Effects of tooth surface modification on planar double-enveloping hourglass worm gear drives

Fayang HE***, Zhaoyao SHI* and Bo YU*

*Beijing Engineering Research Center of Precision Measurement Technology and Instruments, Beijing University of Technology, Pingleyuan 100, Beijing, China, 100124
E-mail: shizhaoyao@bjut.edu.cn

**Shanghai SGR Heavy Industry Machinery Co., Ltd, Jinliu Rd 879, Jinshan District, Shanghai, China, 201506

Received: 18 December 2017; Revised: 4 February 2018; Accepted: 26 March 2018

Abstract

The modification of the planar double-enveloping hourglass worm gear drive can eliminate the phenomenon of the contact line cross and improve the transmission performance. In order to guide the selection of the type and the amount of modification, the equations of the contact zone and rear transition zone on the worm wheel surface are established. According to the position relationship between the contact zone and the rear transition zone on the axial plane and transverse plane, the changes in the tooth surface of the worm wheel obtained by different modification methods are obtained. The tooth surface changes obtained under different modification amounts are further summarized. The structure of the modified worm wheel is finally determined. For Type-II drive processed with the center distance modification, the curvature interference limit line on the surface of the worm wheel can be cut off by the last cutting edge of the hob. The larger the amount of the center distance modification value is, the deeper the depth into the rear transition zone is. Finally, the analysis results were verified by the hobbing experiment and the worm gear drive running test.

Keywords: Hourglass worm gear drive, Modification, Hob, Curvature interference, Transition surface

1. Introduction

The planar double-enveloping hourglass worm gear drives have the larger load capacity and higher transmission efficiency and extensively applied in power transmission. However, there is a twice-contact zone on the tooth surface of the worm wheel. In this zone, the contact lines crosses with each other so that the contact frequency of the two transmission elements is higher and the fatigue pitting resistance may be much lower. This phenomenon can be eliminated through a proper modification, so the modification is necessary. The modification is achieved by changing partial process parameters of the first envelope, so that these parameters are different between the first envelope and the second envelope. These process parameters might be center distance, transmission ratio, relative position, etc. The modified hourglass worm gear drive can still realize the line contact meshing. The center distance modification method is simple and generally adopted.

According to the meshing limit line on the worm wheel teeth surface, the modified drives are divided into two categories(Wu, et al., 1986). When a meshing limit line exists on the worm wheel teeth surface, the worm gear drive is called Type-I drive. When the worm wheel teeth surface has no meshing limit line, the worm gear drive is called Type-II drive. For Type-I drives, a large part of the outlet portion on the worm tooth surface is not engaged into meshing and the effective working length of the surface is short. Moreover, the worm wheel tooth surface consists of a front transition zone and a contact zone and the front transition zone is cut by the first cutting edge of the hob. For Type-II drives, the whole tooth surface of the worm is engaged into meshing and the worm wheel surface consists of a front transition zone, a rear transition zone and a contact zone. Moreover, the front transition zone is cut by the first cutting edge of the hob and the rear transition zone is cut by the last cutting edge of the hob. The contact zone is divided into left part and right part, which are completely separated from each other or partially overlapped. The right boundary of the left contact zone is the contact line at the outlet portion of the hob and the left boundary of the right contact zone is the curvature interference limit line. The zone near the curvature interference limit line shows the poor microscopic engagement quality(Dong 2004).

There are two points of view on the selection of modification. Firstly, it is believed that the oil film strength near
the curvature interference limit line on the worm wheel surface of Type-II drive is low, thus leading to stress concentration (Wu, et al., 1986). According the above point of view, Type-I drive is always preferred and Type-II drive should be avoided as possible. If the curvature interference limit line of Type-II drive is cut off, this type may be adopted. Secondly, other researchers recommended the Type-II drive (Dong 2004; Zhao, et al., 2011; Zhao, et al., 2013). Moreover, the curvature interference limit line can be removed by proper modification and the amount to be removed can be controlled.

For the Type-II drive, there are different views on the selection of the modification value. The modification value of the center distance should be small enough to ensure that the contact line of the outlet portion completely enter the other conjugate zone on the teeth surface of the worm wheel(Zhao, et al., 2011). The curvature interference limit line can be cut off in this way. If the center distance modification value is small and it is difficult to conduct the modification during the machining process, a combined modification of the center distance and transmission ratio might be employed. The larger modification values of the center distance and transmission ratio should be adopted. For the sake of evaluating the effectiveness of undercutting avoidance, the meshing point in the most severe condition is defined (Zhao, et al., 2013). It is pointed out that under the condition that the modification parameters are reasonably chosen, the left and right contact zones will not be overlapped and the rear transition zone will cut off the part with the poor microscopic meshing quality(Feng, et al., 1997). The modification parameters of planar double-enveloping hourglass worm gear drives calculated by empirical formula or general modification formula of Hindley worm gear drives often yield disastrous results (Zhou 2005).

