Atomic Frequency standards Based on Pulsed Coherent Light Storage

Bo Yan, Yisheng Ma and Yuzhu Wang

Key Laboratory for Quantum Optics, Center for Cold Atom Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, 201800, China
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We propose a new scheme of microwave frequency standards based on pulsed coherent optical information storage. Unlike the usual frequency reference where the Ramsey fringe is printed on the population of a certain state, we print the Ramsey fringe on the coherence. Then the coherence is detected in the form of a retrieval light. The central line of the Ramsey fringe can be used as a frequency reference in an absorption-cell-based atomic frequency standard. This scheme is free of light shifts as the interrogating process is separated from the optical pumping processes, and the cavity pulling effect is negligible due to the low Q requirement. Encoding the Ramsey interference into the retrieval light pulse has the merit of high signal to noise ratio and the estimated frequency stability of shot noise limit is about $2 \times 10^{-14}$ in 1 second, this scheme is promising for building small, compact and stable atomic frequency standards.

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Atomic frequency standards are very stable and accurate frequency references widely used in high-speed communication and time keeping system, as well as in fundamental researches[1]. However the applications demand local oscillators with high frequency stability in both the short and medium terms. As such, the passive rubidium frequency standard has received most attention, and that has been reduced to the smallest size while keeping a relatively good frequency stability[2]. But the light shift is still the major effect limiting the frequency stability to meet the goal. In order to improve the frequency stability, many new physical concepts and technologies have been explored, such as electro-magnetic induced transparency (EIT) frequency standards[3], coherent population trapping (CPT) masers[4], pulsed optical pumped (POP) frequency standards[5], and so on. Meanwhile, exciting progresses have been made in the quantum optical information storage based on EIT[6, 7, 8]. By manipulating the strongly coupled excitations of light waves and spin waves slowly propagating together, one can store quantum information into the media and retrieve the signal light by switching on the coupling laser after a short time[9, 10, 11].

In this Letter, combining the above two domains, we have proposed a new scheme of microwave frequency standards based on coherent optical information storage. It provides a new way to improve the frequency stability of atomic frequency standards. The basic idea of the pulsed coherent storage (PCS) frequency standards is illustrated in Fig. 1, and the basic principle can be understood as follows: first, a signal laser pulse and a coupling laser pulse are applied, then shutting down the coupling laser which control the group velocity of coupled excitations (Dark-state Polaritons)[12], and the signal light pulse is coherently converted into a spin wave, and stored into the media. In this case there is no photon existed, then manipulate atom states by two microwave pulses which prints the Ramsey pattern on it. Finally, switching on the coupling laser, the stored signal light pulse is retrieved, and then detected to obtain the Ramsey pattern. The central Ramsey fringe can be used as a frequency reference of cell-atomic frequency standard.

The setup of our proposal shown in Fig. 1 is nearly the same as most groups working on coherent information storage, such as Lukin’s group[13]. The difference is that the cell, which contained Rb atoms and buffer gas, is placed in a cavity. The levels are coupled by the microwave. Ω and Ω are the relaxation rates which include the effect caused by buffer gas in the vapor cell. The typical values are $\gamma_1 = \gamma_2 = 0.5 \times 10^9 s^{-1}$ and $\gamma_3 = \gamma_4 = 300s^{-1}$. And the decay rate of the upper level is $\Gamma = \gamma_1 + \gamma_2 = 1 \times 10^9 s^{-1}$. The assumption of no decay from $|2\rangle$ to $|4\rangle$ is due to theoretical self-consistent, and because the decay from $|2\rangle$ to $|4\rangle$ will not effect the physical result of our theory.

