The proton structure function and
a soft Regge Dipole Pomeron :
a test with recent data

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Abstract A recently published soft Regge Dipole Pomeron model intended for all \( x \) and \( Q^2 \) is proved to give a good agreement with (non fitted) recent HERA data from ZEUS (SVX95) on the proton structure function \( F^p_2(x,Q^2) \) at low \( Q^2 \) and low \( x \). The model also reproduces (without fit) the recently estimated experimental derivatives \( \frac{\partial F^p_2}{\partial \ln Q^2} \) and \( \frac{\partial \ln F^p_2}{\partial \ln(1/x)} \) in a wide \( x \) and \( Q^2 \)-region.

1 Motivations

The proton structure function (SF) is one of the observables most often measured in high energy physics \cite{1}. Consequently a relevant model has to be periodically tested (and eventually updated or abandoned) in the new kinematical ranges of \( x \) (Björken variable) and \( Q^2 \) (virtuality of the photon) investigated by the experimentalists.

Recent measurements \cite{2} of the SF at HERA (from ZEUS 1995 shifted vertex experiment (SVX95)) have motivated us to test our Dipole Pomeron parametrization \cite{3} of the proton structure function \( F^p_2(x,Q^2) \) intended for a wide region of \( Q^2 \) and \( x \). We wish to show that these recent data can be reproduced within an ”old soft Pomeron” framework, which is an ’à la Regge” approach.

Second, we revise a widely extended opinion that a soft Pomeron and more generally an ”à la Regge” approach to deep inelastic scattering (DIS) should be restricted to rather small values of \( Q^2 \). This conclusion is based mainly on the popular Donnachie-Landshoff (DL) model of the Pomeron \cite{4} and its particular parametrization of \( Q^2 \)-dependence of the
residue function[3]. This model was used in [2] and the conclusion was drawn that the Regge theory describes well the data only at very low \( Q^2 \leq 0.65 \text{ GeV}^2 \). We have vice-versa shown that a soft Pomeron contribution (with unit intercept) can be applied to the virtual photoproduction cross-section [3], and that it reproduces well the data on \( F_2^p(x, Q^2) \) in a much wider region of \( Q^2 \) and \( x \).

In the present short note, we emphasize that the fit [3] not only reproduces with high quality also the new data on the SF [2] at low \( Q^2 \) and low \( x \), but is in good agreement with the behavior of the derivatives \( \frac{\partial F_2^p}{\partial \ln Q^2} \) and \( \frac{\partial \ln F_2^p}{\partial \ln (1/x)} \), measured in [2], up to intermediate values of the kinematical variables.

2 The model

Many Pomeron models are on the market in high energy hadron phenomenology. In spite of the quite different \( t \)-dependence of the elastic amplitudes, at \( t = 0 \) they can be combined in two groups:

1. A simple pole in the complex angular momenta (\( j \)-) plane with an intercept \( \alpha_P(0) > 1 \): the DL Pomeron [4] and its generalization [6] with an additional constant term (a preasymptotic simple \( j \)-pole with unit intercept) are examples. Such a Pomeron leads to a total cross-section

\[
\sigma_{\text{tot}}(s) \propto s^{\alpha_P(0)-1},
\]

which at extremely high energies (well beyond the present attainable ones) would violate the Froissart-Martin bound; this could require to be unitarized (for example, by an eikonal method).

2. More complicated singularity in the \( j \)-plane at \( j = 1 \): in these models ([3, 6, 7, 8] and references therein) the Froissart-Martin bound is not violated and asymptotically the total cross-sections behave as

\[
\sigma_{\text{tot}}(s) \propto \ln^\mu(s/s_0), \quad 0 < \mu < 2, \quad s_0 = 1 \text{ GeV}^2.
\]

All these models describe quite well hadronic total cross-sections and \( \gamma p \) inelastic one. The best quality of the description (in the sense of \( \chi^2 \)) was achieved [6] at \( t = 0 \) when \( \mu = 1 \). This corresponds to a double \( j \)-pole in the forward amplitude, \textit{i.e.} to the Dipole Pomeron (DP) model. This model was successfully applied [6] (with its extension to \( Q^2 \neq 0 \)) to DIS with a good description of the SF in a wide region of \( Q^2 \) and \( x \).

