Tunnelling Effects in a Brane System and Quantum Hall Physics

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PACS numbers: 11.27; 73.40.Hm
Keywords: String Theory, Quantum Hall Effect

Abstract  We argue that a system of interacting D-branes, general-
ing a recent proposal, can be modelled as a Quantum Hall fluid. We show that tachyon condensation in such a system is equivalent to one particle tunnelling. In a conformal field theory effective description, that induces a transition from a theory with central charge $c = 2$ to a theory with $c = 3/2$, with a corresponding symmetry enhancement.

1Work partially supported by the E C RTN programme HPRN-CT-2000-00131

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1 Introduction

Recently a pioneering paper has appeared [1], proposing a strict analogy between non perturbative string phenomena and Quantum Hall (QH) physics. The model is based on a particular configuration of D-branes of type II A superstring theory compactified in three space dimensions. It consists of a gas of D0 branes on a spherical D2 brane immersed in the background magnetic field generated by a D6 brane at the center of the sphere. It has been argued that the various interactions among the D-branes lead to a (stable) topological configuration, called the Hall soliton. In particular the system made of D0-D2 branes behaves as an incompressible Hall fluid. It is our strong belief that such an analogy can shed light on old and new phenomena in the Quantum Hall Effect (QHE) (see for instance [2]) and on non perturbative string effects, through the unifying description provided by conformal field theory (CFT). In fact two dimensional CFT methods have already been crucial in the world-sheet description of strings [3] and in an effective description of a QH fluid [4], [5] at Jain hierarchical fillings [6], [7] and also non standard ones ($\nu = m/pm + 2$) [8].

The aim of the paper is to extend the picture of ref.[1] by considering a system of two D2 branes carrying D0 branes on their surface. For that system we argue that tachyon condensation due to strings between the D0 and D2 branes takes places and produces a decrease of the central charge of the (twisted) CFT describing this system from the value $c = 2$ to $c = 3/2$, corresponding to a new stable vacuum. In this process some degrees of freedom drop out, carrying with them a $c = 1/2$ contribution to the original value of the central charge. The fields of the final CFT realize, in a special case, a representation of superconformal algebra, giving the raison d’être of its stability.

The role of the tachyon in the D-brane era of string theories has been recognized soon after the work of Polchinski [9]. Both in the case of D-brane-anti-D-brane system [10] and in the interaction between D-branes [11], which do not form a BPS state, there is an instability. In the closed string channel it is due to a non compensation between the forces due to the exchange of fields of the NS-NS sector and those of the R-R sector; in the dual open string channel, it corresponds to a wrong GSO projection on the spectrum of the open strings stretching between the branes, giving rise to a tachyonic ground state. The
picture changes dramatically if the tachyon field develops a non trivial potential and condensation takes place.

Such an idea originally appeared in the study of bound states of parallel D-branes of different dimensions \[12, 13\]. Successively it was shown that the exceeding energy produced in the formation of a D(p+2)-Dp-branes bound state is compensated by the (negative) value of the tachyon potential, which was evaluated in string perturbation theory \[14\]. More recently Sen \[15\] conjectured that in a D-brane-anti-D-brane system of type II superstrings the negative contribution to the energy density coming from the tachyon potential completely cancels the sum of the tensions of the two branes, giving rise to a configuration with zero energy density equivalent to a closed string vacuum and restoring full space-time supersymmetry. Since then a growing amount of evidence has been presented in favour of it (see for instance refs. \[16, 17, 18\]).

The plan of the paper is as follows. In Section 2, after a short review of the brane configuration proposed in \[1\] to describe the two dimensional electron and magnetic flux system of a QH fluid, we show that the vertex operator formalism allows for the CFT description of the collective excitations, which are relevant both for the brane and Hall physics. In Section 3 a system of two D2-branes is considered and a tunnelling effect due to tachyon condensation is analyzed. As a result we show that the central charge of the CFT \[8\] decreases from the value \(c = 2\) to \(c = 3/2\), with a corresponding enhancement of the chiral algebra, which for a particular value of the filling realizes an \(\mathcal{N} = 2\) super CFT. In Section 4 we present some considerations, which motivated our paper, emphasizing the new aspects of tachyon condensation mechanism mutuated from the QHE analogy. Open problems as well as possible directions for future work are also indicated.

