Too many cooks: Coordinating multi-agent collaboration through inverse planning

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ABSTRACT
Collaboration requires agents to coordinate their behavior on the fly, sometimes cooperating to solve a single task together and other times dividing it up into sub-tasks to work on in parallel. Underlying the human ability to coordinate is theory-of-mind, the ability to infer the hidden mental states that drive others to act. Here, we develop Bayesian Delegation, a decentralized multi-agent learning mechanism with these abilities. Bayesian Delegation enables agents to rapidly infer the hidden intentions of others by inverse planning. These inferences enable agents to flexibly decide in the absence of communication when to cooperate on the same sub-task and when to work on different sub-tasks in parallel. We test this model in a suite of multi-agent Markov Decision processes inspired by cooking problems. To succeed, agents must coordinate both their high-level plans (e.g., what sub-task they should work on) and their low-level actions (e.g., avoiding collisions). Bayesian Delegation bridges these two levels and rapidly aligns agents’ beliefs about who should work on what without communication. When agents cooperate on the same sub-task, coordinated plans emerge that enable the group to achieve tasks no agent can complete on their own. Our model outperforms lesioned agents without Bayesian Delegation or without the ability to cooperate on the same sub-task.

KEYWORDS
coordination; inverse planning; Bayesian inference; multi-agent; reinforcement learning

1 INTRODUCTION
Working together enables a group of agents to achieve together what no individual could achieve on their own [16, 38]. However, collaboration is challenging as it requires agents to coordinate their behaviors. In the absence of prior experience and the social roles, institutions, laws, and norms that constitute human society, we still find ways to negotiate our joint behavior in any given moment to work together with efficiency [29, 39]. Whether we’re writing a scientific manuscript with collaborators or preparing a meal with friends, core questions we must ask ourselves are: how can I help out the group? What should I work on next, and with whom should I do it with? Coordination unfolds over many timescales. We avoid bumping into each other when moving around in a cramped kitchen and also avoid making two desserts instead of a dessert and a main course. These commonsense abilities are at the core of human social intelligence and in order to build social machines we must engineer AI systems that can coordinate with us and with each other as rapidly and as flexibly as people do [20].

Central to this challenge is that agents’ reasoning about what they should do in a multi-agent context requires knowledge about the future actions and intentions of others. When agents, like people, make decisions independently, these intentions are unobserved. Action and interactions can reveal information about intentions, but prediction is difficult because of uncertainty and ambiguity – the same action might result from multiple possible intentions. In humans, the ability to understand intentions from actions is called theory-of-mind (ToM). Humans rely on this ability to cooperate in coordinated ways, even in novel situations [19, 34, 39]. We aim to build agents that have these kinds of abilities and show that they are powerful building blocks for coordinated cooperation.

In this work, we study these abilities in the context of multiple agents cooking a meal together, inspired by the video game Overcooked [14]. These problems have hierarchically organized sub-tasks and share many features with other object-oriented tasks such as construction and assembly. Cooking tasks are challenging because of the sheer variation that is present: no kitchen or recipe is exactly alike, so successful collaboration requires flexible and abstract mechanisms for coordination. These tasks highlight three coordination challenges that any decentralized multi-agent system must grapple with:

(A) Divide and conquer: agents should work in parallel when sub-tasks can be carried out individually.
(B) Cooperation: agents should work together on the same sub-task when most efficient or necessary.
(C) Spatio-temporal movement: agents should avoid getting in each other’s way at any time.

To illustrate, imagine the process required to make a simple salad: first chopping both tomato and lettuce, and then assembling them together on a plate. Two people might collaborate by first dividing the sub-tasks up: one person chops the tomato and the other chops the lettuce. This doubles the efficiency of the pair by completing sub-tasks in parallel (challenge A). Some sub-tasks require two or more agents to work together. If only one person can use the knife but that person can’t reach the tomatoes, then they must cooperate to chop the tomato (challenge B). Whether working apart or together agents must also coordinate their low-level actions in space and time. They need to avoid interfering with others who are working

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We develop working together (challenge C). Different recipes are possible in each environment, allowing for variation in high-level goals while keeping the low-level navigation challenges constant. In (a) Open-Divider, agents can freely move between both sides of the kitchen. In (b) Partial-Divider, agents can move between both sides but must either coordinate their movements through a narrow bottleneck or accelerate completion by passing dishes across the counter. In (c) Full-Divider, agents cannot access all of the objects and coordination is required.

