Modeling of bubble dynamics and heat transfer in polydispersed two-phase turbulent flow in a vertical pipe

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Abstract. The numerical results on the flow, bubble distributions and heat transfer in a bubbly polydisperse upward flow in a pipe are presented. The mathematical model is based on the Eulerian approach with considering the back effects of bubbles on the mean and turbulent characteristics of the carrier fluid phase. The model is coupled with the method of δ-function approximation. The model takes into account the interphase momentum transfer, bubble break-up and coalescence processes. The set of axisymmetrical RANS equations is used for modeling two-phase bubbly flows. Turbulence of the carrier fluid phase is predicted using the Reynolds stress transport model. The effect of variation in the gas volumetric flow rate ratio, inlet mean fluid temperature, and its velocity on the flow structure and heat transfer in the two-phase flow is analyzed. The addition of air bubbles results in a significant increase in the heat transfer rate (up to three times) and the effect augments by increasing the gas volumetric flow rate ratio. The numerical simulations show a good agreement with the experimental and numerical results of other authors.

1. Introduction
Turbulent bubbly flows are widely used in the chemical, pharmaceutical industries, in power and nuclear engineering and etc. Information about the structure, mean, fluctuational characteristics, heat transfer and bubble dispersion over the cross section of the channel (pipe) is necessary both from fundamental and practical points of view [1, 2]. The distribution of dispersed phase across pipe radius has a great influence on the intensity of turbulence of the carrier fluid phase and heat transfer. The gas bubble diameter and local void fraction play a key role in understanding the physical mechanisms affecting the distribution of gas bubbles over the pipe cross-section.

This work is a further development of the Eulerian approach of the authors [3] on the simulation of heat transfer in bubbly polydispersed turbulent flows. The main difference of this work from paper [3] is that evolution of the spectrum of bubble sizes was calculated using the δ–functions method [4] taking into account the effects of gas bubble break-up and coalescence. The model [5] was used for describing the evolution of bubble sizes in paper [3]. The applicability of this method is limited to the equilibrium between bubble break-up and coalescence.

In this paper, the mathematical model for describing flow and heat transfer in the polydispersed gas-liquid turbulent flow with bubble break-up and coalescence is presented. The author predicted results are compared with experimental [6–8] and numerical [4] results.
2. Mathematical model

2.1. Governing equations

The numerical model is based on the Eulerian approach. The fluid phase is treated as a continuum while the gas phase (bubbles) is considered as the dispersed phase. In the two-fluid approach, both phases are considered as interacting continua. This technique involves the solution of a second set of Navier–Stokes-like equations in addition to those of the carrier (fluid) phase. In order to account for the interaction between the phases, that is, momentum transfer and heat and mass transfer, the conservation equations have to be extended by appropriate source/sink terms. The Eulerian approach is based on kinetic equations for a one-point PDF of bubbles coordinates, velocity, and temperature in the turbulent Gaussian fluid flow fields. Properties such as the mass of particles per unit volume are considered as a continuous property and the particle velocity is the averaged velocity over an average control volume. The two-fluid model consists of the set of equations for each phase, including: equation for continuity, two equations for momentum conservation and energy equation. A set of axisymmetric RANS equations is used for the fluid phase. The low-Reynolds-number elliptic blending Reynolds stress model (RSM) by [9] is used in the work. The RSM predicts the turbulent Reynolds stresses directly from partial differential equations and allows us to compute the anisotropic flow. It is modified for the presence of bubbles by the model [10]. To describe mass and momentum transfer between gaseous and carrier phases, the interfacial transfer terms appear in each equation to couple the different phasic effects. It is assumed that there is no a phase change (vapor formation) on the wall surface.

