System Response to Sinusoidal Periodic Signals

Jing-Bo XU and Hong-Tao YU
Information Engineering University, Zhengzhou 450002, China

Keywords: Periodic signal, System, The response.

Abstract. Through to the signal after linear time invariant system (LTI) response analysis method, sine signal are presented through the steady-state response of linear time invariant system after three solving methods, for each method, an example is given to illustrate the solving steps and some problems that should be paid attention to, and the characteristics of each method are summarized, and then compares the advantages and characteristics of three methods, and promote the general solution of the system, periodic signal shows that solving the sine signal through the steady-state response of linear time invariant system is the best method of Fourier series method.

Introduction

Signal through the system after the response contains a zero input response and zero state response [1], which is zero state response system in the absence of the initial state of energy storage or under the condition of zero, only caused by the excitation and response, is applied to the output of the system, incentive in the response signal contains both the motivation and the characteristics of present system. The zero-state response of a system is generally divided into two parts, and its change form is determined by the characteristics of the system itself and the excitation source. When the system is linear and its characteristics can be expressed by a linear differential equation, the form of the zero state response is the sum of several exponential functions plus the term of the same form as the excitation source [2].

The solutions to the zero state response can be divided into time domain analysis method and transformation domain analysis method [3]. The two methods have their own characteristics, and the zero state response can be obtained through a specific way.

Analysis of the Zero State Response of Continuous Linear Time Invariant Systems

Time-Domain Analysis

The first kind of time domain analysis is the classical time domain analysis. The time-domain classical analysis method USES the mathematical classical theory to solve the differential equation of the system. First, the time-domain differential equation of the system is constructed. Then, write the characteristic equation, find the characteristic root, and list the general form of the homogeneous solution; Then, solve for the particular solution of the equation; Finally, the undetermined coefficients of the total solution are obtained by using the initial conditions. The classical time-domain analytical method is clear in thinking, easy to understand but computationally intensive, and difficult to obtain a particular solution for complex input signals.

The second method of time domain analysis is convolution in time domain. The convolution method in time domain is based on the concept of signal decomposition. First, the time-domain differential equation of the system is constructed to obtain the impulse response $h(t)$, and then the convolution of excitation $f(t)$ and impulse response $h(t)$ is used to obtain the zero state $y_{\text{zero}} = f(t) * h(t)$. This method is suitable for analyzing the zero state response of complex input signals. But it can only calculate the zero state, and the convolution integral process is tedious.

Transformation Domain Analysis

The first method of transform domain analysis is the frequency domain analysis based on Fourier
transform. The frequency-domain method firstly constructs the system algebraic equations based on time-domain differential equations. Then, the frequency response \( H(j\omega) \) of the system is obtained. Next, the product of the input signal’s Fourier transform \( F(j\omega) \) and frequency response \( Y_{ss}(j\omega) = F(j\omega)H(j\omega) \) is used to calculate the zero state response and invert it into the time domain. In this method, the convolution integral in time domain is converted to the product in frequency domain, and the differential equation in time domain is converted to the algebraic equation in frequency domain. But the Fourier transform of a signal that does not satisfy the absolutely integrable condition is meaningless or nonexistent.

The second method of transform domain analysis is complex frequency domain analysis based on Laplace transform. The complex frequency domain method first constructs the system algebraic equation according to the time domain differential equation; Then, calculate the frequency response \( H(S) \) of the system; Next, the product of the input signal’s Fourier transform \( F(S) \) and frequency response \( Y_{ss}(S) = F(S)H(S) \) is used to calculate the zero state response and invert it into the time domain. In this method, the convolution integral in time domain is converted into the product of complex frequency domain, and the differential equation in time domain is converted into algebraic equation in complex frequency domain.

The Steady-state Response of a Continuous Linear Time-invariant System to a Sinusoidal Signal

Sinusoidal signals are periodic signals, and according to the definition of periodic signals, sinusoidal signals as excitations operate from minus infinity to the end of the system at infinity. Therefore, different from the general starting signal acting on the system, at this time the system has no zero input response, we only need to focus on the zero state response.

