The structure of pentaquarks $P_c^+$ in the chiral quark model

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The recent experimental results of LHCb collaboration suggested the existence of pentaquark states with charmonium. To understand the structure of the states, a dynamical calculation of 5-quark systems with quantum numbers $IJ^P = \frac{1}{2}(\frac{1}{2})^\pm$, $\frac{3}{2}(\frac{1}{2})^\pm$ and $\frac{1}{2}(\frac{3}{2})^\pm$ is performed in the framework of chiral quark model with the help of gaussian expansion method. The results show that the negative parity states can be bound states while all of the positive parity states are the scattering states. The $P_c(4380)$ state is suggested to be the bound state of $\Sigma_c^* D$. Although the energy of $\Sigma_c^* D^*$ is very close to the mass of $P_c(4450)$, the inconsistent parity prevents the assignment. The calculated distances between quarks confirm the molecular nature of the states. Other five-quark bound states of the combination of $\Sigma_c D$ and $\Sigma_c^* D^*$ are also found in the region about 4.3GeV and 4.5GeV.

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I. INTRODUCTION

Since the report of $\Theta^+(1540)$ by several groups [1,3] about ten years ago, it brought lots of arguments during that time. Although the observation of pentaquark state $\Theta^+(1540)$ was not confirmed by the further experiments [4] (LEPS Collaboration still insisted on the existence of pentaquark $\Theta^+(1540)$ [3]), the study of pentaquark inspires new structures for hadrons, beyond the conventional quark configuration (qq or q¯q). Five-quark components in baryons was also studied which showed that the $qqqqq$ in ground state is more favorable than $qqq$ with $L = 1$ for 1/2− baryons [4].

Recently, the interesting in pentaquark is revived, because LHCb experiment reported the observation of two pentaquark states, denoted as $P_c^+(4380)$ and $P_c^+(4450)$, in the decay of $\Lambda_b^0 \to J/\psi K^- p$ [7]. The masses and widths of these two structures, appeared in the $J/\psi p$ invariant mass, are determined to be $4380 \pm 5 \pm 29$ MeV, 205±18 ± 86 MeV, and $4449 \pm 17 \pm 2.5$ MeV, 39±5 ± 19 MeV. The structure of the pentaquark states comes from the valence structure, $udc\bar{c}$, of $J/\psi p$. The possible quantum numbers $IJ^P$ of these two states are (3/2−, 5/2− ) or (5/2−, 3/2+). In fact, the hidden-charm pentaquark states have been predicted several years ago. In 2010, J. J. Wu et al. predicted several narrow resonances with hidden charm above 4 GeV, $N_c^*(4265)$, $N_c^*(4415)$, and $\Lambda_c^*(4120)$, in the framework of the coupled-channel unitary approach [8]. Z. C. Yang et al. also studied the possible existence of very loosely bound hidden-charm molecular baryons in the one-boson-exchange model, $\Sigma_c D^*$ and $\Sigma_c D$ states are proposed [8]. After the report of LHCb, a lot of theoretical are devoted to explain the nature of the two states. By using the boson exchange model, R. Chen et al. interpreted the two states as the molecular states, $\Sigma_c^*(2455) D^*$ and $\Sigma_c^*(2520) D^*$ with spin-parity $J^P = 3/2^−$ and $J^P = 5/2^+$, respectively [10]. The Bethe-Salpeter equation method was employed to study the $D \Sigma^*$ and $D^* \Sigma_c$ interactions, and the two states $P_c^+(4380)$ and $P_c^+(4450)$ are identified as $\Sigma_c^* D$ and $\Sigma_c(2455) D^*$ molecular states with quantum numbers $J^P = 3/2^−$ and $J^P = 5/2^+$, respectively [11]. In QCD sum rule approach, $P_c^+(4380)$ and $P_c^+(4450)$ were explained as hidden-charm pentaquark states with quantum numbers $J^P = 3/2^−$ and $J^P = 5/2^+$, respectively, by using diquark-diquark-antiquark type interpolating currents [12,13]. By analyzing the reaction $\Lambda_c^0 \to J/\psi K^- p$ with coupled-channel calculation, L. Roca et al. assigned the quantum numbers $J^P = 3/2^−$ to the state $P_c^+(4450)$ and concluded that $P_c(4450)^+$ state is a molecular state of mostly $\Sigma_c D^*$ and $\Sigma_c D^*$ with 3/2− [13]. In the soliton approach, the hidden-charm state with quantum numbers $IJ^P = \frac{1}{2}(\frac{3}{2})^\pm$ was shown to exist and is compatible with $P_c^+(4380)$, but the state with $IJ^P = \frac{1}{2}(\frac{5}{2})^+$ has much higher mass compared with that of $P_c^+(4450)$ [14]. The small mass splitting between $P_c^+(4380)$ and $P_c^+(4450)$ can be understood in the diquark-triquark model by using an effective diquark-triquark Hamiltonian based on spin-orbital interaction [14]. Non-resonance explanations of the structures observed experimentally were also proposed [13,15].

