Quantum Measurements and Information
Restrictions in Algebraic QM

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Abstract

Information-Theoretical restrictions on the systems measurements and the information acquisition are applied to Quantum Measurements Theory. For the measurement of quantum object S by the information system O such restrictions are described by the formalism of restricted states which obtained from the agreement with Schrödinger dynamics and measurement statistics. The analogous restrictions derived in Algebraic QM formalism from the consideration of Segal algebra \( \mathcal{U}_O \) of O observables; the resulting O restricted states \( \{\xi^O_i\} \) set is defined as \( \mathcal{U}_O \) dual space. From Segal theorem for associative (sub)algebras it’s shown that \( \xi^O_j \) describes the random ‘pointer’ outcomes \( q_j \) observed by O in the individual events.

Key Words: Quantum Measurements, Information, Observable algebras

1 Introduction

There are still several unsettled problems concerning the interpretation of Quantum Mechanics (QM), and the majority of them involves to some extent the quantum measurement processes (Jauch, 1968; Aharonov, 1981). The oldest and most prominent of them is probably the State Collapse or Objectification Problem (D’Espagnat, 1990; Busch, 1996). In this paper we shall analyze the quantum measurements within the Information-Theoretical framework which considers some of their important aspects. Indeed, the measurement of any kind is the reception of data about the observed system S parameters by another system O (Observer) (Guilini, 1996; Duvenhage, 2002). Therefore the possible Information-Theoretical restrictions on the information transfer from S to O can influence on the observed measurement effects (Svozil, 1993). In the model regarded here O is the information gaining and utilizing system (IGUS); it processes and memorizes the information acquired as the result of S interactions with the measuring system (MS) which element is O. We assume that QM description is applicable both for a microscopic and macroscopic objects, this is the standard assumption of Quantum Measurement theory, despite that it’s not quite well founded (Busch, 1996). In particular, in our approach O state
is described by the quantum state $\rho$ relative to another observer $O'$ (Rovelli, 1995; Bene, 2000). In this approach $O$ can be either a human brain or some automatic device, in all cases it’s the system which final state correlates with the input data.

Even if S measurement by $O$ is described by the evolution of MS state $\rho_{MS}$ relative to some external $O'$, our aim is to calculate S information acquired by $O$ itself. In general $\rho_{MS}$ ansatz isn’t sufficient for that, and the description of MS state relative to $O$ should be regarded in the self-description framework of Information Theory (Svozil, 1993). Earlier the systems self-description was investigated in the context of the general problem of mathematical Self-reference (Finkelstein, 1988; Mittelstaedt, 1998). It was argued that the self-description of an arbitrary system is always incomplete; this result often interpreted as the analog of Gödel Theorem for Information Theory (Svozil, 1993). Recently the self-description in the measurement process, called also the measurement from inside, was considered in the formalism of inference maps $M_O$ connecting MS and $O$ states $\rho_{MS}, R_O$ (Breuer, 1996). It leads to some general results for the quantum measurements, but by itself doesn’t permit to derive $M_O$ from the first QM principles. In this paper we argue that the inference map or arbitrary systems can be calculated in Algebraic QM framework which exploits Segal algebras of observables - i.e. $C^*$-algebras formalism. (Emch, 1972). In this formalism MS state $\xi^{MS}$ is defined on MS observables algebra $\mathcal{U}$, $O$ state $\xi^O$ is defined on $O$ observables subalgebra $\mathcal{U}_O$ which is the restricted set of MS observables. It will be shown that for MS in the pure state, $\xi^O$ is the stochastic state which describes the random outcomes in the measurement of S states superpositions and this effect can be interpreted as the subjective state collapse observed by $O$. Earlier the analogous inference map was obtained phenomenologically in the formalism of doublet states (Mayburov, 2001, 2004). Alternatively this results can be interpreted also as the consequences of principal restrictions on the information transferred from S to O.

We should stress that the observer’s consciousness doesn’t play any role in our theory and isn’t referred to directly anywhere (London, 1939). The terms ‘perceptions’, ‘impressions’ used here are defined in strictly physical terms. In our model the perception is the acquisition of some information by $O$, i.e. the change of $O$ state; a different $O$ impressions are associated with a different $O$ states; their properties are discussed also in the final chapter. The states of objects defined in $O$ reference frame (RF) (or other $O'$ RF) are referred to also as ‘S state for $O$’.

2 Measurements and Quantum States Restrictions

The essence of the state collapse problem is the possibility of discrimination between pure and mixed states in quantum measurements, so it’s instructive to remind the classification of quantum states. First, there are the individual states - i.e. the quantum states in the individual events, in standard QM they are the pure states - Dirac vectors $|\psi_l\rangle$ in Hilbert space $\mathcal{H}$; see also chap.4 (Primas, 1981). The statistical states describe the properties of quantum ensembles, and are represented by the normalized, positive operators of trace 1 - density matrixes $\rho$ on $\mathcal{H}$. If $|\psi_l\rangle$ composition is known for the given ensemble, it can be described in more detail by the ensemble state (gemenge) represented by the table $W^e = \{\psi_l; P_l\}$, where $P_l$ are the corresponding probabilities (Busch, 1996). Here and below under standard QM we understand Schrödinger formalism which postulates that the quantum states are defined on complex Hilbert space introduced ad hoc (Jauch, 1968). The properties of states in Algebraic QM will be considered in chap. 3.

We shall use the simple MS model which includes the measured state S and IGUS $O$ storing the incoming S information. For the simplicity the detector D is dropped in MS chain, the
role of $O$ decoherence effects will be discussed below. $S$ represented by two-dimensional state vector $\psi_s$, whereas $O$ is described by three-dimensional Hilbert space $\mathcal{H}_O$. Its basis consists of the orthogonal states $|O_{0,1,2}\rangle$, which are the eigenstates of $Q_O$ ‘internal pointer’ observable with eigenvalues $q_{0,1,2}^O$, which for the convinience is taken to be equal to 0, 1, 2 correspondingly. For the simplicity of calculations it assumed that $|O_{1,2}\rangle$ constitutes the basis of two dimensional subspace $\mathcal{H}_O$. Let us consider the measurement of $S$ observable $\hat{S}_z$ for $MS$ initial pure state:

$$\Psi_{MS}^o = \psi_s|O_0\rangle = (a_1|s_1\rangle + a_2|s_2\rangle)|O_o\rangle,$$

(1)

where $|s_{1,2}\rangle$ are $Q$ eigenstates with eigenvalues $q_{1,2}$. In our model $S,O$ interaction $\hat{H}_I$ starts at $t_0$ and finishes effectively at some finite $t_1$; for the suitable $H_I$ ansatz Schrödinger equation for $\Psi_{MS}^o$ would result in $MS$ final state $\rho_{MS}^o$ responding to the state vector:

$$\Psi_{MS} = \sum_i a_i|s_i\rangle|O_i\rangle.$$

(2)

