Spin Fluctuation and Persistent Current in a Mesoscopic Ring Coupled to a Quantum Dot

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We investigate the persistent current influenced by the spin fluctuations in a mesoscopic ring weakly coupled to a quantum dot. It is shown that the Kondo effect gives rise to some unusual features of the persistent current in the limit where the charge transfer between two subsystems is suppressed. Various aspects of the crossover from a delocalized to a localized dot limit are discussed in relation with the effect of the coherent response of the Kondo cloud to the Aharonov-Bohm flux.

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Coherent charge transfer from one region of a system into another in a coupled mesoscopic structure can dramatically change the physical properties of the coupled system. A typical example is a mesoscopic ring coupled to a finite-size wire\footnote{1} or weakly connected to a quantum dot\footnote{2}\footnote{3}. If the charge transfer between the two subsystems is suppressed, these two regions are effectively decoupled from each other in ordinary cases. At very low temperature, however, the spin degree of freedom may play a crucial role in determining the ground state(GS) properties of the coupled system even when the charge fluctuations are suppressed. A system composed of a metallic host and a magnetic impurity offers a good opportunity to study such type of effects. Actually, the Kondo problem, which was first investigated in dilute magnetic alloys\footnote{4}, provides a best known example. Recent experimental efforts have enabled the realization of the Kondo effect in the semiconductor-based QDs\footnote{5}\footnote{6}. In the unitary scattering limit, the Kondo effect gives rise to a perfect transmission\footnote{7}\footnote{8} through the dot. The perfect transmission can be interpreted in terms of a Kondo cloud originating from the itinerant electrons in the leads. Furthermore, using the Aharonov-Bohm(AB) interferometers, phase-coherent Kondo effect has been reported and discussed in Refs.\footnote{9}\footnote{10}. However, transport measurements for such open systems do not provide clear understanding on the phase-coherent property of the GS in closed systems. On the other hand, persistent current(PC) in an isolated AB ring with a coupled dot provides useful information about the effect of spin fluctuations on the GS, which cannot be addressed in open systems. There have been some previous works on the Kondo-assisted PC in a QD embedded in an AB ring\footnote{11}\footnote{12}\footnote{13}.

In this Letter, we investigate the equilibrium properties of a QD side-attached to a mesoscopic ring threaded by an AB flux(see Fig.1). Here, we focus on the effect of spin fluctuations that strongly affect the PC and their phase-coherent properties which are functions of the level spacing, \(\Delta\), and the total number of electrons, \(N_e\). We show that the crossover from the strongly coupling to a localized dot limit introduces various interesting phenomena.

The Hamiltonian of our coupled system is decomposed into the three parts as

\[
H = H_R + H_D + H_T, \tag{1}
\]

where \(H_R\), \(H_D\), and \(H_T\) represent the AB ring, the QD, and the tunneling interaction between the ring and the dot, respectively. The ring is described by an ideal one-dimensional tight-binding model with \(N_R\) lattice sites;

\[
H_R = -t \sum_{j=0}^{N_R-1} \sum_{\sigma} \left( e^{i2\pi/2N_R} c_j^{\dagger} c_{j+1,\sigma} + \text{H.C.} \right), \tag{2}
\]

where \(t\) denotes the hopping integral between the neighboring sites. \(H_R\) can be diagonalized with the eigenvalues \(\varepsilon_m = -2t \cos [(2\pi/N_R)(m+\phi)]\), where \(m\) is an integer within the interval \([- (N_R-1)/2, N_R/2]\) and \(\phi = \Phi/\Phi_0\) with \(\Phi\) and \(\Phi_0 (=\hbar e/c)\) being the magnetic flux and the flux quantum, respectively. The QD, which has a much smaller size compared to the ring, can be described by a spin-degenerate single level with a strong on-site Coulomb repulsion \(U\). At the site “0” of the ring, electrons are allowed to hop to the dot or vice versa. These processes are characterized by a tunnel matrix element \(t’\). In the present study, we adopt the slave-boson(SB) representation for infinite \(U\) in describing the low energy physics in the spin fluctuation limit. In this representation, the electron annihilation operator in the dot, \(d_\sigma\), consists of the SB operator \(b^{\dagger}\) which creates an empty state and a pseudo-fermion operator \(f_\sigma\) which annihilates the singly occupied state with energy \(\varepsilon_D\) and spin \(\sigma\) in the dot; \(d_\sigma = b^{\dagger} f_\sigma\). The Hamiltonians for the dot and for the coupling are given in this representation by...
\[ H_D = \sum_{\sigma} \varepsilon_D \alpha^\dagger_{\alpha} \alpha \sigma, \]  
\[ H_T = -t' \sum_{\sigma} \left( \alpha^\dagger_{0,\sigma} \beta^\dagger \alpha^\dagger \sigma + f^\dagger_{\sigma} \beta \alpha^\dagger_{0,\sigma} \right) \],