In order to get an analytic formula, we calculate the process based on a four-level model: as shown in the right of Fig. 2, the level $|2\rangle$ is the upper level, which can decay to the $|1\rangle$ and $|3\rangle$ levels, but not decay to the level $|4\rangle$. Two laser beams couple the $|2\rangle$ and $|1\rangle$, $|2\rangle$ and $|3\rangle$ levels respectively. They are called the signal laser and the coupling laser. The levels $|3\rangle$ and $|4\rangle$ are coupled by the microwave. $\Omega$, $\Omega$, and $\Omega$ are Rabi frequencies of the coupling laser, the signal laser, and the microwave excitations. $\gamma_1, \gamma_2, \gamma_3$ and $\gamma_4$ are the relaxation rates which include the effect caused by buffer gas in the vapor cell. The typical values are $\gamma_1 = \gamma_2 = 0.5 \times 10^9 s^{-1}$ and $\gamma_3 = \gamma_4 = 300s^{-1}$. And the decay rate of the upper level is $\Gamma = \gamma_1 + \gamma_2 = 1 \times 10^9 s^{-1}$. The assumption of no decay from $|2\rangle$ to $|4\rangle$ is due to theoretical self-consistent, and because the decay from $|2\rangle$ to $|4\rangle$ will not effect the physical result of our theory.

We use the density matrix formalism to deal with this problem, the motion equation is

\[ \dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \ell \rho. \]  

*Electronic address: yzwang@mail.shcnc.ac.cn
Within the rotating wave approximation, the Hamiltonian is[7]

\[
H = -\hbar \begin{pmatrix}
\Delta_c - \Delta_s & \Omega_s & 0 & 0 \\
\Omega_s & \Delta_c & \Omega_c & 0 \\
0 & \Omega_c & 0 & \Omega_m \\
0 & 0 & \Omega_m & -\Delta
\end{pmatrix},
\]  

where \(\Delta_c = \omega_c - \omega_{c3}, \Delta_s = \omega_s - \omega_{21}\) and \(\Delta = \omega_m - \omega_{34}\) are the detunings of the coupling laser, the signal laser and the microwave, respectively. \(\ell\) is the decay matrix which is determined by all the decay rates shown in Fig. 2.

The timing sequences are shown in the Fig. 1. During the first pulse \(t_p\), only the microwave and the coupling laser are on, \(\Omega_c = 0\) and \(\Omega_m \neq 0\). This "state preparation" stage aims to pump all the atoms to the \([1]\) level, and destroy all kinds of coherences which may cause light shifts. There are two pumping rates in this process, the rate of pumping atoms from the \([3]\) level to the \([1]\) level which is characterized by \(\Gamma_p = \Omega_c^2/(2\gamma)\), and the rate of transferring the \([4]\) level to the \([3]\) level which is characterized by the Rabi frequency of microwave \(\Omega_m\). If \(\Omega_m >> \Gamma_p\), the atoms transferred from \([4]\) to \([3]\) will not be pumped to \([1]\) timely. It experiences a damped Rabi oscillation of coherences and populations. The decay rate of the whole system is determined by \(\Gamma_p\). And, if \(\Gamma_p >> \Omega_m\), once upon the atoms in the \([3]\) level are pumped to the \([1]\) level at once, the total decay rate is determined by \(\Omega_m\). So the whole decay rate is

\[
\gamma_d \sim \min(\Omega_m^2/(2\Gamma), \Omega_m).
\]  

In Fig. 3, We plot the dynamics of populations and coherences when \(\Gamma_p = 50000s^{-1}\) and \(\Omega_m = 10^2s^{-1}\). The whole decay rate of coherence is about 20\(\mu s\) as predicted by formula (3). As an example, we choose \(t_p = 500\mu s\), all coherences will be destroyed, and the atoms are pumped to a fixed state,

\[
\rho_{11} = 1, \quad \rho_{31} = \rho_{41} = 0.
\]

After this "state preparation" stage, we switch on the coupling laser and the signal laser. In this way, an EIT state is built. There will be coherence between \([1]\) and \([3]\). Under the assumption \(\Omega_s << \Omega_c, \gamma\), to the first order of \(\Omega_s\), we get a stationary solution

\[
\rho_{31}^0 = \frac{\Omega_s \Omega_c}{(\gamma + i\Delta_s)[\gamma + i(\Delta_s - \Delta_c)] + \Omega_c^2},
\]

where \(\gamma = \gamma_1 + \gamma_2\) is the decay rate of the upper level. In our case, the two-photon resonance condition is fulfilled, and \(\Delta_s << \gamma\). When \(\Omega_m^2 >> \gamma_3\), we have

\[
\rho_{31}^0 = \frac{\Omega_s}{\Omega_c}, \quad \rho_{41}^0 = 0.
\]

It depends only on the Rabi frequency ratio of the signal and the coupling laser.

