The Spin and Flavour Dependence of High-Energy Photoabsorption

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Abstract

We review the present data on high-energy, spin-dependent photoabsorption. We find a strong isotriplet term in \((\sigma_A - \sigma_P)\) which persists from \(Q^2 \sim 0.25\text{GeV}^2\) to high \(Q^2\) polarised deep inelastic scattering. For \(Q^2 \sim 4\text{GeV}^2\) and \(x\) between 0.01 and 0.12 the isotriplet part of \(g_1\) behaves as \(g_1^{(p-n)} \sim x^{-\frac{1}{2}}\), in contrast to soft Regge theory which predicts that \(g_1^{(p-n)}\) should converge as \(x \to 0\). The isotriplet, polarised structure function \(2xg_1^{(p-n)}\) is significantly greater than the isotriplet, unpolarised structure function \(F_2^{(p-n)}\) in this kinematic region. We analyse the low \(Q^2\) photoabsorption data from E-143 and SMC and use this data to estimate the high-energy Regge contribution to the Drell-Hearn-Gerasimov sum-rule.

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1 Introduction

Spin sum-rules for real and deeply virtual photoabsorption provide important constraints for our understanding of the structure of the nucleon. Experimental tests of these sum-rules involve some extrapolation of the measured cross-sections to asymptotic $\sqrt{s}$ at fixed $Q^2$. These extrapolations are, in general, motivated by Regge theory or perturbative QCD.

In this paper we discuss the spin and flavour dependence of high-energy photoabsorption. Polarised deep inelastic scattering experiments at CERN [1, 2], DESY [3] and SLAC [4, 5, 6] have measured the spin asymmetry

$$A_1 = \frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P}$$

over a large range of $x$ and $Q^2$. Here $\sigma_A$ and $\sigma_P$ denote the cross-sections for the absorption of a transversely polarised photon by a nucleon where the photon is polarized anti-parallel $\sigma_A$ and parallel $\sigma_P$ to the nucleon. We concentrate on $A_1$ at small Bjorken $x$ and large $\sqrt{s_{\gamma p}}$. We begin (Section 2) with a brief review of the present status of sum-rules for $(\sigma_A - \sigma_P)$ and spin dependent Regge theory. In Section 3 we present our new results. We examine the present data on $A_1$ and make several interesting observations.

(a) There is a strong isotriplet term in $(\sigma_A - \sigma_P)$ at large $\sqrt{s}$ which persists from low $Q^2 \sim 0.25\text{GeV}^2$ through to polarised deep inelastic scattering.

(b) The isotriplet part of $g_1$ (the nucleon’s first spin-dependent, deep inelastic structure function) behaves as $\sim x^{-\frac{3}{2}}$ at small $x$ (between 0.01 and 0.12) and $Q^2 \sim 4\text{GeV}^2$. This is in contrast to soft Regge theory which predicts that $g_1^{(p-n)}$ should converge as $x \rightarrow 0$ [7, 8].

(c) The isotriplet part of $2xg_1$ is significantly greater than the isotriplet part of the spin-independent structure function $F_2$ for $x$ between 0.01 and 0.12. This result corresponds to the parton-model flavour inequality $(d + \overline{d})^i(x) > (u + \overline{u})^i(x)$.

(d) We analyse the low $Q^2$ ($\sim 0.5\text{GeV}^2$) data from E-143 [3] and SMC [2]. We find that the isosinglet deuteron asymmetry $A_1^d$ is very small and consistent with zero in both experiments (at $\sqrt{s} \simeq 3.5$ and 16.7GeV) whereas the E-143 proton data exhibits a clear positive proton asymmetry $A_1^p$ at $\sqrt{s} \simeq 3.5\text{GeV}$.

Finally, we use the low $Q^2$ data to estimate the high-energy Regge contribution to the Drell-Hearn-Gerasimov sum-rule [9] which is expected to hold at $Q^2 = 0$. We find that a combined estimate of nucleon resonance [10], strangeness [11] and high-energy Regge contributions to the Drell-Hearn-Gerasimov sum-rule is consistent with the theoretical prediction for the fully inclusive sum-rule to within one standard deviation. We conclude (Section 4) by summarising the present status of knowledge about high-energy photoabsorption and the spin and flavour structure of the nucleon at low $Q^2$. 
2 Sum-rules and spin dependent Regge theory

2.1 The asymmetry $A_1$

The spin dependent photoabsorption cross-sections can be expressed as

$$(\sigma_A - \sigma_P) = \frac{4\pi\alpha^2}{mF}(g_1 - \frac{Q^2}{\nu^2}g_2)$$ (2)

and

$$(\sigma_A + \sigma_P) = \frac{4\pi\alpha^2}{mF}F_1.$$ (3)

Here $g_1$ and $g_2$ are the first and second nucleon spin dependent structure functions and $F_1$ is the first spin independent structure function which is measured in unpolarised scattering; $F$ is the photon flux factor which cancels in the asymmetry $A_1$, viz

$$A_1 = \frac{g_1 - \frac{Q^2}{\nu^2}g_2}{F_1} = \frac{g_1 - \frac{2m\nu}{\nu^2}g_2}{F_1}.$$ (4)

In this paper we shall consider two limits

(a) polarised deep inelastic scattering;

(b) high energy photoabsorption at low $Q^2$.

In both these limits

$$A_1 \to \frac{g_1}{F_1}.$$ (5)

There are rigorous sum-rules for $(\sigma_A - \sigma_P)$ in deep inelastic scattering ($Q^2 \to \infty$) and at $Q^2 = 0$.

