Fermi polarons in a driven-dissipative background medium

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The study of the polaron of an open quantum system plays an important role in verifying the effectiveness of an approximate many-body theory and predicting novel quantum phenomena in open quantum systems. In a pioneering work, Piazza et al. (2021) proposed a Fermi-polaron scheme with a lossy impurity, which exhibits a novel long-lived attractive polaron branch in the quantum Zeno limit. However, we have encountered a counterpart problem in which an impurity interacting with an open quantum bath scatters exciting polarons, which is what we focus on. In this work, we determine through analytical research the molecular state under the two limits of vanishingly small and infinitely large dissipation intensities as well as the reason why the dissipation range leads to a decrease in the gap between the molecular state and the molecule-hole continuum in the former case. The spectrum functions of molecular and polaron states with different dissipation ranges and loss rates are investigated. We find the spectral signals of molecular and polaron states will first diffuse and then revive as the dissipation increases. Moreover, we show that the attractive and repulsive polarons respond differently to an increasing dissipation range in our model. Finally, we exhibit the polaron energy, residue, effective mass, and two-body decay for mass-balanced and imbalanced systems. Our results might be useful for cold-atom experiments on open quantum systems.

quantum Zeno dynamics, open systems, quantum statistical methods, decoherence

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1 Introduction

Ever since the 1930s, when the concept of the polaron was proposed as quasiparticles arising from an electron-phonon interaction [1], the mechanism of a mobile impurity interacting with its environment has gained widespread interest. It plays a key role in understanding the low-energy behavior of complex systems, e.g., cuprate superconductors [2], colossal magnetoresistive manganites [3], \(^{3}\)He-\(^{4}\)He mixtures [4], and promising organic semiconductors [5].

Because of its excellent flexibility, the ultracold atomic system has attracted extensive attention in recent years [6, 7]. In particular, research progress on the polaron continues in our times, both experimentally [8-18] and theoretically [19-45]. However, these investigations are restricted to isolated systems. With the development of quantum physics, open quantum systems have become increasingly significant. The nonnegligible coupling of some specific systems with the environment and the decoherence and difficulty of calculating a large, isolated system and removing overwhelming useless information to obtain a specific degree of freedom are two reasons why the theory of the open quantum system plays a crucial role.

Furthermore, dissipation has been proven to be pivotal...
in producing novel quantum phenomena. Many theoretical and experimental breakthroughs have been made. For instance, dissipation can induce a strong correlation in ultra-cold bosonic atoms [46] and control the phase transition from the Mott insulator to the superfluid [47], slow down the relaxation of many-body states to an algebraic law [48], engineer tunable local loss in a synthetic lattice of the momentum state [49], and cause $PT$-symmetry-breaking transitions that create a $PT$-symmetric broken phase that is beyond the quantum Zeno effect [50]. From another perspective, as an effective theory for describing the system undergoing dissipation, the non-Hermitian Hamiltonian approach has aroused great interest in recent years [51], inspiring research regarding skin effects, dynamics, band theory, topological phase-transitions, non-Hermitian linear response theory, non-Hermitian semimetals, dissipation-facilitated molecules, etc.

Nevertheless, to the best of our knowledge, studies on the dissipative polaron [52-54] remain rare. On the one hand, benefiting from simplicity, polaron physics is suitable for checking the consistency of many-body theory and experimental results. As a pioneering work, Piazza’s group [54] considered a polaron scheme in which the impurity atom bears driving and dissipation that do not come from the bath but from other environmental degrees, where they found a quantum Zeno effect in the impurity spectrum function. On the other hand, another important mission of a polaron is to act as a probe of its bath, e.g., a complex open quantum system of concern. On the basis of the importance of many-body theory for an open quantum system and the thirst for knowledge of open quantum systems, which is difficult to calculate, new polaron studies in which the bath is dissipated and driven by its own environment have merits.

In this study, we study a new polaron system in which the bath is Fermi gases suffering external driving and dissipation. To consider the effect of the quantum jump term of the Lindblad master equation (LMEQ), we begin the study with the total action function of the impurity and background gases. With the nonequilibrium Green’s function (GF) method, we present the spectral responses of molecular and polaron states under different system configurations. We also calculate quasiparticle parameters in the mass-balanced and -imbalance polaron setups, which can be realized experimentally. Two analytic results are obtained. First, for a vanishingly small dissipation, we explain why the gap between the molecular state and the molecule-hole continuum decreases with increasing dissipation range. Second, for infinite dissipation, we obtain the dispersion of the bound state as well as the threshold of the molecule-hole continuum. We find that the resonant peaks of molecular and polaron states become diffuse only under moderate dissipation. In contrast, they become well-defined with infinite dissipation. In other words, under this limit, the dissipation range only plays a role in reconstructing the Fermi surface. This mechanism is consistent with the Zeno effect described in refs. [54, 55]. The residue, effective mass, and two-body decay show nonmonotonic character due to the interplay between the intrinsic energy scale of the system and the measurement frequency from the environment. However, they all tend to their respective saturation values under the Zeno limit.

This paper is organized as follows. In sect. 2, we introduce our model and the Keldysh diagrammatic approach to our dissipative polaron problem. In sect. 3, we discuss the numerical results of the molecular and polaron states. In sect. 4, we draw a conclusion and remark on the outlook for this research topic.

