Galaxy alignments: Theory, modelling and simulations

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ABSTRACT

The shapes of galaxies are not randomly oriented on the sky. During the galaxy formation and evolution process, environment has a strong influence, as tidal gravitational fields in large-scale structure tend to align the shapes and angular momenta of nearby galaxies. Additionally, events such as galaxy mergers affect the relative alignments of galaxies throughout their history. These “intrinsic galaxy alignments” are known to exist, but are still poorly understood. This review will offer a pedagogical introduction to the current theories that describe intrinsic galaxy alignments, including the apparent difference in intrinsic alignment between early- and late-type galaxies and the latest efforts to model them analytically. It will then describe the ongoing efforts to simulate intrinsic alignments using both $N$-body and hydrodynamic simulations. Due to the relative youth of this field, there is still much to be done to understand intrinsic galaxy alignments and this review summarises the current state of the field, providing a solid basis for future work.

Subject headings: galaxies: evolution; galaxies: haloes; galaxies: interactions; large-scale structure of Universe; gravitational lensing: weak

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The Universe is filled with galaxies, enormous collections of gas, dust and billions of stars all held together by gravity. Galaxies reside in the centre of dark matter haloes that contain up to 95% of the mass of the galaxy (see e.g. Coupon et al. 2015, and references therein). The current picture of structure formation is hierarchical with galaxies forming through anisotropic collapse of localised overdensities. Larger dark matter haloes are then formed by steady mass accretion of surrounding dark matter and through mergers with other haloes (including the galaxies they contain). These larger dark matter haloes contain many tens or hundreds of galaxies and are known as groups ($\sim 10^{13} M_\odot$) or

1. Introduction
clusters ($\geq 10^{14} M_\odot$), which are the largest gravitationally bound structures in the Universe (see Dodelson 2003 for an introduction to structure formation).

Given that the Universe is homogeneous and isotropic on large scales, it was commonly assumed for some time that the observed galaxies would have random orientations if a large enough sample were considered. However, it is now known that local physical processes have a strong influence on the alignment of galaxies with respect to their surrounding environment and that these alignments can be strongly anisotropic. These intrinsic alignments are not restricted simply to the orientation of the galaxy, but also to the rotational properties of galaxies. The physical processes acting on galaxies depend on many factors including whether the galaxy is early- or late-type (elliptical or spiral disc), red or blue (ellipticals with old stellar populations or spiral discs with active star formation). It is reasonable to assume that these factors may also have an impact on how the galaxy is aligned with surrounding structures (for a more complete introduction to galaxy formation and evolution, see Mo et al. 2010).

Observations are largely focused on investigating the alignments of galaxies, as opposed to dark matter haloes, since these are directly observed by telescopes. However, cosmic structures grow much larger than galactic scales and this growth depends sensitively on the properties of dark matter and dark energy in an expanding Universe. Dark matter particles are only subject to gravity and can be approximated as experiencing no elastic collisions, which would give rise to pressure or viscous forces, in strong contrast to the sub-dominant baryonic component. Dark energy influences the growth of structures through the evolution of the background density with time and the relation between time and the scale factor of the Universe. Computationally, it is far easier to determine the alignments of dark matter haloes in N-body dark matter simulations that include dark energy, since simulating baryons is still computationally expensive and it is unclear how to self-consistently model the dominant physical processes numerically over the large dynamic range involved in the problem at hand. There is a wealth of literature looking at the alignments of dark matter haloes with respect to substructures, other haloes and even the cosmic web. Even though the mass resolution of early simulations restricted the studies to cluster-sized haloes, more recent works have extended this to include group- and galaxy-sized haloes in addition to using semi-analytic models to add some of the key properties of galaxies and hydrodynamics to add baryonic processes to the simulations.

Understanding the alignments of galaxies, groups and clusters is important from a formation and evolution perspective. However, it is essential to galaxy surveys that include weak gravitational lensing as a key cosmological probe\(^1\) for a very different reason. Weak gravitational lensing exploits the correlations between the shapes of distant galaxy images, which have been distorted by the gravitational deflection of light by the intervening matter distribution (see Bartelmann & Schneider 2001 for more information on weak gravitational lensing). This distortion or shear signal can be boosted by the intrinsic alignments of galaxies which mimic the gravitational shear, or diminished by intrinsic alignments of galaxies that are anti-correlated with the gravitational shear signal. If ignored, these can bias the inference on cosmology and represent the largest astrophysical systematic for upcoming weak gravitational lensing surveys (see Kirk et al. 2015). In order to mitigate the signal from these alignments without losing a significant amount of cosmological information, it is first necessary to formulate a reasonably accurate model of the alignments. Only then will it be possible to accurately quantify the effects and mitigate the bias that they induce.

This review is part of a topical volume on galaxy alignments and it provides an introduction to the theory, modelling and simulations of alignments of structures within the Universe. Also in this volume is a basic overview of the galaxy alignments (Joachimi et al. 2015) and a more technical review on observational results, the impact on cosmology and mitigation techniques (Kirk et al. 2015).

This review first defines alignment observables in Section 2, to give context to the alignments being modelled or simulated throughout. The large-, small- and intermediate-scale theories and models for alignments are then reviewed in Section 3. Section 4 summarises the results of alignments in N-body simulations, while Section 5 reviews results from hydrodynamic simulations. Section 6 introduces semi-analytic modelling and this is followed by a roadmap or wish list for future investigations into galaxy intrinsic alignments, Section 7, and final remarks in Section 8.

\(^1\)e.g. the Kilo Degree Survey, KiDS: http://kids.strw.leidenuniv.nl; the Dark Energy Survey, DES: http://www.darkenergysurvey.org, and the Hyper Suprime-Cam Survey, HSC: http://www.naoj.org/Projects/HSC
2. Observables

In this section, the observable quantities that are measured in real data and predicted theoretically are defined. These include galaxy shapes (Section 2.1), the relative angles of interest between the astronomical structures (Section 2.2), and 2-point correlation functions (Section 2.3).

2.1. Galaxy shapes

When observing the sky, it is not possible to observe the full three-dimensional shapes of galaxies. The shapes measured are inherently two-dimensional projections of the three-dimensional shape. While galaxies and dark matter haloes do not in general have elliptical isophotes or isodensity contours, it is common to describe their shapes in terms of the effective ellipticity at some radius (or averaged over a range of radii, as in weak lensing; see Kirk et al. 2015). The observed ellipticity, $\epsilon$, can be split into two components

$$\epsilon \approx \epsilon^i + \gamma,$$

where $\epsilon^i$ is the intrinsic ellipticity that corresponds to the true shape of the light distribution, and $\gamma$ is the gravitational shear, a distortion to the galaxy image produced by intervening matter along the line of sight to the galaxy in question (see Joachimi et al. 2015 for a discussion of the approximation in this equation). There is a fundamental difference between $\epsilon^i$ and $\gamma$ in that $\epsilon^i$ describes the deviation of the true shape of the galaxy itself from circularity, while $\gamma$ is a distortion to the observed galaxy shape, and will depend on the matter distribution through each line of sight.

Ellipticities are tensorial quantities that have, in complex notation, two components (note that Equation 1 is also an equation between complex numbers). It is standard to define a fixed coordinate frame on the sky, with a position angle, $\varphi$, defining the angle of the semi-major axis of the ellipse from one axis of that coordinate frame. If the total magnitude of the ellipticity is $|\epsilon|$ as defined in terms of the semi-minor to semi-major axis ratio $q$ (see, e.g., Bernstein & Jarvis 2002 for many common ellipticity definitions), then the two components of ellipticity in this coordinate system can be defined as $\epsilon_1 = |\epsilon| \cos 2\varphi$ and $\epsilon_2 = |\epsilon| \sin 2\varphi$, with the complex ellipticity denoted $\epsilon = \epsilon_1 + i\epsilon_2$.

The notion of spatial ellipticity correlations and their link to correlation functions is illustrated here with the following model: The ellipticity of every galaxy can be seen as being drawn randomly from some underlying distribution, with a non-vanishing correlation function describing the dependence of the random processes assigning ellipticities to two neighbouring galaxies. If $\epsilon$ and $\epsilon'$ are the ellipticities of galaxies at positions $\alpha$ and $\alpha'$ respectively, their ellipticities are drawn from a multivariate Gaussian distribution $p(\epsilon', \epsilon)$, which can be viewed as a conditional distribution $p(\epsilon'|\epsilon)$, such that the outcome $\epsilon'$ depends on the value of $\epsilon$. As a multivariate distribution $p(\epsilon, \epsilon')$,

$$p(\epsilon, \epsilon') = \frac{1}{\sqrt{(2\pi)^d \det(C_{\epsilon})}} \exp \left[ -\frac{1}{2} (\begin{pmatrix} \epsilon \\ \epsilon' \end{pmatrix} - \begin{pmatrix} \epsilon \\ \epsilon' \end{pmatrix}^*) C_{\epsilon}^{-1} \begin{pmatrix} \epsilon \\ \epsilon' \end{pmatrix}^* \right],$$

this property is encoded in the covariance matrix $C_{\epsilon}$,

$$C_{\epsilon} = \begin{pmatrix} \langle \epsilon \epsilon' \rangle & \langle \epsilon \epsilon'' \rangle \\ \langle \epsilon' \epsilon' \rangle & \langle \epsilon' \epsilon'' \rangle \end{pmatrix}.$$  

(3)

Note that, since $\epsilon$ is a complex number, the covariance $C_{\epsilon}$ is Hermitian, with the asterisk denoting the complex conjugate and the plus sign in Equation (2) denoting the Hermitian conjugate. The two variances $\langle \epsilon^2 \rangle$ and $\langle |\epsilon'|^2 \rangle$ are equal in homogeneous random fields, because the fluctuation properties are identical everywhere. The off-diagonal element is the correlation function $\xi_\epsilon = \langle \epsilon \epsilon'' \rangle$, which is invariant under translation in homogeneous fields. It generally decreases with increasing distance due to the Cauchy-Schwarz inequality, $\xi_\epsilon \leq \langle |\epsilon|^2 \rangle$, and describes how rapidly the

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2 The term gravitational shear could be misleading here since the intrinsic ellipticity, $\epsilon^i$, is also influenced by (tidal) gravitational effects as shown in this section. However, the term gravitational shear for the effects of gravitational lensing has been in use for a long time and it would be unwise to adopt a different convention here. Throughout, the term gravitational shear is synonymous with gravitational lensing.

3 If a two-component ellipticity seems unfamiliar, it is worth considering that the standard geometric representation of an ellipse using an axis ratio and a position angle also requires two numbers.
In this particular convention, the component at 0° of the reference axis (both polar angles are measured in the same coordinate system). The two ellipticity components can then be defined as

\[
\begin{align*}
\epsilon_\varphi &= -|\epsilon| \cos \left[ 2 (\varphi - \varphi_0) \right]; \\
\epsilon_\theta &= -|\epsilon| \sin \left[ 2 (\varphi - \varphi_0) \right],
\end{align*}
\]

where |\epsilon| is the absolute value of the complex ellipticity, \varphi the polar angle of the ellipticity, and \varphi_0 the polar angle of the reference axis (both polar angles are measured in the same coordinate system). \epsilon_\varphi measures the ellipticity component at 0° and 90° from the reference axis, while \epsilon_\theta measures the ellipticity component at 45° and 135° from it. In this particular convention, \epsilon_\varphi > 0 represents tangential alignment, and \epsilon_\varphi < 0 radial alignment, with respect to a fluctuating field loses memory of its value \epsilon at \alpha when increasing the distance to \alpha'. If the random field is isotropic, the random field's correlation properties do not change under rotations, and consequently \epsilon only depends on the distance \theta \equiv |\alpha - \alpha'|. The averaging brackets \langle \ldots \rangle denote ensemble averages over statistically equivalent realisations of the random field \epsilon(\alpha).

### 2.2. Alignment angles and types

There are two common alignments of position angles that are considered theoretically or observationally: the alignment of the shapes of two objects, or the alignment of the shape of an object with the position of another object. In both cases it is common to relate any ellipticity to a reference axis such as the line connecting a pair of objects, which makes this measurement independent of the coordinate system used. The two ellipticity components can then be defined as

\[
\begin{align*}
\epsilon_\varphi &= -|\epsilon| \cos \left[ 2 (\varphi - \varphi_0) \right]; \\
\epsilon_\theta &= -|\epsilon| \sin \left[ 2 (\varphi - \varphi_0) \right],
\end{align*}
\]

Table 1: Intrinsic alignment observable misalignment angles, \theta, for shape (upper case letter) and position (lower case letter) of dark matter haloes (H), central galaxies (C) satellites (or dark matter subhaloes; S) the shape of the satellite distribution (which is defined by the satellite positions; B), a complete galaxy sample (includes centrals and satellites; G), walls (W), filaments (F) and the position of the centre of a void (v). Note that not all of the alignments listed above are shown in Figures 2 to 4. However, if the alignment listed is not explicitly shown, the figure is denoted with a * and it should be straightforward to infer the alignment from the named figure.

| Shape of          | Aligned with                                      | Symbol | Figure(s) | Equation(s) | Section(s) or Paper |
|-------------------|---------------------------------------------------|--------|-----------|-------------|---------------------|
| Halo              | Shape of itself at different radius               | \theta_H(R_1, R_2) | 2a, 10, 15a | (4)         | Sections 4.2.2 and 5.1.1 |
| Halo              | Halo Shape                                        | \theta_H | 3b        | (4)         | Section 4.2.4       |
| Halo              | Halo Position                                     | \theta_B | 3b        | (4)         | Sections 4.2.4 and 4.2.5 |
| Halo              | Sheet/Wall Shape                                  | \theta_W | 4a        | (4)         | Section 2.5         |
| Halo              | Void Position of Centre                           | \theta_V | 4a        | (4)         | Section 2.5         |
| Halo              | Filament Shape                                    | \theta_F | 4b        | (4)         | Section 2.5         |
| Central           | Halo Shape                                        | \theta_H | 3b, 13a   | (4)         | Sections 5.1.1, 5.2.2 and 6 |
| Central           | Central Shape                                     | \theta_C | 3b        | (4)         | Kirk et al. (2015)  |
| Central           | Central Position                                  | \theta_C | 3b        | (4)         | Section 5.2         |
| Central           | Satellite Distribution Shape                      | \theta_B | 3b        | (4)         | Section 5.2         |
| Satellite         | Halo Position of Centre                           | \theta_B | 3a, 12    | (4)         | Sections 4.2.3 and 5.1.2 |
| Satellite         | Halo Shape                                        | \theta_H | 3a        | (4)         | Sections 4.2.4 and 4.2.5 |
| Satellite         | Central Position                                 | \theta_C | 3a        | (4)         | Kirk et al. (2015)  |
| Satellite         | Central Shape                                     | \theta_C | 3a        | (4)         | Kirk et al. (2015)  |
| Satellite         | Satellite Shape                                  | \theta_B | 3b        | (4)         | Kirk et al. (2015)  |
| Satellite         | Satellite Distribution Shape                      | \theta_B | 3b        | (4)         | Kirk et al. (2015)  |
| Galaxy            | Galaxy Shape                                      | \theta_B | 3a, 3b, 8 | (4)         | Sections 5.2.1, 5.2.2 and 6 |
| Galaxy            | Galaxy Position                                  | \theta_B | 3a, 14    | (4)         | Section 5.2.2       |
| Galaxy            | Wall Shape                                        | \theta_W | 4a        | (4)         | Section 5.1.3       |
| Galaxy            | Void Position                                    | \theta_V | 4a        | (4)         | Kirk et al. (2015)  |
| Galaxy            | Filament Shape                                   | \theta_F | 4b        | (4)         | Section 5.1.3       |
Fig. 1.— Parity-even $E$-modes (top row) and parity-odd $B$-modes (bottom row) of the ellipticity field. Measured with respect to a radial line from the centre of the pattern, one observes $\epsilon_+ > 0; \epsilon_\times = 0$ (top left), $\epsilon_+ < 0; \epsilon_\times = 0$ (top right), $\epsilon_+ = 0; \epsilon_\times < 0$ (bottom left), and $\epsilon_+ = 0; \epsilon_\times > 0$ (bottom right), in the convention given by Equation (4).

line connecting the galaxy to a reference point, e.g. the centre of a galaxy cluster$^4$. This is illustrated in Figure 1.

In Figures 2 to 4 a selection of the possible alignments are represented, while additional alignments not explicitly shown in the figures are listed in Table 1. Not all combinations of alignments are useful to measure, so if a possible alignment is absent, it is likely not measured in simulations or observations. Alignments of the spin axis are not shown in Figures 2 and 3 or in Table 1, but they can be inferred easily from the figures.

Alignments can be considered on a wide range of scales. In the following discussion, Table 1 and Figures 2 to 4, (mis)alignment angles, $\theta$, are given two sub-indices for either shape (upper case letter) or position (lower case letter), for a central galaxy (C), satellite galaxy (S), satellite distribution$^5$ (B), dark matter halo (H) or a full galaxy sample (G; which includes both satellite and central galaxies), wall (W), filament (F) or the position of the centre of a void (v). The first and second of these indices denote the object in the first and second columns of Table 1, respectively. A ‘,2’ in the sub-index specifies that the alignment is over two haloes, when it may not otherwise be obvious.

The large-scale structure of the Universe can be classified into four separate components called cosmic web elements - typically it is divided in to clusters, filaments, sheets/walls$^6$ and voids. In the most basic classification of

$^4$There are a variety of sign conventions in the literature. Since gravitational lensing tends to cause tangential alignment and intrinsic alignments are primarily radial, some studies will define the signs such that intrinsic alignments are negative and others such that they are positive. For example, in Figure 8 the latter convention was used.

$^5$Note that the terms satellite and satellite distribution are interchangeable with subhalo and subhalo distribution when measuring dark matter alignments.

$^6$The terms “sheet” and “wall” are used interchangeably throughout the literature and this review.
the large-scale structure, sheets are planes of structure that delineate the edge of voids, which are underdense and often modelled as spherical or ellipsoidal. Filaments are cylindrical structures (to first approximation) and clusters, the nodes where filaments meet, are modelled as ellipsoidal. A more quantitative description of web classification is given in Section 4.1.4.

The smallest scale of an alignment is completely internal to a galaxy system. Figure 2 shows the internal alignments of a dark matter halo at different radii, $\theta_{H}(R_1, R_2)$, where $R_i$ is the radius of the halo shells being aligned. In this example, the inner dark matter halo is being aligned with halo shells at larger radii. This 3D alignment is measured in simulations and the radius may be an absolute radius relative to the halo centre or an isodensity radius; see Section 4.2.2. Dark matter haloes exist over a huge range of scales, but for the purposes of this work, the scales that are interesting range from subhaloes ($\sim 10^{10} M_{\odot}$) through to clusters ($\gtrsim 10^{14} M_{\odot}$). The dominant, most massive galaxy near the centre of a galaxy sized dark matter halo is known as the central galaxy.

It is common for a dark matter halo with a central galaxy to contain a number (varying between zero and tens) of smaller satellite galaxies (each surrounded by a dark matter subhalo) as shown in Figure 3a. These are less massive than the central galaxy and reside in positions throughout the host halo, rather than at the centre. In this figure the smaller ellipses can be considered either a satellite galaxy or a subhalo, depending on the measurement of interest. On larger scales, dark matter group and cluster haloes contain many tens and hundreds of galaxy sized systems. In this case, the objects inside the larger halo (centrals, satellites, subhaloes and galaxy-sized dark matter haloes) are all considered members of the substructure.

The shape of the satellite distribution is defined by identifying the positions of the satellites within the halo and fitting a shape (typically an ellipsoid) to their distribution. In this case, the shape of the individual satellites play no role in defining the shape of the satellite distribution. Figure 3b shows alignments within and between two haloes. The shape of the satellite distribution is sometimes used as a proxy for the shape of the dark matter halo observationally. Consequently, $\theta_{bh}$ and $\theta_{bb}$ are considered equivalent to $\theta_{Hh}$ and $\theta_{HH}$ (although the simulation literature does find differences in the shape and orientation of the satellite distribution compared with the dark matter, see Sections 4.2.3, 5.1.2 and 5.2.2).

Figure 4 shows alignments between dark matter haloes, walls, voids and filaments (the clusters are simply massive dark matter haloes, see Figure 3). Also shown in these figures is the orientation of the dark matter halo spin vector, $\lambda_{H}$, with the cosmic web elements.
Fig. 3.— (Mis)alignments, $\theta$, in one- and two-haloes. Both: Thick solid and dashed lines show the semi-major axes of the dark matter haloes (grey ellipses; H) and central galaxies (larger dark green or blue ellipses; C). Thin solid lines show the semi-major axes of the satellite distribution (light green or blue ellipses; B). Left: One-halo - The thin dashed lines show the semi-major axes of the satellites (small dark green ellipses; S). The thick dotted line shows the vector connecting the satellite and the centre of the dark matter halo. Right: Two-halo - The dotted line shows the vector connecting two central galaxies. See Table 1 for a complete list of alignments.

