Research on GPS Level Fitting Based on EGM2008 Model

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Abstract. In order to improve the accuracy of GPS leveling fitting, a method of leveling fitting based on the EGM2008 gravity field is proposed, which uses the “remove-restore” method. By fitting experiments, the results show that the precision of the “remove-restore” method based on gravity field model is higher than that of the conventional fitting method, and can reach 1 cm.

1. Introduction
GPS leveling fitting is the most commonly used method to determine the normal height of ground points by using GPS positioning technology. However, conventional GPS leveling fitting only uses a simple mathematical model, which can represent the general trend of the whole survey area, but the accuracy is not high.

The order of EGM2008 is completely released to 2159 times, and the order of spherical harmonic coefficient is extended to 2190 times. Compared with the previous gravity field models, this model has higher accuracy. The overall accuracy of this model in China is about 20 cm, and that in North China is about 9 cm. This paper presents a method of GPS leveling fitting by using EGM2008 gravity field model and " remove-restore " method.

2. Principle of " Remove-restore " Method Based on EGM2008

2.1. EGM2008 Model
In April 2008, NGA (National Geospatial Intelligence Agency) released the latest global gravity field model, named EGM2008, which integrates the latest GRACE satellite tracking data, ground gravity data and satellite altimetry data. The maximum order of the model is 2190, and the spatial resolution of the model is 5'(about 9 km). EGM2008 provides the global gravity field model including 2190 orders, the global 5’×5’ grid gravity anomaly, the global 5’×5’, 2.5’×2.5’ grid geoid, and the final achievement of the global 5’×5’ grid vertical deflection. The EGM2008 model expands a harmonic function approaching the zero value of the external gravitational potential of the earth's mass at infinity into a series of 2190 order spherical harmonic functions of integral order which converge theoretically.

According to Bruns formula, the height anomaly at any point on the ground can be obtained.

\[
\zeta_{EGM} = \frac{GM}{\rho \gamma} \sum_{n=2}^{2190} \sum_{m=0}^{n} \left[ C_{nm} \cos(m \lambda) + S_{nm} \sin(m \lambda) \right] \bar{P}_{nm}(\sin \phi)
\]  

(1)
In the formula, \( GM \) is the gravitational constant; \( \rho, \gamma, \lambda \) and \( \phi \) are the geocentric diameter, normal gravity value; longitude and latitude of the points; \( a \) is the ellipsoid radius; \( \mathcal{C}_m \) and \( \mathcal{S}_m \) are the fully normalized potential coefficients; and \( \bar{P}_m(\sin \phi) \) is the fully normalized Association function\(^3\).

2.2. Remove-restore Principle

According to the theory of physical geodesy, elevation anomaly can be expressed as:

\[
\zeta = \zeta^{GM} + \zeta^G + \zeta^T
\]  

(2)

In the formula, \( \zeta^{GM} \) is the long wave term calculated by gravity field model; \( \zeta^G \) is the medium wave part, which can be obtained by solving the boundary value problem of gravity anomaly; \( \zeta^T \) is the short wave part, which can be obtained by solving topographic correction. In this paper, part \( \zeta^G \) and part \( \zeta^T \) are combined and fitted by mathematical model\(^4\).

\[
\zeta = \zeta^{EGM} + \zeta^{res}
\]  

(3)

The idea of "remove-restore" is to remove \( \zeta^{EGM} \) from the elevation anomaly and get \( \zeta^{res} \), and to fit it with known level points. In this paper, quadratic polynomials and polyhedral functions will be used for fitting\(^{5-7}\).

3. Conventional fitting method

3.1. Quadratic polynomial fitting method

Quadratic polynomial fitting method is often used in leveling fitting \(^4\). The expression between elevation anomaly and plane coordinates is as follows:

\[
\zeta = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 x y - \zeta_i
\]  

(4)

In the formula, \( a_0, a_1, a_2, a_3, a_4, a_5 \) are the parameters to be determined.

Therefore, this area requires at least 6 known points. When there are more than 6 known points, the corresponding error equations can be listed:

\[
V_i = a_0 + a_1 x_i + a_2 y_i + a_3 x_i^2 + a_4 y_i^2 + a_5 x_i y_i - \zeta_i
\]  

(5)

The matrix form is:

\[
V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}, \quad A = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}, \quad \zeta = \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_n \end{bmatrix}, \quad X = \begin{bmatrix} 1 & x_1 & y_1 & x_1^2 & y_1^2 & x_1 y_1 \\ 1 & x_2 & y_2 & x_2^2 & y_2^2 & x_2 y_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & y_n & x_n^2 & y_n^2 & x_n y_n \end{bmatrix}
\]  

(6)

The solution of the height anomaly of unknown point is to determine the coefficient of polynomial. Under the condition of \( \sum V^2 = \min \), \( A = (X^T X)^{-1} X^T \zeta \) can be obtained according to the principle of least squares method, and then the height anomaly of any point can be obtained.

