CHAPTER 1

THE PARADIGM OF INFLATION

J. García-Bellido

Departamento de Física Teórica, Universidad Autónoma de Madrid
Cantoblanco, 28049 Madrid, Spain. E-mail: juan.garciaellido@uam.es

The standard model of cosmology is based on the hot Big Bang theory and the inflationary paradigm. Recent precise observations of the temperature and polarization anisotropies in the cosmic microwave background and the matter distribution in large scale structures like galaxies and clusters confirm the general paradigm and put severe constraints on variations of this simple idea. In this essay I will discuss the epistemological foundations of such a paradigm and speculate on its possible realisation within a more fundamental theory.

1. Introduction

Modern cosmology stands today on a firm theoretical framework, based on general relativity and the hot Big Bang theory, which describes with great precision the evolution of the universe from the first fraction of a second to our present epoch. However, this impressive framework was unable to explain the flatness and homogeneity of space, nor the origin of the matter and structures seen in the universe today. The advent of the inflationary paradigm in the 1980’s provided a dynamical mechanism for the generation of both matter and structure in an otherwise flat and homogeneous universe. Its predictions are confirmed today with great accuracy, thanks to a true revolution in cosmological observations, coming mainly from measurements of the anisotropies in the cosmic microwave background (CMB) and the distribution of matter in large scale structures (LSS) like galaxies and clusters of galaxies.

Nowadays, the hot Big Bang theory and the inflationary paradigm constitute the basis of our standard model of cosmology. However, while the theory of the Big Bang is well established, there is no theory of inflation...
yet. At the moment, most people consider inflation as a successful idea, a paradigm, realised in specific models with concrete predictions. I will give here an overview of the present theoretical and observational status of the idea of inflation, within the most general framework, trying to be as much model-independent as possible.

I will first present an epistemological analogy that will help put into context the successes of inflation and the search for a fundamental theory of inflation. I will discuss the assumptions that are used in the construction of the most general models of inflation, with their main caveats. I will then explore its main consequences and predictions, both theoretical and phenomenological. One may think that inflation is such a basic idea that one cannot rule it out. However, I will describe ways in which one could discard not only specific models of inflation but the whole paradigm. To end this broad view of inflation, it might be useful also to state clearly some minor criticisms to the idea of inflation, as well as dispel some false claims that appear in the literature. Finally, although inflation defines a general framework in which most cosmological questions can be addressed, it is certainly not the panacea of all problems in cosmology, so it is worth mentioning those questions inflation cannot answer, at least in its present realisation. At the end, I will give a personal perspective on what I think should be the basic ingredients of a complete theory of inflation.

2. An epistemological analogy

The importance and outreach of a paradigm or a theory is best put into perspective by comparing it with other previous ideas whose epistemology is well known to most researchers in the field. The idea of inflation has sometimes been compared with the gauge principle of the standard model of particle physics. Like the inflationary paradigm, the gauge principle is realised in a specific model (based on a concrete choice of gauge group) with specific predictions that depend on a large number of fine-tuned parameters in order to agree with observations. Fortunately, there is a significant number of independent experiments that have fixed most of these parameters to several decimal places in a consistent framework, at least up to the energies reached with present accelerators. However, if we want to extend the standard model to higher energies, we run into problems of consistency. In order to solve those problems we need to introduce a new theory with many new ingredients. This theory is still uncertain, and will probably require new ideas based on experimental data beyond the present energy scale. Perhaps
with the future particle physics colliders, at higher energies, we may find completely new and unexpected phenomenology. All of this can also be said of the inflationary paradigm and its present realisation in terms of specific models.

However, I feel this is not the best analogy for the inflationary paradigm. What I find more appropriate as an epistemological analogue is Newtonian mechanics. From our modern perspective, we can say that Newton was lucky, he proposed a force law of gravitation without a consistent theoretical framework of space and time, much to the despair of his contemporaries, Leibniz and Berkeley. However, he was smart enough to state no mechanism for this force and propose his laws of celestial mechanics with the statement: “everything works as if...”. What silenced the criticisms made by both physicists and philosophers, like Mach and others, to the idea of absolute space and time was the extraordinary power of Newtonian mechanics for predicting planetary motion.

It was not until the advent of Einstein’s theory of general relativity that this extraordinary coincidence was dispelled, and put Newtonian mechanics in the appropriate perspective as a concrete limit of a more fundamental theory of space and time. I believe the inflationary paradigm is nowadays in a similar situation with respect to a more fundamental theory. Present observations seem to be in complete agreement with the main predictions of inflation, but we still do not understand why it works so well. At a fundamental level, the idea of inflation is outrageously simple, and yet contains all the ingredients necessary to explain a large array of observations, many of which are independent, as well as predicting a surprisingly rich phenomenology. Moreover, most of its predictions were made much before we had any means of making contact with observations. Today, these observations are confirming the general paradigm but still have not singled out any particular model.

3. Basic assumptions

Before describing the successes of the idea of inflation let me begin by stating clearly the basic assumptions that are used in the construction of a generic inflationary model. Variations of this generic model will be discussed below.