2.2 The deep inelastic $g_1$ sum-rule

When $Q^2 \to \infty$, the light-cone operator product expansion relates the first moment of the structure function $g_1$ to the scale-invariant axial charges of the target nucleon by [12, 13, 14]

$$\int_0^1 dx\ g_1^p(x, Q^2) = \left(\frac{1}{12}g_A^{(3)} + \frac{1}{36}g_A^{(8)}\right)\left\{1 + \sum_{\ell \geq 1} c_{NS\ell}g_{2\ell}(Q)\right\}$$

$$+ \frac{1}{g_A^{(0)}}|_{\text{inv}}\left\{1 + \sum_{\ell \geq 1} c_{S\ell}g_{2\ell}(Q)\right\} + O\left(\frac{1}{Q^2}\right).$$ (6)

The flavour non-singlet $c_{NS\ell}$ and singlet $c_{S\ell}$ coefficients are calculable in $\ell$-loop perturbation theory [13]. The first moment of $g_1$ is fully constrained by low energy weak interaction dynamics. The isotriplet axial charge $g_A^{(3)}$ is measured independently in neutron beta decays, the flavour octet axial charge $g_A^{(8)}$ is measured independently in hyperon beta decays, and the flavour singlet, scale invariant axial charge $g_A^{(0)}|_{\text{inv}}$ [15] may be measured independently in elastic neutrino proton scattering [16].

Polarised deep inelastic scattering experiments at CERN [1, 2], DESY [3] and SLAC [4, 5] have verified the Bjorken sum-rule [51] for the isovector part of $g_1$ to
within 15%. They have also revealed a four standard deviations violation of OZI in the flavour singlet axial charge $g_{A}^{0}|_{\mu\nu}$ — for recent reviews see [14, 17]. The small $x$ extrapolation of $g_{1}$ data is presently the largest source of experimental error on measurements of the nucleon’s axial charges from deep inelastic scattering.

2.3 The Drell-Hearn-Gerasimov sum-rule

For real photons, $Q^{2} = 0$, the Drell-Hearn-Gerasimov sum-rule [9] (for reviews see [11, 18]) relates the difference of $\sigma_{A}$ and $\sigma_{P}$ to the square of the nucleon’s anomalous magnetic moment

\begin{equation}
(DHG) \equiv -\frac{4\pi^{2}\alpha\kappa^{2}}{2m^{2}} = \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu}(\sigma_{A} - \sigma_{P})(\nu).
\end{equation}

Here $\nu$ is the energy of the incident photon in the target rest frame, $m$ is the nucleon mass and $\kappa$ is the anomalous magnetic moment.

Charge parity imposes a symmetry constraint on dynamical contributions to the Drell-Hearn-Gerasimov integral. The cross-sections $\sigma_{A}$ and $\sigma_{P}$ are even under charge parity ($C = +1$). They receive contributions from OZI violating processes where the photon couples to the target nucleon via a $C = +1$, colour neutral, gluonic intermediate state in the $t$ channel, for example two gluon exchange. The anomalous magnetic moment $\kappa$ which appears on the left hand side of the Drell-Hearn-Gerasimov sum-rule has charge parity $C = -1$. It is measured in the nucleon matrix element of the vector current. Processes which contribute to $C = +1$ observable but not to matrix elements of the conserved $C = -1$ vector current therefore cancel in the logarithmic Drell-Hearn-Gerasimov integral for the difference of the two spin dependent photoabsorption cross-sections [11, 19].

The theoretical predictions for the isoscalar and isovector parts of the DHG integral $(DHG)_{I=0,1}^{\text{inclusive}}$ are:

\begin{equation}
(DHG)_{I=0}^{\text{inclusive}} = -219 \mu b, \quad (DHG)_{I=1}^{\text{inclusive}} = +15 \mu b.
\end{equation}

The first direct measurements of $\sigma_{A}$ and $\sigma_{P}$ at $Q^{2} = 0$ will soon be available from the ELSA, GRAAL, LEGS and MAMI facilities up to $\sqrt{s_{\gamma p}} \approx 2.5$ GeV.

Multipole analyses [10] of unpolarised pion photoproduction data suggest that the isosinglet part of the Drell-Hearn-Gerasimov sum-rule may be nearly saturated by nucleon resonance contributions with estimates ranging between $-225 \mu b$ and $-222 \mu b$. Estimates of the nucleon resonance contribution to the isovector part of the DHG integral range between $-65 \mu b$ and $-39 \mu b$ — that is, different in sign and a factor of 2-4 bigger than the theoretical prediction for the isovector part of the fully inclusive sum-rule.

To understand the “discrepancy” between the inclusive $(DHG)$ sum-rule and the nucleon resonance contributions to the inclusive sum-rule, it is helpful to express the anomalous magnetic moment $\kappa_{p}$ as the sum of its isovector $\kappa_{3}$ and isoscalar $\kappa_{0}$ parts ($\kappa_{p} = \kappa_{0} + \tau_{3}\kappa_{3}$). We can then write separate isospin sum-rules:

\begin{equation}
(DHG)_{I=0} = (DHG)_{[33]} + (DHG)_{[00]} = -\frac{2\pi^{2}\alpha}{m^{2}}(\kappa_{3}^{2} + \kappa_{0}^{2})
\end{equation}

\begin{equation}
(DHG)_{I=1} = (DHG)_{[30]} = -\frac{2\pi^{2}\alpha}{m^{2}}2\kappa_{3}\kappa_{0}.
\end{equation}
The physical values of the proton and nucleon anomalous magnetic moments \( \kappa_p = 1.79 \) and \( \kappa_n = -1.91 \) correspond to \( \kappa_0 = -0.06 \) and \( \kappa_3 = +1.85 \). If we take the Drell-Hearn-Gerasimov sum-rule as an exact equation and just the pion photoproduction data, then we can invert this equation to extract the following “pion physics” contributions to the anomalous magnetic moment: \( \kappa^{(\pi)}_0 \) between +0.16 and +0.26, and \( \kappa^{(\pi)}_3 \) = +1.86.

