The use of nested sampling in the extraction of polarisation observables at CLAS

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Abstract. The extraction of polarisation observables from photoproduction experiments provides an insight into the spectrum of nucleon resonances and the "missing resonance" problem. Experiments carried out at JLab, Mainz and Bonn cover a wide range of reactions, which will soon result in the first "complete measurement" in pseudoscalar meson photoproduction. Traditionally, these measurements have been analysed using frequentist statistics, where parameters are extracted by fitting distributions. An alternative method is the application of Bayesian statistics, where any existing knowledge about the results can be used in the initial conditions. One such application of this is nested sampling. This work discusses nested sampling and how it can be applied to the extraction of spin observables.

1. Introduction
Quantum Chromodynamics (QCD) provides a reliable description of the strong force in general, but it fails at the mass level of a proton. At such levels, various quark models (such as the symmetric quark model and diquark model) exist to attempt to explain the strong force. Each of these models predicts several mass resonances which need not correspond to one another. In the field of hadron physics, this is known as the ‘missing resonance’ problem and is the main physics motivation behind the work presented in this paper. Pseudoscalar meson photoproduction is one method used to determine these resonances. In order to obtain sufficient information to access these resonances, the measurements of various polarisation observables from experimental data are required. The application of Bayesian analysis to this polarisation observable extraction is relatively new, but it provides several advantages over current methods. The use of nested sampling, a data analysis method derived from Bayesian statistics, is discussed here. In this paper, the background theory behind pseudoscalar meson photoproduction and nested sampling will be discussed initially (Section 2). The relevant experimental work at CLAS will be described in Section 3, and the application of this new analysis program to the physics program at CLAS will be explained in Section 4. Preliminary results using simulated data will be described in Section 5, along with a brief comparison to the current standard, illustrating the benefits and drawbacks of nested sampling. Section 6 will briefly discuss the future developments of the program and Section 7 will concisely summarise and conclude the paper.
2. Background Theory

The quark contributions to the spin of a nucleon are not well understood. There exist several quark models which each attempt to provide an explanation of the spin nature of nucleons. Baryon spectroscopy is an experimental approach used to support or preclude predicted quark models [1]. Each quark model predicts mass resonances at various energies in the nucleon spectrum. There are some mass resonances, however, that are expected for some quark models but not others [1, 2]. If resonances are discovered that conform to one model but not another, for example, it is possible to learn more about the structure of hadrons and to develop a deeper understanding of the strong force at the mass levels of hadrons. Merely examining the graph of a cross-section is not sufficient to learn more information about these resonances, as the peaks are broad and often overlap. One of the methods that can be used to access these resonances is now discussed.

2.1. Pseudoscalar Meson Photoproduction

Pseudoscalar meson photoproduction is used to examine the spectrum of excited nucleon states. A beam of high-energy photons is directed onto a stationary nucleon target. Mesons and the scattered nucleon are detected at points within the detector. From the information detected, it is possible to extract various observables that can be linked to four complex amplitudes which completely describe the pseudoscalar meson photoproduction process. There are, in total, fifteen observables (described in Table I), in addition to the differential cross-section, that can be extracted from variations in experimental set-up. There are three single-spin observables - a photon-beam asymmetry \((B)\), a recoil polarisation \((R)\) and a target polarisation \((T)\). There are four beam-recoil \((BR)\), four beam-target \((BT)\) and four recoil-target \((RT)\) spin polarisations. These observables are all non-independent combinations of the four complex amplitudes and these combinations can be found in Table I. As such, it is necessary to calculate multiple observables in order to extract the amplitudes. Data from suitably arranged experiments must be used to extract these observables [2, 3].

