The probability distribution of 3-D shapes of galaxy clusters from 2-D X-ray images

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ABSTRACT

We present a new method to determine the probability distribution of the 3-D shapes of galaxy clusters from the 2-D images using stereology. In contrast to the conventional approach of combining different data sets (such as X-rays, Sunyaev-Zeldovich effect and lensing) to fit a 3-D model of a galaxy cluster for each cluster, our method requires only a single data set, such as X-ray observations or Sunyaev-Zeldovich effect observations, consisting of sufficiently large number of clusters. Instead of reconstructing the 3-D shape of an individual object, we recover the probability distribution function (PDF) of the 3-D shapes of the observed galaxy clusters. The shape PDF is the relevant statistical quantity which can be compared with the theory and used to test the cosmological models. We apply this method to publicly available Chandra X-ray data of 89 well resolved galaxy clusters. Assuming ellipsoidal shapes, we find that our sample of galaxy clusters is a mixture of prolate and oblate shapes, with a preference for oblateness with the most probable ratio of principle axes 1.4 : 1.3 : 1. The ellipsoidal assumption is not essential to our approach and our method is directly applicable to non-ellipsoidal shapes. Our method is insensitive to the radial density and temperature profiles of the cluster. Our method is sensitive to the changes in shape of the X-ray emitting gas from inner to outer regions and we find evidence for variation in the 3-D shape of the X-ray emitting gas with distance from the centre.

Key words: keyword1 – keyword2 – keyword3

1 INTRODUCTION

It has long been known that non-linear gravitational collapse in the matter dominated Universe, starting with Gaussian random field initial conditions, happens non-spherically, giving rise to Zeldovich pancakes, filaments and galaxy clusters (Zeldovich 1970; Shandarin & Zeldovich 1989), and creating the cosmic web which has been detected in observations (Geller & Huchra 1989; Colless et al. 2001; Gott et al. 2005) as well as simulations (Klypin & Shandarin 1983; Davis et al. 1985; Springel et al. 2005). In particular, we expect that galaxy clusters, the largest collapsed objects in the Universe at the intersection of filaments, will also not be perfectly spherical (Frenk et al. 1988a). Observationally we know that the galaxy clusters are not spherical since their 2-D projections are not circular in optical (Carter & Metcalfe 1980; Binggeli 1982), X-ray (Fabricant et al. 1984; Buote & Canizares 1992, 1996; Kawahara 2010), Sunyaev-Zeldovich (SZ) effect (Sayers et al. 2011), weak gravitational lensing (Evans & Bridle 2009; Oguri et al. 2010, 2012) and strong gravitational lensing (Soucail et al. 1987) data. Further evidence for asphericity of galaxy clusters comes from kinematics of galaxies in the clusters (Skibboe et al. 2012).

Cold dark matter simulations show correlations between the orientations of dark matter halos and the surrounding cosmic web (van Haarlem & van de Weygaert 1993; Splinter et al. 1997; Kasun & Evrard 2005; Bailin & Steinmetz 2005; Altay et al. 2006; Patiri et al. 2006; Aragón-Calvo et al. 2007; Brunino et al. 2007). Taking asphericity into account is also important for accurate mass determinations of the galaxy clusters (Piffaretti et al. 2003; Clowe et al. 2004; Gavazzi 2005; Corless & King 2007; Battaglia et al. 2012) which in turn is important for using the galaxy clusters for precision cosmology (Mantz et al. 2015; Planck Collaboration et al. 2016; de Haan et al. 2016). The shape of the galaxy cluster will also be influenced by the nature of dark matter, for example the self interactions of the dark matter (Peter et al. 2013). The future X-ray (Merloni & German eROSITA Consortium 2012) and Sunyaev-Zeldovich effect surveys (K. N. Abazajian et al. 2016) will yield hundreds of thousands of galaxy clusters making precision cosmology with statistics

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of cluster shapes feasible. The shape of galaxy clusters is therefore emerging as an important observable which can be used to test the ΛCDM cosmology, baryonic physics in the intracluster medium (ICM) and fundamental physics such as the nature of dark matter.

One of the obstacles to using galaxy cluster shapes as a cosmological probe is the fact that we have only 2-D information about these objects. The X-ray and SZ effect (Zeldovich & Sunyaev 1969; Sunyaev & Zeldovich 1972) observations give us 2-D images in X-ray and microwave bands. The optical galaxy surveys also do not give us 3-D information since for most galaxies we do not have absolute distance measurements but only the redshifts. We can therefore only infer the average distance of the cluster in a cosmological model but not the distances to the individual galaxies. Gravitational lensing is also mostly sensitive to the 2-D projected mass distribution. Therefore, before we can use cluster shapes as a cosmological probe, we must solve the problem of inferring 3-D shape of the clusters from 2-D data.

