A tale of two sentiment scales: Disentangling short-run and long-run components in multivariate sentiment dynamics

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Abstract

We propose a novel approach to sentiment data filtering for a portfolio of assets. In our framework, a dynamic factor model drives the evolution of the observed sentiment and allows to identify two distinct components: a long-term component, modeled as a random walk, and a short-term component driven by a stationary VAR(1) process. Our model encompasses alternative approaches available in literature and can be readily estimated by means of Kalman filtering and expectation maximization. This feature makes it convenient when the cross-sectional dimension of the sentiment increases. By applying the model to a portfolio of Dow Jones stocks, we find that the long term component co-integrates with the market principal factor, while the short term one captures transient swings of the market associated with the idiosyncratic components and captures the correlation structure of returns. Finally, using quantile

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regressions, we assess the significance of the contemporaneous and lagged explanatory power of sentiment on returns finding strong statistical evidence when extreme returns, especially negative ones, are considered.

Keywords: Sentiment analysis; dynamic factor models; Kalman filter; expectation maximization; quantile regression

1 Introduction

Nowadays, as Ignacio Ramonet wrote in The Tyranny of Communication, “a single copy of the Sunday edition of the New York Times contains more information than an educated person in the eighteenth century would consume in a lifetime”. This huge amount of information cannot be read by a single person. Recent developments in machine learning algorithms for sentiment analysis help us to categorise and extract signals from text data and pave the way for a new area of research. The use of these new sources of textual data has become popular to analyse the relationship between sentiment and other economic variables using econometric techniques. (Algaba et al., 2016) refer to this new strand of literature as Sentometrics. For instance, (Groß-Klußman and Hautsch, 2011) study the impact of unexpected news on the displayed quotes in a limit order book, (Sun et al., 2016) show that intraday S&P 500 index returns are predictable using lagged half-hour investor sentiment, (Antweiler and Frank, 2004; Borovkova and Mahakena, 2015; Allen et al., 2015; Smales, 2015) study the impact of sentiment on volatilities, (Peterson, 2016) investigates the trading strategies based on sentiment, (Tetlock, 2007; Garcia, 2013) consider the Dow Jones Industrial Average (DJIA) index predictability using sentiment, (Calomiris and Mamaysky, 2019) show how the predictability can be exploited in different markets around the world, (Ranco et al., 2015) analyse the impact of social media attention on market dynamics, (Borovkova, 2015) develops risk measures based on sentiment index, and (Lillo et al., 2015)
show that different types of investors react differently to news sentiment.

The approaches to sentiment analysis can be broadly classified in three categories. The first class is based on (mostly supervised) Machine Learning techniques. Three steps are typically considered. The first one is to collect textual data forming the training dataset. The second step is to select the text features for classification and to pre-process the data according to the selection. The final step is to apply a classification algorithm to the textual data. As an example, (Pang et al., 2002) compare the performance of Naive Bayes, support vector machines, and maximum entropy algorithm to classify positive or negative movie reviews. The second category is the lexicon-based approach. It also typically consists of three steps. The first step is the selection of a dictionary of $N$ words which could be relevant for a specific topic (e.g. the word *great* is considered as a positive word to review a movie). The second step consists in tokenizing the textual data and, for each word in the dictionary, count how many times it appears in the text. This process can be visualized with a vector of length $N$ where the $i$-th element represents the number of times the $i$-th word of the dictionary is mentioned in the text. Finally, a measure takes the vector of length $N$ as an input and gives a quantitative score as an output. One can refer to (Loughran and McDonald, 2011) for a relevant example in the financial literature. The third and last approach is a combination of methodologies coming from the first and second approach. For an overview of textual data treatments and computational techniques, we refer to the review paper (Vohra and Teraiva, 2013) and the book (Liu, 2015).

However, as observed by Zygmunt Bauman in Consuming Life, as the number of information increases also the number of useless information increases, and the noise becomes predominant. Two different non-exclusive methods have been explored in the literature to remove or, at least, mitigate the impact of useless information. In the first case, a general-to-specific approach is used directly on the textual data. The amount of information can be reduced selecting only verified news (i.e. eliminating fake news), considering only the words which are closely related to the topic of interest, considering the importance of any news (e.g. (Da et al., 2011)), selecting only news which appear for the first time (e.g. Thomson Reuters News Analytics engine uses the novelty variable, see (Borovkova et al., 2017)), or
weighting a news by means of a measure of attention (e.g., with the number of clicks it receives when published in a news portal (Ranco et al., 2016)). Obviously, the selection of the relevant data is application-specific. For instance, fake news may be irrelevant to forecast the GDP of a country but may be crucial to forecast the results of an election (e.g. (Allcott and Gentzkow, 2017)).

In a second case, sentiment time series are directly considered, rather than the text source they are built from. The noise is dealt with after the sentiment has been provided exogenously. Various approaches have been proposed to filter the observed noisy sentiment. (Thorsrud, 2018) applies a 60-day moving average, (Peterson, 2016) uses the Moving Average Convergence-Divergence methodology proposed in (Appel, 2003) and (Borovkova and Mahakena, 2013; Audrino and Teterova, 2019; Borovkova et al., 2017) introduce the Local News Sentiment Level model (LNSL), a univariate method which takes inspiration from the Local Level model of (Durbin and Koopman, 2012). In spite of its convenience from a practical perspective, the moving average approach is not statistical sound and the window length is usually chosen following rules of thumb which have been tested empirically but lack a clear theoretical motivation. The methods based on the Kalman-Filter techniques present a natural and computationally simple choice to extract informative signal. Unfortunately, when multiple assets are considered in the analysis, the LNSL model does not exploit the multivariate nature of the data. One goal of the paper is to show that the covariance structure is very informative in sentiment time series analysis.

The first contribution of this paper is to extend the existing time series methods in the latter stream of literature. We propose to model noisy sentiment disentangling two different sentiment signals. In our approach, the observed sentiment follows a linear Gaussian state-space model with three relevant components. A first component, named \textit{long-term sentiment} is modeled as a random walk, the second component is termed \textit{short-term sentiment} and follows a VAR(1) process, and the last component is an i.i.d Gaussian \textit{observation noise} process. We name the novel sentiment state-space model Multivariate Long Short Sentiment (MLSS). We empirically show that the decomposition provides a better insight
on the nature of sentiment time series, linking the long-term sentiment to the long-term evolution of the market – proxied by the market factor – while the short-term sentiments reflect transient swing of the market mood and is more related to the market idiosyncratic components. Specifically, we find that i) the long-term sentiment cointegrates with the first market factor extracted via PCA; ii) the correlation structure of the short-term sentiment explains a significant and sizable fraction of correlation of return residuals in a CAPM model. Finally, we show that multivariate local level model is selected by the data, with respect to alternative nested models, such as the LNSL, and provides a better description of the sentiment series in terms of signal to noise ratios.

The second contribution of the paper is to unravel the relation between news and market returns conditionally on quantile levels. We perform various quantile regressions showing that sentiment has good explanatory power of returns. When contemporaneous effects are considered, the result is expected and holds for all models at intermediate quantile levels. However, when the analysis is focused on abnormal days – i.e. days for which returns belong to the 1% and 99% quantiles – neither the noisy sentiment nor the filtered sentiment from an LNSL model explain the observed market returns. The only model achieving statistical significance is the MLSS. This result shows that it is essential to filter the noisy sentiment according to the MLSS, which exploits both the multivariate structure of the data and disentangles the long- and short-term components. Moreover, a test performed on the single components confirms the intuition that the short-term sentiment is the one responsible for the contemporaneous explanatory power. The empirical evidence in favor of the MLSS becomes even more compelling when lagged relations are tested. When a single lag is considered, i.e. whether yesterday sentiment explains today returns, the significance of all models, but MLSS, drops to zero. This result holds across all quantile levels. Instead, for quantiles smaller than 10% and larger than 90%, the return predictability for the MLSS model is highly significant. As before, the decomposition in two time scales is essential and the short-term component is the one responsible of the effect. The analysis extended including lagged sentiment – up to five days – confirms previous findings by (Garcia, 2013) that past sentiment contributes in shaping present returns. Interestingly, this is true for
quantiles between 5% and 10%, both negative and positive, but neither in the median region nor for extreme days. In light of this findings, we finally investigated whether media and social news immediately digest market returns and whether this relation depends on the sign of returns. Our results provide a clear picture showing that i) the impact of market returns on sentiment is significant up to five days in the future when negative extreme returns – i.e. belonging to quantiles from 1% to 10% – are considered, ii) when positive returns are considered the impact rapidly fades out and is significant only for quantiles smaller than 5%, iii) previous findings become not significant if the MLSS sentiment is replaced by the observed noisy sentiment.

The rest of the paper is organized as follows. In section 2 we develop the multivariate model for the sentiment and discuss the estimation technique. In section 3 we introduce the TRMI sentiment index and describe the data used in the analysis. In section 4 we report the empirical findings and discuss the advantages of the multivariate approach. In section 5 we compare the various techniques and report the performances of the long-short sentiment decomposition in explaining daily returns. Section 6 draws the relevant conclusions and sketch possible future research directions.

2 The Model

Consider $K$ assets and the corresponding $K$ observed daily sentiment series $S^i_t$ where $i = 1, \ldots, K$. The observed daily sentiment $S^i_t$ quantifies the investors and consumers opinions about company $i$. In most cases, the observed sentiment is a continuous number in a compact set.

The Local News Sentiment Level model (LNSL), presented in (Borovkova and Mahakena, 2015) and subsequently used in (Audrino and Tetereva, 2019), reads as follows

$$S^i_t = F^i_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^i_\epsilon),$$

$$F^i_t = F^i_{t-1} + v_t, \quad v_t \sim N(0, \sigma^i_v).$$

(1)
for every $i = 1, \ldots, K$. This model is a univariate specification of the Local Level model of (Durbin and Koopman, 2012). The latent sentiment series $F_i^t$ are considered as slowly changing components, modeled as independent random walks and the parameters $\sigma_i^\epsilon$ and $\sigma_i^v$ are estimated via MLE.

Since the LNSL model does not consider the correlations of the innovations among the $K$ assets, we can easily derive its multivariate version as

$$S_t = F_t + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, R),$$
$$F_t = F_{t-1} + v_t, \quad v_t \sim \mathcal{N}(0, Q).$$

(2)

where $S_t = [S_1^t, \ldots, S_K^t]'$ and $F_t = [F_1^t, \ldots, F_K^t]'$ are $K$ dimensional vectors, $Q$ is a $K \times K$ symmetric matrix and $R$ is a $K \times K$ diagonal matrix. We refer to the multidimensional LNSL model as MLNSL. The synchronous correlation among the innovations of the latent sentiment are described by the covariance matrix $Q$, while the correlations among the observation noises are assumed to be 0. Clearly, the LNSL model is a special case of the MLNSL model when the matrix $Q$ is diagonal. Since the number of parameters for this model scales as $K^2$, the MLE of the MLNSL model is computationally demanding. For this reason, we use the Kalman-EM approach described in (Corsi et al., 2015).

