Measurement of the phase diffusion dynamics in the micromaser

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We propose a realistic scheme for measuring the micromaser linewidth by monitoring the phase diffusion dynamics of the cavity field. Our strategy consists in exciting an initial coherent state with the same photon number distribution as the micromaser steady-state field, singling out a purely diffusive process in the system dynamics. After the injection of a counter-field, measurements of the population statistics of a probe atom allow us to derive the micromaser linewidth. Our proposal aims at solving a classic and relevant decoherence problem in cavity quantum electrodynamics, allowing to establish experimentally the distinctive features appearing in the micromaser spectrum due to the discreteness of the electromagnetic field.

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The micromaser has been the object of continuous interest as a proper tool for the investigation of fundamental questions in cavity quantum electrodynamics. For example, it has permitted the generation of highly pure Fock states, the detection of photon number trapping states, and the possibility of testing new ideas in the physics of quantum information. Nevertheless, some fundamental aspects have remained elusive to the experiments, as the measurement of the micromaser spectrum and its linewidth, whose physical origin is the decoherence of the cavity field induced by a phase diffusion process. Although different proposals have been done in the past, measurements related to the micromaser spectrum remain until now as an experimental challenge for reasons we discuss thoroughly in this work.

In Refs. 1, 2, approximated analytical expressions for the phase diffusion rate were derived from the micromaser master equation (MME), matching reasonably the numerical results. When the micromaser operates at very low temperatures, trapping states of the cavity field occur, inducing sharp minima in the mean photon number. In consequence, as the pumping rate increases, linewidth oscillations are predicted, at strong variance with the monotonical dependence of the Schawlow-Townes laser linewidth. In Refs. 1, 2, a Ramsey-type interferometric scheme was proposed for an experimental measurement of the micromaser linewidth. These techniques require atoms in a coherent superposition of the ground and excited states, which needs further advances in current technology of very high-Q (closed) cavities. Other treatments approached the spectrum problem by investigating the field and atomic correlation functions. Unfortunately, in one way or another, all proposals have failed in approximating the requirements of a feasible experiment.

In this Letter, we consider a realistic measurement of the phase diffusion dynamics of the micromaser and propose different strategies for unveiling its special features.

We propose a realistic measurement of the micromaser linewidth by monitoring the phase diffusion dynamics in the micromaser and propose different strategies for unveiling its special features.

A micromaser consists in a single quantized mode of a high-Q cavity driven by excited two-level atoms crossing the field mode one at a time with pumping rate $r$. The atomic levels are long living Rydberg states that couple strongly to the cavity microwave field. The cavity is in contact with a thermal bath producing mean number of thermal photons $\bar{n}$ and has a linewidth $\gamma = \omega/Q$, where $\omega$ is the angular frequency of the cavity field. However, the decay of the field can be considered as negligible when an atom is flying through, due to its high speed. Each one of the pumping atoms interacts with the cavity field following the Jaynes-Cummings (JC) model, where the evolution of an initial atomic excited state $|e\rangle$ and any field Fock state $|n\rangle$ follows

$$|e\rangle|n\rangle \rightarrow \cos(g\tau\sqrt{n+1})|e\rangle|n\rangle - \sin(g\tau\sqrt{n+1})|g\rangle|n+1\rangle. \quad (1)$$

Here, $g$ is the atom-field coupling strength, $\tau$ is the interaction time and $|g\rangle$ is the atomic ground state. The temporal evolution of the density matrix elements $\rho_{n,m}(t)$ of the cavity field is ruled by the MME

$$\frac{d}{dt}\rho_{n,m} = A_{n,m}\rho_{n-1,m-1} + B_{n,m}\rho_{n,m} + C_{n,m}\rho_{n+1,m+1} \quad (2)$$

where

$$A_{n,m} = r \sin(g\tau\sqrt{n}) \sin(g\tau\sqrt{m}) + \gamma \bar{n}_b \sqrt{nm}$$

$$B_{n,m} = -r[1 - \cos(g\tau\sqrt{n+1}) \cos(g\tau\sqrt{m+1})] - \frac{2}{2}(\bar{n}_b + 1)(n + m) - \frac{2}{2}\bar{n}_b(n + m + 2)$$

$$C_{n,m} = \gamma(\bar{n}_b + 1)\sqrt{(n + 1)(m + 1)}. \quad (3)$$
Here, the first term of $A_{n,m}$ and $B_{n,m}$ are related to the unitary contribution of the Jaynes-Cummings interaction, after tracing out the atomic degrees of freedom, and the remaining terms are associated with the incoherent processes.

