PAPER

An electric concentrator and thermal cloaking device

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Abstract

The concentration and cloaking phenomena of physical fields in Metamaterials has captured the attention of the researchers due to their simplified approaches. However most of the work conducted is focussed on controlling single physical field. Transformation optics has paved the way for developing intelligent bifunctional devices. Bifunctional devices are such controlled devices which execute two different physical functions simultaneously and independently. In this work we have applied the transformation optics theory to design a multilayered two dimensional spherical bifunctional device which behaves like an electric concentrator and thermal invisibility cloak simultaneously. Moreover, we have also observed the normalized behavior of the proposed device. The simulation performance confirms the feasibility of our suggested model.

1. Introduction

Metamaterials are the man-made artificial structures designed under the umbrella of effective medium theory [1] with a wide range of controlled applications [1–20]. From the last decade Metamaterials with transformation optics has attracted interest from researchers due to their exotic properties [5–10]. By the virtue of transformation optics in Metamaterials researchers and engineers have designed the inconceivable electrical and thermal devices exhibiting physical properties like sensing [11], concentration, invisibility cloaking, camouflage [12], controlling Surface Plasmon Polariton (SPP) [13, 14], plasmonics and nanofabrication techniques [15], nano-structured planar lens [16], waveguides [17–19].

Although triggering the properties of single physical phenomenon in metamaterials has shown good progress, but in the recent years bifunctional metamaterials have achieved more success due to simultaneous exhibition of independent physical properties of different physical functions [6–9]. Various scientific groups have successfully demonstrated theoretically and experimentally the controlled devices by manipulating different physical functions [20–28].

Our motivation in this work is to propose a two-dimensional spherical bifunctional device with the help of transformation of coordinates which excites the concentrating and cloaking properties simultaneously by manipulating electric and heat fluxes. We have extended the work on this novel study by proposing a spherical model of the multi-shell structure which simultaneously behaves as an electrical concentrator and thermal cloak. Our model is comprised of three regions of the sphere; inner region is set as vacuum, the region-I and region-II work as an electrical concentrator and thermal cloak respectively and hence a bifunctional device is suggested.

2. Mathematical model of two-dimensional spherical structure

We consider a model of two-dimensional spherical structure composed of two concentric shells having the inner region has radius $R_0$. The region-I, we may call it as first shell lies between radii $R_0$ and $R_1 > R_0$, and the region-II lies between the radii $R_1$ and $R_2 > R_1$. The region-I has the thermal conductivity $\kappa_1$ and electrical conductivity $\sigma_1$. In the region-II thermal and electrical conductivities are known as $\kappa_2$ and $\sigma_2$ respectively. While the thermal
conductivities of the second shell and background materials are kept uniform. This scheme is well depicted in the figure 1.

Laplace’s equation is a second order partial differential equation which is widely used to describe the accurate behavior for the solutions of electrical and thermal problems. Laplace’s equation is used to provide a bridge between physical and virtual domains. It helps in the calculation of materials parameter of TO devices using suitable boundary conditions [29]. Here we use the Laplace equation in spherical coordinates to observe the flow of electrical and thermal conditions. We may model the Laplace equation for electrical flow as

\[ \nabla \cdot (\sigma \nabla V) = 0, \]

where the symbol \( \sigma \) stands for electrical conductivity and the letter \( V \) is representing the potential difference. Similarly, thermal flows can be represented by the equation \( \nabla \cdot (\kappa \nabla T) = 0, \) with the symbol \( \kappa \) representing the thermal conductivity and \( T \) stands for absolute temperature of the model. The general solution of Laplace equations as discussed above in spherical coordinates satisfying the given parameters can be expressed as [30]:

\[
\Phi_i = \sum_{m=1}^{\infty} [A^i_m r^m + B^i_m r^{-m-1}] P_m \cos \theta,
\]

where \( \Phi_i \) represents the electric(heat) conduction in the region \( i \), \( A^i_m \) and \( B^i_m \) are the coefficients of the mth order Legendre polynomial \( P_m \) and can be resolute by using the boundary conditions and \( \theta \) is the angle due to the applied field direction.

Here we apply temperature (voltage) distribution with uniform temperature (voltage) gradient \( \Phi_b \) applied externally. Taking into account that \( \Phi_b \) should tend to \(- \Phi_b r \cos \theta \) when \( r \to \infty \), we only need to consider \( m = 1 \) as discussed in the [25, 31].

