FMCW Radar with Multiple Initial Frequencies for Robust Source Number Estimation

Ryo Saito¹, Katsuhisa Kashiwagi¹, Nobuya Arakawa¹, Shohei Hamada², Koichi Ichige²,a)
¹ Murata Manufacturing Co., Ltd., 1-10-1 Higashikotari, Nagaokakyo-shi, Kyoto 617-8555, Japan.
² Department of Electrical and Computer Engineering, Yokohama National University, 79-5 Tokiwadai, Hodogaya-ku, Yokohama, Kanagawa 240-8501, Japan.
a) koichi@ynu.ac.jp

Abstract: We propose a novel method to estimate source number used in Direction Of Arrival (DOA) estimator for Frequency Modulated Continuous Wave – Multiple-Input Multiple-Output (FMCW-MIMO) radar. The main principle is that the phase of the intermediate signals can be modified by the initial frequency of transmitted signals. We confirmed that our method can distinguish true and spurious sources when the number of sources is larger than the number of true sources.

Keywords: Source number estimation, FMCW-MIMO radar, annihilating filter

Classification: Antennas and Propagation

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1 Introduction

Direction Of Arrival (DOA) estimation is one of the significant tools for high-speed wireless communication. It is well-known that most of accurate DOA estimation algorithms require the source number in advance [1]. Akaike Information Criteria (AIC) [2] and Minimum Description Length (MDL) [3] are known as classical but accurate source number estimation algorithms, however they cannot give accurate estimates when sources are highly correlated. Therefore they cannot be directly applied to Intermediate Frequency (IF) signals of Frequency Modulated Continuous Wave – Multiple-Input Multiple-Output (FMCW-MIMO) radar because they are highly correlated. In such cases, we often set the programmed (pre-determined) source number to be larger than true source number. Once DOAs are estimated, we should distinguish which one is true or spurious sources.

In this paper, we propose a novel method to estimate accurate source number using the principle of IF signals that their phases can be changed by the initial frequency of transmitted (Tx) chirp signals. We use Annihilating Filter (AF) [4] which can estimate DOAs of highly correlated signals, and also use multiple Tx chirps with different initial frequencies. The estimated DOAs by AF are classified into true and spurious sources using the phase variation of IF signals. The true source number can be accurately estimated by calculating the correlation between the initial frequency pattern and the phase fluctuation pattern of an estimated signal. Performance of the proposed method is evaluated through computer simulation.

2 Preliminaries

2.1 Signal Model

We introduce the principle of controlling the phase of IF signal by changing the initial frequency of the Tx chirp in FMCW radar. The IF signal can be obtained by mixing down the Tx signal and the received (Rx) signal. Therefore, the IF signal \( y(t) \) is represented as follows:

\[
y(t) = \frac{A_t \cdot A_r}{2} \exp \left[ j 2\pi \left( \frac{2\mu R}{c} t - \frac{4\mu R^2}{c^2} + \frac{2R}{c} f_{\min} \right) \right],
\]

where \( A_t \) and \( A_r \) are the amplitudes of Tx and Rx signals, \( \mu \) is the chirp gradient, \( R \) is the distance between the target and the radar, \( c \) is the speed of light, and \( f_{\min} \) is the initial frequency of the Tx signal. Note that the third term of the exponent part in (1) becomes a function of \( f_{\min} \).

2.2 DOA and Amplitude Estimation

The principles of estimating DOAs and complex amplitude using AF [4] are also briefly described. Assume an \( M \)-element Uniform Linear Array (ULA) and \( L(<M) \) far-field sources under an Additive White Gaussian Noise (AWGN) environment, where the source number \( L \) is equivalent to the number of objects in radar systems.
Let \( X_m(t) \) be the array input signal at the \( m \)-th element. The AF method [4] estimates DOAs by solving polynomial equation:

\[
H(z) = h_0 z^K + h_1 z^{K-1} + \cdots + h_K,
\]

