Curvaton Scenario with Affleck-Dine Baryogenesis

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Abstract

We discuss the curvaton scenario with the Affleck-Dine baryogenesis. In this scenario, non-vanishing baryonic entropy fluctuation may be generated even without primordial fluctuation of the Affleck-Dine field. Too large entropy fluctuation is inconsistent with the observations and hence constraints on the curvaton scenario with the Affleck-Dine baryogenesis are obtained. We calculate the baryonic entropy fluctuation (as well as other cosmological density fluctuations) in this case and derive constraints. Implications to some of the models of the curvaton are also discussed.
1 Introduction

Recent precise measurement of the cosmic microwave background (CMB) anisotropies by the Wilkinson Microwave Anisotropy Probe (WMAP) [1] has provided detailed informations about the mechanism of generating the cosmic density fluctuations. Importantly, observed CMB anisotropies are highly consistent with those predicted from scale-invariant purely adiabatic primordial density fluctuations. Such a class of primordial density fluctuations is also consistent with the recent results by the Sloan Digital Sky Survey experiments [2]. Among various possibilities, inflation is one of the most famous scenarios of generating cosmic density fluctuations consistent with the observations.

Another class of scenario of generating the scale-invariant adiabatic density fluctuations, however, exists, which is called the “curvaton” scenario [3, 4, 5]. If there exists a scalar field $\phi$ (other than the inflaton) which acquires primordial fluctuation during the inflation, dominant part of the cosmological density fluctuations may originate from the primordial fluctuation of this scalar field. (The scalar field $\phi$ is called the curvaton.) Importantly, this scenario works irrespective of the detailed properties of the curvaton field, i.e., its mass, lifetime, and initial amplitude. Consequently, there are many possible and well-motivated candidates of the curvaton field since, in various scenarios of particle cosmology, many scalar-field condensations which once dominate the universe show up; if those scalars acquire primordial quantum fluctuations, they may play the role of the curvaton. Thus, the curvaton scenario naturally fits into various scenarios of particle cosmology. In addition, with the curvaton scenario, we have a chance to test the properties of the particle responsible for the cosmological density fluctuations by collider experiments; this is the case if the flat direction of a minimal supersymmetric standard model (MSSM) becomes the curvaton, for example. (Remember that it is difficult to construct a model of inflation within the framework of the standard model or the MSSM. #2)

An important aspect of the curvaton scenario is that the universe is reheated by the decay of the curvaton. In some case, reheating temperature at the curvaton decay becomes relatively low and hence it is non-trivial if a viable scenario of baryogenesis can be found. In addition, a large amount of entropy is produced at the time of the decay of the curvaton. As a result, even if non-vanishing baryon asymmetry is generated before the decay of the curvaton, primordial baryon asymmetry may be too much diluted to be consistent with the present value. This may be a problem for the case where, for example, the cosmological moduli fields play the role of the curvaton.

One of the possibilities to generate enough baryon asymmetry with large entropy production is to adopt the Affleck-Dine baryogenesis [9]. Indeed, in [10], it was pointed out that the resultant baryon-to-photon ratio $n_b/n_\gamma$ (with $n_b$ and $n_\gamma$ being the number densities of the baryon and photon, respectively) can be as large as the presently observed value.

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#1 A similar study of the effect of extra scalar field other than the inflaton, see also [6].
#2 In the MSSM, however, it may be possible to use the $D$-flat direction consisting of the up-type squarks as well as the up-type Higgs boson as the inflaton. For details, see [8].
\( \sim O(10^{-10}) \) even with the large late-time entropy production. In the Affleck-Dine scenario, however, baryonic isocurvature fluctuation may arise since, in this scenario, baryon asymmetry is generated from a coherent motion of a scalar field (called Affleck-Dine field). In particular, if the Affleck-Dine mechanism is implemented in the curvaton scenario, correlated baryonic isocurvature fluctuation may arise as we will see below. Thus, it is important to study constraints on this case and see if the curvaton scenario with Affleck-Dine baryogenesis is viable.

In this paper, we consider the curvaton scenario with the Affleck-Dine baryogenesis. In particular we study the possible baryonic entropy fluctuation generated in this framework and how the entropy fluctuation can be suppressed. The organization of the rest of this paper is as follows. In Section 2, we first discuss framework and give the thermal history we consider. Then, we introduce equations by which evolutions of the cosmological density fluctuations are governed. Analytic solutions to those equations for a simple (but well-motivated) case are also discussed. In Section 3, we numerically solve the evolution equations. In particular, we will see that the baryonic entropy fluctuation provides very stringent constraints on the curvaton scenarios with Affleck-Dine baryogenesis. Implications of such constraints for some of the curvaton scenarios are discussed in Section 4. Section 5 is devoted to the summary of this paper.

2 Evolutions of the Fluctuations

2.1 Thermal history

Let us first introduce the framework. In various models of physics beyond the standard model, there are many scalar fields whose potential is approximately parabolic:

\[
V(\phi) = \frac{1}{2} m^2 \phi^2. \tag{2.1}
\]

In the MSSM, for example, \( F \)- and \( D \)-flat directions, whose potential is lifted by the supersymmetry-breaking effects, has parabolic potential. In addition, potential of the cosmological moduli fields are expected to be parabolic when the amplitude of the modulus field is smaller than \( \sim M_* \) (where \( M_* \) is the reduced Planck scale). Another possible candidates of such a scalar field is pseudo-Nambu-Goldstone bosons [11]. In this paper, we assume one of such scalar fields plays the role of the curvaton.

