Abstract

We derive the full analytic expression for the QCD eikonal coupling of a quark-antiquark state to the exchanged gluon-gluon state in the BFKL formalism. The formula is valid for all conformal spin configurations of the $q\bar{q}$ and $gg$ states. In particular, a new selection rule on conformal spins characterizes the non-dominant BFKL components with intercept below the Pomeron in the conformal-invariant framework.

1. The new experimental results on deep-inelastic physics at small $x$ (HERA, Tevatron) have contributed to revive the old but pertinent QCD approach of Lipatov and collaborators [1]. This approach allows one to calculate the resumed perturbative contribution of gluon-gluon states to the hard QCD Pomeron. This contribution is expected to dominate the structure functions at small $x$ in the leading $(\alpha \log 1/x)^n$ orders (LLA approximation). However in all known physical situations, it remains to determine the coupling of this gluon-gluon exchanged state to the $q\bar{q}$ state present in the projectile and/or target wave-function. It is for instance the case in the QCD dipole model [2] of the virtual photon and the proton [3] interacting in deep-inelastic scattering.

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The problem of coupling the BFKL Pomeron to external quarks (or anti quarks) has already been addressed [4, 5]. A first approach considers a local coupling to quark [4], but it explicitly spoils the conformal invariance of the theory. As shown later on [5], conformal invariance can be preserved, provided one uses a wave-function of the incident \( q\bar{q} \) pair obeying gauge invariance properties [6]. A convenient way of satisfying these constraints is to consider the eikonal coupling of the \( q\bar{q} \) state to the two-gluon exchanged state [4, 5, 7]. In particular, the initial dipole states described in the framework of the QCD dipole model [3] satisfies this eikonal prescription.

The aim of our paper is to derive the most general eikonal coupling of a \( q\bar{q} \) state to the BFKL Pomeron. In a first part 2. we recall the properties of the conformal-invariant basis giving a complete description of the BFKL Pomeron states. In a second part 3. we derive the most general expression of the eikonal vertices in terms of this conformal basis and explicitly compute the whole set of components labelled by the conformal spin \( n \) of the gluon-gluon state and the conformal spin \( n' \) of the \( q\bar{q} \) one \((n, n') \in \mathbb{Z}\). We generalize the previous result [7] obtained for \( n = n' = 0 \), and, as shown in section 4. find new selection rules, namely \( n - n' \equiv 0 \) (mod.4). Interestingly enough, this leads to a first secondary conformal-invariant trajectory with vacuum quantum numbers and intercept around \( \frac{1}{2} \) below the Pomeron. The solution is found to be a couple of convergent series in transverse momentum. Section 5. summarizes our results. Appendix A1 gives the derivation of a pseudo orthogonality relation between conformal eigenvectors, which is our main technical tool.

2. We first recall the main results of Lipatov [6] for the scattering amplitudes of colorless objects in QCD in the L.L.A.,

\[
A (s, t) = is \int \frac{d\omega}{2i\pi} s^{\omega} f_{\omega} (q^2); \quad t = -q^2
\]

with

\[
f_{\omega} (q^2) = \int d^2 k \ d^2 k' \ \phi^{(1)} (k, q) \phi^{(2)} (k', q) f_{\omega} (k, k', q)
\]

where \( f_{\omega} (k, k', q) \) can be interpreted as the \( t \)-channel partial-wave amplitude and \( k', k \) are the 2-dimensional transverse components of the exchanged gluon.
momenta (see fig.1). The vertex functions \( \phi^{(1,2)}(k, q) \) characterize the internal structure of the colliding states and can be calculated from perturbation theory in some cases. Our aim is to provide general rules obeyed by the vertex functions in a conformal invariant framework.

