Detection Algorithms on GNSS Millimeter Displacement using Orthogonal Functions*

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This paper presents mathematical algorithms to measure a noise-buried destructive displacement using GNSS (Global Navigation Satellite System) precise positioning and the orthogonal function expansion of its positioning result. In reality, there are 5.25 hundred thousand places in danger of landslide in Japan. The algorithms must retrieve a millimeter-scale displacement in the observation noise, therefore they apply the orthogonal functions from the class of Gegenbauger polynomials and the orthogonal even functions. This method informs the foreboding of a destruction, and will save many lives and infinite property of assets in catastrophic disasters from the present time.

1. Introduction

The MLIT (Ministry of Land, Infrastructure, Transport and Tourism) of Japan has been warning 5.25 hundred thousand places in danger of landslide sediment disasters in Japan [1, 2].

Industrial companies provide GNSS (Global Navigation Satellite System) detection systems and services for this disaster problem. GNSS precise sensors were, however, expensive in past years, and a data-analyzing time is long, and more rapid detection is strongly required. On the other hand, the latest GNSS sensors are low-cost and high performance. Therefore, a new and effective solution has been expected.

In order to respond this solution need, we develop a new principle of mathematical algorithms for detecting a destructive displacement from GNSS precise positioning observation. The algorithms must retrieve a millimeter-scale positional change which is modelled as a step function in observation noise of GNSS precise positioning system.

Some of authors present the effect of this method in the previous articles [3, 4], but we present concrete algorithms for the first time.

2. Models of Problem

A destructive displacement is a discontinuous phenomenon that occurs in a landslide of slopes and a constructional collapse of bridges and buildings. In this section, we define a displacement model and a measurement model for this disaster problem, and show a scope of GNSS techniques which we can use. Japanese GNSS, Quasi-Zenith Satellite System (QZSS)[5] also can be applied to this method.

2.1 Displacement Model

In Japan, more than 20 industrial companies of civil engineering have established the Shamen-net Study Group, and promote to research the displacement measurement using GNSS. They adopt the step function for the destructive displacement model, and the analysis divides each dimension for simplicity [6].

A landslide and a constructional collapse are phenomena which occur in the 3-dimensional space. But generally speaking, a direction of landslide or collapse is known in advance. Because the direction of falling down by the gravity is determined in actual places. Therefore we treat it as 1-dimensional problem in this article.

We define the displacement function $s_D(t)[\text{mm}]$ that a step-shaped displacement occurs at the time $t_D[\text{s}]$ as Eq. (1) and Fig. 1.

$$s_D(t) = \begin{cases} 
\mu_0[\text{mm}] & (t < t_D[\text{s}]), \\
\mu_1 = \mu_0 + \Delta \mu[\text{mm}] & (t \geq t_D[\text{s}]).
\end{cases}$$

(1)

In Fig. 1, $\mu_0[\text{mm}]$ is the position before the displacement, $\mu_1[\text{mm}]$ is the position after the displacement, the start time of measurement is $0[\text{s}]$ the end time is $T[\text{s}]$. The displacement function $s_D(t)$ makes the displacement length $\Delta \mu[\text{mm}]$ at the time $t_D[\text{s}]$. 
2.2 Measurement Model

The measurement data of GNSS positioning \( x(t) [\text{mm}] \) are the sum of the displacement function \( s_D(t) [\text{mm}] \) and the observation noise \( w(t) [\text{mm}] \) shown as

\[
x(t) = s_D(t) + w(t) [\text{mm}].
\]

The problem of this article is to find the occurrence time \( t_D [\text{s}] \) and displacement \( \Delta \mu [\text{mm}] \) by retrieving the displacement function \( s_D(t) [\text{mm}] \) from the measurement data \( x(t) [\text{mm}] \) with observation noise \( w(t) [\text{mm}] \).

This is the so-called second level problem of Van Trees [7] which detects a signal with unknown parameters among noises. In this case, unknown parameters are the occurrence time \( t_D [\text{s}] \) and displacement \( \Delta \mu [\text{mm}] \).

The basis of signal detection and estimation for radars and sonars is given by Harry L. Van Trees [7]. But major countries have deployed new generation GNSS systems by 2020, and the characteristics of millimeter-scale positioning is not known well yet.

In particular, it is a large problem to find rapidly millimeter-scale displacement using the ranging signals transmitted by the navigation satellites which are more distant than 20 thousand kilometers. This study presents the algorithms which make it possible.

