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NONABELIAN $N = 2$ SUPERSTRINGS: HAMILTONIAN STRUCTURE  

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ABSTRACT  

We examine the Hamiltonian structure of nonabelian $N=2$ superstrings models which are the supergroup manifold extensions of $N=2$ Green-Schwarz superstring. We find the Kac-Moody and Virasoro type superalgebras of the relevant constraints and present elements of the corresponding quantum theory. A comparison with the type IIA Green-Schwarz superstring moving in a general curved 10-d supergravity background is also given. We find that nonabelian superstrings (for $d=10$) present a particular case of this general system corresponding to a special choices of the background.

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1 Introduction

In the second paper devoted to nonabelian $N=2$ superstrings [1] we present the basic elements of the corresponding hamiltonian formalism which is a necessary starting point in constructing the full quantum theory of this system.

In Sect. 2 we recapitulate the known facts about ordinary Wess-Zumino-Novikov-Witten (WZNW) sigma models in the hamiltonian approach [5, 4], following a version of the latter employed in [3]. It has an advantage of admitting a straightforward extension to the case of nonabelian $N=2$ superstrings we are interested in. This system is treated in Sect. 3: we find the relevant hamiltonian constraints, show how to divide them into the first and second class ones, establish the structure of the Virasoro and Kac-Moody-type superalgebras generated by these constraints. In Sect. 4 we compare our model with the GS superstring moving in a $N=2$ $d=10$ supergravity background and argue that the former system (for $d=10$) presents a special solution for the latter one (corresponding to the type IIA).

2 WZNW sigma models as sigma models on $G_L \times G_R/G_{\text{diag}}$ coset space. Hamiltonian structure

In this section we recall the basic facts about the hamiltonian formulation of ordinary WZNW sigma models [3, 5, 4]. Analogous techniques will be applied in Sect. 3 to study the hamiltonian structure of nonabelian $N=2$ superstrings.

For reasons to become clear later, we follow the interpretation of WZNW sigma model on the group $G$ as a sigma model whose target space is the symmetric coset space $G_L \times G_R/G_{\text{diag}}$. This means that one originally deals with two sets of group coordinates (parametrizing $G_L$ and $G_R$) related through right gauge transformations of $G_{\text{diag}}$. In a customary approach, only one set of the $G$ valued coordinates is introduced, which corresponds to choosing a particular gauge with respect to the right gauge $G_{\text{diag}}$ transformations.

We start by writing the general action for the WZNW sigma models [5, 6] coupled to 2D gravity

$$A = l_I \left\{ \int d^2 \xi \sqrt{-g} g^{ab} \text{Tr}(\omega_a \omega_b) - 2/3 \int d^3 \xi \varepsilon^{abc} \text{Tr}(\omega_a \omega_b \omega_c) \right\}, \quad (2.1)$$

where $l_I$ is a coupling constant, $\omega_a d\xi^a = U^{-1} \partial_a U d\xi^a$ is a left-invariant 1-form and $U = \exp(x^\mu(\xi^0, \xi^1) R_\mu)$ is a two-dimensional matrix field taking values in some Lie group with the algebra

$$[R_\mu, R_\nu] = t^\lambda_{\mu\nu} R_\lambda, \quad \text{Tr}(R_\mu R_\nu) = -\eta_{\mu\nu} \quad (2.2)$$

(for further notations see our previous paper [1]).

Let us make manifest the $G_L \times G_R/G_{\text{diag}}$ coset structure of the WZNW sigma model with the action (2.1). To this end we put

$$U = U_1 U_2^{-1}, \quad \omega_a = U_2 \omega_a U_2^{-1} \equiv U_2(\omega_a^1 - \omega_a^2) U_2^{-1}, \quad \omega_a^j = U_j^{-1} \partial_a U_j, \quad (2.3)$$
where $U_j = \exp(x^\mu R_\mu)$ belongs to $G_L$ if $j = 1$ and to $G_R$ if $j = 2$ (for convenience we have related these groups to the same set of generators $R_\mu$). Substituting (2.3) into (2.1) we see that the action (defined now on the whole group $G_L \otimes G_R$) is trivially invariant under the right gauge $G_{\text{diag}}$ transformations

$$U_1(\xi) \to U_1(\xi)V(\xi), \quad U_2(\xi) \to U_2(\xi)V(\xi), \quad V(\xi) \in G_{\text{diag}}$$

(2.4)

which may be chosen to gauge away half of the original group coordinates. This explains why we can interpret the above WZNW sigma model as a sigma model on the coset space $G_L \otimes G_R/G_{\text{diag}}$.

The variation of the action (2.1) can be represented as

$$\delta A = (-2l_I)\int d^2\xi \{-1/2\delta(\sqrt{-g}g^{ab})Tr(\omega_a^-\omega_b^-) + \quad + \quad Tr[(\partial_a(P_{ab}^{\omega_1} - P_{+}^{\omega_1}) - P_{a}[\omega_1^\pm, \omega_2^\pm])(\omega^1 - \omega^2)]\},$$

(2.5)

where $\omega^j = U_j^{-1}\delta U_j$, $P_{ab}^\pm = \sqrt{-g}g^{\pm ab} \pm e^{ab}$. It yields the equations of motion in the form

$$\partial_a(P_{-}^{\omega_1^\pm} - P_{+}^{\omega_2^\pm}) + t_{\lambda\nu}^\mu P_{+}^{\omega_1^\nu} \omega_{\lambda\nu}^\mu = 0,$$

$$\omega_{-}^{-\mu}\omega_{b\mu} - 1/2g_{ab}g^{cd}\omega_c^{-\mu}\omega_{d\mu} = 0.$$ 

(2.6)

(2.7)

It is worth mentioning that eqs. (2.6),(2.7) look very similar to the equations of motion of nonabelian superstrings (see eqs. (3.44),(3.46) in [1]). Eqs. (2.6) can be also brought into the form of the current conservation laws

$$\partial_a(P_{-}^{ab}U^{-1}\partial_b U) = 0,$$

$$\partial_a(P_{+}^{ab}U^{-1}\partial_b U) = 0$$

(2.8)

which may be derived directly from the variation (2.5) if we rewrite it as

$$\delta A = (-2l_I)\int d^2\xi \{-1/2\delta(\sqrt{-g}g^{ab})Tr(\omega_a^-\omega_b^-) + \quad + \quad Tr[(\partial_a(P_{-}^{ab}U^{-1}\partial_b U) - \partial_a(P_{+}^{ab}U^{-1}\partial_b U)\delta U_1 U_2^{-1})].$$

(2.9)

From this representation for $\delta A$ it also follows that $A$ is invariant ($\delta A = 0$) if $\delta(\sqrt{-g}g^{ab}) = 0$ and $\delta U_j$ satisfy the constraints

$$P_{+}^{ab}\partial_b(\delta U_2 U_2^{-1}) = P_{-}^{ab}\partial_b(\delta U_1 U_1^{-1}) = 0$$

(2.10)

or, in the conformal gauge $\sqrt{-g}g^{ab} \sim diag(1, -1), \xi_\pm = 1/2(\xi^0 \pm \xi^1),$ 

$$\partial_+ (\delta U_2 U_2^{-1}) = \partial_- (\delta U_1 U_1^{-1}) = 0.$$ 

(2.11)

This amounts to the well-known property that the WZNW model possesses a symmetry under the two sets of local Kac-Moody transformations [5, 6, 7]

$$U_j \to V_j(\xi)U_j, \quad \partial_+ V_2 = \partial_- V_1 = 0.$$ 

(2.12)
Eqs. (2.8) express the fact of conservation of the corresponding currents
\[ J^{1{a}} = -2l_I P^{ab}_+ \partial_b U U^{-1}, \quad J^{2a} = 2l_I P^{ab} U^{-1} \partial_b U. \] (2.13)

It is known that the general solutions of the eqs. (2.8) have the form
\[ U(\xi^0, \xi^1) = U_1(\xi^0) U_2^{-1}(\xi^1), \] (2.14)
where \( \tilde{U}_j \) are arbitrary G-valued functions. In terms of the chiral variables we, correspondingly, have
\[ U_1(\xi^0, \xi^1) = \tilde{U}_1(\xi^0 + \xi^1) V(\xi^0, \xi^1), \quad U_2(\xi^0, \xi^1) = \tilde{U}_2(\xi^0 - \xi^1) V(\xi^0, \xi^1), \] (2.15)
where \( V(\xi^0, \xi^1) \) is an arbitrary gauge \( G_{diag} \) transformation.

