SUPERSYMMETRY AND ELECTRON-HOLE EXCITATIONS
IN SEMICONDUCTORS AT FINITE TEMPERATURE

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Abstract

The fermionic and bosonic electron-hole low lying excitations in a semiconductor are analyzed at finite temperature in a unified way following Nambu’s quasi-supersymmetric approach for the BCS model of superconductivity. The effective lagrangian for the fermionic modes and for the bosonic low lying collective excitations in the semiconductor is no longer supersymmetric in a conventional finite temperature treatment. However the bosonic excitations don’t couple directly to the heat bath and as a result, quasisupersymmetry is restored to the effective lagrangian when a redefinition of the coupling constant associated with the collective excitations is performed. Our result shows that although the mass and coupling parameters are now temperature dependent, the fermion and boson excited states pair together and can still be transmuted into one another.
I. INTRODUCTION

It can be shown that a model with a BCS type of spontaneously symmetry breaking possesses a quasi-supersymmetry if there exists an appropriate mass ratio relation for the bosonic and fermionic excitations of the physical system. Nambu investigated the BCS model of superconductivity in which the masses of the three low energy excitations, the phase (π or Goldstone) collective mode, the fermionic excitation (the quasiparticle) and the amplitude (σ or Higgs) mode are in the ratios 0 : 1 : 2. He showed that the static part of the BCS model can be expressed as an anticommutation of the fermionic composite charges $Q$ and $Q^+$. The $Q$'s are not nilpotent but the equal masses of a pair of fermionic excitations and their corresponding bosonic mode is a clear sign of the built-in quasi-supersymmetry in this model. In our previous work, we invoke the pairing approximation, to make physically equivalent the fermion operators, hole annihilation $\psi_h$ and electron creation $\psi_e^+$, when they act in an appropriate excited state function in the semiconductor. When this equivalence relation holds, it can be shown that the static part of the corresponding effective hamiltonian is truly supersymmetric at $T = 0$. The boson and fermion excitations can be mapped into one another and the model is identical to 1+1 supersymmetric quantum mechanics (SUSY).

In a pure semiconductor, the low lying electronic excitation, as is well known, is a bound electron-hole pair state called exciton. The excitons can be well characterized by their radii. If the exciton radius is of the order of the lattice constant the pair is tightly bound and the exciton state is known as Frenkel exciton. In contrast, in a semiconductor with a large dielectric constant this radius extends over many lattice cells, and as a result the pair is only weakly bound, and known as Wannier exciton. We confine ourselves to this case only.

In our previous work the electron-hole pair, the single-particle states and the phase mode, which is transmuted into a plasmon mode in the presence of Coulomb forces, are described in an unified manner. For a direct gap semiconductor, we show that, similarly to superconductivity, the single fermion and boson masses also define a 0 : 1 : 2 ratio at
zero-temperature.

However, it is known that even if supersymmetry exists at zero-temperature it will be spontaneously broken at finite temperature due to the fact that fermions and bosons are clearly distinguishable from each other when they couple to a heat bath. As a result their effective masses therefore no longer maintain the same value since they obey different statistics \[18\]. So, at a first glance, the model which we investigated in our previous work is not, strictly speaking, supersymmetric at finite temperatures.

Considering that our effective action model describes low energy fermionic and bosonic excitations in the semiconductor, the finite temperature’s main effect is in general terms to reduce the magnitudes of both the low lying excitation energies and the energy gap. The basic physical picture involving the electron, the hole, the exciton and the plasmon excitations remains practically unchanged at \( T \neq 0 \). It is reasonable to expect that the mass relation and the supersymmetry will be maintained in a properly redefined effective action for the physically relevant low lying fermionic and bosonic elementary excitations at finite \( T \).

In the present work, we first derive from first principles, the finite temperature form of the effective action and show that the mass relation between the existing excitations is lost. If this were the case supersymmetry would be spontaneously broken at any finite temperature in the semiconductor. This generalized model however does not reveal the real physical properties of the low energy lying excitations in the semiconductor. We are therefore led to construct a new effective action for the low energy lying excitations of the semiconductor at finite temperatures. When we do this, the model is shown to maintain both its supersymmetry and the mass relations as proposed earlier by Nambu.
II. EFFECTIVE ACTION FOR ELECTRON-HOLE AND THEIR COLLECTIVE EXCITATIONS

Let us review briefly the derivation of the effective action at $T = 0$. Consider for simplicity a two-band direct semiconductor with parabolic conduction and valence bands. At $T = 0$, the valence band is completely full and the conduction band is totally empty. If we excite the semiconductor by laser pumping or by any other means, producing interband transitions between these two bands, the single-particle picture of the resulting low-lying state is basically given in terms of the promotion of the electron to the conduction band and the creation of a hole in the valence band. In reality this single-particle picture is known to be incomplete since electron and hole interact strongly with each other and the resulting pair of particles is associated primarily with a 1s type level inside the energy gap.

