Hyperdeformation in the Cd isotopes: a microscopic analysis

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A systematic search for the nuclei in which the observation of discrete hyperdeformed (HD) bands may be feasible with existing detector facilities has been performed in the Cd isotopes within the framework of cranked relativistic mean field theory. It was found that the $^{96}$Cd nucleus is a doubly magic HD nucleus due to large proton $Z=48$ and neutron $N=48$ shell gaps. The best candidate for experimental search of discrete HD bands is $^{107}$Cd nucleus characterized by the large energy gap between the yrast and excited HD bands, the size of which is only 15% smaller than the one in doubly magic HD $^{96}$Cd nucleus.

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I. INTRODUCTION

Hyperdeformation (HD) is one of critical phenomena in nuclear structure, the study of which will considerably advance our knowledge of nuclei at extreme conditions of very large deformation and fast rotation [1,2]. The studies of HD will also contribute into understanding of the crust of neutron stars, where extremely deformed nuclear structures are expected (see Ref. [3] and references therein). Although some experimental evidences of the existence of HD at low [4,5] and high spin [6,7,8,9] exist, the current experimental knowledge of HD is very limited. New generation of detectors such as GRETA [10] and AGATA [11] will definitely allow to study this phenomenon in more details. However, these detectors will become functional only in the middle of next decade. Thus, it is very important to understand whether new experimental information on HD can be obtained with existing detectors such as GAMMASPHERE [12].

Theoretical efforts to study HD at high spin both in macroscopic+microwave (MM) method and in self-consistent approaches were reviewed in Ref. [1]. Our recent study of HD within the framework of the cranked relativistic mean field (CRMF) theory in the Z=40-58 part of nuclear chart [1] represents the first ever systematic investigation of HD within the self-consistent theory. The general features of the HD bands at high spin have been analysed in Ref. [1]. In particular, it was concluded that the density of the HD states in the vicinity of the yrast line is the major factor which decides whether or not discrete HD bands can be observed. The high density of near-yrast HD states will lead to a situation in which the feeding intensity will be redistributed among many low-lying bands, thus drastically reducing the intensity with which each individual band is populated. For such densities, the feeding intensity of an individual band will most likely drop below the observational limit of the modern experimental facilities. On the contrary, the large energy gap between the yrast and excited HD configurations will lead to an increased population of the yrast HD band, thus increasing the chances of its observation.

The analysis of Ref. [1], based on the energy gap between the last occupied and first unoccupied routhians in the yrast HD configurations, suggests that the density of the HD bands in the spin range where they are yrast is high in the majority of the cases. It also indicates the Cd isotopes as the best candidates for a search of discrete HD bands. However, one has to remember that this type of analysis may be too simplistic because the polarization effects induced by particle-hole excitations are neglected. In particular, it can overestimate the size of the energy gap between the yrast and excited HD configurations.

Realistic analysis of the density of the HD bands should include significant number of the HD configurations calculated in a fully self-consistent manner with all polarization effects included. Such analysis is time-consuming in computational sense and has been performed only for $^{124}$Xe in Ref. [1], but its extension to other nuclei is needed. Thus, the goals of the current manuscript are (i) to perform a fully self-consistent analysis of the density of the HD bands in the Cd isotopes and (ii) to find the best nuclei in which experimental study of discrete HD bands can be feasible with existing experimental facilities.

II. THEORETICAL FRAMEWORK AND THE DETAILS OF THE CALCULATIONS

The calculations in the present manuscript are performed in the framework of the CRMF theory without pairing [13,14] using numerical scheme of Ref. [1]. The CRMF equations for the HD states are solved in the basis of an anisotropic three-dimensional harmonic oscillator in Cartesian coordinates characterized by the deformation parameters $\beta_0=1.0$ and $\gamma=0^\circ$ and oscillator frequency $\hbar\omega_0=41A^{-1/3}$ MeV (see Ref. [1] for details). The truncation of basis is performed in such a way that all states belonging to the shells up to fermionic $N_F=14$ and bosonic $N_B=20$ are taken into account; this truncation scheme provides sufficient numerical accuracy [1]. The Nil1 parametrization of the RMF Lagrangian [15] is used in most of our calculations since it provides a good description of the moments of inertia of the rotational bands in unpaired regime in the SD and ND minima [14,16,17,18], the single-particle energies for the nuclei
around the valley of β stability \[16\] and the excitation energies of the SD minima \[20\]. Other parametrizations such as NL3 \[21\], NLSH \[22\], NLZ \[23\] and NL3* \[24\] are used only to check the size of the HD gaps in the nuclei of interest.

