Polarization Studies: testing explanations of the $B \to \phi K^*$ puzzle and $B \to VT$ decays

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It has been observed in $B \to \phi K^*$ and $B^0(+) \to \rho(+) K^{*0}$ that the fraction of transverse decays, $f_T$, and the fraction of longitudinal decays, $f_L$, are roughly equal, in opposition to the naive expectation, $f_T/f_L \ll 1$. If one requires a single explanation of all polarization puzzles, two possibilities remain within the standard model (SM): penguin annihilation and rescattering. We examine the predictions of these two explanations for $f_T/f_L$ in $b \to d$ decays. We also study polarization observables in $B \to VT$ decays ($V$ is a vector and $T$ is a tensor meson) to probe whether the two SM explanations account for the $f_T/f_L$ ratio in this type of decays and to further investigate the two new-physics scenarios which explain the data in $B \to \pi K$ and the $\phi(p)K^*$ polarization measurements.

1. Introduction

In certain $B \to V_1 V_2$ decays ($V_i$ is a light charmless vector meson) dominated by $b \to s$ penguin transitions in the standard model (SM), an unexpected observation has been made. Considering that the final particles have spin 1, it is straightforward to see that a given $B$ decay into two vector mesons can be analyzed as three separated $B$ decays – one for each polarization of the vector states (one longitudinal, two transverse) – into two spinless particles. Naively, the transverse amplitudes are suppressed by a factor of size $m_V/m_B$ ($V$ is one of the vector mesons) with respect to the longitudinal amplitude. As such, one expects the fraction of transverse decays, $f_T$, to be much less than the fraction of longitudinal decays, $f_L$. However, it has been measured that these two fractions are roughly equal in $B \to \phi K^*$ [4, 5, 7] and $B^{0(+)\to \rho(+) K^{*0}}$ [2, 3] (see Table I for recent measurements).

| Mode               | $B(10^{-6})$ | $f_L$ | $f_T$ |
|--------------------|-------------|-------|-------|
| $\phi K^{*0}$ [4, 5, 6] | 9.5 ± 0.9 | 0.49 ± 0.04 | 0.27 ±0.04 |
| $\phi K^*$ [4, 5, 7] | 10.0 ± 1.1 | 0.50 ± 0.05 | 0.20 ± 0.05 |
| $\rho^+ K^{*+}$ [2, 3] | 9.2 ± 1.5 | 0.48 ± 0.08 |
| $\rho^0 K^{*0}$ [3] | 5.6 ± 1.6 | 0.57 ± 0.12 |
| $\rho^- K^{*-}$ [3] | (<12.0) |
| $\rho^0 K^{*0}_2(1430)^0$ [8] | (3.6 ± 1.8) | (0.9 ± 0.2) |
| $\phi K^*_2(1430)^0$ [7] | 7.8 ± 1.3 | 0.85 ± 0.08 | 0.05 ± 0.05 |

Table I Measurements of the branching fraction $B$, longitudinal polarization fraction $f_L$, and fraction of parity-odd transverse amplitude $f_T$, for $B \to \phi K^*$, $\rho K^*$, and $\phi K^*_2(1430)^0$, expected to proceed through a $b \to s$ transition [3, 8, 4]. Numbers in parentheses indicate observables measured with less than 4σ significance.

The differences between the measurements and the naive expectations could be interpreted in favour of the presence of physics beyond the SM [10], though none of these discrepancies has been statistically significant. On the other hand, two explanations remain as possible solutions within the SM when they are considered one at a time: penguin annihilation [11] and rescattering [12, 13].