Here, we adopt inflation as a solution to the horizon, flatness, and other cosmological problems. Then, the universe starts with the inflationary epoch where the universe is dominated by the potential energy of the inflaton field \( \chi \); thermal history before the inflation is irrelevant for our discussion. During the inflation, we assume that the amplitude of the curvaton is non-vanishing. In addition, we consider the case where the effective mass of the curvaton during the inflation is much smaller than the expansion rate during the
inflation $H_{\text{inf}}$. Then, the curvaton acquires the primordial fluctuation during the inflation

$$\delta \phi_{\text{init}} = \frac{H_{\text{inf}}}{2\pi}. \quad (2.2)$$

(Here and hereafter, the subscript “init” is used for initial values of the quantities at very early epoch when $\phi$ is slowly rolling.) After the inflation, universe is reheated by the decay of the inflaton field and the universe is dominated by radiation. We call this epoch the first radiation-dominated epoch (or RD1 epoch) since, in our scenario, there are two radiation-dominated epochs.

As far as the decay of $\phi$ is negligible, universe consists of two components: radiation generated from the decay products of the inflaton, which we denote $\gamma_\chi$, and the curvaton. After the RD1 epoch, evolution of the energy density of $\gamma_\chi$ obeys the following equation:

$$\dot{\rho}_{\gamma_\chi} + 4H\rho_{\gamma_\chi} = 0, \quad (2.3)$$

where the “dot” denotes the derivative with respect to the time $t$. In addition, (unperturbed part of) $\phi$ obeys the following equation:

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi = 0. \quad (2.4)$$

(As is obvious from the above equations, we have not taken into account of the decay of the curvaton field since the effect of the decay of $\phi$ is negligible for the RD1 and $\phi$-dominated epochs.) Here, $H \equiv \dot{a}/a$ (with $a$ being the scale factor) is the expansion rate of the universe which is given by, when $H$ is much larger than the decay rate of $\phi$,

$$H^2 = \frac{1}{3M_*^2} \left( \rho_{\gamma_\chi} + \frac{1}{2}\phi^2 + \frac{1}{2}m_\phi^2\phi^2 \right). \quad (2.5)$$

From Eq. (2.3), we can see that $\rho_{\gamma_\chi} \propto a^{-4}$. On the contrary, the curvaton starts to oscillate when $H \sim m_\phi$; after this epoch, energy density of the curvaton decreases as $\rho_{\phi} \propto a^{-3}$. Thus, the universe can be dominated by the curvaton if the lifetime of the curvaton is long enough. Hereafter, the $\phi$-dominated epoch is called as $\phi$D epoch. After the $\phi$D epoch, the curvaton decays and the universe is reheated again; we call the second radiation-dominated epoch as RD2 epoch. After the RD2 epoch, the thermal history is the same as that of the conventional big-bang model (as far as the zero-mode is concerned).

The curvaton amplitude $\phi_{\text{eq}}$ at the time of the “radiation-curvaton equality” is easily estimated. (In the following, the subscript “eq” is used for quantities at the time of the radiation-curvaton equality.) The curvaton starts to oscillate when the temperature of the radiation generated by the inflaton decay (which is denoted as $\gamma_\chi$ hereafter) becomes $\sim \sqrt{m_\phi M_*}$ and, after this epoch, the curvaton amplitude is approximately proportional to $a^{-3/2}$. Thus,

$$\phi_{\text{eq}} = \phi_{\text{init}}^4M_*^{-3}. \quad (2.6)$$
Expansion rate at the equality is also easily estimated:

\[ H_{\text{eq}} \sim \frac{m_{\phi} \phi_{\text{eq}}}{M_*} \sim m_{\phi} \left( \frac{\phi_{\text{init}}}{M_*} \right)^4. \]  

(2.7)

Assuming that the energy density of the Affleck-Dine field is always sub-dominant in the RD1 and \( \phi \)D epochs, which we assume in this paper, the above discussion holds even with the Affleck-Dine field. Equation of motion of the Affleck-Dine field is, on the other hand, given by

\[ \ddot{\psi} + 3H \dot{\psi} + \frac{\partial V_{\text{AD}}}{\partial \psi^*} = 0, \]  

(2.8)

where \( V_{\text{AD}} \) is the potential of the Affleck-Dine field.

Evolution of the Affleck-Dine field depends on the detailed shape of its potential. In this paper, to be specific, we consider the case where an \( F \)- and \( D \)-flat direction in the MSSM plays the role of the Affleck-Dine field. In this case, baryon-number violating interaction in the potential of \( \psi \) is assumed to originate from Kähler interaction. Another possibility is with baryon- (or lepton-) number violating superpotential. Then, however, the initial amplitude of the Affleck-Dine field is usually suppressed. Consequently the resultant baryon asymmetry becomes too small to be consistent with the presently observed value if a large amount of entropy is produced by the decay of the curvaton. One of the motivation here to consider the Affleck-Dine baryogenesis is to generate large enough baryon-number asymmetry. In the case of the \( F \)- and \( D \)-flat direction as the Affleck-Dine field, it is known that baryon asymmetry can be large enough even with the large entropy production. Thus, we consider the Affleck-Dine potential of the form:

\[ V_{\text{AD}} = m^2_{\psi} |\psi|^2 + \frac{\lambda m^2_{\psi}}{M_*^2} (\psi^p + \psi^{*p}), \]  

(2.9)

where \( m^2_{\psi} \) is from the supersymmetry-breaking effect and \( \lambda \) is a constant of \( O(1) \). Then, potential of the Affleck-Dine field is dominated by the parabolic term as far as \( |\psi| \lesssim M_* \) and \( \psi \) starts to oscillate when \( H \sim m_{\phi} \). After this epoch, \( |\psi| \) is approximately proportional to \( a^{-3/2} \).

