Bayesian Update of Load Capacity for a Large Steel Piling in a Stratified Soil Profile

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Abstract. This paper applies Bayesian updating of the load capacity of a large steel piling foundation for the high load structure of the Alcântara Wastewater Treatment Plant (WWTP), located near the city of Rio de Janeiro in Brazil. Uncertainty is modeled by a priori and a posteriori distributions of the piling capacity. The a posteriori distribution is determined by updating the a priori distribution using a likelihood function, which incorporates records obtained during pile driving. The Bayesian update was applied to a dataset consisting of 645 steel driven piles. Two pile capacity design models and two different likelihood functions were used to verify their influence on the updated capacity estimates. Static and dynamic test results were compared to the updated estimates. The results demonstrate the ability of the Bayesian update technique to significantly improve the reliability of the entire piling.

Keywords: Bayesian theory, reliability, steel piles.

1. Introduction

Pile capacity predictions usually involve numerous uncertainties due to measurement errors in soil properties, limitations of geotechnical investigation, spatial variability across a site, model simplifications, among others. Huang et al. (2016) also point out that installation processes may also vary from pile to pile and affect the variability of the ultimate pile capacity. Kay (1976) presented an interesting application of probability theory to develop a consistent set of guidelines for safety factor selection for a range of design methods. He demonstrated that such an approach provides the basis for the optimization of the testing procedure and an improved final design. Kay’s 1976 method makes use of the Bayesian approach, which permits the inclusion of subjectively determined aspects of the design in the formal analysis. A sampling from the “state of nature”, according to Kay (1976), allows for a formal reduction in uncertainty through the application of Bayesian theory.

Estimates of pile and pile group reliability based on load test results have been reported by Baecher & Rackwitz (1982), Zhang et al. (2001), Zhang (2004), Zhang et al. (2006, 2010, 2014) and Huang et al. (2016). Guttormsen (1987) applied the wave equation model for obtaining the likelihood distribution function when performing the Bayesian update for offshore piles. One of the strong points of the application of reliability theory is the possibility of combining various methods, as reported by Lacasse et al. (1991) who made use of a pile capacity calculation model and estimation based on some measurement activity to obtain an updated estimate. Vrouwenvelder (1992) emphasized that based on information from testing and monitoring, the engineer may update the estimate of pile capacity. Moreover, as the new estimate is based on more information, the uncertainties are fewer and a corresponding reduction of the safety factor could be justified. Lacasse & Nadim (1994) commented on the importance of adopting rational approaches and well documented projects to account for load capacity uncertainties. Zhang & Tang (2002) applied the Bayesian theory to piling reliability using load test results in the updates. The same approach can have other applications. Baecher & Ladd (1997), for instance, used Bayesian theory in the prediction of clay properties such as undrained strength and overconsolidation ratio. Zhang et al. (2004) applied the Bayesian approach to update empirical predictions with regional information or site-specific observations to effectively reduce the uncertainty associated with the correlations. Goh et al. (2005) used a Bayesian network algorithm to model the relationship between the undrained shear strength of soil, the effective overburden stress and the undrained side resistance alpha factor for drilled shafts. Li et al. (2017) illustrated the application of Bayesian theory to evaluate the spatial variability of soil in slope stability projects.

The objective of this paper is to apply the Bayesian updating technique to update the expected value and variance of the probability distribution of soil resistance to driving for H-section steel piles. For driven piles, the construction processes can be better controlled and the variables due to construction are expected to vary in smaller ranges.
Figure 1 - Relationship between a priori, likelihood and a posteriori distribution of pile capacity (Lacasse & Goulois, 1989; Lacasse et al., 1991).

2. The Bayesian Updating Approach

The Bayesian updating approach, illustrated in Fig. 1, shows the probability density functions used in the updating procedure. Bayesian updating allows for a reliable estimate of the a posteriori pile capacity from a priori and likelihood distribution functions. A detailed description of Bayesian concepts is found in applied statistics books (e.g., Ang & Tang, 1984). Equations 1 and 2 allow the estimation of the expected value and the variance of the updated pile capacity (a posteriori) based on the expected value and the variance of the a priori distribution, as well as the likelihood function.

