Two-dimensional function photonic crystals

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In this paper, we have firstly proposed two-dimensional function photonic crystals, which the dielectric constants of medium columns are the functions of space coordinates $\vec{r}$, it is different from the two-dimensional conventional photonic crystals constituting by the medium columns of dielectric constants are constants. We find the band gaps of two-dimensional function photonic crystals are different from the two-dimensional conventional photonic crystals, and when the functions form of dielectric constants are different, the band gaps structure should be changed, which can be designed into the appropriate band gaps structures by the two-dimensional function photonic crystals.

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1. Introduction

Photonic crystals (PCs) have generated a surge of interest in the last decades because they offer the possibility to control the propagation of light to an unprecedented level [1-4]. In its simplest form, a photonic crystal is an engineered inhomogeneous periodic structure made of two or more materials with very different dielectric constants. PCs important characteristics are: photon band gap, defect states, light localization and so on. These characteristics make it able to control photons, so it may be used to manufacture some high performance devices which have completely new principles or can not be manufactured before, such as high-efficiency semiconductor lasers, right emitting diodes, wave guides, optical filters, high-Q resonators, antennas, frequency-selective surface, optical wave guides and sharp bends [5, 6], WDM-devices [7, 8], splitters and combiners [9, 10], optical limiters and amplifiers [11, 12]. The research on photonic crystals will promote its application and development on integrated photoelectron devices and optical communication.

It is matter of general knowledge that by now the PCs can constructed by varying photonic forbidden band (FB) of a structure in one-, two- and three-dimensional photonic crystals [13-15]. The optical and electronic properties of two- and three-dimensional are intensively studied with the goal of achieving control of electromagnetic propogation, and, especially, a complete photonic band gap in all directions [16-18].

Owing to the PCs periodicity, the plane-wave expansion (PWE) method is conventional for calculating PCs modes and photonic band structures. Its essence is the Fourier expansion of electromagnetic fields and material parameters, which is not only a counterpart of the PWE method for electronic crystals, but it also advantageously follows the classical coupled-wave theory developed for diffraction gratings [19-21].

In Refs. [22-28], we have proposed one-dimensional function photonic crystals, which is constituted by two media $A$ and $B$, their refractive indexes are the functions of space position. Unlike conventional photonic crystal (PCs), which is constituted by the constant refractive index media $A$ and $B$. We have studied the transmissivity and the electric field distribution with and without defect layer. In this paper, we have firstly proposed two-dimensional function photonic photonic crystals, which the dielectric constants of medium columns are the functions of space coordinates $\vec{r}$, it is different from the two-dimensional conventional photonic crystals constituting by the medium columns of dielectric constants are constants. We find the band gaps of two-dimensional function photonic photonic crystals are different from the two-dimensional conventional photonic crystals, and when the functions form of dielectric constants are different, the band gaps structure should be changed, which can be designed into the appropriate band gaps structures by the two-dimensional function photonic crystals.

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2. The plane-wave expansion method of two-dimensional photonic crystals

When the medium constituting photonic crystals is passive, the Maxwell equations are

\[ \nabla \cdot \vec{D} = 0, \]

\[ \nabla \cdot \vec{B} = 0, \]

\[ \nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}, \]

\[ \nabla \times \vec{H} = \varepsilon_0 \varepsilon(\vec{r}) \frac{\partial \vec{E}}{\partial t}, \]

where \( \varepsilon(\vec{r}) \) is the position-dependent dielectric constant.

For the monochromatic plane electromagnetic wave, the electric field and magnetic field intensity are

\[ \vec{H}(\vec{r}, t) = \vec{H}(\vec{r}) e^{-i\omega t}, \]

\[ \vec{E}(\vec{r}, t) = \vec{E}(\vec{r}) e^{-i\omega t}. \]

For the two-dimensional photonic crystals, the medium columns of dielectric constant \( \varepsilon_a(\vec{r}) \) are located in the background of dielectric constant \( \varepsilon_b \), and the medium columns are parallel the z-axis, the cross section form two-dimensional grids. In the following, we should give the eigenvalue equations of \( TM \) and \( TE \) wave.

(1) For the \( TM \) wave, the electric field and magnetic field intensity are

\[ \vec{H}(\vec{r}, t) = (0, 0, H_z(\vec{r})) e^{-i\omega t}, \]

\[ \vec{E}(\vec{r}, t) = (E_x(\vec{r}), E_y(\vec{r}), 0) e^{-i\omega t}, \]

where \( \vec{r} = x\hat{i} + y\hat{j} \).

