Research Article

Finite-Time $H_2/H_\infty$ Control Design for Stochastic Poisson Systems with Applications to Clothing Hanging Device

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In this paper, the design of finite-time $H_2/H_\infty$ controller for linear Itô stochastic Poisson systems is considered. First, the definition of finite-time $H_2/H_\infty$ control is proposed, which considers the transient performance, $H_2$ index, and $H_\infty$ index simultaneously in a predetermined finite-time interval. Then, the state feedback and observer-based finite-time $H_2/H_\infty$ controllers are presented and some new sufficient conditions are obtained. Moreover, an algorithm is given to optimize $H_2$ and $H_\infty$ index simultaneously. Finally, a simulation example indicates the effectiveness of the results.

1. Introduction

It is known to all that stochastic systems have been studied extensively and applied to biological network [1], power systems [2], financial systems [3, 4], and other fields. There are also many other applications of stochastic systems (see, e.g., [5–7]). In the past few decades, stochastic systems driven by Wiener noise have been widely investigated. For example, Shaikin [8] solved the optimization problem for multiplicative stochastic systems with several external disturbances and vector Wiener processes. Xiang et al. [9] introduced the finite-time properties and state feedback $H_\infty$ control problem for switched stochastic systems with Wiener noise. Yan et al. [10] were concerned with finite-time $H_2$ control of the Markovian stochastic systems with Wiener noise. However, in the real world, an actual physical system is inevitably affected by Wiener noise and Poisson jump noise. At present, some achievements have been made in the research of stochastic Poisson systems (see, e.g., [11–14]).

On the other hand, $H_2/H_\infty$ optimization control is one of the most important problems in the controlled system. The $H_2$ optimal control system has good system performance, while the $H_\infty$ control theory can deal with the system robustness problem well. In view of this, Bernstein and Haddad [15] proposed the $H_2/H_\infty$ mixed control problem, which can solve both the problems of system performance and robustness. Since then, the $H_2/H_\infty$ control has been developed and used extensively (see, e.g., [16–19]). Besides, in some engineering research, such as communication system [20–23], robotic operating system [24], and industrial production system [25], more attention should be paid to the system transient performance. In order to describe system transient performance clearly, the concepts of finite-time stability (FTS) and finite-time boundedness (FTB) are proposed, which reflect the specific system behavior in a relatively short time interval. Nowadays, the problems of FTS and FTB have been deeply investigated (see, e.g., [26–36]). In consideration of the merits of FTB and $H_2/H_\infty$ control, the finite-time $H_2/H_\infty$ control for stochastic systems with Wiener noise is first presented in [37], which satisfies both FTB and $H_2/H_\infty$ performance index. However, in many practical systems, it is not only disturbed...
by Wiener noise, but also by Poisson noise. So far, there are few literature studies to investigate this problem of stochastic Poisson systems affected by both Wiener and Poisson noises.

Motivated by aforementioned discussions, the problems of finite-time $H_2/H_{\infty}$ control for stochastic Poisson systems with both Wiener noise and Poisson noise are considered in this paper. The main work of this paper consists of the following three aspects:

(i) Unlike the model considered in [37], this paper studies the model of stochastic Poisson systems with Wiener and Poisson noises. The former considers only Wiener noise, and the latter considers both Wiener and Poisson noises. Moreover, in the former model, the measurement output $y(t)$ is composed of only the state, but the measurement output considered in the latter model is composed of both the state and external interference. The latter model is more general than the former model in [37], which is used to model many real systems.

(ii) The two theorems (Theorems 2 and 4) are obtained to guarantee the existence of state feedback finite-time (SFFT) and observer-based finite-time (OBFT) $H_2/H_{\infty}$ controllers, respectively. The two theorems (Theorems 2 and 4) contain the parameters both $\alpha$ and Poisson jump intensity $\lambda$, which are complex than the corresponding conditions in [37]. By adjusting the two parameters, the most satisfying finite-time $H_2/H_{\infty}$ controllers will be designed.

(iii) A new optimization algorithm constrained by matrix inequality is proposed to demonstrate the relationships among $\alpha$, $\lambda$, and optimal $H_2/H_{\infty}$ index, which is more complex than that in [37].

Notations: the notations presented in this work are standard. For specific contents, one can refer to [37].

2. Preliminaries

Consider a continuous-time stochastic Poisson system

\[
\begin{array}{l}
dx(t) = \begin{bmatrix} A_{11} & B_{11} \\ A_{12} & B_{12} \end{bmatrix} x(t) + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} r(t) \, dt + \begin{bmatrix} A_{11} & B_{11} \\ A_{12} & B_{12} \end{bmatrix} v(t) + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} r(t) \, d\mathcal{W}(t) \\
+ \begin{bmatrix} A_{11} & B_{11} \\ A_{12} & B_{12} \end{bmatrix} v(t) + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} r(t) \, d\mathcal{N}(t), \\
y(t) = C_{11} x(t) + D_{11} r(t), \\
z(t) = C_{12} x(t) + D_{12} v(t), \\
x(0) = x_0 \in \mathbb{R}^n,
\end{array}
\]

where $A_{11}, A_{12}, B_{11}, B_{12}, C_{11}, C_{12}, D_{11}, D_{12}, F_1, F_2$, and $F_3$ are known constant matrices. $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^n$, $z(t) \in \mathbb{R}^n$, and $v(t) \in \mathbb{R}^n$ are the state vector, measurement output, control output, and control input, respectively. $x_0$ is the initial condition of the system. $\mathcal{W}(t)$ presents one-dimensional standard Wiener process and $\mathcal{N}(t)$ is the marked Poisson process with Poisson jump intensity $\lambda$. $r(t) \in \mathbb{R}^p$ is the disturbance input which satisfies the following equation:

\[
\int_0^t r'(s) r(s) \, ds < f, \quad (f > 0).
\]

Next, the definition of mean-square FTB of system (1) is introduced.

