Dynamic discount rate through Ornstein-Uhlenbeck process for mining project valuation

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Abstract. Discounted cash flow (DCF) is a conventional and widely used method for the mine valuation. The reason behind its wide use does not require complex mathematical computations. However, the technique is static; it does not allow the practitioner to react to the market changes. In DCF analysis, one of the important input is the discount rate, which, to some extent, represents the financial risk that the mining company may face in the future. The discount rate is sensitive to market and political uncertainties. Therefore, in fact, it is a dynamic parameter varying over the years. The application of DCF method is quite simple. As the life of mine prolongs, since the effect of discounting grows, DCF may not reflect the actual value of the project. This research proposes a way to find the discount rate for each year. The discount rate is calculated by a weighted average cost of capital formula by incorporating into the interest rate. The interest rate is treated by a stochastic process called Ornstein-Uhlenbeck (OU) process. This approach generates multiple realizations for the discount rate. Therefore, the proposed approach can quantify risks associated with the discount rate. Thus, the management of a mining project can make meaningful decisions. In doing so, a case study is conducted on a mining project to demonstrate the proposed approach.

1. Introduction

The mine valuation comprises four stages: Conceptual study, scoping study or preliminary economic assessment (PEA), pre-feasibility (PFS), and feasibility (FS) studies. In every stage, the accuracy of the project increases. Finally, a decision to go or not go is made. However, the uncertainties cannot be fully eliminated. For example, commodity price, various costs, and interest rate depend upon uncertain future events, and cannot be predicted accurately.

The discounted cash flow (DCF) is a common and widely used technique in business valuation, including mining due to its simplicity. It is also easily understood by stakeholders. The DCF is quite simple, valuing the future expected cash flow into the present time by using specific discount rate. Technically, First of all, we calculate annual expected cash flow by deducting its revenue with its cost. Second, using specific discount rate that represents the risks, we discount back each period of expected cash flow to the present time. Eventually, the summation of each discounted expected cash flow is the DCF value of the project [1]. After that, the net present value (NPV), internal rate of return (IRR), and Payback period methods can be conducted to value a project.

NPV is the most used method in DCF, and they should not be used interchangeably. The DCF is a technique that discounts each year cash flow to the present value with a specific number in percentage.
(discount rate), whereas the NPV is obtained by accumulating all the present value of cash flow each year, then is deducted with its initial investment. Given that discounting the expected cash flows have to be done in advance, and the NPV can only be done afterward.

The discount rate is the core of DCF technique; it represents the risk of the project that may be faced in the future. The risk is strongly associated with market and political changes. Politics in a country may affect the regulation stability. In Figure 1 (a), the risk for some mining countries in the world is shown [2]. It exhibits the countries with a high-risk premium rate. Using a high value of a discount rate may result in an unattractive project. The higher discount rate, the lower NPV, and vice versa [2].

Therefore, the country and market risks should be considered in the discounted rate calculation. Incorporating those risks, weighted average cost of capital (WACC) with capital asset pricing model (CAPM) can be performed. Determining discount rate by WACC is important since it takes debt ratio and interest rate into account. It is significantly important because mining industries are a highly capitalized business.

The interest rate can be treated stochastically using Ornstein-Uhlenbeck (OU) process. The OU process allows the interest rate to change over the time using the average value with the positive standard deviation (volatility) and positive mean reversion rate. In short, the OU process has three main parameters, average value, mean reversion rate, and standard deviation (volatility). It satisfies the interest rate behavior that varies around the average value (Figure 2).
The predicted interest rate is then incorporated into the discount rate formula. In doing so, incorporating interest rate, the WACC with CAPM is conducted. As a result, the prediction of the discount rate is produced for each consecutive year. By applying different discount rate for each year undoubtedly makes mining project valuation close to the reality, reasonable and plausible.

This paper is divided into six sections. In the first section, we discuss briefly the valuation techniques used in mining industries and the reasons for a dynamic discount rate to be applied in real cases. In the second section, the relevant literature is reviewed. The methodology was demonstrated in the third section. In the following section, a numerical calculation is provided. In section five, a case study is demonstrated. Finally, the conclusions and the future work are presented in the sixth section.

