A Study of Size Effect in Shear Resistance of Reinforced Concrete Beams Based on Machine Learning

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Abstract. Size effect is common in structural engineering where a large component fails at lower stress level than a tiny one made of the same material. As a result, the calculation of the loading capacity of a structural component need to be revised when size effect is considered, especially for the reinforced concrete (RC) beam under shear loading. However, due to inadequate theoretical analysis of the size effect in shear failure, the calculation of shear strength in codes all over the world is far from accurate. In this research, the machine learning method, which can efficiently analyse and process data, is applied to study the size effect in shear strength of beams via exploring the experimental data collected worldwide on shear failure of RC beams. The back propagation (BP) neural network model of shear strength of RC beams is built and trained by the data exist, and then give a prediction of shear strength. The values predicted are compared with the values calculated from codes and Bažant size effect formulas and the experimental data, which indicates the feasibility and superiority of machine learning method in dealing with size effect in shear failure.

Keywords. Size effect, shear, machine learning, neural network.

1. Introduction

In classical theories of elasticity and plasticity, it is not amiss with assuming that the strength of a structure is independent of size when geometrically similar structures are in discussion. However, calculation according to this assumption produces results usually unexpectedly different from measurement in engineering practice. In fact, people encountered this type of problems more often with the development of economy and engineering technology nowadays. When designing and using structural parts for mega-structures, especially reinforced concrete beams, the shear strength of beams will decrease with the increase of effective height. This is a typical size effect in reinforced concrete beams under shear loading. Ignoring the size effect in shear failure can lead to unexpected consequence, to name a few, the disaster happened in Ohio Air Force Warehouse and Montreal Slab Bridge in Canada.

Although the existence of size effect has been widely accepted all over the world, as shear failure is complicated in research, no agreement has been reached on the form of size effect formula for shear designing. The calculation of shear strength guided by the codes worldwide is either too conservative or unsafe due to the lack of theoretical analysis of the size effect in shear failure. In fact, it is not easy to get accurate results in analysis of beams shear resistance in spite of size effect. The current mostly cited theories on size effect of concrete’s shear strength are Weibull statistical theory [1], Bažant size
effect law [2] based on fracture energy release rate, and Carpinteri’s [3-4] scaling law via microscale characterization. The theories mentioned above consider the size effect of concrete based on assumptions which might fail at particular situations. Therefore, it is always beneficial to look for a universal approach to study the general size effect of concrete under shear loading.

The inadequate theoretical analysis can be amended by machine learning approach, which can efficiently analyse and process data of experiments. This research attempts to develop a machine learning model of concrete which is trained by the experimental data, and able to predict shear strength of concrete beams with different sizes. The strategy is explained in steps. First, in a data set of shear failure experiments of RC beams collected worldwide, the nominal shear strengths are calculated by removing the contribution of stirrups. After that, the processed data is used to train the BP neural network model of concrete, which will later be applied to predict shear strength of concrete corresponding to an input value of beam size. The predictions are then compared with the calculated values from various codes and the experimental results. Based on the comparison, the model is demonstrated to work better in predicting shear strength of concrete despite size effect. The necessary background of each step is explained in following sections.

2. Size Effect Theories
Currently, the size effect has been considered in the calculation of shear strength, although in different formula. As documented in Yu et al. [5], three major engineering societies developed their own theories.

2.1. JSCE Equation
In 1980s, the Japanese society of civil engineer introduced the following equation to calculate the shear strength of concrete beam,

\[ u = v_0 \theta_{JSCE} \]  

(1)

\[ \theta_{JSCE} = \left( \frac{d_0}{d} \right)^n \]  

(2)

Where \( u \) stands for shear strength, \( v_0 \) and \( d_0 \) are constants for geometrically similar beams, \( n \) is 1/4 for concrete (according to a later calibration, it can be 1/12), \( d \) denote for effective height of the concrete beam (from compression face to the tensile reinforcement centroid).

