Through Their Eyes: Gifted Students’ Views on Integrating History of Mathematics Embedded Videos Into Mathematics Classrooms

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Abstract
This study examined gifted students’ views on integrating the history of mathematics embedded videos into mathematics classrooms. The research was conducted with 30 fifth-grade students who were identified as gifted with the WISC-R Intelligent Test Score and aptitude test. Data were collected through students’ video reflection papers after watching the videos on biographies of mathematicians. Content analysis was conducted to analyze data collected with reflection papers. The findings were grouped as history as a tool and history as a goal. Under the title of history as a tool, the students’ reflections were categorized as history as a cognitive tool and history as a motivational tool. As a cognitive tool, students stated that the history of mathematics embedded videos expanded their knowledge of mathematicians’ early work and their occupations. As a motivational tool, the history of mathematics embedded videos helped raise students’ curiosity about mathematicians and their works, and mathematical concepts, increased their motivation for invention, and developed a positive attitude toward mathematics and learning. On the other hand, under the title of history as a goal, the students stated that the history of mathematics embedded videos broaden their knowledge of mathematicians and their contributions, mathematical concepts, and mathematical evolution. Thus, the videos can be used as enrichment activities for gifted elementary students in mathematics classrooms.

Keywords
gifted students, history of mathematics, mathematics classroom, using videos

Introduction
Gifted learners have different learning characteristics from their peers that need to be considered. They learn quickly, solve problems efficiently and also study abstract concepts applying to other settings (Gross et al., 2001). Such students have also reported their willingness to be involved in more challenging and flexible activities (Willard-Holt, 2003). Additionally, they need opportunities to develop personality attributes such as creativity, curiosity, insight, perseverance, and imagination (Stoltz et al., 2015). These special groups can be clearly distinguished by their strong memory, deep interest, sentimentality, and enthusiasm to engage with novel works from their same-age peers (Davis & Rimm, 2004).

Although teachers believe that not all students learn in the same way and that their needs are different, few teachers address the differences of this special group in their classrooms (Gable et al., 2000). Moreover, many gifted learners spend most of their school time in the regular classroom environment (Gadanidis et al., 2011). However, instructions in a typical classroom environment do not adequately meet those students’ distinct needs (Renzulli, 2011) since it is unlikely to capture the necessities of this unique group of students with regular means of instruction. Ignoring these fundamental desires prevents gifted students from realizing their potential. Achievement scores lower than the expected successes from gifted learners also provide evidence that their learning condition is not suitable for them (Levine, 2003). As a consequence, most of the gifted children fall behind in the class, lose their motivation and get bored (Tomlinson & Kalbfleisch, 1998). Thus, the aim of the present study is to examine gifted students’ views on integrating enrichment activities in mathematics classrooms.

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The Rationale for the Study

Research studies show that 3% to 5% of the population in all communities is gifted (Borland, 2009). If this group does not receive the education that their needs require; they may encounter psychological problems and even become dysfunctional in society (Feldhusen, 1997). Thus, it is necessary for each country to provide educational opportunities that address their needs. If this is realized as expected, these children can use their existing potential through education and can also learn how to use their potential for the sake of the country (Davidson et al., 2004). State differently, providing education according to the needs of each child is important. Hence, the special needs of gifted children should be considered when planning instruction. While distinguishing the necessities of these students, learning activities should be challenging and interesting for gifted students (Van Tassel-Baska, 2003).

Accordingly, it is necessary to plan differentiated learning experiences in typical classroom programs. Tomlinson (2005) states differentiated instruction as a philosophy of teaching that students learn best if their teachers consider the differences according to their interests, learning styles, and readiness. The main goal of differentiated instruction is to take full advantage of each student’s capabilities as an individual and address the needs of diverse learners (Subban, 2006). There are several ways offered to teachers to differentiate the instruction. One of the most widely utilized methods to differentiate teaching gifted students is enrichment. Enrichment provides learners various learning opportunities such as: materials and also deep and extensive learning activities beyond the regular school curriculum (Van Tassel-Baska & Brown, 2007). It also helps the students present their creativity in cognitive processes. Thus, enrichment activities for gifted students should involve appropriate learning experiences and develop the students’ skills (Gavin et al., 2007).

In mathematics, in particular, different types of activities have been prepared to provide extra support for gifted students within classrooms because they get bored from much more review and practice that is offered in many traditional curricula (Dimitriadis, 2011). As far as mathematics education provided for gifted students is concerned, providing an education in the quality of making gifted students think about mathematics, teaching them mathematics in detail, and improving their creativity, meta-cognitive, and abstract skills are necessary (Aygün, 2010). Gifted students should be able to explore the relations of mathematics with other disciplines in daily life using the history of mathematics (Ozdemir, 2016). In that sense, history is a vehicle to reflect the nature of mathematics as a socio-cultural process, and history is also a possible way to conceive and make gifted students understand mathematical concepts (Michalowicz et al., 2000). The use of the history of mathematics can also be beneficial for middle-grade students to cover the misconceptions they may have about what mathematics is and how mathematics evolved throughout history (Furinghetti, 2000). In recent years, the educational value of history has been acknowledged not only in mathematics education but also in the context of science, technology, and engineering education. In addition to using history for conceptual learning of science, technology, engineering, and mathematics (STEM), it has been used to pique students’ curiosity and foster a positive attitude toward such fields (Park & Cho, 2022). Specifically, the integration of the history of mathematics can also make it possible for the way in which such integration opens up different fields of mathematics and expands opportunities to help gifted students to meet their curiosity and intellectual needs (Fauvel & van Maanen, 1997). Thus, the history of mathematics may be used as an enrichment activity (Leng, 2010) for gifted learners to satisfy their specific needs (Yevdokimov, 2007).

Although teachers think that integration of the history of mathematics into the classroom is beneficial, they state that they do not know how to do it in their classrooms (Rogers, 1991). However, there are some ways of integrating the history of mathematics in lessons, such as historical information directly from textbooks, anecdotes, visual materials, games, and outdoor activities (Fauvel & van Maanen, 1997). On the other hand, which method should be used to be more effective while integrating the history of mathematics in mathematics classrooms is a controversial issue. Hence, the appropriate method should be chosen according to the needs of gifted learners.