Substituting Eqs. (7) and (8) into (3) and (4), we obtain

\[ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega \mu_0 H_z, \]

\[ \frac{\partial H_z}{\partial x} = i\omega \varepsilon_0 \varepsilon(\vec{r}) E_y, \]

\[ \frac{\partial H_z}{\partial y} = -i\omega \varepsilon_0 \varepsilon(\vec{r}) E_x. \]

In Eqs. (9)-(11), we take out \( E_x \) and \( E_y \), and obtain the \( H_z \) equation

\[ \frac{\partial}{\partial x} \left[ \frac{1}{\varepsilon(\vec{r})} \frac{\partial H_z(\vec{r})}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{\varepsilon(\vec{r})} \frac{\partial H_z(\vec{r})}{\partial y} \right] = \frac{\omega^2}{c^2} H_z(\vec{r}). \]

In the periodic dielectric of two-dimensional photonic crystals, the dielectric constant \( \varepsilon(\vec{r}) \) satisfy

\[ \varepsilon(\vec{r} + \vec{R}) = \varepsilon(\vec{r}), \]
where \( \vec{R} = l_1 \vec{a}_1 + l_2 \vec{a}_2 \) is the lattice vector, the vectors \( \vec{a}_1 \) and \( \vec{a}_2 \) are the basis vector of cell, and \( l_1 \) and \( l_2 \) are arbitrary integers. We introduce the cell basis vectors \( \vec{b}_1 \) and \( \vec{b}_2 \) of reciprocal space, and reciprocal lattice vector \( \vec{G} \), they are
\[
\vec{a}_i \cdot \vec{b}_j = 2\pi \delta_{ij} \quad (i, j = 1, 2),
\]
\[
\vec{G} = m_1 \vec{b}_1 + m_2 \vec{b}_2 \quad (m_1, m_2 = 0, \pm 1, \pm 2, \cdots).
\]

In the periodic medium, magnetic field intensity \( H_z(\vec{r}) \) is periodic distribution and satisfy the Block law
\[
\left\{
\begin{array}{l}
H_z(\vec{r}) = H_k(\vec{r})e^{i\vec{k} \cdot \vec{r}} \\
H_k(\vec{r} + \vec{R}) = H_k(\vec{r}),
\end{array}
\right.
\]
where \( k \) is the Block wave vector.

