Summary of the CKM 2010 Working Group on Rare Decays

MARTIN GORBahn\textsuperscript{a*}, MITESH PATEL\textsuperscript{b†} AND STEVEN ROBERTSON\textsuperscript{c‡}

\textsuperscript{a} Institute for Advanced Study, Technische Universität München, Lichtenbergstraße 2a, D-85748 Garching, Germany
& Excellence Cluster Universe, Technische Universität München Boltzmannstraße 2, D-85748 Garching, Germany

\textsuperscript{b} Blackett Laboratory, Imperial College London, Prince Consort Rd, London SW7 2BW

\textsuperscript{c} Institute of Particle Physics and Department of Physics, McGill University 3600 rue University, Montréal, Québec Canada H3A 2T8

Rare decays were essential in the discovery of the CKM mechanism of flavour and CP violation and are highly sensitive probes of physics beyond the Standard Model. In this summary the current status and future prospects of experimental measurements and the Standard Model theory predictions of various rare $B$, $D$ and $K$ decay observables are discussed. The specific new physics sensitivities of each mode are also briefly reviewed.

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\*mgorbahn@ph.tum.de
†mitesh.patel@imperial.ac.uk
‡steven@physics.mcgill.ca
1 Introduction

Historically, the study of rare decays was instrumental in the discovery of the Cabibbo Kobayashi Maskawa (CKM) picture of quark mixing and the prediction of the existence and properties of heavy fermions \cite{1, 2}. Recent experimental progress has confirmed the CKM mechanism as the dominant source for flavour and CP violation. In the coming years, the focus will again shift to the discovery of new heavy particles. Rare decays can provide an essential tool in measuring the coupling constants and mixing angles of any new, heavy degrees of freedom, which in turn will reveal the symmetries of the underlying Lagrangian. Observables which can be predicted with high precision, are experimentally accessible, and have an enhanced sensitivity to new physics will play a central role. In the following, the findings of the CKM working group are summarized and the above criteria are discussed mode-by-mode.

In this working group flavour changing neutral current (FCNC) radiative and leptonic decays of $B$, $D$ and $K$ mesons were discussed. At leading loop order all new physics (NP) models with heavy particles discussed here can be described at the mass scale of the decaying meson by the effective Hamiltonian

$$
H_{\text{eff}}^{d_i \rightarrow d_j} = 4\frac{G_F}{\sqrt{2}} V_{t_i} V_{t_j}^* \sum_k (C_{ij}^{k} Q_{k}^{ij} + C_{ij}^{k'} Q_{k'}^{ij}),
$$

(1)

where $V_{ij}$ is the CKM matrix and the operators relevant at tree level are

$$
Q_7^{(ij)} = m_b (\bar{d}_j \sigma_{\mu \nu} P_{R(L)} d_i) F^{\mu \nu}, \\
Q_8^{(ij)} = m_b (\bar{d}_j T^a P_{R(L)} d_i) G^{\mu \nu a}, \\
Q_9^{(ij)} = (\bar{d}_j \gamma_\mu P_{L(R)} d_i) (\bar{\ell} \gamma_\mu \ell), \\
Q_{10}^{(ij)} = (\bar{d}_j \gamma_\mu P_{L(R)} d_i) (\bar{\ell} \gamma_5 \gamma_\mu \ell), \\
Q_9^{(ij)} = m_b (\bar{d}_j \gamma_\mu P_{R(L)} d_i) (\bar{\ell} \gamma_\mu \ell), \\
Q_9^{(ij)} = (\bar{d}_j \gamma_\mu P_{L(R)} d_i) (\bar{\ell} \gamma_5 \ell).
$$

(2)

$P_{L(R)}$ denotes the projector of the left-handed (right-handed) field for the (primed) operator. In the Standard Model (SM) only the Wilson coefficients $C_{ij}^k$ of the unprimed operators are generated. The small size of the light-quark Yukawa couplings suppresses the Wilson coefficients $C_{S,P}$ of the scalar operators even further and renders their contribution negligible.

In extensions of the SM these suppression mechanisms need not be present. A strong SM suppression increases the new physics sensitivity and there are several suppression mechanisms at work: loop suppression is present in all FCNC processes and new heavy degrees of freedom can contribute at the same level as the SM. Helicity suppressed modes are particularly sensitive to flavour changing scalar interactions and CKM suppressed modes to new sources of flavour violation.

In order to be able to exploit this new-physics sensitivity, a precise knowledge of the SM background is needed. To calculate this background, matrix elements of four-
quark operators must be calculated at higher loop order. In this case the current-current operators $Q_{1,q} = (\bar{d}_j \gamma_\mu P_L q)(\bar{q} \gamma_\mu P_L d_i)$ and $Q_{2,q} = (\bar{d}_j \gamma_\mu T^a P_L q)(\bar{q} \gamma_\mu T^a P_L d_i)$ play the most important role (for decays of down-type quarks $q$ represents the up and charm quarks, for charm decays it stands for down and strange quarks – $T$ denotes the Gell-Mann matrices). Their Wilson coefficients are tightly constrained and usually insensitive to physics beyond the SM. However, the calculation of their matrix elements often results in a sizeable uncertainty in the estimation of the SM background. Contributions of higher-order matrix elements can often be absorbed in a redefinition of $C_i$ in terms of effective Wilson coefficients $C_i^{\text{eff}}$.

