Thermal nature of charmonium transverse momentum spectra from Au-Au collisions at the highest energies available at the BNL Relativistic Heavy Ion Collider (RHIC)

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Abstract

We analyze the transverse momentum distribution of $J/\psi$ mesons produced in Au + Au collisions at the top RHIC energy within a blast-wave model that accounts for a possible inhomogeneity of the charmonium distribution and/or flow fluctuations. The results imply that the transverse momentum spectra of $J/\psi$, $\phi$ and $\Omega$ hadrons measured at the RHIC can be described well if kinetic freeze-out takes place just after chemical freeze-out for these particles.

PACS numbers: 25.75.-q, 25.75.Cj, 25.75.Ld
I. INTRODUCTION

Charmonium production is considered as important messenger of the deconfinement transition in heavy-ion collisions. In the original investigation the suppression of charmonium was proposed as a signal of the quark gluon-plasma (QGP) that caused the melting of charmonium \[\text{[1]}\] (for very recent reviews see Refs. \[\text{[2, 3]}\]). Alternatively, the charmonium yield is described as been caused by statistical hadronization at the phase boundary \[\text{[4]}\]. In the corresponding statistical hadronization model (SHM), the $J/\psi$ multiplicities at SPS and RHIC energies are well described for different centrality and rapidity bins; the basic points of the SHM and its current development are presented in reviews \[\text{[6, 7]}\] and references therein. The SHM assumes that the charm quarks are produced in primary hard collisions and that their total number stays constant until hadronization. A crucial hypothesis of the model is thermal equilibration of charm quarks in the QGP, at least near the critical temperature $T_c$. The charmonia are then all produced near the phase boundary.

Because the momentum spectrum of $J/\psi$ is nearly frozen out at $T_c$ (the typical cross section of $J/\psi$ with comoving hadrons is at most a few millibarns \[\text{[9]}\]), the measured momentum spectra of $J/\psi$ mesons contain valuable information on details of the hadronization process in the strongly interacting medium. For instance, if formation of $J/\psi$ proceeds as assumed in the SHM, then the momentum spectrum of $J/\psi$ is expected to be locally equilibrated in the vicinity of $T_c$. On the contrary, if the superdense matter formed in $A+A$ reactions contained a significant portion of $J/\psi$ mesons produced at the early stage of the process in hard nucleon-nucleon collisions, then the final $J/\psi$ spectra could exhibit two components, as, for example, in the model \[\text{[10]}\]: direct $J/\psi$ and secondary thermal $J/\psi$ produced at the stage of recombination of charm quarks. The shape of $J/\psi$ momentum spectra will then be, in fact, nonthermal. Also, in the latter approach, the multiplicities of hadrons with open and hidden charm do not coincide, generally, with those predicted in the SHM.

Recently the PHENIX Collaboration published results for $J/\psi$ production versus centrality, transverse momentum and rapidity in $A+A$ collisions at $\sqrt{s_{NN}} = 200$ GeV \[\text{[11]}\]. The analysis of these data in Ref. \[\text{[6]}\] by means of the hydroinspired blast-wave model \[\text{[12]}\]  

1. Possible thermal features in charmonium and, in particular, $J/\psi$ production in $Pb+Pb$ collisions at SPS energy were first pointed out in Ref. \[\text{[5]}\]. However, thermal production of charm quarks is negligible at RHIC energy and below because of the large charm quark mass \[\text{[4]}\].  

2. Perhaps slightly above $T_c$: the upper limit for the dissociation temperature was recently estimated as $1.2T_c$ \[\text{[8]}\].

\[\text{[1]}\]
demonstrated that a simple blast-wave model parametrization of the freeze-out does not fully account for the results measured by the PHENIX Collaboration on $J/\psi$ transverse momentum spectra despite the success of the SHM in the description of multiplicities. On the contrary, the fit based on the two-component model \[10\] with the blast-wave parametrization of transverse momentum spectra for secondary $J/\psi$ describes well the momentum spectra of $J/\psi$ \[13\]. However, the portion of primordial $J/\psi$ that were not melted in the QGP seems to be too big in this model: about 50%. Note that a good description of $J/\psi$ spectra can also be reached without the assumption of thermal equilibration of the secondary charmonia. If recombination of charm quarks is based on the coalescence mechanism (for review see, e.g., Ref. \[14\] and references therein), then the secondary $J/\psi$ are not thermal even if charm quarks are considered to be thermalized ones \[15\] at the hadronization stage. The latter scenario is realized, for example, in Ref. \[16\] where the recombination coalescence component of $J/\psi$ spectra was calculated based on the kinetic coalescence model of Ref. \[17\]. However, although the recombination contribution matches the low $p_T$ region well \[16\], significant primordial $J/\psi$ production is again required to describe the higher $p_T$ region.

