Hawking temperature of rotating charged black strings from tunneling

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Abstract. Thermal radiations from spherically symmetric black holes have been studied from the point of view of quantum tunneling. In this paper we extend this approach to study radiation of fermions from charged and rotating black strings. Using WKB approximation and Hamilton-Jacobi method we work out the tunneling probabilities of incoming and outgoing fermions and find the correct Hawking temperature for these objects. We show that in appropriate limits the results reduce to those for the uncharged and non-rotating black strings.

Keywords: GR black holes, absorption and radiation processes, quantum black holes

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Dedicated to Professor Asghar Qadir on his sixty fifth birthday.
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1 Introduction

Cylindrically symmetric static solutions of the Einstein-Maxwell equations with a negative cosmological constant represent charged black strings. Soon after the pioneering work [1–3] on cylindrically symmetric black hole solution its charged and rotating versions were found [4, 5]. The rotating version is very similar to the Kerr-Newman black hole in spherical symmetry. These black configurations have been studied for different physical ([6–10]) and thermodynamical [11, 12] properties.

One of the important recent developments in theoretical physics is studying Hawking radiation [13] from the point of view of quantum tunneling from black hole horizons [14–16]. In the so-called Hamilton-Jacobi method the tunneling rate of particles coming out from the black hole horizon is calculated from the imaginary part of the action [17, 18]. This approach has been successfully applied to various spherically and axially symmetric black configurations ([19–31]). In this paper we extend the analysis to tunneling of fermions from anti-de Sitter rotating charged black strings. We use WKB approximation to find the tunneling probabilities of incoming and outgoing charged fermions, and calculate the Hawking temperature. If the charge is taken to be zero one can deduce the temperature for the uncharged rotating black strings. Our results support the recent claim [12] that the Hawking temperature for these strings as calculated earlier [2] is incorrect. Thus we have an instance where the tunneling method confirms the correctness of the Hawking temperature. This was indeed one of the aims and objectives of developing this approach.

The paper is organized as follows. In section 2 we describe the spacetime for rotating charged black strings. Section 3 deals with the solution of charged Dirac equation in the background of these black strings and Hawking temperature is calculated. In the concluding section we show how this temperature reduces to those in the simpler cases of uncharged and non-rotating black strings.

2 Rotating charged black string

Analogous to the Kerr-Newman black hole solution in spherical symmetry, a solution for rotating charged object with cylindrical geometry in the presence of negative cosmological constant was found [4]. Both these solutions have event and Cauchy horizons, closed timelike
curves and timelike singularities. The solution for the black string is given by the line element

\[
ds^2 = -\left(\alpha^2 r^2 - \frac{2G (M + \Omega)}{\alpha r} + 4GQ^2 \frac{\alpha^2 r^2}{\alpha^2 r^2} \right) dt^2 - \frac{16GJ}{\alpha r} \left(1 - \frac{2Q^2}{(M + \Omega) \alpha r}\right) d\theta d\bar{\theta} + \frac{4G (M - \Omega)}{\alpha^4 r} \left(1 - \frac{2Q^2}{(M - \Omega) \alpha r}\right) d\theta^2 + \alpha^2 r^2 dz^2
\]

\[
+ \alpha^2 r^2 - 2G (3\Omega - M) / \alpha r + 4GQ^2 (3\Omega - M) / (M + \Omega) \alpha^2 r^2.
\]

Here

\[
\Omega = \sqrt{M^2 - \frac{8J^2 \alpha^2}{9}}.
\]

We define \(a\) as

\[
a^2 \alpha^2 = 1 - \frac{\Omega}{M},
\]

such that

\[
M + \Omega = 2M \left(1 - \frac{a^2 \alpha^2}{2}\right).
\]