The inflationary paradigm assumes that gravity is described by a classical field theory, usually taken to be general relativity in 4-dimensional space-time. This assumption is not essential, it can be relaxed by including
also a dilaton field, a scalar partner of the graviton field, coupled to matter like in scalar-tensor theories, and also by extending it to possibly extra compactified dimensions.

In order to explain the present homogeneity and flatness on large scales, the only requirement is an early period of accelerated expansion of the universe. In the context of general relativity, this means a negative pressure as the source of gravity. The simplest realisation is an approximately constant energy density, leading to quasi-de Sitter (exponential) expansion of the early universe. It can be parametrized in terms of an effective or fundamental scalar field, called the inflaton field, whose nature is yet unknown. Other species (matter and/or radiation) may be present, but will be diluted away very quickly by the expansion. This inflaton field may evolve slowly down its effective potential, or not. While an approximately constant energy density seems to be required, a slow-roll field is only a simplifying assumption. Non-slow-roll models of inflation exist and for the moment make predictions that are compatible with observations.

All matter fields, as well as fluctuations of the inflaton and the gravitational field, are supposed to be quantum fields evolving in a curved background. Quantum fluctuations of these fields start in the vacuum at small scales. The expansion of the universe takes these inflaton and metric fluctuations out of the de Sitter causal horizon and produces classical metric perturbations at superhorizon scales. Later on, during the radiation and matter eras, the causal horizon grows more quickly than the size of the universe and fluctuations reenter again, giving rise to radiation and matter perturbations as they fall into the potential wells of the metric.

4. General consequences

The main consequences of inflation can be classified in two groups: those that affect the space-time background, and perturbations on this background. Due to the tremendous expansion, inflation predicts that spatial sections must be flat, that is, Euclidean, at least on our local patch. It also predicts that these spatial sections must be homogeneous and isotropic to a high degree. Both homogeneity and flatness were unstable properties under the evolution equations of the Big Bang theory and thus required highly fine-tuned initial conditions. Inflation eliminated the fine-tuning by postulating an early period of acceleration of the universe. The rest is a consequence of this simple hypothesis. For instance, if the universe had a non-trivial topology before inflation, it was redshifted away by the expan-
sion, so that the topological cell today — the geometrical cell whose sides are nontrivially identified — should be much larger than the present Hubble volume. At the moment, this is in perfect agreement with observations of the CMB anisotropies.

Inflation not only explains in a dynamical way the observed homogeneity and flatness, but also predicts the existence of inhomogeneities and anisotropies arising from quantum fluctuations of the inflaton and the metric, and possibly other scalar fields evolving during inflation. General relativity predicts that there must be six physical degrees of freedom associated with a generic metric perturbation: two scalar, two vector (the vorticity and shear fields) and two tensor (the two polarizations of a gravitational wave) components. The perturbed Einstein equations, together with the inflaton field as matter source, imply that the two scalar metric components are linearly dependent, while the vorticity field decays very quickly during inflation. Therefore, we are left with only one scalar (giving rise to curvature perturbations) and two tensor components.

Due to quantum mechanics, superhorizon quantum fluctuations become semi-classical metric perturbations. These perturbations have an amplitude proportional to the approximately constant energy density driving inflation and, therefore, give rise to almost scale-invariant spectra of density perturbations and gravitational waves, that is, fluctuations with approximately the same amplitude on all scales. The density perturbations will later seed the structure we see in our universe, as matter falls into them and evolves through gravitational collapse. Deviations from exact scale invariance — the tilt of the spectrum — characterise a particular model of inflation and thus allow cosmologists to distinguish between them. In some cases, the tilt may also depend on scale, which puts further constrains on models of inflation. On the other hand, the tensor perturbations give rise to a spectrum of gravitational waves that may eventually be detected by laser interferometer gravitational wave observatories. Moreover, since quantum fluctuations arise from essentially free fields during inflation, both primordial spectra are expected to have a statistical distribution that is approximately gaussian.

If there were more than one scalar field evolving during inflation, their quantum fluctuations may have given rise to both energy density and entropy perturbations. The former arise from scalar curvature perturbations, while the latter is responsible for isocurvature perturbations. In some cases, this also implies a small non-gaussianity in the spectrum, and more than one spectral tilt. Unless required by observations, we will use Occam’s razor and assume that inflation is driven by a single scalar field.
5. Observable predictions

The main prediction of inflation is that spatial sections are Euclidean, \textit{i.e.} with negligible spatial curvature, at least within our Hubble volume. This has been confirmed at the 2 percent level by recent measurements of CMB anisotropies, as described below. Inflation also predicts a homogeneous space-time background, with inhomogeneities or anisotropies imprinted by small amplitude quantum fluctuations giving rise to classical curvature perturbations. The amplitude is not fixed by the general paradigm, but can be computed precisely for a given model. In single-field inflation, the amplitude of the curvature perturbation remains constant outside the horizon and, therefore, its amplitude at reentry is precisely given by that at horizon exit during inflation. Thus, a generic prediction of inflation is a primordial spectrum of scale-invariant curvature perturbations on superhorizon scales, that will seed structure once they reenter the causal horizon and matter falls into them, undergoing gravitational collapse. Such a primordial spectrum was postulated in 1970, much before inflation, by Harrison and Zeldovich in order to explain the present distribution of matter, and I think it is extraordinary that such a simple idea as inflation could also explain the origin of this peculiar spectrum in terms of quantum fields in a curved background that, due to the expansion, become classical metric perturbations.

