Trans-Coordinate Physics

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Abstract

Standard practice attempts to remove coordinate influence in physics through the use of invariant equations. Trans-coordinate physics proceeds differently by not introducing space-time coordinates in the first place. Differentials taken from a novel limiting process are defined for a particle’s wave function, allowing the particle’s dynamic principle to operate ‘locally’ without the use of coordinates. These differentials replace the covariant differentials of Riemannian geometry. With coordinates out of the way ‘regional conservation principles’ and the ‘Einstein field equation’ are no longer fundamentally defined; although they are constructible along with coordinate systems so they continue to be analytically useful. Gravity is solely described in terms of gravitons and quantized geodesics and curvatures. Keywords: covariance, invariance, geometry, metric spaces, state reduction; 03.65.a, 03.65.Ta, 04.20.Cv

Introduction

James Clerk Maxwell was the first to use space-time coordinate systems in the way they are used in contemporary physics. They play a role in his formulation of electromagnetic field theory that makes them virtually indispensable. Einstein embraced Maxwell’s methodology but devoted himself to eliminating the influence of coordinates because they have nothing to do with physics. However, the influence of coordinates is not eliminated by relativistic invariance as will be evident below where these space-time representations are removed entirely from physics.

Trans-coordinate physics proceeds on the assumption that space-time coordinates should not be introduced at any level. As a practical matter, and for many analytic reasons, coordinates are very useful and probably always will be.

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But if nature does not use numerical labeling for event identification and/or analytic convenience, and if we are interested in the most fundamental way of thinking about nature, then we should avoid space-time coordinates from the beginning.

Without coordinates the domain of relativity lies solely in the properties of the embedding metric space, and the domain of quantum mechanics resides in properties of local wave functions that are assigned to particles. These two domains overlap 'locally' where Lorentz invariant quantum mechanics is assumed. Photons in the space 'between' massive particles have a reduced function and definition.

As a result, the variables of a particle’s wave packet are wholly contained inside the packet and are coordinate independent. They move with a particle’s wave function in the embedding metric space, but they do not locate it in that space. No particle has a net velocity or kinetic energy when considered in isolation, for these quantities require a coordinate framework for their definition. This alone reveals the radical nature of removing coordinates entirely from physics, and the inadequacy of general relativistic invariance for that purpose.

Another consequence of this program is that energy and momentum are not propagated through the empty space between particles. Although particle energy, momentum, and angular momentum are conserved in local interactions, we say that nature does not provide for the exchange of energy and momentum between separated particles. We are the ones who arrange these transfers through our introduction of regional coordinates that we use to give ourselves the big picture. It facilitates analysis. The organizing power of coordinates and an opportune distribution of matter in space and time often allows us to find a system of coordinates that supports regional conservation; however, we can also find coordinates that do not support conservation. Therefore, regional conservation is coordinate dependent. It is not an invariant idea. It follows from a favorable construction on our part rather than from something intrinsic to the system.

General relativity is a product of energy-momentum conservation that relies on regional coordinates for its meaning. It therefore joins regional conservation principles as something coming from coordinate construction rather than something fundamental. It is found for instance that while the metric tensor $g_{\mu \nu}$

\footnote{A region surrounded by flat space will not conserve energy and momentum if \textbf{no} coordinates are chosen in the region, or if certain discontinuous coordinates are chosen in the region. Here again conservation depends on a coordinate choice or on the choice of a transformation group.}
can be defined at any event inside the wave packet of a massive particle, there is no trans-coordinate continuous function $g_{\mu\nu}$ associated with it. That is, a continuous metric tensor is not physically defined. Therefore derivatives of $g_{\mu\nu}$ at an event are not physically defined. General relativity suffers accordingly.

The separation we establish between quantum mechanics and general relativity avoids a clash of these mismatched disciplines [G. ‘t Hooft (2008); J. Maldacena (2005)], and weighs in favor of quantum mechanics.

And finally, a new definition of state is proposed in this paper. In the absence of regional coordinates there is no common time for two or more particles, so a state definition is proposed that spans the no-mans-land between particles. It is shown in another paper how to write the Hamiltonian for a system of separated particles of this kind [R. A. Mould (2008)]. The new definition of state and the Hamiltonian that applies to it imposes a consistent framework on a system of trans-coordinate particles.

If an atom emits a photon, then the system’s energy and momentum will be locally conserved. If that photon is not subsequently detected in another part of the universe it will essentially disappear from the system because a photon in isolated flight is energetically invisible. This does not violate conservation principles because those principles are satisfied at the emission site.

If the photon is detected somewhere else, then the energy and momentum at the detector site will also be conserved. The difficulty is that the energy emitted by the atom and the energy received by the detector might not be the same, so there is no general basis for claiming that energy conservation holds for the entire two-site system. That’s because nature, we say, does not care about conservation over more than one interaction. It cares only about conservation at individual interactions. However, regional coordinates often make it possible to compose energy differences of this kind in such a way as to validate regional conservation; and hence the great advantage of regional coordinates. They give us a useful analytic tool and a satisfying big picture as well as (sometimes) regional conservation laws. But these laws are not fundamental. They are only products of a fortunate coordinate construction.

This treatment is primarily concerned with electromagnetic interactions.

**Partition Lines**

In Minkowski space one must choose a single world line to define the future time cone of an event $a$. If there is a non-zero mass particle present in the space it should be possible to choose a unique world line at each location inside
the particle’s wave packet that is specific to the particle at that location. That world line corresponds to the direction of square modular flow at that event. The collection of these world lines over the particle’s wave packet can be thought of as the streamlines of its square modular flow in space and time. They will be called partition lines. We also define perpendiculars that are space-like lines drawn through each event perpendicular to the local partition line. We will first develop the properties of partition lines in a 1 + 1 space, and then in 2 + 1 and 3 + 1 spaces.

Figure 1 is a 1 + 1 Minkowski surface with light paths given by 45° dashed lines. Partition lines of an imagined particle wave packet are represented in the figure by the five slightly curved and more-or-less vertical lines. They tell us that the wave packet moves to the left with ever decreasing velocity and that it spreads out as it goes. This description is not trans-coordinate because it is specific to the Lorentz frame in the diagram; but these lines provide a scaffold on which it is possible to hang a trans-coordinate wave function.