2. Literature review
Mining is an upstream industry that experiences cyclicality [3]. The upstream industries receive the market signals later than the downstream industries and also mining industries cannot shut down or reopen the business right away after the signal received. Therefore, a mining company stays to operate while the business is not economically attractive. Realizing that, implementing a correct valuation technique is crucial.

Nowadays there are at least three popular valuation techniques to value a business, which are Multiples, Real Options, and DCF. Multiples are one of the old and quick methods to value a business. It compares fundamental financial variables of one business to the others with the same characteristic [4-6], however, by scrutinizing one mining business characteristic with another is entirely different. It may have the same commodity, but the total reserve, geology risk, or country regulation may be different. That makes the Multiples not an appropriate valuation technique in the mining business.

The other popular technique is Real Options; it recognizes the flexibilities in the future as an additional value [7]. An unattractive current business may have a higher value if the business is predicted to be attractive in the future. In Real Options, two values can be added into the valuation: call option and put option. The call option represents the value of expanding, extending, diversifying, reopening, or deferring the project, while the put option represents the value of contracting, abandoning, selling, shortening the project [8]. Thus, combining DCF and option value increases the project value.

Lastly, the DCF technique, it is a common, widely used, and a standard procedure in modern finance [9]. It is most likely applicable to any business generally. The DCF technique is usually followed by NPV method to decide the project feasibility, NPV is greater than 0 means accept the project, otherwise, reject the project [10]; also the other methods, internal rate of return (IRR) and payback period are the prevalent methods in DCF [11]. In DCF, each year cash flow is discounted back with an arbitrage discount rate that represents the risk to the present time to see the business's present value, but as a consequence discount rate determination comes up as a problem. The higher discount rate exhibits, the more attractive business, and vice versa.

Therefore, overcoming the DCF problem, instead of conducting a static value, this paper applies stochastic process to find a different discount rate for each year. This dynamic discount rate treats the future risk differently year by year. In determining the discount rate, this paper uses the weighted cost of capital (WACC) formula. It considers the capital ratio and historical risk estimation.

A stochastic process is used to describe the fluctuation of a phenomenon over time. In risk quantification, stochastic processes are widely utilized. Geometric Brownian motion and mean reversion are the common processes to be applied. Ornstein-Uhlenbeck (OU) process is one of the stochastic processes that treat the variable as random walk and although there is the fluctuation, eventually it approaches or reverts to the average value, the OU process is also known as mean-reverting process [12]. The interest rate behaves randomly and varies around the average value. Therefore, the OU process is applied. In addition, the OU process is common to use in financial economics, Vasicek [13] applies OU process to forecast the interest rate.

In short, this paper applies the dynamic discount rate to overcome the drawback of DCF technique that uses typically one value of discount rate for the whole year, i.e., static discount rate. The discount rate is forecasted by applying OU process that is believed as the best stochastic process in interest rate
forecasting. Finally, to see the performance of dynamic discount rate, a study case is presented. Needless to say, this methodology can be applied easily in industries to enhance their project valuation.

3. Method
First of all, the OU process is conducted to forecast the future value of interest rate. In this step, the various interest rate for each year is generated. In prior, OU process parameter should be estimated. The OU process is formulated below.

\[ dx_t = \kappa(\mu - x_t)dt + \sigma \sqrt{x_t}dZ_t \]

Where \( \kappa \) is mean reversion rate (\( \kappa \geq 0 \)), \( X_t \) is the initial value, \( \mu \) is the mean value, \( \sigma \) is standard deviation (\( \sigma > 0 \)), and \( Z_t \) is standard Brownian motion. The mean reversion rate is estimated by Autoregressive first order (AR-1) [14], the mean value and standard deviation are estimated by ordinary least square.

