The minimal length uncertainty and the quantum model for the stock market

Pouria Pedram∗
Department of Physics, Science and Research Branch, Islamic Azad University, Tehran, Iran

January 16, 2012

Abstract

We generalize the recently proposed quantum model for the stock market by Zhang and Huang to make it consistent with the discrete nature of the stock price. In this formalism, the price of the stock and its trend satisfy the generalized uncertainty relation and the corresponding generalized Hamiltonian contains an additional term proportional to the fourth power of the trend. We study a driven infinite quantum well where information as the external field periodically fluctuates and show that the presence of the minimal trading value of stocks results in a positive shift in the characteristic frequencies of the quantum system. The connection between the information frequency and the transition probabilities is discussed finally.

Keywords: Econophysics; Quantum finance; Generalized uncertainty relation; Minimal length.

1 Introduction

Econophysics as an interdisciplinary research field was started in mid 1990s by physicists who are interested to apply theories and models originally developed in physics for solving the complex problems appeared in economics, specially in financial markets [1]. Because of the stochastic nature of the financial markets, the majority of tools for market analysis such as stochastic processes and nonlinear dynamics have their roots in statistical physics. Besides statistical physics, other branches of physics and mathematics have a major role in the development of econophysics. The sophisticated tools developed in quantum mechanics such as perturbation theory, path integral (Feynman-Kac) methods, random matrix and the spin-glass theories are shown to be useful for option pricing and portfolio optimization. Among theoretical physics, quantum field theory has a special role to reveal the intricacies of nature

∗p.pedram@srbiau.ac.ir
from quantum electrodynamics to critical phenomena. For instance, it can be used to model portfo-
lios as a financial field and describes the change of financial markets via path integrals and differential
manifolds \[2,3\].

The application of quantum mechanics to financial markets has attracted much attention in recent
years to model the finance behavior with the laws of quantum mechanics and it is becoming now a
rather established fact \[4–10\]. For instance, Schaden, contrary to stochastic descriptions, used the
quantum theory to model secondary financial markets to show the importance of trading in determining
the value of an asset \[11\]. He considered securities and cash held by investors as the wave function to
construct the Hilbert space of the stock market. Another useful application of quantum theory to trading
strategies is quantum game theory which is the generalization of classical game theory to the quantum
domain \[12,13\]. This theory is primarily based on quantum cryptography and contains superimposed
initial wave functions, quantum entanglement of initial wave functions, and superposition of strategies
in addition to its classical counterpart.

At this point, it is worth explaining why quantum mechanics is essential to study the behavior of
the stock market. Classical mechanics which is described by Newton’s law of motion is deterministic
in the sense that it exactly predicts the position of a particle at each instant of time. This is similar
to the evolution of a stock price with zero volatility \(\sigma = 0\) that results in a deterministic evolution
of the stock price. However, in the context of quantum mechanics, the evolution of the position of the
particle has a probabilistic interpretation which is similar to the evolution of a stock price with a non-zero
volatility \(\sigma \neq 0\) \[3\]. Note that there is a close connection between the Black-Scholes-Merton (BSM)
equation \[14,15\] and the Schrödinger equation: The position of a quantum particle is a random variable
in quantum mechanics, and similarly, the price of a security is a random variable in finance. Also,
the Schrödinger equation admits a complex wave function, whereas the BSM equation is a real partial
differential equation which can be considered as the Schrödinger equation for imaginary time. Haven
showed that BSM equation is a special case of the Schrödinger equation where markets are assumed to
be efficient \[16\]. Indeed, various mathematical structures of quantum theory such as probability theory,
state space, operators, Hamiltonians, commutation relations, path integrals, quantized fields, fermions and bosons have natural and useful applications in finance. In the language of Schaden, “The evolution into a superposition of financial states and their measurement by transaction is my understanding of quantum finance” [17].

