Forming of contour-parallel lines family with the detection of their non-working sections

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Abstract. The paper considers family of working contour-parallel lines formation used in the design of the toolpath that processes pocket surfaces. Contour-parallel working lines are the ones from which the lines-noises, i.e. non-working sections, are removed. Non-working sections include self-intersection loops and intersections of oncoming fronts in the case of multiply-connected domains. The spatial geometric model of forming contour-parallel lines is based on the cyclographic mapping. As a tool for detecting non-working sections for the case of oncoming fronts, the method of testing ray is proposed. In case of self-intersections of contour-parallel lines, non-working sections are cut off by the parameter of these lines at the points of self-intersection. At the output of the proposed shaping algorithm of the working contour-parallel lines family, the parametric equations of these lines are formed. The algorithm successfully works for multiply-connected domains with polygonal and curved contours of the boundaries of the domain and islands in it. A comparative evaluation of the proposed method of forming a contour-parallel lines family with trimming non-working sections and the known methods that use the distance function is performed.

1. Introduction

In CAD/CAM systems, calculating the cutting toolpath is an important task. Pocket surfaces are usually processed along contour-parallel paths. To calculate the trajectory of the tool that processes these surfaces, it is necessary to build an OC family (“Offset Curve”) of the initial contour of the pocket surface. OC families of multiply-connected domains are formed when the pocket area includes islands. Optimization of the OC family of multiply-connected domains involves the analysis of OC and the removal of non-working sections of the OC family lines. Non-working sections of the OC family lines are lines-noises. They are formed as loops of self-intersections of the OC of connected domains (local intersections of the OC) and as sections that are formed when intersecting oncoming fronts of the OC (global intersections of the OC) of multiply-connected domains (figure 1). The analysis of the existing optimization methods for the OC family of multiply-connected domains by the criterion of the absence of lines-noises allows to distinguish the following main directions for solving the optimization problem:

1.1. Analysis and trimming non-working sections of the OC family lines by the distance function

The detection of non-working sections of the OC family lines comes down to solving polynomial equations. If the curve of the initial contour of a multiply-connected domain is of an order higher than the second, to find the roots it is necessary to solve equations of high degrees, which leads to a difficult computational problem [1].
1.2. Analysis and trimming non-working sections of the OC family lines using MA as a cutting-off tool

This problem is solved on the plane for a simply-connected domain using MA (“Medial Axis”) [2-4]. On the plane, each MA point is the center of a disk of maximum radius inscribed into the boundary contour. MA in combination with radius function is called “Medial Axis Transformation” (MAT). Analysis and trimming non-working sections of OC is carried out using a complex mathematical apparatus; the computational algorithms are not stable and have a high temporal complexity. The algorithms for finding MA for a domain the boundary of which consists of arcs of circles and line segments work stably. However, if the boundary of the domain has a complex curvilinear shape, the search for MA becomes a difficult task [2-11].

PH-curves (Pythagorean-Hodograph curves) are used as the boundary lines of the domain [1]. The OC family with step \( d \) relative to the boundary curves \( PH \) is determined by the equation:

\[
\bar{r}_d(t) = \bar{r}(t) + \bar{d}_n(t)
\]

in which \( \bar{r}(t) \) is initial contour, \( \bar{d}_n(t) \) is normal distance function. Based on the properties of PH-curves, the lines of the OC family are presented as a set of rational curves. The representation of PH-curves in the Minkowski metric together with the domain decomposition lemma makes it possible to compute the process of trimming the OC family lines for a multiply-connected domain with a curved contour. The procedure for obtaining the cut-off lines of the OC family, i.e., an OC without lines-noises, is carried out in terms of the function of radius of curvature PH-curves and MAT. This allows to obtain the OC family in the form of rational Bezier curves, since the unit normal has a rational dependence on the parameter of the curve \( t \).

In some works [12, 13], a pairwise displacement algorithm for closed two-dimensional point sequence curves for a multiply-connected domain with a curved contour consisting of PS-curves (Point-Sequence curves) is presented. In this approach, the loops of self-intersections and intersections of oncoming fronts are removed by the pairwise detection test of non-working sections of the OC family lines. The algorithm can work in linear time. However, point sequences are input data for the proposed algorithm. This algorithm only works with the PS curve class, which limits its capabilities. The algorithm proposed in the research [14] can automatically connect islands to the external contour at the closest distance. But the total time for calculating the minimum distance between two curves depends on the total number of curves, including contour of domain and contours of islands. The OC families of all islands and the OC contours of the domain are combined into a single connected PS-curve using Delaunay triangulation [15]. The algorithms work in almost linear time, but the result of the proposed algorithm is point data.

