Can Massive Gravitons be an Alternative to Dark Energy?

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Abstract

In this work, we explore some cosmological implications of the model proposed by M. Visser in 1998. In his approach, Visser intends to take in account mass for the graviton by means of an additional bimetric tensor in the Einstein’s field equations. Our study has shown that a consistent cosmological model arises from Visser’s approach. The most interesting feature is that an accelerated expansion phase naturally emerges from the cosmological model, and we do not need to postulate any kind of dark energy to explain the current observational data for distant type Ia supernovae (SNIa).

Keywords: Theory of gravitation, Massive graviton, Cosmology

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1. Introduction

The state of the art in cosmology has led to the following present distribution of the energy densities of the Universe: 4% for baryonic matter, 23 % for non-baryonic dark matter and 73 % for the so-called dark energy (see e.g. [1]). The dark components (matter and energy) have been the focus of studies and a lot of speculations in theoretical and observational astrophysics.

In particular, the dark energy is very curious, not only for its dominant relative energy density, but for its dynamical consequences on cosmic
expansion: it can make the Universe accelerate. Phenomenologically, some equations of state have been proposed in order to explain such a dark energy. The most common example is the cosmological constant (ΛCDM model), which implies on a constant vacuum energy density along the history of the Universe. However, the quantum theory prediction for the vacuum density is 120 orders of magnitude greater than the observational value (see e.g. [2]).

Moreover, ΛCDM model is not satisfactory because it requires a large amount of fine tuning to produce cosmological constant energy density dominant at recent epochs (see e.g. [3]).

Another possibility is a dynamical vacuum or quintessence. In general, the quintessence models involve one [4] or two [5] coupled scalar fields. In these models, the cosmic coincidence, i.e., why dark energy started dominating the cosmic evolution only so recently, has no satisfactory solution and some fine tuning is required.

The Chaplygin gas is another example of dark energy fluid. Its exotic equation of state can be derived from the Nambu-Goto action for ‘D-Branes’ moving on a (d+2) dimensional space-time [6]. The Chaplygin Gas has a dual behavior: in the past it behaves like matter and in recent times like a cosmological constant. So, it can represent dark matter and dark energy with only one equation of state. However, at the time of structure formation the influence of dark energy component is negligible and matter clustering occurs in a similar way as in CDM model [7].

More recently some authors (see, e.g., [3]) have studied the dark energy problem considering a quintessence model with dark matter - dark energy interaction. However, as discussed by Bertolami et al [8], such a kind of coupling between dark matter - dark energy produces a violation of the equivalence principle (EP). Actually, a violation of the EP is reported to be found in other dark energy models [9].

Whatever the dark energy may be, it seems that physics beyond the standard model is necessary.

On the other hand, several studies have been trying to bound the mass of the graviton by many ways. From analysis of the planetary motions in the solar system it was found that we must have \( m_g < 7.8 \times 10^{-55} \text{g} \) [10] in order to respect the accuracy of the observations with the newtonian potential. Another bound comes from the studies of galaxy clusters, which gives \( m_g < 2 \times 10^{-62} \text{g} \) [11]. Although this second limit is more restrictive, it is considered less robust due to uncertainties in the content of the Universe in large scales.
Studying rotation curves of galactic disks, de Araujo and Miranda [12] have found that $m_g \ll 10^{-59}g$ in order to obtain a galactic disk with a scale length of $b \sim 10$ kpc.

Studying the mass of the graviton in the weak field regime Finn and Sutton have shown that the emission of gravitational radiation does not exclude a possible non null rest mass. They found the limit $m_g < 1.4 \times 10^{-52}g$ [13] analyzing the data from the orbital decay of the binary pulsars PSR B1913+16 (Hulse-Taylor pulsar) and PSR B1534+12.

As can be seen, the graviton mass is not observationally excluded. So, it is reasonable to ask if the consideration of mass for the graviton plays some role in cosmology. Do cosmological models exclude a non null graviton mass? Can massive gravitons affect the cosmic dynamics? Is it possible to bound the graviton mass by cosmological observation? Would such a bound be in accordance with other observations? Considering mass for the graviton, do we need to include dark energy in order to explain the cosmological data?

