Threebranes in twelve dimensions

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Abstract

In this note we discuss the theory of super-threebranes in a spacetime of signature (10,2). Upon reduction, the threebrane provides us with the classical representations of the M-2-brane and the type IIB superstring. Many features of the original super (2+2)-brane theory are clarified. In particular, the spinors required to construct the brane action and the (10,2) superspace are discussed.

1 Introduction

Recently we have seen great increases in the understanding of non-perturbative string theory in terms of the eleven dimensional M-theory structure [1, 2]. This understanding has been greatly enhanced by the realisation that many problems may be solved by the introduction of a signature (10,2) ‘F-theory’ [3]. To investigate this proposal further we discuss the formulation of a Green-Schwarz threebrane in twelve dimensions [4].

2 Supersymmetric threebrane in 12D

Traditional supersymmetry p-branes propagate in a superspace which is invariant under the action of some supersymmetric extension of the Poincaré group. In 11D this essentially involves the addition of the term

\[ \{Q_{11}^\alpha, Q_{11}^\beta\} = (\gamma_\rho)^{\alpha\beta} P_\rho + (\gamma^p P_1 P_2)^{\alpha\beta} Z_{p1p2} + (\gamma^{p1...p5})^{\alpha\beta} Z_{p1...p5}, \]  

(2.1)

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where $\gamma^p$ satisfy the 11D Clifford algebra and $Q_{11}$ is a Majorana spinor. This algebra is maximal, possessing $(528+528)$ degrees of freedom, and also reproduces the type IIA and IIB algebras with the help of dimensional reduction and T-duality 4. However, all of these algebras may be obtained via a reduction of another maximal system: the ‘F-algebra’, given by

$$\{Q^\alpha, Q^\beta\}_{(10,2)} = \frac{1}{2!} (CT^{\mu\nu})^{\alpha\beta} Z_{\mu\nu} + \frac{1}{6!} \left( CT^{\mu\nu\rho\sigma\lambda\delta} \right)^{\alpha\beta} Z^{+}_{\mu\nu\rho\sigma\lambda\delta}$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = M_{\nu\sigma} \eta_{\mu\rho} + M_{\mu\rho} \eta_{\nu\sigma} - M_{\nu\rho} \eta_{\mu\sigma} - M_{\sigma\mu} \eta_{\nu\rho},$$

(2.2)

where $Z_{\mu\nu}$ and $Z^{+}_{\mu\nu\rho\sigma\lambda\delta}$ are central charges, $Z^{+}_{\mu\nu\rho\sigma\lambda\delta}$ being self-dual, and $Q^\alpha$ is Majorana-Weyl. This seems to be a natural algebra to study in twelve dimensions 4, and leads to the existence of a threebrane solution. We initially formulate the superspace underlying the F-algebra and then discuss the formulation of the threebrane4.

### 2.1 The Superspace

Given a super Lie group $G$ and some subgroup $H$ we may define a coset supermanifold $M = G/H$. Ordinary superspace is formed by considering the coset of the super Poincaré group by the Lorentz rotations. In 12D we perform an analogous procedure for the F-algebra 2.2. We define

$$G(X, \theta, \omega) = \exp(\theta^\alpha Q^\alpha + X^{\mu\nu} Z_{\mu\nu}) \exp(\frac{1}{2} \omega^{\mu\nu} M_{\mu\nu}).$$

(2.3)

To generate an infinitesimal change in the coordinates we act on the left with an infinitesimal group element $G(\delta X, \delta \theta, \delta \omega)$. From these changes we may generate invariant forms on the superspace

$$\Pi^\alpha = d\theta^\alpha, \quad \Pi^{\mu\nu} = dX^{\mu\nu} - \frac{1}{2} \theta^\alpha (\Gamma^{\mu\nu})^{\alpha\beta} d\theta^\beta.$$  

(2.4)

It is natural to consider the F-algebra as descending from a curved space de Sitter algebra 3, in which the 2-form $Z_{\mu\nu}$ obeys the same algebraic relationships as the Lorentz generator $M_{\mu\nu}$ with itself and $Q^\alpha$. Due to the non-commuting nature of the $Z_{\mu\nu}$ generator in this case, the action of the supergroup on the manifold is highly non-linearly realised. However, a careful analysis of the expansion of the transformed group elements shows that we may obtain a linear realisation of the supersymmetry if one of the following conditions is met:

$$\begin{align*}
(i) \quad (\Gamma_{\mu\nu})^{\alpha[\beta} (\Gamma^{\mu\nu})^{\gamma]\delta} &= 0 \\
(ii) \quad \bar{\Phi} C T^{\mu\nu} \Phi &= 0.
\end{align*}$$  

(2.5)

\footnote{In the following discussion of the properties of the threebrane we set the central 6-form charge to zero for simplicity.}
In case (i) the invariant forms are still written as in (2.4), whereas if the so-called ‘purity’ condition (ii) is met then the spin space becomes trivial: 
\[ \Pi^a \to d\theta^a, \quad \Pi^\mu\nu \to dX^\mu\nu. \] 

