Correlation between LFV and muon \((g - 2)\) in MSSM

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We give a simultaneous analysis of \(g_\mu - 2\) and lepton flavor violation within MSSM. Working on the interaction basis, we give direct relations between \(\delta a_\mu\) and \(\text{Br}(\tau \to \mu \gamma)\) induced by the supersymmetric particles. We find the SUSY parameter space to satisfy the \(g_\mu - 2\) constraint can be considerably different from that given in case of no lepton flavor mixing in the soft SUSY breaking sector.

Recently, the Brookhaven E821 Collaboration announced their new experimental result on muon anomalous magnetic moment, \(a_\mu = (g_\mu - 2)/2\), with improved statistics\(^1\). The present discrepancy between the standard model (SM) prediction and the measurement is \(a_\mu^\text{exp} - a_\mu^\text{SM} = 26(10) \times 10^{-10}\).

In this work we study the SUSY contributions to \(a_\mu\) in the case when considering the lepton flavor mixing in the soft SUSY breaking sector. Besides the numerical studies of lepton flavor mixing effects on \(a_\mu\), we give a thorough analysis of the correlation between the SUSY contributions to \(a_\mu\) and to lepton flavor violation (LFV). We find that in this case the SUSY parameter space may be quite different from that without slepton mixing.

The effective Lagrangian related to \(a_\mu\) and LFV is as follow:

\[
L_{\text{eff}} = e m_i \bar{l}_i \sigma^\alpha \beta F^\alpha \beta (A_{ij}^L P_L + A_{ij}^R P_R) l_j ,
\]

where \(i(j)\) denotes the initial (final) lepton flavor. The \(a_\mu\) is given by

\[
a_\mu = m_\mu^2 (A_{22}^L + A_{22}^R),
\]

while the branching ratio of \(\tau \to \mu \gamma\) is given by

\[
\text{Br}(\tau \to \mu \gamma) = \frac{\alpha e m_\tau}{4} \frac{[|A_{23}^L|^2 + |A_{23}^R|^2]}{\Gamma_\tau},
\]

with \(\Gamma_\tau\) being the tau decay width. We can see that the expressions for \(\delta a_\mu\) and \(\tau \to \mu \gamma\) are closed related.

The SUSY contribution to the form factors \(A_L\) and \(A_R\) is given by the photon-penguin diagrams via exchanging (i) chargino-sneutrino and (ii) neutralino-slepton as shown in FIG. 1. The analytic expressions for \(\delta a_\mu\) can be found in Ref. 2. These expressions are given in the mass eigenstates of the SUSY particles. It is suitable to do numerical calculations on this basis. However, to analyze the physical effects, it is more convenient to work on the interaction basis, which is defined as the basis where the lepton mass matrix and the gauge coupling vertices are diagonal. On this basis the Feynman diagrams are more complicated than those in FIG. 1.

On this basis the sneutrino and slepton mass matrices are generally not diagonal. They can be written in a general form as

\[
M^2_{\tilde{\nu}} = Z_L m^2_{\tilde{\nu}} Z_L^\dagger ,
\]

and

\[
M^2_i = \begin{pmatrix}
Z_L m^2_{\tilde{l}_L} Z_L^\dagger & -m_i (\mu \tan \beta + A_i^\dagger) \\
-m_i (\mu^* \tan \beta + A_i) & Z_R m^2_{\tilde{l}_R} Z_R^\dagger
\end{pmatrix},
\]

where \(Z_L\) and \(Z_R\) are the left- and right-handed mixing matrices. In this work we consider the mixing between the second and the third generations. Thus, \(Z_L\) and \(m^2_L\) are given by(similar for right-handed parameters)

\[
Z_L = \begin{pmatrix}
\cos \theta_L & \sin \theta_L \\
-\sin \theta_L & \cos \theta_L
\end{pmatrix}
\quad \text{and} \quad m^2_L = \begin{pmatrix}
m_2^2 & m_3^2
\end{pmatrix}.
\]

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FIG. 1: Feynman diagrams of the one-loop SUSY contribution to $a_\mu$ (and the process $\tau \to \mu\gamma$) via the exchange of a chargino (left) and via a neutralino (right).