Although the modified drives have been widely used, the modifications of planar double-enveloping hourglass worm gear drives are seldom reported(Dong 2004; Deng, et al., 2013). Moreover, some problems in the selection of the modifications, such as the changes in the tooth surface after the modification, or the removal of curvature interference limit line, were not explored. In this paper, the equations of the contact zone and the rear transition zone of the worm wheel are established and the changes on the tooth surface of worm wheel caused by the modification in the axial plane and transverse plane are analyzed. The study provides the basis for the selection of the type and the amount of modification.

2. Equations of the hourglass worm gear drive

As shown in Fig. 1, the process in which a plane rotates around the axis $Z$ and a hourglass worm rotates around its axial $Z_j$ to form a planar enveloping hourglass worm is called the first envelope. The generating plane is always tangential to the base cone during rotation around axis $Z$. $\beta$ is the inclination angle of the generating plane. The generation process of the worm wheel is called the second envelope in which the worm tooth surface is used as the generating surface. The hourglass worm gear drive composed of the planar enveloping hourglass worm and its enveloped worm wheel is called the planar double-enveloping hourglass worm drive(Maki, et al., 1995; Okamoto and Maki, 1996).

![Fig. 1 Generation process of a planar enveloping hourglass worm](image)

2.1 Equation of hourglass worm

The coordinate system of the first envelope is shown in Fig. 2. $\sigma(O;X,Y,Z)$ and $\sigma_j(O_j;X_j,Y_j,Z_j)$ are static coordinate systems. $\sigma_0(O_0;X_0,Y_0,Z_0)$ is a rotational coordinate system fixed to the generating plane, rotating around axis $Z_0$. $\sigma_i(O_i;X_i,Y_i,Z_i)$ is a rotational coordinate system fixed to the worm, rotating around axis $Z_i$. $Z_j$ is
vertical to $Z$. At the initial moment, $\sigma_0$ and $\sigma_1$ coincide with $\sigma$ and $\sigma_j$, respectively. $X_0$ and $X_1$ are collinear (Litvin 1994; Chen, et al., 2016). $\phi_0$ is the rotation angle of the generating plane, $\phi_1$ is the rotation angle of the worm, $\phi_i = \phi_{j0}$. $i_{0}$ is the transmission ratio of the first envelope, $i_{01} = 1/i_{0}$. $a_1$ is the center distance of the first envelope.

As shown in Fig. 3, $\sigma_j(O_j; X_j, Y_j, Z_j)$ is the reference coordinate system of the generating plane. The coordinates of the point on the generating plane in coordinate system $\sigma_j$ are $(\mu, \nu, \beta)$. The coordinates of the point on the generating plane can be represented in coordinate system $\sigma_0$ as follows:

$$r_{0}(\mu, \nu) = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = \begin{bmatrix} \mu \\ \nu \sin \beta - r_0 \\ \nu \cos \beta \end{bmatrix}$$  

(1)

$$r_1(\mu, \nu, \phi_0, \phi_1, a_1) = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = \begin{bmatrix} \mu \\ \nu \sin \beta - r_0 \\ \nu \cos \beta \end{bmatrix}$$  

(2)

where $M_{1}^{(i)} = -i_{00} \mu \sin \beta$, $M_{2}^{(i)} = i_{00}(r_0 \sin \beta - \nu)$, $M_{3}^{(i)} = \mu \cos \beta - i_{00} a_1 \sin \beta$.

The worm tooth surface $r_0$ produced by the first envelope is provided below:

$$\begin{bmatrix} r_i \\ \phi_i \end{bmatrix} = M_{10}(\phi_0, \phi_1, a_1) r_0(\mu, \nu)$$

(3)

where $M_{10}(\phi_0, \phi_1, a_1)$ represents the coordinate transformation matrix from $\sigma_0$ to $\sigma_j$ (Dong 2004).

The curvature interference limit line function of the first envelope is provided as:

$$\Psi_1 = \mu \cos \beta \sin \phi_0 \cos \phi_1 + \nu \cos \beta \left( \sin \beta \cos^2 \phi_0 + 2i_{00} \cos \beta \cos \phi_0 - i_{00}^2 \sin \beta \right) + \cos \beta \left[ \sin \phi_0 (r_0 \sin \phi_0 - a_1) + i_{00}^2 r_0 \right]$$  

(4)

2.2 Equation of worm wheel contact zone

In the second envelope process, the same coordinate systems in the first envelope are adopted. The difference is
that the generating plane is replaced by the worm wheel. As shown in Fig. 4, \( \sigma_2(O_2;X_2,Y_2,Z_2) \) is the moving coordinate system fixed to the worm wheel. The worm rotates around axis \( Z_1 \), and the worm wheel rotates around axis \( Z_2 \). The rotation angle of worm wheel is \( \theta_2 \) when the worm rotates at the angle \( \theta_1 \). \( \theta_2 = \theta_1 / i_{12} \), \( i_{12} \) is the transmission ratio of the second envelope, \( i_{21} = 1/i_{12} \). The center distance of the second envelope is \( a_2 \).