An unambiguous method of constructing the hamiltonian formalism for WZNW models has been worked out by Witten [5]. This method is based upon the possibility to introduce a nondegenerate symplectic form with the help of which one can define the canonical Poisson brackets. Unfortunately, in the case of nonabelian superstrings [1] the corresponding symplectic form is degenerate and so Witten’s method in its original form fails to be efficient. This is the reason why we prefer here a different hamiltonian approach used in [3]. Moreover, we will treat the variables \( U_1 \) and \( U_2 \) as independent variables connected by the gauge transformation (2.4) (normally, one chooses as the independent variable the principal chiral field \( U = U_1 U_2^{-1} \)). This will allow us to keep manifest the \( G_L \otimes G_R/G_{diag} \) coset structure inherent in the WZNW sigma model and to perform a straightforward extension to the case of nonabelian superstrings.

First of all, we rewrite the action (2.1) as a two-dimensional integral by introducing auxiliary field \( B_{\mu \nu} \) according to
\[ Tr(\omega^j \wedge \omega^j \wedge \omega^j) = d(dx^\mu dx^\nu B_{\mu \nu}), \]
where it is convenient to choose
\[ d(dx^\mu dx^\nu B_{\mu \nu}) = d Tr(\omega^j \wedge \omega^j B^j). \] (2.16)

Of course, the fields \( B^j \) are well defined only locally [5, 8] (these can be explicitly expressed in terms of local coordinates on \( G \)). Then, for the action (2.1) we get the expression
\[ \frac{1}{2l_I} A = -\frac{1}{2l_I} \int d^2 \xi \mathcal{L}(\xi) = \int d^2 \xi \{ -\frac{1}{2} \sqrt{\eta^{ab}} \omega_a^{-\mu} \omega_b^{-\mu} + \varepsilon^{ab} \omega_a^{1\mu} \omega_b^{2\mu} + \frac{1}{6} t_{\lambda \mu \nu} \varepsilon^{a b} (\omega_a^{1\mu} \omega_b^{1\nu} B^{1\lambda} - \omega_a^{2\mu} \omega_b^{2\nu} B^{2\lambda}) \}, \] (2.17)
where \( t_{\lambda \mu \nu} = -\eta_{\lambda \rho} t_{\mu \nu}^{\rho} \). The action (2.17) allows one to deal with the ordinary 2D Lagrangian \( \mathcal{L} \). This Lagrangian gives rise to the following expression for the canonical momentum
\[ P^j_\mu = \frac{\partial \mathcal{L}}{\partial \partial_{\partial x^j \mu}} = 2l_I (-)^j \{ P_\rho^{0b} \omega_{b\rho} - \varepsilon^{0b} \omega_{b\rho} - \frac{1}{3} \varepsilon^{0b} t_{\lambda \mu \nu} \omega_{b\rho}^{2\nu} B^{2\lambda} \} E_\mu^j. \] (2.18)
Here we use the notations $\omega^{j\rho}_a = \partial_a x^{j\mu} E_{\mu}^{j\rho}, P_{+}^{ab}, P_{-}^{ab} = P_{+}^{ab}, P_{-}^{ab}$. We see that the momenta (2.18) are the functions of $B^{j\mu}$ and thus are also well defined only locally. Nevertheless, using (2.18) one can introduce the globally defined chiral combinations of momenta and fields $B^{j\mu}$ which, on the one hand, are the linear combination of the Cartan’s forms and, on the other, form the Kac-Moody-type algebras with respect to the Poisson brackets (see below)

$$J_\mu^j = -\tilde{E}_\mu^{j\rho} P_\rho^j + 2l_I (-)^j \varepsilon_{\rho b} \left( \frac{1}{3} l_{\lambda \mu \nu}, \omega_{b}^{j\nu} B^{j \lambda} - \omega_{b \mu}^{j} \right) = 2l_I (-)^j (P_+^{0b} \omega_{b \mu}^1 - P_-^{0b} \omega_{b \mu}^2).$$ (2.19)

Here we have introduced the inverse matrices $\tilde{E}_\mu^{j\rho}, \tilde{E}_\nu^{j\rho} E_\nu^{j\rho} = \delta_\rho^\nu$. Then, using eq. (2.19) we conclude that

$$J_\mu^1 + J_\mu^2 \approx 0.$$ (2.20)

In the Hamiltonian formalism this gives us the constraint on the Hamiltonian variables which relates the $U_1$ and $U_2$ degrees of freedom. This constraint generates the $G_{diag}$ gauge transformations (2.4),(2.15).

First of all we rewrite eq. (2.7) in terms of the Hamiltonian variables (2.19)

$$2l_I A^j = \frac{1}{4} (J_\mu^j + 4l_I (-)^j \omega_{1 \mu}^j)(J^{j\mu} + 4l_I (-)^j \omega_{1 \mu}^j) \approx 0.$$ (2.21)

The quantities $A^j (j=1,2)$ are the holomorphic and antiholomorphic components of the energy-momentum tensor. Using the definition of the canonical equal time Poisson brackets

$$\{x^{j\mu}(\xi), P_\nu^j(\xi')\} = \delta_\rho^\nu \delta^{ij} \delta(\xi - \xi'),$$ (2.22)

after a straightforward calculation we obtain the following algebra of the variables $J_\mu^j, \omega_{1 \nu}^j$

$$\{J_\mu^j(\xi), J_\mu^j(\xi')\} = t_\mu^{\lambda \nu} J_\lambda^j(\xi) \delta(\xi - \xi') - (-)^j 4l_I \delta(\xi' - \xi) \eta_{\mu \nu},$$
$$\{J_\mu^j(\xi), \omega_{1 \nu}^j\} = \delta(\xi - \xi') t_\mu^{\lambda \nu} \omega_{1 \lambda}^j + \delta(\xi' - \xi) \eta_{\mu \nu},$$
$$\{\omega_{1 \mu}^j, \omega_{1 \nu}^j\} = 0.$$ (2.23)

Thus, the quantities $J_\mu^j(\xi)$ generate two Kac-Moody algebras and, quantizing them $(\{.,\} \to -i[.,.])$, we arrive (choosing an appropriate representation of $G$) at the quantization of the parameter $l_I : l_I = -\frac{N}{16\pi}, \ N \in Z$. To complete an analogy with the nonabelian superstring Hamiltonian structure which will be exposed in the next Section, we give the expressions for the temporal components of the Kac-Moody currents (2.13) in terms of the Hamiltonian variables $(J^{j\mu}, \omega_{1 \mu}^j)$

$$\bar{J}^j(\xi) = U_j (J^{j\mu} + 4l_I (-)^j \omega_{1 \mu}^j) R_\mu U_j^{-1}.$$ (2.24)

