Neutron Densities from a Global Analysis of Medium Energy Proton Nucleus Elastic Scattering

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Abstract

A new method for extracting neutron densities from intermediate energy elastic proton-nucleus scattering observables uses a global Dirac phenomenological (DP) approach based on the Relativistic Impulse Approximation (RIA). Data sets for $^{40}$Ca, $^{48}$Ca and $^{208}$Pb in the energy range from 500 MeV to 1040 MeV are considered. The global fits are successful in reproducing the data and in predicting data sets not included in the analysis. Using this global approach, energy independent neutron densities are obtained. The vector point proton density distribution, $\rho_v^p$, is determined from the empirical charge density after unfolding the proton form factor. The other densities, $\rho_s^n$, $\rho_s^p$, $\rho_v^n$, are parametrized.

This work provides energy independent values for the RMS neutron radius, $R_n$ and the neutron skin thickness, $S_n$, in contrast to the energy dependent values obtained by previous studies. In addition, the results presented in paper show that the expected rms neutron radius and skin thickness for $^{40}$Ca is accurately reproduced. The values of $R_n$ and $S_n$ obtained from the global fits that we consider to be the most reliable are given as follows: for $^{40}$Ca, $3.314 > R_n > 3.310$ fm and $-0.063 > S_n > -0.067$ fm; for $^{48}$Ca, $3.459 > R_n > 3.413$ fm and $0.102 > S_n > 0.056$ fm; and for $^{208}$Pb $5.550 > R_n > 5.522$ fm and $0.111 > S_n > 0.083$ fm. These values are in reasonable agreement with nonrelativistic Skyrme Hartree-Fock models and with relativistic Hartree-Bogoliubov models with density-dependent meson-nucleon couplings. The results from the global fits for $^{48}$Ca and $^{208}$Pb are generally not in agreement with the usual relativistic mean-field models.

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I. INTRODUCTION

Determination of the proton and neutron densities, their root-mean-square radii, $R_p$ and $R_n$, and the neutron skin thickness, $S_n = R_n - R_p$, are critical to understanding many of the bulk properties of matter [1, 2, 3, 4]. Horowitz et al. have pointed out that there are substantial disagreement between theoretical values of $S_n$ [3]. Furnstahl’s recent analysis of neutron radii in the framework of mean-field models shows that relativistic mean-field models overestimate the values of $S_n$ [6]. Parity violation electron scattering may provide the experimental data to resolve the differences in the theoretical values [7], but is also useful to have alternative methods of obtaining neutron densities. Reliable neutron densities are needed for atomic parity violation experiments [5, 8, 9, 10, 11], the analysis of antiprotonic atoms [12], in understanding the surface crust of neutron stars [13], and in extrapolation to proton-rich or neutron-rich nuclei that is important in nuclear astrophysics [14]. In this work, we revisit the analysis of medium energy proton-nucleus elastic scattering data with the goal of obtaining reliable, energy independent neutron densities and the values of $R_n$ and $S_n$. The analysis of elastic electron scattering which has resulted in reliable ground state charge densities has been a guiding light for our work [15].

For a number of years we have used the relativistic impulse approximation (RIA) in the analysis of proton-nucleus elastic and inelastic scattering [16, 17, 18, 19, 20, 21, 22, 23] and the RIA-KDP(Kemmer-Duffin-Petiau) [24, 25, 26] for meson-nucleus elastic scattering [27, 28, 29, 30, 31]. These approaches produce relativistic optical potentials which result in good agreement with medium-energy scattering observables. The input to these calculations are the relativistic densities from Quantum Hadrodynamics (QHD) [32, 33] and the elementary NN amplitudes from Arndt et al. [34]. In recent work we use the modern EFT densities [23, 30, 35, 36, 37].

The seminal analysis of proton-nucleus elastic scattering data done by Ray and Hoffmann used both the RIA and the non-relativistic KMT approach in their fits to get the observables from 300 MeV to 1040 MeV [38]. Unfortunately neither approach produced energy-independent neutron densities. In addition, some of the values of $S_n$ for $^{48}$Ca and $^{208}$Pb were negative, in contradiction to all nuclear structure calculations. Shlomo and Schaff er used the results from an analyses of 1 GeV proton elastic scattering from $^{40}$Ca and $^{48}$Ca to obtain the skin thickness for $^{40}$Ca, $^{42}$Ca, $^{44}$Ca and $^{48}$Ca, see Table 2 in [39]. Starodubsky and Hintz extracted the neutron densities from elastic proton scattering of $^{206,207,208}$Pb at 650 MeV and obtained $S_n$ of $(0.20 \pm 0.04)$ fm for $^{208}$Pb [40]. However, the energy-independence of the neutron densities in the work of Ref. [39] or Ref. [40] was not addressed. Recently Karataglidis et al. have calculated proton and neutron elastic scattering from $^{208}$Pb and $^{40}$Ca targets at three energies 40 MeV, 65 MeV and 200 MeV. They used a model based on coordinate space nonlocal optical potentials using a full folding NN interactions with various Skyrme model ground-state densities [42]. For $^{208}$Pb they found that the SKM* model gave the best agreement with proton and neutron elastic scattering data at 40 MeV, 65 MeV and 200 MeV. Based on this the authors suggest that $S_n$ for $^{40}$Ca is $\sim -0.05$ fm and $^{208}$Pb is $\sim 0.17$ fm [41].

In this paper, a new analysis of proton-nucleus elastic scattering is used to obtain the neutron density. This work is motivated by our considerable experience in obtaining high quality global proton-nucleus optical potentials from 20 MeV to 1040 MeV [44]. The new method meshes the global approach with the RIA and proves to be successful in obtaining energy independent neutron densities. The starting point is the RIA in its simplest form,
which for spin-zero nuclei includes only scalar, vector and tensor terms. The tensor term is very small and is excluded; as was done in the RIA analysis done by Ray and Hoffmann [38].

We find that substantial progress in extracting the neutron densities from proton-nucleus elastic scattering is made by using a global approach focusing on the energy region where the RIA is capable of reproducing experiment very well. We have obtained values of \( S_n \) for \(^{40}\text{Ca} \), \(^{48}\text{Ca} \), and \(^{208}\text{Pb} \) which agree with nonrelativistic Skyrme models [1, 42, 43] and relativistic Hartree-Bogoliubov model extended to include density dependent meson-nucleon couplings [4]. Our results for \(^{48}\text{Ca} \) and \(^{208}\text{Pb} \) are generally not in agreement with relativistic mean-field model, see Ref. [6] and references therein.

The organization of this paper is as follows. Section II describes the global method to obtain the neutron density. Section III discusses the results and the sensitivity of the extracted neutron density, \( R_n \) and \( S_n \) to the input used in the fitting procedure. The values of \( S_n \) and the neutron RMS radii \( R_n \) for \(^{40}\text{Ca} \), \(^{48}\text{Ca} \), and \(^{208}\text{Pb} \) for the various tests of the input to the model are given in this section. The summary and conclusion is in Sect. IV.

