Hypergeometric presentation for one-loop contributing to \( H \to Z\gamma \)

Khiem Hong Phan\textsuperscript{1,2,*} and Dzung Tri Tran\textsuperscript{1,2}

\textsuperscript{1}University of Science Ho Chi Minh City, 227 Nguyen Van Cu, District 5, HCM City, Vietnam
\textsuperscript{2}Vietnam National University Ho Chi Minh City, Linh Trung Ward, Thu Duc District, HCM City, Vietnam
*E-mail: phkhiem@hcmus.edu.vn

Received November 12, 2019; Revised March 14, 2020; Accepted April 4, 2020; Published May 28, 2020

In this paper, new analytic formulas for one-loop contributing to Higgs decay channel \( H \to Z\gamma \) are presented in terms of hypergeometric functions. The calculations are performed by following the technique for tensor one-loop reduction developed in [A. I. Davydychev, Phys. Lett. B 263 (1991) 107]. For the first time, one-loop form factors for the decay process are shown which are valid at arbitrary space–time dimension \( d \).

Subject Index B59, B87

1. Introduction

Among the Higgs (\( H \)) decay processes, the decay channel \( H \to Z\gamma \) is the most important at the Large Hadron Collider (LHC) [1–3]. The channel arises at first from one-loop Feynman diagrams. As a result, the decay width of this channel is sensitive to new physics in which we assume that new heavy particles may exchange in one-loop diagrams. For this reason, theoretical evaluations for one-loop and higher-loop decay amplitudes of \( H \to Z\gamma \) play crucial roles in controlling the standard model (SM) background as well as constraining physical parameters in many beyond standard models (BSM).

There have been many computations for one-loop contributions to \( H \to Z\gamma \) within SM and its extensions [4–21]. The calculations have been performed following the method for tensor one-loop reduction in Ref. [22]. When one-loop contributions to \( H \to Z\gamma \) are evaluated in unitary gauge, the results may meet large numerical cancellations. This is because higher-rank tensor one-loop integrals appear from Feynman loop diagrams with exchanging by vector bosons. To avoid this problem, many of the above references have considered the calculations in the ‘t Hooft–Feynman gauge. In this gauge, we need to handle more Feynman diagrams involving Goldstone bosons. As a result, the calculations are rather complicated. Furthermore, when we consider two-loop or higher-loop corrections to \( H \to Z\gamma \), two-loop and higher-loop Feynman integrals may be evaluated by applying methods from Refs. [23–26]; the resulting integrals may contain the one-loop integrals in the general space–time dimension. These integrals have not been available in previous papers.

In this paper, we apply an alternative approach for evaluating one-loop contributions to \( H \to Z\gamma \). In this calculation, we follow the method for tensor one-loop reduction developed in Ref. [27] in which tensor integrals are decomposed into scalar functions with arbitrary propagator indexes and at a higher space–time dimension \( d > 4 \). Using an integration-by-part method (IBP) [23,25], scalar
One-loop integrals are then expressed in terms of master integrals which can be solved analytically via generalized hypergeometric series. For instance, analytic formulas for the master integrals which are one-loop one-, two-, or three-point functions at general \( d \) appearing in \( H \to Z \gamma \) are provided in this work. Therefore, our methods are easy to apply to \( H \to Z \gamma \) and are expected to be numerically stable in unitary gauge. Furthermore, our analytic expressions for the form factors of the decay process are general as well as valid at an arbitrary space–time dimension.

The layout of the paper is as follows: In Sect. 2, we present a general method for evaluating one-loop Feynman integrals. Using the method, the computations for one-loop contributions to Higgs decay to \( Z \) photon are reported in the Sect. 3. Conclusions are shown in Sect. 4. Several useful formulas used in this calculation and detailed calculations for the process amplitudes are given in the appendices.

2. Method

In this section, we describe a general approach for evaluating one-loop Feynman integrals. In general, tensor one-loop \( N \)-point Feynman integrals with rank \( M \) are defined as follows:

\[
J_{N,\mu_1 \mu_2 \cdots \mu_M}(d; \{v_1, v_2, \cdots, v_N\}) = \int \frac{d^d k}{i \pi^{d/2}} \frac{k_{\mu_1} k_{\mu_2} \cdots k_{\mu_M}}{(k + q_1)^{2 - m_1^2 + i \rho} (k + q_2)^{2 - m_2^2 + i \rho} \cdots (k + q_N)^{2 - m_N^2 + i \rho}^{v_N}}. \tag{1}
\]

Where \( p_i \) (\( m_i \)) for \( i = 1, 2, \cdots, N \) are external momenta (internal masses) respectively. In this convention, \( q_1 = p_1, q_2 = p_1 + p_2, \cdots, q_i = \sum_{j=1}^i p_j \), and \( q_N = \sum_{j=1}^N p_j = 0 \), thanks to momentum conservation. The term \( i \rho \) is Feynman’s prescription and \( d \) is the space–time dimension. One of the physical interests is \( d = 4 + 2n - 2e \) for \( n \in \mathbb{N} \).

Following the method for tensor reduction in Ref. [27], tensor one-loop integrals can be reduced to scalar functions with the shifted space–time dimension as follows:

\[
J_{N,\mu_1 \mu_2 \cdots \mu_M}(d; \{v_1, v_2, \cdots, v_N\}) = \sum_{\lambda, \kappa_1, \kappa_2, \cdots, \kappa_N} \left( \frac{1}{2} \right)^\lambda \left\{ [g]^{\frac{1}{2}} [q_1]^{\kappa_1} [q_2]^{\kappa_2} \cdots [q_N]^{\kappa_N} \right\}_{\mu_1 \nu_2 \cdots \mu_M} \times (v_1)_{\kappa_1} (v_2)_{\kappa_2} \cdots (v_N)_{\kappa_N} J_N(d + 2(M - \lambda); \{v_1 + \kappa_1, v_2 + \kappa_2, \cdots, v_N + \kappa_N\}). \tag{2}
\]

Here \( \lambda, \kappa_1, \kappa_2, \cdots, \kappa_N \) satisfy the following constraints \( 2\lambda + \kappa_1 + \kappa_2 + \cdots + \kappa_N = M, 0 \leq \kappa_1, \kappa_2, \cdots, \kappa_N \leq M \) and \( 0 \leq \lambda \leq \lceil M/2 \rceil \) (integer of \( M/2 \)). The Pochhammer symbol is used as \( (a)_k = \Gamma(a + k)/\Gamma(a) \). The tensor \( \{ [g]^{\frac{1}{2}} [q_1]^{\kappa_1} [q_2]^{\kappa_2} \cdots [q_N]^{\kappa_N} \}_{\mu_1 \nu_2 \cdots \mu_M} \) is symmetric in regard to \( \mu_1, \mu_2, \cdots, \mu_M \). It is formed from \( \lambda \) of metric \( g_{\mu \nu}, k_1 \) of momentum \( q_1, \cdots, \kappa_N \) of momentum \( q_N \). The \( J_N(d + 2(M - \lambda); \{v_1 + \kappa_1, v_2 + \kappa_2, \cdots, v_N + \kappa_N\}) \) with changing space–time dimension to \( d + 2(M - \lambda) \), raising powers of propagators \( \{v_i + \kappa_i\} \) for \( i = 1, 2, \cdots, N \), are scalar one-loop \( N \)-point functions.

