Analysis of Rayleigh-Bénard convection with thermal volumetric radiation using Lattice Boltzmann Formulation

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Abstract
The interactions between Transient Rayleigh-Bénard convection and volumetric radiation are investigated by means of the lattice Boltzmann method (LBM) performed for a two dimensional participating Rayleigh-Bénard cell. Given that, the analysis of the transient convection-radiation finds applications in combustion chambers, rocket propulsion systems, the design of reactors, heat pipes, etc. in this paper, we extended the mesoscopic Lattice Boltzmann model for analyzing the coupled engineering problem Rayleigh-Bénard Convection with thermal radiation. In order to highlight and assess the aim and the computational advantage of computing the radiative information too using the LBM and to demonstrate the workability of the LBM to a such coupled problem in two dimensional media, first, transient Rayleigh-Bénard convection is solved using the lattice Boltzmann method (LBM) and then are compared with those available in the literature. The coupled transient case, Rayleigh-Bénard convection-radiation in participating media is extended, where LBM, is used, both to calculate the volumetric radiative information needed for the energy equation, which is solved using the LBM. Results of this recent approach LBM-LBM work are compared with those available in the literature. In all cases, good agreement has been obtained. Indeed, the recent numerical approach is found to be efficient, accurate, and numerically stable for the simulation of fluid flows with heat and mass transfer in presence of volumetric radiation in participating medium. The steady state stream-functions, isotherms and pressure distribution were compared with results available in the literature. It is found that the recent approach provides accurate results and it is computationally more efficient than others CFD numerical methods which approve the workability of this recent approach and this make it a new potential computational tool for solving a large class of engineering problems.

Key words: Heat transfer, LBM, RTE, Transient RB convection, Volumetric radiation, Participating media

1. Introduction

Understanding unsteady Rayleigh–Bénard convection (RBC) with thermal radiation, is one of the fundamental problems in fluid dynamics. For many of the thermal systems, it is found for example in boilers, furnaces, IC engines, gas turbines, reentry vehicles, etc., its accurate consideration is not only important in their designs, but in their operations too.

The numerical analysis of convection radiation requires solving a set of governing partial differential and integro-differential equations. Knowing that, in any combined mode heat transfer problem involving volumetric radiation, computation of the radiative information is more time-consuming than that for solving the energy equation because of the angular dependence (Mishra et al., 2014a, 2014b), so it is not uncommon to select different methods for
computing the different dependent variables. Thus, development of methods, remain an ongoing phenomenon in the field of radiative heat transfer. For example, in a combined mode transient convection–radiation problem, the radiative information, which appears in the form of the divergence of radiative heat flux in the energy equation can be computed using various methods such as the Monte Carlo method (MCM) (Modest, 2003), the discrete transfer method (DTM) (Cumber, 1995, Mishra et al., 2003), the discrete ordinates method (DOM) (Jamaluddin and Smith, 1988), the finite-volume method (FVM) (Mishra et al., 2014a, Chui et al., 1992, Patankar and Chai, 2000, Mathur and Murthy, 1998, Kim, 2008, Kim and Baek, 2005), the collapsed dimension method (CDM) (Mishra et al., 2003), etc.

However, in multi-dimensional geometry, in a combined mode problem, even with the FVM, the computational time becomes exorbitant (Mishra et al., 2014b). So, efforts toward development of efficient methods continue. In the recent past, the lattice Boltzmann method (LBM) has emerged as an efficient method to analyze a vast range of problems in fluid flow and heat transfer (Chen and Zhang, 1999, Succi, 2001, Wang et al., 2013, Jiaung et al., 2001, Lankadasu and Mishra, 2005, Chaabane et al., 2011a, 2011b, Mishra and Mondal, 2009, Mishra et al., 2009, Chaabane et al., 2011c, Asinari et al., 2010 and Di Rienzo et al., 2011). Unlike conventional methods, which solve the discretized macroscopic Navier-Stokes equations, the LBM uses simple microscopic kinetic models to simulate complex transport phenomena.

So, this surge in applications of the LBM, compared to existing CFD solvers, is owing to its simple calculation procedure, mesoscopic nature, simple and efficient implementation for parallel computation, easy, robust straightforward and efficient handing of complex geometry and boundary conditions, high computational performance with regard to stability, accuracy and precision and memory overhead over other methods (Succi, 2001).

