Kilometer-scale deep under-ice or -water Cherenkov neutrino detectors may detect muon and electron neutrinos from astrophysical sources at energies of a TeV and above. Tau neutrinos are also expected from these sources due to neutrino flavor oscillations in vacuum, and tau neutrinos are free of atmospheric background at a much lower energy than muon and electron neutrinos. Identification of tau neutrinos is expected to be possible above the PeV energy range through the “double bang” and “lollipop” signatures. We discuss another signature of tau in the PeV–EeV range, arising from the decay of tau leptons inside the detector to much brighter muons.

PACS numbers:

I. INTRODUCTION

Kilometer-scale Cherenkov neutrino detectors now planned or under construction, such as IceCube \[^1\] at the South Pole and KM3NeT \[^2\] in the Mediterranean, are poised to detect high energy neutrinos from astrophysical sources such as gamma-ray bursts (GRBs) and active galactic nuclei (AGN). Ultrahigh energy (UHE) cosmic rays with energies exceeding \(\sim 10^{19} \text{ eV}\) are thought to originate from these extraordinary sources of the highest energy \(\gamma\)-rays observed. High energy neutrinos should also be produced in these sources as the result of photons (\(p\gamma\)) and/or proton-proton (\(pp\)) interactions of shock accelerated protons with ambient radiation fields and/or plasma material.

After propagating virtually unimpeded through the universe, high energy neutrinos detected by experiments on Earth may reveal the physical and astrophysical conditions of their sources at energies and distances unmatched by other methods. In addition to measuring the energy and direction of astrophysical neutrinos, future neutrino telescopes will be able to distinguish between the three known flavors of neutrinos (and anti-neutrinos), namely electron, muon and tau neutrinos (\(\nu_e, \nu_{\mu}\) and \(\nu_\tau\)), by looking at the signature(s) of their interactions in the detection media.

Tau neutrinos are not produced in appreciable numbers in astrophysical sources, but will appear in numbers comparable to \(\nu_e\) and \(\nu_{\mu}\) as a consequence of flavor oscillation between their sources and Earth. Astrophysical neutrino sources generically are believed to produce neutrinos through \(\pi^\pm\) (and \(K^\pm\)) decay, leading to a neutrino flavor ratio at production of \(\nu_e : \nu_{\mu} : \nu_\tau = 1 : 2 : 0\). However, neutrino flavor eigenstates \(\nu_\alpha (\alpha = e, \mu, \tau)\) and mass eigenstates \(\nu_j (j = 1, 2, 3)\) are mixed by a unitary matrix \(U\) defined as \(\nu_\alpha = \sum_j U_{\alpha j} \nu_j\). The oscillation probability is given by \(P_{\nu_\alpha \rightarrow \nu_\beta} = |\sum_j U_{\beta j} e^{-i \varphi} U^*_{\alpha j}|^2\), where

\[
\varphi = 6.3 \cdot 10^9 \left(\frac{\delta m^2}{8 \cdot 10^{-5} \text{ eV}^2}\right) \left(\frac{D}{\text{kpc}}\right) \left(\frac{\text{TeV}}{E_\nu}\right)
\]

is the phase of the slower (solar) neutrino oscillation and \(D\) is the distance traveled by the neutrinos before being detected. The oscillation probability reduces to \(P_{\nu_\alpha \rightarrow \nu_\beta} \approx \sum_j |U_{\beta j}|^2 \cdot |U_{\alpha j}|^2\) for \(\varphi \gg 1\), which holds for essentially all astrophysical sources. We use the standard expression for \(U_{\alpha j}\) from Ref. \[^3\] with solar mixing angle \(\theta_\odot \equiv \theta_{12} = 32.5^\circ\) and the atmospheric mixing angle \(\theta_{\text{atm}} \equiv \theta_{23} = 45^\circ\), following the results from the SNO \[^4\] and K2K \[^5\] experiments, respectively. The unknown mixing angle \(\theta_{13}\) and the CP violating phase may be assumed to be zero given the current upper bounds from reactor experiments.