2.3 Scope of GNSS Techniques

In this section, we clarify a scope of GNSS technique which can be applied by these detection algorithms described later.

These algorithms are used to centimeter-class measurement data which include the pseudoranges \( \rho_{i,u}^p(t) \) and the carrier-phases \( \phi_{i,u}^p(t) \) of the ranging signal of L-band \( i \)-th frequency at the time \( t \) from the satellite \( p \) to the user receiver \( u \). When the integer biases of the carriers have been solved, the phase-range \( \Phi_{i,u}^p(t) \) is represented by

\[
\Phi_{i,u}^p(t) = \lambda_i \phi_{i,u}^p(t),
\]

where \( \lambda_i \) is the wave length of the ranging signal of \( i \)-th frequency of L-band. The pseudoranges, the carrier-phases, and the phase-ranges are specified in RTCM Standard 10403.3 [8].

Pseudoranges \( \rho_{i,u}^p \):

\[
\rho_{i,u}^p(t) = \gamma_i^p(t, t - \tau_{i,u}^p) + c (\delta t_u(t) - \delta t_u(t - \tau_{i,u}^p)) + \frac{f_i^2}{j_i^2} \delta I_{i,u}^p(t) + \delta T_{i,u}^p(t) + \epsilon_{i,u}^p(t),
\]

Phase-ranges \( \Phi_{i,u}^p \):

\[
\Phi_{i,u}^p(t) = \gamma_i^p(t, t - \tau_{i,u}^p) + c (\delta t_u(t) - \delta t_u(t - \tau_{i,u}^p)) - \frac{f_i^2}{j_i^2} \delta I_{i,u}^p(t) + \delta T_{i,u}^p(t) + \lambda_i N_{i,u}^p + \epsilon_{i,u}^p(t),
\]

where

- \( \gamma_i^p(t, t - \tau_{i,u}^p) \): geometric ranges,
- \( \delta t_u(t) \): receiver delay times,
- \( \lambda_i \): radio propagation time,
- \( \delta t_u(t) \): satellite delay times,
- \( \delta I_{i,u}^p(t) \): ionospheric delay errors,
- \( \delta T_{i,u}^p(t) \): tropospheric delay errors,
- \( N_{i,u}^p \): integer biases of cycle,
- \( \epsilon_{i,u}^p(t) \): pseudorange measurement noise,
- \( c \): light speed = 299,792,458 [m/s].

The number of frequencies \( i \) and central frequencies \( f_i \) are shown in Table 1.

| \( i \)  | frequency [MHz] | wave length [m] |
|---|---|---|
| 1 | 1575.42 | 0.19 |
| 2 | 1227.60 | 0.24 |
| 3 | 1176.45 | 0.25 |

In GNSS techniques, both RTK (Real-Time Kinematic) and PPP (Precise Point Positioning) can be applied to the above observation Eqs. (4) and (5). We introduce the following four GNSS techniques.

1) RTK GNSS

This technique provides the corrections data in OSR (Observation State Representation) for GNSS precise positioning, that provides the data of the pseudoranges \( \rho_{i,k}^p \) and carrier-phases \( \phi_{i,k}^p \) at the reference stations to user receivers.

The 3GPP (3rd Generation Partnership Project) standards have adopted this technique in its specifications of the release 15 [10]. In Japan, NTT DoCoMo, Softbank Group and the professional distributors provide their services using this technique as of 2020.

2) Network RTK GNSS

This technique provides the correction data of the pseudoranges \( \rho_{i,k}^p \) and carrier-phases \( \phi_{i,k}^p \) at nonphysical and physical reference stations \( k' \), to the user receivers. These corrections are OSRs.

Eqs. (4) and (5) mean the SSR data are equivalent to the OSR data, and both representations can be convertible each other. 3GPP standards have adopted this technique in the release 15 [10]. In Japan, several professional distributors provide their services using...
The standard deviation of the measurement data \( x(t) \) at 1 [Hz] rate for the descriptions of the proposed method. Data A has the displacement of 5[mm] at the time 1201[s] artificially, and Data B does not include the displacement.

When the displacement does not occur, we obtain \( \mu = -6.64[\text{mm}] \) and \( \sigma = 3.82[\text{mm}] \) from all period data of Data B. The standard deviation \( \sigma \) has been considered as constant, and it adopts this value. The rejection region is 5%, therefore a z-testing reference \( z = 1.96 \).