For this sake it is convenient to use the representation in terms of impact parameters \( \rho_i \)

\[
\delta^{(2)}(q-q') f_\omega(k, k', q) = (2\pi)^{-8} \int \prod_{r=1,2} d^2 \rho_r \prod_{r'=1,2} d^2 \rho'_{r'} \times 
\]

\[
f_\omega(\rho_1, \rho_2, \rho'_1, \rho'_2) \exp [ik\rho_1 + i(q-k)\rho_2 - ik'\rho'_1 - i(q'-k')\rho'_2] \quad (3)
\]

Following Ref.[6]:

\[
\delta^{(2)}(q-q') f_\omega(q^2) = \frac{1}{2\pi^6} \int d^2 \rho_1 d^2 \rho_2 d^2 \rho'_1 d^2 \rho'_2 \int \phi^{(1)}(\rho_1, \rho_2, q) \times \hat{\phi}^{(2)}(\rho'_1, \rho'_2, q') f_\omega(\rho_1, \rho_2, \rho'_1, \rho'_2) \quad (4)
\]

where

\[
\hat{\phi} (\rho_1, \rho_2, q) = \int d^2 k \phi(k, q) e^{ik\rho_1} e^{i(q-k)\rho_2}. \quad (5)
\]

Note that, by virtue of gauge invariance

\[
\phi^{(1,2)}(k, q) |_{k=0} = \phi^{(1,2)}(k, q) |_{k=q} \equiv 0. \quad (6)
\]

In the BFKL formalism, using the conformal invariant basis \( E^{n,\nu}(\rho_{i\alpha}, \rho_{j\alpha}) \), where \( \rho_{kl} \equiv \rho_k - \rho_l \), one obtains

\[
f_\omega(\rho_1, \rho_2, \rho'_1, \rho'_2) = \sum_n \int \frac{c(n, \nu) d\nu}{[\omega - \omega(n, \nu)]} \int d^2 \rho_0 E^{n,\nu}(\rho_{10}, \rho_{20}) E^{n,\nu}(\rho_{10}, \rho_{20}), \quad (7)
\]

with

\[
c(n, \nu) = (\nu^2 + \frac{n^2}{4}) \left\{ \left[ \nu^2 + \left( \frac{n-1}{2} \right)^2 \right] \left[ \nu^2 + \left( \frac{n+1}{2} \right)^2 \right] \right\}^{-1}, \quad (8)
\]
where \( E_{n,\nu}(\rho_{i\alpha}, \rho_{j\alpha}) \) are the \( SL(2, \mathbb{C}) \) eigenvectors corresponding to eigenvalues \( \omega(n, \nu) \) defined by the quantum numbers \( \nu \in \mathbb{R} \), the conformal dimension and \( n \in \mathbb{Z} \), the conformal spin.

\[
\omega(n, \nu) = \frac{2\alpha_s N_c}{\pi} \left[ \psi(1) - \Re \left\{ \psi \left( \frac{1 + |n|}{2} + i\nu \right) \right\} \right] \tag{9}
\]

\[
E_{n,\nu}(\rho_{i\alpha}, \rho_{j\alpha}) = \left( \frac{\rho_{ij}}{\rho_{i\alpha}\rho_{j\alpha}} \right)^{\mu+1/2} \left( \frac{\bar{\rho}_{ij}}{\bar{\rho}_{i\alpha}\bar{\rho}_{j\alpha}} \right)^{\tilde{\mu}+1/2} (-1)^{\mu-\tilde{\mu}} \tag{10}
\]

where

\[
\mu = i\nu - \frac{n}{2}; \quad \tilde{\mu} = i\nu + \frac{n}{2}. \tag{11}
\]