Previously, inference of 3-D shapes of galaxy clusters, without assuming spherical or axial symmetry, has been tried by combining many different probes such as X-ray and SZ (Lee & Suto 2004) with lensing data (Limousin et al. 2013), using X-ray spectra (Samsing et al. 2012) and using weak and strong lensing data (Chiu et al. 2018). We propose a new method to infer the distribution of 3-D shapes of galaxy clusters. Our method adds a new tool to the existing toolkit for 3-D shape inference and can serve as an independent check of the results obtained by other methods. Our method is relatively computationally inexpensive and well suited to be applied to large data sets of hundreds of thousands of clusters that will become available with the future SZ and X-ray surveys. As we will see, our method does not need the galaxy clusters or the gas distribution to be ellipsoidal but can handle more general geometries. We will however make the ellipsoidal assumption in this paper for simplicity and also to compare our results with the published results in literature. Similar method has been used by Makarenko et al. (2015) to study the neutral hydrogen gas distribution in the turbulent interstellar medium of the Milky Way.

2 STEREOLOGY OF GALAXY CLUSTERS

The field of Stereology combines the ideas of Geometry and Statistics to obtain information about the 3-D shapes of objects from a small number of 2-D projections or sections (Baddley & Jensen 2004). This approach lends itself naturally to astrophysics where we usually have a single 2-D image or a projection of each of a large number of astrophysical objects belonging to a particular class or population and we want to infer the collective 3-D properties of the population. Following Makarenko et al. (2015), we use the probability distribution of filamentarity (F), a quantity constructed from Minkowski Functionals, to solve the depoprojection problem.

In two dimensions, the morphological properties of any 2-D contour can be completely characterized by three Minkowski Functionals. More generally, in n dimensions there are n+1 Minkowski functionals, a result known as Hadwiger’s theorem (Hadwiger 1957; Schmalzing et al. 1996). These three Minkowski Functionals are enclosed area S, perimeter P and Euler Characteristic of the contour. We will work with simple closed contours (with Euler characteristic = 0) so that the only Minkowski functionals with non-trivial information are the area and the perimeter. We can further combine the perimeter and area to form a quantity called Filamentarity F, which is defined as (e.g. see Bharadwaj et al. 2000):

\[
F = \frac{P^2 - 4\pi S}{P^2 + 4\pi S}.
\]

This definition ensures that 0 \leq F \leq 1 with F \to 0 corresponding to a circle and F \to 1 in the limit S \to 0 i.e. a line segment. When the circle is stretched to an ellipse of increasingly higher eccentricity, its filamentarity keeps increasing. The line segment with the limiting value of F = 1 is not necessarily straight. It should be noted that instead of filamentarity, we can also use ellipticity as a shape descriptor if we confine ourselves to ellipsoidal shapes. We will use filamentarity keeping in mind future applications where we may want to consider non-ellipsoidal shapes.

Let us assume that a galaxy cluster is an ideal ellipsoid with length L, width W and thickness T (L > W > T). To constrain the shape, we need to determine the ratio \( \frac{L}{W} : \frac{W}{T} \equiv \ell : w : 1 \), where we have defined \( \ell = L/T \) and \( w = W/T \). We will work with the ratios \( \ell \) and \( w \), since we are not interested in overall size of the cluster but only its shape. We will first consider a large sample (sample size ~ 150,000 clusters) of isotropically oriented ellipsoids (galaxy clusters) with same shape (\( \ell \) and \( w \)) or equivalently observe the same galaxy cluster from a large number of random observer positions. We use a simple theoretical model for X-ray emission in a galaxy cluster and use it to generate the theoretical X-ray surface brightness map for a given orientation of the cluster. We should emphasize that that our method is not sensitive to the detailed modelling of the cluster, in particular the density and temperature profiles as a function of distance from the centre, but only the 3-D shape of the cluster. We show this model independence explicitly below. We obtain the isocontours of constant X-ray intensity from this X-ray surface brightness map and use them to calculate the filamentarity. Finally, we repeat this for all the clusters of the sample to obtain the Probability Distribution Function (PDF) of filamentarity, \( P(F|\ell, w) \) for a given \( \ell \) and \( w \). Comparing the filamentarity PDF of X-ray images with the theoretical PDFs of different \( \ell \) and \( w \) then gives us information about the population of observed galaxy clusters. Note that by definition, \( L \geq W \geq T \) or \( \ell \geq w \geq 1 \). In particular, \( \ell \approx w \approx 1 \) corresponds to an oblate spheroid while \( \ell > w \approx 1 \) corresponds to a prolate spheroid and \( \ell \approx w \approx 1 \) is a spherical shape.

