Explaining the observed relation between stellar activity and rotation

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ABSTRACT
Observations of late-type main-sequence stars have revealed empirical scalings of coronal activity versus rotation period or Rossby number Ro (a ratio of rotation period to convective turnover time) which has hitherto lacked explanation. For Ro ≫ 1, the activity observed as X-ray to bolometric flux varies as Ro−q with 2 ≤ q ≤ 3, whilst |q| < 0.13 for Ro ≪ 1.

Here, we explain the transition between these two regimes and the power law in the Ro ≫ 1 regime by constructing an expression for the coronal luminosity based on dynamo magnetic field generation and magnetic buoyancy. We explain the Ro ≪ 1 behaviour from the inference that observed rotation is correlated with internal differential rotation and argue that once the shear time-scale is shorter than the convective turnover time, eddies will be shredded on the shear time-scale and so the eddy correlation time actually becomes the shear time and the convection time drops out of the equations. We explain the Ro ≫ 1 behaviour using a dynamo saturation theory based on magnetic helicity buildup and buoyant loss.

Key words: dynamo – magnetic fields – turbulence – stars: activity – stars: magnetic field.

1 INTRODUCTION

Observed relations between coronal activity and rotation period in low-mass stars (Pallavicini et al. 1981; Noyes et al. 1984; Vilhu 1984; Micela et al. 1985; Hartmann & Noyes 1987; Randich 2000; Montesinos et al. 2001; Pizzolato et al. 2003; Wright et al. 2011; Reiners, Schuessler & Passegger 2014; Vidotto et al. 2014) have challenged theorists. A measure of activity is the total X-ray luminosity LX, expressed as

\[
\frac{L_X}{L_\odot} \propto \text{Ro}^{-q},
\]

where LX is the bolometric luminosity, Ro is the Rossby number, Ro ≡ 1/Ωτc = τr/2πτc, where Ω is the surface angular velocity, τr is the rotation period, and τc is the convective turnover time. For Ro ≫ 0.13, the data show that 2 ≤ q ≤ 3, whilst for Ro ≪ 0.13, the data show that |q| < 0.2 (Wright et al. 2011; Reiners et al. 2014).

While τr can be inferred directly from time-series photometry of variability associated with star spots, τc is typically inferred from stellar models using the technique of Noyes et al. (1984). In this approach, τc = δh/ω, where δh is the pressure scaleheight at the base of the convection zone, v is a convective velocity, and δ is a dimensionless mixing-length parameter (e.g. Shu 1992). Stellar model values of δh and v produce a specific colour index such as B − V, that can be compared with observations to obtain τc(B − V).

The ill-constrained δ is typically chosen in the range 1 < δ < 3.

Associating LX with magnetic activity arises from a paradigm in which some fraction of the magnetic field energy created within the star by dynamo action rises buoyantly through the star and ultimately converts some of its energy into accelerated particles that radiate as coronal X-rays (e.g. Schrijver & Zwaan 2000). The connection between X-ray activity, magnetic field generation, and rotation and differential rotation for dynamos (e.g. Moffatt 1978; Parker 1979; Krause & Rädler 1980) has led to the notion that increased activity has something to do with efficiency of dynamo action (Noyes et al. 1984; Montesinos et al. 2001; Wright et al. 2011) but connecting this to a theoretical explanation of equation (1) has been lacking.

Previous efforts to explain equation (1) have focused on the dimensionless dynamo number. In Section 2, we show that such approaches used in the previous work are invalid for Ro < 1. After revising these estimates, we then argue that the more conceptually relevant quantity for connecting X-ray activity with dynamo action is in fact the saturated field strength before magnetic buoyancy ensues, which we derive. The saturated value we derive emerges with a scaling consistent with that shown to match a wide range of simulations (Christensen, Holzwarth & Reiners 2009). In Section 3, we use our results of Section 2 to derive an expression for LX/Lc. We conclude in Section 4.