The bubble-size distribution is the crucial point for simulation of bubbly polydispersed flows. In this paper, only coalescence and breakup processes have been taken. They are the main mechanisms that lead to polydispersed bubble-size distributions. It is assumed that the bubbles hold the spherical shape. The basic mechanism of coalescence is associated with bubble collisions due to their entrainment into the turbulent motion of fluid [11]. It is assumed that only two bubbles collide, which occurs most often in reality. Coalescence consists of three processes: (1) collision of bubbles; (2) formation of a thin liquid film between two colliding bubbles, which traps and drains gradually and (3) process, when liquid film between two bubbles reaches a critical thickness, ruptures and forms one large bubble. Bubble break-up occurs due to their interaction with turbulent eddies, as shown in [11]. The rate of bubble break-up is determined by the interphase forces, which result in deformation and break-up of the bubble. The model of [11] is employed in the paper for modeling the bubble-eddy collision rate.

The equations for the numerical concentration and mass fraction of bubbles (the number of particles per unit volume) are written taking into account the approach [4]. A polydisperse ensemble of bubbles is modeled by a set of monodisperse groups (modes or fractions) in the framework of the δ–approximation, where the continuous density distribution function of mass distribution of the dispersed phase is approximated by the sum of δ–functions [4].

2.2. Numerical implementation

The mean transport equations for both gas and dispersed phases and RSM model are solved using the control volumes method on a staggered grid. The QUICK scheme is used to approximate the convective terms, and the second-order accurate central difference scheme is adopted for the diffusion terms. The velocity correction is used to satisfy continuity through the SIMPLEC algorithm, which couples velocity and pressure.

The results of preliminary calculations for the single-phase flow in a pipe with a length of 150R are used for the gas-phase velocity and turbulence on the pipe edge. These conditions are sufficient to achieve fully developed turbulent gas flow. The symmetry conditions are set on the pipe axis for gas and dispersed phases. No-slip conditions are set on the wall surface for the carrier phase. At the outlet edge, the computational domain condition $\partial \varphi / \partial r = 0$ is set for all variables.
3. The model validation for polydisperse bubbly flow

Validation analysis is divided into two successive parts. In the first part validation based on simulation of polydisperse bubbly flow for measurements conditions of [6] is presented. The second one concerns the comparison with experimental results of [6]. Validations are performed in a pipe with the diameter of 50 mm with taking into account bubble break-up and coalescence process. It should be noted that all experimental regimes [6] are characterized by the position of the peak of the local void fraction in the near-wall zone. The mean bubble diameter in this work is not larger than 3.6 mm.

3.1. Comparison with Hibiki et al. (2001) measurements

The measured results of [6] performed for the upward pipe flow regime are used for the comparison analysis. The measurements were carried out in an acrylic pipe with an inner diameter $2R = 50.8$ mm and length $L = 3061$ mm at distance $x/(2R) = 6, 30.3$ and $53.5$ from the cross section for introducing air bubbles. The temperature of fluid in the experiment was kept constant and equaled $20 \pm 0.2^\circ C$. Measurements of the fluid velocity were carried out using a hot-wire anemometer, and local gas void fraction and bubble size using a special dual sensor. The initial size of the bubbles has been chosen on the basis of estimates of [6] and was constant at the pipe inlet. Further, the bubble diameter is changed both along the length and radius of the pipe due to the break-up and coalescence processes.

The profiles of the mean axial velocity of the carrier fluid phase over the pipe cross section with variations in the gas volumetric flow rate ratios are shown in Figure 1a. Note that the simulations start with distance $x/(2R) = 6$, where the experimental profiles of the mean diameter of bubbles and distribution of their void fraction over the pipe radius are presented. The symbols are the measurement results of [6], the lines are the author predictions. Line 1 in Figure 1a is the simulation for the fluid (water) single-phase flow with other identical parameters. An increase in the gas volumetric flow rate ratio causes an increase in the velocity of fluid over the entire cross section of the pipe, and this effect increases with increasing gas concentration. This is typical both of experiments of [6] and our simulations. The reason is probably the following: an increase in the number of bubbles leads to the growth of their mean diameter. The large gas bubbles carry the fluid phase (liquid) more strongly and the area of the pipe cross-section for the carrier phase decreases also. These two reasons lead to an increase in the mean velocity of the carrier fluid.