2.3. Ellipticity 2-point correlations

When considering sets of objects for which ellipticity correlation functions are to be measured, components of the ellipticity, $\epsilon_+$ and $\epsilon_\times$, are typically defined in a coordinate frame aligned with the separation vector, $\theta$, between the two galaxies. This involves rotating the two original components of ellipticity in the sky frame ($\epsilon_1$ and $\epsilon_2$; see Equation 4). The correlation functions $\xi_\pm$ and $\xi_\times$ of these two new ellipticity components are defined as

$$\xi_\pm(\theta) = \langle \epsilon_+ \epsilon'_\pm(\theta) \rangle \pm \langle \epsilon_\times \epsilon'_\times(\theta) \rangle,$$

while the third correlation $\langle \epsilon_+ \epsilon'_\times(\theta) \rangle = \langle \epsilon_\times \epsilon'_\times(\theta) \rangle$ is parity-odd and expected to vanish due to the parity symmetry of the Universe.

Considering the variance of Fourier modes of the random field, defined in Section 2.1, is a particularly useful concept for homogeneous random fields, because in these cases, the Fourier modes with different wave vectors are uncorrelated, while the variance of equal wave vectors is related to the power spectrum. Constructing power spectra (originally used in the context of polarisation correlations in the cosmic microwave background; see Kamionkowski et al. 1997; Seljak 1997) of the projected galaxy ellipticity field $\epsilon(\alpha)$ yields two parity eigen-modes for the angular power spectrum as a function of multipole $\ell$,

$$C_{E,B}^\epsilon(\ell) = \pi \int_0^\pi d\theta \left[ \xi_+(\theta) J_0(\ell \theta) \pm \xi_\times(\theta) J_4(\ell \theta) \right],$$

with the Bessel functions of the first kind, $J_0$ and $J_4$. The physical interpretation of these parity eigen-modes is illustrated in Figure 1. The generation of parity-odd $B$-modes is a typical feature of intrinsic ellipticity models, but not gravitational lensing (to lowest order).

While the formalism has been outlined for angular correlations of fields at two different positions on the sky, it generalises to correlations of shapes in three dimensions in a straightforward way. Commonly, the correlations of the aligning fields are formulated in three dimensions (using physical separations $r$ derived using known redshifts), though still using the projected (2D) shape. For example, $\xi_{E}(r)$ defines the correlation function of projected galaxy
ellipticities with the positions of galaxy overdensities as a function of 3D separation $r$. However, note that the assumption of isotropy is not a very good one in this case, since it is the unobserved 3D shapes that should correlate with an equal strength to galaxies at separate $r$, not the projected 2D shapes. Moreover, at the stage of expressing the relation between redshifts and distances, complications such as redshift-space distortions enter, causing misestimates of radial distances. Observationally, it is difficult to model the impact of redshift space distortions on 2-point correlation functions, particularly on scales where non-linear density perturbations are important (see, e.g., Kaiser 1987 for a review of redshift space distortions in general, or Singh et al. 2014 for a derivation of their lowest-order impact on intrinsic alignment 2-point correlations). Without a good model, the three dimensional ellipticity correlation functions are difficult to interpret. To avoid both of these complications (anisotropy and redshift-space distortions), a two-dimensional ellipticity correlation function for galaxies as a function of their transverse separation on the sky $r_p$ is commonly calculated,

$$w^g_e(r_p) = \int_{-\Pi_{\text{max}}}^{+\Pi_{\text{max}}} \xi^g_{\epsilon\epsilon}(r_p, \Pi) d\Pi,$$

(7)
determined by projecting the equivalent three-dimensional correlation function $\xi^g_{\epsilon\epsilon} = \langle \epsilon g \rangle$ between ellipticity and the galaxy density along the line-of-sight ($\Pi$ is the separation in the redshift direction). This projected correlation function can similarly be found for other combinations of observables including the ellipticity components $+, x, \epsilon, g$, the galaxy overdensity with respect to the mean, $g = \rho_g/\bar{\rho}_g - 1$, and $\delta$, the matter overdensity with respect to the mean, $\delta = \rho/\bar{\rho} - 1$. The densities $\rho_g$ and $\rho$ are the galaxy density and the matter density respectively.

Identifying intrinsic galaxy alignments is particularly important for upcoming weak lensing surveys, because they mimic the coherent shape distortions resulting from gravitational shear. A correlator of two galaxy ellipticities can be taken directly from the correlation in Equation (1),

$$\langle \epsilon \epsilon' \rangle = \langle \gamma \gamma' \rangle + \langle \epsilon' \epsilon'' \rangle + \langle \gamma \epsilon'' \rangle + \langle \epsilon' \gamma' \rangle.$$

(8)

The left side of the equation is the correlator of the observed ellipticities. GG is the gravitational lensing shear correlation (the signal that is most important in weak lensing analyses; see Joachimi et al. 2015 for a more comprehensive introduction to weak gravitational lensing). II is the correlation between the intrinsic shapes of two galaxies, and GI
is the correlation between the gravitational shear of one galaxy and the intrinsic shape of the other galaxy. Only one of the GI terms is non-zero because any gravitational shear associated with a galaxy closer to the observer can not be correlated with the intrinsic shape of a galaxy further away from the observer\footnote{This may not be true in the presence of photometric redshift errors, where the relative positions of galaxies along the line of sight may be confused.}.

3. Theory and modelling

The alignments of galaxies with cosmic structures is a phenomenon that occurs on a variety of scales. In general these alignments are thought to be sourced through tidal interactions of galaxies with the gravitational field of larger structures. Fundamentally, alignments fall into three regimes: On very large scales above $\sim 10$ Mpc, haloes or spins are tidally aligned with the linearly evolving cosmic large-scale structure (Section 3.1). On scales below $\sim 1$ Mpc, galaxies may align themselves with their host halo and this particular alignment can be described with the halo model (Section 3.2). Intermediate scales of a few Mpc are difficult to grasp due to non-linearly evolving structures and effects due to clustering and a strong peculiar motion contribution to the galaxy redshifts; how to model the influence of these effects on galaxy alignments is a topic of current research (Section 3.3).

There are two theories commonly employed to explain how tidal interactions determine the alignment and hence the shape of a galaxy: stellar ellipsoids of elliptical galaxies may be tidally distorted (Section 3.1.2), and the orientation of stellar discs in spiral galaxies may be determined by the angular momentum direction, which in turn follows from tidal interactions (Section 3.1.3). While tidal interaction processes on large scales may be described by perturbation theory (Section 3.1.1), the shape and orientation of the stellar distribution inside a halo is a more complicated problem that requires numerical simulations to solve (see Sections 4 to 6). It should be emphasised that alignment models, which mediate between an aligning large-scale field and an observable galaxy shape, may be based on fields other than the tidal gravitational fields.

3.1. Large-scale alignments

3.1.1. Tidal interactions of haloes

Tidal interactions of galaxies with the cosmic large-scale structure can be modelled as a perturbative process. Consider the position, $x_i(q, a)$, of a particle as a function of scale factor, $a$, and initial position, $q$. To lowest order, the positions follow straight lines over time, along a direction determined by the gradient, $\partial_i \Psi$, of the displacement potential, $\Psi$, (Zel’dovich 1970),

$$x_i(q, a) = q_i - D_+ (a) \partial_i \Psi(q) ,$$

where $D_+ (a)$ is the growth function. The interaction of an entire halo is obtained by Taylor-expanding the trajectories around the centre of gravity, $\bar{q}$,

$$x_i(q, a) \approx q_i - D_+ (a) \left[ \partial_i \Psi(\bar{q}) + \sum_j (q_j - \bar{q}_j) \partial_i \partial_j \Psi(\bar{q}) \right] ,$$

which reveals the bulk motion of the halo along $\partial_i \Psi(\bar{q})$ and the differential motion of the particles around the centre, which is encoded in the tidal field tensor, $\partial_i \partial_j \Psi(\bar{q}) \equiv T_{ij}(\bar{q})$. The displacement potential is related to the Newtonian gravitational potential, $\Phi$, by a factor of $4\pi G$, where $G$ is Newton’s gravitational constant. It should be noted that in this picture the halo is treated as a test object embedded into a tidal field, whereas in reality the tidal fields themselves determine which particles will ultimately compose the halo.

The haloes principal axis frame defines a coordinate frame into which the tidal field, $\partial_i \partial_j \Phi$, can be decomposed. The component of the tidal field coinciding with the haloes principal axis frame gives rise to an anisotropic deformation as illustrated by Figure 5a. This effect is used to explain the alignment of elliptical galaxies with the large-scale structure and is the basis of the linear alignment model for elliptical galaxies.

If the tidal field tensor does not coincide with the principle axis frame of the halo, a shearing motion is exerted onto the halo, which ultimately leads to angular momentum generation, as shown by Figure 5b. As long as merging
Fig. 5.— Illustration of tidal stretching and torquing. Left: If a halo (green ellipse) is embedded into a tidal gravitational field, it is tidally stretched to a new shape (yellow ellipse): The tidal shear displacement (red arrows) is projected into the coordinate frame defined by the halo principal axes (black arrows), such that the resulting tidal fields cause a contraction of the halo in one direction and an expansion of the halo in the other (green arrows). Right: Embedding the halo (green ellipse) into a tidal field which varies across the halo (blue arrows) gives rise to two effects: First, there is a change in shape of the halo caused by the components of the tidal shear inside the principal axis system of the halo, leading to an anisotropic change in shape (green arrows). Second, there is a shearing deformation of the halo (red arrows), which generates angular momentum.

or accretion do not play an important role, the galactic disc can be expected to form perpendicular to the halo angular momentum direction: This is the picture behind the quadratic alignment of spiral galaxies.

3.1.2. Linear alignment model for elliptical galaxies

Elliptical galaxies are supported by the velocity dispersion of the dark matter particles and of the stars, which can be assumed to be in virial equilibrium. If an elliptical galaxy is embedded in a tidal gravitational field, it will distort the potential of the galaxy’s dark halo and force the distribution of stars to assume a new equilibrium configuration, resulting in a new shape as illustrated by Figure 6. The reaction of an elliptical galaxy to an external tidal field can be considered instantaneous because the dynamical time scale of stars inside the galaxy should be short compared to the time scale on which tidal fields change due to structure formation, such that the distortion of the stellar quadrupole grows with the growth function $D_+$. The intrinsic complex shear, $\gamma'$ (see Joachimi et al. 2015 for an introduction to $\gamma'$), in this model is proportional to the tidal shear field projected onto the sky,

$$\gamma' = -\frac{C_1}{4\pi G} \left( \partial^2_x - \partial^2_y + 2i \partial_x \partial_y \right) S(\Phi),$$

(11)

where $G$ is Newton’s gravitational constant and $x$ and $y$ are Cartesian coordinates in the plane of the sky. Smoothing of the gravitational potential, as indicated by $S(\Phi)$, on the scale of the halo ensures that the halo as a whole reacts to tidal fields and avoids substructures that would be introduced by small-scale fluctuations of $\Phi$. The derivatives $(\partial^2_x - \partial^2_y) S(\Phi)$ and $(2i \partial_x \partial_y) S(\Phi)$ (with the prefactor) give the tangential and cross components of the shear with respect to the $x$ axis. $C_1$ is a constant of proportionality containing information on the strength of the reaction that a galaxy experiences as it adjusts its shape due to an external tidal shear field. Bridle & King (2007) measured this through ellipticity correlations in SuperCOSMOS data (Brown et al. 2002) and found $C_1 \approx 5 \times 10^{-14} (h^2 M_\odot \text{Mpc}^{-3})^{-1}$. The relation between ellipticity and tidal shear is, in the limit of weak fields, a linear one because in the case of strong fields
the galaxy would be tidally disrupted; it is from this linearity that linear alignments derive their name (Catelan et al. 2001). Physically, the dependence of the distorted galaxy shape on the second derivatives of the gravitational potential, corresponds exactly to the tidal stretching illustrated in Figure 5a. In addition, assuming a linear relationship between the quadrupole of the brightness distribution, which is a symmetric tensor of rank 2, and the tidal shear transverse to the line of sight, which is a tensor of the same type, is reasonable motivation for Equation 11 (Hirata & Seljak 2004, 2010)\textsuperscript{8}.

Hirata & Seljak (2004) related the gravitational potential at an early time in the galaxy’s evolution to the matter density contrast, $\delta$, on linear scales via the Poisson equation,

$$\Phi(k) = -4\pi G \frac{\bar{D}(z)}{D(z)} a^2 k^2 \delta(k),$$

(12)

where $\bar{D}(z) \propto (1+z) D_*(z)$ is the scaled growth factor that is normalized to unity during matter domination, $a$ is the scale factor, $k$ is the wave vector and $k$ is the wavenumber of the wave vector. The mean background density, $\bar{\rho}(z) = \Omega_m(z) \rho_{\text{crit}}(z)$, where $\rho_{\text{crit}}(z)$ is the critical density of the Universe, sets the strength of the gravitational potential. The critical density of a smoothed background that produces a spatially flat Universe is,

$$\rho_{\text{crit}}(z) = \frac{3H(z)^2}{8\pi G},$$

(13)

where $H(z)$ is the Hubble constant at redshift $z$.

Hirata & Seljak (2004) assumed that the ‘primordial’ tidal gravitational field acting during the formation of the galaxy determines the galaxy alignment, which would then be frozen in during the subsequent evolution. As this model is mostly applicable to elliptical galaxies, which are believed to undergo dramatic changes in their morphology during mergers at relatively recent times, it may be more valid to instead assume a quasi-instantaneous response to the local tidal field at any given time, which would lead to a different redshift scaling of the alignment signal amplitude. However, the redshift dependence predicted by Hirata & Seljak (2004), which we adopt in the following, is consistent with current observations (Joachimi et al. 2011).

Irrespective of whether the density field is in a linear or non-linear stage of evolution, the link between the observable $\gamma^I$ to the matter density power spectrum $P_{\delta \delta}^\text{lin}(k)$, is best formulated in Fourier space. Due to clustering, the

\textsuperscript{8}Note that Hirata & Seljak (2010) is an updated version of the original Hirata & Seljak (2004) paper that fixes some errors in the equations of the original paper.
density-weighted intrinsic shear, $\tilde{\gamma}'$, is what is actually observed, which is given by

$$\tilde{\gamma}'(k) = \frac{C_1 P}{D} k^2 a^2 \int \frac{\delta^3(k_2)}{k_2^2} \left( \frac{2k_2}{2k_1} \right) \delta_D(k_1) \frac{b_g}{(2\pi)^3} \tilde{\gamma}(k_1) d^3k_1,$$

where $k_2 = k - k_1$, $\delta^3$ is the 3D Dirac delta function and $b_g$ is the linear biasing factor, which relates the relative fluctuation in the number density of galaxies to the local dark matter density. Note that the multiplicative density weighting becomes a convolution in Fourier space with $k_1$ as the integration variable, such that the observable $\tilde{\gamma}'$ depends on $k$. The derivatives of the gravitational potential translate in Fourier space into multiplications with the corresponding components of the wave vector, in this case $k_2$. In this way, the resulting II power spectrum $P_{II}(k)$ of $\tilde{\gamma}'$ is

$$P_{II}(k) = \frac{C_1 P}{D} k^2 a^2 \left\{ P_{\delta\delta}(k) + b_g^2 \int [f_x(k_2) + f_z(k_1)] \times f_x(k_2) \frac{P_{\delta\delta}(k_1) P_{\delta\delta}(k_2)}{(2\pi)^3} d^3k_1 \right\},$$

where $f_x(k)$ is a geometric function that singles out correlations between the E-modes of the ellipticity field (Hirata & Seljak 2004). Note that the B-mode correlations are zero, due to the symmetry of the tidal shear tensor.

The second term in brackets in Equation (15) is caused by the density weighting and is proportional to the square of the linear matter power spectrum, $P_{\delta\delta}(k) P_{\delta\delta}(k_2)$. It is therefore sub-dominant compared to the first term and is usually ignored in the literature when the linear alignment model is applied. Being linear in the tidal field, there is a cross-correlation between alignments and weak gravitational lensing (Hirata & Seljak 2004). While the correlation between shapes in the linear alignment model is necessarily positive, the cross-correlation between the overdensity and the density-weighted intrinsic shear is in general negative because mass overdensities tangentially align the lensed images of background objects while radially aligning local galaxies if the overdensities are massive enough to define the principal axis of the local tidal quadrupole. The corresponding power spectrum reads

$$P_{\delta\delta}(k) = -\frac{C_1 P}{D} a^2 P_{\delta\delta}^{lin}(k),$$

which is important to both weak cosmic shear (Hirata & Seljak 2004; Hirata et al. 2004) and galaxy-galaxy lensing (Blazek et al. 2012).

Correlations of the linear type can be quite long-ranged: Ellipticity auto-correlations (II) have been measured in the Sloan Digital Sky Survey (SDSS; York et al. 2000) to reach out to 30 $h^{-1}$Mpc (Okumura et al. 2009), and cross-correlations between shape and density to almost 100 $h^{-1}$Mpc (e.g. Mandelbaum et al. 2006, and see also Kirk et al. 2015 for a comprehensive list of observations). The shape-density correlations give rise to intrinsic shape-lensing (GI) correlations (see Equation 8), although these have yet to be observed directly.

The linear alignment model is characterized by a single parameter that sets the strength of the external tidal field in relation to the ellipticity of the distorted galaxy. Recent analytical studies found that tidal stretching on a stellar structure in equilibrium may not be strong enough to explain the observed alignments of elliptical galaxies (Camelio & Lombardi 2015). It should be noted that even for the case where the particular model for tidal stretching is incorrect, it may still effectively describe other alignment mechanisms due to its generality.

3.1.3. Quadratic alignment model for spiral galaxies

Spiral galaxy alignments are explained by the alignment of their angular momentum with the tidal field of the large-scale structure, which occurs due to tidal torquing. If the symmetry axis of the galactic disc follows the angular momentum direction of the host halo, the observer will measure ellipticities which depend on the angle of inclination of the galactic disc. In the case where the angular momentum points toward the observer, the galaxy is viewed face on and will have a small ellipticity, in contrast to the case where the angular momentum is perpendicular to the line of sight, implying that the galactic disc is viewed edge on and will have a high ellipticity.

The main uncertainties of this model are the extent to which the angular momentum can be predicted by linear tidal torquing and the orientation of the stellar disc relative to the angular momentum direction of the host halo. In fact, there may be strong deviations in the latter related to merging, which can reorient the angular momentum, and
Fig. 7.— Two haloes (green) are embedded in a gravitational potential (grey sheet). They experience tidal torquing and build up correlated angular momentum directions, due to the long-range correlation of the tidal fields. Galactic discs (blue) will be oriented according to the angular momentum direction, leading to correlated shapes.

dissipative processes, which can destroy angular momentum. Tidal torquing, however, has been shown to predict the angular momentum direction of haloes reasonably well, which matters in this context, but would fail at predicting the correct angular momentum magnitude (Catelan & Theuns 1996).

The tidal torquing mechanism combines halo inertia and gravitational tidal shear to the angular momentum, meaning that the angular momentum resulting from the same tidal fields can be different depending on the halo shape. In order to capture this, Lee & Pen (2000) introduced a model that gives a Gaussian distribution $p(J|T)$ of angular momenta $J$,

$$p(J|T) = \frac{1}{\sqrt{(2\pi)^3 \text{det}(C)}} \exp\left\{-\frac{1}{2} \sum_{a,a'} J_a (C^{-1})_{aa'} J_{a'} \right\},$$  

(17)

conditional on the tidal shear $T$. The conditionality of the Gaussian distribution is incorporated in the covariance matrix $C_{aa'} = \langle J_a J_{a'} \rangle$, which also depends on the tidal shear,

$$\langle J_a J_{a'} \rangle = \langle J^2 \rangle \left\{ \frac{1}{3} \left( \frac{a_T}{3} \delta_{aa'} - a_T \sum_{\sigma} \hat{T}_{a\sigma} \hat{T}_{\sigma a'} \right) \right\},$$  

(18)

where $\delta_{aa'}$ is the Kronecker $\delta$. The model is characterised by the misalignment parameter, $a_T$, which allows the variation between random angular momenta for $a_T = 0$ to maximally aligned angular momenta for $a_T = 3/5$. $\hat{T}$ is the unit-normalised ($\hat{T}_{a\sigma} \hat{T}_{\sigma a} = 1$), traceless ($\text{tr} \hat{T} = 0$) tidal shear tensor, which can easily be derived using

$$\hat{T}_{aa'} = T_{aa'} - \frac{\text{tr}(T)}{3} \delta_{aa'},$$  

(19)

and rescaling $\tilde{T} = T/|T|$ with $|T| = \sqrt{T_{a\sigma} T_{\sigma a}}$. Squaring the tidal shear gives rise to short-ranged correlations in the angular momentum directions, and ultimately in the derived galaxy shapes (Schäfer & Merkel 2012). This picture is illustrated in Figure 7, where two neighbouring haloes are subject to correlated tidal torquing of their motion along the gradient of the gravitational potential. They build up correlated angular momenta, which determines the orientation of their discs and therefore, their shapes.

