A research on short-term hydro-wind economic dispatch problem simulated with a modified differential evolution algorithm

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Abstract. Due to the wide construction of wind power and the difficulty for it to join the power grid, a short-term hydro-wind economic dispatch (WHED) problem is proposed. WHED system contains several wind power units and hydropower plants, which are renewable and clean. Combined with hydropower plants, the wind power units can join the power grid stably. Then, a WHED system with four cascaded hydropower plants and two wind units is established, and a modified differential evolution (DE) algorithm with chaotic perturbation is proposed for optimizing. Finally, two cases are simulated and analysed, the dispatch results show that the presented model and algorithm are feasible and effective.

1. Introduction
In recent years, wind power has been greatly developed[1] and is expected to join the power grid. However, the indeterminacy of wind power may cause power supply fluctuation, even break the power load balance, which makes it difficult for wind power generators feeding electricity to the power grid solely. Hydropower is a widely applied clean energy source in countries with rich water resource. Because the hydropower is stable, controllable and with the ability of peak load regulation, it is compatible that wind power units and hydropower plants join the power grid cooperatively.

In order to arrange the daily operation of wind power and hydropower properly, a short-term hydro-wind economic dispatch (WHED) problem is proposed in this paper. Then, a modified algorithm based on differential evolution[2] (DE) is given to solve the presented problem. Next, a WHED system with 4 hydropower plants and 2 wind power units is built. Finally, two cases are studied to verify the practicability of WHED model and the validity of modified DE, then the optimal results of hydropower and wind power generators are obtained.

2. Problem formulation
The WHED problem is aimed at optimizing generating process, which is formed of hydro discharge flows and planned wind power outputs. This generating process can realize minimum of the total economic cost of the system, while the randomness of wind power and constraints of the system are considered.

2.1. The description of wind power
The uncertainty of wind velocity is the first point with which we should deal. The wind power output relies on the wind velocity, and the stochastic characteristic of wind is commonly described by Weibull distribution[3]. The probability density function (PDF) of Weibull is as follows:

\[ f_w(v) = \left(\frac{k}{c}\right) v^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right], \quad (v > 0) \tag{1} \]

where \( c \) and \( k \) are the radio parameter and scale parameter, respectively; \( v \) is the wind velocity. On the basis of wind velocity PDF, the cumulative distribution function (CDF) of wind velocity \( F_w(v) \) can be described as follows:

\[ F_w(v) = 1 - \exp\left[-\left(\frac{v}{c}\right)^k\right] \tag{2} \]

The wind power output is determined by wind status. For usability, a commonly used liner model[4] is applied to describe the relationship between output and velocity of wind power, which is given as follows:

\[ w = \begin{cases} 
0 & (v < v_m \text{ or } v \geq v_{out}) \\
\frac{w_r}{v_r} \cdot \left(\frac{v-v_m}{v_r-v_m}\right) & (v_r \leq v < v_m) \\
w_r & (v_r \leq v < v_{out})
\end{cases} \tag{3} \]

where \( v_r, v_m \) and \( v_{out} \) are the rated, cut-in and cut-out wind speeds, respectively; \( w_r \) is the installed output of wind unit; \( w \) is the output, and is continuous in \([0, w_r]\).

In terms of Eq. (3), when the wind velocity is in the region of \([v_m, v_r]\), Eq. (1) can be formulated as follows:

\[ F_w(w) = \frac{kp_{w_m}}{c w_r} \left(1 + \frac{pw / w_r - w_{out}}{w_r - w_m}\right)^{k-1} \exp\left[-\left(\frac{1 + pw / w_r - w_{out}}{w_r - w_m}\right)^k\right] \tag{4} \]

where \( p = v_r/v_m - 1 \).

