Abstract In this paper we deal with the initialization problem of a visual-inertial odometry system with rolling shutter cameras. The initialization is a prerequisite to utilize inertial signals and fuse them with the visual data. We propose a novel way to solve this problem on visual and inertial data simultaneously in a statistical sense, by casting it into the renormalization scheme of Kanatani. The renormalization is an optimization scheme which intends to reduce the inherent statistical bias of common linear systems. We derive and present necessary steps and methodology specific for the initialization problem. Extensive evaluations on perfect ground truth exhibit superior performance and up to 20% accuracy gain to the originally proposed Least Squares solution. The renormalization performs similarly to the optimal Maximum Likelihood estimate, despite arriving to the solution by different means. By this, we extend the set of common Computer Vision problems which can be cast into the renormalization scheme.

Keywords Visual-Inertial Odometry Initialization · Renormalization · Rolling-Shutter camera

1 Introduction

Real time pose estimation of a moving camera has been an active topic in Computer Vision and Robotics community for decades, but with the advent of Augmented Reality, the topic experiences a new hype. Augmented Reality draws new requirements on pose estimation performance as on the energy consumption side as well as on the accuracy and robustness. Mobile phones, wearable smart glasses or watches use in-built rigid rigs with a mono or stereo camera and an IMU as de facto a standard hardware. This stems from the fact that combining the two sensor modalities, the visual and inertial one, has been proven to be an ultimate solution towards compensating each others drawbacks. Common development toolkits natively support fusion of both sensors, e.g. ARCore of Google (2018) and ARKit of Apple (2015).

There are two important practical challenges to be considered. First, cameras in mobile devices are in majority rolling shutter cameras. These are cheaper and possess higher dynamic range than standard global shutter cameras. Majority of research has been, however, devoted to the standard global shutter camera models. Rolling shutter geometry has started catching the attention with the rise of mobile phones and smart glasses (Meingast et al. (2003); Hedborg et al. (2012);...
In this paper we aim at estimating the initial velocity and gravity direction of a moving rig, equipped with an IMU sensor. For instance, the IMU delivers zero inertial acceleration signal when it is either static or moving with constant velocity. Without their proper initial conditions, the integration leads to the same static pose. The remedy lies in fusion of the inertial with visual data as the visual cues clearly distinguishes these two cases.

Yet, most systems assume that the mobile device is static at the beginning of its operation and as such, the initial velocity can be set to zero and the initial gravity direction can be deduced from the accelerometer. While this assumption may be safe in many situations, it is violated when triggering the start of a Visual-Inertial Odometry (VIO) system under motion, e.g. on the bicycle or when walking.

1.1 Contribution

In this paper we aim at estimating the initial velocity and gravity direction of a moving rig, equipped with a rolling shutter, optionally stereo, camera and an IMU, as depicted in Fig. 1. First, we consolidate the closed-form minimal solver of Martinelli (2013) and extend it for rolling shutter stereo cameras. Support for partial tracks is considered as well, such the points do not need to be tracked in all the cameras. Second, we propose to reduce the original minimal solver by Shur complement based elimination. Third, the reduced system which is better suited for noise propagation analysis is cast into the renormalization scheme of Kanatani (1996).

The renormalization scheme of Kanatani is a statistical method for certain type of problems. We show that the initialization problem can be brought by the proposed operations into the renormalization scheme. The proposed solution has superior performance to the minimal least squares solver of Martinelli (2013) while both defined on top of the same linear system. The renormalization scheme performs comparably and sometimes outperforms the optimal Maximum Likelihood (ML) estimator which minimizes the re-projection error. Compared to the least squares, the renormalization scheme removes its inherent bias, and explicitly provides covariance of the estimate. The renormalization scheme may suffice to solve the problem in most cases, however, it can initialize ML to enforce faster convergence.

It is known that ML entails statistical bias in the presence of what is known as nuisance parameters. Various studies exist for analyzing and removing bias in the ML solution, e.g. by Okatani and Deguchi (2009). An optimal ML solution is usually given by a nonlinear optimization which is time-consuming when solved by numerical search. It often requires extra nuisance parameters, initial values and solving iteratively large, although sparse, linear systems. On the contrary, the renormalization procedure requires neither a priori knowledge of the initial values nor the noise level which is estimated a posteriori as a result of the renormalization itself. It consists of iterated computations of eigenvalues and eigenvectors of small matrices and bias-correction steps.

This work extends the set of problems in Computer Vision which can be solved by the renormalization scheme. Kanatani et al. (2016) formalized the renormalization scheme for many geometric computations in computer vision, e.g. ellipse, homography, fundamental matrix fitting, triangulation, and 3D reconstruction. These techniques show superior performance under some circumstances on many problems to the Gold Standard Methods of Hartley and Zisserman (2004) and are viable alternatives in many practical use cases.

The paper is structured as follows. First, the related work for VIO initialization and structure from motion systems as well as positioning the renormalization scheme is reviewed in Sec.2. Then, the main concept is presented in Sec.3. Its main parts include geometric relation of a camera and an IMU in Sec 3.1, rolling shutter image formation in Sec. 3.2, the closed-form minimal solver, adjusted for rolling shutter cameras and partial feature tracks in Sec. 3.3, its reduced form in Sec. 3.4 cast as the renormalization scheme outlined and applied in Sec. 3.5. Bundle adjustment is shortly discussed and compared to the renormalization in Sec. 3.6. Finally, an extensive experimental evaluation is presented in Sec. 3 followed by the conclusion in Sec. 4.