Under the assumption that the symmetry axis of the galactic disc follows the angular momentum direction of the host halo, it is possible to write down relationships of the form (Catelan et al. 2001):

$$\epsilon_+ = f(J, J_z)(J_z^2 - J_x^2)$$  

(20)

$$\epsilon_\times = 2f(J, J_z)(J_x J_y),$$  

(21)

and rescaling $\tilde{T} = T/|T|$ with $|T| = \sqrt{T_{a\sigma} T_{\sigma a}}$. Squaring the tidal shear gives rise to short-ranged correlations in the angular momentum directions, and ultimately in the derived galaxy shapes (Schäfer & Merkel 2012). This picture is illustrated in Figure 7, where two neighbouring haloes are subject to correlated tidal torquing of their motion along the gradient of the gravitational potential. They build up correlated angular momenta, which determines the orientation of their discs and therefore, their shapes.

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(20)

$$\epsilon_\times = 2f(J, J_z)(J_x J_y),$$  

(21)
where the constant of proportionality, $C$, and direction. In general, the observed ellipticity should show a scaling behaviour

$$|e| \propto \frac{1 - J^2}{1 + J^2}. \tag{22}$$

A direct connection of the galaxy shape, in analogy to the linear alignment model, would use the quadratic dependence of the shape on tidal shear according to

$$\gamma' = C_2(T_{\alpha\alpha}^2 - T_{\beta\beta}^2, 2T_{\alpha\beta}T_{\beta\alpha}), \tag{23}$$

where the constant of proportionality, $C_2$, captures all the processes related to the angular momentum variance for a given tidal shear and of the inclination of the galactic disc. The effective model in Equation (23) uses the traceless shear $T_{\alpha\alpha'}$ rather than the tidal shear $T_{\alpha\beta}$, because alignment of spiral galaxies is an orientation effect where the absolute value of the tidal shears is not relevant, only the orientation of the tidal shear eigensystem.

Using this model in analogy to the alignment model for elliptical galaxies, the resulting density weighted intrinsic shear $\gamma'$ is (Hirata & Seljak 2004),

$$\gamma'(k) = \frac{C \rho^2}{(2\pi)^3} d^4 \int h_E(\hat{k}_1, \hat{k}_2) \delta(\hat{k}_1') \delta(\hat{k}_2') \left[ \delta_D^{(3)}(\hat{k}_1') + \frac{b_\delta}{(2\pi)^3} \delta(\hat{k}_1') \right] d^3 \hat{k}_1' d^3 \hat{k}_2'. \tag{24}$$

where $\hat{k}_a = k_a/|k_a|$ and $k_1' = k - k_1 - k_2'$. Two of the three powers of $\delta$ are due to the quadratic alignment and one is from the applied density weighting. Similar to the case for linear alignments, there is a geometric factor $h_E$ that depends, in this case, on the directions $\hat{k}$ of the wave vector (Hirata & Seljak 2004). The intrinsic alignment power spectrum $P_{\aleph}(k)$ for the II correlation takes the form

$$P_{\aleph}(k) = \frac{C_2 \rho^2}{D^4} d^4 \left\{ \frac{2}{3} \left[ h_E(\hat{k}_1, \hat{k}_2) \right]^2 \frac{P_{\delta\delta}^{(3)}(\hat{k}_1)P_{\delta\delta}^{(3)}(\hat{k}_2)}{(2\pi)^3} d^3 \hat{k}_1 
+ \frac{2}{3} b_\delta \left[ h_E(\hat{k}_1, \hat{k}_2) + h_E(\hat{k}_2', \hat{k}_3) + h_E(\hat{k}_3', \hat{k}_1) \right]^2 \frac{P_{\delta\delta}^{(3)}(\hat{k}_1)P_{\delta\delta}^{(3)}(\hat{k}_2')P_{\delta\delta}^{(3)}(\hat{k}_3')}{(2\pi)^6} d^3 \hat{k}_1' d^3 \hat{k}_2' \right\}. \tag{25}$$

In this way, the ellipticity is linked to the tidal shear field through the angular momentum direction and it is possible to trace correlations in the ellipticity field back to those in the tidal shear field, which in turn are related to density correlations by the Poisson equation. This has been demonstrated by Crittenden et al. (2001), who computed $E$- and $B$-mode ellipticity correlation functions and showed that they dominate over the lensing signal at redshifts below $z = 0.3$, and are a small but significant contribution to weak lensing spectra at redshifts of unity, under the assumption of perfect alignment between the galaxy and halo angular momenta. The main uncertainties are the misalignment parameter $\alpha_t$, which is measured in numerical simulations by comparing the tidal shear acting on a halo with the resulting angular momentum, and the orientation of the galactic disc relative to the angular momentum direction. This relation is poorly understood, in fact many analytical works assume perfect alignment but allow for a finite thickness of the galactic disc, which dilutes the orientation effect and leads to a less pronounced ellipticity variation when tilting the angular momentum direction (e.g. Crittenden et al. 2001, 2002; Caprano et al. 2013; Merkel & Schäfer 2013)

Following the work of Crittenden et al. (2001), Caprano et al. (2013) computed angular ellipticity power spectra and compared these with weak lensing shear power spectra. The intrinsic $E$-mode (II) spectra resulting from the quadratic alignments was found to be an order of magnitude less than the weak lensing power spectra for lensing surveys reaching unit redshift, while the $B$-mode spectra would be an important test of systematics. Generalising on this, Merkel & Schäfer (2013) computed three-dimensional ellipticity (II) power spectra and found them to differ from weak lensing power spectra in terms of scale dependence and correlations between different Fourier-modes of the weak lensing field.

The density-intrinsic alignment cross-power spectrum is zero in the simple quadratic alignment model due to the assumed Gaussianity of the density field. The correlation would be proportional to three powers of the density field,
two from the alignment model and one from the density field, and is zero, because odd moments of a Gaussian distribution vanish identically. However, there will certainly be some non-zero contribution to the cross-term even in the quadratic alignment model once the third- and higher-order corrections to Equation (23) are taken into account, as well as from non-Gaussianity due to non-linear growth of structure.

3.2. Alignments on scales smaller than host haloes

The current understanding of galaxy alignments within haloes is fundamentally hampered by the limited knowledge of the non-linear evolution and baryonic physics that shapes the haloes themselves and the galaxies they host. Without a physical grounding to study intrinsic galaxy alignments within a halo, the best approach is to describe alignments through the halo model.

The halo model (e.g. Seljak 2000; Cooray & Sheth 2002) is an analytical description of the clustering of dark matter in the Universe, based on the ansatz that all dark matter particles belong to haloes. It further assumes that the mass of a halo is the physical quantity that drives virtually all properties of the haloes (and of the galaxies that inhabit them). This assumption has motivations heavily rooted in the results of N-body simulations. For example, the abundance (see e.g. Tinker et al. 2008), the bias (e.g. Tinker et al. 2010), and the matter density profile (see e.g. Navarro et al. 1997; Dutton & Macciò 2014) are all functions of the halo mass (albeit with some scatter). Perhaps even more interestingly, properties of galaxies such as stellar mass, luminosity, star formation efficiency and size all depend to first order on the mass of their host haloes. Using this model, predictions for the clustering of dark matter, as well as galaxies, can be as accurate as 5-10 % (e.g. Giocoli et al. 2010; van den Bosch et al. 2013). The model has been further improved through incorporation of the potential effects of galaxy formation (e.g. baryonic feedback) into the clustering of dark matter and galaxies (see Fedeli 2014; Fedeli et al. 2014). As the observational and simulated data increase in precision, it has become apparent that the halo formation time plays a similarly important role in shaping the properties of galaxies such as halo mass. This effect is commonly known as assembly bias: at fixed halo mass, the scatter in galaxy properties mainly owes to the different assembly histories of haloes (see e.g. Gao & White 2007; Wang et al. 2013).

The halo model allows for a natural framework to describe galaxy alignments from the largest (tens of Mpc) to the smallest (tens of kpc) scales. This is possible due to the natural halo model separation into intra- and inter-halo properties, commonly named as the ‘one-halo’ (1h) and ‘two-halo’ (2h) terms, respectively. Schneider & Bridle (2010) pioneered the modelling of galaxy alignments in the language of the halo model. As was commonly done for clustering, their formalism explicitly accounted for the distinction between central and satellite galaxies. Furthermore, as is evident from the equations that follow, the large-scale terms are a rescaling (based on galaxy bias) of the matter clustering, whereas the small scale terms are contributions coming from haloes weighted by their relative abundance. Accordingly, the auto- and cross-power spectra of the galaxy alignments read:

\[
P_{1h}(k) = P_{1h,cs}(k) + P_{1h,ss}(k) + P_{2h,cs}(k) + P_{2h,ss}(k) + P_{2h,cs}(k);
\]

\[
P_{1h}(k) = P_{1h,cs}(k) + P_{1h,ss}(k) + P_{2h,cs}(k) + P_{2h,ss}(k) + P_{2h,cs}(k),
\]

where the subscripts ‘c’ and ‘s’ stand for central and satellite galaxies, respectively. Schneider & Bridle (2010) assumed that satellite galaxy ellipticities were uncorrelated with the central galaxy ellipticity. This naturally led to \(P_{1h,cs}(k) = 0\). The remaining II one-halo term is:

\[
P_{1h,ss}(k) = \frac{1}{\bar{n}_g} \int \langle N_g(N_g-1)|M\rangle \tilde{y}^2(M)|w(k|M)|^2 n(M) \, dM,
\]

where \(\bar{n}_g\) is the comoving number density of galaxies, \(n(M)\) is the halo mass function (e.g. Press & Schechter 1974; Bond et al. 1991; Tinker et al. 2008, 2010), and \(\langle N_g(N_g-1)|M\rangle\) is the second moment of the galaxy number distribution at a given halo mass, \(M\). Here, \(w(k|M)\) is the normalized Fourier transform of the radial profile of 3D satellite galaxy ellipticities, whereas \(\tilde{y}(M)\) is the magnitude of the intrinsic shear, \(\tilde{y}^i\), in a halo of mass \(M\).

The model presented by Schneider & Bridle (2010) also assumed that central galaxies acquired their alignment in the same way their host haloes did. Correspondingly, the \(P_{2h,cs}(k)\) equals the power spectrum in Equation (15). This is a manifestation of the fact that on sufficiently large scales, the dominant term for galaxy alignments is the term that
describes the alignment of their host haloes. The term \( P^{2h}_{\text{ILH}}(k) \) is formulated by integrating over the joint probability distribution for two haloes of mass \( M_1 \) and \( M_2 \), with \( \langle N_g|M_1 \rangle \) and \( \langle N_g|M_2 \rangle \) the average number of galaxies in haloes of mass \( M_1 \) and \( M_2 \),

\[
P^{2h}_{\text{ILH}}(k) = \frac{1}{\bar{n}_g} \int \langle N_g|M_1 \rangle \bar{\gamma}(M_1) |w(k|M_1)| n(M_1) \, dM_1 \int \langle N_g|M_2 \rangle \bar{\gamma}(M_2) |w(k|M_2)| n(M_2) \, dM_2 \, P_{\text{hh}}(k|M_1, M_2) ,
\]

where \( P_{\text{hh}}(k|M_1, M_2) = b_h(M_1)b_h(M_2)P_{\text{lin}}^{\text{hh}}(k) \) is the (dark matter) halo-halo power spectrum and \( b_h(M) \) is the halo bias function (Tinker et al. 2010). The two-halo central-satellite term is

\[
P^{2h}_{\text{IICS}}(k) = \frac{C_1 \bar{\rho}_a^2}{D_\gamma} \frac{P_{\text{lin}}^{\text{hh}}(k)}{\bar{n}_g} \int \langle N_g|M \rangle \bar{\gamma}(M)|w(k|M)| b_h(M) n(M) \, dM .
\]

The terms that define the \( P_{\text{gal}}(k) \) power spectrum are:

\[
P^{1h}_{\text{IIC}}(k) = 0 ,
\]

\[
P^{1h}_{\text{ICS}}(k) = \frac{1}{\bar{n}_g} \int M \langle N_g|M \rangle \bar{\gamma}(M) |w(k|M)| u(k|M) n(M) \, dM ,
\]

\[
P^{2h}_{\text{ICS}}(k) = -\frac{C_1 \bar{\rho}_a^2}{D_\gamma} \frac{P_{\text{lin}}^{\text{hh}}(k)}{\bar{n}_g} ,
\]

\[
P^{2h}_{\text{IIC}}(k) = \frac{1}{\bar{n}_g} \int \langle N_g|M_1 \rangle \bar{\gamma}(M_1) |w(k|M_1)| n(M_1) \, dM_1 \int M[u(k|M_2)] n(M_2) \, dM_2 \, P_{\text{hh}}(k|M_1, M_2) ,
\]

where \( u(k|M) \) is the normalized Fourier transform of the radial profile of the matter distribution in a halo of mass \( M \) (e.g. Navarro et al. 1997). Note that \( P^{2h}_{\text{IIC}}(k) = P_{\text{gal}}(k) \) from Equation (16) since on sufficiently large scales, the central term is the dominant one.

Schneider & Bridle (2010) found that the total halo model alignment power spectra are accurately modelled by restricting oneself to the two-halo central correlation on large scales and the one-halo satellite correlation on small scales, i.e.

\[
P_{\text{I}}(k) \approx P^{1h}_{\text{IIC}}(k) + P^{2h}_{\text{ICS}}(k) ;
\]

\[
P_{\text{gal}}(k) \approx P^{1h}_{\text{ICS}}(k) + P^{2h}_{\text{IIC}}(k) .
\]

The light blue dashed curve in Figure 8 shows the satellite contribution derived from \( P^{1h}_{\text{ICS}}(k) \), with the free amplitude contained in \( \bar{\gamma} \) fitted to the data.

### 3.3. Transition to intermediate scales

Alignments on intermediate scales of a few Mpc are difficult to treat, in contrast to alignment models on large scales and alignments inside haloes on small scales. Unifying both regimes would yield a consistent model of alignments over all scales of interest.

Both the linear and quadratic alignment models are derived from (Lagrangian) perturbation theory, as a theory of the tidal interaction of a halo with the gravitational fields generated by the large-scale structure into which the halo is embedded. This naturally implies that the models can only be applied on linear scales above several tens of Mpc. In order to bridge the gap to fully non-linear scales below 1-2 Mpc, where a description according to the halo model would be applicable, it is necessary to modify tidal interactions and take account of the baryonic physics inside the haloes. It is also necessary to have a prescription of clustering and to add observational complications such as redshift-space distortions due to peculiar motions. These intermediate scales, around \( 2 - 10 \) Mpc, are referred to as mildly non-linear, and a number of methods have been proposed as phenomenological solutions (Tinker et al. 2005; van den Bosch et al. 2013). Research using simulations (Heymans et al. 2004) and observations (Mandelbaum et al. 2006) suggest that on these intermediate scales, intrinsic alignments are stronger than predicted by linear theory.
Fig. 8.— Galaxy position ellipticity correlations found in SDSS luminous red galaxies from Singh et al. (2014) in comparison to models based on linear alignments (dark blue line), enhanced by non-linear structures (NLA; red line), and including non-linear clustering effects, biasing and galaxy density weighting (black line), with Gaussian smoothing at $k = 1 h^{-1}$Mpc (solid lines) and without smoothing (dashed lines). The light blue dashed line is the fit to the one-halo (1-halo) term from Schneider & Bridle (2010) and the green line shows a non-perturbative NFW-based one-halo model with linear theory for the two-halo term. The black and green models are reproduced from Blazek et al. (2015). Figure credit: Jonathan Blazek & Sukhdeep Singh.

An immediate solution would be to replace the linear matter power spectrum by a non-linear one, which provides stronger alignments on small spatial scales (Bridle & King 2007). In effect this Non-Linear Alignment (NLA) model asserts that haloes experience stronger tidal fields in non-linearly evolving structures while the linear interaction itself is not changed, leading to increased alignment at non-linear scales.

However, it is possible to imagine that haloes at close separations started interacting directly with each other in a way that would weaken their alignment with the large-scale structure, with the interesting consequence that tidal alignments would be weakened instead of being strengthened in non-linear structures. Nevertheless, the non-linear alignment model is very easy to implement and has proved itself to be more consistent with small-scale observations in comparison to the linear alignment model alone, and thus has remained popular in the literature (Mandelbaum et al. 2011; Kirk et al. 2013; Blazek et al. 2012; Heymans et al. 2013; Chisari et al. 2014; Troxel & Ishak 2014; Hall & Taylor 2014).

The non-linear alignment model fails to explain alignments linear in the tidal field on a number of counts, as pointed out by Blazek et al. (2011, 2012). First, the cosmic density field is in a non-linear stage of evolution on small scales, which can be modeled by Eulerian perturbation theory, as long as the density fluctuations are not too strong. Second, the clustering of galaxies on small scales introduces a non-linear weighting of the ellipticity field on small scales by giving rise to many galaxy pairs with small separations. Third, there are non-linear biases both in the physical distribution of galaxies as well as distortions of the galaxy field due to redshift-space distortions. These effects have an important influence on the shape of the resulting ellipticity correlation function on small scales, most notably on scales
≤10 Mpc, whereas on larger scales the standard linear alignment model is sufficient. It is also common to smooth the tidal field, since fluctuations on scales that are sufficiently small should not have an impact on the observed alignments.

Figure 8 shows the ellipticity-density correlation, $w_{gs}$, for the linear and non-linear alignment (NLA) models and the fit to the one-halo term from Schneider & Bridle (2010) with observational data and best-fit parameters taken from the Singh et al. (2014) SDSS analysis. It also shows two additional models taken from Blazek et al. (2015), who developed analytic models for galaxy intrinsic alignments based on tidal alignment theory. The black line shows the best-fit standard perturbation theory model that contains non-linear contributions including non-linear clustering, halo bias and galaxy density weighting. The green line shows a physically motivated and non-perturbative one-halo model based on the halo profile of Navarro-Frenk-White (NFW$^9$; see Navarro et al. 1996) combined with linear theory for the two-halo term, including a contribution that captures the enhancement of the tidal field in locations where galaxies form. The small wiggles in the green line at small scales are due to the use of observed galaxy clustering, since the model makes a prediction for the alignment per galaxy, rather than the total $w_{gs}$ signal. The figure also shows the effects of smoothing of the tidal field on the linear and non-linear alignment models, where the dashed lines show the models with no smoothing, while the solid lines show the models with smoothing with a Gaussian of width $k = 1 \, h^{-1}\text{Mpc}$. Motivated by observational results, the NFW-based model includes saturation of the alignment (i.e. the alignment signal does not increase any further) per galaxy within the halo virial radius. The linear alignment model matches the observations well on scales $\leq 10 h^{-1}\text{Mpc}$ and the one-halo fit matches the observations on scales $\lesssim 1 h^{-1}\text{Mpc}$, while the non-linear alignment model, both with and without smoothing, provides a reasonable approximation to the intermediate scales. However, the models from Blazek et al. (2015), which include additional non-linear effects, show a much closer match to the data, with the NFW-based one-halo plus linear two-halo model effectively unifying the small, intermediate and large scales.

The alignments of spiral galaxies in non-linear environments are also affected on small scales by galaxy clustering and redshift-space distortions due to peculiar motion, which are partially already included in current analytical models. Because the primary angular momentum build-up of haloes takes place at very high redshifts it might be assumed that the angular momentum direction is conserved, since secondary effects, like the shaping of the ellipticity correlation function due to peculiar motion of galaxies, or the lensing mapping of intrinsically shape-correlated galaxies, has been shown to be small (Giahi & Schäfer 2013; Giahi-Saravani & Schäfer 2014).

However, results from numerical simulations are not conclusive on this issue. Some dark matter simulations confirm this picture (e.g. Porciani et al. 2002), but recent simulations including baryons found spin changes due to the merging of subhaloes (e.g. Dubois et al. 2014; Cen 2014). It seems that ultimately the properties of the galactic disc and its orientation due to an initial tidal interaction and subsequent merging and accretion is very difficult to predict (Dekel et al. 2009; Hahn et al. 2010; Danovich et al. 2014), which could potentially be solved by numerical simulations (see Sections 4.2.2, 5.1.1 and 5.2.1).