Indeed, using the definition (2.19), we find that in terms of the principal chiral field $U = U_1 U_2^{-1}$ the quantities $\bar{J}^j$ are represented as

$$\bar{J}^1(\xi) = -2l_I P_+^{0b} (\partial_b U) U^{-1},$$
$$\bar{J}^2(\xi) = 2l_I P_-^{0b} U^{-1} (\partial_b U).$$ (2.25)
which are just the temporal components of the conserved chiral currents (2.13). These components are often denoted as \(\bar{J}^1 = J_1, \bar{J}^2 = J_2\) [5, 6, 10]. The charges corresponding to \(\bar{J}^1, \bar{J}^2\) generate the left and right global symmetry transformations of the principal chiral field \(U\) (the global limit of transformations (2.12))

\[
U \rightarrow G_L U G_R^{-1}, \quad (U_1 \rightarrow G_L U_1, U_2 \rightarrow G_R U_2).
\]

In the conformal gauge the conservation laws (2.8) and the components (2.25) take the form

\[
\partial_- \bar{J}^1 = \partial_+ \bar{J}^2 = 0
\]

\[
\bar{J}^1 = -2l_I(\partial_+ U U^{-1}), \quad \bar{J}^2 = 2l_I(U^{-1}\partial_- U)
\]

Using (2.22) and (2.23) we get the following commutation relation for \(\bar{J}^j\) [5, 6, 4]

\[
\{\bar{J}^j_\mu(\xi), \bar{J}^j_\nu(\xi')\} = l^j_{i\mu,}, \bar{J}^i_\delta(\xi - \xi') + (-)^j 4l_I \delta^j(\xi' - \xi)\eta_{\mu\nu}.
\]

and

\[
\{\bar{J}^j_\mu(\xi), \bar{J}^j_\nu(\xi')\} = 0, \quad j \neq i
\]

Thus \(\bar{J}^j(\xi)\) represent two mutually commuting Kac-Moody algebras which are just those which generate the transformations (2.12). The energy-momentum components \(A^j\) (2.21) can also be expressed in terms of \(\bar{J}^j_\mu(\xi)\) (2.24), after that they acquire the familiar Sugawara form (no summation over \(j!\))

\[
A^j = -\frac{1}{4}Tr(\bar{J}^j \bar{J}^j) = \frac{1}{4} \bar{J}^j_\mu \bar{J}^j_\mu.
\]

(recall that in the standard notation [11, 6] one denotes \(A^1 = T, A^2 = \bar{T}\)). The quantum versions of these quantities were considered in many papers [5, 6, 10, 12].

In conclusion we stress that the aim of this section was to perform a consistent hamiltonian consideration of the standard WZNW sigma model from a non-standard point of view, namely as a sigma model on the coset space \(G_L \otimes G_R/G_{\text{diag}}\). In the next section we will explore in the same manner the hamiltonian structure of the nonabelian superstrings described by a sigma model on the coset space \(G^* \otimes G/G_+\) [1].

3 Hamiltonian structure of nonabelian \(N = 2\) superstring theory. Generalized Siegel’s algebra

In [1] we have found that the nonabelian \(N = 2\) superstrings are described by the generic action

\[
A = l_I \{ \int d^2\xi ( - \sqrt{-g} g^{ab} \omega_a^{-\mu} \omega_b^{-\nu} + \varepsilon^{ab} \omega_a^{+,\nu} \omega_b^{+,\mu}) - \\
- \frac{2}{3} \int d^3\xi \varepsilon^{abc} Str(\omega_a^1 \omega_b^1 \omega_c^2 - \omega_a^2 \omega_b^2 \omega_c^2) \}
\]
where the notation is explained in the previous section and in [1]. We recall that the model with the action (3.1) is the WZNW sigma model defined on the nonsymmetric target superspace $G_1 \otimes G_2/G_+$ where $G_1$ and $G_2$ are the supergroups generated by two mutually commuting superalgebras dual to each other in Cartan’s sense [9] ($\mu, \nu, \lambda, ... = 1, 2, ..., d; \alpha, \beta, \gamma, ... = 1, 2, ..., D$)

$$\{S_{\alpha}^i, S_{\beta}^j\} = (-)^j \Gamma_{\alpha\beta}^\mu R_{\mu}^j, \quad [R_{\mu}^j, S_{\alpha}^i] = C_{\mu\alpha}^\beta S_{\beta}^j,$$

$$[R_{\mu}^j, R_{\nu}^i] = t_{\mu\nu}^\lambda R_{\lambda}^j. \quad (3.2)$$

with the nondegenerate metrics

$$Str(R_{\mu}^j R_{\nu}^i) = -\eta_{\mu\nu}, \quad Str(S_{\alpha}^i S_{\beta}^j) = (-)^{i+1} X_{\alpha\beta},$$

$$Str(R_{\mu}^j S_{\alpha}^i) = 0. \quad (3.3)$$

Let us note that the supergroups $G_1$ and $G_2$ play for the nonabelian superstring sigma-model the role similar to groups $G_L$ and $G_R$ for WZNW sigma-model (see the previous section). For further convenience, the one-forms $\omega_{a}^{ia} d\xi^{a}$ (defined on the algebras of $G_1$ and $G_2$) will be related to the same superalgebra, $R_{\mu}^1 = R_{\mu}^2 = R_{\mu}, S_{\alpha}^1 = iS_{\alpha}^2 = S_{\alpha},$

$$\omega_{a}^{ia} = U_j^{-1} \partial_a U_j = \omega_{a}^{j\mu} R_{\mu} + \omega_{a}^{j\alpha} S_{\alpha}, \quad (3.4)$$

where $U_j = \exp(i \int x^{\mu} R_{\mu} - (-i)^j \theta^{j\alpha} S_{\alpha})$

$$\{S_{\alpha}, S_{\beta}\} = -\Gamma_{\alpha\beta}^{\mu} R_{\mu}, \quad [R_{\mu}, S_{\alpha}] = C_{\mu\alpha}^{\beta} S_{\beta},$$

$$[R_{\mu}, R_{\nu}] = t_{\mu\nu}^{\lambda} R_{\lambda}. \quad (3.5)$$

Let us mention here that the definition of the coefficients $\omega_{a}^{jA}$ of the Maurer-Cartan forms in this paper is slightly different from the one adopted in [1]. Namely, all the coefficients are the same except for $\omega_{a}^{2\alpha}$ (in [1] we have used $-i \omega_{a}^{2\alpha}$). We make this redefinition to avoid undesirable factors $i$ in the subsequent formulas.

It is easy to see that if we put all the fermionic forms in the action (3.1) equal to zero ($\omega_{a}^{j\alpha} = 0$), we arrive just at the action (2.1). Thus the model based upon the action (3.1) is a superextension of the previously considered WZNW sigma model coupled to two dimensional gravity.

