Semileptonic and radiative $B$ decays circa 2005

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I briefly review the theoretical status of semileptonic and radiative $B$ decays in 2005.

1. Introduction

Semileptonic and radiative beauty decays play an important role in the determination of the CKM matrix elements and in the indirect search for new physics: the former, tree-level dominated, allow for a precise measurement of the CKM elements $V_{cb}$ and $V_{ub}$; the latter, loop-induced, are directly sensitive to new physics contributions, and give also information on $V_{td}$ and $V_{ts}$. These decay modes, all characterized by an electroweak current that probes the $B$ dynamics, have a lot in common and form a remarkable set of interdependent measurements. Their simplicity, however, is only apparent: if one is interested in precision measurements they display all the rich complexity specific of QCD dynamics. The main theoretical divide runs between inclusive and exclusive decays: inclusive decays can be studied using an Operator Product Expansion (OPE), parameterizing the non-perturbative physics in terms of $B$-meson matrix elements of power-suppressed local operators, while exclusive decays require an estimate of the form factors using non-perturbative methods (lattice QCD, sum rules). Having two completely different methods at our disposal represents a huge advantage and makes non-trivial cross-checks possible. See [2] for a review.

2. Exclusive determination of $V_{cb}$

The exclusive determination of $|V_{cb}|$ employs the extrapolation of the $B \rightarrow D^* l\nu$ rate to the kinematic endpoint where the $D^*$ is produced at rest (zero-recoil). In this limit, the form factor $F(1)$ is known, up to corrections suppressed by at least two powers of $\Lambda_{QCD}/m_{c,b}$ that have to be computed, e.g. on the lattice. Since one needs to estimate only the $O(10\%)$ correction to the heavy quark limit, a good accuracy can be reached even with present non-perturbative methods. In fact, current lattice QCD and sum rule results are both consistent with $F(1) = 0.91 \pm 0.04$ [2]. The overall uncertainty is therefore close to 5%: $|V_{cb}^{excl}| = 41.2(1.0)_{ex}(1.8)_{th} \times 10^{-3}$, but the two most precise experimental results, by Babar and Cleo, differ by almost $3\sigma$ [1]. Semileptonic decays to $D$ mesons give consistent but less precise results. Progress is expected especially from unquenched lattice calculations.

3. Inclusive determination of $V_{cb}$

While the non-perturbative unknowns in the exclusive determination of $|V_{cb}|$ have to be calculated, those entering the inclusive semileptonic decay, $B \rightarrow X_c l\nu$, can be measured in a self-consistent way. Indeed, the inclusive decay rate depends only on the hadronic structure of the decaying $B$ meson, but the sensitivity to it is actually suppressed by two powers of $\Lambda_{QCD}/m_b$, as the highly energetic decay products are (generally) unable to probe the long wavelengths characteristic of the $B$ meson. Formally, an OPE allows us to write the differential $B \rightarrow X_c l\nu$ rate as a double series in $\alpha_s$ and $\Lambda_{QCD}/m_b$, whose leading term is nothing but the parton model result. However, the OPE results for the spectra can be compared to experiment only after smearing over a range of energies $\gg \Lambda_{QCD}$ and away from the endpoints. The hadronic mass spectrum, for instance, is dominated by resonance peaks that

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have no counterpart in the OPE: the OPE results have no local meaning.

The observables that can be studied in the OPE include the total rate and moments (weighted integrals) of the lepton energy and hadronic mass spectra, as well as the photon spectrum in radiative decays. They generally are subject to a lower cut on the charged lepton energy and can be written in a way analogous to that of the integrated rate,

$$
\Gamma_{cl} = \frac{G_F^2 m_b^5 \eta_{ew}}{192\pi^3} |V_{cb}|^2 z(r) \left[ 1 + a_1(r) \frac{\mu_\pi^2}{m_\pi^2} + a_2(r) \frac{\mu_\rho^2}{m_\rho^2} + b_1(r) \frac{\rho_D^3}{m_b^3} + b_2(r) \frac{\rho_{D,LS}^3}{m_b^3} + \ldots \right],
$$

where $r = (m_c/m_b)^2$, $z(r)$ is the tree-level expression, $\eta_{ew}$ contains the leading electroweak corrections, the Wilson coefficients $a_i, b_i$ are series in $\alpha_s$, and power corrections up to $1/m_b^3$ have been kept. The parameters entering the predictions are $\alpha_s$, properly defined quark masses $m_{c,b}$, and the $B$ meson matrix elements of the four local operators that appear up to $O(1/m_b^3)$: $\mu_\pi, \mu_\rho, \rho_0$, and $\rho_{D,LS}$. Because they depend on the various parameters in different ways, the moments serve a double purpose: they allow to constrain the non-perturbative parameters and they test the overall consistency of the OPE framework. Effects that cannot be described by the OPE (and therefore violate parton-hadron duality) and higher order power corrections can be severely constrained.

