Dark energy from coexistence of phases

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We suggest that the current acceleration of the universe may be explained by the vacuum energy of a hidden sector which is stuck in a state of equilibrium between phases. The phases are associated to a late-time first-order phase transition, where phase coexistence originates at a temperature \( T_c \sim 10^{-3} \text{eV} \) and lasts until temperature falls below \( T \sim 10^{-4} \text{eV} \). During phase coexistence, the energy density has an effective cosmological constant component with the observed magnitude. This scenario does not require supercooling and may arise naturally in realistic models.

PACS numbers: 98.80.Cq, 05.70.Fh

I. INTRODUCTION

Evidence for the acceleration of the expansion rate of the universe, coming from Type Ia supernovae observations \(^1\), together with large scale structure and CMB measurements \(^2\,3\), are compatible with a flat universe which is composed of 76% dark energy and 24% dark matter, with an energy density of order \((10^{-3}\text{eV})^4\). These observations raise some naturalness problems, which could be summarized in two questions: why is there a tiny, though non-vanishing, cosmological constant? Why do we happen to live in the epoch in which such constant energy density coincides with that of non-relativistic matter?

Many approaches to solve these problems assume that the cosmological constant actually vanishes in the true vacuum due to some as yet unknown mechanism, the observed vacuum energy being caused by a field which is away from the global minimum of its potential. This is the case, e.g., of quintessence models \(^3\), where a field \( \phi \) is slowly rolling towards its minimum. An interesting possibility is that \( \phi \) is a pseudo Goldstone boson (PGB) which arises through the spontaneous breaking of a global symmetry at a high scale \( f \), and acquires a harmonic potential when this symmetry is explicitly broken at a lower scale \( M \). The energy scale \( M \) of the potential is therefore \( \sim \rho_\Lambda^{1/4} \) in order to account for the cosmological constant (this scale may be associated to a neutrino mass), whereas the mass of \( \phi \) is suppressed by the scale \( f \) in order to explain the slow evolution of the field \(^3\).

An interesting alternative to quintessence consists of models in which the field is stuck in a false vacuum minimum. In this case, a splitting \( \varepsilon \) between the false and true vacua accounts for the cosmological constant; therefore, we must have \( \varepsilon \sim \rho_\Lambda \), whereas the scale \( M \) of the potential can be larger. For instance, in models in which \( \phi \) is a PGB, the potential has degenerate minima separated by barriers of height \( \sim M \). The splitting \( \varepsilon \) can then be achieved through higher-order effects \(^3\,4\). Another possibility is that the false vacuum arises from non-perturbative effects \(^5\). In all these cases we will have \( \varepsilon \ll M^4 \). Alternatively, the false vacuum energy scale may be obtained from the confining scale of a hidden gauge theory \(^8\), or from \( \text{TeV} \) scale supersymmetry breaking which is Planck-scale suppressed in a hidden sector \(^2\,10\). In these cases, the false vacuum energy arises from the spontaneous symmetry breaking at the low scale, so the splitting between vacua has the same scale as the potential, \( \varepsilon \sim M^4 \).
A general problem of false vacuum models is the difficulty to accomplish that the field ends up actually in the false vacuum. In the cases in which $\varepsilon \ll M^4$, the false vacuum is stable under quantum tunnelling, because the splitting between minima is much smaller than the barrier which separates them. In these models, the field drops to one of the minima at a high temperature $T \sim M$, when it starts to feel the potential. However, it rolls toward each of the minima with equal probability, and domain walls separating vacua are formed. Then, the regions with the true vacuum grow due to the pressure difference between vacua. As a consequence, either the false vacuum ends up disappearing, or a system of domain walls persists. Inflation is usually invoked to solve this problem, but this approach has nontrivial constraints [6]. On the other hand, in models with a single scale $M^4 \sim \varepsilon$, one expects that thermal effects will favor the false vacuum for $T \gtrsim M$, thus setting the proper initial condition for $\phi$. However, in this case the probability of transition to the true vacuum by quantum tunneling or thermal activation may be too high for the false vacuum to survive long enough.

