Duan, Giedke, Cirac and Zoller and, independently, Simon have recently found necessary and sufficient conditions for the separability (classical correlation) of the Gaussian two-party (continuous variable) states. Duan et al remark that their criterion is based on a “much stronger bound” on the total variance of a pair of Einstein-Podolsky-Rosen-type operators than is required simply by the uncertainty relation. Here, we seek to formalize and test this particular assertion in both classical and quantum-theoretic frameworks. We first attach to these states the classical a priori probability (Jeffreys’ prior), proportional to the volume element of the Fisher information metric on the Riemannian manifold of Gaussian (quadrivariate normal) probability distributions. Then, numerical evidence (indicates that more than ninety-nine percent of the Gaussian two-party states do, in fact, meet the more stringent criterion for separability. We collaterally note that the prior probability assigned to the classical states, that is those having positive Glauber-Sudarshan $P$-representations, is less than one-thousandth of one percent. We, then, seek to attach as a measure to the Gaussian two-party states, the volume element of the associated (quantum-theoretic) Bures (minimal monotone) metric. Our several extensive analyses, then, persistently yield probabilities of separability and classicality that are, to very high orders of accuracy, unity and zero, respectively, so the two quite distinct (classical and quantum-theoretic) forms of analysis are rather remarkably consistent in their findings.

I. INTRODUCTION

Życzkowski, Horodecki, Sanpera and Lewenstein (ZHSL) [1] were the first to raise and address the issue of determining the proportion of quantum states which are — at least, in some appropriate a priori or natural sense — separable, that is, classically correlated. Such composite states can be approximated by convex combinations of product states. Inseparable states are also termed Einstein-Podolsky-Rosen correlated [2].

Though acknowledging the importance and intrinsic interest of the line of investigation opened by ZHSL, Slater [3] was critical of certain aspects of the methodology they employed. He conducted alternative analyses, using normalized volume elements of monotone metrics [4,5] on the quantum states, as candidates for prior probability distributions. It was found (although the various numerical calculations were marked by considerable instabilities [3, Tables 1-4]) that the (much discussed [6,7]) Bures (that is, minimal monotone) metric yielded the highest probabilities of separability for the $2 \times 2$ and $2 \times 3$ systems. (These probabilities — approximately, ten percent in the $2 \times 2$ case — were, nevertheless, substantially less than those arrived at by ZHSL, counterintuitively, to their “surprise”. Making use of “explicit” formulas of Dittmann [8] for the Bures metric, Slater [9] was then able to obtain exact Bures probabilities of separability for certain two-qubit scenarios.)

Both the approaches of ZHSL and Slater relied upon a positive partial transposition condition, shown to be sufficient for separability by Peres [10] and to be necessary as well by Horodecki [1] in the specific instances of the $2 \times 2$ and $2 \times 3$ systems, but not necessary in higher-dimensional systems.

Duan, Griedke, Cirac and Zoller [12] (cf. [14]) have recently found an apparently quite distinct separability criterion for two-party continuous variable (infinite-dimensional) systems. Such systems have served as the basis of many recent protocols for quantum communication and computation [14,15]. Duan et al asserted that it was quite difficult to ascertain if the partial transposition criterion is, in fact, met. However, Simon (in a slightly later prepublication [16], based on an earlier invited talk), was able to accomplish precisely this task, using a geometric interpretation of the partial transposition operator as mirror reflection in phase space.

The criterion of Duan et al — which is the one we study here — is based on the total variance of a pair of Einstein-Podolsky-Rosen type operators $(\hat{u}, \hat{v})$ of the type

$$\hat{u} = \frac{1}{\sqrt{2}} (a \hat{x}_1 \pm 1 \hat{x}_2), \quad \hat{v} = \frac{1}{\sqrt{2}} (a \hat{p}_1 \pm 1 \hat{p}_2),$$

(1)
where $a$ is assumed to be an arbitrary positive number, and $\hat{x}_j, \hat{p}_j (j = 1, 2)$ can be any local operators satisfying the commutators $[\hat{x}_j, \hat{p}_{j'}] = i2\delta_{jj'}$. The new variance criterion provides a sufficient condition for entanglement of any two-party continuous variables states. Furthermore, for those of these states which are Gaussian, this criterion turns out to be necessary as well.

