Discovery of an Excited Pair State in Superfluid $^3$He

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Abstract

Order parameter collective modes are the fingerprint of a condensed phase. The spectroscopy of these modes in superfluid $^3$He and unconventional superconductors can provide key information on the symmetry of the condensate as well as the microscopic pairing mechanism responsible for the ground state and excitation energies. We report the discovery of a new collective mode in superfluid $^3$He-B which we identify as an excited bound state of Cooper pairs. We use interferometry within an acoustic cavity that is very sensitive to changes in the velocity of transverse sound. Our measurements of sound velocity and mode frequency, together with the observation of acoustic birefringence indicate that this new mode is weakly bound with an excitation energy within 1% of the pair-breaking edge of $2\Delta$. Based on the selection rules for coupling of transverse sound to a collective mode in $^3$He-B, combined with the observation of acoustic birefringence near the collective mode frequency, we infer that the new mode is most likely a spin-triplet ($S = 1$), $f$-wave pair exciton ($L = 3$) with total angular momentum, $J = 4$. The existence of a pair exciton with $J = 4$ suggests an attractive, sub-dominant, $f$-wave pairing interaction in liquid $^3$He.
Fifty years ago Bardeen, Cooper and Schrieffer (BCS) published their seminal paper on the theory of superconductivity in metals\(^1\). This theory, combined with developments over the next decade\(^2\)\(^3\)\(^4\)\(^5\), is one of the most successful theoretical achievements of modern physics. The basic feature of BCS theory, the *condensation* of bound pairs of fermions, has impacted research on nuclear structure\(^6\), our understanding of neutron star interiors, the rotational dynamics of pulsars\(^7\), and most recently, the physics of ultra-cold, meta-stable phases of atomic gases\(^8\)\(^9\). In condensed matter, BCS pairing is ubiquitous. It is observed in metals, magnetic materials\(^10\), organic conductors\(^11\), strongly disordered films that are on the verge of becoming insulators\(^12\), and liquid \(^3\)He\(^13\).

Perhaps the most detailed and specific signatures of broken symmetries of the normal state are the collective modes of the pair condensate. These are the dynamical fingerprints of a multi-component order parameter\(^14\). The order parameter collective modes of superfluid \(^3\)He have been extensively studied\(^15\) using acoustic absorption spectroscopy\(^15\)\(^16\)\(^17\)\(^18\)\(^19\)\(^20\)\(^21\)\(^22\)\(^23\). There have been efforts to observe and calculate the spectrum and signatures of such modes in the heavy fermion superconductors, UPt\(_3\)\(^24\)\(^25\), UBe\(_13\)\(^26\)\(^27\), as well as for Sr\(_2\)RuO\(_4\)\(^28\)\(^29\). Recently, observation of the Leggett-mode, i.e. the inter-band Josephson oscillations of a two-band superconductor, was reported in MgB\(_2\)\(^30\). Collective modes of the *amplitude* of the order parameter have also been observed. In NbSe\(_2\) the superconducting state develops below the onset of a charge density wave (CDW) instability. The coupling between the amplitude mode of the CDW and the amplitude mode of the superconducting order parameter produces an infrared active collective mode near the gap edge, \(2\Delta\)\(^31\)\(^32\). However, \(^3\)He is the best example of complex symmetry breaking among the BCS condensates and investigation of its order parameter collective modes holds significance for the understanding of other pairing systems.

Transverse sound has been shown to be a precision spectroscopy of the collective mode spectrum of superfluid \(^3\)He-B\(^16\)\(^17\). Using these techniques we have discovered a new collective mode of the B-phase of superfluid \(^3\)He. Selection rules for the coupling of transverse sound to a collective mode, combined with the observation of acoustic birefringence near the mode frequency, suggest that the most likely candidate for this new mode is the spin-triplet \((S = 1)\), \(f\)-wave pair exciton \((L = 3)\) with total angular momentum, \(J = 4\), predicted by Sauls and Serene\(^33\).

