Full $O(\alpha)$ corrections to $e^+e^- \rightarrow \nu \bar{\nu} H$ by GRACE

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We present the full $O(\alpha)$ corrections to single Higgs production in $e^+e^-$ collision. The computation is performed with the help of GRACE-LOOP where a generalized non-linear gauge fixing condition is implemented. The numerical results are checked by testing their UV and IR finiteness as well as their independence on all five non-linear gauge parameters. We find that for a 500 GeV collider and a light Higgs boson of mass 150 GeV, the total correction is small when the results are expressed in terms of $\alpha$ rather than $G_\mu$. For a higher Higgs boson mass of 350 GeV, the correction is of order $-10\%$.

1. Introduction

Single Higgs production will be one of the most important processes at the future electron-positron linear collider. There are two main mechanisms to this reaction: Higgs-strahlung and $W$ fusion. At tree-level, the calculation is well under control and takes into account the interference effects between Higgs-strahlung and $W$ fusion[1]. The one-loop corrections to the two-body $e^+e^- \rightarrow ZH$ Higgs-strahlung process have been studied both in the standard model[2] and in the minimal supersymmetric standard model[3]. However, despite the fact that for linear collider energies $W$ fusion becomes the dominant Higgs production mechanism a full $O(\alpha)$ calculation has been missing. This is indispensable if one wants to carry a precision measurement program. The loop correction to the $HWW$ vertex has been considered [4] and one could argue that it might constitute a good approximation for the fusion process, but it rests to see how well this approximation fares in comparison with the full calculation. Very recently one-loop radiative corrections to the fusion process have been investigated within supersymmetry[5,6] but again by only taking into account the contribution of the fermions and sfermions to the $H/hWW$ vertex. In this paper, we report the full $O(\alpha)$ corrections consisting of virtual and soft corrections as well as hard photon radiation, to single Higgs production in $e^+e^-$, including both the fusion and Higgs-strahlung processes in the standard model.$^1$

2. GRACE-LOOP system

The Feynman diagrams and the corresponding matrix elements for $e^+e^- \rightarrow \nu \bar{\nu} H$ were generated by GRACE-LOOP[7], an automatic calculation system. In addition, the radiative process $e^+e^- \rightarrow \nu \bar{\nu} H \gamma$ with a hard photon has been calculated by GRACE[8]. The latter process is also necessary to complete the total correction. We deal with two sets of diagrams, the full set and the production set. In the standard model and within the non-linear gauge fixing conditions which will be discussed later, one counts 12 tree-level and 1350 1-loop diagrams (with 98 pentagon graphs). This defines the full set. When the diagrams including scalar/pseudoscalar-electron couplings proportional to the electron mass are removed from the full set, we obtain the production set which consists of 2 tree diagrams and 249 1-loop diagrams (with 15 pentagon graphs). The full set has been used to make careful checks of the numerical calculation in quadruple precision. After checking the system with the full set at some random points in phase space, the phase space integration of the production set is done in dou-

$^1$In the workshop, O. Tarasov also presented the same topics.
ble precision to obtain cross sections and distributions. The production set is sometimes used in quadruple precision to confirm the numerical stability of the result.

The GRACE-LOOP system, which has a model file that describes all the interaction vertices derived from a particular Lagrangian, can generate all the necessary Feynman graphs together with their codes, so that matrix elements can be generated for the calculation of the cross section and event generation. For loop processes, the automatic calculation goes through an intermediate step performing some symbolic manipulation. This involves REDUCE[9] or FORM[10] for handling all the Dirac and tensor algebra in $n$-dimension for all the interference terms between tree-level and 1-loop diagrams. The one-loop diagram contribution from each loop graph $g$ at a phase space point, is defined as

$$d\sigma_g \propto 2\Re (T_{loop}^g \cdot T_{tree}^\dagger)$$  \hspace{1cm} (1)$$

where $T_{tree}$ is the tree-level amplitude summed over all tree-diagrams. $T_{loop}^g$ is the one-loop amplitude contribution of a one-loop diagram $g$. The Feynman trick for the propagator is also automatically applied at this stage. This is then passed to a module of libraries for the loop integration containing the FF[11] packages as well as the in-house reduction formulas. The system together with the one-loop renormalization set-up has been described in [12].

