Towards a fully consistent relativistic quantum mechanics and a change of perspective on quantum gravity.

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Abstract

This paper can be seen as an exercise in how to adapt quantum mechanics from a strict relativistic perspective while being respectful and critical towards the experimental achievements of the contemporary theory. The result is a fully observer independent relativistic quantum mechanics for \( N \) particle systems without tachyonic solutions. A remaining worry for the moment is Bell’s theorem.

1 Introduction

As is often stated, quantum mechanics and general relativity are the great accomplishments of the previous century. However, their unification is particularly troublesome because of the contradictory axioms they are constructed from. General relativity is a theory of space time, reality, objectivity, locality of interaction ... while quantum theory is about space plus time, predicting outcomes of experiments, subjectivity, the instantaneous character of as well measurement and interaction. There has been an immense effort during the last forty years in trying to force general relativity into quantum theory with very little success. In this paper, we boldly address the opposite task and reexamine quantum mechanics in a Bohmian spirit from a strict, die hard relativistic point of view and see how far it brings us. The first obstacle we meet is the multi particle Schroedinger equation and I shall discuss objections against it in section (2) where also new multi particle equations are derived which admit straightforward observer independent relativistic generalizations. The key property is that the single \( N \) particle wave which lives on configuration space cross time is replaced by \( N \) single particle waves which live on space time. Of course our new equations have the correct classical limit. Moreover, interactions between the different waves are mediated by classical gauge fields such as the electromagnetic four vector which implies that there are no instantaneous processes taking place.

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which jeopardize causality. The physical picture we end up with is one of wave packages moving around in space time containing one particle each. The wave packages do not spread faster than with the speed of light and interact through fields which satisfy hyperbolic equations of motion. This weltanschauung is the complete opposite of the one employed in quantum field theory which deals with wave functions on infinite dimensional configuration space. In my opinion, a covariant set of field equations which unite gravitation which quantum mechanics and matter is more in our reach now and I shall present some ideas of how to proceed in that direction in the epilogue.

In section 3, I shall present an elegant solution to the de Broglie mass problem which was already mentioned in the literature \[1\]. Furthermore, in section (3.2) I discuss the same objections against the \(N\) particle Schroedinger equation from a relativistic point of view and present the appropriate relativistic equations. The conclusion from the latter exercise is that it is possible to slightly modify quantum mechanics into an objective, realistic and causal theory. However, as good relativists, we obviously reject spooky action at a distance and therefore subjective entangled states. The price to pay is the rejection of quantum mechanics and some of the interpretations of the measurement results which contain some trivial hidden assumptions which might not be as innocent as they seem. Also, it might be that the assumptions behind some of the Bell inequalities were too naive and this is discussed a bit more at the end of section (3.2). As a matter of notational convenience, Greek letters such as \(\alpha, \beta\) are space indices while \(\mu\) and \(\nu\) denote space time indices. The signature of the metric is taken to be \(+ − − −\) and \(D\) is the dimension of space.

2 Non relativistic Bohm - de Broglie quantum mechanics

In this section, I present the main concepts of Bohmian QM and shall not refrain from going into a rather detailed interpretation. The latter which, in my opinion, was the cause of all confusion. Let us start with the one particle case in \(R^D\), the Schroedinger equation reads:

\[
i\hbar \partial_t \Psi(x^\alpha, t) = -\frac{\hbar^2}{2m} \Delta \Psi(x^\alpha, t) + V(x^\beta, t) \Psi(x^\alpha, t).
\] (1)

Going over to polar coordinates \(\Psi(x^\alpha, t) = R(x^\alpha, t) \exp(iS(x^\beta)/\hbar)\), (1) becomes

\[
\partial_t S + \frac{1}{2m} \left| \nabla S \right|^2 + V - \frac{\hbar^2}{2m} \frac{\Delta R}{R} = 0
\] (2)

\[
\partial_t R^2 + \frac{1}{m} \nabla \cdot \left( R^2 \nabla S \right) = 0
\] (3)

The first equation is of the Hamilton Jacobi type with a dynamical, nonlocal term \(Q_{cl} = -\frac{\hbar^2}{2m} \frac{\Delta R}{R}\), the so called quantum potential. The second equation is
a classical continuity equation for the probability density $R^2$ provided the fluid lines satisfy the differential equation:

$$\dot{x}^\alpha = \frac{1}{m} \partial^\alpha S.$$  \hspace{1cm} (4)

Now, it is easy to check that

$$m \ddot{x}^\alpha = -\partial^\alpha (V + Q)$$  \hspace{1cm} (5)

which is nothing but Newton’s equation.