The equilibrium phase of superfluid \(^3\)He-B is a spin-triplet \((S = 1)\), \(p\)-wave \((L = 1)\)
FIG. 1: Energy levels of the collective modes, $J^\pm$, of superfluid $^3\text{He}-\text{B}$ that have been observed or predicted to couple to either longitudinal or transverse sound and their Zeeman splitting in a magnetic field in the limit of weak quasiparticle interactions. The $4^-$ and $0^+$ modes have not been observed and evidence for the $1^-$ mode is not yet well-established.

condensate. The order parameter has nine complex amplitudes, and correspondingly a spectrum of collective modes\textsuperscript{[15,23]} whose observation has been instrumental in establishing that the dominant pairing interaction is $p$-wave. For pure $p$-wave pairing there are a total of 18 modes corresponding to the number of degrees of freedom of the order parameter. The B-phase has an isotropic gap with magnitude $2\Delta$, where $\Delta/\hbar$ at zero temperature varies from 34 to 97 MHz over the pressure range of the phase diagram. Acoustic techniques are well-suited for these frequencies and, in general, sound can couple effectively to order parameter modes with non-zero frequency at zero wavevector, $\Omega(k = 0) \sim \Delta/\hbar$. These ‘optical’ modes, shown in Fig. 1, correspond to excited pair states with total angular momentum quantum numbers, $J = 0, 1, 2$, etc., each with $2J + 1$ substates, labeled by $-J \leq m_J \leq J$. Additionally, the order parameter modes are classified by their parity under particle-hole symmetry, $J^\pm$, where plus and minus distinguish between modes with even (+) or odd (−) parity under particle-hole transformation, \textit{i.e.} particle ↔ hole conversion. In Fig. 1 we sketch the energy level diagram for the excited pair states in superfluid $^3\text{He}-\text{B}$ that have been observed, or are predicted, to couple to sound\textsuperscript{[15,23]}.

Acoustic waves have a linear dispersion, $\omega = ck$, where $c$ is the sound velocity. The sound
frequency, $\omega$, can be chosen to match an order parameter collective mode, $\Omega_{j\pm,m_J}(T,P)$, which is only weakly dependent on wavevector $k$, but can be tuned by sweeping the pressure or temperature. If there is coupling between sound and the modes they can be identified by divergence in the attenuation and velocity of the propagating sound wave at the crossing point\textsuperscript{15}. We use a unique spectroscopy based on magneto-acoustics that has high spectral resolution\textsuperscript{16,17} and well-defined selection rules for coupling to the order parameter. We constructed a cavity defined by an $AC$-cut quartz piezoelectric transducer and a polished quartz reflector that are separated by $D = 31.6 \pm 0.1 \mu m$; details are given in Sec. I of the supplementary information. Transverse sound is both generated and detected by our transducer. We sweep the $^3$He pressure, holding the temperature near $T \approx 550 \mu K \ll T_c$. The resulting changes in the phase velocity alter the number of half-wavelengths in the cavity and are manifested as acoustic interference producing an oscillatory electrical response, shown in Fig. 2A. The acoustic signal can be represented as $A = A_0 + A_1 \cos \theta \sin(2D\omega/c + \phi)$, where $A_0$ is a smoothly varying background in the absence of cavity wave-interference and is determined by acoustic impedance\textsuperscript{35}; $A_1$ is the amplitude of the oscillatory signal modulation from wave-interference; $\phi$ is a phase angle; and $\theta$ is the angle between the polarization of the sound wave at the surface of the transducer and the direction of linear polarization that the transducer can generate and detect. We apply magnetic fields, $H$, up to 305 G, parallel to the propagation direction, revealing acoustic birefringence where $\theta \propto H$. These acoustic techniques offer a precise means to investigate the order parameter structure of superfluid $^3$He.

Transverse sound does not ordinarily propagate in fluids. However, it was predicted by Moores and Sauls (MS)\textsuperscript{36} to exist in superfluid $^3$He-B as a consequence of coupling to the $J = 2^-, m_J = \pm 1$ modes. The experiments of Lee et al.\textsuperscript{37} confirmed this theory which was later exploited by Davis et al.\textsuperscript{16,17} to investigate the $J = 2^-$ mode with much higher precision than is possible with longitudinal sound\textsuperscript{20}. Furthermore, MS predicted that $^3$He-B becomes circularly birefringent in a magnetic field, where the $J = 2^-, m_J = \pm 1$ modes couple differently to right and left circularly polarized transverse sound waves\textsuperscript{16}, giving them different velocities. Circular birefringence leads to rotation of the plane of polarization of linearly polarized transverse sound by an angle proportional to the component of magnetic field in the propagation direction\textsuperscript{17,18,36}, the acoustic analog of optical Faraday rotation. Selection rules govern the coupling of the collective modes with quantum numbers $J^\pm, m_J$.
to right- and left circularly polarized transverse sound. Circularly polarized transverse waves in $^3$He-B, propagating in the direction of the magnetic field, preserve axial symmetry. As a result, transverse sound couples only to $m_J = \pm 1$ modes. Application of a magnetic field lifts the degeneracy of the $m_J = \pm 1$ states via the nuclear Zeeman energy, producing circular birefringence. Therefore an order parameter collective mode, $\Omega_{J^\pm,m_J}$, that induces acoustic circular birefringence requires $J \geq 1$ with $m_J = \pm 1$. Additionally, MS have shown that in zero field only even angular momentum modes with $m_J = \pm 1$ couple to transverse currents. These selection rules follow from the invariance of the B-phase ground state under joint spin and orbital rotations (i.e. a $J = 0$ ground state) and are applicable for the geometry of the acoustic experiments described here.