In the present article we advocate the local equilibrium picture of $J/\psi$ production at the freeze-out stage near $T_c$ in accordance with the statistical hadronization model \[4, 6, 7\]. While multiplicities in this model were described in Refs. \[6, 7\], our aim here is to demonstrate that transverse momentum spectrum slopes of $J/\psi$ mesons at midrapidity can be described by a hydroinspired parametrization without a significant primordial $J/\psi$ contribution: the latter is questionable taking into consideration recent results \[8\] as for $J/\psi$ melting in the QGP. The article is organized as follows. In Sec. II, we present the results of blast-wave model fits to describe transverse momentum spectra of $\phi$ and $\Omega$ hadrons at RHIC and discuss the problems with the fit of $J/\psi$ spectra. In Sec. III, we briefly analyze the main assumptions of the blast-wave model, to clear up the reasons for the shortcomings in the blast-wave model description of $J/\psi$ transverse momentum spectra, and show that a consistent description of the $\phi$, $\Omega$, and $J/\psi$ transverse momentum spectra can be obtained within a hydroinspired parametrization of transverse momentum spectra that accounts for a possible inhomogeneity of charmonium distribution and/or flow fluctuations. We conclude in Sec. IV. 

3 See also Ref. \[18\], where the $J/\psi$ transverse momentum distribution was calculated in a transport model with both initial production and continuous regeneration of charmonia.
II. BLAST-WAVE MODEL FITS TO TRANSVERSE MOMENTUM SPECTRA
OF $\phi$ AND $\Omega$ AT RHIC AND TRANSVERSE MOMENTUM SPECTRA OF $J/\psi$

Let us start with the assumption that the momentum spectra of $J/\psi$, as well as $\phi$ and $\Omega$ hadrons, are frozen near the phase boundary in RHIC-energy collisions at $T = 160$ MeV. This value is close to the temperature of chemical freeze-out in these collisions [19, 20]. We also note that $T_c$, as obtained for zero-net-baryon-density matter from the most recent lattice QCD calculations is close to this value [21]. We choose $\phi$ and $\Omega$ particles for comparison with $J/\psi$ mesons because these particles are thought to have very small hadronic cross sections and their spectra are not expected to be significantly distorted by feed-down from resonance decays in relativistic heavy-ion collisions. The assumption of coincidence of the kinetic freeze-out of these particles with their chemical freeze-out was supported in Ref. [22] where it was demonstrated that the blast-wave model fit to the transverse mass spectra of these particles at RHIC yields $T \approx 160$ MeV along with an average flow velocity $\langle v \rangle \approx 0.45c$. The latter is lower than that for hadrons strongly interacting in the hadron resonance gas, and, thereby their chemical freeze-out precedes kinetic freeze-out. Coincidence of thermal and chemical freeze-out also underlies the hydroinspired model presented in Ref. [24].

It is noteworthy that the inverse slope parameters for $\Omega$ and $\phi$ transverse momentum spectra do not vary significantly with collision centrality [25–27]. This weak centrality independence can be explained by the independence of the chemical freeze-out temperature from the collision centrality that was observed in the chemical equilibrium model fit to particle number yields [20, 28]. This experimental observation also supports the coincidence of kinetic and chemical freeze-out for these particle species. Note that the insensitivity of the chemical freeze-out temperature to centrality is natural [29] if chemical freeze-out happens at the phase boundary [30] (see also the recent review article [31]).

To calculate transverse momentum spectra of $J/\psi$, $\phi$ and $\Omega$ particles in central collisions we utilize the blast-wave formula [12],

$$
\frac{dN}{p_T dp_T dy} \propto m_T \int_0^R drrI_0 \left( \frac{p_T \sinh y_T(r)}{T} \right) K_1 \left( \frac{m_T \cosh y_T(r)}{T} \right),
$$

where $I_0$ is the modified Bessel function of the first kind.

