Universal Metasurfaces for Complete Linear Control of Coherent Light Transmission

Taeyong Chang, Joonkyo Jung, Sang-Hyeon Nam, Hyeonhee Kim, Jong Uk Kim, Nayoung Kim, Suwan Jeon, Minsung Heo, and Jonghwa Shin*

Recent advances in metasurfaces and optical nanostructures have enabled complex control of incident light with optically thin devices. However, it has thus far been unclear whether it is possible to achieve complete linear control of coherent light transmission, that is, independent control of polarization, amplitude, and phase for both input polarization states, with just a single, thin nanostructure array. Here, it is proved possible, and a universal metasurface is proposed, a bilayer array of high-index elliptic cylinders that possesses a complete degree of optical freedom with fully designable chirality and anisotropy. The completeness of achievable light control is mathematically shown with corresponding Jones matrices, new types of 3D holographic schemes that were formerly impossible are experimentally demonstrated, and a systematic way of realizing any input-state-sensitive vector linear optical device is presented. The results unlock previously inaccessible degrees of freedom in light transmission control.

1. Introduction

Polarization, intensity, and phase are fundamental properties of coherent light, and controlling them with a high degree of freedom is the central objective for numerous optical devices. Controlling all three properties independently for a single input polarization state is a difficult task in itself, but doing so for all possible input polarizations in a precisely designed manner is the ultimate goal: it allows the complete control of coherent light transmission, fully utilizing the vector nature of light. Historically, birefringent materials such as calcites have been used for polarization-dependent light control. Although various crystal classes of natural optical materials possess anisotropy, chirality or both, the effects of these characteristics are not strong enough for a thin slab to achieve the full range of controllability of the optical response. Also, such materials are usually spatially homogeneous and do not allow easy control of transmitted amplitudes. Recent advanced metasurfaces are reanimating the field with their ability to control light at a pixel-by-pixel spatial degree of freedom, allowing novel applications ranging from imaging and communications to quantum optical operations. However, the currently achievable range of light transmission control from a single metasurface is still limited. This limitation translates into restricted accessible sets of Jones matrices, which can quantitatively represent thin optical media in the paraxial regime. In particular, conventional waveplates are restricted to unitary symmetric Jones matrices under a linear polarization basis, that is, they are achiral. This limitation remains the same for many metasurfaces including even the very recently developed. In response, there have been active efforts to surmount this restriction and realize all possible Jones matrices, enabling bespoke control of polarization, amplitude and phase, that is, achieving complete control of light transmission.

First, gaining chirality and breaking the symmetry of the Jones matrix has been attempted by removing the mirror symmetry in the spatial configuration, for example via utilization of 3D chiral (meta-)atoms or metasurfaces and off-normal incidence. Second, the unitary condition, which is associated with the conservation of energy between input and output beams, can be avoided by utilizing gain materials or by incorporating radiative loss through Fano resonances or superposition and translational homogenization of two output states with different phases. However, all the above cases still achieve only subsets of the complete Jones matrix set. Prior theoretical, numerical or experimental demonstrations of pixel-by-pixel controllability of transmitted coherent light are compared in the Venn diagram in Figure 1a, in which white regions indicate categories that have not been fully demonstrated yet. In particular, despite many seminal works on artificial chirality since the early days of metamaterials, it still remains unproven whether a pixel-by-pixel complete control of chirality (i.e., independent control of transmission amplitudes and phases) is possible with a single metasurface, which is currently the critical missing piece in complete designability of Jones matrix and achievement of the full control on coherent light transmission. While utilization of extrinsic chirality by choosing a specific oblique incidence angle on an achiral metasurface may have its own merit, it would be highly desirable if one can realize the full control as an intrinsic chiral property of the metasurface. The incident-angle dependency of the extrinsic chirality makes it difficult to apply the concept in a
pixel-by-pixel manner. Moreover, the Fourier and image planes in typical optical systems are normal to the optical axis and exist in the paraxial regime, which is incompatible with extrinsically chiral systems operating at large incidence angles (Figure S1, Supporting Information). An intrinsic, nanostructure-induced chirality would allow us to avoid this inherent limitation of an extrinsic method and may be applied to diverse optical systems such as optical convolution, optical artificial neural networks, and optical linear transformation systems.

Here, we propose a thin effective optical material composed of a bilayer array of dielectric nanostructures, a universal metasurface, and theoretically and experimentally demonstrate that such a metasurface can embody, at ≈2 µm thickness, any arbitrary passive Jones matrix and gain pixel-wise complete linear control of coherent light transmission as shown in Figure 1a. The theoretical proof is based on a recent report in the field of mathematics on the factorization of an arbitrary unitary matrix. We experimentally demonstrate a new holographic scheme that is impossible with an existing optical medium or metasurface. Furthermore, we theoretically prove that any arbitrary passive vector linear optical device can be constructed systematically by using only such universal metasurfaces and conventional lenses. As an example, using three universal metasurfaces and two lens arrays, we numerically demonstrate parallelized probabilistic linear quantum optical controlled-NOT (CNOT) gates for an array of two single-photon qubits, whose individual spatial channel is functionally equivalent to the CNOT gate implemented previously by bulk optics. The universal metasurfaces provide a full degree of freedom of coherent light transmission control by a thin optical component, and can serve as a new optical platform.

2. Results and Discussion

2.1. Theoretical Background and Design Rule for Universal Metasurfaces

We first prove that an arbitrary 2-by-2 unitary matrix can be decomposed into two unitary symmetric matrices, which is...
the key theoretical foundation for constructing universal metasurfaces. Since unitary symmetric matrices are orthogonally similar to unitary diagonal matrices, multiplication of two arbitrary 2-by-2 unitary symmetric matrices can be expressed as:

$$U_{SB}U_{SA} = \begin{pmatrix}
\cos \theta_A & -\sin \theta_A \\
\sin \theta_A & \cos \theta_A
\end{pmatrix}
\begin{pmatrix}
\cos \theta_B & -\sin \theta_B \\
\sin \theta_B & \cos \theta_B
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
\cos \theta_A & \sin \theta_A \\
-\sin \theta_A & \cos \theta_A
\end{pmatrix}
\begin{pmatrix}
\cos \theta_B & \sin \theta_B \\
-\sin \theta_B & \cos \theta_B
\end{pmatrix}
\begin{pmatrix}
\cos \phi_A & \sin \phi_A \\
-\sin \phi_A & \cos \phi_A
\end{pmatrix}
\begin{pmatrix}
\cos \phi_B & \sin \phi_B \\
-\sin \phi_B & \cos \phi_B
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
\cos \phi_A & -\sin \phi_A \\
\sin \phi_A & \cos \phi_A
\end{pmatrix}
$$