\[
\Phi^2 = M_1^{(2)} \cos \theta_1 - M_2^{(2)} \sin \theta_1 - M_3^{(2)}
\]

where

\[
M_1^{(2)} = \sin \phi_0 \left[ (\sin \beta + i_{10} \cos \beta \cos \phi_0 \mu - a_1 \sin \beta \cos \phi_0) \right] / \sin \phi_0 + \cos \beta \cos \phi_0 (\mu - a_1 \cos \phi_0) .
\]

\[
M_2^{(2)} = \cos \phi_0 \left[ (\sin \beta + i_{10} \cos \beta \cos \phi_0 \mu - a_1 \sin \beta \cos \phi_0) \right] / \sin \phi_0 - \cos \beta \sin \phi_0 (\mu - a_1 \cos \phi_0) .
\]

\[
M_3^{(2)} = i_{12} \cos \beta (i_{10} \mu - i_{21} a_2 \cos \phi_0) .
\]

The worm wheel tooth contact zone surface \( r_z \) obtained by the second envelope is provided as:

\[
\left\{ \begin{array}{l}
r_z = M_{21}(\theta_1, \theta_2, a_2) r_1(\mu, \phi_0) \\
\Phi^2 = M_1^{(2)} \cos \theta_1 - M_2^{(2)} \sin \theta_1 - M_3^{(2)} = 0
\end{array} \right\} \Rightarrow r_z = r_1(\mu, \phi_0, \theta_1)
\]

where \( M_{21}(\theta_1, \theta_2, a_2) \) is the coordinate transformation matrix from \( \sigma_1 \) to \( \sigma_2 \).

The curvature interference limit line function of the second envelope can be obtained as:

\[
\Psi^2 = \frac{\partial \Phi^2}{\partial t} + \omega_2^{(12)} v_1^{(12)} - \omega_1^{(12)} v_2^{(12)} + k_1^{(12)} (v_1^{(12)})^2 + k_2^{(12)} (v_2^{(12)})^2 + 2 \tau_1^{(12)} v_1^{(12)} v_2^{(12)}
\]

where

\[
p_2 = \cos \theta_1 \cdot \mu - a_1 \cos \phi_0 ,
\]

\[
v_1^{(12)} = -(Z_1 \cos \theta_1 - i_{12} Y_1) \cos \phi_0 \cos \phi_1 + (i_{12} X_1 - Z_1 \sin \theta_1) \cos \phi_0 \sin \phi_1 - (Y_1 \sin \theta_1 - X_1 \cos \theta_1 + a_2) \cos \phi_0 ,
\]

\[
v_2^{(12)} = (Z_1 \cos \theta_1 - i_{12} Y_1) \sin \phi_0 \cos \phi_1 - \sin \phi_1 \cos \beta - (i_{12} X_1 - Z_1 \sin \theta_1) \sin \phi_1 \sin \phi_0 \sin \beta + \cos \phi_1 \cos \beta ,
\]

\[
- (Y_1 \sin \theta_1 - X_1 \cos \theta_1 + a_2) \cos \phi_0 \sin \beta
\]

\[
\omega_1^{(12)} = \cos \theta_1 \cos \phi_0 \sin \phi_1 - \sin \theta_1 \cos \phi_1 \cos \phi_1 - i_{12} \sin \phi_0 ,
\]

\[
\omega_2^{(12)} = \sin \theta_1 (\cos \phi_0 \cos \phi_1 \sin \beta - \sin \phi_0 \cos \beta) - \cos \theta_1 (\sin \phi_0 \sin \phi_0 \sin \beta + \cos \phi_1 \cos \beta) - i_{12} \cos \phi_0 \sin \beta .
\]
\[ \begin{align*}
\alpha_1^{(01)} &= i_{10} \sin \varphi_0, \quad \alpha_2^{(01)} = i_{10} \cos \varphi_0, \quad k_1^{(1)} = -\frac{(\alpha_2^{(01)})^2}{\psi^1}, \quad k_2^{(1)} = -\frac{(\alpha_1^{(01)})^2}{\psi^1}, \quad r_{1e}^{(1)} = \frac{\alpha_1^{(01)} \omega_2^{(01)}}{\psi^1}.
\end{align*} \]

The curvature interference limit line of the second envelope is obtained as:

\[ \begin{cases}
\Phi^2 = M_1^{(2)} \cos \theta_1 - M_2^{(2)} \sin \theta_1 - M_1^{(2)} = 0 \\
\psi^2 = \frac{\partial \Phi^2}{\partial \mu} + a_2^{(12)} v_1^{(12)} - a_1^{(12)} v_2^{(12)} + k_1^{(12)} (v_1^{(12)})^2 + k_2^{(12)} (v_2^{(12)})^2 + 2 r_{1e}^{(12)} v_1^{(12)} v_2^{(12)} = 0
\end{cases} \tag{8} \]

According to the above calculations, the equations of the worm wheel left tooth surface contact zone and the curvature interference limit line are obtained.

