Measuring Cold Dark Matter Power Spectrum from Variations of Hubble Flows

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ABSTRACT
When Cold Dark Matter (CDM) power spectrum normalized by COBE results, its amplitude at smaller scales can be parametrized by $\Gamma \sim \Omega_0 h$. The expected variations of Hubble flows in two samples, the sample of 36 clusters in the Mark III catalogue, and the sample of 20 Type Ia supernovae (SNe), are calculated for the power spectrum in critical-density CDM models (including tilted CDM models and vacuum-dominated CDM models). The comparison between the expectations and the real variations in the data offers a bias-free way to constrain the power spectrum. The cluster sample yields $\Gamma \leq 0.30 - 0.88(n_{ps} - 1) + 1.9(n_{ps} - 1)^2$ at 95% C.L., with best fits $\Gamma = 0.15 - 0.39(n_{ps} - 1) + 0.37(n_{ps} - 1)^2$, where $n_{ps}$ is the spectral index of the power spectrum. The Type Ia SN sample yields $\Gamma \leq 0.25 - 0.80(n_{ps} - 1) + 1.6(n_{ps} - 1)^2$ at 95% C.L., strongly favoring lower $\Gamma$'s. The results are inconsistent with a critical-density matter-dominated universe with $n_{ps} \gtrsim 0.8$ and $H_0 \gtrsim 50$ km/sec/Mpc.

Key words: Cosmology: theory, distance scale

1 INTRODUCTION

Variations in Hubble flows (peculiar Hubble flows, or monopolar deviations from a global Hubble flow) are directly tied to the underlying density fluctuation. For example, for a sphere with a uniform density embedded in a homogeneous FRW universe with a critical density, its peculiar Hubble flow under linear theory is

$$\frac{\delta H}{H_0} = -\frac{1}{3} \frac{\delta \rho}{\rho}$$

where $\delta H/H_0$ is the local deviation from a global Hubble flow normalized by the Hubble constant $H_0$, and $\delta \rho/\rho$ is the local density contrast. For realistic $\Omega_0 = 1$ models with gaussian power spectra, the r.m.s. peculiar Hubble flow as measured within a top-hat sphere is, however (Turner, Cen and Ostriker 1992; Shi, Widrow and Dursi 1995)

$$\left(\frac{\delta H}{H_0}\right)^2 \approx -0.6 \left(\frac{T}{T_0}\right)^2$$

Thus, if we know the variations of Hubble flows as a function of scales, we have a grasp of the power spectrum that generates the underlying density fluctuations. Since only peculiar velocities are involved, the approach is free of the bias factor $b$. And ideally, it will not be influenced by structures beyond the sample volume.

Conventionally, the power spectrum $P(k)$ in critical-density CDM models (including tilted CDM models and $\Lambda$CDM Models where $\Lambda$ is the cosmological constant) is parametrized by

$$P(k) = 2\pi^2 \delta^2(3000h^{-1}\text{Mpc})^3 n_{ps} k^3 ps T \times (1 + 3q)$$

where (Bunn and White 1997)

$$\delta = 1.94 \times 10^{-5} \Omega_0^{-0.83} \Omega_0 e^{-0.95(n_{ps} - 1) - 0.169(n_{ps} - 1)^2}$$

and (Bardeen et al. 1986; Sugiyama 1995)

$$T = \frac{\ln(1 + 2.34q)}{2.34q[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^1/4}$$

$$q = \frac{k}{H_0}, \quad \Gamma = \Omega_0 h e^{-\frac{-\sqrt{h}}{0.50_h}/\Omega_0}$$

In the equations, $\Omega_0$ is the matter content of the universe, $\Omega_0$ is the baryonic content of the universe, and $h$ is the Hubble constant in the unit of 100 km/sec/Mpc. Eq. (5) is due to normalization by the COBE results. Spectral index $n_{ps}$ is constrained to between $\approx 0.8$ and 1.1 (Lineweaver and Barbosa 1997). Given the COBE normalization, $\Gamma$ and $n_{ps}$ determine the shape of the power spectrum.

In this article we calculate the expected variations of Hubble flows for real samples using the above form of the CDM power spectrum. We then compare the expectations to real variations and obtain constraints on $\Gamma$ as a function of $n_{ps}$.
2 FORMALISM

A detailed formalism can be found in Shi (1997). Here we only present some essential equations.

