The strongly intensive observable in \( pp \) collisions at LHC energies in the string fusion model

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Abstract. The properties of the strongly intensive variable characterizing correlations between the number of particles in two separated rapidity interval in \( pp \) interactions at LHC energies are studied in the framework of the string fusion model. We perform the MC simulations of string distributions in the impact parameter plane to take into account the experimental conditions of \( pp \) collisions. We account the string fusion processes, leading to the formation of string clusters, embedding a finite lattice (a grid) in the impact parameter plane. As a result, we found the dependence of this variable both on the distance between the centers of the observation windows and their acceptance for the minbias \( pp \) collisions at several initial energies. Analyzing these dependencies we can extract the important information on the properties of string clusters. We show that in \( pp \) collisions at LHC energies the string fusion effects have a significant impact on the behavior of this strongly intensive variable. The role of these effects is increasing with the initial energy and centrality of collisions. In particular, we found that the increase of this variable with initial energy takes place due to the growth of the portion of the fused string clusters in string configurations arising in \( pp \) interactions.

1. Introduction

By now, the string (color flux tubes) model \([1,2]\) has become a standard approach for the description of the soft part of hadronic interactions at high energies. Different versions of the string model are applied in the various MC event generators: PYTHIA, VENUS, HIJING, AMPT, EPOS etc., for a description of soft processes in strong interactions, when the perturbative QCD approach does not works.

Along with this, it was also recognized that at a large string density in nucleus-nucleus collisions the strings start overlap in the impact parameter plain due to finite transverse area of a string, considered as a color flux tube. So one has to take into account the interaction between strings leading to the string fusion processes and the formation of string clusters (“color ropes”) \([3,4]\). In the present paper we’ll argue that the string fusion processes play important role also in \( pp \) interactions at LHC energies.

As one of the tools for the investigation of these string fusion effects the study of the forward-backward (FB) correlations between observables in two separated rapidity intervals was suggested in \([5]\). It is known that the investigations of the long-range rapidity correlations enable to obtain the information on the very initial stages of a hadronic collision \([6]\), and, in particular, on the emerging string configuration.
Unfortunately, back in the work [7], it was shown that the traditional FB correlation coefficient between the charge particle multiplicities, \( n_F \) and \( n_B \), in the observation rapidity windows strongly depends on the event-by-event variance of the number of cut pomerons (strings) in pp collisions, i.e., on the so-called “volume” fluctuation - the trivial fluctuation in the number of sources. From this point of view it is desirable to search for another observables, which is not sensitive to the fluctuation in the number of sources (strings), but is sensitive to the fluctuation of the properties of sources, e.g., to the formation of string clusters by string fusion processes.

We can suppress the influence of these trivial “volume” fluctuations going to the studies of the FB correlation coefficient between the intensive quantities, such as e.g., event-mean transverse momenta \( p_F \) and \( p_B \) of charge particles in the observation rapidity intervals \( \delta y_F \) and \( \delta y_B \) instead of \( n_F \) and \( n_B \), as in [8][9], or defining the more sophisticated correlation observable between \( n_F \) and \( n_B \). In the present paper, we’ll focus on the last approach.

2. The strongly intensive observable \( \Sigma(n_F, n_B) \)

The general methods of the constructing of variables not affected by the “volume” fluctuations (the so-called strongly intensive observables) were developed in the paper [10]. Later it was suggested [11] to study the strongly intensive quantity \( \Sigma(n_F, n_B) \), defined for the charge particle multiplicities, \( n_F \) and \( n_B \), in two observation windows:

\[
\Sigma(n_F, n_B) = \langle (n_F) \omega_{n_B} + \langle n_B \rangle \omega_{n_F} - 2 \text{cov}(n_F, n_B) \rangle / \langle (n_F) + \langle n_B \rangle \rangle, \tag{1}
\]

where the \( \omega_n \equiv D_n/\langle n \rangle \) is a scaled variance and \( \text{cov}(n_F, n_B) \equiv \langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle \). For symmetric reaction and symmetric observation windows, \( \delta y_F = \delta y_B = \delta y \), we have \( \langle n_F \rangle = \langle n_B \rangle = \langle n \rangle \), \( \omega_{n_F} = \omega_{n_B} = \omega_n \) and the definition [11] can be simplified to

\[
\Sigma(n_F, n_B) = \omega_n - \text{cov}(n_F, n_B)/\langle n \rangle = \langle n^2 \rangle - \langle n_F n_B \rangle / \langle n \rangle. \tag{2}
\]