2 $b \to s \ell^+ \ell^-$ and $b \to s\gamma$ decays

The decays $b \to s \ell^+ \ell^-$, where $\ell^+ \ell^-$ is an $e^+e^-, \mu^+\mu^-$ or $\tau^+\tau^-$ pair, are FCNC processes which occur via a $b \to s$ transition through a loop diagram. In the SM the Wilson coefficient $C_7$ plays the dominant role for $b \to s(d)\gamma$ decays. In the case of $b \to s(d)\ell^+\ell^-$ decays the coefficients $C_9$ and $C_{10}$ also contribute significantly. In extensions of the SM the chirality flipped operators $Q_7'$, $Q_9'$ and $Q_{10}'$, as well as the scalar operators can also contribute. A range of observables sensitive to specific combinations of Wilson coefficients of these operators exist. Apart from the integrated branching ratio, charged lepton forward-backward asymmetries ($A_{FB}$) as well as isospin and CP asymmetries can be defined for both the exclusive and inclusive decay modes. The dependence of the $b \to s(d)\ell^+\ell^-$ decay rate on the invariant mass $q^2$ of the lepton pair is described by the differential decay rate. The $q^2$ region dominated by charm-quark resonances is usually excluded and the total branching ratios are given for the regions above or below the resonances. The exclusive decays offer even more observables: in particular the angular analysis of the $B \to K^* \ell^+ \ell^-$ decays, where the $K^*$ polarisation is measured using the $K^* \to K^-\pi^+ \ell^+\ell^-$ decay, leads to 24 observables for each leptonic mode per $q^2$-bin.

Inclusive $B$ decays can be reliably approximated by perturbative calculations. The large bottom-quark mass $m_b$ allows the decay rates to be expanded in terms of the partonic decay rate for $b \to s\gamma$, i.e. $\Gamma(B \to X_s\gamma) = \Gamma(b \to s\gamma) + \mathcal{O}(\Lambda_{QCD}/m_b)$. A similar expansion holds for $b \to s\ell^+\ell^-$. For both partonic decay rates the NNLO calculations are almost complete [3, 4] – see Ref. [5] for a summary of recent developments. The non-perturbative uncertainty of $\Gamma(B \to X_s\gamma)$ was discussed in the talk by Paz [5]: a study of the $\mathcal{O}(\Lambda_{QCD}/m_b)$ uncertainty, including the $Q_7 - Q_8$ contribution, results in an intrinsic 5% non-perturbative uncertainty for the radiative mode. The CP asymmetry in the $b \to s\gamma$ decay modes is small in the SM and a large deviation from zero would be a signal of new physics. The non-perturbative uncertainty of the CP asymmetry was recently studied in [6], where the non-perturbative matrix element of $Q_1 - Q_7$ lead to 2% long-distance pollution, which is larger than previously
Exclusive decays provide an even richer phenomenology, in particular for the many physical observables associated with the $B \rightarrow K^{*}\ell^{+}\ell^{-}$ decay. Yet, a direct calculation of the weak decay matrix elements is mandatory for a precise theory prediction. Here factorisation (and soft collinear effective theory) provides a framework to disentangle perturbatively calculable hard-scattering kernels [7] from form factors and light-cone distribution amplitudes in a power expansion in $\mathcal{O}(\Lambda_{\text{QCD}}/m_{b})$.

The current uncertainty for the form factors of $B$ decays into pseudo-scalar or vector particles, is $12 - 15\%$ [8]. Form factors in the region of small and intermediate $q^2$ are calculated from QCD light-cone sum rules – see Ref. [8] for a summary of recent activities. In the high $q^2$ region improvements can be made using lattice QCD. The recent progress in the calculation of the $B \rightarrow K^{(*)}$ form factors with a moving-non-relativistic QCD action is presented in Ref. [9].

Beyond the leading-order factorisable contributions and hard gluon exchanges there also exist non-factorisable contributions to $B \rightarrow K^{(*)}\ell^{+}\ell^{-}$. A soft gluon which couples to the lepton pair via a virtual charm loop, which is induced by a $Q_{1/2}$ operator insertion, cannot be expressed in terms of a local operator. The influence of the resulting non-local operator [8, 10] is illustrated in Fig. 1, and can be absorbed in a correction to the effective coefficient of the $O_{9}$ operator. The impact on the $A_{FB}$ observable is also shown.