More specifically, Duan et al transform the correlation matrix for the Gaussian two-party state, using a twofold application of local linear unitary Bogoliubov operations (LLUBOs). They then derive an inequality ([12, eq. (16)]) which must be satisfied for separability to hold. (A necessary and sufficient condition for this is that the transformed matrix minus the identity matrix be positive semidefinite.) In our numerical procedures below, we test whether or not this inequality is met. Then, weighting the (systematically generated) states by certain classical ("Jeffreys' prior") and quantum (Bures) measures, we obtain estimates of the proportion of Gaussian two-party states that are separable. As a collateral exercise, we test the initial untransformed matrices to see if they have positive $P$-representations. Again, using the classical and quantum measures as weights, we estimate the proportion of Gaussian two-party states that are classical in character.

II. BURES METRICS FOR CONTINUOUS VARIABLE SYSTEMS

So, it would appear natural, in light of the previous analyses of ZHSL [1] and Slater [3], to address the question of what proportion of the Gaussian two-party continuous variable states are (in)separable. Such states can always be parameterized as thermal squeezed states [17,18]. Twamley [19] has computed the Bures distance between two (one-mode) thermal squeezed states and deduced the statistical distance (Bures) metric of Braunstein and Caves [6].

The computation of Twamley was largely concentrated on determining the fidelity

$$ F(\rho_1, \rho_2) = \text{Tr} \sqrt{\rho_1^{1/2} \rho_2 \rho_1^{1/2}} $$

(the Bures distance itself being given by $2(1 - F(\rho_1, \rho_2))$) between two squeezed thermal states (having density matrices $\rho_1$ and $\rho_2$), where

$$ \rho(\beta, r, \theta) = ZS(r, \theta)T(\beta)S^\dagger(r, \theta) \quad (0 \leq \beta; 0 \leq r; -\pi < \theta \leq \pi) $$

$$ S(r, \theta) = \exp (\zeta K_+ - \zeta^* K_-), \quad T(\beta) = \exp (-\beta K_0), \quad \zeta = re^{i\theta} $$

and

$$ K_+ = \frac{1}{2} a^{\dagger 2}, \quad K_- = \frac{1}{2} a^2, \quad K_0 = \frac{1}{2}(a^{\dagger} a + \frac{1}{2}), \quad [K_0, K_{\pm}] = \pm K_{\pm}, \quad [K_-, K_+] = 2K_0. $$

Here $S(r, \theta)$ is the one-photon squeeze operator, $\hat{a}$ is the single-mode annihilation operator, $Z$ is chosen so that $\text{Tr}(\rho) = 1$, and $(K_0, K_{\pm})$ are the generators of the $SU(1,1)$ group. Twamley was able to express the fidelity [19] in the form [19, eq. (24)]

$$ F(\rho_1, \rho_2) = \frac{\sqrt{2} \sinh \frac{\beta_1}{4} \sinh \frac{\beta_2}{4}}{\sqrt{Y - 1}}, $$

where the $\beta$’s are proportional to inverse temperatures, and $Y$ corresponds to a certain (two-line) trigonometric expression.

Using the results of Twamley, Slater [20, eqs. (8), (9)] expressed the volume element (the "quantum Jeffreys’ prior") of the Bures metric in the form $f(r)g(\beta)$, where

$$ f(r) = \sinh 2r, \quad g(\beta) = \frac{\cosh \frac{\beta}{4} \coth \frac{\beta}{4} \text{sech} \frac{\beta}{4}}{8}. $$

(Subsequently, quantum Jeffreys’ priors have been determined, as well, for the displaced thermal states [21] and for the displaced squeezed thermal states [22]. Following work of Lavenda [23], high-temperature expansions of these priors were considered in [23].)