2 Related Work

2.1 VIO Initialization

The most relevant paper to our method is the closed-form solution for initial velocity and gravity direction by Martinelli (2013). The method assumes a mono global shutter camera and complete tracks. They propose to relate corresponding visual observations through the camera baseline, which linearly depends on the unknown state parameters, that is, the velocity, the gravity in the
IMU frame and the accelerometer bias. Each visual correspondence contributes three linear equations, while the distances between map points and the cameras become unknown parameters too. The resulting linear system is then solved, with an optional constraint on the gravity magnitude. The robustness of the method against biased IMU readings was investigated by Kaiser et al. (2017) and, to account for the gyroscope bias, a non-linear refinement method was proposed. Campos et al. (2019) then built on Martinelli (2013); Kaiser et al. (2017) and improved the method via multiple loops of visual-inertial bundle adjustments and consensus tests. Our solution improves the least square solution and could be directly used for initializing BA of Campos et al. (2019); Huang et al. (2020). Even if the biases are known or ignored, the unknown orientation between camera and IMU system of the device is uncalibrated (Dong-Si and Mourikis (2012); Huang et al. [2020]). In this context, Kneip et al. (2014) suggested using visual SM to obtain camera velocity differences which are then combined with integrated IMU data to recover the scale and gravity direction. The initialization part of Mur-Artal and Tardós (2017) used scaleless poses from ORB-SLAM of Mur-Artal et al. (2015) and then solved several sub-problems to initialize the state and the biases along with the absolute scale. This multi-step solution for the parameter initialization was then adapted in Qin and Shen (2017). The initialization problem becomes harder when the device is uncalibrated (Dong-Si and Mourikis (2012); Huang et al. [2020]). Even if the biases are known or ignored, the unknown orientation between camera and IMU makes the model non-linear and iterative optimization is necessary. Dong-Si and Mourikis (2012) propose two solutions to estimate the unknown orientation, thus allowing solving a linear system which, in turn, initializes a non-linear estimator. Instead, Huang et al. (2020) builds on the multi-step approach of Mur-Artal and Tardós (2017) to jointly calibrate the extrinsics and initialize the state parameters. In a real scenario, however, the joint solution of calibration and initialization problem using only the very first few frames might make the pose tracking algorithm prone to diverge.

It is worth noting that all the above works silently assume that visual observations come from a global-shutter sensor. Consumer devices, however, are mostly equipped with rolling shutter cameras and rolling-shutter effects need to be handled (Hedborg et al. (2012); Ling et al. (2018); Bapat et al. (2018)).

2.2 Renormalization

The renormalization of Kanatani (1996) was at first not well accepted by the computer vision community. This was due to the generally held preconception that parameter estimation should minimize some cost function. Scientists wondered what renormalization was minimizing. In this line of thought, Chojnacki et al. (2001) interpreted renormalization as an approximation to ML. Optimal estimation does not necessarily imply minimizing a cost function and as such the renormalization is an effort to improve accuracy by a direct mean (Kanatani (2014)). The mathematical foundation of the optimal correction techniques of Kanatani et al. (2016) is also discussed in the broader scope of photogrammetric statistical geometric computations by Förstner and Wrobel (2016). It is the non-minimization formalism based on error analysis which intuitive meaning is often difficult to grasp, as we will see in the following.

Regarding re-projection error minimization as the ultimate method, or the Gold Standard, the fact that the accuracy of ML can be improved was rather surprising (Kanatani (2003); Okatani and Deguchi (2009)). For hyperaccurate correction, however, one first needs to obtain the ML solution by an iterative method such as Fundamental Numerical Scheme (FNS) of Chojnacki et al. (2000) on Sampson Error or Heteroscedastic Error-In-Variables (HEIV) method of Leedan and Meer (2000) and also estimate the noise level. However, it is possible to directly compute the corrected solution from the beginning, by modifying the FNS iterations if one adopts the non-minimization approach of geometric estimation of Kanatani (2014).

3 Concept

3.1 Geometry

A 3D point $X_i$ expressed in the local coordinate system of the IMU at time $\tau_i$, projected into the coordinate system of the IMU at time $\tau_0$ reads as

$$X_0 = R_i^0 X_i + t_i^0,$$

where $R_i^0$ and $t_i^0$ stand for the rotation and the translation transformation of the IMU from time $\tau_i$ to time $\tau_0$. Let us assume that a camera attached to the IMU rig
observes the 3D point \( X_0 \) at time \( \tau_i \) as

\[
\lambda_i u_i = K ( R^c_{IMU} R^0_{0} X_0 + R^c_{IMU} t^c_0 + t^c_{IMU} ) \\
\lambda_i R^c_{IMU} R^{DUM}_{0} K^{-1} u_i + t^c_i + R^c_{IMU} t^{DUM}_{0} = X_0
\]  

(2)

where \( R^c_{IMU} \), \( t^c_{IMU} \) is the known fixed relative pose from the camera to the IMU, and \( K \) is the known camera calibration matrix. Image coordinates of the 3D point \( X_0 \), being tracked in multiple views, are denoted \( u_i \) and \( \lambda_i \) are unknown scales, the depths, of their projection rays \( K^{-1} u_i \).

The IMU pose \( R^0_i \), \( t^0_i \) at time \( \tau_i \) is calculated as

\[
R^0_i = \prod_{k=0}^{i-1} R^k_{i+1} = \prod_{k=0}^{i-1} \Omega(\omega_k \Delta \tau) \\
t^0_i = t_0 + i v_0 \Delta \tau + \left( \sum_{k=0}^{i-1} \beta_{k,i} R^0_k a_k + i^2 g_0 \right) \frac{\Delta \tau^2}{2},
\]

(3)

(4)

where

\[
\beta_{k,i} = 2i - 2k - 1.
\]

(5)

The 3 element vector \( a_k \) and \( \omega_k \) is the accelerometer and the gyroscope readout measurements of the IMU at time \( \tau_k \), respectively. The exponential map \( \Omega(\cdot) \) gives a rotation matrix from the argument vector. The time between two IMU samples is denoted by \( \Delta \tau \). Without loss of generality, we set the origin into the coordinate system of the first IMU, thus the translation \( t_0 = 0 \). The initial velocity \( v_0 \) and the gravity vector \( g_0 \) at time \( t_0 \), expressed in the origin, are the unknowns and subjects to estimation. For the sake of simplicity, we assume for now that the measurements are corrected for biases. The incorporation of the biases is proposes later in Sec. \ref{sec:biases}. The biases may vary over time, and can be included in a final non-linear refinement step.

We further assume that the IMU and the cameras are temporarily synchronized.

It is to be noted that unless \( v_0 \) and \( g_0 \) are known, the IMU data cannot be integrated in order to get IMU poses in the above chosen origin. Most visual-inertial systems assume that the camera-IMU rig is static at start and it can be assumed that the initial velocity \( v_0 = 0 \) and the initial gravity \( g_0 \) is determined from the acceleration readout. However, in many practical situations this is violated, the system is in motion at start, e.g. a person is on a bicycle or walks.