Recently, information and communication technology has become a widespread instructional method to develop and replace current delivery means and to improve education for gifted students (Wallace, 2005). The recent research on the use of technology in gifted education has mainly emphasized the effect of technological tools on students’ learning in the fields of critical thinking and adaptation to make the curriculum more challenging (Dixon et al., 2005). In general, gifted students stated their positive opinions related to the use of technology for their learning (Gadanidis et al., 2011).

In addition, the effective use of information and communication technologies in school mathematics and integration of the history of mathematics in classrooms are recommended by the National Council of Teachers of Mathematics (NCTM, 2000) and the Ministry of National Education (MoNE, 2013). However, the history of mathematics has often not been integrated into mathematics classrooms as desired ways. For instance, some mathematics textbooks include, not integrate the history of mathematics with historical notes related to biographies and curiosity in the beginning or at the end of the section of the given mathematics topic (Fasanelli, 2000; Tan-Sisman & Kirez, 2018). In this case, the history of mathematics may seem discrete from the mathematics curriculum and remote from daily classroom activities (Jahneke et al., 1996). On the other hand, a real integration of the history of mathematics into the mathematics classroom enables students to perceive the history of mathematics as the integrating agent that connects mathematics and outdoor activities (Fauvel & van Maanen, 1997). On the other hand, which method should be used to be more effective while integrating the history of mathematics in mathematics classrooms is a controversial issue. Hence, the appropriate method should be chosen according to the needs of gifted learners.
across time and many fields and to see mathematics as a dynamic and creative human enterprise (Tzanakis & Arcavi, 2000). In addition, history of mathematics provides opportunities for students to be challenged more. For example, when they experience the development of mathematics as a process of trial and error, or when they are introduced to more practical applications from the past of the abstract mathematics they learned in school today, they can gain more motivation and develop a more positive attitude toward mathematics (Van den Bogaart-Agterberg et al., 2022). Consequently, integration of the history of mathematics through technological tools can make the learning environment more challenging and interesting for gifted learners and improve their attitudes toward mathematics and increase their motivation to achieve.

In particular, distinct characteristics of the video make it an essential tool for the learning of mathematics. These characteristics include presenting a topic summary in context, especially the historical context, to bring reality into a classroom in the form of case studies and real-world experiences, and most notably, its potential in the affective domain of learning (Esteban et al., 2000). Moreover, videos in the historical context can attract middle school students’ attention in a short time and contribute to their knowledge construction (Hong & Chen, 2016). In line with these, this research study made an attempt to reveal whether the history of mathematics embedded videos can be used as enrichment activities for gifted students. To accomplish this aim, we examined the history of mathematics embedded videos through the eyes of gifted students. As the socio-cultural perspective suggests, knowledge is a process acquired through negotiation of meaning resulting from the social activities of individuals and is covered by the cultural framework in which individuals are involved, the history of mathematics has much to offer to the epistemology of mathematics (Radford, 1997). Historico-epistemological analysis can provide us with interesting information about the development of mathematical knowledge in one culture and across cultures, as well as information on how meanings have emerged and changed, understanding the negotiations and the cultural concepts underlying these meanings is necessary. Thus, the aim of the present study is to examine gifted students’ views on integrating the history of mathematics embedded videos into mathematics classrooms.

While studies encourage the integration of the history of mathematics and instructional technology such as dynamic mathematics software (Zengin, 2018) and e-book (Yevdokimov, 2007) to support mathematics instruction and there is little study on investigating the views of students about integrating the history of mathematics into mathematics lessons through the use of technology, especially for gifted students. There is a need to conduct some research studies on the effectiveness of the history of mathematics as a supportive factor as well (Goktepe & Ozdemir, 2013). Hence, in light of the special needs of gifted learners, the history of mathematics embedded videos can be used as an enriched activity in mathematics classrooms to address the diverse needs of gifted learners. Gardner (1991) supports the idea that the history of mathematics can be used for elementary students to challenge the gifted learner and to concurrently engage students with mathematical or language problems. In this respect, it will be significant to investigate the views of gifted students on the history of mathematics embedded videos in mathematics classrooms. Particularly, the current study aims to clarify the following research question:

- What are gifted students’ views on integrating the history of mathematics-embedded videos into mathematics classrooms?

**Theoretical Framework**

Fried et al. (2016) have attempted to synthesize theoretical frameworks appropriate to the use of history in mathematics education stated that studies on the pedagogical use of history should reflect the characteristics of historical knowledge. Furthermore, Fried et al. (2016) reject the idea of a theoretical framework based on history only as a tool, and according to the “theoretical frameworks for the history of mathematics in mathematics education, as theoretical frameworks, should be driven by questions centered on the historical character of mathematics, on the historical conditioning of our experience of mathematics, and, generally, the meaning of our relationship to the past” (Fried et al., 2016). Hence, the studies of Fried (2001), Guillemette (2017), Jahneke (2014), Jankvist (2009), and Wang et al. (2018) may have promising results in this scope.

According to Jahneke (2014), in the hermeneutic approach, students discover their mathematical skills by experiencing and reflecting the contradiction or similarity of its historical counterparts with modern concepts. Hence, students extend their knowledge of history and their own set of modern conceptualizations. It is equally important to use mathematics itself as a tool (Jankvist, 2009) for reading (modern) mathematics. Furthermore, de Vittori (2018) proposed Balacheff’s CK Lý model (conceptions, knowledge, and concepts) to analyze the use of history in mathematics education at the student level. The connections between the most abstract mathematics (concepts), what is taught (knowledge), and those used/understood by students (conceptions) are shown in that model (Balacheff, 2013). It is important to note that this process is essentially a bottom-up process since the main input is the conception at the student level. The conception level is the main entry point of Balacheff’s model and focuses on what makes students’ tasks interesting (de Vittori, 2018). All of these into account, the theoretical framework proposed by Jahneke (2014) examines the interactions between students’ past and contemporary learning guided in this study.
**The History of Mathematics in Mathematics Education**

There are some discussions about what the role of the history of mathematics can be in mathematics education and how the history of mathematics can be integrated into mathematics classrooms (e.g., Fried, 2001; Jankvist, 2009; Tzanakis & Arcavi, 2000; Wang et al., 2018). A typical argument is based on the idea that using history in mathematics can be a motivating factor to study and learn mathematics for students. In addition to such motive and further affective factors, the history of mathematics may have a cognitive role in promoting the actual learning of mathematics. Jankvist (2009) proposed two sets of categories in order to set the arguments for using the history of mathematics in the classroom (the “whys”) and the diverse methods (illumination, modules, and history-based approaches) to do this (the “hows”). In addition, the arguments for using the history of mathematics in the classroom are separated into two parts: history as a tool and history as a goal (Jankvist, 2009).

