Production of negative parity baryons
in the holographic Sakai-Sugimoto model

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Abstract

We extend our investigation of resonance production in the Sakai-Sugimoto model to the case of negative parity baryon resonances. Using holographic techniques we extract the generalized Dirac and Pauli baryon form factors as well as the helicity amplitudes for these baryonic states. Identifying the first negative parity resonance with the experimentally observed $S_{11}(1535)$, we find reasonable agreement with experimental data from the JLab-CLAS collaboration. We also estimate the contribution of negative parity baryons to the proton structure functions.
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1. Introduction

In the past decade, gauge/gravity dualities inspired by the original Maldacena conjecture \cite{Maldacena:1997re} have been successfully applied to a wide range of problems in Quantum Chromodynamics (QCD) as well as condensed matter theory. For QCD, two classes of models are of particular interest: The phenomenological bottom-up approach based on five-dimensional effective actions, and the more stringent top-down models based on compactifications of ten-dimensional string theories\cite{Witten:1995im}. A prominent and widely studied model of the latter class clearly is the Sakai-Sugimoto model \cite{Sakai:2004cn, Sugimoto:2004in}. Its popularity can be attributed to its close resemblance to large-\(N_c\) QCD, its computational simplicity and the fact that it provides a geometric model that allows to study both the confinement/deconfinement transition (at finite temperature) and chiral symmetry breaking in the same unified framework.

Holographic techniques have also emerged as a very fruitful complementary tool for studying hadronic scattering in QCD where non-perturbative effects become important, namely in the regime of low momentum transfer (\(\sqrt{q^2}\) lower than a few GeVs). In this paper we investigate the production of negative parity baryon resonances in proton electromagnetic scattering within the framework of the Sakai-Sugimoto model. This is a natural continuation of our previous work on positive parity baryonic resonances \cite{OurPreviousWork}. Production of baryonic resonances is a very important and timely problem in hadronic physics for the following reasons: i) Many baryonic resonances are excited nucleon states (\(N^*\)) and their structure is relevant to understand the physics of quark confinement, ii) there is a huge experimental effort at JLab \cite{JLab} to extract the electromagnetic form factors and helicity amplitudes of baryonic resonances in the regime where non-perturbative effects are dominant and perturbative QCD predictions fail.

The paper is organized as follows: In section 2 we introduce the current matrix decomposition for resonance production in proton electromagnetic scattering and present our theoretical results for electromagnetic form factors and helicity amplitudes in the Breit frame. Moreover, we study the contributions of resonance production to the proton structure functions defined in Deep Inelastic Scattering (DIS). Here, we focus on the production of negative parity baryonic resonances and their contributions, but for completeness, we also provide a review of our previous results for positive parity baryon resonances. Section 3 contains a detailed computation of the electromagnetic currents in the holographic Sakai-Sugimoto model and a subsequent calculation of Dirac and Pauli form factors in the holographic setup. This is done in a unified manner for both positive and negative parity baryon resonances. In section 4 we present our numerical results for the generalized form factors, helicity amplitudes and proton structure functions for the special case of negative parity baryon resonances and compare them to experimental results. Section 5 offers some conclusions and an outlook. Appendix A reviews different frames utilized in this article while appendix B gives technical details on the limits relevant to the model at hand.

Previous holographic calculations on electromagnetic form factors of baryons can be found in \cite{PreviousHolographicCalculations}. DIS structure functions from holography were first obtained in \cite{DISHolography}. Further developments include the large \(x\) regime \cite{LargeXRegime} as well as the small \(x\) regime \cite{SmallXRegime}.

\textsuperscript{1}Recommended reviews are \cite{RecommendedReviews} for the bottom-up approach and \cite{RecommendedReviews} for the top-down approach.
2. Form factors and helicity amplitudes

2.1. Dirac and Pauli form factors

We want to describe the electromagnetic interaction of a spin 1/2 baryon in the case where, as a result of the interaction, a spin 1/2 baryonic resonance is produced. This baryonic transition is described by an electromagnetic current evaluated between the initial and final states. In our approach, we embed the electromagnetic current in a vectorial $U(2)$ symmetry present in any effective description of large-$N_c$ QCD with chiral symmetry breaking. Then we define the electromagnetic current as a linear combination of flavour currents:

$$\mathcal{J}^\mu = \sum_a c_a J^{\mu,a}_V, \quad c_0 = 1/N_c, \quad c_3 = 1, \quad c_1 = c_2 = 0.$$  \hspace{1cm} (2.1)

Now we evaluate the flavour currents $J^{\mu,a}_V$ between the initial and final baryonic states.

Positive parity resonances

When the final baryonic state has positive parity we can expand the current matrix element as

$$\langle p_X, B_X, s_X| J^{\mu,a}_V(0)| p, B, s \rangle = \frac{i}{2(2\pi)^3} (\tau^a)_{I_3} I_3 \left( \eta^{\mu
u} - \frac{q^\mu q^\nu}{q^2} \right) \bar{u}(p_X, s_X) \left[ \gamma_\nu F^D_{B_B} (q^2) + \kappa_B \sigma_\nu q^\lambda F^P_{B_B} (q^2) \right] u(p, s),$$  \hspace{1cm} (2.2)

where

$$q^\mu = (p_X - p)^\mu, \quad \kappa_B = \frac{1}{m_B + m_{B_X}},$$

$$(\tau^0)_{I_3} I_3 = \delta_{I_3} I_3, \quad (\tau^a)_{I_3} I_3 = (\sigma^a)_{I_3} I_3, \quad a = (1, 2, 3),$$  \hspace{1cm} (2.3)

and $\sigma^a$ are the Pauli matrices. Here we are using the metric $\eta^{\mu\nu} = \text{diag}(-, +, +, +)$ and we adopt the following convention for spinors and gamma matrices:

$$u(p, s) = \frac{1}{\sqrt{2E}} \left( \frac{f_X}{f_X} \chi_s (\vec{p}) \right), \quad u(p_X, s_X) = \frac{1}{\sqrt{2E_X}} \left( \frac{f_X}{f_X} \chi_{s_X} (\vec{p}_X) \right),$$

$$\gamma^0 = -i \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \gamma^i = -i \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix},$$

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu], \quad \gamma_5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$  \hspace{1cm} (2.4)

where

$$f = \sqrt{E + m_B}, \quad f_X = \sqrt{E_X + m_{B_X}}.$$  \hspace{1cm} (2.5)
In this article we are interested in the case where $I_3^X = I_3 = 1/2$. In this case the baryonic states satisfy the relation [12]:

$$\langle p_X, B_X, S_X | p, B, s \rangle = \delta^3(\vec{p}_X - \vec{p})\delta_{s_X} \delta_{B_X}.$$  \hfill (2.6)

The spinors $\chi_s(\vec{p})$ are defined as the eigenstates of the helicity equation of the initial state:

$$\vec{p} \cdot \vec{\sigma} \chi_s(\vec{p}) = s|\vec{p}|\chi_s(\vec{p}), \quad s = (+, -),$$  \hfill (2.7)

Similarly, the spinors $\chi_{s_X}(\vec{p}_X)$ are defined by the helicity equation for the final state:

$$\vec{p}_X \cdot \vec{\sigma} \chi_{s_X}(\vec{p}_X) = s_X|\vec{p}_X|\chi_{s_X}(\vec{p}_X).$$  \hfill (2.8)

In order to get standard relativistic normalizations we need to transform the spinors and baryon states as [12]

$$u(p, s) \rightarrow \frac{1}{\sqrt{2E}} u(p, s), \quad |p, B, s \rangle \rightarrow \frac{1}{\sqrt{2E(2\pi)^{3/2}}} |p, B, s \rangle.$$  \hfill (2.9)

Using (2.1), (2.2) and (2.9) we obtain for $I_3 = I_3^X = 1/2,

$$\langle p_X, B_X, s_X | J_{\mu}^a(0) | p, B, s \rangle = i \left( \eta_{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \bar{u}(p_X, s_X) \left[ \gamma_\nu F_{BB_X}^D(q^2) + \kappa_B \sigma_\nu \lambda \bar{F}_{BB_X}^P(q^2) \right] u(p, s),$$  \hfill (2.10)

where

$$F_{BB_X}^D(q^2) = \frac{1}{2} \sum_a c_a F_{BB_X}^{D,a}(q^2), \quad F_{BB_X}^P(q^2) = \frac{1}{2} \sum_a c_a F_{BB_X}^{P,a}(q^2),$$  \hfill (2.11)

are the generalized Dirac and Pauli form factors that describe the production of positive parity baryons.

**Negative parity resonances**

A good expansion for the flavor current matrix element in the case when the final baryonic state has negative parity is given by

$$5\langle p_X, B_X, s_X | J_{\nu}^\mu(0) | p, B, s \rangle = \frac{i}{2(2\pi)^3} \left( \tau^a \right)_{I_3^X I_3} \left( \eta_{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \bar{u}(p_X, s_X) \left[ \gamma_\nu \bar{F}_{BB_X}^{D,a}(q^2) + \kappa_B \sigma_\nu \lambda \bar{F}_{BB_X}^{P,a}(q^2) \right] \gamma_5 u(p, s).$$  \hfill (2.12)

Alternatively, if we want to associate the chirality matrix $\gamma_5$ with the final state (which is the non-trivial state) we can write the current matrix element as

$$5\langle p_X, B_X, s_X | J_{\nu}^\mu(0) | p, B, s \rangle = \frac{i}{2(2\pi)^3} \left( \tau^a \right)_{I_3^X I_3} \left( \eta_{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \bar{u}(p_X, s_X) \gamma_5$$
\[ \times \left[ -\gamma_\mu \tilde{F}^{D,a}_{BBX}(q^2) + \kappa_B \sigma_{\nu\lambda} q^\lambda \tilde{F}^{P,a}_{BBX}(q^2) \right] u(p, s). \]  

(2.13)

Transforming the spinors and states as (2.9), we get for \( I_3 = I_3^X = 1/2, \)

\[ 5\langle p_X, B_X, s_X|J^\mu(0)|p, B, s \rangle = i \left( \eta^{\mu\nu} \frac{q^\mu q^\nu}{q^2} \right) \bar{u}(p_X, s_X) \left[ \gamma_\mu \tilde{F}^{D}_{BBX}(q^2) + \kappa_B \sigma_{\nu\lambda} q^\lambda \tilde{F}^{P}_{BBX}(q^2) \right] \gamma_5 u(p, s), \]  

(2.14)

where

\[ \tilde{F}^{D}_{BBX}(q^2) = \frac{1}{2} \sum_a c_a \tilde{F}^{D,a}_{BBX}(q^2), \quad \tilde{F}^{P}_{BBX}(q^2) = \frac{1}{2} \sum_a c_a \tilde{F}^{P,a}_{BBX}(q^2), \]  

(2.15)

are the generalized Dirac and Pauli form factors that describe the production of negative parity baryons.

