The electromagnetic (EM) field pattern created by spectators in relativistic heavy-ion collisions plants a seed of positive (negative) magnetic helicity in the hemisphere above (below) the reaction plane. Owing to the chiral anomaly, the magnetic helicity interacts with the fermionic helicity of the collision system, and causes photons emitted in upper- and lower-hemispheres to have different preferences in the circular polarization. Similar helicity separation for massive particles, due to the global vorticity, is also possible. In this paper, we lay down a procedure to measure the variation of the circular polarization w.r.t the reaction plane in relativistic heavy-ion collisions for massless photons, as well as similar polarization patterns for vector mesons decaying into two daughters. We propose to study the yield differentially and compare the yield between upper- and lower-hemispheres in order to identify and quantify such effects.

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I. INTRODUCTION

In a non-central relativistic heavy-ion collision, an ultra-strong magnetic field will be produced inside the participant zone by energetic spectator protons [1]. On average, the magnetic field is perpendicular to the so-called “reaction plane” (RP) that contains the impact parameter and the beam momenta. This strong magnetic field, coupled with quantum anomalies, will cause a series of novel non-dissipative transport effects that could survive the expansion of the fireball and be detected in experiments. For a review on these subjects, see Ref. [2]. Furthermore, the initial magnetic helicity of the collision system can be quite large, and bears opposite signs in the upper- and lower-hemispheres. It was recently realized [3-6] that owing to the chiral anomaly, the helicity can be transferred back and forth between the magnetic flux and fermions as the collision system evolves, so that the magnetic helicity could last long enough to yield photons with opposite circular polarizations in the hemispheres above and below the RP [3, 7, 8]. A similar asymmetry in photon polarization can also result from the initial global quark polarization [9] which could effectively lead to a polarization of photons [7]. This local imbalance of photon circular polarization could be observed in experiments, e.g., by studying the polarization preference w.r.t the RP for photons that convert into $e^+e^-$ pairs. In this paper, we discuss a few practical thoughts of carrying out such a measurement, first for photons and then for vector mesons.

II. PHOTONS

Photon circular polarization can be measured with dedicated polarimeters [10], by studying the count rate asymmetries with a local magnet. Such a setup is infeasible for large-scale heavy-ion experiments like the Relativistic Heavy Ion Collider or the Large Hadron Collider, as the local magnet will complicate the magnetic field that is used for the track detection of charged particles. Nevertheless, one can still probe photon circular polarization by tracking $e^+e^-$ pairs from photon conversion.

A photon will not decay by itself. However, when a photon is near an atomic nucleus, its energy can be converted into an electron-positron pair. Panels a) and b) in Fig. 1 show the typical conversion process for left and right circularly polarized photons, respectively. Here $\zeta$ is the angle of the electron relative to the positron, when both are projected onto the plane perpendicular to the motion direction of the photon. Owing to the opposite circular polarizations, the preferred direction orderings between the emitted electron and positron are opposite in the two cases.

![Fig. 1: A left (a) or right (b) circularly polarized photon is converted into an electron-positron pair with the help of the recoil of an atomic nucleus.](image-url)
The cross-section for the pair production by completely
circularly polarized high-energy photons is given as [11, 12]
\[ d\sigma \propto 1 \pm a \cdot \sin \zeta, \]  
(1)
where the + and − signs are for right and left circularly
polarized photons, respectively. \( a (\in [0, 1]) \) is a positive
quantity dependent on the photon energy and the ener-
gies and momenta of its two daughters, as well as the
atomic number \( Z \) of the recoiling nucleus.