The resulting astrophysical neutrino flux ratio on Earth is thus generally expected to be \(\nu_e : \nu_{\mu} : \nu_\tau = 1 : 1 : 1\), although this may be modified somewhat by effects such as neutron decay, two-photon annihilation to muon pairs, or muon synchrotron cooling in the source environment \[^6\] and \[^7\]–\[^9\].

Tau neutrinos are particularly interesting because local backgrounds to astrophysical \(\nu_\tau\) signals are low. The possible backgrounds are from UHE cosmic rays interacting in Earth’s atmosphere and producing short-lived charmed mesons which decay as \(D_s \rightarrow \tau \nu_\tau\), known as the “prompt” neutrino flux \[^10\]–\[^13\], and from conventional atmospheric \(\nu_e\) or \(\nu_{\mu}\) produced in these cosmic ray air showers oscillating to \(\nu_\tau\) as they traverse the Earth before being detected.

The prompt atmospheric \(\nu_\tau\) flux is not precisely known due to uncertainties in the extrapolation of the parton distribution functions to low \(x\) and in the composition of the high energy cosmic rays. However, the different models lead to expected rates of \(10^{-3}\) to \(10^{-2}\) prompt tau events above 1 PeV per year in a km\(^3\) neutrino telescope \[^11\]–\[^14\].

As for oscillations to \(\nu_\tau\), the maximum propagation distance \(L \approx 4\) km, the Earth’s diameter, corresponds to a \(\nu_{\mu} \rightarrow \nu_\tau\) oscillation probability \(P \approx 10^{-3}(E_\nu/\text{TeV})^{-2}\) (neglecting matter effects, which would reduce the oscillations), so oscillations will produce a negligible flux of \(\nu_\tau\) for \(E_\nu\) at the PeV scale, even in comparison to the low prompt fluxes. The only remaining source of PeV-scale tau neutrinos is extraterrestrial.
Tau neutrinos are also interesting because of the phenomenon of $\nu_\tau$ regeneration. The high energy neutrino interaction cross section rises approximately linearly with energy, and the mean free path through the Earth becomes shorter than the Earth’s diameter for $E_\nu \gtrsim 100$ TeV. The Earth is thus opaque to $\nu_e$ and $\nu_\mu$ at PeV energies and above, and only horizontal and down-going neutrinos can be observed. For $\nu_\tau$, however, the $\tau^\pm$ produced in a charged current (CC) neutrino interaction will usually decay back to $\nu_\tau$ before losing significant amounts of energy, effectively regenerating the $\nu_\tau$ beam and leaving an upgoing $\nu_\tau$ flux up to the PeV scale.

The best-known signature for detecting $\nu_\tau$ in a water or ice Cherenkov detector is called the “double bang” \[17\]. In these events, a CC neutrino-nucleon interaction $\nu_\tau N \rightarrow \tau X$ produces a hadronic shower (denoted $X$), with the subsequent decay of the $\tau$ lepton producing a second shower, connected to the first by the $\tau$ lepton track. The second shower may also be hadronic, or it may be electromagnetic in the case of $\tau \rightarrow e\nu_\tau \bar{\nu}_e$. The $\tau$ produced in the CC interaction has energy $\langle E_\tau \rangle \approx 0.75E_\nu$ \[18\], and the two showers are separated by the tau decay length $L_\tau = c\tau_\tau \sim 50(E_\tau/\text{PeV})$ m (neglecting energy losses along the track). Due to the short $\tau$ lifetime and wide spacing of the detection elements in kilometer-scale neutrino telescopes, this signature is only expected to be detectable for $\nu_\tau$ with energy $E_\nu \gtrsim \text{PeV}$. Above $\sim 20$ PeV, the typical decay length exceeds 1 km, so both showers usually will not be contained in a kilometer-scale detector; the resulting signature of a tau track and one shower is known as a “lollipop” \[19\].

In this paper, we point out another distinctive signature of extremely high energy (EHE, PeV–EeV) tau leptons, produced by the muonic decay of a $\tau$ inside the instrumented detection volume. Although the muon has lower energy than the parent tau lepton, it will emit more light than the tau. The lepton track will thus appear to suddenly increase in brightness by an amount which should be detectable in a neutrino telescope.

