Cosmological Aspects of the D-brane World

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The D-brane world is an idea that we are living on the D-brane imbedded in a 10- or 11-dimensional spacetime of string theories, aiming at the construction of realistic models from string theories. We investigate the cosmological aspects of the D-brane world, focusing on homogeneous anisotropic cosmology driven by the dilaton and the NS-NS 2-form field which becomes massive in the presence of the D-brane. The dilaton possesses the potential due to the presence of the D-brane, various form field fluxes, and the curvature of extra dimensions. In the absence of stabilizing potential, we found the attractor solutions for this system which show the overall features of general solutions. In the presence of the non-vanishing NS-NS 2-form field, the homogeneous universe expands anisotropically while the D-brane term dominates. The isotropy is recovered as the dilaton rolls down and the curvature term dominates. With the stabilizing potential for the dilaton, the anisotropy developed by the initial NS-NS 2-form field flux is erased as the NS-NS 2-form field begins to oscillate around the minimum, forming the B-matter, and the isotropic matter-dominated universe is obtained.

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I. INTRODUCTION

The idea of the brane world first appeared as an alternative solution to the gauge hierarchy problem by allowing large or warped extra dimensions while we are leaving on a 3-brane to avoid various known experimental bounds [1]. String theories have such structures as D-branes on which open strings can reside and thus provide a framework where realistic brane world models can be constructed. The D-brane world is an idea that we are living on the D-brane imbedded in a 10- or 11-dimensional spacetime of string theories, aiming at the construction of realistic models from string theory. One way to see how realistic the D-brane world models can be is to look into the cosmological aspects of them.

Low energy effective theories derived from the NS-NS sector of string theories contain the gravity, $g_{\mu\nu}$, the dilaton, $\Phi$, and the NS-NS 2-form field, $B_{\mu\nu}$. The existence of the last degree of freedom leads to intriguing implications in string cosmology [2, 3]. In four spacetime dimensions, the massless antisymmetric 2nd-rank tensor field is dual to the a pseudo-scalar (axion) field [3], and the axion–dilaton system is known to develop an unobserved anisotropy in our Universe, which can be diluted away at late times only in a contracting universe [4].

Such a disastrous cosmological situation can be resolved also in string theory which contains another dynamical object called D-brane [5]. The gauge invariance on the D-brane is maintained through the coupling of the gauge field strength to the NS-NS 2-form field [5, 6, 7]. Then, the effective action derived on the D-brane describes the NS-NS 2-form field as a massive and self-interacting antisymmetric tensor field. Cosmological evolution of such a tensor field has been investigated in Ref. [8] assuming that the dilaton is fixed to a reasonable value. Although the time-dependent magnetic $B$ field existing in the early universe develops an anisotropy in the universe, it was realized that the matter-like behavior of the $B$ field (B-matter) ensures a dilution of the anisotropy at late times and thus the isotropy is recovered in reasonable cosmological scenarios [8]. In such sense the effect of antisymmetric tensor field on the D-brane is distinguished from that of field strength of the U(1) gauge field [5].

In this paper, we investigate the cosmological evolution of the dilaton–NS-NS 2-form field system in our Universe which is assumed to be imbedded in the D-brane. In general, the D-brane and the NS-NS 2-form field couple to the R-R form fields. Here, we assume a trivial R-R background and omit them in our analysis. The usual string cosmology with the dilaton suffers from the notorious runaway problem, which is also troublesome in the D-brane universe. In our study, the dilaton obtains two exponential potential terms due to the curvature of extra dimensions $\Lambda$, and the D-brane tension (the mass term of the B-matter) $m_B$. It is interesting to observe that the
dilaton can be stabilized for negative $\Lambda$ which, however, leads to a contracting universe due to the effective negative cosmological constant in our Universe. When $\Lambda$ is positive, the B-matter dominance will be overturned by the $\Lambda$ dominance as the dilaton runs away to the negative infinity. As a consequence of this, the initial anisotropy driven by the B-matter can also be diluted away at late times.

For a realistic low energy effective theory, string theory must be endowed with a certain mechanism generating an appropriate vacuum expectation value for the dilaton. In such a situation, the dilaton is expected to be stabilized at some stage of the cosmological evolution affecting the dynamics of the NS-NS 2-form field. Taking an example of the dilaton stabilization, we will also examine the cosmological evolution of the dilaton–NS-NS 2-form field system in which the essential features of Ref. [8] are reproduced.

This paper is organized as follows. In Section 2, we describe the low energy effective action of the D-brane world and the corresponding field equations. In Section 3, we find analytic and numerical cosmological solutions of the dilaton–NS-NS 2-form field system to observe an intriguing interplay of the curvature $\Lambda$ and the D-brane tension $m_B$. In Section 4, we consider the evolution of the anisotropic universe with the dilaton stabilization which leads to a satisfactory cosmology of the D-brane universe. We conclude in Section 5.