The usage of the LBM to formulate and solve different types of heat transfer problems involving volumetric radiation in different geometries has been extended (Mishra et al., 2014a, Chen and Zhang, 1999, Succi, 2001, Wang et al., 2013, Jiaung et al., 2001, Lankadasu and Mishra, 2005, Chaabane et al, 2011a, 2011b, Mishra and Mondal, 2009, Mishra et al., 2009, Chaabane et al, 2011c). However, in all such problems, although the radiative information was computed using the conventional RTE solvers and in terms of formulation and computational time, the solution of the energy equation by the LBM was encouraging.

However, as the complexity of the problem increases, with LBM as the solvers for, say the momentum and energy equations, and one of the CFD methods for computation of radiative information, for the reasons stated above, the computational time becomes exorbitant (Mishra et al., 2014a, 2014b).

Encouraged by the successful application of the LBM in solving the energy equations of a wide range of problems, more recently, Asinari et al. (Asinari et al., 2010 and Di Rienzo et al., 2011) extended the application of the LBM for the computation of radiative information in the aim to ease above mentioned problem. LBM is extended to solve a benchmark radiative equilibrium problem involving a 2-D rectangular enclosure (Asinari et al., 2010 and Di Rienzo et al., 2011). The LBM was found to have an edge over the FVM and to be computationally much more efficient than conventional CFD methods (Asinari et al., 2010 and Di Rienzo et al., 2011).

A step ahead, to further extend the usage of the LBM, the present work deals with the solution of a combined mode transient Rayleigh Bénard convection and radiation heat transfer problem in a 2D participating rectangular geometry containing an absorbing, emitting and scattering medium in which both the computations of the radiative information and the solution of Navier-Stokes equations are done using the LBM. In solving this set of governing partial differential and integro-differential equations, periodic boundary conditions have been used for both hydrodynamic and thermal fields for the vertical walls and no-slip isothermal boundary conditions were imposed along the bottom and the top boundaries (Mishra et al., 2014a).

2. Formulation and Numerical implementation
2.1 Lattice Boltzmann equation for density and velocity fields

For computation of density and velocity fields, the governing lattice Boltzmann equation is given by (Succi, 2001):

\[
\frac{f_k(\vec{r} + \vec{c}_k \Delta t, t + \Delta t) - f_k(\vec{r}, t)}{\tau_v} + \frac{\Delta t}{\tau_v} \left[ f_k(\vec{r}, t) - f_k^{eq}(\vec{r}, t) \right] F^k = 0 \quad k = 0, ..., b
\]  

(1)

where \(f_k\) are the particle distribution function defined for the finite set of the discrete particle velocity vectors \(\vec{c}_k\). The collision term \(\Omega_k\) on the right-hand side of Eq. (1) uses the so called BGK approximation (Wang et al., 2013, Jiaung et
al., 2001). $f_k^{eq}$ is the local equilibrium distribution function that has an appropriately prescribed functional dependence on the local hydrodynamic properties and $\tau_v$ is the relaxation time. $F$ represents the external force term given by (Mishra et al., 2014b).

$$F = \left[ \frac{\rho \beta_t g (T - T_m) j}{RT} \right] f_k^{eq}$$  (2)

Where the unit vector $\vec{j}$ is in a direction opposite to gravity, $T_m$ is the mean temperature, $g$ is the gravity acceleration, $\beta_t$ is the volumetric thermal expansion coefficient, $R$ is the ideal gas constant and $\rho$ is the density of the fluid at the mean temperature $T_m$.

For the D2Q9 lattice (Figure 1b) used in the present work, the relaxation time $\tau_v$ is defined as (Chaabane et al., 2011a, 2011b):

$$\tau_v = \frac{3\nu}{2c^2 \Delta t}$$  (3)

Where $c$ is the lattice speed $c = \sqrt{3RT}$.

