Thermo-fluid analyses for UCN cryogenic system

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Abstract. The TUCAN collaboration is now developing the new cryogenic system for ultra-cold neutrons experiments (UCN). This cryogenic system requires superfluid helium (He II) around 0.8 K to 1.15 K because cold neutrons are down-scattered by isopure He II with such temperature range to become UCN. Dynamic heat load applied to He II during UCN production will be as high as approximately 10 W. In order to satisfy such requirements, not only $^4$He but also $^3$He has to be circulated through the cryogenic system. There are sixteen heat exchangers for $^4$He and $^3$He. In order to design these heat exchangers properly, thermo-fluid simulations are carried out. Some simulations of them are nontrivial because they include film boiling phenomena in superfluid, molecular gas dynamics effect, transition from single phase flow to two phase flow and discrimination of such kinds of the complicated flow behavior. In addition, Time dependent thermo-fluid behaviors of He-II are also simulated using two fluid model and superfluid turbulent model to clarify the He II behavior. In this presentation, the simulation schemes and some results will be discussed.

1. Introduction

Ultracold neutrons (UCN) are produced by injecting cold neutrons into isopure superfluid helium (He II) near 1 K. Therefore, a heat load of about 10 W is applied to isopure He II during UCN production. From the viewpoint of UCN lifetime, the He II temperature should be kept below 1.15 K during UCN production [1]. Therefore, a heat exchanger (HEX-1) filled with $^3$He kept at 0.8 K is needed to satisfy this requirement. In addition, non-isopure He II with 1.6 K is filled in the 1 K pot as a precooling heat exchanger (HEX-2) for supplied $^3$He to the HEX-1. Therefore, two set of large scale pumping systems for both HEX-1 and HEX-2 are also essential. In addition to them, several counter flow type heat exchangers are also indispensable as introduced in the next subsection 2.1. $^3$He is circulated in the cryogenic system by constructing a closed loop to avoid the loss of $^3$He. On the other hand, $^4$He is supplied from the liquefier installed in TRIUMF.

The design of all heat exchangers needs to be optimized due to the limitations of the cooling capacity of the liquefier and the helium pumping systems for HEX-1 and HEX-2. In addition, Thermo-fluid behaviors of isopure He II are also simulated to understand the time evolution behavior of He II around 0.8 K to 1.2 K, which is mostly composed of superfluid components according to the two fluid model [2], [3].
2. Cryogenic system

2.1. Conceptual flow diagram

Figure 1 shows conceptual flow diagram and overhead view of UCN cryostat. There are four independent circulation lines, isopure helium precooling line (magenta line), atmospheric $^4$He circulation line from 4 K to 300 K (light blue line), subatmospheric $^4$He circulation line from 1.6 K to 300 K (blue line) and $^3$He circulation line (green line). Isopure helium (IPHe II) in the production volume is cooled down by the saturated liquid $^3$He filled in the HEX-1. The typical flow conditions for each line are listed in the table 1. Required $\Delta p_{ret}$ in the table means the requirement value of the pressure drop through the pumping line to obtain saturated liquid $^3$He filled in the HEX-1 with desired temperature.

2.2. Cryogenic requirements

Table 2 summarizes static and dynamic heat load, required isopure He II temperature, mass flow rate of $^4$He and $^3$He. Production volume temperature, $T_{ucn}$ can be evaluated from following

![Conceptual flow diagram and overhead view of UCN cryogenic system.](image)

**Table 1.** Flow condition for each circulation line.

| Circulation line | mass flow rate | temperature range | required $\Delta p_{ret}$ |
|------------------|----------------|-------------------|--------------------------|
| IPHe II          | $0 \sim 0.15$ g/sec | $0.8 \sim 300$ K  | –                        |
| Atmospheric $^4$He | $2.4$ g/sec  | $4.2 \sim 300$ K  | $< 10$ kPa               |
| Decompressed $^4$He | $1.3$ g/sec  | $1.6 \sim 300$ K  | $< 100$ Pa               |
| $^3$He           | $1.1$ g/sec  | $0.8 \sim 300$ K  | $< 100$ Pa               |
Each term in the RHS of (2) is \( \Delta \) respectively.

