Finite Element Modelling and Updating of a Bolted Structure Using Elements Representing the Stiffness Members, Bolts and Affected Areas of the Joints

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Abstract. Efficient and accurate analytical representation of bolted joints in an assembled structure for the prediction of the dynamic behaviour of the structure has been a crucial unsolved issue in the finite element (FE) community. 3D modelling of bolted joints, possessing a large number of degrees of freedom, leads to very high computational time. This paper proposes an efficient representation of bolted joints for the prediction of the dynamic behaviour of a bolted structure. The FE model of the bolted joints was developed using CBUSH element connectors for bolt shanks and CELAS element connectors for the stiffness of the members at the affected areas of the bolted joints. The results of the normal modes analysis of the FE model of the bolted structure were in a very good correlation with experimental modal analysis (EMA) counterparts quantified by a range of MAC values between 0.91 to 0.99 for all modes. Finite element model updating was used to improve the accuracy of the FE model of the bolted joints. The updated FE model of the assembled structure was validated with the physical tested structure. It was found that the comparison of the results has shown good agreement. In addition, the natural frequencies and mode shapes predicted from the proposed model have been found to be more accurate than the results obtained from the previous analytical models reported in the literature on bolted joint modelling of dynamic behaviour investigation. The proposed model can be used for a large and complex structure.

1. Introduction

Bolted joints are extensively used as the main joining method for structures in various industrial sectors due to the low cost and quick assembling work. Furthermore, it also plays a substantial role in obtaining an optimum dynamic response of a bolted structure due to the higher energy dissipation and higher damping ratio [1]–[3]. However, modelling of bolted joints with an efficient and accurate methods for the investigation of structural dynamic behaviour has always been found to be very difficult [4], [5].

It has been found that the stiffness member, bolt modelling and joint affected areas [5]–[9] are the main factors that lead to the development of an efficient FE modelling of the bolted structure. However, very little evidence of research work is available, especially on incorporating all the main factors in the development of an efficient FE model of the bolted structure and on updating the developed FE model using model updating methods.

Normally, a bolted structure comprises several structural members jointed with a series of bolts and nuts. Therefore, a structure with multiple members should have the overall stiffness members...
represented in an FE model by several springs connected in series [10], [11]. In this study, CELAS element connectors are used to represent the overall stiffness members in the FE model of the bolted structure.

There are also various methods for bolt modelling used by researchers [12]–[17]. Elements such as CBAR, CELAS, CBEAM and CBUSH were employed to represent the bolts [7], [18]–[20]. In this work, CBUSH elements are used to represent the bolts. The flexibility of the bolts is calculated based on the Swift’s formula from which the shear stiffness and rotational stiffness components of the CBUSH element are determined [16], [21]–[25].

There are several cases demonstrated in previous studies. The interfaces of the bolted structure were modelled with special elements such as beam-bar elements, contact zone elements, partitioned thin layer elements and connective layer elements [5], [26]–[28]. However, in this work, the contact bodies are defined as sub-parts of the affected areas of the joints [29]. Therefore, the inclusion of the parameters of the joints’ affected areas such as the Young’s modulus, mass density and thickness may lead to an improvement in the prediction of the dynamic behaviour of the bolted structure.

This paper put forward a proposal of an efficient and economical modelling method for the bolted structure focusing specifically on the stiffness members, bolt representation and the joints’ affected areas. Normal modes analysis is performed, and the predicted natural frequencies and mode shapes are compared with the experimental modal analysis (EMA) counterparts. FE model updating is used to update the initial FE model of the bolted structure in the light of experimental data in order to improve the accuracy and reliability of the FE model of the bolted structure.

2. FE Modelling of the Bolted Structure

A bolted structure was developed in this study to investigate the dynamic behaviour of the structure with bolted joints. Figure 1 shows the 3D CAD model of the bolted structure which was developed using NX 11 design software. The bolted structure consists of two equal dimensional plates which are made of steel and jointed together by using stainless-steel bolts and nuts. Each of the plates is termed as Plate A and Plate B respectively. The dimensions of each plate are 380 mm in length, 45 mm in width and 6 mm thick. In addition, there are two cylindrical features in each plate with the diameter of 18 mm and the thickness of 2 mm as shown in Figure 2. The purpose of having the specially designed features is to reduce the contact zones between the assembled plates and they are named as the affected area of the plate [9].

