Photoluminescence signature of skyrmions at $\nu = 1$

T. Portengen, J. R. Chapman, V. Nikos Nicopoulos, and N. F. Johnson

Department of Physics, University of Oxford, Parks Road, Oxford OX1 3PU, United Kingdom

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Abstract

The photoluminescence spectrum of quantized Hall states near filling factor $\nu = 1$ is investigated theoretically. For $\nu \geq 1$ the spectrum consists of a right-circularly polarized (RCP) line and a left-circularly polarized (LCP) line, whose mean energy: (1) does not depend on the electron $g$ factor for spin-$\frac{1}{2}$ quasielectrons, (2) does depend on $g$ for charged spin-texture excitations (skyrmions). For $\nu < 1$ the spectrum consists of a LCP line shifted down in energy from the LCP line at $\nu \geq 1$. The $g$-factor dependence of the red shift of the LCP line determines the nature of the negatively charged excitations.

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The study of the quantum Hall effect has recently focused on filling factors near $\nu = 1$. At $\nu = 1$ the Fermi level is located between the lowest spin-split Landau levels, yielding a fully spin-polarized ground state of the two-dimensional electron system (2DES). The nature of the elementary charged excitations is determined by the interplay of the Zeeman energy $\frac{1}{2}g_e\mu_B B$ and the exchange energy $e^2/\epsilon\ell$, where $g_e$ is the electron $g$ factor ($g_e = -0.44$ in GaAs), $\ell = \sqrt{\hbar c/eB}$ is the magnetic length, $\mu_B$ is the Bohr magneton, and $\epsilon$ is the background dielectric constant. For small ($< 0.02$) values of the parameter $g = \frac{1}{2}|g_e|\mu_B B/(e^2/\epsilon\ell)$ the lowest-energy negatively (positively) charged excitations are not spin-$\frac{1}{2}$ quasielectrons (quasiholes), but objects of much larger spin called skyrmions [1] (antiskyrmions) or charged spin-texture excitations [2]. The gradual spin reversal that characterizes skyrmions increases the Zeeman energy, but reduces the exchange energy due to the near parallelism of neighbouring spins. The spin-$\frac{1}{2}$ quasielectrons can be regarded as spin textures with zero radius [2]. Experimental evidence for skyrmion excitations at $\nu = 1$ has been obtained in NMR Knight-shift studies [3], tilted-field magnetotransport measurements [4], and polarized interband optical transmission spectroscopy [5]. Recent magneto-photoluminescence experiments have studied the optical recombination of 2D electrons with itinerant holes in single heterojunctions [6], with holes confined to the undoped side of a one-side doped quantum well [7], and with holes bound to neutral acceptors in Be $\delta$-doped structures [8]. A clear indication of skyrmions in the photoluminescence (PL) spectrum at $\nu = 1$ has thus far been lacking.

Here we predict a PL signature of skyrmions at $\nu = 1$. The energy of the LCP line exhibits a red shift at $\nu = 1$ as the magnetic field is increased. The magnitude of the red shift does not depend on $g$ if the elementary negatively charged excitations are quasielectrons, but does depend on $g$ if they are skyrmions. Measurements of the variation of the red shift with tilting angle or applied hydrostatic pressure allow the detection of skyrmions using PL spectroscopy.

We consider the interband recombination of electrons in the ground subband of the confining potential with holes in the GaAs valence band, in the presence of a strong $B$
field along the z direction. In the vicinity of $\nu = 1$, transitions are observed between the energy levels shown in the inset of Fig. 1. The main LCP transition is between the $m_s = +\frac{1}{2}$ (spin-up) electron lowest Landau level (LLL) and the $m_j = +\frac{3}{2}$ heavy-hole LLL, and the main RCP transition is between the $m_s = -\frac{1}{2}$ (spin-down) electron LLL and the $m_j = -\frac{3}{2}$ heavy-hole LLL. Since the PL signature of skyrmions is independent of the relative intensities of the LCP and RCP lines, we can for simplicity assume equal populations of the $m_j = +\frac{3}{2}$ and $m_j = -\frac{3}{2}$ hole states prior to recombination.