Based on the theory of QCD, it is possible to excite quark-antiquark pairs from vacuum to form hadronic state. For the light quark-antiquark pair excitation, the effect can be absorbed into the parameters in the quark model description. For the heavy quark-antiquark pair excitation, it is too difficult to occur in the light hadron system and its effect cannot be absorbed by the model parameters. So the states $P_c^+$ reported by LHCb should be genuine pentaquarks. Its study will provide us more information of the underlying fundamental theory of strong interaction, QCD.

The most common approach to multiquark system is quark model. After fifty years development and with the accumulation of experimental data on multiquark states, to tackle the problem of multiquark seriously in the framework of quark model is expected. In the present work, the chiral quark model is used to study the pentaquark states with hidden-charm. Different from other
approaches, no prior spacial structure of the state is assumed, the structure is determined by the system dynamics. For this purpose, a powerful method of few-body system, gaussian expansion method (GEM) \(^\text{[19]}\) is employed to do the calculation. The GEM has been successfully applied to many few-body systems, light nuclei, hypernuclei, hadron physics and so on \(^\text{[19]}\). It suits for both of compact multi-quark systems and loosely bound molecular states.

The structure of the paper is as follows. In section II the quark model, wavefunctions and calculation method is presented. Section III is devoted to the calculated results and discussions. A brief summary is given in the last section.

## II. MODEL AND WAVE FUNCTION

The chiral quark model has acquired great achievement both in describing the hadron spectra and hadron-hadron interaction. Here we apply it to 5-quark system. The details of the model can be found in Ref.\(^\text{[20]}\). The Hamiltonian for multiquark system takes the form

\[
H = \sum_{i=1}^{n} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{j>i=1}^{n} \left[ V_{\text{CON}}(r_{ij}) + V_{\text{OGE}}(r_{ij}) + V_{\chi}(r_{ij}) + V_{\sigma}(r_{ij}) \right],
\]

where \( V_{\text{CON}}(r_{ij}) = \lambda_i^j \cdot \sigma^j \left[ -a_c(1-e^{-\mu r_{ij}}) + \Delta \right], \)

\[
V_{\text{OGE}}(r_{ij}) = \frac{1}{4} \alpha_s \lambda_i^j \cdot \lambda_j^c \left[ \frac{1}{r_{ij}} - \frac{1}{6m_s \sigma_i \cdot \sigma_j} e^{-r_{ij}/r_0(\mu)} \right], \quad r_0(\mu) = r_0/\mu, \quad \alpha_s = \frac{\alpha_0}{\ln(\mu^2/\Lambda_5^2)},
\]

