Thermal Flow for Radiative Ternary Hybrid Nanofluid over Nonlinear Stretching Sheet Subject to Darcy–Forchheimer Phenomenon

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Received 5 June 2022; Revised 21 July 2022; Accepted 22 August 2022; Published 4 October 2022

This study examines the bidimensional nonlinear convective flow of ternary hybrid nanofluid upon a nonlinear stretching sheet. Three types of nanoparticles, namely Cu, TiO₂, Al₂O₃, are suspended in the base fluid taken as water with a new composition Cu + TiO₂ + Al₂O₃/H₂O which is termed as ternary hybrid nanofluid. To stabilize the flow and thermal properties of the new composition, the Brownian as well as thermophoresis properties are incorporated into energy and mass equations. The nonlinear thermal radiations and heat absorption/generation terms are included in the energy equation. The effects of the Darcy–Forchheimer phenomenon are also induced in the momentum equation. The set of model equations has shifted to dimension-free form by employing suitable variables. It has concluded in this study that flow characteristics have been declined with augmenting values of volumetric fractions of solid nanoparticles, porosity, and inertia factors and have upsurge with higher values of thermal and nonlinear thermal Grashof numbers. Thermal characteristics have been observed to be augmented with growth in radiation, Brownian motion, thermophoresis, heat generation/absorption, temperature ratio factors, and volumetric fraction of solid nanoparticles. These effects are more significant for ternary hybrid nanoparticles. Concentration profiles have been declined with higher values of Brownian motion factor, Lewis number, and upsurge with growth in thermophoresis factor. It has also been deduced in this investigation that the thermal flow rate is higher for trihybrid nanofluid than hybrid or traditional nanofluids, and a percentage growth in Nusselt number has been shown through statistical chart in support of this work. Current results have been compared with established results and found a fine agreement amongst all results.

Hindawi
Mathematical Problems in Engineering
Volume 2022, Article ID 3429439, 14 pages
https://doi.org/10.1155/2022/3429439
1. Introduction

Nonlinear stretching sheet has a significant part in different applications at the industrial level for instance cooling bath for the condensation process of metal plate and polymer sheet extrusion through a dye, etc. In the manufacturing of polymer sheet, the melted material is being spread through a slit that is consequently stretching to attain the anticipated wideness of the sheet. The characteristics of the final product depend upon the rate of stretching of the raw material used in the fabrication of the sheet. For such important applications in industry, various studies have been carried out to discuss the fluid flow behavior upon stretching and nonlinear stretching sheets. Using nonlinear stretching sheet for fluid with partial slip by Das [1], he has concluded that with the augmenting values of partial slip and nonlinearity index of the surface, the fluid’s velocity has declined while its mass diffusivity has grown up. Cortell [2] has established that higher values of nonlinearity parameter have increased the skin friction by offering more resistance to fluid flow that has caused a decline in a fluid stream and a growth in the thermal transmission. Fatunmbi et al. [3] have examined the MHD fluid flow on a nonlinear stretched surface. The authors of this work have revealed that the width of the temperature boundary layer has augmented with the upsurge in the values of Eckert and Biot numbers whereas the thermal flow has declined with augmentation in the values of thermal conductance factor and material parameter. Rasool and Wakif [4] have deliberated EMHD mixed convection fluid flow for second-grade fluid towards a Riga plate. Bhatti et al. [5] have discussed thermally the radiated water-based hybrid nanofluid motion upon the nonlinear permeable stretched surface. In this work, the authors have used the Dufour and Soret diffusion effects to the flow problem and have revealed that the thermal and mass diffusivities have augmented with corresponding growth in Dufour and Soret numbers.