Indeed, such a single-scale false-vacuum model will generally have a first-order phase transition at a critical temperature $T_c \sim \rho_A^{1/4} \approx 2 \times 10^{-3} \text{eV}$. Notice that whenever the temperature is close to any scale $M$ at which there is a phase transition (i.e., $T \sim T_c \sim M$), the radiation and vacuum energy densities will be comparable. Indeed, before the phase transition, at $T > T_c$, the system is in the high-temperature phase with $\rho_R \sim T_c^4$ and the field is stuck in the false vacuum with $\rho_A \sim M^4$. This fact is interesting, because the three forms of energy (matter, radiation, and $\Lambda$) happen to be comparable within a relatively short range of temperature scales $\sim (10^{-4} - 1) \text{eV}$. The coincidence of matter and radiation in this range can be easily explained by a dark-matter particle at the electroweak scale, whose relic density is governed by the scale $M_{\text{EW}}^2/M_{\text{Pl}} \sim 10^{-3} \text{eV}$. On the other hand, the single-scale models mentioned above present a potential with non-degenerate minima at the scale $M \sim 10^{-3} \text{eV}$ (which could also be related to the ratio $M_{\text{EW}}^2/M_{\text{Pl}}$). Thus, one would expect a triple coincidence between radiation, matter, and $\Lambda$ at $T \sim 10^{-3} \text{eV}$. In fact, the coincidence is split by dimensionless weak coupling factors $\sim 1/\pi\alpha^2 \sim 10^3$ which enter the dark-matter relic density. Taking into account this correction, the matter-radiation equality naturally becomes $T_{eq} \sim 1 \text{eV}$. Then, within the framework of these models the radiation-$\Lambda$ equality will occur at $T \sim T_c \sim 10^{-3} \text{eV}$ and, if the vacuum energy remains constant, the matter-$\Lambda$ equality will occur at $T \sim 10^{-4} \text{eV}$, in agreement with observation.

For this to happen, however, notice that, although the critical temperature of the phase transition is naturally $T_c \sim 10^{-3} \text{eV}$, the false vacuum should not decay before the temperature has fallen at least to $T \sim 10^{-4} \text{eV}$. Immediately after the transition the field will be in the stable minimum of the free energy. In general, the high-$T$ value of this minimum may differ from its zero-temperature value. However, this minimum now evolves with temperature, and its vacuum energy is relaxed to zero (or to a lower scale value associated with a subsequent phase transition). Furthermore, it is important to notice that the phase transition occurs in a hidden sector, i.e., a system of particles which do not interact with those of the standard model except through gravity. In the discussion above, we have been assuming implicitly that the temperature of this system is the same as that of photons. This is not necessarily the case, since these systems are not in contact. Moreover, the energy density of relativistic particles in a hidden sector is constrained by big bang nucleosynthesis to be less than $0.3\rho_{\nu_e}$, where $\rho_{\nu_e}$ is the density of a single species of left-handed neutrino (see [8] and references therein). Hence, the temperature $T$ of the hidden sector must be lower than that of photons $T_\gamma$ (how much lower, depends on the number of extra species). Since
\(T_\gamma \simeq 2 \times 10^{-4} eV \sim \rho_\Lambda^{1/4}/10\), there seem to be only two possibilities for us to witness the false vacuum: either 1) the hidden sector has a hierarchy \(T_c < \rho_\Lambda^{1/4}/10\), allowing its temperature \(T\) to fulfill the double constraint \(T_c < T < T_\gamma\) [10], so that the phase transition has not occurred yet, or 2) the critical temperature satisfies the more natural relation \(T_c \sim \rho_\Lambda^{1/4}\) but the system is supercooled, which means that the nucleation of bubbles has not begun yet, even though the temperature has fallen well below \(T_c\). In the latter case the supercooling must be of at least an order of magnitude, \(T < T_\gamma \sim T_c/10\) [3, 11]. Both possibilities turn out to be quite unnatural and strongly constrain the model.

In this letter we wish to suggest an alternative scenario for single-scale false-vacuum models. We will study the possibility that the development of the phase transition is very slow, so it is not complete yet, even though bubble nucleation began when the temperature of the hidden sector was \(T \simeq T_c \sim \rho_\Lambda^{1/4}\). In this way, there is no need to force a split between \(T_c\) and \(\rho_\Lambda^{1/4}\), nor to require an excessive amount of supercooling. Since at \(T = T_c\) both the high- and low-temperature phases have non-vanishing potential energy, an average dark-energy component will remain while the two phases coexist. In the next section we describe the scenario, and in section III we compare it with the other alternatives, within the framework of two definite models. Our conclusions are summarized in section IV.

II. THE PHASE-COEXISTENCE SCENARIO

It was pointed out by Witten [12] that a first-order phase transition may occur reversibly in the universe. The essential idea is that the energy (latent heat) that is expelled by the expanding bubbles of low-temperature phase may keep the two phases in equilibrium at \(T = T_c\) until the phase transition is completed. In fact, there is always some supercooling, and the phase transition begins when \(T \lesssim T_c\); however, the entropy that is released in a first-order phase transition can reheat the system up to a temperature \(T\) very close to \(T_c\). Then, the pressure difference between the two phases becomes very small and the phase transition slows down significantly [13]. If the phase transition occurs in a hidden sector, this system will be kept in phase equilibrium at constant \(T \simeq T_c\) while the temperature of photons decreases.

