Establishing the Dark Matter Relic Density in an Era of Particle Decays

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Abstract. If the early universe is dominated by an energy density which evolves other than radiation-like the normal Hubble-temperature relation $H \propto T^2$ is broken and dark matter relic density calculations in this era can be significantly different. We first highlight that for a population of states $\phi$ sourcing an initial expansion rate of the form $H \propto T^{2+n/2}$ for $n \geq -4$, during the period of appreciable $\phi$ decays the evolution transitions to $H \propto T^4$. The decays of $\phi$ imply a source of entropy production in the thermal bath which alters the Boltzmann equations and impacts the dark matter relic abundance. We show that the form of the initial expansion rate leaves a lasting imprint on relic densities established while $H \propto T^4$ since the value of the exponent $n$ changes the temperature evolution of the thermal bath. In particular, a dark matter relic density set via freeze-in or non-thermal production is highly sensitive to the temperature dependance of the initial expansion rate. This work generalises earlier studies which assumed initial expansion rates due to matter or kination domination.
1 Introduction

The dark matter freeze-out paradigm, in particular the WIMP miracle, is prized for its simplicity and predictiveness. However, it is relatively straightforward to arrange for significant deviations in the predictions of freeze-out by changing either the particle physics model, or the cosmological history. For instance, standard freeze-out calculations typically assume that decoupling occurs whilst the energy density of the universe is dominated by radiation in which case the expansion rate of the universe is $H \propto T^2$. Indeed, dark matter freeze-out could occur whilst the universe is dominated by some form of energy other than radiation in which case the usual Hubble-temperature relation $H \propto T^2$ is broken. In particular, one possibility which occurs quite naturally in many Standard Model extensions is the case of an early matter-dominated period for which $H \propto T^{3/2}$, or an era of particle decays leading to significant entropy production in the thermal bath in which case $H \propto T^4$ [1]. The case of dark matter freeze-out during an early period of matter domination was recently highlighted in [2] and freeze-out whilst $H \propto T^4$ was studied in [3–6].

More generally the early universe could be dominated by the energy density of a population of states $\phi$ evolving as an arbitrary power of the scale factor, i.e. $\rho_\phi(T) = \rho_\phi(T_I) a^{4+n}$ with $T_I$ some initial temperature. For $n > 0$ the energy density will eventually redshift to a negligible level and the expansion rate $H \propto T^{n/2+2}$ for $n \geq 0$ is faster than expected from a radiation-dominated universe. Two recent studies by D’Eramo, Fernandez, & Profumo [7, 8] considered the implications for dark matter if the relic density is established during such a period of fast expansion. Note that the case $n = 2$ corresponds to ‘kination domination’, see e.g. [9–12], in which the energy density of the universe is dominated by the kinetic energy of some scalar field (the $\dot{\phi}$ term), for instance the inflaton. Conversely, for $n < 0$, in order to recover the successes of standard cosmology, one requires that the state which dominates the energy density eventually decays. Specifically, the universe should be dominated by the Standard Model radiation bath at temperatures around 10 MeV and below (until matter-radiation equality) so not to spoil the precision predictions of Big Bang nucleosynthesis (see e.g. [13]).
Here we study the implications for the dark matter which freezes out or is produce while some (boson or fermion) state \( \phi \) which dominates the energy density is appreciably decaying and under the assumption that the energy density of \( \phi \) implies an initial expansion rate of the form \( H \propto T^{n/2+2} \). Notably, during this period of particle decays entropy is no longer conserved in thermal bath and this impacts the dark matter relic abundance calculation. In particular, our work generalizes the earlier papers of [3–6] which assume that the initial expansion rate corresponds to an early matter dominated phase. Additionally, this work can also be seen as an extension of dark matter freeze-out or production with a general expansion rate, as studied in [7, 8], to include decays of \( \phi \). This paper is structured as follows: In Section 2 we discuss the formulation of the Boltzmann equation without entropy conservation, generalizing the derivations of [5]. Using these results, in Section 3 we compare different scenarios for setting the dark matter relic abundance and discuss their dependance on the exponent \( n \) of the initial expansion rate. Concluding remarks are given in Section 4.

2 Boltzmann Equations without Entropy Conservation

We start by deriving expressions for the evolution of the different particle populations in the case that the early universe is dominated by a state \( \phi \) implying an initial expansion rate of \( H \propto T^{n/2+2} \). We show that the expansion rate subsequently transitions to \( H \propto T^4 \) and derive an expression for the maximum temperature of the Standard Model radiation. The expressions derived reproduce the results of Giudice, Kolb, & Riotto [5] for matter domination prior to decays (\( n = -1 \)) and Visinelli [14] with kination domination prior to decays (\( n = 2 \)).

2.1 Boltzmann equations

We start in familiar territory by defining the Hubble parameter

\[
H^2 = \frac{8\pi}{3M_{Pl}^2} (\rho_\phi + \rho_R + \rho_X),
\]

where the terms indicate the energy density in \( \phi \), Standard Model radiation, and dark matter \( X \), respectively. The evolution of quantities can be tracked relative to a dimensionless scale factor \( A \) defined as

\[
A \equiv \frac{a}{a_I} = aT_{RH}.
\]

where \( a_I \) is an arbitrary initial reference point which is chosen to be \( a_I = 1/T_{RH} \) and \( T_{RH} \) is the reheat temperature following \( \phi \) decays. Emulating the analysis of [5], we rewrite the relevant variables as dimensionless quantities, but where now the energy density of \( \phi \) evolves as some arbitrary power \( a^{4+n} \)

\[
\Phi \equiv \rho_\phi a^{4+n}T_{RH}^m = \frac{\rho_\phi A^{4+n}}{T_{RH}^4}, \quad R \equiv \rho_R a^4 = \frac{\rho_R A^4}{T_{RH}^4}, \quad X \equiv n_X a^3 = \frac{n_X A^3}{T_{RH}^3}.
\]