2 Model and analytical approach

2.1 Model Hamiltonian and LMEQ

Let us consider a system composed of a fermionic impurity interacting with a two-dimensional, open Fermi bath. The Hamiltonian is expressed as:

$$\mathcal{H} = \mathcal{H}_{\text{imp}} + \mathcal{H}_{\text{bath}} + \mathcal{H}_{\text{int}},$$

(1)

where $\mathcal{H}_{\text{imp}} = \sum_k \epsilon_k c_k^\dagger c_k^\dagger$ and $\mathcal{H}_{\text{bath}} = \sum_j \epsilon_j f_j^\dagger f_j$ are the kinetic Hamiltonians of the impurity and the bath, respectively, the dispersions of which are $\epsilon_k = k^2/2m$, and $\epsilon_j = k^2/2m_j$, respectively.

$$\mathcal{H}_{\text{int}} = g \int \mathcal{d}^{d}r c^\dagger(r)c(r)f^\dagger(r)/f(r)$$

is the $s$-wave contact interaction between the impurity and the bath. The dynamics of the bath can be described by an LMEQ

$$\partial_t \rho_{\text{bath}} = -i[H_{\text{bath}}, \rho_{\text{bath}}(t)] + \mathcal{L}_d \rho_{\text{bath}}(t),$$

(2)

where $\mathcal{L}_d$ is the dissipative Lindblad operator, whose effect on the density operator is $\mathcal{L}_d \rho_{\text{bath}} = \sum_k \gamma(k) D[k] + \Omega(k) P[k] \rho_{\text{bath}}$, describing the loss and recombination of particles at a rate of $\gamma(k)$ and $\Omega(k)$, respectively. The two incoherent processes are induced by the superoperators $D[k] \rho_{\text{bath}} \equiv f_k \rho_{\text{bath}} f_k^\dagger - \frac{1}{2}(f_k^\dagger, \rho_{\text{bath}})$ and $P[k] \equiv -D[k] + D[k]$, where we have chosen a specific pumping behavior for simplicity. The action of $D[k]$ operating on the density matrix $\rho_{\text{bath}}$ results in two processes: the continuous nonunitary evolution coming from the anticommutator term and the quantum jump produced by $f_k \rho_{\text{bath}} f_k^\dagger$. The former process describes the continuous loss of energy and information in the process of decoherence with the environment, and the later process represents the continuous measurement of the system.
In our model, there is no interaction between the bath atoms, so the matrix representing the density operator is block diagonal in the bases of the Fock states for each momentum. Furthermore, the bath should remain in a steady state where the loss and pump are already in equilibrium; therefore, the matrix elements of the density matrix are time-independent and can be determined through the LMEQ as 
\[ \rho_k^{0i} = 1 - \Omega(k)/\gamma(k), \quad \rho_k^{ii} = 0, \quad \text{and} \quad \rho_k^{ij} = \Omega(k)/\gamma(k). \]
In addition, the underlying probability annotation of the density operator requires the ratio \( \Omega(k)/\gamma(k) \) to be confined in the range of \([0, 1]\). For each momentum, the average particle number is determined by this ratio, the momentum distribution of which further establishes the average total particle number.

### 2.2 T-matrix method with the nonequilibrium GF

There are two accepted ways to address a polaron system, the variational ansatz method and the many-body T-matrix method, which were proven to be equivalent by Chevy’s group \cite{21} in 2007. The former method is intuitive and has been extended to the dynamic version \cite{42,56}. Meanwhile, the latter method is slightly more complicated. We need to perturbatively calculate the self-energy and then obtain the spectrum function as well as quasiparticle parameters. However, the T-matrix approach can be conveniently constructed by the nonequilibrium GF to provide a complete description of the open quantum system considering all the effects of the LMEQ without ignoring the quantum jump term \cite{57}, which is the method we adopt in this work.

We start from the full partition function of the system,
\[
\mathcal{Z} = \int \mathcal{D}[e, c, \bar{c}] e^{iS_{\text{imp}} + iS_{\text{int}} + S_{\text{int}}},
\]
where \( C \) is the Keldysh integral contour. Performing the Fermi-Keldysh rotation, the bare impurity action reads
\[
S_{\text{imp}} = \sum_k F_{\mu}(k)G_{0_c}^{-1}\phi_k(k)c_k(k),
\]
where \( \mu, \nu = 1, 2 \) are Keldysh indices, and the Einstein sum convention of repeated indices is contracted here and elsewhere. The matrix of \( G_{0_c}^{-1} \) is upper triangular, so we can straightforwardly read out the bare retarded and advanced GPs,
\[
G_{0_c}^{R/A}(k, \omega) = [\omega - E_c(k) \pm i0^+]^{-1}.
\]
According to the fluctuation-dissipation relation for steady states, the bare Keldysh GF can always be expressed with the corresponding retarded and advanced GPs,
\[
G_{0_c}^{R/A}(k, \omega) = F_{c}^{R/A}(\omega)[G_{0_c}^{R/A}(k, \omega) - G_{0_c}^{A/R}(k, \omega)].
\]
In the impurity limit, where the density of impurity atoms is vanishingly small, we have \( F_{c}^{R/A} = 1 \).