It is important to remark that the temperature \(T\) of the hidden sector must be initially lower than the temperature \(T_\gamma\) of photons. Indeed, the hidden sector has a radiation component which should never dominate the cosmic expansion law. At \(T = T_c \sim 10^{-3} eV\), the energy density of this component is\(^1\) \(\rho_R \sim T_c^4 \sim \rho_\Lambda\). Notice however that, when this equality between \(\rho_\Lambda\) and \(\rho_R\) occurs in the hidden sector, since \(T_\gamma > T_c\), the energy density of photons is \(\rho_\gamma \sim T_\gamma^4 \gg \rho_R\). Then, the phase transition begins, and enters the slow phase-coexistence stage. A vacuum energy density \(\sim (10^{-3} eV)^4\) will remain until the end of the phase transition. In the meantime, the CMB radiation density \(\rho_\gamma\), and the matter density, decrease. When \(T_\gamma\) becomes \(\sim 10^{-3} eV\), the equality \(\rho_\gamma = \rho_\Lambda\) occurs; finally, at \(T_\gamma \sim 10^{-4} eV\) the matter density becomes comparable to \(\rho_\gamma\).

Notice that, even if at late times the species are not in thermal equilibrium with each other, as long as they have thermal distributions they contribute to the finite-temperature

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\(^1\) Notice that we call \(T\) and \(\rho_R\) respectively the temperature and radiation density of the hidden sector. For the CMBR we use instead \(T_\gamma\) and \(\rho_\gamma\).
effective potential. During the phase transition, there is a back-reaction on the particle distributions. The released entropy keeps the kinetic energy of particles high and prevents their temperature from decreasing.

The low-temperature phase has a lower energy density than the high-temperature one. The energy difference is the latent heat $L$, which is liberated as bubbles of the low-temperature phase expand. In general, bubble nucleation begins at a temperature $T_N \lesssim T_c$. If $L$ is comparable to the density $\rho_R$ of the relativistic gas in the hidden sector, the system reheats up to a temperature very close to $T_c$ and remains so for a long time [13]. No further bubbles nucleate; the two phases coexist at a constant temperature $T \simeq T_c$ as bubbles of low-temperature phase slowly expand at the expense of the regions of high-temperature phase [12]. We will assume for simplicity that supercooling and reheating happen in a negligibly short time and consider only the subsequent phase-equilibrium stage [14], which is in general longer. We will thus obtain a lower bound for the total duration of the phase transition.

As we will see, the duration of phase coexistence depends essentially on the amount of latent heat that is released, and on the energy density of radiation. In the early universe, there is a hot plasma with a large number $g_*$ of relativistic species. In general, only a few, say $g$, of them contribute significantly to the effective potential of a field which undergoes a phase transition (e.g., those which have the strongest Yukawa couplings). Thus, the latent heat is proportional to $g$, whereas the radiation density is proportional to $g_*$. As a consequence, the entropy that is liberated in the transition has a relatively small effect on the plasma, due to the large heat capacity of the latter. On the contrary, at late times and low temperatures, only a few species remain relativistic. Furthermore, since the different species are not in thermal equilibrium with each other, it is likely that the released entropy goes only to the $g$ particle species that contribute to the effective potential, and no entropy is transmitted to the rest of the species. Consequently, the released heat will be comparable to the density $\rho_R$ of this sector. Therefore, the occurrence of phase coexistence is favored at later epochs.

### A. The phase transition

We will thus assume that the true vacuum energy vanishes at zero temperature, and consider a $meV$ scale effective potential with a first-order phase transition in a hidden sector with $g$ relativistic degrees of freedom. The general scenario depends only on thermodynamic parameters such as the energy density and the latent heat, so for the moment we will not need to specify any model. Later we will consider a couple of examples.

