Chargino contributions to the CP asymmetry in $B \to \phi K_S$ decay

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Abstract

We perform a model independent analysis of the chargino contributions to the CP asymmetry in $B \to \phi K_S$ process. We use the mass insertion approximation method generalized by including the possibility of a light right-stop. We find that the dominant effect is given by the contributions of the mass insertions $(\delta_{uL})_{32}$ and $(\delta_{uR})_{32}$ to the Wilson coefficient of the chromomagnetic operator. By considering both these contributions simultaneously, the CP asymmetry in $B \to \phi K_S$ process is significantly reduced and negative values, which are within the $1\sigma$ experimental range and satisfy the $b \to s\gamma$ constraints, can be obtained.

The measurement of CP asymmetries in nonleptonic B decays plays a crucial role in testing the CP violation mechanism of the Standard Model (SM) and it is a powerful probe of New Physics (NP) beyond the SM. The CP asymmetries are usually described by the time dependent rates $a_{f_{CP}}(t)$, for $B^0$ and $B^0\bar{B}$ to a CP eigenstate $f_{CP}$

$$a_{f_{CP}}(t) = \frac{\Gamma(B^0(t) \to f_{CP}) - \Gamma(B^0(t) \to f_{CP})}{\Gamma(B^0(t) \to f_{CP}) + \Gamma(B^0(t) \to f_{CP})} = C_{f_{CP}} \cos \Delta M_{B^0_d} t + S_{f_{CP}} \sin \Delta M_{B^0_d} t$$

where $C_{f_{CP}}$ and $S_{f_{CP}}$ represent the coefficients of direct and indirect CP violations respectively, and $\Delta M_{B^0_d}$ is the $B^0$ eigenstate mass difference.

The time dependent CP asymmetry $a_{J/\psi K_S}(t)$ in the B meson decay $B \to J/\psi K_S$ has been recently measured by BaBar and Belle Collaboration, with an average of $S_{J/\psi K_S} =$
\( \sin 2\beta = 0.734 \pm 0.034 \) \cite{1,2}, showing the first evidence of CP violation in B meson system in perfect agreement with the Standard Model (SM) predictions. This is expected, since the SM contribution is at tree-level.

For the decay \( B \to \phi K_S \), where the same weak phase is measured, the situation is qualitatively different. The SM contribution is at one-loop level, and one can expect crucial contributions from New Physics. The branching ratio for \( B \to \phi K_S \) has recently been measured by both BaBar and Belle \cite{3} with an average for the branching ratio of \( BR(B \to \phi K_S) = \left( 8.4^{+2.5}_{-2.1} \right) \times 10^{-6} \) which is slightly different from the SM predictions. However, this is not a signal of a real problem, since the SM evaluation of \( BR(B \to \phi K_S) \) is largely affected by theoretical uncertainties in the evaluation of hadronic matrix elements. On the other hand, the time dependent CP asymmetry in Eq.(1) is less sensitive to these uncertainties, since the hadronic matrix elements almost cancel out in the ratio of rates.

Recently BaBar and Belle Collaborations \cite{2,4} have also measured the time dependent CP asymmetry in \( B \to \phi K_S \) process, reporting an average value of \( S_{\phi K_S} = -0.39 \pm 0.41 \). In the SM, \( S_{\phi K_S} \) is expected to give the same value of \( \sin 2\beta \) as extracted from \( S_{J/\psi K_S} \), up to terms of order \( O(\lambda^2) \) where \( \lambda \) is the Cabibbo mixing. Thus, the comparison of the experimental results for \( S_{J/\psi K_S} \) and \( S_{\phi K_S} \) reveals a 2.7 \( \sigma \) deviation from the SM prediction. If this discrepancy will be confirmed with a better accuracy, it will be a clean signal of NP.

Due to the additional sources of flavor and CP violation beyond the ones of Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix, supersymmetric (SUSY) models are natural candidates for explaining the difference between the CP asymmetries \( S_{\phi K_S} \) and \( S_{J/\psi K_S} \). Recently, the gluino contributions to \( S_{\phi K_S} \) have been analyzed in Refs.\cite{5,6}. In these works, it has been shown that gluino exchanges can explain the experimental results of \( S_{\phi K_S} \) without conflicting the experimental constraints from \( S_{J/\psi K_S} \) and the branching ratio \( BR(b \to s\gamma) \).

The main purpose of this letter is to show that also the chargino contributions to \( S_{\phi K_S} \) can be significant and account for these recent measurements. We perform a model independent analysis by using the well known method of mass insertion approximation \cite{7}, generalized by including the possibility of a light right-stop in the otherwise almost degenerate squark spectrum. In our analysis, we take into account all the operators that contribute to the effective Hamiltonian for \( \Delta B = 1 \) transitions \( H_{\text{eff}}^{\Delta B=1} \) and provide analytical results for the corresponding leading Wilson coefficients.

