Time-domain atom interferometry across the threshold for Bose-Einstein condensation

F. Minardi, C. Fort, P. Maddaloni, M. Modugno, and M. Inguscio

INFM - European Laboratory for Non linear Spectroscopy (LENS) and Dipartimento di Fisica,
Università di Firenze, Largo E. Fermi 2, I-50125 Firenze, Italy.

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We have performed time-domain interferometry experiments with matter waves trapped in an harmonic potential above and below the Bose-Einstein phase transition. We interrogate the atoms according to the method of separated oscillating fields, with a sequence of two radio-frequency pulses, separated by a time delay $T$. We observe the oscillation of the population between two internal Zeeman states, as a function of the delay $T$.

We find a strong depletion of the interference fringes for both the Bose condensates and the thermal clouds above condensation, even at very short times, when the clouds are still overlapping. Actually, we explain the observed loss of contrast in terms of phase patterns imprinted by the relative motion, as a consequence of the entanglement between the internal and external states of the trapped atoms.

The question about the coherence of Bose-Einstein condensates (BECs) [1], and the characterization of their phase properties have drawn considerable attention in the recent literature.

The first evidence of a definite phase for weakly interacting condensates dates back to the experiment of the MIT group [2], where high contrast matter-wave interference fringes were observed in the density distribution of two freely expanding BEC’s. Subsequent experiments performed at JILA [3] measured the relative phase of two condensate in different internal (hyperfine) states experiencing almost the same trapping potential.

Other experiments have further investigated this subject [4][5][6][7]. In particular, the group of W. Phillips has recently measured the evolution of the spatial profile of the phase of BEC’s, by using a Bragg-interferometer [8]. They have shown that a trapped condensate is characterized by a uniform phase, and after release from the trap develops a non-uniform phase profile.

In a recent Letter [9] we have demonstrated an experimental method for a sensitive and precise investigation of the interaction between two Bose-Einstein condensates [10]. In this paper we present an interferometry experiment performed in the time-domain on the same system, which allows to gain a deeper insight into Ramsey interferometry [11] with ultra-cold atoms across the BEC phase transition.

It is well known that Ramsey fringes are readily observable with a sample at room temperature. The reason is that Ramsey signals rely on the persistence of coherence between two distinct atomic levels, no matter what the state of the particles center-of-mass is. This is true only when the internal and external degrees of freedom are uncoupled. Whenever the latter condition fails, a depletion of the Ramsey fringes visibility can occur and has indeed been observed [12]. Thus, the entanglement of external and internal states of atoms trapped in a magnetic potential is the basis for the use of Ramsey method to characterize the phase properties of a condensate and a thermal cloud near BEC. In the JILA experiment [3], such an entanglement caused the observed Ramsey fringes in the population of the two hyperfine levels to undergo a loss of contrast, as a consequence of the reduced spatial overlap between the two BEC’s, due to their mutual repulsion.

In our experiment we assist to a similar depletion but on a much faster time-scale, of the order of tens of $\mu$s, that cannot be explained with the reduction of spatial overlap. We prepare, and subsequently probe the system with a sequence of two identical radio-frequency (rf) pulses, separated by a delay time $T$. The relative phase accumulated by the condensates produces Ramsey fringes in the population of each level, as a function of the delay $T$. We observe a strong reduction of the oscillation amplitude even at very short times, when the condensates are still almost completely overlapping, due to the relative velocity acquired by the condensates. We compare this reduction with that of a cloud of thermal atoms, at a temperature three times larger than the phase-transition temperature $\Theta \sim 3\Theta_c$.

We prepare a condensate of typically $2 \times 10^5$ $^{87}$Rb atoms in the $|F = 2, m_f = 2\rangle$ hyperfine level, confined in a 4-coils Ioffe-Pritchard trap elongated along the $z$ symmetry axis [13]. The axial and radial frequencies for the $|2\rangle$ state are $\omega_{z2} = 2\pi \times 13$ Hz and $\omega_{r2} = 2\pi \times 131$ Hz, respectively, with a magnetic field minimum of 2.86 Gauss. Then, we apply an rf-pulse to split the initial condensate into a coherent superposition of different Zeeman $|m_f\rangle$ sublevels of the $F = 2$ state. The atoms transferred in a different sublevel move away from the $|2\rangle$ equilibrium position with an acceleration that depends on their $m_f$ value. Thus, the state of motion becomes entangled with the internal atomic state.

