Isospin mixing and Coulomb mixing in ground states of even-even nuclei

Bui Minh Loc, Naftali Auerbach, and G. Colò

1School of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel.
2INFN, sezione di Milano, via Celoria 16, I-20133 Milano, Italy
3Dipartimento di Fisica, Università degli Studi di Milano, via Celoria 16, I-20133 Milano, Italy

(Dated: June 13, 2018)

Abstract

In this work, the Coulomb mixing and the isospin mixing in the ground states of even-even nuclei are evaluated in perturbation theory. The calculation of the isospin mixing is performed by using the connection to isovector monopole resonance properties. The uncertainty in the results that depends on different choices of the Skyrme interactions is shown. While Coulomb mixing turns out to be large in the ground states of heavy nuclei, isospin mixing is very small.
I. INTRODUCTION

The best-known part of the nuclear Hamiltonian is the Coulomb interaction between protons $V_C$. As a consequence isospin breaking is dominated by the $V_C$. The parent state $|\pi\rangle$ of the nucleus with isospin $T$ and $T_z = T$ contains the admixtures of states with isospin $T + 1$, 

$$|\pi\rangle = (1 - \sum_\alpha \varepsilon_\alpha^2)^{1/2}|T, T; 0\rangle + \sum_\alpha \varepsilon_\alpha|T + 1, T; \alpha\rangle.$$  \hspace{1cm} (1) 

The total probability $\varepsilon^2 = \sum_\alpha \varepsilon_\alpha^2$ is the isospin mixing. In first-order perturbation theory, the expression for isospin mixing is defined as 

$$\varepsilon_{T+1}^2 = \sum_{\alpha \neq 0} \frac{|\langle T, T; 0|V_C^{(IV)}|T + 1, T; \alpha\rangle|^2}{(E_\alpha - E_0)^2},$$  \hspace{1cm} (2) 

where $|T, T; 0\rangle$ denotes the g.s. at the energy $E_0$, and $\alpha$ are the various quantum numbers needed to specify the states $|\alpha\rangle$ at their energy $E_\alpha$. The Coulomb interaction can be rewritten in terms of isoscalar, isovector, and isotensor parts, but only the isovector part $V_C^{(IV)}$ is kept because the isoscalar part does not contribute to (2) and the isotensor part is small because of the long-range nature of the Coulomb interaction. Note that one needs to indicate $T + 1$ because an isovector operator excites not only $|T + 1, T; \alpha\rangle$, but also $|T, T; \alpha\rangle$. If the $|T, T; \alpha\rangle$ are also taken into account, the admixture is usually much larger. This kind of mixing was defined as the Coulomb mixing \cite{1}:

$$\varepsilon_C^2 = \sum_{\alpha \neq 0} \frac{|\langle 0|V_C^{(IV)}|\alpha\rangle|^2}{(E_\alpha - E_0)^2},$$  \hspace{1cm} (3) 

where the states $|\alpha\rangle$ now include both $T$ and $T + 1$ excitations. Therefore, the Coulomb mixing represents the total change induced by the Coulomb force in the wavefunction of the ground state. This mixing is more general than just isospin mixing because in many instances the effects of the Coulomb force do not lead necessarily to large components that differ in the isospin quantum numbers. In nuclei with $N = Z$, the isospin mixing and the Coulomb mixing are the same, $\varepsilon_C^2 = \varepsilon_{T+1}^2$, because there are only the $T = 1$ states. In a $N > Z$ nucleus, $\varepsilon_C^2 > \varepsilon_{T+1}^2$ as now there are $|T, T; \alpha\rangle$ and $|T + 1, T; \alpha\rangle$ contributing.

In Ref. \cite{1} and references therein, it was shown how the isospin mixing is connected to the notion of the isovector monopole (IVM) resonance that is defined by the operator 

$$Q_0^{(IV)} = \sum_i r_i^2 t_z(i),$$  \hspace{1cm} (4)
where $t_z$ is the $z$-component of isospin operator. Attempts were made to observe the IVM resonance experimentally [2, 3] because it plays an important role in many isospin processes [1]. Isospin mixing, that is the non-conservation of isospin quantum number is a good example. While in the past, the calculations [1] were performed with a few interactions such as SIII, SIV [4], nowadays, there are many Skyrme parameter sets. It is useful to study the dependence of the value of the isospin mixing and also the IVM properties with different modern Skyrme interactions.

