ON THE LIMITATIONS OF NEUTRINO EMISSIVITY FORMULA OF IWAMOTO

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Abstract

The neutrino emissivity from two and three flavour quark matter is numerically calculated and compared with Iwamoto’s formula. We find that the calculated emissivity is smaller than Iwamoto’s result by orders of magnitude when \(p_f(u) + p_f(e) - p_f(d(s))\) is comparable with the temperature. We attribute it to the severe restriction imposed by momentum conservation on the phase space integral. We obtain an alternate formula for the neutrino emissivity which is valid when the quarks and electrons are degenerate and \(p_f(u) + p_f(e) - p_f(d(s))\) is large compared to the temperature.
It has been conjectured that dense stars may consist of quark matter or quark matter core with neutron matter outside [1-5]. Although theoretical understanding of the properties of quark matter is not yet available, various quark models have been used to calculate the equation of state of the quark matter and determine the properties of quark stars [6-9]. Unfortunately, it is found that the properties of quark stars, such as surface gravitational redshift z, moment of inertia I, maximum mass M, radius R and pulsar periods P, are not significantly different compared to those of neutron stars. Therefore it is difficult to distinguish one from the other observationally.

On the other hand Iwamoto [10,11] has proposed that neutrino emissivity ($\epsilon$) could play a significant role in distinguishing between quark and neutron stars because it differs by orders of magnitude for the two. Particularly $\epsilon$ for quark stars is larger by 6-7 orders of magnitude than neutron stars which could lead to faster cooling rate for quark stars, thus reducing their surface temperature. There are, however, a number of other mechanisms [12,13] and modified URCA processes [14] proposed to increase $\epsilon$ for neutron matter.

Iwamoto [10] has derived the formula for $\epsilon$ using apparently reasonable approximations and this formula has been widely used [8,15-17] to calculate $\epsilon$ for two and three flavour quark matter. According to his formula $\epsilon$ is proportional to baryon density ($n_B$), strong coupling constant ($\alpha_c$) and sixth power of temperature (T) for d quark decay. For s quark decay T dependence of $\epsilon$ is same as that for d decay. Furthermore his results imply that electron and quark masses have negligible effect on $\epsilon$ and s quark decay (in case of three flavour quark matter) plays a rather insignificant role.

In the present paper we want to report an exact numerical calculation of $\epsilon$ and a comparison of our results with the Iwamoto formula. Our results show that the Iwamoto formula
overestimates $\epsilon$ by orders of magnitude when $p_f(u) + p_f(e) - p_f(d(s))$ is comparable with the temperature. For reasonable values of $\alpha_c$ and baryon densities, this quantity is much larger than the expected temperatures of neutron stars ($\sim$ few 10ths of MeV) for two flavour quark matter, but is comparable with temperature for three flavour quark matter.

The neutrinos are emitted from the quark matter through reactions

\begin{align*}
    d &\to u + e^- + \bar{\nu}_e \\
    u + e^- &\to d + \nu_e \\
    s &\to u + e^- + \bar{\nu}_e \\
    u + e^- &\to s + \nu_e
\end{align*}

(1)

The equilibrium constitution of the quark matter is determined by its baryon density ($n_B$), charge neutrality conditions and weak interactions given in Eq.(1). Thus, for two flavour quark matter,

\begin{align*}
    \mu_d &= \mu_u + \mu_e (\mu_{\nu_e} = \mu_{\bar{\nu}_e} = 0) \\
    2n_u - n_d - 3n_e &= 0 \\
    n_B &= (n_u + n_d)/3
\end{align*}

(2)

and for three flavour quark matter

\begin{align*}
    \mu_d &= \mu_u + \mu_e (\mu_{\nu_e} = \mu_{\bar{\nu}_e} = 0) \\
    \mu_d &= \mu_s \\
    2n_u - n_d - n_s - 3n_e &= 0 \\
    n_B &= (n_u + n_d + n_s)/3.
\end{align*}

