Periodically-driven facilitated high-efficient dissipative entanglement with Rydberg atoms

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A time-dependent periodical field can be utilized to efficiently modify the Rabi coupling of system, exhibiting nontrivial dynamics. We propose a scheme to show that this feature can be applied for speeding up the formation of dissipative steady entanglement based on Rydberg anti-blockade mechanism in a simplified configuration, fundamentally stemming from a frequency match between the external-field modulation frequency and the systematic characteristic frequency. In the presence of an optimal modulation frequency that is exactly equal to the central frequency of driving field, it enables a sufficient residence time of the two-excitation Rydberg state for an irreversible spontaneous decay onto the target state, leading to an accelerated high-fidelity steady entanglement \( \sim 0.98 \), with a shorter formation time \(< 400 \mu s\). We show that, a global maximal fidelity benefits from a consistence of microwave-field coupling and spontaneous decay strengths, by which the scheme promises a robust insensitivity to the initial population distributions. This simple approach to facilitate the generation of dissipative entangled two-qubit states by using periodic drivings may guide a new experimental direction in Rydberg quantum technology and quantum information.

\section{I. INTRODUCTION}

Dissipative mechanism, in contrast to its natural idea that leads to the destruction of quantum effects de-coherently, can counterintuitively serve as an important resource for implementing quantum information task and controlled quantum state preparation, having been denoted into numerous studies \cite{1–8}. Experimental achievements towards dissipative production of entangled states have been implemented in trapped ions \cite{9,10}, superconducting quantum bits \cite{11,12}, and macroscopic atomic ensembles \cite{13}.

It is remarkable that pioneer works demonstrating preparation of dissipative steady entanglement from an uppermost Rydberg level, were proposed by Saffman \cite{14} and Mølmer \cite{15}, providing great potentials for applications of quantum computation and engineering by Rydberg dissipation \cite{16,17}. However typical time needed for this dissipative preparation has exceeded hundreds of milliseconds due to the use of a high-lying long-lived Rydberg level with its principle quantum number \( n \sim 125 \) e.g. see \cite{18,19}, yet difficult for the experimental measurement. So far great efforts towards ways of accelerated formation of an entangled steady state use Rydberg electromagnetically induced transparency involving a dark eigenstate, which is influenced by a fast-decaying middle state in the presence of Rydberg interactions, promising a rapid entangled-state generation \cite{13}. Nevertheless this approach simultaneously suffering from the complexity of energy levels and laser fields combining with Rydberg interactions \cite{20,22}, still has not been realized in experiment.

Other alternative approaches are proposed by combining an optical cavity to trigger the entanglement formation, because the cavity decay treating as an auxiliary loss channel towards the target state, can permit a reduced stabilization time that is smaller than tens of microseconds depending on the absolute value of cavity mode coupling \cite{23,24}. More recently an intriguing improvement for a ten-times faster generation of steady entanglement in a cavity is suggested by using pulse modulation, achieving an exponentially enhancement for the atom-cavity coupling \cite{22}. While such schemes based on cavity-trapped atoms require a stronger atom-cavity coupling associated with a precise control, remains uneasy for real implementations.

In parallel, periodically driven systems are well-known in quantum physics arising a wealth of versatile quantum phenomena \cite{25,31}. For example a simplest two-level atom system driven by a periodically-modulated field can significantly modify the time evolution of system \cite{32,33}, exhibiting many intriguing effects such as persistent atomic trapping in excited state \cite{34}, an excited two-level emitter \cite{35}, multi-photon resonance and response \cite{36}, maximal population transfer \cite{29}, discrete time crystals \cite{37,38} and so on. In addition, periodically-driven dissipative (open) systems offer various prospects for new-class feature of out-of-equilibrium physics which is inaccessible for equilibrium ensembles \cite{39,41}. Combining periodically-driven with a Rydberg atom array is found to produce the localization of quantum many-body state \cite{40}, and more recently, a new mechanism driven by an amplitude-modulated periodic field to generate dissipative steady-state entanglement in a solid-state qubit system is exploited \cite{42}, giving more perspective for non-trivial periodically-driven Rydberg features.