The current uncertainty in the form factors is the limiting uncertainty in the theory prediction of the radiative and electroweak rare decays. Given the difficulties of making precise theoretical predictions for the exclusive decay modes, attention has therefore focused on observables with a reduced dependence on the form-factors. The zero-crossing point of $A_{FB}$ is the most well known example of a quantity which assumed.

Figure 1: (Left) The charm loop contribution to the Wilson coefficient $C_{9}$ for $B \rightarrow K^{*}\ell^{+}\ell^{-}$ decays. The central values are denoted by the dashed line and the shaded area indicates the estimated uncertainties. (Right) The forward-backward asymmetry $A_{FB}$ with (solid) and without (dashed) the effect of the charm-loop. Taken from Ref. [10].
is almost free of such hadronic uncertainties. However, recent theoretical work has identified a number of additional observables. Quantities such as $A_2^T$ and $A_5^T$, which are defined and discussed in detail in Ref. [11], have precise theoretical predictions (Fig. 2). The $A_2^T$ observable contains almost all the physical information of $A_{FB}$ but is less sensitive to QCD uncertainties and exhibits a much larger sensitivity to right-handed currents.

The B{\textsc{abar}}, Belle and CDF experiments have all measured exclusive $b \rightarrow s \ell^+ \ell^-$ decays [12, 13, 14, 15], where the final lepton pair state comprises electron and muons for the B-factories and only muons at CDF. Events are selected from data samples with integrated luminosities of 349 fb$^{-1}$, 605 fb$^{-1}$ and 4.4 fb$^{-1}$ respectively, corresponding to 384 million $B\bar{B}$ events, 656 million $B\bar{B}$ events and $2 \times 10^{10}$ $b\bar{b}$ events. The measured total branching ratio is in good agreement with theoretical predictions, as is the direct CP asymmetry – see Ref. [15] for a summary table of recent results. Further measurements have therefore focussed on additional observables involving the angular distributions [16, 17, 18] or isospin asymmetries [7, 19]. The B{\textsc{abar}}, Belle and CDF experiments have also analyzed the one-dimensional angular distributions which involve the observables $F_L$, the fraction of $K^+$ longitudinal polarization, and $A_{FB}$, the lepton forward-backward asymmetry.

Fig. 3 shows the B{\textsc{abar}} [12], Belle [13], and CDF [14] results for $F_L$ (left) and $A_{FB}$ (right) as a function of the di-muon invariant mass squared, $q^2$. The figure also shows the SM prediction (lower red curve) [29] and that of a model in which the sign of $C_7^{eff}$ is flipped (upper blue curve) [30, 31]. The measurements from all three experiments are in good agreement. While the measurements are in better agreement with the flipped-sign $C_7^{eff}$ model, they are consistent with the SM prediction. For $B \rightarrow K \ell^+ \ell^-$, $A_{FB}$ is consistent with zero, as expected in the SM.
Figure 3: (Left) Measurements of $F_L$ and (Right) $A_{FB}$ in $B \to K^{(*)}\ell^+\ell^-$ decays by the BABAR (black squares), Belle (brown dots) and CDF (green triangles) experiments. The SM prediction (flipped-sign $C_7^{eff}$ model) is shown by the upper red (lower blue) curve for $F_L$ and the lower red (upper blue) curve for $A_{FB}$. The green shaded regions show the $J/\Psi$ veto used by Babar and the $\Psi(2S)$ veto used by Belle and CDF.

Figure 4: Isospin asymmetry measurements for $B \to K^{(*)}\ell^+\ell^-$ versus $q^2$ from BABAR (black squares, blue dots) and Belle (red triangles, green triangles).

The isospin asymmetry, $A_I$,

$$A_I(q^2) = \frac{dB(B^0 \to K^{(*)0}\ell^+\ell^-)/dq^2 - (\tau_{B^0}/\tau_{B^+})dB(B^+ \to K^{(*)+}\ell^+\ell^-)/dq^2}{dB(B^0 \to K^{(*)0}\ell^+\ell^-)/dq^2 + (\tau_{B^0}/\tau_{B^+})dB(B^+ \to K^{(*)+}\ell^+\ell^-)/dq^2}, \quad (3)$$

is expected to be small in the SM. In particular, in the region $q^2 = 2.7-6$ GeV$^2$/c$^4$, the expectation for $dA_I(q^2)/dq^2$ is $-0.01$ in the case of $B \to K^{(*)}\ell^+\ell^-$ [19]. Fig. 4 shows the BABAR and Belle $A_I$ measurements as a function of $q^2$. Both the $q^2$-integrated isospin asymmetry, and the $A_I$ value for $q^2$ larger than the $J/\psi$ invariant mass-squared, are consistent with the SM prediction. However, below the $J/\psi$ mass-squared, BABAR observes a negative $A_I$ which is $3.9\sigma$ from the SM prediction of $A_I = 0$. For low $q^2$, the Belle results are consistent with both the BABAR measurements and with the SM expectation.