The excited HD configurations were built from the yrast HD configurations obtained in the previous study \[1\] by exciting either one proton or one neutron or both together. Proton and neutron configurations generated in this way are labeled by \(\pi_i\) and \(\nu_j\), where \(i = 0, 1, 2, \ldots\) and \(j = 0, 1, 2, \ldots\) are integers indicating the corresponding configurations. \(\pi_0 \otimes \nu_0\) represents the yrast HD configuration. Total excited configurations \(\pi_i \otimes \nu_j\) are constructed from all possible combinations of proton \(\pi_i\) and neutron \(\nu_j\) configurations excluding the one with \(i = 0\) and \(j = 0\). The selection of excited configurations is also constrained by the condition that the energy gap between the orbital from which the particle is excited and the orbital into which it is excited do not exceed 2.5 MeV in the routhian diagram for the yrast HD configuration. All configurations are calculated in a fully self-consistent manner so that their total energies are defined as a function of spin.

Fig. 1 illustrates the selection of excited configurations. It shows the occupation of the proton and neutron orbitals in the yrast HD configuration in \(^{107}\)Cd. According to our criteria only three proton excitations across the \(Z = 48\) HD gap are considered. On the contrary, more neutron \(ph\)-excitations are allowed across the \(N = 59\) HD shell gap. Table I shows their detailed structure. For example, the \(\nu_1\) configuration is created by exciting one neutron from the \([770\]1/2\(^+\) into \([413\]7/2\(^+\) orbitals. One can notice that we only consider the \(ph\)-excitations between the states which do not have the same combination \((\pi, r)\) of parity \(\pi\) and signature \(r\). The computer code in general can handle the excitations between the states with the same \((\pi, r)\), but the configurations based on such excitations are less numerically stable and require more computational time. Because of this reason and the fact that they do not alter significantly the results for the density of the HD states, it was decided to neglect them in the calculations. However, in the cases of large energy gaps between the yrast and excited HD configurations, they are taken into account.

FIG. 1: Proton (top panel) and neutron (bottom panel) single-particle energies (routhians) in the self-consistent rotating potential as a function of the rotational frequency \(\Omega\). They are given along the deformation path of the yrast HD configuration in \(^{107}\)Cd and obtained in the calculations with the NL1 parametrization of the RMF Lagrangian. Long-dashed, solid, dot-dashed and dotted lines indicate \((\pi = +, r = +i)\), \((\pi = +, r = -i)\), \((\pi = -, r = +i)\) and \((\pi = -, r = -i)\) orbitals, respectively. Solid (open) circles indicate the orbitals occupied (emptied). The dashed box indicates the frequency range corresponding to the spin-range \(I = 55 - 80\) in this configuration. The arrows indicate the particle-hole excitations leading to excited HD configurations.

**TABLE I**: Neutron particle-hole excitations in \(^{107}\)Cd shown in Fig. 1

| label | Excitation |
|-------|------------|
| \(\nu_1\) | \([770\]1/2\(^+\) \(\rightarrow\) \([413\]7/2\(^+\) |
| \(\nu_2\) | \([770\]1/2\(^+\) \(\rightarrow\) \([413\]7/2\(^-\) |
| \(\nu_3\) | \([532\]3/2\(^-\) \(\rightarrow\) \([413\]7/2\(^+\) |
| \(\nu_4\) | \([532\]3/2\(^-\) \(\rightarrow\) \([413\]7/2\(^-\) |
| \(\nu_5\) | \([651\]3/2\(^-\) \(\rightarrow\) \([413\]7/2\(^+\) |
| \(\nu_6\) | \([651\]3/2\(^+\) \(\rightarrow\) \([413\]7/2\(^-\) |

