Fuzzy Paranormal Operators

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Abstract: In this paper, we introduced and discussed about fuzzy paranormal operators. A fuzzy bounded linear operator on a fuzzy Hilbert space is fuzzy paranormal if \( \|T^2 a\| \geq \|Ta\|^2 \) for every unit vector \( a \) in \( \mathcal{H} \). It is easily known that this class includes fuzzy hyponormal operators.

Keywords: Fuzzy Hilbert space, Self-adjoint fuzzy operator, Fuzzy Normal operator, Fuzzy Hyponormal operator, Fuzzy Paranormal operator.

I. INTRODUCTION

Let \( \mathcal{H} \) be a fuzzy Hilbert space and \( FB(\mathcal{H}) \) is the set of all fuzzy bounded linear operators on \( \mathcal{H} \). Biswas [10] first introduced the definition of fuzzy inner product space. In 2009, Goudarzi and Vaezpour [8] has been introduced the definition of a fuzzy Hilbert space. Sudad M Rasheed [4] was first introduced the concept and properties of adjoint fuzzy operator using the triplet \( (\mathcal{H}, \mathcal{F}, \ast) \) and which is a fuzzy Hilbert space. An operator \( T \in FB(\mathcal{H}) \) is a \( \mathcal{F} \)–continuous linear functional, there exist \( T^* \in FB(\mathcal{H}) \) such that \( (Ta, b) = (a, T^* b) \) \( \forall a, b \in \mathcal{H} \). Also \( T \) is a self – adjoint fuzzy operator if \( T = T^* \) and also it commutes with its adjoint fuzzy operator i.e. \( T \cdot T^* = T^* \cdot T \) with this \( T \) is said to be fuzzy normal operator which was introduced by Radharamani et al. [1].

If \( T \in FB(\mathcal{H}) \) is said to be fuzzy unitary operator if \( T \cdot T^* = I = T^* \cdot T \). It is also a fuzzy isometry operator from \( \mathcal{H} \) onto \( \mathcal{H} \). In 2019, Fuzzy hyponormal operators and their properties are studied by Radharamani et al.[3] and investigated many interesting properties of Fuzzy hyponormal operators similar to these of fuzzy normal operators. An operator \( T \) is said to be fuzzy hyponormal if \( T \cdot T^* \geq T \cdot T \). Also fuzzy class of N operators were defined if \( \|T^2 a\| \geq \|Ta\|^2, \forall a \in \mathcal{H}, \|a\| = 1 \).

Now we introduced fuzzy paranormal operator if \( \|T^2 a\| \|a\| \geq \|Ta\|^2, \forall a \in \mathcal{H} \) which is equivalent to \( \|T^2 a\| \geq \|Ta\|^2 \) for every unit vector \( a \) in \( \mathcal{H} \). We have given an example, some lemmas for fuzzy paranormal operator and some properties like, sum and product of fuzzy paranormal operators are also fuzzy paranormal. An operator \( T \) is invertible and fuzzy paranormal, then \( T^{-1} \) also fuzzy paranormal. An operator \( T \) is fuzzy paranormal then its powers also fuzzy paranormal, also an operator \( T \) is fuzzy normal then \( T \) and \( T^* \) are also fuzzy paranormal. We will discuss these in detail.

II. PRELIMINARIES

Definition 2.1: [9] Fuzzy Hilbert space (FH-space)

Let \( (\mathcal{H}, \mathcal{F}, \ast) \) be a FH – space with IP: \( (a, b) = \text{Sup} \{u \in R : \mathcal{F}(a, b, u) < 1\} \) \( \forall a, b \in \mathcal{H} \). If \( T \) is complete in the \( \|\| \), then \( \mathcal{H} \) is called Fuzzy Hilbert space(FH-space).

Definition 2.2: [4] Adjoint Fuzzy operator

Let \( (\mathcal{H}, \mathcal{F}, \ast) \) be a FH – space and let \( T \in FB(\mathcal{H}) \) be \( T_p \) continuous linear functional. Then \( \exists \) unique \( T^* \in FB(\mathcal{H}) \) such that \( (Ta, b) = (a, T^* b) \) \( \forall a, b \in \mathcal{H} \).