4. \textit{N-Body simulations}

On the largest scales, the evolution of the Universe can be investigated analytically and with Gaussian random fields. However, on smaller scales, the Universe can be distinctly non-linear (structures would not collapse without these non-linear effects) and it is not possible to probe the formation and evolution of non-linear structures analytically, which is why an alternative approach had to be found. \textit{N}-body simulations mimic the statistical properties of the Universe by sampling the dark matter density field with discrete particles. These particles are placed in a box and since it is not computationally possible to represent every atom in the Universe with its own particle, each particle represents a sampling of the volume at that particular point in space. The particles are generally very massive, adding together to make the total density of the volume equal to the average density of the Universe. For example, a high resolution, cosmological volume ($\geq 100 \, h^{-3}\text{Mpc}$) simulation today might have a particle mass of $\sim 10^6 \, h^{-1}\text{M}_\odot$. This distribution of particles is imprinted with a theoretical power spectrum of fluctuations from when the Universe was still in a linear state. As the simulation moves forward through time, the dark matter particles act under gravity and evolve over cosmological time. With an appropriate choice of initial condition power spectrum and subsequent

\footnote{Navarro et al. (1996) fitted a universal density profile to dark matter haloes in \textit{N}-body simulations. This profile is now known as the NFW profile.}
dark matter evolution, cosmological $N$-body simulations are able to produce particle distributions that resemble the statistical distribution of matter in our Universe today.

$N$-body simulations are used for alignment studies because they provide a data set with known parameters, which is essential when trying to understand the alignment signals. They can also provide a statistical sample – large numbers of individual dark matter haloes can be identified in a single realization. There are many different measurements made in order to quantify the alignment signal and sometimes many different ways of measuring the same property, and these allow for robustness tests of the results (see Section 4.1). Most importantly, $N$-body simulations are able to provide data on how the shapes, spins and angular momenta of dark matter haloes align over a wide range of scales and in different environments (see Section 4.2).

Another possibility is to investigate a smaller volume simulation with a much higher resolution. This is known as a ‘zoom’ simulation, where a region of interest is selected within a larger-scale simulation and then resimulated with higher resolution. The advantages of this technique are that the large-scale tidal forces are still present to influence the region of interest, but the higher resolution allows a much more detailed study to be performed.

$N$-body simulations alone can not provide a complete picture of the formation and evolution of intrinsic galaxy alignments. The most important limitation is that they do not contain any galaxies. It is also now known that baryons have an impact on the shapes, orientations, spins and angular momenta of dark matter haloes. Investigations of these effects must be studied using hydrodynamic simulations (see Section 5). Despite their limitations, $N$-body simulations will remain an important tool in the understanding of intrinsic alignments. They are much faster to run than their hydrodynamic counterparts and the dark matter physics is well understood (unlike baryon physics). Most importantly, they provide the backbone for the semi-analytic modelling of galaxy properties (see Section 6), which will likely become the standard data sets for the galaxy intrinsic alignment investigations of the future.

4.1. What do people measure?

There are a number of things that must be determined in order to measure the alignments of dark matter haloes in $N$-body simulations. The first step is to identify the haloes within the simulation (Section 4.1.1). It is then possible to determine the halo shapes (Section 4.1.2) and the angular momentum or spin of the halo (Section 4.1.3). Some studies investigate alignments with respect to the environment that the halo resides in, which requires the simulation to be classified into cosmic web elements (Section 4.1.4). With this information in hand, it is possible to measure the alignments of the dark matter haloes (Section 4.1.5). While this section is focused on $N$-body simulations, these measurements are largely the same for hydrodynamic simulations (see Section 5), with the shape of the stellar and/or gas component also being measured.

4.1.1. Dark matter haloes

In order to study the shapes and alignments of dark matter halos in $N$-body simulations, it is useful to provide a formal definition of a halo and some typical methods used to identify them. From a theoretical perspective, a halo is defined as an object whose constituent particles are gravitationally bound. However, this can be difficult to define in practice, given that simulations are discretely sampled by particles, which trace the local density and gravitational potential.

A simple model for the formation of haloes is the spherical collapse model (e.g. Press & Schechter 1974), which describes the formation of a collapsed object by considering the evolution of a sphere of uniform overdensity, $\delta$, in a smooth background. The overdensity initially expands with the expansion of the Universe; however, as it is overdense, the expansion slows, and eventually begins to collapse under its own self-gravity. After a period of relaxation, the halo ultimately reaches virial equilibrium. The overdensity required for a halo to collapse is related to the underlying matter density. For a spatially flat universe today, this collapse overdensity (relative to the critical density) is given by $\Delta_c \approx 178$, which is often rounded up to $\Delta_c = 200$ for simplicity in simulation analyses. It is important to note that the value for $\Delta_c$ is dependent on the cosmology and redshift of the universe (at earlier times $\Delta_c$ will be a smaller number).\footnote{Note that some studies consider overdensities relative to the underlying matter density, rather than the critical density. Such overdensities can be easily scaled, due to the fact that $\Omega_M$ is the ratio between the mean matter density and the critical density.}

Note that some studies consider overdensities relative to the underlying matter density, rather than the critical density. Such overdensities can be easily scaled, due to the fact that $\Omega_M$ is the ratio between the mean matter density and the critical density.
In $N$-body simulations, and indeed in weak lensing mass measurements, the virial mass of a halo, $M_{\text{vir}} = M_{\Delta_c}$, is typically computed within the virial radius, $R_{\text{vir}} = R_{\Delta_c}$, which is the radius at which the density of the halo is $\Delta_c$ times the critical density of the universe.

The simplest method to identify particles that have coalesced into a collapsed halo is to simply group particles together based on their proximity to neighbouring particles. Such a scheme is known as “Friends-of-Friends” (FOF) (e.g. Davis et al. 1985). A typical FOF halo finder will identify all particles that are separated by less than a user-specified linking length, and will group these together as a halo. This type of algorithm is relatively straightforward to implement, and can be run during the evolution of an $N$-body simulation, allowing for a simple way to trace the evolution of structures in such simulations. However, the mass associated with a FOF halo, computed by simply summing the masses of the particles identified as belonging to the halo, is strongly dependent on the choice of linking length, and it is not trivial to relate this mass to something more physically motivated, such as the virial mass.

An alternative approach to halo finding involves the “Bound Density Maximum” (BDM) technique (e.g. Klypin & Holtzman 1997). The basic principle involves identifying local density maxima, and identifying a spherical region around each maximum, within which the mean overdensity is above some pre-defined level. The link between this, and the spherical collapse model, should be immediately evident. In contrast to FOF halos, BDM halos can overlap, and it is therefore straightforward to identify subhaloes within larger structures (i.e. halos that lie entirely within the virial radius of a larger structure).

Most commonly used halo finders use modifications to, or combinations of, the above methods; reviews of contemporary methods can be found in Bett et al. (2007) and Knebe et al. (2011).

### 4.1.2. Dark matter halo shapes

Once the dark matter haloes have been identified, their shapes (halo axes) must be identified in order to measure any shape alignments. The shapes of these haloes can be determined by considering the distribution of their constituent particles. Dark matter haloes are typically modelled as being ellipsoidal, and although this is a simplification of the more complex $N$-body halo shape, this appears to work very well. One common approach to measuring shapes uses the so-called inertia tensor\(^{11}\) for the dark matter particles that have been identified as being a part of the halo (see Section 4.1.1),

$$I_{ij} = \frac{\sum w_n m_n x_{ni} x_{nj}}{\sum w_n m_n}, \quad (32)$$

where $m_n$ represents the mass of the $n^{th}$ particle, $w_n$ represents a radial weight function to be discussed shortly, and $x_{ni}$, $x_{nj}$ represent the position coordinates of the $n^{th}$ particle with respect to the halo centre (where the centre is the location of the gravitational potential minimum), with $0 \leq i, j \leq 2$ for 3D and $0 \leq i, j \leq 1$ for 2D. The eigenvectors, $e_1, e_2, e_3$, of this tensor represent the principal axes of the ellipsoids with the lengths of the principal axes given by the square roots of the eigenvalues $\lambda_i$, such that $\sqrt{\lambda_1} \geq \sqrt{\lambda_2} \geq \sqrt{\lambda_3}$.

The use of $w_n = 1$ results in the so-called unweighted inertia tensor, whereas $w_n = 1/d_n^2$ (where $d_n$ is the weighted distance relative to the centre of the halo) is the reduced inertia tensor, which gives more weight to the centres of haloes. The reduced inertia tensor can be computed using a spherically symmetric weight function proportional to the inverse distance of the particle from the halo centre, i.e. defining $d_n^2 = x_n^2 + y_n^2 + z_n^2$; or, alternatively, using an ellipsoidal weighting function that defines $d_n^2 = x_n^2 + (y_n/q)^2 + (z_n/s)^2$, where $q = b/a$ and $s = c/a$, and $a \geq b \geq c$ are the semi-major, intermediate and semi-minor axes of the halo. When using the ellipsoidal weighting function, the inertia tensor is typically solved for iteratively.

When measuring the shapes of dark matter haloes, it is important to consider the number of particles being used to make the shape measurement. Bett et al. (2007) performed resolution tests on their 3D shape measurements and showed that shapes were biased toward being less spherical (more prolate) when the number of particles in the halo was $\leq 300$. Similarly, Jing (2002) found that their 2D projected shape measurements converged with a minimum of 160 particles. It was not uncommon for some early studies mentioned in this review to include haloes with as few as

\(^{11}\)This equation is actually the quadrupole tensor of the mass distribution, but as it is regularly referred to as the moment of inertia tensor in the literature, this is the convention followed throughout this review.
10 or 20 particles, which offers a reasonable explanation for why some early results may be in contrast to more recent conclusions. Given these known biases, it is remarkable how many early works found results actually in agreement with the current consensus (see Section 4.2).

An alternative shape measurement approach was suggested by Jing & Suto (2002), who noted that while the iterative approach detailed above worked well for low-resolution simulations, it does not converge consistently in the case of high resolution data due to the iterative approach being unstable in the presence of significant substructures (which are more readily resolved in the high-resolution simulations). The alternative approach calculates isodensity surfaces at different overdensities to determine the axes of the halo. To start, the local density is calculated at the position of each particle. A spherically symmetric spline kernel, often used in smoothed particle hydrodynamics calculations (SPH; e.g. Hernquist & Katz 1989; Monaghan & Lattanzio 1985), is adopted,

\[
W(r_s, h_i) = \begin{cases} 
\frac{1 - \frac{3}{2} \left( \frac{r_s}{h_i} \right)^2 + \frac{3}{4} \left( \frac{r_s}{h_i} \right)^3}{\pi h_i^3} & (r_s \leq h_i) \\
\frac{1}{4} \left( 2 - \frac{r_s}{h_i} \right)^3 & (h_i < r_s < 2h_i) \\
0 & \text{otherwise,}
\end{cases}
\] (33)

where \( h_i \) is the smoothing length of the \( i \)th particle. The local density of a particle, \( \rho_i \), is computed by taking the 32 nearest neighbour particles, where \( r_s \) is distance of the neighbour particle from the \( i \)th particle, and applying their contribution to the local density using the weighting in Equation (33). The smoothing length of the particle, \( h_i \), is often set to be half the radius \( r_s \). After the local density of the particles has been calculated, isodensity surfaces can be determined at different overdensities, which correspond to different halo radii, and a triaxial fit can be used to determine the axes.

Another alternative approach to measuring the shape of a dark matter halo is to determine the shape of the potential of the halo (e.g. Kuhlen et al. 2007). The unweighted potential energy tensor, \( W_{ij} = \sum_n x_{ni} \Phi / dx_{ij} \), is related to the (unweighted) kinetic energy tensor,

\[
K_{ij} = \frac{1}{2} \sum_n m v_i v_j ,
\] (34)

where \( v \) is the velocity of the particle, through the tensor virial theorem,

\[
\frac{1}{2} \frac{d^2 I_{ij}}{dt^2} = 2K_{ij} + W_{ij} ,
\] (35)

where \( t \) is time. For a relaxed dark matter halo, the term on the left of this equation should be zero at \( z = 0 \). In this case, the eigenvectors of the diagonalised unweighted kinetic energy tensor (Equation 34) will provide the principal axes of the potential ellipsoid.

The axes derived using any of the shape measurement methods above can also be used to measure the triaxiality of the halo.

\[
\mathcal{T} = \frac{(a^2 - b^2)}{(a^2 - c^2)} = \frac{(1 - q^2)}{(1 - s^2)}
\] (36)

The axis ratio \( s \) is typically referred to as the sphericity of the halo. In general, a halo is oblate when \( \mathcal{T} < \frac{1}{3} \), prolate when \( \mathcal{T} > \frac{2}{3} \) and triaxial otherwise.

4.1.3. Angular momentum and spin

Tidal torque theory presents a framework for how angular momentum is built up in galaxies and dark matter haloes (see Section 3.1.1). Initial angular momentum predictions can be made analytically (e.g. Schäfer 2009), while structure formation is in the linear regime. At the time of halo turnaround, when a halo begins to collapse and the processes acting on it become non-linear, tidal torquing leaves an imprint of the tidal field in which the halo formed. This is the point where analytic models break down and the continued build up of angular momentum, through processes
including mergers and accretion, is best traced through simulations (both N-body and hydrodynamic). The imprint of the early tidal field is expected to remain present through these processes, as has been shown using N-body simulations in Porciani et al. (2002). The large-scale structures of the cosmic web exhibit uniform tidal fields and as a result, the angular momentum vectors of neighbouring haloes within cosmic web elements should show some alignments at small separations. Moreover, haloes should also align in the direction of the large-scale structure.

Similar to the shape measurements, the number of particles in a halo has an affect on the measurement of the angular momentum and spin. Bett et al. (2007) performed resolution tests on their spin measurements and found that the median spin parameter was biased high when the number of particles in the halo was \( \lesssim 300 \). Caution should also be exercised on results from the spin/angular momentum literature that use fewer than 300 particles per halo.

The angular momentum of an N-body dark matter halo can be computed as:

\[
J = \sum_n m_n R_n \times v_n ,
\]  

(37)

where the sum is over the total number of particles, \( N \), identified within a particular halo, \( R_n \) is the radial distance of the \( n^{th} \) particle relative to the halo centre and \( v_n \) is the velocity of the \( n^{th} \) particle relative to the halo centre. Similarly, one can compute the specific angular momentum

\[
j = \frac{1}{N} \sum_n R_n \times v_n ,
\]

(38)

and the cumulative specific angular momentum profile within the halo radius \( R \),

\[
j(\leq R) = \frac{1}{M(\leq R)} \sum_{n \leq R} m_n R_n \times v_n .
\]

(39)

The spin parameter is a dimensionless measure of the amount of rotation in a dark matter halo. Under the standard definition from Peebles (1969), the spin parameter is given by

\[
\lambda = \frac{|J|E^{1/2}}{GM^{3/2}},
\]

(40)

where \( J \) is the magnitude of the angular momentum \( J \), \( E \) is the total energy of the halo, and \( M \) is the halo mass. Due to the difficulty in measuring the total halo energy, Bullock et al. (2001) introduced a modified spin parameter

\[
\lambda' = \frac{J}{\sqrt{2}MVR},
\]

(41)

where \( V \) is the circular velocity at halo radius \( R \) such that \( V^2 = GM/R \).

### 4.1.4 Cosmic web elements

Numerical simulations and large-scale redshift surveys have provided a picture of the large-scale structure of the Universe. This cosmic web is comprised of clusters, filaments, sheets/walls and voids. Topologically, filaments are expected join with clusters and border sheets, which are expected to reside at the boundary of the void surfaces. The method to recover the topology of the large-scale density field must account for the density field being discretely sampled in simulations (and indeed in observations as well), while still providing a consistent classification of web elements.

A number of algorithms exist to define the cosmic web elements in simulations, and three of these are based on the same basic principle. For each, a locally symmetric tensor is diagonalised to give three real eigenvalues, \( \lambda_1 > \lambda_2 > \lambda_3 \) with corresponding eigenvectors \( \hat{e}_1, \hat{e}_2 \) and \( \hat{e}_3 \). Classification into the four different cosmic web types, void, sheet, filament and cluster, is related to the number of eigenvalues above a given threshold \( \lambda_{th} \). The value of \( \lambda_{th} \) differs between studies and is based largely on visual inspection.
This basic procedure to define web elements is performed on the tidal shear tensor (ignoring the factor of $4\pi G$, see Section 3.1.1; e.g. Hahn et al. 2007b,a; Codis et al. 2012; Forero-Romero et al. 2014),

$$T_{ij} = \frac{\partial^2 \Phi}{\partial x_i \partial x_j}, \quad (42)$$

the velocity shear tensor, where the Hubble constant $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ (e.g. Libeskind et al. 2013; Forero-Romero et al. 2014),

$$\Sigma_{ij} = \frac{1}{2H_0} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right), \quad (43)$$

and the field defined by the Hessian of the smoothed density field, $S(\rho)$ (e.g. Aragón-Calvo et al. 2007; Zhang et al. 2009; Trowland et al. 2013),

$$H_{ij} = \frac{\partial^2 S(\rho)}{\partial x_i \partial x_j}. \quad (44)$$

For both $T_{ij}$ and $\Sigma_{ij}$, the classification is such that zero, one, two or three eigenvalues above $\lambda_{th}$ corresponds to void, sheet, filament and cluster respectively. However for $H_{ij}$, the sign of the eigenvalues is opposite. This is due to the potential calculated in $T_{ij}$ being derived from the matter density distribution through the Poisson equation, making $\Phi$ a Fourier transform of the density field. Thus, zero, one, two or three eigenvalues above $\lambda_{th}$ corresponds to cluster, filament, sheet and void respectively.

While the basic premise of these classification schemes remains consistent within the literature, the details of the implementations differ and many works include extensions to the methods to form more robust classifications. The reader is advised to consult the references directly for the fine details of each scheme. Additionally, many alternative classification schemes exist – too many to mention them all here. For a comprehensive list of alternative schemes and the development of the field, see Sousbie et al. (2009) and references therein.

4.1.5. Alignments

The simplest method to measure alignments is to determine a misalignment angle, $\theta$, between the axes (semi-major, intermediate, semi-minor or spin/angular momentum) of the halo or haloes such that $\cos(\theta) = 1$ is a parallel alignment and $\cos(\theta) = 0$ is perpendicular alignment.

When considering the alignments between two haloes in three-dimensions, it is common to give each halo a 3D orientation vector $l$. Again, this orientation can be associated with the direction of any of the axes of the halo or the direction of the angular momentum/spin vector. The 3D unit vector in the direction connecting the two haloes is defined as $\hat{r}$. With these definitions, it is possible to determine whether neighbouring halo axes tend to point toward each other,

$$A_{\theta}(r) = \langle |l_i \cdot l_j| \rangle, \quad (45)$$

and if haloes tend to point in the direction of their neighbours

$$D_{\theta}(r) = \langle |l \cdot \hat{r}| \rangle \equiv \frac{1}{N} \sum_{i,j} |l_i \cdot \hat{r}_{ij}|, \quad (46)$$

Both of these correlations correspond to $\langle |\cos(\theta)| \rangle$, where $\theta$ is the misalignment angle between the two halo axes in Equation (45) or between the halo axis and the vector connecting the two haloes in Equation (46) (e.g. Faltenbacher et al. 2002; Bailin & Steinmetz 2005; Kasun & Evrard 2005). An alternative is to investigate the square of the dot products in Equations (45) and (46), since the halo axes have no preferred direction (i.e. $\pm \cos(\theta)$ are physically identical). This is equivalent to measuring $\langle \cos^2(\theta) \rangle$ (e.g. Hopkins et al. 2005).

It is also common to measure the shape of haloes as viewed in projection on the plane of the sky to match observations. The projected shape of a halo is typically defined by the moment of inertia tensor (Equation 32 and surrounding.
text), in a method that is analogous to calculations of galaxy surface brightness distributions (e.g. Miralda-Escude 1991). An ellipticity can then be defined as
\[ \epsilon = \epsilon_1 + i \epsilon_2, \]
with components given by
\[ \epsilon_1 = \frac{I_{11} - I_{22}}{I_{11} + I_{22}}, \]
\[ \epsilon_2 = \frac{2I_{12}}{I_{11} + I_{22}}, \]
such that
\[ \epsilon_i = 1 - q^2 \frac{\cos(2\varphi), \sin(2\varphi)}{1 + q^2}, \]
where \( q \) is the axis ratio and \( \varphi \) is the position angle of the halo. From these ellipticity measurements, an ellipticity correlation function can be constructed that is analogous to Equation (45) (see also Equation 4),
\[ \xi_{\epsilon\epsilon}(r) = \langle \epsilon_i(x) \cdot \epsilon_j(x + r) \rangle \equiv \langle \epsilon^S \epsilon'^S \rangle, \]
computed over all pairs of haloes with a 3D separation vector connecting the two haloes defined as \( r \) (e.g. Croft & Metzler 2000; Heavens et al. 2000; Jing 2002).

