Systematic analysis of $p_T$-distributions in $p + p$ collisions.

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Abstract

A systematic analysis of transverse momentum distribution of hadrons produced in ultra-relativistic $p + p$ collisions is presented. We investigate the effective temperature and the entropic parameter from the non-extensive thermodynamic theory of strong interaction. We conclude that the existence of a limiting effective temperature and of a limiting entropic parameter is in accordance with experimental data.

1 Introduction

General aspects of strong interactions up to center-of-mass energies $\sqrt{s} \sim 10$ GeV are well understood in terms of a self-consistent theory based on the Boltzmann-Gibbs statistics [1]. Hagedorn’s theory establishes a connection between the mass spectrum of highly excited hadrons and the density of states for fireballs, and provides correct descriptions for transverse momentum distributions and multiplicities of secondaries.

For $\sqrt{s} \geq 10$ GeV, however, theoretical and experimental results diverge. A generalized formalism was proposed taking into account the non-extensive thermodynamics [2, 3]. This generalization recovered the agreement between theory and experiment.

Recently it was shown [4] that a self-consistent theory for hadronic systems based on the non-extensive thermodynamics exists if, for $x \rightarrow \infty$,

$$\rho(x) \rightarrow \gamma x^{-5/2} e^{\beta_q x}$$

(1)
and

\[ \sigma(x) \rightarrow bx^a e^{x^q}, \]

where \( \rho(m) \) is the mass spectrum of hadrons, \( \sigma(E) \) is the density of states for hadronic systems, and \( a \) is given by

\[ a = \frac{\gamma V_o}{2\pi^2 \beta_o^{3/2}}, \]

with \( \gamma \) and \( b \) being constants and \( V_o \) being the interaction volume. Here \( e_q^x \) is the q-exponential function given by

\[ e_q^x = [1 + (q - 1)x]^{\frac{1}{q-1}}. \]

An important consequence of the self-consistency principle is the existence of a limiting effective temperature, \( T_o = 1/\beta_o \), and of a limiting entropic parameter, \( q_o \).

In this work we perform a systematic analysis of experimental data on \( p+p \) collisions to verify if there exist \( T_o \) and \( q_o \) that allow a correct description of all experimental data. The set of experiments used in the present analysis is summarized in Table 1 and covers a wide energy range.

| Experiment     | Energy (GeV) | \(|\eta|\)         |
|----------------|--------------|--------------------|
| CMS (LHC)      | 7000 [5]     | \(|\eta| < 2, 4\)    |
| CMS (LHC)      | 2360 [5]     | \(|\eta| < 2, 4\)    |
| CMS (LHC)      | 900 [5]      | \(|\eta| < 2, 4\)    |
| ALICE (LHC)    | 900 [6]      | \(|\eta| < 0, 8\)    |
| ATLAS (LHC)    | 900 [7]      | \(|\eta| < 2, 5\)    |
| RHIC (BNL)     | 200 [8]      | \(3, 3 < \eta < 5, 0\) |

2 Transverse momentum distribution in non-extensive statistics

A direct method to obtain the effective temperature of the hadronic system produced in hadron-hadron collisions is through the study of the transverse
momentum ($p_T$) distribution of secondaries. According to the non-extensive formalism proposed in Ref. [2], the $p_T$-distribution is given by

$$\frac{1}{\sigma} \frac{d\sigma}{dp_T} = cp_T \int_0^\infty \left[ 1 + (q - 1)\beta \sqrt{p_L^2 + p_T^2 + \mu^2} \right]^{-q/(q-1)} dp_L,$$

where $p_L$ is the longitudinal momentum and $\mu$ is the mass of the hadron.

![Graph showing fittings of Eq. 6 to experimental $p_T$-distributions.](image)

Figure 1: Fittings of Eq. 6 to experimental $p_T$-distributions.

An useful approximation was proposed in Ref. [3] assuming that $\mu/p_L \approx 0$, resulting

$$\frac{1}{\sigma} \frac{d\sigma}{dp_T} = c[2(q - 1)]^{-1/2}B\left(\frac{1}{2}, \frac{q}{q - 1} - \frac{1}{2}\right) u^{3/2}[1 + (q - 1)u]^{-\frac{q}{q - 1} + \frac{1}{q}},$$

where $u = \beta p_T$, and $B(x, y)$ is the Beta-function. The dependence of the distribution on the entropic factor, $q$, and on the temperature, given by $\beta = 1/T$, enable us to obtain both parameters by fitting the above expression to experimental data. This procedure has been already used by many authors [2, 3, 5, 6, 7, 8]. It is important to notice that Equation 6 should be regarded as an approximation to the distribution one would obtain from the Tsallis
entropy by using the thermodynamic relations [4, 9], but to the use made
here the approximation is good enough.

Concerning the approximation on the mass, with the assumption that
\( \mu/p_L \approx 0 \), it should be mentioned that it can be avoided [9] with the restric-
tion that one can not apply the formula obtained for general hadron (\( h^+ \) and
\( h^- \)), but only on specific particle distributions. Since in this work we aim to
make a systematic analysis, and due to the larger number of information on
\((h^++h^-)\)-distributions, we opt to use the approximated formula 6. Even be-
ing a good approximation, it causes the effective temperature obtained to be
slightly shifted to higher values. This effect can be easily observed in Figure
5 of Ref. [9], where it can be seen that the peaks of the \( p_T \)-distributions for
heavier particles are shifted to higher values with respect to that for pions.

