Constraining modified theory of gravity with galaxy bispectrum

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We explore the use of galaxy bispectrum induced by the nonlinear gravitational evolution as a possible probe to test general scalar-tensor theories with second-order equations of motion. We find that time dependence of the leading second-order kernel is approximately characterized by one parameter, the second-order index, which is expected to trace the higher-order growth history of the Universe. We show that our new parameter can significantly carry new information about the non-linear growth of structure. We forecast future constraints on the second-order index as well as the equation-of-state parameter and the growth index.

I. INTRODUCTION

It is one of the biggest challenges of modern cosmology to understand the physical origin of the present cosmic acceleration of the Universe. The origin of the cosmic acceleration is expected to be connected to fundamental theory beyond the current standard model of particle physics. It might eventually require the presence of a new type of energy, usually called dark energy. As another possibility, the accelerated expansion might arise due to a modification of general relativity (GR) on cosmological scales. A variety of theoretical scenarios have been proposed in literature and carefully compared with observational data (see Refs. [1–3] for reviews).

Among many varieties of current cosmological observational data, measuring the growth rate of the density fluctuations, \( f(a) \), is believed to be a powerful tool to test the nature of the dark energy or the modification of the theory of gravity responsible for the present cosmic acceleration. The growth rate of large-scale structure is mainly measured by observing galaxy peculiar velocities along the line of sight through redshift-space distortion (RSD) measurements [4, 5]. To compare the observational data and theoretical predictions efficiently, it should be useful to introduce a phenomenological parameter. A minimal approach to test the theory of gravity from the measurement of the growth rate of large-scale structure is to introduce an additional parameter called gravitational growth index, \( \gamma \), defined through the growth rate [6, 7]:

\[
f(a) = \tilde{\Omega}_m^\gamma(a),
\]

where \( \tilde{\Omega}_m(a) \) denotes the matter density fraction of the total energy density at a cosmic scale factor \( a \). In the standard cosmological model responsible for the present cosmic acceleration, called \( \Lambda \) cold dark matter (\( \Lambda \)CDM) model with GR, we expect the growth index to be approximately constant with \( \gamma \approx 0.545 \). Although the current constraints on the growth index have been reported [8–10], at the moment, there is no evidence for a departure from the standard \( \Lambda \)CDM model. However, since there are numerous different ways of modeling the landscape of cosmological models, it is further required to consider new possible parametrizations as the signature of the modified gravity theory.

In this paper, we focus on the quasi-nonlinearity of the growth of large-scale structure as a way to provide new insight into the modified theory of gravity. As an observable for such a nonlinearity, we investigate the bispectrum of the biased object such as a dark matter halo or galaxy which are frequently discussed as a useful tool to constrain the higher-order statistical nature of cosmological perturbations (see e.g., Ref. [11] for constraining non-local types of primordial non-Gaussianity). Even if the primordial perturbations are Gaussian, the non-zero halo/galaxy bispectrum should be generated from the late-time nonlinear gravitational evolution of the density fluctuations and such a nonlinearity should have new information about the modification of the gravity theory, which would not be imprinted on the growth index in the linear perturbation theory. As examples, Refs. [12, 13] have discussed the bispectrum of the matter density fluctuations in Horndeski theory which has been known as a most general scalar-tensor theory with

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the second-order equations of motion, and they have shown that the deviation from the standard ΛCDM model with GR can be included in a time-evolving coefficient in the kernels of the second-order density perturbations denoted by λ(α).

Here, we study the possibility of future planned galaxy surveys for providing significant information on the modification of gravity theory responsible for the present accelerated expansion of the Universe, based on the matter bispectrum formula derived in Refs. [12, 13]. For this purpose, we first propose a new useful parametrization of the second-order perturbative kernel:

$$\lambda(\alpha) = \tilde{\Omega}^k_{\Delta}(\alpha).$$

(2)

Here ξ is our new parameter, the second-order index, to trace the nonlinear growth history, encompassing deviations in the wide theoretical framework. Actually, there are models in which the expansion history and the growth rate in the linear perturbation theory are same in the fiducial model but the different value of ξ can be obtained. We show that precise measurement of RSDs by future galaxy surveys can distinguish and hopefully exclude these cosmological models.

The paper is organized as follows. In Sec. II, we first give the basic equations for the galaxy bispectrum in redshift space. In Sec. III, following Refs. [12, 13], we review the matter bispectrum in the Horndeski theory of gravity, the most general scalar-tensor theory with second-order equations of motion. In Sec. IV, we estimate the asymptotic values of γ and ξ by introducing the effective field theory parameters which can make the understanding of the signal of the modification of gravity theory easier. In Sec. V, to see the impact of the existence of the new parameter on future galaxy surveys, based on the Fisher analysis, we numerically estimate the expected constraints on the equation-of-state parameters. Finally, Sec. VI is devoted to summary and conclusion.

II. GALAXY BISPECTRUM IN REDSHIFT SPACE

Galaxy redshift survey provides a map of galaxies in redshift space. In this map, the radial position of a galaxy is given by the observed radial component of its relative velocity to an observer, which is a combination of the Hubble recession and the peculiar velocity. Therefore, the mapping from a position \(s\) of a galaxy in real space to a position \(s\) in redshift space is described as

$$s = x + \frac{v_s(a, x)}{aH} \hat{z},$$

(3)

where \(v_s(a, x)\) represents a peculiar velocity of the underlying matter density field along the line-of-sight (we take the line-of-sight direction to be \(\hat{z}\)). \(H\) is the Hubble parameter, and \(a\) is the cosmic scale factor which we use as a time coordinate. This mapping gives us the conversion from the density contrast in real space, \(\delta\), to that in redshift space, \(\delta_s\). Since it is useful to investigate the spatial density distribution in Fourier space, up to the second order in the perturbation theory, we provide the density contrast in redshift space as the Fourier component [14]:

$$\delta_s(a, k) = \int d^3 x e^{-i k \cdot x} e^{-i k \cdot v_s/(\theta(a, x))} [1 + \delta(a, x)] - \delta^3_D(k)$$

$$= [\delta(a, k) + \mu^2 \theta(a, k)]$$

$$+ \int \frac{d^3 k_1 d^3 k_2}{(2\pi)^3} \delta^3_D(k_1 + k_2 - k) \left[ \delta(a, k_1) \delta(a, k_2) \frac{\mu_2}{k_2} \theta(a, k_2) + \frac{k^2 \mu_2^2}{2} \frac{\mu_1}{k_1} \theta(a, k_1) \theta(a, k_2) \right] + \cdots,$$