Definition 1. Given some scalars $b_2 > b_1 > 0$ and $T > 0$ and a matrix $R > 0$, the above stochastic system (1) with $v(t) \equiv 0$ is mean-square FTB w.r.t. $(b_1, b_2, T, R, f)$, if

\[
\mathbb{E} [x'(0) R x(0)] \leq b_1 \implies \mathbb{E} [x'(t) R x(t)] < b_2, \quad \forall t \in [0, T].
\]

Remark 1. From Definition 1, we can know that the concept of FTB describes the specific behavior of the stochastic system (1) in a prescribed time interval.

Lemma 1 (see [38]). Let $\mathcal{T}(t, x) \in C^2(R^1, R^n)$ and $\mathcal{T}(t, x) > 0$. Consider the following system

\[
dx(t) = A_1(x) dt + A_2(x) d\mathcal{W}(t) + A_3(x) d\mathcal{N}(t),
\]

its stochastic differential of $\mathcal{T}(t, x)$ is given by

\[
\mathcal{T}(t, x) = \frac{\partial \mathcal{T}(t, x)}{\partial t} + \frac{\partial \mathcal{T}(t, x)}{\partial x} A_1(x) + \frac{1}{2} A_2(x) \left( \frac{\partial^2 \mathcal{T}(t, x)}{\partial x^2} A_2(x) + \lambda \left[ \mathcal{T}(t, x + A_3(x)) - \mathcal{T}(t, x) \right] \right),
\]
3. Design of SFFT $H_2/H_\infty$ Controller

In this section, a SFFT $H_2/H_\infty$ controller for system (1) is designed. Consider a linear SF controller

$$v(t) = Kx(t),$$

where $K$ is the required SF gain matrix.

Substituting (7) into (1), the following closed-loop system is obtained:

$$
\begin{align*}
\dot{x}(t) &= \left[\bar{A}_{11} x(t) + F_1 r(t)\right]dt + \left[\bar{A}_{12} x(t) + F_2 r(t)\right]dW(t) + \left[\bar{A}_{13} x(t) + F_3 r(t)\right]dV(t), \\
y(t) &= C_{11} x(t) + D_{11} r(t), \\
z(t) &= \bar{C} x(t), \\
x(0) &= x_0 \in \mathbb{R}^n,
\end{align*}
$$

where $\bar{A}_{11} = A_{11} + B_{11} K$, $\bar{A}_{12} = A_{12} + B_{12} K$, $\bar{A}_{13} = A_{13} + B_{13} K$, and $\bar{C} = C + D_z K$.

Next, we choose the following $H_2$ cost function:

$$J_1(x(t), v(t)) = \mathbb{E}\int_0^T [x'(t)G_1 x(t) + v'(t)G_2 v(t)]dt,$$

where $G_1 > 0$ and $G_2 > 0$ are known weighting scalars or positive matrices.

Similarly, substituting the SF controller (7) into (9), the following formula is obtained:

$$J_1(x(t)) = \mathbb{E}\int_0^T [x'(t)G_1 x(t) + x'(t)K^T G_2 K x(t)]dt.$$

Given $\gamma > 0$ and assuming zero initial condition, the control output $z(t)$ and the disturbance input $r(t)$ satisfy the following equation:

$$\mathbb{E}\int_0^T z'(t)z(t)dt < \gamma^2 \mathbb{E}\int_0^T r'(t)r(t)dt.$$

Based on the above preparations, the definition of the SFFT $H_2/H_\infty$ controller is introduced.

**Definition 2.** Given positive scalars $b_1$, $b_2$, $T$, and $f$ and a matrix $R > 0$, if there exist a nonnegative scalar $\alpha$ and two matrices $N \succ 0$ and $K$ such that

$$
\begin{bmatrix}
\mathcal{F}_1 & F_1 & \bar{N}A_{12}' & \sqrt{\bar{N}}(\bar{A}_{13} + I)' \\
* & -\gamma^2 I & F_2 & \sqrt{\bar{N}} F_3' \\
* & * & -\bar{N} & 0 \\
* & * & * & -\bar{N}
\end{bmatrix} < 0,
$$

$$
\begin{bmatrix}
\mathcal{F}_2 & \bar{N}A_{12}' & \sqrt{\bar{N}}(\bar{A}_{13} + I)' \\
* & -\bar{N} & 0 \\
* & * & -\bar{N}
\end{bmatrix} < 0,
$$

$$
\frac{b_1}{\lambda_{\min}(N)} + f \gamma^2 < \frac{b_2}{\lambda_{\max}(N)} e^{-\alpha T},
$$

