Rotating black holes in an expanding Universe from fake supergravity

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Abstract
Using the recipe of Meessen and Palomo-Lozano (2009 J. High Energy Phys. JHEP05(2009)042), where all fake supersymmetric backgrounds of matter-coupled fake $N = 2, d = 4$ gauged supergravity were classified, we construct dynamical rotating black holes in an expanding FLRW Universe. This is done for two different prepotentials that are both truncations of the stu model and correspond to just one vector multiplet. In this scenario, the cosmic expansion is driven by two $U(1)$ gauge fields and by a complex scalar that rolls down its potential. Generically, the solutions of Meessen and Palomo-Lozano are fibrations over a Gauduchon–Tod base space, and we make three different choices for this base, namely flat space, the three-sphere and the Berger sphere. In the first two cases, the black holes are determined by harmonic functions on the base, while in the last case they obey a deformed Laplace equation that contains the squashing parameter of the Berger sphere. This is the generalization to a cosmological context of the usual recipe in ungauged supergravity, where black holes are given in terms of harmonic functions on three-dimensional Euclidean space. The constructed solutions may be instrumental in addressing analytically certain aspects of black hole physics in a dynamical context.

Keywords: black holes, supergravity models, black holes in string theory

1. Introduction

Black holes are the natural test ground for quantum gravity. Much of the current knowledge on quantum effects in strong gravitational fields indeed comes from the study of stationary black holes. However many interesting open questions, such as the validity of the cosmic
censorship conjecture or what happens when black holes collide, are dynamical in nature and thus require the study of time-dependent black hole solutions\(^1\).

One well-known such solution is the McVittie spacetime [1], whose interpretation as a black hole, or a mass particle, in an FLRW Universe has been the subject of some controversy in the literature [2–4]. Another example, which however violates the energy conditions, was constructed by Sultana and Dyer [5] using conformal methods.

Kastor and Traschen (KT) [6] obtained a solution representing an arbitrary number of electrically charged black holes, with charge equal to the mass, in a de Sitter Universe. This solution allows an analytical discussion of black hole collisions and of the issue whether such processes lead to a violation of cosmic censorship [6, 7]. The KT solution is a time-dependent generalization of the Majumdar–Papapetrou (MP) spacetime [8, 9], which describes maximally charged Reissner–Nordström black holes in static equilibrium in an asymptotically flat space. The MP solution is supersymmetric, and the existence of a Killing spinor, satisfying a first order differential equation, explains why one can take arbitrary superpositions of black holes despite the high nonlinearity of Einstein’s equations. Supersymmetry however is only compatible with a negative or vanishing cosmological constant, thus no true Killing spinor can exist in a theory with positive cosmological constant. It was shown in [10] that the KT solution admits instead a fake Killing spinor, i.e., a solution of first order equations which are related to the Killing spinor equations of supergravity but do not come from an underlying supersymmetry.

Maeda, Ohta and Uzawa (MOU) obtained four- and five-dimensional black holes in an FLRW Universe filled with stiff matter from the compactification of higher dimensional intersecting brane solutions [11]. In [12] Gibbons and Maeda presented a class of spacetimes interpolating between the KT and the four-dimensional MOU black holes as solutions to a theory with a Liouville-type scalar potential, later generalized to arbitrary dimension and further analyzed in [13]. In [14] the four-dimensional case was generalized to a scalar potential given by a sum of exponentials and the black holes were shown to admit a fake Killing spinor, explaining the superposition principle observed in the solution.

Only a few time-dependent rotating black hole solutions are known. A spinning generalization of the KT solution in a string-inspired theory was given by Shiromizu in [15]. Five-dimensional multi-centered rotating charged de Sitter black holes were constructed in [16, 17]. A rotating generalization of the five-dimensional MOU solution was obtained in [18] by solving fake Killing spinor equations.

In this paper we will use the classification of all the fake supersymmetric solutions of Wick-rotated\(^2\) \(N = 2, d = 4\) gauged supergravity coupled to (non)abelian vector multiplets given in [19]\(^3\) to build explicit time-dependent black hole solutions. We will restrict ourselves to the case of a single abelian vector multiplet, corresponding to a theory with two \(U(1)\) gauge fields and a single complex scalar field. Unlike what we did in [14], we will not require the scalar to be real (or equivalently imaginary). This will allow us to obtain solutions with rotation and NUT-charge that are generalizations of a subclass of those in [14]. For one choice of the prepotential defining the theory, these can be written in terms of two complex harmonic functions in a form similar to the IWP class of metrics [21, 22], of which they are

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\(^{1}\) Note that the black holes constructed here are comoving with the cosmological expansion, and in general do not describe colliding black holes as such, unless the whole Universe collapses. To get colliding black holes, one would need to allow the moduli (which in this case would be the poles of the various harmonic functions, cf below) to be also time-dependent. It is not clear that such a modified solution would still be fake supersymmetric. We hope to come back to this point in a future publication.

\(^{2}\) In this context, by ‘Wick rotation’ we mean \(g \rightarrow ig\), where \(g\) denotes the coupling constant.

\(^{3}\) For a classification without matter coupling (pure fake \(N = 2, d = 4\) gauged supergravity) see [20].
generalizations. We will also present solutions whose spatial slices have non-flat geometry. If the three-dimensional base space is spherical the solutions are given in terms of functions that are harmonics on the three-sphere.

The paper is organized as follows. In section 2 we briefly review fake $N=2$, $d=4$ gauged supergravity coupled to abelian vector multiplets and present the recipe of [19] to construct fake supersymmetric solutions. In section 3 we consider three different geometries for the three-dimensional base space and obtain some results that are independent of the specific theory (i.e., of the prepotential) under consideration. We also show, for flat or spherical geometry, how to obtain multi-centered solutions. In sections 4 and 5 we obtain explicit solutions for two different choices of the prepotential. In section 6 we conclude with some final remarks.

2. Fake $N=2$, $d=4$ gauged supergravity

2.1. Special geometry

In $N=2$, $d=4$ supergravity coupled to $n_V$ vector multiplets, the complex scalars of the multiplets parametrize an $n_V$-dimensional Kähler–Hodge manifold, which is the base of a symplectic bundle with the covariantly holomorphic sections

$$\mathcal{V} = \left( \frac{\mathcal{L}^A}{\mathcal{M}_A} \right), \quad D_I \mathcal{V} \equiv \partial_I \mathcal{V} - \frac{1}{2} (\partial_I \mathcal{K}) \mathcal{V} = 0,$$ (2.1)

obeying the constraint

$$\langle \mathcal{V}, \tilde{\mathcal{V}} \rangle \equiv \tilde{\mathcal{L}}^A \mathcal{M}_A - \mathcal{L}^A \tilde{\mathcal{M}}_A = -i,$$ (2.2)

where $\mathcal{K}$ is the Kähler potential. We also introduce the explicitly holomorphic section

$$\Omega \equiv e^{-\mathcal{K}/2} \mathcal{V} \equiv \left( \frac{\chi^A}{F_A} \right).$$ (2.3)

If the theory is defined by a prepotential $\mathcal{F}(\chi)$, then $F_A = \partial_A \mathcal{F}$. In terms of the section $\Omega$ the constraint (2.2) becomes

$$\langle \Omega, \tilde{\Omega} \rangle \equiv \tilde{\chi}^A F_A - \chi^A \tilde{F}_A = -i e^{-\mathcal{K}}.$$ (2.4)

The couplings of the vectors to the scalars are determined by the matrix $\mathcal{N}$, defined by the relations

$$\mathcal{M}_A = \mathcal{N}_{A \Sigma} \mathcal{L}^\Sigma, \quad D_I \mathcal{M}_A = \mathcal{N}_{A \Sigma} D_I \tilde{\mathcal{L}}^\Sigma.$$ (2.5)

In a theory with a prepotential, $\mathcal{N}$ is given by

$$\mathcal{N}_{A \Sigma} = \tilde{F}_{A \Sigma} + 2i \frac{\text{Im} (\mathcal{F})_{AA'} \chi^{A'} \text{Im} (\mathcal{F})_{\Sigma \Sigma'} \chi^{\Sigma'}}{\chi^{\Sigma} \text{Im} (\mathcal{F})_{\Omega \Omega} \chi^{\Omega}},$$ (2.6)

where $F_{A \Sigma} = \partial_A \partial_\Sigma \mathcal{F}$.

The bosonic Lagrangian in the case of abelian vector multiplets, and with Fayet–Iliopoulos (FI) gauging of a $U(1)$ R-symmetry subgroup, takes the form

\[\text{Here and in what follows we use the conventions of [19].} \]
\[ e^{-1}L_{\text{box}} = R + 2G_{ij} \partial_a Z^i \partial^a \bar{Z}^j - V + 2 \Im(\mathcal{N})_{12} F_{ab}^A \Sigma^{ab} - 2 \Re(\mathcal{N})_{12} F_{ab}^A \ast \Sigma^{ab}, \]  

(2.7)

with the scalar potential

\[ V = -\frac{g^2}{2} \left[ 4 \left| C_A \mathcal{L}^A \right|^2 + \frac{1}{2} \Im(\mathcal{N})^{-1} \Sigma^{12} C_A C_{\Sigma} \right]. \]  

(2.8)

Here \( g \) denotes the gauge coupling constant, and the FI parameters \( C_A \) determine the linear combination \( C_A A^I \) that is used to gauge the \( U(1) \). Since the matrix \( \Im(\mathcal{N})_{12} \) appears in the kinetic term of the vector fields, it must be negative definite and thus invertible. It can therefore be used as a ‘metric’ to raise and lower \( A, \Sigma, \ldots \) indices.