Let us then consider the model hamiltonian density $\mathcal{H}_f$ given by

$$\mathcal{H}_f = \psi_e^+ e \psi_e + \psi_h^+ e \psi_h - \lambda \psi_e^+ \psi_h^+ \psi_e \psi_h,$$ (2.1)

which describes the low-lying electron-hole excitations in the semiconductor. Here, for convenience, we take equal electron and hole effective masses $m_e = m_h = m$ and, in k-space, $\epsilon = \frac{E_g}{2} + \frac{k^2}{2m}$, where $E_g$ is the energy gap and $\lambda$ is an effective four-fermion coupling constant.

Following standard procedure we define next the path integral $Z$ for these electron and hole states:

$$Z = \int \mathcal{D}\psi_e^+ \mathcal{D}\psi_e \mathcal{D}\psi_h^+ \mathcal{D}\psi_h \exp i \int \mathcal{L}_f$$ (2.2)

where

$$\mathcal{L}_f = \psi_e^+ (i \partial_t - \epsilon) \psi_e + \psi_h^+ (i \partial_t - \epsilon) \psi_h + \lambda \psi_e^+ \psi_h^+ \psi_e \psi_h$$ (2.3)

is the lagrangian density corresponding to the model hamiltonian (2.1). In order to evaluate the path integral $Z$ it is useful to rewrite the quartic interaction in terms of effective quadratic terms. For this we use the Hubbard-Stratonovich transformation [10,11] for the bosonic auxiliary complex fields $\phi, \phi^*$.
\[
\int D\phi D\phi^* \exp \left( -i \int \left[ m_0^2 \phi^* \phi - m_0 \sqrt{\lambda} \psi_e^+ \psi_h^+ \phi - m_0 \sqrt{\lambda} \phi^* \psi_h \psi_e \right] \right) = \text{const} \exp \left( i \lambda \int \psi_e^+ \psi_h^+ \psi_h \psi_e \right). \tag{2.4}\]

When we replace this identity in the path integral, \( Z \) becomes
\[
Z = \int D\psi_e^+ D\psi_e D\psi_h^+ D\psi_h D\phi^* D\phi \exp i \int \mathcal{L}_{\psi-\phi} \tag{2.5}
\]
where
\[
\mathcal{L}_{\psi-\phi} = \psi_e^+(i\partial_t - \epsilon)e + \psi_h^+(i\partial_t - \epsilon)\psi_h + m_0 \sqrt{\lambda} \psi_e^+ \psi_h^+ \phi + m_0 \sqrt{\lambda} \phi^* \psi_h \psi_e - m_0^2 \phi^* \phi \tag{2.6}
\]
Using the Nambu matrix notation for the fermion field \( \Psi \),
\[
\Psi = \begin{pmatrix} \psi_e \\ \psi_h^+ \end{pmatrix},
\]
the lagrangian \( \mathcal{L}_{\psi-\phi} \) becomes
\[
\mathcal{L}_{\psi-\phi} = \Psi^+ \left[ (i\partial_t - \epsilon \tau_3) + m_0 \sqrt{\lambda} \left( \phi \tau_+ + \phi^* \tau_- \right) \right] \Psi - m_0^2 \phi^* \phi + \text{const} \tag{2.7}
\]
where \( \tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2) \) with the \( \tau_i \)‘s \((i = 1, 2, 3)\) being the Pauli matrices. The fermionic lagrangian is in this way transformed into an effective fermion-boson model with a Yukawa-type interaction which is bilinear in the fermion fields.

Substituting this lagrangian in the path integral \( Z \) and integrating it over the fermion fields, it then follows that
\[
Z = \int D\phi D\phi^* \exp -iW(\phi^*, \phi) \tag{2.8}
\]
where \( W \) is now the effective ‘action’ for the bose fields \( \phi \) and \( \phi^* \) given by
\[
W(\phi^*, \phi) = \int m_0^2 \phi^* \phi + iTrln \left[ i\partial_t - \epsilon \tau_3 + m_0 \sqrt{\lambda} \left( \phi \tau_+ + \phi^* \tau_- \right) \right] \tag{2.9}
\]
To determine the low energy and long wavelength dynamics of the bosonic fields \( \phi \) and \( \phi^* \), we will make a gradient expansion in the effective action \( W(\phi^*, \phi) \) to derive their kinetic energy term \([10,11]\). Let us decompose the complex scalar field in the form
\[ \phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \quad \phi^* = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2), \quad (2.10) \]

and define the interaction matrix operator \( M \) as

\[ M = m_0 \sqrt{\lambda}(\phi \tau_+ + \phi^* \tau_-) = m_0 \sqrt{\lambda} \phi \tau, \quad (2.11) \]

where \( \phi \) is the two-component real vector field

\[ \phi = \frac{1}{\sqrt{2}}(\phi_1, -\phi_2, 0). \quad (2.12) \]

It then follows that

\[ W(\phi) = i \text{Tr} \ln[k_0 - \epsilon_k \tau_3] - i \sum_{n=1}^n (i)^n \text{Tr}(G_0 M)^n + \int m_0^2 \phi \phi, \quad (2.13) \]

where

\[ G_0(k) = i \frac{k_0 + \epsilon_k \tau_3}{k_0^2 - \epsilon_k^2 + i\delta} \quad (2.14) \]

is the fermionic Green’s function.