Now it is convenient to introduce the mixed representation of these eigenvectors using a Fourier transform

\[
E_q^{n,\nu}(\rho_{ij}) = \frac{2\pi^2}{b_{n,\nu}} \int d^2\rho_0 \left| \rho_{ij} \right| e^{i\frac{q_+\rho_{ij}}{2}} E_{n,\nu}(\rho_{i0}, \rho_{j0}), \tag{12}
\]

where \( b_{n,\nu} \) is given in Ref. [6]. Note that there exists an analytic expression [8] for \( E_q^{n,\nu}(\rho) \), namely

\[
E_q^{n,\nu}(\rho) = \left( \frac{q}{8} \right)^{\mu} \left( \frac{q}{8} \right)^{\tilde{\mu}} \Gamma(1/2 - \mu) \Gamma(1/2 - \tilde{\mu}) \times \left[ J_{-\mu} \left( \frac{q\rho}{4} \right) J_{-\tilde{\mu}} \left( \frac{q\bar{\rho}}{4} \right) - (-1)^{\mu-\tilde{\mu}} J_\mu \left( \frac{q\bar{\rho}}{4} \right) J_{-\mu} \left( \frac{q\rho}{4} \right) \right]. \tag{13}
\]

The completeness of the conformal basis implies an orthogonality relation in the mixed representation [8]

\[
\frac{1}{4\pi^2} \int \frac{d^2\rho}{|\rho|^2} E_q^{n,\nu}(\rho) \bar{E}_q^{n',\nu'}(\rho) = \delta_{n,n'} \delta(\nu-\nu') + \delta_{-n,n'} \delta(\nu+\nu') (q)^{2\tilde{\mu}} (\bar{q})^{2\mu} e^{i\delta(n,\nu)}, \tag{14}
\]

where the phase \( e^{i\delta(n,\nu)} \) is defined in Ref.[6].

3. Using now the expression (4) in the definition (3) of \( f_\omega(q^2) \) yields after some algebra

\[
\delta^2(q-q') f_\omega(q^2) = (2\pi)^{-8} \sum_n \int d^2\rho_1 d^2\rho_2 d^2\rho_1' d^2\rho_2' d^2kd d^2k' \times \int \frac{c(n, \nu) d\nu}{(\omega - \omega(n, \nu))} \int d^2\rho_0 e^{i(q-q')\rho_0} E_{n,\nu}(\rho_{10}, \rho_{20}) E^{n,\nu}(\rho_{10}, \rho_{20}) \times \phi^{(1)}(k, q) \phi^{(2)}(k', q) e^{i[k\rho_{10}+i(q-k)\rho_{20}-ik'\rho_{10'}-i(q'-k')\rho_{20}']} \tag{15}
\]
The integral over $d^2 \rho_0$ yields the expected $2\pi^2 \delta^{(2)}(q - q')$ and we get

$$f_\omega(q^2) = \sum_n \int \frac{c(n, \nu)}{(\omega - \omega(n, \nu))} V_n^{n, \nu}(q) V_2^{n, \nu}(q), \quad (16)$$

with

$$V_1^{n, \nu}(q) = \frac{1}{(2\pi)^3} \int d^2 \rho_10 d^2 \rho_20 d^2 k \phi^{(1)}(k, q) e^{i[k\rho_10 - (k - q)\rho_20]} E_n^{n, \nu}(\rho_10, \rho_20), \quad (17)$$

and a similar expression for $V_2$. Formula (16) clearly exhibits the factorization between, respectively, the BFKL kernel, the upper, and the lower vertex of the QCD t-channel partial waves.

Let us now discuss the functions $\phi^{(1,2)}$. From now on, we shall assume that the exchanged gluon-gluon state is linked to a quark-antiquark color singlet, through an eikonal current. Under this physical assumption related to a well-known semi-classical description of scattering, the functions $\phi^{(1)}$ and $\phi^{(2)}$ read

$$\phi^{(1)}(k, q) = \int d^2 r \ f_1(r) \left\{ e^{i \frac{q}{2} r} - e^{-i \frac{q}{2} r} \right\} \left\{ e^{i \frac{(q-k)}{2} r} - e^{-i \frac{(q-k)}{2} r} \right\} \quad (18)$$