Table 2. Brief specification of the UCN cryogenic system.

| Specification                      | Value         |
|-----------------------------------|---------------|
| Heat load w/o dynamic load, \( Q_s \) | 1 W           |
| Heat load w/ dynamic load, \( Q_d \) | 10 W          |
| \( T_{UCN} \) w/o dynamic load     | 0.82 K        |
| Allowable isopure He II temperature, \( T_{ucn} \) during UCN production | < 1.15 K |
| Allowable circulated mass flow rate of \(^3\)He, \( \dot{m}_{3He} \) | < 1.1 g/sec |
| Allowable circulated mass flow rate of \(^4\)He, \( \dot{m}_{4He} \) (in total) | < 3.7 g/sec |

Pressure drop in the \(^3\)He pumping duct, \( \Delta p_{3He} \), is quite important to keep temperature of HEX-1 at 0.8 K. It can be expressed as follows.

\[
\Delta p_{3He} \simeq \Delta p_{GL} + \sum_{i \in \{1,4,7\}} \Delta p_{HEX_i} + \sum_j \Delta p_j
\]  

(2)

where \( \Delta p_{GL} \) is the pressure gap corresponding to \( \Delta T_{GL} \). \( \Delta p_{i \in \{1,4,7\}} \) indicate the pressure drop in the HEX-1, HEX-4 and HEX-7 which are schematically described in the flow diagram, respectively. \( \sum_j \Delta p_j \) means summation of pressure drops at the other components. Order of each term in the RHS of (2) is \( \Delta p_{GL} + \sum_{i \in \{1,4,7\}} \Delta p_{HEX_i} \gg \sum_j \Delta p_j \). The mass flow rate of \(^3\)He and \(^4\)He can be obtained from the following equations.

\[
\dot{m}_{3He} \simeq \frac{(Q_s + Q_d)/L_3(T_1 + \Delta T_{GL})}{\epsilon_{JT}}, \quad \dot{m}_{4He} \simeq \frac{Q_{HEX6}/L_4(T_3)}{\epsilon_{loss}} + \frac{Q_{HEX2}/L_4(T_2)}{\epsilon_{JT}}
\]  

(3)

where \( L_3 \) and \( L_4 \) are latent heat of \(^3\)He and \(^4\)He, respectively. \( Q_{HEX6} \) and \( Q_{HEX2} \) are enthalpy difference of \(^3\)He between inlet and outlet in the 4 K reservoir (HEX-6) and 1 K pot (HEX-2), which can be calculated from (5) and (6) introduced in subsection 3.1. \( \epsilon_{JT} \) and \( \epsilon_{loss} \) indicate JT efficiency and transfer loss, respectively.

2.3. Typical heat exchangers

Table 3 shows the typical heat exchangers. "MD", "NB", "NC", "FB", "KC", "TC", "ST", "FC", "2FC" indicate non-equilibrium Molecular gas Dynamics effect [4], "Nucleate Boiling, Natural Convection, Film Boiling, Kapitza Conductance, Thermal Conduction, Superfluid Turbulent flow [5], "Forced Convection, Two phase Forced Convection, respectively. Following notation (coolant | interface | cooling target) indicates brief heat transfer mechanism of the coolant, interface and cooling target fluid, respectively. "G" and "L" means Gas or Liquid, respectively. Pressure of the supplied \(^3\)He is from around 50 kPa to 10 kPa in the precooling HEXs and phase transition from gas phase to liquid occurs in the HEX-2. The practical shape and dimensions of the HEXs are determined from thermo-fluid simulation. The next section shows the numerical procedure for each HEX.
Table 3. Brief specification for typical heat exchangers.