where

\[
\begin{bmatrix}
X_{K+1}(f) & X_K(f) & \cdots & X_1(f) \\
X_{K+2}(f) & X_{K+1}(f) & \cdots & X_2(f) \\
\vdots & \vdots & \ddots & \vdots \\
X_M(f) & X_{M-1}(f) & \cdots & X_{M-K}(f)
\end{bmatrix}
\begin{bmatrix}
h_0 \\
h_1 \\
\vdots \\
h_K
\end{bmatrix} = 0,
\]

with \( K \) being the programmed (pre-determined) source number which satisfy \( L \leq K < M \) and to be assigned in advance. Also the parameter \( X_m(f) \) denotes the Fourier coefficient at the frequency \( f \) at the \( m \)-th element. Note that the value of \( K \) is determined to be enough large, not to miss true sources.

Then the complex amplitude of each source can be retrieved by the linear regression scheme [4] as follows:

\[
\begin{bmatrix}
1 & 1 & \cdots & 1 \\
z_1 & z_2 & \cdots & z_K \\
\vdots & \vdots & \ddots & \vdots \\
z_{M-1} & z_{M-2} & \cdots & z_{M-K}
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_K
\end{bmatrix} =
\begin{bmatrix}
X_1(f) \\
X_2(f) \\
\vdots \\
X_M(f)
\end{bmatrix},
\]

where \( \{z_k\}_{k=1}^K \) and \( \{a_k\}_{k=1}^K \) are the phase difference and the complex amplitude of the \( k \)-th source. Finally \( K \) source candidates are obtained, means that more number of true sources are estimated. Once DOAs are estimated, we should distinguish which one is true or spurious sources.

### 3 Spurious Elimination Using FMCW Signal with Different Initial Frequencies

Our method is to distinguish true and spurious sources using the relationships between the phases of Tx and Rx signals.

Fig. 1 shows a brief overview of the proposed method. Let \( f_{\min}^{(n)} \) be the minimum frequency of \( n \)-th chirp signal, where it is fixed for all \( n \) in the classical FMCW radar. In the proposed method, the minimum frequency \( \{f_{\min}^{(n)}\}_{n=0}^{N-1} \) is changed along with a given frequency pattern as in the right figure in Fig. 1 where \( N \) denotes the number of chirp signals. Means, we set a binary code that represents the relation between neighboring minimum frequencies:

\[
\text{sgn} \left( f_{\min}^{(n+1)} - f_{\min}^{(n)} \right) = 1 \text{ or } -1, \quad \text{for all } n.
\]

Note that spurious signals might have similar DOA values among the different chirp signals. However, we can distinguish true sources from spurious sources because the phases of true sources correspond to (1) but those of the spurious signal do not. Therefore, the true source candidates can be classified by grouping each estimated phase that has similar DOA value among the Tx
signals. In a similar manner with the minimum frequency relation in (5), we encode the phase sequence $p_k(n)$ of the $k$-th candidate ($k = 1, 2, \ldots, K$) to a binary code:

$$\text{sgn} \left( p_k^{(n+1)} - p_k^{(n)} \right) = 1 \text{ or } -1, \quad \text{for all } n. \quad (6)$$

Here the behavior of the minimum frequency in (5) will become similar to that of the phase sequence in (6) if the candidate corresponds to true source, and will be different if the candidate corresponds to spurious. Then the $k$-th candidate can be judged by evaluating the stacked and normalized correlation becomes larger than a certain threshold, i.e.,

$$\frac{1}{N} \sum_{n=1}^{N-1} \text{sgn} \left( f_{\text{min}}^{(n+1)} - f_{\text{min}}^{(n)} \right) \cdot \text{sgn} \left( p_k^{(n+1)} - p_k^{(n)} \right) > \varepsilon \quad (7)$$

where the parameter $\varepsilon$ is a threshold that determines whether it is a true source or a spurious source. Finally the estimated source number is determined by the number of candidates whose stacked correlation in (7) exceeds $\varepsilon$.

