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Jian-qiao Mu
Sichuan Lexi Expressway Co., Ltd.

Tian-tao Li (✉ tiantao.lee@gmail.com)
Chengdu University of Technology

Xiang-jun Pei
Chengdu University of Technology

Run-qiu Huang
Chengdu University of Technology

Fu-an Lan
Sichuan Lexi Expressway Co., Ltd.

Xue-qing Zou
Chengdu Center of Hydrogeology and Engineering Geology

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Jian-qiao Mu, Tian-tao Li, Xiang-jun Pei, Run-qi Huang, Fu-an Lan, Xue-qing Zou

a Sichuan Lexi Expressway Co., Ltd., Chengdu 610000, China
b State Key Laboratory of Geo-Hazard Prevention and Geo-Environment Protection, Chengdu University of Technology 610059, China
c Chengdu Center of Hydrogeology and Engineering Geology Sichuan Bureau of Geology and Mineral Exploration and Development, Chengdu, Sichuan 610081, China

Abstract: With massive engineering projects performed in high and steep mountain areas, the evidence of toppling deformation, which has been an important engineering geological problem in construction, has been exposed and observed in quantities. Three key issues in the early warning of toppling slopes are the boundary condition, evolution mechanism, and deformation stability analysis. This paper investigates an evolution mechanism for timely predicting the occurrence of toppling induced slope failure in rock masses, relates boundary formation and progressive development about toppling fracture planes. By describing an instantaneous toppling velocity field and identifying two possible fracture plane geometries (linear and parabolic), the optimal path of toppling fracture plane is searched via critical toppling heights (i.e., minimum loads) calculation using the upper bound theory of limit analysis. It is interesting to find that no matter what the slope structures and mechanical parameters are, the optimal path of toppling fracture plane is straight and most likely oriented perpendicular to the bedding planes. Hereby, considering structural damage will enable progressive toppling deformation instead of systemic failure, the toppling deformation evolution is probably taking place of a loop following the formation of the first fracture plane due to exceeding slope critical height. In the loop, deformation and column inclination updates due to fracture plane formation and fracture plane inclination increase to adjust the changed

* Corresponding author. State Key Laboratory of Geo-Hazard Prevention and Geo-Environment Protection, College of Environment and Civil Engineering, Chengdu University of Technology, Sichuan, Chengdu 610059, China. Tel.: +86 28 84079791.
E-mail addresses: tiantao.lee@gmail.com (T. Li), 676052478@qq.com (J. Mu).
inclination of columns, as it may take degrees perpendicular to columns. And this progressive formation of ever more inclined fractures plane is what lead to sliding collapse. Altogether we divide the toppling evolution into 5 stages, and define the instability criterion for toppling deformation transform into sliding collapse as the fracture plane inclination being equal to its friction angle. In addition, a PFC2D simulation of the entire slope toppling process is performed to verify this speculative evolution mechanism, and a satisfactory result is acquired. Finally, a deformation calculation model of toppling slopes is proposed for stability analysis in accordance with the instability criterion, which is further applied in a typical toppling case. The findings of this study could lay a foundation for the deformation, stability and early-warning analysis of toppling slopes.

Keywords: Rock slope failure; Toppling; Evolution mechanism; Limit analysis

1. Introduction

As a special deformation mode, toppling slopes are widely distributed in deep-cut canyon areas. The toppling slope is deep-seated, large-scale and occurs over a long timeline. Unlike a landslide, slope toppling does not conform to sliding deformation and does not result in a single fracture plane or an obvious topographic feature. The applicability of analytical models based on the sliding hypothesis in the toppling slope analysis remains to be determined (Huang et al. 2017). Current studies on the stability analysis of toppling slopes are almost always calculated by a static model (Goodman and Bray, 1976; Aydan and Kawamoto, 1992; Adhikary et al. 1997; Zhang et al. 2007; Liu et al. 2009; Amini and Majdi, 2012). However, a reasonable height-width ratio is key to performing accurate calculations for a thin layered toppling slope (Popov, 1976), and the rationality of using a designated failure plane is debatable.

The limit equilibrium method, which ignores the stress-strain relationship of rocks and the evolution of geohazards, cannot be used to predict whether a toppling slope will continuously deform or transform into a large-scale landslide. Regarding the evolution mechanism (Wang et al. 1981; Pritchard and Savigny, 1990; Adhikary et al. 1996, 2007; Goricki and Goodman, 2003; Tosney et al. 2004; Stacey, 2006; Yeung and Wong, 2007; Zheng et al. 2018), there is a lack of consensus. Huang (2008) proposed a 3-stage evolution mode for high rock slopes in 2008 and suggested that stability evaluation is both a strength problem and a deformation problem. Therefore, studying the evolution mechanism of toppling slopes
over their long geological history is a novel approach. In the study of the evolution mechanism of toppling slopes, boundary conditions are necessary and essential. Some studies have been conducted on the boundary conditions of toppling slopes, mostly physical or numerical modeling studies. Aydan and Kawamoto (1992) concluded that the fracture plane was perpendicular to the bedding plane from a base friction model test. According to a centrifuge model test, Adhikary et al. (1997) observed that the angle between the fracture plane and the normal to the rock layers was 12-20°. Furthermore, after summarizing some experimental phenomena, Chen et al. (2016) noted that the fracture plane was discontinuous (Fig. 1). In addition, numerical modeling studies are often performed by setting basal discontinuities or cross-joints to seek the boundary conditions (Ishida et al. 1987; Bovis and Stewart, 1998; Nichol et al. 2002).