In the superfluid state, transverse sound propagates according to the dispersion relation,

$$\left(\frac{\omega}{qv_F}\right)^2_{m_J} = \Lambda_0 + \Lambda_{2^-} - \omega^2 - \Omega_{2^-,m_J}(H)^2 - \frac{2}{5} q^2 v_F^2.$$

This equation can be solved for the phase velocity as shown by the green curve in Fig. 2B, where $v_F$ is the Fermi velocity, $\Lambda_0$ is the quasiparticle restoring force, and the $2^-$ mode frequency is $\Omega_{2^-,m_J}(H) = \Omega_{2^-} + m_J g_2 \gamma_{eff} H$, to first order in magnetic field. The Landé $g$-factor, $g_2^-$, gives the Zeeman splitting of the mode, $\gamma_{eff}$ is the effective gyromagnetic ratio of $^3$He, and the complex wavevector is $q = k + i\alpha$ where $\alpha$ is the attenuation. For substates $m_J = \pm 1$, there is a non-zero magneto-acoustic coupling, $\Lambda_{2^-}$, between the $2^-$ mode and transverse sound. Further details for $\Omega_{2^-,m_J}$, $\Lambda_0$ and $\Lambda_{2^-}$ are given in Sec. II of the supplementary information.

From Eq. 1 it is apparent that the sound velocity diverges for acoustic frequencies approaching the $2^-$ mode, resulting in faster oscillations in the acoustic response as is evident in Fig. 2A. Quantitative comparison with theory can be made by converting the oscillations into the transverse sound phase velocity. In order to obtain absolute values for the velocity we fix one adjustable parameter by comparison of our data with the velocity calculated from Eq. 1 near the $2^-$ mode, as shown in Fig. 2B. The agreement between the calculation, green curve, and the experiment, red circles, is excellent for energies below $\sim 1.8\Delta$. However, above this energy we observe a downturn in the transverse sound velocity. This effect is quite clear in the raw data displayed in Fig. 2C indicated by a decreasing period of the oscillations with increasing energy. This is a classic signature of the approach to an order
FIG. 2: Acoustic cavity response to pressure as a function of energy normalized to the gap energy, at 88 MHz and \(\approx 550 \, \mu K\) in zero magnetic field.  

(A) Interference oscillations obtained from a pressure sweep, where the arrows mark the well-established \(J = 2^−\) mode (black) and a new mode at the gap-edge (green).  

(B) Transverse sound velocities (red circles) from (A) compared with the theory, Eq. 1 (green curve). The black curve is calculated by adding a mode to Eq. 1.  

(C) Detail of (A) near 2\(\Delta\). The green arrow marks where the period of the oscillations goes to zero, indicating the \(2\Delta\)-mode, \(\hbar \Omega_{2\Delta}\).  

(D) The difference between the measured and theoretical (Eq. 1) transverse sound velocities (red circles) and the difference between the phenomenological model and Eq. 1 (black curve).

Parameter collective mode\endnote{15}. In Fig. 2D we show as red circles the difference between the measured sound velocity and the value calculated from Eq. 1, based on coupling only to the \(2^−\) mode. The downturn in velocity is quite evident here. By extrapolating the period of the oscillations in Fig. 2C to zero, i.e. to the point where the velocity diverges indicated by the green arrow, we determine the frequency (excitation energy) of the new collective mode. This procedure is described in detail in Sec. III of the supplementary information. We find the excitation energy of the mode, \(\hbar \Omega_{2\Delta}\), to vary systematically with pressure, but remain within 1% of \(2\Delta\) for pressures from 1 to 20 bar, as shown in Fig. 3A. The precision of the extrapolation is given by the error bars; the accuracy of 1% is determined by the absolute...
temperature scale in the framework of the weak-coupling-plus model for the energy gap.

