Statistical Estimation of Perspective Damage Index for Vibration-Based Structural Health Monitoring

V Pavelko and A Nevsky

Institute of Aeronautics, Riga Technical University, 1A Lomonosovaiela, Riga LV1003, Latvia

Corresponding author: Vitalijs.Pavelko@rtu.lv

Abstract. Statistical analysis of CCD (correlation coefficient deviation) index for the structural damage detection by vibration-based method was performed. Statistic data set was collected in a full-scale test of a large aircraft component. After special pre-processing the most informative narrow frequency band was selected for obtaining the statistic set of CCD between the frequency response functions of intact and pseudo-damaged states of a structure. The two-sample Kolmogorov-Smirnov hypothesis test was used for estimation of a pseudo-damage effect. The stable response of CCD index to a small damage in large-size structure was demonstrated.

1. Introduction

Vibration analysis is one of the oldest methods of non-destructive testing (NDT) of various kinds of mechanical systems (structures). The on-line inspection of wheel pairs of railway cars, checking for no damage to crystal glassware, tuning of stringed musical instruments for a long time has been carried out by a simple and effective hammer test. But interest in vibration-based NDT continues to be high and even increases. First of all, it is due to intensification of research and development in the field of structural health monitoring (SHM) for which the vibration-based NDT is highly adaptable.

The fundamental principle of all vibration-based NDT is: any type structural damage causes some changes of dynamic properties (eigenfrequency spectra, shape of eigenmodes and damping parameters). These changes can be found from the measured dynamic response of a structure and used for damage identification. There are large number of research and developments in mechanical, civil, and aerospace engineering dedicated to vibration-based damage detection and its application for structural health monitoring (SHM). Some corresponding review-information can find in [1-9].

Because the vibration-based NDT is indirect, it does not completely identify the structural damage of a structure: the presence of a damage (detection), its coordinates (location), and the size of the damage or damaged area (quantification). Detection using the shift of eigenfrequencies induced by damage initiation is a simple approach of vibration-based NDT. However, often there are difficulties in practical implementation due to the relatively low sensitivity of the eigenfrequencies to change of the structural stiffness. For example, the allowable fatigue crack in some basic structural element of aircraft cannot be reliably detected by use of a shift of the first (lower) eigenfrequency [10]. In contrast, the mode shape topology and geometry potentially is able to provide also spatial information of structural changes due to damage. It is known that the curvature of mode shape can be more effectively used to identify damage [11, 12]. There are many researches that used the higher modes
shape evolution for damage identification in a beam-type structure. Brief review of earlier works on this field can find in [13]. Frequency response function used in [14] for a structural damage detection and solution some inverse optimization problem for damage localization and quantification. Problem of the crack identification in beam-type structure using the vibration analysis discussed in [15,16] including cases of multiple cracks based [17,18]. In [19] there is given the wild review of researches on the frequency response analysis of beams and plain frames with the viscous-elastic damping in external and internal constraints. The theory of the generalized function is used for deriving of the frequency response function of a beam.

For 2D and 3D structures the problem of damage identification is more difficult that mainly caused by the complexity of adequate geometrical description of the damaged region. An approach of damage identification such type of structure is based to the finite element (FE) simulation of dynamic response and model iterative updating to achieve acceptable correspondence to test data [20-23]. The level set method adapted to structural damage identification in [24] is more effective because it does not require a prior knowledge of the topology of damage distribution. Here the vibration-based inverse problem of structural damage identification is transferred into a generalized shape optimization problem introduced by an evolution type Hamilton–Jacobi equation.

Statistical time series methods realize another popular approach of damage identification using vibration-based SHM. Detailed description of method and review of the essential achievements is given in [25]. Method is based to theory of statistical decisions for selection of the most probable version of the damaged area of possible ones from the predetermined set. Other kind of statistical methods is presented in [26].

Most of the above-mentioned research works introduce elegant mathematical solutions illustrated the fundamental feasibility of damage compete identification (damage presence, position and size) using the vibration-based analysis that is the prerequisite of global SHM system implementation on those bases. However, it can easy estimate that such SHM system is unlikely to be cost-effective and competitive for the large-scale structure. In this respect an aerospace structure is a typical highly loaded one in which a structural damage is allowable, but it should not lead to a significant reduce of strength that means the admissible size of damage must be small, and the monitoring system must be able at least to reliably detect it. Therefore, the local vibration-based SHM system with a restricted aim of damage detection only can be very claimed for many similar applications.