2 RETHINKING KEY DYNAMO QUANTITIES

How and where the dynamo operates in solar-like stars remains an open question (see Charbonneau 2014). Interface dynamos, in
which the shear layer of a tachocline beneath the convection zone dominates the toroidal field amplification by differential rotation (Ω effect) while the convection zone above provides the helical α effect, have been proposed in order to avoid the problem that field strengths greater than a few 100 G might rise too quickly through the convection zone and thus could not easily be generated therein (e.g. Deluca & Gilman 1986; Parker 1993; Thomas, Markiel & van Horn 1995; Markiel & Thomas 1999). The low latitudes of flux emergence and the small tilt angles of loops anchored at sunspots indicate that local field strengths of order 10^6 G is needed to avoid too much deflection of rising flux tubes by the Coriolis force. Such strong fields are most easily anchored in the tachocline. (The field strength in local structures is much larger than that of the spatially averaged mean field.)

However, Brandenburg (2005) emphasizes that there remain plausible arguments that the dynamo could be more distributed in the convection zone, akin to original models (Parker 1955, 1979; Moffatt 1978; Krause & Rädler 1980). Downward turbulent pumping can substantially reduce the rate of buoyant rise of flux tubes (e.g. Hurlburt, Toomre & Massaguer 1984; Tobias et al. 2001; Thomas et al. 2002; Brummell et al. 2008), perhaps obviating one of the motivations for the interface dynamo. Also, regions of strong shear near the tachocline are located at high latitudes, and not low latitudes where sunspots appear. In contrast, the near-surface shear layers are closer to latitudes where the sunspots appear. In addition, surface layers show rotational variations on the solar cycle timescale. Finally, the fact that fully convective M stars have dynamos and activity shows that an interface dynamo is not generally needed, although this does not preclude its existence in higher mass stars.

Here, we revisit previous parameter scalings for both interface and distributed dynamos that are valid for Ro ≫ 1 but have been unwittingly used when Ro ≪ 1. We revise them by taking into account the fact that the eddy correlation time is not the convective turnover time for Ro ≪ 1. We also estimate the saturated magnetic field strength, which we argue to be most important for the activity-Rossby number relation we derived in Section 3.

2.1 Dynamo number for Ro ≪ 1 and Ro ≫ 1

The spherical α − Ω mean-field dynamo equations can be written as (Durney & Robinson 1982; Thomas et al. 1995)

\[ \frac{\partial A}{\partial t} = \alpha B_\varphi + \beta_1 \left( \nabla^2 - \frac{1}{r^2 \sin \theta} \right) A_\varphi \] (2)

and

\[ \frac{\partial B_\varphi}{\partial t} = r \sin \theta (B_\theta \cdot \nabla) \Omega + \beta_\varphi \nabla^2 B_\varphi , \] (3)

where the large-scale magnetic field is \( B = B_\varphi \hat{\varphi} + \nabla \times (A_\varphi \hat{\varphi}) \), \( B_\varphi \) and \( A_\varphi \) are the large-scale toroidal magnetic field and vector potential components, \( B_\varphi = \nabla \times (A_\varphi \hat{\varphi}) \) is the poloidal magnetic field component, and \( (r, \theta, \varphi) \) are spherical coordinates. The angular velocity \( \Omega = \Omega(r, \theta, \varphi) \) in general. In equations (2) and (3), \( \alpha, \beta_1 \) and \( \beta_\varphi \) are, respectively, pseudo-scalar helicity and scalar diffusion transport coefficients (discussed later) that incorporate turbulent correlations of in the electromotive force (EMF) \( (\nu \times B) = \alpha \hat{B} - \beta \nabla \times \hat{B} \).

We remove the \( r \) dependence in equations (2) and (3) by assuming that the poloidal variation of the radial variation (Yoshimura 1975; Durney & Robinson 1982) and \( \hat{A}_\varphi \sin \theta = A(r) e^{ikr} \) and \( B_\varphi = B(r) e^{ikr} \), where \( r_c \) is a radius in the convection zone nearest to where the shear is strongest (i.e. the base for the Sun) and \( k \) is the wavenumber associated with the radius of curvature for quantities dependent on \( \theta \). With a buoyancy loss term added to equation (3), equations (2) and (3) then become (Durney & Robinson 1982)

\[ \frac{\partial A}{\partial t} = \alpha B - \beta_1 k^2 A \] (4)

and

\[ \frac{\partial B_\varphi}{\partial t} = ikr_c \Delta \Omega A - \beta k^2 B - \frac{u_0 B}{L} , \] (5)

where \( L \) is the thickness of the shear layer of differential rotation \( \Delta \Omega \), and \( u_0 \) is a buoyancy velocity. Equations (4) and (5) can be applied to a distributed dynamo or an interface dynamo. For a distributed dynamo, \( L \) is the width of the convection zone. For an interface dynamo, equations (4) and (5) provide a 1D approximation where each layer is separately assumed to have fields that vary slowly in radius (e.g. Thomas et al. 1995), and where \( L \) is the thickness of the shear layer just beneath the convection zone (to be later distinguished from \( L_1 \), the thickness of the α-effect layer above \( r = r_c \)).