Applying the method described above to \( p+p \) collisions we get nice fittings
to the experimental \( p_T \)-distributions, as shown in Figure 1. We observe that
Eq. 6 correctly describes all data for \( p_T \) up to 18 GeV/c. From these fittings
we obtain the results for \( T \) and \( q \) that are shown in Fig 2. The temperature
varies inside a relatively narrow range between 70 MeV and 90 MeV for all
collisions with center-of-mass energy from 0.2 TeV up to 7 TeV. For energies
above 0.9 TeV the temperature can be considered constant with \( T \sim 73 \) MeV.
Also in the case of the parameter \( q \) the variations are relatively small in the
energy range studied, and above 0.9 TeV it is approximately constant with
\( q \sim 1.13 \).

It is important to notice that \( T \) and \( q \) in Eq. 6 are not completely in-
dependent. There is a strong correlation between the two parameters, as can be observed in Fig. 3 where we show the 2D-plot of \( \chi^2 \) distribution for
the fittings shown in Fig 1. We clearly see the ellipses that evidences the
correlation between the two parameters. Due to their correlation, as we vary
\( T \) and \( q \) simultaneously along the line corresponding to the major axis of the
ellipse, the \( \chi^2 \) remains practically unchanged. This means that it is possible
to obtain good \( \chi^2 \) with pairs \((T, q)\) near the optimum point found in the
fitting, indicated by crosses in Figure 3.

A linear behaviour between \( T \) and \( q \) was already predicted by Wilk and
Wlodarczyk [10, 11], who proposed the relation

\[
T = T_o + (q - 1) c ,
\]

where \( c \) is a constant depending on the energy transfer between the source
and its surroundings and on thermodynamical properties of the medium [12].
Since \( T \to T_o \) as \( q \to 1 \), \( T_o \) is considered to be the Hagedorn’s temperature.

Assuming that Eq. 7 establishes a causal relation between \( T \) and \( q \), then
correlation is a necessary consequence [13]. Also, it is possible to obtain
the regression coefficient, $c$, from the correlation. From the ellipses observed in Fig. 3 we obtain $T_0 = (192 \pm 15)$ MeV and $c = -(0.95 \pm 0.10)$ GeV. It is interesting to notice that the value for $T_0$ found here is in agreement with the critical temperature obtained in lattice QCD calculations [14, 15].

3 Testing the constant $T$ hypothesis

Now we slightly modify the fitting procedure by adopting a fixed value for $T$, as suggested by the self-consistency principle [4] and by the results shown in Fig. 2. We used different values for $T$ from 60 MeV up to 120 MeV. Some results are shown in Fig. 4 and we observe good fittings of Eq. 6 to the data in all cases, now with only $q$ as a free parameter.

These results show that we can fit all experimental data for $p + p$, for $\sqrt{s}$
Figure 3: Analysis of the correlation between the parameters $T$ and $q$. $\chi^2$ is shown as a function of $T$ and $q$ and the cross show the position of the best fit values for the parameters.
Figure 4: Typical results for fitting of Eq. 6 to experimental data with only $q$ as a free parameter. The calculations are performed for three different temperatures.

ranging from 0.2 TeV up to 7 TeV with a fixed temperature, $T$, and using only $q$ as free parameter. Also, the new procedure allows us to study $q$ as a function of $\sqrt{s}$ for different values of $T$.

In Fig. 4 we show $q$ vs $\sqrt{s}$ as obtained for different values of $T$. We observe that $q$ increases monotonically with $\sqrt{s}$, the shape being approximately described by a sigmoidal function of the temperature. A sigmoidal behaviour was already conjectured in Ref. [3], but more experimental information is needed before drawing any conclusion, mainly at low energies.

From the results shown in Fig. 4 we see that the best fittings are obtained for $T \approx 80$ MeV, and in Fig. 5 we observe that for $T = 80$ MeV the entropic parameter is approximately constant from $\sqrt{s} = 1$ GeV to 7 GeV, with $q \approx 1.12$.

The results obtained here are in agreement with an analysis performed in [3], where the $p_T$-distributions for different particles produced in $p + p$ collisions at center-of-mass energy of 0.9 TeV were studied, resulting in $T$ and $q$ constant for all particles with $T \approx 75$ MeV and $q \approx 1.15$. There are
Figure 5: The values for $q$ corresponding to the best fit with fixed $T$, for different values of temperature. Lines represent the best fitted curves for a sigmoidal function to the data.

also indications that these results hold for $A + A$ collisions [16].

4 Conclusions

In this work we present a systematic analysis of $p_T$-distributions observed in $p + p$ experiments. It is shown that the experimental data gives support to the hypothesis of a limiting effective temperature, $T$, and a limiting entropic factor, $q$.

In the analysis, we show that $T$ and $q$ are correlated parameters in the fitting procedure of the theoretical transverse momentum distribution to the experimental data. From this correlation, it results that the critical temperature is $T_o = (192 \pm 15)$ MeV, a value in good agreement with lattice-QCD predictions.

The study presented here gives evidences for a limiting effective temperature for the hadronic system formed in hadron-hadron collisions, with $T \approx 80$ MeV, and also of a limiting entropic factor with $q \approx 1.12$. These results are in good agreement with values found in Literature.
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