On quark-hadron duality in the heavy quark sector

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I discuss possible failures of local quark-hadron duality in the system of heavy-light quark mesons. As an example, I consider a correlator of two currents comprising heavy quark operators, and I compare the OPE expression with the result obtained by a complete insertion of hadronic states. After a smearing procedure, OPE and hadronic spectral functions agree with each other. However, the local behaviour of the two functions is different, and the difference manifests itself in a term which is absent in the OPE.

1. Introduction

Quark-gluon/hadron duality represents a basic concept in the theoretical description of inclusive hadronic processes. In general, it means that some hadronic rates can be computed as the corresponding partonic rates, and that high energy processes can be computed in terms of hadronic matrix elements of operators in an expansion, the Operator Product Expansion (OPE), whose leading term is represented by the perturbative QCD expression. It is duality which allows us to extrapolate from the deep Euclidean region, where the OPE is defined, to the Minkowski domain, where physical observables are measured.

An OPE-based approach has been recently proposed for the calculation of semileptonic and nonleptonic decay rates of hadrons containing one heavy quark [1]. The basic idea consists in the expansion in inverse powers of the heavy quark mass $m_Q$. In this approach, however, a distinction must be maintained between semileptonic and nonleptonic decays. In general, hadronic and OPE amplitudes cannot be identical even at very high momentum transfer, due to the different production thresholds of multiparticles and of quarks and gluons. In the case of semileptonic heavy hadron decays this difficulty can be avoided, since one has to integrate over lepton variables, which amounts to a smearing of the OPE width. The equality between smeared OPE and hadronic widths is referred to as global duality (different from local duality, i.e. without smearing). It is generally believed that global duality holds between quark-gluon and hadronic cross sections [2]; for $B$ and $\Lambda_b$ semileptonic decays global duality has been extensively analyzed in [3,4].

For nonleptonic heavy hadron decays there are no lepton momenta to be integrated; therefore, one cannot invoke global duality to prove the identification of OPE and hadronic observables, and therefore local duality must be assumed. In this case, the validity of the OPE-based $1/m_Q$ expansion for the computation of nonleptonic heavy hadron inclusive decay rates appears to be more debatable. Indeed, in [3] it was suggested that the discrepancy between the prediction $\frac{\tau(\Lambda_b)}{\tau(B_d)} > 0.9$ and the experimental result $\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.78 \pm 0.04$ might be solved assuming a violation of local quark-hadron duality, with the appearance of a $\mathcal{O}(\frac{1}{m_Q})$ correction not predicted by OPE.

At the moment, understanding the origin of this possible correction to the OPE is a difficult task. However, it is interesting to investigate possible violations of local duality in some definite model.

Several recent studies can be found in the literature concerning the validity of the quark-gluon/hadron duality in connection with non-perturbative QCD applications in the heavy

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quark sector \([8,12]\). In particular, in \([8]\) the two-point function involving the difference between scalar and pseudoscalar heavy-light currents was considered as a benchmark to investigate global as well as local duality properties. In the chiral limit (for the light flavour) and the infinite mass limit (for the heavy flavour), the coefficients of the OPE for the correlator can be calculated analytically. Then, the spectral function derived form a particular hadronic model (in the time-like region) is expanded in a power series (in the space-like region) and compared with the exact result. From this comparison one can gauge the validity of (local) duality.

Here, I want to discuss how the results are stable against modifications of the hadronic model. Evaluating the two-point function by a complete insertion of states derived from a relativistic constituent quark model, I’ll show that an explicit violation of local duality can be detected, in the form of an unexpected correction to the OPE \([10]\).

2. Example of violation of local duality

I consider the correlator \([8]\)

\[
\Pi(q) = \frac{i}{4} \int dx e^{iqx} [\langle 0|T(J_S(x)J_S^+(0))|0\rangle - \langle 0|T(J_F(x)J_F^+(0))|0\rangle] = \Pi_S(q) - \Pi_F(q),
\]

with \(J_S(x) = \bar{Q}(x)q(x), J_F(x) = \bar{Q}(x)i\gamma_5 q(x), Q(x)\) and \(q(x)\) being heavy and light quark operators, respectively. In the chiral limit, \(m_q \to 0\), \(\Pi(q)\) vanishes in perturbation theory. In the infinite heavy quark mass limit, \(m_Q \to \infty\), it is convenient to choose \(q^2 = (m_Q - \epsilon, 0)\), so that the correlator becomes a function of \(\epsilon\):

\[
\Pi(\epsilon) = \frac{1}{4} \int_0^{+\infty} d\tau e^{-\epsilon \tau} \Phi(\tau) \quad (\epsilon > 0),
\]