It is known that a small-size damage produces a measurable effect to higher modes of a large-scale structure. But usually the group type of higher modes of structure due to structural or modal quasi-periodicity is observed [27]. It is caused high eigenfrequency concentration in separate intervals of the frequency response function. As a result, the direct estimation of damage effect to the separate eigenfrequency shift and the change of mode shape is obstructing and cannot provide the reliable detecting of damage. More perspective is an alternative approach that is used some integral estimate of a damage effect to the dynamic response of structure. In our paper [27] the correlation coefficient deviation (CCD) used for this purpose.

\[
CCD = 1 - C
\]

where

\[
C = \frac{\text{cov}(x,y)}{s_x s_y}
\]

\text{cov}(x,y) is the covariance between two sample random vectors \(x\) and \(y\) those are the frequency response of intact and damaged structures respectively in selected frequency band, and \(s_x, s_y\) are the standard deviations of random vector. It is seen that the \(CCD\) index is equal to zero, if there is not any damage effect, and cannot be more than 1. The larger value of the \(CCD\) index corresponds to higher effect of damage. The efficiency of \(CCD\) index for the vibration-based detection of damage was demonstrated in [27].

Vibration-based detection of a small damage in a large-scale structural component as the fundamental of the local SHM system remains the general objective of presented paper. One of the
most important components of the such system is the CCD index for assessment of structural health. The presented paper is a further development of previous research [27] and is focused to the statistical analysis of the CCD index and to the reliability of a damage detection using this index.

2. Experimental study and statistical analysis

2.1. About Test Setup and Measurement Equipment
Detailed description of the test setup can find in [27]. The helicopter Mi-8 tail beam structure was selected for experimental investigation. It is a 5485mm length full-scale component of the conical form (about 1 m larger base diameter) and the thin-walled Al-alloy structure. The strain gauge technique was used for the dynamic measurement. Two strain gauge rosettes were pasted in two zones of the outer surface of a skin (figure 1). Dynamic strain was measured by the multichannel oscilloscope NI PXIe-4330, 16Ch, 24-Bit, 25 kS/s Bridge Input Module and PC with NI LabVIEW software. The sampling rate 5 kS/s and the length of a record in 5s were accepted as optimal for the data acquisition enough for the aim of experimental study.

The forced vibration of the beam was excited by the mechanical eccentric shaker with control of excitation frequency. In the below presented analysis there are used the dynamic responses of a beam at the cyclic excitation with basic frequency 3.55 Hz (close to the first natural frequency of a beam) and low amplitude vibration in the frequency band of interest (white noise).

![Figure 1](image)

**Figure 1.** A rosette of three strain gauges (a) and a strain gauge rosette 1 with the small pseudo damage (SPD) in zone 2

The technology of pseudo damage was used for damage effect simulation. The small pseudo damage (SPD) was completed as row of eight 6×6×6mm steel blocks (total mass 12 g) and placed in the zone 2 (figure 1, b). Two steel blocks (total mass 2 g) were pasted in the zone 2 and qualified as the large pseudo damage (LPD).

2.2. Some Important Results of Research [27]
The dynamic response in frequency domain was obtained for each stress components (two examples for $\sigma_x$ and $\sigma_y$ in the figure 2) for intact (solid) and structure with a pseudo-damage (dash). It can see that in the frequency band 275-400 Hz there are observed multiple resonances.

The stress component $\sigma_x$ spectrum is especially interesting. In the relatively low frequency band, the frequency response is very weak. And in the above-mentioned frequency band there is a sharp increase in the spectrum. Moreover, for all components of the stress tensor, the response to the appearance of pseudo-damage is clearly observed.
2.3. Data Acquisition and Statistical Analysis

To obtain the test data for the statistical analysis there was carried out on 10 series of measuring of dynamic strains at the checkpoints of the intact structure and in the presence of pseudo-damage. Time record length was chosen to be 1 second at sampling frequency of 5000 points. Pre-processing of each record includes signal centring and filtering in the frequency band 250–400 Hz. The band pass filter was designed by IIR method (Butterworth). Fast Fourier transform (FFT) and light smoothing (with span 7) of frequency response function was done at the final step of pre-processing. As a result, ten samples of the dynamic frequency response were obtained for each of six strain gauges. Example of this type outcome for the longitudinal strain gauge of zone 1 is presented in figure 3.