By the way, the measurement period 1 hour comes from the specification of “GNSS static method” in the MLIT authorized manual of the national public surveying. It means an enough period which GNSS observation becomes stable in precise surveying.

![Sample measurement data](image)

**Fig. 2 Sample measurement data**

In case that the displacement exists, we use Data A. The hypothesis testing is conducted in period 1801-2400[s] after the occurrence time. \( \bar{X}_0[\text{mm}] \) is an average and \( z_0 \) is the z-testing value in this period.

By the way, the sample period 20 minutes comes from the specification of “GNSS shortened static method” and one typical period of GNSS observation.

\[
\bar{X}_0 = -8.92[\text{mm}], \\
|\bar{X}_0 - \mu| = -2.27[\text{mm}], \\
z_0 = 20.58 > z = 1.96.
\]

According to the testing result, the null hypothesis has been rejected, and the displacement occurs. But the problem of averaging which is well-known in the business field, is that “Type I error” which means we judge the displacement exists, easily occurs despite no displacement.

We prove the above fact using Data B with no displacement as follows. In other words, the problem of averaging is to reject null hypothesis \( H_0 \), despite there is not displacement, and we check it.

We take 3 sample-periods from \( t_0[\text{s}] \) to \( t_1[\text{s}] \) as shown in Table 2, and test using Eq. (8).
1200 | −5.69 | 0.952 | 8.62 |
2 | 1201 | 0.623 | 5.65 |
3 | 2401 | 0.189 | 1.71 |

Comparing the reject region 5% and \( z = 1.96 \), the calculation results of the \( z \)-testing value \( z_i \) in respective cases are

\[
\begin{align*}
ze_1 &= 8.62 > z = 1.96, \quad (12) \\
ze_2 &= 5.65 > z = 1.96, \quad (13) \\
ze_3 &= 1.72 < z = 1.96. \quad (14)
\end{align*}
\]

The null hypothesis has been not always accepted, and some cases must reject it. It means the averaging of conventional method easily makes “Type I error.” We could not adopt the averaging of conventional method for this problem. Therefore, new algorithms must be developed.

4. Study for Solution

In order to solve this problem, we study the algorithms to use the orthogonal function expansion for GNSS measurement data, which means that

1. Apply the orthogonal function expansion to the measurement data, and represent the sum of components.
2. Eliminate unwanted components, and remain wanted components for our purpose.
3. Retrieve the occurrence time \( t_D \) and the displacement \( \Delta \mu \) from the wanted components.

4.1 Breakdown by Orthogonal Functions

At first, we expand the measurement data \( x(t) \) to orthogonal functions \( \Psi(t) \), and represent the sum of components such that

\[
\Psi(t) = \sum_{i=0}^{n} a_i \psi_i(t) dt.
\]  

(15)

where \( a_i \) are the coefficients, \( \psi_i(t) \) are the continuous real functions, \( n \) is the order of the orthogonal functions.

In this article, “fit” is used to represent the measurement data as the sum of adequate orthogonal functions. The measurement data \( x(t) \) are represented as the vector \( y \) such that

\[
y \equiv [ x(t_1) \ x(t_2) \ ... \ x(t_n)]^T.
\]  

(16)

The coefficient vector \( a \) consists of the components \( a_i \), which come from the orthogonal function expansion. It is given by

\[
a \equiv [ a_0 \ a_1 \ ... \ a_n]^T.
\]  

(17)

Then a matrix \( G \in \mathbb{R}^{n \times n} \) is defined using \( \psi_i(t) \) such that

\[
G \equiv \begin{bmatrix}
\psi_0(t_1) & \psi_0(t_2) & \cdots & \psi_0(t_n) \\
\psi_1(t_1) & \psi_1(t_2) & \cdots & \psi_1(t_n) \\
\vdots & \vdots & \ddots & \vdots \\
\psi_{n-1}(t_1) & \psi_{n-1}(t_2) & \cdots & \psi_{n-1}(t_n)
\end{bmatrix}.
\]  

(18)

\( a \) is solved using the least square method such that

\[
a = (G^T G)^{-1} G^T y.
\]  

(19)

We use \( a \) in Eq. (19) to eliminate the components of orthogonal functions, and calculate the accumulated residual function \( R_s(t) \), which is described later. The components are represented as Eq. (15) using the orthogonal functions \( \psi_i(t) \).