2.1 The X-ray emission model for galaxy clusters

In galaxy clusters, with typical temperatures \( T \approx 10^7 – 10^8 \) K, the primary emission process is thermal bremsstrahlung (free-free) emission. The total power emitted per unit volume (emissivity integrated over frequency) is given by (Ry-
\[ \epsilon = \sum_i \left( \frac{2\pi k_B}{3m} \right)^{\frac{1}{3}} \frac{\rho_i}{3m c^3} Z_i^2 n_e n_i T^\frac{3}{2} g(z) \propto \sum_i Z_i^2 n_e n_i T^\frac{3}{2} \quad (2) \]

where \( m \) is the mass of electron, \( n_e \) and \( n_i \) are the number densities of electrons and ions of species \( i \) respectively, \( Z_i \) the corresponding charge, \( T \) is the electron temperature and \( \bar{g} \) is the frequency averaged Gaunt factor (\( \bar{g} \sim 1.2 \)), \( k_B \) is the Boltzmann constant, \( \epsilon \) is the charge of the electron, \( b \) is the Planck constant, and \( c \) is the speed of light. Assuming charge neutrality and uniform abundance of elements throughout the cluster gives \( \sum_i Z_i^2 n_i = n_e \) and therefore we can write the X-ray surface brightness \( S_X \) as integral of the emissivity over the line of sight distance \( z \),

\[ S_X = \int \epsilon \, dz = \int n_e^2 T^\frac{3}{2} g(z) \, dz \quad (3) \]

We assume a generalized triaxial Navarro, Frenk & White (NFW) model (Navarro et al. 1996; Jing & Suto 2002) to represent the electron density \( n_e \propto \rho(R) \) of the cluster as

\[ \rho(R) = \frac{\rho_c}{\left(1 + \frac{R}{R_S}\right)^{\gamma + 1}} \quad (4) \]

\[ R^2 = L^2 \left( \frac{x^2}{L^2} + \frac{y^2}{L^2} + \frac{z^2}{L^2} \right) \quad (5) \]

where \( x, y, z \) are the coordinates with origin at the centre of the cluster and \( R = L \) defines the outer boundary of the cluster up to which we integrate the X-ray flux along the line of sight. Since most of the X-ray flux is contributed by high density regions, the integral converges quickly and we do not need to integrate out to a great distance from the cluster centre along the line of sight. Also, we are not interested in absolute magnitude of brightness but only the shapes of the isocontours in an X-ray image. We have explicitly checked that the shapes of isocontours, and hence our results, are not sensitive to how far out we integrate as long as we integrate to a distance greater than the scale radius \( R_S \). We fix the values \( \rho_c = 1 \), \( \gamma = 1 \), and \( R_S = 1 \) in arbitrary units. The profile of electron density (Vikhlinin et al. 2006) usually differs from the dark matter profile which the NFW model represents. Our observable, the filamentarity, which captures only the shape information is relatively insensitive to the exact density profile of gas. We check this explicitly below by using different electron and temperature density profiles from Vikhlinin et al. (2006) obtained from the X-ray observations of different galaxy clusters. This relative insensitivity to the exact density profile is an advantage in our method compared to other methods relying on the complete 3-D modelling of the cluster.

We take a universal temperature profile for galaxy clusters (Loken et al. 2002):

\[ T(R) \propto \left[ 1 + R/a_R \right]^{-\delta}, \quad a_R = 1, \quad \delta = 1.6 \quad (6) \]

