Application of Celestial Mechanics Theory in Spacecraft Orbit Design

Yanxiang Gong *
College of Physics and Electronic Engineering, Taishan University, Taian 271000, China.

*Corresponding author email: yxgong@tsu.edu.cn

Abstract. This paper systematically studies the application of orbital dynamics theory in spacecraft orbit design. In this paper, two linearized equations with J2 relative term perturbation are introduced firstly, and their advantages and disadvantages are compared and analyzed. On this basis, an improved method is proposed and applied to the design of hovering orbit and fast flying orbit, and good results are obtained in some cases. The traditional numerical solution algorithm for ballistic design is innovated and improved, and the improved Newton iterative method is comprehensively applied to ensure that the algorithm can still converge quickly under the condition of inaccurate initial value; The simulation results show that the orbit integrated rapid design method studied in this paper is reliable, fast and accurate, and can be applied to the rapid orbit design of spacecraft.

Key words: Orbital dynamics; Spacecraft; Orbit Design.

1. Introduction
Orbit design is an important link in spacecraft design and development. At the same time, it should comprehensively consider the constraints and influences of the position of measurement and control station, launch site, spacecraft characteristics and various space disturbance factors. The flight environment and stress of aircraft in different flight stages are quite different, so different process constraints and terminal constraints should be considered, and the influence of unpowered taxiing section on flight trajectory should also be considered. Multiple design variables, multiple control constraints and requirements of rapidity and robustness bring great difficulties to trajectory design [1].

Spacecraft's on-orbit operation is affected by various perturbation factors, and the actual orbit and the designed orbit will have different degrees of deviation. When this deviation exceeds the allowable range, it must be controlled. Traditional orbit control is generally aimed at absolute orbit, and its control accuracy is mainly limited by orbit determination accuracy, so the control algorithm is relatively mature [2-3]. However, some satellite formation flying missions need accurate configuration maintenance, and are equipped with high-precision relative measurement equipment such as spaceborne lidar and CCD camera. Under the condition that the measurement accuracy meets the mission requirements, higher requirements are put forward for the control algorithm.

Electric propulsion system has been more and more used for orbit keeping and attitude control of geostationary orbit satellites, and has begun to perform orbit maneuver and orbit transfer tasks as the
main propulsion of deep space probes. In recent years, with the development of complex space operation technologies such as micro-satellite technology, formation technology and on-orbit service, the demand for electric propulsion technology in the aerospace field is increasing [4]. In order to make up for the shortcomings of traditional trajectory design methods, such as high initial sensitivity and slow convergence speed, this paper proposes a fast iterative design method and strategy for spacecraft orbit integration based on orbital dynamics theory. According to the dynamic pressure constraint of the first-stage rocket separation, the altitude and speed of the first-stage rocket separation are determined, and the flight program instructions of the first-stage flight segment are iteratively solved; The method described in this paper has fast convergence and high precision, and can be applied to the rapid orbital mission of spacecraft in the future.

2. Overview of Spacecraft Orbit Design

2.1. Orbit Design

Orbit design of spacecraft mainly includes design of running orbit and design of return orbit [5]. Running orbit refers to the normal orbit of spacecraft, which is possessed by every spacecraft, and return orbit is the orbit for spacecraft to return after completing its space mission, which is generally possessed only by returning spacecraft and manned spacecraft.

(1) Orbit

According to the specific requirements of space missions, the number of six orbits is determined. Generally speaking, the orbit includes transfer orbit, parking orbit, space reference orbit and earth reference orbit.

Transfer orbit refers to the orbit through which spacecraft is transferred from one orbit to another by orbital maneuver.

Parking orbit refers to the orbit where spacecraft temporarily stay in order to transfer to another orbit, also known as the resident orbit.

Space reference orbit refers to the working orbit at a certain position in space, such as Lagrange point orbit for sampling.

Earth reference orbit is the most commonly used orbit at present, which provides necessary coverage for near-earth space and the earth’s surface. According to different characteristics, it includes geostationary orbit, sun-synchronous orbit and regression orbit.

(2) Return to orbit

Returning to orbit can be divided into four different stages [6-7]:

Off-orbit segment: The spacecraft leaves its original orbit under the condition of orbital maneuver by braking rocket. Transition section: refers to the passive orbit section before the spacecraft enters the atmosphere from outer space. In this process, it is generally necessary to make multiple orbital corrections to get into the reentry phase in time and accurately.