As we can see that, Eq. (4) is the continuous part. For the incontinuity parts, \( w = 0 \) and \( w = w_r \), the expressions are formulated as follows:

\[ P_r(w = 0) = P_r(v < v_m) + P_r(v > v_{out}) \]
\[ = 1 - \exp\left[-\left(\frac{v_m}{c}\right)^k\right] + \exp\left[-\left(\frac{v_{out}}{c}\right)^k\right] \tag{5} \]

\[ P_r(w = w_r) = P_r(v_m \leq v \leq v_{out}) \]
\[ = \exp\left[-\left(\frac{v_r}{c}\right)^k\right] - \exp\left[-\left(\frac{v_{out}}{c}\right)^k\right] \tag{6} \]

By the comprehensive of Eqs. (4), (5) and (6), the CDF of \( w \) is derived as follows:

\[ F_w(w) = \begin{cases} 
0 & (w < 0) \\
\frac{kp_{w_m}}{c w_r} \left(1 + \frac{pw / w_r - w_{out}}{w_r - w_m}\right)^{k-1} \exp\left[-\left(\frac{1 + pw / w_r - w_{out}}{w_r - w_m}\right)^k\right] & (0 \leq w < w_r) \\
1 & (w_r \leq w)
\end{cases} \tag{7} \]

2.2. Description of wind power cost

The indeterminacy of wind velocity causes the randomness of wind power, and there will be a gap between the planned and actual output, which will arouse extra expense. In the actual situation, if the power grid obtain less electricity from wind power than planned, it is known as overestimation, and extra cost of backing up power should be added to balance the power load. On the other hand, if the
power grid obtain more electricity from wind power, it is known as underestimation, which leads to electricity waste and incomplete usage of wind power investment, then the wind power owner should be compensated. Based on the gap between the actual and planned output, this paper utilizes a sort of cost computational formulas[3] to calculate the excess cost which is caused by wind indeterminacy. The cost of overestimation for nth wind power generator at rth interval is calculated as:

$$E_{\text{over}}^{n,r} = c_{\text{over}}^{n} \cdot \left[ w_{n,r} \left[ 1 - \exp \left( -\frac{v_{\text{in},n}}{c_n} \right)^{k_n} \right] + \exp \left( -\frac{v_{\text{out},n}}{c_n} \right)^{k_n} \right] + \frac{w_{r,n} v_{\text{in},n} - w_{n,r}}{v_{r,n} - v_{\text{in},n}} \left[ \exp \left( -\frac{v_{\text{in},n}}{c_n} \right)^{k_n} - \exp \left( -\frac{v_{\text{out},n}}{c_n} \right)^{k_n} \right] + \frac{w_{r,n} c_n}{v_{r,n} - v_{\text{in},n}} \left[ \Gamma \left( 1 + \frac{1}{c_n}, \frac{v_{\text{in},n}}{c_n} \right)^{k_n} - \Gamma \left( 1 + \frac{1}{c_n}, \frac{v_{\text{out},n}}{c_n} \right)^{k_n} \right]$$

(8)

The underestimation cost is formulated as:

$$E_{\text{under}}^{n,r} = c_{\text{under}}^{n} \cdot \left[ w_{n,r} \left[ \exp \left( -\frac{v_{\text{in},n}}{c_n} \right)^{k_n} - \exp \left( -\frac{v_{\text{out},n}}{c_n} \right)^{k_n} \right] + \frac{w_{r,n} v_{\text{in},n} - w_{n,r}}{v_{r,n} - v_{\text{in},n}} \left[ \exp \left( -\frac{v_{\text{in},n}}{c_n} \right)^{k_n} - \exp \left( -\frac{v_{\text{out},n}}{c_n} \right)^{k_n} \right] + \frac{w_{r,n} c_n}{v_{r,n} - v_{\text{in},n}} \left[ \Gamma \left( 1 + \frac{1}{c_n}, \frac{v_{\text{in},n}}{c_n} \right)^{k_n} - \Gamma \left( 1 + \frac{1}{c_n}, \frac{v_{\text{out},n}}{c_n} \right)^{k_n} \right] \right]$$

(9)

where $E_{\text{over}}^{n,r}$ and $E_{\text{under}}^{n,r}$ are costs of overestimation and underestimation respectively; $c_{\text{over}}^{n}$ and $c_{\text{under}}^{n}$ are cost coefficients; $v_{\text{in},n}$, $v_{\text{in},n}$ and $v_{\text{out},n}$ are the rated, cut-in and cut-out wind velocities of nth wind power unit; $W_{r,n}$ is installed wind output; $w_{n,r}$ is wind power output; $c_n$ and $k_n$ are the scale factor and shape factor.

