Statistical guided-waves-based structural health monitoring via stochastic non-parametric time series models

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Abstract
Damage detection in active-sensing, guided-waves-based structural health monitoring (SHM) has evolved through multiple eras of development during the past decades. Nevertheless, there still exist a number of challenges facing the current state-of-the-art approaches, both in the industry as well as in research and development, including low damage sensitivity, lack of robustness to uncertainties, need for user-defined thresholds, and non-uniform response across a sensor network. In this work, a novel statistical framework is proposed for active-sensing SHM based on the use of ultrasonic guided waves. This framework is based on stochastic non-parametric time series models and their corresponding statistical properties in order to readily provide healthy confidence bounds and enable accurate and robust damage detection via the use of appropriate statistical decision-making tests. Three such methods and corresponding statistical quantities (test statistics) along with decision-making schemes are formulated and experimentally assessed via the use of three coupons with different levels of complexity: an Al plate with a growing notch, a carbon fiber-reinforced plastic (CFRP) plate with added weights to simulate local damage, and the CFRP panel used in the Open Guided Waves project, all fitted with piezoelectric transducers under a pitch-catch configuration. The performance of the proposed methods is compared to that of state-of-the-art time-domain damage indices (DIs). The results demonstrate the increased detection sensitivity and robustness of the proposed methods, with better tracking capability of damage evolution compared to conventional approaches, even for damage-non-intersecting actuator–sensor paths. In particular, the Z statistic emerges as the best damage detection metric compared to conventional DIs, as well as the other proposed statistics. Overall, the proposed statistics in this study promise greater damage sensitivity across different components, with enhanced robustness to uncertainties, as well as user-friendly application.

Keywords
Guided waves, statistical methods, non-parametric methods, damage detection, stochastic models, time series models

Introduction
In the near future, structural health monitoring (SHM) systems will be capable of implementing all four levels of SHM, namely: damage detection, localization, quantification, and remaining useful life estimation (prognosis),1–5 with sustainable levels of performance in complex components under varying operational and environmental conditions. In order to reach this milestone, a number of common challenges facing current SHM techniques in different fields, that is, aerospace, mechanical, and civil structural systems, needs to be addressed. These challenges originate from the deterministic nature of the majority of the currently employed approaches; that is, they do not account for uncertainty nor allow for the extraction of appropriate confidence intervals for damage detection, localization, and quantification.6–8 This leads to the characterization of those techniques as inefficient in the face of uncertainty, stochastic time-variant, and non-linear structural responses,9–11 as well as incipient damage types and complex failure modes that can be easily masked by the effects of varying
Thus, there lies a need for the development of SHM frameworks where proper understanding, modeling, and analysis of stochastic structural responses under varying states and damage characteristics is achieved for clearing the road towards achieving the aforementioned ultimate goal of SHM systems. Towards this end, many researchers have proposed the use of statistical distributions of damage-related features in devising SHM metrics and corresponding probabilities (also known as statistical or probabilistic SHM), for instance, see Refs. 7, 14–19 for probabilistic damage detection and localization, as well as Refs. 8, 14, 20–22 for probabilistic damage quantification). These approaches promise wider applicability due to the direct extraction of confidence bounds for detection, localization, and quantification, as well as present an alternative approach for SHM reliability quantification without the need for further non-destructive testing, such as that required to obtain the Probability of Detection.2,23–26

In a more specific context, as the most fundamental level of SHM,6,14,23 damage detection has received significant attention throughout the last two decades. Within the framework of active-sensing guided-waves-based SHM, one of the most widely used techniques for damage detection (and quantification) is based on the concept of the Damage/Health Index/Indicator (D/HI),28,29 where features of the signal for an unknown structural state are compared to that coming from the healthy structure.30,31 To this end, the most-widely used DI-based approaches are based on the time delay of specific mode wave packets in the acousto-ultrasound signal, the amplitude/magnitude of the signal, and the energy content of the signals, all used as the features to differentiate between a healthy and a damaged structure.2,30–34 These approaches, therein denoted as conventional DI-based methods, have been used extensively in the literature due to their simplicity (no experience needed in interpretation of results) and the allowance of a damage/no-damage paradigm, which may facilitate the decision-making stage.32 However, there exist a number of challenges facing these types of methods when it comes to damage detection. Namely, their deterministic nature and the time-varying and non-linear structural responses within a structure can limit the applicability of such methods.6,9,10,11 In addition, the effect of complex damage types and their stochastic evolution can be masked under varying operational and environmental states, further inhibiting the effectiveness of such DIs in damage detection.6,12,13 Other issues related to the conventional DI-based approach include the need for user-defined damage thresholds for damage detection35,36 and the phenomenon of saturation.37

As such, the challenges facing the aforementioned methods have been tackled throughout the literature using different approaches, with most of them based on advanced variants of conventional DIs. Although these endeavors span many strategies, the most common approaches either enhance current time-domain DIs (see, for instance, Refs. 2 and 38), use frequency-domain or mixed-domain DIs (see, for instance, Refs. 32, 35, 39), or use advanced signal processing/modeling as a preliminary step before calculating DIs (see Refs. 40–42). Another family of techniques is based on baseline-free approaches (see Refs. 36, 43, 44), which themselves can be further categorized into a number of approaches as will be discussed shortly. Although the proposed enhancements exhibit better damage detection performance compared to conventional DIs, no single technique seems to collectively address the current drawbacks of conventional DIs. The following discussion briefly outlines selected studies from each family of approaches, highlighting the advantages and drawbacks of each family of techniques.

In the context of enhanced time-domain approaches, Janapati et al.32 proposed a DI that depends solely on guided-wave propagation signal normalization and applied it to many identical coupons in order to pinpoint the source of variability between seemingly identical SHM systems, and thus better understand the effect of uncertainties on time-domain DIs. Su and coworkers38 compared three different DIs for fatigue damage characterization within an active-sensing acousto-ultrasonic SHM framework: the traditional time-of-flight delay and energy dissipation indices and a novel index utilizing non-linear features of guided-wave signals, such as the second harmonic generation. They observed higher sensitivities, as well as better damage evolution-tracking using nonlinear features compared to the traditional approaches relying on linear ones. In addition, they concluded that analyzing the time-frequency domain instead of the time domain alone enhances damage detection ability, especially for early stage cracks. Building upon that mixed-domain approach, Jin et al.32 used DIs in both the time and the frequency domains. In addition, in order to address the uncertainties in each individual path DI due to noise and varying conditions, they proposed an arithmetic fusion algorithm, where DIs based on amplitude and energy, both generated in the time and the frequency domains (a total of four DIs), are each summed over all the actuator–sensor path signals coming from a steel plate in order to “visualize” fatigue crack growth. Although these endeavors were capable in addressing certain challenges facing DIs, they are still not probabilistic in nature, and are thus still prone to errors due to uncertainties.

In order to address this, and building upon the fact that the frequency domain may offer a different representation of the signal dynamics, many researchers studied the effect of damage on the energy of wavelets (coming from some type of wavelet transformation of the signals) by using the concept of entropy.45 Basically, the Shannon entropy46 is calculated for windows/parts of the signal and the observed changes are related to damage, as appropriate. As this method is essentially based on the probability distribution of
the energy of each wavelet, it is more effective in capturing uncertainties compared to conventional approaches. However, defining damage/healthy thresholds, as well as the analysis of changes in entropy, are not straightforward and user expertise is required in order to properly detect damage. Another approach was adopted by Qiu and co-workers, where time-domain (as-received signal amplitude) and frequency-domain (frequency-response function) variants of the DI were used as features to develop a Gaussian mixture model for guided-wave signals in the healthy case. Then, upon the acquisition of new feature values, the model is migrated and a statistical technique is used to measure the differences between the baseline and the migrated models. Applying this approach to a real-life fuselage component with a developing fatigue crack, they observed the enhanced sensitivity and better damage evolution tracking. Most importantly, they also concluded the superiority of this technique owing to the lack of requiring user-experience in defining detection thresholds. One drawback to this approach, however, is the complexity in defining the original Gaussian mixture model, which requires many data sets, and multiple steps, including k-means clustering and expectation maximization algorithms.

In another attempt to enhance the detection capability of DIs under uncertainty, several researchers proposed baseline-free techniques, which do not require the presence of pre-sampled reference/healthy signals to compare to. In many of these techniques, either an instantaneous baseline is acquired from an identical path to the one being investigated, or from the reciprocal of that path, or the signal is reversed and analyzed for mode conversions and deviations from the original one, amongst other methods. Although these approaches appear to be more robust to varying conditions owing to the lack of pre-sampled baseline signals, most of them require knowledge of dispersion curves for the components being monitored, and dictate sophisticated actuation strategies. In addition, it has been shown that, depending on the actuator–sensor path from which the signal is coming, the evolution of the DIs can proceed in a manner uncorrelated with damage evolution, which clearly limits the applicability of many of these techniques to specific sensor network designs and simple boundary conditions. Although there are recent studies on averting this latter drawback, these approaches still require experience in defining detection thresholds.