### 2.2. The Breit frame

It is usually convenient to work in the Breit frame where

\[ p^\mu = (E, 0, 0, p), \quad q^\mu = (0, 0, 0, -2xp), \quad p_X^\mu = (E, 0, 0, p(1 - 2x)). \]  

(2.16)

The details of this frame are given in the Appendix. In the Breit frame, we obtain the following helicity equation:

\[ \vec{p}_X \cdot \vec{\sigma} \chi_{sX}(\vec{p}) = (1 - 2x)p \sigma^3 \chi_{sX}(\vec{p}) = s_X(1 - 2x)|\vec{p}| \chi_{sX}(\vec{p}), \]  

(2.17)

Using the relation \(|\vec{p}_X| = |1 - 2x||\vec{p}|\) and (2.8), we identify two situations:

If \( 1 - 2x > 0 \) \rightarrow \( \chi_{sX}(\vec{p}_X) = \chi_{sX}(\vec{p}) \),

If \( 1 - 2x < 0 \) \rightarrow \( \chi_{sX}(\vec{p}_X) = \chi_{-sX}(\vec{p}) \).  

(2.18)

### Positive parity resonances

Using the helicity equations we can calculate the current matrix elements in the Breit frame (for details see [12]). The result is

\[ \langle p_X, B_X, s_X|J^{0,a}_V(0)|p, B, s \rangle = \frac{1}{2(2\pi)^3} \tau_a^{\dagger} I^X_{13} I^X_{3} \chi_{sX}(\vec{p}) \left[ \frac{1}{2(2\pi)^3} \tau_a^{\dagger} I^X_{13} \chi_{sX}(\vec{p}) \right. \]  

\[ \times \left[ \alpha F^{D,a}_{BBX}(q^2) - \beta q^2 \kappa_B F^{P,a}_{BBX}(q^2) \right], \]  

(2.19)

\[ \langle p_X, B_X, s_X|J^{+,a}_V(0)|p, B, s \rangle = -\frac{i}{2(2\pi)^3} \tau_a^{\dagger} I^X_{13} \epsilon^{ijk} \chi_{sX}(\vec{p}) \sigma_k \chi_{s}(\vec{p}) \]  

\[ \times \left[ \beta F^{D,a}_{BBX}(q^2) + \alpha \kappa_B F^{P,a}_{BBX}(q^2) \right], \]  

(2.20)
where
\[
\alpha = \left( \frac{1}{2E} \right) \left( \frac{f}{f_X} \right) \left[ f_X^2 + (1 - 2x)\frac{p^2}{f^2} \right], 
\]
\[
\beta = \left( \frac{1}{2E} \right) \left( \frac{f}{f_X} \right) \left( \frac{1}{2x} \right) \left[ f_X^2 + 2x - 1 \right]. 
\]

**Negative parity resonances**

Note that
\[
\gamma_5 u(p, s) = \frac{1}{\sqrt{2E}} \left( \frac{s|\vec{p}|}{f} \chi_s(\vec{p}) \right) = \frac{1}{\sqrt{2E}} \left( \frac{\tilde{f} \chi_s(\vec{p})}{s|\vec{p}|} \right), 
\]
where
\[
\tilde{f} := \frac{s|\vec{p}|}{f}. 
\]

Therefore, we can recycle the results obtained in the positive parity case by substituting \( f \) by \( \tilde{f} \) in all the calculations. Then it is not difficult to check that in the Breit frame the current matrix element takes the form
\[
5\langle p_X, B_X, s_X | J^{0,a}_V(0) | p, B, s \rangle = - \left( \frac{1}{2x} \right) \frac{1}{2(2\pi)^3} (\tau^a) I_i^X q^i \chi^\dagger_{s_X}(\vec{p}_X) \sigma_i \chi_s(\vec{p}) 
\times \left[ \hat{\alpha} \tilde{F}^{D,a}_{BBX}(q^2) - \hat{\beta} q^2 \kappa_B \tilde{F}^{P,a}_{BBX}(q^2) \right], 
\]
\[
5\langle p_X, B_X, s_X | J^{i,a}_V(0) | p, B, s \rangle = \left( \frac{1}{2x} \right) \frac{q^2}{2(2\pi)^3} (\tau^a) I_i^X q^i \chi^\dagger_{s_X}(\vec{p}_X) \sigma_j \chi_s(\vec{p}) 
\times \left[ \hat{\beta} \tilde{F}^{D,a}_{BBX}(q^2) + \hat{\alpha} \kappa_B \tilde{F}^{P,a}_{BBX}(q^2) \right], 
\]
where
\[
\hat{\alpha} := \left( \frac{f}{f_X} \right) \left( \frac{1}{2E} \right) \left[ f_X^2 + 1 - 2x \right], 
\]
\[
\hat{\beta} := \frac{1}{|\vec{p}|^2} \left( \frac{f}{f_X} \right) \left( \frac{1}{2E} \right) \left( \frac{1}{2x} \right) \left[ f_X^2 - (1 - 2x)\frac{p^2}{f^2} \right]. 
\]

**2.3. Helicity amplitudes**

In order to establish a simple connection between the Dirac and Pauli form factors and the more commonly used helicity amplitudes, we first need to review some Gordon identities. We start with a generalized Gordon identity
\[
p_{\nu}^X \gamma^\nu \gamma^\mu + p_{\nu} \gamma^\mu \gamma^\nu = p_{\nu}^X \left( \frac{1}{2} \{ \gamma^\nu, \gamma^\mu \} + \frac{1}{2} [\gamma^\nu, \gamma^\mu] \right) + p_{\nu} \left( \frac{1}{2} \{ \gamma^\mu, \gamma^\nu \} + \frac{1}{2} [\gamma^\mu, \gamma^\nu] \right)
\]
\[
\begin{align*}
\ &= p^X_\nu (\eta^{\mu\nu} + i\sigma^{\mu\nu}) + p_\nu (\eta^{\mu\nu} - i\sigma^{\mu\nu}) \\
\ &= (p_X + p)^\mu + i\sigma^{\mu\nu} q_\nu. \tag{2.29}
\end{align*}
\]
Evaluating (2.29) on the initial and final spinor and using the Dirac equation we get the Gordon decomposition for positive parity resonances:
\[
\bar{u}(p_X, s_X) \gamma^\mu u(p, s) = -\frac{i}{m_{B_X} + m_B} \bar{u}(p_X, s_X) [(p_X + p)^\mu + i\sigma^{\mu\nu} q_\nu] u(p, s), \tag{2.30}
\]
On the other hand, if we multiply (2.29) by \(\gamma^5\) on the right, evaluate it on the initial and final spinors, and finally use the Dirac equation, we get the Gordon decomposition for the negative parity case,
\[
\bar{u}(p_X, s_X) \gamma^\mu \gamma^5 u(p, s) = -\frac{i}{m_{B_X} - m_B} \bar{u}(p_X, s_X) [(p_X + p)^\mu + i\sigma^{\mu\nu} q_\nu] \gamma^5 u(p, s). \tag{2.31}
\]

**Positive parity resonances**

First we define the \(G_1(q^2)\) and \(G_2(q^2)\) form factors through the vector current decomposition
\[
\langle p_X, B_X, s_X| J^\mu(0)| p, B, s \rangle = i \bar{u}(p_X, s_X) \left\{ \left[ \eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right] \gamma^\nu q^2 G_1(q^2) \\
+ \frac{1}{2} [(p_X^2 - p^2)\gamma^\mu - q_\nu \gamma^\nu (p_X + p)^\mu] G_2(q^2) \right\} u(p, s). \tag{2.32}
\]
Using the Gordon identity (2.30) and the Dirac equation we can rewrite the current as in (2.10). This way we get the Dirac and Pauli form factors in terms of the \(G_1(q^2)\) and \(G_2(q^2)\) form factors:
\[
F^{D}_{BBX}(q^2) = q^2 G_1(q^2) \\
F^{P}_{BBX}(q^2) = -\frac{1}{2} (m_{B_X}^2 - m_B^2) G_2(q^2). \tag{2.33}
\]
According to [36], the transverse helicity amplitude \(A_{1/2}(q^2)\) is defined by
\[
A_{1/2}(q^2) = \sqrt{\frac{E_R - m_B}{2m_B K}} \left[ q^2 G_1(q^2) - \frac{1}{2} (m_{B_X}^2 - m_B^2) G_2(q^2) \right] \\
= \sqrt{\frac{E_R - m_B}{2m_B K}} \left[ F^{D}_{BBX}(q^2) + F^{P}_{BBX}(q^2) \right], \tag{2.34}
\]
where
\[
K = \frac{m_{B_X}^2 - m_B^2}{2m_B}, \tag{2.35}
\]
and \(E_R\) is the proton energy in the resonance rest frame. Details of the resonance rest frame are given in appendix A.2.
The helicity amplitude $A_{1/2}(q^2)$ can be rewritten as \[37\]

$$A_{1/2}(q^2) = \sqrt{\frac{m_B}{m_{BX}^2 - m_B^2}} G_{BBX}^+(q^2),$$

(2.36)

where

$$G_{BBX}^+(q^2) = \frac{\zeta}{m_B} \left[ F_{BBX}^D(q^2) + F_{BBX}^P(q^2) \right],$$

(2.37)

and

$$\zeta := \sqrt{m_{BX}(E_R - m_B)} = \frac{1}{\sqrt{2}} \left[(m_{BX} - m_B)^2 + q^2 \right]^{1/2}.$$  

(2.38)

The longitudinal helicity amplitude $S_{1/2}(q^2)$ is given by \[36\],

$$S_{1/2}(q^2) = \sqrt{\frac{m_B}{m_{BX}^2 - m_B^2}} \sqrt{\frac{q^2}{\zeta}} G_{BBX}^0(q^2),$$

(2.40)

where $\tilde{q}_R$ is the spatial momentum of the virtual photon in the resonance rest frame. According to \[38\], this amplitude can be rewritten as

$$S_{1/2}(q^2) = \sqrt{\frac{m_B}{m_{BX}^2 - m_B^2}} \frac{q^2}{\sqrt{\zeta}} G_{BBX}^0(q^2),$$

(2.41)

Negative parity resonances

In analogy with the previous case we define the $\tilde{G}_1(q^2)$ and $\tilde{G}_2(q^2)$ negative parity form factors through the vector current decomposition as in \[36\],

$$5\langle p_X, B_X, s_X | J^\mu(0) | p, B, s \rangle = -i \tilde{u}(p_X, s_X) \left\{ \left( \gamma^\mu - \frac{q^\mu q^\nu}{q^2} \right) \gamma_\nu q^2 \tilde{G}_1(q^2) 
+ \frac{1}{2} \left[ (p_X^2 - p^2) \gamma^\mu - q_\nu \gamma^\nu(p_X + p)^\mu \right] \tilde{G}_2(q^2) \right\} \gamma_5 u(p, s).$$

(2.42)

Using the Gordon identity (2.31) and the Dirac equation we can rewrite (2.42) as in (2.14). Therefore we obtain the relations

$$\tilde{F}_{BBX}^D(q^2) = -q^2 \tilde{G}_1(q^2)$$
Now let us write the expressions for the helicity amplitudes. According to \[36\], the helicity amplitudes \( A_{1/2}(q^2) \) are given by

\[
\tilde{A}_{1/2}(q^2) = \frac{\sqrt{E_R + m_B}}{2m_B K} \left[ q^2 \tilde{G}_1(q^2) - \frac{1}{2}(m_{BX}^2 - m_B^2) \tilde{G}_2(q^2) \right],
\]

where \( K \) is given by \(2.35\) and \( E_R \) is the proton energy in the resonance rest frame (defined in appendix A.2).