Pure fluids such as oil, ethylene glycol, and water have less thermal conductance which is a constraint in the flow phenomenon of the fluid. For improving the thermal conductivity of base/pure fluid, scientists have introduced different techniques and methods. One of such technique is the suspension of small sized particles, introduced first by Choi and Eastman [6]. Such fluid is termed as nanofluid which is used in industry for cooling purpose, such as coolant of power-plant equipment, coolant of auto-engines, and coolant of different electronic devices and manufacturing processes. Recently, many investigations have been conducted by different researchers for nanofluid flow with main focus on heat transmission. Sheikholeslami and Ganji [7] have discussed water-based copper nanofluid flow through two squeezing plates. The authors of this study have determined the viscosity and thermal conductivity of nanoparticles by employing Brinkman and Garnett model with a fine agreement with other results in literature. It has concluded in this work that the thermal conductance of the fluid has been controlled by the extending values of Eckert number and volumetric fraction of nanoparticles. Hayat et al. [8] have investigated the thermal flow rate for Burger nanofluid flow over a stretching sheet by employing Robin-type conditions and concluded that the thermal flow rate has supported by the growth in thermophoresis and Brownian motion factors. Makinde and Eegunjobi [9] have deliberated the collective influence of magnetic field, changing viscosity, thermophoresis, and Brownian motion for nanofluid with investigation of irreversibility generation for flow system. Nanofluid flow reduces [10–12] the resistance to heat transfer for different flow systems. With the passage of time, researchers came to know that the spread of two different types of nanoparticles in a pure fluid further augments the thermal flow characteristics. They termed such fluid as a hybrid nanofluid. Lund et al. [13] have inspected the dynamics of hybrid nanofluid flow with influence of thermal radiations and viscous dissipation past a stretching surface and have determined two branches of solutions. It has observed by the authors that the upper branch of solution has highlighted to be more applicable due to its stable nature. Khan et al. [14–16] have deliberated an incredible work to discuss the thermal flow improvement for hybrid nanofluid flow by means of different flow geometries and flow conditions. Recently, it has been noticed by the researchers that the suspension of three different types of nanoparticles in a pure fluid can enhance the thermal conductivity of such fluid to the best possible limit. Such fluids are termed as trihybrid nanofluid. Sundar et al. [17] have used ternary hybrid nanofluid flow by suspending the nanoparticles of GO, Fe₃O₄, TiO₂ in ethylene glycol. In this experimental work, the authors have introduced new equations for the estimation of Nusselt number and thermophysical characteristics and have concluded that the thermal performance of the flow system has upsurged by 14.32%. Sahoo [18] has inspected the irreversibility and thermal flow by using different shapes of nanoparticles in trihybrid nanofluid. It has been concluded in this work that, by improving the concentration from 1% to 3% in trihybrid nanofluid, the growth of entropy has jumped up from 12.24%. Manjunatha et al. [19] studied ternary hybrid nanofluid with convective heat transmission over stretched sheet. Different investigations [20–22] have been concluded that the heat flow characteristics of trihybrid nanofluids are much better than other hybrid or traditional nanofluids.

Nonlinear convection flow of fluid plays a vital role in numerous branches of natural science for instance astrophysics, geophysics, and engineering. By taking these important applications into account, Ibrahim and Gamachu [23] have inspected the nonlinear convective fluid flow past a radial stretching surface. It has been concluded in this work that the thermal and mass diffusivities of fluid have boosted up with augmentation in magnetic and thermophoresis factors. Shaw et al. [24] have deliberated the impact of slip condition for a nonlinear convective nanofluid flow past a stretched surface. The authors have determined the numerical solution for the governing partial differential equations and have concluded that higher values of convective factor have upsurged the Sherwood and Nusselt numbers. Waqas et al. [25] inspected MHD fluid flow with nonlinear convection over the convective heated surface with variable thickness. To control the thermal flow of the
system, different flow conditions such as thermal radiations, heat generation/absorption, and Joule heating have been used by the authors. In this work, the thermal profiles have upsurged with growing values of Biot, Eckert numbers, and radiation factor. Hong et al. [26] have inspected numerically the nonlinear mixed convection fluid flow past a cylinder embedded in a permeable medium and have noticed that both Nusselt as well as Sherwood numbers have higher values with growth in the values of convective parameters. Hayat et al. [27] have noticed that the heat transmission has been augmented for higher values of nonlinear convection parameter.

Nonlinear mathematical form of thermal radiation is used in those cases whenever the more general form of the radiation phenomenon for a fluid flow problem is required. Thermal radiations have numerous applications in engineering and material sciences such as manufacturing of glass, polymer fabrication, coolant of atomic reactors, and furnace design. Moreover, thermal radiations also have fruitful applications in space modernization genetic factors such as rockets, rocket streamlines features, propulsion system, and operating system of space craft at the higher temperature. Due to its important applications, many investigations have been conducted [28–30] with the main focus on the utilization of thermal radiations by employing the linear form of Rosseland flow. But for fluid flow systems where higher thermal flow is required, then the linear radiations are not suitable rather nonlinear thermal radiations are employed [31–33] in such cases. Rooman et al. [34] have used nonlinear thermal radiations for MHD nanofluid flow past a porous cylinder. In this study, the authors have determined the irreversibility generation for fluid flow systems and have concluded numerically the influence of various emerging parameters upon different quantities of engineering interest.

From the cited literature, it has been observed that many investigations [35–38] have been conducted for hybrid and traditional nanofluids flow upon linear stretching and nonlinear stretching sheets with some other flow conditions, but no evidence has been found for the flow behavior of trihybrid nanofluid flow upon a nonlinear stretching sheet with nonlinear convection of fluid flow in the presence of nonlinear thermal radiations. The following points will further improve the novelty of the problem:

(i) Three types of nanoparticles namely Cu, TiO₂, Al₂O₃ are suspended in the base fluid taken as water
(ii) Brownian motion and thermophoresis properties are considered for the flow system
(iii) Nonlinear thermal radiations and absorption/generation of heat terms are incorporated in the flow model
(iv) The impact of Darcy–Forchheimer phenomenon is considered for the flow system
(v) The expansion in the sheet is nonlinear which is moving with a nonlinear velocity

