Piezo-optomechanical coupling of a 3D microwave resonator to a bulk acoustic wave crystalline resonator

N. C. Carvalho,1,2 J. Bourhill,2 M. Goryachev,2 S. Galliou,3 and M. E. Tobar2
1) Applied Physics Department, Gleb Wataghin Physics Institute, University of Campinas, 13083-859, Brazil.
2) ARC Centre of Excellence for Engineered Quantum Systems (EQuS), Department of Physics, 35 Stirling Hwy, 6009 Crawley, Western Australia.
3) FEMTO-ST Institute, Universit Bourgogne Franche-Comte, CNRS, ENSMM, 25000 Besancon, France
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We report the observation of coupling between a 3D microwave cavity mode and a bulk mechanical resonator mediated by piezoelectric and radiation pressure effects. The system is composed of a quartz bulk acoustic wave resonator placed inside a microwave re-entrant cavity, which is designed to act as both the electrodes for piezoelectric actuation as well as a 3D resonator. The cavity electromagnetic mode is modulated by a 5 MHz bulk acoustic wave shear mode, which is modeled and experimentally verified using the input-output formalism. Through finite element method simulations, we calculate the various contributions to the electromechanical coupling and discuss the potential of the system to reach high cooperativities as well as suitable applications.

Nowadays, the pursuit of faster, more secure and efficient communication and information processing demands rigorous and diversified development of highly advanced technologies. To this purpose, hybrid systems have been intensely researched, as they combine the advantages of different platforms while avoiding their specific drawbacks. In particular, microwave devices figure as a rich means of photon-phonon coupling, which is a multifold platform for the investigation of fundamental science and applied physical systems. In this sense, electrostriction has been explored using piezoelectric photonic crystals to couple microwave and optical photons, showing that piezomechanics associated to optomechanics is a promising path for the implementation of microwave to optical (MO) interconversion - a convenient route to take advantage of distinct wavelengths in order to promote the optimized information transfer required for modern communication.

In this field of cavity optomechanics, device architectures have been diversified with structures such as membrane-in-the-middle cavities, whispering gallery mode resonators, and photonic crystals demonstrating numerous breakthrough results, such as gravitational wave detection, tests of quantum gravity, ground state cooling and optomechanically induced transparency.

In parallel, Renning et al. demonstrated valuable features of bulk acoustic wave (BAW) resonators for several quantum enabled experiments, with a study of high-coherence phonons driven by optical fields, opening new possibilities for the investigation of quantum mechanics using mesoscopic systems, precision measurements and high-fidelity information processing. BAW devices are phonon-confining structures and an acoustic analog to a Fabry-Perot cavity, where phonons propagating in a plane are reflected at the boundary of the resonator and the external medium. Such phononic cavities are recognized by their potential to exhibit extremely high mechanical quality factors, of greater than billions at cryogenic temperatures, which is a desirable characteristic for experiments demanding large coherence times.

In this paper, we explore the piezoelectricity of a quartz BAW resonator placed into a 3D microwave cavity (MC) in order to achieve coupling between ultra-high-quality-factor-phonons and photons. In the discussed system, the BAW device plays the role of the mechanical resonator and can be excited via piezoelectricity as well as radiation pressure optomechanics. We report experimental results analysed through finite element method (FEM) simulations, which agree with the theoretical model proposed. This investigation provides important information concerning the nature of the coupling of the bulk phonons to the electromagnetic MC...
modes, demonstrating the potential of the device to be implemented in quantum enabled applications and fundamental physics.

\textbf{FIG. 2.} Measured (a) microwave cavity resonance and (b) bulk acoustic resonance.

The BAW resonator is made from pure quartz and is shown in Fig. 1 (a). It has a plano-convex geometry, exaggerated in Fig. 1 (b). A bulk crystalline system with such a design has the key feature of operating in very low-loss regimes when tuned to their thickness modes. Such resonances have a Hermitian-Gaussian distribution due the resonator’s curved shape, avoiding energy leakage through the clamping points. The MC-BAW set up is shown in Fig. 1 (c). The re-entrant microwave mode generated in this cavity geometry has a strong axial electric field in the cavity gap (narrow volume between the top of the cavity central post and the opposite wall). This is an optimal configuration as the field overlaps with the maximum mechanical displacement, improving the interaction between the MC and the BAW mode.