Using the analog of \(2.36\), we get

\[
\tilde{G}_{BBX}^0(q^2) = \sqrt{\frac{E_R + m_B}{m_B K}} \left[ \frac{m_{BX} - m_B}{q^2} \tilde{F}_{BBX}^D(q^2) - \frac{1}{m_{BX} + m_B} \tilde{F}_{BBX}^P(q^2) \right],
\]

where \( \tilde{q}_R \) is the spatial momentum of the virtual photon in the resonance rest frame. Using the negative parity analog of \(2.40\), we get

\[
\tilde{G}_{BBX}^0(q^2) = \sqrt{\frac{q^2}{2m_B}} \left[ \frac{m_{BX} - m_B}{q^2} \tilde{F}_{BBX}^D(q^2) - \frac{1}{m_{BX} + m_B} \tilde{F}_{BBX}^P(q^2) \right].
\]
One usually parametrizes DIS using as dynamical variables the Bjorken parameter \( x = -\frac{q^2}{2p \cdot q} \) and the photon virtuality \( q^2 \). The hadronic tensor can be decomposed in terms of the Lorentz invariant scalar structure functions \( F_1(x, q^2) \) and \( F_2(x, q^2) \):

\[
W^{\mu \nu} = F_1(x, q^2) \left( \eta^{\mu \nu} \frac{q^2}{q^2} \right) + \frac{2x}{q^2} F_2(x, q^2) \left( p^\mu + \frac{q^\mu}{2x} \right) \left( p^\nu + \frac{q^\nu}{2x} \right).
\]

(2.50)

The standard limit of DIS corresponds to the Bjorken limit of large \( q^2 \) and fixed \( x \). In this paper we are interested in the regime of small \( q^2 \) where non-perturbative contributions are relevant (for a review of DIS, see e.g., [39]).

The hadronic tensor for a spin 1/2 baryon, in the case where one particle is produced in the final state, can be written as

\[
W^{\mu \nu} = \frac{1}{4} \sum_{s, s_X} \sum_{m_{BX}} \delta \left[ (p + q)^2 + m_{BX}^2 \right] \left[ \langle p, B, s | J^{\mu}(0) | p_X, B_X, s_X \rangle \langle p_X, B_X, s_X | J^{\nu}(0) | p, B, s \rangle \right.
\]

\[
+ \langle p, B, s | J^{\mu}(0) | p_X, B_X, s_X \rangle s_5 \langle p_X, B_X, s_X | J^{\nu}(0) | p, B \rangle \right].
\]

(2.51)

Note that we are including the contribution from positive parity resonances as well as negative parity resonances. Substituting (2.10) and (2.14) into (2.51) and using some gamma trace identities we obtain the proton structure functions

\[
F_1(q^2, x) = F_1(q^2, x) + \tilde{F}_1(q^2, x),
\]

(2.52)

\[
F_2(q^2, x) = F_2(q^2, x) + \tilde{F}_2(q^2, x),
\]

(2.53)

where

\[
F_1(q^2, x) = \sum_{m_{BX}} \delta \left[ (p + q)^2 + m_{BX}^2 \right] m_B^2 (G_{BBX}^+(q^2))^2,
\]

(2.54)

\[
F_2(q^2, x) = \sum_{m_{BX}} \delta \left[ (p + q)^2 + m_{BX}^2 \right] \left( \frac{q^2}{2x} \right) \left( 1 + \frac{q^2}{4m_B^2 x^2} \right)^{-1}
\]

\[
\times \left[ (G_{BBX}^+(q^2))^2 + 2(G_{BBX}^0(q^2))^2 \right],
\]

(2.55)

are the positive parity contributions to the proton structure functions and

\[
\tilde{F}_1(q^2, x) = \sum_{m_{BX}} \delta \left[ (p + q)^2 + m_{BX}^2 \right] m_B^2 (\tilde{G}_{BBX}^+(q^2))^2,
\]

(2.56)

\[
\tilde{F}_2(q^2, x) = \sum_{m_{BX}} \delta \left[ (p + q)^2 + m_{BX}^2 \right] \left( \frac{q^2}{2x} \right) \left( 1 + \frac{q^2}{4m_B^2 x^2} \right)^{-1}
\]

\[
\times \left[ (\tilde{G}_{BBX}^+(q^2))^2 + 2(\tilde{G}_{BBX}^0(q^2))^2 \right],
\]

(2.57)

are the negative parity contributions to the proton structure functions.

\[\text{For more details see [12]}\]
3. Dirac and Pauli form factors from holography

3.1. Review of the Sakai-Sugimoto model

3.1.1. $D_4 - D_8$ configuration

The Sakai-Sugimoto model \cite{10,11} is the most widely studied string-theoretic model of large-$N_c$ QCD and has been successfully applied to investigate many of its phenomenological aspects. Its holographic limit describes a stable configuration of $D_8 - \overline{D_8}$ branes embedded into Witten’s $D_4$ model \cite{40}. In the following section we will briefly review those features of this model which are important for the investigations carried out in this article. The geometry of Witten’s model is generated by $N_c$ coincident $D_4$ branes with a compact spatial direction $\tau$ in type IIA supergravity with the following metric, dilaton and four-form,

\begin{equation}
    ds^2 = \frac{u^{3/2}}{R^{3/2}} \left( \eta_{\mu\nu} dx^\mu dx^\nu + f(u) d\tau^2 \right) + \frac{R^{3/2}}{u^{3/2}} \frac{du^2}{f(u)} + R^{3/2} u^{1/2} d\Omega_4^2,
\end{equation}

where $u_{KK}$ is the radial position of the tip of the cigar geometry generated by the $D_4$ branes and $R = (\pi g_s N_c)^{1/3} \sqrt{\alpha'}$. To incorporate fundamental (quark and anti-quark) degrees of freedom, one needs to introduce two stacks of $N_f$ coincident $D_8$ and $\overline{D_8}$ flavor branes into the background generated by the $N_c$ $D_4$ branes. The probe condition $N_f \ll N_c$ ensures that the back reaction of the flavor branes on the geometry can be safely neglected. It turns out that the solution to the DBI equations merges the two stacks of $D_8$ and $\overline{D_8}$ branes in the infrared region (small $u$), resulting in a geometrical realization of chiral symmetry breaking $U(N_f) \times U(N_f) \to U(N_f)$. The dynamics of the gauge field fluctuations on the $D_8/\overline{D_8}$ brane embedding is described by the Dirac-Born-Infeld action, which yields a vector meson effective field theory given by a five dimensional $U(N_f)$ Yang-Mills-Chern-Simons theory in a curved background. The details of this construction can be found in the original publications by Sakai and Sugimoto \cite{10,11}. For the present context, cf. our previous work \cite{12}.

3.1.2. Baryons in the Sakai-Sugimoto model

Let us describe the ideas behind the construction of holographic baryons. Recall that, in the confined phase, the Sakai-Sugimoto model reduces to a five-dimensional $U(N_f)$ Yang Mills-Chern Simons (YM-CS) theory. In this article, we restrict ourselves to the $N_f = 2$ case. Then, the $U(2)$ gauge field $A$ can be decomposed as

\begin{equation}
    A = A + \hat{A} \frac{1_2}{2} = A^i \frac{\tau^i}{2} + \hat{A} \frac{1_2}{2} = \sum_{a=0}^{3} A^a \frac{\tau^a}{2},
\end{equation}

where $u_{KK}$ is the radial position of the tip of the cigar geometry generated by the $D_4$ branes and $R = (\pi g_s N_c)^{1/3} \sqrt{\alpha'}$. To incorporate fundamental (quark and anti-quark) degrees of freedom, one needs to introduce two stacks of $N_f$ coincident $D_8$ and $\overline{D_8}$ flavor branes into the background generated by the $N_c$ $D_4$ branes. The probe condition $N_f \ll N_c$ ensures that the back reaction of the flavor branes on the geometry can be safely neglected. It turns out that the solution to the DBI equations merges the two stacks of $D_8$ and $\overline{D_8}$ branes in the infrared region (small $u$), resulting in a geometrical realization of chiral symmetry breaking $U(N_f) \times U(N_f) \to U(N_f)$. The dynamics of the gauge field fluctuations on the $D_8/\overline{D_8}$ brane embedding is described by the Dirac-Born-Infeld action, which yields a vector meson effective field theory given by a five dimensional $U(N_f)$ Yang-Mills-Chern-Simons theory in a curved background. The details of this construction can be found in the original publications by Sakai and Sugimoto \cite{10,11}. For the present context, cf. our previous work \cite{12}.