In the next step, the scalar integrals \( J_N(d; \{v_1, v_2, \cdots, v_N\}) \) are cast into the subset of master functions using IBP [23]. In detail, the operator \( \frac{\partial}{\partial k} \cdot k \) is applied to the integrand of \( J_N(d; \{v_1, v_2, \cdots, v_N\}) \) and \( k \) is set to be the momentum of \( N \) internal lines (\( k = \{k + q_1, k + q_2, \cdots, k + q_N\} \)). As a result, \( J_N(d; \{v_1, v_2, \cdots, v_N\}) \) can be expressed in terms of \( J_N(d; \{1, 1, \cdots, 1\}) \) and \( J_{N-1}(d; \{v'_1, v'_2, \cdots, v'_{N-1}\}) \). In this recurring way [25] we arrive at the master integrals which can be solved analytically. For example, they may be \( J_N(d; \{1, 1, \cdots, 1\}) \) and \( J_{N-L}(d; \{v''_1, v''_2, \cdots, v''_{N-L}\}) \).
with \( L < N \). Recently, scalar one-loop integrals at general \( d \) have been expressed in terms of generalized hypergeometric series [28–31].

In Appendix B, this method is demonstrated in detail for the case of \( H \to Z \gamma \). We show here all the analytic results for the master integrals involving the decay process. In particular, scalar one-loop one-point functions with arbitrary propagator index \( \nu \) are given [32]:

\[
J_1(d; \{ \nu \}; M^2) = (-1)^{\nu} \frac{\Gamma(v - d/2)}{\Gamma(v)} (M^2)^{d/2-v}.
\]

Scalar one-loop two-point functions with general propagator indexes \( \nu_1, \nu_2 \) in the case of \( m_1^2 = m_2^2 = M^2 \) read [33]:

\[
J_2(d; \{ \nu_1, \nu_2 \}; p^2, M^2) = (-1)^{\nu_2} \frac{\Gamma(N_2 - d/2)}{\Gamma(N_2)} (M^2)^{d/2-N_2} \, _3F_2 \left[ \begin{array}{c} \nu_1, \nu_2, N_2 - d/2; \\
\frac{N_2}{2}, \frac{N_2+1}{2}, \\
p^2 \end{array}; \frac{4M^2}{4M^2} \right].
\]

Here \( N_2 = \nu_1 + \nu_2, p^2 = 0, M_H^2, M_Z^2, \) and \( M^2 = m_1^2, M_W^2 \) in this calculation. Other master integrals which are scalar one-loop three-point functions are given:

\[
J_3(d; \{ 1, 1, 1 \}; p_1^2, M_H^2, M^2) = \frac{(d-4)M_H^2}{4(M_H^2 - p_1^2)} (M^2)^{d/2-3} \\
\times \left\{ _3F_2 \left[ 1, 1, 3 - d/2; \frac{M_H^2}{4M^2} \right] - _3F_2 \left[ 1, 1, 3 - d/2; \frac{p_1^2}{4M^2} \right] \right\},
\]

\[
J_3(d; \{ 1, 2, 1 \}; p_1^2, M_H^2, M^2) = \frac{(4-d)(d-4)}{2(M_H^2 - p_1^2)} (M^2)^{d/2-3} \\
\times \left\{ _3F_2 \left[ 1, 2, 3 - d/2; \frac{M_H^2}{4M^2} \right] - _3F_2 \left[ 1, 2, 3 - d/2; \frac{p_1^2}{4M^2} \right] \right\},
\]

\[
J_3(d; \{ 1, 3, 1 \}; p_1^2, M_H^2, M^2) = \frac{(6-d)(d-4)}{16(M_H^2 - p_1^2)} (M^2)^{d/2-4} \\
\times \left\{ _3F_2 \left[ 1, 2, 4 - d/2; \frac{M_H^2}{4M^2} \right] - _3F_2 \left[ 1, 2, 4 - d/2; \frac{p_1^2}{4M^2} \right] \right\},
\]

\[
J_3(d; \{ 2, 2, 1 \}; p_1^2, M_H^2, M^2) = (d-4)(M^2)^{d/2-4} \\
\times \left\{ \frac{(6-d)M_H^2}{16M^2(M_H^2 - p_1^2)^2} _3F_2 \left[ 1, 2, 4 - d/2; \frac{M_H^2}{4M^2} \right] \\
+ \frac{(6-d)[M_H^2 p_1^2 - 2M^2(M_H^2 + p_1^2)]}{16M^2(M_H^2 - p_1^2)^2} _3F_2 \left[ 1, 2, 4 - d/2; \frac{p_1^2}{4M^2} \right] \right\}.
\]
\[ + \frac{(d - 4) M_{\mu}^2}{8(M_{\mu}^2 - p_2^2)} \begin{Bmatrix} 1, 1, 3; \ 1 \ 3/2, 2; \ M_{\mu}^2 \\ 3/2, 2; \ 4M^2 \end{Bmatrix} + \frac{(4 - d) M_2^2}{8(M_{\mu}^2 - p_2^2)} \begin{Bmatrix} 1, 1, 3; \ 1 \ 3/2, 2; \ p_2^2 \\ 3/2, 2; \ 4M^2 \end{Bmatrix} \]  

(8)

where \( p_2^2 = M_Z^2 \), \( 0 \) and \( M_2 = m^2_f \), \( M_2 \) in the present calculation.

We are going to apply this method for evaluating the Higgs decay processes. The first results for one-loop contributions to \( H \rightarrow \gamma \gamma \) have been published in Ref. [33]. In the next section, we show new analytic results for \( H \rightarrow Z \gamma \) by means of a \( _3F_2 \) hypergeometric series.

3. Hypergeometric presentation for one-loop contributing to \( H \rightarrow Z \gamma \)

In unitary gauge, the decay process \( H \rightarrow Z \gamma \) consists of a top loop and a \( W \) boson loop, as shown in Figs. 1 and 2. In general, the total amplitude of the decay \( H \rightarrow Z \gamma \) is expressed in terms of form factors reflecting the Lorentz invariant structure and the content of gauge symmetry as follows:

\[
iA_{H \rightarrow Z \gamma} = iA_{\mu \nu} \epsilon_1^{\mu*}(q_1) \epsilon_2^{\nu*}(q_2) =
\]

\[
= \left( F_{00} g_{\mu \nu} + \sum_{i,j=1}^{2} F_{ij} q_i \cdot q_j + F_5 \times i \epsilon_{\mu \nu \alpha \beta} q_1^{\alpha} q_2^{\beta} \right) \epsilon_1^{\mu*}(q_1) \epsilon_2^{\nu*}(q_2). \]  

(9)

Where \( \epsilon_1^{\mu*} \) and \( \epsilon_2^{\nu*} \) are the polarization vectors of the \( Z \) boson and the photon \( \gamma \) respectively. \( \epsilon_{\mu \nu \alpha \beta} \) is the Levi–Civita tensor. Kinematic invariant variables related to this process are

\[
q_1^2 = M_Z^2, \quad q_2^2 = 0, \quad p^2 = (q_1 + q_2)^2 = M_H^2. \]  

(10)

We also have \( \epsilon_2^{\nu*}(q_2) q_{2, \nu} = 0 \) for the external photon. Following the Ward identity, we confirm that

\[
F_{11} = 0, \quad F_{00} = -(q_1 \cdot q_2) F_{21} = \frac{M_Z^2 - M_H^2}{2} F_{21} \]  

(11)

and \( F_{12,22} \) do not contribute to the total amplitude. Summing all the top-loop diagrams, the result shows that \( F_5 = 0 \). Detailed calculations for the form factors at general \( d \) are presented in Appendix D.

