Signature of heavy sterile neutrinos at CEPC

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Abstract

We study the production of heavy sterile neutrino $N$, $e^+e^- \rightarrow N\nu(\bar{\nu})$, at the Circular Electron Positron Collider (CEPC) and its $ljj$ signal in its decay to three charged fermions. We study background events for this process which are mainly events coming from W pair production. We study the production of a single heavy sterile neutrino and the sensitivity of CEPC to the mixing of sterile neutrino with active neutrinos. We study the production of two degenerate heavy sterile neutrinos in a low energy see-saw model by taking into account the constraints on mixings of sterile neutrinos from the neutrino-less double $\beta$ decay experiment and the masses and mixings of active neutrinos. We show that CEPC under proposal has a good sensitivity to the mixing of sterile neutrinos with active neutrinos for a mass of sterile neutrino around 100 GeV.

Keywords: heavy sterile neutrino, collider signature

I. INTRODUCTION

The establishment of neutrino oscillation and tiny masses of active neutrinos in past decades has raised strong hope that new physics beyond the Standard Model (SM) is possible to exist in leptonic sector of elementary particles. The see-saw mechanism \[1\], as a simple and straightforward extension of neutrinos in the SM, works as a very good mechanism to explain the tiny masses of active neutrinos and is a very good candidate of physics beyond the SM. In see-saw mechanism, several right-handed neutrinos uncharged under the SM gauge groups, hence a type of sterile neutrinos, are introduced with heavy Majorana type masses which violate lepton number. The tiny masses of active neutrinos are understood in low energy scale as the lepton number violating remnant of the Majorana type masses of heavy right-handed neutrinos.

Although see-saw type models are quite interesting models of physics beyond the SM and have fruitful implications, there are very few clues of the mass scale of right-handed neutrinos. In particular, the mass scale of right-handed neutrinos can be much higher than the electroweak scale. Therefore, it is very hard to test such type models in experiments if
such a hierarchy between the mass scale of right-handed neutrinos and the electroweak scale indeed exists. For this reason, a low energy scale see-saw type model \cite{2}, which has right-handed neutrinos at or below the electroweak scale, is quite interesting since it’s possible to be tested in experiments. There are several interesting properties of this low energy see-saw model. For example, one of the right-handed neutrinos can be of keV scale and serves as a good candidate of warm dark matter(WDM) in the universe. Two other right-handed neutrinos in the model are at GeV or hundred GeV scale and are sufficient to generate tiny masses and mixings of active neutrinos measured in neutrino oscillation experiments.

Another interesting property in this type of low energy see-saw model is that the Yukawa couplings of right-handed neutrinos with SM neutrinos can be quite large while they can still give rise to masses and mixing of active neutrinos being consistent with the experimental data in neutrino oscillation and the constraint from neutrino-less double $\beta$ ($0\nu\beta\beta$) experiment \cite{3,4}, in particular when two heavy right-handed neutrinos are degenerate or quasi-degenerate. Consequently, the mixings of right-handed neutrinos with active neutrinos in the SM can be quite large while the masses of right-handed neutrinos are at GeV to hundred GeV scale. This scenario apparently offers great opportunities to search for see-saw type of models of physics beyond the SM in collider experiment.

Experimentally, the single heavy neutrino has been searched for by L3 collaboration at LEP through $N \to eW$ channel \cite{5,6}. Stringent constraint on $|R_{eN}|^2$ has been set for a mass region from 80GeV to 205GeV. Some efforts have been made to study the production and signature of heavy neutrino in $e^+e^-$ or $e^-e^-$ collision processes with both pair and single heavy neutrino productions, and various neutrino decay chains, $lW$, $\nu Z$ and $\nu H$ \cite{7–26}, for a review, see \cite{24}. Currently, new electron-positron colliders, such as CEPC, Future Circular Collider(FCC) and International Linear Collider(ILC), are under proposal. With these colliders, heavy sterile neutrino can be probed to a larger mass range and with better sensitivity on the active-sterile mixing $R_{lN}$. Recently, single heavy neutrino production modes $N\nu$ and $Ne^\pm W^\mp$ at ILC with center of mass energy of 350GeV and 500GeV have been investigated in \cite{21}. A search of long-lived heavy neutrinos with displaced vertices at CEPC, FCC and ILC has been presented in ref. \cite{23}. In our work, we present a detailed study of $e^\pm e^- \to N\nu$ with charged current neutrino decay mode $N \to lW$ at CEPC with center of mass energy $\sqrt{s} = 240$GeV.

In the present article, we are motivated by such kind of possibility and are going to study the signature of this type of right-handed neutrino( or to say sterile neutrino) of hundred GeV masses at CEPC \cite{27}, a collider under proposal. In the next section, we will make a quick review of the low energy see-saw model and describe some basic properties of this model. Then we discuss the collider signatures of a single sterile neutrino of a mass around hundred GeV. For simplicity, we simplify our discussion of collider signature using a single sterile neutrino. We will show that this simplification can be taken as a good simplification for later discussion. Then we come to signatures of low energy see-saw model by including
detailed constraints on the masses and mixings of right-handed neutrinos. We conclude in the last section.

II. GEV SCALE STERILE NEUTRINO AND LOW ENERGY SEE-SAW MODEL

One of major differences between the case of a single GeV scale sterile neutrino and the low energy see-saw type model of GeV scale sterile neutrinos is that for the former the mixings of sterile neutrinos with active neutrino are strongly constrained by $0\nu\beta\beta$ decay experiment \[28\], while for the latter the $0\nu\beta\beta$ constraint can be quite weak and the mixings can be quite large \[3\].

In the presence of one or several sterile neutrinos, active neutrinos in the flavor base $\nu_l (l = e, \mu, \tau)$ are a mixture of the light neutrinos in mass eigenstates $\nu_i (i = 1, 2, 3)$ and heavy sterile neutrinos in mass eigenstates $N_j$,

$$\nu_l = \sum_i U_{li} \nu_i + \sum_j R_{lN_j} N_j,$$

where $U_{ij}$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix, and $R_{lN_j}$ is the matrix element mixing $\nu_l$ with heavy neutrinos $N_j$. For small enough $|R_{lN_j}|$, mixing matrix $U$ can be considered as approximately unitary. Apparently, $\nu_i$ and $N_j$ can all contribute, in virtual intermediate state, to the $0\nu\beta\beta$ decay. It is not hard to see that the contribution of a single GeV scale sterile neutrino to the amplitude of $0\nu\beta\beta$ decay is proportional to $R_{eN}^2 / M_N$. The mixing $R_{eN}$ in this case is constrained to be $|R_{eN}|^2 \lesssim 10^{-5}$ \[28\], unless there are other particles or mechanisms at hand to ease the constraint.

In low energy see-saw type model, at least two heavy sterile neutrinos (right-handed neutrinos) are needed to obtain the correct masses and flavor mixings of active neutrinos \[3\]. In this case, the mixing matrix $R$ is $R = Y_\nu (M^*)^{-1}$ where $Y$ is the Yukawa coupling of neutrinos, $v$ the vacuum expectation value in the SM and $M$ the Majorana mass matrix of sterile neutrinos which can be taken to be real and diagonal in a convenient base. The matrix $M$ is a $2 \times 2$ matrix if considering two heavy sterile neutrinos and a $3 \times 3$ matrix if considering three heavy sterile neutrinos.

