The understanding of the triangle in Lobachevski geometry through local culture

W Widada, D Herawaty, I Hudiria, Y A Prakoso, Y R Anggraeni, and K U Zaid Nugroho
Postgraduate Mathematics Education Program, Universitas Bengkulu. Jl. WR. Supratman, Kandang Limun Bengkulu, Indonesia 38371
Email: w.widada@unib.ac.id

Abstract: A triangle is an abstract object in Lobachevsky's Geometry. The local content is needed as a starting point for learning geometry. This study aims to describe student’s understanding of triangles in Lobachevsky Geometry through local culture. This is an exploratory study. The focus is on analyzing the needs of student’s ability to understand the properties of Lobachevsky's geometry. The subjects of this study were mathematics education students in Bengkulu, Indonesia. In-depth interviews are based on assignments about triangular problems. The problem is taken from the culture of Bengkulu society about "glutinous rice cake". Data were analyzed through a collection of interviews and other physical activity. The results of this study are the subjects can connect the elements together to form a coherent unity based on observations of sticky rice cake. They produced a new product by organizing several elements into different forms or patterns from before, namely triangles with a large number of angles of less than 180. The conclusion of this study is that student’s understanding of triangles in Lobachevsky Geometry has reached the level of creating a new proposition, namely the number of angles in triangles in Lobachevski Geometry is less than 180.

1. Introduction
Geometry is one of the compulsory subjects for mathematics education students. The object is abstract, starting from primitive elements, such as points, lines and fields. Also, some axioms of incidence, and axioms of alignment, as well as other concepts [1]. This makes it difficult for students to understand. Students experience problems in learning geometry [2]. Our experience teaching geometry, students' spatial ability in the development of understanding geometry is low. Students need to know the mistakes made when solving geometry problems. That can be useful to correct his mistakes. There is a correlation between spatial intelligence and students' mistakes in understanding geometry [3]. Furthermore, students with low spatial intelligence make many geometrical concept mistakes. Geometry errors made by students are transformation errors, process skill errors, and coding errors. Students should be given precisely planned geometry learning [4-6].

Learning geometry that utilizes students' native culture based can stimulate their interest in mathematics [5]. For students of any cultural background, the use of Islamic geometric designs in their mathematics classes can increase intercultural awareness as well as awareness of the relationship between mathematics,
art and history [6]. Learning geometry and mathematics in general can be more meaningful, if it starts with something related to local culture [9-12]. The student's thought process in learning geometry with the ethnomathematics approach is an activity based on students' culture of abstraction. Students can build objects about infinite lines that are parallel to certain lines. Encapsulation activity produces correct understanding based on the properties of woven mats [8]. That is an attempt to complete the genetic decomposition of students in understanding mathematical concepts and principles (also geometry) [14, 15], also mathematical problem solving through learning based on ethnomathematics, namely the problem solving ability of students who study ethnomathematics is higher than students who study not ethnomathematics oriented [10]. Ethnomathematics approach makes it easy for students to achieve mathematical concepts. The process of learning mathematics through using culture can achieve the principle of multiplication of two vectors that form a right angle [11]. Mathematical culture-based mathematics learning in Bengkulu can overcome the difficulties of students in understanding the system of linear equations [12]. The process of understanding the concept of graph can be used ethno-mathematical approach, namely the contextual problem of communication culture using mobile phones [13]. Other research results show that students are able to simplify the concept of function through realistic problems based on the "andul dance" culture. Learning realistic mathematics with ethnomathematics approach can be a vehicle for students to simplify the concept of functions to be more meaningful [14].

In learning geometry, students' ability to prove theorem is fairly high ability. One difficult theorem to determine is the Lobachevsky parallel line. Widada et al. [15] found that the students can prove that through the T point that is not located on the g line, there are not many parallel lines with g. Students can compare the structure of deductive Lobachevsky Geometry with Euclid's Geometry. Extended-abstract level students are able to present several elements and pass interdependencies with each other, so that they become integrated entities. Also, it can generalize to new structures [15]. These students are also often referred to as extended-trans students [11, 20]. The student is able to permanently store the schema in long-term memory, and can easily recall the schema in working-memory [17].

Students can represent the existing knowledge in their memory in a mathematical competence. The competency includes a mathematical representation. It is choosing, designing, interpreting, translating between, and using various representations to capture situations, interact with problems, or to present one's work. Also, includes equations, formulas, graphs, tables, diagrams, pictures, textual descriptions and concrete materials [18].