Up to the quartic term in \( \phi \), our effective action can be written as

\[ W(\phi) = \int \phi^a(q) K_{ab}(q) \phi^b(q) + \frac{1}{4} \Lambda(0)_{abcd} \phi^a \phi^b \phi^c \phi^d \quad (2.15) \]

where we use a notation \( \phi = (\phi_1, \phi_2, 0) \),

\[ K_{ab}(q) = \delta_{ab} + \frac{\lambda}{2} \pi_{ab}(q), \quad (2.16) \]

with

\[ \pi_{ab}(q) = i \int \text{Tr}[G_0(k + q) \tau_a G_0(k) \tau_b]; \quad (2.17) \]

represented by the bubble diagram in Fig. 1, and

\[ \Lambda(0)_{abcd} = -im_0^4 \lambda^2 \int \text{Tr}[G_0(k) \tau_a G_0(k) \tau_b G_0(k) \tau_c G_0(k) \tau_d] \quad (2.18) \]

as is shown in Fig. 2. It is easy to show that the linear and cubic term in \( \phi \) in the expansion of \( W(\phi) \) now give zero contributions.
We then have that

\[ \Lambda(0) = -2im_0^4\lambda^2 \int \frac{1}{k(k_0^2 - \epsilon_k^2)^2} \]

(2.19)

and

\[ \pi_{ab}(q) = \pi_{ab}^{(1)}(q) + \pi_{ab}^{(2)}(q) \]

(2.20)

where

\[ \pi_{ab}^{(1)}(q) = -2i \int k_0(k_0 + q_0) - \epsilon_k \epsilon_{k+q} \frac{k_0(k_0 + q_0)}{[(k_0 + q_0)^2 - \epsilon_{k+q}^2][k_0^2 - \epsilon_k^2]} \delta_{ab} = \pi_0^0(q) \delta_{ab} \]

(2.21)

and

\[ \pi_{ab}^{(2)}(q) = -2 \int \frac{(k_0 + q_0)\epsilon_k - k_0 \epsilon_{k+q}}{[(k_0 + q_0)^2 - \epsilon_{k+q}^2][k_0^2 - \epsilon_k^2]} \epsilon_{ab} \]

(2.22)

It is sufficient to expand \( \pi^{(0)} \) and \( \pi_{ab}^{(2)} \) around \( q_0 = 0 \) and \( q = 0 \) to derive the kinetic and the time derivative terms for the bosonic field. Making the expansion of \( \pi^{(0)} \) in terms of the gradients of \( \phi \), we get

\[ K^{(0)}(0) = (1 + \frac{\lambda}{2} \pi^{(0)}(0)) \]

(2.23)

and

\[ K^{(0)}(q) = Zq_0^2 + Iq^2 + \cdots \]

(2.24)

where

\[ \pi^{(0)}(0) = -2i \int \frac{1}{k^2_0 - \epsilon_k^2 + i\delta} \]

(2.25)

\[ Z = -i\lambda \int \frac{k_0^2 + \epsilon_k^2}{k(k_0^2 - \epsilon_k^2)^3} \]

(2.26)

and

\[ I = \frac{-i\lambda}{2m} \int \frac{\epsilon_k}{k(k_0^2 - \epsilon_k^2)^2} + \frac{-i\lambda}{m^2} \left[ 2 \int \frac{\epsilon_k^2 k_0^2}{k(k_0^2 - \epsilon_k^2)^3} + \int \frac{k^2}{k(k_0^2 - \epsilon_k^2)^2} \right] \]

(2.27)
with \( \pi_{ab}^{(2)} \) giving no contribution.

Evaluating the integrals above at zero temperature with the Fermi energy \( \epsilon_F \) as a cut-off parameter, for one-electron states near the bottom of conduction band (i.e. for \( \frac{1}{2} E_g >> \epsilon_F \)) we find

\[
K^{(0)}(0) = \left[ 1 - \frac{\lambda n}{E_g} \left( 1 - \frac{3}{5} \frac{\epsilon_F}{E_g/2} \right) \right]
\]

(2.28)

where \( n = (2m \epsilon_F)^{3/2}/6\pi^2 \) is the free particle density in conduction band,

\[
Z = \frac{-\lambda (2m \epsilon_F)^{3/2}}{6\pi^2 E_g^3} \left( 1 - \frac{9}{5} \frac{\epsilon_F}{E_g/2} \right),
\]

(2.29)

\[
I = \frac{\lambda}{6\pi^2} (2m)^{1/2} \frac{\epsilon_F^{3/2}}{E_g^3} \left( 1 - \frac{12}{5} \frac{\epsilon_F}{E_g/2} \right)
\]

(2.30)

and

\[
\Lambda(0) = -4\lambda m_0^4 Z.
\]

(2.31)

Rescaling the field and coupling constants as

\[
m_0 \varphi = (Z)^{-1/2} \varphi,
\]

(2.32)

\[
\lambda(Z)^{-1} = f^2
\]