The eikonal formulation is such that the QCD gauge invariance relations (6) are automatically fulfilled. $f_1(r)$ is the function which describes the internal structure of the incident state and thus is momentum-independent. In formula (17) for the vertex function $V_1^{n, \nu}(q)$, the integral over $d^2 k$ can thus be easily performed. Indeed

$$\frac{1}{(2\pi)^2} \int d^2 k \ e^{i q \rho_20} e^{i k(\rho_10 - \rho_20)} \left[ e^{i \frac{q}{2} r} + c.c. \right] = e^{i q \rho_20} \left( e^{i \frac{q}{2} r} + c.c. \right) \delta^{(2)}(\rho_10 - \rho_20)$$

$$- e^{i q \rho_20} \left[ e^{-i \frac{q}{2} r} \delta^{(2)}(\rho_10 - \rho_20 + r) + e^{i \frac{q}{2} r} \delta^{(2)}(\rho_10 - \rho_20 - r) \right]. \quad (19)$$

The $\delta^{(2)}(\rho_10 - \rho_20)$ term gives no contribution since $E_n^{n, \nu}(\rho_10, \rho_20)$ vanishes at $\rho_10 = \rho_20$.

The last term in (19) yields after $\rho_10$-integration

$$V_1^{n, \nu}(q) = - \int d^2 r \ f_1(r) \int d^2 \rho_20 \left\{ e^{i q(\rho_20 + r)} \right\} E_n^{n, \nu}(\rho_20 + r, \rho_20) + (r \Rightarrow -r)$$
\[ \begin{align*}
&= -2 \int d^2 r \ f_1 (r) \int d^2 R \ e^{i \eta R} E^{n, \nu} \left( R - \frac{r}{2}, R + \frac{r}{2} \right) \\
&= \frac{b_{n, \nu}}{\pi^2} \int d^2 r \ f_1 (r) \ |r| \ E_q^{n, \nu} (r). \tag{20}
\end{align*} \]

Note that using the eikonal coupling is nothing but projecting \( f_1 (r) \) on the eigenvectors \( E_q^{n, \nu} \) of the mixed representation.

To proceed further it is convenient to expand a generic function \( f_1 (r) \) on the complete basis \( E^{n, \nu} \). Here this function depends on \( r = \sigma_{12} \), where \( \sigma_1 \) (resp. \( \sigma_2 \)), is the quark (resp. antiquark) transverse coordinate.

\[
f_1 (r) = \sum_{n'} \int d \nu' \int \frac{d^2 \sigma_0}{|\sigma_{12}|^2} \ f_1^{n', \nu'} \ E^{n', \nu'} (\sigma_{10}, \sigma_{20}). \tag{21}
\]

where the coefficients \( f_1^{n', \nu'} \) do not depend on \( \sigma_0 \) since \( f_1 \) describes the internal structure of the incoming state independently of the reference coordinate \( \sigma_0 \). Now

\[
\int d^2 \sigma_0 \ E^{n', \nu'} (\sigma_{10}, \sigma_{20}) = |\sigma_{12}| \ E_q^{n', \nu'} (\sigma_{q=0}, \sigma_{20}) \frac{b_{n', \nu'}}{\pi^2}, \tag{22}
\]

with

\[
E_q^{n', \nu'} (\sigma_{12}) |_{q=0} = (\sigma_{12})^{\mu'} (\sigma_{12})^{\bar{\mu}'}, \ \mu' = i \nu' - \frac{n'}{2}, \ \bar{\mu}' = i \nu' + \frac{n'}{2} \tag{23}
\]

which obeys the orthogonality relation (14) for \( q = 0 \), namely

\[
\frac{1}{2\pi^2} \int E_0^{n', \nu'} (\sigma_{12}) E_0^{n, \nu} (\sigma_{12}) \frac{d^2 \sigma_{12}}{|\sigma_{12}|^2} = \delta_{n, n'} \delta (\nu - \nu'). \tag{24}
\]

The coefficients \( f_1^{n', \nu'} \) are readily obtained by inversion

\[
f_1^{n', \nu'} = \frac{1}{b_{n', \nu'}} \int d^2 \sigma_{12} \ f_1 (\sigma_{12}) \ E_q^{n', \nu'} (\sigma_{q=0}, \sigma_{12}) |\sigma_{12}|. \tag{25}
\]