The kinetic viscosity $\nu$ appearing in Eq. (3) is computed from the Prandtl number $Pr = \nu / \alpha$ and Rayleigh number $Ra = g \beta_t (T_h - T_c) H^3 / \alpha \nu$ . $T_h$ is the hot wall temperature, $T_c$ is the cold wall temperature, $\alpha$ is the thermal diffusivity, $g$ is the acceleration due to gravity, $H$ is the height of the cavity. It is to be noted that viscosity is selected to insure that Mach number is within the limit of incompressible flow (Mishra et al., 2014a, 2014b).

The nine velocities $\vec{c}_k$ in the D2Q9 lattice are given by (Fig.1),

$$\vec{c}_0 = (0,0)$$  (4)

$$\vec{c}_k = (\cos((k-1)\pi / 2), \sin((k-1)\pi / 2)) c \quad \text{for} \quad k = 1 - 4$$  (5)

$$\vec{c}_k = \sqrt{2}(\cos((2k-1)\pi / 4), \sin((2k-1)\pi / 4)) c \quad \text{for} \quad k = 5 - 8$$  (6)

The equilibrium function for the density distribution is given by:

$$f_k^{eq} = w_k \rho \left( 1 + \frac{\vec{c}_k \cdot \vec{u}}{c_s^2} + \frac{(\vec{c}_k \cdot \vec{u})^2}{2c_s^4} - \frac{\vec{u} \cdot \vec{u}}{2c_s^2} \right)$$  (7)

c$_s$ is the speed of the sound

$$c_s = \sqrt{RT}$$  (8)

The macroscopic density $\rho$ and the velocity $\vec{u}$ are calculated as follow:

$$\rho(\vec{r},t) = \sum_k f_k(\vec{r},t)$$  (9)

$$\vec{u}(\vec{r},t) = \sum_k \vec{c}_k f_k(\vec{r},t) / \rho(\vec{r},t)$$  (10)
2.2 Lattice Boltzmann Equation for Thermal Field

In order to compute the thermal field, in the presence of volumetric radiation, the governing lattice Boltzmann equation is given by (Chaabane et al., 2011a, 2011b, Mishra and Mondal, 2009, Mishra et al., 2009):

\[
g_k(x + \Delta x, y + \Delta y, t + \Delta t) = (1 - \frac{\Delta t}{\tau_f})g_k(x, y, t) + (\frac{\Delta t}{\tau_f}) g_k^eq(x, y, t) - (\frac{\Delta t}{\rho C_p}) w_k \nabla \overline{q_R^k} \tag{11}
\]

\[
\tau_f = \frac{1}{2} + \frac{3\alpha}{c^2\Delta t} \tag{12}
\]

where \(g_k\) is the particle distribution function denoting the evolution of the internal energy, \(\alpha\) is the thermal diffusivity, \(\tau_f\) is the relaxation time and \(\overline{q_R^k}\) is the radiative heat flux. \(b\) is the number of particle distribution functions considered in a given lattice. Here \(\alpha = k / \rho C_p\) is the thermal diffusivity. \(g_k^eq\) is the equilibrium particle distribution function given by (Mishra et al., 2014b):

\[
g_k^eq = w_k \rho e(1 + \frac{\vec{c}_k \cdot \vec{u}}{c_s^2}) \tag{15}
\]

The internal energy \(e\) is computed from the following equation:

\[
\rho(\vec{r}, t) e = \sum_k g_k(\vec{r}, t) \tag{16}
\]

Temperature is calculated from the equation of state, \(e = RT\)

\[
T(\vec{r}, t) = \sum_k g_k(\vec{r}, t) \tag{17}
\]

The divergence of radiative heat flux appearing in Eq. (11) is given by

\[
\nabla \overline{q_R^k} = k_a [4\pi(\frac{\sigma T^4}{4}) - G] \tag{18}
\]

Where \(k_a\), is the absorption coefficient and \(G\) is the incident radiant energy.

