Accounting for ingrowth of radioactive progeny in dose assessments: generic weighting factors for dose coefficients

Teun van Dillen¹,³, Arjan van Dijk¹, Astrid Kloosterman¹, Federica Russo² and Chantal Mommaert²

¹Centre for Environmental Safety and Security, National Institute for Public Health and the Environment (RIVM), PO Box 1, NL-3720 BA, Bilthoven, The Netherlands
²Bel V, Rue Walcourt 148, B-1070, Brussels, Belgium

E-mail: Teun.van.Dillen@rivm.nl

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Abstract
In this paper, we describe a practical and convenient method to include the contribution of the ingrowth of radioactive progeny in dose assessments of the corresponding parent nuclides. This method modifies the dose coefficients (DCs) of parent nuclides by adding weighted DCs of the corresponding daughter nuclides to them. Based on the decay kinetics of serial nuclear transformations, the progeny weighting factors, with values between 0 and 1, are derived by analysis of the time-integrated activity of each nuclide in the (branched) decay chain headed by a parent nuclide. Using the electronic, nuclear-decay database of Publication 107 of the International Commission on Radiological Protection (ICRP 2008), DC weighting factors for annual dose assessments are calculated for all daughter radionuclides in the decay chains and are tabulated in this paper. Weighting factors based on integration periods other than one year, ranging from 1 h–70 years, are also provided (see the supplementary material). With a priori established weighting factors, dose assessments become significantly simplified by considering the decay kinetics of only the parent nuclides and by applying the modified DCs. This ensures that the ingrowth of progeny is taken into account realistically. In some cases, one requires a conservative estimate of the dose, for instance when dealing with issues of the clearance of materials under regulatory control. Therefore,
we adapted the weighting-factor method to derive conservative DC weighting factors for dose evaluations. These values are calculated for various integration periods and compared with those from an existing method adopted by the Euratom Article 31 Group of Experts and by the International Atomic Energy Agency. Identical progeny weighting factors are obtained for long-lived parent radionuclides, whereas for short-lived parent radionuclides, the new method can yield significantly larger values. For example, the weighting factor of I-131 (daughter of parent Te-131) increases from 0.002 to 1.0 based on an integration period of 1 year. The progeny DC weighting factors, derived based on nuclear transformations in exit-only decay chains, may not always be suitable for use in radiological dose evaluations. For instance, when environmental removal pathways are dominant, the application of these weighting factors may have its limitations. This paper, therefore, provides guidance on the proper selection and application of weighting factors.

Supplementary material for this article is available online

Keywords: ingrowth, radioactive progeny, dose assessment, weighting factor, dose coefficient, parent nuclide, daughter nuclide

(Some figures may appear in colour only in the online journal)

1. Introduction

Dose assessments in radiation protection often involve parent nuclides that decay into radioactive daughter nuclides. It is usually quite straightforward to calculate the dose contribution of the parent nuclides based on a radiological exposure model (public, occupational or medical). However, the contribution related to the (partial) ingrowth and decay of daughter nuclides can be quite complicated, especially when dealing with branched decay chains. Since daughter nuclides may have dose or conversion coefficients comparable to or larger than those of their parent nuclides, such significant contributions cannot be neglected and should therefore be taken into account to avoid any underestimation of the radiological risks.

To estimate dose contributions related to daughter nuclides, one could first calculate the (time-dependent) activities (Bq) or activity concentrations (for example, Bq.g⁻¹ or Bq.cm⁻²) of the daughter nuclides using the well-known Bateman equations (Bateman 1910). These may be modified by the exposure-model’s source terms and removal mechanisms describing the transfer of the radioactive material in the environment. These calculations may then serve as input for the subsequent evaluation of the daughter contributions to the absorbed, equivalent or effective dose of the exposed individual. Such calculations can be done in an exact manner, but will require complicated numerical techniques implementing the exact kinetics of the ingrowth and physical decay of the involved nuclides in the decay chain under consideration. A straightforward and transparent dose estimation based on even a simple radiological dose assessment model can then become quite complicated. Hence, such evaluations may not be preferred for practical reasons.

An alternative and more convenient method is to include the progeny’s contribution into the dose coefficients (internal contamination) or conversion coefficients (external exposure) of the corresponding parent nuclide. Using this practical approach, the dose evaluation itself only considers the activity or activity concentration and its exponential decay kinetics of the
parent nuclide. Instead of using the dose (or conversion) coefficients of only the parent nuclide, one now applies an elevated, total coefficient which includes a proper contribution of the daughter nuclides. This total coefficient is found by adding the coefficients of the daughter nuclides to that of the parent nuclide (head-of-chain, original ancestor) using suitable weighting factors with values between 0 and 1. If these weighting factors are known prior to the assessment, the evaluation of the dose thus remains relatively simple.

In this paper, we describe a method to derive such weighting factors based on the decay kinetics of exit-only, serial nuclear transformations in (branching and merging) decay chains. It is inspired by a method employed in the calculation of dose coefficients for non-human biota (Ulanovsky et al 2008, Ulanovsky and Pröhl 2012, Ulanovsky 2014, ICRP 2017). As this method considers the dose contribution of daughter nuclides, weighting factors are based on the total number of disintegrations within a certain time interval, i.e. the time-integrated activities/activity concentrations. Weighting factors using a default time-integration period of one year for annual dose calculations are tabulated in this paper (appendix A) and its supplementary material, the latter of which is available online at stacks.iop.org/JRP/40/83/mmedia. They are based on the most recent nuclear-decay data compiled in the electronic database of Publication 107 of the International Commission on Radiological Protection (ICRP 2008). Progeny weighting factors for integration periods of 1, 2 and 8 h, 1 and 2 days, 1 week, 1 and 6 months, 1, 2, 5, 10, 50, and 70 years have also been included as supplementary material. Using these values to construct the modified dose (or conversion) coefficients of parent nuclides will realistically take into account the dose contributions related to the ingrowth of their progeny in any exposure model in which the dose is primarily governed by nuclear transformations over the immediate integration period.

In some cases, not only is the realistically estimated dose within the immediate integration period important, but so also is the dose (over a similar integration period) at any time in the future. This may, for instance, occur when dealing with issues of clearance of materials under some form of regulatory control (notification, authorisation). This ‘future’ dose may actually increase over time due to the delayed ingrowth of radioactive progeny, and thus compliance with the dose criteria immediately after clearance would not guarantee future compliance with these criteria. Hence, optimisation of protection against the detrimental effects of exposure to ionising radiation is not guaranteed in the future. To overcome these issues, we also construct a conservative estimate of the dose incurred within the considered (immediate) time-integration interval and derive the corresponding progeny weighting factors for dose coefficients. This dose is evaluated such that it can never be exceeded by the ingrowth of progeny, and therefore the use of these conservative weighting factors should then guarantee future compliance (here, within one century) with the dose criteria as well. This pragmatic method may not always yield an exact answer, but as long as the outcome represents a conservative estimate of the radiological dose, it will usually suffice for radiation-protection purposes.

The (conservative) approach using progeny weighting factors is not new and has been adopted earlier by the Euratom Article 31 Group of Experts (EC 2000) and by the International Atomic Energy Agency (IAEA 2005) who studied the annual effective dose in the context of exemption and clearance. In both publications, these conservative factors have been determined for the radioactive progeny of over 100 parent nuclides using a technique that is based on the activity (concentration) instead of its integrated counterpart as employed in the current study. A detailed comparison of these classical values with the new values based on the alternative method proposed in this paper (conservative approach) shows that identical weighting-factor values are obtained for long-lived parent radionuclides, whereas classical values may be significantly smaller than new values for short-lived radionuclides. Here, ‘short-lived’ and ‘long-lived’ should be regarded with respect to the integration period of the scenario considered in the dose assessment, as will be explained in section 3.4. The
conservative counterparts of the progeny DC weighting factors for dose assessments are also tabulated in this paper and its supplementary material for both the classical and the new method.

Weighting factors (realistic and conservative) for a time-integration period of one year are listed in table A1 of appendix A for the 118 parent radionuclides in IAEA (2004, 2005) and in Council Directive 2013/59/Euratom (EC 2014) that have radioactive progeny. Weighting-factor values for other nuclides and other time-integration periods can be downloaded as supplementary material. A brief description of the numerical implementation of the mathematical methods to calculate the various weighting factors can be found in appendix B of this paper. Guidance on how to perform dose assessments using progeny DC weighting factors and on how to select the proper values is discussed in section 3.5.

2. Methods

2.1. Definition of the generic, progeny weighting factors for DCs

As mentioned in the introduction, the total dose coefficient (internal contamination) or conversion coefficient (exposure to external radiation), $DC_{\text{tot}, P}$, of parent nuclide $P$ (head-of-chain) with $n_D$ progeny nuclides is found by the following equation:

$$DC_{\text{tot}, P} = DC_P + \sum_{j=1}^{n_D} w_{D,j} DC_{D,j},$$

(1)

where $DC_P$ is the dose coefficient of the parent nuclide alone and $DC_{D,j}$ that of progeny nuclide $j \in \{1, 2, \ldots, n_D\}$ in the decay chain headed by parent nuclide $P$, the original ancestor. Progeny nuclides include direct daughter nuclides and progeny nuclides further in the decay chain (daughters of daughters). The dimensionless, progeny DC weighting factors are indicated by $w_{D,j}$ and attain values between 0 and 1. Values for this quantity are derived and tabulated in this paper. For the new method described in the next section, values also depend on the integration period $\tau$ of the dose assessment, and thus $w_{D,j} = w_{D,j}(\tau)$.

The weighting factors are calculated for all progeny nuclides in the entire decay series which may include branching and merging chains. A fictive example of such a complex decay chain is shown in figure 1: the parent (head-of-chain) nuclide and progeny nuclide 3 have more than one mode of decay (branching). Progeny nuclide 2 not only grows directly from the head-of-chain nuclide, but also from progeny nuclide 1 (merging). Similarly, progeny nuclide 4 grows by decay of the two preceding progeny nuclides 2 and 3. Branching ratios indicate the fractions of decays of a nuclide to a daughter nuclide following a certain decay mode. For each parent or daughter nuclide, the sum of its branching ratios always equals 1. Therefore, $v_1 + v_2 + v_3 = 1$ for the decay modes of the parent nuclide and $v_4 + v_5 = 1$ for the decay modes of daughter nuclide 3.

The weighting factors derived in this paper are applicable to dose (conversion) coefficients that do not yet take into account the contribution of radioactive progeny. The dose coefficients for internal contamination (inhalation, ingestion) already include the contribution of radioactive progeny after intake of their corresponding (head-of-chain) parent radionuclide. However, the methods and weighting factors described in this paper remain useful and applicable to these DCs as well, since they take into account the ingrowth of radioactive progeny before and at the moment of intake.
2.2. New method for determining progeny weighting factors

Any evaluation of the absorbed, equivalent or effective dose $D(t)$ over a certain time interval $[0, t]$ should consider the material’s total number of disintegrations $U(t)$ in this interval. For a nuclide $i$, this is given by

$$U_i(t) = \int_0^t A_i(t') \, dt',$$

where, $A_i = A_i(t)$ is the material’s time-dependent activity (Bq) or activity concentration (Bq.g$^{-1}$ or Bq.cm$^{-2}$). In the latter case, $U_i$ denotes the number of disintegrations per g or per cm$^2$ of contaminated material or surface, respectively. The corresponding external-radiation dose (for example, groundshine, cloudshine, submersion, skin contamination, direct exposure from a contaminated source) or the committed dose from the intake of radionuclides (for example, inhalation, ingestion, wound contamination, injection) in Gy or Sv accumulated in this interval is proportional to the number of disintegrations and the dose or conversion coefficient, i.e.

$$D_i(t) = c_i \cdot DC_i \cdot U_i(t) = c_i \cdot DC_i \cdot \int_0^t A_i(t') \, dt'.$$

Here, the constant of proportionality $c_i$ represents the collection of all other relevant parameters in the dose evaluation for the exposure pathway under consideration. Note that symbol $D$ is merely used to indicate a generic dose, but symbols $H$ (equivalent dose) and $E$ (effective dose) could be used as well, depending on the type of dose coefficient one considers.

If parent nuclide $i = P$ heads a decay chain consisting of one or more radioactive progeny nuclides, their dose contributions based on equation (3) should be taken into account. In this paper, we assume that initially the material is only contaminated with the parent nuclide, $A_P(0)$, that there are no additional source terms generating or increasing the number of radionuclides, and that there are no other mechanisms by which activity is removed besides radioactive decay. In other words, we consider exit-only chains based on nuclear transformations: physical decay
and ingrowth. Then, by use of equation (3), the total dose in the immediate time-integration interval \( [0, \tau] \) reads:

\[
D_{\text{tot},p}(\tau) = c_p DC_p U_p(\tau) + \sum_{j=1}^{n_p} c_{D,j} DC_{D,j} U_{D,j}(\tau)
\]

\[
= c_p U_p(\tau) \left[ DC_p + \sum_{j=1}^{n_p} \left( \frac{c_{D,j}}{c_p} \right) \left( \frac{U_{D,j}(\tau)}{U_p(\tau)} \right) DC_{D,j} \right]
\]

\[
= c_p DC_{\text{tot},p}(\tau) U_p(\tau),
\]

with

\[
DC_{\text{tot},p}(\tau) = DC_p + \sum_{j=1}^{n_p} \left( \frac{c_{D,j}}{c_p} \right) \left( \frac{U_{D,j}(\tau)}{U_p(\tau)} \right) DC_{D,j}.
\]

The total dose related to the parent nuclide \emph{including progeny} \( D_{\text{tot},p} \), can thus be written in the same form as equation (3), regarding only the number of disintegrations of the parent nuclide \( (UP) \), but by applying a modified dose coefficient \( DC_{\text{tot},p} \) which properly takes account of the ingrowth of progeny. If we assume that pathway- and scenario-related parameters in the dose evaluation are equal for parent and daughter nuclides, i.e. \( c_p = c_{D,j} \), a comparison of equations (1) and (5) results in:

\[
w_{D,j} = w_{D,j}(\tau) = \frac{U_{D,j}(\tau)}{U_p(\tau)} = \frac{\int_0^\tau A_{D,j}(\tau')d\tau'}{\int_0^\tau A_p(\tau')d\tau'}.
\]

The progeny DC weighting factors \( w_{D,j}(\tau) \) are therefore given by the ratio of time-integrated activities (number of disintegrations) determining the dose components in the immediate time-integration interval \( [0, \tau] \), and they cannot exceed the value of 1: \( 0 \leq w_{D,j} \leq 1 \). Dose evaluations based on weighting factors evaluated by equation (6) may thus provide a realistic estimate of the total dose (parent and progeny) incurred within a period \( \tau \) for the exposure pathways considered.