\[
V_{\chi}(r_{ij}) = v_\chi(r_{ij}) \sum_{a=1}^{3} (\lambda_i^a \cdot \lambda_j^a) + v_K(r_{ij}) \sum_{a=4}^{7} (\lambda_i^a \cdot \lambda_j^a) + v_\eta(r_{ij}) \left[ \cos \theta_P (\lambda_i^8 \cdot \lambda_j^8) - \sin \theta_P \right],
\]

\[
v_{\chi}(r_{ij}) = \frac{g_{\chi}}{4\pi} \frac{m_\chi^2}{12m_i m_j} \left[ \frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2} \right] \left[ Y(m_\chi r_{ij}) - \frac{\Lambda_\chi}{m_\chi} Y(\Lambda_\chi r_{ij}) \right] (\sigma_i \cdot \sigma_j). \quad \chi = \pi, K, \eta
\]

Here \( T_{CM} \) is the center of mass kinetic energy, \( \mu \) is the reduced mass of two interacting quark pair. Because we are interested in the ground state of multiquark system, only the central part of the interaction is given above. All the symbols take their usual meanings. The model parameters of the model are taken from Ref.\(^\text{[20]}\) and are listed in Table I.

The wavefunctions for the system are constructed in the following way. First the 5-quark system is separated into two clusters, one with 3 quarks and another with quark-antiquark. The wavefunctions for these sub-clusters can be easily written down. Then two clusters are coupled and anti-symmetrized (if necessary) to form the total wavefunction of 5-quark system. Clearly, there are other ways to construct the wavefunctions of the system. However, it makes no difference by choosing any one configuration that if we use enough bases for system during the calculation.

For the 5-quark system with quark content \( uud\bar{c} \), there are types of separation, one is \( (ud)(\bar{c}u) + (uc)(\bar{c}d) \) and the other is \( (uud)(\bar{c}\bar{c}) \). Due to the large mass difference between \( c \) quark and \( u, d, s \) quarks, the flavor SU(4) symmetry is strongly broken. Here we just construct the flavor wavefunctions of system based on the flavor SU(2) symmetry. The flavor wavefunctions for the sub-clusters constructed are shown below.

- \( |B_{11}\rangle = uuc, \quad |B_{10}\rangle = \frac{1}{\sqrt{2}}(ud + du)c, \quad |B_{-1}\rangle = ddc, \quad |B_{00}\rangle = \frac{1}{\sqrt{2}}(ud - du)c, \)

\[
|B_{1\frac{1}{2}, \frac{1}{2}}\rangle = \frac{1}{\sqrt{6}} (2uud - udu - dud), \quad |B_{1\frac{1}{2}, -\frac{1}{2}}\rangle = \frac{1}{\sqrt{2}}(ud - du)u, \quad |B_{2\frac{1}{2}, \frac{1}{2}}\rangle = \bar{c}u, \quad |B_{2\frac{1}{2}, -\frac{1}{2}}\rangle = \bar{c}d, \quad |B_{00}\rangle = \bar{c}c.
\]

The flavor wavefunctions for 5-quark system with isospin \( I = 1/2 \) are obtained by the following couplings,

\[
|\chi_{1\frac{1}{2}}^1\rangle = \sqrt{\frac{2}{3}} |B_{11}\rangle |M_{\frac{1}{2} - \frac{1}{2}}\rangle - \sqrt{\frac{1}{3}} |B_{10}\rangle |M_{\frac{1}{2} + \frac{1}{2}}\rangle, \quad |\chi_{1\frac{1}{2}}^2\rangle = |B_{00}\rangle |M_{\frac{1}{2} - \frac{1}{2}}\rangle, \quad (5)
\]

\[
|\chi_{2\frac{1}{2}}^1\rangle = |B_{11}\rangle |M_{\frac{1}{2} + \frac{1}{2}}\rangle, \quad |\chi_{2\frac{1}{2}}^2\rangle = |B_{00}\rangle |M_{\frac{1}{2} - \frac{1}{2}}\rangle, \quad |\chi_{2\frac{1}{2}}^3\rangle = |B_{21}\rangle |M_{00}\rangle.
\]
In a similar way, the spin wavefunctions for 5-quark system can be constructed,