It turns out that

$$\bar{Q}O = \bar{S}_z = |a_1|^2 - |a_2|^2,$$

so $O$ performs the unbiased $Q$ measurement (Von Neumann,1932). To reveal the studied effect, the measurement of incoming $S$ mixture with the same $\bar{S}_z$ should be regarded also, its composition is described by the gemenge $W^s = \{|s_i\rangle, |a_i|^2\}$ to which correspond the density matrix:

$$\rho_s^m = \sum_i |a_i|^2|s_i\rangle\langle s_i|.$$

(3)

Concerning the information recognition by IGUS $O$, at this stage we assume arbitrarily that $|O_{1,2,0}\rangle$ eigenstates corresponds to the certain information patterns $J^O_i$ perceptor by $O$ as $Q_O$ values $q_i^O$. Below more arguments in favor of this assumption will be presented, however it’s worth to notice here that in this case $q_i^O$ are $O$ real properties, so this is quite natural assumption (Busch, 1996). As the example of the simple $O$ toy-model can be regarded the hydrogen-like atom $A_H$ for which $O_0$ is its ground state and $O_i$ are the metastable levels excited by $|s_i\rangle$, for $|s_i\rangle$ superposition it results into the final $S - A_H$ entangled state.

The pure state $\Psi_{MS}$ of (2) describes $MS$ state relative to external observer $O'$, whereas from $O$ ’point of view’ $\Psi_{MS}$ describes the simultaneous superposition (coexistence) of two contradictory information patterns (impressions): $Q_O = q_1^O$ and $Q_O = q_2^O$ perceptor by $O$ simultaneously (Wigner,1961). However, it’s well known that experimentally the macroscopic $O$ would observe at random one of $Q_O$ values $q_{1,2}^O$ in any individual event. It means that $S$ final state is $|s_1\rangle$ or $|s_2\rangle$ for $O$, and $S$ state collapse occurs. $S$ final state is described by $\rho_{s}^m$ which stipulated by the gemenge $W_{mix}^s = \{|s_i\rangle|O_i\rangle, |a_i|^2\}$. In accordance with it one can ascribe to $MS$ the mixed state:

$$\rho_{MS}^o = \sum_i |a_i|^2|s_i\rangle\langle s_i||O_i\rangle\langle O_i|.$$

(4)

relative to $O'$; it differs principally from $\rho_{MS}^o$ of (2). This discrepancy describes the phenomenon of state collapse, it illustrates also the famous Wigner ‘Friend Paradox’ for $O,O'$ (Wigner,1961).

We shall regard here the formalism which temptatively can unify this two $MS$ descriptions ’from outside’ by $O'$ and ’from inside’ by $O$. The first clue prompts the comparison of $O$ average response to $S$ pure and mixed states. Let’s consider for the simplicity only the incoming $\psi_s$ with $r_s = \frac{a_1}{a_2}$ real, and denote them as $\psi_{s}^r$, we introduce also the $z$-symmetric state $\psi_{s}^s$ for which $r_s = 1$. For incoming $\psi_s^r$, one obtains $\bar{S}_z = 2a_1a_2$, $\bar{S}_y = 0$; for $s_1$ mixture $W^s$ of (3) $\bar{S}_z = 0, \bar{S}_y = 0$, so the rate of $S$ state purity $\beta_p$ is characterized by $|\bar{S}_z|$. Meanwhile, beside $\bar{Q}_O = \bar{S}_z$, for any other
O observable $Q'_O \neq F(Q_O)$ the final $\bar{Q}'_O = 0$ independently of the initial $S$ state. Hence the information about the rate of state purity characterized by $\beta_p$ doesn’t transferred to $O$ at all, at least via $O$ expectation values. That’s not quite surprising, because $S_x, S_y$ doesn’t commute, and so from the uncertainty relations follows that the precise $S_z$ measurement would spoil $S_z$ measurement. The analogous results can be obtained for an arbitrary $a_{1,2}$, but in place of $S_z$ some linear combination of $S_x, S_y$ should be used. In Information Theory framework MS and any other measurement set-up can be regarded formally as the information channel transferring the information from $S$ to $O$. Despite that the conclusive picture can be obtained only from the analysis of individual events, this calculations indicates that the information transfer restrictions can be important in Quantum Measurement theory.

From the formal point of view the measurement of an arbitrary system $S'$ is the mapping of $S'$ states set $N_S$ to the given IGUS $O^I$ states set $N_O$ (Svozil,1993). $O^I$ can be considered formally as the subsystem of the large system $S_T = S' + O^I$ with the states set $N_T$ (Mittelstaedt, 1998). In this approach - ‘measurement from inside’, $N_O$ is $N_T$ subset and the inference map $M_T$ of $S_T$ states to $N_O$ defines $O^I$ state $R_O$ called $S_T$ restricted state. The important property of $S_T \rightarrow O^I$ inference map is formulated by Breuer theorem: if for two arbitrary $S_T$ states $\Gamma_S, \Gamma_S'$ their restricted states $R_O, R'_O$ coincide, then for $O^I$ observer this $S_T$ states are indistinguishable (Breuer,1996). Under simple assumptions about $S_T, O^I$ at least several such $S_T$ states should exist. In classical case the origin of this effect is obvious: $O^I$ has less degrees of freedom than $S_T$ and can’t discriminate all possible $S_T$ states, and because of it some number of $S_T$ indistinguishable states always should exist for any classical $S_T$ and $O^I$ (Svozil, 1993). In quantum case the observables noncommutativity and nonlocality introduces some novel features regarded below. Despite that $R_O$ are incomplete $S_T$ states, they are the real physical states for $O^I$ observer - ‘the states in their own right’, as Breuer characterizes them.

The obtained $S', O^I, S_T$ relations are applicable to our MS model which also can be treated as MS measurement from inside. Breuer’s results leaves the considerable freedom for the choice of the inference map $M_T(S_T \rightarrow O^I)$, and doesn’t permit to derive the restriction ansatz for $S_T$ individual states directly. It can be shown that $R^st_O$ - the restricted statistical state coincides with the partial trace of $S_T$ state over $S'$ degrees of freedom; for MS state (2) it is equal to:

$$R^st_O = Tr_s \rho^p_{MS} = \sum |a_i|^2 \langle O_i | O_i \rangle$$  (5)

From that for MS mixture $\rho^m_{MS}$ of (4) the corresponding restricted state is the same $R^st_O = R^st_O$. This equality doesn’t mean the collapse of MS pure state $\Psi_{MS}$, because the collapse appearance should be verified also for MS individual states. For the pure case MS individual state is $\Psi_{MS}$ of (2), for the incoming $S$ mixture - Gemenge $W^s$ of (3) MS state also differs from event to event:

$$\zeta^{MS}(n) = |O_i \rangle \langle O_i | |s_i \rangle \langle s_i |$$  (6)

where the random $l(n)$ frequency is stipulated by the probabilistic distribution $P_l = |a_i|^2$, so in any event $\zeta^{MS}(n)$ differs from MS state (2). It was proposed phenomenologically that for an arbitrary system $S_T$ its restricted individual state is also equal to the partial trace of $S_T$ individual state over $S'$ (Breuer,1996), so for MS pure state:

$$R_O = Tr_s \rho^p_{MS} = \sum |a_i|^2 \langle O_i | O_i \rangle$$  (7)