II. EFFECTIVE FIELD THEORY OF THE D-BRANE WORLD

The main idea of the D-brane world is that we reside on a Dp-brane imbedded in 10- or 11-dimensional spacetime with extra-dimensions compactified. The bosonic NS-NS sector of the D-brane world consists of the U(1) gauge field $A_\mu$ living on the Dp-brane and the bulk degrees including the graviton $g_{\mu\nu}$, the dilaton $\Phi$, and the antisymmetric tensor field of rank-two $B_{\mu\nu}$.

The low energy effective action consists of the bulk action

$$S_{\text{NS}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} e^{-2\Phi} \left[ R + 4(\partial\Phi)^2 - \frac{1}{12} H^2 \right],$$

and thebrane action in p spatial dimensions;

$$S_{\text{Dp}} = -\mu_p \int d^{p+1}x e^{-\Phi} \sqrt{-\det(g + B)}$$

$$+ i\mu_p \int_{p+1} \text{Tr} \left[ \exp(B) \wedge \sum_q C_q \right],$$

where $\kappa_{10}^2 = \frac{1}{2}(2\pi)^{7}\alpha' \gamma$ and $\mu_p = (\pi/\kappa_{10}^2)(4\pi^2\alpha')^{3-p}$. Recall that $\alpha'$ defines the string scale; $m_s = \alpha'^{-1/2}$. Here the spacetime indices are not explicitly expressed and the field strength $H$ of the NS-NS form field is $H_{\mu\rho\sigma} = \partial_{[\mu}B_{\rho\sigma]}$ because of Bianchi identity of the U(1) gauge field. In the presence of the brane, the gauge invariance of $B_{\mu\nu}$ is restored through its coupling to a U(1) gauge field $A_\mu$ and the gauge invariant field strength is

$$B_{\mu\nu} = B_{\mu\nu} + 2\pi\alpha'F_{\mu\nu},$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. (3)

The D-brane and the NS-NS form field on it couple to the R-R form field as shown in the second term of Eq. (2). In this paper, we consider the case that the D-brane and the NS-NS form field are homogeneous in our 4D world and all R-R form fields live in extra dimensions so that their sole effect is the curvature term in the 4D effective action which will be discussed below. Thus all R-R form fields are not considered from now on, assuming the trivial R-R background. We leave the more general case of both the NS-NS and the R-R form field living in our 4D world as our future work.

After the compactification of extra dimensions, the four-dimensional effective action of the bosonic sector in the string frame is

$$S_{\text{E}} = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left[ R - 2(\nabla\Phi)^2 - \frac{1}{12} e^{-4\Phi} H^2 ight]$$

$$- 2\Lambda e^{-2\Phi} m_B^2 e^{3\Phi} \sqrt{1 + \frac{1}{2} e^{-4\Phi} B^2 - \frac{1}{16} e^{-6\Phi} (B\cdot B)^2}.$$ (7)
The field equations derived from the action \( \Sigma \) are

\[
\begin{align*}
\nabla^\lambda H_{\lambda\mu\nu} - 4H_{\lambda\mu\nu} \nabla^\lambda \Phi \\
-m_B^2 \nabla^\lambda F_{\mu\nu} - \frac{1}{2} e^{-4\Phi} \Gamma^\lambda_{\mu\nu} (BB^*) = 0, \tag{8}
\end{align*}
\]

\[
-\nabla^2 \Phi + \frac{\partial V(\Phi)}{\partial \Phi} = 0, \tag{9}
\]

\[
G_{\mu\nu} = \kappa^2 T_{\mu\nu}, \tag{10}
\]

where the dilaton potential is

\[
V(\Phi) = \frac{1}{4} \left[ 2\Lambda e^{2\Phi} + \frac{1}{12} e^{-4\Phi} H^2 + \frac{m_B^2 e^{3\Phi}}{\sqrt{1 + \frac{1}{2} e^{-4\Phi} B^2 - \frac{1}{16} e^{-8\Phi} (BB^*)^2} \right], \tag{11}
\]

and the energy-momentum tensor is given by

\[
\kappa^2 T_{\mu\nu} = -g_{\mu\nu} \Lambda e^{2\Phi} + 2\nabla_\mu \Phi \nabla_\nu \Phi - g_{\mu\nu} (\nabla \Phi)^2 \\
+ \frac{1}{12} e^{-4\Phi} \left[ 3H_{\mu\lambda\rho} H^\lambda_{\nu\rho} - \frac{1}{2} g_{\mu\nu} H^2 \right] \\
+ \frac{1}{2} m_B^2 e^{3\Phi} - g_{\mu\nu} - \frac{1}{2} g_{\mu\nu} e^{-4\Phi} B^2 + e^{-8\Phi} B_{\mu\lambda} B_{\nu} \lambda \frac{1}{\sqrt{1 + \frac{1}{2} e^{-4\Phi} B^2 - \frac{1}{16} e^{-8\Phi} (BB^*)^2}} \tag{12}
\]

In the subsequent sections, we examine the equations of motion and find cosmological homogeneous solutions. Cosmological implications of the D-brane world is of our main interest, including the stabilization of the dilaton and the evolution of the 2-form field.

