Solving the problems of buildings and structures aerodynamics with a vortex method

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Abstract. A mathematical model for calculating wind flows in the space near buildings and structures on the direct solution basis of the flow problem with the use of a vortex approach is presented. Examples of flow calculations in test problems are presented in the wind situation modeling in the flow around groups of buildings and structures.

1. Introduction

Recently, there has been an intensive development and application of mathematical flow modelling methods in the problems of building aerodynamics. The main reason for this is the intensive construction of buildings and high altitude structures, which, moreover, often have a non-standard configuration. There are highly uneven air currents, and both zones of low and high velocities can be observed with respect to the background wind speed in the flow around such buildings. In the case of buildings and structures’ complexes, the aerodynamic interference between different buildings, determined by their mutual arrangement relative to each other, has a significant influence on the formation of the local structure and local characteristics of the wind currents. The whole variety of arising wind situations cannot be exhaustively described on the basis of typical empirical data. Therefore, the wind situation near each such projected building requires a separate study taking into account the location of neighboring projected and existing buildings.

In the process of the wind situation modelling in urban development, we note the following main tasks to be solved: calculation and analysis of wind loads on the buildings and structures under study; calculation and analysis of wind loads acting on buildings; assessment of the wind situation from the life comfort point of view and influence on the ecological state of the nearby territory.

In the case of wind loads calculating on the designed buildings, mathematical modelling simulates the distribution of external pressure along the buildings walls, which serves as a basis for further analysis of wind loads and impacts in accordance with building codes and regulations. These questions are a traditional subject of studying building aerodynamics. Analysis of these issues in the construction of real objects is regulated in sufficient detail.

Recently, issues of comfort in the surrounding area are highlighted. It is possible to single out the following main types of unfavourable situations related to the wind situation in the study area:

- the emergence of stagnant zones and zones with reduced airflow velocities, in comparison with the unperturbed wind speed. There is a poor airing of the territory, the accumulation of...
dust and harmful impurities in these zones. Particularly unfavourable situation is the weak movement of the air flow has a circular character with closed lines of current in such zones. This leads to the a through-flow absence of air through this zone;

- occurrence of the accelerated movement zones of the wind flow near the ground. Wind currents in such zones cause severe discomfort for the people (the effect of a draft). In such zones, there may be increased wind loads on the structures placed: billboards and banners, shopping tents, public transport stops, etc.;
- occurrence of the accelerated movement zones of a wind flow over buildings’ roofs. In these zones wind loads increases, both on the roofs themselves, and on the structures placed on them: billboards, antennas, etc.;
- the formation of vortex flows that can cause discomfort, and can also lead to the raising of dust and harmful impurities from the ground to the upper floors of buildings;
- the unfavourable conditions emergence of snow transport (snowdrift of low structures, roads).

We would like to separately note the problem of calculating and analyzing wind loads acting on surrounding objects. The essence of this problem is that the calculation work and assessment of the wind situation is usually carried out within the design of new facilities. The works are usually financed by the organizations involved in these facilities construction, and the customer requires the calculation and analysis of wind loads specifically for the projected facilities. It does not take into account the fact that the appearance of these new high-rise objects can greatly change the wind impact of the air flow on the existing buildings, buildings and structures built in the previous construction stages. At the same time, as it is already noted, the deceleration of the air flow can be observed, as well as its strong acceleration in the flow around buildings and structures of high altitude. This can lead to the occurrence of wind loads’ existing objects on the walls, significantly exceeding both the values regulated by building codes and rules for free-standing objects, and the values observed in a new building absence.

The authors began the work on the method development for modelling three-dimensional wind flows around buildings and structures in the 90s of the 20th century. At that time, these works were conducted under the leadership of I.K. Lifanov. The basis for the mathematical models being developed was a discrete vortex method in the concept, found by S.M. Belotserkovsky. The theoretical foundations of the mathematical model were presented in [1, 2]. The method provided for the modelling of detached flows within the model of an ideal incompressible fluid. Taking into account the computer technology capabilities of that time, the vortex approach was apparently the only one that allowed one to model three-dimensional flows in such problems.

We should note that later, already in our century, grid methods based on the Reynolds equations and various models of turbulence began to develop widely. Such models, in particular, are implemented in the currently widely used industrial packages ANSYS CFX, ANSIS Fluent, etc. The current state’s analysis of such models is given, for example in [3-5].