We first examine recombination exactly at $\nu = 1$. The ground state of the $N$-electron system before photoexcitation is the filled spin-up LLL. We assume that following photoexcitation, the $(N+1)$-electron system relaxes to its ground state in the presence of the valence hole on a time scale that is short compared to the radiative recombination time. The Hamiltonian for $(N+1)$ electrons in the LLL interacting with a single valence hole in the LLL is

$$H = \frac{1}{2} g_e \mu_B B \sum_{m\sigma} \sum_{m'\sigma'} e^\dagger_{m\sigma} e_{m'\sigma} + \sum_{m\sigma} \varepsilon_{h\sigma} h^\dagger_{m\sigma} h_{m\sigma}$$

$$+ \frac{1}{2} \sum_{\sigma'} \sum_{m'm''m'''} V^{ee}_{mm'm''m'''} e^\dagger_{m\sigma} e^\dagger_{m'\sigma'} e_{m''\sigma'} e_{m''\sigma}$$

$$- \sum_{\sigma'} \sum_{m'm''m'''} V^{eh}_{mm'm''m'''} e^\dagger_{m\sigma} h^\dagger_{m'\sigma'} h_{m''\sigma'} e_{m''\sigma}. \tag{1}$$

Here $e^\dagger_{m\sigma}$ creates an electron with $m_s = \frac{1}{2} \sigma$ ($\sigma = \pm 1$) in the state $\phi_m(r) = (2^{m+1} \pi m!)^{-1/2} r^m e^{-im\phi} e^{-r^2/4}$ ($\ell = 1$), and $h^\dagger_{m\sigma}$ creates a hole with $m_j = \frac{3}{2} \sigma$ in the state $\phi^*_m(r)$. A uniform neutralizing background is added to the Hamiltonian in the usual way.

The energies of the hole states are

$$\varepsilon_{h\sigma} = E_g + \frac{eB}{2\mu c} - \frac{3}{2} \sigma g_h \mu_B B, \tag{2}$$

where we have taken the Fermi level at $\nu = 1$ as the zero of our energy scale. Here $E_g$ is the gap between the ground electron subband and the GaAs valence band, $\mu = m_e m_h / (m_e + m_h)$ is the reduced mass, $m_e$ and $m_h$ are the electron and hole in-plane effective masses, and $g_h$ is the hole $g$ factor. $V^{ee}_{mm'm''m'''}$ and $V^{eh}_{mm'm''m'''}$ are matrix elements of the electron-electron interaction, $V^{ee}(r) = e^2/|r|$, and the electron-hole interaction, $V^{eh}(r) = e^2/|r + zd|$. Here
$d$ is the distance between the planes to which the electrons and the hole are confined, and $\hat{z}$ is a unit vector along $z$. The confinement of the hole along $z$ occurs naturally in a one-side doped quantum well, and is also expected in a single heterojunction [9].

The initial state $|i_{\sigma}\rangle$ prior to recombination is the ground state of the $(N + 1)$-electron system in the presence of a $m_j = -\frac{3}{2}\sigma$ valence hole. Here $\sigma = +1$ for RCP, and $\sigma = -1$ for LCP. Consider recombination in the presence of disorder. The disorder may be caused, for example, by interface roughness or impurities in the GaAs layer. The disorder is likely to localize the hole in a potential minimum, whose location we choose as the origin of our coordinate system. Provided the potential varies slowly on the scale of $\ell$, we may take the hole to be in the $m = 0$ state. The ground state of the $(N + 1)$-electron system in the absence of the hole is known to be a skyrmion [1]. This leads us to consider initial states of the form

$$|i_{\sigma}\rangle = \prod_{m=-1}^{\infty} (-ue_{m+1\downarrow}^\dagger + ve_{m\uparrow}^\dagger)h_{0,-\sigma}^\dagger |\text{vac}\rangle,$$

where $u_{-1} = -1$ and $v_{-1} = 0$. Here $|\text{vac}\rangle$ is the vacuum state with a filled valence band and empty conduction band. The state $|i_{\sigma}\rangle$ describes a skyrmion bound to a localized hole. The parameters $u_m$ and $v_m$ for $m \geq 0$ are determined by minimizing $\langle i_{\sigma}|H|i_{\sigma}\rangle$. The recombination in the absence of disorder is discussed below. The PL signature of skyrmions in the absence of disorder is qualitatively similar to that in the presence of disorder.

The initial states considered in a previous theoretical study of photoluminescence at $\nu = 1$ were excitonic states consisting of a filled spin-up LLL and a spin-down electron bound to a valence hole [10]. In the context of the present model, the excitonic states correspond to a filled spin-up LLL and a spin-down electron bound to a hole in the $m = 0$ state. These states are obtained by setting $u_m = 0$ and $v_m = 1$ for $m \geq 0$ in Eq. (3). The excitonic states do not allow for the formation of a spin texture of nonzero size in the presence of the hole. Hence no general PL signature of skyrmions was obtained in Ref. [10].

