Applications of Chern-Simons Ward Identities to log($T\tau$) Conductivity Calculations of the $\nu = 1/2$ System

Jürgen Dietel
Institut für Theoretische Physik,
Universität Leipzig, Germany

Abstract

We reconsider the theory of the half-filled Landau level with impurities using the Chern-Simons formulation and study Ward identities for the Chern-Simons theory. From these we get conductivity diagrams with impurities which obey the continuity equation. We calculate the conductivity of these diagrams for which we obtain log($T\tau$) divergent conductivity diagrams for low temperature $T$. We compare our result with the experimental values of the low temperature conductivities. Finally we calculate the conductivity for small deviations of the magnetic field $B$ from the value $\nu = 1/2$. In this case we get a singularity in the conductivity.

1 Introduction

In this paper we consider a system of electrons in a strong magnetic field in two dimensions. This system is characterized by the filling factor $\nu$, defined as the electron density divided by the density of a completely filled Landau level. In the case of $\nu \approx 1/2$ the behavior of the system resembles that of a Fermi liquid in the absence of a magnet field or at small magnetic fields. Over past years an intriguing picture has emerged: At $\nu = 1/2$ each electron combines with two flux quanta of the magnetic field to form a composite fermion (CF); these composite fermions then move in an effective magnetic field which is zero on the average. The interpretation of many experiments support this picture. We mention some transport experiments for illustration [1].
Figure 1: Vertex operations for constructing $\Lambda^b$ from a self energy graph $\Sigma^b$. For $\Lambda^b_0$ one has to do vertex operation (a) with a density-vertex between the two Green’s functions. For $\Lambda^b_i$ one has to do vertex operation (a) with a current-vertex between the two Green’s functions as well as the vertex operation (b).

The theoretical framework for the understanding of the $\nu = 1/2$-system was developed by Halperin, Lee and Read [2]. In their theory one has to pay a price to get a mean-field free system for the CFs. The CFs interact via long-ranged gauge interactions. Further in real $GaAs/Al_xGa_{1-x}As$ heterojunctions one also has a large number of Coulomb impurities which has to be integrated in the theory, too. For the case of the Coulomb problem this was done about 20 years ago [6, 7].

Our aim is to calculate the low temperature conductivity of CFs with impurities. In the first section of this work we care about which Feynman graphs should be calculated to get a good conductivity for the $\nu = 1/2$ system. This question will be treated with the help of Ward identities [3] of Chern-Simons systems. In the second section we will apply the results of section 1 to get some Feynman-graphs which should give a good approximation of the conductivity for CFs. At least we will calculate the conductivity of these Feynman graphs at low temperature $T$ and make comparison with experimental results.

2 The Impact of Ward Identities of CFs on the perturbational continuity equation

In this section we try to answer the question which subset of Feynman diagrams should be calculated, in order to get a good estimate for the conductivity of the $\nu = 1/2$-system. To answer this question we consider at first some self energy diagrams $\Sigma^p(k)$ of frequency $k_0$ and impulse $(k_1, k_2)$ (Index $p$ for perturbational). In the following we will construct $T$-product response Feynman-graphs from $\Sigma^p$. $\Lambda_\mu$ (
\( \mu = 0.2 \) is defined as

\[
\langle |T [j_\mu(q), \Psi^*(k) \Psi(k + q)]| \rangle = G(k) \Lambda_\mu(k + q, q)G(k + q) ,
\]

where \( G(k) \) is the Chern-Simons Green's function. \( \vec{j}, \Psi \) are the current operator and field operator of the Chern-Simons theory. In the following one has to notice that the Chern-Simons vertices contain current vertices \((2\vec{k} + \vec{q})/2m\) where \( f_i(q) = i2\pi\delta_{ij} q_j/q^2 \).

We now apply some transformations on \( \Sigma_p \) to get \( \Lambda_\mu \) graphs. We use the notation \( \Lambda_0^p \) for \( \Lambda_0 \) diagrams which originate from \( \Sigma_p \) through the operation (a) in figure 1 for every Green’s function in \( \Sigma_p \). This means that one one has to ‘divide’ every Green’s function in \( \Sigma_p \) making two Green’s function. Similarly \( \Lambda_i^p \) \((i = 1, 2)\) denotes the diagrams which are originating through operation (a), (b) in figure 1. This has to be done for every Green’s function and current vertex in \( \Sigma_p \). In contrast to \( \Lambda_0^p \) operation (a) puts for \( \Lambda_i^p \) a current operator between the two Green’s functions. The vertices in diagram (b) are current CS-vertices. In (b) every current vertex of \( \Sigma_b \) is transformed into a density-density CS current coupling. Then one can derive the following Ward identities

\[
- q_0 (\Lambda^p_0(k, k + q_0e_0) - 1) = \Sigma^p(k + q_0e_0) - \Sigma^p(k) , \tag{2}
\]

\[
q_i \left( \Lambda^p_i(k, k + q_i e_i) - q_i \frac{2k_i + q_i}{2m} \right) = \Sigma^p(k + q_i e_i) - \Sigma^p(k) . \tag{3}
\]

