DOUBLE SCALING VIOLATIONS

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Abstract

We discuss the theoretical implications of the scaling properties of $F_2^p$ at small $x$ observed in recent HERA experiments. We show that low $Q^2$ data display double scaling violations which are adequately described by NL $\ln Q^2$ corrections. Scaling violations due to summations of leading and subleading $\ln \frac{1}{x}$ beyond NLO in $\ln Q^2$ are however disfavoured by the data.

Presented at the Workshop DIS 96,
Rome, April 1996

To be published in the proceedings
The behaviour of the structure function $F_2^p$ as a function of $x$ and $Q^2$ has been determined \cite{1} by the H1 collaboration to an accuracy and with a kinematic coverage which makes precision tests of perturbative QCD now possible. Whereas the bulk of the data confirm the previously established double asymptotic scaling behaviour \cite{2} predicted by perturbative QCD \cite{3} at the leading log level, a study of scaling violations allows one to determine the perturbative mechanism which drives the evolution of $F_2$ as well as to disentangle it from its input nonperturbative shape: in particular, it is now possible to establish the respective roles of leading logs of $Q^2$ and $\frac{1}{x}$ in the QCD evolution equations in the HERA kinematic region.

The contribution of all logarithmic terms of the form

$$\alpha_p^p (\log Q^2)^q (\log \frac{1}{x})^r.$$  \hspace{1cm} (1)

to the evolution of structure functions can be included by solution of appropriate renormalization group equations. Double asymptotic scaling follows from the symmetric summation of all terms with $p = q = r$. The recent H1 data, rescaled by this prediction, are plotted in fig. 1 versus the product of the two scales $\sigma = \sqrt{\ln \frac{t}{t_0} \ln \frac{x_0}{x}}$, in which $\ln F_2$ should grow linearly with a calculable universal slope, and their ratio $\rho = \sqrt{\ln \frac{x_0}{x} / \ln \frac{t}{t_0}}$, of which $F_2$ should be independent (with $t = \ln \frac{Q^2}{\Lambda^2}$). Whereas the data agree very well asymptotically with double scaling, the observed rise of $F_2$ with $\sigma$ appears to be subasymptotically somewhat smaller (i.e. the rescaled $F_2$ drops); further scaling violations are displayed by data with lower values of $Q^2$.

At moderate values of $Q^2$, logarithmic contributions to the evolution equations with $p > q$ in (1) can be important. It turns out that all contributions with $p = r = q + 1$ vanish, hence the most important correction (corresponding to the inclusion of the most singular contribution in $\frac{1}{x}$ to the NL $\ln Q^2$ terms) is given by summing terms with $p = q + 1 = r + 1$. The predicted asymptotic behaviour is still given by a universal double scaling form, but receives now a NL (but scheme independent) enhancement, suppressed by a factor of $\rho\alpha_s(Q^2)$ \cite{4}. This correction thus leads to a rise of $F_2$ at large $\rho$, and a subasymptotic reduction of around 10% in the slope of the rise with $\sigma$ in the HERA region. Indeed, if the data are rescaled with this

\footnote{The first subasymptotic correction \cite{2} to double scaling is also included in the rescaling factor $R_F$.}
NLO prediction (fig. 2) the predominant double scaling violation apparent in fig. 1, namely the drop in $\sigma$, is removed, and the slope of the rise of $F_2^p$ is now in excellent agreement with QCD [1]. The NLO corrections also have the effect of raising somewhat the optimal value of the starting scale.

The remaining scaling violations displayed in fig. 2 could be simply sub-asymptotic corrections, such as contributions to perturbative evolution with no $\ln \frac{1}{x}$ enhancement (i.e. $r = 0$), or the influence of the boundary conditions, or heavy quark thresholds, which can all be kept under control by a full two loop analysis. If we wish to use $F_2^p$ as measured at HERA to perform precision tests of QCD it is important to establish whether this is the whole story, or whether instead part of the observed double scaling violations may be due to higher order corrections in $\ln \frac{1}{x}$, since the treatment of these is subject to sizable uncertainties related to scheme dependence [5].

The summation of $\ln \frac{1}{x}$ contributions can be achieved to all orders [9] by reorganising the perturbative expansion [6], [7] in such a way that $\ln \frac{1}{x}$ is considered to be leading. It is thus possible to define various expansion schemes, each of which will be more accurate in a different kinematic region. Whereas in the usual large-$x$ expansion only $\ln Q^2$ is leading (i.e. in LO $p = q \geq r$, in NLO $p = q + 1 \geq r$ and so on) an expansion more tilted
towards small $x$ (L-expansion) can be constructed by preserving the large $x$ form of the LO anomalous dimensions, but adding to the large $x$ NLO expressions the higher order leading singularities, i.e. terms with $k$ extra powers of $\alpha_s$ accompanied by $k$ powers of $\ln \frac{1}{x}$. It is also possible to define an expansion [9] (double leading or DL expansion) that treats the two logs on the same footing: in LO each power of $\alpha_s$ is accompanied by either of the two logs, while in NLO an overall extra power of $\alpha_s$ is allowed.