**History as a tool.** The history as a tool argument supports the view that history could motivate students and increase their interest and excitement for the subject. It could also render mathematics more approachable to human beings than just abstraction with an evolutionary aspect, i.e., the genetic principle (Jankvist, 2009). In addition to these motivational and affective roles, history can also serve as a cognitive tool to support the actual learning of mathematics.

There are many benefits of integrating the history of mathematics into classrooms for both students and teachers as a cognitive tool. It can assist teachers and students in addressing most of the why questions that may arise in the classroom (Bidwell, 1993). Students who do not accept what teachers say without strong external support or gifted students who are more inquisitive than their peers usually pose such questions (Reimer & Reimer, 1995). For instance, the origins of mathematical computing, notations, and sentences that are currently used can be a source of intriguing questions (Rubinstein & Schwartz, 2000). Therefore, both teachers and students want to learn more about the relevant information on those issues (Fauvel & van Maanen, 1997). In addition, the history of mathematics can enrich the classroom environment with detailed descriptions, examples, and different methods to deliver a subject or to solve problems. Hence, it can help students learn by presenting a different perspective (Tzanakis & Thomaidis, 2000).

In addition, integrating the history of mathematics into mathematics classrooms can motivate students to learn mathematics when they struggle with challenges during the learning process as a motivational tool. Hence, they do not feel lonely and they realize the subject is less scary (Ernest, 1998; Fauvel, 1991). Students can appreciate the power and worth of hard work in addition to the importance of insistence on reaching one’s goal because they can perceive the heuristic approach behind mathematical discoveries (Shostberger, 2000). Students’ common anxieties about mathematics can also be decreased when integrating the history of mathematics into the classroom, and they also feel part of the creative endeavor (e.g., Lefort, 1990; Marshall & Rich, 2000).

The history of mathematics can also be an effective tool to capture students’ interest and curiosity related to mathematics (Barbin, 2000). Thus, many studies support the idea that integrating the history of mathematics in lessons develops a positive attitude toward learning, students’ mathematical thinking, and increases their motivation (e.g., Carter, 2006; Kayan Fadlelmula, 2015). In addition, a real integration of the history of mathematics expands students’ mathematical understanding and stimulates their creativity (Rubinstein & Schwartz, 2000). If the purpose of the history of mathematics is to convey some notions into the mathematics classrooms and to persuade students to learn mathematics in which they are involved, then the argument belongs to history as a tool category (Jankvist, 2009).

**History as a goal.** On the other hand, the aim of the other argument, history as a goal is to demonstrate to the students that mathematics has existed and evolved over space and time; humans have participated in this development, which took place through different cultures, and in turn, affected the shape of mathematics. The history of mathematics serves a purpose all by itself (Jankvist, 2009).

Numerous examples of the importance of innovation and creation in the evolution of mathematics can be provided throughout the history of mathematics (Brizuela, 1997), and students can easily appreciate that mathematics is a creative and cultural human activity. It can also show the relevance of mathematics with all aspects of human life because of its interdisciplinary nature (Ernest, 1998). Additionally, how “notation, terminology, computational methods, modes of expression and representations” (Tzanakis & Arcavi, 2000) have progressed over many times and how the use of mathematical knowledge has changed through times can be better appreciated by students (Grugnetti, 2000). Students may also appreciate the role of social norms and values in the development of mathematics (Fauvel, 1991). Hence, students can understand the foundations of mathematics and can contribute to the body of mathematical knowledge in the future (Calinger, 1996). If the purpose of the history of mathematics is to demonstrate the “developmental and evolutionary aspects of mathematics as a discipline,” then the argument belongs to the history as a goal category (Jankvist, 2009).

In addition to the benefits of integration of the history of mathematics which was mentioned, students get to know some more views about mathematics. For example, they believe that arithmetic, algebra, or geometry are independent branches of mathematics, and mathematical concepts, definitions, terms, or formulas do not change (Dematté & Furinghetti, 1999). Students think that what is written in their course books or what the teachers explain is the most recent
version of a mathematical topic (Tall & Vinner, 1981). They get to realize that mathematics is a developing discipline and is not fixed (Bagni, 2008). Hence, it is important to convince students that they can do something for the future of mathematics.

**Education of Gifted Children in Turkey**

When the history of education for the gifted in Turkey is examined, it could be traced back to the 15th through 18th centuries as ENDERUN, the palace school of the Ottoman Empire. Although ENDERUN School was considered as an example of ‘the first systematic training in the world’ for gifted education, after that period, considerable efforts only emerged in the 1990s in the Republic of Turkey (Akarsu, 2004). At first, the Village Institutes were established in 1940 to raise qualified people for the reconstruction of the country and closed in 1950 because of political reasons. After that, the first Science High School was opened in 1964 in order to support gifted secondary students. Based on the data from the Ministry of National Education (2017), the number of Science High Schools reached three hundred-two. Similar to Science High Schools, the Anatolian High Schools were founded for selected students. However, today many high schools in Turkey turned into Anatolian High Schools. Moreover, New Horizons College was founded in Istanbul between 1991 and 1992 to educate gifted students, but this school did not gain distinct status from the Ministry of National Education, and it was closed after a short time (Akarsu, 2004). Above all, the Ministry of National Education opened Science and Art Centers in 1994 as independent special education centers. The number of these centers has increased since then. They aimed to provide an education for primary or secondary school gifted children. In these centers, individual or group training is provided to gifted students in their spare time outside the school. Furthermore, in recent years, most private schools made some attempts to provide opportunities for gifted students in special and homogeneous schools or classrooms. Also, some private foundations prepared part-time educational programs for gifted students. Thus, nowadays, more attention is given to the education of gifted students in Turkey. However, identifying those students’ needs and preparing activities to satisfy their needs are still the concern of many researchers both in Turkey and abroad.