The MC is made of niobium and, at the operating temperature of 4 K, resonates at 4.1 GHz with intrinsically low-loss regimes when tuned to their thickness modes. Resonance through the clamping points. The MC-BAW system dynamics can be described by the following Hamiltonian:

\[ H = \hbar \omega_{cav} a^\dagger a + \hbar \Omega_m b^\dagger b + \hbar g_{b}^{r} (a^\dagger a) (b^\dagger + b) + \hbar g_{b}^{p} (c^\dagger + c) (b^\dagger + b), \]

where \( a \) (\( a^\dagger \)), \( b \) (\( b^\dagger \)) and \( c \) (\( c^\dagger \)), are the bosonic annihilation (creation) operators of the MC, the acoustic modes and the electrodes driving field, respectively; \( \omega_{cav} \) and \( \Omega_m \) are the MC and BAW resonance frequencies.

The optomechanical and piezomechanical coupling rates are given by \( g_{b}^{r} \) and \( g_{b}^{p} \). The former originates from radiation pressure and must be decomposed as \( g_{b}^{r} = g_{b}^{m} + g_{b}^{p} \), where \( g_{b}^{m} \) corresponds to the thickness movement of the acoustic resonator, which modifies the boundary conditions of the MC field; \( g_{b}^{p} \) relating to the strain-induced change in the refractive index of the BAW resonator, known as the photoelastic effect. For the second interaction involving \( g_{b}^{p} \), the inverse piezoelectric effect produces a deformation of the atomic quartz lattice, also causing a modulation of the refractive index. However, differently from the optomechanical effect, piezoelectricity has a linear relationship with respect to the electric field, as explicitly manifested in Eq. (1).

The three forms of photon-phonon coupling can be calculated by the following overlap field integrals:

\[ g_{mb} = -\frac{\hbar \omega_{cav}}{2} \int_S (\mathbf{u} \cdot \mathbf{n}) (\delta \epsilon_{mb}|\mathbf{E}|^2 - \delta \epsilon_{mb}^{-1} |\mathbf{D}|^2) \, dA, \]

\[ g_{pe} = -\frac{\hbar \omega_{cav}}{2} \int_V \mathbf{E}^\dagger \cdot \delta \epsilon_{pe} \cdot \mathbf{E} \, dV, \]

\[ g_{pzt} = \frac{\hbar \omega_{cav}}{2} \int_V (\mathbf{S}^\dagger \cdot \mathbf{e}_{pzt}^T \cdot \mathbf{E} + \mathbf{E}^\dagger \cdot \mathbf{e}_{pzt} \cdot \mathbf{S}) \, dV, \]

where \( g_{j} = g_{j}^{0}/x_{zp}^{f} \) with \( j = \{ mb, pe, pzt \} \), \( x_{zp}^{f} = \sqrt{\hbar/2m_{eff} \Omega_m} \) is the zero point fluctuations and \( m_{eff} \) is the effective modal mass of the mechanical resonator. Also, \( \mathbf{u} \) and \( \mathbf{n} \) are the mechanical displacement and the unitary normal vectors, \( \delta \epsilon_{mb} = \epsilon_0 (n_2^2 - n_2^2) \) and \( \delta \epsilon_{mb}^{-1} = \epsilon_0 (n_1^2 - n_2^2) \), with \( n_1 \) and \( n_2 \) being the refractive indices of dielectric (in this case quartz) and air, respectively. The electric, electric displacement and strain fields are given by \( \mathbf{D}, \mathbf{E} \) and \( \mathbf{S} \) (complex conjugates: \( \mathbf{E}^\dagger \) and \( \mathbf{S}^\dagger \)), \( \delta \epsilon_{pe} \) stands for the permittivity perturbation tensor, which is the scalar product of the dielectric photoelastic and strain tensors, and \( e_{pzt} \) is the piezoelectric coupling matrix (with transpose \( e_{pzt}^T \)). In this system, the rate equations of the optical and mechanical amplitudes are:

\[ \dot{a} = -(i \Delta + \frac{K_i}{2}) a - ig_{b}^{r} p (b^\dagger a) + \sqrt{\frac{\kappa_{ex}}{2}} a_m, \]

\[ \dot{b} = -(i \Omega_m + \frac{\gamma_i}{2}) b - ig_{b}^{r} a^\dagger a - ig_{b}^{p} p (c^\dagger c), \]
intensity decay rate $\kappa_{ex} = \kappa - \kappa_i (\gamma_{ex})$.

