The Two-Pion Exchange $NN$ Potential in Nuclear Matter and Nuclear Stability

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Abstract

A meson exchange model of the $\pi\pi$ interaction which fits free $\pi\pi$ scattering data is used to calculate the interactions of pions in nuclear matter as a function of nuclear density. Polarization of the nuclear medium by the pions results in a marked increase in the s-wave $\pi\pi$ attraction at low energy. The influence of this effect on the nucleon-nucleon interaction is a corresponding increase with density of the $NN$ central potential due to the exchange of two correlated pions, resulting in an $NN$ interaction which fails to saturate. A possible mechanism for restoring the theoretical stability of nuclear matter is explored and found to be effective.
I. INTRODUCTION

At the level of effective interactions between hadrons, the pion and the $\pi\pi$ interaction occupy a central position. For example, in the two-nucleon system, single-pion exchange provides the longest-range part of the interaction, and two-pion exchange is the agent of the intermediate-range central attraction \[1,2\] which is necessary for nuclear binding. However, in order to provide sufficient attraction, it is essential that the exchanged pions be correlated; that is, that their “in-flight” interaction be taken into account. This, in turn, requires a reliable model for the $\pi\pi$ interaction, the development of which has been the subject of considerable effort \[3–7\].

Hand in hand with the development of a reliable model of the $\pi\pi$ interaction has come the effort to understand what effect the presence of a nuclear medium has on the interaction. This effect is of interest for its consequent influence on the nucleon-nucleon interaction in the presence of other nucleons, as well as, for example, in the analysis of heavy-ion collisions, where it may play a significant role in explaining the spectrum of dilepton production \[8\]. Our focus here will be on the modification of the nucleon-nucleon interaction.

Let us begin by summarizing briefly and qualitatively the results of our and others’ work on the $\pi\pi$ interaction and its modification by the nuclear medium. The Jülich Model \[3,4\] for the free $\pi\pi$ interaction gives a quantitatively accurate description of the interaction in low partial waves over a broad energy range. Subsequent investigations \[9–11\] of the effect of the medium on the $\pi\pi$ interaction through the coupling of pions to nucleon-hole and $\Delta$-hole configurations revealed that almost all of the models typically used for the $\pi\pi$ system led to a considerable build-up of attractive strength near and below the physical $\pi\pi$ energy threshold in the s-wave ($\sigma$ channel), resulting in $\pi\pi$ bound states at subnuclear density. A preliminary estimate \[12\] of the effect of the redistribution of strength on the in-medium nucleon-nucleon interaction indicated only minor alterations of the strength and range of the attraction due to correlated $2\pi$ exchange. Missing from this estimate, however, were the effect of the sub-threshold strength and an accurate treatment of the $2\pi$ propagator.
Further investigation of the medium modification of the $\pi\pi$ amplitude revealed a great sensitivity [7] to the particular scattering equation used to calculate it—that is, to the way in which the interaction is evaluated off-shell. Indeed, the original Jülich Model [3,4] extrapolated to the nuclear medium displayed a $\pi\pi$ pairing instability (i.e., the formation of $\pi\pi$ bound states of less than zero c.m. energy) at only slightly above normal nuclear matter density [6]. This difficulty was eventually traced to a failure of the interaction in the Jülich Model to satisfy low-energy theorems of chiral symmetry, e.g. that the s-wave scattering lengths did not vanish in the limit of zero pion mass. An “improved” model [7], which corrected this deficiency, yet preserved the good description of the $\pi\pi$ data, coupled with a kinematic prescription for continuing the $\pi\pi$ potential off-shell which preserves the scattering length, succeeded in removing the instability or, at least, in delaying its onset to much higher density.

In this report we will investigate how our best model to date of the $\pi\pi$ interaction, extended to take the nuclear medium into account, affects the nucleon-nucleon force. In doing this we will be concerned especially with the implications of the density-dependence that we find in the force for the stability of nuclear matter. We shall see that the conventional methods that we have applied lead to serious problems for the saturation of nuclear matter, and we shall explore one unconventional method which could be a remedy for the problem.

