Almost sure exponential stabilization and suppression by periodically intermittent stochastic perturbation with jumps

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Abstract. The main aim of this article is to examine almost sure exponential stabilization and suppression of nonlinear systems by periodically intermittent stochastic perturbation with jumps. On the one hand, some sufficient criteria ensure almost sure stabilization of the unstable deterministic system by applying exponential martingale inequality with jumps. On the other hand, sufficient conditions of destabilization are provided under which the system is stable by the well-known strong law of large numbers of local martingale and Poisson process. Both the sample Lyapunov exponents are closely related to the control period $T$ and noise width $\theta$. As for applications, the well-known Lorenz chaotic systems and nonlinear Liénard equation with jumps are discussed. Finally, two simulation examples demonstrating the effectiveness of the results are provided.

1. Introduction. Applications of stochastic noise feedback have emerged in a wide range of fields such as plane model [23], engineering [12, 19], finance [9], multi-agent systems [37]. In a string of previous works, the literature on stabilization and destabilization of the Brownian noise has fully spelled out (see, e.g., Appleby [5], Deng et al. [7], Scheutzow [23], Mao [18, 20], Mao et al. [19], Zhang et al. [35]). On the one hand, the approach of input feedback control with white noise has successfully stabilized a deterministic system. On the other hand, some authors have investigated almost sure stabilization and suppression of stochastic systems in terms of coefficient condition (see, e.g., [16, 17, 27, 28, 36]) and the references therein. It should be mentioned that these works in the literature we presented concentrate on the case of SDEs driven by classical Brownian motion. Put differently, the development of the general Lévy noise stabilization and destabilization is rarely tracked synchronously.

Recent years have witnessed Lévy process theory (see, e.g., [1]) as an important branch of modern probability theory, and a great deal of rapid development in both
theory and applications. We are referring here to Lévy processes, which include continuous Brownian motion and Poisson jump process as special cases. In addition, Lévy processes also have many aliases (including almost analogous processes), additive processes, independent incremental processes, infinitely divisible processes, and their associated distributions and so on. Under the real circumstance, one often encounters obstacles influenced by event-driven uncertainties, which can be captured by features of these systems. It has been widely applied in basic mathematics, statistics, economics, finance, insurance, operations research, physics, engineering and other fields. In order to describe stochastic abrupt phenomena, it is quite suitable to introduce SDEs with Lévy noise. For instance, the continuous and discrete coexistence models have attracted widespread attention. From the perspective of modeling, Cont and Tankov [6] introduced financial modeling with jump processes. Patel and Kosko [22] investigated stochastic resonance in continuous and spiking neuron models with Lévy noise. Applebaum [2] further developed stochastic resonance for neuron models to general Lévy noise. Very recently, Gao and Wang [10] were concerned with a stochastic mutualism model under regime switching with Lévy jumps. From the viewpoint of stability, Zhu [32, 33, 34] revealed asymptotic stability in the $p$th moment and Li et al. [15] studied almost sure stability linear SDEs with jumps. In view of stabilization, Applebaum and Siakalli [4] used Lévy noise to establish stochastic stabilization of dynamical systems. Furthermore, Zong et al. [38] examined almost sure and $p$th-moment stability and stabilization of regime-switching jump diffusion systems. For more details, one can refer to [29, 30] and the references therein. Therefore, it is fairly interesting to investigate this problem driven by Lévy process.

Let us turn to the topic of intermittent control. In terms of saving costs, intermittent periodically control, which includes control time and rest time, has gained wide popularity in the control theory. Recently, great progress has been made not only in theory but also in practice. Especially in complex systems, Li and Cao [13] applied the periodically intermittent control to switched networks and switched interval coupled networks respectively. Pinning controllability scheme of directed complex delayed dynamical networks via periodically intermittent control is investigated in [14]. Wan and Cao [25] studied distributed robust stabilization of linear multi-agent systems with intermittent control. For the latest development, one can refer the reader to [8, 11, 21, 26] and the references therein. In this paper, in order to extend the role of intermittent control for stochastic stabilization and suppression, we introduce the continuous Brownian motion and the compensated Poisson integral and periodically intermittent control. We called it periodically intermittent stochastic perturbation with jumps.

Along with the above concerns, it has been well recognized that the nonlinearities and random fluctuation phenomenon are unavoidable in almost all the practical systems and it cannot be completely eliminated. In this paper, we further the works on stabilization of nonlinear systems induced by Lévy noise. More specifically, we are interested in periodically intermittent stochastic perturbation with jumps. Since the quantitative property of nonlinear SDEs with periodically intermittent stochastic noise feedback with jumps is rarely available and still remains open for a while, and recent studies of periodically intermittent stochastic feedback stabilization in [35] give us several motivations, we establish almost sure exponential stabilization and destabilization by adopting the means of exponential martingale inequality, segmented Lyapunov operator, the well-known law of large numbers of Poisson
process and Brownian motion, and by using the method of intermittent stochastic perturbation with jumps. The main highlights of this article are as follows:

- One novel aspect of our methods is that periodically intermittent control, Brownian motion and a compensated Poisson integration (small jumps) are unified. For almost sure stabilization, periodically intermittent stochastic perturbation induced by Brownian motion and the compensated Poisson integral contributes to the stabilization of the system. In the case of destabilization, periodically intermittent stochastic perturbation induced by Brownian motion serves as a destructive factor.

- The well-known Lorenz chaotic systems were stabilized via periodically intermittent control, Brownian motion and a compensated Poisson integration (small jumps) are mutually independent.

This paper is organized as follows. Section 2 begins with an overview of Lévy process and model descriptions. The objective of Section 3 is to present some useful lemmas and to show almost sure exponential stabilization and suppression of nonlinear systems by periodically intermittent stochastic perturbation with jumps. Section 4 shows two applications and numerical results are finally reported. Section 5 summarizes the full paper and puts forward future works as the end of this paper.

**Notations.** Throughout the text for each $x \in \mathbb{R}^n$, we denote $B_c(x) = \{ y \in \mathbb{R}^n : |y - x| < c \}$, $\tilde{B}_c(x) = B_c(x) - \{0\}$. Let $\mathbb{R}^+ = [0, +\infty)$. For a matrix $D$, denote by $\|D\|$ the Hilbert-Schmidt norm, i.e., $\|D\| = \sqrt{\text{trace}(D^T D)}$. Let $C^2(\mathbb{R}^n; \mathbb{R}^+)$. be the family of all nonnegative functions $V(x)$ on $\mathbb{R}^n$ that have continuous partial derivatives w.r.t. $x$ up to the second order. The function $f: \mathbb{R}^+ \to \mathbb{R}^n$ is said to be càdlàg if it is right continuous and left limits with $f(t-) = \lim_{s \uparrow t} f(s)$.

2. Preliminaries. Let $(\Omega, \mathcal{F}, (\mathcal{F}_t, t \geq 0), P)$ be a filtered probability space satisfying the usual condition (i.e., it is right continuous and completeness) which is defined an $m$-dimensional standard $\mathcal{F}_t$-adapted Brownian motion $B_t$. Let $\varphi(t)$ be an $\mathcal{F}_t$-adapted Lévy process with Lévy measure $\nu(\cdot)$. Define

$$N(t, Z) = \sum_{0 \leq s \leq t} I_Z \Delta(\varphi) = \sum_{0 \leq s \leq t} I_Z(\varphi(s) - \varphi(s-)),$$

where $N(\cdot, \cdot)$ defined on $\mathbb{R}^+ \times (\mathbb{R}^n - \{0\})$ is an $\mathcal{F}_t$-adapted Poisson random measure. The compensator $\hat{N}$ of $N$ is defined by $\hat{N}(dt, dz) = N(dt, dz) - \nu(dz)dt$, where a measure $\nu(dz)$, called a Lévy measure on $Z$, satisfies $\int_{R^n - \{0\}} (|z|^2 \land 1)\nu(dz) < \infty$ and $Z$ is a Borel subset of $(\mathbb{R}^n - \{0\})$. Assume that Brownian motion and Poisson random process are mutually independent.