2.2. Fib Model Code
In 2010, The Euro code for concrete shear resistance is derived from modified compression field theory (MCFT) theory, which has the following form,

\[ u = v_0 \theta_{MC} \]  

(3)

\[ \theta_{MC} = \frac{1}{1 + d / d_0} \]  

(4)

Where the parameters are of the same meaning as the JSCE equation.

2.3 ACI-446 Code
The American Concrete Institute adopted the following equation based on fracture mechanics and energy conservation (first law of thermodynamics), namely

\[ u = v_0 \theta \]  

(5)
\[ \theta = \frac{1}{\sqrt{1 + D/D_0}} \]  

(6)

Where \( D_0 \) is a constant for geometrically similar structures, \( D \) is set to be either the total depth of the concrete beam or \( d \).

Derived from different theory, the equations above instruct the calculation of shear strength of concrete beam, although some of them produce results seen unreal in analysis. [5] Besides, as observed in the equations (1)-(6), the size effect factor \( \theta \) should always be a function of \( d/d_0 \). This will be the basic assumption in machine learning model to be developed.

3. Machine Learning Method

In this research, the machine learning method is applied to generate a model, which can predict the shear strength of concrete in a RC beam considering the size effect. In this task, due to the strong nonlinearity of the shear strength \( \nu \) and beam depth \( d \), the back propagation (BP) neural network is employed, which are known for handling nonlinear regression.

3.1. BP Neural Network Algorithm

In machine learning, back propagation computes the gradient of the loss function with respect to the weights of the network for a single input-output data set. It is feasible to use gradient methods for training multi-layer networks, updating weights to minimize loss. And in this research, a straightforward input layer-hidden layer-output layer structure is built, each layer consisting of 10 neurons. Based on the structures depicted in figure 1, the BP algorithm employs gradient searching strategy to reduce the squared error between the expected output and the real one.

![Figure 1. The structure of the neural network generated by Tensorboard.](image)

3.2. TensorFlow System

TensorFlow system is adopted in this research as the platform for BP neural network. This software library has been popular in machine learning applications as a symbolic math reference since 2015 [6]. It has blocks of algorithms and optimization functions which can be called easily by a simple sentence when coding, and therefore is extremely friendly for software developing. The Tensorboard visualization tool is also provided by this system.

In this neural network, the activation function in hidden layer is selected as softplus, which is:
\[ \zeta(x) = \log(1 + e^x) \]  \hspace{1cm} (7)

This function is the smooth version of the mostly used ReLU activation function, possessing many advantages over Sigmoid and Tanh function. To name a few, it converges fast in stochastic gradient descent (SGD) iteration, and performs well in an un-supervised training. In this model, the training steps are set to be 1000, and the velocity is 0.05.

4. Model Calibration and Application

In this section, the machine learning model is trained and applied in prediction of concrete’s shear strength, for some recorded experiments.

4.1. Model Training

As discussed in section 2, the size effect will be considered as a function of \( d / d_0 \). The parameters, such as \( v_0 \) and \( d_0 \), need to be evaluated for the model. First \( v_0 \) is calculated by [7]

\[ v_0 = \rho^{0.37} \sqrt{f'_c \lambda^{0.7}} \]  \hspace{1cm} (8)

where \( \rho \) stands for the reinforcement ratio, \( f'_c \) the compressive strength of concrete and \( \lambda \) the shear span ratio. And \( d_0 \) is calculated by

\[ d_0 = \lambda_0 \cdot d_u \]  \hspace{1cm} (9)

where \( \lambda_0 = 25 \) for beams without stirrups, and \( d_u \) is the size of coarse aggregates. For beams with stirrups, simply replace parameter \( \lambda_0 \) by \( \lambda_0 (1 + \rho_s / \rho) \), where \( \rho_s \) is the ratio of stirrups. Once the parameters are obtained, the data recorded in the experiment, namely \( v_u \) and \( d \) can be transformed to \( v_u / v_0 \) and \( d / d_0 \) for machine learning model training.