Inserting the analytic form (23), we get at once

\[
f_1^{n', \nu'} = \frac{1}{b_{n', \nu'}} \int rdrd\theta \ |r|^{2i\nu' + 1} \ e^{-i \eta' \theta} \ f_1 (r). \tag{26}
\]

Note that in the isotropic case \( f_1 (r) = f_1 (|r|) \). The angular integration yields \( n' = 0 \) and \( f_1^{0, \nu'} \) is nothing but the Mellin transform of \( f_1 (|r|) \).
The final expression for the vertex function \( V_{1,\nu}^{n,\nu}(q) \) reads

\[
V_{1,\nu}^{n,\nu}(q) = 2b_{n,\nu} \sum_{n'} \int d^2 \phi' f_1^{n',\nu'}  \frac{1}{2\pi^2} \int \frac{d^2 r}{|r|^2} E_q^{n,\nu}(r) \bar{E}_{0}^{n',\nu'}(r).
\] (27)

All amounts to compute the pseudo orthogonality relations (i.e. for \( E_q \) and \( E_0 \)), namely

\[
\mathcal{I} = -\frac{1}{2\pi^2} \int \frac{d^2 r}{|r|^2} E_q^{n,\nu}(r) \bar{E}_{0}^{n',\nu'}(r).
\] (28)

Crucial for the vertex evaluation, the calculation of the pseudo orthogonality relation (27) is given in the appendix A1. First, by mere symmetry property, \( E_{l,\lambda}^{l,\lambda}(|r|e^{i\phi}) = (-1)^l E_{l,\lambda}^{l,\lambda}(|r|e^{i(\phi+\pi)}) \) where \( \phi \) is the \( (r,q) \) angle, only the configurations with integer \( \frac{n-n'}{2} \) do contribute, elsewhere \( \mathcal{I} \equiv 0 \). In the latter case, the result reads

\[
\mathcal{I} = \frac{1}{4\pi} (-1)^{\frac{n-n'}{2}} \left[ \frac{q}{8} \right]^{\mu-\mu'} \left[ \frac{q}{8} \right]^{\mu-\mu'} \frac{\Gamma(1-\mu)}{\Gamma(\bar{\mu})} \frac{\Gamma \left( \frac{\mu+\mu'}{2} \right) \Gamma \left( \frac{-\mu+\mu'}{2} \right)}{\Gamma \left( 1-\frac{\mu+\mu'}{2} \right) \Gamma \left( 1-\frac{-\mu+\mu'}{2} \right)}.
\] (29)

4. The result (29) leads to quite noticeable selection rules. Indeed, the integral \( \mathcal{I} \) does exhibit simple poles which are only located at

\[
\mu + \mu' = -2p_1; \quad \mu' - \mu = -2p_2; \quad p_{1,2} \in \mathbb{N}.
\] (30)

At once, using the definition (23) one notes that poles contributing to the \( \nu' \) integral of the vertex (27) have the same imaginary part \( iv' = \pm iv \) and, for the real part, \( \frac{n+n'}{4} \) is an integer. In fact, it is already known [8] that the BFKL Pomeron exchange implies \( n \) even, from symmetry of the reaction. Hence, the relation boils down to \( \frac{n-n'}{4} \) being an integer.

This result is meaningful. The pseudo orthogonality relation (29) tells us that there exists a stringent selection rule, since only spacing by 4 is allowed, namely

\[
n - n' = 0, \pm 4, \pm 8, ..., \pm 4p; \quad p \in \mathbb{N}.
\] (31)

