Is the light neutralino thermal dark matter in the pMSSM ruled out?

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We explore the parameter space of the phenomenological Minimal Supersymmetric Standard Model (pMSSM) with a light neutralino thermal dark matter ($m_{\tilde{\chi}^0_1} \leq m_h/2$) that is consistent with current collider and astrophysical constraints. We consider both positive and negative values of the higgsino mass parameter ($\mu$). Our investigation shows that the recent experimental results from the LHC as well as from direct detection searches for dark matter by the LUX-ZEPLIN (LZ) collaboration rule out the Z-funnel region for the $\mu > 0$ scenario. The same results severely restrict the h-funnel region for positive $\mu$, however, the allowed points can be probed easily with few more days of data from the LZ experiment. In the $\mu < 0$ scenario, we find that very light higgsinos in both the Z and h funnels might survive the present constraints from the electroweakino searches at the LHC, and dedicated efforts from experimental collaborations are necessary to make conclusive statements about their present status.

The R-parity conserved (RPC) scenario of the minimal supersymmetric extension of the Standard Model (MSSM) has been among the most favourable choices for exploring physics beyond the Standard Model (BSM). The RPC-MSSM scenario alleviates the “naturalness” problem [1, 2] in the Standard Model (SM), while also providing a SM-like Higgs boson ($h$) with mass $m_h \sim 125$ GeV and a stable lightest supersymmetric particle (LSP), typically the neutralino $\tilde{\chi}^0_1$, which can be a cold dark matter (DM) candidate. The case of the light neutralino $m_{\tilde{\chi}^0_1} \leq m_h/2$ is of special interest since it is kinematically feasible for the SM Higgs boson to decay invisibly through $h \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1$, thus providing an additional signature for DM in the Higgs sector. Several studies have explored the prospect of a light neutralino DM in the constrained MSSM (cMSSM) and the phenomenological MSSM (pMSSM) considering the various experimental constraints at the time [3–24]. Collider experiments, like ATLAS and CMS, have made available the latest results from searches of heavy Higgs bosons [25], direct searches of charginos and neutralinos [26–29], as well as the invisible decay of the SM Higgs boson [30]. The XENON-1T, PICO-60, PandaX-4T, and LUX-ZEPLIN (LZ) collaborations have also published limits on the DM direct detection (DD) cross-sections – both spin-dependent (SD) and spin-independent (SI) [31–36]. Among these, the results from the LZ collaboration are the most stringent ones for the SI DD cross-sections [35]. In lieu of these new and improved results, it becomes crucial to revisit the MSSM parameter space containing light neutralino DM, which can also contribute to the invisible decay of the Higgs boson.

In this Letter, we study the current status of the light neutralino DM in the MSSM for both positive and negative values of the higgsino mass parameter $\mu$. It is worth noting that the supersymmetric explanation for the discrepancy between the experimentally measured value of the $(g-2)_\mu$ and the SM prediction [37] typically requires $\mu > 0$. Due to the prevalent uncertainties in estimating the hadronic contributions in the SM prediction, we prefer an agnostic attitude towards the sign of $\mu$. As the Large Hadron Collider (LHC) is gearing up for Run-3 and will start collecting data soon, a careful study of the overall status of this scenario is very timely to identify the interesting regions of the parameter space which can be a focal point of the LHC searches at Run-3.

We consider the pMSSM parameter space with ten free parameters defined at the electroweak scale. Our focus is the light neutralino sector with $m_{\tilde{\chi}^0_1} \leq m_h/2$ such that it can contribute to the invisible decay mode of the Higgs boson. The input parameters which capture the physics of the Higgs and electroweakino sectors are: $M_1$, the bino mass, $M_2$, the wino mass ($M_1$ and $M_2$ are collectively referred to as the gaugino masses), $\mu$, the higgsino mass,
tan β, the ratio of the Higgs vacuum expectation values, \( M_A \), the pseudoscalar mass, \( M_{Q_{3L}}, M_{b_R}, M_{b_L} \), the mass of the third generation squarks, \( A_t \), trilinear coupling of the stop, and \( M_3 \), the mass of the gluino. We perform a random scan over ten input parameters for the pMSSM in the following range:

