Biased Discrete Symmetry Breaking and Fermi Balls

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Abstract

The spontaneous breaking of an approximate discrete symmetry is considered, with the resulting protodomains of true and false vacuum being separated by domain walls. Given a strong, symmetric Yukawa coupling of the real scalar field to a generic fermion, the domain walls accumulate a gas of fermions, which modify the domain wall dynamics. The splitting of the degeneracy of the ground states results in the false vacuum protodomain structures eventually being fragmented into tiny false vacuum bags with a Fermi gas shell (Fermi balls), that may be cosmologically stable due to the Fermi gas pressure and wall curvature forces, acting on the domain walls. As fermions inhabiting the domain walls do not undergo number density freeze out, stable Fermi balls exist only if a fermion anti-fermion asymmetry occurs. Fermi balls formed with a new Dirac fermion that possesses no standard model gauge charges provide a novel cold dark matter candidate.
It is well known that spontaneous breaking of a discrete symmetry can produce topological structures composed of different domains separated by topological defects [1, 2, 3]. In the simplest such physical scenario, the topological defects produced are domain walls [1] (transition regions between spatial domains that possess topologically different vacuum orientations), which within the context of cosmological models, have been applied to phenomena ranging from energetically soft topological defects [4] and structurons [5, 6] for the formation of large scale structure [4, 8], to significant deviations from thermal equilibrium at the QCD scale [3], neutrino balls [10, 11], and an origin for cosmological Gamma Ray Bursts [12]. In this paper, the interaction of domain walls with a fermion sector is considered, which suggests the possible production of composite microscopic cosmological relics referred to henceforth as Fermi balls. These Fermi balls, under certain conditions, provide an unusual source for cold dark matter, and may be relics of the seeds for possible structure formation in the cold dark matter scenario.

The simplest model exhibiting topological structure is that of a real scalar field $\varphi$ with a Lagrange density of the form

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{\lambda^2}{8} (\varphi^2 - \varphi_0^2)^2$$

Clearly, equation (1) possess a $Z_2$ symmetry (invariance under $\varphi \to -\varphi$), which if spontaneously broken results in a vacuum expectation value (VEV) for $\varphi$ that has two possible values; $< \varphi > = \pm \varphi_0$. These two VEV's correspond to topologically distinct vacuum orientations (distinct values of the order parameter); here the notion of topologically distinct vacua implies that one vacuum orientation cannot be continuously deformed into the other. Yet due to the $Z_2$ symmetry being exact, neither VEV is preferred, so the determination of the VEV in a particular spatial region is set by random fluctuations in $\varphi$. Thus the spontaneous symmetry breaking results in a randomly generated network of spatial domains of both vacuum orientations that are separated by transition regions called domain walls (topological defects). The form of the domain wall solution is a topological soliton of class $\pi_0$ [13], and is easily obtained from the equation of motion for $\varphi$. The simplest such solution is that of a planar domain wall in the xy plane at $z = 0$ with the boundary conditions $\varphi(z \to \pm \infty) = \pm \varphi_0$, and has the form $< \varphi >= \varphi_0 \tanh(\delta/z)$. Here $\delta = \frac{2}{\lambda \varphi_0}$ is the wall thickness. Typically, $\delta$ is assumed to be small compared to the average radius of curvature of the walls (the thin wall approximation), so that the domain walls can be treated as two dimensional
surfaces. For the planar domain wall, the associated stress-energy tensor is
\[ T^\mu_\nu = \frac{\lambda^2 \varphi^4}{4} \cosh^{-4} \left( \frac{z}{\delta} \right) \text{diag}(1,1,1,0), \]
indicating that the only non zero pressure components are within the plane of the wall, and both are equal to minus the energy density. Due to the form of the stress energy tensor, the surface tension \( (\int T^i_i dz) \) is exactly equal to the surface energy density of the wall \( (\int T^0_0 dz) \), which has the form
\[ \sigma = \frac{2\lambda \varphi^3}{3}. \]  