\[ \langle E(t) \rangle \propto \text{Re}(\hat{a}(t)) = \frac{1}{2} \sum_{n=0}^{\infty} \sqrt{n+1} \rho_{n,n+1}(t) + c.c. \quad (4) \]

This expression shows that the dynamics of the electric field is ruled by the decay of the (first) off-diagonal elements of the field density matrix, determined by the MME of Eq. (2) with $m = n + 1$. The decay of the elements $\rho_{n,n+1}(t)$, while the photon number distribution remains stationary, implies the decay of the cavity field expectation value $\langle E(t) \rangle$ by a phase diffusion process. The physical picture becomes remarkably simple if approximately all elements $\rho_{n,n+1}(t)$ decay exponentially with a similar rate, say $D$. Then,

\[ \langle E(t) \rangle = \langle E(0) \rangle \exp(-Dt) \quad (5) \]

and the micromaser spectrum, i.e., the Fourier transform of $\langle E(t) \rangle$, has a Lorentzian profile with linewidth $D$. Eqs. (4) and (5) are at the heart of the phase diffusion model yielding the micromaser spectrum. In principle, it could be enough to monitor the decay of initial non-vanishing off-diagonal elements $\rho_{n,n+1}(0)$ of the cavity field, created by injecting atoms in a coherent superposition of $|g\rangle$ and $|e\rangle$ or preparing the cavity mode in a coherent state. Unfortunately, this procedure cannot be implemented, as long as we do not have direct experimental access to the cavity field. One of the features that makes the micromaser such an interesting device is that the pumping atoms that are used to build the micromaser field serve also, or could serve, as a quantum probe of the field.

The first step of our scheme is the choice of a suitable initial condition for the cavity field. The micromaser field approaches a stationary state, independent of the initial one, described by a diagonal density matrix with elements $p_{n,n}^{ss} = \rho_{n,n}^{ss}$. If the cavity field is initially prepared in a coherent state $|\alpha\rangle$, such that $\rho_{n,n}^{ss}(0) = p_n^{ss} = |\langle \alpha | \alpha \rangle|^2 \equiv p_n^{ss}$, then, for all practical purposes, the photon statistics remains frozen during the micromaser operation. We choose this initial condition in an effort for singling out the decay of the off-diagonal elements due to phase diffusion from other incoherent processes, allowing to establish a non dissipative decoherence dynamics. This requirement can be satisfied when the steady-state photon statistics $p_n^{ss}$ has a well defined peak, as in the micromaser threshold region, where the maximum amplification of the mean photon number occurs, or to a good approximation in the neighborhood of the alternating trapping state region.

We simulate numerically the system dynamics by a Monte Carlo Wave Function technique \cite{14}, successfully applied to the investigation of other problems in the micromaser \cite{15}. In Fig. 1 we show the field density matrix $\rho_{n,m}(0)$ of the initial coherent state and also the diagonal form of the steady-state at the end of the diffusion process with $\rho_{n,n}^{ss} = \rho_{n,n}^{ss}$, after the progressive vanishing of field coherences. We used typical experimental parameters and an interaction time, $\tau$, such that the micromaser operates near the maximum amplification region.