In order to look for answers to these questions, we have chosen to include the graviton mass adopting the Visser’s approach [14], where the graviton mass appears as an extra term in the Einstein’s equations. It is worth noting that the weak field equations used by Finn and Sutton are the same that come from the Visser’s model when we use the linear approach.

An interesting result that comes out from this model is that the gravitational waves present six polarization modes [15] instead of the two usual polarizations obtained from the General Relativity theory. So, if in the future we would be able to identify the gravitational wave polarizations, we would impose limits on the graviton mass by this way.

This paper is organized as follows: in Section 2 we briefly review the Visser’s approach. Section 3 is devoted to the description of the cosmological model. In Section 4 we discuss how the age of the Universe can constrain the value of the mass for the graviton. Section 5 describes the evolution of the scale factor in the ‘massive cosmology’. In Section 6 we show that a cosmological scenario with massive gravitons can produce a phase of accelerating expansion for the Universe. Section 7 presents a comparison between ΛCDM and our cosmological model using the luminosity distance. Finally, in Section 8 we present our conclusions.
2. The Field Equations

The full action considered by Visser is given by [14]:

\[
I = \int d^4x \left[ \sqrt{-g} \frac{c^4 R(g)}{16\pi G} + \mathcal{L}_{\text{mass}}(g, g_0) + \mathcal{L}_{\text{matter}}(g) \right].
\] (1)

where besides the Einstein-Hilbert lagrangian and the lagrangian of the matter fields we have

\[
\mathcal{L}_{\text{mass}}(g, g_0) = \frac{1}{2} \frac{m_g^2 c^2}{\hbar^2} \sqrt{-g_0} \left\{ (g_0^{-1})^{\mu\nu} (g - g_0)_{\mu\sigma} (g_0^{-1})^\sigma^\rho \right. \\
\times \left( g - g_0 \right)_{\rho\nu} - \frac{1}{2} \left[ (g_0^{-1})^{\mu\nu} (g - g_0)_{\mu\nu} \right]^2 \right\},
\] (2)

where \( m_g \) and \( (g_0)_{\mu\nu} \) are respectively the graviton mass and a general flat metric.

The field equations, which are obtained by variation of (1), can be written as:

\[
G_{\mu\nu} - \frac{1}{2} \frac{m_g^2 c^2}{\hbar^2} M_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu},
\] (3)

where \( G_{\mu\nu} \) is the Einstein tensor, \( T_{\mu\nu} \) is the energy-momentum tensor for perfect fluid, and the contribution of the massive tensor to the field equations reads:

\[
M_{\mu\nu} = (g_0^{-1})^{\mu\sigma} \left[ (g - g_0)_{\sigma\rho} - \frac{1}{2} (g_0)_{\sigma\rho} (g_0^{-1})_{\alpha\beta} \right. \\
\times \left. (g - g_0)_{\alpha\beta} \right] (g_0^{-1})^{\rho\nu}.
\] (4)

Note that if one takes the limit \( m_g \to 0 \) the usual Einstein field equations are recovered.

Regarding the energy-momentum conservation we will follow the same approach of [16] and [17] in such a way that the conservation equation now reads [18]:

\[
\nabla_\nu T_{\mu\nu} = \frac{m_g^2 c^6}{16\pi G \hbar^2} \nabla_\nu M_{\mu\nu},
\] (5)

since the Einstein tensor satisfies the Bianchi identities \( \nabla_\nu G^{\mu\nu} = 0 \).
3. Cosmology with massive gravitons and without dark energy

For convention we use the Robertson-Walker metric as the dynamical metric:

\[ ds^2 = c^2 dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \tag{6} \]

where \( a(t) \) is the scale factor. The flat metric is written in spherical polar coordinates:

\[ ds_0^2 = c^2 dt^2 - \left[ dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]. \tag{7} \]