Now we start our analysis of the SUSY contributions to the time dependent CP asymmetry in \( B \to \phi K_S \) decay. In the following we will adopt the parameterization of the SM and SUSY amplitudes as in Ref.\cite{5}, namely

\[
\left( \frac{A_{\phi K_S}^{SUSY}}{A_{\phi K_S}^{SM}} \right) = R_{\phi} e^{i\phi} e^{i\delta_{12}}
\]  

(2)
where $\theta_\phi$ is the SUSY CP violating phase, and $\delta_{12} = \delta_{SM} - \delta_{SUSY}$ is the strong (CP conserving) phase. In this case, the mixing CP asymmetry $S_{\phi K_S}$ takes the following form

$$S_{\phi K_S} = \frac{\sin 2\beta + 2R_\phi \cos \delta_{12} \sin(\theta_\phi + 2\beta) + R_\phi^2 \sin(2\theta_\phi + 2\beta)}{1 + 2R_\phi \cos \delta_{12} \cos \theta_\phi + R_\phi^2}.$$  \hfill (3)

The most general amplitude for $B \to \phi K_S$ process can be written as

$$\overline{A}(\phi K) = -\frac{G_F}{\sqrt{2}} \sum_{i=1}^{12} \left[ C_i(\mu) + \tilde{C}_i(\mu) \right] \langle \phi \bar{K}^0 | Q_i(\mu) | \bar{B}^0 \rangle,$$  \hfill (4)

where $Q_i$ are the operators which contribute to the effective Hamiltonian for $\Delta B = 1$ transitions and $C_i(\mu)$ are the corresponding Wilson coefficients at energy scale $\mu$. The matrix elements $\langle \phi \bar{K}^0 | Q_i | \bar{B}^0 \rangle$ are calculated in the naive factorization approximation [8], and their expressions can be found in Ref.[5]. In this notation, the $Q_{i=1-10}$ represent the four-fermion operators, and $Q_{11}$ and $Q_{12}$ the magnetic and chromomagnetic dipole operators respectively. The Wilson coefficients $\tilde{C}_i$ are associated to the operators $\tilde{Q}_i$ which are obtained from $Q_i$ by exchanging $\gamma_5 \to -\gamma_5$ in their chiral structure, see Ref.[5] for their definition. In the SM, $\tilde{C}_i$ are chirally suppressed with respect to $C_i$ ones by terms proportional to light quark masses. However, in non-minimal SUSY extensions of the SM they can receive sizeable contributions, for instance from the gluino mediated penguin and box diagrams. On the other hand, the chargino contributions to $\tilde{C}_i$ are always suppressed by Yukawas of the first two generations. Thus, we can safely neglect $\tilde{C}_i$ contributions in our analysis.

The Wilson coefficients $C_i(\mu)$ at a lower scale $\mu \simeq \mathcal{O}(m_b)$ can be extrapolated by the corresponding ones at high scale $C_i(\mu_W)$ as $C_i(\mu) = \sum_j \hat{U}_{ij}(\mu, \mu_W) C_j(\mu_W)$, where $\hat{U}_{ij}(\mu, \mu_W)$ is the QCD evolution matrix and $\mu_W \simeq m_W$. Since the operator $Q_{12}$ is of order $\alpha_s$, we include in our analysis the LO corrections only for the effective Wilson coefficient $C_{12}(\mu)$, while for the remaining ones $C_{i=1-10}(\mu)$ we use the matrix $\hat{U}_{ij}(\mu, \mu_W)$ at NLO order in QCD and QED [9].

As is well known, supersymmetry affects the Wilson coefficients $C_i(\mu)$ only at high scale $\mu \simeq \mu_W$. The chargino contributions to $C_i(\mu_W)$, corresponding to the effective Hamiltonian for $\Delta B = 1$ transitions, have been calculated exactly (at 1-loop) in Refs.[10] and [11]. Here we provide the results for chargino contributions to $C_i(\mu_W)$, evaluated at the first order in mass insertion approximation. By using the notation of Ref.[11] we obtain

$$F_\chi = \left[ \sum_a K_{a2}^* K_{b3}(\delta^{u}_{LL})_{ba} \right] R^L_F + \left[ \sum_a K_{a2}^* K_{33}(\delta^{u}_{RL})_{3a} \right] Y_t R^R_F + \left[ \sum_a K_{32}^* K_{a3}(\delta^{u}_{LR})_{a3} \right] Y_t R^{LR}_F + \left[ K_{32}^* K_{33}(\delta^{u}_{RR})_{33} \right] Y_t^2 R^{RR}_F,$$

\hfill (5)

where for the definition of mass insertions $\langle \delta^{u}_{AB} \rangle_{ij}$ see Ref.[7]. Same notation of Ref.[11] has been used to relate $F$ quantities to the Wilson coefficients $C_{i=1-10}(\mu_W)$, while for the
magnetic and chromomagnetic contributions we have \( C_{11}(\mu_W) = M^\gamma \) and \( C_{12}(\mu_W) = M^g \). Here \( Y_t \) is Yukawa coupling of the top quark and \( F \) refers to the photon-penguins (\( D \)), Z-penguins (\( C \)), gluon-penguins (\( E \)), boxes with external down quarks (\( B^{(d)} \)) and up quarks (\( B^{(u)} \)), magnetic-penguins (\( M^\gamma \)), and chromomagnetic (\( M^g \)) penguin diagrams. We want to stress that there are also contributions from box diagrams mediated by both gluino and chargino exchanges, which affect only \( C_{i=1,2}(\mu_W) \), but their effect is negligible [11], [12] and we will not include them in our analysis.

