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Observation of Transition from Escape Dynamics to Underdamped Phase Diffusion in a Josephson Junction

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We have investigated the dynamics of underdamped Josephson junctions with a relatively small Josephson coupling $E_J$. In addition to the usual crossover between macroscopic quantum tunneling (MQT) and thermal activation (TA) behavior we observe, for the first time, the transition from TA behavior to underdamped phase diffusion. Above the crossover temperature, the threshold for switching into the finite-voltage state becomes extremely sharp. We propose a $(T, E_J)$ phase diagram with various regimes and show that for a proper description of it, dissipation and level quantization in a metastable well are crucial.

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A hysteretic Josephson junction (JJ) switching from its metastable zero-voltage state into a stable finite-voltage state has recently been used as a readout device for superconducting quantum bit systems in many experiments [1]. Switching measurements also enable one to perform conventional large bandwidth current measurements, and recently there have been proposals to use hysteretic JJs as ammeters for studying phenomena like non-Gaussian noise [2]. Often it may be advantageous to reduce the critical current $I_c$ of the detecting junction in order to increase the measurement sensitivity. Yet the physics governing switching phenomena of small $I_c$ junctions ultimately differs from those with large $I_c$ [3]. How far can one reduce $I_c$ while still maintaining the useful features of the detector? Despite a considerable amount of work on junctions with moderate $I_c$ [4], no clear picture based on a systematic study exists as yet of temperature-dependent escape phenomena in such junctions. In this Letter we present such a study.

The dynamical variable of a JJ is the phase difference $\varphi$ of the superconducting wave function in the two electrodes. Within the resistively and capacitively shunted junction (RCSJ) model, the dynamics of $\varphi$ is governed by the Josephson energy $E_J = \hbar I_c/2e$, the charging energy $E_C = e^2/2C_J$ ($C_J$ is the junction capacitance), and a shunt resistance $R$ responsible for dissipation. For later use we also define the bare plasma frequency $\omega_p = \sqrt{8E_J E_C}/\hbar$ and the quality factor $Q = \omega_p R_C$. As we will detail below, the behavior of the junction at a given temperature $T$ can be classified according to the phase diagram of Fig. 1(a). The overdamped case $Q < 1$ with $E_J \approx E_C$ was studied in detail by Vion et al. [3], who uncovered the existence of a phase diffusion regime at finite $T$ with the appearance of a small voltage, prior to switching to a voltage on the order of twice the superconducting gap $\Delta$ [5]. As far as the underdamped case $Q > 1$ is concerned, most experiments were done on junctions with relatively high $I_c$. Depending on $T$ such junctions escape out of the metastable zero-voltage state either via macroscopic quantum tunneling (MQT) or thermal activation (TA) processes, switching directly to the finite-voltage state. In this Letter, we focus on the regime $Q > 1$ with relatively small $I_c$, such that $E_J \sim k_B T \gg E_C$. We show, theoretically and experimentally, that a regime exists where escape does not lead to a finite-voltage state, but rather to underdamped phase diffusion (UPD) [the shaded region in Fig. 1(a)].

According to the RCSJ model, the dynamics of a JJ biased with a current $I$ is that of a particle (mass $m = \hbar^2/8E_C$) whose position is $\varphi$. It moves in a tilted cosine potential $U(\varphi) = -E_J \cos \varphi + (I/I_c)\varphi$ under a viscous force $(-\hbar^2/2m)(\partial^2 \varphi/\partial t^2)$. The bias current renormalizes the plasma frequency, such that $\omega_p = \sqrt{m^{-1} \hbar^2 U(\varphi)/2\pi^2} = \sqrt{8E_J E_C q_0}/\hbar$ with $q_0 = \sqrt{2(1 - I/I_c)}$. The cosine potential has wells where the particle can be localized; $\varphi$ then has constant average value, and the average voltage across the junction is zero. For nonzero $I$ the quantum levels in the potential well are metastable and the particle can escape from a given well either via TA over, or MQT through the barrier. For large junctions $\Gamma_{\text{TA}}(I, T) = \frac{\hbar}{2\pi} e^{-\Delta U/k_b T}$ for the TA escape rate and $\Gamma_{\text{MQT}}(I) = 12\sqrt{6\pi m} \left(\frac{m}{\hbar^2}\right)^{3/2} \sqrt{\Delta U/\hbar \omega_p} \times e^{-36/\Delta U/\hbar \omega_p}$ for the MQT rate [6]. In the cubic approximation the barrier height is $\Delta U = \frac{3}{4} E_J q_0$. Below the crossover temperature $T_0 = \hbar \omega_p/2\pi k_B$ the dominant escape mechanism is MQT. Approximating the total escape rate as $\Gamma_{\Sigma}(I, T) \approx \Gamma_{\text{TA}}(I, T) + \Gamma_{\text{MQT}}(I)$, the escape probability in the time interval $0 \leq t \leq \tau$ can be written as $P_{\Sigma}(I, T) = 1 - e^{-\int_0^\tau \Gamma_{\Sigma}(I, t) dt}$. If dissipation is weak, upon escape from the well the particle moves down the potential and phase is running freely, hence the voltage reaches a finite value,
that are underdamped even at $E_J$ acts as a short. Here we will consider junctions about $2\Delta$. However, if dissipation is strong enough, there is a finite probability that, upon escape from the well, the particle is relocalized in the next well instead of running down the potential: the phase then diffusively moves from one metastable well to another, see Fig. 1(b). In this UPD regime the average voltage across the junction is much smaller than the barrier top, and if the regime the average voltage across the junction is much smaller than the barrier top, and if the escape does not necessarily lead to the transition into a running state. The condition $\Gamma_{\text{TA}}(I_m, T_D) = 1/\tau$ determines the separatrix $E^D_J(T_D)$ between the TA and UPD regions in Fig. 1(a) with current pulses of length $\tau$:

