The study of pressure source term in moving particle semi-implicit (MPS)

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Abstract. In this paper pressure source terms proposed by different researcher are compared in term of pressure stability. In Moving Particle Semi-Implicit (MPS) method, the pressure error is compensated back into pressure Poisson equation as source term in order to enforce the incompressibility. However the non physical pressure fluctuation still remains an open problem in MPS method. In present study the hydrostatic model is used as example to study the pressure stability for each of the pressure source terms. The pressure at the fixed point over time is plotted and the fluctuation from the theoretical pressure is discussed.

1. Introduction

Most of the conventional computational fluid dynamics (CFD) employs finite volume or finite element method. The result generated is highly affected by the quality of the descretized domain, or so called grid. Recently, a new class of solving approach, called particle or meshless method had drawn much research interest. The particle methods are the Largrangian meshless method where particles are free to move in given space. This result in more feasible and effective performance in highly deformed, free surface flow, especially the problem involving complicated boundary and fluid/structure coupling compare with conventional grid based approach.

One of the particle methods, the Moving Particle Semi-Implicit (MPS) is first introduced by Koshizuka [1] for incompressible flow problems. The Navier-Stoke equation is solved by two different stages: the viscous part, which is solved explicitly, and the pressure gradient part, which require Poisson equation of pressure to be solved beforehand.

The density deviation error is compensating back as source term in Poisson pressure equation to take account of the density change after the particle moved. However the original pressure source term in [1] often results in high pressure fluctuation and even lead to solver failure.

In this paper a pressure source terms with different modification will be compared. In the following section, the pressure source terms will be introduced, followed by the investigation on pressure stability for different source terms.

2. Numerical Method

The continuity and Navier Stokes equation are:

\[ \frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \vec{u} = 0 \]  

(1)
\[
\begin{align*}
\frac{D\mathbf{u}}{Dt} &= -\frac{1}{\rho} \nabla P + \nu \nabla^2 \mathbf{u} + \mathbf{g} \\
(2)
\end{align*}
\]

Where \( \nu \) is the kinematic viscosity, \( \rho \) is the fluid density, and \( \mathbf{g} \) represents the gravitational acceleration. The equation (2) is divided into viscous and pressure term, where

\[
\begin{align*}
\frac{D\mathbf{u}}{Dt}^{\text{viscous}} &= \nu \nabla^2 \mathbf{u} + \mathbf{g} \\
\frac{D\mathbf{u}}{Dt}^{\text{pressure}} &= -\frac{1}{\rho} \nabla P \\
(3)
\end{align*}
\]

The pressure term in equation (4) is differentiated to get the Poisson equation, and to be solved implicitly.

\[
\begin{align*}
\nabla \cdot \mathbf{u} &= 0 \\
\frac{D}{Dt}(\nabla \cdot \mathbf{u}) &= \nabla \cdot \frac{D\mathbf{u}}{Dt} = 0 \\
\frac{1}{\rho_0} \nabla^2 P &= \nabla \cdot \frac{D\mathbf{u}}{Dt} = 0 \\
(5)
\end{align*}
\]

In ideal case, the equation (5) is equated to zero when the fluid is incompressible by nature. That is, the volume remains unchanged over time. However as the particle is free to move, the local density may not satisfy the incompressibility all the time. Hence, the error of the density will need to compensate back to equation (5). Assuming the fluid density is proportional to particle number density (PND), the density error is compensated back to equation (5) as:

\[
\frac{1}{\rho_0} \nabla^2 P = -\frac{\rho}{\Delta t^2} \frac{n_i^k - n_0}{n_0} \\
(6)
\]

The left hand side is discretized by diffusion model. The right hand side of the Poisson equation is the deviation of the temporal particle number density (PND) from the standard value. The term is used to maintain the PND to enforce the incompressibility. However the source term also induces large pressure instability, and the pressure at free surface will always be negative hence need special treatment.

