On the velocity of the TE-polarized light wave to propagate through a homogeneous dielectric layer

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Abstract. We present a novel model of elastic scattering of the plane TE-polarized light wave on a homogeneous dielectric layer. This wave is shown can be uniquely decomposed into a coherent superposition of two ‘subprocess’ TE waves to describe transmission and reflection in all spatial regions. Each of them has one incoming and one outgoing waves connected ‘causally’ to each other on the midplane of the layer – namely, with keeping the continuity of the complex-valued electrical field strength and the corresponding energy flow density (averaged over an oscillation period). This model unlike the conventional one fulfills the mandatory physical requirements: in this scattering problem, the velocity of the energy transfer through the layer must be the same on mirror-symmetric planes, as well as it must be always subluminal, including the case of a frustrated total internal reflection (FTIR).

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1. Introduction

Scattering the plane monochromatic light wave on a homogeneous dielectric layer is one of the simplest scattering problems in classical electrodynamics, and, at first glance, its conventional model [1] (named further 'the conventional model of light scattering' (CELS)) provides an exhaustive description of this process. However, the lengthy debate (see, e.g., [2, 3, 4, 5, 6, 7, 8, 9, 10] as well as [11, 12, 13, 14, 15]) around the interpretation of the tunneling-time experiments [2, 3, 4, 5, 6, 9] to observe superluminal group tunneling velocities, predicted, in particular, by this model for a light beam to tunnel through the air gap in the 'glass-air-glass' structure under the condition of a frustrated total internal reflection (FTIR), suggests otherwise.

Analyses of the signal, group, and energy velocity concepts (see, respectively, [5], [8] and Section 6 in [7]) as well as the analysis of the tunneling time experiments (see [8]) show that none of these concepts gives a consistent definition of the tunneling (transit) time. The main difficulty to appear in all known attempts to adapt these wave velocity concepts to timekeeping a non-resonant tunneling is that neither the CELS nor the conventional quantum-mechanical model of a non-resonant tunneling (CQMT) allow tracing the transmission (tunneling) subprocess at all stages of scattering. This gap in the standard description of this phenomenon is filled by (explicit or implicit) "self-evident" assumptions that are erroneous on closer inspection [16].

The most crucial among these assumptions are as follows: (a) the fact that the transmission and reflection subprocesses of a non-resonant tunneling are inextricably intertwined within the CQMT and CELS is construed as the evidence of their indistinguishability at all stages of scattering; (b) in all clock-based timekeeping procedures aimed at measuring the tunneling time, the transmission subprocess is treated as unitary quantum process; (c) contrary to the well-known fact that the incident wave packet does not relate causally to the transmitted one, it is widely accepted that the average starting point of transmitted particles always coincides with that of all scattered particles.

However, our approach [17, 18, 19, 20] refutes all these assumptions: (a) it gives a novel quantum-mechanical model of a non-resonant tunneling, that allow tracing the transmission and reflection dynamics at all stages of scattering; (b) as it turned out, in the time-dependent case the transmission dynamics is nonunitary at the stage when the wave packet interacts with the potential barrier (but the reflection dynamics is always unitary); (c) the centers of 'masses' (CMs) of the transmitted and reflected subensembles of particles as well as the CM of the whole ensemble of scattering particles start, as a rule, from the different spatial points.

This model sheds new light on the Hartman effect. As it turned out, just in the opaque limit the average starting point of transmitted particles coincides with that of all scattered particles. Thus, in fact our approach justifies the appearance, in this limit, of superluminal group tunneling velocities within the CQM and CELS. However, now we meet a cardinally new situation: since the transmission dynamics is nonunitary
at the very stage of scattering, the group tunneling velocity (i.e., the velocity of the CM of the to-be-transmitted wave packet) does not represent the true average velocity of transmitted particles in the barrier region. As was shown in [16] for narrow in $k$ space wave packets, when the (smooth) front part of the to-be-transmitted wave packet crosses the midpoint $x_{\text{mid}}$ of a symmetric potential barrier, this point serves as a source of particles. When its tail part crosses this point, it serves as a sink of particles. In both these cases the wave-packet’s CM is accelerated.