\[
\mu_{Q} = \frac{\sigma_{QL}^{2} \cdot \mu_{QL} + \sigma_{Qr}^{2} \cdot \mu_{Qr}}{\sigma_{QL}^{2} + \sigma_{Qr}^{2}} \tag{1}
\]

\[
\sigma_{Q}^{2} = \frac{\sigma_{QL}^{2} \cdot \sigma_{Qr}^{2}}{\sigma_{QL}^{2} + \sigma_{Qr}^{2}} \tag{2}
\]

where \(\mu_{Q}\) is the expected value of the updated pile capacity obtained a posteriori, \(\mu_{QL}\) and \(\mu_{Qr}\) are, respectively, the expected value of the pile capacity originally predicted (a priori) and that obtained from the measurement activity (likelihood function); \(\sigma_{Q}^{2}\) is the updated variance of the distribution of pile capacity (a posteriori) and \(\sigma_{Qr}^{2}\) and \(\sigma_{QL}^{2}\) are, respectively, the variances of the originally estimated distribution, obtained by means of the available field tests (a priori) and that obtained from the measurements (the likelihood function).

3. The Test Site

The dataset refers to a Wastewater Treatment Plant (WWTP) in Alcantara, located in the lowlands nearby Rio de Janeiro. The piling dataset is related to the representative area delimited by the solid black line and Alcantara river in Fig. 2. The boreholes identified as SP, SPC and SNB in the figure represent different site investigation campaigns. The triangles and squares represent, respectively, the vane tests and CPTu tests. In the hatched area, nearly 4,000 H shaped W200x71 steel piles have been driven as foundation for the structures of the sewage plant. Each pile driven in the representative area had its performance controlled by three procedures: the complete driving log; the final average penetration for the last 10 hammer blows; and the elastic pile rebound for the same 10 blows. The penetration and elastic rebound at pile head were measured during the end of driving by means of the simple and traditional use of pencil and paper form. In the same representative region, 52 dynamic and 2 static loading tests were also performed.

Figure 3 shows: (i) the pile section; (ii) a typical register for the last 10 blows; and (iii) the plugging condition expected to occur at soil-pile during failure of the steel H piles (toe resistance mobilization as fully plugged and skin friction mobilization throughout the entire pile-soil contact). This plugging condition is commonly used in Brazilian, German (Schenck, 1966; EAB, 2008) and French (French Standard, 1993) foundation practice.

3.1. Soil profile and characterization

The soil profile consists of a very soft superficial clay layer with thickness varying from 8 to 12 m. Below the superficial soft layer, the SPT borings showed a fine-to-coarse dense sand, with thickness varying from 4 to 7 m, followed by a silty-sand/sandy-silt gneissic residual soil stratum. The very soft clay layer has been subjected to 8 CPTu tests, 6 vane tests and 5 sampling logs for laboratory tests, with locations also shown in Fig. 2. The laboratory tests consisted of index tests and oedometer tests. The superficial clay characterization and test results for the WWTP site are similar to those described by Almeida et al. (2008). A summary of the main parameters is listed in Table 1.

The mean value and the variance of the standard penetration test “N60” distributions were determined for the representative area.

Spatial variability of the soil profile and geotechnical parameters occurs especially in the more resistant layers, where the CPTu could not penetrate and only the SPT bor...
nings were available. The layers of dense sand and residual gneissic soil are expected to be responsible for nearly the entire pile capacity.

Figure 4 presents the $N_{60}$ values and the statistical distribution for the 46 borings and investigated depths: the mean, the mean minus the standard deviation and the mean plus the standard deviation, represented in three distinct curves. The typical soil profile for the representative area is also shown in the same figure. The superficial soft soil layer presents a zero $N_{60}$ for nearly the entire depth, and therefore the three statistical curves are superimposed on the depth axis. Some very high outlier values of $N_{60}$, representing local discontinuities in the final centimeters of penetration with fractional NSPT (e.g. 20 blows / 5 cm) were removed from the dataset before the statistical calculation. No boreholes penetrated depths beyond 19 m due to the presence of rock fragments in the residual soil. As the steel H-piles have higher impedance than the SPT sampler and are less affected by small rock fragments within the soil, some piles reached depths of 21.5 m. For these depths the $N_{60}$ values were extrapolated by repeating the last $N_{60}$ penetration resistance available from the boreholes.

