Influence of a strong magnetic field on a paramagnetic fluid’s convection in systems with different aspect ratios

L Pyrda 1, A Kraszewska 1 and J Donizak 1

1 AGH University of Science and Technology, Faculty of Energy and Fuels, Al. Mickiewicza 30, 30-059 Kraków, Poland

E-mail: kraszka@agh.edu.pl

Abstract. Influence of a strong magnetic field gradients on a natural convection of a paramagnetic fluid in enclosures with aspect ratios (AR=height/width) 0.5 and 2.0 was investigated. Performed experiments allowed to determine the impact of strong magnetic field on heat transfer and the nature of the flow, showing that strong magnetic field enhances Nusselt number and rapidly changes the character of the flow. Conducted Fast Fourier analysis also allowed to determine the aspect ratio influence on investigated systems.

1. Introduction
Heat transfer due to convection phenomenon was investigated in three dimensional enclosure of various shapes and aspect ratios by numerous researchers over decades, starting from XX century. With introduction of superconducting materials, it was possible to construct superconducting magnets, which allowed obtaining strong and constant magnetic fields. By application of strong magnetic field gradients to non-electrically conductive fluids with paramagnetic or diamagnetic properties, a thermo-magnetic phenomenon could be observed. Thermo-magnetic convection could be utilized in diverse industrial applications: heat exchangers, magnetic mixers and micro gravity conditions.

First reported magnetic convection of paramagnetic fluid was reported by Braithwaite et al., where magnetic field gradient was used both to enhance and to suppress buoyancy-driven convection. Huang and Edwards, provided the theory of magnetically controlled convection in a horizontal paramagnetic fluid layers heated from either above or below [1]. Bednarz et al. performed numerical and experimental analysis of laminar thermo-magnetic convection of paramagnetic fluid inside cubical enclosure [2]. Kenjeres et al. investigated transients and turbulence pockets [3], oscillatory states [4] in thermo-magnetic convection of paramagnetic fluids. Kenjeres et al. performed numerical and experimental study of Rayleigh–Bénard–Kelvin Convection [5]. Pyrda has provided an analysis of unsteady thermal convection of paramagnetic fluid in cubical enclosure under strong magnetic field gradient [6] and application of Fast Fourier Transform [7] in thermo-magnetic convection analysis.

Although magnetic convection and combined thermo-magnetic convection was investigated in last two decades, there is still lack of experimental data including effect of an aspect ratio in considered geometries on thermo-magnetic convection. Present paper will address this matter.

2. Experiment
2.1. Experimental setup
Experimental setup consisted of: considered experimental enclosures with different aspect ratios, superconducting magnet generating strong magnetic field gradients, a power supply, a thermo-stating...
bath and a temperature signal acquisition system. Experimental enclosure centre was placed 0.095 [m] above centre of utilized superconducting magnet, where maximum gradient of magnetic field is present and enhance natural convection due to its orientation and chosen fluid type occur. Schematic diagram of superconducting magnet and position of enclosure is shown in Figure 1. To calculate magnetic field distribution and its gradient, which is responsible for magnetic convection of non-electrically conducting fluid, besides real dimensions of system presented on Figure 1, current density in internal and external coil should be taken into account. For maximum value of magnetic field inside centre of magnet: $|b_0|_{\text{max}}=10$ [T], maximum current was equal to $140.9 \cdot 10^{-6}$ [A/m$^2$] for internal coil and $167.3 \cdot 10^{-6}$ [A/m$^2$] for external coil.

Considered experimental enclosures, presented on Figure 2, possessed aspect ratio AR=0.5 (Enclosure A) and AR=2.0 (Enclosure B), which correspond to height $H=0.016$ [m] and $H=0.064$ [m] with same square base with side length equal to $D=0.032$ [m]. Both where build with same materials and manner: four sidewall were made from acrylic glass. In one of them, an additional holes where made to place five (Enclosure A) or six (Enclosure B) thermocouples at 6 [mm] deep from sidewall for insight of temperature variation in time. Holes position in both enclosures sidewall are presented on Figure 3. Bottom wall was made from cooper plate with additional three holes for thermocouples, necessary to calculate heat transfer ratio. Below bottom wall an electrically insulated nichrome wire was placed as a heater supplied from DC adapter. Top wall was made as well from cooper and above it a cooling chamber was placed, which temperature was maintained at constant value by coolant from thermo-stated water bath.
2.2. Experimental fluid