The idea of the LNSL and MLNSL models is that the latent sentiment is a slowly changing component with a Gaussian disturbance. In their empirical studies, (Audrino and Tetereva, 2019) observe that the signal to noise ratio $\sigma_v^2/\sigma_\epsilon^2$, obtained using the LNSL filter, is very small. This finding points out that the majority of the daily changes in the sentiment series are considered as noise. One possible explanation of this result is that the Local Level specification of these models is not sufficiently rich to capture all the signals from the observed sentiment. Indeed, in newspapers and social media there is a consistent amount of articles and opinions which represent fast trends or rapidly changing consumer preferences. Following the recent strand of literature on persuasion (Gerber et al., 2011; Hill et al., 2013), these fast trends have strong but short-lived effects on consumer preferences. Since the (M)LNSL model interprets the latent sentiment as an integrated series, these signals are
considered as noise.

The main contribution of this paper is to define a new model which disentangles the slowly changing sentiment from a rapidly changing sentiment, that we name short-term sentiment, and the observation noise. In addition, we assume that when we consider a set of firms with common characteristics, e.g. belonging to the same sector, market, or country, their slowly changing components are affected by the same trends and shocks. For this reason, in our model we consider $q \leq K$ common factors driving the slowly changing dynamics and we define these common factors as long-term sentiment.

Specifically, the long-term sentiment $F_t \in \mathbb{R}^q$ follows the dynamics

$$F_t = F_{t-1} + v_t, \quad v_t \overset{d}{\sim} \mathcal{N}(0, Q_{\text{long}}),$$

where $Q_{\text{long}} \in \mathbb{R}^{q \times q}$ is the covariance matrix of the random walk innovations, while the short term sentiment follows the VAR(1) dynamics:

$$\Psi_t = \Phi \Psi_{t-1} + u_t, \quad u_t \overset{d}{\sim} \mathcal{N}(0, Q_{\text{short}})$$

where $\Phi \in \mathbb{R}^{K \times K}$ is the matrix of autoregressive coefficients and $Q_{\text{short}} \in \mathbb{R}^{K \times K}$ is the covariance matrix of the short-term sentiment innovations. In this paper, we force a diagonal structure on $\Phi$, thus neglecting the possible lead-lag effects among sentiments. This restriction has been considered to limit the curse of dimensionality. Adding a third component to deal with the observation error, the Multivariate Long Short Sentiment model (MLSS) for the observed sentiment $S_t$ reads

$$S_t = \Lambda F_t + \Psi_t + \epsilon_t, \quad \epsilon_t \overset{d}{\sim} \mathcal{N}(0, R),$$
$$\Psi_t = \Phi \Psi_{t-1} + u_t, \quad u_t \overset{d}{\sim} \mathcal{N}(0, Q_{\text{short}}),$$
$$F_t = F_{t-1} + v_t, \quad v_t \overset{d}{\sim} \mathcal{N}(0, Q_{\text{long}}),$$

where $R \in \mathbb{R}^{K \times K}$ is the diagonal covariance matrix of the observation noise $\epsilon_t$ and the factor loading matrix $\Lambda$ belongs to $\mathbb{R}^{K \times q}$.
It is worth noticing that, if the observed sentiment $S_t$ lies in a compact set, the LNSL, MLNSL and MLSS models, in their current specification, do not consider the upper and lower bounds. This technical issue can be accounted for with a non linear transformation of the data, e.g., as commonly done for correlation time series, by Fisher transforming the data. In this paper, we do not apply any non linear transformation. As it will be clear from Figure 1 in Section 4, most of the daily sentiment observations are far from the bounds. As we verified, the Fisher transform mildly affects our analysis.

The estimation of the unknown parameters is based on a combination of the Kalman filter with Expectation Maximization (Kalman, 1960; Shumway and Stoffer, 1982; Wu et al., 1996; Harvey, 1990; Banbura and Modugno, 2014; Jungbacker and Koopman, 2008). Given that model (2) is a special case of model (5), in the next session we only consider the estimation procedure of model (5).

### 2.1 Estimation procedure

The estimation of model (5) is performed using the Kalman filter (Kalman, 1960) and the Expectation Maximization (EM) method in (Dempster et al., 1977) and (Shumway and Stoffer, 1982) which was proposed to deal with incomplete or latent data and the intractable likelihood. The EM algorithm is a two-step estimator. In the first step, we write the likelihood considering the latent process as observed. In the second step, we re-estimate the static parameters maximizing the expectation obtained in the first step. This routine is repeated until some convergence criterion is satisfied.

To cast (5) in a standard state-space representation, we use the same procedure of (Banbura and Modugno, 2014) and define the augmented states $\tilde{\Lambda}$, $\tilde{F}$, $\tilde{\Phi}$ and $\tilde{Q}$ s.t.

$$
S_t = \tilde{\Lambda} \tilde{F}_t + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, R),
$$

$$
\tilde{F}_t = \tilde{\Phi} \tilde{F}_{t-1} + v_t, \quad v_t \sim \mathcal{N}(0, \tilde{Q}),
$$

where

$$
\tilde{\Lambda} = \begin{bmatrix} \Lambda & I_K \end{bmatrix} \in \mathbb{R}^{K \times (q+K)}
$$
The EM renders the approach feasible in high dimension. Indeed, while a direct numerical maximization of the likelihood is computationally demanding, the EM algorithm, thanks to the Kalman filtering and smoothing recursions in Appendix A, can be formulated using closed-form equations (35) and (36)-(39) reported in Appendix B. In particular, it allows to disentangle the long-term sentiment $F_t$ and the short-term sentiment $\Psi_t$. To derive the EM steps we consider the log-likelihood $l(S_t, \tilde{F}_t, \theta)$ where $\theta$ denotes the set of static parameters $\tilde{\Lambda}$, $\tilde{\Phi}$, $\tilde{Q}$ and $R$. The EM proceeds in a sequence of the steps:

1. **E-step**: it evaluates the expectation of the log-likelihood using the estimated parameters from the previous iteration $\theta(j)$:

   \[
   G\left(\tilde{\Lambda}(j), \tilde{\Phi}(j), \tilde{Q}(j), R(j)\right) = E\left[l\left(S_t, \tilde{F}_t, \theta(j)\right) | S_1, \ldots, S_T\right].
   \]

   The E-step strongly relies on equations (33). The details are explained in Appendix A.

2. **M-step**: the parameters are estimated again maximizing the expected log-likelihood with respect to $\theta$:

   \[
   \theta(j + 1) = \arg \max_{\theta} G\left(\tilde{\Lambda}(j), \tilde{\Phi}(j), \tilde{Q}(j), R(j)\right).
   \]

   The M-step is performed updating the static parameters following equations (36)-(41). Details are provided in Appendix B.
We initialize the parameters $\theta(0)$ and repeat steps 1 and 2 until we reach the convergence criterion

$$\left| \frac{l(S_t, \tilde{F}_t, \theta(j)) - l(S_t, \tilde{F}_t, \theta(j - 1))}{l(S_t, \tilde{F}_t, \theta(j)) + l(S_t, \tilde{F}_t, \theta(j - 1))} \right| < \frac{\epsilon}{2}. \quad (11)$$

We set $\epsilon = 10^{-3}$.

As observed in (Harvey, 1990), the dynamic factor model (6) is not identifiable. Indeed, if we consider a non-singular invertible matrix $M$, then the parameters $\theta_1 = \{\Lambda, R, Q\}$ and $\theta_2 = \{\Lambda M^{-1}, R, MQM'\}$ are observationally equivalent, then starting from $S_t$ we cannot distinguish $\theta_1$ from $\theta_2$. We solve this identification problem using the approach proposed by (Harvey, 1990), imposing the following restrictions

$$\tilde{Q} = \begin{bmatrix} I_q & 0 \\ 0 & Q_{\text{short}} \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \lambda_{11} & 0 & 0 & \ldots & 0 \\ \lambda_{21} & \lambda_{22} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \lambda_{K1} & \lambda_{K2} & \lambda_{K3} & \ldots & \lambda_{Kq} \end{bmatrix} \quad (12)$$

where $\Lambda$ is the $K \times q$ sub-matrix in (7).

The specifications of $\tilde{\Phi}$, $\tilde{Q}$, $\tilde{F}$ and $R$ in (7), (9) and (10), together with the identification restrictions defined in (12), impose several constraints to the estimations. The EM procedure allows us to impose restrictions on the parameters in a closed-form. According to (Wu et al., 1996) and (Bork, 2009), we get the constrained $\tilde{\Phi}, \tilde{\Lambda}, \tilde{Q}$ and $R$ as:

$$\text{vec}(\tilde{\Phi}_r) = \text{vec}(\tilde{\Phi}) + \left(A^{-1} \otimes Q\right) M (M(A^{-1} \otimes Q)M')^{-1} (k_\phi - M\text{vec}(\Phi)) \quad (13)$$

where $A$ is defined in (35), $M$ is the $f \times 2K(r + K)$ matrix, $f$ is the number of constraints, $k_\phi$ is the $f$ vector containing the constraints values such that $M\text{vec}(\tilde{\Phi}) = k_\phi$. 
Equivalently, for the restricted $\Lambda_r$:

$$\vec(\Lambda_r) = \vec(\Lambda) + (E_1^{-1} \otimes R) G(G(E_1^{-1} \otimes R)G')^{-1}(k_\lambda - G\vec(\Lambda)) \quad (14)$$

where $E_1$ is defined in (35), $G$ is the $s \times Kr$ matrix, $s$ is the number of constraints, $k_\lambda$ is the $s$ vector containing the constraints values such that $G\vec(\Lambda) = k_\lambda$.

$\tilde{Q}$ and $R$ are evaluated using equations (38) and (39) and the restrictions, according to (Wu et al., 1996), can be imposed elementwise.

The final estimation scheme is as follow:

1. Initialize $\tilde{\Lambda} (0), \tilde{\Phi} (0), \tilde{Q} (0)$ and $R (0)$

2. Perform the E-step using the estimations $\tilde{\Lambda} (j), \tilde{\Phi} (j), \tilde{Q} (j)$ and $R (j)$ and the Kalman Smoother [33].

