Analysis On Manufacturing Logistics Network Equilibrium Under the VMI Inventory Management Mode

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Abstract. The network equilibrium theory is used to study the optimization design of manufacturing enterprises in implementation of VMI inventory management model. In particular, we firstly construct a four tier logistics network, composed of raw material suppliers, spare parts suppliers, manufacturers, logistics service providers. And the network system is modeled, and the profit functions of all levels of decision maker are formalized, and then transformed into the equivalent variational inequality form for computation. Finally, a numerical example is given and some valuable research inspiration are obtained.

1. Introduction
Nowadays, although many enterprises have regarded supply chain inventory management as an important part of daily management, the understanding of how to manage the supply chain inventory in Chinese manufacturing enterprises is still not enough. Because the understanding of supply chain management stays on the surface, the ability of enterprises communication and cooperation is poor, the uncertainty of supply chain management is ignored, and the efficiency of information transmission is low, many enterprises are still limited to the traditional inventory management mode, which ensures the continuity and stability of production and operation by increasing total inventory. This kind of inventory management is prone to produce the distorted phenomenon of "bullwhip effect", which makes the total inventory of the whole supply chain greatly increased, and inevitably produces a variety of disadvantages.

Vendor-managed inventory (VMI) model means that under a common framework agreement, the inventory decision-making power of the supply chain is transferred to the supplier, and the supplier carries out the forecast decision according to the information sharing of the downstream enterprises. This inventory management strategy breaks the traditional inventory management model, which makes the whole supply chain minimized and embodies the integrated management idea of the supply chain.

In VMI inventory management, Wei (2012) gave the advantages of the VMI model and the specific ways to implement it, then pointed out the characteristics, application and effects of VIM models in manufacturing enterprises, too[1]. Xiao (2013) developed a Stackelberg game model of a one-supplier and one-retailer supply chain with deteriorating product to investigate how to coordinate the price and service level decisions under VMI and examined system efficiency[2]. Wang et al.(2010)studied the dynamic lot-sizing problem of the replenishment and dispatch policy in the VMI supply chain with time windows in the finite time-horizon, in which considering two transportation modes (by the private logistics, and by third-party logistics), two polynomial algorithm for computing the optimal
solution for each transportation mode were proposed respectively in this study, too[3]. Borade and Sweeney (2015) studied a two-stage series supply chain in which retailers and their suppliers operated VMI in an uncertain demand environment. It is found that the use of genetic algorithm (GA) based decision support system (DSS) in VMI supply chain can achieve significant economic benefits[4]. Feng et al. (2018) examined the impact of inventory inaccuracy and the channel cost sharing rate on the preference for who should undertake inventory management in the supply chain that consists of one supplier and one retailer under consignment contract[5]. Filho et al. (2018) studied the application of a VMI system model in an Animal Nutrition Industry[6].

In the application aspects of network equilibrium, Hu et al. (2012) constructed a four-tier decision makers equilibrium model, which consist of manufacturers, retailers, express service providers, and consumers. In the analysis of the decision-making methods and optimality choices of the four decision makers, the variational inequalities were used to obtain the equilibrium conditions of each subject and the equilibrium decision behavior of the whole system[7]. Zhu (2013) quantitatively analyzed the relationship between carbon emissions and logistics transactions, and the impact of carbon emission intensity on the whole system network was considered in the three-tier logistics network (manufacturer, distributor and retailer)[8]. Yao et al. (2011) researched the supply chain network equilibrium model by using the theories of equilibrium and bilevel programming, and constructed a bilevel supply chain network equilibrium model according to the characteristic of Stackelberg Game in the upper and lower level as well as the Non-cooperative Game in the same level of supply chain network[9]. Zhou and Zhang (2011) developed a global closed-loop supply chain supernetwork equilibrium model consisting of manufactures, retailers, consumers and recyclers, then a finite-dimensional variational inequality formulation was established to get the equilibrium conditions of the model[10]. Fu and Lam (2014) proposed an activity-based network equilibrium model for scheduling daily activity-travel patterns (DATPs) in multi-modal transit networks under uncertainty[11]. Cao et al (2018) focus on the vector traffic network equilibrium problem with demands uncertainty and capacity constraints of arcs has been considered, where the demands were assumed to be a discrete set that consists of finite scenarios[12].

