On the Influence of Gravity on the Propagation of Light
Some Comments on the A. Einstein’s 1911 Paper
(The Mistake Made In 1911)
1911 – 2022

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Abstract

This is a review with some comments on the A. Einstein’s 1911 paper, which he published as one of his many attempts of the general theory of relativity. The main point of the idea is to propose a new approach about light and its motion as well as study the assumption made by A. Einstein concerning the possibility of the variation of the speed of light in presence of Gravitational Field.

§1. A. Einstein’s 1911 paper

When A. Einstein realized, in 1907, the incompatibility of his ideas with the Newtonian’s concepts about gravity he tried to reconcile his ideas with the special relativity and find out a new form to treat gravity using the equivalent principle as a light. Even when H. Minkowski found the geometry meaning of the Lorentz transformation A. Einstein did not use it at all in 1911.

In his article he derived some physical phenomena using the equivalent principle on the propagation of light in a fixed gravitation field. All these consequences were obtained by A. Einstein and showed the importance of the idea between local gravity and non-inertial reference systems. There is not doubt that all these consequences obtained were very unusual during that time, however could there be another form to obtain the same results in particular in relation with the bent and propagation of light in the presence of gravitational field.

The idea in 1911 that was used to derive the bent of light is based on the assumption that the speed of light could change in a presence of gravitational field, is say; the speed is a function of the points in the presence of gravity as Einstein deduced:

\[ c = c_0 \left( 1 + \frac{\phi}{c^2} \right) \]

The equation above allowed A. Einstein to derive the angle of deflection of a light when passes through of gravitational field produces by a heavenly body for instance the Sun. The result obtained was a half of the experimental got in 1916 – 1919 which was predicted using all the frame-work of the general theory of relativity. Go back in 1911 the assumption made about the possibility of the change of the speed of light expresses by the equation deduced by Einstein is a heuristic way, from my point of view, to face the problem due to there is a nice way to possibly obtain the same result and all the physical consequences deduced from it as well.
Let’s now recap to refresh the memory about the assumptions made by A. Einstein in 1911 as we could think were used at that time, this I supposed were made by him and expressed it in his 1911 paper.

1. The Equivalent Principle (Local gravity phenomenon could be mimic as an effects of no-inertial reference systems).
2. The propagation of light in a presence of gravitational field could be treated as wave phenomenon.
3. Static gravitational field (At that moment of time there were not relation between the gravitational potential and the metric tensor $g_{\mu\nu}$ even though H. Minkowski had been published his memory unifying space and time but A. Einstein did not believe that was useful for his seeking).

After being mentioned the assumptions made, is time to analyze what Einstein proposed in 1911 in what is the comments about this issue. In order to do so is necessary to explore how light propagate, so as to obtain the same result as Einstein got.

§2. Propagation of Light

In his article Einstein[1] assumed that the propagation of light through space and in a presence of gravitational field is as a wave phenomenon, so by means of this idea light could be undergo deflection and this could be study using Huyghen’s principle as can be read in his 1911 paper where was obtained a first approximation the deflection or the expected deviation angle $\delta \alpha$. However the same result was obtained many years before him, in 1804 with the idea that really light could propagate as point-like particle with a constant speed of $c$ and using Newton’s ideas of gravity as Einstein did as well.

So the two approaches gave the same value for the deflection angle $\delta \alpha = 0.875$ seconds of arc. Even though Einstein’s approach was in relation of his idea of Equivalent Principle seems more natural the Newtonian’s approach even when the treatment gave the same result however there is nothing new in that. Why both approaches gave the same result? is something interesting to think of. Now taking into account Einstein’s point of view of propagation of light one could ask, Why this result can not be obtained from the Electromagnetism frame-work? if light is not a gravitational phenomenon and if that is possible, which I think it is. It is necessary to modify the general theory of gravity or both but we are getting sidetracked, so let’s continue with the discussion.

Is amazing to think that A. Einstein even after having developed his theory pointed out the idea that the speed of light could be depends on the gravitational phenomenon and which is even more enchanting is that, all the electromagnetism phenomenon does not appear in the general theory and some properties was covered only by the special theory of relativity. The only link as many specialist point out even Einstein did is through the Energy-momentum Tensor, $T_{\mu\nu}$. For instance, can be read in [6] some of the mentioned in the paragraph, which I encourage to the reader to do so.

As Einstein thought in 1911 there is not need to consider Maxwell’s equation in order to calculate the deflection angle and study light propagation in space, just assume that light is an EM wave, even in a presence of gravitational field and from that the only way light can bending (A natural way due to the assertion made then) is under the assumption of the change of the speed of light given by the equation:

$$c = c_0 \left(1 + \frac{\phi}{c^2} \right)$$

Of course the deflection angle is a first order of magnitude, no consider $0(c^4)$ as Einstein did in his calculations of the 1911 paper. However the result of bending light can be obtain without Einstein’s idea of light speed variation. Is also interesting that this idea of possible change in the speed of light is not used to derive the others results as the change of the frequency of light in homogeneous and static gravitational field.
§3. Conservation Energy Law and Light Propagation

Instead of consider a wave light propagation, let’s use the quantum picture of light and parallel to this use the energy conservation applied to conservative system like we are going to study. Let’s imagine a process of interaction between light and the gravitational field in small region of flat space-time where the gravity potential is given and fixed and try to get the same result as Einstein got. By doing use of this idea it is possible to write the following equation:

\[ h\omega + \frac{h\omega}{c^2}\phi = h\omega' + \frac{h\omega'}{c^2}\phi' \]  

(1)