The GFs for the bath are somewhat complicated. The retarded, advanced and Keldysh GFs should be obtained simultaneously by reverting the matrix of \( G_{0_c}^{-1} \), which can be extracted from the dissipative bath action with a Keldysh rotation.

\[
G_{0,f}^{R/A}(k, \omega) = \left[ \omega - E_c(k) + i\frac{\gamma(k)}{2} \right]^{-1},
\]

\[
G_{0,f}^{R}(k, \omega) = F_{0,f}(k, \omega)[G_{0,f}^{R}(k, \omega) - G_{0,f}^{A}(k, \omega)],
\]

where the distribution function \( F_{0,f}(k, \omega) = 1 - 2\eta(k) \), and \( \eta(k) = \Omega(k)/\gamma(k) \) is the average particle number for momentum \( k \).

Next, we introduce an auxiliary molecular field \( \Delta \) to decouple the four-operator interaction term in the total action. By performing a Hubbard-Stratonovich transformation, the interaction part of the action becomes
\[
S_{\Delta,c} = \int d\mathcal{C}(-\Delta c - \bar{c}f\Delta),
\]
and further changes into
\[
S_{\Delta,c} = -\int d\mathcal{C}(1/\sqrt{2}(f_{\mu}\Delta \gamma^{\mu}_{\nu}\gamma^{\nu}_{\mu}\sigma_{\nu}c_{\mu} + h.c.),
\]

with a successive Keldysh rotation to achieve a basis transformation, where \( \mu, \nu, \eta = 1, 2, a = q, cl \) are fermion and boson Keldysh indices, respectively. The elements of \( \gamma^a \) are \( \gamma^I = I \) and \( \gamma^a = \sigma^a \), where \( I \) is the identity matrix and \( \sigma^a \) is the \( a \) component of Pauli matrices. To calculate the self-energy, we integrate out the fermionic bath degrees of freedom and obtain the induced interaction action for the molecule and impurity,
\[
S_{\Delta,c} = -\frac{1}{2} \int d\mathcal{C}(x) \gamma^{\mu}_{\nu}\sigma_{\mu}\gamma^{\nu}_{\mu}(x)G_{0_c}^{R}(x, x')
\]
\times \bar{c}_{\nu}(x')\sigma_{\nu}\gamma^{\nu}_{\mu}(x'c_{\mu}(x'), \eta).
\]

Under the framework of the non-self-consistent T-matrix method \cite{54}, accurate to first order, the real space self-energies are solved as:
\[
\Sigma^{ab}_{\Delta}(x, x') = \frac{i}{2} \gamma^{\mu}_{\nu}\sigma_{\mu}\gamma^{\nu}_{\mu}(x)G_{0_c}^{R}(x, x')\gamma^{\nu}_{\mu}(x'c_{\mu}(x'), \eta).
\]

for the molecule and
\[
\Sigma^{ab}_{c}(x, x') = \frac{i}{2} \sigma_{\nu}\gamma^{\nu}_{\mu}(x)G_{0_c}^{R}(x, x')\gamma^{\nu}_{\mu}(x)\sigma_{\nu}c_{\mu}(x'), \eta.
\]

for the impurity. The self-energy of the molecule and impurity can be expressed by the corresponding retarded, advanced, and Keldysh GFs, respectively.
2.3 The retarded self-energy and spectrum function

To obtain the spectrum functions and quasiparticle parameters, we need to obtain the GFs of the molecule and impurity, which is essential to obtain the corresponding retarded self-energy. In the steady state, the system is time- and space-translational invariant, so all the real space GFs and self-energy can be written out by their Fourier transforms for the relative coordinates, which is more convenient for discussing the quasiparticle properties. Setting $a = q$ and $b = cl$ in eq. (12) and taking the Fourier transform, we obtain the momentum-space retarded self-energy of the molecule state,

$$\Sigma^R_{\Delta}(q) = \frac{i}{2V} \sum_{p_1} \left[ G^R_{0\Delta}(q - p_1) G^A_{0\Delta}(p_1) + G^R_{0\Delta}(q - p_1) G^A_{0\Delta}(p_1) \right].$$

Performing the contour integral for frequency, the retarded self-energy of the molecule becomes

$$\Sigma^R_{\Delta}(q) = \frac{1}{V} \sum_{p_1} \frac{1}{\omega - \epsilon_\Delta(p_1) - \epsilon_\Delta(q - p_1) + i\gamma_\Delta},$$

and the retarded GF of the molecule is written as:

$$G^R_{0\Delta}(q, \omega) = \frac{1}{\frac{1}{\epsilon_\Delta} - \Sigma^R_{\Delta}(q, \omega) + i0^+}.$$  \hspace{1cm} (15)