(2.33)

and

\[
\frac{K^{(0)}(0)}{Z} = m^2,
\]

(2.34)

the effective lagrangian for the bosonic modes in the semiconductor apart from an unimportant constant is

\[
\mathcal{L}_\varphi = (\partial_\mu \varphi^2 - \alpha^2 (\nabla \varphi) \cdot (\nabla \varphi) - f^2 \left( \varphi^2 - \frac{m^2}{2 f^2} \right)^2
\]

(2.35)

which is of a Ginzburg-Landau form, with \( \alpha^2 = \frac{1}{m} \left( \frac{1}{2} E_g - \frac{3}{5} \epsilon_F \right) \).
It should be noted that the contribution from $K^{(0)}(q = 0)$ gives a virtual mass constant and thus there is no spontaneous symmetry breaking unless the coupling constant $\lambda$ exceeds a critical value

$$\lambda_c = \frac{E_g}{n} \left( 1 + \frac{3}{5} \frac{\epsilon_F}{E_g/2} \right). \tag{2.36}$$

Making the substitution of rescaled field $\varphi$ with the previous notation $\phi$ and using $\pi$ as the canonical conjugate to $\phi$, the effective Hamiltonian density $H_{\psi,\phi}$ for the fermionic and bosonic excitations in the semiconductor is

$$H_{\psi,\phi} = \psi^\dagger (\epsilon \tau_3 + f \phi \cdot \tau) \psi + \pi^2 + \alpha^2 (\nabla \phi) \cdot (\nabla \phi) + f^2 (\phi^2 - \left( \frac{m'}{\sqrt{2f}} \right)^2), \tag{2.37}$$

or, equivalently, in terms of $\phi$ and $\phi^*$, we get

$$H_{\psi,\phi} = \psi^\dagger (\epsilon \tau_3 + f \phi \tau_+ + f \phi^* \tau_-) \psi + \pi^* \pi + \alpha^2 (\nabla \phi^*) \cdot (\nabla \phi) + f^2 (\phi^2 - \left( \frac{m'}{\sqrt{2f}} \right)^2). \tag{2.38}$$

This Hamiltonian can be mapped into a supersymmetric quantum mechanical model if we follow the procedure used in our earlier work [5].

### III. EFFECTIVE HAMILTONIAN FOR ELECTRON-HOLE AND THEIR COLLECTIVE MODE EXCITATIONS AT FINITE TEMPERATURE

Let us now approach the finite temperature case. For this, we first derive the temperature dependent effective action for the bosonic field following out our procedure in Sec.II. Finite temperature effects can be obtained by making the field configurations periodic or anti-periodic in time for bosons or fermions respectively, with a period $\beta = 1/T$ (with the Boltzmann constant being equal to one), in the imaginary time formalism. In momentum space what is needed is the replacement of $\int \frac{dk}{2\pi}$ by $i/\beta \sum_n$ where $k_0 = \frac{2\pi i}{\beta}(n + \frac{1}{2})$, for fermions, and $k_0 = \frac{2\pi i}{\beta} n$, for bosons [12]. Making these substitutions we can easily get the finite temperature form of the quantities we are interested in, namely

$$\pi_T^{(0)}(0) = -\frac{(2m \epsilon_F)^{3/2}}{6\pi^2 (E_g/2)} \left[ 1 - \frac{3}{5} \frac{\epsilon_F}{E_g/2} \right] \tanh \left( \frac{E_g}{4T} \right) + \frac{3}{5} \frac{\epsilon_F}{2T} \frac{1}{\cosh^2 \left( \frac{E_g}{4T} \right)}, \tag{3.1}$$
\[ K_T^{(0)}(0) = 1 - \frac{\lambda n}{E_g} \left[ \left( 1 - \frac{3}{5} \frac{\epsilon_F}{E_g/2} \right) \tanh \left( \frac{E_g}{4T} \right) + \frac{3}{5} \frac{\epsilon_F}{2T} \frac{1}{\cosh^2 \left( \frac{E_g}{4T} \right)} \right] , \quad (3.2) \]

where \( n = (2m\epsilon_F)^{3/2}/6\pi^2, \)

\[ Z(T) = -\frac{\lambda}{6\pi^2} \frac{(2m\epsilon_F)^{3/2}}{E_g^3} \left[ \left( 1 - \frac{9}{5} \frac{\epsilon_F}{E_g/2} \right) \left( \tanh \left( \frac{E_g}{4T} \right) - \frac{E_g}{2T} \frac{1}{\cosh^2 \left( \frac{E_g}{4T} \right)} \right) \right] \]

\[ - \left( 1 - \frac{3}{5} \frac{\epsilon_F}{E_g/2} \right) \frac{(E_g/2)^2}{T^2} \frac{\tanh(E_g/4T)}{\cosh^2\left( E_g/4T \right)} + \frac{3}{5} \frac{\epsilon_F(E_g/2)^2}{4T^3} \frac{1 - \sinh^2\left( E_g/4T \right)}{\cosh^4\left( E_g/4T \right)} \right] , \quad (3.3) \]

