Heavy quarks in non-relativistic lattice QCD

JOACHIM HEIN

Department of Physics & Astronomy, University of Glasgow,
Glasgow G12 8QQ, Scotland, UK

I give an overview of phenomenological heavy quark results obtained in NRQCD on the lattice. In particular I discuss the bottomonium and the $b$-light hadron spectrum. I also review recent results on the decay constants $f_B$ and $f_{B_s}$.

1 Introduction

1.1 Heavy quarks on the lattice

Lattice QCD provides an approach to calculate the properties of hadronic bound states of strongly interacting matter from first principles. However when interested in hadronic states involving heavy $c$ and $b$ quarks standard lattice methods would lead to large discretisation errors. This is caused by the Compton wave length of the heavy quark being non-negligible against the lattice spacing. In present simulations the latter is on the order of a few inverse GeV.

However the Compton wave length of the heavy quarks is an irrelevant scale for the dynamics of heavy hadrons, see e.g. the lecture note. One possibility to simulate heavy quarks is the expansion of the heavy quark action around its non-relativistic limit, which is known as non-relativistic QCD (NRQCD). Another cure is the heavy clover approach which is applied by several groups presently. Within this talk we will review some of the phenomenological results obtained in NRQCD.

1.2 Non-relativistic QCD on the lattice

In NRQCD the Hamiltonian of the heavy quark is expanded around its non-relativistic limit

$$H = H_0 + \delta H, \quad H_0 = - \frac{D^2}{2M_Q}. \quad (1)$$

With $H_0$ we denote the leading kinetic term and $\delta H$ stands for the relativistic and spin dependent corrections. In case of a quarkonium system up to order $O(M_Q v_Q^4)$ these corrections read

$$\delta H = -c_1 \frac{(D^2)^2}{8M_Q^3} + c_2 \frac{ig}{8M_Q^3}(\vec{D} \cdot \vec{E} - \vec{E} \cdot \vec{D})$$

1
The coefficients $c_i$ have to be determined, such that $H$ in eqn. (1) matches the Hamiltonian of QCD. So far this has been investigated in lattice perturbation theory. Since NRQCD is non-renormalisable, the $c_i$ will diverge, if the lattice spacing $a$ is sent to zero. Therefore the lattice spacing has to be kept finite and improvement terms have to be added to the action, such that residual discretisation effects become negligible against other sources of error

$$
\delta H_{\text{disc}} = c_5 a^2 \sum_i D_i^4 \frac{\text{D}^2}{24M_Q} - c_6 a \frac{\text{D}^2}{16nM_Q^2}.
$$

The $n$ is a stability parameter used in equation (1). Present calculation use tadpole improved tree-level values for all $c_i$’s. The Hamiltonian (1) leads to a differential equation being first order in time. Therefore the heavy quark propagator $G_t$ can be obtained from an evolution equation

$$
G_{t+1} = (1 - a \frac{1}{2} \delta H - a \frac{1}{2} \delta H_{\text{disc}})(1 - a \frac{1}{2} \delta H_0)^n U_4^+ \\
\quad \cdot (1 - a \frac{1}{2} \delta H - a \frac{1}{2} \delta H_{\text{disc}})^n (1 - a \frac{1}{2} \delta H - a \frac{1}{2} \delta H_{\text{disc}}) G_t.
$$

Thich is numerically quite inexpensive when compared to the matrix inversions associated with a Dirac type Hamiltonian.

2 Heavy quarkonia

2.1 Bottomonium splittings

The radial and orbital splittings in the bottomonium ($\Upsilon$) and charmonium ($J/\Psi$) system are $\approx 500$ MeV, which approximately equals the average kinetic energy of the quarks. One obtains $v_Q^2/c^2 = 0.1$ for the $\Upsilon$ and 0.3 for the $J/\Psi$, which justifies the non-relativistic approach. In the case of heavy quarkonia the Hamiltonian has to be expanded in powers of $v_Q$ instead of $1/M_Q^3$.

In figure (1) we show the dependence of spin independent bottomonium splittings on the lattice spacing $a$. This calculation has been performed in the quenched approximation, which neglects the vacuum polarisation by quark-antiquark pairs. Within the given error bars the lattice result is indeed independent of the lattice spacing. This shows, that within the achieved accuracy continuum results can be obtained at finite $a$. In case of the ratio of the $\chi_b(1P) - \Upsilon(1S)$ energy splitting to the $\rho$-meson mass we observe a clear

\text{\footnote{$\chi_b$ denotes the spin average ($\chi_{b0} + 3\chi_{b1} + 5\chi_{b2})/9$}}
Figure 1: Scaling of spin independent splittings in the quenched approximation. The points with error bars represent the lattice result, the horizontal line the experimental values from PDG.