(3)
The number density of species $i$ is $n_i = g_i p_i^3(i)/(6\pi^2)$ with the degeneracy factor $g_i$ being two for electron and six for quarks. For electrons $\mu_e = \sqrt{p_f^2(e) + m_e^2}$ and for quarks we use [18]

$$\mu_q = \frac{\eta}{x} + \frac{8\alpha_e}{3\pi} \left(1 - \frac{3}{x\eta} \ln(x + \eta)\right) p_f$$  \hspace{1cm} (4)

where $x \equiv p_f(q)/m_q$ and $\eta \equiv \sqrt{1 + x^2}$, $m_q$ being the quark mass. For massless quarks Eq(4) reduces to

$$\mu_q = (1 + \frac{8\alpha_c}{3\pi}) p_f(q)$$  \hspace{1cm} (5)

The neutrino emissivity $\epsilon$ for reactions involving $d(s)$ quarks is calculated by using the reactions in Eq.(1). In terms of the reaction rates of these equations, we get,

$$\epsilon_{d(s)} = A_{d(s)} \int d^3 p_{d(s)} d^3 p_u d^3 p_e d^3 p_\nu \frac{(p_{d(s)} \cdot p_\nu)(p_u \cdot p_e)}{E_u E_{d(s)} E_e}$$

$$\times \delta^4(p_{d(s)} - p_u - p_e - p_\nu) n(p_{d(s)})[1 - n(p_u)][1 - n(p_e)]$$  \hspace{1cm} (6)

where $p_i = (E_i, \vec{p}_i)$ are the four momenta of the particles, $n(p_i) = \frac{1}{e^{(E_i - \mu_i)/T} + 1}$ are the Fermi distribution functions and

$$A_d = \frac{24G^2 \cos^2 \theta_e}{(2\pi)^8}$$  \hspace{1cm} (7)

$$A_s = \frac{24G^2 \sin^2 \theta_e}{(2\pi)^8}$$  \hspace{1cm} (8)

where $G$ is weak coupling constant and $\theta_e$ is Cabibbo angle. For degenerate particles, $(\beta p_f(i) \gg 1)$, Iwamoto has evaluated the integrals in Eq.(5) using certain reasonable approximations and obtained the simple expressions for $\epsilon_d$ and $\epsilon_s$ as given below [11].

$$\epsilon_d = \frac{914}{315} G^2 \cos^2 \theta_e \alpha_c p_f(d) p_f(u) p_f(e) T^6$$

$$\epsilon_s = \frac{457\pi}{840} G^2 \sin^2 \theta_e \alpha_c \mu_s p_f(u) p_f(e) T^6$$  \hspace{1cm} (9)
The approximations involved in obtaining these formulas are

1. neglect of neutrino momentum in momentum conservation,
2. replacing the matrix elements by some angle averaged value, and
3. decoupling momentum and angle integrals.

The expressions for neutrino emissivity as obtained by Iwamoto have been used widely. The temperature dependence of emissivity as obtained by Iwamoto has a physical explanation. Each degenerate fermion gives one power of $T$ from the phase space integral ($\int d^3p_i \rightarrow p_f(i)^2 dE_i \propto T$). Thus one gets $T^3$ from quarks and electrons. Phase space integral for the neutrino gives $d^3p_\nu \propto (E_\nu^2) dE_\nu \propto T^3$. Energy conserving $\delta-$function gives one $T^{-1}$ which is cancelled by one $E_\nu \propto T$ factor coming from matrix element. So finally one gets $\epsilon \propto T^6$.

This argument, however, ignores the fact that $\Delta p_d \Delta p_s = p_f(u) + p_f(e) - p_f(d) - p_f(s)$, which is related to the angle between $\vec{p}_d$, $\vec{p}_u$ and $\vec{p}_e$ could be small and comparable to $T$.

We shall demonstrate below that precisely in this region that the Iwamoto formula fails.

