Ion and Electron Acceleration in Fully Kinetic Plasma Turbulence

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Abstract

Turbulence is often invoked to explain the origin of nonthermal particles in space and astrophysical plasmas. By means of 3D fully kinetic particle-in-cell simulations, we demonstrate that turbulence in low-β plasmas (β is the ratio of plasma pressure to magnetic pressure) accelerates ions and electrons into a nonthermal energy distribution with a power-law energy range. The ion spectrum is harder than the electron one, and both distributions get steeper for higher β. We show that the energization of electrons is accompanied by a significant energy-dependent pitch-angle anisotropy, with most electrons moving parallel to the local magnetic field, while ions stay roughly isotropic. We demonstrate that particle injection from the thermal pool occurs in regions of high current density. Parallel electric fields associated with magnetic reconnection are responsible for the initial energy gain of electrons, whereas perpendicular electric fields control the overall energization of ions. Our findings have important implications for the origin of nonthermal particles in space and astrophysical plasmas.

Unified Astronomy Thesaurus concepts: High energy astrophysics (739); Plasma astrophysics (1261)

1. Introduction

Nonthermal energetic particles are ubiquitous in space and astrophysical environments. The solar corona and wind (McComas et al. 2019), supernova remnants (Reynolds 2008), accretion disk coronae (Yuan & Narayan 2014), jets from supermassive black holes (Blandford et al. 2019), and galaxy clusters (Brunetti & Jones 2014) are just a few examples where the presence of energetic particles, often in the form of power-law distributions, is observed or inferred. However, their origin is still poorly understood. Among the potential processes responsible for their acceleration, turbulence in collisionless plasmas is often invoked as a prime candidate (Melrose 1980; Lazarian et al. 2012; Petrosian 2012).

The idea that magnetized turbulence could lead to a power-law energy distribution of accelerated particles can be traced back to Fermi’s original model of stochastic particle acceleration through repeated interactions with moving magnetized structures (Fermi 1949). Following Fermi’s insight, the development of the quasilinear theory (e.g., Kennel & Engelmann 1966; Hall & Sturrock 1967; Kulsrud & Ferrari 1971; Achterberg 1981; Jaekel & Schlükeiser 1992) and its nonlinear extensions (e.g., Völk 1973; Bieber & Matthaeus 1997; Matthaeus et al. 2003; Shalchi 2006; Yan & Lazarian 2008) has deepened our understanding of particle acceleration in turbulence. On the other hand, these simplified analytical models ignore some important aspects of the physics of turbulence. Above all, they ignore the role of magnetic reconnection, which is an essential component of particle acceleration in a magnetized turbulent cascade (Arzner & Vlahos 2004; Dmitruk et al. 2004; Kowal et al. 2012; Comisso & Sironi 2018, 2019).

In order to address this problem in all its complexity, fully kinetic numerical simulations are necessary. This has recently been possible in the relativistic regime (Zhdankin et al. 2017; Comisso & Sironi 2018; Zhdankin et al. 2018; Comisso & Sironi 2019; Zhdankin et al. 2019; Comisso et al. 2020; Wong et al. 2020; Comisso & Sironi 2021; Nättiäli & Beloborodov 2021; Sobacchi et al. 2021), where the Alfvén speed approaches the speed of light and computational costs are substantially reduced. In the nonrelativistic regime, progress has been made through test-particle simulations in synthetic turbulence (e.g., Michalek & Ostrowsky 1996; Arzner et al. 2006; O’Sullivan et al. 2009; Teraki & Asano 2019) or fluid-type simulations (e.g., Arzner & Vlahos 2004; Dmitruk et al. 2004; Lehe et al. 2009; Kowal et al. 2012; Dalena et al. 2014; Lynn et al. 2014; Beresnyak & Li 2016; González et al. 2017; Isliker et al. 2017; Kimura et al. 2019; Trotta et al. 2020; Sun & Bai 2021; Pezzi et al. 2022) and, more recently, through hybrid fluid-kinetic simulations (e.g., Kunz et al. 2016; Pecora et al. 2018; Arzamasskiy et al. 2019). However, these approaches suffer from important limitations: Test-particle calculations neglect the backreaction of accelerated particles onto the electromagnetic fields, and they need to employ ad hoc prescriptions for particle injection; the hybrid fluid-kinetic approach describes electrons merely as a fluid. The kinetic physics of both ions and electrons is needed to properly account for magnetic reconnection in a collisionless plasma and to self-consistently capture the energization of both species.