Since on this basis all the interaction vertices are diagonal, the LFV effects are transferred to the propagators of sleptons and sneutrinos. The propagator of a scalar is $i\delta a\sqrt{s}/(m^2 s - m^2)$, where $m$ is the corresponding mass. It is easy to get $(M^2_\mu)^{-1} = \left( \begin{array}{cc} \frac{c_2^2}{m_2^2} + \frac{s_2^2}{m_3^2} & \frac{c_{LSL}}{m_2^2 s_3^2} \frac{m_2^2 - m_3^2}{m_2^2 m_3^2} \\ \frac{c_{LSL}}{m_2^2 s_3^2} \frac{m_2^2 - m_3^2}{m_2^2 m_3^2} & s_2^2 \end{array} \right)$.

$\delta a_\mu$ is related to the propagator of $\tilde{\nu}_\mu - \tilde{\nu}_\mu$, which is noted as $P(\tilde{\nu}_\mu - \tilde{\nu}_\mu) = \frac{c_2^2}{m_2^2} + \frac{s_2^2}{m_3^2}$, while $\tau \to \mu\gamma$ is related to the propagator of $\tilde{\nu}_\tau - \tilde{\nu}_\mu$, which is $P(\tilde{\nu}_\tau - \tilde{\nu}_\mu) = \frac{1}{2} \sin 2\theta_L \frac{m_2^2 - m_3^2}{m_2^2 m_3^2}$.

We consider the following two limit cases:

$P(\tilde{\nu}_\mu - \tilde{\nu}_\mu) \to \begin{cases} m_2^2 \approx m_2^2 \approx m_2^2, \\ m_2^2 \gg m_3^2, \frac{\bar{c}_2^2}{m_3^2} \end{cases}$, (8)

while

$P(\tilde{\nu}_\tau - \tilde{\nu}_\mu) \to \begin{cases} m_2^2 \approx m_2^2, \\ m_2^2 \gg m_3^2, \frac{1}{2} \sin 2\theta_L \frac{1}{m_3^2} \end{cases}$, (9)

In the first case with $m_2^2 \approx m_2^2$, we can see that $\delta a_\mu$ does not depend on the mixing angle $\theta_L$ while $\text{Br}(\tau \to \mu\gamma)$ tends to zero. Thus models with gravity or gauge mediated supersymmetry breaking may predict that $\delta a_\mu$ has nothing to do with the mixing angle $\theta_L$ while $\text{Br}(\tau \to \mu\gamma)$ should be very small[2]. Thus the first case is actually the same as the case of no lepton flavor mixing in the soft sector. The second case leads us to the effective SUSY scenario[3], where the first two generations’ sfermions are as heavy as about 20 TeV. In this work we mainly consider the latter case. In this case the two quantities are closely correlated.

The propagator of slepton are approximately given by

$(M^2_\ell)^{-1} \approx \begin{pmatrix} A & C \\ C & B \end{pmatrix}$, (10)

with

$A \approx (M^2_\ell)^{-1}, \ B \approx A(\theta_L \to \theta_R)$, (11)

and

$C \approx m_{\tau} \tan \beta \frac{m_2^2 - m_3^2}{m_2^2 m_3^2} \left[ \frac{1}{2} \sin 2\theta_L \left( \frac{s_3^2}{m_2^2} + \frac{\bar{c}_2^2}{m_3^2} \right) \right] + \frac{1}{2} \sin 2\theta_R \left( \frac{s_3^2}{m_2^2} + \frac{\bar{c}_2^2}{m_3^2} \right) \left( \frac{s_3^2}{m_2^2} + \frac{\bar{c}_2^2}{m_3^2} \right)$, (12)
\[ \tilde{\nu}_\mu(\tilde{\nu}_\tau) \quad \mu \quad M_2 \quad \tilde{\nu}_\mu \]

\[ \tilde{\mu}_R(\tilde{\tau}_R) \quad \tilde{\mu}_R \quad M_1 \quad -\mu \quad \mu_L \]

FIG. 2: Feynman diagram which gives the dominant contribution to \( \delta a_\mu \) (and to the process \( \tau \to \mu \gamma \)) in case of (a) only left-handed slepton mixing and (b) only right-handed mixing. The black dots in the fermion line are mass insertions.