3.2 Rolling shutter image formation

A rolling shutter camera is, in its principle, a moving line camera. When moving along a line, it falls into a class of linear pushbroom cameras, see \cite{Gupta and Hartley1997}. Each scanline is read out one after the other and all of them are stacked into an image buffer. Note that indeed the pose of the IMU in Eq. \ref{sec:imageformation} differs for each \( i \). The readout time of a scanline of the rolling shutter camera is constant even when camera exposure varies. We can therefore safely choose \( \Delta \tau = \tau_{i+1} - \tau_i \) to be exactly the readout time of one line of the camera. The IMU data can be upsampled, e.g. for VGA resolution from a typical sampling IMU rate of 800Hz to 47.6kHz, and integrated, called the interpolate-then-integrate approach. As such, for each scanline of the image, we have one pose, \( R^0_i \) and \( t^0_i \). Alternatively, the integration is performed on the original IMU sample rate and then the poses are interpolated for each scanline, the integrate-then-interpolate approach. We found the first approach to give more accurate results for the initialization problem. If we define \( r_i \) to be the corresponding row index in the image buffer to the \( ith \) sample

\[
r_i = i - i_{r_i=0} = \frac{(\tau_i - \tau_{r_i=0})}{\Delta \tau},
\]

the 3D point \( X_0 \) is observable by the camera row \( r_i \) at the time \( \tau_i \) if and only if

\[
|u_{i,y} - r_i| < 0.5,
\]

where \( u_i = [u_{i,x} \ u_{i,y} \ 1]^\top \). Without loss of generality, in order to simplify the notation, we assume no radial distortion.

3.3 Minimal Solver

Let us assume that a (stereo) camera with the IMU moves and observations of some 3D points in multiple images are available. If \( u_i \) and \( u_{i} \) are the image observations of a point \( X_0 \) in two views, then we can write
Eq. (2) for each point separately. By eliminating $X_0$ we obtain
\[ \lambda_i p_i + t_i^0 + R_i^0 t_c^0 = \lambda_j p_j + t_j^0 + R_j^0 t_c^0, \] (6)
such that $p_i = N(p_i) = N(R_i^0 R_c^0 k^{-1} u_i)$, where $N(x)$ normalizes the vector $x$ to the homogeneous coordinates. Substituting Eq. (11) into Eq. (6) yields
\[
\begin{bmatrix}
\xi_{ij} I & \mu_{ij} I & \kappa_{ij} & p_i & p_j & \cdots & p_l \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\xi_{lk} I & \mu_{lk} I & \kappa_{lk} & p_i & p_k & \cdots & p_l \\
S & \lambda & \lambda & \lambda & \lambda & \cdots & \lambda \\
\end{bmatrix}
\begin{bmatrix}
\lambda_i \\
\mu_i \\
\rho_i \\
\xi_i \\
\eta_i \\
\end{bmatrix}
= 0,
\] (7)
where
\[ \xi_{ij} = (i - j) \Delta \tau, \]
\[ \mu_{ij} = (i^2 - j^2) \frac{\Delta \tau^2}{2}, \]
\[ \kappa_{ij} = R_i^0 t_c^0 - R_j^0 t_c^0 + \sum_{k=0}^{j-1} \beta_{k,i} R_k^0 a_k - \sum_{k=0}^{i-1} \beta_{k,j} R_k^0 a_k \frac{\Delta \tau^2}{2}. \] (8)
In the matrix form, the Eq. (7) can be written as
\[ [S \ P] x = 0, \] (9)
which is a linear equation system. It can be solved, for instance, in the least squares sense. It is worth noting that the error which is minimized by the above least squares solution has a geometric meaning. It relates to the distance between 3D points which are obtained through $\Lambda_i p_i$, as shown in Fig. 2. We tried to formulate the initialization problem on the angular error on projective rays instead of the distance. The angular error is often used in standard epipolar geometry solvers [Hartley and Zisserman (2004)], and also has been used in the relative pose for the rolling shutter camera in Dai et al. [2014]. For static or slow motion the error degenerates as is too sensitive to image noise. Overall, the angular error is inferior to the presented distance based error.

Considering the sparsity of the matrix $P$, a sparse linear solver can be used to solve the system. Each pair of point matches adds three equations and up to two additional unknown $\lambda$’s. In total, the minimal number of point correspondences is 6. The unknown $\lambda$’s are shared between multiple views as shown in Eq. (7). There, for the $(i, j)$ and $(i, k)$ pair, $\lambda_i$ is shared as the corresponding 3D point is projected into three views. This fact allows to stack explicitly the points on an image track, i.e. image projections of the same 3D points, together and thus to better constrain the system. Similar derivations to Eq. (7) for the global shutter camera can be found in Martinelli [2013].

### 3.4 Reduced Minimal Solver

We propose to eliminate the unknown $\lambda$’s from Eq. (9). This can be done with the Schur complement based elimination of the $P$ matrix in Eq. (9) such that it becomes
\[
[S - P(P^T P)^{-1} P^T B] y = 0,
\] (10)
\[
[(I - G) S] y = 0,
\] (11)
\[
B \begin{bmatrix} v_0 \\ g_0 \\ 1 \end{bmatrix} = 0,
\] (12)
where the matrix $B = (I - G) S$ is $3N \times 7$, $N$ is the number of pairs of point matches and $y = [v_0 \ g_0 \ 1]^T$ is the unknown 7 element vector. The matrix $G$ is an idempotent square $N \times N$ projection matrix. Solving the reduced linear problem in Eq. (12) in the least square sense, yields the same result as Eq. (7). Depending on the sparsity of the practical problem, one or another can be faster, and should be chosen accordingly.

The reduced form simplifies the noise analysis in order to arrive to the solver presented in the next. The advantage of the reduced form in Eq. (12) is that the unknown solution vector is fixed in size. It contains the initial velocity and gravity only, and no longer the depth multipliers $\lambda$’s.

At the first look, it might look as if the rolling shutter camera adds difficulties in the equations in comparison to the global shutter case. However, from the geometric point of view, the opposite can be claimed. Each
camera line has a different center of projection and so the rays put into the triangulation equation constrain better the solution. On the other side, the rolling shutter effect adds image artifacts as the long line segments may get projected as bent under a fast motion. However, as we experimentally observed, this is negligible for a feature tracker which uses a small image patch. Overall, the rolling shutter is a beneficial feature which implicitly encodes motion of the camera and makes it directly observable [Bapat et al. (2018)].

Both Eq. (11) and Eq. (12) treat all the data as equally valuable and fall into the class of algebraic least squares estimators. In the next section we propose an improvement by involving a proper noise perturbation analysis to individually weight the point matches.

