An Empirical Analysis of Stochastic Volatility Model Based on SSE 50 ETF Option

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Abstract. Option is an effective financial derivative instrument. It is crucial to price options rationally in the options market. The classical B-S option pricing model yields an option price that deviates from the actual price of the market due to its unduly ideal assumptions. Later scholars proposed the GARCH and SV models to optimize the B-S model. With the SSE 50 ETF option selected as the study object, The GARCH model and the SV model are used to compare the pricing errors of the GARCH model and the SV model. The empirical results show that the SV model is better than the GARCH model in terms of accuracy in any degree of value and option duration, which is suitable for the Chinese options market.

Keywords: SSE 50 ETF Options, GARCH Model, SV Model, Option Pricing

1. Introduction

Through continuous improvement and development, option has become a financial derivatives and investment tools with risk measurement and management, price discovery and other functions in the financial markets. In February 2015, China's financial market entered the options era with the official listing of SSE 50 ETF options on the Shanghai Stock Exchange. It has a profound impact on China's financial market [1].

As an important financial instrument, how to price options appropriately has become the most critical issue in the options market. In 1973, mathematician Black and economist Scholes put forward the B-S pricing formula, which is the first option pricing theory given in a relatively ideal situation. But B-S pricing model puts forward a series of assumptions that are not in line with actual market laws [2]. Therefore, most study of the subsequent option pricing models are based on the B-S model which has loosened assumptions [3]. The AHBS model which imposes an implicit volatility function, and GARCH and SV models with analytical solutions, which loosen the assumption that volatility is constant, have been produced one after another, demonstrating some characteristics that are superior to the B-S pricing model [4].

Finding the suitable option pricing model for China to price the SSE 50 ETF scientifically is of great significance to China's securities market. In this paper, we use software such as Stata to search for an option pricing model more suitable for the domestic option market by comparing and analyzing GARCH model and SV model, using the option data of the SSE 50 ETF in China [1].
2. Data Selection and Processing
In 2015, the SSE launched the SSE 50 ETF option, a stock index option with the SSE 50 ETF as the underlying. It is the first stock index option product in the domestic securities market.

2.1. Data Selection
The empirical analysis of this paper uses data of SSE 50 ETF options. The risk-free rate \( r \) refers to the 2020 HIBOR 1-month rate. The sample interval is from February 1, 2020 to June 30, 2020, a total of 106 trading days, and a total of 15,910 option contract day trading data [5]. In order to make the empirical evidence more valid, the data has been filtered according to the following criteria: (1) This paper only analyzes the SSE 50 ETF call options. (2) Data with zero trading volume is excluded [5]. (3) Excluding data on options with remaining maturities of less than 7 days, which are prone to abnormal trading behavior like speculation or periods of illiquidity [6]. (4) Exclude deep in-the-money options and deep out-of-the-money options with low trading volume and low market price information [6]. (5) Exclude option data that do not meet the following arbitrage limits [7]:

\[
C(t, \tau) \geq \max(S_t - Ke^{-rt}, 0)
\]

After this filter process, the final sample size is 131,792.

Based on the different level of moneyness \( K/S \), the options were classified into in-the-money options \( (ITM, K/S < 0.97) \), at-the-money options \( (ATM, 0.97 < K/S < 1.03) \) and out-of-the-money options \( (OTM, 1.03 < K/S) \).

Depending on the remaining term of the option, it is divided into short-term \( (S\text{term, } \tau < 60) \), medium-term \( (M\text{term, } 60 \leq \tau < 120) \) and long-term options \( (L\text{term, } 120 \leq \tau) \).

2.2. Data Statistics
The result of the filtered SSE 50 ETF options data’s statistical analysis shows in Tables 1 and 2.

| Table 1. Statistical analysis of SSE 50 ETF options data |
|-----------------------------------------------|
| moneyness(K/S) | \( <0.97 \) | (0.97,1.02) | \( \geq1.03 \) | total |
| STerm | 0.0672(21.87%) | 0.1521(16.44%) | 0.2768(9.8%) | 0.1654(48.11%) |
| MTerm | 0.1691(16.92%) | 0.2388(10.65%) | 0.3643(6.29%) | 0.2574(33.86%) |
| LTerm | 0.2033(7.45%) | 0.2982(8.98%) | 0.3893(1.06%) | 0.2969(18.03%) |
| Total | 0.1465(46.24%) | 0.2297(32.07%) | 0.3435(21.69%) | 0.2399(100.00%) |

| Table 2. Implied volatility |
|-------------------------------|
| Moneyness(K/S)(0.97,1.02) | \( \leq0.97 \) | \( \geq1.03 \) | total |
| STerm | 0.4018 | 0.3290 | 0.2961 | 0.3423 |
| MTerm | 0.3281 | 0.2998 | 0.2673 | 0.2984 |
| LTerm | 0.3825 | 0.3357 | 0.2874 | 0.3352 |
| Total | 0.3708 | 0.3215 | 0.2836 | 0.3253 |

As depicted by the statistics in Tables 1 and 2, the implied volatility is significantly skewed as "volatility smile", when the option expiration date is not considered. The implied volatility of the option decreases as the moneyness moves from ITM to ATM to OTM. The independence of the volatility smile phenomenon can be seen in the mean moneyness over time.