Strange quark dynamics are not expected to play an important role in pion-photoproduction. The “pion physics” contribution \( \kappa^{(\pi)} \) to the anomalous magnetic moment is not expected to receive any large contribution from the strangeness part, \( -\frac{1}{3} G_M^s(0) \), of the anomalous magnetic moment \([11]\). (Here \( G_M^s(Q^2) \) is the strangeness magnetic form-factor.) The SAMPLE experiment at Bates have recently measured the \( G_M^s(Q^2) \) at \( Q^2 = 0.1 \text{GeV}^2 \) \([20]\). They find \( G_M^s(0.1 \text{GeV}^2) = +0.23 \pm 0.44 \). If this measurement is extrapolated as a constant to \( Q^2 = 0 \), then the SAMPLE measurement corresponds to a strangeness contribution \(-0.08 \pm 0.15\) to the isoscalar part of the anomalous magnetic moment. Substituting in Eq.(9), we estimate the strangeness contributions to the isovector and isoscalar Drell-Hearn-Gerasimov integrals as \(+20 \pm 38 \mu\text{b}\) and \(\simeq 1 \mu\text{b}\) respectively.

The positive value of \( G_M^s \) measured by SAMPLE suggests that strangeness helps to fill part of the non-resonant contribution to the (DHG) integral. The high-energy part of \( (\sigma_A - \sigma_P) \) is expected to behave according to spin dependent Regge theory.

### 2.4 Spin dependent Regge theory

At large centre of mass energy squared \( s = 2m\nu + m^2 \), soft Regge theory predicts \([6, 8, 21, 22, 23, 24]\)

\[
\left( \sigma_A - \sigma_P \right) \sim N_3 s^{\alpha_1 - 1} + N_0 s^{\alpha_{f_1} - 1} + N_g \ln \frac{s}{\mu} + N_{PP} \frac{1}{\ln \frac{s}{\mu}}. \tag{10}
\]

Here \( \alpha_{a_1} \) and \( \alpha_{f_1} \) are the intercepts of the isovector \( a_1(1260) \) and isoscalar \( f_1(1285) \) and \( f_1(1420) \) Regge trajectories. If we make the usual assumption that the \( a_1 \) and \( f_1 \) trajectories are straight lines running approximately parallel to the \( (\rho, \omega) \) trajectories then they can be continued to arbitrary real \( J \) by

\[
\alpha(t) = \alpha_0 + \alpha't \tag{11}
\]

where \( \alpha' \simeq +0.86 \text{GeV}^{-2} \). If we then average over the masses of the three \( J^{PC} = 1^{++} \) mesons, we find an average intercept \( \alpha_0 = \alpha(0) = -0.5 \). Alternatively, if we assume a linear \( a_1 \) trajectory running through the \( a_1(1260) \) and the listed, but not well established, \( J^{PC} = 3^{++} \) state \( a_3(2050) \) \([25]\), then we find a trajectory with slope \( \alpha_{a_1}' \simeq +0.76 \text{GeV}^{-2} \) and intercept \( \alpha_{a_1} = -0.2 \). Both of these two estimates of the \( a_1 \) intercept lie within the range \((-0.5 \leq \alpha_{a_1} \leq 0)\) quoted in Ref.\([8]\).

The isosinglet part of \( (\sigma_A - \sigma_P) \) receives a contribution which depends on the Lorentz structure of the short range exchange potential \([22]\). If the short range exchange potential which generates the pomeron is a scalar, then it will not contribute to the large \( \sqrt{s} \) limit of \( (\sigma_A - \sigma_P) \). In the Landshoff-Nachtmann approach \([26, 27]\), the soft pomeron is modelled by the exchange of two non-perturbative gluons and transforms as a \( C = +1 \) vector potential with a correlation length of
about 0.1fm. This vector two non-perturbative gluon exchange gives [23] the \((\ln s)/s\) term in Equ.(2). The \(1/\ln^2 s\) term represents any two-pomeron cut contribution to \((\sigma_A - \sigma_P)\) [24].

The coefficients \(N_3, N_0, N_g\) and \(N_{PP}\) in Equ.(10) are to be determined from experiment. Each of the possible Regge contributions in Equ.(10) yield a convergent Drell-Hearn-Gerasimov integral. The mass parameter \(\mu\) is taken as a typical hadronic scale (between 0.2 and 1.0GeV).

It is an open question how far we can increase \(Q^2\) and still trust soft Regge theory to provide an accurate description of \((\sigma_A - \sigma_P)\). The HERA measurements [28, 29] of \((\sigma_A + \sigma_P)\) at low \(Q^2\) suggest that soft Regge theory provides a good description of the large \(\sqrt{s}\) behaviour of the total photoabsorption cross-section up to \(Q^2 \approx 0.5\text{GeV}^2\). The shape of \(g_1\) at small \(x\) is \(Q^2\) dependent, being driven by DGLAP evolution [30, 31, 32, 33] and by the resummation of \(\alpha_s^{l+1} \ln^l x\) radiative corrections [34, 35, 36, 37, 38], which are expected to play an important role at very small \(x\) (below \(x_{\text{min}} \approx 0.005\), the present smallest \(x\) data from SMC [2]).