Comparing with the correlations in Equation (8), the alignments in Equations (45) and (49) are equivalent to the II term and the shape-density alignments in Equation (46) give rise to the GI correlations.

4.2. Results

In this section, the results from the N-body literature are presented by alignment type. Section 4.2.1 starts with the general shapes and spins of dark matter haloes. The section then treats alignments as a function of their scale, starting with dark matter halo internal alignments in Section 4.2.2 and moving to halo-satellite alignments in Section 4.2.3, then halo-halo alignments in Section 4.2.4 before finally presenting dark matter halo alignments with the large-scale structure in Section 4.2.5.

4.2.1. Halo shapes and spins

There is consensus among many N-body simulation studies that dark matter haloes tend to be strongly aspherical, with a preference for prolate over oblate haloes (e.g. Davis et al. 1985; Frenk et al. 1988; Warren et al. 1992; Kasun & Evrard 2005; Shaw et al. 2006; Allgood et al. 2006; Schneider et al. 2012). When considering a full sample of haloes, independent of environment, haloes tend to become more prolate with increasing mass (e.g. Kasun & Evrard 2005; Paz et al. 2006; Hahn et al. 2007a; Schneider et al. 2012). Dark matter haloes also tend to collapse to a shape that is more triaxial and become more spherical over time (Hopkins et al. 2005). There appears to be no significant dependence of shape parameters on environment for massive haloes (\( M_{\text{vir}} > 2 \times 10^{12} h^{-1} M_\odot \)), although environment does play a significant role for less massive haloes in clusters, which tend to be less spherical and more prolate, while haloes in filaments tend to be more oblate (Hahn et al. 2007b). In addition, there is some evidence to suggest that haloes that reside close to clusters and haloes with little substructure are more spherical (Ragone-Figueroa & Plionis 2007). Virialised (relaxed) haloes are more spherical than haloes that have experienced a lot of mergers and contain substructure (which causes them to be more elongated) and haloes in high density environments like clusters will have formed earlier and had more time to relax, resulting in their more spherical nature.

The distribution of halo spin parameters tends to be described as a lognormal distribution (e.g. Davis et al. 1985; Warren et al. 1992; Cole & Lacey 1996; Bailin & Steinmetz 2005; Shaw et al. 2006; Davis & Natarajan 2009),
\[ p(\log \lambda) = \frac{1}{\lambda \sqrt{2\pi} \sigma} \exp \left[ -\frac{\log^2(\lambda/\lambda_0)}{2\sigma^2} \right], \]
where \( \lambda_0 \) is the median spin parameter (the location of the peak), and \( \sigma \) is the scatter in \( \log \lambda \) (Davis et al. 1985; Bullock et al. 2001; Maller et al. 2002; Bailin & Steinmetz 2005; Avila-Reese et al. 2005). This distribution tends to
be rather insensitive to the choice of cosmological parameters, and remains essentially unchanged whether the spin parameter includes all the mass out to the virial radius, or only the mass within a truncated radius e.g. \(0.12R_{\text{vir}}\) (Bailin et al. 2005). The distribution of spins can be modelled equally well by linear tidal-torque theory applied to shells of collapsing material or with a model based on the transfer of the orbital angular momentum of merging satellites to the internal spin of the halo (Maller et al. 2002). Furthermore, Vitvitska et al. (2002) found that a random walk model to build up angular momentum naturally produced a lognormal distribution of the spin parameter, though they only tracked the major progenitor of each halo during its evolution, rather than all the mass in the halo. Hahn et al. (2007b) showed that this lognormal model did not fit the distribution for spin parameters of \(\lambda > 0.1\), while Bett et al. (2007) found the lognormal distribution to be generally a poor fit to the spin distribution of the >10^6 haloes in their catalogue. They suggested that while this fit was sufficient for low numbers of objects, it avoided the very low spin values that were present in the catalogue.

4.2.2. Internal alignments

Central galaxies form in the inner region of a dark matter halo. It is therefore instructive to consider the shape and orientation of the halo at the radius of interest to the galaxy as well as a range of larger radii (and not simply at the virial radius).

The shape of a dark matter halo changes with radius (e.g. Frenk et al. 1988; Dubinski & Carlberg 1991; Warren et al. 1992; Jing & Suto 2002; Bailin & Steinmetz 2005; Allgood et al. 2006; Kuhlen et al. 2007; Schneider et al. 2012). While some early works found that the dark matter haloes are more spherical in the centre (e.g. Frenk et al. 1988), more recent studies agree that dark matter haloes become more spherical with increasing radius. There is, however, some disagreement on how the alignment of a halo changes with radius; i.e. \(\theta_H(R_1, R_2)\), see Figure 2 and Table 1.

For example, Schneider et al. (2012) investigated dark matter haloes using both the Millennium 1 (ME1, Springel et al. 2005) and Millennium 2 (ME2, Boylan-Kolchin et al. 2009) simulations. ME1 has a particle mass resolution of \(8.6\times10^8h^{-1}\,\text{M}_\odot\) and ME2 has a particle mass resolution a little over two orders of magnitude higher at \(6.89\times10^6h^{-1}\,\text{M}_\odot\). This study covered dark matter haloes over a range of masses from \(10^{10} - 2\times10^{14}h^{-1}\,\text{M}_\odot\) and a wide range of halo radii \((0.1R_{\text{vir}} - R_{\text{vir}})\). They used the reduced inertia tensor with an elliptical weighting function to measure the shapes of the haloes at different radii. In addition to showing that dark matter haloes became more spherical with increasing radius, they also showed that the haloes became more spherical with decreasing halo mass at a fixed fraction of the virial radius. Figure 9 shows the internal alignments of individual dark matter haloes as a function of radius and halo mass at a redshift of \(z = 0.5\), which corresponds to an \(R_{\text{vir}}\) or \(R_{\text{14}}\) (see Section 4.1.1). They found that the semi-major axes of the dark matter haloes had a mean misalignment angle of \(\theta_H(R_1, R_2) \approx 20^\circ\) between the inner and outer radii. There was a large scatter on this alignment distribution and while the distribution was skewed toward small misalignment angles, around 25% of the haloes had the semi-major axis of the outer halo perpendicular to the semi-major axis of the inner halo. The authors noted that while the highest halo mass bin that could be investigated with both simulations had good agreement in halo alignments between the two simulations, this agreement decreased with decreasing halo mass bins. This was likely due to substructure contamination in ME1, since its lower resolution made it difficult to accurately remove all substructure particles, likely resulting in the shape measurements spuriously detecting more twisted inner and outer haloes.

Kuhlen et al. (2007) investigated a single very high particle mass resolution (particle mass \(\approx20,000h^{-1}\,\text{M}_\odot\)), isolated Milky-Way scale halo and also used the reduced inertia tensor with an elliptical weighting function to measure the shape of their halo, in addition to measuring the shape with the unweighted kinetic energy tensor (see Equation 34). Both measurements produced a prolate halo, however the velocity shape (from the kinetic energy tensor) was far more spherical than the mass distribution. Additionally, the halo became slightly more spherical in the outer radii in both cases. The semi-major axis of the halo was aligned to \(1^\circ\) at all radii. However, as there was only one halo in this study, it is possible that this is a statistical fluctuation corresponding to one of the (not entirely rare) well-aligned haloes found in Schneider et al. (2012). Additionally, since the halo did not experience any major mergers from \(z = 1.7\), it was not subject to tidal interactions, making it much more likely to be well aligned.

It is still worth remembering that the mass resolution of Kuhlen et al. (2007) was significantly higher than in Schneider et al. (2012), so some of the twisting in the ME2 haloes may be a resolution effect. However, this is unlikely
Fig. 9.— Angle between the major axes at different radii within individual haloes, $\theta_{H}(R_{1}, R_{2})$ (see Figure 2 and Table 1), at $z = 0.5$. The lower resolution simulation, Millennium 1, is shown in red, while the higher resolution simulation, Millennium 2, is shown in blue. Points indicate mean angles at a given radius, while boxes show the 25% – 75% central quartile range with the median of the distribution denoted by a horizontal line within each box. Each panel shows a different dark matter halo mass range; this is shown in the grey box on the right with units of $\log_{10}(M_{200}/h^{-1}M_{\odot})$. The halo radius on the x-axis is normalized by $R_{\text{vir}}$, which is $R_{131}$ at this redshift (see Section 4.1.1). © SISSA Medialab Srl. Reproduced by permission of IOP Publishing from Schneider et al. (2012). All rights reserved.
to account for all of the twisting measured and the general consensus in the field follows the results of Schneider et al. (2012), that dark matter haloes may experience misalignments between the inner and outer halo. This is further confirmed with the addition of baryons (see Section 5.1.1).

It is common to measure the alignment of the halo angular momentum with the halo axes, $\theta_{\lambda H}$. There is a strong consensus that the angular momentum of a halo is aligned parallel with the semi-minor axis and perpendicular to the semi-major axis of the halo mass distribution, as illustrated in Figure 10 (e.g. Barnes & Efstathiou 1987; Warren et al. 1992; Bullock et al. 2001; Bailin & Steinmetz 2005; Avila-Reese et al. 2005; Allgood et al. 2006; Shaw et al. 2006; Ragone-Figueroa & Plionis 2007; Bett et al. 2007; Paz et al. 2008). Bailin & Steinmetz (2005) found a median misalignment of 25$^\circ$ between the semi-minor axis and the angular momentum, and a stronger alignment trend in the central 0.25$R_{\text{vir}}$ with the strength of the alignment increasing as a function of halo mass. There was also evidence to suggest that haloes containing a high level of substructure showed a stronger alignment between their angular momentum and semi-minor axes compared to haloes without substructure (Ragone-Figueroa & Plionis 2007). Dark matter haloes can gain angular momentum through merger activity (e.g. Vitvitska et al. 2002; Maller et al. 2002), and haloes with more substructure have undergone more recent mergers. These mergers occurred preferentially in the plane defined by the halo semi-major/intermediate axes (see Section 4.2.5), adding to the angular momentum in the direction of the semi-minor axis.

Bett & Frenk (2012) investigated significant, rapid changes to the direction of the angular momentum vector, which they termed 'spin flips'. The study focused on Milky Way sized dark matter haloes ($\sim 10^{12-12.5} h^{-1} M_\odot$ at $z = 0$) that were relaxed. They found that these spin flips occurred in dark matter haloes regardless of whether they had experienced a major merger event in their lifetime. Instead, the spin flips were caused by strong tidal forces from events like minor mergers or even flybys of neighbouring haloes. They showed that 10.5% of their haloes had at least one spin flip of $>45^\circ$ within a timescale of 0.5Gyrs and that 10.1% of these occurred without a corresponding major merger event. They also investigated the spin flips in the inner 0.25$R_{\text{vir}}$ and found that 58.5% of the haloes experienced a spin flip of $>45^\circ$ and that all but one of these occurred without a major merger event. This result is

Fig. 10.— Normalized histograms of the cosines of the angle between the angular momentum vector and the major (thick red line), intermediate (medium green line) and minor (thin blue line) axes of the dark matter haloes. A random distribution would be a flat line at $p(\cos \theta) = 1$. Reproduced with permission from Bett et al. (2007).
particularly interesting as spin flips of the inner dark matter halo may disrupt or morphologically transform galactic discs that form in this region (e.g. Okamoto et al. 2005; Romano-Díaz et al. 2009; Scannapieco et al. 2009).

Bullock et al. (2001) considered the alignment between the angular momentum vectors in the inner and outer half-haloes within the virial radius. Between 70% and 90% of the haloes were aligned to a greater degree than \( \cos(\theta_{BH}(R_1, R_2)) = 0.7 \sim 45^\circ \), with most showing significantly lower misalignments. Similar alignment results were seen by Bailin & Steinmetz (2005), who considered the alignments at several reference radii. They found that the alignment became progressively worse as \( R_1 \) and \( R_2 \) were further separated, and that the median misalignment angle between the angular momentum vectors in the innermost and outermost regions was \( \sim 50^\circ \).

Bailin & Steinmetz (2005) also found that the internal alignment of angular momentum was worse for higher mass haloes. This is likely related to hierarchical structure formation; the most massive haloes formed most recently and are likely to have experienced a major merger more recently than smaller mass haloes, which perturbs the angular momentum of the halo.

### 4.2.3. Halo-subhalo alignments

In a Universe where structures form hierarchically (see e.g. Press & Schechter 1974; Bond et al. 1991), it is expected that dark matter haloes will contain substructures or satellites. In the hierarchical formation scenario, at high redshifts low-mass objects begin to collapse. Through continuous accretion of dark matter over time, they increase their mass. They also undergo mergers with other haloes, and in this process the smaller halo will lose its loosely bound outer layers while the denser core survives as a subhalo within its new host halo. The alignment (shape, spin and angular momentum) and distribution of dark matter subhaloes within their host halo has been the subject of many \( N \)-body studies (e.g. Tormen 1997; Knebe et al. 2004; Kuhlen et al. 2007; Faltenbacher et al. 2008; Pereira et al. 2008; Knebe et al. 2008a,b; Angulo et al. 2009; Ocean Wang et al. 2014).

One of the alignment signals investigated is the orientation of the subhalo distribution within the shape of the host dark matter halo, \( \theta_{BH} \), see Figure 3a and Table 1 (e.g. Knebe et al. 2004; Faltenbacher et al. 2008; Kuhlen et al. 2007; Ocean Wang et al. 2014). Most studies used different versions of the inertia tensor (see Equation 32 and the surrounding text) to determine the axes of their host haloes. The exception was Ocean Wang et al. (2014) who determined the axes of the host haloes at given local mass overdensities following the method of Jing & Suto (2002) (see Equation 33 and surrounding text). Most studies determined the shape of the subhalo distribution through the lines connecting the centre of the host halo with each subhalo and calculated the alignment as a shape-position alignment, \( \langle |r \cdot \hat{r}| \rangle \) (Knebe et al. 2004; Faltenbacher et al. 2008; Ocean Wang et al. 2014). While Kuhlen et al. (2007) determined the axes of the subhalo distribution ellipsoid through diagonalising the weighted moment of inertia tensor constructed from the positions of each subhalo, finally treating the alignment as a shape-shape alignment. The resulting alignments determined whether the subhalo distribution ellipsoid (or ellipse if projected on to a 2D observer plane; e.g. Ocean Wang et al. 2014) was random within the host or if it exhibited some anisotropic signal.

Instead of calculating the alignment with the orientation angle of the host halo, Ocean Wang et al. (2014) assumed an orientation for an imagined central galaxy, \( \theta_{HC} \) (see Figure 3a and Table 1). This orientation either followed the inner, intermediate or outer host halo orientations. Their results showed that while the satellite distribution aligned with the “galaxy” semi-major axis at all orientations, the alignments increased in strength for orientations with the inner, intermediate and outer haloes respectively (implying an alignment, \( \theta_{BH} \), with the outer host halo semi-major axis). When comparing with observations, the alignment signal was too strong when the galaxy followed the outer halo orientation, and much closer to the observations when the galaxy followed the inner halo orientation or had a misalignment drawn from a Gaussian distribution with a mean of 0° and standard deviation of 25°.

There are differences within all of these studies on how the subhalo population was selected. Additionally, a number of different halo finding algorithms were employed and each had different particle or mass cutoffs. Regardless of these differences, the consensus is that the orbits of subhaloes within a host halo are strongly anisotropic (not random) and the semi-major axes of the subhalo distribution ellipsoids preferentially align with the semi-major axis of the outer host halo. The question now is why subhaloes exhibit this alignment and whether it is the result of anisotropic infall or if the subhaloes have an isotropic infall and then experience dynamic effects that lock in the alignment. Given that the outer host halo may be significantly misaligned with the inner host halo (see Section 4.2.2), which would
Fig. 11.— Top: Stellar and dark matter position angles (P.A.) vs. time for an eccentric orbit. Grey lines indicate the direction to the cluster centre and dotted vertical lines represent pericentre passages. While the dark matter particles (black dots) appear to follow the orbital motion closely, varying their rotation speed to match the orbital velocity changes, the stellar particles (red circles) appear to be locked in uniform figure rotation. Bottom: Misalignment between the orientation of the dark matter and stars $|\Delta P.A.|$ as a function of orbital time. The misalignment peaks immediately after each pericentre passage, and remains small for the rest of the orbit. © AAS. Reproduced with permission from Pereira & Bryan (2010).

have an impact on the alignment of the subhalo population, this might suggest that dynamical effects are the cause of subhalo distribution alignments. However, haloes merge preferentially along the direction of their host filament (see Section 4.2.5), which would add an element of anisotropic infall.

The alignment of the subhalo distribution with the host halo spin axis, $\theta_{hB}$, was investigated by Angulo et al. (2009). They found that the subhalo distribution was aligned perpendicular to the spin axis. As mentioned in Section 4.2.2, the angular momentum vector is preferentially perpendicular to the major axis of the host halo, indicating that this work is consistent with the results above.

Another quantity that is studied is the alignment of a subhalo with the centre of mass of the host halo, $\theta_{3d}$, see Figure 3a and Table 1 (e.g. Kuhlen et al. 2007; Faltenbacher et al. 2008; Knebe et al. 2008a,b; Pereira et al. 2008). The majority looked at the 3D shapes of the subhaloes but Knebe et al. (2008b) investigated the 2D projection on to the observers plane. The studies all agreed that there is a strong preferential radial alignment of subhaloes with the centre of mass of the host. Faltenbacher et al. (2008) investigated this trend over a wide range of scales and found, unsurprisingly, that this alignment was strongest on smaller scales and dropped off rapidly with increasing radius. The evolution of this alignment showed that while there is a strong preference for a nearly radial alignment, there are some deviations throughout the orbit of the subhalo around its host.

Pereira & Bryan (2010) took a different approach to study the alignments of satellite galaxies within dark matter haloes. They simulated a multi-component, $N$-body (stars+dark matter) satellite, orbiting an external, analytical po-
potential (which simulated the host cluster sized halo), and showed that the stellar component of the satellite reacted significantly slower to the tidal torque from the host halo than the dark matter, producing a radial twisting of the satellite that depended on the eccentricity of the orbit. If the satellite was in a circular orbit, both dark matter and stars were tidally locked (semi-major axis pointing toward the centre of mass) but the stars took approximately twice as long as the dark matter to lock to the external potential. In such orbit, the stars and dark matter were always aligned with each other when measured at the same radius. By contrast, in very eccentric orbits the change in torque was too rapid to tidally lock the stars, which figure-rotated around the potential, while the dark matter maintained a radial alignment except at pericentre where a sudden misalignment occurred (Figure 11, top panel). Additionally, after a brief period of influence from the host halo, the stars and dark matter aligned with each other, except for short periods surrounding the pericentre passage (Figure 11, bottom panel). Overall, Pereira & Bryan (2010) also found that there is a strong preference for nearly radial alignments but they also showed that the magnitude of these deviations depends on the eccentricity of the orbit.

Kuhlen et al. (2007); Pereira et al. (2008) and Pereira & Bryan (2010) used tidal torque theory to explain the motion of a subhalo around the host; this is illustrated in Figure 12. A subhalo is typically be in an eccentric orbit. As it approaches pericentre, the subhalo generally points toward the centre of the host halo mass because it is being tidally torqued in the direction of the potential gradient, which is also close to the direction of motion. As the subhalo reaches pericentre, the torquing is less effective due to its high velocity at this point, resulting in a lower radial alignment signal. The torquing continues throughout the orbit, which keeps the subhalo semi-major axis largely aligned with the centre of mass of the host. The misalignment of the subhalo semi-major axis with the orbital direction initially increases before coming back in to alignment after passing the apocentre. As the subhalo approaches pericentre again, a new cycle begins.

The signals found in the simulations were typically stronger than those in observations and Knebe et al. (2008b) also attempted to explain this through tidal torque theory. Since the subhaloes are subject to strong tidal torquing as
they pass through apocentre, the loosely bound outer particles will be more highly distorted than the inner particles. Consequently, the shape of the outer subhalo becomes more distorted than the inner region and the centre of the subhalo (where the luminous satellite would reside) may have a weaker alignment with the centre of mass of the host.

4.2.4. Halo-halo alignments

Determining whether dark matter haloes have a tendency to align with nearby haloes has been the focus of a number of studies (e.g. Barnes & Efstathiou 1987; Croft & Metzler 2000; Heavens et al. 2000; Onuora & Thomas 2000; Jing 2002; Bailin & Steinmetz 2005; Hopkins et al. 2005; Altay et al. 2006; Schneider et al. 2012). There are two different kinds of alignments measured - the first is a direct alignment, $\theta_{\text{HH}}$, that measures whether neighbouring halo axes tend to point toward each other; see Equation 45 for 3D alignments and Equation 49 for 2D alignments. The second alignment, $\theta_{\text{Hf}}$, determines whether haloes tend to point in the direction of their neighbours; see Equation 46. Both alignments are shown in Figure 3b and Table 1 and are discussed below.