2 The physical picture

Following ref. \[1\], let us first consider a D2-brane wrapped around a cylinder with \(K\) coinciding D6-branes placed along the cylinder axis. All other extra dimensions are compactified. Although in superstring theory it would be consistent to put two NS 5-branes at the two cylinder edges, we will not analyze such setting here, i.e. we will consider an infinitely long cylinder, so that some of the physical quantities which we introduce in the following are to be understood as densities. The \(K\) D6-branes act as a string of monopoles and their magnetic field couples,
through a Chern-Simons interaction term, to the worldvolume U(1) gauge field $A$ of the $D2$-brane. As a result there is an induced electric background charge on the D2-brane given by

$$Q = K$$

(1)

In order to make the D2 electrically neutral, one must add $K$ fundamental strings stretched between the D6 and the D2-brane, whose endpoints are effectively seen as charges in the D2-brane worldvolume theory. At this point $N$ delocalised D0-branes must be disposed on the D2-branes, where they appear as a nonvanishing magnetic flux, to energetically stabilize the system. Indeed it is known that D0 and D6-branes repel each other, so that they can counterbalance the attractive force between D6 and D2 brane due to the tension of the stretched strings, which would otherwise make the system collapse. The D0-branes produce $N$ units of magnetic flux on the D2-brane

$$\frac{1}{2\pi} \int_{D2} F = N$$

(2)

where $F = dA$, is the field strength of the D2-brane gauge field.

The D0-branes then behave as an incompressible fluid [1], that is as a Quantum Hall fluid with filling

$$\nu = \frac{K}{N}$$

(3)

If for example we take $K = N_e$ and $N = (2p + 1)N_e$ the whole system, in the large $N_e$ limit, is stable.

It is well known [4], [5] that the ground state wave function of the Hall system, at filling $\nu = 1/2p + 1$, can be described in terms of correlators of vertex operators $V(z)$ given by

$$V(z) = e^{\frac{1}{2}Q(z)}, \quad l = 1, \ldots, 2p + 1.$$  

(4)

where $Q(z)$ is a chiral scalar Fubini field, compactified on a circle of radius $R$, with $R^2 = 1/\nu = 2p + 1$, and having the standard mode expansion

$$Q(z) = q - ip \ln z + \sum_{n \neq 0} \frac{a_n}{n} z^{-n}$$

(5)
with \(a_n\), \(q\) and \(p\) satisfying the commutation relations 
\[
[a_n, a_{n'}] = n\delta_{n+n',0} \quad \text{and} \quad [q, p] = i.
\]

The vertex operators (4) are the primary fields of a CFT with central charge \(c = 1\) and energy momentum tensor
\[
T = -\frac{1}{2} (\partial z Q)^2.
\] (6)

The (analytic part of the) Laughlin ground state wave function \([19]\) can be written as
\[
<0|V_{2p+1}(z_1)\ldots V_{2p+1}(z_n)|0> = \prod_{i<j=1}^{N} (z_i - z_j)^{2p+1}
\] (7)

It is interesting to notice that the vertex operators (4) fully describe the boundary states in a cylinder geometry \([3]\). In fact, if we glue the cylinder boundaries, so obtaining a torus topology, the ground state wave function of the Hall fluid is degenerate, with degeneracy equal to \(2p+1\), which is the number of the primary fields of the theory. Pictorially, any given (internal) primary field propagates in (proper) time, interacting with external electrons.

We point out that such a dyonic formulation of Laughlin wave function has a remarkable interpretation in the string picture. The \(z_i\) represent the positions, on the D2-brane, of the end points of the fundamental strings between the D2 and the D6; they appear as pointlike charges immersed in a background electric charge density of opposite sign. The D0-branes present on the D2-brane feel the attractive forces of the electric charges, so that the string end points and the D0-branes, surrounding them, tend to form collective states, each having one unit of electric charge and \(2p+1\) units of magnetic flux. As a result the D2-D0 system behaves as a dyonic brane, in perfect analogy to the condensate of a QH fluid at fillings \(\nu = 1/2p+1\) \([19,5]\). Although a full field-theoretical treatment of the formation of such collective states would be required, it is noteworthy that this picture is supported by the analysis of the stability of the Hall soliton made in \([20]\). Further analysis of the dynamics of the string endpoints on the worldvolume of the D2-brane indicates that the system actually has a quantum Hall behaviour for fractional filling factors not too small \((\nu \approx 1/3)\) \([21]\). Using the analogy between the two systems as traced above, the collective excitation of one string endpoints and D0-branes, surrounding it, is thus naturally described by the vertex operator \(V(z_i)\) in eq.(4). In the next section we shall apply this
effective description to study a setting of the Hall soliton model containing two D2-branes.

