Characteristic temperatures and spectral appearance of ULX disks

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Abstract. A standard disk around an accreting black hole may become effectively optically-thin and scattering dominated in the inner region, for high accretion rates (as already predicted by the Shakura-Sunyaev model). Radiative emission from that region is less efficient than blackbody emission, leading to an increase of the colour temperature in the inner region, by an order of magnitude above the effective temperature. We show that the integrated spectrum has a power-law-like shape in the $\sim 1\text{–}5\text{ keV}$ band, with a soft excess at lower energies and a downward curvature or break at higher energies, in agreement with the observed spectra of many ULXs. This scenario offers a physical explanation for the phenomenological “dense corona”, successfully used to fit those spectra. It also relates their fit parameters to physical properties of the accreting black hole, such as mass and accretion rate.

1. Introduction

Fitting colour temperatures to the X-ray spectra of accreting black holes (BHs) has traditionally been a useful, indirect method to estimate BH masses, typically within a factor of 2. This method relies on the assumption that the accretion disk extends down to the innermost stable circular orbit and is optically-thick, emitting a multicolour-blackbody spectrum (Shakura & Sunyaev 1973; Frank, King & Raine 2002). If so, the fitted peak temperature and luminosity give a characteristic inner-disk area and hence BH mass. These conditions are usually verified in Galactic BHs in their high-soft state ($0.01 \lesssim L_x/L_{\text{Edd}} \lesssim 1$).

For ultraluminous X-ray sources (ULXs), the scenario that the disk is optically thick and reaches the innermost stable circular orbit could be a plausible initial assumption, in the absence of dynamic mass determinations. From a characteristic feature in the soft spectrum (“soft excess”), peak disk temperatures $\approx 0.15\text{–}0.20\text{ keV}$ have been inferred and used to suggest masses $\sim 1000M_\odot$ (Miller, Fabian & Miller 2004). A strong objection against this argument is that most of the X-ray flux in ULXs comes out in a harder, power-law-like component, more consistent with inverse-Compton scattering (Stobbart, Roberts & Wilms 2006; Winter, Mushotzky & Reynolds 2006). This suggests that the soft thermal component only traces emission from the outer, optically-thick disk, at radii $R \gtrsim 10R_{\text{ISCO}}$; if so, its temperature and luminosity cannot be directly used to estimate the BH mass. An additional parameter is needed to model the location of the transition radius between the outer, standard disk and the inner, scattering-dominated region. In general, based on a variety of physical arguments, we expect that the transition radius $R_e \propto \dot{m}^\beta R_{\text{ISCO}}$ for $\dot{m} > 1$, where $\dot{m} \equiv \dot{M}/M_{\text{Edd}}$ and $\beta \sim 1$.

2. Alternative physical interpretations

A comparison of X-ray spectral and timing properties of ULXs and of Galactic BHs, in particular XTE J1550–564, has suggested a possible physical interpretation in terms of a standard disk plus Comptonizing corona (Goad et al. 2006; Done & Kubota 2006). Unlike Galactic BHs, however, many ULXs have a second characteristic spectral feature at energies $\sim 5\text{–}10\text{ keV}$. There, the spectrum is not well approximated by a power-law: it shows a downward curvature or break (Stobbart et al. 2006; Roberts 2007). In the disk-corona framework, this can be interpreted as emission from a corona optically-thick to electron scattering ($\tau_e \sim 10$) and with a much lower temperature than is seen in Galactic BHs ($kT \sim kT_{\text{es}} \sim 100\text{ keV}$ in Galactic BHs). In summary, the high-energy break would be related to the coronal temperature, and the soft component would come from the standard disk sticking out outside the corona, at large radii.

A possible difficulty of this model is that, in order to be optically thick, the corona must also be extremely dense. Assuming a characteristic radial size of $\sim 5000\text{ km}$, vertical size $\sim 1000\text{ km}$, $\tau_e \sim 10$ corresponds to $n_e \sim 10^{17}\text{ cm}^{-3}$. This is many orders of magnitude denser than typical coronae in Galactic X-ray binaries. More importantly, one still has to explain how gravitational power is transferred from the underlying disk to the optically-thick corona. For proton densities $n_p \approx n_e \sim 10^{17}\text{ cm}^{-3}$, the thermal energy stored in such a corona is only $\sim 10^{45}\text{ erg}$, to be compared with a power $\sim 10^{40}\text{ erg s}^{-1}$ per-
sistently transferred to the outgoing radiation via inverse Compton scattering. If gravitational power is released in the disk, a very efficient and stable energy transfer mechanism is required to keep the corona energized.