\[
|\chi_{3/2}^1(5)\rangle = \frac{\sqrt{1}}{\sqrt{6}}|\chi_{1}^{1-}(3)|\chi_{10}^1\rangle - \frac{\sqrt{1}}{3}|\chi_{2}^{1-}(3)|\chi_{10}^2\rangle + \frac{\sqrt{1}}{2}|\chi_{3/2}^{1-}(3)|\chi_{10}^3\rangle
\]

\[
|\chi_{3/2}^2(5)\rangle = \frac{1}{3}|\chi_{1}^{1-}(3)|\chi_{10}^1\rangle - \frac{2}{3}|\chi_{2}^{1-}(3)|\chi_{10}^2\rangle - \frac{1}{6}|\chi_{3}^{1-}(3)|\chi_{10}^3\rangle
\]

\[
|\chi_{3/2}^3(5)\rangle = \frac{1}{3}|\chi_{1}^{1-}(3)|\chi_{10}^1\rangle - \frac{2}{3}|\chi_{2}^{1-}(3)|\chi_{10}^2\rangle - \frac{1}{6}|\chi_{3}^{1-}(3)|\chi_{10}^3\rangle
\]

\[
|\chi_{3/2}^4(5)\rangle = \frac{1}{3}|\chi_{1}^{1-}(3)|\chi_{10}^1\rangle - \frac{2}{3}|\chi_{2}^{1-}(3)|\chi_{10}^2\rangle - \frac{1}{6}|\chi_{3}^{1-}(3)|\chi_{10}^3\rangle
\]

with the spin wavefunctions for 3-quark and 2-quark subclusters,

\[
|\chi_{3/2}^1(3)\rangle = \frac{1}{\sqrt{3}}(\alpha\beta\alpha + \alpha\beta\alpha + \beta\alpha\alpha),
\]

\[
|\chi_{3/2}^2(3)\rangle = \frac{1}{\sqrt{3}}(\alpha\beta\alpha + \beta\alpha\beta + \beta\alpha\beta),
\]

\[
|\chi_{3/2}^3(3)\rangle = \frac{1}{\sqrt{3}}(\alpha\beta\beta + \beta\alpha\beta + \beta\alpha\beta),
\]

\[
|\chi_{3/2}^4(3)\rangle = \frac{1}{\sqrt{6}}(2\alpha\beta - \alpha\beta\alpha - \beta\alpha\alpha),
\]

\[
|\chi_{3/2}^5(3)\rangle = \frac{1}{\sqrt{6}}(\alpha\beta\alpha - \beta\alpha\beta - \beta\alpha\alpha),
\]

For the color wavefunction, only the color singlet channel, two clusters are all colorless, is used here. The reason for this simplification comes from that the color singlet channel states are complete when all the excitation of other degrees of freedom are included for the multiquark systems, the energies for the excited states are rather high and have small effect on the group states. Then the color wavefunction of the system is

\[
\chi_c = \frac{1}{\sqrt{6}}(rgb - rbg + gbr - grb + brg - bgr),
\]

As for the orbital wavefunctions, we do not separate the motions of particles in the system into internal and relative ones and freeze the internal motion, the structure of the system is assumed priorly, as the most work did. In the present work, the orbital wavefunctions for each motion of the system are determined by the dynamics of the system, so does the structure. Another reason for not including the hidden-color channels in the calculation is that the direct extension of interactions between quark pairs from colorless states to colorful states are questionable, it will lead to too much bound states [21].