### 2.3 Equation of worm wheel rear transition zone

In the actual processing, the worm wheel tooth surface is cut off by hobbing. For the convenience of chip removal and cutting, the hob shall be made with chip letting grooves and relief angles (Liu, et al., 2017; Mohan and Shunmugam, 2009; Jiang, et al., 2017). An actual hob of planar double-enveloping hourglass worm drive is shown in Fig. 5.

![Fig. 5 Cutting edges of hob](image)

For straight groove hob, the equation of the cutting edge \( r'_1 \) is provided as:

\[ \begin{align*}
&\begin{cases}
r'_1 = r_1(\mu, \varphi_0) \\
Y_1 = \tan \theta' \Rightarrow r'_1 = r_1(\mu, \varphi_0, \theta')
\end{cases}
\end{align*} \tag{9} \]

When \( \theta' = (\alpha + zk / 2 \pi \tau) \times i_{10} \), Equation (9) represents the last cutting edge of the outlet portion side of the hob. \( \alpha \) is the nominal pressure angle of the calculated circle and \( \tau \) is the pitch angle.

The rear transition zone \( r'_2 \) formed by the last cutting edge is expressed as:

\[ r'_2 = M_{12}(\varphi_3, \varphi_4, \alpha_1) r'_1(\mu, \varphi_0, \theta') \Rightarrow r'_2 = r'_2(\mu, \varphi_0, \varphi_3, \varphi_4) \tag{10} \]

where \( \varphi_3 \) is the angle of the rotation of the hob cutting edge; \( \varphi_4 \) is the angle of the rotation of the worm wheel during the hobbing, \( \varphi_4 = \varphi_3 / i_{12} \); \( M_{12}(\varphi_3, \varphi_4, \alpha_1) \) is also the coordinate transformation matrix from \( \sigma_1 \) to \( \sigma_2 \).

The parameters of the unmodified worm gear drive to be analyzed are in Table 1, where \( a_1 = a_2, \quad i_{10} = i_{12} \). Under this parameter, the worm wheel of the unmodified drive consists of the single-contact zone and the twice-contact zone(Dong 2004). There is no front transition zone on the worm wheel tooth surface.

| Table 1  | Parameters of worm gear drive |
|----------|------------------------------|
| Parameters | Values |
| Rotation direction | Right |
| \( a_2 \) (mm) | 180 |
| \( i_{12} \) | 25 |
| \( r_b \) (mm) | 60 |
| \( \beta \) (°) | 13 |
| \( d_1 \) (mm) | 65 |
| \( zk \) | 6 |
| \( n_1 \) | 2 |
Notes: $d_1$ is the diameter of the worm reference circle; $z_k$ is the number of hob’s surrounded teeth; $n_t$ is the number of worm threads.

The center distance modification is performed by changing the center distance of the first envelope. Different types of modified drives are obtained when different values of center distance are taken. The value of center distance modification is $\Delta a = a_1 - a_4$. When $\Delta a = 0$, it is an unmodified drive. When $\Delta a < 0$, it is a Type-I drive. When $\Delta a > 0$, it is a Type-II drive. The worm wheel surfaces obtained under different modification amounts are different.

3. Analysis on worm wheel tooth surface

The contact zone and the rear transition zone on the tooth surface of the worm wheel are complex surfaces. The curvature interference limit line is also a spatial curve. In general, their projections in the axial plane of the worm wheel can reflect the composition of the worm wheel tooth surface. Nevertheless, when the contact zone overlaps with the rear transition zone, it is necessary to perform the phase analysis of the transverse plane of the worm wheel in order to determine the actual structure of the tooth surface.

3.1 Projection on the axis plane

For the Type-II drive, the distribution of the contact zone and the curvature interference limit line in the axis plane of the worm wheel are shown in Figs. 6 and 7.

In the range of worm wheel tooth, when $\Delta a = 0.05$, the distribution of the contact zone on worm wheel tooth surface is shown in Fig. 6. GHFEI constitutes the left contact zone $\Sigma_{2B}$ and CDBAJ constitutes the right contact zone $\Sigma_{2C}$. EF is the outlet portion contact line, which is the right boundary of the left contact zone. CD is the curvature interference limit line, which is the left boundary of the right contact zone. The intersection point K between the outlet portion contact line and the curvature interference limit line is below the root of the worm wheel. EF is on the right side of the CD, and CD is removed. The area near the curvature interference limit line becomes a part of the left contact area.