Figure 1: The Overcooked environments. The starting positions of all movable objects are identical in all three environments; only the counters differ. Different recipes are available in each environment, allowing for variation in high-level goals while keeping the low-level navigation challenges constant. In (a) Open-Divider, agents can freely move between both sides of the kitchen. In (b) Partial-Divider, agents can move between both sides but must either coordinate their movements through a narrow bottleneck or accelerate completion by passing dishes across the counter. In (c) Full-Divider, agents cannot access all of the objects and coordination is required.

1.1 Contributions
We develop Bayesian Delegation, a learning mechanism that enables agents to rapidly infer the sub-tasks of other agents based on theory-of-mind. Bayesian Delegation uses Bayesian inference to navigate uncertainty and computes the likelihood of different intentions by inverse planning. The priors in Bayesian Delegation express different kinds of multi-agent plans including those where both agents cooperate on the same sub-task and those where both agents divide and conquer the sub-tasks. We quantitatively study the behavior and representations of the model in 2D cooking grid worlds inspired by the video game Overcooked. When Bayesian Delegation is used in conjunction with a model-based reinforcement learner, it achieves decentralized coordination across sub-tasks and overcomes the coordination challenges highlighted above. Finally, we empirically demonstrate the effectiveness of the model against lesioned versions of itself.

1.2 Related Work
Our work builds on a long history of using cooking tasks for evaluating multi-agent and AI systems [15, 18, 40]. Most recently, the video game Overcooked has inspired work on single-agent hierarchical reinforcement learning where agents learn and reuse sub-task policies [35]. Most close to this work is Carroll et al. [9] where they study multi-agent coordination challenges in 2D Overcooked-like environments. They evaluate whether deep reinforcement learning agents trained using self-play can coordinate with humans. Their approach requires large amounts of training experience in each specific environment and does not generalize to new situations where data is not available. Finally, they only study one task structure in each of their environments, whereas the abstractions in our agents generalize to multiple tasks and environments including those that they have had no experience with.

Bayesian Delegation is inspired by the cognitive science of how people coordinate their cooperation in the absence of communication [19]. Our approach most closely follows from recent work on Bayesian theory-of-mind (ToM), which closely approximates inferences people make about the latent goals and beliefs that drive the actions of other agents [3, 4, 31, 32], dyads [19, 41, 43], and groups [34]. Finally, our agents use Bayesian Delegation inside a hierarchical planner that build on previous work that links low-level navigation to high-level sub-tasks [1, 2].

Our approach is fully decentralized and so contrasts with existing centralized coordination schemes such as a centralized auctioneer [8, 26] or decentralized consensus approaches (see [8] for a review). It is also inspired by other multi-agent planning approaches in which agents aggregate the effects of others and best respond in a decentralized fashion [10, 11]. We extend this prior work by considering effects over higher order policy objectives such as sub-tasks instead of being limited to observable features (i.e. location). Additionally, our formalism is object-oriented rather than purely spatial which increases planning complexity.

Coordinating multi-agent plans across hierarchies of sub-tasks has its roots in classic formalisms for teamwork [12, 15, 24, 37]. However, these approaches do not easily extend to multi-agent decision making under uncertainty, nor are they easily extended to sequential decision making settings. More recent approaches have studied multi-agent algorithms capable of ad-hoc teamwork [5, 36] including the ability to learn statistical models of others [6, 28]. However, these algorithms are often limited to movement-level coordination tasks, such as chasing and hiding rather than the complexities of coordinated sub-tasks. Finally, there is a growing body of work on multi-agent cooperation in the context of competitive interactions which is complementary to our focus on coordination [17, 19, 21, 22, 33].

2 DECENTRALIZED MARKOV DECISION PROCESSES WITH SUB-TASKS
A decentralized multi-agent Markov decision processes (Dec-MDPs) with sub-tasks is the tuple \((n, S, \mathcal{A}_1, \ldots, n, T, R, \gamma, T)\) where \(n\) is the number of agents. \(S\) is a set of object-oriented states specified by the locations and status of each object and agent in the environment [13]. This environmental state is fully observable to all agents. \(\mathcal{A}_i, \ldots, n\) is the joint action space with \(\mathcal{A}_i\) being the set of actions available to agent \(i\); each agent chooses its own actions independently. \(T(s, a_1, \ldots, n, s')\) is the transition function which describes the probability of transitioning from state \(s\) to \(s'\) after all agents act \(a_1, \ldots, n\). \(R(s, a_1, \ldots, n)\) is the reward function shared by all agents, \(\gamma\) is the discount factor.