II. DYNAMICAL MODELS AND NUMERICAL RESULTS

A. Meson Exchange Model for $N\bar{N} \rightarrow \pi\pi$ and Corresponding $NN$ Potential in Free Space

For the evaluation of the correlated two-pion exchange contribution to the low-energy nucleon-nucleon interaction we adopt the recent approach of the Bonn-Jülich group. It is based on a meson exchange model for the $N\bar{N} \rightarrow \pi\pi$ reaction which by means of a dispersion relation is then transformed to the $NN$ channel. The $N\bar{N} \rightarrow \pi\pi$ model in free space has
been constructed in refs. [13,14] for the so-called pseudophysical region, i.e., well below the physical nucleon-antinucleon threshold, which is the relevant regime for extracting a \( NN \) potential. The corresponding scattering amplitude, \( \tau_{NN\to\pi\pi} \), is composed of a transition Born amplitude \( \tau_{B,NN\to\pi\pi} \) (consisting of nucleon and \( \Delta \) exchange), and a rescattering term accounting for the \( \pi\pi \) final state interaction. Within the Blankenbecler-Sugar (BbS) reduction scheme [15] of the underlying relativistic equation, and after a partial wave expansion, the scattering equation takes the form

\[
\langle 00 | \tau_{JI}^{NN\to\pi\pi}(t', q, p) | \lambda_N \lambda_{\bar{N}} \rangle = \langle 00 | \tau_{JI}^{NN\to\pi\pi}(t', q, p) | \lambda_N \lambda_{\bar{N}} \rangle + \int_0^\infty \frac{k^2 dk}{(2\pi)^2} \langle 00 | M_{\pi\pi}^{JI}(t', q, k) | 00 \rangle G_{\pi\pi}^0(t', k) \langle 00 | \tau_{JI}^{NN\to\pi\pi}(t', k, p) | \lambda_N \lambda_{\bar{N}} \rangle .
\]

where \( \lambda_N \) and \( \lambda_{\bar{N}} \) denote the helicities of nucleon and antinucleon, \( p \) the magnitude of their c.m. momenta, and \( q \) the c.m. momentum of the two pions. \( t' \equiv E^2 \) is the total energy squared of the \( NN \) (\( \pi\pi \)) system, but it will play the role of a t-channel 4-momentum transfer once the analytic continuation to the \( NN \) system is performed. Conservation of G-parity forces the spin-isospin quantum numbers \( JI \) to satisfy \( J+I=\text{even} \). The \( \pi\pi \) invariant scattering amplitude will be discussed in more detail below. The uncorrelated two-pion propagator in free space reads

\[
G_{\pi\pi}^0(t', k) = \frac{1}{\omega_k} \frac{1}{t' - 4\omega_k^2 + i\epsilon}
\]

with \( \omega_k^2 = m_{\pi}^2 + k^2 \). The \( NN \to \pi\pi \) transition amplitude (and the \( \pi\pi \) interaction) also contains the coupling to intermediate \( K\bar{K} \) states, which are not explicitly written in eq. (1), but they are included in all our calculations. The relation of the \( \tau \) amplitude to the in the literature frequently used Frazer-Fulco helicity amplitudes \( f_{J}^{\pm} \) [10] is given by

\[
f_+^{JI}(t') = \frac{p_{on} M_N}{4(2\pi)^2(p_{on} q_{on})^j} \langle 00 | \tau_{NN\to\pi\pi}^{JI}(t', q_{on}, p_{on}) | \frac{1}{2} \frac{1}{2} \rangle \frac{1}{2} \frac{1}{2}
\]

\[
f_-^{JI}(t') = -\frac{p_{on} M_N}{2(2\pi)^2 \sqrt{t}(p_{on} q_{on})^j} \langle 00 | \tau_{NN\to\pi\pi}^{JI}(t', q_{on}, p_{on}) | \frac{1}{2} (-\frac{1}{2}) \rangle \frac{1}{2} \frac{1}{2}
\]

where the on-shell momenta are defined by
In refs. [13,14] the Jülich $\pi\pi$ meson exchange model [3,4] has been employed for the $\pi\pi$ amplitude entering eq. (1). This model gives a satisfactory description of the available $\pi\pi$ scattering data up to rather high energies (beyond 1 GeV) and partial waves up to $J=2$. The $NN \rightarrow \pi\pi$ model is then completed by adjusting the four free parameters of the transition Born amplitudes $\tau_{B,N\bar{N}\rightarrow\pi\pi}$ (two form factor cutoffs and couplings for N and $\Delta$ exchange; see ref. [14] for details) to reproduce the quasi-empirical Frazer-Fulco amplitudes in the pseudophysical region $4m_{\pi}^2 \leq t < 4M_{\bar{N}}^2$ [17]. (See the left column of fig. 1.)