In order to model the sound velocity near the gap-edge, we amend Eq. 1 by adding a new term to represent the coupling of transverse sound to the $2\Delta$ collective mode with a form similar to that of the $2^-$ mode, $\omega^2 \Lambda_{2\Delta}/(\omega^2 - \Omega_{2\Delta}^2)$. The velocity calculated from this model is given by the black curve in Figs. 2B and 2D, describing our data quite well with a coupling strength, $\Lambda_{2\Delta} = 0.18$. Furthermore, we note that the amplitude, $A_1$, of the interference oscillations near the gap-edge decreases in a manner similar to the period. Since the amplitude is proportional to the inverse of the sound attenuation, this is a consistent identification of a collective mode, where it can be shown from Eq. 1 that the attenuation diverges at the mode location, as does the velocity.

We have previously established that an applied magnetic field rotates the plane of polarization of propagating transverse sound in the near vicinity of the $J = 2^-$ mode, $\omega \geq \Omega_{2^-}$. Increasing the frequency above the $J = 2^-$ mode decreases the Faraday rotation rate, which eventually becomes immeasurably small as the coupling to the mode decreases. However, at even higher frequencies near the pair-breaking edge we find that the Faraday rotation reappears (see Fig. 3B) in the same frequency region where we observe the downturn in the velocity of transverse sound. The magnetic field modulates the interference amplitude, $A_1$, by a factor $\cos \theta$, from which we extract the Faraday rotation angle, $\theta$. In magneto-optics, Faraday rotation is parameterized by the Verdet constant, $V = \theta/2DH$. For our magneto-acoustic data we find $V$ to be a monotonically increasing function of frequency, diverging near $\Omega_{2\Delta}$, shown as a green arrow in the inset of Fig. 3C. Aparently the birefringence originates from the $2\Delta$ collective mode. The existence of acoustic Faraday rotation (circular birefringence) requires that the $2\Delta$-mode correspond to an excitation with total angular momentum $J \geq 1$ with substates $m_J = \pm 1$ coupling to transverse currents. It is particularly noteworthy that the divergences in velocity and attenuation occur in both zero and non-zero applied fields at the excitation energy shown in Fig. 3A.

Order parameter modes predicted to lie close in energy to the pair-breaking edge include the $J = 0^+, 1^-, 4^-$ modes (see Fig. 1). Observation of the $0^+$ and $4^-$ modes have not previously been reported. The observation of circular birefringence allows us to rule out the $0^+$ mode. A peak in the longitudinal sound attenuation that appears for non-zero magnetic field was attributed to the coupling of the $J = 1^-, m_J = 0$ mode to the longitudinal current (see Sec. IV of the supplementary information). However, in zero field the $J = 1^-$
FIG. 3: (A) $\hbar \Omega_{2\Delta}$ as a function of pressure in zero magnetic field. The curve is a guide to the eye. (B) The acoustic response as a function of energy offset by magnetic field (right axis) at 88 MHz and $\approx 550$ $\mu$K. Acoustic birefringence rotates the plane of polarization by an angle $\theta$, producing a minimum signal amplitude at $\theta = \pi/2$, indicated by the red arrow for 305 G and $\approx 1.96\Delta$. The blue dashed line represents the predicted field dependence ($g = 0.4$) of the $J = 1^-$, $m_J = -1$ mode. (C) The Faraday rotation angle, $\theta$, is proportional to magnetic field at energies given by solid squares in the inset. Inset: Verdet constant as a function of energy, diverging at $\hbar \Omega_{2\Delta}$, marked by the green arrow. (D) Detail of the 305 G trace from (B).

modes do not contribute to the stress tensor and cannot couple to either the longitudinal or transverse currents. Consequently, neither the $J = 1^-$ nor the $J = 0^+$ modes can account for our observations in zero field. Furthermore, in a non-zero magnetic field the predicted Zeeman splitting of the $J = 1^-$ modes should decrease the energy of the $m_J = -1$ level as shown by the blue dashed line in Fig. 3D, overlapping a region of energy where we have observed acoustic interference oscillations. Since the velocity is not perturbed in this region, it appears that the Zeeman splitting of the $2\Delta$-mode is much smaller than that predicted for the $1^-$ mode and so the $1^-$ mode cannot explain our data in a magnetic field. Finally,
pair breaking is absent for frequencies below a threshold $2\Delta$ in zero magnetic field, and thus cannot account for the downturn in velocity below $2\Delta$. Higher angular momentum modes, in particular modes with $J = 4^-, m_J = \pm 1$, do couple to transverse sound even in zero field. Based on the experimental geometry and the selection rules for transverse sound and acoustic birefringence, the only known theoretical candidate that might account for our observations are the $J = 4^-$ modes. Quantitative predictions for the coupling strength and the magnitude of the Zeeman splitting are needed for a conclusive identification.