The energy range over which this signature is observable is constrained at the lower end by the requirement that a reasonably long tau lepton track be observed prior to the tau decay. At the higher end, the rising rate of tau photonuclear energy loss causes the brightness of the tau to approach that of the daughter muon above EeV energies. It should be noted that these limits apply to the energy of the $\tau$ lepton in the detector; events from higher energy $\nu_\tau$ could be observed if the initial neutrino interaction vertex is some distance from the detector so that the $\tau$ lepton loses energy in stochastic interactions before decaying within the detector.

II. SIGNATURE OF MUONIC DECAY

Identification of tau events through muonic decay, $\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu$, requires the decay to occur within the detector so that the increase in brightness will be observed. The branching ratio for this decay channel is measured to be $\Gamma_\mu = 17.36\%$ \[2\], so only a fraction of tau leptons will manifest themselves via this signature. However, at energies $\gtrsim 20$ PeV, the tau track length $L_\tau$ becomes longer than the geometric scale of the detector and double bangs are no longer visible. Only the lollipop and muonic decay signatures can be used to identify taus in this regime, even though only a fraction $\sim (1 \text{ km})/L_\tau$ of the taus can be tagged by these methods; lollipops are generally considered to be identifiable only when the final, not the initial, bang is observed (see Section IV).

Tau leptons are produced in neutrino $V-A$ interactions, which at the energies of interest produce polarized taus. The spectrum of muon energies from the decay of polarized $\tau$ is $dn/dx = \frac{3}{4}(1 - x^3)$, where $x = E_\mu/E_\tau$ \[13, 20, 21\]. The expected muon energy is thus $\langle E_\mu \rangle = 0.4 E_\tau$.

Depolarization via $\tau$ interactions prior to decay should be small, and would reduce the muon energy only slightly, to $0.35E_\tau$ in the limit of complete depolarization \[20\]. Although the muon has less than half the energy of the tau, it appears brighter because the muon loses energy more rapidly than the tau, as discussed below.

A. Energy Loss Rates

The average energy loss of heavy leptons per unit distance traveled in matter (in $\text{g/cm}^2$) is often approximated as $-\langle dE/dX \rangle \approx a + bE$. The constant part, due to ionization losses, may be calculated using Bethe-Bloch formula, while the radiative part approximately proportional to $E$ is due to a combination of stochastic $e^+e^-$ pair production, bremsstrahlung and photonuclear effects. In the EHE regime, the ionization loss term $a$ is negligible compared to the stochastic losses. The radiative energy loss parameter $b = b(E)$ varies slowly with energy for extremely high energy leptons, primarily due to an increase in the photonuclear energy loss rate $b_{\text{pn}}$ at very high energies. Although the radiative energy losses are in fact due to a series of discrete stochastic events, at high energies these interactions occur frequently enough that they can be considered quasi-continuous, increasing the overall brightness of the lepton track.

While $e^+e^-$ pair production and bremsstrahlung are the dominant energy loss channels for $\mu$ above the TeV scale, for $\tau$ bremsstrahlung is negligible and photonuclear effects dominate at EHE. Tau photonuclear losses are comparable to electromagnetic losses all the way down to the ionization-dominated region below $E_\tau \sim 20$ TeV \[22, 23\].

Photonuclear energy losses by EHE leptons (and to a lesser extent bremsstrahlung losses) are not precisely known. Measurements of photonuclear cross sections from colliders must be extrapolated to very small $x$, and there are several models available in the literature.
Different models of the nuclear structure function are also available. The predicted loss rates for \( \tau \) leptons in ice are shown in Fig. 1. Numerical values for energy loss rates given in this paper were evaluated using the MMC software package, using ice as the default detection medium.

![Photonuclear energy loss per unit energy (\( b_{\text{pn}} \)) for \( \tau \) leptons in ice, according to various models. BB/BS refers to \( \tau \), plus the hard component of \( \gamma \). Kok and ZEUS include the photon-nucleon cross sections of \( \gamma \) and \( \gamma \), respectively, instead of that from \( \gamma \). ALLM91 and ALLM97 refer to \( \gamma \) and \( \gamma \), with BM following \( \gamma \) but using the nuclear structure function from \( \gamma \).