![Figure 1. 3D CAD model of the bolted structure.](image)

![Figure 2. Cross-section of the bolted structure shows the affected areas of the bolted joints.](image)

The material properties of Plate A and Plate B are shown in Table 1. The tabulated values slightly differ from that of the textbook [30]. The values of the material properties used for the plates were systematically adjusted in the previous study using FE model updating [7], [31]–[33]. The use of the FE model updating technique produced accurate modelling of Plate A and Plate B prior to assembling both plates to form a bolted structure. On the other hand, the material for the M10 bolts and nuts are Stainless Steel and the material properties are also tabulated in Table 1 [30].
Table 1. Material properties for Plate A, Plate B and, bolts and nuts.

| Property                        | Plate A | Plate B | Bolts and nuts |
|---------------------------------|---------|---------|----------------|
| Young’s modulus (N/mm²)         | 206,400 | 209,400 | 193,000        |
| Poisson’s ratio                 | 0.302   | 0.316   | 0.27           |
| Mass density (kg/mm³)           | 7.458×10⁻⁶ | 7.45×10⁻⁶ | 7.86×10⁻⁶ |

The mid-surface of the solid geometry of Plate A and Plate B was extracted. The mid-surface which is in the form of 2D thin shell was discretised into finite elements. The FE model of plate A and Plate B was created using 1288 CQUAD8 elements with each element is 5 mm in size. The determination of the element size was selected based on several convergent tests [34]. Figure 3 shows the developed FE model of the bolted structure after the FE models of the plates A and B were assembled together. In the construction of the FE models of the bolted structure, CELAS elements were used to represent the jointed stiffness members. On top of that, the bolt’s shank was represented by CBUSH elements, while the bolt’s heads and nuts were represented by RBE3 elements. The modelling of the full FE model of the bolted structure is illustrated in Figure 4.

![Figure 3. FE model of the bolted structure.](image1)

![Figure 4. Representation of members and bolts by CELAS and CBUSH respectively.](image2)

In this work, the stiffness members was calculated by assuming a uniform pressure with conical envelope under the bolt’s head or nut at the affected area as shown in Figure 5 [10], [11], [35], [36]. The 1st stiffness member and 2nd stiffness member were calculated using equation (1) and equation (2) respectively. The variables of the stiffness member used are tabulated in Table 2. The stiffness values of k₁ and k₂ were used to calculate the overall stiffness members using equation (3). The overall stiffness members were then used as the component of translational stiffness of the CELAS element in the FE modelling of the bolted structure.
Figure 5. Frustums and the conical pressure distribution.

\[ k_1 = \frac{0.577\pi E_1 d}{\ln(1.15t_1 + D_1 - d)(D_1 + d)} = 6.642 \times 10^6 \text{ N/mm} \]  \hspace{1cm} (1)

\[ k_2 = \frac{0.577\pi E_2 d}{\ln(1.15t_2 + D_2 - d)(D_2 + d)} = 6.954 \times 10^6 \text{ N/mm} \]  \hspace{1cm} (2)

\[ k_m = 1/(1/k_1 + 1/k_2) = 3.397 \times 10^6 \text{ N/mm} \]  \hspace{1cm} (3)

### Table 2. Nomenclatures for the calculation of member stiffness.

| Variable | Description               |
|----------|---------------------------|
| \(t_1\)  | Thickness of Plate A      |
| \(t_2\)  | Thickness of Plate B      |
| \(E_1\)  | Young's modulus of Plate A|
| \(E_2\)  | Young's modulus of Plate B|
| \(d\)    | Diameter of the bolt       |
| \(D_1\)  | Diameter of bolt's head    |
| \(D_2\)  | Diameter of nut            |

In this work, bolt’s shank was represented by CBUSH elements. The components of stiffness of a CBUSH element have six degrees of freedom as illustrated in Figure 6. All stiffness components are named as axial stiffness \(k_1\), shear stiffness \(k_2\), \(k_3\) and rotational stiffness \(k_4\), \(k_5\), \(k_6\). The value of each component is defined using the formulae as stated in equation (4) to equation (7). The Swift’s formula \cite{16}, \cite{21}–\cite{25} as shown in equation (6) was used to calculate the flexibility of the bolt. Furthermore, the rotational stiffness around the axis of the bolt, \(k_4\), is fixed at 100 N.mm. Table 3 shows the description of the variables used in the equations while Table 4 is the calculated values for the stiffness components.