The cosmological implication of spontaneous breaking of an exact discrete symmetry, as first analysed by Zel’dovich et al. [1], is the formation of stable domain walls separating protodomains (spatial regions with distinct vacuum orientations) of topologically distinct energetically degenerate ground states. These walls evolve to planar structures that dominate the energy density of the Universe. Clearly, this is in contradiction with our present observations. To avoid this prediction, the self coupling of \( \varphi \) could be fine tuned so to sufficiently delay the wall dominance of the energy density. A more creditable alternative, suggested in [1] is to remove the wall stability by requiring the discrete symmetry to be only approximate. Then, spontaneous symmetry breaking results in topologically distinct ground states that are non degenerate, as the symmetry breaking is biased. This non degeneracy manifests itself in the form of protodomains of true and false vacuum, that are separated by domain walls.

Upon formation, the domain walls evolve in accordance with the protodomain ensemble minimising its energy, so that the wall motion can be described in terms of the pressure imbalance across the domain wall [14]. In a \( \varphi \) self coupling model [14], only the false vacuum volume pressure and the normal component of the wall surface tension contribute to the pressure imbalance. The false vacuum volume pressure is typically constant, and pulls the wall towards the false vacuum protodomain, whilst the normal component of the surface tension acts to straighten the wall, and decreases with decreasing wall curvature. Thus, finite sized false vacuum protodomains (vacuum bags) collapse on themselves, whilst infinite domain walls are pulled toward the false vacuum region [14]. It is this biased discrete symmetry breaking, with its inevitable conversion of false to true vacuum that cause domain walls to disappear, and by which a wall dominated energy density disaster is avoided [2, 15, 16]. Obviously, the degree of biasing between the vacua dictates the average domain wall lifetime, and if their longevity is sufficient for them to dominate the energy density of
the Universe, then power law inflation can be induced [3, 1, 15].

As no $\varphi$ self coupling model of biased discrete symmetry breaking produces stabilised finite size vacuum bags from topological defects, other more novel couplings have been investigated [17, 10, 18], each with there own cosmological implications. The coupling advocated in this paper is one which relies on the presence of fermions strongly coupled to the scalar, with $\varphi$ symmetrically coupled to a fermion via standard Yukawa couplings:

$$L = \frac{1}{2} \bar{\psi} (i \partial - G \varphi) \psi + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{\lambda}{8} (\varphi^2 - \varphi_0^2)^2 + A(\varphi)$$

(3)

The lagrange density now contains both a Yukawa coupling of fermions to the scalar field $\varphi$, and a term $A(\varphi)$ that explicitly breaks the discrete symmetry to an approximate one. The actual form of $A(\varphi)$ is specified only to the extent that the energy difference between the two VEV orientations is $\Lambda$. (For specific examples of $A(\varphi)$ consult [17].) The Yukawa coupling implies that after spontaneous breaking, fermions acquire a mass proportional to $<\varphi>$, and so it is energetically favourable for the fermions to inhabit the domain wall as they become effectively massless there. (In the infinite planar wall there exists an analytic solution for the zero mode of the fermion bound to the domain wall [19].) Thus, any off wall fermions (that are strongly coupled) are swept up by, and reside in the domain wall. Since immediately after the phase transition each fermion will be, on average, within a correlation length of the percolating wall structure, we expect the fermions to be efficiently stuck to the walls. Domain walls quickly become populated with fermions, so that the walls (in the thin wall approximation) are essentially two dimensional surfaces inhabited by a Fermi gas of massless fermions. The associated Fermi gas pressure contributes to the pressure imbalance and acts to modify the wall dynamics. In order to halt the collapse of a finite sized false vacuum protodomain, and give stable false vacuum bubbles, the Fermi gas pressure must cancel the surface tension and false vacuum volume pressure components. This will occur if the energy of a false vacuum protodomain that has accumulated a wall gas of $N$ fermions can be minimised for some finite radius. For a vacuum bag of arbitrary shape, the energy of the bag is

$$E = V \Lambda + S \sigma + E_F$$

(4)