\begin{equation}
    A = A + \hat{A} \frac{1_2}{2} = A^i \frac{\tau^i}{2} + \hat{A} \frac{1_2}{2} = \sum_{a=0}^{3} A^a \frac{\tau^a}{2},
\end{equation}

where $u_{KK}$ is the radial position of the tip of the cigar geometry generated by the $D_4$ branes and $R = (\pi g_s N_c)^{1/3} \sqrt{\alpha'}$. To incorporate fundamental (quark and anti-quark) degrees of freedom, one needs to introduce two stacks of $N_f$ coincident $D_8$ and $\overline{D_8}$ flavor branes into the background generated by the $N_c$ $D_4$ branes. The probe condition $N_f \ll N_c$ ensures that the back reaction of the flavor branes on the geometry can be safely neglected. It turns out that the solution to the DBI equations merges the two stacks of $D_8$ and $\overline{D_8}$ branes in the infrared region (small $u$), resulting in a geometrical realization of chiral symmetry breaking $U(N_f) \times U(N_f) \to U(N_f)$. The dynamics of the gauge field fluctuations on the $D_8/\overline{D_8}$ brane embedding is described by the Dirac-Born-Infeld action, which yields a vector meson effective field theory given by a five dimensional $U(N_f)$ Yang-Mills-Chern-Simons theory in a curved background. The details of this construction can be found in the original publications by Sakai and Sugimoto \cite{10,11}. For the present context, cf. our previous work \cite{12}.
where \( \tau^i \) \((i = 1, 2, 3)\) are Pauli matrices and \( \tau^0 = 1_2 \) is a unit matrix of dimension 2. The equations of motion are given by \([15]\)

\[
-\kappa \left( h(z) \partial_\nu \hat{F}^{\mu \nu} + \partial_z k(z) \hat{F}^{\mu z} \right) + \frac{N_c}{128\pi^2} \epsilon^{\mu \alpha_2 \cdots \alpha_5} \left( F^{a}_{\alpha_2 \alpha_3} F^{a}_{\alpha_4 \alpha_5} + \hat{F}^{a}_{\alpha_2 \alpha_3} \hat{F}^{a}_{\alpha_4 \alpha_5} \right) = 0, \\
-\kappa (h(z) \nabla_\nu F^{\mu \nu} + \nabla_z k(z) F^{\mu z})^a + \frac{N_c}{64\pi^2} \epsilon^{\mu \alpha_2 \cdots \alpha_5} F^{a}_{\alpha_2 \alpha_3} \hat{F}^{a}_{\alpha_4 \alpha_5} = 0, \\
-\kappa k(z) \partial_\nu \hat{F}^{z \nu} + \frac{N_c}{128\pi^2} \epsilon^{z \mu_2 \cdots \mu_5} \left( F^{a}_{\mu_2 \mu_3} F^{a}_{\mu_4 \mu_5} + \hat{F}^{a}_{\mu_2 \mu_3} \hat{F}^{a}_{\mu_4 \mu_5} \right) = 0, \\
-\kappa k(z) (\nabla_\nu F^{z \nu})^a + \frac{N_c}{64\pi^2} \epsilon^{z \mu_2 \cdots \mu_5} F^{a}_{\mu_2 \mu_3} \hat{F}^{a}_{\mu_4 \mu_5} = 0, \\
\tag{3.3}
\]

where \( k(z) = 1 + z^2 \) and \( h(z) = (1 + z^2)^{-1/3} \) are the 5-d warp factors and \( \nabla_\alpha = \partial_\alpha + i A_\alpha \) is the covariant derivative. The baryon in the Sakai-Sugimoto holographic model is represented by a soliton with nontrivial instanton number in the four-dimensional space parameterized by \( x^M \) \((M = 1, 2, 3, z)\). Consequently, the instanton number is interpreted as the baryon number \( N_B \), and reads

\[
N_B = \frac{1}{64\pi^2} \int d^3x dz \epsilon_{M_1 M_2 M_3 M_4} F^a_{M_1 M_2} F^a_{M_3 M_4}. \\
\tag{3.4}
\]

The construction and quantization of solutions to the set of equations \([3.3]\) in the large \( \lambda \) regime was discussed in great detail in the literature \([11,15,12]\). Here we merely state some results that will be important for the purpose of the present work. The resulting baryon eigenstates are characterized by quantum numbers \( B = (l, I_3, n_\rho, n_z) \) in addition to their spin \( s \). For example, the baryon wave functions with quantum numbers \( B_n = (1, +1/2, 0, n) \) are given by

\[
|B_n \uparrow \rangle \propto R(\rho) \psi_{B_n}(Z)(a_1 + ia_2), \\
\tag{3.5}
\]

where

\[
R(\rho) = \rho^{-1 + 2 \sqrt{1 + N_\rho^2/5}} e^{-\frac{M_0}{\sqrt{6}} \rho^2}, \\
\psi_{B_n}(Z) = \left( \frac{(2M_0)^{1/4}}{6^{1/2} \pi^{1/4} 2^{n/2} n!} \right) H_n \left( \sqrt{2M_0} e^{-1/4} Z \right) e^{-\frac{M_0}{\sqrt{6}} z^2}. \\
\tag{3.6}
\]

The mass formula for the baryonic eigenstates (obtained from the quantized Hamiltonian of the system) reads

\[
M = M_0 + \frac{\sqrt{(\ell + 1)^2}}{6} + \frac{2}{15} N_\rho^2 + \frac{2(n_\rho + n_z) + 2}{\sqrt{6}} =: \tilde{M}_0 + \frac{2n_z}{\sqrt{6}}. \\
\tag{3.7}
\]

### 3.2. Electromagnetic currents in the Sakai-Sugimoto model

The holographic currents in the Sakai-Sugimoto model, denoted here by \( J^\mu_{V(SS)} \), can be obtained using the holographic relations \([15]\):

\[
J^\mu_{V(SS)} = -\kappa \left\{ \lim_{z \to \infty} [k(z) F^{cl}_{\mu z}] + \lim_{z \to -\infty} [k(z) F^{cl}_{\mu z}] \right\}, \\
\tag{3.8}
\]

12
where $\mathcal{F}_{\mu
u}^{\text{cl}}$ is the field strength associated with the classical field, cf. (2.80-2.85) in [15].

We define the baryon states as

$$|\vec{p}, B, s, I_3\rangle = \frac{1}{(2\pi)^{3/2}} e^{i\vec{p} \cdot \vec{x}} |n_B\rangle |n_\rho\rangle |s, I_3\rangle_R,$$

$$|\vec{p}_X, B_X, s_X, I_3^X\rangle = \frac{1}{(2\pi)^{3/2}} e^{i\vec{p}_X \cdot \vec{x}} |n_{B_X}\rangle |n_\rho\rangle |s_X, I_3^X\rangle_R.$$  \hspace{1cm} (3.9)

Here we make use of the results and definitions of a recent publication [42], in which a relativistic generalization of baryon states and wave functions was discussed in detail. In particular, the spin and isospin part was defined as

$$|s, I_3\rangle_R = \frac{1}{\sqrt{2E}} \left( f |s, I_3\rangle \right),$$

$$\langle s_X, I_3^X | = \frac{1}{\sqrt{2E_X}} \left( f_X \langle s_X, I_3^X | - \frac{s_X |\vec{p}_X|}{f_X} \langle s_X, I_3^X | \right),$$  \hspace{1cm} (3.10)

where $|s, I_3\rangle$ and $\langle s_X, I_3^X |$ are the non-relativistic initial and final states associated with the spin and isospin operators. From this and (3.8), one gets [12]:

$$\langle J_{V(\Sigma \Sigma)}^{0,0}(0) \rangle = \frac{1}{(2\pi)^3} \frac{N_c}{2} \langle s_X, I_3^X | s, I_3\rangle_R F_{BBX}^1(q^2),$$  \hspace{1cm} (3.11)

$$\langle J_{V(\Sigma \Sigma)}^{i,0}(0) \rangle = \frac{1}{(2\pi)^3} \frac{N_c}{2} \langle s_X, I_3^X | F_{BBX}^1(q^2) \left[ \frac{p^i}{M_0} - \frac{i}{16\pi^2\kappa} \epsilon^{ija} q_j S_a \right] + \frac{q^i}{M_0} F_{BBX}^3(q^2) - \frac{1}{16\pi^2\kappa} F_{BBX}^2(q^2) (q^2 q^a - q^a \delta^{ia}) S_a \rangle |s, I_3\rangle_R,$$  \hspace{1cm} (3.12)

$$\langle J_{V(\Sigma \Sigma)}^{0,c}(0) \rangle = 2\pi^2\kappa \left[ \frac{1}{(2\pi)^3} \langle n_\rho | F_{BBX}^1(q^2) \left[ I^c_{2\pi^2\kappa} + \frac{i}{M_0} \epsilon^{ija} p_i q_j \rho^2 \text{tr}(\tau^c a^i a^{-1}) \right] + \frac{F_{BBX}^2(q^2)}{M_0} (\vec{P} \cdot \vec{q_i} - q^a P_i) \rho^2 \text{tr}(\tau^c a^i a^{-1}) \right] |n_\rho\rangle |s, I_3\rangle_R,$$  \hspace{1cm} (3.13)

$$\langle J_{V(\Sigma \Sigma)}^{i,c}(0) \rangle = 2\pi^2\kappa \left[ \frac{1}{(2\pi)^3} \left[ i F_{BBX}^1(q^2) \epsilon^{ija} q_i + F_{BBX}^2(q^2) (q^i q^a - q^a \delta^{ia}) \right] \langle n_\rho | F_{BBX}^1(q^2) \left[ I^c_{2\pi^2\kappa} + \frac{i}{M_0} \epsilon^{ija} p_i q_j \rho^2 \text{tr}(\tau^c a^i a^{-1}) \right] \right] \langle s_X, I_3^X | \text{tr}(\tau^c a^i a^{-1}) |s, I_3\rangle_R.$$  \hspace{1cm} (3.14)

where

$$F_{BBX}^1(q^2) = \sum_n g_{\nu_n} \langle n_{B_X} | \psi_{2n-1}(Z) | n_B \rangle \frac{q^2 + \lambda_{2n-1}}{\lambda_{2n-1}},$$

$$F_{BBX}^2(q^2) = \sum_n g_{\nu_n} \langle n_{B_X} | \partial_Z \psi_{2n-1}(Z) | n_B \rangle \frac{\lambda_{2n-1}}{\lambda_{2n-1}(q^2 + \lambda_{2n-1})}.$$
\[ F_{BBX}^3(\vec{q}^2) = \sum_n g_{\nu n} \langle n_B | \partial_Z \psi_{2n-1}(Z) \partial_Z | n_B \rangle \overline{\lambda}_{2n-1}(\vec{q}^2 + \lambda_{2n-1}), \] (3.15)

and

\[ S^a = -i4\pi^2 \kappa \rho^2 \text{tr}(\tau^a a^{-1}), \quad I^a = -i4\pi^2 \kappa \rho^2 \text{tr}(\tau^a a^i), \] (3.16)

are the spin and isospin operators. The operators \( a = a_4 + ia_a \tau^a \) represent the SU(2) orientations of the instanton and \( \rho \) is the instanton size. The momentum \( \vec{q} \) is the photon momentum defined by \( \vec{q} = \vec{p}_X - \vec{p} \).