A nice feature in see-saw model is that mixing $R$ is related to $m_\nu$, the mass matrix of active neutrinos responsible for the neutrino oscillation phenomena:

$$(m_\nu)_{ll'} = -v^2 \sum_i Y_{li}^* Y_{l'i}^* M_i^{-1} = -\sum_i M_i R_{lN_i}^* R_{l'N_i}^*,$$

where $M_i$ is the eigenvalue of matrix $M$, that is, we have chosen a base in which $M$ is diagonal. One can see that if a strong cancellation happens between contributions of different sterile neutrinos in \[2\], a mass matrix $m_\nu$ at $10^{-3} - 10^{-2}$ eV scale can be generated for $M_i$ of hundred GeV scale and for pretty large $|R_{lN_i}|$. 
Using mixing matrix $R$, contributions of heavy sterile neutrinos to the amplitude of $0\nu\beta\beta$ decay can be parametrized as follows
\[ \mathcal{A} = F \sum_i R_{eN_i}^2 M_i^{-1}, \] (3)
where $F$ is an overall factor. For two heavy sterile neutrinos $N_1$ and $N_2$, (3) can be rewritten as
\[ \mathcal{A} = \frac{F}{M_1^2}(R_{eN_1}^2 M_1 + R_{eN_2}^2 M_2) + FM_2 R_{eN_2}(\frac{1}{M_2^2} - \frac{1}{M_1^2}), \] (4)
where $M_1$ and $M_2$ are the masses of $N_1$ and $N_2$ respectively. By taking $M_{1,2}$ real in a convenient base, one can see in (2) and (4) that the first term in (4) is of order $10^{-3} - 10^{-2}\text{eV}/M_2^2$ and can be neglected. The second term in (4) can be arbitrarily small if $N_1$ and $N_2$ are quasi-degenerate or degenerate. One can see clearly that the constraint from $0\nu\beta\beta$ decay is no longer strong for two quasi-degenerate heavy sterile neutrinos, which is exactly what happens in low energy see-saw model.

A straightforward consequence of the above discussion about (4) and (2) and the degeneracy of $N_1$ and $N_2$ is that for sterile neutrinos of GeV to hundred GeV mass, large value of $|R_{eN_1}|^2$ is only possible when
\[ R_{eN_1}^2 = -R_{eN_2}, \quad \text{or} \quad R_{eN_1} = \pm i R_{eN_2}. \] (5)

(5) is one of the major relations to be used in later analysis for discussing the collider signal of low energy see-saw model.

Relations among $R_{\mu N_i}$ and $R_{\tau N_i}$ can also be addressed similarly. Using solutions presented for two heavy sterile neutrinos in [3], one can find that $R_{lN_i}$ can be expressed as
\[ R_{lN_1} = \frac{1}{2} e^{\pm i x + |y|}(U_{12}m_2^{1/2}e^{-i\phi_2/2} \mp iU_{13}m_3^{1/2}e^{-i\phi_3/2})(M_1^*)^{-1/2}, \quad R_{lN_2} = \pm i R_{lN_1}, \] (6)
for normal hierarchy(NH) of neutrino masses, and
\[ R_{lN_1} = \frac{1}{2} e^{\pm i x + |y|}(U_{11}m_1^{1/2}e^{-i\phi_1/2} \mp iU_{12}m_2^{1/2}e^{-i\phi_2/2})(M_1^*)^{-1/2}, \quad R_{lN_2} = \pm i R_{lN_1}, \] (7)
for inverted hierarchy(IH) of neutrino masses. $m_{1,2,3}$ are real masses of $\nu_{1,2,3}$, $\phi_{1,2,3}$ the associated Majorana phases in diagonal form of $m_\nu$. For NH, $m_1 = 0$, $m_2 = \sqrt{\Delta m_{32}^2}$, $m_3 = \sqrt{[\Delta m_{32}^2] + \Delta m_{21}^2}$. For IH, $m_3 = 0$, $m_1 = \sqrt{[\Delta m_{32}^2] - \Delta m_{21}^2}$, $m_2 = \sqrt{\Delta m_{32}^2}$. $x$ and $y$ are two real free parameters to parametrize the mass matrix. (6) and (7) are valid for large value of $y$, i.e. for the case that cancellation in (4) is needed to satisfy $0\nu\beta\beta$ constraint.

One can see in (6) and (7) that $|R_{lN_2}|^2 = |R_{lN_1}|^2$ is valid for all flavors of neutrinos $\nu_{\tau,\mu,\nu}$ not just for $l = e$. This is one of the major properties of low energy see-saw model if allowing large mixing of sterile neutrinos with active neutrinos. Using (6) and (7) one can
FIG. 1: \( (|R_{\tau N_1}|^2 + |R_{\mu N_1}|^2)/|R_{e N_1}|^2 \) versus \( |R_{\tau N_1}|^2/|R_{\mu N_1}|^2 \) for NH and IH respectively.

also show the correlation of \( |R_{l N_1}|^2 \) by varying the free Dirac phase in matrix \( U \) and the Majorana phases \( \phi_i \). In Fig. 1 we plot the correlation of \( (|R_{\tau N_1}|^2 + |R_{\mu N_1}|^2)/|R_{e N_1}|^2 \) versus \( |R_{\tau N_1}|^2/|R_{\mu N_1}|^2 \). In our computation we use [29]

\[
\sin^2 2\theta_{12} = 0.846, \quad \sin^2 2\theta_{23} = 0.999, \quad \sin^2 2\theta_{13} = 0.093
\]

and

\[
\Delta m^2_{21} = 7.53 \times 10^{-5} \text{ eV}^2, \quad |\Delta m^2_{32}| = 2.48 \times 10^{-3} \text{ eV}^2
\]

For \( \Delta m^2_{32} \) we have averaged two fit values for NH and IH [29]. One can see in these plots that the mixings of sterile neutrino with \( \nu_\tau \) and \( \nu_\mu \) together are always stronger than the mixing with \( \nu_e \) for NH. For IH, \( |R_{\tau N_1}|^2 + |R_{\mu N_1}|^2 \) can be larger than or smaller than \( |R_{e N_1}|^2 \). On the other hand, the ratio between \( |R_{\tau N_1}|^2 \) and \( |R_{\mu N_1}|^2 \) can be larger than or smaller than one for both NH and IH.

From the above discussions, one can see that a major implication of a low energy see-saw type model with two GeV scale sterile neutrinos and large mixings with active neutrinos is the relation of mixings, such as \( |R_{l N_1}|^2 = |R_{l N_2}|^2 \) and the correlation shown in Fig. 1. For discussion of collider signatures in this low energy see-saw model, one should take these relations into account. However, as a first step towards this goal, we can discuss the signature of a single sterile neutrino with a mass at around 100 GeV. The signature of low energy see-saw model can be obtained by extending the discussion for a single sterile neutrino to two sterile neutrinos and taking into account these relations among mixings described above. A further advantage of first discussing a single sterile neutrino is that the case of a single sterile neutrino may also be valid if other particle or mechanisms, e.g. some scalar particles and Type-II see-saw mechanism, are introduced. So a discussion on the collider signature of a single heavy sterile neutrino is of interests for itself. Needless to say, discussing the signature
of a heavy sterile neutrino together with signature of other particles, e.g. scalar particles in type-II see-saw mechanism, is also of interests. In the present article, we are not going to elaborate on this topic. In the next section, we discuss the signature of a single sterile neutrino with a mass around 100 GeV at CEPC. We come back to the signature of low energy see-saw model in later sections. For previous works on the signature of heavy sterile neutrino on $e^+e^-$ collider, one can see a review in [24]. The present work give a discussion on the signature of heavy sterile neutrino on CEPC within the framework of low energy see-saw model and differs from the previous works in these aspect.