The $B$-factory measurements of $A_{FB}$ will be improved by the LHCb experiment.
Fig. 5 shows the precision on $A_{FB}$ expected with $1 \text{fb}^{-1}$ of integrated luminosity [21]. This dataset will allow the zero-crossing point of $A_{FB}$ to be determined with 13% precision [22]. Observables such as $A_{T}^2$ can be extracted from the projection of the distribution of a single angle. With sufficient experimental data it will be possible to make a full angular fit and improve the precision on this and other observables. This will also allow those observables requiring information from multiple angular distributions to be determined.

Experimental observables for $b \rightarrow s\gamma$ and $b \rightarrow d\gamma$ include the inclusive and exclusive branching fractions, direct CP asymmetry

$$A_{CP} = \frac{\Gamma(b \rightarrow s\gamma) - \Gamma(\bar{b} \rightarrow \bar{s}\gamma)}{\Gamma(b \rightarrow s\gamma) + \Gamma(\bar{b} \rightarrow \bar{s}\gamma)} \quad (4)$$

and the shape of the photon energy spectrum. The photon energy spectrum is not sensitive to new physics, but can be used to determine Heavy Quark Expansion parameters related to the $b$ quark mass and momentum within the $B$ meson. These are used in the extraction of $|V_{ub}|$ and $|V_{cb}|$ from semileptonic $B$ decays. The inclusive $b \rightarrow s\gamma$ branching fraction has been computed to NNLO to a precision of about 7% [3] for $E_{\gamma} > 1.6 \text{ GeV}$, comparable in precision to current experimental determinations.

Inclusive measurements of the $b \rightarrow s\gamma$ branching fraction are based on three distinct techniques: (1) summing many exclusive decay modes, (2) fully inclusive measurements utilising kinematic information to suppress backgrounds, or (3) exclusive reconstruction of the second $B$ meson in the event. Each method has its own strengths and weaknesses and in practice all three are competitive with current $B$-factory data statistics. In general, none of these methods explicitly reject $b \rightarrow d\gamma$, hence this
contribution is usually implicitly included in the branching fraction and $A_{CP}$ definitions. The most precise branching fraction measurement comes from Belle [23] based on 605 fb$^{-1}$, with $B(B \to X_s \gamma) = (3.45 \pm 0.15 \pm 0.40) \times 10^{-4}$ for $E_\gamma > 1.7$ GeV. The combined world average branching fraction is consistent with the NNLO theory prediction at roughly the 1σ level. The BAbAR experiment has recently reported a preliminary determination of $A_{CP}$ based on a fully inclusive method [24] and utilizing a data sample of 347 fb$^{-1}$. To reduce background contamination, the search is restricted to a limited region of the photon energy spectrum, $2.1 < E_\gamma < 2.8$ GeV, resulting in $A_{CP} = 0.056 \pm 0.060 \pm 0.018$. This measurement is consistent with no CP asymmetry and is significantly more precise than previous measurements.

Exclusive and sum-of-exclusive determinations of the $b \to d \gamma$ branching fraction [25, 26, 27] can be compared with the corresponding $b \to s \gamma$ measurements to extract $|V_{td}|/|V_{ts}|$ [28]. This can be compared with the more precise measurement of this quantity from $B_s$ and $B_d$ mixing.

Future $B$-factory facilities offer scope for substantial improvement in inclusive (and semi-inclusive) $B \to X_{s,d} \gamma$ measurements as well as the potential to perform fully inclusive studies of $B \to X_s \ell^+ \ell^-$ and possibly $B \to X_d \ell^+ \ell^-$. Fully inclusive $B \to X_{s,d} \gamma$ measurements are typically limited by the enormous background from continuum events and from photons originating from $\pi^0$ and $\eta$ mesons produced in $B$ decays. Semi-inclusive methods based on summing a large number of exclusive modes are able to more strongly suppress these backgrounds but suffer from large uncertainties related to the normalization of the individual modes and to the modes which are not reconstructed. By requiring exclusive hadronic “tag” reconstruction of the $B$ meson accompanying the $B \to X_{s,d} \gamma$ signal candidate, not only is the continuum background strongly suppressed, but the kinematics of the signal $B$ are fully constrained, resulting in improvements in resolution of the photon energy spectrum and hadronic $X_{s,d}$ system, as well as a strengthening of the $\pi^0$ and $\eta$ veto. In principle this method permits a truly inclusive measurement over the entire $X_{s,d}$ mass range, however current $B$-factory measurements using this method are statistically limited to the extent that they cannot compete with untagged measurements. With the addition of two orders of magnitude of additional data from the new super $B$-factories, this method will have statistical sensitivity comparable to current untagged analyses from BAbAR and Belle, but with substantially reduced backgrounds and systematic uncertainties. In the case of $B \to X_{s,d} \ell^+ \ell^-$, the hadronic tag reconstruction would permit studies based only on the $\ell^+ \ell^-$ kinematics and hence accessing the entire $X_s$ spectrum, however these measurements are not competitive using current $B$-factory data samples. With super $B$-factory data samples of order 100 ab$^{-1}$, these measurements may prove complementary to LHCb exclusive determinations.
3 $B \to K^{(*)}\nu\bar{\nu}$

