High-energy emission from off-axis relativistic jets

E.V. Derishev*, F.A. Aharonian† and Vl.V. Kocharovsky*

*Institute of Applied Physics, 46 Ulyanov st., 603950 Nizhny Novgorod, Russia
†Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, D-69117 Heidelberg, Germany

Abstract.
We analyze how the spectrum of synchrotron and inverse Compton radiation from a narrow relativistic jet changes with the observation angle. It is shown that diversity of acceleration mechanisms (in particular, taking the converter mechanism (Derishev et al. 2003) into account) allows for numerous modifications of the observed spectrum. In general, the off-axis emission in GeV-TeV energy range appears to be brighter, has a much harder spectrum and a much higher cut-off frequency compared to the values derived from Doppler boosting considerations alone. The magnitude of these effects depends on the details of particle acceleration mechanisms, what can be used to discriminate between different models.

One of the implications is the possibility to explain high-latitude unidentified EGRET sources as off-axis but otherwise typical relativistic-jet sources, such as blazars. We also discuss the broadening of beam pattern in application to bright transient jet sources, such as Gamma-Ray Bursts.

INTRODUCTION

Relativistic flows are attractive as a solution of the compactness problem for bright and rapidly variable sources, such as Active Galactic Nuclei (AGNs) and Gamma-Ray Bursts (GRBs). Lorentz boosting lowers both the required comoving number density and the required energy of individual photons, so that the pair-production opacity can be made smaller than one for a source of given size. Often (if not always), the relativistic outflows are in the form of narrow jets, as seen in many AGNs (e.g., Urry & Padovani 1995) and deduced theoretically for GRBs (e.g., Mészáros 2002).

The central dogma for narrow and highly relativistic jets is that they are bright when viewed head-on, but hardly observable off-axis (at the viewing angle much larger than $1/\Gamma$, where $\Gamma$ is the Lorentz factor of a jet) since they appear far dimmer and strongly redshifted as compared to the head-on emission. Indeed, due to aberration of light, isotropic in the jet frame radiation is beamed in the laboratory frame, so that it is essentially confined within an angle $\sim 1/\Gamma$ from the jet’s velocity. This follows from the equation relating polar angles $\theta$ and $\theta'$ of a light beam in the two frames (hereafter, prime denotes the comoving-frame quantities):

$$\cos \theta = \frac{\beta - \cos \theta'}{1 - \beta \cos \theta'},$$

where $\beta$ is the jet velocity divided by the speed of light $c$, the polar axes are directed along the jet’s velocity in the laboratory frame and counter to it in the jet comoving
frame: the latter choice makes Eq. (1) symmetric with respect to $\theta$ and $\theta'$. The frequencies of emission are related as

$$\nu = \Gamma(1 - \beta \cos \theta') \nu' \equiv \delta \nu'. \tag{2}$$

Here $\delta = \Gamma(1 - \beta \cos \theta')$ is the Doppler factor. The observed spectral intensity, i.e., the energy flux per unit solid angle and per unit frequency, is given by

$$I(\nu, \theta) = \delta^n I'(\nu', \theta'), \tag{3}$$

where $n = 2$ for a continuous jet and $n = 3$ for a relativistically moving blob (Lind & Blandford 1985). Below we consider only the continuous jet case. It is convenient to write the asymptotic form of Eqs. (1), (2) and (3) for the case, where $\theta, \theta' \ll 1$:

$$\nu \simeq \frac{\Gamma \theta'^2}{2} \nu', \quad \theta \theta' \simeq \sqrt{\frac{8(1 - \beta)}{\beta}} \approx \frac{2}{\Gamma},$$

$$I(\nu, \theta) \simeq \frac{4}{\Gamma^2 \theta^4} I'(\frac{\Gamma \theta^2}{2} \nu, \frac{2}{\Gamma \theta}). \tag{4}$$

For isotropic in the jet frame emission, i.e., for $I'(\nu', \theta') = I'(\nu')$, the bolometric brightness drops very rapidly with increase of the viewing angle: $\int I d\nu \propto \theta^{-6}$ as long as $\theta \gg 1/\Gamma$. The situation is somewhat complicated if the jet is observed at different angles in the same (narrow) spectral range: hard spectra partially compensate for the decrease of brightness due to frequency shift, but the spectra must be unreasonably hard to reverse the general trend. There is the only way for an off-axis jet not to obey the dogma about darkening and reddening with increasing observation angle: this is possible if the emission produced in the jet has strong anisotropy in the comoving frame.