The authors of this article continue to develop methods for modelling wind flows based on the vortex approach. In the present article, the current state of the vortex method being developed is described. The examples of flow calculations in test problems are presented in the wind situation modelling in the flow around groups of buildings and structures. The currently used mathematical model is described in articles [6-8]. Other variants of vortex methods are described in [9-10].

2. Mathematical model

We use the unsteady vortex flows model for ideal incompressible fluid. Suppose that the buildings surfaces compile given piecewise smooth surface \( \Sigma_i \) which may be closed (the boundary of a solid object), open (a panel or other thin object) or can consist of closed and open components. It is assumed that the wind flow has a given constant speed \( \mathbf{w}_\infty \) at infinity. In the process of the initiation modelling and development of vortex wakes is considered within the assumption that vortex wake appear on bodies in flow on the predefined lines. It is usually assumed that the flow separation occurs at the
edges of buildings. Exterior to bodies and vortex wakes the flow is suggested potential. The no-flow condition is set on the body surfaces.

Let \( L \) be defined flow separation line on the surface of a body in the flow (united separation line in case of body with complex geometry or a system of bodies). Also let it be the same for each particular moment of time \( t \) for vortex wake filling a bounded domain \( D = D(t) \) (see Figure 1).

Figure 1. Modeling of body and the vortex wake.

Let \( \mathbf{w}(x,t) \) be an unknown velocity field where \( x = (x_1, x_2, x_3) \) are points in space, \( t \) is time. The motion of vortex wake is modelled within the Lagrange approach which is based on tracing trajectories of fluid particles. Let \( x(\xi, t) \) be a coordinates array of fluid particle placed in vortex domain \( D \) at the time moment \( t \), \( \xi \) is a Lagrange coordinates of the particle. Motion laws of fluid particles and vorticity \( \omega = \nabla \times \mathbf{w} \) evolution in these particles are described with equations

\[
\frac{\partial x(\xi, t)}{\partial t} = \mathbf{w}, \quad \frac{\partial \psi(\xi, t)}{\partial t} = (\mathbf{w}, \nabla)\omega
\]

where \( \psi(\xi, t) = \omega(x(\xi, t), t) \), the right sides are calculated for \( x = x(\xi, t) \).

The feature of the method developed by authors is that the vortex wake is separated in near and far zones. It is supposed that near zone can be modeled with thin surface of discontinuity of tangential component of velocity field and far zone of vortex wake is modeled as bounded flexible domain with voluminous distribution of vorticity.

Let’s suppose that it is possible to distinguish in vortex wake that part which is close to separation line and to approximate it with a thin surface \( \Sigma_2 = \Sigma_2(t) \) and let other part of vortex wake fill the domain \( \Omega = \Omega(t) \). Then full velocity field can be represented as [11]:

\[
\mathbf{w} = \mathbf{w}_\infty + \mathbf{w}_1 + \mathbf{w}_2 + \mathbf{w}_3, \quad \mathbf{w}_i = \nabla \varphi_i, \quad \varphi_i(x,t) = \int_{x_i} \frac{d}{\partial \mathbf{n}_y} F(x-y)g_i(y,t)d\sigma_y, \quad i = 1, 2,
\]

\[
\mathbf{w}_3(x,t) = \sum_{i=1}^{N\Omega} \omega_i(y,t) \times \nabla (x-y) d\mathbf{y}, \quad F(x-y) = \frac{1}{4\pi |x-y|}, \quad \nabla (x-y) = -\nabla, \quad \mathbf{F}(x-y) = \frac{1}{4\pi \left| x-y \right|^3},
\]

where \( \mathbf{w}_\infty \) is velocity in infinity, function \( \varphi_i(x,t), \quad i = 1, 2, \) is a double-layer potentials with unknown intensities \( g_i(y,t) \), \( \nabla = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3) \), \( \partial/\partial \mathbf{n}_y \) - derivative in the direction of normal to surface vector in point \( y \) calculated as a function of \( y \).