4.2 Identification by Elimination

In Eq. (15), \( a_i \) and \( \psi_i \) come from the characteristics of the original measurement data \( x(t) \). In general, the original data are represented the sum of eigenvalues and eigenvectors of themselves. These are inner-data characteristics of \( x(t) \).

But, it can be said rather difficult to solve from inner-data characteristics of the measurement data. Therefore, we will study the method to mandatorily eliminate specific components by external force, and retrieve a noise-buried displacement. In other words, we will eliminate the components which make the displacement invisible.

The residual function is represented as follows. Our algorithms intentionally draw out specific components as Eq. (21).

\[
r_s(t) = x(t) - \Psi(t)
\]  

(20)

\[
r_s(t) = x(t) - \sum_{i=0}^{n} a_i \psi_i(t).
\]  

(21)

If we can retrieve the step-moved characteristics of the displacement function \( s\phi(t) \) at the occurrence time \( t_D \), for example, as a clear peak, the occurrence time \( t_D \) will be found from the accumulated residual function \( R_s(t) \) which is described later.

4.3 Role of Orthogonal Polynomials

In this section, we use the orthogonal “polynomials” as the function \( \psi_i(t) \). The orthogonal polynomials are generalized as the Jacobi polynomials, and its special example is the Gegenbauer polynomials.

The Gegenbauer polynomials satisfy the following recursive relation [17].

\[
C_0^\alpha(t) = 1, \quad C_1^\alpha(t) = 2t, \quad C_n^\alpha(t) = \frac{1}{n} \{ c_1 C_{n-1}^\alpha(t) - c_2 C_{n-2}^\alpha(t) \}.
\]  

(22)

where

\[
c_1 = 2t(n + \alpha - 1), \quad c_2 = n + 2\alpha - 2.
\]  

(23)

An easy-to-use case is
The displacement function is expressed as
\[ P_a(t) = \frac{1}{2^n} \sum_{i=0}^{n} \binom{n}{i} (t-1)^{n-i} (t+1)^i. \]  

(28)

In this article, we use the Legendre polynomials as one of the Gegenbauer polynomials as an set of orthogonal functions.

The components of orthogonal polynomials do not simply correspond to frequency components as the trigonometric functions. The 0th and 1st order components of Legendre polynomials are a constant and linear function with respect to time. These components have clear physical meanings, and are well-used in various analysis works.

We clarify the merit of the orthogonal polynomials in the following paragraphs. These algorithms are able to find a clear peak at the occurrence time of displacement using 0th and 1st order components of the orthogonal polynomials.

The 0th and 1st order components of the Legendre polynomials are
\[ P_0(t) = 1, \]
\[ P_1(t) = t. \]

(29) (30)

Therefore
\[ P(t) = a_0 P_0(t) + a_1 P_1(t) \]
\[ = a_0 + a_1 t. \]

(31) (32)

Here we put \( \mu_0 = 0 \) and fit \( P(t) \) to the displacement function \( s_D(t) \). The coefficients are given by
\[ a_0 = -\frac{\Delta \mu}{4}, \]
\[ a_1 = \frac{3 \Delta \mu}{2T}. \]

(33) (34)

Then the residual function \( r_s(t) \) is derived from the displacement function \( s_D(t) \) such that
\[ r_s(t) = s_D(t) - P(t). \]

(35)

Dividing cases before and after the time \( t_D[s] \),
\[ r_s(t) = \begin{cases} \frac{\Delta \mu}{4} - \frac{3 \Delta \mu}{2T} t & (t < t_D), \\ \frac{5 \Delta \mu}{4} - \frac{3 \Delta \mu}{2T} t & (t \geq t_D). \end{cases} \]

(36)

Furthermore, we integrate the residual function \( r_s(t) \) with respect to time \( t \) and obtain the accumulated residual functions \( R_s(t) \) such that
\[ R_s(t) = \int_0^t r_s(\tau) d\tau. \]

(37)

In period \( t < t_D \):
\[ R_s(t) = \int_0^t \left( \frac{\Delta \mu}{4} - \frac{3 \Delta \mu}{2T} t \right) d\tau \]
\[ = \frac{\Delta \mu}{4} t - \frac{3 \Delta \mu}{4} t^2. \]

(38) (39)

In period \( t \geq t_D \):
\[ R_s(t) = R_D + \int_{t_D}^t \left( \frac{5 \Delta \mu}{4} - \frac{3 \Delta \mu}{2T} t \right) d\tau \]
\[ = R_D + \frac{5 \Delta \mu}{4} (t - t_D) - \frac{3 \Delta \mu}{4T} (t^2 - t_D^2), \]

(40) (41)

where
\[ R_D = R_s(t_D) = \frac{\Delta \mu}{4} t_D \left( 1 - \frac{3}{T} t_D \right). \]

(42)

Fig. 3 shows an actual example of this calculation. In this case, the parameters are \( \Delta \mu = 5[\text{mm}], T = 60[\text{s}] \) and \( t_D = 30[\text{s}] \).

