Nambu-Goldstone Bosons in CP Violating theory with Majorana Masses

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ABSTRACT

We derive some properties of the Nambu-Goldstone boson coupling in theories that have CP violation and Majorana masses. We show explicitly that its diagonal coupling to a Majorana fermion is pseudoscalar not scalar. This clarifies some confusion in the literature. Some potentially useful off-diagonal properties are also derived. We also show, in the process, that the Goldstone theorem often produces interesting and nontrivial identities in matrix theory which may be hard to prove otherwise.

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Introduction

Theories with Majorana masses and CP violation have been taken more seriously recently because of the experimental evidence for neutrino masses\cite{1,2,3}. They often contain some ingredients of spontaneous symmetry breaking of either gauge or global symmetries, and as a result, some accompanying unphysical Higgs or Nambu-Goldstone (NG) bosons. For example, the theory with spontaneously broken lepton number usually produces a NG boson, usually called majoron. It was well known that the coupling of the NG boson to the fermion should be always pseudoscalar in nature. This is because the degree of freedom of the NG boson usually can be identified as the symmetry generator which is spontaneously broken\cite{4}. Under the symmetry transformation, the associated NG boson is translated by a constant. For the theory to be invariant under this translation, the NG boson has to couple only through its derivative\cite{5}. For the couplings that are diagonal in the fermion flavor, such derivative couplings translate into pseudoscalar coupling using equation of motion. However, there are some recent doubt on the applicability of the above argument to the theory with Majorana masses and CP violation\cite{6}. In this paper we wish to address this issue in detail and reaffirm that the above argument remains true. The conclusion is useful not only for the neutrino sector, but also for the chargino and the neutralino sectors of the supersymmetric theories\cite{7}. For example, in Ref\cite{8}, it was...
shown that the diagonal couplings of the unphysical NG Higgs boson to charginos are pseudoscalar even though the relevant sector of the theory is CP violating with Majorana gaugino masses.

As a result of this analysis, we found that very interesting and nontrivial theorems in matrix algebra can be derived as a direct consequence of the Goldstone theorem.

In the next section we will use the simplest example in $SU(2)_L \times U(1)$ context with one flavor of neutrino species (including a right handed neutrino) to illustrate the pseudoscalar nature of the NG boson coupling. We will prove that the NG boson coupling is pseudoscalar exactly. Then we will generalize it to $N$ flavors of neutrinos. In both cases, the fact the diagonal NG boson coupling has to be pseudoscalar gives rise to some nontrivial matrix theorems. To explore these matrix theorems further, we consider a very general theory with many NG bosons and then formulate a resulting matrix theorem which is simple but very nontrivial. In the appendix we include a general proof of the pseudoscalar nature of the diagonal NG boson couplings for completeness.

2 × 2 matrices of $\nu$ masses and couplings

The model is a combination of the singlet majoron model[9] and the triplet majoron model[10]. Details of the model can be found in Ref.[6]. In addition to the known fermions in the Standard Model (SM), there is an antineutrino (denoted as $\nu^c$) which is a singlet under the gauge group with the lepton number $L = -1$. The scalar bosons include one $SU(2)$ doublet $H$ (SM Higgs), one $SU(2)$ triplet $\chi$ and one gauge singlet $S$. $H$ has hypercharge $Y = -\frac{1}{2}$, lepton number $L = 0$, and $\chi$ has $Y = 1$, $L = -2$, $S$ has $Y = 0$, $L = -2$. Let us start with one generation, the relevant quantum numbers are summarized below

|   | $\nu$ | $\nu^c$ | $H$ | $\chi$ | $S$ |
|---|-------|--------|-----|--------|-----|
| $Y$ | $-\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 1 | 0 |
| $L$ | +1 | -1 | 0 | -2 | -2 |

Redefining each scalar field as deviation from its vacuum expectation value (vev), we write the Yukawa terms in the Hamiltonian after symmetry breaking as

$$\frac{2m_D}{v_H} \nu^c (H^{0*} + v_H) + \frac{M}{v_S} \nu^c (S^* + v_S) + \frac{m}{v_\chi} \nu \nu (\chi^0 + v_\chi) + \text{h.c.}$$  \hspace{1cm} (1)