| HEX No. | (cooling principle) | T.P.C. of coolant \((p, T)\) | Cooling target (temperature range) |
|---------|---------------------|-------------------------------|-----------------------------------|
| HEX-1   | Pool cooling        | saturated liquid \(^3\)He     | isopure He II (L) (0.8 K, 380 Pa) (0.8 K to 1.15 K) |
|         | (MD, NB, NC, KC, TC, ST) |             |                                   |
| HEX-2   | pool cooling        | saturated He II               | supplied \(^3\)He (G/L) (1.6 K, 750 Pa) (2.6 K to 1.6 K) |
|         | (ST, FB, KC, TC, FC, 2FC) |                 |                                   |
| HEX-4   | counter flow        | return \(^3\)He (G) from HEX-1 | supplied \(^3\)He (G) (0.8~4 K, 300~380 Pa) (4.2 K to 2.6 K) |
|         | (FC, KC, TC, FC)    |                                |                                   |
| HEX-6   | pool cooling        | saturated \(^4\)He (L)        | supplied 3He (G) (4.3 K, 110 kPa) (10 K to 4.3 K) |
|         | (NC, NB, FB, KC, TC, FC) |                |                                   |
| HEX-7   | tube in tube counter flow | return \(^4\)He (G) from HEX-6  | supplied 3He (G) (5~300 K, 105~110 kPa) (300 K to 10 K) |
|         | (FC, TC, FC)        |                                |                                   |

3. Simulation Model and Scheme
3.1. General procedure

Design of all heat exchangers inserted in the cryogenic system is performed by finite volume discretization based on following enthalpy based energy balance equation in compressible flow.

\[
\frac{Dh}{Dt} = \frac{\partial p}{\partial t} + u_{\alpha} \frac{\partial p}{\partial x_{\alpha}} - \tau_{\alpha\beta} \frac{\partial u_{\alpha}}{\partial x_{\beta}} - \frac{\partial q_{\alpha}}{\partial x_{\alpha}} + \dot{Q} \tag{4}
\]

where \(\frac{D}{Dt} := \partial_{t} + u_{\alpha} \partial_{x_{\alpha}}\) is convective derivative operator, \(h\) is enthalpy. \(\dot{Q}\) is heat generation per unit volume (W/m\(^3\)). \(q_{\alpha} := -\lambda \partial_{\alpha} T\) is heat flux vector. \(u_{\alpha}\) is velocity vector. \(p\) and \(\tau_{\alpha\beta}\) are pressure and viscous stress tensor, respectively. In the steady state and one dimensional system, (4) integrated in an arbitrary domain, \(x \in [x_1, x_2]\), is rewritten as follows.

\[
\int_{x_1}^{x_2} \rho u A \frac{dh}{dx} dx = \int_{x_1}^{x_2} \frac{\partial p}{\partial x} dx - \int_{x_1}^{x_2} \tau_{xx} \frac{\partial u}{\partial x} dx + \int_{x_1}^{x_2} \frac{\partial}{\partial x} \left( \frac{\lambda}{\partial x} \right) dx + Q^*(x) \tag{5}
\]

where \(A\) is cross sectional area of channel. \(Q^*(x) := \int_{x_1}^{x_2} \dot{Q} Adx\) is the local heat generation rate (W) added in the domain, \(x \in [x_1, x_2]\). It spend quite huge time to evaluate \(Q^*\) from DNS. Instead, \(Q^*\) can be evaluated from following equation with overall heat transfer coefficient, \(h^*\).

\[
Q^*(x) = h^* A^* \Delta T, \quad h^* A^* = \left( \sum_{j=1}^{5} \frac{1}{h_j A_j} + \sum R \right)^{-1} \tag{6}
\]

where \(h_1, h_2\) indicate fluid based heat transfer coefficient such as ”NC”, ”FC”, ”2FC”, ”NB”, ”FB”, ”ST” for hot and cold fluid, respectively. \(h_3, h_4\) indicate Kapitza conductance ”KC” for hot and cold fluid side, respectively. \(h_5\) means effective heat transfer due to thermal conduction ”TC”. \(A_j\) means effective surface area corresponding to \(h_j\). \(R\) is additional thermal resistance such as non-equilibrium molecular gas dynamics effect etc. The temperature dependence of thermo physical properties is also considered by coupling HEPAK and HE3PAK [6]. Due to space limitations, we will only introduce the outline of heat transfer calculation scheme of each heat exchanger while introducing references.
3.2. HEX-7