In order to obtain smoother result, Kondo and Koshizuka [2] had proposed a source term consists of main part and two error compensating parts which multiplied by coefficient to improve the high frequency numerical oscillation of original source term in equation (6).

\[
\frac{1}{\rho_0} \nabla^2 P = -\frac{1}{n_0 \Delta t^2} \left[ (n_i^k - 2n_i^{k-1} + n_i^{k-2}) + \beta (n_i^{k+1} - n_i^{k-1}) + \gamma (n_i^k - n_0) \right] \\
(7)
\]

Where the \( \beta \) and \( \gamma \) are the constant satisfying \( 0 \leq \gamma \leq \beta \leq 1 \). When both \( \beta \) and \( \gamma \) equal to 1, the equation reduced to original source term in equation (6).

Instead of fixed constant throughout the entire calculation for all vertices, Khayyer and Gotoh [3] had proposed the \( \beta \) and \( \gamma \) that depended on instantaneous flow feature. The error compensating term will cancel out each other when the already expanded fluid is being compressed, or vice versa.
3. Result and Discussion

A hydrostatic model is used as test case for the pressure stability calculation. The dimension of the model is follow as in [2]. The initial particle spacing is 0.012 and the time step size is 0.001 s. The weight kernel in present study is same as in [2]. In present study only the different source terms are tested with same condition. Other improvement steps for particular literature such as surface treatment and pressure gradient correction matrix in [3] are not implemented. Figure 1 shows the pressure history at point A with different pressure source term used. Theoretical pressure is plotted as red dashed line.

\[
\beta = \left| \frac{n^k - n^0}{n^0} \right|, \quad \gamma = \left| \left( \frac{\Delta t}{n^0} \left( \frac{Dn}{Dt} \right)^k \right) \right|
\]

Figure 1. Right: Hydrostatic model used. Left: Pressure at point A over time. With source term from (a) equation (6), (b) equation (7), \((\beta, \gamma) = (0.5, 0.1)\), (c) equation (7), \((\beta, \gamma) = (0.5, 0.05)\), (d) equation (7), \((\beta, \gamma) = (1.0, 0.1)\), (e) equation (7), \((\beta, \gamma) = (1.0, 0.2)\), (f) \((\beta, \gamma)\) through equation (8)

From the results obtained, the original source term without any coefficient (equation (6)) give large pressure fluctuation as shows in figure 1(a), and with magnitude of the fluctuation increased as the time progress. This pressure fluctuation eventually leads to solver failure after 0.9s. This is due to the nature of the source term with any small change in position will directly reflect in pressure change without any “buffer”, and cause the rapid change of pressure as the particle moves.
From (b) to (e), Different $\beta$ and $\gamma$ are tested for the source term in equation (7). The pressure history obtained is much closer to theoretical value compared with case (a). It is found that in general the ratio of $1/10$ of $\beta$ and $\gamma$ give better result, with $(\beta, \gamma)$ of $(0.5, 0.05)$ produced most stable result compared with other value. From the result obtained, the larger $\gamma$ is found able to reduce the initial pressure fluctuation. However it will cause higher pressure noise as the time progress.

From the result in (f), the pressure history of Khayyer and Gotoh is surprisingly shows very high pressure fluctuation despite the claim of smoother pressure from the theory and literature. The $\beta$ and $\gamma$ obtained from equation (8) are much smaller than optimal value from Kondo and Koshizuka in [2] and current study as shown in figure 2 below. This may explain the large pressure fluctuation. Another reason of the high pressure fluctuation may be due to different weight function as used in [3], and lack of implementation of pressure gradient correction matrix.

![Figure 2.](image)

**Figure 2.** Compare of the $\beta$ and $\gamma$ from ideal case (blue line) with from equation (8) (red curve). Left: $\beta$ versus time, Right: $\gamma$ versus time

4. Conclusions
The performance of the different pressure source term in the MPS had been compared in term of pressure stability. It is found that the pressure stability can be improved more by fine-tuning the relevant constant in the pressure source term studied, compared with through instantaneous flow feature. Although this may be cause by the lack of implementation of some other improvement steps in the later source term. Even though the smooth pressure can be obtained by current parameter, however the best parameter in one case may not be ideal for other flow type. In future, more investigation will be done in determine the ideal constant for the more violence flow type.

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