2. Formulation of Problem

Take two dimensional nonlinear convective ternary hybrid nanofluid flow upon an expanding surface (see Figure 1). The following assumptions are supported the flow problem:

(i) Three types of nanoparticles namely Cu, TiO₂, Al₂O₃ are suspended in the base fluid are taken as water to form a new composition Cu + TiO₂ + Al₂O₃/H₂O
(ii) To stabilize the flow and thermal properties of this new composition, the Brownian motion, thermophoresis properties, nonlinear thermal radiation, and heat absorption/generation terms are incorporated into the flow model
(iii) The impact of Darcy–Forchheimer phenomenon is also considered for flow system
(iv) The flow occurs at the region y ≥ 0
(v) The flow is established due to the nonlinear expansion of the stretching sheet which appears through the slit x = y = 0
(vi) The expansion in the sheet is nonlinear which is moving with nonlinear velocity U_w = ax^n, where a is constant value, and x is the distance away from the slit while n is nonlinearity index of the stretching sheet
(vii) Concentration and temperature at the surface are C_w, T_w while the corresponding values at free stream are C_∞, T_∞

2.1. Governing Equations. In light of the abovementioned assumptions, the equations that manage flow system can be described as [39, 40]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\]

\[
\left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \nu_{\text{thnf}} \frac{\partial^2 u}{\partial y^2} - \frac{\mu_{\text{thnf}}}{K} u
+ g \left[ \left( \beta_{\text{thnf}} (T - T_\infty) + \beta_{\text{thnf}}^2 (T - T_\infty)^2 \right) \right],
\]

\[
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{k_{\text{thnf}}}{(\rho C_p)_{\text{thnf}}} \frac{\partial^2 T}{\partial y^2}
+ \tau \left[ D_B \frac{\partial^2 C}{\partial y^2} + \frac{(D_T)}{T_\infty} \right]
+ \frac{Q_0}{(\rho C_p)_{\text{thnf}}} (T - T_\infty)
+ \frac{16}{3} \frac{1}{(\rho C_p)_{\text{thnf}}} 
\left( \frac{\sigma^2 T^3}{K} \frac{\partial^2 T}{\partial y^2} \right).
\]
\[
\frac{\partial C}{\partial x} + \nu \frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} D_B + \frac{\partial^2 T}{\partial y^2} \left( \frac{T}{T_{\infty}} \right). \tag{4}
\]

Above \( u, v \) depict the flow elements along \( x \) and \( y \)-axes. Moreover, \( (\rho C_p)_b, \rho_{\text{thnf}}, \mu_{\text{thnf}}, \) and \( \nu_{\text{thnf}} \) are termed as heat capacitance, density, dynamic, and kinematic viscosities for ternary hybrid nanofluid, \( D_B, D_T \) are Brownian and thermophoresis diffusions, \( F \) is nonuniform inertial coefficient, and \( q_0 \) is rate of generation/absorption of heat, while \( \tau \) is heat capacitance ratio.

The above system of equations is subjected to the following conditions at boundaries:
\[
\begin{align*}
    u &= ax^n, \quad v = 0, \quad T = 0 = T_w, \quad C = C_w, \text{at } y = 0, \\
    u \rightarrow 0, T \rightarrow T_{\infty}, C \rightarrow C_{\infty} & \quad \text{when } y \rightarrow \infty. \tag{5}
\end{align*}
\]

To convert the given equation to dimension free form, following set of similar variables will be incorporated [40].
\[
\begin{align*}
    u &= ax^n f' (\eta), \\
    v &= -\left( f (\eta) + \eta \left( \frac{n-1}{n+1} \right) f' (\eta) \right) \sqrt{\frac{a}{2} \nu (n+1) x^{n-1}}, \\
    \Theta (\eta) &= \frac{T - T_{\infty}}{T_w - T_{\infty}}, \quad \Phi (\eta) = \frac{C - C_{\infty}}{C_w - C_{\infty}}, \\
    \eta &= \sqrt{\frac{a}{2} \nu (n+1) x^{n-1} y}.
\end{align*} \tag{6}
\]

In (6) \( \eta \) depicts the similarity variable. Moreover, the flow system for the current investigation is stream lined, which is described by the stream function \( \psi \) and satisfies the mathematical relations \( u = \partial \psi / \partial y, \quad v = -\partial \psi / \partial x \). The mass conservation (1) is satisfied by the stream function, while equations (2)–(4) transformed to
\[
\begin{align*}
    f'' + \frac{\beta_{\text{thnf}}}{\beta_f} \frac{\mu_f}{\mu_{\text{thnf}}} \left[ f f'' - \frac{2n}{(n+1)} f^2 - Fr(f')^2 \right] & = 0, \tag{7}
\end{align*}
\]
\[
\begin{align*}
    - Kr f'' + \frac{\beta_{\text{thnf}}}{\beta_f} \frac{\mu_f}{\mu_{\text{thnf}}} g \left[ (Gr \Theta + Gr^* (\Theta'))^2 \right] & = 0, \\
    \left( \frac{k_{\text{thnf}}}{k_f} \right) \left[ \frac{4}{3} Rd \left( 1 + (1 + \Theta) \Theta \right) \right] \Theta & = 0, \\
    \left( \frac{\rho C_p)_b}{\rho C_p}_{f} \right) \left( \frac{2n}{(n+1)} \right) f \Theta ' + Q Pr \Theta & = 0, \\
    \Phi'' + \left( \frac{Le Pr}{\kappa} \right) f' \Phi' + \frac{Nt}{Nb} \Theta'' & = 0. \tag{9}
\end{align*}
\]