The initial states considered in this work do allow spin textures to form in the presence of the hole. The electron-hole interaction reduces the radius of the spin texture. We have
calculated numerically the magnitude of the spin $S_z$ of the spin texture as a function of the distance $d$ between the electron and hole planes. We find that when $d > \ell$ the ground state corresponds to a spin texture with finite radius ($|S_z| > \frac{1}{2}$), and when $d < \ell$ the ground state corresponds to a spin texture with zero radius ($|S_z| = \frac{1}{2}$). Our results agree with those obtained by other workers [10,11]. As an example, consider a GaAs-AlGaAs heterojunction with $n_s = 10^{11}$ cm$^{-2}$. The magnetic length at $\nu = 1$ is $\ell = 126$ Å. We find $|S_z| = 4.58, 2.94, 1.92, 0.5$ for $d = \infty, 3\ell, 2\ell, \ell$. The wide quantum wells in Ref. [7] have well widths of 400 Å and 500 Å with $n_s = 2.8 \times 10^{11}$ cm$^{-2}$ and $n_s = 1.9 \times 10^{11}$ cm$^{-2}$, respectively. Based on our numerical results, we therefore expect a PL signature of skyrmions in such systems.

We treat the optical recombination in the electric-dipole approximation. The dipole matrix element between the Bloch wavefunctions of the GaAs conduction band and valence band gives rise to the selection rule $M'_J = M_J - \sigma$, where $M_J$ and $M'_J$ are the $z$ components of the total angular momentum before and after the transition. The selection rule for the total angular momentum replaces the usual selection rules for the orbital and spin angular momenta due to the spin-orbit coupling of the GaAs valence band. The $z$ component of the total spin of the $N$-electron state $|f_\sigma\rangle$ after recombination must satisfy the spin selection rule

$$M'_S = \frac{1}{2}N + S_z + \frac{1}{2}\sigma.$$  

This selection rule leads to an important difference between the recombination of a quasielectron and the recombination of a skyrmion. While recombination of a quasielectron leaves either zero (in the case of RCP) or one (in the case of LCP) spin flip in the final state, recombination of a skyrmion leaves a large number of spin flips in the final state. The number of spin flips left is $|S_z + \frac{1}{2}\sigma|$.

The PL spectrum is

$$P_\sigma(\omega) = 2\pi \sum_f |\langle f_\sigma | L_\sigma | i_\sigma \rangle|^2 \delta(E_i - E_f - \omega),$$

where $L_\sigma = \mu_\sigma \sum_m e_m, -\sigma h_{m, -\sigma}$ is the luminescence operator, and $E_i$ and $E_f$ are the energies of the initial and final states. Here $\mu_\sigma$ is the product of the interband dipole matrix
element and the overlap between the electron and hole $z$ wavefunctions. The PL signature of skyrmions occurs in the luminescence energies. We obtain expressions for the moments $\langle \omega^n \rangle = \int d\omega \omega^n P_\sigma(\omega)$ of the luminescence lines by summing over a complete set of final states within the LLL. Following the same algebraic steps as in the derivation of the sum rules for the whole PL spectrum [12], we find

$$\langle \omega^n \rangle = \langle i_\sigma | L^\dagger_\sigma [L_\sigma, H]_n | i_\sigma \rangle,$$

(6)

where $[L_\sigma, H]_n = [[L_\sigma, H]_{n-1}, H]$, and $[L_\sigma, H]_0 = L_\sigma$. The moments do not obey the sum rules for the whole PL spectrum due to the projection of $L_\sigma$ and $H$ onto the LLL. The usefulness of moments was shown by Apalkov and Rashba in the context of the fractional quantum Hall effect [13]. It is convenient to redefine $\langle \omega^n \rangle \equiv \langle \omega^n \rangle / \langle \omega^0 \rangle$.

We calculate the energies of the RCP and LCP lines by evaluating Eq. (6) with $n = 0, 1$ in the initial state given by Eq. (3). Three terms contribute to $\langle \omega_\sigma \rangle$. The first term is $\varepsilon_{h,-\sigma}$. The second term is the energy cost to remove an electron with $m = 0$ and $m_s = -\frac{1}{2} \sigma$ from the spin texture. The third term is the energy of the state $L_\sigma|i_\sigma\rangle$ in the potential $-V_{eh}(r)$ switched on by the removal of the hole. Figure 1 shows the mean energy of the RCP and LCP lines,

$$\langle \omega \rangle = \frac{\langle \omega_+ \rangle + \langle \omega_- \rangle}{2},$$

(7)

as a function of $g$, for various values of $d$. The $g$ dependence of $\langle \omega \rangle$ is a PL signature of skyrmions. By setting $u_m = 0$ and $v_m = 1$ for $m \geq 0$ it can be shown that $\langle \omega \rangle$ does not depend on $g$ when the initial state consists of a quasielectron and a hole. $\langle \omega \rangle$ does depend on $g$ when the initial state consists of a skyrmion and a hole. The value of $g$ below which skyrmions exist in the initial state decreases as the hole approaches the electron plane.