($V =$ the volume of the vacuum bag, $S =$ its surface area, and $E_F =$ the energy of the Fermi gas composed of $N$ wall fermions.) Assuming the wall gas is composed of massless degenerate fermions
with $g=2$ internal degrees of freedom, $E_F$ in the zero temperature limit is

$$E_F = \frac{4\sqrt{\pi}N^{\frac{3}{2}}}{3\sqrt{g}\sqrt{S}}$$

and for a spherical vacuum bag, a stabilised bag is found, with a radius given by

$$N^{\frac{2}{3}} = 6\pi\sqrt{g}\left(R^4\Lambda + 2R^3\sigma\right)$$

This halting of the collapse process is due entirely to the presence of the fermions on the domain wall, and so for spherical false vacuum protodomains, one might expect stabilised false vacuum bags with a bounding outer skin of massless fermions.

But assuming the collapse of false vacuum bags to be completely described by the process of spherical shrinking until the pressure imbalance is nullified is incorrect. The collapse process is driven by a minimisation of the bag energy, to which there are three competing elements: volume energy density splitting, surface tension energy, and surface Fermi gas energy. As the surface tension energy and the energy of the two dimensional Fermi gas, $E_F$, are dependent on the surface area of the bag and not its volume (equation (5)), the vacuum bag energy can be reduced by a decrease in the bag volume, with the surface area held constant. Thus, bags are unstable with respect to “pancake” deformations, implying the bag flattens into a sheet-like structure. In conjunction with this flattening, the vacuum bag lowers its energy by fragmenting into smaller vacuum bags. To see that fragmenting is favoured, consider the energy for an arbitrary vacuum bag, but first neglect the volume contribution. By minimising this energy with respect to the surface area $S$, the energy of the stabilised bag is found to be

$$E \bigg|_{V_A=0} = 3 \left(\frac{4\sigma\pi}{9g}\right)^{\frac{1}{3}} N$$

which is proportional to $N$. This implies that one vacuum bag with a domain wall Fermi gas composed of $N$ fermions is energetically equivalent to two vacuum bags each with $\frac{N}{2}$ fermions on their domain wall, and so vacuum bags may fragment but are not compelled to do so. However, on inclusion of the false vacuum volume energy, minimisation of energy favours bag fragmentation. These facets of the collapse process for a finite sized false vacuum protodomain result in a more involved vacuum bag evolution than the simple shrinkage to a minimal surface area stabilised by
$N$ wall inhabiting fermions, as all three act concurrently. The physical collapse process of a false vacuum bag is one of repeated shrinking, flattening, and fragmenting, that results in numerous smaller vacuum bags.

However, for a sufficiently strong coupling of the fermions to the scalar order parameter the collapse process does not continue ad infinitum, as the soliton origin of the bag structure will eventually arrest the collapse. This onset of the quantum regime is signified by the breakdown of the thin wall approximation, and implies that the domain wall radius of curvature is comparable to the size of the vacuum bag. When this occurs, the vacuum bag is no longer a bubble of false vacuum with a domain wall skin containing a two dimensional Fermi gas, but rather a ball composed almost exclusively of the domain wall, with almost all the interior false vacuum having been destroyed. Such a ball of domain wall still carries the Fermi gas, but now the massless fermions of the Fermi gas constitute a three dimensional Fermi gas inhabiting the interior of the domain wall ball. It is these balls of fermion populated domain wall that we refer to as Fermi balls, and they represent true non topological defects. If the fermions are strongly coupled to the scalar, then the Fermi balls will be stable if the energy invested in the scalar field configuration is less than the total mass the trapped fermions would have to obtain if the wall disappeared. To get a crude estimate of the size of the stabilised Fermi balls, we note that our Lagrangian contains only one dimensional parameter which, in the wall solution, determines its intrinsic thickness. By equating the minimum size of the stabilised Fermi balls $R_{\text{min}}$ to the wall thickness $\delta$, and assuming these stabilised Fermi balls adopt a minimum surface area configuration, the typical stabilised radius (radius at which the collapse process stops) is estimated by