respectively. The infinite \( U \) limit gives rise to the constraint preventing double occupancy in the dot. This emerges in the pseudo-charge constraint \( Q = 1 \), where \( Q = b^\dagger b + \sum_{\sigma} f^\dagger_{\sigma} f_{\sigma} \). One way to impose the constraint is to add a “chemical potential” term \( \lambda Q \) to \( H \), and to project it onto the physical subspace \( Q = 1 \) by taking \( \lambda \to \infty \) at the end of calculation \[13\].

The total Hamiltonian is transformed to the following form by using the diagonalized basis of \( H_R \) as

\[ H = \sum_{m \sigma} \varepsilon_m c^\dagger_{m \sigma} c_{m \sigma} + \sum_{\sigma} \varepsilon_D f^\dagger_{\sigma} f_{\sigma} \]
\[ + \sum_{m \sigma} \left( t_m c^\dagger_{m \sigma} \beta^\dagger \alpha^\dagger \sigma + t^*_m f^\dagger_{\sigma} b c_{m \sigma} \right) \],

where \( t_m = -t' / \sqrt{N_R} \). The effective width of the dot level due to the coupling between the dot and the ring is given by \( \Gamma = \pi N_D(\varepsilon_F) t^2 \). Here, \( N_D(\varepsilon_F) = 1/\Delta \) is the density of states (DOS) at the highest occupied level energy, \( \varepsilon_F \), of the ring. For half-filling \( N_R = N_e \), the effective width becomes \( \Gamma = \pi |t'|^2 / (2t N_e \sin [\pi / N_e]) \).

The partition function \( Z \) of the \( N_e \)-electron system of the Hamiltonian is shown to be expressed as a product \( Z = Z_0 Z_{QD} \), where \( Z_0 \) is the partition function of the \( N_e \) free electrons in the ring and \( Z_{QD} \) corresponds to the contribution from the interacting QD;

\[ Z_{QD} = \int_{-\infty}^{\infty} d\varepsilon \ e^{-\beta \varepsilon} \left( g_0(\varepsilon) + \sum_{\sigma} g_\sigma(\varepsilon) \right) \].

The spectral functions \( g_\sigma(\varepsilon) \) for pseudo-particles are given by imaginary parts of the corresponding retarded Green’s functions: \( g_\sigma(\varepsilon) = -(1/\pi) \text{Im} G_\sigma(\varepsilon) \).

We adopt the leading order \( 1/N_e \)-diagrammatic expansion with \( N_e \) being the magnetic degeneracy. It is well known that this approximation describes well the essence of the Kondo correlation preserving the Fermi liquid properties \[10\]. In the leading order, the Green’s functions of the pseudo-particles are given by

\[ G_f(\varepsilon) = \frac{1}{\varepsilon - \varepsilon_D + i0^+} \]  
\[ G_b(\varepsilon) = \frac{1}{\varepsilon - \Pi_0(\varepsilon) + i0^+} \],

The self-energy of the SB \[18\] is given by

\[ \Pi_0(\varepsilon) = \sum_{m \sigma} f(\varepsilon_m) \frac{|t_m|^2}{\varepsilon - \varepsilon_D + \varepsilon_m} \],

where \( f(\varepsilon_m) \) is the occupation probability of the level \( m \). This leads to the spectral function of the SB of the form

\[ \phi_b(\varepsilon) = Z(E_0) \delta(\varepsilon - E_0) \],

where the renormalization factor \( Z(E_0) \) is given by

\[ Z(E_0) = \left( 1 - \frac{\partial}{\partial \varepsilon} \Pi_0(\varepsilon) \right)^{-1} \]  
\[ _{\varepsilon=E_0} \].