Bilinear form of Weyl fermions are understood to be connected by the Levi-Civita symbol. The neutrino form of Weyl fermions are understood to be connected by the Levi-Civita symbol. The relevant quantum numbers are summarized below

$$\left( \begin{array}{c} \nu \\ \nu^c \end{array} \right) = \left( \begin{array}{cc} m_D \\ m_D \\ m \end{array} \right) \left( \begin{array}{c} \nu \\ \nu^c \end{array} \right) \left( \begin{array}{c} M \\ M \\ M \end{array} \right) \left( \begin{array}{c} \nu \\ \nu^c \end{array} \right) + \text{h.c.}$$  \hspace{1cm} (2)

Two of the three complex masses can be made real by rotating the phases of the neutrino fields, leaving one CP violating mass parameter. The composition of the majoron can be derived from vacuum expectation values as

$$G_M = \sqrt{2/N} \left[ 2v^2_\chi v_H \text{Im}(H^0) + v^2_\chi v_\chi \text{Im}(\chi^0) + (v^2_H + 4v^2_\chi)v_S \text{Im}(S) \right],$$  \hspace{1cm} (3)

where $N = \sqrt{4v^4_\chi v^2_H + v^4_H v^2_\chi + (v^2_H + 4v^2_\chi)^2 v^2_S}$. The majoron-neutrinos coupling terms in the Hamiltonian are then

$$- \frac{i}{\sqrt{2N}} \left( \begin{array}{c} \nu \\ \nu^c \end{array} \right) \left( \begin{array}{c} -m_D v_H v^2_\chi \\ m_D 2v_H v^2_\chi \\ M v_S (v^2_H + 4v^2_\chi) \end{array} \right) \left( \begin{array}{c} \nu \\ \nu^c \end{array} \right) G_M + \text{h.c.}$$  \hspace{1cm} (4)
The above coupling $2 \times 2$ matrix has the simple form $-i(\sqrt{2}N)^{-1}C$, where $C$ is

$$C \equiv \begin{pmatrix} -m v_H^2 & 2m_D v_\chi^2 \\ 2m_D v_\chi^2 & M(v_H^2 + 4v_\chi^2) \end{pmatrix}.$$  \hspace{1cm} (5)$$

The symmetric neutrinos mass matrix $M_n$ can be diagonalized as

$$U^T M_n U = M_{\text{diag}}, \quad M_n = \begin{pmatrix} m & m_D \\ m_D & M \end{pmatrix},$$  \hspace{1cm} (6)$$

where matrix $U$ is unitary and $M_{\text{diag}}$ is a diagonal matrix with real non-negative elements. In the mass eigenstate basis the coupling matrix of the majoron to neutrinos becomes $-i(\sqrt{2}N)^{-1}U^T C U$. The scalar couplings of the majoron are proportional to imaginary parts of diagonal elements of $U^T C U$. One can show directly that this diagonal scalar coupling vanishes by explicitly diagonalizing the mass matrix order by order in $m/M$ (a procedure which was adopted in Ref[6]). We will give such an approximate calculation in the Appendix for comparison with Ref[6]. Here we are going to prove that the diagonal scalar couplings of the majoron are zero using the matrix property without assumption about the size of $m/M$. Define $r = 2v_\chi^2/v_H^2$, we have

$$C = v_H^2 \begin{pmatrix} -m & m_Dr \\ m_Dr & M(1 + 2r) \end{pmatrix} = v_H^2 (1 + r) \begin{pmatrix} -m & 0 \\ 0 & M \end{pmatrix} + v_H^2 r M_n.$$  \hspace{1cm} (7)$$

To prove that the diagonal elements of $U^T C U$ are real, we only need to show that

$$N' \equiv U^T \begin{pmatrix} -m & 0 \\ 0 & M \end{pmatrix} U$$  \hspace{1cm} (8)$$

has real diagonal elements.

$$N'_1 = -m U_{11}^2 + M U_{21}^2,$$  \hspace{1cm} (9)$$

$$N'_2 = -m U_{21}^2 + M U_{22}^2.$$  \hspace{1cm} (10)$$

Denote $m_1, m_2$ as the neutrino mass eigenvalues. Then

$$\begin{pmatrix} m & m_D \\ m_D & M \end{pmatrix} = U^* \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} U^\dagger.$$  \hspace{1cm} (11)$$