For the one-mode case, Paraoanu and Scutaru have expressed the fidelity in the form [25, eq. (31)]
where the $A$'s are $2 \times 2$ correlation matrices, and $P = (\det A_1 - 1)(\det A_2 - 1)$. In the case of two-mode thermal squeezed states with (diagonal) $4 \times 4$ correlation matrices $A_i = D_i$ with $i = 1, 2$, the fidelity was expressed as

$$F(\rho_1, \rho_2) = \frac{2}{\sqrt{\det(A_1 + A_2) + P - \sqrt{P}}} \quad (8)$$

which is the product of the fidelities of the corresponding one-mode thermal states.

It would, then, be of interest, proceeding along such lines, to extend the work of Twamley [14] to obtain the Bures (minimal monotone) metric on the two-party Gaussian states. The volume element of the metric could, then, serve as an a priori measure on these states. We shall seek to do this below but first it seems quite relevant to assess the relative separability/inseparability of these Gaussian states by proceeding in a purely classical/nonquantum fashion. (However, in the present absence of explicit formulas for the Bures metric on the two-party Gaussian states, we must rely upon numerical methods.)

III. CLASSICAL ANALYSIS

In sharp contrast to the quantum state-of-affairs [4], classically there is a unique monotone metric, given by the Fisher information (cf. [23]). The Jeffreys’ prior [27] (of Bayesian theory [28]) is, then, taken to be proportional to the volume element of the Fisher information metric. For the $n$-dimensional Gaussian (multivariate normal) probability distributions (with fixed vectors of means), Jeffreys’ prior is inversely proportional to the $\frac{1}{2}$-power of the determinant of the $n \times n$ covariance matrix $\rho_{nn}$ (This determinant is invariant under symplectic transformations. “The symplectic group acts unitarily and irreducibly on the two-mode Hilbert space” [10].)

To examine the separability properties of the two-party Gaussian states, vis-à-vis this distinguished classical measure, we randomly generated $4 \times 4$ real symmetric matrices ($M$), using uniform random variates drawn from intervals of the form $[0, k]$ for the four diagonal entries of $M$ and intervals of the type $[-l, l]$ for the six distinct off-diagonal entries of $M$. (In an extensive survey article, Holmes [30] has reviewed methods for generating random correlation matrices, that is covariance matrices with all their diagonal entries equal to unity. Let us note, however, that in the physics literature, the terminology “correlation matrix” usually includes what statisticians would more broadly call a variance/covariance matrix.) We, then, tested each such matrix for positive definiteness. If it proved to be positive definite (that is, its four eigenvalues were positive), we accepted it as a covariance matrix of a (zero-mean four-vector) Gaussian probability distribution. We, then, further tested $M$ to see that (in the notation of [12, eq. (8)]), the conditions $n, m \geq 1$ on the entries of the (locally unitarily) transformed Gaussian state were satisfied. If they were, we then examined if the quantity expressed in the form $a_0^2 - \frac{1}{n}$ was exceeded by the total variance of the pair of Einstein-Podolsky-Rosen-type operators stipulated by Duan et al, as required by the uncertainty relation (cf. [27]). (The determination of the parameter $a_0$ requires the solution of a pair of nonlinear simultaneous equations. Simon [14] suggests that, in this respect, his implementation of the Peres-Horodecki separability criterion is simpler than that of Duan et al.) Additionally, we checked (using inequality (16) of [12]) if the total variance exceeded the “much stronger bound” $a_0^2 + \frac{1}{n}$, necessary and sufficient for the separability of the associated Gaussian two-party state.

As an example (Table 1), for the choices $k = 15, l = 15$, we generated ten million $4 \times 4$ real symmetric matrices ($M$). Of these, 39,588 fulfilled the positive definiteness requirement and the inequality conditions (pertaining to the principal minors of $M$) on $n$ and $m$, and passed the test based on the uncertainty relation. Of those, 39,003 exceeded the (more demanding) separability criterion. Assigning the $\frac{1}{2}$-power of the determinant of the covariance matrix as the (Fisher information) weight to each of the 39,588 matrices, and normalizing by the sum of all these weights, we found the relative probability of separability to be .991365. Of the 39,003 separable states, 20,534 still were associated with $a_0^2 - \frac{1}{n}$.

