Z-open sets in a Neutrosophic Topological Spaces

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Abstract-In this paper, introduce a neutrosophic open sets in neutrosophic topological spaces. Also, discuss about near open sets, their properties and examples. Z-open set which is a union of neutrosophic P-open sets and neutrosophic δ of a neutrosophic Z-open set. Moreover, we investigate some of their basic properties and examples of neutrosophic Z-interior and Z-closure in a neutrosophic topological spaces.

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1 Introduction
In mathematics, concept of fuzzy set between the intervals was first introduced by Zadeh [16] in discipline of logic and set theory. The general topology has been framework with fuzzy set was undertaken by Chang [4] as fuzzy topological space. In 1983, Atanassov [2] initiated intuitionistic fuzzy set which contains a membership and non-membership values, Coker [5] created intuitionistic fuzzy set in a topology entitled as intuitionistic fuzzy topological spaces. The concepts of neutrosophy and neutrosophic set was introduced Smarandache [11, 12] at the beginning of 20th century. Salama and Albowli [8] in 2012, originated neutrosophic set in a neutrosophic topological space. Saha [13] defined δ-open sets in fuzzy topological spaces. In 2008, Ekici [6] introduced the notion of e-open sets in a general topology. In 2014, Sevinivasan et. al. [10] introduced fuzzy e-open sets in a topological space along with fuzzy e-continuity. Vadivel et al. [3] studied fuzzy e-open sets in intuitionistic fuzzy topological space. Vadivel et al. [14] introduced e-open sets in a neutrosophic topological space. From 2011, El-Maghrabi and Mubarki [7] introduced and studied some properties of Z-open sets and maps in topological spaces. In this paper, we develop the concept of neutrosophic Z-open sets in a neutrosophic topological spaces and also specialized some of their basic properties with examples. Also, we discuss about neutrosophic Z-interior and Z-closure in neutrosophic topological spaces.

2 Preliminaries
The needful basic definitions & properties of neutrosophic topological spaces are discussed in this section.

Definition 2.1 [9] Let X be a non-empty set. A neutrosophic set (briefly, NS) L is an object having the form \( L = \{(y,\mu_L(y),\sigma_L(y),\nu_L(y)) : y \in X\} \) where \( \mu_L \rightarrow [0,1] \) denote the degree of membership function, \( \sigma_L \rightarrow [0,1] \) denote the degree of indeterminacy function and \( \nu_L \rightarrow [0,1] \) denote the degree of non-membership function respectively of each element \( y \in X \) to the set \( L \) and \( 0 \leq \mu_L(y) + \sigma_L(y) + \nu_L(y) \leq 3 \) for each \( y \in X \).

Remark 2.1 [9] A NS \( L = \{(y,\mu_L(y),\sigma_L(y),\nu_L(y)) : y \in X\} \) can be identified to an ordered triple \( (y,\mu_L(y),\sigma_L(y),\nu_L(y)) \) in \([0,1]\) on \( X \).

Definition 2.2 [9] Let X be a non-empty set & the NS’s \( L \) & \( M \) in the form \( L = \{(y,\mu_L(y),\sigma_L(y),\nu_L(y)) : y \in X\} \), \( M = \{(y,\mu_M(y),\sigma_M(y),\nu_M(y)) : y \in X\} \), then
(i) \( 0_N = (0,0,0,1) \) and \( 1_N = (1,1,0) \),
(ii) \( L \subseteq M \) iff \( \mu_L(y) \leq \mu_M(y), \sigma_L(y) \leq \sigma_M(y) \) & \( \nu_L(y) \geq \nu_M(y) \) for each \( y \in X \),
(iii) \( L = M \) iff \( L \subseteq M \) and \( M \subseteq L \),
(iv) \( 1_N - L = \{(y,\nu_L(y),1-\sigma_L(y),\mu_L(y)) : y \in X\} = L' \),
(v) \( L \cup M = \{(y,\min(\mu_L(y),\mu_M(y)),\max(\sigma_L(y),\sigma_M(y)),\min(\nu_L(y),\nu_M(y))) : y \in X\} \),
(vi) \( L \cap M = \{(y,\min(\mu_L(y),\mu_M(y)),\min(\sigma_L(y),\sigma_M(y)),\max(\nu_L(y),\nu_M(y))) : y \in X\} \).

