Resolving Three Single-Photon Emitters By Only Detecting Third-Order Correlation Functions

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Rayleigh criterion has been continuously broken through with modern detecting instruments, many methods on resolving two emitters below the classical diffraction limit have been developed, one of these methods is detecting high order correlation functions of the emitters. We propose a method to resolve the single-photon emitters with only detecting the highest order correlation functions. We find that the variables that characterize the emitters are contained in the highest order correlation functions. Especially when there are more than two emitters, the spatial distribution of the emitters can be deduced only from the highest order correlation functions, which gives a new vision and a new method on beating classical diffraction limit using high order correlation functions.

I. INTRODUCTION

The limited resolution ability of optical imaging is an ancient and crucial problems in many aspects of science. Nearly a century ago, the lowest central distance between two patterns that the light sources can just be resolved is known as the classical diffraction limit or the Rayleigh criterion. One can not distinguish each pattern from the average image when two patterns are overlapped a lot.

Until now, new methods on beating the classical diffraction limit have attracted much attention in both theory and experiment. In these resolution problems, a common light source is the fluorescence emitter, such as a biological cell, single molecule, or a nitrogen-vacancy center. Because of the antibunching property of a single-photon emitter, in each emission only one photon is emitted. Thus the interval of the emitted photons are much longer than the temporal width of a photon.

The major advantage of single-photon emitter is that the photon number is deterministic during each emission process. Therefore when resolving N single-photon emitters, a common used strategy is detecting the high order correlation function of the emitted photons. Because of the single-photon emitters, the N-th order correlation function contains the information up to N emitters. The image of each emitter can be achieved by combining several orders of correlation functions, on the resolved image the width of each emitter can be decreased to $1/\sqrt{N}$ times.

Although the resolution ability is not strictly defined, whether the emitters can be resolved or not is intuitional from the image. If the intention of resolution is recovering the diffraction pattern of each emitter, for single-photon emitters the image of each emitter can be reconstructed by combining several orders of correlation functions. This does not require any prior knowledge about the type of the pattern, and the variables describing the emitters are obtainable based on the super resolved image of each emitter. If the intention of resolution ability is obtaining the distance between the emitters, based on Tsang the multi-photon absorption is not necessary. When the emitters can be controlled that there is at most one photon emitted, the estimation error of distance is constant by spatial-mode demultiplexing. In fact, the distance can be obtained via the already reconstructed pattern of each emitter. When resolving single-photon emitters, the total number of the emitters is obtainable by the maximal order of detectable correlation functions. With Gaussian approximations, reconstructing the image of an emitter requires two variables: the width and the central position. The resolution ability is reflected by the relative size of distance and the width of a pattern: $s = d/\sigma$, therefore the image can be reconstructed with the knowledge of $s$. Above all, if the prior knowledge about the type of the pattern is known, image reconstruction and $s$-estimation are sufficient to each other.

Besides detecting photon coincide incidence on the same detector positions, high order correlation function on different detector positions carries the information about the emitters too. Hence the property of the emitter is achievable without reconstructing the image of each emitter. This can be used when resolving more than two single-photon emitters, since most of the emitted photons are not collected when detecting the correlation function on the same detector positions once a time. Considering the high order correlation functions under all possible detector positions can be used for obtaining the information about the emitters below the classical diffraction limit.

Inspired by these works, we propose a scheme to resolve the single-photon emitters with only the highest order correlation functions in this paper. We find that the distance between the emitters is obtainable from the correlation functions without image reconstruction. Especially when there are more than two emitters, the spatial structure of the emitters is contained in the highest order correlation functions. Therefore, we can resolve the emitters that have complex spatial structures only based on the knowledge of the highest order correlation functions.

