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To cite this article: Mengqi Zhang and Richard J. A. M. Stevens 2017 J. Phys.: Conf. Ser. 854 012052

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Characterizing the coherent structures in large eddy simulations of aligned windfarms

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Abstract. The present work studies the large coherent structures in large eddy simulations of windfarms using proper orthogonal decomposition (POD) method. In order to evaluate the effect of wind turbines on coherent structures, we consider three cases. One is a reference flow of a neutral atmospheric boundary layer and the other two are periodic and developing aligned windfarms. The number of wind turbines is large, $16 \times 12$ for periodic windfarm, and $12 \times 12$ for developing windfarm. The simulations are run for a long time in order to generate a sufficient database for POD analysis. In all cases, elongated streamwise counter rotating roll structures, covering 1 or 2 turbines in spanwise direction, are identified as the dominant POD mode. Another pattern, varying in streamwise direction, also appears in all the three cases.

1. Introduction
Research on renewable and clean energy receives more and more attentions nowadays due to the deficiency of resources available and the global warming effect. Wind energy is considered as one of the most promising future energy forms [1, 2]. However, the overall efficiency of energy harvesting function in windfarms located around the world remains low because of a lack of an explanation of the detailed physical mechanism at play. In order to better understand the flow dynamics of wind turbine wakes in windfarms, the large coherent structures in windfarms are studied in this work. We simulate the flow using large eddy simulation (LES) and extract the large coherent structures by means of proper orthogonal decomposition (POD) [3] and characterize their features. By doing so, the physical mechanism in windfarms can be further unveiled, paving the way for a better understanding of the energy harvesting process in windfarms.

Windfarm experiments provide authentic and trustworthy data for real cases. A benefit of simulations compared to experiments is that these provide full access to the entire flow field in order to study the main flow structures that are formed. Numerical simulations have been applied to studying the fluid dynamics in windfarm. Windfarm simulations based on Reynolds-Averaged Navier-Stokes (RANS) models are attractive because of the low computational cost [4]. LES on the other hand provide a compromise between computational cost and a more detailed account of the flow dynamics. In LES, the small scale motions are modelled by a subgrid-scale model and the large scale motions are solved directly according to the filtered Navier-Stokes equation. Turbines are modelled using an actuator disk approach or an actuator line approach. Recently, LES has become increasingly popular in the community [2, 4]. In the present paper, an in-house LES code [5, 6] is used to simulate aligned periodic and developing windfarms to investigate the effect of wind turbines on the flow structures formed inside the windfarm.
In order to disclose and characterize the most important flow structures in windfarms, we apply proper orthogonal decomposition (POD) to the turbulent flow fields obtained from LES. Fully developed turbulent flow possesses tremendously many degrees of freedom and a complete account of flow dynamics in all the spatiotemporal scales will be a daunting task. Therefore, a wiser strategy is to describe only the dynamically important degrees of freedom and discard the inessential ones. This is the idea behind mode decomposition methods, such as POD. This technique has been used widely and successfully in many other flow systems, such as wall-bounded flow, free shear flow, and convection, to identify the coherent flow structures, see Refs. [7, 8].

In wind energy research, there are already several works on the POD analysis of flow fields in windfarms or flows behind wind turbines [9–15]. VerHulst & Meneveau [9] presented a comprehensive 3D POD analysis of aligned and staggered windfarms in addition to a neutral atmospheric boundary layer (ABL). They found streamwise counter rotating rolls as the dominant coherent structures creating strong ejection and sweep regions in the flow. It is believed that this mode contributes significantly to the kinetic energy re-distribution above and below the turbines and therefore are strongly related to the energy harvesting process. Other modes, such as streamwise varying structures and modes varying in vertical direction, are also identified. Hamilton et al. [13, 14], when analyzing experimental data of a wind turbine array, applied a double POD (DPOD) analysis to the measurements undertaken at separate streamwise stations. The idea of DPOD is to apply an additional POD analysis to the original POD modes with the same index. One of the merits of DPOD is that the effect of streamwise development, otherwise difficult to be taken into account in the POD analysis of separate measurements, can now be included in DPOD coefficients, but reduced to minimum in DPOD modes. Newman et al. [10] studied the entrainment process in a 3 × 5 array of wind turbines. The Reynolds shear stress is re-constructed at the turbine tip height via POD modes. A characteristic wavelength is identified for each POD mode. They found that lower POD modes (with larger kinetic energy) are associated with larger wavelength. Andersen et al. [15] performed LES of extended windfarms and analyzed POD in vertical planes. They showed that dominant length scales responsible for the entrainment process grow further inside the windfarm, but are restricted by the streamwise turbine spacing. Above the windfarm they observe larger structures.

