Internal Model Control Design Based on Equal Order Fractional Butterworth Filter for Multivariable Systems

KAIYUE LIU AND JUAN CHEN
College of Information Science and Technology, Beijing University of Chemical Technology, Beijing 100029, China
Corresponding author: Juan Chen (jchen@mail.buct.edu.cn)

This research was supported by the National Natural Science Foundation of China (No. 61771034).

ABSTRACT An internal model control with inverted decoupling (ID-IMC) controller design method based on equal fractional Butterworth (EFBW) filter is proposed for Multiple Input-Multiple Output (MIMO) systems with multiple time delays and Right Half Plane (RHP) zeros. There has been finite memory and limited flexibility for multivariable processes developed using a direct ID-IMC method. This paper presents a novel procedure to approximate Butterworth (BW) filters using fractional-order (FO) theories, so that the degree-of-freedom for tunable parameters is increased. The proposed ID-IMC controller cascaded with EFBW filter combines the computational simplicity of the ID-IMC structure with the greater flexibility of the EFBW filter, in conjunction with a better set-point tracking and disturbance rejection performance. Further, the stability analysis of the designed controller is given to ensure the stability of the closed-loop system. Dynamic performance indicators and sensitivity functions are carried out for the time domain and robustness analysis. Two illustrative examples are presented to show the merits of the proposed method.

INDEX TERMS Butterworth filter, fractional order system, internal model control, MIMO systems.

I. INTRODUCTION

Most industrial processes have characteristics of Multiple Input-Multiple Output (MIMO), time delays, couplings and Right Half Plane (RHP) zeros, causing difficulties in feedback controller design. In reference to such problems, many control strategies are proposed, such as PID control [1], [2], LTR method [3], [4], H-infinity [5], [6], internal model control (IMC) [7]–[11], [16]–[18], [20]–[25], etc. Note that the majority of the industrial processes are open-loop stable, thus closed-loop stability of the system is not an issue for the controller design [7]. Instead, a simple control scheme with appropriate number of tuning parameters is rather important, to ensure the controller to be cost-saving and easy to be realized [8]. Based on the aforementioned designing principles, the IMC scheme was developed and widely adopted.

The procedures of the IMC design are as follows: 1. Select a proper block to decouple the system; 2. add a filter to make the system robust. Due to the interactions of the process variables, it is difficult to design a controller for each loop independently [9]. Therefore, many methods for decoupling have been proposed such as simplified decoupling, ideal decoupling, and inverted decoupling (ID) [10]–[15]. The authors in [10] proposed a simplified decoupling IMC scheme for the two area power system, by which the system was decoupled into two independent SISO systems. The simplified decoupling scheme has simple decoupling elements but a complex decoupled process. In [11], the ideal decoupling method is used to decompose MIMO systems, which provides a simple decoupling process, but the decoupling elements are too complicated to be directly used. It is noteworthy that the ID method avoids the disadvantage of the simplified decoupling and achieves the purpose of the ideal decoupling, hence is a better decoupling method for MIMO systems. The authors in [12]–[14] compared the above three decoupling methods, illustrating that the ID method had both simple decoupling elements and simple decoupling process. Moreover, it has additional advantages in dealing with the initialization problem and the saturation phenomenon of manipulated variables [15].

In order to get a causal and stable IMC controller which contains an inverted mathematical model of the process, the ideal mathematical model applied in the IMC controller should not contain RHP zeros and time-delay terms. This
property denotes that the mismatch between the ideal mathematical model and the actual model may always exist, which will deteriorate the performance and robustness of the controller. Furthermore, external disturbances and internal uncertainties are pervasive in the process control. Thus, an extra filter should be added to improve the transient performance and robustness of the system [7].

Due to the fact that the closed-loop performance often can be improved by an appropriate filter in many cases, an IMC design method combined with a filter is worthy of studying. The authors in [16] proposed the decoupling IMC method for MIMO systems, a simple first order low-pass filter was introduced to attain a normal performance requirements. Further, the authors in [17] designed a modified IMC filter for better disturbance rejection ability. However, the order of these filters is integer, which provides less degrees of freedom and worse robustness. Hence, filters designed by fractional calculus, i.e., fractional-order (FO) filters, are studied with the development of the approximate techniques [18]–[33].

In [18]–[33], the procedures for designing the filter with fractional elements were introduced. These filters are based on FO differential equations. The integer-order (IO) filters can be considered as a narrow subset of the FO ones, i.e., FO filters have many characteristics that cannot be achieved by the IO ones [18]. Compared with IO filters, the FO filters have been proven to be advantageous to satisfy the exact specifications and increase the design flexibility. The authors in [19] used a FO filter to enhance the performance of the linear quadratic regulator (LQR) for a simple civil structure. In [20], IMC-FO filter controllers were proposed for PH neutral process and inverted pendulums, where FO filters were used to improve the robustness of the system.

With the advantages of simple structure, more tunable parameters and strong robustness, the IMC scheme combined with a FO filter has been commonly employed in the industry recently [18], [20]–[25]. In [18], [22]–[23], the authors introduced a simple low-pass FO filter, with which an IMC structure was combined to construct the IMC-PID-FO-filter controller for single-input-single-output (SISO) system with time-delay and RHP zeros, which provided strong robustness and more degrees of freedom to meet other specifications. Further, the authors in [24] extended the FO filter scheme to the non-integer system, and by tuning the parameters of the FO filter, a desired setting time and overshoot of the closed-loop step response can be achieved.

Considering that fractional calculus introduced by the implementation of FO filters, as well as the existence of the time-delay and RHP zeros, would affect the stability of the whole system, the authors in [25] discussed the stability condition of the FO filer, and then analyzed the stability domain of the closed-loop system. On the other hand, it is important to determine the transfer function coefficients that yield the desired filter characteristics. Systematic design procedures are widely available for IO filters to determine these coefficients, but the procedures for determining the coefficients of FO filters are not common. Only a few studies focused on the calculation of the coefficients, where FO filters in the type of Chebyshev filter [26], Kalman filter [27], Butterworth (BW) filter [28]–[33] are proposed for an ideal configuration.