The network equilibrium model is widely used in the fields of economy, finance, supply chain and transportation because it can express the complex interaction between the members of the system[7]. At present, scholars have studied VMI from different angles, but few scholars have studied the logistics network of manufacturing industry by equilibrium theory from the perspective of inventory cost. This paper intends to apply the network equilibrium model to the VMI on the basis of the above literature, in the case of the implementation of VMI in manufacturing enterprises, we construct a four-tier logistics network with raw material supplier, component supplier, main manufacturer and logistics service provider. According to the cost and income status of each decision maker, we formalized the profit functions, then the equivalent variational inequality was used to derive the equilibrium conditions of decision-makers and even the whole supply chain.

2. Network equilibrium model of manufacturing logistics under vendor managed inventory

In this section, a four-tier network equilibrium model, consisting of raw materials suppliers, component supplier, main manufacturer and logistics service provider, is constructed. This supply chain includes a series of activities, such as the processing and transportation of raw materials and components, and also the decision-making behavior and interaction effect of many members of the chain for their own interests.

In the case of the implementation of VMI in manufacturing enterprises, this paper studies the logistics network equilibrium problem of manufacturing industry, consisting of $M$ raw materials suppliers, $N$ component suppliers, $O$ main manufacturers and $P$ logistics service providers, in which $i$, $j$, $k$ and $l$ represent raw material suppliers, component suppliers, main manufacturers and logistics service providers, respectively. The network structure diagram as shown in Figure 1.

Under the VMI model, the main manufacturer does not have to bear the cost of inventory, the raw material supplier determines the total quantity of raw materials supplied according to the demand
forecast results, but because of the error of demand forecast and the uncertainty of the market, it is difficult for the raw material suppliers to predict the demand accurately, so raw material suppliers as well as component suppliers may need to pay the shortage cost caused by insufficient supply or the inventory cost caused by oversupply. Some assumptions are first given as follows:

(1) For the main manufacturer, it is assumed that the product is directly oriented to the external demand market (including the demand from the retailer and the individual consumer), and does not bear the cost of the inventory, that is, the product is sold directly. (2) For logistics service provider, it is assumed that it aims at maximizing profit, which determines the transportation price of unit product and the quantity of the products transported from each raw materials supplier and component supplier.

In this paper, the Nash equilibrium theory is used to analyze the optimal decision and equilibrium model of the decision-makers at all levels. When modeling the behavior of the various decision-makers, all the functions are assumed to be differentiable convex functions. The property of convex function determines that the built model can be solved by many existing algorithms, while the model of the non convex function is more complex to design the algorithm. Due to the assumption that all functions are differentiable convex functions and the domain obtained by constraint condition is also obviously convex set, the problem in this paper is essentially a differentiable convex optimization problem which is equivalent to a special monotone variational inequality[13]. Therefore, the equilibrium model among decision-makers can be transformed into the form of variational inequalities.

The relevant variables used in the model and their meanings are shown in Table 1.

![Figure 1. Logistics network structure of manufacturing industry.](image)

| Notations       | Definition                                                                 |
|-----------------|-----------------------------------------------------------------------------|
| $q_i$           | The output of $i$                                                          |
| $q_{ij}$        | Product trading volume between $i$ and $j$                                 |
| $p_{ij}$        | Unit product transaction price between $i$ and $j$                         |
| $C_i(q_i)$      | Production cost of $i$                                                     |
| $C_{ij}(q_{ij})$| The part of transaction cost between $i$ and $j$ that undertaken by $i$    |
| $p_{1ij}$       | Unit product transport price that $i$ delivers products to $j$ via $l$     |
| $h_i$           | Unit inventory cost of $i$                                                 |
| $b_i$           | Unit shortage cost of $i$                                                  |
| $v_i$           | Variable introduced. When $i$ exceeds supply, $v_i=1$, otherwise $v_i=0$. |

The raw materials suppliers $i$

The component suppliers $j$

$g_{0,jk}$: Product trading volume of $j$ and $k$
Unit product transaction price of $j$ and $k$

The manufacturing cost of $j$

The part of transaction cost between $i$ and $j$ that undertaken by $j$

The part of transaction cost between $j$ and $k$ that undertaken by $j$

Unit product transport price that $j$ delivers products to $k$ via $l$

Unit inventory cost of $j$

Unit shortage cost of $j$

Variable introduced, When $j$ exceeds supply, $v_j=1$, otherwise $v_j=0$.