Fig. 1. Example signals used in the proposed method: (top-left) conventional FMCW wave, (bottom-left) proposed FMCW wave, and (right) proposed chirp signal with different initial frequencies, where the initial frequency $f_{\text{min}}^{(0)} = 77$ GHz, 10 MHz frequency difference in each chirp signal, and $N = 16$ chirp signals corresponding to a 16-bit M-sequence. Note that the frequency bandwidth $BW$ and the chirp interval $Tm$ are fixed.

4 Simulation

Performance of the proposed scheme is evaluated through simulation. We assume $3 \times 4$ FMCW-MIMO radar system where its center RF frequency and its bandwidth are 79 GHz and 4 GHz, respectively. We installed three Tx
and four Rx antenna elements on a line with the interval of $2.0\lambda$ between Tx elements and $0.5\lambda$ between Rx elements, which is regarded to be equivalent with a 12-elements ULA. Simulation parameters are as follows: the IF sampling frequency is 2.75 MHz, the number of time samples per a chirp signal is 128, SNR is 20 dB, and the distance between the targets and the radar system is 1.0 m.

### 4.1 Behavior of Phase Characteristics

First we evaluated the phase characteristics in case that the true and programmed source numbers are respectively given as two true sources $L = 2$ and six programmed sources $K = 6$, where the true DOAs are set to $-32$ and $38$ deg.

Fig. 2 shows the behavior of the estimated phase characteristics $p_k^{(n)}$ for $K = 6$ candidates when Tx transmits the chirp pattern in Fig. 1. The signals whose DOA are close to $-32$ and $38$ deg correspond to “candidate 1” and “candidate 2”, respectively.

We see from Fig. 2 that the phase characteristics of the first and second candidates are almost the same with the initial frequency pattern in the right figure of Fig. 1, while the other four phase characteristics are very different from the initial frequency pattern. Therefore, we can easily guess that the stacked correlation in (7) will become large in cases of $k = 1, 2$ and will become small in cases of $k = 3, 4, 5$ and 6.

![Fig. 2.](image)

**Fig. 2.** Example behavior of phase characteristics in case of the true source number $L = 2$ and the programmed source number $K = 6$.

### 4.2 Source Number Estimation by Stacked Correlation

We also evaluate the distribution of the stacked correlation in a different scenario. Fig. 3 shows the stacked histogram of the correlation coefficients in (7) calculated for each candidates with 1,000 Monte-Carlo trials under the conditions in case of five true sources $L = 5$ and six programmed sources $K = 6$. One of the five true sources is SNR = 0 dB with the DOA of $-44$ deg, $-44$ deg,

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and the other four sources are SNR = 10 dB with the DOAs of −25, −8, 14, 26 deg.

We see from Fig. 3 that the coefficients corresponding to the true signals becomes almost one. In contrast, the correlation coefficients calculated from the spurious sources corresponding to “Candidate 6” are distributed around zero. In this situation, the success rate of our method is 94% when we set the value of the threshold $\varepsilon$ to be 0.7. Nevertheless the classical methods AIC and MDL cannot estimate the number of sources at all (the success rate is 0%). This is because five true sources are highly correlated due of the same distance 1.0 m far from the radar system, the sample covariance matrix of the array input signal becomes rank-deficient. Originally AIC and MDL can estimate the number of correlated sources as mentioned in [2],[3], however the specification here is very severe where those methods do not work at all. Indeed AIC and MDL respectively estimated the source number as 1 and 11 (equal to 12 elements minus one) for all the 1,000 trials.

![Fig. 3. The stacked histogram of the correlation coefficient in case of the true source number $L = 5$ and the programmed source number $K = 6$.](image)

5 Conclusion

In this paper, we proposed a novel source number estimation method with multiple chirp signals that have different initial frequencies. We confirmed that the source number could be estimated more accurately compared to the conventional method by exploiting the stacked correlation between the Tx initial frequency and the phase information of the estimated signal.

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