On the basis of Eq.(8) and (9), the total wind power cost in the scheduling period can be expressed as follows:

$$E_W = \sum_{n=1}^{N_n} \sum_{r=1}^{N_r} \left[ E_{\text{over}}^{n,r} + E_{\text{under}}^{n,r} \right]$$

(10)

where $E_W$ is the total wind power cost; $N_W$ is the number of wind power units.

2.3. Objective function and constraints

The WHED problem is aimed at minimizing the total generation cost. The hydropower plants can control the discharge flow by means of reservoirs, and the rainfall and hydrological forecasts within 24 hours are of high accuracy, so there is no extra cost in hydropower generation. Therefore, the total cost of WHED system is the wind power cost, which can be expressed as follows:

$$\min E_{\text{over}} = \min E_W$$

(11)

The output of the system includes two parts: wind power and hydropower. The wind power is discussed above, and a commonly calculation mode for hydropower output is provided as follows:

$$p_{m,i} = \alpha_{1,m} \times (v_{m,i})^2 + \alpha_{2,m} \times (Q_{m,i})^2 + \alpha_{3,m} \times v_{m,i} \times Q_{m,i} + \alpha_{4,m} \times v_{m,i} + \alpha_{5,m} \times Q_{m,i} + \alpha_{6,m}$$

(12)
where $p_{m,t}$ is the power output of the $mth$ hydropower plant at the $tth$ interval; $\alpha_{1,m}$, $\alpha_{2,m}$, $\alpha_{3,m}$, $\alpha_{4,m}$, $\alpha_{5,m}$ and $\alpha_{6,m}$ are the power generation coefficients; $V_{m,t}$ and $Q_{m,t}$ are the storage volume and discharge flow of the hydropower plant.

The optimization of WHED problem should satisfy several constraints to meet the practical conditions, including 8 constraints[5]: system load balance, hydropower output limits, wind power output limits, reservoir storage volumes limits, water discharge limits, water dynamic balance constraints, initial and final reservoir storage volumes constraints.

3. Modified DE algorithm

DE is a commonly used evolutionary algorithm[2], and it can well handle non-linear optimization problems with single objective[6]. DE has been widely put into the use of electrical load dispatch problems[7, 8], and the results were well. In this paper, the chaotic perturbation[9] are utilized to improve the original DE algorithm, and the flow diagram is shown as Fig. 1.

**Figure 1.** The flow diagram of modified DE algorithm

4. Case study

In this section, a WHED system is built, and the modified DE is applied to solve the problem, then, the optimal generation process of hydropower plants and wind power units are obtained.

4.1. The HWED system

In order to verify the practicability of WHED model and the validity of modified DE, a HWED system is built. The system includes 4 hydropower plants and 2 wind power units, the schematic diagram is given in Fig. 2. The detailed parameters of hydropower plants are taken from Ref. [10], and the Weibull distribution parameters are adopted by reference to Ref. [3]. The performance period contains 24 hours. There are two cases are simulated, and the difference of discharge flow limits are given in Table 1.
Wind power

\[ f(v) = \frac{k}{c}(v/c)^{k-1} \exp[-(v/c)^k] \]

Load

Hydro plants

Figure 2. The hydro-wind power system

Table 1. Minimum and maximum discharge flows of hydro plants (m\(^3\)/s).

| case | \( H_1 \) | \( H_2 \) | \( H_3 \) | \( H_4 \) |
|------|-----------|-----------|-----------|-----------|
|      | minimum   | maximum   | minimum   | maximum   | minimum   | maximum   |
| 1    | 0         | 15        | 0         | 15        | 0         | 30        |
| 2    | 5         | 15        | 10        | 30        | 10        | 20        |