### 2.2. Fake Killing spinors

If we perform a Wick rotation on the gauge coupling constant, \( g \to ig/\sqrt{2} \), we obtain a new, non-supersymmetric theory with \( V \to -V \) and a gauged \( \mathbb{R} \)-symmetry\(^5\). The Killing spinor equations, coming from the vanishing of the fermionic supersymmetry variations, become

\[ D_a \epsilon_I = \left[ -2i L_{A^I} \gamma^a_{\mu} - \frac{ig}{4} C_A \mathcal{L}^A a \right] \epsilon_M \epsilon^M, \]

\[ i \mathbb{D} Z^I \epsilon_I = \left[ f^A_{\mu} \mathcal{F}^A \right] \epsilon_M \epsilon^M, \]

(2.9)

where

\[ D_a \epsilon_I \equiv \left( V_a + \frac{1}{2} Q_a - \frac{g}{2} C_A A^I \right) \epsilon_I. \]

\( Q_a = (2i)^{-1} \left( \partial_a Z^j \partial^j \mathcal{K} - \partial_a Z^j \partial^j \mathcal{K} \right) \) is the gauge field of the Kähler \( U(1) \), and

\[ f^A_{\mu} \equiv D_a \mathcal{L}^A = \left( \partial_a + \frac{i}{2} \partial_a \mathcal{K} \right) \mathcal{L}^A. \]

Since these equations do not come from supersymmetry, they are called fake Killing spinor equations, and solutions for which they are satisfied are known as fake supersymmetric.

From the fake Killing spinors one can construct the bilinears

\[ X = \frac{1}{2} \epsilon^M \epsilon_I \epsilon_J, \quad V_a = i \epsilon^M \epsilon_I \epsilon_J, \quad V^a = i (\sigma^I)_{J} \epsilon^M \epsilon_J, \]

(2.10)

and the real symplectic sections of Kähler weight zero

\[ R \equiv \Re(\mathbb{V} / X), \quad I \equiv \Im(\mathbb{V} / X). \]

(2.11)

### 2.3. Fake supersymmetric solutions

In \([19]\), Meessen and Palomo-Lozano presented a general method to obtain fake supersymmetric solutions to fake \( N = 2, d = 4 \) gauged supergravity coupled to nonabelian vector multiplets. We will restrict ourselves here to the case of just abelian multiplets and FI gauging. We will also consider only the timelike case of \([19]\), which means that we take the

\(^5\) Note that the resulting theory is different from the so-called de Sitter supergravities \([23]\). To get the latter, one also takes \( A_a \to iA_a \), which leads to gauge field kinetic terms with the wrong sign, and thus to ghosts. In the theory considered here, the kinetic terms of the gauge fields come with the correct sign. We thank P Meessen for clarifying discussions on this point.
norm of $V$ defined in (2.10) to be positive. With these restrictions, the fake supersymmetric solutions always assume the form [19]

$$ds^2 = 2 |X|^2 (d\tau + \omega)^2 - \frac{1}{2} |X|^2 h_{mn} dy^m dy^n.$$  (2.12)

$$A^A = -\frac{1}{2} R^A V + \tilde{A}^m dy^m.$$  (2.13)

$$Z^A = \frac{L^A}{L^0} = \frac{R^A + iI^A}{R^0 + iI^0},$$  (2.14)

where $V = 2\sqrt{2} |X|^2 (d\tau + \omega)$, $\omega = \omega_m dy^m$ is a 1-form which can in general depend on $\tau$, and $h$ is the metric on a three-dimensional Gauduchon–Tod [24] base space. In particular there must exist a dreibein $W^x$ for $h$ satisfying

$$dW^x = g C_A \tilde{A}^A \wedge W^x + \frac{g}{2\sqrt{2}} C_A I^A e^{xyz} W^y \wedge W^z.$$  (2.15)

Furthermore the following equations must hold:

$$\omega = g C_A \tilde{A}^A \tau + \bar{\omega},$$  (2.16)

$$\tilde{F}^A_{xy} = -\frac{1}{\sqrt{2}} e^{xyz} \hat{D}_z I^A,$$  (2.17)

$$\partial_x I^A = 0, \quad \partial_I I_A = -\frac{g}{2\sqrt{2}} C_A,$$  (2.18)

$$\hat{D}_x^2 I_A - (\hat{D}_x \partial_A) \partial_I I_A = 0,$$  (2.19)

$$\hat{D} \bar{\omega} = e^{xyz} \{ \tilde{I} \partial_I I - \bar{\omega} \partial_I I \} W^y \wedge W^z,$$  (2.20)

with

$$\tilde{F}^A \equiv d\tilde{A}^A, \quad \bar{\omega} \equiv \omega \Bigr|_{\tau=0}, \quad \tilde{I} \equiv I \Bigr|_{\tau=0},$$  (2.21)

$$\hat{D}_m I \equiv \partial_m I + g C_A \tilde{A}^A_m I, \quad \hat{D}_I I \equiv W^m \hat{D}_m I.$$  (2.22)

To obtain a specific solution we will then have to take the following steps.

1. Choose the number of vector multiplets, the real constants $C_A$ and the prepotential $F$. This completely determines the bosonic action and permits to derive the dependence of the $R$’s from the $I$’s, the so-called stabilization equations.

2. Choose a three-dimensional Gauduchon–Tod base space, that is, choose a solution $(W^x, C_A \tilde{A}^A, C_A I^A)$ of equation (2.15).

3. Determine the $I^A$’s and the $\tilde{A}^A$’s that respect the choices of points 1 and 2 and at the same time satisfy equation (2.17).

4. Determine the $I_A$’s and $\bar{\omega}$ from (2.18) and the coupled equations (2.19) and (2.20).

5. Solve the stabilization equations to find the $R$’s and finally write down the metric and the other fields of the solution using (2.16) and $1/|X|^2 = 2 \langle R |I \rangle$.

In the next sections, we will use this procedure to find some solutions to theories with one vector multiplet, so that there will be only one physical scalar $Z^1 \equiv Z$. 


3. Choice of base space

3.1. Flat space

The simplest solution of equation (2.15) is three-dimensional flat space, with

\[ W_m^A = \delta_m^A, \quad C_A \tilde{A} = C_A I^A = 0. \] (3.1)

With this choice for the base space we don’t need to distinguish between \( x, y, z \) and lower \( m, n, p \) indices.

If \( C_0 = C_1 = 0, \quad C_A I^A = 0 \) is automatically satisfied and the section \( I \) is time-independent. Using equation (2.17) and the Bianchi identity \( d\tilde{F}^A = 0 \) it can be seen that the \( I^A \) must be harmonic

\[ I^0 \equiv \sqrt{2} H^0, \quad I^1 \equiv \sqrt{2} H^1. \] (3.2)

Moreover, (2.19) implies that the \( I_A \) are harmonic as well

\[ I_0 \equiv \frac{H_0}{\sqrt{2}}, \quad I_1 \equiv \frac{H_1}{\sqrt{2}}. \] (3.3)

Equation (2.20) becomes

\[ d\tilde{\omega} = \star \left( H_0 dH^0 + H_1 dH^1 - H^0 dH_0 - H^1 dH_1 \right). \] (3.4)

If at least one of the \( C_A \) is nonzero, e.g. \( C_1 \neq 0, \quad C_A I^A = 0 \) implies \( I^1 = \frac{C_0}{C_1} I^0 \). Then, (2.17) and the Bianchi identity \( d\tilde{F}^0 = 0 \) yield

\[ I^0 = \sqrt{2} H_{im}, \quad I^1 = -\sqrt{2} \frac{C_0}{C_1} H_{im}. \] (3.5)

where \( H_{im} \) is a time-independent harmonic function.\(^6\)

Equation (2.19) together with (2.18) implies that the time-independent combination \( I_0 = \frac{C_0}{C_1} I_1 \) is harmonic. It proves convenient to express this defining

\[ I_0 \equiv \frac{C_0}{C_1} \left( I_1 - \frac{1}{2\sqrt{2}} H_1 \right) + \frac{1}{2\sqrt{2}} H_0, \] (3.6)

with \( H_0, H_1 \) harmonic functions independent of \( \tau \). Since there are no further constraints on \( \tilde{t}_1 \), the \( I_A \) can be written as

\[ I_1 = \frac{1}{2\sqrt{2}} \left( \frac{\tau}{t_1} + f \right), \quad I_0 = \frac{1}{2\sqrt{2}} \left[ \frac{\tau}{t_0} + H_0 + \frac{t_1}{t_0} (f - H_1) \right], \] (3.7)

where \( t_A \equiv -(gC_A)^{-1} \) and \( f \) is a generic function of the spatial coordinates.

