Neutrino spin oscillations in external fields in curved space-time

Maxim Dvornikov
Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radiowave Propagation (IZMIRAN),
108840 Troitsk, Moscow, Russia

Abstract

We study spin oscillations of massive Dirac neutrinos in background matter, electromagnetic and gravitational fields. First, using the Dirac equation for a neutrino interacting with the external fields in curved space-time, we rederive the quasiclassical equation for the neutrino spin evolution, which was proposed previously basing on principles of the general covariance. Then, we apply this result for the description of neutrino spin oscillations in nonmoving and unpolarized matter under the influence of a constant transverse magnetic field and a gravitational wave. We derive the effective Schrödinger equation for neutrino oscillations in these external fields and solve it numerically. Choosing realistic parameters of external fields and a neutrino, we show that the parametric resonance can take place in neutrino oscillations. Some astrophysical applications are briefly discussed.

1 Introduction

Cosmic neutrinos play an important role for the evolution of stars, supernovae, and the early universe [1]. Neutrinos have a remarkable property of transitions from one type to another, called neutrino oscillations [2]. For example, neutrino flavor oscillations $\nu_e \rightarrow \nu_\mu$, amplified by the neutrino interaction with background matter, known as the Mikheyev-Smirnov-Wolfenstein (MSW) effect, are the most plausible solution to the solar neutrino problem [3].

There are various types of neutrino oscillations (see, e.g., Ref. [4] for a review). Among them, we can discuss the possibility of transitions between left and right polarized neutrinos of one type, $\nu_L \rightarrow \nu_R$, neglecting the neutrino mixing. This process, named neutrino spin oscillations, results in the effective reduction of an initial flux of left polarized particles since right polarized neutrinos are sterile in the standard model. One can describe neutrino spin oscillations as the precession of the neutrino spin in an external field.

Besides MSW effect, mentioned above, other external fields can influence neutrino oscillations. For example, the gravitational interaction, in spite of its weakness, was found in Ref. [5] to contribute to oscillations of neutrinos. In the present work, we are interested in the influence of gravitational fields on neutrino spin oscillations.

The problem of the evolution of a spinning particle in general relativity was tackled for the first time in Ref. [6]. The resulting evolution equations turn out to be nonlinear, i.e. the particle trajectory is affected by the spin evolution and vice versa. Fortunately, in case of

*maxdvo@izmiran.ru
a point-like elementary particle, such as a neutrino, the particle motion is not influenced by the particle spin. It means that a neutrino follows a geodesic line in an external gravitational field with high level of accuracy.

The equation for the quasiclassical description of the particle spin evolution in a gravitational field was proposed in Ref. [7]. Then, in Ref. [8], this approach was adapted for the studies of neutrino oscillations in a gravitational field. Neutrino oscillations in matter under the influence of an electromagnetic field in curved space-time were studied in Ref. [9] by proposing the generalization of the quasiclassical covariant equation for the neutrino spin evolution in these external fields. The formalism, developed in Refs. [8, 9], was applied in Refs. [10, 11] to study neutrino spin oscillations in various gravitational backgrounds. The method for the description of the particle spin evolution in a gravitational field, based on the analysis of the Dirac equation in curved space-time, was recently developed in Ref. [12].

In the present work, we continue the study of neutrino spin oscillations in external fields in curved space-time. The quasiclassical equation, accounting for the contribution of external fields to the neutrino spin evolution, was derived in Ref. [9] basing on principles of the general covariance. However this approach has to be substantiated by a more rigorous derivation. It deals mainly with the contribution of background matter to the neutrino spin evolution in curved space-time. It is the main motivation of the present work.

Then, in Refs. [8, 9], we considered neutrino spin oscillations in static gravitational backgrounds, like Schwarzschild and Kerr metrics. It is interesting to analyze the dynamics of neutrino oscillations in a time dependent metric such as a gravitational wave (GW). Typically astrophysical neutrinos interact with background matter and an external magnetic field besides a gravitational field. The contributions to neutrino spin oscillations of all these external fields – namely, a background matter, a magnetic field, and a gravitational field (GW) – are accounted for in our study. Note that the evolution of a fermion spin in GW was recently studied in Refs. [13, 14].