By our approach, the group velocity concept is inapplicable (not only in the opaque limit) for revealing the average velocity of transmitted particles at the very stage of scattering. Only the flow velocity concept which is insensitive to the processes taking place at the point $x_{\text{mid}}$ can play this role. As was shown in [16, 18] for a particle tunneling through the rectangular barrier, the dwell transmission time increases exponentially when the barrier width tends to infinity.

Thus, anomalously large velocities of the CM of the wave packet to pass through the opaque potential barrier do not contradict special relativity. This peculiarity of the transmission dynamics results eventually from the interaction between the transmission and reflection subprocesses at the point $x_{\text{mid}}$: being distinguishable at all stages of scattering, they are however not alternative at the very stage of scattering.

Besides, as these subprocesses hide each other, their characteristic times can be measured only indirectly, e.g., on the basis of the Larmor-clock procedure (see [16, 18]). As was shown for symmetric potential barriers, because of the nonunitary character of the transmission subprocess the final readings of the Larmor clock do not display the time of dwelling the transmitted subensemble of electrons in the barrier region. Apart from the dwell transmission time to describe the duration of the Larmor precession of the average spin of transmitted electrons in the barrier region where an infinitesimal magnetic field is switched on, there appears an additional effect on these final readings, which is associated with the midpoint $x_{\text{mid}}$ to connect the transmission and reflection subprocesses. Of course, all these ‘subprocess’ effects, as was stressed above, can be observed only indirectly.

The aim of this paper is to extend the alternative quantum-mechanical model [17] onto the problem of scattering the TE-polarized light wave on a homogeneous dielectric layer, and then, as the first step, to define on the basis of this model the dwell times for the transmission and reflection subprocesses.

2. Setting the problem

Let us consider two homogeneous nonmagnetic ($\mu = 1$) media with the dielectric permittivities $\epsilon_0$ and $\epsilon$: the medium with the refractive index $n$ ($n = \sqrt{\epsilon}$) fills the interval $[0, d]$ on the axis $OZ$, and the background medium with the refractive index $n_0$ ($n_0 = \sqrt{\epsilon_0}$) fills the spatial regions laying outside this interval; $n, n_0 \geq 1; n \neq n_0$. Both these media are assumed to be transparent and non-dispersive.

We assume that the plane light TE wave falls from the left on the interface $z = 0$,
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provided that its wave vector lays in the plane \(YZ\) and the angle between this vector and the axis \(OZ\) is \(\theta\). In this case only the projection \(E_x\) of an electrical field and two projections, \(H_y\) and \(H_z\), of a magnetic field are nonzero. To exploit the analogy between the optical and quantum tunneling problems, it is suitable to write down these quantities, as obeying the same wave equation, in the complex form (see [1]).

Since the structure investigated is nonuniform only in the \(z\)-direction, we have

\[ E_x = U(z)e^{ix}, \quad H_y = V(z)e^{ix}, \quad H_z = W(z)e^{ix}; \]

\[ \chi = kn_{0,y} - \omega t, \quad n_{0,y} = n_0 \sin \theta, \quad k = \omega/c; \]

\(c\) is the speed of light in vacuum. When these complex solutions are known the above searched-for (real) projections of electrical and magnetic fields are simply

\[ \Re\left(Ue^{i\chi}\right), \quad \Re\left(Ve^{i\chi}\right), \quad \Re\left(We^{i\chi}\right). \]

For nonlinear characteristics – the energy density \(w\) and Poynting vector \(S\) – of the TE wave to propagate in the medium with the dielectric permittivity \(\varepsilon\) we have

\[ w = w(0) + w(t), \quad S = S(0) + S(t), \]

\[ w(0) = \frac{1}{16\pi} \left( \epsilon |U|^2 + |V|^2 + |W|^2 \right); \quad w(t) = \frac{1}{16\pi} \Re\left[ (\epsilon U^2 + V^2 + W^2) e^{2i\chi} \right]; \quad S_x^{(0)} = S_z^{(0)} = 0; \quad S_y^{(0)} = -\frac{c}{8\pi} \Re(U^*W); \quad S_z^{(0)} = \frac{c}{8\pi} \Re(U^*V); \]

\[ S_x^{(t)} = -\frac{c}{8\pi} \Re(UWe^{2i\chi}); \quad S_z^{(t)} = \frac{c}{8\pi} \Re(UVe^{2i\chi}). \]