\section{III. COSMOLOGICAL HOMOGENEOUS SOLUTIONS}

\subsection{A. The equations of motion}

In this section we study cosmological solutions of the D-brane universe in the presence of both the dilaton and the NS-NS form field. We assume spatially homogeneous configurations for the NS-NS form field and the dilaton, and look for the time evolution of these fields and the expansion of the universe.

The non-vanishing homogeneous antisymmetric tensor field, in general, implies the anisotropic universe. To consider the simplest form of anisotropic cosmology, we take only a single magnetic component of \( B_{\mu\nu} \) to be nonzero, namely \( B_{12}(t) = B(t) \) and \( B_{01}(t) = B_{23}(t) = B_{31}(t) = 0 \). Then the metric consistent with this choice of field configuration is of Bianchi type I

\[
ds^2 = -dt^2 + \sum_{i=1}^{3} a_i(t)^2 (dx^i)^2. \tag{13}
\]

Then the field equation for \( B \), Eq. \( \Sigma \) becomes

\[
\ddot{B} + \left( \frac{\dot{\alpha}_1}{a_1} + \frac{\dot{\alpha}_2}{a_2} + \frac{\dot{\alpha}_3}{a_3} - 4\dot{\Phi} \right) \dot{B} + \frac{m_B^2 e^{3\Phi} B}{\sqrt{1 + B^2/e^{4\Phi} a_1^2 a_2^2}} = 0, \tag{14}
\]

the dilaton-field equation \( \Sigma \) is

\[
\ddot{\Phi} + \left( \frac{\dot{\alpha}_1}{a_1} + \frac{\dot{\alpha}_2}{a_2} + \frac{\dot{\alpha}_3}{a_3} \right) \dot{\Phi} = -2\dot{\rho}_B - \frac{1}{2} \dot{\rho}_b - \dot{\rho}_b - \dot{\rho}_\Lambda, \tag{15}
\]

and Einstein equations \( \Sigma \) are

\[
\frac{\dot{\alpha}_1}{a_1} + \frac{\dot{\alpha}_2}{a_2} + \frac{\dot{\alpha}_3}{a_3} = \dot{\rho}_\Phi + \dot{\rho}_B + \dot{\rho}_b + \dot{\rho}_\Lambda, \tag{16}
\]

\[
\frac{\dot{\alpha}_2}{a_2} + \frac{\dot{\alpha}_3}{a_3} + \frac{\dot{\alpha}_1}{a_1} = -\dot{\rho}_\Phi + \dot{\rho}_B + \dot{\rho}_b + \dot{\rho}_\Lambda, \tag{17}
\]

\[
\frac{\dot{\alpha}_3}{a_3} + \frac{\dot{\alpha}_1}{a_1} + \frac{\dot{\alpha}_2}{a_2} = -\dot{\rho}_\Phi + \dot{\rho}_B + \dot{\rho}_b + \dot{\rho}_\Lambda, \tag{18}
\]

\[
\frac{\dot{\alpha}_1}{a_1} + \frac{\dot{\alpha}_2}{a_2} + \frac{\dot{\alpha}_3}{a_3} = -\dot{\rho}_\Phi - \dot{\rho}_B + \dot{\rho}_b + \dot{\rho}_\Lambda, \tag{19}
\]

where

\[
\dot{\rho}_\Lambda = \Lambda e^{2\Phi}, \quad \dot{\rho}_\Phi = \dot{\Phi}^2, \quad \rho_B = -\frac{e^{-4\Phi} B^2}{4a_1^2 a_2^2}, \tag{20}
\]

\[
\dot{\rho}_b = \frac{1}{2} m_B^2 e^{3\Phi} \left( 1 + \frac{e^{-4\Phi} B^2}{a_1^2 a_2^2} \right)^{-\frac{1}{2}}, \tag{21}
\]

When \( m_B \neq 0 \), it is more convenient to employ the dimensionless time variable \( t = m_B t \) and to introduce the variables \( \alpha_1 \) and \( b \) defined by

\[
\alpha_1 = \ln a_1, \quad b = \frac{e^{-2\Phi} B}{a_1 a_2}. \tag{22}
\]