The orbital wavefunctions for this purpose is obtained as follows,

\[
\psi_{LMl} = \left[\left(\phi_{n_1, l_1}(\rho)\phi_{n_2, l_2}(\lambda)\right)_{\left|l_1 l_2 \right|} \phi_{n_3, l_3}(R)\right]_{LMl},
\]

where the Jacobi coordinates are defined as,

\[
\rho = x_1 - x_2,
\]

\[
\lambda = x_3 - \left(\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}\right),
\]

\[
r = x_4 - x_5,
\]

\[
R = \left(\frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}\right) - \left(\frac{m_4 x_4 + m_5 x_5}{m_4 + m_5}\right).
\]

To find the orbital wavefunctions, the Gaussian expansion method (GEM) is employed, i.e., each \(\phi\) is expanded by gaussians with various sizes [10]

\[
\phi_{nlm}(r) = \sum_{n=1}^{\infty} c_n N_{nl} r^l e^{-r/r_n^2} Y_{lm}(\hat{r}),
\]

where \(N_{nl}\) is normalization constants

\[
N_{nl} = \left[\left(2l+1\right)2^{l+1} (2\nu_n + l + 1)\right]^{1/2}.
\]
The size parameters \( r_n \) are taken as the geometric progression numbers
\[
\nu_n = 1/r_n^2 \quad r_n = r_{\text{min}}a^{n-1}.
\]
\( c_n \) is the variational parameters, which is determined by the dynamics of the system. Finally, the complete channel wave function for the 5-quark system is written as
\[
\Psi_{JM,i,j,n} = \mathcal{A} \left[ \chi_S^{\nu_i}(5) \psi_L \right]_{JM,j} \chi_X^{\nu_n}
\]
where \( \mathcal{A} \) is the antisymmetry operator of the system, it can be written as
\[
\mathcal{A} = 1 - (15) - (25)
\]
for \((udc)(\bar{c}u)\) case and
\[
\mathcal{A} = 1 - (13) - (23)
\]
for \((uud)(c\bar{c})\) case.

The energy-eigen of the system is obtained by solving the following eigen-equation
\[
H\Psi_{JM} = E\Psi_{JM},
\]
by using variational principle. The eigen functions \( \Psi_{JM} \) are the linear combination of the above channel wavefunctions.

When the angular momenta are not all zero, the calculation of the matrix elements of Hamiltonian is rather complicated. Here a new useful method named the infinitesimally-shifted Gaussian (ISG) are used \[10\]. In this method, the orbital wavefunctions are written as
\[
\phi_{nlm}(r) = N_{nl} \lim_{\epsilon \to 0} \frac{1}{(2\pi)^{3/2}} \sum_{k=1}^{k_{\text{max}}} C_{lm,k} e^{-\nu_n (r - r D_{lm,k})^2}
\]
the coefficients \( C_{lm,k} \) and shift-direction vector \( D_{lm,k} \) are dimensionless. By absorbing the spherical harmonic function into the shifted gaussians, the calculation becomes easy with no tedious angular-momentum algebra required.

\section*{III. RESULTS AND DISCUSSIONS}

In the present calculation, we are interested in the low-lying states of \( uudc\bar{c} \) pentaquark system, so the total orbital angular momentum \( L \) is limited to be 0 and 1. For \( L = 0 \), all of \( l_1, l_2, l_3, l_4 \) are 0 and for \( L = 1 \), only one of \( l_1, l_2, l_3, l_4 \) can be 1. In this way, the total angular momentum \( J \) can take values \( 1/2, 3/2 \) and \( 5/2 \). The possible channels under the consideration are listed in Table II. The single channel and channel coupling calculations are performed in this work. The results are shown in Tables III. The tables gives the eigen-energies of the states (column 3), along with the theoretical (column 4) and experimental thresholds (column 6), the binding energies (column 5) and the corrected energies of the states (column 7), which are obtained by taking the sum of experimental thresholds and the binding energies. Table IV gives the spacial configurations of the states. In the following we analyse the results in detail.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
\( J^P \) & \( (LM_\ell; SM_\ell) \) \\
\hline
\hline
\( \frac{1}{2} - \) & \( (00; \frac{1}{2}^-) \) \\
\hline
\( \frac{1}{2} + \) & \( (\frac{1}{2}; \frac{1}{2}^-) \) \\
\hline
\( 1 + \) & \( (\frac{5}{2}; \frac{1}{2}^-) \) \\
\hline
\hline
\end{tabular}
\caption{The channels under the consideration.}
\end{table}

For the parity negative states, the results are shown in Table III.