Fig. 1 Failure planes of toppling slopes in previous studies

In practical projects, the toppling boundary conditions are determined in different manners, including assuming nearly linear fracture planes or parabolic-like fracture planes (Zhang et al. 2014; Liu et al. 2016; Huang et al. 2017). The insufficient adit exploration depth (approximately 200 m) and nonuniform toppling intensity zoning usually based on the rock quality classification can affect these determinations. Therefore, the main objective of this paper is to search the optimal path of toppling fracture plane and analyze the evolution mechanism as well as prewarning criterion of toppling slope; the 2-dimensional particle flow code PFC2D (Itasca Consulting Group, 2017) is used to verify the study.
Fig. 2 Investigated toppling boundaries in practical projects

2. Limit analysis of a toppling slope

2.1 Core assumptions

The limit analysis theory (Chen, 1975) takes the object as an ideal rigid-plastic material to find the ultimate load using upper or lower bound theory. The upper bound theory in the limit analysis derived by the virtual power principle can be described as follows: the calculated failure load is greater than or equal to its practical ultimate load for all kinematically induced collapse mechanisms:

$$\int_A T_i u_i dA + \int_V F_i u_i dV \geq \int_V \sigma_{ij} \varepsilon_{ij} dV$$

(1)

where $T_i$ and $F_i$ are the surface force and body force that act on the collapse mechanism, respectively; $\sigma_{ij}$ is the internal stress set that reacts to $T_i$ and $F_i$; $u_i$ and $\varepsilon_{ij}$ are the strain sets under $T_i$, $F_i$ and $\sigma_{ij}$; $A$ and $V$ are the area and volume acted on by $T_i$, $F_i$ and $\sigma_{ij}$.

Based on the upper bound theory, a theoretical method to search for the toppling fracture plane is proposed, and the core assumptions in the analysis are as follows:
1. The rock slope is considered an ideal rigid-plastic material and obeys Mohr-Coulomb yield criterion.

2. The analysis of the 2-dimensional slope deformation is simplified to a plane strain problem, and the deformation is compatible.

3. No primary joint or weak plane is included in the analysis model with the exception of the bedding planes, and the fracture plane initiates at the toe of the slope.

2.2 Analysis model

The analysis model for the toppling system is simplified as a series of superposed cantilever beams or rock columns with equal thicknesses (see Fig. 3). The origin of the rectangular coordinates is set at the toe of the slope. Slope function \( f(i) \) in an \( x-y \) rectangular coordinate system can be written as follows:

\[
f(i) = \begin{cases} 
0, & i < 0 \\
\tan \alpha \cdot i \cdot \frac{b \cos \alpha}{\sin(\alpha + \beta)}, & 0 \leq i < \frac{H \sin(\alpha + \beta)}{\sin \alpha b} \\
H, & i \geq \frac{H \sin(\alpha + \beta)}{\sin \alpha b}
\end{cases}
\]

where \( H \) is the slope height; \( b \) is the thickness of the rock columns; \( \alpha \) and \( \beta \) are the slope angle and dip angle of the bedding plane, respectively; \( i \) is the number of rock columns.

Fig. 3 Analysis model for slope toppling

For the convenience of the subsequent analysis and calculation, the slope function is rotated 90-\( \beta \) degrees counterclockwise, so that the \( X \)-axis is perpendicular to the bedding planes, and the \( Y \)-axis is parallel to the bedding planes; thus, slope function \( f(i)' \) in the \( X-Y \) rectangular coordinate system is:

\[
f(i)' = \begin{cases} 
- \cot \beta \cdot i \cdot b, & i < 0 \\
- \cot(\alpha + \beta) \cdot i \cdot b, & 0 \leq i < \frac{H \sin(\alpha + \beta)}{\sin \alpha b} \\
\frac{H}{\sin \beta} - \cot \beta \cdot i \cdot b, & i \geq \frac{H \sin(\alpha + \beta)}{\sin \alpha b}
\end{cases}
\]
By assuming that the fracture plane function is \( g(i) \), the toppling height \( h(i) \) of each rock column and its deformation area \( S_i \) can be represented as follows:

\[
h(i) = f(i)' - g(i)
\]

(4)

\[
S_i = 0.5[h(i) + f(i - 1)' - g(i)] \cdot b
\]

(5)

where \( f(i-1)'-g(i) \) is the relative height or upper section height of the rock columns.