Hence, in Bohmian QM one extends reality by saying that the particle has a well defined position and momentum at each moment in time; the probability density $R^2$ being nothing but a nonlocal guidance mechanism for the particle. This interpretation is somewhat unsatisfying because of the role played by $Q_{cl}$ and we shall improve upon this in section (3.1) where the latter shall be given a space time geometrical meaning. We continue this section by elucidating the geometrical meaning of the argument field $S(x^\alpha, t)$, i.e. a small introduction into the geometry of Hamilton Jacobi theory is given.

Let $\mathcal{L}(x^\alpha, v^\beta, t)$ be a time dependent Lagrangian with corresponding functional

$$I(\gamma, t) = \int_{t_0}^{t} \mathcal{L}(\gamma^\alpha(s), \dot{\gamma}^\beta(s), s) ds$$

on the space of curves $\gamma : [t_0, t_1] \to R^D$ and $t_0 \leq t \leq t_1$. The function $S(x^\alpha, t)$ determines hypersurfaces in $R^{D+1}$ and we can ask how the parameter $S(x^\alpha, t) = \sigma$ changes with $dI(\gamma, t) = \mathcal{L}(\gamma^\alpha(t), \dot{\gamma}^\beta(t), t) dt$. Assuming that an initial surface $S(x^\alpha, t) = \sigma_0$ has been chosen so that $\frac{dI}{d\sigma}|_{\sigma_0} \neq 0$, we can locally choose a sigma orientation such that

$$\frac{d\sigma}{dt} = \partial_1 S(x^\alpha, t) + \partial_3 S(x^\alpha, t) \dot{x}^\beta > 0.$$  \hspace{1cm} (6)

Now, given $d\sigma$ fixed, the curve $(\gamma^\alpha(t), t)$ in $R^{D+1}$ shifts by $(\frac{d\gamma^\alpha(t)}{dt}, \frac{d\gamma^\beta(t)}{dt}, t) d\sigma$. Therefore $\frac{d\gamma^\alpha(t)}{dt}$ is invariant under reparametrisations of $\sigma$ and we can meaningfully ask in which directions $\frac{dI(\gamma^\alpha(t), t)}{d\sigma}$ is extremal, that is:

$$\partial_{\gamma^\alpha} \left( \frac{dI(\gamma^\alpha(t), t)}{d\sigma} \right) = 0.$$  \hspace{1cm} (7)

An elementary calculation reveals that

$$\partial_{\gamma^\alpha} S(\gamma^\beta(t), t) = \frac{\Delta}{\mathcal{L}(\gamma^\beta(t), \dot{\gamma}^\beta(t), t)} p_\alpha$$  \hspace{1cm} (6)

where $\Delta = \frac{d\sigma}{d\tau}(\gamma^\beta(t), \dot{\gamma}^\beta(t), t)$, which is homogeneous in first degree in the velocities, and $p_\alpha = \frac{\partial \mathcal{L}}{\partial \dot{\gamma}^\alpha}$. Now, it is an elementary exercise to show that
$S(x^\alpha, t)$ satisfies the Hamilton Jacobi equation if and only if $\Delta(\gamma^\beta(t), \dot{\gamma}^\beta(t), t) = \mathcal{L}(\gamma^\beta(t), \dot{\gamma}^\beta(t), t)$ for any member $\gamma$ of the congruence determined by equation (6). The latter implies that any action integral of any member of the congruence between $S(x^\alpha, t) = \sigma_1$ equals $\sigma_2 - \sigma_1$. The latter is inviting one to call the surfaces $S(x^\alpha, t)$ satisfying the Hamilton Jacobi equation to be geodetically equidistant. Moreover, it is fairly elementary to show that $S$ satisfies the Hamilton Jacobi equation if and only if any member of its congruence satisfies the Euler Lagrange equations. The reader can find a more extensive treatment in [5].