(a) \( J^P = \frac{1}{2}^- \): For \( N \eta_c, N J/\psi, \Lambda_c D \) and \( \Lambda_c D^* \) states, the single-channel calculation shows that no bound state can be formed. For \( \Sigma_c D, \Sigma_c D^* \) and \( \Sigma_c^* D \) states, bound states with \(-3 \sim -4\) MeV biding energies appear. The channel-coupling is rather weak, it does not push the state \( N \eta_c \) or \( \Lambda_c D \) down enough to form a bound state, and it also does not push the state \( \Sigma_c D \) up above the threshold. So the present calculation shows that there is a resonance \( \Sigma_c D \) with resonance energy 4315 MeV for \( J^P = \frac{1}{2}^- \). The result is in agreement with that of Ref.[9]. Although the energy of \( \Sigma_c D^* \) is very close to that of \( P_c(4450) \), it is difficult to make the assignment because of the different parities.

(b) \( J^P = \frac{3}{2}^- \): The similar results with the case of \( J^P = \frac{1}{2}^- \) are obtained. \( N J/\psi, \Lambda_c D^* \) states are unbound and all \( \Sigma_c D^* \)s are bound states. The channel-coupling pushes down the state \( \Sigma_c^* D \) a little. So a resonance, \( \Sigma_c^* D \), is shown up. After the correction, the resonance energy is 4382 MeV, which is very close the mass of \( P_c^+(4380) \), which was claimed by LHCb collaboration \[1\]. However, the large decay width of \( P_c^+(4380) \) cannot be explained in the present calculation. The decay width of \( \Sigma_c^* D \) state to \( N J/\psi, \Lambda_c D^* \) are estimated to several MeVs due to the weak channel-coupling. Because of the missing of the spin-orbit interaction in the present calculation, the energies of \( N J/\psi, \Sigma_c D^* \) with \( J^P = \frac{3}{2}^- \) are the same as that of \( \Sigma_c D^* \), \( N J/\psi \) with \( J^P = \frac{1}{2}^- \).

(c) \( J^P = \frac{5}{2}^- \): Only one channel: \( \Sigma_c^* D^* \), remains in this case if all orbital angular momenta are set to zero. Bound state is obtained as before. Although the binding energy is small, \(-3\) MeV, the decay width of the state may be small, \( 10 \sim 20 \) MeV, due to the small decay widths of its constituents, \( \Sigma_c^* \) (\( \Gamma_{\Sigma_c^* \to \Lambda_c \pi} \sim 15\) MeV) and \( D^* \) (\( \Gamma_{D^* \to D^* \pi} \sim 1\) MeV). So it is a good candidate of the
1, 2, 3 form a cluster, Σ(1, 2, 3) and 4, 5 are 1.5–2.4 fm. Clearly, quark and 3 is around 0.8 fm, and the distances between quark 4 and 5 are investigated by LHCb collaboration. Nevertheless, the large decay width of P_{c}^{+}(4380), is out of reach of the present picture. The mass of molecule state Σ_{c}^{+}D with J^{P}= \frac{1}{2}^{+} is very close to that of state P_{c}^{+}(4380), a pentaquark announced by LHCb collaboration. The distances between quark pairs suggest a molecular structure for these resonances. A sound interpretation of P_{c}^{+}(4380) is the molecule of Σ_{c}^{+}D with J^{P}= \frac{1}{2}^{+}. However, the large decay width of P_{c}^{+}(4380), 265±18 ± 86 MeV, is out of reach of the present picture. The mass of molecule state Σ_{c}^{+}D with J^{P}= \frac{1}{2}^{+} is also close to that of P_{c}^{+}(4380), another pentaquark reported by LHCb collaboration. Nevertheless, the opposite parity of the state to the P_{c}^{+}(4380) may prevent this assignment. Meanwhile all the positive parity states are all unbound in our calculation.