The same phenomenological spectral features have also been explained in a radically different way. The high-energy curvature and/or break have been interpreted as a very hot inner disk, with colour temperatures $kT_{\text{in}} \approx 1.5–3$ keV; e.g., Stobbart et al. 2006). A possible reason for such hot disks may be that the BH is rapidly spinning. Or, the disk may be non-standard, (e.g., slim disk models: Watarai, Mizuno & Mineshige 2001). In this scenario, ULXs are a natural extension of Galactic BHs, with higher accretion rates and hotter disks (Fig. 3 in Stobbart et al. 2006). The downside of this model is that it does not provide a natural explanation for the soft X-ray component. One possibility is that the soft excess comes from a downscattering outflow (King & Pounds 2003) or from relativistically-blurred emission lines (e.g., Frank et al. 2002) a standard disk is gas-pressure dominated, optically thick ($\tau_{\nu}^{\text{ff}} \gg 1$), and the Rosseland mean absorption opacity $\alpha_{R}^{\text{ff}} \gg \alpha_{\nu}^{\text{ff}}$. In this zone, $I_{\nu} = S_{\nu} = B_{\nu}$, and the spectrum emitted by each annulus is a blackbody.

Moving towards smaller radii, there is a region where the disk is still gas-pressure dominated and optically thick ($\tau_{\nu}^{\text{es}} \gg 1$) but the opacity is dominated by electron scattering ($\tau_{\nu}^{\text{es}} \gg \tau_{\nu}^{\text{ff}} \gg 1$). Deep inside the disk, the radiation is still in thermal equilibrium with the matter ($S_{\nu} = B_{\nu}$), but the emerging spectrum from the disk surface is modified by scattering, such that the outgoing intensity

$$I_{\nu} = \frac{2B_{\nu}}{1 + \sqrt{(\alpha_{\nu}^{\text{es}} + \alpha_{\nu}^{\text{ff}})/\alpha_{\nu}^{\text{es}}}} \approx 2\sqrt{\frac{\alpha_{\nu}^{\text{es}}}{\alpha_{\nu}^{\text{ff}}}} B_{\nu} \approx B_{\nu}$$

(Rybicki & Lightman 1979). From Eq. (7) one can directly calculate the integrated disk spectrum (see Section 4). It is often useful to introduce an approximation of Eq. (7) that does not contain a frequency dependence, by using the Rosseland mean of the absorption coefficient

$$\alpha_{R}^{\text{ff}} \approx 1.7 \times 10^{-25} T^{-7/2} Z^{2} n_{e} n_{i} \nu_{i}^{3} \text{ cm}^{-1},$$

where the frequency-averaged Gaunt factor $\nu_{i}^{3} \approx 1$. Then, the total emitted flux from a unit area on the disk

$$F(R) \approx \frac{4}{3} \frac{\alpha_{R}^{\text{ff}}}{\alpha_{\nu}^{\text{es}}} \sigma T(R)^{4},$$

that is, for a given temperature, the disk is slightly less efficient than a blackbody at radiating the dissipated gravitational power. If we impose that the disk has to radiate the same power, the colour temperature at each radius must increase with respect to the effective temperature:

$$T'(R) \approx 0.81(\alpha_{\nu}^{\text{es}}/\alpha_{R}^{\text{ff}})^{1/8} T_{\text{eff}}(R),$$

where $1 < (\alpha_{\nu}^{\text{es}}/\alpha_{R}^{\text{ff}})^{1/8} \lesssim 2$ as we will show later.

At smaller radii, $R \lesssim 1.1 \times 10^{8} a_{21}^{21/16} n_{16/21} (M/M_{\odot})^{13/21}$ cm (Frank et al. 2002), radiation pressure dominates over gas pressure. In this region, the disk may become optically thin to true absorption ($\tau_{\nu}^{\text{ff}} \ll 1$), but is always optically thick to scattering ($\tau_{\nu}^{\text{es}} \gg 1$) and most importantly, is still effectively optically thick ($\tau_{\nu}^{\text{eff}} \gg 1$). As long as $\tau_{\nu}^{\text{eff}} \gg 1$, Eqs. 7 and 9 are still valid, and the emerging spectrum is still a modified blackbody. Explicitly, for a pure Hydrogen gas, from Eqs. 5 and 6:

$$\alpha_{\nu}^{\text{es}} \approx 1.8 \times 10^{-33} T^{1/2} n_{e} \nu(1 - e^{-h\nu/kT})^{-1},$$

