Light Spectrum and Decay Constants in Full QCD with Wilson Fermions

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We present results from an analysis of the light spectrum and the decay constants $f_\pi$ and $f_{-1V}$ in Full QCD with $n_f = 2$ Wilson fermions at a coupling of $\beta = 5.6$ on a $16^3 \times 32$ lattice.

1. INTRODUCTION

This analysis was performed as part of the SESAM project [1] to investigate Sea quark Effects on Spectrum and Matrix-elements. Configurations were generated using the Hybrid-Monte-Carlo algorithm with the standard Wilson action at a coupling of $\beta = 5.6$ on a $16^3 \times 32$ lattice. We work at three different values of the dynamical quark mass: $\kappa_{\text{sea}} = 0.1560, 0.1570$ and $0.1575$ corresponding to $m_\pi/m_\rho$-ratios of $0.83(1), 0.76(1)$ and $0.71(2)$ [2]. Our lightest quark-mass is approximately equal to the strange quark-mass. Up to now we generated about 8000 trajectories of unit-length with a time-step of $\Delta t = 0.01$. After measuring the integrated autocorrelation time for the plaquette and the pion-correlator at fixed timeslice [3,4] we decided to use configurations separated by 25 trajectories for the present spectrum analysis. Our sample consists of 100, 160 and 100 configurations for our 3 sea-quark values. Since we aim at a final sample of $3 \times 200$ configurations this talk should be considered as a half-time status report.

Quark propagators for the set of valence $\kappa$-values {0.1555, 0.1560, 0.1565, 0.1570, 0.1575} were computed using the standard over-relaxed Minimal Residual Algorithm with smeared sources and local as well as smeared sinks. We used the Wuppertal gauge-invariant gaussian smearing method with $N = 50$ iterations and $\alpha = 4$, fitting the hadron-correlators to single-exponential functions. Throughout this analysis we neglect correlations in time and in $\kappa$. We hope to present a stable correlated analysis on our final sample.

2. CHIRAL EXTRAPOLATIONS WITH FIXED SEA-QUARK MASS

To determine the critical hopping-parameter $\kappa_c$ at a given value of $\kappa_{\text{sea}}$ we fit the pseudoscalar masses according to

$$M_{PS}^2 = bm_q + cm_q^2$$ (1)

with $m_q = 1/2(1/\kappa - 1/\kappa_c)$ being proportional to the quark mass. We find $c$ to be significantly different from zero: a linear fit leads to a decrease of $\kappa_c$ by four standard deviations with respect to the quadratic fit. The resulting values of $\kappa_c$ are shown in table 1 together with their statistical errors. To estimate systematic errors on $\kappa_c$, we exclude the largest quark-mass from our data and find a change in $\kappa_c$ by four standard deviations with respect to the quadratic fit. The resulting values of $\kappa_c$ are shown in table 1 together with their statistical errors. To estimate systematic errors on $\kappa_c$, we exclude the largest quark-mass from our data and find a change in $\kappa_c$ of the order of one standard deviation.

For the chiral extrapolation of vector masses we use two different parametrizations

$$M_V = m_\rho + bm_q + cm_q^2$$ (2)

$$M_V = m_\rho + bm_q + cm_q^{3/2}$$ (3)

which are motivated by chiral perturbation theory. In contrast to the $\Pi$ extrapolation we find that the values obtained from 2- and 3-parameter fits agree within errors; moreover eqs. 2 and 3 yield identical results. In table 1 we quote the values for the $\rho$-masses and the corresponding lattice-spacings as obtained from the

talk presented by U. Glässner
2-parameter-fits. The masses are calculated at $\kappa_c$ because we find the mass-shifts induced by $\Delta \kappa = \kappa_c - \kappa_{\text{light}}$ to be negligible compared to the statistical errors. We will discuss the light quark masses estimated from $\Delta \kappa$ in detail in a forthcoming publication. Figure 1 illustrates the variation of $m_\rho$ with $m_q$ and $\kappa_{\text{sea}}$.