After the EIT state has been built up, we turn off the coupling laser, and the signal light pulse is coherently converted into a spin wave, and stored into the media. We apply two microwave pulses to manipulate the coherences. After that, the coherence between the \([1]\) level and the \([3]\) level becomes

\[
\rho_{31}(T + 2t) = e^{i\Phi(T+2t)-\gamma_3T}[e^{-i\Phi T}(\cos \xi t - i\Delta_{31}\sin \xi t)^2 - e^{i\Phi T}(\frac{\Omega_m}{\xi}\sin \xi t)^2] \rho_{31}^0,
\]

where \(\xi = \sqrt{(\Delta/2)^2 + \Omega_m^2}\). In this way, a Ramsey pattern is printed on the coherence.

In the final signal detection stage, we apply a strong coupling laser pulse to get back the signal light pulse again, i.e., the retrieval of the light pulse stored in the media. The coupling laser can be applied adiabatically or nonadiabatically in certain condition[12, 13], and the retrieval light can keep the spatial shape of \(\rho_{31}\). But different from quantum information storage, what we concern more is the integration of the light intensity over time rather than the shape of the retrieved light pulse, because it is related to the probability of the clock transition. The detected signal is

\[
P = P_0|\rho_{31}(T + 2t)|^2/|\rho_{31}^0|^2.
\]

\(P_0e^{-\gamma_3T}\) is the signal detected when microwave pulses are not applied. It depends to the atom number \(N\), the Rabi frequency of the coupling laser \(\Omega_c\), the way of switching on and off the coupling laser, and so on, can be optimized[15, 16]. But in the identical experiment conditions, \(P_0\) is a constant. In the vicinity of \(\Delta \approx 0\),

\[
P = P_0e^{-2\gamma_3[T - 2\cos^2 \xi t \sin^2 \xi t(1 + \cos \Delta T)]},
\]
exhibits a Ramsey pattern. We plot a numerical result in Fig. 4 when $\Omega_m t = \pi/4$ and $\gamma T = 400\pi$. The full-width at half maximum of the central Ramsey fringe is

$$\Delta\omega_{1/2} = \pi/T. \quad (10)$$

When the microwave pulse is a $\pi/4$ pulse, i.e. $\Omega_m t = \pi/4$, the signal has the maximum signal to noise ratio (SNR). Since the central signal of this Ramsey fringe is zero, it is a homodyne detection. When $\Omega_m t = \pi/2$, the Ramsey pattern vanishes as indicated by equation (9). It can be understood as follow: we detect the mode of $\rho_{41}$, if the pulses are $\pi/2$ pulses, there will be a Ramsey pattern printed on the phase of $\rho_{31}$ which can not be reflected in the intensity detection.

In the scheme of PCS frequency standard there is light shift, but it can be canceled completely. Usually, a pulsed method is used to eliminate the light Stark shift. But there may still be light shift due to ”phase memory”[5]. It is a common problem in many schemes. If we use the Ramsey pattern as a frequency standard, the detection should be done repeatedly and changed the detuning of microwave step by step. After one cycle, the coherence of $\rho_{41}$ will decay exponentially in “state preparation” stage. But if there is some residual coherence of $\rho_{41}$ after the EIT state has been built, it would shift the central frequency of the Ramsey fringe. Consider a residual $\rho_{41}$ exits, so the initiation condition is different from (6), $\rho_{41}^{\prime} \neq 0$, and the detected signal will be

$$P = P_0 \left| \cos^2 \xi t - \sin^2 \xi t e^{i\Delta T} - isin \xi t \cos \xi t \right| \left( 1 + \rho_{41}^{\prime 2} \rho_{31}^{\prime 2} \right). \quad (11)$$