In Brazil, selection of appropriate correction factors is required to convert NSPT into $N_{60}$ according to the actual energy delivered during the SPT test. The correction factor used on the interpreted values was that obtained from Cavalcante et al. (2003, 2004, 2011) based on actual measurements on the Brazilian dataset, indicated below:

$$N_{60} = 137 N_{SPT}$$  \hspace{1cm} (3)

4. The A Priori Mobilized Pile Resistance

As emphasized in the previous section, the piling site has been divided into representative areas characterized by a selected typical soil profile. The results included in the present paper refer to the representative area and typical soil profile shown in Figs. 2 and 4, respectively.
For any calculated model, pile capacity is estimated by Eq. 4:

\[ Q + W = Q_p + Q_l \]  

(4)

where \( Q \) is the pile capacity, \( W \) is the pile weight, \( Q_p \) is the tip load capacity and \( Q_l \) is the load capacity due to lateral friction. The pile weight is often neglected due to its small influence in the overall pile capacity and Eq. 4 is written as:

\[ Q = A_b \cdot q_p + U \sum \tau_i \cdot \Delta_i \]  

(5)

where \( A_b \) is the pile base area, \( q_p \) is the unit tip resistance, \( U \) is the pile perimeter, \( \tau_i \) is the unit shaft resistance, \( \Delta_i \) is the pile length with the same \( \tau_i \) value.

A number of methods can be used in the deterministic analysis of pile capacity. In the present analysis the authors have chosen two distinct methods in order to verify the statistical distribution of the pile capacity for different calculation models and their influence on the a posteriori updated distribution. These methods are described below.

### 4.1. Aoki & Velloso method

The first selected calculation model was that of Aoki & Velloso (1975), widely used in Brazil, which estimates the pile capacity from the results of standard penetration tests. This method is based on empirical correlations between \( q_c \) (from CPT) and \( N_{60} \) (from SPT) established for both sedimentary and residual soil profiles in different regions in Brazil (Aoki & Velloso, 1975, Danziger & Velloso, 1995; Politano et al., 2001). Correction factors are used for different pile types to account for the effects of distinct construction procedures. Comparisons between the estimated pile capacity of this model and that observed from numerous static load tests have shown generally good agreement.

According to Aoki & Velloso (1975), the unit tip resistance and the unit shaft resistance are estimated as:

\[ q_p = \frac{q_c}{F_1} = \frac{k \cdot N_{60}}{F_1} \]  

(6)

\[ \tau_i = \frac{\tau_i}{F_2} = \frac{\alpha \cdot k \cdot N_{60}}{F_2} \]  

(7)

where \( F_1 \), \( F_2 \) are correction factors, \( k \) and \( \alpha \) are dependent on the soil type and \( q_c \) and \( \tau_i \) are related to \( N_{60} \) by means of the correlations presented above. For steel driven piles, \( F_1 \) and \( F_2 \) are 1.5 and 3, respectively. Values of \( k = 220 \), 1000 and 700 kPa and \( \alpha = 4, 1.4 \) and 2.4 \% were used for the soft soil layer (silty-clay), sand layer and residual soil layer (sandy-silt).
4.2. Vesic method

The second method selected for deterministic analysis of pile capacity was the well-known theoretical approach developed by Vesic (1977). Conventional theories consider the tip resistance to be given by the same expression as that used for the load capacity for shallow foundations, excluding the component relative to the ground weight, since this component is very small in this type of foundation, Eq. 8.

\[ q_{p, tri} = C \cdot N_c^* + \sigma'_v \cdot N_a \]  
\[ \sigma'_v = \frac{(1 + 2K_0)\sigma'_0}{3} \]  

were \( \sigma'_v \) is the vertical effective stress and \( \sigma'_a \) is the mean effective stress at the foundation base, \( N_c^* \) and \( N_a \) are the load capacity factors and \( K_0 \) is the soil at-rest coefficient and \( C \) is the cohesion or undrained shear strength.

The \( N_a \) factor can be determined by any method that takes into account soil deformability prior to failure. The soil deformability in a condition of little volume change (dense strata) was estimated, in the present case, with the elastic soil modulus given by Freitas et al. (2012). According to Vesic (1977) the \( N_a \) factor is a function of the friction angle and the reduced stiffness index of the soil.