In this Letter, we investigate the self-consistent acceleration of ions and electrons in a turbulent plasma. We consider a low-β plasma, which is relevant for various space and astrophysical systems, such as the solar corona or accretion disk coronae. We demonstrate that turbulence naturally produces nonthermal tails with a power-law energy range for both ions and electrons. We show that particles are extracted from the thermal pool in regions of high current density associated with magnetic reconnection and that the different acceleration mechanisms responsible for the energization of ions and electrons leave a distinctive signature on their pitch-angle distributions.

2. Numerical Method and Setup

We adopt a first-principles approach by solving the coupled Vlasov–Maxwell system of equations through the particle-in-cell (PIC) method (Birdsall & Langdon 1985) with TRISTAN-MP (Buneman 1993; Spitkovsky 2005). We perform the numerical simulations in a triply periodic cubic domain \( L^3 \) that
is discretized into a regular lattice of $1400^3$ cells. The initial conditions correspond to uniform plasma with total particle density $2n_0$, Maxwellian-distributed ions and electrons of equal temperature $T_0 = T_0 = T_0$, and a uniform guide magnetic field $B_0 = B_0\hat{z}$. Turbulence is seeded by initializing a spectrum of magnetic fluctuations having polarizations transverse to $B_0$ (see Comisso & Sironi 2018, 2019 for details). The rms amplitude of the fluctuations is $\delta B_B = B_0$, where $\delta B = (\delta B^2(t=0))^{1/2}$. The energy-carrying scale is $l = l/3$.

We resolve the electron skin depth $d_e = c/\omega_{pe}$ with 3.3 cells and employ an average of 64 computational particles per cell. In order to capture the full turbulent cascade from MHD scales to electron kinetic scales, we use a reduced ion-to-electron mass ratio of $m_i/m_e = 50$, which gives a domain size $L/d_i = 60$, where $d_i = c/\omega_{pi}$ is the inertial length, with $\omega_{pi} = (4\pi n_0 e^2/m_i)^{1/2}$ indicating the ion plasma frequency. The ratio of electron plasma frequency $\omega_{pe} = (4\pi n_0 e^2/m_i)^{1/2}$ to electron gyrofrequency $\omega_{ce} = eB_0/m_i c$ is $\omega_{pe}/\omega_{ce} = 1$, as expected in the solar corona. Then, the Alfvén speed is $v_A = B_0/(4\pi n_0 m_i)^{1/2} = 0.14 c$. We consider a low-$\beta$ plasma, with $\beta_0$ ranging from 0.32 down to 0.04. Here, $\beta_0 = \beta_0 + \beta_0$, is the initial total plasma $\beta$, with $\beta_0 = \beta_0 = 8\pi n_0 k_B T_0 / B_0^2$. We scan $\beta_0$ by changing the initial plasma temperature $T_0$, which is taken to be $k_B T_0 = (0.08, 0.04, 0.02, 0.01) m_i c^2$, where $k_B$ indicates the Boltzmann constant. The resulting $\beta_0$ values of our simulations are $\beta_0 \in (0.32, 0.16, 0.08, 0.04)$. We take the simulation with $\beta_0 = 0.08$ as the fiducial simulation.

3. Results

In Figure 1(a), we show a volume rendering of the current density (normalized to $J_{rms} = (\overline{J})^{1/2}$) from the fiducial simulation. Sheet-like current density structures are ubiquitous in the turbulent domain. Due to the presence of the mean field $B_0 = B_0\hat{z}$, these structures are mostly elongated along $\hat{z}$. As a result of magnetic reconnection, long and thin current sheets are characterized by the appearance of flux ropes, as shown in Figure 1(b). As we show below, magnetic-field-aligned electric fields ($E_B$) associated with magnetic reconnection play an important role in the early stages of electron energization. Finally, in Figure 1(c), we show the one-dimensional spectra of the magnetic ($\delta B$) and bulk velocity ($\delta V$) fluctuations as a function of $k_z = (k_x^2 + k_y^2)^{1/2}$. Both power spectra approximating follow $P_{\delta B}(k_z) \propto k_z^{-5/3}$ (Goldreich & Sridhar 1995) at $k_z d_i \lesssim 1$. At sub-$d_i$ scales, $P_{\delta B}(k_z) \propto k_z^{-2.8}$ down to $k_z d_i \sim 1$ (e.g., Alexandrova et al. 2009; Chen et al. 2014), and it further steepens for $k_z d_i \gtrsim 1$. A similar trend characterizes the $P_{b}(k_z)$ spectrum, with the main difference that the steeper range occurs already at $k_z d_i \gtrsim 0.3$.

