A pulsational approach to the luminosity of Horizontal Branch stellar structures.

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ABSTRACT

We discuss an alternative approach to constrain the absolute bolometric luminosity of Zero Age Horizontal Branch (ZAHB) structures by using the observational pulsational properties of ab type RR Lyrae stars and theoretical expectations concerning both the relation connecting the pulsational properties of these variables to their evolutionary ones, as luminosity, mass and effective temperature and, also the location in the H-R diagram for the fundamental pulsators instability strip boundaries. Since the main goal of this work is to obtain an evaluation of the ZAHB bolometric luminosity as much as possible independent on stellar evolution theory, we have minimized the use of evolutionary prescriptions, being the only adopted evolutionary input the allowed mass range for fundamental pulsators. Nevertheless, the effects on our final results related to the use of these evolutionary prescriptions have been carefully checked.

In order to test the accuracy of the current framework, we have carefully investigated on the effective temperature scale provided by De Santis (1996) and adopted in the present work. As a result, it has been also found that the pulsational color-effective temperature scale fixed by the De Santis (1996) temperature scale and the relation between intrinsic \((B-V)_0\) color and blue amplitude and metallicity as given by Caputo & De Santis (1992), appears fully consistent with both theoretical color transformations as given by Castelli, Gratton & Kurucz (1997a,b) and Buser & Kurucz (1978) and, semianempirical one as provided by Green (1988). On the contrary, it seems to exist a large discrepancy between this pulsational color-temperature scale and the theoretical one based on the model atmospheres of Kurucz (1992). We have also analized the effects on current analysis due to the adoption of a different effective temperature scale, namely the one provided by Catelan, Sweigart & Borissova (1998).

The reliability of the suggested method to obtain the ZAHB luminosity is shown by applying it to a selected sample of globular clusters (GCs), whose heavy elements abundance covers almost all the complete GCs metallicity range. In order to verify the accuracy of the results obtained by using the fundamental pulsators RR Lyrae stars, a quite similar analysis has been also performed by using both observational and theoretical evidences for first overtone variables.

The results obtained for the ZAHB bolometric luminosities in the adopted sample of clusters have been critically analized and a comparison with evolutionary prescriptions on such quite important quantity as given by recent evolutionary computations has been also performed. The existence of evident mismatches between current results and some evolutionary models has been verified and discussed.

Finally, our investigation on the ZAHB luminosity levels has been extended to field variables, in order to check if it exists a real difference in luminosity between cluster RR Lyrae stars and field ones as suggested by different authors through Hipparcos-based investigations. The comparison between the pulsational properties of field and cluster variables do not show the existence of any significant difference in their intrinsic luminosity, thus providing further support to the results obtained by Catelan (1998).

Key words: stars: distances – stars: evolution – stars: horizontal branch – stars: variables: other – globular clusters: general

1 INTRODUCTION

The distance scale of low-mass, metal-poor stars plays an important role in stellar astrophysics, since the distance evaluations of old stellar systems such as the globular clusters are a fundamental step for providing accurate age determinations (see e.g. the detailed discussion by Renzini 1991) and in turn for providing a firm lower limit to the age of the Universe. The traditional distance ladder for metal-poor populations is the luminosity of Horizontal Branch (HB) stars and in particular of RR Lyrae variables, which are currently considered the natural Population II standard candles.

However, even though large observational and theoretical efforts have been devoted for estimating their ‘true’ luminosity this quantity is still affected by large uncertainties. In particular, as recently suggested by Catelan (1998, and references therein) it seems that the luminosity of HB stars presents a ‘dichotomy’ between a faint and a bright distance scale. The former produces for the galactic GCs short distances and therefore large ages, whereas the latter long distances and small ages. This problem has been widely
discussed in the literature due to the release of the Hipparcos database. In fact, the accurate measurement of stellar parallaxes should allow us to discriminate between the faint and the bright distance scale of RR Lyrae stars.

Unfortunately, this important goal has not been accomplished, since recent investigations on GCs based on Hipparcos parallaxes seem to support both the bright RR Lyrae distance scale (Gratton et al. 1997; Reid 1997, 1998; Chaboyer et al. 1998) and the faint distance scale (Pont et al. 1998). Further support to the the long RR Lyrae distance scale was brought out by recent analysis of different groups of variable stars such as Sx Pheonicis (Mc Namara 1997), Cepheids (Feast & Catchpole 1997, Madore & Freedmann 1997) and Mira (van Leeuwen et al. 1997). Similar conclusions were also reached by new investigations based on the Red Giant Branch Tip (Salaris & Cassisi 1998) and on the SN1987a ring (Panagia 1998; but see also Gould & Uza 1998).

This notwithstanding, accurate analysis of the Hipparcos parallaxes of field RR Lyrae stars support the faint RR Lyrae distance scale (Gratton 1998; Fernley et al. 1998; Tsujimoto, Miyamoto & Yushi 1998). From the above discussion emerges quite clearly that the problem of the luminosity of HB stars has not been properly settled yet. In fact, Gratton (1998) argued that the Hipparcos results could be explained if the luminosity of field and cluster RR Lyrae variables presents an intrinsic difference of the order of 0.2 mag. An accurate investigation of this problem has been recently performed by Catelan (1998), who showed that the pulsational properties of RR Lyrae in the field and in GCs do not support the Gratton’s conclusion concerning the existence of a dichotomy between the two different samples.

The pulsational properties of RR Lyrae variables provide an unique opportunity for testing the prescriptions of both stellar evolution and stellar pulsation theories. In fact, fundamental constraints on the evolutionary properties of HB stars can be derived by adopting the pulsation relation i.e. the relation which supplies the pulsational period as a function of stellar mass, luminosity and effective temperature. The pulsation relation for RR Lyrae variables originally derived by van Albada & Baker (1971) on the basis of linear, radiative, nonadiabatic models has been recently revised by Bono et al. (1997) and by Caputo, Marconi & Santolamazza (1998a) by adopting full amplitude, nonlinear, convective models. Other interesting properties of RR Lyrae pulsation behavior have been brought out by Sandage, Katem & Sandage (1981) who suggested the existence of a tight correlations between temperature and pulsational amplitudes and by Caputo & De Santis (1992, hereinafter CDS92) who supported the evidence of a clear correlation between period, blue amplitude and light-mass ratio.

In this context it is worth mentioning that both periods and amplitudes can be measured with high accuracy, since they are affected neither by distance uncertainties nor by interstellar reddening evaluations. As a consequence, the use of these observables and the comparison between theory and observations can supply independent and useful constraints on the intrinsic luminosity of RR Lyrae variables. A similar approach was adopted by Castellani & De Santis (1994) who showed that the absolute visual magnitude predicted by theoretical models are, within an accuracy of 0.1 mag., in satisfactory agreement with observed values.