For pile-soil skin friction, Vesic (1977) proposes Eq. 10, where \( K_s \) depends on the initial at-rest condition, construction procedures and pile shape.

\[ q_s = K_s \cdot \sigma'_v \]  

For application of the Vesic (1977) method, correlations between the soil parameters and \( N_{60} \) are necessary. The authors made use of Kulhawy & Maine (1990) guidelines with resulting soil strength parameters summarized in Table 2. An overall working platform consisting of a 0.5 m layer of sandy material was used over the whole region due to the low capacity of the soft soil to support the traffic of the machines.

5. The Soil Resistance to Driving

The soil pile resistance mobilized during pile driving differs from the pile capacity estimated for long term conditions, although the calculations are carried out quite similarly. The resistance mobilized during driving, known in the literature as SRD (soil resistance to driving), is that mobilized during pile penetration from the hammer blows (Toolan & Fox, 1977; Stevens et al., 1982; Semple & Gemeinhardt, 1981).

For both methods adopted for \textit{a priori} estimates, the disturbed undrained resistance of the superficial soft layer was used for the unit adhesion. For both the sandy and high permeability soils, resistance to driving is similar to long-term resistance, even though it is known that aging and dissipation of pore pressure can change soil resistance with time even for coarse soils.

Most theoretical methods for pile capacity consider that the skin friction may not indefinitely increase with overburden pressure, but rather a limiting value may be used. The decrease in unit shaft resistance for very deep piles has been analyzed by Lehane \textit{et al.} (1993), Randolph \textit{et al.} (1994) and Jardine & Chow (1996). Although both se-

| Soil                      | \( \gamma_{sat} \) (kN/m^3) | \( \phi' \) (°) | \( c' \) (kPa) |
|---------------------------|------------------------------|---------------|--------------|
| Working platform          | 19                           | 30            | 5            |
| Dense sand                | 20                           | 35-40         | 0            |
| Gneissic residual soil    | 20                           | 35-45         | 0            |

Figure 4 - \( N_{60} \) values - Statistical soil boring profile.

Table 2 - Granular soil parameters.
lected methods for *a priori* estimation do not propose any limit, a $N_{60}$ value of 68 blows for 0.30 m penetration was considered as a limit, as no accurate correlation is available for such higher resistances.

5.1. The dataset

For the final driven depth of each pile of the dataset, the bearing load capacities were calculated using the Aoki & Velloso (1975) and Vesic (1977) methods. Three scenarios of the $N_{60}$ profile were formed: mean (M), mean minus standard deviation (M - D) and mean plus standard deviation (M + D), given the closest borehole as the mean value and the variance of $N_{60}$ for the whole site, according to the curves presented in Fig 4. The calculation results for the predicted pile capacity were assembled, composing a large dataset from which the statistical distribution of the *a priori* values was determined.

Figure 5 compares the *a priori* pile capacity for the 645-pile dataset estimated using both selected methods. The piles were driven to depths varying from 13.5 to 21.5 m, with an average embedded length of 16 m and a standard deviation of 1.5 m, resulting in a COV of 0.09. The Vesic (1977) calculation model resulted in values nearly 37% higher than the Aoki & Velloso (1975) method. Figure 5 shows that for load capacities higher than 2,500 kN for the Vesic method, the M, M-SD and M+SD related values are concentrated over a narrow range. This occurs for the deeper piles embedded into high resistance residual soils, where the $N_{60}$ limit resulted in nearly the same soil resistance independently of the "$N_{60}$ profile" used in the calculation. While in the Aoki & Velloso method the unit tip resistance remains constant and total lateral friction resistance increases with pile penetration in the residual soil layer, the Vesic method considers a decrease in unit tip and friction resistance due to the increase in the confining effective stress for a fixed $N_{60}$. Therefore, a decrease in the calculated soil friction angle occurs, resulting in a decrease in predicted pile capacity. While the values from the Aoki-Velloso method continue to increase with depth at a constant rate, the Vesic method begins to increase at a lower rate when the piles penetrate the residual soil stratum with a fixed $N_{60}$, causing the inflection and concentration of points in the narrow range observed in Fig. 5. On the other hand, some points at the top center of the graph in the same Fig. 5 presented a different behavior, probably due to an over estimation of the friction angle by the Kulhawy & Mayne (1990) correlation for some undisclosed soil stratum.