To generate the X-ray surface brightness map, we start with a uniform grid in the XY plane. We rotate the grid so that the normal to the plane of this grid is aligned with the line of sight direction \( \hat{k} \). For every point on the rotated plane, we calculate the integral \( S_X \) for X-ray intensity using Eq. (3). We then rotate the plane back to the XY-plane. This gives the X-ray surface brightness map in the XY plane for an arbitrary orientation (given by line of sight direction \( \hat{k} \)) of the observer with respect to the cluster. To obtain the isocontours in a computationally efficient manner we do adaptive refinement of the grid. We start with an initial coarse grid and follow the above steps several times, each time successively increasing the resolution in the region close to the desired isocontour. After a small number of refinements, we have the isocontour sampled at high resolution. This is illustrated in Fig. 1. We fit the points on the isocontour with an ellipse using Downhill-simplex algorithm (Nelder & Mead 1965). The filamentarity of the resultant ellipse can be easily calculated using Eq. 1 with \( S = \pi a b \) and \( P \) given with better than a percent accuracy by (Lidstone 1932)

\[ P \approx \pi \left( a + b \right) \left( \frac{1}{1 - \lambda^2} \right)^{1/4} \lambda = \frac{a - b}{a + b} \quad (7) \]

where \( a \) and \( b \) are semi-major and semi-minor axes of the ellipse respectively.

### 2.2 Filamentarity PDF and its model independence

By observing a cluster from random directions, we can build up the probability distribution function of the filamentarity that a random observer would measure. The filamentarity PDF for a fixed \( \ell \equiv L/T \) and \( w \equiv W/T \), \( P(\ell, w) \), depends on the value of \( \ell, w \) of the cluster. Equivalently, if we observe single images of large number of clusters, all of which have the same \( \ell \) and \( w \), then this is the PDF we will get. The PDF is characterized by a sharp peak (Peak Filamentarity, \( F = F_\ell \)) and a sharp cut-off (Cut-off Filamentarity, \( F = F_w \)) which are functions of \( \ell \) and \( w \). If \( L \) and \( T \) are kept constant (i.e. \( \ell \) constant and varying \( w \)), \( F_\ell \) decreases non-linearly.
Comparison of Vikhlinin et al.’s model for A383 with our model

\[ P(F | \ell, w) \text{ for } \ell = 16 \text{ and } w \in \{1, 2, 3, 4, 6, 10\}. \]

The PDF has been obtained using \( \approx 150,000 \) isotropic random projections binned into 120 intervals of \( F \) between \([0, 1]\). The labels refer to the values of \( w \). It can be seen that peak filamentarity \( F_p \) decreases with increase in \( w \).

Comparison of Vikhlinin et al.’s model for A1795 with our model

\[ P(F | \ell, w) \text{ for } \ell = w \in \{16, 16/4, 16/7, 16/10\}. \]

The PDF has been obtained using \( \approx 150,000 \) isotropic random projections binned into 120 intervals of \( F \) between \([0, 1]\). The labels refer to the values of \( \ell = w \). It can be seen that cut-off filamentarity \( F_c \) increases with increase in \( \ell, w \).

Figure 2. Conditional filamentarity PDFs, \( P(F | \ell, w) \), for \( \ell = 16 \) and \( w \in \{1, 2, 3, 4, 6, 10\} \). The PDF has been obtained using \( \approx 150,000 \) isotropic random projections binned into 120 intervals of \( F \) between \([0, 1]\). The labels refer to the values of \( w \). It can be seen that peak filamentarity \( F_p \) decreases with increase in \( w \).

Figure 3. Conditional filamentarity PDFs, \( P(F | \ell, w) \), for \( \ell = w \in \{16, 16/4, 16/7, 16/10\} \). The PDF has been obtained using \( \approx 150,000 \) isotropic random projections binned into 120 intervals of \( F \) between \([0, 1]\). The labels refer to the values of \( \ell = w \). It can be seen that cut-off filamentarity \( F_c \) increases with increase in \( \ell, w \).

with an increase in \( W \). This is illustrated in Fig. 2. On the other hand, if \( L \) and \( W \) are kept constant (i.e. \( \ell \) and \( w \) changing by the same factor), \( F_c \) decreases with an increase in \( T \) as seen in Fig. 3. In reality, we expect different clusters to have different \( \ell \) and \( w \) and the observed filamentarity PDF, \( P(F) \), would be a superposition of conditional PDFs for different \( \ell \) and \( w \),

\[
P(F) = \int d\ell dw P(F, \ell, w)
= \int d\ell dw P(F | \ell, w)P(\ell, w), \tag{8}
\]

where \( P(\ell, w) \) is the PDF of shapes of clusters and \( P(F, \ell, w) \) is the joint PDF. Our goal is to recover the PDF of shapes, \( P(\ell, w) \).