Single-particle orbitals are labeled by \([NnA]\Omega^{\pm}\) where \([NnA]\Omega\) are the asymptotic quantum numbers (Nilsson quantum numbers) of the dominant component of the wave function at \(\Omega = 0.0\) MeV. The superscripts \(\pm\) to the orbital labels are used sometimes to indicate the sign of the signature \(r\) for that orbital \((r = \pm i)\).
E - 0.01I(I+1)[MeV]

Angular momentum I(ℏ)

FIG. 2: (Color online) Energies of the calculated HD configurations in even-even $^{96−106}$Cd nuclei relative to a smooth liquid drop reference $A I(I+1)$, with the inertia parameter $A=0.01$. In each nucleus, the yrast and lowest excited proton configurations are shown by solid lines. Dot-dashed and dotted lines represent the yrast lines at low spin built from normal-deformed (ND) and superdeformed (SD) states, respectively.

### III. DISCUSSION

Figs. 2 and 3 show the density of the HD states in even-even $^{96−108}$Cd and odd mass $^{107,109}$Cd nuclei studied using above outlined procedure. The energy gap between the yrast HD configuration and lowest excited HD configurations is around 1.5 MeV in $^{96}$Cd (Fig. 2a). It is comparable with the energy gap between the yrast and excited SD configurations in doubly magic SD nucleus $^{152}$Dy (Fig. 7 in Ref. [14]). This energy gap in $^{96}$Cd is due to large energy cost of particle-hole excitations across the $Z = 48$ and $N = 48$ HD shell gaps which have similar size (see Fig. 1 and Table I). All that together indicates that the $^{96}$Cd is a doubly magic HD nucleus. Only proton excitations to the [420]1/2− orbital above the $Z = 48$ HD shell gap result in bound excited proton configurations, the excitations to other orbitals located above the $Z = 48$ HD shell gap produce the proton-emitting states. The doubly magic nature of $^{96}$Cd nucleus is confirmed also in the calculations with other RMF parametrizations (Table I). It is interesting to mention that the RMF parametrizations aimed at the description of the nuclei far from stability such as NL3, NL3*, NLSH show larger $Z = 48$ and $N = 48$ HD shells gaps in $^{96,107−109}$Cd than the parametrizations NL1 and NLZ fitted predominantly to β-stability nuclei (Table III).

With increasing neutron number the energy gap between the yrast and excited HD configurations disappears (Fig. 2). This is due to relatively high density of the neutron states above the $N = 48$ HD shell gap (Fig. 1). Indeed, many excited neutron configurations are located below the lowest excited proton configurations (Fig. 2). One can also see that even-even $^{100−104}$Cd nuclei are characterized by appreciable density of the HD states in the vicinity of the yrast HD line (Fig. 2). The analysis of the single-particle structure in these nuclei indicates that similar density of the HD bands is expected also in odd mass nuclei $^{99−105}$Cd. In no way these nuclei have to be considered as good candidates for a search of discrete HD bands since the feeding intensity will be redistributed among many low-lying HD bands. As a result, the feeding intensity of an individual HD band will most likely drop below the observational limit of modern experimental facilities. Although there is some energy
gap between the lowest four HD configurations and other excited configurations in $^{106}$Cd, this nucleus does not appear to be a good candidate for a search of discrete HD bands because the presence of four low-lying HD configurations will lead to a fragmentation of feeding intensity. This is one of possible reasons why the HD bands have not been observed in this nucleus. 

On the other hand, the high density of the HD bands in above discussed nuclei will most likely favor the observation of the rotational patterns in the form of ridge structures in three-dimensional rotational mapped spectra. The study of these patterns as a function of neutron number can provide a valuable information about HD at high spin. 