With the assumption that \( \rho_X = \langle E_X \rangle n_X \), where \( n_X \) is the dark matter number density and \( \langle E_X \rangle \) the expected energy of each dark matter state, one can express the Hubble rate in terms of these dimensionless variables to obtain

\[
H = \frac{T_{RH}^2}{M_{Pl}A^{2+n/2}} \sqrt{\frac{8\pi}{3} \left( \Phi + RA^n + \frac{\langle E_X \rangle X A^{n+1}}{T_{RH}} \right)}. \tag{2.4}
\]
This recovers the form of $H$ in [5] for $n = -1$. For the expansion rate in the form of eq. (2.4), the Boltzmann equations which describe the evolution of the number densities can be expressed in a manner highly reminiscent to those studied in [5, 6]

\[ \dot{\rho}_\phi + (4 + n)H \rho_\phi = -\Gamma_\phi \rho_\phi \]  
\[ \rho_R + 4H \rho_R = \Gamma_\phi \rho_\phi + 2\langle E_X \rangle (n_X^2 - n_{eq}^2) \langle \sigma v \rangle \]  
\[ n_X' + 3H n_X = \frac{b}{m_\phi} \Gamma_\phi \rho_\phi - (n_X^2 - n_{eq}^2) \langle \sigma v \rangle \]

where $m_\phi$ and $\Gamma_\phi$ are the $\phi$ mass and decay rate to Standard Model. Dotted variables indicate differentiation with respect to time, the quantity $b$ parameterises the branching ratio of $\phi$ to dark matter, $\langle \sigma v \rangle$ is the thermally averaged dark matter annihilation cross section, and $n_{eq}$ denotes the equilibrium number density of dark matter which has its usual form. Furthermore, note that the $\phi$ decay rate can be expressed in terms of the reheat temperature after $\phi$ decays

\[ \Gamma_\phi = \sqrt{\frac{4\pi^3 g_*(T_{RH})}{45}} \frac{T_{RH}^2}{M_{Pl}} \]  

We next re-express eqns. (2.5)-(2.7) in terms of the dimensionless units of eq. (2.3). Looking firstly at eq. (2.5), simple substitution yields

\[ \dot{A} \frac{d}{dA} \left( \frac{\Phi}{(AA_I)^{4+n}T_{RH}^n} \right) + (4 + n)H \left( \frac{\Phi}{(AA_I)^{4+n}T_{RH}^n} \right) = -\Gamma_\phi \frac{\Phi}{(AA_I)^{4+n}T_{RH}^n} \]  

After some manipulation, and using that $H = \frac{\dot{a}}{a}$, this can be simplified to

\[ \dot{A} \Phi' = \Gamma_\phi \Phi , \]

where the primed variable indicates differentiation with respect to $A$. Using eq. (2.4) & (2.8) we can express eq. (2.10) in terms of $\Phi'$ as follows

\[ \Phi' = -\sqrt{\frac{\pi^2 g_*(T_{RH})}{30}} \frac{\Phi A^{1+n/2}}{\sqrt{\Phi + RA^n + \frac{\langle E_X \rangle X A^{n+1}}{T_{RH}}}} . \]  

Analogously, eqns. (2.6) & (2.7) can be rewritten in a similar fashion to obtain

\[ R' = \sqrt{\frac{\pi^2 g_*(T_{RH})}{30}} \frac{\Phi A^{1-n/2}}{\sqrt{\Phi + RA^n + \frac{\langle E_X \rangle X A^{n+1}}{T_{RH}}}} + \sqrt{\frac{3}{8\pi}} \frac{2\langle \sigma v \rangle \langle E_X \rangle M_{Pl} A^{n/2-1} (X^2 - X_{eq}^2)}{\sqrt{\Phi + RA^n + \frac{\langle E_X \rangle X A^{n+1}}{T_{RH}}}} \]  

\[ X' = \sqrt{\frac{\pi^2 g_*(T_{RH})}{30}} \frac{b}{m_\phi} \frac{\Phi T_{RH} A^{-n/2}}{\sqrt{\Phi + RA^n + \frac{\langle E_X \rangle X A^{n+1}}{T_{RH}}}} - \sqrt{\frac{3}{8\pi}} \frac{\langle \sigma v \rangle M_{Pl} T_{RH} A^{n/2-2} (X^2 - X_{eq}^2)}{\sqrt{\Phi + RA^n + \frac{\langle E_X \rangle X A^{n+1}}{T_{RH}}}} . \]

Thus the dimensionless versions of eqns. (2.5)-(2.7) are, respectively, eqns. (2.11)-(2.13).
2.2 Radiation temperature maxima

We assume that in the early universe the energy density is dominated by $\phi$ and, moreover, we further suppose that initial radiation bath is negligible. This implies that the initial energy density of $\phi$ can be written $\rho_\phi(a_I) = \frac{3}{8\pi}H_I^2M_{Pl}^2$ where $H_I \equiv H(a_I)$ is the initial expansion rate (throughout we will use the subscript $I$ to mean the value of a give quantity at $a = a_I$).

In terms of dimensionless variables this initial condition is

$$\Phi_I = \frac{3H_I^2M_{Pl}^2}{8\pi T_{RH}^4}, \quad R_I = X_I = 0, \quad A_I = 1. \quad (2.14)$$

One instance in which such initial conditions could arise, for instance, is immediately after inflation, as in the case of kination domination [9–12] corresponding to $n = 2$.

In [5] the authors studied the evolution of the temperature of the Standard Model thermal bath, assuming that prior to the decays of $\phi$ the radiation component is negligible. What was observed is that during the early matter dominated era the temperature rises due to the decays of $\phi$ until it hits some maximum temperature $T_{Max}$ after which the expansion rate transitions to $H \propto T^4$. The bath then cools until the temperature $T_{RH}$ at $H \simeq \Gamma_\phi$ after which $\phi$ decays become negligible and the universe becomes radiation dominated with $H \propto T^2$. We next generalise this analysis to the case that the early universe expansion rate sourced by $\phi$ is an arbitrary power of the temperature as in eq. (2.1).