4.2. Simulation results and analysis

The individuals of the algorithm include the hydropower discharge flows and wind power outputs. The individual can be expressed as follows:

\[
\text{Ind} = \begin{bmatrix}
    w_{1,1} & w_{1,2} & \cdots & w_{1,n_H} & Q_{1,1} & Q_{2,1} & \cdots & Q_{N_W,1} \\
    w_{2,1} & w_{2,2} & \cdots & w_{2,n_H} & Q_{1,2} & Q_{2,2} & \cdots & Q_{N_W,2} \\
    \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    w_{n_H,1} & \vdots & w_{n_H,n_H} & Q_{1,n_H} & Q_{2,n_H} & \cdots & Q_{N_W,n_H} \\
\end{bmatrix}
\]

(13)

The two cases are simulated, and the convergence curves of minimum cost are depicted in Fig. 3. It can be seen that, the cost of the two cases are both converged fast in first 500 iterations, and become slow when it reaches 1200 iterations, the cost performs to be stabilization, which means the algorithm reaches the optimization. In Fig. 3, case 1 can find a cost of 18098.09$, which is lower than case 2 by 1250.33$. It is due to the loose limits of case 1 which can make the optimizing easier.
Figure 3. The convergence curves of minimum cost of case 1 and case 2

Figure 4. Comparison of the hydro outputs between case 1 and case 2

The outputs of hydro plants and wind units in case 1 and 2 are shown in Fig. 5 and Fig. 6 respectively, the comparison of the hydro outputs between the two cases are given in Fig. 4. The detailed optimal hydro-wind power outputs and discharge flows in case 1 and 2 are presented in Fig. 5 and Table 2 respectively. From Fig. 5 and Table 2, we can see that the hydro plants undertake the peak load regulation to make up for the load balance of the power grid, in other words, the hydropower plants are effective by compensating load to the wind power units. It can be seen in Fig. 4, the hydro plants of case 1 generate fewer electricity than those of case 2 when the system load is low, and undertake more load with high system load. The wind power can join the power grid smoothly in company with hydropower, the higher minimum discharge flow in case 2 can increase the average output of hydropower and undertake more parts of the base-load, which can improve the robustness of the power grid, but this will inevitably increase the cost of the power system. The limits of the hydro discharge flow should be determined according to the actual conditions.

Figure 5. The hydro and wind power outputs in case 1

Figure 6. The hydro and wind power outputs in case 2

Table 2. The detailed optimal results of case 1.