Definition 2.3 [8] A neutrosophic topology (briefly, Nt) on a non-empty set \( X \) is a family \( \tau_N \) of neutrosophic subsets of \( X \) satisfying
(i) \( 0_N, 1_N \in \tau_N \),
(ii) \( L_1 \cap L_2 \in \tau_N \) for any \( L_1, L_2 \in \tau_N \),
(iii) \( \cup L_a \in \tau_N \) \( \forall L_a : a \in A \subseteq \tau_N \).

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Then $(X,\tau_0)$ is called a neutrosophic topological space (briefly, $Nts$) in $X$. The $\tau_0$-elements are called neutrosophic open sets (briefly, $Nos$) in $X$. A $Nts$ $C$ is called a neutrosophic closed sets (briefly, $Ncs$) iff its complement $C$ is $Nos$.

**Definition 2.4** [8] Let $(X,\tau_0)$ be $Nts$ on $X$ and $L$ be an $Ns$ on $X$, then the neutrosophic interior of $L$ (briefly, $Nint(L)$) and the neutrosophic closure of $L$ (briefly, $Ncl(L)$) are defined as

$$Nint(L) = \bigcup \{ I : I \subseteq L & I \text{ is a } Nos \text{ in } X \}$$

and

$$Ncl(L) = \bigcap \{ I : L \subseteq I & I \text{ is a } Ncs \text{ in } X \}$$

**Definition 2.5** [1] Let $(X,\tau_0)$ be $Nts$ on $X$ and $L$ be an $Ns$ on $X$. Then $L$ is said to be a neutrosophic regular (resp. pre, semi, $\alpha$ & $\beta$) open set (briefly, $Nros$ (resp. $NPos$, $NSos$, $Naos$ & $N\beta os$)) if $L = Nint(Ncl(L))$ (resp. $L \subseteq Nint(Ncl(L))$, $L \subseteq Nint(Ncl(Nint(L)))$ & $L \subseteq Ncl(Nint(Nint(L)))$).

The complement of an $NPos$ (resp. $NSos$, $Naos$, $Nros$ & $N\beta os$) is called a neutrosophic pre (resp. semi, $\alpha$, regular & $\beta$) closed set (briefly, $NPcs$ (resp. $NScs$, $Naes$, $Nrcs$ & $N\beta cs$)) in $X$.

The family of all $NPos$ (resp. $NPcs$, $NSos$, $NScs$, $Naos$, $Naes$, $Nros$, $N\beta os$ & $N\beta cs$) of $X$ is denoted by $NPOS(X)$ (resp. $NPCS(X)$, $NSOS(X)$, $NSCS(X)$, $NaOS(X)$, $NaCS(X)$, $Na\beta OS(X)$ & $Na\beta CS(X)$).

**Definition 2.6** [14] A set $L$ is said to be a neutrosophic

1. $\delta$-open set (briefly, $N\delta os$) if $L = N\delta int(L)$.
2. $\delta$-semi open set (briefly, $N\delta os$) if $L \subseteq Ncl(N\delta int(L))$.
3. $\delta$-closed set (briefly, $N\delta clos$) if $K \supseteq Ncl(N\delta int(K)) \cap Nint(N\delta clos(K))$.
4. $\delta$-semi closed set (briefly, $N\delta cl$) if $K \subseteq Ncl(N\delta int(K)) \cup Nint(N\delta cl(K))$.

The complement of an $N\delta os$ (resp. $N\delta clos$) is called a neutrosophic $\delta$ (resp. $\delta$-semi) closed set (briefly, $N\delta cs$ (resp. $N\delta Scs$)) in $X$.

The family of all $N\delta os$ (resp. $N\delta Scs$) of $X$ is denoted by $N\delta OS(X)$ (resp. $N\delta SCS(X)$).

**Definition 2.8** [14] A set $K$ is said to be a neutrosophic

1. $e$-open set (briefly, $Nes$) if $K \subseteq Ncl(N\delta int(K)) \cup Nint(N\delta clos(K))$.
2. $e$-closed set (briefly, $Necs$) if $K \supseteq Ncl(N\delta int(K)) \cap Nint(N\delta clos(K))$.

The complement of a $Nes$ is called a $Necs$.