\[ I(T) = \frac{\lambda}{6\pi^2} (2m)^{1/2} \frac{\epsilon_F^{3/2}}{E_g^3} \left[ \left( 1 - \frac{12}{5} \frac{\epsilon_F}{E_g/2} \right) \tanh\left( E_g/4T \right) \right] \]

\[ - \frac{E_g}{4T} \left( 1 - \frac{36}{5} \frac{\epsilon_F}{E_g/2} \right) \frac{1}{\cosh^2\left( E_g/4T \right)} + \frac{3}{5} \frac{\epsilon_F(E_g/2)^2}{T^2} \frac{1}{\cosh^2\left( E_g/4T \right)} \right] , \quad (3.4) \]

and finally

\[ \Lambda_T(0) = -4\lambda m_0^4 \left( Z(T) + \frac{3\lambda n\epsilon_F}{16 \cdot 5E_gT^3} \frac{1 - \sinh^2\left( E_g/4T \right)}{\cosh^4\left( E_g/4T \right)} \right) . \quad (3.5) \]

Now if we rescale the field using

\[ m_0\phi = (-Z(T))^{-1/2} \psi, \quad (3.6) \]

\[ m_0^2\lambda(-Z(T))^{-1} = f_T^2, \quad (3.7) \]

\[ \frac{\Lambda_T(0)}{4m_0^4}(-Z(T))^{-2} = f(T)^2 \quad (3.8) \]

and

\[ \frac{K_T^{(0)}(0)}{Z(T)} = m_T^2 , \quad (3.9) \]

the resulting effective Hamiltonian for the bosonic and fermionic excitations becomes

\[ \mathcal{H}_{\psi-\phi} = \psi^+ (\epsilon(T)\tau_3 + f_T\phi \cdot \vec{\tau}) \psi + \vec{\pi} \cdot \vec{\pi} \]
\[ + \alpha_T^2 (\nabla \phi) \cdot (\nabla \phi) + f(T)^2 \left( \phi^2 - \left( \frac{m_T}{\sqrt{2f(T)}} \right)^2 \right)^2, \quad (3.10) \]

or in our \( \phi \) and \( \phi^* \) notation from Section II,

\[ \mathcal{H}_{\psi - \phi} = \psi^+ (\epsilon(T)\tau_3 + f_T \phi \tau_+ + f_T \phi^* \tau_-) \psi + \pi^* \pi \]

\[ + \alpha_T^2 (\nabla \phi^*) \cdot (\nabla \phi) + f(T)^2 \left( \phi^* \phi - \left( \frac{m_T}{\sqrt{2f(T)}} \right)^2 \right)^2, \quad (3.11) \]

where \( \alpha_T^2 = -\frac{I(T)}{Z(T)} \). In this present form the model can no longer be mapped into 1 + 1 supersymmetric quantum model since the key feature \( f_T = f(T) \) does not hold anymore at any finite temperature. This is crucial for a model to satisfy the requirements for quasisupersymmetry, and hence Nambu’s argument on the realization on the quasisupersymmetry is no longer applicable. This result is not unexpected since in our derivation of the model both fermion and boson are coupled directly to the heat bath. Due to their different statistics they necessarily acquire different effective masses. The supersymmetry of zero temperature model is therefore spontaneously broken by finite temperature effects if we assume that both single particle and electron-hole pair states couple independently to the heat bath. However it is not physically meaningful to do this. As in a superconductor in which the low-lying excitations of the BCS ground state are associated with the breaking of Cooper pairs and not with higher energy paired states, here, near the semiconductor-metal transition, we are in a somewhat similar situation. It is precisely the breaking of electron-hole pair which produces new excited states which are closer in energy to the supersymmetric reference state. The main reason for this is that the original single-particle states are the real fundamental entities in the electron-hole pair. Although the paired state can be described by a boson field the bosonic high energy states are not in reality accessible to the physical system. In other words if we couple both the fermion and boson states directly to the heat bath, the model needs to be corrected further since the higher energy bosonic states have to be excluded from the Hilbert space associated with our effective hamiltonian. One alternative way to achieve this is to consider that the bosonic field only couples indirectly
to the heat bath through the fermionic excitation which feels the presence of a non-zero temperature. If we couple the fermions to the heat bath in the first place we no longer need to do the same with the boson excitations. The bosons will automatically feel the finite temperature effects through the redefined coupling and mass parameters. Let us go back to the original fermionic model at \( T = 0 \):

\[
\mathcal{H}_f = \psi_e^+ \epsilon \psi_e + \psi_h^+ \epsilon \psi_h - \lambda \psi_e^+ \psi_h \psi_e \psi_e .
\] (3.12)

It describes the low-lying electron-hole excitations in the semiconductor with \( \epsilon = \frac{E_g}{2} + \frac{k^2}{2m} \). It is sufficient for us to study what happens to this simple Hamiltonian density at finite temperature. We do not need to care about the precise form of \( \epsilon \) at finite \( T \). Let us denote it by \( \epsilon(T) \). In the imaginary time formalism, the finite temperature effect will come as a result of the loop corrections performed at non-zero temperature. For simplicity, we will only consider the one-loop correction to the four-fermion coupling constant \( \lambda \), as is shown in Fig. 3.