In the representation of geometry, a triangle is a combination of three line spaces that meet at their edges. For example triangle ABC (symbolized by $\Delta ABC$) was $\overrightarrow{AB} \cup \overrightarrow{BC} \cup \overrightarrow{CA}$. At $\Delta ABC$ there are three inner angles viz $\angle ABC$, $\angle ACB$, and $\angle BAC$. We also know that $\angle ABC = \overrightarrow{AB} \cup \overrightarrow{AC}$, with $\overrightarrow{AB}$ is ray AB. In Euclidean Geometry, $\Delta ABC$, applies $m(\angle ABC) + m(\angle ACB) + m(\angle BAC) = 180^\circ$, whereas in Lobachevsky Geometry $m(\angle ABC) + m(\angle ACB) + m(\angle BAC) \leq 180^\circ$ [1]. We will learn that in this research through local culture as an approach to learning geometry. Some previous research results about Lobachevsky's Geometry show that students can summarize two or more features in Lobachevsky's axiom alignment through ethnomathematics in the form of bubu. Students can build objects about infinite lines that are parallel to certain lines [8]. Thus, the focus of this study is to explore students' understanding of triangles in Lobachevsky Geometry.

2. Method

This study aims to describe students' understanding of triangles in Lobachevsky's Geometry based on local culture. That is qualitative research with a case study. It is part of the research development of a grounded mathematics learning model in Bengkulu, Indonesia [19]. The subjects of this study were mathematics education students who were taking micro mathematics teaching classes. Two subjects were chosen from 80 students of mathematics education in Bengkulu, Indonesia. They are students who have the highest ability of initial geometry concepts, based on tests. In collecting data through interviews, we focused on the two students. Subjects were chosen based on the results of the geometry ability test. The two subjects are
those who have the highest ability among the others. We propose one ancestral culture of Bengkulu people, "glutinous rice cake (= Lupis)". Subjects were asked to measure the size of the angles in the triangle formed from Lupis. Measurement was carried out for 5 Lupis from each research subject. Research subjects were interviewed in depth by the research team. Students are asked to arrange the measurement results in a table. Also, asked to describe and make conclusions. Data were analyzed qualitatively through the stages of qualitative data analysis.

3. Results and Discussion
In this study, students are asked to understand the basic concepts of Lobachevsky Geometry. First, they were asked to recall Lobachevsky's Alignment Axiom. Figure 1 is the results of initial tests on eighty mathematics education students in Bengkulu about understanding the Lobachevsky Alignment Axiom (see research results) [13, 19]).

According to Figure 1, there are 10% of students who do not understand, 19% is less-understand, 45% is understand enough, 20% is understand, and only 6% of students are very understand the Lobachevsky Alignment Axiom. This shows a distribution that almost forms a normal curve. 6% of students who are very understanding, have the potential to be able to develop further understanding of geometry. These are extended-trans level students [24, 25]. Students at that level have genetic decomposition with a mature scheme [15, 26]. The researcher chose two students who were at the very level of understanding as research subjects. The two students were Ht and Bg. The two of them were interviewed in depth based on the completed work on triangles in Lobachevsky Geometry. Our focus is on a triangle that begins with something very close to the minds and daily lives of students, namely "Lupis" (see Figure 2).

Figure 1. Percentage level of understanding of Lobachevsky's Axiom Alignment

![Figure 1](image1.png)

Figure 2. Red Triangle Around a Lupis

![Figure 2](image2.png)
Based on the triangular example in Figure 2, the research subjects draw each triangle produced from the circumference of each Lupis. Each subject draws a triangle based on five different Lupis cakes. Then he measured the angle in the triangle. Following are excerpts of interviews with Ht and Bg with the interviewer (R).

R: What can you say from Lupis cake, when viewed from the shape of geometry?
Ht: ... yes ... it is a food that I usually eat and can be bought at the market. I say that Lupis cakes are triangular in shape.
Bg: ... I used to be made by my mother ... it was made from sticky rice, triangular in shape.
R: What is the triangle shape that you produce from the Lupis cake?
Bg: ... the shape is different from the triangle I used to draw. Three-line segments that form a triangle are usually straight-line segments, but the drawing of the triangle that I got from the Lupis cake is the curved line inward.
Ht: ... that's right ... I also got the same thing; the triangle can be seen in this picture (see Figure 3).

Figure 3. Triangles from the circumference of a Lupis cake

Figure 4 is a representation of a triangle obtained from the circumference of a Lupis cake. To further clarify, triangles are given names for each angle being ABC triangles. ∆ ABC becomes a triangle formed from three-line segments, i.e. $\overline{AB} \cup \overline{BC} \cup \overline{CA}$. But each line segment is a line segment that is curved inward. The research subject then drew it as shown in Figure 4.

Figure 4. ABC triangle based on the circumference of a Lupis cake

As shown in Figure 4, each study subject drew five different triangles according to the Lupis he drew. To get an overview of students’ understanding of triangles in Lobachevsky Geometry, the following is an excerpt from the results of our interview with Gb and Ht.