Equation (2.20) becomes

\[ d\tilde{\omega} = \star \left( H_0 - \frac{t_1}{t_0} H_1 \right) dH_{im} - H_{im} d\left( H_0 - \frac{t_1}{t_0} H_1 \right), \] (3.8)

and from (2.19) one gets

\[ \partial_\rho \tilde{\omega}_\rho = t_1 \partial_\rho \partial_\rho f. \] (3.9)

\(^6\) Since \( H_{im} \) is related to the imaginary part \( I^A \), the label ‘im’ stands for ‘imaginary’.
It is always possible to set \( f \) to zero with a shift in the time coordinate, \( \tau = t - tf + t_1H_1 \), and replacing \( \tilde{\omega} \) by \( \tilde{\omega} = \tilde{\omega} - t_1d\tilde{f} + t_1dH_1 \), such that
\[
\ddot{\omega} = \left( H_0 - \frac{t_1}{t_0}H_1 \right) dH_{im} - H_{im} d\left( H_0 - \frac{t_1}{t_0}H_1 \right), \quad \partial_{\rho}\tilde{\omega}_{\rho} = 0,
\]
\[\hat{\omega} = \frac{1}{2\sqrt{2}} \left( \frac{t}{t_1} + H_1 \right). \tag{3.10}\]

An explicit choice for the harmonic functions, best expressed in Boyer–Lindquist coordinates \((r, \theta, \phi)\) with \( x + iy = \sqrt{r^2 + a^2} \sin \theta e^{i\phi} \) and \( z = r \cos \theta \), is
\[H = k + q \Re (V) + Q \Im (V), \tag{3.11}\]
with
\[V = \frac{1}{r - ia \cos \theta}. \tag{3.12}\]
If all the harmonics have this form, (3.10) is solved by
\[
\hat{\omega} = \frac{1}{\Sigma} \left(-\frac{1}{2} a \sin^2 \theta \left( 2 kQ r + q \hat{Q} \right) + \hat{k} q \left( r^2 + a^2 \right) \cos \theta \right) d\phi,
\]
where
\[
\Sigma = r^2 + a^2 \cos^2 \theta, \quad \hat{\phi} = \phi_{im} - \phi_{im'}, \quad \hat{x} = x_0 - \frac{t_1}{t_0} x_1. \tag{3.13}
\]
This choice is also suitable to be generalized to the multi-centered case. To this end, define
\[V(\bar{x}, a) = \frac{1}{\sqrt{x^2 + y^2 + (z - ia)^2}}, \tag{3.15}\]
and consider harmonic functions of the form
\[H = k + \sum_I \left( q_I \Re \left( V_I \right) + Q_I \Im \left( V_I \right) \right), \tag{3.16}\]
with \( V_I \equiv V(\bar{x} - \bar{x}_I, a_I) \), where \( \bar{x}_I \) is an arbitrary point in \( \mathbb{R}^3 \) and the parameter \( a_I \) in general depends on \( I \). As long as the charges are taken to satisfy \( q_{im_I} = \alpha q_I, Q_{im_I} = \alpha Q_I \) for every \( I \), with \( \alpha \) independent of \( I \), (3.10) reduces to
\[
\hat{\omega} = (a\hat{k} - k_{im}) \star_3 \hat{H}, \tag{3.17}
\]
where \( \hat{H} = H_0 - t_1H_1/t_0 \). \( \hat{\omega} \) is thus given by a sum over \( I \) of terms of the form (3.13), with \( q\hat{Q} = 0 \). More explicitly, (3.13) with these charge constraints can be written in Cartesian coordinates and generalized to
\[
\hat{\omega} = -2(a\hat{k} - k_{im}) \sum_I \left[ \frac{\hat{Q}_I \Re \left( V_I \right)}{||\bar{x} - \bar{x}_I||^2 + a_I^2 + 1/||V_I||^2} - \frac{\hat{q}_I \Im \left( V_I \right)}{||\bar{x} - \bar{x}_I||^2 + a_I^2 - 1/||V_I||^2} \right] \\
\cdot a_I \left[ (x - x_I)dy - (y - y_I)dx \right]. \tag{3.18}
\]
3.2. Three-sphere

Since Gauduchon–Tod spaces are actually conformal classes, it would be possible to take any conformally flat three-dimensional manifold as a base space simply by applying a conformal transformation to the quantities in section 3.1 with appropriate conformal weights, leading to a nonzero $C_{\Lambda}A^{\Lambda}$. This would however result in the same four-dimensional solutions expressed in different coordinates.

On the other hand there is a different Gauduchon–Tod structure that can be defined on the same conformal class, giving nonequivalent four-dimensional solutions. Start from a three-sphere, with metric in the form

\[ ds_{S}^{2} = \frac{1}{4} \left[ d\theta^{2} + \sin^{2} \theta \, d\phi^{2} + \left( d\varphi + \cos \theta \, d\theta \right)^{2} \right]. \tag{3.19} \]

and choose the dreibein

\[
\begin{align*}
W^{1} &= \frac{1}{2} \left( \sin \varphi \, d\theta - \sin \theta \cos \varphi \, d\phi \right), \\
W^{2} &= \frac{1}{2} \left( \cos \varphi \, d\theta + \sin \theta \sin \varphi \, d\phi \right), \\
W^{3} &= \frac{1}{2} \left( d\varphi + \cos \theta \, d\theta \right),
\end{align*} \tag{3.20}
\]

that obeys

\[ dW^{a} = -\varepsilon^{a\beta\gamma} W^{\beta} \wedge W^{\gamma}. \tag{3.21} \]

Thus, equation (2.15) is satisfied with

\[ C^{A}_{\Lambda} A^{\Lambda} = 0, \quad C_{\Lambda} I^{\Lambda} = -\frac{2\sqrt{2}}{g}. \tag{3.22} \]

A useful consequence of (3.21) is that with this frame choice we have for the associated spin connection $\omega^{a}_{\beta \gamma} = \omega^{a}_{\gamma \beta} = 2 \varepsilon^{a\beta\gamma}$, where $\omega^{a}_{\beta \gamma} \equiv W^{\mu}_{\alpha} \omega^{\alpha}_{\beta \gamma}$, as can easily be seen from Maurer–Cartan’s first structure equation. This in particular implies that for a scalar function $f$ on the sphere

\[ \partial_{x} \partial_{y} f = V_{m} V^{m} f, \tag{3.23} \]

where $V$ is the Levi-Civita connection associated with the metric (3.19), and

\[ \left[ \partial_{x}, \partial_{y} \right] = 2 \varepsilon^{a\beta\gamma} \partial_{a}. \tag{3.24} \]

From (3.22) it is clear that the ungauged theory, $C_{0} = C_{1} = 0$, is incompatible with this GT-structure, hence at least one of the $C_{\Lambda}$ must be nonzero. If $C_{1} \neq 0$, (3.22) gives

\[ I^{1} = 2\sqrt{2} \, t_{l} - \frac{2}{\sqrt{g}} I^{0}, \]

where the $t_{l}$ were defined in section 3.1. The Bianchi identity $d\tilde{F}^{0} = 0$, using (3.21), immediately implies $\varepsilon^{0\beta\gamma} \partial_{l} \tilde{F}^{\beta\gamma} = 0$. Plugging in the expression for $\tilde{F}^{0}_{\beta\gamma}$ given by (2.17) and using (3.23) one concludes that $I^{0}$ must be harmonic on the sphere

\[ I^{0} = \sqrt{2} H_{m}, \quad I^{1} = \sqrt{2} \left( 2 \, t_{l} - \frac{t_{l}}{t_{0}} H_{m} \right). \tag{3.25} \]

Equations (2.18) and (2.19) again imply that the combination $I_{0} = \frac{t_{0}}{t_{1}} I_{1}$ is harmonic on the base space.
while no additional constraint is imposed on $\tilde{t}_1$, so one has

$$ I_0 = \frac{t}{t_0} \left( I_1 - \frac{1}{2\sqrt{2}} H_1 \right) + \frac{1}{2\sqrt{2}} H_0, \quad (3.26) $$

where a generic function $f$ on $S^3$ was introduced. Equation (2.20) becomes

$$ \omega = \star - - - - + \frac{t}{t_0} H_0 - \frac{t}{t_0} H_1 \right] dH_{im} - H_{im} d\left( H_0 - \frac{t}{t_0} H_1 \right) - 2t_1 df + 2\tilde{\omega}, \quad (3.28) $$

with $\partial_s\tilde{\omega}_s = t_1 \partial_s \partial_s f$ due to (2.19). Setting as before $f = 0$ by taking $\tau = t - t_1 f + t_1 H_1$ and $\tilde{\omega} = \tilde{\omega} + t_1 df - t_1 dH_1$, one gets

$$ I_0 = \frac{1}{2\sqrt{2}} \left( t + H_0 \right), \quad I_1 = \frac{1}{2\sqrt{2}} \left( \frac{t}{t_1} + H_1 \right), \quad (3.29) $$

and $\tilde{\omega}$ satisfies

$$ d\tilde{\omega} = \star \left[ H dH_{im} - H_{im} dH - 2t_1 dH + 2\tilde{\omega} \right], \quad \partial_s\tilde{\omega}_s = V^m \tilde{\omega}_m = 0, \quad (3.30) $$

with $H \equiv H_0 - \frac{t}{t_0} H_1$. If the harmonics are chosen such as to satisfy $dH_{im} \wedge dH = 0$, the simplest solution to these equations is $\tilde{\omega} = \frac{1}{2} H_{im} dH - \frac{1}{2} H dH_{im} + t_1 dH_1$, with $d\tilde{\omega} = 0$, and all other solutions can be obtained by adding arbitrary solutions of $d\omega = 2\star \omega = 0$, which implies $V^m \omega_m = 0$; these are clearly independent of the choice of harmonic functions.