However, as mentioned earlier, the application of the Jülich model to the calculation of s-wave $\pi\pi$ correlations in nuclear matter has been shown to result in an unrealistic $\pi\pi$ pairing instability (pion pair condensation) slightly above saturation density [3]. As we will see in the next section, such a behavior indeed drastically increases the attraction in the intermediate and long range of the $NN$ potential in the nuclear medium, certainly contradicting any mechanism of nuclear saturation. On the other hand, as has been demonstrated in refs. [3,4], the implementation of constraints dictated by chiral symmetry in the $\pi\pi$ interaction significantly suppresses the tendency towards pair condensation. To investigate more quantitatively how the in-medium properties of the central $NN$ potential are affected by the chiral constraints in the $\pi\pi$ sector, we will also perform calculations with a chirally improved version of the Jülich $\pi\pi$ interaction [7]. These “minimal” chiral improvements consist of

(i) the introduction of $\pi\pi$ contact interactions as implicit in the gauged non-linear $\sigma$ model of Weinberg [18]; they ensure that the tree level $\pi\pi$ amplitude satisfies the soft pion theorems; in particular, the contact terms constitute a strong source of subthreshold repulsion in the $\pi\pi$ interaction kernel;

(ii) a slight modification in the off-shell continuation of the interaction kernel of the scat-
tering equation: rather than using the (on-energy-shell) BbS prescription we employ an on-mass-shell prescription; this ensures that the fully iterated $\pi\pi$ amplitude preserves the correct chiral limit for the s-wave scattering lengths (i.e., $a_0^I \to 0$ for $m_\pi \to 0$).

A slight refit of the model parameters gives a similarly good description of the free $\pi\pi$ scattering data to that of the original Jülich model, while the $\pi\pi$ pair condensation in nuclear matter is delayed to much higher density—about $2\rho_0$ [19]. As before, the additional four free parameters in the $NN$ sector are adjusted to reproduce the quasi-empirical Frazer-Fulco amplitudes, cp. the right column in fig. 1 (for consistency, the off-shell continuation of the corresponding transition Born amplitudes $\tau_{B,NN \to \pi\pi}$ is also changed to on-mass-shell).

In the rest of this section we will discuss the procedure for extracting a central $NN$ potential due to correlated two-pion exchange in the $\sigma$ channel. Following refs. [12,13], the relevant spectral function is related to the Frazer-Fulco amplitudes by

$$\eta_{00}(t') = 24\pi \sqrt{\frac{t' - 4m^2_\pi}{t'}} \frac{1}{(t' - 4M^2_N)^2} \left[ |f^{00}_{+,0}(t')|^2 - |f^{00}_{+,0}(t')|^2 \right] \Theta(t' - 4m^2_\pi), \quad (5)$$

where the subtraction of the Born amplitude $|f^{00}_{+,0}|^2$ removes the iterated single-pion exchange part of the $2\pi$ exchange. The $NN$ potential in momentum space is then obtained by means of a dispersion integral along the unitarity cut, the latter starting at the free two-pion threshold:

$$V_{NN\sigma}(t) = -\frac{\kappa}{\pi} \int_{-\infty}^{\infty} \frac{dt'}{t - t'} \eta_{00}(t') P_1,$$

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$$V_{NN\sigma}(t) = -\frac{\kappa}{\pi} \int_{-\infty}^{\infty} \frac{dt'}{t - t'} \eta_{00}(t') P_1,$$
\[ V_{NN}(r) = -\frac{1}{\pi} \int \frac{d^3 k''}{(2\pi)^3} e^{i\vec{k}'' \cdot \vec{r}} \int_{4m^2_{\pi}}^{t_c} dt' \frac{\eta_{00}(t')}{t - t'} \]

\[ = -\frac{1}{\pi} \int_{4m^2_{\pi}}^{t_c} dt' \frac{\eta_{00}(t')}{4\pi} \frac{\exp(-\sqrt{t'} r)}{r}, \tag{7} \]

which is simply a superposition of Yukawa-type potentials with a continuous mass/coupling constant distribution \( \eta_{00}(t') \). Cutting off the integration at at \( t_c = 50m^2_{\pi} \) avoids the extrapolation of our model to physically unreasonable momentum transfers. Due to the presence of the exponential, our numerical results are practically insensitive to moderate variations in \( t_c \).

However, in carrying eqs. (5) and (7) over to the in-medium case the following problem arises: the unitarity cut of the \( 2\pi \) continuum no longer starts at the free \( 2\pi \) threshold \( (E = \sqrt{t'} = 2m_{\pi}) \) but moves all the way down to \( E = 0 \); this is due to the dressing of the pions with \( NN^{-1} \) and \( \Delta N^{-1} \) excitations in nuclear matter, which generates an imaginary part of the two-pion propagator \([11]\),

\[ \text{Im} G_{\pi\pi}(E, k) = -\int_0^E \frac{d\omega}{\pi} \text{Im} D_\pi(\omega, k) \text{Im} D_\pi(E - \omega, k), \tag{8} \]