To proceed, we use a standard substitution \( A(t) = A_0 e^{-t/\tau} \) and \( B(t) = B_0 e^{-t/\tau} \) with \( \omega = \omega_R + \omega_I \), where \( \omega_R \) and \( \omega_I \) are real. We assume that buoyancy kicks in when the field has reached a value beyond that attained in the early-time kinematic growth phase, so we here estimate the growth condition without the last term in equation (5). Equations (4) and (5) have growing solutions when the absolute value of the product of the growth coefficients of the two equations divided by the product of the decay coefficients \( N_0 = \frac{\alpha_0 \tau_c \Delta \Omega}{L \beta k^2} \) exceeds unity. This \( N_0 \) is the dynamo number.

We now specify explicit expressions for \( \alpha_0, \beta_1, \) and \( \beta_\varphi \). An estimate of \( \alpha_0 \) that incorporates the Coriolis force is (Durney & Robinson 1982)

\[ \alpha_0 = \frac{\tau_c}{3} \frac{q_a}{6} \frac{\Omega^2}{r_c \cos \theta} , \] (7)

where \( v \) is the turbulent convection velocity of magnitude \( v \) and \( \tau_c \) is the eddy correlation time (assumed short enough to replace a time-correlation integral e.g. Moffatt 1978, although see Blackman & Field 2002 for a different interpretation of \( \tau_c \) in the convection zone. Note that \( \tau_c \) need not equal \( \tau_e \) since the latter is inferred from the pressure scaleheight and convective velocity (Noyes et al. 1984). The constant \( q_a = [\Omega(r_c)/\Omega][k_{\rm ed}^2/(k_{\rm ed}^2 + k_{\rm ed}^2)] \) accounts for both the factor by which the angular velocity at the base of the convection zone differs from that at the surface, and anisotropy of turbulent wavevectors. We take

\[ \beta_1 = \frac{1}{3} v^2 \tau_c = q_\beta \beta_\varphi , \] (8)

where \( q_\beta \geq 1 \) is a constant.

Equations (7) and (8) apply only for \( Ro \gg 1 \), for which \( \tau_c = \tau_e \). Their invalidity for \( Ro \ll 1 \) is evident from equation (7): when \( \tau_c \gg 1 \), the magnitude of \( \alpha_0 \), which has dimensions of velocity, could otherwise exceed \( v \) (Robinson & Durney 1982). Only \( |\alpha_0| < v \) can be physical, since only a fraction \( \ll 1 \) of the velocity is helical, and the gradient scales entering \( \alpha_0 \) can be no smaller than that of the dominant eddy scale. This restriction on the regime of validity has not been taken into account previously in studies, where \( N_0 \) is presented in the context of activity versus Rossby number relations (e.g. Montesinos et al. 2001; Wright et al. 2011).

However, we now extend the regime of validity by a physically motivated redefinition of \( \tau_c \). A strong surface rotation is plausibly

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also indicative of strong differential rotation within the star and if a convective eddy is shed by shear on a time-scale $\tau_s < \tau_c$, then the shorter shear time-scale $\tau_s$ becomes the relevant eddy correlation lifetime such that $\tau_s \sim \tau_c$. We assume $\tau_s = \tau_c$, so $s$ is a constant that accounts for differential rotation. For $2\pi \tau_s \alpha \lesssim 1$, we have $\tau_s \sim \tau_c \ll \tau_s$, so $\alpha \sim \tau_s \rho v^2$. The transition will occur approximately at $2\pi \tau_s \rho v^2 \sim 1/s$. As discussed further later, we interpret the observational transition where the activity becomes independent of Ro as exactly the transition to this shear-dominated regime. An empirical transition at $2\pi \tau_s \rho v^2 \sim 0.13$ (Wright et al. 2011) would imply $s \simeq 8.3$.