Normal modes analysis was performed on the FE model of the bolted structure. The solution type used is the SOL 103 Real Eigenvalues of the NX11 Simcenter software. The analysis was carried out to predict the first ten natural frequencies and mode shapes of the bolted structure. The predicted results were used as the initial FE results. The initial FE results were then compared with the measured results of the plates obtained from EMA.
Figure 6. Stiffness components of CBUSH element representing bolt.

\[ K_1 = \frac{E_b \times S}{L} \]  
\[ K_2 = K_3 = \frac{1}{C} \]  

\[ C = \frac{1}{E_id} \left( A + Bd \left( \frac{1}{t_1} + \frac{1}{t_2} \right) \right) \]  

\[ K_{5,6} = \frac{1}{\varepsilon} \times \left( \text{Max}(K_2, K_3) \times \frac{(L_{CBUSH})^2}{4} \right) \]  

where \( 1/\varepsilon \) is fixed at 100, \( \text{Max}(K_2, K_3) \) is the maximum value taken from the calculation of \( K_2 \) or \( K_3 \) and \( L_{CBUSH} \) is the length of the CBUSH element.

| Variable | Description |
|----------|-------------|
| \( E_1 \) | Young’s modulus of Plate A |
| \( E_b \) | Young’s modulus of the bolt |
| \( S \) | Section area of the bolt |
| \( L \) | Length of the bolt |
| \( D \) | Diameter of the bolt |
| \( A \) | Constant value used for steel bolt |
| \( B \) | Constant value used for steel bolt |
| \( L_{CBUSH} \) | Length of the CBUSH element |

| Table 3. Nomenclatures for the calculation of stiffness of CBUSH element. |

| Stiffness components | Value |
|----------------------|-------|
| Axial stiffness, \( K_1 \) | \( 9.474 \times 10^5 \) N/mm |
| Shear Stiffness, \( K_{2,3} \) | \( 5.403 \times 10^4 \) N/mm |
| Rotational Stiffness, \( K_4 \) | \( 1.000 \times 10^2 \) N.mm |
| Rotational Stiffness, \( K_{5,6} \) | \( 3.458 \times 10^6 \) N.mm |
3. **Experimental Modal Analysis (EMA)**

EMA is a method to extract the structural dynamic characteristics, which are the natural frequencies and mode shapes, of the actual bolted structure [37]. In this work, the bolted structure was constructed by joining two identically fabricated steel plates using two sets of M10 stainless steel bolts and nuts. The bolted structure was suspended from a test rig by using rubber bands to simulate free-free boundary conditions. The experimental setup of the bolted structure is shown in Figure 7.

The items of test equipment used to measure the dynamic characteristics of the bolted structure are the LMS system with the LMS Test.Lab software, triaxial accelerometers and impact hammer. The impact hammer was used to excite the bolted structure at a fixed point while the accelerometers were roved to 42 pre-determined points of the bolted structure to measure the responses. The excitation needs to be done in Z-direction and Y-direction to successfully extract the first 10 modes. Figure 8 shows the frequency response function of the bolted structure.

![Figure 7. Experimental configuration of the bolted structure.](image)

![Figure 8. Frequency response functions of bolted structure based on the excitation in the directions of Y and Z.](image)

4. **FE Model Updating**

FE model updating is an analytical method from which the accuracy and reliability of a FE model can be improved by systematically adjusting the parameters of the FE model within an identified range [38], [39]. The method was employed in this work in order to reduce the uncertainties introduced in the initial
FE models as a result of invalid assumptions about model properties. The selection of the parameters used for the FE model updating procedure is based on the result of the sensitivity analysis [40].