**Methodology**

In this research study, basic qualitative research methodology is used to examine fifth-grade students’ views on integrating the history of mathematics-embedded videos into mathematics classrooms through video reflection papers.

**Context and Participants**

The study was conducted in a private school where only gifted students attend, located in the capital city of Turkey, Ankara. The participants of the study are 30 fifth-grade gifted students (8 female and 22 male). Identification of those gifted students was made by the WISC-R Intelligent Test score of students (above 120), which is obligatory to enter this school. Also, aptitude tests were applied to select gifted students in special areas such as art and music. However, all students attend the same class regardless of their ability. Among the participants, one male student is talented in art, and two female students are talented in music. Moreover, counseling services conduct tests for students whether they can adapt to the school where only gifted students attend. To conduct the study with such students, parents were informed about the aim and scope of the study in the parent meeting, and parent consent was obtained.

Although the school serves a specific group of students, the mathematics program proposed by the Ministry of National Education is followed. In other words, there is no extra-curriculum followed in the school. Students have four mathematics lessons in a week. In addition, with the 2005 reform (MoNE, 2005) and subsequent revisions (MoNE, 2009, 2013), the use of the history of mathematics has been included in the Turkish middle school (grades 5–8) mathematics curriculum. According to the current curriculum (MoNE, 2013), the history of mathematics can positively change middle grades students’ attitudes toward mathematics and their learning of mathematics. Furthermore, it states that history may demonstrate the discipline of mathematics as a cultural heritage. This study was conducted with fifth-grade students because they are the first to take the current curriculum and updated textbooks, including the history of mathematics.

**Data Sources**

The videos used in this study are featured on the National Educational Television channel (The Turkish Radio and Television Corporation-School) in Turkey. They were developed by a documentary producer and a scriptwriter. The duration of each video is approximately 5 minutes. The developments of mathematics from the rise of mathematics as a science to the present, mathematical developments, the life of famous mathematicians are presented as a documentary. In this study, the use of biographies was not limited to famous mathematicians. The videos of mathematicians whose contributions were important in the development of mathematics are also included in the study (Pazwash & Mavrigian, 1986). There is thirty-one history of mathematics embedded videos. In the scope of the current study, eight different history of mathematics-embedded videos were selected to integrate mathematics instruction by history and mathematics teachers. In the selection process, key components of mathematics curriculum for gifted students such as the opportunity for interdisciplinary connections, a discovery approach that stimulates students to discover concepts and content with greater depth and higher levels of complexity (Johnson, 1993), and topics in
fifth-grade mathematics curriculum were considered. For example, Euclid and Mustafa Kemal Atatürk, founder of the Turkish Republic, videos were watched during the teaching of geometry concepts.

Data Collection Procedure and Data Analysis

Based on the literature, a video reflection paper for this practice was developed by the researchers. The opinions of experts in mathematics education were taken when preparing the form. For this study, students watched the videos and then filled out a video reflection paper. After their reflections were taken, the class discussions took place about the context of the videos. The reflection paper entailed students’ views on the integration of history-embedded videos and what they are interested in the videos. The sample questions from the video reflection paper considered for this study are as follows: What is the most interesting thing while watching the video? What do you learn after watching the video? What do you notice from watching the video? Does the video about the history of mathematics in mathematics lessons contribute to you? If your answer is yes, how?

Item-based in-depth content analysis was used to examine students’ views about integrating the history of mathematics embedded videos in mathematics lessons. To generate the categories, the researchers read through all written responses sentence-by-sentence and identified words or phrases that described the students’ views. In this process, the unit of analysis was students so that their views were categorized through video reflection paper. The responses of one student were compared with those of other students for the same video as well as the same students across other videos. The codes were grouped into categories. At this point, preliminary categories were developed. Students’ responses were initially coded as “learning, curiosity, motivation, attitude, and attention.” Students’ responses were compared across categories in terms of similarities and differences. In the light of these comparisons, the two initial codes, “curiosity” and “attention,” were collapsed into a single theme of “curiosity.” After that, the views on “developmental and evolutionary aspects of mathematics as a discipline” coded under the category of history as a goal similar to Jankvist (2009). Also, the views that cover the cognition dimension and include the phases (learning, knowledge, thinking, and awareness) are coded under the category of history as a cognitive tool. On the other hand, the views that cover the motivation dimension and include the phases (motivation, attitude, interest, attitude) are coded under the category of history as a motivational tool. After that, utilizing the descriptors and sample excerpts from the studies of Jankvist (2009) and Clark (2012), the detailed coding plan based on the data from the present study was made as stated in Table 1. In that table, headings of theme, dimension, and educational values of the history of mathematics of students were formed based on the data of the present study and literature. Then, the researchers revised the categories with students’ responses again and again until the final categories were confirmed (Miles & Huberman, 1994). Next, the codes were generated based on a review of the data and the concepts emerging from the data. Through this form of analysis, students’ views were finally categorized under two main themes: history as a goal and history as a tool which were literature-driven codes. Sub-categories for history as a goal category are: enhance knowledge of mathematicians and their contributions, knowledge of mathematical concepts, and knowledge of mathematical evaluations. On the other hand, sub-categories for history as a tool category are history as a cognitive tool and history as a motivational tool. Also, curiosity on mathematicians and their works, curiosity on mathematical concepts, motivation for invention, and attitude toward mathematics are for history as a motivational tool and enhance the knowledge of mathematicians and their early work and knowledge of mathematicians and their occupations for history as a cognitive tool. Sub-themes, as shown in Table 1, were data-driven codes. The coding was done by researchers. Miles and Huberman’s (1994) formula was used to calculate reliability. The consistency level of the coding was .86. Additionally, the views of students for each sub-category were given in order to reflect their views. Table 1 summarizes the data analysis and gives educational values of the history of mathematics for students for each category.

Findings

This study investigated the views of gifted students on integrating the history of mathematics embedded videos into the mathematics classrooms. After data analysis, the views of the students are presented under two main headings: history as a tool and history as a goal (Jankvist, 2009).