To capture both regimes $\alpha \gg 1$ and $\alpha \ll 1$ based on the physical argument just presented, we write

$$\tau_{ed} = \frac{s \tau_c}{1 + 2\pi \tau_s \rho v^2}.$$  \hspace{1cm} (9)

The dynamo coefficients (and thus the EMF) decrease with shear, consistent with simulations of Cattaneo & Tobias (2014). Using $\tau_c = \tau_s/\Omega$ with equations (7)–(9) in equation (6), along with $\Delta \Omega = \Omega/\tau_c$ and $\kappa r_c \sim 1$, we obtain

$$N_d \sim \frac{3q_s \rho r_c^3 \cos \theta_0}{2Lv^2} = \frac{3q_s \rho r_c^3 \cos \theta_0}{2Lv^2 \tau^2} \left( \frac{2\pi \tau_s \rho v^2}{1 + 2\pi \tau_s \rho v^2} \right)^2,$$  \hspace{1cm} (10)

where $\theta_0$ is the fiducial value for colatitude associated with $s = s(\theta_0)$. For $\tau_s \gg 1$, $\tau_{ed} \sim \tau_c$ from equation (9), and we write $v \tau_{ed} \simeq v \tau_c \sim h_p$, where $h_p$ is a pressure scaleheight satisfying $L_1 \sim \xi h_p$, where the constant $\xi \sim$ few. Thus,

$$N_d(\alpha \gg 1) \sim \frac{3q_s \rho r_c^3 \cos \theta_0}{2Lv^2 \tau^2} \sim \alpha^{\frac{3}{2}},$$  \hspace{1cm} (11)

highlighting the $\alpha^{-\frac{3}{2}}$ scaling as in Montesinos et al. (2001) but with different coefficients in part because we have used a more general formula for $\alpha_0$. Note that equation (11) applies only for $\alpha \gg 1$ as discussed above. If we assume a distributed dynamo, for which $L \sim L_1$ and $q_p = 1$, then

$$N_d(\alpha \gg 1, \text{dist}) \sim \frac{3q_s \rho r_c^3 \cos \theta_0}{2Lv^2 \tau^2}.$$  \hspace{1cm} (12)

For $\alpha \ll 1$, we have $\tau_{ed} \ll \tau_c$ and so $\tau_{ed} \propto \tau_c$ in equation (9). In this regime $\Omega / v \sim 2\pi \tau_s / \tau_{ed}$, where $\tau_{ed}$ is the eddy scale. Equations (9) and (10) then imply that

$$N_d(\alpha \ll 1) \sim \frac{6\pi^2 q_s \rho r_c^3 \cos \theta_0}{Lv^2}.$$  \hspace{1cm} (13)

If we further assume a distributed dynamo so that $L \sim L_1$ and $q_p = 1$, then

$$N_d(\alpha \ll 1, \text{dist}) \sim \frac{6\pi^2 q_s \rho r_c^3 \cos \theta_0}{Lv^2 \tau^2}.$$  \hspace{1cm} (14)

Previous discussions linking activity to dynamos have focused on the Ro dependence of $N_d$ using the Ro $\gg 1$ formulae (Montesinos et al. 2001; Wright et al. 2011) but without making a specific theoretical connection to coronal luminosity. We argue in Section 4 that $N_d$ is not the most important quantity for predicting the activity–Ro relation.

### 2.2 Saturated field strength: estimate and role

Although $N_d$ determines the kinematic cycle period and growth rate, it does not determine the non-linear cycle period (e.g. Tobias 1998) nor the saturated magnetic field strength. The saturated dynamo field strength is in fact commonly imposed by hand (e.g. Markiel & Thomas 1999; Montesinos et al. 2001; Charbonneau 2014). But the saturated field strength is important for determining how much magnetic energy is delivered to the corona and thus the X-ray luminosity averaged over a cycle period.