The total amplitude for this decay process is then cast in the form of

Fig. 1. Feynman diagrams contributing to the \( H \rightarrow Z \gamma \) decay through a top quark loop in unitary gauge.
Fig. 2. Feynman diagrams contributing to the $H \to Z\gamma$ decay through a W boson loop in unitary gauge.

\[
i A_{H \to Z\gamma} = \frac{e^3}{\sin \theta_W M_W} \mathcal{F}_{H \to Z\gamma}(d; M_H^2, M_Z^2, M_W^2, m_f^2) \left[ q_{2,\nu} q_{1,\mu} - (q_1 \cdot q_2) g_{\mu\nu} \right] \epsilon_{1}^{\mu}(q_1) \epsilon_{2}^{\nu}(q_2),
\]

where $\mathcal{F}_{H \to Z\gamma}(d; M_H^2, M_Z^2, M_W^2, m_f^2)$ are form factors which can be derived from $F_{00}$ or $F_{21}$. These form factors are decomposed in terms of $W$-loop and top-loop (including fermion-loop) contributions as follows:

\[
\mathcal{F}_{H \to Z\gamma}(d; M_H^2, M_Z^2, M_W^2, m_f^2) = \cot \theta_W \mathcal{F}_{H \to Z\gamma}^{(W)}(d; M_H^2, M_Z^2, M_W^2) + \sum_f \frac{Q_f N_C}{e} \left( \lambda_1^f + \lambda_2^f \right) \mathcal{F}_{H \to Z\gamma}^{(f)}(d; M_H^2, M_Z^2, m_f^2),
\]

where $\theta_W$ is the Weinberg angle, and $I^f_3, Q_f$ and $m_f$ are the iso-spin, electric charge, and the mass of fermions $f$ in the loops, respectively. $N_C$ is a color factor for the fermions. It becomes 1 for leptons and 3 for quarks. We use the symbolic-manipulation Package-X [34] to handle all Dirac and tensor algebra in $d$ dimensions.

3.1. Form factors
We show two representations for the form factors in terms of $3F_2$ hypergeometric functions in this subsection.

3.1.1. First representation
We first present the form factors which are derived from $F_{00}$ in Eq. (9) in terms of $3F_2$ hypergeometric functions as follows:

\[
\frac{\mathcal{F}_{H \to Z\gamma}^{(W)}(d; M_H^2, M_Z^2, M_W^2)}{\Gamma(2-d/2)} = \frac{(M_W^2)^{d/2-2}}{(4\pi)^{d/2} M_H^2 (M_Z^2 - M_H^2)^2} \times \left\{ (4-d) (M_Z^2 - 4M_W^2)(M_H^2 - M_Z^2) \right\}
\]
Another presentation for the form factors which are obtained from the form factors $F$ confirm that the terms in the curly brackets in the right-hand side of Eqs. (14) and (15) tend to zero as the form factors which have fermion masses smaller than Eq. (A.3) for $3 \rightarrow H → (H → (W → W → W))$. In the limit $d → 4$, we confirm that the terms in the curly brackets in the right-hand side of Eqs. (14) and (15) tend to zero

$$
\left[2M_W^2(M_H^2 − M_Z^2) − M_H^2M_Z^2 + 12M_W^4\right](M_Z^2 + (M_H^2 − M_Z^2) − M_H^2) = 0,
$$

$$
8M_H^2m_t^2 − 8M_Z^2m_t^2 − 8(M_H^2 − M_Z^2)m_t^2 = 0.
$$

It means that the form factors always stay finite in the limit.

### 3.1.2. Second representation

Another presentation for the form factors which are obtained from $F_{21}$ in Eq. (9) are given:

$$
\frac{\mathcal{F}^{(W)}_H → Z \gamma (d; M_H^2, M_Z^2, M_W^2)}{\Gamma (2 − d/2)} = \frac{(M_W^2)^{d/2−2}}{(4\pi)^{d/2} (M_Z^2 − M_H^2)^2}
\times \left\{ (4 − d) M_W^2(M_H^2 − M_Z^2) \left[1, 1, 3 − d/2; \frac{M_H^2}{4M_W^2}\right] − M_H^2 \left[1, 1, 3 − d/2; \frac{M_Z^2}{4M_W^2}\right]\right\}.
$$

The form factors $\mathcal{F}^{(f)}_H → Z \gamma (d; M_H^2, M_Z^2, m_f^2)$ are obtained by replacing $m_t → m_f$ in Eq. (15). For the form factors which have fermion masses smaller than $M_H/2$, the argument of hypergeometric functions $3F_2$ is greater than 1 (or $|M_H^2/4m_f^2| > 1$). We subsequently apply analytic continuation in Eq. (A.3) for $3F_2$ appearing in the form factors $\mathcal{F}^{(f)}_H → Z \gamma (d; M_H^2, M_Z^2, m_f^2)$. In the limit $d → 4$, we confirm that the terms in the curly brackets in the right-hand side of Eqs. (14) and (15) tend to zero
and

\[
\frac{\mathcal{F}_{H\rightarrow Z'}^{(t)}(d; M_H^2, M_Z^2, m_t^2)}{\Gamma (2 - d/2)} = \frac{(m_t^2)^{d/2 - 2}}{(4\pi)^{d/2} (M_Z^2 - M_H^2)^2} \times \left\{ \begin{array}{l}
\frac{4M_H^2 (4m_t^2 - M_H^2)}{3} \left( 3F_2 \left[ 2, 2, 2 - d/2; \frac{M_H^2}{4m_t^2} \right] + 2 \right) 3F_2 \left[ 3, 1, 2 - d/2; \frac{M_H^2}{4m_t^2} \right] \\
+ \frac{4M_H^2 M_Z^2 - 8m_t^2 (M_H^2 + M_Z^2)}{3} \left( 3F_2 \left[ 2, 2, 2 - d/2; \frac{M_H^2}{4m_t^2} \right] - \frac{M_H^2}{6m_t^2} 3F_2 \left[ 3, 2, 2 - d/2; \frac{M_H^2}{4m_t^2} \right] \right) \\
- 16M_H^2 m_t^2 \left( 3F_2 \left[ 2, 1, 2 - d/2; \frac{M_H^2}{4m_t^2} \right] - \frac{M_H^2}{6m_t^2} 3F_2 \left[ 3, 2, 2 - d/2; \frac{M_H^2}{4m_t^2} \right] \right) \\
+ \frac{16M_H^2 m_t^2 (d - 1) - 16M_Z^2 m_t^2}{(d - 2)} \left( 3F_2 \left[ 2, 1, 2 - d/2; \frac{M_H^2}{4m_t^2} \right] - \frac{M_Z^2}{6m_t^2} 3F_2 \left[ 3, 2, 2 - d/2; \frac{M_H^2}{4m_t^2} \right] \right) \\
+ \frac{8M_H^2 m_t^2 d}{(2 - d)} \left( 3F_2 \left[ 1, 1, 2 - d/2; \frac{M_H^2}{4m_t^2} \right] - \frac{M_H^2}{12m_t^2} 3F_2 \left[ 2, 2, 2 - d/2; \frac{M_H^2}{4m_t^2} \right] \right) \\
+ \frac{8M_Z^2 m_t^2 d}{(d - 2)} \left( 3F_2 \left[ 1, 1, 2 - d/2; \frac{M_Z^2}{4m_t^2} \right] - \frac{M_Z^2}{12m_t^2} 3F_2 \left[ 2, 2, 2 - d/2; \frac{M_Z^2}{4m_t^2} \right] \right) \\
+ 8m_t^2 (M_H^2 - M_Z^2) \left( 3F_2 \left[ 2, 1, 2 - d/2; \frac{M_H^2}{4m_t^2} \right] - 3F_2 \left[ 2, 1, 2 - d/2; \frac{M_Z^2}{4m_t^2} \right] \right) \\
+ (d - 4) (M_H^2 - M_Z^2) \left( M_H^2 3F_2 \left[ 1, 1, 3 - d/2; \frac{M_H^2}{4m_t^2} \right] - M_Z^2 3F_2 \left[ 1, 1, 3 - d/2; \frac{M_Z^2}{4m_t^2} \right] \right) \right\}. \\
\right.
\]
In the limit $d \to 4$, we also confirm that the terms in curly brackets on the right-hand side of Eqs. (18) and (19) tend to zero. It means that the form factors always stay finite in the limit.

