Effects of energy-dependent scatterings on fast neutrino flavor conversions

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Neutrino self-interactions in a dense neutrino gas can induce collective neutrino flavor conversions. Fast neutrino flavor conversions (FFCs), one of the collective neutrino conversion modes, potentially change the dynamics and observables in core-collapse supernovae and binary neutron star mergers. In cases without neutrino-matter interactions (or collisions), FFCs are essentially energy-independent, and therefore the single energy treatment has been used in previous studies. However, neutrino-matter collisions in general depend on neutrino energy, suggesting that energy-dependent features may emerge in FFCs with collisions. In this paper, we perform dynamical simulations of FFCs with iso-energetic scatterings (emulating nucleon scatterings) under multi-energy treatment. We find that cancellation between in- and out-scatterings happens in high energy region, which effectively reduces the number of collisions and then affects the FFC dynamics. In fact, the lifetime of FFCs is extended compared to the single-energy case, leading to large flavor conversions. Our result suggests that the multi-energy treatment is mandatory to gauge the sensitivity of FFCs to collisions. We also provide a useful quantity to measure the importance of multi-energy effects of collisions on FFCs.

I. INTRODUCTION

In dense neutrino environments such as core-collapse supernovae (SNe), binary neutron star mergers (BNSMs), and early Universe, refractive effects by neutrino self-interactions can induce collective neutrino flavor conversions [1–4]. Under certain conditions, the growth rate of flavor conversion becomes proportional to the number density of neutrinos, which corresponds to a fast mode or fast flavor conversion (FFC). FFC is triggered when electron neutrino lepton number crossings (ELN crossings) appear in angular distributions of neutrinos [5]. Recent studies have indicated that ELN crossings are ubiquitous in SNe and BNSMs [6–24]. Since neutrino flavor conversions, in general, have a potential to affect the fluid dynamics, nucleosynthesis, and neutrino signals [8, 25–27], FFCs have garnered significant attention recently.

In SNe and BNSMs, neutrinos also undergo matter interactions (or collisions) during propagation through matter. The description of neutrino dynamics involving neutrino transport, flavor conversion, and neutrino-matter collisions requires quantum kinetic treatment. SN- and BNSM simulations with quantum kinetic neutrino transport are, however, a formidable challenge for computational astrophysics. In fact, their global simulations with resolving small-scale variations of FFCs (see, e.g., [28–35]) are intractable at present. Currently feasible approach is either local simulations [28, 29, 31, 33, 34, 36–41] or global simulations with some simplifications and assumptions [26, 42–44].

Recently effects of collisions on FFCs have been also studied numerically [29, 45–49]. Some studies have found enhancement of FFCs by neutral-current-like matter collisions [45–47, 49], whereas others have reported damping effects [29, 48]. Although these results are apparently in conflict with each other, which may be due to different angular distributions of neutrinos [49] or self-consistency issues faced by homogeneous models [50], they are in agreement with a suggestion that collisions have non-negligible roles on FFCs. We also note that a new type of instability of flavor conversion, collisional flavor instability, may occur due to disparity of matter interactions between neutrinos and antineutrinos [51].

FFCs are purely driven by neutrino self-interactions, suggesting that energy dependent features induced by vacuum potential are smeared out. Hence, the energy-integrated QKEs or monochromatic assumption has been commonly used in the previous FFC studies. However, the approximation becomes inappropriate if neutrino-matter interactions are involved. Cross sections of weak current depend on energy, and the energy spectrum of neutrinos is not monochromatic in SNe/BNSMs, indicating that energy-dependent flavor conversions may be induced by collisions. In fact, recent studies showed that vigor of FFCs hinges on the reaction rate [49], and therefore it is of great interest to investigate FFCs with energy-dependent collisions in non-monochromatic neutrino energy spectrum.

In this paper, we perform dynamical simulations of FFCs in homogeneous background with incorporating

1 It should be noted that the vacuum potential plays a role as a seed perturbation to trigger FFCs.
energy-dependent collisions (iso-energetic scatterings). The goal of this paper is to understand the essence of multi-energy effects of iso-scatterings on FFCs. In so doing, a simple setup is adopted in our simulations. We find some qualitative differences of FFCs between single- and multi-energy treatments in the non-linear phase, which is scrutinized in this paper.

The paper is organized as follows. We start with describing basic equations and our numerical solver with a Monte Carlo method in Section II. Before entering into results of multi-energy FFC simulations, we revisit the case of those with energy-independent collisions in Section III A, which provides some clues to interpret multi-energy effects of collisions on FFCs. We then discuss FFCs with energy-dependent collisions in two cases: flat neutrino energy spectrum and non-flat spectrum in Sections III B and III C, respectively. In Section III D, we present an energy-resolution study to assess our proposed diagnostics for quantifying multi-energy effects. Finally, we conclude and discuss our study in Section IV.

II. NUMERICAL SETUP

A. Basic equations

Time evolution of neutrinos in phase space follows the quantum kinetic equations (QKEs),

\[ i \left( \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \rho(\vec{x}, \vec{p}, t) = \left[ \mathcal{H}, \rho(\vec{x}, \vec{p}, t) \right] + i \mathcal{C}[\rho], \]

\[ i \left( \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \tilde{\rho}(\vec{x}, \vec{p}, t) = \left[ \tilde{\mathcal{H}}, \tilde{\rho}(\vec{x}, \vec{p}, t) \right] + i \tilde{\mathcal{C}}[\tilde{\rho}]. \tag{2} \]

In this expression, \( \rho \) (\( \tilde{\rho} \)) describes the density matrix for neutrinos (antineutrinos). The Hamiltonian potentials \( \mathcal{H} \) and \( \tilde{\mathcal{H}} \) include contributions from the neutrino mass difference, the coherent forward scattering with surrounding matters and other neutrinos as detailed in [2, 52]. \( \mathcal{C} \) and \( \tilde{\mathcal{C}} \) represent the collision terms.