Before discussing the causes of the failure of Iwamoto formula, let us first compare our results with the Iwamoto formula and try to find out the specific cases where the deviation is more pronounced. In Figs.1-3 we have plotted $\epsilon$ vs $T$ for 2-flavour d decay, 3-flavour d decay and s decay respectively. For 2-flavour d decay our results ($\epsilon_d$) are in good agreement with the emissivity calculated using Iwamoto formula ($\epsilon_{d(s)}^I$). In Fig.1 curves (a) and (b) are $\epsilon_d$ and $\epsilon_d^I$ respectively, for $\alpha_c = 0.1$ and $n_B = 0.4$. (c) and (d) corresponds to the same but for $\alpha_c = 0.1$ and $n_B = 1.4$. It is evident from the figure that agreement of Iwamoto
results with our calculation is better for higher densities and lower temperatures. Also $\epsilon_d$ is consistently smaller than $\epsilon_{dI}$, the Iwamoto result, in the range of temperatures considered. Corresponding fermi momenta of quarks and electrons are given in Table 1. It is to be noted that all the momenta are much larger than the temperature.

Fig. 2 shows the $\epsilon_d$ for 3-flavour quark matter. It shows that $\epsilon_{dI}$ is 2-3 orders of magnitude higher compared to our results. Here, contrary to the two flavour case, the difference becomes more pronounced at higher densities. Fig. 3 shows the variation of $\epsilon_s$ with temperature. Here again it is clear that $\epsilon_s$ is quite different from $\epsilon_{sI}$ but this difference is less compared to that between $\epsilon_d$ and $\epsilon_{dI}$. For all the cases the difference between our results and those using Iwamoto formula increases at higher temperatures. The fermi momenta of quarks and electron for 3-flavour case are given in Table 2. The study of all the figures and tables above reveals that the cases where Iwamoto formula agrees reasonably well with our results, $\Delta p_d$ ( or $\Delta p_s$ ) is much larger than the temperature. On the other hand when this difference is smaller or comparable with the temperature, the Iwamoto formula overestimates the exact result by order of magnitude. In addition to these, Fig. 3 also shows that our results are about a factor of 2.5 lower than the Iwamoto results even at lower temperatures. We have found that this difference comes from the approximation involved in the calculation of matrix element.

Furthermore Table 2. shows that for three flavour case electron chemical potential (which is same as $p_f(e)$ for massless electrons) becomes small ($<1.\text{MeV}$) for some values of $\alpha_c$, $n_B$ and $m_s$. In these cases, electrons are no longer degenerate. Clearly, for such cases the Iwamoto formula is not applicable. This point is missed in earlier calculations.

Our results have profound implications on neutrino emissivity and quark star cooling rates
because all the earlier calculations have used the Iwamoto formula and predicted large quark
star cooling rates in comparison with the neutron star cooling rates for temperatures less
than 1 MeV. Our results show that, particularly for 3-flavour quark matter, the calculated
emissivity is at least two orders of magnitude smaller than the one given by Iwamoto formula
and therefore, the three-flavour quark star cooling rates are that much smaller. Hence it is
necessary to understand why Iwamoto formula fails.

To investigate the failure of the Iwamoto formula, we consider the integral

\[ I = \int \frac{d^3p_d d^3p_u d^3p_e d^3p_\nu}{\epsilon_d \epsilon_u \epsilon_e} \delta^4(p_{d(s)} - p_u - p_e - p_\nu) n(\vec{p}_d(s))[1 - n(\vec{p}_u)][1 - n(\vec{p}_e)]. \]  

(10)

Here, we have replaced the neutrino emission rate by unity and therefore \( I \) is essentially the
phase space integral. Following the reasoning of Iwamoto, this integral should be proportional
to \( T^5 \). Choosing the coordinate axes such that \( \vec{p}_d \) is along z-axis and \( \vec{p}_u \) is in x-z plane and
using the 3-momentum \( \delta \)–function to perform electron and u-quark angle integrations, we
get

\[ I = 8\pi^2 \int \frac{p_d^2 dp_d p_u^2 dp_u p_e^2 dp_e d^3 p_\nu}{\epsilon_d \epsilon_u \epsilon_e} \frac{\sqrt{1 - x_u^2}}{p_u p_e (\sqrt{1 - x_u^2} (p_d - p_\nu x_\nu) + p_\nu x_\nu \sqrt{1 - x_{\nu}^2} \cos \phi_\nu)} \delta(\epsilon_d - \epsilon_u - \epsilon_e - \epsilon_\nu) n(\vec{p}_d)[1 - n(\vec{p}_u)][1 - n(\vec{p}_e)]. \]  