hold, where $\bar{N} = R^{-1/2}NR^{-1/2}$, $\mathcal{F}_1 = \bar{A}_{11}' \bar{N} + \bar{N}A_{11}' - \lambda \bar{N} - \alpha \bar{N} + \bar{N}C_N \bar{C}_N$, and $\mathcal{F}_2 = \bar{A}_{11}' \bar{N} + \bar{N}A_{11}' - \lambda \bar{N} - \alpha \bar{N} + \bar{N}G_1' \bar{N} + \bar{N}K' \bar{G}_2 \bar{K} \bar{N}$, then $v(t) = K x(t)$ is said to be a SFFT $H_2/H_\infty$ controller and we can get the upper bound of $H_2$ index, that is, $J^*_{\text{state}} = \lambda_{\max}(N^{-1})b_1 e^{\alpha T}$.

**Proof.** Here are three steps to prove Theorem 1.

**Step 1:** prove that system (3) is mean-square FTB.

Obviously,

$$
\begin{bmatrix}
\bar{N}C_N \bar{C}_N & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \geq 0.
$$

Therefore, condition (12) means...
where $\mathcal{F}_1 = \tilde{A}_1 \tilde{N} + \tilde{N} \tilde{A}_1^{-1} - \lambda \tilde{N} - \alpha \tilde{N}$.

Let $\mathcal{F}(x(t)) = x'(t) \tilde{N}^{-1} x(t)$, and applying Lemma 1 for $\mathcal{F}(x(t))$, the $\mathcal{L}_1 \mathcal{F}(x(t))$ of system (8) is given by

$$
\mathcal{L}_1 \mathcal{F}(x(t)) = \begin{bmatrix} Z_1 & Z_2 \\ * & * \end{bmatrix} \begin{bmatrix} x(t) \\ r(t) \end{bmatrix},
$$

where $Z_1 = \tilde{A}_1 \tilde{N}^{-1} + \tilde{N}^{-1} \tilde{A}_1 + A_{12} \tilde{N}^{-1} \tilde{A}_{12} + \lambda (\tilde{A}_{13} + I) \tilde{N}^{-1} (\tilde{A}_{13} + I) - \lambda \tilde{N}^{-1}$, $Z_2 = \tilde{A}_1 \tilde{N}^{-1} F_1 + A_{12} \tilde{N}^{-1} \tilde{F}_2 + \tilde{N}^{-1} \tilde{F}_1$, and $Z_3 = \tilde{F}_1 \tilde{N}^{-1} F_1 + \tilde{F}_1 \tilde{N}^{-1} F_2$.

Pre- and postmultiplying (16) by diag($\tilde{N}^{-1}, I, \tilde{N}^{-1}$), we can get the following inequality:

$$
\mathcal{F}(x(t)) < a \mathcal{F}(x(0)) + \alpha t + \gamma^2 t r'(s) r(s) ds.
$$

Utilizing Gronwall inequality in [26], it follows

$$
\mathcal{F}(x(t)) < \mathcal{F}(x(0)) e^{at} + \gamma^2 e^{at} \int_0^t r'(s) r(s) ds.
$$

On the basis of above conditions, we have

$$
\mathcal{F}(x(t)) = \mathcal{F}[x'(t) \tilde{N}^{-1} \tilde{K} \tilde{N}^{-1} x(t)] \geq \lambda_{\min} \left( \tilde{N}^{-1} \right) E[x'(t) \tilde{N}^{-1} \tilde{K} \tilde{N}^{-1} x(t)],
$$

where $\lambda_{\min} \left( \tilde{N}^{-1} \right) = \lambda_{\min}(N^{-1})$. The pre and postmultiplying (16) by diag($\tilde{N}^{-1}, I, \tilde{N}^{-1}$), we can get the following inequality:

$$
\mathcal{F}(x(t)) < a \mathcal{F}(x(0)) + \alpha t + \gamma^2 t r'(s) r(s) ds.
$$

From (22) to (25), the following inequality is obtained:

$$
\mathcal{F}[x'(t) \tilde{N}^{-1} \tilde{K} \tilde{N}^{-1} x(t)] \leq \lambda_{\max}(N) e^{at} \left( \frac{b_1}{\lambda_{\max}(N)} + f \gamma^2 \right).
$$

According to condition (14), we get that (26) leads to $\mathcal{F}[x'(t) \tilde{K} \tilde{N}^{-1} x(t)] < b_2$ for all $t \in [0, T]$. So, system (8) is mean-square FTB w.r.t. $(b_1, b_2, T, R, f)$.

Step 2: prove that the $H_2$ cost function (10) satisfies $J_1(x(t)) \leq J_1'$ under $r(t) = 0$ condition.
Because (38) and (39), it yields
\[
\mathcal{L}_1 \left[ e^{-at} \mathcal{F}(x(t)) \right] < 0.
\]

From (31), we get
\[
\mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] < \mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] + a \mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] dt.
\]

Combining (33) and (34), it is obtained that
\[
J_1(x(t)) < \mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] + \mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] dt.
\]

Step 3: prove that the non-zero disturbance and the control output satisfy inequality (11).

