Hybrid Expansion Law with Interacting Cosmic Fluids

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Abstract. A theoretical model for hybrid expansion law in a spatially flat Friedmann-Robertson-Walker (FRW) model filled with two interacting perfect fluid is developed. We obtain the behaviour of the energy densities, pressures, and the energy transfer as functions of the cosmological time. This provides a new family of solutions for hybrid expansion law with interacting cosmic fluids.

1. Introduction
The observed late acceleration of the universe [1] could be explained by cosmological models with exchange of energy between two cosmic fluids. The energy transfer between cosmic fluids was first studied by Davidson [3]. Other examples of the physics producing the energy transfer were given by Sistero [4] and McIntosh [5]. For an exhaustive review of interacting fluids in cosmology see [6, 7, 8, 9, 10].

In this paper we analyze a flat FRW model with an interaction between cosmic fluids through a cosmic scale factor given by a hybrid expansion law [13], which mimics the power law and de Sitter cosmologies as special cases, providing a description of the transition from the early decelerating phase to the recent accelerating phase and generating a time dependent deceleration parameter.

Some time ago in Refs. [11, 12] the special case of hybrid expansion law for two fluids cosmological models with and without energy transfer was show, in this works the interaction term was seen as \( Q(t) = 3H\sigma \rho_m \) and where \( \rho_m \) was considered as the energy density of matter. Notice that [13] study the hybrid expansion law whitout interaction and its observational constraints. On the other hand in [14] was studied a spatially homogeneous and anisotropic Bianchi type-V model without energy transfer.

Follow Ref.[2], the goal is to consider a hybrid expansion law and two barotropic perfect fluids without specify the nature of the fluid contents and find the interaction term \( Q(t) \), the energy densities and pressures for the cosmic fluids in this configuration.

This paper is organized as follows. In Section 2, we review briefly the hybrid expansion law in a flat FRW model. In Section 3, we review briefly the interaction for cosmic fluids. In Section 4 we presents the analysis of the interaction found. The conclusions are presented in Section 5.
2. Hybrid expansion law
We consider the cosmological scale factor as a hybrid expansion law [13]

\[ a(t) = a_0 \left( \frac{t}{t_0} \right)^\alpha e^{\beta \left( \frac{t}{t_0} - 1 \right)} \]  

(1)

where \( \alpha \) and \( \beta \) are constants, here \( a_0 \) and \( t_0 \) denote the scale factor and the age of the Universe today, respectively. Setting \( \beta = 0 \) leads the power law cosmology and set \( \alpha = 0 \) the exponential-law cosmology. These cosmologies are the special cases of the Hybrid expansion law cosmology. For any other values of \( \alpha \) and \( \beta \) will have new directions to explore cosmology in the context of the hybrid expansion law [13, 14].

From (1) we can obtain the Hubble parameter and deceleration parameter, these are respectively

\[ H = \frac{\dot{a}(t)}{a(t)} = \frac{\alpha}{t} + \frac{\beta}{t_0} \]  

(2)

\[ q(t) = -\frac{\ddot{a}(t)}{a(t) \dot{a}(t)} = \frac{\alpha t_0^2}{(\beta t + \alpha t_0)^2} - 1 \]  

(3)

if we choose \( \alpha \) and \( \beta \) such that the power law-term dominates over the exponential term in the early universe [13], that is for \( t \sim 0 \), then the cosmological parameters approximate to the following:

\[ a \sim a_0 \left( \frac{t}{t_0} \right)^\alpha, \quad H \sim \frac{\alpha}{t}, \quad q \sim \frac{1}{\alpha} - 1 \]  

(4)

Similarly, if we choose \( \alpha \) and \( \beta \) such that the exponential term dominates at late times [3] such that in the limit \( t \to \infty \) we have

\[ a \to a_0 e^{\beta \left( \frac{t}{t_0} - 1 \right)}, \quad H \to \frac{\beta}{t_0}, \quad q \to -1 \]  

(5)

Notice when \( \alpha \) and \( \beta \) both are non-zero, we have a variable deceleration parameter and we have a transition from deceleration to acceleration for \( \beta > 0 \) and \( 0 < \alpha < 1 \) [13].