In matrix \( C \) we have omitted the terms proportional to \( m_\mu \). From the above expressions we know the propagator of \( \tilde{\mu}_L - \tilde{\mu}_L \) is the same as that of \( \tilde{\nu}_\mu - \tilde{\nu}_\mu \). The most interesting result is the propagator of \( \tilde{\mu}_L - \tilde{\mu}_R \), given by

\[ P(\tilde{\mu}_L - \tilde{\mu}_R) = \frac{1}{4} m_\tau \tan \beta \sin 2 \theta_L \sin 2 \theta_R \left( \frac{m_2^2 - m_3^2}{m_2^2 m_3^2} \right)^2 , \]

which has an \( m_\tau \) enhancement. This term may dominate over others if both the left- and right-handed mixing angles are large.

\( \delta a_\mu \) with only left-handed mixing

When there is only left-handed mixing, the most important contribution to \( \delta a_\mu \) and \( \text{Br}(\tau \to \mu \gamma) \) comes from the diagram in FIG. 2(a), given on the interaction basis. From this diagram we can directly read that

\[ A_R^{23}(c) = \frac{1}{2} \frac{\delta a_\mu(c)}{m_\mu^2} Z_L^{23} . \]

Then we have, assuming \( \theta_L = \pi/4 \), that

\[ \text{Br}(\tau \to \mu \gamma) \approx \frac{\alpha_{\text{em}}}{4} m_\tau^5 |A_R^{23}(c)|^2 / \Gamma_\tau \approx 2.9 \times 10^{-13} |\delta a_\mu|^2 . \]

From the present upper limit of \( \text{Br}(\tau \to \mu \gamma) < 10^{-6} \), we get that

\[ \delta a_\mu < 1.9 \times 10^{-10} , \quad \text{in case of } \theta_R = 0 . \]

From this diagram we also have the conclusion that

\[ \mu M_2 > 0 , \quad \text{in case of } \theta_R = 0 \]

to give positive contribution to \( \delta a_\mu \). The same diagram gives the dominant contribution to \( \delta a_\mu \) in the case of no lepton flavor mixing. Thus the same conclusion of the sign of \( \mu \) is given in that case.

\( \delta a_\mu \) with only right-handed mixing

In case of only right-handed mixing, the chargino-sneutrino diagram gives no contribution to \( \delta a_\mu \). The most important contribution to \( \delta a_\mu \) and \( \text{Br}(\tau \to \mu \gamma) \) comes from the diagram in FIG. 2(b), given on the interaction basis.
If we ignore the mixing between the left- and right-handed sleptons, $Z_R$ is approximately the slepton mixing matrix. From FIG. 2(b) we then have

$$A_{23}^{(n)}(n) \approx \frac{1}{2} \frac{\delta a_{\mu}}{m_{\mu}} \left( \frac{m_{\mu}}{m_{\tau}} \right) Z_{33}^{23} Z_{23}^{-1}.$$  \hspace{1cm} (18)

Then we have, assuming $\theta_R = \pi/4$, that

$$Br(\tau \rightarrow \mu \gamma) \approx \frac{\alpha_{em}}{4} m_{\mu}^5 |A_{23}^{(n)}(n)|^2 / \Gamma_{\tau} \approx 1. \times 10^{11} |\delta a_{\mu}|^2.$$  \hspace{1cm} (19)

From the present upper limit of $Br(\tau \rightarrow \mu \gamma) < 10^{-6}$, we get that

$$\delta a_{\mu} < 32 \times 10^{-10}, \text{ in case of } \theta_L = 0.$$  \hspace{1cm} (20)

This upper bound is much larger than that in case of only left-handed mixing. It is obvious that the factor $\frac{m_{\mu}}{m_{\tau}}$ in Eq. (13), which greatly suppresses $Br(\tau \rightarrow \mu \gamma)$, helps to increase the bound. This factor comes from the Yukawa vertex on the right, where the Higgsino component $H_D$ has to be associated with the muon line since there is only right-handed mixing in the slepton sector. However, in FIG. 2(a), where the charged Higgsino component $H_D$ is associated with the tau line, no such factor helps to suppress $Br(\tau \rightarrow \mu \gamma)$.