Further increase of the neutron number brings the neutron Fermi level to the region of low density of the neutron states characterized by the large $N = 59$ and $N = 61$ HD shell gaps (Fig. 1) with the combined size of these two gaps being around 2.5 MeV (Table I). As a result, the $^{107-109}$Cd nuclei show appreciable energy gap between the yrast and lowest excited HD configurations (Fig. 3). This gap is especially pronounced in the case of $^{107}$Cd for which it is around 1.3 MeV. Note that the size of this gap is defined by the size of the $Z = 48$ HD shell gap, since the lowest excited configuration is based on proton excitation (Fig. 3). Similar or even larger energy gap between the yrast and excited HD configurations is expected in the NLZ, NL3, NL3* and NLSH parametrizations for which the size of the $Z = 48$ and $N = 59$ HD shell gaps is at least 1.7 MeV in $^{107}$Cd (Table I). The energy gaps between the yrast and excited HD configurations at the spins where the HD configurations become yrast are somewhat lower in $^{108,109}$Cd being around 0.9 and 1.1 MeV. This energy gap in $^{108}$Cd is dictated by the size of the $N = 61$ HD shell gap since lowest excited HD configurations are based on neutron excitations. Thus, in $^{108}$Cd it will be smaller (similar) in the case of the NLZ (NL3, NL3*) parametrizations and larger in the NLSH parametrization as compared with the one obtained in the NL1 parametrization (Table II). In the case of $^{109}$Cd, the energy gap between the yrast and excited configurations will be larger (smaller) in the NL3, NL3* and NLSH (NLZ) parametrizations (Table II).

Two factors make the observation of discrete HD bands in $^{108}$Cd with existing facilities less probable than in odd-mass $^{107,109}$Cd nuclei. First, the yrast HD line in this nucleus is built from two signature degenerate configurations (Fig. 3b) in which the last neutron is placed into one of the signatures of the $[413]7/2$ orbital (see Fig. 1 and Ref. [28]). This reduces the feeding intensity of each of these bands by factor of 2 as compared with the case when the yrast HD line is built from single configuration. Second, the energy gap between the yrast and excited HD configurations decreases with increasing spin (Fig. 3c). As a result, further reduction of feeding intensity of the yrast HD bands is expected if the bands are populated at spins higher than the spin at which they be-

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**FIG. 3:** (Color online) The same as in Fig. 2 but for $^{107,108,109}$Cd. The yrast HD line in $^{108}$Cd is built from two signature-degenerated configurations.
these results strongly suggest that the function of spin in yrast and excited HD configurations is more constant as come yrast. On the contrary, the energy gap between the yrast and excited HD configurations is more constant as a function of spin in $^{109}\text{Cd}$ and especially in $^{107}\text{Cd}$. All these results strongly suggest that the $^{107}\text{Cd}$ nucleus is the best candidate for the experimental search of the discrete HD bands. This conclusion is also supported by detailed analysis of the single-particle routhians in the yrast HD configurations of even-even nuclei studied in Ref. [2]; this analysis does not suggest any alternative case which would provide similar or larger gap between the yrast and excited HD configurations in even-even, odd and odd-odd nuclei of the $Z = 40 – 58$ part of the nuclear chart.

The calculated properties of the yrast HD bands in studied nuclei are shown in Fig. 4. The HD shapes undergo a centrifugal stretching that result in an increase of the transition quadrupole moments $Q_t$ with increasing rotational frequency. This process also reveals itself in the dynamic moments of inertia: they increase with increasing rotational frequency in the frequency range of interest. On the other hand, the mass hexadecapole moments $Q_{40}$ do not show a clear trend as a function of rotational frequency and stay nearly constant in the majority of the HD bands. Unpaired band crossings due to interaction of different single-particle orbitals are seen in the configurations of the yrast HD bands in $^{100,102,106}\text{Cd}$ nuclei. For example, the interaction between the $(r = +i)$ signatures of the $\nu[770]1/2$ and $\nu[532]5/2$ orbitals is responsible for the crossing seen at $\Omega_z \approx 1.05 \text{ MeV}$ in the yrast HD band in $^{106}\text{Cd}$. This crossing may be an extra factor (in addition to the density of the near-yrast HD bands) which complicates the observation of the HD bands in $^{106}\text{Cd}$: such bands have not been observed in experiment of Ref. [25].