III. PRODUCTION AND DECAY OF A HEAVY STERILE NEUTRINO

In this section, we discuss the production of a single heavy sterile neutrino at CEPC and its decay. CEPC under proposal plans to run electron positron collision at a center-of-mass energy around 240 GeV and aims at obtaining an integrated luminosity up to 5 ab$^{-1}$ with two interaction points and ten years of operation.

The Feynman diagrams of the production of a heavy Majorana-type sterile neutrino, $N$, are shown in Fig. 2. For simplicity, the heavy neutrino index $j$ will be suppressed in discussion for a single heavy sterile neutrino. The leading contribution to $N$ production is the process $e^+e^- \rightarrow N \nu_l(\bar{\nu}_l)$, the SM process $e^+e^- \rightarrow \nu_l\bar{\nu}_l$ with $\nu_l$ or $\bar{\nu}_l$ replaced by $N$ via its mixing with $\nu_l$. Because of the Majorana nature of $N$, it can mix with both of $\nu_l$ and...
ν_l with the same strength of mixing and can be produced via both of these mixings. These two possibilities are shown in the left and right panels in Fig. 2. As one can see in upper panels of Fig. 2, the production of N can be mediated by a Z boson in s-channel with all type of neutrinos ν_l (¯ν_l) in final state. N production can also be mediated by a W boson in t-channel with ν_e (¯ν_e) in final state, as can be seen in lower panels in Fig. 2. For the same strength of mixings, the t-channel process has a cross section two order of magnitude larger than the s-channel process and hence has a better sensitivities for the mixing R_{eN}.

We calculate the tree-level e^+e^- → Nν_l cross sections with MadGraph [30] and implement the heavy neutrino interactions in FeynRules [33] with the Universal FeynRules Output (UFO) [34] format for the model. The results are shown in Fig. 3. For a heavy neutrino of about 100GeV, the production cross section of σ/|R_{eN}|^2 and σ/|R_{µN}|^2 can reach ∼ 60pb and ∼ 0.8pb for only a single R_{eN} mixing or R_{µN} mixing, respectively.

**Fig. 3:** e^+e^- → Nν cross section at √s = 240GeV with only a single R_{eN} mixing (left) and R_{µN} mixing (right).

Mixing of sterile neutrino N with active neutrinos can lead to decay of N. For m_N, the mass of N, much smaller than m_W, the mass of W boson, the leading decays of N are tree-level three-body decays mediated by off-shell W or Z bosons. Some three-body decay channels of N are quite simple. For example, N → e^-μ^+ν_µ is mediated by an off-shell W boson and is similar to μ → ν_µe¯ν_e, the leptonic decay of μ, except with the presence of a mixing factor |R_{eN}|^2 in decay rate. Some decay channels, e.g. N → ν_ee^-e^+, can be mediated by both off-shell W and Z bosons. But it does not introduce complications in the decay rate. The results are presented in (36), (37), (38), (39), (40), (41), (42) in Appendix.

For m_N much greater than m_W and m_Z, the leading decay of N are two-body decays, N → l^±W^± and N → ν(¯ν)Z. For m_N greater than m_H, the mass of Higgs boson, N can also decay to H via N → ν(¯ν)H. The partial decay widths of the heavy neutrino can be
written as \([3, 21, 35, 36]\)

\[
\Gamma(N \rightarrow l^- W^+) = \frac{g^2}{64\pi} |R_{lN}|^2 \frac{m_N^3}{m_W^2} (1 - \mu_W)^2 (1 + 2\mu_W) \tag{10}
\]

\[
\Gamma(N \rightarrow \nu Z) = \frac{g^2}{64\pi} |R_{lN}|^2 \frac{m_N^3}{m_Z^2} (1 - \mu_Z)^2 (1 + 2\mu_Z) \tag{11}
\]

\[
\Gamma(N \rightarrow \nu H) = \frac{g^2}{64\pi} |R_{lN}|^2 \frac{m_N^3}{m_H^2} (1 - \mu_H)^2 \tag{12}
\]

with \(\mu_i = \frac{m_i^2}{m_N^2} (i = W, Z, H)\). \(W, Z\) or \(H\) eventually decay to fermions. Hence the decay rate to a specific three-body final states can be calculated using (10-12) and the branching ratio of \(W, Z\) or \(H\) to a specific fermion pair. For example, \(\Gamma(N \rightarrow e^- \mu^+ \nu_\mu)\) is obtained using \(\Gamma = G_F^2 m_N^5 / (192\pi^3)\) in (36) up to \(m_N < m_W\); \(\Gamma = \Gamma(N \rightarrow e^- W^+) Br(W^+ \rightarrow \mu^+ \nu_\mu)\) where \(Br(W^+ \rightarrow \mu^+ \nu_\mu)\) is the branching ratio of \(W^+ \rightarrow \mu^+ \nu_\mu\) decay.

![Graph](image_url)

**FIG. 4:** Decay rate of \(N \rightarrow e^- \mu^+ \nu_\mu\) versus \(m_N\) with \(|R_{eN}|^2 = 1\). Line A: calculated using (21); Line B: calculated using \(\Gamma = G_F^2 m_N^5 / (192\pi^3)\) in (36) up to \(m_N < m_W\); Line C: calculated using \(\Gamma = \Gamma(N \rightarrow e^- W^+) Br(W^+ \rightarrow \mu^+ \nu_\mu)\) with \(m_N > m_W\).

For more general value of \(m_N\), in particular for \(m_N \approx m_{W,Z,H}\), the above formulas are not good approximations. Decay rate in more general cases can be calculated by carefully including the propagators of \(W, Z\) and \(H\) bosons into calculation. Four-momentum of the mediated boson can be on-shell for general cases. We take this fact into account and calculate the tree-level decay rate of \(N\) decays with three fermions in final state. In Appendix, we present in detail our result of calculation. One can see that for most cases the decay rate
can be obtained as an analytic function of $m_N$ and the masses and widths of bosons. The most complicated case appears for $N \rightarrow l^-\nu_l$ and $N \rightarrow l^-\bar{\nu}_l$ channels for which $W$ and $Z$ bosons can all mediate. For this particular process, a function $F_S$, shown in (23) and (33), appears which cannot be obtained as an explicit analytic function of $m_N$ and the boson masses. In our analysis we compute $F_S$ numerically.