Although the highlighted approaches show promise for enhancing the damage detection capability of DIs, from the above discussion it can be concluded that no approaches exist capable of overcoming the aforementioned challenges facing DIs with a user-friendly way that can be widely applicable to different sensor networks and component designs. In order to overcome these challenges, the use of non-parametric time series (NP-TS) models within a statistical framework is proposed herein in order to tackle damage detection under uncertainty. NP-TS models have been mainly used in damage detection via vibration-based SHM due to their stochastic nature which inherently accounts for uncertainty and allows for the extraction of theoretical and experimental confidence intervals, avoiding the need for arbitrary, user-defined thresholds, based on statistical decision making schemes. Also, NP-TS frequency-domain representations of system dynamics can exhibit increased sensitivity to damage and entertain simplicity of application requiring little-to-no user experience. Finally, as will be shown herein, NP-TS models can prove superior to conventional DIs in following damage evolution. Thus, the use of stochastic NP-TS representations for guided-waves-based damage detection has the potential to enhance detection performance, that is both sensitivity and robustness, due to their inherent ability to extract statistical thresholds from properly selected SHM metrics, account for uncertainty, as well as simplicity of application.

In a preliminary study, the authors applied NP-TS representations and statistical hypothesis testing (SHT) to an Al coupon and a stiffened panel showing the extraction of estimation confidence intervals from the metrics being used, as well as the enhanced detection capability with these stochastic models compared to DI-based approaches. In the current study, work on NP-TS models for damage detection in active-sensing acousto-ultrasound SHM is significantly expanded, and their performance in detecting damage over three different structural coupons is compared and experimentally assessed with that of two state-of-the-art DIs from the literature. To the author’s best of knowledge, the NP-TS-based damage detection metrics used herein have not been proposed previously within the framework of active-sensing guided-waves-based SHM. The main novel aspects of this study include:

1. The introduction of a novel data-based statistical damage detection framework based on NP-TS models in active-sensing guided-waves-based SHM, as well as the expansion of two previously proposed methods by the authors and co-workers.

2. The application of the proposed methods in two composite panels with different types of simulated damage, as well as in a notched Al coupon.

3. The extraction of statistical confidence intervals and the detection of damage via appropriate SHT schemes, negating the requirement of user-defined thresholds.

4. The proposal of a straight-forward damage detection method in active-sensing guided-waves-based SHM, with the advantage of enhanced detection capability over conventional DI-based approaches, without sacrificing simplicity.

The remainder of this article is organized as follows: The statistical damage detection framework section introduces...
the development of the statistical framework (The general framework), the theory of the utilized stochastic NP-TS representation (Overview of NP-TS representations), the statistics used in this study for damage detection (The single-set F statistic method, The multiple-set modified Fm statistic method, and The multiple-set Z statistic method), as well as briefly presents the literature-based DIs used for comparison in this study (Reference state-of-the-art DIs). Then, the experimental setup, the results and the discussions are presented for every coupon consecutively in the Results and discussion section. Finally, the Conclusions section summarizes this study and proposes extra steps for enhancement of damage detection within active-sensing guided-wave SHM systems.

The statistical damage detection framework

The general framework

The use of statistical methods for damage detection and identification has been previously reported for vibration-based SHM,49,52,53 and only recently for active-sensing guided-wave SHM.7 A typical statistical framework for damage detection, localization, and quantification is shown in Figure 1.7,51 In this framework, \( x[t] \) and \( y[t] \) are the individual actuation and response signals, respectively, for every structural case, indexed with discrete time \( t = 1, 2, \ldots, N \), which can be converted to continuous time through the transformation \( t/C_0 \cdot T_t \), where \( T_t \) is the sampling time for the recorded signals. The subscripts \((o, A, B, \ldots, u)\) indicate the healthy, damage \( A, B, \ldots \), and unknown cases, respectively. In this context, the damage cases labeled as \((A, B, \ldots)\) can resemble different types, sizes, or locations of damage. For each structural case, all actuation \((X)\) and response \((Y)\) signals can be presented as \( Z = (X, Y) \), with \( Z_o, Z_A, Z_B, \ldots, \) and \( Z_u \) indicating the different cases, as before. As a generic representation of the structural state, the subscript \( V \) is used in Figure 1 that can take actual values of \( A, B, \ldots \), and so on.

As shown in Figure 1, the statistical time series framework consists of two phases, namely, the baseline and inspection phase. In the baseline phase, NP-TS models, each producing a characteristic quantity \( Q \), are identified and properly validated for the healthy \((Q_0)\) time series signal, as well as, if available, different predefined damage cases \((Q_A, Q_B, \ldots)\). Then, during the inspection phase, the same NP-TS models are identified for the unknown \((Q_u)\) state of the system. Next, damage detection is achieved through applying appropriate binary statistical hypothesis tests to assess the statistical deviation of the unknown quantity \( Q_u \) from \( Q_0 \) corresponding to the healthy signal (damage detection). Based on data availability, the statistical similarity to one of the damage characteristic quantities \( Q_A, Q_B, \ldots \) with the baseline quantity \( Q_0 \) can enable statistical damage identification/classification. In the present study, this framework is only used for damage detection.

Overview of NP-TS representations

Stochastic NP-TS representations utilize time-domain Auto-/Cross-Covariance Functions (A/CCF) and/or frequency-domain Power-/Cross-Spectral Densities (P/CSD) in order to model a dynamic stationary signal [54, Chapter 2, pp. 39].

![Figure 1. Framework for statistical time series methods for structural health monitoring.](image-url)
As discussed above, frequency-domain models are used in this study. In this context, several estimators have been developed for the PSD (also referred to as Auto Spectral Density) of a sensor excitation and/or response signal, including the periodogram, the Thompson, the Blackman-Tukey, and the Bartlett-Welch (or simply Welch) estimators [55, Chapter 5, pp. 235]. As such estimators are random variables that represent the true PSD of a system, their corresponding statistical properties, such as the mean and variance, allow for the extraction of estimation confidence intervals that can be subsequently used to represent statistical damage thresholds. In this study, the Welch PSD estimate, which is a modified periodogram estimator using a series of overlapping windows [56, Chapter 4, pp. 76] is used for damage detection. For a time series signal \( x[t] \), the frequency-domain \( \omega \) Welch PSD \( (\hat{S}_{xx}(\omega)) \) is based on the averaging of multiple-windowed periodograms using properly selected sample windows \( w[t] \) with 50% overlap and is calculated as follows [57, Chapter 8, pp. 418] (the hat indicates an estimated variable)

\[
\hat{S}_{xx}(\omega) = \frac{1}{KLUT} \sum_{i=0}^{K-1} \left| T \sum_{t=0}^{L-1} w(t) \cdot \hat{x}[t+iD]^{(-\beta SHT)} \right|^2
\]

with

\[
U = \frac{1}{L} \sum_{t=0}^{L-1} w^2[t], \quad \hat{x}[t] = x[t] - \bar{\mu}, \quad N = L + D(K - 1)
\]

and \( N, L, K, D, \) and \( T \) being the total number of signal samples, the size of each window, the number of utilized windows, the number of overlapping data points in each window, and the time period of the signal, respectively. \( \bar{\mu} \) represents the mean of the time series. The estimation statistics, that is, the mean and variance, of the Welch PSD can be described as follows in case the Bartlett window is used [57, Chapter 8, pp. 419]

\[
E\left\{\hat{S}_{xx}(\omega)\right\} = \frac{1}{2\pi LU}S_{xx}(\omega)|W(\omega)|^2
\]

\[
\text{Var}\left\{\hat{S}_{xx}(\omega)\right\} \approx \frac{9}{16} \frac{L}{N} S_{xx}^2(\omega)
\]

where \( W(\omega) \) designates the Fourier transform of the window function. One of the main reasons behind the wide use of the Welch PSD estimator is that it is asymptotically unbiased and consistent. In this study, the Welch PSD estimate is used in developing appropriate statistical quantities, also referred to as test statistics, and corresponding statistical hypothesis tests for damage detection, as described in the following section.

### The single-set F statistic method

Based on the PSD-based NP-TPS method and the corresponding SHT setup presented in Refs. [49, 51], damage can be detected by assessing changes in the Welch PSD of properly determined wave packets/modes from an acoustic-ultrasound time series signal. Thus, the characteristic quantity in this study is \( Q = S_{xx}(\omega) = S(\omega) \). The main idea is based on the comparison of the Welch PSD of the response of the structure in an unknown state, \( S_\delta(\omega) \), to that of the structure in its healthy state, \( S_0(\omega) \). Damage detection can thus be tackled using the following SHT problem.

\[
\begin{align*}
H_0 & : S_\delta(\omega) = S_0(\omega) \\
H_1 & : S_\delta(\omega) \neq S_0(\omega)
\end{align*}
\]

(5)

Again, due to the finite nature of the experimental time series, the true PSD values are unknown, and thus corresponding estimated quantities are used instead \( \hat{S} \). It can be shown that the Welch PSD estimate will have the following property [56, Chapter 3, pp. 46]

\[
2K\hat{S}(\omega) / S(\omega) \sim \chi^2(2K)
\]

(6)

In the above expression, the factor of 2 comes from the fact that every periodogram used in averaging the Welch PSD has a real and a complex component. Consequently, a damage detection statistic following the \( \mathcal{F} \) distribution with \( (2K, 2K) \) degrees of freedom can be developed as follows

\[
F = \frac{\hat{S}_\delta(\omega) / S_\delta(\omega)}{S_0(\omega) / S_0(\omega)} \sim \mathcal{F}(2K,2K)
\]

(7)

In the case of a healthy structure (null hypothesis), \( S_\delta(\omega) \) and \( S_0(\omega) \) coincide, thus

Under \( H_0 \) : \( F = \frac{\hat{S}_\delta(\omega)}{S_\delta(\omega)} \sim \mathcal{F}(2K,2K) \)

(8)

Thus, the above SHT decision-making process can be modified as follows

\[
f_\alpha(2K,2K) \leq F = \frac{\hat{S}_\delta(\omega)}{S_\delta(\omega)} \leq f_{1-\alpha/2}(2K,2K) \quad (\forall \omega),
\]

\[
\Rightarrow H_0 \text{ is accepted (healthy structure)} \quad \text{else} \quad \Rightarrow H_1 \text{ is accepted (damaged structure)}
\]

(9)

where \( \alpha \) is the Type I error (false alarm) probability and \( f_{1-\alpha/2} \) and \( f_{1-\alpha/2} \) designate the \( \mathcal{F} \) distribution’s \( \alpha/2 \) and \( 1 - \alpha/2 \)}
critical points, respectively ($f_\alpha$ is defined such that \(\text{Prob}(F \leq f_\alpha) = \alpha)$.  