As a consequence, for an isotropic vertex \( n' = 0 \), only \( SL(2,\mathbb{C}) \) eigenvectors with \( n = 0 \cdots \pm 4p \) do contribute. Conversely, the \( n = 0 \) component, which is the BFKL Pomeron is coupled only to vertex coefficients with \( n' = 0 \cdots \pm 4p \).
Conformal invariance thus predicts subasymptotic contributions in energy, the first one being of the form (see e.g. formula (1):

\[ A(s, t = 0) \simeq s^{\omega(n=4,\nu=0)}, \]

where the value \( \nu \simeq 0 \) corresponds to the dominant saddle-point at \( t = 0 \) and, from (9), the intercept is given by,

\[ \omega(n = 4, \nu = 0) = \frac{2\alpha_s N_c}{\pi} [\psi(1) - \psi(5/2)]. \] (33)

The high-energy behaviour (32) has to be compared with the dominant BFKL behaviour given by \( s^{\omega(0,0)} \). The difference of intercepts is thus

\[ \omega(0, 0) - \omega(4, 0) = \frac{2\alpha_s N_c}{\pi} [\psi(1/2) - \psi(5/2)] = \frac{16\alpha_s N_c}{3}. \] (34)

Interestingly enough, a phenomenological determination of the BFKL intercept based on proton structure functions gives \( \omega(0, 0) \simeq 0.3 \) and leads to \( \omega(0, 0) - \omega(4, 0) = \frac{3}{4\ln^2} \omega(0, 0) \simeq 1.9 \omega(0, 0) \) of the order 0.6. It is thus likely that conformal invariance selection rules does not contradict some aspects of subasymptotic corrections to the Pomeron.

Using the pseudo-orthogonality relation (29), one finally gets the expression of the vertex functions, namely

\[ V_{1}^{n,\nu}(q) = \frac{b_{n,\nu}}{2\pi} (-1)^{n-n'} \frac{\Gamma(1-\mu)}{\Gamma(\bar{\mu})} \sum_{n'} \int d^{2}\nu' f_{1}^{n',\nu'} \]

\[ \times \left[ \frac{q}{8} \right]^{\bar{\mu}} \left[ \frac{q}{8} \right]^{-\mu} \frac{\Gamma\left(\frac{\mu+\nu'}{2}\right)\Gamma\left(\frac{-\mu+\nu'}{2}\right)}{\Gamma\left(1-\frac{\mu+\nu'}{2}\right)\Gamma\left(1-\frac{-\mu+\nu'}{2}\right)}. \] (35)

In order to take advantage of the simple pole structure of \( I \), we have to deform the imaginary \( z = i\nu' \) integration contour upon the real axis. In order to do so, we have to discuss both the convergence properties of \( I \) for large \( z \) modulus and the analytical structure and convergence properties of the vertex coefficients \( f_{1}^{n',-iz} \). We obtain for large \( |z| \) :

\[ |I| \simeq \left[ \frac{q q}{64} \right]^{-3R_z} \Gamma^4\left(\frac{z}{2}\right) \propto \left\{ \left[ \frac{|z|}{2} \right] \right\}^{-2R_z}. \] (36)
It is clear from (36) that, in absence of a modification of the dominant behaviour of the vertex integrand by the vertex coefficients $f_{1}^{n',-iz}$, the integral contour may be closed on the left of the $z$-plane, picking the two series of pole contributions at $z = i\nu - \frac{n-n'+4p_{1}}{2}$ and $z = -i\nu + \frac{n+n'-4p_{2}}{2}$, provided $\Re z$ be a negative integer. This gives two convergent series for all values of $q$, whose contributions are obtained at the poles fixed by the BFKL dynamics.

Conversely, if the convergence properties of the vertex coefficients $f_{1}^{n',-iz}$ allow a contour deformation to the right of the complex $z$-plane, one obtains another convergent series for all values of $q$, but picking up now the singularities of these vertex coefficients.

An intermediate and interesting alternative is when there is a matching of the large $z$ behaviour of $I$ and $f_{1}^{n',-iz}$. In this case one may deform the contour to the left (resp. to the right), if $q < q_{c}$, (resp. $q > q_{c}$), where the radius of convergence $q_{c}$ is easily determined knowing the behaviour of $f_{1}^{n',-iz}$, compared to that of $I$.