Definition 2.3: [4] Self-Adjoint Fuzzy operator

Let \( (\mathcal{H}, \mathcal{F}, \ast) \) be a FH – space with IP: \( (a, b) = \text{Sup} \{u \in R : \mathcal{F}(a, b, u) < 1\} \) \( \forall a, b \in \mathcal{H} \) and let \( T \in FB(\mathcal{H}) \) Then \( T \) is self–adjoint Fuzzy operator if \( T = T^* \).

Theorem 2.4: [4]

Let \( (\mathcal{H}, \mathcal{F}, \ast) \) be a FH – space with IP: \( (a, b) = \text{Sup} \{u \in R : \mathcal{F}(a, b, u) < 1\} \) \( \forall a, b \in \mathcal{H} \) and let \( T \in FB(\mathcal{H}) \) then \( \|Ta\| = \|T^* a\| \) for all \( a, b \in \mathcal{H} \).

Theorem 2.5:[4]

Let \( (\mathcal{H}, \mathcal{F}, \ast) \) be a FH – space with IP: \( (a, b) = \text{Sup} \{u \in R : \mathcal{F}(a, b, u) < 1\} \) \( \forall a, b \in \mathcal{H} \) and let \( T \in FB(\mathcal{H}) \) then \( T \) is self–adjoint Fuzzy operator.

Definition 2.6: [1] Fuzzy Normal operator

Let \( (\mathcal{H}, \mathcal{F}, \ast) \) be a FH – space with IP: \( (a, b) = \text{Sup} \{u \in R : \mathcal{F}(a, b, u) < 1\} \) \( \forall a, b \in \mathcal{H} \) and let \( T \in FB(\mathcal{H}) \) then \( T \) is said to be Fuzzy Normal operator if it commutes with its (fuzzy) adjoint i.e.\( T \cdot T^* = T^* \cdot T \).

Definition 2.7: [2] Fuzzy isometry operator

Let \( (\mathcal{H}, \mathcal{F}, \ast) \) be an FH-space with IP: \( (a, b) = \text{Sup} \{u \in R : \mathcal{F}(a, b, u) < 1\} \) \( \forall a, b \in \mathcal{H} \) and let an operator \( T \) on a Fuzzy Hilbert space \( \mathcal{H} \) i.e., \( T \in FB(\mathcal{H}) \) then \( T \) is said to be a Fuzzy isometry operator if \( \|Ta\| = \|a\| \) for any \( a \in \mathcal{H} \) i.e., \( (Ta, Tb) = (a, b) \).

Remark 2.8: [1]

Let \( FB(\mathcal{H}) \) the set of all fuzzy linear operators on \( \mathcal{H} \).

Definition 2.8: [2] Fuzzy Unitary operator

Let \( T \in FB(\mathcal{H}) \) is said to be a fuzzy unitary operator if \( T \) is a Fuzzy isometry operator from \( \mathcal{H} \) onto \( \mathcal{H} \).

Definition 2.9: [1] Fuzzy Hyponormal operator

Let \( (\mathcal{H}, \mathcal{F}, \ast) \) be a FH – space with IP: \( (a, b) = \text{Sup} \{u \in R : \mathcal{F}(a, b, u) < 1\} \) \( \forall a, b \in \mathcal{H} \) and let \( T \in FB(\mathcal{H}) \) then \( T \) is a fuzzy hyponormal operator if \( \|T^* a\| \leq \|Ta\| \) \( \forall a \in \mathcal{H} \) and or equivalently \( T \cdot T^* - T^* \cdot T \geq 0 \).

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Theorem 2.11: [1]
Let $T \in FB(\mathcal{H})$ be fuzzy hyponormal iff $\|T^*a\| \leq \|Ta\|$ for all $a \in \mathcal{H}$.

III. MAIN RESULTS OF FUZZY PARANORMAL OPERATORS

Then $T$ is a fuzzy paranormal operator if $\|T^2a\|\|a\| \geq \|Ta\|^2 \forall a \in \mathcal{H}$.

Note:
Let $T \in FB(\mathcal{H})$, then $T$ is a fuzzy paranormal operator if $\|T^2a\| \geq \|Ta\|^2 \forall a \in \mathcal{H}$.

