Postural body sway displacements for quiet standing subjects (measured with a new ultrasonic device) are reported. Two of the well known strategies for balancing, namely ankle and hip movements were probed. The data is modeled using a Fokker-Plank-Langevin stochastic theory. Both analytic and computer simulation techniques are employed. The Kramers transition rate for a fall is expressed as a function of experimental parameters. The root mean square velocity is especially important in determining the fall probability.

The agreement between theory and experiment appears satisfactory. The implications of experimental data on the dynamics of quiet standing are well known [4]: (i) body movement from the ankles, (ii) body movement from the hips, and (iii) stepping motions. Stepping motions will not be discussed in Sec.II. The experimental details concerning the SWA device will be discussed in Sec.II.

Three fundamental movement strategies to maintain a standing balance are well known [4]: (i) body movement from the ankles, (ii) body movement from the hips, and (iii) stepping motions. Stepping motions will not be considered any further in this work which concerns quiet standing. As described in Sec.III, we find very different frequency scales for hip and ankle motions. The ankle motions proceed more slowly than the hip motions by about a factor of ten. Another feature of the ankle motions, also discussed in Sec.III, concerns the stability angle with respect to the vertical direction beyond which a subject, (without bending or stepping) will fall [4]. The falling instability will be modeled by an appropriate potential in Sec.IV. Further, the noise itself will be modelled using the Fokker-Planck stochastic equation [6] with a noise temperature intimately related to the mean square velocity fluctuations of the standing subject.

We have performed computer simulations of quiet standing subjects using the SWA device. Apart from the analytic work on stochastic equations, the computer simulations with data taken on subjects using the SWA device. The agreement and implications of the computer simulations with data taken on subjects using the SWA device are well known [4].

Posture Sway and the Transition Rate for a Fall

R.K. Koleva, A. Widom and D. Garelick
Physics Department, Northeastern University, Boston MA 02115

Meredith Harris
Department of Physical Therapy, Northeastern University, Boston MA 02115

I. INTRODUCTION

A problem of considerable interest in human body motion dynamics is the prediction of the probability to fall in a given environment [4]. Essential information is contained in the small amplitude displacement noise exhibited by the body during quiet standing. Our purpose is to explore how the quiet standing noise data may be used to compute the probability (more precisely the transition rate per unit time) of falling in various external environments.

In the previous work of other groups, the data on body displacement noise were taken using a multicomponent force plate upon which the subject quietly stands [4]. In the present work, we employ a sound wave assessment (SWA) device which measures body displacements using sound wave echoes; e.g., by measuring the time taken for a sound wave pulse to travel from the source to the standing subject and/or the time taken for a sound wave pulse to travel from the standing subject to a receiver. The experimental details concerning the SWA device will be discussed in Sec.II.

The SWA device employs two small ultrasonic transducers each of which can send or receive ultrasonic signals. The first is positioned on a stable laboratory stand. The second is attached with belt to the lower back of quietly standing subject. Each transducer sends sixteen ultrasonic pulses every $3.3 \times 10^{-2}$ sec, which are later detected by the opposite transducer. The distance between the two transducers can then be obtained from the time required for the pulses to travel from the sender to the receiver.

A single transducer system has been used in industry, e.g. by Polaroid for photography applications, in order to determine the distance to an object. The single transducer is used to both emit and detect its own echo. Our use of two transducers improves the accuracy.
by having less air absorption as well as less spatial dispersion (fanning out) of the pulses. Our position measurements are quiet precise; i.e. displacements are measured to within an instrumental accuracy of 0.02 cm in a bandwidth of 12 kHz. This yields a displacement noise error of \( \delta x \approx 2 (\mu m/\sqrt{Hz}) \). Thus, we can record the fine movements of a subject (both towards and away from a static transducer) as a function of time.

