Supersymmetry (SUSY) helps solve the hierarchy problem in high-energy physics and provides a natural groundwork for unifying gravity with other fundamental interactions. While being one of the most promising frameworks for theories beyond the Standard Model, its direct experimental evidence in nature still remains to be discovered. Here we report experimental realization of a supersymmetric quantum mechanics (SUSY QM) model, a reduction of the SUSY quantum field theory for studying its fundamental properties, using a trapped ion quantum simulator. We demonstrate the energy degeneracy caused by SUSY in this model and the spontaneous SUSY breaking. By a partial quantum state tomography of the spin-phonon coupled system, we explicitly measure the supercharge of the degenerate ground states, which are superpositions of the bosonic and the fermionic states. Our work demonstrates the trapped-ion quantum simulator as an economic yet powerful platform to study versatile physics in a single well-controlled system.
Supersymmetry (SUSY) is a quantum mechanical symmetry that unifies the space-time and the internal degree of freedom of elementary particles. In an SUSY theory, bosons and fermions have one-to-one correspondence with the same mass, which leads to a cancellation in the Lagrangian of the Higgs particle and thus helps solve the hierarchy problem or the ultraviolet divergence in Standard Model. However, in reality, this cancellation in Higgs mass is not exact and no SUSY partners of known particles have been discovered at the current energy scales, which requires the spontaneous breaking of SUSY. Since none of known particles have been discovered at the current energy scales, which requires the spontaneous breaking of SUSY, the last term a renormalization constant. This system possesses two supersymmetric points. One is at $g = 0$ and $\omega_s = \omega$, where the system is as simple as a spin and an oscillator without coupling. This becomes evident if we write $\omega_s = 2\beta f - 1$ and $\sigma_s = f + f^\dagger$, where $f$ ($f^\dagger$) denotes the annihilation (creation) operator of a fermionic mode. The excited states $|\uparrow\rangle n$ and $|\downarrow\rangle n+1$ $(n \geq 0)$ are degenerate fermionic and bosonic states $|\uparrow\rangle$ and $|\downarrow\rangle$ have fermionic occupation number of 1 and 0, respectively, with positive energy $E_{n+1} = (n+1)\omega$, while the unique ground state $|\downarrow\rangle 0$ has zero vacuum energy $E_0 = 0$ owing to the cancellation between the spin (fermionic) energy $\omega_s$ and the phonon (bosonic) energy $\omega$. The other more interesting SUSY point appears if we turn on the spin–phonon coupling $g$ and set $\omega_s = 0$ (see Supplementary Note 1). In this special situation, while the Hamiltonian is still supersymmetric, its ground state is not, which indicates a spontaneous SUSY breaking. We connect these two supersymmetric points by the path

$$H = \frac{(1 - r)\omega}{2} \sigma_z + \omega \left( a^\dagger a + \frac{1}{2} \right) + g \sigma_x (a + a^\dagger) + \frac{g^2}{\omega} \sigma_z,$$

where $\sigma_x, \sigma_z$ denote the components of the Pauli spin operator, $a$ ($a^\dagger$) the bosonic annihilation (creation) operator for the phonon mode, and the last term a renormalization constant. This system possesses two supersymmetric points. One is at $g = 0$ and $\omega_s = \omega$, where the system is as simple as a spin and an oscillator without coupling. This becomes evident if we write $\omega_s = 2\beta f - 1$ and $\sigma_s = f + f^\dagger$, where $f$ ($f^\dagger$) denotes the annihilation (creation) operator of a fermionic mode. The excited states $|\uparrow\rangle n$ and $|\downarrow\rangle n+1$ $(n \geq 0)$ are degenerate fermionic and bosonic states $|\uparrow\rangle$ and $|\downarrow\rangle$ have fermionic occupation number of 1 and 0, respectively, with positive energy $E_{n+1} = (n+1)\omega$, while the unique ground state $|\downarrow\rangle 0$ has zero vacuum energy $E_0 = 0$ owing to the cancellation between the spin (fermionic) energy $\omega_s$ and the phonon (bosonic) energy $\omega$. The other more interesting SUSY point appears if we turn on the spin–phonon coupling $g$ and set $\omega_s = 0$ (see Supplementary Note 1). In this special situation, while the Hamiltonian is still supersymmetric, its ground state is not, which indicates a spontaneous SUSY breaking. We connect these two supersymmetric points by the path

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Nevertheless, these two states locate in different symmetry branches of the QRM under parity transform, which commutes with the QRM Hamiltonian. Therefore, the two states will not evolve into each other and the adiabatic condition can still hold. However, there is another problem that as \( r \) increases, the coupling between these two lowest levels by \( H_p \) becomes weaker, which leads to a reduction in the resonant signal even for the ideal state as illustrated in Fig. 2a at large \( r \). Consequently, it is still difficult to accurately determine the energy degeneracy near \( r = 1 \), the nontrivial spontaneous SUSY breaking point. For this purpose, we explicitly measure the energy expectation values of the two ground states at \( r = 1 \).