Predictions for the $J = 4^-$ mode frequency depend on existence of an attractive $f$-wave pairing interaction, subdominant with respect to $p$-wave pairing, as well as the unknown Fermi liquid parameter, $F_{a,s}$. Several experiments, including magnetic susceptibility and $J = 2^\pm$ collective mode spectroscopy, were analyzed to try and determine the $f$-wave pairing interaction, as well as Fermi liquid interactions, in an effort to predict the $J = 4^-$ mode frequency. However, the results of these different analyses are ambiguous owing to imprecision of the Fermi liquid parameters, $F_{a,s}$, as well as non-trivial strong coupling effects. Our observation of a new order parameter collective mode, identified on the basis of selection rules as the $J = 4^-$ mode, provides direct evidence for an attractive $f$-wave pairing interaction. Precise measurements of the mode frequency in this work, combined with a future measurement of the Zeeman splitting, should provide constraints on the microscopic pairing mechanism in $^3$He. Additionally, our results may lead to realization of predictions of mixed symmetry pairing near impurities, surfaces, and interfaces; surface phases with broken time-inversion; and novel vortex phases.

We have discovered an excited pair state in superfluid $^3$He-B near $2\Delta$ from measurements of the divergence of velocity, attenuation, and the magneto-acoustic Verdet constant for transverse sound. Together with selection rules for acoustic birefringence, our experiments indicate that the $2\Delta$-mode has total angular momentum $J \geq 4$ and a frequency within 1% of $2\Delta$ at all pressures. The observed mode is most likely the $J = 4^-$ mode predicted by Sauls and Serene and, as such, provides direct evidence for $f$-wave pairing correlations in superfluid $^3$He. Our results give a more detailed picture of the order parameter structure in superfluid $^3$He-B near the gap-edge than has been possible with longitudinal sound and will serve as a guide to future work on this and other unconventionally paired systems.

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SUPPLEMENTARY INFORMATION

I. EXPERIMENTAL DETAILS

In our experimental arrangement, the cavity spacing of \( D = 31.6 \pm 0.1 \mu m \) was measured \textit{in situ} with longitudinal sound at a temperature of 18 mK, using the 17\textsuperscript{th} transducer harmonic which generates a large amplitude longitudinal signal. We note that the texture of the order parameter must be homogeneously oriented along the sound propagation and magnetic field directions, since the diameter of the transducer (0.84 cm) is much larger than the spacing of the cavity. The frequency and amplitude of the transducer resonances are extremely sensitive to the acoustic conditions of the cavity which modify the electrical impedance that we monitor with a continuous-wave, RF-bridge\textsuperscript{16,17}. We use odd harmonics of our transducer from the 13\textsuperscript{th} to the 29\textsuperscript{th}, corresponding to frequencies from 76 to 171 MHz. Details of cooling and thermometry can be found elsewhere\textsuperscript{17}. We use the weak-coupling-plus model for the gap\textsuperscript{10} as tabulated by Halperin and Varoquaux\textsuperscript{15}, locked to the Greywall temperature scale\textsuperscript{39}. This allows us to account for variations of the transverse sound velocity with changes in both pressure and temperature, although temperature effects are very slight at the low temperatures used in this experiment. The accuracy of the Greywall temperature scale\textsuperscript{39} determines the accuracy of \( \Delta \) in the weak-coupling-plus model and is estimated to be 1%.