The recent observation of GW by the LIGO and Virgo detectors [15] is an unambiguous proof of the validity of the general relativity. The most powerful sources of GW are systems of binary black holes (BHs) [16] and neutron stars (NSs) [17] at the moment of their coalescence. The merging of two compact objects can be a source of both GW and other elementary particles including neutrinos. Moreover, dense background matter and strong magnetic fields are ejected in outer space in the coalescence of BHs or NSs. These factors can affect the propagation and oscillations of emitted neutrinos. There are numerous attempts to carry out multi-messenger observations of both GW and astrophysical neutrinos by the modern neutrino telescopes and cosmic rays experiments [18].

This paper is organized as follows. We start in Sec. 2 with the derivation of the quasiclassical equation for the neutrino spin evolution in external fields in curved space-time basing on the Dirac equation for a massive neutrino accounting for these external fields. Then, in Sec. 3 we apply our results for the description of the neutrino spin evolution in background matter, a transverse magnetic field, and GW. We demonstrate that a parametric resonance can happen in oscillations of realistic astrophysical neutrinos. Finally, in Sec. 4 we discuss our results.
2 Formalism for the description of the neutrino spin evolution

In this section, we derive the covariant quasiclassical equation for the neutrino spin evolution in external fields in curved space-time. For this purpose we analyze the corresponding Dirac equation. Our result is compared with previous findings.

We consider one neutrino eigenstate, which is supposed to be a Dirac particle, and neglect the mixing between different neutrino types. The wave equation for a massive Dirac neutrino with the anomalous magnetic moment, interacting with background matter and the electromagnetic field \( F_{\mu\nu} \) in curved space-time, reads

\[
\left[ i\gamma^\mu \nabla_\mu - \frac{\mu}{2} F_{\mu\nu} \sigma^{\mu\nu} - \frac{V^\mu}{2} \gamma_{\mu} (1 - \gamma^5) - m \right] \psi = 0, \tag{1}\]

where \( \gamma^\mu = \gamma^\mu(x) \), \( \sigma_{\mu\nu} = \frac{1}{2} \left[ \gamma_{\mu}, \gamma_{\nu} \right] \), and \( \gamma^5 = -\frac{i}{4} \gamma_{\mu\nu\rho\sigma} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma / \sqrt{-g} \) is the coordinate dependent Dirac matrices, \( E^{\mu\nu\rho\sigma} = \epsilon^{\mu\nu\rho\sigma} / \sqrt{-g} \) is the covariant antisymmetric tensor in curved space-time, \( g = \det(g_{\mu\nu}) \), \( g_{\mu\nu} \) is the metric tensor, \( \nabla_\mu = \partial_\mu + \Gamma_\mu \) is the covariant derivative, \( \Gamma_\mu \) is the spin connection, \( \mu \) is the magnetic moment of a neutrino, and \( m \) is the neutrino mass.

The effective potential \( V^\mu \) of the neutrino interaction with arbitrarily polarized and moving matter has the form,

\[
V^\mu = \sqrt{2} G_F \sum_f \left( q_f^{(1)} j_f^\mu + q_f^{(2)} \lambda_f^\mu \right), \tag{2}\]

where \( j_f^\mu \) and \( \lambda_f^\mu \) are the hydrodynamic currents and the polarizations of background fermions of the type \( f \), \( G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2} \) is the Fermi constant, and \( q_f^{(1,2)} \) are the constants which are given in the explicit form in Ref. [19]. The contribution of the electroweak interaction of a fermion with background matter to the Dirac equation in curved space-time, as in Eq. (1), was previously considered in Refs. [20,22].