We assume a two flavor system comprised of electron-type neutrinos (\( \nu_e \)) and heavy-leptonic ones (\( \nu_x \)). In this approximation, the density matrix has four independent components, i.e., \( \rho_{ee}, \rho_{ex}, \text{Re}\rho_{ex} \) and \( \text{Im}\rho_{ex} \). We set the mass difference of neutrinos to be 0, and the matter term is also neglected, just for simplicity. It is also assumed that neutrino distributions are spatial homogeneous and axial asymmetry in momentum space. For collision terms, we consider iso-energetic scatterings. The reaction rate depends on neutrino energy, but we assume that it is independent on propagation direction (i.e., isotropic scattering). The rates are also assumed to be the same among all the components of density matrices. Under these assumptions, QKEs can be rewritten as

\[ i \frac{\partial \rho_a(E, \cos \theta, t)}{\partial t} = \left[ \mathcal{H}_{\nu\nu}, \rho_a(E, \cos \theta, t) \right] \\
+ i \int_{-1}^{1} d \cos \theta' \Delta R(E) \rho_a(E, \cos \theta, t), \tag{3} \]

\[ i \frac{\partial \tilde{\rho}_a(E, \cos \theta, t)}{\partial t} = \left[ \tilde{\mathcal{H}}_{\nu\nu}, \tilde{\rho}_a(E, \cos \theta, t) \right] \\
+ i \int_{-1}^{1} d \cos \theta' \tilde{R}(E) \tilde{\rho}_a(E, \cos \theta, t). \tag{4} \]

In this expression, \( \rho_a = \int \rho d\phi \) denotes the azimuthal-angle integrated density matrix. The Hamiltonians by neutrino self-interactions can be written as

\[ \mathcal{H}_{\nu\nu} = \sqrt{2}g \int \int \frac{E_a^2 dE'_a d\cos \theta'}{(2\pi)^2} \times (1 - \cos \theta \cos \theta') \times (\rho_a(E_a, \cos \theta', t) - \bar{\rho}_a(E_a, \cos \theta', t)), \tag{5} \]

\[ \tilde{\mathcal{H}}_{\nu\nu} = \sqrt{2}g \int \int \frac{E_a^2 dE'_a d\cos \theta'}{(2\pi)^2} \times (1 - \cos \theta \cos \theta') \times (\tilde{\rho}_a(E_a, \cos \theta', t) - \tilde{\bar{\rho}}(E_a, \cos \theta', t)). \tag{6} \]

Regarding the reaction kernel \( R(E_a) \), the energy dependence emulates nucleon scatterings, i.e., \( R(E_a) = R_0(E_a/20\text{MeV})^2 \). In this study, we adopt \( \mu = \sqrt{2}gF_n \bar{n}_\nu = 10 \text{ cm}^{-1} \), where \( n_\nu \) denotes the number density of \( \nu_e \). This corresponds to \( n_\nu \sim 1.56 \times 10^{33} \text{ cm}^{-3} \).

We refer [45] to set initial angular distributions of neutrinos. \( \nu_e \) have isotropic angular distributions, whereas antineutrinos (\( \bar{\nu}_e \)) have anisotropic ones,

\[ g_{ee,0}(\cos \theta_e) = 0.5n_\nu, \]

\[ \bar{g}_{ee,0}(\cos \theta_e) = \left[ 0.47 + 0.05 \exp \left( -(\cos \theta_e - 1)^2 \right) \right] n_\nu. \tag{7} \]

where \( g \) is defined as \( g = \int \rho_a E_a^2 dE_a/(2\pi)^3 \). Those for \( \nu_x \) and their antipartners (\( \bar{\nu}_x \)) are set be 0 initially. At the beginning of simulations, we put a small perturbation on off-diagonal components of \( \rho_a \) by \( \text{Im}\rho_{ex} = -\text{Re}\rho_{ex} = 10^{-6}g_{ex} \), to trigger FFCs. Note that the number density of anti-neutrinos \( \bar{n}_\nu \) is \( 1.53 \times 10^{33} \text{ cm}^{-3} \), which is \( \sim 2\% \) smaller than \( n_\nu \).

B. QKE-MC solver

We solve QKEs (Eqs. 3 and 4) using Monte-Carlo technique [46]. Our code is an extended version from our
classical MC neutrino transport solver [53]. This QKE-MC solver has a capability of handling neutrino transport, matter collisions and neutrino flavor conversions self-consistently. Below we describe the essence of our code.

In classical MC methods, each MC particle represents a bundle of neutrinos in a specific flavor state, and the assembly of these particles describes a neutrino distribution function in phase space. On the other hand, if flavor conversions happen, flavor states are no longer constant during propagation. Hence, we add a flavor degree of freedom to each MC particle. More specifically, we define a new matrix, particle-density-matrix (PDM), which is assigned to each MC particle; the assembly of the PDM represents the density matrix of neutrinos.