In fact, curvature perturbations are also responsible for temperature and polarization anisotropies in the CMB. As baryons fall into the potential wells of these perturbations, they induce acoustic oscillations in the plasma, with radiation pressure opposing gravitational collapse. A perturbation of a given scale can only grow after it enters the causal Hubble radius. Since all perturbations of the same wavelength entered simultaneously, they must be in the same phase of the acoustic oscillation at the time of decoupling. This gives rise to spatially coherent oscillations that stand out as peaks in the two-point angular correlation function. This pattern of acoustic oscillations in the power spectrum of temperature anisotropies is a characteristic signature of inflation, and has been clearly seen in the CMB anisotropies by Boomerang and WMAP. Any alternative theory of structure formation, like \textit{e.g.} topological defects, based on an active causal origin of matter perturbations, could not explain the observed pattern of acoustic oscillations and are therefore ruled out. It is extraordinary that the surprisingly simple paradigm of inflation predicted the coherent oscillations much before they could be measured. We are now using the detailed properties of the observed spectrum to determine many cosmological parameters. For
instance, the first acoustic peak in the power spectrum appears at a scale that precisely corresponds to the projected size of the acoustic horizon at decoupling, approximately one degree in the sky today, and indicates that photons have travelled since then in straight lines towards us. This implies that the universe is essentially flat, with deviations less than a few percent, again as predicted by inflation.

Furthermore, according to inflation there will also be large scale perturbations that are still outside the horizon at the time of decoupling and therefore are not enhanced by the acoustic oscillations of the plasma. These superhorizon perturbations are a specific prediction of inflation, and are responsible for the so-called Sachs-Wolfe plateau, arising from photons that are gravitationally redshifted or blueshifted as they escape from or fall into these primordial perturbations. The height of the plateau has direct information about the amplitude of the primordial perturbations and the flatness of the plateau about the tilt of the spectrum. Without an acausal mechanism like inflation that can stretch fluctuations beyond the horizon and imprint a primordial spectrum with superhorizon scales at decoupling, we could not explain the Sachs-Wolfe plateau, observed by the COsmic Background Explorer (COBE) satellite for the first time in 1992. To this plateau contributes not only the (scalar) density perturbations, but also the (tensor) gravitational wave spectrum.

However, the fact that from our particular position in the sky we cannot see more than a few realizations of the gaussian distribution of large scale fluctuations – an example of sample variance that here goes by the name of cosmic variance – means that the small tensor contribution to the Sachs-Wolfe plateau may be hidden inside the cosmic variance of the scalar component, and since gravitational waves are quickly redshifted by the universe expansion after they enter the Hubble radius, their imprint in the temperature power spectrum could be unmeasurable after all. The primordial tensor spectrum is important because it contains complementary information about the inflationary dynamics, and, in particular, its amplitude depends only on the total energy density driving inflation, which is still a great unknown.

Fortunately, the microwave background contains information not only about the energy distribution of photons, a precise blackbody spectrum, but also about their linear polarization. The polarization of the microwave background is a vector field with a gradient and a curl components (called E and B components in analogy with electromagnetism). The amplitude of the polarization spectrum is directly predicted by the size of the temper-
ature fluctuations, since the linear polarization of CMB photons can only arise via Thomson-scattering, from quadrupolar anisotropies in orthogonal directions. This polarization is induced by photons scattering off electrons in the last scattering surface, when the universe was essentially neutral. As a consequence, the polarization spectrum is an order of magnitude smaller than the temperature spectrum, making it even more difficult to observe than the latter.

Only recently, DASI measured for the first time the polarization field at the last scattering surface. However, it was WMAP, with much bigger sky coverage and sensitivity, that measured the precise angular dependence of the polarization field, and confirmed that the temperature and the gradient (E) component of the polarization spectrum were correlated. WMAP gave the multipole expansion of the TE cross-correlation spectrum, with a pattern of peaks matching those predicted long before by the simplest models of single-field inflation. This is without any doubt one of the most important arguments in favour of inflation. While the existence of the CMB polarization spectrum is a generic prediction of Big Bang cosmology at photon decoupling, it is extremely difficult to construct *ad hoc* models of structure formation and yet be in agreement with this pattern of acoustic oscillations in both the temperature and the polarization power spectra.

Moreover, we still have the curl (B) component of the polarization spectrum. This component can only arise from the (tensor) gravitational wave primordial spectrum of perturbations coming from inflation, and has not been detected yet. It may happen that the scale of inflation is high enough for a significant amplitude of tensor perturbations to generate a B polarization field at the level needed for detection in the next generation of CMB satellites like Planck or CMBpol. At this moment, there is a large experimental and theoretical effort searching for ways to extract the rare primordial B-component of the CMB, since it contains crucial information about the scale of inflation that we may not be able to obtain otherwise.