From the equation [1] and taking in consideration the equivalent principle, we can reduce the region of study in such manner that the equation [1] could be written as follows:

\[ -\Delta\omega = \frac{\phi\Delta\omega}{c^2} + \frac{\Delta\omega\Delta\phi}{c^2} + \frac{\omega\Delta\phi}{c^2} \]  

(2)

Keeping in mind , in the equation [2] that we are study small region with a given gravitational field is possible to get:

\[ -\left(1 + \frac{\phi}{c^2}\right)\,d\omega = \frac{\omega\,d\phi}{c^2} \]  

(3)

\[ \omega = \frac{\gamma}{\left(1 + \frac{\phi}{c^2}\right)} \]  

(4)

The equation [4] represents the dependence of the frequency of the photon as a function of the gravitational potential at a point \( x \). From this equation is possible to obtain the significance of the constant \( \gamma \), if we consider the motion of the photon far away of any source of gravity, is say; when \( r \to +\infty \) then \( \phi \to 0 \). Therefore is possible to write the following equation which is one of the many results obtained by A. Einstein in first approximation:

\[ \omega = \frac{\omega_0}{\left(1 + \frac{\phi}{c^2}\right)} \]  

(5)

The equation [5] tell us that far away from any source of gravity the photon carry energy of amount of \( E = h\omega_0 \), which can not be take it out. At first glance only this result is something to put attention on it.\(^1\) All this suggest that really the process is an interaction between the gravitational field and the corpuscular side of the electromagnetism phenomenon, photon, and is not a simple propagation of EM wave in the region which there is gravitational field. To be honest is important to give more thought on this issue and get a final and sound argument on how really light propagate in a presence of gravity but at this point it seems sensible that the conclusion written above is possibly solid. A study of the propagation is crucial in order to comprehend really what is the issue here with electromagnetism and gravity.

§4. Light and the Frequency Shift

In vacuum, light propagate with a constant speed in all possible directions. That is the statement said by A. Einstein in 1905 which was used to build up his theory of special relativity, with experimental support, However in 1911 he changed his mind about it. Even in his general theory of relativity pointed out that in a strong gravitational field this postulate will be not more valid and the speed\(^1\)

\(^1\)Bear in mind that this statement is new, photon could interact with the gravitational field in sense of the Compton Effect but in a more complicated way
should change under those circumstances.

In order to continue the main idea, is reasonable to postulate the following statement and from it deduce all possible physical consequences, so let’s establish that:

Let’s consider that in a weak gravitational field the speed of light should remains constant as well as in vacuum. Following this idea is possible to write an equation which could be consider as identity, some kind of constrain, and a fundamental as well, so we have:

\[ c = \nu_1 \lambda_1 = \nu_2 \lambda_2 = \text{constant} \]

\[ \omega_1 \lambda_1 = \omega_2 \lambda_2 \]

In light of this we can write the two main equations need it to derive some physical conclusions as follows, also considering at the same time equation [4].

\[
\begin{align*}
\omega_1 \lambda_1 &= \omega_2 \lambda_2 = \cdots \\
\omega &= \frac{\gamma}{\left(1 + \frac{\phi}{c^2}\right)} \\
|\phi| &> 0
\end{align*}
\]

(6)

With this, we can obtain some physical results by playing with those equations [6]. The third equation is just the condition that potential gravitational field is negative due to the fact we make at \(+\infty\) the gravitational potential equal to 0. Consider a small region and taking valid, of course, the equivalent principle we are going to try to derive the frequency shift of light using [6].

Imagine two points in a given gravitational field \(\phi\). Let at point \(x_2\) be the potential \(\phi = 0\) and let at point \(x_1\) be the potential \(\phi\) both points lies on the same potential line. Using the conservative property of gravitational field and using the equations [6], so we have:

\[
\frac{\gamma \lambda_2}{\left(1 + \frac{0}{c^2}\right)} = \frac{\gamma \lambda_1}{\left(1 + \frac{\phi}{c^2}\right)}
\]

which can be reduced as follow:

\[
\lambda_2 \left(1 + \frac{\phi}{c^2}\right) = \lambda_1
\]

After some algebraic manipulations and having been done all the intermediate mathematical steps, we can get the follows relationship between \(\lambda\) and the potential gravitational field \(\phi\):

\[
\lambda_2 - \lambda_1 = -\frac{\phi}{c^2} \lambda_2
\]

Considering the third equation in [6], we have:

\[
\frac{\lambda_2 - \lambda_1}{\lambda_2} = -\frac{\phi}{c^2} > 0 \leftrightarrow \lambda_2 > \lambda_1 \leftrightarrow \nu_1 > \nu_2
\]

\[
\frac{\Delta \lambda}{\lambda_2} = -\frac{\phi}{c^2}
\]

(7)

The last equation in [7] is the well-known relationship between the wave-length of the EM, photon, with the gravitational potential. Which is the same mathematical expression A. Einstein got in his 1911 paper. Even though here in this analysis appear the wave-length instead of the frequency, it is easy to demonstrate that both expressions are completely equivalents. So we get the same expression
using a different approach but at the end the physical meaning and process is basically identical, both goes one to one hand.

\[ \frac{\Delta \lambda}{\lambda_2} = -\frac{\phi}{c^2} \]  \hspace{1cm} (8)

The equation [8] shows us the frequency shift of the Electromagnetic phenomenon, or using the corpuscular side; the loss or gain in energy of the photon in a gravitational potential field, either way upward or downward motion.