Further in this section we will often use the supernotation which is much more concise. Instead of $R_{\mu}, S_{\alpha}$ we introduce the generic notation $T_{A}$, where $A$ runs over all possible indices $\mu$ and $\alpha$. We introduce also the supermetric $X_{AB} = Str(T_A T_B) = (-\eta_{\mu\nu}, X_{\alpha\beta})$. Then the commutation relations (3.5) can be rewritten in the condensed form ($t_{\mu\beta}^{\alpha} = C_{\mu\alpha}^{\beta}, t_{\alpha\beta}^{\mu} = -\Gamma_{\alpha\beta}^{\mu}$)

$$[T_A, T_B] = t_{AB}^C T_C \quad (3.6)$$

where the structure constants $t_{BC}^A$ obey the Jacobi identity

$$(-)^{BD} t_{AB}^C t_{DC}^E + (-)^{AB} t_{DA}^C t_{BC}^E + (-)^{DA} t_{BD}^C t_{AC}^E = 0 \quad (3.7)$$
and \( t_{BC}^A = 0 \) if \((A) + (B) + (C) \neq 0 \). Here \((A)\) is the Grassman parity of the generator \( T_A \): \((A) = 0 \) \((\text{mod} \ 2)\) if \( A = \mu \) and \((A) = 1 \) \((\text{mod} \ 2)\) if \( A = \alpha \). It is also useful to introduce the structure constants with the lowered indices \( t_{ABC} = X_{AD}t_{BC}^D \). These constants possess the following symmetry properties

\[
t_{ABC} = (-)^{BC+1}t_{ACB} = (-)^{A}t_{BCA}
\]  

(3.8)

Here and in eqs. (3.7) we put \((-)^A = (-1)^{(A)}\), etc. Using the supernotation one can rewrite the one-forms \( \omega^j = U_j^{-1}dU_j = \omega^a_d\xi^a \) (see equation (3.4)) as follows

\[
\omega^j = \omega^a_dT_d\xi^a = (\partial_0 z^j M E_M^A)T_A d\xi^a
\]  

(3.9)

where \( z^j M = (x^j \mu, \theta^j \alpha) \) are the even and odd parameters of the supergroups \( G_j \) and \( E_M^A \) are the one-forms coefficients which can be identified with supervelbeins. The Maurer-Cartan equations (see (3.15) in [1]) can be rewritten as

\[
\partial_ME_M^A - (-)^{MN}\partial_NE_N^A = (-)^{CN}E_M^C E_N^B t_{BC}^A
\]  

(3.10)

Let us return to studying the system with the action (3.1). Like in the case considered in the previous section it is not so trivial to explore the hamiltonian structure for this action because of nonlocality of the WZ term in eq. (3.1). One might try to treat such an action using Witten’s approach [5]. However, as was mentioned in previous Section, this approach does not apply straightforwardly to our model, because the corresponding symplectic form is degenerate and therefore (as we will see) there appear constraints on the hamiltonian variables. By this reason, it proves more fruitful to use the techniques displayed in the previous section. To this end, it is necessary to rewrite WZ term as a two-dimensional integral by introducing an auxiliary fields \( B^j \) (see eq. (2.16)) defined by the equation (which is valid only locally)

\[
\text{Str}(\omega^j \land \omega^j \land \omega^j) = d\text{Str}(\omega^j \land \omega^j \land B^j).
\]  

(3.11)

Now the action (3.1) is rewritten in the form

\[
A = -\int d^2\xi L(\xi) = l_I \int d^2\xi \{-\sqrt{-g}g^{ab}\omega_a^{-\mu}\omega_b^{-\mu}
+ 2\varepsilon^{ab}\omega_a^{1\mu}\omega_b^{2\mu} + \frac{1}{3}\varepsilon^{ab}t_{ABC}(\omega_a^{1C}\omega_b^{1B}B^{1A} - \omega_a^{2C}\omega_b^{2B}B^{2A})\}. \tag{3.12}
\]

Here \( B^j = B^j A T_A \) and \( L(\xi) \) is the Lagrangian of our model. This Lagrangian results in the following expression for the canonical momentum

\[
P^j_M = \frac{\partial L}{\partial \partial_0 Z^j_M} = (-2l_I)\{(\sim)^{-j}P^{0b}_j\omega^{-\mu}_b E_M^{j\mu} - (-)^j\varepsilon^{0b}\omega_j^i E_M^{j\mu}
- (-)^{\frac{1}{2}}\varepsilon^{0b}t_{ABC}E_M^{jC} \omega_b^B B^{jA}\}, \tag{3.13}
\]

where \( P^{ab}_j \) are the projection operators (for notations see eqs. (2.18)). The canonical momentum \( P^j_M \) and the one-forms \( \omega^j_i \) form the complete set of the hamiltonian variables.
for our model. Unfortunately, the Lagrangian $\mathcal{L}$ is singular and therefore velocities $\dot{z}^jK$ (or the left-invariant forms $\omega^0_jK$) are not expressed in terms of the hamiltonian variables $P^j_M$ and $\omega^i_j$. That is why we will have constraints on these variables. We will discuss these constraints below.

Further for convenience we will reserve the letters $K, L, M, N, ...$ for the indices of the group parameters $z^K$ and for the low indices of the supervielbeins $E^{jA}_K$ while the letters $A, B, C, D, ...$ for the indices of the group generators $T_A$ and for the upper indices of $E^{jA}_K$. In other words $K, L, M, N, ...$ are world indices while $A, B, C, ...$ are the tangent space indices. Keeping in mind this convention we denote the inverse supermatrices for $E^{jA}_K$ as $E^{jM}_{A}$ i.e.

$$E^{jM}_{A} E^{jB}_{M} = \delta^B_A, \quad E^{jA}_{M} E^{jK}_{A} = \delta^K_M (\text{no summation over } j).$$ (3.14)

Now let us note that the dependence of $P^j_M$ (3.13) on the fields $B^{jA}$ means that the expression (3.13) is ill-defined globally. Using the inverse supermatrices $E^{jM}_{A}$ and eq. (3.13) we can introduce the quantities which are linear combinations of the coefficients of the left-invariant forms (3.9) and by construction are independent of $B^{jA}$ (cf. eq. (2.19))

$$J^j_C = -E^{jM}_{A} P^j_M + 2 l_I (-)^{(j)} (1/3 t_{ABC} \omega^i_jB^{jA} + \omega^{jB}_{I} X_{BC})$$

$$= 2 l_I (-)^{(j)} [(P_{-}^{0b} \omega^{1\mu}_b - P_{+}^{0b} \omega^{2\mu}_b) \eta_{\mu C} + \omega^{j_0}_{I} X_{a C}]$$ (3.15)

These quantities are related to the conserved currents appearing in the nonabelian superstring model (see (3.47) in [1]). The independence $J^j_C$ of $B^{jA}$ means that the quantities $J^j_C$ can be defined globally. Now, instead of the complete set of the hamiltonian variables $\{P^j_M, P^j_A, \omega^{j_\mu}_1, \omega^{j_\alpha}_1 (j = 1, 2)\}$ we introduce a new complete set $\{J^j_A, J^j_\mu, \omega^{j_\mu}_1, \omega^{j_\alpha}_1 (j = 1, 2)\}$ where $J^j_A$ are expressed as linear combinations of $P^j_M$ and are well defined globally. As we will see below, our choice of the quantities (3.15) (instead of $P^j_M$) is also convenient in that they form Kac-Moody superalgebras with respect to the Poisson superbrackets.