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FIG. 1. (color online) Schematic diagram of a high-fidelity facilitated entanglement preparation. (a) A detailed two-atom energy-level structure. For each atom, two ground states $|0\rangle$ and $|1\rangle$ are coupled by a continuous microwave field $g$, and $|1\rangle$ is far-off resonantly coupled to the Rydberg state $|r\rangle$ via a periodically-modulated optical field with the Rabi frequency $\Omega(t)$. Assuming that the detuning $\Delta$ can compensate the interaction induced energy shift $U_{rr}$ by satisfying the anti-blockade condition $U_{rr} = 2\Delta$ persistently, $|11\rangle$ is directly coupled to $|rr\rangle$ via a two-photon excitation with an effective Rabi frequency $\Omega_{eff}^2(t)/2\Delta$. (b) The effective five-level diagram in the two-atom base vectors shows the real atom-field interactions and spontaneous decays, where the four singly-excited states (gray shadow area) are safely discarded due to $\Delta \gg \Omega(t)/\sqrt{2}$. $|S\rangle$ is a target dark entangled state, unidirectionally receiving the population from $|rr\rangle$. The amplitude of $\Omega_{eff}(t)$ is illustrated in the inset where blue-solid and black-dashed curves stand for weak and strong frequency modulations, respectively. (c) The experimental setup (proposal, see Section V for detailed experimental parameters). Two atoms are trapped in tightly focused dipolar traps with interatomic distance close to the critical value $R_c$ for van der Waals-type interaction $U_{rr}$. They are persistently driven by a microwave field $g$ as well as a periodically-modulated optical field $\Omega(t)$ realized by an acousto-optical modulator in the experimental implementation.

Utilizing Rydberg anti-blockade mechanism, in the present work we develop a simplified scheme for the dissipative preparation of a maximal steady entangled state $|S\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$, driven by a periodic pump field, representing an exotic and accelerated formation. Remarkably, the existence of an external modulation frequency matching with the characteristic frequency of the effective two-state system, will arise a visible change to the Rabi behavior of the two-excitation Rydberg state in the laser pumping for unitary dynamics, which benefits from a fast decaying onto the target state due to existing a longer residence time. Accompanying with a suitable adjustment for other dissipative rate and microwave coupling strength, the resulting entanglement formation reveals a stepwise acceleration towards the target entangled state with a very high-fidelity $\sim 0.98$, where the required convergence time can be shorter than 400$\mu$s. Because a stepwise-improving entangled-state accumulation rate will appear within these durations, in contrast to the case of a continuous driving where the accumulation rate persists slow. Additionally we demonstrate an easy way for further acceleration by increasing the effective Rabi coupling strength in the pumping and point out a robust insensitivity of our scheme to arbitrary initial conditions under experimental parameters, where the population distributes randomly in different ground states except the target one.

II. THEORETICAL FORMULATION

A. Two-atom Hamiltonian

The relevant energy levels interested have been represented in Fig 1(a) composing two three-level $\Lambda$ interacting atoms. For each atom existing two ground states $|0\rangle$, $|1\rangle$ and one Rydberg state $|r\rangle$, the hyperfine states $|0\rangle$ and $|1\rangle$ suffering from dipole-forbidden are coupled by a continuous microwave field with strength $g$, receiving the population from $|r\rangle$ through stochastic spontaneous emission decays by rate $\gamma$. Population accumulated on
other hyperfine ground states rather than \(|0\rangle\) and \(|1\rangle\), can be re-pumped onto \(|0\rangle\) or \(|1\rangle\) by recycling lasers (not shown) 14. Special attention is paid to the optical transition between \(|1\rangle\) and \(|r\rangle\) driven by a time-dependent field \(\Omega(t)\), accompanied by a big detuning \(\Delta\) (assuming \(\Delta \gg \Omega(t)/\sqrt{2}\)) for realizing a complete suppression to the singly-excited collective states. \(U_{rr}\) describes the two-atom van der Waals(vdWs)-type interaction, making it dynamically compensate the detuning of \(|rr\rangle\) by \(U_{rr} = 2\Delta\), which can facilitate a direct resonant coupling \(\Omega(t)^2/2\Delta\) between \(|11\rangle\) and \(|rr\rangle\) via adiabatically eliminating the middle single-excitation Rydberg states, as shown in Fig.1(b) (shadow box).