Recent calculations of muon bremsstrahlung are in close agreement with each other (although they are about 5\% higher than older calculations), but disagree by as much as 20\% for EHE taus. Because bremsstrahlung is suppressed by 1 \( b_{\text{pn}} \), the differences between models have very little effect on the overall brightness of \( \tau \) tracks.

### B. Photonuclear Interactions

In addition to the uncertainty in photonuclear energy loss rates at high energies noted above, there are several differences between photonuclear and electromagnetic interactions that must be considered.

Both bremsstrahlung and pair production produce electromagnetic showers in the Cherenkov medium, with \( \gamma \)’s converting to \( e^+e^- \) pairs and the \( e^+ \) and \( e^- \) in turn radiating more \( \gamma \).s. Photonuclear interactions, on the other hand, disrupt the nucleon involved and produce showers of hadrons which go on to interact with other nucleons in the medium.

The light produced in a shower comes from the Cherenkov radiation of the many secondary particles involved in the shower. The total light yield is proportional to the integrated track length of the relativistic particles. The yield per unit shower energy is lower in hadronic showers because heavy particles have a higher threshold for Cherenkov radiation, energy is lost to the binding energies of the hadrons involved, and invisible slow neutrons are produced.

The ratio of the light yield per unit energy in hadronic showers to that in electromagnetic showers depends on energy, as well. Neutral \( \pi \) mesons produced in the shower will decay to \( \gamma \) and produce electromagnetic subshowers, but very few hadronic particles are formed in electromagnetic showers. The production of \( \tau^0 \) is thus a “one-way street” carrying energy out of the hadronic sector and into the electromagnetic. Roughly 30\% of the energy in hadrons will go to \( \pi^0 \) in each generation of the shower, so high energy hadronic showers with more generations will be more like electromagnetic showers. For small (\( \sim 10 \) GeV) showers the ratio of light yields is about 65\%, rising to about 85\% for a 10 TeV shower and asymptotically approaching 100\%. In the calculations presented in Sect. IIII, we have assumed an average value of 75\% for the energy deposited in hadronic photonuclear interactions along the \( \tau \) (and \( \mu \)) track.

Another important effect to consider is the energy spectrum of individual stochastic interactions. Relatively large fractions \( \nu \) of the total lepton energy can be deposited in a single photonuclear interaction. In the analysis of Ref. \( \gamma \) in terms of a hard (perturbative) and a soft (non-perturbative) component, the energy spectrum \( v \frac{d\sigma}{dv} \) of the individual photonuclear interactions comprising the perturbative component peaks at \( v \sim 10^{-2} \) for \( \tau \) leptons. Since the radiative energy loss parameter \( b \leq 10^{-6} \text{ g}^{-1} \text{ cm}^2 \), such interactions will be separated by several kilometers.

Most often, such perturbative interactions will not occur along the contained track segment even in a kilometer-scale neutrino telescope. In this case, the perturbative component of the photonuclear term (which in the analysis of Ref. \( \gamma \) dominates above \( \sim 10 \) PeV) should be ignored in estimating the brightness of the tau track, and only the non-perturbative component should be taken into account. If a perturbative interaction does occur within the detector, it will appear as a distinct cascade along the track, and it should still be possible to measure the “baseline” track brightness away from the shower (although the bright shower could confuse event analysis algorithms and thus lower the overall efficiency for detecting such events).

As the tau energy increases over the EHE regime, however, \( v \frac{d\sigma}{dv} \) shifts to lower \( v \) and the total photonuclear energy loss parameter \( b_{\text{pn}} \) increases. Perturbative photonuclear interactions thus become somewhat softer and much more common, with separations of a few hundred meters. In this regime, the quasi-continuous approximation may be appropriate for the perturbative component as well as the non-perturbative part, although the details will depend on the particular detector and event reconstruction algorithm under consideration. In Sect. IIII we present results for the two limiting cases, both of which should produce a detectable signal: first assuming the perturbative photonuclear interactions of the \( \tau \) are very
rare, so that only the non-perturbative component of the photonic interactions contributes to the brightness of the \( \tau \) track; and then assuming the perturbative interactions are also quasi-continuous, increasing the average brightness of the \( \tau \) track and thus decreasing the ratio of the brightness of the \( \tau \) to that of the muon.