Recent work has progressed towards a saturation theory that agrees with simulations of simple helical dynamos when a 20th-century textbook mean-field theory is augmented to include a tracking of the evolution of magnetic helicity (for reviews see Brandenburg & Subramanian 2005; Blackman 2014). When the predominant kinematic driver is kinetic helicity, a key ingredient is that the dynamo $\alpha$ is best represented as the difference $\alpha_0 = -\alpha_M$, where $\alpha_M = (b \cdot \nabla \times b) / \tau_{ed}$, proportional to the current helicity density of magnetic fluctuations. This form emerged from the spectral approach of Pouquet, Frisch & Leorat (1976) and from a simpler two-scale mean-field dynamo approach (Blackman & Field 2002; Field & Blackman 2002). In the Coulomb gauge, $\alpha_M$ is proportional to the magnetic helicity density of fluctuations, a result that is also approximately true in an arbitrary gauge when the fluctuation scale is much less than the averaging scale (Subramanian & Brandenburg 2006).

Saturation in the stellar context might proceed as follows (Blackman & Brandenburg 2003): kinetic helicity initially drives the large-scale helical magnetic field growth, which, to conserve magnetic helicity for large $\alpha_M$, builds up small-scale magnetic helicity of the opposite sign. This grows $\alpha_M$ to offset $\alpha_0$. To lowest order in $\tau_s/\tau_c$, the greatest strength the large-scale helical field can attain before catastrophically slowing the cycle period is estimated by setting $\alpha_0 - \alpha_M = 0$ and using magnetic helicity conservation to connect $\alpha_M$ to the large-scale helical field. The toroidal field is further amplified non-helically by differential rotation above the strength of the mean poloidal field to a value which is limited by magnetic buoyancy. We assume that downward turbulent pumping (e.g. Tobias et al. 2001) hampers buoyant loss only above some threshold field strength (Weber, Fan & Miesch 2013; Mitra et al. 2014) such that $\alpha_M$ can approach the value $\alpha_0$. A dynamo is then maintained with large-field amplification balanced by buoyant loss, coupled to a beneficial loss of small-scale helicity: for dynamos with shear, small-scale helicity fluxes seem to be essential not only for sustaining a fast cycle period but also to avoid catastrophic field decay (e.g. Brandenburg & Sandin 2004; Sur, Shukurov & Subramanian 2007).

The saturation strength of the large-scale poloidal field based on the aforementioned circumstance of $\alpha_M \sim \alpha_0$ is (Blackman & Field 2002; Blackman & Brandenburg 2003)

$$B^2_v \sim 8\pi L_{1} f_b \rho v^2,$$  \hspace{1cm} (15)

where $f_b$ is the fractional magnetic helicity when the initial driver is kinetic helicity. For our present purpose, $f_b = l_{ed} (\mathbf{v} \cdot \nabla \times \mathbf{v}) / v^2 = \frac{q_s \rho r_c^3 \cos \theta_0}{Lv^2 \tau^2}$, where the latter expression follows from equations (7) and (9).

The toroidal field is linearly amplified by shear above this value during a buoyant time loss $\tau_s \sim L_1 / \nu b$, where $\nu b$ is a typical buoyancy speed for those structures that escape. This gives (taking all

$^1$ Magnetic buoyancy could initially source the EMF instead of kinetic helicity. Upon saturation from small-scale twist, buoyancy would still eventually act as a loss mechanism. The connection to flux transport dynamos is beyond our present scope but Karak, Kitchatinov & Choudhuri (2014) present a challenge that such dynamos predict cycle period–Ro trends opposite to both those observed.

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real parts of $B_y$ and $B_x$.

\[
B_y^2 \approx B_x^2 \left(1 + \Omega \tau_{\phi} / r_s \right)^2 \sim B_x^2 \left(\Omega \tau_{\phi} / r_s \right)^2,
\]

which is consistent with $B_y > B_x$ as long as $\Omega \tau_{\phi} / r_s > 1$. If we further assume that $u_0 \approx B_x^2/(12\pi v \rho v)$ from calculations of buoyant flux rise (Parker 1979; Moreno-Insertis 1986; Vishniac 1995; Weber et al. 2013), then $\tau_{\phi} \approx 12\pi L / \rho v / B_x^2$. Using these in equation (16) and solving for $B_y^2 \sim B_x^2$ gives

\[
B_y^2 \sim \left(\frac{12\rho B_x \Omega v L}{8\pi^2 s} \right)^{2/3},
\]