2.3 Lattice Boltzmann Equation for RTE

LBM has emerged as an efficient method to analyze a vast range of problems in fluid flow and heat transfer of a wide range of problems (Wang et al., 2013, Jiaung et al., 2001, Lankadasu and Mishra, 2005, Chaabane et al., 2011a, 2011b, (Mishra and Mondal, 2009), Mishra et al., 2009, Chaabane et al., 2011c), very recently, LBM is extended to solve a benchmark radiative equilibrium problem involving a 2-D rectangular enclosure (Asinari et al., 2010 and Di Rienzo et al., 2011). The LBM was found to have an edge over others CFD numerical approaches. The starting point for deriving the LBM formulation is the Radiative Transfer Equation (RTE). In any direction \(s = (\sin \gamma \cos \delta) \vec{s} + (\sin \gamma \sin \delta) \vec{j} + (\cos \delta) \vec{k}\) the RTE is given by (Asinari et al., 2010 and Di Rienzo et al., 2011).

\[
\frac{dI}{ds} = -\beta I + \beta(1 - \omega)(\frac{\sigma T^4}{4}) + \frac{\beta \omega}{4\pi} G \tag{19}
\]

Where \(G\) is the incident radiation, \(I\) is the intensity, \(\beta\) is the extinction coefficient, \(\omega\) is the scattering albedo and \(s\) is the geometric distance in the direction \(\vec{s}\). Assuming local radiative equilibrium (incident radiation \(G\) equals
volumetric emission $4\pi I_b$ (where $I_b = 4T^4 / \pi$ is the black body intensity), eq.(19) becomes

$$\frac{dI}{ds} = \beta \left( \frac{G}{4\pi} - I \right)$$  \hspace{1cm} \text{(20)}$$

For discrete directions, Eq. (20) is written as

$$\frac{dI_i}{ds} = s_i \nabla I_i = \beta \left( \frac{G}{4\pi} - I_i \right)$$  \hspace{1cm} \text{(21)}$$

Where $I_i$ is the intensity evaluated along the $i$-th azimuthal direction. Azimuthal angle is discretized by introducing a finite number of discrete velocities $V_{\lambda,i} (\lambda = x, y)$, lying on the lattice, whose magnitude is

$$|V_i| = U \left( \sqrt{V_{x,i}^2 + V_{y,i}^2} \right)$$  \hspace{1cm} \text{(22)}$$

Where $U = \Delta x / \Delta t$. The magnitude of the lattice velocities is defined as $V_i = \varepsilon_i U$, where $\varepsilon_i$ is a constant depending on the energy shell of the considered velocity.

In the RTE time dependence of the intensity is usually neglected in radiation problems (Asinari et al., 2010 and Di Rienzo et al., 2011). However it is convenient to consider a pseudo-transient equation as the starting point of the LBM formalism. The transient RTE is rewritten as

$$\frac{1}{c} \frac{\partial I_i}{\partial t} + s_i \nabla I_i = \beta \left( \frac{G}{4\pi} - I_i \right)$$  \hspace{1cm} \text{(23)}$$

Where $c$, is the (fictitious) speed of light. We assume that along each discrete direction, the speed of light to be equal to the corresponding microscopic velocity $c = V_i$. Equation (23) is rewritten as

$$\frac{1}{V_i} \frac{\partial I_i}{\partial t} + s_i \nabla I_i = \beta \left( \frac{G}{4\pi} - I_i \right)$$  \hspace{1cm} \text{(24)}$$

Since we are dealing with a 2D problem, $\partial I_i / \partial z = 0$, equation (24) is as follows

$$\frac{\partial I_i}{\partial t} + V_i \nabla I_i = \frac{DI_i}{Dt} = V_i \beta \left( \frac{G}{4\pi} - I_i \right)$$  \hspace{1cm} \text{(25)}$$

Where $i = 1, \ldots, N_\delta$, $N_\delta$ and $N_\gamma$ are the total number of discrete subdivisions for the azimuthal and polar angles, respectively. $DI_i/Dt$ represent the Lagrangian derivative. Now we can return to the LBM formulation. Equation (24) is rewritten as (Asinari et al., 2010 and Di Rienzo et al., 2011):

$$\frac{1}{\Delta t} [I_i(x_n + V_i \Delta t, t + \Delta t) - I_i(x_n, t)] = \omega_i [I_i^\text{eq}(x_n, t)] - I_i(x_n, t)] + O(\Delta t) + O(V_i \Delta t)$$  \hspace{1cm} \text{(26)}$$