\( E_0 \) is obtained by the self-consistent equation; \( E_0 = \Pi_0(\varepsilon_0) \), and corresponds to the GS energy for \( T = 0 \).

The intrinsic characteristic energy scale of the system, \( T_K \), can be defined by the difference of the GS energy and the lowest excited state energy, in the bulk limit of the ring:

\[ T_K^0 \equiv \varepsilon_D - \varepsilon_F - E_0. \]

This corresponds to the bulk Kondo temperature in our scheme.

The persistent current is defined by the relation

\[ I(\phi, T) = -e \frac{\partial}{\hbar \partial \phi} F(\phi, T), \]

where the free energy is given by \( F = -(1/\beta) \ln Z \). The current can be decomposed into two parts: \( I(\phi, T) = I_0(\phi, T) + I_{QD}(\phi, T) \). \( I_0(\phi, T) \) denotes the current contribution of the \( N_e \) free electrons in the ring which can be expressed as

\[ I_0(\phi, T) = \sum_{m \sigma} f(\varepsilon_m) I_m(\phi). \]

Contribution from each level is given by \( I_m(\phi) = -(2e/h) \Delta \sin([2\pi / N_R](m + \phi)) \) with the oscillation amplitude, \( \Delta = 2\pi t / N_R \). Note that \( \Delta \) corresponds to the mean level spacing in the bulk limit. It has been shown by Loss and Goldbart \[20\] that the behavior of \( I_0 \) for an ideal ring depends on the number of electrons with modulo 4. That is, there exist four different number classes \( i \) \((i = 0, 1, 2, 3) \) for \( I^N_{i} \), with \( N_e \) being defined as \( N_e = 4n - i (n \text{ any positive number}) \).

At low temperature, \( T \ll T_K^0 \), the PC circulating in the ring of the system has the following form:

\[ I(\phi, T) = \sum_{m \sigma} F(\varepsilon_m) I_m(\phi). \]

The effective occupation probability in the state \( m \), \( F(\varepsilon_m) \), is given by

\[ F(\varepsilon_m) = f(\varepsilon_m) \left( 1 - \frac{Z(E_0)}{(\varepsilon_m + E_0 - \varepsilon_D)^2} \frac{|t_m|^2}{(\varepsilon_m + E_0 - \varepsilon_D)^2} \right). \]

Several points can be clarified in terms of Eq. \[14\]. In the empty dot limit \( (\varepsilon_D - \varepsilon_F \gg 2\Gamma \) and \( n_D \rightarrow 0 \) with \( n_D \) being the occupation number of the dot), the second term of Eq. \[14\] is negligible, and \( I \) is equivalent to
In other words, the two mesoscopic regions are effectively decoupled, because charge transfer is absent. However, when $\varepsilon_D$ is lowered to a charge fluctuation regime ($|\varepsilon_D - \varepsilon_F| \lesssim 2\Gamma$), the second term is no longer negligible and gives rise to contributions to the PC induced by the charge transfer. Such an effect of the charge transfer in this geometry were discussed by Büttiker and Stafford [2].