3.5 Renormalization Scheme

The class of problems called geometric fitting, where a geometric relationship in high dimensions is fitted, expressed as an implicit equation has been studied by Kanatani [1994]. Eq. (12) falls into this class. The matrix $B$ in Eq. (12) is parametrized by the IMU sensor data and by the image point correspondences over multiple views $u_{i,j,...}$ via their projective rays $p_{i,j,...}$. We consider short integration time in which the effect of noise on the IMU data is negligible to the noise on the point correspondences. We experimentally verified very little accuracy gain when considering noise in the IMU data. Therefore, in the next we perform noise perturbation analysis when considering noise purely on the point correspondences.

Each image point correspondence pair $u_i \leftrightarrow u_j$ in Eq. (12) contributes three equations to the matrix $B$ and can be written as

$$ (b^{(s)}(u_i, u_j), y) = 0, \quad s = 1 \ldots 3, $$

where $b^{(s)}$ is one row of the matrix $B$ and $(a, b) = a^T b$ stands for the inner product. The three equations are linearly dependent, so we see the same as in Eq. (9) that 6 is the minimal number of point correspondences to guarantee $\text{rank}(B) = 6$.

The coordinates of the image point correspondences $u_i, u_j$ are not perfect. This is caused by the image operations, specifically the feature detection and patch based tracking on noisy image data signal. We model this uncertainty in statistical means. We assume that the observed image point $u_i$ stems from perturbation of the true value $\bar{u}_i$ by independent random Gaussian variable $\Delta u_i$ of zero-mean and with the covariance matrix $\gamma [u_i]$, such that

$$ u_i = \bar{u}_i + \Delta u_i. $$

We experimentally validated that Gaussian noise is a feasible assumption in practical situations with an off-the-shelf feature detector and a feature tracker, see Fig. 3. We assume the covariance matrix is known up to noise level $\sigma$, i.e.

$$ \gamma [u_i] = \sigma^2 \gamma_0 [u_i], \quad (13) $$

where the known normalized covariance matrix $\gamma_0 [u_i]$ describes the orientation dependence of uncertainty in relative terms. The covariance matrix can come from uncertainty of the employed feature detector and the tracker. In all our experiments we assume $\gamma_0[u_i]$ be the identity matrix.

If the observations $u_i, u_j$ are regarded as random variables, their nonlinear mapping $b^{(s)}(u_i, u_j)$, which we write $b_{i,j}$ for short, is also a random variable. Its covariance matrix is

$$ \gamma^{(st)}[b_{i,j}] = \sigma^2 \gamma_0^{(st)}[b_{i,j}], \quad (14) $$

where $s, t = 1 \ldots 3$ to combine mutually the rows of the three equations per correspondence. The covariance matrix is evaluated to first approximation in terms of the Jacobians $J_s$ and $J_t$ of the mapping $b^{(s)}$ as follows

$$ \gamma^{(st)}[b_{i,j}]_{s,t} = J_{ij}^{s} \begin{bmatrix} \sigma^2 \gamma_0 [u_i] & 0 \\ 0 & \sigma^2 \gamma_0 [u_j] \end{bmatrix} J_{ij}^T. \quad (15) $$

If the noise in the $u$-space is assumed Gaussian, the corresponding noise in the transformed $b$-space is no longer Gaussian. However, our numerical experiments have shown that in the noise range of typical feature detector and tracker, i.e. $\sigma \in [0, 0.5]$ pixels, such an assumption is feasible. In order to stay in the safe range, removal of systematic error like outliers prior to estimation is crucial. Correction for higher order noise terms can be omitted, as we observed that the Hyper-renormalization of Kanatani et al. [2016] brings only small accuracy gain for the increased computational burden.

3.5.1 Solver

The standard Least Squares (LS) solution to Eq. (12)

$$ \epsilon^2 = \frac{1}{N} \sum_{a=1}^{N} \sum_{s,t=1}^{3} (b^{(a,s)}(y), b^{(a,t)}(y)) $$

$$ = \left( y, \left( \frac{1}{N} \sum_{a=1}^{N} \sum_{s,t=1}^{3} b^{(a,s)} b^{(a,t)^T} \right) y \right) = (y, M_{ls} y) \quad (16) $$

minimizes the mean square error $\epsilon$. Fig. 4 depicts the geometric meaning of the error. The solution can be obtained as an eigenvalue fit of the $7 \times 7$ matrix $M_{ls}$ as $M_{ls} y = \lambda y$. 
Renormalization for Initialization of Rolling Shutter Visual-Inertial Odometry

Weighting each pair $i, j$ differently, the LS would turn, for small accuracy gain, into iterative re-weighted LS. More importantly, both can be fairly improved by Taubin [1991], as modification of LS and even slightly more by Kanatani [2008]. Our experiments validate what has been demonstrated in the ellipse fitting problem by Kanatani that the error on the estimated entities can be sorted as naïve LS $\rightarrow$ weighted LS $\rightarrow$ Taubin $\rightarrow$ renormalization.

[Taubin 1991] proposed to include higher noise error terms to remove the bias of LS, and such, to first order approximation of the algebraic mean square error it yields a generalized eigenvalue fit. Kanatani further improved upon this idea and proposed to iteratively re-weight the Taubin method, therefore called renormalization [Kanatani 1996]). We present in the following renormalization scheme applied to the initialization of a VIO system.

Renormalization Scheme

1. Let $y_0 = 0$ and $w_0 = \delta_{st}$, $\alpha = 1 \ldots N$, $s, t = 1, 2, 3$, where $\delta_{st}$ is the Kronecker delta, equal 1 if $s = t$ and 0 otherwise.

2. Compute $7 \times 7$ matrices

$$
M = \frac{1}{N} \sum_{\alpha=1}^{N} \sum_{s,t=1}^{3} w_0^{(st)} b_0^{(s)} b_0^{(t)^T} 
$$

$$
N = \frac{1}{N} \sum_{\alpha=1}^{N} \sum_{s,t=1}^{3} w_0^{(st)} y_0^{(st)} [b_\alpha],
$$

where $w_0^{(st)}$ is the element of the matrix $w_0$ at the row $s$ and column $t$.

3. Solve the generalized eigenvalue problem

$$
My = \gamma Ny
$$

and compute the unit eigenvector $y$ for the smallest eigenvalue $\gamma$.

