(y_t^n | s_i^n) \right) \left( \prod_{t=1}^{T_n} p(l_t^n | s_i^n) \right)
\]

(1)

3. Library Implementation and Documentation

PyHHMM implements three different model’s designs depending on the probability distribution that is chosen to manage the observed data: DiscreteHMM.py, GaussianHMM.py, and HeterogenousHMM.py. The three of them have a common parent class _BaseHMM.py that implements general functions for the three models, as the forward and backward methods, the log-probability computation, the training step, or the sampling functions.

The library has dependencies on the Python packages numpy, scipy, scikit-learn, and seaborn. Documentation can be found on the Github repository as well as example notebooks that show simple use-cases to help new users understand the library’s functioning. These notebooks explain how to define the three available models, solve the described problems (training models, estimating data’s likelihood, and decoding observation sequences), utilize the available model selection criteria, and sample data from the trained models. Theoretical properties of PyHHMM’s implemented models and a description of the missing data inference process are also available.

Regarding code quality, unit tests cover the implemented functions. The code is released under Apache-2.0 License and is available at Github, which allows for collaborative development and facilitate the inclusion of prospect new features required by the community. Also, the package can be installed through PyPI.

4. Functionality and Comparison to other Software

Three primary Python HMM implementations were identified: hmmlearn, pomegranate, and pyro. While some of PyHHMM’s features are also available in these libraries, none of these other packages support missing values inference, data heterogeneity, or semi-supervised training, core features while modeling real-world datasets. PyHHMM covers these necessities and some additional functionalities:

- Missing data inference: available models can be trained in the presence of missing observations. Both cases, complete missing and partially missing, are covered by our implementation. Also, PyHHMM can perform missing value inference as a preprocessing technique. For further information regarding the inference process, consult Murphy (2012) and our package’s documentation.
- Semi-supervised training: PyHHMM allows the models to be trained in a semi-supervised manner, fixing discrete observations’ emission probabilities, therefore using discrete sequences as labels. This semi-supervised training allows a guided learning

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3. pip install pyhhmm
4. https://hmmlearn.readthedocs.io/en/latest/index.html
5. https://pomegranate.readthedocs.io/en/latest/
6. https://pyro.ai/examples/hmm.html
process that improves the interpretability of the model parameters: certain states are
associated with particular values of the labels.
• Model selection criteria: Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are implemented to estimate the model’s optimal number of hidden states.
• Diagonal, full, tied, or spherical covariance matrices can model the Gaussian observations’ emission probabilities.
• Synthetic data generation, sampling from the trained models.

5. Basic Example

This section provides a basic example of training a Heterogeneous-HMM over a set of pre-loaded sequences and using the resulting model to perform decoding. Examples of obtaining the AIC of the trained model and how to generate new synthetic samples using the resulting architecture are also included.

```python
>>> from pyhhmm.heterogeneous import HeterogeneousHMM
>>> import pyhhmm.utils as ut

>>> my_hmm = HeterogeneousHMM(n_states=2, n_g_emissions=2,
                             n_d_emissions=2, n_d_features=[2, 2], init_type="random",
                             covariance_type="diagonal", nr_no_train_de=1,
                             verbose=True) # Initializing a HeterogeneousHMM

>>> my_hmm, log_likelihood = my_hmm.train(training_seq, n_init=1,
                                         n_iter=50, thres=0.001, conv_iter=5,
                                         plot_log_likelihood=True) # Training

>>> logL, state_seq = my_hmm.decode([training_seq[0]],
                                         algorithm="viterbi") # Decoding

>>> dof = ut.get_n_fit_scalars(temp_gHmm) # Free parameters of the model

>>> AIC = ut.aic_hmm(log_likelihood, dof) # AIC of the trained model

# Generating new sampled data with the trained model

>>> X, state_sequences = my_hmm.sample(n_sequences=n_sequences,
                                          n_samples=n_samples)
```

6. Summary

PyHHMM is a flexible toolbox currently being used in academia and industry projects. By implementing well-known HMMs algorithms, this library provides a proper solution to several problems, such as obtaining quick predictors for discrete-time stationary processes, generating new synthetic data, or performing missing data inference. Detailed documentation and several examples are available on the project’s site.
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