Unlike traditional Dec-MDPs, the environments we study here have a partially ordered set of sub-tasks \(T = \{T_0, \ldots, T_f\}\). Each sub-task \(T_f\) has preconditions that specify when a sub-task can be started, and postconditions that specify when it is completed. These sub-tasks serve a dual role. First, they provide some structure and guidance when \(R\) is very sparse. Second, they are the targets of high-level coordinated action between agents. In this work, sub-tasks always concerned the “merger” of two objects, that is, to
We now describe the suite of tasks we developed to test our agents. Without centralized learning or any communication side-channels.

When X and Y are brought to the same location. In the cooking environment we study here, the partial order of sub-tasks refers to a “recipe”. Each agent aims to find a policy $\pi_i(s)$ that maximizes expected discounted reward. Although agents fully observe the state of the environment, they do not observe the policies ($\pi_{-i}(s)$ where $-i$ refers to all other agents except $i$) or any other internal representations of the other agents. Agents must plan and act without centralized learning or any communication side-channels.

### Interaction dynamics:

Food1 + Food2 $\rightarrow$ Food.chopped + Knife

$\text{Food1} + \text{Food2} \rightarrow [\text{Food1, Food2}]$

$\text{X} + \text{Y}[\cdot] \rightarrow \text{Y}[\text{X}]$

Table 1: State representation and transitions for the objects and interactions in the Overcooked environments.

| Type       | Location | Status          |
|------------|----------|-----------------|
| Agent      | [x, y]   | [ ]             |
| Plate      | [x, y]   | [ ]             |
| Counter    | [x, y]   | [ ]             |
| Delivery   | [x, y]   | [ ]             |
| Knife      | [x, y]   | N/A             |
| Tomato     | [x, y]   | [chopped, unchopped] |
| Lettuce    | [x, y]   | [chopped, unchopped] |

bring two objects into the same spatial location. For example, $T_i = \text{Merge}(X, Y)$ means that the preconditions are satisfied when X and Y exist in the specified state and the postconditions are satisfied when X and Y are brought to the same location. In the cooking environments we study here, the partial order of sub-tasks refers to a “recipe”. Each agent aims to find a policy $\pi_i(s)$ that maximizes expected discounted reward. Although agents fully observe the state of the environment, they do not observe the policies ($\pi_{-i}(s)$ where $-i$ refers to all other agents except $i$) or any other internal representations of the other agents. Agents must plan and act without centralized learning or any communication side-channels.

### Object state representation:

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Table 1: State representation and transitions for the objects and interactions in the Overcooked environments. The two food items (tomato and lettuce) can be in either chopped or unchopped states. Objects with status [ ] are able to “hold” other objects. For example, an Agent holding a Plate holding an unchopped tomato would be denoted Agent[Plate[Tomato.unchopped]]. Once combined, these nested objects share the same (x, y) coordinates and movement. Interaction dynamics occur when the two objects are in the same (x, y) coordinates.

### 2.1 Coordination in Overcooked

We now describe the suite of tasks we developed to test our agents. These tasks are inspired by the video game *Overcooked*. Each environment is a two-dimensional grid-world kitchen with various cooking objects and foods. Figure 1 shows examples of different kitchens and the coordination challenges generated by different spatial configurations. The state is represented as a list of entities and their type, location, and status [13]. See 1 for a description of the different entities, the dynamics of their interactions, and the statuses that are possible for the Overcooked test suite.

Agents can move north, south, east, west or stay still. All agents move simultaneously. They cannot move through each other, into the same space, or through counters. If they try to do so, then they remain in place instead. The reward function $R$ was specified in terms of costs (negative rewards). The cost of each step is 1 point, and the additional cost of moving or interacting is 0.1 points versus staying still. This cost structure incentivizes the agents to make the recipe as efficiently as possible. The environment terminates after either the agents deliver the finished dish to the star square or 100 time steps elapse.