This paper is organized as follows: In Sec. we firstly investigate the resolution of two single-photon emitters with the help of second-order correlation detections using a two-dimensional detector array, we obtain the lowest bound which is approximately half of the classical diffraction limit. In

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we investigate the resolution ability of three collinear single-photon emitters with third-order correlation detections only, we find that the distance between the emitters is the information that can be extracted from the third-order correlation functions under diverse detectors’ positions. In [4] we further consider the three single-photon emitters are on a two-dimensional plane. We find that by a proper strategy the variables which characterize the spatial structure of the emitters can be obtained by the third-order correlation functions. In [5] we summarize our work and draw our conclusions.

II. THE BEGINNING: RESOLVING TWO SINGLE-PHOTON EMITTERS

When there are two single-photon emitters, named as A and B, suppose the emitters are too far away that the distance between them cannot be directly measured. When the photons arrive at the position of the detectors, they are spread with the patterns named as point-spread-functions (PSF). Assuming the PSF of the photons from A and B are Gaussian distributed, and the width of each PSF is known as $\sigma$. This is practical, for example $\sigma$ can be obtained by a nearby separated celestial body on the image. With Gaussian assumption and $\sigma$ is known, our purpose is to obtain the distance between the emitters, when the magnification ratio of the optical system is known, it is the estimation of the distance between the centers of the PSF.

Our detection scheme is depicted in Fig. 1, the emitted photons are absorbed by a two-dimensional array of single-photon detectors. On the detectors’ array, the PSF of each emitter is two-dimensional, while dimension of X and Y can be written separately. Therefore, in our investigations, dimension Y has no effect on dimension X. Suppose on the plane of the detector-array the PSF from A and B is labeled as: $P_A(x_i)$ and $P_B(x_j)$, where $x_i$ and $x_j$ label the discrete positions of the detectors. Using the Gaussian assumptions, the PSF of each emitter is written as: $P_A(x_i) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{(x-d_i)^2}{2\sigma^2} \right], P_B(x_j) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{(x-d_j)^2}{2\sigma^2} \right]$. Because of the single-photon emitters, the second-order autocorrelation is contributed by the emission of each emitter. Considering the PSF of each emitter, the second order autocorrelation at the detector position $(r_i, r_j)$ is:

$$g^{(2)}(r_i, r_j) = \frac{\langle I(r_i, \tau) I(r_j, 0) \rangle}{\langle I(r_i, 0) \rangle \langle I(r_j, 0) \rangle},$$  \hspace{1cm}(1)$$

where the total intensity $I(r_i, \tau) = I_A(r_i, \tau) + I_B(r_i, \tau)$.

Here we are interested in the numerator part of $g^{(2)}$, which is the coincide rate on two detectors at $r_i$ and $r_j$. Because the photons emitted from A and B are incoherent, in the limit of $\tau \to 0$ the coincide rate at particular two detector position $x_i$ and $x_j$ is:

$$G^{(2)}(\Delta) \propto \eta^2 \left[ P_A(x_i)P_B(x_j) + P_A(x_j)P_B(x_i) \right],$$  \hspace{1cm}(2)$$

where $\eta$ is the detector’s single-photon absorption efficiency, it is related to the wavelength of the photon and the type of the detector and it can be previously obtained. Because the Y dimensional of the PSF is integrated to unity, only the X dimensional part of the PSF is considered.

When we have enough number of coincide events, we can divide these events into several parts based on the separation between the two positions that the coincide occurs. On the X-axis direction, we can label the left-sided coincide position as $x_i$ and the right-sided coincide position as $x_j$. To obtain the coincide number under a certain separation $\Delta$, labeled as $G^{(2)}(\Delta)$, the summation of $G^{(2)}$ under all the positions of $x_i$ which matches $x_j - x_i = \Delta$ is performed, where $\Delta \in [0, +\infty)$. The photons may arrive at the same detector, the detector should have the ability to resolve photon numbers [24]. However our coincide event occurs in the period of $\tau$, when the photons are separated on time scale (such as the constant time difference between the emitters’ excitations), the absorbed photons can be resolve by the detector, thus the coincide event on the same detector can be observed. To reveal the relation between $\Delta$ and the total distance $d_1 + d_2$, $G^{(2)}(\Delta)$ can be obtained by integrating $x_i$ from $-\infty$ to $+\infty$:

$$G^{(2)}(\Delta) \propto \eta^2 \left[ \exp \left( \frac{\Delta d_2}{\sigma^2} \right) + \exp \left( \frac{\Delta d_1}{\sigma^2} \right) \right] \exp \left[ -\frac{(2d_1 + 2d_2 - 2\Delta)^2}{16\sigma^4} \right].$$  \hspace{1cm}(3)$$

Now we have a new distribution respect to detector separation: $\Delta$, since enough coincide events are collected by the detector array, the possibility of $C^{(2)}(\Delta)$ is relative to the summation of $C^{(2)}$ under all possible $\Delta$ values, thus we have the normalized distribution of coincide rate under the separation:

$$C^{(2)}(\Delta) = G^{(2)}(\Delta) / \sum_{\Delta=0}^{+\infty} C^{(2)}(\Delta).$$  \hspace{1cm}(4)$$

The maximal point of $C^{(2)}(\Delta)$ is obtainable by investigating the first derivative of $C^{(2)}(\Delta)$, using Eq. (3) we have the requirement of $\Delta$:

$$\frac{\Delta}{d_1 + d_2} = \tanh \left[ (d_1 + d_2) \frac{\Delta}{2\sigma^2} \right].$$  \hspace{1cm}(5)$$

It can be further verified that when Eq. (5) holds the corre-
Gaussian assumption the Rayleigh criterion is approximately half of the Rayleigh criterion.

## III. RESOLVING THREE COLLINEAR SINGLE-PHOTON EMITTERS

In this section we investigate the resolution ability of three collinear emitters using the triple-photon coincide rates detected by a two-dimensional detector array, our scheme is similar to that depicted in Fig. (1) except there are three emitters on the axis of X. We label the emitters as A, B and C, and the distance between A and B: $d_1$, B and C: $d_2$. Thus the PSF of each emitter can be written as:  

$$
P_A(x, y) = \frac{1}{2\pi\sigma} \exp\left(-\frac{(x+\lambda+d_1)^2+y^2}{2\sigma^2}\right),$$

$$
P_B(x, y) = \frac{1}{2\pi\sigma} \exp\left(-\frac{(x+\lambda)^2+y^2}{2\sigma^2}\right),$$

$$
P_C(x, y) = \frac{1}{2\pi\sigma} \exp\left(-\frac{(x-\lambda+d_2)^2+y^2}{2\sigma^2}\right),$$

where $\lambda$ is the alignment parameter.

The triple-order autocorrelation at the detectors’ position $(r_i, r_j, r_k)$ is:

$$
g^{(3)}(r_i, r_j, r_k|\tau_1, \tau_2) = \langle I(r_i, \tau_1)I(r_j, \tau_2)I(r_k, 0) \rangle / \langle I(r_i, \tau_1)\rangle \langle I(r_j, \tau_2)\rangle \langle I(r_k, 0)\rangle,$$

where the total intensity at time $\tau$ is $I(r, \tau) = I_A(r, \tau) + I_B(r, \tau) + I_C(r, \tau)$.