**ANISOTROPY IN THE JET FRAME: CONDITIONS AND PARAMETERS**

The photons produced by a relativistic particle continue to stream in the direction of the particle’s momentum, hence – in order to have anisotropic emission – the same degree of anisotropy is required for the particle distribution. The origin of every photon can eventually be traced to a charged particle, so that the anisotropic distribution of radiating particles means that there is a current in the jet’s plasma, which should quickly decay due to numerous instabilities. Two conditions have to be met to maintain the anisotropy: there must be a continuous anisotropic supply of relativistic particles and these particles must have a lifetime (with respect to radiative losses) shorter than the isotropisation timescale.

The first condition is almost automatically satisfied at the shock front for the particles advected with the upstream fluid, whereas the second requirement precludes production of such particles via diffusive shock acceleration. Indeed, the diffusive acceleration proceeds through multiple scattering of the accelerated particle downstream and the
resulting passages of the shock in direction from downstream to upstream. In the case of relativistic shock, the particle increases its energy by a factor $\sim 2$ in each shock passage (e.g., Achterberg et al. 2001). The particle should be able to preserve at least a half of its energy over one mean free path to keep accelerating. So, the distribution of diffusively accelerated particles is always close to isotropic in the jet frame.

Anisotropic distribution of radiating particles can be realized in two ways. One is to have these particles pre-injected upstream, as in the case of $e^- e^+$-pair loading ahead of GRB shocks (Madau & Thompson 2000). Another, more general way, is to produce them via non-diffusive converter acceleration mechanism (Derishev et al. 2003), which essentially requires isotropisation of accelerated particles upstream each time before they cross the shock. In what follows, we take an isotropic distribution of particles upstream, created in either way. This distribution is assumed to be a power-law of the particle’s energy $\varepsilon$ ($dN/d\varepsilon \propto \varepsilon^{-s}$) with the cut-off at $\varepsilon_m$. In the comoving frame, this transforms into the power-law injection at the shock with the same index and the cut-off at $\varepsilon'_m \simeq \Gamma \varepsilon_m$. Generalization for the case of arbitrary distribution is straightforward; we skip it for the sake of brevity.

Let $d(\varepsilon')$ be the particle’s mean free path and $\ell(\varepsilon')$ its radiation length, both measured in the jet’s frame. In the majority of cases, the function $d(\varepsilon')$ allows power-law representation:

$$d(\varepsilon') \propto (\varepsilon'/\varepsilon_{\text{cr}})^p,$$

where the critical energy $\varepsilon'_{\text{cr}}$ is defined from equality $d(\varepsilon'_{\text{cr}}) = \ell(\varepsilon'_{\text{cr}})$. For instance, $p = 1$ for a quasi-uniform magnetic field, that includes the case of Bohm diffusion, and $p = 2$ for the case of small-angle scattering. We also introduce the characteristic width of the particle distribution downstream,

$$\phi'(\varepsilon') = \begin{cases} 1, & \varepsilon' < \varepsilon'_{\text{cr}} \\ \left(\frac{\ell}{d}\right)^{1/p} \propto \frac{\varepsilon^{1/p}}{\varepsilon'}, & \varepsilon'_{\text{cr}} < \varepsilon' < \varepsilon'_{\text{cr}2} \\ 1/\Gamma, & \varepsilon' > \varepsilon'_{\text{cr}2} \end{cases},$$

where the second critical energy $\varepsilon'_{\text{cr}2}$ is defined by equality $\phi'(\varepsilon'_{\text{cr}2}) = \Gamma^{-1}$. The equation has simple physical meaning. The sub-critical particles ($\varepsilon' < \varepsilon'_{\text{cr}}$) have enough time to get fully isotropized, whereas for super-critical ones the width of the distribution function is equal to their maximum deflection angle $\phi'$. Above $\varepsilon'_{\text{cr}2}$, the particles lose energy before being significantly deflected, and the distribution function preserves its intrinsic width $1/\Gamma$, which originates due to the Lorentz transformation from the upstream frame.

