THE IMPACT OF UNRESOLVED TURBULENCE ON THE ESCAPE FRACTION OF LYMAN CONTINUUM PHOTONS

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Received 2016 October 10; revised 2016 November 1; accepted 2016 November 1; published 2016 November 14

ABSTRACT

We investigate the relation between the turbulent Mach number ($\mathcal{M}$) and the escape fraction of Lyman continuum photons ($f_{\text{esc}}$) in high-redshift galaxies. Approximating the turbulence as isothermal and isotropic, we show that the increase in the variance in column densities from $\mathcal{M} = 1$ to $\mathcal{M} = 10$ causes $f_{\text{esc}}$ to increase by $\approx 25\%$, and the increase from $\mathcal{M} = 1$ to $\mathcal{M} = 20$ causes $f_{\text{esc}}$ to increase by $\approx 50\%$ for a medium with opacity $\tau \approx 1$. At a fixed Mach number, the correction factor for escape fraction relative to a constant column density case scales exponentially with the opacity in the cell, which has a large impact for simulated star-forming regions. Furthermore, in simulations of isotropic turbulence with full atomic/ionic cooling and chemistry, the fraction of HI drops by a factor of $\approx 2.5$ at $\mathcal{M} \approx 10$ even when the mean temperature is $\approx 5 \times 10^4$ K. If turbulence is unresolved, these effects together enhance $f_{\text{esc}}$ by a factor $> 3$ at Mach numbers above 10. Such Mach numbers are common at high redshifts where vigorous turbulence is driven by supernovae, gravitational instabilities, and merger activity, as shown both by numerical simulations and observations. These results, if implemented in the current hydrodynamical cosmological simulations to account for unresolved turbulence, can boost the theoretical predictions of the Lyman Continuum photon escape fraction and further constrain the sources of reionization.

Key words: galaxies: high-redshift – galaxies: ISM – radiative transfer – turbulence

1. INTRODUCTION

Observations of high-redshift quasars indicate that the universe was reionized by $z = 6$ (Fan et al. 2002, 2006; Becker et al. 2007; Mortlock et al. 2011). Similarly, Planck satellite measurements constrain the reionization optical depth to be $\tau = 0.066 \pm 0.016$, corresponding to a redshift of $z \approx 8.8$ (Planck Collaboration et al. 2016). However, there is a debate as to whether dwarf galaxies or quasars provided the ionizing photons that caused this transition (Volonteri & Gnedin 2009; Grissom et al. 2014). An escape fraction of ionizing photons, $f_{\text{esc}}$, above $10\%$ may be necessary to explain reionization by dwarf galaxies alone (Wise & Cen 2009; Kim et al. 2013), while still higher levels of $z > 9$ ionizing photons are necessary to explain the observed electron optical depth observed by Planck.

Although the correlation between $f_{\text{esc}}$, halo mass, and redshift has been studied by various groups, the results are often inconsistent, with a wide range of $f_{\text{esc}}$ values reported in the literature (see Xu et al. 2016 and references therein). Gnedin et al. (2008), for example, find that $f_{\text{esc}}$ increases in galaxies with higher star formation rates because in these galaxies young stars are found at the edge of the HI disks and are therefore not enshrouded in the disk’s gas. However, they also find that $f_{\text{esc}}$ is less than 0.01 for systems with total masses below $5 \times 10^{10} M_\odot$, a number that is very difficult to reconcile with reionization measurements (Gnedin 2008). Similarly, for high-redshift systems, Wood & Loeb (2000) argued that higher disk densities at high redshifts would lead to a drop in the escape fraction, with $f_{\text{esc}} \approx 0.01$ for sources at $z \approx 10$. On the other hand, recent high-resolution zoom simulations by Kimm & Cen (2014) suggest that runaway OB stars and supernovae may lead to escape fractions as high as $f_{\text{esc}} \approx 14\%$.

A key question regarding these estimates is how unresolved turbulent structures impact the escape fraction of LyC photons. Accounting for these structures would mean capturing the underlying probability distribution function (PDF) of gas density and correcting the column density distribution of gas in the host galaxies containing massive stars. To achieve this, one would need to resolve the driving scale of the turbulence ($L_{\text{drive}}$) by at least 50 elements (e.g., Gray et al. 2015).