where we have used the above expression for $f_s$, equation (15), $\tau_{\phi} \Omega = 2\pi / \nu S$, and equation (9). Equation (17) leads self-consistently to $B_x > B_y$ as long as $\Omega v L / s > B_x^2 / (8\pi \rho v)$, which is satisfied even for slow rotators like the Sun since for $s = 8.3, \Omega / s \sim 2.4 \times 10^{-7}s$, $v \sim 4000 \text{ cm s}^{-1}$, $L \sim 10^{10} \text{ cm}$, and $B_x^2 / 8\pi \rho v^2 \sim \text{from equation (15). Equation (17) also agrees with the Ro \ll 1 \text{ scaling of Christensen et al. (2009) which matches planetary and stellar dynamos simulations since for Ro \ll 1, Ro drops out and } B^2 \text{ becomes independent of } \Omega, \text{ and } B_x^2 \sim \rho v^2 \sim \rho v^3,^{2/3}.$

### 3 X-RAY ACTIVITY AND ROSSBY NUMBER

Overall stellar activity can be gauged by $L_X / L_\odot$, where $L_X$ is assumed to result from dissipation of dynamo-produced magnetic fields that rise into the corona. Each single observation of $L_X$ probes a time-scale short compared to the cycle period and hence can be thought of as taken from an ensemble of luminosities from a distribution over a cycle period. The solar X-ray luminosity varies by a little more than an order of magnitude over a solar cycle (Peres et al. 2004). For very active stars, $\alpha_{opt} \geq 0.1$ (O’Neal et al. 2004). Fig. 1 shows the result of equation (19) for $q \cos \theta \sim 0.1$ and $\Theta = \Theta_1 / (L_X / L_\odot) \sim 6.6 \times 10^{-7}$, for two cases of $\lambda = 0$ and one case of $\lambda = 1/3$, normalized by the average solar value (Peres et al. 2000). In general, $L_X / L_\odot \propto \left(s / \nu s \right)^{2/3}$.

From equation (19) for $\Theta = 1$, the $\lambda = 0$ case gives $q \sim 2$ and the $\lambda = 1/3$ case gives $q = 3$. Larger $\lambda$ would make $q > 3$, whereas the range $2 \leq q \leq 3$ is suggested by observations (Wright et al. 2011). A curve with $\lambda > 0$ can accommodate the higher observed saturation values of $L_X$, compared to the curves of $\lambda = 0$, while still matching the Sun.

Most importantly, note that the expression for $L_X / L_\odot$ in equation (19) becomes independent of Ro for Ro \ll 1, regardless of the specific behaviour in the Ro \gg 1 regime.

### 4 CONCLUSION

Using physical arguments, we have developed a relationship between $L_X / L_\odot$ and Ro. The result accounts for both a transition to Ro quasi-independence at low Ro and a strong inverse dependence at large Ro, in general agreement with observations. Our result that the predicted transition towards Ro independence at low Ro is independent of the specific dependence on Ro for Ro \gg 1. Our emergent saturated field strength $s$ for Ro \ll 1 also agrees with the scalings of Christensen et al. (2009), shown to match a range of planetary and stellar dynamo simulations.

Previous attempts to estimate the activity–Ro number relation have focused on the possible role of the dynamo number but the expressions commonly used are invalid for Ro \ll 1 limit because the convection turnover time is no longer a good approximation for the turbulent correlation time. When eddies are sheared faster than convection can overturn them, the shear time should replace the convection time when estimating correlation times. We have accounted for this using equation (9), which reduces to $\tau_c$ for Ro \gg 1 and to $\tau_c$ for Ro \ll 1. This prescription is widely applicable.

\[2 \text{ in equation (18), } L_X \propto B^4 \propto B_\phi^4 / \rho v^3, \text{ the luminosity associated with the convective heat flux through the convection zone (e.g. Shu 1992). For the Sun, } 2\pi Ro \sim 2 \text{ and } L \sim 2 \tau_c / 5. \]
More fundamentally, the dynamo number is insufficient for capturing activity because it does not determine the saturated magnetic field strength. We estimated the latter using a saturation theory rooted in magnetic helicity evolution, combined with a loss of magnetic field by magnetic buoyancy. The associated magnetic flux provides the source of the X-ray luminosity and, when combined with the generalized $\tau_{\text{ed}}$ just described, culminates in equation (19).

Opportunities for developing more detailed models abound. Observational constraints on $\lambda$ and on the connection between rotation and internal differential rotation would be desirable in testing equation (19).

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