Light meson correlation functions near the deconfining transition on anisotropic lattices

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We study the light hadron correlators near the deconfining transition by extracting the spectral function on quenched anisotropic lattices. We adopt the method successfully applied to the charmonium systems: the use of the smeared operators and the analysis method which is composed of the maximum entropy method and the $\chi^2$ fitting assuming several forms. The numerical simulations are performed on lattices of $a_{s}^{-1} \simeq 2$ GeV and $a_{s}/a_{t} = 4$, with the clover quark around the strange quark mass. Towards applications to the plasma phase, we check the reliability of our procedure by examining how the results for the correlators below $T_c$ ($N_t = 32$) changes under variation of input parameters such as the smearing function.

1. Introduction

The theoretical expectation for properties of quarks and gluons in the deconfinement phase is strongly required for an interpretation of experimental results in the heavy ion collision experiments. Contrary to a naive quark gluon plasma (QGP) picture such as the gas of weakly interacting quarks and gluons, several model calculations inspired by QCD indicate nonperturbative features of QGP. For example, an NJL model calculation suggests that the hadronic excitations survive as soft modes even above the critical temperature \cite{1}.

The lattice QCD can in principle solve the problem in a model independent way, since the dynamical properties of excitation modes can be extracted from the Matsubara Green functions \cite{2}. Although in practice such studies are quite nontrivial even at qualitative level, recent technical development such as anisotropic lattices and computational progress have enabled us to study dynamical properties of hadrons at finite temperature \cite{3,4}. In addition, several procedures such as the maximum entropy method (MEM) \cite{5} have been developed to analyze the spectral function with which we can directly probe the excitation modes of the plasma phase. These procedures have been applied to the charmonium systems and suggest that hadronic excitations survive even above the critical temperature \cite{6,7,8}.

In this work we apply the method which was adopted in Ref. \cite{6} to systems with light quarks. In this report we focus on a check of reliability for applied analysis methods at finite temperature, as a preparation for future applications to the deconfinement phase.

2. Our approach

Here we briefly explain our analysis procedure and its criteria for a reliability, which were in detail described in Ref. \cite{6}.

In order to extract reliable information on the spectral function, we use MEM and $\chi^2$ fit method in a complementary manner. After a rough estimation of spectral function by MEM, we evaluate properties of the mode such as mass and width more quantitatively by the $\chi^2$ fit assuming several fitting forms based on MEM results. As more sophisticated form of the $\chi^2$ fit we adopt the constrained curve fitting (CCF) \cite{9} whose prior knowledge is also estimated with the MEM results.

To verify the reliability of our analysis, we require the following criteria for the extraction methods: (1) The stability of the spectral func-
tion against variations of input parameters or model functions. (2) The stability of the result for the correlators at $T = 0$ under restriction of the degrees of freedom. For the latter, since the $T = 0$ correlators are not ready at present, we instead analyze those below $T_c$ which should produce similar results as at $T = 0$.

In the previous work for the charmonium, in order to satisfy the criteria we employ the smearing operators which enhance the low frequency part of the meson correlators. However the smeared operators may produce artificial peaks in the spectral function [10]. In order to distinguish an artificial peak from the genuine physical one we apply several smearing functions and examine the stability of the results.

3. Numerical results

We use quenched lattices of the sizes $20^3 \times N_t$, where $N_t = 160$ and 32 which roughly correspond to $T \simeq 0$ and $0.9T_c$, respectively. The zero temperature lattice, and the setup of lattice parameters are the same as those used in Ref. [11]. The gauge configurations are generated with the Wilson plaquette action at $\beta = 6.10$ with the renormalized anisotropy $\xi = a_\sigma / a_t = 4$. The spatial cutoff set by the hadronic radius $r_0$ is $a_\sigma^{-1} = 2.030(13)$ GeV. $N_t = 28$ is close to the phase transition. The quark action is the $O(a)$ improved Wilson action [11], with the hopping parameter roughly corresponding to the strange quark mass. We have 500 configurations at $T = 0$ and 1000 configurations at $T > 0$.

We measure the point correlators at $T = 0$ and the smeared correlators at $T \simeq 0.9T_c$. For the latter, the operators are spatially extended by Gaussian functions $\phi(\vec{x}) \propto \exp(a|\vec{x}|^2)$ with $a = 0.16$ and 0.037 which are referred as “smeared” and “half-smeared”, respectively.

First we present a result for the point operators. Using the correlators at $T = 0$ we test the applicability to finite temperatures by varying the number of degrees of freedom. Figure 1 shows the $t_{\max}$ dependence of the MEM results, where the fit ranges are restricted to $t = 1 \sim t_{\max}$. The result at $t_{\max} = 16$ can not reproduce even the position of the lowest peak, while $t_{\max} = 16$ cor-

![Figure 1. $t_{\max}$ dependence of the spectral function for the point correlator in PS channel.](image1)

![Figure 2. The spectral function for the smeared PS correlator at $T \simeq 0.9T_c$. The default model is given by $m(\omega) = m_{DM}\omega^2$.](image2)

responds to the similar situation we encounter at $T \sim 0.9T_c$. This is one of the reasons why we give up to use the point operators to explore finite temperatures.

The MEM result with the smeared PS correlator at $T \simeq 0.9T_c$ are presented in Fig. 2. The lowest and next-lowest peaks are located at almost the same positions as at $T = 0$ which is expected to have a similar result to that at $T \simeq 0.9T_c$. This is encouraging for applications of the MEM to the systems at $T > 0$. Since the results are sensitive to the default model function (as input parameters), it is difficult to obtain quantitative results...
only with MEM.

Based on the MEM results in Fig. 2 we assume multi-peak functions with the Breit-Wigner type forms,

\[ A(\omega) = \sum_{i=1}^{N_{\text{term}}} \frac{\omega^2 m_i^\gamma_i R_i}{(\omega^2 - m_i^2)^2 + m_i^2 \gamma_i^2}, \]

as fitting functions and estimate the prior knowledge of the CCF. In Fig. 3 the fit results for masses and widths of the lowest and next-lowest peaks are displayed, where the results for smeared and half-smeared correlators are plotted with different symbols. Although with a sufficiently large \( N_{\text{term}} \) (\( \geq 4 \)) the results are stable, the differences between the values for the smeared and half-smeared correlators are larger than the statistical errors. Furthermore the results are rather sensitive to the prior knowledge. These results indicate that we need further investigations of CCF to obtain quantitative results with keeping systematic uncertainties under control at finite temperature.

Following our strategy, we have to perform these reliability checks with correlators at \( T = 0 \) and to examine the \( t_{\text{max}} \) dependence of the systematic uncertainties. We also need further examination of the smearing operators, because the suppression of high frequency part of the correlators may not be sufficient with the present smearing operators. This might be a reason that CCF does not work stably at \( T \simeq 0.9T_c \). These problems must be clarified for reliable analysis of the hadronic correlators at finite temperature, in particular in the plasma phase.

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