Before the phase transition, the energy density of the hidden sector is of the form $\rho = \rho_R + \rho_\Lambda$, where $\rho_R = g\pi^2 T^4/30$ corresponds to radiation, and $\rho_\Lambda$ is the constant energy density associated to the false vacuum. The pressure is thus given by $p = \rho R/3 - \rho_\Lambda$. To determine the duration of phase coexistence, we use the adiabatic expansion of the universe [13]. The entropy density of the hidden sector is given by

$$s = s_+ (a_i/a)^3,$$

where $a_i$ is the scale factor at the beginning of the phase transition, and $s_+ = 4\rho R (T_c)/3T_c$ is the entropy density of the high-temperature phase at critical temperature. Normally, the temperature would decrease as entropy dilutes. During a first-order phase transition, however, energy is released in the form of latent heat $L \equiv T_c (s_+ - s_-)$, thus avoiding the
temperature decrease. During phase coexistence, the average entropy density is given by 
\[ s = s_+ + (s_- - s_+) f, \]
where \( f \) is the fraction of volume that has already converted to the low-temperature phase. It follows that
\[ f = \frac{s_+}{s_+ - s_-} \left[ 1 - (a_i/a)^3 \right]. \] (2)

The phase transition completes when \( f = 1 \). Since \( s_- > 0 \), this occurs for a finite \( a = a_i \). However, in the limit \( s_- \to 0 \) the duration of the phase transition becomes infinite, so \( a_i \) can be made arbitrarily large if most particles in the hidden sector lose their entropy. As we shall see, such a strongly first-order phase transition is possible if the particles acquire large masses and become non-relativistic \[ 15 \]. The condition \( s_- \ll s_+ \) implies that the latent heat must be close to its upper bound, which corresponds to \( s_- = 0 \),
\[ L \simeq L_{\text{max}} \equiv 4 \rho_R/3. \] (3)

We have seen that the temperature of photons must have decreased by at least an order of magnitude since the beginning of the phase transition in the hidden sector, so we must require that the transition does not complete before \( a/a_i > 10 \). We thus obtain the condition
\[ s_- < 10^{-3} s_+. \]

In the above we have implicitly assumed that the latent heat is spread out quickly enough to guarantee a uniform temperature at any time. This will be the case if the time scale for bubble growth is much longer than the time required for latent heat to travel the distance between bubbles \[ 16 \]. This condition can be expressed as
\[ df/dt \ll c_s/d, \] (4)
where \( c_s \approx 1/\sqrt{3} \) is the speed of sound in the relativistic gas, and \( d \) is the average bubble separation, which is roughly determined by the number density of bubbles \( n_b \), \( d \sim n_b^{-1/3} \). During phase equilibrium at \( T \simeq T_c \), the number of bubbles is fixed, so \( d \propto a \) and the rhs of Eq. \( 11 \) decreases like \( a^{-1} \). On the other hand, from Eq. \( 2 \), \( df/dt \sim (\rho_R/L) (a_i/a)^3 H \); so, the lhs of Eq. \( 11 \) decreases more quickly than the rhs. Thus, this condition will remain valid in the range of interest if it is fulfilled at the beginning of the phase-equilibrium stage. For \( a = a_i \), Eq. \( 11 \) becomes \( \rho_R/L \ll n_b^{1/3} H^{-1} \). Notice that \( n_b H^{-3} \) is the number of bubbles nucleated inside a causal volume. We certainly have \( n_b H^{-3} > 1 \) if the phase transition has begun (this is the standard condition for a rough estimation of the onset of nucleation). Moreover, when the phase-coexistence stage is reached all bubbles have already nucleated, and this number is in general \( \gg 1 \) (see, e.g., Refs. \[ 13, 16 \]). On the other hand, for \( L \simeq 4 \rho_R/3 \) we have \( \rho_R/L \lesssim 1 \). Hence, the requirement \( 11 \) is generally satisfied.

**B. The equation of state for phase coexistence**

During the phase transition, the temperature and pressure have constant values \( T \simeq T_c \), \( p \simeq p_c \equiv p(T_c) \), so the energy is given by \( \rho = T_c s - p_c \), with \( p_c = \rho_R (T_c)/3 - \rho_\Lambda \). Using again Eq. \( 11 \), we obtain
\[ \rho = T_c s_+ (a_i/a)^3 - p_c. \] (5)

This result can also be obtained by considering the average \( \rho = \rho_- f + \rho_+ (1 - f) \), where \( \rho_+ = \rho_R (T_c) + \rho_\Lambda \), \( \rho_- = \rho_+ - T_c \Delta s \), and \( f \) is given by Eq. \( 2 \). According to Eq. \( 5 \), as far
as the Friedmann equation is concerned, this system can be thought of as being composed of a pressureless fluid, whose density \( \rho_{\text{eff}} = \frac{4 \rho_R}{3} (a_i/a)^3 \) dilutes like matter, plus a constant energy density \( \rho_{\Lambda} = -p_c \). The effective cosmological constant is thus given by

\[
\rho_{\text{eff}}^{\Lambda} = \rho_{\Lambda} - g \pi^2 T_c^4 / 90.
\]

If \( T_c \sim \rho_{\Lambda}^{1/4} \), we will have \( \rho_{\text{eff}}^{\Lambda} > 0 \) provided that \( g \) is not too large. Hence we have a constant \( \rho_{\text{eff}}^{\Lambda} \sim (10^{-3} eV)^4 \) throughout the phase transition.

