Determination of the bearing and roll angle of a moving object using orthogonally elliptically polarized beacon signals received in a circular polarization basis

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Abstract. The possibility of determining bearing and roll angle of a mobile object along orthogonally elliptically polarized beacon signals emitted simultaneously from two spatially separated points in a horizontal plane is investigated. The navigation elements are evaluated by a two-channel receiving system using amplitude-phase processing of the resulting vector signals received on board a mobile object in a circular polarization basis.

1. Introduction
In practical navigation to find the moving object (MO) bearing angle there are usually used amplitude, frequency and time beacon signal characteristics [1, 2]. The roll angle is measures by rather expensive inertial navigation means [3, 4]. Vector properties of the signals in such cases are not considered [5, 6].

In [7] there were investigated cases when the MO bearing angle was measures using beacon circular orthogonal polarized signals. The bearing angle was found at the output of the two canal receiver after amplitude-polarization phase-polarization processing of the vector signals received either in the circular or linear bases.

In [8–10] there was investigated possibility to find simultaneously the MO bearing angle and the angle of roll using linear orthogonal polarized signals [8] and orthogonal elliptically polarized ones [10]. The bearing and roll angles also was measured by the two canal receiver after phase-polarization processing of the vector signals received either in the circular [8, 9] or linear [10] polarization bases.

The correct choice of the polarization basis depends first on the technical realization simplicity and second on the math relations between the MO bearing and roll angles and amplitudes and phases of the orthogonally polarized signals in both bases.

In reality in onboard receiving antennas, more often are used linear or circular polarization basis [8–10]. In [10] there was investigated possibility to use orthogonal elliptically polarized signals to measure the MO bearing and roll angles. Resulting vector signals were received onboard MO in the linear polarized basis. It was found that it is impossible to measure simultaneously and independently the bearing angle and the angle of roll - needed additional information about one of them.

In this paper there is investigated the case of the beacon orthogonal elliptically polarized signals used to measure the MO bearing and roll angles with onboard two-canal receiving system processing amplitudes and phases of vector signals received in the circular polarization basis.
2. The problem

Let's suggest that a beacon emits simultaneously from two points diverse by distance $d$ orthogonal elliptically polarized signals with equal amplitudes, phases, wave length $\lambda$ and equal ellipticity angles $\varepsilon$.

Let's use Jones vector to describe the flat homogeneous, elliptically polarized electromagnetic wave [11]. The resulting wave in the direction $\alpha$ can be written in the linear polarization basis (LPB) in the vector form (time dependence is neglected):

$$
\mathbf{E}_\rho = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos \varepsilon & j \sin \varepsilon \end{pmatrix} \mathbf{E} e^{j\Delta \phi},
$$

where $\Delta \phi = \frac{2\pi d}{\lambda} \sin \alpha$ – is the phase difference of the waves received by MO.

The bearing angle $\alpha$ from (1) is

$$
\alpha = \pm \arcsin \left( \frac{\lambda}{2\pi d} (\Delta \phi) \right) \pm n\pi,
$$

where $n = 0, 1, 2, \ldots$

$1/\sqrt{2}$ in (1) explains the accepted unity intensity of the received wave.

Let's suggest that the resulting wave (1) onboard MO is received with the circular polarization basis (CPB) antenna which corresponds the unity waves of the left and right polarization [11]. Also suggest that the roll angle of MO $\gamma$ is the angle between the right cross MO semi-axe and horizontal plane [7].

The circular polarization basis allows to describe the resulting wave (1) in linear polarization basis as orthogonal circular polarized left $\mathbf{E}_L$ and right $\mathbf{E}_R$ directions of rotation.