\(\Phi(\tau)\) representing the non-local quark condensate

\[
\Phi(\tau) = \langle 0|\bar{q}(\tau)e^{i\tau} \int_0^{+\infty} dy e^{iy}\Lambda_{\nu}(y)|q(0)\rangle >.
\]

When \(\epsilon \gg \Lambda_{QCD}\), the OPE expression for \(\Pi(\epsilon)\)

\[
\Pi_{OPE}(\epsilon) = \frac{\langle 0|\bar{q}q|0\rangle}{4\epsilon} \times \left[ 1 - \frac{m_0^2}{8\epsilon^2} + c_4 \frac{m_4}{\epsilon^4} - c_6 \frac{m_6}{\epsilon^6} + \ldots \right]
\]

is given in terms of the quark condensate \(<0|\bar{q}q|0\rangle = ( -240 \text{ MeV} )^3\) and of the mixed quark-gluon condensate, parametrized by

\[
\frac{m_0^2}{4\epsilon} <0|\bar{q}q\sigma_{\mu\nu}G_{\mu\nu}^a|0\rangle = 0.8 \pm 0.2 \text{ GeV}^2.
\]

The positive coefficients \(c_{2n}\) depend on the actual form of the non-local condensate; notice explicitly the alternating signs in Eq.\([8]\) and the absence of even powers of \(\epsilon^{-1}\).

It is possible to identify a model for \(\Pi(\epsilon)\) having the expansion \([8,12]\):

\[
\Pi(\epsilon) = \frac{\langle 0|\bar{q}q|0\rangle}{4\epsilon} \beta \frac{\epsilon + \Lambda}{2\epsilon}
\]

with \(\Lambda\) a parameter and

\[
\beta(z) = \frac{1}{2} \left[ \psi \left( \frac{z+1}{2} \right) - \psi \left( \frac{z}{2} \right) \right],
\]

\(\psi(z)\) being the logarithmic derivative of the Gamma function. \(\Pi(\epsilon)\) in \([8]\) can be written as

\[
\Pi(\epsilon) = \frac{\langle 0|\bar{q}q|0\rangle}{2\epsilon} \sum_{j=0}^{\infty} \frac{(-1)^j}{\epsilon/\Lambda + 1 + 2j + 1}
\]

and admits the asymptotic expansion

\[
\Pi(\epsilon) \sim \frac{\langle 0|\bar{q}q|0\rangle}{4\epsilon} E_{2n+1} \frac{\Lambda^{2n}}{\epsilon^{2n}}
\]

\((E_{2n} \text{ Euler numbers})\). Comparison of \([8]\) with \([12]\) indicates that the model is able to reproduce the right power structure of \(\Pi(\epsilon)\). The spectral density associated to \([8]\) displays an infinite number of equally spaced poles located along the negative \(\epsilon\) axis, with residues having alternating signs.

The correspondence between \([8]\) and \([12]\), however, follows from the assumed form \([8]\) for the correlator. It would be interesting to investigate the properties of global and local duality using a representation of the correlator obtained from a hadronic model more closely related to QCD.

Let us compute \([8]\) by a complete insertion of hadronic states. We consider the \(N_c \to \infty\) limit, since in this case the only contributions are \(J^P = 0^+\) and \(0^-\) single particle states, contributing respectively to \(\Pi_S\) and \(\Pi_F\). We denote by \(|S_n\rangle >\) and \(|P_n\rangle >\) the states, with masses \(M_{S_n}\) and \(M_{P_n}\) respectively, and we define the
current-particle matrix elements: \(< 0|J_S|S_n \rangle = \frac{M_n^2}{m_Q} f_{S_n, \ell}, \quad < 0|J_P|P_n \rangle = \frac{M_n^2}{m_Q} f_{P_n} \). In the limit \(m_Q \to \infty\) the masses can be written as:

\[ M_{S_n(\ell)} = m_Q + \delta_{S_n(\ell)} + \mathcal{O}(\frac{1}{m_Q}) \quad (9) \]