The family of all $Nes$ (resp. $Necs$) of $X$ is denoted by $NeOS(X)$ (resp. $NeCS(X)$).

3 Neutrosophic Z-open sets in $Nts$

Throughout the sections 3 & 4, let $(X,\tau_0)$ be any $Nts$. Let $K$ and $M$ be a $Ns$’s in $Nts$.

**Definition 3.1** A set $K$ is said to be a neutrosophic

1. $Z$-open set (briefly, $NZos$) if $K \subseteq Ncl(N\delta int(K)) \cup Nint(N\delta clos(K))$.
2. $Z$-closed set (briefly, $NZcs$) if $K \supseteq Ncl(N\delta int(K)) \cap Nint(N\delta clos(K))$.

The complement of a $NZos$ is called a $NZcs$.

The family of all $NZos$ (resp. $NZcs$) of $X$ is denoted by $NZOS(X)$ (resp. $NZCS(X)$).

**Definition 3.2** A set $K$ is said to be a neutrosophic

1. $Z$ interior of $K$ (briefly, $NZint(K)$) is defined by $NZint(K) = \bigcap \{ A : A \subseteq K & A \text{ is a } NZos \text{ in } X \}$.
2. $Z$ closure of $K$ (briefly, $NZcl(K)$) is defined by $NZcl(K) = \bigcap \{ A : K \subseteq A & A \text{ is a } NZcs \text{ in } X \}$.

**Proposition 3.1** The statements hold but the converse does not true.

(i) Every $N\delta os$ (resp. $N\delta cs$) is a $Nos$ (resp. $Ncs$).
(ii) Every $N\delta os$ (resp. $Ncs$) is a $N\delta os$ (resp. $N\delta Scs$).
(iii) Every $Nos$ (resp. $Ncs$) is a $Npos$ (resp. $NPcs$).
(iv) Every $N\delta os$ (resp. $N\delta Scs$) is a $NZos$ (resp. $NZcs$).
(v) Every $N\delta os$ (resp. $NPcs$) is $NZos$ (resp. $NZcs$).
(vi) Every $NZos$ (resp. $NZcs$) is a $Neos$ (resp. $NeCS$).

**Proof.** The proof of (i), (ii) & (iii) are studied in [14, 15].

(iv) $K$ is a $N\delta os$, then $K \subseteq Ncl(N\delta int(K)) \subseteq Ncl(N\delta os) \cup Nint(Ncl(K)) \subseteq Ncl(N\delta int(K)) \cup Ncl(N\delta clos(K)).$  $K$ is a $NZos$.
(v) $K$ is a $N\delta os$, then $K \subseteq Nint(Ncl(K)) \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K)).$ $K$ is a $NZos$.
(vi) $K$ is a $NZos$ then $K \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K)) \subseteq Ncl(N\delta int(K)) \cup Ncl(N\delta clos(K)).$ $K$ is a $Neos$.

It is also true for their respective closed sets.

**Remark 3.1** The diagram shows $NZos$’s in $Nts$. 

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Example 3.1 Let \( Y = \{a, b, c\} \) and define \( Ns \)'s \( Y_1, Y_2, Y_3 \) in \( X \) are
\[
Y_1 = \langle Y, (\mu_a, \mu_b, \mu_c), (\sigma_a, \sigma_b, \sigma_c), (\nu_a, \nu_b, \nu_c) \rangle,
\]
\[
Y_2 = \langle Y, (0.2, 0.3, 0.4), (0.5, 0.5, 0.5), (0.8, 0.7, 0.6) \rangle,
\]
\[
Y_3 = \langle Y, (0.1, 0.1, 0.4), (0.5, 0.5, 0.5), (0.9, 0.9, 0.6) \rangle.
\]
Then we have \( \tau_N = \{0, 1, 2, 3\} \) is a \( Ns \) in \( X \), then
\begin{enumerate}
\item \( Y_3 \) is a \( NPoS \) but not \( Nos \).
\item \( Y_2 \) is a \( NZoS \) but not \( NPos \).
\item \( Y_3 \) is a \( Nos \) but not \( NZos \).
\end{enumerate}