We can spread \( H_z(\vec{r}) \) and \( \frac{1}{\varepsilon(\vec{r})} \) as the Fourier series in the reciprocal space, they are
\[
H_k(\vec{r}) = \sum_{\vec{G}} H_k(\vec{G})e^{i\vec{G} \cdot \vec{r}},
\]
\[
\frac{1}{\varepsilon(\vec{r})} = \sum_{\vec{G}'} \varepsilon^{-1}(\vec{G}')e^{i\vec{G}' \cdot \vec{r}},
\]
substituting Eq. (17) into (16), we have
\[
H_z(\vec{r}) = \sum_{\vec{G}} H_k(\vec{G})e^{i(\vec{k} + \vec{G}) \cdot \vec{r}},
\]
substituting Eqs. (18) and (19) into (12), we get
\[
\frac{\partial}{\partial x} \left[ \sum_{\vec{G}'} \varepsilon^{-1}(\vec{G}')e^{i\vec{G}' \cdot \vec{r}} \sum_{\vec{G}''} i(k_x + G_x') H_k(\vec{G}')e^{i(\vec{k} + \vec{G}') \cdot \vec{r}} \right]
+ \frac{\partial}{\partial y} \left[ \sum_{\vec{G}'} \varepsilon^{-1}(\vec{G}')e^{i\vec{G}' \cdot \vec{r}} \sum_{\vec{G}''} i(k_y + G_y') H_k(\vec{G}')e^{i(\vec{k} + \vec{G}') \cdot \vec{r}} \right]
= -\frac{\omega^2}{c^2} \sum_{\vec{G}} H_k(\vec{G})e^{i(\vec{k} + \vec{G}) \cdot \vec{r}},
\]
i.e.,
\[
\sum_{\vec{G}', \vec{G}''} (k_x + G_x')(k_x + G_x' + G_x'') \varepsilon^{-1}(\vec{G}')e^{i(\vec{k} + \vec{G}' + \vec{G}'') \cdot \vec{r}}
+ \sum_{\vec{G}', \vec{G}''} (k_y + G_y')(k_y + G_y' + G_y'') \varepsilon^{-1}(\vec{G}')e^{i(\vec{k} + \vec{G}' + \vec{G}'') \cdot \vec{r}}
= \frac{\omega^2}{c^2} \sum_{\vec{G}} H_k(\vec{G})e^{i(\vec{k} + \vec{G}) \cdot \vec{r}},
\]
taking $\vec{G'} + \vec{G''} = \vec{G}$, the Eq. (21) can be written as
\[
\sum_{\vec{G}, \vec{G'}} (k_x + G'_x)(k_x + G_x)e^{-1}(\vec{G} - \vec{G'})H_\vec{k}(\vec{G'})e^{i(\vec{k} + \vec{G}) \cdot \vec{r}}
\]
\[
+ \sum_{\vec{G}, \vec{G'}} (k_y + G'_y)(k_y + G_y)e^{-1}(\vec{G} - \vec{G'})H_\vec{k}(\vec{G'})e^{i(\vec{k} + \vec{G}) \cdot \vec{r}}
\]
\[
= \frac{\omega^2}{c^2} \sum_{\vec{G}} H_\vec{k}(\vec{G})e^{i(\vec{k} + \vec{G}) \cdot \vec{r}},
\]  
(22)
since $e^{i(\vec{k} + \vec{G}) \cdot \vec{r}}$ is independent for every $\vec{G}$, the both sides coefficients of Eq. (22) are equal for different $\vec{G}$, there is
\[
\sum_{\vec{G'}} ((k_x + G'_x)(k_x + G_x) + (k_y + G'_y)(k_y + G_y))e^{-1}(\vec{G} - \vec{G'})H_\vec{k}(\vec{G'}) = \frac{\omega^2}{c^2} H_\vec{k}(\vec{G}),
\]  
(23)
i.e.,
\[
\sum_{\vec{G'}} (\vec{k} + \vec{G'}) \cdot (\vec{k} + \vec{G'}) e^{-1}(\vec{G} - \vec{G'})H_\vec{k}(\vec{G'}) = \frac{\omega^2}{c^2} H_\vec{k}(\vec{G}).
\]  
(24)
The Eq. (24) is the eigenvalue equation of $T M$ wave.

(1) For the $T E$ wave, the electric field and magnetic field intensity are
\[
\vec{E}(\vec{r}, t) = (0, 0, E_z(\vec{r}))e^{-i\omega t},
\]  
(25)
\[
\vec{H}(\vec{r}, t) = (H_x(\vec{r}), H_y(\vec{r}), 0)e^{-i\omega t},
\]  
(26)
substituting Eqs. (25) and (26) into (3) and (4), we obtain
\[
\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = -i\omega \varepsilon_0 \varepsilon(\vec{r}) E_z,
\]  
(27)
\[
\frac{\partial H_z}{\partial x} = -i\omega \mu_0 H_y,
\]  
(28)
\[
\frac{\partial E_z}{\partial y} = -i\omega \mu_0 H_x.
\]  
(29)
In Eqs. (27)-(29), we take out $H_x$ and $H_y$, and obtain the $E_z$ equation
\[
\frac{1}{\varepsilon(\vec{r})} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} E_z(\vec{r}) = -\frac{\omega^2}{c^2} H_z(\vec{r}),
\]  
(30)
similarly Eq. (19), $E_z(\vec{r})$ can be written as
\[
E_z(\vec{r}) = \sum_{\vec{G}} E_k(\vec{G}) e^{i(\vec{k} + \vec{G}) \cdot \vec{r}}.
\]  
(31)
By the the same steps of Eqs. (18)-(22), we can obtain the eigenvalue equation of $T E$ wave
\[
\sum_{\vec{G'}} |\vec{k} + \vec{G'}| |\vec{k} + \vec{G}| \varepsilon^{-1}(\vec{G} - \vec{G'}) E_k(\vec{G'}) = \frac{\omega^2}{c^2} E_k(\vec{G}).
\]  
(32)
The Eqs. (24) and (32) are suited to both two-dimensional conventional photonic crystals and two-dimensional function photonic crystals.