It is convenient to separate the effects acting on the acoustic resonator with regard to their origin. Therefore, there are forces due to radiation pressure, $F_{rp}$, piezoelectricity, $F_{pzt}$, and thermal fluctuations, $F_{th}$, all summing up to induce mechanical displacements. It is assumed that the latter is small compared to the former two forcing terms such that it can be ignored. Even though only the optomechanical interaction is able to provide the non-linearity necessary for modulation and feedback between the acoustic and microwave resonators, it is possible to demonstrate that the piezoelectric coupling can enhance the modulation of the photonic field.

If the acoustic pump is set on the mechanical resonance frequency, in the resolved sideband region, it can be shown that the sideband amplitudes are given by:

$$|\alpha_\pm|^2 = \frac{4|\alpha_0|^2|\xi|^2(g_{rp}^2)(g_{pzt}^2)}{(4|\alpha_0|^2 (g_0^2)^2 + \gamma_\xi \kappa / 2)^2 + \gamma_1^2 (\Delta \pm \Omega_m)^2},$$

where $\alpha_0$ is the steady-state amplitude of the electromagnetic drive, $\xi$ the amplitude of the acoustic drive and $\Delta$ stands for the detuning of the electromagnetic pump from the MC resonance frequency.

Here, it’s clear that the response contains combination of radiation pressure and piezoelectric coupling. Thus, a large piezoelectric drive heightens the intensity of the cavity field modulation. The phase relationship between the microwave and acoustic driving fields is also relevant, as it could result in suppression of the mechanical displacement instead of amplification. Interesting outcomes have been observed exploring such destructive and constructive interferences. In this work, however, the driving fields are connected to the same phase reference, therefore such considerations are not necessary.

The electrodes were connected to a vector network analyzer set for impedance analysis. This drove and monitored the acoustic resonance in reflection. Using the scheme in Fig. 3 (d), the cavity electromagnetic field was modulated by the BAW 5 MHz-resonance, generating sidebands at the frequencies $\omega_{cav} \pm \Omega_m$. The microwave photonic field was driven by a two-port system through magnetic loop probes. Thus, the cavity was pumped by a microwave frequency synthesizer, with read out performed by a spectrum analyzer. All the equipment was connected to a frequency reference hydrogen maser and the MC-BAW system was cooled down to cryogenic temperatures in a pulse-tube cryocooler with internal temperature stabilized at 4 K.

Measurements were undertaken to observe the response of the piezo-optomechanical interaction as the MC pump was detuned from the resonance. Fig. 3 (a) displays the data demonstrating higher sideband amplitudes when the pump is tuned on resonance or when the detuning matched the mechanical mode frequency. Fig. 3 (b) shows the amplitude of the sidebands when the synthesizer is tuned to the microwave resonance and its input power swept. Analogously, Fig. 3 (c) shows the amplitude of the sidebands as a function of the power applied to the electrodes, presenting the same linear behaviour as predicted by Eq. 7.

In order to identify the physical nature of the coupling between the electromagnetic mode and the mechanical resonance a FEM model was built and used to evaluate $g_{mb}$, $g_{pe}$ and $g_{pzt}$ through Eqs. (2)-(4). Figs. 3 (a) and (b) present the simulated MC field and the 5 MHz BAW mode.

As the acoustic resonator is a doubly-rotated stress compensated cut of pure quartz, material tensors give rise to the quasi-shear vibrational modes. Hence, such asymmetric geometries can only be reproduced by a 3D model. Tridimensional simulations, though, are very computationally expensive and limit the quality of the FEM mesh due to memory and time consumption. Despite this, the model is still suitable to estimate the order of magnitude of the piezoe-optomechanical coupling.

The piezoelectric component was calculated for two arrangements: first, when the acoustic mode is excited by the electrodes and then when it is resonant with the MC field. Only the first case was experimentally investigated. The second, could be explored using the beating of two pumps detuned from each other by the BAW mode frequency. Fig. 4 (c) shows the vacuum coupling strength due to optomechanical and piezoelectric effects in a frequency range that encompass a series of other shear and longitudinal acoustic modes. Many of these acoustic resonances were observed experimentally in the mechanical spectrum, and some of them were characterized by the simulated third overtone C-mode.
gives yet another advantage: the Teflon spacer can be
placing the electrodes by a two-tone microwave pump,
fact that it is possible to excite acoustic modes by re-
the crystal phonons more efficiently (Fig. 4 (c)). The
or two-tone configuration - we can piezoelectrically drive
sonable cooperativity values with simple optimization.
modest coupling strengths, such devices can achieve rea-
ty used to fit the data. We can show that even with
\[ \gamma = \sqrt{C_{\text{eff}}} \], of the order of MHz, which agrees with the
theory used to fit the data. We can show that even with
modest coupling strengths, such devices can achieve rea-
sonable cooperativity values with simple optimization.
Simulations demonstrate that making the cavity field res-
onant with the BAW mode - using a pump and probe,
or two-tone configuration - we can piezoelectrically drive
the crystal phonons more efficiently (Fig. 4 (c)). The
fact that it is possible to excite acoustic modes by re-
placing the electrodes by a two-tone microwave pump,
gives yet another advantage: the Teflon spacer can be
located at approximately 4.7 MHz (6% deviated from
the measured frequency), with calculated piezomechani-
cal coupling rate of \( g_{\text{eff}} = 4 \times 10^{-5} \) Hz for the case of
electrode driving, which corresponds with the experimen-
tal data (see red curves in Fig. 3).