Following a similar procedure, the retarded self-energy of the impurity is expressed as:

$$\Sigma^R_{\Delta}(k) = -\frac{1}{2V} \sum_{p_1} G^R_{\Delta}(k, p_1) G^A_{0\Delta}(p_1) + G^R_{\Delta}(k, p_1) G^A_{0\Delta}(p_1)$$

$$= \frac{1}{V} \sum_{p_1} \frac{1}{\omega - \epsilon_\Delta(p_1)} G^A_{\Delta}(p_1 + k, \epsilon_\Delta(p_1) + \omega),$$

where $k_\Delta = p_1 + k$. Then, the retarded GF of the impurity is given by

$$G^R_{\Delta}(k) = \frac{1}{\omega - \epsilon_\Delta - \Sigma^R_{\Delta}(k) + i0^+}. \hspace{1cm} (18)$$

The spectrum functions of the molecule and impurity are written as:

$$A^R_{\Delta}(q) = -\frac{1}{\pi} \text{Im}[G^R_{\Delta}(q)], \hspace{1cm} (19)$$

$$A^R_{\Delta}(k) = -\frac{1}{\pi} \text{Im}[G^R_{\Delta}(k)]. \hspace{1cm} (20)$$

The pole of the retarded GF of the impurity determines the polaron energy, that is

$$E_p(k) = \epsilon_\Delta + \text{Re} \Sigma^R_{\Delta}(k, E_p).$$

For our system, $E_p$ always increases with $|k|$, so only the solutions at zero momentum are considered and the equation above develops into $E_p = \text{Re} \Sigma^R_{\Delta}(0, E_p)$. According to Landau-Fermi liquid theory, a polaron state behaves like a quasiparticle with collective energy shift $E_p$, residue $Z$, effective mass $m^*_p$, and two-body decay rate $\gamma_+$, so the asymptotic behavior of the GF in the zero momentum limit becomes

$$G^R_{\Delta}(k, \omega) = \frac{Z}{\omega - \frac{\omega}{2m^*_p} - E_p + i\gamma_+}.$$  \hspace{1cm} (22)

Expanding the retarded self-energy in eq. (18) to the first order at the pole and comparing the GF with that in eq. (22), we have

$$Z = \frac{1}{1 - \frac{\text{Re} \Sigma^R_{\Delta}(0, \omega)}{\text{Im} \Sigma^R_{\Delta}(0, \omega)}}, \hspace{1cm} (23)$$

$$m^*_p = \frac{1 - \frac{\text{Re} \Sigma^R_{\Delta}(0, \omega)}{\text{Im} \Sigma^R_{\Delta}(0, \omega)}}{\text{Im} \Sigma^R_{\Delta}(0, \omega)}, \hspace{1cm} (24)$$

$$\gamma_+ = -2 \text{Im} \Sigma^R_{\Delta}(0, E_p). \hspace{1cm} (25)$$

For two-dimensional Fermi gases, the bare interaction strength above $g$ should be renormalized by the two-body binding energy $E_B$ as:

$$\frac{1}{g} = -\sum_k \frac{1}{E_B + \epsilon_k}, \hspace{1cm} (26)$$

where $\epsilon_k = \frac{\epsilon^2}{2mc}$, and $m_r = \frac{m \cdot m_f}{m \cdot m_f}$ is the reduced mass.

3 Numerical results and discussion

3.1 Dressed molecule state

The molecular state plays a vital role in the scenario of the interaction between the impurity and the bath, not only because introducing an auxiliary field can easily decouple the interaction potential but also because its retarded GF is simply the $T$-matrix characterizing the moving impurity scattered by the bath fermions. From the formula derived above, we could conclude that all the GFs depend on $\gamma(k)$ and the ratio of $\eta(k) = \Omega(k)/\gamma(k)$. For simplicity, we limit the driving and dissipation to the same range depicted by a cutoff momentum $k_\Delta$, within which $\gamma(k) = \gamma_0$ and $\eta(k) = \eta$ are nonzero constants and zero elsewhere. In addition, unless otherwise specified, we set $m_c = m_f = 1/2$ and $k_F = 1$ in the calculations below.

We start by analyzing the parameter regime where the dissipation is vanishingly small. In this condition, the self-energy of the molecular field can be separated into two parts as $\Sigma^R_{\Delta} = \Sigma^R_{\Delta 1} + \Sigma^R_{\Delta 2}$, where the first part is $\Sigma^R_{\Delta 1} = 1/V \sum_{p_1} 1/[(\omega - \epsilon_\Delta(p_1) - \epsilon_\Delta(q - p_1) + i0^+)]$, and the second part is $\Sigma^R_{\Delta 2} = -\eta/V \sum_{p_1<k} 1/[(\omega - \epsilon_\Delta(p_1) - \epsilon_\Delta(q - p_1) + i0^+)]$. Here, we only focus on the molecular bound state, so the energy must be less than the kinetic energy of the center of mass of
the molecule, that is, \( \omega < \min_{p_1} |\varepsilon_f(q/2 + p_1) + \varepsilon_c(q/2 - p_1)| = q^2/2M \), where \( M = m_c + m_f \). In this case, the two integrands in \( \Sigma_{\Delta}^{R,1} \) and \( \Sigma_{\Delta}^{R,2} \) do not have imaginary parts. Setting the upper limit of the integral of \( p_1 \) to \( \Lambda \) in \( \Sigma_{\Delta}^{R,1} \), we conclude that

\[
\Sigma_{\Delta}^{R,1} = -\frac{1}{8\pi} \ln \left| \frac{\tilde{\mu} - \tilde{\omega} + u_q}{u_q - \tilde{\omega}} \right|, \tag{27}
\]

where we have used the notations \( \tilde{\mu} = k_r^2, x = p_i^2/\mu \), \( \tilde{\omega} = \omega/(2\tilde{\mu}) \), \( \omega_s = q^2/2M \), and \( u_q = \omega_s/(2\tilde{\mu}) \) to simplify the expression above. Meanwhile, \( \Sigma_{\Delta}^{R,2} \) evolves into

\[
\Sigma_{\Delta}^{R,2} = \int_0^1 dx \int_0^{2\pi} \frac{\eta/(16\pi^2)}{x + 2u_q - \tilde{\omega} - 2\sqrt{u_q \cos \theta}}. \tag{28}
\]