Another interesting point is that the mass insertion for the neutral component of $\tilde{H}_U \tilde{H}_D$ is $-\mu$, while it is $\mu$ for the same term of the charged component. Thus we have

$$\mu M_1 < 0, \text{ in case of } \theta_L = 0$$  \hspace{1cm} (21)

to give positive contribution to $\delta a_{\mu}$. This means that if we set $M_1$ and $M_2$ have same sign, which is well motivated theoretically, $\mu$ should be negative in this case.

Since we ignore the left-right mixing between the sleptons, the naive bound we get should be examined numerically. In FIG. 3(a) we show the numerical results in this case. If we adopt the GUT motivated relation $M_1 = \frac{\frac{2}{3} \alpha_3}{\alpha_2} M_2 \approx 0.5 M_2$ we have $\delta a_{\mu} < 3 \times 10^{-3}$ to satisfy the $Br(\tau \rightarrow \mu \gamma)$ bound. However, if we relax the above relation and fix $M_1 = 60 GeV$, $\delta a_{\mu}$ can be as large as $\sim 17 \times 10^{-10}$ without violating the bound of $Br(\tau \rightarrow \mu \gamma)$.

**$\delta a_{\mu}$ with both left- and right-handed mixing**
FIG. 4: Feynman diagram which gives the dominant contribution to $\delta a_\mu$ in the case that both the left- and right-handed slepton mixing are large.

In this case we have derived that there is an $m_\tau$ enhancement in the propagator $\tilde{\mu}_L - \tilde{\mu}_R$. The enhancement leads to that the diagram in FIG. 4 may give dominant contribution to $\delta a_\mu$ if both $\theta_L$ and $\theta_R$ are large. However, it seems that there is no obvious term which give dominant contribution to $\text{Br}(\tau \to \mu\gamma)$. If $m_3$ is small, FIG. 4 with $\mu_R$ replaced by $\tau_R$ may dominates other terms. In this case we get a similar limit as that given in the case with only right-handed mixing,

$$\delta a_\mu < 32 \times 10^{-10} \quad \text{in case of no } \theta = 0 \quad (22)$$

However, this bound is very loose because in large parameter space the contribution to $\text{Br}(\tau \to \mu\gamma)$ by exchanging $\chi^-$ is more important than that by exchanging $\chi^0$. We have to study this case numerically.

FIG. 3(b) displays $\delta a_\mu$ and $\text{Br}(\tau \to \mu\gamma)$ with $\theta_L = \theta_R = \pi/4$. If $M_2$ and $\mu$ are both large, there is a large region which can accommodate $\delta a_\mu$ and $\text{Br}(\tau \to \mu\gamma)$ simultaneously. As $\mu$ becomes large, $\text{Br}(\tau \to \mu\gamma)$ decreases while $\delta a_\mu$ increases. This is understood that large $\mu$ enhances the propagator of $P(\tilde{\mu}_L - \tilde{\mu}_R)$ and leads to large chargino mass, which decreases $\text{Br}(\tau \to \mu\gamma)$.

In summary, when both the left- and right-handed slepton mixing is large, SUSY can enhance $\delta a_\mu$ to within the $E821 \pm 2\sigma$ bounds in a large parameter space through the slepton mixing between the second and the third generations. In this case small $\tan \beta$ is slightly favored. Higgsino mass parameter $\mu$ can be either positive or negative depending on the relative sign between $\theta_L$ and $\theta_R$. $\delta a_\mu$ can reach up to $\sim 20 \times 10^{-10}$ even keeping the relation $M_1 \approx 0.5M_2$, which means bino is not necessarily kept very light.

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