The detailed expressions for \( R_F \), including contributions from chargino-gluino box diagrams, are given in the appendix. Here we will just concentrate on the dominant contributions which turn out to be due to the chromomagnetic (\( M^g \)) penguin and Z-penguin (\( C \)) diagrams. In fact, for light SUSY particles (\( \lesssim 1 \) TeV), the contribution from the chromomagnetic penguin is one order and two orders of magnitudes larger than the corresponding ones from Z-penguin and other diagrams, respectively. However, in our numerical analysis we take into account all the contributions.

From Eq. (5), it is clear that \( LR \) and \( RR \) contributions are suppressed by order \( \lambda^2 \) or \( \lambda^3 \). Since we will work in \( \mathcal{O}(\lambda) \) order, we can neglect them and simplify \( F_\chi \) as,

\[
F_\chi = \left[ (\delta^{u}_{LL})_{32} + \lambda(\delta^{u}_{LL})_{31} \right] R^{LL}_F + \left[ (\delta^{u}_{RL})_{32} + \lambda(\delta^{u}_{RL})_{31} \right] Y_t R^{RL}_F. \tag{6}
\]

The functions \( R^{LL}_F \) and \( R^{RL}_F \) depend on the SUSY parameters through the chargino masses (\( m_{\chi_i} \)), squark masses (\( \tilde{m} \)) and the entries of the chargino mass matrix. For instance for \( Z \) and magnetic (chromomagnetic) dipole penguins \( R^{LL,RL}_C \) and \( R^{LL,RL}_{M^{n,g}} \) respectively, we have

\[
R^{LL}_C = \sum_{i=1,2} |V_{i1}|^2 P_{C}^{(0)}(\bar{x}_i) + \sum_{i,j=1,2} \left[ U_{i1}V_{j1}V_{i1}^* V_{j1}^* P_{C}^{(2)}(x_i,x_j) \right] + |V_{i1}|^2 |V_{j1}|^2 \left( \frac{1}{8} - P_{C}^{(1)}(x_i,x_j) \right)
\]

\[
R^{RL}_C = \sum_{i=1,2} |V_{i1}|^2 |V_{j1}|^2 \left[ -\frac{1}{2} \sum_i V_{i1}^2 V_{i1} P_{C}^{(0)}(\bar{x}_i) - \sum_{i,j=1,2} V_{i1}^* V_{j1} (U_{i1}U_{j1}^*) P_{C}^{(2)}(x_i,x_j) \right] + V_{i1}^* V_{j1} P_{C}^{(1)}(x_i,x_j)
\]

\[
R^{LL}_{M^{n,g}} = \sum_i |V_{i1}|^2 x_{wi} P_{M^{n,g}}^{LL}(x_i) - Y_b \sum_i V_{i1} U_{i2} x_{wi} m_{\chi_i} m_b P_{M^{n,g}}^{LR}(x_i)
\]

\[
R^{RL}_{M^{n,g}} = -\sum_i V_{i1} V_{i2} x_{wi} P_{M^{n,g}}^{LL}(x_i), \tag{7}
\]

where \( Y_b \) is the Yukawa coupling of bottom quark, \( x_{wi} = m_{W_i}^2 / m_{\chi_i}^2 \), \( x_i = m_{\chi_i}^2 / \tilde{m}^2 \), \( \bar{x}_i = \tilde{m}_i^2 / m_{\chi_i}^2 \), and \( x_{ij} = m_{\chi_i}^2 / m_{\chi_j}^2 \). The loop functions \( P_{C}^{(1,2)} \), \( P_{M^{n,g}}^{LL,LR} \) are given by

\[
P_{C}^{(1,2)}(x,y) = -2 \left( x \frac{d}{dx} + y \frac{d}{dy} \right) C^{(1,2)}_\chi(x,y),
\]

\[
P_{M^{n,g}}^{LL,LR}(x) = -x \frac{d}{dx} \left( x F_{1,3}^{(3)}(x) + \frac{2}{3} x F_{2}(x) \right), \quad P_{M^{n,g}}^{LL,LR} = -x \frac{d}{dx} \left( x F_{2}(x) \right). \tag{8}
\]
where \( P_C^{(0)}(x) = - \lim_{y \to x} P_C^{(1)}(x, y) \), and the functions \( C^{(1,2)}_\chi(x, y) \) and \( F_i(x) \) can be found in Refs.[10] and [11], respectively. Finally, \( U \) and \( V \) are the matrices that diagonalize chargino mass matrix, defined as \( U^* M_{\tilde{\chi}} V^{-1} = \text{diag}(m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+}) \) where we adopted the notation of Ref.[11] for the chargino matrix \( M_{\tilde{\chi}} \).

Notice that the dependence from Yukawa bottom \( Y_b \) in Eq.(5) leads to enhancing \( C_{12} \) at large \( \tan \beta \). Here, we also considered the case in which the mass of stop-right \((m_{\tilde{t}_R})\) is lighter than other squarks. In this case the functional form of Eq.(5) remains unchanged, while only the expressions of \( R_{F_i}^{RL} \) should be modified by replacing the functions inside \( P^{LL,RL}_{M^{LL,RL}} \) as \(-x_i \frac{d}{dx_i} x_i F_a(x_i) \to \frac{1}{(x_i-1)} [x_iF_a(x_i) - x_iF_a(x_i)]\), with index \( a = 1 - 4 \), \( P_C^{(1,2)}(x_i, x_j) \to \frac{2}{(x_i-1)} [C^{(1,2)}_\chi(x_j, x_i) - C^{(1,2)}_\chi(x_j, x_i)]\), and \( P_C^{(0)}(\bar{x}_i, \bar{x}_it) = \frac{4}{(x_i-1)} [C^{(1)}_\chi(\bar{x}_it, \bar{x}_i) - C^{(1)}_\chi(\bar{x}_i, \bar{x}_i)]\), where \( x_i = \frac{m_i}{\tilde{m}_{\tilde{t}_R}} \) and \( x_i = \frac{m_i}{\tilde{m}_{\tilde{t}_R}} \).