For an unbranched, two-nuclide (parent–daughter) decay chain, the weighting factor based on equation (6) can be written as

\[
w_{D,1}(\tau) = \frac{\lambda_{D,1}}{\lambda_{D,1} - \lambda_p} \left[ 1 - \frac{\Psi(\lambda_{D,1}\tau)}{\Psi(\lambda_p\tau)} \right],
\]

with \( \lambda_p \) and \( \lambda_{D,1} \) the physical decay constants (in s\(^{-1}\), h\(^{-1}\), d\(^{-1}\) or y\(^{-1}\)) of the parent and first daughter nuclide, respectively. The function \( \Psi \) in equation (7) is defined as

\[
\Psi(x) = \begin{cases} 
1 & \text{for } x = 0 \\
1 - \exp[-x] \quad \text{for } x > 0 
\end{cases}
\]

From equation (7), it follows that \( w_{D,1}(0) = 0 \) and \( w_{D,1}(\tau) \to 1 \) for \( \tau \to \infty \). In the latter case, the total number of disintegrations of parent and daughter nuclides is equal and the weighting factor based on equation (6) results in a value of 1. For branched decay, the weighting factor of those daughter nuclides that grow only by \emph{direct} decay from the head-of-chain parent nuclide can be calculated analytically by equation (7) multiplied by the corresponding branching ratio of the parent nuclide. This would, for instance, hold for the progeny nuclides 1 and 3 in the example of figure 1, for example, \( w_{D,3}(\tau) = (\nu_3 \lambda_{D,3}/(\lambda_{D,3} - \lambda_p))[1 - \Psi(\lambda_{D,3}\tau)/\Psi(\lambda_p\tau)] \).
Progeny weighting factors based on equation (6) were first proposed by Ulanovsky et al. (2008, 2012) for application in the calculation of dose coefficients for non-human, terrestrial and aquatic biota. More recently, the method is proposed in ICRP Publication 136 (ICRP 2017). These factors were applied in DCs for terrestrial organisms above radioactive sources in soil (Ulanovsky 2014) and can be calculated by an online calculation tool BiotaDC developed as a complement to ICRP Publication 136 (ICRP 2017, http://biotadc.icrp.org/). We here suggest the same method for use in dose assessments related to the protection of humans against the detrimental effects of exposure to ionising radiation. Weighting-factor values based on the nuclear-decay data in the electronic database from ICRP Publication 107 (ICRP 2008) have been determined for \( \tau = 1, 2 \) and 8 h, 1 and 2 days, 1 week, 1 and 6 months, 1, 2, 5, 10, 50, and 70 years. The results are presented in section 3, appendix A and the supplementary material. Appendix B briefly describes the numerical algorithm used to derive these values.

2.3. Conservative method for determining progeny weighting factors

As a result of the delayed ingrowth of progeny, possibly over many years (for example, Am-241 from Pu-241), the total dose including the contribution of radioactive progeny may increase over time. Compliance with a dose criterion in the immediate interval \([0, \tau]\), i.e. \(D_{\text{tot},P}(\tau) < \text{dose criterion}\), may then not necessarily guarantee compliance with this criterion in a future interval starting at a certain time \(t: [t, t + \tau]\). This may be an issue when a regulatory body or other governmental body needs to take a decision on granting ‘clearance’, by which regulatory control—imposed on a radioactive material or object within a notified or authorised facility or practice—is removed (IAEA 2007).

To overcome these issues, we developed a conservative counterpart of the method developed in section 2.2. Compliance with the dose criteria in the immediate interval \([0, \tau]\) then also guarantees future compliance if these conservative weighting factors are used for the underlying dose assessment. The adapted method investigates the cumulative dose \(\tilde{D}_i(t; \tau)\) related to nuclide \(i\) incurred in an interval \([t, t + \tau]\). By use of equations (2) and (3), this dose can be written as:

\[
\tilde{D}_i(t; \tau) = D_i(t + \tau) - D_i(t) = c_i DC_i \cdot \int_t^{t+\tau} A_i(t') dt' \equiv c_i DC_i \cdot \tilde{U}_i(t; \tau),
\]

and is proportional to the number of disintegrations in the scenario’s integration period, \(\tilde{U}_i(t; \tau) = U_i(t + \tau) - U_i(t)\). For a decay chain headed by nuclide \(i = P\) we can then write, similar to equation (4)

\[
\tilde{D}_{\text{tot},P}(t; \tau) = D_{\text{tot},P}(t + \tau) - D_{\text{tot},P}(t) = c_P DC_{\text{tot},P}(t; \tau) \tilde{U}_P(t; \tau),
\]

with a total dose coefficient equal to

\[
DC_{\text{tot},P}(t; \tau) = DC_P + \sum_{j=1}^{20} \left( \frac{C_{D,j}}{\tilde{U}_j(t; \tau)} \frac{\tilde{U}_{D,j}(t; \tau)}{\tilde{U}_P(t; \tau)} \right) DC_{D,j}.
\]

Defining the progeny weighting factor as \(w_{D,j} = w_{D,j}(t; \tau) = \tilde{U}_{D,j}(t; \tau) / \tilde{U}_P(t; \tau)\) (assuming again that \(c_P = c_{D,j}\)), similar to what was done in section 2.2, has two main problems. First, the weighting factor is not a constant value but is time-dependent, and second, values larger than 1 can be obtained. In fact, the weighting factor will diverge \((w_{D,j} \to \infty)\) for chains not satisfying the conditions of transient or secular equilibrium. To circumvent these problems, we developed a method with subtle adjustments of equations (10) and (11) that results in a more robust definition of a conservative weighting factor. This conservative approach
considers the total dose as the sum of the maximum dose contribution of the parent radionuclide and the maximum dose contributions of each progeny radionuclide. Since these maxima will generally not occur within the same time interval, this method is therefore, by definition, conservative. More specifically, we can write \( D_{\text{tot}}(t; \tau) \leq \max \left[ D_{\text{tot}}(t; \tau) \right] = \max \left[ \tilde{D}_P(t; \tau) + \sum_{j=1}^{n_0} \tilde{D}_{D_j}(t; \tau) \right] \leq \max \left[ \tilde{D}_P(t; \tau) \right] + \sum_{j=1}^{n_0} \max \left[ \tilde{D}_{D_j}(t; \tau) \right] \) where our method considers the conservative, total dose \( D_{\text{tot}}(\tau)_{\text{cons}} \) on the right-hand side. The method is detailed below with the results given by equations (15) and (16).

From equation (9) it follows that, for conservative dose calculations, the dose contribution related to only the parent nuclide is maximum if the number of disintegrations \( \tilde{U}_P(t; \tau) \) within \([t, t + \tau]\) is maximum. This quantity can be written as

\[
\tilde{U}_P(t; \tau) = A_P(0)\Psi(\lambda_P\tau)\exp\left[-\lambda_P t\right],
\]

where \( A_P(0) \) is the initial activity or activity concentration of the parent radionuclide heading the decay chain and where \( \Psi(x) \) is defined in equation (8). \( \tilde{U}_P(t; \tau) \) has its maximum at \( t = 0 \), i.e. the maximum number of disintegrations of parent nuclide \( P \) occurs in the immediate interval \([0, \tau]\), with \( \tilde{U}_P(0; \tau) = U_P(\tau) = A_P(0)\Psi(\lambda_P\tau)\tau \) disintegrations, which is further employed in this approach. However, instead of using the corresponding number of daughter disintegrations, \( \tilde{U}_{D_j}(0; \tau) = U_{D_j}(\tau) \), which would then simply lead to the weighting factor already given by equation (6) in section 2.2, we use the maximum number of daughter disintegrations \( \tilde{U}_{D_j}(t_j^*; \tau) \) occurring within a time interval of duration \( \tau \), starting at some future time \( t_j^* \). In this method, the first 100 years are considered, similar to a classical method for DC weighting factors developed for dose evaluations in the field of exemption and clearance (EC 2000, IAEA 2005). In the classical method, described in section 2.4, one century for consideration of exposure scenarios is regarded to be a reasonable time span, after which significant mixing of the exempted or cleared material may be assumed to occur. Therefore, the value of \( t_j^* \) can be written as

\[
t_j^* = \arg \max_{0<j<100} \tilde{U}_{D_j}(t; \tau) = \min \left\{ 100 \text{ y}, \arg \max_{t \geq 0} \tilde{U}_{D_j}(t; \tau) \right\},
\]

where \( \arg \max_{t \geq 0} \tilde{U}_{D_j}(t; \tau) \) can be found by setting the time derivative of \( \tilde{U}_{D_j}(t; \tau) \) to 0:

\[
d\tilde{U}_{D_j}(t; \tau)/dt = 0.
\]

If the actual maximum of \( \tilde{U}_{D_j}(t; \tau) \) from equation (14) occurs after a century, the value of \( t_j^* \) is truncated at 100 years. The progeny dose contribution is then regarded over the time interval \([100, 100 + \tau]\) y.

Finally, the conservative weighting factor is then defined as (assuming \( c_P = c_{D_j} \)):

\[
\tilde{w}_{D_j}(\tau) = \frac{\tilde{U}_{D_j}(t_j^*; \tau)}{\tilde{U}_P(0; \tau)} = \frac{\tilde{U}_{D_j}(t_j^*; \tau)}{U_P(\tau)} = \frac{\tilde{U}_{D_j}(t_j^*; \tau)}{A_P(0)\Psi(\lambda_P\tau)\tau},
\]

which is similar to equation (6), but with \( \tilde{w}_{D_j}(\tau) = \tilde{U}_{D_j}(0; \tau) \) replaced by its maximum counterpart \( \tilde{w}_{D_j}(t_j^*; \tau) \). The property \( 0 \leq \tilde{w}_{D_j}(\tau) \leq 1 \) still holds, since no progeny radionuclide can have more disintegrations within any arbitrary time interval \([t, t + \tau]\) than the number of transformations of the corresponding head-of-chain parent nuclide within the immediate interval \([0, \tau]\) of equal duration \( \tau \). If \( \tilde{U}_{D_j}(t; \tau) \leq \tilde{U}_P(0; \tau) \) for each \( t \geq 0 \). This is because the maximum number of head-of-chain disintegrations already occurs in interval \([0, \tau]\) and because the transformation of progeny radionuclides can only occur after preceding parent disintegrations.
In summary, as in section 2.2, the dose assessment again explicitly considers the number of parent (head-of-chain) disintegrations \( U_P(\tau) \) within the immediate time interval \([0, \tau]\) as given by equations (4) or (10) with \( t = 0 \), but with \( DC_{\text{tot},P}(\tau) \) (or \( DC_{\text{tot},P}(0; \tau) \) in equation (10)) replaced by its conservative value:

\[
\tilde{DC}_{\text{tot},P}(\tau) = DC_P + \sum_{j=1}^{n_P} \tilde{w}_{D,j}(\tau) DC_{D,j}.
\]

In equation (16), the conservative weighting factors \( \tilde{w}_{D,j}(\tau) \) are given by equation (15). This conservatively estimated dose, \( \tilde{D}_{\text{tot},P}(0; \tau)_{\text{cons}} = DC_{\text{tot},P}(\tau)_{\text{cons}} = \epsilon_P \tilde{DC}_{\text{tot},P}(\tau) U_P(\tau) \), will not be exceeded in the future, at least not within the considered time span of a century.

For an unbranched decay chain, the interval starting time \( t^*_i \) for which the first daughter has a maximum number of disintegrations over duration \( \tau \), is given by

\[
t^*_i = \min \left\{ 100y, \frac{1}{\lambda_{D,1} - \lambda_P} \cdot \ln \left[ \frac{\lambda_{D,1} \cdot \Psi(\lambda_{D,1} \tau)}{\lambda_P \cdot \Psi(\lambda_P \tau)} \right] \right\},
\]

and results in a weighting factor equal to

\[
\tilde{w}_{D,1}(\tau) = \begin{cases} 
\left( \frac{\lambda_{D,1} \cdot \Psi(\lambda_{D,1} \tau)}{\lambda_P \cdot \Psi(\lambda_P \tau)} \right)^{100y - \tau} & \text{for } t^*_i < 100\text{ y} \\
\frac{\lambda_{D,1}}{\lambda_{D,1} - \lambda_P} \left[ \exp[-100\lambda_P \tau] - \exp[-100\lambda_{D,1} \tau] \cdot \frac{\Psi(\lambda_{D,1} \tau)}{\Psi(\lambda_P \tau)} \right] & \text{for } t^*_i = 100\text{ y} 
\end{cases}
\]

In the second part of equation (18), for \( t^*_i = 100\text{ y} \) years, the decay constants \( \lambda_P \) and \( \lambda_{D,1} \) are expressed in units of \( y^{-1} \) (and \( \tau \) should thus be expressed in years as well). Equations (17) and (18) follow directly from equations (13)–(15) by using

\[
\tilde{U}_{D,1}(t; \tau) = \frac{\lambda_{D,1} \rho_P(0) \tau}{\lambda_{D,1} - \lambda_P} \left( \exp[-\lambda_P t] \Psi(\lambda_P \tau) - \exp[-\lambda_{D,1} t] \cdot \Psi(\lambda_{D,1} \tau) \right).
\]

For branched decay, the conservative weighting factor of those daughter nuclides that grow in only by direct decay from the head-of-chain parent nuclide can be calculated analytically by equation (18) multiplied by the corresponding branching ratio of the parent nuclide. For the example presented in figure 1, equation (18) applies to progeny nuclides 1 and 3 (multiplied by \( \nu_1 \) and \( \nu_3 \), respectively).

A description of the numerical evaluation of \( \tilde{w}_{D,j}(\tau) \) based on equations (13) and (15) for any progeny nuclide \( j \) is given in appendix A. Results are presented in section 3, appendix A and the supplementary material.

2.4. Classical, conservative method for determining progeny weighting factors

The classical method of determining conservative progeny weighting factors for dose assessments is briefly described in EC (2000) and IAEA (2005) in the context of exemption and clearance. Such dose evaluations usually consider an integration period of \( \tau = 1 \) year, since compliance with the annual dose constraint is required (for example, \( 10 \mu Sv \cdot y^{-1} \) for the effective dose incurred by a member of the public by exposure to artificial nuclides). The classical method seems similar to the new method described in section 2.3, but is conceptually different as it considers the activity or activity concentration of the parent and progeny nuclides instead of their time-integrated counterparts. This means that the integration period \( \tau \)
of the dose calculation is not explicitly taken into account in the derivation of the DC weighting factor.

