Current-induced domain wall motion in a nanowire with perpendicular magnetic anisotropy

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Abstract

We study theoretically the current-induced magnetic domain wall motion in a metallic nanowire with perpendicular magnetic anisotropy. The anisotropy can reduce the critical current density of the domain wall motion. We explain the reduction mechanism and identify the maximal reduction conditions. This result facilitates both fundamental studies and device applications of the current-induced domain wall motion.

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Spin-polarized electrical currents in ferromagnets can transfer their spin angular momentum to local magnetizations via the s-d exchange interaction and generate torques on local magnetizations. This spin transfer torque (STT) received considerable attention in view of both fundamental physics research and applications.

In a ferromagnetic nanowire, the STT can generate motion of magnetic domain walls (DWs). For conventional metallic ferromagnetic nanowires, which have the in-plane magnetic anisotropy (IMA), experiments found such current-induced DW motion when the current density $J$ in the nanowire is larger than a certain threshold value $J_c$ of the order of $10^8$ A/cm$^2$. This value is too high; at such current densities, the Joule heating generates considerable thermal fluctuations, making fundamental studies of the STT difficult. Furthermore, device applications require $J_c < 10^7$ A/cm$^2$ at room temperature. Thus both for fundamental studies and device applications, it is crucial to reduce $J_c$.

Recently, there are experimental and theoretical indications that $J_c$ may be considerably lower in a metallic nanowire with the perpendicular magnetic anisotropy (PMA). However, it remains unclear how the PMA can lower $J_c$. We aim to answer this question in this Letter.

We consider a nanowire with the wire width $w$ along the y-axis and thickness $t$ along the z-axis ($w > t$). We use the Landau-Lifshitz-Gilbert (LLG) equation with the STT term,

$$\dot{m} = -\gamma m \times H_{\text{eff}} + \alpha m \times \dot{m} + b_J(\hat{J} \cdot \nabla) m - c_J m \times (\hat{J} \cdot \nabla)m,$$

where $m$ is the unit vector of the local magnetization, $\gamma$ is the gyromagnetic ratio, $\alpha$ is the Gilbert damping parameter, $\hat{J}$ is the unit vector of the local current density, and $H_{\text{eff}}$ is the effective magnetic field. $b_J = P \mu_B J/eM_s$ is the magnitude of the adiabatic STT, where $e$ is the electron charge, $P$ is the spin-polarization of the ferromagnet, $\mu_B$ is the Bohr magneton, and $M_s$ is the saturation magnetization. $c_J$ is the magnitude of the nonadiabatic STT with the non-adiabaticity represented by the dimensionless parameter $\beta \equiv c_J/b_J$. $\beta$ is independent of $J$ and estimated to be of the order of $10^{-2}$.

To get an insight into the main physics of the PMA, we first develop an analytical model based on a one-dimensional (1D) approximation. Its results will be later verified by performing the micromagnetic simulations of the LLG equation [Eq. (1)], which are known
to provide a reliable description of nanoscale magnetization dynamics \[19, 20\].

For a ferromagnet with the PMA we have

\[
H_{\text{eff}} = \frac{2A}{M_s} \partial^2 m \partial x^2 + \frac{2K_U M_s}{M_s} \partial z + H_{\text{dipole}},
\]

where \(A\) is the exchange stiffness constant and \(K_U\) is the PMA constant that allows the easy axis (along the \(z\)-axis) to be perpendicular to the wire-plane (\(x-y\) plane). To describe the demagnetization effects, we consider the magnetostatic dipole-dipole interaction field given by

\[
H_{\text{dipole}}(r) = M_s \int d^3 r' N(r - r') m(r'),
\]

where the components of the matrix \(N\) are given by

\[
N_{xx}(r) = -\frac{1 - 3x^2/|r|^2}{|r|^3}, \quad N_{xy}(r) = \frac{3xy/|r|^5}.
\]

Other components of \(N\) are defined in a similar way.