### III. FREQUENCY SCALES FOR HIP AND ANKLE STRATEGIES

Two fundamental body movement strategies \([4]\) (to maintain a standing balance) are apparent in the SWA measurements discussed below. These are the quiet standing ankle movements and hip movements, which occur in widely different frequency bands. The hip motions take place on a time scale of \( \tau_{hip} \sim 2 \text{ sec} \), while the ankle motions take place on a time scale of \( \tau_{ankle} \sim 30 \text{ sec} \). In order to measure ankle motions, the durations of our measurements were rather long, i.e. 3 min, while the shorter runs of 1 min duration were sufficient to study hip motions. The short times scales, of less than 20 sec were sufficient to verify the fractal diffusion estimates of hip motions which have appeared in the pioneering studies \([3,8–10]\) using the force plate technology.

To illustrate the decomposition of movements into ankle motions and hip motions we plot, in FIG.1, the displacement as a function of time for a typical quiet standing subject. One notes the slowly varying drifts back and forth characteristic of ankle motions along with the more rapid (and in this case) much smaller hip motions.

Were the quiet standing subject to move without hip motions, i.e. move rigidly, then the stiff body would be held steady, if and only if the angle \( \vartheta \) formed with the vertical were less then some critical angle \( \vartheta_c \). The geometrical cone \( \vartheta < \vartheta_c \) is called the sway envelop or the cone of stability \([8]\). Typical values for the critical (forward) angle is \( \vartheta_c \approx 8^\circ \) \([14]\). For angles \( \vartheta > \vartheta_c \), the ankles can no longer hold the subject upright. In the absence of a step or other supports (such as arm motions) a fall will take place. If one considers the displacement of the part of the body at a height \( h \) above ground, then \( a = h \tan \vartheta_c \) represents the critical displacement amplitude beyond which the ankle mode is unstable. Such critical displacements are normally measured using standard tests of the subjects forward or backward reach \([12,13]\).

### IV. FOKKER-PLANCK-LANGEVIN THEORY

The Fokker-Planck-Langevin stochastic approach \([3]\) to quiet standing sway processes has been shown to be extremely useful \([14,15]\). In what follows we apply this method to the ankle movements. As discussed in Sec.III, the ankle movements can be described by a slow oscillatory motion modulated by faster random noise.

To mathematically describe the cone of stability, we model the metastable potential by the potential

\[
U(x) = -\left(\frac{m\omega_0^2a^2}{4}\left[1 - \left(\frac{x}{a}\right)^2\right]\right)^2, \tag{2}
\]

where \( \omega_0 \) is the frequency of the ankle oscillation mode, \( m \) is the mass of the mode described by the displacement \( x \), and \( a \) is the critical displacement.

The ankle mode, as shown in the experimental FIG.1, is far from a perfect oscillation. There exists a random force \( f(t) \) and a finite quality factor \( Q \) for the mode which, in the Langevin theory, are intimately related. The equation of motion for the mode may be written as

\[
m \left[ \ddot{x} + \left(\frac{\omega_0}{Q}\right) \dot{x} + \omega_0^2x \right] + U'(x) = f(t) + F(t), \tag{3}
\]

where \( F(t) \) is an applied force, and \( f(t) \) is a “white noise” random force obeying

\[
\langle f(t)f(t') \rangle_{\text{noise}} = \left(\frac{2m\omega_0k_BT_n}{Q}\right) \delta(t-t'), \tag{4}
\]

and where \( T_n \) is noise temperature. The noise temperature is by no means equal to the ambient temperature of the room in which the subject stands. Rather the noise temperature describes all of those fluctuations which couple into the coordinate \( x \). In particular, all of the environmental and internal fluctuations which couple into \( x \) contribute to the root mean square velocity \( V \).
\[ V^2 = \langle \dot{x}^2 \rangle \] (5)

which can be obtained from experimental data such as that pictured in FIG.1. The noise temperature \( T_n \) is here defined in terms of the root mean squared velocity \( V \) via the equipartition theorem \[ \left( \frac{mV^2}{2} \right) = \left( \frac{k_B T_n}{2} \right). \] (6)