To make an explicit choice for $\tilde{\omega}$ and the harmonics it is convenient to work with the usual hyperspherical coordinates

$$ ds^2_3 = d\Psi^2 + \sin^2 \Psi \left( d\Theta^2 + \sin^2 \Theta d\Phi^2 \right). \quad (3.31) $$

In these coordinates the simplest nontrivial choice of harmonic function on $S^3$ is

$$ H = k + q \frac{\cos \Psi}{\sin \Psi}, \quad (3.32) $$

which is singular in the points $\Psi = 0, \pi$. In a neighbourhood of the singularities the metric on $S^3$ is well approximated by the flat metric in spherical coordinates with $\Psi$ playing the role of a radial coordinate, and $H \sim k + \frac{q}{\Psi}$. If all the harmonics are chosen to be of the form (3.32), the minimal $\tilde{\omega}$ becomes

$$ \tilde{\omega} = \frac{1}{2} \frac{\tilde{k} q_{im} - k_{im} \tilde{q} - 2q_1 t_1}{\sin^2 \Psi} d\Psi, \quad (3.33) $$

which is the differential of a harmonic function and as such can be set to zero by a shift in the time coordinate and a redefinition of the harmonics $H_0$ and $H_1$. This is equivalent to taking $\tilde{\omega} = 0$ from the beginning by imposing the constraint

$$ \tilde{k} q_{im} - k_{im} \tilde{q} - 2q_1 t_1 = 0. \quad (3.34) $$

The equation $d\omega = 2\star \omega$, together with (3.21) and (3.24), implies $\partial_s \partial_s \omega_s = -8 \omega_s$, which means that the components of $\omega$ with respect to the dreibein $W^a$ are spherical harmonics on $S^3$ with eigenvalue $1 - n^2 = -8$. Using the well-known expressions for these spherical harmonics and rewriting the one-forms $W^a$ in the coordinates (3.31) it is possible to obtain the most general solution for $\omega$ which is regular on the three-sphere. The metric (3.19) is
obtained by considering $S^3$ embedded in $\mathbb{C}^2$, $|z_1|^2 + |z_2|^2 = 1$, and taking the parametrization

$$z_1 = \cos \frac{\theta}{2} e^{i\frac{\psi+\chi}{2}}, \quad z_2 = \sin \frac{\theta}{2} e^{i\frac{\psi-\chi}{2}}.$$

(3.35)

Comparing this with the usual parametrization for $S^3$ in $\mathbb{R}^4$ one obtains in the coordinates (3.31) the expressions

$$W^1 = -\sin \theta \sin \phi \, d\psi + \sin \psi \left( \sin \psi \cos \theta - \cos \psi \cos \phi \sin \phi \right) d\theta$$

$$W^2 = \sin \theta \cos \phi \, d\psi + \sin \psi \left( \sin \psi \cos \theta + \cos \psi \cos \phi \sin \phi \right) d\phi,$$

$$W^3 = \cos \theta \, d\psi - \sin \psi \cos \theta \sin \theta \, d\theta - \sin^2 \psi \sin^2 \theta \, d\phi,$$

(3.36)

and the most general regular $\omega$ is

$$\omega = (a \cos \phi - b \sin \phi) \left( \sin \theta \, d\psi + \sin \psi \cos \theta \, d\theta - \sin^2 \psi \sin \theta \cos \theta \, d\phi \right)$$

$$- \sin \psi (a \sin \phi + b \cos \phi) (\sin \psi \, d\theta + \cos \psi \sin \theta \, d\phi)$$

$$- c \left( \cos \theta \, d\psi - \sin \psi \cos \theta \sin \theta \, d\theta + \sin^2 \psi \sin^2 \theta \, d\phi \right),$$

(3.37)

where $a$, $b$, and $c$ are constants.

It is also possible to construct multi-centered solutions by taking sums of harmonic functions with singularities in arbitrary points on the three-sphere. Given the standard embedding of $S^3$ in $\mathbb{R}^4$, the harmonic function $\Psi(x) = -\frac{x}{x_1}$ can be written as

$$h = \frac{x_1}{\sqrt{1 - x_1^2}},$$

(3.38)

and the analogous harmonic function with singularities in any couple of antipodal points can be simply obtained by a rotation in $\mathbb{R}^4$ sending the point $(1, 0, 0, 0)$, corresponding to $\psi = 0$, in one of the new points. However in this case one has in general $d\omega \neq 0$, and in order to reinstate $d\omega = 0$ while keeping the possibility of having an arbitrary number of black holes in arbitrary positions and with independent charges one has to impose $q_{\text{im}} = \alpha \tilde{q}$ for each of them, where $\alpha$ is a proportionality constant.

### 3.3. Berger sphere

A more general Gauduchon–Tod space can be defined starting from the Berger sphere [24], which is a squashed $S^3$ or an $\text{SU}(2)$ group manifold with an $\text{SU}(2) \times \text{U}(1)$-invariant metric

$$ds^2 = d\theta^2 + \sin^2 \theta \, d\phi^2 + \cos^2 \mu (d\psi + \cos \theta \, d\phi)^2.$$

(3.39)

Given the well-known expressions for the left-invariant 1-forms

$$\sigma^L_1 = \sin \psi \, d\theta - \sin \theta \cos \psi \, d\phi,$$

$$\sigma^L_2 = \cos \psi \, d\theta + \sin \theta \sin \psi \, d\phi,$$

and for the right-invariant 1-forms

$$\sigma^R_1 = \sin \psi \, d\theta - \sin \theta \cos \psi \, d\phi,$$

$$\sigma^R_2 = \cos \psi \, d\theta + \sin \theta \sin \psi \, d\phi,$$

$$\sigma^R_3 = d\psi + \cos \theta \, d\phi,$$
one can define the dreibein [25]
\[
W^1 = \cos \mu \sigma_1^R \pm \sin \mu \left( \cos \theta \sigma_2^R - \sin \theta \sin \varphi \sigma_3^R \right),
\]
\[
W^2 = \cos \mu \sigma_2^R \mp \sin \mu \left( \cos \theta \sigma_1^R + \sin \theta \cos \varphi \sigma_3^R \right),
\]
\[
W^3 = \cos \mu \sigma_3^R \pm \sin \mu \sin \theta \left( \sin \varphi \sigma_1^R + \cos \varphi \sigma_2^R \right),
\]
that satisfies
\[
dW^x = \pm \sin \mu \cos \mu \sigma_3^L \wedge W^x - \frac{\cos \mu}{2} e^{xyz} W^y \wedge W^z, \tag{3.40}
\]
so that equation (2.15) is satisfied with
\[
F^A = \pm \frac{\sin \mu \cos \mu}{g} \sigma_3^L, \quad C_A F^A = -\frac{\sqrt{2}}{g} \cos \mu. \tag{3.41}
\]
Using Maurer–Cartan’s first structure equation it is possible to see that for a scalar function on the Berger sphere
\[
\partial_\alpha \partial_\beta f \pm 2 \sin \mu \cos \mu \sigma_3^L \partial_\alpha f = V_m V^m f. \tag{3.42}
\]
Again at least one of the $C_A$ must be nonzero. If we assume $C_1 \neq 0$, (3.42) yields
\[
I^1 = \sqrt{2} t_1 \cos \mu - \frac{t_1}{t_0} T^0, \tag{3.43}
\]
where a generic function $f(\theta, \varphi, \psi)$ was introduced. Equation (2.20) becomes
\[
\dd \omega = \sin \mu \cos \mu \sigma_3^L \wedge \dd \omega = \ast \left[ \dd K_{im} - K_{im} \dd \tilde{K} - t_1 \cos \mu \dd f + \cos \mu \dd \omega \right], \tag{3.44}
\]
and from (2.19) we get
\[
V_m \dd \omega^m \mp \sin \mu \cos \mu \sigma_3^L \dd \omega^m = t_1 \left( V_m V^m - \sin^2 \mu \right) f. \tag{3.45}
\]
It is possible to set $f = 0$ by taking $\tau = t - t_1 f + t_1 K_1$ and $\dd \omega = \dd \omega^m + t_1 (f - K_1) \mp \sin \mu \cos \mu \sigma_3^L \dd \omega^m$, where $K_1(\theta, \varphi, \psi)$ satisfies (3.45). In this way
\[
I_0 = \frac{1}{2\sqrt{2}} \left( \frac{t}{t_0} + K_0 \right), \quad I_1 = \frac{1}{2\sqrt{2}} \left( \frac{t}{t_1} + K_1 \right). \tag{3.46}
\]
\[
\dd \omega = \sin \mu \cos \mu \sigma_3^L \dd \omega = \ast \left[ \dd K_{im} - K_{im} \dd \tilde{K} - t_1 \cos \mu \dd f + \cos \mu \dd \omega \right], \tag{3.47}
\]
and from (2.19) we get
\[
V_m \dd \omega^m \mp \sin \mu \cos \mu \sigma_3^L \dd \omega^m = t_1 \left( V_m V^m - \sin^2 \mu \right) f. \tag{3.48}
\]
It is possible to set $f = 0$ by taking $\tau = t - t_1 f + t_1 K_1$ and $\dd \omega = \dd \omega^m + t_1 (f - K_1) \mp \sin \mu \cos \mu \sigma_3^L \dd \omega^m$, where $K_1(\theta, \varphi, \psi)$ satisfies (3.45). In this way
\[
I_0 = \frac{1}{2\sqrt{2}} \left( \frac{t}{t_0} + K_0 \right), \quad I_1 = \frac{1}{2\sqrt{2}} \left( \frac{t}{t_1} + K_1 \right). \tag{3.49}
\]
with \( K_0 \equiv \tilde{K} + \frac{h}{t} K_1 \), and \( \hat{\omega} \) satisfies
\[
\dd \hat{\omega} \pm \sin \mu \cos \mu \alpha^L \wedge \hat{\omega} = \star_3 \left[ \hat{K} dK_{\text{im}} - K_{\text{im}} d\hat{K} - t_1 \cos \mu \, dK_1 + \cos \mu \, \hat{\omega} \right].
\]
\[
\nabla^m \hat{\omega}_{im} \pm \sin \mu \cos \mu \alpha^L \hat{\omega}^m = 0.
\] (3.50)