We also assume that the DW maintains the following shape during the DW motion;

\[
m_z(r,t) = \tanh[(x - q)/\lambda], \quad m_x(r,t) = \cos \psi \sech[(x - q)/\lambda], \quad m_y(r,t) = \sin \psi \sech[(x - q)/\lambda],
\]

where \(\lambda\) is the equilibrium DW width obtained from 1D micromagnetic simulations. In this rigid DW motion approximation \[21\] the DW dynamics is described by two dynamical variables, the DW position \(q(t)\) and the DW tilting angle \(\psi(t)\).

By using the procedure developed by Thiele \[22\], one can then derive, from the LLG equation, the equations of motion for the two collective coordinates \(q\) and \(\psi\),

\[
\lambda \dot{\psi} - \alpha \dot{q} = c_J - (\gamma \lambda/2M_s) f_{\text{pin}},
\]

\[
\dot{q} + \alpha \lambda \dot{\psi} = -b_J - (\gamma \lambda/M_s) K_d \sin 2\psi,
\]

where the pinning force \(f_{\text{pin}}\) is related to the DW energy per unit cross-sectional area \(u_{\text{tot}}\) \((f_{\text{pin}} = -\partial u_{\text{tot}}/\partial q)\) representing pinning potential in the presence of extrinsic defects in a nanowire. Here \(K_d\) is the effective wall anisotropy given by

\[
K_d = K_y - K_x,
\]

where \(K_i = -\frac{M_s^2}{4\pi S} \int d^3 r d^3 r' N_{ii}(r - r') \sech^2 \frac{x}{\lambda} \sech^2 \frac{x'}{\lambda} (i = x, y, z)\) and \(S\) is the cross-sectional area. \(K_d\) represents the magnetostatic energy difference between two types of transverse DWs, the Bloch DW (\(m \parallel e_y\) at the DW center) and the Neel DW (\(m \parallel e_x\) at the DW center).

Before we demonstrate its implications for a general case, we first consider a defect-free nanowire \((f_{\text{pin}} = 0)\) with \(c_J = 0\). \(J_c\) in this case is given \[5\] by \(J_{c}^{\text{in}}\)

\[
J_{c}^{\text{in}} = \frac{e\gamma \lambda}{\mu_B |K_d|}.
\]
Figure 1(a) shows \( w \) and \( t \) dependence of \( J_c^{\text{in}} \). Note that \( J_c \) falls below \( 10^7 \) A/cm\(^2\) in a wide range of \( w \) and \( t \). Since all material parameters used in Fig. 1(a) are similar to those for permalloy except for the PMA constant \( K_U \), this reduction in \( J_c^{\text{in}} \) should be attributed to the PMA. To check the validity of this prediction, we also perform micromagnetic simulations of the LLG equation and excellent agreement is found [Fig. 1(b) upper panel].

This reduction in \( J_c^{\text{in}} \) becomes especially effective when \( w \) is tuned to a \( t \)-dependent special value \( w^*(t) \), at which \( K_d \) reverses its sign [Fig. 1(b) lower panel] and near \( w^*(t) \), \( J_c^{\text{in}} \) \((\propto |K_d|)\) is strongly suppressed. The sign reversal of \( K_d \) implies that \( w^* \) is the equilibrium phase boundary between the Bloch DW and Neel DW. For transverse DWs in an IMA nanowire, on the other hand, \( K_d \) is given by \( K_z - K_y \) and since \( K_z \) is always larger than \( K_y \), \( K_d \) in the IMA case is always positive in a conventional nanowire geometry with \( w > t \). This difference between a PMA nanowire and an IMA nanowire illustrates a crucial role played by the PMA.