Since the functions \(V\) and \(W\) are connected to \(U\) by the relations (see [1])

\[ V(z) = -iU'(z)/k, \quad W(z) = -U(z)n_{0,y}, \quad \]

solving the problem is reduced to finding the function \(U(z)\); hereinafter the prime denotes the derivative on \(z\). In particular, outside and inside the interval \([0, d]\), the initial (three-dimensional) wave equation for \(E_x\) is reduced, respectively, to the one-dimensional equations for the function \(U(z)\),

\[ U'' + k^2 n_{0,z}^2 U = 0, \quad U'' + k^2 (n^2 - n_{0,y}^2) U = 0; \]

where \(n_{0,z} = n_0 \cos \theta\). At the interfaces \(z = 0\) and \(z = d\) the function \(U(z)\) and its first derivative \(U'(z)\) must be continuous. This follows from the boundary conditions for the tangential projections \(E_x, H_y\) and for orthogonal projection \(H_z\), as well as from the relations (3).

Note that in the case of a plane (monochromatic) light wave, apart from the conservation law for the energy of electromagnetic field, which follows from the continuity equation

\[ \frac{\partial w}{\partial t} + \nabla S = 0, \]

we have also the conservation law

\[ S_z^{(0)} = -\frac{c}{8\pi k} \Im(U^*U') = \text{const}; \quad \]
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$S_z^{(0)}$ is the analog to the probability current density in the one-dimensional quantum stationary scattering problem.

Let us write down the solutions to Eqs. (4), making use of the notations of the papers [16, 19]. In the region $z \leq 0$, there are incident and reflected waves

$$U(z) = \exp(i kn_{0,z} z) + b_{\text{out}}(k) \exp(-i kn_{0,z} z);$$

(6)

in the region $z > d$ there is a transmitted wave

$$U(z) = a_{\text{out}}(k) \exp[i kn_{0,z}(z - d)];$$

(7)

inside the layer, for $0 \leq z \leq d$,

$$U(z) = A_f G_1(x - z_c; k) + B_f G_2(z - z_c; k);$$

(8)

$$a_{\text{out}} = \frac{1}{2} \left( \frac{Q}{Q^*} - \frac{P}{P^*} \right), \quad b_{\text{out}} = -\frac{1}{2} \left( \frac{Q}{Q^*} + \frac{P}{P^*} \right);$$

$$A_f = -\frac{P^*}{\kappa} a_{\text{out}}, \quad B_f = \frac{Q^*}{\kappa} a_{\text{out}}; \quad z_c = \frac{d}{2};$$

(9)

$$Q = [G_1'(z) + i kn_{0,z} G_1(z)]_{z = z_c}; \quad P = [G_2'(z) + i kn_{0,z} G_2(z)]_{z = z_c};$$

if $n_{0,y} \leq n$, then

$$G_1 = \sin(\kappa z), \quad G_2 = \cos(\kappa z), \quad \kappa = k \sqrt{n^2 - n_{0,y}^2};$$

in the case of FTIR, i.e., when $n_{0,y} > n$

$$G_1 = \sinh(\tilde{\kappa} z), \quad G_2 = \cosh(\tilde{\kappa} z), \quad \tilde{\kappa} = k \sqrt{n_{0,y}^2 - n^2}.$$  

Here $|a_{\text{out}}|^2 = T$ is the transmission coefficient, $|b_{\text{out}}|^2 = R$ is the reflection coefficient; $T + R = 1$.