**Figure 1:** Geometrical view of flow problem.

In above system of equations, \( Fr = C_L x^n / K^{1/2} \) is inertia coefficient factor, \( Kr = \nu / \rho K \) is porosity factor, \( Gr = g \beta_f (T_w - T_{\infty}) / \rho u_w \) is Grashof number, \( Rd = 4 \eta^2 T^3 / \kappa k \) is radiation parameter, \( Nb = \tau D_B (C_w - C_{\infty}) / \nu \) is Brownian factor, \( Nt = \tau D_T (T_w - T_{\infty}) / \nu T_{\infty} \) is thermophoresis factor, \( Q = Q_w / \rho C_p a \) is heat absorption/generation, \( Pr = \rho C_p v / k \) is Prandtl number, \( Le = \nu / D_B \) is Lewis number, and \( \Theta_w = T_w / T_{\infty} \) is temperature ratio parameter.

The related conditions are transformed as follows:
\[
\begin{align*}
    f (0) &= 0, \quad f' (0) = 1, \quad \Theta (0) = \Phi (0) = 1, \\
    f' (\infty) &\rightarrow 0, \quad \Theta (\infty) \rightarrow 0, \quad \Phi (\infty) \rightarrow 0. \tag{10}
\end{align*}
\]
The base fluid and trihybrid nanofluid characteristics are described as [40, 41]

\[
\frac{\mu_{\text{limf}}}{\mu_f} = \frac{1}{(1 - \phi_{\text{Cu}})^{2.5}(1 - \phi_{\text{TiO}_2})^{2.5}(1 - \phi_{\text{Al}_2\text{O}_3})^{2.5}},
\]

\[
\frac{\rho_{\text{limf}}}{\rho_f} = (1 - \phi_{\text{Cu}}) \left[ (1 - \phi_{\text{TiO}_2}) \left( 1 - \phi_{\text{Al}_2\text{O}_3} \right) + \phi_{\text{Al}_2\text{O}_3} \rho_{\text{Al}_2\text{O}_3} \rho_{\text{TiO}_2} \rho_f \right] + \phi_{\text{Cu}} \rho_{\text{Cu}} \rho_f,
\]

\[
\frac{(\rho c_p)_{\text{limf}}}{(\rho c_p)_f} = \Phi(c) \left[ (1 - \phi_{\text{Cu}}) \left( 1 - \phi_{\text{TiO}_2} \right) \left( 1 - \phi_{\text{Al}_2\text{O}_3} \right) + \phi_{\text{Al}_2\text{O}_3} \left( \frac{(\rho c_p)_{\text{Al}_2\text{O}_3}}{(\rho c_p)_f} \right) + \phi_{\text{TiO}_2} \left( \frac{(\rho c_p)_{\text{TiO}_2}}{(\rho c_p)_f} \right) \right],
\]

\[
k_{\text{limf}} = \left( \frac{k_{\text{Al}_2\text{O}_3} + 2 k_{\text{limf}} - 2 \phi_{\text{Al}_2\text{O}_3}(k_{\text{limf}} - k_{\text{Al}_2\text{O}_3})}{k_{\text{Al}_2\text{O}_3} + 2 k_{\text{limf}} + \phi_{\text{Al}_2\text{O}_3}(k_{\text{limf}} - k_{\text{Al}_2\text{O}_3})} \right),
\]

\[
k_{nf} = \frac{k_{\text{Cu}} + 2 k_f - 2 \phi_{\text{Cu}}(k_f - k_{\text{Cu}})}{k_{\text{Cu}} + 2 k_f + \phi_{\text{Cu}}(k_f - k_{\text{Cu}})}.
\]

The numerical values of above physical properties are expressed in Table 1.