As an example, we compare in Fig. 4 the result computed using analytic formula (21) with known results in low energy region $m_N \ll m_W$ and in high energy limit $m_N > m_W$. $\Gamma(N \rightarrow e^-W^+)$ is calculated using (10). $Br(W^+ \rightarrow \mu^+\nu_\mu)$ is taken as $Br = 0.108$[29]. We can see that in the low energy limit the decay rate agrees with the expected result of tree-level three-body decay. In the high energy limit it agrees with the expectation that it is dominated by the on-shell $N \rightarrow e^-W^+$ decay with a subsequent $W^+ \rightarrow \mu^+\nu_\mu$ decay. In region around $m_W$, (21) gives a smooth transition from low energy behavior to high energy behavior. As a comparison, the result calculated using the two body decay $N \rightarrow e^-W^+$ drops down to zero as $m_N$ approaches $m_W$ from above and is certainly not correct at around threshold. The result given by (21) takes into account the contribution of off-shell boson and removes the ill-behavior at around $m_N \sim m_W$. The plot demonstrates that results presented in Appendix are better to use for studying the signals of sterile neutrino. Tree-level three-body decay rates for general mass $m_N$, presented in appendix, are some of the new results of the present article.

**IV. SIGNAL OF A HEAVY STERILE NEUTRINO AND BACKGROUND**

In this section, we study the process

$$e^+e^- \rightarrow N\nu, N\bar{\nu} \rightarrow ljj\bar{E},$$

the signal of sterile neutrino $N$ due to this process and the associated background.

We simulate the signal and background events with MadGraph [30], and have done the showing and hadronization by using Pythia6 [31]. The results are passed through PGS4 [32] for fast detector simulation.

At CEPC with $\sqrt{s} = 240$ GeV, we adopt the basic cuts (BC) for lepton and jets to trigger the events,

$$p_T^l > 10\text{GeV}, |\eta^l| < 2.5, \Delta R_{ll} > 0.4,$$

$$p_T^j > 10\text{GeV}, |\eta^j| < 2.5, \Delta R_{jj} > 0.4, \Delta R_{lj} > 0.4.$$  

The main backgrounds for the process (13) are $W$ pair production, $e^+e^- \rightarrow W^+W^-$, with one $W$ decaying leptonically and the other $W$ decaying hadronically, and single $W$
production, which decays leptonically. In order to suppress the backgrounds, we set the selection cuts (SC) \[21, 35\],

\[ |M(l, \not{E}) - m_W| > 20 \text{ GeV}, \quad |M(j_1, j_2) - m_W| > 20 \text{ GeV}, \quad (16) \]

and

\[ |M(l, j_1, j_2) - m_N| < 20 \text{ or } 10 \text{ GeV}. \quad (17) \]

Cut (16) is used to exclude background events coming from the decay of on-shell W boson in the background processes. Cut (17) selects events coming from the decay of on-shell N and is used to increase the significance of signal to background ratio.

In Table I we show the efficiency of the cuts for both \( l = e \) and \( l = \mu \) channels. After adding the SC, the signals are survived, but the backgrounds drop several order of magnitude.

| parameters | +cuts (a) | +cuts (b) | +cuts (c) | +cuts (d) | +cuts (e) | significance |
|------------|----------|----------|----------|----------|----------|-------------|
| A \( m_N = 150 \text{GeV}, \quad R_{\mu N} = 0.1 \) | 2.14 | 2.04 | 1.56 | 1.56 | 1.55 | 11.2 |
| \quad | 2.31 \times 10^3 | 2.20 \times 10^3 | 52.4 | 16.3 | 8.05 |
| B \( m_N = 150 \text{GeV}, \quad R_{e N} = 0.02 \) | 7.63 | 7.30 | 5.61 | 5.60 | 5.60 | 18.8 |
| \quad | 2.52 \times 10^3 | 2.37 \times 10^3 | 0.195 \times 10^3 | 76.6 | 38.8 |
| C \( m_N = 90 \text{GeV}, \quad R_{e N} = 0.015 \) | 10.8 | 4.98 | 1.56 | 1.55 | 1.55 | 13.4 |
| \quad | 2.52 \times 10^3 | 2.37 \times 10^3 | 0.195 \times 10^3 | 16.8 | 5.14 |
| D \( m_N = 214 \text{GeV}, \quad R_{e N} = 0.015 \) | 0.852 | 0.827 | 0.243 | 0.242 | 0.241 | 1.75 |
| \quad | 2.52 \times 10^3 | 2.37 \times 10^3 | 0.195 \times 10^3 | 24.9 | 9.26 |

We define the significance \( s \) as

\[ s = \frac{N_s}{\sqrt{N_s + N_b}}, \quad (18) \]

where \( N_s \) and \( N_b \) are the event number of signal and background respectively. In Fig. 5 we plot the significance \( s \) versus \( m_N \) for \( l = e \) with \( R_{e N} = 0.015 \) and \( l = \mu \) with \( R_{\mu N} = 0.1 \), respectively. For the integrated luminosity of 100 fb\(^{-1}\), a heavy neutral neutrino with mass in
the range of $90\text{GeV} \leq m_N \leq 146\text{GeV}$ for the mixing $R_{eN} = 0.015$ is promised to be discovered in $l = e$ channel, and $90\text{GeV} \leq m_N \leq 150\text{GeV}$ for the mixing $R_{\mu N} = 0.1$ is promised to be discovered in $l = \mu$ channel. For the integrated luminosity of $5\text{ab}^{-1}$, the maximal values of heavy neutrino mass can be 235GeV and 205GeV for $l = e$ and $l = \mu$ channel, respectively. One can see that there is a quick drop for heavy neutrino with mass $\lesssim 100\text{GeV}$ for both of $l = e$ and $l = \mu$. This is because for the decay of $N$ of a mass $\lesssim 100\text{ GeV}$, the lepton in $N \rightarrow lW \rightarrow lj\bar{j}$, a decay chain with an almost on-shell W, does not have enough energy and the $p_T$ of $l$ can not be large. This effect of cut on $p_T$ of $l$ can be seen in Table. [I]C. In Fig. 5, one can also see that there is a small peak for a heavy neutrino with a mass around 225GeV. This is because of the cut $|M(l, E) - m_W| > 20\text{GeV}$ to the signal as shown in Table. [I]D.

![Graph](image)

**Fig. 5:** The significance for $l = e$ (left) with $R_{eN} = 0.015$ and $l = \mu$ (right) with $R_{\mu N} = 0.1$. The curves in each plot from up to down correspond to the integrated luminosities $5\text{ab}^{-1}$, $1\text{ab}^{-1}$, $500\text{fb}^{-1}$ and $100\text{fb}^{-1}$.