In a rotating-wave frame, the basic Hamiltonian for the two-atom system is

\[
\hat{H} = \hat{H}_0 \otimes \hat{I} + \hat{I} \otimes \hat{H}_0 + \hat{\nu}
\]

where the single-atom Hamiltonian \(\hat{H}_0\) is

\[
\hat{H}_0 = -\Delta |r\rangle \langle r| + \left(\frac{g}{2}|1\rangle \langle 0| + \frac{\Omega(t)}{2}|1\rangle \langle 1| + H.c.\right) \tag{2}
\]

and \(\hat{I}\) a 3 \(\times\) 3 unit operator. \(\hat{\nu}\) describes the two-atom Rydberg interaction given by \(\hat{\nu} = U_{rr} |rr\rangle \langle rr|\).

Using the two-atom base vectors that contain one doubly-excited state \(|rr\rangle\), four singly-excited states \(|r1\rangle\), \(|r0\rangle\), \(|1r\rangle\), \(|0r\rangle\), and four ground states \(|00\rangle\), \(|11\rangle\), \(|T\rangle\), \(|S\rangle\), the complete decay behavior (dissipation) can be expressed by the regular Lindblad operators,

\[
\begin{align*}
\hat{L}_1 &= \sqrt{3}|11\rangle \langle r1| + (|T\rangle + |S\rangle)(|r0\rangle + |1r\rangle |rr\rangle) \\
\hat{L}_2 &= \sqrt{3}|11\rangle \langle r0| + (|T\rangle - |S\rangle)(|0r\rangle + |r1\rangle |rr\rangle) \\
\hat{L}_3 &= \sqrt{3}|00\rangle \langle r0| + (|T\rangle - |S\rangle)(|1r\rangle + |0r\rangle |rr\rangle) \\
\hat{L}_4 &= \sqrt{3}|00\rangle \langle r1| + (|T\rangle + |S\rangle)(|1r\rangle + |0r\rangle |rr\rangle)
\end{align*}
\]

(iii) Dissipative formation. Owing to the limited lifetime of \(|rr\rangle\), in the effective five-level frame population on \(|rr\rangle\) suffers from a big spontaneous loss, randomly decaying into four ground states. Once it possibly decays into the unique dark state \(|S\rangle\), population will be irreversibly accumulated on it; otherwise they repeatedly experience circular steps (ii) \(\Rightarrow\) (iii) by returning to step (ii) again and again until the system is finally stabilized into the state \(|S\rangle\).

Therefore we stress that, ideally the target entangled state \(|S\rangle\) can be deterministically created for a sufficiently long time, robustly insensitive to arbitrary optical and microwave drivings. In fact this time could be endless which is impossible for real experimental measurement during a finite detection time. So achieving a fast and high-fidelity entanglement depending on a simplified and feasible protocol with Rydberg atoms remains challenge, having attracting numerous efforts in theory 20,22,43. Presently we show a new way by implementing a periodical modulation to the amplitude of \(\Omega(t)\), the Rabi behavior between \(|11\rangle\) and \(|rr\rangle\) will be significantly changed, producing complex multi-frequency oscillations 22. We verify the resulting formation of target ground state \(|S\rangle\) can be dramatically accelerated under an optimization for relevant frequency parameters.

B. Effective five-level system

The physical essence for robustly fast preparation based on a reduced scheme of five relevant energy levels [see Fig. 1(b)], note that the four singly-excited collective states are adiabatically discarded due to the far-off one-photon resonance, can be understood in three steps:

(i) State initialization. Prepare two atoms in ground states e.g. \(|11\rangle\) (robustness to arbitrary initial states will be discussed in section VI).