Chosen paramagnetic non-electrically conducting fluid was made from 50% volume aqueous solution of glycerol with 0.8 [mol/(kg of solution)] concentration of gadolinium nitrate hexahydrate (Gd(NO$_3$)$_3$·6H$_2$O). Experimental fluid was made twice due to year apart between experiments, therefore properties are slightly different – fluid A for Enclosure A and fluid B for Enclosure B. All important, for presented studies, properties of the fluids were measured experimentally and are listed in Table 1.

| Property                        | Symbol | Value          | Unit             |
|---------------------------------|--------|----------------|------------------|
| Heat capacity                   | $c_p$  | $2.92 \times 10^3$ | $2.92 \times 10^3$ [J/kg K] |
| Thermal diffusivity             | $\alpha$ | $9.13 \times 10^{-8}$ | $7.55 \times 10^{-8}$ [m$^2$/s] |
| Thermal expansion coefficient   | $\beta$ | $1.21 \times 10^{-5}$ | $4.13 \times 10^{-6}$ [1/K] |
| Dynamic viscosity               | $\mu$  | $1.30 \times 10^{-2}$ | $1.56 \times 10^{-2}$ [kg/m·s] |
| Thermal conductivity            | $\lambda$ | $0.376$ | $0.376$ [W/m·K] |
| Kinematic viscosity             | $\nu$  | $9.25 \times 10^{-6}$ | $1.10 \times 10^{-5}$ [m$^2$/s] |
| Density                         | $\rho$ | $1411$ | $1418.1$ [kg/m$^3$] |
| Mass magnetic susceptibility    | $\chi_m$ | $2.39 \times 10^{-7}$ | $2.58 \times 10^{-7}$ [m$^3$/kg] |

Due to the narrow temperature range studied, it could be assumed that almost all needed properties were constant, except viscosity. Therefore the dependence of viscosity on the temperature was checked experimentally and approximated function could be written as follows:

for fluid A:

$$\mu_0 = 5.600 \cdot 10^{-6} \theta^2 - 7.354 \cdot 10^{-4} \theta + 2.702 \cdot 10^{-2}$$  \hspace{1cm} (1)

and for fluid B:

$$\mu_0 = 2.078 \cdot 10^{-5} \theta^2 - 1.678 \cdot 10^{-3} \theta + 4.451 \cdot 10^{-2}$$  \hspace{1cm} (2)

2.3. Heat loss estimation

To estimate heat loss at first conduction experiment was performed. Both enclosure, one after another were place upside down in same position as during main experiment. During that stage chosen power level was supplied to electrical heater. After thermal stabilization of system, applied power was measured and temperature on hot and cold wall. This experiment was repeated for various values of
power supplied and together with heat flux calculated from Fourier law allow to estimate total heat loss of system for chosen temperature difference between walls:

\[ Q_{\text{loss}} = Q_{\text{cond}} - Q_{\text{theor\_cond}} \]  

(3)

\[ Q_{\text{theor\_cond}} = D^2 \lambda (\theta_h - \theta_c) H^{-1} \]  

(4)

where: \( \theta_h, \theta_c \) – are temperature of hot and cold walls respectively, \( D \) – enclosure basis side 0.032 [m], \( H \) – enclosure height 0.016 [m] for Enclosure A and 0.064 [m] for Enclosure B.

Heat loss was calculated according to eq. (3) and could be linearly approximated for various heating rates (\( \Delta \theta = \theta_h - \theta_c = 2.5 \div 15 \) [K], where temperature of cold wall was kept at ambient temperature inside magnet), for Enclosure A:

\[ Q_{\text{loss}} = 0.0907 \Delta \theta \]  

(5)

and for Enclosure B:

\[ Q_{\text{loss}} = 0.0831 \Delta \theta \]  

(6)

2.4. Nusselt number calculation

Nusselt number was calculated as a ratio between the net convective heat transfer rate and the net pure conduction contribution with Ozoe and Churchill [8] method to estimate net heat fluxes:

\[ \text{Nu} = \frac{Q_{\text{net\_conv}}}{Q_{\text{net\_cond}}} = \frac{Q_{\text{conv}} - Q_{\text{loss}}}{Q_{\text{cond}} - Q_{\text{loss}}} \]  