From eqs. (3.13) and (3.15) one obtains the following set of the primary constraints

$$D^j_\alpha = -E^{jM}_{A} P^j_M + (-)^{(j)} 2 l_I t_{ABa} \omega^{jB}_{I} B^{jA}$$

$$= J^j_\alpha + (-)^{(j)} 2 l_I \omega^{j_\beta}_1 X_{j_\alpha} \approx 0,$$ (3.16)

$$J^j_\mu = J^j_\mu + J^j_\mu \approx 0.$$ (3.17)

We have also to add the constraint following from the action (3.12) by varying it with respect to the two-dimensional metric $g^{ab}$ (see eq. (2.7))

$$\omega^{\alpha}_{I} \omega^{\beta}_{B} - 1/2 g^{a b} g^{c d} \omega^{\alpha}_{I} \omega^{\beta}_{B} = 0$$ (3.18)

or, in the equivalent form

$$\mathcal{A}^j \sim P^j_0 \omega^{\alpha}_{I} P^j_0 \omega^{\beta}_{B} = 0$$ (3.19)

In terms of the hamiltonian variables (3.13), (3.15) these constraints can be rewritten as

$$2 l_I \mathcal{A}^j = \frac{1}{4}((-)^{(j)} J^j_\mu + 4 l_I \omega^{j}_1) ((-)^{(j)} J^j_\mu + 4 l_I \omega^{j}_1)$$

$$= \frac{1}{4} J^j_\mu J^j_\mu + 2 l_I (-)^{(j)} (\partial_1 z^M P^j_M - \omega^{j_0}_{I} D^j_\alpha) \approx 0$$ (3.20)
We stress that eq. (3.20) is valid only on shell and \( \mathcal{A}^j \) are none other than the components of the energy-momentum tensor (see (2.21)). The constraint (3.17) (cf. eq. (2.20)) is a consequence of our coset construction specific for the action (3.1): it generates gauge transformations forming to the right gauge group \( G_+ \).

Now, in order to divide the constraints (3.16) into the first and second class ones it is necessary to calculate the Poisson brackets of the quantities (3.16). For the variables \( P_i^j(\xi) \) and \( z^N(\xi') \) one postulates the following equal time canonical Poisson superbrackets

\[
\{P_M^i(\xi), z^N(\xi')\} = (-)^{N+1}\{z^N(\xi'), P_M^i(\xi)\} = \delta_M^N\delta(\xi - \xi') \tag{3.21}
\]

Using the definition (3.11) of the auxiliary fields \( B^j \) we find that the quantities (3.15) form two \((j = 1,2)\) mutually commuting Kac-Moody superalgebras with respect to the Poisson superbrackets (3.21)

\[
\{J_A^j(\xi), J_B^l(\xi')\} = t_{AB}^C J_C^j(\xi)\delta(\xi - \xi') + (-)^j4l_1\delta'(\xi' - \xi)X_{AB} \tag{3.22}
\]

\[
\{J_A^j(\xi), \omega^j_{CB}(\xi)X_{CB}\} = \delta(\xi - \xi')t_{AB}^C\omega^j_{CD}(\xi')X_{DC} + \delta'(\xi' - \xi)X_{AB} \tag{3.23}
\]

Note that \( J_A^1 \) and \( J_A^2 \) do not generate independent superalgebras in view of the constraints (3.17) (treated in the weak sense). We stress that the algebra \( \{J_\mu = 1/2(J_\mu^1 - J_\mu^2), J_\alpha^j, \omega^{\mu\nu}_j - \omega^2\mu, \omega^{\alpha\beta}\} \) with respect to the Poisson brackets represents a special case of the Bergshoeff-Sezgin algebra [2]. Now we can use the relations (3.22), (3.23) to obtain the following superalgebra of the primary constraints (3.16),(3.17),(3.20)

\[
\{\mathcal{A}^j(\xi), \mathcal{A}^j(\xi')\} = (-)^j(\mathcal{A}^j(\xi')\delta'(\xi' - \xi) - \mathcal{A}^j(\xi)\delta'(\xi - \xi')) \tag{3.24}
\]

\[
\{\mathcal{A}^j(\xi), J_\mu(\xi')\} = 0 \tag{3.25}
\]

\[
\{J_\mu(\xi), J_\nu(\xi')\} = t_{\mu\nu}^\lambda J_\lambda(\xi)\delta(\xi - \xi') \tag{3.26}
\]

\[
\{J_\mu(\xi), D_\alpha^j(\xi')\} = C_{\mu\alpha}^\beta D_\beta^j(\xi')\delta(\xi - \xi') \tag{3.27}
\]

\[
\{D_\alpha^j(\xi), D_\beta^j(\xi')\} = (-)^{j+1}\Gamma^\mu_{\alpha\beta}(-)^jJ_\mu^1 + 4l_1\omega^{j\mu}(\xi')\delta(\xi - \xi') \tag{3.28}
\]

\[
\{\mathcal{A}^j(\xi), D_\alpha^j(\xi')\} = \frac{1}{4l_1}((-)^jJ^\mu_1 + 4l_1\omega^j_{\alpha\mu})C^\beta_{\mu\alpha}((-)^jD^j_\beta - 4l_1\omega^j_{\beta\gamma})\delta(\xi - \xi') \tag{3.29}
\]

It is seen from eqs. (3.24) - (3.29) that the even quantities \( \mathcal{A}^j(\xi), J_\mu(\xi) \) form the closed subalgebra and thus (3.17), (3.20) can be naively considered as the first class constraints while odd constraints \( D_\alpha^j(\xi) \approx 0 \) at first sight (see eqs. (3.27),(3.28),(3.29)) are the second class. A more careful analysis shows that if there exist the matrices \( \tilde{\Gamma} \) such that

\[
\Gamma^\mu_{\alpha\beta}(\tilde{\Gamma}^{\nu\gamma} + \Gamma^{\nu\gamma}_{\alpha\beta})\delta^\gamma_{\alpha} = \eta^\mu_{\nu}\delta^\gamma_{\alpha}, \tag{3.30}
\]

then the quantities \( D_\alpha^j(\xi) \) are definite combinations of the second and first class constraints in a full analogy with the ordinary superstring theory [13, 14, 15]. We recall here that the action (3.1) possesses \( \kappa \)-supersymmetry if the matrices \( \tilde{\Gamma}^\mu \) exist [1].

To find correct expressions for the first class constraints (which can be linear combinations of the quantities \( \mathcal{A}^j, J_\mu \) and \( D_\alpha^j \)) it is necessary to examine how they evolve with
time. First of all, let us note that the canonical hamiltonian for our model (3.1) can be chosen as a linear combination of the constraints (3.16)

\[ H(\xi_0) = \int_0^{2\pi} d\xi \{ A^j f^j_A(\xi_0, \xi) + D^j_D f^j_D(\xi_0, \xi) + J_\mu f^\mu_J(\xi_0, \xi) \}. \]  

(3.31)

Here the integration goes over the spatial coordinate \( \xi_1 = \xi \) (for simplicity, we assume that all functions are periodical, \( f(\xi) = f(\xi + 2\pi) \)), and \( f_A^j, f_D^j, f_J^\mu \) are Lagrange multipliers, which can be fixed by the condition of compatibility with (3.16), (3.17) and (3.20)

\[ \dot{D}^j_\alpha = \{ D^j_\alpha, H \} \approx 0, \]  

(3.32)

\[ \dot{A}^j = \{ A^j, H \} \approx 0, \]  

(3.33)

\[ \dot{J}_\mu = \{ J_\mu, H \} \approx 0. \]  

(3.34)

Using the commutation relations (3.24 -3.29) one can show that eqs. (3.32 - 3.34) amount to the following consistency conditions

\[ ((-)^j J_j^\mu + 4l I^j_\omega^1_\mu f^j_D) \Gamma^\mu_\alpha_\beta \approx 0 \]  

(3.35)

If \( \Gamma^\mu_\alpha_\beta \) satisfy (3.30), then the general solution to these equations is

\[ f^j_D = -2l I^j_\omega^1_\mu ((\tilde{\Gamma}^\nu)^\gamma_\delta((-)^j J_j^\mu + 4l I^j_\omega^1_\mu) \tilde{\beta}^\gamma_\delta) \]  