Another interesting feature about CMB polarization is that it provides a null test of inflation. We have seen that in the absence of anisotropic stresses acting as sources during inflation, the vector component of metric perturbations is negligible. While the scalar and the tensor components are both symmetric under parity, the E- and the B-components of polarization have opposite parity. Therefore, a robust prediction of inflation is that there should be no correlation between the scalar temperature (T) and the B-component of polarization, nor between the E- and the B-components of polarization. This can be summarised as $\langle BT \rangle = \langle BE \rangle = 0$, and constitute
an important test of inflation. The other four combinations, the three power spectra $\langle TT \rangle$, $\langle EE \rangle$ and $\langle BB \rangle$, and the $\langle TE \rangle$ cross-correlation are predicted by inflation to be non-zero and to possess a precise pattern of acoustic oscillations.

Furthermore, CMB anisotropies are not the only probes of inflationary predictions. The same primordial spectrum of curvature perturbations responsible for temperature anisotropies gives rise, through gravitational collapse of the primordial baryon and dark matter distributions, to the large scale structures like galaxies, clusters and superclusters. These perturbations are probed by the so-called matter power spectrum, that is, the Fourier transform of the two-point correlation function of luminous galaxies in clusters and superclusters, all the way to the horizon in our Hubble volume. If light traces matter then this power spectrum may have a precise relation to the primordial spectrum of curvature perturbations, and thus to inflation and the CMB anisotropies. The primordial adiabatic, gaussian and scale-invariant spectrum of metric perturbations seeds, through gravitational collapse, the measured matter power spectrum. Galaxy surveys both in real and redshift space have increased their covering volume at increasing rate since the first CfA catalogs in the 1970s to the present 2dF Galaxy Redshift Survey and the Sloan Digital Sky Survey, which cover a large fraction of the universe up to redshifts or order $z = 1$ for galaxies, and order $z = 3$ for quasars. These surveys have allowed cosmologists to bring into sharp focus an image of the nearby universe to unprecedented accuracy, confirming basic properties of the power spectrum, like gaussianity and scale invariance, at least on large scales, where the power spectrum has not been strongly distorted by non-linear gravitational collapse.

In the near future, with the advent of the next generation of gravitational wave observatories, it will be possible in principle to detect the approximately scale-invariant spectrum of gravitational waves generated during inflation, seen as a stochastic background in time-correlated detectors across the Earth. However, this background could remain undetectable, even by sensitive gravitational wave observatories, if the scale of inflation is too low. The detection of this new cosmic background, with specific signatures like a small negative tilt, i.e. larger amplitudes at longer wavelengths, may open the possibility of testing the idea of inflation. It happens that in all single field models of inflation, there is a unique relation between the two spectra (scalar and tensor) because the source of these metric perturbations is a single function, the effective inflaton potential. In particular, in models where the inflaton field evolves slowly towards the end of infla-
tion and reheating, the ratio between the tensor and scalar amplitudes is uniquely related with the tilt of the gravitational wave spectrum. This is a prediction that no other theory of structure formation could have imagined a priori, and thus constitutes a test of the whole paradigm of inflation. If confirmed by observations it would have tremendous impact on inflation as a paradigm.

6. The reheating of the universe

The reheating epoch, in which the energy density of the inflaton decayed into radiation, is also a universal feature of inflation. We know that at some point inflation must end, at least in our local patch, since today the universe contains the remnants of the radiation and matter eras. This energy conversion epoch signals the beginning of the hot Big Bang as we know it. However, the details of how it proceeds from the quasi-stationary inflationary regime to the radiation dominated era is rather model-dependent and requires some knowledge of the high energy particle physics model in which inflation is embedded. In some models it occurs in a perturbative way, as the quanta of the inflaton field decay into other field quanta to which the inflaton couples. In other models it occurs in an explosive non-perturbative way, producing a variety of interesting phenomenology like non-thermal phase transitions, with possibly the generation of topological defects, gravitational waves and/or primordial magnetic fields. These features may leave their imprint in the CMB anisotropies or in the stochastic gravitational wave background or even in the primordial intergalactic magnetic fields. Moreover, the far from equilibrium conversion of energy may help generate the matter/antimatter asymmetry at reheating, in a way that is now being explored both analytically and numerically.

Although none of these features are robust predictions of inflation, once observed they could give crucial information about the theory of high energy particle physics in which inflation is embedded, that is, the couplings and masses of the other fields to which the inflaton couples. For instance, the production of topological defects during reheating after inflation is predicted in specific models of inflation. Absence of any signatures from these cosmic defects can be used to impose stringent constraints on those models, but do not rule out other realisations of reheating.

What I think is outstanding of inflation is its ability to create such a novel and rich phenomenology from such a simple hypothesis. For inflation to work, very little has to be put in, and a lot seems to comes out, and
some of its predictions have far reaching consequences.

7. Eternal inflation

Although inflation was invented in order to alleviate the problem of initial conditions of the Big Bang, it is not in itself a theory of initial conditions. In fact, in order for inflation to proceed in the first place it is assumed that it started in a single domain that was sufficiently homogeneous across a causal horizon for the energy density to be treated as approximately constant. Once inflation started, this domain would occupy a large space-time volume due to the rapid expansion and later give rise to the whole large scale structure of space-time.