Denoting the baryon-number of \( \psi \) as \( B_\psi \), relevant part of the baryon-number current is given by

\[ n_b = iB_\psi \left( \psi^* \dot{\psi} - \dot{\psi}^* \psi \right). \]  

(2.10)

Then, we obtain

\[ \dot{n}_b + 3Hn_b = iB_\psi \left( \frac{\partial V_{\text{AD}}}{\partial \psi} \psi - \text{h.c.} \right). \]  

(2.11)

\#3 In our convention, the Affleck-Dine field \( \psi \) is a complex scalar field while the curvaton \( \phi \) is a real scalar field. We assume that the effect of the motion of the curvaton in the phase direction is negligible. (For the effect of the motion of the curvaton in the phase direction, see [12].)
Thus, if the Affleck-Dine field has non-vanishing initial amplitude $\psi_{\text{init}}$, baryon-number density is generated when $\psi$ starts to move. The Affleck-Dine field starts to move when $H \sim m_{\psi}$. Parameterizing the initial value of the Affleck-Dine field as $\psi_{\text{init}} \equiv |\psi_{\text{init}}|e^{i\theta_{\text{init}}}$, the baryon-number density at that moment is estimated as

$$[n_b]_{H \sim m_{\psi}} \sim \left[ B_\psi \text{Im} \left( \frac{\partial V_{\text{AD}}}{\partial \psi} \psi \right) H^{-1} \right]_{H \sim m_{\psi}} \sim \lambda B_\psi \frac{m_{\psi}}{M_*^{p-2}} |\psi_{\text{init}}|^p \sin p \theta_{\text{init}},$$

(2.12)

where coefficients of $O(1)$ is neglected.

First, let us consider the case where the present CMB radiation is generated from the decay product of the curvaton. In this case, the resultant baryon-to-photon ratio depends on the epoch when the slow-roll condition of the Affleck-Dine field breaks down. (Here, we consider the case with $m_{\psi} \lesssim m_{\phi}$; as we will see below, this relation is required in order to suppress the baryonic entropy fluctuation.) To discuss the resultant baryon-to-photon ratio, it is convenient to define

$$\phi_{\text{crit}} = M_* \left( \frac{m_{\psi}}{m_{\phi}} \right)^{1/4}.$$

(2.13)

If $\phi_{\text{init}} \ll \phi_{\text{crit}}$, $H \sim m_{\psi}$ is realized in the RD1 epoch while, for the case of $\phi_{\text{init}} \gg \phi_{\text{crit}}$, the Affleck-Dine field starts to oscillate in the $\phi_D$ epoch.

If the Affleck-Dine field starts to move in the RD1 epoch, background temperature at $H \sim m_{\psi}$ is estimated as $\sim \sqrt{m_{\psi}M_*}$. At this moment, with the condition $m_{\psi} \lesssim m_{\phi}$, amplitude of $\phi$ is given by $\phi \sim \phi_{\text{init}}(m_{\psi}/m_{\phi})^{3/4}$. Then, the ratio $n_b/\rho_\phi$, which is a conserved quantity when $H \lesssim m_{\psi}$, is estimated as

$$\frac{n_b}{\rho_\phi} \sim \frac{\lambda B_\psi |\psi_{\text{init}}|^p \sin p \theta_{\text{init}}}{m_{\phi}^{1/2} m_{\psi}^{1/2} M_*^{p-2} \phi_{\text{init}}^2}.$$

(2.14)

When the curvaton decays, energy density of $\phi$ is converted to that of radiation and hence the baryon-to-photon ratio is estimated as

$$\frac{n_b}{n_\gamma} \sim \frac{\lambda B_\psi |\psi_{\text{init}}|^p \sin p \theta_{\text{init}}}{m_{\phi}^{1/2} m_{\psi}^{1/2} M_*^{p-2} \phi_{\text{init}}^2} T_{\text{RD2}},$$

(2.15)

where $T_{\text{RD2}}$ is the reheating temperature at the time of the curvaton decay. Thus, in this case, the resultant baryon-to-photon ratio depends on the initial amplitude of the curvaton field. If $H \sim m_{\psi}$ is realized in the $\phi_D$ epoch, on the contrary, the situation changes and the expansion rate at the time when $\psi$ starts to move is determined by the energy density of the curvaton. Consequently, at the time of $H \sim m_{\psi}$, amplitude of the curvaton is estimated as

$$[\phi]_{H \sim m_{\psi}} \sim M_* \left( \frac{m_{\psi}}{m_{\phi}} \right).$$

(2.16)
Then, the baryon-to-photon ratio is estimated as
\[
\frac{n_b}{n_\gamma} \sim \frac{\lambda B_\psi |\psi_{\text{init}}|^p \sin p\theta_{\text{init}}}{m_\psi M_*^p} T_{\text{RD2}}.
\] (2.17)

In this case, the resultant baryon-to-photon ratio does not depend on \(\phi_{\text{init}}\).

If the lifetime of the Affleck-Dine field is long enough, the present CMB radiation dominantly originate from the decay product of the Affleck-Dine field. In this case, the expression of the baryon-to-photon ratio changes using the fact that the ratio \(n_b/\rho_\psi\) (with \(\rho_\psi\) being the energy density of the Affleck-Dine field) is a constant after the Affleck-Dine field starts to move. The baryon-to-photon ratio is estimated as
\[
\frac{n_b}{n_\gamma} \sim \left[ \frac{n_b}{\rho_\psi} \right]_{H \sim m_\psi} T_{\text{ADdecay}} \sim \frac{\lambda B_\psi |\psi_{\text{init}}|^{p-2} \sin p\theta_{\text{init}}}{m_\psi M_*^{p-2}} T_{\text{ADdecay}},
\] (2.18)

where \(T_{\text{ADdecay}}\) is the reheating temperature due to the decay of the Affleck-Dine field. Then, the baryon-to-photon ratio is independent of \(\phi_{\text{init}}\) irrespective of the time when the Affleck-Dine field starts to move.