(3.36)

Here \( \tilde{\beta}^\gamma_\delta \) are arbitrary odd functions. Substituting the new expression for the Lagrange multiplier (3.36) into (3.31) we obtain the hamiltonian in the form

\[ H = \int d\xi \{ \tilde{A}^j f^j_A + B^j_\beta \tilde{f}^j_\beta + J_\mu f^\mu_J \}, \]  

(3.37)

where

\[ \tilde{A}^j = A^j - (-)^j D^j_\beta \omega^1_\beta, \]  

(3.38)

\[ B^{j_\beta} = (\tilde{\Gamma}^\nu)^\gamma_\delta((-)^j J_j^\nu + 4l I^j_\omega^1_\nu) D^j_\alpha. \]  

(3.39)

It is easy to check that the quantities \( B^{j_\beta} \) generate, modulo the constraints (3.16), the local fermionic \( \kappa \)-supersymmetry transformations \( (\tilde{\omega}^j = U_j^{-1} \delta U_j) \)

\[ \tilde{\omega}^{1\mu} = \omega^{2\mu} = 0, \]  

\[ \tilde{\omega}^{1\alpha} = P^a_+ P^b_+ \omega^1_\beta (\tilde{\Gamma}^\mu)^a_\beta \kappa^{1\alpha}_b(\xi), \]  

\[ \tilde{\omega}^{2\alpha} = P^a_- P^b_- \omega^2_\beta (\tilde{\Gamma}^\mu)^a_\beta \kappa^{2\alpha}_b(\xi), \]  

(3.40)

\[ \delta(\sqrt{-gg^{ab}}) = -2(P^d_+ P^c_+ \omega^1_\beta (\tilde{\Gamma}^\mu)^d_\gamma \kappa^{1\gamma}_c - P^d_- P^c_- \omega^2_\beta (\tilde{\Gamma}^\mu)^d_\gamma \kappa^{2\gamma}_c). \]

New quantities \( \tilde{A}^j, B^{j_\alpha} \) and \( J_\mu \) are the first class constraints but this set of constraints is not closed with respect to the Poisson brackets and thus must be completed. Commuting these generators among themselves leads to an extended algebra of the first class
constraints which is a generalization of Siegel’s $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$ superalgebra [14] appearing in the ordinary superstring theory.

Now we introduce, besides the forms (3.15), the following currents (cf. the currents (2.24), (2.25))

$$\bar{J}^j(\xi) = \bar{J}^j_A(\xi)X^{AB}T_B = U_j(J^j_AX^{AB}T_B - 4l_1(-)^jU_j^{-1}\partial_1U_j)U_j^{-1} \quad (3.41)$$

Using the field equations following from the action (3.1) (see eqs. (3.47) in [1]) we find that $\bar{J}^j$ in the conformal gauge satisfy the equations (cf. (2.8), (2.27))

$$\begin{align*}
\partial_+(\bar{J}^1) &= 2l_1\partial_1(U_1\omega_-^1S_\beta U_1^{-1}) \\
\partial_+(\bar{J}^2) &= 2l_1\partial_1(U_2\omega_+^2S_\beta U_2^{-1})
\end{align*}$$

Thus we see that $\bar{J}^j$ are the temporal components of the conserved currents associated with the left and right global supersymmetries. The components (3.41) also form Kac-Moody superalgebras with respect to (3.21)

$$\begin{align*}
\{\bar{J}^1_A(\xi), \bar{J}^1_B(\xi')\} &= -i_{AB}^C\bar{J}^1_C(\xi)\delta(\xi' - \xi) - (-)^j4l_1\delta'(\xi' - \xi)X_{AB} \\
\{\bar{J}^1_A(\xi), \bar{J}^2_B(\xi')\} &= -i_{AB}^C\bar{J}^2_C(\xi)\delta(\xi' - \xi) + X_{AB}\delta'(\xi' - \xi). \quad (3.42)
\end{align*}$$

Here $\bar{\omega}^{jA}T_A = \partial_1U_jU_j^{-1}$. Moreover, we have

$$\begin{align*}
\{J^j_A(\xi), \bar{J}^j_B(\xi')\} &= 0 \\
\{J^j_A(\xi), \bar{J}^2_B(\xi')\} &= 0 \\
\{J^1_A(\xi), \bar{J}^2_B(\xi')\} &= 0 \quad (3.44)
\end{align*}$$

We see from (3.22), (3.42) and (3.44) that the superalgebras $J^j_A$ and $\bar{J}^j_A (j = 1, 2)$ constitute a direct sum of four mutually commuting Kac-Moody superalgebras.

Now let us discuss the constraints (3.20), (3.38) in more detail. Using formulas (3.41) we can rewrite the energy-momentum tensor components $\mathcal{A}^j(\xi)$ as

$$\begin{align*}
\mathcal{A}^j &= -\frac{1}{8l_1}(J^j_AX^{AB}J_B^j - J^j_AX^{\alpha\beta}J_B^j) + (-)^j\mathcal{D}_1\omega^{jA}_1, \quad (3.45)
\end{align*}$$

or, for the improved first class constraint $\tilde{\mathcal{A}}^j$ (3.38), (cf. eqs. (2.31))

$$\begin{align*}
\tilde{\mathcal{A}}^j &= -\frac{1}{8l_1}(J^j_AX^{AB}J_B^j - J^j_AX^{\alpha\beta}J_B^j) \\
&= g_1J^j_AX^{AB}J_B^j - (g_2J^j_AX^{AC}J_B^j - g_3J^j_AX^{\mu\nu}J_B^j), \quad (3.46)
\end{align*}$$

where $g_1 = g_2 = g_3 = -1/(8l_1)$. The expression (3.46) gives us the coset version [16] of the Sugawara representation of the classical Virasoro algebra. It is interesting to note that eqs. (3.15), (3.20) and (3.45) imply

$$1/4(J^{\dagger}_A^jX^{AB}J_B^j) - 1/4(J^{\dagger}_A^jX^{AB}J_B^j) = -2l_1(-)^j\partial_1z^{jM}P^j_M, \quad (3.47)$$
where in the right hand side we find the generators of the superstring reparametrization. Now one may canonically quantize the Kac-Moody superalgebras (3.22) and (3.42) using the substitution \([\ldots, \ldots] = i\{\ldots\}\) and following the consideration in refs. [17]. As a result, we obtain (choosing the appropriate representation \(V\) of the superalgebra (3.5)) that the parameter \(l_1\) is quantized, \(l_1 = -\frac{N}{16\pi}, N\) being a positive integer, and the equal time commutators for the Fourier components of \(J\) and \(\bar{J}\) become

\[
\begin{align*}
[J_A^n, J_B^m]_\pm &= t_{AB}^C J_C^{n+m} - (-)^j \frac{Nn}{2} X_{AB} \delta_{n+m,0}, \\
[J_A^n, \bar{J}_B^m]_\pm &= t_{AB}^C \bar{J}_C^{n+m} - (-)^j \frac{Nn}{2} X_{AB} \delta_{n+m,0}, \\
[J_A^n, \bar{J}_B^m]_\pm &= 0,
\end{align*}
\]

where \(J_A^j(\xi) = \frac{i}{2\pi} \sum_n J_A^n \exp(-in\xi), J_B^j(\xi) = -\frac{i}{2\pi} \sum_n J_B^n \exp(-in\xi)\). Now we can define the vacuum \(|0\rangle\) such that \(J_A^n |0\rangle = \bar{J}_A^n |0\rangle = 0\) for \((n > 0, j = 1)\) and for \((n < 0, j = 2)\). To quantize the first class constraints (3.46) it is necessary, in an entire analogy with the consideration in papers [6, 16, 17], to pass to the normal ordering in expression (3.46) and to properly renormalize the constants \(g_1, g_2\) and \(g_3\). Namely, we have to put

\[
g_1 = g_2 = \frac{2\pi}{N + C_V}, g_3 = \frac{2\pi}{N + C_{V^0}}
\]

where \(C_V\) and \(C_{V^0}\) are the quadratic Casimir invariants for the superalgebra (3.5) and its even subalgebra (2.2), respectively,

\[
(-)^B t^B_{DC} t^C_{AB} = C_V X_{DA}, \quad t^\mu_{\nu\lambda} t_{\rho\mu} = C_{V^0} X_{\nu\rho}, \quad X_{AB} = \text{Str}(T_A T_B).
\]