In the model training procedure, the data set collected [5] is input into the model, except for 20 points are selected for testing whether the trained model predict well the shear strength of beams with only known \( d / d_0 \) from 0.1 to 2. The training data and testing data are plotted in figures 2 and 3.

![Figure 2. Data set for model training.](image)

![Figure 3. Data selected for model testing.](image)

After 1000 steps training of the machine learning model, this neural network produced a curve depicted in figure 4. Besides, the loss function values recorded in Tensorboard tool box during training steps are plotted in figure 5, which shows that the loss function converges to 0 after just 200 steps. It proves that \( v_u / v_0 \) has a stable relation to \( d / d_0 \) and the training process is successful. It is also worth mentioning that this model works differently from a traditional designing code. As a typical...
computational black box, it generates no explicit equations for computing the shear strength. And instead, every time after the machine collects an input value of $d/d_0$, it produces a corresponding result of $v_u/v_0$ which can be translated straightforwardly to the shear strength of concrete.

**Figure 4.** The model prediction of shear strength.

**Figure 5.** Loss function values during model training.

4.2. Model Testing

The trained model is now tested by repeating the experiments recorded in figure 3. The recorded values of $d/d_0$ are input into the model, and the output value of $v_u/v_0$ are collected for calculation of $v_u$. The results are documented in the charts in figure 6, together with the calculation from various codes worldwide.

**Figure 6.** Calculated concrete shear strength in selected experiments (testing data set).
In Figure 6, $v_u$ stands for the recorded shear strength in the original data, $ml(v_u)$ the predictions from machine learning model, $ACI(v_u)$ the calculated values according to ACI318-14 while $ACI-B(v_u)$ the model considering Bažant size effect law (ACI-446 proposal), $CSA(v_u)$ the calculation by Canadian Code, $GB(v_u)$ via Chinese Code, $EN(v_u)$ via Euro Code. As demonstrated in the Chart, it is not difficult to see the machine learning model predictions are closer to the test results than any other calculations. And meanwhile, among the present widely used formula, the $ACI$ (ACI-446 proposal) equation considering Bažant size effect law produced results quite close to Canadian Code, and both the two are conservative comparing to the test results. This is a natural observation since the size effect of shear strength is also considered in Canadian Code. The observation directly supported the claim that size effect is a necessary part in predicting shear strength.

Here the goodness of fit equation is introduced to evaluate the calculations listed in Figure 6 by how well they fit the set of measurement,

$$R_{NL} = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum y_i^2}$$  \hspace{1cm} (10)

where $y_i$ stands for the test results, $\hat{y}_i$ a calculation from a model. In this evaluation, for a specific model, the indicator $R_{NL}$ is closer to 1 if the prediction is closer to the measurement. The calculations of $R_{NL}$ are documented in the following table 1:

| Models   | $ml(v_u)$ | $ACI(v_u)$ | $ACI-B(v_u)$ | $CSA(v_u)$ | $GB(v_u)$ | $EN(v_u)$ |
|----------|-----------|------------|--------------|------------|-----------|-----------|
| $R_{NL}$ | 0.797     | 0.281      | 0.411        | 0.408      | 0.462     | 0.316     |

The values of $R_{NL}$ in table 1 show that the machine learning model has the best fit among all the models in predicting the experiment results. Although this may be resulted from the truth that designing codes need to be conservative in engineering practice, it still gives a good reason to introduce machine learning technology into the structural engineering research, since it can help get closer to the real values of material strength.

5. Conclusion
In this research, the machine learning model is applied to study the size effect in shear strength of concrete. This model first gets trained by the collected data of shear failure experiment of concrete beams, and then it is able to predict concrete shear strength according to the input value of beam depth. The model is proved to be effective compared to the mostly used designing codes worldwide in calculating concrete shear strength, and therefore can be introduced in structural engineering for other possible applications.

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