The initial conditions of the model are identical to those assumed in sections 2.2 and 2.3. At $t = 0$, the material is only contaminated with a parent (head-of-chain) radionuclide with an activity or activity concentration $A_P(0)$. Based on nuclear transformations only (i.e. in the absence of other source terms or removal mechanisms), the time dependence of the activity (concentration) of the progeny nuclides is regarded: $A_{D_j}(t)$, with $A_{D_j}(0) = 0$ Bq (or Bq/g). The conservative weighting factors $\tilde{w}_{D_j}$ in equation (16) are then determined as follows. For each progeny nuclide $j$, the maximum value of $A_{D_j}(t)$ is determined within a time span of 100 years (see section 2.3). If an actual maximum occurs within this time frame, at $t = t_{\text{max},j} \leq 100$ y, this absolute, maximum value is used to determine the value of $\tilde{w}_{D_j}$. Although not explicitly stated in EC (2000) and IAEA (2005), and to our understanding, the value of $t_{\text{max},j}$ is thus set at 100 years in case the actual, absolute maximum occurs after the one century time span. The value of the weighting factor is then determined as the activity ratio

$$\tilde{w}_{D_j} = \frac{A_{D_j}(t_{\text{max},j})}{A_P(0)},$$  

where

$$t_{\text{max},j} = \text{arg max}_{0 \leq t \leq 100 \text{ y}} A_{D_j}(t) = \min\left\{100 \text{ y}, \text{ arg max}_{t \geq 0} A_{D_j}(t)\right\},$$  

as described above.

For the first progeny nuclide ($j = 1$) in an unbranched decay chain, which grows in by direct decay from parent nuclide $P$, the value of $t_{\text{max},1}(y)$ can be found analytically by setting the time derivative of $A_{D_1}(t)$ to 0, resulting in (IAEA 2005):

$$t_{\text{max},1} = \min\left\{100 \text{ y}, \frac{\ln (\lambda_{D_1}/\lambda_P)}{\lambda_{D_1} - \lambda_P}\right\}.$$  

The weighting factor $\tilde{w}_{D_1}$ can be by found by inserting equation (22) into equation (20), resulting in:

$$\tilde{w}_{D_1} = \begin{cases} \left(\frac{\lambda_{D_1}}{\lambda_P}\right)^{\frac{\lambda_P}{\lambda_{D_1} - \lambda_P}} & \text{for } t_{\text{max},1} < 100 \text{ y} \\ \left(\frac{\lambda_{D_1}}{\lambda_P}\right) - \frac{\exp[-100\lambda_P] - \exp[-100\lambda_{D_1}]}{\lambda_{D_1} - \lambda_P} & \text{for } t_{\text{max},1} = 100 \text{ y} \end{cases}.$$  

In the second part of equation (23), for $t_{\text{max},1} = 100$ years, the decay constants $\lambda_P$ and $\lambda_{D_1}$ are again expressed in units of $\text{y}^{-1}$. The weighting factor of those progeny nuclides growing in only by direct decay from the head-of-chain parent nuclide can be calculated analytically by equation (23), multiplied by the corresponding branching ratio of the parent–daughter combination. For the example illustrated in figure 1, if $t_{\text{max},3} < 100$ years, the weighting factor of progeny nuclide 3 is $\tilde{w}_{D_3} = \nu_3 \cdot (\lambda_{D_3}/\lambda_P)^{\lambda_P/(\lambda_{D_3} - \lambda_P)}$.

A brief description of the numerical algorithm to calculate the classical weighting factors based on equations (20) and (21) is given in appendix B. Results are presented in section 3, appendix A and the supplementary material. Values can also be found in EC (2000) and IAEA (2005).
3. Results and discussion

We will present the numerical results of the progeny weighting factors for dose or conversion coefficients based on the methods described in sections 2.2 through 2.4. In section 3.1, we start with a simple, fictive example for an unbranched decay chain containing four radionuclides and compare the weighting factors resulting from the three methods. In section 3.2, we continue with a graphical comparison of the three weighting factors by showing contour plots of $w_{Dj}(1 \text{ y})$, $\tilde{w}_{Dj}(1 \text{ y})$ and $\hat{w}_{Dj}$ for the first progeny nuclide in an unbranched chain. Several realistic parent–daughter cases will be compared here as well. Weighting-factor values for 118 progeny-carrying, parent nuclides, presented in appendix A, are described and compared in section 3.3. For these nuclides, weighting factors have also been determined in EC (2000) and IAEA (2005). Values in sections 3.1 through 3.3 are based on an integration period of $\tau = 1 \text{ year}$. Progeny weighting factors for all 783 parent nuclides with at least one radioactive daughter nuclide can be found in the supplementary material for various integration periods ($\tau = 1, 2$ and 8 h, 1 and 2 d, 1 week, 1 and 6 months, 1, 2, 5, 10, 50, and 70 years). In section 3.4, the effect of the integration period $\tau$ on the weighting-factor value is examined. Finally, in section 3.5, we discuss when, how and which weighting factors must be applied in practical dose calculations.

3.1. A simple, fictive example: unbranched decay chain with four long-lived radionuclides

We first consider the example of an unbranched, fictive decay chain as illustrated in figure 2. The nuclides’ half-lives are all significantly larger than the integration period of 1 year (physical decay constants $\ll 0.7 \text{ y}^{-1}$) and each branching ratio equals 1. Initially, a material is only contaminated with parent nuclide $P$ (half-life: 30 years) with an activity concentration of 1.0 Bq g$^{-1}$. Figure 3 shows the time dependence of the activity concentration of the parent nuclide (exponential decay) and of its three radioactive progeny nuclides (daughter, grand-daughter, great-granddaughter) with half-lives of 15, 20 and 90 years, respectively. In this section, we compare the weighting-factor values according to the three methods described in sections 2.2 through 2.4 related to dose evaluations with an integration period of $\tau = 1 \text{ year}$: $w_{Dj}(1 \text{ y})$ and $\tilde{w}_{Dj}(1 \text{ y})$. These are given by equations (6), (15) and (20) and the results are listed in table 1.

In this example for long-lived radionuclides, the ingrowth of daughter nuclides in this first year is small with activity concentrations (after 1 year) of only $4.5 \times 10^{-2}$, $7.7 \times 10^{-4}$ and $2.0 \times 10^{-6}$ Bq g$^{-1}$, respectively. Consequently, the first year weighting factor for realistic dose calculations, $w_{Dj}(1 \text{ y})$, attains small values as well, as seen from the values in the second column of table 1. Depending on the value of the dose coefficient of the progeny nuclides, their corresponding contributions to the annual dose may still be significant compared to that of the parent nuclide: $DC_{P}$ should then be at least several orders of magnitude larger than $DC_{P}$.

The new weighting-factor values for conservative dose calculations, $\tilde{w}_{Dj}(1 \text{ y})$, according to equation (15) in section 2.3, are listed in the third column of table 1. The maximum annual number of daughter disintegrations within a one century time span is reached at $t_j = 29.5$, 59.5 and 100 years for progeny radionuclide $j = 1, 2$, and 3, respectively. The corresponding weighting factors are 1–5 orders of magnitude larger than those for realistic dose calculations for these long-lived radionuclides with half-lives much larger than the period of integration ($\tau = 1 \text{ year}$). Interestingly, in this example, the ‘conservative’ weighting-factor values $\tilde{w}_{Dj}$ following from the classical method (equation (20), section 2.4) are almost equal to—but slightly smaller than—those from the new method, with deviations <2%. In the classical
method, the maximum activity concentration is reached at \( t_{\text{max,j}} \) = 30, 60 and 100 years for progeny radionuclide \( j = 1, 2, \) and \( 3 \), respectively, close to the values of \( t_j^* \). In fact, the maximum number of daughter disintegrations occurs in the time interval located around the

\[ t_j^* \]

...
time at which the activity concentration of the progeny radionuclide reaches its maximum. Note that the absolute maximum of the activity concentration (and thus the annual number of disintegrations) of the third progeny radionuclide \((j = 3)\) occurs after about 133 years (outside the domain of the plot in figure 3). Considering the time span of one century, \(t_{\text{max},3}\) and \(t_i^*\) are clipped to 100 years at which the values of \(\tilde{w}_{D,3}\) and \(\tilde{w}_{D,3}(1 \text{ y})\) are determined. For instance, with \(A_{D,3}(100 \text{ y}) = 0.1548 \text{ Bq.g}^{-1}\), the classical weighting factor is simply \(\tilde{w}_{D,3} = 0.1548\), as follows from equation (20).

### 3.2. Weighting factors for the first progeny radionuclide

In section 3.1, it was shown that, for the fictive decay chain of long-lived radionuclides, the weighting-factor values for conservative dose assessments are similar when comparing the new (proposed) model with the classical model, i.e. \(\tilde{w}_{D,j}(\tau = 1 \text{ y})\) from equation (15) with \(\hat{w}_{D,j}\) from equation (20). To study the similarities and differences in both methods, we calculated these conservative weighting factors for the first progeny radionuclide in the chain of figure 2 \((j = 1)\), direct ingrowth from parent, branching ratio \(= 1\) as a function of the decay constants \(\lambda_{P}\) and \(\lambda_{D,1}\) \((\text{y}^{-1})\). Analytical expressions for these weighting factors are given by equation (23) \((\tilde{w}_{D,1}, \text{ classical method})\) and (18) \((\tilde{w}_{D,1}(\tau = 1 \text{ y}), \text{ new method})\) and are visualised as contour plots in figures 4(a), (b). Iso-curves of the weighting factors are plotted for several values between 0.0001 and 0.9999. The ratio of these weighting factors (new divided by classical value) is shown in figure 4(c), with iso-curves for fixed values of \(10\log[\tilde{w}_{D,1}(\tau = 1 \text{ y})/\hat{w}_{D,1}]\) between 0.0003 (ratio \(\approx 1\)) and 3 (ratio \(\approx 1000\)). The grey-shaded area in (a)–(c) indicates the region in which \(t_{\text{max},1}\) is clipped to a value of 100 years. For comparison, a contour plot of the corresponding weighting factor, \(w_{D,1}(\tau = 1 \text{ y})\), for realistic, annual dose calculations according to equation (7) is shown in figure 4(d). Decay constants in all plots range between \(10^{-4}\) and \(10^{4} \text{ y}^{-1}\).

Comparison of figure 4(a) with 4(b) shows that for long-lived parent nuclides with \(\lambda_{P} < 10^{-1} \text{ y}^{-1}\) (i.e. \(10\log[\lambda_{P}] < -1, T_{1/2,P} > 7 \text{ y}\)), the conservative weighting factor according to the new method does not deviate more than 5% from that according to the classical method. This is confirmed by figure 4(c) with iso-contour values (for the 10-log-ratio) below \(10\log(1.05) = 0.02\). In this region, iso-curves from both methods in figures 4(a) and (b) are close or even coincide. This can be explained by the fact that, for long-lived parent radionuclides, the chain is governed (or limited) by relatively slow decay kinetics at timescales (much) larger than the time-integration period \(\tau\) of the dose calculation. Its overall, slow decay process then causes the number of daughter disintegrations in time interval \([t, t + \tau]\) to be well approximated by \(\tilde{U}_{D,1}(t; \tau) \approx A_{D,1}(t) \cdot \tau\), which is maximum when \(A_{D,1}(t)\) is maximum. This even holds for short-lived daughter nuclides with large values of \(\lambda_{D,1}\), and therefore the new method (section 2.3) approaches the classical method (section 2.4) for long-lived parent radionuclides. This was also observed in the fictive example described in section 3.1, not only for the first progeny radionuclide, but also for following radionuclides in the chain, \(j = 2, 3\) (table 1).

However, for shorter-lived parent nuclides with \(\lambda_{P} > 10^{-1} \text{ y}^{-1}\), deviations much larger than 5% may occur. By comparing the iso-curves in figures 4(a) and (b), we observe that deviations become apparent in the \(\lambda_{P}\)-region between \(10^{-3}\) and \(10^{1} \text{ y}^{-1}\). As seen in figure 4(c), the ratio of these weighting factors increases from 1.05 to 10 (iso-contour curve of 1) in the lower part of this region \((\lambda_{D,1} < 10^{0} \text{ y}^{-1})\), the new method always resulting in the largest value. For \(\lambda_{P} > 10^{1} \text{ y}^{-1}\) and \(\lambda_{D,1} < 10^{0} \text{ y}^{-1}\), the region of no equilibrium, the new method yields weighting-factor values that can be many orders of magnitude larger than those resulting from the classical model. Note that such a drastic increase in the weighting factor would only impact the (conservative) dose calculations considerably if the enhanced product
becomes a significant contribution to the total dose coefficient, with $w_{D,1}(\tau = 1 \text{ y}) \cdot DC_{D,1}$ being at least 10% of the value of $DC_p$.

Finally, in the region $\lambda_P > 10^4 \text{ y}^{-1}$, the weighting factor for conservative dose calculations, $\tilde{w}_{D,1}(\tau = 1 \text{ y})$ in figure 4(b), results in similar values as the weighting factor for realistic dose calculations, $w_{D,1}(\tau = 1 \text{ y})$ in figure 4(d). This is because the value of $t_{10}^* \approx 100 \text{ y}$.

The dashed contour outlines the grey-shaded region for which $t_{\text{max},1} \approx t_{10}^*$.
disintegrations already occurs in the immediate time interval, [0, 1) y, on which the weighting factor for realistic dose calculations is based as well. As can be seen in figures 4(b), (d), the weighting-factor value in this region mainly depends on $\lambda_{D,1}$. Since most parent atoms have decayed within the first year (which amounts to over 14.4 parent half-lives for $\lambda_p > 10^4$ y$^{-1}$), the parent contribution to the dose has reached its saturation value already. The only contribution left is that of the progeny, which depends on the value of $\lambda_{D,1}$ in this region. If also $\lambda_{D,1} > 10^4$ y$^{-1}$, each parent disintegration is followed by a daughter disintegration, and thus the weighting factor attains a value of (close to) 1, as seen in figures 4(b) and (d).