### 3.2. $H \to \gamma\gamma$ reduction

In order to reduce to $H \to \gamma\gamma$, we take $M_H^2 \to 0$, and $\lambda_1^f = eQ_f, \lambda_2^f, \lambda_3^f \to 0$; the total amplitude of the decay $H \to Z\gamma$ is reduced to $H \to \gamma\gamma$. In detail, the results read

$$
\mathcal{F}_{H \to \gamma\gamma}(d; M_H^2, M_W^2, m_f^2) = \mathcal{F}_{H \to \gamma\gamma}^{(W)}(d; M_H^2, M_W^2) + \sum_f N_C Q_f^2 \mathcal{F}_{H \to \gamma\gamma}^{(f)}(d; M_H^2, m_f^2).
$$

Where the form factors are given

$$
\frac{\mathcal{F}_{H \to \gamma\gamma}^{(f)}(d; M_H^2, m_f^2)}{\Gamma(2-d/2)} = \frac{(m_f^2)^{d/2-2}}{(4\pi)^{d/2}} \left\{ - \frac{8m_f^2}{M_H^2} \, 3F_2 \left[ \begin{array}{c} 1, 1, 2 - d/2; \\ 3/2, 1; \end{array} \frac{M_H^2}{4m_f^2} \right] \right.
$$

$$
\quad + (4-d) \, 3F_2 \left[ \begin{array}{c} 1, 1, 3 - d/2; \\ 3/2, 2; \end{array} \frac{M_H^2}{4m_f^2} \right] + \frac{8m_f^2}{M_H^2} \, 3F_2 \left[ \begin{array}{c} 1, 1, 2 - d/2; \\ 3/2, 2; \end{array} \frac{M_H^2}{4m_f^2} \right] \right\}.
$$

and

$$
\frac{\mathcal{F}_{H \to \gamma\gamma}^{(W)}(d; M_H^2, M_W^2)}{\Gamma(2-d/2)} = \frac{(M_W^2)^{d/2-2}}{(4\pi)^{d/2}} \left\{ (4-d) \, 3F_2 \left[ \begin{array}{c} 1, 1, 3 - d/2; \\ 3/2, 2; \end{array} \frac{M_H^2}{4M_W^2} \right] \right.
$$

$$
\quad + \left[ 2 + 4 \frac{M_H^2}{M_W^2} (d-1) \right] \frac{2, 1, 2 - d/2;}{3/2, 2; \frac{M_H^2}{4M_W^2}} - \frac{4}{3} \left[ 2, 1, 2 - d/2; \frac{M_H^2}{4M_W^2} \right] \right\}
$$

To arrive at the last line, we have already used the transformation for hypergeometric functions $3F_2$ in Eq. (A.4). This agrees with the results in Ref. [33].

### 3.3. Numerical results

In numerical results, we set $M_H = 125$ GeV, $M_Z = 91.2$ GeV, $m_t = 173.5$ GeV and $M_W = 80.4$ GeV. Our results are generated by using package $\text{NumEXP}$ [35] for numerical $\epsilon$-expansions of hypergeometric functions. We first confirm two representations for the form factors in Eqs. (14, 15) and (18, 19) at general $d$. It means that we verify numerically the Ward identity at general $d$. In Tables 1 and 2, we show numerical checks for the form factors at general $d$. Two representations for the form factors are in perfect agreement up to the last digit for $3.5 \leq d \leq 5.5$.

We next perform higher-order $\epsilon$-expansion for the form factors in this work up to $\epsilon^5$. We also compare our results with Ref. [19] $(F_{21,W}^{SM})$ at $\epsilon^0$-terms. Our numerical results are shown in Eqs. (25,
Table 1. Numerical confirmations for two representations of the form factors involving top-loop diagrams at arbitrary $d$.

| $d$ | $\mathcal{F}^{(t)}_{H\rightarrow Z\gamma}(d; M_{H}^{2}, M_{Z}^{2}, m_{t}^{2})$ in Eq. (15) | $\mathcal{F}^{(t)}_{H\rightarrow Z\gamma}(d; M_{H}^{2}, M_{Z}^{2}, m_{t}^{2})$ in Eq. (19) |
|-----|-----------------------------------------------------------------|-----------------------------------------------------------------|
| 3.5 | $-0.00117666222408164570889597705142$                           | $-0.00117666222408164570889597705142$                           |
| 4.5 | $-0.0756076123635421878866551078159$                            | $-0.0756076123635421878866551078159$                            |
| 5.0 | $-0.7540013600177827962359989943$                                | $-0.7540013600177827962359989943$                                |
| 5.5 | $-10.6345811567309032438825219401$                               | $-10.6345811567309032438825219401$                               |

Table 2. Numerical confirmations for two representations for the form factors involving $W$-loop diagrams at arbitrary $d$.

| $d$ | $\mathcal{F}^{(W)}_{H\rightarrow Z\gamma}(d; M_{H}^{2}, M_{Z}^{2}, M_{W}^{2})$ in Eq. (14) | $\mathcal{F}^{(W)}_{H\rightarrow Z\gamma}(d; M_{H}^{2}, M_{Z}^{2}, M_{W}^{2})$ in Eq. (18) |
|-----|-----------------------------------------------------------------|-----------------------------------------------------------------|
| 3.5 | $-0.00924203129694608232780754562475$                            | $-0.00924203129694608232780754562475$                            |
| 4.5 | $-0.211488266331639234594811276488$                              | $-0.211488266331639234594811276488$                              |
| 5.0 | $-1.2678629636083047430009124220$                                | $-1.2678629636083047430009124220$                                |
| 5.5 | $-10.804044333273283701507434992$                                | $-10.804044333273283701507434992$                                |

We find a perfect agreement between two results at $\epsilon^{0}$-expansion. It is important to note that the higher-power $\epsilon$-expansions for the form factors in this paper are our first results.

\[
F_{21,W}^{SM} = -0.0418477713507083034768633206537 \epsilon^{0} + \mathcal{O}(\epsilon)\; ; \quad (24)
\]

\[
\mathcal{F}^{(W)}_{H\rightarrow Z\gamma}(d = 4 - 2\epsilon; M_{H}^{2}, M_{Z}^{2}, M_{W}^{2}) = -0.0418477713507083034768633206537 \epsilon^{0} + 0.260913488721110921277821252790 \epsilon^{1} - 0.84941596484283152224099065525 \epsilon^{2} + 1.93196240724203383916822579654 \epsilon^{3} - 3.46717780533875010127157401115 \epsilon^{4} + 5.25914558345954670519178485415 \epsilon^{5}
\]
4. Conclusions

In this paper, we have discussed the alternative approach for evaluating one-loop Feynman integrals. In this method, tensor one-loop integrals are reduced to scalar one-loop functions with the shifted space–time dimension. Scalar one-loop integrals are solved analytically with the help of generalized hypergeometric series. We have applied this method for computing one-loop contributions to Higgs decay to $Z\gamma$. For the first time, we have presented the form factors that are valid in general space–time dimensions. The method can be extended to evaluate one-loop contributions to Higgs decay to $Zf\bar{f}$, $f\bar{f}\gamma$, etc., within the SM and many BSMs.

