Controlled quantum teleportation on the three control pattern via five-qubit cluster state

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Abstract. We proposed a new scheme of controlled teleportation to transmit the arbitrary one qubit state information via five-qubit cluster state channel. The transmission of quantum information from Alice (A) to Charli (C) had been controlled by Bob (B). The distribution of local measurements on Alice's, Bob's and Charli's qubit studied at three patterns; there are AAABC, ABABC, and ABBBC. The important results of this research are the observed differences in output variation and the efficiency of sending arbitrary one qubit information to the control pattern on Bob's Particle via five-qubit cluster state channel.

1. Introduction
Quantum teleportation was first investigated by Benneth et al. [1]. They investigated an arbitrary unknown qubit from Alice to Bob via entanglement channel. After their investigation, quantum teleportation further investigates by the other author since its very important advantages for quantum communication and quantum computation. Not only investigated in theory but also experiment [2,3].

Quantum teleportation via three-qubit entanglement first presented by Karlsson and Bourennne [4]. They send an arbitrary unknown qubit via GHZ state. They called the third side as a supervisor or Controlled Teleportation scheme. This supervisor part named by Charli. The channel was not be transmitted by one side but three sides cooperatively. Bob cannot reconstruct information without permission from Charli. Karlsson et al. provided that this scheme could show the secret sharing and secret splitting of quantum entanglement [5,6]. Besides that, controlled teleportation has been proposed by another author to get the most efficient and optimally controlled quantum teleportation [7,8].

Gao et al. [8] investigated optimal quantum teleportation and showed an analytic expression of the maximal successful probability and exact value from localizable entanglement. Their investigation uses an EPR and GHZ state like the channel and shows that three-qubit state can collapse to an EPR pair by an appropriate measurement on one qubit. They gave the maximal successful probability for controlled teleportation and by channel tripartite entanglement and completing last observation. To confirm that state teleported successfully, we observed by measuring fidelity and probability. The best teleportation reached if the fidelity and probability are 1 [9].

In this paper, we investigated the optimally controlled quantum teleportation of an arbitrary unknown state via five-qubit cluster state with a different pattern of control agents. Like the channel, we used a five-qubit cluster state in (1). Cluster state is used because of its maximally entangled property [10]. We investigated the different pattern of some control agents such as AAACB (one control agent), ACACB (two control agents) and ACCCB (three control agents).
\[ \psi_{12345} = \frac{1}{2} (|00000\rangle + |00011\rangle + |11100\rangle - |11111\rangle ) \] (1)

Alice sends an arbitrary unknown state to Bob by the control Charli in a different pattern. Charli helped Bob to reconstruct measurement from Alice by sending measurement result and unitary operator via classical communication. The unitary operation is defined as Pauli Matrices [11].

\[
I = |0\rangle\langle 0| + |1\rangle\langle 1| = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
Z = |0\rangle\langle 0| - |1\rangle\langle 1| = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
X = |1\rangle\langle 0| + |0\rangle\langle 1| = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\
Y = |0\rangle\langle 1| + |1\rangle\langle 0| = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}
\] (2)

This investigation uses several unsure such Alice as sender, Bob as receiver and Charli as a controller; then we need particle/information to be sent and channel as the main path of teleportation. To teleport information, the channel must be entangled. The measurement used in this scheme is the Bennet and Karlson scheme [1,4]. Alice makes joint state from channel and information. Then, this new state is measured first by Alice. The result of Alice measurement is sent to Charli to be measured. The last measurement is sent to Bob and Bob must build the pure information by using Unitary operator [12].

2. Result and Discussion

2.1 Controlled Quantum Teleportation via AAAACB Pattern

Suppose that Alice wants to teleport one arbitrary state to Bob with controlled by Charli. Alice share qubit 1, 2, and 3 to herself, qubit 4 to Charli and qubit 5 to Bob.

\[ \psi_a = \alpha|0\rangle + \beta|1\rangle \]
\[ \psi_{a12345} = \alpha|0\rangle + \beta|1\rangle \bigotimes \frac{1}{2} (|00000\rangle + |00011\rangle + |11100\rangle - |11111\rangle ) \] (3) (4)

Then, Alice shows von-Neuman measurement four-particle entangled state on her qubits a, 1, 2, and 3 as basis:

\[
\psi_{a12345} = \frac{1}{2} (\alpha|00\rangle + \beta|11\rangle) \bigotimes \frac{1}{2} (|00000\rangle + |00011\rangle + |11100\rangle - |11111\rangle ) \\
+ \frac{1}{2} (\alpha|00\rangle - \beta|11\rangle) \bigotimes \frac{1}{2} (|00000\rangle + |00111\rangle - |11100\rangle + |11111\rangle ) \\
+ \frac{1}{2} (\alpha|00\rangle + \beta|11\rangle) \bigotimes \frac{1}{2} (|00000\rangle - |00111\rangle + |11100\rangle + |11111\rangle ) \\
+ \frac{1}{2} (-\alpha|00\rangle \\
+ \beta|11\rangle) \bigotimes \frac{1}{2} (-|00000\rangle + |00111\rangle + |11100\rangle + |11111\rangle ) \\
\] (5)