Using the integral identity

\[
\int_0^{2\pi} \frac{\theta}{\sin \theta} = \frac{\pi \text{sign}(\Theta)(a^2 - b^2)}{\sqrt{a^2 - b^2}}, \tag{29}
\]

we can determine the second part of the retarded self-energy of the molecular field and then obtain the inverse of the retarded GF of the molecular field.

\[
G_{\Delta}^{R,-1}(\mathbf{q}, \omega) = \frac{1}{g} - \Sigma_{\Delta}^{R}(\mathbf{q}, \omega)
= \frac{1}{8\pi} \ln \left| \frac{E_B/(2\tilde{\mu})}{u_q - \tilde{\omega}} \right|.
\]

Solving the pole of \( G_{\Delta}^{R}(\mathbf{q}, \omega) \), we can obtain the molecular bound state shown as the dashed black lines in Figure 1(a)-(d), which agree with the corresponding numerical results obtained under small \( \gamma_0 \) and the same other parameters. Interestingly, the gap between the boundary of the molecule-hole continuum and the molecular bound state decreases when the dissipation cutoff momentum \( k_r \) increases, which can be understood by the composition of the retarded self-energy when the dissipation is infinitesimal. In this instance, \( \Sigma_{\Delta}^{R,1} \) is the self-energy of the vacuum scattering (the impurity scatters from a single-bath fermion) without dissipation. \( \Sigma_{\Delta}^{R,2} \) comes from a finite density of bath gases and represents the contribution of dissipative bath fermions to the self-energy by participating in the formation of the molecular state. For the vacuum scattering, the T-matrix is given as:

\[
T_0^{-1}(\mathbf{q}, \omega) = -\frac{m_r}{2\pi} \ln \left[ \frac{E}{E_B} \right] - \frac{i\tau}{\pi},
\]

where \( E = \omega - q^2/2M + i\eta' \). Clearly, \( T_0 \) has a pole at \( \omega^0 = q^2/2M + i\eta' - E_B \). When the background participates in the scattering process, the equation for the bound state energy develops into

\[
T_0^{-1}(\mathbf{q}, \omega^0) = \Sigma_{\Delta}^{R,2}(\mathbf{q}, \omega^0).
\]

**Figure 1**  (Color online) Molecular spectral function \( A_{\Delta} \) with different dissipation cutoff momenta \( k_r \) ((a) \( k_r/k_F = 1.0 \); (b) \( k_r/k_F = 2.0 \); (c) \( k_r/k_F = 3.0 \); (d) \( k_r/k_F = 5.0 \)) for a small loss rate \( \gamma_0/E_F = 0.01 \) and with different dissipation strengths \( \gamma_0 \) ((e) \( \gamma_0/E_F = 0.01 \); (f) \( \gamma_0/E_F = 1.0 \); (g) \( \gamma_0/E_F = 5.0 \); (h) \( \gamma_0/E_F = 30 \)) for \( k_r/k_F = 1.0 \). Other parameters are \( \eta = 0.5 \) and \( E_B/E_F = 2 \). In addition, the black dashed lines in (a)-(d) are the analytical results of molecular bound state \( \omega_b \) with \( \gamma_0 = 0 \) and the same other parameters in their respective subplots, which are in perfect agreement with the numerical results at small \( \gamma_0 \). In (h), we show the analytical solution to the lower bound of the molecule-hole continuum at infinite dissipation marked by \( \omega_b \) (solid red line) and \( \omega_\infty \) (dashed red line) to the left and right of their intersection (black dot), respectively (a detailed explanation is provided later in this section). Meanwhile, \( \omega_{\infty}^{\infty} \) (dashed black line) illustrates the analytical solution to the molecular bound state under the same conditions. These analytical results are also consistent with the numerical results at large \( \gamma_0 \).
Because the energy of the bound state must be less than the kinetic energy of the center of mass, we then immediately obtain
\[ \ln[(\omega_0^2 - \mathbf{q}^2/2M)/\omega_0^2] > 0, \] assuming \( \omega_0 > \omega_0 \). Moreover, the right-hand side of eq. (32) grows monotonically with \( k_r \), similar to \( \omega_0 \), which explains why the gap between the energy of the bound state and that of the boundary of the molecule-hole continuum narrows with \( k_r \).