In the presence of the magnetic helicity effect, the
yields of left and right circularly polarized photons can
each be parametrized as a sine wave modulation w.r.t.
the RP, which is similar to the directed flow (\( v \))
each be parametrized as a sine wave modulation w.r.t.
yields of left and right circularly polarized photons can
be obtained from the cross-section for left and right circularly polarized photons. With
this parametrization scheme, the effect of the magnetic helicity aforementioned is strongest in the direction per-
pendicular to the RP (along the direction of the magnetic
field), and weakest in the RP, and so is the difference in
yield between the two groups of photons. The cross-
sections for right and left circularly polarized photons
can be written separately as
\[ d\sigma_R \propto (1 + a \cdot \sin \zeta)(1 + b \cdot \sin \phi), \]  
(2)
\[ d\sigma_L \propto (1 - a \cdot \sin \zeta)(1 - b \cdot \sin \phi), \]  
(3)
where \( \phi \) is the photon’s azimuthal angle relative to the
RP, and \( b (\in [0, 1]) \) represents the magnitude of the mod-
ulation. The total cross-section is the sum of \( d\sigma_R \) and
\( d\sigma_L \):
\[ d\sigma \propto 1 + H \sin \zeta \sin \phi, \]  
(4)
where \( H \equiv ab, H \in [0, 1] \).

\( \zeta, \phi \) are both experimentally measurable quantities,
so in principle \( H \) can be obtained from the cross-section
measurement. A finite \( H \) implies a finite \( b \) or a finite
effect of modulation w.r.t the RP due to the external
magnetic field. Note that only the modulation w.r.t RP
carries the information of the asymmetry between left
and right circularly polarized photons, so other sources of
photons (mostly from hadron decays) do not contribute
to \( H \) as long as they are produced symmetrically w.r.t RP
in terms of the left-right circular polarization. Gener-
ally speaking, photons can have other finite Fourier coeffi-
cients, e.g., \( v_1 \) and \( v_2 \) [13], but they do not couple with
\( H \) as long as we assume those coefficients are the same
for left and right circularly polarized photons.

The (in)efficiency as a function of \( \phi \) angle can be easily
compensated in experiments, as long as the RP distrib-
bution and the azimuthal angle distribution of photons
in the laboratory frame are flattened, which is a com-
mon procedure when analyzing anisotropic flow [13]. We
also assume that the efficiency as a function of \( \zeta \) can be
measured by usual Monte Carlo studies with individ-
ual experiment setups. Taking the difference in yield-
probability between the upper hemisphere and the lower
hemisphere,
\[ \Delta Y^{\gamma}_{\text{up-down}} = \int_0^\pi d\sigma d\phi - \int_{\pi}^{2\pi} d\sigma d\phi \]
\[ \approx 4H \sin \zeta \]  
(5)
and the difference between the left hemisphere and right
hemisphere (separated by \( \phi = \pi/2 \)) would be zero owing
to the symmetry of the system:
\[ \Delta Y^{\gamma}_{\text{left-right}} = \int_{\pi/2}^{3\pi/2} d\sigma d\phi - \int_{-\pi/2}^{\pi/2} d\sigma d\phi = 0, \]  
(6)
which can be regarded as a consistency check in experi-
ments. Note that when there is no signal, \( \Delta Y^{\gamma}_{\text{up-down}} = \Delta Y^{\gamma}_{\text{left-right}} = 0 \), and a finite efficiency \( \epsilon(\zeta) \) will not cause
an artificial, finite \( \Delta Y^{\gamma}_{\text{up-down}} \).

In heavy-ion collider experiments, photons are usually
reconstructed at secondary vertices where they strike de-
tector materials and emit \( e^+e^- \) pairs. The photon sample
is inevitably contaminated by combinatorial background
formed by random \( e^+e^- \) pairs. However, the background
contribution should be symmetric w.r.t both \( \phi = 0 \) and
\( \phi = \pi/2 \) plane, and be canceled in this procedure by tak-
ing the difference in yield between up and down (or left
and right) hemispheres. The same argument also holds
for the study of vector mesons discussed below.

III. VECTOR MESONS

Unlike photons that can be only transversely polarized
(with the spin direction either parallel or anti-parallel to
the momentum direction), vector mesons can be both
longitudinally and transversely polarized. Below, we de-
scribe a procedure to measure a potential difference, if a
similar effect exists, between left and right circular polar-
zations for vector mesons, as well as a way to study the
variation of such a difference w.r.t the RP. The procedure
is applicable to vector mesons decaying into two daugh-
ters, such as \( \omega, \phi, J/\psi \) etc, and applicable to virtual
photons decaying into two leptons by firstly fluctuating
into vector mesons. The earlier the particle is produced,
the better it experiences the magnetic helicity that hap-
pens at the very early time (a few \( fm/c \) of the collision.
In addition, the lighter the particle, the easier for it to be
polarized by the magnetic field. In the end what particle
to choose for this study is a balanced decision between
the particle’s production time, mass, the purity of the
data sample, and the statistics available. The latter two
factors vary from one experiment to another. Note that
although here we mentioned the magnetic helicity, the
system can have helicity separation originated from vor-
ticity [9, 14, 15], which may have an effect on massive
particles.