In the present calculation, the spacial structure of a heavy pentaquark with high spin.

For the parity positive states, one of the orbital angular momenta is 1. Generally it is difficult to form a bound state in this case. Our calculation shows that all the states under investigation are not bounded. So if the state P_{c}^{+}(4450) is identified as a pentaquark state with positive parity, the chiral quark model may be needed to be modified for multi-quark system. The non-resonance explanation of the narrow structure at 4.45 GeV was also compatible with the kinematical effects of the rescattering from χ_{c}P to J/ψp [18].

To find the structure of the resonances obtained in the present work, the distances between any two quarks are calculated. The results are shown in Table IV. From the table, we can see that the distances among quarks 1, 2 and 3 is around 0.8 fm, and the distances between quark 4 and antiquark are 0.6–0.7 fm, while the distances between quarks 1, 2, 3 and 4, 5 are 1.5–2.4 fm. Clearly, quark 1, 2, 3 form a cluster, Σ(∗) and quark 4, 5 form another cluster, D(∗), then two clusters combine to a pentaquark state. To describe P_{c}^{+}(4380) as molecular state of Σ_{c}^{+}D is reasonable.

Fig. 1 shows the correlation functions of two clusters for J^{P}= \frac{1}{2}^{+}. Typical behavior of the wavefunction for bound state is obtained.

### TABLE IV: Summary

| J^{P} | Channel | Eigen Energy E_{th} (Theo.) | Binding Energy | E_{th} (Exp.) | Corrected Energy |
|-------|---------|-----------------------------|----------------|--------------|------------------|
| \( \frac{1}{2} + \) | \( \chi_{1}^{ij} \chi_{j}^{kl} \) | i = 4, 5, j = 3, 4 | 3745 | 0 | 3919(N\eta_{c}) | 3919 |
| \( \frac{1}{2} + \) | \( \chi_{1}^{ij} \chi_{j}^{kl} \) | i = 2, 3, j = 3, 4 | 3841 | 0 | 4036(NJ/ψ) | 4036 |
| \( \frac{1}{2} + \) | \( \chi_{1}^{ij} \chi_{j}^{kl} \) | i = 4, 5, j = 2 | 3996 | 0 | 4151(ΛcD) | 4151 |
| \( \frac{1}{2} + \) | \( \chi_{1}^{ij} \chi_{j}^{kl} \) | i = 2, 3, j = 2 | 4115 | 0 | 4293(ΛcD') | 4293 |
| \( \frac{1}{2} + \) | \( \chi_{1}^{ij} \chi_{j}^{kl} \) | i = 4, 5, j = 1 | 4398 | -4 | 4320(ΣcD) | 4316 |
| \( \frac{1}{2} + \) | \( \chi_{1}^{ij} \chi_{j}^{kl} \) | i = 2, 3, j = 1 | 4518 | -3 | 4462(ΣcD') | 4459 |
| \( \frac{1}{2} + \) | \( \chi_{1}^{ij} \chi_{j}^{kl} \) | i = 1, j = 1 | 4563 | -3 | 4527(ΣcD') | 4524 |