The rare decays $b \to s \nu\bar{\nu}$, like their charged lepton counterparts, provide an interesting potential window into new physics in FCNC processes. In contrast to $b \to s\ell^+\ell^-$, there is no photon penguin contribution and only the $Z$-penguin needs to be considered. Hence, in the SM, the effective Hamiltonian contains only a single dimension-six operator and single complex Wilson coefficient $C_L$. Potential new physics contributions arise in many models including the Minimal SuperSymmetric Model (MSSM), little Higgs and extra dimensions models, and can result in a non-zero $C_R$ and non-SM value of $C_L$. These can be constrained by measurements of a combination of inclusive and exclusive $b \to s \nu\bar{\nu}$ modes [32]. In addition, since the neutrinos are not observed, the experimental signature of $B \to K^{(*)} + E_{\text{miss}}$ does not distinguish between the $\nu\bar{\nu}$ mode or other unobserved final state particles [33]. This can lead to apparent deviations from the SM branching fraction or modifications to the kinematic spectra of the $K^{(*)}$.

Although the inclusive $B \to X_s\nu\bar{\nu}$ processes are theoretically very clean, in practice only the exclusive modes $B \to K^{(*)}\nu\bar{\nu}$ are experimentally accessible at $B$-factories, due to the very limited kinematic information available to constrain these decays. The experimental challenge is to ascertain that the only visible decay daughter of the $B$ is a charged or neutral $K^{(*)}$. At $B$-factories, this is accomplished using exclusive tag reconstruction of the other $B$ meson in the $\Upsilon(4S) \to BB$ event, in one of a large number of either fully-hadronic or semileptonic decay modes. Stringent kinematic constraints on the reconstructed $B$ ensure the high purity of this reconstructed sample, substantially suppressing $q\bar{q}$ continuum backgrounds and mis-reconstructed $B\bar{B}$ decays. Once a clean tag $B$ sample is obtained, the remaining detected particles are hypothesised to originate from a $B \to K^{(*)}\nu\bar{\nu}$ signal candidate. Signal events typically contain only the $K^{(*)}$, plus a small number of low-energy calorimeter energy deposits originating from beam-related backgrounds, hadronic shower fragments (“split-offs”) and similar debris. In the $K^\pm$ and $K^0_s$ modes the Kaon momentum $p^*_{K^{(*)}}$ is the only physical observable, while in the $K^*$ modes the longitudinal or transverse polarization fraction, accessible via the angular distributions of the $K^*$ decay products, is also available. In all modes significant backgrounds from generic $b \to c$ processes contribute at low $p^*_{K^{(*)}}$, so current experimental searches are mainly sensitivity to the high $p^*_{K^{(*)}}$ region and in most cases impose a kinematic requirement that $p^*_{K^{(*)}} > 1.5$ GeV/c. This introduces a modest form factor dependence on experimental limits, but can also significantly impact sensitivity to specific new physics models which predict a softer $p^*_{K^{(*)}}$ spectrum, for example if the missing energy originates from a pair of undetected massive scalar particles. Experimental strategies therefore necessitate a choice between the need to suppress background and the desire to retain the largest possible portion of the $p^*_{K^{(*)}}$ spectrum. A recent $\text{BaBar}$ analysis [34], utilising a Bagged Decision Tree multivariate approach, attempted to address this issue...
by reporting separately a partial branching fraction limit in the \( p_{K^{(*)}}^* > 1.5 \text{ GeV}/c \) region and the low \( p_{K^{(*)}}^* \) region. However, due to the resulting strong dependence of the signal efficiency on \( p_{K^{(*)}}^* \), interpretation of this result in a new physics context remains problematic.

\( \text{BABAR} \) and Belle have published limits on the exclusive modes \( B^+ \to K^+ \nu \bar{\nu}, B^0 \to K^0_\ell \nu \bar{\nu}, B^+ \to K^{*+} \nu \bar{\nu} \) and \( B^0 \to K^{*0} \nu \bar{\nu} \), based on a combination of the hadronic and semileptonic tag reconstruction techniques \([34, 35]\). These searches currently limit the branching fractions to a few times the SM rates and are essentially statistically limited by the low tag reconstruction efficiency. Further improvements in these modes will require additional statistics, either due to substantial improvements in the tag reconstruction method at the current generation of \( B \)-factories, or due to new data recorded at future Super \( B \)-factory facilities in Italy and Japan.