The discount rate can be then calculated by WACC formula [15]. The parameters that are incorporated into the WACC formula have to be calculated in advance using the formula below.

\[
Cd = (r_f + r_p) \ast (1 - t) \]

\[
Ce = r_f + \beta \ast (r_m + r_f) \]

Where:
- \( Cd = \) Cost of debt
- \( Ce = \) Cost of equity
- \( r_f = \) Risk-free rate (interest rate)
- \( r_p = \) Risk premium (reflecting project risk), e.g., BBB company valued by S&P
- \( t = \) The company marginal tax rate
- \( r_m = \) The market rate of return
- \( r_m - r_f = \) Market risk premium

Notice that the risk-free rate is also known as the interest rate that is treated by OU process. Subsequently, result from equation (2) and (3) are then incorporated into the WACC formula below.

\[
WACC = \left[ Cd \ast (1 - T_c) \ast \frac{D}{D+E} \right] + \left[ Ce \ast \frac{E}{D+E} \right] \]

Where \( D \) is total debt, \( E \) is total equity, and \( T_c \) is corporate tax.

Finally, various WACC for each year is obtained. The static DCF calculation is normally produced by conducting formula below.

\[
DCF = \sum_{t=1}^{\infty} \frac{Cash\ Flow}{(1 + WACC)^t} \]

In this paper, dynamic discount rate is proposed, hence, various WACC is applied according to how many years the prediction is. Incorporating different discount rate for each year, thus the DCF formula is modified as shown below.
\[ DCF = \sum_{t=1}^{t=\infty} \frac{\text{Cash flow}}{(1 + WACC_t)^t} \] ................................. (6)

Notice the WACC changes depends on its period t. The further calculation, all three methods, the NPV, IRR, and Payback period can be generated by following equations below soon after DCF calculation is done.

\[ NPV = DCF - \text{Initial investment} \] ................................................................. (7)

\[ IRR = WACC_a + \frac{(WACC_b - WACC_a) \times NPV_a}{NPV_a - NPV_b} \] ................................. (8)

\[ Payback \text{ period} = \text{years before recovery} + \frac{\text{Last minus cumulative DCF}}{\text{First positive DCF}} \] ........... (9)

Where WACC\(_a\) is the lower WACC that results in negative NPV, WACC\(_b\) is the higher WACC that results in positive NPV, NPV\(_a\) is the NPV at WACC\(_a\), and NPV\(_b\) is the NPV at WACC\(_b\).

4. Numerical calculation
The historical data of interest rate is then analyzed to calibrate all the OU process parameter. Ordinary least square (OLS) estimator is applied to estimate the parameters. Especially, the mean reversion rate (κ) is estimated by using AR-1 method. The mean reversion rate (κ) calculation is shown below.

\[ \kappa = -\frac{\ln\beta}{\text{interval}} = -\frac{\ln0.82}{1} = 0.19 \]

The other parameter is the initial value (x\(_t\)), period (T), the time interval (ΔT), the average value (μ) and standard deviation (σ). Both the average value and standard deviation are simply estimated by OLS estimator. The initial value is the latest interest rate (present interest rate), the period is the length of forecasting, and ΔT is the interval between forecasting period. These required parameters for OU process are shown in Table 2.

Table 1. OLS estimator with AR-1 for mean reversion rate (κ) estimation

| Coeff | Std. error | t Stat | P-value |
|-------|------------|--------|---------|
| Intercept | 0.95 | 0.744265 | 1.275877 | 0.212866 |
| Y\(_t-1\) | 0.82 | 0.109357 | 7.527267 | 4.26 x 10\(^{-8}\) |

Regression Statistics

| Multiple R | 0.82 |
| R Square | 0.68 |
| Adj R Square | 0.66 |
| Std. Error | 1.449972 |
| Observations | 29 |

The other parameter is the initial value (x\(_t\)), period (T), the time interval (ΔT), the average value (μ) and standard deviation (σ). Both the average value and standard deviation are simply estimated by OLS estimator. The initial value is the latest interest rate (present interest rate), the period is the length of forecasting, and ΔT is the interval between forecasting period. These required parameters for OU process are shown in Table 2.