3. The Fourier transform of dielectric constant for two-dimensional function photonic crystals

For the two-dimensional function photonic crystals, the medium column dielectric constants are the function of space coordinates $\vec{r}$, it is different from the two-dimensional conventional photonic crystals, which dielectric constants are constant. The dielectric constant of cylindrical medium column can be written as

$$\varepsilon(\vec{r}) = \begin{cases} \varepsilon_a(\vec{r}) & r \leq r_a \\ \varepsilon_b & r > r_a \end{cases},$$

or

$$\frac{1}{\varepsilon(\vec{r})} = \begin{cases} \frac{1}{\varepsilon_a(\vec{r})} & r \leq r_a \\ \frac{1}{\varepsilon_b} & r > r_a \end{cases}.$$  \hspace{1cm} (34)

Eq. (34) can be written as

$$\frac{1}{\varepsilon(\vec{r})} = \frac{1}{\varepsilon_b} + \left( \frac{1}{\varepsilon_a(\vec{r})} - \frac{1}{\varepsilon_b} \right) s(r)$$  \hspace{1cm} (35)

where

$$s(r) = \begin{cases} 1 & r \leq r_a \\ 0 & r > r_a \end{cases}.$$  \hspace{1cm} (36)

The Fourier inverse transform of $\frac{1}{\varepsilon(\vec{r})}$ is

$$\varepsilon^{-1}(\vec{G}) = \frac{1}{V_0} \int_{V_0} d\vec{r} \frac{1}{\varepsilon(\vec{r})} e^{-i\vec{G}\cdot\vec{r}},$$

in the two-dimensional reciprocal space, it is

$$\varepsilon^{-1}(\vec{G}_||) = \frac{1}{V_0^{(2)}} \int_{V_0^{(2)}} d\vec{r}_|| \frac{1}{\varepsilon(\vec{r}_||)} e^{-i\vec{G}_||\cdot\vec{r}_||},$$

where $\vec{G}_|| = m_1 \vec{b}_1 + m_2 \vec{b}_2$, $\vec{r}_|| = x\vec{i} + y\vec{j}$, $V_0^{(2)}$ represents the unit cell area in the two dimensional lattice space.

Substituting Eq. (35) into (38), there is

$$\varepsilon^{-1}(\vec{G}_||) = \frac{1}{V_0^{(2)}} \int_{V_0^{(2)}} d\vec{r}_|| \frac{1}{\varepsilon_b} + \left( \frac{1}{\varepsilon_a(\vec{r}_||)} - \frac{1}{\varepsilon_b} \right) s(\vec{r}_||) e^{-i\vec{G}_||\cdot\vec{r}_||}$$

$$= \frac{1}{\varepsilon_b} \frac{1}{V_0^{(2)}} \delta_{\vec{G}_||,0} + \frac{1}{V_0^{(2)}} \int_{V_0^{(2)}} d\vec{r}_|| \left( \frac{1}{\varepsilon_a(\vec{r}_||)} - \frac{1}{\varepsilon_b} \right) s(\vec{r}_||) e^{-i\vec{G}_||\cdot\vec{r}_||}$$

$$= \frac{1}{\varepsilon_b} \delta_{m,0} \delta_{n,0} + \frac{1}{V_0^{(2)}} \int_{V_0^{(2)}} d\vec{r}_|| \left( \frac{1}{\varepsilon_a(\vec{r}_||)} - \frac{1}{\varepsilon_b} \right) s(\vec{r}_||) e^{-i\vec{G}_||\cdot\vec{r}_||}$$

$$= \frac{1}{\varepsilon_b} \delta_{m,0} \delta_{n,0} + I,$$  \hspace{1cm} (39)

where
\[ I = \frac{1}{V_0^{(2)}} \int_{V_0^{(2)}} \frac{d\vec{r}}{V_0^{(2)}} \frac{1}{\varepsilon_a} \frac{1}{\varepsilon_b} s(\vec{r}) e^{-i\vec{G}_|| \cdot \vec{r}} \]
\[ = \frac{1}{V_0^{(2)}} \int_{V_0^{(2)}} d\vec{r} \frac{1}{\varepsilon_a} \frac{1}{\varepsilon_b} s(\vec{r}) e^{-i\vec{G}_|| \cdot \vec{r}} - \frac{1}{V_0^{(2)}} \int_{V_0^{(2)}} d\vec{r} \frac{1}{\varepsilon_a} \frac{1}{\varepsilon_b} s(\vec{r}) e^{-i\vec{G}_|| \cdot \vec{r}} \]
\[ = I_1 - I_2 \] (40)