Acknowledgements

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2019.346.

Funding

Open Access funding: SCOAP$^3$.

Appendix A. Hypergeometric series

The series of hypergeometric functions $3F_2$ [36] are defined:

$$3F_2\left[ a_1, a_2, a_3; b_1, b_2; z \right] = \sum_{m=0}^{\infty} \frac{(a_1)_m(a_2)_m(a_3)_m z^m}{(b_1)_m(b_2)_m m!}. \tag{A.1}$$

The Mellin–Barnes representation for $3F_2$ is

$$3F_2\left[ a_1, a_2, a_3; b_1, b_2; z \right] = \frac{\Gamma(a_1)\Gamma(a_2)\Gamma(a_3)}{\Gamma(b_1)\Gamma(b_2)} \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} ds \Gamma(-s) \frac{\Gamma(s+a_1)\Gamma(s+a_2)\Gamma(s+a_3)}{\Gamma(s+b_1)\Gamma(s+b_2)} (-z)^s, \tag{A.2}$$
provided that $|\text{Arg}(-z)| < \pi$. The integration contour is chosen in such a way that the poles of $\Gamma(-s)$ and $\Gamma(\cdots + s)$ are well separated. The analytic continuation of $3F_2$ functions is

\[
3F_2 \left[ \begin{array}{c} a_1, a_2, a_3 \\ b_1, b_2 \\ z \end{array} ; \right] = \frac{\Gamma(b_1)\Gamma(b_2)}{\Gamma(a_1)\Gamma(a_2)\Gamma(a_3)} \left\{ \begin{array}{c} \Gamma(a_2 - a_1)\Gamma(a_3 - a_1)\Gamma(a_1) \\ \Gamma(b_1 - a_1)\Gamma(b_2 - a_1)(-z)^{a_1} \end{array} \right\} 3F_2 \left[ \begin{array}{c} a_1, 1 - b_1 + a_1, 1 - b_2 + a_1; \frac{1}{z} \\ 1 - a_2 + a_1, 1 - a_3 + a_1; \frac{1}{z} \end{array} \right] \\
+ \frac{\Gamma(a_1 - a_2)\Gamma(a_3 - a_2)\Gamma(a_2)}{\Gamma(b_1 - a_2)\Gamma(b_2 - a_2)(-z)^{a_2}} 3F_2 \left[ \begin{array}{c} a_2, 1 - b_1 + a_2, 1 - b_2 + a_2; \frac{1}{z} \\ 1 - a_1 + a_2, 1 - a_3 + a_2; \frac{1}{z} \end{array} \right] \\
+ \frac{\Gamma(a_1 - a_3)\Gamma(a_2 - a_3)\Gamma(a_3)}{\Gamma(b_1 - a_3)\Gamma(b_2 - a_3)(-z)^{a_3}} 3F_2 \left[ \begin{array}{c} a_3, 1 - b_1 + a_3, 1 - b_2 + a_3; \frac{1}{z} \\ 1 - a_1 + a_3, 1 - a_2 + a_3; \frac{1}{z} \end{array} \right]. \tag{A.3}
\]

In this work, a useful transformation for $3F_2$ functions is mentioned:

\[
3F_2 \left[ \begin{array}{c} a_1, a_2, a_3 \\ b_1, b_2 \\ z \end{array} ; \right] \equiv \frac{b_1 - a_1}{b_1} 3F_2 \left[ \begin{array}{c} a_1, a_2, a_3 \\ b_1 + 1, b_2 \\ z \end{array} ; \right] + \frac{a_1}{b_1} 3F_2 \left[ \begin{array}{c} a_1 + 1, a_2, a_3 \\ b_1 + 1, b_2 \\ z \end{array} ; \right]. \tag{A.4}
\]

**Appendix B. Calculating master integrals**

Tensor one-loop three-point Feynman integrals with rank $M$ appearing in the process $H \to Z\gamma$ are given as follows:

\[
J_{3,\mu_1\mu_2\ldots\mu_M}(d; \{v_1, v_2, v_3\}) = J_{3,\mu_1\mu_2\ldots\mu_M}(d; \{v_1, v_2, v_3\}; p_2^2, M_H^2, M^2) = \int \frac{d^dk}{i\pi^{d/2}} \frac{k_{\mu_1}k_{\mu_2}\cdots k_{\mu_M}}{P_1^2 P_2^2 P_3^2}, \tag{B.1}
\]

where the inverse Feynman propagators are

\[
P_1 = (k + q)^2 - M^2 + i\rho, \tag{B.2}
\]

\[
P_2 = (k + p)^2 - M^2 + i\rho, \tag{B.3}
\]

\[
P_3 = k^2 - M^2 + i\rho. \tag{B.4}
\]

The related kinematic invariants are $q_1^2 = M_Z^2, q_2^2 = 0,$ and $p^2 = (q_1 + q_2)^2 = M_H^2$. In this paper, $p_2^2 = M_Z^2, 0$ and internal masses $M^2 = m_J^2, M_W^2$.

After presenting tensor one-loop three-point integrals to scalar functions, we next apply IBP for scalar one-loop functions with the general propagator indexes. We then arrive at the following system of equations:

\[
\begin{align*}
(d - 2v_1 - v_2 - v_3)1 - v_1^+1^+3^+ &= v_1(2M^2)1^+ + v_2(2M^2 - q_1^2)1^+ + v_3(2M^2 - q_2^2)1^+, \\
(d - v_1 - 2v_2 - v_3)1 - v_1^+2^- - v_2^-3^+ &= v_1(2M^2 - q_1^2)1^+ + v_2(2M^2)2^+ + v_3(2M^2 - p^2)3^+, \\
(d - v_1 - v_2 - 2v_3)1 - v_1^+3^- - v_2^-3^+ &= v_1(2M^2 - q_1^2)1^+ + v_2(2M^2 - p^2)2^+ + v_3(2M^2)3^+. 
\end{align*}
\]

(B.5)

Here, the standard notation for increasing and lowering operators,

\[
\mathbf{j}^\pm J_3(d; \{v_j\}) = J_3(d; \{v_j \pm 1\}), \tag{B.6}
\]

11/17
is used for \( j = 1, 2, 3 \).

In the following paragraphs, we consider master integrals \( J_3(d; \{v_1, v_2, v_3\}) \) by solving the above system of equations in several special cases. In conclusion, the master integrals shown at Sect. 2 are presented in terms of hypergeometric functions \( _3F_2 \) in this paper.