The next section describes the method of calculation. By connecting the isospin mixing to the IVM resonance, the calculation of isospin mixing can be improved when the isospin properties of the IVM operator [3] are used. In the results, we present 12 Skyrme parameter sets including SIII [4], SGII [6], SKM* [7], SkP, SkI2 [8], SLy4 [9], SkO, SkO’ [10], LNS [11], SK255 [12], BSk17 [13], and SAMi0 [14]. It demonstrates how much the calculation depends on the choice of different Skyrme interactions. We restrict our discussion to even-even nuclei. The isospin mixing and Coulomb mixing were calculated for $N = Z$ including $^{40}$Ca, $^{56}$Ni, and $^{100}$Sn, and $N > Z$ nuclei including $^{48}$Ca, $^{78}$Ni, $^{90}$Zr, $^{120}$Sn, and $^{208}$Pb.

II. METHOD OF CALCULATION

The two-body Coulomb potential is given as

$$V_C = \frac{1}{2} \sum_{i,j}^A \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|} \left( \frac{1}{2} - t_z(i) \right) \left( \frac{1}{2} - t_z(j) \right),$$

(5)

where $t_z$ is the $z$-component of the nucleon-isospin operator, and its eigenvalue is $+\frac{1}{2}$ for neutron and $-\frac{1}{2}$ for proton. In the Hartree-Fock (HF) calculation, one has the one-body Coulomb potential that can be used in Eq. (2) and (3). A simplification is to approximate $V_C^{IV}$ using a homogeneous density distribution,

$$V_C^{(IV)} = \frac{Ze^2}{2R^3} \sum_{i=1}^A (3R^2 - r_i^2)t_z(i),$$

(6)

for $r \leq R$. In this case, the isospin mixing becomes

$$\varepsilon_{T+1}^2 = \left( \frac{Ze^2}{2R^3} \right)^2 \sum_{\alpha \neq 0} \frac{|\langle 0 |Q_0^{(IV)}| T + 1; \alpha \rangle|^2}{(E_\alpha - E_0)^2},$$

(7)

where $Q_0^{(IV)}$ is the $z$-component of the IVM operator and $R = r_0A^{1/3}$. It is pointed out that in this approximation, the results are, of course, affected by the choice of the value $r_0$. 

3
In Eq. (4), the operator $Q_0^{(IV)}$ is part of a s.p. isovector operator $Q_\mu^{(IV)}$ with $\mu = 0, \pm 1$

$$Q_\mu^{(IV)} = \sum_i r_i^2 t_\mu(i),$$  

(8)

where

$$t_{-1} = \frac{t_x - it_y}{\sqrt{2}}; \quad t_{+1} = \frac{-t_x + it_y}{\sqrt{2}}; \quad \text{and} \quad t_0 = t_z.$$  

(9)

When the isovector operator $Q_0^{(IV)}$ is applied in the parent nucleus with $N > Z$, both $|T, T; \alpha\rangle$ and $|T + 1, T; \alpha\rangle$ are excited and the isospin of these states cannot be distinguished when performing complicated HF-RPA calculations involving many orbits and thus many particle-hole (p-h) states. We should mention here that the states constructed of 1p-1h component only do not have good isospin, and in order to have good isospin one has to include certain class of 2p-2h components \[1, 15\]. These components are small and usually are not included. Their effect on the calculation of isospin mixing as performed here is very small. A technique based on the properties of isovector states in nuclei with $N > Z$ (see Fig. \[\text{I}\]) is used to determine separately $|T + 1, T; \alpha\rangle$ for the sum in Eq. (7). First, the calculation using operator $Q_{+1}^{(IV)}$

$$Q_{+1}^{(IV)} = \sum_{i=1}^A r_i^2 t_{+1}(i)$$  

(10)

that excites only $|T + 1, T + 1; \alpha\rangle$ in the nucleus $(N + 1, Z - 1)$ was done. After that, $|T + 1, T; \alpha\rangle$ states in the parent nucleus were obtained by using the fact that their energies $E_0(T + 1; \alpha)$ differ from the energies of $|T + 1, T + 1; \alpha\rangle$ in the nucleus $(N + 1, Z - 1)$ by one Coulomb displacement energy (CDE), $\Delta E_C$, i.e.

$$E_0(T + 1; \alpha) - E_{+1}(T + 1; \alpha) = \Delta E_C.$$  

(11)

We used the notations $E_\mu \ (\mu = 0, \pm 1)$ for the energies in the three nuclei in Fig. \[\text{I}\]. The CDE, $\Delta E_C$, can be obtained from the Skyrme-HF calculation. The transition strengths to various isospin components $T'$ of the $Q_\mu^{(IV)}$ matrix elements are given by the Wigner-Eckart theorem:

$$S_{T'}^{(\mu)}(\alpha) = |\langle T, T; 0|Q_\mu^{(IV)}|\alpha; T', T + \mu\rangle|^2$$

$$= |\langle TT1\mu|T'T + \mu\rangle|^2 \cdot |\langle T; 0||Q^{(IV)}||\alpha; T'\rangle|^2.$$  