Consider some representative situations.

**Synchrotron or self-Compton radiation in the Thomson regime.** The typical frequency of emitted photons is $\nu' \propto \varepsilon'^2$. If these types of emission prevail in radiative losses, then $\ell \propto 1/\varepsilon'$. Consequently, $\phi' \propto \varepsilon'^{-p+1}/p$, that is $\phi' = (\varepsilon'_{\text{cr}}/\varepsilon')^2$ for the Bohm diffusion.

**Self-Compton radiation in the Klein-Nishina regime.** The typical frequency of emitted photons is $\nu' \propto \varepsilon'$. In the case where such losses are prevalent, and the spectrum of radiation being comptonised is a power-law $F'_\nu \propto \nu'^q$, where $-1 < q < 1$, we have
\( \ell \propto \varepsilon'^q \). The width of the particle distribution is \( \phi' \propto \varepsilon'^{\frac{2-p}{p}} \); for the Bohm diffusion \( \phi' = (\varepsilon'/\varepsilon'_{cr})^{q-1} \).

Comptonization of external radiation with a logarithmically narrow spectrum in the Thomson regime. The frequency of emitted photons is given by \( \nu' \propto \varepsilon'^{2}\phi'^2 \). If this is the prevalent type of losses, then \( \ell \propto (\varepsilon'/\varepsilon'_{cr})^{-1} \). Thus, \( \phi' \propto \varepsilon'^{-\frac{p+1}{p+4}} \), i.e., \( \phi' = (\varepsilon'/\varepsilon'_{cr})^{-2/5} \) in the case of Bohm diffusion.

To give an idea of energy scale, the critical energy for electrons, whose acceleration is limited by the synchrotron losses, is

\[
\varepsilon'_{cr} = \frac{3}{2} \left( \frac{m_e c^2}{e B'} \right)^2, \tag{7}
\]

and the associated cut-off frequency of their synchrotron emission is

\[
\nu'_{cr} \simeq \frac{0.5 \, e B'}{\pi \, m_e c} \left( \frac{\varepsilon'_{cr}}{m_e c^2} \right)^2, \quad h\nu'_{cr} \simeq 270 m_e c^2. \tag{8}
\]

The second critical energy is \( \Gamma^{\frac{p+4}{p+1}} \) times larger than \( \varepsilon'_{cr} \).

**OFF-AXIS SPECTRA AND LUMINOSITY**

The majority of particles injected at the shock penetrate deep into downstream, loose energy through synchrotron and other types of radiation, and form a cooling distribution \( dN'/d\varepsilon' = \dot{N}' / \dot{\varepsilon}' \propto \ell \varepsilon^{-s} \). The angle-averaged brightness in the comoving frame can be derived in a straightforward way:

\[
\bar{I}(\nu') \propto \varepsilon' \frac{dN'}{d\varepsilon'} (\frac{d\nu'}{d\varepsilon'})^{-1} \propto \nu^{\frac{2-s-\alpha}{s}}, \tag{9}
\]

where we assumed that the spectrum of an individual particle is monochromatic with frequency \( \nu' \propto \varepsilon'^x \). The index \( x \) is different for different emission types, as discussed in the end of the previous section.

Taking into account that emission produced by the super-critical particles is concentrated within a cone with opening angle \( \phi'(\varepsilon') \), we find the observed brightness at small angles \( 1/\Gamma \ll \theta \ll 1 \):

\[
I(\nu, \theta) \propto \frac{\delta^2}{\phi'^2} \left( \frac{\nu}{\delta} \right)^{\frac{2-s-\alpha}{s}} \simeq \frac{1}{\theta^2} \left( \frac{\nu}{\delta} \right)^{\frac{2-s-\alpha}{s}}, \tag{10}
\]

where \( \theta \) and \( \phi' \) are related as \( \theta \simeq 2/(\Gamma \phi') \) and \( \nu/\delta \) is the function of \( \phi' \).