3. Perform the M-step and evaluate the new estimators $\tilde{\Lambda} (j + 1), \tilde{\Phi} (j + 1), \tilde{Q} (j + 1)$ and $R (j + 1)$ using equations (36)-(39).

4. Use the unrestricted estimations and (13) and (14) to obtain the restricted ones.

5. Repeat 2, 3 and 4 above until the estimates and the log-likelihood reach convergence.

Finally, since the number of long-term sentiment $q$ is considered as known, we select the optimal $q$ using the AIC and BIC indicators.

3 Data

Sentiment analysis is the use of a natural language processing algorithm to extract and quantify subjective information written in a text. In our paper, we use the sentiment data referred to individual companies.

The TRMI sentiment index is constructed using over 700 primary sources, divided in news and social media, and collects more than two millions articles per day. For any article,
a “bag-of-words” technique is used to create a sentiment score, which lies between −1 and +1, a buzz variable \[ \text{Buzz}_{t,s} \], and one or more asset codes, which in our case refer to companies. The time resolution of the sentiment data is one minute.

For any asset \( a \), minute \( s \), and day \( t \) we denote as \( S_{t,s}^{a} \) the sentiment score and as \( \text{Buzz}_{t,s}^{a} \) the buzz variable. Since the following empirical analysis are performed using daily data, we need to aggregate the TRMI series on a daily basis. TRMI user guide suggests to use the following equation

\[
S_{t}^{a} = \frac{\sum_{s=sh^{t-1}}^{sh^{t}} \text{Buzz}_{t,s}^{a} S_{t,s}^{a}}{\sum_{s=sh^{t-1}}^{sh^{t}} \text{Buzz}_{t,s}^{a}} \in [-1, 1],
\]

(15)

where \( S_{t}^{a} \) refers to the daily sentiment at day \( t \), evaluated on a 24-hour window between the selected hour of day \( t - 1 \) (\( sh^{t-1} \)) and the selected hour of day \( t \) (\( sh^{t} \)). Note that the TRMI server provides a daily frequency sentiment, where they use equation (15) with \( sh = 3:30 \text{ PM} \). However, since we want to relate the sentiment series with close to close returns, we construct the daily sentiment series aggregating the high-frequency sentiment according to the trading closing hour of the NYSE (\( sh = 4:00 \text{ PM} \)). For more details, please refer to Peterson, 2016.

For the empirical analysis, we consider the TRMI sentiment index of 27 out of 30 stocks of the Dow Jones Industrial Average (DJIA) over the period 03/01/2006 – 29/12/2017. Since the TRMI index divides the news sentiment from the social sentiment, we have a total of 54 time series. A description of tickers and sectors is reported in table 1. Finally, the MLSS model, in its current specification, does not consider missing values in data, while some of the sentiment time series present missing observations. The EM algorithm is naturally designed to handle missing observations, however, since the number of missing values is small, we fill them using the mean over the last 5 days.

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1. The buzz field represents a sum of entity-specific words and phrases used in TRMI computations. It can be non-integer when any of the words/phrases are described with a minimizer, which reduces the intensity of the primary word or phrase. For example, in the phrase less concerned the score of the word concerned is minimized by “less”. Additionally, common words such as “new” may have a minor but significant contribution to the Innovation TRMI. As a result, the scores of common words/phrases with minor TRMI contributions can be minimized.” See TRMI user guide.

2. We only consider 27 assets because one is missing in the Thomson Reuters dataset and two have an high ratio of missing values at the beginning of the sample.

3. 47 out of 54 sentiment series have less than the 1% of missing observations. All the series have a percentage of missing which is smaller than the 7.5%
In this section, we present the results of the estimation of the MLSS model for the investigated stocks, providing an economic interpretation for the long- and short-term component of the sentiment. In the analyses, we consider separately the case of news and social sentiment indicator.

The first quantity to fix is the number $q$ of long-term sentiment factors. We choose it by using information criteria. Table 2 shows that the AIC and BIC values for the two

| Tickers | Name                  | Sector ticker | Sector name         |
|---------|-----------------------|---------------|---------------------|
| VZ      | Verizon               | COM           | Communication Services |
| CVX     | Chevron               | ENE           | Energy              |
| AXP     | American Express Company | FIN           | Financial           |
| GS      | Goldman Sachs         | FIN           | Financial           |
| JPM     | JPMorgan Chase        | FIN           | Financial           |
| JNJ     | Johnson & Johnson     | HLC           | Health Care         |
| MRK     | Merck                 | HLC           | Health Care         |
| PFE     | Pfizer                | HLC           | Health Care         |
| UNH     | UnitedHealth          | HLC           | Health Care         |
| BA      | Boeing                | IND           | Industrials         |
| CAT     | Caterpillar           | IND           | Industrials         |
| GE      | General Electric      | IND           | Industrials         |
| MMM     | 3M Co                 | IND           | Industrials         |
| UTX     | United Technologies   | IND           | Industrials         |
| XOM     | XOMA Corp             | MAT           | Basic Materials     |
| KO      | Coca-Cola             | NCY           | Consumer Goods      |
| PG      | Procter & Gamble      | NCY           | Consumer Goods      |
| AAPL    | Apple                 | TEC           | Technology          |
| CSCO    | Cisco                 | TEC           | Technology          |
| IBM     | IBM                   | TEC           | Technology          |
| INTC    | Intel                 | TEC           | Technology          |
| MSFT    | Microsoft             | TEC           | Technology          |
| DIS     | Disney                | YCY           | Consumer Cyclical   |
| HD      | Home Depot            | YCY           | Consumer Cyclical   |
| MCD     | McDonalds             | YCY           | Consumer Cyclical   |
| NKE     | Nike                  | YCY           | Consumer Cyclical   |
| WMT     | Wal-Mart              | YCY           | Consumer Cyclical   |

Table 1: List of investigated stocks, their ticker, and the economic sector according to the classification of Yahoo Finance.

4 Empirical analysis
Table 2: AIC and BIC score of the MLSS model (5) estimated on the sentiment news and social data. The quantities in boldface are those corresponding to the minimal value of the score.

| N. of factors | News   | Social  |
|---------------|--------|---------|
|               | AIC    | BIC     | AIC    | BIC     |
| $q = 1$       | –231728| –228968 | –258319| –255559 |
| $q = 2$       | –232108| **229186**| –258507| **255585**|
| $q = 3$       | –232236| –229152 | –258483| –255399 |
| $q = 4$       | –232295| –229048 | –258511| –255264 |
| $q = 5$       | –232317| –228908 | **258525**| –255115|
| $q = 6$       | **232366**| –228794 | –258458| –254886 |

number of factors $q_{\text{news}}$ and $q_{\text{social}}$. As usual the AIC favors the choice of more factors than the BIC and we decide to follow the latter because BIC is more conservative and reduce the dimensionality of the problem. Therefore we select $q_{\text{news}} = 2$ and $q_{\text{social}} = 2$.

Tables 3 and 4 report the values of $\Phi$ and $\Lambda$ with the estimation errors$^4$. Bold values indicate parameters which are significantly different from 0 with a p-value smaller than 0.05. We notice that most of the estimated parameters are statistically significant.

As an illustrative example, Figure 1 shows how the filter works for the Goldman Sachs news sentiment series. We observe that a high proportion of the sentiment daily variation is captured by the filter. In Section 4.3 we quantify more in detail the signal-to-noise ratio.

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$^4$Note that the $\Lambda$ matrices, as imposed in equation (14), have the upper triangular submatrix equal to 0.
of the proposed filter.

The MLSS approach considers two new quantities extracted from the observed sentiment. The first novelty is the long-term sentiment which, by construction, represents the series of common trends in a particular basket of sentiment time series. The second novelty is the multivariate structure of sentiment, extracted using the symmetric matrix $Q_{\text{short}}$. In the next sections, we separately analyse the relation between these two quantities and the stock market prices. To this end, we extract the market factors from the stock prices of these assets. Denote as $r_t \in \mathbb{R}^{27}$ the vector of demeaned close-to-close log-returns and evaluate the unconditional covariance matrix $Q_{\text{ret}}$ and the unconditional correlation matrix $C_{\text{ret}}$. We extract the factor loading matrix $\Lambda^{\text{mrk}} \in \mathbb{R}^{q_{\text{mrk}} \times 27}$ using the PCA on the matrix $C_{\text{ret}}$ and define the return factors $R_t \in \mathbb{R}^{q_{\text{mrk}}}$ as

$$R_t = \Lambda^{\text{mrk}} r_t. \quad (16)$$

We also define the market factors $M_t \in \mathbb{R}^{q_{\text{mrk}}}$ as

$$M_t^{\text{mrk}} = \Lambda^{\text{mrk}} p_t \quad (17)$$

where $p_t \in \mathbb{R}^{27}$ are the log-prices. In the following analysis, we consider $q_{\text{mrk}} = 1$ and define the first market factor as Dow 27.

### 4.1 Long-term Sentiment

We first investigate the economic meaning of the long-term sentiment. Using the Engle-Granger test (Engle and Granger, 1987), we observe that one of the factors of the long-term sentiment is cointegrated with the Dow 27. Figure 2 shows the cointegration relation, pointing out that the main driver of the prices and the driver of the sentiment time series reflect the same common information. The values of the elements of the factor loading matrices $\Lambda^{\text{news}}$ and $\Lambda^{\text{social}}$ reported in Tables 3 and 4 are either positive or negative. This means
| Tickers | $\phi_{news}$ | $\Lambda_{news}$ | Signal to noise |
|---------|--------------|-----------------|----------------|
|        |              |                 | MLSS | MLNSL |
| AXP    | 0.464        | 1.177           | 0.623 | 0.010 |
| JPM    | 0.732        | -0.169          | 0.326 | 0.023 |
| VZ     | 0.682        | 0.545           | 0.431 | 0.029 |
| CVX    | 0.545        | 0.103           | 0.610 | 0.022 |
| GS     | 0.773        | -0.239          | 0.336 | 0.029 |
| JNJ    | 0.407        | 0.851           | 0.788 | 0.010 |
| MRK    | 0.336        | 0.811           | 0.832 | 0.008 |
| PFE    | 0.299        | 0.530           | 1.185 | 0.007 |
| UNH    | 0.374        | 1.177           | 0.574 | 0.009 |
| BA     | 0.585        | 0.376           | 0.896 | 0.033 |
| CAT    | 0.633        | 0.309           | 0.423 | 0.017 |
| GE     | 0.581        | 1.083           | 0.587 | 0.022 |
| MMM    | 0.295        | 0.958           | 0.788 | 0.009 |
| UTX    | 0.331        | 0.422           | 0.690 | 0.011 |
| XOM    | 0.591        | -0.058          | 0.725 | 0.031 |
| KO     | 0.486        | 0.476           | 0.620 | 0.015 |
| PG     | 0.337        | 0.838           | 0.929 | 0.008 |
| AAPL   | 0.593        | 0.221           | 1.736 | 0.096 |
| CSCO   | 0.714        | 1.063           | 0.441 | 0.046 |
| IBM    | 0.603        | 0.754           | 0.853 | 0.040 |
| INTC   | 0.641        | 0.641           | 0.865 | 0.066 |
| MSFT   | 0.651        | 0.858           | 0.668 | 0.053 |
| DIS    | 0.439        | 0.454           | 1.074 | 0.013 |
| HD     | 0.611        | 1.137           | 0.473 | 0.021 |
| MCD    | 0.404        | -0.291          | 1.401 | 0.013 |
| NKE    | 0.368        | 0.664           | 0.783 | 0.010 |
| WMT    | 0.516        | 0.147           | 0.854 | 0.022 |