In Fig. 3, we show that the accumulated residual function \( R_s(t) \) can retrieve the occurrence time \( t_D[\text{s}] \) as a clear peak. Therefore we can utilize it for finding the occurrence time.

In reality, the above algorithms are applied to noise-buried measurement data. The algorithms have a great merit to detect the candidates of the occurrence time. This discussion reveals the effect of the 0th and 1st order components of the orthogonal polynomials.

In addition, we have to consider the role of higher order components than 1st order of the orthogonal polynomials. Such higher order components represent higher power spectrum components. But these components of the Legendre polynomials are different from the Fourier functions. It means higher order components of polynomials do not correspond perfect frequency components. Therefore, we can obtain only candidates of the true occurrence time, and it has be-
come the first step, not a final solution.

Fig. 4 shows an actual example of the accumulated residual function $R_n(t)$. There are several peaks. They can be examples of the candidates $t_c$ of the true occurrence time $t_D$.

![Fig. 4 Example of the accumulated residual function](image)

The above study clarifies that we can obtain the candidates of the occurrence time using the Legendre polynomials. If obtaining the candidates $t_c$ of the occurrence time and eliminating unwanted components, we can identify the occurrence time. This is the next step and it will be discussed in Sec. 5.

4.4 Considering Fourier Series

In this section, we discuss about the use of the Fourier functions. Orthogonal functions are the sum of the 0th and higher order components, and represented by

$$F(t) = \psi_0(t) + \sum_{n=1}^{N} \psi_n(t),$$

(43)

The orthogonal functions are given by

$$p_s\psi_{0}(t) = \frac{a_0}{2},$$

(44)

$$p_s\psi_{n}(t) = a_n \cos \frac{2\pi nt}{T} + b_n \sin \frac{2\pi nt}{T},$$

(45)

where $T$ is the measurement period, and

$$a_n = \frac{2}{T} \int_{0}^{T} x(t) \cos \frac{2\pi nt}{T} \, dt,$$

(46)

$$b_n = \frac{2}{T} \int_{0}^{T} x(t) \sin \frac{2\pi nt}{T} \, dt.$$  

(47)

There are difficulties in the algorithms using the Fourier series. The displacement function $s_D(t)$ and the observation noise $w(t)$ have conspired their power spectrums in the band which is close to the sampling interval 1 [Hz], the upper limit of observation. Therefore it is difficult to distinguish them using the Fourier series.

Furthermore, there is another difficulty. The Fourier series automatically coordinate the phase against our intention. This effect deletes the evidence of the occurrence time $t_D$ by the landslide. It means that we lost the most wanted information for this problem.

According the above two reasons, the Fourier functions are ill-adapted to the problem.

5. Determining the Occurrence Time

We have obtained the candidate $t_c$ of the occurrence time in Sec. 4. All we should do is to clarify how to determine true or not and find the occurrence time $t_D$.

In order to solve the above problem, we re-write a displacement model such as Fig. 5 for the 2nd step of the algorithms, where $t_c$ is the candidate of the true occurrence time $t_D$. Namely if $t_c$ is true, $t_c = t_D$.

![Fig. 5 Displacement model in 2nd step](image)

The measurement data $x(t - t_D)$ can be divided into the even function $x_e(t - t_D)$ and the odd function $x_o(t - t_D)$ on the time $t_D$ such that

$$x(t - t_D) = x_e(t - t_D) + x_o(t - t_D),$$

(48)

where $x_c(t - t_D)$ and $x_o(t - t_D)$ behave as

$$x_e(-t - t_D) = x_e(t - t_D),$$

(49)

$$x_o(-t - t_D) = -x_o(t - t_D).$$

(50)

$x_e(t - t_D)$ and $x_o(t - t_D)$ are orthogonal each other such that

$$\int_{t_D - \Delta T}^{t_D + \Delta T} x_e(t - t_D) x_o(t - t_D) \, dt = 0.$$  

(51)

We can consider that the displacement function $s_D(t)$ is just the odd function, if we put the occurrence time $t_D$ at the origin. Because this displacement is an irreversible phenomenon, and all landslides are irreversible. Therefore we can say they are the odd functions.