As the $\phi$ decay, energy is transferred to the Standard Model bath, we will derive the point $A_{Max}$ at which the maximum temperature of the radiation bath $T_{Max}$ occurs for the more general expansion rate. The dominant contribution comes from $\phi$ at early time, thus we can neglect the second term in eq. (2.12), and using eq. (2.14) we obtain

$$R' = \sqrt{\frac{\pi^2 g_*(T_{RH})}{30}} A^{1-n/2} \Phi_I^{1/2} A^{1-n/2}. \quad (2.15)$$

Further, integrating we obtain the following

$$R = \begin{cases} \sqrt{\frac{\pi^2 g_*(T_{RH})}{30}} \Phi_I \left( \frac{1}{2-n/2} \right) (A^{2-n/2} - 1) & \text{for } n < 4 \\ \sqrt{\frac{\pi^2 g_*(T_{RH})}{30}} \Phi_I \ln(A) & \text{for } n = 4 \end{cases}. \quad (2.16)$$

The temperature is a measure of the radiation energy density, thus we can obtain an expression for the evolution of $T$ as a function of $A$ from the expression

$$\rho_R = \frac{\pi^2 g_*(T)}{30} T^4 = R \left( \frac{T_{RH}}{A} \right)^4. \quad (2.17)$$

Moreover, substituting eq. (2.16) into eq. (2.17) and rearranging, gives the evolution of the temperature (for $n \neq 4$)

$$T = \left( \frac{45}{4\pi^3} \frac{g_*(T_{RH})}{g_*(T)} \right)^{1/8} (H_I M_{Pl} T_{RH}^2)^{1/4} \left[ \frac{A^{-(2+n/2)} - A^{-4}}{2 - n/2} \right]^{1/4}. \quad (2.18)$$

The critical point, with respect to $A$, of the factor in square brackets of eq. (2.18) marks the maximum temperature and the value of the scale factor $A_{Max}$ at which the temperature stops increasing and begins to decrease. For $|n| < 4$ this is given by

$$A_{Max} = \left( \frac{n + 4}{8} \right)^{2/(n-4)}. \quad (2.19)$$
\(\Phi_I = 10^{10} \text{GeV} | H_I = 1 \text{ eV}, T_{RH} = 100 \text{ GeV}\)

\(\Phi_I = 10^{10} \text{GeV} | H_I = 1 \text{ eV}, T_{RH} = 100 \text{ GeV}\)

Figure 1. Assuming that initially the expansion rate is \(H \propto T^{n/2+2}\) and there is negligible energy in the radiation bath or dark matter \(R(a_I) = X(a_I) = 0\), the figure shows the bath temperature \(T\) as function of \(A\) for different values of \(n\). We fix \(\Phi_I = \Phi(a_I) = 10^{10} \text{ GeV}, or equivalently (see eq. (2.14)) this corresponds to, for example, \(H_I = 1 \text{ eV}\) and \(T_{RH} = 100 \text{ GeV}\). The curves follow eq. (2.26). Of the cases shown \(n = -1\) corresponds to a matter-like \(\phi\) (blue, solid), \(n = 0\) is radiation-like \(\phi\) (dashed), and for \(n = 1\) then \(\phi\) redshifts faster than radiation (dotted). Also note that, following eq. (2.19), the maximum temperature drops (and occurs earlier) for increasing \(n\).

For comparison, recall that \(A_I = 1\) corresponds to \(T = T_{RH}\) and \(A > 1\) implies \(a > a_I\). For \(A > A_{\text{Max}}\) the \(A^{-4}\) piece in eq. (2.18) can be neglected and \(T \propto A^{-(2+n/2)}\). Observe that \(A_{\text{Max}}\) is sensitive to the exponent \(n\) of the early universe expansion rate. The temperature extremum \(T_{\text{Max}}\) for \(|n| < 4\) is found at \(A = A_{\text{Max}}\) given by

\[
T_{\text{Max}} = \left(\frac{45 g_*(T_{RH})}{4\pi^3 g_*^2(T_{Max})}\right)^{1/8} (M_{Pl} H_I T_{RH}^2)^{1/4} \left(\frac{2}{4-n}\right)^{1/4} \left[\left(\frac{n+4}{8}\right)^{\frac{4+n}{4-n}} - \left(\frac{n+4}{8}\right)\right]^{1/4}.
\]  

(2.20)

As a reference, taking a few specific values for \(n\), the \(T_{\text{Max}}\) can be approximated as

\[
T_{\text{Max}} \simeq (M_{Pl} H_I T_{RH}^2)^{1/4} \times \begin{cases} 
0.30 & \text{for } n = -1 \\
0.31 & \text{for } n = -2 \\
0.33 & \text{for } n = -3
\end{cases},
\]

(2.21)

where we take \(g_*(T_{RH}) \approx g_*(T_{\text{Max}}) \approx 100\). For example, with reasonable values for \(H_I \sim \text{eV}\) and \(T_{RH} \sim 1 \text{ TeV}\), then \(T_{\text{Max}} \sim 3 \text{ TeV}\) for \(|n| \sim \mathcal{O}(1)\).

Furthermore, we can re-express eq. (2.18) in terms of a normalised function \(f(A_{\text{Max}}) = 1\)

\[
T = T_{\text{Max}} f(A)
\]

(2.22)

for

\[
f(A) \equiv \kappa(T) \left[A^{-(2+n/2)} - A^{-4}\right]^{1/4}
\]

(2.23)

with

\[
\kappa(T) = \left[\frac{g_*(T_{\text{Max}})}{g_*(T)}\right]^{1/4} \left[\left(\frac{4+n}{8}\right)^{\frac{4+n}{4-n}} - \left(\frac{4+n}{8}\right)\right]^{-1/4}.
\]

(2.24)
For reference, a selection of specific values for \( \kappa \) are

\[
\kappa(T) \approx \left[ \frac{g_*(T)}{g_*(T)} \right]^{1/4} \begin{cases} 
\left( \frac{8^6}{3^{3/2}} \right)^{1/20} & \text{for } n = -1 \\
\left( \frac{4^4}{3^3} \right)^{1/12} & \text{for } n = -2 \\
\left( \frac{16^6}{7^3} \right)^{1/28} & \text{for } n = -3
\end{cases}
\]  

(2.25)

Starting from \( a = a_I \) the temperatures increase from a negligible value to \( T_{\text{Max}} \), and then subsequently decreases according to eq. (2.25), as illustrated in Figure 1. The evolution of the bath temperature in Figure 1 assumes that \( \phi \) dominates the energy density, and the evolution will be altered once the energy in radiation becomes comparable to \( \phi \), we denote this \( A \), as we discuss in the next section. Thus for \( A_{\text{Max}} < A < A_\times \) one can approximate the temperature evolution as follows

\[
T \sim \kappa T_{\text{Max}} A^{-(2+n/2)/4}.
\]  

(2.26)