Partition lines pass through every part of the particle’s wave packet and do not cross one another. They are not defined outside of a wave packet. Just as the space is initially given to us in the form of a metric background, any particle is initially given in the form of partition lines with the above characteristics. The interpretation of these lines is given in the next paragraph where values are assigned to them in a way that reflects the intended given conditions. These conditions are not ‘initial’ in the usual temporal sense, but are rather ‘given’ over the space-time region of interest.

Let the third partition line from the left (i.e., the middle line in Fig. 1) portion off 1/2 of the packet, so half of the particle lies to the left of an event such as a in the figure. That is, there is a 0.5 probability that the particle will be found on the perpendicular extending to the left of a. This statement is
assumed to have objective invariant meaning. Of course, the other half of the particle lies to the right of event $a$ on the perpendicular through $a$. The middle partition line is made up of all the events in the wave packet that satisfy this condition, so they together constitute a continuous line to which we assign the value of $1/2$. There is a $0.5$ probability that the particle will be found somewhere on the left side of this line when the included events are all those on both sides of the line.

In a similar way we suppose that the second partition line in Fig. 1 portions off, say, $1/4$ of the packet on the perpendicular to the left of an event $b$, and that the first line portions off $1/100$ of the particle or some other diminished amount. We further assume that the fifth line goes out to $99/100$ of the particle packet, so the entire particle is represented by streamlines that split the particle into objectively defined fractional parts.

When a wave function is finally assigned we will show that its total square modulus remains ‘constant in time’ between any two partition lines in $1 + 1$ space, and is similarly confined in higher dimensions.

**Neighborhoods**

Every event inside the wave packet has a unique time direction defined for it by the partition line passing through the event. This allows us to define unique *inertial* neighborhoods associated with each event.

Consider a flat space inside the wave packet of a massive particle, and assign a Minkowski metric that is intrinsic to that space. Beginning with an event $a$ in Fig. 2a, proceed up the particle’s partition line through $a$ by an amount $-\Delta$ which is the magnitude of the invariant interval from event $a$ to an event $b$. This

![Neighborhoods Diagram](image)

**Figure 2**: Establishing neighborhoods
interval $\mathbf{ab}$ is negative and identifies the chosen time axis inside the particle packet at event $\mathbf{a}$. Then find event $\mathbf{b'}$ by proceeding down the partition line the same invariant interval $-\Delta$. Construct a backward time cone with $\mathbf{b}$ at its vertex and a forward time cone with $\mathbf{b'}$ at its vertex and identify the intersection events $\mathbf{c}$ and $\mathbf{c'}$. Since these events are embedded in a flat space, the positive space-like interval $\mathbf{cc'}$ will pass through event $\mathbf{a}$ and will be bisected by it with

$$\mathbf{ca} = \mathbf{ac'} = \mathbf{cc'}/2 = \Delta > 0$$

For any $\Delta$, all of the events included in the intersection of the light cones of $\mathbf{b}$ and $\mathbf{b'}$ are defined to be a neighborhood of event $\mathbf{a}$. The events along the line $\mathbf{cc'}$ are defined to be a spatial neighborhood of $\mathbf{a}$. The limit as $\Delta$ goes to zero is identical with the limit of small neighborhoods around $\mathbf{a}$.

**Curved Space**

The above considerations for a ‘flat’ space also apply locally in any curved space, so we let the conditions in Fig. 2a be generally valid in the limit as $\Delta \to 0$. Figure 2b shows the resulting Minkowski diagram in the local inertial system with $\hat{x}$ and $\hat{t}$ as the space and time unit vectors in the directions $\mathbf{ac'}$ and $\mathbf{ab}$ respectively.

The unit of these vector directions is given by $\sqrt{\Delta}$ in meters, although we have not established coordinates in those units along those directions. Specifically, we have not established a unique numerical value attached to an event $\mathbf{a}$ or a distant zero-point for that value; so the development so far is consistent with the trans-coordinate (or coordinate-less) aims of this paper.

The unit vectors at event $\mathbf{a}$ will be referred to as the local grid at event $\mathbf{a}$, where the time direction is always along the partition line going through $\mathbf{a}$. These definitions have nothing to do with the curvature of the space in the wave packet at or beyond the immediate vicinity of $\mathbf{a}$. Every event inside a particle packet has a similar local grid. The local grids of other events in the neighborhood of event $\mathbf{a}$ will be continuous with the local grid at $\mathbf{a}$ in this 1+1 space, but not for higher dimensions as we will see.

**The Wave Function**

We specify the quantum mechanical wave function at each event $\mathbf{a}$ in a particle wave packet over the space-time region of interest
\( \varphi(a) \) \hspace{1cm} (1)

which is identified in the manner of Euclid’s geometry since there are no coordinate numbers involved. There are four auxiliary conditions on this function.

**First**: The function \( \varphi(a) \) is a complex number given at event \( a \) that is continuous with all of its neighbors. The units of \( \varphi \) are \( m^{-1/2} \) in this \( 1+1 \) space.

**Second**: Partial derivatives of \( \varphi(a) \) are defined in the limit of small neighborhoods around \( a \) (i.e., for small values of \( \Delta \)).

\[
\frac{\partial \varphi(a)}{\partial x} = \lim_{\Delta \to 0} \frac{\varphi(c') - \varphi(c)}{2\sqrt{\Delta}} \hspace{1cm} (2)
\]

\[
\frac{\partial \varphi(a)}{\partial t} = \lim_{\Delta \to 0} \frac{\varphi(b) - \varphi(b')}{2\sqrt{\Delta}}
\]

The second spatial derivative is then

\[
\frac{\partial^2 \varphi(a)}{\partial x^2} = \lim_{\Delta \to 0} \frac{\varphi(c')/\partial x - \varphi(c)/\partial x}{2\sqrt{\Delta}}
\]

Notice that we have defined derivatives in the directions \( \hat{x} \) and \( \hat{t} \) without using coordinates to ‘locate’ or numerically ‘identify’ events along either of those directions. Only \( \Delta \) intervals between events along the time line are taken from the invariant metric space.

**Third**: The value of \( \varphi \) at event \( a \) is related to its neighbors through the *dynamic principle*. This principle determines how \( \varphi(a) \) evolves relative to its own time against the metric background, and how it relates spatially to its immediate neighbors.