We recall here (see eq. (3.46)) that the classical values for \(g_1, g_2\) and \(g_3\) are \(g_1 = g_2 = g_3 = -1/(8l_1) = 2\pi/N\) (cf. with (3.51)). Only with the choice (3.51) the normally ordered quantities \(\tilde{A}^j\) generate the Virasoro algebras

\[
\left[\tilde{A}^j(\xi), \tilde{A}^\lambda(\xi')\right] = i \left((-)^j \{(\tilde{A}^I(\xi')\delta^\prime(\xi' - \xi) - \tilde{A}^\lambda(\xi)\delta^\prime(\xi - \xi'))\right)
- c \frac{\pi}{24} (\delta''(\xi - \xi') + \delta'(\xi - \xi'))
\]

where the central charge \(c\) is defined according to the coset construction approach [16] as

\[
c = \frac{N}{N + C_V} S\dim(g) - \left(\frac{N}{N + C_V} S\dim(g) - \frac{N}{N + C_{V^0}} \dim(g^0)\right) = \frac{N}{N + C_{V^0}} \dim(g^0)
\]

Here \(S\dim(g) = \dim(g^0) - \dim(g^1) = d - D\) (\(g^0\) and \(g^1\) are the even and odd sectors of the superalgebra (3.5), respectively).

To close this section, we would like to stress that the knowledge of the central charge is important for drawing information about the vanishing of the conformal anomalies.
and about critical dimensions of our nonabelian \( N = 2 \) superstring models. However, the existence of the first and second class constraints relating the hamiltonian variables requires a more careful treatment of the quantum theory (e.g., it is necessary to introduce harmonics and ghosts [15]) and we expect that the corresponding results for ordinary GS superstring [18] will be helpful in this aspect.

4 An interplay between the nonabelian superstrings and the superstrings in a general \( N = 2 \) supergravity background

It is interesting to compare our nonabelian superstring models (for the particular case \( d = 10, D = 16 \)) with the models of superstrings moving in a curved 10-d supergravity background. We will see that our models provide special solutions for \( N = 2 \) supergravity.

First of all, let us bring the action (3.12) into the general form of the action for the superstring evaluating in an arbitrary curved \( N = 2 \) \( 10-d \) superspace [19, 20, 21]

\[
A = \int d^2\xi \left\{ -\frac{1}{2} \eta_{\mu\nu} \sqrt{-g} g^{ab}(\partial_a z^M E_M^\nu)(\partial_b z^N E_N^\mu) \Phi(z(\xi)) + \frac{1}{2} \eta^{ab} \partial_a z^M \partial_b z^N B_{MN} \right\} = -\int d^2\xi \mathcal{L}(z^M, \partial_a z^N) \tag{4.1}
\]

Here \( \Phi(z) \) is the dilaton superfield, which appears in the superfield formulation of the type II supergravities [20, 21, 22]; \( \eta_{\mu\nu} \) is the flat 10-dimensional metric, \( z^M = \{x^\mu, \theta^{1\alpha}, \theta^{2\alpha}\} \) are the coordinates of the coset space \( G_1 \otimes G_2/G_+ \) (we can fix the gauge freedom by the condition \( x^{1\mu} + x^{2\mu} = 0 \) and then put \( x^\mu = x^{1\mu} - x^{2\mu} \); and \( E_M^\mu = (\Phi)^{-1/2}(E_M^{1\mu} - E_M^{2\mu}) \), where

\[
\omega^{A^*}_{\;\;a} = \partial_a z^{M^*} E_{M^*}^{A^*} = \partial_a z^M E_M^{A^*} \tag{4.2}
\]

(see eq. (3.9)). Here and below the primed superindices (\( M', N', \ldots; A', B', \ldots \)) are the ”\( N = 1 \)” ones used to numerate the parameters of the groups \( G_1 \) or \( G_2 \) while the superindices without primes are the ”\( N = 2 \)” ones, which correspond to the coordinates of the coset space \( G_1 \otimes G_2/G_+ \). The extra superfield \( B_{MN} \) possesses the symmetry properties \( B_{MN} = -(-)^{MN} B_{NM} \) and in our case has the following explicit form

\[
B_{MN} = (-)^{MN} E_M^{1\mu} E_N^{2\nu} - E_N^{1\mu} E_M^{2\nu} \eta_{\mu\nu} + 1/3(-)^{(C'+ M + N)t_{A'B'C'}}(E_M^{1C'} E_N^{1B'} E_M^{1A'} - E_M^{2C'} E_N^{2B'} B^{2A'}) \tag{4.3}
\]

Nonlocal fields \( B^{1A'} \) and supervielbeins \( E_N^{1A'} \) have been defined in eqs. (3.11), (4.2). Let us also introduce the two-form \( B \) related to the tensor \( B_{MN} \)

\[
B = dz^M dz^N B_{MN} = 2E_M^{1\mu} E_N^{2\nu} \eta_{\mu\nu} + 1/3 t_{A'B'C'}(E_M^{1C'} E_N^{1B'} E_M^{1A'} - E_M^{2C'} E_N^{2B'} B^{2A'}) \tag{4.4}
\]

\[
dz^M dz^N = -(-)^{MN} dz^N dz^M \tag{4.5}
\]
Here we deal with the one-forms $E^j A' = dz^M E^j M A'$. Then we can define a closed three-form $H$ by

$$ H = dz^M dz^N dz^K H_{MNK} = E^C E^B E^A H_{ABC} = d(B) \tag{4.6} $$

where

$$ E^A \equiv ((\Phi)^{-1/2}(E^1)^{\mu} - E^2\mu), E^{1\alpha}, E^{2\beta}), \quad E^A E^B = (-)^{AB+1} E^B E^A \tag{4.7} $$

Using formulas (3.7), (3.10), (3.11), (4.2) and (4.4) we obtain for the three-form $H$ the following representation (it is useful to rewrite (3.10) as $dE^j A' = 1/2 E^j C' E^B C' t^A_{B'C'}$)

$$ H = (E^1 C' E^1 B' E^2\nu - E^1\nu E^{2 C'} E^{2 B'} ) t^\mu_{B'C'} \eta_{\mu\nu} + 
+ 1/3 t^A_{A'B'C'} (E^1 C' E^1 B' E^1 A' - E^{2 C'} E^{2 B'} E^{2 A'}) = 
= 1/3 \Phi^{3/2} t_{\mu\nu} \epsilon^\alpha_{\nu\mu} \epsilon^\mu + \Phi^{1/2} \Gamma_{\mu,\alpha\beta} (E^\mu E^{2\alpha} E^{2\beta} + E^\mu E^{1\alpha} E^{1\beta}) \tag{4.8} $$