(4)

where \(\delta^3_D\) is a 3-dimensional Dirac’s delta function, \(\mu\) is the cosine of the angle between the wave-vector \(k\) and the line-of-sight direction \(\hat{z}\) (similarly, \(\mu_i\) is the cosine for \(k_i\) with \(i = 1, 2\)). Here \(\delta(a, k)\) and \(\theta(a, k)\) are Fourier components of the density contrast and the scalar velocity divergence, \(\theta(a, x) := -\nabla \cdot v(a, x)/(aH)\) in real space, respectively. For the pressureless non-relativistic matter such as CDM and baryons, the evolution equation for the linear density fluctuations, \(\delta\), does not depend on the wavenumber and hence we can express the time dependence of the matter density fluctuations in the linear theory independent with the wavenumber as

$$\delta(a, k) = D_+(a) \delta_L(k),$$

(5)

where \(D_+(a)\) is a growth factor of the growing mode in the linear theory and \(\delta_L(k)\) is the random initial density perturbations. Furthermore, by using the continuity equation, the scalar velocity divergence, \(\theta\), is given by the logarithmic time derivative of matter density fluctuations as

$$\theta(a, k) = f(a) \delta(a, k),$$

(6)
where \( f(a) := \frac{\text{d} \ln D_+}{\text{d} \ln a} \) is called a linear growth rate. By using above expressions, the expansion forms of \( \delta(a, k) \) and \( \theta(a, k) \) are respectively given as

\[
\delta(a, k) = D_+(a) \delta_L(k) + D_+(a)^2 \int \frac{\text{d}^3k_1 \text{d}^3k_2}{(2\pi)^3}\delta_D^3(k_1 + k_2 - k) F_2(k_1, k_2; a) \delta_L(k_1) \delta_L(k_2) + \cdots, \tag{7}
\]

\[
\theta(a, k) = f(a) D_+(a) \delta_L(k) + f(a) D_+(a)^2 \int \frac{\text{d}^3k_1 \text{d}^3k_2}{(2\pi)^3}\delta_D^3(k_1 + k_2 - k) G_2(k_1, k_2; a) \delta_L(k_1) \delta_L(k_2) + \cdots, \tag{8}
\]

where the functions \( F_n \) and \( G_n \) are the so-called perturbative kernels.

Our interested observable is the galaxy distribution, not the matter density distribution. Here, for simplicity, we introduce a local, nonlinear bias model for galaxies, in which the galaxy density contrast can be expanded as a Taylor series of the underlying dark matter density contrast,

\[
\delta_{\text{gal}}(a, x) = b_1 \delta(a, x) + \frac{1}{2} b_2 \delta^2(a, x) + \cdots, \tag{9}
\]

with the bias parameters \( b_i \).

Then, we can obtain the galaxy density contrast \( \delta_{\text{gal}, s}(a, k) \) in redshift space as

\[
\delta_{\text{gal}, s}(a, k) = D_+(a) Z_1(k ; a) \delta_L(k) + D_+(a)^2 \int \frac{\text{d}^3k_1 \text{d}^3k_2}{(2\pi)^3}\delta_D^3(k_1 + k_2 - k) Z_2(k_1, k_2; a) \delta_L(k_1) \delta_L(k_2) + \cdots. \tag{10}
\]

Here the linear- and second-order RSD kernels, \( Z_i \), are defined as \([14]\)

\[
Z_1(k ; a) = b_1 + f \mu^2, \tag{11}
\]

\[
Z_2(k_1, k_2; a) = b_1 F_2(k_1, k_2; a) + f \mu_1^2 G_2(k_1, k_2; a) + \frac{f_{k_1 k_2} \mu_{k_1} \mu_{k_2}}{2} \left[ \frac{\mu_1}{k_1} (b_1 + f \mu_2^2) + \frac{\mu_2}{k_2} (b_1 + f \mu_1^2) \right] + \frac{1}{2} b_2, \tag{12}
\]

where \( \mu_{k_1} = k_1 \cdot \mathbf{z}/k_1 \) with \( k_{12} = k_1 + k_2 \).

Eq. (10) allows us to calculate the power spectrum and bispectrum of galaxies in redshift space. For simplicity we assume that the initial density field \( \delta_L(k) \) obeys the Gaussian random distribution with

\[
\langle \delta_L(k_1) \delta_L(k_2) \rangle = (2\pi)^3 P_L(k_1) \delta_D^3(k_1 + k_2). \tag{13}
\]

In the leading order of the perturbations, the power spectrum of the redshift-space galaxy is simply described by

\[
\langle \delta_{\text{gal}, s}(a, k_1) \delta_{\text{gal}, s}(a, k_2) \rangle = (2\pi)^3 P_s(k_1 ; a) \delta_D^3(k_1 + k_2). \tag{14}
\]

with

\[
P_s(k ; a) = D_+(a)^2 Z_1^2(k ; a) P_L(k). \tag{15}
\]

We can also calculate the bispectrum of the redshift-space galaxy as

\[
\langle \delta_{\text{gal}, s}(a, k_1) \delta_{\text{gal}, s}(a, k_2) \rangle = (2\pi)^3 B_s(k_1, k_2, k_3; a) \delta_D^3(k_1 + k_2 + k_3), \tag{16}
\]

with

\[
B_s(k_1, k_2, k_3; a) = 2 D_+^4(a) Z_1(k_1 ; a) Z_1(k_2 ; a) Z_2(k_1, k_2; a) P_L(k_1) P_L(k_2) + (\text{cyc}). \tag{17}
\]

Measuring the growth history of cosmological structures can elucidate the nature of the dark energy and test gravity theory on cosmological scales, since the modification of gravity theory typically alters the clustering property of nonlinear structure and the peculiar velocity field. However, there are a wide variety of gravity theories that yield different signatures to the large-scale structure. As shown in Eqs. (15) and (17), the power spectrum and bispectrum is sensitive to the growth of the structure formation through the RSD kernels. Hence, in this paper, we only focus on the general scalar-tensor theory with second-order equations of motion, namely the Horndeski theory, and demonstrate the potential of the redshift-space galaxy bispectrum to constraint the Horndeski theory by future galaxy surveys.
III. NON-LINEAR GRAVITATIONAL GROWTH IN HORNDESKI THEORY

The Horndeski theory is a most general scalar-tensor gravity theory with the second-order equations of motion, which is paid attention as one of attractive modified gravity theories. Therefore, so far, there are many attempts to test the Horndeski theory not only in the Solar system but also in the cosmological context. Up to the second order in cosmological perturbation theory, the modification of gravity theory would be captured in $F_2$ and $G_2$, which are the second-order kernels appearing in a general formula for the galaxy bispectrum in redshift space as shown in Eq. (12). In this section, following a previous work [12], first we briefly review the derivation of the second-order kernels appearing in a general formula for the galaxy bispectrum in redshift space as shown in Eq. (12). which make us investigate the deviation from the standard ΛCDM model with GR easier.

The action of Horndeski theory of gravity is given by [16]

$$S = \int d^4x\sqrt{-g} \sum_{a=2}^{5} \mathcal{L}_a[g_{\mu\nu}, \phi] + S_m[g_{\mu\nu}, \psi_m],$$

where the four Lagrangian $\mathcal{L}_a$ encode the dynamics of the metric $g_{\mu\nu}$ and the scalar field $\phi$. These explicit forms are described by

$$\mathcal{L}_2 = K(\phi, X),$$  
$$\mathcal{L}_3 = -G_3(\phi, X) \Box \phi,$$
$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4X}(\phi, X) \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right],$$
$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{1}{6} G_{5X}(\phi, X) \left[ (\Box \phi)^3 - 3 \Box \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3 \right],$$

with $K$ and $G_a$ ($a = 3, 4, 5$) being an arbitrary function of $\phi$ and $X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi / 2$ and $G_{aX} = \partial G_a / \partial X$. In Eq. (18), $S_m$ describes the matter sector and we assume that matter is universally minimal-coupled to the metric and does not have direct coupling with the scalar field $\phi$.