In our QKE-MC solver, we handle collisions and flavor conversions separately, which is essentially along with an operating-splitting method. Firstly, we determine what particles and their flavor states undergo scatterings \(^2\). We create a new MC particle at each scattering point, and its flight direction is determined probabilistically with a scattering kernel. In the meantime, we compute the self-interaction potential of neutrinos, and then evolve the PDM on each MC particle with fourth-order Runge-Kutta method. It should be mentioned that feedback from collisions is not included here. After updating the PDM, we copy the scattering-experienced PDM from the original MC particle to newly created ones.

Statistical noise, which is inevitably generated by probabilistic computations of collisions in MC methods, is a major concern. This may compromise the accuracy of our simulations. To mitigate the problem without substantially increasing computational costs, we have developed a special technique. Firstly, we discretize the neutrino phase space similar to mesh-based methods. The flight direction of each MC particle is corrected at each timestep to match with the central value of the mesh-point where the particle resides. Although this prescription smears out a small scale structure in neutrino momentum space, it can be controlled by changing the mesh resolution. Secondly, the effective mean free path (EMFP) method is adopted. In essence, we reduce the rate of change in the PDM \((a (0 \leq a \leq 1))\) at each scattering event, meanwhile the scattering length is also reduced by a factor of \(a\). This prescription increases the number of scattering of MC particles (hence, the statistics is improved) with sustaining the physical impact of collisions. It should be mentioned that the mean free time of collisions is much longer than the FFC timescale, implying that the number of scatterings during a single timestep is small. As a result, the numerical cost of computing collisions is subdominant compared to other operations; hence we can maintain the computational cost even if the number of scatterings is increased by the EMFP prescription. We refer readers to [46] for more details of these techniques and a suite of code tests.

In this study, we deploy 128 bins in angular direction of neutrinos and adopt 50,000 MC samples in each angular bin. The control parameter of EMFP method is set to be \(a = 10^{-3}\), which shows a good performance to reduce the statistical noise without increasing numerical cost. In multi-energy cases, we run several simulations with two energy meshes (see Sections III B and III C) and with three ones (Section III D), which are useful to highlight the essential multi-energy effects of collisions on FFCs.

### III. RESULTS

In this study, we have 10 models in total with different numerical setups. In Section III A, we first show results of FCC simulations with energy-independent collisions, in which we adopt a single-energy mesh. We then present results of multi-energy FCC simulations with energy-dependent collisions in Sections III B-III D.

#### A. Energy-independent collisions

Here we revisit effects of energy-independent collisions on FFCs. We consider three cases with different reaction rates: \(R_0 = 0\) (noscat), \(1.25 \times 10^{-5}\) (LR), \(1.25 \times 10^{-4}\) cm\(^{-1}\) (HR), assuming \(R(E_{\nu}) = R_0(E_{\nu}/20\text{MeV})^2\) with \(E_{\nu} = 20\) MeV. These models are referred to E1 models hereafter.

Figure 1(a) portrays time evolution of \(n_x\)’s number density, or \(n_x \equiv \int d\cos\theta_x g_{xx}\) for E1 models. Since \(g_{xx}\) is initially zero (see Section II), the appearance of \(n_x\) exhibits occurrence of flavor conversions. We also note that the trace of density matrix is not affected by flavor conversions, and iso-energetic scattering does not change the total number of neutrinos, implying that \(n_\nu + n_x\) is a conserved quantity in our models. Black line describes the results without collisions (noscat), in which the time evolution of \(n_x\) has a periodic feature with a constant amplitude. This is consistent with previous studies [45], and this feature may be understood through a pendulum model [54, 55]. In cases with collisions, on the other hand, the periodic feature disappears. Although the amplitude of oscillations is smaller than that in the case without collisions, \(n_x\) increases with time, and eventually being saturated at a certain point (see cyan and green lines in Figure 1(a)). In these models, the saturation densities of \(n_\nu\) become higher than the peak of \(n_x\) in the case without collisions, indicating that collisions enhance flavor conversions.

The top panels in Figure 2 show angular distributions of ELN-XLN, or \(L(\cos\theta_{\nu}) \equiv g_{xx}(\cos\theta_{\nu}) - g_{ee}(\cos\theta_{\nu})\). Here XLN denotes a heavy-leptonic neutrino lepton number. As discussed in [42], ELN-XLN angular distributions

\(^2\) We assign a scattering distance to each element of the PDM.
characterize the growth of FFCs. In the case without collisions (left-top panel), the angular distribution of $L$ does not evolve much with time except in the vicinity of ELN-XLN crossing. As a result, the ELN-XLN crossing does not disappear throughout the evolution. In cases with collisions, on the other hand, the angular point where the ELN-XLN crossing occurs changes with time, and eventually the crossing disappears ($L$ becomes positive in all directions). Around the time when the ELN-XLN crossing disappears ($t \sim 4 \times 10^{-7}$, $4 \times 10^{-6}$ s for $E_1^{HR}$ and $E_1^{LR}$ models, respectively), $n_x$ is saturated, indicating that FFCs subside. Since angular distributions of neutrinos are still anisotropic at the saturation time, they continue to be isotropized by collisions. The isotropization does not generate new ELN-XLN crossings, and therefore FFCs are no longer revived.

