A fractional-order Duhem model of rate-dependent hysteresis for piezoelectric actuators

Liu Yang¹, Ruobing Zhong¹, Dongjie Li¹,² and Zhan Li³

Abstract
In this paper, a Fractional-Order Duhem Model (FODuhem) is proposed to describe the rate-dependent hysteresis non-linearity of piezoelectric actuators (PEAs). A fractional-order operator is introduced on the basis of the traditional Duhem model, and the unique nonlocal memory property of the fractional-order operator makes it possible to describe the memory effect inherent in hysteresis. A differential evolutionary algorithm was used to identify the parameters of the FODuhem model. Finally, experimental results clearly show that the FODuhem model can better describe the rate-dependent hysteresis behavior of piezoelectric actuators compared with the conventional Duhem model.

Keywords
Piezoelectric actuator, fractional order, hysteresis nonlinearity, Duhem model

Introduction
Piezoelectric actuators are widely used in high-speed nanopositioning systems,¹-³ micro-vibration isolation,⁴,⁵ and other fields because of their high accuracy, small size, and high speed, but the piezoelectric actuators present significant nonlinearity between the input voltage and output displacement and the inherent hysteresis characteristics of piezoelectric materials, which pose certain difficulties in their applications.

Many control methods have been studied to reduce the adverse effects of this nonlinear hysteresis effect, and hysteresis control can be classified into hysteresis inverse model-based control,⁴-⁶ and hysteresis inverse model-free control,⁷-¹⁰ depending on whether an inverse model is needed to compensate for the hysteresis. Compared to hysteresis-free inverse model control, hysteresis-based inverse model control can better reduce the hysteresis nonlinearity of PEAs. The principle is to model the hysteresis as accurately as possible and then compensate the control system by the inverse of the model, which makes hysteresis modeling particularly important. At the end of the 19th century, physicists P. Duhem and Stefanini proposed the Duhem hysteresis model, which assumes that each state is in equilibrium under a constant input and its output characteristics change when and only when the input signal changes direction, the greatest advantage of this model is that it has a clear mathematical expression, and appropriate adjustment of the model parameters can accurately reflect the different Hysteresis nonlinearity.¹¹ Duhem model is a hysteresis model described by differential equations with explicit functional expressions, which provides convenience for establishing its inverse function.¹²,¹³ Under high-frequency input excitation, Duhem has a large error, especially at the special point \( \dot{u}(t) = 0 \). To compensate for this Gan et al.¹⁴ introduced a trigonometric function based on the Duhem model to compensate for the error at this point by using the special point where the trigonometric function has a derivative of zero. The Duhem hysteresis model takes the most basic form of first-order differential equations, but it is difficult to accurately describe the rate-dependent hysteresis characteristics of the piezoelectric-driven micropositioning platform even though its mathematical expressions contain information on the derivatives of the input signal. Therefore, the Duhem hysteresis model can be regarded as a static rate-independent hysteresis model for the dynamic

¹School of Automation, Harbin University of Science and Technology, Harbin, China
²Key Laboratory of Complex Intelligent Systems and Integration in Heilongjiang Province, Harbin, China
³School of Aeronautics, Harbin Institute of Technology, Harbin, China

Corresponding author: Liu Yang, School of Automation, Harbin University of Science and Technology, 52 Xue Fu Street, Nan Gang District, Harbin, Heilongjiang 150040, China.
Email: yangliuheu@gmail.com

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hysteresis of the piezoelectric-driven micropositioning platform\textsuperscript{15} and the fractional-order operator is introduced in this paper, and the unique nonlocal memory property of the fractional-order operator makes it possible to describe the inherent memory effect of the hysteresis.

