A study of $O(1/m_Q^2)$ corrections for $f_B$ with lattice NRQCD

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We investigate higher order effects in the nonrelativistic expansion of lattice QCD on the heavy-light meson decay constants and some other quantities in order to understand the truncation error of NRQCD. While our numerical results have large $O(a)$ and $O(\alpha_s)$ errors due to the use of Wilson light quark action and the tree-level matching, we find that the truncation error of higher order relativistic corrections are adequately small around the mass of the $b$ quark. We also present a perturbatively matched results through 1-loop level without operator mixing effects.

1. Introduction

It is indispensable for the verification of the Standard Model and the new physics search that weak matrix elements between hadrons involving $b$-quark are calculated with high accuracy. Here, we focus on the heavy-light meson decay constants $f_B$, whose accurate value is needed for the determination of $|V_{ub}|V_{cb}|$ together with $B_B$ parameter. The lattice NRQCD $^3$ enable us to do the direct simulations at the $b$ quark and has yielded remarkable progress on the heavy quark physics. So far the studies including $1/m_Q$ corrections have made it clear that there exists a large $1/m_Q$ correction to the value in the static limit $^3$. In this talk we report on the systematic study of the $1/m_Q^2$ corrections. A preliminary report of an investigation of $O(1/m_Q^2)$ corrections similar to our work has been reported in Ref.$^4$. The action and operators should be matched to the full theory since NRQCD is an effective theory. Recently we have completed the self-energy renormalization constants and the multiplicative part in the renormalization of axial vector current $^5$. In this article we present the results of $f_B$ and some other quantities with and without 1-loop matching.

2. Simulations

Our numerical simulation is carried out with 120 quenched configurations on a $16^3 \times 32$ lattice at $\beta = 5.8$. For the tadpole factor we employ $u_0 = (P_{\text{plaq}})^{1/4}$ with $P_{\text{plaq}}$ the average plaquette, which takes the value $u_0 = 0.867994(13)$ measured during our configuration generation.

For the light quark we use the Wilson quark action with $\kappa = 0.1570, 0.1585$ and $0.1600$. The chiral limit is reached at $\kappa_c = 0.16346(7)$ and the inverse lattice spacing determined from the rho meson mass equals $a^{-1} = 1.714(63)$ GeV. The hopping parameter $\kappa_s$ corresponding to the strange quark is determined from $m_{\phi}/m_{\rho}$ and $m_{K}/m_{\rho}$, which yields $\kappa_s = 0.15922(39)$ and $0.16016(23)$, respectively. We use the factor $\sqrt{1 - \frac{3\mu}{4\kappa_s}}$ as the field normalization for light quark $^3$.

For the heavy quark, we use the lattice NRQCD with the following evolution equation:

$$G_Q(t \geq 0, \vec{x}) = \left(1 - \frac{aH_0}{2n}\right)^n \left(1 - \frac{a\delta}{2}\right) U_4$$

$$\times \left(1 - \frac{a\delta H_2}{2}\right) \left(1 - \frac{aH_0}{2n}\right)^n G_Q(t - 1, \vec{x}) + \delta_{x,0}.$$

Where $H_0$ is defined as follows:

$$H_0 = -\frac{\Delta^{(2)}}{2m_Q},$$

and in order to realize two accuracies we define
two $\delta H$’s as follows:

$$\delta H_1 = -\frac{g \vec{\sigma} \cdot \vec{B}}{2 m_Q}, \quad (3)$$
$$\delta H_2 = -\frac{g \vec{\sigma} \cdot \vec{B}}{2 m_Q} + \frac{ig}{8 m_Q^2} (\vec{\Delta} \cdot \vec{E} - \vec{E} \cdot \vec{\Delta}) + \frac{g}{8 m_Q^2} \vec{\sigma} \cdot (\vec{\Delta} \times \vec{E} - \vec{E} \times \vec{\Delta}) - \frac{(\Delta^{(2)})^2}{8 m_Q^3} + \frac{a^2 \Delta^{(4)}}{24 m_Q} - \frac{a (\Delta^{(2)})^2}{16 m_Q^2}. \quad (4)$$

The symbols $\vec{\Delta}$ and $\Delta^{(2)}$ denote the symmetric lattice differentiation in spatial directions and Laplacian, respectively, and $\Delta^{(4)} \equiv \sum_i (\Delta_i^{(2)})^2$. The field strengths $\vec{B}$ and $\vec{E}$ are generated from the standard clover-leaf operator.