The multiple-set modified \(F_m\) statistic method

In many realistic cases, a single baseline signal may not be representative of the healthy structure, and the average of many signal realizations might be more meaningful. In that case, multiple closely spaced (time-wise) response signal realizations available for each state of the component being monitored under nominally constant environmental/operational conditions can be used to entail some experimental statistics in the estimation of the SHM metric being used. Towards this end, the sample expectation, that is

\[
E\left\{ \tilde{S}_o(\omega) \right\} = \frac{1}{M} \sum_{h=1}^{M} \tilde{S}_o(\omega)
\]  

(10)
can be used in order to “expand” the baseline/healthy estimates for the structure being monitored. In the above expression, \(M\) is the number of healthy data sets used in the estimation of the metric being used. Then, following the aforementioned property of PSD estimates in equation (6), the following expression can be developed

\[
2KME\left( \tilde{S}_o(\omega) \right) / \tilde{S}_o(\omega) \sim \chi^2(2KM)
\]  

(11)

As such, a modified \(F\) statistic can be developed by replacing the Welch PSD estimate with the mean of all PSD estimates of \(M\) number of time-series signals taken for the system under the baseline/healthy state

\[
F_m = \frac{E\left\{ \tilde{S}_o(\omega) \right\} / \tilde{S}_o(\omega)}{\tilde{S}_o(\omega) / \tilde{S}_o(\omega)} \sim \mathcal{F}(2KM,2K)
\]  

(12)

Under the null hypothesis in equation (5), the \(S_o(\omega)\) and \(\tilde{S}_o(\omega)\) coincide

Under \( H_0 \): \( F_m = \frac{E\left\{ \tilde{S}_o(\omega) \right\}}{\tilde{S}_o(\omega)} \sim \mathcal{F}(2KM,2K) \)  

(13)

Thus, the modified decision making scheme with the appropriate confidence levels, can be expressed as follows

\[
f_{\alpha/2}(2KM,2K) \leq F_m < f_{1-\alpha/2}(2KM,2K), \quad (\forall \omega)
\]  

\[
\Rightarrow H_0 \text{ is accepted (healthy)}
\]

Else \( \Rightarrow H_1 \) is accepted (damaged)

(14)

with \(f_{\alpha/2}\) and \(f_{1-\alpha/2}\) designating the \(F\) distribution’s \(\alpha/2\) and \(1 - \alpha/2\) critical points, respectively ($f_\alpha$ is defined such that \(\text{Prob}(F \leq f_\alpha) = \alpha)$.  

The multiple-set \(Z\) statistic method

With the availability of a sufficiently large number of data sets, that is, a large \(M\) in equation (10), \(E\{\tilde{S}_o(\omega)\}\) would approach the true PSD, and according to the Central Limit Theorem (CLT) [58, Chapter 3, pp. 62], \(E\{S_o(\omega)\}\) would also follow a normal distribution with the true PSD being the mean and \(\sigma_0^2\) the variance. Utilizing these statistical phenomena, and based on the \(Z\) statistic developed by Fassois and coworkers [49,51] for the Frequency Response Function of vibration-based SHM signals, a novel \(Z\) statistic is proposed herein utilizing the Welch PSD estimate for many baseline active-sensing acousto-ultrasound SHM signals. The following SHT problem is posed for damage detection in this case

\[
H_o : S_o(\omega) - \tilde{S}_o(\omega) = 0
\]

(null hypothesis – healthy structure)

\[
H_1 : S_o(\omega) - \tilde{S}_o(\omega) \neq 0
\]

(alternative hypothesis – damaged structure)

where both terms in the hypothesis above are the true values of the respective PSDs. As mentioned above, under the assumption of a large \(M\) in equation (10) (many baseline signals used for expectation estimation), the first term in the hypothesis test above \(S_o(\omega)\) can be replaced by the expectation, which would be normally distributed. Additionally, assuming the availability of many data points (i.e., a large \(N\) used for PSD estimation, the second term in the formulation of the hypothesis test \(\tilde{S}_o(\omega)\) can be replaced by an estimate [56, Chapter 3, p. 45], which will also approximate a normal distribution due to the asymptotic properties of \(\chi^2\)-distributed estimates, as also dictated by the CLT [58, Chapter 3, pp. 62]. Under the null hypothesis, both of these terms would follow the same distribution since both would be coming from a healthy structural case. Thus, under the null hypothesis

\[
H_o : E\left\{ \tilde{S}_o(\omega) \right\} - \tilde{S}_o(\omega) \sim \mathcal{N}(0, 2\sigma_o^2(\omega))
\]

(15)

null hypothesis – healthy structure)

where \(\sigma_o^2(\omega)\) can be estimated from the baseline phase and can be assumed to have negligible variability if a large number of signals is used in estimating the value of the PSDs. [49,51] Thus, by defining an appropriate type I error, or false alarm, probability (\(\alpha\), the Welch PSD-based \(Z\) statistic can be expressed as follows

\[
\frac{E\left\{ \tilde{S}_o(\omega) \right\}}{\tilde{S}_o(\omega)} \sim \mathcal{F}(2KM,2K)
\]  

(11)
Table 1. Summary of the different damage detection statistics utilized in this study.

| Quantity | F statistic | Fm statistic | Z statistic |
|----------|-------------|--------------|-------------|
| Property | 2K(\(\bar{S}(\omega)/S(\omega)\)) \(\sim \chi^2(2K)\) | 2KME(\(\bar{S}(\omega)/S(\omega)\)) \(\sim \chi^2(2KM)\) | \(E(\bar{S}(\omega)) - S(\omega) \sim N(0, 2\sigma^2(\omega))\) |
| Test statistic | \(F = \frac{\bar{S}_n(\omega)}{S_n(\omega)}\) | \(F_m = \frac{\bar{E}(\bar{S}_n(\omega))}{\bar{S}_n(\omega)}\) | \(Z = \frac{|E(\bar{S}(\omega)) - \bar{S}(\omega)|}{\sqrt{2\sigma^2(\omega)}}\) |
| Comment | \(K\): Number of non-overlapping segments | \(M\): Number of available baseline data sets | \(\bar{S}(\omega)\): Welch PSD estimate; \(\omega \in [0, 2\pi/T_s]\); frequency in radians per second (\(T_s\) is the sampling time) |

In this notation, \(Y_\omega^n[t]\) and \(Y_\omega^0[t]\) are normalized unknown (inspection) and baseline signals, respectively. The second DI used in this study is the time-domain DI presented by Qiu et al.\(^3\) and used in training their Gaussian mixture models due to its sensitivity to changes in wave form and time of flight. The formulation of that DI is as follows

\[
DI = 1 - \sqrt{\frac{\sum_{i=1}^N Y_\omega^n[i] \cdot Y_\omega^0[i]}{\sum_{i=1}^N Y_\omega^0[i] \cdot \sum_{i=1}^N Y_\omega^n[i]}}
\]  

Results and discussion

In this work, the comparison between state-of-the-art DIs and the proposed NP-TS approaches in damage detection was carried out over three components with different damage cases: a notched Al plate, a Carbon Fiber-Reinforced Plastic (CFRP) coupon with weights taped on the surface to simulate a crack, and the open-source data sets available on the Open Guided-Waves project’s website.\(^1\)

Test case I: Damage detection in an aluminum plate

Test setup, damage types and data acquisition. This first coupon was a 6061 Aluminum 152.4 \(\times\) 254 mm (6 \(\times\) 10 in) coupon (2.36 mm/0.093 in thick) (McMaster Carr) with a 12-mm (0.5-in) diameter hole in the middle, as shown in Figure 2. Using Hysol EA 9394 adhesive, the coupon was fitted with six single-PZT (lead zirconate titanate) SMART Layers type PZT-5A (Acellent Technologies, Inc) as shown in Figure 2(c). To simulate damage, using an end-mill and a 0.8128-mm (0.032-in) hand saw, a notch was generated extending from the middle hole of the coupon with length varying between 2 and 20 mm, in 2-mm increments.

Actuation signals in the form of 5-peak tone bursts (5-cycle Hamming-filtered sine wave) having an amplitude of 90 V peak-to-peak and various center frequencies were generated in a pitch-catch configuration over each sensor consecutively. With a sampling rate of 24 MHz, data was collected using a ScanGenie III data acquisition system.
Preliminary analysis was conducted, and a center frequency of 250 kHz was chosen for the complete analysis presented in this study based upon the best separation between the first two wave packets in various signal paths. All data sets were exported to MATLAB for analysis. 

Tables 2 and 3 summarize the relevant experimental and signal processing details for this coupon.