Charli helped Bob to reconstruct state from Alice measurement by measured his qubit. Charli used qubit \( \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \) and \( \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \). Then, Bob used his qubit 5 to reconstruct information by Unitary Operation sending by Charli. The result of this scheme is shown in Table 1. This table shows the measurement of Bob and the unitary operator send by Charli via classical communication. We defined unitary operation as Pauli Matrices as (2).
Table 1. Measurement by Charli to Bob in AAACB pattern

| No | $|\psi_b\rangle$ | UO |
|----|-----------------|----|
| 1  | $\frac{1}{2\sqrt{2}}(\alpha|0\rangle + \beta|1\rangle)$ | $2\sqrt{2}$ I |
| 2  | $\frac{1}{2\sqrt{2}}(\alpha|0\rangle - \beta|1\rangle)$ | $2\sqrt{2}$ Z |
| 3  | $\frac{1}{2\sqrt{2}}(\alpha|1\rangle + \beta|0\rangle)$ | $2\sqrt{2}$ X |
| 4  | $\frac{1}{2\sqrt{2}}(-\alpha|1\rangle + \beta|0\rangle)$ | $2\sqrt{2}$ ZX |
| 5  | $\frac{1}{2\sqrt{2}}(\alpha|0\rangle - \beta|1\rangle)$ | $2\sqrt{2}$ Z |
| 6  | $\frac{1}{2\sqrt{2}}(\alpha|0\rangle + \beta|1\rangle)$ | $2\sqrt{2}$ I |
| 7  | $\frac{1}{2\sqrt{2}}(-\alpha|1\rangle + \beta|0\rangle)$ | $2\sqrt{2}$ ZX |
| 8  | $\frac{1}{2\sqrt{2}}(\alpha|1\rangle + \beta|0\rangle)$ | $2\sqrt{2}$ X |

Bob can reconstruct the state only if Charli help him by send his measurement and Alice send unitary operation. From Table 1, we can see that AAACB give us eight possible results to establish by Bob in his lab. We can find that the probability of this scheme is perfect teleportation by the formula:

$$P = |\langle \psi_a | \psi_b \rangle|^2$$

(6)

2.2 Controlled Quantum Teleportation via ACACB Pattern

With the same particle, Alice share qubit 2, 3 to Charli, qubit 5 to Bob and the else to herself from the channel (2). Alice performed GHZ measurement for her qubits.

$$\psi_{a12345} = \frac{1}{2}(|000\rangle + |111\rangle)\otimes \frac{1}{2}(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle - \beta|111\rangle) + \frac{1}{2}(|000\rangle - |111\rangle)\otimes \frac{1}{2}(\alpha|000\rangle + \alpha|011\rangle - \beta|110\rangle + \beta|111\rangle) + \frac{1}{2}(|001\rangle + |100\rangle)\otimes \frac{1}{2}(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle - \beta|111\rangle) + \frac{1}{2}(|001\rangle - |100\rangle)\otimes \frac{1}{2}(-\alpha|000\rangle - \alpha|011\rangle + \beta|110\rangle - \beta|111\rangle)$$

(7)

Alice measurement result was sent to Charli and Bob. Then, Charli help Bob reconstruct the particle state. In this pattern, Charli has two control agent to help Bob finish his task. Charli performs Bell-state measurement to his two-qubit in each Alice result.

$$\varphi^+ = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\varphi^- = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\Psi^+ = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
\[ \Psi^- = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle) \]  

(8)

From Bell-state measurement, we get 16 possible result and unitary operation to be transmitted to Bob lab. So he can reconstruct first information state from Alice by control of Charli. The results are shown in Table 2. From Table 2 and by applying Equation (6) we can see that this pattern have perfect probability. Same as its fidelity.

