Scour downstream of a corrugated apron under wall jets

Rakesh Kumar Chaudharya,b,*, Zulfequar Ahmadc and Surendra Kumar Mishraa

a Department of Water Resources Development and Management, Indian Institute of Technology Roorkee, Roorkee, India
b Institute for Integrated Hydro Environment Research, Kathmandu, Nepal
c Department of Civil Engineering, Indian Institute of Technology Roorkee, Roorkee, India

*Corresponding author. E-mail: logtokumarrakesh063@gmail.com; rchaudhary@wr.iitr.ac.in

ABSTRACT

Experiments were performed over smooth and corrugated aprons with different corrugation dimensions to study the scour and flow characteristics under submerged wall jets condition. The scour depth and length are significantly lower for corrugated than smooth rigid aprons. The maximum reductions in scour depth and length are 79 and 83%, respectively. Optimum scour depth and length are found for aspect ratio (ratio of corrugation wave length to amplitude) three for corrugated apron. The factors affecting scour depth and length were analyzed graphically, and empirical equations are proposed for predicting maximum scour depth and length, and the point of maximum scour depth for corrugated aprons. Velocity, turbulence characteristics, and Reynolds stress in scour holes for smooth and corrugated aprons were also studied.

Key words: corrugated apron, Reynolds shear stress, scour, turbulence, wall jets

HIGHLIGHT

• This paper presents the scour downstream of a corrugated apron and flow characteristics under submerged wall jets. Here, scour depth and length reduces more significantly than for the other apron. In this, we have tried to develop an empirical equation on single size sediment considering all the flow parameters and apron parameters. Besides this, we have also conducted study related to turbulence and shear stress and velocity vector profile.

INTRODUCTION

Hydraulic structures constructed on loose soil need protection against erosion occurring in their vicinity. Scour occurs when streamflow shear stress exceeds the critical shear stress of the bed material. Appropriate design of erosion protection works requires thorough study of the scour profile and dimensions, and the flow characteristics within the scour hole. Several studies of scour hole and flow characteristics downstream of sluice gates have been done (Hassan & Narayanan 1985; Chatterjee et al. 1995; Balachandar & Kells 1997, 1998; Balachandar et al. 2000; Dey & Sarkar 2006; Bey et al. 2007, 2008; Farhoudi & Smith 2010; Guan et al. 2014; Melville & Lim 2014; Aamir & Ahmad 2015, 2019; Pandey et al. 2015). Different appurtenances - for example, chutes, sills, and baffle blocks - are generally placed in stilling basins to stabilize flow by creating turbulence to dissipate energy. Peterka (1984) proposed hydraulic jump stilling basins for low to high Froude numbers. Helal et al.(2013) used sills of different heights and in different positions to reduce scour, and found that higher sills and fully covered floors yield less scour, and vice versa.

Pillai & Unny (1964) studied the effects of different baffle block shapes and found that wedge-shaped blocks with a vertex angle of 120° dissipate more energy in a shorter length than rectangular blocks. Helal (2014) found that the use of floor water jets reduces scour depth by 50 to 90% and scour hole length by 42 to 85% compared to floors without water jets. According to Dey & Sarkar (2006), a launching apron (IRC-89 1985) of 18 mm stones downstream of a solid apron on an unconsolidated bed over a length twice the maximum scour depth can reduce scour depth significantly. A comprehensive review of scour due to wall jets was made by Aamir & Ahmad (2016).

The concept of corrugated aprons originated from an experimental study on a circular corrugated culvert in which the flow was similar to open channel turbulent flow (Ead et al. 2000). The velocity decreased appreciably near the surface of a corrugated pipe, indicating that this region could be helpful for fish passage. Later, Rajaratnam & Ead (2002) pioneered the study of flow characteristics of submerged jumps on corrugated
The scour depth and length were reduced for fine sand by 63.4 and 30.2%, respectively, and for coarse sand, by 44.2 and 20.6%, compared to a classical jump, and showed that minimum scour depth occurs at optimal aspect ratio, $S/t = 3$ (where $S$ = corrugation wavelength; $t$ = corrugation amplitude) – see Figure 1. They also showed that scour depth and length vary inversely with mean grain size. Basiouny et al. (2018) studied the effects of a U-shaped corrugated apron on scour downstream of radial gates under submerged jump conditions, and found that average scour depth and length are 44 and 29% less, respectively, than with a smooth apron. Similarly, Chaudhary et al. (2019) studied the scour and flow characteristics downstream of a corrugated apron under free jets.