Using (6) and (7) in the field equations (3) we get the following equations describing the dynamics of the scale factor (taking \( k = 0 \) for simplicity):

\[
\left( \frac{\dot{a}}{a} \right)^2 + \frac{m_g^2 c^4}{8\hbar^2} (a^2 - 1) = \frac{8\pi G}{3c^2} \rho, \tag{8}
\]

and

\[
\frac{\ddot{a}}{a} + \frac{1}{2} \left( \frac{\dot{a}}{a} \right)^2 + \frac{m_g^2 c^4}{8\hbar^2} a^2 (a^2 - 1) = -\frac{4\pi G}{c^2} p, \tag{9}
\]

where as usual \( \rho \) is the energy density and \( p \) is the pressure.

From equation (5) we get the evolution equation for the cosmological fluid, namely:

\[
\dot{\rho} + 3H \left[ (\rho + p) + \frac{m_g^2 c^6}{32\pi G \hbar^2} (a^4 - 6a^2 + 3) \right] = 0, \tag{10}
\]

where \( H = \dot{a}/a \). Considering a matter dominated universe \( (p = 0) \) the above equation gives the following evolution for the energy density:

\[
\rho = \rho_0 \left( \frac{a_0}{a} \right)^3 - \frac{3m_g^2 c^6}{32\pi G \hbar^2} \left( \frac{a^4}{\ell^4} - \frac{6a^2}{5} + 1 \right), \tag{11}
\]

where \( \rho_0 \) and \( a_0 \) are the present values of the energy density and the scale factor respectively. Note that in the case \( m_g \to 0 \) we obtain the usual Friedmann equations.

Now, inserting (11) in the modified Friedmann equation (8) we obtain the Hubble parameter:

\[
H^2(a) = H_0^2 \left[ \Omega_m \left( \frac{a_0}{a} \right)^3 + \frac{1}{140} \left( \frac{m_g}{m_H} \right)^2 (7a^2 - 5a^4) \right], \tag{12}
\]

5
where the relative energy density of the $i$-component is $\Omega_i = \rho_i/\rho_c$ ($\rho_c = 3H^2c^2/8\pi G$ is the critical density) where ‘$i$’ applies for baryonic and dark matter. Here $m_H = hH_0/c^2$ is a constant with units of mass, which we will call ‘Hubble mass’ (as we will see later this constant is very important in the present context).

4. The Age of the Universe Constraining the Mass for the Graviton

The equation (12) for the present time ($a = a_0$) gives:

$$\Omega_m^0 + \frac{1}{140} \left( \frac{m_g}{m_H} \right)^2 \left( 7a_0^2 - 5a_0^4 \right) = 1. \tag{13}$$

Solving this equation to $a_0 = a_0(m_g)$ we get:

$$a_0 = \sqrt{\frac{7}{10}} \left\{ 1 + \left[ 1 - \frac{200}{7} \left( \frac{m_H}{m_g} \right)^2 \left( 1 - \Omega_m^0 \right) \right]^{\frac{1}{2}} \right\}. \tag{14}$$

Thus, if we use (14) in the equation (12) we have the dynamical equations described in terms of the free parameters $\Omega_m^0$, $H_0$ and $m_g$.

Note that in order to have real values for $a_0$, the term in brackets in (14) must satisfy the relation:

$$1 - \frac{200}{7} \left( \frac{m_H}{m_g} \right)^2 \left( 1 - \Omega_m^0 \right) > 0, \tag{15}$$

which lead us to a lower limit for the graviton mass in our model:

$$m_g > \sqrt{\frac{200}{7} \left( 1 - \Omega_m^0 \right) m_H}. \tag{16}$$

If we take, for example, $\Omega_m^0 = 0.27$ we have:

$$m_g > 4.57 m_H. \tag{17}$$

So, the Hubble mass, $m_H$, establishes the magnitude of the expected value of the graviton mass. If we convert this lower limit in a upper limit to the Compton wavelength of the graviton we have:

$$\lambda_g < 0.22 \frac{c}{H_0}. \tag{18}$$
Thus, if the graviton rest mass is non null, its corresponding Compton wavelength must be lower than the observable horizon. But once the Compton wavelength is associated with the range of the interaction, this tell us that the contribution of the mass term can be relevant in the Universe at the present time.