The kitchens have counters that contain food items (e.g., tomatoes and lettuce in various states), knife stations (which are immovable), and plates. Agents interact with objects by moving into them. When an agent interacts with a food item, it picks the object up. When an agent holding a food item interacts with a knife station, the food is transformed into a chopped state. If an agent moves into a counter while carrying an object, it places that object down on the counter. Agents can only carry one object at a time. Sub-tasks in this test suite involve moving the different food items around, processing them, and turning them into a finished recipe. Figure 2 shows the recipes and their partial order of sub-tasks for the test suite. All sub-tasks are expressed by a single operator called $\text{Merge}(X, Y)$, which describes bringing X and Y into the same spatial location. Importantly, $\text{Merge}$ is ambiguous as to both how the merger of the two objects should actually happen (for instance walking around a divider versus passing the object over it) as well as who should do that sub-task. This must be figured out by the agents.

This allows us to test models that can rise to the challenges raised in the introduction. For instance, the salad recipe (which requires combining chopped tomato and chopped lettuce; Figure 2c) can easily be broken down into two disjoint plans (chopping the tomato and chopping the lettuce) that only mesh together later in time when they are merged with a plate and served. The counter layouts also provide opportunities for multiple agents to work together advantageously. For instance, when counters physically separate the agents (Figure 1c), even a simple task of chopping a tomato must be carried out jointly by passing the tomato over the counters between the agents. When counters partially separate the two spaces, a bottleneck is created but so is an opportunity for the agents to work more efficiently together by passing items across the partial barrier (Figure 1b). Thus, the Overcooked environments enable us to study multi-agent coordination across levels of hierarchical planning—agents must coordinate on what part of the total recipe each intends to complete but also have to coordinate their specific actions together to avoid collisions or to pass an object to each other efficiently.

### 3 COMPUTATIONAL MODEL

We introduce a novel learning algorithm for multi-agent coordination based on probabilistic inference over sub-tasks called Bayesian Delegation. At a high-level (Section 3.1), each agent must decide which sub-task they should do next. Bayesian Delegation probabilistically models the unobserved intentions of other agents in order to dynamically decide whether to divide and conquer on different sub-tasks or cooperate on the same one. At a low-level (Section 3.2), agents use model-based reinforcement learning to find approximately optimal policies for a specific sub-task. Planning is decentralized at both levels i.e., each agent plans and learns for itself without sharing any information with others.

#### 3.1 High-level planning (sub-task)

Inferring the sub-tasks others are working on enables each agent to efficiently select a single sub-task when multiple are possible.
Bayesian Delegation works by having each agent maintain and update a belief state over the possible sub-tasks that all agents (including itself) are likely working on based on the evidence that is commonly observed by all. This avoids multiple levels of recursive belief (e.g., level-K and cognitive hierarchy).

Under Bayesian Delegation, $ta$ is the set of all possible allocations of agents to sub-tasks where all agents are assigned to a sub-task. For example, if there are two possible tasks ($\{T_1, T_2\}$) and two agents ($\{i, j\}$), then $ta = [(i : T_1), (j : T_2), (i : T_2), (j : T_1), (i : T_1), (j : T_2)]$ where $i : T_i$ means that agent $i$ is assigned to sub-task $T_i$. Thus $ta$ includes both the possibility that agents will divide and conquer (work on separate sub-tasks) and the possibility that they will cooperate (work on the same sub-task). Each element $ta$ of $ta$ is a high-level plan which "delegates" each agent to a task. If all agents pick the same element, then they will easily coordinate. However, in our environments, agents cannot communicate before or during the task. Thus, each agent $i$ maintains uncertainty about which $ta$ the group is coordinating on, $P(ta)$.

Each time step, the agent selects the most likely sub-task allocation $ta^*$ and plans actions according to that sub-task allocation (see the Section 3.2 for details on how a given $ta$ is grounded into a low-level plan):

$$ta^* = \arg \max_{ta} P(ta|H_{0:T})$$

where $P(ta|H_{0:T})$ is the posterior over $ta$ after having observed a history of actions, $H_{0:T} = \{(s_0, a_0), \ldots (s_T, a_T)\}$ where $T$ is the number of time steps since the start of the episode. This posterior is computed at time step $T$ according to Bayesian inference:

$$P(ta|H_{0:T}) \propto P(ta)P(H_{0:T}|ta)$$

$$= P(ta) \prod_{t=0}^{T} P(a_t|s_t, ta)$$

where $P(ta)$ is the prior over $ta$ and $P(a_t|s_t, ta)$ is the likelihood of actions at time step $t$ for all agents. Note that these belief updates do not explicitly consider what each agent knows about their own sub-tasks at time $T = 1$. Rather, the model only considers the information that is known by all, i.e., the information a a third-party observer would have access to [30]. The likelihood is computed using a soft-max to account for non-optimal and variable behavior:

