2D Lattice of coupled Sinai billiards: metal or insulator at $g \ll 1$?

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Abstract

We investigate the transport in a two-dimensional (2D) lattice of coupled Sinai billiards fabricated on the basis of a high-mobility 2D electron gas in GaAs/AlGaAs heterojunction. For the states with low reduced conductivity $g \ll 1$ an anomalously weak temperature dependence of $g$ was found. The large negative magnetoresistance described by the Lorentz line-shape of the width corresponding to the half magnetic flux quantum through the area of the billiard is observed. In going from $g > 1$ to $g \ll 1$ it strongly increases. The Shubnikov-de Haas oscillations and commensurability magnetoresistance peak are preserved at $g \ll 1$. The data suggest that the system studied behaves more like a metal than an insulator at $g \ll 1$ and is not described by the generally accepted picture of Anderson localization.

Quantum and classical transport in systems with dynamic chaos has been intensively studied during last few years, since the successes of modern semiconductor technology has made it possible to obtain various experimental realizations of such systems with electron billiards as an example. At the present time two varieties of these systems are studied. The first one is unit billiards (regular or chaotic Bunimovich or Sinai billiards) [1–5], while the second one is macroscopic two-dimensional (antidot lattice) [6–10] or one-dimensional [11] Sinai billiards. A number of interesting phenomena resulting from the classical and quantum chaotic electron dynamics was found in these billiards (mesoscopic conductance fluctuations, commensurability magnetoresistance oscillations, weak localization effects, statistical and parametrical correlations of Coulomb blockade peaks).

However up to now, there have been practically no investigations of systems, in which separate electron billiards with reduced conductivity $g = \sigma_{xx}/(e^2/h) \gg 1$ and the size $L < l$ ($l$ is the mean free path) compose a lattice with the coupling between them being weak in the sense that the conductivity of the lattice itself is low $g \ll 1$. The study of such lattice is of interest mainly due to the fact that, on the one hand, this system should have some properties of a unit billiard
(because the coupling is weak), and on the other hand, since the billiards form a regular structure, the systems may exhibit properties caused by that structure.

We report for the first time the results of experimental investigation of this new system with dynamical chaos. The system was built on the basis of a two-dimensional (2D) lattice of closely situated antidots fabricated from a high mobility 2D electron gas in GaAs/AlGaAs heterojunction with a metallic gate evaporated on the top, that permitted us to control the conductivity of the structure in a sufficiently wide range from $g = 0.01$ to $g = 2$.

The square lattice of antidots was fabricated on the basis of a two-dimensional electron gas with electron density $N_S = (2 - 3) \times 10^{11}$ cm$^{-2}$ and mobility $\mu = (3 - 8) \times 10^5$ cm$^2$/V s corresponding to the mean free path $l = (3 - 6)$ $\mu$m by means of electron lithography and subsequent plasma etching. Then the NiAu or TiAu gate was evaporated on the top of the device. We investigated three samples with the lattice period $d = 0.6$ $\mu$m and three ones with $d = 0.7$ $\mu$m. One of the samples had no metallic gate, and its conductivity was controlled using illumination by LED. The lithographic size of antidots $a = 0.2$ $\mu$m was the same for all samples. However, due to the depletion layers the actual size was larger, being approximately equal to the lattice period even before the gate evaporation. The experimental sample was made up of two Hall bars of the length of 100 $\mu$m and the width of 50 $\mu$m. The lattice of antidots was introduced into one of the bars. Measurement were carried out in the temperature range 50 mK–1 K in the magnetic field up to 2 T using ordinary four-terminal scheme at the frequency 6 Hz and with the current 0.01–0.1 nA in order to exclude heating effects.