2.2. Quantities of Interest. For the present model, \( C_f \), \( N_u \), and \( S_h \) are key engineering physical parameters that are described as [40]

\[
C_f = \frac{\tau_w}{\rho U^2_w}, \quad N_u = \frac{\chi q_w}{k(T_w - T_{\infty})}, \quad S_h = \frac{\chi h_w}{D_B(C_w - C_{\infty})},
\]

where \( \tau_w \), \( q_w \), and \( h_w \) are described as

\[
\tau_w = \mu_{\text{limf}} \frac{\partial u}{\partial y} \bigg|_{y=0},
\]

\[
q_w = -k_{\text{limf}} \frac{\partial T}{\partial y} - \frac{16}{3} \left( \frac{\sigma T_{\infty}^4}{k} \frac{\partial T}{\partial y} \right),
\]

\[
h_w = -D_B \frac{\partial c}{\partial y} \bigg|_{y=0},
\]

\[
\text{Re}_x C_{fs} = \frac{\mu_{\text{limf}}}{\mu_f} f^\prime \left|_{0} \right.
\]

\[
\text{Re}_x N_{tu} = \left( \frac{k_{\text{limf}}}{k_f} + \frac{4}{3} R_d \left( \frac{d}{dn} \left[ 1 + \left( 1 + \Theta \right) \Theta(0) \right] \right) \right) \Theta^\prime(0).
\]

\[
\text{Re}_x^3 S_h = \Phi^\prime(0).
\]

Above \( \text{Re}_x = \alpha x^2/\nu \) depicts local Reynolds’s number.

3. Problem Solution

To solve equations (7)–(9) with the help of (10), the widespread method HAM [42, 43] will be employed. This method requires some starting values which are described as follows:

\[
\bar{F}(\eta) = 1 - e^{-\eta}, \quad \Theta(\eta) = \eta e^{-\eta}, \quad \Phi(\eta) = e^{-\eta},
\]

\[
L_{\bar{F}}(\bar{F}) = -f''(0), \quad L_{\Theta}(\Theta) = \Theta''(0), \quad L_{\Phi}(\Phi) = \Phi''(0).
\]

Such that

\[
L_{\bar{F}}(\bar{F} + c_2 \eta + c_3 \eta^2) = 0, \quad L_{\Theta}(\Theta + c_4 + c_5 \eta) = 0, \quad L_{\Phi}(\Phi + c_6 + c_7 \eta) = 0.
\]

The nonlinear operators \( N_{\bar{F}}, N_{\Theta}, \) and \( N_{\Phi} \) are described as

\[
N_{\bar{F}} \left[ \bar{F}(\eta), \Theta(\eta) \right] = \bar{F}_{\eta\eta} + \frac{\mu_{\text{limf}}}{\mu_f} \frac{R_f}{\rho_{\text{limf}}}
\]

\[
+ \left[ \bar{F}_{\eta\eta} - \frac{2n}{(n + 1)} (\bar{F})^2 - F_{\bar{F}} \right]
\]

\[
+ \frac{\mu_f}{\rho_{\text{limf}}} \frac{R_f}{\beta_f} g \left( G \bar{\Theta} + G^2 \bar{\Theta}^2 \right)
\]

\[
- K \bar{F}_{\eta\eta},
\]

\[
N_{\Theta} \left[ \bar{F}(\eta), \Theta(\eta) \right] = \left( \frac{k_{\text{limf}}}{k_f} + \frac{4}{3} R_d \left( 1 + \left( 1 + \Theta \right) \Theta(0) \right) \right) \bar{\Theta}_{\eta\eta}
\]

\[
+ \frac{N_b \bar{F}}{N_b} \bar{\Theta}_{\eta} + N_t \left( \frac{\Theta}{\Theta(0)} \right)^2
\]

\[
+ \frac{(\rho c_p)_{\text{limf}}}{(\rho c_p)_f} \frac{2n}{(n + 1)} \bar{F}_{\eta\eta} + Q \bar{F}_{\eta\eta}.
\]

\[
N_{\Phi} \left[ \bar{F}(\eta), \Phi(\eta) \right] = \Phi_{\eta\eta} + \left( \text{LePr} \frac{\Theta}{\Theta(0)} \right) \bar{\Phi}_{\eta\eta}.
\]
The 0th-order system for equations (7)–(9) is described as

\[
\begin{align*}
(1 - \zeta) F_0(\eta; \zeta) &= p h_n N_0 \tilde{F}(\eta; \zeta), \\
(1 - \zeta) \Theta_0(\eta; \zeta) &= p h_n N_0 \tilde{\Theta}(\eta; \zeta), \\
(1 - \zeta) \Phi_0(\eta; \zeta) &= p h_n N_0 \tilde{\Phi}(\eta; \zeta).
\end{align*}
\]

Corresponding conditions are

\[
\tilde{F}(\eta; \zeta)|_{\eta=0} = 0, \quad \frac{\partial \tilde{F}(\eta; \zeta)}{\partial \eta}|_{\eta=0} = 1, \tilde{\Theta}(\eta; \zeta)|_{\eta=0} = 1, \tilde{\Phi}(\eta; \zeta)|_{\eta=0} = 1,
\]

\[
\frac{\partial \tilde{F}(\eta; \zeta)}{\partial \eta}|_{\eta=\infty} \rightarrow 0, \tilde{\Theta}(\eta; \zeta)|_{\eta=\infty} \rightarrow 0, \tilde{\Phi}(\eta; \zeta)|_{\eta=\infty} \rightarrow 0.
\]

