Transmission of few-cycle pulses by Fabry-Perot interferometer

Elizaveta Buyanovskaya¹ and Egor Kryshkovets²

¹Assistant lecturer, National Research University of Information Technologies, Mechanics and Optics, St. Petersburg 197101, Russia
²Laboratory assistant, National Research University of Information Technologies, Mechanics and Optics, St. Petersburg 197101, Russia
E-mail: lee.buyanovskaya@gmail.com

Abstract. We theoretically model the transmission of ultra-short optical pulses by Fabry-Perot interferometer, taking into account multiple waves re-reflection inside interferometer by examination of two superimposed waves. Based on the analytical and numerical analysis, we calculate the transformations of the optical electric field profiles and optical spectra of the pulses at the output of the interferometer.

1. Introduction
Nowadays high-intensity and extremely short optical pulses containing just few cycles of electromagnetic field are experimentally accessibly [1, 2, 3]. This opens new opportunities for applications taking advantage of strong non-destructive light-matter interactions. One potential possibility is to develop ultrafast optical transistors [4]. Optical switching could be enhanced through wave interactions in resonators [5, 6], and here we theoretically investigate the transmission of extremely short pulses through interferometer. Specifically, we analyze a Fabry-Perot interferometer (IFP) created by a nonlinear dielectric between two metal mirrors [7].

2. Theoretical model of Fabry-Perot interferometers for ultra short pulses
Theoretical investigations of ultra short pulses transmission by IFP were conducted according to technique that takes into account multiple waves re-reflection inside interferometer by examination of just two superimposed waves. We investigate ideal IFP (Fig. 1) that consists of two metal mirrors with reflection coefficient \( R \) spaced at distance \( L \), dispersion and saturation (both mirrors and media inside interferometer) were out of the scope of current study. Here \( z = z'/\lambda, t = t'/T_0 \), where \( T_0 \) is central period of the pulse, \( \lambda \) is central wavelength, \( t, z \) are time and distance in seconds and meters respectively. Interferometer is considered linear if an intensity \( I \) is low and refraction index of the media inside interferometer \( N \) does not depend on it and nonlinear otherwise.

We analyze the wave transmission using short field equations accounting for the cubic nonlinearity of a dielectric material previously derived in [8] by generalizing them to account...
for the boundary conditions on the mirrors:

\[
E^{(0)}_+(0, t) = \sqrt{1 - R} E_{in}(t) + \sqrt{R} E^{(0)}(0, t) \\
E^{(0)}_{refl}(0, t) = \sqrt{R} E_{in}(t) + \sqrt{1 - R} E^{(0)}(0, t) \\
E^{(0)}_-(L, t) = \sqrt{1 - R} E^{(0)}(L, t) \\
E^{(0)}_out(L, t) = \sqrt{1 - R} E^{(0)}_+(L, t)
\]

(1)

Figure 1. Schematic of ideal Fabry-Perot interferometer.

Then by applying method of successive approximations we obtain asymptotic analytical expression that describes linear propagation of the wave (the first term in RHS in (2a)) as well as nonlinear wave self-action (2b) and interaction (2c, 2d) between counter-propagating waves:

\[
E_{out}(L, t) = E^{(0)}_{out}(L, t) + G E^{(1)}_{out}(L, t) = F^{-1}\left\{ \frac{1 - R}{1 - R \exp(-i\omega \cdot 2L)} \right\} + G \sqrt{\frac{1 - R}{R}} \cdot \left[ F^{-1}\left\{ \frac{F\{s_-(L, t) + I_-(L, t)\} + \sqrt{R} F\{s_+(L, t) + I_+(L, t)\}}{R \cdot \exp(-i\omega \cdot 2L) - 1} \right\} + s_-(L, t) + I_-(L, t) \right]
\]

(2a)

where

\[
s_-(L, t) = L \left( \sqrt{\frac{R}{1 - R}} \right)^3 \frac{\partial}{\partial t} \left[ E^{(0)}_{out}(t - L) \right]^3, s_+(L, t) = L \left( \frac{1}{1 - R} \right)^3 \frac{\partial}{\partial t} \left[ E^{(0)}_{out}(t - L) \right]^3
\]

(2b)

\[
I_+(L, t) = \frac{3}{2} \left( \sqrt{\frac{R}{1 - R}} \right)^3 \frac{\partial}{\partial t} \left[ \sqrt{R} \left( E^{(0)}_{out}(t - L) \right)^2 \int_{t-L}^{t+L} E^{(0)}_{out}(\xi - 2L) \, d\xi \right] + R \left( E^{(0)}_{out}(t - L) \right)^2 \int_{t-L}^{t+L} \left( E^{(0)}_{out}(\xi - 2L) \right)^2 \, d\xi
\]