3.3. Nuclide-specific weighting-factor values

Using the nuclear-decay data in the electronic database of ICRP Publication 107 (ICRP 2008), the weighting-factor values according to the three methods described in sections 2.2 through 2.4 have been determined. These numerical values are listed in table A1 of appendix A for an integration period of $\tau = 1$ year for annual dose calculations (note: the classical method does not require an integration period) and for those 118 parent nuclides also included in EC (2000) and IAEA (2005). The set of head-of-chain nuclides and respective progeny nuclides in table A1 differs slightly from that in the IAEA table. This is because some of the combinations parent/progeny in the IAEA table are not listed in the ICRP-107 database, which is the basis for our calculations. These cases generally involve very short-lived, metastable isotopes. An example is Pd-109 $\rightarrow$ Ag-109m. Similarly, some nuclides listed in the table in EC (2000) do not appear in the ICRP-107 database and are therefore not considered in our study, for example, Nb-97m. Weighting factors for all 783 progeny-carrying parent nuclides can be found in the supplementary material, for integration periods of $\tau = 1, 2$ and 8 h, 1 and 2 d, 1 week, 1 and 6 months, 1, 2, 5, 10, 50, and 70 years. These values have been determined by numerically solving the Bateman equations (Bateman 1910) using a matrix-exponential method briefly described in appendix B. Complicated decay schemes containing branching and merging chains, as illustrated in figure 1, are captured adequately by this numerical computer code, written in Fortran-90. Weighting factors for the parent nuclides are 1.0 by definition. Values for realistic dose calculations (method from section 2.2) can easily be verified by using the online calculation tool BiotaDC (ICRP 2017, http://biotadc.icrp.org/), with possibly small differences due to a difference in the numerical algorithm used to solve the Bateman equations and rounding errors. In table A1, the head-of-chain nuclides and all progeny weighting factors are provided in the same order as used in IAEA (2005). The values in this table, and those provided in the supplementary material, can be used for a variety of realistic and conservative dose assessments. Guidance on the selection of the proper set of weighting-factor values and on the subsequent use of these values is provided in section 3.5.

The comparison of conservative weighting factors (classical $\tilde{w}_{D,j}(\tau = 1)$ y) as presented generically in figures 4(a) and (b) for the first progeny radionuclide ($j = 1$) as a function of $\lambda_p$ and $\lambda_{D,1}$, can now be continued on the basis of the numerical results for actual, parent-progeny decay chains with $j \geq 1$. As mentioned earlier, each decay chain headed by a certain parent radionuclide results in a set of conservative weighting-factor values for the corresponding progeny radionuclides in accordance with the methods described in sections 2.3 and 2.4. These values, listed for instance in table A1 (columns 3 and 4, respectively), are used to generate a color-scatter plot for the progeny radionuclides of the database’s 783 parent radionuclides with radioactive progeny. This is shown in figure 5(a), where the new weighting factor $\tilde{w}_{D,j}(\tau = 1)$ y is plotted versus the classical weighting factor $\tilde{w}_{D,j}$, both on a logarithmic scale (range: $10^{-12}$ to $10^9$). On the diagonal line, $y = x$, weighting factors according to both methods are equal (ratio = 1). Each progeny
radionuclide yields a data point in this figure, its color indicating the value of the decay constant \( l_P \) of the corresponding parent (head-of-chain) nuclide, as indicated by the legend. Note that a certain progeny radionuclide may result in several data points here, since it could be a daughter nuclide in decay chains headed by various parent nuclides. These data points may then of course have different values, as well as different colors. For example, Ra-222 could be a direct daughter nuclide \((j = 1)\) in the chain headed by Th-226, but it could also be a granddaughter \((j = 2)\) in the (same) chain, but then headed by U-230. The reason for choosing the decay constant \( \lambda_P \) of the parent radionuclide for color coding is that this is the main parameter determining the ratio of the results of both methods, as already concluded from figure 4(c).

For long-lived parent nuclides (black data points), the progeny weighting-factor values according to both methods are almost identical as data points reside close to or on the line \( y = x \). This is in agreement with the data presented in figure 4(c) for the first daughter nuclide, with the ratio of weighting factors approaching 1 (contour values close to 0) for \( \lambda_P < 10^{-1} \text{ y}^{-1} \), i.e. \( T_{1/2,P} > 7 \) years and thus \( T_{1/2,P} \gg \tau \). If, however, parent radionuclides are short(er)-lived, with \( \lambda_P > 10^0 \text{ y}^{-1} \), i.e. \( T_{1/2,P} < 0.7 \) years and hence \( T_{1/2,P} < \tau \), then the new progeny weighting factor becomes significantly larger than the classical one, as clearly seen in figure 5(a). In fact, we can observe bands of data points of certain colors, each band parallel to the \( y = x \) line and thus with a certain ratio of weighting factors. For example, for \( \lambda_P \approx 10^4 \text{ y}^{-1} \), we observe a corresponding ’red’ band for which the new weighting factor is about four orders of magnitude larger than the classical one.
Since weighting-factor values smaller than $10^{-3}$ will generally not lead to significant dose contributions from radioactive progeny, figure 5(b) shows the same plot as figure 5(a), but zoomed in on the region of weighting factors larger than $10^{-3}$. From this figure, we can conclude that for the progeny of short-lived radionuclides ($T_{1/2,P} < \tau$ or $T_{1/2,P} < \tau$) many classical weighting-factor values between $10^{-3}$ and 1 correspond to a new value close to 1. This can be seen from the pile-up of many data points on the upper, horizontal axis of the plot. Note that these must concern progeny radionuclides for which the product of branching ratios of preceding nuclear transformations leading to that specific progeny radionuclide is (close to) 1 as well. In this context, we should also realise that a new weighting-factor value smaller than 1 might simply be the result of the fact that this product of branching ratios is smaller than 1. Obviously, the largest possible value of a progeny weighting factor is equal to the product of preceding branching ratios.

An example of a nuclide for which the conservative weighting factor drastically changes is that of I-131 as a daughter of parent radionuclide Te-131 (branching ratio = 1). The classical weighting factor amounts to a value of 0.0021, inasmuch as the maximum activity concentration—reached after 3.7 h—is 0.0021 Bq g$^{-1}$, while the new weighting factor attains a value of 1.0 (a ratio of almost 500). The latter can be understood from the fact that, already within the first year, all unstable atoms of both the parent and daughter radionuclide have disintegrated. With an equal number of disintegrations already within the first year, both $w_{D_j} (\tau = 1 \text{ y})$ and $\tilde{w}_{D_j} (\tau = 1 \text{ y})$ attain a value of 1, as follows from equations (6) and (15), respectively. Consequently, the classical method may, in certain cases, significantly underestimate the dose contribution from progeny radionuclides. See section 3.5.2 for guidance on the proper use of weighting factors.

3.4. The influence of the time-integration period $\tau$ of the scenario

In the previous sections, we often mentioned the terms ‘short-lived’ and ‘long-lived’ radionuclides and explained model results, for example, the results in figure 4, in relation to these terms. But how do we define short- and long-lived?

The answer should be directly related to the time-integration period $\tau$ of the scenario considered in the dose assessment. In the previous sections, with $\tau = 1 \text{ year}$ for annual dose calculations, short-lived relates to half-lives shorter or much shorter than a year and long-lived to half-lives longer or much longer than a year. But the question arises as to what happens with the previous results when we vary this integration period, since the proposed, new techniques described in sections 2.2 and 2.3 clearly depend on it, as seen from equations (6) and (15).

To answer this question and to properly visualise the effect of the integration period, we return to the example Te-131/I-131 from section 3.3. The classical, conservative weighting factor for I-131 amounts to a value of 0.0021 (no explicit integration period needed), whereas the new technique results in the maximum possible value of 1.0 for an integration period of $\tau = 1 \text{ year}$. The latter value holds for both the realistic and conservative counterparts, as explained in section 3.3.

Figure 6 shows the new DC weighting-factor values of daughter radionuclide I-131 as a function of $\tau$ for the realistic (dashed curve) and conservative method (solid curve). Using an integration period of $\tau > 56 \text{ days}$ (7 half-lives of I-131) results in realistic and conservative weighting-factor values larger than 0.99. For smaller integration periods, they both decrease with the decreasing value of $\tau$. For example, the conservative value $\tilde{w}_{D_j} (\tau)$ attains a value of 0.453 at $\tau = 1 \text{ week}$ (roughly the half-life of I-131) and 0.004 at $\tau = 1 \text{ h}$. If $\tau$ decreases to an even smaller value of $10^{-3} \text{ day}$ ($\sim \text{ minute}$), much smaller than the half-life of parent nuclide
Te-131, the new, conservative weighting factor $\tilde{w}_{D,1}$ approaches the classical value of $w_{D,1} \approx 0.0021$. Thus, $\tilde{w}_{D,1}(\tau)$ approaches the classical value in the limit for $\tau \to 0$ (such that $\lambda_p \tau \to 0$ and $\lambda_{D,p} \tau \to 0$) which follows directly from equation (14), alternatively written as: $A_{D,j}(t_j^* + \tau) = A_{D,j}(t_j^*)$. For $\tau \to 0$, this condition is met at the absolute, maximum value of the activity concentration $A_{D,j}(t_{\text{max},j})$ of progeny radionuclide $j$ (if this absolute maximum occurs within 100 years). Then, the maximum number of disintegrations, $\tilde{U}_{D,j}(t_j^*; \tau)$, is simply approximated by $A_{D,j}(t_{\text{max},j}) \cdot \tau$, by which equation (15) approaches equation (20), using the fact that $\Psi(\lambda_p \tau) \to 1$ for $\lambda_p \tau \to 0$. Therefore, we find that

$$\lim_{\tau \to 0} \tilde{w}_{D,j}(\tau) = w_{D,j}(\text{classical}),$$

as observed in figure 6 (for $j = 1$), with the solid curve approaching the dotted curve representing the classical, conservative value. Note that the new, realistic weighting factor $w_{D,j}(\tau)$ will approach 0 for $\tau \to 0$, as can be derived directly from equation (6) and by application of L’Hôpital’s rule:

$$\lim_{\tau \to 0} w_{D,j}(\tau) = \lim_{\tau \to 0} \frac{U_{D,j}(\tau)}{U_p(\tau)} = \lim_{\tau \to 0} \frac{\frac{d}{d\tau} U_{D,j}(\tau)}{\frac{d}{d\tau} U_p(\tau)} = \lim_{\tau \to 0} \frac{A_{D,j}(\tau)}{A_p(\tau)} = 0 \cdot \frac{1}{1} = 0.$$  

The latter can be observed in figure 6 as well, with the dashed curve for $w_{D,j}(\tau)$ approaching very small values ($< 10^{-3}$) for small values of $\tau$ ($< 2 \times 10^{-2}$ d).

Nuclide-specific, progeny DC weighting-factor values according to the three methods described in sections 2.2 through 2.4 and for different integration periods ($\tau = 1, 2$ and 8 h, 1 and 2 d, 1 week, 1 and 6 months, 1, 2, 5, 10, 50, and 70 years) are presented as supplementary material to this paper. Guidance on how to apply these values in realistic or conservative dose assessments is provided in section 3.5.
3.5. Guidance on the application of the values

The progeny DC weighting-factor values presented in appendix A of this paper and in the supplementary material can be used to construct a total dose coefficient $D_{\text{tot},P}$ (equation (1)) or $D_{\text{tot},P}$ (equation (16)) for a parent (head-of chain) radionuclide $P$ present in the contaminated material one is exposed to. After the construction of the total dose coefficient, the radiological dose assessment only has to consider the exponential decay kinetics of the parent nuclide. Application of the total-DC value then properly takes into account the dose contribution related to the ingrowth of radioactive progeny, either in a realistic or in a conservative manner. There are three sets of weighting-factor values, based on the methods described in sections 2.2 through 2.4, and two of them depend on the time-integration period $\tau$ of the scenario. Conditions imposed on the use of these values and guidance on the selection of the proper set of values is discussed below.

3.5.1. General conditions for dose assessments using progeny DC weighting factors. The methods described in this paper are solely based on nuclear transformations in exit-only, branching and merging decay chains. Any source term generating radioactivity of the parent radionuclide (i.e. generating $A_P$) is not taken into account in this method. Weighting-factor values derived in this paper may therefore not always be suitable for use in radiological dose-assessments in which such source terms are present. The same holds for dose assessments in which radioactivity is largely removed by other, environmental pathways, such as ventilation, weathering, depletion/deposition, or removal by wipe-off. Note that this does not necessarily mean that these weighting factors cannot be applied in such evaluations, but merely that one should carefully reason whether or not to use these values and what the impact is on the outcome in terms of the level of conservatism.

The weighting-factor values can best be used in scenarios considering exposure to solid materials contaminated with radionuclides, with the activity concentrations only or mainly modified by radioactive decay and ingrowth. Such scenarios are investigated in, for example, EC (2000) and IAEA (2005). Here, even the inhalation pathway is evaluated using weighting factors (even though there is a source term and environmental removal by ventilation) since the airborne activity concentration (of each radionuclide) is regarded as continuously in equilibrium with the solid material’s activity concentration.

Next, DC weighting factors can only be applied if one is using dose coefficients or conversion coefficients (for external radiation) that do not yet incorporate a contribution from the ingrowth of radioactive progeny. Since most dose coefficients available in the literature are only based on the (parent) radionuclide itself, the weighting-factor method can be used for dose evaluations. Note that the dose coefficients for internal contamination, as for instance published in ICRP Publication 119 (ICRP 2012), do contain a contribution from the ingrowth of radioactive progeny once ingested or inhaled. However, since the ingrowth of progeny outside the human body is not taken into account, the weighting factor method may still be applied for these dose coefficients as well.

The weighting-factor method assumes that no progeny radionuclides are present at the start of the exposure scenario, at $t = 0$. But what should one do if a certain progeny radionuclide $D$ is already present at the start of the exposure scenario? Generally, in case the dose contribution from this progeny’s initial activity concentration is expected to be significant, one should always calculate the corresponding dose separately and add it to the dose already evaluated for the head-of-chain parent radionuclide $P$, the latter one calculated under the assumption of the absence of any radioactive progeny in the underlying chain. The additional dose evaluation of that specific progeny radionuclide $D$ should again take into
account its own radioactive progeny, i.e. the descendants of $D$. This means that in the additional dose evaluation, one should treat progeny nuclide $D$ as a new parent radionuclide with its own, underlying subchain and use the progeny DC weighting factors of the subchain now headed by nuclide $D$. Hence, it is important to realise that the weighting-factor value of any progeny radionuclide in the subchain headed by $D$ (the value needed for the additional dose evaluation) may be completely different from that of the same radionuclide in the chain headed by parent $P$ (the value needed for the first calculation related to the main parent $P$).

In some cases, however, the dose evaluation related to parent radionuclide $P$ already properly (and sufficiently) takes into account the dose contribution of (some of) its progeny nuclides with an initial activity concentration. Then, for these progeny radionuclides, no additional or separate evaluation is required with respect to their initial activity concentrations. This is described in more detail in IAEA (2005), but we emphasise that these definitions may change when the time-integration period $\tau$ deviates considerably from 1 year. These issues are outside the scope of the current paper.