The boundary conditions are

\[
\tilde{F}(0) = 0, - F'(0) = 1, \tilde{\Theta}(0) = 1, \tilde{\Phi}(0) = 1, \text{ for } \eta = 0
\]

\[
- F'(\eta) \rightarrow 0, \tilde{\Theta}(\eta) \rightarrow 0, \tilde{\Phi}(\eta) \rightarrow 0, \text{ as } \eta \rightarrow \infty.
\]

Now,

\[
\mathcal{R}_n F(\eta) = F_n^{\prime \prime} + \frac{\rho_m \beta_n \mu_f \alpha}{\rho_f H_{nf}} \sum_{j=0}^{n-1} F_{w-1-j}^{\prime \prime} - \frac{2n}{n+1} \left( \frac{F_{w-1}}{n+1} \right)^2 - F_{w-1}^{\prime \prime} \left\lfloor \frac{1}{n-1} \right\rfloor
\]

\[
+ \frac{\beta_{nf}}{\beta_f} \frac{4}{3} \frac{k_f}{k_f} \sum_{j=0}^{n-1} \left( \frac{1 + (1 + \tilde{\Theta}_j)}{2} \right)^{3/2} \tilde{\Theta}_j + \frac{(\rho C_p)_{nf}}{(\rho C_p)_f} \left( \frac{2n}{n+1} \sum_{j=0}^{n-1} F_{w-1-j} \tilde{\Theta}_j' \right)
\]

\[
+ \frac{4}{3} \frac{k_f}{k_f} \sum_{j=0}^{n-1} \left( \frac{N b \tilde{\Theta}_j' + N t \tilde{\Theta}_j^2 + Q \tilde{\Phi}_j}{N b} \right) = 0,
\]

\[
\mathcal{R}_n \tilde{\Theta}(\eta) = \left( \frac{k_{nf}}{4} \right)^{3/2} \left( \frac{1 + (1 + \tilde{\Theta}_j)}{2} \right)^{3/2} \tilde{\Theta}_j + \left( \frac{\rho C_p)_{nf}}{(\rho C_p)_f} \right) \left( \frac{2n}{n+1} \sum_{j=0}^{n-1} F_{w-1-j} \tilde{\Theta}_j' \right)
\]

\[
+ \frac{4}{3} \frac{k_f}{k_f} \sum_{j=0}^{n-1} \left( \frac{N b \tilde{\Theta}_j' + N t \tilde{\Theta}_j^2 + Q \tilde{\Phi}_j}{N b} \right) = 0,
\]

\[
\mathcal{R}_n \tilde{\Phi}(\eta) = \tilde{\Phi}'' + Le Pr \sum_{j=0}^{n-1} F_{w-1-j} \tilde{\Phi}_j' - \frac{N t}{N b} \tilde{\Phi}_j = 0,
\]

\[
\chi_n = \begin{cases} 
0, & \text{if } \zeta \leq 1, \\
1, & \text{if } \zeta > 1.
\end{cases}
\]
4. Results and Discussions

This study examines nonlinear convection flow of ternary hybrid nanofluid upon a nonlinear stretching sheet. Three different types of nanoparticles have been mixed in the base fluid taken as water to form ternary hybrid nanofluid. Famous Buongiorno model along with nonlinear thermal radiation and heat absorption/generation terms are included in flow model. The effects of Darcy–Forchheimer phenomenon have induced in the momentum equation. After conversion of the modeled equations into dimension-free format, HAM has used for solution of new system of equations. During conversion of the modeled equations, some new factor/parameters have been obtained which will affect the flow system. The influence of these factors upon different profiles of fluid motion will be discussed in the upcoming paragraphs.

4.1. Velocity Characteristics. The influence over fluid’s motion for deviation in different evolving factors has been portrayed in Figures 2–7. The augmenting values of volumetric fractions of solid nanoparticles enhance the density of fluid particles as a result of which more resistance is offered to fluid’s motion. In this process, the skin fraction upsurges while the motion of fluid decays. This variation in the flow characteristics for expansion in volumetric fractions nanoparticles has depicted in Figure 2. As porosity is the characteristics of the porous surface, this property allows the fluid particles to pass through the pores in the surface, due to these pores the motion of the fluid decays. Hence, for greater values of porosity parameter, the flow characteristics observe more resistance which causes a decline in the thickness of the momentum boundary layer as shown in Figure 3. The influence of thermal Grashof and nonlinear thermal Grashof numbers $Gr, Gr^*$ over the flow characteristics is portrayed in Figures 4 and 5. The augmentation in both these numbers enhanced the fluid motion. Actually, the ratio of buoyancy force to viscous forces results in Grashof number. Therefore, the higher values of $GrGr^*$ offer less opposing forces to fluid motion; hence, the augmentation in both these numbers upsurges the fluid motion. With augmenting values of inertial coefficient, the resistance to fluid will upsurge that weakens the layer thickness of momentum boundary. Hence, for higher value of $Fr$, the fluid motion declines as presented in Figure 6. The augmentation in nonlinearity index $n$ has also an adverse effect upon the fluid flow characteristics as depicted in Figure 7.