Thus metric (2.1) can also be written as

\[
ds^2 = -\left(\alpha^2 r^2 - \frac{2M (1 - a^2 \alpha^2 / 2)}{\alpha r} + 4GQ^2 \frac{\alpha^2 r^2}{\alpha^2 r^2} \right) dt^2
\]

\[
- \frac{4aM \sqrt{1 - a^2 \alpha^2 / 2} \Omega}{\alpha r} \left(1 - \frac{Q^2}{M (1 - a^2 \alpha^2 / 2) \alpha r}\right) 2d\theta d\bar{\theta}
\]

\[
+ \left(\alpha^2 r^2 - \frac{4M (1 - 3a^2 \alpha^2 / 2)}{\alpha r} + 4GQ^2 \frac{\alpha^2 r^2}{\alpha^2 r^2} \frac{1 - 3a^2 \alpha^2 / 2}{1 - a^2 \alpha^2 / 2}\right)^{-1} dr^2
\]

\[
+ \left(r^2 + \frac{4Ma^2}{\alpha r} \left(1 - \frac{Q^2}{(1 - a^2 \alpha^2 / 2) M \alpha r}\right) \right) d\phi^2 + \alpha^2 r^2 dz^2.
\]

Here \(-\infty < z < +\infty, 0 < \alpha z < 2\pi\). Defining [4]

\[
\Delta = \alpha^2 r^2 - \frac{b}{\alpha r} + \frac{c^2}{\alpha^2 r^2}, \quad b = 4M \left(1 - \frac{3}{2} \alpha a^2\right),
\]

\[
c^2 = 4GQ^2 \left(\frac{1 - 3a^2 \alpha^2 / 2}{1 - a^2 \alpha^2 / 2}\right), \quad \gamma = \sqrt{\frac{1 - a^2 \alpha^2 / 2}{1 - 3a^2 \alpha^2 / 2}}\text{ and } \omega = \frac{a \alpha^2}{\sqrt{1 - 3d^2 \alpha^2 / 2}}.
\]

the above metric takes the form

\[
ds^2 = -\Delta \left(\gamma dt - \omega d\bar{\theta}\right)^2 + r^2 \left(\gamma d\theta - \omega dt\right)^2 + \frac{dr^2}{\Delta} + \alpha^2 r^2 dz^2.
\]

We choose new angular coordinate given by \(\bar{\theta} = \gamma \theta - \omega t\) such that \(d\theta = d\bar{\theta} / \gamma - \omega dt / \gamma\). With this choice of the coordinate the above metric becomes

\[
ds^2 = -\left(\alpha^2 r^2 - \frac{b}{\alpha r} + \frac{c^2}{\alpha^2 r^2} \right) \left(\frac{dt}{\gamma} - \frac{\omega d\bar{\theta}}{\alpha^2 \gamma}\right)^2
\]

\[
+ r^2 d\bar{\theta} + \left(\alpha^2 r^2 - \frac{b}{\alpha r} + \frac{c^2}{\alpha^2 r^2}\right)^{-1} dr^2 + \alpha^2 r^2 dz^2.
\]
The electromagnetic potential for the black string is given by [4]

\[ A_0 = -\gamma h(r), \quad A_2 = \frac{\omega}{\alpha^2} h(r), \quad A_1 = 0 = A_3. \]

Here \( h(r) = 2\lambda/\alpha r \), where \( \omega \) and \( \gamma \) are constants. In order to obtain the horizon of the black hole we put \( g^{11} = 0 \), so that

\[ \alpha^2 r^2 - \frac{b}{\alpha r} + \frac{c^2}{\alpha^2 r^2} = 0, \]

which on solving yields

\[ r_\pm = \frac{b^{1/3} \sqrt{s} \pm \sqrt{2s^2 - 4q^2 - s}}{2\alpha}. \tag{2.8} \]

Here

\[ s = \left( \frac{1}{2} + \frac{1}{2} \sqrt{1 - 4 \left( \frac{4q^2}{3} \right)^3} \right)^{1/3} + \left( \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4 \left( \frac{4q^2}{3} \right)^3} \right)^{1/3}, \tag{2.9} \]

where \( q^2 = c^2/b^{4/3} \). We put \( F(r) = -(g^{tt})^{-1} \), so that

\[ F(r) = \frac{-\Delta \gamma^2 \alpha^4}{\gamma^4 \alpha^4 + \omega^2 \Delta}, \tag{2.10} \]

and

\[ F_r(r_+) = \frac{\Delta'}{\gamma^2}. \tag{2.11} \]

We also let

\[ g(r) = \Delta, \tag{2.12} \]

so that

\[ g_r(r_+) = \Delta'. \tag{2.13} \]

This notation will be used later, and prime here denotes a derivative with respect to \( r \).