In order to calculate the expectation values of the holographic currents we need the following identities:

\[
\begin{align*}
\langle s_X, I_3^X | s, I_3 \rangle_R &= \frac{1}{2\sqrt{EXE}} (f f X - \frac{ssX}{ff X} | p^X \rangle \delta \chi_{sX} (p^X) \chi_s (\vec{p}) ,
\langle s_X, I_3^X | R \text{tr}(\tau^a a^{-1}) | s, I_3 \rangle_R &= -\frac{1}{3\sqrt{EXE}} (f f X - \frac{ssX}{ff X} | p^X \rangle \tau^c \chi_{sX} (p^X) \sigma^c \chi_s (\vec{p}) ,
\langle s_X, I_3^X | R \iota \chi \chi \rangle | s, I_3 \rangle_R &= \frac{1}{4\sqrt{EXE}} (f f X - \frac{ssX}{ff X} | p^X \rangle \delta \chi_{sX} (p^X) \sigma \chi_s (\vec{p}) ,
\langle s_X, I_3^X | R \iota \chi \chi \rangle | s, I_3 \rangle_R &= \frac{1}{4\sqrt{EXE}} (f f X - \frac{ssX}{ff X} | p^X \rangle (\tau^c)^I_3 I_3 \chi_{sX} (p^X) \chi_s (\vec{p}) ,
\end{align*}
\]

The last identity can be obtained by first noticing that

\[
\text{tr}(\tau^c \delta_\chi) = -\frac{2i}{M_0 \rho^2} \left\{ \left( a_4 \frac{\partial}{\partial a_4} - a_a \frac{\partial}{\partial a_a} \right) \delta^\alpha \chi_s (\vec{p}) \right\} .
\] (3.18)

Using the identities (3.17), we get in the Breit frame

\[
\begin{align*}
\langle J_{V}^{i=0}(0) \rangle &= \frac{N_c}{2(2\pi)^3} \xi \delta I_3^X I_3 \chi_{sX} (p^X) \chi_s (\vec{p}) F_{BBX}^1(\vec{q}^2) ,
\langle J_{V}^{i=0}(0) \rangle &= \frac{N_c}{2(2\pi)^3} M_0 \delta I_3^X I_3 \chi_{sX} (p^X) \chi_s (\vec{p}) \chi_s (\vec{p}) F_{BBX}^1(\vec{q}^2) ,
\langle J_{V}^{i=0}(0) \rangle &= \frac{\xi}{2(2\pi)^3} \left( (\tau^c)^I_3 I_3 \chi_{sX} (p^X) \chi_s (\vec{p}) F_{BBX}^1(\vec{q}^2) ,
\langle J_{V}^{i=0}(0) \rangle &= \frac{\alpha}{2(2\pi)^3} \left( M_0 \right) \langle n \rho | \rho | n \rangle \chi_s (\vec{p}) (\tau^c)^I_3 I_3 \chi_{sX} (p^X) \chi_s (\vec{p}) F_{BBX}^1(\vec{q}^2) ,
\end{align*}
\] (3.19)

when the final state is a positive parity resonance, and

\( 5\langle J_{V}^{i=0}(0) \rangle = 0 \).
\[ 5\langle J_{V(SS)}^{L,0}(0) \rangle = \frac{N_c}{8(2\pi)^3 M_0} \hat{q}^2 \left( \delta^{ia} - \frac{q'^a}{q^2} \right) \delta I_3^I \chi_{s_F}(p \chi) \sigma_a \chi_s(\tilde{p}) \alpha F_{BBX}^2(q^2), \]
\[ 5\langle J_{V(SS)}^{L,0}(0) \rangle = \frac{1}{2(2\pi)^3} (\tau_3 I_3^I I_3 q_3 \chi_{s_F}(p \chi) \sigma_i \chi_s(\tilde{p}) \xi F_{BBX}^2(q^2), \]
\[ 5\langle J_{V(SS)}^{L,0}(0) \rangle = \frac{1}{2(2\pi)^3} (\tau_3 I_3^I I_3 \left( \frac{M_0}{3} \right) \langle n_\rho | \rho^2 | n_\rho \rangle \]
\[ \times \hat{q}^2 \left( \delta^{ia} - \frac{q'^a}{q^2} \right) \chi_{s_F}(p \chi) \sigma_a \chi_s(\tilde{p}) \alpha F_{BBX}^2(q^2), \quad (3.20) \]

when the final state has negative parity. In (3.19) and (3.20) we used the definitions

\[ \xi = \left( \frac{f}{f_X} \right) \left[ f_X^2 + \frac{p^2}{f_2} (2x - 1) \right], \]
\[ \left( \frac{M_0}{3} \right) \langle n_\rho | \rho^2 | n_\rho \rangle = \frac{1}{\sqrt{6M_{KK}}} \left[ 1 + 2 \sqrt{1 + \frac{N_c^2}{5}} \right] = \frac{g_{l=1}}{4m_B}, \quad (3.21) \]

and \( \alpha \) was defined in (2.21).

### 3.3. Dirac and Pauli form factors in the Sakai-Sugimoto model

We are going to use the holographic prescription

\[ \eta_{\mu}(p_X, B_X, s_X | J_{V}^{\mu,a}(0) | p, B, s) = \eta_\mu(p_X, B_X, s_X | J_{V(SS)}^{\mu,a}(0) | p, B, s), \quad (3.22) \]

where \( \eta_\mu = (\eta_0, \vec{\eta}) \) is the polarization of the photon and we choose to work with transverse photons satisfying the relation \( \eta_\mu q^\mu = 0 \) in order to avoid the discussion of current anomalies.

Using (3.22), we can compare, the kinematic currents (2.19), (2.20), (2.25) and (2.26) with the Sakai-Sugimoto currents (3.19) and (3.20). For positive parity resonances we get

\[ F_{BBX}^{D,0}(q^2) = \left[ \frac{\xi \alpha + \beta \alpha \frac{q^2}{2M_0}}{\alpha^2 + \beta^2 q^2} \right] N_c F_{BBX}^1(q^2), \]
\[ F_{BBX}^{P,0}(q^2) = -\frac{1}{\kappa_B} \left[ \frac{\beta \xi - \alpha \frac{q^2}{4M_0}}{\alpha^2 + \beta^2 q^2} \right] N_c F_{BBX}^1(q^2), \]
\[ F_{BBX}^{D,3}(q^2) = \left[ \frac{\xi \alpha + \beta \alpha q^2 \left( \frac{M_3}{3} \right)}{\alpha^2 + \beta^2 q^2} \right] F_{BBX}^1(q^2), \]
\[ F_{BBX}^{P,3}(q^2) = -\frac{1}{\kappa_B} \left[ \frac{\beta \xi - \alpha \frac{q^2 (M_3/3)}{3}}{\alpha^2 + \beta^2 q^2} \right] F_{BBX}^1(q^2), \quad (3.23) \]

where \( \alpha \) and \( \beta \) are given in (2.21), (2.22) and \( \xi \) is given in (3.21). For negative parity resonances, we can write

\[ \tilde{F}_{BBX}^{D,0}(q^2) = x \left( \frac{q^2}{2M_0} \right) \left[ \frac{\hat{\beta} \alpha}{\hat{\alpha}^2 + \hat{\beta}^2 q^2} \right] N_c F_{BBX}^2(q^2), \]

15
The resonances correspond to $\lambda$ elastic case to appendix B. The large $\lambda$ limit in the elastic case, corresponding to $n_x = 0$, $m_{B_X} = m_B$ was already considered in [15]. In the non-elastic case, the positive parity resonances correspond to $n_{B_X} = 2, 4, 6, \ldots$. In this case, we get in the large $\lambda$ limit [12]:

\[
\begin{align*}
F_{BBX}^{D,0}(q^2) &= \left[ \frac{m_B}{E} + O\left(\frac{1}{\lambda N_c}\right) \right] N_c F_{BBX}^{1}(q^2), \\
F_{BBX}^{P,0}(q^2) &= \left[ \frac{g_{l=0}}{2} - \frac{m_B}{E} + O\left(\frac{1}{\lambda N_c}\right) \right] N_c F_{BBX}^{1}(q^2), \\
F_{BBX}^{D,3}(q^2) &= \left[ \frac{m_B}{E} + O\left(\frac{1}{\lambda}\right) \right] F_{BBX}^{1}(q^2), \\
F_{BBX}^{P,3}(q^2) &= \frac{g_{l=1}}{2} \left[ 1 + O\left(\frac{1}{\lambda N_c}\right) \right] F_{BBX}^{1}(q^2).
\end{align*}
\]

For negative parity resonances, we have $n_{B_X} = 1, 3, 5, \ldots$. Using the expansions in appendix B is not difficult to show that in the large $\lambda$ limit the form factors reduce to

\[
\begin{align*}
F_{BBX}^{D,0}(q^2) &= \frac{q^2}{4E} g_{l=0} \left[ 1 + O\left(\frac{1}{\lambda N_c}\right) \right] N_c F_{BBX}^{2}(q^2), \\
F_{BBX}^{P,0}(q^2) &= \left(\frac{1}{x}\right) \frac{q^2}{4E} g_{l=0} \left[ 1 + O\left(\frac{1}{\lambda N_c}\right) \right] N_c F_{BBX}^{2}(q^2), \\
F_{BBX}^{D,3}(q^2) &= \frac{q^2}{4E} g_{l=1} \left[ 1 + O\left(\frac{1}{N_c}\right) \right] F_{BBX}^{2}(q^2), \\
F_{BBX}^{P,3}(q^2) &= \left(\frac{1}{x}\right) \frac{q^2}{4E} g_{l=1} \left[ 1 + O\left(\frac{1}{N_c}\right) \right] F_{BBX}^{2}(q^2).
\end{align*}
\]

4. Numerical results for negative parity baryons

We present in this section our numerical results for the negative parity baryons. These include the wave functions, Dirac and Pauli form factors, helicity amplitudes and their contribution to the proton structure function. We are using the Sakai-Sugimoto parameters $M_{KK} = 949\text{MeV}$ and $\kappa = 7.45 \times 10^{-3}$ [11]. We also choose $M_0 = 940\text{MeV}$, for phenomenological reasons.
4.1. Baryon wave functions

First we present in fig. 1 the results for the wave functions of the first excited baryons with negative parity. These wave functions have quantum numbers $B_n = (1, +1/2, 0, n)$ with $n = 2k - 1$ and are odd functions in the radial coordinate $z$. Table 1 shows the mass spectrum of the first negative parity baryonic resonances. The spectrum of positive parity baryonic resonances can be found in [12].