The resulting \( \lambda_{\text{1-loop}} \) is given in the following form with simple derivation

\[
\lambda_{\text{1-loop}} = \lambda - 2\lambda^2 \int_k \left[ \frac{1}{(k_0^2 - \epsilon_k^2)} + \frac{2\epsilon_k}{(k_0^2 - \epsilon_k^2)^2} \right] ,
\] (3.13)

using the fermionic propagator

\[
G_0(k) = i \frac{k_0 + \epsilon_k \tau_3}{k_0^2 - \epsilon_k^2 + i\delta} .
\]

Performing finite temperature calculation to \( \lambda_{\text{1-loop}} \) in the imaginary time formalism, we get

\[
\lambda_T = \lambda + \frac{\lambda^2}{2T^2} \int_k \frac{1}{\cosh^2(\epsilon_k/2T)} .
\] (3.14)

Evaluating the integral over \( k \), the finite temperature effective four fermion coupling constant is

\[
\lambda_T = \lambda + \frac{\lambda^2 n_T}{2T^2} \left[ \frac{1}{\cosh^2(E_g/4T)} - \frac{3\epsilon_F}{5} \frac{\tanh(E_g/4T)}{T \cosh^2(E_g/4T)} \right] .
\] (3.15)
where \( n_T = (2m \epsilon_F)^{3/2}/6\pi^2 \) and \( \epsilon_F \) and \( E_g \) take their corresponding values at temperature \( T \).

At this stage we can repeat all the computation of Sec.II to derive the effective action for the fermionic and bosonic fields with a temperature dependent coupling constant. It then follows that

\[
K_T^{(0)}(0) = \left[ 1 - \frac{\lambda_T n_T}{E_g} \left( 1 - \frac{3}{5} \frac{\epsilon_F}{E_g/2} \right) \right],
\]

\[
Z_T = -\frac{\lambda_T}{6\pi^2} \left( 2m \epsilon_F \right)^{3/2} \left( E_g^3 \right)^{1/2} \left( 1 - \frac{9}{5} \frac{\epsilon_F}{E_g/2} \right),
\]

\[
I_T = \frac{\lambda_T}{6\pi^2} \left( 2m \right)^{1/2} \left( \epsilon_F \right)^{3/2} \left( E_g^2 \right)^{1/2} \left( 1 - \frac{12}{5} \frac{\epsilon_F}{E_g/2} \right)
\]

and

\[
\Lambda_T(0) = -4\lambda_T m_0^4 Z_T.
\]

Rescaling the field and coupling constants in the same way as before, we get

\[
m_0 \phi = (-Z_T)^{-1/2} \phi,
\]

\[
\lambda_T(-Z_T)^{-1} = f_T^2.
\]

and

\[
\frac{K_T^{(0)}(0)}{Z_T} = m_T^2.
\]

The effective lagrangian for the bosonic modes in the semiconductor apart from an unimportant constant will again have a Ginzburg-Landau form which differs only from its zero \( T \) partner by the fact that the masses and coupling parameters are now temperature dependent.

The final effective Hamiltonian density \( \mathcal{H}_{\psi-\phi} \) for the fermionic and bosonic excitations in the semiconductor at \( T \neq 0 \) is then

\[
\mathcal{H}_{\psi-\phi} = \psi^+(\epsilon_T \tau_3 + f_T \phi \cdot \tau) \psi + \pi \cdot \pi
\]
\[ + \alpha_T^2 (\nabla \phi) \cdot (\nabla \phi) + f_T^2 \left( \phi^2 - \left( \frac{m_T}{\sqrt{2} f_T} \right)^2 \right)^2, \quad (3.23) \]

or, equivalently

\[ \mathcal{H}_{\phi - \pi} = \psi^+ (\epsilon_T \tau_3 + f_T \phi \tau_+ + f_T \phi^* \tau_-) \psi + \pi^* \pi \]

\[ + \alpha_T^2 (\nabla \phi^*) \cdot (\nabla \phi) + f_T^2 \left( \phi^* \phi - \left( \frac{m_T}{\sqrt{2} f_T} \right)^2 \right)^2, \quad (3.24) \]

where we have proceeded with the substitution of rescaled field \( \varphi \) by \( \phi \) with \( \pi \) being its canonical conjugate and with \( \alpha_T^2 = \frac{1}{m} \left( \frac{1}{2} E_g - \frac{2}{\beta} \epsilon_F \right) \).

It should be noted that there is no spontaneous symmetry breaking unless the mass term contributed by \( K^{(0)}_T (q = 0) \) is negative. This means that the coupling constant \( \lambda_T \) should exceed a critical value

\[ \Lambda_c = \lambda_c + \frac{\lambda_n^2}{2 T^2} \left[ \frac{1}{\cosh^2(E_g/4T)} - \frac{3 \epsilon_F}{5 T} \tanh(E_g/4T) \right] = \frac{E_g}{n_T} \left( 1 + \frac{3 \epsilon_F}{5 E_g/2} \right), \quad (3.25) \]

in order for this to occur.