Short-lived parent nuclides (i.e. $T_{1/2,P} \ll \tau$) may have already decayed before actual exposure takes place. Some scenarios, as for instance considered in EC (2000) and IAEA (2005), explicitly incorporate a delay time between the measurement of the activity (concentration) and start-of-exposure. Since all weighting factors in this paper are calculated under the assumption of ‘no delay’ (start of exposure at $t = 0$), the weighting factors in principle do not apply to dose evaluations with a certain delay time. In this case, one should either develop DC weighting factors based on certain delay times (in this case $w_{Dj}$-values $> 1$ may occur, but this discussion is outside the scope of the current paper), or alternatively, one should first estimate the activity concentrations of the progeny nuclides at the start-of-exposure (after the delay) and then proceed with dose evaluations based on DC weighting factors presented in this paper. Note that for long-lived parent radionuclides, the delay time will not have a large influence on the outcome of the dose assessment and the current weighting factors can thus be used safely.

3.5.2. Selection of the proper weighting-factor values. If one decides to use progeny DC weighting factors for a dose assessment (after regarding the limitations and issues described in section 3.5.1), the next step will be to choose the proper values, for which some guidance is given below.

First, one has to decide whether a realistic or conservative dose estimate is required, i.e. whether to use the values of $w_{Dj}$ (section 2.2) or $\tilde{w}_{Dj}$ (sections 2.3 and 2.4), respectively. As an example, for a retrospective evaluation of the radiological dose contributions incurred by exposure to a contaminated source emitting ionising radiation, the assessment is typically based on the best (historical) information possible. Dose reconstruction may then be performed using the realistic weighting-factor values $w_{Dj}(\tau)$ to get the best estimate of the total dose. Other examples of assessments that may employ weighting factors are the scenario-based evaluations of radiological doses in the derivation of limiting values for the activity or activity concentration of radionuclides in materials, as in EC (2000) and IAEA (2005) for purposes of exemption and clearance. Such derived, limiting values are typically rather conservative for protection purposes. For clearance of materials, for instance, not only should the first year dose remain below the prevailing dose criterion, but also the annual dose in the foreseeable future ($\sim$100 y).

By construction (section 2.3), the new, conservative weighting factor $\tilde{w}_{Dj}(\tau)$ may then be the best choice for such calculations. In the case of long-lived parent radionuclides for which clearance issues become relevant, the classical, conservative weighting factor $\tilde{w}_{Dj}$ (section 2.4) could also be applied. In fact, as follows from equation (24) in section 3.4, both conservative weighting factors become similar in this case. When using conservative weighting-factor
values, however, one should also ensure that the resulting assessment is not over-conservative, since conservatism may already be incorporated within the scenario definitions.

Next, one has to select a proper value of the time-integration period $\tau$. If the incurred dose is to be calculated over a time period $T_{\text{ref}}$, then one could set the value of $\tau$ equal to that of $T_{\text{ref}}$. For many dose evaluations, the value of $T_{\text{ref}}$ will be 1 year for annual dose evaluations and for verification of compliance with the annual dose criteria or dose limits. Note that if actual exposure to the source of ionising radiation occurs only during a certain fraction of the time (i.e., actual exposure in $T_{\exp} = f_{\exp} T_{\text{tot}} \leq T_{\text{tot}}$), this so-called duty factor should of course be taken into account in the dose assessment as well (see, for example, van Dillen and van Dijk (2018)).

However, the value of $\tau$ may also be smaller than that of $T_{\text{tot}}$ in case a certain scenario repeats itself several times within the considered time period $T_{\text{tot}}$. Then, the value of $\tau$ is to be set equal to the duration of the repeating scenario, after which one selects the proper values of the weighting factors, $w_{D,j} (\tau)$ or $\tilde{w}_{D,j} (\tau)$ (or alternatively its classical counterpart $w_{D,j}$). After calculating the total dose within the interval of duration $\tau$—again using a suitable value of $f_{\exp} = t_{exp}/\tau$ (i.e., actual exposure time within $\tau$)—one multiplies the outcome with the frequency of occurrence $n$ of the repeating scenario within $T_{\text{tot}}$ to get the final dose result. This result can be used for comparison with a certain dose criterion (note that $T_{\exp} = n t_{exp}$ using these definitions). This may, for example, occur in scenario-based assessments related to the procedure of exemption of a justified practice involving a material contaminated with radionuclides. For example, a worker scenario that repeats itself after a working week for 30 times in a year, then $T_{\text{tot}} = 1 \text{y}$, $\tau = 5 \text{d}$ and the dose outcome (incurred in a working week of 5 days) is multiplied by $n = 30$ to yield the total annual dose for this overall worker scenario, which includes the proper contribution of the ingrowth of radioactive progeny based on $w_{D,j} (\tau = 5 \text{d})$ or $\tilde{w}_{D,j} (\tau = 5 \text{d})$.

Finally, for pragmatic reasons of conservatism, one could decide to explicitly disregard the radioactive decay of parent nuclides in dose assessments, but still include radioactive progeny from ingrowth. The absence of physical decay in dose assessments implies application of the theoretical limit $\tau \to 0$. With $w_{D,j} (\tau \to 0) = 0$ (equation (25)), contribution of progeny is only incorporated when one uses the conservative weighting factors. From equation (24), it follows that we can safely use the classical weighting factors $w_{D,j}$ in this situation. In summary, dose evaluations for parent nuclides that disregard their physical decay are based on a total duration of $T_{\text{tot}}$ with actual exposure in $T_{\exp} = f_{\exp} T_{\text{tot}}$ and may employ the classical, conservative DC weighting factors $w_{D,j}$ to still include contributions from the ingrowth of radioactive progeny.

4. Summary and conclusions

Radiological dose assessments involving parent radionuclides should adequately include the contribution of the ingrowth of the corresponding chain’s progeny nuclides. In this paper, we investigate a practical and convenient method to directly take into account these contributions by modifying the dose coefficients (DC) of the parent (head-of-chain) nuclides. The total, modified DC, $DC_{\text{tot}},$ required for such dose evaluations is then evaluated as the DC of the parent radionuclide to which the DCs of the chain’s radioactive progeny are added using proper, dimensionless weighting factors $w_{D,j} (\tau)$ with values between 0 and 1 (see equations (1) and (16)). The main advantage of this method applying $DC_{\text{tot}}$ is that the dose evaluation itself only requires an explicit assessment of the parent radionuclides, which may result in a significant simplification of its numerical tools.
The resulting weighting factors, \( w_{D_j}(\tau) \), can be used for dose assessments taking into account the ingrowth of daughter nuclides in a realistic manner, as long as the assessed scenario is dominated by nuclear transformations without additional source terms. In the latter case, or in case of removal of radioactivity by other environmental pathways, the weighting factors presented in this paper may not (always) be applicable, as discussed in section 3.5.1.

The weighting-factor method is adapted and extended for use in conservative dose calculations (section 2.3). Such calculations may, for instance, be required in the derivation of limiting values for the activity or activity concentration of radionuclides in materials eligible for clearance (so-called clearance levels). The method not only ensures compliance with the prevailing dose criteria in the immediate time period of duration \( \tau \), but also compliance with these criteria in any future time interval of equal duration (at least within a century). The conservative weighting factors \( \tilde{w}_{D_j}(\tau) \) are defined in almost the same way as \( w_{D_j}(\tau) \), but now with the maximum possible number of progeny disintegrations within a time interval of duration \( \tau \) over the regarded future of 100 years. This is expressed by equation (15), the conservative counterpart of equation (6). These new, realistic and conservative weighting factors are compared with those from a classical model described in EC (2000) and IAEA (2005). Instead of a ratio of time-integrated activities (progeny:parent), the classical, conservative method is based on a ratio of activities or activity concentrations as expressed by equation (20) in section 2.4. These classical conservative weighting factors \( \tilde{w}_{D_j} \) are therefore independent of an integration period.

Weighting factors according to the three methods described above are calculated for all radionuclides with radioactive progeny that are contained within the electronic database from ICRP Publication 107 (ICRP 2008). Based on these nuclear-decay data, weighting-factor values have been determined for integration periods of \( \tau = 1, 2 \) and 8 h, 1 and 2 days, 1 week, 1 and 6 months, 1, 2, 5, 10, 50, and 70 years. These data can be downloaded as supplementary material. For 118 of these parent radionuclides (with progeny)—also included in IAEA (2004, 2005) and in Council Directive 2013/59/Euratom (EC 2014)—progeny DC weighting factors based on \( \tau = 1 \) year are listed in table A1 of appendix A.

The dependence of the various weighting factors has been investigated in terms of parent and progeny decay constants and time-integration period. Comparison of the three methods shows that the new weighting factor \( \tilde{w}_{D_j}(\tau) \) always yields the most conservative value for dose assessments. For long-lived parent radionuclides \( (T_{1/2,P} \gg \tau) \), this factor approaches its classical counterpart \( \tilde{w}_{D_j} \). For short-lived parent radionuclides \( (T_{1/2,P} \ll \tau) \), the conservative values of \( \tilde{w}_{D_j}(\tau) \) approach the realistic values \( w_{D_j}(\tau) \) if the maximum possible number of disintegrations of the \( j \)th progeny nuclide already occurs within the immediate time interval \((0, \tau)\). If, in an unbranched decay chain, that progeny nuclide is short-lived as well \( (T_{1/2,D_j} \ll \tau) \), the new weighting factor will reach a maximum value of 1, while the classical value is often (much) smaller than 1: \( \tilde{w}_{D_j} < \tilde{w}_{D_j}(\tau) \approx w_{D_j}(\tau) \rightarrow 1 \).

The dependence of the various weighting factors on possible source terms or other (environmental) removal factors has not been investigated in this paper. The effect of the possible existence of a delay time before the start of the scenario, by which progeny DC weighting factors may attain values >1, is also outside the scope of this paper. All these effects could be investigated in a future study, but the basic definition of the weighting factors as described here would still hold. However, the generic DC weighting factors derived and listed in this paper and in its supplementary material can still be used for a broad spectrum of dose assessments, as long as the general conditions described in section 3.5.1 are met. To aid radiation-protection experts and scientists in choosing the proper weighting factors, the paper also contains guidance on how to select these values for their dose assessments (section 3.5.2).
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With permission from the International Commission on Radiological Protection (ICRP), RIVM and Bel V make use of the electronic database (file: ICRP-07.NDX) of ICRP Publication 107 (ICRP 2008) to calculate the progeny DC weighting factors compiled in the supplementary data files and the selected values in table A1 in appendix A. Great care is given to their correctness and the numerical accuracy in the calculations. In case inconsistencies in these files are encountered, please contact the authors. RIVM and Bel V accept no liability or claims whatsoever for any direct or indirect damage, including financial or other loss, caused by any errors or incompleteness in the dataset or by misuse or misinterpretation of these weighting-factor data. When using the information from these data files, one should properly refer to the source (article reference) and name of the data file (weightingfactors_SupplementaryMaterial.dat or weightingfactors_supplementary_Excel.dat).

Appendix A. Weighting factor values

Table A1. Weighting-factor values for parent and progeny radionuclides from table II-I in IAEA (2005), based on a scenario-integration time of \( \tau = 1 \) y. Weighting factors for complete decay chains and additional parent radionuclides can be found in the supplementary material. Column 1: head-of-chain, parent nuclides (boldface) and corresponding progeny. Column 2: new weighting factors \( w_{1y,Dj} \) for realistic, first year dose estimates (W-real, equation (6)). Column 3: new weighting factors \( \tilde{w}_{1y,Dj} \) for conservative dose calculations (W-cons, equation (15)). Column 4: classical weighting factors \( w_{0,Dj} \) from the methodology described in EC (2000) and in IAEA (2005) (W-class, equation (20)). Column 5: the ratio of W-cons and W-class. Values of the classical factor have not been copied from IAEA (2005), but have been calculated afresh. Ratios are based on the high precision estimates, not on the truncated values listed in this table. Weighting factors for head-of-chain nuclides are 1 by default.

|         | W-real | W-cons | W-class | cons/class |
|---------|--------|--------|---------|------------|
| Ca-47   | 1      | 1      | 1       | 1          |
| Sc-47   | 1.0000 | 1.0000 | 0.4249  | 2.4        |
| Mn-52m  | 1      | 1      | 1       | 1          |
| Mn-52   | 0.0175 | 0.0175 | 0.0000  | 388        |
| Fe-52   | 1      | 1      | 1       | 1          |
| Mn-52m  | 1.0001 | 1.0001 | 0.8692  | 1.15       |
| Mn-52   | 0.0175 | 0.0175 | 0.0009  | 19         |
| Co-58m  | 1      | 1      | 1       | 1          |
| Co-58   | 0.9717 | 0.9717 | 0.0052  | 188        |
| Co-60m  | 1      | 1      | 1       | 1          |
Table A1. (Continued.)