One of the important roles of collisions on FFC is to broaden the angular region where FFC occurs. As shown in left bottom panel of Figure 2, $g_{ee}$ ($\bar{g}_{ee}$) in $E_1^{noscat}$ model decreases due to FFCs only around the region where $L$ is zero ($\cos \theta_\nu \sim 0.2$). For cases with collisions, on the other hand, FFCs occur in the wider angular range (see middle- and right bottom panels of Figure 2). This is consistent with our observation that the ELN-XLN crossing point moves to forward angular directions due to isotropization by collisions.

To measure the anisotropy of $\nu_e$, we define $\delta$ as

$$\delta = \left| \frac{\int d \cos \theta_\nu \cos \theta_\nu g_{ee} (\cos \theta_\nu)}{\int d \cos \theta_\nu g_{ee} (\cos \theta_\nu)} \right|, \quad (8)$$

and its time evolution is displayed in Figure 3(a). In the case without collisions (black line), $\delta$ has a periodic feature, which is the same trend as $n_x$. Green and cyan lines depict the results for $E_1^{LR}$ and $E_1^{HR}$ models, respectively. In these models, $\delta$ increases due to FFCs until the disappearance of the ELN-XLN crossing. After FFCs are terminated, $\delta$ monotonically decreases by collisions.

As shown in Figure 1(a), we find that the vigor of FFCs is stronger with increasing reaction rate, meanwhile the saturation amplitude and its time become smaller and shorter, respectively; those for $E_1^{HR}$ model ($E_1^{LR}$ model) are $n_x \sim 5.5 \times 10^{32}$ cm$^{-3}$ and $t \sim 4 \times 10^{-7}$ s ($n_x \sim 5.6 \times 10^{32}$ cm$^{-3}$ and $t \sim 4 \times 10^{-6}$ s). This exhibits that the saturation amplitude of FFCs is determined through the competition between the growth rate and the duration of FFCs. This trend needs to be kept in our mind in the following discussions. It should also be mentioned that

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3 There is a caveat, however. In homogeneous models, the appearance of ELN or ELN-XLN crossings does not guarantee the occurrence of FFC. This is due to the fact that the instability would occur in inhomogeneous modes, while flavor conversion may be stable in homogeneous ones. Nevertheless, ELN-XLN angular distributions are still informative to understand qualitative trends of FFCs in homogeneous models. In fact, the disappearance of ELN-XLN angular crossings guarantees that FFCs do not occur in homogeneous models.
our results presented in this section is consistent with previous studies [45, 47, 49]. In the following subsections, we turn our attention to multi-energy effects of collisions on FFCs, which is the subject of this paper.

**B. Energy-dependent collisions: flat neutrino-energy spectrum**

Here, we discuss multi-energy effects of collisions on FFCs with comparing to the case of energy-independent collisions presented in Section III A. For the sake of simplicity, neutrino energies are assumed to be \( E_{\text{low}} = 10 \) MeV or \( E_{\text{high}} = 30 \) MeV. We refer to the models as E2. In this section, we consider the case with flat neutrino-energy spectrum, i.e., \( n_{\text{low}} = n_{\text{high}} = n_{\nu}/2 \) where \( n_{\text{low}} \) and \( n_{\text{high}} \) denote the numbers of neutrinos with the energy of \( E_{\text{low}} \) and \( E_{\text{high}} \), respectively. We note that the case of non-flat energy spectrum will be discussed in the next subsection.

The angular distributions of neutrinos are set to be identical to those used in previous subsection (see Eqs. 7). This guarantees that the neutrino self-interaction potential at the beginning of simulation is identical among all models. We assume that the reaction rate is proportional to \( E_{\nu}^{2} \) emulating the energy dependence of neutral current scatterings (see Eqs. 3 and 4). To discuss multi-energy effects of collisions appropriately, we set the reaction rate so that the energy-integrated rate becomes the same as that of E1HR model. More specifically, we determine \( R_{0} \) with a condition that the energy-averaged reaction rate for \( \nu_{e} \),

\[
\langle R_{\nu e} \rangle = \frac{R(E_{\text{low}})n_{\text{low}} + R(E_{\text{high}})n_{\text{high}}}{n_{\nu}},
\]

becomes \( R_{\text{HR}} = 1.25 \times 10^{-4} \) cm\(^{-1}\), which corresponds to the reaction rate used in E1HR model. The resultant reaction rates at the energy of \( E_{\text{low}} \) and \( E_{\text{high}} \) are \( R(E_{\text{low}}) = 2.50 \times 10^{-5} \) cm\(^{-1}\) and \( R(E_{\text{high}}) = 2.25 \times 10^{-4} \) cm\(^{-1}\), respectively, and the correspond \( R_{0} \) is \( 10^{-4} \) cm\(^{-1}\). We name this model as a E2HR model.

The time evolution of \( n_{x} \) in E2HR model is depicted with a coral line in Figure 1(b). In the early phase, the time evolution of \( n_{x} \) in E2HR model is almost identical to that of E1HR model. After the time of \( t \sim 10^{-7} \) s, differences between the two models become perceptible. The growth rate of \( n_{x} \) in E2HR model becomes smaller than E1HR model, whereas the saturation amplitude of \( n_{x} \) becomes higher. We have already witnessed the same trend when we compared the time evolution of \( n_{x} \) between E1HR and E1LR models (see Section III A). Following the same argument, our result suggests that the frequency of collisions in E2HR model is reduced by multi-energy effects.