Estimating $L_{\text{drive}}$ is not an easy task, as it could be set by many plausible mechanisms, including supernova feedback, gravitational instability, the infall of gas clumps, and thermal instabilities (Wise & Abel 2007; Wise et al. 2008; Green et al. 2010; Krumholz & Burkhart 2016). For example, Chepurnov et al. (2015) measure a driving scale of 2.3 kpc based on the velocity power spectrum of the Small Magellanic Cloud, which implies that large-scale galaxy–galaxy interactions drive the turbulence. However, Martinez et al. (2016) find $L_{\text{drive}} \approx 100$ pc in their supernova-driven vertically stratified medium, consistent with the observations of Dib et al. (2009), which find a $L_{\text{drive}} \approx 100$–800 pc based on the orientation of molecular clouds with respect to the galactic plane. Furthermore, it is expected that the driving scale decreases at higher redshifts because galaxies become smaller (Ferguson et al. 2004) and clumpy in their morphology (Guo et al. 2015). Therefore, it would be rather difficult to claim that turbulence is resolved in simulations of high-redshift galaxy formation.

Failing to resolve this structure would smear out the PDF of the supersonic turbulent gas, which would otherwise approximately follow a lognormal distribution with a variance that is a function of the turbulent Mach number (Vazquez-Semadeni 1994; Ostriker et al. 2001; Hopkins 2013; Nolan et al. 2015). Conservation of mass implies that at higher $\mathcal{M}$, the peak of the density PDF shifts to lower values and the PDF becomes broader. This increase in the variance of the PDF increases the relative fraction of very low column densities, which leads to low column density sightlines that are ideal for the escape of ionizing photons.

Furthermore, recent simulations of non-isothermal isotropic turbulence show that even at temperatures as low as...
$T \approx 5 \times 10^3$ K, the HI fraction drops by a factor of $\approx 2.5$ for the studied range of Mach number $1 < \mathcal{M} < 12$ (Gray & Scannapieco 2016) and more so when the background ultraviolet light is taken into account. Like the increase in the variance of the column depth described above, this drop in the HI mass fraction would increase the probability that a photon would escape a higher Mach number medium due to unresolved turbulence.

In this Letter, we bring attention to these two facts and argue that their combined effect is sufficient to increase current estimates of the escape by a factor of three or more in cases in which the average Mach number of the medium is above $\mathcal{M} \approx 7$. The structure of this work is as follows. In Section 2, we describe a simple model of how the escape fraction scales with Mach number; in Section 3, we describe how this scaling is likely to affect current estimates of $f_{\text{esc}}$ as a function of redshift; and in Section 4, we give conclusions.

2. Modeling a Turbulent Medium

The density distribution in a non-magnetized supersonic box is well approximated by a lognormal distribution where

$$
\sigma_{\ln, p}^2 = \ln(1 + b^2 \mathcal{M}^2),
$$

(Vazquez-Semadeni 1994; Ostriker et al. 2001). The PDF of column densities ($\Sigma$) at the driving scale is also well approximated by a lognormal distribution (Ostriker et al. 2001; Federrath et al. 2010; Burkhart et al. 2015):

$$
p(x, \mathcal{M}) = \frac{1}{(2\pi \sigma_{\ln, \Sigma})^{1/2}} \exp \left[ -\frac{(x - \bar{x})^2}{2\sigma_{\ln, \Sigma}^2} \right],
$$

where $x = \ln(\Sigma/\bar{\Sigma})$. Conservation of mass requires that the mean $\bar{x}$ and dispersion $\sigma_{\ln, \Sigma}$ in $p(x)$ be related by $\bar{x} = -\sigma_{\ln, \Sigma}^2/2$. By assuming a form of power spectrum for density fluctuations, the variance in the two-dimensional projected column density is related to the variance in the three-dimensional density field (Brunt et al. 2010). In this case, Thompson & Krumholz (2016) find the dispersion of the logarithm of column density to be

$$
\sigma_{\ln, \Sigma} \approx \ln(1 + Rh^2\mathcal{M}),
$$

where

$$
R = \frac{1}{2} \left( \frac{3 - \alpha}{2 - \alpha} \right) \left[ 1 - \mathcal{M}^{2(1-\alpha)} \right],
$$

and $\alpha$ is the slope of the turbulent power spectrum $P(k) \propto k^{-\alpha}$ for $1 \leq k \leq \mathcal{M}^2$ and zero otherwise. Here, $k$ is in normalized units such that $k_0 = 1$ corresponds to a mode with with wavelength $\lambda = 2L_{\text{drive}}$. Federrath et al. (2008) showed $b = 1$ for purely compressive forcing ($\Delta \times F = 0$) and $b = 1/3$ for purely solenoidal ($\Delta . F = 0$). Gray et al. (2015) found $b = 0.53$ best fits their simulation result where they relax the assumption of isothermal turbulence and model the cooling and atomic chemistry of the gas in a supersonic turbulent box.