If we now repeat the discussion employed in our \( T = 0 \) treatment of the low-lying excitation and the many-body excited state vector near the semiconductor-metal transition \[ \] this supersymmetry can be made exact even in the presence of a heat bath. From the discussion in the last two sections and from Nambu's argument on the BCS model of superconductivity, it will soon become clear that we have derived a quasi-supersymmetric model for the low energy fermionic and bosonic excitations in a semiconductor at finite temperature \( T \).

\[ \text{IV. STATE VECTOR AND LOW-LYING EXCITATIONS IN A SEMICONDUCTOR} \]

It is well known that the low-lying excitations in a semiconductor are mostly associated with exciton states. Let \( |\Phi_0> \) represent the ground-state of the semiconductor. Then

\[ |\Phi_0> = |0> \otimes \prod_{k \left( k_0 \leq -\frac{1}{2} E_g \right)} \psi^+_c (k_0, k) |0> \equiv \prod_{k \left( k_0 \leq -\frac{1}{2} E_g \right)} \psi^+_c (k_0, k) |0> \quad (4.1) \]
where \( |0 > \) is the vacuum state associated with the conduction and valence bands. Consider next the electron-hole pair orbital state \( \varphi_k \) in which the pair \((k_0, k; k_0, -k)\), representing the electron state \((k_0, k)\) in the conduction band and its corresponding hole partner in the valence band, has equal probability of being either fully or totally empty [5]:

\[
\varphi_k = \frac{1}{\sqrt{2}} [1 + \psi_h^+(k_0, -k)\psi_e^+(k_0, k)] |\Phi_0 > \quad (4.2)
\]

Here \( \psi_e^+ \) acts over the conduction band states while \( \psi_h^+ \) acts over the valence band states only.

If we consider that all excited electron-hole pair states for all values of \( k \) in the two-band semiconductor are arranged in a similar way and are essentially uncorrelated, the state vector \(|\Phi >\) for this total excited state is the linear combination

\[
|\Phi > = \prod_k \varphi_k = \prod_{k(k_0 \geq \frac{1}{2}E_g)} \frac{1}{\sqrt{2}} [1 + \psi_h^+(k_0, -k)\psi_e^+(k_0, k)] |\Phi > \quad (4.3)
\]

At finite T, our excited state can therefore be considered a superposition of states with different numbers of occupied electron-hole pairs. This vector state can be realized if there is not a single electron in the conduction band without its corresponding hole partner in the valence band. That is, for a given \( k_0 \), the interband pair \((k_0, k; k_0, -k)\) has an equal chance of being fully occupied or completely empty.

Following of previous work [5],

\[
\psi_e^+(k_0, k)|\Phi > = \psi_h(k_0, -k)|\Phi > , \quad (4.4)
\]

or, equivalently,

\[
\psi_e(k_0, k)|\Phi > = \psi_h^+(k_0, -k)|\Phi > \quad (4.5)
\]

This means that if we add an extra independent electron to a \( k \)-state of the conduction band we automatically exclude the electron-hole state \((k_0, k; k_0, -k)\), across the energy gap, as a possible state for one existing exciton in the semiconductor. It is thus physically equivalent to add an electron \((k_0, k)\) to the conduction band or to remove a hole from the corresponding state \((k_0, -k)\) in the valence band of the direct gap semiconductor.
Here as before $\psi_e$ and $\psi_h$ satisfy conventional fermion anticommutation relations since when $\psi_e$ and $\psi_h$ operate over a direct product of independent vector spaces, which are associated with the conduction and valence bands of the semiconductor, they commute with each other. It follows from this that for the excited state vector $|\Phi>\rangle$:

$$\{\psi_e(k_0, k), \psi_h(k_0, -k)|\Phi>\rangle = \{\psi_e^+(k_0, k), \psi_h^+(k_0, -k)|\Phi>\rangle = 1 \quad (4.6)$$

These unconventional anticommutation relations allow us to establish the exact supersymmetry of the static part of the effective hamiltonian density for the elementary excitations in the semiconductor when it is restricted to act in excited states such as $|\Phi>\rangle$ just as we did for $T = 0$.

V. SUPERSYMMETRY AND ELECTRON-HOLE EXCITATIONS

In our derivation of the effective Hamiltonian of fermionic and bosonic excitations, the effective potential for bosonic field always take the Ginzburg-Landau form at both zero and finite temperature in the weak coupling regime when the coupling constant exceeds some characteristic critical value. Thus, if we take

$$V^2(\phi^*, \phi) = f^2(\phi^* \phi - R^2)^2, \quad (5.1)$$

the static part of Hamiltonian $H_{st}$ of the fermion-boson low lying excitations in the semiconductor consists of the bosonic kinetic contribution and its potential as well as the fermion-boson interaction term expressed as

$$H_{st} = f' \Psi^+ (\phi \tau_+ + \phi^* \tau_-) \Psi^+ + V^2(\phi^*, \phi) + \pi^* \pi \quad (5.2)$$