In Fig. 6 we plot the potential of probing $R_{lN}$ for a fixed significance $s = 5$ with the integrated luminosities $5\text{ab}^{-1}$, $1\text{ab}^{-1}$, $500\text{fb}^{-1}$ and $100\text{fb}^{-1}$ at CEPC. Using SC $|M(l, j_1, j_2) - m_N| < 10\text{GeV}$, in $l = e$ channel, a heavy neutrino mass of 120GeV with $R_{eN} = 0.0080$ can be discovered for the integrated luminosities $100\text{fb}^{-1}$, and for $5\text{ab}^{-1}$, the mixing as low as $R_{eN} = 0.0030$ for the same mass can be probed. In $l = \mu$ channel, the heavy neutrino of the same mass with $R_{\mu N} = 0.043$ can be discovered for $100\text{fb}^{-1}$, and $R_{\mu N} = 0.016$ for $5\text{ab}^{-1}$. We can have similar results for SC $|M(l, j_1, j_2) - m_N| < 20\text{GeV}$, but the corresponding mixings are a little bigger.
FIG. 6: Sensitivity to $R_{lN}$ ($l = e, \mu$) with significance $s = 5$. The upper plots are for SC $|M(l,j_1,j_2) - m_N| < 10\text{GeV}$ and the lower plots are for SC $|M(l,j_1,j_2) - m_N| < 20\text{GeV}$. The curves in each plot from up to down correspond to integrated luminosities $100\text{fb}^{-1}$, $500\text{fb}^{-1}$, $1\text{ab}^{-1}$ and $5\text{ab}^{-1}$.

V. SIGNAL OF LOW ENERGY SEE-SAW MODEL

In this section we discuss the signature of low energy see-saw model with two heavy sterile neutrinos of mass around 100 GeV.

As discussed in previous section, in the case of large mixing of heavy sterile neutrinos with active neutrinos, not only the masses of these two sterile neutrinos are (quasi)degenerate but also the mixing has a simple relation $R_{lN_2} = \pm i R_{lN_1}$ as shown in (6) and (7). So the signature of the low energy see-saw model discussed here is just the double of the result presented for a single heavy sterile neutrino, except that we need to take into account the correlation of the mixing $R_{lN}$ for different $l$ in the low energy see-saw model, as shown in Fig. 1.

We calculate the signal of $e^+e^- \rightarrow \nu ljj$ events and the related background for $l = e, \mu, \tau$ separately. Then we calculate the significance of $e^+e^- \rightarrow \nu ljj$ events for $l = e, \mu, \tau$ sep-
arately. The total significance is defined as the square root of the sum of the squares of the significances of signals of $l = e$, $l = \mu$ and $l = \tau$, which we call $e + \mu + \tau$ significance. Similarly, we can define $e + \mu$ significance which include signals of $l = e$ and $l = \mu$. For simplicity, we assume 100% efficiency of the identification of \(\tau\) lepton. A realistic efficiency can be put into analysis without difficulty and would give rise to a result in-between the lines of $e + \mu + \tau$ significance and $e + \mu$ significance presented in figures below.

We plot the significance versus the mass of heavy neutrino in Fig. 7 for NH and IH with parameters given in the caption and with integrated luminosity 500 fb\(^{-1}\) as an illustration. In the case of NH, we choose to have $|R_{\mu N}|$ about 10 times larger than $|R_{e N}|$ for $\delta_{\text{CP}} = \pi/2$. $R_{\tau N}$ is of the same magnitude as $R_{\mu N}$, so the dominant decay channels are $\mu$ and $\tau$ channel which dominate the total significance in the figure. For $\delta_{\text{CP}} = -\pi/2$, $|R_{\mu N}|$ is of the same size of $|R_{\tau N}|$, but approximately 2 times larger than $|R_{e N}|$. Furthermore, the backgrounds for $\mu$ and $\tau$ channels are several times smaller than $e$ channel. So, the $\mu$ and $\tau$ channels are still dominant in $e + \mu + \tau$ significance.

As can be seen in Fig. 7, in the case of NH, a heavy sterile neutrino with a mass less than about 152 GeV (\(|R_{e N}| \sim 0.0032, |R_{\mu N}| \sim |R_{\tau N}| \sim 0.034\)) can be discovered for $\delta_{\text{CP}} = \pi/2$. For $\delta_{\text{CP}} = -\pi/2$, a heavy sterile neutrino with a mass less than around 206 GeV (\(|R_{e N}| \sim 0.015, |R_{\mu N}| \sim |R_{\tau N}| \sim 0.028\)) can be discovered. As can be seen in the above example, the case with $\delta_{\text{CP}} = -\pi/2$ has a larger $|R_{e N}|$. This larger value of $|R_{e N}|$ enhances the t-channel production process and gives rise to a larger production rate of heavy sterile neutrino. Meanwhile, the $\mu$ or $\tau$ channel decay of $N$ is still dominating over the $e$ channel, so the significance increases a lot from the case of $\delta_{\text{CP}} = \pi/2$ to the case of $\delta_{\text{CP}} = -\pi/2$.

In the case of IH, we choose to have similar magnitude of $|R_{e N}|$, $|R_{\mu N}|$ and $|R_{\tau N}|$ for both cases of $\delta_{\text{CP}} = \pi/2$ and $\delta_{\text{CP}} = -\pi/2$. All three $e$, $\mu$, and $\tau$ decay channels have comparable contributions to the total significance. For Dirac phase of both cases of $\delta_{\text{CP}} = \pi/2$ and $\delta_{\text{CP}} = -\pi/2$, the magnitude of $|R_{e N}|$ has the same size, and so does $|R_{e N}|^2 + |R_{\mu N}|^2 + |R_{\tau N}|^2$. This leads to the same production rate of $e^+ e^- \rightarrow \nu N$ and the same $ljj$ decays of $N$ for both cases of $\delta_{\text{CP}} = \pi/2$ and $\delta_{\text{CP}} = -\pi/2$. So, the total $e + \mu + \tau$ significances are the same for both cases of $\delta_{\text{CP}} = \pi/2$ and $\delta_{\text{CP}} = -\pi/2$. However, there is a difference between $e + \mu$ significances for these two cases. One can see in Fig. 7 that for IH a heavy neutrino with mass less than about 162 GeV can be discovered. The corresponding mixing parameters in the figure are $|R_{e N}| \sim 0.0086, |R_{\mu N}| \sim 0.0072, |R_{\tau N}| \sim 0.051$ for $\delta_{\text{CP}} = \pi/2$, and $|R_{e N}| \sim 0.0086, |R_{\mu N}| \sim 0.0053, |R_{\tau N}| \sim 0.0071$ for $\delta_{\text{CP}} = -\pi/2$.

In Fig. 8 we also plot the total significance as a function of the Majorana phase $\phi_2$ for a heavy neutrino mass of 150 GeV and integrated luminosity of 500 fb\(^{-1}\) for both NH and IH. The significance depends on both Dirac phase of $\delta_{\text{CP}}$ and Majorana phase $\phi_2$. In the case of NH with $\delta_{\text{CP}} = \pi/2$, there is a bump at $\phi_2 \sim 1.5\pi$ for $e + \mu$ significance. On the other hand, the bump is at around $2\pi$ for $e + \mu + \tau$ significance. This is because $|R_{e N}|^2/\sum |R_{IN}|^2$ increases as $\phi_2$ increases from 0 to $2\pi$, as can be seen in Fig. 9. Since $\sum |R_{IN}|^2$ is a constant
when varying $\phi_2$, as can be easily checked using (6) and (7), $|R_{eN}|^2$ increases as $\phi_2$ increases from 0 to $2\pi$ and peaks at $\phi_2 = 2\pi$. Consequently, the t-channel production process, the dominating production process, increases as $\phi_2$ increases from 0 to $2\pi$. This is why the plot of $e + \mu + \tau$ significance peaks at $\phi_2 = 2\pi$ in the case of NH with $\delta_{CP} = \pi/2$. For $e + \mu$ significance, it is dominated by the $\mu jj$ events, as explained before. As $\phi_2$ increases, $|R_{\mu N}|^2/\sum |R_{N N}|^2$ peaks at $\phi_2 \sim \pi$. For $\phi_2$ larger than around $\pi$, the branching fraction of $N \rightarrow \mu jj$ decay starts to decrease, which is compensated by the increase of the production cross section of $e^+e^- \rightarrow N\nu$. Then, the signature of $\mu jj$ events will increase first and then decrease as $\phi_2$ increases from $\pi$ to $2\pi$. This makes $e + \mu$ significance having a peak at a position less than $2\pi$, as can be seen in Fig. 8. Variation of significance in other cases can be similarly understood.