Recent experimental results \cite{3}, analyzed in the light of up-to-date theoretical predictions \cite{4,5,6}, have led to a big step forward, both in completeness and accuracy. There is a remarkable consistency of a variety of leptonic and hadronic moments, leading to an excellent fit. The values of the quark masses are in agreement with lattice and spectral sum rule determinations, and the other non-perturbative parameters are determined for the first time at the 10-20% level, in agreement with theoretical expectations. Radiative moments from Belle, Cleo, and Babar \cite{7} can be included as well without deteriorating the quality of the fit, that yields $|V_{\text{emt}}^{\text{cl}}| = 41.58(0.45)(e)(0.58)_{\text{th}} \times 10^{-3}$ \cite{8}, in agreement with the exclusive result. The estimate of the theory error (missing higher order perturbative and non-perturbative contributions, intrinsic charm etc.) is particularly delicate; different recipes in different schemes \cite{4,5,6} have led to compatible results. Despite recent progress \cite{8}, higher order perturbative corrections to the Wilson coefficients are the main source of uncertainty: a 1% determination of $|V_{cb}|$ is possible but requires new calculations. It should be stressed that the OPE parameters describe universal properties of the $B$ meson and of the quarks. For example, $m_b$ and $m_b - m_c$ are determined in the fit within less than 40 and 30 MeV, resp. The reach of the new results therefore extends well beyond the $|V_{cb}|$ determination, as is well demonstrated by the case of $|V_{ub}|$.

4. Exclusive determination of $V_{ub}$

The ratio $|V_{ub}/V_{cb}|$ measures the left side of the unitarity triangle, identifying a circle in the $(\bar{c} \bar{b}, \bar{u})$ plane. The determination of $|V_{ub}|$ from $b \to u$ semileptonic decays parallels that of $|V_{cb}|$, but the exclusive determination ($B \to \pi l\nu, B \to \rho l\nu$, etc.) is penalized by the absence of a heavy quark normalization for the form factors at a certain kinematic point. Moreover, if theoretical precision is lower, so is statistics, by about two orders of magnitude. In view of the precision reached by $|V_{cb}|$, a drastic improvement in the determination of $|V_{ub}|$, made possible by the high statistics available, has become the top priority. The relevance of $|V_{ub}|$ is is illustrated for instance in Fig. 1, where the Unitarity Triangle is determined for the first time using tree-level processes only. Comparing a high precision determination of this kind, insensitive to new physics, to the standard one based on loop processes would be very instructive \cite{9}.

In the exclusive case, lattice QCD and light cone sum rules complement each other, but as the first unquenched calculations appear, the error in the high-$q^2$ region accessible to lattice still exceeds 10%, while the $q^2$ extrapolation is well under control, thanks to analyticity and new experimental data \cite{10}. Sum rules prefer a lower value than lattice, $|V_{ub}| = (3.2 \pm 0.1 \pm 0.3) \times 10^{-3}$ (first of \cite{10}) against $|V_{ub}| = (4.1 \pm 0.6) \times 10^{-3}$.
Figure 1. Determination of the Unitarity Triangle using only tree level processes, $|V_{ub}|$ and $\gamma$ from $B \to DK$. 

The goal of lattice simulation is a 5-6% determination within a few years \[12\]. Recent proposals include a new $q^2 = 0$ form factor normalization based on SCET \[13\], and the combination of $B \to K^{*}l^{+}l^{-}$ and $B \to \rho l\nu$ data \[14\]. The latter is not yet competitive and the $|V_{ub}|$ result could depend on new physics in the rare decay.

5. Inclusive determination of $V_{ub}$

The inclusive determination of $V_{ub}$ is strongly affected by the kinematic cuts necessary to isolate $b \to u$ transitions from the dominant $b \to c$ background. In general, cuts placed near the perturbative singularities (typically, the lepton energy endpoint) destroy the convergence of the OPE and introduce a sensitivity to local $b$-quark wave function properties like the Fermi motion of the heavy quark in the $B$ meson. These non-perturbative effects are not suppressed by powers of $1/m_b$: at leading order in this expansion they are described by a single distribution function, often called shape function, whose first moments are given by expectation values of the same local operators we have encountered earlier.