The theory of fractional order calculus is a generalization of conventional integration and differentiation from integer-order to non-integer order. In recent years, various complex systems have been successfully described by fractional order calculus due to its advantage of frequency dependence. In particular, it has many successful applications in the field of system control, highlighting its own unique advantages, irreversibility, and theoretical completeness.\textsuperscript{16} Ding et al.\textsuperscript{17} proposed a fractional-order model approach to describe the broadband hysteresis of piezoelectric actuators, and experiments showed that the fractional-order model can be used in the accurate description and control compensation of nonlinear rate-dependent hysteresis of piezoelectric actuators in the frequency bandwidth range of 1–200 Hz. Kang et al.\textsuperscript{18} proposed a fractional-order based BW model to describe the hysteresis effect, and the comparative experimental results on a PEAs system verified the effectiveness and superiority of the model in asymmetric and rate-dependent hysteresis description as well as model-based feed forward hysteresis compensation. Apparently, previous fractional-order calculus models have been successfully used to describe the nonlinearity of PEAs at some specific frequencies, and most of the existing Duhem models are limited to integer-order differential equations. It is well known that the fractional-order calculus extends the order of classical calculus from the integer domain to the fractional domain, and its unique nonlocal memory effect provides excellent potential for hysteresis modeling applications. In particular, for Duhem models, fractional-order calculus is a good choice for improving the description of rate-dependent hysteresis problems. However, little work has been done to apply fractional-order calculus to the Duhem model for hysteresis modeling of PEAs.

In this paper, a Duhem model based on the combination of fractional order is proposed, which has simple expressions and can accurately describe the rate-dependent hysteresis behavior and thus reduce modeling errors. The parameters of the model can be easily identified by the differential evolution algorithm. Experiments verify the validity of the model.

This paper is organized as follows: In Section 2, the Duhem model and fractional-order calculus are introduced and the FODuhem model is proposed, then in Section 3, the differential evolution algorithm is used to obtain the model parameters, and experiments are performed to prove the validity of the FODuhem model, and finally, the conclusions are given in Section 5.

Improved Duhem model

Review of the convention Duhem model

The Duhem model is a dynamic model and the basic expression of the traditional Duhem model is\textsuperscript{19:}

\[
\dot{\omega}(t) = f_1(\omega, v)\dot{v}(t) + f_2(\omega, v)\dot{v}(t)
\]

where \(\dot{v}(t) = \max[0, \dot{v}(t)], \dot{v}(t) = \min[0, \dot{v}(t)]\)

For the hysteresis effect of PEAs Coleman proposed a simplified Duhem model expressed flowed.

\[
\frac{d\omega}{dt} = \alpha \frac{dv}{dt} [f(v) - \omega] + \frac{dv}{dt} g(v)
\]

Where \(v\) denotes the input voltage signal of the system, that is, the hysteresis input; \(\omega\) denotes the output displacement signal, that is, the hysteresis output; \(\alpha\) denotes the weight coefficient and satisfies \(\alpha > 0, f(\cdot), g(\cdot)\) are the two auxiliary functions in the model. When equation (2) is used to describe the lag of PEAs it can be rewritten as:

\[
\begin{align*}
Y(t) &= X(t) - h(t) \\
X(t) &= ku(t) \\
h(t) &= \alpha\dot{u}(t) - \beta|\dot{u}(t)|\dot{h}(t) + \gamma|\dot{u}(t)|u(t)
\end{align*}
\]

where \(Y(t)\) is the system output, \(X(t)\) is the linear component, \(h(t)\) is the lag component, \(u(t)\) is the input voltage, and \(\dot{u}(t)\) is the derivative of the voltage, and \(\alpha, \beta, \gamma, k\) are the model parameters.

In order to predict the performance of the conventional Duhem model, we conducted the following experiments. \(u(t) = \sin(2\pi \times 10t) + \sin(2\pi \times 50t) + \sin(2\pi \times 100t)\) was chosen as the reference signal to identify the conventional Duhem parameters, and the differential evolution algorithm was used to identify the model parameters \(k, \alpha, \beta\) and \(\gamma\). \(k, \alpha, \beta\) and \(\gamma\) were 0.8517, 0.7802, 1.3288, 0.3152 respectively.

Figure 1 shows the experimental data at different frequencies, and it can be found from Figure 1 that the hysteresis lines are not only symmetric around their center points but also vary with the input frequency. For piezoelectric materials, they have rate-dependent hysteresis behavior, and the hysteresis behavior has memory characteristics. Figure 2 shows the comparison between experimental and Duhem hysteresis loops, and the results show that the conventional Duhem model cannot accurately describe the hysteresis characteristics of piezoelectric materials under high-frequency excitation. Therefore, in order to reduce the modeling error of the Duhem model, it can be improved by increasing the rate dependence of the Duhem model.
The fractional-order, and Re(α) is the real part of α. A commonly used definition of the fractional differointegral is the Riemann-Liouville definition and the R-L definition is as follows:

\[
\mathcal{D}_t^\alpha f(t) = \begin{cases} \frac{d}{dt}^\alpha, & \text{Re}(\alpha) > 0 \\ \int_0^t (dt)^{1-\alpha}, & \text{Re}(\alpha) < 0 \end{cases}
\]

where \( \mathcal{D}_t^\alpha \) is the fractional-order operator, where \( t_0 \) and \( t \) are the lower and upper bounds of the integral, \( \alpha \) is the fractional-order, and \( \text{Re}(\alpha) \) is the real part of \( \alpha \). A commonly used definition of the fractional differointegral is the Riemann-Liouville definition and the R-L definition is as follows:

\[
\mathcal{D}_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \left( \frac{d}{dt} \right)^m f(t) \times \int_0^t (t-\tau)^{-(m-\alpha)} d\tau, m-1 < \alpha < m
\]

(5)

\( \Gamma(\cdot) \) is the gamma function. In this paper, the numerical simulation of the fractional-order operator will be carried out using the approximation calculation defined by G-L, which is defined as follows.

\[
\mathcal{D}_t^\alpha f(t) = \lim_{h \to 0} h^{-\alpha} \times \sum_{j=0}^{(t-\frac{j}{h})} (-1)^j \Gamma(\alpha + 1) \frac{1}{\Gamma(\alpha - j + 1) \Gamma(j + 1)} (t - jh)
\]

(6)

where \( \Gamma(\cdot) \) is the Euler's gamma function, \( n \) is an integer, and \( h \) represents the step size.

Fractional order systems can also be represented by fractional-order linear differentiation, expressed as follows:

\[
a_0 \mathcal{D}_t^{\alpha_0} y(t) + \cdots + a_\alpha \mathcal{D}_t^{\alpha} y(t) = b_0 \mathcal{D}_t^{\beta_0} u(t) + \cdots + b_\alpha \mathcal{D}_t^{\beta} u(t)
\]

(7)

where \( u(t) \) is the input, \( y(t) \) is the output, and \( \mathcal{D}_t^{\alpha} y(t) \) is the \( \alpha \)th-order derivatives of \( y(t) \).

Performing Laplace transformation on (7) under 0 initial conditions, (7) can be rewritten in the form of a transfer function as follows:

\[
G(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^{\beta_0} + \cdots + b_\alpha s^{\beta_\alpha}}{a_0 s^{\alpha_0} + \cdots + a_\alpha s^{\alpha}}
\]

(8)

The non-local memory-dominated nature of the lags can be well described using the memory effects of fractional-order operations, and the lags will be suitable to be described as fractional-order.

The traditional Duhem model mainly consists of a linear component \( X(t) \) with a hysteresis component \( h(t) \). The traditional Duhem model remains unchanged to describe the basic characteristics of the hysteresis behavior of piezoelectric, and it is worth noting that the derivative \( \dot{u}(t) \) of \( u(t) \) has a large impact on the output results of the model when the input frequency is large, so adding the derivative \( \dot{u}(t) \) to the linear part of \( u(t) \), which allows the new model to perform well at high frequencies. Since fractional-order calculus is usually considered an effective choice to describe hysteresis lines with rate-dependent characteristics, in order to describe the hysteresis phenomenon in PEA's with rate-dependent characteristics, the nonlinear hysteresis part \( h(t) \) in (3) is extended from the integer order to the fractional-order, as follows.
\[
\begin{align*}
Y(t) &= X(t) - h(t) \\
X(t) &= ku(t) + pu(t) \\
D^{\alpha} h(t) &= \alpha D^{\alpha} u(t) - \beta |D^{\alpha} u(t)| h(t) + \gamma |D^{\alpha} u(t)| u(t)
\end{align*}
\] (9)

**Experiment and validation**

The experimental setup is shown in Figure 3. The Piezoelectric actuators (max. voltage 120 V, Physik Instrumente (PI) GmbH & Co. KG, Germany) used is driven by a voltage amplifier (model E505.00, PI). Its output displacement is measured in real-time by an internal sensor of the piezoelectric micropositioning stage (model P733.2DD, PI). The sensor output voltage signal is transmitted through a signal conditioner and then acquired by a data acquisition card (Model USB-6346, National Instruments, USA). The voltage control signal is generated by the piezoelectric control module (Model E-509.C2A, PI) and then amplified by a voltage amplifier to drive the piezoelectric ceramic driver. MATLAB/Simulink was used for all experiments.