These two explanations account for a large $f_T/f_L$ in $b \to s$ decays. However, the key point is that a large $f_T/f_L$ is also predicted in certain $b \to d$ decays [14]. The measurement of $f_T/f_L$ in these $b \to d$ decays will allow us to test penguin annihilation and rescattering as the explanations of the observed $f_T/f_L$ ratio in $B \to \phi K^*$ decays [15]. Besides, we also investigate their predictions in $B \to VT$ decays ($T$ is a tensor meson) [16]. Since in this case there are too three polarizations, $f_T/f_L$ can be measured. This ratio has been experimentally determined in $B \to \phi K^*_2$ [4] (see Table I) and it is small. The potential solutions must also explain $f_T/f_L$ in $B \to VT$ decays. In this paper, we study this matter, both within the SM and assuming new physics (NP).

2. SM Explanations of $f_T/f_L$ in $B \to \phi K^*$

We focus on $B \to V_1 V_2$ decays. In this case the amplitude for the process is given in the linear polarization basis by

$$M = A_0 \varepsilon_1^T \cdot \varepsilon_2^L - \frac{1}{\sqrt{2}} A_1 \varepsilon_1^T \cdot \varepsilon_2^T - \frac{i}{\sqrt{2}} A_1 \varepsilon_1^T \times \varepsilon_2^T \cdot \hat{\rho} ,$$

(1)

where the polarizations of the final-state vector mesons ($\varepsilon_i^j$) are either longitudinal ($A_0$), or transverse to their directions of motion and parallel ($A_1$) or perpendicular ($A_2$) to one another. Along this article we also make use of the helicity basis for the transverse polarizations, where $A_2 = (A_2 \pm A_2)/\sqrt{2}$.

The relative fractions into $V$ meson states with longitudinal and transverse polarizations is

$$f_L = \frac{|A_0|^2}{|A_0|^2 + |A_1|^2 + |A_2|^2} , f_T = \frac{|A_1|^2 + |A_2|^2}{|A_0|^2 + |A_1|^2 + |A_2|^2} ,$$

where $f_T = (1 - f_L)$. Moreover, we can define the relative fraction into final states with perpendicular
polarizations as

\[ f_\perp = \frac{|A_\perp|^2}{|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2} . \]

As we noted in the introduction, final-state particles in \( B \to VT \) decays have also three possible polarizations. Therefore, the amplitude for these decays can be decomposed in the same way as in Eq. (1).

We stated above that there are two potential SM solutions for the polarization puzzle observed in \( B \to \phi K^* \). In this section we review these two explanations.

We begin with penguin annihilation in the context of QCD factorization (QCDf) [17]. Normally, annihilation contributions are expected to be small as they are higher order in the \( \alpha_s/m_b \) expansion, and thus ignored. However, within QCDf, it is possible that the coefficients of these terms are large [11]. Penguin annihilation is not calculable in QCDf because of divergences which can be parametrized in terms of unknown quantities chosen to fit the data in \( B \to \phi K^* \). The penguin annihilation amplitude arises only from penguin diagrams with an internal \( f \) quark.

We now discuss the second explanation: rescattering of charm intermediate states can generate large transverse polarization. In a particular picture [12] of this solution, heavy charm mesons rescatter to \( B \to \phi K^* \). Then, if the transverse polarization is not reduced in the scattering process, this mechanism will lead to a large \( f_T/f_L \).

With the previous paragraph we complete our brief review about the different physical origin of the two explanations. We now take a look at the similarities of calculation. In order to see this, consider the penguin contribution \( P_q \) for the decay \( b \to \bar{q}q'q' \) (\( q = d, s \), \( q' = u, d, s \)):

\[ P_q = V_{ub}^* V_{cq} P_u + V_{ub}^* V_{cq} P_c + V_{ub}^* V_{cq} P_t \]

\[ = V_{cb}^* V_{cq} (P_c - P_u) + V_{tb}^* V_{tq} (P_t - P_u) . \]

Both in penguin annihilation and rescattering, the effect of the dominant contribution to the transverse amplitudes is simply the addition of one term in \( P_t \) and \( P_c \) respectively. Below we follow the following prescription: we take into account the additional SM effects by adding a single amplitude to represent the dominant contribution to the transverse amplitudes.