Considering the general case of an atom, for instance in the Sun which emitted a photon and it is received on Earth. We have in this particular case that the gravitational potential at these points are \( \phi_1, \phi_2 \) and using again the equations (1, 2) in [6], we have:

\[ \frac{\gamma \lambda_1}{\left(1 + \frac{\phi_1}{c^2}\right)} = \frac{\gamma \lambda_2}{\left(1 + \frac{\phi_2}{c^2}\right)} \]

which is equal to:

\[ \lambda_2 = \left(1 + \frac{\phi_2}{c^2}\right) \frac{\lambda_1}{1 + \frac{\phi_1}{c^2}} \]  \hspace{1cm} (9)

The above equation [9] can be rearrange as follows and could be compare with the equation in the 1911 paper, which is basically the same expression with the same physical meaning(have been obtained the same result):

\[ \frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{\phi_1 - \phi_2}{c^2} \left(1 + \frac{\phi_1}{c^2}\right) = \frac{\Delta \lambda}{\lambda_1} \]  \hspace{1cm} (10)

If in the mathematical expression [10] we consider that the gravitational potential could be a small quantity, is possible to make the following approximation which results coincides with the limit case found in the general theory of gravity, is say:

\[ \frac{\Delta \lambda}{\lambda_1} \approx \frac{\phi_1 - \phi_2}{c^2} \]

Throughout the paper I don not have mentioned anything about the observer which play a crucial role in the paper published by A. Einstein, however the importance of observers is implicit since to compute all the calculations is essential to know where the radiation is coming from, in other words; which point in a flat space-time is emitting or receiving the photon, EM, so with that information we can get from it a much more valuable physical information.

For instance, let’s imagine that the point called \( x_2 \) is on the surface of Earth and consider also a homogeneous gravitational field as always. Let at \( x_1 \) be the point at which the potential is equal to \( \phi \)
so in this case the equation [8], the right-hand side is negative so we can write it in the form:

\[ \lambda_2 - \lambda_1 = -\frac{\phi}{c^2} \lambda_2 \]

consider the relation:

\[ \lambda_1 = \frac{c}{\nu_1}, \quad \lambda_2 = \frac{c}{\nu_2} \]

\[ \nu_1 - \nu_2 = -\frac{\phi}{c^2} \nu_1 \]

From equation [11] we can obtain the well-known result from the special theory, is say:

\[ \frac{\nu_2 - \nu_1}{\nu_1} = \frac{\phi}{c^2} \]

Whereby a shift of the spectral lines of the source of light would be measured at point \( x_1 \). All the comments is nothing new, however is surprisingly interesting that using the conservation law of energy and the assumption of the constancy of the speed of light in weak gravitational fields led us. Others physical phenomenon can be study as well, using either Einstein’s view of point or the all done above. Which could be sensible to imagine is that all this can be found using the general theory of gravity or the fact that by means which gravity and EM interact is through a intrinsic and complicated fields interaction process rather than just making a conjecture that light propagate as a wave or using the corpuscular side, the quantum aspect.

I will be eager to face this task perhaps in another article. Meanwhile let’s continue with the task of what we are trying to do. All the statement found by Einstein is correct in first order of magnitude as he mentioned in his article. In this approach there is none of that kind of approximation, perhaps will be appear in the computation of the expected deviation angle, but this does not mean that all what the reader could find here is a replacement of Einstein’s ideas it’s rather a support of his point of view in Physics.

§5. Deflection of Light in Flat Space-Time

Let’s calculate first if a light-ray could be trapped by a heavenly body under the studied conditions. Imagine a photon that moves from a region with no gravity presence and it approaches to a heavenly body of mass \( M \) from the equations [6] we can have the total energy of the conservative system as:

\[ E = \hbar \omega - \frac{\hbar \omega}{c^2} \phi \]

\[ \omega = \frac{\omega_0}{\left(1 + \frac{\phi}{c^2}\right)} \]  

(12)

Substituting the second equation of [12] into the first and avoided some steps, we have:

\[ E = \hbar \omega_0 \left(1 - \frac{\phi}{c^2}\right) = \hbar \omega_0 \]

\[ \frac{\nu_2 - \nu_1}{\nu_1} = \frac{\phi}{c^2} \]

(13)

Which clearly means that the light-ray can not be trapped by any gravitational field under the assumption made in this analysis. Since from celestial classical mechanics is well-known that only

6
when the total energy of the system is $E < 0$ should be exists a closed trajectory either a circle or ellipse, therefore by this perspective is possible for light only a small deviation from its initial path. To consider the possibility that light could be trapped by a gravitational field is necessary others considerations coming from like general theory of relativity, electromagnetism or even quantum mechanics.

Is understandable to see that the energy condition $E \geq 0$ permits only a parabolic or hyperbolic trajectory. However between these two options the only one reasonable it is the hyperbolic trajectory due to $E > 0$ and from this we can deduce an expect small angle of deviation $\delta \alpha$. In other words from the analysis done above we arrived to the sound conclusion that light would be undergo a deviation from its original path in a region where there is a weak gravitational field and goes beyond this we could say that in any gravitational field light undergo deflection and its speed, and spite of could be in a mistake, should remains constant which is a counter argument about what A. Einstein wrote in his 1911 paper and some comments he expressed implicitly in the successive developments of his general theory of gravity [6].

§6. Expected Angle of Deviation

From equation [13], we know beforehand the possible trajectory light could take when passes through a heavenly body like the sun, however try to obtain the expected deflection angle is not so easy. So in order to do that, we need to take some concepts from the geometry point of propagation of light as tenable argument, $\lambda \to 0$, under this condition and from equation [6] we can use the eikonal equation to attempt in first approximation $\delta \alpha$.