The frequency histogram for the *a priori* pile capacity was tentatively analyzed by both normal and lognormal distributions. Better results were found for the normal distribution, as presented in Figs. 6a and 6b. The predicted values from the Aoki & Velloso (1975) method are well represented, whereas those of Vesic (1977) are concentrated between the values of 2,500 and 3,000 kN. The behavior of the normal distribution for the second method is probably affected by the abrupt increase in tip resistance when the piles penetrate the residual soil layer. At depths bellow 16 m, the average contribution of the tip resistance to total pile capacity was nearly 30% for the Aoki-Veloso method and 60% for Vesic. This explains the large number of piles with *a priori* capacity ranging from 2,500 to 3,000 kN, calculated using the Vesic model for piles installed through the transition depth where soil profiles move abruptly from the sedimentary to the residual layer.

![Figure 5](image1.jpg)

**Figure 5** - Expected *a priori* values of SRD (soil resistance to driving) estimated using both calculation models for the 645-pile dataset.

![Figure 6](image2.jpg)

**Figure 6** - Histograms for the natural normal distribution curve of SRD (soil resistance to driving): (a) Aoki-Veloso method and (b) Vesic method.
6. Likelihood Function Distribution

The likelihood function is the one that includes on-site results, a sample of the “state of nature”, as reported by Kay (1976). The “state of nature” for the present case was obtained from the registers documented during driving for the entire dataset: the average pile set and rebound for the final ten blows. The average pile set was introduced in the Danish formula (Sorensen & Hansen, 1957) and the pile rebound in the Chellis-Aoki formula (Chellis, 1951; Aoki, 1989) for deterministic calculation of the pile capacity at the end of driving.

6.1. The Danish formula

The Danish formula (Sorensen & Hansen, 1957) is still widely used in Brazil for predicting pile capacity of steel piles. The accuracy of the Danish formula has been checked at several sites, as reported by Olson & Flaate (1967). Danziger & Ferreira (2000) used the formula to compare results to more accurate applications of wave equation programs and found good agreement. The formula is described by the following equations:

\[ R_u = \frac{\eta \cdot W_h \cdot H}{S + 0.5 S_e} \]  
\[ S_e = \sqrt{\frac{2 \eta \cdot W_h \cdot H \cdot L}{A \cdot E}} \]  

where \( R_u \) is the ultimate dynamic pile capacity, \( \eta \) is the driving hammer efficiency, \( W_h \) is the hammer weight, \( H \) is the hammer drop, \( S \) is the pile set per hammer blow, \( S_e \) is the elastic pile rebound per hammer blow, \( L \) is the pile length, \( A \) is the pile end area and \( E \) is the modulus of elasticity of the pile material (Olson & Flaate, 1967).

All the data necessary for the application is known from the dataset, except the system efficiency. A common value used for the efficiency is 0.7. However, as 9 dynamic loading tests were available in the present case study, the measured efficiency from the dynamic tests was used instead. The average measured efficiency was 75 % with a standard deviation of 21 %. Thus, for each pile in the dataset, the capacity mobilized during pile driving was calculated with the Danish formula using the mean (75 %), mean minus the standard deviation (54 %) and mean plus the standard deviation (96 %) of hammer efficiency and those values were included in the dataset forming the first likelihood function.

6.2. Chellis-Aoki rebound formula

The second formula used is based on pile rebound at final penetration. The use of pile rebound was initially proposed by Chellis (1951) and later modified by Aoki (1989). The formula uses the direct relationship between the pile capacity mobilized during driving and the elastic shortening of the pile. The elastic shortening of the pile can be obtained by reducing the part attributable to the soil from the total measured rebound \( K \). The Chellis-Aoki rebound formula is calculated as follows:

\[ R_u = \frac{C_2 \cdot A \cdot E}{\alpha \cdot L} \]  
\[ C_2 = K - C_3 \]  