Taking into account the relations (11) and (2) for $w(0)$, $S_y^{(0)}$ and $S_z^{(0)}$, we obtain

$$w(0)(z) = \frac{1}{16\pi} \left[ \left( n^2 + n_{0,y}^2 \right) |U(z)|^2 + \frac{|U'(z)|^2}{k^2} \right],$$

$$S_y^{(0)} = \frac{cn_{0,y}}{8\pi} |U|^2, \quad S_z^{(0)} = \frac{c}{8\pi \kappa} \Im(U^* U') = \frac{cn_{0,z}}{8\pi} T.$$  

(10)

3. The scattering TE wave as a superposition of two 'subprocess' waves, one being transmitted by the layer and one being reflected by it

As in the quantum case (see [16, 17, 18]), for any value of $k$ there is a unique pair of functions $U_{\text{tr}}(z)$ and $U_{\text{ref}}(z)$ which obey the equation

$$U_{\text{tr}}(z) + U_{\text{ref}}(z) = U(z)$$

as well as possess the following properties: (a) either function unlike $U(z)$ has one outgoing and one incoming wave; (b) the outgoing wave of $U_{\text{tr}}(z)$ coincides with the transmitted wave, and that of $U_{\text{ref}}(z)$ coincides with the reflected one; (c) the incoming wave of either wave function is 'causally' connected at the plane $z = z_c$ to the
corresponding outgoing one—the complex-valued functions $U_{tr}(z)$ and $U_{ref}(z)$ as well as the corresponding energy flow densities are continuous at this plane (but the first derivative of either function is discontinuous here).

For $z \leq 0$

$$U_{tr}(z) = A_{tr}^{in} e^{ik_{no} z} , \quad U_{ref}(z) = A_{ref}^{in} e^{ik_{no} z} + b_{out}(k) e^{-ik_{no} z};$$

for $0 \leq z \leq z_c,$

$$U_{tr}(z) = D_{tr} G_1(z - z_c; k) + B_f G_2(z - z_c; k) \quad U_{ref}(z) = D_{ref} G_1(z - z_c; k);$$

for $z > z_c$

$$U_{tr}(z) \equiv U(z), \quad U_{ref} \equiv 0;$$

$$D_{tr} = -\frac{PQ^*}{P^*Q} A_f, \quad D_{ref} = \frac{1}{k} \left( PA_{ref}^* + P^* b_{out} \right);$$

$$A_{tr}^{in} = a_{out} (a_{out}^* - b_{out}^*), \quad A_{ref}^{in} = b_{out} (a_{out} + b_{out}).$$

As is seen, the found TE waves to describe the transmission and reflection subprocesses in all spatial regions possess the following peculiarities: (i) not only $A_{tr}^{in} + A_{ref}^{in} = 1$, but also $|A_{tr}^{in}|^2 + |A_{ref}^{in}|^2 = 1$; (ii) the to-be-reflected TE wave does not cross the plane $z = z_c$; (iii) despite the fact that the derivative $U_{tr}'(z)$ is discontinuous at the plane $z = z_c$, its absolute value $|U_{tr}'(z)|$ is continuous here, because

$$|U_{tr}(z_c - z)| = |U_{tr}(z - z_c)|, \quad |U_{tr}'(z_c - z)| = |U_{tr}'(z - z_c)|.$$  

These relations follow from the equality $\Re(D_{tr} B_f^*) = \Re(A_f B_f^*)$ which follows, in its turn, from Exps. (11) and (14).

All this means that for the transmitted TE component the real fields $E_{xtr}$ and $H_{ytr}$, as well as the energy density $w_{tr}$ and the Poynting vector $S_{tr}$, are continuous at the plane $z = z_c$:

$$S_{tr} = (0, S_{ytr}, S_{ztr}^x); \quad S_{ytr} = \frac{c n_{0y}}{8\pi} |U_{tr}|^2, \quad S_{ztr}^x = S_{z}^{(0)};$$

$$w_{tr}(z) = \frac{1}{16\pi} \left[ \left( n^2 + n_{0y}^2 \right) |U_{tr}(z)|^2 + \frac{|U_{tr}'(z)|^2}{k^2} \right]$$

(by the real projection $H_{ytr}$ is discontinuous at this plane).