Table 3: Static parameters of model \(\phi\) for news sentiment. Values and standard errors of estimated $\Lambda$ are multiplied by $10^3$. In parenthesis we show the standard error of the estimated parameter. The last two columns show the signal to noise ratio for two competing models.
Table 4: Static parameters of model (14) for social sentiment. Values of Λ are multiplied by $10^3$. In parenthesis we show the standard error of the estimated parameter. The last two columns show the signal to noise ratio for two competing models.
that some firm’s sentiment positively affects the common sentiment factors, while some
firm’s sentiment negatively affects the common sentiment factors. This result per se is not
surprising. However, since one of the factors of the long-term sentiment is cointegrated with
the Dow 27, the common information spreads differently in the market and the sentiment.
Figure 3 shows the standardized weights of the cointegrated factors. The weights of the
market factor are very homogeneous across assets, as shown in the top panel, while the
weights of the second factor of the long-term sentiment are very heterogeneous, as shown
in the bottom panel. For example, the sentiment of United Health and Pfizer are more
representative, in the long-term information of the Dow 27, than the sentiment of General
Electric and Disney. We also check that these weights are not related with the number of
news of a certain asset, nor with the buzz index (results available upon request).

Figure 2: Co-integration between Dow 27, in blue, and the second factor of the news
long-term sentiment, in orange. Time series are scaled.

4.2 Short-term Sentiment

The second novelty of the MLSS model is the multivariate structure of the short-term
sentiment series. The question we want to address in this section is whether the correlation
structure of the short-term sentiment is (linearly) related with the correlation structure
of the daily returns. In the previous section, we observed that one of the factors of the
long-term sentiment is cointegrated with the first market factor. We therefore expect
the short-term sentiment to capture asset-specific features, i.e. they are closely related with the idiosyncratic dynamic of the returns. To test this intuition for the correlation structure, we compare the results of the MLSS model with the results of the MLNSL model which, by construction, does not disentangle the factors from the sentiment series. If the intuition is correct, the correlation matrix of the sentiment extracted using the MLSS model should be linearly related with the return correlations and with the idiosyncratic return correlations. On the contrary, the correlation matrix of the sentiment extracted using the MLNSL model, which only capture the slowly changing component of the sentiment series, which is related with the first market factor, should be linearly related with the returns correlation but it should be mildly correlated with the idiosyncratic return correlations. Finally the correlation matrix of the observed sentiment is considered as a benchmark.

We define $C_{\text{short}}$ as the correlation matrix associated with the covariance matrix $Q_{\text{short}}$, $C_{\text{MLNSL}}$ the correlation matrix associated with the covariance matrix $Q$ of equation (2), $C_{\text{Obs}} = \text{Corr} (\Delta S_t)$ the unconditional correlation of the first difference of the observed

\footnote{We define idiosyncratic returns as the market returns where the first market factor, defined in equation (16), is removed using the factor model (19).}
sentiment, and $C_{\text{ret}}$ the unconditional correlations matrix of the stock returns. We search for a linear element-wise relation between $C_{\text{ret}}$ and $C_{\text{model}}$, where model is one of short, MLNSL, or Obs. The results are reported for the news case only, but the conclusions are similar for the social sentiment.

We perform a standard ordinary least squares estimation on the model

$$\text{vechl}(C_{\text{ret}}) = \alpha + \beta^{\text{model}} \text{vechl}(C_{\text{model}}),$$

where $\text{vechl}(X)$ is the operator which collects the upper diagonal elements of matrix $X$ in a column vector. We compare the results obtained using the MLSS model ($C_{\text{model}} = C_{\text{short}}$), with the results obtained using the MLNSL model ($C_{\text{model}} = C_{\text{MLNSL}}$) and using the Observed sentiment ($C_{\text{model}} = C_{\text{Obs}}$). In addition, since the unconditional correlation between two assets is higher when they belong to the same sector, we separately consider two cases. In the first case, we estimate model (18) considering all the pairs of assets. In the second case, we estimate model (18) considering only the pairs of assets belonging to the same economic sector according to Table 1.

The top left panel of Table 5 shows the results with all the correlation pairs. In the first column we report the $R^2$ of the regression, in the second column we report the F-statistic and the relative p-value is reported in the third column. The regressions with $C_{\text{short}}$ and $C_{\text{MLNSL}}$ have high and significant p-values, while the regression with $C_{\text{obs}}$ is not statistically different from the model with the intercept only. This finding has two implications. The first one is that the sentiment innovations have a similar correlation structure of the returns innovations. In particular, if the returns of two assets are relatively highly correlated, then also the increment of the sentiment of the news about these assets are relatively highly correlated. The second implication is that, if a filtering procedure is not applied on the observed sentiment data, the noise is too large to find significant results. In the top right panel of Table 5 we report the results of the model (18) applied to the pairs of assets belonging to the same sector. We observe that the $R^2$ increases for all models. This result is expected since it is well known that the return correlation is higher and more significant.
between two assets of the same sector. However, even if the $R^2$ increases, the number of pairs decreases. For this reason, the increment in the $R^2$ does not lead to an increment in the $F$-statistic, which fails to reject the null hypothesis for the $C_{\text{obs}}$. This result confirms that the $C_{\text{obs}}$ matrix is not a significant regressor for $C_{\text{ret}}$.

Comparing the top panels of Table 5, we note that the increment in the $R^2$ is higher for the MLSS model rather than the MLNSL model. This evidence is consistent with the intuition that the short-term sentiment series, extracted using the MLSS model, are more related with the idiosyncratic returns. Indeed the correlation induced by the market factor is predominant in the first case, reported in the top left panel, where all the assets are considered, rather than the second case, reported in the top right panel, where the co-movements are not only driven by the first market factor, but they are also driven by sector-specific factors.

Now we extract the Dow 27 return from the asset returns using a 1-factor model. We repeat the analysis comparing the matrices $C_{\text{short}}$, $C_{\text{MLNSL}}$ and $C_{\text{Obs}}$ with the unconditional correlation of the idiosyncratic returns. We extract the market factor $R_t$ defined in equation (16) from the returns using the factor model

$$r^i_t = \alpha^i + \beta^i R_t + z^i_t, \quad \forall i = 1, \ldots, 27$$

| Models | All assets | Same sector |
|--------|------------|-------------|
|        | $R^2$ | $F$-statistic | p-value | $R^2$ | $F$-statistic | p-value |
| MLSS   | 13.77 % | 55.713 | 0.0000 | 37.89 % | 23.182 | 0.0000 |
| MLNSL  | 15.63 % | 64.669 | 0.0000 | 28.78 % | 15.359 | 0.0004 |
| Obs    | 0.95 %  | 3.330  | 0.0689 | 4.19 %  | 1.662  | 0.2052 |
| MLSS   | 11.34 % | 44.659 | 0.0000 | 30.91 % | 17.001 | 0.0002 |
| MLNSL  | 4.31 %  | 15.700 | 0.0001 | 7.50 %  | 3.081  | 0.0873 |
| Obs*   | 1.01 %  | 3.554  | 0.0602 | 4.88 %  | 1.950  | 0.1707 |

Table 5: Results of the linear regression (18) are reported in the top panels. Results of the linear regression (20) are reported in the bottom panels. The left panels show the OLS estimates when all the assets are considered, while the right panels show the OLS estimates when only the correlations between stocks belonging to the same sector are considered. The last row, denoted as Obs*, is the estimation of model (20) using the observed sentiment.
where \( z_t^i \sim N(0, \tilde{Q}_{ret}) \). We then compute the cross-correlation matrix \( \tilde{C}_{ret} \) from the covariance matrix \( \tilde{Q}_{ret} \) and estimate the following model

\[
\text{vechl}(\tilde{C}_{ret}) = \alpha + \beta^\text{model}\text{vechl}(C_{model}).
\] (20)

The bottom panels of Table 5 report the results. In the bottom left panel we show the results for the model (20) where all the correlation pairs are considered. The first evidence is that the MLNSL \( R^2 \) dramatically decreases, while the MLSS \( R^2 \) remains almost the same. This finding suggests that almost all the return correlations explained by the \( C_{MLNSL} \) matrix were associated with the market factor \( R_t \), while the matrix \( C_{short} \), which represents the fast trends on the sentiment data, also captures different dynamics.

In the bottom right panel, we show the results for the model (20) where we consider only the correlation pairs for assets belonging to the same sector. In this case the differences between the MLSS and MLNSL are more severe. Indeed the MLSS model still have an high and highly significant \( R^2 \), while the \( F \)-statistic for the MLNSL model fails to reject the null that \( \beta^\text{MLNSL} \), defined in equation (20), is equal to 0. Again, the model with the observed sentiment has not significant p-values.

As a last observation, we see the different behaviour of the sectors in this regression exercise. Figure 4 reports the scatter plot of the elements of \( C_{short} \) versus the corresponding values of \( C_{ret} \) when the two stocks belong to the same economic sector, characterized by a specific marker. We also superimpose the regression line obtained from equation (18). Note that the behaviour is different among sectors. The financial sector, marked with blue dots, is the one with highest linear relation and the three assets belonging to this sector have all high returns and sentiment correlations. On the contrary, the consumer cyclical sector, marked with garnet-red triangles, has a high dispersion among the correlations of the 5 assets.

In summary, Sections 4.1 and 4.2 support the intuition behind the MLSS model. Indeed, the slowly changing components of the sentiment are effectively captured by the long-term
sentiment. We successfully confirm this hypothesis in Section 4.1. At the same time, the short-term sentiment effectively captures the firm-specific component of the returns. Section 4.2 shows that the MLSS model can capture different features of the returns, while the MLNSL mainly captures the sentiment component associated with the market.