### 3. Checks of the system

The set of input parameters for the calculation is the following. Throughout this paper, the results are expressed in terms of the fine structure constant in the Thomson limit $\alpha_{QED} = 137.0359895$ and the Z mass $M_Z = 91.1876GeV$. The on-shell renormalization program uses $M_W$ as input parameter, nonetheless the numerical value of $M_W$ is derived through $\Delta r[13]$ \footnote{We include NLO QCD corrections and two-loop Higgs effects. We take $\alpha_s(M_Z^2)=0.118$ together with $G_{\mu}=1.16639 \times 10^{-5} GeV^{-2}$.}. $M_W$ thus changes as a function of $M_H$. We take the set $M_u=M_d=58MeV$, $M_s=92MeV$, $M_c=1.5GeV$, $M_\tau=4.7GeV$, which gives a “perturbative” value of $\alpha(M_Z)$ compatible with the current experimentally driven value. We also take $M_{top}=174GeV$. With these values, we derive $M_W=80.3767GeV$ for $M_H=150GeV$, and $M_W=80.3158GeV$ for $M_H=350GeV$. For the Higgs-strahlung subprocess we require a Z-width. We have taken a constant Z-width, $\Gamma_Z=2.4952GeV$. Unless otherwise stated our results refer to the full $e^-e^+ \rightarrow \nu\bar{\nu}H$, summing over all three types of neutrinos with, for electron neutrinos, the effect of interference between fusion and Higgs-strahlung.

The results of the calculation are checked by performing four kinds of tests at some point in phase space. For these tests to be passed one works with the full set in quadruple precision. One first checks the ultraviolet finiteness of the results. The regulator constant $C_{UV} = 1/\varepsilon - \gamma_E + \log 4\pi, n = 4 - 2\varepsilon$ is kept in the matrix elements. When one varies this parameter $C_{UV}$, the ultraviolet finiteness test gives a result that is stable over 30 digits. Infrared (IR) finiteness is checked by introducing a fictitious photon mass $\lambda$, treated in the code as an input parameter. The sum of loop and bremsstrahlung contributions is stable over 23 digits when varying $\lambda$. The third check relates to the independence on the parameter $k_c$ which is a soft photon cut parameter that separates soft photon radiation (analytical formula) and the hard photon performed by the Monte-Carlo integration. Gauge parameter independence of the result is performed as a last check through a set of five gauge fixing parameters. For the latter a generalized non-linear gauge fixing condition[14] has been chosen.

$$\mathcal{L}_{GF} =$$

$$-\frac{1}{\xi_W} \left( \partial_\mu - ie\alpha A_\mu - ig \cos \theta_W \beta Z_\mu \right) W^{\mu+}$$

$$+ \xi_W \frac{g}{2} \left( v + \delta H + i\tilde{\gamma}_3 \chi^+ \right)$$

$$- \frac{1}{2k^2}\left( \partial_\mu Z^{\mu+} + \xi_Z \frac{g}{2 \cos \theta_W} (v + \varepsilon H) \chi^+ \right)^2$$

$$- \frac{1}{2\xi^2} \left( \partial_\mu A^{\mu} \right)^2$$  \hspace{1cm} (2)$$

The $\chi$ represent the Goldstone. We take $\xi_W =$
\(\xi_Z = \xi_A = 1\) so that no “longitudinal” term appears in the gauge propagators. Not only this makes the expressions much simpler and avoids unnecessary large cancellations, but it also avoids the need for high tensor structures in the loop integrals. The use of five parameters is not redundant as often these parameters check complementary sets of diagrams. We introduce a generic notation \(\hat{\zeta}\) to represent either of \(\hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\kappa}, \hat{\epsilon}\). For each \(\hat{\zeta}\) the first check is made while freezing all other four parameters to be 0. We have also made checks with two parameters non-zero. In principle checking for 2 or 3 values of the gauge parameter should be convincing enough. We in fact go one step further and perform a complete test of gauge parameter independence. To achieve this we generate for each non-linear gauge parameter \(\hat{\zeta}\), the values of the loop correction to the total differential cross section as well as the contribution of each one-loop diagram contribution for the five values \(\hat{\zeta} = 0, \pm 1, \pm 2\). A rapid look at the structure of the Feynman rules of the non-linear gauge leads one to conclude that for \(e^+e^- \rightarrow \nu\bar{\nu}H\) each contribution is a polynomial of (at most) third degree in the gauge parameter and thus, that each contribution, \(d\sigma_g\) in Eq.(1) may be written as

\[
d\sigma_g(\hat{\zeta}) = d\sigma_g^{(0)} + \hat{\zeta} d\sigma_g^{(1)} + \hat{\zeta}^2 d\sigma_g^{(2)} + \hat{\zeta}^3 d\sigma_g^{(3)}
\]