### 3 Tunneling probability and Hawking temperature

In this section we work out the probability of fermions to tunnel across the event horizon of rotating charged black string. In order to do this we solve the Dirac equation using WKB approximation. An important outcome of this procedure is the computation of correct Hawking temperature for these black objects. The charged Dirac equation for the field \( \Psi \), and mass, \( m \), and charge, \( q \), of the particle is

\[ i\gamma^\mu \left(D_\mu - \frac{iq}{\hbar} A_\mu \right) \Psi + \frac{m}{\hbar} \Psi = 0. \tag{3.1} \]

Here

\[ D_\mu = \partial_\mu + \Omega_\mu. \]

Also

\[ \Omega_\mu = \frac{1}{2} \Gamma^\alpha_\mu \Sigma_{\alpha\beta}, \Sigma_{\alpha\beta} = \frac{1}{4} t \left[ \gamma^\alpha, \gamma^\beta \right]. \]

Using properties of the commutative brackets we note that \( \Omega_\mu = 0 \), so that

\[ D_\mu = \partial_\mu, \]

\[ \Delta' = \frac{m^2}{\hbar^2} - \frac{\omega^2}{\alpha^2} - \frac{c^2}{\alpha^4}. \]
and eq. (3.1) becomes
\[ \nu \gamma^\mu \left( \partial_\mu - \frac{\nu q}{\hbar} A_\mu \right) \Psi + \frac{m}{\hbar} \Psi = 0. \] (3.2)

Applying summation this takes the form
\[ \nu \gamma^\mu \partial_\mu \Psi + \nu \gamma^r \partial_r \Psi + \nu \gamma^\theta \partial_\theta \Psi + \nu \gamma^\nu \partial_\nu \Psi + \frac{q}{\hbar} A_r \nu \gamma^r \Psi + \frac{q}{\hbar} A_\theta \nu \gamma^\theta \Psi + \frac{m}{\hbar} \Psi = 0. \] (3.3)

We choose the following \( \gamma \)-matrices
\[ \gamma^t = \frac{1}{\sqrt{F(r)}} \left( \begin{array}{cc} 0 & I \\ -I & 0 \end{array} \right), \]
\[ \gamma^\theta = M(r) \left( \begin{array}{cc} 0 & \sigma^2 \\ \sigma^2 & 0 \end{array} \right) + M(r) N(r) \left( \begin{array}{cc} 0 & I \\ -I & 0 \end{array} \right), \]
\[ \gamma^z = \frac{1}{\alpha r} \left( \begin{array}{cc} 0 & \sigma^1 \\ \sigma^1 & 0 \end{array} \right), \]
where \( \sigma^i \) are Pauli matrices, and \( M(r) \) and \( N(r) \) are defined as
\[ M(r) = \frac{\alpha^2 \gamma}{\omega} \left( \frac{\omega^2 - \Delta}{r^2 \alpha^4 \gamma^2} \right)^{1/2}, \]
\[ N(r) = \frac{\gamma}{\Delta^{1/2}}. \]

We assume the following form of Dirac’s field for eq. (3.3)
\[ \Psi(t, r, \theta, z) = \left( \begin{array}{c} A(t, r, \theta, z) \xi \\ B(t, r, \theta, z) \xi \end{array} \right) \exp \left( \frac{t}{\hbar} \gamma^0 \right), \] (3.4)

where \( I \) is the classical action, and \( A \) and \( B \) are arbitrary functions of the coordinates. Substituting this in eq. (3.3) we apply WKB approximation, divide by the exponential term and multiply by \( \hbar \). Thus the resulting equations to leading order in \( \hbar \) take the form
\[ -B \left( \frac{I_t}{\sqrt{F(r)}} + \sqrt{g(r)} I_r - M(r) N(r) I_\theta - \frac{q A_t}{\sqrt{F(r)}} - q A_\theta M(r) N(r) \right) + mA = 0, \] (3.5)
\[ A \left( \frac{I_t}{\sqrt{F(r)}} - \sqrt{g(r)} I_r - M(r) N(r) I_\theta - \frac{q A_t}{\sqrt{F(r)}} - q A_\theta M(r) N(r) \right) + mB = 0. \] (3.6)

