To assessment of stress-strain state in rock continua

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Abstract. The approach for calculation of stress-strain state of mine excavation in rock continua is described in this issue. Boundary element techniques are used here. Proposed method can take into account existing cracks in continua and its propagation under dynamic or static loading.

1. Introduction

The problem of ensuring tunnel stability under mineral deposits development is characterized by steady relevance and escalates as mining operations deepen. In this regard, there is a need for continuous improvement of technologies and means of fastening special workings and methods of their support in difficult mining and geological-mining conditions.

Numerical approach and results of theoretical calculation of deep tunnels by finite element method are described in some other works [1, 2]. There was described surrounding continuum is fractured rocks. However boundary element techniques are more relevant for taking into account conditions on infinity and other specific geological conditions. Soils response of water saturated soils or earth dams [3-7] cannot be used for rock continua.

However, the development of such measures cannot continue without the accumulation of a knowledge base on the complex mechanism of inelastic strain in fractured rocks around the workings.

At present, the stages of elastic strain of fractured rocks around the workings have been investigated in sufficient detail. In some issues, the stages of post-limit strains were wide studied, when a zone of disturbed rocks is formed and develops mainly around circular workings.

Our analysis shows that while the elastic stage of strain is studied in sufficient detail using mathematical models and accounting for the nature of the redistribution of stresses and strains around the mine, the post-limit stages of strain in the host rocks were studied mainly by laboratory-experimental methods based on physical modeling and mine instrumental observations.

In this case, elastic processes give rise to displacements on the working contour which are comparable with insignificant displacements, while displacements measured in large quantities are caused exclusively by irreversible processes: plastic strains, fracture, post-limit strains and irreversible displacement of blocks and pieces of fractured rocks. Recent studies have shown that such displacements occur alternately around the mine, which is a consequence of the self-wedging effect of previously destroyed rocks.
In consideration of mine workings in rock mass is distinguished by the fact that such tunnels should be considered as the cuts in bodies in the state of equilibrium. Structure elements in which the dangerous stresses occur can be reinforced with fastening supports and stringers. But at the same time, it is necessary to take into account emerging and existing cracks in rock mass, its multi-layer nature, initial stresses and strains that could appear as a result of mining.

2. Main part

The boundary element method has proven to be a very convenient tool for solving problems of mining mechanics as it takes into account stresses acting on the infinity.

Simplified approaches like plate models without simplified hypotheses [8-10] as free vibrations cannot be used for analysis.

In practice, calculations of tunnels’ strength are based on building codes. Basically the static and dynamic behaviour of rock mass is described by elasticity.

2.1. Numerical approach

For estimation of stress and strain state of underground caves in rock continua a numerical approach of plane elasticity is used.

Let consider the stress and strain calculation method of tunnel in rock continua. The first stage is calculation only gravity forces. Here rock described as isotopic continua. It is enough as initial data in simplified approach.

The initial data we should have: a cross section of the mine with an indication of the weakened zones, a static stress state on the surface of possible destruction, characteristics of the mechanical properties of the rock and dynamic characteristics of the possible impacts. Deformations are accepted linearly elastic and the strength of the rock mass takes the form of von Mises-Schleicher-Botkin conditions as:

\[ F = \sqrt{I_2^2 + kl_1} - b > 0, \]

here \( I_2 \) is second invariant of the stress deviator, the \( I_1 \) is first invariant of the stress tensor, \( k \) and \( b \) are the rock strength parameters in the weakened zones of the rock continua.

So, follow S.L.Crouch and A.M.Starfield [11,12] general expression for stresses and displacements by using generalized Kelvin’s solution have the form:

\[
\begin{align*}
\sigma_{xx} &= F_x [2(1-\nu)g_{,x} - xg_{,xx}] + F_y [2\nu g_{,y} - yg_{,yy}] \\
\sigma_{yy} &= F_x [2\nu g_{,y} - yg_{,yy}] + F_y [2(1-\nu)g_{,y} - yg_{,yy}] \\
\sigma_{xy} &= F_x [(1-2\nu)g_{,y} - xg_{,xy}] + F_y [2(1-\nu)g_{,x} - yg_{,xy}]
\end{align*}
\]

here

\[
g(x, y) = -\frac{1}{4\pi(1-\nu)} \ln \sqrt{x^2 + y^2}
\]

and it derivatives.

\[
g_{,x} = -\frac{1}{4\pi(1-\nu)} \frac{x}{x^2 + y^2}
\]

\[
g_{,y} = -\frac{1}{4\pi(1-\nu)} \frac{y}{x^2 + y^2}
\]
\[ g_{,xy} = \frac{1}{4\pi(1-\nu)} \frac{2xy}{(x^2 + y^2)^2} \]
\[ g_{,xx} = -g_{,yy} = \frac{1}{4\pi(1-\nu)} \frac{x^2 - y^2}{(x^2 + y^2)^2} \]