\[
30 \text{ GeV} < M_1 < 100 \text{ GeV}, \ 1 \text{ TeV} < M_2 < 3 \text{ TeV},
\]
\[
100 \text{ GeV} < |\mu| < 2 \text{ TeV}, \ 2 < \text{tan} \beta < 50,
\]
\[
100 \text{ GeV} < M_A < 5 \text{ TeV}, \ 3 \text{ TeV} < M_{Q_{3L}} < 20 \text{ TeV},
\]
\[
3 \text{ TeV} < M_{t_R} < 20 \text{ TeV}, \ 3 \text{ TeV} < M_{b_R} < 20 \text{ TeV},
\]
\[
-20 \text{ TeV} < A_t < 20 \text{ TeV}, \ 2 \text{ TeV} < M_3 < 5 \text{ TeV},
\]

while we fix
\[
M_{Q_{1,2L}} = M_{Q_{1,2R}} = M_{t_R} = 5 \text{ TeV}, \ A_{u/d/c/s/b} = 0,
\]
\[
M_{L_{1,2,3L}} = M_{L_{1,2,3R}} = 2 \text{ TeV}, \ A_{e/\mu/\tau} = 0.
\]

Since we are interested in light neutralino with \( m_{\tilde{\chi}^0_1} \leq m_h/2 \), it shall dominantly have bino (\( \tilde{B} \)) component. Therefore, we scan \( M_1 \) in the low mass region. The coupling of \( Z \) and \( h \) bosons to a pair of \( \tilde{\chi}^0_1 \) also depends on its higgsino (\( \tilde{H} \)) and wino (\( \tilde{W} \)) components. Therefore, in order to circumvent the overabundance of \( \tilde{\chi}^0_1 \) as the DM candidate, we require it to have some \( \tilde{H} \) or \( \tilde{W} \) component. We are primarily interested in the higgsino-like next-to-lightest supersymmetric partner (NLSP) in the present work due to the existing stronger limits on wino-like NLSPs. Hence, \( M_2 \) is scanned above 1 TeV, while \( |\mu| \) is varied starting from a comparatively lower value of 100 GeV, considering both positive and negative values of \( \mu \). Previous studies have shown that the DM relic density and direct detection constraints allow only the \( Z \) and \( h \) funnel regions \( [23, 24, 38] \). We have performed a dedicated scan where we dynamically tune the \( M_1 \) parameter to keep \( m_{\tilde{\chi}^0_1} \) within \( m_Z/2 \pm 5 \text{ GeV} \) and \( m_h/2 \pm 3 \text{ GeV} \) to populate the funnel regions sufficiently. Additionally, we extract the pole mass of the top quark, \( M_t \), randomly from a gaussian distribution with a central value of 173.21 GeV and a standard deviation of 0.55 GeV \([39]\). In total, until this point, we scan over a sample of size \( 2 \times 10^8 \) points.

We use \texttt{FeynHiggs 2.18.1} \([40–47]\) to generate the SUSY spectra corresponding to the various sets of input parameters \([48]\), and to calculate the Higgs boson mass, and decays in the Higgs sector. We assume that the lightest CP-even Higgs boson of MSSM is the SM-like Higgs boson, observed by the ATLAS and CMS collaborations, with a combined measured mass \( m_h = 125.09 \pm 0.21 \text{(stat)} \pm 0.11 \text{(syst)} \text{GeV} \([49]\). We conservatively allow for a theoretical uncertainty of 3 GeV and consider the mass range \( 122 \text{ GeV} < m_h < 128 \text{ GeV} \). Our scan starts from very low \( \tan \beta \) values where satisfying the observed Higgs boson mass will require high stop masses and \( A_t \). In order to ensure that the latter is not so large as to lead to color and charged breaking minima (CCB) \([50–52]\), we require \( |X_t| \lesssim \sqrt{6m_{\tilde{t}_1}m_{\tilde{t}_2}} \|[51]\), where \( X_t = A_t - \mu/\tan \beta \) and \( m_{\tilde{t}_{1,2}} \) represents the masses of the two stops. We observe that the CCB condition has little effect on our parameter space.

We apply limits on the partial decay width of the invisible decay of \( Z \)-boson from new physics, \( \Gamma^\text{inv}_{Z} < 2 \text{ MeV} \|[53]\), chargino mass, \( m_{\tilde{\chi}^{\pm}_1} > 103 \text{ GeV} \|[54]\), and cross-section of associated production of neutralinos in final states with jets, \( \sigma(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2 \times \text{Br}(\tilde{\chi}^0_2 \to \tilde{\chi}^{\pm}_1 \pm \text{jets}) \times \text{Br}(\tilde{\chi}^{\pm}_1 \to \chi^{\pm}_1 \pm \text{jets}) < 0.1 \text{ pb} \|[54]\), as obtained from experiments at the LEP.

