Probabilistic model of penetration of biological agents through filtering devices

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Abstract. The human protection issue from biological agents with the help of filtering devices is studied in this article. This issue is actual for all the countries of the world due to the current coronavirus situation. Considering the hypothesis of the ordinariness of the biological agents’ flow and the absence of aftereffects in the filter, the probabilistic model of the penetration of biological agents through filtering respiratory protection devices is designed based on the Kolmogorov differential equation system applied for the adequate protection of medical personnel.

1. Introduction
Diseases that pose a threat to health have always been a particular danger to people [1]. Protecting humanity from various threats has always been a top priority, and now, in light of the Coronavirus COVID-19 pandemic (2019-nCoV), the assessment of filter-type respiratory protection devices is also relevant [2]. Respiratory infections, namely coronavirus infection, are in the first place among all human diseases in terms of social significance, huge damage caused to public health, and the country’s economy. Moreover, one should not forget the mutational variability of the virus, which leads to more and more serious consequences in sick people [3-5].

According to statistics, a significant percentage of those who fell ill and died from coronavirus exposure are medical personnel [6]. As a result, the issue of adequacy of the protection of medical personnel working with patients affected by coronavirus should be given considerable attention. To reduce the risk of disease for medical personnel and to ensure safe work in the red zone, the filtering properties of respiratory protective equipment must meet all safety parameters [7-9].

The work aims to develop a model describing the filtering properties of respiratory protective equipment based on the system of Kolmogorov differential equations.
2. Methods
There are several situations in which medical personnel may be affected by biological agents. But always, and especially when working in medical facilities, the respiratory protection of workers should be kept in mind [10].

To protect the respiratory organs of workers, respiratory protective equipment (RPE) is used. RPE protects the respiratory organs while working with a polluted atmosphere and (or) in conditions of lack of oxygen. RPE prevents dust, chemicals, gases, and aerosols from entering the lungs when extinguishing fires, working at a hazardous production, in conditions of dust storms and smog.

The classification of RPE includes several types of devices that differ in design, purpose, and scope. The RPE includes gas masks, respirators, self-contained breathing apparatus, a set of an additional cartridge, and a hopcalite cartridge.

According to the principle of operation, RPE for the respiratory system is divided into filtering and isolating devices. The peculiarity of the filtering RPE is that they have a filter element, with the help of which the polluted air is cleaned and, after cleaning, it enters the respiratory system. Isolating RPE completely isolate the respiratory organs from the external environment, thus preventing any contact with the atmosphere.

Thus, the RPE used by medical personnel should have filtering properties that correspond to the safety parameters when using them in working with infected people, so that it is possible to avoid the penetration of biological objects through the RPE.

A biological object (e.g., coronavirus) has given discrete characteristics and its slippage through the filtering device is probabilistic [11-12].

Let us use the system of Kolmogorov differential equations which is also called the queuing system of equations [13-14]:

\[
\frac{dp_j(t)}{dt} = \sum_{i=1}^{n} p_i(t) \lambda_{ij}(t) - \sum_{i=1}^{n} \lambda_{ji}(t), \quad j = 1, n, \tag{1}
\]

where \( p_j(t) \) are the probabilities of the device being in state \( s_j \) at the time \( t, j = 1...n \); \( \lambda_{ij}(t) \) is the intensity of transition of a device or process \( S \) from the state \( i \) to the state \( j \). The system of equations (1) is solved with the initial conditions setting the probabilities of the states to the vector \( p(t) \) at the initial time \( t = 0 \). Obviously, for any moment of time the following condition is satisfied:

\[
\sum_{i=1}^{n} p_i(t) = 1. \tag{2}
\]

All states of the system at time \( t \) form a complete group of incompatible states; thus, the sum of the probabilities of these states is equal to one.

The Kolmogorov system of differential equations is composed as follows: on the left side of each equation is the derivative of the probability of the \( i \)-th state; on the right side – the sum of the products of the probabilities of all states by the intensity of the corresponding streams of events, minus the total intensity of all streams that take the system out of the given state, multiplied by the probability of the \( i \)-th state.