It leads to a frequency shift $\Delta \nu$ of the central Ramsey fringe,

$$\tan \Delta \nu T = -\frac{1}{\sin 2\Omega_m t} \frac{2Re(\rho_{31}^{\prime 2} \rho_{41}^{\prime 2})}{|\rho_{31}^{\prime 2} - 2Im(\rho_{31}^{\prime 2} \rho_{31}^{\prime 2}) - |\rho_{31}^{\prime 2}|^2|}.$$ 

In the "state preparation" stage, the residual coherence of $\rho_{41}$ decays exponentially by $e^{-\gamma T}$ where $\gamma$ is determined by formula (2). Since $\gamma_m t_p >> 1$, the residual coherence is very small. Using the data for formula (3), the frequency shift can be evaluated by $\Delta \nu < 10^{-10}/T$, which can be considered completely negligible.

As in the usual frequency standards, there also exits the cavity pulling effect[5]. When the microwave frequency is changed, the amplitude will change. If there is a small difference between the cavity resonance frequency and the atomic transition frequency, the amplitude varies asymmetrically, which causes a frequency shift [16]. And during the free decay time $T$, the atoms in the cavity would induce a microwave field which effects the coherence. It causes a shift of the central Ramsey fringe. The shift can be evaluated of the order [17]

$$\Delta\omega \sim \frac{Q \Delta\omega_c}{T \omega_3}. \quad (12)$$

where $\Delta\omega_c$ is the cavity detuning from $\omega_3$. When a high $Q$ cavity is needed, such as CPT maser, a high cavity is needed to get a high SNR for microwave detection signal, $Q \sim 10^5$[18], the cavity pulling effect should be considered. In our case, we do not need a high $Q$ cavity. $Q$ can be only several hundreds, for example $Q = 300$, the cavity pulling effect is smaller for a factor of $10^3$ than POP scheme.

The most important advantage of our proposal is the optical detection. In the POP experiments, microwave signal is observed to get the Ramsey fringe. But optical detection is much more sensitive than microwave detection. The energy of an optical quantum wins a factor of $10^5$ than a microwave quantum. The SNR of optical detection will be better. So people also want to do optical detection with absorption signal in POP[19]. But because the background of absorption signal always exits because the decay of the population will add a background to signal, a high contrast signal can not be achieved. In our scheme, the decay of the coherence do not add a background, so we can get a high contrast signal.

The contrast $C$, which is an important parameter in evaluating frequency stability, is defined as the signal intensity divided by the background intensity. From equation (9), the contrast is

$$C = \sin^2 2\Omega_m t. \quad (14)$$

When $\Omega_m t = \pi/4$, it has the maximum contrast $C = 1$, when $\Omega_m t = \pi/2, C = 0$, no Ramsey pattern appears as we have already mentioned. The contrast in our scheme as show in formula (14) does not depend on the interrogation time. The ultimate limit of SNR is the shot noise

$$\frac{S}{N} = \sqrt{N_p C}, \quad (15)$$
where $N_p$ is the photon number we detect. The frequency stability of shot noise limit is

$$
\sigma(\tau) = \frac{1}{\pi Q_a S/N} \sqrt{\frac{T_{\text{total}}}{\tau}},
$$

(16)

where $Q_a = \nu / \nu_{1/2}$ is the quality factor of line shape. For a long interrogation time $T$, the quality factor of line shape $Q_a$ become better linear with $T$, but the SNR become worse with $e^{-T/(2\tau_d)}$, where $\tau_d = 1 / \gamma_3$ is the coherence decay time. A trade-off for high stability is achieved by choosing $T = \tau_d \sim 2\tau_d$. For $N_p = 10^9, \nu = 6.8G, \Delta \nu_{1/2} = 200Hz, \Omega_{int} = \pi/4, C = 1, Q_a = 3.4 \times 10^7$, cycle time $T_{\text{total}} = 5ms$, we evaluate the frequency stability of shot noise limit,

$$
\sigma(\tau) = 2.0 \times 10^{-14} \tau^{-1/2}.
$$

(17)