Compared with other types of filters, the BW filter is the best compromise between the attenuation and the phase response. It has no ripple in the pass band or the stop band, and because of this, is sometimes called a maximally flat band filter [28]. Therefore, the BW filter approximation is employed to construct a FOBW filter. Early works can be attributed to the design of the first-order and the second-order FOBW filters in [29]–[30]. The authors in [28] firstly attempted to design FOBW filters in s-plane instead of s-plane, and extended the concept of stability to the design of the FOBW. The authors in [31] further applied this FOBW filter in the audio equalizer to meet the design specifications in an exact manner. Using an approximation of the Laplacian operator, the authors [32] proposed a FOBW representation based on the conventional IOBW filter, which solves the pass-band peaking problem. Recently, the authors in [33] proposed a fixed second-order approximation for the Laplacian operator with the continued fraction expansion (CFE) technique.

The aforementioned literatures [18], [21]–[25] mainly studied the control schemes with traditional IO/FO filters, in practice however, the general industrial process are always described by MIMO systems [8]. In addition, there may be strong couplings and time delays between input and output signals, which further complicate the feedback controller design [16]. Under these circumstances, a simple low-pass IO/FO filter is not enough for an acceptable performance. Therefore, it is justifiable to introduce a FOBW filter in the design process of the ID-IMC controller to attain a desirable closed-loop performance.

This paper provides a positive answer to the above challenging topic on new solutions to an ID-IMC scheme with a FOBW filter, and the control system performance is analyzed. The main objective is not only maintain the system stability, but improve the system robustness and response speed with more design simplicity and flexibility. The proposed scheme has advantages of computational simplicity and a better set-point tracking and disturbance rejection performance, which combines the computational simplicity of the ID-IMC structure with the greater flexibility of the equal fractional BW (EFBW) filter. Further, time domain dynamic performance indicators and robustness sensitivity functions are carried out. Finally, two illustrative examples are presented to show the merits of the proposed method.

The remainder of this paper is organized as follows: Section II deduces the ID-IMC method for multivariable systems with time delays and RHP zeros considering the system realizability requirements. The design procedure of EFBW filter is shown in section III, and the system stability is also discussed. In section IV, performance analysis is carried out to reflect the dynamic performance in the time domain and the robustness in the frequency domain. Simulation examples are carried out in section V. The conclusions are given in Section VI.
II. INTERNAL MODEL CONTROLLER WITH INVERTED DECOUPLING

A. GENERAL EXPRESSIONS FOR MULTIVARIABLE SYSTEMS

Fig. 1 shows the ID-IMC system, where $P(s)$ and $P_m(s)$ represent the controlled process matrix and its process model with $n$ inputs and $n$ outputs. Here $P_m(s)$ is assumed to be identical with $P(s)$ and square, $C(s)$ represents the ID-IMC controller, including a system forward channel matrix $C_d(s)$ and a feedback channel matrix $C_o(s)$. According to the ID scheme [12], $C_d(s)$ must have only one non-zero element in each row which connects the system error $e$ and the controller output $u$. In order to decouple the system, $C_o(s)$ feeds back $u$ towards the controller input, which must have only one zero element in each row corresponding to $C_d(s)$. Thus, there are $n!$ matrix forms in $C_d(s)$ and $C_o(s)$ for multivariable systems. Our task is to achieve the realizable ID-IMC controller and the resultant desired closed-loop transfer function such that the ID-IMC system is decoupled and robust. And the design process of the ID-IMC is as follows.

The closed-loop transfer matrix between the output $y$ and the reference input $r$ can be derived from Fig. 1 as $T(s) = P(s) C(s) [I(s) + (P(s) - P_m(s)) C(s)]^{-1}$, which reduces to $T(s) = P(s) C(s) = \text{diag}(t_1(s), t_2(s), \ldots, t_n(s))$ under the condition $P(s) = P_m(s)$ and $d = 0$. The controller $C(s)$ can be also derived as $C(s) = C_d(s) [I(s) - C_o(s) C_d(s)]^{-1}$. Then the following expression can be obtained

$$C_d^{-1}(s) - C_o(s) = T^{-1}(s) P(s) \quad (1)$$

Assuming the diagonal elements of $C_d(s)$ are non-zero, each element of (1) is expanded as

$$\begin{bmatrix}
1 & \cdots & -c_{1o1} & \cdots & -c_{1o_n} \\
-c_{21} & \ddots & \cdots & \cdots & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
-c_{on1} & \cdots & \cdots & \cdots & 1 \\
\end{bmatrix}$$

For multivariable systems, there are $n!$ matrix forms both in $C_d(s)$ and $C_o(s)$, including the matrix form as seen in (2).

Therefore, the elements in $C_d(s)$ and $C_o(s)$ can be given as

$$c_{dj}(s) = \frac{t_j(s)}{p_{ij}(s)} \quad (3)$$

$$c_{oj}(s) = -\frac{p_{dj}(s)}{t_i(s)} \quad (4)$$

The derivation process shows that $T(s)$ should be given firstly. For a known model $P(s)$, $C(s)$ can be obtained by (3) and (4) under the realizable situation to guarantee a desired system performance. Hence, some realizable conditions and the desired closed-loop transfer matrix will be discussed in section B.

B. REALIZABILITY AND THE CLOSED-LOOP PERFORMANCE

In the controller design process, $C_d(s)$ and $C_o(s)$ must be stable, causal and proper. For MIMO systems with multiple time delays or RHP zeros, direct calculation can lead to pure time-leading and unstable poles [8]. Therefore, this section will discuss the following realizable conditions, where a stable MIMO controlled process with multiple time delays, RHP zeros is considered.