3 Results from experiment

We now examine present experimental data with a view to extracting information about the large \(\sqrt{s}\) behaviour of \(\sigma_A\) and \(\sigma_P\). We begin our discussion with polarised deep inelastic data (Section 3.1) and then consider the low \(Q^2\) measurements from E-143 and SMC (Section 3.2).

3.1 \(g_1\) and \(F_2\)

Polarised deep inelastic data from CERN, DESY and SLAC reveals a strong isotriplet term in \(g_1\) at small \(x\) \((x < 0.15)\). The SLAC E-143 proton [4], E-154 neutron [5] and preliminary E-155 proton [39] data have the smallest experimental error within the particular kinematic region of the various experiments. We combine the E-154 neutron data with the proton data from E-143 and the preliminary E-155 \(x = (0.016, 0.024)\) data points [4]. Assuming that \(g_1^{(p-n)} = (g_1^p - g_1^n)\) has a power behaviour, \(x^\lambda\), at small \(x\) we find a best fit to the isotriplet part of \(g_1\):

\[g_1^{(p-n)} \sim (0.13)x^{-0.49} \quad \text{at} \quad (0.01 < x < 0.123)\]  

with \(\chi^2 = 2.19\) for 6 degrees of freedom – see Fig.1. A similar fit for \(g_1^{(p-n)}\) was obtain by Soffer and Teryaev [40]. The isotriplet part of \(g_1\) behaves as \(x^{-\frac{1}{2}}\) at small \(x\) in the SLAC data corresponding to an effective Regge intercept \(\alpha_{a1}(Q^2) \approx \frac{1}{2}\) at the relatively low deep inelastic \(Q^2 \approx 3 - 5\text{GeV}^2\). This compares with the soft Regge prediction that \(\alpha_{a1}(0)\) is between \(-\frac{1}{2}\) and 0. If we extrapolate Equ.(12) to

\footnote{Our data set consists of E-143 data which was evolved to \(Q^2 = 3\text{GeV}^2\) and E-154 and E-155 data which was evolved to \(Q^2 = 5\text{GeV}^2\) by assuming \(A_1\) is independent of \(Q^2\). DGLAP evolution is expected to induce some \(Q^2\) dependence in the deep inelastic \(A_1\). However, the \(Q^2\) independent hypothesis is consistent with the present deep inelastic data on \(A_1\). Following Soffer and Teryaev [40] we combine the E-143 and E-154 and E-155 data as if they were taken at the same \(Q^2\). The theoretical error induced by this procedure is of the order of 10%; it is small compared to the present experimental error on the data.}
$x = 0$, then the small $x$ (less than 0.12) part of $g_1^{(p-n)}$ contributes 50% of the Bjorken sum-rule.

In Fig. 1 we also show the SLAC data on $g_1^{(p+n)} = (g_1^p + g_1^n)$. We make a fit to this data by assuming a linear combination of a power term $x^\lambda$ and the functional form $(2 \ln \frac{1}{x} - 1)$ which was derived \[23\] in the two non-perturbative gluon exchange model \[27\]. We find:

$$g_1^{(p+n)} \sim -(0.23)x^{-0.56} + (0.28)(2 \ln \frac{1}{x} - 1) \quad \text{at} \quad (0.01 < x < 0.123) \quad (13)$$

with $\chi^2 = 2.95$ for 6 degrees of freedom \[\dagger\]. If we keep only a power term in the fit to the SLAC $g_1^{(p+n)}$, then we find $g_1^{(p+n)} \sim (0.35)x^{+0.36}$ with $\chi^2 = 7.1$ for 6 degrees of freedom. The power exponent $\lambda$ is negative in the fits (12) and (13) in contrast to the soft Regge prediction $\lambda \geq 0$. If we extrapolate Equ.(13) to $x = 0$ and evaluate the small $x$ ($\leq 0.12$) contribution to the first moment of $g_1^{(p+n)}$ then we find -0.21 from the power term and +0.18 from the $(2 \ln \frac{1}{x} - 1)$ term. This result compares with the parton model analysis \[31\] which suggests that the polarised gluon distribution makes a negative contribution to $g_1$ at small $x$.

Whilst Eq.(13) provides a good fit to the data, it should be treated with care. The fitted $g_1^{(p+n)}$ is the sum of two terms with opposite sign, each of which is five times bigger in magnitude than the measured $g_1^{(p+n)}$. As noted in \[12, 13\] the decomposition of $g_1^{(p+n)}$ into the sum of a quark term and a gluonic term is not well constrained at the present time. Our study supports this observation. The data in Fig.1 suggests either that $g_1^{(p+n)}$ changes sign at $x \sim 0.03$ or that it is very close to zero for $x < 0.03$. Let us suppose that $g_1^{(p+n)}$ does change sign around $x \sim 0.03$. The two functional forms that we use in the fit (13) are both positive definite at $x$ below 0.12. The power term has a negative coefficient in Equ.(13) because it grows faster than the logarithmic $(2 \ln \frac{1}{x} - 1)$ term at small $x$. If the $(2 \ln \frac{1}{x} - 1)$ term were replaced in the fit by a different functional form which grows faster with decreasing $x$ than the power contribution $x^{-\frac{1}{2}}$, then the signs of the coefficients of the power and gluonic exchange terms would be reversed. A direct measurement of the sign and shape of the polarised gluon distribution will come from experimental studies of charm \[14\] and di-jet production \[15, 16\] in polarised deep inelastic scattering.