4 \( B_q \to \ell^+ \ell^- \) and \( D \to \ell^+ \ell^- \)

The rare decay \( B_q \to \ell^+ \ell^- \) results in a final-state lepton pair with zero angular momentum. The matrix element of a vector current coupling to the lepton pair vanishes and the decay is dominated by \( Z \)-penguin and electroweak box contributions in the SM. The axial-vector current coupling entails a \( m_\ell^2/M_B^2 \) helicity suppression which strongly suppresses the \( B_q \to \ell^+ \ell^- \) branching ratio. The small size of the bottom-quark and lepton Yukawa couplings renders the Higgs-penguin diagrams negligible in the SM. This suppression mechanism does not hold for generic scalar interactions and the contribution of scalar operators could significantly increase the branching ratio above the SM expectation. Together with the fact that the theory prediction of such branching ratios is under good control, this enhancement makes \( B_q \to \ell^+ \ell^- \) decays an ideal testing ground for new scalar interactions.

For the SM operators the branching ratio of \( B_q \to \ell^+ \ell^- \), \( B(B_q \to \ell^+ \ell^-) \), is given by

\[
\frac{G_F^2 f_{B_q}^2 M_B^5 \tau_{B_q} |V_{tb}^* V_{tq}|^2}{4 \pi} \sqrt{\xi_{lq}} \left[ \xi_{lq} C_S^2 + \left( C_P - \frac{2m_\ell}{M_B^2} C_A \right)^2 \right], \tag{5}
\]

where \( \xi_{lq} = 1 - 4m_\ell^2/M_B^2 \) is a kinematic factor. Numerically, only the Wilson coefficient \( C_A \) is relevant. This coefficient has been computed at NLO in the SM \([36, 37, 38]\). The dependence on \( f_{B_q} \) and the CKM parameters cancel in a combined CKM fit, if the experimental information of \( \Delta M_q \) is also taken into account. The results of the CKMfitter group \([39]\) which were presented at ICHEP10 are

\[
B(B_s \to \mu^+ \mu^-) = 3.073^{+0.070}_{-0.190} \times 10^{-9} \tag{6}
\]

\[
B(B_d \to \mu^+ \mu^-) = 9.87^{+0.25}_{-0.67} \times 10^{-11} \tag{7}
\]
The $B_q \rightarrow \mu^+\mu^-$ decays are sensitive to different extensions of the SM: a modification of the SM $Z$-penguin can be measured using the precise theory prediction of these modes, while the helicity suppression provides an ideal test of flavour changing neutral scalar interactions in the $C^{(l)}_S$ and $C^p_{(l)}$ Wilson coefficients – the extension of Eq. (5) for chirality-flipped operators has a similar structure and is given in Ref. [40].

Potentially large flavour-changing scalar interactions appear in many models with extended Higgs sectors. For example in the Minimal Flavour Violation (MFV) variant of the MSSM the branching ratios could be enhanced by several orders of magnitude. While the ratio $\mathcal{B}(B_d \rightarrow \mu^+\mu^-)/\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ stays constant in models with MFV, this is no longer the case in more generic extensions of the SM. The sensitivity of this ratio to generic soft breaking terms in the MSSM can then be used to distinguish different flavour models or falsify other extensions of the SM [41].

The strongest experimental upper bound on the $B_s$ decay, $\mathcal{B}(B_s \rightarrow \mu^+\mu^-) < 43 \times 10^{-9}$ at 95% CL, comes from the CDF collaboration using 3.7 fb$^{-1}$ of integrated luminosity [42]. The most recent result of the D0 collaboration gives an upper bound of $\mathcal{B}(B_s \rightarrow \mu^+\mu^-) < 51 \times 10^{-9}$ at 95% CL from 6.1 fb$^{-1}$ of data [43]. In the future ATLAS, CMS and LHCb will search for $B_s \rightarrow \mu^+\mu^-$ at the LHC. To measure such a small branching ratio, good control of the background is needed [44]. The main background source for all experiments is the combinatorial double semi-leptonic decay $b\bar{b} \rightarrow \mu^+\mu^- X$, where the two muons accidentally form a secondary vertex. Finally, at hadron colliders only a relative measurement of the branching ratio is possible, where a well known decay channel is used for normalization. If the decay $B^+ \rightarrow J/\psi K^+$ is used as a normalization channel then the uncertainty on the decay constant ratio $f_{B_d}/f_{B_s}$ introduces a parametric uncertainty on the branching ratio of 15%. Two strategies to reduce this uncertainty are described in Ref. [44]. Using the modified frequentist approach the 90% CL limits expected from the LHCb and CMS experiments for 1 fb$^{-1}$ integrated luminosity at a collision energy of 7 TeV are $\mathcal{B}(B_s \rightarrow \mu^+\mu^-) < 7.0 \times 10^{-9}$ and $\mathcal{B}(B_s \rightarrow \mu^+\mu^-) < 15.8 \times 10^{-9}$ respectively [44].