3. Parameter Estimates
In option pricing, the unknown realistic volatility and structural parameters cannot be obtained from observation, so parameter estimation of the option pricing model is required.

For the structural parameters of GARCH model \( (\alpha, \beta, \gamma, \omega) \), the conditional variance \( h_{t+1} \) is an additional parameter, determined by the exponential regression of daily history and the structural
parameter the number of days t. The initial variance $h_0$ is computed from the variance over the past year via daily logistic regression. SV model’s structural parameters are $(\theta, \kappa, \rho, \sigma)$ and volatility parameters is $\nu_t$. In this paper, the parameters are estimated using the day's option market data such that the percentage squared sum of the error between the market price of the option and the model price is minimized [8]. The specific expressions are:

$$\min \sum_{t=1}^{N} \left[ \frac{O_i(t, \tau; K) - O^0_i(t, \tau; K)}{O_i(t, \tau; K)} \right]^2 \quad (t = 1, \ldots, T)$$

Table 3. Parameter estimation

|       | GARCH | SV   |
|-------|-------|------|
| $\alpha$ | 0.00183 | 0.164399 |
| $\beta$  | 0.599728 | 6.974286 |
| $\gamma$ | 0.184573 | 0.225954 |
| $\omega$ | 0.209286 | 1.908213 |

4. Empirical Analysis

GARCH model (abbreviated as G in the following tables) and SV model are compared empirically in terms of in-sample fitted pricing errors and out-of-sample predicted pricing errors. In this section, the mean absolute percentage error (MAPE) and the mean squared error (MSE) measurements, which are more important for option prices, are listed [9].

Mean absolute percentage error is expressed as:

$$\frac{\sum_{n=1}^{N} |\varepsilon_n|}{N}$$

Mean squared error is expressed as:

$$\frac{\sum_{n=1}^{N} (\varepsilon_n)^2}{N}$$

MAPE focuses on the size of the measurement pricing error, and MSE focuses on the amplitude of the measurement pricing error.

4.1. In-sample Fitted Pricing Errors

On the basis of the parameter estimates, this section uses the estimated parameters of the option pricing model for each trading day to determine the option price of the model for that day, then compares the resulting model option price with the actual market option price for that day to calculate the option pricing error for each model [10].

Table 4. In-sample fitted pricing results

|       | <0.94 | 0.94,0.97 | 0.97,1.00 | 1.00,1.03 | 1.03,1.06 | >1.06 | all    |
|-------|-------|-----------|-----------|-----------|-----------|-------|--------|
| MAPE  | G     | 1.000000  | 0.989981  | 0.801342  | 0.758972  | 0.219836 | 0.091265 | 0.643566 |
|       | SV    | 0.696394  | 0.129384  | 0.231394  | 0.051943  | 0.038692 | 0.019745 | 0.194592 |
| MSE   | G     | 0.004355  | 0.017937  | 0.029748  | 0.016134  | 0.012984 | 0.003842 | 0.014167 |
|       | SV    | 0.000031  | 0.000046  | 0.000079  | 0.000015  | 0.000089 | 0.000297 | 0.000117 |

According to Table 4, SV model is better than GARCH model under the MAPE criterion. Among the five types of moneyness, SV model is significantly much better than GARCH model. From the
MSE, the pricing error amplitude of SV model is very small and the pricing error amplitude of GARCH model is quite large, and the difference between the two models is more than 100 times.

By remaining maturity, Tables 5-7 present the pricing error results for GARCH and SV models. Under the MAPE criterion, SV model outperforms GARCH model to a greater extent in both intermediate and long-term options. According to the MSE, SV model has the smallest amplitude of pricing error. After classifying the options according to the moneyness, SV model gives much better results than GARCH model for different moneyness, whether they are short-term, medium-term or long-term options. GARCH model has errors of nearly 100% in both OTM and deep OTM options, which simply does not work as a pricing fit. According to the MSE, the pricing error amplitude of SV model is minimized.

4.2. Out-of-sample Predicted Pricing Errors

In-sample fit pricing results can be biased by potential problems such as overfitting the data. The in-sample fit pricing is only part of the process of determining the effect of sample pricing. An empirical analysis of out-of-sample predictive pricing errors based on the in-sample fit is also needed to examine the model pricing results [10]. When conducting the out-of-sample fit pricing error, the option pricing model parameters in the previous section are still used, which are recorded as the t-th trading day parameters, and the t-th trading day parameters are used to determine the option price of the t+1 trading day model, and the two are compared to calculate the pricing error of SV model. At the same time, the option price of GARCH model at trading day t+1 is calculated using the volatility at trading day t using the parameter estimation model.