The binding energies \(\delta_{S_n(\ell)}\) can be obtained by solving the relativistic wave equation

\[
\begin{align*}
&[\sqrt{-\nabla^2 + m_Q^2} \\
&+ \sqrt{-\nabla^2 + m_n^2 + V(\vec{r})}] \Psi_n(\vec{r}) = M_n \Psi_n(\vec{r})
\end{align*}
\]

where the (central) potential \(V(\vec{r})\) is chosen as \(V(r) = \mu^2 r\), with constant \(\mu\) (string tension). Masses, wavefunctions and current-particle couplings of scalar and pseudoscalar states can be computed applying WKB methods to \([11]\) \([13]\) \([14]\). One gets:

\[
\begin{align*}
\delta^{(n)}_\ell &= \mu \sqrt{\frac{\pi(2n + \ell + \frac{3}{2})}{M_n \ell}} \\
\ell f^{(n)}_\ell &= \sqrt{\frac{3m_Q \delta^{(n)}_\ell \mu}{\pi M_n \ell}} \quad (12)
\end{align*}
\]

Assuming the WKB approximation for the lowest-lying state, an estimate can be derived for the string tension: for \(m_0 = 4.6 - 4.7\) GeV one has \(\mu \approx 300\) MeV.

The choice of considering only single particle states between the currents in the correlator is not too restrictive, as it can be shown by considering the imaginary part of the correlator of scalar currents \(\Pi_S\) \((\Pi_P)\) in Eq.\([1]\). Computing the imaginary part of the quark loop diagram one has, for \(E = -\epsilon \to +\infty\):

\[
\text{Im} \Pi^{\text{OPE}}_S(E) \to \frac{3E^2}{8\pi} \quad (13)
\]

On the other hand, the resonance model gives:

\[
\text{Im} \Pi^{\text{had}}_S(E) = \frac{3\mu^2 E}{8} \sum_{n=0}^{\infty} \delta \left[ E - \mu \sqrt{2\pi(n + 7/4)} \right] \quad (14)
\]

Although the two expressions \([13]\) and \([14]\) look very different, after a smearing procedure \([2]\) obtained by \(\sum_n \to \int dn\), one has

\[
\text{Im} \Pi^{\text{had}}_S(E) = \frac{3\mu^2 E}{8} \int d\epsilon \left[ E - \mu \sqrt{2\pi(n + 7/4)} \right] = \frac{3E^2}{8\pi} = \text{Im} \Pi^{\text{OPE}}_S(E) \quad (15)
\]

Therefore \(\Pi_S(E)\) and \(\Pi_P(E)\) satisfy global duality, at least at the leading order for \(E \to \infty\).

Let us now compute the correlator \([1]\) by the hadronic insertion. We get

\[
\Pi^{\text{had}}_S(E) = \frac{3\mu^3}{8\sqrt{\pi}} \sum_{n=0}^{\infty} (-1)^n \delta \left[ E - \mu \sqrt{\pi(n + 3/2)} \right] \quad (16)
\]

and \((E > 0)\)

\[
R^{\text{had}}(E) = \text{Im} \Pi^{\text{had}}_S(E) = \frac{3\mu^2 E}{8} \sum_{n=0}^{\infty} (-1)^n \delta \left[ E - \mu \sqrt{\pi(n + 3/2)} \right] \quad (17)
\]

From the first four terms in \([4]\) we have:

\[
R^{\text{OPE}}(E) = \text{Im} \Pi^{\text{OPE}}(E) = \frac{3\mu^3}{8\sqrt{\pi}} \sum_{n=0}^{\infty} (-1)^n \delta \left[ E - \mu \sqrt{\pi(n + 3/2)} \right] (18)
\]

with unknown \(c_4\), \(c_6\) \((c_4 = \frac{5}{64}, c_6 = \frac{61}{512}\) in the model \([8]\)). Also in this case the two expressions look very different, but they can be compared after an appropriate smearing; following \([2]\) we consider the smeared function

\[
\tilde{R}^{\text{had}}(E, \Delta) = \frac{\Delta}{\pi} \int dE' \frac{R^{\text{had}}(E')}{(E - E')^2 + \Delta^2} \quad (19)
\]

and a similar expression for \(R^{\text{OPE}}(E, \Delta)\). For \(\Delta \to 0\) on can prove that \(\tilde{R}(E, \Delta) \to R(E)\). Moreover, since

\[
\tilde{R}(E, \Delta) = \frac{1}{2i} [\Pi(E - i\Delta) - \Pi(E + i\Delta)] \quad (20)
\]

for \(\Delta >> \Lambda_{QCD}\) one is far from physical singularities.