Considering the Killing vectors of the background spacetime we employ the following ansatz
\[ I(t, r, \theta, z) = -Et + l \theta + Jz + W(r), \] (3.7)

where \( E \) is the energy of the emitted particles and \( W \) is the part of the action \( I \) that contributes to the tunneling probability. Using eq. (3.7) we find that eqs. (3.5) and (3.6) become
\[ B \left( \frac{E}{\sqrt{F(r)}} \frac{dW}{dr} + M(r) N(r) l + \frac{q A_t}{\sqrt{F(r)}} + q A_\theta M(r) N(r) \right) + mA = 0, \] (3.8)
\[ A \left( \frac{-E}{\sqrt{F(r)}} \frac{dW}{dr} - M(r) N(r) l - \frac{q A_t}{\sqrt{F(r)}} - q A_\theta M(r) N(r) \right) + mB = 0. \] (3.9)
We first calculate the function $W(r)$ for the massless case, i.e., for $m = 0$. From the above equations we can write

$$\frac{dW}{dr} = \frac{E + M(r) N(r) \sqrt{F(r)} l - qA_t - qA_\theta M(r) N(r) \sqrt{F(r)}}{\sqrt{F(r)} g(r)},$$

or

$$W_+(r) = \int \frac{E + M(r) N(r) \sqrt{F(r)} l - qA_t - qA_\theta M(r) N(r) \sqrt{F(r)}}{\sqrt{F(r)} g(r)} dr. \quad (3.10)$$

Now expanding $F(r)$ and $g(r)$ in Taylor’s series near the horizon and neglecting squares and higher powers of $(r - r_+)$ we get

$$g(r) = g(r_+) + (r - r_+) \partial_r g(r_+), \quad (3.11)$$

which can be written as

$$g(r) = (r - r_+) \Delta'. \quad (3.11)$$

Also

$$F(r) = \frac{(r - r_+)}{\gamma^2} \Delta'. \quad (3.11)$$

Using these values in eq. (3.10) we obtain

$$W(r_+) = \int \frac{E + M(r) N(r) \sqrt{F(r)} l - qA_t - qA_\theta M(r) N(r) \sqrt{F(r)}}{(r - r_+) \frac{1}{\gamma^2} \left(2a^2 r_+ + \frac{b}{a r_+} - \frac{2c^2}{a^2 r_+^2}\right)} dr. \quad (3.12)$$

Integrating around the simple pole we get

$$W_+ = \pi i \gamma \left(\frac{E + M(r) N(r) \sqrt{F(r)} l - qA_t - qA_\theta M(r) N(r) \sqrt{F(r)}}{\left(2a^2 r_+ + \frac{b}{a r_+} - \frac{2c^2}{a^2 r_+^2}\right)}\right). \quad (3.12)$$

Similarly

$$W_- = \pi i \gamma \left(\frac{-E + M(r) N(r) \sqrt{F(r)} l - qA_t - qA_\theta M(r) N(r) \sqrt{F(r)}}{\left(2a^2 r_+ + \frac{b}{a r_+} - \frac{2c^2}{a^2 r_+^2}\right)}\right). \quad (3.13)$$

The probabilities of fermions crossing the horizon in each direction are

$$P_{\text{emission}} \propto \exp(-2ImI) = \exp(-2ImW_+)$$

$$P_{\text{absorption}} \propto \exp(-2ImI) = \exp(-2ImW_-).$$

While computing the imaginary part of the action, we note that it is same both for the incoming and outgoing solutions, and so will cancel out. Now the probability of particles tunneling from inside to outside the horizon is given by [17, 18]

$$\Gamma \propto \frac{P_{\text{emission}}}{P_{\text{absorption}}} = \frac{\exp(-2ImW_+)}{\exp(-2ImW_-)}.$$
or
\[ \Gamma = \exp (-4ImW_+) . \quad (3.14) \]

Using the value of \( W_+ \) in this we obtain
\[ \Gamma = \exp \left[ -4\pi \gamma \left( \frac{E + M (r) N (r) \sqrt{F(r)}}{2\alpha^2 r_+ + \frac{b}{\alpha r_+^2} - \frac{2\alpha^2}{\alpha^2 r_+^3}} \right) \right] . \quad (3.15) \]