In paper [13], it was demonstrated that in the model with independent identical strings the observable \( \Sigma(n_F, n_B) \) can be expressed only through the parameters of a single string - the one- and two-particle rapidity distributions arising from decays of a string:

\[
\lambda(y) \equiv dN/dy = \langle \mu \rangle / \delta y = \mu_0, \quad \lambda_2(y_1, y_2) \equiv d^2N/dy_1 dy_2 = \lambda_2(y_1 - y_2). \tag{3}
\]

Instead of the \( \lambda_2(y_1, y_2) \) one usually exploits the two-particle correlation function of a string:

\[
\Lambda(\eta_1, \eta_2) = \lambda_2(\eta_1, \eta_2)/\lambda(\eta_1)\lambda(\eta_2) - 1 = \lambda_2(y_1 - y_2)/\mu_0^2 - 1 = \Lambda(\eta_1 - \eta_2). \tag{4}
\]

The last transitions in these formulas corresponds to the rapidity invariant approximation, which is fulfilled at mid-rapidities at LHC energies. It was shown in [13] that in the model with independent identical strings

\[
\Sigma(n_F, n_B) = \Sigma(\mu_F, \mu_B). \tag{5}
\]

Here the \( \Sigma(\mu_F, \mu_B) \) is the strongly intensive variable \( \Sigma \) defined similarly [1], but for a single source: the \( \mu_F \) and \( \mu_B \) are the number of particles produced in forward and backward rapidity windows from decays of a single string. It’s connected with the string parameters as follows

\[
\Sigma(\mu_F, \mu_B) = 1 + \mu_0 \delta y [J_{FF} - J_{FB}] = \Sigma(\delta y, \Delta y), \tag{6}
\]

where

\[
J_{FB} = \frac{1}{\delta y_F \delta y_B} \int_{\delta y_F} dy_1 \int_{\delta y_B} dy_2 \Lambda(y_1 - y_2), \quad J_{FF} = \frac{1}{\delta y_F^2} \int_{\delta y_F} dy_1 \int_{\delta y_F} dy_2 \Lambda(y_1 - y_2). \tag{7}
\]
For symmetric case it depends on the acceptance of the observation windows, \( \delta y = \delta y_F = \delta y_B \), and the rapidity distance, \( \Delta y \), between the centers of these windows, so \( \Delta y \geq \delta y \).

The formula (5) proves that in the framework of the model with independent identical strings the variable \( \Sigma(n_F, n_B) \) demonstrates the strongly intensive property. Its value is equal to the value for a single string \( \Sigma(\mu_F, \mu_B) \) and does not depend on the number of strings, produced in a given event and their event-by-event fluctuations.

Unfortunately, the string parameters, \( \mu_0 \) and \( \Lambda(\eta_1 - \eta_2) \), extracted in papers [12–15] from the ALICE data [17] on the FB correlations in pp collisions at initial energies 0.9–7 TeV prove to be dependent on the energy. By (5)–(7) this leads to the dependence of the \( \Sigma(n_F, n_B) \) on the collision energy, what can be interpreted as a signature of string fusion processes in pp collisions at LHC energies [13–16].

Really, in the paper [16] it was shown, that in the model with different types of strings the expression (5) for \( \Sigma(n_F, n_B) \) is replaced by

\[
\Sigma(n_F, n_B) = \sum_{\eta=1}^{\infty} \alpha_\eta \Sigma(\mu_F, \mu_B),
\]

where \( \Sigma(\mu_F, \mu_B) \) is a \( \Sigma \) variable for a string cluster, arising due to the fusion of the \( \eta \) initial strings. For each such cluster the formulas (6) and (7) are valid, but with the replacement of the single string parameters to the parameters of the corresponding cluster formed by the \( \eta \) fused initial strings:

\[
\mu_0 \rightarrow \mu_0^\eta, \quad \Lambda(y_1 - y_2) \rightarrow \Lambda_\eta(y_1 - y_2).
\]

The weighting factors \( \alpha_\eta = \langle n \rangle_\eta / \langle n \rangle \) in (8) is an average portion of particles produced from decays of all clusters of a given \( \eta \) type. Note that the first term with \( \eta = 1 \) corresponds to the contribution from single strings.