C. Possible signatures

It is interesting that, although the system is composed only of vacuum energy and radiation, the density \( \rho \) of the hidden sector exhibits a matter component. One may wonder whether it could account for dark matter. Notice however that this effective matter density is not clustered. Such a homogeneous \( \rho_{\text{eff}}^{\Lambda} \) could provide a signature of phase coexistence. However, it is too small to be perceived in current observations. Indeed, it was \( \rho_{\text{eff}}^{\Lambda} \sim \rho_R \sim \rho_{\Lambda} \) at \( a = a_i \). But at that stage the total dark matter density of the universe was \( \rho_{\text{M}}^{\text{tot}} \sim (a_0/a_i)^3 \rho_{\Lambda} \). Since \( a_0/a_i \) is at least \( \sim 10 \), we have \( \rho_{\text{eff}}^{\Lambda} < 10^{-3} \rho_{\text{M}}^{\text{tot}} \).

Although the effective \( \omega(z) \) in this picture is not observably different from constant \( \omega = -1 \), our scenario is characterized by the coexistence of two phases with a constant \( \mathcal{O}(1) \) difference in the value of the vacuum energy. Such kind of inhomogeneity in the cosmological constant will result in specific signatures in large scale structure, which may allow a distinction from \( \Lambda \)CDM and from quintessence. Since these inhomogeneities become important at \( z \sim 1 \), they will not influence structure formation on scales below that of galaxy clusters. If their size is on the cluster scale or above, they may have an effect on cluster abundance, causing a departure from the results of a constant-\( \Lambda \) model. Nevertheless, we expect our scenario to cause deviations no larger than those of models in which dark energy is coupled to matter and clusters in overdense regions. Current data on cluster number counts do not allow to discriminate dark energy models.

Further signatures might be found in the CMB, which may be affected through the integrated Sachs-Wolfe effect. Additionally, there may be gravitational lensing effects. In general, the quantitative effect of the inhomogeneities will depend on the evolution of their amplitude and size. In our picture, the amplitude of the inhomogeneities in \( \rho_{\Lambda} \) is fixed. The size scale is determined by the number of nucleated bubbles \( n_b \), which in turn depends on the nucleation rate, and can be calculated numerically [16]. We shall carry out such analysis elsewhere. Depending on the parameters of the model, a wide range of sizes is possible. Hence, constraints on the range of parameters may be obtained from compatibility with future observations.

III. THE LATE-TIME PHASE TRANSITION

In order to compare the phase-coexistence scenario, in which the hidden sector has a temperature \( T = T_c \sim \rho_{\Lambda}^{1/4} \), to the case in which \( T_c \lesssim T \ll \rho_{\Lambda}^{1/4} \) and to that of supercooling, with \( T \ll T_c \sim \rho_{\Lambda}^{1/4} \), we need to consider a specific model. Several models for late-time phase transitions have been proposed in the literature (see [3-11] and references therein). For our general considerations, a simple model in which a single scalar field \( \phi \) plays the role of an order parameter will suffice.
A. A potential with a negative mass squared

The simplest potential we can consider is one with a negative $\phi^2$ term, so the minimum is away from the origin; thermal effects then move the minimum back to the origin at high temperature. Therefore, we consider the effective potential

$$V(\phi) = -\lambda v^2 \phi^2/2 + \lambda \phi^4/4 + \rho_\Lambda,$$

(7)

where the constant term $\rho_\Lambda \equiv \lambda v^4/4$ makes the energy density vanish in the vacuum. This potential can be regarded as a simplified version of the model considered in Ref. [10], in which two scalar fields have negative squared masses in the scale of $T eV^2/M_P$ from supersymmetry breaking.

The potential (7) does not even possess a metastable vacuum, since the only minimum is $\phi = v$. However, at finite temperature, the effective potential receives temperature-dependent corrections from particles which have gauge or Yukawa couplings with $\phi$. We will assume for simplicity that all the particles in the hidden sector are bosons which couple to $\phi$ through field-dependent masses $m(\phi) = h\phi$, all with the same coupling $h$. The temperature-dependent part of the potential is (at 1-loop order)

$$\Delta V_T(\phi) = \left(gT^4/2\pi^2\right) I(h\phi/T),$$

(8)

where $I(x) = \int_0^\infty dy y^2 \log \left[1 - \exp\left(-\sqrt{y^2 + x^2}\right)\right]$, and $g$ is the number of degrees of freedom in the hidden sector.