Example 3.2:
Let $(\mathcal{H}, \mathcal{F}, \ast)$ be a fuzzy Hilbert space, $\mathcal{H} = l^2$, i.e. $l^2 = \{a = (a_1, a_2, a_3, \ldots) \}_{i=1}^\infty$ $\sum_{i=1}^\infty |a_i|^2 < \infty, a_i \in \mathbb{C}$ for $a \in l^2$, defined $\|a\| = (\sum_{i=1}^\infty |a_i|^2)^{\frac{1}{2}}$. Let $\mathcal{F}: l^2 \times (0, \infty) \rightarrow [0, 1]$ define an operator $T: l^2 \times (0, \infty)$ such that $T(a_1, a_2, \ldots, a_n) = (0, a_1, a_2, \ldots)$ $\forall (a_1, a_2, \ldots) \in l^2$

i) To find $T$ is linear
Take $a = (a_1, a_2, \ldots, b) = (b_1, b_2, \ldots, \in l^2$ and scalar $\alpha$.

$T(a + b) = T(a_1 + b_1, a_2 + b_2, \ldots) = (0, a_1 + b_1, a_2 + b_2, \ldots)$

$T(a + b) = T(a) + T(b)$

$T(\alpha a) = (0, \alpha a_1, \alpha a_2, \ldots)$

$= \alpha(0, a_1, a_2, \ldots)$

$= \alpha(Ta)$

ii) To find $T$ is finite
Take $(a_1, a_2, \ldots, \in l^2$.

$\|T(a_1, a_2, \ldots)\|^2 = \|(0, a_1, a_2, \ldots)\|^2$

$= \sum_{i=1}^\infty |a_i|^2$

$= \|a\|^2$

i.e. $\|T(a_1, a_2, \ldots)\|^2 = \|a\|^2$

$\|T(a)\| = \|a\|$ iff $\|T(a)\| = \|a\|$

$\Rightarrow T$ is finite

$\therefore T \in FB(l^2)$

iii) To find $T$ is fuzzy paranormal operator
Take $(a_1, a_2, \ldots, \in l^2$.

$\|T(a_1, a_2, \ldots)\|^2 = \|(0, a_1, a_2, \ldots)\|^2$

$= \sum_{i=1}^\infty |a_i|^2$

$= \|(a_1, a_2, \ldots)\|^2$

$\Rightarrow \|T(a_1, a_2, \ldots)\|^2 = \|(a_1, a_2, \ldots)\|^2$

iv) Take $(a_1, a_2, \ldots, \in l^2$.

$\|T(a_1, a_2, \ldots)\| = \|(0, a_1, a_2, \ldots)\|$

Let $T^2(a_1, a_2, \ldots) = T(T(a_1, a_2, \ldots)) = (0, 0, a_1, a_2, \ldots)$

$\|T^2(a_1, a_2, \ldots)\| = \|(0, 0, a_1, a_2, \ldots)\|$

$= \sum_{i=1}^\infty |a_i|^2$

v) Take any $(a_1, a_2, \ldots, \in l^2$.

$T(a_1, a_2, \ldots, \in l^2$)

$\|T(a_1, a_2, \ldots)\| = \|(0, a_1, a_2, \ldots)\|$

Definition 3.1:
Let $(\mathcal{H}, \mathcal{F}, \ast)$ be a FH – space with IP: $(a, b) = Sup \{u \in R: \mathcal{F}(a, b, u) < 1\}$ $\forall a, b \in \mathcal{H}$ and let $T \in FB(\mathcal{H})$.

$$\|T(a_1, a_2, \ldots)\|^2 = \|T(0, a_1, a_2, \ldots)\|^2$$

$$= \sum_{i=1}^\infty |a_i|^2$$

From (iv) and (v),

$$\|T^2(a)\| \geq \|T(a)\|^2$$

Thus $T$ is fuzzy paranormal operator.