The element of the process model is given by $p_{ij}(s)$, $(1 < i \leq n, 1 < j \leq n)$. Let the subscript $i$ denote the $i^{th}$ row of a matrix. For the element of the row $i$, if $p_{ik}(s)$ has the minimum time-delay $\tau_{ik}$ and the smallest RHP zero multiplicity $\upsilon_{ik}$, the closed-loop transfer function time-delay $\tau_i$ and multiplicity $\upsilon_i$ should be chosen according to (5) and (6). In addition, to deduce a proper controller, it is necessary to add a filter to the process model. Assuming the $k^{th}$ element $p_{ik}(s)$ of the row $i$ has the minimum filter order $\gamma_{ik}$, the filter order $\gamma_i$ should be chosen according to (7).

$$\tau_{ik} < \tau_i \leq \min(\tau_{ij}), \quad j \neq k \quad (5)$$

$$\upsilon_{ik} < \upsilon_i \leq \min(\upsilon_{ij}), \quad j \neq k \quad (6)$$

$$0 < \gamma_{ik} \leq \min(\gamma_{ij}), \quad j \neq k \quad (7)$$

The diagonal element of the desired closed-loop transfer matrix $t_i(s)$ should be suggested as a transfer function with the minimum time-delay, the smallest RHP zero multiplicity, and the lowest filter order

$$t_i(s) = e^{-\tau_i s} \prod_{x=1}^{N_f} \left( \frac{-s + z_x}{s + z_x} \right)^{\nu_x} \cdot \frac{1}{(r_{i}s + 1)^{\gamma_i}} \quad (8)$$

where $z_x$ is the RHP zero, $N_f$ is the total number of the individual RHP zero in the $i^{th}$ row, $r$ is the time constant, which determines the bandwidth of the closed-loop system.
and affects the system response fastness and robustness, γ
is the filter order whose value depends on the physical real-
izability of the controller. Then based on (3), \( c_{d_k} \) should
be chosen to be non-zero, and \( c_{os} \) should be chosen to be
zero, since \( C_o \) has an opposite signal flow to \( C_d \).

In addition, if \( C_d \) has 2 ≤ \( l \) ≤ \( n \) non-zero elements
in the same column, \( C_d \) is not proper. So it is necessary
to introduce a diagonal matrix \( D(s) \) for realizability, which
becomes a multiplicative factor of \( P(s) \). According to (5)-(7),
each term of \( D(s) \) must be added with the minimum time-
delay, the smallest RHP zero multiplicity and the lowest filter
order, which is defined as

\[
D(s) = \text{diag} \left( e^{-\tau_1 s}, \prod_{x=1}^{N_l} \left( \frac{-s + z_k}{s + z_k} \right)^{u_1}, \frac{1}{(r_1 s + 1)^{q_1}}, \ldots, e^{-\tau_n s}, \prod_{x=1}^{N_l} \left( \frac{-s + z_k}{s + z_k} \right)^{u_1}, \frac{1}{(r_n s + 1)^{q_1}} \right) \quad (9)
\]

As is seen in (8), the filter 1/(\( ts + 1 \))^\( \gamma \) is an important
part in the desired closed-loop transfer matrix, which directly
affects the speed, stability and robustness of the
closed-loop system. In the actual process industry, \( \gamma \)
has often been restricted to be an integer (first-order or second-
order mostly), which results in the lack of the robustness and
tunable parameters. However, the FO filter is more attractive
than the IO one as it has good robustness property as well as
more degrees of freedom to meet other specifications [18].
Therefore, the idea of this paper is to design a controller
with an ID-IMC structure cascaded with a FO filter instead of
traditional IO filter, which would be further discussed in
section III, to meet the closed-loop realizability and improve
the system performance.

According to (3), (4), and (8), the general structure of the
controller is given by

\[
c_{d_k} = e^{-\tau_s} \prod_{x=1}^{N_l} \left( \frac{-s + z_k}{s + z_k} \right)^{u_1} \cdot \frac{1}{(r_1 s + 1)^{q_1}} \\
\text{ID-IMC controller}
\]

\[
c_{os} = e^{-\tau_s} \prod_{x=1}^{N_l} \left( \frac{-s + z_k}{s + z_k} \right)^{u_1} \cdot \frac{1}{H(s)} \\
\text{fractional-order filter}
\]

\[
c_{d_k} = \frac{1}{p_{ji}(s)} \cdot H(s) \quad (10)
\]

\[
c_{os} = \frac{1}{p_{ji}(s)} \cdot H(s) \quad (11)
\]

III. FRACTIONAL-ORDER BUTTERWORTH FILTER

In this section, the FO filter added to the ID-IMC system can
be used for further design. Based on the above advantages of
the FO filter, a BW filter approximation will be employed to
construct the FOBW filter.

A. DEFINITIONS OF THE FRACTIONAL CALCULUS

Fractional calculus is presented as the mathematical back-
ground of the FO filter design. The fractional calculus based
on R-L definition [34] is used and the \( a^{th} \) order integration of
a function \( f(t) \) is defined as

\[
aD_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{a-1}} d\tau \quad (12)
\]

where \( a \) is the initial value and generally a zero initial con-
tion can be assumed, i.e., \( a = 0 \); the subscripts on the left
and right sides of the differential operator \( D \) denote the lower
bound and upper bound of this integral formula; the Euler’s
gamma function is represented by \( \Gamma(\alpha) \), where \( \alpha \in \mathbb{R}^+ \)
and 0 < \( \alpha < 1 \). Similarly, the R-L definition of the \( a^{th} \) order
differentiation of a function \( f(t) \) is

\[
aD_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{f(\tau)}{(t-\tau)^{a-1}} d\tau \quad (13)
\]

Applying the Laplace transformation to (13), yields

\[
\ell \{ f(t) \} = s^\alpha F(s) \quad (14)
\]

where \( \ell \{ \cdot \} \) denotes the Laplace transformation operator
and \( \ell \{ f(t) \} = F(s) \), \( s^\alpha \) is the FO Laplacian operator; \( a \) is an
arbitrary rational or irrational number.