We have used a simple model for the profiles of electron density and temperature. Since we are interested in only the shapes of the isocontours of X-ray surface brightness and not their overall amplitudes, our results are not sensitive to the exact profile. To test this hypothesis, we repeat the calculation using two different density and temperature profiles from Vikhlinin et al. (2006) and compare it with our NFW + universal temperature profile model in Fig. 4. As we can see, there is negligible change (less than the Monte Carlo noise) in the PDF of filamentarity confirming our assertion that the filamentarity PDF is sensitive only to the shape of the cluster.

\[ \text{Comparison of our NFW + universal temperature profile model with temperature and density profile models from Vikhlinin et al. (2006), for two clusters with very different parameter values: A383 and A1795. We have made the comparison for three different shapes: one prolate \( (\ell = w = 16) \), one oblate \( (\ell = 16, w = 1) \) and one intermediate \( (\ell = 6, w = 3) \) shape. We find a general agreement of our model with that of Vikhlinin et al. (2006), differences being less than the Monte Carlo noise.} \]

3 FILAMENTARITY PDF OF CHANDRA X-RAY CLUSTERS

The next step is to obtain the PDF of filamentarity of isocontours of X-ray surface brightness maps from observational data. We use the X-ray data of 89 galaxy clusters from the catalog compiled by Eric Tittley\(^1\) from the Chandra Data Archive (Chaser)\(^2\) for this purpose. We have selected only those galaxy clusters which have signal-to-noise ratio greater than or equal to 3. We use the already processed full image data for our analysis. We do

\[^1\] https://www.roe.ac.uk/~ert/ChandraClusters/
\[^2\] https://cda.harvard.edu/chaser/
not reprocess the data because reprocessing only changes the calibration, which will not affect the shape of the isocontours. The list of galaxy clusters used in this paper is given in Appendix A.

For a given cluster, we subtract the background and carry out convolution of the resultant data using a Gaussian function with the standard deviation \( \sigma = 3 \) pixels to smooth the image and suppress noise. After convolution, we obtain the isocontours corresponding to a given X-ray count and fit it with an ellipse using the Downhill-simplex algorithm. We then do a binary search by looking at isocontours corresponding to larger or smaller X-ray counts until we find the isocontour which encloses the desired fraction of X-ray flux. This algorithm works because the enclosed X-ray flux increases monotonically and X-ray counts decrease monotonically as we move away from the centre of the cluster. In this work, we choose isocontours which enclose 25%, 40%, 60%, 80% and 90% of the total flux respectively. These isocontours laid on top of the X-ray flux maps of Chandra clusters A1835 and A2204 are shown in Fig. 5. We calculate the filamentarity of the best-fit ellipse using Eq. 1 and Eq. 7. We repeat these steps for 89 clusters to get 89 values of filamentarity, which are then binned to obtain the PDFs which are shown in Fig. 6. We see from Fig. 6 that the ellipticity of the clusters is small with the maximum filamentarity smaller than \( \sim 0.15 \). We have binned \( F \) with a bin-width of 1/120 and we also show the Poisson error bars. There is a difference in the tails of the PDFs, indicating that there is a variation in shape as we go from inner part of the cluster to the outer parts.

### 3.1 Results for shapes of Chandra X-ray clusters

Our goal is to recover the shape PDF, \( P(\ell, w) \). To this end, we can bin the shape PDF in bins of \( \ell, w \) and thus reconstruct a discretized form of \( P(\ell, w) \). This is equivalent to approximating \( P(\ell, w) \) by a superposition of Dirac delta distributions,

\[
P(\ell, w) \approx \sum_{i=1}^{n} a_i \delta_D (\ell - \ell_i, w - w_i)
\]  

with the normalization condition,

\[
\sum_{i=1}^{n} a_i = 1.
\]  

(10)

Substituting it in Eq. 8, we get after integrating out the Dirac delta distributions,

\[
P(F) \approx \sum_{i=1}^{n} a_i P(F|\ell_i, w_i)
\]  

(11)

Our problem of finding the shape PDF now reduces to finding the number of bins \( n \), the bin centres \( \ell_i, w_i \), and the relative probability amplitudes of different shapes \( a_i \) which best fit the data.