A similar conclusion was also obtained in Ref. [23] for SPS 158 A GeV energy collisions where blast-wave fits to $J/\psi$ and $\psi'$ mesons and $\Omega$ hyperon spectra in Pb+Pb collisions were performed to show that a good description of these particle spectra by the blast-wave formula with the temperature associated with the chemical freeze-out and a relatively low transverse collective velocity can be obtained.
which follows from boost-invariant parametrizations of freeze-out at
\[ \tau = \sqrt{t^2 - z^2} = \text{const} \]
under the assumption of homogeneous particle number density until \( r = R \), where \( R \) is the transverse size of the system. Here \( I_0 \) and \( K_1 \) represent modified Bessel functions, and \( m_T = \sqrt{p_T^2 + m^2} \). We utilize a constant temperature \( T \) across the freeze-out hypersurface and a linear transverse rapidity profile, \( y_T = y_T^{\text{max}} r/R \), where \( y_T^{\text{max}} = y_T(R) \) is the maximum transverse rapidity. The collective radial expansion velocity \( v \) is then \( v = \tanh y_T \), and \( v_{\text{max}} = \tanh y_T^{\text{max}} \) is the velocity at the surface.

Because we are interested in the description of the transverse spectrum slope, it is convenient to substitute \( r/R \) with \( x \) in Eq. (1) to obtain
\[
\frac{dN}{p_T dp_T dy} \propto m_T \int_0^1 x dx I_0 \left( \frac{p_T \sinh(\alpha x)}{T} \right) K_1 \left( \frac{m_T \cosh(\alpha x)}{T} \right),
\]
with \( \alpha = y_T^{\text{max}} \).

We check first that assuming equal chemical and kinetic freeze-out temperatures is a good approximation for \( \phi \) and \( \Omega \) (\( \Omega \equiv \Omega^+ + \Omega^- \)) particle transverse momentum spectra at midrapidity. The corresponding experimental data for \( Au+Au \) collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) for \( \Omega \) baryons in the 0% – 10% centrality bin are taken from Ref. [25] (STAR Collaboration), and those for \( \phi \) mesons at 0% – 10% centrality from Ref. [26] (PHENIX Collaboration). As shown in Fig. 1, the shapes of the \( \Omega \) and \( \phi \) spectra are reproduced well by the blast-wave formula with fairly realistic parameters at chemical freeze-out: \( T = 160 \text{ MeV} \), \( \alpha = 0.75 \), implying \( v_{\text{max}} \approx 0.635 \). We also show in Fig. 1 the exponential fit to momentum spectra of \( \phi \) mesons in the 0% – 10% centrality bin by the STAR Collaboration [27], which we normalized to PHENIX data points. The STAR Collaboration demonstrated that \( \phi \)-meson momentum spectra are fitted well by an exponential function,
\[
\frac{d^2N}{p_T dp_T dy} \sim \exp \left( -m_T/T_{\text{exp}} \right)
\]
where the slope parameter \( T_{\text{exp}} \approx 359 \text{ MeV} \) for the 0% – 10% centrality bin. Note that the \( dN/dy \) results for \( \phi \) mesons extracted by the PHENIX and STAR Collaborations in this centrality bin are not in agreement, while, as explicitly shown in Fig. 1, there is no discrepancy between the measured spectrum slopes.

\[ ^{5} \text{Note that relatively high collective transverse velocities for } T = 160 \text{ MeV can appear in the course of the evolution as a result of the development of initial transverse velocities at the prethermal partonic stage (with subsequent rapid thermalization) and viscous effects. The latter leads to a decrease in the longitudinal velocity and increase in transverse velocity.} \]
FIG. 1. Transverse momentum data (in arbitrary units) at mid rapidity of $\phi$ mesons and $\Omega$ ($\Omega^+ + \Omega^-$) hyperons in central $Au + Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV RHIC energy along with the results of the blast-wave model, Eq. (2), fit which are indicated by the solid lines, the dashed line represents the exponential, Eq. (3), fit to momentum spectra of $\phi$ mesons by the STAR Collaboration [27], which we normalized to the PHENIX data points. Experimental data for $\phi$ and $\Omega$ are from Refs. [26] and [25] respectively. Statistical error bars are shown for $\phi$ mesons, and total (statistical and systematic) error bars are presented for $\Omega$ hyperons.