We modify for sphere as:

\[
\begin{cases}
\Phi_i \bigg|_{r=R_a} = \Phi_{i+1} \bigg|_{r=R_a} \\
\kappa_i \left. \frac{\partial T_i}{\partial r} \right|_{r=R_a} = \kappa_{i+1} \left. \frac{\partial T_{i+1}}{\partial r} \right|_{r=R_a} \\
\sigma_i \left. \frac{\partial V_i}{\partial r} \right|_{r=R_a} = \sigma_{i+1} \left. \frac{\partial V_{i+1}}{\partial r} \right|_{r=R_a}
\end{cases}
\]

where \( \Phi_i \) is the heat or electric conduction, \( T_i \) is the absolute temperature, \( \kappa_i \) is the thermal conductivity, \( V_i \) is the electric potential and \( \sigma_i \) is the electrical conductivity in the region \( i \).

Setting \( i = 1, 2, 3, 4, m = 1 \) and since \( p_1 = 1 \), equation (1) becomes:

\[
\begin{align*}
\Phi_1 &= A_1^0 r \cos \theta \text{ for the region } 0 < r < R_0 \\
\Phi_2 &= \left[ A_1^0 r + \frac{B_1^0}{r} \right] \cos \theta \text{ for } R_0 < r < R_1 \\
\Phi_3 &= \left[ A_1^0 r + \frac{B_1^0}{r} \right] \cos \theta \text{ for } R_0 < r < R_2
\end{align*}
\]

Figure 1. Schematic diagram of Bifunctional device.
\[ \Phi_2 = \left[ -\Phi_2 r + \frac{B_i^2}{r^2} \right] \cos \theta \text{ for } R_2 < r \]  

(3d)

To avoid any ambiguity, we simplify the coefficients as: \( A_i = A \), \( A_i' = C \), \( B_i = D \), \( A_i = E \), \( B_i = F \), \( B_i' = G \).

Now by using the boundary conditions from equation (2) in equation (3), we have the following two systems of equations for thermal and electrical conductivities respectively:

\[
\begin{align*}
AR_0^2 - CR_0^3 - D &= 0 \\
CR_i^3 - ER_0^3 + D - F &= 0 \\
ER_0^3 + \Phi_0 R_0^3 + F - G &= 0 \\
\kappa_1 CR_0^3 - 2\kappa_1 D &= 0 \\
\kappa_1 CR_i^3 - 2\kappa_1 D - \kappa_2 ER_0^3 + 2\kappa_2 F &= 0 \\
\kappa_2 ER_0^3 - \kappa_2 F - \kappa_3 \Phi_0 R_0^3 + \kappa_3 G &= 0
\end{align*}
\]

(4)

and

\[
\begin{align*}
AR_0^2 - CR_0^3 - D &= 0 \\
CR_i^3 - ER_0^3 + D - F &= 0 \\
ER_0^3 + \Phi_0 R_0^3 + F - G &= 0 \\
\sigma_1 CR_0^3 - 2\sigma_1 D &= 0 \\
\sigma_1 CR_i^3 - 2\sigma_1 D - \sigma_2 ER_0^3 + 2\sigma_2 F &= 0 \\
\sigma_2 ER_0^3 - \sigma_2 F - \sigma_3 \Phi_0 R_0^3 + \sigma_3 G &= 0
\end{align*}
\]

(5)

Solving the systems of equations (4) and (5) for constant coefficients and after tedious manipulation as in the [25], we get the concentration and cloaking conditions for the shell 1 and shell 2 respectively:

\[
\begin{align*}
\sigma_1 &= \frac{2R_0^3 + R_i^3}{2(R_0^3 - R_i^3)} \sigma_b \\
\kappa_1 &= \frac{2R_0^3 + R_i^3}{2(R_0^3 - R_i^3)} \kappa_b
\end{align*}
\]

(6)

\[
\begin{align*}
\sigma_2 &= \frac{2R_0^3 + R_i^3}{2(R_0^3 - R_i^3)} \sigma_b \\
\kappa_2 &= \frac{2R_0^3 + R_i^3}{2(R_0^3 - R_i^3)} \kappa_b
\end{align*}
\]

(7)

The suggested technique is simple and efficient in designing the bifunctional cloak. We have simulated this theoretical model in two cases: without inducting an object in the inner region and with an object in the inner region. Simulations are performed using COMSOL MULTIPHYSICS software. The bottom and upper surfaces of the model are set as insulators, whereas left boundary of the proposed model is used as source and right boundary as sink for both electrical and thermal cases.

3. Electric concentrator

Near-field concentration plays a vital role in binding the energy into a device. Electric concentrators are devised to enhance electric field without distorting the electric field lines outside the concentration region. An ideal concentrator concentrates the electric field into a focused small area without disturbing the external field [27]. Transformation optics through the equations (6) and (7) guide the electric and heat flux to concentrate and cloak respectively in a small region.