To further understand the spin and flavour structure at small $x$ it is helpful to compare the isotriplet parts of $g_1$ and $F_2$. In the parton model

$$2x(g_1^p - g_1^n) = \frac{1}{3}x \left[(u + \bar{u})^\dagger - (u + \bar{u})^\dagger - (d + \bar{d})^\dagger + (d + \bar{d})^\dagger\right] \otimes \Delta C_{NS} \quad (14)$$

and

$$(F_2^p - F_2^n) = \frac{1}{3}x \left[(u + \bar{u})^\dagger + (u + \bar{u})^\dagger - (d + \bar{d})^\dagger - (d + \bar{d})^\dagger\right] \otimes C_{NS}. \quad (15)$$

\[\dagger\] Lower $x$ data is available from SMC \[2\] with larger experimental errors. We note the isotriplet data points $(x, g_1^{(p-n)})$: (0.005, 1.1±1.3), (0.008, 2.4±0.8) and (0.014, 1.5±0.5) and isosinglet points $(x, g_1^{(p+n)})$: (0.005, 0.0±1.3), (0.008, −0.5±0.8) and (0.014, −0.7±0.5) evolved \[41\] to a common $Q^2 = 5$GeV$^2$.\]
Here $u$ and $d$ denote the up and down flavoured quark distributions polarized parallel ($\uparrow$) and antiparallel ($\downarrow$) to the target proton $\uparrow$. $\Delta C_{NS}$ and $C_{NS}$ denote the spin-dependent and spin-independent perturbative QCD coefficients respectively. These coefficients are related (in the $\overline{MS}$ scheme) by \[ \Delta C_{NS}(x) = C_{NS}(x) - \frac{\alpha_s}{2\pi} \frac{4}{3} (1 + x). \] \[ (16) \]

In Fig. 2 we compare the isotriplet part of the SLAC $2xg_1$ data with the NMC measurement \[ [49] \] of the isotriplet part of $F_2$ at $4\text{GeV}^2$. The NMC parametrised their small $x$ data using the fit:

\[
(F_p^p - F_n^n) \sim (0.20 \pm 0.03)x^{0.59 \pm 0.06} \quad \text{at} \quad (0.004 < x < 0.15) \quad (17)
\]

for $Q^2 = 4\text{GeV}^2$. Clearly, the isotriplet part of $2xg_1$ is significantly greater than the isotriplet part of $F_2$

\[
2x(g_p^p - g_n^n) > (F_2^p - F_2^n) \quad (18)
\]

indicating the flavour inequality

\[
(d + \overline{d})^\uparrow(x) > (u + \overline{u})^\uparrow(x) \quad \text{at} \quad (0.01 < x < 0.12). \quad (19)
\]

This inequality includes both valence and sea contributions. It holds both at leading order and, via Equ.(16), also at next-to-leading order. Since the coefficient $C_{NS}$ is greater than $\Delta C_{NS}$ at next-to-leading order, Equ.(16), it follows that the the parton-model inequality (19) is more pronounced at next-to-leading order than at leading order. (The $-\frac{\alpha_s}{2\pi} \frac{4}{3} (1 + x)$ term in Equ.(16) acts to lower $2xg_1^{(p-n)}$ towards $F_2^{(p-n)}$ in Fig.2.) The inequality (18) persists in the data to $x \simeq 0.4$.

One also finds that the measured \[ [49] \] Gottfried integral \[ [50] \]

\[
\int_0^1 dx \left( \frac{F_2^p - F_2^n}{x} \right) = 0.235 \pm 0.026 \quad (20)
\]

at $Q^2 = 4\text{GeV}^2$ is five standard deviations below the theoretical value of the Bjorken sum-rule \[ [51], [13] \]

\[
2 \int_0^1 dx \left( g_1^p - g_1^n \right) = \frac{g_4^{(3)}}{3} \left[ 1 - \frac{\alpha_s}{\pi} - 3.58 \left( \frac{\alpha_s}{\pi} \right)^2 - 20.21 \left( \frac{\alpha_s}{\pi} \right)^3 \right] \quad (21)
\]

\[
= 0.370 \pm 0.008 \quad (22)
\]

at $Q^2 = 4\text{GeV}^2$.

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\[ ^5 \] Equs.(14) and (15) assume that charge symmetry is exact in relating the $u$ and $d$ flavoured distributions in the proton to the $d$ and $u$ distributions in the neutron. Possible charge symmetry violations of up to 3-5% between the unpolarised valence distributions $d_V^p(x)$ and $u_V^p(x)$ are discussed in \[ [17] \].
Table 1: Small $Q^2$ data from E-143 and SMC