Note that in all three cases, one obtains a finite answer for every value of the momentum transfer $q$. In the first two cases, there is a unique regime for the $q$ dependence of the vertex function, which is determined either by BFKL dynamics or by the vertex coefficients. In the third (intermediate) configuration, one observes the existence of two regimes, one for small $q$ with BFKL dynamics and one at large $q$ with vertex dynamics. As an illustration of this discussion, let us consider some physical examples [7] of the function $f_{1}(r)$, corresponding to the simple isotropic case. A typical hadron non-perturbative probability distribution may be of the form $f(r) \propto e^{-Q_{h}^{2}r^{2}}$, where $Q_{h}$ is a typical (small) hadronic scale. Using formula (26), this leads to $f_{1}^{0,-iz} \propto \Gamma(-z)$ at large $z$. Hence, this belongs to the first case (left contour). For a high-$Q^{2}$ virtual photon external state, perturbative QCD leads to $f(r) \propto K_{0,1}(Q_{T})$, and thus to $f_{1}^{0,-iz} \propto \Gamma^{2}(-z) \simeq \Gamma^{4}(-\frac{2}{z})$ at large $z$, so that we are in the third case with a finite radius of convergence. This is the case with two different regimes of vertex $q$-dependence. Notice that, for a non-perturbative type of coupling our conclusions differ from [7], since in this case, we have shown that there is only one, BFKL-dominated, regime instead of two.

5. Let us summarize our main results on the general eikonal coupling of a quark-antiquark state to the BFKL Pomeron:

i) The conformal invariance properties of the BFKL kernel lead to a
general exact solution for the vertex functions using the $SL(2,\mathbb{C})$ expansion of the quark-antiquark probability distribution. It amounts to the projection of this probability distribution on the $SL(2,\mathbb{C})$ eigenvector $E_{q}^{n,\nu}$, see formulae (20,33), where use is made of a pseudo orthogonality relation involving $E_{q}^{n,\nu}$ and $E_{0}^{n,\nu}$, where $q$ is the transverse momentum of the reaction.

ii) Non-trivial selection rules are obtained for the conformal spins $n$ and $n'$, where the former governs the energy dependence of the Pomeron components and the latter is associated with vertex function components. One gets $n-n'=4p$, with $p$ integer.

iii) As a consequence, an isotropic vertex ($n'=0$) leads to a selection rule on the subassymtotic components of the BFKL kernel. Their effective difference of intercept with the Pomeron is given by

$$\omega(0,0) - \omega(4p,0) = \frac{2\alpha_sN_c}{\pi} \left[ \psi(1/2) - \psi \left( \frac{4p+1}{2} \right) \right].$$

Phenomenologically, the first subassymtotic component is around .6 below the BFKL Pomeron, and thus could play a rôle for models using the BFKL formalism. Conversely, the vertex function components coupled to the main BFKL component ($n=0$) are limited to conformal spins $n'=4p$, which may indicate interesting selection rules on the angular momenta of the $q\bar{q}$ states coupled to the QCD Pomeron.

iv) The solution for the vertex can be expressed as a convergent series for all values of $q$. More precisely, In almost all cases, the series is given by the residues of the poles of the BFKL coefficient function $I$, see formula (29). In some cases, e.g. virtual photon QCD vertex, there is a finite radius of convergence $q = q_c$, below which the same is true, while for $q > q_c$ the expansion is given by the residues of the poles related to the quark-antiquark probability distribution in the photon. In this case two regimes are present in the transverse momentum distribution.