To prove these relations we classify the Green’s functions \( G^0_i \) (the free Green’s functions) of the self energy graph \( \Sigma^p \). \( G^0_i \sim G^0_{i_2} \in W_j \in W \) if there is a path in the graph \( \Sigma^p \) which do not pass a vertex, connecting \( G^0_{i_1} \) with \( G^0_{i_2} \). \( G^0_1, G^0_c \in W_{ae} \) denote the Green’s functions which are at the continuation of the outer truncated Green’s functions of the graph \( \Sigma^p \). In the following we show at first relation (3). \( \Sigma(k + q_0e_0) \) is the graph in which one makes the substitution \( G^0_i(k') \rightarrow G^0_i(k' + q_0e_0) \) for every Green’s function in \( \Sigma^p \). Furthermore we denote as \( \Gamma^p_i(k + q, k; k', k' + q) \) the graph in which a Green’s function \( G^0_i \) is eliminated from \( \Sigma^p \). With the help of \( \{G^0_1, G^0_2..\} = W_j \neq W_{ae} \) one gets

\[
\sum_{G^0_i \in W_j} \sum_{k'} \Gamma_i(k, k + q_0e_0; k' + qe_0, k') (G^0_i(k' + q_0e_0) - G^0_i(k')) = 0 . \tag{4}
\]
This is valid because the Green’s functions in \( W_j \neq W_{ae} \) are forming a circle. Furthermore one gets for \( \{ G_0^0, G_0^0 \} = W_{ae} \)

\[
\sum_{G_0^0 \in W_{ae}} \sum_{k'} \Gamma_0^0(k, k + q_0e_0; k' + q_0e_0, k') (G_0^0(k' + q_0e_0) - G_0^0(k'))
\]

\[
= \sum_{k'} \Gamma_0^0(k, k + q_0e_0; k' + q_0e_0, k') G_0^0(k' + q_0e_0) - \Gamma_0^0(k, k + q_0e_0; k' + q_0e_0, k') G_0^0(k')
\]

\[
= \Sigma^p(k + q_0e_0) - \Sigma^p(k) .
\]

With the help of (4), (3), one gets (2).

We now prove the current Ward identity (3). If one has no current vertex in \( \Sigma^p \) one gets immediately (3) in the same manner as (2). Otherwise we denote \( C \) the number of current vertices. Furthermore we denote \( B(G_0^0) (E(G_0^0)) \) as the beginning- (end-) vertex of the directed Green’s function \( G_0^0 \). If \( B(G_0^0) \in C (E(G_0^0) \in C) \), we denote \( \vec{j}(B(G_0^0)) \) \( \vec{j}(E(G_0^0)) \) as the current of the vertex \( B(G_0^0) (E(G_0^0)) \). Relations (4), (3) keep their validity if one makes the following substitutions: Every Green’s function \( G_0^0(k' + q_0e_0) \) \( G_0^0(k' + q_0e_0) \) in the sums of (4), (3) should be replaced by \( G_2^0(k' + q_i e_i) \) \( G_1^0(k' + q_i e_i) \) \( G_2^0(k') \) \( G_1^0(k') \), otherwise by \( G_2^0(k' + q_i e_i) \) \( G_1^0(k') \) \( G_2^0(k') \) \( G_1^0(k') \). The brackets have the meaning of replacing the current \( \vec{j} \) of the vertex \( B(G_i^0) \) \( E(G_i^0) \) in \( \Gamma_0^0 \) by \( \frac{q_i}{2m} e_i \) \( \frac{q_i}{2m} e_i \). With the help of these substitutions in the expressions (4), (3) one immediately gets the current Ward identity (3).

