Abstract—In the last years, the adoption of active systems has increased in many fields of computer science, such as databases, sensor networks, and software engineering. These systems are able to automatically react to events, by collecting information from outside and internally generating new events. However, the collection of data is often hampered by uncertainty and vagueness that can arise from the imprecision of the monitoring infrastructure, unreliable data sources, and networks. The decision making mechanism used to produce a reaction is also imprecise, and cannot be evaluated in a crisp way. It depends on the evaluation of vague temporal constraints, which are expressed on the collected data by humans. Despite fuzzy logic has been mainly conceived as a mathematical abstraction to express vagueness, no attempt has been made to fuzzify the temporal modalities. Existing fuzzy languages do not allow us to represent temporal properties, such as “almost always” and “soon”. Indeed, the semantics of existing fuzzy temporal operators is based on the idea of replacing classical connectives or propositions with their fuzzy counterparts. To overcome these limitations, we propose a temporal framework, FTL (Fuzzy-time Temporal Logic), to express vagueness on time. This framework formally defines a set of fuzzy temporal modalities, which can be customized by choosing a specific semantics for the connectives. The semantics of the language is sound, and the introduced modalities respect a set of expected mutual relations. We also prove that under the assumption that all events are crisp, FTL reduces to LTL. Finally, for some of the possible fuzzy interpretations of the connectives, we identify adequate sets of temporal operators, from which it is possible to derive all the others.

I. INTRODUCTION

In the last years, the adoption of active systems has increased in many fields of computer science. Active systems must automatically react to achieve or maintain their requirements, depending on the information collected from the surrounding environment. Examples of such systems are active databases [1], active sensor networks [2], and smart grids [3]. For instance, smart grids may need to adjust the workload on the appliances (e.g., fridge, oven) available in a building to optimize energy consumption and costs.

Event-driven architectures [4] are a common architectural paradigm to design active systems. This paradigm is based on the idea that the actions the system will perform are generated as a reaction to the events occurred inside and outside the system. In many cases, providing such active functionality requires to materialize the occurrence of other relevant events, according to a set of inference rules. These rules are generally defined by domain experts, and are formalized by designers. Domain experts must provide the set of basic events to be collected, which serve as input to the rules, their inter-relationships, and the parameters of the rules for determining a new event materialization.

However, the collection of data is often hampered by uncertainty and vagueness that can arise from the imprecision of the monitoring infrastructure, unreliable data sources, and networks. The inference rules that are used to produce a reaction are also imprecise. They often depend on the evaluation of untimed or temporal properties that are vague, since they are expressed by humans, and, for this reason, cannot be assessed in a crisp way. For example, a smart grid must satisfy the following property: “all appliances must be available almost always”. This rule is vague since the concept of availability cannot be precisely assessed, because it may depend on the perception of the customers. The temporal period (“almost always”), during which the availability property must be satisfied, is vague as well. For these reasons, it becomes fundamental to identify a suitable formalism to represent vague properties as suitable untimed or temporal formulae.

Fuzzy logic has been conceived as a mathematical abstraction to express vagueness in the satisfaction of formulae. While the propositional fuzzy logic has been deeply investigated, the fuzzy version of the temporal modalities has been often neglected. Few attempts [5], [6], [7], [8], [9] to manage time have been made, but all these approaches just focus on the uncertainty of the information and do not take into account the truth degree of temporal expressions. The semantics of existing fuzzy temporal operators is based on the idea of replacing classical connectives or propositions with their fuzzy counterparts. Existing fuzzy languages do not allow us to represent additional temporal properties, such as “almost always”, “soon”. This kind of modalities may be useful when we need to specify situations when a formula is slightly satisfied, since an event happens a little bit later than expected, when a property is always satisfied except for a small set of time instants, or a property is maintained for a time interval which is slightly smaller than the one
To overcome these limitations, we propose a temporal framework, FTL (Fuzzy-time Temporal Logic), to express vagueness on time. This framework formally defines a set of fuzzy temporal modalities, which can be customized by choosing a specific semantics for the connectives. The semantics of the language is sound, and the introduced modalities respect a set of expected mutual relations. We also prove that under the assumption that all events are crisp, FTL reduces to LTL. Finally, for some of the possible fuzzy interpretations of the connectives, we identify an adequate set of temporal operators, from which it is possible to derive all the others.

The paper is organized as follows. Section II discusses some related work. Section III provides some background knowledge about fuzzy logic and points out its differences w.r.t. probability theory. Section IV presents the FTL framework, by illustrating some interesting properties of the operators it introduces. Section V identifies an adequate set of connectives for some classical interpretations of the connectives. Note that proofs of propositions are given in a sketch form, and minor details are left to the reader. Section VI provides some example of possible FTL specifications in the context of smart grids, and Section VII concludes the paper.

II. RELATED WORK

In computer science, fuzzy logic has been mainly used to represent the uncertainty due to the unpredictability of the environment or the imprecision of the measurements. Many attempts have been made to use fuzzy logic to monitor the satisfaction of temporal properties of the system and/or the environment. For example, Lamine and Kabanza add, for each classic temporal operator (e.g., always, eventually, until, etc.), a corresponding fuzzy temporal one. This operator keeps the same semantics of its crisp counterpart, with the only difference that the Boolean connectives (not, and, or) are replaced with the corresponding operations in the Zadeh interpretation (see operations associated respectively with negation, t-norm and t-conorm in Table I). The authors evaluate a fuzzy proposition over a history (i.e., a sequence of states) and associate a weight with the evaluation made at each state. The weights and the extent to which the history is needed to evaluate a proposition are defined empirically, depending on the application and the properties expressed by the proposition itself. Similarly, Thiele and Kalenka define a fuzzy “interpretation” of the traditional temporal operators. They also introduce proper fuzzy temporal operators to represent the short or long time distance in which a specific property must be satisfied (in the future or in the past). Despite the aforementioned approaches are a first step towards the fuzzification of time, they do not associate a specific fuzzy semantics with the temporal modalities. Instead, temporal modalities have a fuzzy semantics only depending on the interpretation given to their (sub-)argument, which is an untimed fuzzy formula.