The radial profiles of the local void fraction with variations in the gas volumetric flow rate ratio are shown in Figure 1b. The profile of air bubble void fraction has almost uniform distribution in the region of the turbulent flow core and pronounced maximum near the wall. Small bubbles move toward the pipe wall due to the action of lift (Saffman) force, and therefore the local void fraction profile has a pronounced maximum in this area. This conclusion is consistent with recent simulations of [4], where similar numerical results were obtained using other models of break-up and coalescence.

The radial profiles of the Sauter mean diameter of bubbles at distance $x/(2R) = 53.5$ are shown in Figure 3. The dashed lines are the gas bubble diameter predicted using the model of [4, 6]. The mean diameter of bubbles at the inlet to the computational domain is $d_i = 2.4, 2.5, 2.8$ mm according to the value of the fluid phase superficial velocity $J_b = 0.0473, 0.113, 0.242$ m/s, respectively. Further, it changes due to break-up and coalescence both in axial and radial coordinates of the pipe. The growth in gas volumetric flow rate ratios lead to the increase in the size of bubbles over almost all pipe cross section.

The bubble diameter is slightly higher than that in the flow core zone and it is typical both of measurements [6] and our numerical simulations in the near-wall region. This is due to the fact that small bubbles migrate towards the wall due to the action of lift force and they accumulate in the near-wall pipe region. The probability of their coalescence increases in this zone. Our simulations underpredict the bubble diameter in comparison with experimental data of [6] at $x/(2R) = 53.5$. In this case, the balance between break-up and coalescence shifts towards air bubble merging. This is probably due to the fact that the rate of coalescence is proportional to the square of bubbles concentration, and break-up in the first degree is proportional to bubble concentration [4]. This causes an increase in the diameter of gas bubbles.
Figure 1. Radial profiles of axial mean velocity of fluid phase (a) and local void fraction (b) for various gas volumetric flow rate ratios. Symbols are measurements of [6], lines are author predictions. $2R = 50.8$ mm, $Re = 4.9 \cdot 10^4$, $J = 0.986$ m/s, $x/(2R) = 53.5$, $T = T_b = 293$ K. 1 – $J_b = 0$ m/s (single-phase fluid flow $\beta = 0$); 2 – $J_b = 0.0473$ m/s, $\beta = 4.6\%$, $d_1 = 2.4$ mm; 3 – 0.113 m/s, $10.3\%$, 2.5 mm; 4 – 0.242 m/s, $19.7\%$, 2.8 mm.

3.2. Validation by Lucas et al. (2005) experiments

The experiments of [6] were performed in the case when the maximal value of local void fraction is located in the near-wall region of the pipe for all investigated gas volumetric flow rate ratios $\beta$.

In order to carry out the analysis and verification of the model, comparisons are performed with measurement data [7] and computations [4] for the local void fraction profiles over the pipe section (see Figure 2) and bubble diameter distributions (Figure 3).

A vertical pipe was 4 m high with an I.D. of 51.2 mm and upward flow direction. The air bubbles were injected through 19 capillaries uniformly distributed over the pipe cross-section. A grid sensor was used to measure the radial distributions of bubble local void fraction and their size. The additional analysis for the bubbly flow with near-wall peak of local void fraction (MTLOOP074) has been performed. Two other experiments are characterized by the fact that one part of bubbles is located in the axial zone of the pipe, and another one is near the pipe wall (MTLOOP071) and the void fraction peak is observed on the pipe axis (MTLOOP107). The measurements of MTLOOP071 and MTLOOP107 were carried out for almost the same gas volumetric flow rate ratios ($\beta = 12.1$–12.6\%) and significantly different liquid superficial velocities $J_L = 0.255$ and 1.017 m/s and Reynolds numbers, respectively.