(ii) Circular unitary dynamics and microwave coupling. A two-state unitary dynamics leads to exchanged population (excitation and de-excitation) by an engineered Rabi oscillation between \(|11\rangle\) and \(|rr\rangle\), governed by a reduced two-state Hamiltonian

\[
\hat{H}_{eff,uni} = \Omega_{eff}(t)(|11\rangle + |rr\rangle)(\langle 11| + \langle rr|) \tag{4}
\]

where \(\Omega_{eff}(t) = \Omega(t)^2/2\Delta\). Here the driving field \(\Omega(t)\) is time-dependent, accompanied by a microwave coupling persistently transferring populations among \(|11\rangle\), \(|T\rangle\), \(|00\rangle\). Notice that \(|11\rangle\) and \(|00\rangle\) are indirectly coupled via state \(|T\rangle\). This process can be expressed by Hamiltonian \(\hat{H}_{eff,mw}\)

\[
\hat{H}_{eff,mw} = \frac{g}{\sqrt{2}}(|11\rangle \langle T| + |T\rangle \langle 00|) + H.c. \tag{5}
\]

Consequently it is reliable to prepare the maximal entangled state \(|S\rangle\) through a unidirectional spontaneous loss from the doubly-excited state \(|rr\rangle\) as shown by thick red arrow in Fig.1(b). Here the singly-excited states have been safely discarded due to a big detuning \(\Delta\) with respect to \(|r\rangle\), the preparation efficiency for entanglement formation mainly depends on the competition between excitation or de-excitation rates of \(|11\rangle \leftrightarrow |rr\rangle\), and the unidirectional decay process \((\propto \gamma)\) from \(|rr\rangle\) to \(|S\rangle\).

Remarkably, in the presence of a periodical modulation to the amplitude of \(\Omega(t)\), the Rabi behavior between \(|11\rangle\) and \(|rr\rangle\) will be significantly changed, producing complex multi-frequency oscillations 22. We verify the resulting formation of target ground state \(|S\rangle\) can be dramatically accelerated under an optimization for relevant frequency parameters.
III. UNITARY DYNAMICS

A. Frequency Modulation

In step (ii) we consider a reduced subspace with only two states $|11\rangle$ and $|rr\rangle$ (yellow box in Fig. 1(b)), in order to explore the frequency-modulated excitation dynamics. Intuitively when the reduced two-state system is driven via a continuous coupling, the system is a standard Rabi problem, revealing regular single-frequency oscillations (including excitation and de-excitation) of population between two states, where the oscillating frequency is exactly same to the Rabi frequency. A striking difference arises once the driving is modulated to be time-dependent dramatically modifying the Rabi behavior, leading to unexpected dynamics.

First we introduce a cosinoidal driving field $\Omega(t)$, denoted as

$$\Omega(t) = \Omega_0 \cos(\omega t)$$

(6)

focusing on the regime where the external modulation frequency $\omega$ is compatible with the characteristic frequency of system. Such a time-dependent amplitude modulation for the peak Rabi frequency $\Omega_0$ can be experimentally implemented via an acousto-optical modulator triggered via an electronic waveform generator controlling the acoustic profile, outputting shaped pulses. Letting $\omega = 0$ leads to $\Omega(t) = \Omega_0$, standing for the case of a continuous pump, same as considered in previous works.

A brief introduction to the derivation of effective two-state Hamiltonian can be understood in the subspace of $\{|11\rangle, |M\rangle, |rr\rangle\}$ with the singly-excited collective state $|M\rangle = (|1r\rangle + |r1\rangle)/\sqrt{2}$. The original two-state Hamiltonian $H_{uni}$ is

$$H_{uni} = \frac{\Omega(t)}{\sqrt{2}}(|11\rangle \langle M| e^{i\Delta t} + |M\rangle \langle rr| e^{i\Delta t} + H.c.) + \hat{\nu}$$

(7)