Reentry stage: the process from the time the spacecraft enters the atmosphere to the height of 1 ~ 20,000 meters above the ground. This section of the return orbit is the design focus, and the spacecraft must withstand the test of high temperature and large overload.

Landing stage: the process in which a spacecraft uses deceleration-related devices such as parachutes to ensure that the spacecraft finally lands safely on the ground.

Regardless of using the state vector or the number of orbital elements as the representation of orbital state, the essence of the problem of solving and recursion of orbital state is to calculate the ephemeris, that is, to know the motion state of spacecraft at a certain epoch moment, and to find out the motion state of spacecraft at any moment. There are usually two methods to solve the orbital motion state: analytical method and numerical method, but the essence is to calculate the initial value problem of ordinary differential equation composed of the orbital dynamics.
2.2. Orbital dynamics equation

The orbital motion equation is derived from Newton’s law of motion. According to Newton’s second law:

$$\sum \vec{F}_{\text{ext}} = \frac{d \left( m \vec{v} \right)}{dt}$$

(1)

Solve the acceleration of spacecraft and get the second-order differential equation of formula (2):

$$\frac{d^2 \vec{r}}{dt^2} = \sum \vec{F}_{\text{ext}} - \frac{1}{m} \frac{dm}{dt} \frac{d\vec{r}}{dt}$$

(2)

In which $m$ is the mass of spacecraft, $\vec{r}$ is the position vector of spacecraft, $t$ is time, and the first item in the right side of the formula is the sum of accelerations produced by all external forces.

The types of spacecraft forces can be selected by design users, and some of them may change due to different coordinate systems [8]. The following differential equations include all possible stress cases [9]:

$$\rho v_{rel}^2 \frac{C_d A}{m_s} \vec{V}_{rel} - P_{sr} \frac{C_a A}{m_s} \vec{r}_\theta + \frac{\mu}{c^2 r^3} \left( \left( 4 \frac{\mu}{r} - \frac{\mu}{r^3} \right) \vec{r} + 4 \left( \vec{r} \cdot \vec{v} \right) \vec{v} \right) + 2 \left( \vec{\Omega} \times \vec{v} \right) + 2 \frac{\mu}{c^2 r^3} \left( \frac{3}{r^2} \left( \vec{r} \times \vec{v} \right) \left( \vec{r} \cdot \vec{J} \right) + \left( \vec{v} \times \vec{J} \right) \right)$$

(3)

The perturbation factors mainly include the third (N) body gravitational perturbation, non-spherical gravitation, atmospheric resistance, solar radiation pressure, relativity correction, etc.

3. Orbit design of spacecraft based on orbital dynamics theory

3.1. Selection of perigee height

The following factors should be considered in the selection of perigee height:

1. Relationship between orbital height $h_s$ and ground resolution;

$$S_\omega = \frac{h_s}{f \cdot R}$$

(4)

In which: $S_\omega$ is the ground resolution (m/lp), $h_s$ is the orbit height during photography, $f$ is the camera focal length, and $R$ is the comprehensive resolution of photography system (lp/mm).

2. Influence of track height $h_t$ on ground coverage.

$$d = 2 R_k \left[ \arcsin \left( \frac{h_t + R_k}{R_k} \sin \alpha \right) - \alpha \right]$$

(5)

In which $d$ is the ground coverage width and $\alpha$ is the half field angle of the remote sensor.

3. Relationship between orbital altitude and ground stations

The higher the orbit height, the longer the tracking arc of the ground station to space flight. Therefore, in order to ensure the tracking, measurement and control of the spacecraft in orbit, operation and return, the spacecraft orbit must have a certain height.

4. Relationship between the return range and orbital altitude

For the spacecraft to be recovered, there is also a proportional relationship between the return range and the orbital height, that is, the higher the orbit, the longer the return range. In order to ensure the
successful recovery of the spacecraft in a specific area, the orbital height of the ignition point of the brake rocket must also be properly designed.

(5) Relationship between track height and track life

The higher the altitude of near-earth orbit, the smaller the drag perturbation and the longer the orbit life. On the contrary, the lower the orbit height, the greater the drag perturbation and the shorter the orbit life.

(6) Orbit measurement accuracy

There are many error sources that affect the accuracy of orbit measurement, among which atmospheric drag perturbation is one of the main error sources. Therefore, it is a measure to improve the accuracy of orbit measurement by appropriately increasing the orbit height to reduce the drag force.