Therefore, the probability allocated to the separable states was mostly concentrated on the 18,469 states that were nonclassical. It is of interest, in this context, to note that the argument of Duan et al relies upon the transformation using local linear unitary Bogoliubov operations (LLUBO’s) of the covariance matrix associated
with a separable state to a covariance matrix associated with a classical state (that is, one possessing a positive P-representation). Our results, then, clearly indicate that the property of possessing a positive P-representation is not necessarily preserved under the inversion of these LLUBO’s. (Gardiner [32, p. 328] leaves as an exercise to show “that the squeezed state never has a Glauber-Sudarshan P-function for any non-zero \( \zeta \). (It does have of course, a positive P-function.)”)

We have repeated this form of analysis for other choices of \( k \) and \( l \), as well. These results are reported in Table I. The thrust of the results is much the same, although the probability of separability is significantly reduced in the \( k = 30, l = 20 \) analysis. (We are not cognizant of any particular rationale for this observed reduction.) Rather remarkably, the small probabilities of finding a state with a positive Glauber-Sudarshan P-representation all fall within a quite narrow range of probabilities, for the different selections of \( k \) and \( l \).

IV. QUANTUM-THEORETIC ANALYSIS

We would now like to repeat the form of analysis above, but using instead of the volume element of the Fisher information matrix as a measure, the volume element of the Bures (minimal monotone) metric \( B \). To proceed, we are able, using the detailed prescriptions in [33], in particular, eqs. (3.1), (3.9) and (3.10) there, to construct (for a choice of \( k \) and \( l \)) the Gaussian form of the Schrödinger kernel for each (uniformly randomly generated covariance matrix of a Gaussian two-party quantum state) \( M \). (Note that \( G \), in the notation of [33], is the same as \( M^{-1} \).) Then, we approximate this nonnegative-definite kernel by \( m^2 \times m^2 \) matrices (\( \Gamma \)), for various choices of odd \( m \). For the values to substitute into the kernel, we used the intersection points of a regular square \( m \times m \) lattice, with unit spacing, centered at the origin. We computed the \( m^2 \) eigenvalues (\( \lambda \)’s) of \( \Gamma \), and, if they were all positive, normalized them to exactly sum to unity. (If not, we immediately proceeded onto the next randomly generated matrix.) As the volume element \( (V_{Bures}) \) of the Bures metric, we employed [20, eq. (2)]

\[
V_{Bures} \propto (\det \Gamma)^{-\frac{1}{2}} \prod_{1 \leq i < j \leq m^2} 1/(\lambda_i + \lambda_j). \tag{10}
\]

(Let us observe that Krüger [31] has presented a diagonalization of one-mode Gaussian states, based on Mehler’s formula, giving a bilinear generating function for the Hermite polynomials. It should be possible to extend this approach to the two-party Gaussian states, making use of multidimensional Hermite polynomials [34].) The formula (10) is derived from the general formula for the (infinitesimal) Bures distance [3]

\[
d_{Bures}^2(\rho, \rho + d\rho) = \sum_{i,j=1}^{n} \frac{1}{2} |\langle i|d\rho|j \rangle|^2 \frac{1}{\lambda_i + \lambda_j}, \tag{11}
\]

where \( |i \rangle \) denotes the eigenvectors of the \( n \times n \) density matrix \( \rho \) and \( < j | \) the corresponding complex conjugate (dual) vectors.