In the SM the branching ratio of the $D \rightarrow \ell^+\ell^-$ decays is controlled by long distance dynamics. The short distance $Z$-penguin and box diagrams are CKM- and $m_b^2/M_W^2$-suppressed. To a good approximation, the long distance dominated $D \rightarrow \gamma\gamma$ decay gives the sole contribution to the branching ratio $\mathcal{B}(D \rightarrow \mu^+\mu^-) \sim 2.7 \times 10^{-5}\mathcal{B}(D \rightarrow \gamma\gamma)$ [45]. To have a clear signal of new physics the branching ratio has to lie significantly above the SM expectation $\mathcal{B}(D \rightarrow \mu^+\mu^-) \sim (2.7 - 8) \times 10^{-13}$. Such an enhancement is not easily achievable in typical extensions of the SM – see e.g. Ref. [46]. The best experimental bounds come from the Belle collaboration. Using 660 fb$^{-1}$ of data they find $\mathcal{B}(D \rightarrow \mu^+\mu^-) < 1.4 \times 10^{-7}$ and a slightly better bound for the decay into an electron-positron pair [47] – see also Ref. [48] for further information.
5 Lepton flavour and number violating decays

While lepton flavour and lepton number are good quantum numbers within the SM (with massless neutrinos), lepton flavour is known to be explicitly violated by neutrino mixing and there is no fundamental symmetry principle which dictates charged lepton number or flavour conservation in beyond-SM models. Because the SM expectation for these modes is vanishingly small, searches for lepton number violating (LNV) or lepton flavour violating (LFV) decays are very sensitive probes of physics beyond the SM, potentially at mass scales well beyond the electroweak scale. For most models the most sensitive probes are in \( \tau \) or \( \mu \) decays, however LFV and LNV can also be probed in leptonic and semileptonic decays of heavy flavour mesons, providing a complementary approach to the \( \tau \) searches, and in some cases constraining specific scenarios beyond the reach of the \( \tau \) modes.

Experimental searches for LFV and LNV decays in \( B \) and \( D \) meson decays are performed either as incidental additions to related lepton flavour and number conserving searches, or as dedicated searches for specific modes. The former is usually the case for LFV studies involving first and second generation leptons, in particular \( B^0 \to e^\pm \mu^\mp \) and \( B \to K^{(*)}/\pi e^\pm \mu^\mp \), while modes involving \( \tau \) leptons usual require dedicated searches. The most restrictive limits on these modes include

\[
B(B \to K(K^*)e^\pm \mu^\mp) < 5.1(0.38) \times 10^{-7} \quad [49] \quad \text{and} \quad B(B^0 \to e^\pm \mu^\mp) < 6.4 \times 10^{-8} \quad [50].
\]

Dedicated searches have been performed for modes with \( \tau \) leptons: \( B^0 \to \tau^\pm \ell^\mp \) and \( B \to K\tau^\pm \ell^\mp \) (where \( \ell = e, \mu \)). Missing energy associated with the \( \tau \) decay necessitates the use of hadronic tag reconstruction in these modes, resulting in substantially reduced signal sensitivity compared with the \( e - \mu \) modes. Current best limits on these modes are

\[
B(B^0 \to \tau^\pm e^\mp) < 2.8 \times 10^{-5}, \quad B(B^0 \to \tau^\pm \mu^\mp) < 2.2 \times 10^{-5} \quad [51] \quad \text{and} \quad B(B^+ \to K^+\tau^\pm \mu^\mp) < 7.7 \times 10^{-5} \; \text{at 90\% confidence level} \quad [52].
\]

Belle recently reported results of a search for the LFV and LNV modes \( B^+ \to D^- \ell^+ \ell'^+ \) (where \( \ell, \ell' = e, \mu \)), motivated by a model with massive Majorana neutrinos [53], obtaining branching fraction limits ranging from \((1 - 3) \times 10^{-6} \) [54].

6 Rare Kaon decays

The rare \( K \to \pi \nu \bar{\nu} \), \( K_L \to \pi^0 \ell^+ \ell^- \) and \( K_L \to \mu^+ \mu^- \) decays play a central role in constraining and testing new sources of flavour violation. In the SM, light-quark loops are severely suppressed and loops with internal top quarks make the dominant contributions to the branching ratios of these decays. New physics would significantly alter the branching ratios, if the new sources of flavour violation are not suppressed by a coupling constant smaller than

\[
V_{ts}V_{td}^* = \mathcal{O}(\lambda^5).
\]

To exploit this new physics sensitivity a precise calculation of the SM background is needed. The SM short distance contribution to the branching ratio is above 90%
for the $K \to \pi \nu \bar{\nu}$ decays, but below 50% for the other modes. Correspondingly, the uncertainty in the SM prediction for the $K_L \to \pi^0 \ell^+ \ell^-$ and $K_L \to \mu^+ \mu^-$ decays is dominated by the long distance calculations.