$$E^D_J = \frac{3}{2} k_B T_D \left( 1 - \frac{I_m}{I_c} \right)^{-3/2} \ln(\omega_p/2\pi). \quad (1)$$

Similarly, for $T < T_0$, the separatrix between MQT and UPD is found from $\Gamma_{\text{MQT}}(I_m) = 1/\tau$, $E^D_J$ is independent of $T$, and given by Eq. (1) with $T_0$ replacing $T_D$.

We present experimental data of two samples, a dc-SQUID and a single JJ. They were fabricated using standard electron beam lithography and aluminum metallization in a UHV evaporator. The AlOx tunnel barriers were formed by basic room temperature oxidation of Al. The dc-SQUID consists of two wide superconducting planes connected by two short superconducting lines with tunnel junctions in the middle forming the dc-SQUID loop of area $20 \times 39 (\mu \text{m})^2$ (see the inset in Fig. 2). The loop inductance was determined [8] to be around 100 pH, small as compared to the calculated Josephson inductance ($L = \phi_0/2\pi I_c = 400 \text{ pH per junction}$). The dc-SQUID thus behaves almost like a single JJ, whose $I_c$ can be tuned. The other measured sample was a single junction between long inductive biasing lines. The normal state resistances of the dc-SQUID and the single JJ were 1.3 kΩ and 0.41 kΩ, yielding for $I_c$ 199 and 630 nA, respectively. Assuming a specific value of 50 fF/(\mu m)^2, the capacitances of the samples were estimated to be 100 and 130 fF, respectively. Both measured samples had strongly hysteretic $I$-$V$ characteristics with retrapping currents well below 1 nA.

The experimental setup is presented in the inset in Fig. 2. Switching probabilities have been measured by applying a set of trapezoidal current pulses through the sample and by measuring the number of resulting voltage pulses. At the sample stage we used low pass RC filters (surface mount components near the sample). In the measurements on a single junction we used surface mount capacitors ($C_s = 680 \text{ pF}$), but in the dc-SQUID measurements we had $\pi$ filters in series with resistors, with $C_s \sim 5 \text{ nF capacitance to ground}$. The resistors were $R_s = 500 \Omega$ and 680 $\Omega$ in the measurement on a dc-SQUID and on a single junction,
respectively. Bonding wires with inductance of order nH connect the sample to filters.

In Fig. 2 we present the measured cumulative switching histograms (open circles) of the dc-SQUID at different fluxes $\Phi$ and temperatures with $\tau = 200 \mu$s. At the lowest temperatures, the histograms can be well fitted by the MQT model, giving $I_c = 200, 128$, and $55$ nA, for $\Phi/\Phi_0 = 0, 0.28$, and $0.41$, respectively. For $\Phi = 0$ we also plot the escape probability $P_2(I,T)$ defined earlier (dotted blue lines). In Fig. 3 we show the measured histogram position $I_{50%}[P(I_{50%}) = 0.5]$ and the width $\Delta I (= I_{90%} - I_{10%})$ for both samples. The dc-SQUID measurements were done both at negative and positive values of flux in order to ensure that the external flux had not changed. The position in Fig. 3(a) is normalized to the corresponding value of $I_c$ at zero temperature. At low $T$ all the measured data are consistent with MQT results. On increasing $T$ the parameters are constant up to the estimated crossover temperature $T_0$. For $T > T_0$ the width is increasing and the position is moving down as the TA model predicts. The qualitative agreement is good for most of the results up to the temperature $T_D$. At $T_D$, $\Delta I$ starts to decrease abruptly. Moreover, the position $I_{50%}$ saturates at the same value $\approx 0.3I_c$. For the single JJ, the saturation occurs at $\approx 0.3I_c$. The dc-SQUID data measured at $\Phi = \pm 0.41\Phi_0$ show no trace of thermal activation at any temperature, and underdamped phase diffusion prevails down to temperatures below $T_0 \approx 50 \text{ mK}$ where MQT dominates.