Based on the above theory we designed the device simultaneously functioning as concentrator and invisibility cloak. Here we first discuss the Electric concentrator model. In our theoretical model we take electrical conductivity as \( \sigma_b = 111 \text{[S/m]} \) and thermal conductivity as \( \kappa_b = 1250 \text{[W/(m.K)]} \) of the background material. Geometrical parameters are set as \( R_0 = 15 \text{ m} \), \( R_i = 16.5 \text{ mm} \) and \( R_2 = 18 \text{ mm} \). Our proposed model is designed in a way that in the region-I we calculate the thermal conductivity \( \kappa_1 \) and electrical conductivity \( \sigma_1 \) by analytical approach through the equation (6). Then for the region-II we calculate the electrical conductivity \( \sigma_2 \) and thermal conductivity \( \kappa_2 \) from equation (7). This scheme performance demonstrates an efficient bifunctional concentrator and cloak based on the electrical and thermal physical functions simultaneously.
From the figure 2(a) we can observe that electric potential contour is smooth before entering the internal region, then the lines are focused inside the concentration region to enhance the electrical potential, and ultimately become smooth coming out of the internal region without creating any distortion in their original paths. The black stream lines are representing the current density cloaking this specific region without any distortion in the field outside. In the second case we have inducted an elliptical copper object of semi-minor axis $a = 25$ mm and semi-major axis $b = 5$ mm into the cloaked region. In the figure 2(b) we can better perceive the concentration while an object is hidden in the region. The voltage lines concentrate around the hidden object. While coming out of the vacuum shell the voltage lines become smooth.

4. Thermal cloaks

To perform the simulations for thermal cloaking device we keep the geometry of the model same as in the case of electrical concentrator. While we set thermal parameters to the left boundary of the model at temperature 393 K and to the right boundary at the temperature 293 K. Top and bottom surfaces are acting like thermal insulators. Simulating our model for the cases without hidden object and with hidden object, we observed an efficient thermal cloak as shown in the figures 3(a), (b). Heat fluxes can be clearly observed travelling in a smooth way bending around the cloaked region and then attaining the smooth position on their original path. We can observe a few thermal signatures concentrated into the cloaked region, but no distortion is observed in heat fluxes while bending around the cloaked region.

Also, it is clear from the figure 3(b) that the object hidden inside the cloaked region remains unidentified by the heat flux traveling around the vacuum shell. Hence a good cloaking performance is shown by our theoretical model without any minor disturbance.

5. Point source excitations

Point source excitations are performed to validate the theoretical results in the absence of experimental verification. Invisibility cloak simulations performed are based on the homogeneous excitations. We perform this test and results show the good concentrating and cloaking performance in relevance to our previous results of section-I and section-II. We placed a point source near the spherical structure generating the electric and thermal waves as shown in figure 4. Here, the figures 4(a) and (b) represent the electrical concentration and thermal cloaking cases respectively. The normalized voltage and temperature are assigned to point source material. From the figure 4 we can clearly observe the spherical wave patterns generating from the point source in both electric and thermal cases. Figure 4 shows the flow of both types of waves passing outside the
cloaking regions while perfect concentration and cloaking phenomenon can be observed very clearly. We observed an efficient concentrator and perfect cloaking without any disturbance and distortion in waves pattern. Moreover, we have embedded an elliptical object of minor axis of $a = 25$ mm and major axes $b = 5$ mm of copper to observe the thermal cloaking effects. It can be clearly observed from the figure 4(a) the electric potential contours are concentrated inside the vacuum region and from figure 4(b) that no wave can enter the region hiding the object.

6. Conclusion

We have theoretically investigated the bifunctional device through the theory of transformation optics. This device exhibits some novel behavior electrically concentrating and thermally cloaking simultaneously. This work provides the methodology to control the electric and thermal currents at will. However, observing the point source excitation, we noted that the potential energy concentrates into the vacuum shell but could not enter the hidden material while during thermal cloaking no heat flux entered the cloaked region and hence the hidden material could not be detected. This type of devices could be used to isolate an object from external temperature fluctuations by accurately controlling its voltage or to implement a temperature independent voltage sensor.

Figure 3. In (a), A thermal cloak can be observed. Temperature is set from 293 K to 393 K. Black lines represent the conductive heat flux and gray lines represent the temperature contour. In (b) an elliptical copper material is hidden and is cloaked by the device.

Figure 4. Point source test verifying the bifunctional performances of electrical concentrator (a) and heat cloak (b).
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