Authors use numerical model (see [6-8]), where the vortex domain \( \Omega \) is approximated with the system of discrete vortex segments \( l_i \), each of which has the beginning in the point \( x^- \) and the end in the point \( x^+ \) and has some vortex intensity \( \omega_l, \quad l = 1, \ldots, N_\Omega \). Body surface \( \Sigma_i \) is approximated with set
of cells $\sigma_i$, $i = 1, ..., N$, each cell is bounded with closed vortex line with intensity $\omega_i$, $g_i = g_i(t)$. Vortex surface $\Sigma_2 = \Sigma_2(t)$ (vortex sheet) is approximated with a set of moving cells $\sigma_i^2$, $i = 1, ..., N_2$. Velocity field is approximated with expression:

$$\tilde{w}_i(x) \approx \sum_{i=1}^{N} g_i \tilde{W}_i(x), \quad \tilde{w}_2(x) \approx \sum_{i=1}^{N_2} g^2_i \tilde{W}_2_i(x), \quad \tilde{W}_i(x) = \int_{\Delta \sigma_i} \delta(y - x) \nabla \phi (y),$$

$$w_i(x) \approx \sum_{i=1}^{N} \omega_i \left[ (x_i^1 - x_i^2) \times \nabla \phi (x_i^1, x_i^2) \right] \theta_\varepsilon (|x - x_i^1|), \quad x_i = (x_i^1 + x_i^2)/2,$$

$\theta_\varepsilon (|x|)$ is smoothing function ($\theta_\varepsilon (|x|)=1$ outside of $\varepsilon$ -neighbourhood of the point $x=0$, $\theta_\varepsilon (|x|)=0$ when $x=0$, $\varepsilon$ is small parameter). Approximation of vorticity transport laws is assured if following motion equations for the ends of vortex segments are fulfilled in the domain $\Omega$ [12-13]:

$$dx_i / dt = \tilde{w}(x_i, t), \quad i = 1, ..., N_2, \quad dg / dt = 0. \quad (1)$$

Also, similar equations describe the motion of the cells’ vertices that approximate a vortex sheet:

$$dx_{i,l} / dt = \tilde{w}(x_{i,l}, t), \quad i = 1, ..., N_2, \quad dg_i / dt = 0,$$

$x_{i,l}$ is the vertex of the cell $\sigma_i^2$, $l$ is the vertex number in the cell $\sigma_i$ [2].

In the numerical solution of the problem we assume that at each time step the positions of vortex segments that approximates vortex domain $\Omega$, the positions of cells that approximates vortex sheet $\Sigma_2$, and their intensities $\omega_i$ and $g_i^2$ respectively are known. Unknown intensities $g_i$ on the body surface are calculated from the system of linear equations representing the no-flow condition on the body surface, which is checked in the collocation points $x_i \in \sigma_i, l = 1, ..., N$:

$$\sum_{j=1}^{N} g_j \left[ \tilde{W}_j(x_i) n(x_i) \right] = - (w_1 + w_2 + w_3) n(x_i).$$

Then we shift the ends of vortex segments approximating the vortex domain in accordance to equations (1). Here the Euler scheme for time approximation is used. Then the process of a new vortex elements birth is modeled on the body surface and appending of vortex domain by them. These segments appear on predefined separation lines or on the full surface of the body [6-8]. In order to calculate pressure numerical scheme based on the analogue of Cauchy-Lagrange integral for vorticity flows proposed by G.Ya. Dynnikova [14] was developed. The ground surface was accounted by using the reflection method [2].

3. Examples of calculations

In order to assess the computational results reliability in the simulation of flow around buildings and structures, the authors carried out numerous numerical experiments where the flow around various objects was investigated and compared the results with known data. We give some of these examples.

Calculations of the flow around the models of a single building of parallelepiped shape and two buildings of parallelepiped shape located in a tandem are carried out. The conditions of the wind tunnel experiments [15] are reproduced. In the experiments, buildings models are mounted on a flat plate; the incoming flow has the uniform velocity profile with $30 \text{m/s}$ amplitude.

Figure 2 and Table 1 present the results of calculating the flow around a single building. The ground surface is modeled as a flat infinite screen impervious to air. Model parameters are as follows: $D = 0.268 \text{H}, A = 0.319\text{H}, H = 0.665\text{m}$. It is assumed that the flow separation occurs at all the building edges. A grid with 899 rectangular cells is used on the building surface. The vertical wall edges are divided into 20 equal parts, the horizontal edges were divided into 11 and 9 parts in longitudinal and transverse directions, respectively.