Then the equations of motion are written as

\[
\ddot{b} + (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) \dot{b} \\
+ 2\dot{\Phi} + 2(-\dot{\alpha}_1 - \dot{\alpha}_2 + \dot{\alpha}_3 - 2\Phi) \dot{\Phi} \\
+ \dot{\alpha}_1 + \dot{\alpha}_2 + (\dot{\alpha}_1 + \dot{\alpha}_2) \dot{\alpha}_3 + \frac{e^{3\Phi}}{\sqrt{1 + t^2}} b = 0, \tag{23}
\]

\[
\ddot{\Phi} + (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) \dot{\Phi} = -2\dot{\rho}_B - \dot{\rho}_\Lambda - \frac{1}{2} \dot{\rho}_b - \dot{\rho}_b, \tag{24}
\]

\[
\dot{\alpha}_1 \dot{\alpha}_2 + \dot{\alpha}_2 \dot{\alpha}_3 + \dot{\alpha}_3 \dot{\alpha}_1 = \rho_\Phi + \rho_B + \rho_\Lambda + \rho_b, \tag{25}
\]

\[
\dot{\alpha}_1 + \dot{\alpha}_1 (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) = \rho_\Lambda + \rho_b, \tag{26}
\]

\[
\dot{\alpha}_2 + \dot{\alpha}_2 (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) = \rho_\Lambda + \rho_b, \tag{27}
\]
where

\[ \rho_\lambda = \lambda e^{2\Phi}, \quad \rho_\phi = \Phi^2, \]
\[ \rho_B = \frac{1}{4} \left[ \dot{b} + (\dot{\alpha}_1 + \dot{\alpha}_2 + 2\Phi) b \right]^2, \]
\[ \rho_b = \frac{1}{2} e^{3\Phi} (1 + b^2)^{1/2}, \quad \dot{\rho}_b = \frac{1}{2} e^{3\Phi} (1 + b^2)^{-1/2}, \]

where \( \lambda = \Lambda/m_B^2 \). In the subsequent subsections, we look for the solutions to the above field equations for various cases beginning with some simple solutions.

**B. The massless limit \((m_B = 0)\)**

To see the effect of the brane on the spacetime dynamics, let us first consider the limit of \( m_B = 0 \) which corresponds to either the absence of the brane or the limit of vanishing string coupling \( g_s \), namely the usual massless antisymmetric tensor field. In this limit, the equation (14) is easily integrated to yield a constant of motion

\[ \frac{a_3 \dot{B}}{a_1 a_2} = L_3 \text{ (constant)}. \] (29)

With the vanishing potential, \( B \) manifests itself by non-vanishing time derivatives. In the dual variable, it corresponds to the homogeneous gradient along \( x^3 \)-direction. The spacetime evolution with the dilaton rolling in this case was studied in Ref. [13]. Here we have assumed that the dilaton is stabilized by some mechanism. Following Ref. [11], we introduce a new time coordinate \( \eta \) via the relation \( d\eta = L_3dt/a_1a_2a_3 \). Then the equations (10)–(19) can be written as

\[ \alpha'_1 \alpha'_2 + \alpha'_2 \alpha'_3 + \alpha'_3 \alpha'_1 = \frac{1}{4} a_1^2 a_1^2, \] (30)
\[ \alpha''_1 = \alpha''_2 = 0, \] (31)
\[ \alpha''_3 = \frac{1}{2} a_1^2 a_1^2, \] (32)

where the prime denotes the differentiation with respect to \( \eta \).

The solutions for \( \alpha_1 \) and \( \alpha_2 \) are trivial

\[ \alpha_1 = C_1 \eta, \quad \alpha_2 = C_2 \eta, \] (33)

where \( C_{1,2} \) are constants and we omitted the integration constants corresponding simply to re-scaling of scale factors. The \( \alpha_3 \)-equation (32) is also easily integrated to give

\[ \alpha_3 = \frac{e^{2(C_1 + C_2)\eta}}{8(C_1 + C_2)^2} + C_3 \eta. \] (34)

The constraint equation (30) relates \( C_{1,2} \) and \( C_3 \) by \( C_3 = -C_1C_2/(C_1 + C_2) \). Then the relation between \( \eta \) and \( L_3t \) is explicitly given by

\[ L_3t = \int^\eta d\eta \frac{a_1(\eta)a_2(\eta)a_3(\eta)}{\alpha_3(\eta).} \]

**C. The \( B \) oscillation**

Let us now take into consideration the effect of space-filling D-brane. To get sensible solution, we fine-tune the bulk cosmological constant term to cancel the brane tension, that is \( \Lambda = -m_B^2/2 \), so that the effective four-dimensional cosmological constant vanishes. We assume again the dilaton is stabilized in some way and set \( \alpha_1 = \alpha_2 \). Then we can rewrite the full equations as follows

\[ \dot{\alpha}_1^2 + 2\dot{\alpha}_1 \dot{\alpha}_3 = \frac{1}{4} \left[ \dot{b} + 2\dot{\alpha}_1 b \right]^2 \]
\[ + \frac{1}{2} \left( \sqrt{1 + b^2} - 1 \right), \] (38)
\[ \dot{\alpha}_1 + \dot{\alpha}_1 (2\dot{\alpha}_1 + \dot{\alpha}_3) = \frac{1}{2} \left( \sqrt{1 + b^2} - 1 \right), \] (39)
\[ \dot{\alpha}_3 + \dot{\alpha}_3 (2\dot{\alpha}_1 + \dot{\alpha}_3) = \frac{1}{2} \left[ \dot{b} + 2\dot{\alpha}_1 b \right]^2 \]
\[ + \frac{1}{2} \left( \frac{1}{\sqrt{1 + b^2}} - 1 \right). \] (40)