The deviation from a global Hubble flow, \( \delta H = (H_{\text{Local}} - H_0) \), of a sample and its bulk motion \( \mathbf{U} \) are

\[
\delta H = B^{-1} \sum_q S_q r_q - U_i r_i \hat{q}_i,
\]

(6)

\[
U_i = (A - RB^{-1})^{-1} \sum_q S_q \hat{r}_i - B^{-1} \sum_q S_q r_q \hat{r}_q' \hat{r}_q',
\]

(7)

where

\[
A_{ij} = \sum_q \frac{r^2_i r^2_j}{\sigma_q^2}, \quad R_{ij} = \sum_q \sum_{q'} \frac{r_q r_q' r_i r_j}{\sigma_q \sigma_{q'}}, \quad B = \sum_q \frac{r^2_i}{\sigma_q}.
\]

Vector \( \mathbf{r}_q \) is the position of object \( q \) in the sample (with respect to the origin), and \( S_q \) is its estimated line-of-sight peculiar velocity with an uncertainty \( \sigma_q \).

The expectation of the deviation \( \delta H \) consists of two parts: the contribution from the density fluctuations \( \delta H^{(c)} \), and the contribution from noises \( \delta H^{(n)} \).

In linear theory,

\[
\left\langle \left( \frac{\delta H^{(c)}}{H_0} \right)^2 \right\rangle = \Omega_0^{-1.2} \int d^3 k W^i(k) \hat{k}_i \left| \hat{k}_i \right|^2 P(k) \frac{k^2}{k^2},
\]

(9)

where the window function of the sample

\[
W^i(k) = \frac{B^{-1}}{(2\pi)^3/2} \sum_q \frac{r^2_i r^2_j}{\sigma_q^2} e^{i k r_q} - (A - R B^{-1})^{-1} \sum_q \frac{r^2_i r^2_j}{\sigma_q^2} e^{i k r_q} - B^{-1} \sum_q \sum_{q'} \frac{r_q r_q' r_i r_j}{\sigma_q \sigma_{q'}} e^{i k r_q'} \right| \right| \sum_q \frac{r^2_i r^2_j}{\sigma_q^2} e^{i k r_q'}. \]

(10)

Spatial indices \( i, j, l, m \) run from 1 to 3. Identical indices denote summation.

The r.m.s. noise contribution is

\[
\left\langle \left( \frac{\delta H^{(n)}}{H_0} \right)^2 \right\rangle = B^{-1} + B^{-2} (A - R B^{-1})^{-1} R_{il}.
\]

(11)

Since we do not know the precise value of \( H_0 \), we can only deal with relative variations of Hubble flows within a sample. If we denote the expansion rate defined by a subsample with \( n \) nearest objects (relative to us) as \( H_n \), that by another subsample with \( m(< n) \) nearest objects as \( H_m \), and that by the entire sample with \( N \) objects as \( H_N \), then the expected

\[
\left\langle \left( \frac{H_n - H_m}{H_N} \right)^2 \right\rangle \approx \left\langle \frac{\delta H^{(c)}}{H_0} \right\rangle^2 + \left\langle \frac{\delta H^{(m)}}{H_0} \right\rangle^2 - 2 \left\langle \frac{\delta H^{(c)}}{H_0} \right\rangle \left\langle \frac{\delta H^{(m)}}{H_0} \right\rangle + \left\langle \frac{\delta H^{(c)}}{H_0} \right\rangle^2 + \left\langle \frac{\delta H^{(m)}}{H_0} \right\rangle^2 - 2 \left\langle \frac{\delta H^{(c)}}{H_0} \right\rangle \left\langle \frac{\delta H^{(m)}}{H_0} \right\rangle,
\]

(12)

in which

\[
\left\langle \frac{\delta H^{(c)}}{H_0} \right\rangle \left\langle \frac{\delta H^{(m)}}{H_0} \right\rangle \approx \Omega_0^{-1.2} \int d^3 k W^i_n(k) \hat{k}_i W^{i'}_m(k) \hat{k}'_i \frac{P(k)}{k^2}
\]

(13)

and

\[
\left\langle \left( \frac{\delta H^{(c)}}{H_0} \right)^2 \left( \frac{\delta H^{(m)}}{H_0} \right)^2 \right\rangle = \left\langle \left( \frac{\delta H^{(c)}}{H_0} \right)^2 \right\rangle \left\langle \left( \frac{\delta H^{(m)}}{H_0} \right)^2 \right\rangle.
\]

(14)