Obviously such ansatz excludes beforehand any kind of probabilistic $R_O$ behavior. Thereon in any event $n$ for MS Gemenge of (3) its restricted individual state $\zeta^O(n) = |O_i \rangle \langle O_i |$ differs from $R_O$ (below also $\z_1^O = |O_i \rangle \langle O_i |$ notation is used). It follows then, that for the restricted individual
states the main condition of Breuer Theorem is violated. From that Breuer concluded that $O$ can differ pure/mixed states 'from inside' in the individual events, therefore this formalism doesn’t result in the state collapse (Breuer, 1996). However, the formal difference of two restricted states doesn’t mean automatically that these states are physically different. That’s the necessary but not sufficient condition, in general it’s necessary also that some MS parameters measurable by $O$ are different for that states. Otherwise it can turn out that these states are equivalent, and $O$ can’t detect their difference (Mittelstaedt,1998). For this purposes the only parameter i.e. observable value which characterizes it in an individual events is $q^O_i$. In Breuer paper $R_O$ parameters available for $O$ weren’t calculated, without it the conclusion about $R_O,q^O_i$ discrimination seems preposterous; the discussion of this point will be continued in chap. 4. Below by the slight abuse of terminology, MS state restriction to $O$ is called $O$ restricted state.

As was shown above the difference between the initial S pure state and $|s_i⟩$ mixture with the same $S_z$ reflected by $S_{x,y}$ values. As the example, for $ψ^S$ with $a_{1,2} = \frac{1}{\sqrt{2}}$ for pure and mixed state one obtains $S_x = 1, 0$ correspondingly. For MS states the same difference reflected by MS interference term (IT) observable:

$$B = |O_1⟩⟨O_2||s_1⟩⟨s_2| + j.c.$$

which characterizes $O,S$ quantum correlations. Being measured by external $O'$ on $S,O$, it gives $B = 0$ for the mixed MS state of (4), but $B \neq 0$ for the pure MS states (2). For example, for the incoming $ψ^S$, one obtains $B = 1$. However, $B$ value can’t be measured by $O$ 'from inside', so $O,S$ correlations are unavailable for $O$ directly. The principal possibility for $O'$ to measure $B$ and send the information to $O$ doesn’t change the situation, because such $O'$ measurement is incompatible with $S_z$ measurement by $O$. On the whole IT observables are nonlocal and constitutes the special class of observables $\{B^{MS}\}$. Note that $\{B^{MS}\}$ observability excludes the Ignorance Interpretation (II) of pure states, which assumes, in particular, that MS pure state $Ψ_{MS}$ is equivalent to $|s_i⟩|O_i⟩$ gemenge (Busch,1996). As the example, for the symmetric $Ψ_{MS}$ of (2) with $a_{1,2} = \frac{1}{\sqrt{2}}$ II claims that $Q_O$ value in any event is sharp and is equal either to $q^O_1$ or $q^O_2$ with the same probability. Meanwhile in this state $B$ has the eigenvalue $b = 1$ which means that $Q_O$ value $q^O$ is principally uncertain for $O'$: $q^O_1 \leq q^O \leq q^O_2$, and so II is incorrect for $O'$. However such reasoning fails for $O$ observer, because $B$ value is unobservable for him together with other $B^{MS}$ observables. Hence the ‘subjective’ II, in which $Q_O$ value can be sharp for $O$, being simultaneously uncertain for $O'$, can’t be excluded beforehand in self-description approach.

In the regarded approach MS individual state can be rewritten formally in doublet form $\Phi^B(n) = |ϕ^D,ϕ^I⟩$ where $ϕ^D = ρ_{MS}$ is the dynamical state component i.e. MS state for $O'$, the information component $ϕ^I$ describes $O$ subjective information, i.e. $O$ restricted state in the given event $n$. In Breuer theory $ϕ^I = R_O$ is just $ϕ^D$ trace, however in the alternative theory regarded here it describes the independent $O$ degrees of freedom. To agree with the quantum Schrödinger dynamics, any self-description theory should satisfy to the following operational conditions:

i) if an arbitrary system $S'$ doesn’t interact with IGUS $O'$, then for $O^I$ this system evolves according to Schrödinger-Liouville equation (SLE)

ii) If $S'$ interacts with $O^I$ and the measurement of some $S'$ observable occurs, then the quantum dynamics can be violated for $O^I$, but as follows from condition i), in this case $S',O^I$ evolution for the external observer $O'$ should obey SLE.

To reconcile this conditions with the state collapse at the phenomenological level we proposed recently doublet state formalism (DSF) (Mayburov,2001,2004). It’s reviewed here briefly, because it has much in common with the description of measurements in Algebraic QM. For DSF
state \( \Phi = |\phi^D, \phi^I \rangle \) the dynamical component is also equal to QM density matrix \( \phi^D = \rho \) and obeys SLE:

\[
\frac{\partial \phi^D}{\partial t} = [\phi^D, \hat{H}]
\]

therefore for MS the initial \( \phi^D = \rho^D_{MS} \) of (1) evolves at \( t > t_1 \) to \( \phi^D(t) = \rho^D_{MS} \) of (2). \( \phi^I \) describes \( O \) restricted state, so for \( t < t_0 \) the initial \( \phi^I = \langle O_0 | O_0 \rangle \). After S measurement finished at \( t > t_1 \), in DSF its \( \phi^I \) outcome is supposed to be stochastic: \( \phi^I(n) = \phi^I_i \), where \( \phi^I_i = \langle O_i | O_i \rangle \); here \( i(n) \) outcome frequencies are described by the probabilistic distribution with \( P_i = |a_i|^2 \).

Hence such doublet individual state \( \Phi(n) \) can change from event to event, and \( \phi^I(n) \) is partly independent of \( \phi^D \), being correlated with it only statistically. Consequently the ensembles of \( O \) subjective states \( \phi^I \) coincides for the pure and mixed states with the same \( |a_i|^2 \), the conditions of Breuer theorem are fulfilled and the subjective state collapse can be observed by \( O \).

Plainly in this theory the quantum states for external \( O' \) (and other observers) also has the same doublet form \( \Phi' \). In the regarded situation \( O' \) doesn’t interact with MS and so \( O' \) information \( \phi^I \) doesn’t change during S measurement. Consequently, MS state evolution for \( O' \) described by \( \phi^D \), which obeys SLE. Because of it MS state collapse isn’t observed by \( O' \) in agreement with the conditions i, ii; eventually this theory responds to the subjective II regarded above. Witnessing QM Interpretation proposed by Kochen is quite close to DSF but doesn’t exploits the self-description approach (Kochen,1985; Lahti,1990).