In Fig. 10, we present the significance as a function of heavy neutrino mass with integrated lumilosity 5ab$^{-1}$. In the case of NH, a heavy neutrino mass less than about 124GeV ($|R_{eN}| \sim 0.0012$, $|R_{\mu N}| \sim |R_{\tau N}| \sim 0.013$) for $\delta_{CP} = \pi/2$, and 184GeV ($|R_{eN}| \sim 0.0055$, $|R_{\mu N}| \sim |R_{\tau N}| \sim 0.010$) for $\delta_{CP} = -\pi/2$ can be discovered at CEPC. In the case of IH, a heavy neutrino mass less than about 130GeV can be discovered. The corresponding mixing parameters are $|R_{eN}| \sim 0.0034$, $|R_{\mu N}| \sim 0.028$, $|R_{\tau N}| \sim 0.020$ for $\delta_{CP} = \pi/2$, and $|R_{eN}| \sim 0.0034$, $|R_{\mu N}| \sim 0.021$, $|R_{\tau N}| \sim 0.028$ for $\delta_{CP} = \pi/2$.

To conclude, in the low energy see-saw model, due to the correlation of three different $R_{lN}$, sizable $|R_{eN}|$ leads to t-channel production of heavy sterile neutrino and can give rise to a quite large total production cross section of $e^+e^- \rightarrow N\nu$ process. The $N \rightarrow ljj$ events, on the other hand, can be dominated by $\mu jj$ and $\tau jj$ events because $|R_{\mu N}|^2 + |R_{\tau N}|^2$ can be much larger than $|R_{eN}|^2$ as can be seen in Fig. 1. For NH, in particular, $|R_{\mu N}|^2 + |R_{\tau N}|^2$ is always much larger than $|R_{eN}|^2$. In this case, $e + \mu$ significance and $e + \mu + \tau$ significance can be quite different, as can be seen in the left panel of Fig. 8. On the other hand, for IH, $|R_{\mu N}|^2 + |R_{\tau N}|^2$ can be of similar size of $|R_{eN}|^2$ and even much smaller than $|R_{eN}|^2$. In this case, $e + \mu$ significance and $e + \mu + \tau$ significance would not be very different. This is the case for the right panel in Fig. 8. So analyzing the dominating the $N$ decay channel and the difference of $e + \mu$ and $e + \mu + \tau$ significances can give hints on the mass hierarchy of neutrinos. In particular, if the dominating $N \rightarrow ljj$ events are $e jj$ events, it has to be IH.

VI. CONCLUSION

In summary, we have studied the production, decay and signature in $ljj$ events of heavy Majorana-type sterile neutrino of mass around 100 GeV at future CEPC. We study carefully the tree-level decay of heavy sterile neutrinos by carefully taking into account the propagator of bosons, such as $W$ and $Z$. Effects of on-shell and off-shell $W$ and $Z$ bosons are all taken into account by including the width of $W$ and $Z$ in the propagators. We obtain analytic
We choose $e^y = 5000$, $\delta_{CP} = \pm \pi/2$, $\phi_1 = \phi_2 = \phi_3 = 0$ for NH, and $e^y = 1000$, $\delta_{CP} = \pm \pi/2$, $\phi_1 = \phi_2 = \phi_3 = 0$ for IH.

For convenience and for later discussion in low energy see-saw model of heavy sterile neutrino, we have first studied the production of a single heavy sterile neutrino at CEPC and its signature. Although the mixing of a single heavy sterile neutrino with active neutrino is strongly constrained by the $0\nu\beta\beta$ experiment, the study of the signature of a single heavy
\[ NH \delta_{\text{CP}} = \pi/2 \]

\[ se + \mu \times |R_{\mu N^2}|/\Sigma |R_{lN^2}| \]

\[ 20 \times |R_{eN^2}|/\Sigma |R_{lN^2}| \]

\[ 20 \times |R_{\mu N^2}|/\Sigma |R_{lN^2}| \]

\[ \phi^2 (\pi) \]

FIG. 9: The significance \( s \) vs \( \phi_2 \) for NH with \( \delta_{\text{CP}} = \pi/2 \) with \( e^y = 5000 \), \( \phi_1 = \phi_3 = 0 \) and integrated luminosity 500fb\(^{-1}\).

FIG. 10: The significance \( s \) vs \( m_N \) for NH (left) and IH (right) with integrated luminosity 5ab\(^{-1}\).

We choose \( e^y = 1750 \), \( \delta_{\text{CP}} = \pm \pi/2 \), \( \phi_1 = \phi_2 = \phi_3 = 0 \) for NH, and \( e^y = 350 \), \( \delta_{\text{CP}} = \pm \pi/2 \), \( \phi_1 = \phi_2 = \phi_3 = 0 \) for IH.

The sterile neutrino is also of interests for itself, since some other particles or mechanism, e.g. extra scalars or type-II seesaw, may exist to ease the constraint. We have shown that for a single heavy sterile neutrino, an electron positron collider such as CEPC is more sensitive to the mixing of heavy sterile neutrino with electron (anti)neutrino, than the mixing with muon or tau (anti)neutrino. For the former, the production of \( N \) is associated with the production of an electron neutrino or anti-neutrino and can go through t-channel. The cross section of the t-channel process can be two orders of magnitude larger than the cross section of the s-channel process which is responsible for probing the magnitude of the mixing with muon or tau (anti)neutrino. We found that for an integrated luminosity 5 ab\(^{-1}\), CEPC can reach
a $5\sigma$ sensitivity of $R_{eN}$, the mixing of the sterile neutrino with active neutrino, to a value as small as $|R_{eN}| = 10^{-3}$. For the mixing with muon and tau (anti)neutrino $R_{\mu N}$ and $R_{\tau N}$, the $5\sigma$ sensitivity can reach $|R_{\mu N,\tau N}| \approx 10^{-2}$.

We also study the production of heavy sterile neutrinos in a low energy see-saw model and their signature at CEPC. In this model, two heavy sterile neutrinos exist so that an explanation of the masses and mixings of active neutrinos is available using see-saw mechanism. In this model, the mixings of these two heavy sterile neutrinos with active neutrinos, $R_{lN_1}$ and $R_{lN_2}$, are forced to have the same magnitude for all $l$, if we want these mixings to be large. In this case, the masses of these two sterile neutrinos are found to be degenerate or quasi-degenerate if considering into account the constraint from $0\nu\beta\beta$ experiment.