In the high temperature approximation [$m(\phi) \ll T$], the function $I(x)$ is usually expanded in powers of $x$, and the free energy of the hidden sector takes the form

$$V_{\text{high-}T} = \rho_\Lambda - g\pi^2 T^4/90 + \Delta V(\phi, T),$$

(9)

where the field-dependent part

$$\Delta V(\phi, T) = \frac{g h^2}{24} \left(T^2 - T_0^2\right) \phi^2 - \frac{g h^3}{12\pi} T \phi^3 + \frac{\lambda}{4} \phi^4$$

(10)

gives the free energy difference between the two phases. Here, $T_0^2 \simeq 12\lambda v^2/gh^2$. We have kept only the relevant finite-$T$ contributions to $\Delta V$. One of them is of the form $T^2 \phi^2$ and causes the vacuum expectation value of $\phi$ to vanish at high $T$. The other one is of the form $-T\phi^3$ and produces a first-order phase transition. At the critical temperature, $T_c = T_0/\sqrt{1 - gh^4/6\pi^2\lambda}$, the high-temperature minimum $\phi = 0$ becomes degenerate with the low-temperature one, which at $T_c$ is given by $\phi_c \equiv (gh^3/6\pi\lambda) T_c$. For $T_c > T > T_0$ the two minima are separated by a barrier, which at $T = T_0$ disappears, turning the minimum at $\phi = 0$ into a maximum.

For $T > T_c$, the hidden sector is in the phase with $\phi = 0$. In this case, $\Delta V = 0$ in (9) and, as expected, the free energy gives an energy density of the form $\rho = \rho_\Lambda + \rho_R$. Notice that, for $O(1)$ couplings, we have $T_c \sim T_0 \sim v$, and $\rho_\Lambda \sim v^4$, so, naturally, $T_c \sim \rho_\Lambda^{1/4} \sim 10^{-3} eV$. If we require instead that $T_c \ll 10^{-3} eV$, so that a vacuum energy is attained through the condition $T_c < T < T_0$ [10], then necessarily $T_0 < \rho_\Lambda^{1/4}/10$, and we obtain the constraint $\lambda/g^2 h^4 < 10^{-6}$, which is not achieved with natural values of the parameters.

At high temperature the bubble nucleation rate is given by $\Gamma \sim T^4 \exp[\pm F_c(T)/T]$, where $F_c$ is the free energy of the critical bubble that is nucleated. $F_c(T)$ diverges at $T = T_c$, and
attained with an unnaturally large value of the surface tension \( \sigma \). Notice however that, for \( T \approx T_0 \) the nucleation rate is extremely high, \( \Gamma \sim T^4 \), so the phase transition will certainly end before \( T \) gets close to \( T_0 \). Indeed, we may roughly consider that bubble nucleation effectively begins when its rate becomes \( \Gamma \sim H^4 \). Using \( H^2 = 8 \pi \rho / 3 M^2_p \), with \( \rho \sim 10^3 \rho_\Lambda \) for \( T \sim 10^{-3} eV \), we see that the onset of nucleation happens as soon as \( F_c/T \) becomes smaller than \( \simeq 270 \).

If we require supercooling at \( T < T_c \sim 10^{-4} eV \), then we must have, on one hand, \( T_0 < T < T_c/10 \). This condition will be unnatural in any model, since in general \( T_0 \sim T_c \sim \rho_\Lambda^{1/4} \). As we have seen, the condition \( T_0 < \rho_\Lambda^{1/4} / 10 \) already constrains the parameters. If we require also \( T_0 < T_c/10 \) we find for our model the further constraint \( 1 - 10^{-2} < gh^4 / 6 \pi^2 \lambda < 1 \), so the parameters must be fine tuned with a precision of \( 10^{-2} \). This constraint is not compatible with the previous one. Of course, this is due to the simplicity of the model we are considering. Below, we consider a model in which \( T_c \) and \( T_0 \) are independent of each other. Notice, however, that even disregarding one of the two constraints, the other one still implies either unnatural values of the parameters or a fine tuning.