Until now, I have described the observable consequences of a generic model of inflation, \textit{i.e.} what is amenable to direct experimentation in our Hubble volume, but there is also the possibility of studying phenomena beyond our observable patch. The inflationary paradigm has a mechanism for generating metric perturbations that is completely universal, independent of the particular realisation of inflation. The spectral properties of those perturbations depends in a well known way on the precise dynamics associated with a given model, but the mechanism is a generic property of quantum field theory in curved space-times. What is less known is that backreaction of the metric plus inflaton fluctuations on the background space-time makes the inflaton field follow a Brownian motion in which half of the time the inflaton field in a given domain will jump upwards, instead of drifting down the effective potential. In those domains the rate of expansion increases. Since the amplitude of the quantum fluctuation is proportional to the local rate of expansion, the inflaton field in a few domains may continue to jump upwards, driving higher and higher rates of expansion, and therefore those domains will eventually occupy larger and larger volumes. Thus the universe becomes divided into domains in which the inflaton has drifted down the potential towards reheating and others that are still inflating, separated by distances much greater than their causal horizon. This stochastic picture leads to the self-reproduction of the universe and to eternal inflation. We may happen to be living in an island of warmth appropriate for life in an otherwise cold and eternally expanding universe. The island is probably large enough that deviations from homogeneity and flatness are negligible up to many times our present Hubble volume, so there is little chance of observing those inhomogeneous regions, at least in our lifetime as species. However, from the epistemological point of view, it is fascinating to enter-
tain the idea that in causally disconnected regions of the universe we may have independent islands of thermalised regions in all possible stages of their evolution. Eventually all causal domains will end inflation, but at any given time the bulk of the universe is in eternal inflation. From this point of view, there is not just one Big Bang, but infinitely many, corresponding to those events where inflation ends locally and a radiation epoch ensues. This alleviates the problem of initial conditions of inflation: once a given domain allows a tiny burst of inflation, the flame will carry away expanding the rest of the universe forever. Local regions of the universe may collapse after matter domination, while others will expand for ever. Our particular patch, what we call the observable universe, is but a tiny insignificant spot in this metauniverse that encompasses all of space and time. Most probably, this grand picture of the universe will never be amenable to direct experimentation. We may have a hint of its existence from the actual pattern of CMB anisotropies, since the same quantum fluctuations that gave rise to those inhomogeneities is also responsible for the ultra large scale structure of the universe, but we may not be able to disprove it with local observations.

8. Variations on a common theme

Although the paradigm of inflation is based on the simple idea that the early universe went through a quasi-exponential expansion that stretched all scales, there are in principle many ways to realise this paradigm. Our first reaction is, like Newton, to make no hypothesis, but state that \textit{everything works as if} an approximately constant energy density dominated the evolution of the early universe. Usually, this slowly changing energy density is parametrised by a single scalar degree of freedom because it is the simplest possibility, but for no other reason. This is enough to explain all observations, both in the CMB anisotropies and in the LSS distribution of matter. However, one can think of a series of variations on the same theme, for example adding another scalar field – most high energy physics models have not just one but many light scalar degrees of freedom – and induce isocurvature as well as adiabatic perturbations. They would produce specific signatures in the CMB anisotropies that have not been observed, and thus multi-field inflation is essentially ruled out, at least to some degree.

It could also happen that the inflaton field may not be treated as a free field because of its own self-coupling, because it couples to other light fields, or due to its coupling to gravity. In either case, the spectrum of quantum fluctuations is expected to have some, but small, non-gaussian statistics
For the moment, these non-gaussianities have been searched for, both in the CMB and in LSS, without success. They would appear as deviations from the gaussian expectations in the three- and four-point correlation functions of galaxies and CMB anisotropies. A small degree of non-gaussianity is expected at galactic scales due to non-linear gravitational collapse, but for the moment what is observed is compatible with an evolved primordial gaussian spectrum. However, in the future, the information encoded in the non-gaussianities could be crucial in order to distinguish between different models.

On the other hand, a prediction that is quite robust for slow-roll inflation of the power-law type (i.e. with cubic and quartic self-interactions of the inflaton) is a precise relation between the deviations from scale invariance (the tilt of the scalar spectrum) and the duration of inflation, which makes this tilt be negative, and of order a few parts in a hundred. Another prediction of this class of models is that the tilt is also scale-dependent, but the dependence is very weak, of only a few parts in 10000. These two predictions break down in models that are non-slow-roll, which are compatible with the rest of the inflationary phenomenology, but which predict large tilts for both scalar and tensor spectra, as well as a large scale-dependence of those tilts. Since the tilt has not been measured yet with sufficient accuracy, we cannot rule out the possibility of non-slow-roll inflation. Moreover, a large negative tilt would signal the presence of a large tensor (gravitational wave) component which would then be seen in the future CMB satellite experiments. However, this prediction is not universal. There are models of inflation, like hybrid models – that end inflation when the inflaton triggers the breaking of a symmetry associated with another field to which it couples – that can give an essentially flat spectrum, i.e. no tilt within the precision of planned missions, and also no scale-dependence of the tilt, nor a significant tensor component.