So the signature of these two heavy sterile neutrinos are just the double of the signature of a single heavy sterile neutrino discussed above. The major difference compared with the case of a single heavy sterile neutrino is that the mixing $R_{lN_1}$ is no longer arbitrary for different $l$. Instead, values of $R_{lN_1}$ for different $l$ have some correlations. We take these facts into account. We find that the Dirac CP phase $\delta_{CP}$ in the PMNS mixing matrix of active neutrinos and Majorana phases affect the mixing $R_{lN_1}$, and change the relative significance of $e jj$, $\mu jj$ and $\tau jj$ events. So a search for all 3 lepton channels are helpful to constrain the model. With sizable $R_{eN}$, the significance of both $\mu$ and $\tau$ channel will be enhanced, and further constrain $R_{\mu N}$ and $R_{\tau N}$ compared with the case with only a single mixing.

We further note that although our analysis is for CEPC running at 240 GeV, it can also be applied to ILC running at around 250 GeV without much modification [37].

**Acknowledgments**

This research is supported in part by the Natural Science Foundation of China(NSFC), Grant No. 11135009 and No. 11375065, and in part by the Shanghai Key Laboratory of Particle Physics and Cosmology, Grant No. 15DZ2272100. XHW would like to thank Qi-Shu Yan for helpful discussions on MadGraph.

**VII. APPENDIX**

In this section we summarize the tree level decay rate of sterile neutrino decaying to three final fermions through interaction with Z and W bosons induced by mixing with active neutrinos. Effects of on-shell and off-shell Z and W bosons are all taken into account by including the width of W and Z in the propagators. For example, for $N \rightarrow l_1^- l_2^+ \nu_2$ and
\( l_1 \neq l_2 \), the decay rate is obtained as follows

\[
\Gamma(N \rightarrow l_1^+ l_2^- \nu_{l_3}) = |R_{t1,n}|^2 \frac{G_F^2 m_N}{\pi^3} \int_0^{\frac{m_N}{E_1}} dE_1 \int_0^{\frac{m_N}{E_2}} dE_2 \ |X_W|^2 \frac{1}{2} (m_N - 2E_2)E_2, \tag{19}
\]

where \( X_W \) comes from the propagator of \( W \) boson and is

\[
X_W = \frac{m_W^2}{q^2 - m_W^2 + i\Gamma_W m_W}, \tag{20}
\]

where \( q^2 = m_N^2 - 2m_N E_1 \) and \( \Gamma_W \) is the total decay rate of \( W \). \( q = p - p_1 \) is the four momentum of the \( W \) boson where \( p \) and \( p_1 \) are the four momenta of \( N \) and \( l_1 \) respectively. So \( q^2 = m_N^2 - 2m_N E_1 \) when considering the decay of \( N \) at rest and neglecting the mass of \( l_1 \) with \( E_1 \) the energy of \( l_1 \). After performing integration in (19) we can get a formula for the decay rate as a function of \( m_N, m_W \) and \( \Gamma_W \). Similarly we can get formula for other decays through \( Z \) boson exchange.

In the following we summarize the results

1) For \( N \rightarrow l_1^{-} l_2^{+} \nu_{l_3}, N \rightarrow l_1^{+} l_2^{-} \bar{\nu}_{l_2} \) and \( l_1 \neq l_2 \)

\[
\Gamma(N \rightarrow l_1^{-} l_2^{+} \nu_{l_3}) = \Gamma(N \rightarrow l_1^{+} l_2^{-} \bar{\nu}_{l_2}) = |R_{t1,n}|^2 \frac{G_F^2 m_N^5}{\pi^3} F_N(m_N, m_W, \Gamma_W), \tag{21}
\]

where \( F_N \) is a dimensionless function and is given in (31) below.

2) For \( N \rightarrow l^{-} q_1 \bar{q}_2, N \rightarrow l^{+} \bar{q}_1 q_2 \)

\[
\Gamma(N \rightarrow l^{-} q_1 \bar{q}_2) = \Gamma(N \rightarrow l^{+} \bar{q}_1 q_2) = |R_{t1,n}|^2 \frac{G_F^2 m_N^5}{\pi^3} N_C F_N(m_N, m_W, \Gamma_W)|K_{q_1 q_2}|^2. \tag{22}
\]

\( K_{q_1 q_2} \) is the CKM matrix element in \((q_1, q_2)\) entry, \( N_C = 3 \) the number of color degrees of freedom of quarks.

3) For \( N \rightarrow l^{-} l^{+} \nu_{l}, N \rightarrow l^{+} l^{-} \bar{\nu}_{l} \)

\[
\Gamma(N \rightarrow l^{-} l^{+} \nu_{l}) = \Gamma(N \rightarrow l^{+} l^{-} \bar{\nu}_{l}) = |R_{t1,n}|^2 \frac{G_F^2 m_N^5}{\pi^3} \left[ F_N(m_N, m_W, \Gamma_W) + (C_L^2 + C_R^2) F_N(m_N, m_Z, \Gamma_Z) \right. \\
\left. + 2C_L F_S(m_N, m_W, \Gamma_W, m_Z, \Gamma_Z) \right], \tag{23}
\]

where \( C_{L,R} \) is given in (28), \( F_S \) is a dimensionless function and is given below in (33).

4) For \( N \rightarrow \nu_l l^l' \) and \( N \rightarrow \nu_l l^l' \)

\[
\Gamma(N \rightarrow \nu_l l' l') = \Gamma(N \rightarrow \nu_l l' l') = |R_{t1,n}|^2 \frac{G_F^2 m_N^5}{\pi^3} \left( C_L^2 + C_R^2 \right) F_N(m_N, m_Z, \Gamma_Z), \tag{24}
\]

5) For \( N \rightarrow \nu_l q \bar{q} \) and \( N \rightarrow \nu_l q \bar{q} \)

\[
\Gamma(N \rightarrow \nu_l l' l') = \Gamma(N \rightarrow \nu_l l' l') = |R_{t1,n}|^2 \frac{G_F^2 m_N^5}{\pi^3} N_C [(C_L^q)^2 + (C_R^q)^2] F_N(m_N, m_Z, \Gamma_Z), \tag{25}
\]
where \( q = u, d, c, s, b \) for \( m_N < 2m_l \) and \( C^d_{L,R} \) is given in (29) and (30).

6) For \( N \to \nu_l \nu_l \bar{\nu}_l \) and \( N \to \bar{\nu}_l \nu_l \nu_l \), \( l \neq l' \)

\[
\Gamma(N \to \nu_l \nu_l \bar{\nu}_l) = \Gamma(N \to \bar{\nu}_l \nu_l \nu_l) = \left| R_{lN} \right|^2 \frac{C^2_{\nu} m^5_N}{\pi^3} C^2_{\nu} F_N(m_N, m_Z, \Gamma_Z), \tag{26}
\]

where \( C_{\nu} = 1/2 \).