Express $m$ and $M$ as $m_1$ and $m_2$,

$$m = (U_{11}^*)^2 m_1 + (U_{12}^*)^2 m_2,$$  \hspace{1cm} (12)$$

$$M = (U_{21}^*)^2 m_1 + (U_{22}^*)^2 m_2.$$  \hspace{1cm} (13)$$

Unitarity of $U$ gives

$$N'_{11} = -((U_{11}^*)^2 m_1 + (U_{12}^*)^2 m_2) U_{11}^2 + ((U_{21}^*)^2 m_1 + (U_{22}^*)^2 m_2) U_{21}^2$$

$$= ((U_{21} U_{11}^*)^2 - (U_{11} U_{11}^*)^2) m_1,$$  \hspace{1cm} (14)$$

$$N'_{22} = -((U_{11}^*)^2 m_1 + (U_{12}^*)^2 m_2) U_{12}^2 + ((U_{21}^*)^2 m_1 + (U_{22}^*)^2 m_2) U_{22}^2$$

$$= ((U_{22} U_{12}^*)^2 - (U_{12} U_{12}^*)^2) m_2.$$  \hspace{1cm} (15)$$
which are real. So the diagonal scalar couplings of the majoron to the neutrinos are zero even if the couplings are CP violating.

Next consider the off-diagonal terms of the majoron couplings. Note that $U^T C U$ and $v^2_H (1 + r) N'$ have identical off-diagonal components. Using unitarity,

$$ N'_{12} = N'_{21} = (m_1 + m_2) \text{Re}(U^*_{21} U_{22}) + i(m_1 - m_2) \text{Im}(U^*_{21} U_{22}) . $$  

(16)

It means, if the neutrinos are degenerate, the off-diagonal couplings of the majoron are also pseudoscalar. If $(m_1 + m_2) = 0$ then the off diagonal couplings of the majoron are purely scalar. This is not surprising because if $(m_1 + m_2) = 0$ then one can redefine one of the neutrino by $i$, after that it is reduced to the degenerate case and off-diagonal coupling picked up an $i$. These results are consequences of the Goldstone Theorem.

If the global symmetry, say the lepton number in the current example, is an accidental symmetry only valid for renormalizable terms of fields in low energy physics, but broken by higher dimensional operators induced from very short distance physics at the scale $M_X$. In general, the NG boson picks up a tiny mass\[11] which is suppressed by the factor $1/M_X$. CP violation can probably give rise to a scalar diagonal coupling which is also suppressed by $1/M_X$, not by the less reducing factor $1/M$ from the neutrino sector. This mechanism\[12] provides a possible source of a feeble fifth force competing with gravity.

2\(N\times 2N\) matrices of \(N\) generations

If there are \(N\) generations of neutrinos. The above proof can also be applied. We denote the $2N \times 2N$ symmetry mass matrix as

$$ M_n = \begin{pmatrix} \hat{m} & \hat{m}_D \\ \hat{m}_D^T & \hat{M} \end{pmatrix} , $$  

(17)

where $\hat{m}$, $\hat{m}_D$ and $\hat{M}$ are $N \times N$ matrix. We can also find a unitary matrix $U$ to diagonalize $M_n$ such that

$$ U^T M_n U = \begin{pmatrix} \hat{m}_1 & 0 \\ 0 & \hat{m}_2 \end{pmatrix} , \quad U = \begin{pmatrix} \hat{U}_{11} & \hat{U}_{12} \\ \hat{U}_{21} & \hat{U}_{22} \end{pmatrix} , $$  

(18)

where $\hat{m}_1$ and $\hat{m}_2$ are real $N \times N$ diagonal matrices, and $\hat{U}_{11}, \hat{U}_{12}, \hat{U}_{21}, \hat{U}_{22}$ are four $N \times N$ matrix satisfying the unitary conditions,

$$ \hat{U}_{11}^\dagger \hat{U}_{11} + \hat{U}_{21}^\dagger \hat{U}_{21} = 1 , $$  

(19)

$$ \hat{U}_{12}^\dagger \hat{U}_{12} + \hat{U}_{22}^\dagger \hat{U}_{22} = 1 , $$  

(20)

$$ \hat{U}_{11}^\dagger \hat{U}_{12} + \hat{U}_{21}^\dagger \hat{U}_{22} = 0 . $$  

(21)

The majoron coupling matrix in mass eigenstate basis is given by

$$ - \frac{i}{\sqrt{2N}} U^T \begin{pmatrix} -\hat{m} v_H^2 & 2\hat{m}_D v_\chi^2 \\ 2\hat{m}_D^T v_\chi^2 & \hat{M}(v_H^2 + 4v_\chi^2) \end{pmatrix} U . $$  