### 2.2 Adiabatic density fluctuations

Now, we consider the evolution of the density fluctuations. In our scenario, there are two independent sources of the cosmological density fluctuations; one is the primordial fluctuation of the curvaton given in Eq. (2.2) and the other is that of the inflaton. Since we are interested in the case where the curvaton contribution dominates, let us consider the evolution of the cosmological density fluctuations generated from the primordial fluctuation of the curvaton. Effects of the primordial fluctuation of the inflaton will be discussed in Section 4.

Although we are interested in the case where the curvaton and the Affleck-Dine field both exist, we first consider the adiabatic mode (in the \(\phi_D\) epoch and after) neglecting the effects of the Affleck-Dine field. Indeed, behaviors of the adiabatic mode can be understood without taking account of the Affleck-Dine field as far as the energy density of the Affleck-Dine field is always sub-dominant.

In this paper, we use the Newtonian gauge where the line element is described as
\[
\frac{n_b}{n_\gamma} \sim \left[ \frac{n_b}{\rho_\psi} \right]_{H \sim m_\psi} T_{\text{ADdecay}} \sim \frac{\lambda B_\psi |\psi_{\text{init}}|^{p-2} \sin p\theta_{\text{init}}}{m_\psi M_*^{p-2}} T_{\text{ADdecay}},
\] (2.18)

We use the convention and notation of [13] unless otherwise mentioned.

Here, \(a_0\) is some constant and \(x\) is the comoving coordinate. In the RD1 and \(\phi_D\) epochs, it is expected that the temperature of the universe is so high that the momentum-exchange of the relativistic particles are efficient enough. In this case, anisotropic stress vanishes
and $\Phi = -\Psi$. With this relation, relevant part of the equations to be solved are, for the Fourier mode with the (comoving) momentum $k$,

$$\dot{\Psi} + H\Psi = \frac{1}{2M_*^2} \left( \frac{3}{4} a \rho_{\gamma x} \dot{V}_{\gamma x} + \dot{\phi} \delta \phi \right),$$

(2.20)

$$\dot{\delta}_{\gamma x} = 4 \dot{\Psi} - \frac{4 a_0}{3 a} k^2 \ddot{V}_{\gamma x},$$

(2.21)

$$\ddot{V}_{\gamma x} = \frac{a_0}{a} \left( \frac{1}{4} \delta_{\gamma x} + \Psi \right),$$

(2.22)

$$\ddot{\delta} \phi + 3H\dot{\delta} \phi + \left[ \left( \frac{a_0}{a} \right)^2 k^2 + m_\phi^2 \right] \delta \phi = 4\dot{\phi} \dot{\Psi} - 2m_\phi^2 \phi \Psi,$$

(2.23)

where $\delta_{\gamma x} \equiv \delta \rho_{\gamma x} / \rho_{\gamma x}$ with $\delta \rho_{\gamma x}$ being the fluctuation of the energy density of $\gamma x$, $\ddot{V}_{\gamma x} \equiv V_{\gamma x} / k$ with $V_{\gamma x}$ being the velocity of $\gamma x$, and $\delta \phi$ is the fluctuation of the curvaton amplitude.

In order to solve the equations, we have to specify the initial conditions for the fluctuations (as well as those for $\rho_{\gamma x}$ and $\phi$). In order to study the effects of the primordial fluctuation of the curvaton, we concentrate on the mode for which, in the deep RD1 epoch, $\phi$ has non-vanishing fluctuation as given in Eq. (2.2) while $\delta \rho_{\gamma x}$ vanishes.

Importantly, we are interested in the modes related to currently observed cosmological density fluctuations. Those modes reenter the horizon at the time close to the (usual) radiation-matter equality and hence they are superhorizon modes in the RD1 and $\phi D$ epochs. Thus, in discussing their behaviors in the RD1 and $\phi D$ epochs, all the terms proportional to $k^2$ are irrelevant in the evolution equations. Then, solving Eqs. (2.20) – (2.23), fluctuations in the deep RD1 epoch when $H \gg m_\phi$ are given by

$$\Psi_{\text{RD1}}^{(\delta \phi)} \simeq -\frac{1}{14} \frac{\delta \rho_\phi,\text{init}}{\rho_{\gamma x}},$$

(2.24)

$$\delta_{\gamma x, \text{RD1}}^{(\delta \phi)} \simeq -\frac{2}{7} \frac{\delta \rho_\phi,\text{init}}{\rho_{\gamma x}},$$

(2.25)

$$\ddot{V}_{\gamma x, \text{RD1}}^{(\delta \phi)} \simeq -\frac{\sqrt{2} H_0 t \delta \rho_\phi,\text{init}}{35 \rho_{\gamma x}},$$

(2.26)

$$\delta \phi_{\text{RD1}}^{(\delta \phi)} \simeq \left( 1 - \frac{1}{5} m_\phi^2 t^2 \right) \delta \phi,\text{init},$$

(2.27)

where the subscript “RD1” is for variables in deep RD1 epoch, and the superscript “$(\delta \phi)$” is for fluctuations generated from the primordial fluctuation of the curvaton. Here, $H_0$ is the expansion rate at $a = a_0$ (which is, of course, taken at deep RD1 epoch), and

$$\delta \rho_\phi,\text{init} \equiv m_\phi^2 \phi_{\text{init}} \delta \phi_{\text{init}}.$$  

(2.28)

Notice that the relations given in Eqs. (2.24) – (2.27) provide initial conditions for our numerical calculations.
Evolutions of the fluctuations can be studied by solving Eqs. (2.20) − (2.23). Importantly, in the curvaton scenario, fluctuation in the curvaton sector is converted to the adiabatic density fluctuations after the φD epoch. Evolutions of the superhorizon fluctuations are the same as those with the baryonic isocurvature fluctuations [14]; after the φD epoch, \( \Psi^{(\delta \phi)} \) becomes of the order of

\[
S_{\phi \chi}^{(\delta \phi)} \equiv 2 \frac{\delta \phi_{\text{init}}}{\phi_{\text{init}}},
\]