A corrugated apron with triangular profiles is not pragmatic as the pointed crest may change to a sinusoidal form over time due to abrasion. However, sinusoidal profile corrugation is more robust and resists abrasion better than corrugation with triangular profiles. It is noted in this context that there are very few empirical equations for estimating maximum scour depth and length for sinusoidal corrugated aprons, and that not all of the parameters responsible for scour are considered. It is believed that, to date, no previous work has been done on variations in turbulence structures in scour holes due to corrugated aprons under submerged wall jets. The primary objective of this work was to study the variation in scour hole dimension and turbulence, and formulate empirical relationships for maximum scour depth and length, and the point of maximum scour depth.

**MATERIALS AND METHODS**

**Experimental setup**

The experiments were conducted in the Hydraulics Laboratory at the Department of Civil Engineering, Indian Institute of Technology Roorkee, India. The flume length, width, and depth are 10, 0.60, and 0.54 m, respectively. The flume consists of a sluice gate at the upstream end and an electrically operated tailgate downstream. Discharge was regulated with a valve on the inlet. A concrete apron 1.0 m long, 0.6 m wide, and 0.2 m deep was cast under the sluice gate – see Figure 2. A smooth, 0.6 × 0.6 m, acrylic sheet was fixed to the concrete apron,

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**Figure 1** | Corrugation definitions.

**Figure 2** | Side view of the experimental flume.
and a $1.9 \times 0.6$ m mobile sand bed, $0.2$ m deep, was provided downstream of the apron. The sand characteristics were: median size $d_{50} = 1.8$ mm, geometric standard deviation $\sigma_g = 1.15$, sorting coefficient $S_o = 1.06$, coefficient of uniformity $C_u = 1.37$, specific gravity $G = 2.65$, and angle of repose $\phi = 31.5^\circ$.

Corrugated galvanized iron sheet was used to make $0.6$ m long, $La$, with set wavelengths, $S$, and amplitudes, $t$, as shown in Figure 2. The wavelengths selected were 65, 70, 80 and 120 mm, and the amplitudes 15, 20, 25, 30, and 35 mm, in appropriate combinations comprising 8 sets.

**Dimensional analysis**

Taking all basic parameters into account, the maximum scour depth ($d_s$) and length ($L_s$) downstream of a corrugated apron can be expressed as:

$$d_s, L_s = f(g, \rho, \rho_s, \mu, a, y_t, v_1, S, t, d_{50}, La)$$

(1)

where $g$ is the acceleration due to gravity, $\rho$ the density of water, $\rho_s$ the density of sand, $\mu$ the viscosity of water, $a$ the incoming flow depth (sluice gate opening), $y_t$ the tailwater depth, $v_1$ the incoming flow velocity, $S$ the corrugation wavelength and $t$ its amplitude, $d_{50}$ the sediment's median grain size, and $La$ the apron length.

Taking $\rho, v_1,$ and $a$ as repeating variables, and using the Buckingham Pi theorem, dimensional analysis yields:

$$\frac{d_s}{a} = f\left(\frac{v_1}{\sqrt{ga}}, \frac{\rho_s}{\rho}, \frac{\rho a v_1}{\mu}, \frac{y_t}{a}, \frac{S}{a}, \frac{t}{a}, \frac{d_{50}}{a}, \frac{La}{a}\right)$$

(2)

$$\frac{L_s}{a} = f\left(\frac{v_1}{\sqrt{ga}}, \frac{\rho_s}{\rho}, \frac{\rho a v_1}{\mu}, \frac{y_t}{a}, \frac{S}{a}, \frac{t}{a}, \frac{d_{50}}{a}, \frac{La}{a}\right)$$

(3)

**Test procedure**

Initially, the sand bed was covered with plyboard and weights to prevent movement. The discharge was passed, and the tailwater level maintained as needed to ensure that the wall jets were submerged. After flow stabilization, the plyboard was removed gradually to avoid disturbing the sand bed. Scour profiles were traced on paper attached to the flume's glass wall, as shown in Figure 3, for both smooth and corrugated aprons. The scour depth and length at different times were noted. After 360 minutes the scour depth remained constant, the scour hole was stable, and the flow and turbulence characteristics remained the same.

To ensure that the maximum scour bed remained stationary – i.e., was stable – during flow measurement, it was sprayed uniformly, when dry, with water-diluted, synthetic resin and allowed to set for 24 hours.