R: You have drawn each of the five triangles based on the Lupis you have measured. Try to explain your results?
Bg: ... I drew five ABC triangles from each Lupis that I got ...
Ht: ... I also do the same thing even though I produce different ones ...
R: ... what can you do?
Ht: ... yes, I measure with a protractor and I make it in the table ...

The following is the data obtained based on the results of the measurements of the angles in the triangle Bg and Ht respectively as listed in Table 1 and Table 2.

**Table 1. Measurement of Angles in ABC Triangles (by Bg)**

| No. triangle | \(\angle ABC\) | \(\angle BAC\) | \(\angle ACB\) | Sum  |
|--------------|----------------|----------------|----------------|------|
| 1            | 59.8           | 59.9           | 59.6           | 179.3|
| 2            | 59.1           | 58.9           | 59.9           | 177.9|
| 3            | 59.5           | 59.2           | 59.8           | 178.5|
| 4            | 59.9           | 59.8           | 59.9           | 179.6|
| 5            | 59.7           | 59.1           | 58.9           | 177.7|

Table 1 is the measurement results by Subject Bg. The table shows that \(\angle ABC + \angle ACB + \angle BAC \leq 180^\circ\). This is one of the empirical evidences that there are triangles whose large number of angles are less than or equal to 180°. The same thing was also shown by Ht through the measurement of five other Lupis cakes. The measurement results of Ht are shown in Table 2.

**Table 2. Measurement of Angles in ABC Triangles (by Ht)**

| No. triangle | \(\angle ABC\) | \(\angle BAC\) | \(\angle ACB\) | Sum  |
|--------------|----------------|----------------|----------------|------|
| 1            | 59.5           | 59.1           | 59.8           | 178.4|
| 2            | 59.3           | 59.9           | 59.0           | 178.2|
| 3            | 59.1           | 59.3           | 59.1           | 177.5|
| 4            | 59.7           | 59.1           | 59.0           | 177.8|
| 5            | 59.5           | 59.8           | 58.7           | 178.0|

Based on Table 2, from the five Lupis measured by Ht, it is found that the large number of angles in Triangle 1 is equal to 178.4, Triangle 2 is equal to 178.2 and so on Triangle 5 is 178.0. This shows that \(\angle ABC + \angle ACB + \angle BAC \leq 180^\circ\).

Table 1 and Table 2 provide a representation that a triangle has a large number of angles in it. For triangles in a plane geometry system \(\angle ABC + \angle ACB + \angle BAC = 180^\circ\). Thus, there is another system in geometry besides Euclid's Geometry namely Lobachevsky's Geometry. In learning Lobachevsky's Geometry the level of retention is demanded to be more like middle to upper level semi-trans cognitive levels until extended-trans levels [9, 25, 27].

The results of this study indicate that mathematics learning including geometry based on ethnomathematics and connected mathematics approaches have a positive impact on students' cognitive processes. Students are able to find theorems about the large number of angles in triangles of less than one hundred and eighty degrees. It is based on local culture [28-31]. Thus, it is appropriate to recommend that the local cultural approach be a starting-point in learning geometry. This is horizontal mathematical, which is based on informal mathematical towards structuralistic vertical mathematical [32, 33]. This research is one proof that a very abstract theorem can be demonstrated through empirical activity that in the triangle ABC applies \(\angle ABC + \angle ACB + \angle BAC \leq 180^\circ\). Then the vertical mathematical process is carried out, in which students prove the theorem formally, this will be written in further research.
4. Conclusions
Learning geometry is one of the subjects that is difficult to understand. That's because the object is very abstract. This study illustrates that learning geometry can be approached through local culture. *Lupis* cake culture in Bengkulu, Indonesia provides a positive representation for students in starting triangle learning in the Lobachevsky Geometry system. They discovered empirically that in triangles, a large number of inner angles are valid less than or equal to one hundred and eighty degrees. We suggest that learning mathematics which has abstract objects including geometry should start from something rooted in local culture.