By comparing between the bottom-middle panel of Figure 2 and the left-top panel of Figure 4 (displaying \( g_{\nu e} \) and \( g_{\bar{\nu} e} \) as a function of neutrinos directional cosines in momentum space), the trend of weak collisions of E2HR model can also be seen. The isotropization of \( \nu_{e} \) is slower in E2HR model than E1HR model; consequently the \( \delta \)
in E2_HR model also increases more slowly than that in the E1_HR model, which is shown in Figure 3(b). We also find that FFCs in E2_HR model occur at the narrower angular range than in E1_HR model, which is consistent with the trend found in the case of low reaction rate of energy-independent collisions.

One of the keys to understand the difference between the E1_HR and E2_HR models is rolls of collisions on high-energy neutrinos. In the middle and right panels of Figure 4, we display angular distributions of neutrinos at the energies of E_low and E_high, respectively. We note that g_{ee, low} and g_{ee, high} are the energy-integrated angular distributions for low- and high-energy neutrinos; hence, they satisfy the condition of g_{ee} = g_{ee, low} + g_{ee, high}. Since high-energy neutrinos experience collisions more frequently than those in E1_HR model (R(E_{high}) > R(HR)), the angular distribution becomes more isotropic. As a result, the cancellation between in- and out-scatterings happens, which accounts for the effective reduction of collisions in E2_HR model. This feature can also be seen in the time evolution of δ. We display them for E_low and E_high as dashed and dash-dotted coral lines, respectively, in Figure 3(b).

To check if our interpretation, that the scattering-balances happening in high-energy neutrinos is responsible for the difference of FFCs between energy-dependent and independent collisions, is correct, we perform another simulation (E2_HR_f model), in which the angular distribution of high-energy \( \bar{\nu}_e \)'s is set to be isotropic at the beginning of the simulation. We also change the initial angular distribution of low-energy \( \bar{\nu}_e \)'s so as to be the same energy-integrated angular distributions of E2_HR model, which guarantees that the neutrino self-interaction becomes identical to that of E2_HR model. If the cancellation between in- and out-scatterings is a key of multi-energy effects, FFC dynamics in E2_HR_f model further deviates from E1_HR than the case of E2_HR.

As shown in Figure 1(b), the growth rate of \( n_\pi \) for E2_HR_f model (purple) is slower but the saturation amplitude becomes higher than those in E2_HR model (coral), that supports our interpretation. The time evolution of δ displayed in Figure 3(b) also provides another metric for this check. As shown in the purple lines, δ is less energy-dependent up to the time of \( \sim 2 \times 10^{-7} \) s for E2_HR_f model (purple), meanwhile the energy-dependence appears in the earlier phase for E2_HR model (coral). This exhibits that effects of collisions in E2_HR_f model are more suppressed than E2_HR model. In the late phase, on the other hand, the energy-integrated δ gradually increases with time, and eventually it becomes higher than that of E2_HR model. We also display the angular distributions of \( \nu_e \) and \( \bar{\nu}_e \) for E2_HR_f model in bottom panels of Figure 4, and we find that the time evolution is slower than E2_HR model (displayed in the top panels of the same figure). All these results are consistent with our claim that the cancellation between in- and out-scatterings plays a primary role in the deviation of FFCs from those with energy-independent collisions.

FIG. 3. Time evolution of δ for (a) E1, (b) E2_HR and (c) two selected E2_HR_a models. Colors distinguish the models. Solid lines describe δ with energy-integrated angular distributions in eq.8 (Total), while the dashed and dash-dotted lines denote those of low- (E_low) and high-energy neutrinos (E_high), respectively. For the low- (high-) energy case, we replace g_{ee} to g_{ee, low} (g_{ee, high}) in the numerator of Eq.8.
FIG. 4. Angular distributions for E$_{\text{2\,HR}}$ (top), E$_{\text{2\,HR\,f}}$ models (bottom). The energy-integrated distributions are shown in the left panels, and those for low- and high neutrino energy are plotted in the middle and right panels, respectively. Different colors denote the different timesteps, which are the same as those selected in Figure 2 (for E$_{\text{1\,HR}}$ model). Solid and dashed lines describe $\nu_e$ and $\bar{\nu}_e$, respectively.

independent collisions.

To quantify the energy-dependent flavor conversions, we define a transition probability from $\nu_e$ to $\nu_x$ as,

$$\langle P_{ex,i}\rangle = 1 - \frac{\int g_{ex,i}d\cos{\theta}_\nu}{\int g_{ex,0}d\cos{\theta}_\nu}, \quad (10)$$

with $i =$ low or high. We display them for E$_{\text{2\,HR}}$ and E$_{\text{2\,HR\,f}}$ models in the top panel of Figure 5. Solid and dash-dotted lines represent the cases of low- and high-energy neutrinos, respectively. As shown in the figure, the transition probability is always higher in the case of high-energy neutrinos, which is due to effects of collisions; in fact the growth rate of FFCs becomes higher with increasing the reaction rate (see Section III A). It is worthy to note that flavor conversions in high energy region do not stop even if their angular distributions become nearly isotropic. In fact, the saturation time of $\langle P_{ex,i}\rangle$ does not depend on energy. This exhibits that flavor conversions are driven by energy-integrated neutrino distributions, which is consistent with a property of FFCs.

C. Energy-dependent collisions: non-flat neutrino-energy spectrum

It is an intriguing question how the above result (flat neutrino-energy spectrum) is altered in non-flat energy spectrum, i.e., $n_{\text{low}} \neq n_{\text{high}}$. Addressing the question is also important in studying for FFCs in SNe or BNSMs; in fact energy spectrum of neutrinos are non-monochromatic in these environments. In this subsection, we study the case with non-flat neutrino energy spectrum, in which we use E$_{\text{2\,HR}}$ model as a reference to determine the numerical setups. The models which we study in this section are referred to as E$_{\text{2\,HR\,a}}$ models hereafter.