Lemma 3.3:
Let $(\mathcal{H}, \mathcal{F}, \ast)$ be a FH – space with IP: $(a, b) = Sup \{u \in R: \mathcal{F}(a, b, u) < 1\}$ $\forall a, b \in \mathcal{H}$ and let $T \in FB(\mathcal{H})$ be a fuzzy paranormal operator then $\|T^3a\| \geq \|T^2a\|\|Ta\|$ for every unit vector $a \in \mathcal{H}$.

Proof:
For a unit vector $a \in \mathcal{H}$, Let $\|T^3a\| = \|T^2a, T^3a\|$

$= Sup \{u \in R: \mathcal{F}(T^3a, T^3a, u) < 1\}$

$= Sup \{u \in R: \mathcal{F}(T^2a, T^2a, u) < 1\}$

$= Sup \{u \in R: \mathcal{F}(T^4a, T^4a, u) < 1\}$

$= Sup \{u \in R: \mathcal{F}(T^4a, T^4a, u) < 1\}$

$= T^4a, T^3a\|

$\leq \|T^4a\|\|T^2a\|$

$\|T^3a\|^2 \geq \|Ta\|^2\|Ta\|$ [since $T$ is fuzzy paranormal]

$\Rightarrow \|T^3a\| \geq \|T^2a\|\|Ta\|$ [Hence $\|T^3a\| \geq \|T^2a\|\|Ta\|$]

Lemma 3.4:
Let $(\mathcal{H}, \mathcal{F}, \ast)$ be a FH – space with IP: $(a, b) = Sup \{u \in R: \mathcal{F}(a, b, u) < 1\}$ $\forall a, b \in \mathcal{H}$ and let $T \in FB(\mathcal{H})$ be a fuzzy paranormal operator. Then $\|T^{k+1}a\|^2 \geq \|T^ka\|^2\|T^2a\|$ for every positive integer $k \geq 1$ and every unit vector $a \in \mathcal{H}$.

Proof:
Let $T \in FB(\mathcal{H})$ be a fuzzy paranormal operator. By using the induction hypothesis, we will prove the theorem.

For the case $k = 1$, $\|T^2a\|^2 \geq \|Ta\|^2\|T^2a\|$.

Now suppose that $\|T^{k+1}a\|^2 \geq \|T^ka\|^2\|T^2a\|$ is valid for $k$. Then $k = k + 1$.

Let $\|T^{k+2}a\|^2 = \|T^{k+2}a, T^{k+2}a\|$

$= Sup \{u \in R: \mathcal{F}(T^{k+2}a, T^{k+2}a, u) < 1\}$

$= Sup \{u \in R: \mathcal{F}(T^ka, T^ka, u) < 1\}$

$= Sup \{u \in R: \mathcal{F}(T^{k+2}a, T^ka, u) < 1\}$

$= Sup \{u \in R: \mathcal{F}(T^ka, T^ka, u) < 1\}$

$= (T^{2(k+1)}a, T^2a)$

$\leq \|T^{k+1}a\|\|T^2a\|$

Since $\|T^2a\| \geq \|Ta\|^2\|a\|$ $\forall a \in \mathcal{H}$,

$\|T^{k+2}a\|^2 \geq \|T^{k+1}a\|^2\|T^2a\|\|a\|$
Hence the proof is obvious.

**Lemma 3.5:**

Let $T \in FB(H)$ be a fuzzy paranormal operator. Then $T^n$ is also fuzzy paranormal for every integer $n \geq 1$.

**Proof:**

It is sufficient to prove that if $T$ and $T^k$ are fuzzy paranormal then $T^{k+1}$ is also fuzzy paranormal operator.

For every unit vector $a$ in $H$

Let $\|T^{2(k+1)}a\|^2 = \|T^{2(k+1)}a, T^{2(k+1)}a\|$

$= \sup \{u \in R : F(T^{2(k+1)}a, T^{2(k+1)}a, u) < 1\}$

$= \sup \{u \in R : F((T^{2(k+1)})^2a, (T^{2(k+1)})^2a, u, u) < 1\}$

$= \sup \{u \in R : F(T^a, T^a, u, u) < 1\}$

$= \sup \{u \in R : F(T^a)^2a, a, u, u) < 1\}$

$\leq \|T^{2(k+1)}a\|^2 \leq \|T^{2(k+1)}a\|^2$

By the above lemma.