where

\[ I_2 = \frac{1}{V_0^{(2)}} \int_{V_0^{(2)}} d\vec{r} \frac{1}{\varepsilon_b} s(\vec{r}) e^{-i\vec{G}_|| \cdot \vec{r}} \]
\[ = \frac{1}{\varepsilon_b V_0^{(2)}} \int_{V_0^{(2)}} d\vec{r} s(\vec{r}) e^{-i\vec{G}_|| \cdot \vec{r}} \]
\[ = \frac{1}{\varepsilon_b V_0^{(2)}} \int_0^{r_a} rdr \int_0^{2\pi} d\theta e^{-i\vec{G}_|| \cdot r \cdot \cos \theta} \]
\[ = \frac{1}{\varepsilon_b V_0^{(2)}} \int_0^{r_a} rdr \int_0^{2\pi} d\theta e^{i\vec{G}_|| \cdot r \cdot \sin (\theta - \frac{\pi}{2})} \] (41)

where \(|\vec{r}_|| = r|| = r||, |\vec{G}|| = G||, d\vec{r}|| = ds = rdrd\theta\), and \(\theta\) is the included angle of \(\vec{r}||\) and \(\vec{G}||\).

By the formulas
\[ e^{i\omega \cdot \sin \theta} = \sum_{l=-\infty}^{\infty} J_l(\omega) e^{il\theta}, \] (42)
\[ \int_0^{2\pi} d\theta e^{il(\theta - \frac{\pi}{2})} = \begin{cases} 0 & (l \neq 0) \\ 2\pi & (l = 0) \end{cases} \] (43)

and
\[ \int x^m J_{m-1}(x) dx = x^m J_m(x) + c, \] (44)

we have

\[ I_2 = \frac{1}{\varepsilon_b V_0^{(2)}} \int_0^{r_a} rdr \sum_{l=-\infty}^{\infty} J_l(G|| \cdot r) \int_0^{2\pi} d\theta e^{i(l\theta - \frac{\pi}{2})} \]
\[ = \frac{1}{\varepsilon_b V_0^{(2)}} \frac{2\pi r_a}{G||} \cdot J_1(G|| \cdot r_a), \quad (G|| \neq 0) \] (45)
FIG. 1: The band gaps structure of two-dimensional conventional photonic crystals for the triangle lattice, $\varepsilon_b = 1$, $\varepsilon_a = 11.96$, medium column radius $r_a = 0.4a$. (a) TE wave, (b) TM wave.

when $G_{||} \to 0$ ($m \to 0$, $n \to 0$), we have

$$I_2(m = 0, n = 0) = \lim_{G_{||} \to 0} \frac{1}{\varepsilon_b} \frac{1}{V_0^{(2)}} \frac{2\pi r_a}{G_{||}} J_1(G_{||} \cdot r_a)$$

$$= \frac{1}{\varepsilon_b} \frac{1}{V_0^{(2)}} \frac{2\pi r_a}{G_{||}} \lim_{G_{||} \to 0} \frac{J_1(G_{||} \cdot r_a))'}{(G_{||})'}$$

$$= \frac{1}{\varepsilon_b} \frac{1}{V_0^{(2)}} \frac{2\pi r_a}{G_{||} \to 0} \lim J_1'(G_{||} \cdot r_a) \cdot r_a$$

$$= \frac{1}{\varepsilon_b} \frac{\pi r_a^2}{V_0^{(2)}}$$

$$= \frac{f}{\varepsilon_b} \quad (\hat{G}_{||} = 0)$$

(46)

where $J_1'(0) = \frac{1}{2}$ and $f = \frac{\pi r_a^2}{V_0^{(2)}}$ is filling ratio.
FIG. 2: The band gaps structure of two-dimensional function photonic crystals for the triangle lattice, \( \varepsilon_b = 1 \), \( \varepsilon_a = k \cdot r + 11.96 \), function coefficient \( k = 3 \cdot 10^6 \), medium column radius \( r_a = 0.4a \). (a) TE wave, (b) TM wave.

