Factor analysis in the lame problem (the problem of composite cylinders)

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Abstract. The article considers one of the important directions in the repair of cars. The examples discussed relate to the case where the holding force must ensure that the additional element is stationary in the axial direction. This is the case, for example, when restoring cylinder liners, when there are no forces and moments that tend to rotate the additional element in the enclosing part. The probabilistic formulation of the composite cylinder problem described above is also fully applicable to the calculation of these compounds.

1. Introduction
We estimate that more than 70% of the machine and tractor fleet in the Russian Federation is operating outside of the previously established depreciation time limits. This led to an increase in the labor intensity of disassembly and Assembly work by 12 ... 15% when repairing machines and complicating the technology for restoring basic parts that form a movable connection of the "piston-sleeve" type in engines and fixed connections of the "outer ring of the bearing-socket" type in gear box housings, drive axle housings, etc.

A significant factor in stabilizing the cost of repairing machines is the restoration of repair of suitable worn parts, especially expensive basic ones.

New recovery processes developed in the last 25 ... 30 years usually require significant initial costs for expensive equipment and only pay for themselves if the annual recovery programs are large. Therefore, one of the important directions in the repair of cars is the improvement of traditional methods, the possibilities of which are often far from exhausted.

One of such methods is the method of production of additional elements, allowing, on the one hand, to provide the initial fit connection, which is the most important, and on the other, - to manage existing plant process equipment. In the current conditions, for most repair companies, it is economically feasible, and often the most affordable.

Professor V. I. Kazartsev, doctor of technical Sciences, drew attention to the principal possibility of using the method in 1940. In many cases, fixing an additional element in the body part is achieved without the use of any fasteners by placing it in the body with tension. The amount of this tension must ensure that the additional element is stationary in the body (covering) part and does not cause excessive stresses in the material of the connection part.
2. Material and methods

As a theoretical basis for applying the method, it is advisable to use the problem of composite cylinders (the lame problem), first formulated in a deterministic formulation in the early nineteenth century and successfully used so far in the design of workable artillery barrels. In this case, a positive effect is achieved by redistributing the stresses between the inner (most stressed) and outer (least stressed) layers of the barrel material by creating pre-stressed structures from two hollow cylinders nested in each other.

The mathematical apparatus of this problem is described in detail in the technical literature, for example, [8, 9], and others. In some works, it is used to formalize the process of restoring plantings by substituting an additional element [6, 7]. The use of this mathematical apparatus allows you to determine the tension that provides a given value of the holding force (the force that prevents the displacement of the additional element), which guarantees the immobility of its landing in the covering part. The adequacy of this model has been proven by its repeated use in world engineering practice when solving the problem of composite cylinders in various fields of engineering [1, 4].

However, no matter how accurately the deterministic model reflects the actual values of the parameters of the processes it describes, its results will always contain distortions. The main source of these distortions is the error of the original data. The reason for such errors may be measurement errors, heterogeneity of material properties, the inability to accurately withstand the specified dimensions, etc. Therefore, the calculated results are always taken with a certain margin factor that compensates for possible distortions of the results. The value of this margin coefficient is assigned depending on the amount of damage that occurs if an inaccurate value of the calculated result is used. Thus, when calculating a nuclear reactor, it is quite justified to use tenfold or more reserve coefficients, while when calculating any inexpensive construction for household use, this reserve coefficient can be assumed to be one and a half times [2, 3].

In the above cases, use the tasks on composite cylinders to calculate the constructive-technological parameters of the connection "sleeve-sleeve" safety factor to compensate for inaccuracies of inputs such as mechanical properties of the materials being recovered parts and additional elements (elastic modulus, Poisson's ratio, yield stress), linear size (the radius of the bore covering details, the thickness of the walls of the part and the additional element, the length of the billet for its production), the coefficient of friction pair "covering the detail - optional element". As the practice of calculations shows, all these values can vary within a fairly wide range, so the question of assigning a reserve coefficient is quite complex.

In this respect, the principal drawback of the classical detailed formulation of the composite cylinder problem is that it operates with point estimates of the original data. These are either their average values, when the variation can be ignored, or their worst (in the sense of influencing the final result) values, when the variation of the original data cannot be ignored. Both approaches give a point estimate for the result, and in the first case we cannot estimate the probability of bad outcomes, and in the second case we get an estimate for the worst outcome, the probability of obtaining which in the actual conditions of manufacture, restoration or operation of the product is almost zero. This classical formulation of the problem does not allow you to get an interval estimate of the result.

3. Results and discussion

We attempt to eliminate this drawback of the classical formulation of the problem of composite cylinders. The essence of this attempt is that all variables that appear in the model, except for the desired value, are initially considered as random variables with specified probability distributions. The accuracy of these distributions can vary from numerical values of parameters of a particular type of distribution law (the most desirable case) to basic numerical characteristics, such as expectation and variance. The extreme in the range of options for setting the source data is the case when all the variables are set with a single point estimate, for example, the average value. In this case, the model turns into a classical deterministic model [5].