The main manufactures $k$

The unit transaction cost of $k$ and $j$

The Unit order processing cost of $k$

The acceptable price of the unit product in the external market

External market demand function

The logistics service providers $l$

The operating cost of delivering raw material between $i$ and $j$

The operating cost of delivering raw material between $j$ and $k$

2.1. Equilibrium conditions for raw material suppliers

Considering the logistics process of the product from the raw material supplier, the output, production cost and transaction cost of raw material supplier are denoted by $q_i$, $C_i(q_i)$ and $C_j(q_j), \forall j \in n$ respectively. The raw material supplier need to ship product to component supplier through the logistics service provider, with the transportation price of unit product and the amount of the product shipped between raw material supplier $i$ and component supplier $j$ denoted by $P_{ij}$ and $q_{ij}$ respectively (assumed to be paid by the seller). If the supply exceeds the demand, we let $h$ denoted the unit inventory cost of the raw material supplier $i$, on the contrary, we let $b$ denoted the the unit shortage cost (it is assumed that raw material supplier $i$ can replenish the goods in a relatively short period of time when the supply fails to meet the demand, that is the quantity supplied to Parts Supplier $j$ is still $q_{ij}$). In order to simplify the calculation, a 0-1 variable is introduced here:

$$v_j = \begin{cases} 
1 & q_j \geq \sum_{j=1}^n q_{ij} \\
0 & q_j < \sum_{j=1}^n q_{ij} 
\end{cases}$$

The profit function of the raw material supplier is as follows:

$$\text{Max } \sum_{j=1}^n p_{ij}q_{ij} - C_i(q_i) - \sum_{j=1}^n C_j(q_j) - v_1h_1(q_i - \sum_{j=1}^n q_{ij}) - (1 - v_1)b_1(\sum_{j=1}^n q_{ij} - q_i) - \sum_{j=1}^n p_{ij}q_{ij}$$

The Nash equilibrium of the non-cooperative game between raw material suppliers can be transformed into the optimal $(q_i^*, q_j^*)$, which satisfies the following variational inequalities:

$$\sum_{j=1}^n [\partial \frac{C_i(q_i)}{\partial q_i} + v_1h_1 - (1 - v_1)b_1(q_i - q_i^*) + \sum_{j=1}^n \frac{\partial C_j(q_j)}{\partial q_j} + p_{ij} - v_1h_1 + (1 - v_1)b_1 - p_{ij}](q_{ij} - q_{ij}^*) \geq 0 \quad (1)$$

The sum of the raw material supplier’s production $q_i$ has no binding relationship with the total
amount of product supplied to component supplier \( \sum_{j=1}^{n} q_{ij} \), the size of \( q_i \) and \( \sum_{j=1}^{n} q_{ij} \) also determines whether a raw material supplier \( i \) is in excess or out of stock.

2.2. Equilibrium conditions for component suppliers

We assume that the component suppliers \( j \) will purchase \( q_{ij} \) products from the raw material supplier \( i \) at a unit price of \( p_{ij} \), then sell them to the main manufacturer \( k \) at a unit price of \( p_{jk} \) with the quantity of \( q_{0jk} \), and the component suppliers \( j \) need pay the logistics service provider transport price at a unit price of \( p_{2jk} \).

Component supplier \( j \) need to bear their own manufacturing costs and transaction costs with raw material supplier \( i \) and main manufacturer \( k \), denoted by \( C_i(q_i) \) and \( C_j(q_j) \), \( \forall i \in m \) and \( C_{jk}(q_{0jk}) \), \( \forall k \in o \).

In the same way, the sum of the products \( \sum_{k=1}^{o} q_{0jk} \) purchased from the raw material suppliers and the sum of the products \( \sum_{k=1}^{o} q_{0jk} \) supplied to the main manufacturers determine whether the component suppliers are in stock or out of stock.

We let \( h_2 \) and \( b_2 \) denoted the unit inventory cost of the component supplier \( j \) and the the unit shortage cost respectively (the number of products supplied to the main manufacturer is still \( \sum_{k=1}^{o} q_{0jk} \) ).