Similar to (4), (3) one can show with \( q = (q_0, q_1, q_2) \) the following combined Ward identity

\[
-q_0 \Lambda_0^p(k, k + q) + \sum_{i=1}^{2} q_i \Lambda_i^p(k, k + q e_i) + q_0 - \sum_{i=1}^{2} q_i \frac{2k_i + q_i}{2m} = \Sigma^p(k + q) - \Sigma^p(k) .
\]

We now consider our primary problem. It is clear that the response (density or current) to an external \( \vec{A} \)-field should obey the continuity equation. We now define the approximation of the density-current and current-current \( T \)-product as

\[
\langle |j_\mu(q)j_\nu(-q)| \rangle \approx \Pi^p_{\mu\nu}(q) = \int \sum_{k'} G^\Sigma(k' + q) \Lambda^p_{\mu}(k' + q, k') G^\Sigma(k')
\]

\[
\times \left( \delta_{\mu,0} + (1 - \delta_{\mu,0}) \frac{2k'_\mu + q_\mu}{2m} \right) ,
\]

\[
(7)
\]

\[
4
\]
where $G^S(k) = (k_0 - \xi(\vec{k}) - \Sigma^p(k))^{-1}$

Within the help of the combined Ward identities (6), the Kubo-formula and the relation of $T$-products and retarded correlation functions [3], one sees that the continuity equation is valid if $\Pi^p_{\mu\nu}$ fulfills the equation

$$e^2 q_0 \Pi^p_{0i}(q) - e^2 \sum_{j=1}^{2} q_j \Pi^p_{ji}(q) = -i q_i \frac{e^2 n_e}{m},$$

(8)

where $n_e$ is the electron density.

This equation can easily checked with the help of (3). Now we have the problem that the graphs $\Pi^p_{\mu\nu}(q)$ are not symmetric in the coupling of the currents. On one side of $\Pi^p_{\mu\nu}(q)$ one has density-density Chern-Simons current-couplings. These current couplings are missing on the other side of the graphs $\Pi^p_{\mu\nu}(q)$. To get rid of this problem we name $\Lambda^{b,4}_i$ as the members of $\Lambda^b_i$ which are originating through operation (b) in figure 1 from $\Sigma^b$. We now close the open ends of $\Lambda^{b,4}_i$ with a Green’s function $G^0$ and call these graphs $\Lambda^{b,4}_i$. These graphs consists of closed Green’s function loops. To these graphs we apply the operations (a) or (b) of figure 1, respectively, and call these graphs $\Xi_{0i}(q)$ or $\Xi_{ji}(q)$, respectively. Than one sees similar to the proof of (2), (3) that $\Xi_{0i}(q), \Xi_{ji}(q)$ fulfills the equation

$$e^2 q_0 \Xi^p_{0i}(q) - e^2 \sum_{j=1}^{2} q_j \Xi^p_{ji}(q) = 0$$

($\Xi_{0i}(q), \Xi_{ji}(q)$ consists only of closed Green’s function circles). With the definition $\Pi^{b,2}_{\mu\nu} = \Pi^{b}_{\mu\nu} + \Xi^p_{\mu\nu}$ one sees that $\Pi^{b,2}$ consists of current symmetric graphs. Furthermore $\Pi^{b,2}$ fulfills equation (8).

It is also clear that the continuity equation is fulfilled if one only considers the graphs in $\Pi^{b,2}$ which have the same number of vertices.

### 3 The Calculation of the Conductivity

Next we calculate the conductivity of a $\nu = 1/2$ Chern-Simons gas with impurities. The calculable quantity is the conductivity of CFs which is related to the physical conductivity through $\sigma^{CS} = \sigma_{xx}^2 + \sigma_{xy}/\sigma_{xx}$ [2]. For the mean-field Green’s function of CFs in an impurity background one has $G^\pm(\omega, p) = (\omega - \xi(p) \pm i/(2\tau))^{-1}$. In the following calculation we need momentum integrals $\int dp$ of products of such Green’s functions. One can approximate these momentum integrals (also for $k_F l \approx 1$) by extending the range of such integrals to infinity, $\int dp$. We use this approximation
in the following. In doing that we hope to get better results for smaller \( k_F l \) in contrast to the standard \( k_F l \gg 1 \) approximation. To calculate physical quantities also for smaller densities of the \( \nu = 1/2 \)-system is suggestive because one could reduce the effective \( k_F l \) of a \( \nu = 1/2 \)-system in increasing the magnetic field to get a \( \nu = (5/2, 9/2, 13/2..) \)-system. Systems with these fillings behave similar to \( \nu = 1/2 \)-systems with a reduced density \( n_e = (n_e/5, n_e/9, n_e/13..) \).