As is now from geometry light theory there is a relation between these physical magnitudes $\psi$ and $\omega$, is say:

$$\frac{\partial \vec{k}}{\partial t} = -\frac{\partial \omega}{\partial \vec{r}}$$

And the equation

$$\omega = \frac{\omega_0}{\left(1 - \frac{\phi}{c^2}\right)} \approx \omega_0 \left(1 + \frac{\phi}{c^2}\right)$$

(14)

Taking the partial derivative we have:

$$\frac{\partial \omega}{\partial \vec{r}} = \vec{\nabla} \omega = -\frac{\omega_0 GM}{c^2} \frac{\hat{r}}{r^2}$$

(15)

With equations [14] and [15] we can deduce the change in the $\Delta \vec{k}$ very easily, so we get:

$$-\hbar \Delta \vec{k} \cdot \hat{r} = -\frac{GM \hbar \omega_0}{c^2} \int \frac{d\vec{s}}{r^2}$$

$$\hbar |\Delta \vec{k}| = \frac{GM \hbar \omega_0}{c^2 \Delta} \int_0^\pi \sin \phi d\phi = 2 \frac{GM \hbar \omega_0}{c^2 \Delta}$$

(16)

Equating the last two equations of [16], we can get in a first approximation the expected angle as A. Einstein got in his 1911 paper, with the consideration of weak gravitational field but maintaining constant the speed of light in presence in such kind of field. I suggest that this proposal must be a necessary request to Nature in the light that we obtained the same results without the Einstein’s conjecture of variable speed of light as he suggested in his earlier works on general theory of gravity, which have to be called a law of general field geometry gravity. finally we get:
\[
\sin \left( \frac{\delta}{2} \right) = \frac{GM}{c^2 \Delta} \quad \Leftrightarrow \quad \delta \approx 2 \frac{GM}{c^2 \Delta}
\]

A full derivation from the general relativity of the expected deviation angle can be found in [5] which take into account Einstein’s idea on gravity, is say, gravity as geometry model where the potential gravitational are encoded in the metric tensor: \( g_{\mu\nu} \) or in [4] using Newton’s ideas and consider light as flow of particles. In the limit case of weak gravitational field this classical potential are the temporal component of the metric:

\[
g_{00} = 1 + 2 \frac{\phi}{c^2}, \quad |\phi| > 0
\]

of course in first order of approximation. With this the line element \(^2\) of a non-zero mass particle could be written as:

\[
ds^2 \approx \left( 1 + 2 \frac{\phi}{c^2} \right) \, dx^{02} - dr^2
\]

Which the speed of light is a fundamental constant even in the presence of a weak gravitational field. However in the 1911 paper there was not such idea and basically all derivation was done without have been used this idea but in order to obtain the expected deviation angle a variation speed was proposed by him whereby permits him to treat the light propagation as a wave and possible bending was calculated but in first order of magnitude and in unnatural way to got it, neglecting terms like \( \mathcal{O}(c^4) \).

§7. What Does all this mean?
A kind of conclusions of This First Work

First of all, there is nothing new in this paper, apart for the idea of interaction process mentioned many times above, photon-gravity, which are not known today for the physicist community around the world however the main points was to exposure the Einstein’s ideas in 1911 concerning the implicit postulate that speed of light must change in presence of gravitational field. Under this light was possible to compute an expected deviation angle using the wave character of light. In counter of this argument which seems reasonable it was demonstrated such variation of the speed is not a sound argument and is unnecessary in calculate the deviation angle of the light from its original path.

I am not contradicting what Einstein did in 1911 but certainly was not careful in that aspect concerning of the propagation of light as wave, I comprehend that if light propagate as wave the possible mechanism for an expected deviation angle was using the notions coming from wave aspect of the light. Moreover we can found that the same value of the expected deviation angle was computed using the idea of corpuscular aspect and the value was practically the same in order of magnitude. Besides all this, Einstein computation of the deviation angle appears not natural from the physical point of view. Let me put again the equation which is easy to see the major problem with the concept of variation and the necessity of taking only the first order of magnitude and neglect terms of \( \mathcal{O}(c^4) \):

\[
c = c_0 \left( 1 + \frac{\phi}{c^2} \right)
\]

As is easy to see there is not use of electromagnetism theory to predict the deviation angle even in his general theory of relativity is not possible to obtain it using both electromagnetism and gravity, not to consider the \( T_{\mu\nu} \), it is just necessary use gravity as a physical field phenomenon. All the aspects of light in general relativity is absence and only in the special theory some properties are explains and incorporated. Therefore is something to take into consideration and perhaps an indication of the

\(^2\)Could be interesting to derive this result using the main idea developed here as interaction process between the photon and the gravity phenomenon
incompleteness of the general theory of relativity.

So speed of light should remains constant in a weak gravitational field and perhaps in a strong gravity have to remains constant. The reason for what this is not so clear to me now is in relation to the face of many mathematical walls however a possible geometrical constain could gives us an answer to this crucial issue. I comprehend the question is difficult and much more to figure out a possible solution or near solution but if were possible the unification between gravity and electromagnetism a solution will coming from it.

As is known $F_{\mu\nu}$ remains unchanged under the presence of gravity, which means that there is not interaction between these two fields that dominate basically the macro-world. After all this, the assumption made here about leave constant the speed of light was successful, because the same physical phenomena were obtains and even the expected deviation angle. Which indicates that not only in vacuum the speed is a constant, also and presence of gravity should remains constant. Doing this we avoided some major difficulties in deal with the equation [19]. I am not interesting in go beyond what I did, for now, but I hope this will help others to think about what are the main difficulties physics nowadays facing.

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First Paper

§8. Energy and Momentum of Light
Its Minimum Value Energy.