In the piezoelectric coupling rate shown in Fig. 4 (c),
the subscript 1 stands for electrical actuation and 2 for
cavity field drive. These results indicate that, for the
third overtone C-mode, the radiation pressure coupling
is dominated by the photoelastic effect, although this is
not a regular trend for the other modes. With respect to
the piezoelectric effect, coupling resulting from resonantly
excited acoustic modes using the microwave cavity field
is, in general, three orders of magnitude larger than the
excitation with electrodes.

In conclusion, FEM indicates we were able to reach
an effective piezo-optomechanical coupling rate, \( g_{\text{eff}} = \sqrt{g_{0}^{\text{pe}} g_{0}^{\text{pzt}}} \), of the order of \( \mu \text{Hz} \), which agrees with the
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fact that it is possible to excite acoustic modes by re-
placing the electrodes by a two-tone microwave pump,
gives yet another advantage: the Teflon spacer can be
removed. This would permit the niobium cavity to be-
come fully superconducting. Niobium made cavities have
reported electrical Q-factors\(^{20}\) as high as \( 10^{8} \). Then, us-
ing Eq. 7 the piezo-optomechanical cooperativity can be
approximated as:

\[
C_{\text{om,pzt}} = \frac{4(\gamma_{0}^{\text{eff}})^{2}n_{\text{cav}}}{\kappa \gamma}
\]  

where \( \gamma \) is the total mechanical linewidth and \( n_{\text{cav}} \) is
the number of photons inside the cavity. A microwave
cavity field driven at 0.1 \( \mu \text{W} \), operating with a Q-factor of
\( 10^{8} \) and \( g_{0}^{\text{eff}} = \sqrt{g_{0}^{\text{pe}} g_{0}^{\text{pzt}}} \sim 10^{-5} \), could theoretically
achieve \( C_{\text{om,pzt}} > 10^{4} \).

These levels of cooperativity point to an ability to pro-
duce dressed states in the system, and reveal its po-
tential to be employed in a series of applications. Zou
et al.\(^{16}\) proposed a similar device, where an AlN-on-
Si chip is placed in a microwave re-entrant cavity to
enable coherent M-O conversion. In the same direction,
Vainsencher\(^{19}\) and Javerzac-Galy\(^{20}\) have investi-
gated electro-optomechanical devices for use as quantum
transducers. Moreover, Han et al.\(^{21}\) have specifically
worked with bulk acoustic resonators, reaching electro-
mechanical strong coupling, which is also valuable for
quantum memory implementations.

Therefore, the MC-BAW system is a feasible plat-
form for hybrid quantum devices, potentially enabling
quantum state transfer, coupling with superconducting
cubits and tests of fundamental theories\(^{22}\). In this sense,
this piezo-optomechanical device is favored by its high
mechanical frequency-quality-factor product, high power
handling and large effective modal mass. Also, it sup-
ports higher frequency acoustic modes, that can allow
direct ground state cooling via a dilution refrigerator.
Acoustic frequencies approaching 1 GHz have already
been measured in a quartz BAW resonator\(^{23}\). Addition-
ally, other materials with strong photoelastic and piezo-
electric response can be shaped as a confocal resonator
and be explored in the same way.

In summary, we have designed and tested a device able
to sustain piezo-optomechanical coupling between a
microwave 3D cavity and a BAW resonator. This hybrid
device was also theoretically and numerically modelled,
being thoroughly investigated with regard to the nature
of the different coupling mechanisms, namely: piezome-
chanics and optomechanics. It was concluded that the
system can reach cooperativities as high as \( 10^{4} \) with re-
alistic adjustments, demonstrating its potential to inte-
grate systems with applications in MO wavelength con-
version and investigation of nonclassical effects at the
macroscopic scale.

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