In order to simplify the calculation, a 0-1 variable \( v_2 \) is introduced, assume:

\[
v_2 = \begin{cases} 
1 & \sum_{i=1}^{n} q_{ij} \geq \sum_{k=1}^{o} q_{0jk} \\
0 & \sum_{i=1}^{n} q_{ij} < \sum_{k=1}^{o} q_{0jk} 
\end{cases}
\]

Therefore, the profit function of component suppliers can be expressed as:

\[
\text{Max} \sum_{i=1}^{n} p_{ij}q_{ij} - \sum_{i=1}^{n} p_{ij}q_{ij} - \sum_{i=1}^{n} C_{i}(q_i) - \sum_{i=1}^{n} C_{j}(q_j) - \sum_{j=1}^{n} C_{jk}(q_{0jk}) - \sum_{j=1}^{n} p_{2jk}q_{0jk} - v_2 h_2 (\sum_{i=1}^{n} q_{ij} - \sum_{k=1}^{o} q_{0jk}) - (1-v_2) b_2 (q_{0jk} - \sum_{k=1}^{o} q_{0jk})
\]

The Nash equilibrium of the non-cooperative game between the component suppliers can be transformed into the optimal \( (q^*_i, q^*_{0jk}) \), which satisfies the following variational inequalities:

\[
\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial C_{i}(q_i)}{\partial q_{ij}} + \frac{\partial C_{j}(q_j)}{\partial q_{ij}} + p_{ij} + v_2 h_2 - (1-v_2)b_2(q_{0jk} - q_{ij}) + \sum_{j=1}^{n} \sum_{k=1}^{o} \frac{\partial C_{jk}(q_{0jk})}{\partial q_{0jk}} + p_{2jk} - p_{2jk} - v_2 h_2 + (1-v_2)b_2 (q_{0jk} - q^*_{0jk}) \geq 0
\]  \( (2) \)

The explanation of the above variational inequalities is as follows: The first item represents that the shortage cost of the unit product is equal to the sum of the marginal manufacturing cost of the component supplier, the marginal transaction cost between component suppliers and raw material suppliers, the purchase price of the unit product and the inventory cost of the unit product when the volume of transaction between component supplier and raw material supplier is more than zero, the economic meaning of the second item is similar to that of the first item.

2.3. Equilibrium conditions for main manufacturers

The logistics process of the main manufacturer \( k \): the main manufacturer \( k \) purchases \( q_{0jk} \) product from the component supplier \( j \) at the unit price \( p_{jk} \), whether the main manufacturer can achieve a product transaction with the external demand market depends on the relationship between the costs main manufacturer need to bear and the unit product price \( p_k \) that the external demand market can accept, assume that the unit product transaction cost and unit product handling cost taken by the main
manufacturer are $C_{jk}$ and $C_k$ respectively. Because the main manufacturer is directly oriented to the demand market, for the equilibrium problem of the main manufacturers, we refer to the user equilibrium in traffic network equilibrium (Wardrop equilibrium) to express the problem[13]. Wardrop equilibrium is similar to Nash equilibrium. The behavior decision of the main manufacturer satisfies the following function:

$$p_{jk} + C_{jk} + C_k \begin{cases} = p_k & q_{0jk} > 0 \\ \geq p_k & q_{0jk} = 0 \end{cases}$$

And the demand for the external market satisfies the following function:

$$d_k \begin{cases} = \sum_{j=1}^{n} q_{0jk} & p_k > 0 \\ \leq \sum_{j=1}^{n} q_{0jk} & p_k = 0 \end{cases}$$

Therefore, the Nash equilibrium of the non-cooperative game between the main manufacturers can be transformed into the optimal $(q_{0jk}^*, p_k^*)$, which satisfies the following variational inequalities:

$$\sum_{j=1}^{n} \sum_{i=1}^{n} \left[ p_{jk} + C_{jk} + C_k - p_k \right] (q_{0jk} - q_{0jk}^*) + \sum_{k=1}^{n} \left( \sum_{j=1}^{n} q_{0jk} - d_k \right) (p_k - p_k^*) \geq 0 \quad (3)$$

The explanation of the above variational inequalities is as follows: The first item represents that when the main manufacturer achieves equilibrium and the volume of transaction with the external demand market exceeds zero, the costs taken by the main manufacturer are equal to the price of the product in the demand market. The second item represents that when the price of products in the external demand market exceeds zero, the volume of transactions should be balanced, the quantity of products required by the market is exactly equal to the number of completed transaction products.