9. How to rule out inflation

This is a tricky business since inflation is a paradigm with many realisations. Inflation makes space flat and homogeneous, but at the same time imprints space with metric fluctuations that later will seed the local structures we observe, like galaxies and clusters of galaxies. An obvious way to rule out the general idea of inflation would be to find evidence of a large spatial curvature, but the question is on what scale. Since the curvature of space...
depends on the duration of the period of inflation — the longer the flatter — a relatively short period might give rise to a marginally open or closed universe, depending on what sign of curvature they started with. However, curved models of inflation are somewhat fine-tuned and are nowadays ruled out at a high confidence level by the position of the first acoustic peak of the CMB temperature anisotropies.

Another way to rule inflation out is to discover that the universe is topologically non-trivial because, as far as we know from the classical theory of general relativity, the topology of space-time is invariant under the expansion of the universe; the only thing inflation does to topology is to increase the size of the topological cell. For topology to be seen today as correlated patterns in the sky, the cell should have a size smaller than the present Hubble volume. Such patterns have not been seen by the WMAP precision measurements of CMB anisotropies and, therefore, the topological cell, if it exists, must have a size much bigger than our present horizon. Yet another way to rule out the general idea of inflation would be to discover a global rotation of the universe, that is, a privileged axis in the sky. Since inflation expands all scales isotropically, such an axis would stand out very clearly in the CMB sky today, and nothing like it has been observed by WMAP.

The previous discussion has to do with the global background properties of our observable universe. However, what characterises inflation, and allows alternative models to be distinguished, is the predicted spectrum of metric perturbations. Since the source of metric perturbations is a fundamental or an effective scalar field (it could be a condensate of some other field), there can be only two types of perturbations at first order: scalar and tensor metric perturbations. Vector perturbations only appear at second order, and therefore are suppressed, unless there are peculiar additional sources acting during inflation. Thus, a generic prediction of inflation is that the vector perturbation spectrum is absent. If someone would measure a significant primordial spectrum of vector perturbations in the CMB or LSS, one would have to question some of the assumptions of inflation, in particular single-field inflation, although it would probably not rule out the idea altogether. For the moment we have not seen any evidence of a vector perturbation in the CMB. This brings together an immediate consequence with respect to the polarization spectra, as I explained above. In the absence of any vector perturbation, parity is conserved and the cross-correlations $\langle BT \rangle$ and $\langle BE \rangle$ are expected to be zero. But this is a null test, and it may happen that detailed observations of the CMB indicate a non-zero primordial component. In that case, we would have to go back to the drawing board.
Similarly with large tensor perturbations or gravitational waves. If their amplitude is too large, compared with the scalar perturbations, it would imply an origin of inflation close to the Planckian era, and thus the classical description of space-time in terms of the general theory of relativity would not be appropriate. There are models of inflation that consider initial conditions close to the Planck boundary, but their effects are seen on scales much much larger than our Hubble volume, and by the time the fluctuations that gave rise to structure in our observable universe were produced, inflation proceeded well below Planck scale.

Concentrating now on the primordial scalar spectrum, the tilt of the spectrum is not arbitrary. It cannot be too different from scale-invariant \((n = 1)\), or inflation would not have occurred in the first place. A deviation of order 100% larger or smaller than 1 is probably incompatible with most realisations of inflation, since it would require too steep potentials, although there is no general theorem, as far as I know. Typical slow-roll models of inflation predict spectral tilts that are only a few percent away from scale invariance. Some models, like hybrid inflation, are perfectly compatible with exact scale invariance, within experimental errors. In order to get significant tilts one has to go beyond slow-roll, and there are no generic analytical results for these models. One has to study them case by case.

Another key feature of primordial spectra is the scale-dependence of their spectral tilts. Again, most slow-roll models predict no significant scale-dependence, just of order a few parts in 10,000. But there could be specific models with particular features in their effective potential that change significantly at scales corresponding to those seen in the CMB and LSS, which would induce a scale-dependent tilt. In fact, there has been some debate whether WMAP has actually seen such a scale dependence in the CMB anisotropies, but there is no consensus.

In single-field models of inflation, the scalar and tensor spectra are both computed from a single analytical function, the effective scalar potential, and therefore there must be a functional relationship between these two spectra. In slow-roll inflation, it can be cast into a relation between the ratio of the tensor to scalar amplitude and the tensor spectral tilt. Such a relation is very concrete and generic to all single-field models. We know of no other mechanism but inflation which might actually give such a peculiar relation, and for that reason it has been named “the consistency relation” of inflation. If, once the gravitational wave spectrum is discovered, such a relation did not hold, it would signal the demise of slow-roll single-field models. However, deviation from that exact relation could be due to multiple-field models,
which predict a similar but different relation, with one extra parameter, that can also be measured and then checked to hold. If that doesn’t happen either, then probably slow-roll models are not a viable description, and we would need to do fully non-perturbative calculations to find the exact relation between the two spectra. Eventually, whatever the relation, there must be some correlation between them that will allow cosmologists to discard all but a few realizations of inflation. That could be hard but fruitful work.