The nucleon data (see figure 2) cannot be fitted with $c = 0$, but ansätze eqs. 2 and 3 do equally well. The quadratic extrapolation to $\kappa_c$ leads to the values $M_N$ quoted in table 1 together with the values for $M_\Delta$. With the present statistics we do not see any sea-quark dependence of the nucleon mass in physical units.

Table 1
Extrapolation results with fixed sea-quark mass. Errors quoted are purely statistical

| $\kappa_{\text{sea}}$ | $\kappa_C$  | $aM_\rho$  | $a_\rho^{-1}$ [GeV] | $M_N$ [GeV] | $M_\Delta$ [GeV] | $f_\pi$ [MeV] | $f_\Delta^{-1}$ |
|-----------------------|-------------|-------------|---------------------|-------------|------------------|--------------|----------------|
| 0.1560                | 0.16065(8)  | 0.359(8)    | 2.14(5)             | 1.09(7)     | 1.30(9)          | 125(9)      | 0.33(3)        |
| 0.1570                | 0.15987(6)  | 0.341(8)    | 2.23(7)             | 1.14(6)     | 1.40(10)         | 101(8)      | 0.35(2)        |
| 0.1575                | 0.15963(11) | 0.316(10)   | 2.44(8)             | 1.09(10)    | 1.22(8)          | 118(15)     | 0.30(3)        |

Figure 1. Meson extrapolations with fixed sea-quark mass.

Figure 2. Nucleon extrapolations with fixed sea-quark mass.

In fact $\kappa_c$ and $\kappa_{\text{light}}$ still agree within errors.
We now consider the decay-constants, $f_\pi$ and $f_V^{-1}$, defined as

\begin{align*}
<0|A|\pi > Z_k Z_A &= f_\pi m_\pi \\
<0|V_i|\rho > Z_k Z_V &= \epsilon_i f_V^{-1} m_\rho,
\end{align*}

where $A$ and $V_i$ are the local axial and vector currents on the lattice and $Z_k$, $Z_A$ and $Z_V$ the renormalization constants which connect the expectation values of equation 4 and 5 to the continuum. We determine these renormalization constants using tadpole improved perturbation theory \[2,4\]. The $m_q$-dependence of the decay-constants is presented in figure 3. Obviously the statistical accuracy is not yet high enough to resolve a $\kappa_{\text{sea}}$ dependence of $f_\pi$ and $f_V^{-1}$ at the chiral point (see also table 1).

### 3. EXTRAPOLATIONS IN $\kappa_{\text{sea}}$

A consistent method to extrapolate to zero sea-quark mass is to use only the data-points with $\kappa_{\text{sea}} = \kappa_{\text{val}}$ for the extrapolation. In the case of dimensionful observables one might argue about the impact of the varying scale along the trajectory. For this reason we consider only mass-ratios as shown in figure 4. We use linear fits only.

\begin{table}
\centering
\begin{tabular}{|c|c|}
\hline
$M_N/M_\rho$ & 1.6(2) \\
$M_A/M_\rho$ & 1.8(2) \\
$f_\pi/M_\rho$ & 0.14(3) \\
$f_V^{-1}$ & 0.32(6) \\
\hline
\end{tabular}
\caption{Results from extrapolations in $\kappa_{\text{sea}}$.}
\end{table}

The extrapolated values, quoted in table 2, agree (within large errorbars) with those obtained from extrapolations at fixed $\kappa_{\text{sea}}$.

### 4. CONCLUSIONS AND OUTLOOK

On our present statistics, uncorrelated data analyses of $t$- and $\kappa_{\text{val}}$ distributions allow for stable chiral extrapolations in valence quark mass, $m_q$. We find the $\rho$-meson mass to be consistent with a linear chiral ansatz, while the remaining quantities considered exhibit non-linear behaviour in $m_q$. $m_\rho$ varies on the level of 15 % across our range of sea quark masses. Baryon masses and decay constants so far indicate a trend but produce no compelling evidence for such variation.
Chiral extrapolations of our present data in $\kappa_{\text{sea}}$ itself still carry too large uncertainties to be reliable. Better statistics and deeper penetration into the chiral regime are needed to improve on this situation.

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