In this experiment we use a 24 cycles rf-pulse at 2 MHz, which quickly leaves the $|2\rangle$ state with $\sim 41\%$ of the initial number of atoms, transferring an equal part of atoms ($\sim 41\%$) to the $|1\rangle$ state and $\sim 15\%$ of the atoms the $|0\rangle$ state, $\sim 3\%$ to the $|-1\rangle$ and $\sim 0\%$ to the $|-2\rangle$ states.

Even though all the five levels should be taken into
account in the early stages of the evolution (before the atoms in the $|0\rangle$, $|-1\rangle$ and $|-2\rangle$ states leave the trap), the basic features can be explained by considering only the dynamics of the two most populated levels, i.e. $|1\rangle$ and $|2\rangle$. We will discuss later how the other three levels affect the overall behaviour of the system.

We describe our double condensate system by a spinor wavefunction, where the upper and lower components refers to states $|2\rangle$ and $|1\rangle$, respectively. Immediately after the first rf-pulse, the wavefunction can be written as:

$$
\Psi(r; t = 0) = \sqrt{N_0} u(r; 0) \left( \begin{array}{c} 1 \\ i \end{array} \right)
$$

where $\sqrt{N_0} u(r)$ is the equilibrium wavefunction of the $N_0$ atoms of the initial $|2\rangle$ condensate, in the Thomas-Fermi (TF) regime \[13\]. As the rf-pulse is much shorter than the oscillation periods of the harmonic trap, we could safely take the spatial wavefunction $u(r)$ to remain unchanged and flop only the internal state.

In the subsequent free evolution, the relative phase between the two components accumulates with a rate proportional to the difference of chemical potentials $\mu_2 - \mu_1$. Moreover, $u(r; 0)$ is no longer the equilibrium wavefunction for $|2\rangle$ nor for $|1\rangle$: the spatial wavefunctions evolve as dictated by two coupled GP equations \[10\] into $u(r; t)$ and $v(r; t)$, respectively.

By applying a second rf-pulse, identical to the first, after a time delay $T$, we suddenly mix the two components

$$
\psi(r; T) = \frac{\sqrt{N_0}}{2} \left( \begin{array}{c} u(r; T) e^{-i\omega_0 T} + iv(r; T) \\ i u(r; T) e^{-i\omega_0 T} + v(r; T) \end{array} \right)
$$

with $\omega_0 = (\mu_2 - \mu_1)/\hbar$.

We then separate the two internal states and independently count the number of atoms in each, given by the integrated square modulus of the corresponding components, showing an oscillating behaviour at the frequency $\omega_0$ \[13\].

For times $T$ much shorter than the harmonic periods, we can neglect all but the most relevant effect of the spatial wavefunctions evolution: due the differential gravitational “sagging”, the $|1\rangle$ condensate acquires a downward time-dependent momentum $-hq(t)$. In particular, we will also neglect the loss of spatial overlap arising from the relative displacement. Then, taking $u(r; T) = u(r; 0)$, $v(r; T) = iv(r; 0) \exp(-iq(T)y)$ and given the normalization $\int |u(r; 0)|^2 dy = 1$, we have

$$
N_2(T) = N_0 \frac{1}{2} [1 - A_c(T) \cos(\omega_0 T)]
$$

$$
N_1(T) = N_0 - N_2(T)
$$

(3)

with the slow time-dependent oscillation amplitude

$$
A_c(T) = \int |u(r; 0)|^2 \cos(q(T)y) dr.
$$

The amplitude of the quadrature component vanishes because $|u(r; 0)|^2$ is an even function of the $y$ coordinate.

The amplitude $A_c(T)$ decays as the relative velocity increases and $q^{-1}$ becomes of the order of the vertical extension of the original condensate. Eventually, Ramsey fringes are completely washed out. The relative displacement would give the same result, but only at later times.