There is no obvious way of finding solutions to the equations (3.44) and (3.45) that in the limit \( \mu \to 0 \) reduce to harmonic functions of the form given in section 3.2, which is what one would expect for black hole solutions. It is however possible to consider simple solutions given by the trivial choices
\[
\hat{K}_0 = K_1 = 0, \quad \hat{K}_{\text{im}} = k_{\text{im}}, \quad \hat{\omega} = 0,
\] (3.51)
with \( k_{\text{im}} \) constant.

4. The \( F(\chi) = -\frac{1}{4} \chi^0 \chi^1 \) model

Given this prepotential, from (2.4) we can derive the Kähler potential
\[
e^{-K} = \Re (Z),
\] (4.1)
where we fixed \( |\chi^0| = 1 \). The Kähler metric is then
\[
\mathcal{G} = \partial_2 \partial_2 K = \frac{1}{4} \Re (Z)^{-2}.
\] (4.2)

From equation (2.6) one obtains
\[
\mathcal{N} = -\frac{i}{4} \begin{pmatrix} Z & 0 \\ 0 & \frac{1}{Z} \end{pmatrix}
\] (4.3)
and for the scalar potential (2.8) one gets
\[
V = g^2 \left[ \frac{C_0^2}{\Re (Z)} + 4C_0 C_1 + \frac{C_1^2}{\Re (1/Z)} \right]
\] (4.4)
(2.11) leads to
\[
\mathcal{R}^0 = -4\mathcal{I}_1, \quad \mathcal{R}^1 = -4\mathcal{I}_0, \quad \mathcal{R}_0 = \frac{1}{4} \mathcal{I}^1, \quad \mathcal{R}_1 = \frac{1}{4} \mathcal{I}^0,
\] (4.5)
as well as
\[
\frac{1}{2} |\chi|^2 = \langle \mathcal{R} | \mathcal{I} \rangle = \frac{1}{2} \mathcal{I}^0 \mathcal{I}^1 + 8 \mathcal{I}_0 \mathcal{I}_1.
\] (4.6)

4.1. Flat base space

Using the results of section 3.1, one gets in the ungauged case from (4.6)
\[
\frac{1}{2} |\chi|^2 = H^0 H^1 + H_0 H_1,
\] (4.7)
and the solution takes the well-known form [26]

\[ ds^2 = 2 |X|^2 \left( dt + \phi^0 \right)^2 - \frac{1}{2} |X|^2 d\gamma^2, \quad Z = \frac{H_0 - iH_1}{H_1 - iH_0}, \quad (4.8) \]

\[ F^0 = d \left( 2 |X|^2 H_1 \left( dt + \phi^0 \right) \right) - *_3 dH^0, \quad F^1 = d \left( 2 |X|^2 H_0 \left( dt + \phi^0 \right) \right) - *_3 dH^1, \]

with \( \phi^0 \) satisfying (3.4). In the gauged case the solution can be written as

\[ ds^2 = 2 |X|^2 \left( dt + \phi^0 \right)^2 - \frac{1}{2} |X|^2 d\gamma^2, \quad Z = \frac{t/t_0 + H_0 + i\bar{t}_0/H_{im}}{t/t_0 + H_1 - iH_{im}}, \]

\[ F^0 = d \left( 2 |X|^2 \left( \frac{t}{t_0} + H_1 \right) \left( dt + \phi^0 \right) \right) - *_3 dH_{im}, \]

\[ F^1 = d \left( 2 |X|^2 \left( \frac{t}{t_0} + H_0 \right) \left( dt + \phi^0 \right) \right) + \frac{t_1}{t_0} *_3 dH_{im}, \quad (4.9) \]

where

\[ \frac{1}{2} |X|^2 = \left( \frac{t}{t_0} + H_0 \right) \left( \frac{t}{t_1} + H_1 \right) - \frac{t}{t_0} H_{im}^2 \quad (4.10) \]

and \( \phi^0 \equiv \phi^0 - t_1 df + t_0 dH_1 \) satisfies equation (3.10).

Both solutions can also be rewritten in terms of two complex harmonic functions \( H_A \) as follows:

\[ ds^2 = \frac{1}{\Re e \left( H_0H_1 \right)} dt + (\omega)^2 - \Re e \left( H_0H_1 \right) d\gamma^2, \quad Z = \frac{H_0}{H_1}, \]

\[ F^0 = d \left[ \frac{\Re e \left( H_1 \right)}{\Re e \left( H_0H_1 \right)} \left( dt + \omega \right) \right] + *_3 \Im m \left( H_1 \right), \]

\[ F^1 = d \left[ \frac{\Re e \left( H_0 \right)}{\Re e \left( H_0H_1 \right)} \left( dt + \omega \right) \right] + *_3 \Im m \left( H_0 \right), \quad (4.11) \]

where \( \omega \) is time-independent and satisfies

\[ d\omega = *_3 \Im m \left( H_0dH_1 + H_1dH_0 \right). \quad (4.12) \]

In the ungauged case, the only additional constraint on the complex harmonics is that they are independent of time. In terms of the harmonics defined above they are given by

\[ H_0 = H_0 - iH_1, \quad H_1 = H_1 - iH^0. \quad (4.13) \]

In the gauged case the time dependence of the harmonics is completely determined by \( \partial_t H_A = 1/t_A. \) In addition they must satisfy \( \Im m \left( H_0 \right) = -\frac{\omega}{\omega} \Im m \left( H_1 \right), \) and thus

\[ H_0 = \frac{t}{t_0} + H_0 + i\frac{t_1}{t_0} H_{im}, \quad H_1 = \frac{t}{t_1} + H_1 - iH_{im}. \quad (4.14) \]

In this case there is also the additional constraint \( \partial_t \omega_p = 0. \)

---

7 Here one recognizes the substitution principle originally put forward by Behrndt and Cvetic in [27], which amounts to adding a linear time dependence to the harmonic functions in a supersymmetric black hole of ungauged \( N = 2, d = 4 \) supergravity.
Notice that (4.11) reduces to the Israel–Wilson–Perjes \(^{[21, 22]}\) solution for \(H_0 = H_1\). This means in particular that we can recover the Kerr–Newman solution with mass equal to the charge by taking

\[
H_0 = H_1 = 1 + qV \equiv 1 + \frac{q}{r - ia \cos \theta}, \quad \omega = \frac{qa \sin^2 \theta (2r + q)}{r^2 + a^2 \cos^2 \theta},
\]

expressed in Boyer–Lindquist coordinates\(^8\).