(12)
FIG. 1. Isovector states in nuclei with $N > Z$. A single-particle isovector operator $Q^{(IV)}$, has three components, $Q_{-1}^{(IV)}$, $Q_{0}^{(IV)}$, and $Q_{+1}^{(IV)}$. In the parent nucleus, $Q_{0}^{(IV)}$ excites $|T + 1; T; \alpha\rangle$ and $|T, T; \alpha\rangle$. In the analog nucleus $(N - 1, Z + 1)$, $Q_{-1}^{(IV)}$ excites $|T + 1, T - 1; \alpha\rangle$, $|T, T - 1; \alpha\rangle$, and $|T - 1, T - 1; \alpha\rangle$. In the nucleus $(N + 1, Z - 1)$, $Q_{+1}^{(IV)}$ excites $|T + 1, T + 1; \alpha\rangle$ only.

The expression $S_{T'} = \sum_{\alpha} |\langle 0 || Q^{(IV)} || \alpha; T' \rangle|^2$ is the total reduced transition strengths. With $\langle TT10|T + 1 T\rangle^2 = 1/(T + 1)$ we find:

$$\varepsilon_{T+1}^2 = \sum_{\alpha \neq 0} \frac{S_{T+1}^{(+1)}(\alpha)}{(E_\alpha - E_0)^2},$$

(13)

with

$$S_{T+1}^{(+1)}(\alpha) = \frac{1}{T + 1} |\langle T; 0 || Q^{(IV)} || \alpha; T + 1 \rangle|^2,$$

(14)

and $E_\alpha = E_0(T + 1; \alpha) = E_{T+1}(T + 1; \alpha) + \Delta E_C$.

It is useful to recall the isospin properties of the IVM resonance related to the calculation performed here. The total transition strength of $Q^{(IV)}$ is expressed in terms of three reduced transition strengths $S_{T'}$

$$m_\mu(0) = \sum_{T'} \langle TT1\mu|T'T + \mu\rangle^2 S_{T'},$$

(15)

and if $\overline{E}_\mu(T')$ is the centroid energy of states of isospin $T'$ excited by $Q^{(IV)}$, we also have

$$m_\mu(1) = \sum_{T'} \langle TT1\mu|T'T + \mu\rangle^2 S_{T'} \overline{E}_\mu(T').$$

(16)
Using expression (15) and (16), we can obtain $S_T^{(0)}$ and $E_0(T)$ and determine the isospin energy splitting $\Delta E_+$

$$\Delta E_+ = E_0(T + 1) - E_0(T)$$

that relates to the symmetry potential $V_1$ defined by the expression

$$V_1 = \frac{A}{T+1} \Delta E_+.$$  \hspace{1cm} (18)

In practice, the sum in Eq. (3) for the calculation of the Coulomb mixing was obtained from the HF-RPA code following Ref. [16]. The sum in Eq. (2) for the isospin mixing was calculated using the HF-RPA code including the charge-exchange mode (HF-pnRPA) [17].

### III. RESULTS AND DISCUSSION

Although the actual HF Coulomb potential $V_C^{(IV)}$ can be used directly as the “probing” operator, in our calculation the IVM operator $Q_0^{(IV)}$ is also utilized to have the connection between the Coulomb mixing, the isospin mixing and the properties of the IVM resonance. As mentioned above, when the IVM operator is used, the results are affected by the choice of $r_0$. In Fig. 2, the distribution of Coulomb strength evaluated using the $V_C^{(IV)}$ and IVM strength evaluated using the $Q_0^{(IV)}$, are shown for $^{208}$Pb. We can see in Fig. 2 their close similarity. This is the reason why one can use the ratio $\eta$ between the total strength of Coulomb distribution and that of the IVM distribution:

$$\eta = \frac{\sum \langle 0 | V_C^{(IV)} | \alpha \rangle}{\sum \langle 0 | Q_0^{(IV)} | \alpha \rangle}$$  \hspace{1cm} (19)

instead of the factor $\left( \frac{Z q_a^2}{2\hbar^2} \right)^2$ in Eq. (7). Therefore, the uncertainty from the value of $r_0$ is avoided. In addition, it was found that the ratio $\eta$ is close to the value of the factor in Eq. (7) if $r_0 = 1.25$ fm. In the calculation of the Coulomb strength above, the $V_C^{(IV)}$ contains not only the inside part ($r \leq R$) but also the outside part ($r > R$). From the similarity shown in Fig. 2 we can conclude that the outside part ($r > R$) does not contribute much to the result.

Table I shows the values of the Coulomb mixing for $N = Z$ nuclei including $^{40}$Ca, $^{56}$Ni, and $^{100}$Sn. In this case, the Coulomb mixing and isospin mixing are the same because there are only $T = 1$ states. The difference between two different operators is very small, and it
FIG. 2. The distribution of Coulomb strength and IVM strength in $^{208}$Pb. The discrete RPA peaks for both operator were smoothed by using the same Lorentzian averaging with the width of 1 MeV.

allows us to use the $Q_0^{(IV)}$ with the ratio $\eta$ instead of $V_C^{(IV)}$. The difference in the value of the isospin mixing between different Skyrme interactions is not large.