Here, we are mainly interested in the Kondo limit ($\varepsilon_D < -2\Gamma$, and $n_D \to 1$) [21] where the charge transfer is suppressed and the spin fluctuation plays a crucial role. Without loss of generality, we choose the parameters $\varepsilon_D = -0.75$, and $2\Gamma \simeq 0.30$. In fact, the parameters, as long as they are chosen to suppress the charge fluctuation, does not affect the physics in the Kondo limit but only shifts the value of the characteristic energy scale $T_K^0$. For simplicity, we consider only the half-filled case: $N_c = N_R$. Figure 2 shows the numerical result of the PC as a function of the normalized level spacing, $\Delta/T_K^0$. The behavior of $I/I_0$ depends strongly on which number class $N_c$ belongs to, aside from the class dependence of $I_0$ itself which has been discussed in Ref. [21]. The Kondo effect manifested by the second term of Eq. (14) is expected to provide a significant modification on the current. It is instructive to consider the following simple argument: Since the occupation of the dot goes to unity in the Kondo limit, the dot has one electron and the ring $N_c - 1$ electrons. If these two systems are effectively decoupled, the PC of the coupled $N_c$ electron system should be equivalent to that of an ideal ring with $N_c - 1$ electrons. Thus, one may expect that $I^{N_c} \simeq I_0^{N_c - 1}$. However, our result demonstrates that $I^{N_c} = I_0^{N_c}$ in the limit of $\Delta/T_K^0 \to 0$, which is in agreement with the recent result obtained by an exact Bethe ansatz calculation [22]. It is interesting to note that the Kondo impurity does not affect the PC in the continuum limit in spite of the fact that the dot captures one electron from the ring. This result is interpreted as follows. About $N_{eff} \approx T_K^0/\Delta$ electrons in the ring take part in forming a Kondo screening cloud [40]. Our result in the continuum limit shows that the screening cloud exactly compensates the single trapped electron in the dot for a response to the AB flux. In other words, the Kondo cloud plays a role of an extra electron in the ring which participates in the coherent motion.

Our conclusion for the continuum limit may appear in conflict with the result of the electron transmission in a quantum wire with a side-coupled quantum dot [23]. The electron transmission in this open system is shown to be completely suppressed due to a destructive interference between the ballistic and the Kondo channel. We believe that this discrepancy comes from the difference between a closed and an open system. In contrast to the current response to the applied voltage in a quantum wire, the response direction of the PC to the magnetic flux in a ring depends on the symmetry of each eigenstate of the ring. As a result of the alternating sign of the response, the QD makes no net effect on the PC in the limit of $\Delta/T_K^0 \to 0$.

As shown in Fig. 2, the effect of finite level spacing is to reduce the PC by the amount of $\mathcal{O}(\Delta/T_K^0)$ in the $\Delta/T_K^0 \ll 1$ limit. The inset of Fig. 2 shows that $I_{QD}/I_0 = \alpha(\Delta/T_K^0)$ at very small $\Delta/T_K^0$. Note that the numerical coefficient $\alpha \approx 0.5$ is independent of the number class and $\phi$. While this reduction of the current becomes non-linear for odd $N_c$ as $\Delta/T_K^0$ increases, the linear scaling is preserved for a wider range of $\Delta/T_K^0$ for $N_c$ = even. Actually, the reduction of the PC for finite $\Delta$ indicates a weakening of the Kondo screening due to the finite size of the metallic host.

To understand the linear scaling behavior of $I_{QD}/I_0$ more clearly, it is helpful to define the effective “charge” for each level of the ring, $Q_K^{m\sigma}$, which takes part in forming the Kondo cloud:

$$Q_K^{m\sigma}/e \equiv -Z(E_0)\frac{|t_m|^2}{(\varepsilon_m + E_0 - \varepsilon_D)^2}.$$  (15)

It should be noted that the net contribution of the effective charge,

$$\sum_{m\sigma} Q_K^{m\sigma}/e = -n_D,$$  (16)

corresponds to the hole that arises from the local charge trapped by the QD. In the Kondo limit of $n_D = 1$, one can find that

$$Q_K^{m\sigma}/e \sim -\Delta/T_K^0$$  (17)

for $-T_K^0 \lesssim \varepsilon_m - \varepsilon_F$, and $Q_K^{m\sigma}/e \simeq 0$ for $\varepsilon_m - \varepsilon_F \ll -T_K^0$. Therefore, $I_{QD}$ can be estimated as

$$I_{QD} = \sum_{m\sigma} f(\varepsilon_m) \left(\frac{Q_K^{m\sigma}}{e}\right) I_m$$  (18a)

$$\sim -\frac{\Delta}{T_K} \sum_{m\sigma} f(\varepsilon_m) I_m = -\frac{\Delta}{T_K} I_0,$$  (18b)

which explains the linear scaling of $I_{QD}/I_0$.