Between the time when \( T_{\text{Max}} \) is reached and the point of radiation domination, at the earlier of \( T_{\text{RH}} \) and \( T_\times \), the \( \phi \) field energy density scales as \( \rho_\phi = \Phi I T_{\text{RH}}^4 / A^{4+n} \). Since the dominant contribution to the Hubble parameter at early times comes from \( \rho_\phi \), it follows from eq. (2.4) and eq. (2.14) that for \( A_{\text{Max}} < A < A_\times \) then

\[
H^2 \simeq \frac{8\pi}{3M_{\text{Pl}}} \frac{\Phi I T_{\text{RH}}^4}{A^{4+n}} = \left( H_I A^{-(4+n)/2} \right)^2.
\]  

(2.27)

Moreover, using eq. (2.20) and eq. (2.26), we can express \( A \) in terms of the temperature

\[
A^{-(4+n)/2} = \left( \frac{4\pi^3(2 - n/2)^2 g_*(T)}{45g_*(T_{\text{RH}})} \right)^{1/2} \frac{T^4}{H_I M_{\text{Pl}} T_{\text{RH}}^2}.
\]  

(2.28)

Substituting eq. (2.28) into (2.27) give an express for the expansion rate for \( A_{\text{Max}} < A < A_\times \)

\[
H = |4 - n| \left( \frac{\pi^3 g_*(T)}{45g_*(T_{\text{RH}})} \right)^{1/2} \frac{T^4}{M_{\text{Pl}} T_{\text{RH}}^2}.
\]  

(2.29)

Thus the point \( A_{\text{Max}} \) indicates the \( A \) at which the evolution transitions to \( H \propto T^4 \). Interestingly, the form of \( H \) at this stage is independent on the preceding expansion rate apart from the prefactor, however because the values of \( A_{\text{Max}} \) and \( T_{\text{Max}} \) differ the evolution of cosmological abundances still changes for different values of \( n \).

### 2.3 Onset of radiation domination

Since \( \phi \) is decaying eventually radiation will come to dominate the energy density of the universe, indeed this is desirable to match early universe cosmology such as Big Bang nucleosynthesis observations. Due to decays the energy density of \( \phi \), as tracked in dimensionless units by \( \Phi \), changes is described by eq. (2.11). At early time (where \( X \) and \( R \) are negligible), this can be rewritten via separation of variables as follows

\[
\frac{d\Phi'}{\sqrt{\Phi}} = -dA \sqrt{\frac{\pi^2 g_*(T_{\text{RH}})}{30}} A^{1+n/2}.
\]  

(2.30)
Figure 2. (Left). The point $A_x$ at which $R(a_x) = \Phi(a_x)$ as $\Phi_I$ is varied and for different $n$, found by solving the coupled differential eqns. (2.11) & (2.12) with the initial conditions of eq. (2.14). The line styles match Figure 1, with $n = 0$ is dashed and $n = 1$ dotted. The point $A_x$ signifies the breakdown of eq. (2.31), which underlies Figure 1. (Center). For a given value of $T_{RH}$ (the reheating temperature after $\phi$ decay) $A_x$ is associated to a specific temperature $T_x$ via eq. (2.18). Here we show $T_x$ as a function of $\Phi_I$ for $T_{RH} = 1\,\text{TeV}$. The black dashed curve indicates the maximum temperature $T_{Max} \sim 0.3 \times (M_{Pl}H/T_{RH}^{n/3})$. Changes in $T_{RH}$ simply scales the y-axis and the relative orientations of the lines are unchanged. (Right). We illustrate the temperature evolution for two cases with $n = -2$ and $n = -3$, taking $T_{RH} = 1\,\text{TeV}$ and $\Phi_I = 10^{10}\,\text{GeV}$, and we highlight where $T_{Max}$ and $T_x$ occur.

Evaluating this integral from $A_I = 1$ we find

$$\Phi = \begin{cases} 
\Phi_I \cdot \exp \left[ -\sqrt{\frac{\pi^2 g_*(T_{RH})}{30}} \frac{1}{2+n/2} (A^{2+n/2} - 1) \right] & \text{for } n \neq -4 \\
\Phi_I \cdot A^{-\sqrt{\pi^2 g_*(T_{RH})/30}} & \text{for } n = -4 
\end{cases} .$$  \tag{2.31}

Since the energy density in $\phi$ is falling quickly, whilst the radiation component grows gradually, at some point (which we denote $A_x$), the contributions from radiation and $\phi$ become comparable i.e. $\Phi(A_x) \simeq R(A_x)$. Shortly after $A_x$ the universe transitions to radiation domination and the expansion rate transitions to $H \propto T^2$. Importantly, for $A \gtrsim A_x$ then eq. (2.18) (and Figure 1) no longer well describe the evolution, since it is not reasonable to neglect $R$ in the derivation.\footnote{This approximation also breaks down if the value of $X$ grows too large, but since the growth of $X$ depends on the small free parameter $b/m_\phi$, we continue to neglect $X$ in deriving $A_x$.} To find $A_x$ we numerically solve the coupled differential equations eq. (2.11) & (2.12) with the initial conditions $R(a_I) = X(a_I) = 0$. In Figure 2 (left) we show the values of $A_x$ for different values of $n$ (i.e. initial expansion rates), and where $\Phi(a_I) \equiv \Phi_I$ is treated as a free parameter. Fitting to $A_x$ we find the form $A_x = c_n \Phi_I^{m_n}$ where $m_n$ and $c_n$ are constants, for instance, for $n = -1$ then $m_{-1} \approx 0.20$ and $c_{-1} \approx 0.68$.

By inspection of Figure 2 (left) it is seen that for $\Phi_I \gtrsim 10^8$ then eq. (2.31) is valid up to $A \gtrsim 10$, which is the range of Figure 1. The approximation remains good for lower $\Phi_I$ and higher $A$ for larger values of $n$. Moreover, since the expansion rate varies prior and after $A_{Max}$ (the point of transition from increasing to decreasing bath temperature), comparing with eq. (2.19) we note that $n \geq -3$ then $A_{Max} \leq 2$, and $A_{Max} \ll A_x$ for reasonable values of $\Phi_I$ and $n$. In particular, for $\Phi_I \gtrsim 1\,\text{TeV}$ (typically the range of interest) the bath evolves into the decreasing temperature regime prior to $A_x$.\footnote{Changes in $T_{RH}$ simply scales the y-axis and the relative orientations of the lines are unchanged. (Right). We illustrate the temperature evolution for two cases with $n = -2$ and $n = -3$, taking $T_{RH} = 1\,\text{TeV}$ and $\Phi_I = 10^{10}\,\text{GeV}$, and we highlight where $T_{Max}$ and $T_x$ occur.}
For a given $T_{RH}$ we can translate $A_{\times}$ into the corresponding temperature $T_{\times}$ at which the radiation and $\phi$ components become comparable, as shown in Figure 2 (center). The approximate value of $T_{\text{Max}} \simeq 0.3 \times (M_{\text{Pl}} H I T_{\text{RH}}^2)^{1/4}$ is shown as the black dashed line and the separation between $T_{\text{Max}}$ and $T_{\times}$ gives an indication of the length of the period for which the system is dominated by the decaying $\phi$ states. Figure 2 (right) illustrates the temperature evolution and the points at which $T_{\text{Max}}$ and $T_{\times}$ occur for two specific cases. Once the temperature drops below either $T_{\times}$ or $T_{RH}$ the system transitions to radiation domination with $H \propto T^2$. The scenario of interest here is the case in which the dark matter relic density is set prior to the onset of radiation domination, as we discuss in the next section.