**Fourth**: The objective fraction of the particle found between the partition line through event \( c \) in Fig. 2a and a partition line through event \( c' \) is equal to \( f_{cc'} \). In the limit as \( cc' = 2\Delta \) goes to zero the fraction of the particle between differentially close partition lines goes to \( df \). Normalization of \( \varphi(a) \) is strictly ‘local’ and requires

\[
\varphi^*(a)\varphi(a) = \lim_{\Delta \to 0} \frac{f_{cc'}}{2\Delta} \hspace{1cm} (3)
\]

It follows that

\[
\varphi^*(a)\varphi(a) = \varphi^*(b)\varphi(b) = \varphi^*(b')\varphi(b')
\]

because the fractional difference between any two the partition lines is the same over any perpendicular. Therefore, the square modular flow will be *constant in time* between any two partition lines as previously claimed.

These four auxiliary conditions must be satisfied when taken together with the initially given partition lines, but there is no guarantee that there exists a
wave function that qualifies. Finding a solution therefore consists of varying the partition lines (i.e., the given conditions) until a wave function exists that satisfies these conditions.

The choice of a world line based on partition lines is not a coordinate choice, nor is the limiting procedure that follows. So these definitions are not just coordinate invariant, they are fully coordinate free. They allow us to find physically creditable derivatives of any continuous function in a way that is independent of the curvature of the surrounding space, and to found a physics on that basis.

One Particle

Partition lines do not extend beyond the particle, so in the absence of ‘external’ coordinates that do extend beyond the particle (in an otherwise empty space) there is no basis for claiming that the particle has a net velocity, kinetic energy, or net momentum. This will be true of both zero and non-zero mass particles. It is a consequence of a trans-coordinate physics that particles take on these dynamic properties only in interaction with other particles.

A massive particle has an ‘internal’ energy defined at each event in its wave packet, but since that may differ from one event to another there is no single internal energy representing the particle as a whole. Similarly, each part of the particle’s wave packet follows its own world line, so the there is no single world line for the particle as a whole as shown in Fig. 1. It is our claim that nature attends to the particle as a whole by dealing separately with each part. One exception is that the particle as a whole does produce a gravitational disturbance in the background invariant metric that has its origin in the regional distribution of the particle’s internal mass/energy.

Two Particles

Figure 3 shows the partition lines of two separated massive particles where each has its own definition of a grid that is different from the other particle. It is a consequence of the trans-coordinate picture that these particles in isolation will seem to have nothing to do with one another. However, the positional relationship of one to the other is objectively defined in the metric space in the background of both. Every event in the wave packet of each particle has a definite location in the metric space, and that fixes the positional relationship of each part of each particle with other parts of itself and with other particles. In addition, each massive particle produces a gravitational disturbance that has
an invariant influence on the other. That influence is a function of the relative velocity between the two, even though kinetic energy is not defined for either one. Kinetic energy is a coordinate-based idea as has been said, whereas metrical positions and gravitational disturbances in the metric are invariant. We assume that the latter are based solely on graviton activity.

**A Radiation Photon**

The pack of four lines that rise along the light line in Fig. 3 are intended to be the partition lines of a radiation photon that has a group velocity equal to the velocity of light. Photons can have partition lines as do massive particles. They separate the photon into its fractional parts, which is a separation by phase differences. The photon in Fig. 3 is confined to the packet that is distributed over the perpendicular (dashed) light path $l$.

Normally in physics we do not hesitate to use coordinates in empty space, so a photon by itself will be given a period and wavelength relative to that coordinate frame, and hence an energy and momentum. But if coordinates in empty space have no legitimate place in physics, than like any other particle a photon by itself will lack translational variables (e.g., energy and momentum); and since it has no internal energy (i.e., rest mass/energy), the gravitational perturbation of its light line will be zero. There is no photon mass/energy to perturb it. It should also be clear from the diagram in Fig. 3 that the photon bundle has *no definable* wavelength or frequency at event $k$.

Vacuum fluctuations exist in the ‘empty’ space between massive particles and their polarizing effects are physically significant. But if vacuum fluctuation particles are not themselves polarized they will not interact with a passing
Photon (resulting in a scattering of the photon). So the photon cannot use these particle grids to define its period and wavelength. Fluctuation particles do not contribute in any other way to the discussion, so their presence is ignored.

**Information Transfer**

It is the photon’s phases that affect a transfer of energy and momentum from one particle to another. This is shown in Fig. 4 where two particles are narrowly defined to be moving over world lines $w_1$ and $w_2$. The two dashed lines represent the partition lines of a passing photon with ‘relative’ phase differences given by $\delta \pi$. If the photon wave is a superposition of two different frequencies 1 and 2, then $\delta \pi = \delta \pi_1 + \delta \pi_2$.

![Figure 4: Two particles and a photon](image)

A photon interacting with the first particle at event $a$ will have a local energy and momentum given by $e_\gamma(a)$, $p_\gamma(a)$, and as it interacts with the second particle at event $b$ it will have a local energy and momentum given by $e_\gamma(b)$, $p_\gamma(b)$. These quantities are related through the phase relationships that are transmitted between particles, and are articulated in the local grid of the interacting particle.

\[
\begin{align*}
\text{a photon at event } a : & \quad e_\gamma(a) = \hbar \sum_i \omega_i(a) \quad p_\gamma(a) = \hbar \sum_i k_i(a) \\
\text{a photon at event } b : & \quad e_\gamma(b) = \hbar \sum_i \omega_i(b) \quad p_\gamma(b) = \hbar \sum_i k_i(b)
\end{align*}
\]

where $\omega_i(a) = \partial_t \pi_i(a)$ and $k_i(a) = \partial_x \pi_i(a)$. These derivatives refer to the local grid of each event in each particle, and are defined like those in Eq. 2.

**Electromagnetic Variables**

The parallel lines passing by event $k$ in Fig. 3 are lines of constant ‘relative’ phase of the photon. Differential phase changes $\delta \pi$ over a light line like $l$ are
preserved across the length of the photon wave packet. However, since the photon in flight between two particles does not have its own local grid, components cannot be defined for the electromagnetic field any more than can for energy and momentum.

