Vibration Analysis of Gear with Asymmetric Teeth and Double Pressure Angles based on Transient Stiffness and Transfer Error

Huachuan Li
Guangxi Technological College of Machinery and Electricity, Nanning, 530007, China
35095758@qq.com

Abstract. Transient stiffness formula of dynamic load of asymmetric gear is deduced. Considering the effect of transient stiffness and transfer error, the formula of dynamic load of asymmetric gear is deduced and programmed using MATLAB, the variation regulation of dynamic load was obtained. That is the foundation of doing transient EHL analysis of asymmetric gear. The asymmetric gear model program has been made using VBA, and modal analysis of asymmetric gear has been done using ANSYS, which is the foundation of dynamic response of structural calculation and analysis.

1. Introduction
Compared with symmetric gear, asymmetric gear owns stronger bearing capacity, better lubrication condition, small vibration noise, and other advantages. The actual load within the asymmetric gear in operation will change. However, the internal additional load, higher than its nominal value, is the dynamic load of asymmetrical gear. The dynamic load of gear sources from the non-integral values of manufacturing error, stiffness change and overlap ratio [1], and is a periodic time-varying parameter if it is reflected in a systematic dynamic analysis model. In addition, because the errors are inevitable in the processing and installation of gears, a transfer error will be caused in the gear that does not bear load, thus giving rise to an error in the system.

2. Analysis on the Stiffness of the Teeth of Asymmetric Gear
The width of gear is set as b (mm); the total deformation of gear is \( \delta (\mu m) \); the load intensity on gear (i.e. the load on unit width of gear) is \( w (N/mm) \); the flexibility \( q (\mu m \cdot mm/N) \) of gear is as follows.

\[
q = \delta / w
\]  

(1)

The stiffness of gear \( c (N/mm \cdot \mu m) \) can be expressed as follows.

\[
c = 1 / q = w / \delta = F / (b\delta)
\]  

(2)

Gear teeth deformation includes bending, shearing, contact deformation, etc. The total deformation of gear teeth is the sum of these deformations, so the total flexibility is the sum of their flexibilities as follows.
Generally, only a pair of teeth meshes exists near a node, and the numerical values of their stiffness are close to the maximum. Thus, their stiffness is called as single tooth stiffness, and the average stiffness in the end-section meshing process is often called as mesh stiffness. To solve the single tooth stiffness and mesh stiffness, the transient stiffness of gear has to be studied. Transient stiffness is gained from the analysis of teeth deformation (i.e. transient flexibility) at different time. Tooth profiles can be simplified to a ladder-overlapped rectangular conjunctive model in general, and this is called as equivalent tooth profile method.

The bending deformation flexibilities of the rectangular and trapezoidal parts are $q_{Br}$ and $q_{Br}$, respectively; the total flexibility of shearing deformation is $q_{s}$; the flexibility caused by matrix’s elastic inclination is $q_{E}$; the flexibility of the total contact deformation of two gears is $q_{h12}$. Thus, the total flexibility of all deformations of two gears is as follows:

$$q = q_{Br1} + q_{Br2} + q_{s1} + q_{s2} + q_{h12}$$

All flexibilities and stiffness calculation formulas are abridged here.

3. Analysis of Stiffness Calculation Result

A pair of asymmetric involute spur gears is selected for calculation and analysis, and the solving programs are written with MATLAB. The asymmetric gear work conditions are shown in table 1. Gear is with ideal accuracy. It is assumed that the load $F$ is uniformly distributed along the whole tooth width direction, and the normal load of unit tooth width acting on tooth profile is $F/b = 500 \text{ N/mm}$. Five special points are selected from two asymmetric gear meshing lines, and the deformations of driving gear and driven gear when load acts on these give points are shown in figure 1. Single tooth stiffness changing curve is shown in figure 2. However, the single tooth stiffness trend is opposite to the change of the total deformation; the single tooth meshing stiffness on two sides of asymmetric involute gear trends from small to large, begins to change on node, and then goes from large to small. The gear meshing stiffness change curve is shown in figure 3.

| Table 1: Asymmetric Gear Work Conditions |
|------------------------------------------|
| Qty of Gear Teeth:                      |
| $z_1 = 27$, $z_2 = 43$                  |
| Module of gear (mm):                     |
| $m = 4$                                  |
| Tooth Addendum Coefficient:              |
| $h = 1.0$                                |
| Tip Clearance Coefficient:               |
| $c = 0.25$                               |
| Side Pressure Angle of Work Tooth:       |
| $\alpha = 30^\circ$                      |
| Side Pressure Angle of Non-work Tooth:   |
| $\alpha = 20^\circ$                      |
| Elasticity Modulus of Gear Materials (GPa):|
| $E = E_x = 2.06$                        |
| Poisson’s ratio ($\nu$):                 |
| $\nu = 0.3$                             |
| Density (kg/m$^3$):                      |
| $\rho = 7800$                           |
| Power (kW):                              |
| $P = 16$                                |
| Rotation Speed of Small Gear ($\pi$/min):|
| $n_1 = 1100$                            |
4. Analysis of Dynamic Load of Asymmetric Gear Drive

Considering the effect of time-varying stiffness on dynamic load and the manufacturing error of gear, the dynamic load of asymmetric gear drive under the transient change tooth surface load is analyzed and studied in this paper [2].