4. If $y \approx y_0$ up to sign, continue to Step 5. Else, update

$$
\begin{align*}
    w_\alpha &\leftarrow \begin{bmatrix}
    \langle y, v_0^{(11)} | b_\alpha \rangle y & \langle y, v_0^{(12)} | b_\alpha \rangle y & \langle y, v_0^{(13)} | b_\alpha \rangle y \\
    \langle y, v_0^{(21)} | b_\alpha \rangle y & \langle y, v_0^{(22)} | b_\alpha \rangle y & \langle y, v_0^{(23)} | b_\alpha \rangle y \\
    \langle y, v_0^{(31)} | b_\alpha \rangle y & \langle y, v_0^{(32)} | b_\alpha \rangle y & \langle y, v_0^{(33)} | b_\alpha \rangle y 
\end{bmatrix} \\
    y_0 &\leftarrow y
\end{align*}
$$

and go back to Step 2. The expression $[U]_{1:2}$ is the pseudoinverse with truncated rank 2 or 1. The truncation to rank 2 is done iff $\frac{\sigma_2}{\sigma_1} > 0.1$, where $\sigma_1$ and $\sigma_2$ is the first and the second largest singular value of $U$ respectively. Otherwise, the truncation to rank 1 is performed.

5. Return $y$ composed of $v_0$ and $g_0$, its covariance matrix $V_0[y]$, and the noise level $\sigma$

$$
V_0[y] = \frac{\sigma^2}{N} J_H M^{-1} J_H^{T}, \quad \sigma^2 = \frac{y^T M y}{2 - 6/N} \tag{22}
$$

with $J_H$ be the Jacobian of the transformation from a homogeneous to Euclidean vector, see [Förstner and Wrobel 2014]) Eq.10.32.

$$
J_H = \frac{1}{y^T(7)} [y(7) I_6 - y(1:6)].
$$

Justification of estimating the noise level $\sigma$ can be seen in Eq. (6.46) in Kanatani et al. (2016).

The matrix $M$ determines the covariance of the final estimate of $y$, while the matrix $N$ controls the bias of the $y$. The contribution of the renormalization scheme is in the derivation of the matrix $N$ such that when combined with the matrix $M$ it compensates for the statistical bias which is inherent in Least Squares solution [Kanatani 2008]).

Least Squares choose $y$ which minimizes the cost function $c^2$ in Eq. (17). In renormalization scheme there is no explicit cost function which is minimized. The estimated $y$ is obtained by solving set of equations in order to reduce the dominant bias of optimally weighted Least Squares, such that it reaches Kanatani-Cramer-Rao lower bound [Kanatani 1996]).

3.5.2 Jacobians

In order to compute the covariance matrix $V_0^{(st)}[b_{ij}]$ in Eq. (15), the Jacobian matrices $J_0^{(s)}$ and $J_0^{(t)}$ need to be computed. Each $7 \times 4$ Jacobian matrix is composed of four elements

$$
J_{ij}^{(s)} = J^{(s)}_{ij} \begin{bmatrix} J_{ij}^{(s2)} & J_{ij}^{(s1)} \end{bmatrix}. \tag{23}
$$

The first Jacobian captures the transformation of the point from homogeneous coordinates to the calibrated ray,

$$
J_{ij}^{(s1)} = \begin{bmatrix} f_j I_2 & 0 \\
0 & f_i I_2 \end{bmatrix}, \tag{24}
$$

where $f_i$ stands for the focal length of the camera which observes $u_i$ and $I$ for the identity matrix.

The second Jacobian captures rotation of the vector. Denoting $\hat{R}_i = R_0^{(i)} R_0^{(i)T}$ and $\hat{R}_j = R_0^{(j)} R_0^{(j)T}$, then

$$
J_{ij}^{(s2)} = \begin{bmatrix} \hat{R}_{ij}^{(i)} & 0 \\
0 & \hat{R}_{ij}^{(j)} \end{bmatrix}, \tag{25}
$$

where $\hat{R}_{ij}^{(i)}$ is $3 \times 2$ matrix composed of the first two columns of the rotation matrix $\hat{R}_i$. 


The third Jacobian captures the transformation to homogeneous coordinates. This would not be in general needed, however, from computational point of view, one avoids the need of derivative \( w.r.t. \) the \( \tilde{p}_z \). Introducing this extra non-linearity seems in practice not affecting the solution. Considering Eq. (26), then

\[
\begin{bmatrix}
\frac{1}{\tilde{p}_{z,2}}[I_2 - \tilde{p}_i,(1:2)] & 0 \\
0 & \frac{1}{\tilde{p}_{z,2}}[I_2 - \tilde{p}_i,(1:2)]
\end{bmatrix}_{4 \times 6},
\]

where \( \tilde{p}_i = [\tilde{p}_{i,x} \tilde{p}_{i,y} \tilde{p}_{i,z}]^\top \), and \( \tilde{p}_i = N(\tilde{p}_i) = [p_{i,(1:2)}^T]_i \).

The fourth Jacobian captures the Schur elimination of Eq. (12).

\[
J_{ij}^{(s)} = \begin{bmatrix}
\frac{\partial b_{ij}^{(s)}}{\partial p_{i,x}} & \frac{\partial b_{ij}^{(s)}}{\partial p_{i,y}} & \frac{\partial b_{ij}^{(s)}}{\partial p_{j,x}} & \frac{\partial b_{ij}^{(s)}}{\partial p_{j,y}}
\end{bmatrix}_{7 \times 4}.
\]

This Jacobian requires to access the whole matrix \( \mathcal{B} \). Let us further investigate the partial derivative \( w.r.t. \) to the first component \( p_{i,x} \)

\[
\frac{\partial \mathcal{B}}{\partial p_{i,x}} = \frac{\partial \mathcal{G}}{\partial p_{i,x}} S,
\]

as outcome from derivative of Eq. (11). It is analogous for the rest three components. For any non-singular square matrix \( \mathcal{A} \) the following holds (Golub and van Loan 2013)

\[
\frac{\partial \mathcal{A}^{-1}}{\partial \alpha} = -\mathcal{A}^{-1} \frac{\partial \mathcal{A}}{\partial \alpha} \mathcal{A}^{-1}.
\]

This allows to split inversion and derivative of the matrix. It can be computed only once as it is independent on the \( \alpha \). Since \( \mathcal{G} \) is a regular idempotent projection matrix, then we can apply it to get

\[
\frac{\partial \mathcal{G}}{\partial p_{i,x}} = \frac{\partial \tilde{P}}{\partial p_{i,x}} \tilde{P}^\top + \tilde{P} \frac{\partial \tilde{P}^\top}{\partial p_{i,x}} = \tilde{P} \frac{\partial (\tilde{P}^\top \tilde{P})}{\partial p_{i,x}} \tilde{P}^\top,
\]

where \( \tilde{P} = (\tilde{P}^\top \tilde{P})^{-1} \). The factors \( \tilde{P} \tilde{P}^\top, \tilde{P} \tilde{P}^\top, \tilde{P}^\top \tilde{P}^\top \) are computed only once for all the correspondences. The three partial derivatives are correspondence dependent as they depend on \( i \) and \( j \). Since the matrix \( \tilde{P} \) is very sparse and linear in \( p \), the derivative matrices contain only few 1’s depending how often \( u_t \) appears in the correspondence pairs. Overall, using factorization in Eq. (29) and sparse matrix calculus, the total Jacobian in Eq. (23) can be calculated very efficiently.