In Fig. 1 we plot the experimental data of the oscillation amplitudes for the fraction $N_2/(N_1 + N_2)$ versus the time delay $T$. To obtain each data point, we have sampled one oscillation period around the reported $T$ value. We have taken about 10 points per period and each point is the average of few acquisitions. To compare the experimental points with the values predicted by Eq. (4), we rescale the latter by a suitable factor, chosen to match the data around $T = 0$. As for the damping time, we find a satisfactory agreement between theory and experiment.

Actually, for the double condensate we can refine the above model to include the mean-field repulsion in the wavefunctions evolution: to this end, we numerically integrate two coupled Gross-Pitaevskii (GP) equations, according to the model described in Ref. \[10\]. On general ground, in the first stages of the evolution the phase of each component can be written as a quadratic form in the spatial coordinates \[13\], describing the mean-field expansion/contraction of the condensates in the Thomas-Fermi regime \[13\] and the motion along the vertical direction $y$. The results of the numerical simulations, in the time range considered here, show the following: (i) the phase of the $|2\rangle$ condensate remains almost uniform, and only for later times ($t \simeq 0.5$ ms) develops a negative curvature, due to the fact that the condensate is contracting after the initial transfer of atoms to the other levels; (ii) the phase of the $|1\rangle$ condensate is dominated by a linear term

$$
\phi_1(r; t) \simeq \frac{m}{\hbar} (v_0(t) + \delta v(t)) y
g \sin \left( \frac{\omega_{\perp 2} t}{\sqrt{2}} \right)
$$

(5)

where $v_0(t)$ is the velocity acquired during the fall in the trapping potential

$$
v_0(t) = -\frac{g}{\sqrt{2} \omega_{\perp 2}} \sin \left( \frac{\omega_{\perp 2} t}{\sqrt{2}} \right)
$$

(6)

and $\delta v(t)$, which is negligible for $t < 0.5$ ms, is due to the mutual repulsion between the two condensates. Thus, the GP simulations confirm that, at short times, the simple model adopted above correctly describes the basic features of the system.

The phase term \[4\] is responsible for washing out completely the Ramsey fringes even at very short times, when the condensate are still almost completely overlapping (as shown in Fig. 3 the overlap between the two wavepackets is substantial even at $T$ as long as 0.5 ms). Physically, across the spatial extension of the wavepacket there are regions of alternated positive and negative interference, with a vanishing net transfer of atoms.
For a thermal cloud we can consider the system to be in a given quantum state and then take an ensemble average over all the accessible states. This way, we need only to replace the time-dependent amplitude \( A_\text{th}(T) \) with

\[
A_\text{th}(T) = \frac{1}{N_0} \sum_{\{n\}} \int f_{\{n\}} \cdot |\Psi_{\{n\}}(r)|^2 \cos(q(T)y)dr
\]  

(7)

where \( f_{\{n\}} = [\exp((\epsilon_{\{n\}} - \mu)/k\Theta) - 1]^{-1} \) is the Bose mean occupation number of the harmonic trap eigenstate \( \Psi_{\{n\}} = \psi_{n_1}(x)\psi_{n_2}(y)\psi_{n_3}(z) \) with energy \( \epsilon_{\{n\}} \), \( \mu \) is the chemical potential, given by the normalization \( \sum_{\{n\}} f_{\{n\}} = N_0 \), and \( \Theta \) the temperature. By carrying out the integration over \( x, z \) and the sum over the corresponding quantum numbers \( n_1, n_3 \), we find

\[
A_\text{th}(T) = \frac{(k\Theta)^2}{N_0 h^3 \omega_x \omega_z} \sum_{n_2} g_2(\exp(\mu - n_2 \hbar \omega_y/k\Theta)) \times \int |\psi_{n_2}(y)|^2 \cos(q(T)y)dy,
\]

(8)

the \( g_2(x) = \sum_{l=1}^{\infty} x/l^2 \) function being the result of replacing the discrete sum over \( n_1, n_3 \) with a double integral. In principle, one should allow for the rf-pulses to act differently on the different harmonic oscillator eigenstates as the detuning varies. This would introduce \( \{n\} \)-dependent weights in Eq. (7). However, we have verified that for the relevant levels at \( \Theta \approx 0.4 \mu \text{K} \), these weights are equal within a few percent.