Besides, we must demand that \( \Gamma \sim H^4 \) still at \( T < 10^{-4} eV \). With \( \rho \sim \rho_\Lambda \), this requires \( F_c/T > 270 \), which is quite a large value for \( F_c \) if we take into account that \( T \ll T_c \). The free energy \( F_c \) is difficult to estimate analytically. In the thin wall approximation, it is given by \( F_c(T)/T \sim (16 \pi / 3) \sigma^3 T_c^4 / L^2 (T_c - T)^2 \), where \( \sigma \) is the bubble wall tension. Although \( F_c \rightarrow \infty \) for \( T \rightarrow T_c \), in the case \( T \ll T_c \), a large enough value of this quantity can only be attained with an unnaturally large value of the surface tension \( \sigma \). In fact, the thin wall approximation is no longer valid in this limit, and a numerical calculation is required. For a simple potential of the form of Eq. (10), such calculation has been performed [17, 18]. In the case \( T_0 \ll T_c \), the free energy of the critical bubble is given by

\[
F_c(T)/T = 7.7 \left( g^{1/4} / \lambda^{3/4} \right) \alpha^{3/2} f(\alpha),
\]

where \( \alpha = (T^2 - T_0^2) / T^2 \). In Eq. (11) we have used the constraint \( gh^4 / 6 \pi^2 \lambda \simeq 1 \) to eliminate the parameter \( h \). In Ref. [17], it was found that the function \( f(\alpha) \) is fit by the expression

\[
f(\alpha) \simeq 1 + \frac{\alpha}{4} \left( 1 + \frac{2.4}{1 - \alpha} + \frac{0.26}{(1 - \alpha)^2} \right)
\]

with an accuracy of 2%. Assume for definiteness that \( g \sim 10 \). Then, from Eq. (11) it is not difficult to find the constraints imposed by the condition \( F_c/T > 270 \). On one hand, if \( \lambda = O(1) \), such high value of \( F_c/T \) is attained only for \( T > 10T_0 \), so \( T_0 \) must be less than \( 10^{-2} T_c \). This increases the fine tuning on the parameters. On the other hand, for \( T \sim T_0 \) the condition is reached for unnaturally small values of the parameter \( \lambda \) (e.g., \( T \simeq 1.5 T_0 \) requires \( \lambda < 10^{-2} \)).

Notice that immediately after the completion of the phase transition there is still a non-vanishing vacuum energy. Indeed, the value \( \phi_c = \phi(T_c) \) of the free energy minimum at the critical temperature is displaced from the zero-temperature (true vacuum) value \( \phi(T = 0) = v \). However, after the phase transition \( \phi(T) \) evolves with temperature and the energy density \( \rho_{\text{vac}}(\phi) \) does not behave like a cosmological constant. During the phase coexistence period, instead, as one phase is converted into the other, the values \( \phi = 0 \) and \( \phi = \phi_c \) coexist at \( T_c \), and one expects that the average vacuum energy remains larger than a certain value \( \rho_{\text{vac}} = O(\rho_\Lambda) \). Indeed, since \( \rho_\Lambda \sim \lambda v^4 \) and \( T_c^4 \sim T_0^4 \simeq (12 \lambda / g)^2 v^4 / h^4 \), Eq. (6) gives a positive \( \rho_{\text{eff}} \sim \lambda v^4 \) as long as the coupling \( h \) is not too small (i.e., if \( h \simeq 1 \)).
According to Eq. (3), the period of phase coexistence can be long enough if the latent heat is close to its maximum value \( L_{\text{max}} \approx \rho R \). The latent heat \( L \equiv T \partial \Delta V / \partial T |_{T=T_c} \) is readily calculated for the potential (10). For \( T_c \sim T_0 \) and \( \phi_c \sim v \), we have \( L \approx \lambda v^2 \phi_c^2 \), so in order to accomplish \( L \sim \rho R \) we obtain the condition \( \phi_c/T_c \gtrsim 1 \), i.e., the phase transition must be strongly first-order. This is achieved for strong couplings \( h \gtrsim 1 \). Notice that within this approximation the latent heat \( L \) is unbound. In fact, for \( h\phi/T > 1 \) the high-temperature approximation (10) breaks down. If we use the exact one-loop result (8), the latent heat is close to its maximum value \( L \). In Ref. [8] it is shown that in this case the supercooling condition \( T/\rho \) is again achieved with a strong coupling \( h \), since the order parameter \( \phi_c/T_c \) increases with \( h \). Perturbativity imposes a generic upper limit \( h \lesssim \sqrt{4\pi} \). However, this limit depends on the details of the model [13].

B. A potential with Coleman-Weinberg symmetry breaking

Alternatively, one can obtain a strong phase transition by considering a gauge theory with a strong coupling scale \( \sim \rhoA^{1/4} \). A model in which a hidden \( SU(2) \) Yang-Mills theory has a chiral phase transition at a scale \( \Lambda_{SU(2)} \sim 10^{-3} \text{eV} \) has been considered by Goldberg [8]. The model has \( g = 34 \) d.o.f., comprising the \( SU(2) \) gauge fields and 8 Weyl doublets. In Ref. [8] it is shown that in this case the supercooling condition is \( T/\rhoA^{1/4} \lesssim 10^{-2} \).