(2c)

\[
I_-(L, t) = \frac{3}{2} \left( \sqrt{\frac{1}{1 - R}} \right)^3 \frac{\partial}{\partial t} \left[ \left( E^{(0)}_{out}(t + L) \right)^2 \int_{t-L}^{t+L} E^{(0)}_{out}(\tau - 2L) \, d\tau \right] + E^{(0)}_{out}(t + L) \int_{t-L}^{t+L} \left( E^{(0)}_{out}(\tau - 2L) \right)^2 \, d\tau
\]

(2d)

\[ G = 4n_2 I/N_0 \text{ (here } n_2 \text{ is coefficient of nonlinear refraction index), } F\{\} \text{ and } F^{-1}\{\} \text{ are direct and inverse Fourier transform respectively.} \]
Figure 2. Temporal (above) and spectral (down) profiles of transmitted single-cycle radiation in linear case.

3. Simulation of ultra short pulses dynamics through Fabry-Perot interferometer

We perform numerical simulations for transmission of the Gaussian pulse

\[ E_{in}(t) = E^{(0)}(t) = E_0 \exp \left( -\frac{t^2}{\tau_0^2} \right) \sin(2\pi t) \]

for various normalized pulse durations \( \tau_0 = \frac{t_0}{T_0} \) (\( t_0 \) is pulse duration in fs), distances \( L \) and coefficients \( R \) for linear and nonlinear cases separately.

On the Fig. 2 typical temporal electric field profile and corresponding spectrum for different \( L \) for linear case are presented. From the figure it can be seen that in result of transmission of ultra short pulses by IFP we observe, as expected, modulated spectrum that in time domain corresponds to the sequence of subpulses. When the distance between mirrors is less than or commensurable with dimensions of single-cycle pulse in media in spectrum domain we observe bell-shaped structure more narrow than spectrum of initial pulse. In the nonlinear case (Fig. 3) the simulations predict enhanced generation of triple, quintuple and higher order harmonics.

4. Summary

In this paper we analyze the wave transmission using model that is formulated directly for the electric field without resorting to a slowly varying amplitude approximation and accounts for the cubic nonlinearity of a dielectric [9]. This allows us to analyze the dynamics of extremely short pulses, with durations down to a single field oscillation. We use method of successive approximations in order to find approximate analytical solutions, while taking into account nonlinear wave self-action and interaction between counter-propagating waves. Then based on the analytical and numerical analysis, we calculate the transformations of the optical electric field profiles and optical spectra of the pulses at the output of the interferometer.

It was shown that by varying interferometer parameters we can control transmitted spatiotemporal few-cycle structure. In result of transmission of ultra short pulses by linear IFP we observe modulated spectrum that in time domain corresponds to the sequence of subpulses that is defined by a resonator round-trip time. When the distance between mirrors is less than
or commensurable with dimensions of single-cycle pulse in media in spectrum domain we observe bell-shaped structure more narrow than spectrum of initial pulse. Maximum of this structure is shifted to the short wavelength range with distance $L$. In time domain the single field structure with number of oscillations increasing from one and half to ten and more with coefficient $R$ is formed. Energy of transmitted radiation is changing quasi periodically.

In the nonlinear case the simulations show enhanced generation of triple, quintuple and higher order harmonics, which is facilitated by multiple reflections and the shift of the spectrum maximum to the short wavelength region for the pulses with ultra broad spectrum [10]. The integral spectral density of transmitted radiation with decreasing distance $L$ is changing quasi periodically like in linear case. These results indicate a possibility to create an optical transistor that works as an amplifier of few-cycle optical pulses.

Acknowledgments

Authors wish to acknowledge helpful discussions with Prof. Sergei Kozlov and financial support from Government of Russian Federation (Grant 074-U01) and from Russian Foundation for Basic Research (Grant 14-02-31367).

References

[1] Nazarkin A 2006 Phys. Rev. Lett 97 163904
[2] Brabec Th and Krausz F 1997 Phys. Rev. Lett. 78 3282
[3] Kozlov S and Samartsev V 2013 Fundamentals of femtosecond optics (Woodhead Publishing)
[4] Smith S P 1986 Applied Optics 25 1550
[5] Renger J, Quidant R and Novotny L 2011 Optics Express 19 1777
[6] Webb R P, Manning R J, Maxwell G D and Poustie A J 2003 Electronics Letters 39 79
[7] Fabry C and Perot A 1897 Ann. Chim. Phys. 12 459
[8] Buyanovskaya E and Kozlov S 2007 J. Exp. and Theor. Phys. Lett. 86 297
[9] Bespalov V, Kozlov S, Shpolyansky Yu and Walmsley I A 2002 Phys. Rev. A 666 013811
[10] Drozdov A, Kozlov S, Sukhorukov A and Kivshar Yu 2012 Phys. Rev. A 86 053822