We consider 6 models with $n_{\text{low}}, n_{\text{high}} = 1:1, 2:1, 5:1, 10:1, 50:1$ and $100:1$, satisfying a condition of $n_{\text{low}} + n_{\text{high}} = n_\nu$, initially. We note that the model with $n_{\text{low}} = n_{\text{high}}$ corresponds to E$_{\text{2\,HR}}$ model. In the following discussions, we use the degree of neutrino-number-asymmetry ($\beta \equiv n_{\text{low}}/n_{\text{high}}$) to specify models. As with the case of flat energy spectrum, neutrino energies are set to be $E_{\text{low}} = 10$ MeV and $E_{\text{high}} = 30$ MeV; we control the coefficient $R_0$ so that the average reaction rates become $R_{\text{HR}} = 1.25 \times 10^{-4}$ cm$^{-3}$ (see Eq. 9). We note that $R_0$ increases with $\beta$, since the dominance of low-energy neutrinos requires higher $R_0$ so as for $\langle R_{\text{ee}}\rangle$ to be $R_{\text{HR}}$. The initial angular distributions of $g$’s are adopted from Eqs. 7, and the shapes are energy-independent. The setup parameters are summarized in Table I.

We find that all models have essentially the same trend as that found in the flat spectrum; the time evolution of FFCs tends to deviate towards the case with lower reaction rate in energy-independent collisions. More specif-
and out-scatterings at high-energy neutrinos.

Although the overall trend is the same as the case of flat energy spectrum, the detailed feature of FFCs depends on the spectrum. To measure the sensitivity quantitatively, we use the absolute difference of $\Delta n_x$ from that of E1\_HR model, i.e., $\Delta n_x \equiv |n_x - n_{x,E1}|$, where $n_{x,E1}$ denotes the $n_x$ for E1\_HR model. In Figure 6, we display them (normalized by $n_{x,E1}$) as a function of $\beta$ at $t = 2 \times 10^{-7}$ and $1 \times 10^{-6}$ s. As shown in the figure, $\Delta n_x$ is a non-monotonic dependence on $\beta$. This can be understood by considering the two opposite limits: $n_{low} \to 0$ ($\beta \to 0$) and $n_{high} \to 0$ ($\beta \to \infty$). Both cases are identical to E1\_HR model, implying that $\Delta n_x$ should vary non-monotonically with $\beta$. We also find that $\Delta n_x$ has a single peak, which is located at $1 \lesssim \beta \lesssim 10$. Below, we attempt to understand the reason why $\Delta n_x$ becomes the largest at $\beta = O(1)$.

There are mainly two important elements to understand this trend. First, multi-energy effects of collisions should hinge on the dispersion of neutrino energy spectrum. One can expect that they tend to be larger for neutrinos with a broad energy spectrum. On the other hand, the dispersion of neutrino energy spectrum is not enough to quantify multi-energy effects of collisions. In fact, there are no effects if reaction rates do not have the

| $R_{0}/[10^{-6}\text{cm}^{-1}]$ | $n_{\text{low}}/n_{\nu}$ | $n_{\text{high}}/n_{\nu}$ | $\beta \cdot R_{\text{low}}/[10^{-4}\text{cm}^{-1}]$ | $R_{\text{high}}/[10^{-4}\text{cm}^{-1}]$ | $\chi^2$ |
|----------------|----------------|----------------|-----------------|-----------------|--------|
| E2\_HR(1:1)   | 1.00           | 1/2            | 1/2             | 1               | 1.0     | 1.06   |
| E2\_HR\_a(2:1)| 1.36           | 2/3            | 1/3             | 2               | 1.0     | 1.23   |
| E2\_HR\_a(5:1)| 2.14           | 5/6            | 1/6             | 5               | 0.5     | 4.82   |
| E2\_HR\_a(10:1)| 2.89          | 10/11          | 1/11            | 10              | 0.72    | 6.50   |
| E2\_HR\_a(50:1)| 4.32          | 50/51          | 1/51            | 50              | 1.08    | 9.72   |
| E2\_HR\_a(100:1)| 4.63         | 100/101        | 1/101           | 100             | 1.16    | 10.4   |

FIG. 5. Time evolution of $\langle P_{xx} \rangle$ for E2\_HR (top) and E2\_HR\_a models (bottom). Colors distinguish the models. Solid and dash-dotted lines denote the cases of low- and high-energy neutrinos, respectively.

FIG. 6. $\beta$ dependence of $\Delta n_x$ (normalized by $n_{x,E1}$). Results at two time snapshots are displayed: $t = 2 \times 10^{-7}$ (black) and $1 \times 10^{-6}$ s (coral).
energy dependence (see in Section III A). We develop a new diagnostics to quantify multi-energy effects, which satisfies above considerations. As shown below, this diagnostics suggests that multi-energy effects become the highest at $\beta = \mathcal{O}(1)$.