Pre- and postmultiplying (12) respectively by \( \text{diag}\{\hat{N}^{-1}, I, \hat{N}^{-1}, \hat{N}^{-1}\} \), and then using Schur complement, we have
\[
\begin{bmatrix}
\mathcal{F}_4 & \mathcal{F}_5 \\
\mathcal{F}_6 & \mathcal{F}_7
\end{bmatrix} < 0,
\]

where
\[
\mathcal{F}_k = A_{11}^{-1} + \hat{N}^{-1}A_{11} + \hat{N}^{-1}A_{12} + \hat{N}^{-1}A_{11}^{-1}A_{11} + \lambda(A_{11} + I)\hat{N}^{-1}(A_{11} + I) + \hat{C}_i \hat{C}^\top - \lambda \hat{N}^{-1} - \hat{N}.
\]

Combining (17), (36), we get
\[
\mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] < \mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] + \mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] dt.
\]

Pre- and postmultiplying (37) by \( e^{-ut} \), one has
\[
e^{-ut} \mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] < a e^{-ut} \mathcal{F}(x(t)) + e^{-ut} \left[ 2 \gamma^2 r^T(t) r(t) - z^T(t) z(t) \right].
\]

By applying Lemma 1, we obtain
\[
\mathcal{L}_1 \left[ e^{-ut} \mathcal{F}(x(t)) \right] = -a e^{-ut} \mathcal{F}(x(t)) + e^{-ut} \mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right].
\]

According to (38) and (39), it yields
\[
\mathcal{L}_1 \left[ e^{-ut} \mathcal{F}(x(t)) \right] < 0.
\]

Because \( e^{-ut} \) is between 0 and 1, for (40), we have
\[
\mathcal{L}_1 \left[ e^{-ut} \mathcal{F}(x(t)) \right] < 0.
\]

Integrating from 0 to \( t \) on both sides of (41), then taking mathematical expectation, the following inequality can be obtained under zero initial condition:
\[
e^{-ut} \mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] < 0.
\]

We know that \( e^{-ut} \mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] > 0 \), so it yields
\[
\int_0^t e^{-ut} \mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] dt < 0.
\]

This completes the proof.

It is obvious that conditions (12)–(14) are not linear matrix inequalities. In order to simplify the solving process, the following theorem is given.

**Theorem 2.** Given positive scalars \( b_1, b_2, T, \) and \( f \) and a matrix \( R > 0 \), if there exist two scalars \( m > 0 \) and \( \alpha > 0 \) and two matrices \( N > 0 \) and \( Y \) such that
\[
\begin{bmatrix}
\mathcal{F}_4 & \mathcal{F}_5 & \mathcal{F}_6 & \mathcal{F}_7 \\
\mathcal{F}_6 & \mathcal{F}_7 & -\hat{N} & 0 \\
\mathcal{F}_7 & -\hat{N} & 0 & 0
\end{bmatrix} < 0,
\]

then
\[
\begin{bmatrix}
f \gamma^2 - b_2 e^{-ut} \sqrt{\mathcal{F}_8} \\
\mathcal{F}_4 \mathcal{F}_5 \mathcal{F}_6 \mathcal{F}_7
\end{bmatrix} < 0,
\]

\[
\mathcal{L}_1 \left[ e^{-at} \mathcal{F}(x(t)) \right] < \mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] - m e^{-ut}.
\]

\[
\int_0^t e^{-ut} \mathcal{L}_1 \left[ \mathcal{F}(x(t)) \right] dt < 0.
\]

Proof. Let \( Y = K \hat{N} \), inequalities (12) and (13) can be obtained from (44) and (45), respectively, and (14) in Theorem 1 can be obtained from (46) and (47) easily. This ends the proof.

**Remark 3.** In Theorem 2, when \( \alpha \) is fixed, (44)–(47) can be treated as LMIs which are easy to solve.
4. Design of OBFT $H_2/H_{\infty}$ Controller

In some practical cases, not all states can be measured directly. Therefore, the design of OBFT $H_2/H_{\infty}$ controller is necessary. Typically, an OB dynamic controller is given by

$$\begin{align*}
\dot{x}(t) &= [A_{11} \bar{x}(t) + B_{11}v(t) + L(y(t) - C_{11} \bar{x}(t))]dt, \\
v(t) &= K\bar{x}(t), \\
\bar{x}(0) &= 0,
\end{align*}$$

(48)

where $\bar{x}(t) \in \mathbb{R}^n$ is the estimation of $x(t)$ and $L$ is the desired estimator gain.

Substituting the OB controller (48) into system (1), we will obtain the following closed-loop system:

$$\begin{align*}
\dot{\bar{x}}(t) &= \bar{A}_{11} \bar{x}(t) + \bar{W}_1 r(t) + \bar{W}_2 r(t) \dot{\bar{r}}(t) + \bar{W}_3 r(t) \dot{\bar{r}}'(t), \\
\bar{z}(t) &= H\bar{x}(t),
\end{align*}$$