### 2.4. Equilibrium conditions for logistics service providers

Logistics service providers play an intermediary role in the whole network, the logistics service cost of logistics service provider $l$ is a continuous differentiable convex function of the number of products transported, we assume that the price that the logistics service provider charges for the same product is the same, and the price charges for different products are different. The logistics service will ship a quantity of $q_{ij}$ products from the raw material suppliers $i$ to the component suppliers with a unit transport price of $p_{ij}$, in this process the operation cost of logistics service providers denoted by $C_i(q_{ij})$. Then the logistics service will ship a quantity of $q_{0jk}$ products from the raw material suppliers $i$ to the component suppliers with a unit transport price of $p_{zjk}$, in this process the operation cost of logistics service providers denoted by $C_i(q_{0jk})$. The aim of logistics providers is to optimize the profit function:

$$\max \sum_{i=1}^{n} \sum_{j=1}^{n} \left[ p_{ij} q_{ij} - C_i(q_{ij}) \right] + \sum_{j=1}^{n} \sum_{k=1}^{n} \left[ p_{zjk} q_{0jk} - C_i(q_{0jk}) \right]$$

Therefore, the Nash equilibrium of the non-cooperative game between the logistics service providers can be transformed into the optimal $(q_{ij}^*, q_{0jk}^*)$, which satisfies the following variational inequalities:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \left( \frac{\partial C_i(q_{ij})}{\partial q_{ij}} - p_{ij} \right) (q_{ij} - q_{ij}^*) + \sum_{j=1}^{n} \sum_{k=1}^{n} \left( \frac{\partial C_i(q_{0jk})}{\partial q_{0jk}} - p_{zjk} \right) (q_{0jk} - q_{0jk}^*) \geq 0 \quad (4)$$

The meaning of the variational inequality above: if the marginal operation cost of the logistics service provider $l$ is equal to its corresponding logistics quotation, the logistics service provider will accept the delivery, if the marginal operation cost exceeds its corresponding logistics quotation, there will be zero transaction flow.
3. Numerical Example

In this section, we will verify the accuracy of the model proposed above through a given numerical example. Analysis from the angle of network equilibrium model, the equilibrium optimization of the whole network is to get a set of optimal \((q_1^*, q_j^*, q_{0,jk}^*, p_k^*)\), which satisfies the sum of variational inequalities (1) (2) (3) (4)\(^7\), that is, under the following conditions (see (5)), the model reaches equilibrium.

\[
\sum_{i=1}^n \left[ \frac{\partial C_i(q_i)}{\partial q_i} + v_i h_i - (1 - v_i) b_i \right] (q_i - q_i^*) +
\sum_{j=1}^n \sum_{k=1}^n \sum_{l=1}^n \left[ \frac{\partial C_j(q_j)}{\partial q_j} - \frac{\partial C_j(q_{0,jk})}{\partial q_{0,jk}} + \frac{\partial C_j(q_{0,jk})}{\partial q_{0,jk}} + v_j h_j - (1 - v_j) b_j - v_i h_i + (1 - v_j) b_i \frac{\partial C_j(q_{0,jk})}{\partial q_{0,jk}} + \frac{\partial C_j(q_{0,jk})}{\partial q_{0,jk}} \right] (q_{0,jk} - q_{0,jk}^*)
\]
\[
+ \sum_{j=1}^n \sum_{k=1}^n \left[ \frac{\partial C_k(q_{0,jk})}{\partial q_{0,jk}} - v_j h_j - (1 - v_j) b_j + C_j + C_k - p_k \right] (q_{0,jk} - q_{0,jk}^*) +
\sum_{k=1}^n \left( \sum_{j=1}^n \sum_{k=1}^n (q_{0,jk} - d_j)(p_k - p_k^*) \right) \geq 0
\]

For the convenience of calculation, a simple model \((i=2, j=2, k=2, l=2)\) is given, the functions of the enterprises at all levels are set as shown in Tables 2 and 3. (Some data are referred to literature conducted by\(^7\))