The FO system can be described by the FO differential equation as

\[
b_{m}D_t^\alpha y(t) + \cdots + b_{2}D_t^{\alpha} y(t) + b_{1}D_t^{\alpha} y(t) + b_{0}D_t^{\alpha} y(t)
\]

\[
= a_{n}D_t^{\alpha} x(t) + \cdots + a_{j}D_t^{\alpha} x(t) + a_{0}D_t^{\alpha} x(t)
\]

(15)

where \( a_{j}, (i = 0, 1, \ldots, n) \) and \( b_{i}, (i = 0, 1, \ldots, m) \) are any
rational numbers.

Then the FO transfer function is

\[
G(s)
\]

\[
= \frac{Y(s)}{X(s)}
\]

\[
= b_{m}s^m + b_{m-1}s^{m-1} + \cdots + b_{1}s + b_{0}
\]

\[
a_{n}s^{n} + a_{n-1}s^{n-1} + \cdots + a_{1}s + a_{0}
\]

(16)

Since the FO calculus has the advantages of the infinite
memory performance and more abundant information, FO
controllers have better performances that the IO ones cannot
achieve. Fig.2 and Fig.3 illustrate the fractional differential
of the unit step signal and the sinusoidal signal.

As can be seen from Fig.2, the 0.5th order differential-
response of the unit step signal is no longer a pure
impulse signal. Because the value of the fractional differential
approaches infinity at the initial moment, and then a slowly
gradient process at the later moment, the fractional calculus
is generally thought to have memory ability.

Fig.3 shows a three-dimensional fractional differential dia-
gram of the sinusoidal function \( y(t) = \sin((3t + 1)) \) under
the different orders, it is can be seen that the value of this
fractional derivative sinusoidal function alternates between
sinusoidal and cosine function signals. Hence, the fractional
calculus contains more information.
Let $\lambda = s^\alpha$, then (17) can be rewritten as

$$
G(\lambda) = \frac{\sum_{i=0}^{m} b_i \lambda^k}{\sum_{j=0}^{n} a_j \lambda^p}
$$

(18)

After defining the equal order fractional system, the general procedure of the EFBW filter will be given below.

Considering a normalized low-pass BW filter [35] whose magnitude equation was introduced as

$$
H(\omega) = \frac{1}{\sqrt{1 + \omega_n^2}}
$$

(19)

where $N$ is a positive integer that denotes the filter order, $\omega_n = \omega/\omega_c$ is the normalized frequency, and $\omega_c$ is the cutoff frequency.

The magnitude squared function of the characteristic polynomial is given in terms of (19) as

$$
|D(j\omega)|^2 = 1 + \omega_n^{2N}
$$

(20)

In order to obtain the generalized condition of the EFBW filter design instead of the IO filter, in the above equation, $|D(j\omega)|^2$ becomes a FO function by the following expression

$$
|D(j\omega)|^2 = 1 + \omega_m^{2m}
$$

(21)

where $m$ is any real positive number.

The general form of the low-pass FO filter transfer function is given as

$$
H(s) = \frac{d}{s^{\alpha+\beta} + ax^\alpha + bx^\beta + c}
$$

(22)

where the coefficients $a$, $b$, $c$, and $d$ are constants, and $\alpha$, $\beta \in (0, 2]$ denote the fractional orders of the filter.

The main purpose is that the transfer function of the FO low-pass filter is approximated as BW filter by calculating the coefficients $a$, $b$, $c$, and $d$.

When $s = j\omega$, the characteristic polynomial of (22) can be obtained as

$$
D(j\omega) = \left(\omega^{\alpha+\beta} \cos\left(\frac{\alpha + \beta}{2}\pi\right) + a\omega^\alpha \cos\left(\frac{\alpha\pi}{2}\right) + b\omega^\beta \cos\left(\frac{\beta\pi}{2}\right) + c\right)
$$

$$
+ j\omega^{\alpha+\beta} \sin\left(\frac{\alpha + \beta}{2}\pi\right) + a\omega^\alpha \sin\left(\frac{\alpha\pi}{2}\right) + b\omega^\beta \sin\left(\frac{\beta\pi}{2}\right)
$$

(23)

The magnitude squared function of characteristic polynomial $|D(j\omega)|^2$ is deduced as

$$
|D(j\omega)|^2 = \omega^{2(\alpha+\beta)} + a^2 \omega^{2\alpha} + \left(2b\omega^{\alpha+\beta} + 2ac\omega^\alpha\right) \cos\left(\frac{\alpha\pi}{2}\right)
$$

$$
+ \left(2a\omega^{2\alpha+\beta} + 2bc\omega^\beta\right) \cos\left(\frac{\beta\pi}{2}\right)
$$

(21)

\[+2ab\omega^\alpha + \beta \cos \left(\frac{\alpha - \beta}{2}\pi\right)\]
\[+2\cos^\alpha + \beta \cos \left(\frac{\alpha + \beta}{2}\pi\right) + b^2\omega^{2\beta} + c^2\]  
(24)

To approximate the BW filter response, applying (21) into (24) yields
\[|D(j\omega)|^2 = c^2 + \omega^{2(\alpha + \beta)}\]  
(25)

By comparing (21) with (25), the cutoff frequency is obtained as
\[\omega_c = c^{1+\beta}\]  
(26)