In order to simplify our analysis, we consider first the case where a mass insertion per time, switching off all the others. Thus, there is only one SUSY CP phase which factorizes in the SUSY amplitude, and so \( \theta_\phi \) in Eq. (2) can be identified with the corresponding \( \text{arg}[(\delta_{1B})_{ij}] \).

We present our numerical results in Figs. 1-3, where the CP asymmetry \( S_{\Phi K_S} \) is plotted versus the SUSY CP violating phase. In this analysis we worked at fixed values of \( \tan \beta \) and scanned over all the relevant SUSY parameters - \( \tilde{m} \), the weak gaugino mass \( \tilde{M}_2 \), the \( \mu \) term, and \( m_{\tilde{t}_R} \) - and required that they satisfy the present experimental lower mass bounds, namely the lightest chargino \( m_\chi > 90 \text{ GeV} \), heavy squarks \( \tilde{m} > 300 \text{ GeV} \), and light right-stop \( m_{\tilde{t}_R} > 150 \text{ GeV} \). In addition, we scanned over the real and imaginary part of the corresponding mass insertions, by requiring that the \( b \to s\gamma \) and \( B - \bar{B} \) mixing constraints are satisfied. In our calculation we have used the formula of the branching ratio (BR) \( b \to s\gamma \) at the NLO in QCD, as provided in Ref.[13]. Indeed, the BR of \( b \to s\gamma \) can be easily parametrized in terms of the SUSY contributions to Wilson coefficients \( C_{11} \) and \( C_{12} \) at \( \mu_{\text{UV}} \) scale given in Eq. (5). For this parametrization, we used the central values of the SM parameters as provided in Ref.[13], and the low energy renormalization scale fixed at \( \mu = m_b \).

In Figs.1 and 2 we show the effects of one mass insertion per time, \( (\delta_{LL}^u)_{32} \) and \( (\delta_{RL}^u)_{32} \), evaluated at \( \tan \beta = 40 \). In all these plots, the red points are allowed by all experimental constraints, while light-blue points correspond to the points disallowed by \( BR(b \to s\gamma) \) constraints at 95% C.L. , namely \( 2.0 \times 10^{-4} < BR(b \to s\gamma) < 4.5 \times 10^{-4} \). In order to get the maximum effect for the negative values of CP asymmetry, we fixed the strong CP conserving phase \( \delta_{12} \) to be zero. We have not shown the contributions of the other mass insertions since they are sub-leading, being suppressed by terms of order \( \lambda \).

As we can see from the results in Figs.1-2, there is no chance with only one mass insertion to achieve negative values for the CP asymmetry. The main reason for \( (\delta_{LL}^u)_{32} \) is due to the \( b \to s\gamma \) constraints which are particularly sensitive to \( \tan \beta \), while this is
not the case for \((\delta^u_{RL})_{32}\). Clearly, we have considered also different values of \(\tan \beta\), and we found that the allowed regions in the scatter plots are not very sensitive to \(\tan \beta\).

In Fig. 3 we show another example, where we take simultaneously both the mass insertions \((\delta^u_{LL})_{32}\) and \((\delta^u_{RL})_{32}\) per time, but assuming that their CP violating phase is the same. As can be seen from Fig. 3 there are points, allowed by \(b \to s\gamma\) constraints, which can fit inside the 1σ experimental region.

In order to understand the behavior of these results, it is very useful to look at the numerical parametrization of the ratios of amplitudes in terms of the relevant mass insertions. Indeed, we would like to show that the main contribution to the SUSY amplitude is provided by the chromomagnetic dipole operator. For example, with \(M_2 = 200 \text{ GeV}\), \(\mu = 300 \text{ GeV}\), \(m_{\tilde{q}} = 400 \text{ GeV}\), \(m_{\tilde{t}_R} = 150 \text{ GeV}\), and \(\tan \beta = 30\), we find \(R^{RL}_C \simeq -0.033\), \(R^{LL}_{M_0} \simeq -0.068\), while for all the other ones \(R^{AB}_F \simeq O(10^{-3})\), and the amplitudes ratio \(R_A = \frac{A_{\chi^0}^{A_{\chi^0}}}{A_{\chi^0}^{A_{\chi^0}}\chi^0}\) is given by

\[
R_A \simeq 0.37(\delta^u_{LL})_{31} + 1.64(\delta^u_{LL})_{32} - 0.05(\delta^u_{RL})_{31} - 0.21(\delta^u_{RL})_{32}. \tag{9}
\]

Now, if we switch off the chromomagnetic dipole operator, the coefficients of the mass insertions \(\delta^u_{LL}\) are significantly reduced, while the coefficients of \(\delta^u_{RL}\) are slightly changed and \(R_A\) takes the form

\[
R_A \simeq -0.0031(\delta^u_{LL})_{31} - 0.014(\delta^u_{LL})_{32} - 0.045(\delta^u_{RL})_{31} - 0.20(\delta^u_{RL})_{32}. \tag{10}
\]