Working in a rotating-wave frame with respect to the rotational transformation $\hat{U} = e^{-i\Omega_{rr}(|rr\rangle \langle rr|)},$ the above Hamiltonian can be re-expressed in a more concise form

$$\hat{H}_{eff,uni} = \frac{\Omega(t)^2}{2\Delta}(|11\rangle + |rr\rangle)\langle|11\rangle + |rr\rangle| - \frac{\Omega(t)^2}{\Delta} |M\rangle \langle M|,$$

(8)

where $U_{rr} = 2\Delta$ and $\Delta \gg \Omega(t)/\sqrt{2}$ are assumed. Comparing (4) and (8), we ignore the second term $-\frac{\Omega(t)^2}{\Delta} |M\rangle \langle M|$ owing to its decoupling effect of state $|M\rangle$. The effective coupling strength $\Omega_{eff}(t)$ between $|11\rangle$ and $|rr\rangle$ can be expressed as

$$\Omega_{eff}(t) = \frac{\Omega_0^2}{4\Delta} + \frac{\Omega_0^2}{4\Delta} \cos(2\omega t)$$

(9)

which includes base frequency component $\omega_0 = \frac{\Omega_0^2}{2\Delta}$ and the real-time modulated component $\omega_0 \cos(\omega t)$, giving to the peak amplitude belonging to $[0, 2\omega_0]$ as shown in the inset of Fig.1(b). If $\omega \gg \omega_0$, the modulation term adds a fast high-frequency oscillation to the base frequency $\omega_0$ whose effect can be averagely canceled within a sufficient time, i.e. $\int_0^\infty \frac{\omega_0^2 \cos(2\omega t) dt}{\omega^2} = \sqrt{\omega_0^2} \rightarrow 0$, arising a dominant base frequency $\omega_0$ only; otherwise the non-negligible modulated component will induce several modulated-frequency sidebands with separation $\omega$ to the central frequency $\omega_0$, revealing complex multi-frequency dynamical behavior.

B. Frequency Spectrum analysis

By numerically solving the master equation $\dot{\rho}_{uni} = i[\rho_{uni}, H_{eff,uni}]$, we detect the observable quantity $P_{rr}(t)$

$$P_{rr}(t) = \langle rr| \hat{\rho}_{uni}(t)| rr\rangle$$

(10)

for the time-dependent population probability on state $|rr\rangle$ where $\hat{\rho}_{uni}(t)$ is the density matrix of two-state subspace, as globally plotted in Figure 2(a) versus time $t$ and the relative modulation frequency $\omega/\omega_0$. Besides Fig 2(b) describes a global view for the frequency spectrum of dynamics by implementing a Fourier trans-
form $F_{rr}(\omega) = \int_{-\infty}^{+\infty} P_{rr}(t)e^{-i\omega t}dt$. From Eq. (3) and Fig. 2(b) it is obvious that the presence of modulation $\omega$ can add frequency sidebands to the base frequency $\omega_0$ which is also the characteristic frequency of system, giving rise to the multiple frequencies $\omega_n = \omega_0 \pm n\omega$ with $n = 0, 1, 2, \ldots$, symmetrically located with respect to $\omega_0$. As $\omega$ increases the dynamics become regular with one dominant base frequency $\omega_0$ because a high-modulation-frequency $\omega$ will cause an average cancellation to the accumulated quantity by the frequency modulation $(\omega_0/2\omega)\sin(2\omega t)_{t_{-\infty}}^{+\infty} \rightarrow 0$ as $\omega \gg \omega_0$. The inset of Fig. 2(b) comparably shows the Rabi oscillation behavior of $\Omega_{eff}$ under weak (blue-solid) and strong (black-dashed) modulation frequencies.

As far as we know when the modulation is exactly an integral multiple of characteristic frequency $\omega_0$, i.e. $\omega = n\omega_0$, the system exists a dramatic frequency match, revealing unexpected behavior. To show this, we select $n = 0, 1, 2$ and represent the corresponding unitary dynamics and frequency spectrum in Fig. 3(a-b). Except for $\omega = 0$ that it reveals a complete single-frequency Rabi oscillation with frequency $2\omega_0$ ($\Omega_{eff} = 2\omega_0$), the external frequency modulation $\omega_{\neq 0}$ will arise sideband frequencies aside from the base frequency $\omega_0$, with a tunable separation $\omega$. The resulting unitary dynamics changes significantly by meeting different frequency matching conditions.