The Friedmann and matter conservation equations in the Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological background can be written in the standard manner [15]:

$$H^2 = \frac{1}{3M^2} (\rho_m + \rho_{DE}),$$
$$\dot{\rho}_m + 3H\rho_m = 0,$$

with

$$M^2 = 2 \left( G_4 - 2XG_{4X} + XG_{5\phi} - \dot{\phi} H XG_{5X} \right),$$

and

$$\rho_{DE} = 2XK_X - K - 2XG_{3\phi} + 6H\dot{\phi} (XG_{3X} - G_{4\phi}) + 12H^2 X (G_{4X} + 2XG_{4XX} - G_{5\phi} - XG_{5\phi X}) - 4XH \dot{\phi} (3G_{4\phi X} - H^2 (G_{5X} + XG_{5XX})).$$

where the scale $M$ is consider to be about Planck mass scale, and $\rho_m$ is the background energy density of matter.

Let us derive the evolutilonal equations governing the density perturbations in the flat FLRW with Horndeski theory. Throughout this paper, we work in the Newtonian gauge which is defined as

$$ds^2 = -(1 + 2\Phi(t, x)) dt^2 + a^2(t) \left( 1 - 2\Psi(t, x) \right) dx^2,$$

and the perturbation of the scalar field is described by

$$\phi(t, x) \rightarrow \phi(t) + \delta \phi(t, x).$$

We are interested in the behavior of the gravitational and scalar fields on subhorizon scales sourced by a nonrelativistic matter overdensity. Therefore, in the derivation of the basic equations, we ignore time derivatives in the effective equations, while keeping spatial derivatives. We will keep the nonlinear term schematically written as $(\nabla^2 \epsilon)^n$, where
\( \nabla \) and \( \epsilon \) stand for the spatial derivatives and any of \( \Phi \), \( \Psi \), and the perturbation of the scalar field \( \delta \phi \), respectively. Under these assumptions, the basic equations up to the second-order (i.e., \( n = 1, 2 \)) perturbations are given by [12, 17]

\[
\nabla^2 (\mathcal{F}_T \Psi - G_T \Phi + A_1 Q) = \frac{B_1}{2a^2H^2} Q^{(2)} + \frac{B_3}{a^2H^2} (\nabla^2 \Psi \nabla^2 Q - \nabla_i \nabla_j \Phi \nabla^i \nabla^j Q),
\]

where the time-dependent coefficient \( \tau \) and \( Q \) can be expressed as

\[\nabla^2 (G_T \Psi + A_2 Q) = \frac{a^2}{2} \rho_m \delta - \frac{B_2}{2a^2H^2} Q^{(2)} - \frac{B_3}{a^2H^2} (\nabla^2 \Psi \nabla^2 Q - \nabla_i \nabla_j \Psi \nabla^i \nabla^j Q), \]

\[\nabla^2 (A_0 Q - A_1 \Psi - A_2 \Phi) = -\frac{B_0}{a^2H^2} Q^{(2)} + \frac{B_1}{a^2H^2} (\nabla^2 \Psi \nabla^2 Q - \nabla_i \nabla_j \Psi \nabla^i \nabla^j Q),\]

\[-\frac{k^2}{a^2H^2} \epsilon(t, k) = \kappa_c(t, k) \delta(t, k) + \int \frac{d^3k_1 d^3k_2}{(2\pi)^3} \delta^2 \delta^{(3)}(k_1 + k_2 - k) \gamma_{2, \epsilon}(k_1, k_2; t) \delta(t, k_1) \delta(t, k_2) + \cdots,\]

up to the second order in the matter density perturbations. Each time dependent coefficient in the linear term, \( \kappa_c \), can be expressed as

\[\kappa_\Phi(t) = \frac{\rho_m(A_0 F_T - A_1^2)}{2H^2(A_0 G_T + 2A_1 A_2 G_T + A_2^2 F_T)},\]

\[\kappa_\Psi(t) = \frac{\rho_m(A_0 G_T - A_1 A_2)}{2H^2(A_0 G_T + 2A_1 A_2 G_T + A_2^2 F_T)},\]

\[\kappa_Q(t) = \frac{\rho_m(A_1 G_T - A_2 F_T)}{2H^2(A_0 G_T + 2A_1 A_2 G_T + A_2^2 F_T)}.\]

Moreover, comparing the second-order contributions of Eqs. (29)-(31) with Eq. (32), we obtain the nonlinear interaction term for \( \Phi \) as

\[\gamma_{2, \Phi}(k_1, k_2; t) = \tau_\Phi(t) \left(1 - \frac{(k_1 \cdot k_2)^2}{k_1^2 k_2^2}\right),\]

where the time-dependent coefficient \( \tau_\Phi \) is

\[\tau_\Phi(t) = \frac{H^2}{\rho_m} \left(2B_0 \kappa_Q^3 - 3B_1 \kappa_\Phi \kappa_Q^2 - 3B_2 \kappa_\Phi \kappa_Q - 6B_3 \kappa_\Phi \kappa_Q \kappa_\Phi \kappa_Q\right).\]

In the similar way, we can obtain \( \kappa \) and \( \tau \) for \( \Psi \) and \( Q \). However, since we consider matter which is minimal-coupled to the metric, the effect of the gravity theory on the evolution of the matter perturbations appears only through the gravitational potential \( \Phi \) as same as in the GR case. Therefore, only \( \kappa \) and \( \tau \) for \( \Phi \) are required to evaluate the evolution of the matter perturbation in the second-order perturbation theory. Note that, although these coefficients, \( \kappa \) and \( \tau \), should be in general treated as the scale-dependent terms when the scalar potential terms are taken into account to accommodate chameleon-type models such as \( f(R) \) gravity [19], we only consider the Vainshtein-type models in which \( \kappa \) and \( \tau \) are shown to depend only on the time.

Next we consider the matter perturbation evolution. In the case of minimal-coupling, the evolution equations for the matter perturbations are same as in the case of GR. Then, the modification of the gravity theory in the growth of the matter density fluctuations would appear through \( \kappa_\Phi \) and \( \gamma_{2, \Phi} \) (or \( \tau_\Phi \)) given in the above discussion. Here, we focus on the galaxy bispectrum which can probe the nonlinear (second-order) gravitational evolution of the matter density fluctuations, and it is described by the second-order perturbative kernels \( F_2 \) and \( G_2 \) defined in Eqs. (7) and (8).