![Figure 1: (Normalized) wave functions $\Psi_{B_{2k-1}}(z)$ for the first six parity odd baryon states.](image)

4.2. Dirac and Pauli Form factors

In the previous section we extracted from holography the Dirac and Pauli form factors that describe the production of negative parity baryons. Interestingly, our results (3.29) show that the Dirac and Pauli form factors depend on only one form factor $F_{BBX}^2(q^2)$ defined by (3.15). This is a feature that has also appeared in previous holographic approaches to
Table 1: Some numerical values for the masses of negative parity baryon states

emission\(^3\) The form factor \(F_{BBX}^2(\vec{q}^2)\) in (3.15) can be written as

\[
F_{BBX}^2(\vec{q}^2) = \sum_n g_{v^n B_0 B_i} \frac{1}{\lambda_{2n-1}} \langle n_{B_X} | \partial_Z \psi_{2n-1}(Z) | n_B \rangle ,
\]

are the effective couplings between a vector meson, a negative parity baryon and the proton. We show in table 2 our numerical results for these effective couplings. Identifying the first negative parity resonance with the experimentally observed \(S_{11}(1535)\), our numerical result for the coupling constant \(g_{v^1 B_0 B_1} = -1.889\) should be useful to describe the decay of \(S_{11}(1535)\) into a \(\rho\) meson and a proton. This result is compatible with recent analysis from experimental data [45] where \(0.79 < |g_{v^1 B_0 B_1}| < 2.63\). The vector meson squared masses \(\lambda_{2n-1}\) and decay constants \(g_{v^n}\) are also shown in table 2.

| \(n\) | 1   | 2   | 3   | 4   | 5   | 6   | 7  | 8  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|
| \(\lambda_{2n-1}\) | 0.6693 | 2.874 | 6.591 | 11.80 | 18.49 | 26.67 | 36.34 | 47.49 |
| \(g_{v^n B_0 B_1}/\sqrt{M_{KK}}\) | 2.109 | 9.108 | 20.80 | 37.15 | 58.17 | 83.83 | 114.2 | 149.1 |
| \(g_{v^n B_0 B_3}\) | -1.889 | 1.182 | -0.562 | 0.1381 | 0.04057 | -0.05213 | 0.01239 | 0.009893 |
| \(g_{v^n B_0 B_5}\) | 1.038 | -0.841 | 0.6132 | -0.3325 | 0.08703 | 0.04209 | -0.05382 | 0.01409 |
| \(g_{v^n B_0 B_7}\) | -0.6432 | 0.5892 | -0.5217 | 0.3802 | -0.1907 | 0.02706 | 0.05097 | -0.04458 |
| \(g_{v^n B_0 B_9}\) | 0.429 | -0.4239 | 0.4223 | -0.3644 | 0.2416 | -0.09421 | -0.01629 | 0.05276 |
| \(g_{v^n B_0 B_9}\) | -0.3005 | 0.3132 | -0.3386 | 0.3273 | -0.2571 | 0.1417 | -0.0266 | -0.0404 |

Table 2: Coupling constants between vector mesons and baryons when the initial state is the proton and the final state has negative parity.

The Dirac and Pauli form factors depend on the magnetic \(g_I\) factors whose numerical values in the Sakai-Sugimoto model are given by

\[
g_{I=0} \approx 1.684, \quad g_{I=1} \approx 7.031.
\]
parity baryon states. We show our results for the first three excited states in figure 2. As a general feature, the form factors go to zero as $q^2 \to 0$, reach a maximum and then decay for large $q^2$. Note that some of the form factors are non-positive.

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{form_factors.png}
\caption{Dirac and Pauli form factors $\tilde{F}_{B_0B_{2j-1}}^D(q^2)$ for the first three negative parity baryon states. The momentum transfer $q^2$ is given in $(\text{GeV})^2$.}
\end{figure}

4.3. Helicity amplitudes: comparison with JLab-CLAS data

In the large $\lambda$ limit, the transverse helicity amplitudes take the form

\begin{align}
\tilde{G}_{BBX}^{+}(q^2) &\approx -\sqrt{2} \left[ \tilde{F}_{BBX}^D(q^2) + \frac{m_{BX} - m_B}{m_B} \tilde{F}_{BBX}^P(q^2) \right], \\
\tilde{A}_{BBX}^{1/2}(q^2) &\approx \frac{e}{\sqrt{2(m_{BX} - m_B)}} \tilde{G}_{BBX}^{+}(q^2),
\end{align}

Unfortunately, in the large $\lambda$ limit we cannot say too much about the longitudinal helicity
amplitudes because we obtain
\[
\tilde{G}_{BBX}^0(q^2) \approx \sqrt{q^2} \left[ \frac{m_{BX} - m_B}{q^2} \tilde{F}_{BBX}^D(q^2) - \frac{1}{2m_B} \tilde{F}_{BBX}^P(q^2) \right] \approx 0,
\]
\[
\tilde{S}_{BBX}^{1/2}(q^2) \approx e \sqrt{\frac{m_B}{q^2}} \tilde{G}_{BBX}^0(q^2) \approx 0.
\] (4.5)

This result seems to be consistent with the fact that the experimental data available for these helicity amplitudes indicates a strong contribution from meson clouds [46]. This kind of effect necessitates the investigation of loop corrections of order $1/\lambda$ in electromagnetic scattering. The $1/\lambda$ corrections would not only modify our results but also the standard results on the elastic electromagnetic form factors.

Some of the meson cloud contributions to the helicity amplitudes $\tilde{A}_{BBX}^{1/2}$ and $\tilde{S}_{BBX}^{1/2}$ for the resonance $S_{11}(1535)$ were calculated by the EBAC group [49], fitting the dynamical coupled-channel model [48] with experimental data. The EBAC result [49] was displayed nicely in [50], where the authors removed the meson cloud contributions from the dressed helicity amplitudes and presented the EBAC bare helicity amplitudes. In figure 3, we present our result for the transverse helicity amplitude $\tilde{A}_{BBX}^{1/2}(q^2)$ for the first negative parity resonance and also show the EBAC results [49,50] and recent experimental data from the JLAB-CLAS collaboration [46] for comparison. As discussed in section 4.2, this resonance can be identified with the experimentally observed $S_{11}(1535)$. Despite the limitations of our model (the large $\lambda$ limit), we find good agreement with the EBAC results for bare helicity amplitudes and reasonable agreement with JLAB-CLAS experimental data. This is to be expected since the EBAC data is available for $q^2 \leq 1.5(\text{GeV})^2$, which coincides with the regime of validity of the Sakai-Sugimoto model where we expect our results to be reliable, whereas the JLAB-CLAS data extends to higher $q^2$ beyond the regime of validity. Furthermore, the EBAC and JLAB-CLAS results clearly demonstrate the importance of $1/\lambda$-corrections from meson cloud contributions in the non-perturbative regime, which means that we need to go beyond tree level to get a better than qualitative agreement with experimental results for helicity amplitudes. Nevertheless, this is a very encouraging result in view of our long-term project of investigating resonance production in holographic models.

In figure 4 we show our results for $\tilde{G}_{BB_1}^0(q^2)$. Note that, in our model, $\tilde{A}_{BBX}^{1/2}(q^2) \rightarrow 0$ as $q^2 \rightarrow 0$, contrary to the experimental results reported in [51], p. 1155,
\[
\tilde{A}_{BB_1}^{1/2\exp.}(q^2) = 0.09 \pm 0.03 \text{ (GeV)}^{-1/2}.
\] (4.6)

This can be understood from the fact that the baryons in our model are very massive and thus stable in the large $\lambda$ limit. Therefore the currents that we construct do not account for the decay of baryonic resonances.

\[\text{See [47] for a discussion regarding pion loop corrections in baryon electromagnetic form factors.}\]
Figure 3: Helicity amplitude $\tilde{A}_{BB_1}^{1/2}(q^2)$ (in units $10^{-3}(\text{GeV})^{-1/2}$) plotted versus $q^2$ in $(\text{GeV})^2$. The JLAB-CLAS experimental data (red dots) was taken from ref. [46], while the EBAC bare amplitude results (blue triangles) were taken from [49,50].

Figure 4: Helicity amplitude $\tilde{G}_{BB_1}^{+}(q^2)$ (in units $10^{-3}(\text{GeV})^{-1/2}$) plotted versus $q^2$ in $(\text{GeV})^2$.

4.4. The proton structure function

4.4.1. A first approximation

Assuming approximate continuity of the mass distribution, we can now approximate the delta distributions in the following way:

$$
\sum_{B_X} \delta[m_{B_X}^2 - s] = \sum_{n} \delta[m_n^2 - m_{\bar{n}}^2] = \int dn \left[ \left| \frac{\partial m_n^2}{\partial n} \right| \right]^{-1} \delta(n - \bar{n})
$$

$$
= \left[ \left| \frac{\partial m_n^2}{\partial n} \right| \right]_{n=\bar{n}}^{-1} =: f(\bar{n}),
$$

(4.7)
with the definition
\[ s := -(p + q)^2 = m_{B_0}^2 + q^2 \left( \frac{1}{x} - 1 \right). \] (4.8)

Therefore we have to evaluate the Regge trajectory of the baryon spectrum in order to calculate \( \frac{\partial m_n^2}{\partial n} \). We find from (3.7)
\[ \frac{\partial m_n^2}{\partial n} = \left( \frac{4}{\sqrt{6}} \tilde{M}_0 M_{KK} + \frac{4}{3} n M_{KK}^2 \right), \] (4.9)

where \( \tilde{M}_0 \) can be chosen to match, e.g. the proton mass \( m_{B_0} \) and \( n := n_z \).

Using the approximation (4.7) we get in the large \( \lambda \) limit the structure functions
\[ \tilde{F}_1(q^2, x) \approx f(\bar{n}) m_B^2 \left( \tilde{G}^+_{BB_B}(q^2) \right)^2, \]
\[ \tilde{F}_2(q^2, x) \approx f(\bar{n}) \left( \frac{q^2}{2x} \right) \left( 1 + \frac{q^2}{4m_B^2 x^2} \right)^{-1} \left( \tilde{G}^+_{BB_B}(q^2) \right)^2. \] (4.10)

We plot in figures 5 and 6 the structure functions obtained from (4.10) as a function of \( q^2 \) and \( x \). We also demonstrate the violation of the Callan-Gross relation at intermediate values of \( x \) in [7].