Historically, Schroedinger knew that the Hamilton Jacobi function determines a family of surfaces which are geodetically equidistant and crossed orthogonally by the specified congruence of solutions to Hamilton’s equations [4]. Therefore, it appeared natural to him that these surfaces correspond to the phase factor of a monochromatic wave which consequently has to satisfy the Schroedinger equation; $\hbar$ being needed for dimensional reasons only. In those days, one was aware of the wave-particle aspect of any substance and since monochromatic waves would not be physically adequate (substance would interfere periodically at arbitrary large spatial separations) it deemed necessary to include an amplitude. Scaling invariance of the Schroedinger equation with respect to this amplitude hinders it from having a physical meaning and consequently a probability interpretation becomes natural. This is all very well for one particle: one started from a natural idea inspired by classical mechanics, saw that it needed extension in order to correspond to localized substance in reality and then became immersed in the casino game. Moreover, at this stage there is no real conflict with Einstein’s principles of locality and causality since it is possible to give a geometrical meaning to the one particle wave function assuming a Bohmian point of view. This can be realized by taking a Weyl geometrical approach as followed by Santamato, leaving the relativistic Klein Gordon equations intact [2] [3]. He writes down an action principle for a relativistic particle and a connection in an external electromagnetic field and background space time. It turns out that the connection $\Gamma$ is semi-metrical and that the gauge freedom in $\Gamma$ (which determines the curvature $R$ of the connection in a flat background) is dynamically determined such that $R$ equals the quantum potential. Another approach consists in asserting that quantum mechanics really changes the conformal factor of the space time metric which results in a modification of the continuity equation for the probability density in $D > 1$.

In my personal opinion, quantum mechanics went in the wrong direction when Schroedinger wrote down his equations for a system of $N$ particles. In doing so, he started from the Hamilton Jacobi equations for the composite system of particles, interacting instantaneously through global forces. Thereby, he completely ignored the lesson Einstein presented in his theory of general relativity that interaction occurs causally and moreover is mediated by fields satisfying hyperbolic partial differential equations, a comment which was also made by Louis de Broglie [4] (p. 140). Another valuable lesson of that theory was that a
conserved energy has only a global meaning in a static, asymptotically flat space
time and therefore is an highly inappropriate concept to start from. Inevitably,
this brought Schroedinger to the conclusion that the wave function has to live
in configuration space cross some time, instead of in real physical space time,
making it devoid of any realistic interpretation\(^1\). However, what is not well
stressed at all in the standard literature, is that the probabilistic interpretation
of the multi particle wave function is quite unsatisfying too. If I were a real
Newtonian, I would demand that the fact that the probability of having some
particle at time \(t\) present is one, is independent of the consideration of other
particles. This is in conflict with the contemporary dogma which only allows a
particle to exist in space \textit{given} that all other particles do. Therefore, it seems to
me that each particle should have its own wave function with the corresponding
continuity equation. In the remaining part of this paper, I will gradually
restore a realistic interpretation of the \(N\) particle quantum mechanical system
and, as highlighted in section (3.2), such enterprise is intimately connected to
the principle of relativistic invariance. The first step consists in recognizing that
indeed each particle has its own wave function living in space time and that the
latter are interacting causally through gauge fields. Next, a natural space time
geometrical meaning is given to the quantal guidance mechanism and this is
discussed in more detail in section (3.1). Therefore, the only probabilistic rem-
nant in the theory concerns the unknown initial positions of the particles. The
measurement process is, in this context, a multi particle (scattering) problem
and deeper insight into this subject will require a lot of future computational
work.