3. Method of the problem solution and the main relations

Let's find the amplitudes $A_L$, $A_R$ and phases $\Psi_L$, $\Psi_R$ of the components $\mathbf{E}_L$, $\mathbf{E}_R$ and their dependence on the MO bearing $\alpha$ and roll $\gamma$ angles. To describe the interaction of the resulting wave (1) with elements of the onboard antenna receiving waves in circular polarization basis can be used Jones vectors and matrices [11]. So the Jones vector of the resulting wave with projects in linear polarization basis (time dependence is neglected):

$$
\begin{pmatrix} \mathbf{E}_L \\ \mathbf{E}_R \end{pmatrix} = [Q][R(\pm \gamma)] \begin{pmatrix} \mathbf{E}_L \\ \mathbf{E}_R \end{pmatrix},
$$

where $[R(\pm \gamma)] = \begin{pmatrix} \cos \gamma & \mp \sin \gamma \\ \pm \sin \gamma & \cos \gamma \end{pmatrix}$ – operator of the roll angle rotation $\pm \gamma$;

$\pm \gamma$ – positive MO roll angle $\gamma > 0$ - the right MO cross semi-axe is lower the horizontal plane;

$\pm \gamma$ – negative MO roll angle $\gamma > 0$ - the right MO cross semi-axe is above the horizontal plane,

$$
[Q] = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix}
$$

– operator of the basis transformation from the linear polarized one into the circular polarized [11].

Components $\mathbf{E}_L$ and $\mathbf{E}_R$ at the inputs of the two canal receiver can be calculate using (3) for the roll angles $\pm \gamma$. 


Complex elements (4) and (5) are projections of the elliptically polarized resulting wave (1) on
the orts of the circular polarization basis.

Amplitudes $A_L$, $A_R$ and phases $\Psi_L$, $\Psi_R$ of the signals (4) and (5) at the two canal receiver
output for the roll angles $\pm \gamma$ are:

$$
E_L(\pm \gamma) = \frac{\sqrt{2}}{2} \left[ \cos \gamma \cos \left( \frac{\pi}{4} - \varepsilon \right) + \sin (\Delta \phi \mp \gamma) \sin \left( \frac{\pi}{4} - \varepsilon \right) \right] - 
$$

$$
- j \left[ \pm \sin \gamma \cos \left( \frac{\pi}{4} - \varepsilon \right) + \cos (\Delta \phi \mp \gamma) \sin \left( \frac{\pi}{4} - \varepsilon \right) \right],
$$

$$
E_R(\pm \gamma) = \frac{\sqrt{2}}{2} \left[ \cos (\Delta \phi \pm \gamma) \cos \left( \frac{\pi}{4} - \varepsilon \right) \pm \sin \gamma \sin \left( \frac{\pi}{4} - \varepsilon \right) \right] + 
$$

$$
+ j \left[ \sin (\Delta \phi \pm \gamma) \cos \left( \frac{\pi}{4} - \varepsilon \right) - \cos \gamma \sin \left( \frac{\pi}{4} - \varepsilon \right) \right].
$$

Ratio $A_R/A_L$ and the phase difference $\Psi_{RL}(\pm \gamma) = \Psi_R(\pm \gamma) - \Psi_L(\pm \gamma)$ of $E_L$ and $E_R$ at the output
of the two canal receiver with linear amplitude characteristic and linear detector are:

$$
A_R = \frac{\sqrt{2}}{2} \sqrt{1 - \cos 2\varepsilon \sin \Delta \phi},
$$

$$
\Psi_L(\pm \gamma) = - \arctg \frac{\sin \left( \frac{\pi}{4} - \varepsilon \right) \pm \sin \gamma \cos \left( \frac{\pi}{4} - \varepsilon \right)}{\cos \gamma \cos \left( \frac{\pi}{4} - \varepsilon \right) + \sin (\Delta \phi \mp \gamma) \sin \left( \frac{\pi}{4} - \varepsilon \right)},
$$

$$
A_R = \frac{\sqrt{2}}{2} \sqrt{1 - \cos 2\varepsilon \sin \Delta \phi},
$$

$$
\Psi_R(\pm \gamma) = \arctg \frac{\sin \left( \frac{\pi}{4} - \varepsilon \right) \pm \sin \gamma \cos \left( \frac{\pi}{4} - \varepsilon \right)}{\cos \Delta \phi \mp \gamma \cos \left( \frac{\pi}{4} - \varepsilon \right) \pm \sin \gamma \sin \left( \frac{\pi}{4} - \varepsilon \right)}.
$$