It has been found that in Horndeski theory these kernels are provided in the more suggestive forms as [12, 13]

\[F_2(k_1, k_2) = \left(1 + \frac{k_1 \cdot k_2(k_1^2 + k_2^2)}{2k_1^2 k_2^2}\right) - \frac{2}{7} \lambda(t) \left(1 - \frac{(k_1 \cdot k_2)^2}{k_1^2 k_2^2}\right),\]

\[G_2(k_1, k_2) = \left(1 + \frac{k_1 \cdot k_2(k_1^2 + k_2^2)}{2k_1^2 k_2^2}\right) - \frac{4}{7} \lambda(t) \left(1 - \frac{(k_1 \cdot k_2)^2}{k_1^2 k_2^2}\right),\]
The second-order perturbations, we further define new two functions as (21):

\[ \alpha \]

The time-dependent coefficient \( \lambda(t) \) obeys a following second-rank differential equation [13] (see also [12]):

\[
\frac{d^2 \lambda}{d \ln a^2} + \left( 2 + \frac{d \ln H}{d \ln a} + 4f \right) \frac{d \lambda}{d \ln a} + (2f^2 + 4f) \lambda = \frac{7}{2} (f^2 - \tau_\Phi). \tag{41}
\]

One can clearly see that \( \lambda \) can be induced by \( \kappa_\Phi \) and \( \tau_\Phi \). In the above equation, \( f \) is the linear growth rate and the evolution equation for \( f \) is given by

\[
\frac{df}{d \ln a} + \left( 2 + \frac{d \ln H}{d \ln a} \right) f + f^2 - \kappa_\Phi = 0. \tag{42}
\]

This means that the precise measurement of the linear growth rate, \( f \), can test the information captured in \( \kappa_\Phi \). On the other hand, a new coefficient \( \lambda \) appearing in the second-order kernels depends on not only \( \kappa_\Phi \) but also \( \tau_\Phi \) which could have new information about the modification of gravity theory. Thus, in this sense, we would like to stress that the precise investigation of the nonlinear growth of matter density fluctuations by using galaxy bispectrum would give new insight into the gravity theory. Although \( \lambda \) is shown to go to unity in the limit of the Einstein-de Sitter Universe, the large deviation from unity can be possible in the Horndeski theory, as seen in the subsequent section. Once the Horndeski functions \( G_a \) are given as the underlying model, we can solve the equations for the growth rate and the coefficient of the second-order kernel by substituting Eqs. (33)-(35) and (37) into Eqs. (41) and (42).

### IV. ANALYTIC EVALUATION

As shown in the previous section, the growth rate \( f \) and the coefficient of the second-order kernel \( \lambda \) would be good candidates to capture the signature of the modification of gravity theory. The general solutions of \( f \) and \( \lambda \) are formal and the attempting to fit observations of the growth history for each model is rather complicated. One alternatively considers the simple characterization stimulating physical intuition.

#### A. EFT parametrization of Horndeski gravity

First, instead of considering the explicit form of the Horndeski function \( G_a \), we will introduce the EFT parameters to specify the cosmological information. In Ref. [15], the authors identified the minimum number of the functions that fully specify these linear perturbations in the Horndeski class of gravity theory. The minimum set to specify the total amount of cosmological information up to the linear order is four functions of time that can be labeled \( \{\alpha_M, \alpha_T, \alpha_B, \alpha_K\} \) in addition to the background expansion history \( H(t) \) and the effective Planck mass \( M(t) \). These \( \alpha_i \) are related to the Horndeski variables as [15, 20]

\[
HM^2 \alpha_M = \frac{d}{dt} \left( 2G_4X - 2G_{5\phi} - \left( \frac{\phi}{H} - \dot{H} \right) G_{5X} \right), \tag{43}
\]

\[
M^2 \alpha_T = 2X \left[ 2G_{4X} - 2G_{5\phi} - \left( \frac{\phi}{H} - \dot{H} \right) G_{5X} \right], \tag{44}
\]

\[
HM^2 \alpha_B = 2\phi \left( XG_{3X} - G_{4\phi} - 2G_{4\phi X} \right) + 8H \left( G_{4X} + 2XG_{4XX} - G_{5\phi} - XG_{5\phi X} \right)
+ 2\phi H^2 \left( 3G_{5X} + 2XG_{5XX} \right), \tag{45}
\]

\[
H^2 M^2 \alpha_K = 2X \left( K_X + 2XK_{XX} - 2G_{3\phi} - 2XG_{3\phi X} \right) + 12\phi H \left( G_{4X} + XG_{3XX} - 3G_{4\phi} - 2XG_{4\phi X} \right)
+ 12H^2 \left( G_{4X} + 8XG_{4XX} + 4X^2G_{4XX} - G_{5\phi} - 5XG_{5\phi X} - 2X^2G_{5\phi XX} \right)
+ 6\phi H^3 \left( 3G_{5X} + 7XG_{5XX} + 2X^2G_{5XX} \right). \tag{46}
\]

The \( \alpha \) functions represent the linear freedom of the Horndeski class of models. Moreover, to generalize this fact to the second-order perturbation theory in the subhorizon scales, two other functions of time are required [21]. To fit the second-order perturbations, we further define new two \( \alpha \) functions as

\[
M^2 \alpha_{V1} = -2X \left( G_{4X} + 2XG_{4XX} - G_{4\phi} + 2\phi G_{5\phi} - XG_{5\phi X} + \phi HXG_{5XX} \right), \tag{47}
\]

\[
M^2 \alpha_{V2} = 2\phi HXG_{5X}, \tag{48}
\]
where $\alpha_{V1}$ and $\alpha_{V2}$ represent the amplitude of the Vainshtein screening mechanism [17, 22–24]. To derive the effective theory in terms of the scalar field perturbations, we simply consider the form of the scalar field:

$$\phi(t) \rightarrow \phi = t + \delta \phi(t, x).$$

(49)

Using these $\alpha$ functions, the coefficients in the basic equations (29)-(31) can be rewritten as

$$G_T = M^2,$$

(50)

$$F_T = M^2 \left(1 + \alpha_T\right),$$

(51)

$$A_0 = M^2 \left[H + \frac{3}{2} \Omega_m - \frac{(HM^2 \alpha_B)}{2HM^2} - \left(\alpha_M - \alpha_T + \frac{1}{2} \alpha_B\right)\right],$$

(52)

$$A_1 = M^2 (\alpha_M - \alpha_T),$$

(53)

$$A_2 = \frac{1}{2} M^2 \alpha_B,$$

(54)

$$B_0 = \frac{1}{2} M^2 \left[\alpha_{V1} - \frac{\left(M^2 \alpha_{V1}\right)}{HM^2} + \alpha_M + \alpha_B - \frac{3}{2} \left(\alpha_T - \alpha_{V2} + \frac{1}{M^2} \left(M^2 \alpha_{V2}\right)\right)\right],$$

(55)

$$B_1 = \frac{1}{2} M^2 \left[\alpha_T - \alpha_{V2} + \frac{1}{M^2} \left(M^2 \alpha_{V2}\right)\right],$$

(56)

$$B_2 = M^2 (\alpha_{V1} + \alpha_{V2}),$$

(57)

$$B_3 = \frac{1}{2} M^2 \alpha_{V2},$$

(58)

where we have introduced the fractional matter density defined as

$$\tilde{\Omega}_m = \frac{\rho_m}{3M^2 H^2}.$$  

(59)

As pointed out in Ref. [15], the sound speed of the scalar field perturbations is sufficiently close to the speed of light, the quasi-static approximation is valid and $\alpha_K$ does not appear the equations for the matter density fluctuations.