II. RIA GLOBAL METHOD FOR EXTRACTING THE NEUTRON DENSITY

The RIA nuclear reaction formalism is used as the basis of global fits to medium-energy proton nucleus elastic scattering data. The input to the RIA consists of the Arndt NN amplitudes [34] and the point proton density, which is fixed from the charge distribution obtained from electron nucleus scattering. The neutron vector density, and the scalar proton and neutron densities are parametrized, resulting in good fits of \( p + A \) elastic scattering data between 500 MeV and 1040 MeV. Using the RIA as the basis for the global fits is a new approach and our results shows that it is a valid method for extracting neutron densities, \( R_n \) and \( S_n \).

In the global approach used in this work the form of the RIA vector and scalar and tensor optical potentials are given by

\[
U_s(r) = -\frac{P_{\text{lab}}}{(2\pi)^2m} \sum_{j=p,n} \int_0^q 4\pi q^2 dq \frac{R(q)}{R(0)} j_0(qr) F^j_s(q) \tilde{\rho}^j_s(q),
\]

\[
U_v(r) = -\frac{P_{\text{lab}}}{(2\pi)^2m} \sum_{j=p,n} \int_0^q 4\pi q^2 dq \frac{R(q)}{R(0)} j_0(qr) F^j_v(q) \tilde{\rho}^j_v(q),
\]

\[
U_t(r) = -\frac{P_{\text{lab}}}{(2\pi)^2m} \sum_{j=p,n} \left[ r \int_0^q 4\pi q^2 dq \frac{R(q)}{R(0)} j_0(qr) F^j_t(q) \tilde{\rho}^j_t(q) + \int_0^q 4\pi q^2 dq \frac{d}{dq} \left( \frac{R(q)}{R(0)} F^j_t(q) \right) j_1(qr) \tilde{\rho}^j_t(q) \right],
\]

where the fourier transforms of the density form factors are

\[
\tilde{\rho}^j_s(q) = \int d^3r' e^{i\vec{q} \cdot \vec{r}'} \rho^j_s(r'),
\]

\[
\tilde{\rho}^j_v(q) = \int d^3r' e^{i\vec{q} \cdot \vec{r}'} \rho^j_v(r'),
\]
\[
\tilde{\rho}_j^t(q) = \frac{1}{q^3} \int d^3 r' e^{i\vec{q} \cdot \vec{r}'} \rho_j^t(r').
\] (6)

The subscripts s,v,t refer to Lorentz scalar, vector (time-like), and tensor quantities. The superscript n and p refer to neutrons and protons, \( F(q) \) are the invariant NN amplitudes, and \( R(q) \) is the kinematical factor required to obtain the invariant NN amplitude in the Breit frame [38]. The value of the upper limit on the momentum transfer, \( \bar{q} \), is determined by the available on–shell NN data. Many studies conducted have demonstrated that higher order corrections to this first order RIA-Dirac optical model approach are negligible at the energies being studied in this paper. See Ray et al. for more details of the RIA-Dirac calculations used for elastic scattering observables [16].

Elastic \( p + A \) data between 500 MeV and 1040 MeV for \( ^{40}\text{Ca}, ^{48}\text{Ca} \) and \( ^{208}\text{Pb} \) form the data set. The quality of the fits are good and the predictions of data not in the data set is used to verify the procedure. However, unlike the usual RIA, where we have generally used scalar, vector and tensor densities from the results of relativistic effective field theory (EFT) calculations, we use the RIA as a basis for extracting the vector neutron density as well as the two scalar densities using a global fitting procedure.

Next we describe the treatment of the vector and scalar densities. The point proton density is fixed by using the results from electron scattering as follows,

\[
\rho_p^v(r) = \frac{1}{(2\pi)^3} \int d^3 q e^{-i\vec{q} \cdot \vec{r}} \tilde{\rho}_p^v(q)
\] (7)

where,

\[
\tilde{\rho}_p^v(q) = \frac{\int d^3 q e^{i\vec{q} \cdot \vec{r}} \rho_p(r)}{G(q)}.
\] (8)

Here \( \rho_p(r) \) is the experimental charge density, \( G(q) \) is the proton form factor. The point proton density, \( \rho_p^v(r) \) is normalized to \( Z \) and the neutron density \( \rho_n^v(r) \) is normalized to \( N \).

The next step is to consider the model that will be used. After several years of testing a number of different forms used to parametrize the neutron vector and scalar densities, we have chosen to model the vector neutron density and the scalar proton and neutron densities using the cosh form (COSH) used in many of our global fits [44]. We have found the cosh form parametrization produced more stable results for \( R-n \) and \( S_n \) than the three parameter fermi (3PF) or the sum of gaussian (SOG) parametrizations. This stability of the cosh was understood from the results from many global fits using the all three forms. In global fits we used four or five different momentum transfer ranges. For example, if five momentum transfer ranges are used the fits are done from 0.0 fm\(^{-1}\) to 1.5 fm\(^{-1}\) to 0.0 fm\(^{-1}\) to 3.5 fm\(^{-1}\) in steps of 0.5 fm\(^{-1}\). In future work we will consider other parametrizations.

The form of the cosh model in this work the superscript V stands for volume and S for surface terms,

\[
f_V(r, R, a) = \frac{\{\cosh[R/a] - 1\}}{\{\cosh[R/a] + \cosh[r/a] - 2\}} \] (9)

\[
f_S(r, R, a) = \frac{\{\cosh[R/a] - 1\}\{\cosh[r/a] - 1\}}{\{\cosh[R/a] + \cosh[r/a] - 2\}^2}.
\] (10)
The vector neutron density is
\[ \rho^n_v(r, R_b, a_b) = \rho^B(r, R_b, a_b) - \rho_p^p(r). \] (11)

The density \( \rho^B(r, R_b, a_b) \) is given by
\[ \rho^B(r, R_b, a_b) \propto f^V(r, R_b, a_b) + \alpha f^S(r, R_b, a_b), \] (12)
and \( \rho^B(r, R_b, a_b) \) is normalized to \( A \). There are two geometric parameters, \( R_b \) and \( a_b \), and the parameter, \( \alpha \).

The scalar proton and neutron densities are,
\[ \rho^p_s(r, R^p_s, a^p_s) \propto f^V(r, R^p_s, a^p_s) + \beta f^S(r, R^p_s, a^p_s) \] (13)
and
\[ \rho^n_s(r, R^n_s, a^n_s) \propto f^V(r, R^n_s, a^n_s) + \gamma f^S(r, R^n_s, a^n_s). \] (14)

Each of these densities contains three parameters, two geometry parameters, \( R^p_b \), \( R^n_b \), \( a^p_b \), \( a^n_b \) and the parameters \( \beta \) and \( \gamma \). The 10th parameter, \( P_{10} \), searched is given by \( \int d^3r \rho^p_s = P_{10} Z \). A similar 11th parameter given by \( \int d^3r \rho^n_s = P_{11} N \) could also have been searched. However, the searches were more stable if the ratio \( \frac{P_{10}}{P_{11}} \), i.e., the ratio of the volume integrals per particle of the scalar proton and neutron densities from any one of the EFT models given by Rustnak et al. [36, 37]. Three different EFT densities are used, two are point coupling models, VA3 and FZ4, and one is a meson model, MA4. The parameter \( P_{10} \) is not sensitive to the EFT model chosen and the final difference and the scalar proton and neutron densities are not sensitive to the EFT model chosen even though the values of \( S_n \) for \( N \neq Z \) in these models differ widely. For example, values of \( S_n \) for \( ^{208}\text{Pb} \) are 0.332 fm for VA3, 0.259 fm for MA4 and 0.160 fm for FZ4.