In DSF \( |O_i\rangle \) constitutes the preferred basis (PB) in \( \mathcal{H}_O \); this problem called also the basis degeneracy is well-known in Quantum Measurement Theory (Lahti,1990; Elby,1994). In its essence, the theory consistency demands that the final MS states decomposition should be unique, but this isn’t the case for \( \Psi_{MS} \) of (2). In DSF PB problem acquires the additional aspects related to the information recognition by \( O \). The plausible explanation of PB appearance prompts \( O \) decoherence - i.e. \( O \) interaction with environment \( E \), which is practically unavoidable in lab. conditions (Zurek,1982). Such interaction results in the final entangled \( S,O,E \) state which decomposed on some orthogonal \( O \) basis \( |O_i^E\rangle \) (Guilini,1996). Tuning the \( H_{O,E} \) interaction parameters, \( |O_i^E\rangle \) basis can be made equivalent to \( |O_i\rangle \) basis. For the initial MS state \( \Psi_{MS}^I \) of (1) it results in the final MS-E state:

\[
\Psi_{MS+E} = \sum a_i |s_i\rangle |O_i\rangle |E_i\rangle
\]

where \( |E_i\rangle \) are final E states. It was proved that such triple decomposition is unique, even if \( |E_i\rangle \) aren’t orthogonal (Elby,1994). This measurement scheme denoted as MS+E will be considered below together with MS model. In other aspects the decoherence doesn’t change our measurement model; its most important role is the unambiguous definition of \( O \) PB. In fact \( \mathcal{H}_O \) symmetry is broken dynamically by \( H_{O,E} \) interaction which makes majority of \( O \) states unstable. As will be shown in Algebraic QM PB is defined by \( S,O \) interactions only and is independent of \( O \) decoherence, however the account of decoherence effects is necessary in any consistent measurement theory.

### 3 Quantum Measurements in Algebraic QM

Now the quantum measurements and \( O \) self-description for the finite quantum systems will be regarded in Algebraic QM formalism; analogously to DSF it should also satisfy to the conditions i) - ii) formulated above. Besides the standard quantum effects, Algebraic QM describes successfully the phase transitions and other nonperturbative phenomena which standard QM fails to incorporate (Emch,1972). Consequently, there are the serious reasons to regard Algebraic
QM as the consistent generalization of standard QM. Algebraic QM was applied extensively to the superselection models of quantum measurements, in which the detector D or environment E are regarded as the infinite systems with \( m, V \to \infty \) (Primas,1990; Guilini,1996). The algebraic formalism of nonperturbative QFT was used also in the study of measurement dynamics in some realistic systems (Mayburov,1998; Blanchard,2003).

What is the fundamental QM entity - the Hilbert space of states or the algebra of observables is the problem disputed since the time of QM formalism construction (Von Neuman, 1932; Emch,1972). In standard QM the fundamental structure is the Hilbert space of states \( \mathcal{H} \) on which an observables - Hermitian operators ( or POV) are defined (Busch,1996). However, it was found that for some nonperturbative systems the structure of states set differs principally from \( \mathcal{H} \), and the axiomatics of standard QM becomes preposterous (Bratelli,1981). In distinction, the fundamental structure of Algebraic QM is Segal algebra \( \mathcal{U} \) of observables which incorporates the main properties of any regarded system \( S_f \) (Emch, 1972). From the mathematical point of view the duality between the operators algebra and the states set is more natural, than the states set priority postulated in standard QM (Bratelli, 1981). In its framework it’s more convenient technically to deal with \( C^* \)-algebra \( \mathcal{C} \) for which \( \mathcal{U} \) constitute the subset. \( \mathcal{C} \) and \( \mathcal{U} \) elements in Algebraic QM are the linear operators for which the sum \( A + B \) and product \( A \cdot B \) defined. Roughly speaking, \( \mathcal{C} \) is the complex algebra for which \( \mathcal{U} \) is the subset of its real (Hermitian) elements. For any system \( \mathcal{C}, \mathcal{U} \) are in the unambiguous correspondence: \( \mathcal{C} \leftrightarrow \mathcal{U} \), and below their use is equivalent in this sense. \( S_f \) states set \( \Omega \) defined by \( \mathcal{U} \) via the notorious GNS construction; in its framework \( \Omega \) is the vector space dual to the corresponding \( \mathcal{C} \) (Bratelli,1981). Such states are called here the algebraic states \( \varphi \) and are the normalized, positive, linear functionals on \( \mathcal{U} \): for any observable \( A \in \mathcal{U}, \forall \varphi \in \Omega ; \) its expectation value \( \bar{A} = \langle \varphi; A \rangle \). Eventually \( \varphi \) states are the analog of QM density matrixes \( \rho \). The pure states - i.e. \( \Omega \) extremal points are regarded as the algebraic individual states (AIS) \( \xi_i \), their set which is a convex shell denoted \( \Omega^p \); the further consideration of AIS properties is given below in chap.4 (Emch,1972; Primas,1990). All other elements \( \varphi \in \Omega \) are treated as the algebraic mixed states, despite that their physical meaning isn’t settled finally (Primas, 1981). Independently of it any algebraic state \( \varphi \) can be constructed operationally as \( \xi_i \) ensemble, the ensemble states \( W^A \) are defined analogously to QM ansatz given in chap.2. Here only a finite-dimensional \( S_f \) will be considered, in this case both regarded QM formalisms are principally equivalent, but Algebraic QM is more convinient for our problems, because in its framework the correspondence between states and observables is more straightforward. In this case \( \varphi \) states set \( \Omega \) is isomorphic to \( S_f \) set of QM density matrixes \( \rho \).

Concerning the application of Algebraic QM to the measurements, in particular for our MS scheme, \( O \) self-descripton restrictions can be formulated as the restrictions on the set of MS observables available for \( O \). The situations in which for some system \( S_f \) only some restricted linear subspace \( \mathcal{M}_R \) or subalgebra \( \mathcal{U}_R \) of algebra \( \mathcal{U} \) is available for the observation were extensively studied in (Emch, 1972). In this case the restricted algebraic states \( \varphi_R \) can be constructed starting from the expectation values of \( A_R \in \mathcal{U}_R \):

\[
\bar{A}_{R} = \langle \varphi; A_R \rangle = \langle \varphi_R; A_R \rangle
\]  

(11)

\( \varphi_R \) doesn’t depend on any \( A' \notin \mathcal{U}_R \), thereby \( \forall \varphi_R, \langle \varphi_R; A' \rangle = 0 \). In Algebraic QM any classical system \( S^c \) described by the commutative system of observables \( \{ A^c_i \} \) which constitute the associative Segal algebra \( \mathcal{U}^c \). Conversely any associative Segal (sub)algebra \( \mathcal{U}' \) is isomorphic to the algebra \( \mathcal{U}^c \) which describes some \( S^c \), its \( \varphi^c \) states set \( \Omega^c \) is isomorphic to the set \( \Omega^c \) of the classical statistical \( S^c \) states \( \varphi^c \) (Segal,1947). The corresponding AIS - i.e. the pure states corresponds to the classical individual states \( \xi^c_i \) - points in \( S^c \) phase space. For the self-description
the most important is the case when \( U' \) is elementary, i.e. includes only \( I \) - unit operator and the single \( A \neq I \) (it’s also called A-subsystem). Then \( \xi^\epsilon_i = \delta(q^A - q^A_i) \) corresponds to \( A \) eigenvalues \( q^A_i \) spectra. Consequently, even if quantum \( S_f \) is described by nonassociative \( U \), it includes the subalgebras \( U' \in U \) for which the restricted states are classical. Hence in Algebraic QM \( S_f \) state can be described formally as the multiplet, each member of which is the state defined on the particular (sub)algebra analogously to DSF state regarded above.