Recently, Zhang and Huang have proposed a new quantum financial model in econophysics and defined wave functions and operators of the stock market to construct the Schrödinger equation for studying the dynamics of the stock price [18]. They solved the corresponding partial differential equation of a given Hamiltonian to find a quantitative description for the volatility of the Chinese stock market. In their formalism, the wave function $\psi(\wp, t)$ is considered as the price distribution, where $\wp$ denotes the stock price and $t$ is the time. There, the stock price is approximately considered as a continuous-variable. However, the stock price is actually a discrete variable and admits a non-zero minimal price length $(\Delta \wp)_{\text{min}} \neq 0$ which depends on the stock market’s local currency. In this paper, we incorporate the fact of discrete nature of the stock price with the quantum description of the stock market. We show that the uncertainty relation between the price and its trend and the form of the Hamiltonian should be modified to make the quantum formulation consistent with discrete property of the stock price. Note that, Bagarello has also tried to present quantum financial models which describe quantities which assume discrete values [6–10]. However, Bagarello’s approach is mainly based on the Heisenberg approach rather than on the Schrödinger equation.

2 The Quantum Model

Before applying quantum theory to finance we need to identify the macro-scale and micro-scale objects of the stock market. Since the stock index is based on the share prices of many representative stocks, it is meaningful to consider the stock index as a macro system and take every stock as micro-scale object [18]. Note that each stock is always traded at a certain price which shows the particle behavior. Also, the stock price always fluctuates in the market which is the wave property. Therefore, because of this wave–particle duality, we can consider the micro-scale stock as a quantum system. Now we can
construct the quantum model for the stock market based on the postulates of quantum mechanics.

First, we introduce the wave function $\psi(\wp_0, t)$ as the vector in the Hilbert space which describes the state of the quantum system. More precisely, $\psi(\wp_0, t)$ is the state vector $|\psi, t\rangle$ in price representation i.e. $\psi(\wp_0, t) \equiv (\wp_0 |\psi, t\rangle)$. Also, we take the modulus square of the wave function as the price distribution and demand that the superposition principle of quantum mechanics also holds

$$|\psi\rangle = \sum_n c_n |\phi_n\rangle,$$

(1)

where $|\phi_n\rangle$ is the possible orthonormal basis states of the stock system and $c_n = \langle \phi_n |\psi\rangle$. Therefore, the state of the stock price before trading should be a superposition of its various possible states with different prices so-called a “wave packet”. We can consider a trading process, buy or sell at some price, as a physical observation or measurement. So the trading process projects the state of the stock to one of the possible states with a definite price where $|c_n|^2$ denotes its probability. In other words, we can interpret $|\psi(\wp_0, t)|^2$ as the probability density of the stock price versus time, namely,

$$P(t) = \int_a^b |\psi(\wp_0, t)|^2 d\wp_0,$$

(2)

which shows the probability of the stock price between $a$ and $b$ at time $t$.

At this point, we introduce the Hermitian operators correspond to the financial observables in the Hilbert space. The price operator $\hat{\wp}_0$ measures the price of a state vector similar to the position operator $\hat{x}$ in quantum mechanics. Similarly, we define $T_0 = m_0 \frac{d\wp_0}{dt}$ as the rate of price change which is related to the trend of the price in the stock market where $m_0$ is the mass of stock. As indicated by Baaquie, a comparison between the Black-Scholes Hamiltonian and the Schrödinger equation shows that the volatility is the analog of the inverse of mass i.e. $m_0 = \sigma^{-2}$ (see Sec. 4.4 of Ref. [3]). Also, the intensity of the price movement can be considered as the energy of the stock or the associated Hermitian Hamiltonian which results in the following Schrödinger equation

$$\hat{H}\psi(\wp_0, t) = i\frac{\partial}{\partial t}\psi(\wp_0, t),$$

(3)

\footnote{Note that this definition is only valid at the continuous price limit. As we shall show in the next section, when we consider the minimal price length, $T$ cannot be written simply as the derivative of the price with respect to time. However, it can be expressed in terms of such operators.}
where the Hamiltonian is a function of price, trend, and time and generates the temporal evolution of the quantum system.

The trade of a stock can be considered as the basic process that measures its momentary price. This measurement can only be performed by changing the owner of the stock which represents the Copenhagen interpretation of a quantum system [11]. Therefore, a measurement may change the outcome of subsequent measurements so that it cannot be described by ordinary probability theory. Indeed, we can never simultaneously know both the ownership of a stock and its price. The stock price can only be determined at the time of sale when it is between traders. Moreover, the owners decide to sell or buy the stock at higher or lower prices that determines the trend of the stock price. So, in the quantum domain, the stock price and stock trend operators satisfy the following uncertainty relation

\[
\Delta \rho_0 \Delta T_0 \geq \frac{1}{2},
\]

where \( T_0 = -i \partial / \partial \rho_0 \). However, as we show in the next section, this relation should be modified when we take into account the discrete nature of the stock price.