Example 3.2 Let \( Y = \{a, b, c\} \) and define \( Ns \)'s \( Y_1, Y_2, Y_3 \) in \( X \) are
\[
Y_1 = \langle Y, (\mu_a, \mu_b, \mu_c), (\sigma_a, \sigma_b, \sigma_c), (\nu_a, \nu_b, \nu_c) \rangle,
\]
\[
Y_2 = \langle Y, (0.4, 0.6, 0.5), (0.5, 0.5, 0.5), (0.6, 0.4, 0.5) \rangle,
\]
\[
Y_3 = \langle Y, (0.6, 0.4, 0.5), (0.5, 0.5, 0.5), (0.4, 0.6, 0.6) \rangle,
\]
\[
Y_4 = \langle Y, (0.4, 0.5, 0.5), (0.5, 0.5, 0.5), (0.6, 0.4, 0.5) \rangle.
\]
Then we have \( \tau_N = \{0, 1, 2, 3, 4\} \) is a \( Ns \) in \( X \), then \( Y_3 \) is a \( Ns \) but not \( Nsos \).

The other implications are shown in [14].

Theorem 3.1 Let \( (X, \tau_N) \) be a \( Ns \). Then if \( M \in N\delta OS(X) \) and \( M \in NZOS(X) \), then \( H \cap M \) is \( NZo \).

Proof. Suppose that \( H \in N\delta OS(X) \). Then \( H = N\int(H) \). Since \( M \in NZOS(X) \), then \( M \subseteq Ncl(N\int(H)) \cup N\int(Ncl(M)) \) and hence
\[
H \cap M \subseteq N\int(H) \cap (Ncl(N\int(M)) \cup N\int(Ncl(M)))
\]
\[
= (N\int(H) \cap Ncl(N\int(M))) \cup (N\int(H) \cap N\int(Ncl(M)))
\]
\[
\subseteq N\int(N\int(H) \cap N\int(M)) \cup N\int(N\int(H) \cap Ncl(M)) \subseteq N\int(N\int(H) \cap M) \cup N\int(N\int(H) \cap M).
\]
Thus \( H \cap M \subseteq N\int(N\int(H) \cap M) \cup N\int(N\int(H) \cap M) \). Therefore, \( H \cap M \) is \( NZo \).

Proposition 3.2 Let \( (X, \tau_N) \) be a \( Ns \). Then the closure of a \( NZo \) set of \( X \) is \( NSo \). ■

Proof. Let \( H \in NZOS(X) \). Then
\[
Ncl(H) \subseteq Ncl(N\int(H)) \cup N\int(Ncl(H))\]
\[
\subseteq N\int(N\int(H)) \cup N\int(N\int(H)) = N\int(N\int(H)).
\]

Therefore, \( Ncl(H) \) is \( NSo \).

Theorem 3.2 The statements are true. ■

(i) \( NPcl(K) \supseteq K \cup Ncl(N\int(K)) \).

(ii) \( NP\int(K) \subseteq K \cap N\int(Ncl(K)) \).

(iii) \( N\delta Sc(K) \supseteq K \cup N\int(N\delta cl(K)) \).

(iv) \( N\delta S\int(K) \subseteq K \cap N\int(N\delta cl(K)) \).

Proof. (i) Since \( NPcl(K) \) is \( NPcs \), we have
\[
Ncl(N\int(K)) \subseteq Ncl(N\int(NPcl(K))) \subseteq NPcl(K).
\]
Thus \( K \cup Ncl(N\int(K)) \subseteq NPcl(K) \).
The other cases are similar.

**Theorem 3.3** Let \( K \) is a \( NZos \) iff \( K = NPint(K) \cup N\delta Sint(K) \).

**Proof.** Let \( K \) is a \( NZos \). Then \( K \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K)) \). By Theorem 3.2, we have
\[
NPint(K) \cup N\delta Sint(K) = K \cap (N\delta int(Ncl(K))) \cup (K \cap Ncl(N\delta int(K))) = K \cap (Nint(Ncl(K))) \cup Ncl(N\delta int(K)) = K.
\]
Conversely, if \( K = NPint(K) \cup N\delta Sint(K) \) then, by Theorem 3.2
\[
K = NPint(K) \cup N\delta Sint(K)
\]
and hence \( K \) is a \( NZos \).

**Theorem 3.4** The union (resp. intersection) of any family of \( NZOS(X) \) (resp. \( NZCS(X) \)) is a \( NZOS(X) \) (resp. \( NZCS(X) \)).