Other works have a slightly different objective. They use fuzzy temporal logic to express uncertainty about the time in which some specific events may occur and the temporal relationships among events and states. Dutta defines the occurrence of an event as the possibility of its occurrence in any time interval. This way the authors can evaluate a set of temporal relations between a pair of events: if an event precedes/follows another one, the degree an event overlaps another one, or whether an event immediately follows another one. Similarly, Dubois and Prade represent dates as a possibility distribution. Hence, it is possible to express different situations: whether a date is precisely known or not (i.e., it is within an interval), whether a date is fuzzily known (i.e., the interval boundaries that contain the date are not clearly known), or whether a date is attached to an event that may not occur. From this representation the authors use fuzzy sets to represent time points that are possibly/necessarily after or before a date, and use fuzzy comparators to express relations between time instants. Finally, Moon et al. do not consider uncertainty on the time instants, but fuzzify temporal events and states and define an order relation among events and states, represented as a directed graph.

In requirements engineering fuzzy logic has been adopted to perform tradeoff analysis among conflicting functional requirements. In particular, aggregation functions are used to combine correlated requirements into high-level ones. Fuzzy logic has been also exploited to express uncertain requirements. Liu et al. introduce a methodology to elicit non-functional requirements through fuzzy membership functions that allow one to represent the uncertainty about the human perception. RELAX is a notation to express uncertain requirements, whose assessment is affected by the imprecision of measurement. Finally, FLAGS extends traditional LTL by adding new operators to represent transient/small violations in the temporal domain. Its main purpose is providing a notion of satisfaction level of requirements in the temporal domain. In particular, the authors use this approach to tolerate small deviations of the satisfaction of the requirements during or within a temporal interval. Despite the purpose of FLAGS is similar to our approach, the syntax and the semantics of the FLAGS language are not formally described, and the relations among temporal operators are not even provided.

III. BACKGROUND

This section provides a general definition of fuzzy logic, and points out the differences between a fuzzy and a probabilistic approach for the evaluation of temporal properties. Finally, the section introduces the formalism proposed in and discusses its limitations.
A. General formalization of fuzzy logic

The term “fuzzy” has been explicitly used for the first time in Zadeh’s seminal work [14] about fuzzy sets, where he presented the theory of classes with unsharp boundaries. In this work, the logical formalism of fuzzy sets shares the same syntax of Propositional Logic (PL), but its formulae may have a truth value comprised between 0 and 1. Conjunction and disjunction are interpreted as min and max operations, respectively.

As Zadeh pointed out [15], two main directions in fuzzy logic have to be distinguished. In a broad sense, fuzzy logic has been used to support fuzzy control and to express the vagueness of natural languages, without demonstrating its formal properties. In a narrow sense, “fuzzy logic is a logical system which is an extension of multivalued logic and is intended to serve as a logic of approximate reasoning”. In this paper, we use the term “fuzzy logic” to refer both to the Zadeh Logic [14] (which in computer science it is often called “Fuzzy Logic”) and each continuous t-norm fuzzy logic [16]. Despite the Zadeh Logic has been heavily applied in soft computing, it has no strong logical characterization. Instead, for t-norm fuzzy logics, it is often possible to provide an axiomatization and some completeness results.

We conceive a fuzzy logic as a many-valued logic [17], whose formulae may have a truth value comprised between 0 and 1 and the semantics of the connectives satisfies some monotonicity laws. The semantics of a fuzzy logic must also be coherent with PL, which means that fuzzy logic and PL must share the same syntax, fuzzy logic must reduce to PL when all predicates assume value 0 or 1, and conjunction and disjunction must be commutative and associative connectives. The semantics of the conjunction (\(\land\)), disjunction (\(\lor\)), negation (\(\neg\)), and implication (\(\Rightarrow\)), is inferred by considering respectively a continuous t-norm (\(\otimes\)) [13], its associated t-conorm (\(\oplus\)), a negation function (\(\ominus\)), and an implication function (\(\odot\)). In the case of a t-norm fuzzy logic, the negation is the pseudo-complement (i.e., \(\ominus\alpha = \max\{\beta \in [0,1] \mid \alpha \otimes \beta = 0\}\)), while the implication function becomes the residuum of the t-norm (i.e., \(\alpha \odot \beta = \max\{\gamma \in [0,1] \mid \alpha \otimes \gamma \leq \gamma\}\)). In the rest of the paper we will refer to these functions as the interpretation of connectives. Note also that the family of (continuous) t-norm fuzzy logics is infinite, as demonstrated by the infinite number of useful properties of the connectives of a fuzzy logic, while Table I provides the interpretation of these connectives for the Zadeh Logic, and three other well-known t-norm fuzzy logics.

Table I summarizes some useful properties of the connectives of a fuzzy logic, while Table II provides the interpretation of these connectives for the Zadeh Logic and three other well-known t-norm fuzzy logics.

Once identified an interpretation of the connectives, the evaluation of a (fuzzy) formula can be represented as a function \(v_i\) from the set of well-formed formulae to \([0,1]\), which extends the interpretation \(i : AP \rightarrow [0,1]\) used to evaluate an atomic proposition in \(AP\).

The following proposition describes some well-known properties of t-norms and t-conorms.

**Proposition 1.** Let \(\otimes\) be a t-norm and \(\oplus\) be a t-conorm, \(\alpha, \beta \in [0,1]\), and \(d^+, d^- : [0,1]^2 \rightarrow [0,1]\) be the drastic sum and the drastic product defined respectively by:

\[
d^+(\alpha, \beta) = 1 \Leftrightarrow \alpha + \beta > 0,
\]

\[
d^-(\alpha, \beta) = 1 \Leftrightarrow \alpha \cdot \beta = 1.
\]

Then

\[
\max\{\alpha, \beta\} \leq \alpha \oplus \beta \leq d^+(\alpha, \beta),
\]

\[
d^-(\alpha, \beta) \leq \alpha \otimes \beta \leq \min\{\alpha, \beta\}.
\]

For a continuous t-norm it is possible to define two connectives called lattice (or weak) conjunction (\(\land^\wedge\)) and lattice disjunction (\(\lor^\wedge\)). The semantics of these connectives is given by:

\[
p \land^\wedge q = p \land (p \Rightarrow q),
\]

\[
p \lor^\wedge q = ((p \Rightarrow q) \Rightarrow q) \land^\wedge ((q \Rightarrow p) \Rightarrow p).
\]

Nevertheless, they reduce respectively to the max and min operations, as stated in the following well-known proposition.

**Proposition 2.** Let \(p_\alpha, p_\beta \in AP\) such that \(\overline{i}(p_\alpha) = \alpha\), and \(\overline{i}(p_\beta) = \beta\), then, for each continuous t-norm:

\[
v_i(p_\alpha \land^\wedge p_\beta) = \max\{\alpha, \beta\},
\]

\[
v_i(p_\alpha \lor^\wedge p_\beta) = \min\{\alpha, \beta\}.
\]

B. Fuzzy Logic and Probability

Fuzzy logic and probability have been usually conceived as similar disciplines. However, the nature of fuzzy logic and probability are totally different both on the ontological and epistemological level. These disciplines deal with two different topics. Probability focuses on observable events whose occurrence is uncertain, while fuzzy logic deals with vague events that cannot be clearly assessed.