### 3 The Generalized Uncertainty Principle

According to the above uncertainty relation, in principle, we can separately measure the price and the trend with arbitrary precision. However, since the price is a discrete variable there is a genuine lower bound on the uncertainty of its measurement. Thus, the ordinary uncertainty principle should be modified to so-called generalized uncertainty principle (GUP). Here we consider a GUP which results in a minimum price uncertainty

\[
\Delta \rho \Delta T \geq \frac{1}{2} \left(1 + \beta_0 (\Delta T)^2 + \zeta\right),
\]

where \( \beta_0 \) and \( \zeta \) are positive constants which depend on the expectation value of the price and the trend operators. In ordinary quantum mechanics \( \Delta \rho \) can be made arbitrarily small as \( \Delta T \) grows correspondingly. However, this is no longer the case if the above relation holds. For instance, if \( \Delta \rho \) decreases and \( \Delta T \) increases, the new term \( \beta_0 (\Delta T)^2 \) will eventually grow faster than the left-hand side and \( \Delta \rho \) cannot
be made arbitrarily small. Now the boundary of the allowed region in $\Delta \varphi \Delta T$ plane is given by

$$\Delta T = \Delta \varphi \beta_0 + \frac{1}{\beta_0} \sqrt{(\Delta \varphi)^2 - (1 + \zeta) \beta_0},$$

(6)

which yields the following minimal price uncertainty

$$(\Delta \varphi)_{\text{min}} = \sqrt{(1 + \zeta) \beta_0}.$$  

(7)

The above uncertainty relation can be obtained from the deformed commutation relation

$$[\varphi, T] = i(1 + \beta_0 T^2),$$

(8)

Because of the extra term $\beta_0 T^2$, this relation cannot be satisfied by the ordinary price and trend operators since they obey the canonical commutation relation $[\varphi_0, T_0] = i$. However, we can write them in terms of ordinary operators as

$$\varphi = \varphi_0,$$

(9)

$$T = T_0 \left(1 + \frac{1}{3} \beta_0 T_0^2\right),$$

(10)

It is easy to check that using this definition, Eq. (8) is satisfied to first-order of GUP parameter i.e.

$$[\varphi, T] = i(1 + \beta_0 T^2) + O(\beta_0^2),$$

(11)

where $\varphi$ and $T$ are given by Eqs. (9) and (10). Note that since $[\varphi, T] \neq i$, we cannot further consider the generalized trend operator $T$ as the derivative with respect to price $^2T = -i \frac{\partial}{\partial \varphi_0} + \frac{\beta_0}{3} \frac{\partial^3}{\partial \varphi_0^3}$. Now using Eqs. (8) and (10) and $\Delta \varphi \Delta T \geq (1/2)([\varphi, T])$ we find $\zeta = \beta_0 \langle T \rangle^2$. So using Eq. (7) the absolutely smallest uncertainty in price is

$$(\Delta \varphi)_{\text{min}} = \sqrt{\beta_0},$$

(12)

when the expectation value of the trend operator (or $\zeta$) vanishes, namely $\langle T \rangle = 0 = \zeta$. We can interpret $\Delta \varphi_{\text{min}}$ as the minimal price length and indicates that we cannot measure the price of a stock with
uncertainty less than \((\Delta \varphi)_{\text{min}}\) which agrees with discrete nature of the stock price. It is worth mentioning that, the generalized uncertainty relation also appears in the context of quantum gravity where there is a minimal observable length proportional to the Planck length \([19–23]\). In the string theory one can interpret this length as the length of strings.

Since \(\varphi\) and \(T\) do not exactly satisfy Eq. (9), our approach is essentially perturbative. Obviously, this procedure affects all Hamiltonians in quantum financial models. To proceed further, let us consider the following Hamiltonian:

\[
H = \frac{T^2}{2m} + V(\varphi, t),
\]

which using Eq. (9) can be written as

\[
H = H_0 + \beta_0 H_1 + \mathcal{O}(\beta_0^2),
\]

where \(H_0 = \frac{T_0^2}{2m} + V(\varphi_0, t)\) and \(H_1 = \frac{T_0^4}{3m}\). So the corrected term in the modified Hamiltonian is only trend dependent and is proportional to \(T_0^4\). In fact, the presence of this term leads to a positive shift in energy spectrum.