(1)

where $\theta_A, \theta_B, \phi_{AM,AM},$ and $\phi_{BM,BM} \in \mathbb{R}$. Since a 2-by-2 unitary matrix has four real degrees of freedom while the above parameterization has six real parameters, the mapping between two unitary symmetric matrices to one unitary matrix would be surjective. Thus, we choose $\phi_{BM} = 0$ and $\phi_{BM} = \pi$ (i.e., a “half-wave retarder”) to simplify the problem but note that it is not a unique way:

$$U_{SB}U_{SA} = \begin{pmatrix}
\cos 2\theta_B & -\sin 2\theta_B \\
\sin 2\theta_B & \cos 2\theta_B
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
\cos \theta_A & -\sin \theta_A \\
\sin \theta_A & \cos \theta_A
\end{pmatrix}
\begin{pmatrix}
\cos \theta_B & \sin \theta_B \\
-\sin \theta_B & \cos \theta_B
\end{pmatrix}
\begin{pmatrix}
\cos \phi_A & \sin \phi_A \\
-\sin \phi_A & \cos \phi_A
\end{pmatrix}
\begin{pmatrix}
\cos \phi_B & \sin \phi_B \\
-\sin \phi_B & \cos \phi_B
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
\cos \phi_A & -\sin \phi_A \\
\sin \phi_A & \cos \phi_A
\end{pmatrix}
$$

(2)

where $\theta_B = 2\theta_B - \theta_A$. The last form of $U_{SB}U_{SA}$ in Equation (2) is a general factorization of an arbitrary 2-by-2 unitary matrix[16] and reveals the bijective mapping between one 2-by-2 unitary matrix and two 2-by-2 unitary symmetric matrices under the $(0,\pi)$-phase constraint for one of the symmetric matrices. Physically, this suggests we can realize metasurfaces with arbitrary unitary Jones matrices (unitary metasurfaces) by forming a tandem structure made of two metasurfaces with arbitrary unitary symmetric Jones matrices (unitary symmetric metasurfaces). The schematic of unit structure of the unitary metasurface in Figure 1b shows the geometric parameters used as design variables. These structural parameters are directly related to the parameters in Equations (1) and (2) ($\theta_A, \theta_B, \phi_{AM}, \phi_{BM}$, and $\phi_{BM}$). This cascade principle does not rely on a near-field coupling between the top and bottom layers, so the optical properties of the bilayer unitary metasurface are robust to lateral misalignment between the layers (Figure S2, Supporting Information).

Once all unitary Jones matrices become accessible, it is relatively straightforward to form universal metasurfaces with access to the full set of all possible passive Jones matrices including amplitude controllability: for example, one can utilize a transversal homogenization of unitary metasurfaces (i.e., introducing a super-cell composed of two kinds of unitary unit cells)[6] such as $A = U\Sigma V^\dagger = \frac{1}{2}(U_1 + U_2)V^\dagger = \frac{1}{2}(U_1 + U_2)$ for an arbitrary passive (i.e., the maximum singular value is one) Jones matrix $A$ where $U$, $V$, $U_1$ and $U_2$ are unitary matrices, $\Sigma$ is a diagonal matrix showing singular values, and $D_1$ and $D_2$ are unitary and diagonal matrices. In this scheme, the undesired portion of the energy is reflected or deflected into higher diffraction orders other than the main transmitted beam that we are interested in.[10,12] Thus, the main loss mechanism is radiative rather than absorptive. If desired, one can eliminate these unwanted scattered waves using spatial frequency filters such as an aperture placed on a Fourier plane. We use such transversal homogenization as the basis for our universal metasurfaces targeting 915 nm wavelength and fabricate them by depositing silicon and dielectric encapsulation layers as shown in Figure 1c,d (Experimental Section).

The preceding discussion provides mathematical completeness of realizable arbitrary passive Jones matrix with the proposed universal metasurfaces. Now, we provide systematic design strategy for unitary metasurfaces and universal metasurfaces with a closed-form solution. For unitary metasurfaces, one can predict the output polarizations and phases based on such structural parameters by first reformulating the Jones matrix with input-output polarization pairs and corresponding phase advances. Without loss of generality, we can represent an arbitrary Jones matrix in the form $Q = W P S \chi \chi \chi (\psi_{w}, \psi_{w})$, where $|R\rangle = |1; i\rangle / \sqrt{2}$ (right-handed circularly polarized state, RCP) and $|L\rangle = |1; -i\rangle / \sqrt{2}$ (left-handed circularly polarized state, LCP) are used as a basis for the input polarization states, and the corresponding output polarization states are denoted with $|w\rangle$ and $|w\rangle$. We now find an analytic relationship between the geometric parameters ($\theta_A, \theta_B, \phi_{AM},$ and $\phi_{BM}$) and output polarization states and phases ($2\psi, 2\chi, \phi_1,$ and $\phi_2$) where $2\psi$ and $2\chi$ are the spherical coordinates of $|w\rangle$ on the Poincaré sphere). $Q$ can also be represented as:

$$Q = \begin{pmatrix}
\cos \psi & -\sin \psi \\
\sin \psi & \cos \psi
\end{pmatrix}
\begin{pmatrix}
\cos \chi & \sin \chi \\
-i \sin \chi & -i \cos \chi
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
\cos \phi_{w} & \sin \phi_{w} \\
-\sin \phi_{w} & \cos \phi_{w}
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
1 & i \sqrt{2} \\
1 & -i \sqrt{2}
\end{pmatrix}
$$

(3)

where $\phi_1 = \phi_1 - \phi_1^\dagger (i = 1, 2), \phi_2 = \angle(\cos \psi \cos \chi - \sin \psi \sin \chi)$ and $\phi_2' = \angle(\cos \psi \sin \chi + \sin \psi \cos \chi)$. By equating Equations (2) and (3), analytic relations between design parameters and output polarization including phases can be obtained (Text S1, Supporting Information):

$$\begin{align*}
\theta_A &= \frac{\phi_1 - \phi_1^\dagger}{2} + \frac{\pi}{4}, \\
\theta_B &= \frac{\psi + \phi_2 - \phi_2^\dagger}{2} + \frac{\pi}{4}, \\
\phi_{AM} &= \frac{\phi_1 + \phi_2'}{2} + \frac{\pi}{4}, \\
\phi_{BM} &= \frac{\phi_1' + \phi_2'}{2} - \frac{\pi}{4}.
\end{align*}$$

(4)

We designed unitary metasurfaces for target Jones matrices based on this solution. We note that the resulting output polarization states and phases are regularly spaced for regular arrays of geometric parameters under the incidence of circularly
polarized states, without any abrupt changes. This simplifies the design process and also hints at the relative robustness of the design against possible imperfection in fabrication. In addition, even though the analytic solution in Equation (4) is expressed for circular polarization incidence \((Q = WPC^\dagger)\), one can use it for any desired input polarization pair \((E^\dagger = [|e\rangle, |e\rangle_\perp]^\dagger)\) with any desired target output polarization pair \((W^\dagger)\) and phase pair \((P^\dagger)\) such that \(Q^\dagger = W^\dagger P^\dagger E^\dagger C^\dagger = W^\dagger P^\dagger C^\dagger\) where \(W^\dagger\) and \(P^\dagger\) can be calculated from \(WPC^\dagger\).