In the sequel, we study periodically intermittent stochastic perturbation system with jumps described as

$$\begin{cases} 
\text{dy}(t) = f(y(t-))dt + g(y(t-))dB_t \\
+ \int_{|z| \leq H} H(y(t-), z)\hat{N}(dt, dz), \\
t \in [t_0 + lT, t_0 + lT + \theta), \\
\text{dy}(t) = f(y(t))dt, \\
t \in [t_0 + lT + \theta, t_0 + (l + 1)T), l = 0, 1, 2, \cdots
\end{cases}
$$

with initial value $y(t_0) = y_0 \in \mathbb{R}^n$, where $f: \mathbb{R}^n \to \mathbb{R}^n$, $g: \mathbb{R}^n \to \mathbb{R}^{n \times n}$, $H: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$, $\epsilon \in (0, +\infty]$ is the maximum allowable jump size, $T > 0$ denotes the control period and $\theta > 0$ is the noise width satisfying $0 < \theta < T$. For $V \in$
Proof. For the detailed proof, see (Applebaum, Theorem 5.2.9, [1]).

Assumption 2.2. There exists a positive constant such that

\[ V(y,f(y),y), \quad t \in [t_0 + IT, t_0 + IT + \theta), \quad t \in [t_0 + l IT + \theta, t_0 + (l + 1)T), \quad l = 0, 1, 2, \cdots. \]

Let us introduce two space \( P_1(T) \) and \( P_2(T, Z) \). The space \( P_1(T) \) is the linear space of all predictable mappings \( F : [0, T] \times \Omega \rightarrow \mathbb{R}^n \) for which \( P \left( \int_0^T |F(s)|^2 ds < \infty \right) = 1 \). The space \( P_2(T, Z) \) is defined as the linear space of all predictable mappings \( H : [0, T] \times Z \times \Omega \rightarrow \mathbb{R}^n \) which satisfy

\[ P \left[ \int_0^T \int_Z |H(s, z)| \nu(dz) ds < \infty \right] = 1. \]

Lemma 2.1. (Exponential martingale inequality, [1], pp. 287-289) Assume that \( a \) and \( \gamma \) are two positive constants. Let \( g \in P_1(T), H \in P_2(T, Z) \), then we have

\[ P \left( \sup_{0 \leq t \leq T} \left[ \int_0^t g(s) dB_s - \frac{a}{2} \int_0^t |g(s)|^2 ds + \int_0^t \int_{|z| \leq c} H(s, z) \tilde{N}(ds, dz) \right] - \frac{1}{a} \int_0^t \int_{|z| \leq c} \left\{ e^{aH(s, z)} - 1 - aH(s, z) \right\} \nu(dz) ds > \gamma \right) \leq e^{-\gamma}. \]

Proof. For the detailed proof, see (Applebaum, Theorem 5.2.9, [1]).

Assumption 2.2. There exists a positive constant \( \rho_k(k \geq 1) \) such that

\[ |f(x) - f(y)| \leq \rho_k |x - y|, \quad |g(x) - g(y)| \leq \rho_k |x - y| \]

and

\[ \int_{|y| \leq c} |H(x, z) - H(y, z)| \nu(dz) \leq \rho_k |x - y|, \]

for \( x, y \in \mathbb{R}^n \) with \( |x| \vee |y| \leq k \).

Assumption 2.3. There exists a nonnegative Lyapunov function \( V \in C^2(\mathbb{R}^n; \mathbb{R}^+), \) such that \( \inf_{|y| > k} V(y) \rightarrow \infty, \) as \( k \rightarrow \infty, \) and for some constant \( C_0 \) and all \( y \in \mathbb{R}^n, \)

\[ \mathcal{L}V(y) \leq C_0 V(y). \]

Remark 1. Under the Assumptions 2.2 and 2.3, it is easy to conclude that system (1) has a unique global solution \( y(t) \) (see, Lemma 4.7, [38]). We will denote the solution by \( y(t; t_0, y_0) \) when we need to emphasize the initial data \( y_0 \) at time \( t_0 \).

Suppose that \( f(0) = 0, g(0) = 0, \) and \( H(0, z) = 0 \) for all \( |z| < c, \) then (1) has a unique solution \( y(t) = 0 \) for all \( t > t_0 \) corresponding to the initial value \( y(t_0) = 0, \) which is called the trivial solution.

Definition 2.4. The trivial solution of (1) is called to be almost surely (a.s. for short) exponentially stable if there exists a constant \( \lambda > 0 \) such that

\[ \limsup_{t \rightarrow \infty} \frac{1}{t} \log |y(t)| < -\lambda, \quad \text{a.s.} \]

for all \( y_0 \in \mathbb{R}^n. \)

Assumption 2.5. We always assume that \( H(y, z) \) satisfies

\[ \nu\{ |z| < c, \text{there is } y \neq 0 \text{ such that } y + H(y, z) = 0 \} = 0. \]
Lemma 2.6. (Lemma 2.4, [4]) There exists $\rho_k > 0$ for any $k > 0$ such that
\[
|f(y)| + \|g(y)\| + 2 \int_{|z| < c} |H(y, z)| \left( \frac{|y| + |H(y, z)|}{|y + H(y, z)|} \right) \nu(dz) \leq \rho_k |y|, \quad \text{if } |y| \leq k.
\]
(2)

Then
\[ P(y(t) \neq 0 \text{ on } t \geq t_0) = 1, \text{ for all } y_0 \neq 0. \]

Proof. See Applebaum and Siakalli [3]. \hfill \Box

3. Stabilization and suppression.

3.1. Stabilization of noise. In this paper, let us first recall some results of [4] which will be useful in the forthcoming Theorem. The two auxiliary statement is Lemma 3.1-3.2 below and we put forward it without proof as it is straightforward.

Lemma 3.1. For any constant $a \in \mathbb{R}$, then $e^a - 1 - a \geq 0$.

Lemma 3.2. For any constant $a > 0$ and $0 < b < 1$, then $a^b < 1 + b(a - 1)$.

Lemma 3.3. (Lemma 3.3, [3]) Let $V(y) \in C^2(\mathbb{R}^n; \mathbb{R}^+).$ Define, for any $\alpha \in (0, 1)$ and $z \in \mathcal{B}_c$,
\[
J(t, \alpha) = \frac{1}{\alpha} \int_{t_0}^t \int_{|z| \leq c} \left[ \left( \frac{V(y(s-) + H(y(s-), z))}{V(y(s-))} \right)^\alpha - 1 - \alpha \log \left( \frac{V(y(s-) + H(y(s-), z))}{V(y(s-))} \right) \right] \nu(dz) ds,
\]
then, $\lim_{\alpha \rightarrow 0} J(t, \alpha) = 0$.

Proof. For any $z \in \mathcal{B}_c$, let
\[
u_\alpha(z) = \frac{1}{\alpha} \left[ \left( \frac{V(y + H(y, z))}{V(y)} \right)^\alpha - 1 - \alpha \log \left( \frac{V(y + H(y, z))}{V(y)} \right) \right].
\]
By Lemma 3.1 and taking $a = \log \left( \frac{V(y + H(y, z))}{V(y)} \right)^\alpha$ we deduce that
\[
\left( \frac{V(y + H(y, z))}{V(y)} \right)^\alpha - 1 - \alpha \log \left( \frac{V(y + H(y, z))}{V(y)} \right) \geq 0.
\]
On the other hand, using Lemma 3.2, we can verify that
\[
u_\alpha(z) \leq \frac{1}{\alpha} \left[ 1 + \alpha \left( \frac{V(y + H(y, z))}{V(y)} - 1 \right) - 1 - \alpha \log \left( \frac{V(y + H(y, z))}{V(y)} \right) \right]
\]
\[
= \frac{V(y + H(y, z))}{V(y)} - 1 - \log \left( \frac{V(y + H(y, z))}{V(y)} \right).
\]
It follows from dominated convergence theorem and Lemma 3.3 (see Applebaum and Siakalli [3]) that
\[
\lim_{\alpha \rightarrow 0} J(t, \alpha) = \int_{t_0}^t \int_{|z| \leq c} \lim_{\alpha \rightarrow 0} \frac{1}{\alpha} \left[ \left( \frac{V(y(s-) + H(y(s-), z))}{V(y(s-))} \right)^\alpha - 1 - \alpha \log \left( \frac{V(y(s-) + H(y(s-), z))}{V(y(s-))} \right) \right] \nu(dz) ds = 0.
\]
Therefore, we obtain the desired assertion. \hfill \Box
We now give general hypothesis which enables us to prove almost sure exponential stability results.