The Coulomb potential is kept in our HF and RPA calculation. One can argue that the Coulomb potential should not be included in this calculation. The code [16] we use allows us easily to include or exclude the Coulomb potential consistently (in both HF and RPA). We find that this uncertainty, in this case, is not large even in $^{100}$Sn (see Table I and Table II). We prefer to use the results of the HF and RPA included Coulomb potential as inputs into the calculation of isospin and Coulomb mixing because they are more realistic and can be compared to experiment.

In the case of nuclei that have $N > Z$, the Coulomb mixing and isospin mixing are different. Among $N > Z$ nuclei, $^{78}$Ni is an interesting nucleus because $T = 14$ is large while $Z = 20$ is relatively small. As one expects, the isospin mixing is strongly reduced by the factor $1/(T + 1)$. In most nuclei, the Coulomb potential can be treated in perturbation theory. When $Z$ becomes large and the Coulomb potential becomes very strong such as in the case of Oganneson $^{302}_{118}$Og, the perturbation theory in the calculation of the Coulomb mixing is not correct. However, the isospin mixing in $^{302}_{118}$Og ($T = 33$) is still small, around 2%, because of the factor $1/(T + 1)$. It is useful to remind that the uncertainties caused by other sources besides the Coulomb interaction are expected to be an order of magnitude
TABLE I. The Coulomb mixing (%) of $N = Z$ nuclei including $^{40}$Ca, $^{56}$Ni, and $^{100}$Sn. The Coulomb potential is included in the HF-RPA calculation.

| No. | Int.   | $^{40}$Ca |   | $^{56}$Ni |   | $^{100}$Sn |   |
|-----|--------|-----------|---|-----------|---|-----------|---|
|     |        | $Q_0^{(IV)}$ | $V_C^{(IV)}$ | $Q_0^{(IV)}$ | $V_C^{(IV)}$ | $Q_0^{(IV)}$ | $V_C^{(IV)}$ |
| 1   | SIII   | 0.96      | 0.68 | 1.55      | 1.22 | 5.44      | 4.54 |
| 2   | SGII   | 1.09      | 0.79 | 1.85      | 1.46 | 6.58      | 5.51 |
| 3   | SKM*   | 1.12      | 0.78 | 1.82      | 1.42 | 6.44      | 5.34 |
| 4   | SKP    | 1.15      | 0.81 | 2.00      | 1.56 | 6.82      | 5.69 |
| 5   | SkI2   | 0.87      | 0.62 | 1.43      | 1.11 | 5.34      | 4.46 |
| 6   | SLy4   | 1.05      | 0.77 | 1.78      | 1.43 | 6.17      | 5.27 |
| 7   | SKO    | 0.90      | 0.62 | 1.31      | 0.98 | 5.04      | 4.13 |
| 8   | SKO’   | 1.06      | 0.73 | 1.45      | 1.13 | 5.64      | 4.68 |
| 9   | LNS    | 1.15      | 0.81 | 1.90      | 1.49 | 6.64      | 5.47 |
| 10  | SK255  | 0.89      | 0.62 | 1.55      | 1.17 | 5.37      | 4.40 |
| 11  | BSK17  | 1.02      | 0.70 | 1.54      | 1.23 | 5.68      | 4.75 |
| 12  | SAMi0  | 1.01      | 0.74 | 1.73      | 1.36 | 6.13      | 5.15 |

TABLE II. The Coulomb mixing (%) of $N = Z$ nuclei including $^{40}$Ca, $^{56}$Ni, and $^{100}$Sn. The Coulomb potential is excluded in the HF-RPA calculation.

| No. | Int.   | $^{40}$Ca |   | $^{56}$Ni |   | $^{100}$Sn |   |
|-----|--------|-----------|---|-----------|---|-----------|---|
|     |        | $Q_0^{(IV)}$ | $V_C^{(IV)}$ | $Q_0^{(IV)}$ | $V_C^{(IV)}$ | $Q_0^{(IV)}$ | $V_C^{(IV)}$ |
| 1   | SIII   | 1.06      | 0.80 | 1.69      | 1.42 | 5.54      | 4.94 |
| 2   | SGII   | 1.21      | 0.94 | 2.01      | 1.68 | 6.70      | 5.97 |
| 3   | SKM*   | 1.23      | 0.93 | 1.97      | 1.64 | 6.53      | 5.76 |
| 4   | SLy4   | 1.17      | 0.92 | 1.95      | 1.67 | 6.40      | 5.79 |
| 5   | SAMi0  | 1.12      | 0.88 | 1.86      | 1.55 | 6.21      | 5.53 |

smaller, thus they are smaller than the difference between results obtained with different choices of the Skyrme interactions.