**Damage detection results.** In order to assess the performance of the proposed approach, a simple isotropic Al coupon was initially used. Figure 3 panels a and b, respectively, show one indicative full response signal and its corresponding first-arrival wave packet off of sensor 6 when sensor 2 was actuated (refer to Figure 2 for sensor numbering) under different notch sizes. Because this is a damage-intersecting path, a gradual decrease in signal amplitude, with a slight delay, can be observed with increasing notch size. This is expected since the notch scatters the wave, decreasing the amount of energy going through to sensor 6 as scattering increases.

Figure 3(c) shows the evolution of the two chosen state-of-the-art DIs for the first-arrival wave packet. As shown, although the DIs closely follow damage for notch sizes more than 8 mm, it might be difficult to detect damages up to 8 mm in size, given the proximity of the DI values for the healthy case and the damaged cases. Without prior experience with these types of materials/components, assigning a threshold between a healthy component and a damaged one might be challenging in that range of damages. As the length of the analyzed signal increases, the DIs become more sensitive to small damages as shown in Figure 3(d). However, it can be observed that the DIs do not follow the increase in notch size uniformly even for a damage-intersecting path like path 2–6. Exploring a damage-non-intersecting path (Figure 4), the DIs fail to follow damage evolution to a greater extent, with fluctuations being observed as notch size increases. Such fluctuations in the DIs can be mistaken for a change in conditions surrounding the component, which would make the task of damage detection and threshold identification even more challenging.
Figure 3. Indicative signals from the Al coupon for path 2–6 (damage-intersecting) under different notch sizes: (a) full signal; (b) first-arrival wave packet; (c) single wave packet damage indices—the dashed lines designate the upper and lower 95% confidence bounds for the Janapati et al. (blue) and Qiu et al. (red) damage indices; (d) full-signal damage indices.

Figure 4. Indicative signals from the Al coupon for path 6–3 (damage-non-intersecting) under different notch sizes: (a) full signal; (b) a single wave packet; (c) single-wave packet damage indices—the dashed lines designate the upper and lower 95% confidence bounds for the Janapati et al. (blue) and Qiu et al. (red) damage indices; (d) full-signal damage indices.
Figure 5 presents indicative results of applying the proposed framework to the response signal from path 2–6 in the Al coupon2 (see Table 9 in the Appendix 1 for summary results). Figure 5(a) shows the evolution of the Welch PSD of the signals as notch size increases, with the red and the black dashed lines indicating the theoretical (estimation uncertainty) and the experimental 95% confidence intervals of the healthy PSD, respectively. The first aspect to be observed in this figure is that, using the theoretical estimation confidence intervals, notch sizes more than 2 mm can be detected with 95% confidence, and all damage sizes are detected when the experimental 95% confidence levels are considered. Although the latter result is expected due to the nominally controlled lab environment significantly inhibiting change in the Welch PSD over multiple healthy signals, the former observation shows the enhanced detection capability of the frequency-domain PSD compared to time-domain DIs for damages of this type in Al. Furthermore, in contrast to the DIs in Figure 3, the PSDs evolve uniformly with damage, which hints on the enhanced damage quantification capability of these techniques. Thus, the Welch PSD emerges as a better metric when it comes to damage detection and quantification for the case at hand. Applying the SHT methods developed in The general framework (Figure 5(b)–(d)), one can assess the difference between both approaches in a statistical way. As shown in Figure 5(b), the F statistic is capable of only detecting the last three damage cases (10–18 mm) with 95% confidence. Although this performance is somewhat similar to that of the DIs, an advantage in the proposed approach is the extraction of confidence intervals directly from the SHM metric being used, without the need for user experience for defining damage thresholds. The $F_m$ statistic (Figure 5(c)) does a slightly better job by detecting the 8-mm damage with 95% confidence, which is attributed to the inclusion of experimental statistics into the definition of this metric. Examining the $Z$ statistic (Figure 5(d)), one can observe that all damage cases are detected with 95% confidence. Furthermore, the effect of damage on the $Z$ statistic is again uniform, indicating the superior performance of this statistic in damage quantification compared to the conventional time-domain DI approach. Figure 6(a) shows indicative Welch PSD estimates for the full signal (using a window size equal to the width of a single wave packet). As shown, although all damage cases are detected considering the experimental confidence intervals (black dashed lines), all damage cases lie within the 95% healthy estimation.
confidence intervals (red dashed lines). This deterioration in detection might be attributed to the non-stationary behavior of the full signal compared to a single wave packet, which is a disadvantage when it comes to the Welch PSD estimate since it is generally designed for stationary signals. The same argument can be said for the $F$ and $F_m$ statistics (Figure 6(b) and (c)) which consider the estimation confidence bounds in their formulation. This is, however, not the case for the $Z$ statistic (Figure 6(d)), which depends on the experimental confidence bounds, where all damage sizes can be detected with 95% confidence.

Moving on to the damage-non-intersecting path (path 6–3), Figure 7 shows indicative results for a single wave packet (see Table 10 in Appendix 1 for summary results). As shown in Figure 7(a), using the PSD’s theoretical estimation confidence intervals (red dashed lines), all damages are deemed healthy with 95% confidence. Because a single wave packet can be considered stationary in this case, this is attributed to the wide nature of the estimation uncertainty when the PSD of a deterministic signal is being estimated, as is the case in this study. However, just like the DIs, all damages are detected with 95% confidence when the experimental uncertainty is being considered. Again, being based on the theoretical confidence intervals, both the $F$ and $F_m$ statistics also indicate all damages as healthy with 95% confidence (with the exception of the 14 mm case for the $F_m$ statistic), as shown in Figure 7(b) and (c), respectively. On the other hand, the $Z$ statistic (Figure 7(d)) detects all damage cases with 95% confidence due to the fact that its formulation is based on the effective representation of the experimental uncertainty. A similar trend can be observed when the entire signal is considered in the analysis, as shown in Figure 8.

A number of conclusions can be drawn from these observations when it comes to comparing the proposed statistics to the DIs. First, for the notched Al coupon, the Welch PSD, the $F$ and $F_m$ statistics can be used as a preliminary step in differentiating between damage-intersecting and non-intersecting paths owing to their uniform evolution with damage size, as well as high detection performance, in the former case compared to the latter. DIs, on the other hand, do not show a clear distinction between both types of paths due to exhibiting uniform evolution, or the lack thereof, with damage size in both cases. Second, the sensitivity of the $Z$ statistic appears to be the same as the DIs because of both being based on the experimental confidence intervals (also, see Figure 21 in Appendix I). However, an advantage of the $Z$ statistic over the DI is that it allows the extraction of confidence bounds at a predetermined type I error probability. Thus, the extracted damage detection thresholds emerge
from the formulation of the SHM metric itself, and do not require prior experience with such materials and damages, or physics-based modeling. In contrast, the DIs require complex approaches in order to set accurate thresholds, and do not entail any theoretical distribution on the signals, from which thresholds can emerge naturally. It is important to note that this advantage prevails even if the $Z$ statistic is compared with frequency-based DI formulations due to the additional underlying assumptions in the $Z$ statistic; this can be evident when comparing, for instance, the results of the $Z$ statistic with those of the Welch PSD estimate itself. In addition, in the realm of statistical methods, the proposed metrics also hold an advantage when compared with typical statistical outlier analysis techniques, since the latter exhibit a binary nature and do not provide useful information about the evolution of damage size.59

**Test case II: Damage detection in CFRP plate**

**Test setup, damage types, and data acquisition.** The second coupon used in this study was a CFRP coupon (ACP Composites) having the same dimensions as the Al coupon with multiple 0/90 unidirectional CF plies. This coupon was also fitted with 6 single-PZT SMART Layers type PZT-5A (Acellent Technologies, Inc) as shown in Figure 9, with 0.2 mm (0.00787 in) in thickness and 3.175 mm (1/8 in) diameter. Damage was simulated by attaching from one up to six 3-g weights to the surface of the coupon next to each other using tacky tape to simulated local damage of increasing size. The same actuation and data acquisition properties were used for this coupon as with the Al one. Also, similar to the case of the Al coupon, the actuation center frequency of 250 kHz was chosen for the analysis presented herein. Tables 4–6 summarize the experimental and signal processing details for this coupon.

**Damage detection results.** Figure 10 panels a and b present, respectively, the signals and the first discernible wave packet, obtained at sensor 4 when sensor 3 was actuated under different damage sizes, where damage size here indicates the number of taped weights. Figure 10(c) and (d) show the DIs for a single and double wave packet lengths, respectively. As shown, the CFRP coupon exhibits complex wave propagation patterns (such as increased attenuation

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**Figure 7.** Indicative results from applying the proposed non-parametric time series approach to the first-arrival wave packet from path 6–3 in the Al coupon under different damage sizes: (a) Welch PSD—the red and the black dashed lines indicate the theoretical (estimation uncertainty) and the experimental 95% confidence bounds of the healthy PSD, respectively; (b) $F$ statistic; (c) $F_m$ statistic; (d) $Z$ statistic.
and/or non-uniform wave propagation throughout the coupon) compared to an Al coupon originating from the anisotropic properties of CFRP laminate, the single-wave packet DIs completely fail to follow damage evolution and can further only detect the last damage case (6 weights) within the 95% experimental healthy confidence intervals for both DI formulations as shown in Figure 10(c). This performance is slightly enhanced when analyzing two wave packet lengths (Figure 10(d)). Figure 11 shows the same 4 plots for a damage non-intersecting path (path 1–4). As shown in panel c, the performance of the DIs substantially deteriorates with a decrease in the value of both DIs with increasing simulated damage size up to 4 weights. A similar trend is observed in Figure 11(d) when considering two wave packet lengths for the analysis. Furthermore, although some damage cases fall outside the 95% confidence bounds (4 weights for single-wave packet DIs and 5 and 6 weights for two-wave packet DIs), the general trend is a reduction in the values of the DIs which can again be mistaken with changing environmental or operational conditions over an otherwise healthy component. Thus, in terms of damage detection, the DIs offer poor performance for the CFRP coupon with the simulated damage used in this study.