Therefore, the FO filter at the cutoff frequency should meet the frequency response of BW filter, i.e., the remaining terms of (24) are equal to zero.
\[a^2\omega^{2\alpha} + \left(2b\omega^{2\beta} + 2ac\omega^\beta\right)\cos\left(\frac{\alpha\pi}{2}\right)\]
\[+\left(2\omega^{2\alpha + \beta} + 2bc\omega^\beta\right)\cos\left(\frac{\beta\pi}{2}\right)\]
\[+2ab\omega^\alpha + \beta \cos \left(\frac{\alpha - \beta}{2}\pi\right)\]
\[+2\cos^\alpha + \beta \cos \left(\frac{\alpha + \beta}{2}\pi\right) + b^2\omega^{2\beta} = 0\]  
(27)

To simplify the calculation and analysis, the variables \(u = a\omega^\alpha/c\) and \(v = b\omega^\beta/c\) are selected to replace the previous variables. Then (27) can be written as
\[u^2 + v^2 + 2(u+v)\cos\left(\frac{\beta\pi}{2} + \cos\left(\frac{\alpha\pi}{2}\right)\right)\]
\[+2uv\cos\left(\frac{\alpha - \beta}{2}\pi\right) + 2\cos\left(\frac{\alpha + \beta}{2}\pi\right) = 0\]  
(28)

Note that the solution of (28) depends on not only \(u\) and \(v\) but also \(\alpha\) and \(\beta\), where the increasing system degrees of freedom can make the filter design much more complex. Hence, an equal fractional order is taken into account to simplify the design.

When \(\alpha = \beta\), (25) and (26) are equal to
\[|D(j\omega)|^2 = c^2 + \omega^{4\alpha}\]  
(29)
\[\omega_c = c^{1+2\alpha}\]  
(30)

Let \(\kappa = a + b\), \(\eta = \kappa\omega^\alpha/c\), and \(\xi = c/\kappa^2\), then (27) will be
\[\xi\eta^{2\alpha} + \left(\frac{2\xi\cos(\alpha\pi) + 1}{2}\right)\eta^\alpha + 1 = 0\]  
(31)

The four degrees of freedom restricted by \(u\), \(v\), \(\alpha\), and \(\beta\) reduces to two from (31) that \(\eta\) and \(\xi\) should be solved. The expression between \(\eta\) and \(\xi\) is given by
\[\xi = \frac{1}{\eta^{2\alpha}}\]  
(32)

The solution of (31) is divided into three different cases by analyzing the range of \(\alpha\), which is given by
\[
\eta = \begin{cases} 
0 & 0 < \alpha < 0.5 \\
1 & 0.5 \leq \alpha < 1.5 \\
\frac{1}{2\cos(0.5\alpha\pi) + \sqrt{2}} & 1.5 \leq \alpha < 2.0 
\end{cases}
\]  
(33)

Consequently, if the fractional order \(\alpha\) is known, \(\eta\) can be calculated. And then \(a\), \(b\), and \(c\) can be computed at the interesting fixed cutoff frequency, thus one can obtain the EFBW filter. Fig.4 shows the frequency characteristics diagram of the BW filter in the 2nd, 2.4th, and 3rd order.

It can be seen from the magnitude characteristics diagram that the 2.4th FOBW filter fulfills a better approximation of the ideal response than the 2nd order one, and from the phase characteristics diagram that the phase-lag of the 2.4th order FOBW is smaller than the 2nd one. Therefore, the FOBW filter can achieve a good trade-off between the system response and the phase characteristics.

### C. Stability of the Equal Order Fractional System

**Theorem 1** [7]: Assume a perfect model \(P(s) = P_{m}(s)\), and \(P(s)\) is stable, the IMC control system is internally stable (IS) if and only if \(C(s)\) is stable.

Since the stability of \(C(s)\) is characterized by the stability of the EFBW filter to be designed, according to Theorem 1, one can conclude that the IMC control system is IS if the EFBW filter is stable. The stability condition of the EFBW filter is given in Theorem 2.

**Theorem 2**: For a system with the basic order \(\alpha\), the stability region of the equal order fractional system is on the left side of \(\Gamma_1\) and \(\Gamma_2\) in \(\lambda\) plane, and the slope of \(\Gamma_1\) and \(\Gamma_2\) are \(\pm \alpha \pi / 2\). The stable condition can be expressed as \(|\arg(\lambda_i)| > \alpha \pi / 2\), where \(\lambda_i\) is the characteristic root of \(G(\lambda)\). The stable region is illustrated in Fig.5.

When \(\alpha = 1\), the system is IO, the curves of the stability region become imaginary axis, which is completely consistent with the stability conclusion of the IO system.
IV. CONTROL SYSTEM PERFORMANCE ANALYSIS

A. TIME DOMAIN PERFORMANCE ANALYSIS

In order to better analyze the step response transient characteristics of the control system, it is necessary to examine the time domain performance indicators that are divided into two types: the setting time $t_s$ (5%) and the overshoot $\sigma$ (%).

B. ROBUSTNESS ANALYSIS

Sensitivity function denote by $S(j\omega)$ is an ideal index that measures the closed-loop control system robustness to parametric uncertainties and mismatches quantitatively, and represents the ability to the external disturbance rejection. Let $M_s := \max_{0<\omega<\infty} |S(j\omega)|$ define the maximum sensitivity, which is introduced as the robustness performance index.

The control system structure is shown in Fig.6.

As seen in Fig.6, $T(s)$ contains two parts: the control matrix $C(s)$ and the process model $P_m(s)$, which is expressed as

$$T(s) = C(s)[I - P_m(s)C(s)]^{-1} \quad (34)$$

The sensitivity function is defined as

$$S(s) = (I + L(s))^{-1} \quad (35)$$

where $L(s) = P(s)T(s)$ is the open-loop transfer matrix.