|         | W-real | W-cons | W-class | cons/class |
|---------|--------|--------|---------|------------|
| Co-60   | 0.1228 | 0.1228 | 0.0000  | 32 618     |
| Zn-69m  | 1      | 1      | 1       | 1          |
| Zn-69   | 0.9997 | 0.9997 | 0.8210  | 1.22       |
| Sr-85m  | 1      | 1      | 1       | 1          |
| Sr-85   | 0.8485 | 0.8485 | 0.0006  | 1360       |
| Sr-87m  | 1      | 1      | 1       | 1          |
| Rh-87   | 0.0000 | 0.0000 | 0.0000  | 2157       |
| Sr-90   | 1      | 1      | 1       | 1          |
| Y-90    | 0.9896 | 0.9990 | 0.9979  | 1.00       |
| Sr-91   | 1      | 1      | 1       | 1          |
| Y-91m   | 0.5825 | 0.5825 | 0.4617  | 1.26       |
| Y-91    | 0.9867 | 0.9867 | 0.0066  | 149        |
| Sr-92   | 1      | 1      | 1       | 1          |
| Y-92    | 1.0001 | 1.0001 | 0.3153  | 3.2        |
| Y-91m   | 1      | 1      | 1       | 1          |
| Y-91    | 0.9867 | 0.9867 | 0.0006  | 1680       |
| Y-93    | 1      | 1      | 1       | 1          |
| Zr-93   | 0.0000 | 0.0000 | 0.0000  | 596        |
| Nb-93m  | 0.0000 | 0.0000 | 0.0000  | 597        |
| Zr-93   | 1      | 1      | 1       | 1          |
| Nb-93m  | 0.0206 | 0.9619 | 0.9617  | 1.00       |
| Zr-95   | 1      | 1      | 1       | 1          |
| Nb-95m  | 0.0108 | 0.0108 | 0.0091  | 1.19       |
| Nb-95   | 0.9766 | 0.9771 | 0.4825  | 2.0        |
| Zr-97   | 1      | 1      | 1       | 1          |
| Nb-97   | 1.0001 | 1.0001 | 0.8157  | 1.23       |
| Mo-93   | 1      | 1      | 1       | 1          |
| Nb-93m  | 0.0186 | 0.8566 | 0.8563  | 1.00       |
| Mo-99   | 1      | 1      | 1       | 1          |
| Tc-99m  | 0.8773 | 0.8773 | 0.6899  | 1.27       |
| Tc-99   | 0.0000 | 0.0000 | 0.0000  | 92         |
| Mo-101  | 1      | 1      | 1       | 1          |
| Tc-101  | 1.0000 | 1.0000 | 0.3731  | 2.7        |
| Tc-96m  | 1      | 1      | 1       | 1          |
| Tc-96   | 0.9800 | 0.9800 | 0.0079  | 125        |
| Tc-97m  | 1      | 1      | 1       | 1          |
| Tc-97   | 0.0000 | 0.0000 | 0.0000  | 3.0        |
| Tc-99m  | 1      | 1      | 1       | 1          |
| Tc-99   | 0.0000 | 0.0000 | 0.0000  | 1009       |
| Ru-97   | 1      | 1      | 1       | 1          |
| Tc-97   | 0.0000 | 0.0000 | 0.0000  | 87         |
| Ru-103  | 1      | 1      | 1       | 1          |
| Rh-103m | 0.9875 | 0.9875 | 0.9808  | 1.01       |
| Ru-105  | 1      | 1      | 1       | 1          |
| Rh-105  | 1.0000 | 1.0000 | 0.0932  | 11         |
| Ru-106  | 1      | 1      | 1       | 1          |
| Rh-106  | 1.0000 | 1.0000 | 1.0000  | 1.00       |
| Pd-103  | 1      | 1      | 1       | 1          |
| Rh-103m | 0.9988 | 0.9988 | 0.9849  | 1.01       |
|        | W-real | W-cons | W-class | cons/class |
|--------|--------|--------|---------|------------|
| Ag-108m | 1      | 1      | 1       | 1          |
| Ag-108  | 0.0870 | 0.0870 | 0.0870  | 1.00       |
| Cd-115  | 1      | 1      | 1       | 1          |
| In-115m | 1.0000 | 1.0000 | 0.7967  | 1.26       |
| In-115  | 0.0000 | 0.0000 | 0.0000  | 114        |
| Cd-115m | 1      | 1      | 1       | 1          |
| In-114m | 1      | 1      | 1       | 1          |
| In-114  | 0.9675 | 0.9675 | 0.9673  | 1.00       |
| In-115m | 1      | 1      | 1       | 1          |
| In-115  | 0.0000 | 0.0000 | 0.0000  | 1354       |
| Sn-113  | 1      | 1      | 1       | 1          |
| Sn-125  | 1      | 1      | 1       | 1          |
| Sb-125  | 0.2145 | 0.2189 | 0.0091  | 24         |
| Te-125m | 0.0389 | 0.0469 | 0.0019  | 25         |
| Sb-125m | 1      | 1      | 1       | 1          |
| Te-125m | 0.1832 | 0.2114 | 0.1946  | 1.09       |
| Te-123m | 1      | 1      | 1       | 1          |
| Te-123  | 0.0000 | 0.0000 | 0.0000  | 2.4        |
| Te-127m | 1      | 1      | 1       | 1          |
| Te-127  | 0.9756 | 0.9756 | 0.9565  | 1.02       |
| Te-129  | 1      | 1      | 1       | 1          |
| Te-129m | 1      | 1      | 1       | 1          |
| I-129   | 0.0000 | 0.0000 | 0.0000  | 5234       |
| Te-131  | 1      | 1      | 1       | 1          |
| I-131   | 1.0000 | 1.0000 | 0.0021  | 468        |
| Xe-131m | 0.0118 | 0.0118 | 0.0000  | 1545       |
| Te-131m | 1      | 1      | 1       | 1          |
| Te-131  | 0.2220 | 0.2220 | 0.2090  | 1.06       |
| I-131   | 1.0000 | 1.0000 | 0.1106  | 9.0        |
| Xe-131m | 0.0118 | 0.0118 | 0.0005  | 22         |
| Te-132  | 1      | 1      | 1       | 1          |
| I-132   | 1.0000 | 1.0000 | 0.8976  | 1.11       |
| Te-133  | 1      | 1      | 1       | 1          |
| I-133   | 1.0000 | 1.0000 | 0.0096  | 105        |
| Xe-133m | 0.0288 | 0.0288 | 0.0001  | 463        |
| Xe-133  | 1.0000 | 1.0000 | 0.0011  | 876        |
| Te-133m | 1      | 1      | 1       | 1          |
| Te-133  | 0.1750 | 0.1750 | 0.1134  | 1.54       |
| I-133   | 1.0000 | 1.0000 | 0.0383  | 26         |
| Xe-133m | 0.0288 | 0.0288 | 0.0003  | 105        |
| Xe-133  | 1.0000 | 1.0000 | 0.0051  | 198        |
| Te-134  | 1      | 1      | 1       | 1          |
| I-134   | 1.0000 | 1.0000 | 0.3268  | 3.1        |
| I-123   | 1      | 1      | 1       | 1          |
| Te-123  | 0.0000 | 0.0000 | 0.0000  | 458        |
|        | W-real | W-cons | W-class | cons/class |
|--------|--------|--------|---------|------------|
| I-131  | 1      | 1      | 1       | 1          |
| Xe-131m| 0.0118 | 0.0118 | 0.0035  | 3.3        |
| I-133  | 1      | 1      | 1       | 1          |
| Xe-133m| 0.0288 | 0.0288 | 0.0062  | 4.6        |
| Xe-133 | 1.0000 | 1.0000 | 0.1139  | 8.8        |
| I-135  | 1      | 1      | 1       | 1          |
| Xe-135m| 0.1657 | 0.1657 | 0.1453  | 1.14       |
| Xe-135 | 0.9991 | 0.9991 | 0.3087  | 3.2        |
| Cs-135 | 0.0000 | 0.0000 | 0.0000  | 924        |
| Cs-134m| 1      | 1      | 1       | 1          |
| Cs-134 | 0.2849 | 0.2849 | 0.0002  | 1779       |
| Cs-137 | 1      | 1      | 1       | 1          |
| Ba-137m| 0.9440 | 0.9440 | 0.9440  | 1.00       |
| Ba-131 | 1      | 1      | 1       | 1          |
| Cs-131 | 1.0000 | 1.0000 | 0.3998  | 2.5        |
| Ba-140 | 1      | 1      | 1       | 1          |
| La-140 | 1.0000 | 1.0000 | 0.7354  | 1.36       |
| Ce-143 | 1      | 1      | 1       | 1          |
| Pr-143 | 1.0000 | 1.0000 | 0.0783  | 13         |
| Ce-144 | 1      | 1      | 1       | 1          |
| Pr-144m| 0.0098 | 0.0098 | 0.0098  | 1.00       |
| Pr-144 | 1.0000 | 1.0000 | 0.9996  | 1.00       |
| Nd-147 | 1      | 1      | 1       | 1          |
| Pm-147 | 0.2232 | 0.2282 | 0.0109  | 2.1        |
| Sm-147 | 0.0000 | 0.0000 | 0.0000  | 23         |
| Nd-149 | 1      | 1      | 1       | 1          |
| Pm-149 | 1.0000 | 1.0000 | 0.0290  | 34         |
| Pm-147 | 1      | 1      | 1       | 1          |
| Sm-147 | 0.0000 | 0.0000 | 0.0000  | 1.14       |
| Eu-152 | 1      | 1      | 1       | 1          |
| Gd-152 | 0.0000 | 0.0000 | 0.0000  | 1.03       |
| Eu-152m| 1      | 1      | 1       | 1          |
| Gd-152 | 0.0000 | 0.0000 | 0.0000  | 652        |
| Dy-166 | 1      | 1      | 1       | 1          |
| Ho-166 | 1.0000 | 1.0000 | 0.5801  | 1.72       |
| Er-171 | 1      | 1      | 1       | 1          |
| Tm-171 | 0.3025 | 0.3027 | 0.0004  | 680        |
| W-187  | 1      | 1      | 1       | 1          |
| Re-187 | 0.0000 | 0.0000 | 0.0000  | 256        |
| Re-186 | 1      | 1      | 1       | 1          |
| Os-186 | 0.0000 | 0.0000 | 0.0000  | 68         |
| Os-191m| 1      | 1      | 1       | 1          |
| Os-191 | 1.0000 | 1.0000 | 0.0313  | 32         |
| Os-193 | 1      | 1      | 1       | 1          |
| Ir-193m| 0.0035 | 0.0035 | 0.0003  | 11         |
| Pt-193m| 1      | 1      | 1       | 1          |
| Pt-193 | 0.0135 | 0.0137 | 0.0002  | 58         |
| Pt-197m| 1      | 1      | 1       | 1          |
| Pt-197 | 0.9670 | 0.9670 | 0.0620  | 16         |
| Radioactive Isotope | W-real | W-cons | W-class | cons/class |
|---------------------|---------|---------|---------|------------|
| Hg-197m             | 1       | 1       | 1       | 1          |
| Hg-197              | 0.9140  | 0.9140  | 0.1874  | 4.9        |
| At-211              | 1       | 1       | 1       | 1          |
| Po-211              | 0.5820  | 0.5820  | 0.5820  | 1.00       |
| Bi-207              | 0.0087  | 0.0087  | 0.0000  | 833        |
| Ra-225              | 1       | 1       | 1       | 1          |
| Ac-225              | 1.0000  | 1.0000  | 0.4430  | 2.3        |
| Fr-221              | 1.0000  | 1.0000  | 0.4430  | 2.3        |
| At-217              | 1.0000  | 1.0000  | 0.4430  | 2.3        |
| Bi-213              | 0.9999  | 0.9999  | 0.4430  | 2.3        |
| Po-213              | 0.9790  | 0.9790  | 0.4337  | 2.3        |
| Th-209              | 0.0209  | 0.0209  | 0.0093  | 2.3        |
| Pb-209              | 0.9999  | 0.9999  | 0.4430  | 2.3        |
| Th-226              | 1       | 1       | 1       | 1          |
| Ra-222              | 1.0000  | 1.0000  | 0.9198  | 1.09       |
| Rn-218              | 1.0000  | 1.0000  | 0.9198  | 1.09       |
| Po-214              | 1.0000  | 1.0000  | 0.9198  | 1.09       |
| Pb-210              | 0.0307  | 0.0307  | 0.0000  | 11 734     |
| Bi-210              | 0.0301  | 0.0307  | 0.0000  | 11 762     |
| Po-210              | 0.0162  | 0.0290  | 0.0000  | 11 890     |
| Th-229              | 1       | 1       | 1       | 1          |
| Ra-225              | 0.9411  | 0.9999  | 0.9999  | 1.00       |
| Ac-225              | 0.9016  | 0.9999  | 0.9999  | 1.00       |
| Fr-221              | 0.9016  | 0.9999  | 0.9999  | 1.00       |
| At-217              | 0.9016  | 0.9999  | 0.9999  | 1.00       |
| Bi-213              | 0.9013  | 0.9998  | 0.9998  | 1.00       |
| Po-213              | 0.8825  | 0.9789  | 0.9789  | 1.00       |
| Th-209              | 0.0188  | 0.0209  | 0.0209  | 1.00       |
| Pb-209              | 0.9008  | 0.9998  | 0.9998  | 1.00       |
| Pa-230              | 1       | 1       | 1       | 1          |
| Th-230              | 0.0000  | 0.0000  | 0.0000  | 15         |
| U-230               | 0.0840  | 0.0840  | 0.0282  | 3.0        |
| Th-226              | 0.0840  | 0.0840  | 0.0282  | 3.0        |
| Ra-222              | 0.0840  | 0.0840  | 0.0282  | 3.0        |
| Rn-218              | 0.0840  | 0.0840  | 0.0282  | 3.0        |
| Po-214              | 0.0840  | 0.0840  | 0.0282  | 3.0        |
| Pb-210              | 0.0022  | 0.0026  | 0.0002  | 14         |
| Bi-210              | 0.0021  | 0.0026  | 0.0002  | 14         |
| Po-210              | 0.0011  | 0.0024  | 0.0002  | 15         |
| Ra-226              | 0.0000  | 0.0000  | 0.0000  | 15         |
| Rn-222              | 0.0000  | 0.0000  | 0.0000  | 15         |
| Po-218              | 0.0000  | 0.0000  | 0.0000  | 15         |
| Pb-214              | 0.0000  | 0.0000  | 0.0000  | 15         |
| Bi-214              | 0.0000  | 0.0000  | 0.0000  | 15         |
| Pa-233              | 1       | 1       | 1       | 1          |
| U-233               | 0.0000  | 0.0000  | 0.0000  | 9.4        |
| Th-229              | 0.0000  | 0.0000  | 0.0000  | 9.4        |
| Ra-225              | 0.0000  | 0.0000  | 0.0000  | 9.4        |
| Ac-225              | 0.0000  | 0.0000  | 0.0000  | 9.4        |
|     | W-real | W-cons | W-class | cons/class |
|-----|--------|--------|---------|------------|
| Fr-221 | 0.0000 | 0.0000 | 0.0000 | 9.4        |
| At-217 | 0.0000 | 0.0000 | 0.0000 | 9.4        |
| Bi-213 | 0.0000 | 0.0000 | 0.0000 | 9.4        |
| Po-213 | 0.0000 | 0.0000 | 0.0000 | 9.4        |
| Tl-209 | 0.0000 | 0.0000 | 0.0000 | 9.4        |
| Pb-209 | 0.0000 | 0.0000 | 0.0000 | 9.4        |
| U-230   | 1      | 1      | 1      | 1          |
| Th-226 | 1.0000 | 1.0000 | 0.9930 | 1.01       |
| Ra-222 | 1.0000 | 1.0000 | 0.9930 | 1.01       |
| Rn-218 | 1.0000 | 1.0000 | 0.9930 | 1.01       |
| Po-214 | 1.0000 | 1.0000 | 0.9930 | 1.01       |
| Pb-210 | 0.0282 | 0.0304 | 0.0025 | 12         |
| Bi-210 | 0.0276 | 0.0304 | 0.0025 | 12         |
| Po-210 | 0.0141 | 0.0290 | 0.0024 | 12         |
| U-231   | 1      | 1      | 1      | 1          |
| Pa-231 | 0.0000 | 0.0000 | 0.0000 | 60         |
| Ac-227 | 0.0000 | 0.0000 | 0.0000 | 60         |
| Fr-223 | 0.0000 | 0.0000 | 0.0000 | 60         |
| Th-227 | 0.0000 | 0.0000 | 0.0000 | 6.9        |
| Ra-223 | 0.0000 | 0.0000 | 0.0000 | 10         |
| Rn-219 | 0.0000 | 0.0000 | 0.0000 | 10         |
| Po-215 | 0.0000 | 0.0000 | 0.0000 | 10         |
| Bi-211 | 0.0000 | 0.0000 | 0.0000 | 10         |
| Po-211 | 0.0000 | 0.0000 | 0.0000 | 10         |
| Tl-207 | 0.0000 | 0.0000 | 0.0000 | 10         |
| U-232   | 1      | 1      | 1      | 1          |
| Th-228 | 0.1614 | 0.9072 | 0.9028 | 1.00       |
| Ra-224 | 0.1571 | 0.9072 | 0.9028 | 1.00       |
| Rn-220 | 0.1570 | 0.9072 | 0.9028 | 1.00       |
| Po-216 | 0.1570 | 0.9072 | 0.9028 | 1.00       |
| Pb-212 | 0.1565 | 0.9072 | 0.9028 | 1.00       |
| Bi-212 | 0.1565 | 0.9072 | 0.9028 | 1.00       |
| Po-212 | 0.1002 | 0.5811 | 0.5783 | 1.00       |
| Tl-208 | 0.0562 | 0.3260 | 0.3245 | 1.00       |
| U-233   | 1      | 1      | 1      | 1          |
| Th-229 | 0.0000 | 0.0094 | 0.0094 | 1.00       |
| Ra-225 | 0.0000 | 0.0094 | 0.0094 | 1.00       |
| Ac-225 | 0.0000 | 0.0094 | 0.0094 | 1.00       |
| Fr-221 | 0.0000 | 0.0094 | 0.0094 | 1.00       |
| At-217 | 0.0000 | 0.0094 | 0.0094 | 1.00       |
| Bi-213 | 0.0000 | 0.0094 | 0.0094 | 1.00       |
| Po-213 | 0.0000 | 0.0092 | 0.0092 | 1.00       |
| Tl-209 | 0.0000 | 0.0002 | 0.0002 | 1.00       |
| Pb-209 | 0.0000 | 0.0094 | 0.0094 | 1.00       |
| U-236   | 1      | 1      | 1      | 1          |
| Th-232 | 0.0000 | 0.0000 | 0.0000 | 1.00       |
| Ra-228 | 0.0000 | 0.0000 | 0.0000 | 1.01       |
| Ac-228 | 0.0000 | 0.0000 | 0.0000 | 1.01       |
|          | W-real | W-cons | W-class | cons/class |
|----------|--------|--------|---------|------------|
| Th-228   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Ra-224   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Rn-220   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Po-216   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Pb-212   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Bi-212   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Po-212   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Tl-208   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| U-237    | 1      | 1      | 1       | 1          |
| Np-237   | 0.0000 | 0.0000 | 0.0000  | 37         |
| Pa-233   | 0.0000 | 0.0000 | 0.0000  | 37         |
| U-233    | 0.0000 | 0.0000 | 0.0000  | 38         |
| Th-229   | 0.0000 | 0.0000 | 0.0000  | 38         |
| Ra-225   | 0.0000 | 0.0000 | 0.0000  | 38         |
| Ac-225   | 0.0000 | 0.0000 | 0.0000  | 38         |
| Fr-221   | 0.0000 | 0.0000 | 0.0000  | 38         |
| At-217   | 0.0000 | 0.0000 | 0.0000  | 38         |
| Bi-213   | 0.0000 | 0.0000 | 0.0000  | 38         |
| Po-213   | 0.0000 | 0.0000 | 0.0000  | 38         |
| Tl-209   | 0.0000 | 0.0000 | 0.0000  | 38         |
| Pb-209   | 0.0000 | 0.0000 | 0.0000  | 38         |
| U-239    | 1      | 1      | 1       | 1          |
| Np-239   | 1.0000 | 1.0000 | 0.0067  | 150        |
| Pu-239   | 0.0000 | 0.0000 | 0.0000  | 15536      |
| U-235    | 0.0000 | 0.0000 | 0.0000  | 15614      |
| U-240    | 1      | 1      | 1       | 1          |
| Np-240m  | 1.0001 | 1.0001 | 0.9598  | 1.04       |
| Np-240   | 0.0011 | 0.0011 | 0.0009  | 1.23       |
| Pu-240   | 0.0001 | 0.0001 | 0.0000  | 431        |
| U-236    | 0.0000 | 0.0000 | 0.0000  | 433        |
| Th-232   | 0.0000 | 0.0000 | 0.0000  | 435        |
| Np-237   | 1      | 1      | 1       | 1          |
| Pa-233   | 0.8934 | 1.0000 | 1.0000  | 1.00       |
| U-233    | 0.0000 | 0.0004 | 0.0004  | 1.00       |
| Th-229   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Ra-225   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Ac-225   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Fr-221   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| At-217   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Bi-213   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Po-213   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Tl-209   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Pb-209   | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Np-239   | 1      | 1      | 1       | 1          |
| Pu-239   | 0.0000 | 0.0000 | 0.0000  | 107        |
| U-235    | 0.0000 | 0.0000 | 0.0000  | 108        |
| Np-240   | 1      | 1      | 1       | 1          |
| Pu-240   | 0.0001 | 0.0001 | 0.0000  | 5885       |
| U-236    | 0.0000 | 0.0000 | 0.0000  | 5915       |
|  | W-real | W-cons | W-class | cons/class |
|---|---|---|---|---|
| Th-232 | 0.0000 | 0.0000 | 0.0000 | 5945 |
| Pu-236 | 1 | 1 | 1 | 1 |
| U-232 | 0.0052 | 0.0407 | 0.0361 | 1.13 |
| Th-228 | 0.0006 | 0.0401 | 0.0356 | 1.13 |
| Ra-224 | 0.0006 | 0.0401 | 0.0356 | 1.13 |
| Rn-220 | 0.0006 | 0.0401 | 0.0356 | 1.13 |
| Po-216 | 0.0006 | 0.0401 | 0.0356 | 1.13 |
| Pb-212 | 0.0006 | 0.0401 | 0.0356 | 1.13 |
| Bi-212 | 0.0006 | 0.0401 | 0.0356 | 1.13 |
| Po-212 | 0.0004 | 0.0257 | 0.0228 | 1.13 |
| Po-212 | 0.0004 | 0.0257 | 0.0228 | 1.13 |
| Th-208 | 0.0002 | 0.0144 | 0.0128 | 1.13 |
| Pu-237 | 1 | 1 | 1 | 1 |
| Np-237 | 0.0000 | 0.0000 | 0.0000 | 5.6 |
| Pa-233 | 0.0000 | 0.0000 | 0.0000 | 5.6 |
| U-233 | 0.0000 | 0.0000 | 0.0000 | 5.6 |
| Th-229 | 0.0000 | 0.0000 | 0.0000 | 5.7 |
| Ra-225 | 0.0000 | 0.0000 | 0.0000 | 5.7 |
| Ac-225 | 0.0000 | 0.0000 | 0.0000 | 5.7 |
| Fr-221 | 0.0000 | 0.0000 | 0.0000 | 5.7 |
| At-217 | 0.0000 | 0.0000 | 0.0000 | 5.7 |
| Bi-213 | 0.0000 | 0.0000 | 0.0000 | 5.7 |
| Po-213 | 0.0000 | 0.0000 | 0.0000 | 5.7 |
| Tl-209 | 0.0000 | 0.0000 | 0.0000 | 5.7 |
| Pb-209 | 0.0000 | 0.0000 | 0.0000 | 5.7 |
| Pu-238 | 1 | 1 | 1 | 1 |
| U-234 | 0.0000 | 0.0002 | 0.0002 | 1.01 |
| Th-230 | 0.0000 | 0.0000 | 0.0000 | 1.01 |
| Ra-226 | 0.0000 | 0.0000 | 0.0000 | 1.02 |
| Rn-222 | 0.0000 | 0.0000 | 0.0000 | 1.02 |
| Po-218 | 0.0000 | 0.0000 | 0.0000 | 1.02 |
| Pb-214 | 0.0000 | 0.0000 | 0.0000 | 1.02 |
| Bi-214 | 0.0000 | 0.0000 | 0.0000 | 1.02 |
| Po-214 | 0.0000 | 0.0000 | 0.0000 | 1.02 |
| Pb-210 | 0.0000 | 0.0000 | 0.0000 | 1.02 |
| Bi-210 | 0.0000 | 0.0000 | 0.0000 | 1.02 |
| Po-210 | 0.0000 | 0.0000 | 0.0000 | 1.02 |
| Pu-239 | 1 | 1 | 1 | 1 |
| U-235 | 0.0000 | 0.0000 | 0.0000 | 1.01 |
| Pu-240 | 1 | 1 | 1 | 1 |
| U-236 | 0.0000 | 0.0000 | 0.0000 | 1.01 |
| Th-232 | 0.0000 | 0.0000 | 0.0000 | 1.01 |
| Pu-241 | 1 | 1 | 1 | 1 |
| Am-241 | 0.0008 | 0.0303 | 0.0295 | 1.02 |
| Np-237 | 0.0000 | 0.0000 | 0.0000 | 1.03 |
| Pu-242 | 1 | 1 | 1 | 1 |
| U-238 | 0.0000 | 0.0000 | 0.0000 | 1.00 |
| Pu-243 | 1 | 1 | 1 | 1 |
| Am-243 | 0.0001 | 0.0001 | 0.0000 | 1225 |
Table A1. (Continued.)