Table 5. Short-term options’ in-sample fitted pricing results ($\tau < 60$)

| MAPE  | G  | 0.100000 | 0.999981 | 0.761387 | 0.628971 | 0.189836 | 0.053275 | 0.605575 |
|-------|----|----------|----------|----------|----------|----------|----------|----------|
| SV    | 0.226572 | 0.199681 | 0.061347 | 0.051943 | 0.036312 | 0.018745 | 0.099145 |

Table 6. Medium-term options’ in-sample fitted pricing results ($60 \leq \tau < 120$)

| MAPE  | G  | 0.100000 | 0.999756 | 0.792337 | 0.706971 | 0.073975 | 0.642079 |
|-------|----|----------|----------|----------|----------|----------|----------|
| SV    | 0.110172 | 0.056181 | 0.031361 | 0.052943 | 0.028382 | 0.018735 | 0.049629 |

Table 7. Long-term options’ in-sample fitted pricing results ($120 \leq \tau$)

| MAPE  | G  | 0.999999 | 0.989626 | 0.792437 | 0.761952 | 0.272396 | 0.107397 | 0.653967 |
|-------|----|----------|----------|----------|----------|----------|----------|----------|
| SV    | 0.058017 | 0.034781 | 0.029981 | 0.052943 | 0.028382 | 0.018735 | 0.049629 |

Table 8. Out-of-sample predicted pricing results

| MAPE  | G  | 0.000000 | 0.999964 | 0.798234 | 0.736972 | 0.229516 | 0.072265 | 0.639491 |
|-------|----|----------|----------|----------|----------|----------|----------|----------|
| MSE   | G  | 0.013982 | 0.045125 | 0.058928 | 0.039512 | 0.023611 | 0.005892 | 0.031175 |

Table 8 presents the pricing error results for SV model and GARCH model. Under the MAPE criterion, SV model is better than GARCH model, but neither fits well. From the MSE, SV pricing error has the smallest amplitude, less than 5 parts per million, while GARCH model has the largest
amplitude and exceeds one percent. SV model is much better than GARCH model when the sample is classified according to the moneyness, and the fit is quite accurate in both ATM and ITM options under the MAPE criterion. GARCH model works quite poorly.

Classifying the sample by remaining maturity, Tables 9, 10 and 11 present the pricing error results of SV and GARCH models. Under the MAPE criterion, SV model outperforms GARCH model in short-term options, and SV model works best in medium-term and long-term options, its pricing accuracy gradually increases with the remaining maturity, which is the same condition as the in-sample fit. The MSE shows that the pricing error amplitude of SV model is very small.

The short-term, medium-term and long-term options have a consistent performance after the moneyness classification: Under the MAPE criterion, SV model accuracy is much higher than GARCH model, especially in the deep OTM, ATM and deep ITM. From the MSE, SV model pricing error amplitude is much smaller than GARCH model.

Table 9. Short-term options’ out-of-sample predicted pricing results (τ < 60)

| MAPE  | G     | 0.94,0.97 | 0.97,1.00 | 1.00,1.03 | 1.03,1.06 | >1.06   | all     |
|-------|-------|-----------|-----------|-----------|-----------|---------|---------|
|       | 1.000000 | 0.999998 | 0.741783  | 0.738821  | 0.179121  | 0.04595 | 0.617612|
|       | 0.286262 | 0.299681  | 0.061347  | 0.437210  | 0.110951  | 0.018998| 0.202408|

Table 10. Medium-term options’ out-of-sample predicted pricing results (60 ≤ τ < 120)

| MAPE  | G     | 0.94,0.97 | 0.97,1.00 | 1.00,1.03 | 1.03,1.06 | >1.06   | all     |
|-------|-------|-----------|-----------|-----------|-----------|---------|---------|
|       | 1.000000 | 0.999753 | 0.804726  | 0.716932  | 0.249182  | 0.089125| 0.643286|
|       | 0.205812 | 0.107681  | 0.066316  | 0.096215  | 0.047241  | 0.031598| 0.092477|

Table 11. Long-term options’ out-of-sample predicted pricing results (120 ≤ τ)

| MAPE  | G     | 0.94,0.97 | 0.97,1.00 | 1.00,1.03 | 1.03,1.06 | >1.06   | all     |
|-------|-------|-----------|-----------|-----------|-----------|---------|---------|
|       | 0.999998 | 0.997998 | 0.793483  | 0.758247  | 0.308631  | 0.105975| 0.660722|
|       | 0.158937 | 0.069456  | 0.058921  | 0.068743  | 0.050045  | 0.042918| 0.074836|

In summary, the results of SV model are much better than GARCH model. SV model works well in any moneyness and option duration, while GARCH model only has practical use in deep ITM options. As the moneyness changes, that is, from OTM to ITM, the pricing errors of both SV model and GARCH model are getting smaller and smaller in general. SV model accuracy is always higher, improving as the moneyness changes. Finally, in both ITM and deep ITM options, the lower the remaining maturity, the smaller the pricing errors in both SV and GARCH models overall.

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