### B. Approximate expressions

Next, we introduce phenomenological parameters that describe non-standard cosmological models. For the growth rate $f$, the gravitational growth index, $\gamma$, is known as the simplest parametrization:

$$f(a) = \tilde{\Omega}_m'(a).$$

(60)

One can capture the signature of the deviations from the general relativity up to the linear order through the difference in $\gamma$ from the standard value $\gamma_{GR+\Lambda CDM} \approx 6/11$. Let us derive the analytic formula of the gravitational growth index for the Horndeski class of gravity theory [7]. Substituting this parametrization for $f(a)$ into Eq. (42) yields

$$\gamma \tilde{\Omega}_m \frac{d \ln \tilde{\Omega}_m}{d \ln a} + \left(2 + \frac{d \ln H}{d \ln a}\right) \tilde{\Omega}_m^2 + \Omega_m^{2\gamma} - \kappa \phi = 0.$$  

(61)

To evaluate the value of $\gamma$, it is required to specify the expansion history of the Universe. Therefore we rewrite the Friedmann and matter conservation equations in terms of $\Omega_m$ as

$$\frac{\rho_{\text{DE}}}{3M^2 H^2} = 1 - \tilde{\Omega}_m,$$

(62)

$$\frac{d \ln H}{d \ln a} = -\frac{3}{2} \left[1 + w_{\text{DE}} \left(1 - \tilde{\Omega}_m\right)\right],$$

(63)

$$\frac{d \ln \tilde{\Omega}_m}{d \ln a} = 3w_{\text{DE}} \left(1 - \tilde{\Omega}_m\right) - \alpha_M,$$

(64)
where $w_{DE}$ is an effective equation of state for the dark energy component, which is defined as $w_{DE} := p_{DE}/\rho_{DE}$ with $p_{DE} := -M^2(3H^2 + 2\dot{H})$. We assume that the Universe can be well described by the ΛCDM model in the deep matter dominated era, hence $w_{DE}$ and $\kappa_{\Phi}$ can be approximated in the following expanded forms:

$$w_{DE} = \sum_{n=0}^{\infty} \frac{1}{n!} w^{(n)} \left(1 - \tilde{\Omega}_m\right)^n,$$

(65)

$$\kappa_{\Phi} - \frac{3\tau_1}{2} \tilde{\Omega}_m = \sum_{n=1}^{\infty} \frac{1}{n!} \kappa_{\Phi}^{(n)} \left(1 - \tilde{\Omega}_m\right)^n.$$

(66)

For $\alpha_{\Omega}$, we take the following parametrization as

$$\alpha_{\Omega} = c_{\Omega} \left(1 - \tilde{\Omega}_m\right).$$

(67)

Combining Eqs. (61)-(67), the leading order equation in the limit of $1 - \tilde{\Omega}_m \ll 1$ leads to the approximate value of the gravitational growth index

$$\gamma \approx \frac{3 - 3w^{(0)} - 2\kappa_{\Phi}^{(1)}}{5 - 6w^{(0)} + 2c_{\Omega}}.$$

(68)

This result is a generalization of the well-known formula for the $w$CDM model [7]. We note that in the case of the ΛCDM model with GR as our fiducial model, we take $w^{(0)} = -1$, $\kappa_{\Phi}^{(1)} = c_{\Omega} = 0$ and the standard result $\gamma = 6/11 \approx 0.545$ can be recovered. Current observations give the stringent constraint on $\gamma$. In particular, the deviation from the standard value is allowed only below 10%. Hence, the only small deviations of $(w^{(0)} + 1)$, $c_{\Omega}$ and $\kappa_{\Phi}^{(1)}$ are possible.

Similarly, for the coefficient of the second-order kernel, $\lambda$, we propose the following approximated form as the simplest characterization with the second-order index $\xi$:

$$\lambda(t) = \tilde{\Omega}_m^\xi(t).$$

(69)

Applying this, we rewrite Eq. (41) to

$$\tilde{\Omega}_m^\xi \left[\left(\frac{d \ln \tilde{\Omega}_m}{d \ln a}\right)^2 + \xi \frac{d^2 \ln \tilde{\Omega}_m}{d \ln a^2} + \left(2 + \frac{d \ln H}{d \ln a} + 4\tilde{\Omega}_m^\gamma\right) \xi \frac{d \ln \tilde{\Omega}_m}{d \ln a} + \left(2\tilde{\Omega}_m + \kappa_{\Phi}\right)\right] = \frac{7}{2} \left(\tilde{\Omega}_m^2 - \tau_\Phi\right).$$

(70)

Since $\tau_\Phi$ reduces to zero in the limit of the ΛCDM+GR, we can expand

$$\tau_\Phi = \sum_{n=1}^{\infty} \frac{1}{n!} \tau_\Phi^{(n)} \left(1 - \tilde{\Omega}_m\right)^n.$$

(71)

Substituting this expression and Eqs. (63)-(67) into Eq. (70), we find that the leading value of the second-order index can be estimated as

$$\xi = \frac{-3 + 6\gamma + 2\kappa_{\Phi}^{(1)} + 7\tau_\Phi^{(1)}}{(7 - 6w^{(0)} + 2c_{\Omega})(1 - 3w^{(0)} + c_{\Omega})}.$$

(72)

In the case of the ΛCDM model with GR, $\tau_\Phi^{(1)} = 0$ and we have $\xi = 3/572 \approx 0.00524$, suggesting that $\lambda$ can be well approximated by unity in this limit [25]. Once the underlying theory describing the dark energy or the modification of gravity theory is given, we immediately calculate the second-order index $\xi$ through Eq. (72) as well as the gravitational growth index $\gamma$, Eq. (68).

In the above discussion, we have obtained approximated forms of $\gamma$ and $\xi$ which capture the modification of gravity theory, in terms of $w_{DE}$, $\kappa_{\Phi}$, $\tau_\Phi$ and an EFT function $\alpha_{\Omega}$. Since $\kappa_{\Phi}$ and $\tau_\Phi$ are written in terms of the EFT $\alpha$ functions introduced in the previous subsection, the phenomenological parameters $\gamma$ and $\xi$ are approximately written with the EFT functions. We take a following parametrization for all $\alpha$ functions, which is suggested in Ref. [15]:

$$\alpha_i = c_i \left(1 - \tilde{\Omega}_m\right),$$

(73)
where \(c_i\) are constant characterizing the amplitude of the modification. Hence the remaining nontrivial parameters can be reduced to \(\{c_B, c_M, c_T, \xi_1, \xi_2\}\).

Using the concrete expression for the \(\alpha\) functions in Eq. (73), we can apply the formalism derived in the previous subsection to calculate \(\gamma\) and \(\xi\). Therefore, we first explicitly expand the quantities in terms of \(1 - \tilde{\Omega}_m\) as

\[
\kappa_\Phi - \frac{3}{2} \tilde{\Omega}_m = \sum_{n=1}^{\infty} \frac{1}{n!} \kappa_\Phi^{(n)} \left(1 - \tilde{\Omega}_m\right)^n,
\]

\[
\kappa_Q = \sum_{n=0}^{\infty} \frac{1}{n!} \kappa_Q^{(n)} \left(1 - \tilde{\Omega}_m\right)^n.
\]

By using these constant parameters, we then obtain the leading-order corrections:

\[
\kappa_\Phi^{(1)} = \frac{3}{2} \left(\frac{c_T + \left(c_B + 2c_M - 2c_T\right)^2}{6(1 + w^{(0)}) - 6c_B w^{(0)} + 2c_B c_M - c_B + 4c_M - 4c_T}\right),
\]

\[
\kappa_Q^{(0)} = \frac{6(1 + w^{(0)}) - 6c_B w^{(0)} + 2c_B c_M - c_B + 4c_M - 4c_T}{3(c_B + 2c_M - 2c_T)},
\]

\[
\kappa_\Phi^{(1)} = -\frac{c_B}{2} \kappa_Q^{(0)},
\]

and

\[
\tau_\Phi^{(1)} = \frac{1}{3} \left(\kappa_Q^{(0)}\right)^2 \left(\frac{c_B + c_M - \frac{3}{2} c_T}{2} \kappa_Q^{(0)} - \frac{9}{4} c_T\right) + \frac{1}{3} \left(\kappa_Q^{(0)}\right)^2 \left[1 + 3w^{(0)} - c_M\right] \kappa_Q^{(0)} - \frac{9}{2} \left[\frac{1}{2} - 3w^{(0)} + c_M\right] \kappa_Q^{(0)} + 3 \right] \xi_2.
\]