Many studies showed that the semi-major axes of group and cluster sized haloes ($M_h \geq 10^{13} \ h^{-1} M_\odot$) preferentially point toward each other in a 3D analysis (e.g. Barnes & Efstathiou 1987; Faltenbacher et al. 2002; Kasun & Evrard 2005; Hopkins et al. 2005). There was evidence showing this signal to be significant out to 30 $h^{-1}$Mpc in the simulations (e.g. Faltenbacher et al. 2002; Hopkins et al. 2005; Kasun & Evrard 2005). Bailin & Steinmetz (2005) investigated the alignments, $\theta_{\text{HH}}$, using predominantly galaxy mass haloes and measured an alignment signal that was lower in amplitude than the works above, which they attributed to the lower mass range of their haloes. This assumption was confirmed in a later investigation by Schneider et al. (2012) that measured alignment over a wide range of halo masses from subhaloes to clusters ($10^{10} < M_h < 2 \times 10^{14} \ h^{-1} M_\odot$). This study showed that there was a tendency for neighbouring halo semi-major axes to point toward each other in all mass ranges, but the strength of the alignment was a strongly increasing function of halo mass.

In similar studies, Croft & Metzler (2000); Heavens et al. (2000) and Jing (2002) investigated projected 2D axis alignments of cluster sized dark matter haloes. Projections provide a lower limit on the alignment signal (since projections necessarily lose information about the system). However, Jing (2002) showed that the results in Croft & Metzler (2000) and Heavens et al. (2000) further underestimated the correlations as their shape measurements included haloes with as few as 20 particles, which could lead to a factor of two underestimate of ellipticity correlation. Using a minimum of 160 particles for their final shape measurements, Jing (2002) showed that the ellipticity correlations had a high enough amplitude to contaminate both deep and wide weak lensing surveys.

A number of works considered the alignment, $\theta_{\text{Hf}}$, of the orientation of a halo with the direction to the centre of mass of a neighbouring halo, (e.g. Onuora & Thomas 2000; Faltenbacher et al. 2002; Hopkins et al. 2005; Kasun & Evrard 2005; Altay et al. 2006; Schneider et al. 2012). This alignment signal tended to be stronger than the halo axis alignments (e.g. Faltenbacher et al. 2002; Hopkins et al. 2005; Kasun & Evrard 2005; Schneider et al. 2012), and was also a strongly increasing function of halo mass (e.g. Bailin & Steinmetz 2005; Schneider et al. 2012). The signal was detected out to $\geq 100 \ h^{-1}$Mpc for group and cluster mass haloes ($M_h \geq 2 \times 10^{13} \ h^{-1} M_\odot$) (e.g. Faltenbacher et al. 2002; Hopkins et al. 2005; Kasun & Evrard 2005). Hopkins et al. (2005) also studied cluster halo alignments as a function of redshift and showed that the alignments were greater at early times and that aligned halo pairs were more likely to be connected by a filament than unaligned halo pairs. They found this result was consistent with the anisotropic merging and infall scenario, where haloes form through mergers and accretion of dark matter traveling coherently along the direction of the large-scale filaments, such that the alignments should be present from the time of halo formation.

Lee et al. (2005) attempted to provide an alternative explanation to the anisotropic infall model, arguing that the model is merely qualitative and that primordial alignments should be damped over time by non-linear processes. Anisotropic infall has been used to explain the observational result that brightest cluster galaxies and dark matter haloes have their semi-major axes preferentially aligned. To mimic this scenario in the absence of baryons, they identified the largest central subhalo within the cluster-sized dark matter haloes to represent the central galaxy and investigated the alignment, $\theta_{\text{SH}}$. In their alternative model, interaction between the host halo tidal field and the subhalo should account for the observed alignments, assuming that the subhalo angular momentum is aligned parallel to the subhalo semi-minor axis. Their simulations showed that the semi-minor axes of the subhaloes were preferentially perpendicular to the semi-major axis of the host haloes, in agreement with observations and the predictions of their
alternative model. However, in order to rule out the anisotropic infall model, they conceded that they needed to measure the angular momentum alignments directly but their simulations lacked the resolution required to do this.

When investigating spin-spin alignments, \( \theta_{s_{1}s_{2}} \), tidal torque theory suggests that neighbouring haloes should experience some alignment of their spin vectors. Trowland et al. (2013) showed that neighbouring halo spins had a weak parallel alignment to each other, but only for halo separations under \( 0.3 h^{-1}\text{Mpc} \). On these scales, only subhaloes in massive clusters would be able to exhibit alignments. The scale of this halo separation is so small that this is likely why other lower resolution studies did not detect any clear alignment signal (Barnes & Efstathiou 1987; Porciani et al. 2002; Faltenbacher et al. 2002; Bailin & Steinmetz 2005). By contrast, Hatton & Ninin (2001) found significant but weak alignments on scales from 1 – 30 \( h^{-1}\text{Mpc} \) and Hahn et al. (2007b) also found a correlation in alignments of spins for massive haloes in clusters. However, in an extension of this work using a different selection criteria for their haloes, Hahn et al. (2007a) found no halo-halo spin correlation. They used a more stringent selection criteria to remove haloes that had not yet relaxed. These systems were mostly two close neighbours that were either in the process of merging or had been spuriously linked into a single halo. They also removed the haloes where the distance of the most bound particle from the centre of mass exceeded a quarter of the largest distance between the outermost particle in the halo and the centre of mass (since the most bound particle in a relaxed halo would normally reside close to the centre of mass). This cleaned catalogue modified the spin parameter distribution, and hence reversed their earlier detection of spin alignments. Since halo selection criteria can have a significant effect on results, this, and the fact that they measured the spins of haloes with as few as 20 particles, may offer reasonable explanations for why Hatton & Ninin (2001) found alignments in their simulations.

4.2.5. Halo-LSS alignments

\( N \)-body simulations on a cosmological scale have shown that structure formation (i.e. gravitational collapse) occurs first in sheets. Filaments then form and matter in these filaments collapses into dark matter haloes. These dark matter haloes move along the filaments toward intersections between multiple filaments that are knots of potential minima where cluster haloes are formed. See Section 4.1.4 for more information and details on some cosmic web classification schemes.

There are a number of studies that measure the shape alignment of a halo with the surrounding large-scale structure; \( \theta_{HF}, \theta_{HF}^{\perp}, \theta_{5H} \) (e.g. Patiri et al. 2006; Hahn et al. 2007a; Brunino et al. 2007; Cuesta et al. 2008; Zhang et al. 2009; Paz et al. 2011; Libeskind et al. 2013; Forero-Romero et al. 2014). The consensus is that shape alignments with the LSS are stronger and more robust to measurement algorithm and simulation than angular momentum/spin correlations. There is a consistently strong parallel alignment of the semi-major axis of a halo with its host sheet, filament or cluster or perpendicular to the radial direction of the centre of voids (the semi-minor axis of the haloes tends to be parallel to the radial direction of voids) in all but the lowest mass haloes. This alignment increases in strength with increasing halo mass. Forero-Romero et al. (2014) traced the shape of the structure using both the tidal shear field (which they term the T-web; Equation 42) and velocity field (V-web; Equation 43). However, they did not divide their simulation into distinct web elements using a threshold \( \lambda_{th} \), instead choosing to classify the alignments directly with respect to the eigenvectors \( \hat{e}_{1} \) and \( \hat{e}_{3} \). For simplicity of reporting results, they defined a strong alignment with \( \hat{e}_{3} \), a strong alignment \( \theta_{HF} \) (see Figure 4b), with a filament, while a strong alignment with \( \hat{e}_{1} \) is an anti-alignment, \( \theta_{HF}^{\perp} \) (see Figure 4a and Table 1), with a sheet. This choice has the benefit of being independent of \( \lambda_{th} \) for each classification scheme, however the signal may become diluted when mixing of the signal occurs between environments. Despite this unusual classification scheme, their results for the tidal shear field matched previous works. However in the momentum-based velocity field measurement, they found an anti-alignment of haloes with sheets for haloes \( M_{h} > 10^{12} h^{-1}\text{M}_{\odot} \).

Many works investigated the spins of a haloes as a function of environment (e.g. Aragón-Calvo et al. 2007; Hahn et al. 2007a,b; Brunino et al. 2007; Sousbie et al. 2008; Zhang et al. 2009; Codis et al. 2012; Trowland et al. 2013; Libeskind et al. 2013; Aragon-Calvo & Yang 2014; Forero-Romero et al. 2014). There is consensus that this measurement is far less robust than the alignments with halo shape and results are more dependent on measurement algorithm.

\(^{12}\text{Note that the subscript ‘H’ in this alignment is the cluster cosmic web element and the ‘S’ represents the galaxy-sized and smaller halo substructure within the cluster.}\)
While this simulation includes hydrodynamics, the results are focused on dark matter haloes and appear to be independent of the baryons, hence its simulation and environment definition. There is little N-body simulation literature for angular momentum alignments around voids, \( \theta_{hiw} \), directly, although comparisons may be made with alignments of sheets since these reside at the boundaries of the void surfaces and in the case where the void is modelled as spherical, the alignment would simply be perpendicular to the sheet alignment. Heymans et al. (2006); Patiri et al. (2006) and Brunino et al. (2007) investigated the void alignments directly and concluded that there were no angular momentum alignments around voids and the orientations of haloes, was random. By contrast Cuesta et al. (2008) did find alignments, where the angular momentum of haloes with sheets, there is a trend for the orientations of haloes was random. By contrast Cuesta et al. (2008) found the semi-minor axis of their haloes was preferentially perpendicular to the semi-minor axis in this work. Given these discrepancies, further investigations are required to clarify the alignments of angular momentum around voids.

When considering the alignment, \( \theta_{hiw} \), of the angular momentum of haloes with sheets, there is a trend for the angular momentum to be aligned parallel to the sheet (e.g. Aragón-Calvo et al. 2007; Hahn et al. 2007a; Zhang et al. 2009). Table 2: Angular momentum alignment with the cosmic web, where \( \hat{e}_1 \) (\( \hat{e}_2 \)) is the major (minor) eigenvector of the corresponding tensor (velocity/shear). Summary of theoretical results provided by similar analysis methods. (- -)++ indicates a strong (anti-)alignment and (-) indicates a weak (anti-)alignment. Reproduced with permission from Forero-Romero et al. (2014).

| Author | Web Method | Spatial Scale \( h^{-1} \) Mpc | Along | Alignment | Mass dependence |
|--------|------------|---------------------------------|-------|-----------|-----------------|
| Forero-Romero et al. (2014) | T-Web | 0.5 – 1 | \( \hat{e}_1 \) (filament) | none | > 10^{12} h^{-1}M_{\odot} |
| | | | \( \hat{e}_1 \) (filament) | none | < 10^{12} h^{-1}M_{\odot} |
| | | | \( \hat{e}_1 \) (wall) | none | > 10^{12} h^{-1}M_{\odot} |
| | | | \( \hat{e}_1 \) (wall) | none | < 10^{12} h^{-1}M_{\odot} |
| Forero-Romero et al. (2014) | V-Web | 0.5 – 1 | \( \hat{e}_1 \) (filament) | none | > 10^{12} h^{-1}M_{\odot} |
| | | | \( \hat{e}_1 \) (filament) | none | < 10^{12} h^{-1}M_{\odot} |
| | | | \( \hat{e}_1 \) (wall) | none | > 10^{12} h^{-1}M_{\odot} |
| | | | \( \hat{e}_1 \) (wall) | none | < 10^{12} h^{-1}M_{\odot} |
| Libeskind et al. (2013) | V-Web | 1 | filament | none | > 10^{12} h^{-1}M_{\odot} |
| | | | filament | none | < 10^{12} h^{-1}M_{\odot} |
| | | | wall | none | all masses |
| Trowland et al. (2013) | Hessian density | 2 – 5 | filament | none | > 5 \times 10^{12} h^{-1}M_{\odot} |
| | | | filament | none | < 5 \times 10^{12} h^{-1}M_{\odot} |
| Codis et al. (2012) | Morse Theory & T-Web | 1 – 5 | filament | none | > 10^{12} h^{-1}M_{\odot} |
| | | | filament | none | < 10^{12} h^{-1}M_{\odot} |
| | | | wall | none | all masses |
| Zhang et al. (2009) | Hessian density | 2.1 | filament | none | if anticorrelated with shape |
| | | | filament | none | if correlated with shape |
| Aragón-Calvo et al. (2007) | Hessian density | - | wall | ++ | > 10^{12} h^{-1}M_{\odot} |
| | | | wall | ++ | < 10^{12} h^{-1}M_{\odot} |
| | | | - | - | > 10^{12} h^{-1}M_{\odot} |
| | | | - | - | < 10^{12} h^{-1}M_{\odot} |
| Hahn et al. (2007a) | Tidal Web | 2.1 | filament | ++ | none |
| | | | wall | ++ | > 10^{12} h^{-1}M_{\odot} |
| | | | wall | ++ | < 10^{12} h^{-1}M_{\odot} |

\footnote{While this simulation includes hydrodynamics, the results are focused on dark matter haloes and appear to be independent of the baryons, hence its inclusion in this section.}

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There is also a trend for the alignment, $\theta_{\text{lin}}$, of the angular momentum of high mass haloes to be perpendicular to a filament, while lower mass haloes align parallel to the filament (e.g. Aragón-Calvo et al. 2007; Hahn et al. 2007b; Sousbie et al. 2008; Codis et al. 2012; Trowland et al. 2013; Aragon-Calvo & Yang 2014) and the mass where this transition occurs is around $M_{\text{crit}}^S \approx 5 \times 10^{12} h^{-1} M_{\odot}$ (e.g. Hahn et al. 2007b; Aragón-Calvo et al. 2007; Codis et al. 2012; Trowland et al. 2013). Codis et al. (2012) investigated the redshift dependence of the transition mass and found that the halo transition mass $M_{\text{crit}}^S$ decreased with increasing redshift such that

$$M_{\text{crit}}^S \approx M_0^S (1 + z)^{-\gamma_S}, \quad \gamma_S = 2.5 \pm 0.2. \quad (51)$$

Looking at the alignments in terms of the halo model, Paz et al. (2008) found similar results. They calculated the dark matter correlation function parallel and perpendicular to the angular momentum of the haloes and found the two-halo term aligned preferentially perpendicular to the distribution of the large-scale structure at high masses while low mass haloes may preferentially point parallel to the mass distribution on large scales. This inversion occurred around masses of $5.5 \times 10^{12.5} h^{-1} M_{\odot}$, consistent with the findings above.

In contrast to these findings Libeskind et al. (2013) disputed that the high-mass halo transition occurs only in filaments and showed evidence for the transition occurring in all environments, with the mass at which the transition occurs decreasing with environment type from clusters to voids (e.g. in voids, haloes of intermediate mass transition their angular momentum). In further contrast, Forero-Romero et al. (2014) found results that only broadly matched many of these earlier works. For web elements defined in the V-web, they found that the angular momentum of haloes with masses above $10^{12} h^{-1} M_{\odot}$ tended to align along walls, without any clear trend with respect to filaments, while the alignment signal disappeared for halo masses below $10^{11} h^{-1} M_{\odot}$. In the T-web, they similarly found no evidence for any alignment of haloes with masses below $10^{12} h^{-1} M_{\odot}$, and only a weak signal for alignment at higher mass, with a trend for the angular momentum to lie perpendicular to filaments. The discrepancies in this work may be the result of the chosen web classification scheme diluting an already weak signal even further by mixing environments, or the alignments could have a high sensitivity on small scales to the method used to construct the cosmic web (including numerical choices for interpolating the relevant fields). Forero-Romero et al. (2014) provided a helpful comparison of angular momentum alignments found in these similar works, which is reproduced in Table 2.

5. Hydrodynamic simulations

A significant shortcoming of $N$-body simulations in the study of galaxy dynamics or intrinsic alignments is that these simulations are concerned only with the evolution of structure under gravity. The particles used in the simulations interact only gravitationally, and any effects relating to baryon or gas physics are completely ignored. Hydrodynamic simulations address this issue by treating the evolution of the gaseous component of the Universe using the methods of computational fluid dynamics. This approach enables the complex interactions of the different baryonic components (gas, stars, etc.) to be treated self-consistently with the dark matter, and on a much smaller scale. This enables simulations of the formation of galaxies within dark matter halos, and the ability to probe the complex physical processes that give rise to the visual properties of galaxies that we observe today.

Hydrodynamic simulations are a relatively new way to explore the intrinsic alignments of galaxy shapes and spins, in part because they are quite computationally intensive. The obvious advantage of using hydrodynamic simulations is that $N$-body simulations require some sort of semi-analytic model to identify the positions and orientations of galaxies (see Section 6), whereas in hydrodynamic simulations, the process of galaxy formation is naturally included. The disadvantage of using hydrodynamic simulations is that the relevant sub-grid physics that is needed to form realistic galaxies is not yet known. The term “sub-grid physics” is meant to include all physical processes that take place on a smaller scale than the resolution of the simulation, and therefore must be included with some model. This includes some important aspects of star formation, accretion onto supermassive black holes, radiative heating/cooling, and feedback from supernovae (important at low masses) and some other mechanisms at high masses (e.g., from AGN). While there are claims in the literature of increasingly realistic galaxy populations using hydrodynamic simulations (e.g.

\[^{14}\text{Note that the terms "hydrodynamic" and "gasdynamic" are used interchangeably in the literature, although hydrodynamic is adopted in this review.}\]
Khandai et al. 2014; Vogelsberger et al. 2014), the results still show some discrepancies with reality that require caution with their usage. For example, the results of the sub-grid physics are often tuned to match particular observables (e.g. the luminosity function) at a particular redshift (typically redshift zero), so extrapolation to different redshifts should therefore be treated with caution, as observations at high redshift are limited and matching these results is not guaranteed. Ideally, a full exploration of intrinsic alignments in hydrodynamic simulations would involve a comparison between several independent simulations with different implementations of sub-grid physics, to check how the results for galaxy intrinsic alignments depend on the details of galaxy formation and feedback. Unfortunately, such a comparison is not yet possible due to the tremendous expense of large-volume and high-resolution hydrodynamic simulations.

Detailed theoretical understanding of the (expected) shape and angular momentum/spin alignments between galaxies inside haloes and their host haloes (and combinations thereof) is fundamentally limited due to the not-well-understood role that gravitational collapse of the halo and baryonic physics has on the shapes of galaxies. In particular the feedback processes between galaxy and cluster-scale physics and the surrounding dark matter haloes is not well understood. It is also important to note that while spin alignments are likely the dominant alignment mechanism for rotationally-supported galaxies, the study of spin alignments may omit some critical details for intrinsic alignment contamination of weak lensing measurements, given that observed intrinsic alignments are dominated by red galaxies, which are pressure-supported and thus subject to intrinsic alignments due to different physics (e.g., tidal effects).

5.1. Small-volume hydrodynamic simulations

In this section, small-volume and zoom hydrodynamic simulations are discussed.

5.1.1. Internal alignments

One very interesting study made possible with hydrodynamic simulations is how the baryonic galaxy aligns with its host dark matter halo, $\theta_{CH}$, and whether baryons have an effect on halo shape and alignment.

Broadly, studies agree that the addition of baryons significantly affects the shape of the host dark matter halo. Most agree that a galaxy forming in the centre of the halo will cause the halo to be more spherical overall and there is a consensus that haloes with galaxies tend toward oblateness, rather than the prolateness seen in $N$-body only simulations (e.g. Kazantzidis et al. 2004; Bailin et al. 2005; Berentzen & Shlosman 2006; Gustafsson et al. 2006; Abadi et al. 2010; Deason et al. 2011). However, since these studies used different sized haloes or simulations that had different feedback effects, it is difficult to make direct comparisons. For example Kazantzidis et al. (2004) ran Adaptive Refinement Tree $N$-body + hydrodynamic simulations of cluster sized haloes and showed that the presence of baryons significantly increased the sphericity of the host dark matter haloes with a magnitude that decreased at larger radii such that the sphericity was largely indistinguishable from dark matter only haloes at the virial radius. Bailin et al. (2005) found very similar results, an increase in halo sphericity that decreases with increasing radius, using seven different implementations of hydrodynamics in the simulations and employing a multi-mass resimulation technique to generate high resolution galactic sized discs. Abadi et al. (2010) ran $N$-body + hydrodynamic simulations of galactic sized discs with radiative cooling but neglected stellar feedback. By calculating equipotential axial ratios they found haloes that were significantly rounder than their dark matter only counterparts at all radii out to nearly $2R_{\text{vir}}$, which may be due to strong overcooling due to neglecting stellar feedback. By contrast, Deason et al. (2011) used simulations including stellar feedback and used the galaxy density to characterise the halo shapes (which are typically flatter than equipotential surfaces). The haloes in their sample were almost spherical and slightly more oblate in the inner regions, while their dark matter only haloes tended toward more prolate in the inner regions. Thus, they agreed that baryon physics significantly affects the shape of the halo in the inner regions but they did not see significant changes in sphericity in the outer regions between the $N$-body and hydrodynamic simulations.