(11)

where \( x_\nu = \cos \theta_\nu \) and \( x_u = \cos \theta_u \) is determined by solving

\[ p_u x_u = p_d - p_\nu x_\nu - [p_e^2 - p_\nu^2 (1 - x_u^2) - p_\nu^2 (1 - x_\nu^2) - 2p_u p_\nu \sqrt{(1 - x_u^2)(1 - x_\nu^2) \cos \phi_\nu} ]^{1/2}. \]  

(12)

The integral in eq(11) above is restricted to the momenta \( |p_i - p_f(i)| \) few times \( T \) due
to Fermi distribution functions and the energy \( \delta \)–function. Now, if we neglect the neutrino
momentum in the $\delta-$functions, we get, $x_u = (p_d^2 + p_u^2 - p_e^2)/2p_d p_u$ and the factor in the
square brackets of eq(11) becomes $1/p_d p_u p_e$.

Two points should be noted at this stage.

1. Generally, $x_u$ is close to unity, so that $1 - x_u^2$ is small. But, if $\Delta p_d$ is of the order of
$T$, $\sqrt{1 - x_u^2} p_d$ can be comparable with $T$ and $p_\nu$ and therefore $p_\nu$ cannot be neglected
in the momentum $\delta-$functions. Particularly, the denominator in the square bracket
of eq(11) cannot be approximated by $p_e p_d \sqrt{1 - x_u^2}$. Thus, if $p_d \sqrt{1 - x_u^2} < p_\nu$, one
would get a power of $T$ from the denominator and $I$ will not be proportional to $T^5$.

2. Secondly, the momenta may differ from the corresponding Fermi momenta by few times
$T$ in the integral. When $\Delta p_d \sim T$, there are regions in $p_d p_u p_e-$space where $x_u > 1$
and the rest of the integrand is not small. Clearly, these regions must be excluded
from the integration as these values of $x_u$ are unphysical. If one does not put this
restriction, as is done when one factorises angle and momentum integrals, the phase
space integral will be overestimated (and wrong) when $\Delta p_d \sim T$.

The above discussion clearly shows why the integral in eq(11) should not be proportional
to $T^5$ when $\Delta p_d \sim T$. In order to demonstrate this point, we have calculated the integral
in eq(11) numerically and compared with the approximation where the neutrino momenta
are neglected and the restriction imposed by $x_u$ condition is not imposed. The calculation
is done for $\alpha_c = 0.1$ and for two-flavour case. The results are shown in Fig(4). In this figure
we also show the result for a case where the electron mass is taken to be 25 MeV. This is of
course unphysical, but by adjusting the electron mass we can reduce $\Delta p_d$. The figure clearly
shows that the approximate value of $I$ is proportional to $T^5$ where as the exact integral is
smaller than the approximate one at large T. Furthermore, for 25 MeV electron mass, the
departure from $T^5$ sets in at smaller value of the temperature. This clearly shows that the
departure is dependent on the value of $\Delta p_d$. Here we would like to mention that for some
values of $\alpha_c$ and $m_s$, $p_f(e)$ is small and is of the order of T. This implies that electrons are
no longer degenerate and deviation from the Iwamoto result is most pronounced.

In eq(11) we have dropped the matrix element of the weak interaction in the emissivity
calculation (eq(6)). So, the discussion of preceding paragraphs apply to the emissivity
calculation as well. Therefore it is now clear why the Iwamoto formula fails when $\Delta p_d$ (or
$\Delta p_s$ in case of weak interactions involving strange quarks) is close to the temperature of the
quark matter. We would like to note that the failure of $T^6$ dependence of the emissivity
essentially arises from invalid approximations in the phase space integration. Since similar
arguments are used to obtain the neutrino emissivity of neutron matter, it is possible that
the emissivity calculated for neutron matter may also be overestimated. We are investigating
this point.