Is very known since 1900 – 1905 is possible to associate to light both energy which is a function of its frequency $E = h\nu$ and momentum $E = pc$. The second equation is consequence of the fact that rest-mass of the photon is zero so basically is one of many results from the special theory of relativity, is say, come from the relativistic equation between energy and momentum $E^2 = p^2c^2 + m_0^2c^4$. What perhaps kind of new is what was mentioned in the article [3] about a minimum value concerning the energy of the photon far away from gravitational field or others kind of interactions, classically speaking of course.

This minimum value energy it will be denoted as:

\[
\omega = \frac{\omega_0}{\left(1 + \frac{\phi}{c^2}\right)} \quad |\phi| > 0
\]

(18)

Where now:

$E = h\omega_0 = h\nu_{\text{min}}$, $\phi \to 0$ when $r \to \infty$

So the idea is that this value frequency $\nu_{\text{min}}$ can not be taken it out from the photon, in other words the energy associated with it is the minimum value possible that the photon could has, isolated from any kind of interactions. It can be summarize saying that the photon can not loss all its energy, so the limit value is: $E = h\nu_{\text{min}}$. For simplicity it will noted as: $E_0 = h\nu_0$. Concerning how possible is to obtain this value experimentally is a difficult issue to think of. In the other hand we could possible think about its consequences for instance in the cosmology aspects. How the expansion of the universe could affects this value energy of the photon? Would be exists some kind of constraints due to this value energy and could affect to the expansion of Universe? This point turns out to be interesting.

Assuming the two relations related with light we can deduce the Doppler effect with the aid of the
Lorentz energy transformation:

\[ E = \gamma (E' + \beta cp') \]

Putting the follows identity

\[ E' = h\nu', \ E = h\nu \text{ and } p = \frac{E'}{c} \]

We have

\[ \nu = \gamma (\nu' + \beta \nu') \]  

(19)

From equation [19] we get the well-known result:

\[ \nu' = \nu \sqrt{\frac{1-\beta}{1+\beta}} \]  

(20)

Which is nothing new but the derivation is much easier than others. What seem to me interesting is the used of the idea of the quantum side of the electromagnetic phenomenon. Having in mind that what was obtained above is just the classical Doppler effect which are very well studied. A second derivation is given by the follows procedures:

If we imagine two reference system being one of them a stationary observer whose the emission source of light is located and the other observer is moving with a constant speed along the \( x \)-axis, we can compute the relation between these measurements of the two wavelengths by the following equation:

\[ \frac{1}{\lambda} = \gamma \left( \frac{1}{\lambda'} + \frac{\beta}{c} \nu' \right) \]

\[ \lambda \nu = \lambda' \nu' = \text{Constant} \]  

(21)

Using the second equation [21] as a constrain, we can obtain:

\[ \nu' \gamma \left( 1 + \frac{u}{c} \right) \lambda = \lambda \nu \]

\[ \beta = \frac{u}{c} \]

We have

\[ \nu' = \nu \sqrt{\frac{1-\beta}{1+\beta}} \quad u > 0, u < 0 \]  

(22)

From this point of view as always happen in any physical model, there is a kind of problem so in this case there is a big problem when the velocity \( u \) tends to \( c \) is say, \( u \to c \). Therefore as consequence of this, the energy can goes up at any value or goes down up to be zero and those kind of thing are very problematic and nonphysical. Of course this is not new but the issue here is how avoid these nonphysical phenomena. As I can see these situations is very common in many physics areas, which surprised me strongly. In this particular case concerning the motion of light we have the 4—four quantity vector \( dk^\alpha = 0 \), this condition have to be modified when there is a gravitational field in the region where light it moves.

Let’s now clarify the meaning of the constraint condition related with the constancy of speed of light, which was used in the article [3] to predict a possible variation of the wavelength of the light moving through a homogeneous and static gravitational field.

\[ \lambda_0 \omega_0 = \lambda_1 \omega_1 = \lambda_2 \omega_2 = \cdots = \text{Constant} \]  

(23)
Now equation [18] tell us something interesting. These equation can be used to obtain the shift in the frequency of the light, however there is no mention about whether or not the speed undergo a variation during its motion, as Einstein thought it could be possible and uses it in order to obtain the expected deviation angle as can be read in his 1911 article, on the other hand the same result is possible using the fact that actually the propagation of light is an interaction process with gravity of course that a full description perhaps would be given using quantum electrodynamics (QED) but that idea is far from the scope of this article.

Equation [23] is basically a constrain imposed to the motion of light therefore along its path this must be satisfied, resulting that \( c \) remains constant during its motion at every moment of time and points. Also can be applied to different observers due to the speed of light is a constant in special theory of relativity, is say:

\[
\lambda \omega = \lambda' \omega' = \lambda'' \omega'' = \cdots
\]  

Meanwhile equation [18] can be applied only in two points, where is the source of light, the emitted, and in the arrived point. Nevertheless, this equation together with equation [24] can describe all the physical process of the propagation maintaining constant always the speed of light. Of course not all physical process can describe using this two ideas due that there is not use of the electromagnetism ideas, which is an covariant-theory concerning the Lorentz transformation. At this point I understand the issue about whether or not light it could undergo variation in its speed however is much natural and advantage suggestion that this should be remains constant if this is also true in the general theory in its full frame-work is something that we need to think of. Concerning this issue, I can say that also in the general theory of relativity equation [23] could be modified or by some reason could be remains in that form. For me is amazingly that the electromagnetism is left out somehow by the theoretical frame-work of the general theory of relativity which its the foundation and only the tensor energy-momentum is the unique link between them, \( G_{\nu \mu} = \gamma T_{\nu \mu} \). As we have mentioned in the first article.