For universal metasurfaces of arbitrary Jones matrix \(A\) for any desired input polarization pair with any desired target output polarization pair, amplitude pair, and phase pair (subject to passiveness), one can find \(U_1\) and \(U_2\) such that \(A = U_1U_2^\dagger\) and apply above design strategy to find all geometric parameters for the realization. For simplicity of the proof-of-concept, we composed square clusters with 3 to 15 unit structures in each direction to minimize the change in the optical properties of unit structures due to near-field coupling in heterogeneous configurations. One can further reduce the size of clusters by optimizing the shape of each elliptic cylinder to compensate for near-field effects.

2.2. Numerical Validations for Unitary Metasurfaces and Universal Metasurfaces

Finite-difference time-domain (FDTD) simulations were employed to confirm the generality of the proposed unitary metasurfaces (Figure 2). Based on Equation (3), output polarization states and phase delays under two orthogonal input polarization states (e.g., RCP and LCP) for a unitary metasurface can be represented as \(|w\rangle, \phi_1\) and \(|w\rangle_\perp, \phi_2\). Using unitary metasurfaces with proper geometric parameters, simulation results show that transmission phases can be independently controlled over the entire phase space for the incidence of (without loss of generality) RCP and LCP (Figure 2a), while the output polarization states are intentionally kept constant for all cases (Figure 2b) (Experimental Section). We show corresponding experimental results in Figure S3, Supporting Information. Note that the target output polarization states were arbitrarily chosen by a random number generator to demonstrate the generality of the result. This is a prominent difference between unitary symmetric and general unitary metasurfaces and reveals the continuous controllability of chirality by general unitary metasurfaces. There is no set of unitary symmetric Jones matrices that can provide an arbitrary pair of transmission phases for an arbitrary unitary conversion of polarization states except when output states are exact conjugations of input states.\(^{6d,e}\) However, input–output polarization pairs (e.g., RCP–\(|w\rangle\) and LCP–\(|w\rangle_\perp\)) of unitary metasurfaces have no such restriction. Moreover, with the universal metasurface, the range of achievable chirality extends to independent attenuation control for each polarization and realizes full degrees of freedom in coherent light transmission control, filling all previously unrealized instances of passive Jones matrices. We show controllable optical activity and circular dichroism in Figure S4, Supporting Information, as explicit examples of chirality control.

To show the arbitrary controllability of a Jones matrix of a universal metasurface, we first randomly generate a Jones matrix and implement it with a universal metasurface (Figure 2c). We then arbitrarily shift one individual matrix component of the given Jones matrix in Figure 2c while keeping the other components constant and try to find universal metasurfaces for

---

**Figure 2.** Numerical validation of unitary and universal metasurface. a,b) Arbitrary wave retardation for two arbitrary orthogonal input–output polarization pairs based on a full degree of freedom of chirality and anisotropy of unitary metasurfaces. a) Transmission phases for RCP (LCP) input, \(\phi_1\) (\(\phi_2\)). The red circles indicate target phases. The blue dots are phases retrieved from FDTD simulations. b) Output polarization states for RCP (left) and LCP (right) inputs on the Poincaré sphere. The red (blue) dot shows target (retrieved) polarization states. c,d) Elements of the Jones matrices on a complex plane. The gray markers show original elements being shifted. The dashed lines indicate unit magnitude.
the resulting Jones matrices as well (left and right panels in Figure 2d). For all cases, the target Jones matrices and those retrieved from the FDTD simulation of the corresponding universal metasurfaces are well matched (Experimental Section). These are randomly generated examples and illustrate the generality of the proposed universal metasurface. We show experimental results of independent controllability of amplitude and phase for a randomly chosen output polarization pair of universal metasurfaces in Figure S5, Supporting Information.

2.3. Demonstration of Complete Linear Control of Coherent Light Transmission

We experimentally demonstrate complete vectorial wavefront manipulation enabled by unitary and universal metasurfaces through four examples (Figures 3 and 4). Throughout this paper, we visualize polarization states and intensity measured with full-stokes polarimetry (Figure S6, Supporting Information) with false colors by directly mapping the Poincaré sphere into the CIELAB color space (Figure 3a). For example, red, green, white and black indicate x-, y-polarized, RCP and LCP states, respectively.

For a unitary metasurface with spatially varying structures, output polarization states with different phase delays under two orthogonal input polarization states (e.g., RCP and LCP) can be represented as spatially dependent forms, $|u|\langle r|, \phi_i\langle r| \text{ and } |w|\langle r|$, respectively. With the full set of unitary Jones matrices available, one can arbitrarily design the output polarization state pairs in a pixel-by-pixel manner (i.e., vector profiles), with independent control of the phase delay for each output polarization (also in a pixel-wise manner), which was impossible for previous metasurfaces. Figure 3b shows two orthogonal Poincaré beams generated with independently controlled phase profiles under RCP and LCP Gaussian beams illumination (Experimental Section). The demonstration in Figure 3b directly contrasts with the example in ref. [6d], which shows correlated phase profiles for two orthogonal output vector beams. In a similar manner, we show two independent holographic images whose polarization states are not conjugations of corresponding input states (Figure 3c), which is in direct contrast to ref. [6e].

Universal metasurfaces can provide even more generalized vectorial wavefront manipulation. In Figure 3d, we show two independent vectorial holographic images under two orthogonal input states that require arbitrary controllable asymmetric and non-unitary Jones matrices (Experimental Section). This is in contrast to refs. [6f,g,19], which show the generations of vectorial holographic images for a single input polarization state, and ref. [5b], which can be interpreted as providing vectorial holographic images for different input polarization states that cannot be arbitrary and are not independent of each other.