Figure 12(a) shows the estimated Welch PSD for a single wave packet from path 3–4 under different damage cases. Note that, as mentioned, due to the nature of the actuation signal, the theoretical confidence intervals are too wide to detect any of the simulated damages and thus they are not shown here. Figure 12(b) shows the Z statistic for that path, from which it can be concluded that the cases of 3–6 weights are all damage cases with 95% confidence. Thus, the Z statistic surpasses the DIs in detection performance for this damage-intersecting path. Examining a longer signal length for the analysis, it can be seen that both the Welch PSD estimate and the Z statistic (Figure 12(c) and (d), respectively) show more sensitivity to damage, with the former detecting damage sizes as small as 3 weights and the latter detecting ones as small as 2 weights. Moving onto the damage-non-intersecting path (1–4), it can be seen that the Welch PSD estimates (Figure 13(a) and (c)) fail to detect almost any of the damages with 95% confidence levels. The same can be said for the Z statistics, as shown in Figure 13(b) and (d), with the exception of detecting the 2- and 4-weight cases for the single-wave packet Z statistics. This reduction in sensitivity for damage-non-intersecting paths can be attributed to the effect of damage on the signal, as well as the relatively wide variability in the baseline signal amplitude, which in turn leads to widening the 95% healthy confidence bounds.

For this reason, another experiment was taken out where the baseline data acquisition process was more restrictive.
Table 5. Details of the second experimental data set for the CFRP coupon.

| Structural state | Number of data sets | Total added weight* (g) |
|------------------|---------------------|-------------------------|
| Healthy          | 20\(^b\)            | 0                       |
| 1 steel weight   | 20                  | 3                       |
| 2 steel weights  | 20                  | 6                       |
| 3 steel weights  | 20                  | 9                       |
| 4 steel weights  | 20                  | 12                      |
| 5 steel weights  | 20                  | 15                      |
| 6 steel weights  | 20                  | 18                      |

*Sampling frequency: \(f_s = 24\) MHz. Center frequency range: [50 : 50: 750] kHz. 
Number of samples per data set \(N = 8000\). 
CFRP: carbon fiber-reinforced plastic. 
*Weight of tacky tape not considered here. 
\(^b\)M = 20 in equation (10). 

Table 6. Parameters used in estimating the Welch PSD for the CFRP coupon data sets.

| Number of DFT Points (NFFT) | 2000 |
|-----------------------------|------|
| Window type                 | Hamming |
| Frequency resolution        | \(\Delta f = 12\) kHz |
| Sampling frequency          | 24 MHz |
| Single wave packet          |       |
| Data length                 | \(N = 500\) samples (\(\sim 20\) \(\mu s\)) |
| Segment length              | 100   |
| No of non-overlapping segments | 9   |
| Double-wave packet length   |       |
| Data length                 | \(N = 800 \sim 1000\) samples (\(\sim 40\) \(\mu s\)) |
| Segment length              | 500   |
| No of non-overlapping segments | 3   |

CFRP: carbon fiber-reinforced plastic.

Figure 9. The carbon fiber-reinforced plastic coupon used shown with 6 added 3-g weights as simulated damage (the largest damage size of this test case). The arrows indicate the paths used in this study.

Table 4. Details of the first experimental data set for the CFRP coupon.

| Structural state | Number of data sets | Total added weight* (g) |
|------------------|---------------------|-------------------------|
| Healthy          | 20\(^b\)            | 0                       |
| 1 steel weight   | 1                   | 3                       |
| 2 steel weights  | 1                   | 6                       |
| 3 steel weights  | 1                   | 9                       |
| 4 steel weights  | 1                   | 12                      |
| 5 steel weights  | 1                   | 15                      |
| 6 steel weights  | 1                   | 18                      |

*Sampling frequency: \(f_s = 24\) MHz. Center frequency range: [50 : 50: 750] kHz. 
Number of samples per data set \(N = 8000\). 
CFRP: carbon fiber-reinforced plastic. 
*Weight of tacky tape not considered here. 
\(^b\)M = 20 in equation (10).
Figure 10. Indicative signals from path 3–4 in the carbon fiber-reinforced plastic coupon under different simulated damage sizes (number of attached weights): (a) full signal; (b) single wave packet; (c) single wave packet damage indices—the dashed lines designate the upper and lower 95\% confidence bounds for the Janapati et al. (blue) and Qiu et al. (red) damage indices; (d) damage indices for double the wave packet length.

Figure 11. Indicative signal from path 1–4 in the carbon fiber-reinforced plastic coupon under different simulated damage sizes (number of attached weights): (a) full signal; (b) single wave packet; (c) single wave packet damage indices—the dashed lines designate the healthy upper and lower 95\% confidence bounds for the Janapati et al. (blue) and Qiu et al. (red) damage indices; (d) damage indices for double the wave packet length.
Although both the DI and the Z statistics almost consistently follow damage size evolution for the damage-intersecting path (path 3–4), the Z statistic shows better detection capability for the damage-non-intersecting path, detecting all damages with 95% confidence.

In order to assess the performance of all 4 metrics (the DI, F, Fm, and Z statistics) at different alpha (false alarm levels), that is, at different confidence intervals, the corresponding receiver operating characteristics (ROC) curves were explored for different signal lengths (also, see Tables 11 and 12 in Appendix 2 for summary results). Figure 16(a) shows the ROC for the 4 metrics at alpha levels ranging from $1 \times 10^{-6}$ to 1, as applied to a single wave packet off of the damage-intersecting path 3–4. As shown, because this is a damage-intersecting path, 3 out of the 4 metrics exhibit perfect detection performance with an area under the ROC curve equal to 1. Also, although the performance of the Fm statistic does not seem to be better than the worst statistical estimator (the dashed line), the F statistic shows optimal performance as the alpha levels change, in contrast to its weak detection performance at an alpha level of 0.05, as mentioned in the discussion of Figure 12. Moving onto two wave packets of the same path (Figure 16(b)), one can observe that the Z statistic outperforms all other metrics in damage detection. In addition, the F statistic outperforms the DI metric, which hints on the advantages of using frequency-domain approaches and statistical hypothesis tests instead of time-domain approaches. For the damage-non-intersecting path 1–4, it can be observed that the Z statistic outperforms the DI for both: a single- (Figure 16(c)) and two- (Figure 16(d)) wave packet lengths. Thus, it can be concluded that the Z statistic emerges as the best damage detection statistic in this study for the CFRP coupon investigated herein.

**Test case III: Damage detection in the open guided-waves CFRP panel**

**Test setup, damage types, and data acquisition.** The third test case used in this study was the CFRP panel utilized in the Open Guided-Waves project, which had a quasi-isotropic construction with layup [45/0/ − 45/90/ − 45/0/ − 45/90]. The panel had the dimensions of 500 × 500 mm (19.69 × 19.69 in) and a thickness of 2 mm (0.079 in). During the fabrication process of the panel, 12 PZT sensors, 5 mm (0.2 in) in diameter and 0.2 mm (0.0079 in) in thickness, were co-bonded on the panel. To simulate damage, a 10-mm diameter, 2.35-mm-thick (0.0925 in) Al disk (0.5 g) was...
consecutively attached using tacky tape on 28 different locations on the panel grouped into seven groups. Figure 17(a) shows a schematic of the CFRP panel, where the sensor and damage locations are shown. Also, the inset in Figure 17(a) shows the simulated damage on one of the locations.

Each sensor was consecutively actuated using a 5-peak tone burst signal (5-cycle Hanning-filtered sine wave) having an amplitude of ±100 V. Response signals were sampled over the remaining sensors at a sampling rate of 10 MHz. Three sets of 20-baseline (healthy) signal realizations per sensor were recorded, with damage-state signals recorded after each 20-signal set. The following discussion elaborates on the data acquisition process. First, 20 healthy signals were recorded forming the first baseline set for each sensor (so a total of 20 × 12 signals for all 12 sensors). After that, one signal was recorded at each sensor for each damage location, with the weight (damage) on locations D1/C0. This formed the first set of 11 damage-state signals for each sensor (one signal per sensor for each damage location—12 signals for each damage location for all 12 sensors) After that, the second baseline set was acquired (healthy signals 21–40), followed by recording a single damage signal per weight location for locations D12/C0 (second set of damage-state signal—9 signals per sensor for all damage locations in this set). Finally, after the third baseline set was acquired (healthy signals 41–60 for each sensor), signals were recorded with the weight at locations D21/C0 (third set of damage-state signals). This resulted in 60 baseline realizations and 28 damage signals per sensor (one signal for each damage location for each sensor). Other data sets were also recorded that are not used in this study. Tables 7 and 8 summarize the experimental and signal processing details, respectively, of this panel, and the readers are directed to the original study1 for more information on test setup. For ease of comparison of the proposed damage detection methods, only the response signals for the actuation with 260 kHz center frequency were chosen for analysis in this study.