where \( R_u \) is the pile mobilized capacity, \( C_2 \) is the elastic shortening of the pile shaft, \( C_3 \) is the soil rebound for which Aoki (1989) suggests a value of 2 or 3 mm or a value close to the pile set, \( A \) is the pile section area, \( E \) is the modulus of elasticity of the pile material, \( L \) is the pile length and \( \alpha \) depends on the pile transfer between toe and friction resistance. If only the pile tip resists the load, the \( \alpha \) value is 1, whereas if the tip resistance is zero, \( \alpha \) is 0.5. For typical cases, in which part of the pile capacity is due to the tip and part comes from friction resistance, Aoki (1989) suggests \( \alpha = 0.7 \) as an approximate value. As the pile capacity had already been predicted, the authors made use of the range of \( \alpha \) estimated from the \textit{a priori} pile capacity calculations. The first 8-12 m of the soil profile is formed by soft clay, so almost 90 % of the bearing capacity is mobilized in the last few meters. Therefore, \( \alpha \) does not vary significantly (\( \mu = 0.9 \), COV = 0.025) and no bias is expected due to correlation between \textit{a priori} and likelihood functions.

Figure 7a presents the histograms associated with the likelihood function obtained from the Danish formula and
Fig. 7b for that related to Chellis-Aoki formula. Figure 7 also illustrates that the expected value of mobilized pile capacity from the statistical distribution of both selected likelihood functions was very close, as well as the COV. In fact, both results came from the same dataset, from different registers but from the same site, thus consisting of the same “state of nature”.

Figure 8 compares the pile capacity estimated with the two selected likelihood functions. In spite of the scatter, which is reasonable due to uncertainty in the hammer efficiency, elastic rebound of the soil and actual drop height, most of the results concentrate close to the 45° line between ±30%.

7. The A Posteriori Distribution

Equations 1 and 2 were used to update the SRD value for each pile estimated a priori by using both likelihood functions, resulting in the a posteriori SRD estimates. For each pile, the mean values in Eqs. 1 and 2 were obtained as the specific a priori prediction and estimates from the dynamic formulas. The variance values were derived from the statistical distribution of the whole dataset, containing all analyzed piles and the uncertainties involved. Every pile has its own updated estimates and the set of all pile forms the a posteriori distribution.

Figures 9a and 9b present the histograms associated with the a posteriori distribution obtained from the Aoki-Velloso method of SRD: (a) updated with Danish formula and (b) updated with Chellis-Aoki formula.

Figures 10a and 10b represent a posteriori distribution histograms obtained from the Vesic method updated with the Danish and Chellis-Aoki formulas respectively. The a posteriori pile capacity always has a value between that obtained a priori and the one corresponding to the likelihood function, moving closer to the distribution of lower variance. The coefficient of variation of the a posteriori distribution is always lower than that of the other distributions since the a posteriori distribution includes information from both a priori and likelihood function, reducing the uncertainty of the estimate.
Figure 11 summarizes the updating procedure for both pile capacity methods (Aoki & Velloso and Vesic). Figure 11a shows the expected values \textit{a posteriori} updated using the Danish formula as the likelihood function, whereas in Fig. 11b the Chellis-Aoki formula was applied. Unlike the \textit{a priori} estimates, where the Vesic method resulted in capacities 37\% higher than those from Aoki-Velloso, for the \textit{a posteriori} estimates this difference is very low, 7.9\%, when updated with Chellis-Aoki, and 10\% when updated with the Danish formula. After the updates, the uncertainties evidenced in the \textit{a priori} comparisons were not observed. The update greatly reduced the uncertainty between the different calculation methods, reducing the uncertainty to a significantly lower level.

8. Load Test Results

The pile load test results are summarized in Tables 3 and 4. Table 3 includes the results from nine dynamic tests and Table 4 the results of two slow maintained static load tests. Pile 148 suffered structural damage during the dynamic test and its results were not included in the following analysis. The conventional pile capacity from the static load tests was obtained through extrapolation according to Van der Veen (1953), a common method used in Brazil when physical failure is not reached during the load test. The load settlement curve of pile E-106 presented a nearly linear behavior up to the maximum testing load. That is the reason why the authors did not use the extrapolated capacity for this pile, consistent with Van der Veen (1953).

The \textit{a priori} and \textit{a posteriori} estimates of pile capacity are now compared to the load tests (8 dynamic and 1 static), as presented in Table 5 and Fig. 12. The mobilized pile resistance in the dynamic test was interpreted with CAPWAP, as presented by Rausche et al. (1972). In Fig. 12a the \textit{a priori} pile capacity estimated using the Aoki-Velloso method is represented in the vertical axis, with the statistical distribution indicated by its statistical range and the expected value by the middle point. Figures 12b and 12c compare the \textit{a posteriori} expected values updated with the Chellis-Aoki and Danish dynamic formulas.