| Observable | Type | Amplitude Combination |
|------------|------|-----------------------|
| \(B\)     | Single | \(|a_1|^2 + |a_2|^2 - |a_3|^2 - |a_4|^2\) |
| \(R\)     | Beam-target | \(2\Re(a_1a_3^* + a_2a_4^*)\) |
| \(T\)     | Beam-recoil | \(-2\Re(a_1a_3^* - a_2a_4^*)\) |
| \(E\)     | Beam-target | \(2\Im(a_1a_3^* + a_2a_4^*)\) |
| \(F\)     | Beam-target | \(2\Im(a_1a_3^* - a_2a_4^*)\) |
| \(G\)     | Beam-target | \(2\Im(a_1a_3^* + a_2a_4^*)\) |
| \(H\)     | Beam-target | \(-2\Re(a_1a_3^* - a_2a_4^*)\) |
| \(C_x\)   | Beam-recoil | \(-2\Im(a_1a_3^* - a_2a_4^*)\) |
| \(C_z\)   | Beam-recoil | \(2\Re(a_1a_3^* + a_2a_4^*)\) |
| \(O_x\)   | Beam-recoil | \(2\Re(a_1a_3^* - a_2a_4^*)\) |
| \(O_z\)   | Beam-recoil | \(2\Im(a_1a_3^* + a_2a_4^*)\) |
| \(T_x\)   | Target-recoil | \(2\Re(a_1a_3^* - a_2a_4^*)\) |
| \(T_z\)   | Target-recoil | \(2\Im(a_1a_3^* + a_2a_4^*)\) |
| \(L_x\)   | Target-recoil | \(-2\Im(a_1a_3^* + a_2a_4^*)\) |
| \(L_z\)   | Target-recoil | \(2\Re(a_1a_3^* + a_2a_4^*)\) |
2.2. Nested Sampling

Nested sampling is a modern model comparison technique based on the principles of Bayesian statistics. Its application to hadron spectroscopy is relatively new, yet it can offer several advantages over the current data analysis method.

Bayesian statistics involves making an estimation of the results prior to any calculations. This 'guess' is used to form a distribution with an easily determined mean and variance, known as the 'Prior'. It is important to point out that with an increasing number of iterations, the choice of prior becomes less and less significant [5]. Bayes' Theorem is then used to combine the prior distribution with the data and produce a posterior distribution, the statistics of which determine the resulting mean, variance, etc [5].

The idea of Bayesian statistics can be expressed in the form of a simple equation [6]:

\[
Prior \times Likelihood \propto Posterior
\]  

The prior is a distribution, or set of points, that act as an initial starting point; an estimation or expectation of the results. Each point has an associated likelihood determined by a likelihood function, which is generally dependent on data. For example, in an effort to determine the x-coordinate of an object, the prior would consist of a set of possible x-coordinates. The likelihood associated with each point describes how likely that point is to be the x-coordinate of the object.

The nested sampling algorithm determines the point with the lowest likelihood and overwrites it with a copy of another point, selected randomly. This new point is then changed slightly, based on a predetermined explore function, and the algorithm ensures that the likelihood of the changed point is greater than that of the overwritten point. This process is iterated through for either a predetermined number of iterations or until some termination condition is met [5, 6].

The following diagram shows the process of nested sampling pictorially. In this example, an object is placed at \( x = 3 \). The prior consists of a set of x values distributed uniformly on the interval (0,5). The likelihood of the object being found at a given x-position is defined by the function below.

\[
L = 6x - x^2 - 2
\]  

The distribution of the points after each iteration of the nested sampling algorithm is shown in each line of Figure 4.
3. Experiment at CLAS
The CEBAF Large Acceptance Spectrometer (CLAS) detector is located in Hall B of the Thomas Jefferson National Accelerator Facility (JLab) in Newport News, Virginia. The detector has a radius of approximately 5m and an acceptance of almost $2\pi$. This acceptance makes the CLAS detector well suited for exclusive reactions [7]. The CLAS Collaboration runs a variety of experiments using this detector that pertain to nucleon structure and hadron spectroscopy. Particularly relevant to the work presented here is the excited nucleon ($N^*$) program [2]. The $N^*$ program includes several pseudoscalar meson photoproduction experiments. The reaction channel most related to this work is described by the following equation:

$$\gamma p \rightarrow K^+\Lambda \rightarrow K^+\pi^- p$$

(3)

In this reaction, a high energy photon beam is incident on a stationary proton target. The resulting $\pi$ mesons and the scattered proton hit the detector, which records values such as energy, position along detector and timing. This information is then used to calculate more useful physical quantities, including mass, momentum and angular momentum distributions [2]. The raw data is processed and useful four-vectors are produced. The data analysis described here is used on processed data.