**Proof.** Let \( \{K_a : a \in \tau_N\} \) be a family of \( NZos \)'s. For each \( a \in \tau_N \), \( K_a \subseteq Ncl(N\delta int(K_a)) \cup Nint(Ncl(K_a)) \).
\[
\bigcup_{a \in \tau_N} K_a \subseteq \bigcup_{a \in \tau_N} Ncl(N\delta int(K_a)) \cup Nint(Ncl(K_a))
\]
\[
\subseteq \bigcup_{a \in \tau_N} Ncl(N\delta int(\cup K_a)) \cup Nint(Ncl(\cup K_a))
\]
The other case is similar.

**Remark 3.2** The intersection of two \( NZos \)'s need not be \( NZos \).

**Example 3.3** Let \( Y = \{a, b\} \) and define \( N\delta s \)'s \( Y_1, Y_2, Y_3 \) in \( X \) are
\[
Y_1 = \big\{ (Y, (\mu_a, \mu_b, \sigma_a, \sigma_b), (\nu_a, \nu_b)), \big(0.2, 0.1, 0.7, 0.5\big) \big\}
\]
\[
Y_2 = \big\{ (Y, (\mu_a, \mu_b, \sigma_a, \sigma_b), (\nu_a, \nu_b)), \big(0.3, 0.5, 0.7, 0.2\big) \big\}
\]
\[
Y_3 = \big\{ (Y, (\mu_a, \mu_b, \sigma_a, \sigma_b), (\nu_a, \nu_b)), \big(0.1, 0.2, 0.1, 0.1\big) \big\}
\]
Then we have \( \tau_N = \{0_N Y_1, 1_N\} \) is a \( N\delta s \) in \( X \), then \( Y_2 \cap Y_3 \) are \( NZos \) but \( Y_2 \cap Y_3 \) is not \( NZos \).

**Proposition 3.3** Let \( K \) is a \( NZos \), that is
\[
K \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K)) = 0_N \cup Nint(Ncl(K)) = Nint(Ncl(K))
\]

Hence \( K \) is a \( NPos \).

(ii) Let \( K \) be a \( NZos \), that is
\[
K \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K)) = Ncl(N\delta int(K)) \cup 0_N = Ncl(N\delta int(K))
\]

Hence \( K \) is a \( N\delta Sos \).

(iii) Let \( K \) be a \( NZos \) and \( N\delta cs \), that is
\[
K \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K)) = Ncl(N\delta int(K)) \cup Nint(Ncl(K)) = Ncl(N\delta int(K)).
\]
Hence \( K \) is a \( N\delta Sos \).

(iv) Let \( K \) be a \( N\delta Sos \) and \( Ncs \), that is
\[
K \subseteq Ncl(N\delta int(K)) \subseteq Ncl(N\delta int(K)) \cup Nint(Ncl(K)).
\]
Hence \( K \) is a \( NZos \).

**Theorem 3.5** Let \( K \) be a \( NZcs \) (resp. \( NZos \)) iff \( K = NZcl(K) \) (resp. \( K = NZint(K) \)).

**Proof.** Suppose \( K = NZcl(K) = \big\{ A : K \subseteq A & A \text{ is a } NZcs \big\} \). This means \( K \in \bigcap \{ A : K \subseteq A & A \text{ is a } NZcs \} \) and hence \( K \) is \( NZcs \).

Conversely, suppose \( K \) be a \( NZcs \) in \( X \). Then, we have \( K \in \bigcap \{ A : K \subseteq A & A \text{ is a } NZcs \} \). Hence, \( K \subseteq A \) implies \( K = \bigcap \{ A : K \subseteq A & A \text{ is a } NZcs \} = NZcl(K) \).

Similarly for \( K = NZint(K) \).

**Proposition 3.4** Let \( K \) and \( L \) are in \( X \), then
(i) \( NZcl(K) = NZint(K), NZint(K) = NZcl(K) \).
(ii) \( NZcl(K \cup L) \supseteq NZcl(K) \cup NZcl(L), NZcl(K \cap L) \subseteq NZcl(K) \cap NZcl(L) \).
(iii) \( NZint(K \cup L) \supseteq NZint(K) \cup NZint(L), NZint(K \cap L) \subseteq NZint(K) \cap NZint(L) \).