3.6 Renormalization vs. Bundle Adjustment

In order to demonstrate performance of the renormalization \( w.r.t. \) to the optimal Maximum Likelihood estimator, we employ the Bundle Adjustment (BA) framework. We first use the solution of Eq. (11) in Eq. (7) to compute \( \lambda \)'s and we then average the multiple reconstructions per point to estimate the initial points \( X \). The BA algorithm minimizes the total re-projection error \( \sum_i ||u_i - \hat{u}_i(\cdot)||^2 \), where \( \hat{u}_i(\cdot) \) is the resulting non-linear function of \( v_0, g_0 \) and the auxiliary variable \( X \), while index \( i \) runs over all the observations. Note that our goal is to use a standard framework as a baseline to evaluate the performance of the proposed estimator. Therefore, we stay with the same and necessary parameters of velocity and gravity and we do not augment the set of unknowns with the sensor biases. To refine the parameters, the Levenberg-Marquardt algorithm is used, similar to the visual BA framework of Lourakis and Argyros (2005).

The renormalization, despite not being an optimal ML estimator, can in practical situations well replace BA, as will be demonstrated in Sec. 4. The accuracy of both methods is very comparable, but the computational burden differs. There are multiple advantages of the renormalization over BA, as the renormalization

- does not need auxiliary variables to be introduced as are the 3D points \( X \) for BA,
- needs no initial conditions. Renormalization in its first iteration starts with the Taubin (1991) method and then iteratively renormalizes the matrices. BA needs a good starting point,
- solves in each iteration a generalized eigenvalue problem of size \( 7 \times 7 \) which is very fast and can be solved within microseconds. BA solves iteratively a linear system of normal equations of the matrices \( \sim 400 \times 400 \) in case \( \sim 200 \) feature points are tracked. Despite sparsity of the problem, the computational time by two magnitudes higher, and goes to milliseconds,
- converges in no more than 5–10 iterations. BA needs up to five times more iterations,
- provides the covariance matrix of \( v_0 \) and \( g_0 \) explicitly without any extra computations and this is directly encoded in the matrix \( \mathcal{M} \) in Eq. (18). BA computes the covariance matrix implicitly,
- provides an estimate of the noise level on the feature points in Eq. (22). In BA one cannot explicitly estimate the noise level.
3.7 Accelerometer and Gyroscope Bias

Recall that we assume a calibrated device and any estimated offset has been removed from the IMU data. However, a small and slowly varying bias may still be present, while it can be modeled as a constant offset owing to the short integration time. For completeness, we show how the biases can be added.

As shown in [Martinelli 2013], a constant accelerometer bias can be modeled in a linear way. Such a bias can be likewise inserted into the solver of Eq. (7), that is, \( \dot{\kappa}_{ij} \) can be replaced by

\[
\dot{\kappa}_{ij} = \kappa_{ij} + \zeta_{ij} \mathbf{e}_a
\]

(30)

where

\[
\zeta_{ij} = \sum_{k=0}^{i-1} \beta_{ki} \mathbf{R}_k^0 \sum_{k=0}^{j-1} \beta_{kj} \mathbf{R}_k^0 \frac{\Delta \tau^2}{2},
\]

(31)

and \( \mathbf{e}_a \) denotes the accelerometer bias. It is straightforward to show that \( \zeta_{ij} \rightarrow \mu_{ij} \mathbf{I} \) when the system does not rotate. As a result, \( \mathbf{e}_a \) is not always identifiable, and separable from \( \mathbf{g}_0 \).

Instead, \( \kappa_{ij} \), \( \mathbf{p}_i \) and \( \mathbf{p}_j \) depend on the gyroscope bias in a non-linear way. The small bias magnitude let us though use a first-order approximation, that is, \( \kappa_{ij} \) can be now replaced by

\[
\dot{\kappa}_{ij} \simeq \kappa_{ij} + \frac{\partial \kappa_{ij}}{\partial \mathbf{e}_w} \mathbf{e}_w
\]

(32)

and likewise

\[
\dot{\mathbf{p}}_i \simeq \mathbf{p}_i + \frac{\partial \mathbf{p}_i}{\partial \mathbf{e}_w} \mathbf{e}_w,
\]

(33)

where \( \mathbf{e}_w \) is the constant gyroscope bias and \( \frac{\partial \mathbf{p}_i}{\partial \mathbf{e}_w}, \frac{\partial \mathbf{p}_j}{\partial \mathbf{e}_w} \) are the respective Jacobians. Note that the gyroscope bias directly affects the rotation, that is, biased gyroscope data is integrated in Eq. (12). In order to compute \( \frac{\partial \mathbf{x}}{\partial \mathbf{e}_w} \), some useful properties of the exponential map, see [Forster et al. 2017], can be used, thus leading to the following approximations

\[
\frac{\partial (\mathbf{R}_i^0 \mathbf{t}_i^0)}{\partial \mathbf{e}_w} \simeq -\mathbf{R}_i^0 [\mathbf{t}_i^0] \times \frac{\partial \mathbf{R}_i^0}{\partial \mathbf{e}_w}
\]

(34)

and

\[
\frac{\partial \left( \sum_{k=0}^{i-1} \beta_{ki} \mathbf{R}_k^0 \mathbf{a}_k \right)}{\partial \mathbf{e}_w} \simeq -\sum_{k=0}^{i-1} \beta_{ki} \mathbf{R}_k^0 [\mathbf{a}_k] \times \frac{\partial \mathbf{R}_i^0}{\partial \mathbf{e}_w},
\]