3. Interaction for cosmic fluids
We consider the flat FRW metric given by

\[ ds^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right) \]  

(6)

the Einstein’s field equations for the metric (6) and the universe filled with two fluids \( \rho_1 \) and \( \rho_2 \) are

\[ 3H^2 = \kappa(\rho_1 + \rho_2) \]  

(7)

\[ -2\frac{\ddot{a}(t)}{a(t)} - H^2 = \kappa(P_1 + P_2) \]  

(8)

where \( \kappa = 8\pi G = 1 \).

We assume that the perfect fluids interact through the interaction term \( Q \) according to

\[ \dot{\rho}_1 + 3H(\rho_1 + P_1) = Q \]  

(9)

\[ \dot{\rho}_2 + 3H(\rho_2 + P_2) = -Q \]  

(10)
Note that if \( Q > 0 \) we have there is a transfer of energy from the fluid \( \rho_2 \) to the fluid \( \rho_1 \). If \( Q = 0 \) we have two noninteracting fluids, and then each fluid satisfies the standard balance equation separately.

Now we consider barotropic equations of state given by \( P_1 = \omega_1 \rho_1 \) and \( P_2 = \omega_2 \rho_2 \), where \( \omega_1 \neq \omega_2 \). So, the balance equations (8) and (9) are

\[
\dot{\rho}_1 + 3H \rho_1 (1 + \omega_1) = Q \quad (11)
\]
\[
\dot{\rho}_2 + 3H \rho_2 (1 + \omega_2) = -Q \quad (12)
\]

4. The Interaction \( Q \) term

Taking into account \( Q \neq 0 \), the Friedmann equation (7), the equations (11) and (12) we have the solutions for the energy densities

\[
\rho_1(t) = \frac{3 \left[ \frac{\alpha}{t} + \frac{\beta}{t_0} \right]^2 (1 + \omega_2) - 2 \frac{\alpha}{t} \left[ \frac{\alpha}{t} + \frac{\beta}{t_0} \right]}{\kappa(\omega_2 - \omega_1)} \quad (13)
\]

and

\[
\rho_2(t) = \frac{3 \left[ \frac{\alpha}{t} + \frac{\beta}{t_0} \right]^2 (1 + \omega_1) - 2 \frac{\alpha}{t} \left[ \frac{\alpha}{t} + \frac{\beta}{t_0} \right]}{\kappa(\omega_1 - \omega_2)} \quad (14)
\]

where the interaction \( Q \) term takes the form

\[
Q = \frac{4 \frac{\alpha}{t^3} + 9 \left[ \frac{\alpha}{t} + \frac{\beta}{t_0} \right]^3 (1 + \omega_1)(1 + \omega_2) - 6 \frac{\alpha(\omega_1 + \beta)(2 + \omega_1 + \omega_2)}{t_0 \kappa(\omega_2 - \omega_1)}}{\kappa(\omega_2 - \omega_1)} \quad (15)
\]

Now we considerer the case \( \beta = 0 \), which give the results obtained by [2], that is, for the Hubble parameter and the deceleration parameter, we have

\[
H = \frac{\alpha}{t} \quad (16)
\]

and

\[
q = \frac{1}{\alpha} - 1 \quad (17)
\]

For the energy densities of the fluids \( \rho_1 \) and \( \rho_2 \)

\[
\rho_1(t) = \frac{3 \left[ \frac{\alpha}{t} \right]^2 (1 + \omega_2) - 2 \frac{\alpha}{t^3}}{\kappa(\omega_2 - \omega_1)} \quad (18)
\]

and

\[
\rho_2(t) = \frac{3 \left[ \frac{\alpha}{t} \right]^2 (1 + \omega_1) - 2 \frac{\alpha}{t^3}}{\kappa(\omega_1 - \omega_2)} \quad (19)
\]

finally the interaction \( Q \) term

\[
Q = \frac{4 \frac{\alpha}{t^3} + 9 \left[ \frac{\alpha}{t} \right]^3 (1 + \omega_1)(1 + \omega_2) - 6 \frac{\alpha^2(2 + \omega_1 + \omega_2)}{t^3 \kappa(\omega_2 - \omega_1)}}{\kappa(\omega_2 - \omega_1)} \quad (20)
\]
5. Conclusions
We have shown that through of a hybrid expansion law with two interacting perfect fluid, we found the interaction term $Q$, which gives us the energy transfer between two cosmic fluids and without specify the nature of the fluid contents. We also show that in the special case $\beta = 0$, we recover the results obtained by [2].

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