For example, the statement “tomorrow there will be a power outage” is uncertain, since it is not possible to know the truth value of the formula. However, by applying the probability theory (e.g., by analyzing the frequency of power outages during the last month), it is possible to state that, for example, the probability that the aforementioned statement will be true is 3.8%. Still, when a direct observation can be performed (i.e., tomorrow), it is possible to assess whether an outage took place or not and, indeed, the probability value can collapse either to 0 or 1.

Instead, the statement “tomorrow the number of power outages will be low” is not tractable from a probabilistic point of view, because the nature of the event itself is not clearly measurable, since the concept of “low” has not been defined in a observable way. In this case, we are not facing the problem of uncertainty of an event, but the vagueness of its definition. Indeed, assigning the truth degree of 0.038
to the aforementioned statement means that tomorrow the smart grid will face a “high number of outages”. Even a direct observation of the number of outages will not cause this value to collapse to 0 or 1.

C. Fuzzy Linear-time Temporal Logic

This section briefly describes FLTL (Fuzzy Linear-time Temporal Logic) [6], which is an extension of Zadeh Logic with temporal operators. FLTL has the same syntax of LTL. In particular, let \( \Phi \) be the set of well formed formulae and \( AP \) the set of propositional letters, then \( \varphi \in \Phi \) if and only if

\[
\varphi := p \mid \neg \varphi \mid \varphi \land \varphi \mid X \varphi \mid G \varphi \mid \varphi U \varphi,
\]

where \( p \in AP \). The semantics of a formula \( \varphi \in \Phi \) is defined w.r.t. a linear time structure \( \pi_\sigma = (S, w_0, w, L) \), where \( S \) is a set of states, \( w_0 \) is the initial state, \( w \in w_0 S^\omega \) is an infinite path, and \( L : S \to [0, 1] ^ {AP} \) is a fuzzy labeling function. The evaluation \( v(\varphi, w^i) \) of a formula \( \varphi \in \Phi \) along the path \( w \) from the \( i \)-th instant is a real number in \([0, 1]\) recursively defined by:

\[
v(p, w^i) = L(w_i)(p),
v(\neg \varphi, w^i) = 1 - v(\varphi, w^i),
v(\varphi \land \psi, w^i) = \min\{v(\varphi, w^i), v(\psi, w^i)\},
v(X\varphi, w^i) = v(\varphi, w^{i+1}),
v(G \varphi, w^i) = \min\{v(\varphi, w^i), v(G \varphi, w^{i+1})\},
v(\varphi U \psi, w^i) = \max\{v(\psi, w^i), \min\{v(\varphi, w^i), v(\varphi U \psi, w^{i+1})\}\}.
\]

It is easy to see that FLTL extends LTL in the sense that if for all \( s \in S \) and \( p \in AP \) is \( L(s)(p) \in \{0, 1\} \), then \( v(\varphi, w^i) = 1 \iff w^i \models \varphi \).

Note that FLTL cannot represent the vagueness in the temporal dimension. Fuzzyfication just addresses Boolean connectives and keeps a crisp semantics for the time (always/never). For example, when we evaluate the “globally” (always) operator, it may not be suitable to consider the minimum truth value encountered. For instance, this semantics does not allow us to tolerate transient violations that take place for a few number of times compared to a long time interval. For example, if we want to assess the truth of the statement “this week no power outage happened”, we must consider that even one power outage is enough to negatively affect the truth value of this formula, and we cannot tolerate a few power outages. Furthermore, even if this semantics allows us to express statements about the future, such as “tomorrow power outages will take place”, we cannot express statements such as “soon a power outage will happen”.

For these reasons, the language we propose in this paper, although partially inspired by FLTL, introduces a completely new approach to the fuzzification of the temporal domain.

IV. FTL: Fuzzy-time Temporal Logic

In this section we describe the syntax and semantics of FTL, which is our fuzzy-time temporal logic.
A. Syntax

FTL extends LTL in order to deal with fuzzyness on time. Let $AP$ be a numerable set of atomic propositions, $\neg, \land, \lor, \Rightarrow$ be the (fuzzy) connectives, and $O$ and $T$ be the sets of unary and binary (fuzzy) temporal modalities. Then, $\varphi$ belongs to the set $\Phi$ of well-formed FTL formulae (from now on, simply $\Phi$ formulae) if it is defined as follows:

$$\varphi := p \mid \neg \varphi \mid \varphi \sim \varphi \mid \varphi \lor \varphi \mid \varphi T \varphi,$$

where $p \in AP$, $\sim$ is a binary connective, $O \in O$, and $T \in T$. As unary operators we consider $X$ (next), $\text{Soon}$ (soon), $F$ (eventually), $G_i$ (eventually in the next $t$ instants), $G$ (always), $G_t$ (always in the next $t$ instants), $A_G$ (almost always), $A_G^i$ (almost always in the next $t$ instants), $L_t$ (lasts $t$ instants), $W_i$ (within $t$ instants), where $t \in \mathbb{N}$. Binary operators are $U$ (until), $U_t$ (bounded until), $A_U$ (almost until), and $A_U^i$ (bounded almost until). We admit the use of $\lambda^j(\cdot)$ as a shorthand for $j$ applications of $\wedge$. For example, $X^3(\cdot) \equiv X(X(\cdot))$. Conventionally we also set $X^n \varphi \equiv \varphi$. From now on, operators $\text{Soon}, A_G, A_G^i, L_t, W_i, A_U$, and $A_U^i$ will be indicated as “almost” operators.

