Study on Mathematical Model of Desiccant Wheel

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Abstract. In the process of dehumidification and regeneration, a complicated heat and mass transfer process occurred between air and hygroscopic materials in desiccant wheel. The performance of dehumidifier is affected by the air inlet state parameters, regeneration air inlet state, air flow, regeneration air flow, and wheel speed and many other factors. Based on the conservation of water mass and energy in the gas region of the desiccant wheel and the conservation of water mass and energy in the solid region, a differential equation describing the absorption and regeneration processes in the wheel is established. In addition to necessary boundary conditions and supplementary equations, a complete mathematical model of desiccant wheel is developed. Numerical solution of mathematical model can be obtained by using finite difference method, which can be used in calculation of actual process and optimization analysis of wheel dehumidification combined air conditioning system.

1. Introduction
The wheel dehumidifier is composed of desiccant wheel, driving device, fan, filter, regenerative heater and so on. The main component is a continuously rotating honeycomb desiccant wheel. Desiccant wheel is divided into two areas by a partition containing a highly sealed packing: one is an air-treated dehumidification sector (270°); the other is a regeneration sector (90°) for air regeneration. In the process of dehumidification and regeneration, a complicated heat and mass transfer process occurred between air and hygroscopic materials.

2. Overview
The change of air state in engineering is approximately an isenthalpic process. After the parameters of solid absorbent material of the dehumidifier and structural parameters of desiccant wheel are determined, the performance of dehumidifier is still affected by the air inlet state parameters, regeneration air inlet state, air flow, regeneration air flow, and wheel speed and many other factors. Dehumidification performance curves given in the product samples are generally only data under certain conditions, which are not complete. As a key component of wheel dehumidifier, mathematical model of desiccant wheel can make up for the above deficiencies.

3. Mathematical Model

3.1 Classification of mathematical models
The mathematical models of desiccant wheel can be divided into two categories. One type is the correlation expression of dehumidifier performance fitted by a large number of experimental test data[1]. When establishing this mathematical model, experimental work is very heavy, and the
application of mathematical model is limited when extending from experimental area to non-experimental area. Another type of mathematical model is based on the conservation of the water mass in the gas region of microelement body in the desiccant wheel, the conservation of water mass in the solid region, the conservation of energy in gas region, and the conservation of energy in solid region. It consists of a set of differential equations describing the absorption and regeneration processes in the wheel, plus necessary boundary conditions and supplementary equations[2-5]. At present, the latter type of mathematical model is the mainstream, and mathematical model in this paper belongs to the latter type.

3.2 Fundamental governing equation

The complicated heat and mass transfer process in the desiccant wheel is simplified as follows: ① The seal between dehumidification area and regeneration area is intact, and there is no leakage. ② The heat dissipation of the wheel housing is ignored. ③ Because rotating speed is very low, the effect of centrifugal force on heat and mass transfer is ignored. ④ Momentum change in air and water is ignored, that is, pressure does not change along the axis. ⑤ Hygroscopic material is evenly distributed in the wheel. ⑥ Heat conduction and mass diffusion of hygroscopic material along the φ direction is small compared with the effect of convective heat transfer and mass transfer, which can be ignored. ⑦ Since it takes much less time for air flow to reach equilibrium than for heat and mass balance, it is considered that air flows steadily in the flow channel. ⑧ Air is a complete gas with constant specific heat capacity.

Figure 1. Schematic diagram of microelement body of desiccant wheel

Figure 1 is the schematic diagram of microelement body of the desiccant wheel at the cylindrical coordinates r, φ, Z. The 0 ≤ φ < 2π−φR region is dehumidification region. The 2π−φR ≤ φ < 2π region is regeneration region. ① The change of the moisture of air itself in the micro-body in the time dτ is

$$\rho_s f_s \frac{1}{2} R^2 d\phi dZ \frac{D}{d\tau} d\tau.$$ ② In the time of dτ, water content that enters air in the microelement body from Z direction is

$$m_i \frac{1}{2} R^2 d\phi dD d\tau.$$ ③ In the time of dτ, water content of air flowing out of microelement body from the Z direction is