**Proof.**
(i) The proof is directly from definition.
(ii) $K \subseteq K \cup L$ or $L \subseteq K \cup L$. Hence $NZcl(K) \subseteq NZcl(K \cup L)$ or $NZcl(L) \subseteq NZcl(K \cup L)$. Therefore, $NZcl(K \cup L) \supseteq NZcl(K) \cup NZcl(L)$. The other one is similar.

(iii) $K \subseteq K \cup L$ or $L \subseteq K \cup L$. Hence $NZint(K) \subseteq NZint(K \cup L)$ or $NZint(L) \subseteq NZint(K \cup L)$. Therefore, $NZint(K \cup L) \supseteq NZint(K) \cup NZint(L)$. The other one is similar.

Remark 3.3 The equality of (ii) in Proposition 3.4 cannot be true in the given example.

Example 3.4 Let $Y = \{a,b,c,d\}$ and define $N$'s $Y_1, Y_2, Y_3 \& Y_4$ in $X$ are

\[
Y_1 = \langle Y, (\mu_a, \mu_b, \mu_c, \mu_d), (\sigma_a, \sigma_b, \sigma_c, \sigma_d), (\nu_a, \nu_b, \nu_c, \nu_d) \rangle
\]

\[
Y_2 = \langle Y, (\mu_a, \mu_b, \mu_c, \mu_d), (\sigma_a, \sigma_b, \sigma_c, \sigma_d), (\nu_a, \nu_b, \nu_c, \nu_d) \rangle
\]

\[
Y_3 = \langle Y, (\mu_a, \mu_b, \mu_c, \mu_d), (\sigma_a, \sigma_b, \sigma_c, \sigma_d), (\nu_a, \nu_b, \nu_c, \nu_d) \rangle
\]

\[
Y_4 = \langle Y, (\mu_a, \mu_b, \mu_c, \mu_d), (\sigma_a, \sigma_b, \sigma_c, \sigma_d), (\nu_a, \nu_b, \nu_c, \nu_d) \rangle
\]

Then we have $r_N = \{0, Y_1, Y_2, Y_3 \cap Y_2, Y_4\}$ is a $Nts$ in $X$, then $NZcl(Y_3 \cup Y_4) = \emptyset$ $NZcl(Y_3) \cup NZcl(Y_4)$.

Proposition 3.5 Let $K$ be a neutrosophic set in a neutrosophic topological space $X$. Then $NZint(K) \subseteq NZint(K) \subseteq K \subseteq NZcl(K) \subseteq Ncl(K)$.

Proof. It follows from the definitions of corresponding operators.

Theorem 3.6 Let $K$ and $L$ in $X$, then the $NZint$ sets have

(i) $NZint(0_N) = 0_N$, $NZint(1_N) = 1_N$.

(ii) $NZint(K)$ is a $NZcs$ in $X$.

(iii) $NZint(K) \subseteq NZcl(L)$ if $K \subseteq L$.

(iv) $K \subseteq NZcl(K)$.

(v) $K$ is $NZc$ set in $X$ $\Leftrightarrow NZcl(K) = K$.

(vi) $NZint(NZint(K)) = NZint(K)$.

Proof. The proofs (i) to (iv) and (vi) are directly from definitions of $NZcl$ set.

Let $K$ be $NZc$ set in $X$. By using Proposition 3.4, $K$ is $NZo$ set in $X$. By Proposition 3.4, $NZint(K) = K \Leftrightarrow NZcl(K) = K$.

Theorem 3.7 Let $K$ and $L$ in $X$, then the $NZint$ sets have

(i) $NZint(0_N) = 0_N$, $NZint(1_N) = 1_N$.

(ii) $NZint(K)$ is a $NZos$ in $X$.

(iii) $NZint(K) \subseteq NZint(L)$ if $K \subseteq L$.

(iv) $NZint(NZint(K)) = NZint(K)$.

Proof. The proofs are directly from definitions of $NZint$ set.

Proposition 3.6 If $K$ and $L$ in $X$, then (i) $NZcl(K) \supseteq K \cup NZcl(NZint(K))$.

(ii) $NZint(K) \subseteq K \cap NZint(NZcl(K))$.

(iii) $NZint(NZcl(K)) \supseteq NZint(NZint(NZcl(K)))$.