Flow data were measured using a Nortek Vectrino, 5 cm, down-looking, acoustic doppler velocimeter with four probes. The sampling frequency and time were adjusted to 50 Hz and 3 to 5 minutes, respectively. Data with signal-to-noise ratios exceeding 15, and correlation values exceeding 70 were only taken after filtration.

Temporal variations in the scour profile, at $t = 2, 5, 15, 30, 45, 60, 120,$ and 360 minutes, were drawn on tracing paper as scour proceeded – see Figure 3.
Figure 4 is a definition diagram of a scour hole downstream of the corrugated apron under submerged wall jet conditions. The scour profile was two-dimensional (Figure 5).

Figure 4 | Definition diagram of scour downstream of a corrugated apron under submerged wall jet conditions.

Scour profiles and hydraulic parameters were measured for at least four sets of upstream gate opening for every corrugated apron model. The discharge measurements were made with an ultrasonic flow meter fitted on the inlet pipe. Flow depth was measured with a pointer gauge to an accuracy of 0.1 mm. The hydraulic parameters and scour dimensions for each set of runs are summarized in Tables 1 and 2.

Table 1 | Submerged wall jet conditions: smooth apron

| Runs | V (m/s) | a (m) | Y_l (m) | d_s (m) | l_s (m) | F |
|------|---------|-------|---------|---------|---------|---|
| 1B   | 2.778   | 0.015 | 0.165   | 0.135   | 1.12    | 7.24 |
| 2B   | 2.083   | 0.020 | 0.165   | 0.125   | 1.05    | 4.70 |
| 3B   | 1.667   | 0.025 | 0.165   | 0.110   | 0.97    | 3.37 |
| 4B   | 1.190   | 0.035 | 0.165   | 0.101   | 0.92    | 2.03 |
| 5B   | 0.926   | 0.045 | 0.165   | 0.095   | 0.89    | 1.39 |
| 6B   | 2.111   | 0.015 | 0.165   | 0.088   | 0.93    | 5.50 |
| 7B   | 1.583   | 0.020 | 0.165   | 0.076   | 0.79    | 3.57 |
| 8B   | 1.267   | 0.025 | 0.165   | 0.064   | 0.70    | 2.56 |
| 9B   | 0.905   | 0.035 | 0.165   | 0.055   | 0.645   | 1.54 |
| 10B  | 0.792   | 0.040 | 0.165   | 0.049   | 0.57    | 1.26 |
| 11B  | 1.667   | 0.025 | 0.190   | 0.062   | 0.84    | 3.37 |