In order to proceed, we generate 100 random values of \( (\ell, w) \) taken from a uniform distribution. By definition, \( \ell \geq w \geq 1 \). Hence, we take the value of \( \ell \) from a uniform random distribution [1, 2.6] and the value of \( w \) from a uniform random distribution [1, \( \ell \)]. The upper limit for \( \ell \), 2.6, corresponds to \( F_i \approx 0.15 \) and therefore covers the observed range of \( F \). We generate the conditional PDFs, \( P(F|\ell, w) \), for each of the 100 randomly sampled \( (\ell, w) \). This is suffi-
where $F_j$ is the $j$th bin and $\sigma_j$ is the Poisson error in the corresponding bin given by $\sigma_j = \sqrt{\hat{n}_{\text{obs}}(j)/n_{\text{total}}} \times \hat{n}_{\text{min}}$, where $n_{\text{obs}}$ is the observed count in that bin, $n_{\text{total}}$ is the total number of clusters in the data set and $n_{\text{min}}$ is the total number of bins of filamentarity.

We first consider the simplest model that all clusters share the same shape, i.e. $n = 1$. The Observational PDF has $F_0 \approx 0$ and $F_2 \approx 0.06 - 0.13$, depending on the per cent flux enclosed. We vary $\ell_i, \ell_1$ to fit the $P(F|\ell_1, \ell_1)$ that best fits the observed $P(F)$. However, the lowest $\chi^2/d.o.f$ for this model comes out to be $\approx 1.5 - 3.5$, which is not satisfactory. Thus, we see immediately that the shapes of the galaxy clusters must vary from cluster to cluster.

We next perform a $\chi^2$-fitting of $P(F)$ to the observed PDF by fitting the $n - 1$ variables $a_i$ for each combination of $\ell_i, \ell_1$ for different $n$, starting with $n = 2$ and choose the combination $\ell_i, \ell_1$ that gives the least $\chi^2$. Note that one of the $a_i$ is fixed by the normalization condition, $\sum a_i = 1$. We are therefore doing a model selection, with different $\ell_i, \ell_1, n$ corresponding to a different model for $P(F)$, while $a_i$ are the parameters of the model which are being fit for each model. Our model selection consists of a brute force search for the best-fitting $\ell_i, \ell_1$ by repeating the fit for every possible set of $\ell_i, \ell_1$. Thus, we have a total of $10^6 \times 6$ combinations to fit. We find the $\chi^2$ for each of these sets and take the combination with the minimum value $\chi^2_{\text{min}}$ as the best-fitting model. We repeat the whole procedure for $n = 2, 3, 4$ and find $\chi^2_{\text{min}}$ in each case. The results are tabulated in Table 1.

To summarize, our method is accomplishing three things simultaneously:

(i) We find the optimal number of bins, $n$, demanded by the data into which to divide the filamentarity PDF, $P(F)$.
(ii) For each $n$, we find the best-fitting model corresponding to different values of $\{\ell_i, \ell_1\}$.
(iii) For each model we find the best-fitting parameters $a_i$ by solving the linear minimization problem.

We observe that $\chi^2/d.o.f$ decreases progressively from $n = 1$ to $n = 4$, for each enclosed per cent flux. However, $\chi^2/d.o.f$ becomes $\lesssim 1$ in every case for $n = 2$. This means that $n = 2$ is the optimal number of parameters. Thus, a superposition of 2 cluster shapes is adequate to describe the data.

The best-fitting parameters and model for $n = 2$ is shown in Table 2 and Fig. 7.

We can define a triaxiality parameter $T$ to classify the shapes of the clusters:

$$T = \frac{L^2 - W^2}{L^2 - T^2} = \frac{\ell^2 - w^2}{\ell^2 - 1}$$

(13)

For a purely oblate shape, $L = W$ or $T = 0$. For a purely prolate shape, $W = T$ or $T = 1$. We classify the shapes as nearly oblate ($0 < T \lesssim 0.33$), triaxial ($0.33 \lesssim T \lesssim 0.67$) or nearly prolate ($0.67 \lesssim T < 1$) (Warren et al. 1992). The results of this classification are shown in Table 3. We see that the higher-weighted shape is prolate for the innermost part while it has preference towards oblateness for the outer parts. Lower-weighted shape is prolate in the inner parts and triaxial in the outer parts.