So $T^{k+1}$ is also fuzzy paranormal operator.

**Theorem 3.6:**

Let $T \in FB(H)$ be a self-adjoint fuzzy operator then $T$ is a fuzzy paranormal.

**Proof:**

For any $a$ in $H$ with $\|a\| = 1$, we know that $T$ is a self-adjoint fuzzy operator.

Let $\|Ta\|^2 = (Ta, Ta)$

$= \sup \{u \in R : F(Ta, Ta, u) < 1\}$

$= \sup \{u \in R : F(T^a, T^a, u) < 1\}$

$= \sup \{u \in R : F((TTa)a, a, u) < 1\}$

$= \sup \{u \in R : F(T^a, a, u) < 1\}$

$\leq \|T^a\|^2 \leq \|T^2a\|^2$

By the above lemma.

So $T$ is a fuzzy paranormal operator.

**Theorem 3.7:**

Let $T \in FB(H)$ be a fuzzy paranormal operator and self-adjoint fuzzy operator. Then $T^*$ is fuzzy paranormal.

**Proof:**

For any $a$ in $H$, $\|a\| = 1$

Let $\|T^*a\|^2 = (T^*a, T^*a)$

$= \sup \{u \in R : F(T^*a, T^*a, u) < 1\}$

$= \sup \{u \in R : F((TT^*a)a, a, u) < 1\}$

$= \sup \{u \in R : F(T^*a, a, u) < 1\}$

$\leq \|T^*\|^2 \leq \|T^2a\|^2$

By the above lemma.

So $T^*$ is a fuzzy paranormal operator.

**Theorem 3.8:**

Let $S$ and $T \in FB(H)$ is a fuzzy paranormal operator and self-adjoint fuzzy operator. Then $S + T$ and $ST$ are also a fuzzy paranormal operator.

**Proof:**

For every unit vector $a$ in $H$, we know that $\|T^2a\| \geq \|T^2a\|^2, \|S^2a\| \geq \|S^2a\|^2$ and $S = S^*, T = T^*$.

i). To prove that $S + T$ is a fuzzy paranormal operator.

Let $\|(S + T)a\|^2 = \|(S + T)a, (S + T)a\)$

$= \sup \{u \in R : F((S + T)a, (S + T)a, u) < 1\}$

$= \sup \{u \in R : F((S + T)^2a, (S + T)^2a, u, u) < 1\}$

$= \sup \{u \in R : F((S + T)^2a), (S + T)^2a, u, u) < 1\}$

$\leq \|(S + T)^2a\|^2$

Implies that $\|(S + T)a\|^2 \leq \|(S + T)^2a\|^2$

Therefore $S + T$ is a fuzzy paranormal operator.

ii). To prove that $ST$ is a fuzzy paranormal operator.

Let $\|(ST)a\|^2 = \|(ST)a, (ST)a\)$

$= \sup \{u \in R : F((ST)a, (ST)a, u) < 1\}$

$= \sup \{u \in R : F((ST)^2a, (ST)^2a, u, u) < 1\}$

$= \sup \{u \in R : F((ST)^2a), (ST)^2a, u, u) < 1\}$

$\leq \|(ST)^2a\|^2$

Implies that $\|(ST)a\|^2 \leq \|(ST)^2a\|^2$

Therefore $ST$ is a fuzzy paranormal operator.

**Theorem 3.9:**

Let $T \in FB(H)$ is a fuzzy normal operator. Then $T$ is a fuzzy paranormal operator.

**Proof:**

For every unit vector $a$ in $H$

Let $\|Ta\|^2 = (Ta, Ta)$

$= \sup \{u \in R : F(Ta, Ta, u) < 1\}$

$= \sup \{u \in R : F(T^a, T^a, u) < 1\}$

$= \sup \{u \in R : F((TTa)a, a, u) < 1\}$

$= \sup \{u \in R : F(Ta, a, u) < 1\}$

$\leq \|T^2a\|^2 \leq \|T^2a\|^2$

i.e. $\|Ta\|^2 \leq \|T^2a\|^2$

Implies that $\|Ta\|^2 \leq \|T^2a\|^2$

Therefore $T$ is a fuzzy paranormal operator.