Quite recently, De Santis (1996, hereinafter DS96) provided an accurate revision of the RR Lyrae temperature scale adopted in previous investigations and suggested a slight change in the zero point of the magnitude scale for these variables. However, Caputo et al. (1998b) by investigating the pulsational and evolutionary properties of RR Lyrae variables in M5 found a sizable discrepancy between recent theoretical prescriptions and observations. According these authors, this discrepancy could be explained, without changes in the adopted evolutionary scenario, by slightly shifting the position in the H-R diagram of the instability strip toward higher effective temperatures.

The evaluation of HB luminosity based on stellar models has been recently reviewed by Cassisi et al. (1998a,b). In these papers, it has been thoroughly discussed the dependence of HB luminosity on the input physics adopted for computing stellar models. However, the current observational scenario does not supply sound constraints on HB magnitude and in turn on theoretical predictions based on different physical assumptions (De Boer, Tucholke & Schmidt 1997; Cassisi et al. 1998a, 1998b). As a consequence, new and independent approaches are necessary for testing the accuracy of the theoretical scenario for low-mass helium burning stars and for assessing their intrinsic properties.

The main goal of this investigation is to analyse the intrinsic luminosity of HB stars by adopting the pulsational characteristics of fundamental RR Lyrae variables (hereinafter $RR_{ab}$) in a sample of galactic GCs. It is worth stressing that for comparing theory and observations we did not transform into the observable plane theoretical observables, but we directly estimated the ZAHB bolometric magnitude. As a consequence, this approach overcomes the uncertainties which affect the bolometric corrections based on static atmosphere models. The theoretical framework and the method adopted for deriving the ZAHB bolometric magnitude are outlined in the §2.

In §3, we briefly review our approach for estimating the effective temperatures of $RR_{ab}$ stars on the basis of their pulsational properties. In this section we also compare our results with the most recent theoretical color-temperature relations and discuss the evaluation of the interstellar reddening for the selected clusters. The results of the application of our method are presented in §4. Finally, observational data are compared with theoretical predictions and a critical analysis of the aftermaths of this investigation is presented.

2 THE THEORETICAL FRAMEWORK AND THE METHOD.

In this section, we discuss the theoretical framework and the procedure adopted for providing an empirical evaluation of the ZAHB luminosity in a sample of galactic globular clusters.

Since we are interested in obtaining an independent measurement of the intrinsic luminosity of the ZAHB for providing firm constraints on the reliability of the current evolutionary scenario, we have reduced as much as possible the use of evolutionary models. In fact, our analysis relies only on the estimate of the stellar masses along the HB which produce fundamental pulsators. However, it will
be shown that our results are largely unaffected by this approach. In every case, it will be accurately discussed the uncertainty related to the use of these evolutionary evidences.

In the literature to account for both evolutionary and pulsational properties of RR Lyrae stars it has been generally adopted the pulsation relation provided by van Albada & Baker (1971). However, recent theoretical investigations strongly support the evidence that sound estimates of both the pulsation period and the modal stability of RR Lyrae stars can only be obtained in a nonlinear regime (Bono & Stellingwerf 1994). On the basis of an extensive grid of RR Lyrae full amplitude, nonlinear and time-dependent convective models Bono et al. (1997) have revised the pulsation relation by van Albada & Baker. Therefore, the pulsation relation for fundamental pulsators, we adopt in the following, is:

$$\log P = 11.627 + 0.823 \cdot \log L - 0.582 \cdot \log M - 3.506 \log T_e$$

where $P$ is the fundamental period (days), $T_e$ is the effective temperature and $L$ and $M$ are the luminosity and the stellar mass in solar units (see Bono et al. (1997) for more details).

Since the Horizontal branch is not really horizontal, we need to fix an effective temperature to which we will refer to when estimating the ZAHB luminosity. For this goal we decide to adopt an effective temperature equal to $T_e = 3.85$ and, define as $L_{zahb}^{3.85}$ the ZAHB luminosity level at this effective temperature. Then we write the luminosity of each variable as:

$$\log L = \log L_{zahb}^{3.85} + \Delta \log L_{zahb}$$

where $\Delta \log L_{zahb}$ is the difference in luminosity between the individual variable and the ZAHB at $T_e = 3.85$. With simple algebraic substitutions, equation 1) can be rewritten (see also Sandage 1981) as follows:

$$\log P + 0.823 \cdot \Delta \log L_{zahb} = \log P + 0.33 \cdot \Delta M_{bol}^{zahb} = 11.627 + 0.823 \cdot \log L_{zahb}^{3.85} - 0.582 \cdot \log M - 3.506 \log T_e$$

where the symbols have their usual meaning, and the quantity $\log P + 0.33 \cdot \Delta M_{bol}^{zahb}$ corresponds to the fundamental reduced period (Sandage 1981).

The method, we have developed for estimating the bolometric ZAHB magnitude, is based on the use of the equation 2) in order to evaluate the relation between the fundamental reduced period and the effective temperature, as a function of the variable mass (see below) and $L_{zahb}^{3.85}$, and on the comparison between this relation and the observational prescriptions on these same quantities as provided by a sample of $RR_{ab}$ variables in selected GCs. In section IV, this procedure will be outlined in more detail and it will be shown that this method allows us to obtain a straightforward estimate of the ZAHB luminosity (i.e. $L_{zahb}^{3.85}$).

In order to compare theory and observations, we need to transform the equation 2) into the observable plane, so we adopt:

$$\Delta M_{bol}^{zahb} = \Delta V^{zahb} + \Delta BC$$

where we have defined the difference in bolometric correction between the individual variable and the ZAHB at $T_e = 3.85$ as $\Delta BC = BC_{bol}^T - BC_{3.85}$. Obviously the bolometric correction in the previous relation has to be evaluated by adopting the surface gravity and effective temperature values of the variable star and of the fictitious star located on the ZAHB at $T_e = 3.85$. However, it is worth noting that, in the CMD region populated by RR Lyrae stars, the allowed gravity range is quite narrow, as it can be easily tested by using any set of evolutionary models available in the literature and a theoretical evaluations of the instability strip boundaries. However, in order to validate our assumption - i.e. to neglect the gravity changes within the instability strip - we performed a test by adopting several sets of bolometric corrections (see below). Interestingly enough we found that the bolometric correction is sensitive to surface gravity values but the quantity $\Delta BC = BC_{bol}^T - BC_{3.85}$ presents a negligible dependence on gravity in the range of effective temperatures and luminosities typical of $RR_{ab}$ stars.

Finally we have adopted the following relation for $\Delta BC$ as a function of the effective temperature, derived by adopting the Buser & Kurucz (1978) model atmospheres:

$$\Delta BC = -5.252 \cdot \log^2 T_e + 41.636 \cdot \log T_e - 82.454$$

The standard deviation of this relation is $\sigma = \pm 0.004$, while the correlation coefficient $r \approx 1.0$ and it is correct in the ranges: $6000 \leq T_e (K) \leq 7500, 2.5 \leq \log g \leq 3.0$ and metallicity $0.002 \leq Z \leq 0.002$. It is important to notice that the use of a different BC scale does not substantially change the dependence of $\Delta BC$ on temperature as given by equation 3).