To prepare these two states, we fix \( \omega_z = 0 \) in Eq. (1) and gradually turn up \( g \) from zero to \( g_m \) (see Methods). The two initial ground states \( |\uparrow\rangle|0\rangle \) and \( |\downarrow\rangle|0\rangle \) will then adiabatically evolve into the ground states at the spontaneous SUSY breaking point \( |\psi_{g+}\rangle = (|\uparrow\rangle - g_m/\omega) \pm i|\downarrow\rangle g_m/\omega \sqrt{2} \), which are ideally two Schrödinger’s cat states.

Now to evaluate the expectation value of the Hamiltonian \( H = \omega (a^\dagger a + 1/2) + g_m \sigma_y (a + a^\dagger) + g_m^2/\omega \) we only need to measure the average phonon number \( \langle a^\dagger a \rangle \) and a spin–phonon coupling term \( \langle \sigma_y (a + a^\dagger) \rangle \). The former can be derived by the standard procedure of first resetting the spin state to \( |\downarrow\rangle \) and then driving the phonon blue-sideband to fit the phonon number population from the spin dynamics, as shown in Fig. 3a, b. As for the second term, we apply a spin-dependent force \( H_{SDF} = (-\Omega_p/2) \sigma_y (a + a^\dagger) \) and measure the evolution of \( \sigma_y (t) \) by observing that \( e^{iH_{SDF}t} \sigma_y e^{-iH_{SDF}t} \) has a linear term in \( t \) as \( \Omega_p \sigma_y (a + a^\dagger)^t \). Therefore, after preparing \( |\psi_i\rangle \), we adjust the parameters of the bichromatic Raman laser beams to create this spin-dependent force and fit \( \sigma_y (t) \) to extract the slope at \( t = 0 \) (see Methods), from which we obtain \( \langle \sigma_y (a + a^\dagger) \rangle \), as shown in Fig. 3c. Combining these two results, we obtain \( E_+ = 2\pi \times (7.3 \pm 1.9) \) kHz and \( E_- = 2\pi \times (5.2 \pm 2.1) \) kHz. The two levels are degenerate within about one standard deviation.

In quantum field theory, \( E_0 > 0 \) has the nontrivial meaning of a positive vacuum energy. However, in quantum mechanics, a
Fig. 4 Measuring ground state supercharges. We reconstruct the phonon density matrix with the spin projected to $\sigma_z = \pm 1$ or $\sigma_y = \pm 1$ by maximum-likelihood estimation. a-d The measured spin dynamics under blue-sideband (blue dots) and red-sideband (red dots) driving and those predicted by the best-fitted density matrix (blue and red curves) for $|\psi_y\rangle$ after four typical phonon displacements with $\sigma_y$ projected to $\pm 1$. Each data point is the average of 400 experimental shots, half of which have a $\sigma_y$ gate before the measurement sequence to remove the off-diagonal spin terms. e-h Similar plots for $|\psi_x\rangle$ after four typical phonon displacements with $\sigma_x$ projected to $\pm 1$. Here, we choose $\omega = 2\pi \times 10$ kHz and $g_m = 2\pi \times 5.43$ kHz. The displacement operators $D(\beta_i)$ are characterized by $\beta_i = i(2\pi \hbar \omega_i)/\hbar$ where $\beta = 0.687$, $N = 12$ and $j = 0, 1, ... , N = 1$. The complete data for all the displacements and both $|\psi_x\rangle$ and $|\psi_y\rangle$ are presented in Supplementary Note 2. From the fitted density matrices, we get $(\sigma_x \otimes A) = 0.201 \pm 0.013$ and $(\sigma_x \otimes B) = 0.300 \pm 0.009$ for $|\psi_x\rangle$ and $(\sigma_x \otimes A) = -0.163 \pm 0.017$ and $(\sigma_x \otimes B) = -0.349 \pm 0.011$ for $|\psi_y\rangle$ where error bars are estimated by Monte Carlo sampling (see Methods).