B. Semantics

The semantics of a formula $\varphi$ is defined w.r.t. a linear time structure $(S, s_0, \pi, L)$, where $S$ is the set of states, $s_0$ is the initial state, $\pi$ is an infinite path $\pi = s_0 s_1 \cdots \in S^\omega$, and $L : S \to [0, 1]^{|AP|}$ is the (fuzzy) labeling function that assigns to each state an evaluation for each atomic proposition in $AP$. $\pi^i$ indicates the suffix of $\pi$, by starting from the $i$-th position and $s^i$ is the first state of $\pi^i$. Besides, we adopt an avoiding function $\eta : \mathbb{Z} \to [0, 1]$. We assume that $\eta(i) = 1$ for all $i \leq 0$, and $n_\eta \in \mathbb{N}$ exists such that $\eta$ is strictly decreasing in $\{0, \ldots, n_\eta\}$ and $\eta(n') = 0$ for all $n' \geq n_\eta$. Function $\eta$ expresses the penalization assigned to the number of events we want to ignore in evaluating the truth degree of a formula that contains an “almost” operator. For example, we interpret the formula “almost always $p^n$” as “always $p$ except for a small number of cases”, and we penalize the evaluation of the formula according to the number of avoided events. Hence, the evaluation of a formula that contains the operator $A_G$ realizes a tradeoff between the number of avoided events, and the penalization assigned to this number. Since we are dealing with a multi-valued logic, it makes no sense to define a crisp satisfiability relation. Instead, to define the semantics of a formula $\varphi$ along a path, we express a fuzzy satisfiability relation as $\models \subseteq S^\omega \times F \times [0, 1]$, where $\pi \models \varphi = \nu \in [0, 1]$ means that the truth degree of $\varphi$ along $\pi$ is $\nu$. We say that two formulae $\varphi$ and $\psi$ in $\Phi$ are logically equivalent, in symbols $\varphi \equiv \psi$, if, and only if, $\pi \models \varphi = (\pi \models \psi)$ for each linear time structure, and for each avoiding function.

The truth degree of a formula is defined, as usual, recursively on its structure. Let $p \in AP$ and $\pi^i$ be a path, then:

$$(\pi^i \models p) = L(s^i)(p),$$

$$(\pi^i \models \neg \varphi) \equiv (\pi^i \models \varphi),$$

$$(\pi^i \models \varphi \land \psi) = (\pi^i \models \varphi) \land (\pi^i \models \psi),$$

$$(\pi^i \models \varphi \lor \psi) = (\pi^i \models \varphi) \lor (\pi^i \models \psi),$$

$$(\pi^i \models \varphi \Rightarrow \psi) = (\pi^i \models \varphi) \lor (\pi^i \models \psi),$$

where $p \in AP$, $i \in \mathbb{N}$, and $\land, \lor, \Rightarrow, \Rightarrow, \Rightarrow, \Rightarrow$ are the operations, between real numbers, defining the chosen semantics of the connectives ($\neg$, $\land$, $\lor$, $\Rightarrow$).

We are now able to introduce the semantics of FTL temporal operators.

**Next:** Operator “next” ($X$) has the same semantics of its corresponding LTL operator $X$:

$$(\pi^i \models X \varphi) = (\pi^{i+1} \models \varphi).$$

**Soon:** Operator “soon” ($\text{Soon}$) extends the semantics of the “next” operator, by tolerating at most $n_\eta$ time instants of delay. In other words, the greater the number of tolerated instants, the greater the penalization will be.

$$(\pi^i \models \text{Soon} \varphi) = \bigoplus_{j=i+1}^{i+n_\eta} (\pi^j \models \varphi) \cdot \eta(j - i - 1).$$

**Proposition 3.** From the monotonicity of the t-conorm $\oplus$ (see Table 10) it naturally follows that

$$(\pi^i \models X \varphi) \leq (\pi^i \models \text{Soon} \varphi).$$

**Eventually:** Operator “eventually” ($F$) and its bounded version ($F_i$) also maintain the same semantics of their corresponding LTL operator $F$. Namely,

$$(\pi^i \models F \varphi) = \bigoplus_{j=i}^{i+t} (\pi^j \models \varphi),$$

$$(\pi^i \models F_i \varphi) = \bigoplus_{j=i}^{i+t} (\pi^j \models \varphi) = \lim_{t \to +\infty} (\pi^i \models F_i \varphi).$$

First, observe that for $F_i$ the equivalences $F_0 \varphi \equiv \varphi$ and $F_i \varphi \equiv \varphi \lor X F_{i-1} \varphi$ hold, for $t \geq 0$. The semantics of $F$ requires a passage to the limit, whose existence is ensured by the fact that the sequence $(\pi^i \models F_i \varphi)_{i \in \mathbb{N}}$ is increasing, as the t-conorm $\oplus$ is monotonic. These facts are summarized in the following proposition.

**Proposition 4.** For all $\varphi \in F$ and $t \leq t'$:

$$(\pi^i \models \varphi) \leq (\pi^i \models F_i \varphi) \leq (\pi^i \models F_t \varphi) \leq (\pi^i \models F \varphi).$$

**Within:** Operator “within” ($W_t$) is inherently bounded, and its semantics is defined by

$$(\pi^i \models W_t \varphi) = \bigoplus_{j=i}^{i+t+n_\eta-1} (\pi^j \models \varphi) \cdot \eta(j - t - i).$$

Formula $W_t p$ states that subformula $p$ is supposed to hold in at least one of the next $t$ instant or, possibly, in the next
Proposition 5. The semantics of operator $W_t$ can be expressed by only using operators $X$ and $\Sigma$. More formally, for all $\varphi \in F$ and $t \in \mathbb{N}$:

$$W_t \varphi \equiv F_t \varphi \vee X^{t+1} \text{ Soon } \varphi,$$

and

$$W_0 \varphi \equiv \text{ Soon } \varphi.$$

Corollary 6. For all $\varphi \in F$ and $t \in \mathbb{N}$

$$(\pi^t \models W_t \varphi) \geq (\pi^t \models F_t \varphi),$$

$$\lim_{t \to +\infty} (\pi^t \models W_t \varphi) = (\pi^t \models F \varphi),$$

Proof: The first property follows immediately from the previous proposition. For the second property, observe that

$$(\pi^t \models F_{t+n} \varphi) \geq (\pi^t \models \text{ Soon } \varphi),$$

and then actually

$$(\pi^t \models F_{t+n} \varphi) \geq (\pi^t \models W_t \varphi) \geq (\pi^t \models F_t \varphi),$$

and applying the squeeze theorem we have the thesis. ■

Always: Operator “always” ($G$) and its bounded version ($G_t$) extend the semantics of their corresponding LTL operator $G$. Namely,

$$(\pi^t \models G_t \varphi) = \bigotimes_{j=t}^{t+t} (\pi^j \models \varphi),$$

$$(\pi^t \models G \varphi) = \bigotimes_{j=t}^{t+t} (\pi^j \models \varphi) = \lim_{t \to +\infty} (\pi^t \models G_t \varphi).$$

As for $F$, observe that for $G_t$ the equivalences $G_0 \varphi \equiv \varphi$ and $G_t \varphi \equiv \varphi \land XG_{t-1} \varphi$ hold, for $t \geq 0$. Similarly to $F$, the semantics of $G$ also requires a passage to the limit, whose existence is ensured by the fact that the sequence $G_t(\varphi)_{t \in \mathbb{N}}$ is decreasing, as the t-norm $\otimes$ is monotonic (see Table 1). These facts are summarized in the following proposition.