$$m_i \frac{1}{2} R^2 d\phi \left( D + \frac{\partial D}{\partial Z} dZ \right) d\tau.$$ ④ In time of dτ, moisture transferred from adsorbent surface to air mass transfer process is

$$K_v F_v \frac{1}{2} R^2 d\phi dZ (D_w - D) d\tau.$$ ⑤ In the time of dτ, moisture in the air entering micro-element body from φ direction due to rotation is

$$\int_0^\phi (\omega r d\theta f_s \rho_s D d\tau) = f_s \rho_s \omega \frac{1}{2} R^2 dZ d\tau.$$ ⑥ In the time of dτ, water content of air in the microelement body due to the rotation from φ direction is...
\[ \int_0^R \sqrt{\omega rdrdZf_r} \left( D + \frac{\partial D}{\partial \phi} \right) d\phi = f_r \rho_t \omega R^2 dZ \left( D + \frac{\partial D}{\partial \phi} \right) d\tau. \]

It can be deduced from the conservation of mass.

\[ \frac{\partial D}{\partial \tau} + \omega \frac{\partial D}{\partial \phi} + m_i \frac{\partial D}{\partial Z} = \frac{K_y F_v}{\rho_i f} (D_w - D) \quad (1) \]

Similarly, the equation for energy conservation in air is as follows.

\[ \frac{\partial t}{\partial \tau} + \omega \frac{\partial t}{\partial \phi} + m_i \frac{\partial t}{\partial Z} = \alpha F_v (t_w - t) \quad (2) \]

Mass conservation equation of water in the adsorbent is as follows.

\[ \frac{\partial W}{\partial \tau} + \omega \frac{\partial W}{\partial \phi} = D_e (1 - f_i) \frac{\partial^2 W}{\partial Z^2} + \frac{K_y F_v (D - D_w)}{M_w} \quad (3) \]

Energy conservation equation of adsorbent is as follows.

\[ \frac{\partial t_w}{\partial \tau} + \omega \frac{\partial t_w}{\partial \phi} = \frac{(1 - f_i) \lambda}{M_w (c_{pu} + Wc_{ps})} \frac{\partial^2 t_w}{\partial Z^2} + \frac{F_v}{M_w} \left[ \alpha (t - t_w) + K_y (D - D_w) \right] \quad (4) \]

In the above 4 equations, the air moisture content \( D_w \) at adsorbent surface is involved, and its value can be determined by the following formula.

\[ D_w = \frac{0.622 \varphi P_s}{(B - \varphi P_s)} \quad (5) \]

\[ \ln P_s = (-5800.22)/T_w + 1.3915 - 0.04860247 T_w + 0.41765 \times 10^{-4} T_w^2 - 0.144521 \times 10^{-7} T_w^3 + 6.545967 \ln T_w \quad (6) \]

\[ \frac{W}{W_{max}} = \frac{\varphi}{\gamma (1 - \gamma)} \varphi \quad (7) \]