Where the relaxation frequency, $\omega_i = \beta V_i$. In the LBM terminology, we have $I_i$ is the particle distribution function. $I_i^\text{eq}$ is the equilibrium distribution function defined as (Di Rienzo et al., 2011):

$$I_i^\text{eq} = \frac{G}{4\pi} = \frac{1}{4\pi} \int_{\Omega=0}^{4\pi} I(x_n, t, \delta, \gamma)d\Omega = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi I(x_n, t, \delta, \gamma) \sin \gamma d\gamma d\delta$$  \hspace{1cm} \text{(27)}$$

Where $G$ is the incident radiation which in the LBM is computed numerically as

$$G = \int_0^{2\pi} \int_0^\pi I(x_n, t, \delta, \gamma) \sin \gamma d\gamma d\delta = 4\pi \sum_{i=1}^{N\delta} W_{gi} I_i$$  \hspace{1cm} \text{(28)}$$

With

$$W_{gi} = \frac{1}{4\pi} \int_0^\pi \sin \gamma d\gamma \int_{\delta-\Delta\delta/2}^{\delta+\Delta\delta/2} d\delta \equiv \frac{\Delta\delta}{2\pi}$$  \hspace{1cm} \text{(29)}$$

It is shown that, $D_2Q_0$ LBE system is adequate for modeling fluid mechanics problems in two dimensions, but in
the case of RTEs a higher number of angular discretization schemes may be required where the non-diffusive transport mechanism of radiation is important. Therefore, higher order angular discretization scheme should be adopted. A detailed formulation is provided in (Mishra et al., 2014b), and is not repeated here for the sake of brevity.

The expression for the radiative heat flux is given by

\[ q_R = \int_{\delta-\Delta\delta/2}^{\delta+\Delta\delta/2} I(n,s) \sin \gamma d\gamma d\delta \]  
\[ (30) \]

For a diffuse-gray boundary having temperature \( T_b \) and emissivity \( \varepsilon_b \), the boundary intensity \( I_b \) is computed from

\[ I_b = \frac{\varepsilon_b \sigma T_b^4}{\pi} + \left( \frac{1 - \varepsilon_b}{\pi} \right) \int_{|n.s|<\delta} I(n,s) |n.s| d\gamma d\delta \]  
\[ (31) \]

3. Results and discussion

To ensure that the LBM-LBM code is working properly, the problem of unsteady Rayleigh-Bénard without and with volumetric radiation in a rectangular participating cavity is modelled and compared with the results reported by Wang 2013 et al. (Wang et al., 2013, Asinari et al., 2010 and Di Rienzo et al., 2011, Mishra et al., 2014a, 2014b).

Periodic boundary conditions have been used for both hydrodynamic and thermal fields for vertical walls and no-slip isothermal boundary conditions were imposed along the bottom and the top boundaries in order to solve this set of governing partial differential and integro-differential equations.

3.1 Radiation validation

First, we validate the recent approach for solving the volumetric radiation information in a two dimensional enclosure containing an absorbing, emitting and scattering medium presented by A. F. Di Rienzo and P. Asinari et al. (Asinari et al., 2010 and Di Rienzo et al., 2011) with the Lattice Boltzmann Method. The same benchmark considered in Di Rienzo et al., 2011 is revalidated in the present work with LBM method. In figure 2, the distribution of the dimensionless temperature in the square enclosure is given.
3.2 Rayleigh-Bénard convection without radiation

The results for the natural convection without volumetric radiation in a rectangular cavity with LBM are validated against the results available in literature (Wang et al., 2013). Contours of flow fields (the stream-function, the isotherms, and the pressure) of the Rayleigh-Bénard convection in the rectangle for Pr=0.71 and Ra=5000 are depicted in figure 3. It is seen that the present results are in very good agreement with those available in literature.
3.3 Rayleigh-Bénard convection with radiation

As it can be seen in previous results, in each case, the results are considerably consistent with those in the literature and these comparisons corroborate the employed numerical method, which can produce reliable results. Having validated convection, radiation separately, we are now at a stage to present our brand results of LBM-LBM for a Rayleigh-Bénard convection with volumetric radiation. Besides, our recent approach will be validated against results presented in (Mishra et al., 2014b) where momentum and energy equations are formulated and solved using the lattice Boltzmann method (LBM) but the volumetric radiative information needed in the energy equation is computed using the Finite Volume Method (FVM). Figure 4 shows the streamlines and isotherms contours for Pr=0.71, Ra=25000, β=1, θ = 0, time step=20000 and the value of the convection-radiation parameter RC defined as \( k /\sigma HT_0^3 \) was taken to be 250. In all cases a good agreement was achieved.