We quantify the impact of multi-energy effects on FFCs with a new variable, $\chi$, in which we compute the difference between the average energy of neutrinos and reaction-weighted one. More specifically, $\chi$ is defined as,

$$\chi = \left| \frac{\langle E_{\nu} \rangle - \langle R E_{\nu} \rangle}{\langle E_{\nu} \rangle + \langle R E_{\nu} \rangle} \right|,$$

where

$$\langle E_{\nu} \rangle = \frac{\int d^3 p E_{\nu} \rho}{\int d^3 p \rho},$$

$$\langle R E_{\nu} \rangle = \frac{\int d^3 p R E_{\nu} \rho}{\int d^3 p \rho}.$$

For monochromatic neutrinos, we obtain $\langle E_{\nu} \rangle = \langle R E_{\nu} \rangle$, leading to $\chi = 0$, which is consistent with no multi-energy effects. In the case with non-monochromatic energy spectrum of neutrinos but no energy dependence in reaction rates, we again obtain $\chi = 0$, guaranteeing no multi-energy effects. For energy-dependent reactions, $\chi$ has, in general, a non-zero value, and it becomes higher for broader energy spectrum. This is in line with what we expect in multi-energy effects.

We apply the $\chi$ diagnostics to our E2 models. The $\langle E_{\nu} \rangle$ and $\langle R E_{\nu} \rangle$ can be written as

$$\langle E_{\nu} \rangle = \frac{E_{\text{low}} + E_{\text{high}}}{\beta + 1},$$

$$\langle R E_{\nu} \rangle = \frac{E_{\text{low}}^3 + E_{\text{high}}^3}{E_{\text{low}}^2 + E_{\text{high}}^2},$$

and we can obtain $\chi$ from Eq. 11. $\chi$ for all E2 models are listed in Table I. We find that $\chi$ becomes maximum around $\beta \sim 5$, which is consistent with the peak at $\beta \sim \mathcal{O}(1)$ in Figure 6, exhibiting that $\chi$ captures the trend of multi-energy effects of collision qualitatively.

It should be noted that the $\chi$ diagnostics is not capable of determining the exact value of $\beta$, which has the maximum of $\Delta n_{\nu}$. In fact, $\Delta n_{\nu}$ at $t = 1 \times 10^{-6}$ s becomes the largest at $\beta = 2$ among our models, which is different from the $\chi$ diagnostics. This is due to the fact that there are a number of complexities that affect $\Delta n_{\nu}$ through non-linear interactions between FFCs and collisions. This can be seen in the time evolution of $\delta$, which is displayed in Figure 3(c). In the figure, we display results of $\beta = 2$ and 5 models in navy and dark-green lines, respectively. As shown by the solid lines, $\delta$ for the energy-integrated angular distributions in the $\beta = 5$ model is slightly lower than that in the $\beta = 2$ one until $t \sim 5 \times 10^{-7}$ s. This indicates that the deviation from E1 model (the difference from the cyan line in Figure 3(c)) is higher for $\beta = 5$ model than $\beta = 2$ one, which is consistent with the $\chi$ diagnostics. In the late phase ($t \geq 5 \times 10^{-7}$ s), however, the time evolution of $\delta$ becomes very complex; $\delta$ in the $\beta = 5$ model becomes higher than the $\beta = 2$ one in $5 \times 10^{-7} \lesssim t \lesssim 8 \times 10^{-7}$ s, but the order is again reversed after $t \sim 8 \times 10^{-7}$ s. To capture these temporal features, more elaborate diagnostics needs to be developed. It should be emphasized, however, that the deviation of the peak $\beta$ is a factor of a few in the $\chi$ diagnostics, suggesting that the diagnostics is still informative to exhibit the dependence of $\Delta n_{\nu}$ on $\beta$ qualitatively.

Finally, we show the time evolution of $\langle P_{\nu} \rangle$ in the bottom panel of Figure 5. The figure illustrates the impact of energy dependence of collisions on FFCs in models with non-flat energy spectrum. This leads us to a robust conclusion that multi-energy effects of collisions induce energy-dependent features in FFCs, and the mixing degree sensitively depends on neutrino energy spectrum.

D. $\chi$ diagnostics in three energy meshes

One of the virtues of $\chi$ diagnostics is that it can be applied to arbitrary energy spectra of neutrinos (see Eqs. 11-13). Although it is useful for the study of SNe and BNSMs, we need to assess how well the diagnostics can quantify multi-energy effects of collisions on FFCs in cases with $N_{\nu} \geq 3$, where $N_{\nu}$ denotes the number of energy meshes. In this section, we run another FFC simulation with three energy meshes to check this concern.

The initial condition is set in the same manner as in the case of E2 models. Neutrino energies are assumed to be 6.67, 20.0 and 33.3 MeV, and the energy spectrum is flat. The initial angular distributions of $g$’s are adopted from Eqs. 7, and the shapes are energy-independent. We take $R_0 = 9.64 \times 10^{-5}$ cm$^{-1}$ so that the average reaction rates become $R_{\text{HR}} = 1.25 \times 10^{-4}$ cm$^{-1}$ ($E_3\text{HR}$). In $E_3\text{HR}$ model, $\chi$ becomes 0.186, which is similar to that of $E_2\text{HR}$ model ($\chi = 0.167$).

In Figure 7, we compare the time evolution of $\Delta n_{\nu}$ for $E_3\text{HR}$ model to those in $E_1\text{HR}$ and $E_2\text{HR}$ models. We find that $E_3\text{HR}$ model is almost identical to $E_2\text{HR}$ one. It is consistent with the $\chi$ diagnostics, lending confidence to its applicability in the case with $N_{\nu} = 3$.