Table 2. Measurement by Charli to Bob in ACACB Pattern

| No | \(|\psi_b\rangle\) | UO |
|----|----------------|----|
| 1  | \(\frac{1}{4}(\alpha|0\rangle + \beta|1\rangle)\) | 4 I |
| 2  | \(\frac{1}{4}(\alpha|0\rangle - \beta|1\rangle)\) | 4 Z |
| 3  | \(\frac{1}{4}(\alpha|1\rangle + \beta|0\rangle)\) | 4 X |
| 4  | \(\frac{1}{4}(\alpha|1\rangle - \beta|0\rangle)\) | 4 XZ |
| 5  | \(\frac{1}{4}(\alpha|0\rangle + \beta|1\rangle)\) | 4 I |
| 6  | \(\frac{1}{4}(\alpha|0\rangle - \beta|1\rangle)\) | 4 Z |
| 7  | \(\frac{1}{4}(\alpha|1\rangle + \beta|0\rangle)\) | 4 X |
| 8  | \(\frac{1}{4}(\alpha|1\rangle - \beta|0\rangle)\) | 4 XZ |
| 9  | \(\frac{1}{4}(\alpha|1\rangle + \beta|0\rangle)\) | 4 ZX |
| 10 | \(\frac{1}{4}(\alpha|1\rangle + \beta|0\rangle)\) | 4 X |
| 11 | \(\frac{1}{4}(\alpha|0\rangle + \beta|1\rangle)\) | 4 I |
| 12 | \(\frac{1}{4}(\alpha|0\rangle - \beta|1\rangle)\) | -4 I |
| 13 | \(\frac{1}{4}(\alpha|1\rangle - \beta|0\rangle)\) | -4 X |
| 14 | \(\frac{1}{4}(\alpha|1\rangle - \beta|0\rangle)\) | 4 ZX |
| 15 | \(\frac{1}{4}(\alpha|0\rangle - \beta|1\rangle)\) | 4 Z |
| 16 | \(\frac{1}{4}(\alpha|0\rangle - \beta|1\rangle)\) | 4 I |

2.3 Controlled Quantum Teleportation via ACCCB Pattern

This pattern uses most number of control agent in this scheme. With same particle and channel, Alice sends qubit 1 to herself, qubit 3,4,5 to Charli and qubit 5 to Bob. With the same step, Alice performs Bell-state measurement by (10 - 13), and Charli performs GHZ measurement. So, Bob receives results such as shown in Table 3. Table 3 shows us that Bob gets 16 possible state and unitary operation to reconstruct particle state from Alice. Addition, by Equation (6) we can see that this scheme also perfect teleportation with probability and fidelity and probability are 1. We have to remember that this pattern has the most number of control agents with three control agents. We can see that there are no significant differences with the ACACB pattern before. It shows that in
teleporting one qubit information by using controlled agent give the perfect teleportation in all pattern.

Table 3. Measurement by Charli to Bob in ACCCB Pattern

| No | $|\psi_b\rangle$ | UO |
|----|-----------------|----|
| 1  | $\frac{1}{4}(\alpha|0\rangle + \beta|1\rangle)$ | 4 I |
| 2  | $\frac{1}{4}(\alpha|0\rangle - \beta|1\rangle)$ | 4 Z |
| 3  | $\frac{1}{4}(\alpha|1\rangle + \beta|0\rangle)$ | 4 X |
| 4  | $\frac{1}{4}(\alpha|1\rangle - \beta|0\rangle)$ | 4 XZ |
| 5  | $\frac{1}{4}(\alpha|0\rangle + \beta|1\rangle)$ | 4 I |
| 6  | $\frac{1}{4}(\alpha|0\rangle - \beta|1\rangle)$ | 4 Z |
| 7  | $\frac{1}{4}(\alpha|1\rangle + \beta|0\rangle)$ | 4 X |
| 8  | $\frac{1}{4}(\alpha|1\rangle - \beta|0\rangle)$ | 4 XZ |
| 9  | $\frac{1}{4}(\alpha|1\rangle + \beta|0\rangle)$ | 4 X |
| 10 | $\frac{1}{4}(\alpha|1\rangle + \beta|0\rangle)$ | 4 Z |
| 11 | $\frac{1}{4}(\alpha|0\rangle - \beta|1\rangle)$ | 4 Z |
| 12 | $\frac{1}{4}(\alpha|0\rangle - \beta|1\rangle)$ | 4 Z |
| 13 | $\frac{1}{4}(\alpha|1\rangle - \beta|0\rangle)$ | 4 X |
| 14 | $\frac{1}{4}(\alpha|1\rangle - \beta|0\rangle)$ | 4 XZ |
| 15 | $\frac{1}{4}(\alpha|0\rangle - \beta|1\rangle)$ | 4 Z |
| 16 | $\frac{1}{4}(\alpha|0\rangle - \beta|1\rangle)$ | 4 I |

3. Conclusion

Three patterns of quantum teleportation of an arbitrary unknown qubit via five-qubit cluster state have been investigated to complete [8]. From this investigation, we can see that all of the pattern show perfect probability because of its cluster state channels and control agent. Its complete [8] result in optimally controlled teleportation. Our investigation shows that we can reach optimally controlled teleportation by use maximally entangled channel with an appropriate pattern in the controlled scheme.

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