![Asymmetric gear deformation curve when mesh position changes](image1)

*Fig.1: Asymmetric gear deformation curve when mesh position changes*

![Single tooth stiffness curve changing with mesh position](image2)

*Fig.2: Single tooth stiffness curve changing with mesh position*
4.1. Motion differential equation

Gear system is a multi-variant vibration system, as shown in figure 4. It is assumed that the impact caused by errors on gear tooth surface is \( F(t) \); the system's damping coefficient is \( \gamma \); the tooth stiffness is \( c \). Therefore, according to the Newton's second law, the system's motion equation can be expressed as follows.

\[
\begin{align*}
\bar{m}_1 \ddot{x}_1 &= -c(x_1 - x_2) - \gamma (\dot{x}_1 - \dot{x}_2) + F(t) \\
\bar{m}_2 \ddot{x}_2 &= -c(x_2 - x_1) - \gamma (\dot{x}_2 - \dot{x}_1) - F(t)
\end{align*}
\]  

(5)

In equation (5), \( \bar{m}_1, \bar{m}_2 \) are the unit tooth width equivalent qualities of the driving and driven gears on the meshing line. Thus, the following equation can be gained after equation (5):

\[
\frac{\bar{m}_1 \bar{m}_2}{\bar{m}_1 + \bar{m}_2} (\ddot{x}_1 - \ddot{x}_2) + c(x_1 - x_2) + \gamma (\dot{x}_1 - \dot{x}_2) = F(t)
\]  

(6)

Order the comprehensive equivalent quality (induction quality) of gear is as follows.
Thus, the following equation can be established

$$\dot{m}_{red} \ddot{x} + r \dot{x} + cx = F(t) \quad (7)$$

### 4.2 Analysis of forced vibration

From equation (7), it can be seen that the motion equation of gear vibration is similar to the forced vibration equation of a single degree-of-freedom system. It is assumed that the mesh stiffness of gear is $c_v$. Therefore, according to vibration theory, the system's natural frequency is as follows.

$$\omega_n = \sqrt{\frac{c_v}{m_{red}}} \quad (8)$$

The units of $c_v$ and $m_{red}$ are used as the common units in engineering, and then the rotation speed of driving gear in the system's resonance can be gained. That is, the critical speed is as follows.

$$n_{ei} = \frac{30 \times 10^3}{\pi} \sqrt{\frac{c_v}{m_{red}}} \quad \text{r/min} \quad (9)$$

Frequency ratio $N$ is critical speed ratio, defined as the ratio between driving gear rotation speed and critical speed, and used for characterizing the motion conditions of the system. That is, the system is in the critical state if $N<1$; it is beyond the critical state if $N>1$; it is in resonance if $N=1$. The calculation of gear's dynamic load in ISO is started from the amplitude-frequency characteristics curve of gear, and then solves conversion coefficient through the amplitude-frequency characteristics curve. After amplification factor is gained through the amplitude-frequency characteristics curve, the dynamic load coefficient $k_v$ can be determined according to the following equation.

$$K_v = 1 + \frac{W_p}{W} + \frac{W_f}{W} + \frac{W_k}{W} \quad (10)$$

In equation (10), $W$ is static load; $W_p$, $W_f$ and $W_k$ are the dynamic loads caused by base pitch error, tooth profile error and mesh stiffness, respectively. The amplification factors caused by the base pitch error in main vibration region and supercritical region are $C_{v2}$ and $C_{v6}$; the amplification factors caused by the mesh stiffness in main vibration region and supercritical region are $C_{v3}$ and $C_{v4}$.

It is assumed that the base pitch error of gear is $f_{pb}$; tooth profile error is $f_f$; the tooth deformation caused by work load is $f$; the amount of running-in of tooth is $y_a$; the amount of addendum tip relief is $C_a$; single tooth stiffness is $c'$; effective tooth deformation is $f_e$.

In above equations, there are

$$B_p = \frac{c f_{pb} \text{eff}}{K_a F_s / b} \quad B_f = \frac{c f_{f} \text{eff}}{K_a F_s / b}$$
When gear accuracy is below level 6, Bk=1 can be used. $f_{p_{\text{eff}}} = f_{pb} - y_a$, and $y_a$ is the amount of running-in of tooth; $f_{f_{\text{eff}}}$ is effective tooth profile error ($f_{f_{\text{eff}}} = f_f - y_a$; $f_k = |\delta - C_a|$); $\delta$ is the tooth deformation caused by work load.