A brief review shows the obvious need to develop a model for shaping the OC family with simpler algorithms for trimming non-working sections for multiply-connected domains with curved contours.

2. Problem definition

To propose a geometric model of shaping the OC family with linear algorithms for trimming their non-working sections.

3. Geometric model of shaping of the OC family based on the cyclographic mapping
3.1. Analysis and trimming non-working sections of oncoming lines of the OC families of the domain contour and island contours

According to the cyclographic method of forming concept, OC families are obtained by dissecting α-surfaces by a horizontal planes pencil along the z axis with a step \( z_j = \text{const} \) [16]. The section lines form the LOC (“Level Offset Curves”) families related to the islands and the contour of the domain belonging to the same level plane in the plane ray. OC families are formed by orthogonal projection of the LOC family onto the \( z = 0 \) plane (figure 2). To determine non-working sections of lines of OC family, the analysis of LOC families of domain contour and LOC of oncoming island contours is made. Lines of the LOC family in the ray planes intersect at the points \( A_j \in \text{MAT} \) (figure 2a). Points \( A_j \) divide the lines of the LOC family into working and non-working sections. MAT line in the cyclographic model is a spatial curve, which is formed as a result of pairwise intersection of α-surfaces, constructed from composite contours of domain and islands in it. MAT line, α-surfaces constructed from compound contours of domain and islands form α-shell in space (figure 3a). On α-shell, working sections of lines of LOC family are formed in planes of their level. If line segment does not fall on α-shell, then it is a non-working one. α-shell, as a geometric object, is not modeled, but is used to determine the signs showing that the lines of the LOC family belong to it.

![Figure 2](image)

**Figure 2.** Construction of the OC family for a multiply-connected domain: a) OC family construction sequence; b) the formation of LOC.

\( A_j \in \text{MAT} \) points divide the lines of the LOC family into sections. To analyze and identify working and non-working sections, testing ray is used as a tool. The operation of the testing ray is demonstrated with the following example. For analysis, the family of LOC intersecting α-surfaces of the domain contour and the island contour in each ray plane of horizontal planes is considered (figure 3b).
Figure 3. The fragment of the testing ray operation: a) α-shell, b) formation of LOC families, c) trimming a non-working section of the LOC family line.

The lines of the families of the LOC contour and the LOC of the island intersect at points 1 and 2. The analysis of the line of the LOC family of the island is performed. At the intersection points this line is divided into sections (1-2) and (2-1). Having chosen the direction of bypassing this line clockwise, for example, we analyze sections (1-2) and (2-1). The section (1-2) is considered. The testing ray $r_1$ from anywhere in the section (1-2) is formed. Ray $r_1$ crosses the line of the LOC family of the domain contour at one point, which means the section (1-2) is in the zone bounded by the line of the LOC family of the domain contour. Thus, the section (1-2) belongs to the α-shell and therefore it is a working section. The section (2-1) is considered. The testing ray $r_2$ from anywhere in the domain (2-1) is formed. Ray $r_2$ crosses the line of the LOC family of the domain contour at two points. Thus, the section (2-1) is not in the zone bounded by the line of the LOC family of the domain contour. Therefore, the section (2-1) does not belong to the α-shell. The section (2-1) does not belong to the α-shell. The section (2-1) is non-working and must be cut off.

All possible intersections of the lines of the LOC families of the oncoming fronts with the cut-off of non-working sections are analyzed. In the general case, if a testing ray formed from any point on a segment of a line of the LOC family has an odd number of points of intersection with the line of the LOC family of the oncoming front, then the segment belongs to the α-shell. Thus, it is a working section. If the testing ray formed from any point on the line segment of the LOC family has an even number of intersection points, or does not intersect the line of the LOC family of the oncoming front, then the section does not belong to the α-shell; therefore, it is a non-working section.

An enlarged algorithm for analyzing global intersections of lines of the OC family is given [17].

3.2. Analysis and trimming self-intersecting loops of the LOC family lines

Self-intersecting loops are formed when the evolute involved in the formation of the α-surface has a singular point (figure 4a). The self-intersection points $A_i$ are determined from the equation $\vec{r}(t_a) = \vec{r}(t_b)$, where $t_a$ and $t_b$ are the value of the parameter $t$ of the $i$-th segment $\vec{r}(t)$ of the domain or island contour, where $t \in [0,1]$ (the solution of this vector equation is two roots $t_a=a$ and $t_b=b$). Then, to eliminate the self-intersection loop, the line of the LOC family is divided into three sections with the parameters $t \in [0,a]; t \in [a,b]$ and $t \in [b,1]$. The section of the line of the LOC family with the parameter $t \in [a,b]$ is cut off. The working section of the line of the LOC family is composed of two segments $LOC = LOC_a \cup LOC_b$. 
Figure 4. The sequence of analysis and exclusion of local intersections: a) self-intersection loop, b) final result.