C. Radiative Decays

Tau decays to \( \mu \) are sometimes accompanied by initial-state or final-state radiation. This radiation follows a \( 1/k \) spectrum at low energies (below the \( \tau \) mass scale), and is measured to occur with a branching fraction of 0.36\% with a threshold of \( E_\gamma > 10 \) MeV in the \( \tau \) rest frame. These photons will be boosted by a factor of up to \( \gamma (1 + \beta) \), depending on the emission angle of the photon in the \( \tau \) rest frame, so that for \( E_\gamma = 1 \) PeV a photon with 10 MeV in the rest frame will have at most \( \sim 20 \) TeV in the detector frame. Neutrino telescopes generally have an energy threshold of several TeV to a few tens of TeV for reconstruction of cascades (e.g. \( \nu_\tau \), CC events), so 20 TeV is a reasonable benchmark for the energy at which a shower from initial-state or final-state radiation might be noticeable along a lepton track. For \( \tau \) tracks at the PeV scale, we therefore expect a sizeable shower at the decay vertex in 2\% or fewer of observed tau decays. Because of the higher boost factor, noticeable showers will be about twice as common for EeV \( \tau \) decays. It is not clear that these showers would be distinguishable from the background of stochastic energy losses, but it might be helpful to treat sizeable showers along an observed track as candidate positions for tau decay vertices.

III. DETECTABILITY

An EHE \( \tau \) decaying to \( \mu \) will appear as a track which suddenly increases in brightness. The magnitude of the increase, taking into account the most probable fraction of the tau energy carried by the muon, the average energy loss rates of the two leptons, and the relative light yields of hadronic and electromagnetic showers, is shown in Fig. 2. Losses to ionization, pair production and bremsstrahlung are also included; the bremsstrahlung model of \[34\] is used, but the results are nearly independent of the choice of bremsstrahlung model.

Although the IceCube collaboration has not published any estimate of track energy resolution for the IceCube detector, a resolution of \( \sigma (\log_{10} E_\mu) \simeq 0.3 \), corresponding to a factor of 2 in \( E_\mu \), was claimed for AMANDA-II \[11\]. (In the radiative-dominated regime, the brightness scales approximately linearly with muon energy, so the resolution in brightness should be the same as the energy resolution.) One would expect IceCube to do at least this well, given the larger detector volume and better optical module electronics.

The ANTARES collaboration expects a track energy resolution of \( \sigma (\log_{10} E_\mu) \simeq 0.3–0.4 \) for PeV muons \[42\] (resolution at higher energies was not given). A review of the literature did not produce published track energy resolutions for the Baikal, NESTOR, or NEMO detectors. These smaller detectors (like AMANDA) do not have the effective volume to yield significant event rates at and above the PeV scale in any case, but the energy resolution for \( \mathrm{km}^3 \) detectors in water should be comparable to, or better than, the ANTARES resolution.

It appears that, for both ice and water \( \mathrm{km}^3 \) neutrino telescopes, the expected track energy resolution should be sufficient to distinguish the brightness of the initial \( \tau \) track from that of the final \( \mu \) track. We note that the energy resolutions given above refer to measuring a single energy for a through-going track, rather than trying to make separate energy measurements for different segments of a track. However, in a \( \mathrm{km}^3 \) detector, the observed tracks will be several times longer than those visible in smaller instruments such as AMANDA or ANTARES, so enough information should be recorded for measurements of comparable accuracy.