Fig. 6 Contact zone on the worm wheel left tooth surface when $\Delta a = 0.05$

With the increase in $\Delta a$, the worm outlet portion contact line $EF$ moves to the left and the curvature interference limit line $CD$ moves to the right (Feng, et al., 1997). The intersection point of the two lines moves out from the root to the tip of the worm wheel. When $\Delta a = 0.3$, the intersection point K between the outlet portion contact line and the curvature interference limit line is near the tooth tip, as shown in Fig. 7. Therefore, CK is removed and CKE is still a part of the left contact area. To determine whether KD is also removed, it is necessary to analyze the position relationship between the contact zone and the rear transition zone on the transverse plane.

Fig. 7 Contact zone on the worm wheel left tooth surface when $\Delta a = 0.3$

3.2 Projection on the transverse plane

To analyze the relationship between the contact zone and the rear transition zone on the transverse plane of the
worm wheel, it is necessary to calculate the circular arc length. As shown in Fig. 8, Left Tooth Surface 1 is the tooth surface of the unmodified worm wheel calculated from the meshing principle. Left Tooth Surfaces 2, 3, and 4 are the tooth surfaces in the array. A cylindrical surface is established around the axis $Z_2$ of worm wheel with a radius $r$. $L$ is the intersection line between the cylindrical surface and the Tooth Surface 1.

![Fig. 8 Worm wheel intersection line](image)

The intersection equation of the line $L$ is provided as follows:

$$X_2^2 + Y_2^2 = r^2$$

(11)

When $r = d_z/2$, the cylindrical surface corresponds to the pitch cylinder of the worm wheel. $d_z$ is the diameter of the worm wheel reference circle. When $Z_2$ is a constant, perpendicular to the axial direction of the worm wheel to do a transverse plane. The circular arc length of the intersection point between the line $L$ and the transverse plane to the plane $X_2OZ_2$ is determined as:

$$l = -r \cdot \arctan \frac{Y_2}{X_2}$$

(12)

For the Type-II drive, the cylindrical surface intersects with the contact zone and the rear transition zone respectively, and the arc length of the intersection of the contact zone and the rear transition zone needs to be calculated separately. Based on the Eqs. (6), (11) and (12), the circular arc length of the contact zone can be calculated. With $Z_2$ as the abscissa and $l$ as the ordinate, the circular arc length of the contact zone is shown in Fig. 9.

![Fig. 9 Circular arc length of the contact zone](image)

For the contact zone of unmodified worm wheel, the arc length corresponds to the red line in Fig. 9. The right part of the arc is a straight line and the left part is a concave curve. The straight line is tangential to the concave curve. For Type-I drive, when $\Delta a = -0.3$, the arc length is shown in the black line. It is above the unmodified worm wheel. For Type-II drive, when $\Delta a = 0.3$, the arc length is shown as the black dotted line. It is below the unmodified worm wheel and consists of two discontinuous parts. The two discontinuous parts will be connected together by the rear transition zone. Based on the Eqs.(10), (11) and (12), the circular arc length of the rear transition zone can be calculated.

### 3.3 Comparative analysis

Phase analysis indicates that the surface far away from the worm wheel tooth will be removed and that the surface near the worm wheel tooth will be remained. It is assumed that when $Z_2 = 0$, the cylinder is intersecting with Type-II drive Tooth Surface 1 and the unmodified drive Tooth Surface 1 respectively at Point S and Point T. As shown in Fig.
When the deformation $\Delta l = l_s - l_r$ is negative, the cutting depth of Point S is bigger than that of Point T. Point S is closer to the worm wheel entity than Point T, so Point T is removed.

In order to analyze the position relationship between the intersection points, the arc length of the point on the unmodified worm wheel are subtracted from the arc length of the point on the modified worm wheel to obtain the amount of the deformation on the transverse plane.

With $Z_2$ as the abscissa and the amount of the deformation $\Delta l$ as the ordinate. Deformation of Type-I drive is plotted in Fig. 11. For Type-I drive, the entire tooth surface of the worm wheel is the contact zone. The smaller amount of modification means the more convex deformation curve and the shallower worm. Due to the existence of meshing limit line, the last cutting edge of the hob is not engaged into cutting.

For Type-II drive, as shown in Fig. 12, the black line indicates the amount of the deformation in the contact zone and the blue line is the amount of the deformation in the rear transition zone. The left of the blue line is intersects with the left contact zone at Point $E$, and the right of the blue line intersects with the right contact zone at Point $M$. Point $C$ is on the curvature interference limit line. $EM$ is smaller than $CM$, so $CM$ is removed. When $\Delta a = 0.3$, the amount of the deformation of Type-II drive is $G_1E,M_1A_1$. When $\Delta a = 0.05$, the amount of the deformation is $G_4E,M_4A_4$. The points $C$ formed the curvature interference limit line $CD$ when the radius $r$ taking different values. Point $C$ is removed.