Where

\[
I_1 = \frac{1}{V_0^{(2)}} \int_{V_0^{(2)}} r d\vec{r} \frac{1}{\varepsilon_a(r, \theta)} e^{-i\vec{G}_{||} \cdot \vec{r}} e^{-i\vec{G}_{||} \cdot \vec{r}}
\]

\[
= \frac{1}{V_0^{(2)}} \int_0^{r_a} r dr \int_0^{2\pi} \frac{1}{\varepsilon_a(r, \theta)} e^{i\varepsilon_a(r, \theta) \cdot \sin(\theta - \frac{\pi}{2})} d\theta
\]

\[
= \frac{1}{V_0^{(2)}} \int_0^{r_a} r dr \int_0^{2\pi} d\theta \frac{1}{\varepsilon_a(r, \theta)} \sum_{l=-\infty}^{\infty} J_l(\varepsilon_a(r, \theta)) e^{i\theta \cdot \frac{\pi}{2}}
\]

\[
= \frac{1}{V_0^{(2)}} \int_0^{r_a} r \frac{1}{\varepsilon_a(r)} \sum_{l=-\infty}^{\infty} J_l(\varepsilon_a(r)) dr \int_0^{2\pi} d\theta e^{i\theta \cdot \frac{\pi}{2}}
\]

\[
= \frac{2\pi}{V_0^{(2)}} \int_0^{r_a} r \frac{1}{\varepsilon_a(r)} J_0(\varepsilon_a(r)) dr, \quad (\vec{G}_{||} \neq 0)
\]

In Eq. (47), we consider \( \varepsilon_a(r, \theta) = \varepsilon_a(r) \).

When \( G_{||} = 0 \), as \( J_0(0) = 1 \), we have

\[
I_1 = \frac{2\pi}{V_0^{(2)}} \int_0^{r_a} r \frac{1}{\varepsilon_a(r)} dr, \quad (\vec{G}_{||} = 0),
\]

(substituting \( I_1, I_2 \) and \( I \) into Eq. (39), we obtain

\[
\varepsilon^{-1}(\vec{G}_{||}) = \begin{cases} 
\frac{1}{\varepsilon_a}(1 - f) + 2f \int_0^{r_a} r \frac{1}{\varepsilon_a(r)} dr & (\vec{G}_{||} = 0) \\
\frac{2f}{\varepsilon_a} \int_0^{r_a} r \frac{1}{\varepsilon_a(r)} J_0(\varepsilon_a(r)) dr - 2f J_0(\varepsilon_a(r_a)) \frac{\varepsilon_a}{G_{||} r_a} & (\vec{G}_{||} \neq 0)
\end{cases}
\]
FIG. 3: The band gaps structure of two-dimensional function photonic crystals for the triangle lattice, \( \varepsilon_b = 1 \), \( \varepsilon_a = k \cdot r + 11.96 \), function coefficient \( k = 45 \cdot 10^6 \), medium column radius \( r_a = 0.4a \). (a) TE wave, (b) TM wave.

when \( \varepsilon_a(r) = \varepsilon_a \), the \( \varepsilon_a \) is a constant, the Eq. (49) becomes

\[
\varepsilon^{-1}(G_{||}) = \begin{cases} 
\frac{1}{\varepsilon_b} + \frac{(\varepsilon_a - \varepsilon_b)f}{2f(\frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_b})G_{||} r_a} & (G_{||} = 0) \\
\frac{1}{\varepsilon_a} + \frac{(\varepsilon_a - \varepsilon_b)f}{2f(\frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_b})G_{||} r_a} - \frac{2}{\varepsilon_b} & (G_{||} \neq 0) 
\end{cases}
\]

The Eq. (50) is the dielectric constant Fourier transform of two-dimensional conventional photonic crystals. So, the two-dimensional conventional photonic crystals is the special case of two-dimensional function photonic crystals.

4. Numerical result

In this section, we report our numerical results of band structures for the two-dimensional function photonic crystals. In order to compare the band gaps structures of the two-dimensional conventional and function photonic crystals, we firstly calculate the band gaps structures of the two-dimensional conventional photonic crystals. The structure is the triangle lattice, and the cylindrical medium column are located in the air, its dielectric constant \( \varepsilon_a = 11.96 \) and medium column radius \( r_a = 0.4a \), where \( a = 10^{-6}m \) is lattice constant. The band gaps structures of \( TE \) and \( TM \) wave are shown in FIG. 1. FIG. 1 (a) and (b) are the band gaps structures of \( TE \) and \( TM \) wave, respectively. In the frequency range of 0 – 0.8 (in unit of \( a/2\pi c \)), there are six narrow band gaps for the \( TE \) wave, and there are three band gaps, including a width band gap in the frequency range of 0.2 – 0.38 for the \( TM \) wave.