(11)

and

$$\tau_{\nu}^{\text{es}} \approx (H/cos i) \sqrt{\alpha_{\nu}^{\text{es}} \alpha_{\nu}^{\text{ff}}} \approx 1.6 \times 10^{-8} (H/cos i) \times n_{e}^{3/2} \nu^{-2}(1 - e^{-h\nu/kT})^{1/2}. \quad (12)$$

### 3. Modified blackbody spectra

In most ULXs, both scenarios (cool disk with a hotter Comptonised component, or a hot disk with a soft excess) are consistent with the observed spectra. Before we can use the peak disk temperature as a mass indicator, we need to determine whether the disk is associated with the soft or the hard components. Here, we suggest that both interpretations are in a sense correct, and follow simply from the standard disk model (Shakura & Sunyaev 1973).

The thermal emission spectrum of an accretion disk is successfully approximated with a multicolor blackbody model, provided that two assumptions are verified: the disk is optically-thick in the vertical direction, and all the gravitationally powered dissipated during the infall is radiated. Then (see, e.g., Frank et al. 2002 for details),

$$I_{\nu}(R) = B_{\nu}[T(R)],$$

$$F(R) = \sigma T^{4}(R) \equiv \frac{3GM\dot{M}}{8\pi R^{3}} \left[ 1 - \left( \frac{R_{\text{ISCO}}}{R} \right)^{1/2} \right],$$

$$F_{\nu}^{\text{obs}} = \frac{2\pi \cos i}{d^{2}} \int_{R_{\text{ISCO}}}^{R_{\text{out}}} I_{\nu}(R) RdR$$

where $D(R)$ is the power dissipated per unit disk area, $d$ is the distance to the source, and $F_{\nu}^{\text{obs}}$ is the observed flux. Before substituting Eqs. 1 and 2 into Eq. 3, one has to make sure that the disk is optically thick at all $R$. The effective optical depth (Rybicki & Lightman 1979)

$$\tau_{\nu}^{\text{eff}} \approx \sqrt{\tau_{\nu}^{\text{ff}}(\tau_{\nu}^{\text{ff}} + \tau_{\nu}^{\text{es}})} \approx (H/cos i) \sqrt{\alpha_{\nu}^{\text{ff}}(\alpha_{\nu}^{\text{ff}} + \alpha_{\nu}^{\text{es}})},$$

where $H(R)$ is the disk thickness, takes into account both the (frequency-dependent) free-free absorption coefficient

$$\alpha_{\nu}^{\text{ff}} \approx 3.7 \times 10^{8} T^{-1/2} Z^{2} n_{e} n_{i} \nu^{-3}(1 - e^{-h\nu/kT}) g^{\text{ff}} \text{ cm}^{-1}$$

(5)

(where the Gaunt factor $g^{\text{ff}} \approx 1$), and the electron scattering coefficient

$$\alpha_{\nu}^{\text{es}} = n_{e} \sigma_{T} \approx 6.7 \times 10^{-25} n_{e} \text{ cm}^{-1}.$$
Applying standard-disk solutions for the radiation-pressure dominated region (Shakura & Sunyaev 1973; Frank et al. 2002), we can approximate

\[ T(R) \approx 2.3 \times 10^5 \alpha^{-1/4} m^{-1/4} \nu^{-3/4} \text{ K,} \]

\[ H(R) \approx 2.5 \times 10^6 \dot{m} m \left(1 - r^{-1/2}\right) \text{ cm}, \]

\[ n_e(R) \approx 6.1 \times 10^{17} \alpha^{-1} m^{-2} m^{-3/2} \]

\[ \times \left(1 - r^{-1/2}\right)^{-2} \text{ cm}^{-3}, \]

where we have defined \( m = M/M_\odot, r = R/R_{\text{ISCO}}. \) Inserting Eqs. 13–15 into Eqs. 11 and 12, we obtain:

\[ \alpha^{\text{es}} \approx 2.0 \times 10^5 \nu_{\text{keV}}^3 \left(1 - e^{-\nu/kT}\right)^{-1/8} \]

\[ \times m^{-1/8} m^{-2} r^{15/8} \left(1 - r^{-1/2}\right)^2 \]

\[ \tau_{\nu}^{\text{eff}} \approx 2.3 \times 10^{-3} \nu_{\text{keV}}^{-3/2} \left(1 - e^{-\nu/kT}\right)^{1/2} \alpha^{-23/16} \]

\[ \times \nu^{-7/16} m^{-2} r^{39/16} \left(1 - r^{-1/2}\right)^{-3} \left(1/\cos i\right). \]