(35)

where

\[
\frac{\partial \mathbf{R}_i^0}{\partial \mathbf{e}_w} \simeq \sum_{k=0}^{i-1} \mathbf{R}_{k+1} \mathbf{J}_k \Delta \tau
\]

(36)

with \( \mathbf{J}_k \) being the right Jacobian of \( \mathbf{SO3} \) at \( \omega_k \) (see Eq. (8) in [Forster et al. 2017]). The notation \( [\cdot] \) denotes the skew symmetric matrix. Based on Eq. (31), the Jacobian \( \frac{\partial \mathbf{p}_i}{\partial \mathbf{e}_w} \) can be computed by

\[
\frac{\partial \mathbf{p}_i}{\partial \mathbf{e}_w} \simeq -\mathbf{J}_N \mathbf{R}_i^0 \mathbf{R}_N^{-1} \mathbf{u}_1, \frac{\partial \mathbf{R}_0^0}{\partial \mathbf{e}_w},
\]

(37)

where \( \mathbf{J}_N \) is the Jacobian of the transformation \( \mathcal{N}(\mathbf{x}) \) and is given by the first block of \( \mathbf{J}_N^{(i3)} \) in Eq. (29).

When both the biases need to be modeled, Eq. (30) can be combined with Eq. (32), while the cross dependence of biases can be ignored.

Adding biases into the renormalization scheme by involving the above equations is rather straightforward. In short, in case of the accelerometer bias, the matrix \( \mathbf{S} \) in Eq. (7) would contain three additional columns before the last column of \( \kappa \)'s. The solution vector \( \mathbf{x} \) would contain the unknown \( \mathbf{e}_a \). As entries into these columns do not depend on \( \mathbf{u}_i \), nothing substantial changes. In case of the gyroscope bias, three extra columns would be again added into the matrix \( \mathbf{S} \) and the unknown \( \mathbf{e}_w \) into the solution vector \( \mathbf{x} \). The entries into \( \mathbf{S} \) now depend on \( \mathbf{u}_i \), see Eq. (33), so the matrix \( \mathbf{B} \) in Eq. (12) has different form. Its partial derivative in Eq. (28) needs to take into account the derivative of the matrix \( \mathbf{S} \) as well. The Jacobian in Eq. (29) changes to size \( 10 \times 4 \). If both biases are considered, the size of the Jacobian is \( 13 \times 4 \).

[Kaiser et al. 2017] tested the robustness of [Martinelli 2013] against biased IMU readings. As far the accelerometer bias is concerned, when it is identifiable, the initialization remains unaffected. In particular, their experiments show that even large unrealistic bias magnitudes can be well compensated. Therefore, we only expect a minor refinement through the renormalization scheme.

On the contrary, the initializer may be affected from a gyroscope bias when its magnitude is relatively large and the integration time is long ([Kaiser et al. 2017]). However, the rolling-shutter camera allows short integration times and the initializer would not benefit much from modeling a gyroscope bias of low magnitude.

4 Experiments

4.1 Synthetic Data

In this section we perform quantitative synthetic analysis to investigate influence of noise on the final estimate of \( \mathbf{v}_0 \) and \( \mathbf{g}_0 \). In order to get realistic data with ground truth (GT) structure and poses, we process data from Snap Spectacles glasses with IMU BM160. States resulting from a Kalman filter on visual-inertial data play the role of GT states and high-order splines on
the IMU data provide ideal gyroscope and acceleration readings, such that a continuous integrator perfectly interpolates between the states. The IMU data are then sampled at 800Hz and finally, noise and time varying biases are added based on the calibrated variances of the used device. The device was moved forward 9 m along a straight trajectory with repeatedly changing viewpoint rotation from left to right. The shape of the trajectory and velocity can be seen in Fig. 5(c). We simulate a stereo camera with a baseline of 14 cm attached to the IMU. During the data acquisition the glasses were shortly static at the beginning such that we could safely initialize the IMU with the static motion assumption. It allows to integrate the signals to get the ground truth poses.

To produce the image correspondences, we generate 50 random feature points in the first left stereo image, assign them random depths in the range [1, 15] m and project the 3D points into the other views. We then perturb the feature points with Gaussian noise $\sigma = \{0, 0.1, \ldots, 0.5\}$ pixels, the accelerometer with standard deviation of 0.005 $\text{ms}^{-2}$ and the rotations $R^0_0$ computed from the gyroscope data with 0.02 $\text{deg}$ at random orientation. At each $\sigma$ we repeat 100 random realizations. The evaluated errors are defined as the norm on the velocity difference vector and the angle between the gravity vectors, i.e.

$$\epsilon_v = \|v_0 - v_0^{GT}\|_2,$$

$$\epsilon_g = \angle(g_0, g_0^{GT})$$

for one realization. We repeat the same procedure for each five-tuple of stereo images, which is slid along the whole sequence at 22 consecutive camera positions. In all the experiments, we use consecutive five stereo cameras at 10fps. To show the final statistics, at each $\sigma$ we compute mean and standard deviation.

Recall that the visual constraints are fed into the solver in Eq. (4) as image pairs, as shown for $ij, ik, kl$. Let us denote the cameras in the order: first left, first right, second left, second right and so on as $\{l_1, r_1\}, \{l_2, r_2\}, \{l_3, r_3\}, \{l_4, r_4\}, \{l_5, r_5\}$. Then we feed the following camera pairs into the matrix: $l_1 - r_2, l_1 - r_3, l_1 - r_4, l_1 - r_5, l_2 - r_3, l_2 - r_4, l_2 - r_5, l_3 - r_4, l_3 - r_5, l_4 - r_5$. We experimented with various combinations, and chose this as a trade-off between speed and accuracy. In case of a mono camera, the links would be between $l$ cameras only.

The results can be seen in Fig. 4. We compare three methods, (i) the Least Squares of Sec. 5.3, LS, (ii) the proposed renormalization of Sec. 3.5, RNM, (iii) Bundle Adjustment as ML with Levenberg-Marquardt, BA, detailed in Sec. 3.6, initialized by LS. Initializing BA by RNM rapidly speeds up the convergence, but does not improve the accuracy. For the two latter methods, RNM and BA, we can compute standard deviations of the estimated $v_0$ and $g_0$ from the theoretical covariance matrices. For RNM, see Eq. (22), for BA, see Eq. (A6.10) (Hartley and Zisserman (2004)). They both require knowing the noise level $\sigma$, see Eq. (38). To fairly compare, we set it to the ground truth $\sigma$ at which is the corresponding simulation performed. However, we confirmed that the estimated noise level $\sigma$ by the renormalization in Eq. (22) is very tight to the ground truth. As can be seen, the theoretical values are very well aligned to the empirical ones and can be well utilized in practice, to know how much to trust the final estimate.