### 4.3 Signal-to-noise ratio and comparison with MLNSL

Finally, we compare how the MLSS model fits the data with respect to the MLNSL model using the likelihood ratio test. Since the MLNSL model is nested into the MLSS model, we use the $\chi^2$ distribution to test the null hypothesis (the MLSS model does not fit the data better than the MLNSL) against the alternative hypothesis (the MLSS model fits the data better than the MLNSL). The null hypothesis is rejected with a $p$-value smaller than 0.01 for both news and social sentiment.

In the last columns of Tables 3 and 4 we report the signal to noise ratio for each asset obtained using the MLSS model and the signal to noise ratio obtained using the MLNSL model. The signal to noise ratio for the MLSS model, using the same notation of equation
is evaluated as
\[
\text{stn}(i)_{\text{MLSS}} = \frac{\text{Var}(\Lambda(i, \cdot)v_t) + \text{Var}(u_t^i)}{\text{Var}(\epsilon_t)} = \frac{\sum_{j=1}^{q}(\Lambda(i, j))^2 + Q_{\text{short}}(i, i)}{R(i, i)}
\] (21)

while the signal to noise ratio for the MLNSL model, using the notation of equation (2), is evaluated as
\[
\text{stn}(i)_{\text{MLNSL}} = \frac{\text{Var}(v_t^i)}{\text{Var}(\epsilon_t)} = \frac{Q(i, i)}{R(i, i)}
\] (22)

When the MLSS model is estimated, the signal to noise ratio is on average around 0.8 for the news sentiment and around 0.4 for the social sentiment. On the contrary, when the MLNSL model is estimated, the signal to noise ratio decreases to an average of 0.03 in the news sentiment and 0.02 in the social sentiment.

Thus our proposed MLSS model has a signal to noise ratio approximately twenty times larger than the MLNSL. Our result also points out that the noise in social media is generally higher than the noise in newspapers.

5 Contemporaneous and lagged relations between returns and sentiment

In the last section, we assess the explanatory power of the sentiment with respect to the market returns using the different filters presented in the previous sections. In particular, we show that both the extraction of long-term and short-term sentiment components and the multivariate specification of the model are crucial ingredients to capture the synchronous and lagged effects.

We consider the asset prices \(P_t^i\) for the 27 stocks of the Dow 30 and construct the equally weighted portfolio
\[
M_t = \frac{1}{27} \sum_{i=1}^{27} P_t^i
\] (23)
as a representative portfolio and denote with \(r_t^M\) its log-returns. We consider a representative portfolio for two reasons. Firstly, (Beckers, 2018) shows that the returns predictability
using sentiment indicators is higher when using market indexes rather than single stocks. Secondly, using a representative portfolio we can compare different filtering techniques which do or do not consider the multivariate structure.

We define $S_{t, \text{news}} = \frac{1}{27} \sum_{i=1}^{27} S_{i, \text{news}, t}$ and $S_{t, \text{social}} = \frac{1}{27} \sum_{i=1}^{27} S_{i, \text{social}, t}$ as the sentiment associated to the representative portfolio. We consider five different filtering techniques defined as follows.

1. $S_t^{\text{MLSS}}$ is the filtered signal obtained using the MLSS model in equation (5). The resulting filtered quantities are 4 long-term sentiment factors $F_t^{\text{MLSS}}$, 2 for the news and 2 for the social sentiment, and 54 short-term sentiment series $\Psi_t^{\text{MLSS}}$, 27 for the news and 27 for the social sentiment. We compute the cross-sectional average for the news short-term sentiment $\Psi_{t, \text{news}}^{\text{MLSS}}$ and social short-term sentiment $\Psi_{t, \text{social}}^{\text{MLSS}}$. As a final result, we define

   $$S_t^{\text{MLSS}} = \left[ \Delta F_{t, \text{news}}^{\text{MLSS}}, \Delta F_{t, \text{social}}^{\text{MLSS}}, \Psi_{t, \text{news}}^{\text{MLSS}}, \Psi_{t, \text{social}}^{\text{MLSS}} \right]' \in \mathbb{R}^6.$$

2. $S_t^{\text{LSS}}$ is the filtered signal obtained applying the MLSS model directly to the univariate series $S_{t, \text{news}}$ and $S_{t, \text{social}}$. For identifiability reasons, the number of common factors is one. The motivation behind this model is to test whether a simple cross-sectional average of sentiment time series can be an effective proxy of the sentiment of the representative asset. This approach intentionally neglects the multivariate structure of the sentiment and treats it as a non relevant feature. A similar reasoning has been used in (Borovkova et al., 2017). The resulting filtered quantities are 2 long-term sentiment factors $F_t^{\text{LSS}}$, one for the news and one for the social sentiment, and 2 short-term sentiment series $\Psi_t^{\text{LSS}}$, one for the news and one for the social sentiment. The final model reads

   $$S_t^{\text{LSS}} = \left[ \Delta F_{t, \text{news}}^{\text{LSS}}, \Delta F_{t, \text{social}}^{\text{LSS}}, \Psi_{t, \text{news}}^{\text{LSS}}, \Psi_{t, \text{social}}^{\text{LSS}} \right]' \in \mathbb{R}^4.$$

3. $S_t^{\text{MLNSL}}$ is the filtered signal obtained using the MLNSL model in equation (2) from
the 54 observed sentiment time series. The resulting filtered quantities are 54 filtered sentiment series \( F_{t}^{MLNSL} \), 27 for the news and 27 for the social sentiment. We compute the cross-sectional average for the news sentiment \( \bar{F}_{t}^{MLNSL,news} \) and social sentiment \( \bar{F}_{t}^{MLNSL,social} \). As a final result, we define

\[
S_{t}^{MLNSL} = \left[ \Delta \bar{F}_{t}^{MLNSL,news}, \Delta \bar{F}_{t}^{MLNSL,social} \right]' \in \mathbb{R}^2.
\]

4. \( S_{t}^{LNSL} \) is the filtered signal obtained applying the LNSL model, introduced by (Borovkova and Mahakena, 2015) and presented in equation (1), to \( \bar{S}_{t}^{news} \) and \( \bar{S}_{t}^{social} \). As for the LSS model, the motivation behind this choice is to test whether the multivariate structure of sentiment is a relevant feature or not. We obtain 2 filtered sentiment series \( \bar{F}_{t}^{LNSL} \), one for the news and one for the social sentiment. We then define

\[
S_{t}^{LNSL} = \left[ \Delta \bar{F}_{t}^{LNSL,news}, \Delta \bar{F}_{t}^{LNSL,social} \right]' \in \mathbb{R}^2.
\]

5. \( S_{t}^{obs} \) only considers the observed sentiment \( \bar{S}_{t}^{news} \) and \( \bar{S}_{t}^{social} \)

\[
S_{t}^{obs} = \left[ \Delta \bar{S}_{t}^{news}, \Delta \bar{S}_{t}^{social} \right]' \in \mathbb{R}^2.
\]

In summary, the five models allow us to separate the effect of the different components. The MLSS model exploits all the possible information from the multidimensional time series and all the relevant common factors are considered. The average across assets is computed at a later stage on the short-term sentiment. For this reason, it does not affect the long-term components. The LSS model computes the cross-section average as a first step and does not exploit the multidimensional structure. Then, both the short-term and long-term components are different from the one of the MLSS model. The MLNSL and LNSL models differ only on the step of the aggregation. The first model applies the filter on the multivariate time series, while the second model applies the filter on the aggregated time series. Finally, the Obs model works as a benchmark.
5.1 Quantile regression

In this section, we investigate the contemporaneous and lagged relation among sentiment and market returns. In the recent literature, (Garcia, 2013) for the DJIA and (Smales, 2014) for the gold futures, found that the reaction to news is more pronounced during recessions. For this reason, we use the quantile regression in place of a simple linear regression to obtain a more comprehensive analysis of the relationship between variables.

5.1.1 Contemporaneous effects

In the first analysis, we consider the following quantile regression

\[ r^m(\tau) = \alpha(\tau) + \beta^{\text{model}}(\tau) S^{\text{model}}_t \]  

(24)

where model denotes one of the five filtering models presented above. According to (Koenker and Machado, 1999), we can compare the explanatory power of a selected model according to the \( R^1 \) measure. In particular, if we consider the functional expression for the quantile regression

\[ \hat{V}(\tau) = \min_{(\alpha, \beta)} \sum_{t=1}^T \rho_\tau (r^m_t - \alpha - \beta S_t) , \]  

(25)

where \( \rho(u) = u(\tau - I_{u<0}) \), we can define the quantile \( R^1 \) measure as

\[ R^1(\tau) = 1 - \frac{\hat{V}(\tau)}{\tilde{V}(\tau)} \]  

(26)

where \( \tilde{V}(\tau) \) is evaluated restricting equation (25) with the intercept parameter only. In contrast to the \( R^2 \) measure of the linear models, \( R^1(\tau) \) is a local measure of goodness of fit and only applies to a particular quantile. In addition, (Koenker and Machado, 1999) show that using \( \hat{V} \) we can test the significance of the \( \beta^{\text{model}} \) parameters. Considering \( \beta^{\text{model}} = 0 \) as the null hypothesis and \( F \) as the probability distribution of the i.i.d. residuals \( \{u_t\} \), the statistic

\[ L_T(\tau) = \frac{2(\tilde{V}(\tau) - \hat{V}(\tau))}{\tau(1-\tau)s(\tau)} \rightarrow \chi^2_q \]  

(27)
Table 6: The $R^1(\tau)$ measure across the value $\tau$. We denote with *** the significance at 1%, ** the significance at 5% and * the significance at 10%

| $\tau$ quantiles | MLSS | LSS  | MLNSL | LNSL | Obs  |
|------------------|------|------|-------|------|------|
| 0.01             | 16.2%*** | 6.1%** | 1.4%  | 1.6% | 0.6% |
| 0.05             | 9.2%***  | 4.0%*** | 2.8%*** | 2.7%** | 1.7%*** |
| 0.10             | 7.1%***  | 4.3%*** | 3.5%*** | 3.2%** | 2.5%*** |
| 0.33             | 2.2%***  | 1.8%*** | 1.9%*** | 1.7%** | 1.0%*** |
| 0.50             | 1.1%***  | 1.1%*** | 1.2%*** | 1.0%** | 0.7%*** |
| 0.66             | 0.5%***  | 0.9%*** | 1.3%*** | 0.8%** | 0.7%*** |
| 0.90             | 1.2%***  | 1.7%*** | 1.5%*** | 0.8%** | 0.8%*** |
| 0.95             | 2.9%***  | 2.3%*** | 1.9%*** | 1.0%*** | 1.0%*** |
| 0.99             | 10.2%*** | 4.6%**  | 0.9%  | 0.6% | 1.5% |

where $q$ is the dimension of $\beta_{\text{model}}$ and $s(\tau) = 1/f(F^{-1}(\tau))$.