(1) Function settings of raw material suppliers

| Raw material supplier | \(i=1\) | \(i=2\) |
|-----------------------|---------|---------|
| Production cost \(C_i(q_i)\) | \(C_i(q_1) = 2.5q_1^2 + q_1q_2 + 2q_1\) | \(C_i(q_2) = 2.5q_2^2 + q_1q_2 + 2q_2\) |
| Transaction cost associated with component supplier \(j\) \(C_j(q_j)\) | \(j=1: C_{11}(q_{11}) = q_{11}^2 + 2q_{11}\) | \(j=1: C_{21}(q_{21}) = q_{21}^2 + 2q_{21}\) |
| | \(j=2: C_{12}(q_{12}) = q_{12}^2 + 2q_{12}\) | \(j=2: C_{22}(q_{22}) = q_{22}^2 + 2q_{22}\) |
| Unit inventory cost | \(h_1 = 10\) | \(h_2 = 10\) |
| Unit shortage cost | \(b_1 = 20\) | \(b_2 = 20\) |

\(q_1^*\) and \(q_2^*\) are not related to \(q_{0,jk}\) here.

(2) Function settings of component suppliers

Let \(Q_1 = q_{11} + q_{21}\), \(Q_2 = q_{12} + q_{22}\) denote the amount of raw materials accepted by component suppliers 1 and 2 respectively.

| Component suppliers | \(j=1\) | \(j=2\) |
|---------------------|---------|---------|
| Production cost \(C_j(q_j)\) | \(i=1: C_i(q_{1i}) = 2.5Q_{1i}^2 + Q_{1i}Q_{2i} + 2Q_{1i}\) | \(i=1: C_j(q_{2j}) = 2.5Q_{2j}^2 + Q_{2j}Q_{22} + 2Q_{2j}\) |
| | \(i=2: C_i(q_{2i}) = 2.5Q_{2i}^2 + Q_{2i}Q_{22} + 2Q_{2i}\) | \(i=2: C_j(q_{2j}) = 2.5Q_{2j}^2 + Q_{2j}Q_{22} + 2Q_{2j}\) |
| Transaction cost associated with raw material supplier \(i\) \(C_j(q_j)\) | \(i=1: C_{11}(q_{11}) = q_{11}^2 + q_{11}\) | \(i=1: C_{12}(q_{12}) = q_{12}^2 + q_{12}\) |
| | \(i=2: C_{21}(q_{21}) = q_{21}^2 + q_{21}\) | \(i=2: C_{22}(q_{22}) = q_{22}^2 + q_{22}\) |
| Transaction cost associated with main manufacture \(k\) \(C_j(q_{0,jk})\) | \(i=1: C_i(q_{0i}) = q_{0i}^2 + q_{0i}\) | \(i=1: C_j(q_{0j}) = q_{0j}^2 + q_{0j}\) |
| | \(i=2: C_{12}(q_{01}) = q_{01}^2 + q_{01}\) | \(i=2: C_{22}(q_{02}) = q_{02}^2 + q_{02}\) |
| Unit inventory cost | \(h_1 = 10\) | \(h_2 = 10\) |
| Unit shortage cost | \(b_1 = 20\) | \(b_2 = 20\) |

(3) Function settings of main manufactures
Unit transaction cost: \( C_{jk} = 10 \); Unit manufacturing cost: \( C_{k} = 15 \);

Market demand function: \( d_k = -p_k + 200 \);

Function settings of logistics service providers

Operating costs \( C_i(q_{ij}) : C_i(q_{ij}) = 1.5 \left( \sum_{i=1}^{2} \sum_{j=1}^{2} q_{ij} \right)^2 + 10 \); 

Operating costs \( C_j(q_{0jk}) : C_j(q_{0jk}) = 1.5 \left( \sum_{j=1}^{2} \sum_{k=1}^{2} q_{0jk} \right)^2 + 10 \);

In this paper, we use the Matlab tool and the projection gradient algorithm[14] [15] to solve the model, the equilibrium solutions are as follows:

Total output of raw material suppliers: \( q_1 = 0.59 \), \( q_2 = 0.70 \). Quantity of products that raw material suppliers supplied to component suppliers: \( q_{11} = 0 \), \( q_{12} = 0.57 \), \( q_{21} = 0.72 \), \( q_{22} = 0.01 \). Quantity of products that component suppliers supplied to main manufactures: \( q_{011} = 0 \), \( q_{012} = 8.24 \), \( q_{021} = 0 \), \( q_{022} = 7.40 \). The prices that external demand markets can accept: \( p_1 = 135.79 \), \( p_2 = 158.18 \).