### 3. \( B \to \rho \rho \) Decays

Both penguin annihilation and rescattering explain the \( f_T/f_L \) ratio in the \( b \to s \) decay of \( B \to \phi K^* \) by modifying the penguin amplitude. A similar modification must appear in some \( b \to d \) decays. Then, the question that we intend to reply in this section is how their effects appear in \( B \to \rho \rho \) decays.

Within the diagrammatic approach [18], the three \( B \to \rho \rho \) amplitudes are given mainly by three diagrams: the color-favored and color-suppressed tree amplitudes \( T \) and \( C \), and the gluonic penguin amplitude \( P \),

\[ -\sqrt{2} A(B^+ \to \rho^+ \rho^0) = T_\tau + C , \]

\[ -A(B^0_d \to \rho^+ \rho^0) = T_\tau + P + R , \]

\[ -\sqrt{2} A(B^0_d \to \rho^0 \rho^0) = C - P - R . \]

Here \( R \) stands for the single extra term arising from the new penguin annihilation or rescattering contribution. Some immediate conclusions can be extracted from the above equations. First, since a modification of \( P \) is involved, \( f_T/f_L \) in \( B^+ \to \rho^+ \rho^0 \) will not be affected. This agrees with observation (see Table [11]). Second, in order to calculate \( f_T/f_L \), it is necessary to estimate the size of \( R \). As discussed earlier, rescattering and penguin annihilation affects the penguin amplitude \( P \); thus, \( |R| \sim |P| \). This shows that \( f_T/f_L \) is expected to be small in \( B^+ \to \rho^+ \rho^0 \), since it is proportional to \( |R|^2/|P|^2 \sim |P|^2/|P|^2 \). This also agrees with observation (Table [11]). Finally, \( f_T/f_L \) can be large in \( B^0_d \to \rho^0 \rho^0 \) since the contributions to the transverse and longitudinal polarizations are the same size. It will be interesting to measure this precisely.

| Mode       | \( \mathcal{B}(10^{-6}) \) | \( f_T \) |
|------------|---------------------------|---------|
| \( \rho^0 \rho^+ \) | 18.2 ± 3.0 \( \times 10^{-6} \) | 0.912 \( \pm 0.044 \) |
| \( \rho^+ \rho^- \) | 24.2 \( \pm 3.0 \) \( \times 10^{-6} \) | 0.976 \( \pm 0.028 \) |
| \( \rho^0 \rho^0 \) | 1.07 \( \pm 0.35 \) \( \times 10^{-6} \) | 0.86 \( \pm 0.12 \) |

Table II. Measurements of the branching fraction \( \mathcal{B} \) and longitudinal polarization fraction \( f_L \) for \( B^+ \) and \( B^0_d \) meson decays into \( \rho \rho \) final states. Numbers in parentheses indicate observables measured with less than \( 4\sigma \) significance.

There are some further tests to be performed. Since there is only one added amplitude, one has \( |A_+ (B^0_d \to \rho^0 \rho^0)| = |A_- (B^0_d \to \rho^0 \rho^0)| \), and similarly for \( A_- \) and \( A_+ \) (where \( A_\pm \) are given in the helicity basis and \( A_\pm \) stands for the corresponding amplitudes in the CP-conjugated decay). If this is not found, penguin annihilation and rescattering will be ruled out. Another way to probe the SM explanations is by using SU(3) since the extra transverse amplitudes, \( |R| \) (\( b \to d \)) and \( |R'| \) (\( b \to s \)), are related by flavor symmetries. This allows us to estimate \( f_T/f_L \) in \( B^0_d \to \rho^0 \rho^0 \) from \( B^+ \to \rho^+ K^{*0} \) decays. If we assume that SU(3) is an exact symmetry, we obtain an explicit relation between \( |R| \) and \( |R'| \): \( R = |V_{td}(cd)/V_{ts}(cs)| R' \) in penguin annihilation (rescattering). Thus, we can obtain the \( f_T/f_L \) ratio in \( B^0_d \to \rho^0 \rho^0 \) by using experimental data on \( B^+ \to \rho^+ K^{*0} \) decays, and we find