In the following, we should calculate the band gaps structures of the two-dimensional function photonic crystals. The structure is the triangle lattice, and the cylindrical medium column are located in the air, its dielectric constant is the function of space coordinate, it is \( \varepsilon_a(r) = kr + 11.96(0 \leq r \leq r_a) \) and medium column radius \( r_a = 0.4a \), the coefficient \( k \) is called function coefficient, when \( k = 0 \), it is conventional photonic crystals, when \( k \neq 0 \), it is function photonic crystals. In FIG. 2 (a) and (b), we give the band gaps structures of \( TE \) and \( TM \) wave when \( k = 3 \cdot 10^6 \). In FIG. 2 (a), in the frequency range of 0 – 0.8, there are six band gaps for the \( TE \) wave, which are wider than the FIG. 1 (a) band gaps (conventional photonic crystals). In FIG. 2 (b), there are two band gaps for the \( TM \) wave, and they are narrower than the FIG. 1 (b) band gaps (conventional photonic crystals). Near the 0.3 frequency, the band gaps of \( TE \) and \( TM \) wave
FIG. 4: The band gaps structure of two-dimensional function photonic crystals for the triangle lattice, $\varepsilon_b = 1$, $\varepsilon_a = k \cdot r + 11.96$, function coefficient $k = -3 \cdot 10^6$, medium column radius $r_a = 0.4a$. (a) $TE$ wave, (b) $TM$ wave.

appear overlap. In the other frequency interval, the band gaps of $TE$ and $TM$ wave without overlap, which can be designed the polarization selection device. In FIG. 3 (a) and (b), we give the band gaps structures of $TE$ and $TM$ wave when the function coefficient $k = 45 \cdot 10^6$. In the frequency range of $0 - 0.8$, there are nine band gaps for the $TE$ wave, and two band gaps for the $TM$ wave. Comparing with FIG. 2, we can find the band gaps numbers of $TE$ wave increase and red shift, and $TM$ wave band gaps become narrow and red shift with the function coefficient $k$ increasing. Comparing with the conventional photonic crystals FIG. 1, we can find the band gaps numbers of $TE$ wave increase, become wider and red shift, and the $TM$ wave band gaps numbers decrease, become narrow and red shift. In FIG. 4 (a) and (b), we give the band gaps structures of $TE$ and $TM$ wave when the function coefficient $k = -3 \cdot 10^6$. In the frequency range of $0 - 0.8$, there are seven band gaps for the $TE$ wave, and two band gaps for the $TM$ wave. Comparing with the conventional photonic crystals FIG. 1, we can find the band gaps numbers of $TE$ wave increase and become wider, and the $TM$ wave band gaps numbers decrease and become narrow. In FIG. 5 (a) and (b), we give the band gaps structures of $TE$ and $TM$ wave when the function coefficient $k = -45 \cdot 10^6$. In the frequency range of $0 - 0.8$, there are three band gaps for the $TE$ wave, and one band gap for the $TM$ wave. Comparing with FIG. 4, we can find the band gaps numbers of $TE$ wave decrease, and $TM$ wave band gaps numbers decrease and become narrow with the function coefficient $k$ decreasing. Comparing with the conventional photonic crystals FIG. 1, we can find the band gaps numbers of $TE$ wave decrease, and become wider, and the $TM$ wave band gaps numbers decrease and become narrow. Comparing with FIG. 4 and FIG. 2, FIG. 5 and FIG. 3, we can find the band gaps of function coefficient $k < 0$ are wider than $k > 0$. So, we can obtain the appropriate band gaps structures by selecting the different function from of dielectric constant.

5. Conclusion

In this paper, we have firstly proposed two-dimensional function photonic crystals, and calculate the band gaps structures of $TE$ and $TM$ wave, and find the band gaps of two-dimensional function photonic crystals are different from the two-dimensional conventional photonic crystals. when the functions form of dielectric constants and the crystal structure are different, the band gaps structure should be changed, which can be designed into the appropriate band gaps structures by the two-dimensional function photonic crystals.
FIG. 5: The band gaps structure of two-dimensional function photonic crystals for the triangle lattice, \( \varepsilon_b = 1 \), \( \varepsilon_a = k \cdot r + 11.96 \), function coefficient \( k = -45 \cdot 10^6 \), medium column radius \( r_a = 0.4a \). (a) \( TE \) wave, (b) \( TM \) wave.

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