$$P(a_t|s_t, ta) = \exp(\beta \cdot Q^*_{IQA}(s,a))$$

where $Q^*_{IQA}(s,a)$, the expected future reward of actions towards the completion of sub-task $ta$, is computed by low-level planning (described in detail in Section 3.2 and $\beta$ controls the degree to which an agent believes other agents are optimizing. When $\beta \rightarrow 0$, the agent believes that others are acting randomly. When $\beta \rightarrow \infty$, the agent believes that others are perfectly maximizing with respect to a model of others. Intermediate values of $\beta$ interpolate between these extremes. Since the likelihood is computed by planning the approach to posterior inference is called inverse planning.

By adding further structure to the prior, $P(ta)$, we can improve the computational efficiency and bias agents towards more efficient coordination structures. First, all $ta$ that do not have their preconditions met are given 0 probability; this means they do not need to be accounted for in the Bayesian inference. Second, in order to help break symmetries at $ta$ and bias agents towards more efficient divisions of work, each $ta$ is given normalized prior probability inversely-proportional to estimated value. While the low-level planners can estimate these values exactly, for computationally efficiency we initialize the probability of each $ta$ to be inversely proportional to the distance between the two objects specified in the Merge operator. Finally, $ta$ does not include allocations where some agents are assigned to do nothing; if there is only one available sub-task then all other agents are assigned to do it jointly. Figure 3 shows an example of the dynamics of $P(ta)$ during agent interaction. The figure illustrates how Bayesian delegation enables agents to dynamically align their beliefs about who is doing what (i.e., assign high probability to a single $ta$).

Figure 2: Visual descriptions of the sub-tasks and example partial orderings. All sub-tasks are written in the form of the Merge operator, which is completed by bringing the first object argument into spatial contact with the second object argument. This either transforms the objects (as in the case of chopping) or combines them into a single object, with one embedded in the other. In (a) *Tomato*, the task is to take an unchopped tomato and then chop, plate, and deliver it. In (b) *Tomato+Lettuce*, the task builds on *Tomato* but also requires chopping, plating, and delivering a piece of lettuce. In (c) *Salad*, the two chopped foods are combined on a single plate and delivered. The example plans show one possible ordering for how agents might finish the recipes but other permutations are possible.
3.2 Low-level planning (action)

High-level planning lets agents to dynamically learn to coordinate their sub-tasks with others while low-level planning grounds those sub-tasks into context-sensitive actions and movements. Low-level planning also provides the critical likelihood for Bayesian delegation (see Equation 2). Low-level planning takes the $ta$ selected by Bayesian Delegation and generates a sequence of actions for the agent to execute while modeling the movements of other agents across time and space. In this work, we used bounded real-time dynamic programming (BRTDP), an algorithm for model-based reinforcement learning, and extend it to a multi-agent setting [27].

Each sub-task $T_i \in ta$ induces a new multi-agent MDP with a reward function $R_{T_i}$ just for completing that sub-task in as few time steps as possible. BRTDP updates lower and upper bounds on $V^*_{T_i}(s)$, the value function of the state under the optimal policy for that sub-task $\pi^*_T(s)$. The state-action values with respect to $V_{T_i}$ are defined by lower and upper bounds:

$$Q^L_{T_i}(s, a) = R_{T_i}(s, a) + \sum_{s' \in S} T(s'|s, a, h_{T_i}(s')) V^L_{T_i}(s')$$

$$Q^H_{T_i}(s, a) = R_{T_i}(s, a) + \sum_{s' \in S} T(s'|s, a, h_{T_i}(s')) V^H_{T_i}(s')$$

Since the state representation is object-oriented (see Table 1) and the sub-tasks are expressed in terms of $Merge(X, Y)$, we used efficient deterministic path-finding algorithms (Dijkstra’s) and the Manhattan distance between objects to infer soft spatial geometry into the bounds of BRTDP.

The lower-bound was initialized to the Manhattan distance between objects (which ignores barriers). The upper-bound was the sum of the shortest-paths between objects which ignores the possibility of more efficiently passing objects. While BRTDP and these heuristics are useful for the specific spatial environments and sub-task structures we develop here, it could be replaced with any other algorithm for finding an approximately optimal single-agent policy for a given sub-task. At each time step, agents select the action with the highest value for their sub-task.