In the present study, signals from simulated damages in the same damage group (see Figure 17(a) and Table 7) were...
treated as different realizations of single damage in the vicinity of that group on the CFRP panel. In addition, for all the detection metrics in this study, each damage group was analyzed against its corresponding baseline data set only (the healthy data set immediately preceding that damage group in the data acquisition process) for accuracy. Actuator-sensor path 3–12 was used to demonstrate the performance of the different detection techniques proposed herein. As such, damage groups 2 and 3 (see Table 7) were considered as different realizations of signals for two damages intersected by the path (with the first baseline set used for comparison). On the other hand, damage groups 7 and 8 were treated as different realizations of signals for two damages not intersected by the signal path (with the third baseline set used for comparison).

**Damage detection results.** Figure 17(b) and (c) show the complete signal and the first-arrival wave packet, respectively, for signal path 3–12 on the CFRP panel when the Al weight was on locations 5–11 (damage-intersecting case). It is worth noting that, examining other paths (not shown here), the packet shown in panel c is actually a combination of two wave packets merged together, as can also be observed from the number of cycles in the shown packet. Even though this limits the analysis to only this single wave structure, this path was still chosen as it directly intersects with certain damage location groups almost completely, and almost completely misses other damage location groups, which allows for the analysis of detection performance for both types of paths.

Figure 17(d) shows the values of the DI formulated by Janapati et al. for the first 20 baseline signals and the signals corresponding to damage locations 5–11. As shown, within the 95% healthy confidence bounds (dashed blue lines), only damages at locations 5–7 are detected, while the rest of the damage cases are considered healthy with 95% confidence. Noting that the healthy signals in each baseline set were taken under controlled temperatures, it can be again concluded that the DI lacks robustness to uncertainties even in controlled environments, where the values of the DI still fluctuate even for the healthy case, producing wide healthy bounds that affect detection performance. Figure 18 shows the
Figure 15. Indicative Z statistic results for the second acquired carbon fiber-reinforced plastic coupon data set shown in Table 5: (a) single-wave packet Z statistics for path 3–4; (b) double-wave packet damage indices for path 3–4; (c) single-wave packet Z statistics for path 1–4; (d) double-wave packet Z statistics for path 1–4. In all plots, the dashed red lines are the healthy 95% confidence bounds.

Figure 16. Receiver operating characteristic plots comparing the different damage detection methods for the new data set of the carbon fiber-reinforced plastic coupon under the effect of the first simulated damage (1 weight): (a) path 3–4 one wave packet; (b) path 3–4 two wave packets; (c) path 1–4 one wave packet; and (d) path 1–4 two wave packets. In all subplots, 15 out of 20 healthy signals were used for calculating the mean in estimating the $F_m$ and the Z statistics.
Figure 17. (a) A schematic of the carbon fiber-reinforced plastic panel used in the Open Guided Waves open source data project\textsuperscript{1} with all the simulated damage locations. The inset shows a snapshot of part of the actual panel with damage location markings and the Al weight used to simulate damage on one of the locations. The arrow indicates the path used in the analysis presented herein; (b) indicative signals from path 3–12 for the healthy case, as well as when the Al weight (simulated damage) was on locations D5-10 (damage-intersecting case); (c) the first arrival wave packets; (d) the Janapati et al. damage indice for the first arrival wave packets—the dashed blue lines are the upper and lower 95% confidence bounds for the Janapati et al. damage indice as applied to the damage indice values of corresponding 20-baseline signal data set.

Table 7. Summary of experimental details for the CFRP panel [1].

| Structural state\textsuperscript{a} | Number of data sets | Data set label |
|-----------------------------------|---------------------|----------------|
| Healthy (weight unattached)       | 20\textsuperscript{b} | First baseline set |
| Weight on D1-4                    | 4                   | Damage group 1  |
| Weight on D5-8                    | 4                   | Damage group 2  |
| Weight on D9-11                   | 3                   | Damage group 3  |
| Healthy (weight unattached)       | 20\textsuperscript{b} | Second baseline set |
| Weight on D12                     | 1                   | Damage group 4  |
| Weight on D13-16                  | 4                   | Damage group 5  |
| Weight on D17-20                  | 4                   | Damage group 6  |
| Healthy (weight unattached)       | 20\textsuperscript{b} | Third baseline set |
| Weight on D21-24                  | 4                   | Damage group 7  |
| Weight on D25-28                  | 4                   | Damage group 8  |

\textsuperscript{a}Weight was attached to one location (e.g. D4) at a time within each damage group.

\textsuperscript{b}M = 20 in equation (10).
same set of figures for the third baseline set (healthy signals 41–60) and the signals corresponding to damage locations 21–28 (damage-non-intersecting case). As shown, the DI fails to detect any of the damage cases with 95% confidence.

Examining the Welch PSD estimates for the damage-intersecting case, Figure 19(a) shows that all damage cases are detected with 95% confidence. This immediately shows the advantage of this frequency-domain metric when it comes to damage detection. Figure 19(b) shows the Z

statistic for that case, where again all damage cases are detected with 95% confidence. Also, it can be observed that there are no false alarms in this case, whereas there was at least one false alarm event with the DI (see Figure 17(c)). Figure 19(c) presents the Welch PSD estimate for the damage-non-intersecting set of signals. As shown, at least 5 out of the 8 damage cases were detected with 95% accuracy. Examining the Z statistic, it can be observed that only one damage case is detected with 95% confidence, while the rest are deemed healthy. It is worth noting that neither of the other two statistics (the F and Fm statistics) detected any of the damage cases with the set confidence bounds for the damage-intersecting and non-intersecting cases. Again, this called upon the exploration of the effect of different confidence intervals (manifested in different alpha false alarm levels in the statistical hypothesis tests) in order to conclusively assess the performance of the different detectors proposed herein.

Figures 20(a) and (b) show the ROC plots for the damage-intersecting case and the damage-non-intersecting case, respectively. In constructing each plot, detection statistics from all corresponding damage locations were used, and only corresponding baseline groups were considered in each case. As shown in the damage-intersecting case (panel a), the Z,

Table 8. The Welch PSD estimation parameters for the OGW coupon data sets.

| Parameter                        | Value          |
|----------------------------------|----------------|
| Number of DFT Points (NFFT)     | 2000           |
| Window type                      | Hamming        |
| Frequency resolution Δf         | 5 kHz          |
| Sampling frequency               | 10 MHz         |
| Single wave packet               |                |
| Data length                      | N = 360 samples (~40 μs) |
| Segment length                   | 40             |
| No of non-overlapping segments   | 17             |

**Figure 18.** (a) Indicative signals from path 3–12 for the healthy case, as well as when the Al weight (simulated damage) was on locations D21-28 (damage-non-intersecting case); (b) the first arrival wave packets; (c) the Janapati et al. damage indice for the first arrival wave packets—the dashed blue lines are the upper and lower 95% confidence bounds for the Janapati et al. damage indice as applied to the damage indice values of corresponding 20-baseline signal data set.
$F$, and $F_{in}$ statistics all outperform the DI in overall detection performance, with larger areas under the curves. For the damage-non-intersecting case (panel b), although the $F$, $F_{in}$ statistics and the DI show similar performance, the $Z$ statistic surpasses all of them, even for low alpha levels (wider confidence bounds). Tables 13 and 14 in Appendix 3 also present summary detection results. All of these results show the superiority of the $Z$ statistic when it comes to damage detection.
Conclusions
In this study, three frequency-domain damage detection metrics based on stochastic NP-TS representations were developed and compared with state-of-the-art DIs as applied to three different test cases: a notched Al plate, a CFRP coupon with stacked weights, and the CFRP panel with different weight locations used in the Open Guided-Waves project. It was shown that, although the DIs can accurately detect damage and follow damage evolution in the isotropic Al coupon case, it fails to do either in the CFRP coupon for 95% healthy confidence bounds. In addition, it also shows poor detection performance for the different damage cases of the CFRP panel at the same confidence levels. Examining the $F$ and $F_m$ statistics, because their detection thresholds are either solely dependent on the theoretical estimation confidence bounds of the Welch PSD estimator ($F$ statistic), or dependent on the theoretical estimation intervals with the incorporation of some experimental statistics ($F_m$ statistics), their detection performance at 95% confidence can, in some cases, be even worse than the DIs. However, for different confidence levels, both, especially the $F$ statistic, exhibit a detection performance more or less similar to that of the DIs, as shown in the different ROC plots in this study. On the other hand, the $Z$ statistic outperforms all other detectors used in this study for all three test cases, for both damage-intersecting and non-intersecting paths. In addition, it also better follows the evolution of damage for the Al and CFRP coupons in the damage-intersecting case compared to the DIs, which hints on its damage quantification capabilities.

Overall, it can be concluded from this study that, for the three test cases studied herein, methods based on frequency-domain non-parametric statistical time series models show greater sensitivity to damage, even when used to analyze damage-non-intersecting signals, compared to time-domain DI-based approaches, especially in materials exhibiting non-linearities and anisotropic behavior such as composites. This was clearly demonstrated when constructing 95% healthy confidence bounds accounting for the same experimental uncertainties in both approaches. In addition, the proposed approaches show increased robustness to uncertainty with less fluctuation in the values of the metrics for the healthy test cases compared to the time-domain-based DIs. Thus, NP-TS representations emerge as sources of constructing accurate and robust metrics that promise enhancement in damage detection performance for SHM systems.

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Notes
1. Matlab version R2018a; function pwelch.m (window size: 100 for single wave packet analyses and 500 for the full signal/two-wave packet analyses; NFFT: 2000; Overlap: 50%).
2. The computational time to calculate the Welch-based PSD estimates as well as the three statistics combined is on the order of 300 $\mu$s, while that of calculating the DI values is on the order of 180 $\mu$s.