If the fracture plane intersects the slope surface, the total number of unstable rock columns in the system can be obtained, and its implicit expression is shown as follows:

\[
f(n)' - g(n) = 0
\]

(6)

where \( n \) is the total number of unstable rock columns.

### 2.3 Instantaneous velocity field description

According to the upper bound theory of limit analysis, it is necessary to describe an instantaneous and allowable velocity field for the toppling system virtual power equation. Define that the first rock column freely topples with an instantaneous angular velocity \( \omega_1 \). The interrelationships among the rock columns are considered to be similar at their tops during deformation, which implies that the tangential velocities of the bedding plane vertices are identical, while the interbedded dislocations occur in the radial direction and form imbricates (see Fig. 4).

![Fig. 4 Instantaneous velocity field for the toppling deformation](image-url)

Hence, the general formula of the angular velocity for each rock column to calculate the toppling power calculation is as follows:

\[
\omega_i = \omega_1 \prod_{j=2}^{i} \frac{h(j-1)}{f(j-1)'-g(j)}
\]

(7)

where \( \omega_1 \) and \( \omega_i \) are the instantaneous angular velocities.
2.4 External power

Without considering other external forces (i.e., earthquakes), the body force that acts on the slope system is only due to gravity. External power $W_r$ of gravity in the described velocity field can be described as follows:

$$W_r = \frac{1}{4} \gamma \cos \beta \sum_{i=1}^{n} S_i \omega_i [h(i) + f(i-1)' - g(i)]$$  \hspace{1cm} (8)

where $\gamma$ is the unit weight of the rock columns.

2.5 Internal energy consumption

In the process of toppling deformation, the internal energy consumption of the system is mainly due to compression along the neutral planes, tension between the neutral planes and resistance caused by the interbedded dislocations, as shown in Fig. 5.

![Fig. 5 Internal energy consumption of each rock column](image)

(1) Compression zone

If the neutral plane coefficient of the rock columns is 1, the compression zone width of each rock column is $b/2$. Chen (1975) proposed the following internal energy consumption per unit of compressed volume when bending a rock column:

$$\dot{D} = 2c \dot{\varepsilon} \tan \left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$  \hspace{1cm} (9)

where $\dot{D}$ and $\dot{\varepsilon}$ are the internal energy consumption per unit of compressed volume and its maximum compressive strain, respectively. Therefore, the internal energy consumption $W_c$ of the compression zone in the toppling system can be derived as follows:
\[ W_c = cb_c \tan \left( \frac{\pi}{4} + \frac{\varphi}{2} \right) \sum_{i=1}^{n} \omega_i \]  

where \( c \) and \( \varphi \) are the cohesion and friction angle of the rock material, respectively, and \( b_c \) is the compression zone width of each rock column.

(2) **Tension zone**

Similarly, the tension zone width of each rock column is \( b/2 \). Since the pure tensile state, internal energy consumption \( W_t \) of the tension zone in the toppling system can be expressed as follows:

\[ W_t = \frac{1}{2} \sigma_t b_t \sum_{i=1}^{n} \omega_i \]  

where \( \sigma_t \) is the tensile strength of the rock material; \( b_t \) is the tension zone width of each rock column.

(3) **Interbedded dislocation**

When the total number of unstable rock columns in the system is \( n \), the resistances caused by interbedded dislocations are \( n-1 \). According to the described velocity field, 3 interbedded dislocation modes can be determined:

(a) If \( h(i) = f(i) - g(i+1) \), \( \omega_i = \omega_{i+1} = \omega_1 \), and the interbedded dislocation mode is only dislocation, as shown in Fig. 6(a). The internal energy consumption \( W_{(\omega)} \) of this mode can be considered a simple shear power, see Eq. (12).

(b) If \( h(i) > f(i) - g(i+1) \), \( \omega_i < \omega_{i+1} \), and the interbedded dislocation mode is dislocation coupled with tension, as shown in Fig. 6(b). Since the tensile strength of bedding planes is 0, no internal energy consumption occurs during the tension process, but the interbedded resistances will be attenuated by tension. Hence, the internal energy consumption \( W_{(\sigma)} \) of this mode can be determined with Eq. (13).