4.3. Analysis of calculation results
A pair of asymmetric involute gear spur gear drives is selected for calculation and analysis, and the asymmetric gears drives work conditions are shown in table 1. In this paper, the programs are written for the dynamic load of this pair of asymmetric involute gear spur gear drives. The change law of tooth surface dynamic load in a tooth meshing process is shown in figure 5. The maximum dynamic load value appears in the single tooth meshing area.

(1) Along with the increase of mesh stiffness in the double teeth meshing area, the gear drive dynamic load will decrease. Instead, along with the decrease of mesh stiffness in the single tooth meshing area, dynamic load will increase.

(2) Because of the effect of tooth profile error, tooth surface dynamic load fluctuates along the meshing line, and the maximum dynamic load value appears in the single tooth meshing area.

5. Analysis of Asymmetric Gear Vibration Mode
The natural frequency and vibration mode of asymmetric gear directly affect the drive process and noise of the gear; dynamic response is an integrated embodiment of the effects of inherent characteristics and exciting force [3].

In this paper, through secondary development software VBA of INVENTOR, the parameterized modeling program is developed for asymmetric gear, making the 3d mode of asymmetric gear easy to be generated. Subsequently, the mode can be input in ANSYS, laying a solid foundation for modal analysis.

(1) Establishment of Motion Differential Equation.
According to vibration mechanics and finite element theory, the motion differential equation of cylindrical gear is as follows.

$$[M]\ddot{\{X\}} + [C]\dot{\{X\}} + [K]\{X\} = \{f(t)\}$$

In equation (11), $\{f(t)\}$ is the exciting force vector of structure; $[M], [C], [K]$ are total mass matrix, damping matrix and stiffness matrix; $\{\dot{X}\}, \{\dot{X}\}, \{X\}$ are acceleration vector, speed vector and displacement vector of structure.

The motion equation of un-damped free vibration is as follows.

$$[M]\ddot{\{X\}} + [K]\{X\} = \{0\}$$

Its corresponding characteristic equation is as follows.

$$([K] - \omega^2[M])\{X\} = \{0\}$$

In equation (13), $\omega$ is the natural frequency of system; $\{X\}$ is the characteristic vector (modal shape) of system.

(2) Analysis of Finite Element Mode Result
The vibration of structure can be expressed as the linear combination of all-order natural vibration modes. In this paper, the first five order natural frequencies and vibration modes of the asymmetric
gear are calculated. The above finite element mode is called into ANSYS. After calculation, the first five order natural frequencies and vibration models of cylindrical gear are shown in table 2. In figure 6, the effect of the size of side pressure angle of work tooth of asymmetric gear on the natural frequency of the gear is shown. Because of the length limit of this paper, vibration mode diagram is abridged here. Calculation shows that the structure mode distribution is mainly torsional vibration and bending vibration, and also the relative stress near dedendum circle is very high, and is the weak link of the structure.

(3) Analysis of Results
First, asymmetric gear can be accurately described by the established mathematical model; through the ANSYS post-processing program, vibration mode and animation can be visually shown. The analysis shows that asymmetric gear vibrates mainly along peripheral direction.

Second, with vibration model figure, the dynamic characteristics and weak links of asymmetric gear can be visually analyzed, providing a theory basis for the experiment, design and maintenance of the dynamic performance of gear.

Third, through the finite element modal analysis of asymmetric gear, the natural frequency and vibration model of the gear were determined, preventing the emergence of resonance and harmful vibration mode in the working of system.

Fourth, as shown in figure 6, along with the continuous increase of the side pressure angle of the work tooth of asymmetric gear (i.e. the increase of the asymmetric coefficient of asymmetric gear), the natural frequencies of all orders of asymmetric gear continues to increase.

![Fig.5: Change law of tooth surface dynamic load different work tooth side pressure angle](image1)

![Fig.6: Natural frequency of asymmetric gear at](image2)
Table 2: First five order natural frequencies and vibration modes of asymmetric gear

| Order | Natural Frequency (Hz) | Vibration Mode |
|-------|------------------------|----------------|
|       |                        | Peripheral Vibration | Torsion | Torsion | Torsion |
| 1     | 19.951                 |                |          |          |          |
| 2     | 22.057                 |                |          |          |          |
| 3     | 22.065                 |                |          |          |          |
| 4     | 23.523                 | Peripheral Vibration |          |          |          |
| 5     | 25.065                 |                |          |          |          |

6. Conclusion
In this paper, the formula for solving the dynamic load on the asymmetric gear meshing line is deduced; the effects of tooth time-varying mesh stiffness, tooth error and base pitch error on the asymmetric gear drive dynamic load are studied. Also, the program for the solving formula is developed with MATLAB respectively, can effectively solve the stiffness and dynamic load of asymmetric gear, and also lays a solid foundation for the subsequent EHL analysis of asymmetric involute gear. Meanwhile, through the application of ANSYS in the modal analysis of the asymmetric involute gear, the natural frequencies and vibration modes of all orders of asymmetric tooth are solved. Thus, the emergence of resonance and harmful vibration mode in the working of system can be prevented, laying a foundation for the dynamic response calculation and analysis of the structural system.

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