Proposition 7. For all $\varphi \in F$ and $t \leq t'$:

$$(\pi^t \models G \varphi) \leq (\pi^t \models G_t \varphi) \leq (\pi^t \models G_{t'} \varphi) \leq (\pi^t \models G_{t+1} \varphi) = (\pi^t \models \varphi \land X \varphi) \leq (\pi^t \models G_0 \varphi) = (\pi^t \models \varphi).$$

From propositions 3, 5 and 7 we can immediately obtain the following corollary.

Corollary 8. For all $\varphi \in F$ and $t, t' \in \mathbb{N}$:

$$(\pi^t \models G \varphi) \leq (\pi^t \models F_t \varphi),$$

$$(\pi^t \models G_t \varphi) \leq (\pi^t \models F_{t'} \varphi),$$

$$(\pi^t \models G_t \varphi) \leq (\pi^t \models W_t \varphi).$$

Almost always: Operator “almost always” ($AG$) and its bounded version ($AG_t$) allow us to evaluate a property over the path $\pi^t$, by avoiding at most $n_\eta$ evaluations of this property, and, at the same time, introducing a penalization for each avoided case. Let $I_t$ be the initial segment of $\mathbb{N}$ of length $t + 1$, i.e., $I_t = \{0, 1, \ldots, t\}$, and $P^h(I_t)$ the set of subsets of $I_t$ of cardinality $k$, then

$$(\pi^t \models AG_t \varphi) = \max_{j \in I_t} \max_{h \in P^h(I_t)} \bigotimes (\pi^{t+h} \models \varphi) \cdot \eta(j),$$

$$(\pi^t \models AG \varphi) = \lim_{t \to +\infty} (\pi^t \models AG_t \varphi).$$

As we will see later, the sequence $(\pi^t \models AG_t(\varphi))_{t \in \mathbb{N}}$ is not monotonic. Nevertheless, we can still prove that the semantics of $AG$ is well-defined.

Proposition 9. Given $\varphi \in F$, it is possible to recursively define $n$ propositional letters $p_0, \ldots, p_{n-1}$, such that

$$(\pi^t \models AG \varphi) = \max_{j \leq n-1} \{GP_j \cdot \eta(j)\}.$$

Proof: Let define $p_0$ as:

$$\forall i \in \mathbb{N}, \ (\pi^t \models p_0) = (\pi^t \models \varphi).$$

Then, for all $0 < m \leq n_\eta$, we recursively obtain $p_m$ from $p_{m-1}$ in the following way. Let $m$ be the minimum in $\mathbb{N} \cup \{\infty\}$, such that for all $k \in \mathbb{N}$, $(\pi^k \models p_m) \leq (\pi^k \models p_{m-1})$.

Then, let set

$$\begin{cases}
(\pi^t \models p_m) = (\pi^t \models p_{m-1}), & j < h; \\
(\pi^t \models p_m) = (\pi^{t+1} \models p_{m-1}), & j \geq h.
\end{cases}$$

Hence, for all $t \geq j$

$$(\pi^t \models G_{t-j}p_j) = \max_{H \in P^{t-j}(I_t)} \bigotimes (\pi^{t+h} \models \varphi).$$

The first term corresponds to choose $H = I_t \setminus \{h_1, \ldots, h_j\}$. The converse inequality also holds, since it derives from the monotonicity of the operation $\otimes$. Then, passing to the limit

$$\lim_{t \to +\infty} (\pi^t \models AG_t \varphi) = \lim_{t \to +\infty} \max_{j \leq n} \{GP_j \cdot \eta(j)\} = \max_{j \leq n} \{GP_j \cdot \eta(j)\},$$

and, indeed, we have the thesis. ■

Note that the maximum in the definition above can be expressed in each fuzzy logic we are considering. Indeed, in the Zadeh Logic the maximum is simply the (standard) $\lor$, and in a t-norm fuzzy logic it is the lattice disjunction $\lor ^w$. We decide to use the maximum to find the best matching between the number of avoided cases, and the penalization due to $\eta$. Indeed, if we define the semantics of $AG$ via the (strong) disjunction as

$$(\pi^t \models AG_t \varphi) = \bigoplus_{j=0}^{t} \bigoplus_{H \in P^{t-j}(I_t)} \bigotimes (\pi^{t+h} \models \varphi) \cdot \eta(j).$$
and consider the Łukasiewicz’s interpretation for the connective $\lor$, then a formula $\mathcal{AG}_t p$ will often evaluated to 1 due to the high number of considered cases, and (almost) independently from the evaluations of $p$.

In the following proposition we show how to reduce the complexity of the evaluation of operator $\mathcal{AG}_t$, by exploiting the monotonicity of the t-conorm.

**Proposition 10.** It is possible to evaluate the truth degree of formula $\mathcal{AG}_t p$ by performing $O(n_\eta(\log(t) + 1))$ comparisons, $O(t)$ applications of the norm $\otimes$, and $O(n_\eta)$ multiplications.

**Proof:** We consider the same technique applied in the proof of Proposition 9. Let $(a_k)_{k \leq n}$ be a finite sequence of indices such that $\forall k \leq n, a_k \leq t$, and $\forall h \leq k \leq n$, $(\pi^{nh} \models p) \leq (\pi^{a_k} \models p)$, then

$$(\pi^i \models \mathcal{AG}_t p) = \max_{j \leq n_\eta} \left\{ \bigotimes_{h \leq n_\eta} \left( (\pi^{i+h} \models p) \cdot \eta(j) \right) \right\} = \max_{1 \leq j \leq n_\eta} \left\{ \left( (\pi^i \models G_t p) , \bigotimes_{h \leq n_\eta} \left( (\pi^{i+h} \models p) \cdot \eta(j) \right) \right) \right\}. \quad (2)$$

Finding the indices $a_i$ requires at most $O(n_\eta \log(t))$ comparisons (for example applying the heuristic algorithm), and extra $O(n_\eta)$ comparisons are used to evaluate the maximum. $O(t)$ applications of $\otimes$ are needed, observing that the operation is associative, and, indeed, the value obtained at one step can be used for calculating the value for the following step.