Table 6 shows the values of the $R^1(\tau)$ measure for different values of $\tau$. It is worth to notice that the quantile regressions are highly significant for every model, except for the 0.01 and 0.99 quantiles, where they are only significant for the MLSS and LSS models. There are three important findings. The first one is that, for any model, the values of $R^1$ are higher in the tails and lower close to the median. The results are not symmetric around the median. The lower quantiles, which correspond to highly negative returns, have higher $R^1$ than the corresponding $R^1$ in the higher quantiles. This suggests that the sentiment series are powerful explanatory variables in bad times. This conclusion is in accordance with the results in [Garcia, 2013], which shows that investors’ sensitivity to news is most pronounced going through hard times. The second result is that the models which exploit the multivariate structure (MLSS and MLNSL) produce higher $R^1$ measures than the corresponding models which apply the cross-sectional averaging procedure on the sentiment series (LSS and LNSL models, respectively). This result confirms that the cross-sectional dependence structure is helpful in extracting a sensible signal. The last result is that the MLSS and LSS models, excluding few values around the median, have higher $R^1(\tau)$ values than other models. This suggests that disentangling the long-term and short-term sentiment component is the most important feature to capture the contemporaneous relation with market returns. In particular, the MLSS model, which exploits both the
Table 7: p-values expressed in % for the statistics in equation (29) and equation (28).

| $\tau$ quantiles | $L_i^{ST}$ | $L_i^{LT}$ |
|-------------------|------------|------------|
| 0.01              | 0.005%     | 76.313%    |
| 0.05              | 0.000%     | 2.052%     |
| 0.10              | 0.000%     | 3.381%     |
| 0.33              | 0.000%     | 3.257%     |
| 0.50              | 0.000%     | 7.668%     |
| 0.66              | 1.487%     | 20.078%    |
| 0.90              | 0.189%     | 0.309%     |
| 0.95              | 0.007%     | 0.922%     |
| 0.99              | 0.006%     | 22.903%    |

Table 7 reports the p-values of the statistics (29) and (28) and shows that the short-term sentiment is highly significant at any level of $\tau$, while the long-term sentiment has lower separation in two components and the multivariate structure, strongly outperforms the benchmark model which solely uses the observed noisy sentiment.

A further advantage of the long-short decomposition is that we can properly assess the relative contribution of the two components. In particular, we use equation (27) to test the significance of the parameters in the MLSS model. Considering the $S_{MLSS} = [\Delta F_{MLSS}^t, \Psi_{MLSS}^t]$, the significance of the parameter $\beta^{LT} \in \mathbb{R}^4$ and $\beta^{ST} \in \mathbb{R}^2$ can be tested using

$$\tilde{V}^{LT}(\tau) = \min_{(\alpha, \beta^{LT})} \sum_{t=1}^{T} \rho_{\tau} \left( r_{t}^m - \alpha - \beta^{LT} \Delta F_{MLSS}^t \right)$$

and

$$\tilde{V}^{ST}(\tau) = \min_{(\alpha, \beta^{ST})} \sum_{t=1}^{T} \rho_{\tau} \left( r_{t}^m - \alpha - \beta^{ST} \Psi_{MLSS}^t \right),$$

which lead to the statistics

$$L_i^{LT}(\tau) = \frac{2(\tilde{V}^{ST}(\tau) - \hat{V}(\tau))}{\tau(1-\tau)s(\tau)} \rightarrow \chi^2_4.$$  

and

$$L_i^{ST}(\tau) = \frac{2(\tilde{V}^{LT}(\tau) - \hat{V}(\tau))}{\tau(1-\tau)s(\tau)} \rightarrow \chi^2_2.$$  

Table 7 reports the p-values of the statistics (29) and (28) and shows that the short-term sentiment is highly significant at any level of $\tau$, while the long-term sentiment has lower...
p-values. In particular, the short-term sentiment, which captures rapidly changing trends, is significant for extreme returns ($\tau = 0.01$ or $\tau = 0.99$) while the long-term sentiment is not. This result suggests that extreme market swings can be explained by unexpected and short-lasting news. Moreover, it further supports the importance of disentangling sentiment components which are sensitive to different time scales.

These findings, together with those for the short-term and long-term sentiment presented in the previous section, show very strong contemporaneous relation between sentiment and market returns. We can look at these results as a sanity check of our approach. Indeed, since we are not claiming that sentiment causes returns or vice versa, it is reasonable to expect a significant contemporaneous relation at daily time scale. The sentiment explains returns and this could be simply due to the fact that the news, from which sentiment is computed, report and comment about the market performance. What is more promising is that the $R^1$ measure increases with the complexity of the model, and this is especially true for extreme market events – where the observed sentiment is not significant. Then, we conclude that an essential ingredient of the analysis is the combination of a multivariate model with the separation of sentiment in two components, the stochastic long-run trend (long-term sentiment) common to all assets and a fast changing and asset-specific trend (short-term sentiment).

5.1.2 Lagged relations

To further support our previous findings in favor of the Multivariate Long-Short Sentiment model, we now consider lagged quantile regressions. The goal is to assess the statistical significance, if any, of the lagged explanatory power of sentiment on market returns. Again, if this explanatory power exists, we test for the relative contribution of the multivariate and the long-short sentiment specifications. Since the future returns can depend on present observations, we use the lagged returns as a control variable. In the same fashion of (Tetlock, 2007; Garcia, 2013), we use an $h$-day lag operator $\mathcal{L}_h(x_t) = [x_{t-1}, \ldots, x_{t-h}]$.

As a first step, we consider $h = 1$. Adapting equation (25) to the present case, we evaluate the $R^1(\tau)$ statistic and test the significance using the $\chi^2$-test. Table 8 reports the
Table 8: The $R^1(\tau)$ measure across the value $\tau$ for the one-lag quantile regression. We denote with $***$ the significance at 1%, $**$ the significance at 5% and $*$ the significance at 10%

values and significance of the $R^1$ measure. What we observe is unexpected but extremely promising for the Long-Short modeling approach. The significance of the noisy sentiment drops to zero for all quantile levels. Filtering the time series is essential to recover predictability. However, filtering alone is not sufficient. Indeed, the predictability neither of the LSNL model nor of the multivariate extension MLSNL is statistically significant. Significance is recovered only when the filtered sentiment is decomposed into the short-run and long-run components. This is true for extreme returns, both positive and negative.

The result is stronger when the LSS model is replaced by the MLSS, meaning that the cross-sectional dependence is an important ingredient to enhance predictability. As done before, we can further test whether the main contribution comes from the short-term or from the long-term component. To test this hypothesis we proceed as for the contemporaneous regression separating the long-term and short-term sentiment contributions from the quantile regression. We report the p-values of the test statistics in Table 9. The contribution given by the short-term sentiment is strongly significant, in particular for extreme quantiles. On the contrary, the long-term sentiment is not significant in 6 out of 9 quantiles.

The results support the intuition that, if today a very high or very low return appears, it can be partially explained by the yesterday’s rapidly changing mood, while the permanent trend in the sentiment series have almost no impact.
| τ quantiles | $L_{t-1}^{ST}$ p-values | $L_{t-1}^{LT}$ p-values |
|-----------|------------------------|------------------------|
| 0.01      | 0.020%                 | 67.688%                |
| 0.05      | 0.000%                 | 66.738%                |
| 0.10      | 0.003%                 | 73.881%                |
| 0.33      | 5.069%                 | 66.666%                |
| 0.50      | 16.360%                | 59.465%                |
| 0.66      | 7.692%                 | 7.411%                 |
| 0.90      | 0.000%                 | 0.011%                 |
| 0.95      | 0.000%                 | 1.789%                 |
| 0.99      | 0.001%                 | 57.969%                |

Table 9: p-values expressed in % for the statistics $L_{t-1}^{ST} \sim \chi_2^2$ and $L_{t-1}^{LT} \sim \chi_4^2$ defined in a similar fashion to equations (29) and (28).

The experiments performed in the contemporaneous and one-lag cases show that the MLSS model is the best model to capture the return variations. For this reason, for the multi-period analysis we will only consider the MLSS model. Considering a general $h$, we wonder if extra lags can add explanatory power to the regression exercise. Using the functional form which follows

$$\hat{V}_{h,\text{MLSS}}(\tau) = \min_{(\alpha^0, \alpha^1, \beta^1 \in \mathbb{R}, \beta^2 \in \mathbb{R}^{6(h-1)})} \sum_{t=h+1}^{T} \rho_{\tau} \left( r_{t}^m - \alpha^0 - \alpha^1 r_{t-1}^m - \beta^1 S_{t-1}^{\text{MLSS}} - \beta^2 L_{h-1}^{\text{MLSS}}(S_{t-1}^{\text{MLSS}}) \right),$$

we separate the contributions given by the first and higher order lags. Under the null hypothesis that $\beta^2 = 0$, the statistic

$$L_{t-h}^{h,\text{MLSS}}(\tau) = \frac{2(\hat{V}_{1,\text{MLSS}}(\tau) - \hat{V}_{h,\text{MLSS}}(\tau))}{\tau(1-\tau)s(\tau)} \rightarrow \chi^2_{6(h-1)}.$$  \hspace{1cm} (30)

Following (Tetlock, 2007; Garcia, 2013), we fix a maximum number of $h = 5$ and Table 10 reports the p-values for the different values of $h$.

The $h$-lagged sentiment series are uninformative in the median region, where the 1-lag sentiment have less explanatory power too. However, in agreement with (Garcia, 2013), the lagged sentiment remains informative for few days and, in our case, this is true for the 5%, 10%, 90%, and 95% quantile levels. It is worth noticing that the 1% and 99% quantiles are unaffected by higher-order lags. This shows that, in case of very good or very bad days,
Table 10: p-values expressed in % for the statistics defined in equation (30) for different values of $h$. Bold values correspond to $\beta^2$ significantly different from zero.

| $\tau$ | $h = 2$ | $h = 3$ | $h = 4$ | $h = 5$ |
|--------|---------|---------|---------|---------|
| 0.01   | 18.133% | 29.136% | 57.652% | 72.784% |
| 0.05   | **0.618%** | **0.946%** | **4.317%** | **3.009%** |
| 0.10   | **0.907%** | **0.773%** | **4.341%** | **1.968%** |
| 0.33   | 65.530% | 47.389% | 74.932% | 74.071% |
| 0.50   | 62.489% | 70.078% | 80.725% | 90.581% |
| 0.66   | 43.722% | 53.518% | 52.853% | 74.962% |
| 0.90   | **4.831%** | **0.662%** | **0.063%** | **0.208%** |
| 0.95   | 12.800% | **2.504%** | **0.628%** | **2.468%** |
| 0.99   | 38.580% | 71.448% | 81.945% | 87.196% |

the returns are strongly driven by very fresh news ($h = 1$) while the older news have no informative power.