3.2. Control method

According to different control mechanisms, there are two ways to realize the manual freezing track: multiple pulse control and continuous low thrust control. The latter is generally realized by electric thruster. Compared with chemical thruster, the specific impulse of electric thruster is an order of magnitude higher, so it has less fuel consumption and can effectively prolong the life of satellite. It is possible in engineering to use the electric propulsion technology which can work continuously for a long time to realize the artificial freezing orbit for a long time.

However, according to the data available at present, there are few studies on the artificial freezing orbit by continuous thrust at home and abroad. Literature [10] studies a continuous thrust control scheme to eliminate perigee amplitude perturbation. This scheme considers all the perturbations of \( J_2 \) term on perigee amplitude, such as periodic term and long-term term, which requires excessive fuel consumption, and does not consider the long-term influence of the control scheme on the number of other tracks. Reference [11] proposes to eliminate perigee amplitude and angle perturbation by applying radial constant control force.

On the basis of references [10] and [11], this paper puts forward two control strategies of applying continuous constant small thrust only in the circumferential direction or both in the radial and circumferential directions, and carries out optimization design to minimize fuel consumption on the premise of ensuring that this control method will not affect the long-term changes of other track elements.

To analyze the long-term influence of perturbation function on the number of orbital elements, the average element method is generally used. The average root number method was first proposed by Kozai according to the idea of average method in nonlinear mechanics, aiming at the \( J_2, J_3, J_4 \) perturbation of the main harmonic term. The mean value \( \overline{F} \) of any function \( F(\delta, t) \) in a movement period \( T \) is defined as:

\[
\overline{F} = \frac{1}{T} \int_0^T F(\delta, t)dt
\]  

Then the short-period term \( F_s \) can be expressed as \( F_s = F - \overline{F} \), \( \overline{F} \) can be decomposed into \( F_e \) related to \( a, e, i \) and \( F_i \) related to \( \Omega, \omega \), and the corresponding perturbation function can be decomposed into three parts with different properties.

\[
F(\delta, t) = F_e + F_i + F_s
\]  

The \( J_2 \) term perturbation function can be expressed as:

\[
R = -\frac{3\mu_e J_2 a_E^2}{2r^3} \left(3\sin^2 \varphi - 1\right)
\]
$a_e$ is radius of the earth, $\varphi$ is geocentric latitude and $r$ is geocentric distance. According to formulas (6)-(7), the long-term term $R_c$ corresponding to formula (8) is calculated and substituted into Lagrange perturbation equation, and the long-term influence of $J_2$ term perturbation on the number of orbital elements can be obtained. As shown in formula (9) [12].

$$
\begin{align}
\dot{\alpha}_{J_2} &= 0 \\
\dot{e}_{J_2} &= 0 \\
\dot{i}_{J_2} &= 0 \\
\dot{\Omega}_{J_2} &= -\frac{3J_2a_e^2}{2p^2} n \cos i \\
\dot{\omega}_{J_2} &= -\frac{3J_2a_e^2}{2p^2} n \left(2 - \frac{5}{2} \sin^2 i\right) \\
\dot{M}_{J_2} &= -\frac{3J_2a_e^2}{2p^2} n \left(2 - \frac{5}{2} \sin^2 i\right) \sqrt{1 - e^2}
\end{align}
$$

In which $(a, e, i, \Omega, \omega, M)$ is the orbital element, $p = a(1 - e^2)$ is the half path, and $n = \sqrt{\frac{\mu}{a^3}}$ is the average angular velocity.

Let the perigee amplitude perturbation be zero, that is

$$\dot{\omega} = \dot{\omega}_{\text{avg}} + \dot{\omega}_{J_2} = 0$$

It is not difficult to get:

$$\dot{\omega}_{\text{avg}} = -\dot{\omega}_{J_2}$$

3.3. Numerical solution method and improvement

3.3.1. Traditional Newton iterative method and its improvement. The general idea of the traditional orbit design of spacecraft is as follows: in order to meet the terminal constraint $F(x) = 0$, the program angle parameters and shutdown time (i.e., iterative control quantity $x$) of multiple points are iteratively calculated by Newton iterative method, so that the iterative goal can meet the requirements.

The traditional Newton iterative method can only be applied to the case where the dimensions of iterative control quantity and terminal constraint quantity are equal. This paper presents a generalized Newton iterative method, which can be applied to the case where iterative control quantity is more than terminal constraint quantity.