(2.29)

If the initial amplitude of the curvaton is much smaller than \( M_* \), \( \phi \) starts to oscillate when the universe is dominated by radiation. In this case, \( S_{\phi \chi}^{(\delta \phi)} \) becomes the entropy fluctuation between components generated from the decay products of \( \phi \) and those generated from \( \chi \) (like \( \gamma \chi \)). Since the curvaton can be identified as a non-relativistic component once it starts to oscillate, situation is the same as the models with isocurvature fluctuation in non-relativistic component. Then we obtain

\[
\Psi^{(\delta \phi)}_{\text{RD2}} = \frac{10}{9} \Psi^{(\delta \phi)}_{\phi \text{D}} = -4 \frac{\delta \phi_{\text{init}}}{9 \phi_{\text{init}}},
\]

(2.30)

where the subscripts “\( \phi \text{D} \)” and “RD2” are quantities in the \( \phi \text{D} \) and RD2 epochs, respectively.

### 2.3 Entropy fluctuation

Now, we discuss the entropy fluctuation in the baryonic sector, which is defined as

\[
S_{b \gamma} = \frac{\delta(n_b/n_\gamma)}{(n_b/n_\gamma)}.
\]

(2.31)

Although the precise calculation of the entropy fluctuation will be done in the next section with numerical calculations, it is instructive to discuss the basic behavior of the entropy fluctuation. Indeed, in some case, baryonic entropy fluctuation associated with the Affleck-Dine field can be analytically estimated.

Here, we consider the case where the energy density of the Affleck-Dine field is subdominant in the RD1 and \( \phi \)D epochs. In such a case, Eqs. (2.15) and (2.17) are applicable, from which we can estimate the entropy fluctuation generated in our scenario. As can be seen in Eq. (2.14), if \( \phi_{\text{init}} \ll \phi_{\text{crit}} \), \( n_b/n_\gamma \) depends on \( \phi_{\text{init}} \). Thus, if the curvaton amplitude has primordial fluctuation, it also generates the entropy fluctuation between the baryon and the photon. Using Eq. (2.31),

\[
S_{b \gamma}^{(\delta \phi)} \simeq -2 \frac{\delta \phi_{\text{init}}}{\phi_{\text{init}}} \text{ for } \phi_{\text{init}} \ll \phi_{\text{crit}}.
\]

(2.32)

On the contrary, when \( \phi_{\text{init}} \gg \phi_{\text{crit}} \), \( n_b/n_\gamma \) is independent of \( \phi_{\text{init}} \), and we obtain

\[
S_{b \gamma}^{(\delta \phi)} \simeq 0 \text{ for } \phi_{\text{init}} \gg \phi_{\text{crit}}.
\]

(2.33)
Importantly, shape of the CMB angular power spectrum depends on the relative size between the adiabatic and entropy fluctuations. Thus, for our following discussions, it is convenient to define

\[ \kappa_b \equiv \frac{S_{b\gamma}^{(\delta \phi)}}{\Psi_{RD2}^{(\delta \phi)}}. \]

(2.34)

If the curvaton starts to oscillate in the deep RD1 epoch, which is realized when \( \phi_{\text{init}} \ll M_* \), metric perturbation is also calculated as given in Eq. (2.30) and obtain

\[ \kappa_b \approx \begin{cases} 
9/2 & : \phi_{\text{init}} \ll \phi_{\text{crit}} \\
0 & : \phi_{\text{init}} \gg \phi_{\text{crit}}.
\end{cases} \]

(2.35)

As mentioned before, the CMB angular power spectrum obtained by the WMAP is highly consistent with the prediction of the purely adiabatic primordial density fluctuations. Thus, if \( |\kappa_b| \) is too large, resultant CMB angular power spectrum becomes inconsistent with the observations. Indeed, size of the correlated isocurvature fluctuation is constrained to be \( |\kappa_b| \lesssim 0.5 \).

(2.36)

With this constraint, it is obvious that the case with \( \phi_{\text{init}} \ll \phi_{\text{crit}} \) is excluded. Consequently, if the Affleck-Dine mechanism is implemented in the curvaton scenario, one of the following two conditions should be satisfied: (i) \( \phi_{\text{init}} \gtrsim M_* \), or (ii) \( m_\psi \ll m_\phi \). In order to be more quantitative, in the next section, we use numerical method to calculate \( \kappa_b \) (and other quantities) and derive the constraints.

Notice that, in the extreme case where the Affleck-Dine field eventually dominates the universe after the decay of the curvaton, all the components in the universe are generated from the Affleck-Dine field. In this case, entropy fluctuation vanishes and the cosmic density fluctuations are purely adiabatic. (See Eq. (2.18).) Then, \( \kappa_b = 0 \) and the resultant cosmic density fluctuations become consistent with the observations. Thus, hereafter, we concentrate on the cases where the present CMB radiation (as well as other components in the universe) are generated from the decay product of \( \phi \).\(^\#5\)

Before closing this section, we comment on the effects of the primordial fluctuation of the Affleck-Dine field. If the effective mass of the Affleck-Dine field is much smaller than the expansion rate during the inflation, it is expected that the Affleck-Dine field also acquires the primordial fluctuation \( \delta \psi_{\text{init}} \). Since the resultant baryon-to-photon ratio depends on the initial amplitude of \( \psi \), such a primordial fluctuation results in extra entropy fluctuation in the baryonic sector. Assuming no correlation between the primordial fluctuations of