Following the idea of the unchanged of speed of light, from the computations made in the article [3] we expect a variation in the wavelength apart of a frequency shift variation in the motion of light in a gravitational field which was not considered by Einstein’s 1911 paper. This variation as was calculated is:

\[
\frac{\Delta \lambda}{\lambda_2} = -\frac{\phi}{c^2}
\]  

However, What does mean such expected variation of the wavelength? Is more common to find out a change in the frequency but equation [25] could be used to calculate the speed of light which value we already known, \( c \). A single source is not enough to do this so we need two source of light with the same wavelength and frequency and compare the displacement of the two wavelength at one point in the gravitational field. Doing the computation to this, we can get:

\[
\frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{\phi_1}{c^2}
\]  

And concerning frequency, we have:

\[
\frac{\nu_1 - \nu_2}{\nu_2} = \frac{\phi_1}{c^2}.
\]  

As can be see in figure (1) we divide it into two components with different values of the potential gravitational but with the condition that the \( \nu \) and \( \lambda \) of both sources is the same. Of course, we found that the result depends only on the \( \phi_1 \) potential. Therefore we can expect a little bit displacement on the wavelength at the point of potential in question. All this is undoubtedly very well-known but
what is differ is the use of the equation:

$$\omega = \frac{\gamma}{1 + \frac{\phi}{c^2}}$$

And the constrain:

$$\lambda \omega = \lambda_1 \omega_1 = \cdots = \text{Constant}$$

What I need to stress and clarify in more detail is the physical concepts contain in equation [1] as we used it here in this context, because I am afraid that could be misunderstanding or could be wrong or either both.

So in order to do that let me first of all, I need to clarify in a plain form what was the real purpose in propose this approach even when could be possible wrong or simply is a wrong manipulation of the idea what I had in mind. In other way the physical ideas which is based on could be wrong.

Even though equation [18] is the limit case obtained from general theory of relativity, the derivation was made in a different way. Using the conservation energy law and the propose that really the propagation of light is an interaction between gravity and using the corpuscular side of the electromagnetic field, was able to arrive at the same equation, however in order to use it is necessary to impose the constraint, is say; the constancy of the speed of light as main requirement.

Concerning this, I would like to draw the attention to a particular article found in [2, 283 – 295] which gave a demonstration of the Lorentz’s transform without using the propagation signal like light as Einstein did in his special theory of relativity. Of course, this is not a contradiction of Albert’s theory and ideas. All the demonstration in[2] is concerning that using only classical ideas is possible to derive the Lorentz’s transformation but what I would like to stress here is the following: If we request to Nature that all observers are equivalents in the sense as Einstein taught us, the only sensible conclusion is that the speed of light have to be constant for all observers as well as the inertial ones.

Let me copy the transforms as general as possible I can, which I think contain some interesting physical aspects :

$$x = \frac{x' + ut'}{\sqrt{1-Du^2}}$$

$$t = \frac{Dux' + t'}{\sqrt{1-Du^2}}$$

Where $D$ is:

$$\frac{1 - d^{-2}(u)}{u^2} = \frac{1 - d^{-2}(v)}{v^2} = D = \text{Constant}$$

Therefore if we take into consideration that, this transformation laws must be satisfied by the electromagnetic phenomena and the only possible and sensible choice is that the speed of light have to be constant, i.e,$D = \frac{1}{c^2}$. However, in spite of this I feel the necessity of a reason, a physical argument how this it so, is say; why in Nature exists a limit of information transmission from one point to another even if this is a local phenomenon and can not be applied to a global structure.
Whether the constancy of the speed of information transmission remains constant a more large scale or not, is something that the current theories can not be give an answer to it as far I know and understand.

Which is surprisingly interesting is how A. Einstein in his 1911 paper [1] did the suggestion about the variation of the speed of light without gave it a sound physical argument. At least all the above written point out that the speed of light is essentially a constant magnitude but I am not talking in relation of its numerical value but rather the fact that the constancy of the information transmission from one point in space-time to another so based on this, the following equation has not physical meaning:

\[ c = c_0 \left(1 + \frac{\phi}{c^2}\right) \]

Moreover is possible that if there is a defensible unification at least in first order between the electromagnetic and gravitational fields, the constancy of information transmission should remains as a firm foundation to build up such frame-work for unify the two tangible fields that physicists know very well in some degree. To finish this part of the article is important to say that the motion of any body on the gravitational field is along a curve so called the geodesic, so light as the general theory say must be follow the same path which is determined by the source of gravity field. Of curse this is very understood by all the physics community. Therefore sooner or later in future paper will be necessary to talk about the energy-momentum tensor, which is symmetry, adding some useful term, and satisfy the condition: \( \partial_\mu T^{\alpha\nu} = 0 \).

\[ T^\alpha_\nu = \frac{\delta L}{\delta \partial_\mu \phi} \partial_\nu \phi - \delta^\alpha_\nu L \]

Which is in general not a symmetry energy-momentum tensor but for the moment is not necessary to talk about it in great detail because this will take us beyond the scope of this little article.

§9. Equivalence Principle and the Motion of Light
The Quantum Aspect of the Electromagnetic Field

Without doubt the equivalence principle is the foundation of the general theory of gravity with some limitations of course, however the relation with the constancy of the speed of light is hidden in some manner. When Albert Einstein made the thought experiment to show the possibility of bending light, the equivalence principle play a secondary role since the effect of bending light is a consequence of its finite speed. On the other hand the equivalence principle showed that this physical process should be happen also in a gravitational field, that is what really is all about the relation between the constancy of light and the principle mentioned. If had not finite value the speed of light none of those effects will occur in Nature so was necessary the idea to pass all this as possible tenable effects in a presence of gravity.