RESULTS AND DISCUSSION

Scouring

The scour profiles with the submerged wall jets had two parts; that is, scour hole and dune – see Figure 5. Dunes formed due to flow reversal under submerged wall jet conditions. Initially, just after the removal of the ply board cover, the flow exerts shear stress to the mobile bed, and scour of that starts rapidly both vertically and horizontally. Flow separation occurred at the end of the apron after development of a level difference between the apron and scour bed. The split flow reattached at the deepest point of the scour hole, before leaving the hole. Here the sediment transport mode was suspended and bed load transport. Initially the flow began digging the hole
| Runs | S (m) | t (m) | V (m/s) | a (m) | y (m) | d (m) | l (m) | F    |
|------|-------|-------|---------|-------|-------|-------|-------|------|
| 1C   | 0.12  | 0.035 | 2.083   | 0.020 | 0.165 | 0.086 | 0.71  | 4.70 |
| 2C   | 0.12  | 0.035 | 1.667   | 0.025 | 0.165 | 0.079 | 0.59  | 3.37 |
| 3C   | 0.12  | 0.035 | 1.190   | 0.035 | 0.165 | 0.059 | 0.32  | 2.03 |
| 4C   | 0.12  | 0.035 | 0.926   | 0.045 | 0.165 | 0.048 | 0.27  | 1.39 |
| 5C   | 0.12  | 0.030 | 2.083   | 0.020 | 0.165 | 0.107 | 0.61  | 4.70 |
| 6C   | 0.12  | 0.030 | 1.667   | 0.025 | 0.165 | 0.094 | 0.47  | 3.37 |
| 7C   | 0.12  | 0.030 | 1.190   | 0.035 | 0.165 | 0.081 | 0.38  | 2.03 |
| 8C   | 0.12  | 0.030 | 0.926   | 0.045 | 0.165 | 0.065 | 0.35  | 1.39 |
| 9C   | 0.07  | 0.015 | 2.083   | 0.020 | 0.165 | 0.087 | 0.71  | 4.70 |
| 10C  | 0.07  | 0.015 | 1.667   | 0.025 | 0.165 | 0.079 | 0.59  | 3.37 |
| 11C  | 0.07  | 0.015 | 1.190   | 0.035 | 0.165 | 0.069 | 0.38  | 2.03 |
| 12C  | 0.07  | 0.015 | 0.926   | 0.045 | 0.165 | 0.057 | 0.35  | 1.39 |
| 13C  | 0.07  | 0.020 | 2.083   | 0.020 | 0.165 | 0.064 | 0.55  | 4.70 |
| 14C  | 0.07  | 0.020 | 1.667   | 0.025 | 0.165 | 0.045 | 0.34  | 3.37 |
| 15C  | 0.07  | 0.020 | 1.190   | 0.035 | 0.165 | 0.036 | 0.21  | 2.03 |
| 16C  | 0.07  | 0.020 | 0.926   | 0.045 | 0.165 | 0.024 | 0.19  | 1.39 |
| 17C  | 0.07  | 0.025 | 2.083   | 0.020 | 0.165 | 0.081 | 0.57  | 4.70 |
| 18C  | 0.07  | 0.025 | 1.667   | 0.025 | 0.165 | 0.071 | 0.49  | 3.37 |
| 19C  | 0.07  | 0.025 | 1.190   | 0.035 | 0.165 | 0.058 | 0.41  | 2.03 |
| 20C  | 0.07  | 0.025 | 0.926   | 0.045 | 0.165 | 0.045 | 0.38  | 1.39 |
| 21C  | 0.08  | 0.020 | 2.083   | 0.020 | 0.165 | 0.081 | 0.57  | 4.70 |
| 22C  | 0.08  | 0.020 | 1.667   | 0.025 | 0.165 | 0.071 | 0.49  | 3.37 |
| 23C  | 0.08  | 0.020 | 1.190   | 0.035 | 0.165 | 0.058 | 0.41  | 2.03 |
| 24C  | 0.08  | 0.020 | 0.926   | 0.045 | 0.165 | 0.045 | 0.38  | 1.39 |
| 25C  | 0.07  | 0.020 | 2.111   | 0.015 | 0.165 | 0.045 | 0.36  | 5.50 |
| 26C  | 0.07  | 0.020 | 1.583   | 0.020 | 0.165 | 0.036 | 0.25  | 5.57 |
| 27C  | 0.07  | 0.020 | 1.267   | 0.025 | 0.165 | 0.058 | 0.29  | 2.56 |
| 28C  | 0.07  | 0.020 | 0.905   | 0.035 | 0.165 | 0.021 | 0.15  | 1.54 |
| 29C  | 0.07  | 0.020 | 0.792   | 0.040 | 0.165 | 0.016 | 0.105 | 1.26 |
| 30C  | 0.065 | 0.015 | 2.111   | 0.015 | 0.165 | 0.076 | 0.83  | 5.50 |
| 31C  | 0.065 | 0.015 | 1.583   | 0.020 | 0.165 | 0.062 | 0.62  | 3.57 |
| 32C  | 0.065 | 0.015 | 1.267   | 0.025 | 0.165 | 0.052 | 0.54  | 2.56 |
| 33C  | 0.065 | 0.015 | 0.905   | 0.035 | 0.165 | 0.048 | 0.43  | 1.54 |
| 34C  | 0.065 | 0.015 | 0.792   | 0.040 | 0.165 | 0.032 | 0.35  | 1.26 |
| 35C  | 0.08  | 0.020 | 2.111   | 0.015 | 0.165 | 0.071 | 0.76  | 5.50 |
| 36C  | 0.08  | 0.020 | 1.583   | 0.020 | 0.165 | 0.058 | 0.55  | 3.57 |
| 37C  | 0.08  | 0.020 | 1.267   | 0.025 | 0.165 | 0.046 | 0.45  | 2.56 |
| 38C  | 0.08  | 0.020 | 0.905   | 0.035 | 0.165 | 0.032 | 0.39  | 1.54 |
| 39C  | 0.08  | 0.020 | 0.792   | 0.040 | 0.165 | 0.027 | 0.31  | 1.26 |
| 40C  | 0.12  | 0.035 | 2.111   | 0.015 | 0.165 | 0.063 | 0.64  | 5.50 |
| 41C  | 0.12  | 0.035 | 1.583   | 0.020 | 0.165 | 0.052 | 0.49  | 3.57 |
| 42C  | 0.12  | 0.035 | 1.267   | 0.025 | 0.165 | 0.042 | 0.40  | 2.56 |
| 43C  | 0.12  | 0.035 | 0.905   | 0.035 | 0.165 | 0.035 | 0.32  | 1.54 |
| 44C  | 0.12  | 0.035 | 0.792   | 0.040 | 0.165 | 0.021 | 0.25  | 1.26 |
| 45C  | 0.07  | 0.030 | 2.080   | 0.020 | 0.165 | 0.060 | 0.46  | 4.70 |