For the classical system \( S^c_f = S^c + O^c \) described by some \( U^c \), the self-description restrictions for IGUS \( O^c \) are simple and straightforward - the restricted \( S^c_f \) states depend only on those \( S^c_f \) coordinates \( \{x^O_{ij}\} \), which are \( O \) internal degrees of freedom (Breuer, 1996). They constitute the subalgebra \( U^c_O \in U^c \). In practice \( O^c \) effective subalgebra \( U^c_O \in U^c_O \) which really defines the measurements can be even smaller, because some \( x^O_{ij} \) can be uninvolved directly into the measurement process. From the analogous reasons for the quantum localized IGUS \( O \) its subalgebra \( U^c_R \) of MS algebra \( U \) also should include only \( O \) internal (local) observables. It corresponds to the locality principle acknowledged in Quantum Physics. Really \( O \) perception of any other MS observable \( A_{MS} \notin U^c_R \) involves the instant measurement of \( S \) which can be miles away from \( O \) at that time, the example is IT B of (8). Any effective \( O \) subalgebra \( U^c_O \in U^c_R \), their states sets are denoted \( \Omega^o_O, \Omega^c_R \) correspondingly. Concerning with \( O \) individual states the main assumption of our theory is as follows: given the subalgebra \( U^c_A \in U \), in any individual event \( n \) an arbitrary MS state \( \xi^{MS}(n) \) induces some restricted AIS \( \xi^A(n) \) spanned on \( U^c_A \). This hypothesis seems to us quite natural and below the additional arguments in its favor will be presented for the particular \( U^c_A \).

MS is described by \( U \) Segal algebra of MS observables which defines \( \varphi^{MS} \) set \( \Omega \). In case of \( O \) decoherence \( MS + E \) involves \( U_{MS,E} \) algebra correspondingly. \( O \) subalgebra \( U^c_R \) includes \( I \) and all \( O \) internal observables, so it means that \( \Omega^c_R \) is isomorphic to \( O \) statistical states \( \rho^O \) set \( \nu^O_O \). Then \( O \) AIS set \( \Omega^c_R \) is isomorphic to \( \mathcal{H}_O \), and any \( O \) AIS \( \xi^R_i \) should correspond to some state vector \( |O^R_i\rangle \in \mathcal{H}_O \). We don’t study here \( \xi^R \) states in detail, because the effective \( O \) subalgebra \( U^c_O \) differs from \( U^c_R \), and our aim is to calculate \( O \) states defined on it. To define \( U^c_O \), remind that in the used MS dynamics of \( S \) measurement for any final \( \Psi_{MS} \) only \( Q^O \neq 0 \) in MS final state, \( Q^O = Q \); for any other \( Q^O \neq F(Q^O) \) one obtains \( Q^O = 0 \). Therefore for any corresponding AIS \( \xi^O \in \Omega^O_O \) it follows that \( \langle \xi^O; Q^O \rangle = 0 \). Any \( \varphi^O \in \Omega^O_O \) is the convex state of \( \{\xi^O_i\} \), hence \( \langle \varphi^O, Q^O \rangle = 0 \). It means that \( O \) effective subalgebra \( U^c_O \) is equal to the elementary \( U^c_R \), which includes only \( Q^O \) and \( I \). Really, only in this case \( \langle \varphi^i; Q^O \rangle = 0 \) for all \( \varphi^i \in \Omega^R_i \) defined on \( U^c_R \); each \( \varphi^i \) corresponds to \( \varphi^O \) with the same \( Q^O \) and vice versa; so \( \varphi^O \) set \( \Omega^O_O \) is isomorphic to \( \Omega^c_R \) (Segal, 1947). There is no other \( U^c_R \) subalgebras with such properties, and that settles \( U^c_O \) finally as the classical algebra of single ‘pointer’ observable \( Q^O \). Consequently such \( U^c_O \) defines also PB for \( O \) states unambiguously, and so \( O-E \) decoherence only can only duplicate it; more detailed consideration of the decoherence effects is given below.

### 4 Algebraic QM Restrictions and Self-description

Now the relation between the pure and individual states is regarded in more detail, because it will be used below in the construction of restricted \( O \) states in Algebraic QM - AIS. Despite that mathematically this question is quite plain, it’s considered here in most detailed way, because in the discussion of QM foundations it became the source of many confusions (Bene, 2000; Breuer, 1996). In this consideration of individual quantum states i.e. the states distinguishable in the individual events we shall follow the approach of (Primas, 1990). As was mentioned in chap. 2, the physically different states of any kind can be operationally discriminated by the
particular experiment which puts in correspondence to this states some measured parameters values. For QM statistical states it can be the parameters of experimental distributions which can be expressed via the expectation values of some observables. For the finite-dimensional nondegenerate \( S_f \) any pure state \( \psi_a \) is the eigenstate of some observable \( A \) with the eigenvalue \( q_{a}^{A} \), in this case \( q_{a}^{A} \) corresponds to some real \( S_f \) property (Busch, 1996). It permits \( \psi_a \) discrimination from any other pure state \( \psi_b \) in a single event by the objective (non-collapse) \( A \) measurement, hence any \( \psi_a \) is the individual state. There is no other classes of states which satisfies to this conditions, in particular, the convex states can’t be an individual states. To illustrate it, let’s regard as the example S state \( \psi_s \) of (1), for an arbitrary \( a_i \) it is the eigenstate of some operator \( \vec{S}\vec{n}_i \), where \( \vec{n} \) direction defined by \( a_i \). Now let’s consider the convex state \( \phi_z = \sum w_i |s_i \rangle \langle s_i | \) with \( w_i \geq 0 \), in this framework it has no real properties. Hence it’s impossible to prove that \( \phi_z \) is the superposition of states with \( s_z = \pm 1 \), alike the real property \( s_x = 1 \) demonstrates it for \( S_x \) eigenstate \( \psi_s^{\pm} \) with \( a_{1,2} = \frac{1}{\sqrt{2}} \). \( \phi_z \) is smeared on \( s_z \) axe, i.e. \( -1 \leq s_z \leq 1 \), but the same inequality is true for \( |s_{1,2} \rangle \) states also. Consequently by no means \( \phi \) can be discriminated from \( |s_{1,2} \rangle \) in the individual events, and because of it \( \phi \) can’t be the feasible individual state. In this framework the ansatz for the individual states is defined by the operational conditions only, and hence will be principally the same in Algebraic QM - i.e. AIS should be also the pure states. For the restricted algebraic states this ansatz doesn’t change principally, as shown below for \( \mathcal{U} \) its derivation is even more simple and straightforward.