Equations (1) ~ (7): \( D \) is the mass of dry air containing water vapor in unit mass, that is, moisture content of air, \( \text{kg} \cdot \text{kg}^{-1} \). \( D_w \) is moisture content of air at the surface of adsorbent, \( \text{kg} \cdot \text{kg}^{-1} \). \( m_i \) is mass air flow through unit cross-section of wheel, \( \text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1} \). \( f_i \) is the ratio of air flow area to total cross-sectional area of wheel on air flow cross section, \( \text{m}^2 \cdot \text{m}^{-2} \). \( F_i \) is surface area of the adsorbent in the unit volume wheel, \( \text{m}^2 \cdot \text{m}^{-3} \). \( K_y \) is mass transfer coefficient driven by moisture content difference, \( \text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1} \). \( \lambda \) is density. \( \rho_t \) is density, \( \text{kg} \cdot \text{m}^{-3} \). \( t \) is air temperature, \( \text{C} \). \( t_w \) is the temperature of adsorbent, \( \text{C} \). \( c_{pu} \) and \( c_{ps} \) are specific pressure heat capacity of air and water vapor, respectively, \( \text{J} \cdot \text{kg}^{-1} \cdot \text{C}^{-1} \). \( D_e \) is effective diffusion coefficient of adsorbent, \( \text{m}^2 \cdot \text{s}^{-1} \). \( \omega \) is rotational angular velocity of wheel, \( \text{rad} \cdot \text{s}^{-1} \). \( c_{pu} \) and \( c_{ps} \) are specific pressure heat capacity of adsorbent and water respectively, \( \text{J} \cdot \text{kg}^{-1} \cdot \text{C}^{-1} \). \( \lambda \) is thermal conductivity of adsorbent, \( \text{W} \cdot \text{m}^{-1} \cdot \text{C}^{-1} \). \( B \) is atmospheric pressure, \( \text{Pa} \). \( P_s \) is saturated vapor pressure of water when adsorbent wall temperature is \( t_w \). \( \varphi \) is the relative humidity. \( W \) is mass of adsorbent absorbed water per unit mass, \( \text{kg} \cdot \text{kg}^{-1} \). \( a \) is heat transfer coefficient between air and adsorbent, \( \text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-1} \). \( M_w \) is the mass of adsorbent contained in the unit volume runner, \( \text{kg} \cdot \text{m}^{-3} \). \( W_{max} \) is maximum adsorption rate of adsorbent. \( \gamma \) is the shape factor of adsorption characteristics, which is an important parameter to characterize isothermal adsorption equilibrium relationship of adsorbent. Subscript \( i \) is 1 and 2, 1 means humid air, 2 means regeneration air.

Equations (1) to (4) reflect the laws of heat and mass transfer between air and hygroscopic material during adsorption-desorption process. There are 5 unknowns in the 4 equations, which are
\(D, t, W, t_s, D_w, W\). Equations (5) to (7) are three supplementary equations with two unknowns \(P_s\) and \(\phi\). The above 7 equations are solved with 7 unknowns and equations can be solved.

### 3.3 Boundary conditions
- For dehumidification zone \((0 \leq \phi < 2\pi - \phi_R)\), there are boundary conditions.
  \[
  D_{\text{inlet}} = D_1, t_{\text{inlet}} = t_1 \tag{8}
  \]
- For regeneration zone \((2\pi - \phi_R \leq \phi < 2\pi)\), there are boundary conditions.
  \[
  D_{\text{inlet}} = D_2, t_{\text{inlet}} = t_2 \tag{9}
  \]
- For \(Z = 0\), there are the following periodic boundary conditions.
  \[
  \begin{align*}
  D(0, Z, \tau) &= D(2\pi, Z, \tau) \\
  t(0, Z, \tau) &= t(2\pi, Z, \tau) \\
  W(0, Z, \tau) &= W(2\pi, Z, \tau) \\
  t_w(0, Z, \tau) &= t_w(2\pi, Z, \tau)
  \end{align*} \tag{10}
  \]

### 3.4 Initial conditions
- For adsorbent, there are initial conditions.
  \[
  W(\phi, Z, 0) = W_0, t(\phi, Z, 0) = t_0 \tag{11}
  \]
- For air, there are initial conditions. According to assumption condition 7, air flows in the flow channel steadily. Therefore, initial conditions of \(D\) and \(t\) are taken as their inlet conditions.

### 3.5 Mathematical Model
The above basic control equations (1) to (4), supplementary equations (5) to (7), boundary conditions (8) to (10), and initial conditions (11) constitute a complete mathematical model of desiccant wheel.

### 4. Conclusion
- Based on the conservation of water mass and energy in the gas region of the desiccant wheel and the conservation of water mass and energy in the solid region, a differential equation describing the absorption and regeneration processes in the wheel is established. In addition to necessary boundary conditions and supplementary equations, a complete mathematical model of desiccant wheel is developed.
- Numerical solution of mathematical model can be obtained by using finite difference method, which can be used in calculation of actual process and optimization analysis of wheel dehumidification combined air conditioning system.

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