### 2.2 The threebrane

The basic threebrane action in the new superspace is defined in exact analogy with the traditional Green-Schwarz string and membrane:
\[ S = \int d^4\xi \sqrt{\det(\Pi^\mu\nu_i \Pi^\mu\nu_i)}, \] 

where the \( \Pi^\mu\nu_i \) are the pullback of the forms (2.4) to the (2+2)-dimensional worldvolume. By construction, this algebra is manifestly spacetime supersymmetric, whereas in order for worldvolume supersymmetry to occur we must obtain a matching of bosonic and fermionic excitations on the brane. To see that this is the case even for the F-algebra, note that a superspace consists of the disjoint union of a bosonic piece \( B \) and a fermionic piece \( F \). The supersymmetry generator \( Q \) maps \( B \) onto a proper subspace of \( F \) and vice-versa. Due to the group structure of the superspace this implies that the mapping is a bijection, and therefore the bosonic and fermionic degrees of freedom must match up. The \((2+2)(=3)\)-brane has eight transverse scalar fields propagating on the worldsheet, whereas the on-shell Majorana-Weyl spinors have sixteen real degrees of freedom. In order to obtain a match we must project out half of the remaining spin degrees of freedom. For super-Poincaré brane theories, the additional fermionic excitations are lost due to a \( \kappa \)-symmetry of the full action consisting of a Dirac term of type (2.7) plus a Wess-Zumino integral \[5\]. No such symmetry may occur in the 12D theory \[4\]. However, to obtain a linear realisation of the superspace theory of the underlying \textit{de Sitter} algebra we need to make a projection of the spinor operators such that (2.5) holds. This has the same effect as a \( \kappa \)-symmetry and gives us the required \((8+8)\) worldvolume degrees of freedom. Note that since branes are microscopic objects, and therefore insensitive to large scale structure, we choose to keep this restriction in the flat space limit.

#### 2.2.1 Reduction to lower dimensions

We now consider the reduction of the 12D threebrane. We reduce on the timelike direction 12 to 11D and on a null torus to 10D. Reduction to 10D and 11D requires us to act on the spinor indices with the projectors \( P_{10} = \frac{1}{2}(1 + \Gamma_0 \ldots \Gamma_9) \) and \( P_{11} = \frac{1}{2}(1 + \Gamma_0 \ldots \Gamma_9 \Gamma_{11}) \) respectively. It is a simple matter to show that one obtains two 10D spinors of the \textit{same} chirality and one 11D spinor after these reductions. Hence the theories are projected down onto the IIB and
11D sectors of M-theory. We now consider the reduction of the action (2.7), concentrating on the circle compactification to 11D. Upon reduction we find that the $Z^{p12} \rightarrow P_{11}^p$ and $Z^{pq} \rightarrow Z_{11}^{pq}$, in an obvious notation. There are two ways in which the superspace forms will reduce. In the case for which the first equation of (2.5) holds then the forms possess a torsion term as in (2.4) and reduce as follows

$$\Pi^{p12} = dX^{p12} - \frac{1}{2} \theta_\alpha (\Gamma^{p12})^{\alpha\beta} d\theta_\beta \longrightarrow \Pi^p = dX^p - \frac{1}{2} \theta_\alpha (\gamma^p)^{\alpha\beta} d\theta_\beta ,$$

(2.8)

where we set the superspace terms $Z_{11}^{pq}$ equal to zero as usual. Thus we obtain the usual Green-Schwarz actions in lower dimensions. In the case for which the spinors are restricted so as to satisfy the equation $\tilde{\mathcal{P}} \mathcal{C} \Gamma^{i\mu} \mathcal{P} = 0$ the superspace is trivially realised; there then exists a very simple (2+2)-brane action which possesses a manifest supersymmetry between the bosons and fermions:

$$S = \int d^4\xi \sqrt{\det(\Pi^A_i \Pi^B_j \mathcal{G}_{AB})} ,$$

(2.9)

in which the superspace ‘metric’ is defined by the union of the flat spacetime metric $\eta$ and the charge conjugation matrix $C$, as $\mathcal{G} = \text{diag}(\eta, C)$. The forms in the action are now trivial: $\Pi^A = (dX^\mu, d\theta^\alpha)$. In this situation it is less clear that the action will reduce to the correct super-Poincaré invariant action under dimensional reduction. However, when one performs a superspace compactification then it becomes clear that a natural torsion term of the form $\theta_\alpha (\gamma^\mu)^{\alpha\beta} d\theta_\beta$ may be introduced into the expression for $\Pi^\mu$ [4]. Thus we do still recover the traditional Green-Schwarz actions in lower dimension. However, the addition of this torsion term is not compulsory and thus reduction may also lead to a torsion-free superspace sector of M-theory. A discussion of such supersymmetries is presented elsewhere [8].

3 Conclusion

We have outlined the formulation of the three-brane in twelve dimensions. Upon reduction we find that the superspaces associated with the F-algebra reduce to those for the Poincaré algebra, and that the threebrane action reduces to that for a usual Green-Schwarz string or membrane. In addition, reduction of the 12D spinors leads to one spinor in 11D and two spinors of the same chirality in 10D. Thus we obtain the classical representations of the type IIB string and the M-2-brane, directly from twelve dimensions.

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