(49)

and then we get the closed-loop cost function

$$J_2(\bar{x}(t)) = \mathcal{E}^T \bar{x}(t) \mathcal{E}(t) dt,$$

(50)

where

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ \bar{x}(t) \end{bmatrix},$$

$$\bar{A}_{11} = \begin{bmatrix} A_{11} & B_{11}K \\ LC_{11} & A_{11} + B_{11}K - LC_{11} \end{bmatrix},$$

$$\bar{W}_1 = \begin{bmatrix} F_1 \\ LD_{11} \end{bmatrix},$$

$$\bar{A}_{12} = \begin{bmatrix} A_{12} & B_{12}K \\ 0 & 0 \end{bmatrix},$$

$$\bar{W}_2 = \begin{bmatrix} F_2 \\ 0 \end{bmatrix},$$

$$\bar{A}_{13} = \begin{bmatrix} A_{13} & B_{13}K \\ 0 & 0 \end{bmatrix},$$

$$\bar{W}_3 = \begin{bmatrix} F_3 \\ 0 \end{bmatrix},$$

$$H = \begin{bmatrix} C_{12} K' D_{12}' \\ G_1 \\ 0 \end{bmatrix},$$

$$\Xi = \begin{bmatrix} K' G_1 K \end{bmatrix}.$$

(51)

Assuming that the initial state is zero, the control output $\bar{z}(t)$ and the arbitrary nonzero disturbance input $r(t)$ satisfy the following equation:

$$\mathcal{E}^T \bar{z}'(t) \bar{z}(t) dt < \mathcal{E}^T \bar{z}(t) \bar{z}(t) dt.$$

(53)

Then, we give the definition of OBFT $H_2/H_{\infty}$ control.

**Definition 3.** Given positive scalars $b_1$, $b_2$, $T$, and $f$ and a matrix $\bar{R} > 0$, if there exist a nonnegative scalar $\beta$ and a positive matrix $P$ such that

$$\begin{bmatrix}
\bar{X}_1 \bar{P} W_1 \bar{A}_{12} \bar{P} \sqrt{\lambda} (\bar{A}_{13} + I) \bar{P} \\
\ast & -\gamma^2 I & W_2^T \bar{P} & \sqrt{\lambda} W_3^T \bar{P} \\
\ast & \ast & -\bar{P} & 0 \\
\ast & \ast & \ast & -\bar{P}
\end{bmatrix} < 0,$$

(54)

$$\bar{A}_{11} \bar{P} + \bar{P} \bar{A}_{11} + \bar{A}_{13}^T \bar{P} \bar{A}_{12} + \lambda (\bar{A}_{13} + I)^T \bar{P} (\bar{A}_{13} + I) - \lambda \bar{P} - \beta \bar{P} + \Xi < 0,$$

(55)

$$\lambda_{\max}(P) b_1 + f \gamma^2 < \lambda_{\min}(P) b_2 e^{-\beta T},$$

(56)

hold, where $\bar{P} = \bar{R}^{1/2} \bar{P} \bar{R}^{1/2}$ and $\bar{X}_1 = \bar{A}_{11}^T \bar{P} + \bar{P} \bar{A}_{11} + H' H - \beta \bar{P} - \lambda \bar{P}$, then (48) is said to be an OBFT $H_2/H_{\infty}$ controller and we can get the upper bound of $H_2$ index, that is, $J_2^* = \lambda_{\max}(P) b_1 e^{\beta T}$.

**Proof.** Here are three steps to prove the theorem.

Step 1: prove that system (49) is mean-square FTB.
Let \( \mathcal{F}(\bar{x}(t)) = \bar{x}'(t)P \bar{x}(t) \) where \( P > 0 \). Applying generalized Itô formula for \( \mathcal{F}(\bar{x}(t)) \), the \( L_3 \mathcal{F}(\bar{x}(t)) \) of system (49) is given by

\[
L_3 \mathcal{F}(\bar{x}(t)) = \begin{bmatrix} \bar{x}(t) \\ r(t) \end{bmatrix} \begin{bmatrix} \bar{Z}_1 & \bar{Z}_2 & \bar{Z}_3 \\ * & * & * \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ r(t) \end{bmatrix},
\]

(57)

where \( \bar{Z}_1 = \bar{A}_1'P + P\bar{A}_1 + \bar{A}_1'P\bar{A}_1 + \lambda(\bar{A}_1 + I)P(\bar{A}_1 + I) - \lambda \bar{P} \), \( \bar{Z}_2 = \bar{P}W_1 + \bar{A}_1'PW_2 + \lambda(\bar{A}_1 + I)\bar{P}W_3 \), and \( \bar{Z}_3 = W_2\bar{P}W_2 + \lambda W_3\bar{P}W_3 \).

Note that

\[
\begin{bmatrix} H' & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} H' & H' \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \geq 0.
\]

(58)

Therefore, inequality (54) means

\[
[\mathcal{H}_2 \bar{P}W_1 + \bar{A}_1'\bar{P} \sqrt{1}(\bar{A}_1 + I)^\prime \bar{P}] \\
\begin{bmatrix} -\gamma^2 I & -\bar{P} \\ -\bar{P} & 0 \end{bmatrix} < 0.
\]

(59)

where \( \mathcal{H}_2 = \bar{A}_1'\bar{P} + \bar{P}\bar{A}_1 - \bar{\beta} \bar{P} - \lambda \bar{P} \).