In empty space the electromagnetic potential of a radiation photon is normally given by a fourvector $A^\mu(a)$, where the d’Alembertian operating on $A^\mu(a)$ is equal to zero. However, trans-coordinate physics cannot use the d’Alembertian in empty space although the photon’s behavior there is lawful – it follows a dynamic principle of some kind. Where a grid exists we can give analytic expression to the dynamic principle; but where there is no grid we must settle for another kind of description. All we can do in this case is notice the physical manifestations of the dynamic principle, and there are just four in $3 + 1$ space. First, different relative phases appear on different parallel layers along a light line as in Figs. 3 and 4. There is a definite phase relationship between any two of these layers. Second, the probability that a photon goes into a particular solid angle from an emission site $a$ depends on the distribution given by an atomic decay at $a$, or by the interaction of $A_\mu(a)$ with the current $j(a)$ at that site. The only mid-flight indication of the strength of a signal in a given solid angle is the probability of a photon emission in that direction. Third, we say that the magnitude of $A_\mu$ arriving at a material target is determined by that probability – rather than probability determined by magnitude. In the case of a single photon (or for any definite number of photons) the components of $A_\mu$ at a material destination are indeterminate, and the magnitude of the transmission diminishes with square distance from the source by virtue of the constancy of photon number in a solid angle. The fourth property provides for Huygens’ wavelets. So far we have considered a photon as moving undeflected in an outward direction from a source along a light cone. We now say that an event such as $k$ in Fig. 3 acts as a point source of radiation in all directions. The wavelet from $k$ has the same (relative) phase as event $k$, and it reradiates the “probability intensity” at $k$ uniformly in all directions with a velocity $c$. Two wavelets that arrive at a third event $m$ have a definite phase difference that produces interference there.

Notice that a Huygens’ electromagnetic wavelet is a ‘scalar’ like the primary wave that gives rise to it. The vector nature of an EM wave does not appear until it interacts with matter, and only then when an indefinite number of photons are phased in such a way as to make that happen.
Photon Scattering

If a photon scatters at an event a inside the wave packet of a particle, the grid for that purpose will be the particle’s grid at a. There will be no quantum jump or wave collapse in a scattering of this kind. Instead, some fraction of the particle p and photon γ will evolve continuously into a scattered wave that consists of a correlated particle p’ and a photon γ’. Energy and momentum will be defined for each of the four particles p, p’, γ, and γ’ that are mapped together on that common grid of p at event a, and the dynamic principles of these particles (plus their interaction) will insure that total conservation applies to all four. Each component of the scattered wave of p’ will also have a grid that is well defined at event a, and is a Lorentz transformation away from the grid of p. Energy and momentum will be conserved on the grid of each component of p’. The velocity of any component of p’ relative to p is not explicitly given in the trans-coordinate case; however, it is implicit in the Lorentz transformation that is required to go from the locally evolving grid of p to that of p’.

Virtual Photons

So far we have talked about radiation photons that travel at the velocity of light. Virtual photons (in a Coulomb field) do not bundle themselves into wave packets, so they do not have a ‘group’ velocity that requires the identification of a world line over which the group travels. It makes no sense to say that they travel over light lines. It may therefore be possible to give the virtual photon a local grid in the same way that we created a grid for particles with non-zero mass. Its vector nature would then be more evident. However, we choose not to do that. It is unnecessary and would put the virtual photon grid in competition with the particle grid during an interaction between the two. That would necessitate a choice between one or the other in any case; so all photons will be considered gridless in this treatment – just like radiation photons. They all lack internal energies. They also lack translational variables such as energy and momentum when in transit between particles; and they acquire these values only when they overlap the charged particles with which they interact. We say in effect that there is no fundamental difference between ‘near’ field photons and ‘far’ field photons in an electromagnetic disturbance.
Gravity

If a photon in transit (radiation or virtual) has no frequency or translational energy $hν$, it will not have a weight in the presence of a gravitating body or create a curvature in the surrounding metric space. However, massive objects having rest energy do create curvatures in their vicinity in which light line geodesics are well defined. We claim that radiation photons follow these geodesics without themselves contributing to the curvature of space. Although photons in transit are massless and hence weightless, they nonetheless behave as though they are attracted to gravitational masses.

This does not mean that current photon trajectories are in error, or that particle masses have to be adjusted. The mass of an electron found from the oil drop experiment is currently assumed to include the mass of the accompanying electromagnetic field. From a trans-coordinate point of view the electric field surrounding a charged particle is not defined, so this experiment reveals the ‘bare’ mass of the electron. The mass of the Sun obtained from the period of a planet is normally assumed to include the mass of the radiation field surrounding the sun. From a trans-coordinate point of view the radiation field is not defined, so this calculation reveals the ‘bare’ mass of the sun – that is, the total number of each kind of solar particle times its mass. These changes will not result in observational anomalies in particle theory or astronomy, for we have no way to separately weigh the electromagnetic field of a charged particle, or to count the number of particles in the Sun.

Binding Energy

Even in coordinate language we are able to give up the idea of electromagnetic field energy, so the binding energy of particles in a nucleus can be considered a property of the particles themselves. Imagine two positive particles of rest mass $m_0$ that approach one another in the center-of-mass system with kinetic energy $T$. The momentum of one of these particles decreases as a result of virtual photon exchange; however, its energy will not change. A virtual photon leaving one particle will carry away a certain amount of energy, but that energy is restored in equal amount by the virtual photon that is received from the other particle. This means that the net energy of the advancing particle will be unchanged during the trip. When the particle reaches the point at which it has lost all of its kinetic energy and has combined with the other particle due to nuclear forces, we would say that the initial kinetic energy of one of them has
become its binding energy $BE$, where

$$E = BE + m_0c^2 = T + m_0c^2$$

As the particle moves inward its energy square $E^2 = P^2 + m_0^2c^4$ remains constant while $P^2$ goes to zero. Therefore $E^2$ becomes identified with an increased mass $M^2c^4$ giving $E = Mc^2 = BE + m_0$. Then

$$\text{Binding energy} = Mc^2 - m_0c^2$$

In relativity theory a particle’s (relativistic) mass is a function of kinetic energy. We can also say it is a function of an interaction with other particles, thereby avoiding any notion of ‘field’ energy.