As expected, LS is by far the worst estimation, fairly improved by the renormalization, and very slightly polished by ML. Moreover, renormalization returns estimate of the noise level of the feature detector / tracker. Due to perturbation with the ideal Gaussian noise, the minimization of the re-projection error is a perfect Max-
imum Likelihood estimate. Using real data, this might be slightly violated and the BA is not ML in its strict sense. We will see in the next that this may result in RNM sometimes outperforming BA.

4.2 Rendered Data

We perform qualitative comparison on realistic rendered image data, as this gives us perfect ground truth to compare to. We deploy Unreal Engine of Epic Games (2019) for rendering the images. We obtained the trajectories and the IMU data the same way as described in the previous section, for various types of walking trajectories of Snap Spectacles glasses. We simulate two virtual VGA rolling shutter cameras with noisy sensors, with the baseline of 14 cm, and readout time of 10 ms. As an input into the initializer, the IMU data is perturbed by Gaussian noise and biases on accelerometer and gyroscope with random walk noise. The features are detected by the FAST corners of Rosten et al. (2010) and further tracked by the ECC tracker of Evangelidis and Psarakis (2008). We use RANSAC with the minimal solver of Sec. 3.4 to prune outliers. To confirm the feasibility of the statistical assumption, we plot the error distribution of the tracked features which can be obtained through known depth values of the rendered images. As the Fig. 3 depicts, the distribution is Gaussian with subpixel accuracy.

We report quantitative results in Tab. 1 for the six sequences, shown in Fig. 4, Fig. 5 and Fig. 6. In Tab. 1 the "forward" trajectory is shown in Fig. 5 the "loop" trajectory in Fig. 6. The "fast shaking" trajectory is 0.5 m wide left-right shaking motion with rapid acceleration and average velocity of 0.9 ms⁻¹. The "for/backward" trajectory is 14 m straight forward, followed by 180° turn and back to the start with the average speed 1.8 ms⁻¹. We capture typical motions of a person when wearing smart glasses.

We present detailed qualitative results for two sequences. The first sequence, SUBWAYTRAIN is a forward 9 m long sequence inside a static subway train, see Fig. 5. The second sequence, TRAPCAM is a loop shaped 25 m long sequence outdoors, see Fig. 6.

The results confirm the observation from the Synthetic experiment that LS method can be improved by the renormalization RNM which is comparable and sometimes better to ML estimation of BA. In most cases, the initial velocity \( v_0 \) and gravity \( g_0 \) are both improved w.r.t. the LM, and this by roughly 20% and 8%, respectively. This is a significant improvement.

4.3 Real Data

We use real data from the Snap Spectacles glasses, as stereo images as well as IMU readings. We do not possess ground truth for these sequences. Instead, we run a typical VIO system based on the temporal Extended Kalman Filter, similar to Mourikis and Roumeliotis (2007). The filter framework fuses inertial and visual data in iterative updating procedure for maximum a posteriori probability of a linear dynamical system. The filter uses a strong prior that the sequences are static at the beginning, copes with a rolling shutter stereo camera and optimizes also for both accelerometer and gyroscope biases. For the proposed solver, though, we do not include the biases as we found that their magnitude is quite low in the used device.
The first is the OfficeLoop, a loop-shaped 25 m long sequence in a typical open space office. Since there is only negligible drift between the end and starting position, we can consider the used VIO as a reasonably accurate baseline to compare to. The second WALK is a forward 36 m long sequence outdoors. Both sequences are acquired during a walk.

The results align with the previous experiments; the renormalization RNM is similar to ML estimation of BA, both outperforming the Least Squares LS. As mentioned, we do not have the ground truth and the comparison for these sequences might not be representative. What should be noticed and taken from these results is that the renormalization and ML estimator perform very similar to each other. They, however, arrive to the final solution by different means.

5 Conclusion

We presented a novel way to solve the initialization problem of the inertial-visual odometry system. We derived a novel solver through proper statistical modeling and we cast the problem into the renormalization scheme of Kanatani. We incorporated proper noise propagation thus yielding a solution which exhibits higher accuracy over the original Least Squares solution. The extensive evaluation shows that the renormalization scheme performs very closely to the ML estimator which is statistically optimal in case of Gaussian noise. As such, the renormalization can serve to get good initial point for the ML, or fully replace it, as the additional improvement is rather small at the cost of higher computational cost.

With this paper, we extend the set of problems in Computer Vision which can be beneficially solved by the renormalization scheme. As the set of problems where the renormalization improves Gold Standards grows, the renormalization scheme is slowly finding its way into the Computer Vision community.

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Table 1 Quantitative results. Each column shows the name of the sequence, type of the motion, mean of the absolute distance error on the initial velocity / mean of the angular error on the gravity to the baseline Least Squares method LS in Eq. 35. For the RNM and BA improvements in percentage are shown.

| Sequence          | Type         | LS [m s⁻¹ / deg] | RNM [% / %] | BA [% / %] |
|-------------------|--------------|------------------|-------------|------------|
| SubwayTrain       | forward      | 0.15 / 0.125     | 15 / 5      | 15 / 33    |
| TrapCam           | loop         | 0.059 / 0.81     | 9 / 8       | 18 / 5     |
| StorageHouse      | fast shaking | 0.089 / 1.28     | 24 / 6      | 16 / 5     |
| SNESD                | loop         | 0.027 / 0.47     | 35 / 12     | 42 / 21    |
| SeaSideTown       | forward      | 0.057 / 1.01     | 16 / 15     | 8 / 17     |
| SpaceStation      | for/back-ward| 0.065 / 0.38     | 26 / 7      | 22 / 12    |

Fig. 6 TrapCam sequence. Left stereo image (a) at the start and (b) in the middle of the sequence. See Fig. 5 for the remaining caption.
Fig. 7 Example images of the rendered sequences.

(a) (b)

Fig. 8 OfficeLoop sequence. Left stereo image (a) at the start and (b) in the middle of the sequence. Trajectory is the same as in Fig. 6(c). See Fig. 5 for the remaining caption.

(c)

(d)

Fig. 9 Walk sequence. Left stereo image (a) at the start and (b) in the middle of the sequence. See Fig. 5 for the remaining caption.

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