Fig. 1(a) shows the set of temperature dependences of the conductivity $g(T)$ for the sample AG219 with the period $d = 0.6$ $\mu$m ($\mu = 7 \times 10^5$ cm$^2$/V s at $N_S = 2 \times 10^{11}$ cm$^{-2}$ in the unpatterned part of the sample) for different values of the gate voltage. It is seen that for $g > 1$ conductivity is practically temperature independent. More exactly, weak logarithmic decrease of $g$ is observed typical for the weak localization effects. It becomes more noticeable for lower values of $g$, but it still remains weak even for $g \ll 1$, and is well described by the power law dependence $g(T) \propto T^a$ with $a < 1$ for all of the tested samples. Specifically, for the dependences of Fig. 1(a) $a = 0.1 - 0.27$ at $g \ll 1$. It should be mentioned that the value of $a$ is larger for the samples with lower mobility of the initial 2D electron gas. As an example, the same set of dependences $g(T)$ for sample AG35 with the period $d = 0.7$ $\mu$m ($\mu = 3 \times 10^5$ cm$^2$/V s at $N_S = 2 \times 10^{11}$ cm$^{-2}$) is presented in the Fig. 1(b). It is seen that they are characterized by two times higher magnitude of $a$ for the same values of $g$. Hence it may be inferred that the enhancement of fluctuation potential leads to stronger temperature dependence. The behavior of $g(T)$ described above is significantly different from that for the unpatterned 2D electron gas both in silicon MOS-transistors [12] and in AlGaAs/GaAs heterojunction [13], as well as for antidot lattices with short period [14,15]. In all of these cases at $g \sim 1$ the transition from the weak logarithmic dependence (weak localization regime) to
the strong exponential one (strong localization regime) is observed. In our case the weak logarithmic decrease of \( g \) (for \( g > 1 \)) is followed by a weak power law, that has not been observed in other 2D systems before.

We now turn to description of the influence of magnetic field. It is well known that in AlGaAs/GaAs heterojunctions at \( g < 1 \) under the influence of magnetic field a transition from an insulator to the quantum-Hall-liquid is observed. This transition is characterized by the critical point \( B_c \) and \( g_c \approx 0.5 - 1 \) [13]. A similar transition was recently observed in the triangular antidot lattice with a small period \( d = 0.2 \, \mu \text{m} \) [15]. Our samples exhibit a radically different picture. It is seen from Fig. 2 that for all values of \( g \) (\( B = 0 \)) in the magnetic fields about \( B \approx 1 \, \text{T} \) the transition takes place from weak power-law dependence \( g(T) \) to no temperature dependence at all. Moreover, this transition is of different kind, for there is no critical point, and the metallic behavior extends for \( g \ll 1 \). Fig. 2(a) also shows an interesting picture in weak magnetic field. For all values of \( g \), the negative magnetoresistance (NMR) is observed for \( B < 0.05 \, \text{T} \), followed by a peak at \( B \approx 0.2 \, \text{T} \) corresponding to the condition \( 2R_c = d \). This peak is well-known for the antidot lattices at \( g > 1 \) and originates from the so-called pin-ball trajectory that surrounds an antidot not colliding with it. It is seen from Fig. 2(b) that the second commensurate peak is observed at \( g \approx 1 \). The second peak corresponds to the condition \( 2R_c = (\sqrt{2} - 1)d \). We have recently established that this peak is associated with the non-colliding trajectory inside the billiard [16]. The positions of the commensurability peaks shows that we really deal with the lattice of closely situated antidots with \( d \approx a \) and \( d \gg d - a \).

The Sinai billiards between the antidots have the area \( S = d^2(1 - \pi/4) \) and contain a large number of electrons \( N \gg 1 \). In our case we have correspondingly \( S = 0.5 \, \mu \text{m}^2 \), \( N \approx 70 \) for \( d = 0.7 \, \mu \text{m} \), and \( S = 0.36 \, \mu \text{m}^2, N \approx 50 \) for \( d = 0.6 \, \mu \text{m} \). It is also important that both the main commensurability peak and NMR are conserved at the transition from \( g > 1 \) to \( g \ll 1 \). The peak position is slightly shifted towards lower magnetic fields with decreasing \( g \). This indicates a small change of electron density (from \( 1.4 \times 10^{11} \) to \( 0.9 \times 10^{11} \, \text{cm}^{-2} \)) in the billiard within the whole range of \( g \).