To find the spin connection in Eq. (1) we choose a locally Minkowskian frame \( x^\mu \rightarrow \bar{x}^a \): \( \eta_{ab} = e_a^\mu e_b^\nu g_{\mu\nu} \), where \( e_a^\mu = \partial x^\mu / \partial \bar{x}^a \) are the vierbein vectors and \( \eta_{ab} = \text{diag}(+1,-1,-1,-1) \) is the metric tensor in Minkowski space. In such a frame, \( \Gamma_\mu \) is defined as \( \Gamma_\mu = -\frac{1}{4} \sigma_{ab} \omega_{ab\mu} \), where \( \omega_{ab\mu} = e_a^\nu e_{bc\mu} \) are the components of the connection one-form, \( \sigma_{ab} = \frac{1}{2} \left[ \gamma_a, \gamma_b \right] \), \( \bar{\gamma}^a = e_a^\mu \gamma^\mu \) are the constant Dirac matrices given in the chosen frame, and the semicolon stays for a covariant derivative.

The Dirac operator in the vierbein frame takes the form, \( i\gamma^\mu \nabla_\mu = i\bar{\gamma}^a \partial_a + \frac{1}{4} \bar{\epsilon}^{a\mu\nu\rho\sigma} \bar{\gamma}^\mu \bar{\gamma}^\nu \bar{\gamma}^\rho \bar{\gamma}^\sigma - i \bar{\epsilon}^{abcd} \bar{\gamma}_a \bar{\gamma}_b \bar{\gamma}_c \bar{\gamma}_d \gamma^5 \), where \( \epsilon^{abcd} \) is the absolute antisymmetric tensor in Minkowski space, having \( \epsilon^{0123} = +1 \), and \( \gamma^5 = i \bar{\gamma}^0 \bar{\gamma}^1 \bar{\gamma}^2 \bar{\gamma}^3 \), we rewrite the Dirac Eq. (1) in the form

\[
\left[ \bar{\gamma}^a \left( i\partial_a + \frac{i}{2} \gamma_{ab\mu} \gamma^\mu \right) + \bar{\gamma}^a \gamma^5 \frac{1}{4} \bar{\epsilon}^{abcd} \gamma^{cd} - \frac{\mu}{2} \bar{f}_{ab} \gamma^{ab} - \frac{\nu^a}{2} \left( 1 - \gamma^5 \right) - m \right] \psi = 0. \tag{4}\]

Here \( f_{ab} = e_a^\mu e_b^\nu F_{\mu
u} \) and \( \nu^a = e^a_\mu V^\mu \) are the corresponding objects in the vierbein frame.

The axial-vector contribution \( \sim \bar{\gamma}^a \gamma^5 \) of a gravitational field to the evolution of a fermion in Eq. (1) was previously obtained in Refs. [23,24]. However, the vector contribution, \( \frac{1}{2} \bar{\gamma}^a \gamma_{ab\mu} \gamma^\mu \),
is omitted in these works. For example, in Ref. [24], the incorrect analogue of Eq. (3), in which only the term $-i\varepsilon^{abce}_{d}\gamma_{a}\gamma^{5}$ is accounted for, is used. The correct form of the Dirac equation, coinciding with Eq. (4), in a vierbein frame is derived in Ref. [25].

The vector contribution of the gravitational interaction acts as the effective electromagnetic field $q_{\text{eff}}A_{\text{eff}}^{a} = -\frac{i}{2}\gamma^{abce}_{d}\eta_{bc}$. However, the vector potential of this electromagnetic field is imaginary. It makes the Hamiltonian of the Dirac Eq. (4) nonhermitian. The problem of the nonhermicity of Hamiltonians of fermions in curved space-time was discussed earlier in Refs. [26,27]. For instance, following the approach of Ref. [28], a nonunitary transformation of the wave function, which recovers the hermicity of the Hamiltonian, was proposed in Ref. [27].