It should be stressed that more extensive tests are obviously needed to assess the applicability of the $\chi$ diagnostics to more general cases such as neutrino radiation fields in SNe and BNSMs. We leave this important task for future works.

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4 The slow but long-time increase of $\delta$ in the $\beta = 2$ model displayed in Figure 3(c) may be interpreted through $R_{\text{low}}$. Since $R_{\text{low}}$ of $\beta = 2$ model is lower than that of $\beta = 5$ one, the life time of FFCs in the $\beta = 2$ model becomes longer and the total amount of flavor conversions becomes higher than those in $\beta = 5$ model (see also Section III A).
Recent studies have indicated that fast neutrino flavor conversion (FFC), one of neutrino oscillations induced by neutrino self-interaction, occurs in SNe or BNSMs. However, there is still little known about the non-linear dynamics. One of unresolved issues is the interplay between FFCs and neutrino-matter collisions, which is the main focus of this paper.

FFCs are essentially energy-independent; hence, almost all previous studies have taken the monochromatic or energy-integrated assumption. On the other hand, neutrino-matter collisions are in general energy-dependent, suggesting that the energy-integrated assumption may discard some potentially important effects of collisions on FFCs.

Motivated by this fact, we perform dynamical simulations of FFCs with iso-energetic scatterings by our QKE-MC solver [46]. We start with revisiting the case of energy-independent collisions in Section IIIA. The $n_x$ evolution without collisions has a periodic feature with a constant amplitude, while $n_x$ increases with small oscillations, and eventually it reaches the saturation at a certain time in the case with collisions. There are also clear differences in angular distributions of neutrinos between with and without collisions. In the case without collisions, FFCs occur in a very narrow angular region, which is due to the fact that the ELN-XLN angular crossing point is almost constant with time. In cases with collisions, on the other hand, the crossing point becomes dynamical due to effects of collisions; consequently, FFCs occur in wider angular regions than the case without collisions. We also find that FFCs in higher-reaction rate model grow faster, but they are saturated at earlier time. As a result, the saturation value of $n_x$ tends to be smaller with increasing the reaction rate.

In Section IIIB, we investigate effects of energy-dependent collisions on FFCs under the assumption of flat neutrino-energy spectrum. To capture the qualitative trend, we study the impact with two energy meshes. The energy dependence of reaction rate is proportional to $E^2$, emulating neutrino-nucleon scatterings ($R \propto E^2$). Our simulations show the clear difference between energy-independent and dependent collisions; the latter tends to be similar evolution as that with a lower reaction rate in the energy-independent collision. This is because high-energy neutrinos experience collisions more frequently, implying that their angular distributions become more isotropic. As a result, in- and out-scatterings are cancelled out each other, which accounts for reducing the number of collisions effectively. We also find that the transition probability ($P_{\nu_\alpha}$) depends on neutrino energy, exhibiting the importance of multi-energy treatment in studying FFCs with collisions.

In Section IIIC, we discuss the case of non-flat neutrino-energy spectrum. The overall trend is in line with the case of flat energy-spectrum; in fact, FFCs slowly evolve with time, but its lifetime becomes longer due to the effective reduction in the number of collisions. On the other hand, the detailed feature of FFCs depends on the energy-spectrum, and we find that the large deviation from the case with energy-independent collisions happens for $\beta(\equiv n_{\text{low}}/n_{\text{high}}) \sim O(1)$. To understand the trend of deviation, we propose a new diagnostics with $\chi$ (see Eqs. 11-13), in which we quantify multi-energy effects of collisions on FFCs by the difference between the average-energy of neutrinos and reaction-weighted ones. Although the $\chi$ diagnostics is not capable of capturing all features of multi-energy effects (such as temporal features), the diagnostics is in reasonable agreement with the numerical results. In Section IID, we further assess the capability of the diagnostics by comparing to another FFC simulation with three energy meshes, and we confirm that the numerical results are consistent with the diagnostics.

Another thing we do notice from the present study is that the transition probability of flavor conversion is energy-dependent. This also sensitively depends on the neutrino-energy spectrum. Our finding opens a new path to create energy-dependent flavor conversions driven by FFCs, which should be distinct from the slow mode.

Obviously, we imposed many assumptions and simplifications in the present study. Although these conditions are useful to highlight effects of energy-dependent collisions on FFCs, they should be relaxed for more realistic situations. The top priority would be to get rid of homogeneous condition. As is well known, the homogeneous condition restricts the dynamics of FFCs [28, 29, 31, 33], and more importantly, it also faces a self-consistency issue to consider effects of collisions on flavor conversions [50]. It is also an intriguing question how other collision terms, (emissions, absorptions, inelastic scatterings, and pair processes) create energy-dependent features in FFCs. We leave the investigation on these remaining issues to future works.
It should be stressed, however, that our main conclusion, emergence of energy-dependent FFCs induced by collisions, would not be changed in more realistic situations. Hence, it is particularly interesting to delve into FFCs in high-energy neutrinos with energy-dependent collisions; the reason is as follows. In classical neutrino transport, those neutrinos tend to be in thermal/chemical equilibrium with matter due to strong neutrino-matter coupling. Since the equilibrium state hinges on a neutrino flavor, the competition between FFCs and neutrino-matter interactions would happen. This competition may induce unexpected phenomena in both neutrino-flavor conversion and fluid dynamics. Addressing this issue would be important to gauge the sensitivity of FFCs to SNe and BNSMs. We also leave this exploration to future works.

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