Our methodology is closer to [9] with a longer simulation time and a larger computational domain. The results presented here complement the literature with new significances and are believed to be interesting to the researchers in windfarm, particularly those using POD as a probing tool. The present paper is organized as follows. In section 2 we introduce the used POD formalism and describe the LES setup. Section 3 presents the averaged velocity fields and the corresponding POD analysis. Finally in section 4, we conclude the paper and discuss possible future works.

2. Problem formulation

2.1. Proper orthogonal decomposition (POD)

POD is a powerful post-processing tool for revealing coherent structures in a flow system [3, 9, 16]. It can be interpreted as a singular value decomposition of the flow fields in a series of snapshots, obtained either numerically or experimentally. We present below its mathematical formulation. Other interpretations of POD and its mathematical formulations also exist. One can refer to [9, 17] in similar contexts of numerically simulated ABL and windfarms or [7, 18] for a general introduction to this technique. As noted by VerHulst & Meneveau [9], even though periodic boundary conditions are assumed in periodic windfarm, its POD modes need not be Fourier modes since the presence of turbines renders the flow inhomogeneous. This issue has also been discussed in general by George [19].

From the numerical results of LES, we first evoke Reynolds’ decomposition of a certain flow variable \( g_{\text{total}}(x, t) = G(x) + g(x, t) \) to subtract the mean from total fields. \( g(x, t) \) can represent
either velocity fields or flux fields. In the following, we take the streamwise, spanwise and vertical velocities \((u, v, w)\), respectively) as an example. Using the flow data, we formulate a matrix \(A\) as

\[
A = \begin{bmatrix}
\vdots & \vdots & \ldots & \vdots & \vdots \\
\vdots & \vdots & \ldots & \vdots & \vdots \\
\vdots & \vdots & \ldots & \vdots & \vdots \\
\end{bmatrix}_{3N_{xyz} \times N_t}
\]  

(1)

where \(N_{xyz} = N_x \times N_y \times N_z\) and \(N_x, N_y, N_z\) are the grid resolutions in Cartesian coordinate and \(N_t\) is the number of snapshots recorded. The multiplier 3 accounts for three velocity components considered. By this arrangement, each row of \(A\) represents a snapshot of the flow fields and each column of \(A\) stands for the time history of a certain flow variable at a grid point in the flow. We apply singular value decomposition to matrix \(A\), yielding

\[
A = U \Sigma V^\dagger.
\]  

(2)

The superscript \(^\dagger\) represents transpose conjugate of a matrix. The unitary matrices \(U\) (of size \(N_t \times N_t\)) and \(V\) (of size \(3N_{xyz} \times 3N_{xyz}\)) are respectively the left and right singular vectors of \(A\). \(\Sigma\) is a rectangular diagonal matrix whose diagonal entries are the singular values. Their physical interpretations will be explained below. In order to obtain these matrices, one can resort to the eigenvalue analysis by forming the correlation matrix \(C = A^\dagger A\), whose dimension is \(3N_{xyz} \times 3N_{xyz}\) in this case. The symmetric matrix \(C\) satisfies the following relation

\[
C = A^\dagger A = V \Sigma^2 V^\dagger.
\]  

(3)