Thus, to develop the approach for the description of neutrino evolution in external fields in curved space-time based on Eq. (5) one has to choose the vierbein vectors satisfying the condition $\gamma^{abc}_{d}\eta_{bc} = e_{a}\mu; \mu = 0$. We suppose that this condition is fulfilled. Thus, we conclude that the neutrino bispinor obeys the following wave equation in general external fields:

\[
\left[i\bar{\gamma}^{a}\partial_{a} + \bar{\gamma}^{a}\gamma^{5}\frac{1}{4}\varepsilon_{abcd}\gamma^{c}\gamma^{bd} - \frac{\mu}{2}f_{ab}\sigma^{ab} - \frac{v^{a}}{2}\bar{\gamma}_{a} + \frac{v^{a}}{2}\bar{\gamma}_{a}\gamma^{5} - m\right]\psi = 0,
\]

(5)

where

\[v_{5}^{a} = v^{a} + \frac{1}{2}\varepsilon^{abcd}\gamma_{c}\gamma_{d},\]

(6)

is the effective axial-vector field.

The covariant equation for the quasiclassical evolution of the neutrino spin $s^{a}$ in general external fields in Minkowski space is derived in Ref. [19]. Using Eqs. (5) and (6), as well as applying the formalism of Ref. [19], one gets that $s^{a}$ obeys the equation,

\[
\frac{d}s^{a}/d\tau = 2\mu \left(f^{ab}s_{b} - u^{a}f^{bc}u_{b}s_{c}\right) + \varepsilon^{abcd}v_{b}u_{c}s_{d} + G^{ab}s_{b},
\]

(7)

where

\[G^{ab} = \left(\gamma^{ab} + \gamma^{cab} + \gamma^{bca}\right)u_{c},\]

(8)

is the antisymmetric tensor, $G^{ab} = -G^{ba}$, which incorporates the influence of the gravitational field on the neutrino spin evolution, $u^{a}$ is the neutrino four velocity, and $\tau$ is the proper time in the vierbein frame.

In general situation, the particle four velocity evolves under the influence of a gravitational field in a vierbein frame as [7]

\[
\frac{du^{a}}{d\tau} = \tilde{G}^{ab}u_{b}, \quad \tilde{G}^{ab} = \gamma^{abc}u_{c}.
\]

(9)

One can see that Eqs. (7)-(9) are inconsistent with the requirement that $s^{a}u_{a} = 0$ in the course of the evolution of both $s^{a}$ and $u^{a}$. Thus, we have to choose such a vierbein frame in which $u^{a} = \text{const}$. In this case, Eqs. (7) and (8) correctly describe the neutrino spin evolution. It is such a frame, which is chosen, e.g., in Refs. [8,9], where we studied the neutrino circular motion around Schwarzschild and Kerr BHs.

The first two terms in left hand side of Eq. (7) reproduce the contributions of the electromagnetic field and the electroweak interaction with matter to the neutrino spin evolution, when a particle moves in a curved space-time, first derived in Ref. [9] basing on principles of the general covariance. Now we rederive these results starting from the more fundamental Dirac
Eq. (1). The third term $G_{ab} s_b$ is different from that proposed in Refs. [7–9]: $d s^a / d \tau = \tilde{G}_{ab} s_b$, where $G_{ab}$ in given in Eq. (9).

It is convenient to rewrite Eq. (7) using the invariant three vector of the polarization $\zeta$, which fixes the neutrino spin states in the particle rest frame. With help of the results of Refs. [8, 9], we get that

$$\frac{d \zeta}{d t} = \frac{2}{\gamma} (\zeta \times G),$$

where

$$G = \frac{1}{2} \left[ b_g + \frac{(e_g \times b_g)}{1 + u^0} \right] + \frac{1}{2} \left[ u \left( v^0 - \frac{(v u)}{1 + u^0} \right) - v \right] + \mu \left[ u^0 b - u (u b) + (e \times u) \right].$$

Here we represent $G_{ab} = (e_g, b_g), u^a = (u^0, u), v^a = (v^0, v), f_{ab} = (e, b), \gamma = U^0$, and $U^\mu$ is the neutrino four velocity in the world coordinates. Note that the evolution of the neutrino spin in Eq. (10) is with respect to the time $t$ of the observer in the original world coordinates.