**Parameter identification**

The sinusoidal input voltage and output displacement response data were recorded using sinusoidal signals at different frequencies from 1 to 50 Hz. To obtain the parameters of the established hysteresis model, the seven parameters of the hysteresis model in equation (9) were obtained using a differential evolution algorithm. And the fractional-order operator was simulated numerically using the high precision approximation calculation defined by G-L. The flow chart of the differential evolution algorithm is shown in Figure 4.

**Generation of initial populations**

Set the evolutionary generation of the population G, the population size Np, and the dimension of the search space D. Randomly generated individuals within the D dimension can be expressed as

\[X^G_G = (X^G_{i1} \cdots X^G_{iD})(i = 1, 2 \cdots Np)\]

Set the range of individual variables to

\[X_{\text{max}} = [X_{\text{max1}} \cdots X_{\text{maxD}}], X_{\text{min}} = [X_{\text{min1}} \cdots X_{\text{minD}}]\] According to equation (11) the initial generated population is

\[P^G = (X^G_{1}, \cdots X^G_{Np})\]

\[X^G_i = X^G_{\text{min}} + \text{rand}(X^G_{\text{max}} - X^G_{\text{min}})\] (10)

where rand is a random number between [0, 1].

**Mutation operation**

After population initialization, the vector of randomly determined individuals is \(X^G_{p1}, X^G_{p2}\). Operate \(X^G_i\) as a variance vector according to the following equation denoted as \(H^G = (H^G_{11} \cdots H^G_{1D})\).
HG exchanging the information of the variant individual
new individual, and the selection operation is as in (13).

one with the higher fitness is selected as the evolved
individual in the G generation.




The new test individual vector \( V^G_i \) is obtained by
exchanging the information of the variant individual
\( H^G_i \) with the target individual \( X^G_i \) as follows

\[
V^G_i = \begin{cases} 
  h^G_{ij}, \text{rand} & \leq CR \text{ or } j = j_{\text{rand}} \\
  x^G_{ij}, \text{otherwise} 
\end{cases}
\]

(12)

where CR is the crossover probability, \( j = (j = 1 \ldots D) \)

is the component dimension, and \( j_{\text{rand}} \) is a positive integer
between 0 and \( D \).

Select operation

The vector \( V^G_i \) is compared with the vector \( X^G_i \), and the one with the higher fitness is selected as the evolved
new individual, and the selection operation is as in (13).

\[
X^G_{i+1} = \begin{cases} 
  V^G_i, f(V^G_i, \ldots, V^G_D) < f(X^G_i, \ldots, X^G_D) \\
  X^G_i, f(X^G_i, \ldots, X^G_D) \geq f(V^G_i, \ldots, V^G_D) 
\end{cases}
\]

(13)

The parameters to be identified are \( k, \alpha, \beta, \gamma, p, \lambda_1, \lambda_2 \)
in (9) using root mean square error (RMSE) to describe
the accuracy of parameter identification. The formula
is as follows.

\[
\min_{k, \alpha, \beta, \gamma, p, \lambda_1, \lambda_2} J(X) = \frac{1}{N} \sum_{i=1}^{N} (y(i) - y(i))^2 \times 100\%
\]

(14)

where \( y(i) \) is the output displacement of the
Piezoelectric actuators measured by experiment at the \( i \)th moment, \( y(i) \) is the output displacement of the model at the \( i \)th moment, and \( N \) is the total number of samples. \( f \) is the fitness function and \( f = 1/J \).

In DE, the parameters to be set are maximum num-
ber of iterations \( G_m \), initial population number \( N_{pop} \),
variability \( F \), crossover probability \( CR \), and dimension
\( D \). For the population number, it is generally taken to
be about 10 times of the identification parameter, so
\( N_{pop} = 70 \). The \( G_m \) depends on the convergence, and is
taken to be 200 in this paper. The crossover probability
\( CR \) is generally chosen from 0.6 to 0.9, and \( CR \) is taken
as 0.9 in this paper. \( D \) is the number of dimensions, so
\( D = 7 \). The parameters of the FODuhem model and
Duhem model were identified using a swept signal of 1–
50 Hz as the input signal, and the identification results
are shown in Table 1.