5. References.
[1] Prenowitz W and Jordan M 1989 *Basic Concept of Geometry* (Boston: Aedsley Hoyse Publishers. Inc.)
[2] De Villiers M 2012 Some Reflections on the Van Hiele theory *Natl. Math. Congr.* 21–3
[3] Kristayulita K, Nusantara T, Maelasari E and Jupri A 2017 Students ’ Errors in Geometry Viewed from Spatial Intelligence Students ’ Errors in Geometry Viewed from Spatial Intelligence *J. Phys. Conf. Ser. (international Conf. Math. Sci. Educ.* 895
[4] Clements D H 2015 Teaching and learning geometry https://www.researchgate.net/publication/258933229 Teach.
[5] Widada W, Agustina A, Serlis S, Dinata B M and Hasari S T 2019 The abstraction ability of students in understanding the concept of geometry The abstraction ability of students in understanding the concept of geometry *J. Phys. Conf. Ser.* 1318 1–7
[6] Karssenberg G 2014 LEARNING GEOMETRY BY DESIGNING PERSIAN MOSAICS 1
[7] Widada W, Herawaty D, Nugroho K U Z and Anggoro A F D 2019 The ability to Understanding of the Concept of Derivative Functions for Inter-Level Students During Ethnomathematics Learning *J. Phys. Conf. Ser.* 1179 1–6
[8] Herawaty D, Khrisnawati D, Widada W and Mundana P 2020 The cognitive process of students in understanding the parallels axiom through ethnomathematics learning *IOP Conf. Ser. J. Phys. Conf. Ser.* 1470 012077 doi10.1088/1742-6596/1470/1/012077 1470 1–8
[9] Widada W 2017 Beberapa Dekomposisi Genetik Siswa dalam Memahami Matematika *J. Pendidik. Mat. Raflesia* 1 44–54
[10] Nugroho K U Z, Widada W and Herawaty D 2019 The Ability To Solve Mathematical Problems Through Youtube Based Ethnomathematics Learning *Int. J. Sci. Technol. Res.* 8 1232–7
[11] Widada W, Herawaty D, Beka Y, Sari R M and Riyani R 2020 The mathematization process of students to understand the concept of vectors through learning realistic mathematics and ethnomathematics *IOP Conf. Ser. J. Phys. Conf. Ser.* 1470 012071 doi10.1088/1742-6596/1470/1/012071 1470 1–10
[12] Widada W, Herawaty D, Rahman M H, Yustika D and Elsa P 2020 Overcoming the difficulty of understanding systems of linear equations through learning ethnomathematics *IOP Conf. Ser. J. Phys. Conf. Ser.* 1470 012074 doi10.1088/1742-6596/1470/1/012074 1470 1–14
[13] Widada W, Herawaty D, Andriyani D S, Marantika R and Yanti I D 2020 The thinking process of students in understanding the concept of graphs during ethnomathematics learning *IOP Conf. Ser. J. Phys. Conf. Ser.* 1470 012072 doi10.1088/1742-6596/1470/1/012072 1470 1–8
[14] Herawaty D, Widada W, Adhiyta A, Sari R D W and Novianita L 2020 Students ’ ability to simplify the concept of function through realistic mathematics learning with the ethnomathematics approach *IOP Conf. Ser. J. Phys. Conf. Ser.* 1470 012031 doi10.1088/1742-6596/1470/1/012031 1470 1–8
[15] Widada W, Herawaty D, Jumri R and Wulandari H 2020 Students of the extended abstract in proving Lobachevsky’s parallel lines theorem *IOP Conf. Ser. J. Phys. Conf. Ser.* 1470 012098 doi10.1088/1742-6596/1470/1/012098 1470 1–10
[16] Widada W, Efendi S, Herawaty D and Nugroho K U Z 2020 The genetic decomposition of students
about infinite series through the ethnomathematics of Bengkulu, Indonesia *IOP Conf. Ser. J. Phys. Conf. Ser.* 1470 012078 doi10.1088/1742-6596/1470/1/012078 1470 1–9

[17] Campos I S, Almeida L S, Ferreira A I, Martinez L F and Ramalho G 2012 Cognitive processes and math performance: a study with children at third grade of basic education *Eur J Psychol Educ*

[18] Turner R 2010 Identifying cognitive processes important to mathematics learning but often overlooked *Res. Conf. 2010* 56–61

[19] Widada W, Herawaty D, Falaq A, Anggoro D, Yudha A and Hayati M K 2019 Ethnomathematics and Outdoor Learning to Improve Problem Solving Ability *Adv. Soc. Sci. Educ. Humanit. Res. Vol.* 295 295 13–6

[20] Widada W, Herawaty D, Ma’rifah N and Yunita D 2019 Characteristics of Students Thinking in Understanding Geometry in Learning Ethnomathematics *Int. J. Sci. Technol. Res.* 8 3496–503

[21] Widada W and Herawaty D 2017 Dekomposisi Genetik tentang Hambatan Mahasiswa dalam Menerapkan Sifat-sifat Turunan *J. Didakt. Mat.* 4 136–51

[22] Andriani D, Widada W, Herawaty D, Ardy H, Nugroho K U Z, Ma’rifah N, Anggreni D and Anggoro A F D 2020 Understanding the number concepts through learning Connected Mathematics (CM): A local cultural approach *Univers. J. Educ. Res.* 8 1055–61

[23] Treffers A 1991 Didactical background of a mathematics programm for primary education *L. Streefland (ed.), Realistic Mathematics Education in Primary School, CD-β Press / Freudenthal Institute. Utrecht University*, (Utrecht: Freudenthal Institute)