For \( k = 15, l = 15 \) and \( m = 3, 5, 7, 9, 11 \), we ran extensive analyses. All these yielded probabilities of separability and classicality essentially unity and zero, respectively. However, since in the computation of the Bures volume (10), long strings of products are computed, having very large and small outcomes, we did not have as much confidence in the results, from a numerical point of view, as we would hope. Therefore, we sought, additionally, to pursue an approach which it was believed should be rather robust [35] in this regard. We chose the parameters \( k = 15, l = 15 \) and \( m = 5 \). But now rather than using a square \( 25 \times 25 \) grid, we used five rectangular (but not, in general, square) \( 25 \times 25 \) grids. The five points (set to be the same along both axes), were now chosen randomly from the interval [-2,2]. Then, with each of these five random rectangular grids, we associated a \( 25 \times 25 \) matrix, by substituting the values at the intersection points of the grids into the Gaussian kernel, as described in [36]. The eigenvalues (\( \lambda \)’s) and the Bures volumes (10) were computed for each of the five cases. (If any of the eigenvalues were nonpositive, we immediately discarded the present randomly generated covariance matrix from consideration and proceeded onto the examination of the next one.) The five volume elements were ordered by magnitude, and we chose for our subsequent computations of probabilities of separability and classicality, two robust estimators of the true volume. One was the median of the five figures, and the other was the trimmed mean, obtained by averaging the second, third and fourth largest values (thus, of course, fully ignoring the smallest and largest estimates of the volume element).

Following this scheme, we generated 18,600,000 random real symmetric matrices. Of these, 74,431 were positive definite and did not violate the uncertainty relation. From them, we discarded 4,943, since negative eigenvalues
appeared in at least one of the five $25 \times 25$ matrices generated from the random grids. Of the remaining 69,488, there were 68,952 that were separable and 38,620 that were classical in nature. Applying the two (median and trimmed mean) robust estimators of the Bures volumes $\lambda_i$ as weights to these covariance matrices, we obtained for both estimators (to many places of precision), probabilities of separability equal to unity and probabilities of classicality that were zero. So, our robust calculations simply served to reconfirm our earlier (nonrobust) ones. In parallel with these computations, we recomputed the two probabilities, using the Fisher information metric. The probability of separability was now .86397 and that of classicality, .00968. We suspect that the rather noticeable changes from Table I are a consequence of the necessity to discard roughly seven percent of the covariance matrices. But we would think that if this discarding also affects the results based on the Bures metric, it should tend to similarly both decrease the probability of separability and increase that of classicality, effects, which if present at all, are imperceptibly small in nature.

In a study of finite-dimensional quantum systems, analogous to the investigation here (of infinite-dimensional systems), Slater found that the semiclassical probability of separability (based, in part, on the Fisher information metric) of the $2 \times 2$ systems was approximately thirty-six percent, while the quantum-theoretic (Bures/minimal monotone) probability was roughly ten percent. The use of other monotone metrics, such as the Kubo-Mori and maximal ones $\lambda_i$, led to further reductions in this probability. (Of the monotone metrics, the Bures/minimal one has been found to be the least noninformative and the maximal monotone metric, the most.) Proceeding along such lines, we repeated our form of analysis for $k = 15$, $l = 15$, $m = 7$, based on the single uniformly-spaced square grid for both these alternative forms of metrics. This involved replacing the term $(\lambda_i + \lambda_j)$, which is of course proportional to the arithmetic mean of the two eigenvalues, in (1) by $(\lambda_i \lambda_j)/(\log \lambda_i - \log \lambda_j)$ which is proportional to the logarithmic mean, for the Kubo-Mori case, and by $\lambda_i \lambda_j/(\lambda_i + \lambda_j)$, which is proportional to the harmonic mean, for the maximal monotone scenario. However, our initial analyses produced numerical errors (divisions by zero), and we have not yet pursued the matter further.

V. CONCLUDING REMARKS

Życzkowski has argued that if one controls for the participation ratio, $R(\rho) = 1/\text{Tr}(\rho^2)$, of density matrices $\rho$, then, the probability of separability should be essentially invariant for a broad class of natural measures on the space of $\rho$’s, that is, “the link between the purity of the mixed states and the probability of entanglement is not sensitive to the measure chosen”. (The participation ratio is unity for a pure state, and greater than unity for a mixed/impure state.) The results reported here seem even stronger in this regard, since both the classical (Fisher information) and quantum-theoretic (Bures) measures used give quite similar results in toto, without the need to control for the degree of purity.