In the frequency domain, the value of the maximum sensitivity function $M_s$ should be as small as possible, regardless of the system input signal and external disturbance. The anti-disturbance ability of the closed-loop system can be directly analyzed by the sensitivity function curve.

V. EXAMPLES

To illustrate the effectiveness of the proposed method, the simulation study is carried out on two IO processes. All the time-domain responses involving fractional calculus are obtained with simulation using Oustaloup’s approximation [36] in appropriate frequency range.

Example 1: Consider the two-input two-output (TITO) heavy oil fractionation tower model [37] with multiple time delays

$$P(s) = \begin{bmatrix} p_{11}(s) & p_{12}(s) \\ p_{21}(s) & p_{22}(s) \end{bmatrix} = \begin{bmatrix} 4.05e^{-27s} & 1.77e^{-28s} \\ 27s + 1 & 60s + 1 \\ 5.39e^{-18s} & 5.72e^{-14s} \\ 50s + 1 & 60s + 1 \end{bmatrix} \quad (36)$$

It is a multivariable large time-delay system. The EFBW filter extracted by (29)-(33) is used here, where the basic order is $\alpha = 0.7$ at the fixed cutoff frequency $\omega_c = 5$. Then the EFBW filter is obtained as

$$H(s) = \frac{9.55}{s^{1.4} + 1.564s^{0.7} + 9.55} \quad (37)$$

Let $\lambda = s^{\alpha} = s^{0.7}$, then $H(\lambda) = 9.55/(\lambda^2 + 1.564\lambda + 9.55)$, its characteristic roots are $\lambda_{1,2} = -0.782 \pm j2.99$. The stability region is achieved by $\Gamma_1$ and $\Gamma_2$, whose slopes are $\pm 7\pi/20$. Fig.7 shows the root locus of the EFBW filter, where the roots are distributed in the stability region, implying that the EFBW filter and the whole closed-loop system are stable.

From (36), the minimum time-delay terms in each row of the process model transfer matrix are $p_{11}(s)$ and $p_{22}(s)$, then
based on (3), the elements $c_{d_{11}}(s)$ and $c_{d_{22}}(s)$ in the forward channel matrix $C_d(s)$ are not zero. Hence, the elements of the IMC matrix $C(s)$ based on ID are chosen as the form of (2).

Based on (8) and (37), $T(s)$ is divided into

$$T(s) = \frac{\omega_1}{s^2 + \frac{\omega_1}{C}}$$

Substituting (38) and (39) into (3) and (4), yields

$$e^{-t_1 s} \cdot \frac{-s + z_1}{s + z_1} \cdot H(s)$$

$$e^{-t_2 s} \cdot \frac{-s + z_2}{s + z_2} \cdot H(s)$$

It can be known from (40) that $c_{o_{12}}(s)$ and $c_{o_{21}}(s)$ are non-realizable. To satisfy the physical realizability of the system, the diagonal matrix $D(s)$ is introduced by (9) as

$$D(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

After multiplying the process model of (36) by (41), the new process model $P_D(s)$ is obtained as

$$P_D(s) = P(s) \cdot D(s)$$

The final forward channel transfer matrix $C_d(s)$ and the feedback channel transfer matrix $C_o(s)$ are obtained by (43) and (44) as shown at the bottom of the next page.

Following the above steps, with the fixed cutoff frequency $\omega_c = 5$, the IOBW filter in 2nd order is given as

$$H(s) = \frac{25}{s^2 + 7.071s + 25}$$

$C_d(s)$ and $C_o(s)$ deduced by the IOBW filter in 2nd order are given (46) and (47), as shown at the bottom of the next page.
be noted that the IOBW filter converge more slowly to the target value than the proposed EFBW filter. 

As seen in Fig.9, the controller output curve used by the EFBW filter is smoother and has less chattering compared with the IOBW filter. The sensitivity function curves of the proposed method and the method proposed by [38] are shown in Fig.10. It can be observed from Fig.10 that $M_1$ of the proposed method is smaller, which indicates that the proposed method has better disturbance rejection ability. Besides, $M_2$ with the EFBW filter is smaller than that with the IO one.

**Example 2:** Consider the Tyreus distillation column model [37] whose transfer matrix is $s$

$$P(s) = \begin{bmatrix} 1.986e^{-0.71s} & -5.24e^{-60s} & -5.984e^{-2.24s} \\ 66.7s + 1 & 400s + 1 & 14.29s + 1 \\ -0.0204e^{-0.59s} & 0.33e^{-0.68s} & -2.38e^{-0.42s} \\ (7.14s + 1)^2 & (2.38s + 1)^2 & (1.43s + 1)^2 \\ -0.374e^{-7.75s} & 11.3e^{-3.79s} & 9.811e^{-1.59s} \\ 22.22s + 1 & (21.74s + 1)^2 & 11.36s + 1 \end{bmatrix}$$

(48)

The proposed EFBW filter is used, where the order is $\alpha = 1.2$ and the cutoff frequency $\omega_c = 3$. The EFBW filter obtained by (29)-(33) is

$$H(s) = \frac{14}{s^{1.4} + 7.606s^{1.2} + 14}$$

(51)

Further, the stability analysis will be discussed here. Let $\lambda = s^\alpha = s^{1.2}$, and then the characteristic equation of $H(s)$ is $\lambda^2 + 7.606\lambda + 14 = 0$, the characteristic roots are $\lambda_1 = -3.12$.
and $\lambda_2 = -4.48$. The stability region achieved by $\Gamma_1$ and $\Gamma_2$, whose slopes are $\pm 3\pi / 5$. Fig. 11 shows that the roots are distributed in the stability region, which means that the EFBW filter and the resulting closed-loop system are stable.