As follows from Segal theorem, for the obtained \( \mathcal{U} \) subalgebra the algebraic \( O \) states \( \varphi^O \in \Omega_O \) are isomorphic to the classical \( q_i^O \) probabilistic distributions; meanwhile \( \xi^O - \Omega_O \) extremal points are the positive states:

\[
\xi^O_i = \delta(q^O - q_i^O) \tag{12}
\]

which corresponds to the classical pointlike states. In particular, such \( \xi^O_i \) appears in \( |s_i \rangle \) measurement by \( O \) as the restriction of MS final state \( \xi_{\text{MS}}^{\ell} \sim |s_i \rangle |O_i \rangle \). \( \xi^O \) are \( O \) individual states, their distinction can be revealed in the single event from the the difference of \( Q_O \) eigenvalues \( q_i^O \). The existence of such restricted \( O \) states at least for some MS states restriction is important for our formalism, because it permits to analyze any other restricted states (if they exist) by the comparison with \( \xi^O \). Following the consideration of chap. 2, we concede that \( O \) perceives this states as the information pattern \( J^O_i = q_i^O \).

The incoming S mixture - \( |s_i \rangle \) gemenge \( W^s \) results in MS algebraic final state \( \varphi^\text{mix} \) which is equivalent to \( \rho_{MS}^{m} \) of (4); \( O \) restricted state \( \varphi^O_{\text{mix}} \) is defined from the relation for \( Q^O_O \), in particular :

\[
Q_O = \langle \varphi_{\text{mix}}^O; Q_O \rangle = \langle \varphi_{\text{mix}}^O; Q_O \rangle = \sum |a_i|^2 q_i^O
\]

which results in the solution \( \varphi_{\text{mix}}^O = \sum |a_i|^2 \varphi_i^O \), where \( \varphi_i^O = \xi_i^O \) of (12). From the correspondence of MS state \( \xi_{\text{MS}}^{\ell} \) and \( O \) state \( \xi^O \) in the individual events, the restricted algebraic state \( \varphi_{\text{mix}}^O \) represents in this case the statistical mixture of AIS \( \xi^O_i \) described by \( O \) ensemble state

\[
W_{\text{mix}}^O = \{ \xi^O_i; P_i = |a_i|^2; i = 1, 2 \} \tag{13}
\]

If MS final state is the pure state \( \xi_{\text{MS}}^{\ell} \) which corresponds to \( \Psi_{\text{MS}} \) of (2), then MS algebraic state is \( \varphi_{\text{MS}}^O = \xi_{\text{MS}}^{\ell} \), and it results in the same \( Q^O_O \) value as the regarded mixture. Therefore its \( O \) restricted algebraic state coincides with the mixed one: \( \varphi^O = \varphi_{\text{mix}}^O \) for the same \( |a_i| \).

To illustrate the derivation of MS restriction to \( O \) for individual states, let’s consider it for \( \xi_{\text{MS}}^{\ell} \) which is equivalent to \( \Psi_{\text{MS}} \) of (2) for \( z \)-symmetric initial state \( \psi^s_{\ell} \) with \( a_{1,2} = \frac{1}{\sqrt{2}} \). Let’s express the properties which MS restricted state - \( \xi^O_s \) should possess via its relations with the
observables $Q_O$ and $B$ of (8), their values (in general uncertain) are denoted as $\bar{q}_O, \bar{b}$. In $O'$ RF $\xi^{MS}_{s}$ is $B$ eigenstate with the eigenvalue $\bar{b} = 1$ which is called IT property; $Q_O$ obeys the inequality: $\bar{q}_1^O \leq \bar{q}_O \leq \bar{q}_2^O$, it is called the spectral property. Taken together this properties indicates that for $O'$ $\bar{q}_O$ is located within the interval $[\bar{q}_1^O, \bar{q}_2^O]$, and so is principally uncertain inside it, as $\bar{b} = 1$ value evidences. $\xi_s^O$ is defined on $U_O = \{I,Q_O\}$, therefore the spectral property also holds for $O$. Now because $B \notin U_O$ and so is unavailable for $O$, for $\xi_s^O$ IT property can be dropped. Without it the spectral property alone means that $\bar{q}_O$ is localized in $[\bar{q}_1^O, \bar{q}_2^O]$ interval, but $\bar{q}_O$ can be either uncertain or sharp in its limits. Meanwhile $\xi_{1,2}^O$ states with the sharp $\bar{q}_O = q_{1,2}^O$ possess that only property which MS restricted state should have. Therefore the solution for $\xi_s^{MS} \rightarrow \xi_s^O$ restriction in the individual event $n$ can be formally written as:

$$\xi_s^O(n) = \xi_{1,2}^{O} \text{ or } \xi_s^{O}$$

(14)

which means that in each event $\xi_1^O$ or $\xi_2^O$ appears at random. To reproduce the correct expectation values $\bar{q}_i^O$, the corresponding $\xi_i^O$ probabilities are $P_i^O = |a_i|^2$. Then for any $\xi^{MS}$ there is always the probabilistic ensemble $W^O = W^O_{mix}$ of (13) which describes consistently the properties of MS restricted state. One must also show that no alternative nonprobabilistic solution $\xi^O_f$ for MS restriction exists. $\xi^O_f$ should have correct the expectation values $\bar{q}_i^O$, the only nonstochastic state $\xi^O_f \in \Omega_O$ which obeys this conditions is $\xi^O_f = \sum |a_i|^2 \xi^O_i$, the analog of Breuer state $R_O$. Such $\xi^O_f$ should be a nonlocalized state on $Q_O$ for which $\bar{q}_1^O \leq \bar{q}_O \leq \bar{q}_2^O$, i.e. $\bar{q}_O$ is uncertain. As was explained above, in that case some observable $B' \in U_O$ should exist which eigenvalue or an expectation value $\bar{q}_O$ for $\xi^O_f$ would reveal its difference from $\xi^O_i$ states and demonstrate $\bar{q}_O$ genuine uncertainty, i.e the simultaneous presence in several $Q_O$ points. Yet $U_O$ doesn’t include any other observables beside $Q_O$, thereon $\xi^O_f$ isn’t feasible as $O$ individual state and $\xi^O_s$ is the only suitable solution.