It would be of interest to examine the cases of the $n$-party Gaussian states ($n > 2$), as well, in particular to ascertain if the a priori probability of separability decreases from its apparent high value for $n = 2$, but neither sufficient nor necessary conditions for separability appear to have been developed for them. Also, it would be desirable to conduct a classical (Fisher information metric) analysis of the one-mode thermal squeezed (Gaussian) states. This result could then be compared with the quantum-theoretic one of Twamley, based on the Bures (minimal monotone) metric.

As a possible caveat here, it should be pointed out that in [24], for the squeezed thermal states, the one-dimensional marginal $g(\beta)$ of the Bures volume element was found to be improper/unnormalizable over the full range of possible values of $\beta \in [0, \infty]$, as well as $f(r)$ over $r \in [0, \infty]$. This would strongly suggest that the volume element of the Bures metric (for which no explicit formula is presently available) for the Gaussian two-party states is also improper. (A similar conclusion would appear to hold in terms of Jeffreys’ prior.) Therefore, in some strict sense, it may be inappropriate to speak in terms of the probability of separability or classicality for the Gaussian two-party states in toto, but rather one would have to restrict oneself to some subset of these states, for which the corresponding volume element is, in fact, normalizable.

Although, of course, for the Gaussian one-party states, the concept of separability is not applicable, one may still inquire concerning the associated probability of classicality. We have investigated this question, using the Jeffreys’ prior, and were able to determine formally that in the limit as one considers all Gaussian one-party states, the probability of classicality converges to zero. This, of course, conforms strongly to our results for the Gaussian two-party states.

We would also like to point out the interesting question of whether or not the necessary separability (entropic) criterion of Cerf and Adami carries over from discrete to continuous systems. Our preliminary analyses, in the context of this study, using the general formula for the entropy of a quantum Gaussian state, appear to indicate
that it does not. This may not be altogether surprising, as “one may even claim that discrete entropies and continuous entropies are fundamentally different in their nature and, to some extent, in their meanings” [45, p. 33] (cf. [46]).

The statistical/Bayesian interpretation of the main results here would appear to be that it should take a very considerable amount of evidence (that is, a large number of repeated measurements) to convince one that an initially unknown Gaussian two-party state is, in fact, inseparable or classical in nature, the a priori assumption being (in the absence of any specific information available) that the state is, in all probability, both separable (classically correlated) and nonclassical.

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TABLE I. Probabilities of separability and classicality for Gaussian two-party states based on the volume element of the Fisher information metric, that is, Jeffreys' prior, for various choices of $k$ and $l$. These parameters determine the ranges $[0, k]$ and $[-l, l]$, of the diagonal and off-diagonal entries, respectively, of uniformly randomly generated real symmetric matrices. If these matrices are positive definite and do not violate the uncertainty relation, their aggregate separability and classicality properties are studied and tabulated here.

| random matrices | $k$  | $l$  | insep. and sep. | separable | classical | $\text{prob}_{\text{sep}}$ | $\text{prob}_{\text{classical}}$ |
|-----------------|------|------|-----------------|-----------|-----------|----------------|----------------|
| 500,000         | 10   | 5    | 58,835          | 57,663    | 27,646    | .993330        | .000001470     |
| 1,900,000       | 500  | 250  | 119,447         | 119,447   | 116,781   | 1.000000       | .000004259     |
| 5,200,000       | 20   | 10   | 650,718         | 648,475   | 451,820   | .998716        | .000003094     |
| 8,100,000       | 30   | 20   | 316,319         | 315,622   | 235,459   | .938819        | .000001701     |
| 10,000,000      | 15   | 15   | 39,558          | 39,003    | 20,534    | .991365        | .000003027     |