All the changes in the spectrum as compared to the head-on emission are due to the factor \( \phi'^{-2} \), which is a rising function of frequency in the range between \( \nu'_{cr} = \delta \nu'_{cr} \) and the cut-off \( \nu_m = [\varepsilon'(\phi')/\varepsilon'_{cr}]^{\frac{1}{p'}} \nu'_{cr} \). The difference between spectral indices in the head-on
and off-axis emission depends on the type of this emission, but the off-axis spectrum is always much harder.

As an example, let us consider the synchrotron emission in some detail. In this case, the spectral index between $\nu_{cr}$ and $\nu_m$ increases by 2 and 1.5 for the Bohm diffusion and the small-angle scattering, respectively. For an injected particle distribution with indices $s$ smaller than 13/3 or 10/3 (the Bohm diffusion and the small-angle scattering, respectively), that includes the typical assumption $s \simeq 2$, the resulting spectrum formally appears to be harder than the low-frequency asymptotic for the synchrotron emission of an individual particle. This means that in the above frequency range, as well as immediately below $\nu_{cr}$, the spectrum is determined by the low-frequency emission of the most energetic particles.

It is noteworthy that the cut-off frequency of the synchrotron emission observed at an angle $1/\Gamma \ll \theta \ll 1$ to the jet’s axis can be expressed via the radiative length:

$$\nu_m(\theta) \simeq \nu_{cr} \left( \frac{\ell(\epsilon')}{\ell(\epsilon'_{cr})} \right)^{2/p}. \quad (11)$$

Here $\epsilon(\theta)$ is the energy of particles whose beam-pattern width is equal to $\theta$. In the case of synchrotron losses, where $\ell \propto \epsilon^{-1}$, the dependencies of the observed cut-off frequency on the viewing angle are $\nu_m \propto \phi^{-1}$ for the Bohm diffusion and $\nu_m \propto \phi^{-2/3}$ for the small-angle scattering. They are much weaker than the relation $\nu_{max} \propto \phi^{-2}$, which stems from the Doppler factor alone. Moreover, prevalence of the inverse Compton losses cancels out or even reverses the above dependencies.

**CONCLUSIONS AND IMPLICATIONS**

The jet sources, which are observed off-axis owing to the effect of beam pattern broadening should exhibit very hard spectra. Indeed, the emission with broadened beam pattern is produced by high-energy particles which cool radiatively over a distance smaller than their mean free path. For such particles the deflection angle (and hence the width of the beam pattern) is a function of their energy. An observer situated at a large angle to the jet axis effectively sees the particle distribution devoid of its low-energy part, whose emission can only be seen at smaller viewing angles. Therefore, the off-axis emission is the hardest possible – it is essentially as hard as the spectrum of an individual particle.

The effect of beam-pattern broadening opens an interesting possibility for observation of the off-axis blazars and GRBs. The harder spectrum at large viewing angles can explain the phenomenon of unidentified gamma-ray sources and may even give rise to orphan GRB afterglows in the TeV range. In addition, a population-synthesis survey of off-axis sources will be able to reveal the details and relative importance of the electron cooling processes.
ACKNOWLEDGMENTS

E.V. Derishev acknowledges the support from the Russian Science Support Foundation. This work was also supported by the RFBR grant no. 02-02-16236, the President of the Russian Federation Program for Support of Leading Scientific Schools (grant no. NSh-1744.2003.2), and the program "Nonstationary Phenomena in Astronomy" of the Presidium of the Russian Academy of Science.

REFERENCES

1. Achterberg A., Gallant Y.A., Kirk J.G., Guthmann A.W., 2001, MNRAS 328, 393
2. E.V. Derishev, F.A. Aharonian, V.V. Kocharovsky, Vl.V. Kocharovsky, Phys. Rev. D 68, 043003 (2003).
3. K.R. Lind and R.D. Blandford, ApJ 295, 358 (1985).
4. P. Madau and C. Thompson, ApJ 534, 239 (2000).
5. P. Mészáros, Annu. Rev. Astron. Astrophys. 40, 137 (2002).
6. C.M. Urry and P. Padovani, PASP 107, 803 (1995).