| $x$ | $Q^2$ | $s$ | $A_1^p$ | $A_1^d$ |
|-----|-------|-----|---------|---------|
| SLAC E-143 | | | | |
| 0.035 | 0.32 | 9.7 | 0.053 ± 0.030 | -0.020 ± 0.032 |
| 0.035 | 0.65 | 18.8 | 0.069 ± 0.018 | +0.039 ± 0.046 |
| 0.050 | 0.37 | 7.9 | 0.110 ± 0.033 | +0.004 ± 0.034 |
| 0.050 | 0.79 | 15.9 | 0.117 ± 0.019 | +0.023 ± 0.034 |
| 0.080 | 0.42 | 5.7 | 0.095 ± 0.037 | +0.031 ± 0.040 |
| 0.080 | 0.71 | 9.0 | 0.129 ± 0.038 | -0.010 ± 0.043 |
| 0.125 | 0.47 | 4.2 | 0.110 ± 0.048 | +0.022 ± 0.057 |
| CERN SMC | | | | |
| 0.0009 | 0.25 | 278 | 0.001 ± 0.069 | -0.058 ± 0.055 |
| 0.0011 | 0.30 | 273 | 0.016 ± 0.085 | +0.004 ± 0.067 |
| 0.0011 | 0.34 | 309 | 0.196 ± 0.111 | +0.056 ± 0.084 |
| 0.0014 | 0.38 | 272 | 0.139 ± 0.044 | -0.045 ± 0.041 |
| 0.0017 | 0.46 | 271 | 0.076 ± 0.053 | -0.088 ± 0.050 |
| 0.0019 | 0.55 | 290 | 0.037 ± 0.057 | +0.013 ± 0.055 |
| 0.0023 | 0.58 | 252 | 0.020 ± 0.040 | +0.114 ± 0.042 |
| 0.0025 | 0.70 | 280 | 0.025 ± 0.044 | -0.099 ± 0.046 |

3.2 $A_1$ at low $Q^2$

The E-143 and SMC experiments have measured $A_1$ for both proton and deuteron targets over a wide range of $Q^2$, including between 0.25 GeV$^2$ and 0.80 GeV$^2$. We list this low $Q^2$ data in Table 1.

The low $Q^2$ data has the following general features. First, the isoscalar deuteron asymmetry $A_1^d$ is very small and consistent with zero in both the E-143 and SMC low $Q^2$ bins. Second, there is a clear positive proton asymmetry in the E-143 data, signalling a strong isotriplet term in $(\sigma_A - \sigma_P)$ at $s \approx 12\text{GeV}^2$. The SMC $A_1^p$ data is less clear: combining the SMC low $Q^2$ $A_1^p$ data yields a positive value for $A_1^p$. However, the majority of these SMC points are consistent with zero and more precise data are needed to resolve this value.

Due to the wide separation in $s$ range measured in E143 and SMC, we combine the low $Q^2$ data to obtain one point corresponding to each experiment. This is shown in Table 2. We make two cuts:

(a) keeping $\sqrt{s} \geq 2.5\text{GeV}$ to ensure that our data set is well beyond the resonance region and including all such data that the mean $Q^2$ is kept below 0.5GeV$^2$ for each experiment. (In practice, this amounts to a common $Q^2$ cut of 0.7GeV$^2$ and yields a mean $Q^2 = 0.45\text{GeV}^2$ for each experiment.)
Table 2: \( A_1 \) at large \( s \) and low \( Q^2 \)

| Cuts | \( \langle Q^2 \rangle \) | \( s \) | \( A_1^p \) | \( A_1^d \) |
|------|-----------------|-----|---------|---------|
| (a)  | \( \langle Q^2 \rangle \leq 0.5\text{GeV}^2, s \geq 7\text{GeV}^2 \) | 0.45 | 12 | 0.077 ± 0.016 | +0.008 ± 0.022 |
|      |                 | 0.45 | 278 | 0.064 ± 0.024 | −0.013 ± 0.020 |
| (b)  | \( Q^2 \leq 0.8\text{GeV}^2, s \geq 4\text{GeV}^2 \) | 0.53 | 10 | 0.098 ± 0.013 | +0.016 ± 0.016 |
|      |                 | 0.45 | 278 | 0.064 ± 0.024 | −0.013 ± 0.020 |

(b) including all data at \( \sqrt{s} \geq 2\text{GeV} \) and \( Q^2 \leq 0.8\text{GeV}^2 \).

The HERMES experiment hope to measure the low \( Q^2 \) \( A_1^p \) at \( \sqrt{s} \approx 7\text{GeV} \) over the same range of \( Q^2 \) and with similar accuracy to the SLAC experiments [52].

In what follows, we work with Cut (a). This choice of cut is a compromise between keeping \( Q^2 \) as low as possible and including the maximum amount of data. The choice \( Q^2_{\text{max}} \approx 0.5\text{GeV}^2 \) is motivated by the HERA data [28, 29] on \((\sigma_A + \sigma_P)\) which rises with increasing \( \sqrt{s} \) according to soft Regge theory up to \( Q^2 \approx 0.5\text{GeV}^2 \). At larger \( Q^2 \) the data exhibits evidence of \( Q^2 \) dependence in the effective Regge intercepts for high-energy, virtual photoabsorption.