5.1.3 Market absorption of news

The 1-lag and the $h$-lag sentiment series contain useful information to explain future returns. The market does not immediately digest all the news in the newspapers and social media but it takes few days to do it. It is worth to investigate whether, at the same time, the opposite relation may hold. In particular, do media and social networks immediately digest returns that occurred in the previous days? Is this reaction dependent on the sign of returns? In order to answer these questions, we define the non-causal quantile regression

$$\hat{V}^{h,\text{MLSS}}(\tau) = \min_{(\alpha^0,\alpha^1,\beta^1 \in \mathbb{R}^p)} \sum_{t=1}^{T} \rho_\tau \left( r^m_t - \alpha^0 - \beta^1 S^\text{MLSS}_{t-h} \right),$$

where $h$ can assume positive and negative values. From the $R^1(\tau)$ of this model, we can unravel the interplay among professional and social media news, and returns in the financial market. For $h > 0$, we measure the impact of the news on future returns, while for $h < 0$, we assess how fast past returns are digested in newspapers and social media. When $h = 0$, we evaluate to the immediate impact of the news on daily returns. The two cases $h = 0$ and $h = 1$ have been studied in the previous sections. In figure 5, we show the evolution of $R^1(\tau)$ compared to the evolution of $R^1(1-\tau)$. In this manner, we observe how the market predicts...
or digests returns of comparable absolute value but opposite sign. We shrink to zero all values of $R^1(\tau)$ which are not significant with a $p$-value smaller than 0.05. For $h \geq 0$, the $R^1$ measures are statistically significantly and slightly increase as they approach $h = 1$. At this stage, the behavior of $R^1(\tau)$ is not different for values of $\tau < 1/2$ and $\tau > 1/2$. However, for $h \leq 0$, when a return appears, the response to a positive or a negative value is very different. For $h = 0$, $R^1(\tau)$ increases when the return is negative, confirming that, as reported in Table 6, the current news have an higher explanatory power on the negative rather then positive returns. However, as mentioned previously, it is not possible to assess the causal relation among news and returns. Then, the $R^1(\tau)$ measure reaches its maximum when $h = -1$, suggesting that newspapers and social media keep talking about the previous day market performance. This is especially true when the previous day return is negative. The impact of negative returns on sentiment slowly decreases but remains significant. On the contrary, when the return is positive, the effect is milder. This general picture is stronger for extreme quantile levels, e.g. $\tau = 0.01, 0.05$, and becomes negligible when we move to the median region of returns. Summarizing, positive returns are rapidly digested from newspaper and social media, while the echo of negative market performances persists for few days. As a final test of reliability of our approach, we perform the same analysis using the noisy sentiment $S^{Obs}$ instead of the filtered signal $S^{MLSS}$. Figure 6 reports the results. No clear patterns arise and the $h$-lag observed sentiment has no statistical significance in explaining contemporaneous and future returns. Again, the proper filtering of sentiment time series appears essential to extract some meaningful information from the mood of the market.

6 Conclusions

In this paper, we presented a novel way to filter sentiment time series. The approach is very general and encompasses previous models discussed in the literature. Using a dynamic factor model, we were able to identify two different sentiment components. The first one, named long-term sentiment and modeled as a random walk, captures the common trends
Figure 5: $R^1$ measure of model (31) for different $h$ and $\tau$. Top left panel: $R^1(0.99)$ in orange and $R^1(0.01)$ in blue. Top right panel: $R^1(0.95)$ in orange and $R^1(0.05)$ in blue. Bottom left panel: $R^1(0.9)$ in orange and $R^1(0.1)$ in blue. Bottom right panel: $R^1(0.66)$ in orange and $R^1(0.33)$ in blue. The x-axis labeling in terms of $d = -h$ allows to have past sentiment on the left and future sentiment on the right.

Figure 6: Same analyses as in Figure 5 but the filtered sentiment signal $S^{MLSS}$ is replaced by the noisy sentiment signal $S^{Obs}$. 

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which drive the long-term dynamics. The second component, dubbed short-term sentiment and modeled as a VAR(1) process, captures short-term swings of market mood. An extensive empirical section investigates the different features of the two sentiment components. In a first analysis, we pointed out that one of the long-term sentiment factors co-integrates with the first principal component of the market. Quite surprisingly, the structure of the sentiment factor loadings does not mimic the typical uniform profile of the market factor. Some assets are over-expressed and contributes to the factor with a positive or negative sign, while others are under-expressed. Concerning the short-term sentiment, its multivariate dependence structure explains a sizable fraction of the residual covariance in a single factor market model. This result suggests that the short-term component captures transient and rapidly changing trends associated with the idiosyncratic components of the market. In a second analysis, based on quantile regression, we showed that the Multivariate Long-Short Sentiment model provides the highest explanatory power of lagged and contemporaneous returns. Essential to achieve statistical significance are the multivariate nature of the approach and the separation of the sentiment signal in a long and a short component. In particular, disentangling the short-term sentiment is crucial to capture the behavior of extreme returns. In a final analysis, we observed that newspapers and social media differently react to negative and positive returns. Specifically, they can effectively explain abnormal returns from one to five days in advance, but they almost immediately digest the positive market realizations while they echo negative realizations for several days to come.

It is worth noting that (Tetlock, 2007) and (Garcia, 2013) reported results similar to ours for the unfiltered sentiment focusing on period before 2007. Using the TRMI dataset, (Beckers, 2018) showed that the forecasting power on returns of the sentiment dropped dramatically after 2007. Our results suggest that the filtering procedures are more important nowadays than in the past. In light of these findings, as a future perspective, we plan to investigate the effectiveness of an asset allocation strategy based on the filtered sentiment signal.
References

Algaba, A., Ardia, D., Bluteau, K., Borms, S., Boudt, K., 2016. Econometrics meets sentiment: An overview of methodology and applications. Available at SSRN 2652876.

Allcott, H., Gentzkow, M., 2017. Social media and fake news in the 2016 election. Journal of Economic Perspectives 31, 211–36.

Allen, D., McAleer, M., Singh, A., 2015. Machine news and volatility: The dow jones industrial average and the trna real-time high-frequency sentiment series. Handbook of High Frequency Trading , 327–344.

Antweiler, W., Frank, M.Z., 2004. Is all that talk just noise? The information content of internet stock message boards. The Journal of Finance 59, 1259–1294.

Appel, G., 2003. Become your own technical analyst: How to identify significant market turning points using the moving average convergence-divergence indicator or macd. The Journal of Wealth Management 6, 27–36.

Audrino, F., Tetereva, A., 2019. Sentiment spillover effects for US and European companies. Journal of Banking & Finance 106, 542 – 567.

Banbura, M., Modugno, M., 2014. Maximum likelihood estimation of factor models on datasets with arbitrary pattern of missing data. Journal of Applied Econometrics 29, 133–160.

Beckers, S., 2018. Do social media trump news? The relative importance of social media and news based sentiment for market timing. The Journal of Portfolio Management 45, 58–67.

Bork, L., 2009. Estimating US monetary policy shocks using a factor-augmented vector autoregression: an EM algorithm approach. Available at SSRN 1358876.

Borovkova, S., 2015. The role of news in commodity markets. Available at SSRN 2587285.
Borovkova, S., Garmaev, E., Lammers, P., Rustige, J., 2017. SenSR: A sentiment-based systemic risk indicator. Technical Report 553. De Nederlandsche Bank.

Borovkova, S., Mahakena, D., 2015. News, volatility and jumps: the case of natural gas futures. Quantitative Finance 15, 1217–1242.

Calomiris, C.W., Mamaysky, H., 2019. How news and its context drive risk and returns around the world. Journal of Financial Economics 133, 299–336.

Corsi, F., Peluso, S., Audrino, F., 2015. Missing in asynchronicity: A Kalman-EM approach for multivariate realized covariance estimation. Journal of Applied Econometrics 30, 377–397.

Da, Z., Engelberg, J., Gao, P., 2011. In search of attention. The Journal of Finance 66, 1461–1499.

Dempster, A.P., Laird, N.M., Rubin, D.B., 1977. Maximum likelihood from incomplete data via the EM algorithm. Journal of the Royal Statistical Society. Series B (Methodological), 1–38.

Durbin, J., Koopman, S.J., 2012. Time series analysis by state space methods. Oxford University Press.

Engle, R.F., Granger, C.W.J., 1987. Co-integration and error correction: Representation, estimation, and testing. Econometrica 55, 251–276.

Garcia, D., 2013. Sentiment during recessions. The Journal of Finance 68, 1267–1300.

Gerber, A.S., Gimpel, J.G., Green, D.P., Shaw, D.R., 2011. How large and long-lasting are the persuasive effects of televised campaign ads? Results from a randomized field experiment. American Political Science Review 105, 135–150.

Groß-Klußman, A., Hautsch, N., 2011. When machines read the news: Using automated text analytics to quantify high frequency news-implied market reactions. Journal of Empirical Finance 18, 321–340.
Harvey, A.C., 1990. Estimation, prediction and smoothing for univariate structural time series models. Cambridge University Press. pp. 168 – 233.

Hill, S.J., Lo, J., Vavreck, L., Zaller, J., 2013. How quickly we forget: The duration of persuasion effects from mass communication. Political Communication 30, 521–547.

Jungbacker, B., Koopman, S.J., 2008. Likelihood-based analysis for dynamic factor models. Tinbergen Institute, Tinbergen Institute Discussion Papers.

Kalman, R.E., 1960. A new approach to linear filtering and prediction problems. Transactions of the ASME–Journal of Basic Engineering 82, 35–45.

Koenker, R., Machado, J.A.F., 1999. Goodness of fit and related inference processes for quantile regression. Journal of the American Statistical Association 94, 1296–1310.

Lilly, F., Micciché, S., Tumminello, M., Piilo, J., Mantegna, R.N., 2015. How news affect the trading behavior of different categories of investors in a financial market. Quantitative Finance 15, 213–229.

Liu, B., 2015. Sentiment analysis: Mining opinions, sentiments, and emotions. Cambridge University Press.

Loughran, T., McDonald, B., 2011. When is a liability not a liability? Textual analysis, dictionaries, and 10-ks. The Journal of Finance 66, 35–65.