In order to plan in the presence of other agents, agents must address the two types of low-level coordination problems: (1) avoiding collisions and getting in the way of other agents while working on distinct sub-tasks, and (2) solving a sub-task cooperatively with another agent [34]. Given a sampled $ta$, each agent $i$ has a hypothesis about the sub-tasks carried out by the other agents $T_{-i}$. In the case where $T_i \neq T_{-i}$, agent $i$ creates simple models of the others performing $T_{-i}$ and best responds. This approach is similar to level-K or cognitive hierarchy [19, 34, 42]. This process starts by finding non-strategic policies (level-0) of others $\pi^0_{T_{-i}}(s)$, which assume that all other agents except for $-i$ are static. These level-0 models are then used to reduce the multi-agent transition function to a single agent problem $T'$ where the transitions of the other agents are assumed to follow the level-0 policies:

$$T'(s'|s, a_{-i}) = \sum_{a_{-i}} T(s'|s, a_{-i}, a_i) \prod_{A \in -i} \pi^0_{T_{-A}}(s).$$

Replacing the dynamics of the other agents with $\pi^0$ returns a single-agent MDP with transformed transition function $T'$. Running BRTDP on this transformed environment finds an approximately optimal level-1 policy $\pi^1_T(s)$ for agent $i$ that best responds to the level-0 models of the other agents.

When the sampled $ta$ specifies that agents are to work together on the same sub-task $i$, $T_i = T_{-i}$, agents cooperate to solve the same sub-task. In this case, an agent simulates a fictitious centralized agent that controls the actions of all agents working together on the same sub-task [19]. This gives the same sub-task problem as before but with a transformed action space. For instance if both $i$ and $j$ are working on $T_i$ then $A^* = a_i \times a_j$. By transforming the action space in this way, joint policies $\pi^1_T(s)$ can be found by single-agent planners such as BRTDP. Once the agent solves for the cooperative plan, it can find its own role in that cooperation by playing the action assigned to it under $\pi^1_T(s)$. This kind of cooperative planning enables emergent cooperative behavior – agents pass objects across the counters when efficient even though there was nothing about passing encoded into the environment. Since different agents may find different joint policies, there are no guarantees of perfect decentralized coordination. We note that these two mechanisms for low-level planning mirror those developed in the composable team hierarchy framework of Shum et al. [34]. In that work, these two modes of planning were called the Best-Response and Joint-Planning operators.

4 EXPERIMENTAL RESULTS

We conduct a series of computational experiments to test the agent’s ability to generate coordinated behavior in the suite of cooking tasks we described earlier. Agents play with each other in each of the 9 combinations of kitchens and recipes (3 kitchens x 3 recipes). We compared performance by measuring the time to complete the full recipe, the sub-tasks, and the number of collisions between agents. Our aim is to investigate in what way and to what degree Bayesian Delegation and cooperative joint planning is important for successful multi-agent coordination. To quantitatively measure the impact of these elements, we create “lesions” or alternative models that are missing either one or both of these two components.

The first lesioned model (NJP) has the ability to do Bayesian Delegation but cannot do cooperative joint planning. It does not consider cases where both agents are assigned to the same sub-task. The second lesioned model (NJP+NBD) cannot use cooperative joint planning and also has no Bayesian delegation. It makes no inferences or predictions about others. This mimics an agent without any social intelligence. It does not maintain any beliefs over possible sub-tasks that the other agents are working on. Instead, at each time step, each agent greedily chooses a sub-task for itself out of all currently available sub-tasks.

To contrast these lesions from the complete model, let us consider the example from Section 3.1 with two possible tasks $[(T_1, T_2)]$ and two agents $[(i, j)]$. The full model would propose $ta = [(i : T_1, j : T_2), (i : T_2, j : T_1), (i : T_1, j : T_1), (i : T_2, j : T_2)]$ where $i : T_1$ means that agent $i$ is assigned to sub-task $T_1$. NJP does not allow for joint planning, and thus $ta = [(i : T_1, j : T_2), (i : T_2, j : T_1)]$; note that the options where both $i$ and $j$ are assigned to the same sub-task $T$ do not appear in this lesioned $ta$. Lastly, NJP+NBD makes no predictions about other agents. Each agent plans possible allocations only for itself. For instance, agent $i$’s model would propose $ta = [(i : T_1), (i : T_2)]$. Note that $j$ does not appear here because this lesion does
not make inferences over other agents. These lesions enable us to highlight the coordination failures that arise when either joint planning or Bayesian Delegation are unavailable. We have also tried running the low-level planner without any high-level planning, but it does not converge within our computational budget due to reward sparsity.