To evaluate $(\sigma_y \otimes A)$, in principle we can decompose the spin–phonon density matrix as $\rho = \langle 1 \rangle \otimes \rho_{11} + \langle 1 \rangle \otimes \rho_{1\bar{1}} + \langle \bar{1} \rangle \otimes \rho_{1\bar{1}} + \langle \bar{1} \rangle \otimes \rho_{11}$ where $\rho_a = \langle a^\dagger a| \otimes \rho_{a\bar{a}}$ are operators on the phonon states. Then we have $(\sigma_y \otimes A) = \text{tr}[\rho_{11} - \rho_{1\bar{1}}]A]$. Naively, to obtain $\rho_{11}$ and $\rho_{1\bar{1}}$ separately, we can first measure $\sigma_y$ and then reset the spin state as an ancilla for quantum state tomography on the phonon state, conditioned on the measurement outcome $\sigma_y = \pm 1$. However, in ion trap experiments, this measurement of the spin state typically takes hundreds of microseconds to accumulate sufficient photon counts, during which the decoherence of the phonon state may not be ignored. This suggests that we shall incorporate the spin states into the tomography of the phonon states. Rather than resetting the spin state to $|\bar{1}\rangle$, we remove its off-diagonal term by applying random $\sigma_z$ operations but keep the population in the $|1\rangle$ and $|\bar{1}\rangle$ basis (see Methods). Then we follow the standard procedure of phonon quantum state tomography by applying displacements in different directions and blue/red-sideband driving\textsuperscript{22,31}. Through maximum-likelihood estimation (MLE)\textsuperscript{32}, we can reconstruct the joint spin–phonon density matrix in the diagonal spin basis, that is, $\sigma_z$ and $\sigma_y$. Some typical fitting results for various displacement directions and blue/red sidebands are presented in Fig. 4a–d. Similarly, by first rotating the $\sigma_z$ basis to the $\sigma_y$ basis, we can reconstruct the partial information required to evaluate $(\sigma_y \otimes B)$, as shown in Fig. 4e–h. Combining the two results, we get $Q_y = 0.501 \pm 0.016$ and $Q_x = -0.512 \pm 0.020$ after correcting a phase in the phonon state due to the slow drift of the trap frequency (see Methods and Supplementary Note 2). The measured supercharges are significantly away from zero, which suggests that the ground states are superposition of bosonic and fermionic states rather than an SUSY-invariant vacuum. Note that here the error bars only account for the statistical errors of MLE in finding the best-fitted density matrices for the prepared states (see Methods), while imperfect state preparation and measurement can still result in a systematic deviation from the ideal values of $\pm 0.707$. For example, a slow fluctuation in the trap frequency of $2\pi \times 0.5 \text{ kHz}$ during the joint tomography, which takes tens of minutes, can already reduce the supercharges to about $\pm 0.5$ (see Methods). Also note that, while here we are only interested in the $\sigma_z$ and $\sigma_y$ parts of the spin for measuring supercharges, our method can be extended to include the $\sigma_x$ part as well. Combining these results altogether one will be able to perform full quantum state tomography of the joint spin–phonon system.

To sum up, we have demonstrated the quantum simulation of an SUSY QM model using a single trapped ion. By measuring the energy spectrum of the QRM along a carefully chosen path, we illustrate the characteristic properties in the degeneracy of the energy levels, whether the SUSY spontaneously breaks or not. We adiabatically prepare the degenerate ground states at the spontaneous SUSY breaking point and explicitly measure the expectation value of the energy and the supercharge, from which the spontaneous SUSY breaking can be verified experimentally. Recently, quantum simulation using up to 16 ions and 16 phonon...
modes has been demonstrated, which should allow the demonstration of some lattice quantum field theory models using spin-boson-coupled systems, similar to ref. for spin models. To follow this direction to further explore the SUSY theory, suitable SUSY lattice models need to be designed to fit into the ion trap setup, and suitable observables should be chosen to characterize its properties: while the joint quantum state tomography scheme we develop here can directly be generalized to the multi-ion case to reconstruct the joint state of any particular collective phonon mode with any particular spin, and provide some useful information, in general the full tomography of all the spins and all the phonon modes will be exponentially more challenging. Nevertheless, as we show in this experiment, partial information about the system may suffice to demonstrate the spontaneous SUSY breaking or other properties of concern. Our work showcases the suitability of ion trap as an economic but powerful platform for simulating diverse physics through its unparalleled controllability.