Finally, as mentioned in the text, for the isospin mixing, only the $T + 1$ states are taken
TABLE III. Coulomb mixing $\varepsilon_C^2$ (%) and isospin mixing $\varepsilon_{T+1}^2$ (%) of $N > Z$ nuclei including $^{48}\text{Ca}$ ($T = 4$), $^{78}\text{Ni}$ ($T = 14$), $^{90}\text{Zr}$ ($T = 5$), $^{120}\text{Sn}$ ($T = 10$), and $^{208}\text{Pb}$ ($T = 22$).

|       | $^{48}\text{Ca}$ | $^{78}\text{Ni}$ | $^{90}\text{Zr}$ | $^{120}\text{Sn}$ | $^{208}\text{Pb}$ |
|-------|------------------|------------------|------------------|------------------|------------------|
|       | $\varepsilon_C^2$ | $\varepsilon_{T+1}^2$ | $\varepsilon_C^2$ | $\varepsilon_{T+1}^2$ | $\varepsilon_C^2$ | $\varepsilon_{T+1}^2$ |
| 1 SIII | 1.14 0.10 | 3.83 0.04 | 4.13 0.52 | 10.85 0.23 | 29.90 0.29 |
| 2 SGII | 1.33 0.12 | 4.28 0.05 | 4.98 0.66 | 11.66 0.32 | 34.38 0.41 |
| 3 SKM* | 1.35 0.12 | 4.66 0.05 | 4.91 0.62 | 12.10 0.30 | 35.57 0.36 |
| 4 SKP | 1.52 0.11 | 5.53 0.04 | 5.18 0.58 | 12.41 0.28 | 38.50 0.32 |
| 5 SkI2 | 1.20 0.09 | 4.71 0.03 | 4.13 0.53 | 11.03 0.25 | 32.36 0.32 |
| 6 SLy4 | 1.37 0.12 | 4.45 0.04 | 4.73 0.56 | 11.36 0.25 | 33.64 0.29 |
| 7 SKO | 1.32 0.05 | 6.19 0.02 | 4.02 0.41 | 10.50 0.23 | 32.75 0.26 |
| 8 SKO' | 1.20 0.07 | 4.79 0.03 | 4.36 0.49 | 11.69 0.25 | 34.64 0.28 |
| 9 LNS | 1.43 0.12 | 4.85 0.04 | 5.10 0.63 | 12.86 0.29 | 38.04 0.34 |
| 10 SK255 | 1.24 0.08 | 4.64 0.03 | 4.13 0.49 | 10.84 0.24 | 31.94 0.28 |
| 11 BSki7 | 1.18 0.10 | 4.40 0.04 | 4.30 0.53 | 11.38 0.23 | 33.24 0.28 |
| 12 SAMi0 | 1.35 0.11 | 4.27 0.05 | 4.72 0.60 | 10.93 0.30 | 31.85 0.37 |

into account and the isospin properties of the IVM resonance are useful for the calculation. Therefore, the properties of the IVM resonance are shown in Tables IV-VIII for $^{48}\text{Ca}$, $^{78}\text{Ni}$, $^{90}\text{Zr}$, $^{120}\text{Sn}$, and $^{208}\text{Pb}$, respectively.

In Tables IV-VIII the average energy of the transition strength distribution is $E_0 = m_0(1)/m_0(0)$. $S_T$ and $S_{T+1}$ are the total transition strength to the $|T, T; \alpha\rangle$ and $|T+1, T; \alpha\rangle$, respectively. $\Delta E_C$ is the direct CDE. $\overline{\Delta E_+}$ given by Eq. (17) is the difference in energy between $|T, T; \alpha\rangle$ and $|T+1, T; \alpha\rangle$, and $V_1$ is the symmetry potential defined in Eq. (18). These values can be compared to the work in Ref. 5 where the Green function method was employed using the SIII Skyrme interaction.

