On the formation of membership functions for fuzzy sets in the logical representation of knowledge

Vladimir V Serov
Russian state social university, str. 4, p. 1, Wilhelm Peak, Moscow, 129226, Russia

E-mail: vvs030948@mail.ru

Abstract. The problems of using characteristic membership functions to describe fuzzy knowledge in the form of a logical calculus of fuzzy first-order predicates are considered. Some possible types of functions and methods for their preparation are described. The possibility of using these methods for the development of expert and intelligent systems in various subject areas, such as enterprise management, recognition of the structures of polyatomic molecules, is shown.

1. Introduction
In expert systems, for formalizing knowledge, network, frame, and production models are often used structurally integrated into a hierarchical network. Moreover, local knowledge (microknowledge) has a logical character in the form of IF THEN THAT, i.e. production systems. For their formalization, Boolean algebra, or the first-order predicate calculus, is used. A predicate is a propositional function P (x₁, x₂, ..., xₙ) defined on the individual variables x₁, x₂, ..., xₙ, the range of which is made up of true or false statements (1 or 0).

In practice, knowledge has the fundamental property of fuzziness, uncertainty due to the complexity of formalization objects [1]. For an adequate representation of knowledge, a formal logical system was developed - applied calculus of fuzzy predicates [2,4,5,6], combining the descriptive capabilities of the theory of fuzzy sets and deductive calculus of predicates. For predicates, variables, and constants, characteristic functions of fuzzy sets are introduced, the truth of which is in the range of values from 0 to 1.

For the practical application of this theory and methods, the main issues are the development of a system of predicates that adequately reflect the domain, and a description of the membership functions of fuzzy predicates, variables, and constants. This requires the joint efforts of specialists both in the subject field and in knowledge engineering [2,4,5,6]. The development of a predicate system for a specific subject area and its subsequent structuring is a problem for specialists (experts).

2. Results
Two approaches are possible to describe membership functions. The first is expert judgment. The specific form of membership functions is determined on the basis of various additional assumptions about the properties of these functions (piecewise linear approximation, exponential, quadratic, such characteristics as symmetry, monotonicity, continuity of the first derivative, etc.), taking into account the specifics of the uncertainty, the real situation and the number degrees of freedom in functional dependence.
Figures 1-4 show some types of membership functions.

**Figure 1.** \( \mu_1 (x,a,b) = 0 \) if \( x \leq a \); \( (x-a)/2 \) if \( a < x \leq (a+b)/2 \); \( 1 - 2(x-a)^2/((b-a)^2) \) if \( (a+b)/2 < x < b \); \( 1 \) if \( x \geq b \).

**Figure 2.** \( \mu_2 (x,a,b,c) = \mu_1 (x,a,b) \) if \( x < b \); \( 1 \) if \( b \leq x \leq c \); \( 1 - \mu_1 (x,c,c+b-a) \) if \( x > c \).

**Figure 3.** \( \mu_3 (x,a,b,c) = 0 \) if \( x \leq a \); \( (x-a)/(c-a) \) if \( a < x \leq c \); \( (b-x)/(b-c) \) if \( c < x < b \); \( 0 \) if \( x \geq b \).

**Figure 4.** \( \mu_4 (x,a,b,c,d) = 0 \) if \( x \leq a \); \( (x-a)/(c-a) \) if \( a < x < c \); \( 1 \) if \( c \leq x \leq d \); \( (b-x)/(b-d) \) if \( d < x < b \); \( 0 \) if \( x > b \).

The second approach is statistical methods — correlation, regression, factor analysis, etc. The disadvantage is the requirement for special user training. In addition, powerful modern statistical packages are too “heavyweight” for mass practical application. Membership functions play a decisive role both in the representation of knowledge and in assessing the truth of decisions in fuzzy-logical systems.

If we consider knowledge as a system of axioms \( F_1, F_2, \ldots, F_n \) of applied logical calculus, then problems \( G \) are represented by statements (theorems) that need to be proved. When using the methods of proving theorems in predicate calculus on the principle of Robinson resolutions, logical expressions
(formulas) are reduced to the preceding Second approach involves the application of training procedures. To study in normal form (PNF), the known identities of predicate logic are used for transformation, then existence quantifiers are excluded by sculmerization, universal quantifiers are omitted, conjunction signs are replaced by commas, as a result we have many clauses S. In the applied logical calculus, x_i u t_i - are terms, and moreover if x_i is constant, then t_i is also constant. In the general case, x_i and t_i are sets of odd subsets with membership functions \{\mu_a(t_i)\} and \{\mu_a(x_i)\}. Then substitution is a finite set of pairs of the form \alpha = \{(t_i/x_i, \{\mu_a(t_i)\}+\mu_b(t_i)/\mu_c(x_i))\},...,(t_n/x_n, \{\mu_d(t_n)\}+\mu_e(t_n)/\mu_f(x_n))\). Let C_1 and C_2 be two clauses that do not have common variables, L_1 and L_2 be two letters with truth functions \mu_1 and \mu_2 b C_1 and C_2. If L_1 and L_2 have the most common unifier \alpha, then the disjoint, \alpha_1=[\mu_1+\mu_2](C_1 \land \alpha_1)\lor(C_2 \land \alpha_2) will be called the resolvent C_1 and C_2 in the calculus of fuzzy predicates.