Ratio $A_R/A_L$ and the phase difference $\Psi_{RL}(\pm \gamma) = \Psi_R(\pm \gamma) - \Psi_L(\pm \gamma)$ of $E_L$ and $E_R$ at the output
of the two canal receiver with linear amplitude characteristic and linear detector are:

$$
\frac{A_R}{A_L} = \sqrt{1 - \cos 2\varepsilon \sin \Delta \phi},
$$

$$
\Psi_{RL}(\pm \gamma) = \arctg \frac{\sin \left( \frac{\pi}{4} - \varepsilon \right) \pm \sin \gamma \cos \left( \frac{\pi}{4} - \varepsilon \right)}{\cos \gamma \cos \left( \frac{\pi}{4} - \varepsilon \right) + \sin (\Delta \phi \mp \gamma) \sin \left( \frac{\pi}{4} - \varepsilon \right)},
$$

where $n = 0, 1, 2, \ldots$

From (10) and (11) follows that in general case when the beacon simultaneously emits
orthogonal elliptically polarized waves, the ratio $A_R/A_L$ (10) depends only on the illuminated waves
eLLipticity angle $\varepsilon$ and their phase difference $\Delta \phi$ at the receiving point on MO and does not depend on
MO angle of roll $\gamma$. But the phase difference $\Psi_{RL}$ (11) depends both on the illuminated waves
eLLipticity angle $\varepsilon$, their phase difference $\Delta \phi$ and on the MO angle of roll. So in general case it is
possible to find only the MO bearing angle if known are angle of ellipticity $\varepsilon$ of illuminated
orthogonal polarized waves and the amplitudes ratio (10).

Let’s investigate dependence (10), (11) on the illuminated wave ellipticity angle.

Suggest that the beacon illuminates orthogonal linear polarized waves, if in (10) and (11) $\varepsilon = 0$:

$$
\frac{A_R}{A_L} = \left| \eta \left( \frac{\pi}{4} \Delta \phi \right) \right|
$$

and

$$
\Psi_{RL} = \Psi_R - \Psi_L = \pm 2\gamma.
$$
\[
\Delta\phi = \pm \left( \frac{\pi}{2} - 2 \arctg \frac{A_r}{A_l} \pm 2n\pi \right),
\]

where \( n = 1, 2, 3 \ldots \)

\[
\gamma = \pm \frac{\Psi_{rl}}{2} [rad].
\]

In (14) it was shown that the phase difference \( \Delta\phi \) of orthogonal linear polarized waves at the MO receiving point can be found using \( A_r / A_l \) at the receiver output and the MO angle of roll \( \gamma \) using the measures phase difference \( \Psi_{rl} \) (15) between orthogonal circular polarized signals \( E_L \) and \( E_R \). These two estimates does not depend from each other.

MO bearing angle \( \alpha \) can be found by putting (14) into (2):

\[
\alpha [rad] = \pm \arcsin \left[ \frac{\lambda}{\pi d} \left( \frac{\pi}{4} - \arctg \frac{A_r}{A_l} \right) \right] \pm n\pi.
\]

If the two-channel receiver amplitude characteristic is logarithmic and detector is linear (16) can be transformed:

\[
\alpha [rad] = \pm \arcsin \left[ \frac{\lambda}{\pi d} \left( \frac{\pi}{4} - \arctg \left( 10^{\frac{\Psi_{rl}}{10}} \right) \right) \right] \pm n\pi.
\]

One can see in (16) and (17) that if \( A_r / A_l = 1 \), \( \alpha = 0 \), if \( A_r / A_l > 1 \), \( \alpha < 0 \), and if \( A_r / A_l < 1 \), \( \alpha > 0 \).

(15), (16) and (17) are correct if \( \varepsilon = 0 \), which prove the results published in [7].