Figure 2 shows the shape of the building and the sensors’ location where the pressure was measured in the experiment [15]. The numbers indicate the points of the pressure coefficient measurement. Table 1 shows the values of the external pressure coefficient $C_p = 2(p - p_\infty) / (\rho w_\infty^2)$.
obtained at these points in the calculation and in the experiment, where $p$ is the pressure in the considered point, $p_\infty$ is the pressure at infinity, $\rho$ is the air density.

The calculation of the pressure distribution in the vortex zone is much more complicated than outside this zone. Figures 3, 4 show the results of calculating the flow around two buildings of rectangular shape located in a tandem. Model parameters are as follows: $L=0.6\,H_2$, $D_1=0.314\,H_1$, $A_1=0.361\,D_1$, $D_2=0.268\,H_2$, $A_2=0.319\,H_2$, $H_3=1.117\,H_1$, $H_2=0.665\,m$.

In this case, the rear with respect to the wind flow the building is in the zone of the vortex trail that appears behind the front building. Figure 4 shows the averaged time values of the external pressure coefficient at the calculated points which positions are marked by crosses in Figure 3 on the walls of the buildings in comparison with the data of the physical experiment [15]. In Figure 4, the relative height of the point $\bar{h}=h/H_i$ is marked along the $x$ axis, where $H_i$ is the height of the building, $i=1,2$ is the building number, $h$ is the height of the point location above the ground level.

Table 1. Pressure coefficient values on the surface of the single building of a parallelepiped shape.

| Point number | calculation | experiment | Point number | calculation | experiment |
|--------------|-------------|------------|--------------|-------------|------------|
| $C_p$        |             |            | $C_p$        |             |            |
| front        | 1           | 0.97       | 2            | 0.95        | 0.95       |
| wall         | 3           | 1.04       | 4            | 1.04        | 1.02       |
| $C_p$        | 5           | -0.68      | 6            | -0.63       | -0.75      |
| left wall    | 7           | -0.62      | 8            | -0.62       | -0.69      |
| back wall    | 8           | -0.68      | 10           | -0.67       | -0.70      |
| right wall   | 13          | -0.68      | 14           | -0.63       | -0.75      |
| left wall    | 15          | -0.62      | 16           | -0.62       | -0.69      |
| back wall    | 16          | -0.62      | 17           | -0.62       | -0.60      |
| left wall    | 18          | -0.62      | 19           | -0.60       | -0.54      |

Figure 2. The shape of the building and the pressure sensor locations.

Figure 3. Two building configuration (a) and the computed shape of vortex structures (b).
With the application of the developed software system which is based on the models described in [2, 6-8] over the past 20 years, more than 100 works have been performed to assess the wind situation around the designed complexes of buildings in the Moscow City. Figure 5, a shows the distribution of the wind velocity amplitude at the ground level obtained in the calculation of the flow around a buildings’ complex. Figure 5, b shows the distribution of the external pressure coefficient distribution over the surface of buildings, in the case when the incoming flow velocity profile is uniform. Note that in a case of a real non-uniform incoming velocity profile we can estimate the wind loads on the building walls by multiplying the pressure coefficient by a factor that corresponds to the value of the incoming velocity at a given height.

4. Conclusions
The application of the vortex method based on the model of ideal incompressible fluid to the problems of buildings and structures aerodynamics is described. The hypothesis is used that the viscosity of air is essential only in a thin boundary layer on the surface of buildings. In this case, the viscosity affects on the external flow and the aerodynamic forces through the formation of flow separation. In the case of flow around buildings and structures, the position of the flow separation lines coincides with edges formed by walls and roofs. In this way, it is possible to construct a very economical and effective model describing the air flow. The conducted testing showed that the model allows calculating the wind loads with a sufficient accuracy and describing the main features of the airflow around the buildings.

The considered vortex method is significantly less expensive computationally than grid methods based on turbulence modeling. On the other hand, this vortex method has some drawbacks, namely, the dissipation of vortex structures at large distances is not modeled and it is necessary to empirically determine the position of the flow separation line on a smooth surface when flow around buildings and structures of a curved shape is simulated. The method also does not allow directly modeling the flow around buildings if the incoming velocity profile is not uniform. In this case, the engineering formulas are used to calculate the wind loads depending on the altitude.
Figure 5. Distribution of the wind velocity amplitude relative to the incoming velocity (a) and pressure coefficient on the building surfaces (b).

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