As mentioned above we found that the model used in this paper was the best of many different models we tried. We will investigate other models in future work using this global approach.

III. FITTING PROCEDURE AND RESULTS

It has long been known that using a global approach has been very useful in obtaining NN amplitudes and the NN phase shifts. This is one of the reasons for using a global approach when the data set used is large and usually correlated. In this work we use elastic \( p + \text{A} \) data between 500 MeV and 1040 MeV for \( ^{40}\text{Ca}, \, ^{48}\text{Ca} \) and \( ^{208}\text{Pb} \). For \( ^{40}\text{Ca} \) there are five energies; 497.5 MeV [45, 46], 613 MeV [47], 650 MeV [48], 797.5 MeV [49, 50], and 1040 MeV [51, 52]. For \( ^{208}\text{Pb} \) there are five energies; 497.5 MeV [43, 53], 613 MeV [47], 650 MeV [54, 55], 797.5 MeV [56, 57], and 1040 MeV [58]. However for \( ^{48}\text{Ca} \) only three energies; 497.5 MeV [45], 797.5 MeV [59], and 1044 MeV [51, 52] are available. In order to make predictions of data not included in the data set we remove one energy. For \( ^{40}\text{Ca} \) and \( ^{208}\text{Pb} \) the 650 MeV data has been excluded, and for \( ^{48}\text{Ca} \) 1044 MeV data has been excluded. The quality of the global fits to the data are good and the predictions for data not in the global data sets verify the procedure.

In previous Dirac phenomenological (DP) global fitting, the data sets were cut at 100 degrees in the center of mass or at momentum transfer \( q \) at 3.0 fm\(^{-1} \), whichever came first.
In this work we do global fits using a variety of momentum transfer values as discussed below. In addition, as is discussed later, we also test the sensitivity of the input to the model.

In the global fitting we have used five momentum transfer ranges from 0.0 fm$^{-1}$ to 1.5 fm$^{-1}$ in steps of 0.5 fm$^{-1}$. Or four momentum transfer ranges from 0.0 fm$^{-1}$ to 2.0 fm$^{-1}$ to 0.0 fm$^{-1}$ to 3.5 fm$^{-1}$ again in steps of 0.5 fm$^{-1}$. The values of $S_n$ and $R_n$ are expected to change with the value of the momentum transfer range used as the data sets are changed. We find that the values of $S_n$ and $R_n$ for a given momentum transfer range (we use five momentum transfer ranges or four momentum transfer ranges) are clustered in a reasonably well constrained set of values for $S_n$ and $R_n$, we call these stable results. The momentum transfer range for 0.0 fm$^{-1}$ to 1.5 fm$^{-1}$ is almost always the outlier, which is understandable as the data set is small and does not have the diffraction structure needed when fitting proton scattering from nuclear targets. Of course in any such global approach the data sets are usually correlated. This is certainly true as the data sets sets with different momentum transfer ranges do overlap. Thus a statistical analysis to obtain a mean value and a standard deviation can not generally be used. However, we can determine the range of the values of $S_n$ and $R_n$ for each fit from every momentum transfer range, and obtain the values for $S_n$ and $R_n$ which could be shown as a scatter plot, as is usually done when data sets used are correlated. We have obtained the range of values for $S_n$ and $R_n$ for every test considered, while these ranges can not be interpreted as resulting from a statistical analysis we found that doing such an analysis gives a useful guide to present the results.

There are a number of features in the fitting procedures which could produce changes in the neutron density, $R_n$ and $S_n$. As mentioned above, two ranges of momentum transfer cuts are used. In the following sensitivity tests we use both ranges. Generally the smaller range produces the smallest range of values for $R_n$ and $S_n$ so in most of the figures we show the larger range values for $R_n$ and $S_n$ which gives the most conservative results.

We test the results obtained when the following features of the model used in the fitting procedures are changed. First the three different EFT models used in fixing the 10th parameter are considered \cite{36, 37}. To investigate the effect of the $G(q)$ used in obtaining the point proton density we use two different forms they are identified as $G_1(q)$ from \cite{60} and $G_2(q)$ from \cite{61}. The set chosen for the Arndt NN amplitudes is input to the fitting procedure, and we use sets FA00 and SM86 to find the sensitivity of the fits to this input \cite{34}. The data sets included in the fits have been changed to see if the results change significantly. Finally, we use the three different charge density models used in the analysis of electron-nucleus elastic scattering data; the three parameter fermi (3PF), the sum of gaussian (SOG) and the fourier-bessel (FB) \cite{62}. Thus each target we have done global fits which have used effective field theory densities MA4, FZ4 and VA3, and for each three charge distributions are used. In addition two different form factors $G_1(q)$ and $G_2(q)$ and two sets of Arndt amplitudes, set FA00 and set SM86. Figures 1-5 show the results of these tests of the global fitting procedure.

The results for the $R_n$ and $S_n$ are given in three tables, found in the Appendix; Table I for $^{40}$Ca, Table II for $^{48}$Ca and Table III for $^{208}$Pb.

There are three general cases we use in testing the global fits: Case 1 uses Arndt amplitudes set FA00 and the $G_1(q)$ form factor; Case 2 uses Arndt amplitudes set FA00 and the $G_2(q)$ form factor; and Case 3 uses Arndt amplitudes set SM86 and the $G_1(q)$ form factor. In every case, we obtain good global fits for MA4, FZ4 and VA3 for every charge distribution used; SOG, 3PF, and FB. As discussed above the momentum cuts ranges used
are 0.0 fm$^{-1}$ to 1.5 fm$^{-1}$ to 0.0 fm$^{-1}$ to 3.5 fm$^{-1}$ in steps of 0.5 fm$^{-1}$ or we remove the 0.0 fm$^{-1}$ to 1.5 fm$^{-1}$ set. For each case the one with five momentum transfer ranges or four momentum transfer ranges we calculate the mean and standard deviation for $R_n$ and $S_n$ for every global fit, this gives us an average over all EFT models for a given charge distribution. This is denoted by $AVE_{EFT}$ in the tables. This is not to be taken as a statistical error, it is however a convenient way to show the range of the values in a consistent way. We do this rather than remove every outlier. Then for each case we show the combined values of the range for $R_n$ and $S_n$ for that case, this gives the average over all charge distributions as well as over all EFT models. These results are denoted as $AVE_{CD}$ and $AVE_{EFT}$ in the tables. Finally, we calculate the ranges of $R_n$ and $S_n$ for all three cases combined for each target.