Comparing to the case of no modulation, it is observable that for $\omega = \omega_0$ (black-solid) the population dynamics $P_{rr}(t)$ benefits from a sufficiently longer residence time sustaining on the uppermost two-excitation state $|rr\rangle$, promising an efficient population dissipation onto the target state through spontaneous emission as long as the decay rate $\gamma$ is suitable. Hence the frequency match offers an essential reason for the accelerated dissipative formation in step (iii). Furthermore as $\omega$ increases, the system again tends to a single-frequency oscillation with the oscillating frequency $\omega_0$, revealing regular modulated Rabi behavior. For example, if $\omega = 2\omega_0$ (red-dashed) $P_{rr}(t)$ suffers from a faster excitation and de-excitation processes without any stagnation on the upper state, leading to the population exchange in step (ii) between $|11\rangle \leftrightarrow |rr\rangle$ in a repeated cycle. The resulting dissipative entanglement preparation is quite inefficient.

Based on the above analysis, we will adopt this optimal resonant modulation frequency $\omega = \omega_0$ to study the accelerated entanglement preparation.
FIG. 5. (color online) The fidelity of target entangled state $F_S = P_S(t = 100)$ versus the simultaneous adjustments for microwave coupling $g$ and spontaneous decay $\gamma$. A global maximum of $F_S$ is denoted in the plot. Here $\omega_0(\omega_0^{-1})$ is the frequency(time) unit.

IV. FACILITATION FOR ENTANGLEMENT FORMATION

The unitary dynamic evolution of the reduced two-state model can be guided to solve the complete master equation $\dot{\rho} = -i[H, \rho] + \mathcal{L}[\rho]$ for the entire system, with the basic two-atom Hamiltonian $\hat{H}$ [see Eq. (1)] as well as the Lindblad dissipation operators, described by

$$\mathcal{L}[\rho] = \sum_{i=1}^{4} [\hat{L}_i \rho \hat{L}_i^\dagger - \frac{1}{2} \{\hat{L}_i^\dagger \hat{L}_i, \rho\} + \rho \hat{L}_i^\dagger \hat{L}_i]$$

(11)

presenting the four spontaneous decay channels as shown in Fig.1(a). According to the above discussion it is confirmed that $|S\rangle$ is an absolute unique steady state, arising the system ideally staying on that state as long as the evolution time is sufficient. However, due to the competition between unitary dynamics and spontaneous dissipative process, the formation time can be endless which is far beyond the detection time in real implementation. Remember the typical time for entanglement formation basing on similar schemes is more than tens of milliseconds by using continuous driving with higher Rydberg levels $n > 100$. Here, with the help of a periodic amplitude modulation to the pump laser it is observed that a clear entanglement facilitation can be verified, accompanied by a reduced detecting time $\sim 100/\omega_0$.

Figure 1 comparably exhibits the real-time dynamics for population on target state $|S\rangle$, defined by $P_S(t) = \langle S|\hat{\rho}(t)|S\rangle$, with the modulation frequency $\omega = \omega_0$ (red-solid) or without modulation $\omega = 0$ (green-dashed). Clearly without periodical modulation, $P_S(t)$ represents a smoothly increasing curve but saturating towards 0.96 for $t \rightarrow \infty$, see the inset inside for an extensive range within $t \in [0, 10^4]/\omega_0$. However with $\omega = \omega_0$, $P_S(t)$ exhibits a fast stepwise increase, even catching up with the case of $\omega = 0$ at $t = 14.3/\omega_0$, finally being stabilized to be as high as 0.98 which benefits from a significantly shortened time. Because for $\omega = 0$ it is impossible to reach 0.98 during the finite detection time. The reason accounting for this facilitation can be understood by finding longer durations $\tau$ in every oscillation period on state $|rr\rangle$ that is able to provide enough time of dissipation into the target state. As indicated clearly in the inset below, during each duration time $\tau$ ($P_{rr}(\tau) = 1.0$), $P_S(t)$ exhibits a dramatic enhancement especially at the initial time when the sufficient population can be exchanged between $|11\rangle$ and $|rr\rangle$.