4.2. Thermal Characteristics. The influences upon thermal characteristics correspondent to variations in various emerging parameters are depicted in Figures 8–13. The growth in the source for heat absorption or generation causes a progression in the strength of thermal boundaries. In this physical process, more heat transmission occurs that upsurge the thermal profiles as depicted in Figure 8. Such growth in thermal profiles is more significant for trihybrid nanoparticles. A growth in quantity of solid nanoparticles’ volumetric fractions offers more resistance to fluid’s motion.
due to higher densities of fluid particles. In this phenomenon, supplementary heat is transferred from hotter to colder zone. Therefore, the thermal characteristics grow up with higher values of $\phi_{Cu}, \phi_{TiO_2}, \phi_{AlO_3}$ nanoparticles as depicted in Figure 9. The higher values of radiation factor $Rd$ also support the heat transmission in the fluid flow motion due to which thermal strength of the boundary enhances. So, for augmentation in $Rd$, the thermal characteristics grow up as depicted in Figure 10. The influence of temperature ratio factor $\Theta_w$ over the thermal profiles is portrayed in Figure 11 since $\Theta_w$ presents the ratio of temperatures at surface of medium to its values at free stream. So augmentation in $\Theta_w$ causes growth in thermal flow at the surface of the medium that strengthen the thermal layer at the boundaries. Hence, higher values of $\Theta_w$ result an upsurge in the values of thermal profiles as presented in Figure 11. The boosting values of Brownian motion factor $Nb$ cause a corresponding growth in the random motion of solid nanoparticles. In this process, the internal energy of solid nanoparticles is transformed to heat/kinetic energy that again strengthen the thermal boundary layer. So, for growth in $Nb$, the values of thermal characteristics upsurge as depicted in Figure 12. Similarly, for expansion in thermophoresis factor $Nt$, an additional heat flow occurs from region of greater concentration to a zone of lower concentration. Therefore, growth in $Nt$ corresponds to an intensification in thermal characteristics as portrayed in Figure 13.

4.3. Concentration Characteristics. Variations in concentration characteristics for different values of emerging factors have portrayed in Figures 14–16. The influence of higher values of Lewis number $Le$ upon $\Phi(\eta)$ is presented in Figure 14 since $Le$ is the ratio of kinematic viscous forces to diffusion of solutal. So augmenting values of $Le$ specify that the solutal diffusions retarded swiftly and cause a reduction in the values of concentration profiles as depicted in Figure 14. For growth in the Brownian motion factor $Nb$, the random motion amongst the nanoparticles upsurges due to which less mass diffuses and weakens the strength of solutal concentration. In this phenomenon, the concentration characteristics retarded as depicted in Figure 15 since the mathematical value of thermophoresis factor is depicted as $Nt = \tau D_f(T_w - T_{\infty})/uT_{\infty}$. It is quite significant that $Nt$ is the force that is produced due to temperature gradient to its value at free stream so an augmentation in $Nt$ describes that more fluid’s particles move from hotter zone to a colder one and strengthen the concentration layer thickness. Hence, augmentation in $Nt$ will enhance the concentration characteristics as depicted in Figure 16.

4.4. Percentage variations in Nusselt Number. From Figure 17 appended, it has noticed that for a variation in solid nanoparticles from 0.01 to 0.03, the values of Nusselt number augment from 3.71% to 11.27% in case of Cu-nanoparticles, while in case of Cu + TiO$_2$ nanoparticles the thermal flow rate rises from 4.32% to 13.46%. At the same interval, the thermal flow rate rises from 5.53% to 18.45% for trinanoparticles Cu + TiO$_2$ + Al$_2$O$_3$. Hence, it is
Figure 8: $\Theta(\eta)$ Vs variations in $Q$.

Figure 9: $\Theta(\eta)$ Vs variations in $\phi_{Cu},\phi_{TiO_2},\phi_{Al_2O_3}$.

Figure 10: $\Theta(\eta)$ Vs variations in $Rd$.

Figure 11: $\Theta(\eta)$ Vs variations in $\Theta_w(\eta)$.

Figure 12: $\Theta(\eta)$ Vs variations in $Nb$.

Figure 13: $\Theta(\eta)$ Vs variations in $Nt$. 
Figure 14: $\Phi(\eta)$ Vs variations in $Le$. 

Figure 15: $\Phi(\eta)$ Vs variations in $Nb$. 

Figure 16: $\Phi(\eta)$ Vs variations in $Nt$. 
quite visible that thermal flow rate is higher for trihybrid nanofluid than hybrid or traditional nanofluid. This is the main theme of current work which signifies it from the work established in the literature.