The same arguments are applicable for an arbitrary $a_i$, the only difference is that another IT $B^a \in \{B^{MS}\}$ is involved in the derivation; eventually the same unique solution $\xi_s^O$ of (13) exists, and $W^O$ with the given $|a_i|^2$ describes its ensemble. In general he nonexistence of any $O$ individual states $\xi^O_f \neq \xi^O_i$ directly follows from $U_O$ elementarity, because as was shown, all individual states should be the eigenstates of some observables and in this case it can be only $Q_O$. Consequently in Algebraic QM $\xi^{MS} \rightarrow \xi^O$ inference map is stochastic and results in the subjective state collapse observed by $O$. MS AIS can be expressed as the doublet $\xi^{MS}, \xi^{O}$, which corresponds to DSF dynamical and information components. Correspondingly MS evolution operator $\hat{Z}(t)$ is also the multiplet (doublet) which includes the unitary component for $\xi^{MS}$ and nonunitary stochastic one for $\xi^O$. In the regarded situation $O'$ doesn’t interact with MS, and so analogously to DSF formalism $O'$ information doesn’t change during S measurement. Because of it MS state collapse isn’t observed by $O'$ in agreement with the conditions i, ii of chap.2. Note that the same results can be obtained if in place of $U_O$ we shall exploit MS restriction to $U_R$ - the subalgebra of all $O$ observables.

For illustration let’s regard the opposite hypothesis: namely that $O$ percepts pure MS state $\xi^{MS}_s$ as $Q_O$ ‘pointer superposition’ and it differs from $O$ perception of regarded MS mixture; the analogous situations were widely discussed (Von Neumann,1934; Wigner,1961). If to assume that $O$ perception doesn’t violate QM laws, in particular, that all $O$ observations are related to some $O$ observables, then it should be some $O$ observable $G_O$ - ‘the number of $Q_O$ peaks’ for which $G_O = 1,2$ for MS mixture and pure state correspondingly. But such $O$ observable doesn’t exist not only for $U_O$, but even for $U_R$ subalgebra.

Note that MS symmetric individual states $\xi^{MS}_{s_i}$ possess the reflection symmetry $Q_O \rightarrow -Q_O$, but no $O$ states $\xi_i^O$ have that property. In $C^*$algebras formalism of Quantum Field Theory such
symmetry reduction results in the phenomena of Spontaneous Symmetry Breaking; in particular, it leads to the randomness of outcomes for some models of measurements in the collective systems (Guilini, 1996; Mayburov, 1998). The self-description approach permits to extend such randomness mechanism on the finite systems measurements. Its most outstanding feature is the simultaneous coexistence of two individual states $\xi_{MS}^S, \xi_i^O$ which seems contradictive. From the mathematical point of view the Algebraic QM contains the generic structure - the restricted AIS set $\Omega_R^I$ defined on the elementary subalgebra $U_R^I \in U$ (in our model $U_R^I = U_O$). $\Omega_R^I$ extremal points are treated as the individual states. Hence in this theory the quantum state reduction results from the reduction of a system algebra to its associative subalgebra. Eventually MS, O states are defined on different algebras which corresponds to observations in $O, O'$ RFs, and so they are unitarily nonequivalent.

If to analyze obtained results in the Information-Theoretical framework, remind that the difference between the pure and mixed MS states reflected by the expectation values $B$ of (8) and other IT observables. Therefore O possible observation of S pure/mixed states difference would mean that $O$ can acquire the information on $B$ expectation value. But $B \notin U_O$ because of it $S, O$ IT correlations are unobservable from inside by $O$ (Mittelstaedt, 1998). Therefore the information which describes the difference of the pure and mixed S states principally can’t be transferred in such MS scheme from $S$ to IGUS $O$ for the consequent processing. It corresponds with Wigner conclusion that the perception by $O$ of the superposition of two contradictive impressions is nonsense and should be excluded in the consistent theory (Wigner, 1961). Our calculations can be regarded as the kind of ‘no-go’ theorem which prohibit such superpositions observations. In this context the obtained $\xi_i^O$ state describes the upper limit of $S$ information available to $O$. Note that the formal impossibility of pure/mixed states discrimination for the restricted set of observables was considered earlier (Busch, 1996), but its applicability to the feasible measuring schemes wasn’t proved.

In practice it’s possible that $O$ effective subalgebra is larger than $U_O$, but this case demands more complicated calculations which we plan to present in the forthcoming paper. In Algebraic QM the only important condition for the classicality appearance is $U_O$ associativity but it feasible, in principle, also for the complex IGUS structures. As was shown the formalism of Algebraic QM extracts PB $\xi_i^O \sim \{|O_i\} \in H_O$ even without the account of E decoherent interaction, but only from the uniqueness $O$ subalgebra $U_O$ for a given $S, O$ interaction. The $O$-$E$ decoherence, in fact, only duplicates this effect resulting in the optimal case in the same PB solution. If to consider MS+$E$ system and its $U_{MS,E}$ algebra, the effective $O$ information subalgebra will be the same $U_O$ considered above. Therefore $O$ subalgebra and its states set properties can’t depend directly on the surrounding $E$ properties, and the final $O$ states structure is analogous to the obtained above.

As was shown in chap. 2 standard QM doesn’t contain the unambiguous restriction ansatz for individual states, therefore some additional assumptions are necessary to introduce it, if one prefers to work inside its framework. From our analysis it’s sensible to assume that for $MS \to O$ restriction to any $|\Psi_{MS}\rangle$ corresponds some $O$ individual state in any event. Undoubtedly any pure state $\psi_j^O \in H_O$ is $O$ individual state with some real property. Meanwhile from the arguments given in the beginning of the chapter, Breuer state $R_O$ of (7) is the convex state and so can’t be $O$ individual state. But this is the only nonprobabilistic solution which can dispatch the correct $Q_O, Q_i^O$ expectation values. Hence they can be restored only by the random mixture of several $\psi_j^O$ states, and the stochastic ensemble of $|O_i\rangle$ states with the probability $P_i = |a_i|^2$ is the only solution. Note that it reproduces the results of our DSF ansatz which earlier was obtained phenomenologically. Eventually under the simple minimal assumptions the effect of state collapse can be obtained in the standard QM framework, yet it seems that Algebraic QM
on whole proposes more consistent approach to the problem.

For the regarded simple MS model Algebraic QM formalism in many aspects is analogous to Orthomodular Algebra of propositions or Quantum Logics (Jauch, 1968; Emch, 1972). Its possible application to the systems self-description and the measurement from inside deserves detailed investigation, here only few notices are given. If one assumes that in this framework the measurement restrictions also are defined by the set of O local observables - i.e propositions available for O observations, then the results can be comprehended easily for the case when only single observable QO is available. In this case the corresponding lattice L^O of propositions includes only QO projectors set \{P_i^O\}, (together with \oplus, I) and hence is atomic \sigma-continuous Boolean lattice. Then it follows that the set of restricted O individual states \eta^O which corresponds to this lattice is isomorphic to classical O states set \{ξ^O\} (Jauch, 1968). For any other state \eta^Q_R no proposition from L^O can be put in correspondence, and therefore such state isn’t feasible. In general the search of alternative and more simple formalism of self-description deserves the thorough investigation. Really the only principal feature of Algebraic QM, which was used in our approach, is the strict correspondence of the restricted states and the set of local observables, but it doesn’t demand that such set should constitute Segal algebra.