Next we consider a general case with \( f_{\text{pin}} \neq 0 \) and \( c_J \neq 0 \). After some calculation, one can obtain an upper bound \( J_c^{\text{up}} \) of \( J_c^{\text{in}} \) \[23\],

\[
J_c^{\text{up}} \equiv \max \left[ \min(J_c^{\text{in}}, J_c^{\text{ex}}), \alpha \beta J_c^{\text{ex}} \right],
\]

where \( J_c^{\text{ex}} \equiv (\gamma \lambda e/2P\mu_B)(f_{\text{pin}}^{\text{max}}/\beta) \) and \( f_{\text{pin}}^{\text{max}} \) represents the maximum value of \( f_{\text{pin}} \). The dashed line in Fig. 1(c) shows \( J_c^{\text{up}} \) as a function of \( f_{\text{pin}}^{\text{max}} \) for a PMA nanowire. For the case \( J_c^{\text{in}} = 1.6 \times 10^6 \) A/cm\(^2\), Fig. 1(c) also shows \( J_c \) determined from numerical simulations of Eqs. 3 and 11 with the pinning potential energy \( u_{\text{tot}} \) modelled by a finite ranged harmonic potential [Fig. 1(c) inset]. A few remarks are in order. Firstly, both \( J_c \) and \( J_c^{\text{up}} \) exhibit plateaus near \( J_c^{\text{in}} \) in a wide range of \( f_{\text{pin}}^{\text{max}} \). Secondly, \( J_c \) depends on \( \beta \) only in the weak pinning regime \((f_{\text{pin}}^{\text{max}}/2M_s < 1 \) Oe\) and the \( \beta \) dependence essentially disappears in the intermediate (plateau) and strong (above plateau) pinning regimes. This behavior is consistent with the prediction of Eq. 7. Thirdly, a recent experiment \[24\] with a PMA nanowire finds the depinning magnetic field of about 500 Oe for a field-driven DW motion. When this value is used as an estimation of \( f_{\text{pin}}^{\text{max}}/2M_s \), one finds \( J_c^{\text{up}} \sim J_c^{\text{in}} \sim 10^6 \) A/cm\(^2\). Thus Fig. 1(c) demonstrates that the reduction of \( J_c^{\text{in}} \) via the PMA indeed leads to the reduction of \( J_c \). As a comparison, results for an IMA nanowire are also given in Fig. 1(c). Differences from the PMA case are evident.

Next we present micromagnetic simulation results of the LLG equation. Various sources
of $f_{\text{pin}}$ are examined. Figure 2(a) shows $J_c$ obtained from the 1D LLG equation for a situation where the magnitude of the PMA constant $K_U$ fluctuates from its bulk average value $K_{U,0}$ with the maximum deviation given by $V_0$. Note that the result is remarkably similar to that in Fig. 1(c). In good quality PMA samples, $V_0/K_{U,0}$ is reported [25] to be less than 0.1, for which we obtain $J_c \approx 10^6 \text{ A/cm}^2$. Figure 2(b) shows the effect of a notch investigated with the two-dimensional (2D) LLG equation. In a wide range of $w$, $J_c$ falls below $10^7 \text{ A/cm}^2$ despite the notch formation. Note that for $w \geq 80$ nm, $J_c$ decreases as the notch depth $\delta w$ increases. This strange behavior is not due to the locally enhanced current density near the notch, since this effect should be stronger for $w \leq 80$ nm. Instead it is due to the fact that $J_{c_\text{in}}$ is determined by an effective wire width that a DW senses. When $J_c$ is plotted as a function of $w - \delta w/2$, an estimation of the effective width, this strange behavior disappears and $J_c$ is now almost independent of $\delta w$, in agreement with the prediction $J_{c_{\text{up}}} = J_{c_{\text{in}}}$ in the plateau range in Fig. 1(c). Figure 2(c) shows $J_c$ for a PMA nanowire with the edge roughness and with the PMA fluctuations. Although values of $J_c$ are somewhat scattered with the realizations of the randomness, $J_c$ still remains below $10^7 \text{ A/cm}^2$ in a wide range of the average width $w_{\text{ave}}$ [26].