7) For \( N \to \nu_l \nu_l \bar{\nu}_l \) and \( N \to \bar{\nu}_l \nu_l \nu_l \)

\[
\Gamma(N \to \nu_l \nu_l \bar{\nu}_l) = \Gamma(N \to \bar{\nu}_l \nu_l \nu_l) = \left| R_{lN} \right|^2 \frac{C^2_{\nu} m^5_N}{\pi^3} 4C^2_{\nu} F_N(m_N, m_Z, \Gamma_Z). \tag{27}
\]

Couplings \( C_L, C_R \) etc. which appear in expressions above, are given as

\[
C_L = -\frac{1}{2} + \sin^2 \theta_W, \quad C_R = \sin^2 \theta_W, \tag{28}
\]

\[
C^u_L = \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W, \quad C^u_R = -\frac{2}{3} \sin^2 \theta_W, \tag{29}
\]

\[
C^w_L = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W, \quad C^w_R = \frac{1}{3} \sin^2 \theta_W. \tag{30}
\]

For mass \( m_N, m_X \) and decay rate \( \Gamma_X \), the function \( F_N \) used above is

\[
F_N(m_N, m_X, \Gamma_X) = \frac{m_X^4}{96m_N^8} \left\{ -2m_N^2(m_N^2 - m_X^2) + (A_X + C_X \Gamma^2_X m^2_X) \frac{1}{\Gamma_X m_X} \left[ \arctan \left( \frac{m_N^2 - m_X^2}{\Gamma_X m_X} \right) - \arctan \left( \frac{-m_X^2}{\Gamma_X m_X} \right) \right] - \frac{1}{2} (B_X + 2\Gamma^2_X m^2_X) \ln \left( \frac{\Gamma^2_X m^2_X + (m_N^2 - m_X^2)^2}{\Gamma^2_X m^2_X + m_X^4} \right) \right\} \tag{31}
\]

where

\[
A_X = (m_N^2 - m_X^2)^2(m_N^2 + 2m_X^2), \quad B_X = 6(m_N^2 - m_X^2)m_X^2, \quad C_X = 3(m_N^2 - 2m_X^2). \tag{32}
\]

Function \( F_S \) in (23) is given as

\[
F_S = \frac{1}{m_N} \int_0^{m_N} dE_1 \int_{\frac{m_N}{2} - E_1}^{m_N} dE_2 (X_W X^*_Z + X^*_W X_Z) \frac{1}{2} (m_N - 2E_2) E_2, \tag{33}
\]

where

\[
X_Z = m_Z^2 \frac{m^2_Z}{q_3^2 - m^2_Z + i\Gamma_Z m_Z}. \tag{34}
\]

\( q_3^2 = m_N^2 - 2m_N E_3 \) with \( E_3 = m_N - E_1 - E_2 \) when considering the decay of \( N \) at rest and neglecting the mass of final fermions. (33) can not be obtained as an explicit function of \( m_N, m_W \) and \( m_Z \). We can calculate this function numerically.
For $N \rightarrow \nu H$ decay, the effect described here can be similarly obtained with the introduction of a function $F_N(m_N, m_H, \Gamma_H)$. For example, for $N \rightarrow \nu f \bar{f}$ and $N \rightarrow \bar{\nu} f \bar{f}$

$$
\Gamma(N \rightarrow \nu f \bar{f}) = \Gamma(N \rightarrow \bar{\nu} f \bar{f}) = \frac{g^2 m_N^7 |R_{1N}|^2 y_f^2}{16\pi^3 m_W^2 m_H^4} N_f F_N(m_N, m_H, \Gamma_H),
$$

(35)

where $y_f$ is the Yukawa coupling of fermion $f$, $N_f = 1$ for $f$ being a lepton and $N_f = 3$ for $f$ being a quark. Interference of $N$ decay through $Z$ boson and $H$ boson vanishes. Since the Yukawa coupling to fermion $f$ is always small for $f = b, c, s, d, u$ and leptons, inclusion of $N$ decay through the neutral Higgs boson does not change significantly the signature of sterile neutrino $N$ discussed in this article, as long as we are not going to concentrate on the signature of $N$ coming from $N \rightarrow \nu \bar{b}b$ and $N \rightarrow \bar{\nu} \bar{b}b$ decay.

In low energy limit $m_N^2 \ll m_W^2$, we have $|X_W| \approx |X_Z| \approx 1$, the above equations of decay rate, (21), (22), (23), (24), (25), (26), (27), can be simplified to be as follows.

1) For $N \rightarrow l_1 l_2^\pm \nu_{l_2}$, $N \rightarrow l_1^+ l_2^- \bar{\nu}_{l_2}$ and $l_1 \neq l_2$

$$
\Gamma(N \rightarrow l_1^- l_2^+ \nu_{l_2}) = \Gamma(N \rightarrow l_1^+ l_2^- \bar{\nu}_{l_2}) = |R_{1N}|^2 \frac{G_F^2 m_N^5}{192 \pi^3},
$$

(36)

2) For $N \rightarrow l^- q_1 \bar{q}_2$, $N \rightarrow l^+ \bar{q}_1 q_2$

$$
\Gamma(N \rightarrow l^- q_1 \bar{q}_2) = \Gamma(N \rightarrow l^+ \bar{q}_1 q_2) = |R_{1N}|^2 \frac{G_F^2 m_N^5}{192 \pi^3} N_C |K_{q_1 q_2}|^2.
$$

(37)

3) For $N \rightarrow l^- l^+ \nu_l$, $N \rightarrow l^+ l^- \bar{\nu}_l$

$$
\Gamma(N \rightarrow l^- l^+ \nu_l) = \Gamma(N \rightarrow l^+ l^- \bar{\nu}_l) = |R_{1N}|^2 \frac{G_F^2 m_N^5}{192 \pi^3} [(1 + C_L)^2 + C_R^2],
$$

(38)

4) For $N \rightarrow \nu_l \bar{\nu}_l'$ and $N \rightarrow \bar{\nu}_l \nu_l'$

$$
\Gamma(N \rightarrow \nu_l \bar{\nu}_l') = \Gamma(N \rightarrow \bar{\nu}_l \nu_l') = |R_{1N}|^2 \frac{G_F^2 m_N^5}{192 \pi^3} (C_L^2 + C_R^2).
$$

(39)

5) For $N \rightarrow \nu q \bar{q}$ and $N \rightarrow \bar{\nu} q q$

$$
\Gamma(N \rightarrow \nu q \bar{q}) = \Gamma(N \rightarrow \bar{\nu} q q) = |R_{1N}|^2 \frac{G_F^2 m_N^5}{192 \pi^3} N_C [(C_L^q)^2 + (C_R^q)^2].
$$

(40)

6) For $N \rightarrow \nu_l \nu_l \bar{\nu}_l$ and $N \rightarrow \bar{\nu}_l \nu_l \nu_l$, $l \neq l'$

$$
\Gamma(N \rightarrow \nu_l \nu_l \bar{\nu}_l) = \Gamma(N \rightarrow \bar{\nu}_l \nu_l \nu_l) = |R_{1N}|^2 \frac{G_F^2 m_N^5}{192 \pi^3} C^2_{\nu},
$$

(41)

7) For $N \rightarrow \nu_l \nu_l$ and $N \rightarrow \bar{\nu}_l \nu_l$

$$
\Gamma(N \rightarrow \nu_l \nu_l) = \Gamma(N \rightarrow \bar{\nu}_l \nu_l) = |R_{1N}|^2 \frac{G_F^2 m_N^5}{192 \pi^3} 4 C^2_{\nu}.
$$

(42)
In all these results, the masses of the final fermions have all been neglected.

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