This construction suggests the more general form (3.11) for the harmonics, with \(\omega\) given by (3.13). With these choices the gauged solution explicitly reads

\[
ds^2 = \frac{\Sigma}{\Delta}dr^2 + \frac{\Sigma}{\Delta} \left[ -a \sin^2 \theta \left( 2 \tilde{k}\tilde{Q} r + \tilde{q}\tilde{Q} \right) + 2 \tilde{k}q \left( r^2 + a^2 \right) \cos \theta \right] dt d\phi \]

\[
- \frac{\Delta}{\Sigma} \left( r^2 + a^2 \right) d\phi^2 - \frac{\Delta}{\Sigma} d\theta^2 + \left[ \frac{1}{4\Delta} \left( -a \sin^2 \theta \left( 2 \tilde{k}\tilde{Q} r + \tilde{q}\tilde{Q} \right) + 2 \tilde{k}q \left( r^2 + a^2 \right) \cos \theta \right)^2 \right] d\rho^2,
\]

\[
A^0 = \frac{\Sigma}{\Delta} \left( \Sigma \left( t/t_1 + k_1 \right) + q_1 r + Q_1 a \cos \theta \right) dt
\]

\[
- \frac{1}{2} \left[ \frac{\Sigma}{\Delta} \left( \Sigma \left( t/t_1 + k_1 \right) + q_1 r + Q_1 a \cos \theta \right) \left( 2 \tilde{k}\tilde{Q} r + \tilde{q}\tilde{Q} \right) - 2Q_{im} r \right] \frac{a \sin^2 \theta}{\Sigma} d\phi
\]

\[
+ \left[ \frac{\Sigma}{\Delta} \left( \Sigma \left( t/t_1 + k_1 \right) + q_1 r + Q_1 a \cos \theta \right) \tilde{k}q - q_{im} \right] \frac{\left( r^2 + a^2 \right) \cos \theta}{\Sigma} d\rho,
\]

\[
A^1 = \frac{\Sigma}{\Delta} \left( \Sigma \left( t/t_0 + k_0 \right) + q_0 r + Q_0 a \cos \theta \right) dt
\]

\[
- \frac{1}{2} \left[ \frac{\Sigma}{\Delta} \left( \Sigma \left( t/t_0 + k_0 \right) + q_0 r + Q_0 a \cos \theta \right) \left( 2 \tilde{k}\tilde{Q} r + \tilde{q}\tilde{Q} \right) + 2 \frac{\tilde{k}}{t_0} Q_{im} r \right] \frac{a \sin^2 \theta}{\Sigma} d\phi
\]

\[
+ \left[ \frac{\Sigma}{\Delta} \left( \Sigma \left( t/t_0 + k_0 \right) + q_0 r + Q_0 a \cos \theta \right) \tilde{k}q + \frac{t_1}{t_0} q_{im} \right] \frac{\left( r^2 + a^2 \right) \cos \theta}{\Sigma} d\rho,
\]

\[
Z = \frac{\Sigma \left( t/t_1 + k_1 \right) + q_1 r + Q_1 a \cos \theta \left( \Sigma k_{im} + q_{im} r + Q_{im} a \cos \theta \right)}{\Sigma \left( t/t_1 + k_1 \right) + q_r + Q_1 a \cos \theta - i \left( \Sigma k_{im} + q_{im} r + Q_{im} a \cos \theta \right)},
\]

\(^8\) One might ask whether the solution (4.11) has a minimal fake gauged supergravity limit. However, it is easy to see that requiring the scalar \(Z\) to be constant implies \(t_1 \to \infty\) with \(t_0/t_1\) fixed, and thus \(g \to 0\), which brings us back to the ungauged case. This is consistent with the fact that (for nonvanishing rotation) the Kerr–Newman–de Sitter solution can never admit fake Killing spinors, as can be seen by analytically continuing the BPS condition (3.27) of \(\,[28]\,\) for the Carter–Plebański solution with \(\Lambda < 0\), whose KNdS limit cannot be taken. We thank M Nozawa for pointing out this.
where
\[
\Delta = \left[ \sum \left( \frac{I}{l_0} + k_0 \right) + q_0 r + Q_0 a \cos \theta \right] \left[ \sum \left( \frac{I}{l_1} + k_1 \right) + q_1 r + Q_1 a \cos \theta \right] - \frac{l_1}{l_0} \left[ \Sigma x_{im} + q_{im} r + Q_{im} a \cos \theta \right]^2,
\]
(4.20)
\[
\Sigma = r^2 + a^2 \cos^2 \theta, \quad \vec{x}_y = \vec{x}_{y_{im}} - x_{im} \vec{y}, \quad \vec{x} = x_0 - \frac{l_1}{l_0} x_1.
\]
(4.21)

It can be seen from these expressions that the constant \( k \) in \( \omega \) represents essentially a NUT charge.

### 4.2. Spherical base space

Using the results of section 3.2, the complete solution can be written in terms of harmonic functions \( H_{im}, H_0, H_1 \) on \( S^3 \) and a time-independent one-form \( \hat{\omega} \) as
\[
d s^2 = 2 |X|^2 (d \tau + \hat{\omega})^2 - \frac{1}{2} |X|^2 dS_3^2,
\]
\[
F^0 = d \left[ 2 |X|^2 \left( \frac{I}{l_0} + H_0 \right) \right] (d \tau + \hat{\omega}) - *_3 dH_{im},
\]
\[
F^1 = d \left[ 2 |X|^2 \left( \frac{I}{l_1} + H_1 \right) \right] (d \tau + \hat{\omega}) + \frac{l_1}{l_0} *_3 dH_{im},
\]
\[
Z = \frac{t_I/I_0 + H_0 - 2 t_1 + 2 t_1 H_{im}/l_0}{t/I_1 + H_1 - i H_{im}},
\]
(4.22)

where
\[
\frac{1}{2} |X|^2 = \left( \frac{I}{l_0} + H_0 \right) \left( \frac{I}{l_1} + H_1 \right) + H_{im} \left( 2 t_1 l_1 - \frac{l_1}{l_0} H_{im} \right),
\]
(4.23)

and \( \hat{\omega} \) satisfies (3.30). In particular the harmonics can be taken to be of the form (3.32), with \( \hat{\omega} \) as in section 3.2. The curvature scalars \( R, R_{\mu\nu}, R^{\mu\nu} \) and \( \bar{R}_{\mu\nu\rho\sigma} \) are singular for \( \psi = 0 \), but not in the points \( \psi = 0, \pi \) unless \( q_0 q_1 = \frac{h}{2} a^2 \).

Note finally that the scalar field (4.22) assumes the constant value \( Z = t_I/I_0 \) (where the potential (4.4) has an extremum\(^9\)) if \( t_I H_0 = t_1 H_1 \) and \( H_{im} = 0 \). In this case, \( \vec{H} = 0 \) and \( \hat{\omega} = t_1 dH_1 \). If we take \( \omega = 0 \) and define a new time coordinate \( \tau \) by \( t + t_1 H_1 = t_0 t_1 \sinh \tau \), the metric becomes
\[
d s^2 = t_0 t_1 \left[ d \tau^2 - \cosh^2 \tau dS_3^2 \right],
\]
(4.24)

and the gauge field strengths \( F^A \) vanish, so that the solution is dS\(_4\). For \( \omega \neq 0 \), one gets a deformation of dS\(_4\) with nonzero \( F^A \). This is what happens also in the ‘asymptotic’ limit \( \vec{\Psi} \sim \pi/2 \) of the solution with the explicit choice (3.32) and with \( t_0 k_0 = t_1 k_1, k_{im} = 0 \).

\(^9\) We assume \( t_I/I_0 > 0 \).
4.3. Berger sphere

For this base space, the results of section 3.3 imply that the complete solution can be written in the form

\[ ds^2 = 2 |X|^2 \left( dt \pm \sin \mu \cos \mu \sigma^i t + \hat{\omega} \right)^2 - \frac{1}{2 |X|^2} ds_3^2, \]

\[ F^0 = d \left[ 2 |X|^2 \left( \frac{t}{t_0} + K_1 \right) \left( dt \pm \sin \mu \cos \mu \sigma^i t + \hat{\omega} \right) \right] \]
\[ - \frac{1}{t_0} \left[ dK_{im} \pm \sin \mu \cos \mu \sigma^i \ K_{im} \right], \]

\[ F^1 = d \left[ 2 |X|^2 \left( \frac{t}{t_0} + K_0 \right) \left( dt \pm \sin \mu \cos \mu \sigma^i t + \hat{\omega} \right) \right] \]
\[ + \frac{f_1}{t_0} \left[ dK_{im} \pm \sin \mu \cos \mu \sigma^i \left( K_{im} - t_0 \cos \mu \right) \right], \]

\[ Z = \frac{t_0 t_1 + K_0 - i_a \cos \mu + i_t K_{im}/t_0}{t/t_1 + K_1 - iK_{im}}, \quad (4.25) \]

where

\[ \frac{1}{2 |X|^2} = \left( \frac{t}{l_0} + K_0 \right) \left( \frac{t}{l_1} + K_1 \right) + K_{im} \left( t_1 \cos \mu - \frac{f_1}{t_0} K_{im} \right), \quad (4.26) \]

the functions \( K_0 \) and \( K_1 \) satisfy (3.45), \( K_{im} \) satisfies (3.44), and the time-independent one-form \( \hat{\omega} \) is a solution of (3.50).