(Continued.)
vertically and horizontally. The eroded sediment was moved downstream to leave the scour hole, while some sediment moved back, upstream of the scour hole, due to flow reversal. As the flow depth increased, the shear stress exerted by the bottom layer of fluid became much lower than that of the top layer.

Figure 5 shows the scour profiles for a particular flow condition for a smooth apron – run 5B. Initially, the scour profile’s vertical and horizontal dimension increase rapidly for some time and then decrease to equilibrium. Scour length exceeds scour depth since the fluid’s shear stress decreases with increasing water depth in the scour hole. The figure also shows that the rate of increase in dune height is less than that of scour depth, and most of the scoured material goes over the dune crest and downstream, widening the dune base. Figure 6 shows the scour profiles for a corrugated apron with \( S = 120 \) mm and \( t = 35 \) mm (run 3C). The nature of the curves is similar to those in Figure 5 but the scour dimensions are smaller than those arising from a smooth apron.

**Temporal variation of maximum scour depth**

The time of maximum scour depth was measured for different sets of smooth and corrugated aprons for the same median sediment size, \( d_{50} = 1.80 \) mm, and compared with the equation given by Sumer *et al.* (1993) – Equation (4). Sumer *et al.* proposed an instantaneous scour depth \( d_{st} \) at any time \( t' \) as:

\[
d_{st} = d_{sa}(1 - e^{-t/T})
\]

where \( d_{sa} \) is the maximum scour depth and \( T \) a substantial time. Initially, scour depth and length increase rapidly, then they decrease. This is shown in Figure 7 for both smooth and corrugated aprons (runs 5B, 2C, 22C, 14C,
and 10C) at constant Froude number $F = 3.36$. The time required to develop substantial scour depth, known as the time scale, is represented by $T$ and can be estimated from the $d_s$ versus $t'$ curve by measuring the slope of the tangent to the curve, $d_s(t')$ at $t' = 0$. For sediment with $d_{50} = 1.80$ mm, $T = 25$ minutes. Scour depth variation with time is shown in Figures 7 and 8. The observed data are in good agreement with Equation (4).

Figure 8 shows the variation of scour depth with time in normalized form, for smooth and corrugated aprons. As can be seen, 50 to 60% of the scour depth is attained within 1 to 10% of time $t'$. 

Factors affecting maximum scour depth and length

Froude number

Figure 9 shows the variation in normalized scour depth with Froude number for six different corrugated aprons and one smooth one. The normalized scour depth $d_{s}/a$ increases with increasing Froude number. Figure 10 shows that normalized maximum scour length is proportional to Froude number, and that, for the smooth
apron, the maximum scour depth and length are greater than those for corrugated aprons for the same Froude number. The smallest scour depth and length are found when \( S = 70 \text{ mm} \) and \( t = 25 \text{ mm} \).

**Aspect ratio, \( S/t \)**

Five corrugated aprons having 70 mm wavelength and different amplitudes, \( t = 15, 20, 25, 30, \) and \( 35 \text{ mm} \), were used to determine the best corrugation dimension to minimize scour. Aspect ratio (\( S/t \)) variation with normalized maximum scour depth is shown in Figure 11. As \( S/t \) increases, the maximum scour depth decreases, to \( S/t = 3.0 \). Further increases in \( S/t \) causes \( ds/a \) to increase – i.e., \( S/t = 3.0 \) gives the minimum normalized scour depth in the range of \( S/t \) tested. Similar results were found for normalized maximum scour length – Figure 12. Analysis of the 52 runs using corrugated aprons and 11 using a smooth one, shows that the maximum reductions in scour depth and length – both values calculated – were 78.95 and 83.15%, respectively.

![Figure 8](image1.png)  
**Figure 8** | Evolution of scour depth with time in normalized form.