If we assume a realistic shunt impedance $R(\omega_p) \approx 500 \Omega$ (the value of the surface mount resistors), we obtain $Q \approx 4$ at $I_c = 200$ nA and $C_J = 100 \text{ fF}$, which yields $I_m \approx 0.35I_c$, like in the experiment. In the diagram of Fig. 1 we also present $I_c$ of the dc-SQUID at fluxes 0, $\pm 0.28\Phi_0$, and $\pm 0.41\Phi_0$ by horizontal dashed lines. It can be seen that the intersections of the dashed lines and the boundary of the UPD regime are very close to the experimental values of $T_D$. The saturation of the histograms and their reentrant steepness is thus a manifestation of the crossover from TA escape into UPD due to dissipation. In the case of a single JJ we obtain $I_m = 0.3I_c$ and $T_D = 650 \text{ mK}$, yielding $Q \approx 4.4$. This corresponds to $R(\omega_p) = 230 \Omega$, somewhat smaller than the anticipated value $R(\omega_p) \approx 680 \Omega$.

Figure 2 shows that the standard TA and MQT models cannot account for our observations. Except for the data at...
the lowest temperatures, the measured histograms deviate from $\mathcal{P}_2(I, T)$. Dissipation alone cannot explain the difference. The basic TA model yields a width $\Delta I \propto T^{2/3}$ [6] and it can be seen in Figs. 2 and 3 that the dc-SQUID has weaker temperature dependence even well below $T_D$. In the dc-SQUID there are just few energy levels in the well and thus the assumptions of continuous energy spectrum are not valid [6]. Within the semiclassical model of Larkin and Ovchinnikov (LO) [9], the total escape probability is calculated using $P_{LO}(\tau) = \sum_1^l \rho_1(\tau)$, where $\rho_1(\tau)$ is the probability of finding the particle in a state $k$ after the current pulse of length $\tau$. The kinetic equation can be written as $\frac{d\rho_k}{dt} = \sum_j (\gamma_{jk} \rho_j - \gamma_{kj} \rho_k) - \Gamma_k \rho_k$. We take into account transitions $\gamma_{jk}$ between neighboring levels and tunneling out, $\Gamma_k$. The relaxation rates between levels $j$ and $j-1$ are well approximated by $\gamma_{j-1,j} = \frac{\hbar}{2 \pi} |I_t|^2 \times \int_0^t e^{\lambda(t)} \rho(0) dt$, where $\lambda$ is the transition matrix. $I_t$ is set to zero in the beginning, and $\rho(0)$ is Boltzmann distributed.

The effect of the relocating dissipation must be taken into account also in the quantized energy level model. Using again $E_F = 8E_j/Q$, and the fact that the energy difference between the two successive maxima is $-2\pi E_j I_c / I_\epsilon$, the level energy $E$ must satisfy

$$\Delta U - E < E_j (2\pi I_c / I_\epsilon - 8/Q) \quad (2)$$

to allow switching into the free running state. If (2) is violated, the corresponding tunneling rate is set to zero [10]. Note that (2) yields the condition $I > I_c$ if we set $\Delta U = E$, but in Eq. (2) we assume that after tunneling the starting point is not at the potential maximum. The solid red lines in Fig. 2 present results of simulations with quantized energy levels and dissipation for the data of the dc-SQUID at zero and $\pm 0.28 q_0$ fluxes. At $\pm 0.14 q_0$ $I_c$ is so small that the escape probability is large even at zero bias (except at the lowest temperatures). This means that the phase is moving constantly rather than infrequently escaping from a localized state, and thus our model does not work anymore. The only fitting parameter was $Q$, and the fitted values were in a very reasonable range. At $\Phi = 0$ we find $Q = 6$ at the lowest temperature, and it decreases with increasing temperature up to 4 at 325 mK. At $\pm 0.28 q_0$ $Q = 3$ to 4, again decreasing with temperature. In Fig. 3 we present $I_{\text{peak}}$ and $\Delta I$ of the simulated histograms. We present also the simulated parameters for single junction ($Q = 4$) [5]. In the measurements on a single junction, the number of energy levels is large ($\approx 20$) and the their separation is smaller than their width. Our model assumes well-separated levels and thus the agreement with a simple TA model is better for a single junction at $T < T_D$, especially in Fig. 3(a), whereas reentrant steepness of the histograms could not be explained with the basic TA model. The agreement between simulation and measurements on a dc-SQUID is excellent. The position and the width of the measured histograms coincide and, in particular, at higher temperatures the simulated histograms for both samples start to get steeper again. At higher temperatures the upper energy levels, whose escape rate is significant with smaller potential tilting angles, are populated as well. The histograms thus peak at smaller currents and the condition (2) is not necessarily fulfilled anymore. What remains in the measured (and simulated) histograms is the escape from the states at the tail of the Boltzmann distribution above the dissipation barrier.

In summary, our measurements demonstrate for the first time how a Josephson junction can transit from the macroscopic quantum tunneling through thermal activation into underdamped phase diffusion under the variation of temperature and magnetic flux. A model including a phase diffusion barrier and level quantization yields a quantitative fit of the observations.

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Note added.—Since the submission of this manuscript, similar experimental results were reported in [11]. Their interpretation differs from ours in details.

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