In Tables IV-VIII the values of $\hbar \omega = 41 \times A^{-1/3}$ are given to describe the $A^{-1/3}$ behavior of the energy of the IVM- $E_0$, from our calculation. This behavior was also obtained in the hydrodynamical model or other collective models. Due to the Pauli blocking of the excess neutrons, it is expected that $S_{T+1}/S_T < 1$. Indeed, the p-h excitations involving a proton transformed into a neutron in an excess neutron orbit are forbidden by the Pauli principle.
Not so in the excitations represented by $S_T$. There one can have p-h components in which the proton is placed in the orbits occupied by the excess neutrons. However, it is different in the case of $^{90}$Zr. A plausible explanation is in the following. In $^{90}$Zr the excess neutrons occupy mostly the $1g_{9/2}$ orbit. There are no $j = 9/2$ orbit below the $1g_{9/2}$ and therefore one cannot occupy this orbit if one constructs a $J^\pi = 0^+$ state. Thus the amount of p-h components for the $T$, and $T + 1$ is the same and there should be little difference between $S_T$ and $S_{T+1}$. In $\Delta E_C$, only the direct part which contributes more than 90% to the CDE was included. Other effects: exchange term, finite proton size effect, and vacuum polarization \[18\] were not taken into account. All these corrections are small and contribute only a few percents to the CDE. It makes the value of the direct CDE quite acceptable for the purpose of our study. The value of $V_1$ obtained from a single-particle-symmetry potential is about 100 MeV \[19\]. In our calculation, $V_1$ is also around 100 MeV.

TABLE IV. Isospin properties of the IVM resonance for $^{48}$Ca ($\hbar\omega = 11.28$ MeV). $E_0$ is the average energy of the strength distribution. $S_T$ and $S_{T+1}$ are the reduced transition strength to $|T, T; \alpha\rangle$, and $|T + 1, T; \alpha\rangle$, respectively. $\Delta E_C$ is the direct term of the CDE. $\Delta E_+$ is the energy difference between $|T, T; \alpha\rangle$ and $|T + 1, T; \alpha\rangle$. $V_1$ is the symmetry potential as defined in the expression \[18\].

|   | $E_0$   | $S_T$ | $S_{T+1}$ | $\Delta E_C$ | $\Delta E_+$ | $V_1$ |
|---|--------|-------|-----------|--------------|-------------|------|
| 1 | 34.79  | 149.27| 125.68    | 7.27         | 11.06       | 106.14|
| 2 | 32.87  | 153.12| 113.98    | 7.33         | 6.88        | 66.01 |
| 3 | 32.54  | 156.72| 125.68    | 7.23         | 9.60        | 92.17 |
| 4 | 30.00  | 160.48| 123.00    | 7.23         | 11.15       | 107.02|
| 5 | 33.08  | 141.64| 91.56     | 7.10         | 7.43        | 71.34 |
| 6 | 30.57  | 148.43| 123.51    | 7.22         | 10.19       | 97.80 |
| 7 | 32.60  | 144.58| 72.78     | 6.96         | 15.84       | 152.08|
| 8 | 32.32  | 138.67| 81.67     | 7.14         | 11.75       | 112.77|
| 9 | 33.14  | 134.72| 102.85    | 7.52         | 9.78        | 93.86 |
|10 | 34.65  | 153.56| 100.39    | 7.08         | 11.77       | 112.99|
|11 | 32.83  | 139.00| 110.69    | 7.31         | 11.32       | 108.63|
|12 | 32.28  | 154.64| 109.01    | 7.20         | 8.05        | 77.25 |
TABLE V. The same as in Table IV but for $^{78}$Ni ($\hbar \omega = 9.60$ MeV).

|   | $E_0$ | $S_T$ | $S_{T+1}$ | $\Delta E_C$ | $\Delta E_+$ | $V_1$ |
|---|-----|-----|-----|--------|--------|-----|
| 1 | SIII | 30.55 | 382.04 | 186.47 | 8.90 | 18.19 | 118.23 |
| 2 | SGII | 28.86 | 385.18 | 186.63 | 8.96 | 14.34 | 93.22 |
| 3 | SKM* | 28.47 | 400.08 | 175.45 | 8.85 | 16.29 | 105.91 |
| 4 | SKP  | 26.14 | 405.03 | 165.44 | 8.88 | 18.40 | 119.57 |
| 5 | SkI2 | 27.01 | 387.49 | 123.12 | 8.64 | 19.35 | 125.79 |
| 6 | SLy4 | 27.10 | 373.59 | 168.29 | 8.87 | 17.50 | 113.74 |
| 7 | SKO  | 25.51 | 412.48 | 117.36 | 8.60 | 26.73 | 173.73 |
| 8 | SKO' | 26.77 | 365.85 | 112.93 | 8.80 | 21.14 | 137.41 |
| 9 | LNS  | 29.24 | 341.93 | 121.66 | 9.18 | 17.14 | 111.43 |
| 10 | SK255 | 29.37 | 401.49 | 120.28 | 8.64 | 19.59 | 127.33 |
| 11 | BSk17 | 28.01 | 364.18 | 155.15 | 8.93 | 15.02 | 129.29 |
| 12 | SAMi0 | 28.96 | 388.83 | 196.12 | 8.87 | 15.02 | 97.66 |

IV. CONCLUSION

The isospin mixing in the ground state is small, especially in heavy nuclei. It does not strongly depend on the choice of the Skyrme interactions. It is clear that the formalism of isospin is not only useful but also powerful in nuclear physics, where many examples of isospin symmetry can be found. In particular, this is true in the ground states where isospin mixing does not exceed a few percents. This does not mean that the isospin non-conserving interaction, the Coulomb force plays a minor role in forming the nucleus, as can be seen in the large Coulomb mixing we calculated.