The update form of control quantity of traditional Newton iteration method is as follows:

$$x^{i+1} = x^i - \left[F'(x^i)\right]^{-1} F(x^i)$$

In which: $F'(x)$ is the value of Jacobi matrix of $F(x)$ at $x$. When the dimension of $x$ is more than that of $F(x)$, it can be updated according to the following formula:

$$x^{i+1} = x^i - G^T (GG^T)^{-1} F(x^i)$$

In the process of trajectory iterative design, the monotone decreasing of terminal constraint deviation can adopt the following improved iterative updating formula of control quantity:

$$x^{i+1} = x^i - \eta G^T (GG^T)^{-1} F(x^i)$$
In which $\eta$ is the relaxation factor. The calculation criteria are as follows:

$$\eta = \max_{i} \left\{ \frac{\| F(x^i) \|}{\| F(x^{i+1}) \|} \right\} \text{ s.t. } 0 < \eta < 1$$

(15)

In order to ensure that the control variables $x^i$ of each iteration are within a reasonable range in the process of trajectory iterative design, it is necessary to give the reasonable range of control variables before trajectory design, that is, the feasible range $[x_{\text{min}}, x_{\text{max}}]$. In order to ensure that the iterative control quantity always meets the process constraint requirements in the iterative process, the trigonometric function control quantity conversion method which limits the maximum and minimum values of the iterative control quantity is adopted, and its formula is [13]:

$$x_k = \frac{1}{2} \left( x_{\text{min}} + x_{\text{max}} \right) \sin \phi + \frac{1}{2} \left( x_{\text{min}} + x_{\text{max}} \right)$$

(16)

In the process of trajectory design, $x^i$ can be kept within a limited range by directly iterating the variable $x^i$, thus ensuring the reasonable convergence of the algorithm.

3.3.2. **Rapid iterative design process of spacecraft orbit integration.** Orbit integrated design of spacecraft is complex, and it is necessary to design parameters such as launch azimuth, vertical flight time, extreme angle of attack, turning rate, pitch angle instruction of upper stage and taxiing time quickly and accurately to ensure that process constraints and orbit accuracy meet the requirements of orbit entry. In order to ensure accurate and reliable convergence of the algorithm, a multi-round and multi-level solution strategy is proposed in this paper. The basic idea is: to meet the terminal constraints of different flight segments, the flight program instructions of this segment are iteratively designed according to the terminal constraints of different flight segments, so as to ensure the reliability of the algorithm. The specific steps are as follows:

1. According to the mission requirements, the mission requirements of spacecraft into orbit are given, as well as the orbital semi-major axis, eccentricity, orbital inclination angle, right ascension of ascending intersection point and angular distance near the center point.
2. According to the dynamic pressure and attitude requirements of the first separation, the height, velocity and inclination angle requirements of the first separation point are given;
3. According to the process constraint requirements, the reasonable value range $[x_{\text{min}}, x_{\text{max}}]$ of each variable to be designed is given;
4. According to the historical flight experience data, the initial approximation of each variable to be designed is given;
5. The vertical flight time, extreme angle of attack and turning rate are iteratively designed by using the orbital dynamics equation to ensure that the constraint conditions of the first-order separation terminal are met;
6. Using orbital dynamics equation, the upper pitch angle instruction and taxiing time are iteratively designed to ensure that the mission requirements of semi-long axis, eccentricity, right ascension of ascending intersection point and angular distance near center point of spacecraft orbit are met;
7. Judging whether the track inclination meets the accuracy requirements, and if so, completing the ballistic/track integrated design work; if the accuracy requirements are not met, Newton method is used to update the launch azimuth angle, and step 5 is returned until the orbit and flight program parameters that meet all the requirements of orbit accuracy are designed.

4. **Simulation experiment and results**

For convenience of comparison, the orbital elements of reference satellite orbits are the same as that in references [10] and [11], as shown in Table 1.
Table 1. The orbital elements of initial orbits of reference satellites

| a (km) | e  | i (deg) | ω (deg) | Ω (deg) | f (deg) |
|-------|----|---------|---------|---------|---------|
| 15000 | 0.2| 30      | 0       | 0       | 0       |

Figure 1. Influence of inclination angle on fuel consumption

The burn up of spacecraft orbit is discussed when different orbital inclination angles are used. Fig. 1 shows the fuel consumption for one month (30 days) of artificial freezing of orbit under different inclination angles in \( e = 0.02, a = 15000 \text{ km} \). It can be seen from fig. 1 that the fuel consumption required for manually freezing the track near the critical track inclination angle is small.