By substituting these into Eqs. (68) and (72), we can evaluate the indices of the gravitational growth and the second-order kernel, \(\gamma\) and \(\xi\), in terms of constant parameters \(\{w^{(0)}, c_B, c_M, c_T, \xi_1, \xi_2\}\), while the explicit expressions are complicated. To confirm the validity of the approximated expression for our new parameter \(\xi\), at least, we have to check the constancy of \(\xi\). In Fig. 1, we numerically evaluate \(\xi_{\text{eval}}(t) = \ln \lambda(t)/\ln \tilde{\Omega}_m(t)\) with various values of the \(\alpha\) functions. We found that the deviations from the constant value of \(\xi_{\text{eval}}\) in most cases are less than 10%, though in some cases the constancy of \(\xi\) seems to be eventually violated in low redshifts. Hence we conclude that we can neglect the time-dependence on \(\xi\) as far as future surveys observing high redshifts are considered.
Let us consider a specific cosmological model to investigate the gravitational growth index $\gamma$ and the second-order index $\xi$. We assume the $\Lambda$CDM cosmology as the background Hubble expansion, namely $w(0) = -1$. In particular, when we take the small braiding limit, namely $c_\beta \to 0$ the situation is found to be drastically simplified (see also Ref. [21]). The parameters in Eqs. (76)-(79) reduce to

$$
\kappa_\phi^{(1)} = \frac{3}{2} c_M, \quad \kappa_Q^{(0)} = \frac{3}{2}, \quad \kappa_\phi^{(1)} = 0, \quad \tau_\phi^{(1)} = \frac{9}{8} (c_M - 1) c_{V1} - c_M.
$$

(80)

Surprisingly, these variables depend only on $c_M$ and $c_{V1}$. Hence $\gamma$ and $\xi$ also depend only on $c_M$ and $c_{V1}$ as

$$
\gamma(c_\beta = 0) = \frac{3(2 - c_M)}{11 + 2 c_M},
$$

(81)

$$
\xi(c_\beta = 0) = \frac{3}{8(4 + c_M)(13 + 2 c_M)} \left[ \frac{(8 + c_M)(1 - 26 c_M)}{11 + 2 c_M} + 21 (c_M - 1) c_{V1} \right].
$$

(82)

From the above expression, we can easily find that by taking $c_M \to 0$ the growth index $\gamma$ goes to $6/11$ which is just the value in the standard $\Lambda$CDM model with GR. On the other hand, our new parameter $\xi$ depends not only on $c_M$ but also on $c_{V1}$, and hence we could realize the large $\xi$ even in the case which can not be distinguished with the standard $\Lambda$CDM model with GR up to the linear growth of density fluctuations. In Appendix B, we construct the explicit model in which there is observably a large deviation for $\xi$ but $\gamma$ remains the standard one.

V. FISHER ANALYSIS

Let us numerically investigate the expected constraints on the second-order index $\xi$ as well as the gravitational growth index $\gamma$, based on the Fisher analysis. To evaluate the expected constraints from the galaxy bispectrum measured in future galaxy surveys, we calculate the Fisher matrix for the bispectrum, which is obtained by summing over all possible triangular configurations. The explicit expression is written as

$$
F_{\alpha\beta} = \sum_{k_1,k_2,k_3=k_{min}}^{k_{max}} \frac{\partial B_{obs}^\alpha}{\partial \theta^\alpha} \cdot \left[ \text{Cov}^{-1}(B_{obs}^\alpha, B_{obs}^\beta) \right] \cdot \frac{\partial B_{obs}^\beta}{\partial \theta^\beta},
$$

(83)

where $\theta^\alpha$ are free parameters to be determined by observations. The marginalized expected $1\sigma$ error on parameter $\theta^\alpha$ from the Fisher matrix is estimated to be $\sigma(\theta^\alpha) = \sqrt{(F^{-1})_{\alpha\alpha}}$. Assuming the Gaussian covariance, we obtain the covariance matrix as [26–28]

$$
\text{Cov}(B_{obs}^\alpha, B_{obs}^\beta) = \frac{s_B V_{\text{survey}}}{N_t} \left( P_s(k_1) + \frac{1}{n_g} \right) \left( P_s(k_2) + \frac{1}{n_g} \right) \left( P_s(k_3) + \frac{1}{n_g} \right),
$$

(84)

where $s_B$ is the symmetric factor describing the number of a given bispectrum triangle ($s_B = 6, 2, 1$ for equilateral, isosceles and general triangles, respectively) and the quantity $N_t = V_R/k_F^3$ denotes the total number of available triangles with $k_F = 2\pi/V_{\text{survey}}^{1/3}$ and $V_R = 8\pi^2 k_1 k_2 k_3 k_F^3$ being the fundamental frequency and the volume of the fundamental cell in Fourier space, respectively. The maximal wavelength is chosen so that $k_{max} = \pi/(2R_{min})$ with $\sigma(R_{min}, z) = 0.5$ [26]. To parametrize the background evolution, we take the constant dark energy equation-of-state parameter for simplicity. Throughout this paper, our fiducial model is the $\Lambda$CDM cosmological model with a nearly scale-invariant primordial scalar perturbations; $\Omega_{m,0} = 0.318$, $\Omega_{b,0} = 0.495$, $\Omega_{\Lambda,0} = 0.6817$, $w_{DE} = -1$, $h = 0.67$, $n_s = 0.9619$, and $\sigma_8 = 0.835$.

We apply our Fisher matrix analysis to future redshift surveys conducted by Euclid and the SKA. Our forecast is performed for the parameter set: \{w_{DE}, \gamma, \xi, b_1(z), b_2(z)\}. We adopt the predicted number density of galaxies, $n_g(z)$, as a function of redshift, given in Table 3 of Ref. [29] for Euclid and in Table 1 of Ref. [30] for the SKA1MID and SKA2, respectively. Considering the 5, 17, and 14 redshift bins for SKA1MID, SKA2, and Euclid, in total, we include 10, 34, 28 nuisance parameters to model the bias parameters as well as three parameters characterizing the growth history. The fiducial values of $\gamma$ and $\xi$ are $11/6$ and $3/572$, respectively. The fiducial values of the linear and nonlinear bias parameters are obtained from the dark matter halo bias, because galaxies are formed in dark matter halos. To evaluate galaxy bias parameters, we follow the procedure of the halo bias parameters $b_{\ell}^h(M, z)$ ($\ell = 1, 2$) given in Ref. [31], that is, $b_\ell = \frac{1}{n_g} \int_{M_{min}} M_{max} dM \frac{dn}{dT} b_{\ell}^h (N)_M$. Here we adopt the Sheth-Tormen mass function $dn/dM$ [32], and the halo occupation distributions $\langle N \rangle_M$ in Ref. [33] with the fitting parameters given in Ref. [34], and we obtain the minimum mass $M_{min}$ from $n_g = \int_{M_{min}} M_{max} dM \frac{dn}{dT} \langle N \rangle_M$ for a galaxy number density $n_g$. 