At the lower end of the EHE scale considered here (a few PeV), the \( \tau \) track will only be visible if the neutrino interaction vertex is contained within the detector; even then, the \( \tau \) track will be quite short. Reconstruction of such events will be further complicated by the hadronic shower produced at the \( \nu N \) vertex, which will have typically an energy of 0.25\( E_\nu \). The ability to measure the energy of a short track segment within such a shower must be determined with a detailed and detector-specific Monte Carlo study, but it seems likely that a \( \tau \) track with a length of at least 200 m will be required. This would correspond to a neutrino energy threshold of around 5
PeV for detection of $\tau$ using this signature, comparable to but slightly higher than the threshold for the double bang signature. The advantage of the muonic signature is that it should be remain detectable to very high energies, at least up to the EeV scale.

As discussed in Sect. [11][13] the curves in Fig. 2 assume that the light emission from photonuclear interactions is quasi-continuous, which may not be a valid assumption. Figure 3 shows, for the model of [25], the effect of assuming to the contrary that hard photonuclear interactions are sufficiently rare that they do not contribute to the measured brightness of the underlying track. This assumption would hold if no such interactions occurred within the detector volume. Alternatively, an analysis searching for muonic tau decays could attempt to identify bright showers along the observed track, and measure the “baseline” track brightness away from such showers; because very hard stochastic interactions are more common for $\tau$ than for $\mu$, such an approach should heighten the contrast between $\mu$ and $\tau$ tracks. To our knowledge, no study of the ability of any neutrino telescope to resolve showers along a track in this manner has yet been published.

IV. DISCUSSION

The detection of tau neutrinos offers an excellent method for detecting astrophysical neutrinos, since there is essentially no atmospheric tau neutrino background. However, the classic double bang signature of tau neutrinos is only expected to be detectable over a single decade of energy, from a few PeV to perhaps 20 PeV. The possibility of tagging tau events at higher energies by detecting only the final bang, known as a lollipop event, has already been pointed out. In this paper, we have shown that it should also be possible to tag taus which decay to muons.

We note that this signature of $\tau$ decaying to $\mu$ was considered in Ref. [22], which concluded that the increase in brightness would not be detectable in a neutrino telescope. Two factors lead us to the opposite conclusion: the typical fraction of energy transferred to the $\mu$ is somewhat higher than they assumed, and the fact that a $\tau$ loses energy primarily through photonuclear interactions reduces the brightness of the $\tau$ track relative to that of a $\mu$ losing energy at the same rate.

We also note the observation made in Ref. [28] that the double bang and lollipop signatures are not completely free of experimental backgrounds, because muons which decay in flight can mimic these signatures. The authors of Ref. [28] estimate a rate of up to 50 km$^{-3}$ yr$^{-1}$ of these events. By contrast, there is no apparent physical background to a track which suddenly increases in brightness as in the $\tau \rightarrow \mu \nu \nu$ decay signature. Experimental backgrounds will of course arise due to the intrinsic variations of lepton energy deposition and photon detection which contribute to the detector energy resolutions quoted above. Because these variations depend on the particular detector and analysis techniques used, Monte Carlo studies will be needed to quantify the background levels to be expected in actual experiments.

At lower (TeV–PeV) energies than considered here thus far, $\tau \rightarrow \mu$ might also be detectable if the neutrino interaction vertex occurs within the detector, because the two neutrinos produced in the $\tau$ decay would carry off approximately half of the lepton energy. Although the $\tau$ track would not be observed directly, the presence of the tau could be inferred on a statistical basis from the lower-than-expected energy of the muon track, when compared to the energy of the hadronic shower at the $\nu N$ vertex. However, this signature would suffer from the same backgrounds as the ‘inverted’ lollipop (in which the tau production vertex and tau track are observed, rather than the tau track and decay vertex). These signatures can be faked by a $\mu N$ CC interaction with high $y$, where less than the mean energy is transferred to the outgoing $\mu$, or by CC $\nu_e$ or NC $\nu_x$ interactions where secondary lower-energy $\mu$ are produced via $\pi^\pm$ decay in the hadronic shower.

We believe the signature of muonic tau decay will be useful in identifying astrophysical tau neutrino events in the coming generation of kilometer-scale Cherenkov neutrino telescopes such as IceCube. This signature may be particularly important in the energy region above a few tens of PeV, where the classic double-bang signature is no longer observable.
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