**B.1. Case 1:** \( v_1 = v_2 = v_3 = 1 \)

\[
J_3(d; \{1, 2, 1\}; p_2^2, M_{H}, M^2) = \frac{2}{(p_2^2 - M_{H}^2)} \left[ J_2(d; \{2, 1, M_{H}^2, M^2\}) - J_2(d; \{2, 1, p_2^2, M^2\}) \right], \tag{B.7}
\]

\[
J_3(d; \{2, 1, 1\}; p_2^2, M_{H}^2, M^2) = \frac{(d - 4)M_{H}^2}{2M^2(p_2^2 - M_{H}^2)} J_3(d; \{1, 1, 1\}; p_2^2, M_{H}^2, M^2) + \frac{2}{(p_2^2 - M_{H}^2)} J_2(d; \{2, 1, 0, M^2\})
\]

\[
+ \frac{M_{H}^2(4M^2 - M_{H}^2)}{M^2(M_{H}^2 - p_2^2)^2} J_2(d; \{2, 1, M_{H}^2, M^2\}) + \frac{p_2^2M_{H}^2 - 2M^2(p_2^2 + M_{H}^2)}{M^2(M_{H}^2 - p_2^2)^2} J_2(d; \{2, 1, p_2^2, M^2\}). \tag{B.8}
\]

\[
J_3(d; \{1, 1, 2\}; p_2^2, M_{H}^2, M^2) = \frac{(d - 4)p_2^2}{2M^2(p_2^2 - M_{H}^2)} J_3(d; \{1, 1, 1\}; p_2^2, M_{H}^2, M^2) + \frac{2}{(M_{H}^2 - p_2^2)} J_2(d; \{2, 1, 0, M^2\})
\]

\[
+ \frac{p_2^2M_{H}^2 - 2M^2(M_{H}^2 + p_2^2)}{M^2(M_{H}^2 - p_2^2)^2} J_2(d; \{2, 1, M_{H}^2, M^2\}) + \frac{p_2^2(4M^2 - p_2^2)}{M^2(M_{H}^2 - p_2^2)^2} J_2(d; \{2, 1, p_2^2, M^2\}). \tag{B.9}
\]

**B.2. Case 2:** \( v_1 = 1, v_2 = 2, v_3 = 1 \)

\[
J_3(d; \{1, 3, 1\}; p_2^2, M_{H}^2, M^2) = \frac{1}{2(p_2^2 - M_{H}^2)} \left[ J_2(d; \{2, 2, M_{H}^2, M^2\}) - J_2(d; \{2, 2, p_2^2, M^2\}) \right]
\]

\[
+ \frac{1}{(p_2^2 - M_{H}^2)} \left[ J_2(d; \{3, 1, M_{H}^2, M^2\}) - J_2(d; \{3, 1, p_2^2, M^2\}) \right]. \tag{B.10}
\]

\[
J_3(d; \{2, 2, 1\}; p_2^2, M_{H}^2, M^2) = \frac{(4 - d)}{2M^2(M_{H}^2 - p_2^2)} J_3(d; \{1, 1, 1\}; p_2^2, M_{H}^2, M^2)
\]

\[
+ \frac{M_{H}^2(4M^2 - M_{H}^2)}{2M^2(M_{H}^2 - p_2^2)^2} \left[ J_2(d; \{2, 2, M_{H}^2, M^2\}) + 2J_2(d; \{3, 1, M_{H}^2, M^2\}) \right]
\]

\[
+ \frac{p_2^2M_{H}^2 - 2M^2(M_{H}^2 + p_2^2)}{2M^2(M_{H}^2 - p_2^2)^2} \left[ J_2(d; \{2, 2, p_2^2, M^2\}) + 2J_2(d; \{3, 1, p_2^2, M^2\}) \right]
\]

\[
+ \frac{(6 - d)M_{H}^2}{M^2(M_{H}^2 - p_2^2)^2} J_2(d; \{2, 1, M_{H}^2, M^2\}) + \frac{(d - 5)M_{H}^2 - p_2^2}{M^2(M_{H}^2 - p_2^2)^2} J_2(d; \{2, 1, p_2^2, M^2\}). \tag{B.11}
\]
Table C3. Couplings involving the decay $H \rightarrow Z\gamma$. In our notation, $\lambda'_1 = eQ_i$, $\lambda'_2 = \frac{g}{2\cos\theta_W} (l_i^3 - 2Q_i \sin^2\theta_W - 2Q_i \sin\theta_W \cos\theta_W)$, and $\lambda'_3 = -\frac{g}{2\cos\theta_W} l_i^3$, where $l_i^3$ and $Q_i$ are iso-spin and electric charge of top quarks in the loops. The term $i\lambda'_1\gamma^\mu$ should be the coupling of photons to top quarks.

| Vertices Couplings |  \\
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $HW_i W_\nu$       | $igM_W g^{\mu\nu}$  | $Z_{\mu}(k_1) W_{\nu}(k_2) W_{\nu}(k_3)$ | $-ig \cos\theta_W \left[ (k_1 - k_2)_\mu g^{\alpha\nu} + (k_2 - k_3)_\mu g^{\beta\nu} + (k_3 - k_1)_\mu g^{\gamma\nu} \right]$ | $A_{\mu}(k_1) W_{\nu}(k_2) W_{\nu}(k_3)$ | $-ie \left[ (k_1 - k_2)_\mu g^{\alpha\nu} + (k_2 - k_3)_\mu g^{\beta\nu} + (k_3 - k_1)_\mu g^{\gamma\nu} \right]$ | $A_{\mu}(Z_{\nu} W_{\nu} W_\beta)$ | $-ig \cos\theta_W \left[ 2g^{\alpha\beta} g^{\nu\rho} - g^{\alpha\nu} g^{\rho\beta} - g^{\alpha\rho} g^{\nu\beta} \right]$ | $HZ_{\mu} Z_{\nu}$ | $i gM_Z g^{\mu\nu} / \cos\theta_W$ | $H_{tt}$ | $-igm_t/2M_W$ | $tuZ_{\mu}$ | $i (\lambda'_1 + \lambda'_2) \gamma^\mu + i \lambda'_1\gamma^\mu \gamma^\nu$ | $t\mu A_{\mu}$ | $i eQ_i \gamma^\mu$ |

\begin{align}
J_3(d; \{1, 2, 2\}; p^2, M_H^2, M^2) &= \frac{(d - 4)}{2M^2(M_H^2 - P_1^2)} J_3(d; \{1, 1, 1\}; p^2, M_H^2, M^2) \\
&+ \frac{p^2 M_H^2 - 2M^2(M_H^2 + p^2)}{2M^2(M_H^2 - P_1^2)^2} \left[ J_2(d; \{2, 2\}, M_H^2, M^2) + 2J_2(d; \{3, 1\}, M_H^2, M^2) \right] \\
&+ \frac{p^2 (4M^2 - P_1^2)}{2M^2(M_H^2 - P_1^2)^2} \left[ J_2(d; \{2, 2\}, p^2, M^2) + 2J_2(d; \{3, 1\}, p^2, M^2) \right] \\
&+ \frac{(d - 5)p^2 - M_H^2}{M^2(M_H^2 - P_1^2)^2} J_2(d; \{2, 1\}, M_H^2, M^2) + \frac{(6 - d)p^2}{M^2(M_H^2 - P_1^2)^2} J_2(d; \{2, 1\}, p^2, M^2). \quad (B.12)
\end{align}