3.2 Multiplanar Function Fitting Method

The idea of polyhedral function is that any mathematical surface or irregular smooth surface can be obtained by adding several regular mathematical surfaces. In Cartesian coordinate system, the general formula of many functions is:

\[
\zeta_{(x,y)} = f(x,y) = \sum_{i=1}^{\infty} \beta_i F(x, y, x_i, y_i)
\]  

(7)
In the formula, $\beta_i$ is the coefficients to be determined; $F(x, y, x_i, y_i)$ is the quadratic kernel functions of $x$ and $y$, and the core point is at $(x_i, y_i)$; $\zeta(x, y)$ can be expressed as the sum of quadratic polynomials.$^8$ The selection of quadratic kernels can be arbitrary. For ease of calculation, symmetrical positive hyperboloid functions are often used to represent:

$$F(x, y, x_i, y_i) = ((x-x_i)^2 + (y-y_i)^2 + \delta)^k$$

(8)

In the formula, $\delta$ is called smoothing factor and can be any positive number; when $k=1/2$, the kernel function is a positive hyperboloid function; when $k=-1/2$, it is an inverted hyperboloid function.

With $n$ known point, select $m$ point as the core point, and make $Q_i = F(x, y, x_i, y_i)$, the formula (7) can be expressed as follows:

$$\zeta(x, y) = \sum_{i=1}^{n} \beta_i Q_i$$

(9)

Then the error equation is:

$$
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & \cdots & Q_{1m} \\
Q_{21} & \cdots & Q_{2m} \\
\vdots & \ddots & \vdots \\
Q_{n1} & \cdots & Q_{nm}
\end{bmatrix}
\begin{bmatrix}
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_m
\end{bmatrix}
\begin{bmatrix}
\zeta_1 \\
\zeta_2 \\
\vdots \\
\zeta_n
\end{bmatrix}
$$

(9)

According to the principle of least squares, it can be obtained:

$$\beta = (Q^T Q)^{-1} Q^T \zeta$$

(10)

4. Analysis of Level Fitting Experiments

4.1. Experimental area

The experimental area is hilly terrain and the area is about 10 km$^2$. There are 20 known GPS points in the experimental area, and the survey area is a third-class leveling survey. Points 1 to 12 are selected as fitting points and 13 to 20 are checkpoints$^{[9-11]}$.

4.2. Experimental scheme

- Scheme 1: EGM2008 model and quadratic polynomial fitting method of "remove-restore" method. The EGM2008 model is used to solve the residual elevation anomaly, and the quadratic polynomial fitting method is used to fit the residual elevation.
- Scheme 2: EGM2008 Model and "remove-restore" Method of Polyhedral Function Method. The EGM2008 model is used to solve the residual elevation anomaly, and the residual elevation is fitted by polyhedral function method.
• Scheme 3: The method of quadratic polynomial is used to fit the elevation anomaly directly.
• Scheme 4: The method of polyhedral function is used to fit the elevation anomaly directly.

4.3. Analysis of Level Fitting Accuracy

Table 1. Fitting Residual Table

| Number | Scheme 1/m | Scheme 2/m | Scheme 3/m | Scheme 4/m |
|--------|------------|------------|------------|------------|
| 1      | 0.000      | 0.000      | 0.000      | 0.000      |
| 2      | 0.001      | 0.004      | -0.020     | -0.002     |
| 3      | -0.005     | -0.006     | 0.001      | 0.001      |
| 4      | 0.001      | 0.000      | -0.006     | -0.006     |
| 5      | 0.006      | 0.003      | 0.006      | 0.006      |
| 6      | -0.003     | -0.014     | 0.001      | 0.001      |
| 7      | 0.021      | 0.008      | 0.032      | 0.032      |
| 8      | 0.007      | -0.001     | -0.008     | -0.008     |
| 9      | 0.025      | 0.009      | 0.029      | 0.029      |
| 10     | 0.019      | 0.001      | 0.009      | 0.009      |
| 11     | 0.012      | 0.001      | 0.021      | 0.021      |
| 12     | 0.027      | 0.000      | 0.026      | 0.026      |
| 13     | 0.059      | 0.004      | 0.053      | 0.053      |
| 14     | 0.047      | 0.005      | 0.046      | 0.046      |
| 15     | 0.003      | 0.005      | -0.003     | -0.003     |
| 16     | 0.002      | -0.017     | -0.009     | -0.009     |
| 17     | 0.002      | 0.000      | -0.007     | -0.007     |
| 18     | 0.022      | 0.001      | 0.038      | 0.038      |
| 19     | 0.042      | -0.003     | 0.040      | 0.040      |
| 20     | 0.045      | 0.015      | 0.052      | 0.052      |

The fitting residuals are shown in the table below:

Table 2. Fitting Accuracy Table

| Programmes | Programme 1 | Programme 1 | Programme 1 | Programme 1 |
|------------|-------------|-------------|-------------|-------------|
| Internal accuracy /m | 0.015 | 0.006 | 0.017 | 0.008 |
| External accuracy /m  | 0.037 | 0.010 | 0.039 | 0.020 |

The internal coincidence accuracy of scheme 1 is 0.015 m, scheme 2 is 0.006 m, scheme 3 is 0.017 m and scheme 4 is 0.008 m.

The external coincidence accuracy of scheme 1 is 0.037 m, scheme 2 is 0.010 m, scheme 3 is 0.039 m and scheme 4 is 0.020 m.

In summary, the polyhedral function with EGM2008 model has the highest internal and external coincidence accuracy, reaching 1 cm.

5. Conclusion

With the idea of "remove-restore" method, this paper presents a method based on the EGM2008 gravity field model, and uses the polyhedral function method to fit the residual part of elevation anomaly to complete the transformation from normal height to normal height. The "remove-restore" method can also achieve better accuracy with the few known GPS point. Examples show that within a certain range, a small number of GPS points can be used as known points to realize the transformation of geodetic height to normal height in this area, and the fitting results can be applied to large-scale surveying and mapping.

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