Abstract

We show that axion domain walls gain spontaneous magnetization in early universe by trapping either electrons or positrons with their spins polarized. The reason is that the walls produces an attractive potential for these particles. We argue that the wall bounded by an axionic superconducting string leaves a magnetic field after its decay. We obtain a field $\sim 10^{-23}$ Gauss on the scale of horizon at the recombination.

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Magnetic fields in galaxies or stars are present in our universe. Nevertheless, their origin is still unknown although there are several cosmological origins proposed \[1\]–\[5\]; it could arise during quark-hadron phase transition \[1\], electro-weak phase transition \[2,3\], inflation \[4\] e.t.c.. It is generally assumed that dynamo process \[6\] amplifies the seed of small primordial magnetic field \(\sim 10^{-18}\) Gauss generated by these origins to the observed one \(10^{-6}\) Gauss in galaxy.

Previously we have discussed \[3\] a magnetic field associated with domain walls, which gain ferromagnetism owing to fermion zero modes \[7\] bounded to the walls. But the zero modes does not necessarily exist in any realistic models. In this letter we show that axion domain walls, although they do not possess such zero modes, become ferromagnetic by trapping either electrons or positrons in the hot universe. The walls leave dipole magnetic fields after their decay with various strengths and sizes, e.g. a field of \(\sim 10^{20}\) Gauss on scales of \(\sim 10^{-3}\) cm at the temperature \(\sim 100\) MeV. These dipole fields generate a large scale magnetic field. The field on the scale of horizon is estimated to possess its strength \(10^{-23}\) Gauss at the recombination of photons and electrons. This magnetic field is a candidate of primordial magnetic fields which lead to galactic or intergalactic magnetic field at present.

Let us begin to briefly explain axion domain walls \[8\]. The axion is a Goldstone mode \[9\] of Pecci-Quinn global U(1) symmetry \[10\] which is broken spontaneously with the energy scale \(f_{PQ} = 10^{10} \sim 10^{12}\) GeV. This massless axion gains a mass \(m_a\) through effects of QCD instantons due to anomaly \[11\] of the symmetry. Since the axion field \(a\) is essentially the phase \(\theta\) of a Higgs field \(\sigma \sim f_{PQ} e^{i\theta} (a = f_{PQ} \theta)\), the vacuum \(\theta = 0\) of the axion is degenerate with the vacuum \(\theta = 2\pi\). This fact leads to domain walls between these degenerate vacua. Namely the domain walls are produced during QCD phase transition at which QCD instantons work effectively. Besides the domain wall solitons, string solitons associated with the breaking of the U(1) symmetry are produced at the temperature \(f_{PQ}\). In general they are superconducting \[12\]. Since we consider an axion model with the color anomaly of the Pecci-Quinn symmetry equal to one, domain walls are surrounded by these strings. Furthermore, they decay soon after their production without dominating the universe. As we
show below, however, the walls leave magnetic fields with various sizes and strengths. This is because the walls gain magnetic moments by trapping either electrons or positrons with their spins polarized. It turns out that in the decay of the walls the superconducting strings play important roles on producing the magnetic fields. (For definiteness we use numerical values such that $f_{PQ} = 10^{12}$ GeV and $m_a = 10^{-5}$ eV throughout the paper.)

In order to see the spontaneous magnetization of the wall we first show that the axion domain wall generates an attractive potential for electrons and positrons with a particular polarization. Suppose an axial vector coupling between electron field $\psi$ and axion field $a$,

$$L_{int} = g \partial_\mu a \psi \gamma_5 \gamma^\mu \psi$$

(1)

where $g$ is the coupling constant whose value is given by $cf_{PQ}^{-1}$, with a numerical positive constant $c$; the value of $c$ depends on models of the axion. Then, we can see easily that the wall produce a spin dependent potential for electrons and positrons. Assuming that the flat wall is located at $x_3 = 0$, we rewrite Dirac equation of $\psi = \begin{pmatrix} u \\ v \end{pmatrix}$ such that

$$\varepsilon u = \left( -\frac{\partial^2}{2m} + g \partial_3 a \sigma_3 \right) u \quad \varepsilon v = \left( -\frac{\partial^2}{2m} - g \partial_3 a \sigma_3 \right) v$$

(2)

with energy $E = m + \varepsilon \ ( E = -m - \varepsilon )$; $m$ is the mass of electron. We have taken the nonrelativistic limit so that only large component $u \ ( v )$ is exhibited; small component $v \ ( u )$ is given by $v = -i \partial_3 u/2m \ ( u = i \partial_3 v/2m )$.