Thus, for the case in which the applied force is written \( F(t) = m\beta(t) \), the random force is written \( f(t) = m\alpha(t) \) and the potential \( U(x) = m\phi(x) \), the Langevin equation reads

\[ \ddot{x} + \left( \frac{\omega_0}{Q} \right) \dot{x} + \omega_0^2 x + \phi'(x) = \alpha(t) + \beta(t), \] (7)

where

\[ \phi(x) = - \left( \frac{\omega_0^2 a^2}{4} \right) \left[ 1 - \left( \frac{x}{a} \right)^2 \right]^2, \] (8)

and

\[ \langle \alpha(t)\alpha(t') \rangle_{\text{noise}} = \frac{2\omega_0 V^2}{Q} \delta(t - t'). \] (9)

Eqs.(7), (8) and (9), with a hip modulation given by

\[ \beta(t) = \beta_{\text{max}} \cos(\omega_{\text{hip}} t) \] (10)

can be used to perform a computer simulation of the ankle mode of motion.

For the healthy quiet standing subject, shown in FIG.1, an output of the random motion simulation is shown in FIG.2. The qualitative similarity between a number of quiet standing subjects and simulations allow us to conclude that the Langevin theory of postural sway is reasonable.

The Langevin theory can also be analytically expressed as an equation for the probability \( P(x, v, t)dx dv \) for the subject to have a velocity in the interval \( dv \) and a displacement in the interval \( dx \). For example, without the hip modulation, the Fokker-Planck equation reads

\[ \left( \frac{\partial P}{\partial t} \right) + v \left( \frac{\partial P}{\partial x} \right) = \frac{\partial}{\partial v} \left\{ \left( \frac{\omega_0 v}{Q} \right) P + \phi'(x) P \right\} + \left( \frac{\omega_0 V^2}{Q} \right) \left( \frac{\partial^2 P}{\partial v^2} \right). \] (11)

The Fokker-Planck and the Langevin formulations of the problem are equivalent. The former is useful for analytical calculations while the later is useful for computer simulations.

V. POSTURAL SWAY DATA

Not all of the quiet standing subjects measured exhibited large undamped ankle mode oscillations. For some quiet standing subjects the ankle movements were strongly damped. Shown in FIG.3 is an example of a subject with suppressed ankle movements. The hip oscillations are still clearly visible.
The computer simulations for the overdamped case proceed exactly as previously described. However the quality factor of the mode is taken to be much smaller than for the underdamped case. For the quiet standing subject, shown in FIG.3, an output of the random motion simulation is shown in FIG.4. We again conclude, for the overdamped case, that the Langevin theory of postural sway is reasonable.

\[ \langle \Delta x(\tau)^2 \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{t_0}^{t_0 + (T/2)} |x(t + \tau) - x(t)|^2 dt. \]  

In physical systems the time \( T \) of a run is finite. In our case \( T = 180 \) sec. We can then plot \( \langle \Delta x(\tau)^2 \rangle \) on the interval \( 0 < \tau < 140 \) sec.

For the underdamped experimental data of FIG.1, we show in FIG.5 a plot of \( \langle \Delta x(\tau)^2 \rangle \). The ankle movements show an oscillation in a very clear form when the averaging procedure of Eq.(12) is performed. The hip movement modulations are largely smoothed away by this same averaging procedure.

The data produced by the computer simulations can also be averaged according to Eq.(12) and then compared with mean square fluctuations taken from the experimental data. The hip movement modulations are barely noticeable in \( \langle \Delta x(\tau)^2 \rangle \).