We are interested in the numerator part of $g^{(3)}$, which is the triple-photon coincide rates where the photons arrive at the detector array are: $r_i, r_j$ and $r_k$. Because the Y dimensional part of the PSF is integrated to unity, our investigation can be simplified on dimension X only. Therefore in the limit of $\tau_{1,2} \to 0$, at $x_i, x_j$ and $x_k$ the triple-photon coincide rate can be written as:

$$
C^{(3)} / \eta^3 \propto P_A(x_i) [P_B(x_j)P_C(x_k) + P_B(x_k)P_C(x_j)] + P_A(x_i) [P_B(x_k)P_C(x_j) + P_B(x_j)P_C(x_k)] + P_A(x_k) [P_B(x_i)P_C(x_j) + P_B(x_j)P_C(x_i)].
$$

When enough samples are collected, we can divide the coincide event based on the separations between $x_i, x_j$ and $x_k$, for three points at least two separations are needed, named as $\delta_1$ and $\delta_2$ where $\delta_{1,2} \in [0, +\infty)$. Since $i, j, k$ are just labels for coincide positions, relative to X-axis direction we label $x_i$ as the left-sided position $x_j$ as the middle position and $x_k$ as the right-sided position, therefore $\delta_{1,2}$ can be defined as $\delta_1 = x_j - x_i$, $\delta_2 = x_k - x_j$. To obtain the coincide numbers under a given $\delta_{1,2}$, a summation under all possible positions of $x_i$ is performed. For the purpose of simplicity, we firstly investigate the formation of coincide rate under $\sigma = 1$, $d_1 = d_2 = d$ and $\lambda = 0$, after integrating on all possible $x_i$ values we have the formation of coincide rate, $C^{(3)}(\delta_1, \delta_2)$:

$$
C^{(3)}(\delta_1, \delta_2)/\eta^3 \propto e^{d(\delta_1+\delta_2)} + e^{3d(\delta_1+\delta_2)} + e^{d(\delta_1+2\delta_2)} + e^{d(2\delta_1+\delta_2)} + e^{d(\delta_1+2\delta_2)} + e^{d(3\delta_1+2\delta_2)} + e^{d(2\delta_1+3\delta_2)} + e^{d(3\delta_1+2\delta_2)} + e^{-d^2-2d\delta_1-2d\delta_2-\delta_1^2/3-\delta_1\delta_2/3-\delta_2^2/3}.
$$

The upper figure reveals the distribution of coincide events $c^{(2)}(\Delta)$ based on Eq. (3) and Eq. (4). The lower figure reveals the left side and the right side of the equality in Eq. (5), the intersection is where the peak of $c^{(2)}$ occurs.

In the left figure, the middle figure reveals the function of $c^{(2)}(\Delta)$ with $\Delta = 0$ and the right figure reveals the left side and the right side of the equality in Eq. (6), the intersection is where the peak of $c^{(2)}$ occurs.
Now we have a new distribution with respect to $\delta_1$ and $\delta_2$, with enough collecting samples we can normalize $C^{(3)}$ on all possible variables of $(\delta_1, \delta_2)$, thus we have the normalized distribution of $C^{(3)}(\delta_1, \delta_2)$:

$$e^{(3)}(\delta_1, \delta_2) = C^{(3)}(\delta_1, \delta_2) / \sum_{\delta_1, \delta_2=0}^{+\infty} C^{(3)}(\delta_1, \delta_2).$$ (10)

To find out the maximal position of $e^{(3)}$, we can evaluate the function: $f(\delta_1, \delta_2) = \sqrt{\sum_{k=1,2} (\partial C^{(3)} / \partial \delta_k)}$, the values of $(\delta_1, \delta_2)$ on the zero point of $f(\delta_1, \delta_2)$ is the candidate position for the peak of $C^{(3)}(\delta_1, \delta_2)$. In fact, the coincide rate obtained by Eq.(9) and Eq.(10) is only related to the relative separation: $\delta_1$ and $\delta_2$, namely, if all the spatial position that has non-zero photon absorption are considered, changing the relative position between the PSF and the reference point makes no difference to $C^{(3)}$, therefore for simplicity we set $\lambda = 0$.

Here, we numerically investigate the resolution ability of emitter A, B and C based on $C^{(3)}(\delta_1, \delta_2)$ and $f(\delta_1, \delta_2)$. Firstly we investigate the distance between A and B. B and C: $d_1 = 2$, $d_2 = 2.5$, under $\lambda = 0$.