(22)

Using the same trick as above, we only need to show that

$$ N' = U^T \begin{pmatrix} -\hat{m} & 0 \\ 0 & \hat{M} \end{pmatrix} U $$  

(23)
We express \( \hat{m} \) and \( \hat{M} \) as \( \hat{m}_1 \) and \( \hat{m}_2 \),

\[
\hat{m} = \hat{U}^* \hat{m}_1 \hat{U}^\dagger_{11} + \hat{U}^* \hat{m}_2 \hat{U}^\dagger_{12} ,
\]

\[
\hat{M} = \hat{U}^* \hat{m}_1 \hat{U}^\dagger_{21} + \hat{U}^* \hat{m}_2 \hat{U}^\dagger_{22} .
\]

Then, using the unitary conditions (19-21), we obtain

\[
\hat{N}'_{11} = -\hat{U}^T_{11} \hat{m} \hat{U}^\dagger_{11} + \hat{U}^T_{21} \hat{M} \hat{U}^\dagger_{21} ,
\]

\[
= \hat{m}_1 - (\hat{m}_1 \hat{U}^\dagger_{11} \hat{U}^\dagger_{11} + \hat{U}^T_{11} \hat{U}^\dagger_{11} \hat{m}_1) .
\]

Similarly,

\[
\hat{N}'_{22} = \hat{m}_2 \hat{U}^\dagger_{22} \hat{U}^\dagger_{22} + \hat{U}^T_{22} \hat{M} \hat{U}^\dagger_{22} - \hat{m}_2 .
\]

In terms of the mass eigenvalues \( (\hat{m}_1)_i \) and \( (\hat{m}_2)_i \), we have

\[
(\hat{N}'_{11})_{ij} = +((\hat{m}_1)_i \delta_{ij} - (\hat{m}_1)_i + (\hat{m}_1)_j) \text{Re}(\hat{U}^\dagger_{11} \hat{U}_{11})_{ij} - i((\hat{m}_1)_i - (\hat{m}_1)_j) \text{Im}(\hat{U}^\dagger_{11} \hat{U}_{11})_{ij} ,
\]

\[
(\hat{N}'_{22})_{ij} = -((\hat{m}_2)_i \delta_{ij} + (\hat{m}_2)_i + (\hat{m}_2)_j) \text{Re}(\hat{U}^\dagger_{22} \hat{U}_{22})_{ij} + i((\hat{m}_2)_i - (\hat{m}_2)_j) \text{Im}(\hat{U}^\dagger_{22} \hat{U}_{22})_{ij} .
\]

No dummy index summation occurs in above expression. It is obvious that the diagonal components of both \( \hat{N}'_{11} \) and \( \hat{N}'_{22} \) are real. The off-diagonal block is

\[
\hat{N}'_{12} = \hat{N}'_{21}^T = \hat{m}_1 \hat{U}^\dagger_{21} \hat{U}^\dagger_{22} + \hat{U}^T_{21} \hat{U}^\dagger_{22} \hat{m}_2
\]

\[
(\hat{N}'_{12})_{ij} = ((\hat{m}_1)_i + (\hat{m}_2)_j) \text{Re}(\hat{U}_{21}^\dagger \hat{U}_{22})_{ij} + i((\hat{m}_1)_i - (\hat{m}_2)_j) \text{Im}(\hat{U}_{21}^\dagger \hat{U}_{22})_{ij} .
\]

These results all can be understood through arguments similar to that in the last section.

**General \( N \times N \) mass matrix**

In this section, we demonstrate how the Goldstone theorem can be used to prove a matrix theorem.

Theorem : Given any complex symmetric matrix of dimension \( N \times N \),

\[
M = \begin{pmatrix}
  m_{11} & m_{12} & \cdots \\
  m_{12} & m_{22} & \cdots \\
  \vdots & \vdots & \ddots \\
  m_{N1} & m_{N2} & \cdots & m_{NN}
\end{pmatrix},
\]

we decompose \( M \) as the sum of \( \frac{1}{2} \mathbf{C}_i \),

\[
\mathbf{C}_i = \begin{pmatrix}
  \cdots & m_{i1} & \cdots \\
  0 & \vdots & 0 \\
  \cdots & m_{i-1,i} & \cdots \\
  2m_{ii} & m_{i,i+1} & \cdots & m_{iN} \\
  \vdots & \cdots & \cdots & \cdots \\
  \cdots & 0 & \cdots
\end{pmatrix}
\]