With the trivial choices (3.51) the solution reduces to

\[ ds^2 = \frac{t_0 t_1}{t^2 + 4 \alpha_0 t_1} \left( dt \pm \sin \mu \cos \mu \sigma^i t \right)^2 - \frac{t_0^2 + \alpha_0 t_1}{t_0 t_1} ds_3^2, \]

\[ A^4 = \pm \frac{t}{t_1} \sin \mu \left( \frac{t_1^2 \cos \mu}{t^2 + \alpha_0 t_1} - \frac{\alpha_1}{t_0 t_1} \right) \sigma^i t, \quad Z = \frac{t_1 t - i a_1}{t_0 t - i a_0}, \quad (4.27) \]

with

\[ \alpha_0 = t_1 k_{im}, \quad \alpha_1 = t_0 t_1 \cos \mu - \alpha_0 = t_0 t_1 \cos \mu - t_1 k_{im}. \quad (4.28) \]

Imposing \( \alpha_0 = \alpha_1 \), the scalar becomes constant and one obtains a solution of Einstein–Maxwell–de Sitter theory already found by Meessen [29]. This can be seen as a deformation of \( dS_4 \), which is recovered for \( \mu = 0 \).

5. The \( F(\chi) = -\frac{1}{8} \chi^2 \) model

Using (2.4) this prepotential leads to the Kähler potential

\[ e^{-K} = \Im \left( Z \right), \quad (5.1) \]

where we took \( |\chi^0| = 1 \), and to the Kähler metric

\[ G = \partial_2 \partial_2 K = \frac{3}{4} \Im \left( Z \right)^{-2}. \quad (5.2) \]
The vectors’ kinetic matrix is, according to equation (2.6)

\[
\mathcal{N} = \frac{1}{4} \begin{pmatrix}
-Z \Re (Z)^2 - \frac{i}{2} |Z|^2 \Im (Z) & \frac{3}{2} Z \Re (Z) \\
\frac{3}{2} Z \Re (Z) & -3Z + \frac{i}{2} \Im (Z)
\end{pmatrix},
\tag{5.3}
\]

and from (2.8) one gets the scalar potential

\[
V = \frac{4}{3} g^2 \frac{C^2}{\Im (Z)}.
\tag{5.4}
\]

It is worth noting that for the choice \(C_1 = 0\) (and \(C_0\) arbitrary) the potential vanishes (so-called flat gauging), and the fake supersymmetric solutions constructed here are also solutions to the equations of motion of the corresponding ungauged supergravity.

Requiring \(\Re (Z), \Im (Z) \neq 0\) and \(\langle \mathcal{R} | I \rangle > 0\) the stabilization equations give

\[
\begin{align*}
\mathcal{R}^0 &= \frac{1}{2S} \left[ (I^1)^3 + 4I^0 I_1 I^1 + 4I_0 (I^0)^2 \right], \\
\mathcal{R}^1 &= -\frac{2}{9S} \left[ 16I^0 (I_1)^2 + 3I_1 (I^1)^2 - 9I_0 I^0 I^1 \right], \\
\mathcal{R}_0 &= \frac{2}{27S} \left[ 16 (I_1)^3 - 27(I_0)^2 I^0 - 27I_0 I_1 I^1 \right], \\
\mathcal{R}_1 &= \frac{1}{6S} \left[ 4(I^2)^2 I^1 - 12I_0 I^0 I_1 - 9I_0 (I^1)^2 \right],
\end{align*}
\tag{5.5}
\]

with

\[
S \equiv \sqrt{-4(I_0 I^0)^2 + \frac{4}{3}(I_1 I^1)^2 + \frac{128}{27}I^0 (I_1)^3 - 21I_0 (I^1)^3 - 8I_0 I^0 I_1 I^1},
\tag{5.6}
\]

and

\[
\frac{1}{2} |X|^2 = \langle \mathcal{R} | I \rangle = S.
\tag{5.7}
\]

### 5.1. Flat base space

Using again the results of section 3.1 the solution in the gauged case can be written in terms of harmonic functions \(H_0, H_1\) and \(H_\text{im}\) and a time-independent one-form \(\omega\) as

\[
\begin{align*}
d\xi^2 &= S^{-1}[(dt + \omega)^2 - S d\eta^2], \\
Z &= \frac{T^1 + i_1 S / I_0}{T^0 - i S}, \\
F^0 &= \left( \frac{H_\text{im} T^0}{S^2} (dt + \omega) \right) - \star_3 dH_\text{im}, \\
F^1 &= \left( \frac{H_\text{im} T^1}{S^2} (dt + \omega) \right) + \frac{i_1}{I_0} \star_3 dH_\text{im}.
\end{align*}
\tag{5.8}
\]
with

\[
S = \sqrt{H_{im}H_0 \left[ \mathcal{T}^0 + \left( \frac{t_1}{t_0} \right)^3 H_{im}^2 \right] + H_{im}H_i \left[ \mathcal{T}^1 = \frac{4}{27} H_1^3 \right]},
\]

\[
\mathcal{T}^0 = H_{im} \left[ \left( \frac{t_1}{t_0} \right)^3 H_{im} - H_0 + \frac{t_1}{t_0} H_i \right],
\]

\[
\mathcal{T}^1 = \frac{4}{9} H_1^2 + \frac{1}{3} \left( \frac{t_1}{t_0} \right)^2 H_{im}H_1 + \frac{t_1}{t_0} H_{im}H_0,
\]

\[
H_0 = \frac{t}{t_0} + H_0, \quad H_i = \frac{t}{t_i} + H_i,
\]

(5.9)

while \( \omega \) solves equation (3.10).

In the case \( C_0 = 0 \ (t_0 \to \infty) \) and with the convenient redefinitions \( H_i \to 3/2 H_i, \ t_i = 3/2 t_i \) the solution simplifies to

\[
ds^2 = S^{-1}(dt + \omega)^2 - Sd\theta^2, \quad Z = -\frac{H_1^2}{H_0H_{im} + iS},
\]

\[
F^0 = -d \left( \frac{H_0H_{im}^2}{S^2} (dt + \omega) \right) - \star_3 dH_{im}, \quad F^1 = d \left( \frac{H_{im}H_1^2}{S^2} (dt + \omega) \right),
\]

(5.10)

where

\[
S = \sqrt{H_{im}H_1^3 - H_{im}^2 H_0^2}.
\]

(5.11)

With the choice (3.11) and (3.13), this can be explicitly written as

\[
ds^2 = \frac{\Sigma^2}{\Delta} dr^2 + \frac{\Sigma}{\Delta} \left[ -a \sin^2 \theta \left( 2 \hat{k}Q r + \hat{q}Q \right) + 2 \hat{k}q \left( r^2 + a^2 \right) \cos \theta \right] d\varphi
\]

\[
- \frac{\Delta}{\Sigma} \left( r^2 + a^2 \right) dr^2 - \frac{\Delta}{\Sigma} d\theta^2
\]

\[
+ \left[ \frac{1}{4\Delta} \frac{a \sin^2 \theta \left( 2 \hat{k}Q r + \hat{q}Q \right) + 2 \hat{k}q \left( r^2 + a^2 \right) \cos \theta}{\left( r^2 + a^2 \right)} \right]^2 d\varphi^2,
\]

(5.12)

\[
A^0 = \frac{\Sigma}{\Delta^2} \left( \Sigma k_0 + q_0 r + Q_0 a \cos \theta \left( \Sigma k_{im} + q_{im} r + Q_{im} a \cos \theta \right) \right)^2 dt
\]

\[
+ \left[ \frac{\Sigma}{2\Delta^2} \left( \Sigma k_0 + q_0 r + Q_0 a \cos \theta \right) \left( \Sigma k_{im} + q_{im} r + Q_{im} a \cos \theta \right) \right]^2
\]

\[
\times \left( 2 \hat{k}Q r + \hat{q}Q \right) + Q_{im} r \right] \cdot \frac{a \sin^2 \theta}{\Sigma} d\varphi - \left[ \frac{\Sigma}{\Delta^2} \left( \Sigma k_0 + q_0 r + Q_0 a \cos \theta \right) \right.
\]

\[
\times \left( \Sigma k_{im} + q_{im} r + Q_{im} a \cos \theta \right) \left( \hat{k}q + q_{im} \right) \cdot \frac{\left( r^2 + a^2 \right) \cos \theta}{\Sigma} d\varphi,
\]

(5.13)
\[ A^1 = \frac{\Sigma}{\Delta}(\Sigma k + q_m + Q_{im} a \cos \theta)[(\Sigma (t/\bar{t}_1 + k_1) + q_r + Q_1 a \cos \theta)^2 \]
\[ \cdot \left[ dt - \frac{1}{2} \sum \left( 2\tilde{k}\tilde{Q} + \tilde{Q}\right) a \sin^2 \theta + kq \left( r^2 + a^2 \right) \cos \theta \right] \varphi. \] (5.14)

\[ Z = -\frac{\Sigma (t/\bar{t}_1 + k_1) + q_r + Q_1 a \cos \theta}{(\Sigma k + q_m + Q_{im} a \cos \theta) (\Sigma k_0 + q_0 r + Q_0 a \cos \theta) + i\Delta}. \] (5.15)

where

\[ \Delta = \left\{ \left[ \Sigma (t/\bar{t}_1 + k_1) + q_r + Q_1 a \cos \theta \right]^2 \left[ \Sigma k + q_m + Q_{im} a \cos \theta \right] \right. \]
\[ - \left[ \Sigma k_0 + q_0 r + Q_0 a \cos \theta \right]^2 \left[ \Sigma k + q_m + Q_{im} a \cos \theta \right] \right\}^{1/2}. \]

\[ \Sigma = r^2 + a^2 \cos^2 \theta, \quad \tilde{x}\tilde{y} = x_0 y_{im} - x_{im} y_0. \] (5.16)