The results for momentum spectra of $J/\psi$ with the same set of parameters are shown in Fig. 2 together with the data measured by the PHENIX Collaboration [11] for two centrality bins. One notices significant deviations for the data points at low momentum, leading to a excessively high effective temperature.

To see what blast-wave model parameters are required to describe the low-$p_T$ region of $J/\psi$ spectra, we performed a blast-wave model fit to the low-momentum part of $J/\psi$ spectra at the same temperature, $T = 160$ MeV, but considered the maximum transverse rapidity $\alpha$ in Eq. (2) to be a free parameter. The results are demonstrated in Fig. 2 where one can see that the blast-wave model with $\alpha = 0.55$ (implying $v_{\max} \approx 0.5$) yields a good description.

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6 We show the data points of noncentral collisions (20% − 40% centrality bin) just for comparison.
FIG. 2. Transverse momentum data for $J/\psi$ mesons (in arbitrary units) at midrapidity in $Au+Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV RHIC energy along with the results of the blast-wave model, Eq. (2), fit with the same parameters as in Fig. 1 (solid lines). Dashed lines represent the blast-wave model, Eq. (2), fit results with the "soft" transverse flow parameter: see text for details. Experimental data for $J/\psi$ are from Ref. [11] for two centrality bins. Statistical and systematic uncorrelated errors are represented as error bars.

of the low-momentum part of the $J/\psi$ spectrum. These findings, at first sight, might imply a contradiction with the blast-wave model fits to $\phi$ and $\Omega$ transverse momentum spectra where the same temperature but another surface velocity was obtained. Consequently, the question arises: Can $J/\psi$, $\phi$ and $\Omega$ transverse momentum spectra be coherently described within a hydroinspired parametrization of freeze-out with reasonable parameters?

III. HYDROINSPIRED PARAMETRIZATION OF $J/\psi$ TRANSVERSE MOMENTUM SPECTRUM FREEZE-OUT AT THE RHIC

To understand why a blast-wave model, Eq. (1), description with parameters fitted to $\phi$ and $\Omega$ transverse momentum spectra does not reproduce the $J/\psi$ spectra properly, let us
look carefully over the main assumptions used in the comparison to spectra by employing Eq. (1). They are the following: the freeze-out isotherm is $\tau = \text{const}$ and the transverse rapidity profile at the isotherm is linear. Further assumptions are on-mass shell distribution functions, absence of flow fluctuations, a homogeneous particle number density, $n = \sqrt{n^\mu n^\mu}$, at freeze-out, and absence of resonance feed-down. Here we briefly discuss these assumptions to clarify whether giving up debatable ones can result in consistent description of $\phi$, $\Omega$, and $J/\psi$ transverse momentum spectra if kinetic freeze-out takes place just after chemical freeze-out for these particles.

First note that a hypersurface of constant proper Bjorken time, $\tau = \sqrt{t^2 - z^2} = \text{const}$ in general does not correspond to any isotherm, and a linear transverse rapidity profile is questionable. However, based on estimates of corresponding quantities in viable scenarios of hydrodynamic evolution and results of correspondingly modified blast-wave model calculations (see, e.g., Ref. [20] for blast-wave model calculations with different velocity profiles), one can conclude that the spectra discussed are little affected by the shape of the flow velocity profile and by deviations of the chemical freeze-out isotherm from the $\tau = \text{const}$ hypersurface. We hence keep these assumptions unchanged. Actually, the corresponding hypersurface and velocity profile depend on the whole dynamical history of the hydrodynamically expanding system - initial conditions, equation of state, etc. - and can be calculated in a true dynamical model only. It will be the subject of follow-up work.