In the following we limit our calculation to the case of impurities with a \( \delta \)-correlation \([2]\). It could be shown \([5, 10]\) that one gets the same results in the case of RMF-scattering for \( k_F l \gg 1 \). Further we will only discuss the particle-hole channel conductivity graphs because we don’t expect any weak localization correction due to the magnetic broken time reversal symmetry \([2]\).

In the Coulomb gauge \((\text{div}\, \vec{A} = 0)\), the gauge interaction between CFs at \( \omega \tau \ll 1 \) and \( q l \ll 1 \) is described by

\[
D^{-1}_{\mu\nu} = \left( \begin{array}{cc}
\frac{m}{2\pi} Dq^2 - i\omega & -i\frac{q}{4\pi} \\
-\frac{i}{4\pi} & -i\gamma_q |\omega| + \chi_q q^2
\end{array} \right), \tag{9}
\]

where \( D = \frac{1}{2} v_F^2 \tau \) is a diffusion coefficient, \( \chi_q = \frac{1}{24\pi m} + \frac{V_q}{(4\pi)^2} \) is an effective diamagnetic susceptibility given in terms of the pairwise electron potential \( V_q \), \( \gamma_q = \frac{mD}{2\pi} \) is proportional to the CF mean free path \( l = v_F \tau \) and \( m \) stands for the effective mass.

Comparing the gauge interaction functions \([2, 10]\) with the gauge interaction functions in \([2, 10]\) one sees that we get a difference by a factor \( 1/2 \) in \( \chi_q \). In \([4]\) we made an exact calculation of the polarisator \( \Pi_{11} \) without impurities. In this polarisator one also gets half the value of the magnetic susceptibility in \([2]\). So we believe that our calculation is correct. As in \([10]\) we will discuss in the following the two regimes

\[
V_q \approx V_0 = 2\pi^2 \frac{e^2}{\kappa} \quad \text{and} \quad V_q = 2\pi^2 \frac{e^2}{q^2}.
\]

We now generate the conductivity diagrams from the left hand side self energy diagrams in figure 2. These diagrams are also the starting point in the case of the Coulomb gas \([4, 7]\). Then one gets among others the diagrams in the right hand side of figure 2. In \([7]\) was shown that the conductivity of the diagrams not listed in figure 2 adds to zero. This can also be shown for our \( k_F l \approx 1 \) approximation. This is also significant for CFs. We now invert the matrix \([2]\). So we get

\[
D_{00} = \frac{-i|\omega| + Dq^2}{\frac{m}{2\pi} Dq^2 - \frac{q^2}{16\pi^2} - i|\omega| + \chi_q q^2}, \tag{10}
\]

\[
D_{11} = \frac{1}{-i\gamma_q |\omega| + \chi_q q^2 - \frac{q^2}{16\pi^2} - i|\omega| + \chi_q q^2} \approx \frac{1}{-i\gamma_q |\omega| + \chi_q q^2} \theta(Dq^2 - |\omega|), \tag{11}
\]
Figure 2: The relevant conductivity diagrams discussed in the text. On the left hand side one sees the self energy diagram from which the conductivity diagrams are extracted through the formalism of section 2.

where $\chi' = \chi - \frac{1}{\pi m}$.

One can further see from (9) that the vertex $D_{01}$ isn’t as much divergent as $D_{00}$, $D_{11}$. Because of this one can show, that this vertex gives no $\log(T \tau)$-term in the conductivity. We now calculate the graphs (a), (b) of figure 2. At first we will calculate the graphs (a), (b) with the $D_{00}$-vertex. To get the divergent part of the graphs one has to put the vertex correction $\Gamma(\epsilon, \omega, q) = \left(\frac{m \tau}{2} q^2 - i |\omega|\right)^{-1}$ for $\epsilon(\epsilon + \omega) < 0$ at the endpoints of the vertex $D_{00}$. One can then use the analysis of the Coulomb problem [7]. After integration one gets for $D\gamma_q \gg \chi_q$

$$\delta \sigma_{ii}^{D_0} \approx \frac{1}{\pi h} \left(\frac{(k_F l)^2}{\frac{\pi}{2} + (k_F l)^2}\right) + \frac{2}{3} \frac{(k_F l)^2}{\frac{\pi}{2} + (k_F l)^2} \log \left(\frac{3}{2} (k_F l)^2\right) \log(T \tau), \quad (12)$$

where $l = (h k_F \tau)/(2\pi m)$.