\[ |A_T (B^+ \to \rho^+ K^{*0})|^2 = (5.10 \pm 1.14) \times 10^{-16} \text{ GeV}^2 , \]

\[ |A_L (B^0_d \to \rho^0 \rho^0)|^2 = (2.10 \pm 0.81) \times 10^{-16} \text{ GeV}^2 , \]
leading to
\[ f_t/f_L(B_d^0 \rightarrow \rho^0 \rho^0) = |V_{td}/V_{ts}|^2 (2.43 \pm 1.08) \, . \]
This result agrees with data taken directly from \( B_d^0 \rightarrow \rho^0 \rho^0 \) (see Table III):
\[ f_t/f_L(B_d^0 \rightarrow \rho^0 \rho^0) = (1 - f_L)/f_L = 0.16 \pm 0.15 \, . \]

The agreement is good because of the large errors, making the hypothesis of no violation of SU(3) a consistent assumption at this accuracy level. Equally, the measurement does not give a definite answer as to whether \( f_t/f_L \) is large or small. Another point is related to this: if central values are taken, \( f_t/f_L \) is not large after all. This shows that \( f_t/f_L \) is not guaranteed to be large in \( B_d^0 \rightarrow \rho^0 \rho^0 \). The reason for this is that, due to the additional amplitude \( C, f_t \) can be big, making \( f_t/f_L \) small. There is a further complication: if \( C \) contributes significantly to the transverse polarization, as it could be the case in QCDf, then a test of the explanations might be performed by means of a time-dependent angular analysis (see [13] for details). The lesson here is that it is best to consider \( b \rightarrow d \) decays for which \( f_t/f_L \) is expected to be large and which receive only one dominant contribution to the transverse polarization.

4. U-spin Pairs

We have previously stressed the idea of measuring \( f_t/f_L \) in \( b \rightarrow d \) decays. But this raises the question: how do we choose the \( b \rightarrow d \) decay to study? U-spin symmetry can help us to investigate this issue. Pairs of \( B \) decays which are related by U-spin are given in Ref. [22]. In \( B \rightarrow VV \) form, these are

1. \( B_d^0 \rightarrow K^{*+} \rho^- \) and \( B_s \rightarrow \rho^+ K^{*-} \),
2. \( B_s \rightarrow K^{*+} K^{*-} \) and \( B_d^0 \rightarrow \rho^+ \rho^- \),
3. \( B_d^0 \rightarrow K^{*0} \rho^0 \) and \( B_s \rightarrow K^{*0} \rho^0 \),
4. \( B^+ \rightarrow K^{*0} \rho^0 \) and \( B^+ \rightarrow K^{*0} K^{*+} \),
5. \( B_s \rightarrow K^{*0} \bar{K}^{*0} \) and \( B_d^0 \rightarrow \bar{K}^{*0} K^{*0} \).

In all cases, the first decay is \( \Delta S = 1 \) (\( b \rightarrow s \)); the second is \( \Delta S = 0 \) (\( b \rightarrow d \)). The procedure here is to measure the polarizations in the \( b \rightarrow s \) decay, and compare them with the measurements in the corresponding \( b \rightarrow d \) decay.

As noted in the past section, the best \( b \rightarrow d \) decays to be considered in the tests are those with an expected large \( f_t/f_L \) ratio and with only one contribution to the transverse amplitude. Given this, the best possibilities are the last two pairs: (i) \( B^+ \rightarrow K^{*0} \rho^+ \) (\( b \rightarrow s \)) and \( B^+ \rightarrow K^{*0} K^{*+} \) (\( b \rightarrow d \)) and (ii) \( B_s \rightarrow K^{*0} \bar{K}^{*0} \) (\( b \rightarrow s \)) and \( B_d^0 \rightarrow K^{*0} K^{*0} \) (\( b \rightarrow d \)).