### Table 3 - Dynamic test results.

| Pile | Embedded length (m) | Final set (mm/10 blows) | Transferred energy (kJ) | Tip resistance (%) | CAPWAP capacity (kN) |
|------|---------------------|-------------------------|-------------------------|------------------|----------------------|
| 113  | 17.4                | 5                       | 38.2                    | 21.0             | 2,850                |
| 114  | 16.4                | 10                      | 46.3                    | 27.4             | 2,790                |
| 115  | 16.4                | 5                       | 43.0                    | 26.7             | 2,770                |
| 116  | 16.1                | 5                       | 38.7                    | 28.8             | 2,950                |
| 148* | 17.4                | 20                      | 23.1                    | 7.4              | 1,450                |
| 156  | 17.9                | 10                      | 22.6                    | 15.4             | 2,530                |
| 240  | 18.8                | 1                       | 29.8                    | 16.9             | 2,400                |
| 260  | 17.6                | 1                       | 28.8                    | 14.0             | 2,320                |
| 265  | 17.7                | 9                       | 25.6                    | 8.8              | 2,310                |

Note: * Pile was damaged during the tests.
respectively. Similar results are found when the estimated \textit{a priori} values are calculated using the Vesic method. Figures 12b and 12c illustrate the decrease in uncertainty, clearly observed by the relative reduction of the statistical range for each tested pile. The comparison of Fig. 12a with Figs. 12b and 12c demonstrates that the update with both the Danish and the Chellis-Aoki formulas were quite efficient in estimating the expected pile capacity within a narrow range and approximating the expected value to the results of the pile load tests. With the exception of pile E-106, all the other tests presented in Figs. 12b and 12c indicated measured pile capacities higher than the updated. Since the tests were performed with an interval of at least 190 days after the end of driving, the higher pile capacity can be attributed both to a higher hammer energy during the tests, compared to the final driving, and also to an increase in soil resistance over time due to pore pressure dissipation. The extent of the pile capacity increase, due to both higher energy (which can cause a higher mobilized resistance) and an increase in soil resistance over time was nearly 30\% and 19\% for the Aoki & Velloso and Vesic methods, respectively.

Table 4 - Static tests results.

| Pile | Embedded length (m) | Maximum testing load (kN) | Maximum testing displacement (mm) | Permanent displacement (mm) | Van der Veen extrapolated capacity (kN) |
|------|---------------------|---------------------------|----------------------------------|-----------------------------|----------------------------------------|
| 172  | 16.9                | 1,400                     | 14.87                            | 1.25                        | 2,160                                   |
| 106  | 15.7                | 1,000                     | 8.09                             | 1.91                        | *                                       |

Note: * No Van der Veen extrapolated capacity was considered for pile 106.

Figure 12 - Comparisons between estimates of SRD (soil resistance to driving) using Aoki-Velloso method and load pile tests: (a) \textit{a priori} estimate (b) \textit{a posteriori} estimate updated with Chellis-Aoki formula and (c) \textit{a posteriori} estimate updated with Danish formula.

Table 5 - Statistical distributions of the database.

| Method                  | $Q_{rupt}$ expected value (kN) | Standard deviation (kN) | COV  |
|-------------------------|---------------------------------|-------------------------|------|
| Calculations\(^1\)      |                                 |                         |      |
| AV                      | 1,665                           | 448                     | 0.27 |
| Vesic                   | 2,262                           | 701                     | 0.31 |
| Dynamic Formulas\(^1\) |                                 |                         |      |
| Chellis-Aoki            | 1,909                           | 318                     | 0.17 |
| Danish                  | 2,072                           | 371                     | 0.18 |
| Updated by Chellis-Aoki |                                 |                         |      |
| AV                      | 1,830                           | 210                     | 0.11 |
| Vesic                   | 1,974                           | 254                     | 0.13 |
| Updated by Danish\(^1\) |                                 |                         |      |
| AV                      | 1,922                           | 223                     | 0.12 |
| Vesic                   | 2,136                           | 257                     | 0.12 |
| Dynamic Load Tests\(^2\)|                                 |                         |      |
| CAPWAP                  | 2,615                           | 239                     | 0.09 |