So, there is a unique way to decompose the scattering TE-polarized light wave into two causally evolving components to describe the transmission and reflection subprocesses. Now, when the dynamics of each subprocess is known in all spatial regions, we can proceed to studying their temporal aspects.

4. Transmission and reflection dwell times

By the analogy with quantum-mechanical approach [16, 17, 18], we define here the velocity $v_{tr}(z)$ ($v_{tr} = (0, v_{ytr}^r, v_{ztr}^r)$) of the light component to propagate through the layer as the ratio of the Poynting vector to the energy density at the point $z$ (see Exps. (17)):
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$v_{tr}(z) = S_{tr}(z)/w_{tr}(z)$. Thus, the time $\tau_D^v$ to describe the duration of the transmission of the light wave through the layer can be defined as follows

$$\tau_D^v = \int_0^d \frac{dz}{v_{tr}(z)} = \frac{1}{S_{tr}} \int_0^d w_{tr}(z)dz$$

(18)

hereinafter, this quantity will be referred to as the transmission dwell time.

Note that the CEMT deals with the Buttiker dwell time $\tau_D^S$, $\tau_D^S = \frac{1}{S_{inc}} \int_0^d w_{0}(z)dz$, $S_{inc}^z = \frac{cn_0, z}{8\pi}$, (19) as well as with the dwell time $t_gS$ $t_gS = \frac{1}{S_{0}} \int_0^d w_{0}(z)dz$ (20) whose physical meaning is obscure (see [16]).

Then, considering (6)-(8) and (11)-(15), for the transmission time we obtain the following expressions. For $n_{0,y} \leq n$

$$\tau_D^v = \frac{k^2}{4\kappa^3 cn_0,z} [(n^2 - n_{0,y}^2)n_{0,y}^2 \sin(2\kappa d) + 2n^2(n^2 + n_{0,y}^2 - n_{0,y}^2)\kappa d]$$

(21)

for $n_{0,y} > n$ (the FTIR case)

$$\tau_D^v = \frac{k^2}{4\kappa^3 cn_0,z} [(n_0^2 - n^2)n_{0,y}^2 \sinh(2\kappa d) - 2n^2(n^2 + n_{0,z}^2 - n_{0,y}^2)\kappa d]$$

(22)

Figs. 1 and 2 show the dependence $\tau_D^v/\tau_{free}$ on $kd$ for the case when one medium is vacuum, and another is glass; $\tau_{free} = d/c$. As it follows from Exp. (22) (see also the curve 4 on Fig. 2) obtained for $n_0 = 1, 5$ and $n = 1$), in the case of FTIR ($\theta > 41, 8^\circ$) this quantity exponentially increases when $kd \to \infty$.

![Figure 1](image-url)  
**Figure 1.** The dependence of $\tau_D^v/\tau_{free}$ on $kd$ for $n_0 = 1$ and $n = 1, 5$: (1) $\theta = 0^\circ$; (2) $\theta = 15^\circ$; (3) $\theta = 30^\circ$; (4) $\theta = 45^\circ$. 
When \( n_{0,y} \leq n \) the transmission (energy flow) velocity does not exceed \( c \) too (see Fig. 1). As it follows from (21), in the limit \( kd \to \infty \)

\[
\frac{\tau_{tr}^r}{\tau_{free}} = \frac{\kappa^2 + k^2 n_{0,z}^2}{2\kappa k n_{0,z}} \cdot \frac{n^2}{\sqrt{n^2 - n_{0,y}^2}} \geq 1.
\]

Note that \( \tau_{tr}^r \) does not yield a full information about the velocity of the energy propagation inside the layer, since \( \tau_{tr}^r \) depends only on the \( z \)-projection \( v_{tr,z} \). Therefore a more detailed analysis of this question is done in Section 5.