The current study clearly shows that the polarization effects in time-even and time-odd mean fields have an important impact on the density of the HD states and especially on the energy gap between the yrast and excited HD states. The latter quantity is appreciably smaller (by up to $\sim 0.5 \text{ MeV}$; compare Figs. 2 and 3 with Table II) than the respective HD shell gap in the routhian diagram.

![FIG. 4: (Color online) Dynamic moments of inertia $J^{(2)}$ (panel (a)), transition quadrupole moments (panel (b)), and mass hexadecapole moments $Q_{40}$ (panel (c)) of the yrast HD bands in the nuclei under study. All these bands have the proton $\pi 6^2$ configuration, their neutron configurations are shown in the right panel. The configuration of the yrast HD band in $^{106}\text{Cd}$ is shown in panel (a).]

| TABLE II: The size of the $Z=48$, $N=59$, and $N=61$ HD shell gaps (in MeV) obtained with different parametrizations of the RMF Lagrangian for the yrast HD configurations in $^{96,107,108,106}\text{Cd}$. They are given at rotational frequency $\Omega_z = 1.00 \text{ MeV}$ approximately corresponding to the spin at which the HD bands become yrast. The lowest (among the different parametrizations) value of the shell gap is shown by bold style. '59+61' line shows the combined size of the $N = 59$ and $N = 61$ HD shell gaps. |
|---|---|---|---|---|---|---|
| Nucleus | Gap $\Omega_z=1.00 \text{ MeV}$ | NL1 | NLZ | NL3 | NL3* | NLSH |
| $^{96}\text{Cd}$ | $Z=48$ | 1.75 | 1.93 | 2.43 | 2.27 | 2.71 |
| | $N=48$ | 2.00 | 2.07 | 2.59 | 2.44 | 3.03 |
| $^{108}\text{Cd}$ | $Z=48$ | 1.62 | 1.66 | 2.23 | 1.99 | 2.06 |
| | $N=59$ | 1.30 | 1.70 | 1.50 | 1.46 | 1.20 |
| | $N=61$ | 1.20 | 0.74 | 1.20 | 1.19 | 1.50 |
| | 59+61 | 2.50 | 2.44 | 2.70 | 2.65 | 2.70 |
| $^{107}\text{Cd}$ | $Z=48$ | 1.70 | 1.73 | 2.22 | 2.18 | 2.27 |
| | $N=59$ | 1.89 | 2.16 | 2.08 | 2.04 | 1.74 |
| $^{109}\text{Cd}$ | $Z=48$ | 1.52 | 1.61 | 1.89 | 1.84 | 1.54 |
| | $N=61$ | 1.37 | 1.16 | 1.83 | 1.74 | 2.16 |
The role of time-odd mean fields in the definition of the energy gap between the yrast and excited HD configurations is quite complicated. This is illustrated by the fact that the energy gap between the yrast HD and the lowest excited proton and neutron HD configurations is larger by $\approx 0.2$ MeV in the calculations without NM than in the ones with NM at spins where the HD configurations become yrast ($I \approx 67h$). This fact reflects two different mechanisms by which the time-odd mean fields affect the relative energies of different rotational bands. In the first mechanism, the angular momentum content of the single-particle orbitals is modified in the presence of time-odd mean fields, see Ref. [30] for details. There are two important consequences of this mechanism. First, the same total angular momentum of the system is built at rotational frequency which is by $\approx 25\%$ lower in the calculations with NM than in the calculations without NM. Second, the changes of the single-particle angular momenta of the single-particle orbitals surrounding the HD gaps of interest (the $\pi[420]1/2$ and $\pi[541]1/2$ orbitals for proton subsystem and $\nu[413]7/2$ and $\nu[651]3/2$ for neutron subsystem (Fig. 1)) induced by NM modify the single-particle energies of these orbitals. As a result, these gaps are smaller by $\approx 0.12$ MeV in the calculations with NM at $I = 67h$. The second mechanism is related to additional binding due to time-odd mean fields. The time-odd mean fields are stronger in the excited HD configuration than in the yrast HD configuration. Thus, additional binding due to NM is stronger in excited HD configuration than in the yrast HD configuration. This also leads to the decrease of the energy gap between the yrast and excited HD configurations in the calculations with NM as compared with the ones without NM.