![Figure 5](image)

Figure 5: Structure functions \( F_{1,2}(q^2) \) for \( x = 0.3 \) (orange, solid), \( x = 0.1 \) (red, dashed) and \( x = 0.01 \) (green, dotted).

4.4.2. A realistic approach near a resonance peak

The results shown in figures 5 and 6 were obtained using the naive approximation (4.7). Alternatively, if we are only interested in the region of \( q^2 \) where a resonance is produced we can approximate the Delta distribution by a Lorentzian function [37]:
\[ \delta[m_{B_X}^2 - s] \approx \frac{\Gamma_{B_X}}{4\pi m_{B_X}} \left[ (\sqrt{s} - m_{B_X})^2 + \frac{\Gamma_{B_X}^2}{4} \right]^{-1}, \] (4.11)

where \( \Gamma_{B_X} \) is the decay width of the resonance \( B_X \). Identifying the first negative parity baryonic resonance \( B_1 \) with the experimentally observed \( S_{11}(1535) \) and using the decay
Figure 6: Structure functions $\Bar{F}_{1,2}(x)$ for $q^2 = 3(\text{GeV})^2$ (purple, solid), $q^2 = 2(\text{GeV})^2$ (blue, dotdashed), $q^2 = 1(\text{GeV})^2$ (green, dotted) and $q^2 = 0.5(\text{GeV})^2$ (red, dashed).

Figure 7: Callan-Gross ratio $R_{CG}(x)$ for $q^2 = 3(\text{GeV})^2$ (purple, solid), $q^2 = 2(\text{GeV})^2$ (blue, dotdashed), $q^2 = 1(\text{GeV})^2$ (green, dotted) and $q^2 = 0.5(\text{GeV})^2$ (red, dashed).

width $\Gamma_{B_1} = 150$ Mev, estimated from experimental data in [51], we obtain the results for the structure functions shown in Figure 8. Note that the structure functions have improved by an order of magnitude. Unfortunately, we cannot follow this procedure for the higher resonances because there are no experimental results available for the decay widths.

The results for the proton structure functions obtained in this paper represent only a small fraction of possible final states, namely single final states with spin $\frac{1}{2}$ and negative parity. If we include the contribution from final states with positive parity [12] as well as final states with higher spin\footnote{Usually one expects a high contribution coming from the production of $\Delta$ resonances. See [18] for $\Delta$ resonances in the Sakai-Sugimoto model and [19] for higher spin resonances.} and pion production relevant in this kinematical regime, we should get a better/more complete picture of the proton structure functions and significantly improve the comparison with experimental data.
Figure 8: Structure functions $F_{1,2}(x)$ for $q^2 = 3(\text{GeV})^2$ (purple, solid), $q^2 = 2(\text{GeV})^2$ (blue, dotdashed), $q^2 = 1(\text{GeV})^2$ (green, dotted) near the first negative parity resonance $B_1$.

5. Conclusions and Outlook

In this article we have presented a treatment of non-elastic proton electromagnetic scattering for the special case when baryonic resonances of negative parity are produced as the single-particle final state of the scattering process. We have in turn applied the Sakai-Sugimoto model of holographic baryons in the large $\lambda$ limit to compute the relevant form factors and proton structure functions. Our numerical results show good agreement with available experimental data. One should, however, keep in mind the limitations of the (holographic) description of baryons in large-$N_c$ QCD [52], which fully apply to the non-relativistic (large $\lambda$) model discussed herein as well. It would be very interesting to calculate $1/\lambda$- and other corrections to the current model and to study other scattering processes within the Sakai-Sugimoto model. Finally, it would be fruitful to investigate baryons and their resonance production in more recent holographic models, e.g., [53,54,55]. We leave this for future work.

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A. Some frames in inelastic scattering

A.1. The Breit frame

Consider the scattering between a virtual photon and a hadron in the hadron rest frame. After two rotations we can set the spatial momentum of the photon to the $x^3$ direction so

$$p^\mu = (m_B, 0, 0, 0)$$
\[ q^\mu = (q_0, 0, 0, q_3), \quad (A.1) \]

and we choose \( q_3 > 0 \). The virtuality and Bjorken variable are in this frame given by

\[ Q^2 = q_3^2 - q_0^2, \quad x = -\frac{Q^2}{2m_B q_0}. \quad (A.2) \]

Now we perform a boost in the \( x^3 \) direction so that

\[ p'^\mu = (\gamma m_B, 0, 0, -\beta \gamma m_B), \quad q'^\mu = (\gamma q_0 - \beta \gamma q_3, 0, 0, -\beta \gamma q_0 + \gamma q_3). \quad (A.3) \]

The Breit Frame is defined by the condition \( q'_0 = 0 \) so that

\[ \beta = \frac{q_0}{q_3} = \frac{q_0}{\sqrt{q_0^2 + Q^2}}, \quad \gamma = \frac{\sqrt{q_0^2 + Q^2}}{Q}, \quad q'_3 = Q, \quad (A.4) \]

and we arrive to

\[ p'^\mu = (\sqrt{m_B^2 + p^2}, 0, 0, p), \quad q'^\mu = (0, 0, 0, Q), \quad (A.5) \]

with

\[ p = -\frac{Q}{2x}. \quad (A.6) \]

**A.2. The resonant rest frame**

In the resonant frame we have

\[ p^\mu = (E_R, -\vec{q}), \quad q^\mu = (m_{Bx} - E_R, \vec{q}_R), \quad (p + q)^\mu = (m_{Bx}, 0). \quad (A.7) \]

We can write the energy and the momentum squared in terms of the squared masses and virtuality

\[ E_R = \frac{1}{2m_{Bx}} [q^2 + m_{Bx}^2 + m_B^2], \quad |\vec{q}_R|^2 = (E_R - m_B)(E_R + m_B). \quad (A.8) \]

**B. Expansions at large \( \lambda \)**

The relevant large \( \lambda \) expansions for the non-elastic case are given by

\[ q^2 \sim \mathcal{O}(1), \quad m_B \sim \mathcal{O}(\lambda N_c) \quad , \]

\[ m \sim \mathcal{O}(\lambda N_c) \quad , \]

\[ E \sim \mathcal{O}(\lambda N_c) \quad , \]

\[ p \sim \mathcal{O}(\lambda N_c). \]
\[ m_{Bx} = m_B + 2 \frac{n_x M_{KK}}{\sqrt{6}} = m_B \left[ 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right) \right], \]

\[ x = \left( \frac{\sqrt{6}}{4} \right) \frac{q^2}{m_B M_{KK} n_x} \left[ 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right) \right] = \mathcal{O} \left( \frac{1}{\lambda N_c} \right), \]

\[ E = m_B \sqrt{1 + \frac{2 n_x^2 M_{KK}^2}{3 q^2}} \left[ 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right) \right], \]

\[ \langle \rho^2 \rangle = \frac{g_{t=1}}{4m_B} = \mathcal{O}(N_c), \quad \alpha = 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right), \]

\[ \beta = \frac{1}{2m_B} \left[ 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right) \right] = \mathcal{O} \left( \frac{1}{\lambda N_c} \right), \]

\[ \hat{\alpha} = \left( \frac{f}{f_x} \right) \left( \frac{1}{2E} \right) \left( \frac{f_x^2}{f^2} + 1 - 2x \right) = \frac{1}{E} \left[ 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right) \right], \]

\[ \hat{\beta} = \frac{2x}{q^2} \xi = \left( \frac{2x}{q^2} \right) \frac{m_B}{E} \left[ 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right) \right] = \mathcal{O} \left( \frac{1}{\lambda N_c} \right), \]

\[ \xi = \frac{m_B}{E} + \mathcal{O} \left( \frac{1}{\lambda N_c} \right), \quad \alpha^2 + \beta^2 q^2 = 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right), \]

\[ \xi + \beta \alpha \frac{q^2}{4M_0} = \frac{m_B}{E} + \mathcal{O} \left( \frac{1}{\lambda N_c} \right), \]

\[ -\frac{1}{\kappa_B} \left( \beta \xi - \alpha^2 \frac{M_0}{4M_0} \right) = \frac{g_{t=0}}{2} - \frac{m_B}{E} + \mathcal{O} \left( \frac{1}{\lambda N_c} \right), \]

\[ \xi + \beta \alpha \frac{q^2}{4M_0} \langle \rho^2 \rangle = \frac{m_B}{E} + \mathcal{O} \left( \frac{1}{\lambda} \right), \]

\[ -\frac{1}{\kappa_B} \left[ \beta \xi - \alpha^2 \frac{M_0}{3} \langle \rho^2 \rangle \right] = \frac{g_{t=1}}{2} \left[ 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right) \right], \]

\[ \hat{\alpha}^2 + \beta^2 q^2 = \frac{4x^2}{q^2} \left[ 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right) \right], \]

\[ x \left( \frac{q^2}{2M_0} \right) \hat{\beta} = \frac{x^2}{E} g_{t=0} \left[ 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right) \right], \]

\[ x \left( \frac{1}{2M_0 \kappa_B} \right) \hat{\alpha} = \frac{x}{E} g_{t=0} \left[ 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right) \right], \]

\[ 2x \left[ \frac{M_0}{3} \langle \rho^2 \rangle \hat{\beta} \alpha q^2 - \hat{\xi} \right] = \frac{x^2}{E} g_{t=1} \left[ 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right) \right], \]

\[ 2x \left( \frac{1}{\kappa_B} \right) \left[ \frac{M_0}{3} \langle \rho^2 \rangle \hat{\alpha} + \hat{\xi} \right] = \frac{x}{E} g_{t=1} \left[ 1 + \mathcal{O} \left( \frac{1}{\lambda N_c} \right) \right]. \]
[3] J. Erlich, “How Well Does AdS/QCD Describe QCD?,” Int. J. Mod. Phys. A 25, 411 (2010) [arXiv:0908.0312 [hep-ph]].

[4] S. J. Brodsky and G. de Teramond, “AdS/QCD and Light Front Holography: A New Approximation to QCD,” Chin. Phys. C 34, 1 (2010) [arXiv:1001.1978 [hep-ph]].

[5] U. Gursoy, E. Kiritsis, L. Mazzanti, G. Michalogiorgakis and F. Nitti, “Improved Holographic QCD,” Lect. Notes Phys. 828, 79 (2011) [arXiv:1006.5461 [hep-th]].

[6] K. Peeters, M. Zamaklar, “The String/gauge theory correspondence in QCD,” Eur. Phys. J. ST 152, 113-138 (2007). [arXiv:0708.1502 [hep-ph]].