A note about the lower bound on the graviton mass. Since we are considering only matter in the context of a massive gravity theory, the lower bound on the graviton mass is a direct consequence of the fact that we have $\Omega_{\text{total}} \simeq 1$ today. Once matter contributes only with 27% of the total relative energy density, and the radiation contribution is negligible, we could not have a zero graviton mass without considering another kind of dominating component, but such idea is not the aim of the present paper.

Once we have an analytical description of the dynamics in this scenario we can impose limits on the parameters in many ways. One of them can be the age of the Universe.

The time scale to the age of the Universe is related to the Hubble parameter, namely, $t_H = H_0^{-1}$, which is about 13 Gy as given by the current observational data [1].

So, we hope that a consistent cosmological model can give ages of this order, which contemplates the age of the oldest stars and the time to structure formation.
In order to calculate the age of the Universe in our model we just solve the integral:

\[ t_U = \int_0^{a_0} \frac{da}{aH(a)}. \]  

(19)

Substituting (12) in (19) we obtain the age of the Universe as shown in Figure 1 to some values of \( \Omega^0_m \). We can see that the age of the Universe is very closely related to the mass of the graviton. This gives us a very restrictive upper limit. The effect of taking different values of \( \Omega^0_m \) is to shift the curve upward or downward, but the qualitative behavior is the same in all cases.

If we take, for example, \( t_H = 13 \text{ Gy} \) and \( \Omega^0_m = 0.27 \) we obtain the limit:

\[ m_g h_0^{-1} \lesssim 2.24 \times 10^{-65} \text{ g}, \]  

(20)

where the Hubble parameter is given by \( H_0 = 100h_0 \text{ kms}^{-1} \text{Mpc}^{-1} \) and \( h_0 \) is a dimensionless constant, which parameterizes the uncertainty in the measurement of \( H_0 \). This limit for \( m_g \) is about 10 orders of magnitude more restrictive than the limits obtained from the orbital motion in solar system and about 3 orders more restrictive than the inferences from galaxy clusters.

Considering the lower limit obtained previously, the mass of the graviton in this model must be in the interval:

\[ 1.73 \times 10^{-65} \text{ g} < m_g h_0^{-1} \lesssim 2.24 \times 10^{-65} \text{ g}, \]  

(21)

for \( \Omega^0_m = 0.27 \). This upper limit depends on the value taken for the age of the Universe, while both upper and lower limits depend on the value of \( \Omega^0_m \).

5. Past and Future

One can determine how the Universe evolves by integrating equation (12), which yields \( a(t) \) as shown in Figure 2.

In the past and in the present the curve \( a(t) \) behaves like the evolution of the Friedmann models with a cosmological constant. However, the future is drastically different. The massive term contributes in such a way that in the past it generates a repulsion, becoming attractive in the future and leading the Universe to the so called big crunch. Moreover, although the evolution is qualitatively the same for any value of the graviton mass, it is notable the strong dependence between the time of evolution and \( m_g \). This emphasizes again the restricted values to \( m_g \) which we are considering.
In the Figure 3 we can see the evolution of the Hubble parameter. In the past the function behaves like the Friedmann models and in the future $H(z) \rightarrow 0$. This shows us that the massive contribution becomes important in the late time for the history of the Universe. In fact, if we take $a \rightarrow 0$ in equation (12) we see that the massive term is negligible when compared to the radiation and matter contribution. That is very important in order to have no change in the expansion rate of the Universe, for example, in the nucleosynthesis era (a detailed study of this issue will appear elsewhere).