Figure 2. The dynamic response of stress component \( \sigma_x \) (a) and \( \sigma_y \) (b) in the frequency band 0-400Hz in the zone 1

Figure 3. Frequency response of the strain gauge sg1 after filtering and FFT (intact structure)

The data set for final statistical analysis contents two set for each of the strain gauges. The set size is ten observations, each of which represents the frequency response in the band 250-400 Hz and has a size of 500 points.

The matrix \( A_k(500,10) \) contents ten observations of the intact structure frequency response measured by the strain gauge \( k \), and the matrix \( B_k(500,10) \) is the same for pseudo-damaged structure.
The final statistical analysis consists three steps.

Step 1: Analysis of the inter-sensor correlation in a separate loading session. In such case, the dynamic response of all strain gauges is caused by the same external load. This means that in a linear system, the strain components must be strictly proportional to each other, and the correlation coefficient between them must be 1. Deviation from 1 may be caused by the influence of the measurement accuracy and/or nonlinearities of a structure.

The correlation coefficient $C_{km}^{(n)} = \text{corcoef}(A_k(:,n), A_m(:,n))$ between random variable $A_k(:,n)$ and $A_m(:,n)$ of observation (test option) $n$ was calculated for intact structure, and similarly, for pseudo-damaged structure $C_{km}^{(n)} = \text{corcoef}(B_k(:,n), B_m(:,n))$. For strain gauges of zone 1 $k = 1$ and $m = 2, 3$, and for zone 2 $k = 4$ and $m = 5, 6, n = 1, 2, ..., 10$.

Outcome of this analysis is presented in the Table 1.

| State of structure | Parameter value of parameter | $C_{12}$ | $C_{13}$ | $C_{45}$ | $C_{46}$ |
|--------------------|-----------------------------|---------|---------|---------|---------|
| Intact             | Mean                        | 0.9999  | 0.9999  | 0.9999  | 0.9999  |
|                    | m                           | 0.9998  | 0.9998  | 0.9999  | 0.9999  |
| Pseudo-damaged     | Mean                        | 0.9999  | 0.9999  | 0.9999  | 0.9999  |
|                    | m                           | 0.9998  | 0.9999  | 0.9997  | 0.9997  |

The mean value of correlation coefficient for all data set is equal to 0.99997, and the minimum is 0.99987. It means that the inter-sensor correlation in a separate loading session is very close, the accuracy of the measurements is relatively high, and the effect of nonlinearities is insignificant.

Step 2: Estimation of load scattering effect to the dynamic response of a separate strain gauges and the distribution law of CCD. The mean vectors of all observations for each strain gauge was obtained for both intact and pseudo-damaged state of a structure.

$$\bar{A}_k = \sum_{n=1}^{10} A_k(:,n) \quad \bar{B}_k = \sum_{n=1}^{10} B_k(:,n)$$

The correlation coefficients

$$C_{k0}^{(n)} = \text{corcoef}(A_k(:,n), \bar{A}_k)$$

and the correlation coefficient deviation (CCD)

$$CCD_{k0}^{(n)} = 1 - C_{k0}^{(n)}$$

can be introduced as the signatures of deviation of any observation from averaged value due to the specific load at given state of a structure. The mean values of the CCD for all six strain gauges are presented in the Table 2.

| State of structure | sg1   | sg2   | sg3   | sg4   | sg5   | sg6   |
|--------------------|-------|-------|-------|-------|-------|-------|
| Intact             | 0.04147 | 0.03588 | 0.06115 | 0.10023 | 0.04093 | 0.05373 |
| Pseudo-damaged     | 0.21180 | 0.10863 | 0.06877 | 0.13966 | 0.04489 | 0.05579 |
Thus, estimates of the effect of load variation to CCD index were obtained for two states of system.
At the same time, the installation of pseudo-damages led to an increase of CCD for two sensors in the
LPD zone, which indicates the effect of a configuration change on the dispersion of CCD due to the
variation in load.

The null hypothesis that the data in vector-column in $CCD_0$ and $CCD_1$ matrices is from a
population with a normal distribution in seven tests from twelve were rejected by the Anderson-
Darling test. Thus, there is not reliable evidence about normal distribution of considered random
variables.