We now consider the PL spectrum slightly away from $\nu = 1$. For $\nu > 1$ the ground state prior to photoexcitation already contains a small number of skyrmions. Provided their density is small, the skyrmions can be considered as noninteracting. (We do not consider filling factors further away from $\nu = 1$, where interactions between skyrmions can give rise
to the formation of a Skyrme crystal [14].) The initial state prior to recombination contains an additional skyrmion and a valence hole. One skyrmion recombines with the hole, leaving behind enough spin flips to satisfy the spin selection rule. The final state also contains the remaining skyrmions. Since only one skyrmion is involved in the recombination, the PL spectrum is similar to that at \( \nu = 1 \).

The PL spectrum at \( \nu < 1 \) is qualitatively different from that at \( \nu \geq 1 \). Consider the filling factor at which the ground state before photoexcitation contains a single antiskyrmion. We denote this filling factor by \( \nu = 1^- \). The initial state prior to recombination consists of a filled spin-up LLL and a valence hole. Since the initial state contains no spin-down electrons, the RCP line is missing from the PL spectrum. The final state after LCP recombination has a quasihole in the spin-up LLL. Because of the spin selection rule, the excitation left in the final state cannot be a large-spin antiskyrmion. Thus we expect no PL signature of antiskyrmions at \( \nu < 1 \).

We calculate the energy of the LCP line at \( \nu = 1^- \) by evaluating Eq. (6) with \( n = 0, 1 \) in the initial state \( |i_{\pm}\rangle = \prod_{m=0}^{\infty} e^\dagger_m h^\dagger_{0,1} |\text{vac}\rangle \). The LCP line at \( \nu = 1^- \) is shifted down in energy from the LCP line at \( \nu = 1 \). The red shift does not depend on \( g \) when the initial state at \( \nu = 1 \) consists of a quasielectron and a valence hole. The red shift does depend on \( g \) when the initial state at \( \nu = 1 \) consists of a skyrmion and a valence hole. The red shift can be understood by comparing the final states after LCP recombination at \( \nu = 1^- \) and \( \nu = 1 \) [15]. While the final state at \( \nu = 1^- \) contains a free quasihole, the final state at \( \nu = 1 \) contains a quasihole bound to a skyrmion. The red shift is the binding energy of the skyrmion-quasihole pair. For \( \nu < 1^- \), the initial state contains a small density of antiskyrmions in addition to the valence hole. We argue that because of the Coulomb repulsion, and the compressibility of the electron system at \( \nu < 1 \), the valence hole avoids the antiskyrmions and recombines with a spin-up electron, leaving a quasihole in the final state. Thus a small density of antiskyrmions does not affect the recombination. The PL spectrum is similar to that at \( \nu = 1^- \).

We now discuss the PL spectrum in the absence of disorder. For a translationally invari-
ant system, a skyrmion-hole pair has the center-of-mass momentum $k$ as a good quantum number [10]. By analogy with a magnetoexciton in the LLL [17], we expect the dispersion of a skyrmion-hole pair to have an absolute minimum at $k = 0$. The initial state in the absence of disorder is then a skyrmion-hole pair with $k = 0$. Because of the spin selection rule, the final state contains $|S_z + \frac{1}{2}\sigma|$ spin waves with total momentum $k = 0$. When $|S_z + \frac{1}{2}\sigma| > 1$ there is a continuum of final states. To estimate the mean luminescence energy, we calculate the energy of the transition to the final state with $|S_z + \frac{1}{2}\sigma|$ spin waves, each of momentum $k = 0$. We expect this final state to have a large oscillator strength. By Larmor’s theorem, the energy of this final state is $-g_e\mu_B B|S_z + \frac{1}{2}\sigma|$. Neglecting the binding energy of the skyrmion-hole pair ($d = \infty$), we find

$$\langle \omega \rangle = E_g + \frac{eB}{2\mu_c} + E_{sk} - S_z g_e\mu_B B.$$  

(8)

Here $E_{sk}$ is the energy of the skyrmion. The red shift of the LCP line is $E_{sk} - S_z g_e\mu_B B + (\pi/2)^{1/2} e^2/\epsilon\ell$. For a spin texture with radius $\lambda > 0$, the mean luminescence energy and the red shift depend on $g$. For $\lambda \gg \ell$ the explicit $g$ dependence can be obtained using $S_z = -\pi(\lambda/\ell)$ and Eq. (7) of Ref. [1]. For a spin texture with zero radius the mean luminescence energy and the red shift do not depend on $g$. This can be seen by setting $E_{sk} = -\frac{1}{2}g_e\mu_B B$ and $S_z = -\frac{1}{2}$.