| Hours | Hydro output (MW) | Discharge flow (m³/s) | Wind output (MW) |
|-------|-------------------|----------------------|-----------------|
|       | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $w_1$ | $w_2$ |
| 1     | 76.58  | 49.01  | 19.66  | 131.88 | 8.24  | 6.00  | 24.59  | 6.00  | 51.08  | 51.79  |
| 2     | 81.81  | 50.18  | 29.14  | 129.03 | 9.05  | 6.00  | 22.12  | 6.00  | 54.94  | 54.91  |
| 3     | 81.46  | 51.31  | 30.73  | 125.74 | 8.99  | 6.00  | 20.82  | 6.00  | 55.59  | 55.16  |
| 4     | 63.07  | 52.94  | 21.63  | 121.63 | 6.26  | 6.00  | 21.85  | 6.00  | 45.23  | 45.50  |
| 5     | 72.83  | 54.51  | 35.57  | 115.82 | 7.58  | 6.00  | 18.53  | 6.00  | 50.55  | 50.72  |
| 6     | 78.57  | 55.51  | 38.78  | 133.10 | 8.56  | 6.00  | 17.57  | 6.00  | 56.84  | 57.20  |
| Hours | Hydro output (MW) | Discharge flow (m³/s) | Wind output (MW) |
|-------|------------------|-----------------------|------------------|
|       | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $w_1$ | $w_2$ |
| 1     | 69.31 | 41.60 | 0.03  | 181.00| 7.13  | 5.00  | 27.15 | 10.00| 44.05 | 44.01 |
| 2     | 80.96 | 43.29 | 0.08  | 173.72| 8.83  | 5.00  | 26.20 | 10.00| 50.94 | 51.01 |
| 3     | 81.70 | 44.91 | 3.22  | 165.69| 8.96  | 5.00  | 24.76 | 10.00| 52.23 | 52.25 |
| 4     | 60.23 | 46.95 | 0.06  | 156.42| 5.86  | 5.00  | 24.28 | 10.00| 43.16 | 43.18 |
| 5     | 73.08 | 48.87 | 14.14 | 144.83| 7.55  | 5.00  | 21.52 | 10.00| 49.67 | 49.41 |
| 6     | 73.42 | 50.22 | 26.26 | 164.34| 7.66  | 5.00  | 18.68 | 10.00| 53.05 | 52.71 |
| 7     | 83.75 | 68.55 | 33.64 | 181.15| 9.46  | 7.41  | 16.03 | 10.00| 66.29 | 66.61 |
| 8     | 85.69 | 77.51 | 33.12 | 195.10| 9.97  | 9.00  | 16.14 | 10.00| 74.23 | 74.35 |
| 9     | 86.70 | 79.20 | 32.93 | 245.80| 10.27 | 9.49  | 15.97 | 13.59| 77.98 | 77.40 |
| 10    | 85.85 | 78.38 | 30.86 | 273.13| 10.11 | 9.49  | 16.61 | 15.81| 75.63 | 76.15 |
| 11    | 82.40 | 72.78 | 29.38 | 277.54| 9.32  | 8.52  | 17.31 | 16.02| 68.72 | 69.18 |
| 12    | 84.01 | 76.37 | 30.20 | 278.53| 9.46  | 9.11  | 17.47 | 16.14| 70.63 | 70.26 |
| 13    | 82.33 | 74.41 | 30.18 | 277.08| 9.11  | 8.86  | 17.99 | 15.97| 67.89 | 68.11 |
| 14    | 82.39 | 75.00 | 33.87 | 282.35| 9.01  | 9.05  | 17.56 | 16.61| 68.08 | 68.31 |
| 15    | 83.49 | 76.83 | 36.84 | 287.85| 9.06  | 9.40  | 17.10 | 17.31| 67.53 | 67.46 |
| 16    | 82.40 | 77.09 | 39.83 | 289.03| 8.78  | 9.49  | 16.65 | 17.47| 65.89 | 65.75 |
| 17    | 82.34 | 78.86 | 43.21 | 292.86| 8.72  | 10.01 | 15.80 | 17.99| 66.53 | 66.21 |
| 18    | 83.35 | 80.10 | 45.32 | 298.01| 8.88  | 10.66 | 15.54 | 18.73| 66.42 | 66.80 |
| 19    | 72.54 | 73.37 | 46.40 | 291.86| 7.26  | 9.83  | 15.81 | 18.01| 57.97 | 57.86 |
| 20    | 71.32 | 75.10 | 48.96 | 299.91| 7.09  | 10.60 | 15.04 | 19.35| 57.45 | 57.25 |
| 21    | 73.46 | 77.19 | 52.00 | 300.90| 7.42  | 11.56 | 11.23 | 19.97| 58.12 | 58.34 |
| 22    | 53.93 | 62.99 | 54.04 | 289.13| 5.00  | 8.81  | 11.17 | 18.77| 45.16 | 44.76 |
| 23    | 54.31 | 61.55 | 55.91 | 285.05| 5.00  | 8.52  | 11.40 | 18.66| 41.40 | 41.78 |
| 24    | 55.64 | 77.40 | 58.00 | 289.98| 5.10  | 12.20 | 12.28 | 20.00| 49.63 | 49.36 |

Table 3. The detailed optimal results in case 2.
5. Conclusions
The modified DE algorithm has been successfully applied to solve the WHED problem. To enhance diversity of DE, the chaotic perturbation has been added. Two cases of the daily scheduling problem with 4 cascaded hydropower plants and 2 wind units are provided to verify the performance of the modified DE for solving the WHED problem. As seen from the numerical simulation, the higher minimum discharge flow limit can improve the robustness of the power grid, but will inevitably increase the cost. As it turned out that the model and algorithm presented above are feasible and effective.

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