### 4 The Schrödinger equation

Many factors impact the price and the trend of the stock in the financial markets. These factors include political environment, market information, economic policies of the government, psychology of traders, etc. So it is difficult to construct a Hamiltonian which contains all effective variables. However, we can write a simple Hamiltonian to model the fluctuation of the stock price with ideal periodic external factors.

In most Chinese stock markets there is a price limit rule: the rate of return in a trading day compared with the previous day’s closing price cannot be more than \(\pm 10\%\). So the stock price fluctuates between the price limits or in a one-dimensional infinite well (particle in a box). The size of the box is \(d_0 = \tilde{\varphi} \times 20\%\) where \(\tilde{\varphi}\) is the previous day’s closing price. Now if we use a transformation of coordinate \(\varphi' = \varphi_0 - \tilde{\varphi}\), the infinite square well will be symmetric with width \(d\) and we can define the absolute return as \(r = \varphi'/\tilde{\varphi}\). So
the rate of the return is the new coordinate variable and the well’s width becomes $d = 20\%$. Now we can write the GUP corrected Hamiltonian inside the well in the absence of external factors approximately as

$$\hat{H} = -\frac{1}{2m} \frac{\partial^2}{\partial r^2} + \frac{\beta}{3m} \frac{\partial^4}{\partial r^4},$$  \hspace{1cm} (15)$$

where $m = m_0 / \bar{\wp}^2$, $\beta = \beta_0 / \bar{\wp}^2$, and it is valid to first-order of GUP parameter \([14]\). This Hamiltonian has exact eigenvalues and eigenfunctions \([22]\):

$$\phi_n(r) = \sqrt{\frac{2}{d}} \sin \left( \frac{n\pi(r + d/2)}{d} \right),$$  \hspace{1cm} (16)$$

$$E_n = \frac{n^2\pi^2}{2md^2} + \frac{\beta n^4\pi^4}{3md^4},$$  \hspace{1cm} (17)$$

where $n = 1, 2, 3, \ldots$. To write the total Hamiltonian we need to add the potential which describes the effects of information on the stock price. The market information usually results either in the increase of the stock price or in the decrease of the stock price. Here, similar to Ref. \([18]\), we consider a periodical idealized model which represents the two types of information. This form of potential also appears for a charged particle moving in an electromagnetic field with the difference that the information play the role of the external fields. So up to the dipole approximation we can write the GUP corrected Hamiltonian of this coupled system as

$$\hat{H} = -\frac{1}{2m} \frac{\partial^2}{\partial r^2} + \frac{\beta}{3m} \frac{\partial^4}{\partial r^4} + \lambda r \cos \omega t,$$  \hspace{1cm} (18)$$

where $\omega$ is the frequency of information and $\lambda$ denotes the amplitude of the information field. The first two terms of the above equation represent the GUP corrected kinetic energy of the stock return and the last term corresponds to the potential energy due to presence of information in the stock market. Note that, the choice of the Hamiltonian in \([18]\) is not the only one and we can replace $\cos(\omega t)$ with $\sin(\omega t)$, as well as with some other periodic functions.

To find the temporal evolution of the wave function in price-representation, we need to solve the following Schrödinger equation

$$i \frac{\partial}{\partial t} \psi(r, t) = \left[ -\frac{1}{2m} \frac{\partial^2}{\partial r^2} + \frac{\beta}{3m} \frac{\partial^4}{\partial r^4} + \lambda r \cos \omega t \right] \psi(r, t).$$  \hspace{1cm} (19)$$
To solve this equation, we can use the perturbative procedure that is also used in Ref. [9] in connection with stock markets. Since the exact solutions for $\lambda = 0$ is presented in Eqs. (16) and (17), we can expand the solutions in terms of these state vectors [24]

$$\psi(r,t) = \sum_{n} c_n(t) e^{-iE_n t} \phi_n(r),$$  \hspace{1cm} (20)

where

$$c_n(t) = c_n(0) - i\lambda \sum_{k} \langle n|r|k \rangle \int_{0}^{t} dt' c_k(t') \cos \omega t' e^{-i(E_k - E_n)t'}. \hspace{1cm} (21)$$