In this work, the SOL 200 Model Update of NX 11 Simcenter 3D software was utilised in the effort to update the FE models of the bolted structure. The first ten natural frequencies were incorporated in the objective function to improve the correlation between the predicted and measured natural frequencies. The objective function used in this work is as in equation (8) [32], [41]:

$$\min \sum_{i=1}^{m} W_i \left( \frac{\omega_i^n}{\omega_i^e} - 1 \right)^2$$

(8)

where $\omega_i^n$ is the $i$-th predicted frequency, $\omega_i^e$ is the $i$-th measured frequency, the weightage, $W_i$ is set to unity and $i = 1, \ldots, 10$.

5. Results and Discussion

In this work, the natural frequencies and mode shapes of the bolted structure were obtained experimentally and numerically using the EMA and FE method respectively. Differences in the results between EMA and FE method were reduced using the FE model updating method through which the selected parameters of the initial FE model are systematically adjusted to match the EMA results as close as possible.

The 10 modes of the natural frequencies and mode shapes obtained from EMA and the initial FE model of the bolted structure are tabulated in Table 5. The comparison of the natural frequencies obtained from EMA and the initial FE shows that the total error is 14.59%. Meanwhile, the quantification of mode shapes through the Modal Assurance Criterion (MAC) shows a range of values between 0.91 to 0.99 for all modes. This is an indication of a very good correlation between EMA and initial FE results.

| Mode | I. EMA (Hz) | II. Initial FE (Hz) | III. Error between I & II (%) | IV. MAC |
|------|-----------|------------------|-------------------------------|------|
| 1    | 75.08     | 75.29            | 0.28                          | 0.98 |
| 2    | 200.77    | 202.44           | 0.83                          | 0.94 |
| 3    | 404.93    | 401.63           | 0.81                          | 0.99 |
| 4    | 469.31    | 446.94           | 4.77                          | 0.95 |
| 5    | 630.78    | 632.63           | 0.29                          | 0.97 |
| 6    | 674.39    | 661.27           | 1.95                          | 0.96 |
| 7    | 1034.64   | 1019.16          | 1.50                          | 0.94 |
| 8    | 1053.23   | 1067.47          | 1.35                          | 0.91 |
| 9    | 1276.95   | 1278.41          | 0.11                          | 0.96 |
| 10   | 1435.09   | 1473.76          | 2.69                          | 0.94 |

Total Error 14.59

A list of the potential updating parameters is tabulated in Table 6. Based on the sensitivity analysis of the potential updating parameters, the natural frequencies of the bolted structure are most sensitive to the shear stiffness in x-axis and y-axis of the CBUSH element representing the bolt’s shank. On top of that, the thickness, Young’s modulus and mass density of the affected areas of Plate A and Plate B may also affect the natural frequencies. The results of the sensitivity analysis of the potential updating parameters are shown in Figure 9.
Table 6. Description of the potential updating parameters.

| Parameter ID | Description                                           | Unit   |
|--------------|-------------------------------------------------------|--------|
| K1 CELAS     | Stiffness of joint members                           | N/mm   |
| tAA          | Thickness of affected area of Plate A                 | mm     |
| tAB          | Thickness of affected area of Plate B                 | mm     |
| EAA          | Young’s modulus of affected area of Plate A          | MPa    |
| EAB          | Young’s modulus of affected area of Plate B          | MPa    |
| pAA          | Mass density of affected area of Plate A             | kg/mm³ |
| pAB          | Mass density of affected area of Plate B             | kg/mm³ |
| νAA          | Poisson’s ratio of affected area of Plate A          | unitless |
| νAB          | Poisson’s ratio of affected area of Plate B          | unitless |
| K1 CBUSH     | Axial stiffness of bolt                               | N/mm   |
| K2 CBUSH     | Shear stiffness (x-axis) of bolt                      | N/mm   |
| K3 CBUSH     | Shear stiffness (y-axis) of bolt                      | N/mm   |
| K4 CBUSH     | Rotational stiffness around the axial axis of bolt    | N.mm   |
| K5 CBUSH     | Rotational stiffness around x-axis of bolt            | N.mm   |
| K6 CBUSH     | Rotational stiffness around y-axis of bolt            | N.mm   |

Figure 9. Sensitivity matrix of potential updating parameters.