Further increase of $\Delta$ results in a various kinds of crossover behaviors in $I/I_0$ around $0.05 \lesssim \Delta/T_K^0 \lesssim 1.0$, as shown in Fig. 2. For a given AB flux ($\phi = 0.23$), $I^{N_c}/I_0^{N_c}$ for the class 3 rapidly increases, whereas its direction is reversed for the class 1. In contrast, the current displays a slow crossover for even $N_c$. Note that the behavior of the crossover depends not only on the number class but also on the value of $\phi$. However, in general, the crossover can be explained in terms of a continuous evolution from a delocalized Kondo to a localized dot limit. The crossover can be clearly seen by considering the limit of very large level spacing, $\Delta/T_K^0 \gg 1$. In this limit, only the topmost level of the ring contributes to the modification of the current, and one can write
as $I_{QD}^{N_e}(\phi) = -n_D I_F(\phi)$, where $I_F$ denotes the current component of the highest occupied level, $m = F$. Since $n_D \simeq 1$ in our case, we obtain the relation

$$I^{N_e}(\phi) \simeq I_0^{N_e-1}(\phi). \quad (19)$$

This result is exactly what one would expect for an effectively decoupled system. Therefore, as $\Delta/T_K^d$ increases, the behavior of the current evolves from $I^{N_e} = I_0^{N_e}$ of the delocalized dot limit to $I^{N_e} = I_0^{N_e-1}$ of the localized dot limit.

The number class dependence of the PC for an ideal ring [20] allows to understand the PC behavior of our dot limit. This result is exactly what one would expect for an effectively decoupled system. Therefore, as $\Delta/T_K^d$ increases, the behavior of the current evolves from $I^{N_e} = I_0^{N_e}$ of the delocalized dot limit to $I^{N_e} = I_0^{N_e-1}$ of the localized dot limit.

The number class dependence of the PC for an ideal ring [20] allows to understand the PC behavior of our system as a function of $\Delta/T_K^d$ for a given AB flux. For instance, in the range of $0 < \phi < 0.25$, class 1 undergoes a transition to class 2 as $\Delta/T_K^d$ is increased. This changes the direction of the current, since $I_0^{N_e-1}/I_0^{N_e} < 0$ in this region of $\phi$. Class 3 transforms into class 0 with an enhanced value of $I/I_0$ since $I_0^{N_e-1}/I_0^{N_e} > 1$. All the other cases for any number class and $\phi$ can be understood in the same way.

We have investigated the effect of spin fluctuations on the equilibrium properties of a coupled mesoscopic ring-quantum dot system. The phase-coherent response of the strongly correlated ground state to the AB flux was shown to depend on the ratio of the finite level spacing of the ring to the Kondo temperature. A crossover from a region of $0 < \phi < \pi/2$ to $\pi/2 < \phi < \pi$ for a given AB flux. For

$K$ to 0, the direction of the current, since $I_0^{N_e-1}/I_0^{N_e} > 1$. All the other cases for any number class and $\phi$ can be understood in the same way.

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FIG. 1. Left : An Aharonov-Bohm ring threaded by the magnetic flux $\Phi$ and laterally coupled to a quantum dot. The coupling between the dot and the site “0” of the ring is characterized by a tunnel matrix element $t'$. Right : Schematic energy diagram of the system.
FIG. 2. Normalized persistent currents as a function of the energy level spacing of the ring for $\phi = 0.23$ and $T = 0$. The other parameters are chosen as $\varepsilon_D = -0.75$, $t' = \sqrt{0.3}$, and $t = 1.0$ ($\Gamma \simeq 0.15$). With these parameters, the Kondo correlation energy is achieved at $T^K_0 = 0.16 \times 10^{-3}$. The currents for different number classes are denoted by $0(\odot), 1(\oplus), 2(\bigstar)$, and $3(\times)$, respectively. In the inset, the dot contribution of the current is shown in a linear scale.