Such expansion schemes will be adequate provided $x$ is small enough. They can then be matched to the large $x$ expansion by imposing continuity of anomalous dimensions and coefficient functions at a reference value $x = x_0$. This introduces an expansion scheme ambiguity which, once the expansion to be used in the small $x$ region has been chosen, is parametrized by $x_0$. [10] If we take for simplicity $x_0$ to be $Q^2$-independent, this simply means that the log which is being summed is actually $\ln \frac{x_0}{x}$ when $x \leq x_0$, so as $x_0 \to 0$ all schemes reduce to the standard loop expansion. It should then be possible to determine $x_0$ by comparing the computed scaling violations to the data.

†The NLO anomalous dimensions in this scheme are known explicitly in the quark sector [6], but in the gluon sector they can be fixed by choosing factorization schemes in which momentum is conserved [8].
Figure 3: The $\chi^2$ of the three parameter fit (166 d. f.) as a function of $x_0$ in MS (a) and DIS (b) schemes. The solid and dashed lines correspond to the DL expansion with standard and $Q_0^{11}$ factorization, and the dotted line to the L expansion.

Because of the importance of leading $\ln \frac{1}{x}$ effects already at one and two loops in the HERA region, one might naively expect the optimal value of $x_0$ to be large enough that $\ln \frac{1}{x}$ effects beyond two loops are already relevant there. Previous data did not allow a determination of $x_0$, essentially because even in the DL scheme the dominant asymptotic behaviour is still given by double scaling. The recent data, however, besides being very precise, extend well in the subasymptotic (low $x$ and low $Q^2$) region and are thus more sensitive to $x_0$.

In order to search for higher order logarithms we perform fits to the $F_2$ data using two free parameters, namely the exponents $\lambda_q$ and $\lambda_g$ which characterize the small-$x$ behaviour $x^{\lambda_i}$ of the input singlet quark and gluon distributions. A full set of NLO fits can then be performed with different values of $x_0$ and in a variety of factorization and small $x$ expansion schemes.

The dependence of the $\chi^2$ of these fits on the value of $x_0$ is displayed in fig. 3. The values of $\lambda_q$, $\lambda_g$ and even $\alpha_s(M_Z)$ are refitted independently at each $x_0$. The quality of the fit always gets monotonically worse as $x_0$ is increased: the data generally favour a very small value of $x_0$. The deterioration of the fit is less pronounced in factorization schemes (such as $Q_0^{11}$ compared to standard factorization) or expansion schemes (such as L compared to DL) which are closer to the conventional large $x$ two loop expansion. We conclude

‡More details of the fitting procedure are given in ref. 12.
that the recent data \cite{1} seem to dislike scaling violations which go beyond standard two-loop effects.

We thus focus on the case $x_0 = 0$ (two loops). The results of the fit as the starting scale is varied, but with $\alpha_s$ kept fixed at its best-fit value $\alpha_s(M_Z) = 0.122$ \cite{2} are shown in fig. 4. The data turn out to require soft boundary conditions, with a somewhat softer gluon distribution, and a moderately rising quark. As the starting scale $Q_0$ is raised, the gluon becomes more singular and eventually leads the quark. The quality of the fit is largely independent of $Q_0$ within a broad range of values. However, it deteriorates rapidly if $Q_0$ is too high, because the assumed power-like behaviour of the input spoils double scaling, or if it is too low, because the perturbatively generated rise becomes too strong. The best-fit prediction (with $Q_0 = 2$ GeV) is compared directly to the data in fig.2.

We can also look for higher twist contributions by repeating the fits with $F_2$ reparametrized as $F_2^{HT} = F_2^{LT}(1 + C_{HT}/Q^2)$. Taking $C_{HT}$ to be $x$ independent its best-fit value turns out to be $C_{HT} = 0.2 \pm 0.2$ GeV$^2$.

In conclusion, our analysis of the 1994 H1 data suggests that this is an ideal place to perform high-precision tests of QCD: due to the smallness of contributions related to higher logs of $\frac{1}{x}$ and higher twists, the usual NLO perturbative expansion is perfectly adequate. The absence of scaling violations related to higher logs of $\frac{1}{x}$, as expressed in the unnaturally small value of $x_0$, is however unexpected, suggesting that our understanding of the way these logs should be summed is as yet incomplete.
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