When the scale factor reaches its maximum value (the turning point $z_{\text{turn}}$), the Hubble parameter goes to zero $H(z_{\text{turn}}) = 0$. The smaller $m_g$ is, the greater $z_{\text{turn}}$ is. This indicates the relation between the turning point and the range of the gravity interaction, once the Compton wavelength is inversely proportional to the mass of the particle.

![Figure 2: Evolution of the normalized scale factor with time (in units of the Hubble time) for different values of $m_g$.](image)

6. Accelerating Universe?

In order to study the evolution of the second derivative of the scale factor we will use the definition of the dimensionless desaccelerating parameter:

$$q \equiv -\frac{\ddot{a}a}{a^2}.$$ (22)
Figure 3: Evolution of the Hubble parameter as a function of the redshift for different values of $m_g$. We take $\Omega_m^0 = 0.27$.

Applying this to the equations (8) and (9) we get:

$$q(a) = \frac{H_0^2}{H^2} \left[ \frac{1}{2} \Omega_m^0 \left( \frac{a_0}{a} \right)^3 + \left( \frac{m_g}{m_H} \right)^2 \left( \frac{3a_0^4}{14} - \frac{a_0^2}{5} \right) \right].$$  

(23)

The evolution of the desaccelerating parameter as a function of the redshift is shown in Figure 4.

Note that an accelerating expansion at the present time is closely related to the value of the graviton mass. In the past, as expected, the Universe has a desaccelerated phase. The fast growth of $q(a)$ in the future is explained by the reversion in the expansion as discussed above. When the Universe is contracting, the values of $q(a)$ are the same as in the expansion phase, which lead us to conclude that the Universe has two accelerating phases in this model: the first when it expands and the second when it contracts.

In order to verify the dependence between the present value of the desaccelerating parameter ($q_0$) and the value of $m_g$ we just set $a = 1$ in the equation (23), giving:

$$q_0 = \frac{1}{2} \Omega_m^0 + \left( \frac{m_g}{m_H} \right)^2 \left( \frac{3a_0^4}{14} - \frac{a_0^2}{5} \right).$$  

(24)

The curve $q_0 \times m_g$ is shown in Figure 5. If we assume $\Omega_m^0 = 0.27$, for example, we have a present accelerating expansion for $m_g h_0^{-1} < 1.80 \times 10^{-65}$ g.
It is interesting to observe that this result is compatible with older Universes and greater lifetimes, which indicates that the most probable value for $m_g$ would be closer to the lower limit (see equation (21)).

Figure 4: Evolution of the desaccelerating parameter $q(z)$ for different values of $m_g$. We take $\Omega_m^0 = 0.27$

Figure 5: Relation between the current value of the desaccelerating parameter and the graviton mass for different values of $\Omega_m$. 
7. The Luminosity Distance

In order to test this cosmological model, we calculate the luminosity distance and compare it with the ΛCDM best fit of the type Ia supernovae.

The luminosity distance to a flat Universe \((k=0)\) is given by (see, e.g. [19]):

\[
d_L(z) = (1 + z)cH_0^{-1} \int_0^z \frac{dz'}{h(z')}, \tag{25}
\]

where \(h(z) = H(z)/H_0\) and \((1 + z) = a^{-1}\).

In the ΛCDM model the equation (25) gives us:

\[
d_{\Lambda CDM} = (1 + z)cH_0^{-1} \int_0^z \frac{dz'}{\sqrt{\Omega_0^m(1 + z')^3 + \Omega_0^\Lambda}}. \tag{26}
\]

To the massive model we can combine (12) with (25) to obtain:

\[
d_{\text{mass}} = (1 + z)cH_0^{-1} \int_0^z dz' \sqrt{\Pi(z')} , \tag{27}
\]

where:

\[
\Pi(z') \equiv \frac{(1 + z')^4}{\Omega_0^m(1 + z')^7 - M(z')} , \tag{28}
\]

and

\[
M(z') \equiv \left( \frac{m_g}{m_{H}} \right)^2 \left[ \frac{a_0^4}{14} - \frac{a_0^2}{10}(1 + z')^2 \right] .
\]

If we use the relation between \(a_0\) and \(m_g\), given by (14), the luminosity distance in our model has the same number of free parameters as in the ΛCDM model.