It is worth mentioning that the chromomagnetic contributions are sensitive to the value of \(\tan \beta\). Indeed, the contribution coming from \(R^{LL}_{M_0}\) in Eq.(7) is enhanced by \(\tan \beta\), due to the term proportional to \(Y_b\). For instance, for \(\tan \beta \sim 10\), the value of \(R^{LL}_{M_0}\) is reduced to \(R^{LL}_{M_0} \simeq -0.023\), while \(R^{RL}_C\) is slightly increased to \(R^{RL}_C \simeq -0.033\) and the amplitudes ratio becomes

\[
R_A \simeq 0.12(\delta^u_{LL})_{31} + 0.54(\delta^u_{LL})_{32} - 0.05(\delta^u_{RL})_{31} - 0.21(\delta^u_{RL})_{32}. \tag{11}
\]

Furthermore it is remarkable to notice that, with heavy SUSY particles \((M_{\tilde{q}} \sim 1 \text{ TeV})\), the \(Z\)-penguin diagram would provide the dominant contributions to \(F_\chi\), since \(R^{RL}_C\) tends to a constant value of order \(-0.05\). This effect clearly shows the phenomena of non-decoupling of the chargino contribution to the \(Z\) penguin, as discussed for instance in Ref.[14].

Finally, we stress that the contribution of \((\delta^u_{LL})_{32}\) to the chromomagnetic dipole operator, which leads to the dominant contribution to \(S_{\phi K^0}\), is strongly constrained by \(b \to s\gamma\) (which is particularly sensitive to \(C_{11}(\mu_W)\)). This is due to the fact that \((\delta^u_{LL})_{32}\) gives almost the same contribution to both \(C_{11}(\mu_W)\) and \(C_{12}(\mu_W)\), as can be seen from Eq.(7). Notice that this is not the case for gluino exchanges, since there the contributions to the chromomagnetic dipole operator are enhanced by color factors with respect to the magnetic dipole ones, allowing large contributions to \(C_{12}\) while respecting the \(b \to s\gamma\) constraints [15]. Regarding the effects of \((\delta^u_{RL})_{31}\) and \((\delta^u_{LL})_{31}\), their contributions to \(S_{\phi K^0}\) is quite small since they are mostly constrained by \(\Delta M_B\) and \(\sin 2\beta\) [16].
For the above set of input parameters, the $b \rightarrow s \gamma$ limits impose $|\langle \delta^u_{LL} \rangle_{32}| < 0.58$. Thus, the maximum individual mass insertion contributions are given by $\left| \frac{A_{SUSY}}{A_{SM}} \right| < 0.31$ and $\left| \frac{A_{SUSY}}{A_{SM}} \right| < 0.21$. This shows that after imposing the $b \rightarrow s \gamma$ constraints, the contribution from $\langle \delta^u_{LL} \rangle_{32}$ is of the same order as the contribution from $\langle \delta^u_{RL} \rangle_{32}$.

Since the ratio $R_\phi \equiv |A_{SUSY}/A_{SM}| < 1$, one can expand the expression of $S_{\phi K_S}$ in Eq. (3) in terms of $R_\phi$ and gets the following simplified formula

$$S_{\phi K_S} = \sin 2 \beta + 2 \cos 2 \beta \sin \theta_\phi R_\phi,$$

(12)

which shows that with $R_\phi \sim 0.4$ and even if $\sin \theta_\phi \sim -1$, one can reduce $S_{\phi K_S}$ from the SM prediction $sin 2 \beta$ to 0.2 at most and it is not possible with one mass insertion contribution to reach negative CP asymmetry. However, by considering the contributions from both $\langle \delta^u_{LL} \rangle_{32}$ and $\langle \delta^u_{RL} \rangle_{32}$ simultaneously, $R_\phi$ can become large and values of order $S_{\phi K_S} \simeq -0.2$ can be achieved.

It is worth mentioning that we have also considered the BR of $B^0 \rightarrow \phi K^0$ decay and ensured that the SUSY effects do not violate the experimental limits observed by BaBar and Belle [3].

Finally, let us emphasize that generally in supersymmetric models the lighter chargino is expected to be one of the lightest sparticles (for instance, in Anomaly Mediated SUSY breaking models it is almost degenerate with the lightest one). Thus, it can be expected to contribute significantly in the one-loop processes. Although the gluino contribution to the studied asymmetry can be very large, on the other hand gluino in many models is one of the heaviest SUSY partners and thus its contribution may be reduced essentially.

To conclude, we have studied the chargino contributions to the CP asymmetry $S_{\phi K_S}$ and showed that, although the experimental limits on $b \rightarrow s \gamma$ impose stringent constraints on the parameter space, it is still possible to reduce $S_{\phi K_S}$ significantly and negative values within the 1$\sigma$ experimental range can be obtained.