We note that the typical scale of spatial variation of $\theta$ is given by the axion mass $m_a$; it increases from 0 to $2\pi$ along $x_3$ axis. Thus it turns out that the potential $g \partial_\mu a \sigma_3$ for both of electrons and positrons is attractive for states with spin down, and repulsive for states with spin up. It has approximately the width of $m_a^{-1} \sim 1$ cm and the depth of $m_a \sim 10^{-5}$ eV when the constant of $c$ is order of one. Hence the potential depth is quite shallow but the width is so large that the potential can accommodate bound states for electrons and positrons. Obviously, spins of these particles bounded to the wall are aligned. This causes the ferromagnetism of the axion domain wall.

It seems that in the universe with its temperature $\sim 100$ MeV the states with such small binding energies are irrelevant for the property of the wall. But we should note that
there exists a small fraction of the particles occupying the bound states even in such hot universe; there is tiny but nonvanishing probability of a particle occupying the state. This small fraction of the particles gives rise to the magnetic property of the wall whose relevant energy scale is much smaller than the temperature.

Secondly we show that for a strong magnetic field either electrons or positrons have no bound states even in such attractive potential. For the purpose let us see the energy spectra of these particles with spins down under the magnetic field imposed perpendicular to the wall. When the field is pointed to the direction along positive $x_3$ axis, the binding energies of these particles are found easily,

$$\varepsilon_{k,n} = -m_a + \frac{k_3^2}{2m} + \omega n$$ for electron

$$\varepsilon_{k,n+1} = -m_a + \frac{k_3^2}{2m} + \omega(n + 1)$$ for positron

where $n$ is integer ($\geq 0$) and $\omega$ is the cyclotron frequency ($\omega = eB/m$); $k_3$ is momentum along $x_3$ axis. For convenience we have simplified the attractive potential by a square well potential with a width equal to $m_a^{-1}$ and a depth equal to $m_a$. The difference in the energy spectra between electrons and positrons comes from the difference in the directions of the magnetic moments of these particles in the attractive potential of the wall.

We note that when binding energies are positive ($\varepsilon \geq 0$), the particles are not bounded. Such localized states are quite unstable against any small perturbations, e.g. couplings with scattering states of the particles, oscillation modes of the wall e.t.c.. Especially thermal fluctuations would destroy such unstable states. Thus the states with $\varepsilon > 0$ must decay. Hence for the sufficiently strong magnetic field $eB \geq m_a m$, positrons do not form bound states on the wall since the lowest energy $\varepsilon_{k=0,1} ( = -m_a + \omega$ ) of positrons becomes positive. Similarly, when the magnetic field is pointed to the direction along the negative $x_3$ axis, electrons do not form bound states.

If the magnetic field is generated by these polarized electrons or positrons themselves bounded to the wall, it means that the wall gains magnetization spontaneously. As will be shown, when the temperature is higher than 180 MeV, the wall gains the spontaneous
magnetization with sufficiently strong magnetic field.