extending below \( E = 2m_{\pi} \). Consequently, \( \text{Im} M_{\pi\pi} \) also becomes nonzero below threshold. On the other hand, an extension of eq. (5) below \( t' = 4m^2_{\pi} \) is not obvious since already the \textit{free} Frazer-Fulco amplitudes do not vanish below threshold. In ref. \([12]\) the subthreshold contribution to the medium modified \( \eta_{00} \) was neglected by employing eq. (3) with in-medium \( f^0_+ \) amplitudes. However, since we intend to study the impact of in-medium \( \pi\pi \) effects such as possible bound state formation on the \( N\bar{N} \) potential, the subthreshold strength in the \( \pi\pi \) amplitude has to be accounted for. This can be achieved by formulating a scattering equation for \( N\bar{N} \to \pi\pi \to N\bar{N} \) and relating the imaginary part of \( M_{NN} \) in the pseudophysical region to \( \eta_{00} \). For an in- and outgoing \( N\bar{N} \) pair with momenta \( \vec{n}, -\vec{n} \) and \( \vec{p}, -\vec{p} \), respectively (\textit{cp.} fig. 2), and total energy \( \sqrt{t'} \) one obtains in the BbS framework:
\[ M_{\bar{N}N}(t', \bar{p}, n) = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \tau_B^*(t', \bar{p}, q) G_{\pi\pi}^0(t', q) \tau_B(t', \bar{q}, n) \]
\[ + \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \tau_B^*(t', \bar{p}, q) G_{\pi\pi}^0(t', q) \int \frac{d^3q'}{(2\pi)^3} M_{\pi\pi}(t', \bar{q}, \bar{q}') G_{\pi\pi}^0(t', q') \tau_B(t', \bar{q}', n), \quad (9) \]

where \( \tau_B^* \) is the hermitian conjugate of \( \tau_B \). The factors of 1/2 account for the appearance of closed loops of identical bosons. A partial wave expansion leads to

\[ \langle \Lambda | M^{JI}_{\bar{N}N}(t') | \Sigma \rangle = \frac{1}{2} \int_0^\infty \frac{q^2dq}{(2\pi)^3} \langle 00 | \tau_B^{JI}(t', q, p) | \Lambda \rangle^* G_{\pi\pi}^0(t', q) \langle 00 | \tau_B^{JI}(t', q, n) | \Sigma \rangle \]
\[ + \frac{1}{2} \int_{0}^{\infty} \frac{q^2dq}{(2\pi)^3} \langle 00 | \tau_B^{JI}(t', q, p) | \Lambda \rangle^* G_{\pi\pi}^0(t', q) \]
\[ \times \int_{0}^{\infty} \frac{q^2dq'}{(2\pi)^2} M_{\pi\pi}^{JI}(t', q, q') G_{\pi\pi}^0(t', q') \langle 00 | \tau_B^{JI}(t', q', n) | \Sigma \rangle \]
\[ \equiv \langle \Lambda | M^{JI, bare}_{\bar{N}N}(t') | \Sigma \rangle + \langle \Lambda | M^{JI, rescat}_{\bar{N}N}(t') | \Sigma \rangle \quad \quad \quad (10) \]

with \( \Sigma = \sigma \bar{\sigma}, \Lambda = \lambda \bar{\lambda} \) denoting the \( N\bar{N} \) helicity states. For our purpose we need the on-shell values of \( M_{\bar{N}N} \) in the pseudophysical region, therefore \( n = p = \sqrt{t'/4 - M_N^2} \) and consequently the transition Born amplitudes \( \tau_B^{JI} \) are purely imaginary. Thus the combination \( \tau_B^{JI}(t')^* \tau_B^{JI}(t') \) is always real, so that the imaginary part of \( M_{\bar{N}N} \) is solely generated by on-shell intermediate \( \pi\pi \) states. Therefore, in free space, it vanishes below \( \sqrt{t'} \leq 2m_\pi \). On the other hand, in the nuclear environment an in-medium amplitude \( ImM_{\pi\pi} \) that does not vanish below threshold naturally generates a corresponding nonzero \( ImM_{\bar{N}N}(\sqrt{t'} \leq 2m_\pi) \); i.e., within the framework of the scattering eq. (10) the inclusion of the subthreshold region is well defined. It remains to relate \( ImM_{\bar{N}N} \) to the spectral function \( \eta_{00} \). From eqs. (11),(3),(5), (10) one finds

\[ Im \left[ \langle + + | M^{00, rescat}_{\bar{N}N}(t') | + + \rangle \right] = \frac{4\pi}{3} \eta_{00}(t') \langle + + | P_1 | + + \rangle. \quad \quad \quad (11) \]

With

\[ \langle + + | P_1 | + + \rangle = (\bar{u}_\lambda(\bar{p})1^P u_\lambda(-\bar{p})) (\bar{v}_\sigma(-\bar{n})1^n u_\sigma(\bar{n})) \]
\[ = \frac{t'-4M_N^2}{4M_N^2}, \quad \quad \quad (12) \]
we arrive at
\[
\eta_{00}(t') = \frac{4M^2_N}{t' - 4M^2_N} \frac{3}{4\pi} Im \left[ (+ + |M_{NN,\text{rescat}}^{00}(t')| + +) \right],
\]
which now allows the desired extension of \( \eta_{00} \) below \( \sqrt{t'} = 2m_\pi \).