**Methods**

**Experimental setup.** Our experimental setup is a single $^{171}$Yb$^+$ ion in a linear Paul trap. The spin state is encoded in the $|1\rangle = |S_{1/2}, F = 0, m_F = 0\rangle$ and the $|0\rangle = |S_{1/2}, F = 1, m_F = 0\rangle$ levels of the ion with atomic transition frequency around $\omega_0 = 2\pi \times 12.643$ GHz. We exploit a radial oscillation mode with secular frequency around $\omega_{21} = 2\pi \times 3.35$ MHz.

We control the physical laser beams with a specially designed repetition rate of 20 kHz and a bandwidth of about 100 GHz to manipulate the ion through Raman transition. Two acousto-optic modulators (AOMs) are used to fine-tune the frequency, the phase and the amplitude of the laser beams. Before each experiment, we initialize the phonon state to $|0\rangle$ through sideband cooling using 355 nm laser. Then we initialize the spin state by optical pumping of 369 nm laser with a 933 nm laser to repump the population leaked to $|2\rangle$ levels. More details about our setup can be found in our previous work.

**Simulating quantum Rabi model.** We follow the scheme of refs. to simulate the QRM for an ion with qubit frequency $\omega_a$ in a harmonic trap with trap frequency $\omega_b$. By applying bichromatic Raman laser beams along the x direction, we get red (blue) sideband Hamiltonian $H_{\text{sideband}} = \Omega t \sigma_a \cos(\Omega t \sigma_b x - \omega_{21} t + \delta t)$ with small detuning $\delta = \omega_a - \omega_b - \omega_b$. $\delta t = \omega_a - \omega_b - \Omega t$ is the desired value, and $\Omega = 2 \pi \times 20$ kHz is the Lamb-Dicke parameter, we get the QRM Hamiltonian of Eq. (1) in the interaction picture of $H_0 = (\omega_a - \omega_b)\sigma_0/2 + (\omega_b - \omega_a^2) a$.

**Measuring excitation spectrum.** After preparing the ground state of the system (see below), we apply a weak probe $H_p = \Omega t \sigma_0 \cos(\Omega t \sigma_b x - \omega_{21} t + \delta t)$ with $\Omega = 2 \pi \times 0.17$ kHz (much smaller than the typical energy scale $\omega_a$ and $\omega_b$ of the system) and a duration $\tau = 1000 \mu s$ to measure the excitation spectrum of the system, whose peaks give the energy difference to the excited states. Note that the QRM Hamiltonian is realized in an interaction picture as mentioned above, in which the effective probe frequency is $\omega_p = \omega_b - \omega_a - \Omega t$. This is what we plot as the horizontal axes in Fig. 2c, d.

**Adiabatic evolution.** In principle, we can use arbitrary adiabatic passages to prepare the ground states so long as the adiabatic condition is satisfied. However, the change in the AC Stark shift must be taken into account when we tune the laser frequency or strength we use. Now we can adiabatically turn down the phonon frequency following an exponential function $g(t) = g_0 e^{-t/\tau}$ with $\tau = 40 \mu s$ and an overall quench time of 400 μs. From numerical simulation, the non-adiabatic excitation can be below 1% for $\tau$ ranging from 0 to 0.9.

As for the degenerate ground states used for measuring the average energy and the supercharge, we observe that in the case of $r = 1$, the driving on the blue and the red sidebands are balanced. Our 355 nm pulsed laser has a specially designed repetition rate such that in this situation the AC Stark shift from these two sidebands largely cancel each other. For coupling as large as $2 \pi \times 10$ kHz, the measured AC Stark shift is still below 100 Hz. Therefore, here we simply perform a linear quench in the coupling g as $g(t) = g_0 \tau / t$ with $\tau = 200 \mu s$.

**Measuring expectation value of Hamiltonian.** The Hamiltonian at the spontaneous SUSY breaking point $r = 1$ contains a phonon term $a^\dagger a$ and a spin–phonon coupling term $\sigma_0 \sigma_+(a + a^\dagger)$. The phonon term can be measured by fitting the phonon number distribution through the spin dynamics under a blue-sideband driving. Details can be found in our previous work.