This fact represents the characteristics of the displacement function $s_D(t)$. It becomes an odd function on the occurrence time $t_D$ shown as

$$s_D(-t) - \frac{\mu}{2} = -s_D(t) - \frac{\mu}{2}.$$  

(52)

In other words, $s_D(t)$ never have the even components primitively. Therefore if we eliminate the even components, it is able to obtain the information of the displacement $s_D(t)$. From these discussions, we will precisely eliminate unwanted components using the orthogonal even functions $E(t - t_c)$, which have the following characteristic, and remain components of $s_D(t)$ only.

$$E(-t - t_c) = E(t - t_c).$$  

(53)

We calculate using actual data, and use the measurement data, which are artificially added a step of 5[mm] to PPP-RTK data utilizing QZSS CLAS, mentioned in Sec. 2.3.
On the stage obtaining the candidates \( t_c \), we eliminate 0th and 1st components of the Legendre functions, similarly to Sec. 4.3. We use the cosine functions, to eliminate the unwanted even components for determining the true occurrence time. The cosine functions are used as the orthogonal even functions \( E(t-t_c) \) to eliminate the observation noise \( w(t) \).

\[
E(t-t_c) = \sum_{n=0}^{N} a_n \cos \frac{2\pi(n+1)(t-t_c)}{T} \tag{54}
\]

In this case, the order must cover the frequency band, which covers the disturbance of ionospheric delay. In general, the ionospheric disturbance has the variance of approximately 1 minute. Therefore we take the order which covers this disturbance. The numerical example is 35th order in maximum.

We integrate a residual function, which eliminates the orthogonal functions, and calculate the accumulated residual function. This result is shown as Fig. 6. It shows a clear peak when the candidate is true.

Fig. 6 Clear peak appearance in a true case only

According to this result, if the candidate is false, we cannot find the clear peak. On the other hand, if the candidate is true, we can find the clear peak. Therefore it is able to determine true or not of a candidate \( t_c \). After determining the occurrence time \( t_D \), we can calculate the length of the displacement \( \Delta \mu \) by taking an average difference before and after the occurrence time \( t_D \).

The study of this section has clarified that we can determine the candidate is true or not, using the elimination of the observation noise \( w(t) \) from the measurement data \( x(t) \) with the candidates \( t_c \) of the occurrence time \( t_D \).

These algorithms are to fit the even functions and eliminate the unwanted noise \( w(t) \) from the original data \( x(t) \). By using this method, we can determine the true occurrence time \( t_D \) and retrieve the millimeter-scale displacement \( \Delta \mu \).

6. Integration of Algorithms

We can derive a new and practical solution by integrating the algorithms described in this article. The algorithms consist of two steps. The 1st step is to find the candidates \( t_c \) of the occurrence time \( t_D \).

The 2nd step is, using the result of the 1st step, to determine the occurrence time \( t_D \) and displacement \( \Delta \mu \). We summarize 2-step algorithms in the following sections.

6.1 Candidate of the Occurrence Time

The 1st step is the following procedure to find \( t_c \).
1) Take the measurement data \( x(t) \) during the time \( T \). For example, \( T=3600[s] \).
2) Fit the Legendre polynomials to the measurement data \( x(t) \).
3) Eliminate the components of Eq. (56) from the original measurement data \( x(t) \).
4) Integrate the residual function \( r_s(t) \) from the start time 0 to \( t \), and obtain the accumulated residual function \( R_s(t) \).
5) We can find a number of clear peaks, and make them the candidate \( t_c \) of the occurrence time.

6.2 Occurrence Time and Displacement

The 2nd step is the following procedure to find \( t_D \) and \( \Delta \mu \).
1) Take the measurement data \( x'(t) \) during \( t_c - \Delta T_c \) to \( t_c + \Delta T_c \). For example, \( \Delta T_c=600 [s] \).
2) Eliminate 0th and 1st terms of the Legendre polynomials
3) Fit the cosine function to \( x''(t) \), and eliminate the lower order components, and obtain the residual function \( r_s'(t) \). For example, \( N = 35 \).
4) Integrate \( r_s'(t) \) from \( t_c - \Delta T_c \) to \( t \), and obtain the accumulated redial function \( R_s'(t) \).
5) Check \( R_s'(t) \), and find the clear peaks at the candidate \( t_c \). If so, \( t_c \) is the true value \( t_D \). If not a clear peak, reject the candidate \( t_c \).
6) Take the average of \( x(t) \) before and after \( t_D \) respectively, and calculate the difference, and
obtain the displacement $\Delta \mu$.