ACKNOWLEDGMENTS

The authors thank to Nguyen Van Giai for discussions and Vladimir Zelevinsky for discussions made possible by the travel grant from the US-Israel Binational Science Foundation (2014.24). This work was supported by the US-Israel Binational Science Foundation, grant 2014.24.
TABLE VI. The same as in Table IV but for $^{90}$Zr ($\hbar \omega = 9.15$ MeV).

|     | $\bar{E}_0$ | $S_T$ | $S_{T+1}$ | $\Delta E_C$ | $\Delta E_+$ | $V_1$ |
|-----|-------------|-------|-----------|--------------|-------------|-------|
| 1   | 33.53       | 425.52| 430.42    | 12.22        | 4.24        | 63.66 |
| 2   | 31.45       | 426.10| 444.33    | 12.38        | 3.52        | 52.74 |
| 3   | 31.53       | 435.10| 445.68    | 12.25        | 3.98        | 59.64 |
| 4   | 29.61       | 431.82| 424.44    | 12.28        | 5.30        | 79.56 |
| 5   | 31.89       | 380.95| 368.23    | 12.04        | 3.12        | 46.82 |
| 6   | 29.96       | 405.62| 405.58    | 12.25        | 4.90        | 73.44 |
| 7   | 32.08       | 377.30| 293.82    | 11.85        | 3.41        | 51.18 |
| 8   | 30.98       | 375.09| 330.47    | 12.08        | 3.71        | 55.64 |
| 9   | 32.08       | 371.34| 385.09    | 12.70        | 4.82        | 72.28 |
| 10  | 33.93       | 412.47| 400.59    | 12.01        | 4.60        | 69.04 |
| 11  | 31.61       | 387.62| 380.95    | 12.34        | 4.32        | 64.73 |
| 12  | 31.78       | 424.63| 426.62    | 12.23        | 3.59        | 53.78 |

TABLE VII. The same as in Table IV but for $^{120}$Sn ($\hbar \omega = 8.31$ MeV).

|     | $\bar{E}_0$ | $S_T$ | $S_{T+1}$ | $\Delta E_C$ | $\Delta E_+$ | $V_1$ |
|-----|-------------|-------|-----------|--------------|-------------|-------|
| 1   | 29.86       | 804.40| 469.01    | 13.97        | 8.73        | 95.20 |
| 2   | 28.25       | 798.77| 499.43    | 14.12        | 6.87        | 74.98 |
| 3   | 28.21       | 818.03| 492.10    | 13.92        | 7.51        | 81.94 |
| 4   | 26.77       | 795.35| 466.98    | 14.00        | 8.39        | 91.48 |
| 5   | 27.78       | 735.27| 402.59    | 13.73        | 8.03        | 87.55 |
| 6   | 26.96       | 756.12| 427.30    | 13.97        | 8.91        | 97.18 |
| 7   | 28.10       | 695.72| 401.01    | 13.95        | 8.53        | 93.08 |
| 8   | 27.04       | 709.41| 405.85    | 13.99        | 8.98        | 97.98 |
| 9   | 28.70       | 701.41| 414.99    | 14.37        | 8.49        | 92.57 |
| 10  | 30.11       | 777.41| 453.56    | 13.67        | 8.43        | 91.93 |
| 11  | 27.87       | 740.77| 404.41    | 14.03        | 9.12        | 99.45 |
| 12  | 28.70       | 788.05| 490.95    | 14.08        | 7.21        | 78.63 |
[1] N. Auerbach. Coulomb effects in nuclear structure. *Physics Reports*, 98(5):273 – 341, 1983.

[2] A. Erell, *et al.*. Measurements on isovector giant resonances in pion charge exchange. *Phys. Rev. C*, 34:1822–1844, Nov 1986.

[3] M. Scott, *et al.*. Observation of the Isovector Giant Monopole Resonance via the $^{28}\text{Si}^{(10}\text{Be},^{10}\text{B}^*[1.74\text{ MeV}]$) Reaction at 100 AMeV. *Phys. Rev. Lett.*, 118:172501, Apr 2017.

[4] M. Beiner, H. Flocard, Nguyen Van Giai, and P. Quentin. Nuclear ground-state properties and self-consistent calculations with the Skyrme interaction: (I). Spherical description. *Nuclear Physics A*, 238(1):29 – 69, 1975.

[5] N. Auerbach and A. Klein. A microscopic theory of giant electric isovector resonances. Nuclear Physics A, 395(1):77 – 118, 1983.