with $\pi$ and $\pi^*$ being the canonical conjugate to $\phi$ and $\phi^*$ respectively and $f' = f$ in the case of (2.38) and (3.24). Clearly, at tree level, the amplitude $\sigma$-mode is such that $m_\sigma^2 = 4R^2 f^2$ while the $\Theta$-field is the corresponding Nambu-Goldstone mode with $m_\Theta = 0$. In the semiconductor there are therefore two kinds of bosonic excitations, the radial $\sigma$ oscillations and the massless $\Theta$-phase mode. The fermionic mode is a single-particle excitation. In the presence of the
energy-gap the fermion mass is finite and equal to \( f'R \). In contrast, the \( \sigma \)-mode, which is directly associated with the electron-hole pair, has a mass of \( 2fR \), while the \( \Theta \)-mode, which measures essentially the phase coherence of the electron-hole pair and which couples to transverse excitations, has zero mass. In the presence of the Coulomb field the electron-hole pair form an exciton state and the massless \( \Theta \)-mode is transmuted into a massive plasmon mode. If we take into consideration the existence of the fermionic mode with mass \( fR \) as in the case of (2.38) and (3.24), we establish the mass ratios \( 0 : fR : 2fR \) for the fermion-boson low lying excitations of the semiconductor at both zero and finite temperature. This is, of course, the same mass ratio observed by Nambu in the BCS superconductor \([1,2]\). Hence at non-zero temperature, we can still define the fermionic composite charge operators \( \hat{Q} \) and \( \hat{Q}^+ \)

\[
\hat{Q}^+ = \pi^+ \psi_e - iV \psi_h^+ \\
\hat{Q} = \pi \psi_e^+ + iV \psi_h
\]  

(5.3)  

(5.4)

In terms of these operators it then follows that

\[
\{\hat{Q}, \hat{Q}^+\} = \pi^+ \pi + V^2 + f \phi \psi_e^+ \psi_h^+ + f \phi^* \psi_h \psi_e = \mathcal{H}_{st}
\]

(5.5)

In order to guarantee the nilpotency of the charge operators we next impose a restriction in the state vectors they should act. For this reason we limit the action of the operators \( \hat{Q} \) and \( \hat{Q}^+ \) to the symmetric electron-hole excited states such as \( |\Phi\rangle \). Invoking the anticommutation relations \([1,4]\), we can establish an exact supersymmetry and a non-trivial mapping to \( SU SY \) quantum mechanics. Following our previous treatment of this problem one possible representation for the composite operators is then

\[
\hat{Q} = (\pi - iV)\tau_+ \\
\hat{Q}^+ = (\pi - iV)\tau_-
\]

(5.6)  

(5.7)

with \( \pi = -i\partial/\partial \rho \). In this way the static part of the effective hamiltonian \( \mathcal{H}_{st} \) reduces to
\[ \mathcal{H}_{st} = \{\hat{Q}, \hat{Q}^+\} = \pi^2 + V^2(\rho) + \frac{\partial V(\rho)}{\partial \rho} \tau_3 , \]  

(5.8)

where \( \partial V/\partial \rho = 2f\rho \).

In this form \( \mathcal{H}_{st} \) is exactly supersymmetric and is equivalent to \( 1 + 1 \) SUSY quantum mechanics. The operators \( \hat{Q} \) and \( \hat{Q}^+ \) are now nilpotent and the fermionic and bosonic components of the model have both, at tree level, an equal mass of \( 2fR \) at both \( T = 0 \) and non-zero \( T \).

**VI. CONCLUSION**

Using the framework of Nambu for the BCS model of superconductivity, we generalized our previous work to take explicit account of finite temperature effects in our supersymmetric description of elementary excitations in a semiconductor. As expected, despite the presence of a non-zero energy gap, if we couple fermions and bosons directly to heat bath, supersymmetry is spontaneously broken. This is simply due to the fact that in this case we can no longer maintain the special mass relation \( 0 : 1 : 2 \) between the existent fermionic and bosonic elementary excitations in the physical system. However, we argue that the electron-hole pair state, as the Cooper pair in a superconductor, should not be considered as a real boson having an infinite number of energy states simply due to the presence of a heat bath. The excited state which is closer in energy to the uncorrelated pair state is precisely the one in which at least one pair breaking is allowed. Therefore the higher energy bosonic state are not physically meaningful for the effective hamiltonian of the elementary excitations in the semiconductor. These states are automatically ruled out if we assume that both fermion and boson excitations are constituted by the fermionic single-particles states which couple directly to the heat bath. The excitations will therefore feel the temperature effects only through the new temperature dependent mass and coupling parameters and supersymmetry can be preserved if \( \lambda \) exceeds a critical value. The mapping to \( 1 + 1 \) SUSY quantum mechanics is then shown to follow the same \( T = 0 \) scheme shown in \( \Box \) and the mass parameter ratios are essentially preserved at finite temperature.
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FIGURE CAPTIONS

**Fig.1** - Diagram which generates the boson field free effective lagrangian.

**Fig.2** - Diagram which defines the boson field self-interaction.

**Fig.3** - Diagram which defines the one-loop correction for four-fermion coupling constant.
Fig. 3