The last calculation, which combines all of the values of $R_n$ and $S_n$ for every test made, gives us the most conservative results. From these final test cases, shown at the bottom of the tables in the appendix, we get the following values using the five momentum transfer ranges for $^{40}$Ca, 3.350 > $R_n$ > 3.300 fm and −0.008 > $S_n$ > −0.080 fm; for $^{48}$Ca, 3.505 > $R_n$ > 3.421 fm and 0.148 > $S_n$ > 0.058 fm; for $^{208}$Pb, 5.589 > $R_n$ > 5.513 fm and 0.156 > $S_n$ > 0.076 fm. As mentioned earlier if we use the four momentum transfer ranges results are generally more clustered and we obtain: for $^{40}$Ca, 3.350 > $R_n$ > 3.304 fm and −0.006 > $S_n$ > −0.080 fm; for $^{48}$Ca, 3.485 > $R_n$ > 3.417 fm and 0.129 > $S_n$ > 0.053 fm; and for $^{208}$Pb, 5.561 > $R_n$ > 5.513 fm and 0.130 > $S_n$ > 0.074 fm. The values for $R_n$ and $S_n$ for both of the final case results are quite similar.

However, the authors found that the global fits using the Arndt NN amplitude set FA00 and the form factor $G_1(q)$ gave the most stable results i.e. the values did not vary very much, see Tables I, II and III in the Appendix. This is especially true for using the SOG charge distribution. In this case the values of $R_n$ and $S_n$ using the five momentum transfer ranges are: for $^{40}$Ca, 3.318 > $R_n$ > 3.308 fm and −0.059 > $S_n$ > −0.069 fm; for $^{48}$Ca, 3.498 > $R_n$ > 3.412 fm and 0.141 > $S_n$ > 0.055 fm; and for $^{208}$Pb, 5.602 > $R_n$ > 5.512 fm and 0.164 > $S_n$ > 0.074 fm. The values for $R_n$ and $S_n$ using the four momentum transfer ranges are: for $^{40}$Ca, 3.314 > $R_n$ > 3.310 fm and −0.063 > $S_n$ > −0.067 fm; for $^{48}$Ca, 3.459 > $R_n$ > 3.413 fm and 0.102 > $S_n$ > 0.056 fm; and for $^{208}$Pb, 5.550 > $R_n$ > 5.522 fm and 0.111 > $S_n$ > 0.083 fm.

The results using the form factor $G_2(q)$ are quite similar as well as the case using form factor $G_1(q)$ but with the NN set SM86. These values as well as the values using the obtained from the final case values are in agreement with nonrelativistic Skyrme models $^{[1]}$, $^{[42]}$, $^{[43]}$, the relativistic Hartree-Bogoliubov model extended to include density dependent meson-nucleon couplings $^{[4]}$ and the recent analysis of antiproton atoms $^{[12]}$. Our results are generally not in agreement with relativistic mean-field models.

Next we discuss the results of each test of the model separately. Figure 1 shows the effect of the two different momentum transfer ranges as well as the difference in the three EFT models used to fix parameter 10. The filled boxes (the smaller momentum transfer range) and diamonds (the larger momentum transfer range) are the values from the three different EFT models used. The charge density used in this case was the SOG for all targets, the proton form factor $G_1(q)$, and the set FA00 Arndt NN amplitudes. This figure shows that the results from the two different momentum ranges overlap and that the EFT model used does not affect the results. The figure also shows several theoretical results from Refs. $^{[1]}$, $^{[2]}$, $^{[6]}$, $^{[36]}$, $^{[37]}$, $^{[39]}$, $^{[40]}$, $^{[41]}$. The comparison between theory and range of the values from the fits shows that all theoretical $S_n$ values agree with the results of the global fits for $^{40}$Ca. However, the theoretical $S_n$ values for $^{48}$Ca and $^{208}$Pb are somewhat larger than the
global fits.

Next we consider the results when two different proton form factors, \( G_1(q) \) from Ref. [60] and \( G_2(q) \) from Ref. [61], are used in obtaining the vector point proton density for a given charge distribution model. The five momentum transfer ranges, the values for the \( AV E_{EFT} \) models for the SOG model charge distribution and set FA00 Arndt NN amplitudes are used and shown in Figure 2.

Investigating the sensitivity to the set of Arndt NN amplitudes used is done by comparing sets FA00 and SM86. The larger momentum transfer range, the averaged \( EFT \) models for the SOG model charge distribution, and the proton form factor \( G_1(q) \) are used and shown in Figure 3.

In order to check the sensitivity due to the data sets included in the fit we have done fits using only two data sets (497.5 MeV and 797.5 MeV) for \(^{40}\)Ca and \(^{208}\)Pb. The results agree very well with the same case using four data sets as is shown in Fig. 4. The values of \( S_n \) for the two data set case are for \(^{40}\)Ca, \(-0.054 > S_n > -0.066 \) fm; and for \(^{208}\)Pb \(0.166 > S_n > 0.088 \) fm. These results agree very well with the values for the four data set case \(^{40}\)Ca, \(-0.059 > S_n > -0.069 \) fm and \(^{208}\)Pb \(0.164 > S_n > 0.074 \) fm. This encourages us to extend our global procedure to nuclei that have at least two data sets in the medium energy range.

The sensitivity of \( S_n \) and \( R_n \) to the three different charge distributions, three parameter fermi (3PF), sum of gaussian (SOG) and fourier-bessel (FB) obtained from Ref. [62] which are used in the fitting procedure is shown in Fig. 5 and given in detail in Tables I, II and III. The five momentum transfer ranges, the proton form factor \( G_1(q) \) and the set FA00 Arndt NN amplitudes are used. For \(^{208}\)Pb and \(^{48}\)Ca there is little difference in the values of \( S_n \) and \( R_n \) for the different charge densities. We note that the RMS radii for \(^{208}\)Pb and \(^{48}\)Ca for these three charge density are about the same [62]. Two of the three \(^{40}\)Ca charge distributions also have almost identical charge RMS radii, 3PF 3.482(25)fm, SOG and 3.479(3)fm but the RMS radius for the FB, 3.450(10)fm, is considerably smaller (see Ref. [62]). The result is that for \(^{40}\)Ca the difference in the value of \( S_n \) is pronounced. As shown in Fig. 5 the FB charge distribution has range of \( S_n \) values that goes from positive to negative but the range of the \( S_n \) values 3PF and SOG are all negative. We attribute this to the smaller RMS radius for the FB charge distribution. The values of \( R_p \), \( R_n \) and \( S_n \) should not depend heavily on the momentum transfer ranges, the set of amplitudes, the form factors, and the charge distribution. In fact, the only case that does not overlap is the FB case for \(^{40}\)Ca. The results for all three of the charge distributions are given in Tables I-III in the Appendix.