Knebe et al. (2010) performed a similar analysis by comparing a dark matter only simulation with a hydrodynamic simulation with the same initial conditions. This study included 3 Milky Way sized galaxy haloes with their corresponding substructures. They tested the shapes of the dark matter haloes and subhaloes in their simulations and found the haloes to be more spherical in the hydrodynamic simulation, in agreement with the general consensus above. However, the subhaloes showed no shape differences between the two simulations, indicating that the addition of gas
physics had no influence on the subhalo population. They suggested that the lack of shape changes may be due to the low mass nature of the subhaloes and that there may be more obvious changes for cluster sized host haloes with large galaxy sized subhaloes.

Bailin et al. (2005) tested whether the orientation of the inner halo was being driven by the galactic disc, by generating rings of test particles tilted by $10^\circ$ with respect to the disc plane and comparing the gravitational torque on the test particles due to the dark matter and the baryons. For test particles in rings $< 0.1R_{\text{vir}}$, the baryons and dark matter exerted torques of comparable magnitude, while the dark matter dominated the torque at larger radii. Therefore, it was concluded that the alignments at inner radii were due to simultaneous evolution of the halo and disc, rather than the disc driving the halo orientation, or vice versa, and that the orientation of the inner and outer halo in the hydrodynamic simulations was uncorrelated. Tidal torque theory suggests that correlations between a galaxy and its host halo are expected during the formation epoch (e.g. Heavens & Peacock 1988). However, while the alignment of a galaxy and its host halo is initially very close, at later times the outer halo will continue to accrete material. This could alter the net angular momentum or shape of the outer halo, causing the misalignments. This distinction is important when considering the alignment of satellite populations and their intrinsic alignment signal.

Thinking about how galaxy angular momentum aligns with its host halo, Bailin et al. (2005); Bett et al. (2010) and Deason et al. (2011) all showed that the alignment of the angular momentum vector of the galactic disc with the semi-minor axis of the host halo was close at $R < 0.1R_{\text{vir}}$. Bailin et al. (2005) and Deason et al. (2011) also showed that the halo could be significantly misaligned at $R > 0.1R_{\text{vir}}$ with misalignments of $\theta_{\text{halo}} > 45^\circ$ at $R_{\text{vir}}$.

van den Bosch et al. (2002); Chen et al. (2003); van den Bosch et al. (2003) and Sharma & Steinmetz (2005) made measurements of the misalignments, $\theta_{\text{halo}}$ at $R_{\text{vir}}$ and found an average misalignment of between $20^\circ$ and $36^\circ$ for their standard baryonic simulations. In a similar study, Hahn et al. (2010) found that while the alignment of the angular momentum for both the central disc galaxy gas and stars was close to the angular momentum of the inner dark matter halo at $\theta_{\text{halo}} \sim 18^\circ$, there could be significant misalignments, $\theta_{\text{halo}} \sim 50^\circ$, with the angular momentum of the whole dark matter halo. This result appeared to have no dependence on environment density or stellar or halo mass.

There is a known problem in hydrodynamic simulations where cooling gas forming the central galaxies has an angular momentum that is too low to produce extended galaxies (e.g. Navarro & Benz 1991). This may be partially due to numerical limitations, but uncertainties in the baryonic feedback processes are certain to contribute significantly (e.g. Maller & Dekel 2002). While the feedback processes are being investigated separately, van den Bosch et al. (2003) tested the effects of artificially pre-heating the baryonic component of the intergalactic medium, and found that this pre-heating decoupled the baryons from the dark matter, resulting in significantly higher misalignments between the angular momentum of both the gas and the dark matter halo of $58^\circ$ on average.

Romano-Díaz et al. (2009); Scannapieco et al. (2009) and Cen (2014) investigated spin orientation changes over time. When investigating a $\sim 10^{12} M_\odot h^{-1}$, somewhat isolated (no major mergers after $z = 1.5$) central galaxy, Romano-Díaz et al. (2009) showed that the spin alignments of the dark matter halo (within $R_{\text{vir}} \sim 400$ kpc), stars and gas (within the inner $8$ kpc) changed frequently over time and even experienced $\sim 180^\circ$ flips over short timescales and at times with no major mergers (Figure 13a). When also limiting the dark matter to the inner $8$ kpc, they found that at $z \geq 3$ major merger activity caused all alignments to change frequently. From $3 \geq z \geq 0.5$, the spins of the dark matter, stars and gas were well aligned but at $z \leq 0.5$, the alignments flipped when cold gas became concentrated in the central regions and had a spin in the opposite direction to the stars and dark matter (Figure 13b). Scannapieco et al. (2009) found that misalignments between the stellar disc and newly accreted cold gas could cause the stellar disc to lose part of its mass or become completely destroyed (but later regrow).

Cen (2014) simulated and investigated spin alignments in more than 300 galaxies between $M_* = 10^{10} - 10^{12} M_\odot$. They calculated the orientations of the specific angular momentum (Equation 39) for both gas and stars within the galaxy radius over time and showed that orientations were dependent on galaxy type (elliptical or spiral), environment, luminosity and redshift. They found that the spin alignments decreased with increasing redshift, increased with increasing stellar mass and were larger for elliptical than spiral galaxies.

It is clear that frequent changes in spin direction over time have a large effect on the final shape and orientation of a galactic disc. Mergers and cold gas accretion cause more stochastic and rapid orientation changes, while the large-scale tidal field slowly acts to bring galaxy spins back into alignment. The question remains whether steady
anisotropic infall of gas and stars along filaments and sheets could also act as a mechanism to align the spins of haloes along their structures.

5.1.2. Satellite alignments

The alignment of the distribution of satellite galaxies in host dark matter haloes, $\theta_{BH}$, was extensively examined in Libeskind et al. (2007) and Deason et al. (2011). The method of analysis was similar in both studies although Libeskind et al. (2007) had 9 parent haloes (with only three of these having 11 or more satellites), while Deason et al. (2011) had 431 parent haloes (of which 80 had 10 or more satellites) and they only included late-type disc galaxies (the galaxies were classified dynamically as dispersion-supported spheroids or rotationally supported discs) in the analysis. Both computed the shape of the satellite distribution by diagonalising the unweighted inertia tensor (see Equation (32) and subsequent explanations). They both found that the distribution of ten or eleven brightest (most massive) satellites was significantly flatter (more planar) than the underlying dark matter distribution. They also found that satellite distribution preferentially aligned with the plane perpendicular to the semi-minor axis of the host dark matter halo. Deason et al. (2011) showed that accretion of satellites occurred preferentially along the major axis of the dark matter halo and suggested that the anisotropic satellite distribution was due to anisotropic infall. This distribution was preserved because present day satellites had been in orbit for far less time than the dark matter halo, which would have undergone relaxation and phase mixing at an earlier time. Thus, the satellites had only ever experienced a static dark matter potential, which is reflected in the preserved anisotropic spatial distribution. Deason et al. (2011) went on to show that the satellite distribution showed no preferential alignment relative to the galaxy. In fact, they found that 20% of their systems lay within $10^\circ$ of perpendicular to the plane of the galaxy. Similarly, Libeskind et al. (2007) found two of their three galaxies with eleven or more satellites were within $20^\circ$ of perpendicular to the plane of the galaxy, making these simulations consistent with the observed distribution of the Milky Way satellites.

In a separate analysis that accounted for all satellites in the systems, Deason et al. (2011) showed that the shape of the full satellite distribution continued to be much flatter than the shape of the dark matter halo, which is in contrast to Libeskind et al. (2007) who found that the full satellite distribution matched the shape of the dark matter halo well and the distribution did not exhibit the flattening found when only considering the eleven brightest satellites. However,
it is important to remember that there were only three systems in the analysis by Libeskind et al. (2007), and all galaxy types were considered, making a direct comparison between the two studies difficult. Deason et al. (2011) also showed that the orientation of the satellite distribution shapes showed only a weak bias toward the plane of the galaxy but much stronger alignments relative to the plane perpendicular to the semi-minor axis of the host halo. The alignment of the satellite distribution was significantly more influenced by the dark matter host halo, so in the presence of misalignments between the inner and outer halo, the orientation of the galactic disc is largely decoupled from the satellites.

Knebe et al. (2010) investigated the alignment of subhaloes with the centre of mass of the host dark matter halo, $\theta_{3h}$, in both a hydrodynamic and dark matter only simulation with the same initial conditions. Both simulations were a $2\,h^{-1}$Mpc resimulated region in a $64\,h^{-1}$Mpc parent simulation. They found that the subhaloes pointed preferentially toward the centre of the host halo in both simulations and that the addition of baryonic physics had no effect on the result. This is unsurprising given the subhaloes did not change shape with the addition of baryonic physics (as mentioned in Section 5.1.1).

5.1.3. **Large-scale structure Alignments**

There is some question of what the inclination of the baryonic components of galaxies is with respect to the large-scale structure. Navarro et al. (2004) used N-body/hydrodynamic simulations to produce four disc galaxies and investigated their alignments with sheets, $\theta_{GW}$. They found that the major axes of some disc galaxies tended to be highly inclined relative to the plane of the large-scale (several Mpc) two-dimensional sheet that the galaxy was formed in. This was likely due to coherence in anisotropies across a large number of scales present during the expansion and contraction of the protogalactic material causing some galactic discs to be highly inclined relative to the large-scale two-dimensional structure where they were embedded. Investigating alignments with respect to filaments, $\theta_{GF}$, Hahn et al. (2010) sampled $\sim 100$ galactic discs spanning two orders of magnitude in stellar and halo mass. They found a strong alignment of the major axis of low mass ($< 4 \times 10^{11}\,h^{-1}M_{\odot}$) galactic discs along the direction of the filament at $z = 1$. This alignment decreased with increasing density, which may be due to non-linear effects like torques from ram pressure or accretion/mergers. They did not see this alignment at $z = 0$, however this may be a numerical effect in their simulations. While admitting some statistical limitations in their studies, they also found that the most massive disc galaxies were aligned with their major axis pointing along the direction of a filament.

5.2. **Cosmological-volume hydrodynamic simulations**

When studying galaxy alignments, the use of hydrodynamic simulations over cosmological volumes is obviously highly desired. Large volume simulations enable a statistical approach to galaxy alignments by providing huge catalogues of galaxies. At the same time, if we want to use hydrodynamical simulations to predict intrinsic alignments on the scales that are relevant for lensing, up to tens of Mpc, then the minimum simulation volume is $\sim 100\,h^{-1}$Mpc.

There are a number of recent papers on intrinsic alignments in hydrodynamic simulations in cosmological volumes, including Bryan et al. (2013); Tenneti et al. (2014, 2015); Dubois et al. (2014); Welker et al. (2014) and Codis et al. (2014). Bryan et al. (2013) used the OverWhelmingly Large Simulations (OWLS) (Schaye et al. 2010) to study the effects of baryonic cooling on the spins and alignments of dark matter haloes, rather than studying the baryonic components of the galaxies themselves.

5.2.1. **Alignments with spin and angular momentum**

Dubois et al. (2014); Welker et al. (2014) and Codis et al. (2014) all focused on the issue of spin alignments using the Horizon-AGN simulation (Dubois et al. 2014) in a 100 $h^{-1}$ Mpc box. Using a catalogue of $\sim 150$, 000 galaxies, Dubois et al. (2014) investigated a snapshot at $z = 1.83$ and focused on the orientation of the spin of the galaxies, with respect to the direction of the filaments, $\theta_{GF}$. They found that more massive galaxies have their spin perpendicular to the nearest filament while lower mass galaxies have their spin parallel to the nearest filament. The mass of this transition for the stellar disc was around $M_*= 3 \times 10^{10} M_{\odot}$, which is comparable to a dark matter halo transition mass at $z = 1.83$ of $M_h = 5 \times 10^{11} M_{\odot}$. This is consistent with the redshift evolution of the transition mass found in Codis et al. (2012) (see Section 4.2.5. The study went on to determine spin orientations with respect to the nearest filament.
for a large number of galaxy properties. The overall findings formed a picture that old, massive, red, metal-rich, and dispersion-dominated (elliptical) galaxies have their spins preferentially perpendicular to the nearest filament, while young, low-mass, blue, metal-poor, and centrifugally supported (disc) galaxies have their spins preferentially aligned parallel to the filament.

The Horizon-AGN simulation was run with an adaptive mesh refinement code that uses a Cartesian-based Poisson solver (Teyssier 2002) to compute the forces. A common issue in such a solver is forces grid-locking on the Cartesian axes of the box due to a numerical anisotropy in the force calculation (see e.g. Hockney & Eastwood 1981). Dubois et al. (2014) found that gaseous disc spins of low-mass galaxies ($M_{\text{gas}} \sim 10^9 h^{-1} M_\odot$) in the simulation were preferentially aligned with one of the Cartesian axes, while higher mass galaxies did not show any grid-locking. The cosmic web elements in the simulation did not experience any grid-locking so the overall effect of the grid-locked galaxies may make the measured alignments with the filaments more noisy. Codis et al. (2014) presented evidence that if there is no spatial correlation between the grid-locking experienced by different galaxies, then the grid-locking would not significantly affect their results. However, a comparison of all of the grid-based solver studies with a solver that does not use a grid-based technique (like smoothed particle hydrodynamics (SPH) for example) would be required to test these assumptions and assess the full impact of the grid-locking on the results.

Welker et al. (2014) used the same simulations as Dubois et al. (2014) between $1.2 < z < 3.8$ in a study of the effect of mergers on galaxy spin changes. They showed that mergers along filaments caused galaxies to have a spin perpendicular to the filament and the more mergers that contributed to the galaxy mass, the stronger the perpendicular alignment. By contrast, in the absence of (successive) mergers, the galaxy spins would (re)align with the direction of the host filament.

Addressing the concerns of weak gravitational lensing, Codis et al. (2014) used the spin of the stellar component of the galaxy as a proxy for ellipticity and investigated 160,000 simulated galaxies at $z = 1.2$. When determining the apparent axis ratio for their projected galaxies, they ignored the disc flattening (which they set to zero) due to resolution limitations in the simulation, which has the effect of maximising the alignment signal measured and giving upper limits to the predictions. To investigate the GI (Equation 8) alignment signal, where correlations occur between the gravitationally sheared background galaxy and the intrinsically oriented foreground galaxy, they determined the galaxy population that was likely to experience coherent alignments as a result of the local tidal field. Similar to Dubois et al. (2014), they found that low-mass galaxies were preferentially aligned parallel to filaments and higher mass ($M_\star > 4 \times 10^{10}$) galaxies were preferentially aligned perpendicular to the filaments. Additionally, red galaxies showed more random correlations with the surrounding tidal field than blue galaxies. Part of this may be related to the fact that red galaxies are typically more massive than blue galaxies and at $z = 1.2$, the red galaxies in the simulation had a mass around the transition mass (some above, some below). The random orientations detected in this study might be expected to become more correlated at later times as more of the population exceeds the transition mass. However, it is also important to note that massive red galaxies are observed to be dispersion-dominated (see Kirk et al. 2015) and the spin of such a galaxy is not expected to be strongly correlated with its shape, meaning that the physical mechanism causing the observed alignments of massive red galaxies is unrelated to the spin. Further investigation would require use of the actual galaxy shapes rather than using spin as a proxy for the shape.

When investigating the II (Equation 8) alignment in 3D orientations, Codis et al. (2014) found a strong correlation between the spins of blue galaxy pairs out to a distance of $10 h^{-1}$ Mpc, while red galaxy pairs and red-blue pairs showed no correlations. There was also a correlation of the spins between low- and intermediate-mass galaxies, while massive galaxies had a signal consistent with zero at all separations. These conclusions were unchanged when projecting the spin onto the plane of the sky.

Observational studies of spin alignments have shown some correlation between the spins of disc-like galaxies (e.g., Lee & Erdogdu 2007; Lee 2011), but all were for $z < 0.2$, while this Horizon-AGN simulation study was performed at $z = 1.2$. Observations of shape alignments across a wide range of redshifts show red elliptical galaxies dominating the intrinsic alignment signal with blue disc-dominated galaxies being consistent with zero alignments (see Kirk et al. 2015, for a comprehensive overview of observational studies). At later times in the simulation, it will be interesting to directly compare the amplitudes of the signal with observations. For these to agree, it is possible that late-time effects that reduce the correlation of disc-dominated galaxy spins will be necessary. However, the alignments of
dispersion-dominated red ellipticals would be better captured through study of the shapes of the galaxies directly.

5.2.2. Alignments with shape and position

Unfortunately, any reliable measurement of the shape of a distribution of particles requires the distribution to be sampled with a large number of particles. Convergence tests in, e.g., Tenneti et al. (2014) suggested that hundreds of (but preferably 1000) star particles are necessary to accurately infer galaxy shapes and therefore measure intrinsic alignments\(^\text{15}\) (with a similar requirement applying to the number of dark matter particles, when studying dark matter halo alignments). This leads to stringent requirements on resolution; for example, a study of intrinsic alignments of galaxies with stellar mass of \(M_\star \geq 10^{10} M_\odot\) requires a star particle mass of \(10^7 h^{-1} M_\odot\). Relevant to the understanding of intrinsic alignments is the study of the shapes and alignments of galaxies with respect to the (mostly dark) matter distribution in their host haloes. In this case, a stochastic misalignment, \(\theta_{\text{GH}}\) (see Table 1), between galaxies and their host haloes suppresses the galaxy intrinsic alignments with respect to those from \(N\)-body simulations.

Tenneti et al. (2014) used the MassiveBlack-II SPH simulation (MB-II; Khandai et al. 2014), with star formation and AGN feedback, to study the shapes and alignments of galaxies with >1000 particles (Figure 14). In a similar study, Velliscig et al. (2015) used the Evolution and Assembly of GaLaxies and their Environments (EAGLE; Schaye et al. 2015; Crain et al. 2015) and cosmo-OverWhelmingly Large Simulations (cosmo-OWLS; Schaye et al. 2010; Le Brun et al. 2014; McCarthy et al. 2014) to study the alignment and shape of dark matter, stellar, and hot gas distributions with >300 particles. Both investigations found that misalignments, \(\theta_{\text{GH}}\), were larger for lower-mass haloes than for higher-mass haloes and Tenneti et al. (2014) showed that the misalignments were only weakly dependent on redshift (see Figure 15a). Velliscig et al. (2015) showed that, in the EAGLE simulation, the median misalignment angle was reduced to \(\sim 25 - 30^\circ\) in the most massive haloes \((10^{13} < M_{200}/[h^{-1} M_\odot] < 10^{15})\) and both Velliscig et al. (2015) and Tenneti et al. (2014) showed that the misalignment angles were systematically reduced at all masses when considering 2D, rather than 3D, misalignment angles. Some observational studies (e.g. Heymans et al. 2004; Okumura et al. 2009) have assumed a Gaussian functional form for the distribution of the 2D stochastic misalignment angle between galaxies and their host haloes. However, both Tenneti et al. (2014) and Velliscig et al. (2015) showed that the probability distributions of the misalignment angle were not Gaussian (Figure 15). Figure 15b shows an example of a non-Gaussian distribution of misalignment angles for dark matter haloes, which are potential hosts of Luminous Red Galaxies (LRG), from Velliscig et al. (2015) compared with the assumed Gaussian functional forms from Okumura et al. (2009). These results show the importance of exercising caution when employing oversimplified assumptions about the stochastic misalignment between galaxies and haloes and when interpreting the measurements of galaxy intrinsic alignment correlation functions.

Velliscig et al. (2015) calculated the misalignment of the baryonic components (hot gas and stars) of the galaxies with their host haloes as a function of halo mass, radius, and galaxy type. Overall, galaxies were found to align well with the inner host haloes. However, the stellar distributions exhibited a median misalignment of \(\sim 45 - 50^\circ\) with respect to the outer host haloes. One simulation, calibrated to reproduce the stellar mass function at redshift zero, was used for comparison with the simulations that were generated using different feedback models. Importantly, the simulations that did not reproduce the basic properties of galaxy populations, such as abundance (number density per comoving volume) and galaxy size, exhibited differences in the median misalignment angle between galaxies and their host haloes of the order of \(10^\circ\) and this could be as large as \(20^\circ\) for the case where AGN feedback was neglected in the simulation.