We urge the measurement of \( f_t/f_L \) in these pairs of decays.

The explanations of \( f_t/f_L \) in \( B \rightarrow \phi K^* \) then make three predictions:

- \( f_t/f_L \) is expected to be large in both the \( b \rightarrow s \) and the corresponding \( b \rightarrow d \) decay.
- \( |A_+| \) and \( |\bar{A}_-| \) are expected to be equal in both the \( B \) and \( \bar{B} \) decays, and similarly for \( A_- \) and \( A_+ \).
- \( R' \) and \( R \) can be extracted from the \( b \rightarrow s \) and \( b \rightarrow d \) decays, respectively. These should be related by SU(3) (including SU(3) breaking).

If any of these predictions fail, penguin annihilation and rescattering are ruled out in the U-spin limit or for small U-spin breaking.

Since the ratio of \( f_t/f_L \) in these pairs of decays measures SU(3) breaking, an additional test can be made. If one ignores SU(3) breaking, one has the following prediction
\[ (f_t/f_L)_{b \rightarrow s} = (f_t/f_L)_{b \rightarrow d} \, . \] (2)

The breaking of SU(3) in the above equation is model-dependent. If it is found experimentally that the above relation is broken badly, then particular models of penguin annihilation and rescattering will have to invent a mechanism to generate large SU(3)-breaking effects or they will be ruled out. In other words, Eq. (2) can be used to constrain those specific models.

Up to here, we have not distinguished penguin annihilation and rescattering since their effects are very similar. We will see now that there is a possible way to differentiate them. As noted above, rescattering involves only a change to \( P_r \), while penguin annihilation involves only \( P_t \). However, the weak phase of these pieces in \( b \rightarrow d \) decays is different: \( \beta \) (rescattering) \( \sim 0 \), \( \phi \) (penguin annihilation) \( \sim -\beta \). If this weak phase can be measured, one could distinguish penguin annihilation and rescattering. The measurement of that phase can be made by performing a time-dependent angular analysis of the \( b \rightarrow d \) decay. One has to focus on observables which provide information about the relative phase of the transverse amplitudes in the direct and the \( CP \)-conjugated decays (see [15] for a more detailed discussion). Of the decays pointed out as those which satisfy the necessary requirements to carry out meaningful tests, there is only one for which a time-dependent angular analysis can be done: \( B_d^0 \rightarrow \bar{K}^{*0} K^{*0} \).

5. \( B \rightarrow VT \) Decays

So far we have been discussing \( B \rightarrow VV \) decays. In the current section we concentrate on \( B \rightarrow VT \) processes. As mentioned in the introduction, these decays
are also analyzed in terms of three polarizations and the $f_\tau/f_L$ ratio can be measured.

The various explanations must account for the $f_\tau/f_L$ data in both $B \to V_1 V_2$ and $B \to V T$ decays. We examine this question now, both in the SM and with NP.

We first briefly review the SM (naive) prediction for $f_\tau/f_L$. In the large-energy effective theory \cite{23}, it can be shown that the $B \to T$ form factors are expressible in terms of two universal quantities. Depending on the relative magnitude of these quantities, three different scenarios are possible. It is found that two of them predict $f_\tau/f_L \ll 1$, whereas the other $f_\tau/f_L \sim 1$. However, since the last case is in contradiction with the experimental results for $B \to \phi K_2^*$ (see Table I), we conclude that the SM naively predicts $f_\tau/f_L \ll 1$ in $B \to V T$ decays.