\(^\#5\)In our discussion, for simplicity, it is assumed that the energy density of the Affleck-Dine field is always sub-dominant. Our results are, however applicable to the case where the energy density of the Affleck-Dine field becomes comparable to that of the curvaton at some epoch as far as the present CMB radiation (as well as other components in the universe) are generated from the decay product of \( \phi \).
the curvaton and the Affleck-Dine field, \( \delta \psi_{\text{init}} \) generates uncorrelated baryonic entropy fluctuation. Using Eq. (2.15) with \( \phi_{\text{init}} \sim M_* \), which is required in order to suppress \( \kappa_b \) when \( m_{\phi} \sim m_{\psi} \) as will be seen in the next section, we obtain

\[
S_{b\gamma}^{(\delta \psi)} \sim \frac{\delta \psi_{\text{init}}}{\psi_{\text{init}}}, \tag{2.37}
\]

where \( S_{b\gamma}^{(\delta \psi)} \) is the baryonic entropy fluctuation associated with the primordial fluctuation of the Affleck-Dine field. Defining

\[
\kappa_b^{(\text{uncorr})} \equiv \frac{S_{b\gamma}^{(\delta \psi)}}{\Psi^{(\delta \phi)}_{\text{RD2}}}, \tag{2.38}
\]

we obtain \( \kappa_b^{(\text{uncorr})} \sim \phi_{\text{init}}/\psi_{\text{init}} \). Uncorrelated entropy fluctuation is constrained by the WMAP results and \( \kappa_b^{(\text{uncorr})} \lesssim 3 \). This constraint can be evaded if the initial amplitude of the Affleck-Dine field is not much smaller than that of the curvaton. Of course, if \( \psi \) acquires effective mass as large as the expansion rate during the inflation, \( \delta \psi_{\text{init}} \) vanishes and there is no (uncorrelated) baryonic entropy fluctuation. Hereafter, we consider the case where the effects of the uncorrelated baryonic entropy fluctuation becomes not important, and concentrate on the effects of correlated baryonic entropy fluctuation.

3 Numerical Results

3.1 Metric perturbation

So far, we have seen that the case with \( \phi_{\text{init}} \ll \phi_{\text{crit}} \) is inconsistent with the observations. One possibility of realizing \( \phi_{\text{init}} \gtrsim \phi_{\text{crit}} \) when \( m_{\psi} \sim m_{\phi} \) is to adopt \( \phi_{\text{init}} \gtrsim M_* \). In this case, (a short period of) inflation may occur due to the energy density of the curvaton field and Eq. (2.30) becomes unapplicable. In addition, if \( \phi_{\text{init}} \sim \phi_{\text{crit}} \), analytic estimation of the precise value of \( \kappa_b \) is difficult. Thus, in this section, we use numerical method to calculate \( \Psi \) and \( S_{b\gamma} \) associated with the primordial fluctuation of the curvaton.

The first step is to calculate the metric perturbation generated from \( \delta \phi_{\text{init}} \). We use the initial conditions given in Eqs. (2.24) – (2.27) in the deep RD1 epoch, and numerically solve Eqs. (2.20) – (2.23) as well as the equations for the zero-mode from the RD1 epoch to \( \phi_{\text{D}} \) epoch. We checked that \( \Psi \) becomes constant in the \( \phi_{\text{D}} \) epoch. Metric perturbation in the RD2 epoch is evaluated with the relation \( \Psi^{(\delta \phi)}_{\text{RD2}} = \frac{10}{9} \Psi^{(\delta \phi)}_{\phi_{\text{D}}} \).

In Fig. 1 we plot \( \Psi^{(\delta \phi)}_{\text{RD2}} \) normalized by \( S_{\phi_{\chi}}^{(\delta \phi)} \equiv 2 \delta \phi_{\text{init}}/\phi_{\text{init}} \) as a function of \( \phi_{\text{init}} \). We have checked that the ratio \( \Psi^{(\delta \phi)}_{\text{RD2}}/S_{\phi_{\chi}}^{(\delta \phi)} \) is independent of \( m_{\phi} \). As we have discussed in the previous section, the ratio \( \Psi^{(\delta \phi)}_{\text{RD2}}/S_{\phi_{\chi}}^{(\delta \phi)} \) becomes \( -\frac{2}{3} \) when \( \phi_{\text{init}} \ll M_* \). (See Eq. (2.30).) In this case, the curvaton becomes (almost) non-relativistic matter in the deep RD1 epoch. Once \( \phi_{\text{init}} \) becomes close to \( M_* \), on the contrary, \( H \sim m_{\phi} \) is realized when the energy density of \( \phi \) becomes comparable to that of radiation. Then, \( \Psi^{(\delta \phi)}_{\text{RD2}} \) deviates from the result given in Eq. (2.30) and, as seen in Fig. 1, \( |\Psi^{(\delta \phi)}_{\text{RD2}}/S_{\phi_{\chi}}^{(\delta \phi)}| \) becomes larger than \( \frac{2}{9} \).
Figure 1: $\Psi^{(\delta\phi)}_{\text{RD2}}$ normalized by $S_{\phi \chi}^{(\delta\phi)} \equiv 2\delta\phi_{\text{init}}/\phi_{\text{init}}$ as a function of $\phi_{\text{init}}$.

### 3.2 Baryonic entropy fluctuation

Now, we consider the baryonic entropy fluctuation and discuss constraints on the curvaton scenario with the Affleck-Dine baryogenesis. For this purpose, we numerically solve Eqs. (2.20) – (2.23) simultaneously with the evolution equation of the Affleck-Dine field $\psi$.