Table 2. The Interest rate OU process parameters

| Parameters | x\(_t\) | κ | M | σ | T | ΔT |
|------------|--------|---|---|---|---|----|
| Value      | 3.75   | 0.19 | 6.26 | 0.21 | 8 | 1 |

Next, the OU process then can be performed by incorporating all those estimated parameters into the equation (1). The OU process results in the future interest rate for given time period. Table 3 provides the outcome of one simulation from 2018 to 2025. Many probable realizations can be generated through the proposed approach.
Table 3. Interest rate value in the future

| Period | Time | Random Numbers | Interest rate prediction (%) |
|--------|------|----------------|-----------------------------|
|        |      | Uniform | Normal |                     |
| 2017   | 0    |          |        | 3.75                |
| 2018   | 1    | 0.12    | -1.18  | 3.77                |
| 2019   | 2    | 0.30    | -0.53  | 4.05                |
| 2020   | 3    | 0.53    | 0.06   | 4.50                |
| 2021   | 4    | 0.19    | -0.87  | 4.46                |
| 2022   | 5    | 0.70    | 0.53   | 5.04                |
| 2023   | 6    | 0.01    | -2.18  | 4.28                |
| 2024   | 7    | 0.75    | 0.67   | 4.95                |
| 2025   | 8    | 0.00    | -2.63  | 4.00                |

These interest rate predictions are then incorporated into the WACC formula, equation (4), to generate the discount rate from 2018 to 2025. Note, for further research, different random number can be applied.

5. Results and discussion

To calculate the discount rate by WACC, a case study is conducted on coal mining industry. Also, Indonesia’s benchmark thermal coal reference price (harga batubara acuan or HBA) in December 2017 [16] is applied.

First of all, all the assumptions are defined as follows to calculate the expected cash flow of the project for each year. The assumptions and expected cash flow are presented in Table 4 and Table 5.

Table 4. Study case assumptions

| Variables          | Value       |
|--------------------|-------------|
| Ore tonnage (tonne)| 2,000,000   |
| Waste tonnage (tonne)| 12,000,000 |
| Production escalation (%) | 0.00        |
| Mining cost ($)    | 5.00        |
| Processing cost ($)| 15.00       |
| Cost escalation per year (%) | 8%          |
| Initial investment ($) | 150,000,000|

Table 5. Expected cash flow

|                  | 2017   | 2018    | 2019    | 2020    | 2021    | 2022    | 2023    | 2024    | 2025    |
|------------------|--------|---------|---------|---------|---------|---------|---------|---------|---------|
| Revenue          | 0      | 188,080 | 188,080 | 188,080 | 188,080 | 188,080 | 188,080 | 188,080 | 188,080 |
| Cost             | 0      | 100,000 | 108,000 | 116,640 | 125,971 | 136,049 | 146,933 | 158,687 | 171,382 |
| Liabilities      | 0      | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       |
| Initial inv.     | 150,000| 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       |
| Net CF           | -150,000| 88,080  | 80,080  | 71,440  | 62,109  | 52,031  | 41,147  | 29,393  | 16,698  |

The WACC calculation in table 6 below requires some assumptions, which are risk premium (rp) is inferred as 3% that supposed to be company’s risk by credit rating company (e.g., S&P), the corporation tax (Tc) is defined as 21%, debt ratio is assumed 0 (self-funded), beta value (β) is assumed 1.4 (regression
slope between historical data of the company’s IRR and its market rate of return), and market risk premium \((rm - rf)\) is defined as 10%. 