The next question is: which are the penguin annihilation and rescattering predictions for $f_\tau/f_L$? In order to answer this question, we must establish whether or not the individual explanations depend on the final-state particles. If they do not, then the prediction for $f_\tau/f_L$ in $B \to \phi K_2^*$ will be the same as that in $B \to \phi \rho^*$, which is in disagreement with experiment. The calculation of penguin annihilation does depend on the final-state wave function. Thus, it is possible that $f_\tau/f_L$ is small in $B \to \phi K_2^*$ for the three cases discussed above, in agreement with experiment. Within the rescattering solution, it is again possible to choose parameters in order to obtain a small $f_\tau/f_L$ in $B \to \phi K_2^*$. These arguments can be extended to any other $B \to V T$ decay. Therefore, since there is a new set of parameters for each final state, and it is virtually impossible to calculate the values of the parameters, we conclude that both penguin annihilation and rescattering are viable, but not very convincing.

Since the CP measurements in many penguin decays that proceed through $b \to s$ transitions \cite{24} and the polarization measurements in some $B \to V_1 V_2$ ($b \to s \bar{s}$) to be in conflict with naive SM expectations decays, it is not unreasonable to attempt to understand the data assuming new physics. The important question to ask is then the following: can we find a unified new-physics explanation for all the discrepancies so far reported in measurements of pure-penguin or penguin-dominated decays? And, what would we expect for $B \to V T$ decays if NP is assumed? After considering a general parametrization of NP as in Ref. 10, it appears that the NP scenario is the same as that of the SM – the prediction of $f_\tau/f_L$ in $B \to \phi K_2^*$ depends on the values of unknown parameters and with the proper choice it can be made consistent with the experimental results (see 16 for details). However, the difference is that, with penguin annihilation and rescattering, the parameters are essentially incalculable, while the NP prediction depends on form factors. Although the values of these form factors are not very well known at the moment, they can be calculated. We strongly urge that the $B \to T$ form factors be computed. In other decays like $B \to \rho K_2^*$, the prediction for $f_\tau/f_L$ is the same as that of the SM since the NP does not affect these processes. It will be important to measure the polarization particularly in $B_d^0 \to \rho^0 K_2^0$ and $B^+ \to \rho^0 K_2^+$ decays in order to test the SM and this type of NP.

6. Conclusions

We have seen that penguin annihilation and rescattering are two possible SM solutions to the polarization puzzles in $B \to \phi K^*$ and $B_0^0 \to \rho^0 \rho^0$, it may be possible to test these explanations by comparing $B_d^0 \to \rho \rho^0$ with $B^+ \to K^{*0} \rho^+$ and see if flavor SU(3) is respected. We have examined other $b \to d$ decays related by U-spin to certain $b \to s$ decays. Two promising pairs are: (i) $B^+ \to K^{*0} \rho^+ (b \to s)$ and $B^+ \to K^{*0} K^{-0} (b \to d)$ and (ii) $B_s \to K^{*0} \bar{K}^{0*} (b \to s)$ and $B_d^0 \to K^{*0} K^{0*} (b \to d)$. A large $f_\tau/f_L$ is predicted by penguin annihilation or rescattering in these decays. We have also mentioned that it is possible to distinguish penguin annihilation from rescattering by performing a time-dependent angular analysis of $B_d^0 \to K^{*0} K^{*0}$. This is difficult experimentally, but it may be possible at a future machine.

Furthermore, we have analyzed the predictions of the SM (naive and extended) and NP in $B \to V T$ decays. The SM naively reproduces the polarization measurements in $B \to \phi K_2^*$ and predicts in general a small $f_\tau/f_L$ ratio. The polarization predictions of both penguin annihilation and rescattering are not certain. That is, the predictions depend on a new set of parameters for each final state. It is therefore possible that both explanations agree with the $f_\tau/f_L$ measurements in $B \to \phi K^*$ and $B \to \phi K_2^*$. Finally, in Ref. 10 it was found that only two new-physics operators can account for the discrepancies in both the $\pi K$ data and the $\phi(\rho) K^*$ polarization measurements. We have mentioned that the prediction of these operators can account for the small $f_\tau/f_L$ ratio observed in $B \to \phi K_2^*$. This prediction can be tested by explicit computations of the $B \to T$ form factors.

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