The problem, however, could be observational: the foregrounds to the CMB. In order to measure the tensor spectrum we need to detect the B-component of polarization, which may be swamped by contamination from a distorted E-component due to gravitational lensing in the intervening mass distribution between us and the last scattering surface. Moreover, on small scales it may be difficult to disentangle the primordial spectrum from the observed spectrum of perturbations. For instance, we know that non-linear gravitational collapse induces both vorticity and non-gaussianities and that gravitational lensing induces BE cross-correlation. In order to disentangle foregrounds from the CMB anisotropies, we may have to know more about the local and global distribution of matter in structures like galaxies, clusters and superclusters.

Finally, most realizations of inflation describe the universe dynamics with an effective scalar field that is essentially free during inflation, only its mass is relevant. In that case, the ground state of this quantum field gives rise to a gaussian spectrum of metric fluctuations, with only second order deviations from gaussianity. Such a prediction is very robust, at least in single-field models of inflation, and therefore, if a significant non-gaussianity were observed in the CMB anisotropies or LSS, it could signal multiple-field inflation, which could be contrasted with other predictions like isocurvature signatures; or it could indicate a significant contribution from postulated modifications of gravity on large scales. In any case, ruling out specific models of inflation leave fewer possibilities and allow cosmologists to close in on the best model, which may eventually lead to a final theory of inflation.

10. Honest criticisms and false claims

There are some features of inflation that are directly related with its status as a paradigm, i.e. not yet a theory. The main criticism that inflation has received over the years is that there is no unique model, but a variety of single- and multi-field models, some with fundamental descriptions, others
purely phenomenological. I think this is an honest criticism; in the absence of a fundamental theory of inflation we have to deal with its effective realisation in terms of specific models. However, the models are fully predictive, they give concrete values for all observables, and can be ruled out one by one. It is possible that in the near future, with more precise cosmological observations, we may end up with a narrow range of models and eventually only one agreeing with all observations.

There are, however, some basic properties of inflation we still do not comprehend. In particular, we ignore the energy scale at which inflation occurred. It has often been claimed to be associated with the scale of grand unification, in which all fundamental forces but gravity unified into a single one, at around $10^{16}$ GeV, but this is still uncertain. The only observable with direct information about the scale of inflation is the amplitude of the gravitational wave spectrum, but we have not measured that yet and it may happen that it is too low for such a spectrum to be seen in the foreseeable future. This would be a pity, of course, but it would open the door to low scale models of inflation and a rethinking of the finetuning problems associated with them. A related criticism is that of not knowing whether the inflaton field is a low energy effective description of a more fundamental interaction or a genuine scalar field. We still have not measured any fundamental scalar field, and it could be that the inflaton is actually a geometrical degree of freedom related to the compactification of some extra dimensions, as recently proposed in the context of some string theory realizations of inflation.

Moreover, the issue of reheating of the universe remains somewhat speculative, even though a lot can be said in generic cases. For instance, there are low energy models of hybrid inflation that could have taken place — albeit with some finetuning — at the electroweak scale. In that case, it is expected that the inflaton will be some scalar degree of freedom associated with electroweak symmetry breaking and thus in the same sector as the Higgs. The gravitational wave component of the spectrum would be negligible, but at least one could explore the reheating mechanism by determining the couplings of the inflaton to other fields directly at high energy particle accelerators like the Large Hadron Collider at CERN.

On the other hand, inflation has also been for many years the target of unfounded criticisms. For instance, it has been claimed for years that inflation requires extremely small (finetuned) couplings in order to agree with CMB observations. Although this is true for some models, this is not a generic feature of inflation: hybrid inflation models agree very well with
observations and do not require unnaturally small couplings and masses. Another common error is to claim that inflation is not predictive because of its many models. This is false, each specific model makes definite predictions that can be used to rule the model out. In fact, since the idea was proposed in the 1980’s, the whole set of open inflation models has been ruled out, as well as models within a scalar-tensor theory of gravity. Furthermore, multi-field models of inflation have been recently constrained by observations of the temperature and polarization spectra, which do not seem to allow a significant isocurvature component to mix with the adiabatic one.

Another false claim which is heard from time to time is that inflation always predicts a red tilt, with a smaller amplitude at smaller wavelengths, but this is not true. Again, hybrid models of inflation, which are based on the idea of spontaneous symmetry breaking in particle physics, predict a slightly blue-tilted spectrum, and sometimes no tilt at all. Associated with this claim, there was the standard lore that the ratio of the tensor to scalar component of the spectrum was proportional to the tilt of the scalar spectrum. This is incorrect even in slow-roll models of inflation, being proportional to the tensor tilt, not the scalar one, which makes the prediction of the tensor amplitude more difficult.