All experiments are replicated across 50 random seeds. Agents are allowed 100 time steps before an episode terminates. If the pair does not complete the task, then they receive the maximum completion time of 100. We run BRTDP until the bounds converge ($\alpha = 0.01, \tau = 2$ see McMahan et al. [27] for usage) or for a maximum of 100 trajectories each with up to 75 roll-outs. The softmax used in the Bayesian inference uses $\beta = 0.3$.

Figure 3: Dynamics of the belief state, $P(\alpha)$ for each agent during Bayesian delegation with the Tomato+Lettuce recipe (Figure 2b) on Partial-Divider (Figure 1b). During the first 5 time steps, only Merge(Tomato.unchopped, Knife) and Merge(Lettuce.unchopped, Knife) are nonzero because their preconditions are met. The two sub-tasks can be completed in four possible ways: (1) agent-1 chops the tomato and agent-2 chops the lettuce in parallel, (2) agent-2 chops the tomato and agent-1 chops the lettuce in parallel, (3) they jointly chop the tomato, or (4) they jointly chop the lettuce. By the third time step, both agents are aligned on first chopping the tomato together. Once this sub-task is complete, Merge(Tomato.chopped, Plate[]) becomes available (row 3). The alignment between the two agents continues, even though there is never any communication or prior agreement on what sub-task each agent should be doing or when.

Figure 4 shows the empirical results for completion time. The full model performs well on all recipes and kitchen layouts, rarely needing more than half the maximum time and was systematically more efficient than all three alternatives. For recipe Tomato on Open-Divider, all three models are comparable in terms of performance since both the high-level and low-level planning problems are quite simple and there is plenty of space to maneuver. However, when faced with more complex recipes on the same level (top row), NJP and NJP+NBD take significantly longer to finish. We also describe these differences qualitatively. For recipe Tomato+Lettuce, NJP frequently fails to complete the recipe because the agents are unable to jointly plan. As a result, they often simultaneously yield to each other (assuming the other will pass through) and are unable to accomplish the joint goal. For recipe Salad, both NJP and NJP+NBD rarely finish. While they do attempt to complete tasks in parallel (e.g. one agent chops the tomato while the other chops the lettuce), combining the two separate ingredients requires them to work together, which they cannot do since they lack joint planning.

In Partial-Divider (middle row), NJP and NJP+NBD take significantly longer than the full model, especially for the Tomato+Lettuce recipe. This is because completion time multiplies with the number of sub-tasks, and each task must be done individually by one of the two agents. If an agent plans to chop a tomato, for instance, they must walk all the way around the divider in order to get from the tomato to the knife, as opposed to directly passing across the

| Time Steps | 1000 | 2000 | 4000 | 6000 | 8000 | 10000 |
|------------|------|------|------|------|------|-------|
| Agent-1    | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0   |
| Agent-2    | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0   |

Figure 4: Performance of Bayesian Delegation model in terms of number of time steps needed to complete each task on each kitchen-recipe composition (lower is better). The row shows the kitchen and the column shows the recipe. The full model is compared to (and systematically outperforms) two lesioned models described in the text. The error bars show the standard error of the mean, and all simulations are run with the same 50 random seeds.
divider with a joint plan. For the Salad recipe, the agents are unable to finish because they get stuck attempting to cross to opposite sides of the kitchen at the same time and are caught in the bottleneck. There, the lack of joint planning prevents them from passing objects over the counter and causes collisions.

Without the ability to reason about and for other agents, levels like Open-Divider and Partial-Divider result in frequent spatial miscoordination, where agents cannot move out of each other’s way. This is especially evident in the performance of NJP on recipe Tomato+Lettuce, where the two agents must deliver two separate dishes. However, once one agent places its finished item on the delivery square, it blocks the only access point to the delivery location. Because these lesioned agents cannot plan jointly, the agent does not move out of the way to allow the other agent to deliver its completed plate thereby preventing full completion of the task. In the Full-Divider level (bottom row), NJP and NJP+NBD are unable to complete any recipe because the physical barrier between the two agents requires joint planning in order to successfully complete any task.