**Theorem 3.10:**

Let $T \in FB(H)$ is a fuzzy paranormal operator and a fuzzy hyponormal operator. Then $T$ is a fuzzy paranormal operator.

**Proof:**

For every unit vector $a$ in $H$

Let $\|Ta\|^2 = (Ta, Ta)$

$= \sup \{u \in R : F(Ta, Ta, u) < 1\}$

$= \sup \{u \in R : F(T^a, T^a, u) < 1\}$

$\geq \sup \{u \in R : F((TT^a)a, a, u) < 1\}$

$\geq \|T^a, T^a, a, a\|

Since $\|T^2a\| \geq 0 \forall a \in H$.
Theorem 3.11:
Let $T_n \in FB(\mathcal{H})$ is a sequence of fuzzy paranormal operator and $T_n \rightarrow T$. Then $T$ is a fuzzy paranormal operator.

Proof:
For every unit vector $a$ in $\mathcal{H}$
\[
\|T_n a\|^2 = (Ta, a) = 0
\]
Hence $T$ is a fuzzy paranormal operator.

Theorem 3.12:
Let $T \in FB(\mathcal{H})$ is a fuzzy paranormal operator and $S$ is unitarily equivalent to $T$. Then $S$ is a fuzzy paranormal operator.

Proof:
For every unit vector $a$ in $\mathcal{H}$
\[
\|S a\|^2 = (S^* S a, a) = (T a, a) = 0
\]
Hence $S$ is a fuzzy paranormal operator.

Theorem 3.13:
An operator $T \in FB(\mathcal{H})$ is an invertible and fuzzy paranormal operator. Then $T^{-1}$ is also a fuzzy paranormal operator.

Proof:
For every unit vector $a$ in $\mathcal{H}$
\[
\|T^{-1} a\|^2 = (T^{-1} a, a) = 0
\]
Hence $T^{-1}$ is also a fuzzy paranormal operator.

Theorem 3.14:
If $T^* T \geq (T^* T)^2$, then $T$ is a fuzzy paranormal operator.

Proof:
For every $a$ in $\mathcal{H}$
\[
\|T^* T a\|^2 = (T^* T a, a) = 0
\]
Hence $T$ is a fuzzy paranormal operator.

Theorem 3.15:
An operator $T \in FB(\mathcal{H})$ is fuzzy paranormal if and only if $T^* T - 2kT^2 + k^2 \geq 0$ for all $k \in R$.

Proof:
For every $a$ in $\mathcal{H}$
\[
\|T^* T a - 2kT^2 + k^2\|^2 = 0
\]
Hence $T$ is a fuzzy paranormal operator.
Since $T$ is a fuzzy normal operator. We know that $T^* T = T T^*$ iff $\|T^* a\| = \|T a\|$ for every unit vector $a$ in $H$.

Let $\|T^* a\|^2 = (T^* a, T^* a)$

$= \sup \{ u \in R : F(T^* a, T^* a, u) < 1 \}$

$= \sup \{ u \in R : F((TT^*) a, a, u) < 1 \}$

$= \sup \{ u \in R : F(T (T^* T) a, a, u) < 1 \}$

$= (T^* T a, a) \leq \|T^* T a\| \| T a \|$\n
$\|T^* a\|^2 \leq \|T^* T a\| \| T a \|$\n
Implies that $\|T^* a\|^2 \leq \|T^* T a\|$. Therefore $T$ is fuzzy paranormal.

**Theorem 3.17**: Let $T \in FB(\mathcal{H})$ is a fuzzy paranormal operator commutes with a fuzzy isometry operator $S$. Then $TS$ is a fuzzy paranormal operator.

**Proof:**

Let $TS$ for any real number $k$. To prove $S$ is a fuzzy normal operator. That is to prove $A^* A - 2k A A^* A + k^2 \geq 0$.

Now $A^* A^2 - 2k A A^* A + k^2$

$= (T^* T)^2 (S^* S)^2 - 2k (S^* S)(T^* T) + k^2$

$= (S^* S)^2 (T^* T)^2 - 2k (S^* S)(T^* T) + k^2$

$= T^* T S^* S - 2k T S^* S + k^2$

Since $T$ is a fuzzy paranormal operator commutes with an fuzzy isometry operator $S$.