(34)
where \( C_i \) is nonzero only along the \( i \)'th row and \( i \)'th column. Let \( U \) be the unitary matrix that diagonalizes the symmetric complex matrix \( M \), that is, \( U^T M U = M_D \) where \( M_D \) is a real diagonal matrix. Then the diagonal elements of the matrix \( U^T C U \) are also real, for general real linear combination of \( C_i \), that is,

\[
C = \sum_{i=1}^{N} a_i C_i = \begin{pmatrix}
2a_1 m_{11} & (a_1 + a_2)m_{12} & \cdots & (a_1 + a_N)m_{1N} \\
2a_2 m_{22} & (a_2 + a_1)m_{21} & \cdots & (a_2 + a_N)m_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
2a_N m_{N1} & (a_N + a_1)m_{N1} & \cdots & a_N m_{NN}
\end{pmatrix},
\]

with arbitrary real coefficients \( a_1, \ldots, a_N \).

We also know that if two of eigenvalues of \( M_D \) are equal, say \( (M_D)_{ii} = (M_D)_{jj} \), then the correspondent term \((U^T C U)_{ij}\) is real. If \((M_D)_{ii} = -(M_D)_{jj}\), then the correspondent term \((U^T C U)_{ij}\) is pure imaginary.

The proof of this seemingly very nontrivial theorem follows immediately from the Goldstone theorem. Consider a Lagrangian with \( U(1) \times U(1) \cdots \times U(1) \) symmetry. For each \( U_i(1) \), there are one Weyl fermion \( \psi_i \) which carries a unit of its quantum number and one scalar \( H_i \) with \( -2 \) units of its quantum number. In addition, for each pair of \((U_i(1), U_j(1))\) there is one scalar, \( h_{ij} \) carrying the quantum number \((-1, -1)\). There are \( \frac{1}{2} N(N - 1) \) independent \( h_{ij} \) as \( h_{ij} \equiv h_{ji} \) and \( i \neq j \). The Lagrangian of Yukawa couplings can be written as

\[
\mathcal{L} = \sum_{i=1}^{N} g_i \psi_i H_i + \sum_{i,j=1}^{N} f_{ij} \psi_i \psi_j h_{ij} + \text{h.c.}
\]

where \( f_{ij} = f_{ji} \). The couplings can be complex. After symmetry breaking, scalars develop vev’s, which are real when phases are absorbed into couplings with redefinition of fields \( \langle H_i \rangle = v_i, \langle h_{ij} \rangle = v_{ij} \). We define \( m_{ii} = g_i v_i, m_{ij} = f_{ij} v_{ij} \) (no summation). The resulting complex symmetric Majorana mass matrix is exactly in the form of \( M \) above.

The spectrum has \( N \) NG bosons

\[
G_i = -\frac{\sqrt{2}}{N_i} (2v_i \text{Im}(H_i) + \sum_{j=1}^{N} v_{ij} \text{Im}(h_{ij}))
\]

where \( N_i = \sqrt{4v_i^2 + \sum_{j=1}^{N} v_{ij}^2} \). The Yukawa coupling of \( G_i \) is \(-i(\sqrt{2}N)^{-1}C_i \). The conclusion follows immediately from the Goldstone theorem. The explicit proof is very similar to those in the last two sections. Basically, we need to show \( C_i - M \) after mass diagonalization has real diagonal entries. Note that the simple example of the one generation model described by Eq. (1) corresponds to the present case of \( N = 2 \) with \( U_1(1) \) generated by the quantum number \( Y \), and \( U_2(1) \) generated by \( Y + L/2 \).

**Conclusion**

In this paper, we basically reaffirm that the diagonal Goldstone boson coupling should be pseudoscalar even when there are CP violation and Majorana masses in the system. Such
a property is probably known, however, it is still quite complicated to work it out explicitly considering that there exists confusion in the literature\cite{6}. The pseudoscalar character of the unphysical Higgs couplings to chargino and neutralinos were not worked out explicitly until recently\cite{8}. This nontrivial character implies that the Goldstone theorem often results in matrix theorems which are very nontrivial unless one uses the Goldstone theorem to motivate it.