By utilizing Schur complement, (59) can be converted into

\[
\begin{bmatrix} \bar{Z}_1 - \beta \bar{P} & \bar{Z}_2 \\ * & -\gamma^2 I + \bar{Z}_3 \end{bmatrix} < 0.
\]

(60)

Combining (57) and (60), we get

\[
L_3 \mathcal{F}(\bar{x}(t)) < 2\mathcal{F}(\bar{x}(t)) + \gamma^2 r'(t)r(t).
\]

(61)

Integrating from 0 to \( t \) on both sides of (61), then taking mathematical expectation, the following inequality is obtained:

\[
\mathbb{E}\mathcal{F}(\bar{x}(t)) < \mathbb{E}\mathcal{F}(\bar{x}(0)) + \beta \int_0^t \mathbb{E}\mathcal{F}(\bar{x}(s))ds + \gamma^2 \int_0^t \mathbb{E}r'(s)r(s)ds.
\]

(62)

According to Gronwall inequality, it yields

\[
\mathbb{E}\mathcal{F}(\bar{x}(t)) < \mathbb{E}\mathcal{F}(\bar{x}(0))e^{\beta t} + \gamma^2 e^{\beta t} \int_0^t \mathbb{E}r'(s)r(s)ds.
\]

(63)

According to known conditions, it yields

\[
\mathbb{E}\mathcal{F}(\bar{x}(t)) \geq \lambda_{\min}(P)\mathbb{E}\left[\bar{x}'(t)\bar{R}^{1/2}\bar{P}^{1/2}\bar{x}(t)\right],
\]

(64)

\[
\mathbb{E}\mathcal{F}(\bar{x}(0))e^{\beta t} \leq \lambda_{\max}(P)\mathbb{E}\left[\bar{x}'(0)\bar{R}^{1/2}\bar{P}^{1/2}\bar{x}(0)\right]e^{\beta t}
\]

\[
\leq \lambda_{\max}(P)b_1 e^{\beta t},
\]

\[
\gamma^2 e^{\beta t} \int_0^t \mathbb{E}r'(s)r(s)ds < e^{\beta t} f y^2.
\]

(65)

From (63) to (66), we obtain

\[
\mathbb{E}[\bar{x}'(t)\bar{R}\bar{x}(t)] < \frac{\lambda_{\max}(P)b_1 e^{\beta t} + f y^2 e^{\beta t}}{\lambda_{\min}(P)}.
\]

(67)

According to (56) and (67), we get \( \mathbb{E}[\bar{x}'(t)\bar{R}\bar{x}(t)] < b_2 \) for all \( t \in [0, T] \). So, system (49) is FTB w.r.t. \((b_1, b_2, T, \bar{R}, f)\).

Step 2: prove that the H₂ cost function (50) satisfies \( J_2(\bar{x}(t)) \leq J_2^1 \) under \( r(t) = 0 \) condition.

When \( r(t) = 0 \), we get that the \( L_3 \mathcal{F}(\bar{x}(t)) \) of system (49) is given by

\[
L_3 \mathcal{F}(\bar{x}(t)) = \bar{x}'(t)\left[\bar{A}_1'\bar{P} + \bar{P}\bar{A}_1 + \bar{A}_1'\bar{P}\bar{A}_1 + \lambda(\bar{A}_1 + I)\bar{P}(\bar{A}_1 + I) - \lambda \bar{P}\right]\bar{x}(t).
\]

(68)

According to (55), we have

\[
L_4 \mathcal{F}(\bar{x}(t)) - \beta \mathcal{F}(\bar{x}(t)) + \bar{x}'(t)\bar{x}(t) < 0.
\]

(69)

Integrating from 0 to \( t \) on both sides of (69), then taking mathematical expectation, it yields

\[
\mathbb{E}\mathcal{F}(\bar{x}(t)) + \beta \int_0^t \mathbb{E}\mathcal{F}(\bar{x}(s))ds + \gamma^2 \int_0^t \mathbb{E}r'(s)r(s)ds.
\]

(70)

From (70), we have

\[
\mathbb{E}\mathcal{F}(\bar{x}(t)) < \mathbb{E}\mathcal{F}(\bar{x}(0)) + \beta \mathbb{E}\int_0^t \mathcal{F}(\bar{x}(s))ds.
\]

(71)

\[
\mathbb{E}\int_0^t \bar{x}'(t)\bar{x}(t)ds < \mathbb{E}\mathcal{F}(\bar{x}(0)) + \beta \mathbb{E}\int_0^t \mathcal{F}(\bar{x}(s))ds.
\]

(72)

Using Gronwall inequality for (71), one has

\[
\mathbb{E}\mathcal{F}(\bar{x}(t)) < \mathbb{E}\mathcal{F}(\bar{x}(0))e^{\beta t}.
\]

(73)

From (72) and (73), we have
complexity in theorem 3 is difficult to solve, we transform the inequality (53). Let $\Sigma_3 \mathcal{T} (\tilde{x}(t)) < \beta \beta \mathcal{T} (\tilde{x}(0)) + \gamma \gamma' t(t) - \gamma' (t) \tilde{x}(t)$.

Repeating the proof process of Step 3 in Theorem 1, it yields

$$\mathcal{G} \int_0^t \gamma' (s) \xi(s) ds < \psi' \mathcal{G} \int_0^t \gamma' \xi(t) ds.$$  (77)

This completes the proof.