In order to verify the applicability of the integrated orbit rapid design method for spacecraft, this paper carries out simulation analysis based on the mission of rapid orbital spacecraft. The spacecraft was first transported to a low orbit by booster rocket, then transferred by 2000 N small thrust engine, and finally entered the 650 km near-earth orbit. The dynamic pressure limit of interstage separation is ≤ 25 kPa, and the design speed reaches the minute level.

Using the ballistic/orbit integrated rapid design method described in this paper, the vertical flight time, turning rate, taxiing flight time and orbit transfer time are designed rapidly, and the results are shown in Figure 2 ~ Figure 3.

Figure 2. Time-height curve
The results in fig. 2 ~ 3 show that the orbit integrated rapid design method of spacecraft studied in this paper can complete the orbit design within 25s, with the design accuracy higher than 0.4m, and the method has fast convergence and high precision, which can be applied to the rapid orbit entry mission of spacecraft in the future.

5. Summary
On the basis of establishing the orbit dynamics equation with comprehensive perturbation factors, the orbit state recursive algorithm and attitude recursive algorithm are studied in detail, which lays a foundation for the modeling and design of the system. The influence of J2 term on the absolute orbit of satellite is complex, and there is no analytical expression. It is difficult to describe the influence of relative motion by precise analytical method, and the approximate solution can only be obtained by linearization method. The simulation example shows that compared with the existing research methods, the orbit integrated rapid design method of spacecraft proposed in this paper can save fuel. This design method can complete orbit design in 25s, and the design accuracy is higher than 0.4m. The method has fast convergence and high precision, and can be applied to the rapid orbit mission of future spacecraft.

References
[1] Lee K W, Singh S N. Adaptive and Supertwisting Adaptive Spacecraft Orbit Control Around Asteroids[J]. Journal of aerospace engineering, 2019, 32(4): 1-14.
[2] Kobayashi M, K Yamada. Spacecraft orbit around two fixed bodies[J]. Acta Astronautica, 2019, 160.
[3] Liu Wei, Liu Siqing, Gong Jiancun, et al. Analysis of Effect of Gauss Distribution Density Error on Space Craft Orbit [J]. Research on Space Debris, 2019, 19(02): 10-17.
[4] Zhang J, Liu W, Cao X. Exploration of Low-Energy Earth-Moon Transfer Orbit Based on Crossing Orbit of Double Three-Body System[J]. Journal of Physics: Conference Series, 2021, 1739(1):012051 (7pp).
[5] Rubinraut A. Mercuryplane—A Spacecraft for Regular Delivery of Astronauts onto the Mercury[J]. Advances in Aerospace Science and Technology, 2020, 05: 71-84.
[6] Huo Y, Li Z, Zhang F. Fast and Accurate Spacecraft Pose Estimation From Single Shot Space Imagery Using Box Reliability and Keypoints Existence Judgments[J]. IEEE Access, 2020, 8:216283-216297.
[7] Kayama Y, Takahashi S, Kawakatsu Y. Multi-Impulse Transfer from Mars Approach to Martian Moon's Orbit[J]. Aerospace Technology Japan, The Japan Society for Aeronautical and Space Sciences, 2019, 18:1-8.
[8] Joseph, John, Bevelacqua, et al. Commentary regarding "on-orbit sleep problems of astronauts
and countermeasures"[J]. Military Medical Research, 2018, 5:38.

[9] Liu L. Introduction to Aerospace Dynamics[M]. Nanjing: Nanjing University Press, 2006, 17-20

[10] Christopher C W, Vikram K. Eliminating Perigee Rotation in J2 Perturbed Orbits with a Constant Radial Acceleration[C]. AIAA Guidance, Navigation and Control Conference and Exhibit, Hilton Head, Carolina, 2007.

[11] Zhou Jiangbin, Yuan Jianping, Luo Jianjun. Research on the radial low thrust control strategy of frozen orbit with arbitrary orbital elements [J]. Acta Astronautica Sinica, 2008, 29 (5): 1536-1539.

[12] Wang G. B. Research on spacecraft orbit design and control method based on continuous low thrust [D]. National University of Defense Technology, 2011.

[13] Kwl A, Sns B. Quaternion-based adaptive attitude control of asteroid-orbiting spacecraft via immersion and invariance[J]. Acta Astronautica, 2020, 167: 164-180.