The most general type of vectorial wavefront manipulation based on a universal metasurface is illustrated in Figure 4. Based on the electromagnetic equivalence principle, an arbitrary set of sources inside a closed surface can be replaced with equivalent surface sources with exactly the same resulting fields in the outside volume. Hence, if one can realize, with a single metasurface, any combination of two completely unrelated arbitrary tangential field profiles on the transmission-side of an infinite plane for two incident uniform plane waves with different polarization states, this means that it is possible to obtain any combination of two unrelated 3D electromagnetic field profiles on the transmission-side that would be obtainable by two sets of virtual sources in the incident-side semi-infinite space, using just the given metasurface and two plane waves (Figure 4a,b). This is different from typical volumetric holography because the phase and polarization profiles in addition to the intensity profiles, can be exactly designed, leading to potential for new applications. As an illustrative example, we designed a universal metasurface that produces a 3D spiral pattern with radial polarization under RCP incidence and produces the letters “KAI” and “ST” on different transverse planes with gradually varying polarization states following the circumference of the $S_{1}$-$S_{3}$ plane of a Poincaré sphere under LCP incidence (Figure 4c) (Experimental Section). Experimental results show good agreement with numerical predictions (Figure 4d; Figure S7, Supporting Information). We note that the result in Figure 4d is enabled by the arbitrary polarization, amplitude, and phase controllability of the universal metasurface. In particular, it is revealed that both full control of the target 3D holographic patterns and design of their point-by-point polarization states are possible, in addition to control of their intensity and
phase patterns, all independently for the two polarization states of the incident beam, which was previously unheard of for a single metasurface. The demonstration in Figure 4d contrasts with ref. [20], which shows a single scalar (spatially homogeneous polarization state) 3D hologram, ref. [19b], which shows a 3D vectorial hologram with a single input polarization state, and ref. [6i], which can be interpreted as providing 3D vectorial holograms for different input polarization states that cannot be arbitrary and are not independent of each other. We compare all experimental data in Figures 3 and 4 with numerical calculations in Figure S7, Supporting Information, and show measured raw data in Figure S8, Supporting Information.

2.4. Proposal for Arbitrary Linear Optical Transformation

With the proposed universal metasurfaces, one can construct a complete vector linear optical platform that can perform arbitrary vector linear operations on incident beams. It was previously shown that arbitrary scalar (i.e., polarization-insensitive) linear optical transformation can be constructed using only diffractive optical components and lenses,[15,21] Mathematically, this is equivalent to the proof of decomposability of an arbitrary matrix representing a linear operator \( L : X \rightarrow X' \) into diagonal matrices (\( D \)) and discrete Fourier transform matrices (\( F \)), where \( X \) is a scalar field vector space. We expand this result and prove that an arbitrary passive vector linear optical transformation (an arbitrary passive matrix for \( L \otimes P \rightarrow (X \otimes P)' \), where \( P \) is the polarization state vector space) can be constructed using only universal metasurfaces (\( D^{(2)} \); block diagonal matrices with 2-by-2 sized blocks representing the Jones matrix at each spatial position) and conventional lenses (\( F^{(2)} = F \otimes [1,0; 0,1] \)) (Text S2, Supporting Information).

As a specific example of such vectorial linear optical devices, we propose a probabilistic linear quantum optical CNOT gate composed of three universal metasurfaces and two lenses (Figure 5a). The metasurface-based platform can be parallelized in 2D arrays (Figure 5b) and also enables manipulation of transverse modes, which is challenging with photonic integrated circuits. A transmission matrix of the CNOT gate

---

**Figure 4.** Experimental demonstrations of two independent 3D vector field profiles. a) Two solutions of Maxwell’s equation in a source-free volumetric region outside the enclosed surface, \( S \). b) Two electromagnetic field profiles outside \( S \) remain unchanged by realizing the same tangential electric fields at \( S \), which can be done by a universal metasurface under illuminations of two plane waves with different polarization states. c) Schematic of a representative demonstration. Line segments and ellipses represent local polarization profiles. d) Two measured independent 3D vector field profiles. The top and bottom panels of each input state are profiles measured at different transverse planes, \( z_1 = 480 \ \mu m \) and \( z_2 = 960 \ \mu m \), respectively.
in ref. [17] represented in $X \otimes P$ basis can be decomposed as follows:

$$
\frac{1}{\sqrt{3}} \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 1 & 1 \\
0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1
\end{pmatrix} = D^{(2)} F^{(2)} D^{(1)} F^{(3)} D^{(1)}
$$

(5)

and we design a metasurface-based CNOT gate using this factorization and assume ideal universal metasurfaces (Experimental Section). The designed CNOT gate has additional controllability on the orbital angular momentum (OAM) of photons\(^{[22]}\) compared to that of the previous implementation in ref. [17], and this enables rotation-tolerant qubit encoding.\(^{[3a]}\)

We estimate the performance of the CNOT gate with Fourier optics analysis (Figure 5c–f) (Experimental Section, more details about the designed metasurfaces are described in Figure S9, Supporting Information). The calculated input–output field profiles (Figure 5c) and the resultant transmission matrix (Figure 5d) confirm the intended behavior of the device. The calculated transmission matrix is very similar to Equation (5), although the overall magnitude decreases by a factor of 1.42, mainly due to the discrepancy between the mathematical discrete Fourier transform and the optical Fourier transform performed by conventional lenses (Experimental Section). The calculated truth table also verifies proper CNOT gate functionality (Figure 5e). The density matrix of the output state for the input state $(|0\rangle - |1\rangle)/\sqrt{2}$ is shown in Figure 5f, and the fidelity with state $(|01\rangle - |10\rangle)/\sqrt{2}$ becomes close to 100%, confirming the ability of the proposed CNOT gate to create an entangled pair.

3. Conclusion

We have achieved complete pixel-wise control of transmitted coherent light by devising a systematic way to realize the most general chiral and anisotropic metasurfaces that can implement arbitrary passive Jones matrices in a pixel-wise manner. First, we theoretically proved that a bilayer array of nanostructures is sufficient to cover the full set of passive Jones matrices. The extended degree of freedom of Jones matrices that universal metasurfaces provide makes possible unprecedented optical functionalities. We presented examples of new types of holographic imaging schemes, each of which breaks limitations of holographic images generated by conventional metasurfaces especially for the independent controllability of output polarizations, amplitudes and phases for two different input polarization states. By fabricating bilayer arrays of silicon posts, we demonstrated that all three aspects—polarization, amplitudes and phases—of two different 3D light field distributions are independently controllable as long as they are legitimate solutions of Maxwell's equations in a source-free half-space. In addition, we proposed a new optical platform composed of universal metasurfaces and lenses that can realize arbitrary passive vector linear optical devices, including an array of probabilistic linear quantum optical CNOT gates.