The reflection dwell time \( \tau_{D}^{ref} \) is defined similarly –

\[
\tau_{D}^{ref} = \frac{1}{S_z^{ref}} \int_0^{z_c} w_{ref}(z)dz; \quad S_z^{ref} = \frac{cn_{0,z}}{8\pi} R,
\]

\[
w_{ref}(z) = \frac{1}{16\pi} \left[ (n^2 + n_{0,y}^2) |U_{ref}(z)|^2 + \frac{|U_{ref}'(z)|^2}{k^2} \right].
\]

For \( n_{0,y} \leq n \), with considering Exps. (11)-(15), we obtain

\[
\tau_{D}^{ref} = \frac{2n_{0,z}}{ck} \frac{n^2 \kappa d - n_{0,y}^2 \sin(\kappa d)}{(n^2 - n_{0,y}^2) \cos(\kappa d) + n^2 + n_{0,z}^2 - n_{0,y}^2}; \tag{23}
\]

for \( n_{0,y} > n \)

\[
\tau_{D}^{ref} = \frac{2n_{0,z}}{c\tilde{k}} \frac{n_{0,y}^2 \sinh(\tilde{k}d) - n_{0,y}^2 \tilde{k} d}{(n_{0,y}^2 - n^2) \cosh(\tilde{k}d) - (n^2 + n_{0,z}^2 - n_{0,y}^2)}.
\]

From Exps. (23) it follows that, under the conditions of FTIR, the function \( \tau_{D}^{ref}(d) \) saturates in the limit \( d \to \infty \). However, this fact does not say that we deal with the Hartman effect, because the reflection time depends not only on the average velocity of reflected particles, but also on the average depth of their penetration into the layer. The above fact means simply that in the case of FTIR this depth tends to some fixed value when \( d \to \infty \).
5. On the velocity of the energy transfer

A detailed analysis of the velocity of the energy transfer for the light component to pass through the layer has been carried out by the example of the above two media – vacuum and glass. Figs. 3-5 present numerical results obtained for the case when the interval $[0, d]$ is filled with glass. Fig. 3 shows the function $v_{tr}(z) = \left|v_{ty}(z)\right|$ within the layer. Fig. 4 displays the $z$-dependence of the angle $\Theta$ ($\Theta = \arctan(S_{ytr}/S_{ztr})$) to characterize the direction of propagation of the transmitted TE component.

As is seen from Figs. 3 and 4, both the functions – $v_{tr}(z)$ and $\Theta(z)$ – reach their maximal values on the set of points, that includes the boundary points $z = 0$ and $z = d$. 
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Figure 5. The functions $v_{tr}(z)/c$ (firm line) and $v_{tot}(z)/c$ (dashed line) for $\theta = 89.1^\circ$; the values of remaining parameters are the same as for Fig. 3.

When $\theta$ increases, these functions vary more rapidly. In this case $\Theta(z) \leq \theta$ for $n_0 < n$.

It should be noted that the refraction angle $\Theta_\infty(\theta)$ to characterize scattering the plane light wave on the semi-infinite dielectric medium does not appear in the considered problem where the same dielectric fills the layer of a finite width. For a given $\theta$ the function $\Theta(z)$ oscillates inside the layer around the value to be approximately equal to $\Theta_\infty(\theta)$.

In the limit $\theta \to 90^\circ$, the function $v_{tr}(z)$ tends to zero at the points of minimum. This is seen from the numerical results presented on Fig. 3 as well as on Fig. 5 where, in addition to $v_{tr}(z)$, we show also the function $v_{tot}(z)$: $v_{tot}(z) = |v_{tot}(z)|$; $v_{tot}(z) = S(0)/w(0)(z)$. At the point $z = 0$ this function like $v_{tr}(z)$ is discontinuous, but its discontinuity is so small in this case that it is unapparent on the figure.

Fig. 5 shows explicitly the qualitative difference between the behaviour of the functions $v_{tr}(z)$ and $v_{tot}(z)$ near the interfaces $z = 0$ and $z = d$. Due to (16) $v_{tr}(z_c - z) = v_{tr}(z - z_c)$. However, the ”velocity” $v_{tot}(z)$ to underlie the concept of the dwell time (20) does not obey this requirement. This fact presents one more argument in favor of our approach.

Indeed, in the symmetrical structure to consist of transparent homogeneous media, all reflection symmetric points are physically equivalent. Thus, the tunneling velocity must represent the even function of $z - z_c$. The reflected wave must not affect the tunneling velocity.