The radial profiles of local void fraction distributions for the conditions of [7] (symbols 1) and predictions of [4] (lines 2 and 3) are shown in Figure 2. All simulations were carried out for 4 $\delta$-functions, similarly to [4]. The initial distribution (5) of bubble sizes, numerical and mass concentration of bubbles were also set on the basis of data of [4]. Predictions [4] (red 6 and blue 7 columns) were performed using two different models of coalescence and break-up. It is seen that the mathematical model of the polydisperse bubbly flow developed by the authors of the paper predicts correctly the distributions of gas bubbles for the experimental conditions MTLOOP074 over the pipe radius (see Figure 2a). As shown below in terms of the MTLOOP074 conditions, the mean size of bubbles is smaller than for the MTLOOP107 conditions (see Figure 3). Therefore, as mentioned above, small bubbles migrate to the wall due to the action of lift force. Similar results were obtained in [4] for one of two models of coalescence and break-up.
Figure 2. The radial profiles of local void fraction in bubbly upward flow in the pipe. Symbols (1) are the experiments of MTLOOP071 (a) and MTLOOP107 (b), lines are numerical simulations: 2 and 3 are predictions of [4] using two different break-up and coalescence models, 4 is author’s simulations. $2R = 51.2$ mm, $Re = 4.5 \times 10^4$, $T = T_b = 303$ K, $X = 3$ m.
(a): $J_L = 0.255$ m/s, $J_b = 0.0368$ m/s, $\beta = 12.6\%$; (b): $1.017$ m/s, $0.14$ m/s, $\beta = 12.1\%$.

Figure 3. The bubble size distributions in bubbly upward flow in the pipe experiments for experimental conditions of [7] MTLOOP074 (a) and MTLOOP107 (b). (a): $J_L = 1.017$ m/s, $J_b = 0.0368$ m/s, $\beta = 3.5\%$; (b): $1.017$ m/s, $0.14$ m/s, $\beta = 12.1\%$. 1–4 are four modes of bubble size, 5 are inlet distributions of bubble diameter for four $\delta$-functions, 6 and 7 are simulations of [4], 8 are author’s simulations, 9 are measurements of [7].

3.3. Comparison with Ganchev and Peresad’ko (1985) measurements
The results of a comparative analysis of the case of the downward bubbly flow with interfacial heat transfer are presented in Figure 4. The experiments of [8] and numerical results of [13] are used for analysis, where $h_0$ and $C_f0$ are the heat transfer and wall friction coefficients in a single-phase fluid (water) flow under other identical conditions. It should be noted that the measurements of [8] were performed at heat flux density $q_w = \text{const}$, but the value of the heat flux supplied to the wall surface was not written. It has been found in our previous paper [13] that the best agreement was obtained at $q_w = 18$ kW/m$^2$. The dashed line shows the results of numerical simulations of [13] for the monodisperse flow regime. There is a good agreement between the results of simulations and measurements on the magnitude of the parameter of heat transfer enhancement ratio.
Figure 4. Heat transfer enhancement ratio in bubbly downward flow vs wall friction modification ratio. \( \beta = 10\% \), \( d = 3.5 \) mm. Symbols 1 are measurements of [8], lines are predictions: 2 – numerical results of [13], 3 – author simulations.

Conclusion
The mathematical model is based on the use of Eulerian approach, considering the back effect of bubbles on the mean characteristics and turbulence of the carrier phase. Bubble dynamics is described taking into account the changes in the gas bubble density, break-up and coalescence processes. Turbulence of the carrier fluid phase is predicted using the model of Reynolds stress transport. The presence of bubbles in the carrier fluid flow leads to the additional turbulence production by vortex formation after the bubbles.

It is shown that the developed model allows adequate prediction of the parameters of two-phase flow and distributions of bubbles and local void fraction across the pipe radius in a turbulent bubble and descending flow in the range of flow rate ratios \( \beta = 3.5–19.7\% \).

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