These ideas are peculiar to the center-of-mass coordinate system but are not correct from a trans-coordinate point of view. Fundamentally there is no energy associated with the particle as a whole. There is only the time derivative of $\varphi$ at each separate event inside the particle’s wave packet. There is also no kinetic energy of the particle or binding energy of a captured particle. The ‘correct’ trans-coordinate account of a coulomb interaction is given below.

**Virtual Interaction**

The virtual (Coulomb) interaction cannot be thought of as a single virtual photon interacting with a single charged particle because that is not energetically possible. However, the interaction is *continuous* like Compton scattering; so in spite of the fact that the theory is based on photons the interaction does not manifest itself as discrete quantum jumps. A particle in a Coulomb interaction is therefore continuously receiving and transmitting equal amounts of energy, which means that it undergoes a change of momentum with no change of energy. The resulting behavior of the charged particle is given by a continuum of particle grids along its partition line that are related by infinitesimal Lorentz transformations. Energy and momentum are conserved on any one of these grids. Since each particle is well localized in the background metric space, predictable continuous transformations of the world line of each event in the packet are *all that is necessary* to determine the packet’s complete behavior. Nature is not concerned with the coordinate-based energies of the previous section – and does not need to be.
Regional Coordinates and Conservation

Trans-coordinate physics does not provide for energy and momentum conservation in the region between particles. We cannot assign frequency or wavelength to a radiation photon in an otherwise empty space as we have seen, so we cannot say that it carries energy $h\nu$ or momentum $h/\lambda$ from one part of space to another. Also, a massive particle has no velocity or acceleration when it is considered in isolation. It moves into its future time cone over the invariant metric background following its dynamic principle, but that path does not break down into spatial and temporal directions relative to which the wave packet can be said to be moving with a kinetic energy or velocity $v$. So it cannot be said to carry a net momentum $mv$.

Regional conservation of these quantities is therefore related to the possibility of system-wide coordinates that we construct. Having done that we can define a metric tensor throughout the region. That is, from the background invariant metric it is generally possible to find the continuous metric tensor $g_{\mu\nu}$ that goes with the chosen coordinates. If that tensor is time independent then energy will be conserved in the region covered by those coordinates. If it is independent of a spatial coordinate such as $x$, then momentum in the $x$-direction will be conserved in the region covered by the coordinates. If the metric is symmetric about some axis (in $3 + 1$ space) then angular momentum will be conserved about that axis [R. A. Mould (2002)]. It is therefore useful for us to construct system-wide coordinates in order to take advantage of these regional conservation principles. It is important to remember however that we do this, not nature. Nature has no need to analyze as we do over extended regions. For the most part it only performs on a local platform.

If there is a difference in energy between $e_\gamma(a)$ and $e_\gamma(b)$ in Eq. 4, it is possible that the photon in Fig. 4 is Doppler shifted because of a relative velocity between the two particles, or that particle #2 is at a different gravitational potential than particle #1. When a coordinate system is chosen the velocity of one particle is decided relative to the other particle, and only then will the extent of the Doppler influence be determined. Only then will it be clear how the organizing power of a coordinate system makes use of gravity to explain the non-Doppler difference between $e_\gamma(a)$ and $e_\gamma(b)$. 
Trans-Coordinate Tensors

Every event in a massive particle wave packet has a grid associated with it. In 3 + 1 space the spatial part is a three dimensional grid. When this is combined with the metric background the metric tensor $g_{\mu\nu}$ is determined at each event. One can therefore raise and lower indices of vectors inside the wave packet in the trans-coordinate case.

However, we do not assign derivatives to $g_{\mu\nu}$ because it is not a uniquely continuous function. For a given $g_{\mu\nu}(a)$ at event $a$ there are an infinite number of ways that a continuous $g_{\mu\nu}$ field ‘might’ be applied in the region around $a$, corresponding to the infinite number coordinate systems that ‘might’ be employed in that region. But if we do not attach physical significance to coordinates, then physical significance cannot be attached to a continuous metric tensor. Derivatives of that tensor are therefore not defined in trans-coordinate physics. This applies to the derivatives in Eq. 2 as well as to the covariant derivatives of Riemannian geometry. Therefore, Christoffel symbols are not defined in trans-coordinate physics.

It follows that the Riemann and Ricci tensors and the field equation of general relativity are also not fundamentally defined. Like energy and momentum conservation from which it is derived, the gravitational field equation is a regional creation of ours that is analytically useful and that gives us a satisfying big picture – but that is all. Of course the ‘curvature’ is objectively defined everywhere because it follows directly from the invariant metric background in which everything is embedded.

We can be guided by our experience with general relativity when choosing the most useful coordinate system in a given region of interest. A metric tensor can then be defined; and from the symmetry of its components, energy, momentum, and angular momentum conservation can be established over the region. However, there is no assurance that one can always find an agreeable system, for general relativity does not guarantee that the chosen coordinates will conserve energy, momentum, or angular momentum without introducing special pseudo-tensors that are devised for that purpose [L. Landou and E. Lifshitz (1971)].

Gravitons

If general relativity is not fundamental then gravitons must be the exclusive cause of gravitational effects. The geodesics that result from graviton interactions between massive particles are not the smooth curves of general relativity,
but are quantized by discrete graviton interactions. The wider effect of gravitons is to bend the background metric space between geodesics. Their influence will spread through the invariant metric space; and as a result, the curvature produced by gravitons will follow the average curvature of general relativity except that it will have the jagged edge of quantization. General Relativity is therefore a science that only approximates the underlying reality. It is a science we initiate when we introduce the coordinates that permit the definition of metric tensor derivatives and allow the formulation of Einstein’s field equation.

**Internal Coordinates**

In addition to regional coordinates that cover the space between particles, we want to give ourselves an internal picture of the particle. We want the wave function $\varphi(a)$ in Eq. 1 in a form that permits analysis. To do this starting at event $a$, integrate the minus square root of the metric along the partition line going through $a$ and assign a time coordinate $t_a$ with an origin at $a$. Then integrate the square root of the metric over the perpendicular going through event $a$ and assign a space coordinate $x_a$ with an origin at $a$. The coordinates $x$ and $t$ may be extended over the entire object yielding a wave function that can be written in the conventional way $\varphi(x,t)$. These internal coordinates will have the same status as external coordinates. They are only created by us for the purpose of analysis.