In this global analysis we obtain many densities from the various cases used in checking the results of the fits. Figures 6 and 7 show the vector point proton density distribution, \( \rho_n^p \) and \( r^2 \rho_n^p \) and the vector point neutron density distribution, \( \rho_n^n \) and \( r^2 \rho_n^n \) for \(^{40}\)Ca, \(^{48}\)Ca, and \(^{208}\)Pb. The SOG charge distribution and the \( G_1(q) \) form factor were used to obtain the vector point proton density. The neutron density also depends on the charge distribution chosen and the form factor used to obtain the vector point proton density as well as the other input to a given global fit, the NN amplitudes, and the EFT density used to fix the ratio of \( P_{10} \) to \( P_{11} \). In Figs. 6 and 7 the Arndt NN amplitude set FA00, the SOG charge distribution, the momentum transfer range was 0.0 fm\(^{-1}\) to 3.0 fm\(^{-1}\), proton from factor \( G_1(q) \), and all three EFT cases, MA4, FZ4 and VA3 are used. It is clear that the neutron densities overlap showing that the use of different EFT cases does not have any significant impact on the global fit. Tables of the densities and the parameters are available from the authors [63].

This same input as in Figs 6-7 is also used in the Figs. 8-9 which shows the vector proton
FIG. 1: The ranges, shown as bars, of the skin thickness for $^{40}\text{Ca}$, $^{48}\text{Ca}$ and $^{208}\text{Pb}$ from the global fits using the SOG charge distribution, set FA00 Arndt NN amplitudes, proton form factor $G_1(q)$, and the three EFT models, MA4, FZ4 and VA3 are shown by the filled boxes, for the four momentum transfer ranges and the filled diamonds for the five momentum transfer ranges. Several theoretical $S_n$ values from Refs. [1, 3, 4, 6, 36, 37, 39, 40, 41] are also shown by using various symbols.

FIG. 2: The skin thickness with different proton form factors. Results for $^{40}\text{Ca}$, $^{48}\text{Ca}$ and $^{208}\text{Pb}$ from the global fitting procedure shows the $AVE_{EFT}$ models values for the SOG charge distribution, set FA00 Arndt NN amplitudes, the five momentum transfer ranges. We compare $G_1(q)$ from Ref. [60] shown as filled boxes and $G_2(q)$ from Ref. [61] shown as filled diamonds. The same theoretical values for $S_n$ shown in Fig. 1 are also shown.
FIG. 3: The skin thickness for $^{40}$Ca, $^{48}$Ca and $^{208}$Pb due to different sets of Arndt NN amplitudes. The global results shown are the $AV_EFT$ models for SOG charge distribution, the five momentum transfer ranges, and proton form factor $G_1(q)$. Set FA00 is shown as filled boxes and set SM86 is shown as filled diamonds. The same theoretical values for $S_n$ shown in Fig. 1 are also shown.

FIG. 4: The skin thickness for $^{40}$Ca, $^{48}$Ca and $^{208}$Pb when different data sets included in the fits, one has four energies and the other has two energies. The results of the global procedure uses the $AV_EFT$ models and the SOG charge distribution, the five momentum transfer ranges, the proton form factor $G_1(q)$, set FA00 Arndt NN amplitudes. The results for four-energy sets are shown by filled boxes and the results for two-energy data sets by filled diamonds. The data set for $^{48}$Ca has only three energies so only has two data sets are used in the fits. The same theoretical values for $S_n$ shown in Fig. 1 are also shown.
FIG. 5: The figures shows the change in the skin thickness for $^{40}\text{Ca}$, $^{48}\text{Ca}$ and $^{208}\text{Pb}$ due to different charge distribution models. The global fits use set FA00 Arndt NN amplitudes, the five momentum transfer ranges, and the proton from factor $G_1(q)$. The results for the $AVE_{EFT}$ SOG are shown by filled boxes, the $AVE_{EFT}$ 3PF are shown by filled diamonds and the $AVE_{EFT}$ FB are shown by filled circles. The results for all effective field theory cases and for all three charge distributions i.e. the results from the final cases are given by the open squares. The same theoretical values for $S_n$ shown in Fig.1 are also shown.
FIG. 6: The vector point proton density distribution shown by the solid curve. The three neutron density distribution densities using the EFT densities, MA4, FZ4 and VA3 are shown by the dashed line for case MA4, the dots for case FZ4, and the dotdashed for case VA3. All density are obtained from the global fit as discussed in the manuscript.
FIG. 7: The vector point proton density distribution shown by the solid curve. The three neutron density distribution densities using the EFT densities, MA4, FZ4 and VA3 are shown by the dashed line for case MA4, the dots for case FZ4, and the dotdashed for case VA3. All density are multiplied by the radius squared and obtained from the global fit as discussed in the manuscript.
point density and the scalar proton $\rho_p^s$ and $r^2\rho_p^s$, and in Figs. 10-11 for the vector point density and the scalar neutron $\rho_n^s$ and $r^2\rho_n^s$. Figures 8-11 show that the phenomenology gives sensible scalar densities that are not unphysical.

Figures 12 and 13 show the observables for one of the energies included in the global fit for $^{40}$Ca and a prediction of an energy not included in the global fit. Figure 12 shows the fit for $^{40}$Ca at 497.5 MeV and Fig. 13 shows the prediction for the $^{40}$Ca 650 MeV spin observables; no cross section data is available. Figures 14 and 15 show the observables for one of the energies included in the global fit for $^{48}$Ca and the results from a prediction of an energy not included in the global fit. Figure 14 shows the fit for $^{48}$Ca at 497.5 MeV and the prediction of Q at that energy. Figure 15 shows the prediction for $^{48}$Ca at 1044 MeV. Figures 16 and 17 show the observables for one of the energies included in the global fit for $^{208}$Pb and the results from a prediction of an energy not included in the global fit. Figure 16 shows the fit for $^{208}$Pb at 497.5 MeV and Fig. 17 shows the prediction for the 650 MeV observables for this target. The observables shown in Figs. 12-17 all used the Arndt NN amplitude set FA00, the SOG charge distribution, the proton form factor $G_1(q)$, the momentum transfer range from $0.0 \text{ fm}^{-1}$ to $3.0 \text{ fm}^{-1}$, and the EFT case MA4.

While the figures of the observable are small we have magnified them and we find that the heights of the diffractive maxima and the angular positions of the minima and maxima are very well reproduced at each energy with no systematic energy dependent discrepancies. These are the most critical features of the data which determine rms radii. In fact, precision fits to $A_y$ and Q are not as important for determining neutron radii but our fits are quite reasonable. The magnified figures are available from the authors [63].

We also predict the total neutron cross section and the proton reaction cross section. In Fig. 18 the predicted total neutron cross sections for $^{40}$Ca and $^{208}$Pb is compared with the experimental values from Finlay et al. [64]. The predicted proton reaction cross sections for the same two targets are given in Fig. 19 with the experimental values [65] and the predictions are reasonable.

All of the results confirm that the global approach produces good fits to the data, that predictions are quite acceptable and that the neutron densities are energy independent. These densities should be useful input to a large number of nuclear reactions. They provide the empirical values of $R_n$ and $S_n$ that are needed for testing theoretical models.