The shape function is universal: in principle it can be extracted from the photon spectrum in $B \to X_s\gamma$ or studying the differential distributions in $B \to X_u l\nu$, although the two processes are different at subleading order in $1/m_b$ and $\alpha_s$. The shape function gets renormalized by perturbative effects: disentangling the latter from non-perturbative contributions is not trivial and has been done in different ways \[15,16\].

Different cutting strategies have been proposed: cuts on the hadronic invariant mass $M_X < M_D$, on the electron energy, on the $q^2$ of the lepton pair and combinations thereof, on the light-cone variable $P_+ = E_X - |\vec{P}_X|$; each has peculiar experimental and theoretical systematics \[17\], though the uncertainty on leading and subleading shape functions plays often a central role. Eventually, the variety of complementary approaches that have been developed will be extremely useful. Recent theoretical work has focused on the optimization of the cuts, subleading non-perturbative effects \[18\], the resummation of Sudakov logs, the role of the radiative decay spectrum in constraining the shape function, etc. \[19\].

The latest HFAG average of inclusive determinations, $|V_{ub}| = 4.38(33) \times 10^{-3}$ \[1\] agrees with the exclusive determination based on lattice. The error is close to 8% and dominated by theoretical systematics. The progress since last year is significant and due to: i) the implementation of the constraints on $m_b$ and on the moments of the shape function derived from $b \to c l\nu$ spectral moments; ii) larger statistics and better knowledge of the charm background, that has allowed to cut in a milder way, to the relief of theorists.

Further improvement can be expected from high statistics data. A more precise determination of the radiative spectrum and better experimental constraints on the Weak Annihilation (WA) contributions \[21\] are needed. Particularly promising are new analyses based on fully reconstructed events that allow high discrimination of charmed final states. They allow the measurement of $b \to u$ decay distributions well beyond the kinematic cuts on $b \to c$ \[22\]. Of course for milder cuts the experimental error tends to increase, while the sensitivity to the shape func-
tion decreases: the balance between theoretical and experimental errors can be optimized. The $b \to u$ differential distributions and their truncated moments will help constraining the shape function(s), Weak Annihilation (WA) contributions, and the heavy quark parameters [23].

6. Radiative decays

While the V-A current involved in the semileptonic $B$ decays is conserved, that is not the case of the tensor current that induces radiative decays. As a consequence, these decays depend logarithmically on the electroweak scale at which the current is generated by $W$ and top quark loops: in addition to the $b$ mass and the scale of QCD, a third scale $\sim M_W \gg m_b \gg \Lambda_{QCD}$ must be taken into account. The large logs $L = \log M_W/m_b$ are resummed in the context of an effective theory, at leading order in $1/M_W$: the resummation of $O(\alpha_s^n L^n)$ term corresponds to the leading order (LO) expression, of $O(\alpha_s^n L^{n-1})$ terms to the NLO, etc. Apart from this complication, and from additional ones due to the presence of charm loops [24], the OPE for inclusive radiative decays is analogous to the one for semileptonic ones. The NLO calculation was practically completed in 1996 [25] and involves $O(\alpha_s)$ matrix elements. Electroweak effects and power corrections are also known [26,24]. The main theoretical uncertainty in the NLO analysis of the Branching Ratio (BR) is related to the mass of the charm quarks circulating in the $O(\alpha_s)$ loops of the matrix element $\langle Xs\gamma|(s_Lc_L)(c_Lb_L)|b \rangle$ [27]. This matrix element vanishes at LO, and the charm dependence is an $O(\alpha_s)$ effect. The natural scale at which $m_c$ should be normalized is therefore undetermined without and $O(\alpha_s)$ calculation. The matter is quite relevant: the numerical difference between $m_c(m_c) \approx 1.25$ GeV and $m_c(m_b) \approx 0.85$ GeV is large enough to shift the BR by more than 10%. Using $m_c(m_b/2)$ as central value, the BR of $B \to Xs\gamma$ for $E_\gamma > 1.6$ GeV is $(3.60 \pm 0.30) \times 10^{-4}$, with an error close to 8% [27], dominated by the charm scale uncertainty. In practice, a lower cut on the photon energy ($E_\gamma < 1.8$-1.9 GeV) is always applied to avoid background. A detailed knowledge of the tail of the spectrum is therefore required for the extrapolation to a region where the OPE can be trusted. The same problem also affects the moments of the photon spectrum that are employed in the HQE fit [10]. Both perturbative hard gluon emission and the tail of the shape function concur to form the tail of the photon spectrum. As already mentioned, disentangling them is not straightforward. The transition from the shape function dominated region to the local OPE has been studied by Neubert, who found that it can be described by a multi-scale non-local OPE [28]. He noticed that the cut $E_\gamma < E_{cut}$ effectively introduces two new scales that may be relevant besides $m_b$: $\Delta = m_b - 2E_{cut}$ and $\sqrt{m_b \Delta}$. According to this picture, the perturbative tail of the spectrum receives contributions at the scale $\Delta$, and may be subject to larger higher order perturbative corrections, even for $E_{cut} \leq 1.8$ GeV, simply because $\Delta$ is then close to 1 GeV. This view has been criticized [29]. Certainly, the results of a new calculation of the dominant $O(\alpha_s^2)$ effects in the photon spectrum [30] show only very small deviations from the LO plus BLM perturbative spectrum considered in [10], excluding large corrections in the standard picture. Moreover, the partial BR calculated in the multiscale OPE, $\text{BR}(E_\gamma > 1.6$ GeV$)= (3.47^{+0.33+0.32}_{-0.40-0.29}) \times 10^{-4}$ [28] has a central value very close to that of [27]. The values of $m_b$ and $\mu_b^2$ extracted from the first and second photonic moments in [9] following [16] are also very close to those extracted from the same data using the multiscale OPE [31]. One would conclude that the local OPE is likely to provide a good description of the spectrum in the intermediate region $E_\gamma \sim 1.6-1.8$ GeV of phenomenological interest, and that there is no need to enlarge substantially the theoretical error on the partial BR given in [27].