The expected exponential decay of $\langle E(t) \rangle$, Eq. (6), induced by the decay of the coherences $\rho_{n,n+1}(t)$, Eq. (5), is well reproduced in the numerical simulations as shown in Fig. 2. Consequently, the phase diffusion rate can be easily derived; in this example $D/\gamma \approx 0.049$, in good agreement with approximated analytical expressions \cite{5}. This behaviour, however, can be neither directly observed, nor even inferred from measurements on the populations of outgoing atoms. Actually, from the JC dynamics \cite{13}, exemplified in Eq. (6), the probability $p_e$ that any excited atom, at any moment of the diffusion process, is observed out of the cavity still in the excited state is

\[ p_e = \sum_{n=0}^{\infty} p_n^{ss} \cos^2(g\tau \sqrt{n + 1}). \quad (6) \]

In conclusion, no information on field coherences can be
derived from this kind of atomic population measurements, even if the field coherences are nonvanishing and evolving in time. In order to extract the required information, we suggest a strategy based on the following steps during the diffusion dynamics: i) to interrupt the flux of the pump atoms at any time \( t \) in the transient regime; ii) to apply a "counter-field" \( | - \alpha \rangle \); iii) to interrogate the back-shifted cavity field by a single probe atom.

\[ \rho_n(t) = \rho_{n,n}(t) = \langle n|\hat{D}(-\alpha)\rho(t)\hat{D}(\alpha)|n \rangle = \sum_{i,j=0}^{\infty} \langle i|\hat{D}(-\alpha)|i\rangle\langle j|\hat{D}(\alpha)|n \rangle \rho_{i,j}(t). \]

When \( t = 0 \), the photonstatistics \( \rho_n(0) \) is simply the one associated with the vacuum state, while, for arbitrary \( t \), \( \rho_n(t) \) can be obtained numerically [6]. Now, an excited probe atom is injected in the cavity, and the probability of finding it in the excited state after a transit time \( \tau_p \) is

\[ \tilde{\rho}_e(t, \tau_p) = \sum_{n=0}^{\infty} \tilde{\rho}_n(t) \cos^2(g\tau_p\sqrt{n + 1}). \]

Now we are left with a kind of inverse problem in cavity quantum electrodynamics, namely, to extract from the measured atomic statistics - our *scattering data* - the information on the decay of coherences \( \rho_{n,n+1}(t) \) which is hidden in \( \rho_n(t) \). We indicate two solutions to this problem. The first one exploits the time dependence of the photonstatistics \( \rho_n(t) \). Eq. (8) shows that \( \tilde{\rho}_n(t) \) reflects the time dependence of the unshifted field density matrix, \( \rho_{i,j}(t) \). If the probe atom is injected in the cavity at times \( t \geq D^{-1} \), all coherences \( \rho_{i,i+k}(t) \) for \( k \geq 2 \) have already vanished. Hence, the only contributions to \( \tilde{\rho}_n(t) \) in Eq. (7) are given by the terms with \( j = i \), involving the diagonal elements \( \rho_{i,i}(t) = \rho_i^s \), and the terms with \( j = i \pm 1 \), involving the coherences \( \rho_{i,i+1}(t) = \rho_{i,i+1}(0) \exp(-Dt) \) and their complex conjugates. Hence, for times \( t \geq D^{-1} \), we can rewrite the atomic probability of Eq. (8) as

\[ \tilde{\rho}_e(t, \tau_p) = K_{\tau_p} \exp(-Dt) + \tilde{\rho}_e(\infty, \tau_p) \]

where

\[ K_{\tau_p} = 2Re\sum_{n=0}^{\infty} \sum_{i=0}^{\infty} \rho_{i,i+1}(0)\langle n|\hat{D}(-\alpha)|i\rangle(i + 1|\hat{D}(\alpha)|n) \times \cos^2(g\tau_p\sqrt{n + 1}) \]

and

\[ \tilde{\rho}_e(\infty, \tau_p) = \sum_{n=0}^{\infty} \sum_{i=0}^{\infty} \rho_i^s\langle n|\hat{D}(-\alpha)|i\rangle^2 \cos^2(g\tau_p\sqrt{n + 1}). \]

Eq. (9) shows that \( \tilde{\rho}_e(t, \tau_p) \) as a function of \( t \) is ruled by the phase diffusion rate \( D \), providing a simple and direct link between the experimental data and the micromaser linewidth. Remark that \( K_{\tau_p} \) and \( \tilde{\rho}_e(\infty, \tau_p) \) can be calculated directly from the initial conditions and experimental parameters. In Fig. 3, we show \( \tilde{\rho}_e(t, \tau_p) \) as obtained from the numerical simulations of the whole scheme, using the MME (2), with \( \tau_p = \tau \). In this example, \( \tilde{\rho}_e(t) \) shows a minimum that is followed, for \( t \geq D^{-1} \), by an exponential approach to the asymptotic value \( \tilde{\rho}_e(\infty, \tau) \). Actually, in Fig. 3 we also show that, as predicted by Eq. (8), the behavior of \( \tilde{\rho}_e(t, \tau) \) for times \( t \geq D^{-1} \) is perfectly fitted by an exponential law. This allows us to derive the phase diffusion rate \( D/\gamma \approx 0.047 \), quite close to the value we calculated from Fig. 2.