The method of analyzing local intersections based on a cyclographic mapping is described in detail in the research work [17]. It should be noted that the input data of the proposed algorithm are arrays of points of the contours of the domain and islands. A discrete set of points is interpolated by a closed curve line. Interpolation can be performed by segments of Bezier curves of the third degree, fractional rational Bezier curves of the second degree, or a contour of segments of second-order curves is constructed from an array of points [18].

The output data of the algorithm are the parametric equations of the working lines of the $OC$ families:

\[
OC_{\text{island}(j)}^{(1)}: \quad \mathbf{r}_{\text{island}(j)}^{(1)} = \mathbf{r}_{\text{island}(k_{1},j)}^{(1)}(t_1^{(1)}) \cup \mathbf{r}_{\text{island}(k_{2},j)}^{(1)}(t_2^{(1)}) \cup \ldots \cup \mathbf{r}_{\text{island}(k_{n},j)}^{(1)}(t_n^{(1)}),
\]

\[
OC_{\text{island}(j)}^{(2)}: \quad \mathbf{r}_{\text{island}(j)}^{(2)} = \mathbf{r}_{\text{island}(k_{1},j)}^{(2)}(t_1^{(2)}) \cup \mathbf{r}_{\text{island}(k_{2},j)}^{(2)}(t_2^{(2)}) \cup \ldots \cup \mathbf{r}_{\text{island}(k_{n},j)}^{(2)}(t_n^{(2)}),
\]

\[
OC_{\text{island}(j)}^{(N)}: \quad \mathbf{r}_{\text{island}(j)}^{(N)} = \mathbf{r}_{\text{island}(k_{1},j)}^{(N)}(t_1^{(N)}) \cup \mathbf{r}_{\text{island}(k_{2},j)}^{(N)}(t_2^{(N)}) \cup \ldots \cup \mathbf{r}_{\text{island}(k_{n},j)}^{(N)}(t_n^{(N)}),
\]

\[
OC_{\text{domain}(j)}: \quad \mathbf{r}_{\text{domain}(j)} = \mathbf{r}_{\text{domain}(k_{1},j)}(t_1) \cup \mathbf{r}_{\text{domain}(k_{2},j)}(t_2) \cup \ldots \cup \mathbf{r}_{\text{domain}(k_{n},j)}(t_n),
\]

where:

the parametric equations of the 1st segment of the $OC$ family line of the first island with the parameter $t_1^{(1)} \in [0,1]$ in the $j$-th level plane are:

\[
\mathbf{r}_{\text{island}(k_{1},j)}^{(1)}(t_1^{(1)}) = \{x_{\text{island}(k_{1},j)}^{(1)}(t_1^{(1)}), y_{\text{island}(k_{1},j)}^{(1)}(t_1^{(1)})\};
\]

parametric equations of the last segments of the $OC$ family line of the first island with a parameter $t_n^{(1)} \in [0,1]$ in the $j$-th level plane are:

\[
\mathbf{r}_{\text{island}(k_{1},j)}^{(1)}(t_n^{(1)}) = \{x_{\text{island}(k_{1},j)}^{(1)}(t_n^{(1)}), y_{\text{island}(k_{1},j)}^{(1)}(t_n^{(1)})\};
\]

parametric equations of the first segment of the $OC$ family line of the last islands with a parameter $t_1^{(N)} \in [0,1]$ in the $j$-th level plane are:

\[
\mathbf{r}_{\text{island}(k_{1},j)}^{(N)}(t_1^{(N)}) = \{x_{\text{island}(k_{1},j)}^{(N)}(t_1^{(N)}), y_{\text{island}(k_{1},j)}^{(N)}(t_1^{(N)})\};
\]

parametric equations of the last segment of the $OC$ family line of the last islands with a parameter $t_n^{(N)} \in [0,1]$ in the $j$-th level plane are:

\[
\mathbf{r}_{\text{island}(k_{1},j)}^{(N)}(t_n^{(N)}) = \{x_{\text{island}(k_{1},j)}^{(N)}(t_n^{(N)}), y_{\text{island}(k_{1},j)}^{(N)}(t_n^{(N)})\};
\]

parametric equations of the 1st segment of the $OC$ family line of the contour of the domain with parameter $t_1 \in [0,1]$ in the $j$-th plane are:
The main problem of the proposed optimization method for
optimizing OC families is the preparation of initial data.
In the work presented, several types of curves were used to form the initial contours of the domain and islands: Bezier curves of the third degree, fractional rational Bezier curves of the second degree, and second-order curves. Obviously, use of PH-curves for forming the initial contours will improve the proposed method for optimizing OC families.

6. References
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