From now on, the reader should refrain from any Newtonian thought and reason
as a proper relativist (or at least switch off his/her Galilean preconceptions). As
a warmup, consider a system of two particles whose wave functions \(\Psi_i(x^\alpha, t_0)\)
have disjoint support and spreads \(\Delta E_i\), \(\Delta x^\alpha_i\) of the order \(\sqrt{\hbar}\) around center values \(\bar{E}_i\) and \(\bar{x}_i^\alpha\) respectively. Although the two particles might interact after we
have prepared the system, it is very reasonable to think that they are independent initially\(^2\). Therefore, we can speak of the separate energies of the particles
(and they could be determined after some time by letting the particles hit a
screen and by measuring the impact factors) which logically implies the need
for two Galilean time parameters. One can formalize this intuition by studying
the classical problem of \(N\) particles with charge \(e_j\) and mass \(m_j\) interacting

\(^1\)Bohm, quite incomprehensively, does give a realistic meaning to this wave!
\(^2\)Although, ardent quantum physicists of the environmental decoherence clan might question this. However, I have never seen any detailed microscopic model for long range entan-
glement between the subsystem under study and the environment. One typically proposes,
however, the most convenient possible effective interaction Hamiltonian. Moreover, this atti-
dude is highly counterintuitive since one would expect the gun of particle 2 to shelter it from
any long range influence from particle 1 as well as to destroy any preexisting correlations.
through the Coulomb force. In this case, the Hamiltonian is:

$$\mathcal{H}(x_1^{\alpha_1}, \ldots, x_N^{\alpha_N}, p_{\alpha_1}, \ldots, p_{\alpha_N}) = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \sum_{i<j}^{N} \frac{\alpha e_i e_j}{r_{ij}}$$

with $\alpha$ a positive coupling constant. However, this theory neglects the facts that interactions between the different particles occur with finite speed and that the couplings between the particles and the fields generated by the others are local as well. Taking into account the latter considerations results in the following correct theory:

$$\mathcal{H}_i(x_i^{\alpha}, p_{\alpha}, t) = \frac{|p_i| - \frac{e}{c} \sum_j A_j(x_i^{\alpha}, t)|^2}{2m_i} + e_i \sum_j \phi_j(x_i^{\alpha}, t)$$

$$\partial_{\mu} F^{\mu\nu}(x^{\alpha}, t) = 4\pi e^{-1} J_i^{\nu}(x^{\alpha}, t)$$

$$\partial_{\mu} J_i^{\mu}(x^{\alpha}, t) = 0$$

where the $\phi_j$ and the $A_j^\alpha$ are the field and vector potentials respectively, $A_j^\mu = (\phi_j, A_j)$ is the $j$'th gauge four vector and $F_{\mu\nu}^j = \partial_{\mu} A_j^\nu - \partial_{\nu} A_j^\mu$ is the $j$'th field strength. The $N$ Hamilton functions enable one to write down the orbits of the particles as explicit functions of time and the electromagnetic fields of the other particles, given the initial positions and momenta, i.e. $x_i^\alpha(t) \equiv x_i^\alpha(A_j^\nu(t))$. However, strictly speaking, the four currents are

$$(J_i^{\mu}(x^{\alpha}, t)) = e_i \left( \frac{p^i(t) - \frac{e}{c} \sum_j A_j(x_i^{\alpha}(t), t)}{m_i} \right) \delta^{(3)}(x^{\alpha} - x_i^{\alpha}(t))$$

and these obviously satisfy the conservation laws in the distributional sense on shell (that is when the Hamilton equations are satisfied). Now, it becomes very clear why the $j$'th particle determines its own wave function satisfying a one particle Schröedinger equation with a time dependent Hamiltonian. Therefore, the new $N$ particle quantum mechanical equations read:

$$\partial_j S_j(x_j^{\alpha}, t_j) + e_j \sum_i \phi_i(x_j^{\alpha}, t_j) - \frac{\hbar^2}{2m_j} \frac{\Delta_j R_j(x_j^{\alpha}, t_j)}{R_j(x_j^{\alpha}, t_j)} + \frac{\nabla_j S_j(x_j^{\alpha}, t_j) - \frac{e}{c} \sum_i A_i(x_j^{\alpha}, t_j))^2}{2m_j} = 0$$

$$\partial_j R_j^2(x_j^{\alpha}, t_j) + \frac{\nabla_j \left( R_j^2(x_j^{\alpha}, t_j) \left( \nabla_j S_j(x_j^{\alpha}, t_j) - \frac{e}{c} \sum_i A_i(x_j^{\alpha}, t_j) \right) \right)}{m_j} = 0$$