(c) If \( h(i) < f(i) - g(i+1) \), \( \omega_i > \omega_{i+1} \), and the interbedded dislocation mode is dislocation coupled with compression, as shown in Fig. 6(c). The internal energy consumption \( W_{(c)} \) of this mode can be considered the simple shear combined with simple compression (see Eq. (14)).
Interbedded dislocation modes at different relative heights

\[ W_{f(a)} = c_j \sum_{i=1}^{n-1} h(i) \cdot (\omega_i b_i + \omega_{i+1} b_c) \]  

\[ W_{f(b)} = c_j \tan \left( \frac{\pi}{4} + \frac{\theta_i}{2} \right) \sum_{i=1}^{n-1} h(i) \cdot (\omega_i b_i + \omega_{i+1} b_c) \]  

\[ W_{f(c)} = W_{f(a)} + c \tan \left( \frac{\pi}{4} + \frac{\theta_i}{2} \right) \sum_{i=1}^{n-1} \omega_i b_c \]  

where \( c_j \) and \( \phi_i \) are the cohesion and friction angle of bedding planes, respectively; \( \tan \left( \frac{\pi}{4} + \frac{\theta_i}{2} \right) \) is a reduction factor for the attenuated interbedded resistances; \( \theta_i \) is the angle between tension velocity \( \delta v \) and shear velocity \( \delta u \), and its tangent value can be proposed by the improved Coulomb model (Chen, 1975):

\[ \tan \theta_i = \frac{\delta v}{\delta u} = \tan \phi + \frac{\omega_i b_c}{2h(i) \cdot (\omega_i b_i + \omega_{i+1} b_c)} \]  

### 2.6 Ultimate load computation

If the external power of Eq. (8) is equal to the internal energy consumption of Eqs. (10)-(14), the following upper bound theory expression of the toppling slope can be established, as shown in Eq. (16).

Where the ultimate load is expressed as the critical toppling height \( H_{cr} \), which is implicit in the equation. Therefore, once the fracture plane function \( g(i) \) is determined, the critical toppling height can be computed by eliminating the instantaneous angular velocity \( \omega_i \) assumed on both sides of the expression, and different fracture plane geometries corresponds to different critical toppling heights.

\[ W_f = W_c + W_t + W_{f(a)}/(b)\]/(c) \]  

Thus, when the real slope height is less than the critical toppling height, the slope has not yet undergone toppling deformation. When the real slope height is equal to or even exceeds the critical toppling height, the slope will undergo or has undergone toppling deformation.
3. Fracture plane search

Due to the complexity of Eq. (16), it is too difficult to find the minimum solution of critical toppling height by direct differentiation, where the critical toppling height $H_{cr}$ as the dependent variable and the fracture plane geometry as the independent variable. In this paper, 2 possible fracture plane geometries (line and parabola) are assumed, and Mathematica (Mathematica; Wolfram Research 2014) is applied to search for the optimal fracture plane that corresponds to the minimum solution of critical toppling height under certain slope structures and mechanical parameters via trial and error.

3.1 Straight line

The linear fracture plane function $g(i)$ can be used as a single-parameter equation for the straight line inclination:

$$g(i) = \tan m \cdot i \cdot b$$  (17)

where $m$ is the fracture plane function coefficient in the $X$-$Y$ rectangular coordinate system (Fig. 7).

When $m = 0$, $h(i) = f(i)' - g(i+1)$, and dislocation occurs only between the bedding planes (Fig. 6(a)). When $m > 0$, $h(i) > f(i)' - g(i+1)$, and dislocation coupled with tension occurs between the bedding planes (Fig. 6(b)). However, when $m < 0$, $h(i) < f(i)' - g(i+1)$, and dislocation coupled with compression occurs between the bedding planes (Fig. 6(c)).

Moreover, since the fracture plane should intersect with the slope surface, and the angular velocity should be positive, the value range of the straight line inclination must satisfy the following conditions:

$$-\cot \beta < \tan m < -\frac{\cot(\alpha + \beta)}{2}$$  (18)

where $\alpha$ and $\beta$ are the slope angle and dip angle of the bedding plane, respectively.
For the optimal fracture plane search, either the slope structure or mechanical parameters should be designated in advance to compute and compare the critical toppling heights by changing the straight line inclination.

1. Consistent slope structure, changing mechanical parameters: the slope structure is fixed as a Hoek and Bray (1977) classical model, where the slope angle is 56°, and the dip angle is 60°. In addition, it is reasonable to divide the mechanical parameters into 2 categories (i.e., corresponding to the rock mass and bedding plane) to avoid the dimensional disaster, and the designed mechanical parameters are shown in Tables 1 and 2.

Table 1 Rock mass mechanical parameters

| Rock mass classification | γ (kN/m³) | Unit weight | Shear strength Friction angle φ (°) | Cohesion c (MPa) | Tension στ (MPa) | Thickness b (m) |
|--------------------------|-----------|-------------|-----------------------------------|-----------------|----------------|----------------|
| I                        | 27.0      | 55          | 2.00                              | 5.0             | 1.0            |                |
| II                       | 26.5      | 50          | 1.50                              | 2.0             | 0.5            |                |
| III                      | 25.5      | 39          | 0.70                              | 0.8             | 0.3            |                |
| IV                       | 24.5      | 27          | 0.20                              | 0.3             | 0.1            |                |

Table 2 Bedding plane mechanical parameters

| Number | Bedding plane type | Shear strength Friction angle φ (°) | Cohesion c (MPa) | Tension στ (MPa) |
|--------|--------------------|-----------------------------------|-----------------|----------------|
| 1      | Cementation        | 35                                | 0.20            | 0.0            |
| 2      | Rigidness          | 29                                | 0.10            | 0.0            |
| 3      | Softness           | 24                                | 0.08            | 0.0            |

Mathematica is used to determine the computed critical toppling heights for Hoek and Bray model by changing the straight line inclination for different mechanical parameters, and the results are listed in Table 3.