II. CALCULATION OF THE TRANSVERSE SOUND VELOCITY

In order to calculate the transverse sound velocity from Eq. 1, which appears in Figs. 2B and 2D, we use the full expression for the \( J = 2^- \) mode frequency in a magnetic field given by Moores and Sauls\textsuperscript{36} \( \Omega_{2-,m_J}(H)^2 = \Omega_{2-}^2 + 2m_Jg_2-\gamma_{eff}H\omega \), for \( m_J = \pm 1 \). Additionally, we used the quasiparticle restoring force for transverse sound\textsuperscript{36} \( \Lambda_0 = \frac{F_1^s}{15}(1-\lambda)(1+\frac{F_2^s}{\gamma})/(1+\lambda\frac{F_2^s}{\gamma}) \), which includes all quasiparticle interaction terms\textsuperscript{15}, \( F_l^s \), for \( l \leq 2 \). Similarly \( \Lambda_2^- = \frac{2F_2^s}{75}\lambda(1+\frac{F_2^s}{\gamma})^2)/(1+\lambda\frac{F_2^s}{\gamma}) \). The Tsuneto function \( \lambda(\omega,T) \) can be thought of as a frequency dependent superfluid density\textsuperscript{36}, that depends on the gap amplitude, \( \Delta \). Therefore, at low temperatures the quasiparticle term is small and \( \Lambda_2^- \) is enhanced. In our calculations of velocity we use \( \lambda(\omega,T) \) adapted to incorporate the weak-coupling-plus gap as described above\textsuperscript{10}. The
FIG. S1. Period of acoustic energy oscillations from the acoustic cavity interference of Fig. 3B as the acoustic frequency approaches the $2\Delta$ mode, green arrow, for zero magnetic field (red circles) and 305 G (black squares). We identify $\hbar\Omega_{2\Delta}$ from the extrapolation to zero (where the velocity diverges). Both traces point to the same energy within experimental resolution. For comparison the energy for the collective mode $J = 1^-$, $m_J = -1$ in $H = 305$ G with a Zeeman splitting of $g = 0.4$ is shown as a black dashed line as expected theoretically. The absence of data over a small range near 1.95 for the 305 G trace corresponds to the region where Faraday rotation has decreased the amplitude of the acoustic oscillations and the period is not reliably determined.

dispersion relation, Eq. 1, holds in the long wavelength limit, $kv_F \ll \omega$. Consequently, our estimation of the coupling strength of the $2\Delta$-mode may be modified in a full $q$-dependent analysis.

III. LOCATING THE NEW MODE

As the acoustic frequency approaches that of an order parameter collective mode, the sound velocity and attenuation diverge. In order to pinpoint the frequency of the mode we plot the inverse of the signal oscillation amplitude (attenuation) and the oscillation period (velocity) and extrapolate to zero. In Fig. S1 we show this procedure for the oscillation period for two experiments with zero magnetic field and $H = 305$ G. The curves are fits to guide the eye. Nonetheless it is clear that the mode frequency can be precisely determined. This is the method used to obtain Fig. 3A. Moreover, the $2\Delta$ mode does not appear to
have a Landé $g$-factor that is large enough to appear directly as a shift of the data trace in $H = 305$ G. As a point of comparison, if the Landé $g$-factor were to be of the magnitude theoretically predicted for the $J = 1^-$ mode a shift in frequency indicated by the dashed line would be expected\textsuperscript{12}.

IV. LONGITUDINAL SOUND AND THE $J = 1^-$ MODE

Ling \textit{et al.}\textsuperscript{19} reported the existence of a mode, with a splitting which they ascribed to $J = 1^-, m_J = \pm 1$. These signatures in longitudinal sound attenuation appeared only in the presence of an applied magnetic field, which in their case was oriented transverse to the sound propagation direction. Since then it has been shown\textsuperscript{18,41} that only the $m_J = 0$ mode can couple to longitudinal sound, contrary to what was originally proposed\textsuperscript{19,42}. Additionally, Ashida \textit{et al.}\textsuperscript{18} find that the anomalies observed in longitudinal sound attenuation in non-zero field\textsuperscript{19} can be accounted for in terms of pair-breaking coupled to $J = 1^-, m_J = 0$ and $J = 2^-, m_J = 0, \pm 2$ modes, in some combination, but that the analysis is complicated by the inhomogeneous texture associated with the experimental conditions of a transverse magnetic field. From their analysis it appears that identification of the $J = 1^-$ mode is not yet well-established. A comparable calculation has not been performed for transverse sound; however, on symmetry grounds the $J = 1^-$ mode cannot couple to transverse sound in zero magnetic field, and in a non-zero field the coupling to transverse sound is indirect via the $J = 2^-$ mode. We estimate the coupling strength for this process to be very weak compared to what we measure for the $2\Delta$ mode providing additional evidence that the $J = 1^-$ mode is not responsible for our observations.