The original 4-component heavy quark spinor $h$ is related to two 2-component spinors $Q$ and $\chi$ through Foldy-Wouthuysen-Tani (FWT) transformation,

$$h(x) = R \left( \begin{array}{c} Q(x) \\ \chi^+(x) \end{array} \right), \quad (5)$$

where $R$ is an inverse FWT transformation matrix which has $4 \times 4$ spin and $3 \times 3$ color indices.

We prepare two $R$’s as before. After discretization, $R$ at the tree level is written as follows:

$$R_1 = 1 - \frac{\vec{\gamma} \cdot \vec{\Delta}}{2 m_Q}, \quad (6)$$
$$R_2 = 1 - \frac{\vec{\gamma} \cdot \vec{\Delta}}{2 m_Q} + \frac{\Delta^{(2)}}{8 m_Q^2} + \frac{g \vec{\Sigma} \cdot \vec{B}}{8 m_Q^2} - \frac{ig \gamma_4 \vec{\gamma} \cdot \vec{E}}{4 m_Q^2}. \quad (7)$$

where $\vec{\Sigma} \equiv \text{diag.}\{\sigma^1, \sigma^2, \sigma^3\}$. We apply the tadpole improvement[4] to all link variables in the evolution equation and $R$ by rescaling the link variables as $U_\mu \rightarrow U_\mu/a_0$.

We performed two simulations using following two sets:

set $I \equiv \{ \delta H_1, R_1 \}$ and set $II \equiv \{ \delta H_2, R_2 \}. \quad (8)$

set I and II are fully consistent through $O(1/m_Q)$ and $O(1/m_Q^3)$, respectively. Furthermore $\delta H_2$ has the leading relativistic correction to the dispersion relation, which is an $O(1/m_Q^3)$ term, and the terms improving the discretization errors appearing in $H_0$ and time evolution are also included.

Figure 1. $1/M_P$ dependence of $f_P M_P^{1/2}$ with set I(open symbols) and set II(solid symbols). Circles are the results with tree-level matching and squares and triangles are with 1-loop matching on the scale of $q^2=\pi/a$ and $1/a$, respectively.

For the heavy quark mass and the stabilizing parameter, we use $(am_Q, m) = (10.0, 1), (5.0, 2), (2.6, 2), (2.1, 3), (1.5, 3), (1.2, 3)$ and $(0.9, 4)$, which cover a mass range between $4m_b$ and $m_c$.

Our strategy is to study systematically the effects of an inclusion of $O(1/m_Q^2)$ terms by comparing two results from these simulations. All of our errors are estimated by a single elimination jack-knife procedure.

3. Results

3.1. decay constants

Fig2 shows the $1/M_P$ dependence of $f_P \sqrt{M_P}$. Where $M_P$ is a pseudoscalar heavy-light meson mass. Two vertical lines represent the mass regions corresponding to the B and D meson. Circles(open for set I and filled for set II) are the results with tree-level matching. From more quantitative study, we can find that within tree-level analysis the relative magnitude of $(1/m_Q^2)$ correction is about 3%, conservatively 6%, around the B meson region[6].

3.2. other quantities

We also performed a similar investigation on $M_{P_0} - M_P$ and $f_{P_s}/f_P$, where $P_s$ denotes a pseudo-scalar heavy-strange meson. Those results are shown in Fig3 and 4. One can see from these figures that there is no significant difference between the results from the two sets over almost
all mass region.

3.3. perturbative matching

Recently we have completed the self-energy renormalization constants and the multiplicative part in the renormalization of axial vector current with set I\(^5\). Since the scale \( q^* \) where the strong coupling constant should be estimated is still undetermined, we estimate the matching factor at \( q^* = 1/a \) and \( \pi/a \) and show the results with the factor in Fig. 2.

4. Summary

We presented the effects of \( O(1/m_Q^2) \) correction to the heavy-light meson decay constant with tree-level matched lattice NRQCD and Wilson quark action in a quenched approximation. While the \( O(1/m_Q) \) correction to the decay constant in the static limit is significant\(^3\), we find in our systematic study that the \( O(1/m_Q^2) \) correction is sufficiently small for B meson, so that there will be no need for incorporating \( O(1/m_Q^3) \) corrections unless an accuracy of better than 5% is sought for. Our examination of other physical quantities in the same respect also provides encouraging support to this statement. We have thus shown using our highly improved lattice NRQCD that the relativistic error, which has been one of the largest uncertainties in lattice calculations of the B meson decay constant, is well under control.

Numerical calculations have been done on Paragon XP/S at INSAM (Institute for Numerical Simulations and Applied Mathematics) in Hiroshima University.

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