While both HI and dust can absorb the LyC photons, dust is found to have a negligible impact on the final escape fraction of photons in the presence of neutral hydrogen (Gnedin et al. 2008). This is because the dust opacity is in general much less than HI ($\tau_{\text{dust}} \ll \tau_{\text{HI}}$) for ionizing photons and for lines of sight where $\tau_{\text{dust}} > 1$, the hydrogen opacity is already so large that it dominates. To model the effect of increasing the Mach number on the escape fraction of LyC photons, we therefore take the distribution of HI column densities, multiply by the opacity of each column that is directly proportional to the column density, and integrate over all the column densities:

$$
\text{Esc}(\mathcal{M}, \tau_{\text{mean}}) = \int_{-\infty}^{+\infty} p(x, \mathcal{M}) e^{-\tau_{\text{HI}}} dx,
$$

where $\tau_{\text{HI}} = \kappa_{\text{HI}} \Sigma = \kappa_{\text{HI}} \langle \Sigma \rangle e^x$. $\kappa_{\text{HI}}$ is the hydrogen opacity per unit mass in $\text{cm}^2 \text{g}^{-1}$ at frequency ($\nu$). The escape fraction is a function of both $\mathcal{M}$ and $\tau_{\text{mean}}$ (the product of $\kappa_{\text{HI}} \langle \Sigma \rangle$). We compute the integral as a function of these two parameters and report the relative enhancement of escape fraction as compared to the case assuming a constant column density of $\langle \Sigma \rangle$. Only a specific range in $x$ makes most of the contribution to the integral because the PDF drops at either too high or too low column densities. Moreover, high column densities suffer from large absorption that makes them irrelevant for contributing to the escape fraction. This point is illustrated in Figure 1.

It should be noted that in our calculations we have assumed the scale height ($H$) of the galaxy in the question is comparable or less than the driving scale of turbulence. If, on the other hand, the scale height of the system is larger than the driving scale ($H > L_{\text{drive}}$), then the PDF of column densities would be the product of $N$ lognormal PDFs characterized by $\sigma_{\ln, \Sigma} = \sqrt{N} \sigma_{\ln, \Sigma}^\text{drive}$ and $\langle \Sigma \rangle = N \langle \Sigma \rangle^\text{drive}$, where $N = H/L_{\text{drive}}$.

Figure 1 shows the function $p(x)$ for three different Mach numbers as well as the normalized escape fraction computed for different ranges of the lower limit on $x$. The lower panel plot indicates to what lower limits in column densities one

![Figure 1](image-url)
Figure 2. Enhancement of the escape fraction of LyC photons as a function of $M$ relative to fixed column density of $\Sigma$. The three curves correspond to three values of $b$: 0.33, 1, and 0.53, which correspond to an isothermal medium with solenoidal forcing, an isothermal medium with compressive forcing, and a non-isothermal medium with solenoidal forcing, respectively. The escape fraction is modeled based on Equation (5), assuming a turbulent power spectrum with $p(k) = k^{-\alpha}$ and $\alpha = 2.5$. The escape fraction is increased by 25% at $M \approx 10$ and by 50% at $M \approx 20$ for the case of $b = 1/2$, which is the case when detailed chemistry is calculated for the turbulent gas. This calculation does not take into account the reduction of the HI fraction in turbulent media. We have assumed $\Sigma_i(\Sigma) = 1$ in this calculation.

Figure 3. Enhancement of the escape fraction of LyC photons as a function of $\tau_{\text{mean}}$ (mean opacity) relative to fixed column density of $\Sigma$. The three curves correspond to three values of $M$: 5, 10, and 20, and we have considered $b = 1/2$ for all the curves. The escape fraction is modeled based on Equation (5), assuming a turbulent power spectrum with $p(k) = k^{-\alpha}$ and $\alpha = 2.5$. Resolving turbulence becomes more crucial for more opaque cells in a simulation and this can potentially have a large impact on the escape of ionizing photons in star-forming cells that are generally located in highly opaque regions.

what we have assumed in our calculations (Gray et al. 2015; Gray & Scannapieco 2016).

Other studies have considered the impact of self-gravity and thermal energy from supernovae on the turbulent density distribution. Slyz et al. (2005), like Gray et al. (2015), found that the density PDF is lognormal when one includes cooling and turbulent driving. They also showed that the addition of self-gravity causes the PDF to exhibit a high-density tail, corresponding to collapsing gravitationally bound structures (Klessen 2000; Federrath & Klessen 2013). However, this has little effect on the low-density end that determines $f_\text{esc}$. When thermal feedback is included in their isotropic simulations, however, the PDF becomes bimodal as most of the volume becomes heated to high temperatures. Although, in a real galaxy, much of this hot gas is likely to be vented away in an outflow, its presence will nevertheless cause $f_\text{esc}$ to increase even more than in our estimates above. Thus, while various pieces of physics can affect the density PDF in a supersonic turbulent gas, the results show these changes will generally enhance the low-density part of the density PDF and therefore will further enhance the escape fraction.