First we examine the evolution of \( b(t) \) and scale factors qualitatively. Suppose \( b \) starts to roll from an initial value \( b_0 \), while the universe is isotropic in the sense that \( \dot{\alpha}_{10} = \dot{\alpha}_{30} \). We assume initially \( a_{10} = a_{30} = 1 \) (\( \alpha_{10} = \alpha_{30} = 0 \))
and \( \dot{B}_0 = 0 \) so that \( b_0 = B_0 \) and \( \dot{b}_0 = -2\alpha_{10} B_0 \). While \( b \) is much larger than unity, the rapid expansion of \( \alpha_1 \) due to the large potential proportional to \( m_B^2 b \) drives \( b \) in feedback to drop very quickly to a small value of order one. A numerical analysis shows that this happens within \( m_B t < 2 \) up to reasonably large value of \( b_0 \) for which the numerical solution is satisfied. The behavior of \( b(t) \) after this point is almost universal irrespective of the initial value \( b_0 \) if it is much larger than unity.

Once \( b \) becomes smaller than unity, the quadratic term of mass dominates over the expansion and \( b \) begins to oscillate about \( b = 0 \). Then the expansion of the universe provides the slow decrease of the oscillation amplitude. The situation is the same as that of the coherently oscillating scalar field such as the axion or the moduli in the expanding universe. For small \( b \), the energy-momentum tensor of the oscillating \( B \)-field is given by \( T^\nu_\mu = \text{diag}[ -\rho, p_1, p_2, p_3 ] \) where

\[
\rho = \frac{1}{4} \left( b + 2\alpha_1 b \right)^2 + \frac{1}{2} m_B^2 \left( \sqrt{1 + b^2} - 1 \right) \\
\approx \frac{1}{4} \left( b^2 + m_B^2 b^2 \right), \tag{42}
\]

\[
p_1 = p_2 = -\frac{1}{4} \left( b + 2\alpha_1 b \right)^2 - \frac{1}{2} m_B^2 \left( \frac{1}{\sqrt{1 + b^2}} - 1 \right) \\
\approx -\frac{1}{4} \left( b^2 - m_B^2 b^2 \right), \tag{43}
\]

\[
p_3 = \frac{1}{4} \left( b + 2\alpha_1 b \right)^2 - \frac{1}{2} m_B^2 \left( \sqrt{1 + b^2} - 1 \right) \\
\approx \frac{1}{4} \left( b^2 - m_B^2 b^2 \right). \tag{44}
\]

With the expansion of the universe neglected, the equation of motion for \( b \), Eq. 41 is then approximated by

\[
\ddot{b} + m_B^2 b \approx 0. \tag{45}
\]

Since the oscillation is much faster than the expansion, we can use the time-averaged quantities over one period of oscillation for the evolution of spacetime. The equation 43 gives the relation \( \langle \dot{b}^2 \rangle = \langle m_B^2 b^2 \rangle \). Thus, the oscillating \( B \)-field has the property \( p_1, p_2, p_3 \approx 0 \) and behaves like homogeneous and isotropic matter. This justifies the name of \( B \)-matter. Therefore, after \( b \) begins to oscillate, the isotropy of the universe is recovered.

D. Attractor solutions

Now, we consider the generic case that both the dilaton and the NS-NS 2-form field evolve in time. The dilaton in the system of equations 22–27 has the exponential potential up to the correction due to the NS-NS 2-form field. It is well-known that the scalar field with the exponential potential possesses the scaling solution in which the energy density of the scalar field mimics the background fluid energy density 12–13. This scaling solution is also an attractor, so that the late time behavior of the solutions are universal irrespective of initial conditions. This is an attractive property of the exponential potential. For the potential of the form \( V(\Phi) = V_0 e^{\beta \Phi} \), there is an attractor solution

\[
\Phi = -\frac{2}{\beta^2} \ln \left[ \frac{\beta V_0^{1/2} t}{\sqrt{2(12 - \beta^2)}} \right], \quad \alpha = \left( \frac{2}{\beta} \right)^2 \ln t, \tag{46}
\]

for \( 0 \leq \beta < \sqrt{12} \). The scale factor obeys the power-law time-dependence, implying that the rolling of \( \Phi \) constitutes the matter having an equation of state \( p = \rho \) where \( \omega = \beta^2/6 - 1 \) varies from \(-1\) to \(+1\) for the aforementioned range of \( \beta \).