Also we stress the importance of competition among the unitary dynamics, spontaneous dissipation and microwave coupling that leads to an accelerated entanglement formation. Therefore we globally change the rates of dissipation and microwave coupling, in order to see the essential importance of frequency match to the fidelity of entanglement, characterized by $F_S = \langle S|\hat{\rho}(t = 100/\omega_0)|S\rangle$. In Fig 5 by varying $\gamma$ and $g$ within a same range $0.1, 0.2\text{ MHz}$ there exists a global maximum region with $F_S \approx 0.98$ persistently, where the values of $\gamma$ and $g$ have a best matching. In contrast, beyond that $F_S$ reveals a considerable reduction, especially for a small $g$ value arising a weaker transfer rate among $|11\rangle$, $|T\rangle$, $|00\rangle$ that can not catch up with the (de-)excitation and dissipation rates, leading to a very poor entanglement production $F_S < 0.5$. In other words, the realization of an accelerated entanglement formation needs an optimal modulation frequency $\omega$ implemented by an external pump laser, together with a perfect frequency consistence between the dissipative rate $\gamma$ and microwave coupling strength $g$. 

FIG. 6. (color online) Time-dependent population of the target state $P_S(t)$ versus $t(\mu$s) for different $\omega_0$ and $\Delta$ values. Note that the conditions of $U = 2\Delta$ and $\omega = \omega_0$ are kept.
V. FEASIBILITY AND WAYS FOR FURTHER FACILITATION

In experiment the configuration like Fig.1(a) can be implemented in two rubidium atoms where the hyperfine energy levels are $|1\rangle = |S_{1/2}, f = 1, m = 0\rangle$, $|0\rangle = |S_{1/2}, f = 2, m = 0\rangle$ that can be excited to the Rydberg state $|r\rangle = |100s\rangle$ via a two-photon transition mediated by $e.g.$ $|5p_{3/2}\rangle$. The effective two-photon Rabi frequency is achieved to be a reasonable value $\Omega_0 = 2\pi \times 2.5$MHz $^{[27]}$, with which the ground state $|1\rangle$ is coupled to the $|r\rangle$ by a big detuning $\Delta = 2\pi \times 36$MHz, giving rise to the characteristic frequency of system $\omega_0 = \Omega_0^2/4\Delta = 0.273$MHz. In the real implementation two atoms can be trapped separately with an interatomic distance $R_c = 9.5\mu m$ for realizing a considerable vdWs interaction $U_{vdw} = 2\pi \times 72$MHz, driven by a periodically time-dependent laser beam $\Omega(t)$ with amplitude $\Omega_0$ and modulation frequency $\omega = \omega_0 = 0.273$MHz. Under an optimization for other frequency parameters we have $\gamma = 2\pi \times 17$kHz $^{[53]}$ and $g = 2\pi \times 36.9$kHz that enable a well coincidence with the unitary dynamics, the fidelity for entangled state formation can attain as high as $F_S = 0.981$ within a much shortened time $T_S = 366.3$µs. Notice that here these values $\gamma$ and $g$ can also be tunable in a global maximum region, e.g. $\gamma = 2\pi \times 10$kHz and $g = 2\pi \times 34.8$kHz, leading to $F_S = 0.977$ within a same time $T_S$.