FIG. 2: Forecast $1\sigma$ and $2\sigma$ marginalized contours in $(\gamma, \xi)$ plane for SKA1MID(blue), SKA2(red) and Euclid(green), marginalizing over the equation-of-state parameter and bias parameters. For comparison, we also plot the kinetic gravity braiding model (purple boxes), and the large $\xi$ model (orange triangle).

TABLE I: Summary of the $1\sigma$ constraints on the equation-of-state parameter, the gravitational growth index $\gamma$ and the second-order index $\xi$ marginalized over the linear and nonlinear bias parameters.

| survey    | $\Delta w_{DE}$ | $\Delta \gamma$ | $\Delta \xi$ |
|-----------|------------------|------------------|--------------|
| SKA1MID   | 0.135            | 0.067            | 0.060        |
| SKA2      | 0.0085           | 0.0087           | 0.0094       |
| Euclid    | 0.016            | 0.021            | 0.018        |

In Fig. 2, we show the $1\sigma$ and $2\sigma$ confidence regions of the gravitational growth index $\gamma$ and the second-order index $\xi$. The results of our Fisher analysis marginalizing over the bias parameters are summarized in Table I. For comparison, we also plot the predicted values of $\{\gamma, \xi\}$ for the kinetic gravity braiding model [35] ($n = 1, 2, 3$) as purple boxes and the large $\xi$ model with $p = 1$, derived in Appendix B as orange triangle. Although the constraint from galaxy bispectrum on the gravitational growth index $\gamma$ is relatively weaker than the expected constraints by galaxy power spectrum, the precise measurement conducted by future galaxy surveys can constrain $\xi$ significantly. In particular, we can distinguish the models in which the expansion history and the linear growth rate are almost same as the fiducial mode but the different nonlinear evolution is given.

VI. CONCLUSION

In this paper, we have discussed the potential power of the bispectrum of biased objects as a possible new probe to test the theory of gravity beyond the linear-order perturbation. To investigate the impact of the galaxy bispectrum, we have performed the generalization of the redshift-space galaxy bispectrum to the wider class of gravity theory based on the standard cosmological perturbation theory. Since the modification of gravity theory typically changes the clustering property of large-scale structure, measuring the galaxy bispectrum induced by the late-time nonlinear gravitational evolution of the density fluctuations can be used to test the gravity theory through the evolution of the linear growth rate and the second-order kernels. Among them, in order to focus on the time-evolving coefficient $\lambda$ in the second-order kernel, we have introduced the second-order index $\xi$ defined in Eq. (69), as a good candidate to
catch the higher-order nature of modified gravity theory. We analytically obtained the expression of $\xi$ in terms of the parameters characterizing the growth history of the Universe \[ \text{Eq. (72)} \] as well as the gravitational growth index $\gamma$. We then found that the second-order index $\xi$ can carry new information about the growth of structure that is not included in the linear perturbation theory.

As a specific model of modified gravity theory, we have applied the result to the most general scalar-tensor theory with second-order equations of motion, namely the Horndeski theory. There are models in which the Hubble expansion and the gravitational growth index are completely same as the ΛCDM model but the different value of $\xi$ can be obtained. Finally, we have performed the Fisher matrix analysis to show that the future precise measurements of galaxy bispectrum can significantly constrain $\xi$ as well as the equation-of-state parameter and the gravitational growth index.

In this paper, we have made several simplified assumptions. We have considered only the constant $\gamma$ and $\xi$ as the asymptotic values in high redshifts. Although many modified gravity theories are known to be well described by constant $\gamma$ and $\xi$, in some cases, it would be useful to consider the time-dependence of them. Particularly we found that the time-dependence of $\xi$ generally becomes large in lower redshifts. When we consider a near-future survey that covers a comparative low-redshift depth, the effect of the time-dependence should be included to model the growth-history of the Universe precisely. We hope to address these issues in more realistic situations.

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Appendix A: Coefficients

In this Appendix, we summarize the definitions of the coefficients in the field equation presented in Sec. III. The coefficients in the linear part are defined by

$$A_0 = \frac{\dot{\Theta}}{H^2} + \frac{\Theta}{H} + \mathcal{F}_T - 2\dot{G}_T - 2\frac{\dot{\Theta}}{H} + \rho_m \frac{\rho_m}{2H^2}, \quad (A1)$$
$$A_1 = \frac{\ddot{G}_T}{H} + \dot{G}_T - \mathcal{F}_T, \quad (A2)$$
$$A_2 = \dot{G}_T - \frac{\Theta}{H}. \quad (A3)$$

where

$$\mathcal{F}_T = 2 \left[ G_4 - X \left( \phi G_{5X} + G_{5\phi} \right) \right], \quad (A4)$$
$$\mathcal{G}_T = 2 \left[ G_4 - 2XG_{4X} - X \left( H\phi G_{5X} - G_{5\phi} \right) \right], \quad (A5)$$
$$\Theta = -\dot{\phi}XG_{3X} + 2HG_4 - 8HXG_{4X} - 8HX^2G_{4XX} + \phi G_{4\phi} + 2X\phi G_{4\phi X}$$
$$- H^2\dot{\phi} \left( 5XG_{5X} + 2X^2G_{5XX} \right) + 2HX \left( 3G_{5\phi} + 2XG_{5\phi X} \right). \quad (A6)$$

The coefficients in the higher-order parts are given by

$$B_0 = \frac{X}{H} \left[ \phi G_{3X} + 3 \left( \ddot{X} + 2HX \right)G_{4XX} + 2XXG_{4XXX} - 3\dot{\phi}G_{4X} + 2\phi XG_{4\phi XX} + \left( \dot{H} + H^2 \right)\phi G_{5X} + \phi \left( 2HX + (\dot{H} + H^2)X \right)G_{5XX} + H\phi X\dot{X}G_{5XX} \right]$$
$$- 2 \left( \ddot{X} + 2HX \right)G_{5\phi X} - \phi XG_{5\phi XX} - X \left( \ddot{X} + 2HX \right)G_{5\phi X}, \quad (A7)$$
$$B_1 = 2X \left[ G_{4X} + \phi (G_{5X} + XG_{5XX}) - G_{5\phi} + XG_{5\phi X} \right], \quad (A8)$$
$$B_2 = -2X \left( G_{4X} + 2XG_{4XX} + H\phi G_{5X} + H\phi XG_{5XX} - G_{5\phi} - XG_{5\phi X} \right), \quad (A9)$$
$$B_3 = H\phi XG_{5X}. \quad (A10)$$
Appendix B: Large $\xi$ model