Now we show the spontaneous magnetization of the wall by calculating free energy of the gas of electrons bounded to the wall. The free energy $F_M$ of the system under the magnetic field is defined as

$$F_M = -\beta^{-1} \sum_{\varepsilon_{k,n} \leq 0} \log (1 + e^{-(m + \varepsilon_{k,n})\beta})$$

where we have taken account of only contribution of electrons bounded to the wall, since positrons have no bound states under the sufficiently strong magnetic field. Summation, $\sum_{\varepsilon_{k,n} \leq 0}$ implies $N_d \sum_{n=0} \int m_a^{-1} dk_3/(2\pi)$ with the condition that $\varepsilon_{k,n} \leq 0$; $N_d$ is the degeneracy of each Landau level given by $eBS/2\pi$ where $S$ is the surface area of the wall. $\beta^{-1}$ is the temperature of the universe. We have assumed the chemical potential to be vanishingly small. Noting that only states relevant in the summation are states of the lowest Landau level, we find that $F_M$ is approximately given by

$$F_M = -\frac{m_a^{-1}N_d2\log 2\sqrt{2mm_a}}{2\pi\beta} = -\frac{eBS\log 2\sqrt{2m/m_a}}{2\pi^2\beta}$$

in the limit of the temperature being much higher than the mass of electron, $\beta^{-1} >> m$.

Thus magnetization of the wall is

$$M = -\frac{m_a}{S} \frac{\partial F_M}{\partial B} = \frac{e\log 2\sqrt{2mm_a}}{2\pi^2\beta}$$

This magnetization induces a current $J$ at the boundary of the wall; $J = M/m_a$. Then, the current induces a magnetic field which is not uniform; but it is approximated to $B_0 = J/R$ where $R$ is the radius of the wall. We identify this magnetic field $B_0$ with the field $B$ in the free energy $F_M$. In this way we obtain the magnetic field generated by polarized electrons bounded to the wall,

$$B = \frac{e\sqrt{2}\log 2\sqrt{m}}{2\pi^2R\beta\sqrt{m/m_a}}$$

For the consistency this magnetic field has to satisfy a condition, $eB \geq mm_a$. Otherwise the assumption that only particles bounded to the wall are electrons and that positrons have
no contribution to the free energy, does not hold. Therefore when the condition is satisfied, 
the wall may gain the spontaneous magnetization by trapping either electrons or positrons. 
As have been shown in numerical simulations [13], energetically important domain walls 
are ones with boundaries of horizon size. Thus we equate $R$ with the distance to the horizon. Noting that $R$ is about equal to $0.3M_{pl}\beta^2/\sqrt{f}$ ( $M_{pl}$ is Planck mass and $f$ is 
dynamical massless degrees of freedom at temperature $\beta^{-1}$ ) [14], we find from eq(7) that 
the consistency condition,

$$B \geq m_{m}/e \sim 230\text{Gauss} \tag{8}$$

is satisfied when the temperature is higher than 180 MeV. Therefore the wall gain the 
magnetization $M$ spontaneously when it is produced during QCD phase transition ( $\sim 200$ 
MeV ). Note that $m_{m}$ in the above formulae represents the depth of the potential when the 
coupling constant $g$ in eq(1) between the axion and electron is the order of $f_{PQ}^{-1}$. On the 
other hand if $g$ is the order of $\alpha g$ ( $\alpha = e^2/4\pi$ ) as predicted in a model of a hadronic axion 
[8], then the depth of the potential $m_{a}$ is replaced by $\alpha m_{a}$. Thus the minimum temperature 
needed for the appearance of the magnetization is given approximately by 15 MeV.

In order to establish the fact that the spontaneous magnetization actually arises, we 
have to make sure that the above free energy $F_{M}$ is lower than that of the state with no 
magnetization. Such a free energy is given by

$$F_{0} = -2\beta^{-1}\sum_{\varepsilon_{k} \leq 0} \log(1 + e^{-\varepsilon_{k}\beta}) \sim -2\beta^{-1}\log 2 \sum_{\varepsilon_{k} \leq 0} = -\frac{2S \log 2 m_{m} \sqrt{2m_{m}}}{3m_{a}\pi^{2}/\beta} \tag{9}$$

