Mutual or Unrequited Love: Identifying Stable Clusters in Social Networks with Uni- and Bi-directional Links

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Abstract. Many social networks, e.g., Slashdot and Twitter, can be represented as directed graphs (digraphs) with two types of links between entities: mutual (bi-directional) and one-way (uni-directional) connections. Social science theories reveal that mutual connections are more stable than one-way connections, and one-way connections exhibit various tendencies to become mutual connections. It is therefore important to take such tendencies into account when performing clustering of social networks with both mutual and one-way connections.

In this paper, we utilize the dyadic methods to analyze social networks, and develop a generalized mutuality tendency theory to capture the tendencies of those node pairs which tend to establish mutual connections more frequently than those occur by chance. Using these results, we develop a mutuality-tendency-aware spectral clustering algorithm to identify more stable clusters by maximizing the within-cluster mutuality tendency and minimizing the cross-cluster mutuality tendency. Extensive simulation results on synthetic datasets as well as real online social network datasets such as Slashdot, demonstrate that our proposed mutuality-tendency-aware spectral clustering algorithm extracts more stable social community structures than traditional spectral clustering methods.

1 Introduction

Graph models are widely utilized to represent relations among entities in social networks. Especially, many online social networks, e.g., Slashdot and Twitter, where the users’ social relationships are represented as directed edges in directed graphs (or in short, digraphs). Entity connections in a digraph can be categorized into two types, namely, bi-directional links (mutual connections) and uni-directional links (one-way connections). Social theories [28] and online social network analysis [2, 6, 28] have revealed that various types of connections exhibit different stabilities, where mutual connections are more stable than one-way connections. In other words, mutual connections are the source of social cohesion [3,4] that, if two individuals mutually attend to one another, then the bond is reinforced in each direction.

Studying the social network structure and properties of social ties have been an active area of research. Clustering and identifying social structures in social networks is an especially important problem [8,17,24] that has wide applications, for instance, community detection and friend recommendation in social networks. Existing clustering...
methods [21,29] are originally developed for undirected graphs, based on the classical spectral clustering theory. Several recent studies (see, e.g., [10,21,27,29]) extend the spectral clustering method to digraphs, by first converting the underlying digraphs to undirected graphs via some form of symmetrization, and then apply spectral clustering to the resulting symmetrized (undirected) graphs. However, all these methods have two common drawbacks, which prevent them from obtaining stable clusters with more mutual connections. First, these methods do not explicitly distinguish between mutual and one-way connections commonly occurring in many social networks, treating them essentially as the same and therefore ignoring the different social relations and interpretations these two types of connections represent (see Section 2 for more details). Second, by simply minimizing the total cross-cluster links (that are symmetrized in some fashion), these methods do not explicitly account for the potential tendencies of node pairs to become mutually connected. As a simple example, Fig. 1 shows two groups of people in a network, where people in the same group tend to have more mutual (stable) connections, and people across two groups have more one-way (unstable) connections. When using the traditional spectral clustering method, as shown in Fig. 1(a), group B will be partitioned into two clusters, due to its strict rule of minimizing the total number of across cluster edges. On the other hand, the correct partition should be done as shown in Fig. 1(b), where the majority of mutual (stable) connections are placed within clusters, and one-way (unstable) connections are placed across clusters.

In this paper, we propose and develop a stable social cluster detection algorithm that takes into account the tendencies of node pairs whether to form mutual (thus stable) connections or not, which can result in more stable cluster structures. To tackle this clustering problem, we need to answer the following questions: 1) how to track and evaluate the tendencies of node pairs to become mutual (stable) relations? and 2) how to cluster the entities in social networks by accounting for their mutuality tendencies so as to extract more stable clustering structures?

To address these questions, we utilize dyadic methods to analyze social networks, and develop a generalized mutuality tendency theory which better captures the tendencies of node pairs that tend to establish mutual connections more frequently than those occur by chance. Using these results, we develop a mutuality-tendency-aware spectral clustering algorithm to detect more stable clusters by maximizing the within-cluster mutuality tendency and minimizing the cross-cluster mutuality tendency. Our contributions are summarized as follows.