We can easily see that each column of \(V\) spans the whole flow fields \((u, v, w)\) and it is called POD mode of the flow. The eigenvalues in \(\Sigma^2_{3N_{xyz} \times 3N_{xyz}}\) (equal to the singular values squared \(\lambda^2\)) represent the kinetic energy contained in each POD mode of velocity fields. When \(N_{xyz}\) is relatively small, the above procedure is feasible; however, for large-scale numerical simulations as we do in this paper, \(N_{xyz}\) is of order of tens of millions, rendering the computation of equation (3) almost impossible due to the memory constraints. In practice, the correlation matrix \(C\) is usually formulated alternatively as \(C = AA^\dagger\) and one poses the corresponding eigenvalue problem as

\[
C = AA^\dagger = U \Sigma^2_{N_t \times N_t} U^\dagger.
\]  

(4)

After solving this eigenvalue problem, we can obtain \(U\) and \(\Sigma_{N_t \times N_t}\). In addition, by recalling equation (2), we obtain POD modes in \(V\) according to \(\Sigma^{-1} U^\dagger A = V^\dagger\). From this equation, it is clear that \(U\) can be viewed as temporal coefficients for the snapshots in \(A\), constituting the POD modes in \(V\). Finally, as mentioned above, \(U\) and \(V\) are unitary matrices; hence, the columns in \(U\) (or \(V\)) form an orthonormal set, a property which we use as an \textit{a posteriori} check for all the results shown below.

### 2.2. Setup in large eddy simulation

In this work we simulated a neutral ABL, an aligned periodic windfarm, and an aligned developing windfarm, see table 1. Turbines are modelled using an actuator disk model [20]. We use a constant thrust coefficient \(C_T = 0.75\) for all turbines. It is assumes that turbines are operating in regime II [21], in which the thrust coefficient is constant as function of the wind speed. The constant thrust coefficient is a result of the blade rotation frequency that is adjusted appropriately to the incoming wind. According to Porté-Agel et al. [22] the turbines operate most of the time in this regime.

The resolution for all simulations is \(512 \times 256 \times 128\) in respectively streamwise \((x)\), spanwise \((y)\) and vertical \((z)\) directions in a fixed computational domain of \(4\pi H \times 2\pi H \times H\), where \(H\) is
the reference length in the vertical direction. This resolution is considered sufficient to simulate the large scale flow structures accurately in LES. The turbine diameter is $D = 0.1H$. As a result, each turbine has 5 grid points along its diameter. This grid resolution for turbines is comparable to the simulations performed in [9], and similar resolution guideline is put forward in Refs. [22,23].

We simulated a periodic windfarms with $16 \times 12$ turbines in streamwise and spanwise directions, respectively. The used turbines spacing in streamwise direction is $s_x = 7.85D$ and in spanwise direction $s_y = 5.24D$. This gives a turbine density $s_{xy} = \sqrt{s_x s_y} = 6.41D$. For the developing windfarm, we use the concurrent precursor inflow method, introduced in [23], to simulate simultaneously the flow dynamics in both inflow field and windfarm. In this case, we consider a windfarm of $12 \times 12$ turbines. The turbine spacings are the same as the periodic windfarm, see table 1. In this study we only consider aligned wind farms.

The numerical methods employed in LES are explained in [5,6]. Time step in the computations is adjusted according to the CFL condition. We checked that the used sampling frequency is higher than the frequency scale of most energetic events in the auto-spectral density of streamwise velocity $u$. This ensures that all the essential dynamics are captured. The databases generated in the simulations are relatively large. For example, the size of database for the aligned periodic windfarm is about 2 terabytes.

3. Results

In this section, we present the POD analysis results. We first show the averaged fields which are subtracted from the total field. Then the results of POD analysis are given including the eigenvalue distributions and the shapes of POD modes for ABL and periodic and developing windfarms. The convergence has been checked by applying POD to 75% of the snapshots used in the main analysis and we found no significant differences.