In our analysis, entropy fluctuation is evaluated by taking the derivative numerically; denoting the resultant baryon and curvaton number densities with the initial amplitude $\phi_{\text{init}}$ as $n_b(\phi_{\text{init}})$ and $n_\phi(\phi_{\text{init}}) \equiv \rho_\phi(\phi_{\text{init}})/m_\phi$, respectively, the entropy fluctuation between the baryon and the radiation is calculated as

$$S_{b\gamma}^{(\delta\phi)} = \frac{n_b(\phi_{\text{init}} + \delta\phi_{\text{init}})/n_\phi(\phi_{\text{init}} + \delta\phi_{\text{init}}) - n_b(\phi_{\text{init}})/n_\phi(\phi_{\text{init}})}{n_b(\phi_{\text{init}})/n_\phi(\phi_{\text{init}})}. \quad (3.1)$$

With the entropy fluctuation in the baryonic sector, we can calculate the $\kappa_b$ parameter. The result is shown in Fig. 2 on the $\phi_{\text{init}}$ vs. $m_\phi$ plane. The metric perturbation $\Psi^{(\delta\phi)}_{\text{RD2}}$ is determined by $\phi_{\text{init}}$ (with $S_{\phi \chi}^{(\delta\phi)}$ being fixed), and has relatively mild dependence on $\phi_{\text{init}}$. Thus, Fig. 2 primarily shows the behavior of baryonic entropy fluctuation in this model. With this in mind, the behavior of $\kappa_b$ can be easily understood. The important feature is that $S_{b\gamma}^{(\delta\phi)} \to 0$ as $\phi_{\text{init}}$ or $m_\phi$ becomes large enough. This is because, as explained in the previous section, baryonic entropy fluctuation is suppressed if the Affleck-Dine field starts to oscillate after the $\phi$D epoch is realized. In other words, if the expansion rate at the time of the radiation-curvaton equality is larger than $m_\psi$, resultant entropy fluctuations become adiabatic. Since $H_{\text{eq}} \propto m_\phi\phi_{\text{init}}^4$, $m_\phi$ or $\phi_{\text{init}}$ is required to be large enough for suppressing baryonic entropy fluctuations. As a result, $\kappa_b$ becomes close to zero when the combination $m_\phi$ or $\phi_{\text{init}}$ becomes large.

With the constraint on the $\kappa_b$ parameter from the WMAP (2.36), we obtain stringent constraint on the initial amplitude and the mass of the curvaton field. In particular, for
Figure 2: Contours of constant $\kappa_b$ on $m_\phi/m_\psi$ vs. $\phi_{\text{init}}$ plane. The contours correspond to $\kappa_b = 0.5$, 0.05, and 0.005 from below. We take $m_\psi = 100$ GeV. The parameter region below the contour for $\kappa_b = 0.5$ is inconsistent with the WMAP result.

a fixed value of the initial amplitude $\phi_{\text{init}}$, we obtain an lower bound on $m_\phi$. Implications of such a constraint will be discussed in the next section.

4 Implications

In the previous sections, we have studied general constraints without specifying the detailed properties of the curvaton field. Consequently, we have seen that the expansion rate at the time of the radiation-curvaton equality should be larger than the mass of the Affleck-Dine field; otherwise, the Affleck-Dine field starts to move in the RD1 epoch and too large baryonic entropy fluctuation is generated. This fact has serious implications to some of the cases.

In particular, it is important to consider the case where the mass of the curvaton (as well as that of the Affleck-Dine field) is from the effect of the supersymmetry breaking. In this case, mass of the curvaton is expected to be close to that of the Affleck-Dine field and we obtain serious constraint on the initial amplitude of the curvaton.

One of the important cases is that the cosmological modulus field plays the role of the curvaton. The moduli fields in the superstring theory acquire masses from the effects of the supersymmetry breaking and hence their masses are expected to be of the order of the gravitino mass. In addition, their interactions are expected to be suppressed by inverse powers of the gravitational scale $M_*$ and hence, once their amplitudes become smaller than $M_*$, their potentials are well approximated by the parabolic ones. Thus, if one of such moduli fields has non-vanishing amplitude as well as the primordial fluctuation, it
may play the role of the curvaton.

If the mass of the modulus field (which is close to the gravitino mass) is of $O(100 \text{ GeV})$ as in the case of the supergravity-induced supersymmetry breaking, its lifetime becomes much longer than 1 sec. With such a long lifetime, decay of the curvaton occurs after the big-bang nucleosynthesis (BBN) and it spoils the success of the BBN [15]. If the modulus mass becomes larger than $\sim (10 - 100) \text{ TeV}$, on the contrary, the cosmological modulus field may decay before the BBN and it may be a viable candidate of the curvaton. In the scenario of the anomaly-mediated supersymmetry breaking [16], for example, this may be the case. In addition, in the anomaly-mediated models, Wino may become the lightest superparticle; importantly, non-thermally produced Winos from the decay of the cosmological modulus field (i.e., the curvaton) can become the cold dark matter [17]. In such a model, baryon asymmetry of the universe should be generated with very low reheating temperature and with very large dilution factor; as a candidate of the scenario of baryogenesis in such a situation, the Affleck-Dine mechanism has been known to be promising [10].

Although a sizable hierarchy is possible between $m_\phi$ and $m_\psi$ in the anomaly-mediated scenarios, we still obtain remarkable constraint on $\phi_{\text{init}}$. Even for the case with relatively large hierarchy between $m_\phi$ and $m_\psi$ as $m_\phi/m_\psi = 10^2$ ($10^3$), for example, $\phi_{\text{init}}$ should be larger than $1.1M_* (0.7M_*)$; otherwise, too large baryonic entropy fluctuation is generated as seen in Fig. 2. For the case of the anomaly-mediated supersymmetry breaking, the mass of the Affleck-Dine field, which is given by the masses of the MSSM particles, is suppressed by the loop-induced factor of order $\sim 10^{-2} - 10^{-3}$ compared to the gravitino mass. Even in that case, $\phi_{\text{init}} \sim M_*$ is required in order to suppress the baryonic entropy fluctuations.