3 Neutrino spin oscillations in a gravitational wave

In this section, we apply the results of Sec. 2 to study neutrino spin oscillations in matter and a magnetic field under the influence of a plane GW. Some astrophysical application are also briefly considered.

Let us take that a plane GW propagates of along the $z$-axis. Choosing the transverse-traceless gauge, we get that the metric has the form [29],

$$d s^2 = g_{\mu \nu} dx^\mu dx^\nu = dt^2 - \left( 1 - h_+ \cos \phi \right) dx^2 - \left( 1 + h_+ \cos \phi \right) dy^2 + 2 dx dy h_\times \sin \phi - dz^2,$$

where $h_+$ and $h_\times$ are the dimensionless amplitudes of two independent polarizations of the wave, $\phi = (\omega t - k z)$ is the phase of the wave, $\omega$ is frequency of the wave, and $k$ is the wave vector. In Eq. (12), we use Cartesian world coordinates $x^\mu = (t, x, y, z)$. In the following, we shall consider GW with the circular polarization in which $h_+ = h_\times = h$. To discriminate between left and right polarizations of the wave we introduce the parameter $\epsilon = \pm 1$ in the phase of the wave $\phi \rightarrow \epsilon (\omega t - k z)$.

Neutrino spin evolution is governed by Eq. (10). To proceed in the study of neutrino spin oscillations we should find all the parameters in Eq. (11) in the vierbein frame. One can check that the following vierbein vectors:

$$e_0^\mu = (1, 0, 0, 0),$$
$$e_1^\mu = \frac{1}{\sqrt{1 + h}} \left( 0, - \sin \frac{\phi}{2}, \cos \frac{\phi}{2}, 0 \right),$$
$$e_2^\mu = \frac{1}{\sqrt{1 - h}} \left( 0, \cos \frac{\phi}{2}, \sin \frac{\phi}{2}, 0 \right),$$
$$e_3^\mu = (0, 0, 0, 1),$$

(13)
diagonalize the metric in Eq. (12). Note that Eq. (13) exactly accounts for the amplitude of the wave.

First, we have to check the hermicity of the Hamiltonian of Eq. (4). Using Eqs. (12) and (13), one gets that $e_\alpha^\mu ; \mu = 0$. Thus Eqs. (12) and (5) coincide.
We consider the situation when neutrinos are emitted by the same source of GWs. It is a reasonable situation when we study oscillations of astrophysical neutrinos. In this case, $U = dx/ds = (0, 0, U_z)$. Using Eq. (9), one can show that the acceleration of such neutrinos in the vierbein frame is absent: $\frac{d}{ds} = G^{ab}u_b \equiv 0$. It means that the change of the polarization ($\zeta u$) is entirely defined by the neutrino spin evolution in Eq. (10). It should be also noted that $u_z = U_z = \text{const}$ and $u^0 = U^0 = \text{const}$.

Moreover we suppose that, besides GW, a neutrino interacts with nonmoving and unpolarized matter, i.e. $V^0 \neq 0$ and $V = 0$ in Eq. (2). We also take that a constant uniform magnetic field transverse to the neutrino motion is present in the world coordinates $x^\mu$. For example, we suppose that $B = (B, 0, 0)$. In this situation, $v_0 = V_0$, $b_x = \frac{B}{\sqrt{1 - h}} \sin \frac{\phi}{2}$, $b_y = -\frac{B}{\sqrt{1 + h}} \cos \frac{\phi}{2}$, and $e = 0$.