Defining the DP model for DIS, we start from the expression connecting the transverse cross-section for the \( (\gamma^*p) \) interaction to the proton structure function \( F_2^p \)

\[
\sigma_T^{\gamma^*p}(W, Q^2) = \frac{4\pi^2\alpha}{Q^2}(1 + \frac{4m_p^2x^2}{Q^2}) \frac{1}{1 + R(x, Q^2)} F_2^p(x, Q^2),\quad (1)
\]

where \( \alpha \) is the fine structure constant, \( m_p \) is the proton mass, \( R \) is the ratio of longitudinal to transverse cross-sections. We have approximated \( R(x, Q^2) = 0 \), due to its experimental smallness. The center of mass energy \( W \) of the \( (\gamma^*p) \) system obeys

\[
W^2 = Q^2 \left( \frac{1}{x} - 1 \right) + m_p^2 \quad (2)
\]
Here we only mention that they vary between the constants $G \leq \text{good agreement for } 0$.

We support the point of view that there is just one “bare” Pomeron (two Pomerons, “soft” and “hard”, are considered in [9]). This unique Pomeron is universal and factorizable compared to the whole set of fitted and non fitted data in Fig. 1. The total real (adding the new (44) values of $F$) set of data completed by the forthcoming 96-97 HERA results). We have proved that (we postpone an update of this model, fixing some parameters and refitting the others with published values and do not perform any new adjustment in order to introduce no confusion)

This is not probably the most economical set within this framework, however we keep these.

We choose the DP model defined in details in [3] with the 23 parameters (see Table 2 in [3]).

Let us make a few comments on the chosen Pomeron model.

We support the point of view that there is just one ”bare” Pomeron (two Pomerons, ”soft” and ”hard”, are considered in [9]). This unique Pomeron is universal and factorizable i.e. it is the same in all processes; only the vertex functions depend on which are the interacting particles. It follows from these special requirements that the Pomeron trajectory should be independent of $Q^2$. One should mention that this approach differs from other ones where an ”effective” Pomeron with a $Q^2$-dependent intercept [11, 12, 13] has been chosen. The hard Pomeron or BFKL Pomeron [14] with a quite large intercept is only an approximation to a true Pomeron. A growth of the total cross-sections means that in $j$-plane a true Pomeron is harder than a simple pole singularity at $j = 1$.

The Dipole Pomeron defined by (6),(7) obeys the following specificities: it is universal, it has a $Q^2$-independent intercept $\alpha_P(0) = 1$, it does not violate, at least explicitly, the unitarity restrictions on the amplitude.

### 3 The proton SF and total ($\gamma p$) cross section results

We choose the DP model defined in details in [3] with the 23 parameters (see Table 2 in [3]). This is not probably the most economical set within this framework, however we keep these published values and do not perform any new adjustment in order to introduce no confusion (we postpone an update of this model, fixing some parameters and refitting the others with a set of data completed by the forthcoming 96-97 HERA results). We have proved that adding the new (44) values of $F_2$ does not change the quality of the fit in [3], we found : $\chi^2 = 1321$ for 1209 data points which becomes $\chi^2 = 1341$ for 1253 data, in practice leaving unchanged $\chi^2/d.o.f \simeq 1.1$.

The $F_2(x, Q^2)$ results are plotted versus $x$ for the experimental $Q^2$ bins of [4] (low $Q^2$) and compared to the whole set of fitted and non fitted data in Fig. 1. The total real ($\gamma p$) cross section versus the c.m squared energy $W^2$ is shown in Fig. 2. These figures show the good agreement for $0 \leq Q^2 \leq 17 \text{ GeV}^2$; higher $Q^2$ values (where DP model also reproduce well the data) are discussed in [3].

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4If Pomeron is a sum of two terms (as in (7)) then at least the leading one at $W \gg m_p$ should satisfy factorization.
4 The $Q$-slope as a function of $x$

The $Q$-slope

$$B_Q(x, Q^2) = \frac{\partial F^p_2(x, Q^2)}{\partial \ln(Q^2)}$$

depends on the two independent variables $x$ and $Q^2$. However to compare with experiment the $Q$-slope has been calculated for the set [2] ($x_i, < Q^2 >_i$; $i=1,24$) of strongly correlated variables which includes $x$ up to 0.2. The results are shown in Fig. 3; the agreement is good up to $x \sim 0.1$.