In Figure 6 we plot some values of the graviton mass for the limits given by (21). We also plot the curve for the ΛCDM model, which is the best fit for the most recent observational data from the recession of type Ia Supernovae, i.e., \(\Omega_0^m = 0.27\) and \(\Omega_0^\Lambda = 0.73\) [20]. The comparison between the model and the SNIa data can be seen in Figure 7 where we use the definition of the distance modulus:

\[
\mu \equiv m - M = 5 \log(d_L) - 5 + A , \tag{29}
\]

where \(m\) and \(M\) are the apparent and absolute magnitude respectively and \(A\) is the absorption in magnitude.
Figure 6: Luminosity distance (in units of $cH_0^{-1}$) as a function of the redshift for a ΛCDM model and for the massive model.

Figure 7: Distance modulus as a function of the redshift to the massive model compared with the SNIa data from Astier et al. (2006).

As can be seen, there are not significant differences in the curves $d_L \times z$, when we change the graviton mass in that range we have considered. Furthermore, we can give a value to $m_g$ which describes exactly the same curve given by the ΛCDM model, this value is around $m_g h_0^{-1} = 1.95 \times 10^{-65}$ g. From Figure (5) we see that for this value of the graviton mass we have a positive desaccelerating parameter in the present ($z = 0$), so we can fit the
observational data for SNIa without a present acceleration in the expansion of the Universe, although we have an inevitable accelerating expansion era in the near past.

Again, we would like to emphasize that there are no significant changes in the curve $d_L \times z$ when we use other values of $m_g$, which are in the established limit. In such a way there are some possible values of $m_g$, which give current acceleration and other that give deceleration. For example, taking the values used in the Figure 7, the lower one ($1.74 \times 10^{-65}$ g) gives current deceleration ($q_0 > 0$), and the greater one ($2.23 \times 10^{-65}$ g) gives a current cosmic acceleration ($q_0 < 0$).

In view of the above results we may conclude that it is not possible to decide between the massive model and ΛCDM model only by the type Ia Supernovae observations, in particular for the small redshifts in which the Supernovae have been observed ($z \lesssim 1$).

8. Conclusions

We have shown that within the context of a classical gravity theory with massive gravitons, we can obtain a consistent cosmological model which has acceptable values for the age of the Universe, furthermore, it can fit the present cosmological SNIa data without any kind of dark energy.

The possible values for the graviton mass in this theory are in accordance with all established limits.

We identify that there is degeneracy between the ΛCDM model and the massive model when we analyze the luminosity distance versus redshift. Such a degeneracy could be removed, in principle, by other tests of the massive model such as structure formation or analyzing the power spectrum of the cosmic microwave background radiation (CMBR).

In future works we will study in detail the nucleosynthesis era within the context of the massive model, as well as to provide a statistical approach to the analysis of the SNIa data within this model.

Once the graviton mass is introduced via an effective tensor as given by (3), we hope that the primordial graviton production would be different from those models which consider general relativity. So, the future detection of primordial gravitational waves will provide a way to test this alternative gravity theory.

It is worth stressing that Gabadadze and Gruzinov (2005) have analyzed the instabilities of the background and ghosts produced by massive gravitons.
in 4D Minkowski space-time. They conclude that a natural way to account for a massive graviton on flat space is to invoke theories with extra dimensions. However, Visser’s model is truly continuous with general relativity (GR) in the limit of vanishing graviton mass. Together with Pauli-Fierz (PF) massive term, Visser’s theory is the simplest attempt to incorporate mass for the graviton in a ghost-free manner. Moreover, the van Dam-Veltman-Zakharov discontinuity (vDVZ) present in the PF term can be circumvented in Visser’s model by introducing a non-dynamical flat-background metric [23].

Thus, Visser’s model can help us to understand the influence of the mass for the graviton in cosmology and maybe it could shed some light on the following question: what is the dark energy?

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