First study of $B \to \pi$ semileptonic decay form factors using NRQCD

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We present a quenched calculation of the form factors of the semileptonic weak decay $B \to \pi l\bar{v}$ with $O(1/m_Q)$ NRQCD heavy quark and Wilson light quark on a $16^3 \times 32$ lattice at $\beta = 5.8$. The form factors are evaluated at six heavy quark masses, in the range of $m_Q \sim 1.5 - 8$ GeV. $1/m_Q$ dependence of matrix elements are investigated and compared with HQET predictions. We observe clear signal for the form factors near $q_{max}^2$, even at the $b$-quark mass range. $f_0(q_{max}^2)$ is compared with $f_B/f_{\pi}$ based on the soft pion theorem and significant difference is observed.

1. Introduction

Lattice study of $B$ decay matrix elements is important for the determination of Cabibbo-Kobayashi-Maskawa matrix elements, and for investigations of applicability of Heavy quark effective theory (HQET) which is extensively applied to phenomenological studies. In this work, we calculate $B \to \pi$ form factors using the heavy quark described by $O(1/m_Q)$ NRQCD and Wilson light quark $[1]$. $G_\varphi(t+1) = \left(1 - \frac{1}{2n} H_0 \right)^n U_4 \left(1 - \frac{1}{2n} H_0 \right)^n G_\varphi(0)$

\begin{equation}
G_\varphi(t+1) = \left(1 - \frac{1}{2n} H_0 \right)^n U_4 \left(1 - \frac{1}{2n} H_0 \right)^n G_\varphi(t).
\end{equation}

\begin{equation}
H_0 = -\frac{1}{2m_Q} \Delta^{(2)}, \quad \delta H = -\frac{1}{2m_Q} \vec{\sigma} \cdot \vec{B},
\end{equation}

where $\Delta^{(2)}$ denotes the lattice Laplacian and $B$ is the chromomagnetic field. The stabilizing parameter $n$ should satisfy $n > 3/2m_Q$.

For the heavy quark, eight values of mass and stabilizing parameter are used: $(m_Q, n) = (5.0, 1), (2.6, 1), (2.1, 1), (2.1, 2), (1.5, 2), (1.2, 2), (1.2, 3),$ and $(0.9, 2)$. $m_Q = 2.6$ and $0.9$ roughly correspond to the $b$- and $c$-quark masses. The mean-field improvement $[2]$ is applied to the heavy quark evolution equation with $u_0 = \langle \frac{1}{3} U_{\text{plaq}} \rangle^{1/4} = 0.867994(13)$.

The matrix elements are extracted from three point correlation functions,

\begin{equation}
\langle \mu | \langle x_f | \sum_{\bar{t}} \langle \bar{x}_f, t | O_B(x_f) V_\mu^\dagger (x_\pi) O_\pi (t_0, 0) \rangle \rangle.
\end{equation}

We use 20 rotationally nonequivalent sets of $(\vec{p}, \pi)$ with $|\vec{p}|, |\vec{k}| \leq \sqrt{3} \cdot 2\pi/16$. The source and the current operators are set on the time slices $t_s = 4$ and $t_s = 14$ respectively. The matrix elements are extracted in the region $t_f = 23 - 28$.

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We estimate the effect of perturbative corrections to the heavy quark self-energy and the current. In some cases, we use larger values of \( n \) for the perturbation than those in the simulation: \((m_Q, n) = (5.0, 1), (2.6, 2), (2.1, 2), (1.5, 3), (1.2, 3), \) and \((0.9, 6)\). This is because of the singularities encountered in the perturbative expressions for some set of \((m_Q, n)\) with small \( n \). The multiplicative part of the current renormalization constant is calculated with massless Wilson quark for vanishing external momenta. Two scales \( q^* = \pi/a \) and \( 1/a \) are considered to define the expansion parameter \( g_V^2 \).

3. Results

It is useful to define following quantity.

\[
\hat{V}_\mu(\vec{p}, \vec{k}) = \frac{\langle \pi(\vec{k}) | V_\mu | B(\vec{p}) \rangle}{\sqrt{2} E_\pi(k) \sqrt{2} E_B(p)}
\]  

This expression can be entirely composed of numerical results, without any assumption such as a dispersion relation. It is also convenient for a comparison with HQET predictions. According to the heavy quark symmetry, for \( \vec{p} = 0 \), \( \hat{V}_\mu \) takes constant value in the leading order of \( 1/m_Q \):

\[
\hat{V}_4(\vec{p} = 0, \vec{k}) = \hat{V}_4^{(0)} [1 + c_4^{(1)}/m_B + \cdots],
\]

\[
\hat{V}_k(\vec{p} = 0, \vec{k}) = \frac{\hat{V}_k^{(0)}}{k} [1 + c_k^{(1)}/m_B + \cdots],
\]

where \( \vec{k} = 2 \sin(k_i/2) \). Upper two of Figure 1 show the results of \( \hat{V}_4 \) for \( \vec{p} = \vec{k} = 0 \) and \( \hat{V}_k \) for \( |\vec{k}| = 2\pi/16 \) in the case of \( \kappa = 0.1570 \). They are evaluated at three renormalization scales, mean-field tree, \( q^* = \pi/a \) and \( 1/a \). Both \( \hat{V}_4 \) and \( \hat{V}_k \) less depend on \( m_B \) in comparison with \( f_B \) case. The spacial component of \( \hat{V}_4 \) is more affected by the perturbative corrections than the temporal one is. It is also predicted that

\[
\hat{V}_p(\vec{p} = 0, \vec{k}) = \lim_{\vec{p}^2 \to 0} \frac{\vec{p} \cdot \vec{V}(\vec{p}, \vec{k})/\vec{p}^2}{\sqrt{2} E_\pi(k) \sqrt{2} E_B(p)}.
\]

We extrapolate \( \hat{V}_p \) at finite \( \vec{p} \) to \( \vec{p} = 0 \) linearly in \( \vec{p}^2 \) to determine \( \hat{V}_p(\vec{p} = 0, \vec{k}) \). \( \hat{V}_p(\vec{p} = 0, \vec{k}) \) multiplied by \( m_B \) is also displayed in Figure 1. Contrary to the cases of \( \hat{V}_4 \) and \( \hat{V}_k \), \( O(1/m_B) \) effect is significant for \( \hat{V}_p \).

The matrix elements are expressed in terms of two form factors, \( f^0 \) and \( f^+ \):

\[
\langle \pi(k) | V_\mu | B(p) \rangle = \left( p + k - \frac{m_B^2 - m^2}{q^2} \right) \frac{f^+(q^2)}{q}.
\]
Finally, we consider the implication of soft pion theorem \[ \text{[7,6]} \]. For the massless pion limit, \( f^0(q^2_{\text{max}}) \) should equal to \( f_B/f_\pi \). This relation is examined in Figure 3, using our result on \( f_B \) determined with slightly different form of NRQCD \[ \text{[5]} \]. Significant difference is observed in large \( m_B \) region. Similar result is obtained in the work using Fermilab action for the heavy quark \[ \text{[8]} \]. The origin and physical meaning of this discrepancy remains as a future problem.

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