**Theorem 1.** Suppose that, by definition, the resolvent \alpha_1=L_1 \lor \alpha_1, C_2=\lor \alpha_1, C_0=\lor \alpha_1, then \mu_0 and \mu_2 have the most common unifier \alpha, since 0 \leq \mu_0 \leq 1, \mu_2 \leq \mu_2. Theorem 2. Suppose we are given two clauses C_1 and C_2 that do not have common variables, their truth functions \mu_1 and \mu_2, and there exists a permutation \alpha, such that C_1=\lor \alpha_1, \alpha_2=\lor \alpha_2, \alpha \leq \alpha_2, \alpha_1 \leq \alpha_2, \alpha \leq \alpha_2, \alpha \leq \alpha_2. If at least one of the values of the membership functions is \mu_1 \geq \mu_1 \lor \mu_2 \geq 0.5, then for the truth function \mu_0 of the resolvent C_0 the inequality \mu_0 \leq \min(\mu_1, \mu_2) holds.

As a rule, the knowledge system uses information with membership function values greater than 0.5 and close to 1; therefore, Theorem 2 is of great practical importance for solving qualitative problems, because serves as a justification for a higher value of the truth functions of results in comparison with Theorem 1. Decisions obtained for knowledge with truth less than 0.5 usually have no positive practical value. For the practical application of this theory and methods, the main issues are the development of a system of predicates that adequately reflect the subject area.

As already noted, for the practical application of this theory and methods, the main issue is the development of a predicate system that adequately displays the subject area. For example, to manage the activities of one of the small enterprises, a predicate system was chosen

- **PRODUCTION** (code, number)
- **GOODS** (code, quantity)
- **RAW-MATERIALS** (cipher number)
- **SUPPLY** (code, supplier, quantity)
- **TRANSPORT** (type, code, quantity)
- **PAYMENT** (customer, type, amount, time)

For an expert system in the field of recognition of molecular structures by molecular spectroscopy [4]

1. Predicates characterizing the IR spectrum.
   - I1 (a, b, x1, y1, z1) - the spectrum contains a type a vibration band of group b with frequency x1, intensity y1, half-width z1.
   - Intensity and half-width are specified by integers in the interval (0.5), frequency values by positive integers, variable a is defined on the set of possible types of vibrations (s- symmetrical stretching vibrations, p-plane deformation, etc.).
   - I2 (a, b, x2, n) - the spectrum contains n split vibrational bands of type a of group b with a frequency of x2.
2. Predicates characterizing the structure of a molecule.
   - R1 (r1) - the molecule contains a fragment of r1.
   - R2 (r1, r2, n) - the molecule contains a fragment of r1 and, through n bonds from it, a fragment of r2.
3. Predicates characterizing additional conditions.
As for membership functions, they are determined by expert methods and have a piecewise linear shape (figure 4).

3. Conclusions
From the examples given it is clear that it is hardly possible to apply formal methods to formulate a predicate system, but with some assumptions it is possible to use statistical methods to form and describe membership functions.

References
[1] Zadeh L A 1965 Fuzzy sets Information and Control 8(3) 338-53
[2] Serov V V, Elyashberg M E and Gribov L A 1988 System-related matters in the theory of solving qualitative problems of molecular spectroscopy Journal of Molecular Structure 178 1-21
[3] Miroshnik M A et al 2015 Designing artificial intelligence systems using fuzzy logic Radio Engineering 182 42-50
[4] Serov V V 2019 On the possibility of scientific knowledge formalization by fuzzy logic methods J. Phys.: Conf. Ser. 1399 033043
[5] Serov V V 2019 Application of fuzzy logic for an enterprise production activity management IOP Conf. Ser.: Earth Environ. Sci. 315 032002
[6] Elyashberg M E, Serov V V and Gribov L A 1987 Artificial intelligence systems for molecular spectral analysis Talanta 34 21-30