**Assumption 3.4.** Assume that a function $V \in C^2(\mathbb{R}^n; \mathbb{R}^+)$ and there exist some constants $p > 0$, $k_1 > 0$, $k_2 \in \mathbb{R}$, $k_3 \geq 0$, $k_4 \geq 0$ and $k_5 > 0$ such that

(i) $k_1|y|^p \leq V(y)$,

(ii) $V_y(y)f(y) \leq k_2V(y)$,

$$\frac{1}{2}\text{trace}[g^T(y)V_{yy}(y)g(y)] + \int_{|z| \leq c}[V(y + H(y, z)) - V(y) - V_y(y)H(y, z)]\nu(dz) \leq k_3V(y)$,

(iii) $|V_y(y)g(y)|^2 \geq k_4(V(y))^2$,

(iv) $\int_{|z| \leq c}[\log \left(\frac{V(y + H(y, z))}{V_y(y)}\right) - \frac{V(y + H(y, z)) - V(y)}{V_y(y)}] \nu(dz) \leq -k_5$.

**Remark 2.** From the logarithmic inequality $\log(y) \leq y - 1$ for $y > 0$, we conclude that the (iv) in Assumption 3.4 is reasonable.

After above preparations, we now are in a position to state the main results in this section.

**Theorem 3.5.** Let $V \in C^2(\mathbb{R}^n; \mathbb{R}^+)$. Under Assumptions 2.2 and 3.4, the system (1) is said to be almost surely exponentially stable if $[k_4 + 2k_5 - 2k_3]R^+ - 2k_2 > 0$ hold.

**Proof.** An application of Itô’s formula with jump to $\log(V(y(t)))$ show that for any $t \geq t_0$,

$$\begin{align*}
\log(V(y(t))) & = \log(V(y_0)) + \int_{t_0}^t \frac{1}{V(y(s-))}[V_y(y(s-))f(y(s-))]ds \\
& + \int_{t_0}^t \text{trace}[g^T(y(s-))V_{yy}(y(s-))g(y(s-))]ds \\
& - \frac{1}{2} \int_{t_0}^t \frac{|V_y(y(s-))g(y(s-))|^2}{(V(y(s-)))^2}ds + M(t) \\
& + \int_{t_0}^t \int_{|z| \leq c}\left[\log(V(y(s-)) + H(y(s-), z))) - \log(V(y(s-))) - \frac{V_y(y(s-))H(y(s-), z)}{V(y(s-))}\right] \nu(dz)ds \\
& = \log(V(y_0)) + \int_{t_0}^t \frac{1}{V(y(s-))}\nu(V(y(s-)))ds \\
& - \frac{1}{2} \int_{t_0}^t \frac{|V_y(y(s-))g(y(s-))|^2}{(V(y(s-)))^2}ds + M(t) + I(t),
\end{align*}$$

where

$$\begin{align*}
M(t) & = \int_{t_0}^t \frac{1}{V(y(s-))}V_y(y(s-))g(y(s-))dB_s \\
& + \int_{t_0}^t \int_{|z| \leq c}[\log(V(y(s-)) + H(y(s-), z))) - \log(V(y(s-)))]\tilde{N}(ds, dz)
\end{align*}$$

and

\[ I(t) = \int_{t_0}^{t} \int_{|z| \leq c} \left[ \log(V(y(s)) + H(y(s), z)) - \log(V(y(s))) + 1 - \frac{V(y(s)) + H(y(s), z)}{V(y(s))} \right] \nu(dz)ds. \]

An application of exponential martingale inequality yields that

\[
P \left\{ \sup_{t_0 \leq t \leq t_0 + (t+1)T} \left( M(t) - \frac{\alpha}{2} \int_{t_0}^{t} \frac{|V(y(s))|g(y(s))|^2}{(V(y(s)))^2} ds - \frac{1}{\alpha} \int_{t_0}^{t} \int_{|z| \leq c} \left[ e^{\log \left( \frac{V(y(s)) + H(y(s), z)}{V(y(s))} \right)} \right] \right. \\
-1 - \frac{1}{\alpha} \log \left( \frac{V(y(s)) + H(y(s), z))}{V(y(s))} \right) \left. \right) \nu(dz)ds \right\} \leq e^{-\alpha^2t}.
\]

Consequently, for almost all \( \omega \in \Omega \), \( t_0 \leq t \leq t_0 + (t+1)T \), using Borel-Cantelli Lemma, we have

\[
M(t) \leq \frac{\alpha}{2} \int_{t_0}^{t} \frac{|V(y(s))|g(y(s))|^2}{(V(y(s)))^2} ds + \alpha t + \frac{1}{\alpha} \int_{t_0}^{t} \int_{|z| \leq c} \left[ e^{\log \left( \frac{V(y(s)) + H(y(s), z)}{V(y(s))} \right)} \right] \right. \\
-1 - \frac{1}{\alpha} \log \left( \frac{V(y(s)) + H(y(s), z))}{V(y(s))} \right) \left. \right) \nu(dz)ds. \tag{4}
\]

It follows from (3) and (4) that

\[
\log(V(y(t))) \leq \log(V(y_0)) + \int_{t_0}^{t} \frac{\mathcal{L}V(y(s))}{V(y(s))} ds + I(t) + J(t, \alpha) - \frac{1}{2} (1 - \alpha) \int_{t_0}^{t} \frac{|V_y(y(s))|g(y(s))|^2}{(V(y(s)))^2} ds + \alpha t, \tag{5}
\]

where

\[
J(t, \alpha) = \frac{1}{\alpha} \int_{t_0}^{t} \int_{|z| \leq c} \left[ e^{\log \left( \frac{V(y(s)) + H(y(s), z)}{V(y(s))} \right)} \right] \right. \\
-1 - \frac{1}{\alpha} \log \left( \frac{V(y(s)) + H(y(s), z))}{V(y(s))} \right) \left. \right) \nu(dz)ds.
\]

So, for \( t \in [t_0 + lT, t_0 + (l+1)T + \theta) \), it follows from (5) that

\[
\log(V(y(t))) = \log(V(y_0)) + \int_{t_0}^{t_0 + \theta} \frac{\mathcal{L}V(y(s))}{V(y(s))} ds + \int_{t_0 + \theta}^{t_0 + T} \frac{\mathcal{L}V(y(s))}{V(y(s))} ds \\
+ \int_{t_0 + T}^{t_0 + T + \theta} \frac{\mathcal{L}V(y(s))}{V(y(s))} ds + \int_{t_0 + T + \theta}^{t_0 + 2T} \frac{\mathcal{L}V(y(s))}{V(y(s))} ds \\
+ \int_{t_0 + 2T}^{t_0 + 2T + \theta} \frac{\mathcal{L}V(y(s))}{V(y(s))} ds + \cdots + \int_{t_0 + lT}^{t} \frac{\mathcal{L}V(y(s))}{V(y(s))} ds.
\]
\[-\frac{1}{2}(1 - \alpha) \left( \int_{t_0}^{t_0 + \theta} \frac{V_y(y(s^-))g(y(s^-))^2}{V(y(s^-))^2} \, ds \right)
+ \int_{t_0 + \theta}^{t_0 + T} \frac{V_y(y(s^-))g(y(s^-))^2}{V(y(s^-))^2} \, ds
+ \int_{t_0 + \theta}^{t_0 + T + \theta} \frac{V_y(y(s^-))g(y(s^-))^2}{V(y(s^-))^2} \, ds
+ \int_{t_0 + \theta + T}^{t_0 + 2T} \frac{V_y(y(s^-))g(y(s^-))^2}{V(y(s^-))^2} \, ds
+ \int_{t_0 + \theta}^{t_0 + T + \theta} \frac{V_y(y(s^-))g(y(s^-))^2}{V(y(s^-))^2} \, ds
+ \cdots + \int_{t_0 + T + \theta}^{t_0 + T + 2T} \frac{V_y(y(s^-))g(y(s^-))^2}{V(y(s^-))^2} \, ds\]

\[+ \cdots + \int_{t_0 + T + 2T}^{t_0 + TL} \frac{V_y(y(s^-))g(y(s^-))^2}{V(y(s^-))^2} \, ds\]}