Obviously, with a change in the initial energy and centrality of pp collisions the portion of the clusters, corresponding to the different number of fused strings, will change, leading to the change of the weights \( \alpha_\eta \) and hence by (8) to the change of the observable \( \Sigma(n_F, n_B) \). So, strictly speaking, if we take into account the possibility of the formation of the sources of different types, the observable \( \Sigma(n_F, n_B) \) loses its strongly intensive property, starting to depend through the weight \( \alpha_\eta \) on the collision details.

3. Modeling string configurations in pp collision

For the Monte-Carlo generation of the string configurations arising in pp collision we have used the approach developed earlier in the paper [18]. In this approach we suppose that at given value of the impact parameter \( b \) for the number of cut pomerons, \( N \), we have event-by-event the poissonian distribution with the parameter \( \bar{N}(b) = N_0 \exp(-b^2/2r_0^2) \), discarding the cases with \( N = 0 \). The last condition reflects the fact that we take into account only inelastic (non-diffractive, ND) pp collisions.

We will consider that each cut pomeron corresponds to the formation of two initial strings [12], \( N_{str} = 2N \). To take into account the string fusion effects by the MC calculations later we need to simulate not only the total number of strings in each event, but also their distribution in the transverse plane. According to [18], we suppose that these strings are distributed in the transverse plane with the probability density:

\[
w_{str}(s, b) \sim T(s - b/2)T(s + b/2)/\sigma_{pp}(b),
\]

where the \( w_{str}(s, b) \) is the probability density of finding a string at the point \( s \) of the transverse plane in the case of the ND pp collision at the impact parameter \( b \), the \( \sigma_{pp}(b) \) is the probability...
Table 1. The non-diffractive cross section, the multiplicity density at mid-rapidity and the mean number of initial strings in pp collisions at different initial energies.

| √s (GeV) | σ_{th}^{ND} (mb) | σ_{MC}^{ND} (mb) | dN^{ND}/dy | \langle N_{str} \rangle |
|----------|------------------|------------------|-------------|---------------------|
| 60       | 24.9             | 24.9             | 2.44        | 4.2                 |
| 900      | 39.9             | 39.9             | 3.76        | 7.8                 |
| 7000     | 52.5             | 52.4             | 5.44        | 13.4                |
| 13000    | 56.5             | 56.6             | 6.03        | 16.0                |

of such interaction. Here the $T(s)$ is the partonic profile function of a nucleon, for which we use the simplest gaussian distribution with some parameter $r_0$.

As it was shown in [18], in this model along with MC simulations we can analytically calculate all quantities of physical interest. So, for the cross-section of the ND pp interaction we have

$$
\sigma^{ND} = 2\pi r_0^2 \left[ \Phi(N_0) + \gamma + \ln N_0 \right], \quad \Phi(x) = \int_x^\infty \frac{e^{-t}}{t} dt,
$$

where the $\gamma = 0.577...$ is the Euler constant. In the paper [18] it was also shown that the present approach leads to the same distribution for the number of cut pomerons in minbias pp inelastic events as in the Gribov-Regge approach (see e.g. [19]). This enables to connect the model parameters $N_0$ and $r_0$, with the parameters of the pomeron trajectory and its couplings to hadrons. We have used the following numerical values for these parameters:

$$
\Delta = 0.2, \quad \alpha' = 0.05 \text{ GeV}^{-2}, \quad \gamma_{pp} = 1.035 \text{ GeV}^{-2}, \quad R_{pp}^2 = 3.3 \text{ GeV}^{-2}, \quad C = 1.5,
$$

which gives an appropriate description the non-diffractive cross section and the multiplicity density at mid-rapidities in pp collisions (see the Table 1). In (12) $\Delta$ and $\alpha'$ are the intercept and the slope of the pomeron trajectory. The parameters $\gamma_{pp}$ and $R_{pp}$ characterize the coupling of the pomeron trajectory to the initial hadrons. The quasi-eikonal parameter $C$ is related to the small-mass diffraction dissociation of incoming hadrons.