3.1. Averaged fields

In figure 1, the averaged fields of streamwise velocity $U$, spanwise velocity $V$ and vertical velocity $W$ are shown for periodic windfarm with $16 \times 12$ turbines and in figure 2 for developing windfarm with $12 \times 12$ turbines. The flow velocity is normalized by $U_a = \sqrt{U_{hub}^2 + V_{hub}^2 + W_{hub}^2}$, where $U_{hub}, V_{hub}, W_{hub}$ are the averaged velocities in the horizontal plane at hub height ($z/H = 0.1$). Figure 1(a) shows the mean $U$ field in the horizontal $xy$ plane at hub height and reveals the structure of wind turbine wakes. Panel (b) displays $V$ field, where one can observe regions of positive and negative velocities around the turbine positions. Physically, the wind turbines, modelled as actuator disks, act as blockage in the flow field. Due to the continuity of velocity field, fluid will flow around the turbines. Therefore, in the $xy$ plane, some fluid has a positive $V$ velocity and some fluid has a negative velocity. This results in the flow pattern shown in figure 1(b). The same figure also displays blue (small negative $V$) and green (small positive $V$) regions behind the turbines, showing up as patches in larger regions of small positive and negative $V$ velocities, respectively. When a blue wake region lies in the larger region of negative $V$, it disappears due to the superposition of two small negative $V$ values in the visualization. The same applies to the green positive wake regions. We note that the velocity in these regions

| Types of flow           | Turbine layout | $s_x$    | $s_y$    | $s_{xy}$ | No. of snapshots |
|-------------------------|----------------|----------|----------|----------|------------------|
| ABL                     | 16 x 12        | 7.85D    | 5.24D    | 6.41D    | 5300             |
| Aligned periodic        | 12 x 12        | 7.85D    | 5.24D    | 6.41D    | 5467             |
| Aligned developing      | 12 x 12        | 7.85D    | 5.24D    | 6.41D    | 6248             |

Table 1. Cases considered for ABL and windfarms in the present paper. Horizontal area $4\pi H \times 2\pi H$ is fixed for all the simulations. Grid resolution is also the same $512 \times 256 \times 128$. 
Figure 1. Averaged velocity fields for aligned periodic windfarms. The velocities have been normalized by $U_a$ (see the text for its definition). (a) normalized $U$ field in $xy$ plane at the hub height of $z/H = 0.1$. (b) normalized $V$ field in $xy$ plane at the hub height of $z/H = 0.1$. (c) normalized $U, V, W$ fields averaged over all the $xz$ planes intersecting the turbines. The colorbars have been adjusted to facilitate comparison and show the mean features.

is small, i.e. about 1% of the reference velocity $U_a$.

By the same argument of flow continuity, in $xz$ plane of $W$ velocity, some fluid flows upwards and some downwards around the turbines. This is illustrated in the lower panel of figure 1(c), which is averaged over all the $xz$ planes intersecting the turbines. The other two panels in figure 1(c) plot the velocity components $U$ and $V$. Because of the imposed periodic condition in spanwise direction, the averaged $V$ field is supposed to be vanishing. The $V$ field in panel (c) reveals a small (around 0.1% of $U_a$) residual velocity, which is probably caused by numerical uncertainty, a relatively large sampling intervals, and the limited simulation length.

Figure 2 shows averaged velocity fields for a developing windfarm. The number of wind turbines in this case has been reduced to $12 \times 12$ due to a fringe region in the rear part of windfarm domain, which is used to create the inflow condition [23]. This part of the computational domain has also been discarded in POD analysis and is thus not shown in the figures below. One of the differences between the periodic and developing windfarms is the non-periodicity in streamwise direction in the developing case. In panels (a,b), the $U$ and $V$ velocities in $xy$ plane manifest similar flow patterns in turbine scale as those in periodic windfarm, but in the scale of ABL, one can clearly see the streamwise development of $U$ velocity. This non-periodicity in the streamwise direction is also clear in $xz$ plane, see the figures in panel (c), which is again related to the lack of periodicity condition in the streamwise direction in developing windfarm.