Despite of the acknowledged achievements of Algebraic QM its foundations are still discussed and aren’t settled finally. In particular, it’s still unclear whether all the algebraic states \varphi correspond to the physical states, this problem discussed thoroughly in (Mayburov, 2005). This question is important by itself and can be essential for our formalism feasibility (Primas, 1983). We admitted also that for MS arbitrary ξ^MS some O restricted AIS responds in any event. It agrees with the consideration of the restricted states as the real physical states, however, this assumption needs further clarification.

5 Discussion

in this report the information-theoretical restrictions on the quantum measurements were studied in the simple selfdescription model of IGUS O. Self-description theory shows that by itself O inclusion as the quantum object into the measurement scheme doesn’t result in the state collapse appearance (Breuer,1996). At the phenomenological level the appearance of the state collapse described by DSF which exploits the doublet states Φ ansatz, where one of its components \phi^I corresponds to O subjective information. Algebraic QM presents the additional arguments in favor of this collapse mechanism; its formalism reflects the strict correspondence between the states and observables - the principal distinction of QM from Classical Waves theory. In our approach O subjective state after the measurements is defined on the set of O internal observables U^O. Yet the observable B, which value characterizes the pure/mixed states difference, or more precisely the class of IT observables doesn’t belongs to U^O, hence O internal state can’t react on this difference. In this paper Algebraic QM was applied for the simple measurement model, but if this formalism universality will be proved, it would mean that the proposed measurement theory follows from the established Quantum Physics realm (Emch,1972). From the formal point of view the only novel feature of our approach is the use of Segal algebra for the individual O states R^O(n) calculations. Note also that no compelling mathematical arguments proves that for the individual states R^O ansatz in standard QM framework should be chosen the same QM formulae (7) which used for the statistical restricted states R^st (Lahti,1990).

Our theory demonstrates that the probabilistic realization is generic and unavoidable for QM and without it QM supposedly can’t acquire any operational meaning. Wave-particle dualism was always regarded as characteristic QM feature but in our theory it has straightforward
correspondence. Algebraic QM approach stresses also the dual character of quantum measurement: this is the interaction of studied S with IGUS O and in the same time the information acquisition and recognition by IGUS.

On the whole the decoherence of macroscopic objects is a very important effect, under the realistic conditions the rate of E atoms interactions with macroscopic detector D is very high, and because of it in a very short time \( t_d \) S,D partial state \( \rho_{SD} = T_{TEPSDE} \) becomes approximately equal to the mixed one, as \( \rho_p \) nondiagonal elements become very small. This fact induced the claim that the objective state collapse can be completely explained by detector state decoherence without the observer’s inclusion, but it was proved to be incorrect (D’Espagnat,1990). The further development of decoherence approach which accounts the observer was proposed in Zurek ‘Existential interpretation (Zurek,1998). IGUS O regarded as the quantum object and included in the measurement chain; the memorization of input S signal occurs in several binary memory cells \( |m_{1,2}^0 \rangle \) (chain) which are the analog of the brain neurons. O memory state suffers the decoherence from surrounding E ’atoms’ which results in the system state analogous to (10). Under practical conditions, the decoherence time \( t_d \) is also small and for \( t \gg t_d \) S,O partial state \( \rho_p \) differs from the mixture very little. From that Zurek concludes that O percepts input pure S signal as the random measurement outcomes. However the system S,O,E is still in the pure state even at \( t \gg t_d \), and there is IT observable \( B \) analogous to (8) which proves it. Therefore in standard QM framework it’s incorrect to claim that IGUS perceives random events. The regarded IGUS model doesn’t differs principally from our MS scheme, hence in Algebraic formalism IGUS subjective perception is described by O restricted state \( \xi^O \) defined on \( \mathcal{U}_O \) which describes the random outcomes for the input pure S state. Consequently Algebraic QM application to Zurek IGUS model leads to the results which are equivalent to Existential Interpretation of QM.

From the formal point of view all the experiments in Physics at the final stage include the human subjective perception which simulated by O state in our model. The possible importance of observer in the quantum measurement process was discussed first by London and Bauer (London,1939). They supposed that Observer Consciousness (OC) due to ‘introspection action’ violates in fact Schrödinger equation and results in the state reduction. In our theory the process of O perception doesn’t violate Schrodinger evolution relative to external RF \( O' \). In principle, Self-description Theory permits to regard the relation between MS, IGUS O states and O subjective information (impression), and one can try to extend it on the human perception. This is the separate, important problem which is beyond our scope and here we consider briefly only some its principal points. Basing on obtained results, our approach to it formulated here as the semiquantitative Impression Model (IM). Following Algebraic QM approach, IM assumes that O perception affected only by O internal states defined on O observables subalgebra \( \mathcal{U}_O \). For O perception the calibration assumption introduced: for any Q eigenstate \( |s_i \rangle \) after S measurement finished at \( t > t_1 \) and O 'internal pointer’state is \( |O_i \rangle \) observer O have the definite impression \( J^O \) corresponding to \( J^O_i = q^O_i \) eigenvalue - the information pattern percepted by O. It settles the hypothetical correspondence between MS quantum dynamics model and the human perception. Impression \( J^O \) is O subjective information which isn’t dynamical parameter and its introduction can’t have any influence on the theory dynamics. Furthermore, it’s sensible to assume that if S state is the superposition \( \psi_s \) then its measurement by O also results in appearance for each individual event n of some definite and unambiguous O impression - the information pattern \( J^O = \{ q^O_{\text{sup}}(n) \} \) which is expressed as some finite sequence of real numbers. In Algebraic formalism the corresponding O subjective information - impression in the individual event is equal to \( J^O(n) = q^O_i \) and can be consistently defined in this ansatz for our IM as the stochastic state appearing with probability \( |a_i|^2 \). In Algebraic QM framework they corresponds to the restricted O AIS \( \xi^O_i \) defined on \( \mathcal{U}_O \). Eventually in this approach IM doesn’t need to exploit Von
Neuman psychophysical parallelism hypothesis (Von Neumann, 1933).

To conclude, the quantum measurements were studied within the Information-Theoretical framework and the self-description restrictions on the information acquisition are shown to be important in Quantum Measurement Theory. Algebraic QM represents the appropriate formalism of systems self-description, in particular, IGUS $O$ observable algebra $\mathcal{U}_O$ defines $O$ restricted states $\xi_O^O$ set $\Omega_O$. The appearance of stochastic events stipulated by MS individual states restriction to $\xi_O^O$ states and results in the state collapse observation by $O$. The regarded IGUS model is quite simple and on the whole doesn’t permit us to make any final conclusions at this stage. Yet the obtained results evidences that the IGUS information restrictions and its interactions with the observed system should be accounted in Quantum Measurement Problem analysis (Zurek, 1998).

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