In addition, noting that the optimal dynamics of $P_S(t)$ as displayed in Fig.3(red solid) reveals a stepwise-increasing behavior, dominantly decided by the increasing rate during the duration time $\tau$ when $P_{rr}(t) = 1$ in the case of unitary dynamics. As a result an intuitive way for further facilitation to entanglement is increasing the effective coupling strength $\omega_0$ between $|11\rangle$ and $|rr\rangle$, accelerating the accumulated rate of population on the target state during each residence time $\tau$. As represented in Fig. 4 if $\Omega_0$ increases or $\Delta$ decreases, leading to a stronger coupling $\omega_0$ for the excitation and de-excitation transitions during the laser pumping process, $P_S(t)$ can achieves a further accelerated growth, quickly saturating to a high-fidelity value $\sim 0.98$ within a shorter time, which provides an easy facilitation optimization in this scheme when other relevant frequency parameters are already suitable.

VI. ROBUST INSENSITIVITY TO INITIALIZATION

For exploring the relation of initial population distribution and the final fidelity of the target state we show the values of $F_S$ under different initial populations $P_{00}(0)$, $P_r(0)$ and $P_{11}(0)$, meeting the normalized condition $P_{11}(0) + P_r(0) + P_{00}(0) = 1.0$. It needs to point out that in plotting Figure 7 we have calculated an average $F_S$ value covering a duration of 100µs for overcoming the slight population fluctuations in its dynamics towards the saturation, which come from a modified Rabi oscillation as described in Fig.4.

For atom initially occupying a determined state $|11\rangle$ i.e. $P_{11}(0)=1.0$ the unitary dynamics in step (ii) between $|11\rangle$ and $|rr\rangle$ is straightforward, leading to a maximal average fidelity $F_S \sim 0.9767$, as indicated by a dark red dot in Fig.7. However, in an opposite case if atoms are entirely prepared in $|00\rangle$ that promises a fully indirect transfer to $|11\rangle$ mediated by $|T\rangle$, a very small decrease by an amplitude of $\Delta F_S \sim 0.001$ is observed for the average fidelity, strongly proving the robustness of fidelity insensitivity against initialization.

Otherwise when the initial population is prepared in different ground states except the target state $|S\rangle$, a continuous coupling by the microwave field among $|11\rangle$, $|T\rangle$, $|00\rangle$ would suffer from a persistent population conversion towards $|11\rangle$, causing a gradual varying of average fidelity between the former two cases. Intuitively e.g., for a given $P_{00}$ value, $F_S$ continuously grows with the increase of $P_{11}$ which is directly connected to $|rr\rangle$.

The above results again stress that our scheme is flexible and fully insensitive to the system initialization, promising a feasible way for the generation of steady entanglement in current experimental environment.

VII. CONCLUSION

Depending on a simple two A-type Rydberg atoms, we show that when a periodically-modulated pump laser is used one can achieve an accelerated formation of dissipative entangled steady state, arising an unprecedented facilitation mechanism that has never been considered in
previous similar schemes. The merit of our scheme lies on the chosen of an external modulation frequency that well agrees with the characteristic frequency of system, resulting in a dramatic modulation to the behavior of unitary dynamics in the optical pumping process. The modified Rabi oscillation behavior benefits from a longer residence time on the two-excitation Rydberg state, promising a fast decaying onto the target entangled state when the spontaneous emission rate and the microwave transferring rate are both tuned to be consistent at the same time. Under optimization we successfully raise the fidelity to \( \sim 0.98 \) with a formation time shorter than \( 400 \mu s \). Comparably if no modulation is carried out it is found that the fidelity persists \( \sim 0.96 \) for a sufficient time of tens of milliseconds, un-enabling the realization of a higher entanglement within a limited detection time. Additionally we propose the way for further accelerating the convergence time by enhancing the effective coupling strength in the optical pump, and put forward to discuss the robust insensitivity of fidelity against the preparation strength in the optical pump, and put forward to discuss the robust insensitivity of fidelity against the preparation of arbitrary initial population on three ground states except the target state.

For realizing a fast and high-fidelity dissipative steady entanglement, our new proposal offers one-step closer to this goal, simultaneously overcoming the obstacles from a complex energy-level structure (EIT approaches) or a long formation time (traditional Rydberg anti-blockade approaches) in a number of previous works, which may provide new perspectives for experimentalists to create maximal and deterministic entangled state via dissipation in interacting Rydberg atom systems.

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