### Table 6. WACC calculation

| Year | Cost of debt (%) | 2018 | 2019 | 2020 | 2021 | 2022 | 2023 | 2024 | 2025 |
|------|------------------|------|------|------|------|------|------|------|------|
| 2018 | 3.87             | 4.87 | 5.07 | 5.40 | 5.37 | 5.79 | 5.24 | 5.72 | 5.04 |
| 2019 | 3.87             | 4.87 | 5.07 | 5.40 | 5.37 | 5.79 | 5.24 | 5.72 | 5.04 |
| 2020 | 3.87             | 4.87 | 5.07 | 5.40 | 5.37 | 5.79 | 5.24 | 5.72 | 5.04 |
| 2021 | 3.87             | 4.87 | 5.07 | 5.40 | 5.37 | 5.79 | 5.24 | 5.72 | 5.04 |
| 2022 | 3.87             | 4.87 | 5.07 | 5.40 | 5.37 | 5.79 | 5.24 | 5.72 | 5.04 |
| 2023 | 3.87             | 4.87 | 5.07 | 5.40 | 5.37 | 5.79 | 5.24 | 5.72 | 5.04 |
| 2024 | 3.87             | 4.87 | 5.07 | 5.40 | 5.37 | 5.79 | 5.24 | 5.72 | 5.04 |
| 2025 | 3.87             | 4.87 | 5.07 | 5.40 | 5.37 | 5.79 | 5.24 | 5.72 | 5.04 |

Those WACC values are implemented in DCF calculation through discount rate. The WACC represents the risk of the project based on its business risk and debt ratio. It makes the WACC equal to discount rate, see table 7. Notice that the WACC is equal to cost of equity, it is due to the company capital is 100% self-funded (no debt).

### Table 7. Discount rate prediction

| Year | Disc. rate (%) |
|------|----------------|
| 2018 | 17.77          |
| 2019 | 18.05          |
| 2020 | 18.50          |
| 2021 | 18.46          |
| 2022 | 19.04          |
| 2023 | 18.28          |
| 2024 | 18.95          |
| 2025 | 18.00          |

Using varying discount rates in table 7, then incorporate it into DCF formula in equation (6), subsequently, discounted cash flow for each year can be generated. Table 8 presents the annual cumulative discounted cash flow.

### Table 8. Discounted cash flow calculation (in thousand)

| Year | Net CF | 2017 | 2018 | 2019 | 2020 | 2021 | 2022 | 2023 | 2024 | 2025 |
|------|--------|------|------|------|------|------|------|------|------|------|
| 2017 | -150,000 | 88,080 | 80,080 | 71,440 | 62,109 | 52,031 | 41,417 | 29,393 | 16,698 | 106,683,970 |
| 2018 | 88,080 | 80,080 | 71,440 | 62,109 | 52,031 | 41,417 | 29,393 | 16,698 | 106,683,970 | 106,683,970 |
| 2019 | 71,440 | 62,109 | 52,031 | 41,417 | 29,393 | 16,698 | 106,683,970 | 106,683,970 | 106,683,970 |
| 2020 | 62,109 | 52,031 | 41,417 | 29,393 | 16,698 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 |
| 2021 | 52,031 | 41,417 | 29,393 | 16,698 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 |
| 2022 | 41,417 | 29,393 | 16,698 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 |
| 2023 | 29,393 | 16,698 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 |
| 2024 | 16,698 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 |
| 2025 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 | 106,683,970 |

Finally, the NPV, IRR, and payback period calculation can be conducted by applying equation (7), (8), and (9), respectively. According to previous calculations, the NPV, IRR, and payback period of the project is presented in Table 9. Also, please note that green colored rows and columns in table 3, 6, 7, 8, and 9 may change by the effect of the random number.

### Table 9. Value of a coal mining project

| Project Value | NPV | $106,683,970 |
|---------------|-----|-------------|
| IRR           | 23.2%|
| Payback Period (year) | 2.41|
6. Conclusion
To conclude, this research demonstrates the dynamic discount rate to evaluate the project risk. Thus, the value of a mining project can be increased. Overcoming market and political uncertainties, the interest rate is treated as random walk by conducting OU process and is then incorporated into discount rate calculation. As the future works, this research can be extended to the other valuation technique (e.g., real options valuation) and applying a simulation technique for value at risk calculation.

Acknowledgment
The authors would like to say thank you to Indonesia Endowment Fund for Education (LPDP), Ministry of Finance of the Republic of Indonesia who supports the study in this research.

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