The behavior of NMR is shown in Fig. 3 in more detail. It is characterized by two distinguishing features: (i) for all states with \( 0.05 < g < 2 \) NMR is cut off at the same magnitude of magnetic field \( B \approx 0.05 \, \text{T} \); (ii) NMR noticeably increases with decreasing \( g \) (Fig. 3(a) shows that at \( g = 0.05 \) it reaches a considerable value about 40%), and its temperature dependence becomes stronger. For the states with the highest resistivity it was more stronger than \( g(T) \)). For \( g > 1 \) NMR can be attributed to the effects of weak localization in open chaotic billiards [2], because it has relatively small amplitude and Lorentzian line-shape. The behavior of NMR for \( g \ll 1 \) is surprising. It increases by an order of magnitude reaching a considerable value comparable to the total resistance of the sample, while the line-shape of NMR is described by a Lorentz curve of the same width. The width is equal to \( \Delta B_1 = 27 \pm 2 \, \text{mT} \) for the sample AG219. It corresponds to a half magnetic flux quantum through the area of the billiard that
is equal to $d^2(1 - \pi/4)$. The fact that the width is determined by the magnetic flux quantum is well seen from the comparison of NMR for the samples with two different periods 0.6 $\mu$m and 0.7 $\mu$m. As it is clearly seen from the Fig. 3(c) the width of NMR curve for the period 0.7 $\mu$m equals $\Delta B_2 = 20 \pm 2$ mT, that is $\Delta B_1 / \Delta B_2 = (0.7)^2 / (0.6)^2$. Thus at $g \ll 1$ we observe NMR which is very similar to weak localization NMR in chaotic open billiards [2]. But the value of NMR is much larger. It is necessary to add that the following two conditions should be satisfied in order to observe the effects described above: (i) the influence of the fluctuation potential should be as low as possible, and (ii) the area of billiard should be relatively large. That is because one or both these conditions were not met that these effects were not observed in [13,14,17] (except for the commensurability peak observed in [14] at $g \ll 1$, that might still have been of different nature, for the temperature dependence was exponentially strong).

Let us discuss the results obtained. First turn to the temperature dependence at the transition from $g > 1$ to $g \ll 1$. It differs from the accepted picture of the metal-insulator transition in 2D and 3D electron systems. This picture is based on the concept of Anderson localization of electrons, be it the model of minimal metallic conductivity (MMC) or the scaling theory (ST) [18]. According to it at $k_F l \sim 1$ or $g \sim 1$ in a macroscopic 2D or 3D system the transition should occur from the metallic behavior (as in MMC model) or from the weak localization (as in ST) to the strong localization characterized by the exponential temperature dependence of the activation type (hopping conductivity) or of the Mott type (variable range hopping conductivity). In real systems the transition can be quite complicated, but for $g \ll 1$ the state with hopping conductivity is always realized [12,13]. We know only one work [19] in which the linear temperature dependence for $g \ll 1$ was observed in thin In$_2$O$_3$ films. Recently a similar system was considered theoretically in [20] within the model of metal grains with $g \gg 1$ coupled by tunneling in a way that the conductivity of the macroscopic sample was low $g \ll 1$. At $g \ll 1$, due to inelastic electron-electron scattering the linear dependence $g(T)$ was obtained in [20] in a wide temperature range. The dependence changes to the exponential one at $T \ll e^2/C$, where $C$ is the capacitance of a metal grain. Our results are not in agreement with the predictions of this theory. Firstly, in our case $g(T)$ is weaker than linear. Secondly, it is observed at $T \ll e^2/C$, because the Coulomb energy for our samples $e^2/2C \approx 15$–25 K. This means that the low conductivity of the lattice of the Sinai billiards can not be explained by the model of metal lakes coupled by weak tunnelling junctions. The effects of Coulomb blockade are not manifested in the experiment, because one observe no features in $g(V_g)$ dependence. So we have to assume that at $g \ll 1$ the coupling between the billiards is stronger than that provided by tunneling. The behavior of the lattice in the magnetic field supports this assumption. In Fig. 2(a) one can see the Shubnikov–de Haas oscillations, which give the electron density in the lattice saddle points connecting the billiards. It coincides with the density determined from the Hall effect that should give the concentration in these points [21]. The
electron density in these points weakly changes with the strong change of $g$. It decreases only by about 30% while $g$ drops by a factor of 30, and its magnitude, equal to $4.1 \times 10^{10}$ cm$^{-2}$, is only three times less than $N_S$ inside the billiard even for the state with the lowest value of $g$. This means that the Fermi level in the saddle point lies several meV above the barrier at $g \ll 1$, and an electron should ballistically move through the "bottle-neck" to go from one billiard to another. The behavior of commensurability peak and of NMR support this picture. Hence, we should come to a paradoxical conclusion that 2D lattice of Sinai billiards coupled via conducting "bottle-neck" can have very low conductivity $g \ll 1$ but simultaneously exhibits the properties typical for metallic ballistic systems rather then for the insulators. This conclusion is in a drastic contradiction with the standard picture of Anderson localization. Let us discuss possible reasons for such a situation and consider it first from the weak localization side. The transition from the weak to strong localization, caused, for example, by the decrease of temperature, should be accompanied by the increase of the phase coherence length. Due to this increase, the number of localized trajectories should increase until at $T = 0$ all the trajectories become localized. In the system investigated this increase can be at least hampered, because an electron can loose the phase coherence inside a billiard before it leaves for the other billiard through the "bottle-neck". A simple estimation gives for the dwell-time of the electron inside a billiard $\tau = 10^{-8}$ s. The collimation effects can only increase this time. The estimation shows that $\tau$ can be larger than the time of phase coherence, which is of the order of $\tau_{\varphi} = 10^{-9}$ s at 40 mK. This means that even at the lowest available temperature an electron can loose the phase coherence on the length scale $L = d$. Then the resistance is the classical sum of the resistances of individual billiards, which can yield any value of $g$. The situation discussed is to some extent similar to that considered in [20]. In contrast to [20] in our system the quantum dots are separated by the conducting "bottle-neck" instead of the tunnel barrier as in [20]. Obviously, this should lead to the weaker temperature dependence of the conductivity, that is just observed in our experiments. Nevertheless, it is not clear what happens at $T \to 0$, because the Coulomb blockade effects are not observed in our case. The description of the system from the strong localization side is complicated, because one can not start from the ground state of electron in the well, for it represents a multilevel system with the large ($\sim 100$) number of electrons. In any case the description of 2D lattice of coupled Sinai billiards and the phenomena described in the present work presents a challenge to the modern theory of quantum transport in the condensed matter.