Now we can find the components of the vector $\Omega = G/\gamma$, which defines the neutrino spin evolution, as

$$
\begin{align*}
\Omega_x &= \frac{\mu B}{\sqrt{1 - h}} \sin \frac{\phi}{2}, \\
\Omega_y &= -\frac{\mu B}{\sqrt{1 + h}} \cos \frac{\phi}{2}, \\
\Omega_z &= \frac{V_0 U_z}{2U^0} + \frac{\epsilon (kU_z - \omega U^0)}{4U^0 \sqrt{1 - h^2}}.
\end{align*}
$$

Using Eqs. (10), (11) and (15), the evolution of the neutrino polarization can be rewritten using the effective neutrino wave function $\nu^T = (\nu_R, \nu_L)$, which obeys the Schrödinger equation,

$$
\frac{d\nu}{dt} = H_{\text{eff}} \nu, \quad H_{\text{eff}} = (\sigma \cdot \Omega),
$$

where $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices.

It is convenient to modify the effective wave function as $\nu = \exp \left[-i\sigma_3 \left(\dot{\phi}t + \pi/4\right)\right] \tilde{\nu}$, where $\dot{\phi} = \epsilon (\omega - kU_z/U^0)$. Taking into account that $h \ll 1$, we get that

$$
\frac{d\tilde{\nu}}{dt} = \tilde{H}_{\text{eff}} \tilde{\nu}, \quad \tilde{H}_{\text{eff}} = \begin{pmatrix}
\frac{-V^0/2}{\mu B \left[1 - \epsilon \text{e}^{-i\dot{\phi}/2}\right]} & \\
\mu B \left[1 - \epsilon \text{e}^{-i\dot{\phi}/2}\right] & V^0/2
\end{pmatrix}.
$$

In Eq. (17) we assume that neutrinos are ultrarelativistic, i.e. $U_z = \beta U^0 \approx U^0$, where $\beta$ is the neutrino velocity.

Now it is interesting to compare neutrino spin oscillations in a plane electromagnetic wave, studied in Refs. [30–32], with oscillations in a plane GW with the circular polarization. Unlike an electromagnetic wave, GW cannot induce neutrino spin flip. If $B = 0$, there are no transitions $\nu_L \leftrightarrow \nu_R$. This result coincides with the finding of Ref. [13].

GW can only influence the resonance condition in neutrino oscillations. Indeed, choosing $\dot{\phi}$ in Eq. (17) in a certain way, we can reach a significant enhancement of the probability of neutrino spin oscillations $\nu_L \to \nu_R$, $P_{\text{L} \to \text{R}}(t) \equiv |\nu_R|^2 = |\tilde{\nu}_R|^2$, calculated using the solution of Eq. (17). This phenomenon is known as the parametric resonance in neutrino oscillations.
For the first time, the parametric resonance in neutrino oscillations in matter with harmonically varying density was studied in Ref. [33]. One can see in Eq. (17) that the action of GW on neutrino oscillations is the effective harmonic modulation of the transverse magnetic field. It is this manifestation of the parametric resonance, which was discussed in Ref. [34], where neutrino spin and spin-flavor oscillations in inhomogeneous electromagnetic fields were studied.

Using the results of Ref. [34], we suppose that

\[ \dot{\phi} = 2\Omega, \]

where \( \Omega = \sqrt{(\mu B)^2 + V_0^2}/4 \) is the frequency of the neutrino spin precession at the absence of GW. If \( h \ll 1 \), Eq. (17) can be reduced to Mathieu equation. The solution of such equation can be represented in terms of special functions only. That is why we analyze here only numerical solutions of Eq. (17) to highlight the manifestation of the parametric resonance.

We suppose that a beam of left polarized neutrinos is produced at the distance \( z_0 = 10^{-2} \text{au} \) from merging BHs. Here 1 au = 1.5 \times 10^{13} \text{cm} \) is the astronomical unit. These BHs are taken to emit GW in the same direction as the neutrino beam. We assume that, at \( z_0 = 10^{-2} \text{au}, h = 10^{-4} \). If we observed the merging of these BHs at the distance \( z \sim 1 \text{Gpc} \), the amplitude of the produced GW would be \( h_{\text{obs}} = 5 \times 10^{-21} \), which is very close to values observed by the modern GW detectors [15].