3 Implications for Dark Matter

In the preceding section we studied the behavior of the temperature and the expansion rate for the case of a period in the early universe in which the expansion follows some general power law and while $\phi$ is decaying. We consider next the implications for dark matter, in particular, how the predicted dark matter relic density depends on the exponent $n$ in the initial expansion rate $H \propto T^{n/2+2}$. We will break the discussion into the following cases:

§ 3.1: Freeze-in: Thermal production without chemical equilibrium.

§ 3.2: Freeze-out during reheating: Thermal production without chemical equilibrium.

§ 3.3: Non-thermal production.

3.1 Freeze-in

First we assume that $X$ particles are always non-relativistic and do not reach chemical equilibrium at early time ($X \ll X_{\text{eq}}$). The case of thermal production of non-relativistic dark matter without reaching chemical equilibrium with the thermal radiation bath is an instance of the dark matter freeze-in scenario formulated more generally in [15] and developed in e.g. [16–21]. Thus we consider the evolution of $X$ following eq. (2.13), for now taking $b = 0$, in which case

$$X' = \sqrt{\frac{3}{8\pi}} \langle \sigma v \rangle M_{\text{Pl}} T_{\text{RH}} \Phi_{I}^{-1/2} A^{n/2-2} X_{\text{eq}}^2,$$

(3.1)
in terms of the equilibrium distribution given by

$$X_{\text{eq}} \equiv a^3 n_{X}^{\text{eq}} = \frac{A^3}{T_{\text{RH}}^3} g \left( \frac{M_X T}{2\pi} \right)^{3/2} e^{-\frac{M_X}{T}},$$

(3.2)

where $g$ is the number of internal degrees of freedom of the dark matter state. Using eq. (2.26) and substituting eq. (3.2) into eq. (3.1) we obtain

$$X' = (\langle \sigma v \rangle)^{2/3 M_X^2 \kappa^3 T_{\text{Max}}^3 / 8\pi^3 H I T_{\text{RH}}^3} A^{(20+n)/8} e^{-\frac{2M_X A^{(4+n)/8}}{\kappa T_{\text{Max}}}}.$$

(3.3)

Expressing the cross section in terms of the s and p-wave pieces $\langle \sigma v \rangle = \alpha_s/M_X^2 + \alpha_p T/M_X^2$, and integrating (neglecting the temperature dependence in $\kappa$, i.e. $\kappa = \kappa_{RH} \equiv \kappa(T_{RH})$) gives

$$X_{\infty} = 2^{-(n+28)/(n+4)} \frac{g^2 (\kappa_{RH} T_{\text{Max}})^{(4n+40)/(n+4)}}{n+4 \pi^3 H I T_{\text{RH}}^3 M_X^{24/(n+4)} \Gamma \left( \frac{n+28}{n+4} \right) \left( \alpha_s + \frac{n+4}{12} \alpha_p \right)}.$$

(3.4)
where here $\Gamma$ indicates the gamma function. Thus for $b \approx 0$ (more precisely provided that the first term of the Boltzmann equation for $X$, eq. (2.13), can be safely neglected) and assuming that the dark matter is non-relativistic and does not enter chemical equilibrium

$$\rho_X(T_{RH}) = M_X n_X(T_{RH}) = M_X X_\infty \frac{T_{RH}^3}{A_{RH}^3}. \quad (3.5)$$

Furthermore, it is known that at the point of reheating $H \approx \Gamma_\phi$ the energy density for radiation is

$$\rho_R(T_{RH}) = \frac{\pi^2 g_*(T_{RH}) T_{RH}^4}{30} \quad (3.6)$$

and thus comparing the ratio of energy densities now and at reheating we have

$$\frac{\rho_X(T_{now})}{\rho_R(T_{now})} = \frac{T_{RH} \rho_X(T_{RH})}{T_{now} \rho_R(T_{RH})} = \frac{M_X}{T_{now} A_{RH}^3 \pi^2 g_*(T_{RH}) X_\infty}. \quad (3.7)$$

Substituting eq. (3.4) it follows that

$$\frac{\rho_X(T_{now})}{\rho_R(T_{now})} = \frac{30 \times 2^{\frac{4n+28}{n+4}} M_X}{A_{RH}^3 \pi^5 g_*(T_{RH}) T_{now}(n+4)} g^2 \left(\frac{\kappa_{RH} T_{Max}}{H} \frac{4^{n+10}}{n+4} \Gamma \left(\frac{n+28}{n+4} \right) \left(\alpha_p + \frac{n+4}{12} \alpha_p \right) \right). \quad (3.8)$$