Mismatch to the experimental result. This is expected to be caused by the different running of the strong coupling $\alpha_s$ in the quenched theory and the real world, where vacuum polarisation is present. This expectation is supported by the ratio of the splittings $[\Upsilon(2S) - \Upsilon(1S)]/[\chi_b(1P) - \Upsilon(1S)]$ being closer to the experimental outcome. In this case both quantities probe similar momentum scales.

For spin dependent quantities the equation (4) is not improved to the same level with respect to higher order relativistic and discretisation corrections as for spin independent ones. Therefore at this level one observes some scaling violation in the bottomonium fine and hyperfine structure. Also the agreement of the $\chi_b$ fine structure with experiment is not as good as for the radial and orbital splittings. Improvement needs the inclusion of the higher order terms and a better determination of the $c_i$’s in the Hamiltonian.

2.2 Partly unquenching

The effect of quenching in quarkonium spectroscopy has been investigated by two groups. Due to algorithmic reasons, both studies use two flavours of dynamical quarks. In figure 2 we compare the outcome for the spin independent spectrum to the quenched approximation. Within error bars no significant sea quark effect can be shown.

2.3 Heavy hybrid mesons

Hybrid mesons denote colour neutral states consisting of gluonic excitations as well as a quark-antiquark pair, see e.g. the review by F. Close.
special interest are so called exotic states with quantum numbers unavailable to $q\bar{q}$-mesons, which are hoped to be the first identified experimentally.

So far exploratory studies have been undertaken to investigate the $b\bar{b}g$ spectrum in NRQCD. These use $H_0$ in eqn. (4) only. In this approximation one obtains two sets of degenerate states, which we denote by their cubic group representation $T_1^{-+}$ and $T_1^{++}$. States below the $BB^{**}$-threshold are expected to be stable. Using the $B_J^*(5732)$ for this threshold one obtains 11.01 GeV. With the $a$-value from figure 2 one gets for the masses

$$M(T_1^{++}) = 11.09(10) \text{ GeV}, \quad M(T_1^{-+}) = 11.15(5) \text{ GeV}. \quad (5)$$

The result $M(T_1^{++}) = 10.82(25) \text{ GeV}$ is in agreement with the above. With this size of an error the question of stability of these states can not be answered.

3 Heavy light systems

3.1 Heavy light spectrum

The physics of heavy light mesons is quite different from the physics of heavy quarkonia. For the former in the limit $M_Q \to \infty$ the heavy quark $Q$ decouples
Figure 3: Scaling of $B$-meson splittings. Please note the different nature of these quantities. From the left we give an example for a spin dependent, a flavour dependent and a radial splitting. Experimental results apart from the $B(2S)$ are from the PDG. The $B(2S)$ is a recent result from DELPHI.

Figure 4: Spectrum of mesons and baryons containing one heavy $b$-quark in the quenched approximation. Dashed lines indicate experimental results from the PDG and dotted lines recent results from DELPHI.

from the dynamics and becomes a static colour source. The properties of such a state would be determined entirely by the light quarks and the glue. In NRQCD this leads to different power counting rules. The corrections in eqn. (2) have to be ordered in powers of $(\Lambda_{QCD}/M_Q)$.

Figure 3 shows results obtained at two different values of the lattice spacing in the quenched theory. No sign of large lattice spacing dependence can be observed for any of the quantities. For the spin independent results one observes reasonable agreement with the experimental result, however the results for the $B_s$-hyperfine is in clear disagreement. Here the discussion of section 2.1 also applies. The quenched approximation seems to have an effect as well, since the heavy clover approach with different systematic errors, delivers a similar reduction of the hyperfine.

Figure 4 gives an overview over the hadron spectrum containing one heavy $b$-quark. Apart from the above discussed hyperfine splittings one observes good agreement with experimental results, including heavy baryons.
Because of the small leptonic branching fraction, the pseudoscalar decay constant $f_B$ is hard to measure experimentally, however its knowledge is required in the determination of e.g. the bag parameter in $B$-$\bar{B}$ mixing.