(7)

Assuming that convection heat flux was equal to heater heat flux (\( Q_{\text{net\_conv}} = Q_{\text{heater}} \)) and applying eq. (3) and (4) to eq. (7) it could be rewritten in following form:

\[ \text{Nu} = \frac{Q_{\text{heater}} - Q_{\text{loss}}}{Q_{\text{theor\_cond}}} = \frac{IU - Q_{\text{loss}}}{D^2 \lambda (\theta_h - \theta_c) H^{-1}} \]  

(8)

The Prandtl number definition was:

\[ \text{Pr} = \frac{\nu}{\alpha} \]  

(9)

The thermal Rayleigh number was defined as:

\[ \text{Ra}_T = \beta g \Delta \theta H^3 \]  

(10)

where \( \beta \) is the thermal expansion coefficient, \( g \) is the gravitational vector, \( \Delta \theta \) is the temperature difference between horizontal thermally active walls, \( H \) is the enclosure height, \( \nu \) is the kinematic viscosity coefficient and \( \alpha \) is the thermal diffusivity.

The magnetic Rayleigh number was defined as:

\[ \text{Ra}_M = \left(1 + \frac{1}{\beta \theta_0}\right) \left(\frac{\gamma \beta g \Delta \theta H^3}{2 \nu \alpha}\right) \]  

(11)

The magnetization number:

\[ \gamma = \frac{\chi_m |b_0|^2}{\mu_0 g H} \]  

(12)

where: \( \mu_0 = 4\pi \times 10^{-7} \) [H/m].

The thermo-magnetic Rayleigh number was defined as:

\[ \text{Ra}_{TM} = \text{Ra}_T + \text{Ra}_M = \left(\frac{\beta g \Delta \theta H^3}{\nu \alpha}\right) \left(1 + \frac{\gamma}{2} + \frac{\gamma}{2 \beta \theta_0}\right) \]  

(13)
2.5. Signal analysis

Temperature signals, recorded during experiments, enabled investigation of the flow structure. This was obtained through Fast Fourier Transform (FFT), which computes discrete Fourier transform (DFT) of a sequence:

\[ F_n = \sum_{i=0}^{N-1} x_i e^{-\frac{2\pi i n}{N}} \]  

(14)

where \( x_i \) is a sequence of length \( N \), and \( F_n \) is its discrete Fourier transform.

Calculated results, in the form of amplitude:

\[ \text{Amplitude} = \sqrt{\text{Re}^2 + \text{Im}^2} / n \]  

(15)

versus frequency, were used to analyze the fluid behaviour in thermo-magnetic convection in enclosures with two different aspect ratios.

3. Results

Figures 4-6 present results of FFT in the form of amplitude versus frequency diagrams. All of the thermocouple signals (see Figures 2 and 3 for placement of the thermocouples in experimental enclosures) were analyzed.

Figure 4 shows results of flow analysis for both experimental enclosures for 3[K] temperature difference between thermally active walls. For natural convection case (\(|b_{0,\text{max}}|=0\) [T]) fluid flow in Enclosure A is steady and no fluctuations of the temperature can be observed, while in Enclosure B small fluctuations of amplitude 0.00578 [-], 0.00496 [-] and 0.00287 [-] for characteristic frequency 0.6766 [Hz] and 0.00396 [-] and 0.12422 [Hz] appear for two thermocouples placed in bottom left and bottom central positions. After applying magnetic induction to both systems, flow becomes faster and temperature oscillations are more frequent. For \(|b_{0,\text{max}}|=5\) [T] in enclosure A oscillation of temperature signal has characteristic frequency equal to 0.04218 [Hz]. In enclosure B, at the same magnetic induction value, all but one thermocouples have characteristic oscillations as for natural convection case. One thermocouple, placed in bottom left corner of the enclosure shows different behaviour – many peaks of relatively high amplitude can be observed. This character of fluid behaviour could indicate that a vertical structure appeared in experimental enclosure and was not a part of general fluid flow. With further increase of magnetic induction up to maximum value of 10 [T] oscillation amplitudes slightly increased and became more intensive than results for natural convection case, which confirms intensification impact of magnetic field to paramagnetic fluid flow.