At numerical calculation after some transformations for constant displacements and stresses on each boundary element we have:
\[ u_x = \frac{P_x}{2G} \left[ (3-4\nu) f + yf_{,y} \right] + \frac{P_y}{2G} \left[ -yf_{,x} \right] \]
\[ u_y = \frac{P_x}{2G} \left[ -yf_{,x} \right] + \frac{P_y}{2G} \left[ (3-4\nu) f - yf_{,y} \right] \]
\[ \sigma_{xx} = P_x \left[ (3-2\nu) f_{,x} + yf_{,y} \right] + P_y \left[ 2yf_{,y} - yf_{,xx} \right] \]
\[ \sigma_{yy} = P_x \left[ 2(1-\nu) f_{,y} - yf_{,xy} \right] + P_y \left[ (1-2\nu) f_{,x} - yf_{,xy} \right] \]
\[ \sigma_{xy} = P_x \left[ 2(1-\nu) f_{,y} - yf_{,xy} \right] + P_y \left[ (1-2\nu) f_{,x} - yf_{,xy} \right] \]

here \( P_i \) is fictitious loads and
\[ f(x, y) = -\frac{1}{4\pi(1-\nu)} \left( y \arctg \frac{y}{x-a} - \arctg \frac{y}{x+a} \right) \]
\[ - (x-a) \ln \sqrt{(x-a)^2 + y^2} + (x+a) \ln \sqrt{(x+a)^2 + y^2} \]
\[ f_{,x} = \frac{1}{4\pi(1-\nu)} \left[ \ln \sqrt{(x-a)^2 + y^2} - \ln \sqrt{(x+a)^2 + y^2} \right] \]
\[ f_{,y} = -\frac{1}{4\pi(1-\nu)} \left[ \arctg \frac{y}{x-a} - \arctg \frac{y}{x+a} \right] \]
\[ f_{,xy} = \frac{1}{4\pi(1-\nu)} \left[ \frac{y}{(x-a)^2 + y^2} - \frac{y}{(x+a)^2 + y^2} \right] \]
\[ f_{,xx} = f_{,yy} = \frac{1}{4\pi(1-\nu)} \left[ \frac{x-a}{(x-a)^2 + y^2} - \frac{x+a}{(x+a)^2 + y^2} \right] \]

At numerical realisation we have to solve system of algebraic linear equations \([11, 12]\). Gauss method was used here.

If the surfaces of the possible destruction are marked accurately enough, then the strength condition is taken in the form of a Mohr-Coulomb and it failure is unacceptable.

2.2. Results of calculation

The stress state of the rock mass is, as a rule, a state of compression. In many problems of applied mechanics, a body is initially considered to be free of stresses. In mining mechanics associated with underground mining, it is necessary to postulate the initial stress state of the rock mass. This initial state is violated after the formation of production. Thus, the total stresses are represented as the sum of the initial stresses and their changes due to the development as:
\[ \sigma_{ij} = \sigma_{ij}^{0} + \sigma_{ij}', \]

here \( \sigma_{ij}^{0} \) are initial stresses and \( \sigma_{ij}' \) are additional ones.

In this issue the initial stresses accepted like \([13]\):
\[ \sigma_y = -\mu \sigma_x; \quad \sigma_x = \frac{\nu}{(1-\nu)} \sigma_y \]
Based on the methods proposed in [11], the problem of tunneling is considered, taking into account the stage-by-stage excavation of the rock mass which initially is in the stress state (under its own weight) and the effect of parallel tunneling on the stress-strain state of structure. The following initial data were adopted: $E = 10\text{E}6$ kPa, $\nu=0.2$, $\gamma=270$ kPa. The length of each tunnel is 6 m and the height is 10 m, the distance between the tunnels is 6 m. The soil layer is 100 m apart from the base of the tunnel. Boundary forces conditions are assumed to be zero at the entire boundary.

On figures 1 and 2 are shown the distribution of main stresses in the rock mass.

![Figure 1](image1.png)

**Figure 1.** Tunnel cut in a rock mass ($\sigma_x$ stress isolines)

![Figure 2](image2.png)

**Figure 2.** Tunnel cut in a rock mass ($\sigma_y$ stress isolines)
Figures 3 and 4 show the distribution of stresses in the rock mass and the stress state in the rock during the cutting of a parallel tunnel and its effect on the strength of overall structure.

As can be seen, when excavating a parallel tunnel, the stresses around the mine increase, in this case, up to 20%. But it should be borne in mind that the existence of horizontal thrust and the possible anisotropy of rock mass were not taken into account. In general, as can be seen the stress concentration in tunnel corners is significantly high compared to other sections and here the likelihood of a crack and its initiation (propagation) is maximal.
3. Result and discussion
Analysis of the calculation results shows that the rock mass displacements appear vertically into the depth of the working, the rock mass displacements along the working contour increase with the transition of corner elements to an inelastic state. So type of boundary problem can be solved by the step-by-step approach. Here is possible to trace the changes in the stress-strain state around the working. Our results show the nature of destruction of the host rocks and their displacements on the working contour.

4. Conclusion
Applied boundary element techniques can allow us to define stress strain state and to find path for strengthening of mine excavated structure in rock continua.

This approach allows us the calculation of such tunnels by models taking into account the properties of the rock, its fissuring, fracturing and layering. Moreover, this path can take into account condition on infinity and anisotropy also. It is vital for precise calculation even at static loading.

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