From (12) one sees that in the case $(k_F l) \gg 1$, $\sigma_{ii}^{D_0}$ goes to the conductivity of the Coulomb problem [6]. We now calculate the conductivity of the $D_{11}$-vertex. To calculate the diagrams (a), (b) of figure 2 one needs the quantities $J^\pm_i(\vec{q}) = \sum_{\vec{p}} G^2_{\pm}(p) G^\pm(p) \hat{\vec{e}}_i \times \hat{\vec{q}} = (\mp i \epsilon_F \tau^2 - \tau/2)(\hat{\vec{e}}_i \times \hat{\vec{q}})$. Similar for the graphs (c), (d) one needs the quantity $J_i(\vec{q}) = \sum_{\vec{p}} G^\pm(p) G^\pm(p) (\hat{\vec{e}}_i \times \hat{\vec{q}}) = \tau(\hat{\vec{e}}_i \times \hat{\vec{q}})$. One now has to discuss the different frequency combinations of (a), (b). We make in the following the convention $\Omega > 0$. Because of the frequency constraints of the pole of the particle-hole channel $(1/\tau)\Gamma(\epsilon, \omega, q)$ one has some frequency constraint on the graphs to get this pole. Due to the different prefactor of the first terms $J^\pm_i$ of $J^\pm$ one immediately gets for the nonvanishing $(J^\pm_i)^2$-terms in $\sigma^{D_{11.a}} + \sigma^{D_{11.b}}$ the two frequency combinations $\epsilon < 0, \epsilon + \Omega > 0, \epsilon + \omega + \Omega > 0, \epsilon + \omega < 0$ and $\epsilon < 0, \epsilon + \Omega < 0, \epsilon + \omega + \Omega > 0, \epsilon + \omega > 0$. In each one of these two frequency combinations one gets $\sigma^{D_{11.a}} + \sigma^{D_{11.b}} = 2\sigma^{D_{11.a}}$. For the diagrams (a) (b) with reversed Green’s function direction one gets two similar frequency combinations of additive $\sigma$ from (a), (b). Then one can carry out the $\Omega$-integration and gets for
the conductivity of the diagrams (a), (b) of current-combination $(J_i^\pm)^2$

$$\delta\sigma_{ii}^{D_{11}} \approx -\frac{i2e^2}{2\pi} \int_{\Omega} \frac{d\omega}{(2\pi)^2} \int \frac{dq}{m\tau^2} \frac{(4J^\pm_i(q)J^\pm_i(q))}{|Dq^2 - i\omega + \Omega|} \theta(Dq^2 - |\omega|) \log(1 + iDq^2/\omega^2).$$

(13)

At finite temperature $T \gg \Omega$ we can calculate the frequency-integral in $\omega$ by using imaginary frequencies with an $\omega$ integral cut-off at low frequencies $\frac{1}{T}$. For $V_q = (2\pi)^2\frac{e^2}{\kappa}$ one gets

$$\delta\sigma_{ii}^{D_{11}} \approx \frac{2}{\pi h} \left( \log(k_F l) + \frac{3}{8\pi} \right) \log(T\tau).$$

(14)

For $V_q = (2\pi)^2\frac{e^2}{\kappa}$ we make a partial fraction decomposition of the denominator of (13) and get three additive terms ($\Omega \to 0$) $1/2(\pm 8\pi i|\omega|\gamma_q + \sqrt{i|\omega|/D}) (i|\omega|/D)^{-\frac{3}{2}} (q + \sqrt{i|\omega|/D})^{-1}$ and $iD/\omega|(-8\pi i|\omega|\gamma_q + q)^{-1}$. Then one sees that the integral (13) results in a finite value for $T \to 0$. Next we calculate $\sigma_{D_{11},a}^i + \sigma_{D_{11},b}$ for the current combinations $(J_2^\pm)^2$ and $J_1^\pm J_2^\pm$. For example for the $(J_2^\pm)^2$ combination one gets for the conductivity to be nonvanishing the two regimes $\epsilon > 0$, $\epsilon + \Omega < 0$ and $\epsilon < 0$, $\epsilon + \Omega > 0$. Integrating out the $\Omega$-integral one gets two terms $\Omega \int d\omega$, $\int d\omega \omega$. The $\log(T\tau)$-singularity is cancelled for the $\Omega \int d\omega$-terms of the two frequency ranges and the remaining two $\int d\omega \omega$-integrals are infinite. These infinities are cancelled by similar infinities of the graphs (c), (d). Furthermore one can see with the help of $J^\pm$, $J$ that the $\Omega \int d\omega$ integrals are cancel for every single graph (c), (d). In summary one can see, that the sum of the conductivities of the graphs (a), (b) for $(J_2^\pm)^2$, $J_1^\pm J_2^\pm$ and of the graphs (c), (d) is zero for $T \to 0$. One sees from the above analysis that the typical $\rho \rho$ part of the CS-current is important to get a finite result for lower $k_F l$.