**B. The \( \sigma \) channel of the \( NN \) Potential in Nuclear Matter**

As suggested in ref. [12], we restrict the medium effects in the \( NN \) potential to be solely generated by the medium modifications in the \( \pi\pi \) interaction \( M^{00}_{\pi\pi} \) entering the rescattering term in eq. (10). Both the transition Born amplitude \( \tau^{00}_{B} \) and the explicitly-appearing two-pion propagator \( G^0_{\pi\pi} \) remain in their vacuum form; the inclusion of medium effects in these two quantities would require further assumptions concerning the treatment of nucleons in the nuclear medium within a \( NN \) scattering equation (e.g., Bethe-Goldstone equation). Furthermore, we want to stay as close as possible to the spirit of the one-boson exchange picture, and compare the results to our earlier findings. The medium modifications of \( M^{00}_{\pi\pi} \) are induced by dressing the intermediate pion propagators with the standard p-wave nucleon-nucleonhole (\( NN^{-1} \)) and delta-nucleonhole (\( \Delta N^{-1} \)) excitations [20]. In refinement of the three-branch model utilized in refs. [10,12], we here retain the full off-shell propagation dynamics of the pions in nuclear matter by means of a numerical treatment as described in detail in refs. [3,11].

Our in-medium results for the \( \pi\pi \) amplitude \( ImM^{00}_{\pi\pi} \), the spectral function \( \eta_{00} \) of ‘\( \sigma \)’ exchange and the corresponding central part of the coordinate \( NN \) potential, eq. (7) (with the lower integration limit set to zero) are displayed in fig. 3 for both the BbS Jülich \( \pi\pi \) model (upper panels) and its chirally improved version (lower panels). In order to provide a more realistic context for the attractive contribution of the \( 2\pi \) exchange, we supplemented this contribution to the \( NN \) central potential with a zero-width \( \omega \) exchange characterized by
\[
\eta_\omega(t') = \pi g^2_{NN,\omega} \delta(t' - m^2_\omega)
\]
(14)
with $g^2_{NN\omega}/4\pi=20$, as in the Bonn potential \cite{[4]}. (Due to the absence of a form factor, the potential which results from eq. (14) is substantially stronger, especially at short range, than the $\omega$ exchange of the Bonn potential.) The features found in $ImM^{00}_{\pi\pi}$ (left panels) are essentially reproduced by the $'\sigma'$ spectral function (middle panels): in the BbS Jülich model these are an appreciable accumulation of strength around the $2\pi$ threshold region evident already at half normal nuclear matter density ($\rho = 0.5\rho_0$), together with a suppression in the higher energy range. At $\rho = \rho_0$ a strong $\pi\pi$ bound state emerges that absorbs the major part of the strength in the entire scalar-isoscalar channel. In terms of a one-boson exchange picture this corresponds to a $\sigma$-like particle with a mass of about 100 MeV! The consequences for the central part of the $NN$ potential are obvious: a dramatic increase in the attraction by more than an order of magnitude, even invading very short distances. This is at variance with the results obtained earlier \cite{[12]} with the same model for the free $NN\rightarrow\pi\pi$ interaction. The differences stem from the full off-shell treatment of the in-medium pion propagation in connection with the inclusion of the subthreshold region in the present work.

With the chirally improved Jülich model the medium effects are much less pronounced (lower panels in fig. 3): the chiral constraints inhibit the development of $\pi\pi$ bound states at moderate densities—up to $\rho \simeq 1.6\rho_0$. However, a significant shift of strength to lower energies in both $ImM^{00}_{\pi\pi}$ and $\eta_{00}$ seems inevitable. This results in a smaller, but still appreciable increase in attraction in $V_{NN}^{C}(r)$ which, when incorporated into nuclear many-body calculations, is most likely incompatible with saturation. Thus we have to conclude that even though the chiral constraints on the $\pi\pi$ interaction substantially improve the situation, there remain important deficiencies in our microscopic description of the in-medium $NN$ potential.