IV. SUMMARY AND CONCLUSIONS

This paper reports a new method for extracting neutron densities from intermediate energy elastic proton-nucleus scattering observables. Neutron densities are needed for atomic parity violation experiments, the analysis of antiproton atoms, the experiments parity violation elastic electron scattering, and theoretical nonrelativistic and covariant mean-field models. It is interesting to note that our results for $S_n$ are in agreement with work using antiprotonic atoms, see Fig. 4 in Ref. [12].

The approach uses a global analysis, similar to the global fits using a Dirac phenomenology, but in this case it is based on the relativistic impulse approximation (RIA). The input to the procedure are the vector point proton density distribution, $\rho_p^v$, which is determined from the empirical charge density after unfolding the proton form factor and the Arndt NN amplitudes. The other densities, $\rho_n^v$, $\rho_p^s$, $\rho_n^s$, are parametrized. The neutron densities for $^{40}$Ca, $^{48}$Ca and $^{208}$Pb obtained from the global fits have been tested for sensitivity to the input. Different proton form factors, different charge densities, different sets of the Arndt
FIG. 8: The vector point proton density distribution shown by the solid curve. The three scalar proton density distribution densities using the EFT densities, MA4, FZ4 and VA3 are shown by the dashed line for case MA4, the dots for case FZ4, and the dotdashed for case VA3. All density are obtained from the global fit as discussed in the manuscript.
FIG. 9: The vector point proton density distribution shown by the solid curve. The three scalar proton density distribution densities using the EFT densities, MA4, FZ4 and VA3 are shown by the dashed line for case MA4, the dots for case FZ4, and the dotdashed for case VA3. All density are multiplied by the radius squared and obtained from the global fit as discussed in the manuscript.
FIG. 10: The vector point proton density distribution shown by the solid curve. The three neutron scalar density distribution densities using the EFT densities, MA4, FZ4 and VA3 are shown by the dashed line for case MA4, the dots for case FZ4, and the dotdashed for case VA3. All density are obtained from the global fit as discussed in the manuscript.
FIG. 11: The vector point proton density distribution shown by the solid curve. The three neutron scalar density distribution densities using the EFT densities, MA4, FZ4 and VA3 are shown by the dashed line for case MA4, the dots for case FZ4, and the dotdashed for case VA3. All densities are multiplied by the radius squared and obtained from the global fit as discussed in the manuscript.
FIG. 12: Results of the global analysis for $^{40}$Ca at 497.5 MeV.
FIG. 13: Prediction for $^{40}$Ca at 650 MeV.
FIG. 14: Results of the global analysis for $^{48}$Ca at 497.5 MeV.
FIG. 15: Prediction for $^{48}$Ca at 1044 MeV.
FIG. 16: Results of the global analysis for $^{208}$Pb at 497.5 MeV.
FIG. 17: Prediction for $^{208}\text{Pb}$ at 650 MeV
FIG. 18: Predicted total cross sections for $^{40}$Ca and $^{208}$Pb shown as circles are compared with the experimental values shown as diamonds from Ref. [64].
FIG. 19: Predicted proton reaction cross sections are shown as circles for $^{40}$Ca and $^{208}$Pb compared with the experimental values shown as diamonds from Ref. [65]. The dashed lines are the results of the EDAI global fit and the dotted lines are the EDAD fit 3 from Ref. [44].
NN amplitudes, different EFT models, different data sets included in the fits, and differences in the ranges of the momentum transfer used in the fits have been investigated. With the exception of the fourier-bessel charge density for $^{40}$Ca all of these tests produce values for $R_n$ and $S_n$ that overlap. The prediction of data sets not used in the fits are well reproduced and the calculated observables are in good agreement with data.

In conclusion, we have obtained values of $R_n$ and $S_n$ for $^{40}$Ca, $^{48}$Ca, and $^{208}$Pb. We have obtain good global fits for MA4, FZ4 and VA3 for every charge distribution used; SOG, 3PF, and FB. Both five momentum transfer ranges and four momentum transfer ranges were used. As discussed earlier, for each case we calculate the mean and standard deviation for $R_n$ and $S_n$ for every global fit, this gives us an average over all EFT models for a given charge distribution. This is denoted by $AVE_{EFT}$ in the tables. As mentioned several times this is not to be taken as a statistical analysis, but as a way to show the range of the values in a consistent way, they are shown as bars in the figures. We do this rather than remove outliers. Then for each case we show the combined values of the range for $R_n$ and $S_n$ for that case, this gives the average over all charge distributions as well as over all EFT models. These results are denoted as $AVE_{CD}$ and $AVE_{EFT}$ in the tables. We then calculate the ranges of $R_n$ and $S_n$ for all three cases combined for each target. From these final test cases, shown at the bottom of the tables in the appendix, using the five momentum transfer ranges we obtain the following values: for $^{40}$Ca, $3.350 > R_n > 3.300$ fm and $-0.008 > S_n > -0.080$ fm; for $^{48}$Ca, $3.505 > R_n > 3.421$ fm and $0.148 > S_n > 0.058$ fm; for $^{208}$Pb, $5.589 > R_n > 5.513$ and $0.156 > S_n > 0.076$ fm. In conclusion, the RMS neutron radius, $R_n$ and the neutron skin thickness $S_n$ obtained from the global fits that we consider to be the most reliable ranges of our results are given as follows: for $^{40}$Ca, $3.314 > R_n > 3.310$ fm and $-0.063 > S_n > -0.007$ fm; for $^{48}$Ca, $3.459 > R_n > 3.413$ fm and $0.102 > S_n > 0.056$ fm; and for $^{208}$Pb $5.550 > R_n > 5.522$ fm and $0.111 > S_n > 0.083$ fm.

The authors have found that the global fits are more stable when using the Arndt NN amplitude set FA00 rather than the Arndt NN amplitude set SM86. This might be expected as the FA00 analysis is more recent. Both of the form factors used, $G_1(q)$ and $G_2(q)$ when using FA00 were the most stable, see Tables I, II and III in the Appendix. The impact on the fits due to the charge distribution input, SOG, 3PF and FB, as shown in Fig. 5 clearly showed that the cases using the SOG and 3PF charge distributions were stable, i.e. the ranges overlapped, for all targets, however, for $^{40}$Ca the results for the FB charge distribution gave very different results, see Table I. We also found that the fits using Arndt NN amplitude set FA00, the $G_1(q)$ form factor, and the SOG charge distribution produced the most stable results. In this case the values of $R_n$ and $S_n$ using the five momentum transfer ranges are: for $^{40}$Ca, $3.318 > R_n > 3.308$ fm and $-0.059 > S_n > -0.069$ fm; for $^{48}$Ca, $3.498 > R_n > 3.412$ fm and $0.141 > S_n > 0.055$ fm; and for $^{208}$Pb $5.602 > R_n > 5.512$ fm and $0.164 > S_n > 0.074$ fm. These values, as well as the values using the most conservative errors and the results that we consider most reliable, are in agreement with nonrelativistic Skyrme models $^{[1, 12, 43]}$, relativistic Hartree-Bogoliubov model extended to include density dependent meson-nucleon couplings $^{[4]}$ and results from a recent analysis of antiprotonic atoms $^{[12]}$. Our results for $^{48}$Ca and $^{208}$Pb are generally not in agreement with relativistic mean-field models, see Ref. 6 and references there in.