The next topic, we wish to address, is the one corresponding to the method adopted to evaluate the visual magnitude of the ZAHB from the observations of RR Lyrae variables and non-variable HB stars in globular clusters.

In order to estimate the ZAHB visual magnitude, we have adopted the same operative approach described by Sandage (1990) by taking into account the lower envelope of the HB star distribution. It could be raised the question that in the previous relations we have adopted the visual magnitude of the ZAHB at $T_e = 3.85$ while now we adopt the ‘generic’ lower envelope of the HB star distributions. Even though these definitions seem to be substantially different, any test performed by adopting the grids of evolutionary models (Dorman, Rood & O’Connell 1993; Caloi, D’Antona & Mazzitelli 1997; Cassisi et al. 1998ab) and the sets of color-temperature relations and of bolometric corrections available in the literature, discloses that within the instability strip the dependence of $M_V$ (ZAHB) on the effective temperature is quite negligible. In fact, we found that $M_V$ (ZAHB) changes by only 0.006 mag when moving from $T_e \approx 6700K$ to $\approx 7100K$.

As a relevant point, we note that our method is not affected by uncertainties on stellar photometry such as the zero point and the calibration procedure, since we use the apparent visual magnitude of the ZAHB as a reference magnitude from which to measure the mean visual magnitude of each variable. In fact our approach relies only on the difference in the V magnitude between each ab RR Lyrae star and the ZAHB at approximately $\log T_e = 3.85$.

As we have underlined at the beginning of this paper, the main goal of our work is to obtain an evaluation of the
luminosity level of the HB stars in GCs as much as possible independent on the theoretical evolutionary scenario. As it can be easily recognized in all previous relations which represent our working framework, do not appear quantities related to evolutionary theory but the mass of the variable.

It is well known that the mass of an RR Lyrae variable can not be directly evaluated unless the star is a double pulsators and, also in this case such evaluation is based on theoretical pulsational models (Kovacs, Buchler & Marom 1991). For long time, it existed a large discrepancy between the pulsational and evolutionary determinations of double mode RR Lyrae stars mass, but recently the introduction of new radiative opacity evaluations from the Livermore Laboratory (OPAL opacity, Rogers & Iglesias 1992) has allowed to make insignificant this discrepancy.

In the present work, we adopt evolutionary determinations of the pulsator masses as provided by our own stellar computations (Bono, Cassisi & Castellani 1998). Since as it is evident, we can not assign a mass to the whole sample of variables in a GC, we use only an evaluation on the lower and upper mass ($M_{\text{RR min}}$ and $M_{\text{RR max}}$ respectively) producing ab RR Lyrae stars. Due to the dependence on the metallicity of these parameters, we have analyzed their variations when changing the heavy elements abundance in a quite large range, namely $0.0001 \leq Z \leq 0.003$.

For each fixed metallicity, we have adopted as $M_{\text{RR min}}$ and $M_{\text{RR max}}$ the lower and larger mass respectively which spend in the instability strip as fundamental pulsators at least the 5% of their central He burning evolutionary lifetime. In order to adopt a homogeneous pulsational context the boundaries of the instability strip were fixed according to the prescriptions by Bono et al. (1997) i.e., the same set of RR Lyrae models adopted for the pulsational relation.
The Horizontal Branch luminosity level.

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The Horizontal Branch luminosity level.

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Figure 2. Comparisons in the \((B - V) - A_B\) plane between the intrinsic colors of M3 RR Lyrae stars as obtained by the CDS92 relation (solid line) and the one obtained by using the \(T_{\text{eff}}\) scale of De Santis (1996) and theoretical (panel a: CGK97 - panel c: BK78 - panel d: K92) or semi-empirical (panel b: Yale) color - temperature relations.

Figure 3. As figure 2, but for the ab RR Lyrae stars in M15.

3 THE EFFECTIVE TEMPERATURE SCALE.

The analytical relation which supplies the effective temperature as a function of period and blue amplitude \((A_B)\), we adopt in this investigation, is the one provided by DS96 which corresponds to an updated version of the relation given by Castellani & De Santis (1994):

\[
\log T_e = -0.1094 \cdot \log P + 0.0134 \cdot A_B + 3.770
\]

with a probable error equal to \(\pm 0.003\). This relation has been derived by relying on the RR Lyrae database of Lub (1977) and, by correcting its zero point in order to achieve agreement with the Baade-Wesselink data for field variables (Carney, Storm & Jones 1992, hereinafter CSJ92) (see DS96 for more details on this point).

The accuracy of this or a similar relation for deriving the effective temperature of RR Lyrae stars has been recently questioned by Walker (1998) due to the large number of parameters that are involved and to the small number of calibrating stars. However, this is not the case for equation 4), since it has been obtained by using a quite large sample (about 70 objects) of RR Lyrae stars.

Both the reliability and accuracy of this effective temperature scale has already been extensively discussed by Castellani & De Santis (1994) and, due to the revision, further emphasized by DS96. Therefore, this topic will not be again reviewed here and we address the interested reader to the quoted papers. However, it is worth remembering that CDS92, through an accurate analysis of the Lub's (1977) data for field RR Lyrae stars with well-determined reddening, have clearly shown the existence of a tight correlation between the intrinsic color of the variable and its blue amplitude and metal abundance:

\[
(B - V)_0 = -0.0775 \cdot A_B + 0.005 \cdot [Fe/H] + 0.434
\]

with a standard deviation equal to \(\sigma = \pm 0.015\) mag.

Obviously, the coupling of equations 4) and 5) provide a color - temperature scale, so it could be interesting to check the consistency between these two equations by comparing it with the color - temperature relation provided by theoretical model atmospheres.

Being aware of the problems still existing with theoretical model atmospheres (Castelli 1998, private communication), we have tried to perform this comparison by using as many as possible independent sets of transformations: Buser...
To perform the check we have used the observational data for \(ab\) RR Lyrae stars in 3 globular clusters, namely M3, M15 (Sandage 1990) and M68 (Walker 1994). The procedure is quite simple: i) we derive the effective temperatures of the variable stars by using their period and B amplitude (equation 4); ii) their intrinsic \((B - V)\) colors are then estimated by adopting an average value for the gravity i.e. \(\log g = 2.75\) and by using a color - temperature relation (see below) for the suited metal abundance \([\text{Fe/H}] = -1.3\) for M3 and \([\text{Fe/H}] = -2.0\) for M15 and M68). These color estimations are then compared with the ones provided by equation 5) in the \((B - V)_0 - A_B\) plane, as shown in figures 2) to 4).