History as a Tool

History as a tool category was divided into two sub-categories: history as a cognitive tool and history as a motivational tool. Mathematicians and their early work, mathematicians, and their occupations are presented under the title of history as a cognitive tool. Curiosity about mathematicians and their works, curiosity about mathematical concepts, motivation for invention, and attitude toward mathematics are presented under the title of history as a motivational tool. Details will be given below.

History as a Cognitive Tool

This code represented gifted students’ acquired knowledge and understanding through the history of mathematics embedded videos with an emphasis on mathematicians and their early work, and mathematicians and their occupations sub-titles are presented in the following. Under the title of history as a cognitive tool, mathematicians and their early
work and mathematicians and their occupations sub-titles are presented in the following.

Mathematicians and their early work. Analysis of the data revealed that the history of mathematics embedded videos broadens students’ horizon of knowledge of mathematicians and their works. About 29 answers (97%) of students showed that they acquire more information on mathematicians’ life and their works. Students generally expressed their views about the works of mathematicians when they were young. For example,

I learned that although Pascal did not know anything about geometry, he drew circles and equilateral triangles when he was 12 years old. Even he found that the sum of angles of a triangle is equal to the sum of two right angles. (Student 2)

In elementary school, the mathematics teacher of Gauss asked him and his classmates to calculate the sum of the numbers from 1 to 100. The question was called ‘busy work’ by the mathematics teacher. However, Gauss reached the solution rather quickly by finding a pattern. Hence, Gauss found the ‘Gauss method’ when he was only ten years old. We use this method frequently nowadays. (Student 5)

In addition, students enhance their knowledge of mathematicians and whose famous anecdotes. About 12 students (40%) stated that anecdotal facts in the videos are attractive for them. For example, the quotation “Let no man ignorant of geometry enter here” written above the door of Plato’s Academy in Athens, where Euclid studied, attracts many students’ attention. Another example that attracts many students’ attention was from the quotation of Atatürk as follows:

“Our true mentor in life is science.” (Student 18)

Hence, students appreciated that mathematicians emphasized the importance of science and mathematics in life. Data analysis revealed that students mentioned expanding the knowledge of mathematicians and their occupations. Those reflections were also categorized under the sub-title of history as a cognitive tool.

Mathematicians and their occupations. When students’ reflections were examined, sixteen students (53%) pointed out that mathematicians are interested in many occupations. The following are some examples of how students expressed their views:

I learned that Omer Khayyam is not only a mathematician but also a famous poet with his rubaiies. (Student 6)

In fact, Fermat was a lawyer. He was interested in mathematics and listened to mathematics lessons secretly behind the door when he was at university. (Student 17)

Atatürk was a versatile leader. He was interested in not only politics and the military but also science and mathematics. (Student 22) Even in war, Atatürk used mathematics to calculate the result of war successfully, and I learned how much importance he gives to mathematics. (Student 6)

I learned that Pythagoras is also a philosopher. He is interested in astronomy, physics, philosophy, mathematics, and music at the same time. (Student 10)

Based on the verbatim above, it can be concluded that in addition to the reflections that are categorized as a cognitive

| Theme | Dimension | Educational values of history of mathematics of students | Subthemes |
|-------|-----------|--------------------------------------------------------|-----------|
| History as tool | “History is a cognitive tool in supporting learning of mathematics” | “History can improve learning by providing a different point of view” | Mathematicians and their early work |
| | “History is a motivating factor for students in their learning of mathematics” | “History makes mathematics more concrete and gives students more insight” | Mathematicians and their occupations |
| History as a goal | “Learning aspects of the history of mathematics serves a purpose in it of itself” | “History increases students’ interest for learning” | Curiosity toward mathematicians and their works |
| | | “History sustains the students’ interest and excitement in mathematics” | Curiosity on mathematical concepts |
| | | “History motivates students” | Motivation for invention |
| | | “History makes mathematics lessons less frightening, more enjoyable and exciting” | Attitude toward mathematics |
| | | “History enables brighter learners to look further” | Mathematical evaluation |
| | | “Students derive comfort from history” | Mathematical concepts |
| | “History focuses on the developmental and evolutionary aspects (e.g., evolution of mathematical techniques and notation) of mathematics as a discipline”; | “History can improve learning by providing a different point of view” | |
| | “Human beings have taken part in the evolution of mathematics” | “History makes mathematics more concrete and gives students more insight” | |
| | “Mathematics has evolved through many different cultures throughout history” | “History increases students’ interest for learning” | |
| | “Different cultures have had an influence on the shaping of mathematics and vice versa” | “History sustains the students’ interest and excitement in mathematics” | |
tool, students reflect on their views which can be accepted as a motivational tool.

**History as a motivational tool.** This code represented the history of mathematics embedded videos raise gifted students’ motivation to learn mathematicians, their works, and mathematical concepts and also increase gifted students’ attitudes toward mathematics. Under the title of history as a motivational tool, curiosity toward mathematicians and their works, curiosity on mathematical concepts, motivation for invention, and attitude toward mathematics sub-titles are presented in the following.

**Curiosity toward mathematicians and their works.** When students’ views about the sub-category were considered, twenty-nine students (90%) reported that integration of the history of mathematics-embedded videos into mathematics instruction raised their curiosity. Data analysis revealed that there was considerable emphasis on their curiosity about mathematicians and their works. The videos arouse their curiosity to seek some questions. For example, almost all students stated that although Fermat found many important theorems, they wonder why he gained the title of “Amateur Mathematician.” In particular, students got curious about the work of mathematicians stated as in the following quotes as questions:

- Does the book of Euclid come down today as in the original copy of the book? (Student 11)
- Why could Omer Khayyam not announce the Pascal triangle to the world before Pascal? (Student 12)
- Why could Fermat not announce the solution of the last theorem? (Student 9)
- As mentioned, computers have come from calculators. In that case, were computers which we use in daily life most of the time invented by Pascal? (Student 30)
- They also stated their curiosity about which steps mathematicians follow to do their study. For example,
  - I wonder how Fermat’s Last Theorem was proven? (Student 14)
  - How did Atatürk translate French mathematics terms into Turkish? I really want to know (Student 18)
- In addition to this, four students (13%) expressed their curiosity about the life of other mathematicians and their works, the childhood of mathematicians, the difficulties they experienced in their life, and their family life. For instance,
  - I wonder about other mathematicians’ life and studies. (Student 23)
  - I am worried about what mathematicians did when they were the same age as us. (Student 10)

**Curiosity on mathematical concepts.** More specifically, seven students (23%) explained their curiosity in expanding their mathematical knowledge on specific topics. For instance,

- I want to search on the history of mathematics and the origin of geometry. After watching the videos, I am curious about such issues. (Student 17)
- I wonder about higher-level topics in geometry and want to learn numbers in more detail since I do not have detailed knowledge of these issues. (Student 23)
- I want to learn more information on rational numbers, irrational numbers, and prime numbers because we do not learn these concepts in the fifth grade. However, I wonder about such topics after being mentioned in the videos. (Student 8)
- I got curious about the exponential numbers, Binomial Theorem, and probability after watching the videos on Pascal and his studies. Especially, I wonder about the real-life applications of these concepts. (Student 4)

The above verbatim could be accepted as evidence that participants want to learn the history of mathematics and more advanced topics in mathematics.