In this section, we try to construct the model in which there is observably large deviation for the second order index $\xi$ but the gravitational growth index $\gamma$ remains the close value to the standard one. To proceed the model building, we consider the subclass of Horndeski theory with the shift-symmetry, namely $K_\phi = G_1, \phi = 0$, and $G_5 = 0$. In this setup, the EFT parameters reduce to

\begin{align}
M^2 &= 2(G_4 - 2XG_{4X}) , \\
HM^2_\alpha &= -2\dot{X}(G_{4X} + 2XG_{4XX}) , \\
M^2_\alpha &= 4XG_{4X} , \\
HM^2_\alpha &= 2\dot{\phi}XG_{3X} + 8HX(G_{4X} + 2XG_{4XX}) , \\
H^2M^2_\alpha &= 2X(K_X + 2XK_{XX}) + 12\dot{\phi}HX(G_{3X} + XG_{3XX}) \\
&+ 12H^2X(G_{4X} + 8XG_{4XX} + 4X^2G_{4XX}) , \\
M^2_\alpha &= -2X(G_{4X} + 2XG_{4XX}) , \\
\rho^2_\alpha &= 0 .
\end{align}

The gravitational field equations are given by [17]

\begin{align}
\mathcal{E} &= -\rho_m , \quad \mathcal{P} = 0 ,
\end{align}

where

\begin{align}
\mathcal{E} &= 2XK_X - K + 6H\dot{\phi}XG_{3X} - 6H^2\left[G_4 - 4X(G_{4X} + XG_{4XX})\right] , \\
\mathcal{P} &= K - \dot{\phi}XG_{3X} + 2(3H^2 + 2\dot{H})G_4 - 4\left[(3H^2 + 2\dot{H})X + H\dot{X}\right]G_{4X} - 8HXG_{4XX} .
\end{align}

Here $\rho_m$ is the nonrelativistic matter energy density. Assuming the presence of the shift symmetry of the scalar field, $\phi \to \phi + \text{const.}$, we obtain the conservation equation of the Noether current, which is given by

\begin{align}
\dot{J} + 3HJ &= 0 ,
\end{align}

where the Noether current is defined as

\begin{align}
J &= \dot{\phi}K_X + 6HXG_{3X} + 6H^2\dot{\phi}(G_{4X} + 2XG_{4XX}) .
\end{align}

We clearly find that its solution is given by $J \propto 1/a^3$, implying that $J$ approaches zero as the universe expands. Therefore, we take $J = 0$ as the simplest attractor solution throughout the paper. With the attractor condition and the $\alpha$ functions defined in Eqs. (B2)-(B7), the gravitational equations of motion can become the more suggestive form:

\begin{align}
3M^2H^2 &= \rho_m - K , \\
M^2 \left(2\dot{H} + 3H^2\right) &= -K + \frac{1}{2} \frac{d\ln X}{d\ln a} H^2M^2_\alpha ,
\end{align}

which immediately implies that the corresponding dark energy model is given by

\begin{align}
\rho_{DE} &= -K , \\
p_{DE} &= K - \frac{1}{2} \frac{d\ln X}{d\ln a} H^2M^2_\alpha .
\end{align}

The attractor solution $J = 0$ provides the conditions for the $\alpha$ functions:

\begin{align}
J\dot{\phi} &= 2XK_X + 3H^2M^2(\alpha_\alpha + 2\alpha_{V1}) = 0 , \\
\dot{J}\dot{\phi} &= M^2H^3 \left[\frac{1}{2} \frac{d\ln X}{d\ln a} \alpha_K + 3\frac{d\ln H}{d\ln a} \alpha_B\right] = 0 .
\end{align}

Hence, the equation-of-state parameter for the dark energy can be written as

\begin{align}
w_{DE} &= -1 + \frac{1}{1 - \Omega_m \frac{d\ln H}{d\ln a} \alpha_B} .
\end{align}
Substituting (B19) into Eqs. (63) and (64), we then have

$$\frac{d \ln H}{d \ln a} = -\frac{3}{2} \tilde{\Omega}_m \frac{1}{1 + \alpha_{BK}}, \quad (B20)$$

$$\frac{d \ln \tilde{\Omega}_m}{d \ln a} = -3 \left(1 - \tilde{\Omega}_m\right) - 3 \tilde{\Omega}_m \frac{\alpha_{BK}}{1 + \alpha_{BK}} - \alpha_M, \quad (B21)$$

where $\alpha_{BK} = 3\alpha_B^2/2\alpha_K$ and Eq. (B19) can be given by

$$w_{DE} = -1 - \tilde{\Omega}_m \frac{\alpha_{BK}}{1 + \alpha_{BK}}. \quad (B22)$$

It leads that the asymptotic value of $w_{DE}$ at the early stage of the Universe is obtained as

$$w^{(0)} = -1 - \frac{3c_B^2}{2c_K}. \quad (B23)$$

Using these solutions, we would like to construct the explicit model to realize the large $\xi$ with small deviation of $\gamma$. Let us assume the following form of the Horndeski functions (see also [36, 37] for the extended Galileon model):

$$K = -c_2 M_4^2 \left(\frac{X}{M_4^2}\right)^{p_2}, \quad G_3 = c_3 M_3 \left(\frac{X}{M_3^2}\right)^{p_3}, \quad G_4 = \frac{M_4^2}{2} - c_4 M_4^2 \left(\frac{X}{M_4^2}\right)^{p_4}. \quad (B24)$$

We search for a tracker solution, which is characterized by the condition

$$H \dot{\phi}^{2p} = \text{const.}, \quad (B25)$$

where $q$ is assumed to be real constant. When we choose the following powers, all terms in Eq. (B12) are proportional to $\dot{\phi}^{2p}$:

$$p_2 = p, \quad p_3 = p + q - \frac{1}{2}, \quad p_4 = p + 2q. \quad (B26)$$

With them, the attractor condition $J = 0$ gives the relation between $c_i$ and $M_i$. Eqs. (B17) and (B18) yield the relation between $\alpha$ functions:

$$\alpha_B = 2p \left(1 - \tilde{\Omega}_m\right) - 2 \alpha_{V1}, \quad \alpha_K = 6q \alpha_B. \quad (B27)$$

Moreover, Eqs. (B2) and (B3) can be rewritten as

$$\alpha_M = \frac{3}{2q} \tilde{\Omega}_m \frac{\alpha_{V1}}{1 - \alpha_{BK}}, \quad \alpha_T = \frac{2 \alpha_{V1}}{1 - 2p - 4q}. \quad (B28)$$

Combining these, we can write down the explicit form of $c_i$ in terms of $p$, $q$ and $c_B$ as

$$c_K = 6qc_B, \quad c_M = \frac{3}{2q} \left(p - \frac{1}{2}c_B\right), \quad c_T = \frac{2}{1 - 2p - 4q} \left(p - \frac{1}{2}c_B\right), \quad c_{V1} = p - \frac{1}{2}c_B, \quad c_{V2} = 0. \quad (B29)$$

Here, we consider the negligible braiding case as a specific example, that is $c_B \to 0$. In this case, $c_K$ gives no contributions to the density perturbations in the small scale limit and there is no dependence on $c_T$ in the case of $c_B \to 0$, as shown in the previous section. Since $c_M = 3p/2q$ and $c_{V1} = p$, the leading order index of the growth and the second-order kernel can be rewritten in terms of $p$ and $q$. In particular, when we consider the large hierarchy $|p/q| \ll 1$, Eqs. (81) and (82) are given by

$$\gamma \approx 0.545, \quad \xi \approx 0.005 - 0.151p. \quad (B30)$$

We found that there is one parameter family of the model in which the gravitational growth index $\gamma$ is the same as the standard $\Lambda$CDM+GR value while the second-order index $\xi$ can have the large deviation from the standard one.

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