![Fig. 10 Analysis of intersection point](image1)

![Fig. 11 Deformation of Type-I drive](image2)

![Fig. 12 Deformation of Type-II drive](image3)
For Type-II drive, with the decrease of $\Delta a$, the curvature interference limit line is gradually replaced by the left contact zone which participates in the meshing. With the increase of $\Delta a$, the curvature interference limit line is gradually replaced by the rear transition zone which does not participate in the meshing. The larger modification amount indicates the more concave deformation curve, the longer track in the axial direction of the worm wheel and the larger part of the right contact zone to be removed.

When different values of $r$ are selected, Point $M$ on the different cylindrical intersection line can be obtained. The connected line $MN$ among these points constitutes the actual left boundary of the right contact zone of the worm wheel. Due to the discontinuous hobbing surface, the intersection lines of the track surfaces formed by the adjacent two cutting edges of the hob can be seen on the surface. When the number of the chip letting grooves is large, the intersection line is dense and the tooth surface is relatively smooth. The distribution of the intersection lines is similar to that of the contact lines (Qin, et al., 1994). The contact line $EF$ of the outlet portion is regarded as the intersection line between the track surfaces of the last cutting edge and its former edge. The intersection line between the rear transition zone formed by the last cutting edge of the outlet portion side and the right contact region is $MN$. When $\Delta a = 0.3$, the tooth surface of the worm wheel after cutting is shown in Fig. 13. $EFMN$ is the rear transition zone.

As the $\Delta a$ increases, when the curvature interference limit line is on the right side of the outlet portion contact line, the curvature interference limit line and its vicinity are also removed. For Type-II drive, when $\Delta a > 0$, the curvature interference limit line is removed. The result of the center distance modification is similar to that of the transmission ration modification (Dong 2004). Therefore, it is unnecessary to use the transmission ration modification or the combined modification when the modification effect can be achieved by the single center distance modification. The center distance modification is also applicable to the other double-enveloping hourglass worm drives with the cone, double cone, or dual torus as the generating surface.

4. Verification experiment

In order to verify whether the curvature interference limit line has been removed, the experiment was conducted with a Type-II drive and the modification amount was set as $\Delta a = 0.3$. The hob adopted the inlaid structure. The chip letting grooves are the straight. The materials of the hob, the worm wheel and the worm are shown in Table 2.

| Items         | Materials      |
|---------------|----------------|
| cutter body   | 40Cr           |
| cutting edge  | W18Cr4V        |
| worm wheel    | ZCuSn10Pb1     |
| worm          | 20CrMnTi       |

The test was conducted at Shanghai SGR Heavy Industry Machinery Co., Ltd. The worm with the center distance of 180.3 mm for the first envelop was processed on the CNC four-axis grinding machine (He, et al., 2015; Chen, et al., 2015). The worm was mounted on the spindle. The grinding head was mounted on the rotary table and the rotary table was installed on the platform which can move along the worm axis and radial direction. The platform carried the arc interpolation motion and the center of arc interpolation was coincided with the center of the main base circle. At the same time, the spindle and rotary table rotated the corresponding angles according to the requirements of the transmission ratio. In this way, the four-axis combined grinding was carried out.

The worm was measured on the special measuring instrument after grinding (Shi, et al., 2015; Luo, et al., 2015). The helix error of the worm was 22.4um and the profile error was 11.2um. They all met the accuracy requirement of...
Class 6 in JB/T9051-2010 where the helix tolerance was 25um, the profile tolerance was 13um. After worm’s grinding, the tooth surface of the hob was ground. To ensure the same accuracy between the worm and the hob without changing the process parameters. The structure of the hob is shown in Fig. 14.

After grinding the relief angles of the hob, the hob and the worm wheel were installed on the Y3180 hobbing machine, as shown in Fig. 15. The position of the hob was adjusted along the axis direction of the worm wheel so that the axis of the hob was coplanar with the middle plane of the worm wheel and adjusted in its axis direction so that the middle plane of the hob was coplanar with the axis of the worm wheel.

After adjusting the relative position of the hob, the second enveloping was performed on the hobbing machine. The hob speed was 80 rpm and the feed rate was 0.4 mm/r. The center distance between the center of the hob and the center of the worm wheel was constantly decreased until it was reduced to 180 mm. The worm wheel tooth after the actual processing is shown in Fig. 16.

Tooth Surface 1 and Tooth Surface 3 were hobbed with the same hob thread, so the two tooth surfaces were the same. The rear transition zones of Tooth Surface 1 and Tooth Surface 3 are different because of the view of angle. It is a continuous surface in the middle of the worm wheel tooth surface and shows the significant differences compared with the contact zone on both sides. If the curvature interference limit line exists, there will be a step at the limit line. The left side of the step is a smaller transition zone and the right side is the contact zone. This is inconsistent with the actual result, indicating that the curvature interference limit line has been removed. Tooth surface distribution in Fig. 16 is consistent with that in Fig. 13.