Proof. (i) By Theorem 3.6 $K \subseteq NZcl(K) \rightarrow (1)$. Again using Theorem 3.6, $NZint(K) \subseteq K$. Then $NZcl(NZint(K)) \subseteq NZcl(K) \rightarrow (2)$. By (1) and (2) we have, $K \cup NZcl(NZint(K)) \subseteq NZcl(K)$.

(ii) By Theorem 3.6, $NZint(K) \subseteq K \rightarrow (1)$. Again using Theorem 3.6, $K \subseteq NZcl(K)$. Then $NZcl(K) \subseteq NZint(NZcl(K)) \rightarrow (2)$. By (1) and (2) we have, $NZint(K) \subseteq K \cup NZint(NZcl(K))$.

(iii) By Theorem 3.6, $NZcl(K) \subseteq Ncl(K)$, we get $NZcl(NZcl(K)) \subseteq NZcl(Ncl(K))$. Hence (iii).

(iv) By (i), $NZcl(K) \supseteq K \cup NZint(NZcl(K))$. We have, $NZint(NZcl(K)) \supseteq K \cup NZint(NZcl(K))$. Since $NZint(K) \cup \subseteq NZcl(K)$, $NZcl(K) \cup NZint(NZint(NZcl(K))) \supseteq NZcl(NZint(K))$.

(v) Conclusion

We have studied about neutrosophic $Z$-open set and neutrosophic $Z$-closed set and their respective interior and closure operators of neutrosophic topological space in this paper. Also studied some of their fundamental properties along with examples in $Nts$. Also, we have discussed a near open sets of neutrosophic $Z$-open sets in $Nts$. In future, we can be extended to neutrosophic $Z$ continuous mappings, neutrosophic $Z$-open mappings and neutrosophic $Z$-closed mappings in $Nts$.  

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References

1. I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala, *On some new notions and functions in neutrosophic topological spaces*, Neutrosophic Sets and Systems, 16 (2017), 16-19.
2. K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20 (1986), 87-96.
3. V. Chandrasekar, D. Sobana and A. Vadivel, *On Fuzzy e-open Sets, Fuzzy e-continuity and Fuzzy e-compactness in Intuitionistic Fuzzy Topological Spaces*, Sahand Communications in Mathematical Analysis (SCMA), 12 (1) (2018), 131-153.
4. C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl., 24 (1968), 182-190.
5. D. Coker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy sets and systems, 88 (1997), 81-89.
6. Erdal Ekici, *On e-open sets, DP-sets and DP*-sets and decomposition of continuity*, The Arabian Journal for Science and Engineering, 33 (2A) (2008), 269-282.
7. A. I. El-Magharabi and A. M. Mubarki, *Z-open sets and Z-continuity in topological spaces*, International Journal of Mathematical Archive, 2 (10) (2011), 1819-1827.
8. A. A. Salama and S. A. Alblowi, *Neutrosophic set and neutrosophic topological spaces*, IOSR Journal of Mathematics, 3 (4) (2012), 31-35.
9. A. A. Salama and F. Smarandache, *Neutrosophic crisp set theory*, Educational Publisher, Columbus, Ohio, USA, 2015.
10. V. Seenivasan and K. Kamala, *Fuzzy e-continuity and fuzzy e-open sets*, Annals of Fuzzy Mathematics and Informatics, 8 (2014), 141-148.
11. F. Smarandache, *A Unifying field in logics: neutrosophic logic, neutrosophy, neutrosophic set, neutrosophic probability*, American Research Press, Rehoboth, NM, (1999).
12. F. Smarandache, *Neutrosophy and neutrosophic logic*, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA (2002).
13. Supriti Saha, *Fuzzy δ-continuous mappings*, Journal of Mathematical Analysis and Applications, 126 (1987), 130-142. [14] A. Vadivel, C. John Sundar and P. Thangaraja, *Neutrosophic e-open sets in a topological spaces*, Submitted.
14. V. Venkateswara Rao and Y. Srinivasa Rao, *Neutrosophic pre-open sets and pre-closed sets in Neutrosophic topology*, International Journal of Chem Tech Research, 10 (10) (2017), 449-458.
15. L. A. Zadeh, *Fuzzy sets*, Information and control, 8 (1965), 338-353.