Another assumption, namely, the on-mass shell approximation of the distribution function, may be questioned for finite-width particles such as $J/\psi$, although the free width of the $J/\psi$ is less than 100 keV. The main reason for the higher effective temperature of $J/\psi$ spectra in the low-momentum region (see Fig. 2) compared with the effective temperatures of $\phi$ and $\Omega$ spectra in the same region (see Fig. 1) is the larger mass of the $J/\psi$ particle. An explanation of this fact can be based on the simple approximation of the integrand in Eq. (2). Then one may surmise that an appropriate off-mass-shell hydroinspired parametrization of $J/\psi$ spectra that includes low invariant mass contributions could improve the description of the low-momentum part of the spectra. Most naturally, such a parametrization - namely, a Breit-Wigner formula for the spectral function instead of the $\delta$ function approximation - appears because of the finite width of $J/\psi$ meson. However, the corresponding calculations of the $J/\psi$ spectra with the same hydrodynamical parameters of the blast-wave model as for $\Omega$ and $\phi$ particles do not result in a significant improvement in the description of $J/\psi$
spectra if experimental conditions are properly taken into account. Namely, as reported by
the PHENIX Collaboration [11], the J/ψ spectra at midrapidity are measured by counting
lepton pairs in the invariant mass range $2.9 \text{ GeV} \leq M \leq 3.3 \text{ GeV}$, and such a narrow invari-
ant mass window accompanied by the rather low decay width, $\Gamma_{\text{tot}} = 93.2 \times 10^{-6} \text{ GeV}$, does
not result in noticeable deviation from on-mass shell calculations.

The next assumption, namely, a neglect of flow fluctuations in the blast-wave model,
Eq. (1), is also debatable. Owing to the finite size of the systems, large fluctuations are
expected in the initial stage of the nuclear collisions, even for a fixed impact parameter, and
various event generators do show such effects. While a very early thermalization in rela-
tivistic heavy-ion collisions is doubtful now [33] (for review see also Ref. [34] and references
therein), the initial inhomogeneities can result in fluctuating initial conditions for the sub-
sequent hydrodynamical expansion. Because the hydrodynamical equations are nonlinear,
the event average of any hydrodynamical parameter (in other words, average over solutions
of hydrodynamical equations) is quite different from that for a smooth initial configuration
and results in large differences in spectra from hydrodynamical calculations with averaged
initial conditions. This fact was pointed out and explicitly demonstrated in event-by-event
hydrodynamic simulations based on smoothed particle hydrodynamics code (see Ref. [35]
for review and Ref. [36] for recent results). Fluctuations of transverse flows in noncentral
collisions also are considered in Ref. [37].

Therefore, the collective flow fluctuations are rather natural and can have some effect
on the particle momentum spectra. To study it, let us consider a blast-wave model with
bulk matter flow fluctuations. The transverse spectra of the blast-wave model averaged over
ensemble of the fluctuations are assumed to have the form:

$$\frac{dN}{p_T dp_T dy} \propto m_T \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha G(\alpha) \int_0^1 dx I_0 \left( \frac{p_T \sinh(\alpha x)}{T} \right) K_1 \left( \frac{m_T \cosh(\alpha x)}{T} \right). \quad (4)$$

We consider here two simple and, in some sense, opposite cases of the distribution of $\alpha$.
First, we assume a distribution that is flat in $\alpha$, that is, $G(\alpha) = 1$, with the lower and
upper limits $\alpha_{\min}$ and $\alpha_{\max}$ considered to be free fit parameters. Second, we assume a
Gaussian form for the distribution of hydrodynamical velocities. With $v_{\max} = \tanh \alpha$, we
obtain: $G(\alpha) = \exp \left( -\frac{(\tanh \alpha - \tanh \alpha_0)^2}{\alpha^2} \right) \equiv \exp \left( -\frac{(v_{\max} - v_{\max}^0)^2}{\delta v_{\max}^2} \right)$, with $\alpha_{\min} = 0$ ($v_{\min} = 0$) and
$\alpha_{\max} = \infty$ ($v_{\max}^0 = 1$). Note that, for such a distribution, there is no explicit cutoff of high
$\alpha$ contributions; the convergence of the integral takes place because the main contribution
at high $\alpha$ happens at small $x \sim 1/\alpha$, leading to a cutoff factor $1/\alpha^2$. The results for $G(\alpha) = 1$ with $\alpha_{\text{min}} = 0.23 \ (v^\text{min}_{\text{max}} \approx 0.23)$ and $\alpha_{\text{max}} = 1.0 \ (v^\text{max}_{\text{max}} \approx 0.76)$, and for $G(\alpha) = \exp \left( \frac{-(\tanh \alpha - \tanh \alpha^0)^2}{\delta^2} \right)$ with $\alpha^0 = 0.55 \ (v^0_{\text{max}} \approx 0.5)$ and $\delta = 0.18$ are displayed in Figs. 3 and 4 for $\Omega$, $\phi$, and $J/\psi$ transverse momentum spectra, respectively. One can see from these figures that the blast-wave model supplemented with appropriate fluctuations of the hydrodynamical flow yields a significantly improved description of the shapes of transverse momentum distributions for $\phi$, $\Omega$, and $J/\psi$ particles.