For typical Galactic BHs in a high state, the innermost part of the disk falls into the radiation-pressure, electron-scattering-dominated, effectively-thick regime. For \( M = 10M_\odot, R = 2R_{\text{ISCO}}, \dot{M} = \dot{M}_{\text{Edd}}, \) viscous parameter \( \alpha = 0.1, \nu = 1 \text{ keV} \) (so that \( 1 - \exp(-\nu/kT) \approx 1 \) across the 0.3–10 keV band), we see from Eq. 16 that the colour correction \( f_\nu(R) \approx 0.81(\alpha^{\text{es}}/\alpha^{\text{eff}})^{1/8} \approx 2, \) only weakly dependent on frequency (within the X-ray band), accretion rate, BH mass and radius. In fact, \( 1.5 \lesssim f_\nu \lesssim 2.5 \) throughout the X-ray-emitting regions, for the characteristic range of physical parameters typical of the high/soft state in Galactic BHs. That is why \( f_\nu(R) \) can be approximated with a constant “hardening factor” \( f \approx 1.7–2.5 \) (Shimura & Takahara 1995; Shafee et al. 2006; Shrader & Titarchuk 2003).

The previous results depend critically on the condition \( \tau_{\nu}^{\text{eff}} \gtrsim 1. \) From Eq. 17, using the same set of parameters suitable for Galactic BHs in a high state, we see that \( \tau_{\nu}^{\text{eff}} \approx 5 \) in the innermost region, at the Eddington accretion rate; at larger radii, \( \tau_{\nu}^{\text{eff}} \gg 1. \) So, the optically-thick condition is always verified for Galactic BHs, but the X-ray-emitting region gets close to becoming effectively thin. A small increase in viscosity, BH mass and/or accretion rate above Eddington will lead to the formation of a radiation-pressure dominated, effectively-thin region in the inner disk (\( \tau_{\nu}^{\text{eff}} \lesssim 1, \) with \( \tau_{\nu}^{\text{eff}} \ll 1 \) and \( \tau_{\nu}^{\text{eff}} \gtrsim 1 \)). If that happens, Eq. 7 is no longer applicable. Note that an increase of the accretion rate leads to a reduction of the effective optical depth in the radiation-dominated zone (perhaps counter-intuitively). This is essentially because \( n_e \propto \dot{m}^{-2}. \) Conversely, in the gas-pressure dominated disk, an increase in \( \dot{m} \) leads to an increase in optical depth.

In the region where the disk becomes effectively thin, the specific intensity emitted by an annulus is

\[ I_\nu \approx \left(1 - e^{-\tau_{\nu}^{\text{eff}}}\right) \frac{\alpha^{\text{eff}} B_\nu}{\alpha^{\text{es}}} \frac{\alpha B_\nu H}{\cos i} B_\nu \ll B_\nu. \]

\[ f_\nu(R) \approx \frac{\alpha B_\nu H}{\cos i} \left(\nu_{\text{keV}}/kT\right)^{-1/4} \approx 21.0 \nu_{3/4}^{3/4} (1 - e^{-\nu/kT})^{-1/4} \left(1/\cos i\right)^{1/4} \]

\[ \times \alpha^{15/32} \dot{m}^{7/32} \mathrm{r}^{-27/32} \left(1 - r^{-1/2}\right)^{3/4}. \]

Let us consider a hypothetical ULX with a mass \( m = 100 \) and an accretion rate \( \dot{m} \gtrsim 1. \) Then, the inner disk is effectively thin for “reasonable” choices of \( \alpha, \) and the transition radius between thin and thick regions is

\[ r_c \lesssim 12.1 \nu_{3/4}^{23/32} \dot{m}_{1/3}^{32/39} \left(1 - r^{-1/2}\right)^{48/39} \left(\cos i\right)^{16/39} \lesssim 10^{-0.8}. \]

For \( r > r_c, \) the spectrum is a disk-blackbody, slightly modified by a hardening factor \( \approx \text{constant} \) over the frequencies and radii relevant to the X-ray spectral observations, and most likely \( \lesssim 2. \) For \( r < r_c, \) the hardening factor increases sharply towards the innermost stable circular orbit, reaching \( f_{\text{keV}} \approx 4.3 \dot{m}^{3/4} \sim 10 \) in the brightest region of the disk. Although the effective optical depth is \( < 1 \) in this region, the optical depth to scattering alone

\[ \tau_{\nu}^{\text{es}} \approx 6.7 \times 10^{-25} \frac{n_e H}{\cos i} \approx \frac{\nu^{3/2}}{\alpha \dot{m} (1 - r^{-1/2}) \cos i} \gtrsim 10. \]

\[ F_\nu^{\text{obs}} = \frac{2\pi \cos i}{d^2} \int_{R_{\text{ISCO}}}^{R_{\text{out}}} I_\nu(R) RdR \]

\[ = 4\pi h \cos i \nu^3 \int_{R_{\text{ISCO}}}^{R_{\text{out}}} \frac{RdR}{F^4(R)} \left\{ e^{h\nu/kT(R)}T(R) - 1 \right\}. \]