As depicted by the numerical result in Row (b) of Fig.3, when $d_1 = 2$ and $d_2 = 2.5$ the PSF of each emitter can hardly be resolved from the average image, as the revealed by left-sided figure. Combining the middle and the right-sided figures, the corresponding values of $\delta_{1,2}$ to the peak of $C^{(3)}$ is $\delta_1 \approx 2$ and $\delta_2 \approx 2.5$. These results reflect that the distance values can be estimated by the optimal $\delta_1$ and $\delta_2$ which corresponds to the peak position of the coincide distribution. Although the estimation may be biased when $d_{1,2}$ is too small (like the situation with two emitters), because the peak position is determined by the zero-point of $f(\delta_1, \delta_2)$ and the relationship between $d_{1,2}$ and $\delta_{1,2}$ can be complicated, however it still gives approximated values of $d_{1,2}$ under $d_{1,2} \geq 2$, which is the capability that can not be provided by the average image alone.

FIG. 3. (Color online) The resolution ability of three collinear emitters with the help of third-order correlation distributions. (a) From left to right, the first figure is the image of three emitters with $d = 2$, while the mis-alignment parameter $\lambda = 0$. The second figure reveals the coincide rate $C^{(3)}(\delta_1, \delta_2)$ based on Eq.(9). The third figure reveals the function of $f(\delta_1, \delta_2)$. (b) From left to right, the first, the second and the third figure reveal the image, the $C^{(3)}(\delta_1, \delta_2)$ and the $f(\delta_1, \delta_2)$, respectively. With the distance between A and B: $d_1 = 2$, B and C: $d_2 = 2.5$, under $\lambda = 0$.

FIG. 4. (Color online) The resolution ability of three emitters on a two-dimensional plane. The difference in the arrangements of the emitters is revealed in the context in Fig.3. In both Row (a) and Row (b), from left to right, the first figure is the image of three emitters. The second figure is the normalized distribution of $D^{(3)}(\theta_1, \theta_2)$ as revealed by Eq.(12). The third figure is the distribution of coincide rate with respect to separations under the optimal values of angles.
IV. RESOLVING THREE SINGLE-PHOTON EMITTERS ON TWO DIMENSIONS

In the previous section, we showed the enhanced resolution ability of three collinear emitters by third-order correlation detections. In reality, the three emitters can be placed on a two-dimensional plane, the relative positions of the emitters is related to not only distances but also angles. Therefore, to resolve the emitters on two-dimensional plane by third-order correlations, the distances between the triggered detectors are represented by two-dimensional vectors.

Here we still use the two-dimensional detector array depicted in Fig. (1) to perform the third-order correlation detections, while the form of \( g^{(3)} \) is expressed by Eq. (7), with the separations between \( r_{i,j,k} \) described by two-dimensional vectors, the triple-photon coincide rate is described by Eq. (8).

The total third-order coincide events is divided based on the relative positions between the triggered detectors. For example, when the triggered detectors are at positions \( r_{i,j,k} = (x_i, j, k) \), the distances between \( i,j \) and \( j,k \) can be expressed by two vector: \( (\rho_1, \theta_1) \) and \( (\rho_2, \theta_2) \):

\[
\begin{align*}
x_j &= x_i + \rho_1 \cos(\theta_1), \quad y_j = y_i + \rho_1 \sin(\theta_1), \\
x_k &= x_j + \rho_2 \cos(\theta_2), \quad y_k = y_j + \rho_2 \sin(\theta_2),
\end{align*}
\]

where \( \rho_1, \rho_2 \in [0, +\infty) \) and \( \theta_1, \theta_2 \in [-\pi/2, \pi/2] \) for covering all possible events under the integration on \((x_i, y_i)\).