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Using the Γ doubling formula

\[
\Gamma\left(\frac{\mu}{2}\right) = \frac{\pi}{\Gamma\left(\frac{\mu+1}{2}\right) \Gamma\left(\frac{\mu-1}{2}\right)}
\]

yields

\[
I = \left(\frac{1}{4\pi^2}\right)^n \frac{b_{n,\nu}}{2\pi^2} \int d^2 \rho d^2 R e^{iq \cdot R} \rho^\mu \rho^\mu' \rho^\nu \rho^\nu' \left[ R^2 - \frac{\rho^2}{4} \right]^{-\left(\mu + \frac{1}{2}\right)} \left[ R^2 - \frac{\rho^2}{4} \right]^{-\left(\nu + \frac{1}{2}\right)},
\]

for \( n - n' \) even, and is 0 for \( n - n' \) odd. The change of variable \( \rho^2 = 4R^2r \) yields

\[
I = (-1)^n \frac{b_{n,\nu}}{\pi^2} \int d^2 r r^{\frac{\mu + \mu'}{2} - 1} r^{\frac{\mu + \mu'}{2} - 1} [1 - r]^{-\left(\mu + \frac{1}{2}\right)} [1 - r]^{-\left(\nu + \frac{1}{2}\right)}
\]

\[
\times 2^{(\mu + \mu' + \nu' + \nu)} \int d^2 \bar{r} e^{iq \cdot \bar{R}} R^{\mu - \nu - 1} \bar{R}^{\nu' - \mu' - 1}.
\]

Using now known mathematical identities \[6, 9, 10\], one writes

\[
\int d^2 r r^{\frac{\mu + \mu'}{2} - 1} r^{\frac{\mu + \mu'}{2} - 1} [1 - r]^{-\left(\mu + \frac{1}{2}\right)} [1 - r]^{-\left(\nu + \frac{1}{2}\right)}
\]

\[
\times \sin \frac{\pi}{2} (\mu + \mu') \sin \frac{\pi}{2} (-\mu + 1/2)
\]

\[
\Gamma\left(\frac{\mu + \mu'}{2}\right) \Gamma\left(-\mu + 1/2\right) \Gamma\left(\frac{\mu + \mu' + 1}{2}\right) \Gamma\left(-\nu + 1/2\right)
\]

\[
\frac{\Gamma\left(\frac{\mu + \mu'}{2}\right) \Gamma\left(-\mu + 1/2\right)}{\Gamma\left(\frac{\mu + \mu' + 1}{2}\right)} \frac{\Gamma\left(\frac{\mu + \mu' + 1}{2}\right) \Gamma\left(-\nu + 1/2\right)}{\Gamma\left(\frac{\mu + \mu' + 1}{2}\right)},
\]

and

\[
\int d^2 Re^{iq \cdot R} R^{\mu - \mu' - 1} \bar{R}^{\nu - \nu' - 1} = \left[ \frac{q}{2} \right]^{\frac{\bar{\mu} - \bar{\nu}}{2}} \left[ \frac{q}{2} \right]^{\frac{\mu - \mu'}{2}}
\]

\[e^{i\frac{\pi}{2}(\mu - \nu - \mu' - \nu')} \sin \pi (\bar{\nu} - \bar{\mu}) \Gamma (\nu' - \mu) \Gamma (\mu' - \mu).
\]

Using the Γ doubling formula \( \frac{\Gamma(\mu' - \mu)}{\Gamma\left(\frac{\mu' - \mu}{2}\right)} = \frac{2^{\mu' - \mu - 1}}{\sqrt{\pi}} \Gamma\left(\frac{\mu' - \mu}{2}\right) \) and the definition \[1\] of \( b_{n,\nu} \), one gets at last the equation (29) of the text. namely

\[
I = \frac{1}{4\pi^2} (-1)^{n-n'} \frac{b_{n,\nu}}{8} \left[ \frac{q}{2} \right]^{\frac{\bar{\mu} - \bar{\nu}}{2}} \left[ \frac{q}{2} \right]^{\frac{\mu - \mu'}{2}} \frac{\Gamma(1 - \mu)}{\Gamma(\bar{\mu})} \frac{\Gamma\left(\frac{\mu + \mu'}{2}\right)}{\Gamma\left(1 - \frac{\mu + \mu'}{2}\right)} \frac{\Gamma\left(\frac{\mu + \mu'}{2}\right)}{\Gamma\left(1 - \frac{\mu + \mu'}{2}\right)}.
\]
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