For $t \in \left[ t_0 + IT + \theta, t_0 + (l + 1)IT \right)$, the following result can be shown by using similar steps:

\[
\log(V(y(t))) \leq \log(V(y_0)) + \int_{t_0}^{t_0 + T + \theta} \frac{\mathcal{L}V(y(s^-))}{V(y(s^-))} \, ds
+ \int_{t_0 + T + \theta}^{t_0 + T + 2T} \frac{\mathcal{L}V(y(s^-))}{V(y(s^-))} \, ds
+ \int_{t_0 + T + 2T + \theta}^{t_0 + T + 3T} \frac{\mathcal{L}V(y(s^-))}{V(y(s^-))} \, ds
+ \cdots + \int_{t_0 + T + lT + \theta}^{t_0 + T + (l + 1)T} \frac{\mathcal{L}V(y(s^-))}{V(y(s^-))} \, ds\]
almost surely exponential stabilization based on Mao’s ideal (see, [18]). Of course, Poisson integral to stabilize a nonlinear system, and establish a class of theories on mittent stochastic perturbation induced by Brownian motion and the compensated.

By above inequality and condition (i) of Assumption 3.4, it follows that

\[ \lim_{\alpha \to 0} J_{l_0, t_0 + T} + J_{l_0 + T, t_0 + T + \theta} + \cdots + J_{l_0 + T, t_0 + T + \theta} = 0. \] (8)

Now let \( \alpha \to 0 \). Owing to \( \alpha > 0 \) is sufficient small, \( J(t, \alpha) = 0 \) as \( \alpha \to 0 \) in Lemma 3.3 and we have \( \lim_{\alpha \to 0} J_{l_0, t_0 + \theta} = \lim_{\alpha \to 0} J_{l_0 + T, t_0 + T + \theta} \cdots = \lim_{\alpha \to 0} J_{l_0 + T, t_0 + T + \theta} = 0. \)

The inequality (8) implies

\[ \frac{\log(V(y(t)))}{t} \leq \frac{\log(V(y_0)) + k_3(l + 1)\theta}{t_0 + lT} + \frac{k_2(t - t_0)}{t} - \left( \frac{1}{2} k_4 + k_5 \right) \frac{l\theta}{t_0 + (l + 1)T}. \] (9)

By above inequality and condition (i) of Assumption 3.4, it follows that

\[ \lim_{t \to \infty} \frac{1}{l} \log |y(t)| < -\frac{|k_4 + 2k_5 - 2k_3|}{2p} \frac{\theta}{T} - \frac{k_2}{2p}, \quad a.s. \] (10)

As a consequence, the required result holds. □

We shall close this Theorem 3.5 by the following remarks and discuss the relation between Theorem 3.5 and (Theorem 4.4.2, [24]) (Theorem 3.1, [3]).

**Remark 3.** The main results of Theorem 3.5 are to apply the periodically intermittent stochastic perturbation induced by Brownian motion and the compensated Poisson integral to stabilize a nonlinear system, and establish a class of theories on almost surely exponential stabilization based on Mao’s ideal (see, [18]). Of course,
it makes the analysis more difficult owing to the discontinuity of its sample paths. And quite remarkably, the results in [35] are covered by Theorem 3.5.

**Remark 4.** When \( \theta \to T \), the system (1) will become general SDEs with small jumps (e.g., see [3] and [4]). For this case, condition (ii) in Assumption 3.4 becomes inequality (10) will be reduced to

\[
\limsup_{t \to \infty} \frac{1}{t} \log |y(t)| < -\frac{k_4 + 2k_5 - 2(k_2 + k_3)}{2p}, \quad \text{a.s.}
\]

**Remark 5.** When \( \theta \to 0 \), the system (1) will reduce to ODE system. By similar analysis, we conclude that the ODE is exponentially attractive if \( k_2 < 0 \).

Let us consider a multidimensional semi-linear case:

\[
\begin{aligned}
\quad & dy(t) = f(y(t-))dt + \sum_{i=1}^{m} A_i y(t-))d\tilde{B}_i^t + \int_{|z| \leq c} C(z)y(t-)d\tilde{N}(dt,dz), \\
& \quad t \in [t_0 + IT, t_0 + IT + \theta], \\
& dy(t) = f(y(t))dt, \quad t \in [t_0 + IT + \theta, t_0 + (l+1)T], l = 0, 1, 2, \cdots ,
\end{aligned}
\]

where \( A_i \in \mathbb{R}^{n \times n} \), \( C(z) \) does not have eigenvalue of \(-1\), \( B(t) \) is \( m \)-dimensional Brownian motion with \( B_t = (B_1^t, \cdots, B_m^t)^T \) and \( \tilde{N}(dt,dz) \) is the compensated Poisson process with \( N(dt,dz) = N(dt,dz) - \nu(dz)dt \). Assume that \( B(t) \) and \( N(dt,dz) \) are independent.

**Remark 6.** It is obvious that \( g(y) = (A_1y, A_2y, \cdots, A_my) \) from (11). Since \( A_i \) is a constant matrix, then \( g \) satisfies Lipschitz condition. The existence and uniqueness of the solutions of (11) can be satisfied.

**Assumption 3.6.** Suppose that there exist some constants \( \gamma_1 \in \mathbb{R}, \gamma_i > 0 \) \((i = 2, 4, 6), \gamma_i \geq 0 \) \((i = 3, 5)\) such that the following conditions

\[
\begin{aligned}
g^Tf(y) & \leq \gamma_1|y|^2, \quad \sum_{i=1}^{m} |A_i y|^2 \leq \gamma_2|y|^2, \quad \sum_{i=1}^{m} |y^T A_i y|^2 \geq \gamma_3 |y|^4, \\
\int_{|z| \leq c} [y^T(1 + ||C(z)||)y] \nu(dz) & \leq \gamma_4 |y|^2, \quad \int_{|z| \leq c} [y^T C(z) y] \nu(dz) \geq \gamma_5 |y|^2, \\
\int_{|z| \leq c} \left[ \log(|y + C(z)y|^2) - \log(|y|^2) - \left( \frac{|y + C(z)y|^2}{|y|^2} \right) + 1 \right] \nu(dz) & \leq -\gamma_6 \end{aligned}
\]

are satisfied for any \( y \in \mathbb{R}^n \) and \( \gamma_2 + \gamma_4 - 1 - 2\gamma_7 > 0 \).

**Corollary 1.** Under Assumptions 2.5, 3.6 and Lemma 2.6, then we have

\[
\limsup_{t \to \infty} \frac{1}{t} \log |y(t; t_0, y_0)| \leq -\frac{[2\gamma_3 + \gamma_6 - \gamma_2 - \gamma_4 + 1 + 2\gamma_7] \theta}{2} - 2\gamma_7, \quad \text{a.s.}
\]

Furthermore, if \([2\gamma_3 + \gamma_6 - \gamma_2 - \gamma_4 + 1 + 2\gamma_7] \frac{\theta}{T} - 2\gamma_7 > 0\) holds, then the trivial solution of system (11) is almost surely exponentially stable.