Of importance is to stress that $v_{tr}(z) = c/n_0$ outside the interval $[0, d]$. Inside this interval the function $v_{tr}(z)$ varies. However its values do not exceed here the limiting velocity $c$. For $n > n_0$ the velocity $v_{tr}(z)$ takes its maximal value at those points $z$ where $\sin(\kappa z) = 0$. This set is always nonempty, as it contains the boundary points $z = 0$ and
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Figure 6. The dependence of $v_{tr}$ on $z$ for $n_0 = 1, 5$ and $n = 1; kd = 10$; the values of the angle $\theta$ are the same as for Fig. 1.

Figure 7. The dependence of the angle $\Theta$ on $z$ for the same parameters as for Fig. 6.

$z = d$. At any point of this set

$v_{tr} = v_{tr}^{(1)} = \frac{c}{n} \cdot \frac{2n_0n}{n_0^2 + n^2} < \frac{c}{n}$.

Figs. 6-8 present numerical results for the case when glass and vacuum traded places in the considered structure. Figs. 6 and 7 show, respectively, the functions $v_{tr}(z)$ and $\Theta(z)$. Fig. 8 shows the functions $v_{tr}^{(2)}(z)$ and $v_{tot}^{(2)}(z)$ in the case $n < n_{0,y}$.

Note that for $n_0 > n \geq n_{0,y}$ the velocity $v_{tr}$ takes maximal value at those points $z$ where $\cos(\kappa z) = 0$:

$v_{tr} = v_{tr}^{(2)} = \frac{c}{n} \cdot \frac{2k^2n_0zn\sqrt{\kappa^4 + k^4n_{0,y}^2n_{0,z}^2}}{k^6n_{0,z}^2n^2 + (\kappa^4 + k^4n_{0,y}^2n_{0,z}^2)} \leq \frac{c}{n}$. 
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If $n = n_{0,y}$, then the function $v_{tr}(z)$ reaches the maximal value $c/n$ at the point $z = z_c$. It is the only case when the speed of light in the medium that fills the finite layer $[0, d]$, approaches the one in the same medium, but filling the infinite space.

In the case of FTIR, the maximal value $v_{tr}(z_c)$ diminishes when the angle $\theta$ increases; if $\theta$ exceeds some critical value, the function $v_{tr}(z)$ reaches its maximal value $v^{(1)}_{max}$ at the boundary points $z = 0$ and $z = d$. As regards the point $z_c$, in the limit $\theta \to 90^\circ$, $v_{tr}(z_c) = c/n_0$ (see Fig. 8). When $n_0 > n$ the inequality $\Theta(z) \geq \theta$ holds (see Fig. 7).

6. Conclusion

A new model of scattering the plane TE-polarized light wave on a homogeneous dielectric layer has been developed. It is shown that this wave can be uniquely represented as the superposition of two coherently evolved components to describe the stationary transmission and reflection subprocesses in all spatial regions. Either of these components possesses one incoming and one outgoing waves, joined on the midplane of the layer with keeping the continuity of the complex-valued electrical field strength as well as the corresponding energy flow density averaged over the period of oscillations. As was shown, for the structure to possess the mirror symmetry, with the symmetry plane $z = z_c$, the transmission velocity introduced within this model on the basis of the concept of energy flow velocity represents an even function of $z - z_c$. In the case of FTIR the transmission (tunneling) velocity increases exponentially when the width of the air gap, in the ‘glass-air-glass’ structure, tends to infinity.

Of course, this research should be continued. Of importance is also to study the time-dependent dynamics of both subprocesses on the basis of the group velocity
On the velocity of the TE-polarized light wave to propagate through a homogeneous dielectric layer concept, as well as to elaborate for this scattering problem the Larmor timekeeping procedure, which would allow one to indirectly measure the characteristic times of both subprocesses. We have to stress once more that direct measurements of their characteristics in the case of a non-resonant tunneling is impossible in principle, because each of them creates an unremovable context for another.

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