We show that a key outcome of the turbulent cascade is the generation of nonthermal particles for both ions and electrons. This is illustrated in Figure 2, where we show the time evolution of the ion and electron energy spectra $f_i(\varepsilon) = dN_i(\varepsilon)/d\varepsilon$ (compensated by $\varepsilon$ to emphasize the particle content) in the fiducial simulation. Here, $\varepsilon = m_i c^2/(3 - 1)\varepsilon$ is the particle kinetic energy ($s = i$ or $e$ for ions or electrons), with $\gamma = (1 - v^2/c^2)^{-1/2}$ indicating the particle Lorentz factor. As a result of turbulent field dissipation, the spectral peak shifts to higher energies. The shift is bounded by the magnetic energy available per ion-electron pair, $\Delta \varepsilon_{\delta B} = \delta B^2/8\pi n_0 = (\delta B_0/B_0)^2 k_B T_0 / \beta_0 = 0.5 m_i c^2$. In addition, a significant number of particles increase in energy by $\Delta \varepsilon \gg \Delta \varepsilon_{\delta B}$. At late times, when most of the turbulent energy has decayed, the spectra of both ions and electrons extend well beyond the peak into a nonthermal tail of high-energy particles that can be described by a power law $f_i(\varepsilon) d\varepsilon \propto \varepsilon^{-\gamma} d\varepsilon$ for $\varepsilon_{min} < \varepsilon < \varepsilon_{max}$ and a cutoff for $\varepsilon \gtrsim \varepsilon_{max}$.

The slope $\gamma$ of the power law depends on $\beta_0$. This is shown in the insets (top for ions, bottom for electrons) of Figure 2. To facilitate the comparison of the different $\beta_0$ cases, we normalize $\varepsilon$ by the mean energy per particle after turbulent dissipation, $\varepsilon_{\text{th},i} = \varepsilon_{\text{th},i} + \kappa \Delta \varepsilon_{\delta B} = (3/2 + 2\kappa/\beta_0) k_B T_0$, where $\varepsilon_{\text{th},i} = (3/2) k_B T_0$ is the initial thermal energy and $\kappa$ is the fraction of magnetic energy transferred to a given species.
(we take $\kappa = 1/3$ for both ions and electrons because about two-thirds of the turbulent magnetic energy is equally distributed between ions and electrons by the end of the simulations). As observed in simulations of low-$\beta$ magnetic reconnection (Dahlin et al. 2015; Li et al. 2019; Zhang et al. 2021), the spectrum hardens with decreasing $\beta_0$, with the ion slope approaching $p \approx 3.5$ and the electron slope approaching $p \approx 4.2$ for $\beta_0 = 0.04$.

An important difference between energized ions and electrons is encoded by the distribution of the pitch angle $\alpha$, i.e., the angle between the particle velocity and the local magnetic field. In Figure 3(a), we show the probability density functions (PDFs) of the pitch-angle cosine $\cos \alpha = v \cdot B / (|v| |B|)$. While ions exhibit roughly isotropic PDFs, electrons are highly anisotropic with PDFs strongly peaked at $\cos \alpha = \pm 1$, i.e., electrons move mostly along the magnetic field lines. The different $\beta_0$ cases display similar behavior, apart from the fact that electrons have higher PDF peaks near $\cos \alpha = \pm 1$ for lower $\beta_0$. As we show below, this trend is due to the fact that electrons gain more energy from $E_B^0$ as $\beta_0$ decreases.

The PDFs shown in Figure 3(a) are dominated by particles near the peak of $\epsilon_f(\epsilon)$ because these particles control the number census. To characterize the anisotropy of higher-energy particles, we construct PDFs of $\cos \alpha$ for particles in different energy ranges. Figure 3(b) shows these PDFs for the fiducial simulation. At moderate energies, $\epsilon \lesssim \epsilon_{th,0} + \Delta \epsilon_{th}$, ions and electrons display PDFs that are similar to those in Figure 3(a). However, at higher energies the PDFs peak at intermediate values between $\cos \alpha = 0$ and $\cos \alpha = \pm 1$. This is particularly prominent for electrons, which lose the $B$-field alignment that characterizes low and moderate energies.