We assume that coalescing BHs are surrounded by the electroneutral hydrogen plasma with the electron number density \( n_e = 10^{18} \text{cm}^{-3} \). In this case, the effective potential of neutrino interaction with matter reads \( V_0 = \sqrt{2G_F n_e} \). Note that the existence of accretion disks with much higher densities is predicted in some models of gamma ray bursts [35]. We also suppose that the constant magnetic field \( B = 1 \text{kG} \) is present in the system. This strength of the magnetic field is very moderate. The neutrino magnetic moment is taken to be \( \mu = 10^{-14} \mu_B \), where \( \mu_B \) is the Bohr magneton. The chosen value of \( \mu \) does not exceed the theoretical upper bound on magnetic moments of Dirac neutrinos derived in Ref. [36].

Neutrinos are taken to have the mass \( m \sim 1 \text{eV} \) [37] and the energy \( E = 1 \text{keV} \). Then, neglecting the dispersion of GW, i.e. at \( \omega = k \), we get that \( \dot{\phi} = \omega m^2/2E^2 \). To fulfill the resonance condition in Eq. (18) for the chosen parameters of neutrinos and external field, we have to set \( \omega = 5.6 \times 10^2 \text{s}^{-1} \). If one observed GWs emitted by the merging of BHs from the distance \( z \sim 1 \text{Gpc} \), the GW frequency would be red-shifted to \( \omega_{\text{obs}} = \omega/(1 + z) = 4.5 \times 10^2 \text{s}^{-1} \), where \( z \approx 0.2 \) [38]. This value of \( \omega_{\text{obs}} \) is again very close to GW frequencies observed by the current GW detectors [15].

In Fig. 1 we show the transition probability \( P_{L\rightarrow R}(z) \) versus the distance traveled by the neutrino beam \( z \sim t \) for spin oscillations \( \nu_L \rightarrow \nu_R \) obtained using the numerical solution of Eq. (17). The function \( P_{L\rightarrow R}(z) \) is rapidly oscillating. That is why we represent it only in the inset in Fig. 1 for small \( 0 < z < 5 \times 10^2 \text{au} \). In main Fig. 1 we show only the upper and lower envelope functions, which are built with help of the spline interpolation of the maxima and minima of \( P_{L\rightarrow R}(z) \), respectively, as well as the averaged transition probabilities \( \bar{P}_{L\rightarrow R} \). To highlight the action of GW on neutrino spin oscillations, we show the upper envelope function of \( P_{L\rightarrow R}(z) \), as well as \( \bar{P}_{L\rightarrow R} \) at \( h = 0 \), i.e. for neutrino spin oscillations only in matter and the transverse magnetic field.

One can see in Fig. 1 that, at \( z \approx 4 \times 10^6 \text{au} \), the upper envelope function reaches the unit value and \( \bar{P}_{L\rightarrow R} \sim 0.75 \). It is the manifestation of the parametric resonance, which happens even for small \( h = 10^{-4} \). The upper envelope function and \( \bar{P}_{L\rightarrow R} \) for \( h = 0 \) do not exceed 0.5...
Figure 1: The probability $P_{L \rightarrow R}(z)$ for transitions $\nu_L \rightarrow \nu_R$ versus the distance $z = t$, passed by the neutrino beam, built on the basis of the numerical solution of Eq. (17). Red and green lines are the upper and lower envelope functions. Blue lines are the averaged transition probabilities. Solid lines correspond to $h = 10^{-4}$ and dashed lines to $h = 0$. The inset represents the transition probability, shown by the black line, for $0 < z < 5 \times 10^2$ au.

and 0.25, respectively. It means that the transition probability cannot be amplified to high values without GW.