**Motivation for invention.** Analysis of findings revealed that integration of the history of mathematics-embedded videos into mathematics classrooms is also an effective tool for motivating students to achieve and discover. Four students (13%) reported that they were motivated to make inventions after they watched videos. For instance,

- I must not give up studying for invention after watching the video on Pythagoras. (Student 16)
- There are many inventions to be discovered in the world. (Student 17)
- If Pythagoras found the connection between music and mathematics using the sound of a hammer, we can find a connection between mathematics and so many things in our life. (Student 10)
- I must observe the things around me carefully to invent something. (Student 28)

Students appreciate the importance of hard work to reach one’s goal. Hence, they stated that they do not give up after having encountered the first obstacles. For example, most of the students found it interesting that Euclid wrote the book of Elements when he was 40 years old. The following anecdote between King and Euclid also made them realize that learning geometry takes time.
When King Ptolemy I asked if there was a shorter path to learning geometry than Euclid’s Elements, Euclid replied there is no royal road to geometry (Student 1).

Hence, students think that learning geometry is not an easy task, and they should try to succeed in geometry. Moreover, six students (20%) stated that they were motivated to invent something at an early age. There are examples as follows:

Mathematicians did not give up studying. If we study hard, we can achieve everything now. (Student 10)

We can make an invention at an early age because Gauss found the Gauss Method when he was ten years old. (Student 5)

Pascal invested first numerical calculator to make easier the work of his father at a young age. Like this, things we made to help our family may affect all humanity. (Student 14)

Moreover, three students (10%) feel they are not alone when they are faced with many problems. Famous mathematicians also experienced so many problems in their life. For example,

No matter what happens, we should support our ideas like Galileo Galilei. (Student 10)

I am afraid of no one. If I am afraid, I may not be as famous as Euclid. (Student 1)

On the other hand, mathematicians were not perfect in every aspect of life. Thus, it motivates students to achieve despite the difficulties. For instance,

Although Fermat was disorganized, he was very successful. So, I can also achieve something (Student 19)

Even if we have difficulties, the things we can accomplish are endless. (Student 24)

Fermat did not hold regular notes and neglected to publish their inventions. His attitude led to the fact that Descartes came into prominence as the founder of analytic geometry. Hence, I appreciate the importance of note-taking to succeed. (Student 20)

The last category that could be categorized under the history as a motivational tool sub-title is the attitude toward mathematics.

After watching videos, I see that people work hard for mathematics, and mathematics is very important in our life. Also, mathematics helps us in every aspect of our life. I get to learn more about mathematics, and I like mathematics. (Student 26)

Thanks to mathematics, our life is now much easier. The videos contribute to understanding the desire for mathematics inside me. (Student 19)

I developed a positive attitude toward the history of mathematics after watching videos. (Student 24)

The views of gifted students on integrating the history of mathematics embedded videos into mathematics classrooms under the sub-title of history as a cognitive tool and history as a motivational tool are presented above. Data analysis revealed that students’ views could also be categorized under the title of history as a goal.

History as a Goal

The analysis of the data revealed that gifted students’ views grouped not only history as a tool but also history as a goal categorization. This code represented the history of mathematics-embedded videos show the development of mathematical techniques and notations, the “developmental and evolutionary aspects of mathematics” (Jankvist, 2009), the relationship between mathematics and other disciplines to gifted students. The views of gifted students about the history of mathematics embedded videos were divided into three sub-categories: mathematicians and their contribution, mathematical evolution, and mathematical concepts.

Mathematicians and their contributions. When students’ view about the sub-category was taken into consideration, twenty-nine students (97%) expressed their views on mathematicians’ contributions. After watching the videos, students tried to make a connection between the past and the present. Students stated that the mathematical concepts or materials they use in their classroom and daily life have been in existence for a quite long time take their attention. For instance,

In our geometry lessons, we learn the topics in the book of Elements written by Euclid so many years ago. Although most of the theorems have changed recently, the studies of Euclid have not been changed. (Student 20)

I learned that Atatürk contributed to mathematics by writing the book “Geometry.” He brought new geometric terms in Turkish. Hence, we understand mathematics better now. (Student 23)

I found out that Pascal invented the first numerical calculator and introduced it in 1645. (Student 15)
I noticed that Omer Khayyam finds the acute, right, and obtuse angle and also works on algebra. (Student 23)

Moreover, nine students (30%) emphasized that they learned that many mathematicians contributed not only to mathematics but also other sciences. For example,

I learned that Galileo Galilei made many contributions to physics and astronomy as well as mathematics. (Student 8)

Students expressed their views by giving more specific examples. For instance,

Galileo Galilei postulated, “earth rotates around the sun.” (Student 22)

The first telescope done by Galileo Galilei can enlarged 30 times. (Student 7)

**Mathematical evolution.** Another sub-category of history as a goal is that students learn the “developmental and evolutionary aspects of mathematics.” The reflections of students showed that they acquired knowledge of how mathematics emerges and develops in time and space. Integrating the history of mathematics-embedded videos into the classroom enabled students to recognize and acknowledge the social, cultural, political, and economic aspects of mathematical evolutions in addition to the significant roles of different cultures in the development of mathematics. Twenty students (60%) expressed their views on mathematical evolution. For example,

The King of Egypt, Ptolemy I was interested in science and literature. He opened a mathematics school in Alexandria. He gave importance to science at that time. Hence, Alexandria became a city of science, and scientists all around the world came there. (Student 19)

On the contrary, Galileo Galilei postulated the idea that the earth rotates around the sun. The matter was investigated by Inquisition and they concluded that it could only be supported as a possibility, not as an established fact. He later defended his views in the book “Dialogue.” The Inquisition pushed him to retract (say he was wrong) under the threat of execution and to ban the publication of his works. Galileo spent the rest of his life under house arrest. (Student 9)

Besides, students stated that the history of mathematics establishes a connection among past, present, and future. For instance, students clarified that some problems like Fermat’s Last Theorem take centuries to solve.