The desired value is also considered random, but its probability distribution is considered unknown and is found in the course of solving the problem. In this case, the desired value is considered as a
function of random arguments, which are all the other variables appearing in the model. In this study, using the example of restoring the cylinder liners of YAMZ-240 diesels by setting the rolling bushings, we consider the case when the desired value is the spread of the holding force. Therefore, the result of the simulation is a probability distribution of the holding force value, depending on the accepted structural and technological parameters of the sleeve-sleeve connection, that is, in this case, we consider the holding force as a function of random arguments [10, 11].

Thus, the conceptual essence of this approach can be expressed in General terms by the following formula:

\[ f(U) = \psi(r_p, l, b, E_1, E_2, \mu_1, \mu_2, f_{tr}) \]

where \( f(U) \) – density of the holding force distribution;
\( r_p \) – radius of the sleeve boring;
\( l \) – length of the tape for the roll-up sleeve;
\( b \) – tape thickness;
\( E_1, E_2 \) – elastic modulus of sleeve and sleeve materials;
\( \mu_1, \mu_2 \) – Poisson's ratio of sleeve materials;
\( f_{tr} \) – friction coefficient of the sleeve-sleeve pair.

Finding the analytical expression of distribution law of function of random variable, even for simple cases due to a number of strict requirements (monotonic ascending or descending, uniqueness, differentiability, the existence of analytical expressions for the inverse functions etc.). Analysis of the mathematical apparatus of the lamb problem shows that in our case it is not possible to meet all these requirements and that the only way to solve this problem at least numerically is to use the Monte Carlo method [12].

For the practical implementation of the proposed approach, we developed and tested a probabilistic model of the composite construct "sleeve-folding sleeve" [13]. The following values can be used as the required unknowns (singly or in any combination):

- contact pressure in the sleeve-sleeve connection;
- voltage at any point of the structure;
- holding force the friction force that prevents the mutual displacement of the sleeve and sleeve;
- linear dimensions of structural parts that change as a result of external and internal loads.

The following variables appear in the model as factors on which the required values depend:

- is the radius of the bore of the sleeve under the sleeve;
- length of the blank made of steel tape for folding the insert sleeve;
- thickness of the steel strip for making the roll-up sleeve;
- elastic modulus of the sleeve material;
- is the modulus of elasticity of the material of the sleeve;
- is the Poisson's ratio of the sleeve material;
- Poisson's ratio of the sleeve material;
- coefficient of friction of the sleeve-sleeve pair.

The modeling algorithm is based on the application of the famous Monte Carlo method and includes repeated repetition of the following procedures:

- generation of random (more precisely, pseudo-random) values of the set of factors included in the model;
- calculation of these generated values of the desired values by the accepted physical relationship (solution of the lamb problem);
• accumulation of statistical information for each such implementation of the process;
• at the end of the specified number of implementations, the static characteristics of the results found are calculated.

On the basis of this algorithm, we developed a computer program for calculating the structural and technological parameters of the composite "sleeve-sleeve" structure and provided calculations on the example of restoring the YAMZ-240 diesel cylinders.

This model served as a kind of "experimental setup" for studying the influence of the above factors on the dispersion of the holding force and other parameters depending on the set values of the design and technological parameters. In this regard, in the course of numerical experiments on the model, the question arose about the purpose of rational equations of varying factors.

Undoubtedly, the best solution to this problem is to use methods of experiment planning that have been repeatedly tested by the world practice of scientific research.

Experiments on this model allowed us to establish some previously known regularities concerning the spread of the values of the holding force depending on the magnitude of the variation of the factors that form it. As a measure of the spread, we have adopted the value of the standard deviation of the estimated parameter.

It is established that the main contribution to the spread of calculated parameters, including the holding force, is made (in descending order of importance) by the following factors:

• tolerance for the bore radius of the sleeve;
• the tolerance on the thickness of the tape;
• tolerance for the length of the blank for the sleeve.

The tolerance for all other factors does not have a significant impact on the spread of the calculated parameters, and therefore is excluded from further consideration.

It is also established that the spread of the holding force practically does not depend on the value of its calculated value and, other things being equal, depends only on the size of the tolerances for the above three factors. This is a very significant conclusion, since it makes it possible to significantly simplify the formula for describing the dependence of the holding force spread (as well as other calculated parameters) on the factor tolerance value. In this regard, to evaluate the parameters of the response surface.

Experimentation on the model allowed us to select the type and determine numerical estimates of the parameters of this dependence.

The most suitable turned out to be the dependence of the form

\[ U = A \times X^{b1} \times Y^{b2} \times Z^{b3} + C, \]

where \( U \) — spread (standard deviation) of the calculated parameter;
\( X \) — tolerance for the length of the tape, mm / 100;
\( Y \) — tolerance for the bore radius of the sleeve, mm / 100;
\( Z \) — the tolerance on the thickness of tape, mm / 100;
\( A, b_1, b_2, b_3 \) и \( C \) — dependency parameters.