3. TURBULENT MACH NUMBER AT HIGH REDSHIFTS

Regardless of the detailed physics, the efficacy of small-scale turbulent structures in facilitating reionization will depend on turbulent Mach numbers being large in high-redshift galaxies. Yet there are several theoretical and observation clues that this may indeed be the case. From a theoretical point of view, Wise et al. (2008) found that turbulent Mach number rises to $M \approx 2-4$ in simulations of galaxies at high redshifts, when the halos become gravitationally unstable and start to collapse. Greif et al. (2008) show that inflow of cold gas along high-redshift cosmic filaments is supersonic, leading to a high level of $M \approx 1-5$ turbulence in accreting galaxies. Sur et al. (2016) discuss the turbulent velocity dispersions expected to be caused by gravitational instabilities in rotationally supported structures, showing they are likely to be much greater at high redshifts.
From an observational standpoint, at high redshifts, smaller sizes and higher temperatures can create extreme environments that lead to highly supersonic structures (Chabrier et al. 2014) with \(10 < M < 100\). Disk settlement happens at redshifts \(z < 1.2\) (Kassin et al. 2012) and the structure of galaxies at higher redshifts is clumpy in nature (Guo et al. 2012, 2015; Mandelker et al. 2014; Moody et al. 2014) with line of sight velocity dispersions typically five times higher than in the local universe (Green et al. 2010). These clumpy structures indicate that disks at high redshifts are not stable, which leads to a highly turbulent interstellar medium (ISM).

A highly turbulent ISM is also observed at \(z \approx 2.3\) in the lensed star-forming galaxy SMM J2135-0102 (Swinbank et al. 2011). The measured mid-plane hydrostatic pressure of SMM J2135-0102 is estimated to be \(P_{\text{tot}}/k_B \approx (2 \pm 1) \times 10^7\ \text{K cm}^{-3}\), which is \(\approx 1000\) higher than what is found for the Milky Way and only comparable to local ultraluminous infrared galaxies (Downes & Solomon 1998). In fact, Swinbank et al. (2011) find that supersonic turbulence is dominant on all scales down to \(\approx 100\) smaller than kinematically quiescent ISM of the Milky Way and a supersonic turbulent theory can explain such observations (Rathborne et al. 2014). Strong lensing studies of the ISM of high-redshift galaxies appear to be the path forward to test the theories of turbulent structure at redshifts relevant for reionization.

4. CONCLUSIONS

We have shown that the escape probability of LyC photons increases significantly with Mach number in supersonic turbulent gas. This is due to two reasons. First, at higher Mach numbers, the column density PDF becomes broader, characterized by the relation between the dispersion of the PDF and the fraction increases for the regions with high opacity. Second, simulations of supersonic turbulent flow that track the detailed abundance of the elements through chemical networks show a drop within a factor of 2–3 in the fraction of neutral hydrogen at \(M \approx 10\) even at temperatures \(\approx 5 \times 10^3\ \text{K}\). These two effects combined can enhance the escape fraction of LyC photons by a factor of more than 3 in flows with \(M \approx 10\), values expected to be typical in high-redshift galaxies.

The impact of resolving the density PDF on the escape fraction increases for the regions with high opacity (Figure 3). However, it should be noted that star formation happens in dense regions and therefore the sources of ionization do not experience the average column density of the galaxy they reside in. If a star is born in a high-density region, an estimate of the eddy turnover timescale can be used to determine the escape of LyC photons. The turbulent velocity and cell size \(t_{\text{edd}} \approx 10^2\ \text{pc} \left(\frac{\Delta v}{100\ \text{pc}}\right) \left(\frac{10\ \text{km s}^{-1}}{v_{\text{esc}}}ight) \approx 10^3\ \text{yr}\) should be compared with the lifetime of the star \(t_{\text{age}}\). In the case \(t_{\text{edd}} > t_{\text{age}}\), then it would not be physical to replace the column density of the cell with its equivalent supersonic turbulence box. The turbulent velocity and cell size are related through the assumed supersonic power spectrum.

Currently, the detailed chemistry of species in an isotropic turbulent box is carried out up to \(M \approx 10\). It can be that at higher Mach numbers we might see an even more dramatic drop in the neutral hydrogen fraction that would enhance the escape of LyC photons. Separately, a more careful study of the driving scale of turbulence in high-redshift galaxies needs to be conducted to determine the minimum spatial resolution needed to capture the turbulent structure of their ISM and its impact on the escape fraction of Lyman continuum photons.

We are thankful to Romeel Dave and Phil Hopkins for insightful conversations. This work was supported by the National Science Foundation under grant AST14-07835 and by NASA under theory grant NNX15AK82G.

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