Since the departure from the Iwamoto formula arises from the fact that $T/\Delta p_d$ (or $T/\Delta p_s$
) is not small, it may be possible to fit the numerically calculated $\epsilon$ with a function of the
form $\epsilon_I f(x)$, where $x = T/\Delta p_d$ ($T/\Delta p_s$ for strange sector). The function $f(x)$ should be
such that for small values of $x$ it should approach unity. Choosing $f(x) = \frac{1}{1 + ax + bx^2 + cx^3}$,
we have fitted the calculated $\epsilon$ for a number of values of $n_B$, $\alpha_c$ and $m_s$ and obtained the
values of $a$, $b$ and $c$. The quality of fit is shown in Fig.5. The values of $a$, $b$ and $c$ are
$-2.0$, 110, and 30, respectively. Here we would like to point out that since, for $s$- decay, as
mentioned above, there is a difference of a factor of 2.5 in Iwamoto and our results even at
lower temperatures, the data points for $s$- decay, in Fig.5., have been scaled accordingly.
To summarise, we have demonstrated that the Iwamoto formula of neutrino emissivity fails when \( T/\Delta p_d \) (or \( T/\Delta p_s \)) is not small. The formula fails because the neglect of neutrino momentum and factorisation of angle and momentum integrals is not valid. We propose an alternate formula which is obtained by fitting the numerically calculated \( \epsilon \).
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FIGURE CAPTIONS

FIG.1. Two flavour $d$-decay for $\alpha_c = 0.1$; (a) Iwamoto results for $n_B = 0.4 \text{fm}^{-3}$, (b) Our results for $n_B = 0.4 \text{fm}^{-3}$ ($\Delta p_d = 6.78$), (c) Iwamoto results for $n_B = 1.4 \text{fm}^{-3}$, (b) Our results for $n_B = 1.4 \text{fm}^{-3}$ ($\Delta p_d = 10.29$).

FIG.2. Three flavour $d$-decay for $\alpha_c = 0.1$ and $s$ quark mass is 150 MeV; (a) Iwamoto results for $n_B = 1.4 \text{fm}^{-3}$, (b) Our results for $n_B = 1.4 \text{fm}^{-3}$ ($\Delta p_d = 0.067$), (c) Iwamoto results for $n_B = 0.4 \text{fm}^{-3}$, (b) Our results for $n_B = 0.4 \text{fm}^{-3}$ ($\Delta p_d = 0.39$).

FIG.3. Three flavour $s$-decay for $\alpha_c = 0.1$ and $s$ quark mass is 150 MeV; (a) Iwamoto results for $n_B = 1.4 \text{fm}^{-3}$, (b) Our results for $n_B = 1.4 \text{fm}^{-3}$ ($\Delta p_s = 1.613$), (c) Iwamoto results for $n_B = 0.4 \text{fm}^{-3}$, (b) Our results for $n_B = 0.4 \text{fm}^{-3}$ ($\Delta p_d = 9.719$).

FIG.4. Two flavour phase space integrals for $\alpha_c = 0.1$ (a) Without restriction on $\cos \theta_u$ for both electron mass $m_e=0.0$ and 25 MeV, (b) Exact integral for $m_e=0.0$, (c) Exact integral for $m_e=25$ MeV.