For a large energy \(E\) the two expressions \(\tilde{R}^{\text{OPE}}\) and \(\tilde{R}^{\text{had}}\) should coincide regardless of \(\Delta\). Such a request implies:

\[
< 0|\bar{q}q|0 > = -3\frac{0.51}{2} \frac{\mu^3}{\sqrt{\pi}} \quad (21)
\]

e.g. \(\mu = 317\) MeV, in agreement with the value derived from the spectrum. For smaller values of
For $E$, i.e. $1 < E < 20$ GeV, the numerical results concerning $\tilde{R}^{\text{had}}(E, \Delta)$ and $\tilde{R}^{\text{OPE}}(E, \Delta)$ are reported in Fig. 1, where the ratio $P = \tilde{R}^{\text{had}} / \tilde{R}^{\text{OPE}}$ is plotted as a function of $E$ for several values of $\Delta$. As expected, the agreement between $\tilde{R}^{\text{had}}$ and $\tilde{R}^{\text{OPE}}$ improves with increasing $\Delta$; in particular, for $\Delta = 3.0$ GeV the difference does not exceed 20%. For any value of the smearing parameter $\Delta$ the ratio $P$ tends to unity for large energies. Therefore, the resonance model satisfies the requirement of global duality: the smeared imaginary parts of the correlator, when computed by OPE or by hadronic states, agree with each other.

Concerning local duality, the expressions are significantly different, since

$$\Pi^{\text{had}}(\epsilon) = -3 \frac{0.51 \mu_3^3}{8\sqrt{\pi} \epsilon} \left( 1 - \frac{\tilde{m}_0}{\epsilon} + 0.93 \frac{\tilde{m}_0^2}{\epsilon^2} + ... \right).$$

The factor multiplying $\epsilon^{-1}$ coincides numerically with $\frac{<0|\bar{q}q|0>}{4 \epsilon}$ for $\mu = 317$ MeV (eq. [22]). As for the mass parameter $\tilde{m}_0$, numerically we find $\tilde{m}_0 = 560$ MeV. In conclusion one has:

$$\Pi^{\text{had}}(\epsilon) = \frac{0|\bar{q}q|0>}{4 \epsilon} \left( 1 - \frac{\tilde{m}_0}{\epsilon} + 0.93 \frac{\tilde{m}_0^2}{\epsilon^2} + ... \right).$$

The comparison between (23) and (1) shows a violation of local duality, which manifests itself in the form of an unexpected term in the asymptotic expansion in powers of $\frac{1}{\epsilon}$.

3. Conclusions

In the case of a simple correlator of quark currents, using a particular quark model and in the $N_c \to \infty$ limit, I have shown an example of violation of local duality which occurs in a condition where global duality is verified. The difference between the hadronic and the OPE spectral functions is made evident by a term which is absent in the expansion predicted by OPE.

Of course, the calculation of the correlators required for the evaluation of the $B_d$ and $\Lambda_b$ inclusive decay widths, is, beyond our present possibilities. However, the example supports the conjecture that the $\Lambda_b$ lifetime problem can be explained by the presence of a $1/m_Q$ correction not included in the usual OPE expansion.

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Discussions

B. Blok (Technion, Haifa)
How are your results model dependent?

P. Colangelo
As discussed, there is a dramatic difference between two choices of the hadronic representations of the correlator (1). Therefore, the results are model dependent, although the model I presented seems rather realistic and QCD-inspired. The main point is that it allows to detect a mechanism of violation of local quark-hadron duality which is similar to what is needed to explain the $\Lambda_b$ lifetime problem.

M. Neubert (CERN, Geneva)
In my opinion, the violation of global duality found in your model ($\sim 10-40\%$) are surprisingly large. Is the $E$-dependence of these violations in accordance of general expectations as discussed, e.g., in the previous talk by B. Blok?

P. Colangelo
The numerical details can be ascribed to some features of the model, for example to the assumption of vanishing width of the resonances. What, in