References
1. Moll J, Kathol J, Fritzen C-P, et al. Open guided waves: online platform for ultrasonic guided wave measurements. Struct Health Monit 2019; 18: 1–12.
2. Janapati V, Kopsaftopoulos F, Li F, et al. Damage detection sensitivity characterization of acousto-ultrasound-based structural health monitoring techniques. Struct Health Monit 2016; 15(2): 143–161.
3. Qiu L, Liu M, Qing X, et al. A quantitative multidamage monitoring method for large-scale complex composite. Struct Health Monit 2013; 12(3): 183–196.
4. Romano F, Ciminello M, Sorrentino A, et al. Application of structural health monitoring techniques to composite wing panels. J Compos Mater 2019; 53(25): 3515–3533.
5. Das S and Saha P. A review of some advanced sensors used for health diagnosis of civil engineering structures. Measurement 2018; 129: 68–90.
6. Farrar CR and Worden K. An introduction to structural health monitoring. Philosophical Trans R Soc A: Math Phys Eng Sci 2007; 365: 303–315.
7. Amer A and Kopsaftopoulos FP. Probabilistic active sensing acousto-ultrasound SHM based on non-parametric stochastic representations. In: Proceedings of the vertical flight society 75th annual forum & technology display, Philadelphia, PA, USA, 13–16 May 2019.
8. Amer A and Kopsaftopoulos FP. Probabilistic damage quantification via the integration of non-parametric time-series and gaussian process regression models. In: Proceedings of the 12th international workshop on structural health monitoring (IWSHM 2019), Palo Alto, CA, USA, September 2019.
9. Kopsaftopoulos F, Nardari R, Li Y-H, et al. A stochastic global identification framework for aerospace structures operating under varying flight states. *Mech Syst Signal Process* 2018; 98: 425–447.

10. Spiridonakos MD and Fassois SD. “Non-stationary random vibration modelling and analysis via functional series time-dependent ARMA (FS-TARMA) models—a critical survey.” *Mech Syst Signal Process* 2014; 47(1–2): 175–224.

11. Zhang QW. Statistical damage identification for bridges using ambient vibration data. *Comput Struct* 2007; 85: 476–485.

12. Ahmed S and Kopsaftopoulos FP. Uncertainty quantification of guided waves propagation for active sensing structural health monitoring. In: Proceedings of the vertical flight society 75th annual forum & technology display, Philadelphia, PA, USA, May 2019.

13. Ahmed S and Kopsaftopoulos FP. Investigation of broadband high-frequency stochastic actuation for active-sensing SHM under varying temperature. In: Proceedings of the 12th international workshop on structural health monitoring (IWSHM 2019), Palo Alto, CA, USA, September 2019.

14. Zhao J, Gao HD, Chang GF, et al. Active health monitoring of an aircraft wing with embedded piezoelectric sensor/actuator network: I. Defect detection, localization and growth monitoring. *Smart Mater Struct* 2007; 16(4): 1208–1217.

15. Flynn EB, Todd MD, Wilcox PD, et al. Maximum-likelihood estimation of damage location in guided-wave structural health monitoring. In: Proceedings of the royal society A, Burlington, VT, 2011, 467(2133), pp. 2575–2596.

16. Todd MD, Flynn EB, Wilcox PD, et al. Ultrasonic wave-based defect localization using probabilistic modeling. In: American institute of physics conference proceedings, Burlington, VT, May 2012.

17. Haynes C, Todd M, Flynn E, et al. Statistically-based damage detection in geometrically-complex structures using ultrasonic interrogation. *Struct Health Monit* 2012; 12(2): 141–152.

18. Ng C-T. On the selection of advanced signal processing techniques for guided wave damage identification using a statistical approach. *Eng Struct* 2014; 67: 50–60.

19. Mujica LE, Ruiz M, Pozo F, et al. A structural damage detection indicator based on principal component analysis and statistical hypothesis testing. *Smart Mater Struct* 2013; 23(2): 025014.

20. Peng T, Saxena A, Goebel K, et al. A novel Bayesian imaging method for probabilistic delamination detection of composite materials. *Smart Mater Struct* 2013; 22: 125019–125028.

21. Yang J, He J, Guan X, et al. A probabilistic crack size quantification method using in-situ Lamb wave test and Bayesian updating. *Mech Syst Signal Process* 2016; 78: 118–133.

22. He J, Ran Y, Liu B, et al. A Lamb wave based fatigue crack length estimation method using finite element simulations. In: The 9th international symposium on NDT in aerospace, Xiamen, China, November 2017.

23. MIL-HDBK-1823A. “MIL-HDBK-1823A” Nondestructive Evaluation System Reliability Assessment. Greene County, OH: Department of Defense, 2009.

24. Gallina A, Packo P and Ambrozinski L. “Model assisted probability of detection in structural health monitoring”. In Stepinski T, Uhl T and Staszewski W (eds). *Advanced structural damage detection: from theory to engineering applications*. Hoboken, NJ: John Wiley and Sons, Ltd., 2013, pp. 382–407.

25. Jarmer G and Kessler SS. “Application of Model Assisted Probability of Detection (MAPOD) to a Guided Wave SHM System”. In: Chang F-K and Kopsaftopoulos F (eds). *Structural health monitoring 2017: Real-time material state awareness and data-driven safety assurance—proceedings of the 12th international workshop on structural health monitoring (IWSHM 2017)*, USA, 2017: Stanford University.

26. Moriot J, Quagebeur N, Duff AL, et al. “A model-based approach for statistical assessment of detection and localization performance of guided wave-based imaging techniques”. *Struct Health Monit* 2017; 17(6): 1460–1472.

27. Giurgiutiu V. Flutter prediction for flight/wind-tunnel flutter test under atmospheric turbulence excitation. *J Intell Mater Syst Struct* 2005; 16(4): 291–305.

28. Ihn J-B and Chang F-K. Detection and monitoring of hidden fatigue crack growth using a built-in piezoelectric sensor/actuator network: I. Diagnostics. *Smart Mater Struct* 2004; 13: 609–620.

29. Ihn J-B and Chang F-K. Detection and monitoring of hidden fatigue crack growth using a built-in piezoelectric sensor/actuator network: II. Validation using riveted joints and repair patches. *Smart Mater Struct* 2004; 13: 621–630.

30. Jin H and Chang F-K. Pitch-catch active sensing methods in structural health monitoring for aircraft structures. *Struct Health Monit* 2008; 7(1): 5–19.

31. Giurgiutiu V. Piezoelectric wafer active sensors for structural health monitoring of composite structures using tuned guided waves. *J Eng Mater Technol* 2011; 133(4): 041012.

32. Jin H, Yan J, Li W, et al. Monitoring of fatigue crack propagation by damage index of ultrasonic guided waves calculated by various acoustic features. *Appl Sci* 2019; 9: 4254.

33. Xu B, Zhang T, Song G, et al. Active interface debonding detection of a concrete-filled steel tube with piezoelectric technologies using wavelet packet analysis. *Mech Syst Signal Process* 2013; 36: 7–17.

34. Nasrollahi A, Deng W, Ma Z, et al. Multimodal structural health monitoring based on active and passive sensing. *Struct Health Monit* 2018; 17(2): 395–409.

35. Qiu L, Yuan S, Bao Q, et al. Crack propagation monitoring in a full-scale aircraft fatigue test based on guided wave-Gaussian mixture model. *Smart Mater Struct* 2016; 25: 055048.
36. Wang F, Huo L and Song G. A piezoelectric active sensing method for quantitative monitoring of bolt loosening using energy dissipation caused by tangential damping based on the fractal contact theory. *Smart Mater Struct* 2018; 27: 015023.

37. Castro E, Moreno-García P and Gallego A. Damage detection in CFRP Plates using spectral entropy. London: Shock and Vibration, 2014, pp. 1–8.

38. An Y-K, Giurgiuţiu V and Sohn H. Integrated impedance and guided wave based damage detection. *Mech Syst Signal Process* 2012; 28: 50–62.

39. Su Z, Zhou C, Hong M, et al. Acousto-ultrasonics-based fatigue damage characterization: linear versus nonlinear signal features. *Mech Syst Signal Process* 2014; 45: 225–239.

40. Su Z and Ye L. Lamb wave-based quantitative identification of delamination in CF/EP composite structures using artificial neural algorithm. *Compos Struct* 2004; 66: 627–637.

41. Song G, Gu H and Mo Y-L. Smart aggregates: multifunctional sensors for concrete structures—a tutorial and a review. *Smart Mater Structures* 2008; 17: 033001.

42. Tibaduiza DA, Mujica LE, Rodellar J, et al. Structural damage detection using principal component analysis and damage indices. *J Intell Mater Syst Struct* 2016; 27(2): 233–248.

43. Lizé E, Rébillat M, Mechbal N, et al. Optimal dual-PZT sizing and network design for baseline-free SHM of complex anisotropic composite structures. *Smart Mater Struct* 2018; 27: 115018.

44. Hua J, Cao X, Yi Y, et al. Time-frequency damage index of broadband lamb wave for corrosion inspection. *J Sound Vibration* 2020; 464: 114985.

45. Ibáñez F, Baltazar A and Mijárez R. Detection of damage in multiwire cables based on wavelet entropy evolution. *Smart Mater Struct* 2015; 24: 085036.

46. Shannon CE. A mathematical theory of communication. *Bell Syst Tech J* 1948; 27: 379–423.

47. Rojas E, Baltazar A and Loh KJ. Damage detection using the signal entropy of an ultrasonic sensor network. *Smart Mater Struct* 2015; 24: 075008.