Appendix C. One-loop amplitudes for $H \rightarrow Z\gamma$

We present detailed calculations for the decay amplitude of $H \rightarrow Z\gamma$ in unitary gauge in this appendix. All couplings involving the decay process are listed in Table C3. The decay amplitude $H \rightarrow Z\gamma$ of top-loop diagrams is expressed as follows:

$$iA_{H \rightarrow Z\gamma}^{(T)} = -\frac{eQ_i g m_t^2}{(4\pi)^{d/2} M_W} (\lambda'_1 + \lambda'_2) \int \frac{d^d k}{i\pi^{d/2}} \frac{\epsilon_{\alpha\beta\gamma\delta}(q_1) \epsilon_{\gamma\delta\alpha\beta}(q_2)}{P_1 P_2 P_3} \left\{ 16 k^\mu k^\nu + 8k^\nu q_1^\mu + 16 k^\mu q_2^\nu + 4q_1^\mu q_2^\nu - g^{\mu\nu} \left[ 8(k \cdot q_2) + 4k^2 + (2M_H^2 - 2M_Z^2 - 4m_t^2) \right] \right\}, \quad (C.1)$$

where the coefficient factors are written in terms of the master integrals:

\begin{align}
\int \frac{d^d k}{i\pi^{d/2}} \frac{k^\mu k^\nu}{P_1 P_2 P_3} &= -\frac{g^{\mu\nu}}{2} J_3(d + 2; \{1, 1, 1\}; M_Z^2, M_H^2, m_t^2) \\
&+ q_1^\mu q_1^\nu J_3(d + 4; \{1, 3, 1\}; M_Z^2, M_H^2, m_t^2) \\
&+ q_2^\mu q_2^\nu J_3(d + 4; \{2, 2, 1\}; M_Z^2, M_H^2, m_t^2) + J_3(d + 4; \{1, 3, 1\}; M_Z^2, M_H^2, m_t^2), \quad (C.2)
\end{align}

\begin{align}
\int \frac{d^d k}{i\pi^{d/2}} \frac{k^\mu q_1^\nu}{P_1 P_2 P_3} &= q_1^\mu q_1^\nu J_3(d + 2; \{1, 2, 1\}; M_Z^2, M_H^2, m_t^2), \quad (C.3)
\end{align}
\[
\int \frac{d^d k}{i\pi^{d/2}} \frac{k^\mu q_2^\nu}{P_1 P_2 P_3} = q_2^\mu q_1^\nu J_3(d + 2; \{1, 2, 1\}; M_{\tilde{Z}}, M_{\tilde{H}}, m_1^2),
\]
(C.4)
\[
\int \frac{d^d k}{i\pi^{d/2}} \frac{q_1^\mu q_2^\nu}{P_1 P_2 P_3} = q_2^\mu q_1^\nu J_3(d; \{1, 1, 1\}; M_{\tilde{Z}}, M_{\tilde{H}}, m_1^2),
\]
(C.5)
\[
\int \frac{d^d k}{i\pi^{d/2}} \frac{k \cdot q_2}{P_1 P_2 P_3} = \frac{1}{2} \left[ J_2(d; \{1, 1\}; M_{\tilde{Z}}, m_1^2) - J_2(d; \{1, 1\}; M_{\tilde{Z}}, m_1^2) \right],
\]
(C.6)
\[
\int \frac{d^d k}{i\pi^{d/2}} \frac{k^2}{P_1 P_2 P_3} = J_2(d; \{1, 1\}; M_{\tilde{Z}}, m_1^2) + m_1^2 J_3(d; \{1, 1, 1\}; M_{\tilde{Z}}, M_{\tilde{H}}, m_1^2).
\]
(C.7)

For the $W$ boson loop contributions, the decay amplitude is written
\[
i A_{H \rightarrow Z\gamma}^{(W)} = \frac{ie g^2 \cos \theta_W}{(4\pi)^{d/2} M_W^5} \int \frac{d^d k}{i\pi^{d/2}} \epsilon_1^{\mu*}(q_1) \epsilon_2^{\nu*}(q_2)
\]
\[
\times \left[ i A_1 g^{\mu\nu} + i A_2 k^\mu k^\nu + i A_3 k^\mu q_1^\nu + i A_4 q_2^\mu q_1^\nu + i A_5 q_2^\mu q_1^\nu + i A_6 q_1^\mu q_2^\nu + i A_7 q_1^\nu q_2^\mu \right],
\]
(C.8)

where the coefficient factors are presented in terms of the master integrals in detail:
\[
\int \frac{d^d k}{i\pi^{d/2}} (i A_1 g^{\mu\nu}) = \int \frac{d^d k}{i\pi^{d/2}} \left\{ \frac{1}{P_1 P_2 P_3} \left[ 2M_W^4 \left( M_{\tilde{Z}}^2 - 4M_W^2 \right) \left( M_{\tilde{H}}^2 - M_{\tilde{Z}}^2 \right) \right]
\]
\[
+ \frac{1}{P_2 P_3} \left[ 2M_W^4 \left( M_{\tilde{Z}}^2 - M_{\tilde{H}}^2 \right) + M_{\tilde{H}}^2 M_W^2 M_{\tilde{Z}}^2 - \frac{M_{\tilde{H}}^4 M_W^2}{2} \right]
\]
\[
+ \frac{1}{P_2 P_3} \left[ - P_1 \left( M_{\tilde{H}}^2 M_W^2 + 2M_W^4 \right) + 2M_W^2 (1 - d) \right] + \frac{1}{P_1} \left( - 2M_W^2 \right)
\]
\[
+ \left( \frac{1}{P_2} + \frac{1}{P_3} \right) \left[ \frac{M_W^2}{2} (2P_1 - P_2 - P_3) + M_{\tilde{W}}^2 \left( 2M_{\tilde{W}}^2 + M_{\tilde{H}}^2 - M_{\tilde{Z}}^2 \right) \right] \right\} g^{\mu\nu}
\]
(C.9)
\[
\int \frac{d^d k}{i\pi^{d/2}} (i A_2 k^\mu k^\nu) = \int \frac{d^d k}{i\pi^{d/2}} \left\{ \frac{k^\mu k^\nu}{P_1 P_2 P_3} \left[ 4M_W^4 \left( M_{\tilde{H}}^2 - M_{\tilde{Z}}^2 \right) - 2M_W^2 M_{\tilde{H}}^2 M_{\tilde{Z}}^2 + 8M_W^6 (d - 1) \right]
\]
\[
+ \frac{k^\mu k^\nu}{P_1 P_2 P_3} \left( 2M_W^2 M_{\tilde{Z}}^2 \right) \right\} g^{\mu\nu}
\]
(C.10)
\[
\int \frac{d^d k}{i \pi^{d/2}} (i A_3 k^\mu q_1^v) = \int \frac{d^d k}{i \pi^{d/2}} \left\{ k^\mu q_1^v \left( 4M_W^4 - 2M_W^2 M_Z^2 \right) - k^\mu q_1^v \left( \frac{M_H^2 M_Z^2}{2} + 7M_W^4 \right) + \frac{M_W^2}{2} \left( \frac{k^\mu q_1^v}{P_2} + \frac{k^\mu q_1^v}{P_3} \right) \right\}
\]
\[
= \left\{ \left( 2M_W^2 M_Z^2 - 4M_W^4 \right) J_2(d; \{1, 1\}; M_Z^2, M_W^2) - \left( \frac{M_H^2 M_Z^2}{2} + 7M_W^4 \right) J_2(d + 2; \{2, 1\}; M_Z^2, M_W^2) + \frac{M_W^2}{2} J_1(d; \{1\}; M_W^2) \right\} q_1^\mu q_1^v, \quad (C.12)
\]