With internal coordinates we can integrate across one of the perpendiculars to find the width of the wave packet. It should also be possible to integrate the square modulus over a perpendicular to find the total normalization. That total will be equal to 1.0 if $df$ is equal to the fraction of the particle sandwiched between two differentially close partition lines as claimed. We can also use internal coordinates to give expression to the internal variables of a particle, such as its total internal energy and net momentum.

**Three and Four Dimensions**

Imagine that a particle’s wave packet occupies the two-dimensional area shown on the space-like surface in Fig. 5. The surface is divided into a patchwork of squares, each of which is made to contain a given fraction of the particle, like 1/100th of the particle. Each of these squares has four distinguishable crossing points or corners. A similar two-dimensional scaffold is constructed on all of the space-like surfaces through which the particle passes in time, thereby creating
a continuous 2 + 1 scaffold. Each of the enclosed areas generated in this way is required to contain 1/100 of the particle, and its corners will constitute the partition lines of the particle. As in the 1 + 1 case, these lines may be thought of as streamlines of the square modular flow of the particle through time. In the limit as this fraction goes to zero, partition lines pass through each event on the space-like surface in the figure and they do not cross one another.

It is possible to find the direction of the partition line through an event \( a \) without having to erect a system-wide scaffolding like that of Fig. 5. Any small neighborhood of \( a \) has a probability that the particle will be found within it; and that probability will be ‘minimal’ when the partition line going through \( a \) coincides with the preferred direction of time for that neighborhood.

Space-time directions are chosen for a given partition line in a way that is similar to the procedure in Fig. 2. Starting with an event \( a \) in Fig. 6a, move up its partition line a metrical distance \(-\Delta \) to event \( b \). Then find \( b' \) by proceeding down the partition line the same invariant interval \(-\Delta \). Construct a backward time cone with \( b \) at its vertex and a forward time cone with \( b' \) at its vertex and identify the closed two-dimensional loop intersection shown in Fig. 6a in the limit as \( \Delta \) goes to zero. In the local inertial system, two perpendicular unit vectors \( \hat{x} \) and \( \hat{y} \) are chosen along the radius of the circle of radius \( \Delta \) that spans the spatial part of the local grid at event \( a \). For any \( \Delta \), choose a space-like line beginning at \( a \) that is aligned with \( \hat{x} \) and extends to the circumference of the circle in Fig. 6a. It intercepts the circle at the event we call \( c' \) in Eq. 2. The space-like line that begins with \(-\hat{x} \) intercepts the circle at the event we call \( c \) in Eq. 2. These space-like lines do not have to be ‘straight’, so long as they are initially aligned with the unit vector and intercept the circle in only one place.

The spatial grids of nearby events such as \( a \) and \( a' \) in Fig. 6b do not have to line up in any particular way. Even if they are in each other’s spatial neighborhood for some value of \( \Delta \), \( \hat{x} \) and \( \hat{x}' \) will generally point in different directions.

In 3 + 1 space the intersection of a backward and forward time cone will
Figure 6: Establishing space-like unit vectors

produce a spherical surface like the one pictured in Fig. 6c. In this case choose four mutually perpendicular unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$, and $\hat{t}$ to form the local grid at event $a$. As before, the orientation of the spatial part of these grids is of no importance. They may be arbitrarily directed because their only purpose is to locally define all three spatial derivatives of the function $\varphi$. That function is continuous throughout the wave packet in any direction; therefore, it does not matter which grid orientation is chosen at any event for the purpose of specifying the function and its derivatives there. The Dirac solution has four components $\varphi_\mu$ where each satisfies all of the above conditions in the $3+1$ directions.

Since every event on the surface of the sphere in Fig. 6c locates a partition line, the event $a$ is enclosed by a sphere with a differential volume $d\Omega$ that contains a differential fraction $df$ of the entire particle, where

$$\varphi^*(a)\varphi(a)d\Omega = df$$

which normalizes the $3+1$ wave function.

**Applying the Dynamic Principle ($3+1$)**

The third condition on a wave function $\varphi(a)$ in Eq. 1 requires that the dynamic principle applies throughout the space. This can be done in the $3+1$ space of an event $a$ by using the grid defined in Fig. 6c. Since we can do this at any event and for any orientation of the grid, we state the more general form of the third condition:

*The wave function $\varphi(a)$ of a particle at any event $a$ is subject to a dynamic principle that is applied locally to any four mutually perpendicular space-time directions centered at $a$, where time is directed*
along the partition line through $a$. This principle determines how
$\varphi(a)$ evolves relative to its own time against the metric background,
and how it relates spatially to its immediate neighbors.

The continuity condition applies to the function $\varphi$ along any finite segment of
line emanating from any event.

Atoms and Solids

Consider how all this might apply to a hydrogen atom. Each massive particle
carries a local grid that is independently defined at each event in its wave packet.
This insures separate normalization at each event for each particle. The proton
and electron grids may overlap but they need not be aligned because the particles
do not directly interact. They are connected through the Coulomb field by
virtual photons that carry no grid of their own. There are two interactions, one
involving a virtual photon and the event grids of the proton, and one involving
a virtual photon and the event grids of the electron. These are described in the
section “Virtual Interaction”.

In the non-relativistic case both particles can be covered by a single common inertial frame in which the total energy and momentum is conserved. It does
no harm and it facilitates analysis to imagine that each grid in the system is
aligned with this common coordinate frame. The time $t$ assigned to each proton
grid and the time $t'$ assigned to each electron grid are then set equal to each
other and to the time of the common inertial frame. The retarded interaction
$j_\mu A^\mu$ at each end of the interaction will then give the Coulomb intensity of
$(e^2/4\pi r)\delta(t - t')$ where $r$ is the distance between the particles in the common
frame [R. P. Feynman et al. (1995)]. Relativistic corrections to this occur when
the spatial components of the current fourvectors are taken into account.