Pang, B., Lee, L., Vaithyanathan, S., 2002. Thumbs up?: Sentiment classification using machine learning techniques, in: Proceedings of the ACL-02 conference on Empirical methods in natural language processing-Volume 10, Association for Computational Linguistics. pp. 79–86.

Peterson, R., 2016. Trading on Sentiment: The Power of Minds Over Markets. John Wiley & Sons, Ltd.

Ranco, G., Alekovski, D., Caldarelli, G., Grcar, M., Mozetic, I., 2015. The effects of Twitter sentiment on stock price returns. PLOS ONE 10, e0138441.
Ranco, G., Bordino, I., Bormetti, G., Caldarelli, G., Lillo, F., Treccani, M., 2016. Coupling news sentiment with web browsing data improves prediction of intra-day price dynamics. PLOS ONE 11, e0146576.

Shumway, R.H., Stoffer, D.S., 1982. An approach to time series smoothing and forecasting using the EM algorithm. Journal of Time Series Analysis 3, 253–264.

Smales, L.A., 2014. News sentiment in the gold futures market. Journal of Banking & Finance 49, 275 – 286.

Smales, L.A., 2015. Asymmetric volatility response to news sentiment in gold futures. Journal of International Financial Markets, Institutions and Money 34, 161–172.

Sun, L., Najand, M., Shen, J., 2016. Stock return predictability and investor sentiment: A high-frequency perspective. Journal of Banking & Finance 73, 147 – 164.

Tetlock, P.C., 2007. Giving content to investor sentiment: The role of media in the stock market. The Journal of Finance 62, 1139–1168.

Thorsrud, L.A., 2018. Words are the new numbers: A newsy coincident index of the business cycle. Journal of Business & Economic Statistics, 1–17.

Vohra, S., Teraiya, J., 2013. A comparative study of sentiment analysis techniques. Journal JIKRCE 2, 313–317.

Wu, L.S.Y., Pai, J.S., Hosking, J., 1996. An algorithm for estimating parameters of state-space models. Statistics & Probability Letters 28, 99–106.
A Filter and Smoother recursions

In this section, we report Kalman Filter and Smoother recursions ancillary to the EM algorithm. The derivation of the formulas which follow can be found in (Shumway and Stoffer, 1982).

Starting from system 6, we calculate recursively the Kalman Filter as:

\[
\begin{align*}
\hat{F}_{t|t-1} & = E \left[ \hat{F}_t | S_1, \ldots, S_{t-1} \right] = \Phi \hat{F}_{t-1|t-1} \\
\hat{P}_{t|t-1} & = E \left[ \left( \hat{F}_t - \hat{F}_{t|t-1} \right) \left( \hat{F}_t - \hat{F}_{t|t-1} \right)' | S_1, \ldots, S_{t-1} \right] = \Phi \hat{P}_{t-1|t-1} \Phi' + Q \\
K_t & = \hat{P}_{t|t-1} \hat{\Lambda}' \left( \hat{\Lambda} \hat{P}_{t|t-1} \hat{\Lambda}' + R \right)^{-1} \\
\hat{F}_t & = \hat{F}_{t|t-1} + K_t \left( S_t - \hat{\Lambda} \hat{F}_{t|t-1} \right) \\
\hat{P}_t & = \hat{P}_{t|t-1} - K_t \hat{\Lambda} \hat{P}_{t|t-1}
\end{align*}
\]

where we take \( \hat{F}_{0|0} = \mu \) and \( \hat{P}_{0|0} = \Sigma \). Now, using backward recursions \( t = T, \ldots, 1 \) we derive the Smoother as

\[
\begin{align*}
J_{t-1} & = \hat{P}_{t-1|t-1} \hat{\Phi}' \left( \hat{P}_{t|t-1} \right)^{-1} \\
\hat{F}_{t-1|T} & = \hat{F}_{t-1|t-1} + J_{t-1} \left( \hat{F}_{t|T} - \Phi \hat{F}_{t-1|t-1} \right) \\
\hat{P}_{t-1|T} & = \hat{P}_{t-1|t-1} + J_{t-1} \left( \hat{P}_{t|T} - \hat{P}_{t|t-1} \right) J_{t-1}' \\
\hat{P}_{t-1,t-2|T} & = \hat{P}_{t-1|t-1} J_{t-2}' + J_{t-1} \left( \hat{P}_{t|T} - \Phi \hat{P}_{t-1|t-1} \right) J_{t-2}'
\end{align*}
\]

where \( \hat{P}_{T,T-1|T} = \left( I - K_T \hat{\Lambda} \right) \Phi \hat{P}_{T-1|T-1} \).
The log-likelihood of the model (6) is

\[ l \left( S_t, \tilde{F}_t, \theta (j) \right) = \log f(\tilde{F}_0) + \sum_{t=1}^{T} \log f(\tilde{F}_t | S_{t-1}) + \sum_{t=1}^{T} \log f(S_t | \tilde{F}_t) \]

\[ = -\frac{1}{2} \log |\Sigma| - \frac{1}{2} \left( \tilde{F}_0 - a \right) \Sigma^{-1} \left( \tilde{F}_0 - a \right)' \]

\[ - \frac{T}{2} \log |\tilde{Q}| - \frac{1}{2} \sum_{t=1}^{T} \left( \tilde{F}_t - \tilde{\Phi} \tilde{F}_{t-1} \right) \tilde{Q}^{-1} \left( \tilde{F}_t - \tilde{\Phi} \tilde{F}_{t-1} \right)' \]

\[ - \frac{T}{2} \log |R| - \frac{1}{2} \sum_{t=1}^{T} \left( S_t - \tilde{\Lambda} \tilde{F}_t \right) R^{-1} \left( S_t - \tilde{\Lambda} \tilde{F}_t \right)' \]

where \( a \) and \( \Sigma \) are the parameters s.t. \( \tilde{F}_0 \sim \mathcal{N}(a, \Sigma) \).

E-step

The objective function to maximize is, from Shumway and Stoffer (1982),

\[ G \left( a, \Sigma, R, \tilde{Q}, \tilde{\Lambda}, \tilde{\Phi} \right) = E_m \left[ \log f | S_1, \ldots, S_T \right] , \]

where \( E_m \) denotes the conditional expectation relative to a density containing the \( m \)th iterate values \( a(m), \Sigma(m), R(m), \tilde{Q}(m), \tilde{\Lambda}(m) \) and \( \tilde{\Phi}(m) \).

Using now the Kalman smoother (33) we can derive

\[ E \left[ \left( S_t - \tilde{\Lambda} \tilde{F}_t \right) \left( S_t - \tilde{\Lambda} \tilde{F}_t \right)' | S_1, \ldots, S_T \right] = \left( S_t - \tilde{\Lambda} \tilde{F}_t | T \right) \left( S_t - \tilde{\Lambda} \tilde{F}_t | T \right)' + \tilde{\Lambda} P_t | T \tilde{\Lambda}' \]

\[ E \left[ \left( \tilde{F}_t - \tilde{\Phi} \tilde{F}_{t-1} \right) \left( \tilde{F}_t - \tilde{\Phi} \tilde{F}_{t-1} \right)' | S_1, \ldots, S_T \right] = \left( P_t | T \tilde{F}_t | T \tilde{F}_t | T \tilde{F}_t | T \tilde{F}_t | T \tilde{F}_t | T \right) \tilde{\Phi}' + \tilde{\Phi} P_{t-1} | T \tilde{\Phi}' \]

\[ + \tilde{\Phi} P_{t-1} | T \tilde{F}_{t-1} | T \tilde{\Phi}' - P_{t-1} | T \tilde{\Phi}' \]

\[ - \tilde{F}_{t-1} | T \tilde{F}_{t-1} | T \tilde{\Phi}' - \tilde{\Phi} P_{t-1} | T \tilde{F}_t | T \tilde{\Phi}' - \tilde{\Phi} P_{t-1} | T \tilde{F}_t | T \tilde{\Phi}' \]
lead to

\[
G \left( a, \Sigma, R, \tilde{Q}, \tilde{\Lambda}, \tilde{\Phi} \right) = -\frac{1}{2} \log |\Sigma| - \frac{1}{2} \text{tr} \left\{ \Sigma^{-1} \left[ P_{0|T} + \left( \tilde{F}_0 - a \right) \left( \tilde{F}_0 - a \right)' \right] \right\} \\
- \frac{T}{2} \log |Q| - \frac{1}{2} \text{tr} \left\{ Q^{-1} \left( C - B\tilde{\Phi}' - \tilde{\Phi}B' + \tilde{\Phi}A\tilde{\Phi}' \right) \right\} \tag{34}
\]

\[
- \frac{T}{2} \log |R| - \frac{1}{2} \text{tr} \left\{ R^{-1} \left( E_3 - \tilde{\Lambda}E_2' - E_2\tilde{\Lambda}' + \tilde{\Lambda}E_1\tilde{\Lambda}' \right) \right\} ,
\]

where

\[
A = \sum_{t=1}^{T} \left( \tilde{F}_{t-1|T} \tilde{F}_{t-1|T}' + P_{t-1|T} \right),
\]

\[
B = \sum_{t=1}^{T} \left( \tilde{F}_{t|T} \tilde{F}_{t-1|T}' + P_{t-1|T} \right),
\]

\[
C = \sum_{t=1}^{T} \left( \tilde{F}_{t|T} \tilde{F}_{t|T}' + P_{t|T} \right),
\]

\[
E_1 = \sum_{t=1}^{T} P_{t|T} + \tilde{F}_{t|T} \tilde{F}_{t|T}',
\]

\[
E_2 = \sum_{t=1}^{T} S_t \tilde{F}_{t|T},
\]

\[
E_3 = \sum_{t=1}^{T} S_t S_t'.
\]

**M-step**

The resulting update equations are

\[
\Lambda(m+1) = E_2 E_1^{-1} \tag{36}
\]

\[
\tilde{\Phi}(m+1) = BA^{-1} \tag{37}
\]

\[
Q(m+1) = \frac{1}{T} \left( C - B\tilde{\Phi}(m+1)' - \tilde{\Phi}(m+1) B' + \tilde{\Phi}(m+1) A\tilde{\Phi}(m+1)' \right) \tag{38}
\]

\[
R(m+1) = \frac{1}{T} \left( E_3 - \tilde{\Lambda}(m+1)E_2' - E_2\tilde{\Lambda}(m+1)' + \tilde{\Lambda}(m+1)E_1\tilde{\Lambda}(m+1)' \right) \tag{39}
\]

\[
a(m+1) = \tilde{F}_{0|T} \tag{40}
\]

\[
\Sigma(m+1) = Z \tag{41}
\]

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For simplicity, in our estimations we impose $\tilde{F}_0 = 0$. 