We found this type of particular solutions of the Eqs. 22–27, which can be found when we have a single exponential term in the potential, that is, for the case of \( \Lambda = 0 \) and for the case of \( m_B = 0 \). For both cases we start from an ansatz of the form

\[
\alpha_1 = \gamma_1 \ln t, \quad \alpha_3 = \gamma_3 \ln t, \quad \Phi(t) = \gamma_4 \ln t + \Phi_0, \quad b = \text{constant}, \tag{47}
\]

where we suppressed constant terms in \( \alpha_1 \) and \( \alpha_3 \) which correspond to simple rescaling of coordinates. For the case of \( \Lambda = 0 \), we obtain two distinguished solutions

\[
\alpha_1(t) = \alpha_3(t) = \frac{4}{9} \ln t, \quad \Phi(t) = -\frac{2}{3} \ln t + \frac{2}{3}, \quad b = 0, \tag{48}
\]

and

\[
\alpha_1(t) = \frac{10}{21} \ln t, \quad \alpha_3(t) = \frac{3}{7} \ln t, \quad \Phi(t) = -\frac{2}{3} \ln t + \frac{2}{3} \ln \left( \frac{8 \cdot 5^{1/4}}{21} \right), \quad b = \pm \frac{1}{2}. \tag{49}
\]

The first solution is nothing but the solution 10 with \( \beta = 3 \). The second solution has the non-vanishing NS-NS form field. For the case of \( m_B = 0 \), we introduce a new rescaled dimensionless time variable \( \tilde{t} = \Lambda^{1/2} t \) instead of \( t \) and then get the continuous set of solutions from Eqs. 22–27

\[
\alpha_1(t) = \alpha_3(t) = \ln t, \quad \Phi(t) = -\ln \tilde{t} + \frac{1}{2} \ln 2, \quad b = \text{arbitrary constant}. \tag{50}
\]

\( \Phi(t) \) and \( \alpha(t) \) are same as those in Eq. 40 with \( \beta = 2 \), while we have the non-vanishing \( B \)-field condensate.

These solutions are the solutions to the Eqs. 22–27 for the specific initial conditions. However, the importance of these solutions, as noted in the paragraph above, arises from the fact that they are attractors, which means that after enough time the solutions with different initial conditions approach these solutions. We will confirm this through numerical analysis in the next subsection.

The solution 13 applies for the brane tension dominated case where the dilaton potential is approximated
by \( V(\Phi) = \frac{1}{2} m_{\Phi}^2 e^{3\Phi} \). The evolution of the dilaton under this potential produces matter with the equation of state \( p = \frac{1}{3} \rho \).

Once the antisymmetric tensor field is turned on, the anisotropy appears as in the solution \([10]\). The measure of anisotropy is

\[
\frac{\dot{\alpha}_3}{\dot{\alpha}_1} = \frac{9}{10}.
\]

This result is contrasted with that in Ref. \([8]\) where the dilaton potential possesses the minimum for \( \Lambda = 0 \), but the value

\[
\tilde{\Phi}_0 = \frac{1}{3} \tilde{\rho},
\]

to specify the solution. Among these, the initial values \( \dot{\Phi}_0 \), \( \dot{B}_0 \), \( \dot{B}_0 \), \( \dot{\alpha}_{10} \), \( \dot{\alpha}_{30} \), \( \dot{\alpha}_{30} \) to specify the solution. Among these, \( \dot{\alpha}_{10} \) can always be set to zero by coordinate rescaling. \( \dot{\alpha}_{10} \) must obey the constraint equation \([24]\), but this does not fix the ratio \( \dot{\alpha}_{10}/\dot{\alpha}_{30} \). We choose the isotropic universe as a natural initial condition which leads to \( \dot{\alpha}_{10} = \dot{\alpha}_{30} \).

Since the dilaton potential is composed of exponential terms, the shift of the dilaton field by a constant can be traded for the redefinition of mass scale. We use this property to take the initial value of the dilaton to be zero without loss of generality. In our numerical analysis, we take the dimensionless time variable as \( \tilde{t} = \tilde{m}t \) where \( \tilde{m} = m_B e^{\tilde{\Phi}_0} \) and use the variable \( \tilde{\Phi} = \Phi - \tilde{\Phi}_0 \) with its initial value \( \tilde{\Phi}_0 = 0 \). This means that the proper time scale for the cosmological evolution is not \( m_B^{-1} \), but \( \tilde{m}^{-1} \). For convenience’s sake, we take the initial time as \( \tilde{t}_0 = 1 \). The other mass scale \( \Lambda \) is also affected by this shift and we can treat it by replacing the parameter \( \lambda \) with \( \tilde{\lambda} = \lambda e^{-\tilde{\Phi}_0} \).