In the presence of dissipation, the molecular bound state shows obvious nonmonotonic behavior. Under moderate dissipation strength, the spectral function presents a certain degree of broadening, as shown in Figure 1(f) and (g). However, it becomes well defined again when \( \omega_0 \) becomes sufficiently large, e.g., \( \omega_0 = 30\Omega_F \), as shown in Figure 1(h), indicating the revival of the bound state. The competition between the increase in the argument and the decrease in the modulus of the self-energy by the growth of dissipation prompts this nonmonotonicity. For each channel within \( k_r \), when the dissipation increases slightly from zero, the imaginary part of the self-energy increases substantially, which is particularly obvious for small momentum channels. However, when the dissipation is very large, the contribution of the summation terms to the self-energy in all the dissipative channels will eventually be suppressed by the divergent denominator in eq. (15), resulting in the attenuation of all the channels within \( k_r \).

In particular, if \( \gamma_0 \) is infinite, the retarded molecular self-

energy is given by

\[ \Sigma_{\text{R}}(\mathbf{q}, \omega) = \sum_{\mathbf{p_1}} \frac{1}{\mathbf{V}} \frac{1}{\omega - \varepsilon_{\mathbf{p_1}} - \varepsilon_{\mathbf{q}} + i0^+}. \]

In this limit, the dissipative boundary takes a similar role as the Fermi surface. In other words, effective “Pauli blocking” is reproduced in this situation where the T-matrix develops into

\[ T(q, \omega) = T_0 \left( \frac{1}{1 - \frac{1}{2} \sqrt{(z - \varepsilon_f)^2 - 4\varepsilon_f(q)^2 \frac{k_r^2}{2m}}} \right), \]

specifying \( z = \omega - \frac{\hbar^2 k_r^2}{2m} + i0^+ \), and \( z = \text{sign} [\text{Re} (z - \varepsilon_f(q))] \).

Proceeding to the next step, we analytically solve the dispersion of the bound state as

\[ \omega_b^{\text{ext}}(q) = \frac{1}{2} \varepsilon_f(q) \left( \varepsilon_f(q) - \frac{4 k_r^2}{2m} - E_b^2 \right) + \frac{k_r^2}{2m}, \]

which is plotted in Figure 1(h) with a dashed black line and is very consistent with the numerical result under large \( \gamma_0 \). Meanwhile, the minimum value of \( \frac{\pi^2}{M} + \pi^2 \) with the confinement \( \int \frac{\mathbf{p}}{M} > k_r \) determines the threshold of the molecule-hole continuum. This optimization problem needs to be discussed in different situations. On the one hand, if \( k_r < \frac{\pi}{2}, \) then \( \omega_b(q) = \frac{\pi^2}{M} \), on the other hand, if \( k_r > \frac{\pi}{2}, \) then \( \omega_b(q) = \frac{4 k_r^2}{2m} \). The two curves intersect at the \( (2k_r, 2k_r^2) \), as is shown in Figure 1(h). When the dissipation is infinite, the behavior of the molecular state is identical to that in the scattering by the impurity and Fermi sphere. In fact, this phenomenon can also be interpreted as a Zeno effect, where the dissipation processes can be considered continuous measurement [54]. A similar scenario has been found by Ueda’s group [55] in a non-Hermitian BCS superfluid with complex-valued interaction, which is due to inelastic scattering between fermions. In their work, the superfluid gap never collapses, but it is enhanced by the dissipation as a result of the Zeno effect. Actually, in the impurity limit, our molecular bound state can be considered a counterpart of the population-balanced Fermi superfluid state.

We further characterize the scattering by analyzing the phase of the T-matrix. We start from the scenario in the absence of dissipation. In the case of vacuum scattering where \( \eta = 0 \), the many-body dynamics reduces to two-body dynamics. The aforementioned dispersion of the pole of \( T_0 \), i.e., \( \omega_0 \), generates an excitation with energy \( -E_b \) at zero momentum, which is confirmed by a sharp peak and a \( \pi \) jump, in the process of crossing the energy of the bound state, in Figure 2(a1) and (a2), respectively, with dotted red lines. The phase is zero in the regime without a state, i.e., \( \omega = -E_b \). It maintains this value within the gap between the bound and

\[ \omega_b(q) = \frac{4 k_r^2}{2m} \]

Figure 2: (Color online) Spectrum function of the molecular bound state at zero momentum, i.e., \( \Delta_b(0, \omega)E_b \), and the phase of the T-matrix, i.e., \( \text{Arg}[T]/\pi \) for different average particle numbers \( (a1), (a2); \eta = 0 ; (b1), (b2); \eta = 0.5 \) and different dissipations \( (\gamma_0 = 0, \text{dotted red line}, \gamma_0 = \Omega_F) \), dashed blue line; \( \gamma_0 = 30\Omega_F \), solid black line) as a function of energy. The arrow indicates the bound state \( \omega_b(q) = 0E_F \) and the threshold of the continuum state \( \omega_b = 2E_F \) for the infinite loss rate case. In each subplot, we have set \( m = m_F \), \( E_b = 2E_F \), and \( k_r = k_F \) in the calculation.
continuous regimes and then begins to decay monotonically when $\omega > 0$ in that the continuous state emerges. If $\eta$ is greater than zero, then the bath states within $k$, have a certain probability of being occupied, and then, on average, the density of the bath gases is finite. Therefore, the formation of a dressed molecular state is more inclined to deduct the bath states within $k$, resulting in a blueshift of the bound state energy, as is shown in Figure 1(e) with the dashed blue line while consistent with Figure 2(b1). Notably, although the continuum state still arises at $\omega = 0$, which is the minimum total kinetic energy of a molecule without any constraints, the phase shows nonmonotonicity and has a peak at $\omega > 0$ (see the details below).