**Proof.** Let \( V(y) = |y|^2 \). With assumptions given by (12), (13) and (14), we have

\[
V_y(y)f(y) \leq 2\gamma_1 |y|^2, \quad \frac{1}{2} \text{trace}[g^T(y)V_{yy}(y)g(y)] = \sum_{i=1}^{m} |A_i y|^2 \leq \gamma_2 |y|^2,
\]
\[ |V_y(y)g(y)|^2 = 4|y^T g(y)|^2 = 4 \sum_{i=1}^{m} |y^T A_i y|^2 \geq 4\gamma_3 |y|^4, \]

\[ \int_{|z| \leq c} [V(y + H(y, z)) - V(y) - V_y(t, y)H(y, z)]\nu(dz) \leq (\gamma_4 - 1 - 2\gamma_5)|y|^2 \]

and

\[ \int_{|z| \leq c} \left( \log \left( \frac{V(y + H(y, z))}{V(y)} \right) - \left( \frac{V(y + H(y, z))}{V(y)} - V(y) \right) \right) \nu(dz) \]

\[ = \int_{|z| \leq c} \left[ \log(|y + C(z)y|^2) - \log(|y|^2) - \left( \frac{|y + C(z)y|^2}{|y|^2} \right) + 1 \right] \nu(dz) \leq -\gamma_6. \]

In view of Theorem 3.5, it is easy to deduce that

\[ \limsup_{t \to \infty} \frac{1}{t} \log |y(t)| \leq -\frac{2\gamma_3 + \gamma_6 - \gamma_2 - \gamma_4 + 1 + 2\gamma_5}{2} \frac{\theta}{T} - 2\gamma_1, \text{a.s.} \]

This completes this proof. \(\Box\)

Now the stability of system (11) are discussed as follows:

**Case 4.1.** Stabilization from only periodically intermittent stochastic perturbation induced by the Brownian motion if

\( (2\gamma_3 - \gamma_2) \frac{\theta}{T} - 2\gamma_1 > 0. \)

**Case 4.2.** Stabilization from only periodically intermittent stochastic perturbation induced by the compensated Poisson integral if

\( (\gamma_6 - \gamma_4 + 1 + 2\gamma_5) \frac{\theta}{T} - 2\gamma_1 > 0. \)

**Case 4.3.** Stabilization from a mixture of periodically intermittent stochastic perturbation induced by the Brownian motion and the compensated Poisson integral if

\( (2\gamma_3 + \gamma_6 - \gamma_2 - \gamma_4 + 1 + 2\gamma_5) \frac{\theta}{T} - 2\gamma_1 > 0. \)

**Remark 7.** In Corollary 1, it is worth mentioning that **Case 4.3** showcases that an unstable system is stabilized by a mixture of the compensated Poisson jumps and Brownian motion. However, Zhang et al. [35] studied stabilization via the periodically intermittent stochastic perturbation induced by Brownian motion, and it is regarded as a special case of these results.

### 3.2. Suppression of noise.

From **Case 4.2**, an unstable system can be stabilized by a compensated Poisson integral, i.e.

\( (\gamma_6 - \gamma_4 + 1 + 2\gamma_5) - 2\gamma_1 > 0, \text{ as } \theta \to T. \)

It is well known that Brownian motion can destabilize a stable ODE system.

**Question.** Can we use periodically intermittent stochastic perturbation induced by Brownian motion to suppress a stable stochastic system with jumps?

This is the main task of this section. A single compensated Poisson process enables the readers to choose \( \bar{\pi} \) flexibly according to their needs from a wide range \((0, \infty)\). In what follows, we demonstrate that a stochastic system with jumps is
destabilized by periodically intermittent stochastic perturbation induced by Brownian motion:

\[
\begin{align*}
\text{Assumption 3.7.} \quad & \text{Assume that there exists a function } V \in C^2(\mathbb{R}^n; \mathbb{R}^+) \text{ and some constants } p > 0, \ k_2 \in \mathbb{R}, \ k_i > 0 \ (i = 1, 3, 4, 5) \text{ and } k_6 > 1 \text{ such that for all } y \neq 0, \\
& \quad (i) \ V(y) \leq k_1 |y|^p, \\
& \quad (ii) \ V_y(y)f(y) \geq k_2 V(y), \ \text{trace} \left[ g^T(y)V_{yy}(y)g(y) \right] \geq k_3 V(y), \\
& \quad (iii) \ |V_y(y)g(y)|^2 \leq k_4 (V(y))^2, \ |V_y(y)H(y)| \leq k_5 V(y), \\
& \quad (iv) \ \log(V(y + H(y))) - \log(V(y)) \geq \log k_6, \\
\end{align*}
\]

Theorem 3.8. Under Assumption 3.7. Then

\[
\lim_{t \to \infty} \inf_t \frac{\log |y(t)|}{t} \geq \frac{(k_3 - k_4) \pi^2 + 2k_2 - 2\pi k_5 + 2\pi \log k_6}{2p}, \quad a.s. \tag{16}
\]

for \( y_0 \neq 0 \). If \((k_3 - k_4) \pi^2 + 2k_2 - 2\pi k_5 + 2\pi \log k_6 > 0\), then almost all the sample paths of \( y(t) \) tends to infinity, and in this case we say the trivial solution of the system (15) is almost surely exponentially unstable.

Proof. Fix \( y_0 \neq 0 \), then \( y(t) \neq 0 \) for all \( t \geq t_0 \). Using Itô’s formula to \( \log(V(y(t))) \) that

\[
\log(V(y(t))) = \log(V(y_0)) + \int_{t_0}^t R_1(s)ds + \int_{t_0}^t R_2(s)ds \\
- \frac{1}{2} \int_{t_0}^t R_3(s)ds - \pi \int_{t_0}^t R_4(s)ds + \int_{t_0}^t R_5(s)dN(s) \\
+ \int_{t_0}^t \frac{1}{V(y(s-))} V_y(y(s-))g(y(s-))dB_s, \tag{17}
\]

where

\[
R_1(s) = \frac{1}{V(y(s-))}[V_y(y(s-))f(y(s-))],
\]

\[
R_2(s) = \frac{1}{2V(y(s-))} \text{trace}[g^T(y(s-))V_{yy}(y(s-))g(y(s-))],
\]

\[
R_3(s) = \frac{1}{\left(V(y(s-)) \right)^2} (V_y(y(s-))g(y(s-)))^2,
\]

\[
R_4(s) = \frac{1}{V(y(s-))} [V_y(y(s-))H(y(s-))]
\]

and

\[
R_5(s) = \log(V(y(s-)) + H(y(s-))) - \log(V(y(s-))).
\]