3 Tachyon condensation and tunnelling effects

Let us now consider a configuration of two coaxial cylindrical branes, D2 and D2’, with K D6-branes still sitting along the axis of the cylinder. The K D6-branes induce a background charge on both D2-branes and fundamental strings can be attached to them. The set of fundamental strings is now richer, since it contains also strings stretching between D2 and D2’. When the two D2-branes are superimposed to form a bound state there is an enhancement of the symmetry and the D2-brane world-volume theory is described by a $U(2)$ (Super) Yang-Mills gauge theory. Indeed, short strings between D2 and D2’ can be identified with the non diagonal sector of $U(2)$. (Delocalised) D0-branes, bound to the D2-branes, are represented by a non vanishing magnetic flux given by eq. (2). More generally when $m$ D2-branes form a bound state the integer value $N$ of the flux classifies the $U(m)$ bundles, and using the decomposition $U(m) = (U(1) \times SU(m))/\mathbb{Z}_m$, which corresponds to separating the center of mass (charged sector) from the remaining degrees of freedom (neutral sector), standard arguments [22] show that the neutral sector must have a $\mathbb{Z}_m$ twist, and there is just one current carrying charge and not $m$.

For $m$ D2 branes we consider a realization of an abelian orbifold CFT in terms of $m$ scalar fields with central charge $c = m$, by an inductive procedure acting on a $c = 1$ CFT with a single scalar field [6]. The $m$ scalars can be organized into an untwisted (charged) field and $m - 1$ twisted (neutral) ones. Such a construction has been successively applied to the analysis of paired and parafermionic states of a Quantum Hall Fluid, with non standard fillings $\nu = m/(pm + 2)$, in [8]. The system of two D2-branes we are considering, forming a (quasi-)bound state is expected to be described by a $c = 2$ effective CFT of paired states. More explicitly, starting from a single chiral boson $Q(z)$, given in (5), we get the untwisted field

$$X(z) = \frac{1}{2} (Q(z) + Q(-z)),$$

compactified on a circle of radius $R^2 = 2/m = 1$, representing the charged
sector, and a twisted field

\[ \phi(z) = Q(z) - X(z) = \frac{1}{2} (Q(z) - Q(-z)) \]  \hspace{1cm} (9)

which satisfies the boundary conditions

\[ \phi(e^{\pi i} z) = -\phi(z) \]  \hspace{1cm} (10)

and describes the neutral sector (with no zero-mode). Correspondingly, the Virasoro generator is split in two terms \[23\] both contributing with \( c = 1 \) to the central charge. They are:

\[ T_X(z) = -\frac{1}{2} (\partial_z X(z))^2 \]  \hspace{1cm} (11)

and

\[ T_\phi(z) = -\frac{\partial_z \phi(z)}{4^2} + \frac{1}{16z^2} \]  \hspace{1cm} (12)

Notice the second term in (12), which is typical of twisted scalar fields \[24\]. The primary fields \( V(z) \) of the theory are then written as composite operators

\[ V(z) = U^{(\alpha)}(z)\psi(z), \]  \hspace{1cm} (13)

where the \( U^{(\alpha)}(z) = \frac{1}{\sqrt{2}} e^{i\alpha X(z)} \) with \( \alpha^2 = 2 \), while \( \psi(z) \), the neutral sector, is built out from the twisted scalar fields as \( \psi(z) = \frac{1}{\sqrt{2}} e^{i\sqrt{2}/2 \phi(z)} \). Moreover, the highest weight states (h.w.s.) of the neutral sector contain two types of chiral operators: one which does not change the boundary conditions

\[ \psi_s(z) = \frac{1}{2\sqrt{z}} \left( e^{i\sqrt{2}/2 \phi(z)} + e^{i\sqrt{2}/2 \phi(-z)} \right) \]  \hspace{1cm} (14)

and the other which does,

\[ \psi_u(z) = \frac{1}{2\sqrt{z}} \left( e^{i\sqrt{2}/2 \phi(z)} - e^{i\sqrt{2}/2 \phi(-z)} \right). \]  \hspace{1cm} (15)