We also include the flavor physics constraints on various observables, like the branching fractions of processes \( b \to s\gamma, B_s \to \mu^+\mu^- \), and \( B \to \tau\nu \) which are required to satisfy \( 3.00 \times 10^{-4} < \text{Br}(b \to s\gamma) < 3.64 \times 10^{-4} \|[55]\), \( 1.66 \times 10^{-9} < \text{Br}(B_s \to \mu^+\mu^-) < 4.34 \times 10^{-9} \|[56]\), and \( 0.78 < \text{Br}(B \to \tau\nu)_{\text{obs}}/\text{Br}(B \to \tau\nu)_{\text{SM}} > 1.78 \|[57]\), respectively. We use \texttt{MicrOMEGAS 5.2.13} \([58–63]\) to calculate both the LEP and flavor physics observables.

Additionally, we apply the limits from signal strength measurements of the SM Higgs boson implemented in \texttt{HiggsSignal 2.6.2} \([64–66]\), as well as limits from heavy Higgs searches at the colliders using the \texttt{HiggsBounds 5.10.0} \([67–71]\) package. The recent search of heavy Higgs bosons decaying to \( \tau \) leptons at ATLAS \([25]\) excludes a large part of high \( \tan \beta \) region for \( M_A \lesssim 1 \text{ TeV} \). The parameter space must also satisfy the recent limit on the invisible branching fraction of the SM Higgs boson, \( \text{Br}(h \to \text{invisible}) < 0.11 \|[72]\). We refer to all the constraints related to the Higgs bosons together as the “Higgs constraints” hereafter for simplicity, including the SM-like Higgs mass constraint, constraints from \texttt{HiggsSignal 2.6.2, HiggsBounds 5.10.0}, and the invisible decay of the SM-like Higgs boson.

The lightest supersymmetric particle (LSP), \( \tilde{\chi}^0_1 \), is
FIG. 1. Scaled SI DM-nucleon cross-section \((\sigma_{SI} \times \xi)\) for \(\mu > 0\) (upper) and \(\mu < 0\) (center), along with scaled SD DM-neutron cross-section \((\sigma_{SDn} \times \xi)\) for \(\mu < 0\) (lower) as a function of the mass of the LSP neutralino DM in the region of parameter space satisfying LEP, flavor, Higgs constraints, relic density, DM DD constraints from the XENON-1T, PICO-60 and PandaX-4T experiments ("Before LZ" in yellow circles), "After LZ" in light green circles, and from electroweakino searches. The DD limits, typically derived by assuming that a single DM candidate constitutes the entire relic, will weaken in the scenario where the neutralino DM is underabundant by a factor of \(\xi = \Omega_{LSP}/0.120\).

Moreover, we must consider the results from direct electroweakino searches at the LHC. We use the SModelS 2.2.1 package to implement the electroweakino search constraints on our scanned parameter space. This version of SModelS includes results from the recent search for electroweinkinos in the leptonic final states at CMS [26] and ATLAS [27] and in the hadronic final states at ATLAS [28], all of which play significant roles in excluding a large range of \(m_{\tilde{\chi}^\pm_1}, m_{\tilde{\chi}^0_2}\) and \(m_{\tilde{\chi}^0_3}\), especially with the ATLAS analysis extending the sensitivity to high masses with the hadronic final states.

We apply the constraints on our scanned parameter space in three steps – first “Before LZ” with constraints from LEP, flavor, Higgs constraints, relic density, and the DD experiments XENON-1T, PICO-60, and PandaX-4T, then with the constraint from the LZ experiment, denoted as “After LZ”, and lastly, we add the electroweakino constraints from the LHC. We identify interesting regions of the parameter space surviving all the constraints and perform dedicated scans over these regions again with an additional sample of size \(\sim 10^8\), which makes the total size of our scan \(3 \times 10^8\). The upper panel of Fig. 1 shows the scaled (with \(\xi\)) SI DD cross-sections for the allowed parameter space with \(\mu > 0\) after applying all the constraints from “Before LZ” in yellow, “After LZ” in light green and from electroweakino con-