48. Qiu J, Li F, Abbas S, et al. A baseline-free damage detection approach based on distance compensation of guided waves. *J Low Frequency Noise, Vibration Active Control* 2019; 38: 1132–1148.

49. Kopsaftopoulos FP and Fassois SD. Vibration based health monitoring for a lightweight truss structure: experimental assessment of several statistical time series methods. *Mech Syst Signal Process* 2010; 24: 1977–1997.

50. Kopsaftopoulos FP and Fassois SD. A functional model based statistical time series method for vibration based damage detection, localization, and magnitude estimation. *Mech Syst Signal Process* 2013; 39: 143–161.

51. Fassois SD and Kopsaftopoulos FP. Statistical time series methods for vibration based structural health monitoring. In: Ostachowicz W and Guemes A (eds) *New trends in structural health monitoring*. Vienna: Springer; 2013, pp. 209–264, chap. 4.

52. Kopsaftopoulos FP and Fassois SD. Experimental assessment of time series methods for structural health monitoring (SHM). In: Proceedings of the 4th European workshop on structural health monitoring (EWSHM), Cracow, Poland, 2008.

53. Kopsaftopoulos FP and Fassois SD. “Vibration based health monitoring for a thin aluminum plate – a comparative assessment of statistical time series methods”. In: Proceedings of the 5th European workshop on structural health monitoring (EWSHM), Sorrento, Italy, 2010.

54. Box GEP, Jenkins GM and Reinsel GC. *Time series analysis: forecasting & control*. 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, 1994.

55. Manolakis D, Ingle VK and Kogon SM. *Statistical and adaptive signal processing: spectral estimation, signal modeling, adaptive filtering and array processing*. New York: Artech House, 2005.

56. Kay SM. *Modern spectral estimation: theory and application*. New Jersey: Prentice-Hall, 1988.

57. Hayes MH. *Statistical digital signal processing and modeling*. New York: John Wiley and Sons, 1996.

58. Bendat JS and Piersol AG. *Random data: analysis and measurement procedures*. 3rd ed. New York: Wiley-Interscience, 2000.

59. Bull LA, Rogers TJ, Wickramarachchi C, et al. Probabilistic active learning: an online framework for structural health monitoring. *Mech Syst Signal Process* 2019; 134(1–2): 106294.
Appendix 1

Damage detection summary results: Notched Al coupon

Table 9. Damage detection summary results at an $\alpha$ value of 95% for path 2–6 (single wave packet) in the Al plate (damage presented in units of mm).

| Method      | False alarms (%) | Missed damage (%) |
|-------------|------------------|-------------------|
|             | 2    | 4    | 6    | 8    | 10   | 12   | 14   | 16   | 18   | 20   |
| DI$^a$      | 6    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |
| $F$ statistic$^a$ | 0    | 100  | 100  | 100  | 100  | 0    | 0    | 0    | 0    | 0    |
| $F_m$ statistic$^b$ | 0    | 100  | 100  | 100  | 0    | 0    | 0    | 0    | 0    | 0    |
| $Z$ statistic$^b$ | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    |

False alarms presented as percentage of 400 test cases for the DI and $F$ statistic, and as percentage of 20 data sets for the $F_m$ and $Z$ statistics. Missed damages presented as percentage of 20 test cases.

$^a$All 20 baseline data sets were used as reference signals consecutively.

$^b$15 out of 20 baseline data sets were used to calculate the baseline mean.

Figure 21. Receiver operating characteristics plots comparing the different damage detection methods for the notched Al coupon with a notch size of 2 mm: (a) path 2–6 wave packet; (b) path 2–6 full signal; (c) path 6–3 wave packet; (d) path 6–3 full signal.
Table 10. Damage detection summary results at an $\alpha$ value of 95% for path 6–3 (single wave packet) in the Al plate (damage presented in units of mm).

| Method          | False alarms (%) | Missed damage (%) |
|-----------------|------------------|-------------------|
|                 |                  | 2     | 4     | 6     | 8     | 10    | 12    | 14    | 16    | 18    | 20    |
| $D_{A,d}^{2}$   | 5.25             | 99.75 | 84.5  | 94.5  | 99.25 | 93.75 | 53.5  |
| $F_{A}^{b,c,d}$ | 25               | 65    | 5     | 5     | 0     | 0     | 0     |
| $F_{m}^{c,e}$   | 25               | 40    | 10    | 60    | 0     | 0     | 0     |
| $Z_{A}^{c,e}$   | 0                | 25    | 10    | 30    | 10    | 0     | 0     |

False alarms presented as percentage of 400 test cases for the DI and $F$ statistic, and as percentage of 20 data sets for the $F_m$ and $Z$ statistics. Missed damage presented as percentage of 20 test cases.

All 20 baseline data sets were used as reference signals consecutively.

15 out of 20 baseline data sets were used to calculate the baseline mean.

Appendix 2

Damage detection summary results: Carbon fiber-reinforced plastic coupon

Table 11. Damage detection summary results at multiple $\alpha$ values for path 1–4 (single wave packet) in the CFRP plate.

| Method          | False alarms (%) | Missed damage (%) |
|-----------------|------------------|-------------------|
|                 |                  | 1 weight | 2 weights | 3 weights | 4 weights | 5 weights | 6 weights |
| $D_{A,d}^{2}$   | 5.25             | 99.75     | 84.5      | 94.5      | 99.25      | 93.75      | 53.5      |
| $F_{A}^{b,c,d}$ | 25               | 65        | 5         | 5         | 0          | 0          | 0         |
| $F_{m}^{c,e}$   | 25               | 40        | 10        | 60        | 0          | 0          | 0         |
| $Z_{A}^{c,e}$   | 0                | 25        | 10        | 30        | 10         | 0          | 0         |

False alarms presented as percentage of 20 test cases. Missed damage presented as percentage of 20 test cases. CFRP: carbon fiber-reinforced plastic.

$^a$ $\alpha = 95\%$.

$^b$ $\alpha = 1\%$.

$^c$ $\alpha = 10\%$.

$^d$ All 20 baseline data sets were used as reference signals consecutively.

$^e$ 15 out of 20 baseline data sets were used to calculate the baseline mean.

Table 12. Damage detection summary results at multiple $\alpha$ values for path 3–4 (single wave packet) in the CFRP plate.

| Method          | False alarms (%) | Missed damage (%) |
|-----------------|------------------|-------------------|
|                 |                  | 1 weight | 2 weights | 3 weights | 4 weights | 5 weights | 6 weights |
| $D_{A,d}^{2}$   | 5.25             | 13.25     | 0         | 0         | 0         | 0         | 0         |
| $F_{A}^{b,c,d}$ | 95               | 65        | 5         | 5         | 0         | 0         | 0         |
| $F_{m}^{c,e}$   | 30               | 60        | 0         | 0         | 0         | 0         | 0         |
| $Z_{A}^{c,e}$   | 0                | 0         | 0         | 0         | 0         | 0         | 0         |

False alarms presented as percentage of 20 test cases. Missed damage presented as percentage of 20 test cases. CFRP: carbon fiber-reinforced plastic.

$^a$ $\alpha = 95\%$.

$^b$ $\alpha = 1\%$.

$^c$ $\alpha = 10\%$.

$^d$ All 20 baseline data sets were used as reference signals consecutively.

$^e$ 15 out of 20 baseline data sets were used to calculate the baseline mean.
Appendix 3

**Damage detection summary results: OGW carbon fiber-reinforced plastic panel**

Table 13. Damage detection summary results at multiple $\alpha$ values for path 3–12 (damage-intersecting case) in the CFRP panel.1

| Method       | False alarms (%) | Missed damage (%) |
|--------------|------------------|-------------------|
|              |                  | D5/6/7/8          | D9/10/11          |
| $D_i^{a,c,2}$| 7.5              | 51.25             | 100               |
| $F_{statistic}^{b,c}$ | 0                | 75                | 100               |
| $F_{m,statistic}^{b,d}$ | 0                | 75                | 33                |
| $Z_{statistic}^{a,d}$ | 0                | 0                 | 0                 |

*False alarms presented as percentage of 20 test cases. Missed damages presented as percentage of all test cases per damage group.*

1 CFRP: carbon fiber-reinforced plastic.

2 $\alpha = 95\%$.

3 $\alpha = 80\%$.

All 20 baseline data sets were used as reference signals consecutively.

15 out of 20 baseline data sets were used to calculate the baseline mean.

Table 14. Damage detection summary results at an $\alpha$ value of 95% for path 3–12 (damage-non-intersecting case) in the CFRP panel.1

| Method       | False alarms (%) | Missed damage (%) |
|--------------|------------------|-------------------|
|              |                  | D21/22/23/24      | D25/26/27/28      |
| $D_i^{a,c,2}$| 7.5              | 100               | 93.75             |
| $F_{statistic}^{b,c}$ | 0                | 100               | 75                |
| $F_{m,statistic}^{b,d}$ | 0                | 100               | 75                |
| $Z_{statistic}^{a,d}$ | 0                | 50                | 75                |

*False alarms presented as percentage of 20 test cases. Missed damages presented as percentage of all test cases per damage group.*

1 CFRP: carbon fiber-reinforced plastic.

2 $\alpha = 95\%$.

3 $\alpha = 70\%$.

All 20 baseline data sets were used as reference signals consecutively.

15 out of 20 baseline data sets were used to calculate the baseline mean.