In the fermionized version one can see that they correspond to \( c = 1/2 \) Majorana fermions, with periodic (Ramond) or anti-periodic (Neveu-Schwarz) boundary conditions \[24\]. The corresponding bosonized energy-momentum tensors are

\[ T_{\psi_s}(z) = -\frac{1}{8} (\partial \phi)^2 - \frac{1}{32z^2} \cos(2\sqrt{2} \phi) + \frac{1}{32z^2} \]  \hspace{1cm} (16)
and

\[ T_{\phi\phi}(z) = -\frac{1}{8}(\partial\phi)^2 + \frac{1}{32z^2}\cos(2\sqrt{2}\phi) + \frac{1}{32z^2}. \]  

(17)

The description given above can be easily generalized to the entire series with filling \( \nu = 2/2p + 2 \), which is equivalent to add \( 2p \) elementary fluxes (D0-branes) to the system. The corresponding CFT has still \( c = 2 \) but \( \alpha \) has now the values \( \alpha_l = l/2p + 2 \), with \( l = 1, \ldots, 2(2p + 2) \).

Returning to the brane setting, we now argue that there is a tunnelling phenomenon due to D0-branes which can migrate from one D2-brane to the other. To better understand the following physical picture, it may be useful to recall the role played by the tachyon in the formation of a D0-D2 bound state. It is well known [11] that the original (non BPS) configuration of a D0-brane superimposed to a D2-brane relaxes to a (BPS) state of minimal energy consisting of the D2-brane with one unit of magnetic flux. Note that the transition from a non BPS state, represented by a long supermultiplet of the space-time supersymmetry, to a BPS state, which is a short multiplet, implies the disappearance (trasformation) of some of the degrees of freedom. As we mentioned in the introduction, the binding energy can be ascribed to the tachyon potential which generates a new minimum of lower energy [14], [15]. This mechanism is also responsible for the potential barrier which prevents the D0-branes from escaping from the D2-brane in the Hall soliton [1].

Let us now consider the two D2-branes with a small but finite separation. A D0-brane located in between feels an attractive force due to the tachyonic strings going from the D0 to any D2. Thus, the dynamical behaviour of the D0-brane is determined by a non trivial tachyon potential with two minima, symmetric with respect to the center of mass of the two D2-branes and separated by a potential barrier which becomes lower when the D2-branes distance decreases. In this configuration the D0-brane undergoes a tunnelling effect, and we expect that only symmetric states are selected out. In our effective CFT description this is due to the cosine term in the energy momentum tensor (16), which produces a lowering of the vacuum energy of the Ramond sector, \( \text{i.e.} \) of the \( (Z_2 \text{ invariant}) \) degrees of freedom which survive the tunneling effect, leading then to a CFT with total central charge \( c = 3/2 \).

The new vacuum is annihilated by the total Virasoro generator \( L_0 = L_0^X + L_0^0 \), and for the special case of \( p = 1 \), corresponding to a filling \( \nu = 1/2 \), it is also
annihilated by the fermionic zero modes of h.w.s. with conformal dimension \( h = 3/2 \). In that case the resulting CFT is actually \( \mathcal{N} = 2 \) superconformal theory, with the supercurrent given by

\[
J(z) = \partial X(z) + \frac{i}{\sqrt{2}}(\theta G^-(z) + \bar{\theta} G^+(z)) + \theta \bar{\theta} T(z)
\]  

(18)

where \( G^\pm(z) = \mathcal{U}_c^{\pm2}\psi_m(z) \), \( T(z) = T_X(z) + T_{\psi_s}(z) \) and \( \theta \) and \( \bar{\theta} \) are Grassmann variables. That is a quite important result, since as a consequence of the tunnelling, a new stable D0-D2 bound state is formed, described by a superconformal theory, which then guarantees the stability of the system. We consider that as a strong support for the consistency of our proposal.