From (2), we also have the following corollary.

**Corollary 11.** For all $\varphi \in F$ and $t \in \mathbb{N}$:

$$(\pi^i \models \mathcal{AG}_t \varphi) \geq (\pi^i \models G_t \varphi),$$

$$(\pi^i \models \mathcal{AG} \varphi) \geq (\pi^i \models G \varphi).$$

Observe that in general it is not possible to establish a priori which inequality holds between $(\pi^i \models \mathcal{AG}_t \varphi)$ and $(\pi^i \models \mathcal{AG}_{t'} \varphi)$, with $t \neq t'$, as this also depends on function $\eta$. For example, let us consider a predicate $p$ together with an avoiding function $\eta$, whose behaviors are described in Table III.

If we consider the Zadeh interpretation of connectives, then $(\pi^0 \models \mathcal{AG}_1 p) = 0.1$, $(\pi^0 \models \mathcal{AG}_2 p) = 0.3$, and $(\pi^0 \models \mathcal{AG}_3 p) = 0.06$, and the sequence $(\pi^i \models \mathcal{AG}_t p)_{t \in \mathbb{N}}$ is not monotonic.

**Lasts:** Operator “lasts” ($\mathcal{L}_t$) is bounded, and expresses a property that lasts for $t$ consecutive instants from now, possibly avoiding some event at the end of the considered time interval. The semantics of this operator is defined as follows:

$$(\pi^i \models \mathcal{L}_t \varphi) = \max_{0 \leq j \leq \min(t,n_\eta-1)} \left\{ (\pi^{i-j} \models G_{t-j} \varphi) \cdot \eta(j) \right\}.$$  

**Proposition 12.** Let $\varphi \in F$ and $t \in \mathbb{N}$, then the sequence $(\pi^i \models \mathcal{L}_t \varphi)_{t \in \mathbb{N}}$ is decreasing, and its limit is $(\pi^i \models G \varphi)$. Moreover, the following inequalities hold:

$$(\pi^i \models \mathcal{L}_t \varphi) \leq (\pi^i \models \mathcal{L}_{t+1} \varphi) \leq (\pi^i \models \mathcal{AG}_t \varphi).$$

**Proof:** The fact that the sequence $(\pi^i \models \mathcal{L}_t \varphi)_{t \in \mathbb{N}}$ is decreasing follows immediately from the definition and from Proposition 10. Moreover, again from definition

$$(\pi^i \models \mathcal{L}_t \varphi) \leq (\pi^i \models \mathcal{L}_{t+1} \varphi) \leq (\pi^i \models \mathcal{G}_{t+1} \varphi),$$

and then passing to the limit the first part follows. The inequality $(\pi^i \models \mathcal{L}_t \varphi) \leq (\pi^i \models \mathcal{AG}_t \varphi)$ is a direct consequence of Proposition 10.

**Until:** The semantics of operator “until” ($\mathcal{U}_t$) and its bounded version ($\mathcal{U}_t$) naturally extends the one assigned to the corresponding LTL operator $\mathcal{U}$, for $t > 0$:

$$(\pi^i \models G_0 \varphi) \implies (\pi^i \models \mathcal{U}_t \varphi) = \max_{1 \leq j \leq t} ((\pi^i \models \varphi) \otimes (\pi^i \models G_j \varphi)),$$

$$(\pi^i \models \mathcal{U}_t \varphi) = \lim_{t \rightarrow +\infty} (\pi^i \models \varphi \mathcal{U}_t \varphi),$$

Analogously to $\mathcal{AG}$, the maximum is used to find the best matching between the evaluation of $\varphi$ and $\psi$.

**Proposition 13.** The semantics of operator $\mathcal{U}_t$ is well-defined. Moreover, $(\pi^i \models \mathcal{U}_t \varphi) \leq (\pi^i \models \mathcal{F} \varphi)$.

**Proof:** For the first part, it suffices to prove that the sequence $(\pi^i \models \mathcal{U}_t \varphi)_{t \in \mathbb{N}}$ is increasing. This is obvious as, for all $t > 0$:

$$(\pi^i \models \mathcal{U}_t \varphi) = \max\{(\pi^i \models G_{t-1} \varphi), (\pi^i \models G_t \varphi \land \mathcal{X} \varphi)\}.$$  

For the second part, let $p \in AP$ such that $\forall j \geq i, (\pi^j \models p) = 1$. Then $(\pi^i \models \varphi \mathcal{U}_t \varphi) \leq (\pi^i \models p \mathcal{U}_t \varphi)$, and from Proposition 11 we have

$$(\pi^i \models \varphi \mathcal{U}_t \varphi) \leq (\pi^i \models \mathcal{U}_t \varphi) \leq \max_{j \geq i} (\pi^j \models \varphi) \leq (\pi^i \models \mathcal{F} \varphi).$$

In particular, for all $t \in \mathbb{N}$, we can write

$$(\pi^i \models \varphi) = (\pi^i \models \varphi \mathcal{U}_0 \varphi) \leq (\pi^i \models \varphi \mathcal{U}_t \varphi) \leq (\pi^i \models \varphi \mathcal{U}_t \varphi) \leq (\pi^i \models \mathcal{F} \varphi).$$  

\hfill \blacksquare
Almost until: Operator “almost until” ($\mathcal{A}U$) and its bounded version ($\mathcal{A}U_t$) are obtained by the previous ones, by replacing operator $G_t$ with its relaxed version $AG_t:\$

$$
(\pi^i \models \varphi \mathcal{A}U_0 \psi) = (\pi^i \models \psi),
$$
$$
(\pi^i \models \varphi \mathcal{A}U_t \psi) = \max_{i \leq j \leq t+1} \left( (\pi^i \models \psi) \otimes (\pi^i \models AG_{j-1} \varphi) \right),
$$
$$
(\pi^i \models \varphi \mathcal{A}U \psi) = \lim_{t \to +\infty} (\pi^i \models \varphi \mathcal{A}U_t \psi),
$$

for $t > 0$. Similarly to $U$, we can state the following.

**Proposition 14.** The semantics of operator $\mathcal{A}U$ is well-defined. Moreover, for all $t \in \mathbb{N}$

$$
(\pi^i \models \psi) = (\pi^i \models \varphi \mathcal{A}U_0 \psi) \leq (\pi^i \models \varphi \mathcal{A}U_t \psi) \leq (\pi^i \models \varphi \mathcal{A}U \psi). \quad (4)
$$

**Proof:** As for $U$, we can observe that for all $t > 0$,

$$
(\pi^i \models \varphi \mathcal{A}U_t \psi) = \max\{ (\pi^i \models \varphi \mathcal{A}U(t-1) \psi), (\pi^i \models AG(t-1) \varphi \land \chi \psi) \}.
$$

The sequence $(\pi^i \models \varphi \mathcal{A}U_t \psi)_{t \in \mathbb{N}}$ is increasing and the semantics of $\mathcal{A}U$ is well-defined. The latter part follows from Corollary 11.