Here $\Gamma_{\mu,\alpha\beta} = t_{\mu\alpha\beta} = -\eta_{\mu\nu} t^\nu_{\alpha\beta}$. For the components of $H$ we have

$$ H_{\mu\nu\lambda} = 1/3 \Phi^{3/2} t_{\mu\nu\lambda}, \quad H_{\mu(\alpha)(\beta)} = H_{\mu(\alpha)(\beta 2)} = \Phi^{1/2} \Gamma_{\mu,\alpha\beta} \quad \text{The remaining components vanish.} $$

Let us now recall the definition of the torsion

$$ T^C_{AB} = (E^M_A \partial_M E^N_B - (-)^{AB} E^M_B \partial_M E^N_A) E^C_N = 
= (-)^{NB} E^N_A E^M_B (\partial_M E^C_N - (-)^{MN} \partial_N E^C_M) \tag{4.10} $$

The factor $(-)^{NB}$ in (4.10) ensures the symmetry properties $T^C_{AB} = (-)^{1+AB} T^C_{BA}$. Substituting (3.10), (4.7) into (4.10) we get

$$ T^\mu_{AB} = (-)^{N(B+C')} E^N_M E^M_B \Phi^{-1/2}(E^C_M E^1 B' - E^{2 C'} E^{2 B'}) t^\mu_{B'C'} + 
+ (-)^{NB} E^N_A E^M_B ((-1/2) \Phi^{-1} \partial_M \Phi E^\mu_N + (-)^{MN} 1/2 \Phi^{-1} \partial_M \Phi E^\mu_M) \tag{4.11} $$

Introducing the supermatrices $e_i^{(i C')}$ we can rewrite (4.11), (4.12) in the form

$$ T^\mu_{AB} = (-)^{A(B+C')} t^\mu_{B'C'} \{ e_i^{(i 1 C')} e_A^{(1 B')} - e_i^{(i 2 C')} e_A^{(2 B')} \} \Phi^{-1/2} + 
+ 1/2 (\delta^\mu_B E^M_M \partial_M \ln \Phi - \delta^\mu_A E^M_B \partial_M \ln \Phi) \tag{4.13} $$

$$ T^{\alpha i}_{AB} = (-)^{A(B+C')} t^{\alpha i}_{B'C'} e_i^{(i C')} e_A^{(1 B')} \tag{4.14} $$

Introducing the supermatrices $e_i^{(i C')}$ into the multi-indices $(i C')$ and $(i B')$ run over $(\mu, \alpha i)$, for each $i$, while $A, B, \ldots$ over $(\mu, (\alpha 1), (\beta 2))$. This means that the supermatrices $e_i^{(i C')}$ are rectangular. Taking into account the definition of $E^M = (\Phi^{-1/2}(E^1)^{\mu} - E^2\mu), E^{1\alpha}, E^{2\beta})$ we find

$$ e_i^{(1 \mu)} - e_i^{(2 \mu)} = \delta^\mu_\beta \Phi^{1/2} \tag{4.15} $$

$$ e_i^{(i \alpha)} = \delta^\alpha_\beta \delta_i^j, \quad e_i^{(i \alpha)} = 0 \tag{4.15} $$
Therefore the supermatrices $e_{B}^{(IC')}$ are not independent and we can express the torsion components (4.13), (4.14) in terms of a single supermatrix, for example in terms of $e_{A}^{(IC)} \equiv e_{A}^{C}$. Thus, for the components of torsion we obtain from eqs. (4.13), (4.14) and (4.15)

$$
T_{\nu\lambda}^{\mu} = t_{\nu\lambda}^{\mu}(e_{\nu}^{\alpha} \delta_{\lambda}^{\alpha} + e_{\lambda}^{\alpha} \delta_{\nu}^{\alpha}) + \Phi^{1/2} t_{\nu\lambda}^{\mu} + \\
+ 1/2(\delta_{\lambda}^{\alpha} E_{\nu}^{M} - \delta_{\nu}^{\alpha} E_{\lambda}^{M}) \partial_{M} \ln \Phi,
$$

(4.16)

$$
T_{\nu(\alpha)}^{\mu} = t_{\nu\lambda}^{\mu} e_{(\alpha)}^{\lambda} - 1/2 \delta_{\nu}^{\alpha} E_{(\alpha)}^{M} \partial_{M} \ln \Phi,
$$

(4.17)

$$
T_{\nu(\alpha2)}^{\mu} = t_{\nu\lambda}^{\mu} e_{(\alpha2)}^{\lambda} - 1/2 \delta_{\nu}^{\alpha} E_{(\alpha2)}^{M} \partial_{M} \ln \Phi,
$$

(4.18)

$$
-T_{(\alpha1)(\beta1)}^{\mu} = T_{(\alpha2)(\beta2)}^{\mu} = \Gamma_{\alpha\beta}^{\mu}, T_{(\alpha1)(\beta2)}^{(\alpha1)} = 0,
$$

(4.19)

$$
T_{(\alpha1)}^{(\alpha2)} = 0, T_{(\beta2)(\gamma2)}^{(\alpha1)} = 0,
$$

(4.20)

$$
T_{(\alpha1)}^{(\beta)} = t_{\beta\mu}^{\alpha} e_{\mu}^{\nu},
$$

(4.21)

$$
T_{(\beta1)(\gamma1)}^{(\alpha1)} = t_{\mu}^{\alpha} e_{(\beta1)}^{\mu} - t_{\beta\mu}^{\alpha} e_{(\gamma1)}^{\mu},
$$

(4.22)

$$
T_{(\beta1)(\gamma2)}^{(\alpha1)} = -t_{\beta\mu}^{\alpha} e_{(\gamma2)}^{\mu}.
$$

(4.23)

The remaining components have a similar form. Comparing (4.19) with the constraints on the torsion components $T_{(\alpha)(\beta j)}^{\mu}$ of the type $IIA$ and $IIB$ supergravities [20, 21, 22] we conclude that our non-abelian superstring model belongs to the type $IIA$. We remind here that for the type $IIA$ we have the constraints like (4.19) while for the type $IIB$ [20, 21, 22] the following ones

$$
T_{(\beta j)(\alpha1)}^{\mu} = 0, T_{(\alpha1)(\beta2)}^{\mu} = T_{(\beta2)(\alpha1)}^{(\beta2)} = \Gamma_{\alpha\beta}^{\mu}.
$$

(4.24)

5 Conclusion

In this paper we have constructed the hamiltonian formulation for the nonabelian $N=2$ superstring models proposed in [1]. This allowed us to deduce some elements of the corresponding quantum theory. It seems that the complete quantum consideration will require the whole arsenal of the methods worked out for the covariant quantization of Green-Schwarz superstring [13, 15, 18]. In particular, the relevant ghosts and harmonic variables are to be introduced (see [15, 18]).

In refs. [19, 20, 21, 22] it has been shown, within the Lagrangian approach, that the requirement of $\kappa$-supersymmetry in the sigma models of the type (4.1) leads to the constraints on the supergravity background which are precisely the same as those imposed in the standard superspace formulation of 10-d supergravity. Using our approach one can easily derive an analogous result in the framework of the Hamiltonian formalism.

It could be very interesting to apply our methods to the case of nonabelian superstring model based on the supergroup $Osp(2 \mid 1)$, the structure constants of which satisfy (3.30). This model can be interpreted as a Green-Schwarz superstring moving in 3-dimensional curved superspace which is coset space $Osp(2 \mid 1) \otimes Osp(2 \mid 1)^*/Sp(2)$.

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