The Mathematical Route Model Based on Analytic Hierarchy Process

Zeen Pan, Chuanmengzhen Yue and Yanbing Zhang

College of Information Science and Engineering, East China University of Technology, Shanghai, 200237

Abstract. In 2017, a devastating hurricane attacked the United States territory of Puerto Rico and caused severe damage. In the meanwhile, it set great obstacles to the aid work. Therefore, the non-government organization puts forward one possible method that we would use the drones to succour the disaster area. Hence, our model aims to develop a comprehensive program to tackle this issue. Our model has three parts. First of all, we compare the weighting average of all requirements and then select the drones with high performance via the analytic hierarchy process. After considering the distribution of the hospital, we formulated two possible transporting strategies and the corresponding transporting sub-route, including star routes and the ring routes. In the third part, we obtain some intuitional constraints from the routes via some mathematical definitions, directly showing the performance of the drone under the effect of these external causes.

1. Introduction

The hurricane is a kind of tropic storm, moving from the tropic and to the temperate zone, during which it could land on the offshore city and cause devastating disaster. For instance, 2017, Puerto Rico suffered the most devastating hurricane attack to ever. The massive damage and the flood cut off the road, making it nearly impossible for emergency services ground vehicles to plan and navigate their routes. The hurricane is irresistible which decide what we should do is enforcing the rebuilding the disaster area, alleviating the loss of the estate. Therefore, our model aims to develop a comprehensive plan to solve this problem based on analytic hierarchy process (AHP). AHP is a simple, flexible and practical multi-criteria decision-making method proposed by Professor T. L. Saaty, an American operational researcher, in the early 1970s. A simple method for making decision on some complicated and vague problems is especially suitable for those problems which are difficult to be quantitatively analyzed. Therefore, this paper uses the analytic hierarchy process to establish the model.

2. The basic selection model

The function of this basic model is preliminarily selecting the drones with high performance to form a drone fleet (we put the impact of the route and the detail of the hospital to this problem on the secondary place) we use the principle of the analytic hierarchy process to solve this issue. If the factors of the analyze system could be stratified to the first level, the middle level and the last level. The basic process of the analytic hierarchy process:
3. **Mathematical route model**

We focus on the distribution of hospitals and formulated two possible transporting route and the corresponding transporting strategies.

1) The star route: the drone will back to the departure place on the same line of its trip to the destination.

2) The ring route: the drone will back to the departure place on the other airline.

The star route: other drones do not need to pass this point, when the drones have met its requirements, which highlight the advantage of speed.

The ring route: it can meet the demand of several points in an only flight, which highlights the advantage of the massive carrying capacity.

The request of ring network: from the perceptual intuition, only the areas with high density of the stations can form a ring network.

Evidently all the possible flight routes are the combination of the star route, and the ring network route. We give mathematical expressions for the basic judgment.
Table 1. Symbol table.

| Symbol | Definition                                                      |
|--------|----------------------------------------------------------------|
| $S$    | The assumed distance between two stations                      |
| $S_f$  | The maximum flight distance of the drone                       |

After judging the position relationship among circles, we can know that:
If there are no points of departure between two stations, then:

$$S_f > S$$

Then we can draw the conclusion: it does not have to exist a point of departure between two stations while the flight distance of the drone is farther than the distance of two stations.

4. The impact of external cause model

Next, we will consider the impact of external factors on the performance of the drone, the criterion as follow:

Compare the drones’ ratios of the total required payload of all stations during one flight to the maximum payload, in order to evaluate the level of the impact of external cause on the performance of the carrying capacity which is the maximum weight in the first part.

$$\mu = \frac{\text{Required payload}}{\text{Maximum payload}}$$

\(\mu\) is the load utilization ratio, we will consider the route between area a and b, and the route between d and e. the distance between a and b = 41.9928 km, the distance between d and e = 60.6982 km.
The departure place does not have to exist between the two stations, while the maximum flight distance of the drone farther than the distance of two stations. Hence there must be a departure place between area a and b, and there must be a departure place between d and e, when we chose c type drone. Hence the flight route must be the star network route.

According to the optimum scheme of preliminary route and what we define, we try to analyze the impact of external causes on the performance of the drones:

Since the requirement of medicine packages are not too much in A and E, we will dispatch the C type drones to deliver the packages. There is only a MED1 to be delivered, so choosing a C type drone flies to E point is not economic. Its load utilization ratio: \(\mu = \frac{2}{14} = \frac{1}{7}\); the A point need a MED3 and the MED1, its load utilization ratio is \(\mu = \frac{5}{14}\); according to that, we see that the load utilization ratio of C type is quite low. The loading capacity should not be the priority factor after we use the F type drone, instant, the flight speed, maximum flight distance and whether equipped with video (every fleet have to be equipped with a type of drone with video to ensure the reconnaissance work) become more significant. Hence we have to change the weight of the factors; means rebuild a new Pairwise comparison matrix of the criterion level.

As with the analytic hierarchy process of the primary model, we will calculate the weight of each type of the drone:

$$Y_1 = 0.1155, Y_2 = 0.2163, Y_3 = 0.1913, Y_4 = 0.1080$$

$$Y_5 = 0.1035, Y_6 = 0.1487, Y_7 = 0.1167$$

$$CR_m = 0.0113,$$

$$CR = CR_p + CR_m = 0.0032 + 0.0113 = 0.0145 < 0.1,$$

Where, \(CR_m\) is the consistency ratio of the scheme level.

In the meanwhile, we make more clear pictures to prove the necessity of selecting the B type drone.
5. Conclusion
Our model aims to consider the performance of the drone, the mode of the route and the impact of the external cause on the performance of drone step by step, in order to obtain the optimum selection of the type of the drones and the optimum scheme of preliminary route via finite correction processes (modify the type of the drones and the route planning that under the original assumption via some feedback).

References
[1] G Dantzig, J. Ramser. The truck dispatching problem [J]. Management Science, 1959, 6:80-91.
[2] M. W. P. Savelsbergh. Local search in Routing Problems with Time Windows [J]. Annals of Operations Research. 1985, 16(4):285-305.
[3] Wren A. Holliday A. Computer scheduling of vehicles from one or more depots to a number of delivery points [J]. Journal of the Operation Research Society, 1972, 23(3):333-344.
[4] Renaud J, Laporte G, Boctor F E. A tabu search heuristic for the multi-depot vehicle routing problem [J]. Computer Ops Res. 1996.23(3):229-235.