A viable DM candidate in the MSSM, having a thermal freeze-out production in the early Universe. In the standard cosmology, we require the relic density of the LSP \(\Omega_{LSP}\) to be equal to the observed DM relic density as measured by the PLANCK collaboration \(\Omega_{DM}\) = 0.120 ± 0.001 [73], which assuming a 2σ interval can vary from 0.118-0.122. Lifting up the requisite that the neutralino LSP forms 100% of the observed DM relic owing to the possibility of multicomponent DM, we can modify the relic density constraint to \(\Omega_{LSP} \lesssim 0.122\). MicrOMEGAS 5.2.13 is used to compute the relic density of \(\tilde{\chi}^0_1\).

In addition to the relic density constraint, we take into consideration the results from the current DD experiments. These experiments constrain the spin-dependent DM-neutron (SDn) and DM-proton (SDp) as well as the spin-independent DD cross-sections of the lightest neutralino LSP \((\tilde{\chi}^0_1)\) as a function of its mass. We use MicrOMEGAS 5.2.13 to compute these cross-sections and then compare them with the 90% confidence level (CL) upper limits quoted by the XENON-1T (SI [31] and SDn [32]), PICO-60 (SDp [33]), PandaX-4T (SI [34] and SDn [36]), and LZ (SI [35]) experiments. The DD limits, typically derived by assuming that a single DM candidate constitutes the entire relic, will weaken in the scenario where the neutralino DM is underabundant by a factor of \(\xi = \Omega_{LSP}/0.120\).
FIG. 2. Allowed parameter space for $\mu > 0$ (top) and $\mu < 0$ (bottom) after satisfying the LEP, flavor, Higgs constraints, relic density DD constraints from the XENON-1T, PICO-60 and PandaX-4T experiments (“Before LZ” in yellow circles), “After LZ” in light green circles, overlayed with additional constraints from electroweakino searches at the LHC (in dark green circles) in the $m_{\tilde{\chi}_1^0}$-$m_{\tilde{\chi}_2^0}$ plane.

FIG. 2 reveals a major difference between the $\mu > 0$ and the $\mu < 0$ scenarios. The LZ limit excludes lighter higgsinos ($\lesssim 300$ GeV) for $\mu > 0$, while they survive when $\mu < 0$. This happens due to a cancellation between the contributions of the two CP-even neutral Higgs bosons ($h$ and $H$) involving diagrams with down-type quarks to the SI DD cross-section when $\mu < 0$ in the MSSM. On the other hand these two contributions interfere constructively for $\mu > 0$, increasing the SI DD cross-sections. Among the light higgsinos allowed by LZ, the electroweakino searches allow points in a very narrow region of parameter space in both the $Z$ and $h$-funnels for $\mu < 0$, many of which have very small $R$-values, as shown in Fig. 1 of Supplemental Material. It is interesting to note that if future DD experiments discover a light DM in the $Z$-funnel, it will mostly indicate a negative value of $\mu$. An observation of DM signal in the $h$-funnel from DD experiments would require an additional observation of a signal in the collider experiments to shed light on the sign of $\mu$—observation of light higgsinos will hint towards negative $\mu$, however, heavy higgsinos will not be able to lift this ambiguity. Moreover, we find that the al-
lowed points in each of these regions are clustered around specific tanβ ranges — low tan beta values of 3-10 in the h-funnel for μ > 0 while for μ < 0, tanβ ∼ 3-18 in the Z-funnel, ∼ 3-6 for heavy higgsinos in the h-funnel, and ∼ 16-50 for light higgsinos in the h-funnel.