Experimental validation

To verify the validity of the FODuhem model, five sets of
experiments were conducted using sinusoidal input signals
\( u(t) = \sin(2\pi ft) \) with different frequency are applied
to the PEA system. Each set of experiments was conducted
using the above selected excitation signals to drive the
Piezoelectric actuators and record the output displacement.
After obtaining the data, MATLAB/Simulink was used to obtain the displacement predicted
by conventional Duhem and FODuhem model. Finally,
the comparison results are obtained. Figures 5 to 9
shows the comparison of the experimental simulation of
traditional Duhem model and FODuhem model. Figures 5(a) to 9(a) gives the modeling error, and
Figures 5(b) to 9(b) gives the comparison of each model
with the experimental hysteresis loop, it can be clearly
seen that the experimental results of FODuhem model
are closer to the experimental data. The modeling error
of the FODuhem model is significantly smaller than
that of the conventional Duhem modeling error. For
quantitative comparison, the root mean square error as
well as the relative error of the FODuhem model are
presented in Table 2, the calculation formula is shown
in (14), (15).

\[
E_{RE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y(i) - y(i))^2} \times \frac{100\%}{N} 
\]

(15)

where \( y_i(i) \) is the output displacement of the
Piezoelectric actuators measured by experiment at the \( i \)th moment, \( y(i) \) is the output displacement of the model at the \( i \)th moment, and \( N \) is the total number of samples.

It can be seen that the traditional Duhem model
cannot accurately describe complex hysteresis loops
with asymmetric and rate-dependent behavior. The
model output has a large error with the experimental
results. It is worth noting that the hysteresis shape is
basically unchanged at low frequencies, and the
fractional order operator will have a larger effect at high
frequencies, so the error at higher frequencies will be
smaller than the error at low frequencies. The RE error
decreases sharply from 5% to 31% as the input fre-
cquency increases. In contrast, the FODuhem model
performs better in describing asymmetric and rate-




| Table 1. Identified parameters of FODuhem model and Duhem model. |
|-----------------|-----------------|-----------------|
| Parameters      | Duhem model     | FODuhem model   |
| \( k \)         | 17.2468         | 2.3580          |
| \( \alpha \)     | 17.9154         | 1.4209          |
| \( \beta \)      | 0.6799          | 0.4682          |
| \( \gamma \)     | -12.4472        | 0.7280          |
| \( p_1 \)        | -0.00057433     | -0.0235         |
| \( \lambda_1 \)  | 0.0499          |                 |
| \( \lambda_2 \)  |                 |                 |
dependent hysteresis. Therefore, the proposed FODuhem model can solve the nonlinear rate-dependent behavior of piezoelectric hysteresis better and with high accuracy, which verifies the validity and feasibility of the model.

**Conclusions**

In order to improve the modeling accuracy, this paper proposes a new FODuhem model to describe the rate-dependent hysteresis characteristics of PEAs. A fractional-order operator is introduced in the FODuhem model instead of integer-order differentiation to enhance the Duhem rate-dependent characteristics, and the derivatives of the input signal allows FODuhem to better describe the hysteresis behavior of PEAs at high frequencies, and the adaptive DE algorithm is used to identify the model parameters. Comparative experimental results on PEAs systems validate the superiority of the developed model in the description of rate-dependent hysteresis. Based on these advantages, the proposed modeling approach offers a wide range of possibilities for model-based control.
Figure 7. Comparison of experimental results with FODuhem model and Duhem model under $f = 20$Hz: (a) comparison of experimental hysteresis loop with the hysteresis loop of FODuhem model and Duhem model and (b) comparison of the error of FODuhem's model with the error of Duhem's model.