$$R_{\text{min}} \sim \frac{2}{\lambda \varphi_0} \quad (8)$$

The radius $R_{\text{min}}$ is small, indicating the collapse of false vacuum protodomains produces in a mist of tiny Fermi balls distributed throughout the 3-space. This mist of Fermi balls should be considered as possible cosmological relics, since their stability against further collapse may be assured by energetic considerations, and Fermi ball annihilation is ruled out if the fermions are Dirac particles with conserved fermion number.

Yet biased spontaneous symmetry breaking doesn’t necessarily result in the formation of finite sized false vacuum protodomains. The nature of the protodomain structure at formation depends
on the degree of anomalous breaking $\Lambda$; for $\Lambda$ small compared to the potential barrier, a percolating domain wall structure \cite{20, 3, 8} is expected, whilst a $\Lambda$ comparable to the barrier height implies the formation of finite sized false vacuum bags. For the dynamical evolution of percolating domain walls, the analysis and conclusions differ little from that of Gelmini et. al. \cite{14}, who show that although there are several different cosmological scenarios, in which the domain walls straighten out on various scales, the false vacuum volume pressure eventually dominates the pressure imbalance. This causes the domain walls to be driven inward on the false vacuum protodomain structure, inducing a “melting” of the false vacuum. Once the false vacuum volume pressure becomes dominant, the conversion of false to true vacuum is relentless, and eventually leads to a fragmentation of the percolating domain wall structure into finite sized false vacuum bags. This fragmentation is essentially the conversion of topological defects to nontopological ones, and is a result of the system’s desire to minimise its energy. Inclusion of a strong coupling to a fermion sector causes a modification to the constraints on $\Lambda$ that define the different dynamical regimes (The surface tension $\sigma$ is replaced by $\sigma - P$ to account for the two dimensional Fermi gas pressure $P$.), but the conclusions of \cite{14} remain unaltered. This implies that irrespective of the protodomain structure formed at symmetry breaking, finite sized false vacuum bags are eventually produced, which in turn evolve into the Fermi ball structures discussed above.

Thus, biased discrete symmetry breaking with strongly coupled Dirac fermions may result in a mist of nontopological objects (Fermi balls) comprised of a superposition of the massless fermions and a local deformation of the order parameter $< \varphi >$. These Fermi balls are expected to be approximately spherical, with a radius $R_{\text{min}}$ (equation (8)). Their stability against further collapse is assured by sufficiently strong spinor scalar coupling, but stability under fermion anti-fermion annihilation has not been addressed. Such annihilations could significantly affect the Fermi ball lifetime.

A strong Yukawa coupling implies that after the symmetry breaking, the fermions collect on the domain walls, thereby enhancing the fermion anti-fermion annihilation rate. Fermion anti-fermion annihilations reduce the Fermi gas pressure, so destabilising the false vacuum bag so that collapse continues until the pressure balance is restored. Thus, confinement of the fermions to the domain wall prohibits freeze out of the number density of fermions, and so Fermi balls can exist only if there is a net fermion anti-fermion asymmetry.
Given that Fermi balls are produced, equations (6) and (8) imply that they would be composed of approximately 50 fermions, independent of the symmetry breaking scale, and possess a mass of the order of $100\varphi_0$ GeV. This suggests that a Fermi ball would appear as a very heavy slow moving particle, which if the individual wall fermion had electric charge, would carry a charge in the order of $10 - 50$ times the electron charge. Such objects therefore have characteristics similar to either heavy ions or nuclearities [21], and so analysis of the Fermi ball stopping power [22] and the negative results of nuclearite searches by collaborations such as MACRO [23] can place a constraint on the relation between the Fermi ball mass and number density.