For each contribution \(d\sigma_g\), it is a straightforward matter, given the values of \(d\sigma_g\) for the five input \(\hat{\zeta} = 0, \pm 1, \pm 2\), to reconstruct \(\sigma_g^{(0,1,2,3)}\). For each set of parameters we automatically pick up all those diagrams that involve an explicit dependence on the gauge parameter. The number of diagrams in this set depends on the parameter chosen. In some cases a huge number of diagrams is involved. For the process at hand this occurs with the parameter \(\hat{\delta}\) where over 1000 diagrams contribute to the sums, as shown in Table 1, and thus almost the entire set of diagrams, is involved in the check.

We then verify that the differential cross section is independent of \(\hat{\zeta}\)

\[
d\sigma = \sum_g d\sigma_g(\hat{\zeta}) = \sum_g d\sigma_g^{(0)}. \tag{4}
\]

In order to check this, we introduce the following notation for each parameter,

\[
S/M^{(i)} = \frac{\sum_g d\sigma_g^{(i)}}{\text{Max}_g(|d\sigma_g^{(i)}|)}, \quad i = 1, 2, 3. \tag{5}
\]

As seen from Table 1 agreement within 20 to 30 digits is observed. (The agreement gets better if one artificially gives the electron mass a higher value.) The gauge parameter dependence check not only tests the various components of the input file (correct Feynman diagrams for example) but also the symbolic manipulation part and most important of all the correctness of all the reduction formulae and the proper implementation of all the \(n\)-point functions. This is quite useful when one deals with 5-point functions as the case at hand.

4. Numerical results

The results we show here is based on the production set with all 3 neutrinos species and include the hard bremsstrahlung part. Integration
over all photon energies and angles is thus performed. The effect of radiative corrections are presented in the $\alpha$-scheme.

The overall correction to the total cross section at the linear collider energy ($\sqrt{s}=500\text{GeV}$), is small for a light Higgs mass of order 150 GeV as shown in Table 2. The radiative correction factor $\Delta$ is defined as

$$\Delta \equiv \frac{\sigma_{O(\alpha)}}{\sigma_{\text{tree}}} - 1.$$  

The magnitude of the correction increases steadily for higher Higgs masses. For example, for a Higgs mass of 350 GeV at a center-of-mass energy of 500 GeV the correction reaches $-10\%$. This $-10\%$ correction is distributed almost evenly over the range of scattering angles of the Higgs boson, see Fig.1.

The distribution of the Higgs boson energy is, however, affected by the $O(\alpha)$ corrections only in the region of the resonance of the $Z$-boson. The resonance structure of the tree level is distorted in the form of a radiative tail as shown in Fig. 2.

5. Summary

GRACE/LOOP has been applied to calculate the full $O(\alpha)$ corrections to single Higgs production in the standard model. One-loop amplitudes of the full 1350 diagrams were generated by this system. The non-linear gauge fixing condition has been introduced and shown to be a very powerful tool to check the consistency of the full set of amplitudes and the system itself. At $\sqrt{s}=500$ GeV, the correction is around $-10\%$ for a 350 GeV Higgs mass. The radiative tail was observed for the energy distribution of the Higgs boson. It means that for more realistic predictions, one has somehow to include multiple photon emissions.

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Table 2
Higgs mass dependence for the tree total cross sections, the full $O(\alpha)$ corrected one and the radiative correction factor at $\sqrt{s} = 500$ GeV.

| $M_H$ (GeV) | $\sigma_{\text{tree}}$ (fb) | $\sigma_{O(\alpha)}$ (fb) | $\Delta$ (%) |
|-----------|-----------------|-----------------|---------|
| 150       | 61.12           | 60.99 ±0.07     | −0.2    |
| 200       | 37.33           | 37.16 ±0.04     | −0.4    |
| 250       | 21.17           | 20.63 ±0.02     | −2.5    |
| 300       | 10.76           | 10.30 ±0.01     | −4.2    |
| 350       | 4.603           | 4.184 ± 0.004   | −9.1    |

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