Table 3 Computed critical toppling heights for Hoek and Bray model

| Rock mass classification | Bedding plane type | Critical toppling height Hcr (m) |
|--------------------------|--------------------|---------------------------------|
|                          | m=10°              | m=5°              | m=2°              | m=0°              | m=-2°              | m=-5°              |
| I                        | Cementation        | 166                | 135               | 122               | 115                | 182                | 278                |
|                          | Rigidness          | 123                | 99                | 89                | 84                 | 147                | 243                |
|                          | Softness           | 115                | 93                | 84                | 78                 | 140                | 237                |
| II                       | Cementation        | 138                | 113               | 102               | 96                 | 144                | 207                |
|                          | Rigidness          | 87                 | 71                | 64                | 60                 | 106                | 171                |
|                          | Softness           | 78                 | 63                | 57                | 53                 | 98                 | 164                |
| III                      | Cementation        | 132                | 109               | 99                | 94                 | 108                | 127                |
|                          | Rigidness          | 71                 | 59                | 53                | 50                 | 67                 | 90                 |
### Table 4 Contour plots of the critical toppling heights

| Thickness (m) | Slope angle (°) | Critical height (m) |
|---------------|-----------------|---------------------|
|               | 40              |                     |
| 0.1           | 80              | ![Contour plotted](image) |
|               | 60              | ![Contour plotted](image) |
|               | 80              | ![Contour plotted](image) |
|               | 0.3             |                     |
|               | 80              | ![Contour plotted](image) |
|               | 60              | ![Contour plotted](image) |
|               | 40              | ![Contour plotted](image) |
|               | 0.5             |                     |
|               | 80              | ![Contour plotted](image) |
|               | 60              | ![Contour plotted](image) |
|               | 40              | ![Contour plotted](image) |
|               | 1.0             |                     |
|               | 80              | ![Contour plotted](image) |
|               | 60              | ![Contour plotted](image) |
|               | 40              | ![Contour plotted](image) |

Tables 3 and 4 show that regardless of the mechanical parameters or slope structures, the straight line inclination corresponding to the minimum toppling height is 0. Due to the optimal control (Berkovitz,
1974), it can be certain that toppling most likely occurs when the linear fracture plane is perpendicular to the bedding plane; alternatively, when toppling occurs, the most favorable linear fracture plane is perpendicular to the bedding plane.

### 3.2 Parabola

Assuming a parabolic fracture plane, the single-parameter equation for the quadratic coefficient is as follows:

\[ g(i) = m \cdot (i \cdot b)^2 \]  

where \( m \) (\( m > 0 \)) is the fracture plane function coefficient in the \( X-Y \) coordinate system.

The optimal solution for the quadratic coefficient of the parabolic fracture plane is still performed by the semigraphic method, and the calculation process is not repeated. The calculation results show that the critical height is minimized when the parabolic quadratic coefficient approaches 0, i.e., infinitely near the normal line of the bedding plane, as shown in Fig. 8.

![Fig. 8 Minimum critical height in relation to the parabolic fracture plane](image)

In conclusion, no matter what the slope structures and mechanical parameters are, the optimal path of toppling fracture plane is straight and most likely oriented perpendicular to the bedding planes.

### 4. Evolution mechanism analysis

#### 4.1 Evolution process speculation

Toppling deformation is a dynamic evolution process with an inherent mechanism. Based on the fracture plane initiation characteristics, a toppling evolution mechanism along with an instability criterion is proposed here and divided into 5 stages as follows:
Toppling deformation requires a certain external force, so there is a critical height $H_{cr}$ at which the slope reaches the critical state. When the real slope height is less than this critical height, the slope will not topple and will maintain its intact slope and rock mass structure characteristics, as shown in Fig. 9(a).

(b) Initial toppling damage

Due to a long-term geological history of valley cutting, the slope height will increase. When the slope height reaches its critical toppling height $H_{cr}$, the slope will topple, and the formed initial fracture plane is a straight line perpendicular to the bedding planes (see Fig. 9(b)). However, the gentle fracture plane angle in this stage is usually not sufficiently steep to cause sliding failure, and toppling deformation will slowly progress.

(c) Time-dependent deformation

Because of the toppling fracture plane must take degrees perpendicular to columns, the fracture plane inclination will increase to adjust the decreasing changed inclination of columns. Thus forms a loop, which corresponds to the time-dependent deformation stage, as shown in Fig. 9(c). Notably, this progressive development of toppling fracture plane is not conducive to maintaining the slope stability.