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**Appendix**

**Approximation for $2 \times 2$ Majoron Model**

Below we work out the detailed calculation of the case studied in Ref.\cite{6} for the neutrino mass matrix of size $2 \times 2$ of one generation. Physical CP phase $\phi$ appears in $M = |M| e^{i\phi}$ of Eq. \cite{6} in the basis that $m$ and $m_D$ are real. We assume $|M| \gg m_D \gg m > 0$ and $m_D^2/|M|$ the same order as $m$.

First, we rotate phases of the two diagonal elements of $M_n$ in opposite directions so that the resulting imaginary parts are equal. Then, leaving temporarily the unit matrix with imaginary coefficient, we easily diagonalize the remaining real symmetric matrix. Putting back the imaginary part, we make the final phase rotations to obtain the physical real diagonal mass matrix. These steps are summarized as

$$U \simeq \begin{pmatrix} e^{\frac{i\phi}{2}} & 0 \\ 0 & e^{-i\frac{\theta}{2}} \end{pmatrix} \begin{pmatrix} 1 - im \sin \phi/|M| & 0 \\ 0 & 1 + im \sin \phi/|M| \end{pmatrix} \times \begin{pmatrix} 1 & m_D/|M| \\ -m_D/|M| & 1 \end{pmatrix} \begin{pmatrix} e^{-\frac{i\theta}{2}} & 0 \\ 0 & 1 - im \sin \phi/|M| \end{pmatrix},$$

with $e^{i\theta} = m \cos \phi - m_D^2/|M| + im \sin \phi$. We find that

$$U \simeq \begin{pmatrix} (1 - im \sin \phi/|M|) e^{-\frac{i\phi}{2} (\theta - \phi)} & m_D/|M| e^{i\frac{\theta}{2}} \\ -m_D/|M| e^{-\frac{i\phi}{2} (\theta + \phi)} & e^{-i\frac{\theta}{2}} \end{pmatrix},$$

$$M_{\text{diag}} \simeq \begin{pmatrix} m_1 & 0 \\ 0 & |M| \end{pmatrix}, \quad \text{with } m_1 = \sqrt{m^2 \sin^2 \phi + (m \cos \phi - m_D^2/|M|)^2}.$$

Denote the two neutrino mass eigenstates as $\psi_l$ and $\psi_h$. The majoron-neutrinos coupling terms become

$$-\frac{i}{\sqrt{2}N} \begin{pmatrix} \psi_l \\ \psi_h \end{pmatrix} U^T C U \begin{pmatrix} \psi_l \\ \psi_h \end{pmatrix} G_M + \text{h.c.}$$

(40)
Following the usual procedure, we pair up each left-handed Weyl field $\psi$ with its complex conjugated field $\psi^\dagger$ to form a four-component Dirac field $\Psi$. In term of the heavy and light Dirac fields $\Psi_l, \Psi_h$, the interaction is described by

$$\frac{1}{\sqrt{2N}} (\bar{\Psi}_l \bar{\Psi}_h) [\text{Im}(U^T C U) + \text{Re}(U^T C U) i\gamma_5] \left( \begin{array}{c} \Psi_l \\ \Psi_h \end{array} \right) G_M.$$  \hfill (42)

In the leading order, we obtain

$$ (U^T C U)_{11} = -v_H^2 m_1 \quad , \quad (U^T C U)_{22} = (v_H^2 + 4v_{\chi}^2)|M| ,$$  \hfill (43)

with no imaginary parts to the accuracy of our expansion. This confirms that the diagonal scalar couplings of the majoron are zero, contrary to the conclusion in Eq.(16) of Ref.\cite{6}.

**General Proof of Pseudoscalar Nature**

The pseudoscalar nature of the Goldstone boson when coupled to the fermion is very generic, not limited to the neutrino models discussed above. It applies to the Goldstone boson coupling to chargino and neutralino as well. Below, we work out the detail in the basis of $\psi_R$ and $\psi_L$.