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**Appendix**

Here we provide the analytical results for the the expressions $R_F$ and $\bar{R}_F$ appearing in Eq.(5), which are given by

$$R^{LL}_D = \sum_{i=1,2} |V_{i1}|^2 x_{W_i} P_D(x_i)$$
\[ R_{DL}^{RL} = - \sum_{i=1,2} V_{i2}^* V_{i1} \ x_{wi} \ P_D(x_i) \]

\[ R_{DL}^{RR} = \sum_{i=1,2} |V_{i2}|^2 \ x_{wi} \ P_D(x_i) \]

\[ R_{DL}^{LR} = (R_{DL}^{RL})^* \]

\[ R_{DL}^{LL} = \sum_{i=1,2} |V_{i1}|^2 \ P_C^{(0)}(x_i) + \sum_{i,j=1,2} [U_{i1} V_{i1}^* U_{j1}^* V_{j1}^* P_C^{(2)}(x_i, x_j) \]

\[ + |V_{i1}|^2 |V_{j1}|^2 \left( \frac{1}{8} - P_C^{(1)}(x_i, x_j) \right) \]

\[ R_{DL}^{RL} = - \frac{1}{2} \sum_{i=1,2} V_{i2}^* V_{i1} \ P_C^{(0)}(\bar{x}_i) - \sum_{i,j=1,2} V_{i2}^* V_{i1} \left( U_{i1} U_{j1}^* P_C^{(2)}(x_i, x_j) \right) \]

\[ + V_{i1} V_{j1} \ P_C^{(1)}(x_i, x_j) \]

\[ R_{DL}^{RR} = (R_{DL}^{RL})^* \]

\[ R_{DL}^{LL} = \sum_{i,j=1,2} V_{j2}^* V_{i1} \left( U_{i1} U_{j1}^* P_C^{(2)}(x_i, x_j) + V_{i1} V_{j1} P_C^{(1)}(x_i, x_j) \right) \]

\[ R_{B^u}^{RL} = 2 \sum_{i,j=1,2} V_{i1} V_{i1}^* U_{i1} U_{j1}^* x_{wj} \sqrt{x_{ij}} \ P_{B^u}^n(\bar{x}_j, x_{ij}) \]

\[ R_{B^u}^{RL} = -2 \sum_{i,j=1,2} V_{i1} V_{i1}^* U_{i1} U_{j1}^* x_{wj} \sqrt{x_{ij}} \ P_{B^u}^n(\bar{x}_j, x_{ij}) \]

\[ R_{B^u}^{LL} = 2 \sum_{i,j=1,2} V_{i2} V_{j2}^* U_{i1} U_{j1}^* x_{wj} \sqrt{x_{ij}} \ P_{B^u}^n(\bar{x}_j, x_{ij}) \]

\[ R_{B^u}^{RR} = 2 \sum_{i,j=1,2} V_{i2} V_{j2}^* U_{i1} U_{j1}^* x_{wj} \sqrt{x_{ij}} \ P_{B^u}^n(\bar{x}_j, x_{ij}) \]

\[ R_{B^d}^{LL} = \sum_{i,j=1,2} |V_{i1}|^2 |V_{j1}|^2 x_{wj} \ P_{B^d}^d(\bar{x}_j, x_{ij}) \]

\[ R_{B^d}^{RL} = - \sum_{i,j=1,2} V_{i2}^* V_{i1} |V_{j1}|^2 x_{wj} \ P_{B^d}^d(\bar{x}_j, x_{ij}) \]

\[ R_{B^d}^{RR} = - \sum_{i,j=1,2} V_{i2}^* V_{i1} |V_{j1}|^2 x_{wj} \ P_{B^d}^d(\bar{x}_j, x_{ij}) \]

\[ R_{M_{\gamma, g}}^{LL} = \sum_{i} |V_{i1}|^2 x_{wi} P_{M_{\gamma, g}}^{LL}(x_i) - \sum_{i} V_{i1} U_{i2} x_{wi} \frac{m_{\chi}}{m_{b}} P_{M_{\gamma, g}}^{LR}(x_i) \]

\[ R_{M_{\gamma, g}}^{LR} = - \sum_{i} V_{i1}^* V_{i2} x_{wi} P_{M_{\gamma, g}}^{LL}(x_i) + \sum_{i} V_{i2} U_{i2} x_{wi} \frac{m_{\chi}}{m_{b}} P_{M_{\gamma, g}}^{LR}(x_i) \]
\[ R_{M^{+},g}^{LL} = - \sum_i V_{i1}V_{i2}^* x_{Wi} P_{M^{+},g}^{LL}(x_i) \]

\[ R_{M^{+},g}^{RR} = \sum_i |V_{i2}|^2 x_{Wi} P_{M^{+},g}^{LL}(x_i) \]  

(13)

where \( x_{wi} = m_W^2/m_{\chi_i}^2 \), \( x_i = m_{\chi_i}^2/\bar{m}^2 \), \( \bar{R}_i = \bar{m}^2/m_{\chi_i}^2 \), and \( x_{ij} = m_{\chi_i}^2/m_{\chi_j}^2 \). The expressions for the functions \( P_{E,D,C}^{(u,d)}, P_{B}^{u,d}, P_{M^{+},g}^{LL}, \) and \( P_{A^{+},g}^{RR} \), are given in the next subsection.