5 The $x$-slope and the ”effective intercept” as functions of $Q^2$

The $x$-slope

$$B_x(x, Q^2) = \frac{\partial \ln F^p_2(x, Q^2)}{\partial \ln(1/x)}$$

is also a function of two variables; the quantity currently replacing $B_x$ in experimental papers is the ”effective power” $\Delta_{eff}$ (sometimes denoted as $\lambda_{eff}$) in the low $x$ - fixed $Q^2$ approximation of the structure function

$$F^p_2 \propto x^{-\Delta_{eff}}.$$

This power is currently connected to the Pomeron effective intercept ($\alpha(0) = 1 + \Delta_{eff}$). Actually, from a phenomenological point of view, one can extract $\Delta_{eff}$ at fixed $Q^2$ depending on $x$ assuming a parametrization

$$F^p_2(x, Q^2) = G(Q^2) \left( \frac{1}{x} \right)^{\Delta_{eff}(x,Q^2)}.$$

Strictly speaking, however, the identification $B_x = \Delta_{eff}$ is possible only when the $x$–independence of the $x$-slope is a model assumption (one may see [3] for a discussion of the slopes). In general, this is not the case. From that point of view, it would be interesting and important to have ”measured” values for $\Delta_{eff}$ at fixed $Q^2$ an at different $< x >$ : it will allow to study the $x$-dependence of the effective Pomeron intercept.

The comparison between the calculated value of the $x$-slope and the experimental effective power $\lambda_{eff}$ is given in the last figure (Fig. 4) for the set of kinematical variables [2] ($< x >_i, Q^2_i$; $i=1,30$) including ZEUS and fixed target E665 results. The agreement is good in the whole experimental range : up to $Q^2 \sim 250 \text{ GeV}^2$.

6 Conclusion

We have proved that a soft dipole Pomeron model [3] (”à la ” Regge) not only reproduces with a high quality the new data on the proton structure function [2] at low $Q^2$ and low $x$, but also is in good agreement with the measured slopes [2] $\frac{\partial F^p_2}{\partial \ln Q^2}$ and $\frac{\partial \ln F^p_2}{\partial \ln(1/x)}$ up to $Q^2 \sim 250 \text{ GeV}^2$ and $x \sim 0.1$.

Such a success in reproducing the data is due not only to an appropriate choice of the asymptotic Pomeron contribution but is due also to the preasymptotic terms (chosen constant here) in the Pomeron and $f$-reggeon. A more detailed discussion of preasymptotics properties of the model is in [3].

This enforces our belief that a universal -factorizable- Pomeron with a $Q^2$ independent intercept $\alpha(0) = 1$ may be successful in Deep Inelastic Scattering not only at low $Q^2$ and $x$.
(as the DL Pomeron does). On the basis of the results from [3] and the present paper, we claim that the area of validity of a Regge approach is much wider (especially in $Q^2$) than usually assumed and can be extended up to rather high values of $Q^2$ (may be up to a few hundreds GeV$^2$).

Acknowledgments E.M. would like to thank the IPNL for the kind hospitality and financial support provided to him during this work. It is a pleasure to thank E. Predazzi for a critical reading of the manuscript and illuminating comments.

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Fig. 1 Experimental data for the proton structure function $F_2^p(x, Q^2)$ at low $Q^2$ compared to the results within the Dipole Pomeron model (shown are the 44 recent ZEUS SVX 95 data -non fitted- and the other fitted data).
Fig. 2. Experimental fitted data for the photoproduction total cross-section and predictions in the Dipole Pomeron model.
Fig. 3. $Q$-slope $B_Q(x, < Q^2 >)$: experimental points from [2] as a function of $x$ (for the indicated $< Q^2 >$ values). The continuous line is the prediction for the Q-slope $\frac{dF_P}{d\ln Q^2}$ calculated (not fitted) within the Dipole Pomeron model.
Fig. 4. Experimental effective power $\lambda_{\text{eff}}(<x>, Q^2)$: data from [2] as a function of $Q^2$ (for the indicated $<x>$ values). The continuous line is the predictions for the $x$-slope $\frac{\partial \ln F_p}{\partial \ln (1/x)}$ calculated (not fitted) in the Dipole Pomeron model.