(d) Limit equilibrium state

When the fracture plane continues to develop, the slope will be in a limit equilibrium state when the fracture plane inclination is equal to its friction angle (see Fig. 9(d)). This condition indicates that the slope is about to lose its stability and slide.

(e) Turning into a landslide

After the limit equilibrium state, the slope state will transform from toppling deformation to sliding failure. At this stage, the toppling fracture plane evolves into a sliding surface, which causes the rock masses to slide and forms a landslide, as shown in Fig. 9(e).
The speculated evolution process indicates that the prerequisite for slope toppling is gravity load, which is controlled by the slope height. Toppling deformation occurs only when the slope height reaches its critical toppling height. Moreover, the instability criterion of the slope for the transition from toppling deformation to sliding failure is when the inclination angle of the toppling fracture plane is equal to its friction angle; when this criterion is satisfied, the formation of a large-scale landslide is inevitable.

### 4.2 Simulation verification

To verify the speculative toppling evolution mechanism, the entire slope toppling process is simulated using PFC2D (Itasca 2008). PFC2D uses the discrete element method (DEM) to simulate continuous interactions and motions among spherical particles in order to calculate their location and relative displacement and the amount of overlap of particles at each time step. González et al. (2002) indicated the following: (1) PFC can simulate the bonding and separation of rock, and (2) using the contact between wall elements and ball elements, PFC is able to simulate collisions and friction in landslide behavior. In recent years, the software has been widely used in slope deformation and motion process simulation (Feng et al. 2016; Wei et al. 2019).

The designed simulation model generated by explosive repulsion is 110 m high and 213 m wide, the effective slope height is 70 m, the slope angle is 70°, the bedding plane dip angle is 65°, and the bed thickness is 2 m. A marker bed and 5 monitor particles are set to observe the entire movement process (Fig. 10).
The regular particle contacts are defined as parallel bonds, while the contacts of the bedding planes are defined as smooth joints. The microparameters are checked by trial and error considering the physical and mechanical properties of the rocks and bedding planes (Table 5).

**Table 5** Microparameters of the particles

| Item                          | Value     | Item                          | Value     |
|-------------------------------|-----------|-------------------------------|-----------|
| Particle density $\rho$ (kg/m$^3$) | 2800      | Parallel bond stiffness ratio $knb/ksb$ | 1.2       |
| Porosity $n$                  | 0.15      | Parallel bond cohesion $pb\_coh$ (MPa) | 10        |
| Particle contact modulus $E_c$ (GPa) | 20        | Parallel bond friction angle $pb\_fa$ (°) | 45        |
| Particle stiffness ratio $kn/ks$ | 1.2      | Parallel bond tensile strength $pb\_ten$ (MPa) | 7         |
| Particle friction coefficient $\mu$ | 0.9      | Smooth joint cohesion $sj\_coh$ (MPa) | 0.3       |
| Parallel bond modulus $E_b$ (GPa) | 18        | Smooth joint friction angle $sj\_fa$ (°) | 17        |

Before the simulation, the displacement boundary is constrained, and high material parameters are set to achieve the initial equilibrium under gravity (9.81 m/s$^2$). After the initial balance, the material parameters in Table 5 are updated to obtain the required calculation model; in Fig. 11, the slope displacement contour display ranges in the results of the entire slope toppling process are fixed to 0-18 m.
(a) Undeformed slope                      (b) Initial fracture plane formation

(c) Discontinuities offset to the free           (d) Limit equilibrium state

(e) Slope failure                          (f) Final stable state

**Fig. 11** Complete slope toppling simulation

Fig. 11 shows that the initial formed fracture plane presents a straight line almost perpendicular to
the bedding plane and does not correspond to an immediate instability. Meanwhile, the simulated
evolution process is perfectly consistent with the speculation in section 4.1, where the discontinuities
gradually offset to the free state until the slope reaches the limit equilibrium state. When the displacement
exceeds the 18-m threshold, the deformation is difficult to control. The colors of the displacement field
in the simulation represent multiple stages of fracture deformation. As a matter of fact, Goricki and
Goodman (Goricki and Goodman, 2003) found a similar evolution process in their published article.

5. Deformation stability analysis

For toppling slopes, we believe that the structural damage will result in continuous toppling
deformation instead of systemic failure, and the key of early-warning control lies in the deformation
stability analysis. According to the toppling evolution process, a deformation calculation model for the
prewarning value $y_{max}$ is proposed, and this prewarning value is applicable from the current state to the
limit equilibrium state, as shown in Fig. 12.
This deformation stability evaluation can be performed in four steps: rotation angle determination, rock column selection, prewarning value calculation and comparative analysis with the actual displacement.