The global fit based on the RIA is a new tool for obtaining the neutron density. The values of $R_n$ and $S_n$ obtained are robust. This quality is verified in several ways as discussed in Section III. For example, we checked the sensitivity due to the data sets included in the
fit by doing global fits using only two data sets (497.5 MeV and 797.5 MeV) for $^{40}$Ca and $^{208}$Pb as well as four data sets. The values of $S_n$ for the two data set case are: for $^{40}$Ca $-0.054 > S_n > -0.066$ fm; and for $^{208}$Pb $0.166 > S_n > 0.088$ fm. These results agree well with $S_n$ for the four data set case: for $^{40}$Ca $-0.059 > S_n > -0.069$ fm; and for $^{208}$Pb $0.164 > S_n > 0.074$ fm. These results, shown in Fig. 4 motivates us to use the global procedure for all nuclei that have at least two data sets in the medium energy range.

This work provides energy independent values for $R_n$ and $S_n$, in contrast to the energy dependent values obtained by previous studies. In addition, the results presented in paper show that the expected rms neutron radius and skin thickness for $^{40}$Ca is accurately reproduced. The values of $R_n$ and $S_n$ obtained from the global fits that we consider to be the most reliable are given as follows: We plan to extend our work to additional nuclei and will continue to investigate different models and procedures. The goal is to continue to improve quality of neutron densities that result from our global fits.

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### TABLE I:

| Range (fm$^{-1}$) | $R_n$ (fm) | $S_n$ (fm) |
|------------------|------------|------------|
| **40$^{Ca}$ Case 1** |            |            |
| $G_1(q)$         | $AV E_{EFT}$ |            |
| SOG              | 2.0-3.5    | 3.314 > $R_n$ > 3.310 | $-0.063 > S_n > -0.067$ |
|                  | 1.5-3.5    | 3.318 > $R_n$ > 3.308 | $-0.059 > S_n > -0.069$ |
|                  |            | 3PF        | $AV E_{EFT}$ |
|                  | 2.0-3.5    | 3.328 > $R_n$ > 3.304 | $-0.051 > S_n > -0.075$ |
|                  | 1.5-3.5    | 3.328 > $R_n$ > 3.304 | $-0.051 > S_n > -0.075$ |
|                  |            | FB         | $AV E_{EFT}$ |
|                  | 2.0-3.5    | 3.373 > $R_n$ > 3.349 | $0.028 > S_n > 0.004$ |
|                  | 1.5-3.5    | 3.371 > $R_n$ > 3.347 | $0.025 > S_n > 0.001$ |
|                  |            | $AV E_{CD}$ | $AV E_{EFT}$ |
|                  | 2.0-3.5    | 3.354 > $R_n$ > 3.306 | $0.002 > S_n > -0.076$ |
|                  | 1.5-3.5    | 3.352 > $R_n$ > 3.306 | $0.000 > S_n > -0.076$ |
| **40$^{Ca}$ Case 2** |            |            |
| $G_2(q)$         | $AV E_{EFT}$ |            |
| SOG              | 2.0-3.5    | 3.314 > $R_n$ > 3.300 | $-0.070 > S_n > -0.084$ |
|                  | 1.5-3.5    | 3.315 > $R_n$ > 3.299 | $-0.069 > S_n > -0.085$ |
|                  |            | 3PF        | $AV E_{EFT}$ |
|                  | 2.0-3.5    | 3.317 > $R_n$ > 3.299 | $-0.070 > S_n > -0.088$ |
|                  | 1.5-3.5    | 3.324 > $R_n$ > 3.296 | $-0.062 > S_n > -0.090$ |
|                  |            | FB         | $AV E_{EFT}$ |
|                  | 2.0-3.5    | 3.341 > $R_n$ > 3.333 | $-0.012 > S_n > -0.020$ |
|                  | 1.5-3.5    | 3.340 > $R_n$ > 3.332 | $-0.013 > S_n > -0.021$ |
|                  |            | $AV E_{CD}$ | $AV E_{EFT}$ |
|                  | 2.0-3.5    | 3.333 > $R_n$ > 3.301 | $-0.027 > S_n > -0.087$ |
|                  | 1.5-3.5    | 3.334 > $R_n$ > 3.302 | $-0.026 > S_n > -0.086$ |
| **40$^{Ca}$ Case 3** |            |            |
| $G_1(q)$         | $SM_{86}$  |            |
| SOG              | 2.0-3.5    | 3.344 > $R_n$ > 3.308 | $-0.033 > S_n > -0.069$ |
|                  | 1.5-3.5    | 3.341 > $R_n$ > 3.305 | $-0.030 > S_n > -0.072$ |
|                  |            | 3PF        | $AV E_{EFT}$ |
|                  | 2.0-3.5    | 3.332 > $R_n$ > 3.302 | $-0.047 > S_n > -0.077$ |
|                  | 1.5-3.5    | 3.330 > $R_n$ > 3.292 | $-0.049 > S_n > -0.087$ |
|                  |            | FB         | $AV E_{EFT}$ |
|                  | 2.0-3.5    | 3.374 > $R_n$ > 3.340 | $0.028 > S_n > -0.006$ |
|                  | 1.5-3.5    | 3.371 > $R_n$ > 3.335 | $0.025 > S_n > -0.011$ |
|                  |            | $AV E_{CD}$ | $AV E_{EFT}$ |
|                  | 2.0-3.5    | 3.356 > $R_n$ > 3.308 | $0.002 > S_n > -0.070$ |
|                  | 1.5-3.5    | 3.354 > $R_n$ > 3.304 | $-0.001 > S_n > -0.075$ |
| **40$^{Ca}$ for all three cases** |            |            |
|                  | 2.0-3.5    | 3.350 > $R_n$ > 3.304 | $-0.006 > S_n > -0.080$ |
|                  | 1.5-3.5    | 3.350 > $R_n$ > 3.300 | $-0.008 > S_n > -0.080$ |
| Range (fm⁻¹) | $R_n$ (fm) | $S_n$ (fm) |
|-----|-----|-----|
| **48Ca Case 1** |
| | | |
| $G_1(q)$ | **FA00** | |
| SOG | $AV E_{EFT}$ | |
| 2.0-3.5 | $3.459 > R_n > 3.413$ | $0.102 > S_n > 0.056$ |
| 1.5-3.5 | $3.498 > R_n > 3.412$ | $0.141 > S_n > 0.055$ |
| 3PF | $AV E_{EFT}$ | |
| 2.0-3.5 | $3.456 > R_n > 3.412$ | $0.087 > S_n > 0.043$ |
| 1.5-3.5 | $3.501 > R_n > 3.409$ | $0.132 > S_n > 0.040$ |
| FB | $AV E_{EFT}$ | |
| 2.0-3.5 | $3.460 > R_n > 3.420$ | $0.113 > S_n > 0.073$ |
| 1.5-3.5 | $3.499 > R_n > 3.417$ | $0.152 > S_n > 0.070$ |
| $AV E_{CD}$ | $AV E_{EFT}$ | |
| 2.0-3.5 | $3.459 > R_n > 3.415$ | $0.103 > S_n > 0.055$ |
| 1.5-3.5 | $3.499 > R_n > 3.413$ | $0.142 > S_n > 0.054$ |
| **48Ca Case 2** |
| | | |
| $G_2(q)$ | FA00 | |
| SOG | $AV E_{EFT}$ | |
| 2.0-3.5 | $3.453 > R_n > 3.407$ | $0.089 > S_n > 0.043$ |
| 1.5-3.5 | $3.498 > R_n > 3.404$ | $0.134 > S_n > 0.040$ |
| 3PF | $AV E_{EFT}$ | |
| 2.0-3.5 | $3.448 > R_n > 3.404$ | $0.072 > S_n > 0.028$ |
| 1.5-3.5 | $3.500 > R_n > 3.398$ | $0.124 > S_n > 0.022$ |
| FB | $AV E_{EFT}$ | |
| 2.0-3.5 | $3.458 > R_n > 3.414$ | $0.103 > S_n > 0.059$ |
| 1.5-3.5 | $3.511 > R_n > 3.407$ | $0.157 > S_n > 0.053$ |
| $AV E_{CD}$ | $AV E_{EFT}$ | |
| 2.0-3.5 | $3.454 > R_n > 3.408$ | $0.092 > S_n > 0.040$ |
| 1.5-3.5 | $3.503 > R_n > 3.403$ | $0.139 > S_n > 0.037$ |
| **48Ca Case 3** |
| | | |
| $G_1(q)$ | **SM86** | |
| SOG | $AV E_{EFT}$ | |
| 2.0-3.5 | $3.506 > R_n > 3.476$ | $0.150 > S_n > 0.120$ |
| 1.5-3.5 | $3.505 > R_n > 3.463$ | $0.148 > S_n > 0.106$ |
| 3PF | $AV E_{EFT}$ | |
| 2.0-3.5 | $3.487 > R_n > 3.447$ | $0.119 > S_n > 0.079$ |
| 1.5-3.5 | $3.484 > R_n > 3.446$ | $0.116 > S_n > 0.078$ |
| FB | $AV E_{EFT}$ | |
| 2.0-3.5 | $3.516 > R_n > 3.486$ | $0.169 > S_n > 0.139$ |
| 1.5-3.5 | $3.515 > R_n > 3.473$ | $0.167 > S_n > 0.125$ |
| $AV E_{CD}$ | $AV E_{EFT}$ | |
| 2.0-3.5 | $3.509 > R_n > 3.465$ | $0.158 > S_n > 0.100$ |
| 1.5-3.5 | $3.504 > R_n > 3.458$ | $0.152 > S_n > 0.094$ |
| **48Ca for all three cases** |
| | | |
| | | |
| | | |
| 2.0-3.5 | $3.485 > R_n > 3.417$ | $0.129 > S_n > 0.053$ |
| 1.5-3.5 | $3.505 > R_n > 3.421$ | $0.148 > S_n > 0.058$ |
### TABLE III:

| Range (fm$^{-1}$) | $R_n$ (fm) | $S_n$ (fm) |
|------------------|-----------|-----------|
| 208$^\text{Pb}$ Case 1 |
| $G_1(q)$ | $AV E_{EFT}$ | |
| SOG | 5.550 | 0.111 | |
| $R_n > 5.522$ | $S_n > 0.083$ | |
| 2.0-3.5 | 5.602 | 0.164 | |
| 1.5-3.5 | $R_n > 5.512$ | $S_n > 0.074$ | |
| 3PF | $AV E_{EFT}$ | |
| 2.0-3.5 | 5.580 | 0.155 | |
| 1.5-3.5 | $R_n > 5.548$ | $S_n > 0.123$ | |
| FB | $AV E_{EFT}$ | |
| 2.0-3.5 | 5.546 | 0.108 | |
| 1.5-3.5 | $R_n > 5.518$ | $S_n > 0.080$ | |
| $AV E_{CD}$ | $AV E_{EFT}$ | |
| 2.0-3.5 | 5.565 | 0.135 | |
| 1.5-3.5 | $R_n > 5.523$ | $S_n > 0.085$ | |
| 208$^\text{Pb}$ Case 2 |
| $G_2(q)$ | $AV E_{EFT}$ | |
| SOG | 5.545 | 0.102 | |
| $R_n > 5.517$ | $S_n > 0.074$ | |
| 2.0-3.5 | 5.599 | 0.156 | |
| 1.5-3.5 | $R_n > 5.507$ | $S_n > 0.064$ | |
| 3PF | $AV E_{EFT}$ | |
| 2.0-3.5 | 5.576 | 0.146 | |
| 1.5-3.5 | $R_n > 5.538$ | $S_n > 0.108$ | |
| FB | $AV E_{EFT}$ | |
| 2.0-3.5 | 5.537 | 0.095 | |
| 1.5-3.5 | $R_n > 5.511$ | $S_n > 0.069$ | |
| $AV E_{CD}$ | $AV E_{EFT}$ | |
| 2.0-3.5 | 5.593 | 0.151 | |
| 1.5-3.5 | $R_n > 5.499$ | $S_n > 0.057$ | |
| 208$^\text{Pb}$ Case 3 |
| $G_1(q)$ | $AV E_{EFT}$ | |
| SOG | 5.538 | 0.100 | |
| $R_n > 5.498$ | $S_n > 0.060$ | |
| 2.0-3.5 | 5.554 | 0.115 | |
| 1.5-3.5 | $R_n > 5.502$ | $S_n > 0.063$ | |
| 3PF | $AV E_{EFT}$ | |
| 2.0-3.5 | 5.584 | 0.158 | |
| 1.5-3.5 | $R_n > 5.532$ | $S_n > 0.106$ | |
| FB | $AV E_{EFT}$ | |
| 2.0-3.5 | 5.532 | 0.094 | |
| 1.5-3.5 | $R_n > 5.502$ | $S_n > 0.064$ | |
| $AV E_{CD}$ | $AV E_{EFT}$ | |
| 2.0-3.5 | 5.551 | 0.113 | |
| 1.5-3.5 | $R_n > 5.503$ | $S_n > 0.065$ | |
| 208$^\text{Pb}$ for all three cases |
| 2.0-3.5 | 5.561 | 0.130 | |
| 1.5-3.5 | $R_n > 5.513$ | $S_n > 0.074$ | |
| 1.5-3.5 | 5.589 | 0.156 | |
| $R_n > 5.513$ | $S_n > 0.076$ | |

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