Because the nonlinear problem of inequalities (54)–(56) in Theorem 3 is difficult to solve, we transform the inequalities (54)–(56) into LMIs.

**Theorem 4.** Given positive scalars $b_1, b_2, T,$ and $f$, if there exist two positive scalars $\beta$ and $\zeta$ and three matrices $\bar{P}_{ii} > 0$, $\bar{P}_{ii} > 0$, and $M$ such that

$$\begin{bmatrix}
\Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\
\Sigma_{22} & \Sigma_{23} & 0 \\
0 & \Sigma_{33}
\end{bmatrix} < 0,$$  (78)

$$\begin{bmatrix}
Y_{11} - (\beta + \lambda) \bar{P}_{11} & Y_{12} \\
* & Y_{22} - (\beta + \lambda) \bar{P}_{22}
\end{bmatrix} < 0,$$  (79)

$$e^{\sigma T} (\zeta b_1 + f \gamma) \gamma - b_2 < 0,$$  (80)

$$I < \text{diag}(P_{11}, P_{22}),$$  (81)

hold, where $\Sigma_{11} = A_{11} \bar{P}_{11} + \bar{P}_{11} A_{11} + A_{12} \bar{P}_{11} A_{12} + C_{11} C_{12} + \lambda (A_{11} + I) \bar{P}_{11} (A_{11} + I) - (\beta + \lambda) \bar{P}_{11},$ $\Sigma_{12} = C_{11} M' + \bar{P}_{11}$ $B_{11} K + A_{12} \bar{P}_{11} B_{12} K + C_{12} D_{12} K + \lambda (A_{11} + I) \bar{P}_{11} B_{13} K,$ $\Sigma_{22} = (A_{11} + B_{11} K)' \bar{P}_{22} + \bar{P}_{22} (A_{11} + B_{11} K) + K' B_{12}' \bar{P}_{11} B_{12} K + \lambda K' B_{12}' \bar{P}_{11} B_{12} K + \lambda K' B_{12}' \bar{P}_{11} B_{12} K + \lambda \bar{P}_{22} = D_{11} - C_{11} M' + K' D_{12} K + (\beta + \lambda) \bar{P}_{22},$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} E_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = M D_{11} + K' B_{12}' \bar{P}_{11} F_2 + K' B_{12}' \bar{P}_{11} F_3,$ $\Sigma_{33} = -G_1,$ $G_1 = 5, G_2 = 4, D_{11} = 8, D_{12} = 10, b_1 = 1, b_2 = 4, T = 1, R = I, f = 0.4,$ $I = 0.4, \lambda = 2.5.$

6.1. Design of SFT $H_2/H_\infty$ Controller. By using the above algorithm in Section 5, the relationships of $\alpha$ and $\xi$ (Figure 1), $a$ and $\gamma$ (Figure 2), and $\xi$ and $\gamma$ (Figure 3) are derived, respectively. It can be seen from Figure 1 that the value of $\xi$ increases with the increase of $\alpha$. Besides, it is obvious that...
Algorithm 1: Optimization algorithm.

\[
\begin{align*}
\text{Step 1:} & \quad \text{given } b_1, b_2, R, T, f, \text{ and } \lambda. \\
\text{Step 2:} & \quad \text{take an appropriate step size } d_\alpha \text{ for } \alpha, \text{ and then the values of } \alpha \text{ are expressed as } \alpha_i. \\
\text{Step 3:} & \quad \text{let } i = 1. \\
\text{Step 4:} & \quad \text{if } \alpha_i \text{ makes the following problems min}_{x \in (37)-(40)} (\xi = \text{feasible}, \text{ then store } \alpha_i \text{ into } U(i), \xi_{\text{min}} \text{ into } V(i), \text{ and } \gamma_{\text{min}} \text{ into } W(i), \text{ and let } \alpha_{i+1} = \alpha_i + d_\alpha, \text{ loop. Otherwise, go to Step 5.} \\
\text{Step 5:} & \quad \text{exit.}
\end{align*}
\]

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{$\xi$ versus $\alpha$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{$\gamma$ versus $\alpha$.}
\end{figure}

\[\xi = 1 \text{ when } \alpha = 0 \text{ and } \xi \approx 3.0344 \text{ when } \alpha = 1.11, \text{ that is, the minimum and maximum values of } H_2 \text{ performance index are } 1 \text{ and } 3.0344, \text{ respectively. Also, the range of } \alpha \text{ is } [0, 1.11].\]

As you can see from Figure 2, the value of $\gamma$ decreases first and then increases with the increase of $\alpha$. When $\alpha = 0.85$, $\gamma$ can take the minimum value of 0.8688; at this point, we can get the optimal value of $H_\infty$ performance index. When $\alpha = 1.11$, $\gamma$ can take the minimum value of 0.8822. Also, $\alpha$ can be taken within $[0, 1.11]$.

In fact, Figure 3 reflects the relation between $\xi$ and $\gamma$. As shown in Figure 3, with the increase of $\xi$, the value of $\gamma$ decreases first, and at the point of $\xi = 2.3397$, the value of $\gamma$ begins to increase. From Figures 1 to 3, we can see how to choose the right state feedback finite-time $H_2/H_\infty$ controller. If the cost problem is mainly considered, a smaller $\alpha$ can be selected. If the ability to suppress interference is mainly considered, we need to refer to Figure 2 to select the appropriate $\alpha$.