![Figure 9](image2.png)  
**Figure 9** | Variation of normalized maximum scour depth with Froude number for different sets of corrugated and smooth aprons.
Figure 10 | Variation of normalized maximum scour length with Froude number for different sets of corrugated and smooth aprons.

Figure 11 | Variation of normalized maximum scour depth, $d_s/a$, with aspect ratio, $S/t$.

$S/a$ and $t/a$

Figure 13 shows that normalized maximum scour depth increases with normalized wavelength ($S/a$) for corrugated aprons, where other parameters remain constant. For a constant amplitude ($t$) and Froude number, normalized maximum scour depth increases with the apron’s corrugation wavelength $S$. This is because the number of corrugations decreases with increasing $S$ and the apron becomes flatter, for the same $t$, resulting in lower value of apron roughness. Similar results were found for normalized maximum scour length – Figure 14.

Figures 15 and 16 show a decreasing trend of normalized scour depth with increasing normalized amplitude, $t/a$, up to a certain value of $t/a$; after which it increases with increasing corrugation amplitude for the same wavelength and Froude number. This is because the corrugated apron’s roughness decreased when the amplitude increased beyond the optimum, for constant wavelength.

Normalized tailwater depth $y/a$

Tailwater depth has a significant effect on maximum scour depth and length. Generally, greater tailwater depth causes a decrease in maximum scour. The experiment showed, however, that the normalized maximum scour depth increases with increasing normalized tailwater depth up to a critical value, after which an increase in tailwater depth reduces maximum scour. Figures 17 and 18 show the variation of normalized maximum scour depth...
Figure 12 | Variation of normalized maximum scour length, $L_s/a$, with aspect ratio, $S/t$.

Figure 13 | Variation of normalized maximum scour depth, $d_s/a$, with $S/a$, for different $F$, $t/a$ and $y/t/a$.

Figure 14 | Normalized maximum scour length, $L_s/a$, as a function of $S/a$. 
**Figure 15** Normalized maximum scour depth, $d_s/a$, as a function of $t/a$.

**Figure 16** Normalized maximum scour length, $L_s/a$, as a function of $t/a$.

**Figure 17** Normalized maximum scour depth, $d_s/a$, as a function of $y_t/a$. 
and length with normalized tailwater depth. The curve does not sag like that reported by Dey & Sarkar (2006), which was on a smooth apron and at higher Froude number than in this study. This is due to variations in the flow field and scour process with increasing tailwater depth. Also, the flipping of the jet was not prominent at higher tailwater depths.

Proposed equation for maximum scour depth and length, and point of maximum scour depth

Keeping the effect of various parameters on maximum scour depth and length in mind, Equations (2) and (3) are simplified as:

\[
d_s/a = f\left(F, \frac{y}{a}, \frac{s}{a}, \frac{t}{a}, \frac{d_{50}}{a}, \frac{L_s}{a}\right)
\]

(5)

\[
L_s/a = f\left(F, \frac{y}{a}, \frac{s}{a}, \frac{t}{a}, \frac{d_{50}}{a}, \frac{L_s}{a}\right)
\]

(6)

As \(d_{50}\) and \(L_s\) are constant, these terms are omitted. Multiple nonlinear regression analysis for the experimental data given in Table 2 yields Equations (7) and (8):

\[
\frac{d_s}{a} = 0.427F^{1.618} \left(\frac{s}{a}\right)^{1.309} \left(\frac{t}{a}\right)^{-0.852} \left(\frac{y}{a}\right)^{-0.963}
\]

(7)

\[
\frac{L_s}{a} = 0.49F^{0.91} \left(\frac{s}{a}\right)^{1.597} \left(\frac{t}{a}\right)^{-1.29} \left(\frac{y}{a}\right)^{0.26}
\]

(8)

Here, \(R^2\) for Equations (7) and (8) comes out to be 0.92, 0.88. \(R^2\) is an important statistical measure, it denotes how well the data fits the regression model. Values closer to 1 indicate the regression model is good.

Figure 19(a) and 19(b) show comparisons of maximum scour depth and length, \(d_s/a\) and \(L_s/a\), respectively, obtained using Equations (7) and (8) with experimental data.

The maximum scour depth location was also analyzed against maximum scour depth for corrugated and smooth aprons. The variation is shown in Figure 20. The distance to the point of maximum scour increases linearly with increasing maximum scour depth.