After the anticlockwise motion of fluid has evolved observed at an early stage, convective cells are fully formed and their effect is visible in the isotherms and streamlines. As shown in figure 4, hot fluids near the bottom wall flow upward and increase the temperature in the central portion of the cavity, while cold fluids near the top wall flow downward and decrease the temperature near the side boundaries. For a Rayleigh number Ra=25000, two trends were observed for the temperature distribution: enhanced mixing of the hot and cold fluids, and an increase in the temperature gradients near the bottom and top boundaries. Both trends enhance the heat transfer in the cavity.
Fig. 4: Streamlines and isotherms for $Pr=0.71$, $Ra=25000$, $\beta=1$, $\omega=0$, time step=20000 and $RC=250$, (a) reference (Mishra et al., 2014b), (b) present work.
boundary layers are
To investigate radiation effects on the flow field, we study the particular case of occur in a region close to the horizontal cold wall, indicating a high temperature gradient there.
steady state become almost parallel to the hot and cold wall in the presence of radiation. We notice that dense isotherms inside the cavity which drives the Rayleigh-Bénard convection, and the energy in the flow field can be transferred more efficiently than for a single mode heat transfer. As a result, the flow is stabilized. This results in a reduction in the temperature gradient inside the cavity which drives the Rayleigh-Bénard convection. With radiation, isotherms present a significant change as the convection radiation parameter RC increase. And this confirms the known theoretical result that isotherms at steady state become almost parallel to the hot and cold wall in the presence of radiation. We notice that dense isotherms occur in a region close to the horizontal cold wall, indicating a high temperature gradient there.
To investigate radiation effects on the flow field, we study the particular case of $Ra=25000$ with a value of convection radiation parameter RC=250. In general, the horizontal side wall creates a resistance to the convection flow as boundary layers are formed near the horizontal side walls. The no-slip condition near the side wall causes a hindrance to the flow and the viscous forces increase, resulting in a higher critical Rayleigh number for the onset of convection. Radiation results in heat transfer from the horizontal sidewalls into the fluid domain. This results in a reduction in the

Fig. 5: Isotherms, Horizontal velocity, vertical velocity, streamlines and pressure profiles for $Pr=0.71$, $Ra=25000$, $\beta=1$ at time step=20000 for RC=0 (a) and (b) RC=250.
temperature gradient inside the cavity which drives the Rayleigh-Bénard convection. Thus, the onset of convection is delayed due to radiation. So, as the convection radiation parameter $RC$ increases from 0 to 250, the effect of radiation increases and the flow is therefore stabilized by the presence of the radiative source. In addition to the qualitative information about temperature and velocity fields provided in the last section, a quantitative overview of temperature variations along the channel depth $y/H$ at $x/H=1/4$ is highlighted for $Ra=25000$ in figure 6. Results were found in good agreement with literature (Mishra et al., 2014b).

Fig. 6: Variations of temperature along the y direction at $x=X/4$ for extinction coefficient $\beta=1$, scattering albedo $\omega = 0$, convection-radiation parameter $RC= 250$ for $Ra = 25000$ at time step = 20000

4. Conclusions

Transient Rayleigh-Bénard convection in the presence of volumetric radiation in a rectangular cavity containing an absorbing, emitting and scattering medium was analysed using the recent approach LBM-LBM.

Without the effect of radiation, the results for Rayleigh-Bénard convection were compared with those available in literature. The results were found in good agreements. With the effects of radiation, the non-hybrid code was validated for different cluster values with the results available in literature. All results were compared with the results of literature and were found to provide accurate results. In the presence of radiation, we provide also the isotherms, streamlines, pressure distributions. The results found using the new algorithm LBM-LBM which, to the knowledge of the authors, is applied at the first time to such coupled heat flow process, were in good agreement with those of literature.

This non coupled and non-hybrid numerical approach has the advantage of simplicity in parallel computing platforms and can be extended to other complex engineering heat and flow transfer problems including more sophisticated geometry.

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