### 3.2 Comparison with previous results

We compare our results with the results obtained by Limousin et al. (2013) and Chiu et al. (2018). Limousin et al. (2013) used triaxial model fitting to obtain the 3-D shape of 4 strong lensing clusters by combining X-ray, SZ and lensing data. Chiu et al. (2018) obtained the shapes of galaxy clusters using strong and weak lensing data. For

| Enclosed flux | $\ell_1$ | $w_1$ | $a_1$ | $\ell_2$ | $w_2$ | $a_2$ |
|--------------|--------|------|------|--------|------|------|
| 25%          | 1.24   | 1.07 | 0.55 | 1.81   | 1.34 | 0.45 |
| 40%          | 1.33   | 1.24 | 0.74 | 1.80   | 1.25 | 0.26 |
| 60%          | 1.41   | 1.30 | 0.75 | 1.99   | 1.62 | 0.25 |
| 80%          | 1.41   | 1.30 | 0.84 | 2.04   | 1.52 | 0.16 |
| 90%          | 1.41   | 1.30 | 0.81 | 1.97   | 1.51 | 0.19 |

| Flux | $T_1$ | $T_2$ |
|------|------|------|
| 25%  | (1.24,1.07) | (1.81,1.34) |
| 40%  | (1.33,1.24) | (1.80,1.25) |
| 60%  | (1.41,1.30) | (1.99,1.62) |
| 80%  | (1.41,1.30) | (2.04,1.52) |
| 90%  | (1.41,1.30) | (1.97,1.51) |

Table 3. Same as Table 2, but showing the shape classification of the clusters as prolate, triaxial or oblate.
most of the galaxy clusters, Chiu et al. (2018) are only able to constrain one of the axes ratios and provide only lower limits on the second axes ratio when they do not use any shape priors from simulations. They have given their results in the form of $T/L$ and $W/L$, which we convert to $\ell$ and $w$ for comparison, and tabulate the results in Table 4. We also show the axes ratios obtained in Limousin et al. (2013) and Chiu et al. (2018) along with our best-fitting PDFs in the $\ell-w$ plane in Fig. 8 and 9. We should emphasize that the points corresponding to Limousin et al. (2013) and Chiu et al. (2018) results are the $\ell, w$ values for individual clusters whereas the two points corresponding to our work are the two points on the shape PDF, $P(\ell,w)$, of the 89 Chandra clusters with the numbers referring to the relative probability amplitude of these points, $a_i$. Also our points refer to the shapes at different distances from the centre for which we use the fraction of X-ray flux enclosed by an isocontour on the X-ray surface brightness maps of the clusters. The earlier works obtain a single average shape for the cluster. Our results show that the shape of the halo is different as we move from the inner regions to the outskirts of the cluster.

In the innermost part of the cluster (25% enclosed flux), we find that the cluster shapes are predominantly prolate. There is preference towards oblateness as we move to the outskirts. The points from Limousin et al. (2013) are clustered together and are in rough agreement with our results. The points from Chiu et al. (2018) are more spread out and lean towards prolate shapes. However, we note that the Chiu et al. (2018) are measuring the shapes of dark matter halos while we are measuring the shape of the baryons. Taken together, these results may point towards a difference in shape of baryons and dark matter, however we need more data to make any definite conclusion.

4 CONCLUSIONS

We have presented a powerful new method to infer the PDF of shapes, $P(\ell,w)$, using only 2-D images of clusters of galaxies. To illustrate our method we have used X-ray images from publicly available Chandra data. Our method can also be applied to optical as well as SZ data. The main requirement for our method is a large sample of images, since we directly infer the PDF of shapes of the whole population rather than the shapes of individual clusters. We have also shown that our method is relatively insensitive to the density and temperature profiles of the clusters and thus does not require detailed 3-D modelling.

We find general consistency with the existing results of Limousin et al. (2013), who use data sensitive to baryons, such as X-rays and SZ effect, given the small statistical samples but differs significantly from the analysis of Chiu et al. (2018) who use only lensing data and are therefore sensitive to dark matter distribution. Our main results are that the shape of X-ray emitting gas is prolate for the innermost parts of the cluster, but shows a preference towards oblateness in the outer parts. The shapes of the haloes in dark matter simulations show preference towards prolateness in the inner parts(Frenk et al. 1988b; Dubinski & Carlberg 1991; Warren et al. 1992; Bailin & Steinmetz 2005; Bett et al. 2007), which is consistent with our results in the innermost parts. In these simulations, the haloes are found to be more oblate in outer parts, though prolateness is still predominant, except for Dubinski & Carlberg (1991) who find no preference towards prolate or oblate shape in the outskirts. Thus, our results in the outer parts are in contrast with the halo shapes found in dark matter simulations. However, it should again be noted that these simulation results are for the shapes of dark matter haloes, while we are measuring the shape of baryons.