FIG.5. $\frac{\epsilon_{d(x)}}{\epsilon_{d(x)}}$ is plotted against $x$ where $x = \frac{T}{\Delta p_d(x)}$. The fitted function is $f(x) = 1 + ax + bx^2 + cx^3$ where $a = -2.0$, $b = 110$, and $c = 30$. 
Table 1. Baryon number density \( n_B \), Fermi momenta of u-quark \( p_f(u) \), d-quark \( p_f(d) \), and electron \( p_f(e) \) for different \( \alpha_c \), where \( \Delta p_d = p_f(u) + p_f(e) - p_f(d) \).

| \( \alpha_c \) | \( n_B \) (\( fm^{-3} \)) | \( p_f(u) \) (MeV) | \( p_f(d) \) (MeV) | \( p_f(e) \) (MeV) | \( \Delta p_d \) (MeV) |
|---------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.1           | 0.60            | 357.80          | 449.20          | 99.15           | 7.75            |
|               | 1.00            | 424.22          | 532.52          | 117.56          | 9.20            |
|               | 1.40            | 474.57          | 595.79          | 131.51          | 10.29           |
| 0.05          | 0.60            | 357.71          | 449.25          | 95.43           | 3.89            |
|               | 1.00            | 424.11          | 532.65          | 113.15          | 4.61            |
|               | 1.40            | 474.45          | 595.87          | 126.57          | 5.15            |
Table 2. Baryon number density $n_B$, Fermi momenta of u-quark $p_f(u)$, d-quark $p_f(d)$, s-quark $p_f(s)$ and electron $p_f(e)$ for different $m_s$ and different $\alpha_c$, where $\Delta p_d = p_f(u) + p_f(e) - p_f(d)$ and $\Delta p_s = p_f(u) + p_f(e) - p_f(s)$

| $m_s$ (MeV) | $\alpha_c$ | $n_B$ (fm$^{-3}$) | $p_f(u)$ (MeV) | $p_f(d)$ (MeV) | $p_f(s)$ (MeV) | $p_f(e)$ (MeV) | $\Delta p_d$ (MeV) | $\Delta p_s$ (MeV) |
|------------|-----------|-----------------|---------------|---------------|---------------|---------------|----------------|----------------|
| 150.0      | 0.60      | 356.99          | 360.06        | 353.86        | 3.33          | 0.26          | 6.46           |                |
|            | 1.00      | 423.26          | 424.81        | 421.69        | 1.69          | 0.14          | 3.26           |                |
|            | 1.40      | 473.49          | 474.27        | 472.72        | 0.84          | 0.06          | 1.61           |                |
| 0.05       | 1.00      | 423.26          | 430.29        | 415.98        | 7.33          | 0.30          | 14.61          |                |
|            | 1.40      | 473.49          | 479.49        | 467.34        | 6.25          | 0.25          | 12.40          |                |
| 200.0      | 0.60      | 356.99          | 365.89        | 347.62        | 9.28          | 0.38          | 18.65          |                |
|            | 1.00      | 423.26          | 430.29        | 415.98        | 7.33          | 0.30          | 14.61          |                |
|            | 1.40      | 473.49          | 479.49        | 467.34        | 6.25          | 0.25          | 12.40          |                |
| 0.05       | 1.00      | 423.26          | 430.29        | 415.98        | 7.33          | 0.30          | 14.61          |                |
|            | 1.40      | 473.49          | 479.49        | 467.34        | 6.25          | 0.25          | 12.40          |                |
| 0.60       | 1.00      | 423.26          | 430.29        | 415.98        | 7.33          | 0.30          | 14.61          |                |
|            | 1.40      | 473.49          | 479.49        | 467.34        | 6.25          | 0.25          | 12.40          |                |
| 200.0      | 0.60      | 356.99          | 365.89        | 347.62        | 9.28          | 0.38          | 18.65          |                |
|            | 1.00      | 423.26          | 430.29        | 415.98        | 7.33          | 0.30          | 14.61          |                |
|            | 1.40      | 473.49          | 479.49        | 467.34        | 6.25          | 0.25          | 12.40          |                |
| 0.05       | 1.00      | 423.26          | 430.29        | 415.98        | 7.33          | 0.30          | 14.61          |                |
|            | 1.40      | 473.49          | 479.49        | 467.34        | 6.25          | 0.25          | 12.40          |                |
| 0.60       | 1.00      | 423.26          | 430.29        | 415.98        | 7.33          | 0.30          | 14.61          |                |
|            | 1.40      | 473.49          | 479.49        | 467.34        | 6.25          | 0.25          | 12.40          |                |