By repeatedly substituting this expression back into right hand side, we obtain an iterative solution

$$c_n(t) = c_n^{(0)} + c_n^{(1)} + c_n^{(2)} + \ldots, \hspace{1cm} (22)$$

where, for instance, $c_n^{(0)} = c_n(0)$ and the first-order term is

$$c_n^{(1)}(t) = -i\lambda \sum_{k} \langle n|r|k \rangle \int_{0}^{t} dt' c_k(0) \cos \omega t' e^{-i(E_k - E_n)t'}. \hspace{1cm} (23)$$

If we take the initial wave function as the ground state of unperturbed Hamiltonian (a cosine distribution to simulate the state of stock price in equilibrium) i.e. $\psi(r,0) = \langle r|1 \rangle = \sqrt{\frac{2}{d}} \cos \left( \frac{\pi r}{d} \right)$, we have $c_n(0) = \delta_{1n}$ which results in [9]

$$c_n^{(1)}(t) = -i\lambda \langle n|r|1 \rangle \int_{0}^{t} dt' \cos \omega t' e^{-i(E_1 - E_n)t'}, \hspace{1cm} (24)$$

where $\langle n|r|1 \rangle = -\frac{8nd}{(n^2 - 1)^2\pi^2}$ for $n$ even and $\langle n|r|1 \rangle = 0$ for $n$ odd. By evaluating the time integral we get

$$c_n^{(1)}(t) = \lambda \frac{4nd}{(n^2 - 1)^2\pi^2} \left( \frac{e^{iE_n - E_1 + \omega} - 1}{E_n - E_1 + \omega} + \frac{e^{iE_n - E_1 - \omega} - 1}{E_n - E_1 - \omega} \right), \hspace{1cm} (25)$$

for $n$ even and $c_n^{(1)}(t) = 0$ for $n$ odd. As this equation shows, at the characteristic frequency $\omega_n = E_n - E_1$ we observe a large transition probability from ground state to $(n - 1)$th excited state, namely

$$\omega_n = \omega_0 \left( 1 + \frac{4}{3} \beta m \frac{n^2 + 1}{n^2 - 1} \omega_n^0 \right), \hspace{1cm} (26)$$
where \( \omega_0 \) are the characteristic frequencies at the continuous limit. In the Chinese stock market the average stock price is approximately 10 Yuan and \((\Delta \phi)_{\text{min}} = 0.01\) Yuan. To find \( m \) we need to calculate the mean daily volatility from annual volatility given by

\[
\sigma_{\text{daily}} = \sqrt{\frac{1}{252}} \sigma_{\text{annual}},
\]

since there are 252 trading days in any given year. For instance, the annual volatility of Chinese stock market during 2001–2002 was about 0.3% [25] which results in \( m = \sigma_{\text{daily}}^{-2} \simeq 3 \times 10^3 \). So we obtain \( \beta_0 = (\Delta \phi)^2_{\text{min}} = 100\beta \simeq 10^{-4} \) and \( \omega_n^0 \simeq 4 \times 10^{-3}(n^2 - 1) \) s\(^{-1} \). In the presence of the minimal trading value we observe a frequency dependent positive shift in the characteristic frequencies proportional to \( \omega_n^0 \) as \( \omega_n \simeq \omega_n^0 \left[ 1 + 4 \times 10^{-3} \left( \frac{n^2+1}{n^2-1} \right) \omega_n^0 \right] \) or

\[
\omega_n \simeq \omega_n^0 \left( 1 + 4 \times 10^{-3} \omega_n^0 \right),
\]

for large \( n \). Since \( \omega_n > 4 \times 10^{-3} \) s\(^{-1} \) if a single cycle of information fluctuation is larger than 25 minutes there is no large transition probability to other states and the probability density of the rate of stock return approximately maintains its shape over time (see Fig. 2 of Ref. [18] for \( \omega = 10^{-4} \) s\(^{-1} \)).

Note that, in quantum gravity, it is usually assumed that the minimal length is of the order of the Planck length \( \ell_{\text{Pl}} \sim 10^{-35} \) m. However, the existence of this infinitesimal length is not yet confirmed by the experiment. On the other hand, in quantum finance the minimal trading value is not too small which makes essentially detectable effects. In other words, the application of GUP in quantum description of finance is more meaningful than in quantum physics.