The above inertial system is one that we impose on the atom. By itself, the
system operates on the basis of individual event grids alone. A photon passing
over the atom will interact with each separate event in the proton wave function
throughout its volume, and with each separate event of the electron throughout
its volume. Energy and momentum conservation is required at each site, but
the system will not support conservation unless the interaction Hamiltonian
[R. A. Mould (2008)] includes the entire system in a ‘single’ interaction. It is the
interaction Hamiltonian that makes the difference between particles in a single
interaction that conserves energy and momentum, and particles in separate
interactions that may or may not conserve these quantities. In the atomic case
the dynamic principle for the entire atom provides the unity that can give rise to a quantum jump that carries the product $pe\gamma$ of the proton, the electron, and the photon, into a new product $p' e' \gamma'$, conserving energy and momentum in the process.

In the case of macroscopic crystals, metals, and other stationary solid forms in a flat space, each event in each particle wave packet has its own space-time grid and is separately normalized. However, they are all interactively aligned to such an extent that we can usually impose a single common coordinate system. We require the coordinates of this system to co-move with the average density of matter in the solid. If that system has the right symmetry properties it will insure macroscopic energy, momentum, and angular momentum conservation.

**Containers**

Let the central region of the hollow spherical container in Fig. 7 be a general relativistic space of unknown curvature. The center of the sphere is initially empty (suppressing vacuum fluctuations). A massive object leaves event a and at some later time arrives at event b. At each event along the way it is propelled by its dynamic principle into its forward time cone; and since the resulting path of the packet cannot be broken down into spatial and temporal parts, its velocity, energy, momentum, and distance traveled on that path are not determined. The particle will have ‘internal’ energy and momentum that are derived from internal coordinates, but these will not be its ‘translational’ energy and momentum in the usual sense going from a to b. A radiation photon will not even have these internal properties over its path; for it will only acquire the energy and momentum in Eq. 4 when it encounters a particle in the container wall.

We can certainly construct a common coordinate system over this system, extending the co-moving coordinates of the solid into the center of the sphere.

![Figure 7: Particle in container](image)
We will then know how far the object goes and its velocity along the way. If the metric of that system is time independent, then total energy will be conserved throughout the trip from event $a$ to event $b$. Although we can usually cover the system with extended coordinates and a metric, there is no guarantee that resulting system will conserve total energy and momentum without introducing the pseudo-potentials of L. Landou and E. Lifshitz (1971).

A Gaseous System

The introduction of many gas particles in the space of Fig. 7 does not change anything of substance. Molecular collisions occurring on the inside surface of the container and between molecules are distinct physical events. But we still do not have a natural basis for ascribing a numerical distance between any of these collisions or the molecular velocities between them.

Molecular collisions are here assumed to be electromagnetic in nature. Parts of the colliding molecules may or may not overlap, but they each (i.e., the internal parts of each) maintain their separate grids for the purpose of normalization. These grids do not compete with one another during a collision because the interaction between them is conducted through virtual photons, and these are declared to be gridless.

States

In coordinate physics we normally define a physical ‘state’ across a horizontal plane at some given time. This definition identifies an origin of coordinates relative to which the system’s particles are located at that time. That scheme will not work in the trans-coordinate case because the “same time” for separated particles is undefined. Indeed, the time of a single particle at a single location is undefined. The meaning of state must therefore be revised.

The state of a system of three particles is now given by

$$\Psi(a, b, c) = \phi_1(a)\phi_2(b)\phi_3(c)$$

where $a$, $b$, and $c$ are events anywhere within each of the given wave functions, subject only to the constraint that each event has a space-like relationship to the others. Each of these three functions is defined relative to its own local grid and is related to its time-like successors through its dynamic principle. These events are connected by the space-like line in Fig. 8, thereby defining the state
Ψ of the particles that are specified along their separate world lines \( w_1, w_2, \) and \( w_3. \)

A successor state can be written

\[
\Psi'(a', b', c') = \phi_1(a')\phi_2(b')\phi_3(c')
\]  

(5)

where events \( a', b', \) and \( c' \) in the new state must also have space-like relationships to each other; and in addition, they are required to be in the forward time cones of events \( a, b, \) and \( c \) respectively. These events lie along a space-like line in Fig. 8 giving the state function \( \Psi' \). Equation 5 does not say that each event has advanced by the same amount of time. It says only that each particle has advanced continuously along its own world line (i.e., along its own partition line) under its own dynamic principle, and has reached the designated ‘primed’ events.

We might also let \( b'' \) replace event \( b' \), where \( b'' \) has space-like relationships to \( a' \) and \( c' \) and is in the forward time cone of event \( b \). The resulting state \( \Psi''(a', b'', c') \) is not the same as \( \Psi'(a', b', c') \), but it is just as much a successor of the initial state \( \Psi(a,b,c) \). Also, \( \Psi'' \) is a successor of \( \Psi' \) because \( b'' \) is a successor of \( b' \). This definition of state is far more general than the coordinate based (planar) definition, giving us an important degree of flexibility as will be demonstrated below and in another paper [R. A. Mould (2008)]. The Hamiltonian for this kind of state can be defined in such a way as to establish the conservation of probability current flow, as is also shown in this reference.

**An Application**

Consider the case of an atom emitting a photon that is captured by a distant detector. The initial spontaneous decay of the atom can be written in the form

\[
\varphi = (a_1 + a_0\gamma)D
\]  

(6)
where $a_1$ is the initial state of the atom, $a_0$ is its ground state, $\gamma$ is the emitted photon, and $D$ is a distant detector that is not involved in the decay. At this point we do not specify specific events or use the new definition of state. In response to the dynamic principle, the probability current flows from the first component in Eq. 6 to the second component inside the bracket, so the first component decreases in time and the second component increases in such a way as to conserve square modulus as shown in Ref. 3. At some moment of time a stochastic choice occurs and the state undergoes a quantum jump from $\varphi$ to $\varphi'$ conserving energy and momentum and giving

$$\varphi' = a_0\gamma D$$  

(7)

that describes the state of the system during the time the photon is in flight from the atom to the detector. When the photon interacts with the detector the equation of state becomes

$$\varphi'' = a_0(\gamma D + D'')$$  

(8)

where $D''$ is the detector after capture. The atom $a_0$ is not a participant in this interaction. Again, probability current flows from $\gamma D$ to $D''$ and this, we assume, results in another stochastic hit conserving energy and momentum and yielding

$$\varphi''' = a_0D''$$

When the new definition of state is applied to this case Eq. 6 is written

$$\varphi(a, c) = [a_1(a) + a_0(a)\gamma(a)]D(c)$$  

(9)

where the atom and the photon overlap at event $a$. The photon uses the grid of the atom at event $a$ to evaluate its frequency and wavelength, whereas the detector uses its own grid. Nonetheless, the dynamic principle in the form of the Hamiltonian defined in Ref. 3 applies to this interaction equation that is local to event $a$.