11. What inflation cannot answer

Although inflation answers many fundamental questions of modern cosmology, and also serves as the arena where further questions can be posed, it is by no means the panacea of all problems in cosmology. In particular, inflation was not invented for solving, and probably cannot solve by itself, the problem of the cosmological constant. This problem is really a fundamental one, having to do with basic notions of gravity and quantum physics. It may happen that its resolution will give us a hint to the embedding of inflation in a larger theory, but I don’t think inflation should be asked responsible for not explaining the cosmological constant problem. The same applies to the present value of cosmological parameters like the baryon fraction, or the dark matter content of the universe, or the rate of expansion today. Those are parameters that depend on the matter/energy content after reheating and their evolution during the subsequent hot Big Bang eras. At the moment, the only thing we know for sure is that we are made of baryons, which contribute only 4% of the energy content of the universe. The origin of the dark matter and the cosmological constant is a mystery. In fact, the only thing we know is that dark matter collapses gravitationally and
forms structure, while vacuum energy acts like a cosmic repulsion, separating these structures at an accelerated rate. They could both be related to modifications of gravity on large scales — associated with the mysterious nature of the vacuum of quantum field theories — but for the moment this is just an educated guess.

Inflation can neither predict the present age of the (observable) universe, nor its fate. If the present observation of the acceleration of the universe is due to a slowly decaying cosmological constant, the universe may eventually recollapse. Nor can inflation predict the initial conditions of the universe, which gave rise to our observable patch. Quantum gravity is probably behind the origin of the inflationary domain that started everything, but we still do not have a credible quantum measure for the distribution of those domains.

12. Conclusions and Outlook

The paradigm of inflation is based on the elegantly simple idea that the early universe went through a quasi-exponential expansion that stretched all scales, thus making space flat and homogeneous. As a consequence, microscopic quantum field fluctuations are also stretched to cosmological scales and become classical metric perturbations that seed large scale structure and can be seen as temperature anisotropies in the CMB.

The main predictions of this paradigm were drawn at least a decade before observations of the microwave background anisotropies and the matter distribution in galaxies and clusters were precise enough to confirm the general paradigm. We are at the moment improving our measurements of the primordial spectrum of metric fluctuations, through CMB temperature and polarization anisotropies, as well as the LSS distribution of matter. Eventually, we will have a direct probe of the energy scale at which inflation took place, which may open the possibility of constructing a complete theory of inflation.

For the moment, all cosmological observations are consistent with the predictions of inflation, from the background space-time to the metric and matter fluctuations. It is remarkable that such a simple idea should work so well, and we still do not know why. From the epistemological point of view, one could say that we are in a situation similar to Newtonian mechanics in the nineteenth century, before its embedding in Einstein’s theory of general relativity. We can predict the outcome of most cosmological observations within the standard model of cosmology, but we do not yet have
a fundamental theory of inflation.

Such a theory will probably require some knowledge of quantum gravity. This has led many theoretical cosmologists into speculations about realisations of inflation in the context of string theory or M-theory. This is certainly an avenue worth pursuing but, for the moment, their predictions cannot be distinguished from those given by other models of inflation. Eventually the higher accuracy of future CMB measurements and LSS observations will allow cosmologists to discard most models of inflation but a few, in the search for the final theory of inflation. It may require completely new ideas and perhaps even an epistemological revolution, but I am sure that, eventually, we will be able to construct a consistent theory of inflation based on quantum gravity and high energy particle physics.

Acknowledgments

It is always a pleasure to thank Andrei Linde for many enlightening discussions on the paradigm of inflation and its far reaching consequences. This work was supported by the Spanish Ministry of Science and Technology under contract FPA-2003-04597.

References

1. A.D. Linde, *Particle Physics and Inflationary Cosmology*, (Harwood Academic Press, New York, 1990).
2. A.D. Linde, “The self-reproducing inflationary universe”, *Scientific Am.* November, 32 (1994).
3. A.H. Guth, *The inflationary universe*, (Perseus Books, Reading, 1997).
4. J. García-Bellido, “The origin of matter and structure in the universe”, *Phil. Trans. R. Soc. Lond.* 357, 3237 (1999).
5. A.R. Liddle & D.H. Lyth, *Cosmological Inflation and Large Scale Structure*, (Cambridge U. Press, Cambridge, 2000).
6. Boomerang home page, [http://oberon.roma1.infn.it/boomerang/](http://oberon.roma1.infn.it/boomerang/)
7. Wilkinson Microwave Anisotropy Probe home page, [http://map.gsfc.nasa.gov/](http://map.gsfc.nasa.gov/)
8. 2dF Galaxy Redshift Survey home page, [http://www.mso.anu.edu.au/2dFGRS/](http://www.mso.anu.edu.au/2dFGRS/)
9. Sloan Digital Sky Survey home page, [http://www.sdss.org/sdss.html](http://www.sdss.org/sdss.html)
Juan García-Bellido

Born in 1966 in Madrid, Spain, Juan García-Bellido obtained his PhD in 1992 from the Autónoma University of Madrid. He was a Postdoctoral Fellow at Stanford University (1992-94), PPARC Fellow of the Astronomy Centre at the University of Sussex (1995-96), Fellow of the TH-Division at CERN (1996-98), a Royal Society University Research Fellow at Imperial College (1998-99), and since then Professor at the Institute of Theoretical Physics in Madrid. He has published around 70 papers in theoretical physics and cosmology; attended, organised and gave lectures at international conferences and summer schools around the world, and is referee of the most prestigious journals in the field. Aged 38, he is married to a particle physicist and has two kids, a girl and a boy, aged 7 and 2, respectively. Scientific interests include the early universe, black holes and quantum gravity.