For the FE model updating of the bolted structure, genetic algorithms were used as the optimizer. The comparison of results of EMA and the updated FE models is tabulated in Table 7 with several sets of updating parameters. The recorded total error has reduced to 14.39% using the shear stiffnesses $K_2$ CBUSH and $K_3$ CBUSH as the updating parameters. The total error has decreased to 14.24% with the Young’s modulus of the affected areas of Plate A and Plate B. Further reduction in total error to 14.10% can be achieved with all selected updating parameters were included in the updating procedure. However, it was found that the lowest total error of 14.08% can be achieved when the mass densities of the affected areas of Plate A and Plate B were excluded. This comparison process to select appropriate updating parameters was also demonstrated in previous researches [5], [41].

The updated parameters resulted from the FE model updating procedure which gave the lowest total error are shown in Table 8. It was discovered that there is a substantial increase in the shear stiffnesses $K_2$ CBUSH and $K_3$ CBUSH of the bolt’s CBUSH element. There is also significant increase in Young’s modulus and small increment of thickness of the affected areas of Plate A and Plate B. These results showed that the bolts and nuts used to assemble the plates contributed significantly to the dynamic behaviour of the assembled structure as well as the affected areas of the Plate A and Plate B.
Table 7. Comparison of EMA and updated FE results.

| Mode | EMA (Hz) | FE (Hz) | Error between I & II (%) | FE (Hz) | Error between I & IV (%) | FE (Hz) | Error between I & VI (%) | FE (Hz) | Error between I & VIII (%) |
|------|----------|---------|--------------------------|---------|-------------------------|---------|-------------------------|---------|---------------------------|
| 1    | 75.08    | 75.29   | 0.28                     | 75.31   | 0.31                    | 75.36   | 0.37                    | 75.34   | 0.35                      |
| 2    | 200.77   | 202.44  | 0.83                     | 202.45  | 0.84                    | 202.46  | 0.84                    | 202.45  | 0.84                      |
| 3    | 404.93   | 401.66  | 0.81                     | 401.75  | 0.79                    | 402.00  | 0.72                    | 401.89  | 0.75                      |
| 4    | 469.31   | 447.80  | 4.58                     | 448.58  | 4.42                    | 449.23  | 4.28                    | 449.29  | 4.27                      |
| 5    | 630.78   | 632.63  | 0.29                     | 632.68  | 0.30                    | 632.84  | 0.33                    | 632.77  | 0.32                      |
| 6    | 674.39   | 661.31  | 1.94                     | 661.44  | 1.92                    | 661.83  | 1.86                    | 661.72  | 1.88                      |
| 7    | 1034.64  | 1019.23 | 1.49                     | 1019.46 | 1.47                    | 1020.13 | 1.40                    | 1019.86 | 1.43                      |
| 8    | 1053.23  | 1067.50 | 1.35                     | 1067.58 | 1.36                    | 1067.81 | 1.38                    | 1067.74 | 1.38                      |
| 9    | 1276.95  | 1278.41 | 0.11                     | 1278.66 | 0.13                    | 1279.43 | 0.19                    | 1279.12 | 0.17                      |
| 10   | 1435.09  | 1473.85 | 2.70                     | 1473.93 | 2.71                    | 1473.97 | 2.71                    | 1473.97 | 2.71                      |
| Total Error | 14.39 | 14.24 | 14.10 | 14.08 | 14.08 |

Table 8. Updated parameters for the assembled structure with bolted joints.

| Parameter ID | Initial Value | Updated Value | Unit |
|--------------|---------------|---------------|------|
| K2_CBUSH     | 540,314       | 556,125       | N/mm |
| K3_CBUSH     | 540,314       | 661,007       | N/mm |
| EAA          | 206,400       | 216,196       | MPa  |
| EAB          | 209,400       | 217,519       | MPa  |
| tAA          | 2             | 2.11          | mm   |
| tAB          | 2             | 2.08          | mm   |

6. Conclusions
The work of developing and investigating an efficient, economic and reliable FE model of a bolted structure using the FE model updating method in light of the EMA results has been presented. It was found that the updated parameters of the physical and material properties of the FE model which are the stiffness members, bolts and joints’ affected areas, have a significant influence towards the accuracy of the prediction of the dynamic behaviour of the bolted structure. Therefore, an accurate FE model can be confidently used for any subsequent analysis.

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