4.5. Table Discussion. In Table 1, the numerical values of thermophysical properties have been expressed while, to ensure the validity of the current work, a comparison amongst the current result and those already established has been made as depicted in Table 2. A fine agreement has been found amongst these results.

5. Conclusions

This study investigates the nonlinear convective flow of ternary hybrid nanofluid upon a nonlinear stretching sheet. Three types of nanoparticles are suspended in the base fluid to form a ternary hybrid nanofluid. To stabilize the flow and thermal properties of the new composition, the Brownian motion and thermophoresis properties are incorporated into energy and mass equations. Nonlinear thermal radiation and heat absorption/generation terms are also included in the energy equation. The effects of the Darcy–Forchheimer phenomenon have been induced in the momentum equation. The dimensionless form of modeled equations has been solved using HAM. It has deduced in this study that is as follows:

(i) With higher values of volumetric fractions of nanoparticles, the dense behavior of fluid upsurges due to which more resistance is experienced by fluid motion. In this process, the fluid motion declines while thermal transmission grows up.

(ii) Flow characteristics have declined with augmenting values of porosity, inertia factors, and nonlinearity index.

(iii) It has also been pointed out in this study that the motion of the fluid has upsurged with higher values of thermal and nonlinear thermal Grashof numbers.

(iv) Thermal characteristics are observed to be augmented with growth in radiation, Brownian motion, thermophoresis, heat generation/absorption, and temperature ratio factors. These effects are more significant for ternary hybrid nanoparticles.

(v) Diffusion of mass has declined with a higher value of Brownian motion factor and Lewis number and upsurges with growth in thermophoresis factor.

(vi) A percentage growth in the heat transfer rate has also been presented through a statistical chart, which depicts that for a variation in solid nanoparticles from 0.01 to 0.03 the values of Nusselt number augment from 3.71% to 11.27% in case of Cu-nanoparticles, while in case of Cu + TiO₂-nanoparticles, the thermal flow rate rises from 4.32% to 13.46%. At the same interval, the thermal flow rate rises from 5.53% to 18.45% for trinanoparticles Cu + TiO₂ + Al₂O₃.

(vii) It has been deduced in this investigation that the thermal flow rate is higher for trihybrid nanofluid than for hybrid or traditional nanofluids.

(viii) In the future, the impact of microrotation will be added to the mathematical modelling of current work.

Table 2: Comparison of the present work with the published work considering common parameters and actual Prandtl number of water

| n  | −θ(0) Hady et al. [44] | −θ(0) Jafar et al. [45] | −θ(0) Present result |
|----|------------------------|--------------------------|----------------------|
| 1  | 3.445933000            | 3.445935000              | 3.44593110           |
| 2  | 3.886934200            | 3.886875200              | 3.88673210           |
| 3  | 3.998769324            | 3.998634210              | 3.99852091           |
| 4  | 4.217054210            | 4.217004210              | 4.21696321           |

Figure 17: Percentage changes in Nusselt number for variations in ϕ_{Cu}, ϕ_{TiO₂}, ϕ_{Al₂O₃}. 

Mathematical Problems in Engineering 11
Nomenclature

Symbol: Description

\( u, v \): The velocity components (m.s\(^{-1}\))

\( x, y \): Cartesian coordinates (m)

\( T \): Temperature of ternary hybrid Nano fluid (K)

\( T_w \): Wall temperature (K)

\( T_\infty \): Ambient temperature (K)

\( Rd \): Radiation parameter

\( Nb \): Brownian motion parameter

\( Nt \): Thermophoresis factor

\( \nu_{thnf} \): Kinematic viscosity of ternary hybrid nanofluid (m\(^2\).s\(^{-1}\))

\( Le \): Lewis number

\( \Theta_{thnf} \): Temperature ratio parameter

\( f' \): Dimensionless velocity

\( \psi \): Stream function

\( Fr \): Inertia coefficient factor

\( Kr \): Porosity factor

\( (pc_p)_{thnf} \): Heat capacitance of ternary hybrid nanofluid (J.m\(^{-3}\).K\(^{-1}\))

\( Gr \): Thermal Grashof numbers

\( Gr^* \): Nonlinear thermal Grashof numbers

\( C_{fx} \): Skin friction coefficient

\( Nu_x \): Nusselt number

\( Re \): Reynolds number

\( \alpha_{thnf} \): Thermal diffusivity of ternary hybrid nanofluid (m\(^2\).s\(^{-1}\))

\( Pr \): Prandtl number

\( \infty \): Ambient condition

\( \eta \): Similarity variable

HAM: Homotopy analysis method

EMHD: Electro-magnetohydrodynamics

MHD: Magneto-hydrodynamics.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The author (Z. Raizah) extend her appreciation to the Deanship of Scientific Research at King Khalid University, Abha, Saudi Arabia, for funding this work through the Research Group Project under Grant Number (RGP.2/54/43). The authors would like to thank the Deanship of Scientific Research at Umm Al-Qura University for supporting this work by Grant Code: (22UQU433137DSR77).

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