In conclusion, we investigate a new type of dynamic chaos system — 2D lattice of coupled Sinai billiards. We find that its transport properties are very unusual. They characterize the system as a metal rather than an insulator at $g \ll 1$. The whole set of experimental data (temperature dependence of conductivity, NMR, and commensurability peak of magnetoresistance) is in contradiction with the standard picture of Anderson localization, and testifies
that the system possesses quite unusual transport properties the description of which requires new theoretical approaches.

Note added. Since the completion of this paper, D.P.Druist et al [22] have reported the observation of metallic behavior at $g \ll 1$ in a completely different system — a layered three-dimensional semiconductor structure — under the conditions of IQHE.

We would like to thank M.Entin, V.Falko, A.Charplik, and E.Baskin for useful discussion and V.Alperovich for reading the manuscript. M.V.Budantsev acknowledges "Mission scientifique" from French Embassy in Moscow for the support. This work was supported by RFFI through Grant No 96-02-287 and by NATO Linkage through grant No HTECH.LG.971304.

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Figure captions

Fig. 1
Temperature dependences of the conductivity at the transition from $g > 1$ to $g \ll 1$: (a) sample AG219, (b) sample AG35. Solid lines are $g \sim T^\alpha$.

Fig. 2
Magnetoresistance (MR) traces for different values of the conductivity and at different temperatures (sample AG219): (a) $g = 0.05$, (b) $g = 0.18$, (c) $g = 2.2$. (d) Schematic view of antidot lattice (black points show the etched regions, broken lines show the boundary of depletion region, 1 - an electron trajectory around antidot, 2 - an electron trajectory between antidots).

Fig. 3
(a,b,c) NMR curves for the sample AG219. (d) Experimental and calculated NMR curves for the antidot lattices with two different periods $d = 0.6 \, \mu m$ (sample AG219) and $d = 0.7 \, \mu m$ (sample AG35). Solid lines are the experimental curves, broken lines are the calculated Lorenz ones, $B_{1/2}$ is the width of the Lorenz curves.
Fig. 1
Fig. 2
Figure 3

Graphs showing the behavior of \( \rho_{xx} \) (in units of \( h/e^2 \)) as a function of magnetic field \( B \) at different temperatures and layer thicknesses.

(a) \( T=60 \text{ mK} - 1.2 \text{ K} \)

(b) \( T=60 \text{ mK} - 1.1 \text{ K} \)

(c) \( T=60 \text{ mK} - 1.1 \text{ K} \)

(d) \( T=60 \text{ mK} \)

- For \( B_{1/2} = 27 \text{ mT} \) with \( d = 0.6 \mu \text{m} \)
- For \( B_{1/2} = 20 \text{ mT} \) with \( d = 0.7 \mu \text{m} \)