Let \( M(t) = \int_{t_0}^t \frac{1}{V(y(s-))} V_y(y(s-))g(y(s-))dB_s. \)
For \( t \in [t_0 + lT, t_0 + lT + \theta) \), an application of assumption 3.7 show that
\[
\log(V(y(t)))
\]
\[
= \log(V(y_0)) + \int_{t_0}^{t_0 + \theta} (R_1(s) + R_2(s))ds + \int_{t_0 + T}^{t_0 + T + \theta} (R_1(s) + R_2(s))ds + \int_{t_0 + T + \theta}^{t_0 + 2T} (R_1(s) + R_2(s))ds + \cdots + \int_{t_0 + lT + \theta}^{t_0 + T + \theta} (R_1(s) + R_2(s))ds
\]
\[
- \frac{1}{2} \left( \int_{t_0}^{t_0 + \theta} R_3(s)ds + \int_{t_0 + \theta}^{t_0 + T} R_3(s)ds + \int_{t_0 + T}^{t_0 + 2T} R_3(s)ds + \cdots + \int_{t_0 + lT}^{t_0 + T} R_3(s)ds \right)
\]
\[
- \bar{\pi} \left( \int_{t_0}^{t_0 + \theta} R_4(s)ds + \int_{t_0 + \theta}^{t_0 + T} R_4(s)ds + \int_{t_0 + T}^{t_0 + 2T} R_4(s)ds + \cdots + \int_{t_0 + lT}^{t_0 + T} R_4(s)ds \right)
\]
\[
+ M(t) + \left( \int_{t_0}^{t_0 + \theta} R_5(s)dN(s) + \int_{t_0 + T}^{t_0 + \theta} R_5(s)dN(s) + \int_{t_0 + T + \theta}^{t_0 + 2T} R_5(s)dN(s) + \cdots + \int_{t_0 + lT + \theta}^{t_0 + T + \theta} R_5(s)dN(s) \right)
\]
\[
\geq \log(V(y_0)) + \left( k_2 + \frac{1}{2} k_3 \right) \theta + k_2(T - \theta) + \left( k_2 + \frac{1}{2} k_3 \right) \theta
\]
\[
+ k_2(T - \theta) + \left( k_2 + \frac{1}{2} k_3 \right) \theta + \cdots + \left( k_2 + \frac{1}{2} k_3 \right) (t - t_0 - lT)
\]
\[
- \frac{1}{2} \left( k_3 \theta + 0 + k_3 \theta + 0 + k_3 \theta + 0 + k_3 \theta + \cdots + k_4(t - t_0 - lT) \right)
\]
\[
- \bar{\pi} \left( k_5 \theta + k_5(T - \theta) + k_5 \theta + k_5(T - \theta) + k_5 \theta + k_5(T - \theta) + k_5 \theta + \cdots + k_5(t - t_0 - lT) \right) + M(t) + (N(t_0 + \theta) - N(t_0)) \log k_6
\]
\[
+(N(t_0 + T) - N(t_0 + \theta)) \log k_6
\]
\[
+(N(t_0 + T + \theta) - N(t_0 + T)) \log k_6
\]
\[
+(N(t_0 + 2T) - N(t_0 + T + \theta)) \log k_6
\]
\[
+ (N(t_0 + 2T + \theta) - N(t_0 + 2T)) \log k_6
\]
\[
+ \cdots + (N(t) - N(t_0 + lT)) \log k_6
\]
\[
\geq \log(V(y_0)) + \left( k_2 + \frac{1}{2} k_3 \right) (l\theta + t - t_0 - lT) + k_2 l(T - \theta) + M(t)
\]
\[
- \bar{\pi} k_5(t - t_0) - \frac{1}{2} k_4(l\theta + t - t_0 - lT) + (N(t) - N(t_0)) \log k_6
\]
\[
\geq \log(V(y_0)) + \frac{1}{2} k_2 \theta + (k_2 - \bar{\pi} k_5)(t - t_0) - \frac{1}{2} k_4(l + 1) \theta
\]
\[
+ M(t) + (N(t) - N(t_0)) \log k_6.
\]
In a similar manner, if \( t \in [t_0 + lT + \theta, t_0 + (l + 1)T) \), we show
\[
\log(V(y(t))) \geq \log(V(y_0)) + \frac{1}{2} k_3 t \theta + (k_1 - \bar{\pi} k_5)(t - t_0).
\]
This, together with (18) and (19), yields
\[
\frac{\log(V(y(t)))}{t} \geq \log(V(y_0)) + \left( k_2 - \bar{\pi} k_5 \right)(t - t_0) + \frac{M(t)}{t} + \frac{1}{2} k_4 (l + 1) \theta + M(t) \left( N(t) - N(t_0) \right) \frac{\log k_6}{t}. \tag{19}
\]

The well-known strong law of large numbers of for the Brownian motion and Poisson process show that
\[
\lim_{t \to \infty} \frac{M(t)}{t} = 0, \ a.s. \quad \text{and} \quad \lim_{t \to \infty} \frac{\tilde{N}(t)}{t} = \bar{\pi}, \ a.s. \tag{21}
\]

By condition (i) of Assumption 3.7, (20) and (21),
\[
\liminf_{t \to \infty} \frac{1}{t} \log |y(t)| \geq \frac{(k_3 - k_1) \frac{\theta}{T} + 2k_2 - \bar{\pi} k_5 + \bar{\pi} \log k_6}{2p}, \ a.s. \tag{22}
\]
as required. \( \square \)

We consider linear stochastic perturbation as follows
\[
\begin{cases}
\frac{dy(t)}{dt} = f(y(t-))dt + \sum_{i=1}^{m} A_i y(t-)_i dB_i^0 + C y(t-)d\tilde{N}(t), \\
\end{cases}
\]
\[\frac{dy(t)}{dt} = f(y(t))dt + C y(t-)d\tilde{N}(t), \quad t \in [t_0 + lT, t_0 + (l + 1)T) \quad l = 0, 1, 2, \ldots , \tag{23}
\]
where \( A_i \in \mathbb{R}^{n \times n} \), \( C \) is a symmetric positive definite matric, \( B(t) \) is \( m \)-dimensional Brownian motion with \( B_l = (B_l^1, \ldots , B_l^n)^T \) and \( \tilde{N}(t) \) is the a single compensated Poisson process with \( \tilde{N}(t) = N(t) - \bar{\pi} t \). Assume that \( B(t) \) and \( N(t) \) are independent.

**Corollary 2.** Assume that there exist three constants \( \xi \in \mathbb{R} \), \( \eta_1 > 0 \) and \( \eta_2 \geq 0 \) such that \( y^T f(y) \geq \xi |y|^2 \),
\[
\sum_{i=1}^{m} |A_i|^2 \geq \eta_1 |y|^2 \tag{24}
\]
and
\[
\sum_{i=1}^{m} |y^T A_i|^2 \leq \eta_2 |y|^4. \tag{25}
\]
for all \( y \in \mathbb{R}^n \). Then the solution of system (23) satisfies
\[
\liminf_{t \to \infty} \frac{1}{t} \log |y(t)| \geq \Lambda, \ a.s \tag{26}
\]
for any \( y_0 \in \mathbb{R}^n \), where
\[
\Lambda = \left( \frac{\eta_1}{2} - \eta_2 \right) \frac{\theta}{T} + \xi - \bar{\pi} \lambda_{\text{max}} + \bar{\pi} \log(1 + \lambda_{\text{min}}),
\]
\(\lambda_{\min}\) and \(\lambda_{\max}\) denote the minimum and maximum eigenvalues of the matrix \(C\), respectively. Particularly, if \(\Lambda > 0\) holds, then the trivial solution of system (23) is almost surely exponentially unstable.

**Proof.** Let \(V(y) = |y|^2\). Direct computations show that

\[
\lambda_{\min}|y|^2 \leq y^TCy \leq \lambda_{\max}|y|^2,
\]

\[
|g(y)|^2 = \sum_{i=1}^{m}|A_i|y|^2 \geq \eta_1|y|^2,
\]

\[
|y^Tg(y)|^2 = \sum_{i=1}^{m}|y^TA_iy|^2 \leq \eta_2|y|^4.
\]

and

\[
\log(|y + Cy|^2) - \log(|y|^2) = \log \left(\frac{|y|^2 + 2y^TCy + |Cy|^2}{|y|^2}\right) \\
\geq \log(1 + 2\lambda_{\min} + \lambda_{\max}^2) \\
= 2 \log(1 + \lambda_{\min}).
\]

Hence, Assumption 3.7 is easily examined and the required assertion follows from Theorem 3.8. \(\Box\)

**Remark 8.** When \(\theta \to T\), the system (23) will degenerate into general SDEs with Poisson noise (e.g., see [4] and [24]). From Applebaum and Siakalli (Theorem 7.2, [4]), we know inequality (31) degenerate into

\[
\limsup_{t \to \infty} \frac{1}{t} \log |y(t)| \geq \frac{\eta_1}{2} - \eta_2 + \xi - \bar{\pi}\lambda_{\max} + \bar{\pi}\log(1 + \lambda_{\min}), \text{ a.s.}
\]

**Remark 9.** When \(\theta \to 0\), the system (23) will become a stable stochastic system with Poisson jumps as follows

\[
dy(t) = f(y(t))dt + Cy(t-)d\tilde{N}(t).
\]

If \(y^Tf(y) \leq \xi|y|^2\) (\(\xi \in \mathbb{R}\)) and \(\bar{\pi}\lambda_{\min} - \xi - \bar{\pi}\log(1 + \lambda_{\max}) > 0\) hold, then by Theorem 7.1 (see, [4]), the solution of (27) is almost surely exponentially stable.

**Remark 10.** The Corollary 2 manifests that the stochastic noise (Brownian motion) plays the dominating role in determining the almost surely exponentially unstable. To date, we have not been able to determine whether Poisson noise can destabilize the ODE system. But, Poisson noise contributes to stabilize an unstable system (see, Example 4.1) and to make a system more stable when it is already stable. Here is an example.