When comparing the lesioned agents on task performance we find that NJP+NBD is either comparable or better than NJP. In the kitchen-recipe compositions where NJP+NBD performs better than NJP (e.g., Tomato+Lettuce on Open-Divider and Partial-Divider), one possibility is that acting completely greedily is more effective. In both lesions, agents do not determine or plan for sub-tasks that require cooperation between themselves and their partner, because they lack joint planning capabilities. In NJP+NBD experiments, agents plan only for themselves. Neither agent stops unless there is a physical collision as long as there are unfinished sub-tasks. In contrast, agents under NJP still perform inference over the sub-tasks of other agents. This sometimes results in deadlock due to each agent yielding to the other without a way to break the symmetry. Finally, in the cases where the lesioned models were able to complete Partial-Divider, they collided at higher rates with each other while the full model was able to better coordinate with its partner and avoid collisions (Figure 6). Together, these results highlight the importance of giving agents the ability to combine joint planning with sub-task coordination enabled by Bayesian delegation.

Figure 5 shows the rate that the different models complete sub-tasks over time. On Full-Divider, both lesions fail to complete any sub-tasks because joint planning is required in all cases. When NJP and NJP+NBD do make progress, they often fail to finish all sub-tasks and are systematically slower than the full model on the ones they do complete. NJP and NJP+NBD are comparable on most tasks and levels except for the case of Tomato+Lettuce in Partial-Divider. There, NJP+NBD actually outperforms NJP, likely due to the same scenario where each agent attempts to yield to others described above.

5 DISCUSSION

We developed a new set of spatial and object-oriented cooking challenges that require a variety of coordinated strategies to successfully complete. In particular, the environments are highly similar, but subtle changes in the recipe or changes in spatial layout call for very different efficient multi-agent strategies. We developed an abstract approach inspired by cognitive science, Bayesian Delegation, which rises to these challenges and learns to coordinate in a decentralized way. Bayesian Delegation enables coordination through two key mechanisms:

1. Inverse planning that enables agents to rapidly infer what high-level sub-tasks other agents are doing. This allows for each agent to figure out when they should be helping out the other agents on a single task and when they should work in parallel to more rapidly finish the next sub-task.
2. Joint planning enables agents to mesh their intentions together and flexibly find coordinated low-level policies that
complete sub-tasks in ways that neither agent could achieve on their own.

Our agents reflect many natural aspects of human cooperation, such as the emergence of and convergence of joint behavior when joint planning is deemed better than planning alone [38]. These behaviors come quite intuitively to humans even in novel situations and Bayesian delegation enables these abilities in computational agents.

While Bayesian delegation solves some aspects of commonsense coordination, there are still limitations which we hope to address in future work. One challenge is that when agents jointly plan for a single sub-task, they currently have no way of knowing when they have completed their individual “part” of the joint effort. For instance, in the case where one agent needs to pass lettuce and tomato across the divider for the other to chop it, after dropping off the lettuce, the first agent is currently unable to reason that it has fulfilled its role in that joint plan and can move on, i.e., that the rest of the sub-task depends only on the actions of the other agent. Currently, our agents considers sub-tasks active as long as their post-conditions remain unsatisfied. If agents were able to recognize when their sub-tasks were finished with respect to themselves, then they would be able to coordinate even more efficiently and flexibly. This opens the possibility of looking ahead to future sub-tasks that will need to be done even before their preconditions are satisfied. For example, once an agent passes off a tomato to another to chop, the first agent can go and get a plate in anticipation of also passing that over even before the chopping has begun.

Finally, the algorithms presented here scale poorly with the number of agents. In some sense this is a natural trade-off, as Bayesian Delegation through inverse planning requires computing policies not just for oneself but also for each other agent. As one scales up the number of agents (as in human coordination), other less flexible but more efficient mechanisms will also play a crucial role. Over time, people build up and establish behavioral norms and conventions which yield coordination without sophisticated agent modeling [7, 25, 44]. Roles often emerge between people that spend significant time together [29]. For instance in Partial-Divider a pair of agents could break the symmetry by converging on a norm where one person always yields to the other or in Open-Divider a pair of agents might decide to always move in a clockwise direction to minimize the probability of collisions [9, 23]. Models that allow for these kinds of subtle norms and roles to emerge are needed for agents to form longer term collaborations that persist beyond a single short interaction. Such representations are essential for building AI agents that successfully and efficiently partner with human teams and with each other.

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