[7] J. Erdmenger, N. Evans, I. Kirsch, E. Threlfall, “Mesons in Gauge/Gravity Duals - A Review,” Eur. Phys. J. A35, 81-133 (2008). [arXiv:0711.4467 [hep-ph]].

[8] S. S. Gubser, A. Karch, “From gauge-string duality to strong interactions: A Pedestrian’s Guide,” Ann. Rev. Nucl. Part. Sci. 59, 145-168 (2009). [arXiv:0901.0935 [hep-th]].

[9] J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, “Gauge/String Duality, Hot QCD and Heavy Ion Collisions,” arXiv:1101.0618 [hep-th].

[10] T. Sakai and S. Sugimoto, “Low energy hadron physics in holographic QCD,” Prog. Theor. Phys. 113, 843 (2005) [arXiv:hep-th/0412141].

[11] T. Sakai and S. Sugimoto, “More on a holographic dual of QCD,” Prog. Theor. Phys. 114, 1083 (2005) [arXiv:hep-th/0507073].

[12] C. A. B. Bayona, H. Boschi-Filho, N. R. F. Braga, M. Ihl and M. A. C. Torres, “Generalized baryon form factors and proton structure functions in the Sakai-Sugimoto model,” Nucl. Phys. B. 866, 124 (2013) [arXiv:1112.1439 [hep-ph]].

[13] I. Aznauryan, V. D. Burkert, T. -S. H. Lee and V. I. Mokeev, “Results from the N* program at JLab,” J. Phys. Conf. Ser. 299, 012008 (2011) [arXiv:1102.0597 [nucl-ex]].

[14] D. K. Hong, M. Rho, H. U. Yee and P. Yi, “Dynamics of baryons from string theory and vector dominance,” JHEP 0709, 063 (2007) [arXiv:0705.2632 [hep-th]].

[15] K. Hashimoto, T. Sakai, S. Sugimoto, “Holographic Baryons: Static Properties and Form Factors from Gauge/String Duality,” Prog. Theor. Phys. 120, 1093-1137 (2008). [arXiv:0806.3122 [hep-th]].

[16] K. -Y. Kim and I. Zahed, “Electromagnetic Baryon Form Factors from Holographic QCD,” JHEP 0809, 007 (2008) [arXiv:0807.0033 [hep-th]].

[17] Z. Abidin and C. E. Carlson, “Nucleon electromagnetic and gravitational form factors from holography,” Phys. Rev. D 79, 115003 (2009) [arXiv:0903.4818 [hep-ph]].

[18] H. R. Grigoryan, T. -S. H. Lee and H. -U. Yee, “Electromagnetic Nucleon-to-Delta Transition in Holographic QCD,” Phys. Rev. D 80, 055006 (2009) [arXiv:0904.3710 [hep-ph]].
[19] J. Park and P. Yi, “A Holographic QCD and Excited Baryons from String Theory,” JHEP 0806, 011 (2008) arXiv:0804.2926 [hep-th].

[20] H. C. Ahn, D. K. Hong, C. Park and S. Siwach, “Spin 3/2 Baryons and Form Factors in AdS/QCD,” Phys. Rev. D 80, 054001 (2009) arXiv:0904.3731 [hep-ph].

[21] J. Polchinski, M. J. Strassler, “Deep inelastic scattering and gauge / string duality,” JHEP 0305, 012 (2003). [hep-th/0209211].

[22] C. A. Ballon Bayona, H. Boschi-Filho, N. R. F. Braga, “Deep inelastic scattering from gauge string duality in the soft wall model,” JHEP 0803, 064 (2008). arXiv:0711.0221 [hep-th].

[23] C. A. Ballon Bayona, H. Boschi-Filho, N. R. F. Braga, “Deep inelastic scattering from gauge string duality in D3-D7 brane model,” JHEP 0809, 114 (2008). arXiv:0807.1917 [hep-th].

[24] B. Pire, C. Roiesnel, L. Szymanowski, S. Wallon, “On AdS/QCD correspondence and the partonic picture of deep inelastic scattering,” Phys. Lett. B670, 84-90 (2008). arXiv:0805.4346 [hep-ph].

[25] C. A. Ballon Bayona, H. Boschi-Filho, N. R. F. Braga, M. A. C. Torres, “Deep inelastic scattering for vector mesons in holographic D4-D8 model,” JHEP 1010, 055 (2010). arXiv:1007.2448 [hep-th].

[26] R. C. Brower, J. Polchinski, M. J. Strassler, C. -ITan, “The Pomeron and gauge/string duality,” JHEP 0712, 005 (2007). [hep-th/0603115].

[27] Y. Hatta, E. Iancu, A. H. Mueller, “Deep inelastic scattering at strong coupling from gauge/string duality: The Saturation line,” JHEP 0801, 026 (2008). arXiv:0710.2148 [hep-th].

[28] C. A. Ballon Bayona, H. Boschi-Filho, N. R. F. Braga, “Deep inelastic structure functions from supergravity at small x,” JHEP 0810, 088 (2008). arXiv:0712.3530 [hep-th].

[29] L. Cornalba and M. S. Costa, “Saturation in Deep Inelastic Scattering from AdS/CFT,” Phys. Rev. D 78, 096010 (2008) arXiv:0804.1562 [hep-ph].

[30] L. Cornalba, M. S. Costa, J. Penedones, “Deep Inelastic Scattering in Conformal QCD,” JHEP 1003, 133 (2010). arXiv:0911.0043 [hep-th].

[31] R. C. Brower, M. Djuric, I. Sarcevic, C. -ITan, “String-Gauge Dual Description of Deep Inelastic Scattering at Small-x,” JHEP 1011, 051 (2010). arXiv:1007.2259 [hep-ph].

[32] Y. Hatta, E. Iancu and A. H. Mueller, “Deep inelastic scattering off a N=4 SYM plasma at strong coupling,” JHEP 0801, 063 (2008) arXiv:0710.5297 [hep-th].
[33] C. A. B. Bayona, H. Boschi-Filho and N. R. F. Braga, “Deep inelastic scattering off a plasma with flavour from D3-D7 brane model,” Phys. Rev. D 81, 086003 (2010) [arXiv:0912.0231 [hep-th]].

[34] E. Iancu and A. H. Mueller, “Light-like mesons and deep inelastic scattering in finite-temperature AdS/CFT with flavor,” JHEP 1002, 023 (2010) [arXiv:0912.2238 [hep-th]].

[35] Y. Y. Bu and J. M. Yang, “Structure function of holographic quark-gluon plasma: Sakai-Sugimoto model versus its non-critical version,” Phys. Rev. D 84, 106004 (2011) [arXiv:1109.4283 [hep-th]].

[36] I. G. Aznauryan, V. D. Burkert and T. -S. H. Lee, “On the definitions of the gamma* N → N* helicity amplitudes,” [arXiv:0810.0997 [nucl-th]].

[37] C. E. Carlson and N. C. Mukhopadhyay, “Bloom-Gilman duality in the resonance spin structure functions,” Phys. Rev. D 58, 094029 (1998) [hep-ph/9801205].

[38] P. Stoler, “Baryon form-factors at high Q**2 and the transition to perturbative QCD,” Phys. Rept. 226, 103 (1993).

[39] A. V. Manohar, “An introduction to spin dependent deep inelastic scattering,” [arXiv:hep-ph/9204208].

[40] E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2, 253 (1998) [arXiv:hep-th/9802150].

[41] H. Hata, T. Sakai, S. Sugimoto, S. Yamato, “Baryons from instantons in holographic QCD,” Prog. Theor. Phys. 117, 1157 (2007). [hep-th/0701280 [hep-th]].

[42] H. Boschi-Filho, N. R. F. Braga, M. Ihl and M. A. C. Torres, “Relativistic baryons in the Skyrme model revisited,” Phys. Rev. D 85, 085013 (2012) [arXiv:1111.2287 [hep-th]].

[43] C. A. Ballon Bayona, H. Boschi-Filho, N. R. F. Braga and M. A. C. Torres, “Form factors of vector and axial-vector mesons in holographic D4-D8 model,” JHEP 1001, 052 (2010) [arXiv:0911.0023 [hep-th]].

[44] C. A. B. Bayona, H. Boschi-Filho, M. Ihl and M. A. C. Torres, “Pion and Vector Meson Form Factors in the Kuperstein-Sonnenschein holographic model,” JHEP 1008, 122 (2010) [arXiv:1006.2363 [hep-th]].

[45] J. -j. Xie, C. Wilkin and B. -s. Zou, “On the Coupling Constant for N*(1535)N(rho),” Phys. Rev. C 77, 058202 (2008) [arXiv:0802.2802 [nucl-th]].

[46] I. G. Aznauryan et al. [CLAS Collaboration], “Electroexcitation of nucleon resonances from CLAS data on single pion electroproduction,” Phys. Rev. C 80, 055203 (2009) [arXiv:0909.2349 [nucl-ex]].
[47] A. Cherman, T. D. Cohen and M. Nielsen, “Model Independent Tests of Skyrmions and Their Holographic Cousins,” Phys. Rev. Lett. 103, 022001 (2009) [arXiv:0903.2662 [hep-ph]].

[48] A. Matsuyama, T. Sato and T. -S. H. Lee, Phys. Rept. 439, 193 (2007) [nucl-th/0608051].

[49] B. Julia-Diaz, H. Kamano, T. -S. H. Lee, A. Matsuyama, T. Sato and N. Suzuki, Phys. Rev. C 80, 025207 (2009) [arXiv:0904.1918 [nucl-th]].

[50] G. Ramalho and M. T. Pena, Phys. Rev. D 84, 033007 (2011) [arXiv:1105.2223 [hep-ph]].

[51] J. Beringer et al. [Particle Data Group Collaboration], Phys. Rev. D 86, 010001 (2012).

[52] E. Witten, “Baryons and branes in anti-de Sitter space,” JHEP 9807, 006 (1998) [arXiv:hep-th/9805112].

[53] S. Kuperstein and J. Sonnenschein, “A New Holographic Model of Chiral Symmetry Breaking,” JHEP 0809, 012 (2008) [arXiv:0807.2897 [hep-th]].

[54] A. Dymarsky, S. Kuperstein and J. Sonnenschein, “Chiral Symmetry Breaking with non-SUSY D7-branes in ISD backgrounds,” JHEP 0908, 005 (2009) [arXiv:0904.0988 [hep-th]].

[55] M. Ihl, A. Kundu and S. Kundu, “Back-reaction of Non-supersymmetric Probes: Phase Transition and Stability,” arXiv:1208.2663 [hep-th].