Both Tenneti et al. (2014) and Velliscig et al. (2015) used only the unweighted inertia tensor defined in Section 4.1.2. However, a subsequent study by Tenneti et al. (2015) noted that this might have an impact on the conclusions, as shape parameters and alignment angles significantly depend on the way they are computed. For instance, Tenneti et al. (2015) included a study of how the results depended on the use of unweighted or reduced inertia tensors; the latter weighting more the inner regions of galaxies. However, weighting star particles by luminosity rather than stellar mass did not lead to a noticeable change.

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\(^{15}\)There are measurements in the literature that use as few as 50 particles. The results in this paper suggest that those results should be approached with caution.
Fig. 14.— *Top:* Snapshot of the MB-II SPH simulation in a slice of thickness $2\ h^{-1}\text{Mpc}$ at redshift $z = 0.06$. The bluish-white colours represent the density of the dark matter distribution and the red lines show the direction of the major axis of ellipse for the projected shape defined by the stellar component. The length of the lines is linearly proportional to the semi-major axis length. *Bottom Left:* Dark matter (shown in grey) and stellar matter (shown in red) distribution in the most massive group at $z = 0.06$ of mass $7.2 \times 10^{14}\ h^{-1}M_\odot$. The blue and black ellipses show the projected shapes of dark matter and stellar matter of subhaloes respectively. *Bottom Middle:* Dark matter and stellar matter distribution in a group of mass $3.8 \times 10^{12}\ h^{-1}M_\odot$. *Bottom Right:* Dark matter and stellar matter distribution in a group of mass $1.1 \times 10^{12}\ h^{-1}M_\odot$. *Reproduced with permission from Tenneti et al. (2014).*

Tenneti et al. (2015) took the work in Tenneti et al. (2014) a step further and measured the intrinsic alignment 2-point correlation functions in the MB-II simulation. These included measurements of position-angle statistics and projected
correlation functions like $w_{g+}$ (Equation 7) for galaxies as a function of mass, luminosity, and stellar mass, with tabulated fits to the non-linear alignment model (see Section 3.3) that demonstrate the scalings with these parameters. A preliminary study was made of colour-dependence, with blue galaxies having lower intrinsic alignment amplitudes than red galaxies; however, the colours of the galaxies were not fully realistic, so a more detailed study would be required to understand these results. Comparison of $w_{g+}$ for central and satellite galaxies within fixed subhalo mass ranges suggests that at lower mass, central and satellite galaxies have similar small-scale alignments ($< 1 \, h^{-1}\text{Mpc}$) but satellites have smaller (but non-zero) large-scale alignments. At intermediate subhalo masses, $10^{11.5} < M < 10^{13} \, h^{-1}\text{M}_\odot$, central and satellite subhaloes were found to have similar intrinsic alignments on all scales, which is somewhat in tension with observations (see Kirk et al. 2015). The $w_{g+}$ predictions for a massive galaxy sample were reasonably consistent with SDSS luminous red galaxy measurements, which provided some validation that the simulation included the relevant processes for galaxy intrinsic alignments of at least massive galaxies.

Dong et al. (2014) investigated the alignments of the satellite distribution using the positions of satellites within their host halos. An SPH simulation was used with a $100 \, h^{-1}\text{Mpc}$ box, including the effects of gas cooling, star formation, and stellar feedback (although no AGN feedback, which is necessary to avoid over-cooling). They investigated the alignment of central galaxies with the positions of satellites, finding stronger alignments both for red centrals and red satellites, consistent with the observational picture (see Kirk et al. 2015 for more details). Interestingly, they found a physical reason for this effect, which was that the red satellites stayed closer to the inner regions of the dark matter halo, which correlated strongly with the central galaxy shape (unlike the outer regions of the dark matter halo, which were less correlated with the central galaxy shape). They found trends with the host halo mass and with satellite galaxy metallicity, providing predictions that could be tested with future datasets.
6. Semi-analytic modelling

It will remain impossible for the foreseeable future to run hydrodynamic simulations with the volumes, resolutions, and number of realisations required for direct, robust and precise, measurements of intrinsic alignments. An established way to link the output of pure $N$-body simulations to observables of galaxies is the use of analytic prescriptions that take the properties of dark matter haloes as input. They can be informed by simplified and parametrised physical models or by effective relations determined from higher resolution or hydrodynamic simulations.

Such semi-analytic models of galaxy formation and evolution are routinely used to predict observables like galaxy clustering and luminosity functions (see Bower et al. 2006 for a review). These semi-analytic models of galaxies have been embedded in a number of $N$-body simulations in order to investigate satellite alignments in haloes (e.g. Zentner et al. 2005; Kang et al. 2005; Agustsson & Brainerd 2006, 2010; Kang et al. 2007; Metz et al. 2008; Bailin et al. 2008; Faltenbacher et al. 2009; Libeskind et al. 2009)\textsuperscript{16}. The requirement for realistic galaxies lies in the fact that observations have indicated that satellite alignments appear to be different for a range of galaxy types; e.g. early- and late-type host galaxies, red and blue host galaxies, satellites with low and high star formation rates (see Kirk et al. 2015, for further discussion on observational satellite alignment results). If the central galaxy is naively assumed to follow the shape of the host halo, the semi-analytic model predicts a larger alignment signal than the observations (e.g. Agustsson & Brainerd 2006; Kang et al. 2007). It is also interesting to note that, in order to correctly reproduce magnitudes and colours, the models have to take into account a notion of size such as the scale length of discs. Therefore it seems plausible to extend the semi-analytic approach to the shapes and orientations of galaxies.

The simplest ansatz assumes that galaxies are homologous with the underlying dark matter halo (in 3D or in projection), which may be applicable to elliptical galaxies. In their simulation analysis, Heavens et al. (2000) proposed additionally a model for spiral galaxies, which are assumed to be thin discs perpendicular to the angular momentum of the host dark matter halo. Heymans et al. (2004) extended the spiral model in two ways: (a) they now assumed a thick disc via rescaling all ellipticities by a constant factor 0.73 which was estimated from observed ellipticity distributions, and (b) they allowed for a misalignment between the angular momentum of the galaxy and the halo, randomly drawn from a distribution fitted to the results from the simulations of van den Bosch et al. (2002).

Heymans et al. (2006) made the first attempt to model a realistic galaxy population with intrinsic alignments based on the conditional luminosity function by Cooray & Milosavljević (2005), which provides the halo occupation with elliptical or spiral galaxies of a certain luminosity for a given redshift and halo mass. Using only haloes with occupancy one, i.e. central galaxies, Heymans et al. (2006) assigned elliptical galaxies the shape of the parent halo and modelled spiral galaxies as in Heymans et al. (2004). The resulting ‘mix’ model of elliptical and spiral galaxies produced an II signal that is in good agreement with the SDSS main sample measurements of Mandelbaum et al. (2006). Heymans et al. (2006) also found a clear GI signal that showed no redshift evolution and increased with host halo mass.

While the halo occupation formalism captures the average alignment properties of galaxies with a certain halo mass, a full semi-analytic model is capable of tracking any dependencies on the merger history of an individual halo. Thus, only the latter can account for correlations between alignment and galaxy luminosity, colour, morphological type, etc. induced by the evolution of that galaxy. Joachimi et al. (2013a,b) complemented the Millennium Simulation and the Durham semi-analytic models (Bower et al. 2006) with a range of prescriptions for galaxy shapes.

Elliptical galaxies were still assumed to follow the shape of their haloes; however, while the aforementioned works did not specify how the halo ellipticity was measured, Joachimi et al. (2013a,b) considered explicitly an ellipticity based on the simple inertia tensor (see Equation 32 and following text) and a rescaled version approximating the reduced inertia tensor, the latter generally producing more circular galaxy shapes. Spiral galaxies were modelled again as a thick disc, but the misalignment distribution was updated to one from Bett (2012) and the disc was approximated as an opaque cylinder with two different values of thickness. Satellites were included, but as no subhalo properties were available, their shapes were assumed to be determined by the same prescription as the respective central galaxy type, in some cases with rescalings based on the observations by Knebe et al. (2008a). All satellites point towards the centre of the host, with misalignment distributions taken from Knebe et al. (2008a) and Bett (2012).

\textsuperscript{16}Some of these studies use a halo occupation distribution (HOD; e.g. Berlind & Weinberg 2002) approach to reproduce observed galaxy properties. This is a more statistical approach to matching galaxy properties than semi-analytic modelling.
These models were confronted with intrinsic ellipticity distributions for various galaxy samples extracted from the COSMOS survey, as well as the GI correlation measurements of Joachimi et al. (2011) and Mandelbaum et al. (2011). No combination of models was able to fit all observables simultaneously, although the semi-analytic predictions generally reproduced at least the correct trend and order of magnitude (see Figure 16), despite the numerous simplistic assumptions. A notable exception is the GI correlation function $w_{\delta+}$ for early-type galaxies with $M_r < -19$ for which all models strongly over-predict the amplitude. This could indicate a failure of the alignment or galaxy evolution models for satellite galaxies, which dominate these samples, or suggest that the linear luminosity scaling of $w_{\delta+}$ found by Joachimi et al. (2011) cannot be extrapolated to fainter magnitudes.

There are clear avenues for improving semi-analytic models of galaxy shapes and orientations, such as updating scaling and alignment relations from state-of-the-art simulations (e.g. Tenneti et al. 2014) and iterating on the alignment mechanisms, e.g. by including information on the local tidal shear tensor instead of just halo shape. The requirements on the underlying N-body simulations are quite demanding, especially on the mass resolution, which should at least allow for about 300 particles (ideally more than $10^5$ particles) per halo for which shapes and angular momenta are to be measured (Bett et al. 2007; Joachimi et al. 2013a; Tenneti et al. 2014). If these halo properties are determined only for the central part of haloes, then the same requirement will translate into even lower particle mass for the simulation. For sufficiently small scales and high precision the impact of baryons on the properties of the dark matter halo will become relevant, making an analytic link between luminous and dark matter alignments even more challenging.
Fig. 17.— Simulation specifications critical for halo/galaxy alignment studies. Plotted are the particle mass and simulation box length for simulations underlying a selection of alignment works, with the publication year of said works given by the colour coding. Dots (triangles) represent N-body (the gas/stellar component of hydrodynamic) simulations. Dashed lines link the gas and dark matter parts of a given simulation. The black solid (dotted) line connects points at which one expects to find $10^3$ ($10^4$) haloes of a given mass that are sampled with 300 (1000) particles each. Multiples of ten for these halo masses are indicated by the crosses. Areas to the top and right of a given point on the lines cover simulations that fulfill the minimum (solid) or strict (dotted) criteria for accurate alignment measurements of haloes of that mass, as illustrated by the grey areas for a Milky Way sized halo ($10^{12} M_\odot$).

**7. Roadmap/wish list**

When planning to connect theoretical predictions and observations, what would an idealized roadmap or wish list look like?

In an ideal scenario with unlimited resources, simulations would be run in a large ($\approx 4$Gpc) box with a very high particle mass resolution ($\approx 10^6 M_\odot$). Suites of hydrodynamic simulations would be generated using many different simulation codes for comparison and a wide exploration of baryonic physics and feedback effects would be included to gauge their impact on the simulations and to compare with observations for accuracy. In reality, these simulations would be impossible to run, using far more CPU hours, memory, disk space and time than are available in the world today. Given a realistic landscape, the following list represents tasks that would be useful to pursue for future galaxy alignment analyses.

**7.1. Simulations with sufficient resolution and numbers of haloes**

Simulations are currently still the best way to test intrinsic alignment models and mitigation techniques. The first task on this wish list is to determine the size and resolution that simulations should have for future intrinsic

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17The large box size would account for the largest scales in the models and the high mass resolution would resolve the shapes and details of individual galaxies with high precision.
alignment studies. When investigating the properties of \( N \)-body haloes, Bett et al. (2007) found that in order to mitigate numerical artifacts, each halo required greater than 300 particles. However, a more recent study on hydrodynamic simulations found that results converged when haloes had \( \sim \)1000 particles (Tenneti et al. 2014). A simulation with \( 10^3 \) haloes resolved will provide a minimum statistical sample, while \( 10^4 \) haloes will generate a more statistically significant result. These numbers are roughly informed by the galaxy sample sizes of observational intrinsic alignment studies (e.g. Joachimi et al. 2011), where a few thousand galaxies are typically required for a detection, while a few ten-thousand galaxies allow for the definition of subsamples and higher signal-to-noise. Figure 17 shows a census of published simulations that have been used for halo/galaxy alignment studies to date. Only simulations with a fixed resolution, to allow for a direct and fair comparison, are included (zoom simulations and resimulations are excluded). The area above and to the right of the solid (dotted) line is where a simulation would need to lie to meet the minimum (strict) criteria for performing galaxy alignment studies of galaxies in haloes of a given mass. The example shown in this figure is for studies wishing to resolve Milky Way sized haloes \( (10^{12} h^{-1} M_\odot) \) and currently only two of the simulations published meet the strict criteria required for intrinsic alignment studies. Therefore, when planning simulations for future intrinsic alignment studies, this figure can be used to determine minimum box size and particle resolution requirements to ensure that the simulations produce meaningful results. Needless to say, the simulations described in the following tasks in this wish list should all conform to the requirements outlined here.

7.2. A mapping between hydrodynamic and dark matter only simulations

The addition of baryons to a simulation has an impact on the dark matter distribution, as was shown in Section 5.1. In order to quantify these effects, the second task in this wish list is to provide a mapping between hydrodynamic and dark matter only simulations. This will require a detailed comparison between a hydrodynamic simulation and an dark matter only simulation with exactly the same initial conditions. The resulting simulations should contain the same dark matter haloes in general\(^{18}\), which will enable a quantitative comparison. A detailed analysis of dark matter halo and subhalo shape and orientation misalignments between the two simulations as a function of radius from the halo centre should be performed to help identify whether a mapping from the hydrodynamic simulation to the dark matter only simulation can be made that will enable more precise semi-analytic models.

7.3. Accurate galaxy properties in hydrodynamic simulations

Beyond the technicalities involved in the estimation of galaxy shapes and misalignments in Sections 4 and 5, the open challenge in studying galaxy alignments with hydrodynamic simulations remains the fact that the same processes that determine galaxy shapes and mutual alignments are connected (if not exactly the same) to those that give rise to several other observed scaling relations. Unfortunately, hydrodynamic simulations have not yet converged on an implementation of the physics behind these processes and often they fail in reproducing basic observables such as the number density of galaxies as a function of their luminosity or the mass-size relation. Although these simulations are ideal tools to highlight possible hidden connections between different processes, they have not yet reached the maturity to have predictive power. The third task in this wish list is the most challenging and time consuming - to produce hydrodynamic simulations with more realistic galaxy properties.

For galaxy alignment studies, there is good evidence to suggest that the alignment signal is a function of halo mass (e.g. Schneider et al. 2012), but observations provide luminosity or stellar mass. However, see Singh et al. (2014) who investigated galaxy intrinsic alignments as a function of halo mass as determined from simultaneous galaxy weak lensing measurements. On the largest scales, theory appears to match the observed galaxy alignments reasonably well (Section 3.1; Figure 8) and they appear to be well modelled phenomenologically on the smallest scales (Section 3.2; Figure 8). Therefore, if hydrodynamic simulations are able to match the observed stellar and halo masses, matching the galaxy alignment signal to observations should be possible. However, the intermediate scales are still largely unknown and poorly constrained (Section 3.3) and the transition from the linear regime to the non-linear (high density) regime is where simulations will provide significant insight to the understanding of galaxy alignments (see Figure 12 in Joachimi

\(^{18}\)Some differences between the hydrodynamic and dark matter only simulation haloes should be expected and may arise if the halo finders split a structure into different haloes or some haloes fall below the minimum particle threshold between the realisations etc.
et al. 2015). It is also clear that the environment (local and large-scale) should have an impact on intrinsic alignments. As an initial request, the simulation should match the abundances of galaxies as a function of stellar mass, type (early- and late-type) and colour. As an additional step, it would be desirable to match the size and ellipticity distributions\footnote{In order to match ellipticity distributions, hydrodynamic simulations would need to resolve the thickness of the discs accurately, which would require resolving interstellar medium processes accurately. It is not clear that this final point will be possibly in the simulations, so this task may have to remain on the "wish" list.} of galaxies to observations. These tasks are time consuming, challenging and in some cases may not be possible, but this is where effort should be focused to ensure continued progress in understanding how intrinsic alignments form and evolve.

7.4. \textbf{High precision semi-analytic simulations}

The hydrodynamic simulations outlined in Section 7.3 will be essential to providing insight into how galaxies form and evolve and in modelling the galaxy alignment signal. However, generating these simulations will use enormous computing resources – both CPU hours and disk space – so it will not be possible to generate the large suites of simulations that are required for upcoming weak gravitational lensing surveys. Therefore, a faster way to generate simulations that contain realistic intrinsic alignments is required. The final task in this wish list is to take the galaxy alignments and galaxy properties derived from these increasingly realistic hydrodynamic simulations in Section 7.3 and use the mapping identified in Section 7.2 to populate \textit{N}-body simulations. The semi-analytic simulations in Section 6 used theoretical models of alignments to populate the \textit{N}-body simulations. This task represents the next generation of semi-analytic simulation that contains the increasingly realistic galaxy properties and alignments derived from hydrodynamic simulations, likely in the form of a probability distribution to account for the generally stochastic nature of the processes involved. Another option to populate the \textit{N}-body simulations may be to use machine learning algorithms that use hydrodynamic simulations or, use current observations of intrinsic galaxy alignments, as a training set. This task should be completed concurrently with the previous tasks. For each advance in knowledge coming from a hydrodynamic simulation, many \textit{N}-body simulations may be populated for testing and development of mitigation techniques.

8. \textbf{Concluding Remarks}

While the title of this review refers to galaxies, as they are the primary observable accessible to large-scale structure observations, this review has shown that, within a theoretical and simulation perspective, it is possible to measure the shapes and orientations of structures over a wide range of scales, from satellite galaxies and subhaloes through to the large-scale structures of the cosmic web.

Although a reasonably consistent picture exists for alignments of dark matter haloes in \textit{N}-body simulations, these are proving to be of limited use in understanding the alignments of galaxies, which remain the focus of interest for both galaxy formation and evolution and weak gravitational lensing studies. To this end, future efforts would be best spent on understanding alignments through the theory, modelling and simulations (both hydrodynamic and semi-analytic) of galaxies within their host dark matter haloes.

The theories and models of the alignments of galaxies span large and small scales for both early- and late-type galaxies, with reasonable success. However, even for samples of bright elliptical galaxies, for which detailed and high-signal-to-noise measurements exist, the intermediate-scales between $1 - 10 \ h^{-1}\text{Mpc}$ are still proving difficult to model accurately. Work is ongoing in this area and it is expected that a model that accounts for clustering, halo bias and redshift-space distortions will effectively unify the small, intermediate, and large scales. In the case of disc galaxies, observations generally yield marginal detections at best, so that only future data will be able to tell if the prevailing models based on tidal torque theory provide good descriptions of a variety of alignment processes.

Simulations also provide much needed insight into galaxy intrinsic alignments on all scales. Only with suites of \textit{N}-body and hydrodynamic simulations will it be possible to quantify alignment correlations in the non-linear regime, investigate the link between the morphology of bright and dark matter, and establish the physical mechanisms behind galaxy alignments – whether these are driven by the well-understood tidal stretching and torquing paradigms, or by
purely non-linear causes such as vorticity effects. It is, however, important to be realistic about the limitations of simulations. For example, the sub-grid physics in hydrodynamic simulations are still poorly understood and represent a large area of uncertainty in the simulations. Additionally, reproducing realistic ellipticity distributions in hydrodynamic simulations may never be possible due to resolution effects. This is not a reason to disregard simulations; it is simply a reminder to use caution when interpreting the results presented, particularly when the studies do not adhere to the resolution requirements outlined in Section 7.1.

The continued study of alignments is important for both understanding galaxy formation and evolution and for mitigating the intrinsic alignment effect in cosmological weak lensing surveys. In particular, upcoming billion dollar-class telescope missions like Euclid\(^20\) (Laureijs et al. 2011), the Large Synoptic Survey Telescope (LSST\(^21\); LSST Science Collaboration et al. 2009) and the Wide Field InfraRed Survey Telescope (WFIRST\(^22\); Spergel et al. 2015) can only reach their full potential if the effects of intrinsic alignments can be mitigated from the observations. Continued efforts into modelling the mildly non-linear scales and pursuit of the roadmap outlined here will both increase our understanding of the formation and evolution of intrinsic alignments on all scales and also provide data sets with known parameters to develop and test mitigation techniques.

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\(^21\)LSST: http://www.lsst.org/lsst

\(^22\)WFIRST: http://wfirst.gsfc.nasa.gov
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