The 0-1 variable: \( v_1 = 1 \), that is, the supply of raw material suppliers is greater than (or equal to) the demand of component suppliers, this example is the basic equilibrium state of supply and demand; \( v_2 = 0 \), the supply of parts suppliers can not meet the demand of the main manufacturers, that is, supply exceeds demand.

4. Parameter Analysis

Under the VMI model, many factors can affect the whole manufacturing logistics network equilibrium, when making decisions, raw material suppliers and component suppliers must consider various factors such as external market demand, market uncertainty and so on, the costs of decision-makers at all levels will also affect the equilibrium of the system.

In this example, we assumed the unit inventory cost \( h \) of raw material suppliers and component suppliers is 10 (the main manufacturer does not bear the cost), to observe the change of the transaction volume and the equilibrium price in system equilibrium when the \( h \) increases from 0 to 30, and import the obtained data into the Origin tool, a contrastive analysis between total output of raw material suppliers, quantity of products that raw material suppliers supplied to component suppliers, quantity of products that component suppliers supplied to main manufactures has obtained, as shown in Figure 2.

![Figure 2. Changes in the volume of transactions with the unit inventory costs.](image-url)
between component suppliers and main manufacturers \( q_{0jk} \) is decreasing with the increase of \( h \). It means when the system adopt VMI, with the increase of inventory cost, the system will reduce the transaction volume, once the suppliers are in short supply, so as to achieve the overall optimization of the system and achieve the network equilibrium of the whole manufacturing logistics system.

From the perspective of supply and demand, with the increase of unit inventory cost, \( q_i \) and \( q_{ij} \) always coincide in the left chart of Figure 2. For raw material suppliers, the supply and demand with component suppliers always be balanced; In the right picture, \( q_{ij} < q_{0jk} \), which shows that component suppliers have been in a state of short supply, but with the increase of unit inventory cost \( h \), the shortage of component suppliers will be alleviated under the condition of system equilibrium.

Meanwhile, we can observe the changes of the acceptable price of the external demand market and the sum of system transaction volume when the equilibrium is achieved, as shown in Figure 3.

From the left figure in Figure 3, as the unit inventory cost \( h \) increases, the price of products in the external demand market of supply chain is decreasing gradually, which corresponds to the decline of the transaction volume when system is in equilibrium expressed in the right figure in Figure 3, indicating that with the increase of unit inventory cost, the sum of transaction volume in the whole manufacturing logistics system and the price of product in demand market are reduced, the system reaches another equilibrium state. This fully illustrates the impact of unit inventory cost on transaction volume and price of manufacturing logistics system, and also draws some inspirations: under the premise of system network equilibrium, reducing unit inventory cost can also be a means to increase the market transaction volume and raise market price.

### Figure 3. Changes of equilibrium price and total transactions with the change of unit inventory cost

5. **Summary and Prospect**

Under VMI, raw material suppliers and component suppliers with inventory decision-making power may have inventory costs arising from oversupply or shortage costs arising from undersupply, hence, 0-1 variables are introduced into the model for selection. Consider that the main manufacturers need no inventory and directly face the external demand market, a network equilibrium model of manufacturing logistics is constructed, the variational inequality method is used to describe the optimal choice of the decision-makers in the model. Finally, a numerical example is used to verify the correctness of the model, and the related parameters are analyzed.

The equilibrium state of the system is influenced by many factors, in this paper, by controlling the changes in unit inventory cost, it is found that the total amount of transaction in the whole manufacturing logistics system is reduced and the price of the demand market is reduced with the increase of the cost of the unit inventory, however, there is no negative correlation between the unit inventory cost and the output of the raw material suppliers. the raw material suppliers maintain the stable output to seek the supply balance with the component suppliers, and the component suppliers
reduce the shortage by reducing the trading volume with the main manufacturers, which further illustrates that under the VMI inventory management model, the optimal decisions made by two-level suppliers is to coordinate the quantity and price balance from the perspective of seeking the overall optimization of the manufacturing logistics system.

In the future research, we need to further clarify other factors that affect the equilibrium state of the network system, and analyze its influence on the equilibrium state of the system, and further consider the complex inventory model and the network equilibrium model of the manufacturing logistics system under the replenishment strategy to make it closer to the real transaction environment.

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