There are other contributions which come from box diagrams, where both chargino and gluino are exchanged \((B_g^{u,c})\), and cannot be expressed in the same form of Eq.(5). We provide below the results for these contributions, which affect only the Wilson coefficients \( C_{1,2}^{(u,c)}(\mu_W) \) as

\[ C_1^{(u,c)}(\mu_W) = \frac{\alpha_s(m_W)}{16\pi} \left( 14 - B_g^{(u,c)} \right) \]

\[ C_2^{(u,c)}(\mu_W) = 1 + \frac{\alpha_s(m_W)}{48\pi} B_g^{(u,c)} \]  

(14)

where

\[ B_g^{u} = \left[ \sum_a K_{a2} K_{23}(\delta_{LL}^{u})_{12a} \right] R_{g}^{LL}(u) + \left[ \sum_a K_{a2} K_{33}(\delta_{LL}^{u})_{23a} \right] R_{g}^{RL}(d) + \left[ \sum_a K_{a2} K_{33}(\delta_{LL}^{u})_{32a} \right] R_{g}^{RL}(u) \]

\[ B_g^{c} = \left[ \sum_a K_{a2} K_{23}(\delta_{LL}^{c})_{12a} \right] R_{g}^{LL}(u) + \left[ \sum_a K_{a2} K_{33}(\delta_{LL}^{c})_{23a} \right] R_{g}^{RL}(d) + \left[ \sum_a K_{a2} K_{33}(\delta_{LL}^{c})_{32a} \right] R_{g}^{RL}(u) \]  

(15)

and the functions \( R_i \) are given by

\[ R_{g}^{LL}(u) = 4x_{Wg} \sum_{i=1,2} \left[ |V_{i1}|^2 P_B^{d}(z_i, y) + 2U_{i1}V_{i1}^* \left( \frac{m_{\chi_i}}{m_{\bar{g}}} \right) P_B^{u}(z_i, y) \right] \]

\[ R_{g}^{LL}(d) = 4x_{Wg} \sum_{i=1,2} \left[ |U_{i1}|^2 P_B^{d}(z_i, y) + 2U_{i1}V_{i1}^* \left( \frac{m_{\chi_i}}{m_{\bar{g}}} \right) P_B^{u}(z_i, y) \right] \]

\[ R_{g}^{RL} = -4x_{Wg} \sum_{i=1,2} \left[ V_{i1}V_{i2}^* P_B^{d}(z_i, y) + 2V_{i2}U_{i1}^* \left( \frac{m_{\chi_i}}{m_{\bar{g}}} \right) P_B^{u}(z_i, y) \right] \]  

(17)

(18)

(19)

with \( x_{Wg} = m_W^2/m_{\bar{g}}^2, z_i = m_{\chi_i}^2/m_{\bar{g}}^2, \) and \( y = \bar{m}^2/m_{\bar{g}}^2. \) In obtaining the above results in Eqs.(15)-(16) we neglect terms of order of \( \mathcal{O}(Y_b). \)
Loop functions

Here we provide the expressions for the loop functions of penguin $P_{D,E,C}$, box $P_{B}^{(u,d,\bar{d})}$, and magnetic– and chromomagnetic–penguin diagrams $P_{M_{+,-}}^{LL}$, and $P_{M_{-,-}}^{LR}$ respectively, which enter in Eqs. (13), (19)

\begin{align*}
P_D(x) &= \frac{2x(-22+60x-45x^2+4x^3+3x^4-3(3-9x^2+4x^3)\log x)}{27(1-x)^5} \\
P_E(x) &= \frac{x(-1+6x-18x^2+10x^3+3x^4-12x^3\log x)}{9(1-x)^5} \\
P_C^{(0)}(x) &= \frac{x(3-4x+x^2+2\log x)}{8(1-x)^3} \\
P_C^{(1)}(x,y) &= \frac{1}{8(x-y)} \left[ \frac{x^2(x-1-\log x)}{(x-1)^2} - \frac{y^2(y-1-\log y)}{(y-1)^2} \right] \\
P_C^{(2)}(x,y) &= \frac{\sqrt{xy}}{4(x-y)} \left[ \frac{x(x-1-\log x)}{(x-1)^2} - \frac{y(y-1-\log y)}{(y-1)^2} \right] \\
P_B^u(x,y) &= \frac{-y-x(1-3x+y)}{4(x-1)^2(x-y)^2} - \frac{x(x^3+y^3x+y^2)\log x}{2(x-1)^3(x-y)^3} \\
&\quad + \frac{xy\log y}{2(x-y)^3(y-1)} \\
P_B^d(x,y) &= \frac{x(3y-x(1+x+y))}{4(x-1)^2(x-y)^2} - \frac{x(x^3+(x-3)x^2y+y^2)\log x}{2(x-1)^3(x-y)^3} \\
&\quad + \frac{xy^2\log y}{2(x-y)^3(y-1)} \\
P_{M_{+}}^{LL}(x) &= -x\frac{d}{dx}\left(xF_1(x) + \frac{2}{3}xF_2(x)\right) \\
P_{M_{+}}^{LR}(x) &= -x\frac{d}{dx}\left(xF_3(x) + \frac{2}{3}xF_4(x)\right) \\
P_{M_{+}}^{LL}(x) &= -x\frac{d}{dx}(xF_2(x)) \\
P_{M_{+}}^{LR}(x) &= -x\frac{d}{dx}(xF_4(x)) \\
\end{align*}

where the functions $F_i(x)$ are provided in Ref. [10].