| \( \frac{1}{2} + \) | \( \chi_{2}^{ij} \chi_{j}^{kl} \) | i = 3, 4, j = 3, 4 | 3841 | 0 | 4036(NJ/ψ) | 4036 |
| \( \frac{1}{2} + \) | \( \chi_{2}^{ij} \chi_{j}^{kl} \) | i = 3, 4, j = 2 | 4115 | 0 | 4293(ΛcD') | 4293 |
| \( \frac{1}{2} + \) | \( \chi_{2}^{ij} \chi_{j}^{kl} \) | i = 3, 4, j = 1 | 4518 | -2 | 4462(ΣcD') | 4460 |
| \( \frac{1}{2} + \) | \( \chi_{2}^{ij} \chi_{j}^{kl} \) | i = 2, j = 1 | 4444 | -3 | 4385(ΣcD) | 4382 |
| \( \frac{1}{2} + \) | \( \chi_{2}^{ij} \chi_{j}^{kl} \) | i = 1, j = 1 | 4564 | -2 | 4527(ΣcD') | 4525 |
| \( \frac{1}{2} + \) | \( \chi_{3}^{ij} \chi_{j}^{kl} \) | i = 1, j = 1 | 4563 | -3 | 4527(ΣcD') | 4524 |

![Fig. 1: The correlation functions of ΣcD*, ΣcD and ΣcD* with J^{P}= \frac{1}{2}^{+}.](image-url)
TABLE IV: Distances between any two quarks (unit: fm).

| $J^P$ Channel | $r_{12}$ | $r_{13}$ | $r_{14}$ | $r_{15}$ | $r_{34}$ | $r_{35}$ | $r_{45}$ |
|---------------|---------|---------|---------|---------|---------|---------|---------|
| $\frac{1}{2}^-$ $\chi_{1/2}^{\sigma_i} \chi_{j}^{f}$, $i = 4, 5$, $j = 1$ ($\Sigma_c D$) | 0.8 | 0.7 | 1.5 | 1.6 | 1.9 | 1.8 | 0.6 |
| $\chi_{1/2}^{\sigma_i} \chi_{j}^{f}$, $i = 2, 3$, $j = 1$ ($\Sigma_c D^*$) | 0.8 | 0.7 | 1.7 | 1.6 | 2.1 | 1.8 | 0.7 |
| $\chi_{1/2}^{\sigma_i} \chi_{j}^{f}$, $i = 1$, $j = 1$ ($\Sigma_c^* D^*$) | 0.9 | 0.8 | 1.6 | 1.6 | 2.0 | 1.7 | 0.7 |
| $\frac{1}{2}^-$ $\chi_{5/2}^{\sigma_i} \chi_{j}^{f}$, $i = 3, 4$, $j = 1$ ($\Sigma_c D^*$) | 0.8 | 0.7 | 1.9 | 1.7 | 2.3 | 2.0 | 0.7 |
| $\chi_{5/2}^{\sigma_i} \chi_{j}^{f}$, $i = 2$, $j = 1$ ($\Sigma_c D^*$) | 0.9 | 0.8 | 1.5 | 1.8 | 2.1 | 2.0 | 0.6 |
| $\chi_{5/2}^{\sigma_i} \chi_{j}^{f}$, $i = 1$, $j = 1$ ($\Sigma_c^* D^*$) | 0.9 | 0.8 | 2.1 | 2.1 | 2.7 | 2.3 | 0.7 |
| $\frac{1}{2}^-$ $\chi_{5/2}^{\sigma_i} \chi_{j}^{f}$, $i = 1$, $j = 1$ ($\Sigma_c^* D^*$) | 0.9 | 0.8 | 1.9 | 1.8 | 2.4 | 2.0 | 0.7 |

5-quark system is not assumed in advance. Although the two colorless sub-clusters are used, the internal structures of the sub-clusters are not fixed. Generally the state in this approach will have smaller energy than it in other approaches, because of the larger space.

As a preliminary work, the spin-orbit and tensor interactions are not included in the calculation. For parity negative states, their effects are expected to be zero or small. For parity positive state, it will play a minor role. To understand the nature of $P_c^c$=(4450) in quark model approach, the calculation with including the spin-orbit interaction is needed, which in progress in our group.

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