The resonance condition of the parametric resonance in Eq. (18) is very sensitive to the change of $\dot{\phi}$ or, equivalently, to different frequencies $\omega$ of GW. To illustrate the dependence of the dynamics of neutrino oscillations for different $\dot{\phi}$, in Fig. 2, we show the upper and lower envelope functions, as well as the averaged transition probabilities, versus the distance passed by the neutrino beam. We compare the case when the resonance condition in Eq. (18) is fulfilled with the small deviations from this condition, $\dot{\phi} = 2\Omega \pm h\Omega/4$. One can see in Fig. 2 that in both cases the upper envelope function and $\bar{P}_{L \rightarrow R}$ for $\dot{\phi} \neq 2\Omega$ have smaller values than these for $\dot{\phi} = 2\Omega$. Moreover, comparing Figs. 2(a) and 2(b), one can see that the transition probability behaves differently in decreasing and increasing of $\omega$.

4 Discussion

In the present work, we have studied the neutrino spin evolution in background matter and an external electromagnetic field in curved space-time. This study was motivated by the necessity for the substantiation of the quasiclassical equation for the neutrino spin evolution, which was proposed in Ref. [9]. In Sec. 2, we have rederived this covariant equation starting from the Dirac equation for a massive neutrino interacting with external fields in curved space-time.

The form of the tensor $G_{ab}$ in Eq. (8), which incorporates the contribution of the gravitational interaction to the neutrino spin evolution, is different from that found in Refs. [7,9]. Nevertheless, as shown in Ref. [24], this difference is not important for the frequency of the neutrino spin precession in a gravitational field. The contributions of background matter and an electromagnetic field to the neutrino spin evolution Eq. (17) coincide with those proposed in Ref. [9].

Then, in Sec. 3, we have applied the developed formalism for the description of neutrino
spin oscillations in matter and the constant transverse magnetic field under the influence of a plane GW. We have derived the effective Schrödinger equation and demonstrated that GW can induce the parametric resonance in neutrino spin oscillations.

It should be noted that the contribution to the neutrino spin evolution from the proper gravitational field of a compact rotating object depends on the distance between a neutrino and this object as $1/r^3$ [8]. The amplitude of GW falls with the distance as $1/r$. Hence, despite the effect of the proper gravitational interaction on the spin precession is stronger than that of GW at short distances, neutrino spin oscillations can be more significantly affected by GW at larger distances where the direct gravitational interaction vanishes.

Considering GW emitted by coalescing BHs, we have demonstrated that the transition probability of neutrino spin oscillations can potentially reach great values if we choose realistic parameters of external fields at the initial distance between the neutrino beam and the point where BHs merge. We have also studied the dynamics of neutrino spin oscillations for different frequencies of GW; cf. Fig. 2.

Since the amplitude of GW is small, the parametric amplification of neutrino oscillations takes place if a neutrino passes a large distance; cf. Fig. 1. One cannot expect that the matter density and the strength of the external magnetic field are unchanged at such distances. Thus the description of neutrino oscillations in Sec. 3 is valid only at limited distances of the neutrino beam propagation. In the opposite case, one has to account for the dependence of external fields on coordinates. However, in this situation, the model of the parametric resonance becomes invalid. We plan to analyze neutrino spin oscillations at large distances from coalescing BHs in one of our forthcoming works.

Figure 2: The behavior of neutrino spin oscillations for different $\dot{\phi}$. The upper (red lines) and lower (green lines) envelope functions for $P_{\nu_L \rightarrow \nu_R}(z)$, as well as the averaged transition probabilities (blue lines), versus the distance travelled by the neutrino beam built on the basis of the numerical solution of Eq. (17). Solid lines correspond to the valid resonance condition in Eq. (18) and dashed lines to small deviations $\dot{\phi}_\pm$ from Eq. (18). (a) $\dot{\phi}_+ = 2\Omega + h\Omega/4$. (b) $\dot{\phi}_- = 2\Omega - h\Omega/4$. 

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\[ P_{\nu_L \rightarrow \nu_R}(z) \]

\[ z, \text{ au} \times 10^6 \]
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