A linear sigma model gives an effective potential of the form

\[
V(\phi, T) = A (T^2 - T_0^2) \phi^2/2 + \lambda \phi^4 \left[ \log(\phi/\phi_0) - 1/4 \right],
\]

(13)

where \( A \) depends on the number of fermions; in this case, \( A \approx 24\sqrt{\lambda} \) (see [8] for details). Notice that the temperature-dependent term here corresponds to the first term in the high-temperature expansion (10). Furthermore, the role of \( T_0 \) in the dynamics of the phase transition is similar to the one it played in the previous model: there is a barrier between the two vacua for \( T_0 < T < T_c \). So, in order to achieve the desired supercooling, the condition \( T_0 \ll T_c \) must be imposed. There is no reason for such a hierarchy, and in general it will be difficult to realize in a realistic model [8]. Nevertheless, in Eq. (13) the parameter \( T_0 \) can be set to 0, since the symmetry is already broken spontaneously in the Coleman-Weinberg manner by the last term. Setting \( T_0 = 0 \), the vacuum energy is given by the difference \( V(0) - V(\phi_0) \) at \( T = 0 \). It is related to \( T_c \) by \( \rhoA = (4.4T_c)^4 \). It can be then shown that the required supercooling is attained with \( \lambda \leq 0.01 \).

On the other hand, the latent heat is easily calculated for the free energy (13); we obtain \( L = AT_c^2 \phi_c^2 \). Here, the critical temperature and order parameter are related by \( \phi_c^2 = A (T_c^2 - T_0^2)/\lambda \). Setting \( T_0 = 0 \), we find \( L = AT_c^4/\lambda \approx 560T_c^4 \), which gives the ratio \( L/\rhoR \approx 50 \). Obviously, this value of \( L \) is unrealistically large. This means that the high-temperature approximation is not valid, so expression (8) should be used for the finite-\( T \) correction. This fact indicates that supercooling demands an extremely strong phase transition.

Since the supercooling requirement implies such a large \( L \), we infer that the conditions for phase coexistence will be less stringent in this model. If supercooling is not needed, the
restrictions $T_0 \ll T_c$, $\lambda \ll 1$ can be relaxed. Without entering into the details of the model, we can consider the condition $L/\rho_R \sim 1$ to get an idea of the requirements of a long enough phase-coexistence stage. Letting $T_0 \neq 0$, we have the condition $50 (T_c^2 - T_0^2) / T_c^2 \sim 1$, which is independent of $\lambda$ and is satisfied even for $T_0$ very close to $T_c$. This shows that the phase coexistence scenario arises naturally in this model.

The comparison between the supercooling and phase-coexistence scenarios is transparent in this case because the latent heat is proportional to the difference $T_c^2 - T_0^2$, which is required to be unnaturally large in the supercooling scenario. Thus, a large amount of supercooling is clearly more difficult to attain than a sizeable latent heat, and the phase coexistence scenario is favored. However, to get a precise comparison of the two scenarios a numerical calculation of the phase transition would be suitable. We shall perform such calculation elsewhere. In any case, both supercooling and phase-coexistence may occur in general, and both effects contribute to delay the completion of the phase transition.

IV. CONCLUSIONS

We have pointed out that a late-time first-order phase transition may enter a long phase-equilibrium stage, thus avoiding the need of excessive supercooling to account for the durability of a false vacuum phase. Basically, this occurs if the cooling method used by the universe (namely, the adiabatic expansion of a relativistic gas) fails in taking away the latent heat associated to the transition. Thus, the main requirement for this scenario is that the latent heat must be of the order of the energy density of radiation. We have shown that if the temperature scale of the phase transition is $T_c \sim 10^{-3}$eV, the current equation of state for phase coexistence is essentially $\omega = -1$, and the dark energy density has the correct magnitude. The distinctive inhomogeneities in $\rho_{\text{vac}}$ during phase coexistence may leave an imprint on large scale structure. We have argued that a long period of phase coexistence may arise naturally in the context of a particle physics theory, in contrast to the supercooling case. In general, the condition $L \sim \rho_R$ is achieved in theories with a strong coupling $h \gtrsim 1$.

Acknowledgements

I thank C. Biggio, E. Massó, G. Zsembinszki, and especially J. Garriga for helpful conversations. I also thank D. Scott for useful comments.

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