Figure 2 shows the overview and schematic flow behavior of $^3$He and $^4$He gas. Both fluids can be regarded as in the gas phase forced flow heat transfer as shown in the table 3. Therefore $h_1$ and $h_2$ can be calculated from laminar and turbulent forced convection heat transfer correlation such as Dittus-Boelter correlation [7], [8]. Kapitza resistance, $h_3$, $h_4$, can be negligible. Recently, a prototype of the HEX-7 was fabricated and tested in KEK using $^4$He instead of $^3$He to check the numerical validities.

3.3. HEX-6

HEX-6 has almost same structures as 1 K pot (HEX-2) shown in the Figure 1. HEX coil is filled in the 4 K reservoir tank. Thermo physical condition of supplied $^3$He is gas single phase. Therefore the heat transfer coefficient of $^3$He, $h_1$, can be calculated from Dittus-Boelter correlation as well. On the contrary, Heat transfer characteristics of $^4$He about non-boiling natural convection, nucleate boiling and film boiling of He I in addition to critical heat flux from nucleate boiling to film boiling has to be considered to calculate $h_2$ [9], [10], [11], [12], [13]. Effect of the Kapitza resistance can be almost negligible in this HEX but is considered in the simulation.

3.4. HEX-2

The brief structure is shown in the Figure 1. Supplied $^3$He gas condenses from gas to liquid phase in the HEX-2. Thermo-physical condition of upstream and downstream side is gas single phase and liquid single phase flow, respectively. Therefore $h_1$ has to be considered not only using the Dittus-Boelter correlation but also two phase flow heat transfer characteristics [14], [15], [16], [17] in addition to calculation of quality distribution, $\chi(x)$. The heat transfer coefficient for the He II bath is evaluated by considering film boiling heat transfer, critical heat flux from non-boiling to film boiling region in addition to the superfluid counter flow [18], [19], [20]. Since the Kapiza resistance can not be ignored, we measured the Kapitza conductance of the copper actually used. Then $h_3$ and $h_4$ is calculated from the extrapolation of experimental data based on Khalatnikov theory [21], [22].

3.5. Pressure drop of HEX-4, HEX-5, HEX-1 and film burner

In order to calculate the pressure drops, following the compressible form of the mass conservation equation, the momentum conservation equation, the energy conservation equation coupled with
The equation of state are solved instead of solving (5) and (6).

\[ \partial_t \Lambda + \nabla \cdot \Pi = \Gamma \]  

\[ pv = zRT \]  

\[ \Lambda = \begin{pmatrix} \rho \\ \rho u \\ \rho E \end{pmatrix}, \quad \Pi = \begin{pmatrix} \rho u \\ \rho uu - T \\ \rho E u - T \cdot u + q \end{pmatrix}, \quad \Gamma = \begin{pmatrix} 0 \\ \rho f \\ \rho u \cdot f \end{pmatrix} \]  

where \( T, u, q \) and \( f \) is stress tensor, velocity vector, heat flux vector and external force vector, respectively. \( E \) is total energy. If the flow becomes turbulent, the above equation system is solved by applying two different simulation schemes such as LES with Smagorinsky model and \( k - \epsilon \) model derived from renormalization group theory [23]. If the compressibility can be negligible, the incompressible Navier-Stokes equation coupled with solenoidal condition of velocity field is also applied to reduce the simulation load [24].