1. **Rotation angle determination:** by defining the inclination angle of the initial fracture plane as $\theta_1$, the inclination angle of the current fracture plane as $\theta_2$, and the friction angle of the critical sliding
surface in the limit equilibrium state as $\theta_3$, we can obtain the rotation angle $\theta_{cr}$ of the toppling slope from the current state to the limit equilibrium state as follows:

$$\theta_{cr} = \theta_3 - \theta_2$$  \hspace{1cm} (20)

Where $\theta_2$ can be investigated by systematic geological exploration, preferably adit exploration. $\theta_3$ should be estimated by the geological strength index (GSI) method (Hoek et al. 2013).

2. Rock column selection: By selecting the top rock column as the analysis object, we obtain its cantilever length $l_{\text{max}}$ by the corresponding geometric relationships:

$$l_{\text{max}} = H \frac{\sin(\alpha - \theta_2)}{\sin \alpha}$$ \hspace{1cm} (21)

where $H$ is the slope height, and $\alpha$ is the slope angle. During the actual practice, the monitoring point position could be selected as the analysis object.

3. Prewarning value calculation: because the toppling fracture plane must take degrees perpendicular to columns, the rotation angle of the top rock column from the current state to the limit equilibrium state is also $\theta_{cr}$. Based on the cosine law, its prewarning value $y_{\text{max}}$ can be calculated:

$$y_{\text{max}} = l_{\text{max}} \cdot \sqrt{2(1 - \cos \theta_{cr})} \hspace{1cm} (22)$$

4. Comparative analysis with the actual displacement: in the comparison of the calculated prewarning value $y_{\text{max}}$ with the monitored cumulative displacement, if the cumulative displacement is less than prewarning value $y_{\text{max}}$, the slope remains in a stable state; however, if the cumulative displacement is close to or even exceeds prewarning value $y_{\text{max}}$, a disaster prewarning should occur.

Therefore, regardless of how a toppling slope develops, it will not turn into a landslide as long as the deformation does not reach its threshold, even if a sudden increase in amount of deformation occurs due to the addition of sudden external factors such as rainfall or earthquakes.

6. Example application

The Xingguangsanzu deformation body in the Xiluodu reservoir region is a typical toppling slope (Zhang et al. 2015). Its upstream and downstream areas are deeply cut by 2 gullies and present a “three-empty state,” which forms a 1500-m-long and 950-m-high rock slope. From June 2014 to February 2017, the maximum cumulative deformation monitored by the Global Navigation Satellite System (GNSS) was 1130.7 mm (at TP01). A panoramic view of the slope, several signs of deformation from exploration adits, and an engineering geological profile are shown in Fig. 13:
(a) Panoramic photo of the investigated toppling slope with rock deformation. (i) A reverse fissure due to toppling in PD01; (ii)–(iv) rock toppling developed in PD07, PD02 and PD03, with V-shaped fracture opening formed in hard rock layers or flexible bending formed in soft rock layers.
Based on the investigated rock deformation characteristics (Table 6), 3 toppling intensities (intense toppling, weak toppling, and parent rock zones) are divided by 2 estimated toppling fracture planes, which connect the signs of rock toppling exposed in the adits. Meanwhile, 2 connected straight fracture planes have measured dips of 28.7° and 12.5° and are almost perpendicular to the bedding planes, which is consistent with the above analysis conclusion. Combined with the slope ground fissure distribution, signs and rates of deformation, we preliminarily determine that the slope remains stable.

**Table 6** Toppling intensity grading system for the toppling slope

| Toppling intensity | Intense toppling (Zone A) | Weak toppling (Zone B) | Parent rock (Zone C) |
|--------------------|---------------------------|------------------------|----------------------|
|                    | ● Intense deformation accompanied by tensile fractures in the rock strata. | ● Rock toppling is developed and occasional dislocation. | ● Rocks generally maintain a good integrity, localized tensile deformation due to rotation of the overlaying rock columns. |
|                    | ● Compression-shear fractures formed in rock layers and often causing dislocations. | ● V-shaped fracture opening formed in hard rock layers or the rock columns in the lower portion of the slope. | |
|                    | ● Localized surface rock fragments detaching from the columns and layers. | ● Joint slip occurs in the rock strata without any fracture openings crossing the rock columns. | |

**Fig. 13** Overview of the Xingguangsanzu slope

(b) Cross-section profile of the upstream slope ridge in the maximum deformation direction
slipping.

| Dip angle | Rock mass structure | Stress relaxation | Weathering intensity | Rock mass structure | Stress relaxation | Weathering intensity |
|-----------|---------------------|------------------|----------------------|---------------------|------------------|----------------------|
| 30-50°    | Cataclastic         | Strongly released | Highly weathered     | Interlocked         | Slightly released | Moderately weathered |
|          | with local interlocked |                  |                      | with local cataclastic |                |                      |
| 60° average |                    |                  |                      | Interlocked         | Slightly released | Slightly weathered  |
| Above 75° |                    |                  |                      | Stratified          |                  | or completely intact |
|          |                    |                  |                      |                     |                  |                     |

To further analyze the slope deformation stability, the GSI method is applied to quantitatively estimate the current fracture plane mechanical parameters (i.e., intense toppling boundary). The estimated equivalent friction angle is 31.69°, which is also the critical sliding surface angle. If the monitoring point positions are selected as the targets, the prewarning value $y_{\text{max}}$ of the slope can be calculated by Eqs. (20)-(22), and the slope deformation stability analysis can be determined as follows.