Here we note that the probabilities do not violate unitarity by exceeding the value of 1. This is because besides the spatial contribution there is a temporal part also which contributes both to emission and absorption probabilities \cite{32-34} and we obtain a correct value of \( \Gamma \). When we compare this with \( \Gamma = \exp (-\beta E) \), where \( \beta = 1/T_H \), we note that
\[ \beta = \frac{-4\pi \gamma}{2\alpha^2 r_+ + \frac{b}{\alpha r_+^2} - \frac{2\alpha^2}{\alpha^2 r_+^3}} , \quad (3.16) \]

so that the Hawking temperature for rotating charged black strings comes out to be
\[ T_H = \frac{1}{4\pi}\left( \frac{2\alpha^2 r_+ + \frac{b}{\alpha r_+^2} - \frac{2\alpha^2}{\alpha^2 r_+^3}}{1} \right) . \quad (3.17) \]

Using the values of \( \gamma, b \) and \( c^2 \) this takes the form
\[ T = \frac{1}{2\pi}\sqrt{\frac{1 - \frac{1}{2}\alpha^2 a^2}{1 - \frac{3}{2}\alpha^2 a^2}} \left[ \alpha^2 r_+ + \frac{2M (1 - \frac{3}{2}\alpha^2 a^2)}{\alpha^2 r_+^2} - \frac{4Q^2}{\alpha^2 r_+^3} \left( 1 - \frac{3}{2}\alpha^2 a^2 \right) \right] . \quad (3.18) \]

To recover Hawking temperature for the non-rotating case, we put \( a = 0 \) to get
\[ T_H = \frac{1}{2\pi}\left( \frac{2M}{\alpha r_+^2} - \frac{4Q^2}{\alpha^2 r_+^3} \right) , \quad (3.19) \]

which is in agreement with the result derived in refs. \cite{5, 12, 31}. For the massive case we multiply eq. (3.8) by \( A \) and eq. (3.9) by \( B \) to obtain
\[ AB \left( E - \sqrt{g(r)} F(r) \frac{dW}{dr} + M (r) N (r) \sqrt{F(r)} \right) + \sqrt{F(r)} mA^2 = 0 , \quad (3.20) \]
\[ AB \left( -E - \sqrt{g(r)} F(r) \frac{dW}{dr} - M (r) N (r) \sqrt{F(r)} \right) - \sqrt{F(r)} mB^2 = 0 . \quad (3.21) \]

Subtracting eq. (3.21) from eq. (3.20) gives
\[ \frac{A}{B} = \frac{1}{m\sqrt{F(r)}} \left( E + M (r) N (r) \sqrt{F(r)} + qA_t + qA_\theta M (r) N (r) \sqrt{F(r)} \right) \pm \sqrt{\left( E + M (r) N (r) \sqrt{F(r)} + qA_t + qA_\theta M (r) N (r) \sqrt{F(r)} \right)^2 + m^2 F(r)} . \quad (3.22) \]
We see that
\[ \lim_{r \to r_+} \left( \frac{A}{B} \right) = \begin{cases} 0 & \text{for upper sign (+)} \\ -\infty & \text{for lower sign (−)} \end{cases} . \]
At the horizon either \( A = 0 \) or \( B = 0 \). For \( A \to 0 \), we solve eq. (3.9) for \( m \) and insert in eq. (3.8) to get after simplification
\[
\left( \frac{dW_+}{dr} \right) = \frac{E + M(r)N(r) + lqA_l + qA_\theta M(r)N(r)}{\sqrt{F_r(r)g_r(r)(r - r_+)} \left( 1 + A^2/B^2 \right)} \left( \frac{1 + A^2/B^2}{1 - A^2/B^2} \right). \tag{3.23}
\]
Similarly, for \( B \to 0 \) we have
\[
\left( \frac{dW_-}{dr} \right) = -\frac{E + M(r)N(r) + lqA_l + qA_\theta M(r)N(r)}{\sqrt{F_r(r)g_r(r)(r - r_+)} \left( 1 + A^2/B^2 \right)} \left( \frac{1 + A^2/B^2}{1 - A^2/B^2} \right). \tag{3.24}
\]
Integrating eq. (3.23) around the contour and using the values of \( F_r(r) \) and \( g_r(r) \) from eqs. (2.11) and (2.13) yields
\[
W_+ = \frac{\pi i \gamma (E + M(r)N(r) + lqA_l + qA_\theta M(r)N(r))}{\left( 2\alpha^2 r_+ + \frac{b}{\alpha r_+^2} - \frac{2c^2}{\alpha^2 r_+^3} \right)}. \tag{3.25}
\]
Similarly, from eq. (3.24) we get
\[
W_- = \frac{\pi i \gamma (-E + M(r)N(r) + lqA_l + qA_\theta M(r)N(r))}{\left( 2\alpha^2 r_+ + \frac{b}{\alpha r_+^2} - \frac{2c^2}{\alpha^2 r_+^3} \right)}. \tag{3.26}
\]
Thus in this case we obtain
\[
\Gamma = -4\pi \left( \gamma \frac{(E + M(r)N(r) + lqA_l + qA_\theta M(r)N(r))}{\left( 2\alpha^2 r_+ + \frac{b}{\alpha r_+^2} - \frac{2c^2}{\alpha^2 r_+^3} \right)} \right). \tag{3.27}
\]
This gives the Hawking temperature as
\[
T_H = \frac{1}{4\pi \gamma} \left( 2\alpha^2 r_+ + \frac{b}{\alpha r_+^2} - \frac{2c^2}{\alpha^2 r_+^3} \right). \tag{3.28}
\]
Using the values of \( \gamma, b \) and \( c^2 \) we get the same result as in eq. (3.18).