3.2. POD analysis

In this section, we show large coherent structures educed by POD analysis from LES results. Compared to [9], whose computation domain is around $\pi H \times \pi H \times H$ with $4 \times 6$ turbines, our domain for the aligned case is 8 times larger ($4\pi H \times 2\pi H \times H$) with more turbines ($16 \times 12$). Therefore, by comparing the results in these two works, one can note the effects of computational
Figure 2. Averaged velocity fields for aligned developing windfarms. The velocities have been normalized by $U_a$ (see the text for its definition). (a) normalized $U$ field in $xy$ plane at the hub height of $z/H = 0.1$. (b) normalized $V$ field in $xy$ plane at the hub height of $z/H = 0.1$. (c) normalized $U, V, W$ fields averaged over all the $xz$ planes intersecting the turbines. The colorbars have been adjusted to facilitate comparison.

domain and turbine layout on the POD results of windfarms and hopefully find new features of POD structures. Besides, we do not shift the velocity fields in spanwise direction to expand the database as [9] did, even though such a practice is reasonable based on the periodicity in the problem. Instead, we simulate the flow system long enough to create sufficiently many snapshots in terms of convergence and sampling. As mentioned above, we will present POD results of ABL, periodic and developing aligned windfarms below.

Firstly, the eigenvalues in POD analysis, which represent the kinetic energy of POD modes, are shown in figure 3. They are normalized by the largest eigenvalue in (a) and cumulated with a normalization of the maximum total sum in (b). From panel (a), we can see that the three cases all show a trend of decaying magnitudes of eigenvalues as the index of POD mode increases. The smallest eigenvalues are 2 orders smaller than the largest one, indicating that these higher POD modes bear less dynamic significance, at least from the point of view of kinetic energy composition. We thus concentrate on the first POD modes which have more kinetic energy. Panel (b) shows the cumulative eigenvalues normalized by the maximum sum. The trend is similar to the results in VerHulst & Meneveau [9].

Figure 4 (a) displays POD modes for the streamwise velocity in a horizontal $xy$ plane at $z/H = 0.1$ for the ABL case. One can easily see that the dominant pattern of POD modes is the elongated streamwise rolls that vary slightly in spanwise direction. The vertical extent of these streamwise rolls can be of the order of the whole computational height as shown in panel (b). Aside from this structure, we see that POD modes 6 and 7 show another pattern which varies in the streamwise direction. In panel (b), the first POD modes of $u, v, w$ velocities are plotted in the $yz$ plane at $x/H = 1.963$. For $u$, the above discussed roll structures are clearly seen in this figure. For the $v$ and $w$ components, one can observe that there exists a counterclockwise
Figure 3. The distributions of eigenvalues solved in equation 4 for the velocity fields in ABL and aligned periodic and developing windfarms, respectively. (a) normalized eigenvalues by the largest one ($\lambda_1^2$), (b) normalized cumulative eigenvalues by the maximum total sum.

Figure 4. (a) POD modes of $u$ component for ABL at $z/H = 0.1$. (b) The first POD modes for $u, v, w$ in $yz$ plane at $x/H = 1.963$. Each POD mode is normalized by its maximum absolute value.

roll at around $0 < y/H < 1.5$ and a clockwise roll at around $4.5 < y/H < 6$ and these two counter rotating rolls are at the same time modulated in the streamwise direction. These results altogether depict the rolls pattern seen in POD mode 1 of panel (a).

Figure 5 shows the POD results for an aligned periodic windfarm. Unlike ABL, now the roll structures in panel (a) are more clearly delimited. We can see that each roll structure covers one or two turbine spacing in spanwise direction, spanning roughly $2\pi H \times 1/12 = 0.523H$ to $2\pi H \times 2/12 = 1.047H$. Another feature of aligned periodic windfarm that differs from ABL is that the streamwise varying pattern appears at an earlier POD mode. This indicates that turbines can redistribute the kinetic energy across POD modes and lead to rearrangement of POD modes. This streamwise varying pattern has also been discussed in [9] for periodic windfarms. However, the structures shown here, covering $4 - 6H$ in the streamwise direction (like the ones in POD mode 5), are much larger than theirs in size. Their streamwise varying structures seem to be confined between the turbines in the streamwise direction, while ours span across several turbines. Here we also mention that Newman et al. [10] and Andersen et al. [15] showed with POD analysis...
in a vertical 2D plane that inside the windfarm the important characteristic length scales are restricted by the turbine spacing, but that this is not the case above the turbine tip height. When we look at the flow structures at the turbine tip height we also find shorter characteristic length scales, in agreement with the aforementioned studies.

