A Hamiltonian lattice study of the two-dimensional Wess-Zumino model
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We investigate a Hamiltonian lattice version of the two-dimensional Wess-Zumino model by Quantum Monte Carlo simulations. In order to study the pattern of supersymmetry breaking, we measure the ground state energy and the correlation length along a trajectory approaching the continuum limit. The algorithm is very effective in measuring the ground state energy, and adequate for the correlation length.

1. INTRODUCTION

Numerical simulations of lattice field theories are usually performed in the Lagrangian formulation. Nonetheless, we think there are very good reasons to develop numerical simulation techniques for the Hamiltonian approach\textsuperscript{[1]}: powerful many-body techniques are available\textsuperscript{[2]}, which allow the direct computation of the vacuum wave function properties; fermions are implemented directly and need not be integrated out; properties like the mass spectrum are more immediate. Finally, universality checks between the Lagrangian and the Hamiltonian formalism are very welcome.

2. THE MODEL

We study the Hamiltonian lattice version of the two-dimensional Wess-Zumino model described in Refs.\textsuperscript{[3,4]}; we only wish to highlight here the main features of the formulation.

In the Hamiltonian formalism, since $H$ is conserved, it is possible to preserve exactly a 1-dimensional subalgebra of the original supersymmetry algebra, i.e., we can write $H = Q^2$, where $Q$ is a fermionic charge. This subalgebra is enough to guarantee some of the most important property of supersymmetry, including a non-negative spectrum, and pairing of fermionic and bosonic states of nonzero energy; spontaneous breaking of supersymmetry is equivalent to a strictly positive ground-state energy $E_0$; the full supersymmetry algebra is recovered in the continuum limit together with Lorentz invariance.

In order to obtain a Hamiltonian free of fermion sign problems, and therefore amenable to Quantum Monte Carlo methods, we adopt free boundary conditions, with lattice size $L \equiv 2 \pmod{4}$.

The model is parametrized by a prepotential $V(\phi)$, an arbitrary polynomial in the bosonic field. The two-dimensional Wess-Zumino model is superrenormalizable; fields do not renormalize, and only $V(\phi)$ needs to be normal ordered.

In strong coupling at leading order, the model reduces to independent copies of supersymmetric quantum mechanics, one for each site; supersymmetry is broken if and only if the degree of the prepotential $V$ is even\textsuperscript{[5]}. In weak coupling, on the other hand, supersymmetry is broken at tree level if and only if $V$ has no zeroes. The predictions of strong coupling and weak coupling are quite different, and it is interesting to study the crossover from strong to weak coupling.

3. MONTE CARLO SIMULATIONS

We perform our simulations using the Green Function Monte Carlo (GFMC) algorithm\textsuperscript{[2]}. A discussion of GFMC in the context of the present problem can be found in Ref.\textsuperscript{[4]}; we only wish to remark the main features of the algorithm: the aim is to generate a stochastic representation of the ground-state wavefunction, which is then
used to compute expectation values of observables. Statistical fluctuations are reduced with the help of a guiding wavefunction, whose free parameters are determined dynamically during the simulation. In order to keep the variance of observables finite as the simulation proceeds, it is necessary to simulate a population of $K$ walkers (field configurations at fixed time), and extrapolate the results to $K \to \infty$.

We focus on the case $V = \lambda_2 \phi^2 + \lambda_0 \phi$; strong coupling always predicts supersymmetry breaking; weak coupling predicts unbroken supersymmetry for $\lambda_0 < 0$; according to Ref. [9], unbroken supersymmetry should be accompanied by a nonzero $\langle \phi \rangle$ (parity breaking).

We presented preliminary results, obtained by GFMC, for intermediate couplings in Ref. [3]; our aim is to extend the study towards the continuum limit and to larger lattices.

Perturbative computations show that

$$\lambda_2 \approx a\lambda_2^{\text{ren}}, \quad \lambda_0 \approx a\lambda_0^{\text{ren}} + a\lambda_2^{\text{ren}} \frac{1}{2\pi} \ln (aM), \quad (1)$$

where $\lambda_i$ is the adimensional lattice bare coupling, $\lambda_i^{\text{ren}}$ is the renormalized (continuum) coupling, with dimension of $m^3$, defined at the mass scale $M$, and $a$ is the lattice spacing. We study, as $\lambda_2 \to 0$, the trajectory

$$\lambda_0 = \frac{\lambda_2}{2\pi} \ln(4\lambda_2), \quad (2)$$

corresponding to a perturbative RG trajectory (1); the effect of $\lambda_0$ is small in the range we considered, therefore we expect Eq. (2) to be a reasonable approximation to a true RG trajectory.

We estimate the correlation length from the exponential decay of the connected correlation function $G_d = \langle \phi_n \phi_m \rangle_c$ averaged over all $n,m$ pairs with $|m-n| = d$, excluding pairs for which $m$ or $n$ is closer to the border than (typically) 8.

In our formulation, fermions are staggered and even/odd $d$ correspond to different channels.

We begin with the discussion of the case $V = 0.35 \phi^2 + 0.02$, for which we have obtained the statistics of $4 \times 10^6$ GFMC iterations. The even-$d$ channel is plotted in Fig. 1; it is very difficult to extract a correlation length, presumably because $\phi$ has a very small overlap with the lightest state of the channel, and the value of $\xi$ quoted in Fig. 1 should be considered tentative. The odd-$d$ channel, plotted in Fig. 2, is much cleaner, and it is possible to estimate $\xi$ with a good precision.

For the other values of $\lambda_2$, the situation is similar but with larger errors; we have a statistics of at least $10^6$ iterations, which we are increasing to $4 \times 10^6$. The values of $\xi_{\text{odd}}$ follow nicely the expected behavior $\xi \propto \lambda_2$, as shown in Fig. 3; the entire range $0.088 \leq \lambda_2 \leq 0.35$ seems to be in the scaling region, with $\lambda_2 = 0.5$ a borderline case. The values of $\xi_{\text{even}}$ have very large errors, and it is hard to draw any conclusion from them.

We measure the ground state energy $E_0$ along the trajectory (3); the measurements have a very small statistical error, ranging from 1% for $\lambda_2 = 0.044$ (where $E_0/L \approx 10^{-3}$) to 0.1% for $\lambda_2 = 0.5$. We extrapolate to $L \to \infty$ and $K \to \infty$ fitting $E_0/L$ to the form

$$E_0/L = \mathcal{E}_0 (1 + c_L / L + c_K / K). \quad (3)$$

$E_0/L$ is plotted in Fig. 4; it seems to behave
Figure 2. Connected correlation of $\phi$ at odd distance for $V = 0.35 \phi^2 + 0.02$: the solid line is an exponential fit from distance 3 to 15; $K = 100$ data are consistent with the $K = 200$ data shown.

$\propto \lambda_2^{5/3}$, while naïve scaling would predict $\propto \lambda_2^2$. The value of $E_0/L$ (disregarding this puzzling exponent) and the lack of any signal for a breakdown of parity (like a double-peaked distribution of $\phi$) strongly hint that the trajectory (2) belongs to the phase with broken supersymmetry and zero $\langle \phi \rangle$. We are repeating the computation for trajectories with smaller $\lambda_0$.

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Figure 3. Correlation length in the odd distance channel vs. $\lambda_2$: the solid line is a scaling fit.

Figure 4. Ground-state energy density vs. $\lambda_2$. The dashed line is the result of a fit; the solid line shows the naïve scaling behavior.