|       | W-real | W-cons | W-class | cons/class |
|-------|--------|--------|---------|------------|
| Np-239 | 0.0001 | 0.0001 | 0.0000  | 1225       |
| Pu-239 | 0.0000 | 0.0000 | 0.0000  | 1231       |
| U-235  | 0.0000 | 0.0000 | 0.0000  | 1237       |
| Pu-244 |        |        |         |            |
| U-240  | 0.9965 | 0.9988 | 0.9988  | 1.00       |
| Np-240m| 0.9965 | 0.9988 | 0.9988  | 1.00       |
| Np-240 | 0.0011 | 0.0011 | 0.0011  | 1.00       |
| Pu-240 | 0.0001 | 0.0105 | 0.0105  | 1.00       |
| U-236  | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Th-232 | 0.0000 | 0.0000 | 0.0000  | 1.02       |
| Am-241 |        |        |         |            |
| U-232  | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Am-242 |        |        |         |            |
| U-238  | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Am-242m| 0.9929 | 0.9954 | 0.9954  | 1.00       |
| Cm-242 | 0.4040 | 0.8101 | 0.8084  | 1.00       |
| Np-238 | 0.0045 | 0.0045 | 0.0045  | 1.00       |
| Pu-238 | 0.0012 | 0.3465 | 0.3450  | 1.00       |
| Pu-242 | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| U-234  | 0.0000 | 0.0001 | 0.0001  | 1.01       |
| Th-230 | 0.0000 | 0.0000 | 0.0000  | 1.02       |
| Am-243 |        |        |         |            |
| U-235  | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Cm-242 |        |        |         |            |
| U-238  | 0.0049 | 0.0097 | 0.0049  | 1.97       |
| U-234  | 0.0000 | 0.0000 | 0.0000  | 1.98       |
| Th-230 | 0.0000 | 0.0000 | 0.0000  | 1.99       |
| Cm-243 |        |        |         |            |
| U-235  | 0.0000 | 0.0011 | 0.0011  | 1.01       |
| Am-243 | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Np-239 | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| U-235  | 0.0000 | 0.0000 | 0.0000  | 1.02       |
| Cm-244 |        |        |         |            |
| U-235  | 0.0000 | 0.0000 | 0.0000  | 1.03       |
| Th-232 | 0.0000 | 0.0000 | 0.0000  | 1.03       |
| Cm-245 |        |        |         |            |
| U-235  | 0.0238 | 0.9857 | 0.9855  | 1.00       |
| Am-241 | 0.0000 | 0.1194 | 0.1187  | 1.01       |
| Np-237 | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Cm-246 |        |        |         |            |
| U-235  | 0.0000 | 0.0002 | 0.0002  | 1.01       |
| Th-232 | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Cm-247 |        |        |         |            |

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| Element   | W-real | W-cons | W-class | cons/class |
|-----------|--------|--------|---------|------------|
| Pu-243    | 0.9992 | 1.0000 | 1.0000  | 1.00       |
| Am-243    | 0.0000 | 0.0094 | 0.0094  | 1.00       |
| Np-239    | 0.0000 | 0.0094 | 0.0094  | 1.00       |
| Pu-239    | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| U-235     | 0.0000 | 0.0000 | 0.0000  | 1.02       |
| **Cm-248**| 1      | 1      | 1       | 1          |
| Pu-244    | 0.0000 | 0.0000 | 0.0000  | 1.00       |
| U-240     | 0.0000 | 0.0000 | 0.0000  | 1.00       |
| Np-240m   | 0.0000 | 0.0000 | 0.0000  | 1.00       |
| Np-240    | 0.0000 | 0.0000 | 0.0000  | 1.00       |
| Pu-240    | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| U-236     | 0.0000 | 0.0000 | 0.0000  | 1.02       |
| Th-232    | 0.0000 | 0.0000 | 0.0000  | 1.02       |
| **Bk-249**| 1      | 1      | 1       | 1          |
| Cf-249    | 0.0011 | 0.0036 | 0.0025  | 1.43       |
| Cm-245    | 0.0000 | 0.0000 | 0.0000  | 1.44       |
| Pu-241    | 0.0000 | 0.0000 | 0.0000  | 1.44       |
| Am-241    | 0.0000 | 0.0000 | 0.0000  | 1.45       |
| Np-237    | 0.0000 | 0.0000 | 0.0000  | 1.46       |
| **Cf-248**| 1      | 1      | 1       | 1          |
| Cm-244    | 0.0212 | 0.0614 | 0.0431  | 1.42       |
| Pu-240    | 0.0000 | 0.0002 | 0.0001  | 1.43       |
| U-236     | 0.0000 | 0.0000 | 0.0000  | 1.44       |
| Th-232    | 0.0000 | 0.0000 | 0.0000  | 1.44       |
| **Cf-249**| 1      | 1      | 1       | 1          |
| Cm-245    | 0.0000 | 0.0074 | 0.0074  | 1.01       |
| Pu-241    | 0.0000 | 0.0060 | 0.0059  | 1.01       |
| Am-241    | 0.0000 | 0.0004 | 0.0004  | 1.01       |
| Np-237    | 0.0000 | 0.0000 | 0.0000  | 1.02       |
| **Cf-250**| 1      | 1      | 1       | 1          |
| Cm-246    | 0.0001 | 0.0028 | 0.0027  | 1.03       |
| Pu-242    | 0.0000 | 0.0000 | 0.0000  | 1.03       |
| U-238     | 0.0000 | 0.0000 | 0.0000  | 1.04       |
| **Cf-252**| 1      | 1      | 1       | 1          |
| Cm-248    | 0.0000 | 0.0000 | 0.0000  | 1.14       |
| Pu-244    | 0.0000 | 0.0000 | 0.0000  | 1.14       |
| U-240     | 0.0000 | 0.0000 | 0.0000  | 1.14       |
| Np-240m   | 0.0000 | 0.0000 | 0.0000  | 1.14       |
| Np-240    | 0.0000 | 0.0000 | 0.0000  | 1.14       |
| Pu-240    | 0.0000 | 0.0000 | 0.0000  | 1.15       |
| U-236     | 0.0000 | 0.0000 | 0.0000  | 1.15       |
| Th-232    | 0.0000 | 0.0000 | 0.0000  | 1.16       |
| **Cf-253**| 1      | 1      | 1       | 1          |
| Cm-249    | 0.0031 | 0.0031 | 0.0031  | 1.02       |
| Es-253    | 0.9969 | 0.9969 | 0.3415  | 2.9        |
| Bk-249    | 0.4766 | 0.5044 | 0.0433  | 12         |
| Cf-249    | 0.0004 | 0.0019 | 0.0001  | 14         |
| Cm-245    | 0.0000 | 0.0000 | 0.0000  | 14         |
| Pu-241    | 0.0000 | 0.0000 | 0.0000  | 14         |
Appendix B. Brief description of the numerical algorithm to derive weighting-factor values