4 Discussion

Hawking radiation can be viewed as a process of quantum tunneling from black hole horizons by using the techniques of quantum field theory on curved spacetimes. Mathematically this is achieved by the Hamilton-Jacobi method by using WKB approximation and complex path integration. The particles traverse trajectories which are forbidden classically. These paths are given in terms of the imaginary part of their classical action.

We have considered cylindrically symmetric solutions of the Einstein-Maxwell equations which are anti-de Sitter type. We have studied radiation of charged fermions from these black

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strings. The method yields the tunneling probability of the particles crossing the horizon, and the Hawking temperature for rotating charged black string which is given by

\[
T = \frac{1}{2\pi} \sqrt{\frac{1 - \frac{1}{2}\alpha^2 a^2}{1 - \frac{3}{2}\alpha^2 a^2}} \left[ \alpha^2 r_+ + \frac{2M (1 - \frac{3}{2}\alpha^2 a^2)}{\alpha^2 r_+^2} - \frac{4Q^2}{\alpha^2 r_+^3} \left(1 - \frac{3}{2}\alpha^2 a^2\right)\right]. \quad (4.1)
\]

We see that from this formula we can recover the temperature for simpler cases. For example, if we put charge, \( Q = 0 \), we obtain the result for the uncharged rotating black string as

\[
T = \frac{1}{2\pi} \sqrt{\frac{1 - \frac{1}{2}\alpha^2 a^2}{1 - \frac{3}{2}\alpha^2 a^2}} \left[ \alpha^2 r_+ + \frac{2M (1 - \frac{3}{2}\alpha^2 a^2)}{\alpha^2 r_+^2} \right]. \quad (4.2)
\]

It has been noted recently \cite{12} that the temperature for this case as given earlier \cite{2} is incorrect. Here it is confirmed by the tunneling procedure, and this is one of the successes of this method that it provides the correct Hawking temperature. This was also an important motivation behind developing this approach.

Further, putting \( a = 0 \) in eq. (4.2) yields temperature for the non-rotating case as

\[
T = \frac{1}{2\pi} \left( \alpha^2 r_+ + \frac{2M}{\alpha r_+^2} - \frac{4Q^2}{\alpha^2 r_+^3} \right), \quad (4.3)
\]

which is consistent with the literature \cite{5, 12, 31}. Taking \( Q = 0 = a \) recovers the formula for non-rotating uncharged black string

\[
T = \frac{1}{2\pi} \left( \alpha^2 r_+ + \frac{2M}{\alpha r_+^2} \right). \quad (4.4)
\]

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