It is worth noting the fine agreement achieved when comparing the data obtained by adopting the relation provided by CD92 with the colors obtained by using both the theoretical color-temperature scales by BK78 and CGK97 and also with the Yale semi-empirical one. The average difference is equal to 0.005 mag in the worst case (panel a in fig. 3). On the contrary, it is important to notice the large discrepancy obtained when using the theoretical color-\(T_{\text{eff}}\) relation from K92. In this case the average difference is of the order of \(\approx 0.04\) mag. These results led some support to the following conclusions:

- \(T_{\text{eff}}\) scales;
- it seems to exist a large discrepancy between the color\(-T_{\text{eff}}\) scale based on model atmospheres from K92 and our ‘pulsational’ scale based on equations 4) and 5). This result is a clear evidence that the reddening scale of Lub (1977) on which rely both equation 4) and 5) is not consistent with the color-temperature scale of K92. It is evident that such occurrence could be explained as a drawback of the K92 model atmospheres or alternatively as due to a problem in the Lub’s reddening scale.

In this context, it is worth noticing that recently several accurate theoretical and observational investigations have provided clear indications about the presence of significant problems in comparing theory and observations when using the color-temperature relation as given by Kurucz (1992).

In particular, in a detailed analysis of the pulsational properties of RR Lyrae stars in GCs, Bono et al. (1997) have clearly shown that the use of the K92 color-temperature scale produces, for each given amplitude, effective temperatures about 300K higher than predicted by the theory. When considering the fine agreement that the quoted authors, achieved as far as it concerns the Bailey (blue amplitude versus period) diagram, one is strongly forced to consider this result as a clear evidence for some problems in the K92 color-temperature relation. This question has been also addressed by Silbermann & Smith (1995) by comparing the \((V - R)\) color of a sample of RR Lyrae stars with theoretical expectations. In addition, it is important to remember that an accurate analysis on the possible drawbacks in model atmosphere computations and, more in detail, on the K92 calculations, have been provided by CGK97 and by Castelli (1998, private communications).

Taking into account such evidences, we suggest that our result is a further indication against the use of the K92 color-temperature scale, at least when working on RR Lyrae variables.

It is also worth noticing that the satisfactory agreement achieved between the results provided by equations 4) and 5) and the Yale, BK78 and CGK97 color-temperature scales, can be regarded as a positive check on the consistency and reliability of the pulsational effective temperature scale adopted in the present work.

Up to now we discussed the accuracy of the color - temperature scale defined by equations 4) and 5), now we briefly address a question concerning the use of mean colors and effective temperatures. The relation between effective temperature, period and blue amplitude (equation 4) has been obtained by adopting for each variable in the Lub’s sample the mean \((B - V)\) color, i.e. the time average along a full pulsational cycle of the color curve (see DS96 for more details on this point). At the same time, the effective temperature adopted by Bono et al. (1997) for deriving the pulsational equation 1) is the ‘static’ equivalent temperature. As a consequence, it could be claimed that we are dealing with two ‘different’ and, un-consistent effective temperature definitions. However, DS96 has already shown the quite fine agreement between the effective temperature evaluations provided by equation 4) and the static temperatures from CS92.

Concerning this point, it is also worth noticing that the comparison (see for instance Bono, Caputo & Stellingwerf...
Figure 5. Comparison in the \((B - V) - AB\) plane between static colors as obtained by using the K92 (open circles) and CGK97 (full circles) color - temperature scales, corresponding to the pulsational models of Bono et al. (1997) for different assumptions on the luminosity level but for the same pulsator mass \(0.65M_\odot\) and metallicity \(Z=0.001\). The CDS92 relation is also displayed.

The Horizontal Branch luminosity level.

Before closing this section, we wish to adopt the previously discussed relation between intrinsic color and pulsational properties of the variables in order to evaluate the interstellar reddening for a sample of clusters. In fact, even though our approach for estimating the HB luminosity does not depend on the reddening, the color - temperature scale, we derived, can be easily adopted for providing independent estimates of this parameter.

The sample of GCs taken into account, is the same adopted for the further investigation on the HB luminosity level and, it will be discussed in the next section.

The procedure adopted to derive the reddening is quite simple: we compare in the \((B - V) - AB\) plane the observational data for the sample of ab RR Lyrae stars in each cluster with the reddening free colors provided by the equation 5) by using the observational blue amplitudes and the cluster metallicity \([Fe/H]\) as given by Carretta & Gratton (1997, hereinafter CG97). The amount of color shift which is need in order to achieve a satisfactory agreement between observations and the CDS92 relation, provides a fine evaluation of the cluster reddening. In figures 6) to 8), we show the comparison between the observed color of the RR Lyrae stars and the dereddened color as provided by the CDS92 relation for the clusters: M3, M15 and M68, respectively.

The reddening evaluations obtained with this method and their maximum errors, are listed for all clusters in our database in Table 1) together with other relevant cluster parameters (see below).

In passing, we note that our evaluation of the M68 reddening is in fine agreement with the estimate recently provided by Gratton et al. (1997) on the basis of their accurate Strömgren photometric analysis of field stars projected on the sky near a selected sample of GCs.

4 THE ZAHB LUMINOSITY LEVEL.

4.1 Fundamental pulsators.

In order to apply the method for estimating the ZAHB luminosity we selected a sample of galactic GCs which is characterized by homogeneous photometric observations for both non-variable HB stars and RR Lyrae variables and that covers a wide range of metal contents. On the basis of these requirements we selected the following clusters: M3, M15, M68, NGC6171, NGC1851, NGC6362 and NGC6981. The data for both variable and non-variable stars in M3, M15, NGC6171 and NGC6981 have been collected from the work of Sandage (1990); for M68, NGC1851 and NGC6362 we have used the data from Walker (1994), Walker (1998) and Brocato et al. (1998), respectively. Unfortunately for some of these clusters only the photographic photometry of static and variable HB stars is available in the literature. However, as already mentioned, the lack of accurate CCD data is not a strong limit to the application of our method. In fact, our analysis relies on the difference in magnitude between the RR Lyrae stars and the ZAHB. This means that the ZAHB luminosities we derive are not affected by any uncertainty on the true zero point of the photometry.
Table 1. Selected data for the sample of galactic globular clusters.

| Cluster          | [Fe/H]     | \( V_{zahb} \) | \( \log L_{zahb}^{1.85} \) | E(B-V) |
|------------------|------------|----------------|-----------------------------|--------|
| NGC1851          | -1.08      | 16.13 ± 0.025  | 1.645 ± 0.010               | 0.045 ± 0.016 |
| NGC4590 (M68)    | -1.99      | 15.70 ± 0.02   | 1.70 ± 0.02                 | 0.043 ± 0.014 |
| NGC5272 (M3)     | -1.34      | 15.73 ± 0.02   | 1.645 ± 0.010               | 0.012 ± 0.010 |
| NGC6171          | -0.87      | 15.85 ± 0.10   | 1.54 ± 0.04                 | 0.363 ± 0.023 |
| NGC6362          | -0.96      | 15.33 ± 0.03   | 1.615 ± 0.015               | 0.070 ± 0.013 |
| NGC9081          | -1.30      | 17.07 ± 0.03   | 1.635 ± 0.010               | 0.070 ± 0.015 |
| NGC7078 (M15)    | -2.12      | 15.92 ± 0.05   | 1.68 ± 0.02                 | 0.074 ± 0.014 |