The polarization of vector mesons can be studied with
the distribution of its decay products in azimuthal and
polar angles (for example, for $J/\psi$ particle see [16][18]) in a chosen coordinate system (frame). The four popular choices of frame are the particle rest frame [16], the Gottfried-Jackson frame [16][19], the center-of-mass helicity frame [16], and the Collins-Soper frame [20]. However, no discussion is given in [16] [17] to distinguish the left and right circular polarizations, both belonging to transverse polarization.

Our discussion will be limited to the center-of-mass helicity frame only. In the helicity frame [16], the orientation of the polar axis is set to be the opposite of the direction of motion of the interaction point (the flight direction of the quarkonium itself in the center-of-mass of the colliding beam). The illustration of the helicity frame and the definition of polar angle ($\beta$) is given in Fig. 2.

![Fig. 2: Illustration of the helicity frame (left), in which $b_1$ and $b_2$ are beam directions in the quarkonium rest frame, and the orientation of the polar axis ($Z_{HX}$) is set to be the opposite of the direction of motion of the interaction point. The polar angle ($\beta$) is defined as the angle between the polar axis and the positive lepton (right).](image)

As a side remark, the definition of transverse and longitudinal polarization for vector mesons is commonly used, but counterintuitive. The definition is originated in analogy to a photon which is said to be *transversely* polarized with spin projection $J_z = \pm 1$, owing to the fact that the EM field carried by a photon oscillates in the transverse plane with respect to its momentum, while its spin is aligned along the momentum. *Longitudinal* polarization means $J_z = 0$. We respect this convention in this paper.

### A. Asymmetry in yield w.r.t RP for left and right-circular polarization only

Without loss of generality, let’s assume the vector meson under discussion is a $J/\psi$ particle decaying into a $e^+e^-$ pair. In the helicity frame, the polar angle distributions of positrons for left-handed, longitudinal and right-handed $J/\psi$ can be written as [21]

\[
P_{T_l} \propto \frac{1}{4}(1 - \cos\beta)^2
\]

\[
P_L \propto \frac{1}{2}\sin^2\beta
\]

\[
P_{T_r} \propto \frac{1}{4}(1 + \cos\beta)^2.
\]

If one assumes the left-right symmetry and introduces $\lambda_\beta \in [-1, 1]$, with $\lambda_\beta = 1(-1)$ representing the complete transverse (longitudinal) polarization, the polarization-summed cross-section can be expressed as

\[
P_{pol_{sum}} \propto \frac{1 + \lambda_\beta}{2}(P_{T_l} + P_{T_r}) + \frac{1 - \lambda_\beta}{2}P_L
\]

\[
\propto 1 + \lambda_\beta\cos^2\beta,
\]

with which we have recovered the formula to describe $J/\psi$ polarization in the polar angle in Refs. [16] [17].

Now we relax the assumption of the left-right symmetry, and introduce $x = A\sin\phi$, where $\phi$ is $J/\psi$’s azimuthal angle relative to the RP, and $A \in [0, 1]$ represents the asymmetry in the emission of left and right circularly polarized $J/\psi$ w.r.t RP,

\[
P_{pol_{sum}} \propto \frac{1 + \lambda_\beta}{2}(1 - x)P_{T_l} + \frac{1 + x}{2}P_{T_r} + \frac{1 - \lambda_\beta}{2}P_L.
\]

With this definition, when $x$ is 1 (-1), $J/\psi$’s are produced 100% with right (left) circular polarization. After normalization in $\beta$ space, $P_{pol_{sum}}$ is written in full form as :

\[
P_{pol_{sum}} = \frac{3}{32}(5 + \lambda_\beta + 4A(1 + \lambda_\beta)\sin\phi\cos\beta + 3(3\lambda_\beta - 1)\cos2\beta)
\]