Finally, figure 6 shows the POD analysis results for developing aligned windfarm. From this
figure, due to the development of flow in $x$, one can note the relatively stronger non-periodicity of POD structures in the streamwise direction contrary to the results shown above. Nevertheless, the dominating pattern in this case still assumes similar roll structures as in the ABL and the periodic windfarm. Also, the spanwise size of the rolls appears similar to the ones observed in the periodic windfarm, covering 1 or 2 turbines in spanwise direction. The streamwise rolls start to diminish around POD mode 5 and for POD mode 8, its pattern resembles the streamwise varying pattern appearing in the other two flow cases. The shapes of POD modes 6 and 7 seem to lie in between the elongated streamwise structures and the streamwise varying structures.

4. Conclusion and discussions

In this work, we have performed LES of an ABL and aligned periodic and developing windfarms, resulting in databases of a very long simulation time and sufficient number of snapshots. We then applied POD analysis to the LES results and depicted the features of the dominant flow patterns in turbulent flow of windfarms.

We first showed the averaged fields for both periodic and developing windfarms. The turbines, modelled as actuator disks facing the atmospheric flow, drag the velocity fields and force the fluid particles to flow around them. This results in a $U$ velocity field that clearly reveals the wind turbine wakes, a $V$ velocity that has small positive and negative regions in $xy$ plane, and a $W$ velocity with positive and negative velocities around the turbines in $xz$ plane. The developing windfarm clearly demonstrates streamwise development compared to the periodic windfarm, whose averaged fields are periodic in the streamwise direction.

By comparing POD results of windfarm to ABL, one can easily see the influence of wind turbines on the large coherent structures. For example, the turbines confine the coherent structures in between them, such as the streamwise constant rolls structures and the streamwise varying structures. The roll structures are thus more clearly shaped in the windfarms and consistently span one or two turbine spacing (that is $0.523H$ to $1.047H$) in spanwise direction. In [9], whose computation domain is smaller than ours, the spanwise length of the rolls in POD mode 1 is around $\pi H/2 = 1.571H$, indicating that the size of roll structures is dependent on computational domain and turbine layout. Also, one recognizes that the streamwise varying structures seem not to be exclusive for windfarms and also appear in ABL.

By comparing periodic windfarm to developing windfarm, we conclude that the structures do not change significantly between the two except for the streamwise non-periodicity expected for the developing windfarms. Also, there seem to be more elongated roll structures in the first several POD modes of developing windfarms. In the periodic windfarm the shorter roll structures appear at a lower POD mode. This indicates that these shorter roll structures are more important for the periodic windfarm.

The results presented in this paper concern only aligned windfarms. It is interesting to study in addition the effect of staggering of the wind turbines on the large coherent structures in both periodic and developing windfarms. Besides, other quantities, such as energy flux, and their coherent structures deserve to be further investigated. Finally, it is planned to explore the coherent structures in the spectral space after the database of physical flow is Fourier Transformed in order to describe their features at a certain frequency, which might be interesting for the frequency-wise flow control in windfarm. These analyses are currently undertaken and will be reported properly in the future.

Acknowledgement

This work is part of the Shell-NWO/FOM-initiative Computational sciences for energy research of Shell and Chemical Sciences, Earth and Live Sciences, Physical Sciences, FOM and STW and an STW VIDI grant (No. 14868). This work was carried out on the national e-infrastructure
of SURFsara, a subsidiary of SURF cooperation, the collaborative ICT organization for Dutch education and research.

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