To better account for the contribution from fluctuations on very small scales at which the window functions are choppy and random (see figure 1) and density fluctuations become non-linear, we use the variation of \( \delta H \) as a measure of the amplitude of density fluctuations, and represent the small-scale contribution by a one-dimension gaussian random motion with a r.m.s. velocity \( \sigma_* \). The expectation is then

\[
\Sigma_{nm} = \left\langle \left( \frac{\delta H^{(c)}}{H_0} \right) \left( \frac{\delta H^{(m)}}{H_0} \right) \right\rangle = \left\langle \left( \frac{\delta H^{(c)}}{H_0} \right)^2 \right\rangle \left\langle \left( \frac{\delta H^{(m)}}{H_0} \right)^2 \right\rangle.
\]

(15)

where \( \delta_{nm} \) is the Kronecker delta function. Thus, given \( \Delta H_n = \delta H_n - \delta H_{n-1} \) from a real sample \( D \), the probability of a critical-density CDM model with a particular set of \( \Gamma \) and \( \eta_{100} \) is

\[
P(M|D) \propto \frac{1}{|\Gamma|^{1/2}} \exp \left[ - \frac{1}{2} (\Delta H)^T (\Sigma)^{-1} (\Delta H) \right].
\]

(16)

3 RESULTS

The formalism is applied to the cluster sample that yields the template Tully-Fisher relation in the Mark III catalogue (Willick et al. 1997). Only subsamples with no less than 15 clusters are included in calculations, because smaller subsamples are susceptible to large effects of non-linearities. This cut-off does not significantly affect the final results due to the large uncertainties of \( \delta H \) in the small subsamples. The integrals over the window functions are done by sampling about 40000 K's in space. The window functions are truncated at \( \pi/k \approx 6h^{-1} \) Mpc. To include the effect of non-linearities below the truncation scale, a \( \sigma_* = 300 \) km/sec is added quadratically to the uncertainty in distance of each cluster to fully represent the uncertainty of its radial peculiar velocity. The truncation scale is verified not to significantly influence our final result, but the magnitude of \( \sigma_* \) does have an appreciable impact. A smaller \( \sigma_* \) will increase the contrast of the probability distribution \( P(M|D) \), thus tighten the resultant constraints. At the moment, a \( \sigma_* \) of 300 km/sec is quite a safe assumption to make (Bahcall and Oh 1996).

The test is also applied to the sample of 20 Type Ia SNe of Riess, Press and Kirshner (1996). Subsamples run from the 6 nearest Type Ia SNe to the full sample of 20 SNe.
Figure 1. $|W^k|^2$ of the cluster sample in question. Different curves are for different directions in the $k$-space.

Truncation of window functions is made at $\pi/k \approx 4h^{-1}$ Mpc. The host galaxies of the SNe are assumed to have a $\sigma_\star = 480$ km/sec. This corresponds to a r.m.s. gaussian random motion of 830 km/sec for host galaxies, which roughly matches the velocity dispersion of galaxies in clusters. It is also the maximal $\sigma_\star$ allowed by the sample, because any addition of distance uncertainties will result in a dispersion of radial peculiar velocities that is larger than that of data. Therefore, the constraint we obtain will be rather conservative.

Figure 2 shows the probability distribution (with an arbitrary normalization) of $\Gamma$ when $\Omega_0 = 1$ and $n_{ps} = 1$, for the two samples. After the same distribution function is calculated for other $\Omega_0$ (in which case $\Lambda = 1 - \Omega_0$) and $n_{ps}$, we find that at 95% C.L., the cluster sample yields $\Gamma \leq 0.30 - 0.88(n_{ps} - 1) + 1.9(n_{ps} - 1)^2$, with best fits $\Gamma = 0.15 - 0.39(n_{ps} - 1) + 0.37(n_{ps} - 1)^2$; the Type Ia SN sample yields $\Gamma \leq 0.525 - 0.80(n_{ps} - 1) + 1.6(n_{ps} - 1)^2$ at 95% C.L., strongly favoring lower $\Gamma$'s. These results are accurate for critical-density CDM models in the range $0.3 \leq \Omega_0 \leq 1$ to $\approx 10\%$, because the factor $H_0^{2.6}$ in peculiar velocities is mostly offset by the $\Omega_0$ dependence in the COBE normalization eq. [4]. The results indicate that a critical-density matter-dominated universe with $H_0 \geq 50$ km/sec/Mpc and $n_{ps} \gtrsim 0.8$ cannot fit the variations of Hubble flows in the data. (The constraints will not change significantly if one includes the tensor contribution to the COBE normalization of $P(k)$, but will tend to relax when one decreases $P(k)$ on $\sim 100h^{-1}$ Mpc scales.) With the number of Type Ia SNe with distance measurements doubling very soon, we can only expect that the limit on $\Gamma$ becomes more secure and interesting.

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