Using the form of $A_{RH}$ from eq. (2.26) we can re-express the dark matter relic abundance as

$$\frac{\Omega_X h^2}{\Omega_R h^2} = \frac{30 \times 2^{-\frac{n+28}{n+4}} g^2}{\pi^5(n+4) H I g_*(T_{RH}) T_{now}(n+4)} \Gamma \left(\frac{n+28}{n+4} \right) \left(\frac{\kappa_{RH} T_{Max}}{H} \frac{4^{n+10}}{n+4} \Gamma \left(\frac{n+28}{n+4} \right) \left(\alpha_p + \frac{n+4}{12} \alpha_p \right) \right), \quad (3.9)$$

in terms of the observed fractional energy densities $\Omega_{R,X}$ for radiation and dark matter. One could further rewrite eq. (3.9) in terms of $T_{RH}$ by substituting the form of $T_{Max}$ from eq. (2.20). For $n = -1$ this scenario is studied in ‘Case A’ of [5], and eq. (3.9) generalises this to other values of the exponent $n$, reproducing the earlier result for $n = -1$. Additionally note that the point of peak dark matter production $A_*$ can be found by looking at where the derivative in eq. (3.3) vanishes. For the s-wave case this occurs for $^2$

$$A_* = \left[\left(\frac{20+n}{4+n}\right) \frac{\kappa_{Max}}{2M_X} \right]^\frac{\pi}{4n+4} \simeq (\Phi_I)^\frac{1}{4n+4} \left[0.3 \cdot \frac{T_{RH}}{2M_X} \left(\frac{20+n}{4+n}\right) \right]^\frac{\pi}{4n+4}, \quad (3.10)$$

where in the final equation we have used eqns. (2.14) & (2.25). Note that $A_*$ depends strongly on $T_{RH}/M_X$ but is relatively insensitive to $\Phi_I$. In order for the relic density to be described by eq. (3.9), i.e. while $H \propto T^4$, it is required that $A_{Max} < A_* < A_X$.

Recall from Figure 2 (left) that $A_X \sim O(10)$, thus dark matter production occurs prior to the onset of radiation domination for $A_* < A_X \sim O(10)$. Typically it can be arranged that $A_* \sim O(10) < A_X$ by choosing an appropriate $\Phi_I$. In Figure 3 (left) we highlight parameter values in which $A_{Max} < A_* < A_X$ for the case of $n = -2$ as $H_I$ is varied. If $\Phi_I$ is taken to be relatively large then the dark matter mass should be fairly heavy to reproduce the observed relic density. Additionally, note that since trans-Planckian $\Phi_I$ are unreasonable this places an additional restriction on the parameter space.

$^2$Note taking $n = -1$ we find that the prefactor is $19/3$ which is slightly different from derived in [5] which give the prefactor as $17/2$. We believe the authors use different criteria and approximations.
Thus the calculation is largely unchanged from earlier studies, differing only in the factor \( \frac{T}{m} \) (matching the left panel), outside of this region the relic density curve is unreliable and we indicate this by dashing the line. Note that \( T_{RH} \sim 1 \text{ TeV} \) and \( H_I \sim 10^{-4} \text{ GeV} \) corresponds to \( \Phi_I \sim 10^{17} \text{ GeV} \).

In Figure 3 (right) we show the parameter region \( A_{Max} < A_s < A_X \) along with curve for which the observed dark matter relic density is reproduced (\( \Omega_X = \Omega_{Obs} \)) for a specific example taking \( n = -2 \) and \( H_I = 10^{-4} \text{ GeV} \) with couplings \( \alpha_s = 10^{-18} \) and \( \alpha_p = 0 \). We highlight that generally for the relic density curve to align with the region \( A \) diminutive couplings are needed \( \alpha \ll 1 \), however this is actually fortuitous since such feeble coupling strengths are required in freeze-in models in order to ensure that the dark matter remains out of equilibrium \cite{15} (as assumed for this case). In the next subsection we shall derive the condition on \( \alpha \) under which \( X < X_{eq} \) at all times.

### 3.2 Freeze-out during reheating

Next we consider the case in which the dark matter reaches chemical equilibrium and then freezes out while \( H \propto T^4 \). The point of freeze-out can be defined implicitly by

\[
\frac{n_X^{eq}(T_F)}{\langle \sigma v \rangle} = H(T_F).
\]

Using that \( n_X^{eq} = X^{eq} A^{-3} T_{RH}^3 \) and eq. (3.2) we can rewrite the left-hand side of the above equation in terms of \( T_F \) and for the right-hand side we substitute eq. (2.29) to obtain

\[
\frac{g}{\sqrt{8\pi}} \langle \sigma v \rangle (M_X T_F)^{3/2} \exp \left( \frac{-M_X}{T_F} \right) = \left| 4 - n \right| \left( \frac{\pi^3 g_s^2(T_F)}{45 g_s(T_{RH})} \right)^{1/2} \frac{T_F^4}{M_{Pl} T_{RH}^2}.
\]

Thus the calculation is largely unchanged from earlier studies, differing only in the factor \( \left| n - 4 \right| \), and the freeze-out temperature is analogous to as derived in \cite{5}, given by

\[
x_F = \ln \left( \frac{3gM_{Pl} T_{RH}^2 g_s(T_{RH})^{1/2}}{4 - n \sqrt{5 \cdot 8\pi^3 g_s(T_F) M_X^2}} \left( \alpha_s x_F^{5/2} + \alpha_p x_F^{3/2} \right) \right).
\]
Note that, as usual, because of the insensitivity of the logarithm dependences for a large range of reasonable parameter values \( x_F \sim \mathcal{O}(10) \). Comparing \( T_F \sim M_X/\mathcal{O}(10) \) to \( T_* \) in Figure 2 (centre) one finds that \( T_* \sim (\mathcal{O}(100) - \mathcal{O}(1000)) \) GeV therefore for \( M_X \gtrsim 1 \) TeV then typically \( T_F > T_* \). Thus for a large range of parameters dark matter freeze-out can occur well before the transition to radiation domination, while \( H \) is described by eq. (2.29).

The dark matter abundance remains constant after the point of reheating at \( H \simeq \Gamma_\phi \), but to ascertain the abundance of dark matter at reheating it is necessary to evolve the freeze-out abundance from \( T_F \) to \( T_{RH} \) as follows

\[
\rho_X(T_{RH}) = \left( \frac{a(T_{RH})}{a(T_F)} \right)^{-3} \rho_X(T_F) = \left( \frac{g_*(T_{RH})}{g_*(T_F)} \right)^2 \left( \frac{T_{RH}}{T_F} \right)^8 \rho_X(T_F) ,
\]

where we use that the ratio of FRW scale factors can be replaced by the ratio of dimensionless \( \Lambda \) factors. It follows that the dark matter relic density is given by

\[
\frac{\Omega_X h^2}{\Omega_R h^2} = |4 - n| \frac{5\sqrt{5}}{4\sqrt{\pi}} \frac{\sqrt{g_*(T_{RH})}}{g_*(T_F)} \frac{T_{RH}^3}{T_{now}^3 M_{Pl} \alpha_s x_F^{-4} + \alpha_p x_F^{-5}/5} .
\]

Notably, the abundance is essentially insensitive to the expansion rate prior to reheating in the case that dark matter freeze-out is non-relativistic and in thermal equilibrium.