To measure the spin–phonon coupling term, we observe that $\sigma_z(t) = U(t)|\psi(t)\rangle = \cos(\Omega t (a + a^\dagger))\rho_0 + \sin(\Omega t (a - a^\dagger))\rho_1$, where $\Omega t (a + a^\dagger)$ is the evolution under a spin-dependent force. Therefore, we measure $\sigma_z(t)$ under the spin-dependent force and fit its slope at $t = 0$ to get $\sigma(z + a + a^\dagger)$. This can in principle be obtained by measuring only the initial part of the evolution, which, however, would require high measurement accuracy. Here, we observe that for the ideal ground states $|\psi_g\rangle = (|+\rangle - |\rangle)\sqrt{2}/\sqrt{2}$, we have $|\psi(t)\rangle = e^{-i\theta \Omega^2 t/(2\pi \times 3.35 \hspace{1mm} MHz)} - \sin(\Omega t^2 (a - a^\dagger))$ using the fact that displacement operator $D(\beta) = e^{-i\beta \sigma_0/2} e^{i\beta \sigma_b x/2}$ and thus $\langle a|D(\beta)\rho_0|a\rangle = e^{-i\beta\Omega^2 t/(2\pi \times 3.35 \hspace{1mm} MHz)}$. Considering experimental imperfections, we fit the measured spin-up state population as $P_x(t) = |\langle\psi(t)\rangle|^2 = e^{-i\beta\Omega^2 t/(2\pi \times 3.35 \hspace{1mm} MHz)}$ with two fitting parameters $\beta$ and $A$, as shown in Fig. 3c. After fitting these two parameters, we can compute the slope at $t = 0$ and evaluate the spin–phonon coupling term using $\Omega = 2 \pi \times 8.1$ kHz.

**Partial spin-phonon state tomography.** Rather than first projecting the spin state to $|\sigma_z = -1\rangle$ and then performing the quantum state tomography for the phonon state conditionally, here we reconstruct the the desired $\sigma_z$ and $\rho_1$ altogether. Different from the standard procedure for phonon state tomography where the spin state needs to be discarded, here we only remove the off-diagonal terms of the spin state by inserting a $\sigma_z$ gate on half of the measurements. Then we follow the steps of the partial quantum state tomography by applying phonon displacements in various directions and measure the evolution of spin population under a driving on the blue or red sidebands. Theoretically, after removing the spin coherence between $|\uparrow\rangle$ and $|\downarrow\rangle$, the spin-up state population for any quantum state under the blue or red-sideband driving can be given by

$$P_{\pm}(t) = \frac{1}{2} \left[ 1 + \frac{2}{\sqrt{\gamma^2 + 1}} \left( 1 - \frac{\gamma^2}{1 + \gamma^2} + \frac{\gamma^2}{\sqrt{\gamma^2 + 1}} \right) \right]$$

and

$$P_{\pm}(t) = \frac{1}{2} \left[ 1 + \frac{2}{\sqrt{\gamma^2 + 1}} \left( 1 - \frac{\gamma^2}{1 + \gamma^2} + \frac{\gamma^2}{\sqrt{\gamma^2 + 1}} \right) \right]$$

In principle, by fitting these curves, one can get complete information about $P^\dagger_\pm$ and $P^\dagger_\mp$, the diagonal elements in the spin–phonon joint density matrix. Then by applying different phonon displacements, the desired density matrices $\rho^\dagger_1$ and $\rho^\dagger_2$ can be obtained.

In practice, this procedure of first extracting the coefficients $P^\dagger_\pm$ and $P^\dagger_\mp$ and then solving the original density matrix can be sensitive to noise in the measurement, and the obtained matrix may not satisfy the physical constraints such as being positive semidefinite with a trace of one. Therefore, here we use maximum-likelihood estimation to find the most probable density matrix that can generate all the measured blue/red-sideband evolutions. We use the Hamiltonian $H_0 = (\Omega_0/2)\sigma_0 (a^\dagger + a) + \sigma_0 (a^\dagger + a)$ and $H_1 = (H_0/2)\sigma_+(a^\dagger + a)$ to drive the blue or red sidebands along with a phonon displacement $\beta$ at the same time. We use $N = 12$ displacements $D(\beta)$ with $\beta = e^{i\theta \Omega^2 t/(2\pi \times 3.35 \hspace{1mm} MHz)}$. $\beta = 0.687$ and $\gamma = 0.1$, $\cdots$, $N - 1$, and we set a phonon cutoff of $n_{\text{cut}} = 7$ for the reconstructed density matrices with only $10^7$ population outside the truncated Hilbert space for the ideal states.

The projection to $|\sigma_z = \pm 1\rangle$ can be obtained in a similar way by first rotating the $\sigma_z$ basis to the $\sigma_z$ basis. The complete data and fitting results are presented as Supplementary Figs. 1–4 in Supplementary Note 2.