For each cluster in our sample in order to apply the method outlined in section II, we have to preliminary estimate the ZAHB visual magnitude, and also the expected range of fundamental pulsator mass. In the following, we discuss for each single cluster the values adopted for these important parameters:

NGC1851: for this cluster, a metallicity evaluation is not present in the CG97 compilation so we have adopted the estimate provided by Zinn & West (1984). However, in order to be consistent with the CG97 metallicity scale adopted for all other clusters, we have taken into account the relation relating the Zinn & West (1984) scale with the CG97 one (the equation 7 in CG97), so finally we have adopted \([Fe/H] = -1.08\), which corresponds to \( Z \approx 0.002 \). This occurrence allows us by using theoretical stellar models of suitable metallicity to obtain a mass interval for \( ab \) RR Lyrae stars in this cluster equal to: \( 0.61 \leq M_{RRab}/M_\odot \leq 0.67 \). Concerning the visual magnitude of the ZAHB, it has been estimated by adopting the lower envelope of the observed distribution. In figure 9, we have plotted for each cluster in our sample the observed distribution of stars on the HB and marked the adopted visual magnitude of the ZAHB (see also Table 1). By adopting this approach we estimate for the visual magnitude of the ZAHB \((V_{zahb})\) for the cluster NGC1851, a value equal to 16.13 ± 0.025 mag.

M68: in this case the metallicity measurement provided by CG97 is equal to \([Fe/H] \approx -1.99\), i.e. \( Z \approx 0.0002 \). From data in figure 1, this mass range for \( ab \) RR Lyrae stars has been evaluated: \( 0.70 \leq M_{RRab}/M_\odot \leq 0.80 \). By analyzing the photometric data from Walker (1994), we estimate a value for the ZAHB visual magnitude equal to 15.70 ± 0.02 mag.

M3: the heavy element abundance of this cluster according to the CG97 metallicity scale is equal to \([Fe/H] = -1.34\) which corresponds to about \( Z \approx 0.001 \). Therefore, by accounting for the theoretical evidences shown in figure 1), we estimate that the most suitable fundamental pulsator mass range is: \( 0.64 \leq M_{RRab}/M_\odot \leq 0.70 \). The adopted value for \( V_{zahb} \) is 15.73 ± 0.02 mag.

NGC6171: by adopting \([Fe/H] = -0.87\) from the stellar models for \( Z = 0.003 \), one obtains a mass range for the fundamental mode variables equal to: \( 0.60 \leq M_{RRab}/M_\odot \leq 0.635 \). From the data plotted in figure 9), the most suitable estimate for \( V_{zahb} \) is of about 15.85 ± 0.10 mag.

NGC6362: for this cluster CG97 report a metallicity value equal to -0.96 dex, which corresponds to \( Z \approx 0.002 \). Therefore we adopt for this cluster a RR Lyrae mass range equal to: \( 0.61 \leq M_{RRab}/M_\odot \leq 0.66 \). As far as it concerns the visual magnitude of the ZAHB from the observed distribution of HB stars we derive a value of 15.33 ± 0.03 mag.

NGC6981: since the metallicity of this cluster is equal to \( Z \approx -1.30 \), we adopt the same mass range adopted for M3 and, by analyzing the observed distribution of HB stars in the CM diagram we obtain a value for \( V_{zahb} \) of about 17.07 ± 0.03 mag.

M15: since the metallicity of this cluster is quite similar to the M68 metallicity, the same mass range for the fundamental pulsators has been adopted. From the data in figure 9), we estimate a value for \( V_{zahb} \) equal to 15.92 ± 0.05 mag.

In section II, we have already defined the fundamental reduced period: \( \log P_{red} = \log P + 0.33 \cdot \Delta M_{bol}(ZAHB) \). Now, we compare the observational data for the \( ab \) RR Lyrae variables in each individual cluster with the prescription provided from equation 2) in the \( \log P_{red} - \log T_e \) diagram. Since these theoretical expectations depend on the bolometric magnitude of the ZAHB at \( \log T_e = 3.85 \) (according to our definition), this comparison supplies a straightforward evaluation of the ZAHB luminosity. More in detail, the procedure adopted for estimating this quantity is the following:

- since equation 2) depends on the mass of the pulsators by adopting both the lower and upper limit of the
The Horizontal Branch luminosity level.

**Figure 9.** The distribution in C-M diagram of the HB stars (open circle: non-variable stars; full circle: RR Lyrae stars) for all selected clusters. In each panel, the adopted visual magnitude of the ZAHB with the associated error is shown.

*$RR_{ab}$* mass range, we obtain two different solutions for the behavior of the reduced period as a function of the effective temperature, both depending on the ZAHB luminosity level;

- then the ZAHB luminosity is estimated by properly fitting the lower and upper boundaries of the RR Lyrae distribution in the reduced period - temperature plane. This approach is outlined in figure 10.

The results listed in Table 1 support the following conclusions:

- *i)* the bolometric ZAHB luminosity decreases significantly: $\Delta \log L_{ZAHB} \approx 0.14$, when increasing the metallicity from the most metal-poor cluster (M15) to the more metal-rich one (NGC6171) in our sample;

- *ii)* the ZAHB luminosity levels evaluated for the subsample of metal-poor clusters (M15 and M68) appear in fine agreement. The same outcome applies for intermediate metallicity clusters such as NGC6981 and M3. This result can be regarded as an evidence of the reliability of our approach.

- *iii)* the method, we developed, allows to estimate the bolometric ZAHB luminosity at a fixed effective temperature with an high accuracy, and indeed in the worst case the uncertainty on $\log L_{ZAHB}$ is of the order of 0.04.

On the basis of the ZAHB luminosities we estimated for these clusters, we can transform the observational data into the theoretical plane, thus suppling an independent approach to compare theory and observations. We perform this comparison in the last section.

4.2 First overtone pulsators.

The evaluations of the ZAHB luminosity discussed in the previous subsection are based on $RR_{ab}$ variables. In order to provide an independent test on the accuracy of the adopted method, we undertake a similar investigation but
by adopting first overtone variables (RR\textsubscript{c}).