To estimate the spin asymmetry at \( Q^2 = 0 \) we shall assume that the large \( \sqrt{s} \) \( A_1 \) is approximately independent of \( Q^2 \) between \( Q^2 = 0 \) and \( Q^2 \approx 0.5 \text{ GeV}^2 \). Since the E-143 data has the lowest experimental error and shows a clear positive signal in \( A_1^p \) at low \( Q^2 \) we choose to normalise to E-143. For the total photoproduction cross-section we take

\[
(\sigma_A + \sigma_P) = 67.7\, s^{+0.0808} + 129\, s^{-0.4545} \tag{23}
\]

(in units of \( \mu\text{b} \)), which is known to provide a good Regge fit for \( \sqrt{s} \) between 2.5GeV and 250GeV [23]. (Here, the \( s^{+0.0808} \) contribution is associated with pomeron exchange and the \( s^{-0.4545} \) contribution is associated with the isoscalar \( \omega \) and isovector \( \rho \) trajectories.) Multiplying \( A_1^p \) by the value of \((\sigma_A + \sigma_P)\) at \( \sqrt{s} = 3.5\text{GeV} \), we estimate

\[
(\sigma_A - \sigma_P) \simeq +10\mu\text{b} \quad \text{at} \quad (Q^2 = 0, \sqrt{s} = 3.5\text{GeV}). \tag{24}
\]

The small isoscalar deuteron asymmetry \( A_1^d \) indicates that the isoscalar contribution to \( A_1^p \) in the E-143 data is unlikely to be more than 30%. In Fig. 3 we show the asymmetry \( A_1^p \) as a function of \( \sqrt{s} \) between 2.5 and 250 GeV for the four different would-be Regge behaviours for \((\sigma_A - \sigma_P)\): that the high energy behaviour of \((\sigma_A - \sigma_P)\) is given.
(a) entirely by the \((a_1,f_1)\) terms in Equ.(2) with Regge intercept either (1) \(-\frac{1}{2}\) (conventional) or (2) \(+\frac{1}{2}\) (motivated by the observed small \(x\) behaviour of \(g_1^{(p-n)}\)),

(b) by taking 2/3 isovector (conventional) \(a_1\) and 1/3 two non-perturbative gluon exchange contributions at \(\sqrt{s} = 3.5\text{GeV}\),

(c) by taking 2/3 isovector (conventional) \(a_1\) and 1/3 pomeron-pomeron cut contributions at \(\sqrt{s} = 3.5\text{GeV}\).

(In Fig.3 we take the mass parameter in the Regge fit, Equ.(10), as \(\mu^2 = 0.5\text{GeV}^2\).)

The combined E-143 and SMC data are consistent with 
\(A_{1}^{p}\) being constant in \(\sqrt{s}\). They are consistent at the level of two standard deviations with \((\sigma_A - \sigma_P) \sim s^{-\frac{1}{2}}\). However, given the large experimental error on the SMC data it is not possible to draw any meaningful conclusion at the present stage.

The small isosinglet asymmetry \(A_{1}^{d}\) is consistent with the symmetry constraints from charge parity that we discussed in Sect.2.3. Furry’s theorem tells us that two (non-perturbative) gluon exchange does not contribute to the anomalous magnetic moment. It follows that this exchange yields zero contribution to the (DHG) integral. The contribution of such processes to \((\sigma_A - \sigma_P)\) is either exactly zero or it changes sign (at least once) so that they yield a vanishing contribution to (DHG). The Regge contribution \((\ln s)/s\) is positive definite at large \(\sqrt{s}\). The sign of this gluonic exchange contribution to \((\sigma_A - \sigma_P)\) is unknown at energies below the Regge region. The small value of \(A_{1}^{d}\) in the E-143 and SMC low \(Q^2\) data is consistent with a vanishing \(C = +1\) gluonic exchange contribution to \((\sigma_A - \sigma_P)\) at \(Q^2 = 0\).

To estimate the high-energy Regge contribution to the Drell-Hearn-Gerasimov integral we fit a Regge form \((\sigma_A - \sigma_P) \sim s^{\alpha - 1}\) through the E-143 value, Equ.(24), and allow \(\alpha\) to vary between \(\pm \frac{1}{2}\). This leads to an estimate of the Regge contribution to (DHG) of \(+25 \pm 10\mu\text{b}\) from \(\sqrt{s} \geq 2.5\text{GeV}\) taking into account the error on \(A_{1}^{p}\). If we allow a maximum 30\% contribution from the two-pomeron cut in the E-143 \((\sigma_A - \sigma_P)\) we find a contribution to (DHG) which is at the upper limit of this range. (We note, however, that there is no evidence for any two-pomeron contribution in present deep inelastic data [3].)

4 Conclusions

We have examined the present data on polarised photoabsorption for \(Q^2\) between 0.25 and 5\text{GeV}^2.

We find two interesting results in the isotriplet channel. First, soft Regge theory predicts that the isotriplet \((\sigma_A - \sigma_P) \sim s^{\alpha_{a1} - 1}\) at large \(s\) and that \(g_1^{(p-n)} \sim x^{-\alpha_{a1}}\) at small \(x\), where \((-\frac{1}{2} \leq \alpha_{a1} \leq 0\)). Polarised deep inelastic data suggests that \(g_1^{(p-n)} \sim x^{-\frac{1}{2}}\) for \(x\) between 0.01 and 0.12 and \(Q^2 \simeq 4\text{GeV}^2\). That is, \(\alpha_{a1}(Q^2)\) at \(Q^2 \simeq 4\text{GeV}^2\) has the opposite sign to the soft Regge prediction. It is a challenge for future polarisation experiments to map the \(Q^2\) dependence of the effective \(a_1\) intercept: either to observe it change sign at a particular value of \(Q^2\) or, if does not, to observe a Regge trajectory that is either nearly flat or non-linear at \(t \to 0^+\).
Second, we find the novel result that the isotriplet part of the polarised $2xg_1$ is significantly greater than the isotriplet part of the unpolarised $F_2$ at small $x$.

In the isosinglet channel, the sign of the gluonic exchange contribution to the first moment of $g_1$ is not well constrained at the present time. The sign that we extract from the fit to $g_1^{p+n}$ is sensitive to the functional form that is assumed for this term in the fit.