If this is a sound argument also for the general theory of relativity, we need to think more, however is plausible that will be reliable also. We also need to consider the quantum theory where the classical notions undergo a fuzzy understanding, philosophically speaking at this moment and consider the level of math up to now exists a unification theory could be possible. The particular relation established between the non-inertial reference system with gravity was a good idea, however there is not inclusion of the electromagnetism phenomenon and that is a weak point. On the other hand the equivalence principle was not intended to describe the electromagnetism, is an axiom to study gravity with all the ideas which came from special theory of relativity.

So all the consequences that arises like the expected deviation angle of light, the shift frequency
is basically a result to consider the speed of light or the information transmission as a fundamental constant independent of the inertial observers, the same argument is valid in a small region of a manifold where the special theory is remains valid. Of course, nothing of this is new but it’s important to consider that a possible unification will need a revise of the main concepts in which is the founded the modern physics.

However, how about the quantum aspect of the electromagnetic field? I’m not talking about QED which a very sophisticated subject, what I’m referring is what A. Einstein pointed out in 1905. Due to the rest-mass of the photon is zero, we can use the relativistic equation

\[ E^2 - p^2c^2 = m^2c^4 \]

and find an equation between the energy and the momentum, so we have:

\[ E = pc \quad (28) \]

From the point of view of the electromagnetism, there is a associated momentum with the electromagnetic wave and perhaps equation [28] is an interesting relation between the special theory and the electromagnetism. Besides that from the 1905 Einstein’s paper we know that the energy of a single photon is in relationship with one of the characteristic of electromagnetic wave, the frequency \( \nu \) in the follows way:

\[ E = h \nu \quad (29) \]

If we have a sound reason to equate both equation [28] and [29] then, we have:

\[ p = \frac{h \nu}{c} = \hbar |\vec{k}| \quad (30) \]

So far, nothing new however this idea of light as a particle moving with a constant speed \( c \), and the assumption of the interaction between gravity and the photon gave the same result got by A. Einstein in 1911 paper. Therefore we can made the suggestion about the interaction between the photon and the gravitational potential field at point \( x \) by means the following equation:

Basic interaction term \( \to \frac{h \omega}{c^2} \phi \quad (31) \)

If we not taking into account other kind of possible interactions with the photon in the gravitational field, is normal to think that equation [31] is the most simple and natural one to postulate. As consequence that \( \omega \) is a function of \( \phi \) we expect a deviation of the photon from its original path, that was the proposal made in the article [3] in order to derive the equation:

\[ \omega = \frac{\gamma}{\left(1 + \frac{\phi}{c^2}\right)} \]

Of course, there is not doubt that this isn’t new however what is new is the manipulations made with this idea and the ability to uses it in order to obtain useful physical results as long as we accept the fact that conservation of energy for the case of the photon interacting with the gravitational field is valid as we used it in the last paper. Now, go beyond this proposal and try to use more sophisticated math and physics concepts is for now not possible at least for me. But is useful to think that perhaps some applications in Cosmology is feasible, how affects the expansion of the universe, a possible minimum energy value of the photon? In other words nowadays there are many defiance in physics.
§10. Closing Remarks

The moving of the photon in a gravitational field must be an interaction process, so It’s necessary to demonstrate this assertion somehow. Why must be in this manner? Other considerations like the propagation as a wave in a gravitation field yields us to consider possibly the variation of the speed of light, where this speed must be a fundamental constant in Nature and besides that makes more simple some physical formulations. Even though the speed must be a fundamental constant in Nature, will be necessary try to find out a feasible justification about this constancy, I’m not talking about the number assigned to the speed but rather the reason why there exists a limitation in the information transmission from one point to another, perhaps a geometrical reason will be possible or could be found when the unification between the well-known physical fields happen. Try to point out another form to visualize the physical phenomena is very important more than ever because physics without new augment glasses could be a wrong way to follow.

A possible minimum value energy of the photon could gives us perhaps a new insight concerning the expansion of the universe. However, take this statement seriously needs more physical justification rather than a mathematical frame-work because nowadays most theoretical ideas does not have a good physical reason to consider it as nice and elegant way to follows. Photon can not have either higher value as much as we want or a lower value than a minimum one, what is left pending now is to figure out that and apply it to cosmology for instance. As we know the task is not easy and as consequence I’ll try to study more deeply this issue but always taking into consideration the basis ideas where the frame-work were constructed. At least in mine mind there is the idea,that really the motion of the photon in a gravitational field is an interaction process so to go beyond this and apply more physics concepts I need to clarify this mechanism and consider quantum ideas. So in some point of this research I need to see how this process could be derived from QED. So, the process of interaction between gravitational field and others fields is the idea and keeping always constant the speed of light could be the two guides in order to explore what we can get from this but now in most higher use of physical concepts.

§11. Corrections to Some Equations of the Article [3]

In the previous article [3], we had been established that the energy of a photon in a gravitational field as follows:

\[ \mathcal{E} = \hbar \omega + \frac{\hbar \omega}{c^2} \phi \quad (32) \]

From equation [32] we obtained the relation between the frequency at two point where exists a gravitational field and deduced from it the wavelength displacement as well as the frequency shift without using the suggestion of the variation of the speed of light. Whether this is a useful concept or not I can not tell anything concerning this issue but turns out to be more easy to keep constant the speed of light as a feasible argument. Also we take advantage of the idea of interaction to write a energy conservation law along the path followed for the photon, is say:

\[ \mathcal{E}_1 = \hbar \omega_1 + \frac{\hbar \omega_1}{c^2} \phi_1 = \hbar \omega_2 + \frac{\hbar \omega_2}{c^2} \phi_2 = \hbar \omega_3 + \frac{\hbar \omega_3}{c^2} \phi_3 = \cdots = \text{Constant} \]