I learned that Fermat’s Last Theorem was first conjectured by Pierre de Fermat in the margin of a copy of Arithmetica in which he claimed that he proves the theorem but space is not enough to write it. The first successful proof was released in 1994 by Andrew Wiles after around 400 years of effort by mathematicians. (Student 19)

The last category that could be categorized under the history as a goal title is mathematical concepts. This category concerns aspects of mathematics in the evolution of mathematical techniques and notation.

**Mathematical concepts.** Students reflected their views on the history of mathematics-embedded videos, which helped them gain a rich and deep knowledge related to the development of mathematical concepts because the videos on biographies of ancient mathematicians made them appreciate the connections between life and mathematics utilizing daily life context. They expressed that all videos they watched were informative and interesting because they did not learn much of the information in the context of the videos previously. Students reflected the history of mathematics is a way of understanding the various connections that exist among the various mathematical areas and between mathematics and other subjects. Nine students (30%) thought that the videos increased their knowledge of mathematical concepts. They appreciated in which areas some mathematical concepts are used. For instance,

I learned that free-fall velocity is proportional to the square of time. (Student 7)

I learned that “x” represents an unknown thing. (Student 3)

On the other hand, the connection between mathematics with other mathematical areas such as geometry, algebra, and arithmetic was also emphasized by students as in the following examples:

The Pythagorean Theorem is a relation among three sides of a right triangle. It states that “the square of the hypotenuse is equal to the sum of the squares of the other two sides.” The theorem can be written as an equation in connection with the squares of side length. This shows us the relationship between arithmetic, algebra, and geometry. (Student 4)

Gauss found connections among prime number and triangle, rectangle, and pentagon and also created seventeen-sided polygons using such connections. To sum up, he mastered the relations between numbers and geometry. (Student 28)

**Discussion and Implications**

This study investigates the views of gifted students on integrating the history of mathematics embedded videos. The findings of this study revealed that the history of mathematics could be used both as a tool and as a goal, which matches with Jankvist’s (2009) categorization. Although Jankvist (2009) supported the idea that illumination approaches are more suitable for using history as a tool, the findings of the present study revealed that the illumination approach could be used both history as a tool and history as a goal. Analysis of data further revealed that under the title of history as a
tool, the history of mathematics can be used as a cognitive and motivational tool.

The findings of the current study revealed that the integration of history of mathematics embedded videos not only expand gifted students’ mathematical knowledge but also increase their motivation pointing to the wealth of human-made activities in mathematics. The literature also supports that integration of the history of mathematics makes the mathematics lesson more meaningful and interesting and thus increases students’ motivation to learn (Carter, 2006; Tözlüyurt, 2008). For instance, in the Gauss example, students not only learned how their frequently used method to find the sum of numbers was found but also appreciated the invention of something at an early age. When viewed from this aspect, the findings of this study are consistent with previous studies since students found the history of mathematics embedded videos more informative and interesting. This may be a result of clarifying most of the why and how questions in their mind, such as the origin of mathematical formulas, terms in their textbooks, and their applications in real life and other fields. From the students’ point of view, the integration of the history of mathematics into the classroom affected gifted students’ views both cognitively and affectively. The findings of this study validated that the integration of history of mathematics into mathematics lessons enhance curiosity, motivation, and attitudes especially for gifted students consistent with previous research (Carter, 2006; Marshall & Rich, 2000; Tözlüyurt, 2008; Van den Bogaart-Agterberg et al., 2022).

Moreover, integrating the history of mathematics embedded videos into mathematics classrooms increased students’ curiosity about mathematicians and their works. They were curious about some questions, such as why Omer Khayyam could not announce the Pascal triangle to the world before Pascal. Also, they wondered about which steps mathematicians follow to conduct their study and the difficulties they experienced in their life. In particular, they wondered about what mathematicians did when they were the same age as them, and they were more curious about expanding their mathematical knowledge on specific mathematical concepts. Hence, gifted students associated the mathematicians’ lives with their lives and wanted to do something based on the steps mathematicians follow. Similar to the study (Calinger, 1996), the findings of this study indicated that gifted students appreciated the foundations of mathematics and wanted to contribute to the body of mathematical knowledge in the future.

From another point of view, the findings of the study revealed that gifted students expanded their knowledge of mathematical concepts. They also learned how mathematical concepts were connected with other mathematical areas. Hence, they were able to see the developments of mathematical concepts from ancient times to the present. Furthermore, they were able to analyze the different characteristics of mathematical concepts shown throughout the centuries with today’s perspective. These findings support the idea of Jahnke (2014) by “reflecting on the contrast between modern concepts and their historical counterparts” (p.86). However, students emphasized that before the videos, they did not know much about mathematicians and their contributions. This might be a consequence of students’ lack of information about mathematicians and their contributions since little emphasis is given to the history of mathematics in mathematics classrooms. TIMSS’s (1999) findings support this idea that 3% of the 638 mathematics lessons included the history of mathematics, and the time devoted to the history of mathematics was limited to only 3 minutes (Hiebert et al., 2003). Similar to findings from the studies (Fasanelli, 2000; Smestad, 2000) in the international arena, Baki and Butuner (2013) also stated that the history of mathematics is generally referred to as the inclusion of mathematicians’ short biographies and pictures of mathematicians in Turkish elementary mathematics textbooks. In addition, the current analysis of six Turkish middle school mathematics textbooks from fifth to eighth grade indicated that out of 1995 total pages of textbooks, the history of mathematics takes place only 35 pages of the books (Tan-Sisman & Kirez, 2018). Hence, fifth-grade students have little chance to see the rationale behind the development of mathematical concepts, rules, and formulas through the historical perspective of mathematics (Tan-Sisman & Kirez, 2018). But, through videos, fifth-grade gifted students had the opportunity to learn the “developmental and evolutionary aspects of mathematics” as a purpose. This may be the reason why gifted students reflected their views of integration of the history of mathematics into mathematics classrooms in a positive way.