Polarization-resolved PL spectra from GaAs-AlGaAs heterojunctions and one-side doped quantum wells were reported in Ref. [7]. The ground subband emission from heterojunctions and wide quantum wells shows a LCP line and a RCP line on the low-field side of $\nu = 1$, and a LCP line on the high-field side of $\nu = 1$. The LCP line on the high-field side is shifted down in energy from the LCP line on the low-field side by 0.4 meV in wide quantum wells and by 2 meV in single heterojunctions. Unpolarized PL spectra from GaAs-AlGaAs heterojunctions reported in Ref. [3] show two lines on the low-field side and one line on the high-field side of $\nu = 1$. The lower-energy line on the low-field side is red shifted by 1 meV as the field increases through $\nu = 1$.

The observations of Refs. [3,7] are consistent with our theoretical description of lum-
nescence near \( \nu = 1 \). The calculated red shifts are larger than the observed red shifts, partly due to our neglect of the finite extent of the \( z \) wavefunctions. The observation of a red shift as such provides no clue about the nature of the negatively charged excitations at \( \nu = 1 \). This is because a red shift is expected both for skyrmions and for quasielectrons. It is the \( g \)-factor dependence of the red shift that allows the distinction to be made. Two methods of varying \( g \) have been employed in recent magnetotransport measurements. In Ref. [4] \( g \) was varied by tilting the total magnetic field \( B_{\text{tot}} \) away from the normal to the heterojunction plane, while keeping the perpendicular field \( B_\perp \) (and hence the filling factor) constant. While the electron spin couples to \( B_{\text{tot}} \), the orbital motion is determined by \( B_\perp \). As a result, the effective \( g \) factor is \( g_e/\cos \theta \), where \( \theta \) is the tilting angle. An additional \( \theta \) dependence of the PL energies arises from the deformation of the \( z \) wavefunctions by the parallel component \( B_\parallel \) of the magnetic field. The \( \theta \) dependence of \( \langle \omega \rangle \) may therefore be less suitable as a PL signature of skyrmions. The red shift remains unaffected by \( B_\parallel \) because the \( z \) wavefunctions are deformed equally on either side of \( \nu = 1 \). Figure 2 shows the variation of the red shift with tilting angle for a GaAs-AlGaAs heterojunction with \( n_s = 10^{11} \text{ cm}^{-2} \). Tilting the field by 60° increases the red shift by 0.6 meV. Such an increase should be readily observable experimentally.

In Ref. [18] \( g \) was varied by applying hydrostatic pressure. The applied pressure reduces the magnitude of \( g_e \) through the mixing of the GaAs conduction band and the spin-orbit split valence band. Reference [18] claims \( g_e = -0.43 + 0.020p \) for pressures \( p \) from 0 to 10 kbar. The applied pressure also changes other bandstructure parameters. However, as these parameters change equally on either side of \( \nu = 1 \), the red shift acquires no additional pressure dependence. Figure 2 shows the variation of the red shift with applied pressure for a GaAs-AlGaAs heterojunction with \( n_s = 10^{11} \text{ cm}^{-2} \). Applying a pressure of 10 kbar reduces the red shift by 0.4 meV. The problem of the decrease of \( n_s \) with applied pressure [18] may be overcome by reducing the field (keeping \( \nu = 1 \) fixed) and dividing the red shift by \( e^2/\epsilon \ell \) to correct for the pressure dependence of \( \ell \). The division by \( e^2/\epsilon \ell \) also corrects for the small pressure dependence of \( \epsilon \).
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FIGURES

FIG. 1. Mean energy of the RCP and LCP luminescence lines as a function of the $g$ factor, for various values of the separation $d$ between the electron and hole planes. Parameter values are for a GaAs-AlGaAs heterojunction with $n_s = 10^{11}$ cm$^{-2}$. The gap value $E_g = 1509$ meV was taken from Ref. [6]. Curves are displaced for clarity. Inset: energy levels between which RCP and LCP transitions occur.

FIG. 2. Variation of the red shift of the LCP line with tilting angle $\theta$ and applied hydrostatic pressure $p$. The separation between the electron and hole planes is $d = 400$ Å. Remaining parameter values as in Fig. 1. Inset: dependence of the RCP and LCP luminescence energies on the magnetic field.
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