We now compare our results with the experimental results of Rokhinson et al [3]. They get a $\log(T)$ dependence for $\sigma_{xx}^{CS}$ for temperature $T < 500\text{mK}$. So we use $V_q = 2\pi e^2/\kappa$ to compare our results with their measurements. With $\sigma_{xx}^{CS} = \sigma_0^{CS} + \lambda_0^{CS}\log(T\tau)$ they get $\lambda_{xx} = (0.1, 0.4, 1.6)$ for Drude conductivities $\sigma_0^{CS} = e^2/(2h)(k_F l) = (5.4e^2/h, 16.0e^2/h, 39.0e^2/h)$. From (12), (14) we get for the theoretical value $\lambda_{th}, \lambda_{th} = (1.81, 2.53, 3.1)$. We see from these results that $\lambda_{th}$ is getting better for larger $k_F l$.

An improvement of the above calculation would be the calculation of the conductivity of diagrams which are originating through the operations of section 2 from the corresponding Hartree self energy diagrams of figure 2. For the $D_{00}$-vertex this
was done by Castellani et al. [12]. They calculated higher order $D_{00}$-vertex graphs and got for $k_F l \gg 1$, $\sigma_{xx}^{CS} = e^2/(\pi h)(2 - 2 \log(2)) \log(T\tau)$. This is only a small correction to (12) and (14) for $k_F l \gg 1$. Further one sees from $D_{11}(0,q) \sim 1/q^2$ that the $D_{11}$-Vertex can not give any contribution to $\sigma_{xx}^{CS}$ in Hartree-diagrams [8]. At least one could calculate the CS-conductivity from the diagrams of figure 2 for small deviations of the magnetic field $B_{1/2}$. This means that the CFs are subject to an effective magnetic field $B_d = B - B_{1/2}$. For $\omega_d^2 \tau \ll 1$, $k_F l \gg 1$ one gets with the help of the formalism of Houghton et al. [9], $J_{1,1}^\pm(\vec{q}, B_d) = i\epsilon \tau^2(1/2 - (\omega_d^2 \tau)^2)(\vec{e} \times \vec{q}) - (\omega_d^2 \tau)(\vec{e} \cdot \vec{q})$. Also one gets for the magnetic correction to the diffusion constant $D$, $D(B_d) = D(1 + (\omega_d^2 \tau)^2)$ (this should also be considered in $\gamma_q$). In this $k_F l \gg 1$ limit, the only relevant graphs in figure (2) are (a), (b). Because the second term in $J_{1,1}^\pm(\vec{q}, B_d)$ has the same behaviour as $J_{2}^\pm$ for $B_d = 0$ one sees from the above analysis that the conductivity $\sigma_{xx}(B_d) + \sigma_{xx}^b(B_d)$ is infinite for $|\omega_d^2 \tau| > 0$. Since the graphs (c),(d) of figure 2 give only lower order terms in $k_F l$ this singularity can not be cancelled. So the next stage of improvement should be to consider Feynman-graphs which make $\sigma_{xx}(B_d)$ finite for $|\omega_d^2 \tau| > 0$. This should also give better results for $\sigma_{xx}(0)$. Further we have to remark that we don’t use a density of states correction at the fermi-level due to $B_d \neq 0$ in the above formulas because the general statement of an infinite conductivity is still correct. At least we have to remark that Khveshchenko [10,11] calculated the conductivity of the graphs (a), (b) of figure 2 for $k_F l \gg 1$. He comes to other results than (14) and also for the conductivity of $V_q = 2\pi e^2/q$ in the large $k_F l$ limit. For example the difference to (14) is a factor 1/4. This is exakt the result one gets if one assumes frequency constraints through vertex corrections $\Gamma$ at the endpoints of the $D_{11}$ vertex as in the case of the $D_{00}$ vertex [6]. These Feynman graphs have a vanishing conductivity because of the current couplings at the endpoints of the $D_{11}$ vertices. If one calculates the graphs (a), (b) without vertex corrections, one gets another result, as we have shown in this paper.

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