We note that the performance of devices based on universal metasurface can be further enhanced with optimization. We
utilized a direct design method, where the proper structural parameters can be derived from analytic expressions based on the desired Jones matrix at each region of the metasurface. This method is conceptually clear and computationally very efficient. However, one can also utilize numerical optimizations considering near-field coupling between different unit structures and its effect on the transmitted amplitude and phase for each polarization. While doing so globally over the entire metasurface area is computationally very intensive, there are recent proposals toward fast optimization methods that are potentially applicable to large-area metasurfaces, such as gradient-based optimization\cite{23} or machine learning-based approaches.\cite{22,23}

In addition, we would like to emphasize that the universal metasurfaces presented here provide full controllability for both input polarization channels for the first time, compared to existing metasurfaces giving full controllability only for one of the two possible input polarization channels. This could lead to a new way of controlling 3D light fields in real time. Overall, we expect that the fully extended set of optical properties provided by the universal metasurfaces suggested here may find uses in many different fields.

4. Experimental Section

Fabrication of Universal (Including Unitary) Metasurfaces: Amorphous silicon (a-Si) thin film (thickness of 790 nm) was deposited on the fused silica substrate with plasma-enhanced chemical vapor deposition (PECVD) for the lower metasurface layer. In order to do aligned electron-beam (e-beam) lithography for a lower and an upper layer, an align-key was fabricated on the a-Si layer using additional e-beam lithography followed by metal deposition using e-beam evaporation (20 nm of Cr followed by 50 nm of Au) and lift-off. The lower layer of a cylinder array (square lattice of 450 nm period) was defined by aligned e-beam lithography using an align-key. Alumina hard mask (70 nm thickness) was realized with e-beam evaporation deposition and lift-off. Pseudo-Bosch dry etching process was utilized to realize the vertical side-wall slope of nano-pillar structures. The remaining alumina hard mask was utilized as a part of cylinders. The entire lower layer was encapsulated and planarized by 1.3 um thickness of SU-8 polymer by spin coating and hardening. Additional a-Si layer thin film (thickness of 650 nm) was deposited with radio-frequency (RF) sputtering deposition for the upper metasurface layer. The upper metasurface layer was fabricated as a similar process for the lower layer.

Finite-Difference Time-Domain (FDTD) Simulation of Unitary Metasurface to Optimize Geometric Parameters: A bilayer of ideal elliptic cylinder arrays (schematics in Figure 1b) composed of realistic materials was assumed for a unitary metasurface in FDTD simulations. Throughout this study, commercial software from Ansys Lumerical Inc. was utilized for FDTD simulations. Based on the ellipsometry measurement, a complex refractive index of \(n_{\text{Si}} = 3.43 + 0.01i\) where \(n_{\text{Si}}\) and \(k_{\text{Si}}\) are the real and imaginary index of crystalline silicon was assumed for PECVD a-Si, and complex refractive index of 3.7 + 0.05i was assumed for RF sputtered a-Si at 915 nm wavelength. A refractive index of 1.56 was assumed for the hardened SU-8 polymer. All electromagnetic field data were obtained for 915 nm wavelength. First, a square lattice with 450 nm x 450 nm period, 790 nm thickness for layer A, and 650 nm thickness for layer B were chosen for best performance based on the FDTD simulation results. Next, optimised pairs of \(I_{\text{AM}}\) and \(I_{\text{BW}}\) were found which realize every 36 combinations of \(\phi_{\text{AM}} = \{-9\pi/12, -5\pi/12, -\pi/12, 3\pi/12, 7\pi/12, 11\pi/12\}\) and \(\phi_{\text{BW}} = \{-10\pi/12, -6\pi/12, -2\pi/12, 2\pi/12, 6\pi/12, 10\pi/12\}\) for the 790 nm-thick for PECVD a-Si elliptic cylinders by sweeping fine grid of all \(I_{\text{AM}}\) and \(I_{\text{BW}}\) combinations in the FDTD simulations of SU-8-encapsulated layer A. Similarly, an optimized pair of \(I_{\text{AM}}\) and \(I_{\text{BW}}\) were found which realize \(\phi_{\text{AM}} = 0\) and \(\phi_{\text{BW}} = -\pi\) for the 650 nm-thick RF sputtered a-Si elliptic cylinders (global phase was omitted for both layer A and layer B).

Comparison between Target Optical Properties and FDTD Simulation Results for Unitary Metasurfaces in Figure 2a,b: For the FDTD simulation data in Figure 2, both elliptic cylinders in layer A and layer B were assumed to be composed of PECVD a-Si in order to investigate for an achievable performance limit for the unitary metasurfaces (RF sputtered a-Si was more absorptive than PECVD a-Si). Based on the analytic solution in Text S1, Supporting Information, 36 unitary metasurfaces (each composed of unit structures with an identical shape, i.e., spatially homogeneous) were designed to have the same polarization conversion but to have all different accumulated phase pairs. Designed output polarization states were \([\ell_{\text{1}}]\) and \([\ell_{\text{2}}]\) under the incidence of right- and left-handed circular polarization states (RCP and LCP), respectively, where the spherical coordinate of \([\ell_{\text{1}}]\) on the Poincaré sphere was \((2\mu, 2\eta) = (2\pi/3, -\pi/6)\).

Comparison between Target Jones Matrices and FDTD Simulations Results for Universal Metasurfaces in Figure 2c,d: Similar to the FDTD simulations for Figure 2a,b, both elliptic cylinders in layer A and layer B were assumed to be composed of PECVD a-Si. A 15-by-15 cluster of unitary metasurface unit structures (in length in length) as one part of unit structure of universal metasurface was utilized to minimize near-field effects in this simulation. As stated in the main text, clusters size can be reduced without deterioration of performance by another optimization for shapes of elliptic cylinders to compensate for near-field effects, which is not in the scope of this study. Since an arbitrary Jones matrix could be realized whose maximum singular value was less than 0.85 with a-Si in the FDTD simulations as stated in the Text S1, Supporting Information, this constraint was applied when random and modified target Jones matrices were chosen.

Generation of Two Orthogonal Poincaré Beam with Independent Phase Profiles Shown in Figure 3b: Spatially varying polarization states of two Poincaré beams were designed based on ref. \cite{18} for RCP and LCP input. Phase profiles were designed such that one output beam had a divergence angle of 1.4° and deflection angle of 5° toward the +y direction, and another output beam had a deflection angle of 0.7° and deflection angle of 10° toward the +x direction. The input Gaussian beams were focused at the metasurface where the beam waist was \(\approx 37.5 \mu m\) for both RCP and LCP incidence. The output beam profiles were measured at the far-field at a finite distance using a convex lens. Spatially varying polarization states of Poincaré beams were quantitatively retrieved with standard full-Stokes polarimetry as described in Figure S6, Supporting Information.