Whether the relic density is set via freeze-out or freeze-in depends on if the dark matter enters equilibrium. Specifically, for \( X_\infty \lesssim X_{eq}(T_*) \), where \( T_* \) is the temperature of dominant particle production given by eq. (3.14), the dark matter will remain out of equilibrium at all times and the production rate sets the relic density (the freeze-in scenario). Thus there is a critical value of the coupling \( \alpha_s^{(crit)} \) above which the inequality \( X_\infty \lesssim X_{eq}(T_*) \) is violated. From eqns. (2.26) & (3.10) we have that \( A_s \) corresponds to a temperatures \( T_* \) which for s-wave is

\[
T_* \simeq M_X \left( \frac{8 + 2n}{20 + n} \right) .
\]

Applying the criteria that for \( \alpha = \alpha_s^{(crit)} \) then \( X_\infty = X_{eq}(T_*) \), it follows that for the case with \( \langle \sigma v \rangle \simeq \alpha_s/M_X^2 \) the critical coupling \( \alpha_s^{(crit)} \) is given by

\[
\alpha_s^{(crit)} = \frac{2\pi^3 M_X^3 (20 + n) \frac{3(12 - n)}{2n + 1} (4 + n) \frac{5n - 28}{2n + 1} |4 - n| g_*(T_*)}{\sqrt{15} g \Gamma \left( \frac{28 + n}{4 + n} \right) \epsilon \frac{2^{2n + 4}}{n!} M_{Pl} T_{RH}^2} .
\]

Thus for \( \alpha_s < \alpha_s^{(crit)} \) the relic density is set by freeze-in, as in Section 3.1, whereas for \( \alpha_s > \alpha_s^{(crit)} \) freeze-out dynamics determines the dark matter relic density.

### 3.3 Non-thermal production

The scenario of non-thermal production is important for \( b \neq 0 \), that is a significant (possibly dominant) population of dark matter is produced directly from \( \phi \) decays [6]. The case of non-thermal production without chemical equilibrium is described by eq. (2.13) with the second (\( b \)-independent) term neglected

\[
X' = \sqrt{\frac{\pi^2 g_*(T_{RH})}{30} \frac{b}{m_\phi} \frac{\Phi_{RH} \Lambda^{-n/2}}{\sqrt{\Phi + R \Lambda^n + \langle E_X \rangle X \Lambda^{n+1}}}} .
\]
Integrating (for $n \neq 2$) from $A_I$ to $A_{RH}$ and applying the boundary conditions of eq. (2.14) we find the total population of dark matter produced due to $\phi$ decays

$$X_{RH} \equiv X(T_{RH}) \simeq -\frac{2\eta}{n-2} \sqrt{\frac{\pi g_*(T_{RH}) H_I M_{Pl}}{80 T_{RH}}} \left( A_{RH}^{1-n/2} - 1 \right),$$

where we define $\eta \equiv b/m_\phi$ which parameterises the $\phi$-dark matter branching fraction. For $n = 2$ then rather $X(T_{RH}) \propto \ln(A_{RH})$, this case was studied in [14] and we will not discuss it further here. Using the above equation and eq. (3.5) & (3.6) it follows that

$$\frac{\rho_{X_b}(T_{now})}{\rho_R(T_{now})} \sim \frac{15 M_X H_I M_{Pl} \eta}{\sqrt{5}(2-n)\pi^{3/2}g_*(\sqrt{T_{RH}})T_{RH}^4 T_{now}} \left( A_{RH}^{2-n} - 1 \right) \frac{T_{RH}^3}{A_{RH}^3}. \tag{3.20}$$

Further, using eq. (2.26) to replace $A_{RH}$ this can be rewritten to obtain an expression for the dark matter relic abundance

$$\frac{\Omega_X}{\Omega_R} \sim \frac{15 M_X H_I M_{Pl} \eta}{\sqrt{5}(2-n)g_*(\sqrt{T_{RH}})\pi^{3/2}T_{RH}^3 T_{now}} \left[ \kappa T_{Max}^{\frac{4(2-n)}{n+4}} - 1 \right] \left( \frac{T_{RH}}{\kappa T_{Max}} \right)^{\frac{24}{n+4}}. \tag{3.21}$$

Similar to previously, the above result generalises expressions in Case 3 of [6] from $n = -1$ to general $n$.

Note that the preceding calculation assumes the ordering $A_{Max} < A_{RH} < A_X$ where $A_{RH} \equiv A(T_{RH})$. We can obtain an expression for $A_{RH}$ from eqns. (2.8) & (3.16) as follows

$$A_{RH} \sim (\sqrt{\Gamma_\phi M_{Pl}/\kappa T_{Max}})^{-8/(4+n)}.$$ Note that $A_{RH}$ depends on $\Gamma_\phi$ and since the other quantities do not depend on $\Gamma_\phi$ the above inequality involving $A_{RH}$ is typically not constraining. We also note here that Big Bang nucleosynthesis constraints imply a limit $T_{RH} \gtrsim 10\text{ MeV}$. 

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**Figure 4.** Plot shows lines in the $T_{RH}$-$M_X$ plane for which the dark matter relic density is reproduced via non-thermal production, following eq. (3.21). Taking three different exponents of the initial expansion rate $n = 1$ (dotted), $n = 0$ (dashed) and $n = -1$ (solid) and, parameterising the $\phi$-dark matter branching fraction in terms of $\eta \equiv b \cdot \text{GeV}/m_\phi$, we show three different values $\eta = 0.5, 10^{-4}, 10^{-7}$. The plot fixes the initial Hubble rate to be $H_I = 10^{-9}$. The right panel shows an enlargement of the dashed rectangle of the left panel and illustrates the difference between $n = 0, 1,$ and $-1$. 