One might first ask to what extent one can rely on the treatment of the pion propagation in nuclear matter. The model of p-wave $\Delta N^{-1}$ and $NN^{-1}$ excitations has been proven successful in different branches of nuclear pion physics \cite{[21],[22]}. However further refinements, which potentially reduce the rather pronounced softening of the in-medium pion dispersion
relation, seem possible. Among them are:

- the inclusion of s-wave $\pi N$ interactions leading to a suppression of the pion self-energy [23]. For low-momentum/energy pions especially such contributions might be significant, as the free $\pi N$ amplitude is also known to obey soft pion theorems; however, those will not be included in our present study;

- Pauli blocking effects in the $2\pi$ propagator when both pions are simultaneously excited into a $NN^{-1}$ (or $\Delta N^{-1}$) mode, which amounts to taking into account certain exchange diagrams. A preliminary estimate of this effect indicated it to be rather small [24];

- a density dependent increase of the short-range correlation parameters $g'_N$ [25] (so far we used constant values of $g'_{NN}=0.8$ and $g'_{N\Delta}=g'_{\Delta\Delta}=0.5$); this point will be addressed at the end of the next section.

One might also ask if the $\pi\pi$ interaction kernel (pseudopotential) itself undergoes substantial medium modifications, in particular concerning the exchanged mesons (most importantly the $\rho$ meson). On the $\pi\pi$ level, this is just the analog of the modification of the ‘$\sigma$’ exchange in the $NN$ potential discussed above. In the next section we will pursue such a possibility in terms of the Brown-Rho scaling hypothesis [26], which asserts that, e.g., vector meson masses decrease with increasing nucleon density.

**C. Impact of Chiral Symmetry Restoration on the Central $NN$ Potential**

As has been proposed by Brown and Rho [26] the (partial) restoration of spontaneously broken chiral symmetry in hot/dense matter leads to an approximately universal decrease of most physical quantities like masses, coupling constants, form factor cutoffs, etc. The corresponding “BR scaling” conjecture as a function of temperature $T$ and nuclear matter density $\rho$ reads:

$$\Phi(\rho, T) = \frac{m_V^*}{m_V} = \frac{m_\sigma^*}{m_\sigma} = \frac{M_N^*}{M_N} = \frac{f_\pi^*}{f_\pi} = \frac{\Lambda^*}{\Lambda} = \ldots \text{ etc. } ,$$

(15)
where the asterisks indicate in-medium values. Due to the Goldstone boson nature of the pion, its mass is excluded from this relation. The scale factor $\Phi(\rho, T)$ is governed by the decrease of the chiral quark condensate, conservatively estimated to be \[27\]

$$\Phi(\rho, T) \simeq \left( \frac{\langle 0|\bar{q}q|0\rangle(\rho, T)}{\langle 0|\bar{q}q|0\rangle^0} \right)^{\frac{1}{3}}.$$ \hspace{1cm} (16)

At zero temperature, and to leading order in density, the reduction of the quark condensate can be related to the pion-nucleon sigma term by

$$\frac{\langle 0|\bar{q}q|0\rangle(\rho)}{\langle 0|\bar{q}q|0\rangle^0} = 1 - \frac{\Sigma_{\pi N}}{m^2_\pi f^2_\pi} \rho.$$ \hspace{1cm} (17)

With $\Sigma_{\pi N}=45$ MeV one obtains $\Phi(\rho = \rho_0)=0.87$, which is compatible with QCD sum rule analyses of vector meson masses \[28\] giving

$$\frac{m_V(\rho)}{m_V(0)} = 1 - C \frac{\rho}{\rho_0}$$ \hspace{1cm} (18)

with $C=0.18\pm0.06$. In the following we will assume $\Phi(\rho)$ to drop linearly according to eq. (18) with $C=0.15$.

In our present context, BR scaling enters on the level of the $\pi\pi$ interaction kernel $V_{\pi\pi}^{00}$ (pseudopotential) for both t-channel $\rho$ exchange (decreasing $m^*_\rho$, $\Lambda^*_\rho$) as well as $\pi\pi$ contact interactions (decreasing $f^*_\pi$, $m^*_\rho$), the latter appearing only in the chirally improved version. Since the KSFR relation \[29\] is supposed to hold also in the medium,

$$2g_{\pi\pi\rho}^2(f^*_\pi)^2 = (m^*_\rho)^2,$$ \hspace{1cm} (19)

the $\pi\pi\rho$ coupling constant is not affected. The many-body excitations ($NN^{-1}$ and $\Delta N^{-1}$) of the in-medium single-pion propagator are always evaluated in terms of effective baryon masses

$$M^*_N = M_N(1 - 0.2\rho/\rho_0)$$

$$M^*_\Delta = M_\Delta - (M_N - M^*_N),$$ \hspace{1cm} (20)
where the second relation accounts for the fact that the $\Delta$-$N$ mass difference in nuclei is not seen to alter very much. Within the uncertainties, this agrees with eq. (18).