with $\varepsilon_{k} = -m_{a} + \vec{k}^{2}/2m$, where we have taken both contributions from electrons and 
positrons, and have taken the limit of the temperature being much larger than $m$; $\vec{k}$ is 
3-momentum. Comparing this free energy $F_{0}$ with the above one $F_{M}$ in eq(7) we find that 
$F_{0} > F_{M}$ when a condition, $eB > 4m_{m}/3$, is satisfied; the condition is roughly the same as 
the above condition eq(8). Therefore spontaneous magnetization of the axion domain wall 
of horizon size arises when the temperature is higher than about 180 MeV. In the case the 
magnetic field stronger than 200 Gauss is generated.
It seems apparently strange that the spontaneous magnetization arises in the side of higher temperature than the critical one. In general, ordered states appear in the side of lower temperature than the critical one, possessing smaller entropies than those of disordered states. In our case, however, the entropy of the state with the magnetization is larger than that of the state without the magnetization, when $B$ is larger than a critical value given in eq(8). This is because the number of the bound states, especially the states of the lowest Landau level increase with the magnetic field ( the degeneracy $N_d$ increases with $B$ ) and consequently the entropy becomes large. This magnetic field is proportional to the magnetization which increases with the temperature: The thermal fluctuation leads to large orbital angular momentum and hence large magnetic moment. ( Note that states with various angular momenta are degenerate in a Landau level. ) Thus when the temperature becomes large, the magnetic field increases and the entropy becomes large. Eventually it dominates over the entropy of the state without the magnetization and consequently the spontaneous magnetization can arise.

As we have shown, the axion domain walls trap either electrons or positrons with their spins polarized and gain the magnetization. Although they also gain electric charges, the charges are screened immediately due to large electric conductivity of the universe. Thus charge neutrality is kept.

Finally we discuss magnetic fields left after the decay of the walls. We easily understand that during the decay of the walls, the magnetic field becomes strong as the radius of the walls decreases. To see it we note that the walls are surrounded by superconducting strings which must carries the boundary current $J = M/m_a$. This current is supposed to be carried by fermion zero modes [13,12] on the strings. Then as the number of the zero modes ($\sim JR$ ) is conserved, the current increases as the radius $R$ of the string decreases. The strings shrink with increasing the current until the current is saturated [12] with its maximal value $q m_f / \pi$; $q$ and $m_f$ are the electric charge and the mass of the fermion making zero modes on the strings. Subsequently, the strings shrink without increasing the current but by emitting the fermion zero modes. Therefore, when we start with, for example, a magnetic field $10^2$
Gauss of the walls on the scale of horizon $R_h \sim 10^6$ cm at the temperature 100 MeV, the field $B_c = 10^2(R_h/L_c)^2 \sim 10^{20}$ Gauss on the scale of $L_c$ is achieved when the current is saturated with the radius of the strings being equal to $L_c$. $L_c$ is determined by $MR_h/m_a = q m_f L_c/\pi$ (conservation of the zero modes); $L_c \sim 10^{-3}$ cm with use of $m_f = 10^{12}$ GeV and $q = 1$ (this is a typical energy scale $f_{PQ}$ of the fermion mass which is generated through Higgs mechanism associated with Pecki-Quinn symmetry). For smaller radius R of the strings, stronger magnetic fields ($\propto 1/R$) arise on smaller scales of $R$. On the other hand the large scale magnetic field should be determined by these randomly oriented dipole fields. Thus they produce a magnetic field $B(L)$ on the scale of $L$ such that $B(L) = B_c(L_c/L)^{3/2}$ [16]. The field evolves to the field $B_{re}(L_{re}) = B(L)(1\text{eV}/100\text{MeV})^2$ on the scale of $L_{re} = L(100\text{MeV}/1\text{eV})$ at the recombination ($1\text{eV}$). Note that the magnetic flux is conserved in the early universe owing to large electric conductivity; $B a_{RW}^2 = \text{constant}$ ($a_{RW}$ is the cosmic scale factor in Robertson-Walker metric). As we are concerned with the field on scales of horizon size $L_h \sim 0.1\text{Mpc}$ at the recombination, we find that $B_{re}(L_h) = 10^{-23}$ Gauss. Similarly we obtain the magnetic field $\sim 10^8$ Gauss on scales of $\sim 10^4$ cm at the nucleosynthesis. This is sufficiently small not to affect seriously the production of light elements [17]. In this way the ferromagnetic axion domain walls generates the primordial magnetic fields.

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