Nonlinear regression analysis between the normalized distance, \(X_m/a\) (point of maximum scour), and normalized maximum scour depth, \(d_s/a\), which yields the relationships in Equation (9)

\[
\frac{X_m}{a} = 2.67 \left(\frac{d_s}{a}\right)^{0.936}
\]

(9)

Similarly, \(R^2\) for Equation (9) comes out to be 0.86
Figure 21 is a comparison of the location of maximum scour, $X_m/a$, computed using Equation (9) with observed experimental data.

Equations (7)–(9) are valid for the range $1.26 \leq F \leq 5.5$, $1.56 \leq s/a \leq 8.0$, $0.33 \leq t/a \leq 1.75$, and $3.56 \leq y_t/a \leq 11.00$.

**FLOW CHARACTERISTICS**

**Distribution of velocity vectors**

The time-averaged velocity vector plot over the smooth and corrugated aprons, in the middle of a scour hole along the central flow-line, is shown in Figures 22 and 23. The magnitude of the velocity vector is computed as:

$$U = \sqrt{u^2 + v^2}$$

(10)
and the vector direction as:

\[ \alpha = \tan^{-1} \frac{v}{u} \]
where

\[ u = \frac{1}{n} \sum_{i=1}^{n} u_i , \quad v = \frac{1}{n} \sum_{i=1}^{n} v_i \quad (11a) \]

\( u_i, v_i \) are individual velocity components at points along the stream-line and vertical directions, and \( n \) is the total number of velocity data points at that point.

The velocity over the smooth apron increases more rapidly than inside the scour hole, showing that the boundary layer growth rate on the smooth apron is slower than in the scour hole. This trend is the same for both smooth and corrugated aprons. Comparing the velocity on smooth and corrugated aprons shows that the rate of increase in velocity is higher on the smooth apron than the corrugated one, confirming that the boundary layer growth rate is lower over a smooth apron than a corrugated one.

Figures 22 and 23 show that the flow velocity is high near-surface of the smooth apron, but decreases upward and, finally, the flow direction reverses; that is, it has created a rolling type of flow near the surface. This exists as far as the point of maximum scour depth from the beginning of the apron, beyond which it vanishes – see Figure 22. The velocity vector direction at the apron edge’s near-surface follows the scour hole slope. Similarly, the direction of the velocity vector near the scour bed follows the hole’s bed slope up to the initial bed level, where the flow direction become nearly horizontal and the flow accelerates as it leaves the scour hole.

A surface roller also formed near the end of the corrugated apron – see Figure 23 – but it is shorter than in the smooth apron case. The velocity vector plot at the near edge of the corrugated apron points upward along the crest facing, which may be due to the shape of the edge of corrugation. The scour hole is not pronounced for either smooth or corrugated aprons, and there is no flow recirculation zone in the near-bed of the scour hole.

**Turbulence intensity distribution**

Flow characteristics in the scour hole and on rough beds have been reported by Dey & Sarkar (2008), Dey et al. (2011), Sarkar & Dey (2008). The vertical distribution of normalized turbulence intensity for both smooth and corrugated aprons is shown in Figures 24 and 25. Horizontal normalized turbulence intensity

\[ TI_u = (\frac{u' u}{V})^{0.5} / V \quad (11b) \]

where \( u' \) = fluctuation of \( u \) from the mean velocity at that point in the X direction = \( u_i - \bar{u} \) and \( V \) = average approach velocity of the jet; that is, the jet velocity at the sluice gate.

Similarly, vertical normalized turbulence intensity \( TI_v = (\frac{v' v}{V})^{0.5} / V \).

**Figure 24** | Vertical distribution of \( TI_u \) for a smooth apron (Run 11B).
where \( v' \) = fluctuation of \( v \) from mean in the Y direction = \( v_i - \bar{v} \).

Horizontal normalized turbulence intensity \( T_{I_h} \) increases gradually from the bottom of the apron at (0,0) for a short distance along the Y-axis and then falls slightly until it reaches a constant value. This trend is the same for both smooth and corrugated aprons. Similarly, \( T_{I_u} \), near-bed is higher for smooth than corrugated aprons. The turbulence intensity, \( T_{I_u} \) for corrugated and smooth aprons near the scour bed is low, and increases gradually along the vertical Y-axis. In total, the magnitude of turbulence intensity is higher for smooth than corrugated aprons at every measurement location, as shown in Figures 24 and 25.