The rare decay $K_L \to \mu^+ \mu^-$ was important in establishing the GIM mechanism. No single photon penguin contributes to the decay. Accordingly, at leading order in the electroweak couplings, only $Z$-penguin and box diagrams can contribute. In these diagrams light quark dynamics are suppressed by a power-like GIM mechanism. At the next order in $\alpha$ the two-photon penguin results in a long distance contribution, where the absorptive part saturates the measured branching ratio and the dispersive part can only be estimated with large uncertainties [55]. Even with these large theoretical uncertainties, important constraints on flavour changing scalar operators result [56] from the measurement $\mathcal{B}(K_L \to \mu^+ \mu^-) = 6.84(11) \times 10^{-9}$ [57].

The rare $K_L \to \pi^0 \ell^+ \ell^-$ decays are CP violating at leading order in the electroweak interactions. The direct CP violating contribution is dominated by top-quark loops in the SM and is correspondingly sensitive to short distances. There is a long distance pollution via indirect CP violation through $K_L - K_S$ mixing and the CP conserving two-photon contribution. Contrary to the $K_L \to \mu^+ \mu^-$ decay, these long distance effects can be calculated in chiral perturbation theory. A combined measurement could be used to disentangle short- and long-distance contributions and would be sensitive to the scalar operators of Eq. (2) or tensor operators [56].

Rare Kaon decays with neutrinos in the final state are exceptionally clean. Only $Z$-penguin and box diagrams contribute and light-quark effects are suppressed by a quadratic GIM mechanism. The branching ratio of the $K^+ \to \pi^+ \nu \bar{\nu}$ decay is given,

$$\kappa_+(1 + \Delta_{\text{EM}})
\left[
\left(\frac{\text{Im}\lambda_t}{\lambda^5} X_t\right)^2 + \left(\frac{\text{Re}\lambda_c}{\lambda^5} (P_c + \delta P_{c,a}) + \frac{\text{Re}\lambda_t}{\lambda^5} X_t\right)^2\right], \tag{8}
$$

where the normalisation factor $\kappa_+$ is extracted from $K_{l3}$ decays including isospin breaking corrections and the corresponding photon cutoff dependence denoted by $\Delta_{\text{EM}}$ [58]. The factors $P_c$ and $\delta P_{c,a}$ comprise the short- and long-distance effects of light quarks – for the most recent theoretical estimates see Refs. [59, 60]. The Wilson coefficient $X_t$ is the top-quark contribution of $C^d_t$ times $2\pi\sin^2\theta_W/\alpha$. It is now known to high accuracy, after the inclusion of two-loop electroweak corrections removed the electroweak scheme dependence of the electroweak input parameters [61, 62]. This error reduction is even more important for the CP violating neutral decay mode whose branching ratio depends only on the top-quark contribution. The errors in the SM theory prediction $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu}) = (7.81^{+0.80}_{-0.71} \pm 0.29) \times 10^{-11}$ and $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu}) = (2.43^{+0.40}_{-0.37} \pm 0.06) \times 10^{-11}$ are dominated by the parametric uncertainties (first error), while the intrinsic theory error (second error) is less than 4% for both modes.

The precise theory prediction and the $V_{ts} V_{td}$ suppression imply a high sensitivity to new physics with generic flavour violation. Effects of new heavy particles modify
\( X_t = 2\pi \sin^2 \theta_W /\alpha (C_L^{sd} + C_R^{sd}) \), and correspondingly the branching ratios of the neutral and charged decay modes. Correlations between the two branching ratios can then be used to discriminate between various new physics models e.g. Randal-Sundrum type models, theories with extra fermions or the MSSM [41]. Interactions of additional light particles could also be tested via the \( K \rightarrow \pi + \) nothing signals – for details see Ref. [33].

The NA62 experiment aims to measure 80 \( K^+ \rightarrow \pi^+ \nu\bar{\nu} \) events with signal acceptance and background both of the order of 10% [63], using an 75GeV, 800MHz beam with \( \sim 6\% \) \( K^+ \). Several background rejection methods have to be employed to measure a branching ratio of \( \mathcal{O}(10^{-10}) \): a cut on the missing mass of the reconstructed candidates can be performed by measuring the momentum of the incoming \( K^+ \) momentum with the so-called Gigatracker, and the momenta of the daughter particles with a straw-chamber magnetic spectrometer. Further background from leptonic and semileptonic Kaon decays is removed using a Cherenkov detector, while a photon-veto system should remove decay modes with \( \pi^0 \)s and/or radiative photons.

### 7 Conclusions

Recent progress from experiment and theory in the field of rare decays has confirmed that flavour and CP violation is well described by the CKM mechanism. However, at present, only a limited number of theoretically clean observables have been probed by experiments. Current and future experiments at CERN, KEK and SuperB will change this and allow precision tests of flavour violation even beyond the Standard Model. Rare decays will therefore continue to be an important tool for understanding fundamental interactions.

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