Generation of Two Independent Scalar (i.e., Homogeneous Polarization State Over the Entire Image Area) Holographic Images Shown in Figure 3c: For each target far-field image, spatially varying output phase profiles \(\exp(i\phi[n])\) using Gerchberg–Saxton (GS) algorithm were found for \(\phi[n] = \pi n, n = 1, 2\) for each image. Spatially identical output polarization state under RCP input was set to \([\ell_{\text{1}}]\) whose spherical coordinate on the Poincare sphere was \(2\mu = \pi/6\) and \(2\eta = \pi/12\). Then, spatially varying unitary Jones matrices for two independent scalar holographic images as \(U(r) = \begin{bmatrix} [\ell_{\text{1}}] \mid [\ell_{\text{2}}] \\ \exp[i\phi[1](r)] \mid 0 \\ 0 \mid \exp[i\phi[2](r)] \end{bmatrix} \) were constructed, which generated scalar holographic images of a rose and butterfly under RCP and LCP incidence, respectively. The input Gaussian beams were focused at the metasurface where the beam waist was \(\approx 300 \mu m\) for both RCP and LCP incidence. The output scalar holographic images were measured at the far-field at a finite distance using a convex lens. Output polarization states were quantitatively retrieved with standard full-Stokes polarimetry as described in Figure S6, Supporting Information.

Generation of Two Independent Vectorial Holographic Images Shown in Figure 3d: For each target far-field image, using the modified GS algorithm based on ref. \cite{19a}, spatially varying output polarization states and phases at the metasurface plane were found, \(E[n], E[\phi[n]], C[n], C[\phi[n]], \) where \(E[n], E[\phi[n]], C[n], C[\phi[n]], \) are \(n = 1, 2\) for each image, and...
as $J(r) = \frac{1}{\sqrt{2}} \left[ \begin{array}{c} E_I^\| (r) \exp[i\psi_I^\| (r)] \\ E_I^\perp (r) \exp[i\psi_I^\perp (r)] \\ 0 \\ \exp[i\psi_I^\perp (r)] \end{array} \right] \times \left( \begin{array}{c} 1 \\ 1 - i \\ i \\ 1 + i \end{array} \right)$ were constructed where the normalization factor $J_0$ is a maximum singular value of $J(r)$ over the entire $r$. A universal metasurface realizing this $J(r)$ generated vectorial holographic images of the rose and butterfly under RCP and LCP incidence, respectively. That singular values of Jones matrices at various spatial positions on the metasurface might vary was noted because an inner product of two polarization states was not necessarily the same for various spatial positions.

The input Gaussian beams were focused at the metasurface where the beam waist was $\approx 300 \mu m$ for both RCP and LCP incidence. The output vectorial holographic images were measured at the far-field at a finite distance using a convex lens. Spatially varying polarization states were quantitatively retrieved with standard full-Stokes polarimetry as described in Figure S5, Supporting Information.

Generation of Two Independent 3D Vector Field Profiles Shown in Figure 4d: For each target 3D vector field profile, spatially varying output polarization states, amplitude and phases were found by adding complex conjugation of electric field pattern radiation from all Gaussian beams with desired polarization states whose waist $(2-5 \mu m)$ were at the desired intensity hotspots. The input Gaussian beams were slightly focused at the metasurface, where the beam waist was $\approx 1 \mu m$ for both RCP and LCP incidence. The output 3D vector field profiles were measured with an imaging system composed of an objective lens and a tube lens. Images at different transverse planes were obtained by varying the position of the metasurface along the optic axis of the imaging system. Spatially varying polarization states of 3D vector field profiles were quantitatively retrieved with standard full-Stokes polarimetry as described in Figure S6, Supporting Information.

Design of a Metasurface-Based Probabilistic Linear Optical Controlled-NOT (CNOT) Gate and Fourier Optic Calculation for Figure 5c: A transmission matrix of a CNOT gate in ref. [17] represented in $\mathbb{X} \otimes \mathbb{P}$ basis can be factorized in the form of Equation (5) using a nonlinear least-square fitting algorithm provided by commercial software as follows:

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} = D^{(3)} F^{(1)} F^{(2)} T$$

where $a_I$ to $a_3$ are $0.8095 + 0.5405i, 0.6208 + 0.7839i, -0.5724 - 0.3822i, 0.5543 - 0.4390i, 0.2336 - 0.3628i, 0.9271 + 0.3749i$, and $-0.6625 + 0.7491i$, respectively. Singular values of all 2-by-2 blocks (Jones matrices) were equal or less than 1, which means the block diagonal matrices could be realized with universal metasurfaces with arbitrary passive Jones matrices. However, the retrieved transmission matrix of a designed optical system based on the Equation (6) using Fourier optic calculation (described at the end of this section) became

$$\begin{pmatrix} 0.5733 & 0 & 0 & 0 \\ 0 & -0.5732 & 0.3653 & 0.3653 \\ 0 & 0.3653 & 0.5733 & 0 \\ 0 & 0.3653 & 0 & 0.5733 \end{pmatrix}$$

(smaller than 1/10000), which was significantly different from the target transmission matrix. This difference was due to the discrepancy between the mathematical discrete Fourier transform with $F^{(1)}$ and the optical Fourier transform with a lens. While the mathematically discrete Fourier transform with $F^{(1)}$ inherently assumed transversal periodic copies of input states in the spatial domain, there was no signal except around the optic axis for an optical system. With the trial and error scheme, the following factorization was considered:

$$\begin{pmatrix} 0.7099 & 0 & 0 & 0 \\ 0 & -0.7099 & 1.115 & 1.115 \\ 0 & 1.115 & 0 & 0.7099 \\ 0 & 0.7099 & 0 & 0 \end{pmatrix} = D^{(3)} F^{(1)} D^{(2)} F^{(2)} T$$

smaller than 1/10000, which was the desired value, although the overall magnitude was decreased by a factor of 1.42.