Let us consider the application of the Kolmogorov equations on the example of filtering biological objects on the corresponding devices [15]. The main are three states of the filtration process \( S \):

- \( s_1 \) – the biological object is in a free state before the filtering device;
- \( s_2 \) – the biological object is inside the filtering device in a connected state;
- \( s_3 \) – the biological object is behind the filtering device after the slip.

Figure 1 presents three states of the filtration process \( S \).
The system of equations (1) in such formulation of the problem will have the following form:

\[
\begin{align*}
\frac{dp_1(t)}{dt} &= -\lambda_{12}(t)p_1(t) + (\lambda_{21}(t) + \lambda_{23}(t))p_2(t) + \lambda_{32}(t)p_3(t), \\
\frac{dp_2(t)}{dt} &= \lambda_{12}(t)p_1(t) - (\lambda_{21}(t) + \lambda_{23}(t))p_2(t) + \lambda_{32}(t)p_3(t), \\
\frac{dp_3(t)}{dt} &= \lambda_{12}(t)p_1(t) + (\lambda_{21}(t) + \lambda_{23}(t))p_2(t) - \lambda_{32}(t)p_3(t).
\end{align*}
\] (3)

This system of equations should be solved taking into account the vector of initial states, which components are equal: \(p_1(0) = 1, p_i(0) = 0, p_1(0) = 1, i = 2...3\). Any of the equations of the system (3) can be replaced by the normalization equation:

\[
p_1(t) + p_2(t) + p_3(t) = 1
\] (4)

To solve the system (3), it is necessary to know the dependences of transition intensities \(\lambda_{ij}(t)\) on time, which can be estimated by the results of corresponding experiments based on the statistical determination of probabilities.

3. Results

The solution of the system of equations (3) makes it possible to trace the change of probabilities of staying of the biological object in the corresponding state at the initial stage of the filtering device operation. After a certain interval of time (theoretically \(t \to \infty\)) a stationary air purification mode will occur, at which the probabilities of states will be set constant, independent of time.

The system of equations (1) will take the form:

\[
\sum_{i=1}^{3} p_j(t)\lambda_{ij}(t) - p_j(t) \sum_{i=1}^{3} \lambda_{ji}(t) = 0, \quad j = 1, 3.
\] (5)

To obtain a nontrivial solution (5), condition (4) should be used instead of one of the equations (5).

The limit probability \(p_i\) of state \(s_i\) can be considered as the average relative time spent by a biological object in this state. The probability can be expressed by the average time \(t_i\) of a single stay of a particle in state \(s_i\) and the average time \(r_i\) of a single stay of a particle outside the state \(s_i\) as a ratio:

\[
p_i = \frac{t_i}{t_i + r_i}.
\] (6)

Figure 1. Three states of the filtration process S.
The solution of the system of equations (5) with the required $p_3 = 0$ (since we do not want the biological agent to penetrate behind the filtering device) gives the probability values expressed through the intensity of transitions:

$$
\begin{align*}
    p_1(t) &= \frac{\lambda_{21}(t) + \lambda_{23}(t)}{\lambda_{12}(t) + \lambda_{21}(t) + \lambda_{23}(t)}, \\
    p_2(t) &= \frac{\lambda_{21}(t)}{\lambda_{12}(t) + \lambda_{21}(t) + \lambda_{23}(t)}, \\
    p_3(t) &= 0.
\end{align*}
$$

(7)

Thus, we achieve the desired result – the absence of penetration of the biological agent behind the filtering device, and, consequently, the protection of medical personnel from infection while performing their duties.

4. Conclusion

Protecting humanity from various mass diseases and threats is a top priority for all the current research. Providing the medical personnel who take the blow in dealing with patients infected with airborne infections with adequate protective equipment, namely, respiratory protection, is especially important. Therefore, a model describing the filtering properties of respiratory protective equipment was constructed based on Kolmogorov equations. This model can help in the installing appropriate respiratory protective equipment, which will be used when working in a dangerous zone, including patients with coronavirus.

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