Although it is not the aim of this paper to make a detailed comparison of our physical picture with the usual analysis of tachyon condensation in open strings as a boundary CFT (see [18] and references therein), some remarks are now in order. Let us consider a string scalar field, \( Y(\sigma) \) with \( \sigma \in [0, \pi] \), interacting with a boundary tachyon potential of the form

\[
\lambda \cos(\beta Y(\sigma))|_{\sigma=0},
\]  

(19)

where \( \lambda \) and \( \beta \) are real parameters. Boundary CFT analysis [19] shows that if the perturbation is relevant, as in the tachyon case, the coupling constant \( \lambda \) flows to an IR fixed point, pinning \( Y(0) \) to the values \( Y_k = k\pi/\beta \) with integer \( k \), corresponding to the minima of the potential, which are to be identified with the positions of D-branes along \( Y \). If we now consider \( Y \) as the radial coordinate of the open tachyonic strings of our model, we notice that the minima of the tachyon potential have also a continuous degeneracy, which is due to the global symmetries of the system, in fact, there is no dependence on the variables \( z \), tangential to the D2-branes. The \( z \) are then continuous moduli parametrizing the vacua \( Y(z) = Y_k \). In a low energy approximation, the dynamics reduces to the motion on the moduli space and we can add (at lowest order in derivatives) a kinetic term of the form \((\partial Y(z)/\partial z)^2\). In this way we can reconstruct the effective CFT of the neutral sector, which then plays a double role: on one side it describes a boundary interaction for the fundamental strings ending on the D2-branes, on the other side, it is a "bulk" effective theory on the D2 brane world volume.

As we have seen, a cosine term of the form (19), with the correct periodicity value, automatically appears in the energy-momentum tensor (14) of the
(twisted) neutral sector. In QH physics, such an interaction term goes under the name of interlayer one particle tunnelling (see for instance [27] and references therein), and $\lambda$ in (19) is the tunnelling amplitude. Other attempts to describe the transition from $c = 2$ to $c = 3/2$ have been developed for the QH fluid. In particular the models proposed in [27], [28] share with our analysis the relevance of tunneling processes in determining such a transition, as well as the correct reduction of degrees of freedom (i.e. the elimination of one Majorana fermion).

For $p > 1$ the theory is not supersymmetric, but nonetheless it is invariant under an enlarged symmetry, whose role, both in string theory and in the QHE, is under study.

4 Conclusions and outlook

The Quantum Hall soliton introduced in [1] is a powerful source of inspiration for those who think that the two fields of string theory and Quantum Hall physics have many contact points, through the unifying description provided by CFT. In that framework the vertex operators give a simple interpretation of the collective excitations relevant to the physics of the QHE, both for Jain hierarchical fillings and for non-standard ones. We have pushed forward the analogy between string theory and the QHE by analyzing tachyon condensation in a non BPS system of D-branes. In fact we find that this phenomenon is analogous to the tunnelling between two layers in the QHE. The effective CFT description given in [8] fits very well in this setting and shows that tachyon condensation leads to a new stable vacuum with higher symmetry.

The main novelty of the mechanism of tachyon condensation proposed here is the role played by the magnetic flux (carried by the D0-branes), and the setting of two (or more generally $m$) D2-branes. As it has been discussed, the description of the dyonic Laughlin particles in terms of vertices for the Hall plateaus is crucial for understanding the analogy between the brane setting and the QH system. Moreover, for the setting of at least two (or $m > 2$) D2-branes one can give a description in terms of Fubini scalar fields (and corresponding vertices) with different boundary conditions: the field with periodic b.c. (the charged sector) and the one (or $m - 1$) with twisted $Z_2$ ($Z_m$) phases (the neutral sector). Altogether they build up the complete set of vertices (h.w.s) of the relevant CFT. In this context the role of the $Z_2$ ($Z_m$) symmetry acquires a
peculiar relevance; such a discrete symmetry couples the charged and the neutral vertices, hinting to a more general description provided by Matrix Theory with $U(2) (U(m))$ symmetry \cite{29}. Nevertheless, an interesting result has already been found: for the setting of two interacting D2-branes we have obtained an effective description in terms of a two-dimensional $\mathcal{N} = 2$ super CFT. The generalization to the case of $m > 2$ should reveal new interesting aspects of the effective CFT description of $m$ D2-brane setting.

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