| Cluster     | $\ell$ | $w$  | $T$ |
|-------------|-------|-----|-----|
| Abell 1835  | 1.69  | 1.20| 0.76| prolate |
| Abell 383   | 1.82  | 1.29| 0.74| prolate |
| Abell 1689  | 1.79  | 1.34| 0.64| triaxial|
| MACS 1423   | 1.61  | 1.16| 0.78| prolate |

Table 4. List of galaxy clusters for which shapes have been obtained by fitting cluster models to multiple data-sets (Reference L2013: Limousin et al. (2013), Reference C2018: Chiu et al. (2018))
Comparison of our work with previous results
(at 25% enclosed flux)

Comparison of our work with previous results
(at 40% enclosed flux)

Comparison of our work with previous results
(at 60% enclosed flux)

Comparison of our work with previous results
(at 80% enclosed flux)

Comparison of our work with previous results
(at 90% enclosed flux)

Figure 8. Comparison of the results obtained by us with the results of Limousin et al. (2013) and Chiu et al. (2018) in the ($\ell, w$) plane. The blue square points obtained in this work are the approximation for the shape PDF, $P(\ell, w)$, of 87 – 89 Chandra clusters with the numbers next to them the relative probability amplitude of the corresponding shape. The triangles and diamonds on the other hand are the axes ratios of individual clusters from previous works. The shaded light-green region is excluded by definition of $\ell, w$.

Figure 9. Same as 8, but for 80% and 90% enclosed flux. Analysis for these two cases has been performed with 78 – 81 clusters.

We expect that the future X-ray and SZ surveys to image hundreds of thousands of galaxy clusters Merloni & German eROSITA Consortium (2012); K. N. Abazajian et al. (2016). With large statistical samples, detailed statistical comparisons of observations with simulations, including evolution of shape with redshifts and dependence on mass, should become possible with our method.

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| S.No. | Object name       | Observation ID |
|-------|-------------------|----------------|
| 1     | A3532             | 10745          |
| 2     | 3C348             | 1625           |
| 3     | A3921             | 4973           |
| 4     | 4C55.16           | 1645           |
| 5     | cl1212+2733       | 5767           |
| 6     | cl0809+2811       | 5774           |
| 7     | cl0318-0302       | 5775           |
| 8     | cl0302-0423       | 5782           |
| 9     | 2PIGGz0.061 J0011.5-2850 | 5797   |
| 10    | 2PIGGz0.058 J2227.0-3041 | 5798   |
| 11    | A3102             | 6951           |
| 12    | cl1349+4918       | 9396           |
| 13    | AWM4              | 9423           |
| 14    | A0013             | 4945           |
| 15    | A0068             | 3250           |
| 16    | A0193             | 6931           |
| 17    | A0209             | 3579           |
| 18    | A0795             | 11734          |
| 19    | A0586             | 19962          |
| 20    | A0697             | 4217           |
| 21    | A0744             | 6947           |
| 22    | A0773             | 5006           |
| 23    | A0801             | 11767          |
| 24    | A0907             | 3185           |
| 25    | A0963             | 903            |
| 26    | A1068             | 1652           |
| 27    | A1201             | 4216           |
| 28    | A1204             | 2205           |
| 29    | A1361             | 2200           |
| 30    | A1413             | 1661           |
| 31    | A1423             | 11724          |
| 32    | A1446             | 4975           |
| 33    | G125.70+53.85     | 15127          |
| 34    | A1650             | 6356           |
| 35    | A1664             | 7901           |
| S.No. | Object name       | Observation ID |
|-------|-------------------|----------------|
| 71    | A2717             | 6974           |
| 72    | A3112             | 2216           |
| 73    | A3528s            | 8268           |
| 74    | A3558             | 1646           |
| 75    | A4059             | 5785           |
| 76    | ACT J0616-5227    | 13127          |
| 77    | AS1063            | 18611          |
| 78    | AWM7              | 908            |
| 79    | cl0056+4107       | 5759           |
| 80    | cl1120+4318       | 5771           |
| 81    | Cygnus A          | 360            |
| 82    | ESO3060170-B      | 3188           |
| 83    | MACSJ2311.5+0338  | 3288           |
| 84    | PKS1404-257       | 1650           |
| 85    | SERSIC 159-03     | 1668           |
| 86    | A383              | 2321           |
| 87    | A1689             | 7289           |
| 88    | MACS-J0329.6-0211 | 3582           |
| 89    | RXJ1347.5-1145    | 3592           |