Another assumption used in Eq. (1), namely, that of homogeneous particle number density across the system at the isotherm hypersurface, is rather natural for bulk-matter particles consisting of quarks that are produced in QGP, as well as in the initial collisions, but can be questioned for heavy rare particles like $J/\psi$. The reasons are the following. Because the charm quarks are produced in primary binary nucleon-nucleon collisions only and there is no charm quark production during the evolution of the system, the initial distribution of charm quarks follows the distribution of initial binary collisions. Then the

![Graph of transverse momentum spectra for $\phi$ and $\Omega$.]
charm quark density can be more strongly peaked in the center of the system than the density of light-flavor and strange quarks that initially follow the participant density distribution. Hence the effective transverse size of a volume occupied by $J/\psi$ mesons at the hadronization hypersurface can be less than that for bulk-matter particles.

For simplicity, we consider the particle number density of $J/\psi$ in two extreme scenarios: first, the $J/\psi$ mesons are assumed to follow the density distribution of bulk-matter particles, as in fact considered in the previous section, and second, we assume that the spatial extension of $J/\psi$ in the transverse direction coincides with the initial spatial distribution of charm quarks determined by the geometry of binary nucleon-nucleon collisions. The latter is examined here.

The initial spatial distribution of charm quarks in the transverse direction as determined by the transverse distribution of binary nucleon-nucleon collisions in $Au+Au$ collisions, and

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It was recently demonstrated [38] that a difference in the initial distribution of charm quarks compared to the light flavors survives the hydrodynamical evolution and is still significant at decoupling, thereby resulting in the reduction of the mean transverse momentum of the $D$-meson spectra.
at impact parameter $b = 0$, which we assume here for simplicity, is

$$n(r) \propto T_A^2(r), \quad (5)$$

where the nuclear thickness distribution $T_A(r)$ is obtained from a Woods-Saxon distribution,

$$T_A(r) = \int_{-\infty}^{+\infty} dz \rho \left( \sqrt{r^2 + z^2} \right). \quad (6)$$

The Woods-Saxon distribution of nucleons in the $Au$ nucleus is defined by the formula

$$\rho(\sqrt{r^2 + z^2}) = \rho_0 \left[ 1 + \exp \left( \frac{\sqrt{r^2 + z^2} - c}{a} \right) \right]^{-1} \quad (7)$$

with $c = 6.38 \text{ fm}$, $a = 0.535 \text{ fm}$ \[39\], and $\rho_0$ given by the normalization condition.

The transverse momentum spectrum of the blast-wave model of $J/\psi$ distributed according to this initial spatial distribution of charm quarks is

$$\frac{dN}{p_T dp_T dy} \propto m_T \int_0^1 xdT_A^2(xR) I_0 \left( \frac{p_T \sinh(\alpha x)}{T} \right) K_1 \left( \frac{m_T \cosh(\alpha x)}{T} \right). \quad (8)$$

Then, before calculating the transverse momentum spectra of $J/\psi$ by means of Eq. (8), one needs to define the transverse size of the bulk-matter distribution $R$. We estimate this value to be between 7 and 8 fm, and Fig. 5 presents the $J/\psi$ spectrum calculated with the hydroinspired parametrization, Eq. (8), for both $R$ values. A good description of the spectrum at low $p_T$ spectra is obtained, this is not surprising because, as noted at the end of the previous section, to fit the low-$p_T$ part of the $J/\psi$ spectrum one needs to utilize, for $J/\psi$ mesons, a lower value of the maximum transverse velocity than for bulk-matter particles at the same isotherm.

The last assumption of the blast-wave model, Eq. (1), discussed here is the neglect of the resonance feed-down. While the measured inclusive particle spectra in general contain contributions from resonance decays, for the spectra discussed in this article the resonance feed-down is rather small and one can expect that generalization of the blast-wave model to include the resonance feed-down does not seriously influence the fitted parameters. In any case, feed-down contributions to $J/\psi$ spectra will, except for LHC energies where feed-down from $B$-meson decay is likely significant, coming from excited charmonia. Although this contribution can be up to 40% \[2\], we expect little modification of the spectral shapes,

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8 See also Ref. [20] where it was demonstrated that resonance decays have no significant effect on the kinetic freeze-out parameters extracted by means of the blast-wave model.
as the charmonia masses are all very large compared to masses of hadrons made of light and strange quarks. A comprehensive treatment of resonance feed-down within a dynamical hydrokinetic model of the evolution of the fireball and of particle production will be provided in a forthcoming work.