Taking into account the modulation w.r.t. the RP due to the elliptic flow ($v_2$) [19], the final probability in yield becomes

\[
y(\phi, \beta) = \frac{1}{2\pi}(1 + 2v_2\cos2\phi)P_{pol_{sum}}.
\]

In Fig. 3, $y(\phi, \beta)$ is shown for three different $A$ values. For a given $\beta$, a finite $A$ value will distort the originally (when $A = 0$) symmetric $J/\psi$ production w.r.t. $\phi = 0$. In principle, the $J/\psi$ yield measured in $(\phi, \beta)$ spaces, if fitted with Eq. [14], can be used to extract $A$.

Taking the difference in probability between the upper and lower hemispheres,

\[
\Delta Y_{up-down} = \int_{-\pi/2}^{\pi/2} y(\phi)\,d\phi - \int_{-\pi/2}^{\pi/2} y(\phi)\,d\phi
\]

\[
= \frac{A}{4\pi}(3 - 2v_2)(1 + \lambda_\beta)\cos\beta.
\]
FIG. 3: Probability distribution $y(\phi, \beta)$ for $A = 0$ (top), $A = 0.5$ (middle) and $A = 1$ (bottom). For all three cases, $\lambda_\beta$ and $v_2$ are -0.1 and 0.1, respectively.

Note that Eq. (15) shows that $\Delta Y_{J/\psi}^{up-down}$ could still be finite even if the transverse and longitudinal polarizations have equal possibility ($\lambda_\beta = 0$). Again the difference between the left and right hemispheres (separated by $\phi = \pi/2$) is

$$\Delta Y_{J/\psi}^{left-right} = \int_{\pi/2}^{3\pi/2} y(\phi) d\phi - \int_{-\pi/2}^{\pi/2} y(\phi) d\phi = 0.$$  

(16)

When the detection efficiency is finite, Eq. (14) can be simply multiplied by $\epsilon(\beta)\epsilon(\phi)$, where $\epsilon(\beta)$ and $\epsilon(\phi)$ are efficiency functions of $\beta$ and $\phi$, respectively. With a finite efficiency, $\Delta Y_{J/\psi}^{up-down}$ does not take the simple form of Eq. (15) any more. However, similar to the case of photons, a finite efficiency $\epsilon(\beta)$ will not cause an artificial, finite signal in $\Delta Y_{J/\psi}^{up-down}$. Here we assume that the (in)efficiency as a function of $\phi$ angle can be easily compensated in experiments, as long as the RP distribution and the azimuthal angle distribution of particles of interest in the laboratory frame are flattened, which is a common practice when analyzing anisotropic flow \cite{13}.

**B. Additional asymmetry in yield w.r.t RP between longitudinal and transverse polarizations**

A more general physics case would not exclude the possibility that the portions of the longitudinal and transverse polarizations also vary w.r.t. the RP. To take this effect into account, one can introduce the RP dependence of $\lambda_\beta$ in Sec. III A as:

$$\lambda_\beta(\phi) = \lambda_\beta^0(1 + 2u \sin \phi).$$  

(17)

The corresponding $\Delta Y_{J/\psi}^{up-down}$ becomes

$$\Delta Y_{J/\psi}^{up-down} = \frac{3 - 2v_2}{8\pi} \left[ 2A(1 + \lambda_\beta^0 \cos \beta + \lambda_\beta^0 u(1+3\cos 2\beta) \right]$$  

and $\Delta Y_{J/\psi}^{left-right}$ remains zero owing to the symmetry w.r.t $\phi = \pi/2$ plane.

**IV. DISCUSSION AND SUMMARY**

We have shown that for massless photons, as well as vector mesons decaying into two daughters, the difference in the circular polarization preference between upper- and lower-hemispheres could be feasibly measured. We propose a scheme to carry out the measurement that quantifies the effect. In practice, there are other complications that need to be considered in experiments. For example, $\pi_0$ decays may dominate the photon production, effectively diluting the signal. In the end it is a competition between the strength of the signal and the dilution effect. Similar dilution effects can happen to vector mesons due to the background contamination. The correction for dilution depends on the detailed experimental setup, which varies from one experiment to another, and is beyond the scope of this paper.

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