4. Comparison of Nested Sampling to Current Method
The current method of data analysis in this area involves fitting a sinusoidal curve to an asymmetry distribution [7]. Data is collected when the photon beam is polarised parallel and perpendicular to the horizontal axis of the CLAS coordinate system [7]. The asymmetry of this data is then calculated based on the equation below.

$$A = \frac{\sigma^\| - \sigma^\perp}{\sigma^\| + \sigma^\perp}$$

(4)

The data are binned and fitted with a $\cos(2\phi)$ curve. For illustrative purposes, this method was used to analyse simulated data. The resulting plots are shown in Fig 2. The observable is then determined by the fitted coefficient.
Figure 2. a) Simulated data from parallel polarised beam; b) Simulated data from perpendicular polarised beam; c) Asymmetry fit, with small number of events (1000).

This approach, although straightforward, has several limitations. Only one observable can be extracted at a time, and often the value of an observable can be dependent on a previous measurement of another polarisation observable. There is an element of information loss, as the data are binned in histograms. Also, it is not straightforward to constrain observables to the physical region using this method.

A generic nested sampling program was created using the ROOT Framework and C++. In principle, nested sampling can be used to extract multiple observables simultaneously from one data set (and in fact, this has been naively done, but for simplicity will not be discussed further). Initially, a simplified likelihood function ([9]) involving only one observable was used in order to determine the feasibility of this new approach. An event generator was used to create data. The data files containing all relevant information (polarisation, angles) were then passed to the nested sampling program, and the results were compared to the input data entered into the event generator. When a small number of events are generated, the nested sampling program still provided a sensible result (Fig 3a). Using a higher number of events yielded a much more precise result (Fig 3b). There was no information lost due to binning as nested sampling uses an event-by-event likelihood function. The observables can be constrained easily, and any new information gained about the observables can be easily taken into account. As mentioned previously, it is also possible to extract multiple observables simultaneously - therefore, the extraction of one
observable does not depend on any previously extracted values. The main drawback of this method is the amount of time required to run the program. When running with 10,000 events over 100,000 iterations, the program takes several hours to return the result(s). The old method takes a fraction of this amount of time to run.

5. Developments
The nested sampling program is still very much in development. Currently, a more realistic likelihood function is being applied with the aim of analysing experimental data by the end of the calendar year. Work on extraction of multiple observables simultaneously is already well into development and the initial results are encouraging. The slow run-time is also being improved through the implementation of GPU programming and data parallelisation. Studies into similar problems have shown speed-ups of up to three orders of magnitude [8]. This is still in its early stages, however, and the improvements to be made with this method are hardware-dependent.

As well as improvements to the program itself, new applications are being investigated. The nested sampling program was designed to be as generic as possible, with only small portions of code to be specific to a particular problem. Simplified data from a Deeply Virtual Compton Scattering (DVCS) experiment have been analysed using this new method and have shown promising results. Further analysis of DVCS data will be carried out in order to demonstrate the applicability of nested sampling in other areas of study.

6. Conclusions
Missing resonances are crucial in developing a deeper understanding of the structure of nucleons. Pseudoscalar meson photoproduction can be used to find these resonances. To do this, the measurement of polarisation observables is essential. Nested sampling can be used to extract these polarisation observables and offers some important advantages over current methods. A new data analysis method based on Bayesian statistics has been under development. It has been applied to simulated pseudoscalar meson photoproduction reaction data and has shown promising results. Nested sampling uses an event-by-event likelihood function, meaning no loss of information due to binning. It is possible to extract several observables from the same data set simultaneously, and the results do not depend on any previously measured observables. Physical constraints can be taken into account easily, as well as any important or new information. Issues with the time required to run the code are being improved through the use of new computer technology. Once fully functional, nested sampling should provide a powerful analysis tool that can be easily applied to many areas of research.

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