In FIG.6 we have plotted \( \langle \Delta x(\tau)^2 \rangle \) for the simulation in FIG.2. We note that the oscillations for underdamped ankle movements are clearly present although the amplitude of the oscillation are high when compared with the experimental FIG.5. However, we still conclude that the Langevin model is qualitatively reasonable.
to an experimental $\langle \Delta x(\tau)^2 \rangle$ shown in FIG.7. Notice for the overdamped case the absence of oscillations in the ankle motion. The behavior of $\langle \Delta x(\tau)^2 \rangle$ on the time scale shown is qualitatively not very far from simple Brownian diffusion.

The Fokker-Plank-Langevin theory is mainly of qualitative significance when comparing simulations to experimental data.

### VI. KRAMERS “ESCAPE RATE” FOR A FALL

Although the form of the metastable potential in Eq.(2) is commonly used, the potential has not yet been shown to be unique. The qualitative features required are a minimum near $x = 0$ and a barrier to falling near $x = a$. Eq.(2) is plotted in FIG.9.

A quiet standing subject exhibits oscillations about the potential minimum. The random force at a noise temperature $T_n$ will at some time push the displacement to values $|x| > a$ over a potential maxima at which time there is a fall. The potential barrier protecting the subject from a fall is given by

$$\Delta U = U(|a|) - U(0) = \left( \frac{m\omega_0^2a^2}{4} \right).$$  \hspace{1cm} (13)

Neglecting hip motion modulations, the Kramers equation [7] for the escape rate contains the Boltzman factor for overcoming a barrier; It is

$$\Gamma_K = \left( \frac{\omega_0Q}{\pi \sqrt{2}} \right) e^{-\Delta U/k_B T_n}.$$  \hspace{1cm} (14)

In terms of experimental parameters

$$\Gamma_K = \left( \frac{\omega_0Q}{\pi \sqrt{2}} \right) \exp \left( -\left( \frac{\omega_0a}{2V} \right)^2 \right).$$  \hspace{1cm} (15)

We used the parameters in Eq.(15) to describe twenty healthy subjects (between sixteen and forty five years of age).
age) in street clothing with eyes wide open. The reach test length scale $a$ was calculated from the critical angle $\theta_c$ given in the literature [11]. We find the Kramers predicted “times for a fall” to be anywhere from one week to three years. The time shortens considerably as the mean square velocity increases. Of course, this prediction by no means implies that these subjects will all experience a fall within the next three years! The fall transition rate of our model will be further discussed in the next and concluding section.

VII. DISCUSSION

We have exhibited measurements of displacement noise with quiet standing subjects along with computer simulations based on a Fokker-Planck-Langevin model. For all ranges of the quality factor $Q$, from underdamped $Q >> 1$ to overdamped $Q << 1$, the Langevin equation computer simulations were in qualitative agreement with the data. In this regard, one should note that the same subject when measured on different days can exhibit both overdamped and/or underdamped behavior, as well a somewhat different effective noise temperatures $T_n$, i.e. a somewhat different root mean square velocity $V$.

Since the Kramers transition rate for a fall depends sensitively on the root mean square velocity $V$, it follows that a persons transition rate for a fall can vary from day to day depending on the “noise temperature”. For example, on a day when a person is tired we conjecture that a fall is more likely than on a day when the person is wide awake. However, we presently have no data with which to prove this conjecture.

While the above Fokker-Planck-Langevin theory seems to be qualitatively correct, the theory overestimates the displacement noise amplitudes and thereby also overestimates the transition rate for a fall. Apart from leaving out the step strategy for avoiding a fall, we believe that we have perhaps also underestimated the subtlety of the hip strategy. Thus far, the hip motions have been described by an added modulation force in our computer simulations, and this inclusion gives at least qualitative agreement with experimental data. Nevertheless, the hip modulations might be more strongly correlated with the center of mass body coordinate $[17,18]$ than is presently being included. Further improvements on the present model are presently being pursued on both the experimental and theoretical level.

[1] T.E. Prieto, J.B. Myklebust, and B.M. Myklebust, IEEE Trans. Rehab. Eng. 1, 26 (1993).