In order to achieve rotation-tolerant qubit encoding, the basis rather than the $\mathbb{X} \otimes \mathbb{P}$ basis where $|l \rangle$ and $|r \rangle$ are Laguerre–Gaussian modes of zero radial index with $-1$ and $+1$ azimuthal index, respectively, were considered. By merging 4-plates (ref. [22]) to both endmost universal metasurfaces, one channel of the proposed CNOT gate array did the same role in ref. [17] for the $\mathbb{X} \otimes \mathbb{P}$ basis. That this qubit encoding did not increase system complexity because 4-plate merged universal metasurfaces, $D^{(3)} = T D^{(4)}$, were still single universal metasurfaces where $T$ is a transmission matrix of a 4-plate, was noted. The $T$ designed to make the orbital angular momentum (OAM) of all states within the CNOT gate to be zero. The angular momentums were restored at the exit of the CNOT gate because of $T$. Ideal universal metasurfaces were designed, that is, spatially varying arbitrary passive Jones matrices, and an overall optical system including lenses considering target maximum numerical aperture. The maximum numerical aperture was set to be a low enough value, 0.08, in order to allow a larger universal meta-atom size that was easier to optimize and experimentally realize in future works. Accordingly, target beam waist, a single port size (Figure 5a) and lens focal length to be 9 mm as in Figure 5a. Designed metasurfaces are represented in Figure S5, Supporting Information.

The output field profile for various input states represented in Figure 5c using Fourier optics considering polarization states, that is, Jones vectors were calculated. For a given input field profile (spatially varying Jones vectors), the field profiles right before the first lens by multiplying designed Jones matrices of the frontal universal metasurface ($D^{(3)}$) were first calculated followed by free-space propagation to the first lens. After applying the phase profile of the first lens, another free-space propagation to the next metasurface was considered. A similar process could be done until the field profile after the last metasurface were obtained. Large enough zero-padding for the field profiles were considered, which determined enough transversal device area to avoid crosstalk in case of integration (Figure 5b).
Retrieval of the Transmission Matrix, Truth Table, and the Density Matrix of a Highly Entangled State for the Proposed CNOT Gate (Figure 5d–f): In order to retrieve the transmission matrix in the $XO\otimes(|R\rangle\otimes|L\rangle\otimes|\theta\rangle\otimes|\phi\rangle)$ basis, each element of the transmission matrix with mode overlap integral, $t_j = \int E_j(|r\rangle)^\ast(E_j(|l\rangle)) \, dx \, dy$, where $E_j$ and $E_l$ are basis mode field profiles and $E_j(|r\rangle)$ is an output field profile under $E_j$ incidence based on Fourier optics calculations, were calculated, which are shown in Figure 5c. Output port number was flipped in the $x$-direction, as shown in Figure 5a, due to the intrinsic image inversion property of the 4f system. This result is shown in Figure 5d.

As long as transmission matrix and logical qubit basis are known to be given, truth table and output qubit states (and density matrices) can be calculated, as in an example in ref. [1b]. Rotation-tolerant logical qubit encoding was set such that single-photon states in port 1 and port 2 as a control and a target qubit, respectively: $|R\rangle\otimes|L\rangle\otimes|\theta\rangle\otimes|\phi\rangle$ state as $|0\rangle$ and $|1\rangle$, respectively, as shown in Figure 5c.

First, the truth table was retrieved based on the transmission matrix. The relationship between creation operators for the input and output state becomes:

\[
a_{in}^\dagger \rightarrow t_1 a_{out} + t_2 b_{out} + t_3 c_{out} + t_4 d_{out} \\
b_{in}^\dagger \rightarrow t_1 a_{out} + t_2 b_{out} + t_3 c_{out} + t_4 d_{out} \\
c_{in}^\dagger \rightarrow t_1 a_{out} + t_2 b_{out} + t_3 c_{out} + t_4 d_{out} \\
d_{in}^\dagger \rightarrow t_1 a_{out} + t_2 b_{out} + t_3 c_{out} + t_4 d_{out}
\]

where $a_{in}^\dagger$, $b_{in}^\dagger$, $c_{in}^\dagger$, and $d_{in}^\dagger$ are creation operators for single-photon input (output) states (1, 0, 0, 0)$^\dagger$, (0, 1, 0, 0)$^\dagger$, (0, 0, 1, 0)$^\dagger$, and (0, 0, 0, 1)$^\dagger$ in $XO\otimes|\theta\rangle\otimes|\phi\rangle$ basis, respectively. Output qubit state under input qubit state of $|pq\rangle = |p\rangle c |q\rangle\overline{|q\rangle}^\dagger$, $p, q = 0 \text{ or } 1$, can be calculated with Equation (8). For example, an output qubit state under the input state of $|00\rangle_\text{in}$ becomes $(t_1 a_{out} + t_2 b_{out} + t_3 c_{out} + t_4 d_{out}) (t_1 a_{out} + t_2 b_{out} + t_3 c_{out} + t_4 d_{out})$. Among 16 terms, eight terms representing single-photon outcomes per each port (control and target) were considered successful operations, which becomes $(t_1 t_2 + t_1 t_3 |00\rangle_{out} + t_2 t_4 + t_3 t_4 |01\rangle_{out} + t_2 t_3 |10\rangle_{out} + t_1 t_4 |11\rangle_{out})_{out}$. Therefore, the complex amplitude of a truth table becomes:

\[
C_{TT} = \begin{pmatrix}
 t_1 t_2 + t_1 t_3 & t_1 t_4 + t_2 t_3 & t_2 t_4 & t_3 t_4 \\
t_1 t_2 + t_1 t_3 & t_1 t_4 + t_2 t_3 & t_2 t_4 & t_3 t_4 \\
t_1 t_2 + t_1 t_3 & t_1 t_4 + t_2 t_3 & t_2 t_4 & t_3 t_4 \\
t_1 t_2 + t_1 t_3 & t_1 t_4 + t_2 t_3 & t_2 t_4 & t_3 t_4
\end{pmatrix}
\]

The probability of a truth table could be calculated by the element-wise square of each element’s magnitude. Each column with its magnitude in order could be normalized to obtain conditional probability upon success. This conditional probability of truth table is represented in Figure 5e. For the proposed CNOT gate, the probability of success (normalization value of each column) was $\frac{1}{2} \times \frac{1}{\sqrt{2}}$ (same for all columns), where $\frac{1}{\sqrt{2}}$ was the fundamental probability limit.