Note: \(^1\)645 piles; \(^2\)8 piles.
The decrease in uncertainty due to Bayesian updating can be interpreted in terms of the coefficient of variation (COV). Table 5 shows the variability ranges for each method including the expected value (mean), variance and COV with respect to all 645 piles and 8 dynamic load tests. There is a very significant decrease in the variability of the expected capacities due to updating. The \textit{a priori} pile capacity estimated using the Aoki & Velloso and Vesic methods presented COV of 0.27 and 0.31, respectively. The likelihood functions (dynamic formulas) presented lower uncertainties as they made use of the sample from the “state of nature”. Such “state of nature” uncertainty produced COV of 0.18 and 0.17 for the Danish and Chelli-Aoki formulas, respectively. After Bayesian updating, COV fell to 0.12 and 0.11 for the Aoki & Velloso method updated with Danish and Chellis-Aoki likelihood functions, respectively. For the Vesic method, COV fell to 0.12 and 0.13 using the Danish and Chellis-Aoki likelihood functions, respectively. The COV of the dynamic load tests, 0.09, was very close to that for the \textit{a posteriori} estimates. Therefore, the Bayesian update was quite effective in improving the reliability of the estimated pile capacity, reducing the uncertainty to a range very close to that obtained with dynamic tests.

Figure 13 summarizes the Bayesian procedure applied in this paper using the Aoki & Velloso method. Similar results were obtained for the Vesic method. Figure 13a shows the three statistical normal curves for the \textit{a priori} (Aoki & Velloso method), likelihood (Danish formula) and \textit{a posteriori} (Aoki & Velloso updated by Danish) distributions. Figure 13b presents the Aoki & Velloso \textit{a priori} distribution using the Chellis-Aoki formula for the likelihood and \textit{a posteriori} distributions. The flattened shape of the \textit{a priori} distribution is attributed to its high variance due to uncertainties involving the limitation of the calculation methods, spatial variability and limitations of the SPT tests. Both likelihood functions are narrower due to the lower variance and consequently lower uncertainty. The \textit{a posteriori} distributions are even narrower reflecting lower variance, and therefore even lower uncertainty. As expected, the mean \textit{a posteriori} pile resistance is between the mean values for the \textit{a priori} and likelihood distributions, being

![Figure 13 - Comparison between \textit{a priori}, likelihood and \textit{a posteriori} normal distributions of SRD (soil resistance to driving) for the Aoki-Velloso method: (a) with respect to Danish likelihood distribution and (b) with respect to Chellis-Aoki likelihood distribution.](image)
closer to the likelihood mean value due to its lower variance. Figure 13 is a real representation of Fig. 1 and shows the proper functioning of the procedure applied in this paper.

9. Conclusions

This paper presents the application of the Bayesian updating procedure to a comprehensive steel piling dataset in a sedimentary soil profile in Rio de Janeiro, Brazil.

Two methods for estimating the a priori pile capacity were used, as well as two likelihood functions, for updating the soil resistance mobilized by the piling at the end of driving. The updating procedure was able to practically eliminate significant model uncertainties of the a priori predictions.

Although different, the Chellis-Aoki and Danish likelihood functions revealed very close statistical distribution and were both efficient in reproducing a sampling from the “state of nature.” The functions also demonstrated their efficiency in obtaining an improved a posteriori estimate of pile capacity including a much lower uncertainty range.

The pile capacity a posteriori always had a value between that obtained a priori and the one corresponding to the likelihood function, thus closer to the distribution of lower variance. The coefficient of variation of the a posteriori distribution was always lower than that of the other distributions, since the a posteriori distribution includes information from both (a priori and “likelihood function”), reducing the uncertainty of the estimate.

The updated estimates were then compared to the results of dynamic and static load tests. The comparisons indicated measured pile capacity nearly 30 % and 19 % higher than the updated for the Aoki-Velloso and Vesic methods, respectively. The higher values measured may be attributed to set up effects and a higher energy adopted in the tests compared to final driving.

The Bayesian updating technique can be applied to a single pile, a group of piles, as in the present paper, as well as to a combination of estimated pile capacity and static or dynamic load tests. The practical application to different pile types, sites and various soil formations can produce relevant statistical information concerning the accuracy of different geotechnical calculation methods. Focus on continuous application can contribute significantly to a more rational design and better quality control systems, with resulting improvement to piling safety.

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