The physical meaning of the right side of the formula is obvious: the first term represents the part of variation that depend on the selected factors; the second summand is independent from them, the proportion of variation of the estimated parameter, i.e. the share of variation that is determined not by the scientists in the model factors.

The value of the parameters of this formula is determined by the least squares method based on the initial data generated from the simulation results with variations of all three factors within the boundaries actually observed in the recovery practice. The results of these calculations for the height of the bushing belt 100 mm are shown in table 1.
In the literature on experiment planning, for example, for a visual representation of the response function, it is recommended to build its two-dimensional cross-sections with fixed values of the target function and several factors.

By entering a symbol \( d_\text{e} = X^{b_1} \cdot Y^{b_2} \cdot Z^{b_3} \), formula (1) can be rewritten as:

\[
U = A \cdot d_\text{e} + C
\]

**Table 1.** Parameters of dependence (1) for a belt height of 100 mm.

| Tape Thickness, mm | C     | b1   | b2   | b3   | A     |
|-------------------|-------|------|------|------|-------|
| 0.5               | 134.8 | 0.025| 0.575| 0.364| 176.08|
| 0.6               | 130.5 | 0.027| 0.597| 0.329| 205.85|
| 0.7               | 126.1 | 0.026| 0.609| 0.304| 236.77|
| 0.8               | 124.5 | 0.025| 0.620| 0.290| 264.43|

In this case, the value \( d \) can be interpreted as some generalized equivalent tolerance that replaces the combination of the actual values of the three factors that form the value of the output value (in our case, the spread). This method of replacing a certain set of variables with a single value is often used to facilitate data manipulation and can significantly simplify the mathematical apparatus. For example, in the resistance of materials, this technique is used when introducing the concept of equivalent stress, when instead of three values of the main stresses, one (equivalent) value is used, calculated according to special rules, which uniquely identifies the dangerous stress state of the material.

Using an equivalent tolerance, it is not difficult to construct nomograms to determine the required coefficient of retention force margin, which provides a given unheard-of level of rejection. The term margin factor in this article refers to the ratio of the calculated (average) value of the holding force as the minimum allowable value equal to 3450 Newtons. So, if the calculated value of the holding force is 8000 Newtons, only the margin factor is 2.32. If the margin factor is 1.75, the calculated holding force is 4060. The term marriage in this context refers to a situation where, due to insufficient tension, the holding force is less than the minimum allowable value.

To build a nomogram, you need to fix the equivalent tolerance at some level and, using this value, calculate the spread. This standard deviation determines the family of laws of the normal distribution of the holding force corresponding to this equivalent tolerance. After that, going through the calculated values of the holding force with a certain step, you can determine for each of them the probability that the holding force does not exceed the minimum allowable value, i.e., the probability of marriage. This probability is calculated using the normal distribution table, or using available computer programs, such as EXCEL. Then a curve is plotted in the coordinates "margin coefficient - acceptable level of marriage", and the margin coefficient is calculated as the ratio of the calculated value of the holding force to its minimum allowable value.

By repeating this procedure for other equivalent tolerance values, you can obtain a family of curves with the required step of changing the equivalent tolerance. Using such a nomogram, you can find the third one by two known values. For example, given the values of the equivalent tolerance and the margin factor, you can determine the probability of marriage at such tolerances and such an estimated (average) holding force.

Finally, setting the probability of marriage and equivalent to the tolerance, it is possible to find the required factor of safety.

In the case where large tolerance fields lead to an unacceptably high level of defects and it is not possible to tighten the tolerances (technically impossible or costly), knowing the dependence of the parameters of the law of distribution of the holding force on the size of the tolerances allows you to professionally solve the question of splitting parts into selective groups.
4. Conclusion
The examples discussed in this article concern the case when the holding force must ensure that the additional element is stationary in the axial direction. This is the case, for example, when restoring cylinder liners, when there are no forces and moments that tend to rotate the additional element in the enclosing part. However, there are a large number of connections where such forces and moments cannot be ignored. An example here is the fit of the upper connecting rod head bushing, the fit of the outer rings of rolling bearings in transmission housings, and so on.

The probabilistic formulation of the composite cylinder problem described above is also fully applicable to the calculation of these compounds. The changes consist only in the fact that the retention moment, rather than the retention force, should be considered as a criterion for the validity of the restored compound. The latter is easily found as the product of the holding force on the radius of boring of the covering part.

In addition, it should be borne in mind that in the case of calculating the holding moment, the value of the holding force must be calculated using a different value of the coefficient of friction, since the direction of mutual movement of the covering part and the additional element in this case changes. Consequently, the orientation of the machining traces changes in relation to the direction of displacement of the rubbing surfaces. This leads to a change in the coefficient of friction, since according to the literature, the coefficient of friction, among other factors, depends on the direction of the marks (strokes) left on the surface by the cutting tool during machining.

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