Equation 7 for the proton in flight is then

$$\varphi'(a, k, c) = a_0(a)\gamma(k)D(c)$$  

(10)

where the energy of the atom and the detector are given by their time derivatives at events $a$ and $c$, but there is no energy associated with the independent photon in this equation. The function $\gamma(k)$ is of the form $\exp[i\theta(k)]$ where $k$ is the event appearing in Fig. 3, so frequency and wavelength are not given. The photon’s
Hamiltonian applied to this equation equals zero. Equation 9 applies so long as the photon is located on a definite partition line of the atom; but the moment the photon event appears apart from the atom, Eq. 10 will apply.

Equation 8 using the above state definition is

\[ \varphi''(a, c) = a_0(a)[\gamma(c)D(c) + D''(c)] \]

where the photon overlaps the detector at event \( c \). In this case the photon uses the grid of the detector at event \( c \) to evaluate its frequency and wavelength, and the energy of the atom is given by its time derivative on the grid of event \( a \).

Here again the dynamic principle applies to this interaction equation that is local to event \( c \).

Actually the atom should be written as a product of the proton \( p \) and the electron \( e \) giving \( a = pe \). In the parts of the atom where the proton and the electron do not overlap, Eq. 9 could be written as either

\[ \varphi''(a, b, c) = [p_1(a)e_1(b) + p_0(a)e_0(b)\gamma(a)]D(c) \]

or

\[ \varphi''(a, b, c) = [p_1(a)e_1(b) + p_0(a)e_0(b)\gamma(b)]D(c) \]

Both equations are correct. They both describe the interaction of the photon on different grids associated with different parts of the atom, where the dynamic principle applies in each case.

Equations of this kind are used more extensively in Ref. 3, and the rules that govern them are given in the Appendix of that reference.

### Unifying Features

The most important non-local unifying feature of a trans-coordinate system is the *invariant metric space* in which everything is embedded. Another important unifying feature is the *dynamic principle* applied to each particle by itself and to any system of particles as a whole.

*Non-local correlations* are another unifying features of the functions generated by the dynamic principle. These qualify the location of one particle relative to the location of another particle; so the equation of state of two particles \( p_1 \) and \( p_2 \) is written \( \Phi = p_1p_2(a,b) \), rather than \( \Phi = p_1(a)p_2(b) \). These particles have their separate grids as always, to which the dynamic principle separately applies as always. The difference is that the range of \( b \) depends on the value of \( a \) and visa versa, and their joint values determine \( \Phi \). This function is local to both events \( a \) and \( b \), so it is a bi-local function.
The fourth unifying feature is the collapse of the wave function over finite regions of space.

Modified Hellwig-Kraus Collapse

A local quantum mechanical measurement can have regional consequences through the collapse of a wave function. The question is: How can that superluminal influence be invariantly transmitted over a relativistic metric space?

Hellwig and Kraus answered this question by saying that the collapse takes place across the surface of the backward time cone of the triggering event [K. E. Hellwig and K. Kraus (1970)]. The Hellwig-Kraus collapse has been criticized because it appears to result in causal loops [Y. Aharonov and D. Z. Albert (1981)], but the situation changes dramatically with the new trans-coordinate definition of state. We keep the idea that the influence of a collapse is communicated along the backward time cone; however, the state of the system that survives a collapse (i.e., the finally realized eigenstate) is not defined along a “simultaneous” surface. The increased flexibility of the new state definition allows the remaining (uncollapsed) state to retain its original relationship with the event that initiates the collapse. When this program is consistently carried out causal loops are eliminated, even in a system of two correlated particles. I will not elaborate on this idea in this paper but it is demonstrated in detail in [R. A. Mould (2008)].

Another Approach

Invariance under coordinate transformation is not discussed at any length in this paper because coordinates are not introduced in the first place; but it should be noted that the idea of coordinate invariance is limited. General relativity is not truly independent of coordinates because it does not include all possible coordinates in its transformation group. It does not include ‘discontinuous’ coordinate systems, many of which are capable of uniquely identifying all the events in a space-time continuum – as is claimed to be the purpose of a space-time coordinate system. For example, imagine Minkowski coordinates in which the number 1.0 is added to all irrational numbers but not to rational numbers. This system is perfectly capable of systematically and uniquely identifying all of the events in a space-time continuum, but it is thoroughly discontinuous in a way that prevents it from being useful to general relativity. It only takes one example of unfit coordinates to disqualify invariance as a fundamental requirement in
physics, and there are many discontinuous coordinates like this one. Of course one can always reject coordinates that don’t work in the desired way on the basis of the fact that they don’t work in the desired way. But that avoids the issue. The point is that the influence of unnatural identification labels cannot be eliminated from physics through an invariance principle that affects only a sub-set of unnatural identification labels. Another approach is indicated.

References

Aharonov, Y., Albert, D. Z. (1981) *Phys. Rev. D* **24**, 359

Feynman, R. P., Morinigo, F. B., Wagner, W. G. (1995) *Feynman Lectures on Gravitation*, B. Hatfield, ed., Addison-Westley, New York, 33

Hellwig, K. E., Kraus, K. (1970) *Phys. Rev. D* **1**, 566

Landou, L. and Lifshitz, E. *The Classical Theory of Fields*, Pergamon Press, New York, (1971) p. 316

Maldacena, J. (2005) "The Illusion of Gravity", *Sci. Am.* Nov, 56

Mould, R. A. (2002) *Basic Relativity*, Springer, New York 2002) Eq. 8.66

Mould, R. A. (2008) “Trans-Coordinate States”, [arXiv:0812.1937](https://arxiv.org/abs/0812.1937)

't Hooft, G. (2008) “A Grand View of Physics”, *Int’l J. Mod. Phys.* **A23** 3755, sect 3:[arXiv:0707.4572](https://arxiv.org/abs/0707.4572)