Bono et al. (1997) have provided a relation, quite similar to the equation 1), but suitable for first overtone pulsators:

\[
\log P = 10.789 + 0.800 \cdot \log L - 0.594 \cdot \log M - 3.309 \cdot \log T_e
\]  

where the symbols have their usual meaning and the units are the same as in equation 1). By following the same definitions adopted in section II, one obtains:

\[
\log P + 0.32 \cdot \Delta M_{\text{Bol}}^{zahb} = 10.789 + 0.800 \cdot \log L^{zahb} + -0.594 \cdot \log M - 3.309 \log T_e
\]  

Since the relation 4) has been obtained by adopting a sample of field RR\textsubscript{ab} Lyrae stars, it can not be used for first overtone pulsators. So in order to estimate the effective temperature of RR\textsubscript{c} Lyrae stars, we have adopted a different approach:

\textit{i)} by using the reddening evaluation obtained by using the RR\textsubscript{ab} Lyrae stars (see section III and data in Table 1), we have derived the true (B - V) color of the sample of first overtone pulsators in the selected clusters;

\textit{ii)} from the (B - V)$_0$ color the effective temperature of the variable has been estimated by using a theoretical color-temperature relation (as given, for instance, by CGK97 or by BK78 or Yale), since in section III we have shown the consistency between these temperature scale and the pulsational one corresponding to equations 4) and 5).

This method has been applied to the variables in two clusters, namely, M15 and M68. In the case of M15 we have taken into account 26 RR\textsubscript{c} Lyrae stars (Sandage 1990), while for M68 we have excluded from our analysis all variables affected by Blazko effect (see Walker 1994 for more details) and then we have recovered only 15 variables.

As far as it concerns the mass range for first overtone pulsators, from data in figure 1) it has been obtained a mass interval: 0.67 \leq M_{RRc}/M_\odot \leq 0.75, for both clusters (their metallicity is quite similar). The result of this investigation
as already discussed in section II, a shift of the whole mass range suitable for fundamental pulsators of $\Delta M = 0.01 M_\odot$ gives $\Delta \log L_{3.85} \approx 0.004$. It is worth remembering that the effective temperature of stellar models, which is important in order to estimate the time spent inside the instability strip by each model, is usually strongly affected by all the physical inputs (as low temperature opacities and superadiabatic region treatment) which determine the stellar outer layers structure. On this concern, we wish to emphasize that all evolutionary computations adopted in our analysis have been performed by using the most updated stellar physics as far as concerns stellar matter opacity (see Bono et al. 1998 for more details), while for the treatment of the superadiabatic layers it has been adopted the mixing length calibration provided by Salaris & Cassisi (1996).

Nevertheless, it is important to verify if the estimations of fundamental pulsators mass ranges ($\Delta M_{RRab}$) at the various metallicities, obtained by using our own evolutionary computations are consistent with the results provided by stellar evolutionary computations on literature. More in detail, we have compared our results with the ones obtained when using the Castellani, Chieffi & Pulone (1991) models or the Caloi, D’Antona & Mazzitelli (1997) ones.

The comparison with the HB models of Castellani et al. (1991) has shown the existence of a quite good agreement, for instance at $Z=0.001$, we obtain a mass range equal to $0.64 \leq M_{RRab}/M_\odot \leq 0.70$ which has to be compared with the range $0.64 \leq M_{RRab}/M_\odot \leq 0.69$, one obtains when using the Castellani et al. (1991) models. As far as it concerns the comparison with the HB models of Caloi et al. (1997), in the limit of the too coarse grid of models adopted by these authors, a satisfactory agreement is obtained for all the metallicities for which is possible to perform the comparison;

c) as already discussed, the relation connecting the pulsational periods to stellar masses, luminosities and effective temperatures as given by Bono et al. (1997), represents a significant improvement in comparison with the original relation provided by van Albada & Baker (1971). However, it could be interesting to verify the effect on the present analysis due to the use of the van Albada & Baker’s relation. Bono et al. (1997) have already realized a comparison between their own relation and the van Albada & Baker’s one: due to the use of more reliable nonlinear pulsational models they find periods smaller than linear periods (see their figure 6). By taking into account this occurrence, one can easily verify that the use of the van Albada & Baker’s relation in order to constraint the ZAHB luminosity level in GCs has the effect to produce an average reduction on the evaluation of $\log L_{3.85}$ of about $0.015 - 0.02$. It is also worth noting that recently Caputo, Marconi & Santolamassa (1998) have revised the original Bono et al. (1997) relation by including the dependence on the heavy elements abundance. We have also investigated if the use of this revised relation could significantly affect our results: it has been obtained that for the metal-poor clusters there are no effects on the evaluation of the ZAHB luminosity and, that for metal-rich GCs the change on $\log L_{3.85}$ estimate is, in the worst case, of the order of $\approx -0.01$;

d) concerning the metallicity of the cluster, it is worth noticing that the only point where the heavy elements abundance

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5 DISCUSSION AND CONCLUSIONS

In the previous sections, it has been shown that the method, we developed, to derive from the pulsational properties of RR Lyrae stars in GCs, an estimate of the bolometric magnitude of the ZAHB at $T_e = 3.85$ allow us to evaluate this important quantity with an high accuracy: the average uncertainty on $\log L_{3.85}^{zahb}$ is equal to $\approx 0.02$. Therefore, till now it can be considered one of the most reliable approaches to constrain the HB luminosity level in GCs.

Nevertheless, it is important to provide a deeper insight on this method by showing the main possible error sources in order to better quantify its accuracy. The most important uncertainties can derive from: the evaluation of the visual magnitude of the ZAHB in GCs, the estimate of the suitable mass range for fundamental pulsators, the adopted relation between pulsational period and the evolutionary properties of the variable, the metallicity of the clusters and the adopted RR Lyrae temperature scale. On this concern, it is quite easy to verify the following indications:

a) an uncertainty on $V_{zahb}$ of about 0.025 mag produces an error on $\log L_{3.85}^{zahb}$ of about 0.01;

b) as already discussed in section II, a shift of the whole mass range suitable for fundamental pulsators of $\Delta M = 0.01 M_\odot$ gives $\Delta \log L_{3.85} \approx 0.004$. It is worth remembering that the effective temperature of stellar models, which is important in order to estimate the time spent inside the instability strip by each model, is usually strongly affected by all the physical inputs (as low temperature opacities and superadiabatic region treatment) which determine the stellar outer layers structure. On this concern, we wish to emphasize that all evolutionary computations adopted in our analysis have been performed by using the most updated stellar physics as far as concerns stellar matter opacity (see Bono et al. 1998 for more details), while for the treatment of the superadiabatic layers it has been adopted the mixing length calibration provided by Salaris & Cassisi (1996).

Nevertheless, it is important to verify if the estimations of fundamental pulsators mass ranges ($\Delta M_{RRab}$) at the various metallicities, obtained by using our own evolutionary computations are consistent with the results provided by stellar evolutionary computations on literature. More in detail, we have compared our results with the ones obtained when using the Castellani, Chieffi & Pulone (1991) models or the Caloi, D’Antona & Mazzitelli (1997) ones.