The equation [9] in the previous article there was a error in the subscript, so it must have been as follows:

\[ \frac{\lambda_1 - \lambda_2}{\lambda_2} = \frac{\phi_1 - \phi_2}{c^2} = \frac{\phi_2 - \phi_1}{c^2}, \ |\phi| > 0 \]

Submitted date: (September 16, 2022)

Second Article
§12. The Line Element for Light in a Weak Gravitational Field
A Little Introduction

As we presented in the last paper [3] concerning a certain comments about the A. Einstein’s 1911 paper [1], it has be proved there that a energy consideration and the guide idea of interaction process between the motion of the photon in a weak gravitational field led us to the same conclusion that Albert got and published in 1911. In the article it was also argued that the suggestion of the changed of the speed of light wasn’t necessary and it was established an ideal energy equation which led us to the relationship between the angular frequency and the potential of the gravitational field, is say:

\[ \omega = \frac{\gamma}{\left(1 + \frac{\phi}{c^2}\right)} \]

\[ |\phi| > 0, \phi \to 0 \text{ at } r \to \infty \]  

(33)

Using the following equatio:

\[ E = h\omega + \frac{h\omega}{c^2}\phi \]

What was left out in the paper is about the line element of the photon, It’s know that this must be zero, \( ds = 0 \) so if the idea presented has a physical meaning at least this result could be derived from it. I comprehend that this must be uncommon in some sense but if the idea of a real interaction is true it is necessary to study all its consequences, at least in the classical physics and is a good idea to do this however more profound study of the idea presented in the paper have to be a must.

§13. Energy and Line Element of the Photon

If we ask ourselves about the motion of the photon, we speak a single photon but we know that is for the sake of simplification, it’s possible to established a relation between the energy due to the changed in the angular frequency and the changed in the gravitational potential along the path followed for the photon through its motion, is say:

\[ h\omega = -\frac{h\omega}{c^2}d\phi \]

(34)

Taking into consideration that the photon is moving along a curve line in the space, we can divide the right-hand side of the equation [34] by the arc element, \( ds \), covered in the time \( t \) where the arc element is not the four-dimensional arc element,\( ds^2 = g_{\nu\mu}dx^\nu dx^\mu \), defined in the general theory of relativity, in other words:

\[ d\omega = -\frac{\omega}{c^2} \frac{d\phi}{ds}dt \]

(35)

Where

\[ ds^2 = dx_1^2 + dx_2^2 \]

Considering the equations [35] and the [33], we can firstly differentiate the first equation of [33] to
obtain a relation between $d\phi$ and $d\omega$ and substitute it in the equation [35], so:

$$
\begin{align*}
    d\omega &= -\gamma \frac{d\phi}{c^2 \left(1 + \frac{\phi}{c^2}\right)^2}, \\
    \omega &= \frac{\gamma}{\left(1 + \frac{\phi}{c^2}\right)} \\
    ds &= \left(1 + \frac{\phi}{c^2}\right) c dt
\end{align*}
$$

Paying attention that:

$$
    ds^2 = dx_1^2 + dx_2^2
$$

Taking the square of both sides of equation [36] and compare them, we have then:

$$
    dx_1^2 + dx_2^2 = \left(1 + \frac{\phi}{c^2}\right)^2 c^2 dt^2 \approx \left[1 + 2\frac{\phi}{c^2} + O(c^4)\right] c^2 dt^2
$$

So, we can rearrange the equation [37] in order to encounter the very well-known line element of the path followed by the light in the space-time framework nearby in a weak gravitational field. This result found here by itself is very interesting and in my consideration have to be study whether is true or not, however this is only my humble opinion and this is what I could point out, to be demised by researchers in this field with more knowledge in experience. So, we have definitely:

$$
    dx_1^2 + dx_2^2 = \left(1 + 2\frac{\phi}{c^2}\right) c^2 dt^2
$$

Or in more familiar way:

$$
    \left(1 + 2\frac{\phi}{c^2}\right) c^2 dt^2 - dx_1^2 - dx_2^2 = 0
$$

Which is basically:

$$
    ds^2 = 0
$$

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Third Article

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I’d like to address the issue that may be the derivation has not the force of a prove in physics but is a first insight about how could be obtain the line element for the light in space-time, of course that this not contradict anything done in the previous article, version 3
§14. Final Comments About this Little Research

In 1911 Albert Einstein took the advantage concerning the development of the new theory of gravity however he not realized all the useful mathematical tools in his time like the unification of the space-time due to H. Minkoswi. Apart from that his idea of possible variation of the speed of light was the only light through many years of seeking until he arrived with the right solution, he finally realized that was necessary to take all the components of the metric tensor into account, $g_{\nu\mu}$. And wrote a several papers concerning the meaning of this geometrical quantity, however Riemann in the early development of his geometry pointed out the same ideas as Einstein found in 1916 approximately.

So it’s was not totally new but what was new is the fact that Einstein was able to formulate a gravity theory and not only to point out the possibility. Even his idea of variation of the speed of light was a possible solution for him in those years 1907 – 1912, and the final result of the theory showed up that was not necessary and perhaps unnatural to suggest that, but the question remains until now, is say, Why does exists such limit in the information transmission in Nature? The main purpose and idea to write this was not to contradict anyone whom are expertise in the field of gravity but rather to point out that all the problems nowadays physics is facing is due that there’s not a clearly understanding about what’s really is space and time as well.

It’s necessary new ideas and therefore new approach to the solution, so I’d like to try to find some different approaches to the solutions and I’ll be eager to prove it whether I’m wrong or not. Please any comments about this research and the proposed idea let me know it ASAP.

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