Alternatively, Fermi balls could be composed of a new Dirac fermion that possesses no standard model gauge charges. Fermi balls would then be neutral, heavy, and non-relativistic, and due to their absence of gauge charges, would interact extremely weakly with standard model matter; barring new couplings, the only interaction (apart from the gravitational one) would be via couplings of the real scalar field $\varphi$ to the standard Higgs fields. Thus, the heavy non-relativistic neutral Fermi balls would constitute an ideal candidate for cold dark matter. This suggests a possible constraint on these neutral Fermi balls, as gravity results in an accumulation of Fermi balls around massive objects such as the sun. Gravitationally bound Fermi balls may orbit through or within the solar interior, thereby transporting energy away from the solar core by their weak scattering from solar core baryons (protons). If such heat diffusion is sufficiently efficient, the gravitationally bound Fermi balls become incompatible with the standard solar model.

Assuming the Fermi balls are the sole source of dark matter and that their contribution is such that the Universe attains closure density ($\Omega = 1$), the magnitude of the luminosity diffusion, as a function of the Fermi ball mass, can be evaluated. The analysis is based on the work of Press and Spregel [24, 25], which deals with the solar capture and the subsequent luminosity transport of cosmions [26]. For this closure density scenario, with gravitationally bound Fermi balls in approximate thermal equilibrium with the solar core, Figure 1 shows a contour plot of the luminosity transported by them relative to the solar luminosity, as a function of the Fermi ball mass relative to the proton mass, and the Fermi ball-baryon cross section relative to a fiducial cross section of reference [25]. A relative luminosity contour of unity is used to restrict the relative cross section and relative mass of the Fermi balls (which in turn can be related to $\varphi_0$), as a relative luminosity of unity or greater implies that for fixed total energy transport, the Fermi ball transport would more than halve the
Figure 1: A contour plot of the relative solar luminosity carried by the Fermi balls, as a function of $\alpha = \frac{m_{FB}}{m_p}$, the Fermi ball mass relative to the proton mass, and $\beta = \frac{\sigma_{FB}}{\sigma_c}$, the Fermi ball-baryon cross section, relative to the fiducial cross section $\sigma_c \equiv \frac{m_p}{M_\odot} R_\odot = 4.0 \times 10^{-36}$ cm$^2$. The contour shown is that of relative luminosity of unity, and the excluded region is where the relative luminosity is greater than 1.

core temperature gradient, in contradiction with the solar model [25]. The restriction on parameter space isn’t particularly severe, considering that neutral Fermi balls are expected to have extremely weak non-gravitational interactions, and so would free stream through the sun.

Finally, if Fermi ball closure density is assumed, the fermion anti-fermion asymmetry required just prior to the biased spontaneous symmetry breaking in order to produce Fermi balls can be estimated. The constraint of closure density sets a restriction on the present day Fermi ball number density, which is in turn related to the relative fermion anti-fermion asymmetry just prior to symmetry breaking, defined by

$$B = \frac{n - \bar{n}}{n}$$  \hspace{1cm} (9)

(here $n$ and $\bar{n}$ represent the number density of fermions and anti-fermions). The present day Fermi ball number density is obtained from the number density of excess fermions at the symmetry
breaking by evolving this number density forward to the present day, and then dividing this number density by the number of fermions in a typical Fermi ball. From this, the constraint on the relative fermion asymmetry is found to be of order

$$B \sim \frac{10^{-7}\text{GeV}}{\varphi_0}$$

which for a breaking scale of $\varphi_0 = 1\text{GeV}$ implies an asymmetry of $10^{-7}$, which is of similar magnitude to the baryon asymmetry at the 1GeV scale.

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