3.6. Time dependent behavior of He II

In such a sub-Kelvin to around 1 K region, we need to be aware of the possibility of ballistic heat conduction instead of the turbulent flow model. This is because the pipe diameter, \( D \), can be on the same order as the mean free path of the normal component, \( \lambda_n \), in such temperature regime. However, in the case of UCN, since Knudsen number, \( Kn := \lambda_n / D \ll 0.1 \), continuum assumption is satisfied rather than ballistic. In addition, some discriminant based on motion of superfluid component and normal fluid one indicate that fluid behavior becomes superfluid turbulent state rather than superfluid laminar state (e.g., [5], [25]). Therefore in addition to the two fluid model, Gorter-Mellink turbulent model is coupled with two fluid model to express the mutual friction. The total density almost keeps constant. Therefore solenoidal condition, \( \nabla \cdot u = 0 \), is satisfied. Then pressure based algorithm such as fractional step [26] is applied to the simulation using parallel computing techniques.

4. Results

Figure 3 shows the streamwise direction of supplied \(^3\)He temperature obtained from numerical simulation on the HEX-2, 4, 6 and 7, respectively. The cooling channel lengths of each heat exchanger obtained from the simulations are 8 m, 26 m, 12 m and 18 m, respectively. Table 4 shows the comparison the numerical results with corresponding experimental results. The words ”exp” and ”num” in the Table 4 indicate experimental results and numerical results, respectively. Concerning the HEX-7, numerical simulation results such as inlet and outlet temperatures and pressures agree well with the experimental results.

Figure 4 shows the model and simulation results examples on the pressure drop through the HEX-4. When the fins have holes for better gas permeability (case-2), the pressure drop
Table 4. Comparison the numerical results with experimental ones.

| Run-No | $\dot{m}_{3He}$ (g/sec) | $\dot{m}_{4He}$ (g/sec) | $T_{in3}$ (K) (exp/num) | $T_{out3}$ (K) (exp/num) | $T_{in4}$ (K) (exp/num) | $T_{out4}$ (K) (exp/num) |
|--------|----------------------|----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| RUN-1  | 0.28                 | 0.47                 | 296/291               | 6.4/7.6               | 6.1/6.1               | 181/176               |
| RUN-2  | 0.28                 | 0.36                 | 295/293               | 6.7/11.4              | 5.9/6.1               | 225/223               |
| RUN-3  | 0.14                 | 0.36                 | 294/289               | 6.2/6.2               | 6.0/6.1               | 123/115               |
| RUN-4  | 0.14                 | 0.28                 | 294/292               | 6.4/6.4               | 6.0/6.1               | 150/148               |
| RUN-5  | 0.14                 | 0.20                 | 294/292               | 7.4/8.4               | 6.2/6.1               | 206/204               |

is reduced, but the enthalpy recovery efficiency from the return gas is reduced in case-2. The optimal diameter, coil pitch and effective coil length to satisfy the specification listed in the Table 1 and 3 is 280 mm, 18 mm and 26 m, respectively.

Figure 5 shows the time evolution of the isopure He II temperature during 60 sec in the worst case that the HEX-1 does not work well such that it behaves adiabatically. In such a worst case, production volume temperature exceeds the allowable temperature of 1.15 K around 20~30 seconds after the start of the UCN production. In addition, two fluid behavior under the 0.8 K to 1.1 K is completely different from that in the case of 1.8 K, it means that the normal component propagates toward the bulk like a kind of shock wave, and its velocity is drastically faster than the superfluid component. However there is still remaining the possibility that Gorter-Mellink equation may be modified such as deviation from $m = 3$ or 3.4 due to the peculiarity of such flow, and this is a future subject so far.

5. Conclusions
The UCN cooling system requires isopure He II with a temperature range of 0.8 K to 1.15 K. In order to realize this, thermo-fluid simulations were carried out to optimize the structures of the heat exchangers. Time dependent behavior of He II under 10 W heat load were predicted from the numerical simulation based on Landau two fluid model and Gorter-Mellink turbulent model. The validity of the Gorter-Mellink equation in He II with very small Knudsen numbers

Figure 4. Flow patterns in the HEX-4 in the case of w/o hole (left) and 6 holes/pitch (right).
and less than 1 K is further to be studied. The validity of the calculation results of HEX7 was confirmed from the experiments. The validity of the other simulation models and results will be verified by fabrication and testing of the prototype of other HEXs in this fiscal year.

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