Repeating the in-medium calculations as described in the previous section with BR scaling assumed, the following picture emerges (fig. 4): in the BbS Jülich model (upper panels) the dropping $\rho$ mass leads to an increased attraction in the low-energy $\pi\pi$ interaction which reinforces the shift of strength to (very) low energies in the spectral function $\eta_{00}$. On the other hand, with the chirally improved version (lower panels), the simultaneous increase of repulsion in the $\pi\pi$ contact interactions (being proportional to $(f_\pi^*)^{-2}$) counterbalances the (attractive) $t$-channel $\rho$ exchange. The corresponding central $NN$ potentials, supplemented with $\omega$ exchange according to eq. (14) with $m_\omega^*$ from eq. (18), are displayed in the right panel of fig. 4: the BbS Jülich model still leads to an undesirable strong increase in attraction, whereas the chirally improved version results in a nice compensation between correlated $2\pi$ (‘$\sigma$’) and $\omega$ exchange. It is worthwhile to mention here that a very similar behavior is obtained if one replaces the correlated $2\pi$ exchange by a sharp (zero-width) effective $\sigma$ exchange, as employed in earlier versions of the Bonn potential, and scales its mass $m_\sigma^*$ according to eq. (18). In other words, the implementation of BR scaling in our dynamical model for correlated $2\pi$ exchange reproduces an effective scaling of a fictitious $\sigma$ meson in a nontrivial way, thereby generating a considerable stabilization of the central $NN$ potential in nuclear matter.

Note that once a universal scaling is established, its effects are not unexpected. A simple, non-relativistic model with static central potentials of Yukawa form derived from $\sigma$ and $\omega$ exchange will saturate at some density if all effective masses—meson and nucleon—are scaled uniformly downwards. All that is required is that the $\omega$ potential be stronger than the $\sigma$ potential at short range, and that the $\omega$ mass be larger than the $\sigma$ mass. We make no claim that the $2\pi$ exchange interaction that we have calculated (supplemented with $\omega$ exchange) will saturate at the correct energy and density—only that it will saturate if BR scaling is implemented in our framework. Quantitative predictions of the saturation energy and density will clearly depend on the constants of the model, such as the $\omega$ coupling constant.
and form factor, the rate of change with density of the vector meson masses (and related quantities, cp. eq. (15)), as well as a possible density dependence of the Migdal parameters $g'$ entering the pion selfenergy, e.g.:

- when increasing the BR scaling factor from $C = 0.15$ to $C = 0.22$ [31], and repeating our calculations as described above, the repulsive $\omega$ exchange dominates the attraction generated by correlated $2\pi$ exchange: with increasing density the minimum in $V_{NN}^C(r)$ gradually moves upwards in both energy and distance, eventually resulting in an entirely repulsive potential at densities around $2\rho_0$;

- on the other hand, a density dependence of the short-range correlation parameters is found to have only minor impact on our results; in an exploratory calculation we chose

$$
\begin{align*}
g'_NN(\rho) &= 0.6 + 0.2\rho/\rho_0 \\
g'_N\Delta(\rho) &= g'_\Delta(\rho) = 0.33 + 0.17\rho/\rho_0 ,
\end{align*}
$$

which is a somewhat weaker increase than suggested in ref. [31], but coincides with our values used above at $\rho=\rho_0$. Using again $C = 0.15$ in our meson exchange potentials, the resulting in-medium $NN$ potential from $\omega$ and correlated $2\pi$ exchange exhibits only marginal changes: compared to the results shown in the lower right panel of fig. 4, the long range attraction decreased by about 5% at $\rho=2\rho_0$.

### III. Summary and Conclusions

We have seen that a realistic model of the $\pi\pi$ interaction, extrapolated by standard techniques to account for interactions with a nuclear medium, immediately leads to serious difficulties in the form of a marked shift in (attractive) interaction strength in the $\sigma$ channel to low energies. This effect is directly traceable to the polarization of the medium by the pion—through $\Delta$-nucleonhole and nucleon-nucleonhole excitations, which makes the sub-threshold energy region $E < 2m_\pi$ accessible. This leads to an amplification of any attractive interaction strength in that energy range and results, ultimately, in a $\pi\pi$ pairing instability.
The situation improves when certain constraints dictated by chiral symmetry are imposed on the effective $\pi\pi$ interaction: i.e., employing a (broken) chirally symmetric interaction kernel as well as enforcing the correct chiral limit of the scattering length on the full amplitude. In that case the sub-threshold strength of the interaction is suppressed and the pairing instability does not set in until well above normal nuclear matter density. As we remarked in the introduction, this shifts the problem; it does not cure it. The in-medium nucleon-nucleon central interaction due to correlated $2\pi$ exchange still grows increasingly attractive with density. In the absence of mechanisms which increase the short-range repulsion as the density of the medium increases, nuclear matter will fail to saturate.