Representative benchmarks from each of the allowed regions of the parameter space are presented in the Supplemental Material. These benchmarks have very small uncertainty in the Higgs boson mass (≪ O(1) GeV), and have SModelS R-values below 0.5. They are also allowed when tested with CheckMATE 2 [86], another package that implements the constraints from electroweakino searches. We find that the Tevatron limits for light charginos [87] are also not sensitive to these benchmarks. To estimate the prospects for probing the region with light charginos and neutralinos at the LHC, we perform an analysis of the low mass higgsino-like electroweakinos in the leptonic 3l + E_T final state at √s = 14 TeV using the XGBOOST [88] framework. We study the process pp → χ^+_1χ^-_1χ^0_2, χ^+_1 → W^±χ^0_1, χ^0_2 → f fχ^0_1 with mχ^±_1 = 125.1 GeV, mχ^0_2 = 129.9 GeV, mχ^0_3 = 133.5 GeV, and mχ^0_1 = 44.6 GeV (benchmark 2 from Table I in Supplemental Material) where f is an SM fermion, considering 11 possible SM backgrounds for this process. The XGBOOST model, trained with 21 kinematic variables, is used to discriminate the signal benchmark from each background class by computing the significance of observing the signal over the background events. At the √s = 14 TeV LHC with 137 fb^-1 of integrated luminosity (L), we expect to observe 763 signal events and 987 background events for a threshold of 0.9 on our XGBOOST output. Adding a 20% (50%) systematic uncertainty translates to a significance (using the formula in Ref. [89]) of 3.1 (1.3). We present our results for √s = 14 TeV to make it easier to translate to the case of Run-3 (√s = 13.6 TeV) and HL-LHC (√s = 14 TeV) as the cross-sections are not expected to change much. We find that the result sensitively depends on the systematic uncertainty, which can have a significant impact for light electroweakinos.

In summary, this letter shows that the current experiments, especially the recent results from electroweakino searches at the LHC and dark matter DD measurements at the LZ, have severely constrained the μ > 0 scenario, with the Z-funnel being completely excluded, and only very heavy higgsinos allowed in the h-funnel. For the μ < 0 scenario, the allowed parameter space consists of either higgsinos heavier than ∼ 850 GeV in the h-funnel or restricted to a narrow region of light higgsinos having mass of 120-155 GeV in the Z and h-funnels, for a light neutralino thermal DM in the pMSSM with 10 free parameters. Light right-handed staus, still allowed by the LHC, can have a mild affect on the relic density of the lightest neutralino, which is investigated in detail in our future work (Ref. [83]). Moreover, presence of light staus can affect the collider constraints on higgsinos, in regions of the parameter space where the latter decay into the former with significant branching fractions.

The current status of light higgsinos in the mass range of 125-150 GeV from the present electroweakino constraints is not completely clear in the pMSSM, since the experimental results are presented for simplified models. Therefore, experimental collaborations must zoom in on this region and provide definitive answers, covering all possibilities of light higgsinos (including multiple decay modes), which can be an interesting target for Run-3 of LHC. Such light higgsinos are also motivated by naturalness arguments. To conclude, at present, we are at a very exciting juncture where the experiments lined up in the near future might exclude the possibility of a light neutralino thermal DM in the MSSM altogether, or we might be very close to starting to observe the first direct hints of new physics at the LHC.

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SUPPLEMENTAL MATERIAL

Analyses sensitive for light higgsinos and benchmarks

FIG. 1. For the lighter higgsinos allowed from SModelS, the range of $M_{\tilde{\chi}^0_2}$ mass for $\mu < 0$ in the $Z$ funnel (left) and in the $h$ funnel (right) where the different colors depict the most sensitive analysis for each point with the $R$-values of the analysis in the y-axis.

We have found from our exhaustive scan that for $\mu > 0$, we can have allowed points only in the $h$ funnel with $\mu \gtrsim 850$ GeV. For $\mu < 0$, we have three allowed regions — very small $\mu$ (125-145 GeV) in the $Z$ funnel, very small $\mu$ (140-155 GeV) and large $\mu \gtrsim 850$ GeV in the $h$ funnel. For the lighter higgsinos, we present all the sensitive analyses, as given by the SModelS package along with their $R$-values in Fig. 1 for the $Z$-funnel (left) and $h$-funnel (right) in the negative $\mu$ scenario.

We present a benchmark point from each of the allowed regions in Table I along with the scaled spin-independent DD cross-section for these points. We observe that the points in the $h$ funnel of both positive and negative $\mu$ have DD cross-sections very close to the present LZ bound (at most 12-13% lower than the current limit) and, therefore, can be probed with only a few more days of LZ data. The benchmark from the $Z$ funnel in $\mu < 0$ has a very small SI DD cross-section and can be probed with the LHC electroweakino searches dedicated for light higgsinos.