Now we need three initial values \( \dot{\Phi}_0 \), \( \dot{B}_0 \), and \( \dot{B}_0 \) to fix the functional form of the solution. The initial values \( \dot{b}_0 \) and \( \dot{b}_0 \) are related to \( \dot{B}_0 \) and \( \dot{B}_0 \) by \( \dot{b}_0 = B_0 = B_0 \) and \( \dot{b}_0 = B_0 - 2(\dot{\alpha}_{10} + \dot{\Phi}_0)B_0 \).

2. Numerical solutions for \( B \neq 0 \)

We turn on the NS-NS 2-form field along the \( x^3 \) direction, \( B_{12} = B \neq 0 \). The spacetime becomes anisotropic, \( \alpha_1(t) = a_2(t) \neq a_3(t) \), in general. The solutions are classified by the signature of \( \Lambda \). For \( \Lambda < 0 \), the solution becomes singular. Here we skip the description of such singular solutions which are not suitable for the evolution of our Universe.

For \( \Lambda > 0 \), the evolution is divided into two stages. In the first stage where \( \rho_b \) is dominant, the solution approaches an attractor \([13]\) of the \( \Lambda = 0 \) case. The approached value of \( b \) is either \( +\frac{1}{2} \) or \( -\frac{1}{2} \) depending on the initial conditions. If we look at the evolution of each component of energy density, the kinetic energy of the dilaton \( \rho_b \) catches up the potential energy \( \rho_b \) and the ratio of them becomes constant. This is a characteristic feature of the scaling solution \([13]\). The kinetic energy of \( B \) field is kept much smaller than both of them, but the anisotropy is still maintained due to the difference between \( \rho_b \) and \( \tilde{\rho}_b \). In the second stage where \( \rho_b \) is dominant, it approaches another attractor \([50]\). Thus the universe recovers the isotropy. The final value of \( b \) is a certain constant which is determined by initial conditions and can differ from \( \pm \frac{1}{2} \). Numerical solutions for a few initial conditions are shown in Figure \([11]\). The kinetic energy of the dilaton \( \rho_b \) catches up the potential energy \( \rho_b \) in the first stage and \( \Lambda \) in the second stage. The ratios, \( \rho_b/\rho_b \) and \( \rho_b/\rho_b \), approach constants in each stage. The kinetic energy of \( B \) field is kept much smaller as in \( \Lambda = 0 \) case.

IV. THE DILATON STABILIZATION

The vacuum expectation value of the dilaton determines both the gauge and gravitational coupling constants of the low energy effective theory. Therefore, the dilaton must be stabilized at some stage of the evolution for the action \([7]\) to have something to do with the reality. In this section, we study the cosmological evolution when the dilaton is stabilized. As for the correct mechanism of dilaton stabilization, the consensus has not been made yet. Our goal here is to illustrate an example of the dilaton stabilization and look into the effect of it on the dynamics of the NS-NS 2-form field and the cosmological evolution, since the overall features of which are insensitive to the detailed mechanism of stabilization.

Our starting point is the dilaton potential \([11]\). This potential possesses the minimum for \( \Lambda < 0 \), but the value
FIG. 1: Numerical solutions for $B \neq 0$ and $\Lambda > 0$. When $\lambda = 10^{-4}$, the solutions of four initial conditions are given: $\Phi_0 = -7/4 \cdot 5^{1/4}$, $b_0 = 1/2$, $B_0 = 0$ for the solid curves, $\Phi_0 = 0$, $b_0 = 1$, $B_0 = 0$ for the dotted curves, $\Phi_0 = 0$, $b_0 = 10$, $B_0 = 0$ for the dashed curves, and $\Phi_0 = -\sqrt{3}/2$, $b_0 = 1$, $B_0 = 0$ for the dash-dotted curves. The lower-right panel shows the evolution of each component of energy density for initial conditions $\Phi_0 = -\sqrt{3}/2$, $b_0 = 1$, $B_0 = 0$.

of the potential at the minimum is negative and needs to be set to zero by fine-tuning of the constant shift. However, the constant shift of the potential has no motivation in the context of string theory. Instead, we introduce a term $\frac{1}{4}m^2_F e^{-3\Phi}$ in the potential, which can arise from the effect of various form field fluxes in extra dimensions [14]. Thus, our potential for the dilaton for $B = 0$ looks like

$$V_F(\Phi) = \frac{1}{4} \left( m^2_B e^{3\Phi} + 2\Lambda e^{2\Phi} + m^2_F e^{-3\Phi} \right). \quad (52)$$