**Calibrate phonon state rotation error due to trap frequency drift.** In Supplementary Fig. 5 of Supplementary Note 2, we present the fidelity between the reconstructed density matrices and the ideal ones (with the off-diagonal spin terms discarded as described in the main text), versus a rotation angle $\theta$ applied on the phonon state $R(\theta) = e^{i\theta a^\dagger + a}$. As we can see, while $|\psi_1\rangle$ has the highest fidelity near $\theta = 0$, the best result for $|\psi_1\rangle$ is achieved at a finite rotation angle of about $\theta = 1.7\pi$ (or $\theta = -0.26\pi$). This may come from the slow drift in the trap frequency of our setup between the measurements of $|\psi_1\rangle$ and $|\psi_2\rangle$, which in turn would result in a drift in the rotating speed of the interaction picture. For this case this cause an effective rotation in the phonon state. As we present in Supplementary Fig. 6 of Supplementary Note 2, the supercharges will also be changing under the phonon rotation. The maximizer of the supercharge (strictly...
speaking, its magnitude) is close to that of the fidelity, but a small discrepancy exists. Here we use the $\theta$ for the highest average state fidelity to calibrate the phonon rotation in $|\psi_{+}\rangle$ with no such noise correction. Since it is measured earlier and the bath parameters can be smaller. After this correction, the measured supercharges in the main text can be obtained. Also note that once the optimal $\theta$ for $|\psi_{+}\rangle$ is determined, it is fixed in the data processing. That is, we keep using the same $\theta$ when estimating the error when Monte Carlo sampling as shown in the next section, rather than optimizing a $\theta$ for each simulated result.

**Error estimation for maximum-likelihood method.** We use Monte Carlo sampling to estimate the error in the reconstructed density matrix obtained from maximum-likelihood estimation and that in the computed supercharge. These measurement outcomes are computed from the raw experimental data whether the spin is in $|\uparrow\rangle$ or $|\downarrow\rangle$ for each displacement and the red-blue-sideband driving at each time point, which is repeated for 400 times (200 times with $\sigma$ operations and 200 times without to remove the spin coherence). Note that for each data point, the number of spin-up events follows a binomial distribution whose parameter $p$ can be estimated using the measured frequency $P$. Therefore, we can generate new sets of measurement outcomes following these binomial distributions, and then use them to find the most likely density matrix and the corresponding supercharges. We repeat this procedure for 100 times and use the standard deviation of the simulated results to estimate the error of the measurement outcomes.

**Estimation of preparation and measurement errors for ground state supercharges.** The error bars for the measured supercharges using Monte Carlo sampling from the uncertainty from the best fit for the adiabatically prepared ground states. If the state preparation and measurement by themselves are imperfect, clearly there will be a deviation from the ideal values of $\pm \sqrt{2}$. For the adiabatic state preparation, we can numerically integrate the master equation for the joint spin–phonon state tomography. If such a fluctuation $\Delta_\omega$ occurs when measuring the blue/red-sideband dynamics for the $N = 12$ displacement directions, effectively the phonon state will experience a random rotation of $\Delta_\omega$, $\pi$ where $\tau = 200 \mu$s is the adiabatic evolution time. Such a random rotation among different displacement directions for the same ground state $|\psi_{\pm}\rangle$ or $|\psi_{\uparrow}\rangle$, cannot be corrected by a constant rotation angle $\theta$ as done in the previous sections for a fixed drift between the measurements of $|\psi_{\pm}\rangle$ and $|\psi_{\uparrow}\rangle$. To estimate its effect, we purposely apply a rotation $\Delta_\omega$, $\pi$ to the ideal ground state and calculate the supercharges. It turns out that a fluctuation of $2 \pi \times 0.5 \text{kHz}$ already reduces the supercharges to about $0.52$. In principle, such a slow fluctuation can be suppressed if we calibrate the trap frequency before measuring each data point, which however is too time-consuming for this experiment.

**Data availability**

All the data generated and analyzed in this study have been deposited in the Tsinghua cloud database without access code (https://cloud.tsinghua.edu.cn/f/d0675aad336e4d8be19a8867d=1). Contact the corresponding author with any further questions.

**Code availability**

The code that support the findings of this study is available from the corresponding authors upon reasonable request.

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**Author contributions**

L.M.D. conceived and supervised the project. T.K.D. did the theoretical calculation. M.L.C., Q.X.M., W.D.Z., Y.J.Y., L.H., Z.C.Z. carried out the experiment. M.L.C., Y.K.W., L.M.D. wrote the manuscript.
Competing interests
The authors declare no competing interests.

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