[6] Nguyen Van Giai and H. Sagawa. Spin-isospin and pairing properties of modified Skyrme interactions. *Physics Letters B*, 106(5):379 – 382, 1981.

**TABLE VIII.** The same as in Table IV but for $^{208}\text{Pb}$ ($\hbar\omega = 6.92\text{ MeV}$).

|   | $\bar{E}_0$ | $S_T$ | $S_{T+1}$ | $\Delta E_C$ | $\Delta \bar{E}_+$ | $V_1$ |
|---|---|---|---|---|---|---|
| 1 | SIII | 28.19 | 2081.68 | 1164.34 | 19.34 | 9.87 | 89.25 |
| 2 | SGII | 26.35 | 2104.85 | 1271.02 | 19.52 | 7.90 | 71.45 |
| 3 | SKM* | 26.50 | 2130.51 | 1221.69 | 19.36 | 8.81 | 79.69 |
| 4 | SKP | 25.04 | 2065.16 | 1111.53 | 19.46 | 10.59 | 95.75 |
| 5 | SkI2 | 25.37 | 1972.33 | 1012.50 | 19.00 | 8.64 | 78.12 |
| 6 | SLy4 | 25.29 | 1954.69 | 1025.64 | 19.39 | 10.85 | 98.12 |
| 7 | SKO | 25.28 | 1918.29 | 895.07 | 19.08 | 9.28 | 83.90 |
| 8 | SKO’ | 24.94 | 1878.92 | 928.52 | 19.28 | 10.06 | 91.01 |
| 9 | LNS | 26.85 | 1839.60 | 1034.58 | 19.93 | 10.29 | 93.03 |
| 10 | SK255 | 27.89 | 2064.87 | 1106.52 | 18.98 | 10.00 | 90.46 |
| 11 | BSk17 | 26.04 | 1918.62 | 986.63 | 19.50 | 10.48 | 94.80 |
| 12 | SAMi0 | 26.87 | 2077.67 | 1215.95 | 19.45 | 8.10 | 73.26 |
[7] M. Brack, C. Guet, and H.-B Håkansson. Selfconsistent semiclassical description of average nuclear properties—a link between microscopic and macroscopic models. *Physics Reports*, 123(5):275 – 364, 1985.

[8] J. Dobaczewski, H. Flocard, and J. Treiner. Hartree-Fock-Bogolyubov description of nuclei near the neutron-drip line. *Nuclear Physics A*, 422(1):103 – 139, 1984.

[9] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer. A Skyrme parametrization from subnuclear to neutron star densities Part II. Nuclei far from stabilities. *Nuclear Physics A*, 635(1):231 – 256, 1998.

[10] P.-G. Reinhard, D. J. Dean, W. Nazarewicz, J. Dobaczewski, J. A. Maruhn, and M. R. Strayer. Shape coexistence and the effective nucleon-nucleon interaction. *Phys. Rev. C*, 60:014316, Jun 1999.

[11] L. G. Cao, U. Lombardo, C. W. Shen, and Nguyen Van Giai. From Brueckner approach to Skyrme-type energy density functional. *Phys. Rev. C*, 73:014313, Jan 2006.

[12] B. K. Agrawal, S. Shlomo, and V. Kim An. Nuclear matter incompressibility coefficient in relativistic and nonrelativistic microscopic models. *Phys. Rev. C*, 68:031304, Sep 2003.

[13] S. Goriely, N. Chamel, and J. M. Pearson. Skyrme-Hartree-Fock-Bogoliubov Nuclear Mass Formulas: Crossing the 0.6 MeV Accuracy Threshold with Microscopically Deduced Pairing. *Phys. Rev. Lett.*, 102:152503, Apr 2009.

[14] X. Roca-Maza, G. Colò, and H. Sagawa. New Skyrme interaction with improved spin-isospin properties. *Phys. Rev. C*, 86:031306, Sep 2012.

[15] N. Auerbach and A. Yeverechyahu. The isospin splitting of isovector monopole excitations. Nuclear Physics A, 332(1):173 – 182, 1979.

[16] Gianluca Colò, Ligang Cao, Nguyen Van Giai, and Luigi Capelli. Self-consistent RPA calculations with Skyrme-type interactions: The Skyrme rpa program. *Computer Physics Communications*, 184(1):142 – 161, 2013.

[17] G. Colò. Charge-changing Random Phase Approximation code (pnRPA). (unpublished).

[18] Naftali Auerbach, Jörg Hüfner, A. K. Kerman, and C. M. Shasin. A Theory of Isobaric Analog Resonances. *Rev. Mod. Phys.*, 44:48–125, Jan 1972.

[19] A. Bohr, B. Mottelson. Nuclear Structure, volume I, page 148. Benjamin, NY, 1969.