Since complex amplitudes of input–output qubit states transformations (i.e., complex amplitudes of a truth table) were known, output states $|\psi\rangle$ as well as density matrix $|\psi\rangle\langle\psi|$ using this relation could be directly calculated. As an example in Figure 5f, the output state under the input state $(|0\rangle - |1\rangle)|0\rangle \sqrt{2}$ could be calculated as $|\psi\rangle = C_{TT} (|0\rangle - |1\rangle)|0\rangle \sqrt{2}$ and could be normalized with its magnitude. The result density matrix $|\psi\rangle\langle\psi|$ is shown in Figure 5f, and fidelity with $(|0\rangle - |1\rangle)|0\rangle \sqrt{2}$ can be calculated as $\frac{1}{2} \sqrt{1 - 10|1\rangle |0\rangle \langle0| \langle1|}$, which gives almost 100% as noted in the main text.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

Acknowledgements

The authors thank Michael H. Hwang (SEMCRON) for the highly reliable electron beam lithography. This work is supported by National Research Foundation of Korea (NRF) grants funded by the Korea government (MSIT) (NRF-2018M3D1A1058998, NRF-2021R1A2C2008687, NRF-2021M3H4A1A04086555), and a KAIST Grand Challenge 30 Project (KC30) grant funded by KAIST (N12120118).

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

T.C. and J.J. contributed equally to this work. T.C. conceived the idea and J.S. supervised the project. T.C., J.J., S.N., S.J., and M.H. conducted the theoretical analyses. J.J. performed the numerical simulation. T.J., J.J., and U.K. fabricated the samples. T.C., J.J., and H.K. characterized them optically. J.J., T.C., and J.S. prepared the manuscript.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Keywords

Jones matrices, metasurface-based vector linear optics, polarization-multiplexed vectorial holographies, unitary metasurfaces, universal meta-surfaces

Received: May 5, 2022
Revised: August 15, 2022
Published online: October 3, 2022

[1] a) R. Simon, N. Mukunda, Phys. Lett. A 1990, 143, 165; b) P. Kok, W. J. Munro, K. Nemoto, T. C. Ralph, J. P. Dowling, G. J. Milburn, Rev. Mod. Phys. 2007, 79, 135.
[2] a) N. A. Rubin, G. D'Aversa, P. Chevalier, Z. Shi, W. T. Chen, F. Capasso, Science 2019, 365, eaax1839; b) H. Kwon, E. Arabi, S. M. Kamali, M. Faraji-Dana, A. Farao, Nat. Photonics 2020, 14, 109.
[3] a) V. D'Ambrosio, E. Nagali, S. P. Walborn, L. Aolita, S. Slussarenko, L. Marrucci, F. Sciarrino, Nat. Commun. 2012, 3, 961; b) N. Bozinovic, Y. Yue, Y. Ren, M. Tur, P. Kristensen, H. Huang, A. E. Willner, S. Ramachandran, Science 2013, 340, 1545.
[4] a) R. C. Devlin, A. Ambrosio, N. A. Rubin, J. P. B. Mueller, F. Capasso, Science 2017, 358, 896; b) T. Stav, A. Faerman, E. Maguid, D. Oren, V. Kleiner, E. Hasman, M. Segev, Science 2018, 361, 1101.
[5] a) C. Menzel, C. Rockstuhl, F. Lederer, Phys. Rev. A 2010, 82, 053811; b) N. A. Rubin, A. Zaidi, A. H. Dorrah, Z. Shi, F. Capasso, Sci. Adv. 2021, 7, eabg7488.
[6] a) E. Plum, V. A. Fedotov, N. I. Zheludev, Appl. Phys. Lett. 2009, 94, 131901; b) N. Yu, P. Genevet, M. A. Kats, F. Aieta, J.-P. Tetienne, F. Capasso, Z. Gaburro, Science 2011, 334, 333; c) L. Huang, X. Chen, H. Mühlenbernd, H. Zhang, S. Chen, B. Bai, Q. Tan, G. Jin, K.-W. Cheah, C.-W. Qiu, J. Li, T. Tzentgraf, S. Zhang, Nat. Commun. 2013, 4, 2808; d) A. Arbabi, Y. Horie, M. Bagheri, A. Faraon, Nanotechnology 2015, 10, 937; e) J. P. Balthasar Mueller, N. A. Rubin, R. C. Devlin, B. Groever, F. Capasso, Phys. Rev. Lett. 2017, 118, 113901; f) Z.-L. Deng, J. Deng, X. Zhuang, S. Wang, K. Li, Y. Wang, Y. Chi, X. Ye, J. Xu, G. P. Wang, R. Zhao, X. Wang, Y. Cao, X. Cheng, G. Li, X. Li, Nano Lett. 2018, 18, 2885; g) Q. Song, A. Baroni, P. C. Wu, S. Chenot, V. Brandli, S. Vézian, B. Damilano, P. de Miery, S. Khadir, P. Ferrand, P. Genevet, Nat. Commun. 2021, 12, 3631; h) J. K. Gansel, M. Thiel, M. S. Rill, M. Decker, K. Bade, V. Saile, G. von Freymann, S. Linden, M. Wegener, Sci. Rep. 2018, 8, 12324; i) A. H. Dorrah, N. A. Rubin, A. Zaidi, M. Tamagnone, F. Capasso, Adv. Photon. 2021, 12, 1513.

[7] J. K. Gansel, M. Thiel, M. S. Rill, M. Decker, K. Bade, V. Saile, G. von Freymann, S. Linden, M. Wegener, Science 2009, 325, 1513.

[8] a) M. Decker, M. Ruther, C. E. Kriegler, J. Zhou, C. M. Soukoulis, S. Linden, M. Wegener, Opt. Lett. 2009, 34, 2501; b) K. Tanaka, D. Arslan, S. Fasold, M. Steinert, J. Sautter, M. Falkner, T. Pertsch, M. Decker, I. Staudte, ACS Nano 2020, 14, 15926; c) N. Liu, H. Liu, S. Zhu, H. Giessen, Nanophotonics 2009, 3, 157; d) C. Menzel, C. Helgert, C. Rockstuhl, E. B. Kley, A. Tünnemann, T. Pertsch, F. Lederer, Phys. Rev. Lett. 2010, 104, 253902; e) Y. Zhao, M. A. Belkin, A. Alù, Nat. Commun. 2013, 4, 870; f) Y.-W. Huang, N. A. Rubin, A. Ambrosio, Z. Shi, R. C. Devlin, C.-W. Qiu, F. Capasso, Opt. Express 2019, 27, 7469; g) A. Overvig, N. Yu, A. Alù, Phys. Rev. Lett. 2021, 126, 073001; h) A. Overvig, A. Alù, Adv. Photonics 2021, 3, 026002.

[9] a) E. Plum, V. A. Fedotov, N. I. Zheludev, Appl. Phys. Lett. 2008, 93, 191911; b) Z. Shi, A. Y. Zhu, Z. Li, Y.-W. Huang, W. T. Chen, C.-W. Qiu, F. Capasso, Sci. Adv. 2020, 6, eaba3367; c) J. Zhang, J. A. Fan, Nano Lett. 2019, 19, 5366.