Figure 8. Comparison of experimental results with FODuhem model and Duhem model under $f = 50$Hz: (a) comparison of experimental hysteresis loop with the hysteresis loop of FODuhem model and Duhem model and (b) comparison of the error of FODuhem's model with the error of Duhem's model.
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ORCID iDs

Ruobing Zhong https://orcid.org/0000-0002-4468-055X
Dongjie Li https://orcid.org/0000-0002-1005-7890
Zhan Li https://orcid.org/0000-0002-7601-4332

Table 2. The simulation errors of the FODuhem model and Duhem model.

| Frequency | $E_{\text{RMSE}}$ (%) | $E_{\text{RE}}$ (%) |
|-----------|------------------------|---------------------|
| 5 Hz      | 3.59                   | 6.37                |
| 10 Hz     | 3.44                   | 6.15                |
| 20 Hz     | 3.29                   | 5.59                |
| 50 Hz     | 3.22                   | 5.9                 |
| 100 Hz    | 6.44                   | 12.34               |

3. Orszulik RR and Shan J. Output feedback integral control of piezoelectric actuators considering hysteresis. *Precis Eng* 2017; 47: 90–96.
4. Tang H and Li Y. Feedforward nonlinear PID control of a novel micromanipulator using Preisach hysteresis compensator. *Robot Comput Integr Manuf* 2015; 34: 124–132.
5. Qin Y, Tian Y, Zhang D, et al. A novel direct inverse modeling approach for hysteresis compensation of piezoelectric actuator in feedforward applications. *IEEE ASME Trans Mechatron* 2013; 18(3): 981–989.
6. Li Z, Shan J and Gabbert U. Inverse compensation of hysteresis using Krasnoselskii-Pokrovskii model. *IEEE ASME Trans Mechatron* 2018; 23: 966–971.
7. Xu Q. Identification and compensation of piezoelectric hysteresis without modeling hysteresis inverse. *IEEE Trans Ind Electron* 2013; 60(9): 3927–3937.
8. Li Z, Xue S, Lin W, et al. Training a robust reinforcement learning controller for the uncertain system based on policy gradient method. *Neurocomputing* 2018; 316: 313–321.
9. Li Z, Liu HHT, Zhu B, et al. Robust second-order consensus tracking of multiple 3-DOF laboratory helicopters via output feedback. *IEEE ASME Trans Mechatron* 2015; 20(5): 2538–2549.
10. Li Z, Liu HHT, Zhu B, et al. Nonlinear robust attitude tracking control of a table-mount experimental helicopter using output feedback. *IEEE ASME Trans Mechatron* 2015; 62(9): 5665–5676.
11. Feng Y, Rabbath CA, Chai T, et al. Robust adaptive control of systems with hysteretic nonlinearities: a Duhem hysteresis modelling approach. In: AFRICON, 23–25 September 2009, pp.130–135. IEEE.

12. Zhou M and Wang J. Research on hysteresis of piezoceramic actuator based on the Duhem model. Sci World J 2013; 2013(7): 814919.

13. Chen Q, Yang ZX and Fu Z. A systematic identification approach for biaxial piezoelectric stage with coupled Duhem-type hysteresis. Compel 2021; 40(3): 358–372.

14. Gan J, Mei Z, Chen X, et al. A modified Duhem model for rate-dependent hysteresis behaviors. Micromachines 2019; 10(10): 1–9.

15. Wang G and Chen G. Identification of piezoelectric hysteresis by a novel Duhem model based neural network. Sens Actuators A Phys 2017; 264: 282–288.

16. Chen YQ, Xue D and Dou H. Fractional calculus and biomimetic control. In: IEEE international conference on robotics & biomimetics, 22–26 August 2004, pp.901–906. IEEE.

17. Ding C, Cao J and Chen Y. Fractional-order model and experimental verification for broadband hysteresis in piezoelectric actuators. Nonlinear Dyn 2019; 98(4): 3143–3153.

18. Kang S, Wu H, Li Y, et al. A fractional-order normalized Bouc–Wen model for piezoelectric hysteresis nonlinearity. IEEE ASME Trans Mechatron 2022; 27: 126–136.

19. Coleman BD and Hodgdon ML. A constitutive relation for rate-independent hysteresis in ferromagnetically soft materials. Int J Eng Sci 1986; 24(6): 897–919.

20. Zhao C, Xue D and Chen Y Q. A fractional order PID tuning algorithm for a class of fractional order plants. In: IEEE international conference on mechatronics & automation, 29 July–01 August 2005, pp.216–221. IEEE.

21. Jingzhuo S, Juanping Z, Jingtao H, et al. Identification of ultrasonic motor’s nonlinear Hammerstein model. J Control Auto Electr Syst 2014; 25(5): 537–546.