Step 3: Features extraction. The damage index CCD corresponding to the strain gauge $k$ frequency
response in the observation $n$ is defined by equation (5)

$$CCD_k^{(n)} = 1 - C_k^{(n)}(5)$$

The correlation coefficient $C_k^{(n)}$ between the dynamic response of the pseudo-damaged structure in
the frequency band of interest $B_k(\cdot, n)$ measured by the strain gauge $k$ in the observation (test option)
n, and the average response $\tilde{A}_k$ of this strain gauge in the intact structure.

First of all, a test decision for the null hypothesis that the random vectors $B_k(\cdot, n)$ and $\tilde{A}_k$ are from
the same continuous distribution was done using the two-sample Kolmogorov-Smirnov test. Data set
of this test decision for pseudo-damaged structure defined by equation (5) are represented in the Table
3.

For five tests ($sg1, sg2, sg3$ – zone 1 of LPD and $sg5, sg6$ – zone 2 of SPD) the null hypothesis
was rejected at the 5% significance level. Only for one test ($sg4$ - zone 2 of SPD) null hypothesis was
not rejected. This means that the CCB index uniquely detects the appearance of a LPD. Effect of
the SPD to CCD is significantly weaker that bear witness to the effectiveness of the proposed index.

| Number of test | sg1  | sg2  | sg3  | sg4  | sg5  | sg6  |
|---------------|------|------|------|------|------|------|
| 1             | 0.7010 | 0.3969 | 0.3682 | 0.1957 | 0.0928 | 0.1214 |
| 2             | 0.4260 | 0.2185 | 0.4995 | 0.1307 | 0.0549 | 0.2303 |
| 3             | 0.2003 | 0.2029 | 0.4052 | 0.0886 | 0.0580 | 0.1033 |
| 4             | 0.3606 | 0.1160 | 0.4448 | 0.1663 | 0.0475 | 0.0664 |
| 5             | 0.5063 | 0.2449 | 0.3808 | 0.2812 | 0.1146 | 0.2016 |
| 6             | 0.3484 | 0.3547 | 0.3445 | 0.2120 | 0.0795 | 0.1439 |
| 7             | 0.3124 | 0.1567 | 0.4091 | 0.0979 | 0.0592 | 0.1317 |
| 8             | 0.6085 | 0.3026 | 0.3686 | 0.3237 | 0.1437 | 0.1437 |
| 9             | 0.6496 | 0.8709 | 0.4902 | 0.1227 | 0.0410 | 0.1022 |
| 10            | 0.3208 | 0.1594 | 0.3042 | 0.1674 | 0.1243 | 0.2415 |
| Mean          | 0.4506 | 0.3023 | 0.4013 | 0.1786 | 0.0815 | 0.1486 |

Comparison of the mean value of CCD indices of intact and pseudo-damaged structures is
represented in Fig. 4. This index of intact structure is associated mainly with the variation of the
external load in different test options, but the index of pseudo-damaged structure mainly caused by a
pseudo-damage effect. It is seen that for all three strain gauges ($sg1, sg2,$ and $sg3$) located in the zone
1 the significant increment of CCD index is observed that due to presence of the large pseudo-damage.
The small pseudo-damage in zone 2 also induces increasing of CCD index of all three strain gauges
($sg4, sg5,$ and $sg6$), but effect of SPD is much less.
3. Conclusions

This research is focused to the problem of reliability of vibration-based detection of a small damage in a large-scale structural component by using the correlation coefficient deviation (CCD) index. The CCD index is an integral estimate of evolution of the frequency response function caused by structural component degradation.

Statistical analysis of CCD index for the structural damage detection by vibration-based method was performed. Statistical data set was collected in a full-scale test of a large aircraft component. In each test the same (nominally) load was reproduced and contained the cyclic excitation with basic frequency 3.55 Hz (close to the first natural frequency of a structure) and low amplitude vibration in the frequency band of interest (close to the white noise). Special pre-processing was contained a procedure of measured sample centring and Fourier transformation in wide frequency band for extracting of the most informative narrow frequency band in which there is a dense set of higher eigenfrequencies. After pass band filtering and Fourier transformation of filtered signals there was obtained the statistic set of CCD between the frequency response functions of intact and pseudo-damaged states of a structure. It is shown that the statistical dispersion of the CCD estimate due to load variation is relatively small and about the same for the intact and damaged state of the structure. The two-sample Kolmogorov-Smirnov hypothesis test was used for estimation of a pseudo-damage effect. The stable response of CCD index to a small damage in large-size structure was demonstrated. Moreover, the growth trend of the CCD index with an increase in the mass of pseudo-damage is observed.