In order to ensure that the working length of the hob after chipping letting groove is longer than that of the worm, the number of the surrounded teeth of the hob is one tooth more than that of the worm according to the experience. In order to reduce the impact of manufacturing errors and installation errors, the inlet and outlet portions of the worm are processed with tip relief, both at the length of $\tau/4$. The theoretical tooth surface of the worm wheel after running is shown in Fig. 17. $G'H'E''I''$ constitutes the left contact zone $\Sigma_{2B}$ and the $MNB'$ constitutes the right contact zone $\Sigma_{2C}$. The left region of the $GI'$ and the right region of the $MB'$ form the front transition zone. $EFNM$ forms the rear transition zone. After assembling the worm gear pair, with Mobil Glygoyle 320 as lubricating oil, the running
test was carried out according to JBT 5558-91. The worm wheel tooth surface obtained after running is shown in Fig. 18. The theoretical designed tooth surface was consistent with the tooth surface obtained after running, indicating that there was no curvature interference in the worm gear drive processed with the center distance modification.

5. Conclusions

For the planar double-enveloping hourglass worm gear drive processed with a center distance modification, the cutting depth of the worm wheel of Type-I drive is smaller than that of the unmodified drive, whereas the cutting depth of the worm wheel of Type-II drive is bigger than that of the unmodified drive. The larger modification value may lead to the more significant difference between the surface of the modified worm wheel and that of the unmodified worm wheel.

For the planar double-enveloping hourglass worm gear of Type-II drive processed with a center distance modification, the curvature interference limit line on the surface of the worm wheel is removed. As the amount of modification increases, the replacement region of the curvature interference limit line is gradually changed from the left contact zone to the rear transition zone.

The center distance modification can meet the requirements, it is not required to carry out the combined modification. The study results are of the great significance to other types of hourglass worm gear drives.

Nomenclature

\[ \sigma, \sigma_i \] Static coordinate systems
\[ \sigma_0, \sigma_1, \sigma_2 \] Rotational coordinate systems
\[ \sigma \] Reference coordinate system
\[ \mu, V \] Variable parameters, (mm)
\[ a_1 \] Center distance of the first envelope, (mm)
\[ a_2 \] Center distance of the second envelope, (mm)
\[ \beta \] Inclination angle of the generating plane, (°)
\[ r_b \] Radius of the main base circle, (mm)
\[ \Phi^1 \] Meshing function of the first envelope
\[ \Phi^2 \] Meshing function of the second envelope
\[ \Psi^1 \] Curvature interference limit line function of the first envelope
\[ \Psi^2 \] Curvature interference limit line function of the second envelope
\[ r_n \] Position vector of the generating plane
\[ r \] Position vector of the worm surface
\[ r' \] Position vector of the cutting edge
\[ r_2 \] Position vector of worm wheel tooth surface
Position vector of the rear transition zone
\[ r'_2 \]
Coordinate transformation matrices
\[ M_{10}, M_{21} \]
Rotation angle of the generating plane, (°)
\[ \phi_0 \]
Rotation angle of the worm, (°)
\[ \phi_1 \]
Rotation angle of worm wheel, (°)
\[ \theta_1 \]
Rotation angle of worm of the second envelope, (°)
\[ \theta_2 \]
Rotation angle of the hob cutting edge, (°)
\[ \phi_3 \]
Rotation angle of the worm wheel during the hobbing, (°)
\[ i_{10} \]
Transmission ratio of the first envelope
\[ i_{12} \]
Transmission ratio of the second envelope
\[ d_1 \]
Diameter of the worm reference circle, (mm)
\[ z_k \]
Number of hob’s surrounded teeth
\[ n_1 \]
Number of worm threads
\[ \alpha \]
Nominal pressure angle, (°)
\[ \tau \]
Pitch angle, (°)
\[ d_2 \]
Diameter of the worm wheel reference circle, (mm)
\[ l \]
Circular arc length, (mm)
\[ r \]
Radius of the cylindrical surface, (mm)

References

Chen, Y. H., Chen, Y., Wang, J. G. and Zhang, G. H., Manufacturing and measuring investigation of crown worm tooth surface, Journal of Advanced Mechanical Design Systems and Manufacturing, Vol.9, No.1(2015), DOI: 10.1299/jamdsm.2015jamdsm0003.

Chen, Y. H., Chen, Y., Luo, W. J. and Zhang, G. H., A novel backlash-adjustable and wear-compensable hourglass worm drive: computerized design, simulation of meshing and stress analysis, Journal of Advanced Mechanical Design Systems and Manufacturing, Vol.10, No.2(2016), DOI: 10.1299/jamdsm.2016jamdsm0029.