![Figure 4](image-url)
Figure 5 presents results for temperature difference of 5 [K] between bottom and top walls. As before in Enclosure A fluid is stable and no temperature oscillation can be observed. Different phenomena occurs in Enclosure B – many temperature fluctuations appear on amplitude vs. frequency diagram and the maximum amplitude has a value of 0.10239 for characteristic frequency 0.00888 [Hz]. After applying magnetic induction at \(|b_0|_{\text{max}}=5\) [T] in smaller enclosure many small fluctuations appear with maximal amplitude values of 0.05466 and 0.05109 for frequencies 0.00333 [Hz] and 0.00666 [Hz] respectively. For larger enclosure fluid flow changes significantly. The thermocouple placed in bottom left corner shows different behaviour from the rest – it fluctuates with high amplitudes, while every other thermocouple show periodic fluctuations of small amplitudes. Further increase of magnetic induction up to 10 [T] causes unification of temperature field in Enclosure B with aspect ratio 2.0 – signal from thermocouple in bottom left corner changes and shows behaviour similar to other thermocouples. Probably high magnetic field caused dissipation of vertical structure that appeared in experimental enclosure for lower magnetic induction values.

![Graphs showing FFT amplitude vs. frequency for Enclosures A and B with different magnetic inductions.](image)

**Figure 5.** FFT amplitude versus frequency for a) Enclosure A with AR=0.5 and b) Enclosure B with AR=2.0 for \(\Delta \theta =5\) [K].

Amplitude vs. frequency diagrams for temperature difference between thermally active walls of 11 [K] are shown in Figure 6. Natural convection case shows results similar to \(\Delta \theta =5\) [K]. In Enclosure A flow is stable and temperature signals show no fluctuations. In Enclosure B flow field oscillates significantly – the maximum amplitude 0.18652 appears for frequency 0.00111 and more high amplitudes occur for a series of characteristic frequencies. Increasing magnetic induction to 5 [T] significantly changes the character of the flow in Enclosure A with aspect ratio 0.5. Many periodic oscillations appear on the spectrum for all thermocouples but one. Thermocouple placed in bottom right corner shows different behaviour – oscillations with high amplitude indicating that a structure separate from the main flow appeared in the enclosure. For the same magnetic induction in higher enclosure flow field is more uniform but higher temperature oscillations appear for a thermocouple placed in right bottom corner, as in Enclosure A. Further increasing the magnetic induction causes unification of the temperature field. For both enclosures temperature oscillations decreased, and while in Enclosure B fluid structure which appeared in bottom right corner dissipated, in Enclosure A periodic temperature oscillations in this region suggest that the structure is still present.
Results of heat transfer analysis are presented in Figure 7 as a ratio \( \frac{Nu}{Nu_0} \) versus thermo-magnetic Rayleigh number, where \( Nu_0 \) is Nusselt number value for natural convection case (\( |b_0|_{\text{max}} = 0 \) [T]) enlisted in Table 2.

### Table 2. Nusselt number values for natural convection measurements

| Case     | \( \Delta \theta \) [K] | Value   |
|----------|--------------------------|---------|
| AR=0.5   | 3                        | 3.458   |
|          | 5                        | 2.954   |
|          | 11                       | 3.618   |
| AR=2.0   | 3                        | 13.447  |
|          | 5                        | 14.498  |
|          | 11                       | 18.820  |

With increasing value of magnetic induction in the centre of the magnet enhancement of heat transfer was observed for every case. The largest growth in Nusselt number occurred for a small enclosure (AR=0.5) for \( \Delta \theta = 5 \) [K], where the relative value of \( \frac{Nu}{Nu_0} \) is about 3.73. Slightly lower rise appeared for tall enclosure, where the increase of heat transfer in the form of \( \frac{Nu}{Nu_0} \) for \( |b_0|_{\text{max}} = 10 \) [T] and \( \Delta \theta = 5 \) [K] equals 3.37.
4. Conclusions
An experimental analysis of thermo-magnetic convection of paramagnetic fluid in two enclosures with different aspect ratios was conducted. Flow behaviour was investigated based on temperature variations and Fourier analysis. Application of strong magnetic field gradients strongly influence fluid flow which could be observed as grow of frequency of temperature signal, its amplitude and finally thermal heat flux presented as Nusselt number. Heat transfer enhancement with application of strong magnetic field gradient was significant for both enclosures. Presented approach could be applied in similar system with lack of classical visualization methods.

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