**Table 7** Deformation stability analysis for the toppling slope

| Monitoring point | Inclination angle $\theta_2$ (°) | Equivalent friction angle $\theta_3$ (°) | Rotation angle $\theta_r$ (°) | Cantilever length $l$ (m) | Prewarning value $y_{\text{max}}$ (m) | Cumulative deformation $y$ (m) |
|------------------|---------------------------------|---------------------------------|--------------------------------|--------------------------|---------------------------------|-------------------------------|
| TP01             | 28.7                            | 31.69                           | 2.99                          | 78.91                    | 4.12                            | 1.13 (Until Feb. 2017)        |
| TP02             | 62.55                           | 3.26                            | 0.74                          | 62.55                    | 3.26                            | 0.74 (Until Sept. 2016)       |

**Fig. 14** Deformation stability analysis of the slope

Comparing the prewarning values and monitored cumulative deformations in Fig. (14), the...
monitored cumulative deformations have not yet reached the prewarning values, which indicates that the slope remains in a stable state, which is consistent with our qualitative judgment.

7. Discussion and conclusions

Unlike the lengthy and complicated iterative calculation of the traditional rigid body limit equilibrium method (Goodman and Bray, 1976), the proposed method is simple and can be combined with monitoring data for stability prediction. However, since no flexible bending is considered in the proposed method, the calculated prewarning value is relatively conservative and may be more suitable for hard rock slopes with large stiffness. The applicability to soft rock slopes and complex lithological associations remains to be improved. Meanwhile, it is important to study the mechanical parameter evaluation methods for toppling fracture planes, which directly control the accuracy of the early-warning analysis.

A good understanding of the evolution mechanism in the process of toppling deformation for early-warning analysis is proposed, and its reasonability is checked by simulating the entire slope toppling process. In this paper, the limit analysis method can describe the external power and internal energy consumption at the ultimate loads using a toppling velocity field. The optimal control theory confirms the optimal boundary condition of the toppling slopes with the minimum toppling height (ultimate load).

The consistently perpendicular arrangement of the fracture planes and bedding planes is the key to the study of the toppling evolution. Admittedly, an idealized toppling evolution is rare in nature because rocks include primary joints and weak planes. Therefore, this study could lay a foundation to guide the toppling slope stability prediction.

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Conflict of interest
The authors declared that they have no conflicts of interest in this work and no commercial or associative interests that represent a conflict of interest in connection with the work submitted.

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Figures

Figure 1

Failure planes of toppling slopes in previous studies
Figure 2

(a) Nearly linear fracture planes (b) Parabolic fracture planes Investigated toppling boundaries in practical projects
Figure 3

Analysis model for slope toppling

Figure 4
Instantaneous velocity field for the toppling deformation

Neutral plane

Rigid

Compression $(\pi/2 + \varnothing)$

Tension

Dislocation $h_i$

Block $i$

Figure 5

Internal energy consumption of each rock column
Figure 6

(a) Only dislocation (b) Dislocation coupled with tension (c) Dislocation coupled with compression
Interbedded dislocation modes at different relative heights

Figure 7

Analysis model for linear fracture planes
Figure 8

Minimum critical height in relation to the parabolic fracture plane
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(a) Original intact slope (b) Initial toppling damage (c) Time-dependent deformation (d) Limit equilibrium state (e) Turning into a landslide Speculated toppling evolution mechanism
Figure 10

PFC2D model for slope toppling
Figure 11

(a) Undeformed slope  (b) Initial fracture plane formation  (c) Discontinuities offset to the free  (d) Limit equilibrium state  (e) Slope failure  (f) Final stable state Complete slope toppling simulation
Figure 12

(a) Deformation calculation model (b) Deformation stability prediction Early-warning analysis for toppling slopes
Figure 13

(a) Panoramic photo of the investigated toppling slope with rock deformation. (i) A reverse fissure due to toppling in PD01; (ii)~(iv) rock toppling developed in PD07, PD02 and PD03, with V-shaped fracture opening formed in hard rock layers or flexible bending formed in soft rock layers. (b) Cross-section profile of the upstream slope ridge in the maximum deformation direction Overview of the Xingguangsanzu slope Note: The designations employed and the presentation of the material on this map do not imply the
expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.

Figure 14

Deformation stability analysis of the slope