$= T^* T - 2k T S^* S + k^2$ [ by using theorem 2.15]

$A^* A - 2k A A^* A + k^2 \geq 0$

$\Rightarrow (T^* T - 2k T S^* S + k^2) \geq 0$

Hence $TS$ is a fuzzy paranormal operator.

**IV. CONCLUSION**

The conclusion that can be taken is a new idea of fuzzy paranormal operator in fuzzy Hilbert space, example and properties of fuzzy paranormal operators. Including its relationship with self-adjoint fuzzy operator, fuzzy normal operator and fuzzy hyponormal operators. In future, we hope it is very useful to find many types of fuzzy paranormal operators.

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**REFERENCES**

1. A. Radharamani et al., “Fuzzy Normal Operator in fuzzy Hilbert space and its properties”, IOSR Journal of Engineering, vol.8(7), 2018, 1-6.
2. A. Radharamani et al., “Fuzzy Unitary Operator in Fuzzy Hilbert Space and its Properties”, International Journal of Research and Analytical Reviews (IJRAR), vol.5(4), 2018, 258-261.
3. A.Radharamani,A.Brindha, “Fuzzy Hyponormal operator in Fuzzy Hilbert space”, International journal of mathematical archive(BMA), vol.10(1), 2019, pp 6-12.
4. Sudhakar Rashheed, “Self-adjoint Fuzzy Operator in Fuzzy Hilbert Space and its Properties”, Journal of Zankoy Sulaimani, vol.19(1), 2017, 233-238.
5. K. Katsaras, “Fuzzy topological vector space-II, Fuzzy Sets and Systems”, vol.12, 1984, 143-154.
6. C. Felbin, “Finite Dimensional Fuzzy Normed Linear Space, Fuzzy Sets and Systems”, vol.48, 1992, 239-248.
7. J.K. Kohli and R. Kumar, “Linear mappings, Fuzzy linear spaces, Fuzzy inner product spaces and Fuzzy Co-inner product spaces”, Bull. Calcutta Math. Soc., vol.87, 1995, 237-246.
8. M. Goudarzi and S.M. Vaezpour, “On the definition of Fuzzy Hilbert spaces and its Applications”, J. Nonlinear Sci. Appl., vol.2(1), 2009, 46-59.
9. P. Majumdar and S.K. Samanta, “On Fuzzy inner product spaces”, J. Fuzzy Math., vol.16(2), 2008, 377-392.
10. R. Biswas, “Fuzzy Inner Product Spaces & Fuzzy Norm Functions”, Information Sciences, vol.53, 1991, 185-190.
11. R. Saadati and S.M. Vaezpour, “Some results on fuzzy Banach spaces”, J. Appl. Math. and computing, vol.17(1), 2005, 475-488.
12. S.C. Cheng, J.N. Mordeson, “Fuzzy linear operators and Fuzzy normed linear spaces”, Bull. Cal. Math.Soc, vol.86, 1994, 429-436.
13. Yongfusu, “Riesz Theorem in probabilistic inner product spaces”, Inter.Math.Forum, vol.2(62), 2007, 3073-3078.
14. T. Bagand S.K. Samanta, “Finite Dimensional fuzzy normed linear spaces”, J.Fuzzy Math., vol.11(3), 2003, 687-705.
15. P.J.Dowari, N.Goswami, “A study of paranormal operators on Hilbert spaces”, International Journal of Advanced Information Science and Technology, vol.5(8), 2016, 69-74.
16. Takayuki Furuta, “On the class of paranormal operators”, J. Appl. Math. and computing, vol.43, 1997, 594-598.
17. N.L. Braha, M.Lohaj and F.H.Masevci and S.H.Lohaj, “Some properties of paranormal and hyponormal operators”, Bull. Math. Anal.and Appl., vol.1(2), 2009, 23-35.
18. Gunawan, D.A.Yuwantiningish and M.Muhammad, “Expansion of Paranormal Operator”, IOP conferences: Journal of Physics, 2019.