The fermion mass matrix $M$ defined in the weak basis is diagonalized by bi-unitary transformation,

$$U'MU^\dagger = M_D , \quad -\mathcal{L}_M \supset \bar{\psi}_R M \psi_L = \bar{\Psi}_R M_D \Psi_L.$$  \hfill (44)

Different choices of vev’s correspond to various rotations $e^{iT\phi}$ generated by the generator $T$ of the spontaneously broken symmetry. With the overall physics unchanged, the physical diagonal mass matrix $M_D$ maintains basis independent, if we impose the following substitutions,

$$U^\dagger \rightarrow e^{i\phi T_L} U^\dagger , \quad U' \rightarrow U'e^{-i\phi T_R}.$$  \hfill (45)

The invariance $\delta(U'MU^\dagger) = 0$ gives

$$U' (\delta M) U^\dagger = -[(\delta U') M U^\dagger + U'M (\delta U^\dagger)] = -[(\delta U') U'^\dagger M_D + M_D (U\delta U^\dagger)] .$$  \hfill (46)

For an infinitesimal $\phi$, we have

$$\delta M = i\phi (Tv) \frac{d}{dv} M , \quad \delta U' = -i\phi U'R , \quad \delta U^\dagger = i\phi T_L U^\dagger .$$  \hfill (47)

Therefore,

$$Y \equiv [U'R U'^\dagger M_D - M_D (U'T_L U^\dagger)] , \quad U'(Tv) \frac{d}{dv} M U^\dagger = -Y .$$  \hfill (48)

As $M_D$ is diagonal and $U'R U'^\dagger$, $UT_L U^\dagger$ are Hermitean, the diagonal entries of above expression $Y$ is purely real.
The NG boson corresponding to the generator $T$ is

$$G = (Tv)_i \text{Im} \sqrt{2} \Phi_i / N , \text{ or, } \sqrt{2} \text{Im} \Phi_i = (Tv)_i G/N + \cdots ,$$

(49)

with the normalization $N = |Tv|$. Here we adopt the convention that $v_i$ are all real. This can always be achieved by field redefinition, sometimes inducing complex couplings. Its Yukawa terms in the Lagrangian become

$$\mathcal{L} \supset -\bar{\Psi}_R U'(i \text{Im} \Phi_i \frac{d}{d v_i} M) U \dagger \Psi_L = -i(\sqrt{2}N)^{-1} G \bar{\Psi}_R U'(Tv)_i \frac{d}{d v_i} M U \dagger \Psi_L , \quad (50)$$

and

$$\mathcal{L} \supset (\sqrt{2}N)^{-1} G [i \bar{\Psi}_R Y \Psi_L - i \bar{\Psi}_L Y \dagger \Psi_R] = -(\sqrt{2}N)^{-1} G \bar{\Psi}_i Y_i i \gamma_5 \Psi_i + \cdots . \quad (51)$$

This proves the diagonal coupling of NG boson is pseudoscalar without assuming CP conservation.

In the following, we provide another way to understand this result by the current conservation. The divergence of the $T$ current is zero because of symmetry.

$$J^{T,\mu} = \bar{\psi}_L T \gamma^\mu \psi_L + \bar{\psi}_R T \gamma^\mu \psi_R + \Phi \dagger \gamma^\mu T \Phi , \quad (52)$$

$$J^{T,\mu} = \bar{\Psi}_L \gamma^\mu U_T U \dagger \Psi_L + \bar{\Psi}_R \gamma^\mu U'_T U'_\dagger \Psi_R - 2(N/\sqrt{2}) \partial^\mu G + \cdots . \quad (53)$$

In the low energy limit,

$$\partial_\mu [\bar{\Psi}_L \gamma^\mu U_T U \dagger \Psi_L + \bar{\Psi}_R \gamma^\mu U'_T U'_\dagger \Psi_R] = (\sqrt{2}N) \partial_\mu \partial^\mu G . \quad (54)$$

Making use of the equation of motion, $i \not\partial \Psi_{L,R} = m \Psi_{R,L}$, we obtain

$$((\sqrt{2}N) \partial_\mu \partial^\mu G = -i \bar{\Psi}_R [U'_T U'_\dagger M_D - M_D (U T_U \dagger)] \Psi_L + \text{H.c.} \quad (55)$$

$$\partial_\mu [i \bar{\Psi}_R Y \Psi_L - i \bar{\Psi}_L Y \dagger \Psi_R] \quad (56)$$

Thus, this equation can be effectively generated by

$$\mathcal{L}_{\text{eff.}} = \frac{1}{2} (\partial G)^2 + \bar{\Psi}_i \not\partial \Psi - [\bar{\Psi}_R M_D \Psi_L + \text{H.c.} - (\sqrt{2}N)^{-1} G [i \bar{\Psi}_R Y \Psi_L - i \bar{\Psi}_L Y \dagger \Psi_R] . \quad (57)$$

So we have consistent results from two different approaches.

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