7. Experiments

In this sections, we show numerical examples of experiment. These experiments were conducted in Tokyo, Japan. As GNSS technique, we used RTK mentioned in Sec. 2.3 for checking the algorithms. Our equipment made an artificial input of displacement, and measured this displacement using the mathematical algorithms in this article.

7.1 Experiment 1

In Table 3, we show the conditions of this experiment. Fig. 7 shows detected clear peaks at the occurrence time. Table 4 shows the result of these displacement measured values, where the columns of UTC present the starting time of the experiment in Coordinated Universal Time.

| No. | Item     | Description             |
|-----|----------|-------------------------|
| 1   | Place    | 35.679°N/139.147°E     |
| 2   | Date     | 2018-02-22              |
| 3   | Equipment| Javad Alpha             |
| 4   | Direction| Easting                |
| 5   | Input    | 7.5 mm                  |

Table 3 Condition of Experiment 1

![Fig. 7 Detection of the occurrence time in Experiment 1](image)

Table 4 Results of Experiment 1

| Case | UTC  | True value | Measured value |
|------|------|------------|----------------|
| 1-A  | 00:00| 7.5 mm     | 8.5 mm         |
| 1-B  | 06:00| 7.5 mm     | 5.8 mm         |

The measured values could be detected closer displacement to the true value in both cases. Such measured values have been enough accurate that an administrator of the infrastructures release a warning to citizens in actual services.

7.2 Experiment 2

In Table 5, we show the conditions of this experiment.

| No. | Item     | Description             |
|-----|----------|-------------------------|
| 1   | Place    | 35.679°N/139.147°E     |
| 2   | Date     | 2018-02-22              |
| 3   | Equipment| Javad Alpha             |
| 4   | Direction| Easting                |
| 5   | Input    | 5 mm                    |

![Fig. 8 Detection of the occurrence time in Experiment 2](image)

Table 5 Condition of Experiment 2

| Case | UTC  | True value | Measured value |
|------|------|------------|----------------|
| 2-A  | 00:20| 5.0 mm     | 4.7 mm         |
| 2-B  | 06:20| 5.0 mm     | 7.5 mm         |

The measured values could be detected closer displacement to the true value in all of the cases. Such measured values have been enough accurate that an administrator of the infrastructures release a warning to citizens in actual services.

7.3 Experiment 3

In Table 7, we show the conditions of this experiment. Fig. 9 shows detected clear peaks at the occurrence time. Table 8 shows the result of these displacement measured values.

| No. | Item     | Description             |
|-----|----------|-------------------------|
| 1   | Place    | 35.679°N/139.147°E     |
| 2   | Date     | 2018-02-22              |
| 3   | Equipment| Javad Alpha             |
| 4   | Direction| Easting                |
| 5   | Input    | 2.5 mm                  |

![Fig. 9 Detection of the occurrence time in Experiment 3](image)

Table 7 Condition of Experiment 3

| Case | UTC  | True value | Measured value |
|------|------|------------|----------------|
| 3-A  | 00:20| 2.5 mm     | 2.5 mm         |
| 3-B  | 06:20| 2.5 mm     | 2.5 mm         |

The measured values could be detected closer displacement to the true value in all of the cases. Such measured values have been enough accurate that an
There are other techniques of GNSS precise positioning which are described in Sec. 2.3. Application of this algorithm to these techniques are our future works.

8. Conclusion

In this article, we show mathematical algorithms to measure a noise-buried destructive displacement using GNSS precise positioning and the orthogonal function expansion of its positioning result.

The major application is the detection of a landslide which is one of the most intimidating disasters. In reality, there are 5.25 hundred thousand points assessed disaster dangerous places in Japan.

This mathematical algorithm to retrieve millimeter-scale displacement in observation noise of GNSS precise positioning. Principally speaking, these algorithms apply onto RTK, PPP and applied GNSS precise positioning in not only Japan but also other regions. This method will inform the foreboding of a destructive landslide, and will save many lives and infinite property of assets in catastrophic disasters [1,2].

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