In order to obtain reliable results the inclusion of operator renormalisation and mixing is crucial. In figure 5 we compile a scaling plot for $f_B$. The plot shows good scaling as the lattice spacing is changed. We quote the result of reference 26, which is in good agreement with the findings of reference 27:

$$f_B = 147(11)(^{+8}_{-12})(9)(6) \text{ MeV}, \quad f_{B_s} = 175(8)(^{+7}_{-10})(11)(^{+7}_{-0}) \text{ MeV}. \quad (6)$$

These results also agree with those of other lattice calculations using Dirac type Hamiltonians 29.

4 Conclusion and outlook

We discussed phenomenological heavy quark results including spectroscopy and the decay constants $f_B$ and $f_{B_s}$. Future work has to improve on spin dependent splittings and quenching effects have to be further addressed.

Acknowledgements

It is a pleasure to thank Christine Davies, Arifa Ali Khan, Sara Collins and Achim Spitz for suggestions and support while preparing this talk. A Marie Curie research fellowship by the European commission under ERB FMB ICT 961729 is gratefully acknowledged.
References

1. C. Davies, in Computing Particle Properties, editors H. Gausterer, C.B. Lang, (Springer Verlag, Berlin, Heidelberg 1998), hep-ph/9710394.
2. B.A. Thacker, G.P. Lepage, Phys. Rev. D 43, 196 (1991).
3. G.P. Lepage, et al., Phys. Rev. D 46, 4052 (1992).
4. A. El-Khadra, A. Kronfeld, P. Mackenzie, Phys. Rev. D 55, 3933 (1997).
5. C. Morningstar, Phys. Rev. D 50, 5902 (1994).
6. G.P. Lepage, P.B. Mackenzie, Phys. Rev. D 48, 2250 (1993).
7. C. Davies, et al., Phys. Rev. D 58, 054505 (1998), hep-lat/9802024.
8. R. Barnett, et al., Phys. Rev. D 54, 1 (1996).
9. UKQCD collaboration, R.D. Kenway, Nucl. Phys. B (Proc. Suppl.) 53, 206 (1997); H.P. Shanahan et al., Phys. Rev. D 55, 1548 (1997); P. Randall, Ph.D. thesis, University of Edinburgh, 1997.
10. H. Trottier, Phys. Rev. D 55, 6844 (1997).
11. T. Manke, et al., Phys. Lett. B 408, 308 (1997).
12. H. Trottier, G.P. Lepage, Nucl. Phys. B (Proc. Suppl.) 63, 865 (1998).
13. C. Davies, et al., Phys. Rev. D 56, 2755 (1997).
14. SESAM collaboration, N. Eicker, et al., Phys. Rev. D 57, 4080 (1998).
15. C. Davies, et al., unpublished notes.
16. Achim Spitz (SESAM collaboration), doctoral thesis, Bergische Universität Wuppertal, 1998.
17. F.E. Close, Nucl. Phys. B (Proc. Suppl.) 63, 28 (1998).
18. T. Manke, et al., Phys. Rev. D 57, 3829 (1998).
19. S. Collins, Nucl. Phys. B (Proc. Suppl.) 63, 335 (1998).
20. A. Ali Khan, et al., Phys. Rev. D 53, 6433 (1996).
21. A. Ali Khan, et al., in preparation; A. Ali Khan et al, preprint, hep-lat/9809140.
22. J. Hein, et al., in preparation; J. Hein, et al., Nucl. Phys. B (Proc. Suppl.) 63, 347 (1998).
23. UKQCD collaboration, P. Boyle, in preparation; P. Boyle, Nucl. Phys. B (Proc. Suppl.) 63, 314 (1998).
24. DELPHI collaboration, DELPHI Note 95-107, contribution to EPS '95; DELPHI Note 96-93 CONF 22, contribution to ICHEP '96.
25. C. Morningstar, J. Shigemitsu, Phys. Rev. D 57, 6741 (1998).
26. A. Ali Khan, et al., Phys. Lett. B 427, 132 (1998).
27. K-I. Ishikawa, et al., preprint, hep-lat/9809152.
28. J. Hein, et al., preprint, hep-lat/9809051.
29. T. Draper, talk given at Lattice '98 in Boulder, Colorado, July 1998, Nucl. Phys. B (Proc. Suppl.) to appear.