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The explanation of Figure 4: The plot illustrates the dependence of the dark matter relic density on the initial Hubble rate $H_I$ and the mass $M_X$ of the dark matter candidate. The lines on the plot correspond to different values of the parameter $\eta$, which is related to the branching fraction $b$ and the mass $m_\phi$ of the particle responsible for dark matter. The initial Hubble rate is fixed at $H_I = 10^{-9}$, and three different exponents $n$ are considered: $n = 1$ (dotted lines), $n = 0$ (dashed lines), and $n = -1$ (solid lines). Each line represents a constraint on the parameter space $(M_X, T_{RH})$ for which the dark matter relic density is reproduced via non-thermal production. The plot shows that for certain values of $n$, the relic density is not constrained by Big Bang nucleosynthesis, as indicated by the limit $T_{RH} \gtrsim 10\text{ MeV}$. The dashed rectangle on the left panel highlights the region of parameter space where the dark matter relic density is dominated by non-thermal production, and the right panel provides a magnified view of this region, demonstrating the effect of varying $n$. 

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**Equations Reference:**

1. $X_{RH} \equiv X(T_{RH}) \simeq -\frac{2\eta}{n-2} \sqrt{\frac{\pi g_*(T_{RH}) H_I M_{Pl}}{80 T_{RH}}} \left( A_{RH}^{1-n/2} - 1 \right),$
2. $\rho_{X_b}(T_{now})/\rho_R(T_{now}) \sim \frac{15 M_X H_I M_{Pl} \eta}{\sqrt{5}(2-n)\pi^{3/2}g_*(\sqrt{T_{RH}})T_{RH}^4 T_{now}} \left( A_{RH}^{2-n} - 1 \right) \frac{T_{RH}^3}{A_{RH}^3},$
3. $\frac{\Omega_X}{\Omega_R} \sim \frac{15 M_X H_I M_{Pl} \eta}{\sqrt{5}(2-n)g_*(\sqrt{T_{RH}})\pi^{3/2}T_{RH}^3 T_{now}} \left[ \kappa T_{Max}^{\frac{4(2-n)}{n+4}} - 1 \right] \left( \frac{T_{RH}}{\kappa T_{Max}} \right)^{\frac{24}{n+4}}.$
In Figure 4 we illustrate some example parameter ranges which reproduce the observed dark matter relic density for the case of non-thermal production without chemical equilibrium with \( n = 0, 1, -1 \). In particular, we highlight that changes in \( n \) has a modest impact on the appropriate \( T_{\text{RH}} \) required to reproduce the relic density, however there is a great degree of freedom in \( \eta \equiv b/m_\phi \) which can lead to substantially larger impact on the required value of \( T_{\text{RH}} \) needed to reproduce the observed dark matter relic density.

Note that if the branching fraction of \( \phi \) decays, controlled by \( b \) is sufficiently large, the dark matter will enter equilibrium in which case the contribution from non-thermal production is reduced due to dark matter annihilations. The production of dark matter due to \( \phi \) decays can maintain the dark matter at an equilibrium abundance past \( T \sim M_X \) and dark matter only freezes out at \( T \sim T_{\text{RH}} \), when dark matter production ceases. Thus the period of freeze-out is during the era of radiation domination. As argued in [6] (Case 4), this leads to a scaling of the radiation dominated relic density \( \Omega_{\text{RD}} \) due to the difference in the entropy density between \( T_{\text{RH}} \) and the radiation domination freeze-out temperature \( T_{\text{FO}} \). Specifically, the dark matter relic abundance will be \( \Omega_X \sim (T_{\text{FO}}/T_{\text{RH}})^2 \Omega_{\text{RD}} \). Since the freeze-out occurs during radiation domination, the relic abundance will be largely insensitive to the temperature dependence of the initial expansion rate.

4 Concluding Remarks

A myriad of scenarios exists in which the early universe is not immediately radiation dominated but goes through periods with expansion rates different to the commonly assumed relationship \( H \propto T^2 \). In this work we have focused on a previously unstudied case in which the early universe is dominated by some state \( \phi \) which leads to a general expansion rate of the form \( H \sim T^{2+n/2} \), but due to the fact that \( \phi \) is decaying there is a subsequent transition to \( H \sim T^4 \). Notably, the form of the initial expansion rate leaves a lasting imprint on relic densities established while \( H \propto T^4 \), because the value of the exponent \( n \) (for \( H \sim T^{2+n/2} \)) changes the temperature evolution of the Standard Model thermal bath. While freeze-out during reheating (§3.2) is largely insensitive to the initial expansion rate, the abundances of dark matter produced via freeze-in (§3.1) or non-thermal production without equilibrium (§3.1) are sensitive to the value of the exponent \( n \).

The prospect of the dark matter relic abundance being established during a period of entropy injection following an early matter dominated era was originally studied in influential papers of Giudice, Kolb & Riotto [5] and Gelmini and Gondolo [6]. This was later adapted to the specific case of a period of decays following kination domination by Visinelli [14]. In this work we have further generalised to the case in which the initial epoch has a general expansion rate of the form \( H \sim T^{2+n/2} \). We have highlighted how the choice of \( n \) propagates into the cosmology of the era of significant entropy injection from \( \phi \) decays and into calculations of the dark matter relic density. Reassuringly, for \( n = -1 \) we reproduce the expressions of [5, 6] and for \( n = 2 \) we reproduce the results of [14].

Before closing it is worth highlighting that an explicit example of scenarios with expansion rates of the form \( H \sim T^{2+n/2} \) was constructed in [7], in which the energy density of the universe is dominated by the contributions from a real scalar field with a potential

\[
V(\phi) \propto \frac{n-2}{(n+4)^2} \exp(-\phi \sqrt{n+4}) \ .
\]  

Furthermore, a range of expansion rates of the form \( H \sim T^{2+n/2} \) can arise from scalars with periodic potentials [22, 23].
We have explored here the implications for dark matter in the case that a population \( \phi \) sources an initial expansion rate of the form \( H \propto T^{2+n/2} \) and assuming the dark matter relic density is established while \( \phi \) is decaying. Interestingly, the temperature dependance of the initial expansion rate can significantly impact the form of the dark matter relic density. Since such variant cosmologies can alter the predicted dark matter relic density they have previously been used to adjust the freeze-out abundance or evade experimental constraints, see e.g. \[6, 11, 12, 24–31\]. It would be interesting to examine how these specific particle physics models (such as the bino, neutralino, and Higgs portal) vary in the context of the generalise scenario outlined here. Moreover, in future work we plan to explore to what extent the temperature dependance of the early expansion rate imprints on cosmological parameters and observables, such as deviations in the matter power spectrum \[32–34\], and whether there is a degeneracy between the initial temperature dependence of \( H \) (the value of \( n \)) and the magnitude of the initial expansion rate \( H_I \).

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