Note that the mean multiplicities in the Table 1 are calculated already with taking into account the string fusion effects, as it is explained in the next section with $\mu_0 = 0.7$. To check the MC algorithm we have calculated the non-diffractive cross section both analytically by formula (11), $\sigma_{th}^{ND}$, and by MC simulations as $\sigma_{MC}^{ND} = S_b n_{sim}(N = 0)/n_{sim}(N \geq 0)$, where the $n_{sim}(N = 0)$ is a number of simulation without cut pomerons, the $n_{sim}(N \geq 0)$ is a total number of simulation and $S_b$ is an area of the impact parameter simulations.

4. Accounting the string fusion effects

In our MC model to account the string fusion effects we introduce a finite transverse lattice (a grid) [20,21] with the cells, which area is equal to the transverse area of a single initial string, $\sigma_{str} = \pi r_{str}^2$. We use $r_{str} = 0.2 \text{ fm}$ in our calculation [22].

Generating the event we at first generate the impact parameter $b$, then the number of pomerons, $N$, and hence the number of strings, $N_{str} = 2N$, as it was described in the previous section. After that we distribute these strings in the transverse plane according to the density given by the formula (10). We consider that all $\eta_i$ strings, which centers occur within the same $i$-th cell, are fused into one string cluster.

In the accordance with the string fusion model prescriptions [4] we suppose the following dependence of the average number of particles, $\langle \mu \rangle_{\eta_i}$, produced from the decay of this string
Figure 1. The strongly intensive observable \( \Sigma(n_F, n_B) \) for pp collisions as a function of the rapidity distance \( \Delta y \) between the centers of the FB observation windows, for two widths of windows: \( \delta y = 0.2 \) (a) and \( \delta y = 0.4 \) (b), and for two initial energies: 0.9 TeV (dashed lines) and 7 TeV (solid lines), calculated for particles with transverse momenta in the interval 0.3–1.5 GeV/c, as in the experimental analysis in [17].

…

The values of the parameters \( y^\text{corr}_\eta = 2.7 \) and \( \Lambda_0^\eta = 0.8 \) were chosen to obtain a correspondence with the values of the \( \Sigma(n_F, n_B) \) obtained in \([13, 15]\). Note that in these papers the \( \Sigma(n_F, n_B) \) was calculated on the base of the string pair correlation function, \( \Lambda(y_1 - y_2) \), extracted in [12] from the ALICE data [17] on the FB correlations in the approximation of identical strings.
The $\Sigma(n_F, n_B)$ calculated by formula (8) with the listed parameters is presented in the figure as function of the rapidity distance $\Delta y$ between the centers of the observation windows for two values of the window acceptance, $\delta y=0.2$ and 0.4, and for two initial energies, 0.9 and 7 TeV.

5. Conclusion
In present research we performed the MC simulations of string distributions in the impact parameter plane for $pp$ interactions at few initial energies. We took into account the string fusion processes leading to the formation of string clusters embedding the finite lattice (the grid) in transverse plane. Making the assumption on the properties of the string cluster correlation functions we calculate the strongly intensive observable $\Sigma(n_F, n_B)$ for the minbias $pp$ collisions at few initial energies and analyze its dependence on the distance between the centers of the observation windows and their width.

We show that the increase of the $\Sigma(n_F, n_B)$ with initial energy of $pp$ collision originates from the growth of the portion of the fused string clusters in arising string configurations. In the case with different emitting clusters this observable loses its strongly intensive properties. It becomes equal to the combination of the ones for different clusters with the weights depending on details of the collision - its energy and centrality. Nevertheless analyzing these dependencies of the $\Sigma(n_F, n_B)$ we can extract the important information on the characteristics of string clusters.

Similarly to the considered growth of the observable $\Sigma(n_F, n_B)$ with energy, we expect also the enhance of the string fusion effects in the central $pp$ collisions. The analysis of the arising dependence of this observable on the $pp$ collision centrality is now in progress.

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