The presence of time-odd mean fields reveals itself also in the energy splitting of the opposite signatures of the $\nu[770]1/2$ orbital visible at $\Omega_z = 0.0$ MeV (Fig. 1): the occupied orbital is more bound than unoccupied one in the RMF theory (Ref. [14]).

When considering theoretical predictions one has to keep in mind that they are subject of the errors in the description of the energies of the single-particle states, which exist in the RMF theory at spherical shape [29], normal deformation [10] and quite likely at superdeformation [28]. The extrapolation from spherical and normal deformation towards HD is itself a potential source of errors since it is not know how well the response of the mean field (or the single-particle potential and liquid drop in the MM method) to the extreme elongation of the nucleus is reproduced in model calculations. Such errors are not restricted to the self-consistent models; they are also expected in the phenomenological potentials (used in the MM method) which describe single-particle energies at normal deformation better than self-consistent models. However, several facts support the results and interpretations given above. First, all RMF parametrizations used in this study lead to the same HD configurations in $^{96,107−109}$Cd nuclei which become yrast at similar spins (see Ref. [1] for comparison of the results obtained with NL1 and NL3) and to similar sizes of the proton and neutron HD shell gaps (Table III). Second, the large size of the $Z = 48$ and $N = 59$ (and especially of combined neutron $59 + 61$ gap) HD shell gaps reduces the importance of the errors in the description of the energies of specific single-particle states. Third, the MM results of Ref. [31] suggest similar conclusions for the nuclei around $^{108}$Cd. Indeed, large $Z = 48$ shell gap and low density of the single-particle states in the vicinity of the $N = 59$ and $N = 61$ HD shell gaps is clearly visible in Figs. 4 and 5 of Ref. [31]. The $N = 59$ and $N = 61$ shell gaps are separated by the signature-degenerated $7/2^+$ state (Fig. 5 in Ref. [31]). Thus, similar to our case, the yrast HD line in $^{108}$Cd will be formed from two signature degenerated configurations in the MM calculations.

**IV. CONCLUSIONS**

In summary, a systematic analysis of hyperdeformation in the Cd isotopes has been performed in the cranked relativistic mean field theory. The density of the HD states has been analysed with the goal to find the best cases for experimental search of the discrete HD bands. Our analysis indicates $^{96}$Cd as a doubly magic HD nucleus in this part of nuclear chart; its magicity is due to large $Z = 48$ and $N = 48$ HD shell gaps. However, experimental study of HD in this nucleus is problematic with existing facilities due to its $N = Z$ status. The low density of the neutron single-particle states in the vicinity of the $N = 59$ and $61$ HD shell gaps and sizable $Z = 48$ HD shell gap lead to appreciable gaps between the yrast and excited HD bands in $^{107−109}$Cd nuclei, thus offering better opportunities to observe discrete HD bands. Among these three nuclei, the best candidate for observing the discrete HD bands with existing facilities is $^{108}$Cd nucleus. The MM calculations of Refs. [31, 32] indicate that the fission barriers are sufficiently large in the nuclei around $^{108}$Cd so that the HD minimum could survive fission for a significant range of angular momentum. The stability of the HD minimum is defined by its depth, the fission barrier height and the height of the barrier between the HD and normal-deformed/superdeformed minima [2, 32]. Our study clearly indicates that the HD minimum is localized in the potential energy surface. However, future studies of the HD in this mass region have to provide more quantitative answers on these properties of the HD minima in a fully self-consistent framework.

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