When dissipation is applied to the background gas, the spectral function of the bound state exhibits a greater broadening and blueshift than its corresponding lossless case. Meanwhile, the $T$-matrix in the gap regime is no longer a real one because of the finite dissipation, resulting in the $\pi$ platform evolvolving into a broad peak (shown as a dashed blue line in Figure 2(a2) and (b2)). Interestingly, regardless of whether the dissipation or the average particle number is nonzero, the phase of the $T$-matrix is nonmonotonic in the continuous regime, particularly their peaks are in the same position, which coincides with where the continuous state begins to emerge when the dissipation is infinite. This commonality is observed because the final effect is to reconstruct the same Fermi surface when increasing the dissipation strength or the average number of particles. In their respective limiting cases, the effective “Pauli blocking” will push the threshold of the continuous state to the optimal value obtained in the subspace: $k_f > k_r$.

The numerical solutions with very large dissipation are also shown by solid black lines in Figure 2 for comparison with the analytical results with infinite dissipation. Eq. (35) clearly shows that the bound state at zero momentum is $\omega_{EF}^2 = \hbar^2 k_r^2 / 2m_r - E_B$, whether $\eta$ is zero or finite, which is just at the position of the phase jump of the corresponding numerical results. To address the threshold of the continuum state, we use the condition that the bath atom cannot be scattered into the dissipation subspace: $k_f < k_r$. The momentum of bath atom is written as $k_r = \sqrt{m_r q}$ - $k_{ol}$, where $q$ is the center of mass momentum, and $k_{ol}$ is the relative momentum. Then, the effective “Pauli blocking” requires

$$\sqrt{2m_r E_{EF}} \cdot \frac{m_f}{M} q > k_r,$$

that is,$$E_{EF} > \omega_{EF} - \frac{\hbar^2 q^2}{2M} = (k_r + \frac{m_f}{M} q)^2 / 2m_r.$$

Here, $\omega_{EF}$ at $q = 0$ shows no difference with the outcomes of numerical simulation with large dissipation, e.g., $\gamma_0 = 30E_F$.

### 3.2 Polaron state

In our system, the impurity does not directly participate in the nonunitary evolution, but it is affected by dissipation through the scattering with the intermediate molecular state. In this section, we primarily consider the minimum energy state of polarons that occur at $k = 0$. We can intuitively observe the excitations due to the spectral function of the impurity, the resonant peaks of which correspond to polaron states. In Figure 3, we connect $E_B$ with $a_{2D}$ by $E_B = 1/m a_{2D}^2$ [29], and present the contour plot of $A_r(k = 0, \omega)$. For small $\gamma_0$ and $k_r = k_F$, the attractive branch is well-defined with weak interaction. When the interaction strength increases, a repulsive polaron emerges, and the spectrum weight transfers from the attractive polaron to the repulsive polaron. Finally, the repulsive polaron becomes long-lived and stable with strong interaction. This scenario is basically identical to that when the background gases have a Fermi surface [29]. The polaron spectral function shows that they first diffuse and converge with increasing dissipation (see the second row in Figure 3), which can be attributed to the Zeno effect, as was mentioned in the previous section. When $k_r$ exceeds $k_F$, the dissipation will affect the higher energy excitations, which can be corroborated by the obvious weakening of the repulsive polaron in Figure 3(b), (c), and (d) (see explanations below). In addition, the molecule-hole continuum spreads out more with increasing $k_r$.

We see the different responses of the two excitations to $k_r$ more clearly from the frequency sweep with a fixed $E_B$ in Figure 4(a). The attractive and repulsive polarons show different behavior as $k_r$ increases: although the energies of both polarons are redshifted, the spectral signal of the former polaron is enhanced due to the weakening of the latter polaron. On the one hand, these frequency shifts in common can be confirmed by calculating the positions of polaron states, which are obtained by solving Eq. (21) for $k = 0$ in our treatment (see the intersections of the straight dashed line $g(\omega) = \omega$ and the self-energy curves in Figure 4(b)). On the other hand, $k_F$ acts as a Fermi surface to some extent. In this picture, the particle-hole excitations around it dominate the formation of polarons. Therefore, with increasing $k_r$, the repulsive polaron is easier to couple with the virtual molecular state; that is, it will be more dissipated. Meanwhile, the attractive branch becomes increasingly like a bare impurity, which is reflected in the increase in two-body decay and the decrease in effective mass, as is shown in Figure 5(c) and (d). Thus, the spectral weight of the repulsive polaron is continuously shifted to that of the attractive one.