This potential has a global minimum for any value of $\Lambda$ and $m_F$. To have sensible cosmology, the potential at the minimum must be zero. For $\Lambda < 0$, this can be done through a fine-tuning of the parameters in the potential

$$\mu^2 = \frac{1}{5} \left( -\frac{5}{3} \lambda \right)^6, \quad (53)$$

where $\mu^2 = m^2_F/m^2_B$. Then the minimum is located at

$$\Phi_F = \ln \left( -\frac{5}{3} \lambda \right). \quad (54)$$

The shape of this fine-tuned potential for $\lambda = -0.1$ is shown in Figure 2(a). Now the equations 23 and 27 are modified accordingly

$$\dot{\Phi} + (\ddot{\alpha}_1 + \ddot{\alpha}_2 + \ddot{\alpha}_3) \Phi = -2\rho_F - \rho_\Lambda - \frac{1}{2} \dot{\rho}_B - \dot{\rho}_\Lambda + \frac{3}{2} \rho_F, \quad (55)$$
The potential $V(\Phi)$ for $\lambda = -0.1$. The minimum of the potential is at $\Phi_m = -\ln 6$ with $V(\Phi_m) = 0$. (b), (c) and (d) show the numerical solutions of $\Phi(t)$, $b(t)$, and $\dot{\alpha_i}(t)$, respectively, for $\lambda = -0.1$ and initial conditions $\Phi_0 = 0$, $\dot{\Phi}_0 = 0$ and $b_0 = 1$. $\Phi(t)$, $b(t)$, and $\dot{\alpha}_i(t)$ approach $\Phi_m$, 0, and 2/3, respectively.

\begin{align}
\alpha_1 + \dot{\alpha}_1(\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) &= \rho_\Lambda + \rho_b + \rho_F, \\
\alpha_3 + \dot{\alpha}_3(\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) &= 2\rho_B + \rho_\Lambda + \dot{\rho}_b + \rho_F,
\end{align}

where $\rho_F = \frac{1}{2}\mu^2 e^{-3\Phi}$, while the equation for $b$ is not changed.

With the stabilizing potential for the dilaton, the cosmological evolution is completely changed. The dilaton and the NS-NS 2-form field rapidly come to the oscillation about the potential minimum $\Phi = \Phi_F$ and $b = 0$. Oscillating $\Phi$ and $B$ fields behave like ordinary matter satisfying the equation of state $p = 0$. The universe becomes isotropic and matter dominated. The numerical solutions for the stabilizing dilaton potential with $\lambda = -0.1$ are plotted in Figure 2 (b) to (d). One can see the oscillation of $\Phi$ and $b$, and that both $\dot{\alpha}_1$ and $\dot{\alpha}_3$ approach to $2/3t$, indicating the matter domination. Damping of $\Phi$ and $b$ oscillations is due to the expansion of the universe.

### V. CONCLUSION

We investigated cosmology of a four-dimensional low energy effective theory arising from the NS-NS sector of string theory with a D-brane which contains the dynamical degrees of freedom such as the gravity, the dilaton, and the antisymmetric tensor field of second rank, coupling to the gauge field strength living on the brane. The
dilaton gains a potential in the presence of the D-brane, the fluxes of various form fields and the curvature of extra dimensions. The NS-NS 2-form field becomes massive in the presence of the D-brane. The dynamics of the system crucially depends on the curvature parameter $\Lambda$ and the brane tension parameter $m_B$ through which the dilaton obtains the potential of the form $\Lambda e^{2\Phi} + \frac{1}{3} m_B^2 e^{3\Phi}$. Here, the latter becomes the effective mass of the 2-form field.

When the 12-component $B(t)$ of the 2-form field is turned on, the universe undergoes an anisotropic expansion described by the Bianchi type-I cosmology. We found the attractor solutions showing the overall features of general solutions and confirmed it through numerical analysis. The dilaton $\Phi(t)$ runs to the negative infinity settling to a logarithmic decrease in time. When the brane tension term $\frac{1}{3} m_B^2 e^{3\Phi}$ dominates, the anisotropy due to the 2-form field flux is sustained. If there is a positive curvature term $\Lambda e^{2\Phi}$, it dominates finally over the brane tension term as the dilaton rolls down to the negative infinity. Then the expansion of the universe turns to be isotropic and linear in time. Accordingly, $B(t)$ decreases inversely proportional to time.

For sensible phenomenology and cosmology, the dilaton must be stabilized. In order to study the dynamics of the 2-form field (B-matter) and the stabilized dilaton system, we adopted a dilaton mass term of the form $m_B^2 e^{-\Phi}$. Then the dilaton potential has a global minimum and the cosmological constant of our Universe can be fine-tuned to a desired value with negative $\Lambda$. With this stabilizing potential, we obtain a reasonable cosmology from an initially anisotropic universe resulting from the 2-form field flux. While the dilaton evolved to a stabilized value, $B(t)$ shows an oscillatory matter-like behavior (B-matter), and the universe expands as in the usual matter-dominated era recovering the isotropy.

Finally, there are other dynamical degrees of freedom in the D-brane world, such as the R-R form fields, which are not included in the present work. Investigating the cosmological implications of them is left for the future work.

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