The comparison with the HB models of Castellani et al. (1991) has shown the existence of a quite good agreement, for instance at $Z=0.001$, we obtain a mass range equal to $0.64 \leq M_{RRab}/M_\odot \leq 0.70$ which has to be compared with the range $0.64 \leq M_{RRab}/M_\odot \leq 0.69$, one obtains when using the Castellani et al. (1991) models. As far as it concerns the comparison with the HB models of Caloi et al. (1997), in the limit of the too coarse grid of models adopted by these authors, a satisfactory agreement is obtained for all the metallicities for which is possible to perform the comparison;

c) as already discussed, the relation connecting the pulsational periods to stellar masses, luminosities and effective temperatures as given by Bono et al. (1997), represents a significant improvement in comparison with the original relation provided by van Albada & Baker (1971). However, it could be interesting to verify the effect on the present analysis due to the use of the van Albada & Baker’s relation. Bono et al. (1997) have already realized a comparison between their own relation and the van Albada & Baker’s one: due to the use of more reliable nonlinear pulsational models they find periods smaller than linear periods (see their figure 6). By taking into account this occurrence, one can easily verify that the use of the van Albada & Baker’s relation in order to constraint the ZAHB luminosity level in GCs has the effect to produce an average reduction on the evaluation of $\log L_{3.85}$ of about $0.015 - 0.02$. It is also worth noting that recently Caputo, Marconi & Santolamassa (1998) have revised the original Bono et al. (1997) relation by including the dependence on the heavy elements abundance. We have also investigated if the use of this revised relation could significantly affect our results: it has been obtained that for the metal-poor clusters there are no effects on the evaluation of the ZAHB luminosity and, that for metal-rich GCs the change on $\log L_{3.85}$ estimate is, in the worst case, of the order of $\approx -0.01$;

d) concerning the metallicity of the cluster, it is worth noticing that the only point where the heavy elements abundance

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Figure 11. As figure 10, but for first overtone pulsators in M15 and M68.

is shown for both clusters in figure 11.

We have obtained a luminosity level of the ZAHB equal to $\log L_{3.85}^{zahb} = 1.67$ and 1.69 for M15 and M68 respectively, and then in fine agreement, within the associated uncertainties, with the previous evaluations based on the pulsational properties of $RRab$ Lyrae stars. This occurrence can be clearly considered an additional support to the reliability of our working framework and of the accuracy of the adopted method.

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The Horizontal Branch luminosity level.

11
plays a role is related to the choice of the most suitable mass range for RR Lyrae stars. However, if one takes into account the discussion at the point b), the evidences shown in figure 1) and also that the mean uncertainty in the CG97 metallicity scale (≈ 0.10 – 0.15 dex), it is easy to conclude that the uncertainty on the cluster metallicity does not affect significantly the evaluation of \( L_{3,85}^{\text{zahb}} \). However, it is important to notice that in all previous discussion we have used the spectroscopical measurement of \([Fe/H]\) as representative of the true cluster metallicity but, as it is well known (see, e.g. the review by Wheeler, Sneden & Truran 1989), there are clear observational evidence that the \( \alpha \) – elements are enhanced in GCs stars.

So it is interesting to evaluate the effect of the \( L_{3,85}^{\text{zahb}} \) measurements due to an \( \alpha \) – elements enhancement. Due to the lack of recent \( \alpha \) – element enhancement measurements for the selected clusters, we have adopted for all clusters (irrespective of their \([Fe/H]\) value) a mean enhancement \([\alpha/Fe] = 0.30\) (Gratton et al. 1997), obtaining an average decreasing of \( \log L_{3,85}^{\text{zahb}} \) of the order of 0.004 – 0.005;

e) as far as it concerns the RR Lyrae temperature scale, the accuracy of the \( T_{\text{eff}} \) scale, adopted in the present work, has been discussed in detail by DS96 and briefly reviewed in section III. Therefore, now we do not repeat this analysis. However, there is an important question on the temperature scale which we now wish to address concerning the possibility that our scale (as given by the equation 4) can be dependent on the stellar luminosity. It is evident that this is a quite important question for our investigation in fact if the adopted temperature scale should be dependent on the luminosity of the RR Lyrae stars, this occurrence could produce a not accurate estimate of the ZAHB luminosity.

On this point, Catelan (1998) and Catelan, Sweigart & Borissova (1998, hereinafter CSB98) have recently claimed that ‘a relationship involving only the equilibrium temperature, blue amplitude and metallicity would be safer to adopt in period-shift analysis than CSJ92’s (depending also on the pulsational period) since period shifts caused by luminosity variations could easily be misinterpreted as being due to temperature variations’. So we have decided to check if the adopted temperature scale could be dependent on the luminosity. For this aim, we have taken into account the sample of 17 field \( ab \) type RR Lyrae, studied by using the Baade-Wesselink method by CSJ92, and investigated if the difference between the temperature, provided by equation 4), and the one given by CSJ92 (their table 4) depends on the measured luminosity of these stars. The result of such comparison is shown in figure 12)

It is evident from this plot the absence of any correlation between the effective temperature residuals and the luminosity. Such occurrence clearly allows us to be confident in the use of DS96’s temperature scale in our analysis. However, to provide a more deep investigation on the accuracy of our method, we have decided to repeat our investigation by using a different temperature scale and, being aware of the warning from Catelan (1998), the period independent CSB98 scale has been adopted.

It is worth stressing that for all clusters in our sample, the estimates of the absolute bolometric magnitude of the ZAHB \( \log L_{3,85}^{\text{zahb}} \) are in fine agreement with the one determined by using the temperature scale provided by the equation 4). This occurrence provides a further plain evidence of the reliability of both our temperature scale and our global approach. The results of this investigation for the case corresponding to the cluster M3 (but the same results have been achieved for the other clusters) have been plotted in figure 13). From this figure, one can obtain the following indications:

i) the estimated ZAHB luminosity level is equal to the one obtained by adopting the \( T_{\text{eff}} \) given by equation 4;
ii) the observational points, corresponding to the stars at the lower effective temperatures inside the instability strip, as given by the CSB98 temperature scale are not in satisfactory agreement with the pulsational theory prescriptions. In fact, it is worth noting that the slope of the observational data is different from the theoretical one. In our belief, this occurrence has to be related to the fact that the calibrating stars used by CSB98 for deriving their relation, do not cover the full expected range of RR Lyrae effective temperatures.