To further describe the properties of the attractive polaron and compare them with the dissipation-free case (at this time, the lifetime of the attractive polaron is infinite, i.e., $\gamma_+ = 0$),
we present the polaron energy, quasiparticle residue, effective mass, and two-body decay rate as functions of \( \gamma_0 \) for the mass-balanced case in Figure 5. These quantities initially change vs. \( \gamma_0 \) and gradually reach their saturation values, which can be explained in terms of the Zeno effect, as those in the previous section. The polaron energy displays a redshift for specific \( \gamma_0 \) with added \( k_e \), showing consistency with Figure 4. The quasi-particle ratio \( Z \), effective mass \( m'_e/m_e \), and two-body decay \( \gamma' \) show nonmonotonic behavior vs. \( \gamma_0 \). The trend change positions of the three curves are basically identical, and they are all at the dissipation of intermediate intensity, i.e., \( \gamma_0/E_F \sim 2 \) for our setup. To understand these results more clearly, we exhibit the two partial derivatives \( \partial \text{Re} \Sigma^0_e(0, \omega)/\partial \omega \big|_{\omega=E_F} \) and \( \partial \text{Re} \Sigma^0_e(0, \omega)/\partial \omega \big|_{\omega=E_F} \) in Figure 5(e) and (f). Then, on the basis of eqs. (23) and (24), the behaviors of \( Z \) and \( m'_e/m_e \) are unambiguous. These results are consistent with the point of view in this paper; that is, only moderate dissipation is detrimental to the formation of polarons. Because only the bath will be dissipated, when \( \gamma_0 \sim E_k \), the properties of the system are as follows: (1) the proportion of impurity excitation in the polaron increases; (2) the effective mass approaches unity; and (3) the lifetime of the polaron is greatly compressed. These results help us to observe an attractive polaron in a dirty environment, e.g., a background gas strongly coupled with its environment.

### 3.3 Experimental relevance

Finally, we demonstrate the quasiparticle properties of the experimentally realizable systems. Because the impurity is a single atom, either a Bose or Fermi impurity can be a candidate. Here, we consider one type of Fermi-Fermi mixture and two types of Bose-Fermi mixtures, as shown in Figure 6. In the small impurity density limit, the quasiparticle properties of the minority can be detected using an RF spectrum or a Raman spectrum.

![Figure 3](image-url) (Color online) Polaron spectral function, i.e., \( A_e(\omega = 0, \omega) \) as a function of the interaction strength parameter, i.e., \( \ln(k_F r_{220}) \) with different values of \( k_e \) ((a), \( k_e/k_F = 1.0 \)); (b), \( k_e/k_F = 1.5 \); (c), \( k_e/k_F = 2.0 \); (d), \( k_e/k_F = 2.5 \)) for \( \gamma_0 = 0.1 E_F \), and with different values of \( \gamma_0 \) ((e), \( \gamma_0/E_F = 0.1 \)); (f), \( \gamma_0/E_F = 1.0 \); (g), \( \gamma_0/E_F = 5.0 \); (h), \( \gamma_0/E_F = 30 \) for \( k_e/k_F = 1.0 \). The average particle number is \( \eta = 0.5 \).

![Figure 4](image-url) (Color online) Polaron spectral function ((a) and self-energy (b) with different values of \( k_e \) (\( k_e/k_F = 0.75 \): blue line; \( k_e/k_F = 1.0 \): red line; \( k_e/k_F = 1.25 \): yellow line; \( k_e/k_F = 1.5 \): black line). Other parameters are \( E_F = 2 E_F \), \( \gamma_0 = 0.1 E_F \), and \( \eta = 0.5 \). The black dashed line is the function of \( g(\omega) = \omega \).
4 Conclusion and outlook

In summary, we use non-self-consistent $T$-matrix theory to calculate the polaron problem in a driven-dissipative bath. The nonequilibrium GF is adopted to include the effect of the quantum jump term in the LMEQ. We illustrate the nontrivial interplay among dissipation range, dissipation strength, and interaction. In particular, we analytically specify that the gap between the molecular and continuum states decreases monotonically with the dissipation range in the limit of vanishingly small dissipation strength. Furthermore, we obtain the analytical dispersion of the bound state and the threshold of the continuum state when the dissipation strength is infinitely large. We numerically show that the molecular and polaron spectrum functions first diffuse and then converge as the dissipation strength increases, which are attributed to the quantum Zeno effect. Meanwhile, the positions of the attractive and repulsive polarons display a redshift when the dissipation range increases, and the two branches exhibit opposite behavior in this process—we see an increase and a decrease in the weights of the attractive and repulsive polarons, respectively. The quasiparticle parameters, such as the residue, effective mass, and two-body decay, are also investigated and show nonmonotonic behavior due to the interplay between the intrinsic energy scale of the system and the measurement frequency from the environment. However, they finally tend to their respective saturation values in the quantum Zero limit. Our model can be implemented with recent experimental technology [47-50].

On the basis of the technical methods used in this work, we can further study experimentally realizable systems, e.g., the exciton-polariton system in quantum wells embedded in an optical microcavity. The leakage of photons from the cavity and the decay of excitons via radiative and nonradiative processes make the platform a natural open quantum system [58-60]. How to use the nonequilibrium Green’s function to characterize natural open systems is reserved for future research. Upon completion of this work, we became aware of two related works [61,62] that explored the signatures and tunability of dissipative polarons in transition metal dichalcogenide materials, providing new insights for our future work.

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