Before considering further relations among operators, note that for each class of operators, a different avoiding function can be considered. For example, we may prefer to tolerate a long delay in evaluating $W_t$ operator, but we accept to tolerate only a few number of avoided events in evaluating $AG_t$. In this case, we can define two functions, $\eta_W$ and $\eta_G$, such that for all $i \in \mathbb{N}$, $\eta_W(i) \geq \eta_G(i)$. However, we leave this issue for a future investigation.

As a final remark, notice that the semantics we have chosen for our operators is arbitrary, and many other variants can be proposed. However, the properties above show that our choice is reasonable. For example, the “almost” operators are more lax than the traditional ones, since their evaluation has a greater value, exactly as one would expect.

V. REDUCTIONS AND EQUIVALENCES

This section prove that, under the assumption that all events are crisp, FTL reduces to LTL, and provides a set of interesting relations between the operators of FTL. Finally we also provide some possible adequate set of connectives, from which it is possible to infer all the others.

Reduction to LTL: We can prove that, in some sense, the semantics of FTL extends LTL, as stated in the following proposition and theorem.

**Proposition 15.** Let $p, q \in AP$ such that for all $j \geq i$,

$$
(\pi^i \models p), (\pi^j \models q) \in \{0, 1\}, \text{ then }
$$

$$
(\pi^i \models Fp) = 1 \iff \pi^i \models Fp,
$$
$$
(\pi^i \models Gp) = 1 \iff \pi^i \models Gp,
$$
$$
(\pi^i \models pUq) = 1 \iff \pi^i \models pUq.
$$

Proof: It follows, through straightforward calculation, by applying the boundary value in Table 1.

**Theorem 16.** Let for all $p \in AP$ and $i \in \mathbb{N}$, $\pi^i \models p \in \{0, 1\}$, and $\eta(1) = 0$. Then FTL reduces to LTL.

**Proof:** First notice that, by definition, $\texttt{Soon}$ reduces to $X, W_t$ to $F, AG_t$ and $L_t$ to $G_t$, and $\mathcal{A}U_t$ to $\mathcal{U}_t$. Then, the thesis follows by applying an argument similar to the one used in the previous proposition.

**General relations:** The relations between some of FTL operators are shown in Figure 1. Moreover, as shown in the following proposition, their values coincide only in a special case.

**Proposition 17.** Let $\varphi \in F$ and $i \in \mathbb{N}$, then $(\pi^i \models F\varphi) = (\pi^i \models G\varphi)$ if, and only if, $(\pi^j \models \varphi)$ is constant for all $j \geq i$.

**Proof:** For the first implication, observe that if $(\pi^j \models \varphi)$ is constant for all $j \geq i$, then for all $j \geq i$, $(\pi^j \models F\varphi) = (\pi^j \models G\varphi) = (\pi^i \models \varphi)$. Conversely, suppose $h, k \geq i$ exist such that $(\pi^h \models \varphi) = a < (\pi^k \models \varphi) = b$. Then from Proposition 11 it follows that:

$$
(\pi^i \models G\varphi) \leq \min_{j \geq i} (\pi^j \models \varphi) \leq a < b
$$

$$
\leq \min_{j \geq i} (\pi^j \models \varphi) \leq (\pi^i \models F\varphi).
$$

**Adequate sets:** An adequate set of connectives for a given logic is a subset of its connectives that is sufficient to equivalently express any formula of the logic. For example, it is well known that $X$ and $U$, together with $\land$ and $\lor$, form an adequate set of connectives for LTL. Clearly, adequate sets also depend on the interpretation of the connectives. So we denote by $\text{FTL}(Z)$, $\text{FTL}(G)$, $\text{FTL}(L)$, and $\text{FTL}(II)$ the logics whose semantics is based on Zadeh, Gödel-Dummett, Łukasiewicz, and Product interpretation, respectively.

Before finding an adequate sets of connectives for $\text{FTL}(Z)$, $\text{FTL}(G)$, $\text{FTL}(L)$, and $\text{FTL}(II)$, we need to introduce the extra operators $\circ_j$, for $1 \leq j < \eta_i$, whose semantics is

$$
(\pi^i \models \circ_j \varphi) = (\pi^i \models \varphi) \cdot \eta(j).
$$

**Proposition 18.** Let $\varphi \in F$, then in $\text{FTL}(Z)$ and $\text{FTL}(L)$

$$
G\varphi \equiv \neg F\neg \varphi \text{ and } F\varphi \equiv \neg G\neg \varphi.
$$
Adequate sets for FTL(Z), FTL(G), FTL(L), and FTL(II).

| Logic       | Adequate set                        |
|-------------|-------------------------------------|
| FTL(Z)      | \(\land, \neg, \land U, \land \lor, \circ \otimes \ldots, \circ \otimes^{n-1} \) |
| FTL(G)      | \(\land, \Rightarrow, \land U, \land \lor, \circ \otimes \ldots, \circ \otimes^{n-1} \) |
| FTL(L)      | \(\land, \Rightarrow, \land U, \land \lor, \circ \otimes \ldots, \circ \otimes^{n-1} \) |
| FTL(II)     | \(\land, \Rightarrow, \land U, \land \lor, \land U, \land \lor, \circ \otimes \ldots, \circ \otimes^{n-1} \) |

**Proof:** Simply observe that, in the considered logics, \(\varphi \land \psi \equiv \neg(-\varphi \lor \neg \psi)\), and \(\varphi \lor \psi \equiv \neg(-\varphi \land \neg \psi)\).

**Theorem 19.** Let \(\{T, p_1, \ldots, p_n\} \subseteq AP\), with \(\pi^I \models T = 1\), \(\pi^I \models p_j = \eta(j)\), for all \(i \in \mathbb{N}\), and \(1 \leq j < n\). Then FTL(Z), FTL(G), FTL(L), and FTL(II) admit a finite set of adequate connectives. Some of the possible adequate sets are presented in Table IV.