We analyse the scenario where at $t = t_0$ a material is only contaminated with head-of-chain radionuclide $P$ with initial activity (concentration) $A_P(0)$. The corresponding decay chain of this nuclide consists of $n_P + 1$ radionuclides and the time-rate of change of activity $A_i$ of any

| Table A1. (Continued.) | W-real | W-cons | W-class | cons/class |
|-------------------------|--------|--------|---------|------------|
| Am-241                  | 0.0000 | 0.0000 | 0.0000  | 14         |
| Np-237                  | 0.0000 | 0.0000 | 0.0000  | 14         |
| **Cf-254**              | 1      | 1      | 1       | 1          |
| Cm-250                  | 0.0000 | 0.0000 | 0.0000  | 4.2        |
| Bk-250                  | 0.0000 | 0.0000 | 0.0000  | 4.2        |
| **Cf-250**              | 0.0000 | 0.0000 | 0.0000  | 4.2        |
| Pu-246                  | 0.0000 | 0.0000 | 0.0000  | 4.2        |
| Cm-246                  | 0.0000 | 0.0000 | 0.0000  | 4.3        |
| Pu-242                  | 0.0000 | 0.0000 | 0.0000  | 4.3        |
| **Es-253**              | 1      | 1      | 1       | 1          |
| Bk-249                  | 0.5047 | 0.5138 | 0.0516  | 10.0       |
| Cf-249                  | 0.0005 | 0.0019 | 0.0002  | 12         |
| Cm-245                  | 0.0000 | 0.0000 | 0.0000  | 12         |
| Pu-241                  | 0.0000 | 0.0000 | 0.0000  | 12         |
| Am-241                  | 0.0000 | 0.0000 | 0.0000  | 13         |
| Np-237                  | 0.0000 | 0.0000 | 0.0000  | 13         |
| **Es-254**              | 1      | 1      | 1       | 1          |
| Bk-250                  | 0.9997 | 0.9998 | 0.9963  | 1.00       |
| Cf-250                  | 0.0299 | 0.0739 | 0.0485  | 1.52       |
| Cm-246                  | 0.0000 | 0.0002 | 0.0002  | 1.53       |
| Pu-242                  | 0.0000 | 0.0000 | 0.0000  | 1.54       |
| **U-238**               | 0.0000 | 0.0000 | 0.0000  | 1.55       |
| **Es-254m**             | 1      | 1      | 1       | 1          |
| Fm-254                  | 0.9800 | 0.9800 | 0.7830  | 1.25       |
| Bk-250                  | 0.0032 | 0.0032 | 0.0026  | 1.25       |
| Cf-250                  | 0.0503 | 0.0506 | 0.0003  | 151        |
| Cm-246                  | 0.0000 | 0.0001 | 0.0000  | 155        |
| Pu-242                  | 0.0000 | 0.0000 | 0.0000  | 155        |
| **U-238**               | 0.0000 | 0.0000 | 0.0000  | 156        |
| **Fm-254**              | 1      | 1      | 1       | 1          |
| Cf-250                  | 0.0515 | 0.0515 | 0.0000  | 1826       |
| Cm-246                  | 0.0000 | 0.0001 | 0.0000  | 1874       |
| Pu-242                  | 0.0000 | 0.0000 | 0.0000  | 1885       |
| **U-238**               | 0.0000 | 0.0000 | 0.0000  | 1896       |
| **Cf-251**              | 1      | 1      | 1       | 1          |
| Cm-247                  | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Pu-243                  | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Am-243                  | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Np-239                  | 0.0000 | 0.0000 | 0.0000  | 1.01       |
| Pu-239                  | 0.0000 | 0.0000 | 0.0000  | 1.02       |
| **U-235**               | 0.0000 | 0.0000 | 0.0000  | 1.02       |
member $i$ can be described with the following equation, also known as the Bateman equation (Bateman 1910):

$$\frac{\partial}{\partial t} A_j = \sum_{j=0}^{n_0} M_{ij}A_j \text{ with } M_{ij} \equiv \nu_{ij}\lambda_i,$$

where index $j$ runs over all nuclides in the chain (including the head-of-chain $j = 0$, indicated by ‘P’; see section 2.1 for notation). In this equation, $\lambda_i = \ln 2/T_{1/2,i}$ is the decay constant of nuclide $i$, with $T_{1/2,i}$ its half-life. Furthermore, $\nu_{ij}$ is defined as the fraction of parent nuclide $j$ that upon decay forms daughter $i$. This means that for all direct parent nuclides $j$ of nuclide $i$, $\nu_{ij}$ is the branching ratio, and that $\nu_{ij} = 0$ for all combinations of radionuclides that are not directly related to each other. For $j = i$, we define $\nu_{ij} = -1$, such that pure radioactive decay of nuclide $i$ itself is captured by the same conceptual formulation: per disintegration, 100% of the decaying parent is lost.

The general solution of the set of $n_0 + 1$ differential equations given by equation (B1) for activity concentration $A_i(t)$ at time $t$ is then found from the set of initial concentrations $A_j(0)$ via:

$$A_i(t) = \sum_{j=0}^{n_0} H_{ij}(t)A_j(0),$$

with $H_{ij}(t)$ the components of the following matrix-exponential:

$$H(t) \equiv \exp [Mt],$$

and with matrix $M$ consisting of components $M_{ij}$. Note that the exponential of matrix $Mt$ as in equation (B3) does not simply consist of the exponential of its elements, i.e. $H_{ij} \neq \exp [M_{ij}t]$, see, for example, Moler and Van Loan (2003). Instead, to estimate the matrix-exponential components $H_{ij}$, we adopt the numerical implementation in FORTRAN-90 by Alan Miller as provided by Blevins (2002). We evaluate the evolution of the activity (concentration) $A_i(t)$ for all nuclides in the decay chain over a period of 100 years. This is numerically done as follows.

The entire time period of 100 years is subdivided in short time intervals. At the start of each time interval, the time is reset to $t = 0$, after which the values of $A_i(t)$ at the end of the short time interval are calculated by equation (B2). These values then serve as initial activity concentrations $A_i(0)$ in the subsequent (short) time interval, which is again subjected to equation (B2). Only in the first time interval, we have $A_{i=0}(0) = A_P(0) = 0$, whereas $A_j(0) = 0$ for all progeny $j = 1, 2, \ldots, n_P$, as follows directly from the initial conditions. Hence, $A_j(0)$ for $j = 1, 2, \ldots, n_P$ can attain values larger than 0 in any subsequent time interval. This interval-procedure is repeated until the end of the final time interval reaches 100 years.

Transformation matrices $H$ from equation (B3) for the entire family of head-of-chain plus all progeny are constructed for four different time intervals: (1) 1 s, (2) 1 min, (3) 1 h, and (4) 1 day. Following the aforementioned procedure, these are used to estimate the activities of all nuclides in a decay chain at: (1) all seconds in the first 3 min, (2) all minutes in the first 3 h, (3) all hours in the first week, and (4) all days in 100 years, respectively. The different timescales are used to facilitate the discrimination of different timescales in the dynamical effects.

For some head-of-chain nuclides, the set of progeny nuclides involves a wide range of half-lives. This gives a matrix $M$ with a high condition number, which can lead to numerical problems. To avoid such problems, we eliminate all 39 nuclides with a half-life shorter than
10 s and let its parents decay directly to its children. These so-called ‘orphan nuclides’ are taken into account afterwards by setting the corresponding DC weighting factors of these nuclides equal to the sum of the products of the weighting factors of their direct parents and the branching ratios associated with the transitions to these orphan nuclides, for instance:

$$w_{D,j,\text{orphan}} = \sum_{k \in \text{direct parents of } j} \nu_{jk} w_{D,k}$$ \hspace{1cm} (B4)

In case the head-of-chain, parent radionuclide concerns an orphan nuclide, no weighting factors for the corresponding progeny have been determined. The shortest integration step in our algorithm is 1 s, which is therefore ten times smaller than the shortest half-life of the nuclides that are explicitly regarded (the non-orphan nuclides). This relation meets the requirement set for this type of calculation as suggested by Tachihara and Sekimoto (1999, p 1181) to ensure sufficient accuracy of the numerical integration.

The activities of all progeny at the given points in time are used as the basis to calculate the different weighting factors according to their different definitions in sections 2.2 through 2.4. The maximum activity within the first 100 years is used to estimate the classical, conservative weighting factor according to equation (20). For all selected time-integration periods τ, the new, conservative weighting factor is estimated by equation (15), where τ* is estimated by searching for the point in time τ* where the activity of the progeny equals that at τ* + τ (condition given by equation (14), alternatively written as: A_{D,j}(t_\tau^* + \tau) = A_{D,j}(t_\tau^*)) for daughter nuclides that grow only by direct decay from the head-of-chain parent nuclide P. These two weights are also calculated via analytic solutions (23) (classical) and (18) (new), both multiplied with the branching ratio related to this specific nuclear transformation. The new method applied to the first, immediate time-integration interval [0, τ) (for realistic dose calculations), is estimated by equation (6), i.e. by the ratio of time-integrated activities (= ratio of the number of disintegrations) over this interval.

All dose (activity) integrations are performed as follows: once the starting time of an integration period has been fixed (0 or τ*), the activity (concentration) in the first hour of the interval is integrated with steps of 1 s, the rest of the first day of the interval is integrated with steps of 1 min and the rest of the integration is performed with steps of 1 h. To this end, new estimates of all activity concentrations are constructed on the fly of the integrations, as the set of activities that was constructed to find τ* often lack the required time resolution close to τ*.

When available, the estimates for the weighting factors from EC (2000) and IAEA (2005) are provided along with our calculations in the supplementary material. This analysis is performed for all progeny associated with all 1252 radionuclides in the electronic database of Publication 107 of the ICRP (2008). Several checks were performed on the output of the numerical model to ensure validity and consistency. More specific information on (the use of) the supplementary data files is given in their header.

**ORCID iDs**

Teun van Dillen © https://orcid.org/0000-0002-8754-0071

**References**

Bateman H 1910 Solution of a system of differential equations occurring in the theory of radioactive transformations Proc. Camb. Philos. Soc. 15 423–7

Blevins J 2002 Alan Miller’s Fortran software (Accessed 9 September 2019) (http://jblevins.org/mirror/amiller/dmexp.f90)
EC 2000 Practical use of the concepts of clearance and exemption—part I: guidance on general clearance levels for practices European Commission, Radiation Protection 122-I (Luxembourg: Office for official publications of the European Communities) (Accessed 9 September 2019) (https://ec.europa.eu/energy/sites/ener/files/documents/122_part1.pdf)

EC 2014 European council directive 2013/59/EURATOM of 5 december 2013 laying down basic safety standards for protection against the dangers arising from exposure to ionising radiation, and repealing directives 89/618/euratom, 90/641/euratom, 96/29/euratom, 97/43/euratom and 2003/122/euratom OJ L 13, 17.1.2014 (L 13/1) (Luxembourg: Official Journal of the European Union) (Accessed 9 September 2019) (https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32013L0059&from=EN)

IAEA 2004 Application of the concepts of exclusion, exemption and clearance Safety Standards Series No. RS-G-1.7 (Vienna: International Atomic Energy Agency)

IAEA 2005 Derivation of activity concentration values for exclusion, exemption and clearance Safety Reports Series No. 44 (Vienna: International Atomic Energy Agency)

IAEA 2007 IAEA Safety Glossary: Terminology used in Nuclear Safety and Radiation Protection 2007 edn (Vienna: International Atomic Energy Agency)

ICRP 2008 Nuclear decay data for dosimetric calculations. ICRP Publication 107 Ann. ICRP vol 38(3) (Amsterdam: Elsevier)

ICRP 2012 Compendium of dose coefficients based on ICRP Publication 60. ICRP publication 119 Ann. ICRP vol 41(Suppl.) (Amsterdam: Elsevier)

ICRP 2017 Dose coefficients for nonhuman biota environmentally exposed to radiation. ICRP publication 136 Ann. ICRP vol 46(2) (London: Sage Publications Ltd)

Moler C and Van Loan C 2003 Nineteen dubious ways to compute the exponential of a matrix, twenty-five years later SIAM Rev. 45 3–49

Tachihara H and Sekimoto H 1999 Exact error estimation for solutions of nuclide chain equations J. Nucl. Sci. Technol. 36 1176–85

Ulanovsky A, Pröhl G and Gómez-Ros J M 2008 Methods for calculating dose conversion coefficients for terrestrial and aquatic biota J. Environ. Radioact. 99 1440–8

Ulanovsky A and Pröhl G 2012 Dosimetry for reference animals and plants: current state and prospects. Proceedings of the first ICRP symposium on the international system of radiological protection Ann. ICRP 41 218–32 (Accessed 9 September 2019)

Ulanovsky A 2014 Absorbed doses in tissue-equivalent spheres above radioactive sources in soil Radiat. Environ. Biophys. 53 729–37

van Dillen T and van Dijk A 2018 SUDOQU, a new dose-assessment methodology for radiological surface contamination J. Radiol. Prot. 38 1147–203