5.1 Comparison between field and cluster RR Lyrae stars

In his accurate analysis on field RR Lyrae stars, whose parallaxes have been recently provided by the Hipparcos mission, Catelan (1998) has clearly shown that it does not exist any evidence for a mean luminosity difference between field RR Lyrae stars and GCs variables. This result has been confirmed also by the work of Carney & Lee (1998 - see the note added in proof in the Catelan’s paper). Since the main goal of the present investigation is to evaluate the ZAHB luminosity level from the pulsational properties of RR Lyrae stars, it is natural to extend our investigation to field variables in order to eventually provide further support to the results obtained by Catelan (1998). By courtesy of Dr. M. Catelan, we have been provided with his original list of field RR Lyrae stars and related pulsational properties, which corresponds to a sub-sample of the field variables, belonging to the original list of stars adopted by Tsujimoto et al. (1998) in their Hipparcos-based investigation. As far as it concerns the selection criteria adopted by Catelan (1998) to choose this sample of variables, we refer the interested reader to the quoted paper.

The procedure adopted to perform the comparison between GCs RR Lyrae stars and field variables is quite similar to the one used by Catelan (1998): we have split the data for field variables in two different subsamples corresponding to two different metallicity ranges, with an average metallicity equal to \( \approx -2.0 \) and \( \approx -1.3 \), respectively. Then we have compared the pulsational properties of field variables in each range of metallicity with the properties of RR Lyrae stars in GC of suitable metallicity. In figure 14a), it is shown the comparison between variable in the cluster M3 and field RR Lyrae stars filling in the ‘metal-rich’ range; while figure 14b) shows the same comparison but between RR Lyrae stars in M15 and M68 and field stars in the ‘metal-poor’ range.

From the results shown in these figures, it is evident that there is no clear difference in the pulsational properties of GCs RR Lyrae stars and field ones in both (at least) explored metallicity range, which could be related to the existence of a real difference in luminosity between GCs and field variables. This result clearly provides further support to the Catelan’s (1998) investigation.

5.2 Comparison with theoretical results.

Since we have been able to evaluate the bolometric magnitude of the ZAHB for different clusters, it is now obvious to compare present results with the theoretical prescriptions on this important quantity. This comparison has been performed in figure 15, where we have plotted the evaluations on \( \log L_{Zahb}^{*} \) as obtained in the present work and also the most recent evolutionary theoretical evidences. In particular, we have considered the ZAHB stellar models from Castellani et al. (1991), Dorman, Rood & O’Connell (1993), Caloi et al. (1997), Cassisi & Salaris (1997), Cassisi et al. (1998a,b), Salaris & Weiss (1998) and Vandenberg (1998). All these models have been computed in a canonical evolutionary framework, but the Cassisi et al.’s (1998a,b) ones which have been computed for both a canonical scenario and also a scenario accounting for element (Helium + heavy elements) diffusion. We refer to the quoted papers for details on the evolutionary computations performed by the various authors. It is worth noticing that the ZAHB models of Cassisi & Salaris (1997) are full consistent with the evolutionary computations used in the previous sections.

We have already discussed the effect on the evaluation of the ZAHB luminosity due to a possible \( \alpha \)–elements enhancement in the heavy elements distribution and shown that it is very little. Nevertheless, when comparing observational evidences with theoretical results, one has to pay attention to adopt self-consistent metallicity evaluations. For such reason and also being aware for the lack of accurate \( \alpha \)–elements enhancement measurements (consistent with the [Fe/H] metallicity scale of Carretta & Gratton 1997) for the clusters in our sample, we have made two different assumptions by adopting both \([\alpha/Fe] = 0.0 \) and \( 0.30 \) (following the suggestion given by Gratton et al. 1997). The comparison between theory and present results for both assumptions on the \( \alpha \)–elements enhancement, has been per-
deviant result, it is still possible to assess that theory and observations, the decrease of the ZAHB luminosity - at a theoretical models underestimate, in comparison with the preliminary results, one is facing on with the evidence that the However, if further investigations should support this preliminary agreement at least up to a metallicity of the order of \( \approx -0.9, -0.8 \) dex;

c) it is worth noting the satisfactory agreement which has been achieved for both assumptions on the \( \alpha \)-elements enhancement between the values of \( \log L_{3.85}^{\text{zahb}} \) for the selected clusters and the evolutionary prescriptions as given by Cassisi & Salaris (1997) and Vandenberg (1998) concerning both the slope and the absolute values;

d) recently Cassisi et al. (1998a,b), in order to thoroughly investigate the effects on several important evolutionary quantities of the most updated evaluations on the several physical inputs adopted in stellar computations, have provided a complete set of evolutionary models for both H and He burning phase in a canonical scenario and also by accounting for element diffusion. The results shown in figure 15a), seem to indicate that the ZAHB luminosity levels provided by these models appear to be ruled out by the present investigations. The situation is slightly better when considering models accounting for element diffusion and when assuming for the clusters an \([\alpha/\text{Fe}]\) value larger than zero. Such occurrence could be regarded as an indication of the fact that one (or more than one) updated physical inputs (as for instance, nuclear cross sections, neutrino energy losses, conductive opacity and so on) adopted in these computations, is still affected by a large uncertainty.

Nevertheless, we think that, due to the large uncertainties on the observational measurements of both \([\text{Fe}/\text{H}]\) and the heavy elements distribution, there are not yet clear evidences that this is the case. Even if, a deeper insight on this topic is clearly out of the aim of the present work, we wish only to notice, as a warning, that it has been recently discussed (Fiorentini, Lissia & Ricci 1998) the possibility that the element diffusion coefficients adopted in the evolutionary codes, could be affected by a large uncertainty and it has been also shown by Castellani & Degl’Innocenti (1998) that these uncertainties can produce significant changes on the ZAHB luminosity. Therefore, we think that it does still exist in the parameters (adopted in stellar computations) space the possibility to obtain a better agreement between present results and these evolutionary computations.

As a final point, we wish to notice that the present results allow us to obtain relevant informations on both the slope and the zero point of the relation between the absolute visual magnitude of the RR Lyrae stars and the metallicity on a ground quite independent from similar analysis already performed on this concern. However, a deep insight on this topic is out of the aim of the present work so we will address it in a forthcoming paper (Cassisi & De Santis 1998).

It is evident that safer conclusions about the real luminosity level of the ZAHB could be obtained only when the sample of GCs with high accuracy photometry for both RR Lyrae stars and non-variable HB structures and a fine measurement of the RR Lyrae pulsational properties, will be larger. Since in the present work it has been clearly shown that the analysis of the pulsational properties of RR Lyrae stars allows us to evaluate the ZAHB luminosity with high accuracy and with an approach largely independent from evolutionary computations, this occurrence is strongly desired in order to finally achieve a large consensus on the most important Population II distance scale.
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Figure 2)
Figure 3)
Figure 4)
Figure 5)
M3

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure6}
\caption{Figure 6)
\end{figure}

\textbf{dashed} – CDS92’s relation

\textbf{solid} – CDS92’s relation shifted by +0.012
Figure 7)
Figure 8)
Figure 10)
Figure 11)
Figure 12)
Figure 14)
Figure 15)