IV. RESULTS AND DISCUSSION

We have analyzed central $J/\psi$, $\phi$, and $\Omega$ transverse momentum spectra at midrapidity within a blast-wave model and demonstrated that, while the high-$p_T$ part of the spectrum of $J/\psi$ is well described by fit parameters obtained for a description of $\phi$ and $\Omega$ spectra, the low-$p_T$ experimental transverse momentum spectrum of $J/\psi$ exhibits an effective temperature lower than that calculated with the fit parameters for $\phi$ and $\Omega$ spectra. In our opinion, the reason for this is grounded in some specific assumptions of the blast-wave model rather than in significant nonthermal contributions to $J/\psi$ spectra. To demonstrate this we have developed and presented a generalized blast-wave model.
First, we have demonstrated that a blast-wave model modified to account for collective flow fluctuations results in good agreement with data over a wide $p_T$ region for $J/\psi$, $\phi$ and $\Omega$ transverse momentum spectra. However, good agreement with $J/\psi$ data is reached only if rather high flow fluctuations take place near the chemical freeze-out isotherm. Note that in a very recent paper [40] it is estimated that much lower flow fluctuations, following from the fluctuations of the transverse size of the initial source, are consistent with recently observed [41] event-by-event fluctuations of the average transverse momentum of bulk-matter particles in $Au+Au$ collisions at RHIC energies. If this result is confirmed, it rules out the interpretation that the low-$p_T$ peculiarities of $J/\psi$ transverse momentum spectra are a result of hydrodynamic flow fluctuations.

Another generalization of the blast-wave model considered in this article is based on the fact that, because charm quarks initially follow the density distribution of primary binary nucleon-nucleon collisions, the shape of $J/\psi$ density at freeze-out differs from the particle density of uniform bulk matter because of a higher concentration of charm quarks in the center of the system at the hadronization hypersurface. The spectrum of $J/\psi$, calculated with the density distribution of primary binary collisions and with a conservative estimate of the transverse size of the volume occupied by the bulk matter, demonstrates a fairly good agreement with $0\%−20\%$ $J/\psi$ transverse momentum data until $p_T = 3.5$ GeV, and the curve calculated is below the data points for higher $p_T$. Note that high-$p_T$ $J/\psi$ can be emitted mostly by corona and can be, in fact, the result of charmonium production in nucleon-nucleon collisions (see Ref. [6]). Because the fraction of participating nucleons contained in the corona region increases when the centrality decreases, in noncentral collisions one can expect a higher effective temperature of the $J/\psi$ spectra than in central collisions; indeed Fig. 5 shows this effect.

Finally, we conclude that the present data at the top RHIC energy are compatible with the picture in which $J/\psi$, $\phi$, and $\Omega$ momentum distributions are frozen simultaneously with chemical composition at $T = 160$ MeV. We have demonstrated that a consistent description of the $\phi$, $\Omega$, and $J/\psi$ transverse momentum spectra can be obtained within

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9 It is worthy of note, also, that the magnitude of event-by-event elliptic flow fluctuations in $Au+Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV [42] was found to be in agreement with predictions based on spatial fluctuations of the participating nucleons in the initial nuclear overlap region.

10 The relatively high corona contribution is, perhaps, the reason why the $D^0$-meson spectra measured by the STAR Collaboration in the $0\%−80\%$ centrality bin [43] can be described by the relatively high radial flow velocity at the surface, 0.6, and the freeze-out temperature, 170 MeV (see Ref. [44]).
a hydroinspired parametrization of common freeze-out for these particles if nonhomogeneity
of the charmonium distribution and/or significant flow fluctuations are assumed.

ACKNOWLEDGMENTS

This work was supported by the Bilateral Grant DLR (Germany) - MESU (Ukraine)
for the UKR 06/008 Project, Agreement No. M/26-2008, and by a Ukrainian-French grant
(DNIPRO Program), Agreement with MESU No. M/4-2009.

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