For the purpose of high measuring precision, the size of each detector in the detector array should not be too large. Therefore \( \rho_1, \rho_2 \) and \((x_i, y_i)\) can be considered as continuous variables. As \( g^{(3)} \) has four variables, finding the four optimal variables simultaneously is too computational costly. Hence we firstly try to find out the optimal angles under all possible distance values, then continue to find out the optimal separations based on the knowledge of optimal angles.

In our numerical simulations we investigate two kinds of the emitters’ arrangements. The first kind is when the emitters are on the vertexes of an equilateral triangle, the separations between \( r_{i,j,k} \) described by two-dimensional vectors, the triple-photon coincidence rate is expressed by two-dimensional vectors.

The variables that characterize the structure of the emitters can be achieved by only detecting the third-order correlation functions with \((\theta_1, \theta_2)\) under all possible detector positions and all possible separations of \((\rho_1, \rho_2)\), such distribution is achievable when enough samples of third-order correlations are collected. The normalized distribution of \( D^{(3)}(\theta_1, \theta_2) \) is depicted in the middle figure in Row (a) of Fig (4). The optimal value of \((\theta_1, \theta_2)\) is approximately \((0.35\pi, -0.35\pi)\), which is close to the real value that \((\pi/3, -\pi/3)\) with \(6\%\) estimation error. Under the optimal angles, the distribution with respect to \((\rho_1, \rho_2)\) can be investigated, as the right-sided figure in Row (b) of Fig. (4) depicts. Clearly, the distance between emitter A and C, C and B can be obtained by the optimal values of \( \rho_1 = \rho_2 \approx 2.00\).

In the above numerical investigations, we assume that the separation between the emitters are twice as the width of the approximated Gaussian distribution: \(d/\sigma = 2\). The resolving capability of high order correlation detections require \(s = d/\sigma\) is not too small, for example with only two emitters \(s\) should be greater than \(\sqrt{2}\). When a third emitter is introduced, since the resolution capability depends on whether the optimal values exist, the critical distance is depend on how the emitters are arranged on the two-dimensional plane. From the numerical results depicted in Fig. (4), the variables that characterize the structure of the emitters can be achieved by only detecting the third-order correlation functions with \(s = 2\).

V. SUMMARY AND CONCLUSIONS

In this paper, we propose that the information of the emitters is obtained by the highest order of correlation functions, while the emitters can not be resolved from the image of the first-order correlation function.

In our investigations, the variables that characterize the emitters is not based on a direct measurement, they are based on the final distribution of high order correlations when enough samples of correlation functions are collected. Thus the variables such as distance or angles of the emitters are indirectly obtained by the optimal values which characterize the distributions. Unlike two emitters that the relationship revealed by Eq. (5) is obtained analytically, for three emitters the relationship between the optimal values and the real values which describe the emitters is determined by the actual arrangement of the emitters, which is not the prior knowledge.
Therefore, the estimation error is the estimation bias based on the relationship between the distribution’s peak and the variables that characterize the distribution.

Compared to detect high order correlation functions on the same position, the correlation functions on various detector positions can be detected using a two-dimensional detector array. For precisely obtaining the high order distributions, the number of the detectors is not limited. Therefore enormous data samples should be collected, during the collecting process, a computer program should be used to classify each detection based on the relative positions of the triggered detectors, such classification criteria should cover all the possible measuring outcomes without duplications.

The advantage of using detector array is that the high order correlations on different positions can be detected, which gives clues about the emitters without reconstructing the image of each emitter. This is useful especially when there are several single-photon emitters located in a small area, since reconstructing the image of each emitter is based on the correlation function on the same position, using a two-dimensional detector array is more efficient for photon collections.

In conclusion, we conclude that three single-photon emitters can be resolved by only detecting third-order correlation functions. The variables that characterize the emitters’ spatial distribution is obtainable by finding the optimal positions from the distributions of third-order correlation functions. The only reconstructed image is the distribution of third-order correlation function, and no post-processing of the collected data is performed.

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