We note here that Eq. (7) can be rewritten

\[
A_\text{th}(T) = \frac{1}{N_0} \int n(r) \cos(q(T)y)dr
\]

(9)

where \( n(r) \) is the spatial density. In this form, it is evident that the thermal cloud behaves as a coherent wavepacket, the reason being that the displacement occurring between the two rf-pulses is much less than the thermal coherence length \( (\lambda_\text{th} = h/\sqrt{2\pi m k\Theta}) \).

We point out that for the non-condensed sample, where typical densities are a factor 30 lower than those of BEC, we neglect the atom-atom interactions, which justifies the above single-particle analysis. In Fig. 2 we plot the amplitude of the observed \( N_2/(N_2 + N_1) \) oscillation for several values of the time delay \( T \) between the two rf-pulses and we compare with the corresponding predictions given by Eq. (6), rescaled to match the experimental values for \( T \to 0 \). As for the damping time, we have again a satisfactory agreement; thus, we believe that the cause of the Ramsey fringes loss of contrast is well understood.

However, the experimental values are about a factor 2 less than those given by Eq. (6). As we already noticed, part of the atoms end up in the \( m_f = 0, -1, -2 \) Zeeman sublevels after the second rf-pulse. Two are the main consequences: (i) the peak-to-peak oscillation amplitude of the \( N_2/(N_1 + N_2) \) fraction cannot exceed 0.87, both for the condensates and the thermal clouds; (ii) the oscillation waveforms deviate from a pure sinusoidal behaviour.

To conclude, we point out that our system is suitable to study the possibility of a revival of the Ramsey fringes after the condensates have been spatially well separated. As the center-of-mass of the \( |1\rangle \) condensate undergoes harmonic oscillations around its equilibrium position \( \{1\} \), it comes back to rest at its initial position. According to the above description one should expect a revival of the oscillations in the relative population when the condensate \( |1\rangle \) and \( |2\rangle \) come to overlap again with almost vanishing velocity. The numerical solution of the GP equations of the two-level model shows indeed that over time scales of tens of ms the Ramsey fringes are characterized by collapse and revival. [17]

To summarize, we have studied the Ramsey interference of atomic clouds across the BEC phase-transition, with a system where the two involved states have equilibrium positions far apart. We have shown that the phase pattern imprinted on the moving wavepacket by its acquired velocity washes out the Ramsey oscillations of the fractional populations well before that the spatial overlap decreases. By repeating the same experiment on a thermal cloud at three times the condensation temperature, we have observed that the same mechanism is responsible for an even faster damping of oscillation contrast. In this respect, there is no substantial difference from a thermal cloud and a Bose condensate. The analogy with light optics is straightforward: indeed, to observe interference between two paths we need only the difference between the paths length not to exceed the coherence length.

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\* Dipartimento di Fisica, Università di Padova, Via F. Marzolo 8, I-35131 Padova, Italy.

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[17] We defer a more complete discussion on this point to a forthcoming publication.

![FIG. 1. Peak-to-peak amplitude of the oscillating observable $N_2/(N_2+N_1)$, i.e. the $|2⟩$ condensate population normalized to the total number of atoms in $|2⟩$ and $|1⟩$: experiment and calculation (solid line).](image1)

![FIG. 2. Peak-to-peak amplitude of the oscillating observable $N_{2z}/(N_{2z}+N_1)$, for thermal atoms at $\Theta = 0.4 \mu K \approx 3\Theta_c$: experiment and calculation (solid line).](image2)

![FIG. 3. Overlap of the two condensates at the second rf-pulse. Density profiles along the vertical axis $y$ of condensates $|2⟩$ (dashed-line) and $|1⟩$ (solid line) before the pulse, and of condensate $|1⟩$ (dotted line) immediately after. The curves are obtained by solving the GP equations for the two-state model in Ref. [14]. Lengths are given in units of $a_{\perp_2} = [\hbar/(m\omega_{\perp_2})]^{1/2} = 0.94 \mu m; T = 0.5 \mu s$.](image3)