In order to compare different experiments, the cut rates are usually extrapolated to a conventionally defined total rate [32]. The NLO result $(3.73 \pm 0.30) \times 10^{-4}$ [27] can then be compared with the experimental world average: $3.39^{+0.30}_{-0.27} \times 10^{-4}$ [11]. No deviation from the SM is observed, but in view of the final accuracy expected at the $B$ factories the theoretical prediction has to be improved. That is the aim of the NNLO calculation currently under way [30,33]. To date, the
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Table 1
Summary of the main theoretical limitations.

| process       | quantity       | TH error | needs                     | goal |
|---------------|----------------|----------|---------------------------|------|
| $B \to D^* l\nu$ | $|V_{cb}|$       | $\sim 4\%$ | unquenching, analytic work | 1\% |
| $B \to X_c l\nu$ | $|V_{cb}|$       | $\sim 1.5\%$ | new pert calculations     | <1\% |
| $B \to \pi(\rho) l\nu$ | $|V_{ub}|$       | 10-15\% | 2-loop lattice matching etc. | 6\% |
| $B \to X_s l\nu$ | $|V_{ub}|$       | $\sim 6 - 7\%$ | more data/synergy with th | < 5\% |
| $B \to X_s \gamma$ | BR             | $\sim 10\%$ | NNLO                     | <5\% |
| $B \to \rho^0 \gamma / B \to K^* \gamma$ | $|V_{td}/V_{ts}|$ | 10-20\% | lattice SU(3) breaking etc | ?    |

The missing pieces of this challenging enterprise include the four loop anomalous dimension matrix and the finite parts of the three loop matrix elements with charm loop.

The high precision of inclusive radiative $b \to s$ decays is not yet matched by the theoretical understanding of exclusive radiative $B$ decays [2], despite some recent progress [34]. Because of the difficulty of measuring inclusively $b \to d\gamma$, the ratio of $b \to d\gamma$ over $b \to s\gamma$ exclusive modes is extremely interesting. $B \to (\rho, \omega) \gamma$ has just been measured for the first time [35]. The ratio $B \to \rho^0 \gamma / B \to K^* \gamma$, in particular, is hardly affected by WA contributions, while the SU(3) breaking effects can be estimated on the lattice and using light cone sum rules; the error is 10-20% at most [36]. In this way, we can extract $|V_{td}/V_{ts}|$ before $\Delta M_s / \Delta M_d$ is measured. While the 2004 preliminary result showed an interesting deviation from the global UT fit, the recent BELLE update has restored consistency with the SM and damped enthusiasm [35,20].

Finally, it should also be mentioned that the rare leptonic transitions $b \to s l^+ l^-$ complement radiative decays in constraining new physics [37]. The inclusive decay $B \to X_s l^+ l^-$, in particular, has reached experimental and theoretical maturity with theoretical errors comparable to $B \to X_s \gamma$ [38].

7. Conclusions

The joint theoretical and experimental effort to study semileptonic and radiative decays to high precision has led to relevant progress in the determination of the CKM matrix elements and in testing the Standard Model. Table 1 summarizes the present theoretical uncertainty and outlines the main ingredients necessary to improve on it for the various processes. While no deviation from the Standard Model has been so far uncovered, more theoretical work is needed to make the most of the wealth of data coming from the $B$ factories.

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