Deng, X. Q, Wang, J. G., Horstemeyer, M. F., Modification design method for an enveloping hourglass worm gear with consideration of machining and misalignment errors, Chinese Journal of Mechanical Engineering(English Edition), Vol.26, No.5(2013), pp.948-956, DOI: 10.3901/CJME.2013.05.948.

Deng, X. Q., Zhu W. B., Chen, Y. H., Chen, S. A., and WANG J. G., Optimal Design for an End Face Engagement Worm Gear with Multiple Worm-Wheel Meshing, Chinese Journal of Mechanical Engineering(English Edition), Vol.30, No.1(2017), pp.144-151, pp: 10.3901/CJME.2016.1025.126

Dong, X. Z., Design and modification of hourglass worm drives(in Chinese), Beijing, China Machine Press(2004), pp.67-117.

Feng, X. Y., Dong, X. Z. and Zhang, D. H., The Best Modification Method of Planar Double Enveloping Worm(in Chinese), Journal of Mechanical Transmission, Vol.21, No.1(1997), pp.14-18, DOI: 10.16578/j.issn.1004.2539.1997.01.005.

He, F. Y., Shi, Z. Y., Li, Z. S., and Yu, B., Hourglass worm multi-axis linkage machining position calibration method(in Chinese), China Patent Licensing(2015).

Jiang, Y. Y., Deng, X. Q., Parametric study on the cylindrical roller enveloping end-face internal engagement worm gear, Manufacturing and measuring investigation of crown worm tooth surface, Journal of Advanced Mechanical Design Systems and Manufacturing, Vol.11, No.2(2017), DOI: 10.1299/jamdsm.2017jamdsm0024.

Litvin, F. L., Gear geometry and applied theory, New Jersey. PTR Prentice Hall (1994), pp.616-625.

Liu, G.Y., Wei, W. J., Dong, X. Z., Rui, C. J., Liu, P. Y. and Li, H. T., Relief grinding of planar double-enveloping worm gear hob using a four-axis CNC grinding machine, International Journal of Advanced Manufacturing Technology, Vol.89, No.9-12(2017), pp.3631-3640, DOI: 10.1007/s00170-016-9325-6.

Luo, W. J., Chen, Y. H. and Zhang, G. H., Helix error testing and tracing on planar enveloping hourglass worm tooth surface(in Chinese), Southwest Jiaotong University, Vol.50, No.2(2015), pp.279-285, DOI: 10.3969/j.issn.0258-2724.2015.02.011.

Maki, M., Okamoto, K. and Midorikawa, I., A study on the hourglass worm gearing whose wheel has the helical
teeth(1st report), Transactions of the Japan Society of Mechanical Engineers, Part C, Vol.6, No.1582(1995), pp.362-366.

Mohan, L. V. and Shunmugam, M. S., Geometrical aspects of double enveloping worm gear drive, Mechanism and Machine Theory, Vol.44, No.11 (2009), pp.2053-2065.

Okamoto, K. and Maki, M., A study on the hourglass worm gearing whose wheel has the helical teeth(2nd report), Transactions of the Japan Society of Mechanical Engineers, Part C, Vol.62, No.601(1996), pp.3642-3646.

Qin, D. T., Tang, J. P. and Zhang, G. H., Geometric research on running-in properties of cone-generated hourglass worm gearing(in Chinese), Mechanical Transmission Vol.18, No.3(1994), pp.46-47, DOI: 10.16578/j.issn.1004.2539.1994.s1.023.

Shi, Z. Y., Yu, B., Ye, Y. and Yan, L., Hourglass hob measuring machine(in Chinese), Optics and Precision Engineering, Vol.23, No.10(2015), pp.2827-2834, DOI: 10.3788/OPE.20152310.2827.

Shi, Z. Y., Yu, B. and He, F. Y., Precision measurement of planar double-enveloping hourglass worms, Measurement, Vol.91(2016), pp.177-185, DOI: 10.1016/j.measurement.2016.05.021.

Wu, H. Y., Zhao, Y. X. and Qi, L., Worm drives design(in Chinese), Beijing, China Machine Press(1986), pp.155-172.

Zhao, Y. P., Kong, J. Y., Li, G. F. and Wu, T. C., Tooth flank modification theory of dual-torus double-enveloping hourglass worm drives, Computer Aided Design, Vol.43, No.12(2011), pp.1535-1544, DOI: 10.1016/j.cad.2011.06.024.

Zhao, Y. P. and Zhang, Y. M., Determination of the most dangerous meshing point for modified-hourglass worm drives, Journal of Mechanical Design, Vol.135, No.3(2013), DOI: 10.1115/1.4023281.

Zhou, L. Y., Modification principle and manufacture technology for hourglass worm drives(in Chinese), Changsha, National University of Defense Technology Press(2005), pp.373.