Using the example of the hardening factor plotted in Figure 1 (top panel), where we have assumed that \( f = 1 \) for \( r > r_c, \) and applying Eq. 22, we obtain the X-ray spectrum plotted in the bottom panel of Figure 1. A detailed study of the emitted spectra is left to a paper currently in preparation. Here we simply want to point out the main qualitative result: the spectrum has a power-law-like shape for energies \( 1.5 \text{ keV} \lesssim E \lesssim 6 \text{ keV}, \) a downward curvature and break above \( 6 \text{ keV}, \) and a “soft excess” with a characteristic temperature \( T_c \sim 0.2 \text{ keV}. \) The latter component corresponds to the disk-blackbody emission from the outer disk (\( R > R_c \)), where \( f \sim 1. \) The high-energy
A hardening factor that increases sharply at small radii (top panel), as we suggest is the case in ULX disks, produces an emitted X-ray spectrum (bottom panel) with three characteristic features: a power-law-like component, a soft excess at low energies, and a curvature or break at high energies.

Fig. 1. A hardening factor that increases sharply at small radii (top panel), as we suggest is the case in ULX disks, produces an emitted X-ray spectrum (bottom panel) with three characteristic features: a power-law-like component, a soft excess at low energies, and a curvature or break at high energies.

break corresponds to the maximum colour temperature in the effectively-thin, scattering-dominated inner disk: \( T \approx f T_{\text{eff}} \approx 6 T_{\text{eff}} \). This result is already well known (Shimura & Takahara 1995; Fig. 12 in Shakura & Sunyaev 1973), but its implications for ULX studies are not well explored. In between the two thermal features, the slope of the power-law-like section of the spectrum depends on how steeply the hardening factor increases towards the centre. A slightly steeper slope is obtained when the frequency dependence in the hardening function \( f_\nu \) is also taken into account. Additional steepening of the spectrum results from non-radiative contributions (outflows, Poynting flux) to the cooling flux from the inner region. Increasing the range of values of the hardening factor (for example, by increasing the viscosity or the accretion rate) moves the soft excess and high-energy break below and above the observed X-ray band, respectively, creating an apparently “pure power-law” spectrum.

5. Conclusions: standard disk or corona?
We have shown that the accretion disk in BHs with \( M \gtrsim 10 M_\odot, \dot{M} \sim \text{a few } \dot{M}_{\text{Edd}} \) is expected to develop a scattering-dominated, effectively-thin inner region (\( R \lesssim 10 R_{\text{ISCO}} \)), with a corresponding increase in the colour temperature, to compensate for the decreased emission efficiency. The predicted spectrum is very similar to what is observed from many ULXs at \( L_X \sim 10^{40} \text{ erg s}^{-1} \). Phenomenologically, those spectra have been successfully fitted with an optically-thick disk-blackbody emission from the outer disk, plus a dense scattering corona in the inner region, “covering” the inner disk. We have shown here that the characteristic scattering opacity, size and temperature observationally inferred for those “coronae” are the same as predicted for a radiation-pressure dominated standard disk, for accretion rates and BH masses consistent with ULX parameters. So, we argue that there is no need to postulate an ad-hoc corona covering or replacing the inner disk. It is the standard disk itself that becomes a hot, scattering-dominated environment at small radii, and produces the observed spectra. The advantage of seeing it as the inner disk rather than a corona is that we do not have to worry about how energy is transferred from one to the other, or how their interface looks like. Moreover, in the scattering disk scenario, the peak temperature, optical depth and emitted flux are not just empirical parameters: they are predictable functions of \( M, \dot{M}, \alpha \), known from standard-disk models. This provides the tools to determine the physical properties of the accreting BHs in ULXs from their fitted spectral parameters, and relate their spectral evolution to changes in the accretion rate. In this scenario, both the cool (\( T \sim 0.2 \text{ keV} \)) and the hot component (\( T > \sim 5 \text{ keV} \)) come from the disk.

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