**Proof:** It mainly follows from propositions [5,18] and from the definition of the operators. Moreover, observe that in FTL(Z) and FTL(G), \(F \varphi \equiv \land U \varphi\) and \(G \varphi \equiv \lor \land \top\). While \(F\) and \(G\) are dual in FTL(Z), this does not hold in FTL(G), because of the different interpretation of the negation. Observe that in Product Logic, \(\lor\) cannot be expressed in terms of \(\land\), while this is possible in Gödel-Dummett and Łukasiewicz logics (see [16]). Note that the adoption of the adequate sets in Table IV can possibly cause a super-exponential blow-up of the length of the formulas. For example, formula \(\mathcal{A}\mathcal{G}_t p\), is equivalent in FTL(II) to a formula of length \(O(3^{2^{2t+1}} \cdot t)\) that only contains connectives \(\land, \Rightarrow, \land U\).

**VI. EXAMPLES OF PROPERTIES AND SPECIFICATIONS**

This section illustrates how FTL can be adopted in practice to formalize a set of properties of a smart grid. Smart grids must maximize the availability of appliances and optimize the consumption of energy. Metering data regarding the energy consumption are periodically computed and are used by the Energy Management System (EMS) to balance the work load of the appliances. In particular, the EMS sends proper operational control data to the appliances to schedule their tasks and tune their functioning in order to avoid outages. To this aim, we may need to express some statements about the amount of energy consumed and the availability of appliances. Furthermore, we may need to tolerate a few number of outages or some cases in which the appliances are temporarily unavailable. Our example defines a set of formulae, under the assumption that the smart grid controls a single appliance \(N_1\). However, provided formulae can be easily modified to cover the cases when more than one appliance must be controlled.

The first property, which may be necessary to evaluate, is “\(N_1\) must be available almost always during the day”. Let \(\pi\) be the path of the daily minutes, and consider a (fuzzy) predicate \(a\) that measures whether the availability of \(N_1\) is high. More precisely, \((\pi^I \models a)\) expresses the truth degree of proposition “at the \(i\)-th minute of the day, the availability of \(N_1\) is high”. Availability is, in general, measured as the time difference between the instant when a request is issued and the instant when the appliance is active. This time difference can be estimated in seconds and this makes reasonable the choice of minutes as time granularity. Using this definition of availability, we can evaluate predicate \(a\) as follows. If \(A_t\) is the actual time delay of the \(i\)-th minute, \(M_t\) the mean delay of the \(i\)-th minute of the day computed daily over the last month, and \(\sigma^2\) the variance, let \(\Delta_i = A_t - M_t\), then

\[(\pi^I \models a) = \left\{ \begin{array}{ll}
\min\{1, \frac{1}{t^2}\sigma^2 \}, & \Delta_i \geq -\frac{3}{2}\sigma^2; \\
0, & \text{otherwise.}
\end{array} \right.\]

As avoiding function we can choose \(\eta(n) = e^{-(n/20)^2}\), if \(n \leq 20\), and 0 otherwise. The evaluation of formula \(G_{1440^0} a\) along \(\pi\) gives a value corresponding, at most, to the worst time difference. Formula \(\mathcal{A}\mathcal{G}_{1440^0} a\), instead, can be used when we want to tolerate the cases when the availability of \(N_1\) is fine, except for at most 20 minutes during the day. Indeed, if the availability is below the average for no more than 4 minutes, then the evaluation of \(\mathcal{A}\mathcal{G}_{1440^0} a\) is, at least, \(e^{-(16/20)^2} \sim 0.53\), independently from the value of the worst minute of the day. Observe that, if we consider the mean delay calculated all over the day, we may obtain less expressive results, since in case of one big delay, the evaluation of the daily availability will dramatically decrease.

We can also consider the crisp propositions \(d\) and \(c\). The former is satisfied if new metering data are available, while the latter is satisfied if an operational control signal is sent by the EMS to \(N_1\). If we want to evaluate the property “as soon as new metering data are available, a new operational control data must be sent by the EMS to \(N_1\)”, we can formalize it as \(d \Rightarrow W_1 c\) (or by \(d \Rightarrow Soon c\), if we do not evaluate the formula from the first second), which allows to tolerate small delays in the transmission of operational control data. Instead, we cannot tolerate small delays by using LTL, since the same proposition would be expressed as \(d \Rightarrow c\) or \(d \Rightarrow Xc\).

Furthermore, let \(s a\) be a crisp proposition whose evaluation is 0, if the appliance is disconnected. Hence, if \(p\) is the (fuzzy) proposition “the energy consumption is moderate”, then \(\pi^I \models s U_{1440} p\) is the truth value of proposition “there is no outage in the day until the energy consumption is moderate”. In case we decide to relax our requirement to “the outages of the day are negligible until the energy consumption is moderate”, we can express this requirement as \(s A U_{1440} p\). The choice of operator \(A U_{1440}\) is suitable because \(\mathcal{A}\mathcal{G}_{1440} s\) allows us to neglect a few number of outages of the appliance during the day.

Finally, the choice of a specific interpretation for the connectives is highly important to get more precise results, although all the inequalities we proved are still valid independently of the interpretation. If we consider formula
\( \mathcal{A}G_{1440} s \) ("the daily number of outages is negligible"), then for the evaluation of formula \( \mathcal{A}G_{1440} s \lor \mathcal{A}1440 \), \( \mathcal{A}G_{1440} s \) it is quite natural to choose the Zadeh or Gödel-Dummett interpretation, instead of the Łukasiewicz interpretation (namely, the truncated sum of their evaluations). As a matter of fact, the predicates of this formula do not "saturate", i.e., a long sequence of days with many outages cannot be equivalent to a day with no outages. Still, the Łukasiewicz interpretation defines a substructural logic in which idempotency of \( \bot \) fails, and can be useful once we are interested in entailment.

VII. Conclusions

This paper introduces FTL, a fuzzy-time temporal logic to express vagueness on time. The semantics of the temporal operators provided by FTL is highly flexible, since it allows us to select a particular interpretation for the connectives, which best suits the kind of property to be formalized. We prove that FTL extends LTL, since, under the assumption that all events are crisp, FTL reduces to LTL. We show that the temporal operators introduced by our logic respect a set of interesting relations, and we also identify adequate sets of connectives. As future work, we are investigating a verification technique [23] for checking the truth degree of the FTL formulae on an automata-based model of the system under analysis. This technique modifies the traditional reachability analysis, according to the peculiarities of the FTL language. Moreover, considering that FTL is particularly suitable for describing requirements of active system, in which vagueness is often embedded with uncertainty, we are planning to investigate the relationship between FTL and probabilistic languages.

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