We have already enumerated some of the possible sources for this additional repulsion. It is also possible—indeed, almost certain—that a fuller treatment of the chiral dynamics in the medium would result in modifications of the effective $\pi\pi$ interaction beyond our rather simple enforcement of the chiral constraints. For example, our way of imposing the scattering length constraint guarantees only that the first two terms in the momentum expansion of the $\pi\pi$ amplitude are correct; higher-order terms are generated solely by the fit of the model to the free $\pi\pi$ data and are probably not consistent with chiral symmetry \[32\]. However, such a calculation over the broad energy and density ranges required for our purposes is far beyond our means, so that, for the present, we have to limit our investigations to what can be done with the best available effective model.

Our search for an effective mechanism for providing the additional repulsion needed to stabilize nuclear matter led us to consider the Brown-Rho scaling scenario \[26\] in which, e.g., vector meson masses decrease with increasing density. We found that a consistent scaling of masses, coupling constants, etc. does, indeed, lead to sufficient repulsion in the nucleon-nucleon interaction at higher densities which counteracts the growing attraction due to $\sigma$ channel $2\pi$ exchange. While it is clear that BR scaling is an efficient mechanism—that is, in our approach it works to stabilize nuclear matter, its dynamical origins are not clear \[33\]. Thus, we are left with at least two important questions:
• Do effects which we have so far neglected in our treatment of pion propagation sufficiently “stiffen” the in-medium pion dispersion relation and thereby provide the repulsion necessary for the saturation of nuclear matter?

• Is BR scaling contained in a more complete conventional approach to the dynamics of pions in nuclear matter, or is it a consequence of QCD in the intermediate energy range which cannot be derived from effective interactions involving a limited set of mesons and nucleons?

These two questions are, in fact, facets of the same question. If the answer to the first is ‘yes’, then we would expect the inclusion of the neglected processes to produce effects which resemble BR scaling. In that case, the scaling would be simply an economical way to express these effects. If the answer is ‘no’, then it may be necessary to promote the BR scenario from an hypothesis to a rule for constructing meson exchange models of hadronic interactions at non-zero density, and to seek its origin in QCD at a different level from effective meson exchange interactions. The answer to these questions appears to be worth some effort.

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Figure Captions

Figure 1: Our fit to the quasi-empirical Frazer Fulco amplitudes [17] in free space (open squares: real part; solid squares: imaginary part) employing two different models for the underlying $\pi\pi$ interaction, namely the BbS Jülich model [4,14] (left column) and a chirally improved version [7] (right column).

Figure 2: Our meson exchange model for the correlated two-pion exchange contribution to the nucleon-nucleon potential;
upper panel: microscopic model for $N\bar{N} \rightarrow \pi\pi \rightarrow N\bar{N}$ scattering with in- and outgoing $N,\bar{N}$ 4-momenta $n = (E_n, \vec{n})$, $\bar{n} = (E_{\bar{n}}, -\vec{n})$ and $p = (E_p, \vec{p})$, $\bar{p} = (E_{\bar{p}}, -\vec{p})$, respectively;
lower panel: corresponding $N-N$ potential for in- and outgoing nucleon 4-momenta $p_1 = (E_{p_1}, \vec{k})$, $n_1 = (E_{n_1}, -\vec{k})$ and $p_2 = (E_{p_2}, \vec{k'})$, $n_2 = (E_{n_2}, -\vec{k'})$, respectively.
The time direction in both diagrams is pointing from left to right.

Figure 3: Imaginary part of the $\pi\pi$ scattering amplitude (left panels) and $N\bar{N}$ spectral function (middle panels) in the $\sigma$ channel (JI=00) as well as the corresponding central $N-N$ potential (right panels), supplemented with (zero-width) $\omega$ exchange, at various nuclear matter densities (full lines: free space; dashed lines: $\rho/\rho_0=0.5$; dashed-dotted lines: $\rho/\rho_0=1$; dotted lines: $\rho/\rho_0=1.14$ in the upper panels and $\rho/\rho_0=1.9$ in the lower panels); the upper panels show the results for the BbS Jülich model, the lower ones for the chirally improved version.

Figure 4: Same as fig. 3, but including the BR scaling as described in the text with a scaling factor $C=0.15$ (cp. eqs. (15), (18)).
Fig. 2

\[ \text{Relation} \]

\[ \text{Dispersion} \]
Fig. 3
Fig. 4