| $\mu > 0$ | $\mu < 0$ |
|---|---|
| **h-funnel** | **Z-funnel** | **h-funnel** |
| $M_t = 173.21, M_1 = 62.5, M_2 = 2000, \mu = 1000, \tan\beta = 5, M_A = 3000, M_{\tilde{\chi}_{2L}} = 10000, M_{\tilde{\chi}_{2R}} = 10000, \tilde{A}_t = 10000, M_3 = 3000$ | $M_t = 173.21, M_1 = 44, M_2 = 2000, \mu = -124, \tan\beta = 5, M_A = 3000, M_{\tilde{\chi}_{2L}} = 10000, M_{\tilde{\chi}_{2R}} = 10000, \tilde{A}_t = 10000, M_3 = 3000$ | $M_t = 173.21, M_1 = 68, M_2 = 2000, \mu = -150, \tan\beta = 50, M_A = 3000, M_{\tilde{\chi}_{2L}} = 5000, M_{\tilde{\chi}_{2R}} = 5000, \tilde{A}_t = -5000, M_3 = 3000$ |
| $125.38 \pm 0.97$ | $125.88 \pm 0.96$ | $125.67 \pm 0.63$ |

TABLE I. Parameters corresponding to four benchmark points satisfying all the present constraints from the $\mu > 0$ and $\mu < 0$ scenarios along with their scaled SI DD cross-sections. The mass of the Higgs boson $M_h$ and the uncertainty in $M_h$ computed by FeynHiggs ($\Delta^{FH}_{M_h}$) are also shown.
Details of the XGBOOST analysis

Here, we discuss the details of our analysis for the light higgsino benchmark. We study 11 possible SM backgrounds for this process: \( lll\nu \) \((l \equiv e, \mu, \tau)\), \( ZZ, t\bar{t}, VVV \), \( Wh, Zh \), ggF and VBF production of \( h \) with \( h \to ZZ^*, t\bar{t}h, t\bar{t}W \), and \( t\bar{t}Z \). We restrict to the leptonic final state which is cleaner for a lighter benchmark, such as ours. We perform an analysis of the \( 3l + \not{E}_T \) final state where we require exactly three leptons satisfying \( p_T > 25, 25, 20 \) GeV and \( |\eta| < 2.4 \), and we have put a veto on \( b \)-jets with \( p_T > 30 \) GeV and \( |\eta| < 2.5 \). In our signal benchmark, since we do not have any on-shell \( Z \)-boson, we also veto events where the invariant mass of a pair of same flavor opposite sign (SFOS) leptons lie within 10 GeV window of \( m_Z = 91.2 \) GeV. After these preselections, we train our signal and background samples using XGBOOST with a set of the following variables:

- Transverse momenta \( (p_T) \) of the three leptons
- Transverse mass \( (M_T) \) and contransverse mass \( (M_{CT}) \) of each of the three leptons with the \( \not{E}_T \)
- Minimum and maximum values of \( \Delta R \) between opposite sign lepton pairs along with their \( \Delta \eta \) values
- Invariant mass of the opposite sign lepton pairs with minimum and maximum \( \Delta R \)
- Missing transverse momentum
- Number of jets in the event with the \( p_T \) of the two leading jets
- Scalar sum of \( p_T \) of all the jets in the event \( (H_T) \)
- Invariant mass of the three leptons

Following are the hyperparameters of the XGBOOST model:

- \texttt{`objective’:’multi:softprob’, ‘colsample_bytree’:0.3, ‘learning_rate’:0.1, ‘num_class’:12, ‘max_depth’:7, ‘alpha’:5, ‘eval_metric’:’mlogloss’, ‘num_round’:1000, ‘early_stopping_rounds’:3}

Prospects at Future Lepton Colliders

The future lepton colliders like ILC and CEPC will be crucial for precision measurements of Higgs boson. The projected upper limit on the invisible branching of the Higgs boson is 0.4% at ILC [90] and 0.3% at CEPC [91]. Although these can probe a significant part of the allowed parameter space in the \( \mu < 0 \) case, we still have regions with \( \text{Br}(h \to \text{invisible} < 0.003) \) in both the \( Z \) and \( h \)-funnels. In the \( \mu < 0 \) case, the partial decay width of the \( Z \) boson to \( \tilde{\chi}_1^0 \) \((\Gamma_{\text{inv}})\) is always less than 0.1 MeV for the allowed parameter region that we obtain. Therefore, we do not expect the Giga-Z option of ILC, which is expected to have a modest improvement over LEP [92], to be sensitive to this region.