![FIG. 2. Exponential decay of the expectation value of the cavity field due to phase diffusion Re(\( \tilde{\alpha} \)) vs. dimensionless time \( \gamma t \) from the MME (same parameters as in Fig. 1).](image1)

![FIG. 3. Solid line: probability \( \tilde{\rho}_e \), from Eq. (8), that an excited probe atom is still excited out of the cavity vs. \( \gamma t \). Dashed line: fit from Eq. (8) for \( t > D^{-1} \), with \( K_{\tau_p} = -0.283 \), \( \tilde{\rho}_e(\infty, \tau_p) = 0.551 \), and \( D/\gamma = 0.047 \). Same parameters as in previous figures.](image2)

Another solution to our problem exploits the fact that the interaction time \( \tau_p \) of the probe atom, injected in the final stage of the measurement, may be different (in fact shorter) than the interaction time \( \tau \) of the pump atoms. The crucial point is provided by the expression of the mean photon number of the back-shifted field.

\[ \langle \hat{a}^\dagger \hat{a} \rangle = \sum_{n=0}^{\infty} \rho_{n,n+1} \cos^2(g\tau_p\sqrt{n + 1}) = \sum_{n=0}^{\infty} \rho_i^s \cos^2(g\tau_p\sqrt{n + 1}). \]
\[ \langle \tilde{N}(t) \rangle = \text{Tr}[\hat{a}^\dagger \hat{D}(-\alpha) \rho(t) \hat{D}(\alpha)] = \text{Tr}[(\hat{a}^\dagger - \alpha^\ast)(\hat{a} - \alpha) \rho(t)] = 2|\alpha|^2[1 - \exp(-Dt)]. \]  

Eq. (12) shows that the mean photon number of the back-shifted field grows up exponentially just with the phase diffusion rate \( D \). The velocity of the probe atom should be small enough, i.e., \( 2g\tau_p \sqrt{n + 1} \ll 1 \) for any \( n \) such that \( \tilde{p}_n(t) \) is appreciable throughout the diffusion process. In this case, the probability for the outgoing probe atom to be in the ground state, \( \tilde{p}_g(t, \tau_p) = 1 - \tilde{p}_e(t, \tau_p) \), can be well approximated by

\[ \tilde{p}_g(t, \tau_p) \sim (g\tau_p)^2(\langle \tilde{N}(t) \rangle + 1). \]  

Replacing \( \langle \tilde{N}(t) \rangle \) of Eq. (12) in Eq. (13) provides another simple and direct link between measured atomic populations and the phase diffusion rate. For example, using the same micromaser parameters as in Figs. 1 and 2, the result of Eq. (13) turns out to hold [10] for probe atom interaction times \( \tau_p \) on the range \( 0 < g\tau_p < 0.15 \). This means that the velocity of the probe atom should be greater than the velocity previously chosen for the micromaser operation \( g\tau \sim 0.5 \), so that the atomic statistics exhibit an exponential behavior that will allow to monitor the phase diffusion dynamics. We remark that the criterion based on Eq. (13) holds even in the initial stage of the diffusion process, \( t < D^{-1} \), where our first criterion failed.

We have presented a scheme for the measurement of the micromaser phase diffusion dynamics and its associated decay rate \( D \), that is, the micromaser linewidth. We have shown, following two different strategies, how this dynamics can be obtained from measurable statistics of probe atoms. An experimental implementation of this scheme is currently planned. We expect that the proposed scheme helps to unveil experimentally the distinctive properties of the micromaser spectrum and phase diffusion dynamics, when compared with the conventional Schawlow-Townes laser linewidth, due to the discrete character of the quantized electromagnetic field.

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