The significant energy dependence of the electron’s pitch-angle distribution is clearly displayed in Figure 3(c), where we show $(\sin^2 \alpha)$ as a function of $\epsilon$ (normalized by $\epsilon_m$) for different $\beta_0$ values. Contrary to ions, the electron $(\sin^2 \alpha)$ deviates significantly from the expected mean for an isotropic distribution, $(\sin^2 \alpha) = 2/3$ (compare with the black dashed line) and attains a minimum at $\epsilon/\epsilon_m \sim 2$ (as in pair plasma turbulence; Comisso & Sironi 2019; Comisso et al. 2020; Comisso & Sironi 2021). The value of $(\sin^2 \alpha)$ increases for higher energies and eventually $(\sin^2 \alpha) \sim 2/3$ for $\epsilon \gg \epsilon_m$ as a consequence of pitch-angle scatterings off the turbulent fluctuations.

To understand the difference between the energization of ions and electrons and its effect on pitch-angle anisotropy, we tracked the trajectories of a random sample of $\sim 10^8$ particles in each of the simulations. Figure 4(a) shows the time evolution of the kinetic energy for five representative ions and five representative electrons that eventually populate the high-energy end of the nonthermal tail in the fiducial simulation. These particles experience a sudden “injection phase,” i.e., an episode of rapid energy gain that brings them to energies much higher than the initial thermal energy (Comisso & Sironi 2018, 2019), followed by a more gradual energization phase akin to the stochastic Fermi acceleration mechanism (Fermi 1949).
energy ions (ε < 0.5 m_ec^2) have a rate of increase of the particle kinetic energy (averaged over △t = 150 ω_pe^-1) that satisfies the empirical threshold △ε/△t ≥ 0.2 R_{rec}v_A^2 m_ec^2 ω_pe, where R_{rec} = 0.1 is the normalized collisionless reconnection rate (Comisso & Bhattacharjee 2016; Cassak et al. 2017). For most particles, t_{inj} falls in the range v_A/τ ~ 0.5–2. One can see that the PDF of electrons at injection peaks at |Jp|/|J_{max}~ 2.5, while the PDF of ions at injection peaks at a lower value of |Jp|/|J_{max}~ 1. This should be contrasted with the PDF of the entire population of particles at a generic time, which peaks at |Jp|/|J_{max}~ 0. Therefore, particle injection is associated with locations of high current density (current sheets).

The injection phase of ions and electrons is governed by different energization mechanisms (E_β versus E_∥). In order to distinguish the role of the electric field parallel (∥) and perpendicular (⊥) to the local magnetic field, we compute W_β(t) = g ∫ t^t_0 E_i(t’) · v(t’) dt’ and W_∥(t) = g ∫ t^t_0 E_i(t’) · v(t’) dt’ for all the tracked particles. This allows us to construct the distributions f_k(W_β, W_∥/W_{tot}), where W_{tot} = W_β + W_∥ is the work done by the total electric field. These distributions are shown in Figure 4(c) for ions (left) and electrons (right) from the fiducial simulation. Ions gain energy almost entirely via E_∥. In contrast, the injection of electrons is controlled by the E_β field associated with reconnecting current sheets, while E_∥ energization from scatterings off the turbulent fluctuations becomes progressively more important as the electron energy increases.

The observed trend is robust across the different simulations. This is illustrated in Figure 4(d), where we show the median of f_k(W_β/W_{tot} | W_{tot}) for various β_0 values. The contribution of E_∥ consistently increases with energy in the range of the high-energy nonthermal tail for both ions and electrons. However, low-energy electrons need to be first accelerated via E_β up to the characteristic energy ε/ε_β ~ 2 (Figure 3(c)) associated with magnetic reconnection. This is also the reason why the electron pitch-angle distribution develops a strong anisotropy, with min(sin^2 α) attained at the same energy ε/ε_β ~ 2. For electrons, the initial energy gain via E_β increases with decreasing β_0, which is also consistent with the observed increase in pitch-angle anisotropy.

4. Summary

In summary, we have demonstrated with 3D fully kinetic simulations that magnetized plasma turbulence is a viable mechanism for ion and electron acceleration. Both species develop nonthermal power-law tails, but ions attain harder nonthermal spectra and reach higher energies than electrons. For both species, the power-law slope becomes harder when decreasing the plasma β, i.e., the ratio of plasma pressure to magnetic pressure. We show that the energization of electrons is accompanied by a significant energy-dependent pitch-angle anisotropy, with most electrons moving parallel to the local magnetic field, while ions stay roughly isotropic. We demonstrate that particle injection from the thermal pool occurs in regions of high current density. Parallel electric fields associated with magnetic reconnection are responsible for the initial energy gain of electrons—which also imprints into their pitch-angle anisotropy—whereas perpendicular electric fields control the overall energization of ions. The results of our first-principles simulations shed light on the origin of nonthermal particles in space and astrophysical systems.
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