Time Domain Analysis of Sinusoidal Signal Passing through the System

In order to facilitate analysis without loss of generality, it is assumed that the input sinusoidal excitation signal \( f(t) = 2\cos2\pi t, \quad -\infty < t < \infty \), and the impulse response \( h(t) \) of the physically achievable LTI system is set as \( h(t) = e^{-\epsilon(t)} \). According to \( y_{ss}(t) = f(t) * h(t) \), it can be obtained that:

\[
y_{ss}(t) = f(t) * h(t) = e^{-\epsilon(t)} \int_{-\infty}^{\infty} \frac{e^{\tau(t\tau)} + e^{(t\tau)}}{2} d\tau = \frac{2}{5} \cos 2\pi t, \quad \frac{4}{5} \sin 2\pi t = \frac{2\sqrt{5}}{5} \cos(2\pi - \frac{\pi}{2}) \quad , \quad -\infty < t < \infty
\]

It can be seen from equation (1) that sinusoidal excitation signal is still a sinusoidal signal of the same frequency after passing through the causal LTI system, and such output is called steady-state output or steady-state response, which can usually be expressed as \( y_{ss}(t) \). The method of time domain convolution starts with signal decomposition and obtains the system response according to the characteristics of linear time invariant system.

Frequency Domain Analysis of Sinusoidal Signals Passing through the System

The signal and the impulse response of the system are still the same as the hypothesis in 3.1. According to the convolution property of the Fourier transform, it can be known that:

\[
f(t) = 2\cos 2\pi t \iff F(\omega) = 2\pi[\delta(\omega + 2) + \delta(\omega - 2)]; h(t) = e^{-\epsilon(t)} \iff H(\omega) = \frac{1}{j\omega + 1}
\]

\[
Y_{ss}(\omega) = F(\omega)H(\omega)
\]

\[
= 2\pi[\delta(\omega + 2) + \delta(\omega - 2)] \frac{1}{j\omega + 1} - \frac{2\pi}{5} [\delta(\omega + 2) + \delta(\omega - 2) - 2\delta(\omega - 2) + 2\delta(\omega + 2)] y_{ss}(t) = \frac{2\sqrt{5}}{5} \cos(2\pi - \frac{\pi}{2}) \quad , \quad -\infty < t < \infty
\]

The analysis method in frequency domain USES Fourier transform and its properties to transform the convolution operation in time domain into multiplication operation in frequency domain.
Fourier Series Analysis of Sinusoidal Signals Passing through the System

The periodic signal is a signal that may have both Fourier series and Fourier transform. By using Fourier series for analysis, the signal and the impulse response of the system are still the same as the assumption in 3.1:

Since the zero state response of the periodic signal \( e^{j\omega t} \) after passing through the system:

\[
y_s(t) = h(t) * e^{j\omega t} = e^{j\omega t} \int_{-\infty}^{\infty} h(\tau) e^{-j\omega \tau} d\tau = e^{j\omega t} H(\omega) |_{\omega = \omega_0}
\]  

(3)

So the periodic signal \( f(t) = 2\cos(2t) = e^{j2t} + e^{-j2t} \) goes through the system

\[
y_s(t) = e^{j2\omega} H(\omega) |_{\omega = 2} + e^{-j2\omega} H(\omega) |_{\omega = -2} = e^{j2\omega} |H(2)| e^{j(2\omega-\theta)} + e^{-j2\omega} |H(-2)| e^{-j(2\omega-\theta)} = 2|H(2)| \cos(2t + \theta(2))
\]  

(4)

Using equation (3), when the system function is \( H(\omega) = \frac{1}{j\omega + 1} \), its modulus is \( |H(\omega)| |_{\omega = 2} = \frac{\sqrt{5}}{5} \) and its amplitude is \( \theta(\omega) |_{\omega = 2} = -\theta + \frac{\pi}{2} \),

\[
y_s(t) = \frac{2\sqrt{5}}{5} \cos(2t - \theta + \frac{\pi}{2}) , \quad -\infty < t < \infty
\]  

(5)

By comparing different methods of equations (1), (2) and (5), the results are the same. Fourier series method according to the steady-state response of the sine signal through system is still the same frequency sine signals, in the process of calculation, only need the system function modules and the picture of Angle in the signal frequency value calculation, and plug in the input signal expression, to change the input signal of the mold and painting Angle: system function module and modular multiplication of excitation signal, the system function of the amplitude and Angle of the excitation signal and adding together, the expression of the output response signals can be. The method is simple and the physical concept is very clear.

Conclusion

In this paper, the characteristics of the steady-state response of sinusoidal signal passing through the system and the solving methods are analyzed through examples: time-domain convolution method, Fourier transform method and Fourier series method. In this paper, the characteristics of each method are analyzed, and the advantages and disadvantages of the three methods are compared.

References

[1] Signal and linear system analysis [M]. Wu dazheng. The fourth edition. Beijing: higher education press, 2008:47-49.
[2] China’s science popularization [J]. Zhang haixia. Beijing: baidu baike, 2018.
[3] Signal and linear system analysis [M]. Guan zhizhong. Fifth edition. Beijing: higher education press, 2011:26-27.