Next, substituting $\alpha = 0$ into Theorem 2, we get
\[
N = \begin{bmatrix} 0.8673 & -0.0135 \\ -0.0135 & 0.8946 \end{bmatrix}, \quad Y = \begin{bmatrix} -0.0366 & 0.0770 \end{bmatrix}, \quad m = 0.5237.
\]

Then, we get the controller gain matrix as follows:
\[
K = \begin{bmatrix} -0.0409 & 0.0854 \end{bmatrix}.
\]

Because the state $x(t)$ in this example is two-dimensional, we make $x(t) = [x_1(t) x_2(t)]^T$. Figure 4 describes the trajectories of $x_1(t)$, $x_2(t)$, and $E[x^2(t)R_x(t)]$ with stochastic fluctuation driven by both Wiener and Poisson noises in Figures 5 and 6 versus the dimensionless time $\lambda t$. From Figure 4, we can see that the trajectory of $E[x^2(t)R_x(t)]$ does not exceed $b_1 = 4$ in the time interval $\lambda T = 2.5$. Obviously, when the time interval is $T = 1$, the trajectory does not exceed the given range, so we conclude that system (8) is mean-square FTB w.r.t. (1, 4, 1, 0.4).

Among them, we assume that $r(t) = \sin t \left( \int_0^t \sin^2 t \text{d}t < f = 0.4 \right)$.

6.2. Design of OBFT $H_2/H_\infty$ Controller. As in the case of state feedback, similar results can be obtained in the case of observer-based finite-time $H_2/H_\infty$ control. The relationships of $\beta$ and $\xi$ (Figure 7), $\beta$ and $\gamma$ (Figure 8), and $\xi$ and $\gamma$ (Figure 9) are derived, respectively. It can be seen from Figure 7 that the value of $\xi$ increases with the increase of $\beta$. Besides, it is obvious that $\xi = 1$ when $\beta = 0$ and $\xi = 2.5857$ when $\beta = 0.95$, that is, the minimum and maximum values of $H_2$ performance index are 1 and 2.5857, respectively.

Also, the range of $\beta$ is $[0, 0.95]$. As you can see from Figure 8, the value of $\gamma$ decreases first and then increases with the increase of $\beta$. When $\beta = 0.72$, $\gamma$ can take the minimum value of 1.1456, and at this point, we can get the optimal value of $H_\infty$ performance index. Besides, the maximum value of $H_\infty$ performance index is 1.1650 when $\beta = 0.95$. Also, the range of $\beta$ is $[0, 0.95]$.

In fact, Figure 9 reflects the relation between $\xi$ and $\gamma$. As shown in Figure 9, with the increase of $\xi$, the value of $\gamma$ decreases first, and at the point of $\xi = 2.0544$, the value of $\gamma$ begins to increase. From Figures 7 to 9, we can see how to
choose the right OBFT $H_2/H_\infty$ controller. If the cost problem is mainly considered, select the smaller $\beta$ with reference to Figure 7. If the ability to suppress interference is mainly considered, we need to refer to Figure 8 to select the appropriate $\beta$. 
Substituting $\beta = 0$ into Theorem 4, we have

$$P_{11} = \begin{bmatrix} 1.3326 & 0.0458 \\ 0.0458 & 1.5739 \end{bmatrix},$$

$$P_{22} = \begin{bmatrix} 1.4821 & -0.0298 \\ -0.0298 & 1.7362 \end{bmatrix},$$

$$M = \begin{bmatrix} -0.0326 \\ 0.1348 \end{bmatrix},$$

$$\zeta = 2.7271.$$

Then, we obtain the following observer gain matrix:

$$L = \begin{bmatrix} -0.0205 \\ 0.0773 \end{bmatrix}.$$  \hfill (87)

Because the state $\tilde{x}$ in this example is two-dimensional, we make $\tilde{x} = [\tilde{x}_1 \tilde{x}_2]$. Figure 10 describes the trajectories of $\tilde{x}_1, \tilde{x}_2$, and $E[\tilde{x}'(t)R\tilde{x}(t)]$ with stochastic fluctuation driven by both Wiener and Poisson noises in Figures 5 and 6 versus the dimensionless time $\lambda t$. From Figure 10, it is obvious that the trajectory of $E[\tilde{x}'(t)R\tilde{x}(t)]$ does not exceed $b_2 = 4$ in the time interval $\lambda T = 2.5$. Obviously, when the time interval is $T = 1$, the trajectory does not exceed the given range, so we draw a conclusion that system (8) is mean-square FTB w.r.t. $(1, 4, 1, 1, 0.4)$. Among them, we assume that $r(t) = \sin t (\int_0^1 \sin^2 t dt < f = 0.4)$.

## 7. Conclusions

In this paper, state feedback and observer-based finite-time $H_2/H_\infty$ controllers for stochastic Poisson systems have been designed, respectively. Two sufficient conditions for guaranteeing the existence of controllers have been proposed and converted to matrix inequality constrained optimization problems, and an algorithm for all theorems has been provided to derive the optimal $H_2$ index and $H_\infty$ index under the condition of the finite-time boundedness.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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