It is a very high performance for compact atomic clocks, because the stability limit is comparable with the typical performance of H-masers.\(^{20}\)

Usually, the laser noise will effect the coherence stored in the media, the retrieval light intensity, consequently the frequency stability. Fortunately, in our scheme, the laser noise will not limit the stability. The coherence, from formula \(^{10}\), is determined by the Rabi frequency ratio, and is not effected by the laser amplitude fluctuations because $\Omega_s$ and $\Omega_c$ fluctuate with the same rate as come from one laser beam, and the ratio is constant. In the detection stage, The retrieval window in Rb is several hundreds MHz, so the frequency noise is not a problem. And there is a saturation effect when the coupling laser is week, the integration of retrieval light intensity increases as the power of the coupling laser increases. But when the power of the coupling laser increases to a certain value, the integration of retrieval light becomes a constant. So a strong coupling laser will ensure the amplitude noise not a problem. The frequency stability of our scheme is much immune to laser noise.

In conclusion, we have proposed a pulsed frequency standard based on coherent storage and retrieval. The difference between usual frequency references and our scheme is that: Usually, one prepare atoms in a certain state, and use double microwave pulses to manipulate the population of this state, which print the Ramsey fringes on the population, and finally detect the residual population. In our scheme, we prepare atoms in a state with coherence, and use double microwave pulses to manipulate the coherence, which also print the Ramsey fringes on the coherence. Finally we detect the coherence in the form of the retrieval light and also get the Ramsey fringes. This PCS scheme has several advantages: a) light shift due to phase memory is eliminated; b) it only needs a low Q cavity, the cavity pulling effect can be neglected; c) the detection is a homodyne light detection, making the signal contrast quite high; d) the stability is quite immune to the laser noise. This method can also be applied to other elements and cold atoms. It is very promising to build compact atomic clocks with high frequency stability.

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Coupling laser pulses
Signal laser pulse
Microwave pulses
Detection window
Rb cell placed in a shield microwave cavity
Optical detection

FIG. 1: The setup of our proposal and the timing sequences of our scheme. PBS: polarization beam splitter. AOM: acousto-optic modulator. The Rb cell is placed in a shield microwave cavity. AOM are used to adjust the two-photon detuning caused by the C-field. The total cycle time is $T_{\text{total}}$, EIT preparation time is $t_b$, microwave pulse time $t$, and interrogation time $T$.

$$m_f = -2 \quad -1 \quad 0 \quad 1 \quad 2$$

$5P_{1/2}$

$5S_{1/2}$

$\sigma_+$

$\sigma_-$

$\Omega_\text{m}$

$F=2$

$F=1$

$\gamma_3$ and $\gamma_4$ are the decay rates of hyperfine coherence and population difference, respectively. $\gamma_1$ and $\gamma_2$ are the decay rates from the level $|2\rangle$ to the level $|1\rangle$ and the level $|3\rangle$, respectively. We assume there is no decay from the level $|2\rangle$ to the level $|4\rangle$.

FIG. 2: The energy levels used in PCS. In the left, it is the levels used in Rb; In the right, it is the four-level model: $\gamma_3$ and $\gamma_4$ are the decay rates of hyperfine coherence and population difference, respectively. $\gamma_1$ and $\gamma_2$ are the decay rates from the level $|2\rangle$ to the level $|1\rangle$ and the level $|3\rangle$, respectively. We assume there is no decay from the level $|2\rangle$ to the level $|4\rangle$.

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FIG. 3: In the "state preparation" stage, all atoms will be pumped to state $|1\rangle$, and destroy all kinds of coherence after a few hundreds microsecond. We plot the decay of populations and coherences when $\Gamma = 1 \times 10^9 s^{-1}$, $\Gamma_p = 50000 s^{-1}$ and $\Omega_m = 10^5 s^{-1}$.

FIG. 4: The Ramsey pattern of the detected retrieval light when $\Omega_m t = \pi/4$ and $T = 400\pi/\Gamma$. The minimum signal is zero.