Light right-stop

Here we generalize the above formulas for the case in which the right-stop is lighter than other squarks. Notice, that this will modify only the expressions of $R_F^{RL}$ and $R_F^{RR}$, since the light right-stop does not affect $R_F^{LL}$. In the case of $R_F^{RR}$ the functional forms of $R_F^{RR}$ remain unchanged, while the arguments of the functions involved are changed as $x_i \rightarrow x_{it}$ and $\bar{x}_i \rightarrow \bar{x}_{it}$. In the case of $R_F^{LL}$ and $R_F^{RL}$ the analytical expression of loop functions of
penguin $P_{D,E,C}$, box $P_B^{(u,d,g)}$, and magnetic and chromomagnetic penguin diagrams $P_{M_{r,g}}^{LL}$ and $P_{M_{r,g}}^{LR}$ respectively, should be changed as follows

\[
P_D(x_i, x_{it}) = \frac{2}{(x_t - 1)} \left[ x_{it} D_{\chi}(x_{it}) - x_i D_{\chi}(x_i) \right]
\]
\[
P_E(x_i, x_{it}) = \frac{2}{(x_t - 1)} \left[ x_{it} E_{\chi}(x_{it}) - x_i E_{\chi}(x_i) \right]
\]
\[
P_{C^{(1,2)}}(x_i, x_{it}, x_j, x_{jt}) = \frac{2}{(x_t - 1)} \left[ C_{\chi}^{(1,2)}(x_{jt}, x_{it}) - C_{\chi}^{(1,2)}(x_j, x_i) \right]
\]
\[
P_{C^{(0)}}(\bar{x}_i, \bar{x}_{it}) = \frac{4}{(x_t - 1)} \left[ C_{\chi}^{(1)}(\bar{x}_{it}, \bar{x}_i) - C_{\chi}^{(1)}(\bar{x}_i, \bar{x}_i) \right]
\]
\[
P_B^{(u)}(\bar{x}_j, \bar{x}_{jt}, x_{ij}) = \frac{1}{2(x_t - 1)} \left[ B_{\chi}^{(u)}(\bar{x}_{jt}, \bar{x}_j, x_{ij}) - B_{\chi}^{(u)}(\bar{x}_j, \bar{x}_j, x_{ij}) \right]
\]
\[
P_B^{(d)}(\bar{x}_j, \bar{x}_{jt}, x_{ij}) = -\frac{1}{2(x_t - 1)} \left[ B_{\chi}^{(d)}(\bar{x}_{jt}, \bar{x}_j, x_{ij}) - B_{\chi}^{(d)}(\bar{x}_j, \bar{x}_j, x_{ij}) \right]
\]
\[
P_{M_{r,g}}^{LL}(x_i, x_{it}) = \frac{1}{(x_t - 1)} \left[ x_{it} \left( F_1(x_{it}) + \frac{2}{3} F_2(x_{it}) \right) - x_i \left( F_1(x_i) + \frac{2}{3} F_2(x_i) \right) \right]
\]
\[
P_{M_{r,g}}^{LR}(x_i, x_{it}) = \frac{1}{(x_t - 1)} \left[ x_{it} \left( F_3(x_{it}) + \frac{2}{3} F_4(x_{it}) \right) - x_i \left( F_3(x_i) + \frac{2}{3} F_4(x_i) \right) \right]
\]
\[
P_{M_{r,g}}^{LL}(x_i, x_{it}) = \frac{1}{(x_t - 1)} \left[ x_{it} F_2(x_{it}) - x_i F_2(x_i) \right]
\]
\[
P_{M_{r,g}}^{LR}(x_i, x_{it}) = \frac{1}{(x_t - 1)} \left[ x_{it} F_4(x_{it}) - x_i F_4(x_i) \right]
\]

where $x_i = m_{\chi_i}^2/\tilde{m}^2$, $\bar{x}_i = \tilde{m}^2/m_{\chi_i}^2$, $x_{it} = m_{\chi_i}^2/m_{\chi_i}^2$, $\bar{x}_{it} = m^2_{\chi_i}/m^2_{\chi_i}$, $x_{ij} = m^2_{\chi_i}/m^2_{\chi_j}$ and $x_t = m^2_{\chi_t}/\tilde{m}^2$. The functions $D_{\chi}, C_{\chi}, E_{\chi}, C^{(1,2)}_{\chi}, B_{\chi}^{(u,d)}$ and $F_i$ are provided in Ref.[11] and Ref.[10] respectively.

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Figure 1: The mixing CP asymmetry as function of $\text{arg}[(\delta_{LL}^u)_{32}]$, for $\tan \beta = 40$, and with the contribution of one mass insertion $|(\delta_{LL}^u)_{32}|$. Red points correspond to $|(\delta_{LL}^u)_{32}|$ that satisfy all the experimental bounds. The light blue points are not allowed by $BR(b \rightarrow s\gamma)$. The strong phase $\delta_{12}$ is fixed at $\cos \delta_{12} = 1$.

Figure 2: As in Fig. 1, but for the mass insertion $(\delta_{RL}^u)_{32}$.
Figure 3: The mixing CP asymmetry as function of $\arg[(\delta^u_{LL})_{32}] = \arg[(\delta^u_{RL})_{32}]$, for $\tan \beta = 40$, and with the contribution of two mass insertions $|\langle \delta^u_{RL}\rangle_{32}|$ and $|\langle \delta^u_{LL}\rangle_{32}|$. Red points correspond to $|\langle \delta^d_{LL}\rangle_{32}|$ and $|\langle \delta^d_{RL}\rangle_{32}|$ that satisfy all the experimental bounds. The light blue points are not allowed by $BR(b \to s\gamma)$. The strong phase $\delta_{12}$ is fixed at $\cos \delta_{12} = 1$. 