Another case – the beacon illuminates simultaneously orthogonal circular polarized waves. Putting \( \varepsilon = \pi / 4 \) in (10) and (11) we have

\[
A_r / A_l = 1
\]

and

\[
\Psi_{rl} = \Delta\phi \pm 2\gamma [rad].
\]

The analyze of (18) and (19) shows that the ratio \( A_r / A_l \) is constant and does not depend on \( \alpha \) and \( \gamma \). While the phase difference \( \Psi_{rl} \) (19) of \( E_L \) and \( E_R \) depend on \( \Delta\phi \) (both on the MO bearing and the angle of roll). To get the correct information one of them should be known. For example, if the MO receiving antenna is gyrostabilized according to the angles of pitch and roll [3,4], \( \gamma = 0 \) in (19) and having (2) in mind we have

\[
\alpha [rad] = \pm \arcsin \left[ \frac{\lambda}{2\pi d} \left( \Psi_{rl} \right) \right] \pm n\pi.
\]

which prove the results in [8].

If MO moves along the equal signal direction corresponding the perpendicular in the center of the base \( d \) between the sources of orthogonal polarized waves, the MO antenna is not stabilized according to the angle roll \( \gamma \) and in (19) \( \Delta\phi = 0 \)

\[
\gamma = \pm \frac{\Psi_{rl}}{2} [rad].
\]

So (20) and (21) are correct if the beacon illuminates orthogonal circular polarized signals \( \varepsilon = \pi / 4 \).

4. Conclusion

In general case when the beacon illuminates orthogonal elliptically polarized signals with known ellipticity angle \( \varepsilon \) and onboard MO the signals are received on the circular polarization basis the ratio \( A_r / A_l \) of the signals with right and left polarization depend only on the MO bearing \( \alpha \) and it could be found without knowledge about the angle of roll.
The phase difference depends both on the MO bearing and the angle of roll. To get the correct information about one of them should be known.

In particular case when the beacon illuminates orthogonal linear polarized signals and onboard MO the signals are received on the circular polarization basis the ratio $A_R/A_L$ of the signals with right and left polarization depend only on the MO bearing $\alpha$ and their phase difference $\Psi_{RL}$ depends only on the MO $\gamma$. No a priori information is needed to find one of them.

When the beacon illuminates orthogonal circular polarized signals and onboard MO the signals are received on the circular polarization basis the ratio $A_R/A_L$ of the signals with right and left polarization does not depend on the navigation elements $\alpha$ and $\gamma$. Their phase difference $\Psi_{RL}$ depends both on the MO $\alpha$ and its roll $\gamma$. There is needed a priori information about one of them.

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References
[1] Yarlykov M 1985 Statistical theory of radio navigation (Moscow: Radio i svayaz')
[2] Shirman Ya et al 2007 Radioelectronic systems: bases of construction and theory: a Handbook. (Moscow: Radiotekhnik)
[3] Pe'lpov D 1982 Gyroscopic orientation and stabilization systems (Moscow: Mashinostroenie)
[4] Smirnov E 2004 Gyroscopic navigation system (Sankt-Peterburg: El'mor)
[5] Bogorodskij V, Kanarejkin D, Kozlov A 1981 Polarization of scattered and intrinsic radio emission of the earth's covers (Leningrad: Gidrometeoizdat)
[6] Kozlov A, Logvin A, Sarychev V 2005 Polarization of radio waves. Polarization structure of radar signals (Moskow: Radiotekhnik)
[7] Gulko V, Mescheryakov A 2017 Proceedings of the XXIII International Scientific and Technical Conference "Radiolocation, navigation, communications" 822–826
[8] Gulko V, Mescheryakov A 2017 Russian Physics Journal 60(6) 972–977
[9] Gulko V, Mescheryakov A 2018 Proceedings of the XXIII International Scientific and Technical Conference "Radiolocation, navigation, communications" 251–255
[10] Gulko V, Mescheryakov A 2018 Doclady TUSUR 21 7–11
[11] Azzam R, Bashara N 1981 Ellipsometry and Polarized Light (Moscow: Mir)