Based on (8) and (51), $T(s)$ is divided into

$$t_1(s) = e^{-t_{d_1}s} \cdot \prod_{i=1}^{N} \left( \frac{-s + z_i}{s + z_i} \right)^{v_1} \cdot H(s)$$

$$= \frac{1}{s^{2.4} + 7.606s^{1.2} + 14}$$

$$t_2(s) = e^{-t_{d_2}s} \cdot \prod_{i=1}^{N} \left( \frac{-s + z_i}{s + z_i} \right)^{v_2} \cdot H(s)$$

$$= \frac{1}{s^{2.4} + 7.606s^{1.2} + 14}$$

$$t_3(s) = e^{-t_{d_3}s} \cdot \prod_{i=1}^{N} \left( \frac{-s + z_i}{s + z_i} \right)^{v_2} \cdot H(s)$$

$$= \frac{1}{s^{2.4} + 7.606s^{1.2} + 14}$$

It can be seen from (50) that the delay terms of $p_{11}(s)$, $p_{22}(s)$, and $p_{33}(s)$ are the smallest, so the elements $c_{d_{11}}(s)$, $c_{d_{22}}(s)$, and $c_{d_{33}}(s)$ of the forward channel matrix $C_d(s)$ are not zero. By using (3) and (4), the forward channel transfer matrix $C_d(s)$ and the feedback channel transfer matrix $C_o(s)$ are obtained as

$$c_{d_{11}}(s) = \frac{t_1(s)}{p_{11}(s)} = \frac{14 (66.7s + 1)}{1.986 (s^{2.4} + 7.606s^{1.2} + 14)}$$

$$c_{d_{22}}(s) = \frac{t_2(s)}{p_{22}(s)} = \frac{0.33 (s^{2.4} + 7.606s^{1.2} + 14)}{14 (11.36s + 1)}$$

$$c_{d_{33}}(s) = \frac{t_3(s)}{p_{33}(s)} = \frac{9.811 (s^{2.4} + 7.606s^{1.2} + 14)}{14 (11.36s + 1)}$$

Note that the denominator orders of $c_{o_{12}}(s)$, $c_{o_{13}}(s)$, $c_{o_{22}}(s)$, $c_{o_{23}}(s)$, $c_{o_{31}}(s)$, and $c_{o_{32}}(s)$ are lower than the numerator orders in (56), which means the controller is not proper. So the diagonal matrix $D(s)$ must be introduced as a multiplicative factor by using (9), which is expressed as

$$D(s) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s^{1.4} + 1 \\ s^{1.4} + 1 \\ s^{1.4} + 1 \end{bmatrix}$$

Then multiplying $D(s)$ by $P_F(s)$, a new process model transfer matrix $P_D(s)$ is obtained as (58), shown at the bottom of the next page.

The final forward channel transfer matrix $C_{d}(s)$ is obtained as $C_d(s) = \text{diag} \left( C_{d_{11}}(s), C_{d_{22}}(s), C_{d_{33}}(s) \right)$, where

$$C_{d_{11}}(s) = \frac{7.049 (66.7s^{2.4} + s^{1.4} + 66.7s + 1)}{s^{2.4} + 7.606s^{1.2} + 14}$$

$$C_{d_{22}}(s) = \frac{42.42 (5.66s^{3.4} + 4.76s^{2.4} + 5.66s^2 + s^{1.4} + 4.76s + 1)}{s^{3.4} + s^{2.4} + 7.606s^{2.2} + 7.606s^{1.2} + 14s + 14}$$

$$C_{d_{33}}(s) = \frac{1.427 (11.36s^{2.4} + s^{1.4} + 11.36s + 1)}{s^{2.4} + 7.606s^{1.2} + 14}$$

and the feedback channel transfer matrix is

$$C_o(s) = \begin{bmatrix} 0 & C_{o_{12}}(s) & C_{o_{13}}(s) \\ C_{o_{21}}(s) & 0 & C_{o_{23}}(s) \\ C_{o_{31}}(s) & C_{o_{32}}(s) & 0 \end{bmatrix}$$
where

\[
C_{012}(s) = \frac{0.367e^{-59.2s}(s^{2.4} + 7.606s^{1.2} + 14)}{400s^{2.4} + s^{1.4} + 400s + 1}
\]

\[
C_{013}(s) = \frac{0.427e^{-1.7s}(s^{2.4} + 7.606s^{1.2} + 14)}{14.29s^{2.4} + s^{1.4} + 14.29s + 1}
\]

\[
C_{021}(s) = \frac{0.00146(s^{2.4} + 7.606s^{1.2} + 14)}{50.9796s^{3.4} + 14.28s^{2.4} + 50.9796s^{2} + s^{1.4} + 14.28s + 1}
\]

\[
C_{023}(s) = \frac{0.17(s^{2.4} + 7.606s^{1.2} + 14)}{2.0449s^{3.4} + 2.86s^{2.4} + 2.0449s^{2} + s^{1.4} + 2.86s + 1}
\]

\[
C_{031}(s) = \frac{0.0267e^{-5.99s}(s^{2.4} + 7.606s^{1.2} + 14)}{22.22s^{2.4} + s^{1.4} + 22.22s + 1}
\]

\[
C_{032}(s) = \frac{0.807e^{-1.94s}(s^{2.4} + 7.606s^{1.2} + 14)}{472.63s^{3.4} + 43.48s^{2.4} + 472.63s^{2} + s^{1.4} + 43.48s + 1}
\]

Following the above steps, with the fixed cutoff frequency \(\omega_c = 3\), the IOBW filter in 3rd order is given as

\[H(s) = \frac{27}{s^3 + 6s^2 + 18s + 27}\]

(62)