**Example 3.9.** The following nonlinear Liénard equation (see, e.g., [31]) is described as follows:

\[
x'' + f(x)x' + x = 0,
\]

where \(f \in C(\mathbb{R}, \mathbb{R})\). Suppose that \(f(0) > 0\). Let equation (28) translate into the following second-order system:

\[
\begin{cases}
\quad dx_1(t) = x_2(t)dt, \\
\quad dx_2(t) = (-x_1(t) - f(0)x_2(t))dt + (f(0) - f(x_1))x_2(t)dt,
\end{cases}
\]

in which \(x(t) = (x_1(t), x_2(t))^T, f(x) = \frac{3+x^2}{1+x^2}\), or their vector form:

\[
dx(t) = Dx(t)dt + G(x(t))dt,
\]
where
\[ D = \begin{pmatrix} 0 & 1 \\ -1 & -f(0) \end{pmatrix}, \quad G(x(t)) = \begin{pmatrix} 0 \\ (f(0) - f(x_1(t)))x_2(t) \end{pmatrix}. \]

It is not difficult to check that \( D \) is stable and \( G(x) = o(|x|) \) as \(|x| \to 0\). Consequently, the equation (29) is uniformly asymptotic stability and the state \( x(t) \) of (29) is shown Figure 1. Let us present nonlinear Liénard equation with jumps as follows:
\[ \frac{dy(t)}{dt} = f(y(t))dt + \sigma \tilde{y}(t-)dB_1^t + Cy(t-)d\tilde{N}(t), \quad t \in [t_0 + lT, t_0 + lT + \theta), \]
\[ \frac{dy(t)}{dt} = f(y(t))dt, \quad t \in [t_0 + lT + \theta, t_0 + (l + 1)T), \quad l = 0, 1, 2, \cdots, \]
where \( \tilde{y}(t) = (y_2(t), -y_1(t), \cdots, y_{2p}(t), -y_{2p-1}(t))^T \) for all \( t \geq t_0 \). According to (24) and (25), we have
\[ \sum_{i=1}^{m} |A_i y|^2 = \sigma^2 |y|^2 \quad \text{and} \quad \sum_{i=1}^{m} |y^T A_i y|^2 = 0. \]
Hence,
\[ \Lambda = \frac{\sigma^2 \theta}{2} + \xi - \bar{\pi}\lambda_{\text{max}} + \bar{\pi}\log(1 + \lambda_{\text{min}}). \]

In particular, if \( \Lambda > 0 \) holds, then system (31) is almost surely exponentially unstable. For more schemes, one can refer to Mao [18], Sinkalli [24] and Zhang et al. [35].

4. Applications and examples. In this section, two examples are presented to illustrate the effectiveness of the results in Theorems 3.5 and 3.8. The well-known Lorenz chaotic system is shown as follows:

\[
\begin{cases}
    dx_1(t) = -\rho_1x_1(t)dt + x_2(t)x_3(t)dt, \\
    dx_2(t) = -\rho_2x_2(t)dt + \rho_2x_3(t)dt, \\
    dx_3(t) = -x_1(t)x_2(t)dt - x_3(t)dt.
\end{cases}
\] (32)

As a matter of convenience, system (32) is rewritten as
\[ dx(t) = f(x(t))dt, \]
where \( x(t) = (x_1(t), x_2(t), x_3(t))^T, \) \( f(x(t)) = (-\rho_1x_1(t) + x_2(t)x_3(t), -\rho_2x_2(t) + \rho_2x_3(t), -x_1(t)x_2(t) + \rho_3x_2(t) - x_3(t))^T. \) The system (32) is chaotic as shown in Figure 3.

![Figure 3. The state \( x(t) \) of the system (32).](image1)

![Figure 4. The state \( x(t) \) of system (33).](image2)

**Example 4.1.** Let the Lévy measure \( \nu \) satisfy \( \nu(dz) = \frac{dz}{1+z^2} \) and take \( c = 1. \) The corresponding stochastically perturbed system is denoted as follows:

\[
\begin{cases}
    dx(t) = f(x(t-))dt + \sum_{i=1}^{m} a_i x(t-)dB_i^t + \int_{|z|\leq 1} bx(t-)\tilde{N}(dt,dz), \\
    t \in [t_0 + lT, t_0 + lT + \theta), \\
    dx(t) = f(x(t))dt, \quad t \in [t_0 + lT + \theta, t_0 + (l+1)T), \quad l = 0, 1, 2, \cdots.
\end{cases}
\] (33)
For any $x(t) \in \mathbb{R}^3$, we now verify
\[
x^T(t)f(t, x(t)) = x_1(t)[-\rho_1 x_1(t) + x_2(t)x_3(t)] + x_2[-\rho_2 x_2(t) + \rho_3 x_2(t)] \\
+ \rho_2 x_3(t)] + x_3[-x_1(t)x_2(t) + \rho_3 x_2(t) - x_3(t)]
\]
\[
\leq -\rho_1 x_1^2(t) + \left(-\rho_2 + \frac{1}{2}\rho_2 + \frac{1}{2}\rho_3\right)x_2^2(t) \\
+ \left(\frac{1}{2}\rho_2 + \frac{1}{2}\rho_3 - 1\right)x_3^2(t)
\]
\[
\leq \gamma_1 x^2(t),
\]
\[
\sum_{i=1}^{m}|A_i|^2 = \sum_{i=1}^{m}|a_i|^2 = \sum_{i=1}^{m}|a_i|^2|z|^2, \sum_{i=1}^{m}|x^T A_i x|^2 = \sum_{i=1}^{m}|a_i|^2|z|^4,
\]
\[
\int_{|z| \leq 1} x^T (1 + ||C(z)||^2 x)|\nu(dz) = \int_{|z| \leq 1} x^T (1 + |b|^2 x)|\nu(dz) = \frac{\pi}{4} (1 + |b|^2)|z|^2
\]
\[
\int_{|z| \leq 1} \left(\log (1 + b)^2 - (1 + b)^2 + 1\right) \frac{dz}{1 + z^2} = -\frac{\pi}{2} \left((1 + b)^2 - \log (1 + b)^2 - 1\right),
\]
which implies that conditions of Corollary 1 with
\[
\gamma_1 = \max \left\{-\rho_1, \left(-\rho_2 + \frac{1}{2}\rho_2 + \frac{1}{2}\rho_3\right), \left(\frac{1}{2}\rho_2 + \frac{1}{2}\rho_3 - 1\right)\right\},
\]
\[
\gamma_2 = \sum_{i=1}^{m}|a_i|^2, \gamma_3 = \sum_{i=1}^{m}|a_i|^2, \gamma_4 = \frac{\pi}{2} (1 + |b|^2), \gamma_5 = \frac{\pi}{2} b,
\]
\[
\gamma_6 = \frac{\pi}{2} x \left((1 + b)^2 - \log (1 + b)^2 - 1\right). \text{ Design } m = 3, b = 2, a_i = 7.7 \ (i = 1, 2, 3), \theta = 1, T = 5, \rho_1 = \frac{8}{3}, \rho_2 = 10, \rho_3 = 28. \text{ Hence, } 2\gamma_3 + \gamma_6 - \gamma_2 - \gamma_4 + 1 + 2\gamma_5 \frac{\theta}{T} - 2\gamma_1 = 7.7^2 \times 3 + \frac{\pi}{2} (\ln 9 - 1) + 2\pi + 1 \times \frac{1}{3} - 36 = 0.0262 > 0
\]
as required. Consequently, all the hypothesis of Corollary 1 are fulfilled. As a result, periodically intermittent stochastic perturbation contributes to almost sure exponential stabilization of system (33) and the numerical result is shown in Figure 4.

**Figure 5.** The exponential dynamical behaviors of state $x(t)$ of the system (34).

**Figure 6.** The state $x(t)$ of system (34).
To proceed, in order to show the advantages of the proposed results in Theorem 3.8, we further consider a stable system (30) in Example 3.9.