As another point to discuss, through videos, the findings of this study revealed that gifted students acquired knowledge of how important developments came about by learning the history of the related mathematics topics. They recognized that mathematics was created by human beings as a result of their needs and also were more eager to learn since the history of mathematics revealed long traditions, different cultures, peoples’ emotions, and development. For example, they stated that in the time of Ptolemy I, he gave importance to science and mathematics. They also emphasized how social, cultural, political, and economic issues affect mathematical evolutions. On the other hand, the video regarding the life and works of Galileo served as examples of expectation, failure, insistence, and eventual success. Similar to the study of Jankvist (2009), findings from this study support the idea that the history of mathematics may help students learn the origin of mathematics and the reasons why they learn mathematics.

As understood from the reflections of students, they appreciated the interdisciplinary connection between mathematics and other fields like physics. Similar to findings from Ozdemir (2016), students had an opportunity to realize where mathematics is used and the interdisciplinary connection between mathematics and other fields. For example, the students were able to see the connection between physics and
mathematics after watching the video on Galileo Galilei. In addition, in the present study, gifted students realized that mathematics has existed and evolved throughout the ages. For example, students stated their views about the first numerical calculator invention in 1645 and the first successful proof of Fermat’s Last Theorem in 1994. Hence, they expanded the knowledge that human beings participated in this evolution which took place through different cultures and, in turn, affected the shape of mathematics.

Different learning characteristics of gifted students make classroom activities more difficult. They may be bored and lose their interest easily (Tomlinson & Kalbfleisch, 1998). It may also be difficult for teachers to keep them in regular classrooms. Similar to the study of Gardner (1991), findings from this study supported the idea that the history of mathematics-embedded videos can be used to support elementary gifted students in mathematics classrooms through raising their curiosity, motivating them, and capturing their attention. In addition, the context of the videos used in the present study addressed the key components of mathematics curriculum for gifted students proposed by Johnson (1993), such as the opportunity for interdisciplinary connections, a discovery approach that stimulates students to discover concepts and content with greater depth and higher levels of complexity. In the present study, the students found the videos more entertaining, realistic, and detailed and then these videos expanded their knowledge of mathematicians and their contributions. Thus, the videos can be an alternative way to satisfy the needs of gifted learners considering their readiness, learning style (audial, verbal, and visual), and interest. Consequently, the history of mathematics embedded videos can be an effective method to enrich mathematics instruction for gifted learners.

The current mathematics curriculum in Turkey (2013 and 2017) emphasizes some of the goals which could increase the success of mathematics education. One of the goals concerning the use of the history of mathematics (MoNE, 2009) is to make students understand the historical progress of mathematics as well as the historical progress of the human mind. In this way, students may see the whole picture of mathematics and also its position in their life and may expand their understanding of mathematics. Another goal of the program is to increase students’ positive attitudes toward mathematics and help them gain self-confidence. For this reason, the findings of the current study supported that the history of mathematics-embedded videos could be used in mathematics classrooms to support the goals of the mathematics program regarding the history of mathematics. State differently, watching videos on biographies of mathematicians can be an effective way to draw students’ attention to the subject. Also, the concept can be taught in association with the lives and works of mathematicians or anecdotes related to mathematicians. Especially, at the elementary level, teachers can use facts from the history of mathematics to spark students’ interest in the concept they teach. The use of famous historical anecdotes is widely accepted to be an effective method for removing monotony and making the instruction more interesting (Ho, 2008). The reflections of students in this study also supported that idea.

As a result of this study, there are multiple implications for both gifted students and mathematics education. Considering the implications with regard to gifted education, the history of mathematics embedded videos could be used as enrichment activities for gifted students since such videos promoted appropriate learning environments to meet their interest, curiosity, and intellectual needs consistent with the previous research (Fauvel & van Maanen, 1997). Besides, the current study provides teachers with an example of how to integrate the history of mathematics for gifted students in a practical setting. On the other hand, the findings of the current study contribute to mathematics education not only by providing the history of mathematics embedded videos as the conceptual learning tool in mathematics and other disciplines with their evaluation but also by enhancing curiosity, motivation, and attitude toward mathematics. Hence, through the use of history of mathematics videos, the findings from previous study (Park & Cho, 2022) are validated for gifted students.

Furthermore, the findings of the present study were limited to gifted students in homogeneous classrooms and selected to school according to WISC-R Test and aptitude test. Although this study was conducted in a school where only gifted students enrolled, in future studies, such videos can be used in regular classrooms in which gifted students attend. It may help to create a learning environment where such students can fully improve their skills and interests while maintaining their sense of belonging in the classroom. Hence, teachers may take the overall dynamics of the class into account and provide a working environment in which all students completely improve their skills and interests within the boundaries of the classroom. In addition, the findings of the given study were limited to the videos in which all mathematicians are male. Thus, further studies may include female mathematicians and may discuss the role of gender.

Besides, studies could be conducted with gifted students in homogeneous and heterogeneous classrooms and could be compared to their peers. Similar research studies can also be conducted in different countries, and cultural differences can be discussed in further studies. In this way, whether the history of mathematics embedded videos can be used as enrichment activities for gifted students can be discussed in different settings. In addition, future studies can use other methods such as plays, projects for students, or previous problems while integrating the history of mathematics into classrooms and discussing their effect on students.

Last but not least, the findings of the present study were limited to 30 fifth-grade gifted students who attended a homogenous private school in Turkey and eight different history of mathematics-embedded videos. However, the aim is not to make a generalization. Instead, the focal point in this study is to investigate the views of gifted students on the
history of mathematics embedded videos in mathematics classrooms. Thus, further research could be conducted with a larger sample and alternative videos in order to generalize the findings to the other settings.

Declaration of Conflicting Interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) received no financial support for the research, authorship, and/or publication of this article.

Ethical Approval
We confirm that all original research procedures were consistent with the principles of the research ethics published by the American Psychological Association.

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