In the case of flat gauging, \( C_1 = 0 \) (which is inequivalent to \( C_0 = 0 \) for this model), the results of section 3.1 are still valid provided one exchanges \( 0 \) and \( 1 \) indices everywhere. Redefining \( H_t \rightarrow H_t^* \), the solution simplifies to

\[ ds^2 = S^{-1}(dt + \omega)^2 - Sd\tilde{x}\tilde{y}^2, \quad Z = -\frac{H_t - i U^*}{H_{im}}, \]

\[ F^0 = -d\left( \frac{H_{im}}{U^2} (dt + \omega) \right), \quad F^3 = d\left( \frac{H_t}{U^2} (dt + \omega) \right) - \ast_3 dH_{im}. \] (5.17)

with

\[ S = H_{im} U^* = H_{im} \sqrt{3H_t^2 - 2H_0 H_{im}}. \] (5.18)

Since the potential vanishes for \( C_1 = 0 \), this is also a (non-supersymmetric) time-dependent solution of ungauged supergravity.

The metric with the same harmonic functions and \( \omega \) as before can again be written in the form (5.12), but where now

\[ \tilde{x}\tilde{y} = 3(x_1 y_{im} - x_{im} y_1), \quad \Delta = \left[ \Sigma k + q_m + Q_{im} a \cos \theta \right] \tilde{\Delta}, \]

\[ \tilde{\Delta} = \left[ 3 \left[ \Sigma k + q_r + Q_1 a \cos \theta \right]^2 - \left[ \Sigma (t/\bar{t}_0 + k_0) + q_0 r + Q_0 a \cos \theta \right] \right. \]
\[ \cdot \left[ \Sigma k + q_m + Q_{im} a \cos \theta \right] \right\}^{1/2}. \] (5.19)

while the other fields read

\[ A^0 = -\frac{\Sigma}{\tilde{\Delta}} \left[ \Sigma k + q_m + Q_{im} a \cos \theta \right] \]
\[ \cdot \left[ dt + \frac{1}{2} \sum \left( 2\tilde{k}\tilde{Q} + \tilde{Q}\right) a \sin^2 \theta - 2kq \left( r^2 + a^2 \right) \cos \theta \right] \varphi. \] (5.20)
\[ A^1 = \frac{\Delta}{\Sigma} \left( 2\Sigma k_1 + q_1 r + Q_1 a \cos \theta \right) dt \]
\[- \frac{1}{2} \left[ \frac{\Sigma}{\Delta} \left( 2\Sigma k_1 + q_1 r + Q_1 a \cos \theta \right) \left( 2k\tilde{Q}r + \tilde{q}Q \right) - 2Q_{\text{im}} r \right] \frac{a \sin^2 \theta}{\Sigma} d\varphi \]
\[ + \left[ \frac{\Sigma}{\Delta^2} \left( \Sigma k_1 + q_1 r + Q_1 a \cos \theta \right) \kappa q - q_{\text{im}} \right] \left( r^2 + a^2 \right) \cos \theta \frac{1}{\Sigma} d\varphi, \quad (5.21) \]
\[ Z = -\frac{\Sigma k_1 + q_1 r + Q_1 a \cos \theta - i\Delta}{\Sigma k_1 + q_{\text{im}} r + Q_{\text{im}} a \cos \theta}. \quad (5.22) \]

### 5.2. Spherical base space

Using the results of section 3.2, the complete solution can be written as

\[ dx^2 = S^{-1} (dt + \dot{\omega})^2 - S dx_3^2, \quad Z = -\frac{T^1 - i\tilde{S}H_{\text{im}}}{T^0 + i\tilde{S}H_{\text{im}}}, \]
\[ F^0 = -d \left[ \frac{T^0}{S^2} (dt + \dot{\omega}) \right] - \star_3 dH_{\text{im}}, \quad F^1 = d \left[ \frac{T^1}{S^2} (dt + \dot{\omega}) \right] + \frac{t_1}{t_0} \star_3 dH_{\text{im}}, \quad (5.23) \]

where

\[ S = \sqrt{-H_0 \left( T^0 + \Omega_{\text{im}}^3 \right) + H_1 \left( T^1 - \frac{4}{27} H_{\text{im}} H_{\text{im}}^2 \right)} . \]
\[ T^0 = \Omega_{\text{im}}^3 + H_{\text{im}} H_{\text{im}} H_1 + H_{\text{im}}^3 H_0, \quad T^1 = \frac{4}{9} H_{\text{im}} H_{\text{im}}^2 + \frac{1}{3} H_{\text{im}}^2 H_1 - H_{\text{im}} \Omega_{\text{im}} H_0, \]
\[ H_A = \frac{t}{t_A} + H_A, \quad \Omega_{\text{im}} = 2t_1 - \frac{t_1}{t_0} H_{\text{im}}. \quad (5.24) \]

and \( \dot{\omega} \) satisfies (3.30). An explicit solution can be obtained with harmonics of the form (3.32), obeying the constraint (3.34), and \( \dot{\omega} \) given by (3.37).

### 5.3. Berger sphere

Making use of the results of section 3.3 the complete solution can be written as

\[ dx^2 = S^{-1} \left( dt \pm \sin \mu \cos \mu \sigma_3 t + \dot{\omega} \right)^2 - S dx_3^2, \quad Z = -\frac{\mathcal{T}^1 - i\mathcal{S}K_{\text{im}}}{\mathcal{T}^0 + i\mathcal{S}K_{\text{im}}}, \]
\[ F^0 = -d \left[ \frac{T^0}{S^2} \left( dt \pm \sin \mu \cos \mu \sigma_3 t + \dot{\omega} \right) \right] - \star_3 \left[ dK_{\text{im}} \pm \sin \mu \cos \mu \sigma_3 K_{\text{im}} \right], \]
\[ F^1 = d \left[ \frac{T^1}{S^2} \left( dt \pm \sin \mu \cos \mu \sigma_3 t + \dot{\omega} \right) \right] - \star_3 \left[ dK_{\text{im}} \pm \sin \mu \cos \mu \sigma_3 K_{\text{im}} \right]. \quad (5.25) \]
where

\[
S = \sqrt{-\mathcal{K}_0 \left( \mathcal{T}^0 + \mathcal{K}_m^3 \right) + \mathcal{K}_1 \left( \mathcal{T}^1 - \frac{4}{27} \mathcal{K}_m \mathcal{K}_0^2 \right)},
\]

\[
\mathcal{T}^0 = \mathcal{K}_m^3 + \mathcal{K}_m \mathcal{K}_0 \mathcal{K}_1 + \mathcal{K}_m \mathcal{K}_2, \quad \mathcal{T}^1 = \frac{4}{9} \mathcal{K}_m \mathcal{K}_0^2 + \frac{1}{3} \mathcal{K}_m \mathcal{K}_1 - \mathcal{K}_m \mathcal{K}_0.
\]

\[\mathcal{K}_\lambda = \frac{\mathcal{T}_\lambda}{\mathcal{T}_A}, \quad \mathcal{K}_m = t_1 \cos \mu - \frac{t_1}{t_0} \mathcal{K}_m. \quad (5.26)\]

Here the functions \( \mathcal{K}_0 \) and \( \mathcal{K}_1 \) satisfy equation (3.45), \( \mathcal{K}_m \) obeys (3.44), and the time-independent one-form \( \hat{\omega} \) is a solution of (3.50).

6. Conclusions

In this paper, we used the results of [19], where all solutions to matter-coupled fake \( N = 2, d = 4 \) gauged supergravity admitting covariantly constant spinors were classified, to construct dynamical rotating black holes in an expanding FLRW Universe. This was done for two different prepotentials that are both truncations of the stu model and correspond to just one vector multiplet. The cosmic expansion was thereby driven by two \( U(1) \) gauge fields and by a complex scalar that rolls down its potential. We considered three different choices for the Gauduchon–Tod base space over which the four-dimensional geometry is fibered, namely flat space, the three-sphere and the Berger sphere, and saw how the usual recipe in ungauged supergravity, where extremal black holes are given in terms of harmonic functions on three-dimensional Euclidean space, generalizes to a cosmological context. Some possible extensions and questions for future work are:

- study more in detail the physics of the constructed solutions, for instance the presence of trapping horizons [30], and see whether a first law of trapping horizons [31] holds;
- extend the analytic studies of nonrotating black hole collisions in de Sitter space performed in [6, 7] to the more general solutions considered here, and see how the results depend on the rotation, the cosmological scale factor different from dS, and the spatial curvature of the underlying FLRW cosmology.

We hope to come back to these points in a future publication.

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