In fact, if the curvaton field has an initial amplitude as large as $\sim M_*$, it becomes non-trivial if the inflaton contribution to the cosmic density fluctuations is negligible. Importance of the inflaton contribution can be seen by comparing the metric perturbations generated from the fluctuations of the inflaton and the curvaton. The curvaton contribution is given in Eq. (2.30) while the inflaton contribution is given by

$$\Psi_{\text{RD}2}^{(\text{inf})} = \frac{2}{3} \left[ \frac{3H^2_{\text{inf}}}{V'_{\text{inf}}} \times \frac{H_{\text{inf}}}{2\pi} \right]_{k=aH},$$

where $V_{\text{inf}}$ is the inflaton potential, $V'_{\text{inf}} \equiv \partial V_{\text{inf}}/\partial \chi$, and the superscript “(inf)" is for variables generated from the primordial fluctuation of the inflaton field. If $|\Psi_{\text{RD}2}^{(\delta \phi)}| \gg |\Psi_{\text{RD}2}^{(\text{inf})}|$, the curvaton contribution dominates. The relative size depends on the model of the inflation.

For example, for the case of the chaotic inflation with inflaton potential of the form $V_{\text{inf}} \propto \chi^q$, we obtain

$$\frac{\Psi_{\text{RD}2}^{(\text{inf})}}{\Psi_{\text{RD}2}^{(\delta \phi)}} = \left[ \frac{3}{2q} \frac{\phi_{\text{init}} \chi}{M_*^2} \right]_{k=aH_{\text{inf}}},$$

where $\phi_{\text{init}}$ is the initial value of the curvaton field.
where we have used Eq. (2.30) as an approximation. (In fact, as shown in Fig. 1, \(|\Psi_{\text{RD2}}^{(\delta\phi)}|\) becomes larger than the value given in Eq. (2.30) if the initial amplitude of the curvaton becomes comparable to \(M_*\). Then, the ratio becomes smaller.) Using the fact that the inflaton amplitude is \(\sim 15M_*\) when the COBE scale exits the horizon, \(\Psi_{\text{RD2}}^{(\text{inf})}\) becomes comparable to or larger than \(\Psi_{\text{RD2}}^{(\delta\phi)}\) for \(q = 2 - 6\) even if \(\phi_{\text{init}} \sim M_*\). Thus, sizable fraction of the cosmological density fluctuation originates from the inflaton fluctuation; with the chaotic inflation, the inflaton contribution is non-negligible unless there is a large hierarchy between the masses of the curvaton and the Affleck-Dine field. In other words, if we consider the case where \(m_{\phi}\) is close to \(m_{\psi}\), \(\phi_{\text{init}} \sim M_*\) is required and the cosmic density fluctuations may not be dominated by the curvaton contribution.

Assuming no correlation between the inflaton and curvaton fields, the CMB angular power spectrum has the form

\[
C_l = C_l^{(\delta\phi)} + C_l^{(\text{inf})},
\]

where \(C_l^{(\delta\phi)}\) and \(C_l^{(\text{inf})}\) are the CMB angular power spectra for the cases where the primordial fluctuations of the curvaton and inflaton dominates, respectively. As we mentioned, \(C_l^{(\text{inf})}\) may become comparable to (or even larger than) \(C_l^{(\delta\phi)}\). Since the density fluctuations related to the primordial fluctuation of the inflaton field are adiabatic, constraints on \(\phi_{\text{init}}\) (for fixed value of \(m_{\phi}/m_{\psi}\)) can be relaxed as far as the scale dependence of the primordial density fluctuation is negligible. On the contrary, however, if \(\Psi_{\text{RD2}}^{(\text{inf})}\) becomes comparable or larger than \(\Psi_{\text{RD2}}^{(\delta\phi)}\), significant amount of the cosmic density fluctuations are generated from the primordial fluctuation of the inflaton. In this case, of course, it becomes difficult to relax the observational constraints on the inflaton potential with the curvaton, which has been one of the important motivation of the curvaton scenario.

Of course, the chaotic model is not the only possibility of the inflation and, in other case, inflaton contribution to the total cosmic density fluctuations may become minor even though \(\phi_{\text{init}} \sim M_*\). Model of the inflation with large \(H_{\text{inf}}\) and small \(\Psi^{(\text{inf})}\) is considered, for example, in [18].

5 Summary

In this paper, we have discussed the curvaton scenario with Affleck-Dine baryogenesis. Even with a large entropy production due to the decay of the curvaton field, Affleck-Dine mechanism may be able to generate large enough baryon asymmetry and hence, for some of the curvaton scenarios, Affleck-Dine mechanism is a prominent candidate of the scenario of baryogenesis. We have seen, however, that baryonic entropy fluctuation is induced if the Affleck-Dine field starts to move in the RD1 epoch. Since large baryonic entropy fluctuation is inconsistent with the observation (in particular, with the results of the WMAP), this provides constraints on the scenario; as we have seen, in order to evade the constraint, mass of the curvaton or the initial amplitude of the curvaton should be large enough (for fixed value of \(m_{\psi}\)).
We have also discussed implications of such constraints on some scenario of the curvaton. One of the cases where the Affleck-Dine scenario is preferred is that the cosmological modulus field plays the role of the curvaton; in such a case, baryon asymmetry should be generated with large amount of entropy production and with very low reheating temperature. Assuming the anomaly-mediated supersymmetry breaking, the ratio $m_\phi/m_\psi$ is expected to be $10^2 - 10^3$. With such a ratio, we have seen that the initial amplitude of the curvaton is constrained to be larger than $\sim M_*$ in order not to generate too large baryonic entropy fluctuation.

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