Research results can be applied in practical implementation of a local SHM system of a structure dynamically loaded in operation. In the local zone of a large structure with high risk of failure one or a few sensors should be embedded in structure for output measurement. The procedure of measurement, results processing, and estimation described above is simply realised in operation with high level of automatization. The highly limited area of a large structure, the small number of sensors, the operational modal analysis (OMA) as a basic technique, the reliable CCD index of damage detection promise to obtain the simple, low-cost, and effective local SHM system of a large-size structural component.

4. References

[1] Doebling S. W., Farrar C. R., and Prime M. B., 1998 Shock Vib. Dig., 30(2), 91–105
[2] Conte J.P., He X., Moaveni B., Masri S.F., Caffrey J.P., Wahbeh M., Tasbihgoo F., Whang D.H., and Elgamal A. 2008 Journal of Structural Engineering, ASCE, 134(6), 871-1066
[3] B. Peeters, J. Maeck, and G. De Roeck. 2001 Smart materials and Structures, 10, 518–527
[4] Schubel P.J., Crossley R.J., Boateng E.K.G., Hutchinson J.R. 2013 Renewable Energy, 51, 113
[5] He K., and Zhu W. D. 2014ASME J. Vib. Acoust., 136(3), 034503
[6] Soln H., Farrar C., Hunter N. and Worden K. A. Review of Structural Health Monitoring Literature: 1996-2001. Los Alamos National Laboratory report, (LA-13976-MS) (2003).
[7] Cardin E. P. and Fanning P. 2004 Structural Health Monitoring, 3, 355-377
[8] Farrar C. R., Doebling S. W. and Nix D. A. 2001 Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 359(1778), 131-149
[9] Jun Lia and Hong Hao. 2016 Structural Monitoring and Maintenance, 3(1), 33-49
[10] V.Pavelko, G.Shakhmansky. 1971 Trans. of Riga Institute of Civil Aviation, 191, 18-23
[11] Pandey A. K., Biswas M., and Samman M. M. 1991 J. Sound Vib., 145(2), 321–332
[12] Ratcliffe C. P. 2000 ASME J. Vib. Acoust., 122(3), 324–329
[13] R.D. Adams, P. Cawley, C.J. Pye, B.J. 1978 Journal of Mechanical Engineering Science, 20, 93
[14] U. Lee, J. Shin. 2002 Computers Structures, 80, 117-132
[15] J. Kim, N. Stubbs. 2003 Journal of Sound and Vibration, 259, 145-160
[16] A. Morassi. 2001 Journal of Sound and Vibration, 242, pp. 577-596
[17] A. Morassi, M. Rollo. 2001 Journal of Vibration and Control, 7, 729-739
[18] A. Labib, D. Kennedy, C. Featherston. 2014 Journal of Sound and Vibration, 333, 4991-5003
[19] G. Failla. 2016 Journal of Sound and Vibration, 360, 172-202
[20] X.J. Wang, H.F. Yang, Z.P. 2010 AIAA Journal, 48, 1108-1116
[21] A. Teughels, J. Maeck, G.D. Roec. 2002 Computers and Structures, 80, 1869-1879 (2002).
[22] B. Jaishia, W.X. Ren. Journal of Sound and Vibration, 290, 369-387
[23] X.J. Wang, H.F. Yang, Z.P. Qiu. 2010 AIAA Journal, 48 (2010), 1108-1116
[24] W. Zhang, Z. Du, G. Sun, X. Guo. 2017 Journal of Sound and Vibration, 386, 100–115
[25] S. D. Fassois and F. P. Kopsaftopoulos. 2013 in. New Trends in Structural Health Monitoring, edited by W. Ostachowicz, J. Güemes, CISM International Centre for Mechanical Sciences, 209-264
[26] Y. Xia, H. Hao. 2003 Journal of Sound and Vibration, 263, 853-870 (2003).