5 Conclusions

We have studied the effects of the discreteness of the stock price on the quantum models for the stock markets. In this formalism, the minimum trading value of every stock is not zero and the stock price and its trend satisfy the generalized uncertainty relation. This modifies all Hamiltonians of the stock markets and adds a term proportional to the fourth power of the trend to the Hamiltonians. For the quantum model proposed by Zhang and Huang where there is a price limit rule and the information has a
periodic fluctuation, we obtained the characteristic frequencies of the quantum system. If the frequency of information fluctuation coincides with $\omega_n$, we have a large transition probability to $(n - 1)$th excited state. We also showed that the discrete nature of the stock price results in a positive frequency dependent shift in characteristic frequencies where for the Chinese stock market we have $\omega_n > 4 \times 10^{-3} \text{s}^{-1}$.

References

[1] R.N. Mantegna and H.E. Stanley, An Introduction to Econophysics: Correlations and Complexity in Finance, Cambridge University Press, Cambridge, 1999.

[2] K. Ilinski, Physics of Finance, Wiley, New York, 2001.

[3] B.E. Baaquie, Quantum Finance, Cambridge University Press, Cambridge, 2004.

[4] C. Ye and J.P. Huang, Non-classical oscillator model for persistent fluctuations in stock markets, Physica A 387 (2008) 1255-1263.

[5] A. Ataullah, I. Davidson, and M. Tippett, A wave function for stock market returns, Physica A 388 (2009) 455-461.

[6] F. Bagarello, Stock markets and quantum dynamics: a second quantized description, Physica A 386 (2007) 283-302.

[7] F. Bagarello, An operatorial approach to stock markets, J. Phys. A 39 (2006) 6823-6840.

[8] F. Bagarello, The Heisenberg picture in the analysis of stock markets and in other sociological contexts, Qual. Quant. 41 (2007) 533-544.

[9] F. Bagarello, A quantum statistical approach to simplified stock markets, Physica A 388 (2009) 4397-4406.

[10] F. Bagarello, Simplified stock markets described by number operators, Rep. Math. Phys. 63 (2009) 381-398.
[11] M. Schaden, Quantum finance, Physica A 316 (2002) 511-538.

[12] D. Meyer, Quantum strategies, Phys. Rev. Lett. 82 (1999) 1052–1055.

[13] J. Eisert, M. Wilkens, and M. Lewenstein, Quantum games and quantum strategies, Phys. Rev. Lett. 83 (1999) 3077–3080.

[14] F. Black and M. Scholes, The pricing of options and corporate liabilities, Journal of Political Economy, 81 (1973) 637–654.

[15] R.C. Merton, Continuous Time Finance, Blackwell, 1990.

[16] E. Haven, A discussion on embedding the Black-Scholes option pricing model in a quantum physics setting, Physica A 304 (2002) 507–524.

[17] M. Schaden, review of the book “Interest Rates and Coupon Bonds in Quantum Finance” by B.E. Baaquie, Am. J. Phys. 78 (2010) 654–656.

[18] C. Zhang and L. Huang, A quantum model for the stock market, Physica A 389 (2010) 5769-5775.

[19] A. Kempf, G. Mangano, and R.B. Mann, Hilbert space representation of the minimal length uncertainty relation, Phys. Rev. D 52 (1995) 1108–1118.

[20] D. Amati, M. Ciafaloni, and G. Veneziano, Can spacetime be probed below the string size?, Phys. Lett. B 216 (1989) 41–47.

[21] S. Das and E.C. Vagenas, Universality of quantum gravity corrections, Phys. Rev. Lett. 101 (2008) 221301 [4 pages], [arXiv:0810.5333].

[22] P. Pedram, A class of gup solutions in deformed quantum mechanics, Int. J. Mod. Phys. D 19 (2010) 2003–2009, [arXiv:1103.3805].

[23] P. Pedram, On the modification of the Hamiltonians’ spectrum in gravitational quantum mechanics, Erouphys. Lett. 89 (2010) 50008 [5 pages], [arXiv:1003.2769].
[24] D.J. Griffiths, Introduction to Quantum Mechanics, Prentice Hall, Upper Saddle River, NJ, 1995.

[25] Y. Xu, Diversification in the Chinese stock market, Working Paper, School of Management, The University of Texas at Dallas and Shanghai Stock Exchange (2003).