The final forward channel transfer matrix \(C_d(s)\) is obtained as

\[C_d(s) = \text{diag}(C_{d11}(s), C_{d22}(s), C_{d33}(s))\],

where

\[
C_{d11}(s) = \frac{13.595(66.7s^2 + 67.7s + 1)}{s^3 + 6s^2 + 18s + 27}
\]

\[
C_{d22}(s) = \frac{81.81(5.66s^3 + 10.42s^2 + 5.76s + 1)}{s^3 + 6s^2 + 18s + 27}
\]

\[
C_{d33}(s) = \frac{2.752(11.36s^2 + 12.36s + 1)}{s^3 + 6s^2 + 18s + 27}
\]

(63)

and the feedback channel transfer matrix is

\[
C_o(s) = \begin{bmatrix}
0 & C_{012}(s) & C_{013}(s) \\
C_{021}(s) & 0 & C_{023}(s) \\
C_{031}(s) & C_{032}(s) & 0
\end{bmatrix}
\]

(64)

where

\[
C_{012}(s) = \frac{0.194e^{-59.2s}(s^{3} + 6s^{2} + 18s + 27)}{400s^{3} + 801s^{2} + 402s + 1}
\]

\[
P_F^D(s) = P_F(s) \cdot D(s)
\]

\[
= \begin{bmatrix}
1.986e^{-0.8s} \\
(66.7s + 1)(s^{1.4} + 1) \\
-0.0204e^{-0.68s} \\
(7.14s + 1)^2(s^{1.4} + 1) \\
-0.374e^{-7.84s}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
-5.24e^{-60s} \\
(400s + 1)(s^{1.4} + 1) \\
0.33e^{-0.68s} \\
(2.38s + 1)^2(s^{1.4} + 1) \\
11.3e^{-3.79s}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
-5.984e^{-2.5s} \\
(14.29s + 1)(s^{1.4} + 1) \\
-2.38e^{-0.68s} \\
(1.43s + 1)^2(s^{1.4} + 1) \\
9.81e^{-1.85s}
\end{bmatrix}
\]

(58)

The setting times and the overshoots of the proposed method and the method in [38] are illustrated in Table 2. For Loop1, Loop 2 and Loop 3, the setting times using the proposed method are 51.95s, 151.92s, and 252.8s, which are shorter than the other methods, and the overshoots of the proposed method are smaller than the ID-IMC based on the 2nd IOBW filter. Therefore, the proposed method based on the EFBW has a better dynamic performance.

The system output response curves and controller output curves are shown in Fig.12 and Fig.13. Unit step changes
TABLE 2. Dynamic performance indicator in the time domain.

|        | Loop1 | Loop2 | Loop3 | Loop4 | Loop5 | Loop6 |
|--------|-------|-------|-------|-------|-------|-------|
| 1.4th order | 51.95 | 151.92 | 252.8 | 24.1  | 13.6  | 11.2  |
| 2nd order  | 53.31 | 154.54 | 255.59| 76.2  | 75.1  | 74.3  |
| Juan [38]| 96.32 | 208.1  | 306.73| 0     | 0     | 0     |

FIGURE 13. The controller output.

FIGURE 14. The sensitivity function.

are sequentially added in the individual loops of the setpoint inputs at $t = 50s$, $t = 150s$, and $t = 250s$. It is shown from Fig.12 (a) that the proposed controller has better step responses compared to [38]. Besides, Fig.12 (b) shows the magnified diagrams of the step response curves with the 2.4th and 3rd order BW filters. It can be observed that the EFBW filter has a faster response than the IOBW filter with less overshoot and buffeting. Fig.13 shows that the controller output of the EFBW filter is smoother.

Fig.14 shows the sensitivity function curves of the proposed method and the method proposed by [38]. It can be observed that the $M_{r}$ of the proposed method using the EFBW filter is the smallest, which indicates that the proposed method has a stronger disturbance rejection ability.

VI. CONCLUSION
This paper presents a novel procedure to approximate BW filters using FO theories, with which the ID-IMC control scheme is combined, so that the degree-of-freedom for tunable parameters is increased. The proposed controller has the advantages of computational simplicity of the ID-IMC structure as well as the greater flexibility of the EFBW filter. By applying the proposed controller, a better set-point tracking and disturbance rejection performance can be guaranteed. Stability analysis of the designed controller is given to ensure the stability of the closed-loop system, where the dynamic performance indicators and sensitivity functions are carried out for time domain and robustness analysis. Two illustrative examples are presented, showing that a minimum overshoot and setting time can be achieved.

ACKNOWLEDGMENT
This research was supported by the National Natural Science Foundation of China (No. 61771034).

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KAIYUE LIU was born in Liaocheng, Shandong, China, in 1991. She received the B.S. degree in automation from the Tianjin University of Science and Technology, Tianjin, in 2014, and the M.S. degree in control science and engineering from the Tianjin University of Technology, Tianjin, in 2017. She is currently pursuing the Ph.D. degree with the School of Information Science, Beijing University of Chemical Technology. She has published one article in the International Conference, Internal Model Control With Improved Butterworth Filter Based on Inverted Decoupling for Multivariable Systems with the IEEE SMC, in 2017. Her research interests include internal model control and fractional-order system control.

JUAN CHEN was born in Harbin, Heilongjiang, China, in 1961. She received the B.S. degree in automation from the College of Northeast Heavy Machinery, Yanshan University, Qinhuangdao, China, in 1983, the M.S. degree in control science and engineering from the Harbin Institute of Technology, Harbin, in 1999, and the Ph.D. degree in control science and engineering from the Beijing University of Chemical Technology, Beijing, China, in 2006. She is currently a Professor and a Doctoral Tutor with the School of Information Science, Beijing University of Chemical Technology. She has authored over 80 articles. She holds over three invention patents. Her research interests include control theory and advanced control method and fractional-order control systems.

Dr. Chen received awards and honors, include the Henan Province Science and Technology Progress Award, in 1987, and the National Fund for Natural Science Fund Project Award, in 2014 and 2018.