**Example 4.2.** Let us consider periodically intermittent stochastic perturbation system induced by Brownian motion as follows

\[
\begin{cases}
    dx(t) = Dx(t-)dt + G(x(t-))dt + Ax(t-)dB(t) + Cx(t-)d\tilde{N}(t), \\
    t \in [t_0 + lT, t_0 + lT + \theta), \\
    dx(t) = Dx(t)dt + G(x(t))dt + Cx(t-)d\tilde{N}(t), \\
    t \in [t_0 + lT + \theta, t_0 + (l + 1)T), l = 0, 1, 2, \ldots.
\end{cases}
\]  

(34)

In (34), the corresponding parameters are

\[
A = \begin{pmatrix}
    0 & 10 \\
    -10 & 0
\end{pmatrix}, \quad C = \begin{pmatrix}
    2.7182 & 0 \\
    0 & 1.7182
\end{pmatrix};
\]

\[
\theta = 1, \ T = 4, \ \bar{\pi} = 4. \quad \text{Direct computations show that} \ x^T(Dx + G(x)) \geq -3x^2, \\
\eta_1 = 100, \ \eta_2 = 0, \ \lambda_{\text{max}} = 2.7182, \ \lambda_{\text{min}} = 1.7182, \ \Lambda = (\eta_1 - \eta_2) \bar{\pi} + \xi + \bar{\pi}\lambda_{\text{max}} + \\
\bar{\pi}\log(1 + \lambda_{\text{min}}) = 2.6271 > 0. \quad \text{Corollary 2 shows that system (34) is almost surely exponentially unstable (see Figure 6).}
\]

5. **Conclusions and future works.** This paper is devoted to study almost sure exponential stabilization and suppression with Brownian motion, the compensated Poisson jumps process and periodically intermittent control, respectively. We called it periodically intermittent stochastic perturbation with jumps. Exponential martingale inequality have been used to derive sufficient conditions for almost sure stabilization and strong law of large numbers of local martingale and Poisson process for almost sure exponential destabilization. As for application, two classes of nonlinear stochastic systems induced by periodically intermittent stochastic noise have been discussed. Finally, some examples are given to support the results obtained. Based on analysis above, the following problems will be of interest in the future.

Since Lévy noise plays a significant role in practical systems whose structure is subject to the fluctuations and large disasters, hence, complex systems (including multi-agent networks, ecosystems) by the periodically intermittent stochastic noise with jumps will be our future work.

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**REFERENCES**

[1] D. Applebaum, *Lévy Processes and Stochastic Calculus*, Cambridge University Press, 2009.
[2] D. Applebaum, Extending stochastic resonance for neuron models to general Lévy noise, *IEEE Trans. Neural Netw.*, 20 (2009), 1993–1995.
[3] D. Applebaum and M. Siakalli, Asymptotic stability of stochastic differential equations driven by Lévy noise, *J. Appl. Probab.*, 46 (2009), 1116–1129.
[4] D. Applebaum and M. Siakalli, Stochastic stabilization of dynamical systems using Lévy noise, *Stoch. Dyn.*, 10 (2010), 509–527.
[5] A. D. Appleby, X. Mao and A. Rodkina, Stabilization and destabilization of nonlinear differential equations by noise, *IEEE Tran. Automat. Control*, 53 (2008), 683–691.
[6] R. Cont and P. Tankov, *Financial Modelling with Jump Processes*, Chapman & Hall/CRC Financial Mathematics Series, Chapman & Hall/CRC, Boca Raton, FL, 2004.
[7] F. Deng, Q. Luo and X. Mao, Stochastic stabilization of hybrid differential equations, *Automatica J. IFAC*, 48 (2012), 2321–2328.
[8] Y. Feng, X. Yang, Q. Song and J. Cao, Synchronization of memristive neural networks with mixed delays via quantized intermittent control, *Appl. Math. Comput.*, **339** (2018), 874–887.

[9] R. Fernholz and I. Karatzas, Relative arbitrage in volatility-stabilized markets, *Ann. Financ.*, **1** (2005), 149–177.

[10] H. Gao and Y. Wang, Stochastic mutualism model under regime switching with Lévy jumps, *Phys. A*, **515** (2019), 355–375.

[11] B. Guo, Y. Wu, Y. Xiao and C. Zhang, Graph-theoretic approach to synchronizing stochastic coupled systems with time-varying delays on networks via periodically intermittent control, *Appl. Math. Comput.*, **331** (2018), 341–357.

[12] R. Hasminskii, *Stochastic Stability of Differential Equations*, Monographs and Textbooks on Mechanics of Solids and Fluids: Mechanics and Analysis, 7, Sijthoff & Noordhoff, Alphen aan den Rijn—Germantown, Md., 1980.

[13] N. Li and J. Cao, Intermittent control on switched networks via $\omega$-matrix measure method, *Nonlinear Dynam.*, **77** (2014), 1363–1375.

[14] S. Li, J. Cao and Y. He, Pinning controllability scheme of directed complex delayed dynamical networks via periodically intermittent control, *Discrete Dyn. Nat. Soc.*, (2016), 10 pp.

[15] C. Li, Z. Dong and R. Situ, Almost sure stability of linear stochastic differential equations with jumps, *Probab. Theory Related Fields*, **123** (2002), 121–155.

[16] X. Mao, Stochastic stabilization and destabilization, *Systems Control Lett.*, **48** (2002), 619–624.

[17] X. Mao, Almost sure exponential stability of stochastic delay differential equations with Markovian switching, *J. Franklin Inst.*, **352** (2015), 4515–4527.

[18] P. Wang, Y. Hong and H. Su, Stabilization of stochastic complex-valued coupled delayed systems with Markovian switching via periodically intermittent control, *Nonlinear Anal. Hybrid Syst.*, **29** (2018), 395–413.

[19] F. Wu and S. Hu, Suppression and stabilisation of noise, *Internat. J. Control*, **82** (2009), 2150–2157.

[20] F. Wu, G. Yin and Z. Jin, Kolmogorov-type systems with regime-switching jump diffusion perturbations, *Discrete Contin. Dyn. Syst. Ser. B*, **21** (2016), 2293–2319.

[21] Y. Xu, X. Wang, H. Zhang and W. Xu, Stochastic stability for nonlinear systems driven by Lévy noise, *Nonlinear Dynam.*, **68** (2012), 7–15.

[22] Y. Xu, H. Zhou and W. Li, Stabilization of stochastic delayed systems with Lévy noise on networks via periodically intermittent control, *Internat. J. Control*, (2018).

[23] G. Yin, Y. Talafha and F. Xi, Stochastic Liénard equations with random switching and two-time scales, *Comm. Statist. Theory Methods*, **43** (2014), 1533–1547.

[24] Q. Zhu, Asymptotic stability in the $p$th moment for stochastic differential equations with Lévy noise, *J. Math. Anal. Appl.*, **416** (2014), 126–142.

[25] Q. Zhu, Razumikhin-type theorem for stochastic functional differential equations with Lévy noise and Markov switching, *Internat. J. Control*, **90** (2017), 1703–1712.

[26] Q. Zhu, Stability analysis of stochastic delay differential equations with Lévy noise, *Systems Control Lett.*, **118** (2018), 62–68.
[35] B. Zhang, F. Deng, S. Peng and S. Xie, Stabilization and destabilization of nonlinear systems via intermittent stochastic noise with application to memristor-based system, *J. Franklin Inst.*, **355** (2018), 3829–3852.

[36] S. Zhu, K. Sun, S. Zhou and Y. Shi, Stochastic suppression and almost surely stabilization of non-autonomous hybrid system with a new general one-sided polynomial, *J. Franklin Inst.*, **354** (2017), 6550–6566.

[37] X. Zong, T. Li and J. Zhang, Consensus conditions for continuous-time multi-agent systems with additive and multiplicative measurement noises, *SIAM J. Control Optim.*, **56** (2018), 19–52.

[38] X. Zong, F. Wu, G. Yin and Z. Jin, Almost sure and $p$th-moment stability and stabilization of regime-switching jump diffusion systems, *SIAM J. Control Optim.*, **52** (2014), 2595–2622.

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