**Bed shear stress**

Under equilibrium conditions, the hydrodynamic forces acting on sediment particles in the scour hole are lift, \( F_l \), and drag, \( F_d \), exerted by the fluid, and the submerged weight, \( W \), of the particle. Considering these forces, the critical shear stress in a streamwise sloping bed is given by Chiew & Parker 1994):

\[
\frac{\tau'_c}{\tau_c} = \cos \theta \left( 1 + \frac{\tan \phi}{\tan \theta} \right)
\]

where \( \tau'_c \) is the critical shear stress on a bed sloping at angle \( \theta \) and \( \tau_c \) = the critical shear stress on a horizontal bed computed using the Shields diagram (1936). Dimensionless critical shear stress is given as:

\[
\frac{\tau_c}{(\gamma_s - \gamma_f) d} = 0.06 \text{ for coarser particles and turbulent flow}
\]

where, \( \gamma_s \) and \( \gamma_f \) are the specific weights of the sediment and water, respectively.

The experimental bed shear stress is computed using Reynolds stress distribution (Dey & Sarkar 2008). Reynolds stress \( \tau_{xy} \) is computed as \( \overline{i \overline{u'v'}} \). For a sloping bed,

\[
[\tau_{xy}]_\theta = \cos \theta \mu \overline{u'v'}
\]

\( \theta \) is computed by drawing a tangent to the equilibrium bed profile and a horizontal line from each point on the profile at 0.1 m intervals. For experimental bed shear stress computation, points more than 10 mm from the bed are taken. Threshold bed shear stress and Reynolds stress are computed using Equations (12) and (13), respectively.

Figure 25 shows that the threshold bed shear stress decreases with distance along the flow direction. The experimental bed shear stress computed from the Reynolds stress distribution shows that, initially, it decreases on the
smooth apron along the flow line. As it leaves the apron, the shear stress increases, due to the change from a smooth to a rough surface, after which it continues to decrease. For a corrugated apron, the Reynolds stress developed is about five or six times higher than on a smooth apron, because of the corrugations roughness. The Reynolds stress increases again after leaving the apron, due to the change in surface roughness from the apron to the sand bed. After that, it continues to decrease – see Figure 26. The value of the threshold bed shear stress in the scour hole computed using Equation (12) depends on the scour bed slope. Figures 26 and 27 show that the threshold bed shear stress increases with increasing scour bed slope, and, for a horizontal bed, its value is 1.748 Pa computed using the Shields diagram (1936).

**CONCLUSIONS**

Experiments performed using smooth and corrugated aprons under submerged wall jets, with different corrugation dimensions, led to a number of observations:

- Under submerged wall jet conditions, use of a corrugated apron downstream of a sluice gate reduces scour significantly compared to a smooth apron.

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**Figure 26** | Comparison of threshold bed shear stress and Reynolds stress downstream of a smooth apron (Run 11B).

**Figure 27** | Comparison of threshold bed shear stress and Reynolds stress downstream of a corrugated apron (Run 52C).
The optimum aspect ratio, S/t, is 3, when scour depth and length are minimized.

The maximum reductions in scour depth and length were 78.95 and 83.15%, respectively, and the minimum reductions achieved by any of the corrugated aprons tested were 12.73 and 10.75%.

The maximum scour depth and length are proportional to the Froude number of the issuing jet; that is, as the incoming flow jet’s Froude number increases, scour depth and length increase, and vice versa.

The maximum scour depth and length increase with increasing wavelength, S, for constant amplitude, t, of corrugated aprons.

The variations in d_s/a and L_s/a with normalized amplitude, t/a, for constant S/a for a corrugated apron, at constant tailwater depth, produce a sagging curve.

Below a critical value of tailwater depth, at which maximum scour depth and length occur, the scour depth and length decrease with increasing tailwater depth. It is valid with in the tested range of tailwater depth and Froude number.

The rate of velocity decay in the scour hole is higher than over the apron and increases with increasing scour hole dimensions.

The turbulence of flow approaching the end of a smooth apron is higher than for a corrugated apron, which governs the scour hole dimensions, as indicated by both the turbulence intensities and the Reynolds shear stress observed.

The Reynolds shear stress at the end of the smooth apron is higher than the threshold bed shear stress and decreases faster at the stage of development of maximum scour. However, for corrugated aprons, the threshold bed shear stress at the end of the apron is higher than the Reynolds shear stress and the smooth apron’s Reynolds shear stress is also greater than that of the corrugated apron.

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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