Here we remark that all demonstrations for the reduction of $J_c$ assume the proper tuning of $w$ and $t$ to achieve the reduced $J_{c_{\text{in}}}$. A recent experiment on the PMA nanowire [24] found $J_c = 1.0 \times 10^8 \text{ A/cm}^2$ without such tuning. We suggest that the tuning of $w$ and $t$ can reduce $J_c$. Another experiment [17] found indications of the enhanced STT efficiency in a PMA spin valve. However the measurement was still restricted to the thermally assisted creep regime with extremely low DW velocity (average $v_{\text{dw}} < 10^{-8}$ m/s). According to our calculation (not shown), much higher velocity ($v_{\text{dw}} \sim 10$ m/s) can be achieved at $J \sim 10^7 \text{ A/cm}^2$ if $w$ and $t$ are properly tuned. Finally the report [27] of the reduced $J_c$ in ferromagnetic semiconductors is yet limited to low temperatures ($\sim 100$ K) while the reduction scheme presented in this Letter does not require low temperature operation.

In summary, we have clarified the mechanism by which the PMA can drastically reduce $J_c$. The geometrical tuning is important to maximize the reduction by the PMA. When properly tuned, the dependence of $J_c$ on $\beta$ and the DW pinning force $f_{\text{pin}}$ is very weak. This result solves the large thermal fluctuation problem and also makes feasible nanoscale magnetoelectronic devices [8, 9] based on the current-induced DW motion.

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[26] In Figs. 2(b) and (c), $J_c$ is minimized around $w - \delta w/2$ (or $w_{ave}$) near 72 nm, which is somewhat smaller than the value ($\sim 78$ nm) in Fig. 1(b). We attribute this difference to the pinning-induced DW deformation.
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FIG. 1: (Color online) (a) $J^\text{in}_c$ from Eq. (6) as a function of $w$ and $t$. (b) Upper panel: $J^\text{in}_c$ defined in Eq. (6) vs $J_c$ obtained from micromagnetic simulations of the 1D and 2D LLG equation. Lower panel: $K_d/M_s^2$ as a function of $w$. $t = 10$ nm in both panels. (c) The dashed (dash-dotted) line shows $J^\text{up}_c$ [Eq. (7)] for a PMA (IMA) nanowire with $\beta = \alpha$ and $J^\text{in}_c = 1.6 \times 10^6 (3.15 \times 10^9)$ A/cm$^2$. Symbols shows $J_c$ obtained from numerical simulations of Eqs. (3) and (4). The result is in reasonable agreement with $J^\text{up}_c$ for $\beta = \alpha$. Inset: Spatial profile of $u_{\text{tot}}$ with $q_0 = 3\lambda$. The following parameters are used: $\alpha = 0.02$, and $P = 0.7$, $A = 1.3 \times 10^{-6}$ erg/cm, $K_U = 1.5 \times 10^6 (0 \times 10^6)$ erg/cm$^3$, and $M_s = 400(800)$ emu/cm$^3$ for PMA (IMA) nanowire.
FIG. 2: (Color online) Micromagnetic simulation results of the LLG equation. (a) Effects of the magnitude fluctuation of the PMA constant $K_U$ for $w = 77$ nm and $t = 10$ nm. Upper inset: Spatial profile of $K_U$ with $q_0 = 37.5$ nm. (b) Effects of a notch. Inset: Schematic of a notch. (c) Combined effects of edge roughness (2.5 nm for each edge) and PMA fluctuations (Gaussian magnitude fluctuations of 5% and direction fluctuations of 5° for each cell of size 2.5 nm × 2.5 nm × t nm). For each $w_{\text{ave}}$, three realizations of the randomness are considered (The dotted line is a guideline). $\beta = \alpha$ and $t = 10$ nm. All other parameters are the same as those in Fig. 1.