At the present time, there is no data on $(\sigma_A - \sigma_P)$ at $Q^2 = 0$. Experiments with real photons are planned or underway at the ELSA, GRAAL, LEGS and MAMI facilities to measure the $(\sigma_A - \sigma_P)$ up to $\sqrt{s_{cp}} \leq 2.5\text{GeV}$. They will make a direct measurement of nucleon resonance and strangeness production contributions to the Drell-Hearn-Gerasimov sum-rule. One future possibility to measure high energy $\gamma p$ collisions is polarised HERA [15]. If realised, this facility could measure $A_1^p$ to an accuracy of 0.0003 at $\sqrt{s}$ between 50 and 250 GeV assuming an integrated luminosity $\mathcal{L} \simeq 500\text{pb}^{-1}$. Details of the detector acceptances and the expected asymmetries are given in [53].

Whilst we wait for direct measurements of $(\sigma_A - \sigma_P)$, the Drell-Hearn-Gerasimov sum-rule allows us to deduce a consistent picture of the spin structure of the nucleon at $Q^2 = 0$.

(a) Multipole analyses of (unpolarised) pion photoproduction data suggest that the isosinglet part of the DHG sum-rule (-219 $\mu$b) is nearly fully saturated by nucleon resonance contributions (to within a few percent), whereas resonance contributions to the isotriplet part of the sum-rule are a factor of 2-4 times bigger and have the opposite sign to the theoretical prediction for the isotriplet part of the fully inclusive sum-rule (+15 $\mu$b).

(b) Pion photoproduction data does not include any net strangeness production in the final state. Strangeness contributions to (DHG) can be quantified through the strangeness magnetic moment $G_M^s(0)$ which is measured in parity violating elastic $\vec{e}\vec{p}$ scattering [11]. If one assumes that $G_M^s(Q^2)$ is independent of $Q^2$ between zero and $0.1\text{GeV}^2$, then the recent SAMPLE measurement of $G_M^s(0.1\text{GeV}^2) \simeq 0.23 \pm 0.44$ corresponds to a strangeness contribution $+20 \pm 38\mu$b in the isovector (DHG) integral and $\simeq 1\mu$b in the isoscalar (DHG) integral.

(c) Low $Q^2$ measurements of the spin asymmetry $A_1$ reveal a small isosinglet deuteron asymmetry (consistent with zero) and a strong isotriplet term in $(\sigma_A - \sigma_P)$ at $\sqrt{s} \simeq 3.5\text{GeV}$ and $Q^2$ between 0.25$\text{GeV}^2$ and 0.5$\text{GeV}^2$. If we assume that $A_1$ is independent of $Q^2$ between $Q^2 = 0$ and $\simeq 0.5\text{GeV}^2$, then we can estimate the Regge contribution to the Drell-Hearn-Gerasimov integral as $+25 \pm 10\mu$b. This contribution is predominantly isotriplet.

Both the strangeness and the Regge contributions appear to reconcile part of the “discrepancy” between the nucleon resonance contribution to the isovector part of the DHG integral and the theoretical prediction for the isovector part of the fully inclusive sum-rule. They appear to yield only a very small contribution to the isoscalar part of (DHG) which multipole analyses suggest is nearly fully saturated by nucleon resonance contributions. Combining the above estimates of nucleon
resonance, strangeness and Regge contributions, which are extracted from three independent experiments, the net contribution to each of the isovector and isoscalar parts of the Drell-Hearn-Gerasimov sum-rule is consistent with the theoretical prediction for the fully inclusive (isospin dependent) sum-rules to within one standard deviation. Clearly, each of these contributions should be measured directly in a real photon beam DHG experiment. However, even at this preliminary stage, the available experimental evidence is consistent with the validity of the DHG sum-rule.

High-energy polarised $\gamma p$ collisions continue to offer up many unexpected surprises. Experiments with polarised real photons are just beginning; the $Q^2$ dependence of the spin dependent part of the high-energy photoabsorption cross-section, through the study of $\alpha_{a_1}(Q^2)$ and $\alpha_{f_1}(Q^2)$, would open up a new window on the spin structure of the nucleon.

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Figure 1: The SLAC data on $g_1$. The upper curve shows the fit (12) to the isotriplet $g_1^{(p-n)}(x)$. The lower curve shows the fit (13) to the isosinglet $g_1^{(p+n)}(x)$ at $Q^2 \simeq 4 \text{GeV}^2$. 
\[ 2 \times (g_1^p - g_1^n) \]
\[ (F_2^p - F_2^n) \]

Figure 2: Comparison of the isotriplet parts of the polarised \(2xg_1\) (SLAC) and unpolarised \(F_2\) (NMC) at \(Q^2 \simeq 4\text{GeV}^2\).
Figure 3: The real photon asymmetry $A_1^p$ as a function of $\sqrt{s}$ for different Regge behaviours for $(\sigma_A - \sigma_P)$: given entirely by (1a) the $(a_1, f_1)$ terms in Equ.(2) with Regge intercept either $-\frac{1}{2}$ (conventional) or (1b) $+\frac{1}{2}$; (2) by 2/3 isovector (conventional) $a_1$ and 1/3 two non-perturbative gluon exchange contributions at $\sqrt{s} = 3.5\text{GeV}$; (3) by 2/3 isovector (conventional) $a_1$ and 1/3 pomeron-pomeron cut contributions at $\sqrt{s} = 3.5\text{GeV}$. 