\[
\int \frac{d^d k}{i \pi^{d/2}} (i A_4 q_2^\mu k^v) = \int \frac{d^d k}{i \pi^{d/2}} \left\{ q_2^\mu k^v \left( 4M_W^4 - 2M_W^2 M_Z^2 \right) - \left( \frac{M_H^2 M_Z^2}{2} + 7M_W^4 \right) J_3(d + 1; \{2, 1, 1\}; M_Z^2, M_H^2, M_W^2) + \frac{M_W^2}{2} J_1(d; \{1\}; M_W^2) \right\} q_2^\mu q_1^v, \quad (C.13)
\]

\[
\int \frac{d^d k}{i \pi^{d/2}} (i A_5 q_2^\mu q_1^v) = \int \frac{d^d k}{i \pi^{d/2}} \left\{ q_2^\mu q_1^v \left( 4M_W^4 - 2M_W^2 M_Z^2 \right) - \left( \frac{M_H^2 M_Z^2}{2} + 7M_W^4 \right) J_3(d + 1; \{2, 1, 1\}; M_Z^2, M_H^2, M_W^2) \right\} q_2^\mu q_1^v, \quad (C.14)
\]

\[
\int \frac{d^d k}{i \pi^{d/2}} (i A_6 q_1^\mu k^v) = \int \frac{d^d k}{i \pi^{d/2}} \left\{ q_1^\mu k^v \left( 4M_W^4 - 2M_W^2 M_Z^2 \right) - \left( \frac{M_H^2 M_Z^2}{2} + 7M_W^4 \right) J_3(d + 1; \{2, 1, 1\}; M_Z^2, M_H^2, M_W^2) \right\} q_1^\mu q_1^v, \quad (C.15)
\]
Appendix D. Form factors at general $d$

The form factors in Eq. (9) are

\[
F_{00}^{(t)} = -\frac{eQ_{t}g_{t}^{2}}{(4\pi)^{d/2}M_{W}} \left( \lambda_{1}^{(t)} + \lambda_{2}^{(t)} \right) \left[ 2M_{Z}^{2} - 2M_{H}^{2} \right] J_{3}(d; \{1, 1, 1\}; M_{Z}^{2}, M_{H}^{2}, m_{t}^{2}) - 8J_{3}(d + 2; \{1, 1, 1\}; M_{Z}^{2}, M_{H}^{2}, m_{t}^{2}) - 4J_{2}(d; \{1, 1\}; M_{H}^{2}, m_{t}^{2}),
\]

(D.1)

\[
F_{11}^{(t)} = -\frac{eQ_{t}g_{t}^{2}}{(4\pi)^{d/2}M_{W}} \left( \lambda_{1}^{(t)} + \lambda_{2}^{(t)} \right) \left[ 16J_{3}(d + 4; \{1, 3, 1\}; M_{Z}^{2}, M_{H}^{2}, m_{t}^{2}) + 8J_{3}(d + 2; \{1, 2, 1\}; M_{Z}^{2}, M_{H}^{2}, m_{t}^{2}) \right],
\]

(D.2)

\[
F_{21}^{(t)} = -\frac{eQ_{t}g_{t}^{2}}{(4\pi)^{d/2}M_{W}} \left( \lambda_{1}^{(t)} + \lambda_{2}^{(t)} \right) \left[ 16J_{3}(d + 4; \{2, 2, 1\}; M_{Z}^{2}, M_{H}^{2}, m_{t}^{2}) + 8J_{3}(d + 2; \{1, 2, 1\}; M_{Z}^{2}, M_{H}^{2}, m_{t}^{2}) + 4J_{3}(d; \{1, 1, 1\}; M_{Z}^{2}, M_{H}^{2}, m_{t}^{2}) \right],
\]

(D.3)

\[
F_{S}^{(t)} = 0,
\]

(D.4)

and

\[
F_{00}^{(W)} = \frac{eg_{W}^{2} \cos \theta_{W}}{(4\pi)^{d/2}M_{W}^{2}} \left\{ 2M_{W}^{4}(M_{Z}^{2} - 4M_{H}^{2})(M_{H}^{2} - M_{Z}^{2})J_{3}(d; \{1, 1, 1\}; M_{Z}^{2}, M_{H}^{2}, M_{W}^{2}) + \left[ 2M_{W}^{4}(M_{Z}^{2} - M_{H}^{2}) + M_{H}^{2}M_{W}^{2}M_{Z}^{2} - 4M_{W}^{6}(d - 1) \right]J_{3}(d + 2; \{1, 1, 1\}; M_{Z}^{2}, M_{H}^{2}, M_{W}^{2}) + \left[ 2M_{W}^{4}(M_{H}^{2} - M_{Z}^{2}) - M_{H}^{2}M_{W}^{2}M_{Z}^{2} + 4M_{W}^{6}(d - 1) \right]J_{2}(d + 2; \{2, 1\}; M_{H}^{2}, M_{W}^{2}) \right\},
\]

(D.5)

\[
F_{11}^{(W)} = \frac{eg_{W}^{2} \cos \theta_{W}}{(4\pi)^{d/2}M_{W}^{2}}
\]
\[
F_{21}^{(W)} = \frac{e g^2 \cos \theta_W}{(4\pi)^{d/2} M_W^2} \left\{ 4 M_W^4 \left( M_H^2 - M_Z^2 \right) J_3 \left( d; \{1,1,1\}; M_Z^2, M_H^2, M_W^2 \right) + \left[ 2 M_W^2 \left( M_H^2 - M_Z^2 \right) - M_H^2 M_W^2 M_Z^2 + 8 M_W^6 (d - 1) \right] J_3 \left( d + 2; \{1,2,1\}; M_Z^2, M_H^2, M_W^2 \right) \right\}.
\]

(D.6)

\[
F_{21}^{(W)} = \frac{e g^2 \cos \theta_W}{(4\pi)^{d/2} M_W^2} \left\{ 4 M_W^4 \left( M_H^2 - M_Z^2 \right) J_3 \left( d; \{1,1,1\}; M_Z^2, M_H^2, M_W^2 \right) + \left[ 2 M_W^2 \left( M_H^2 - M_Z^2 \right) - M_H^2 M_W^2 M_Z^2 + 8 M_W^6 (d - 1) \right] J_3 \left( d + 2; \{1,2,1\}; M_Z^2, M_H^2, M_W^2 \right) \right\}.
\]

(D.7)

References

[1] V. M. Abazov et al. [DØ Collaboration], Phys. Lett. B 671, 349 (2009).
[2] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 726, 587 (2013).
[3] M. Aaboud et al. [ATLAS Collaboration], J. High. Energy Phys. 1710, 112 (2017).
[4] R. N. Cahn, M. S. Chanowitz and N. Fleishon, Phys. Lett. B. 82, 113 (1979).
[5] L. J. Slater, Generalized Hypergeometric Functions (Cambridge University Press, Cambridge, 1966).