We still need $N$ Maxwell’s equations for the electromagnetic fields, that is we need to identify the correct electric currents. Obviously, the probability currents
determine the charge currents with charge densities $\rho_j = e_j R_j^2$ and spatial currents $\vec{j}_j = e_j R_j^2 \frac{\Psi_j(x_j^a, t_j) - \frac{2}{m_j} \sum_i A_i(x_i^a, t_i)}{m_j}$ . The latter is equivalent to stating that the possibility of the particle being at some place influences the dynamics of the other particles and the quantum mechanical continuity equations imply that $\partial_\mu j^{\mu}_j = 0$. Therefore, the remaining equations are (in cgs units):

$$\partial_\mu F^{j \mu \nu} = 4\pi c^{-1} j^{\nu}_j .$$

The above system obviously produces good Newtonian laws with the correct classical limits. Restricting to the two particle case, it becomes possible to ask questions like: given the fact that particle one passed at time $t_1$ through the region $O_1$ of space, what is the probability that particle two shall pass through $O_2$ at a later time $t_2 > t_1$? Or, assuming that particle one shall pass through $O_1$ in five seconds from now, what is the probability that the second particle goes through $O_2$ now? Obviously, performing an experiment to check the above assertions will change the particles momentum and therefore the outcome of the result, but that does not invalidate the possibility for addressing such questions. So in this interpretation, God does still play dice but in real physical space time and not in configuration space.

2.1 Comments

Verschelde kindly pointed out to me that the formulae I end up with coincide - in the case of vanishing vector potentials - with those of Hartree theory, the latter being known not to give correct results for the Helium atom\(^3\). Does this mean that (a) my reasoning is abundant and (b) one of my assumptions is incorrect? The answer to both arguments is clearly no for me, since (a) all derivations of Hartree assume the multi particle Schroedinger equation to be correct and set up a global variational principle to approximate its solution by a product state while I simply get my formulas in a much more elegant way by rejecting the $N$ particle equation. Concerning (b): let me note that spin is not included yet in my theory, something which is likely to be done by making use of the ideas behind Einstein Cartan theory instead of spinors. One reason for this is that anti-symmetrization is not going to lower the energy for the Helium atom since the lowering terms in Hatree Fock only involve wave functions of equal spin. Moreover, anti-symmetrization would not be allowed in my theory a priori since we have previously rejected the notion of one wave function for $N$ particles. Further evidence against the Pauli-Dirac description of spin is provided by the fact that the spin statistics theorem is essentially a kinematical result while one would intuitively expect it to be of a dynamical nature \[^4\]. Although the topological observations involved concerning the swap operation on the space of $\epsilon$-localized basis states - which is mathematically a

\[^4\]In case the vector potentials do not vanish, one typically resorts to an operator formalism for the latter which coincides with my ansatz in the first order when doing perturbation theory around the stationary solution.
principal $U(1)^4$ fiber bundle over (non-relativistic) configuration space modulo permutation operations corresponding to identical particles - are cute, they are themselves questionable and have no real physical underpinning. Therefore, we need to find a spin dynamics which results in a repulsion of equally spinning and attraction of opposite spinning particles. The energy lowering effect in Helium could therefore be accomplished by a specific anti-symmetric tensor field (a spin field if you want) which obeys the above dynamical rules and this brings us into the realms of Einstein Cartan theory. Moreover, gravitation is expected to indirectly play - say by gravitationally induced electromagnetism - a significant role too.

So, the above result has to be interpreted as follows: given flat space time physics, taken into account that a proper form of spin has not been included yet and assuming that the description of a particle by a mathematical point is fairly accurate in these circumstances, then it follows that the above theory is the preferred candidate from the relativistic point of view; the latter which is derived from first principles in section (3.2). This, by itself, is a worthwhile result in my opinion since it clearly indicates the direction in which a (deterministic?) reformulation of quantum mechanics could be found.

3 Relativistic Bohm - de Broglie quantum mechanics for a zero spin particle revisited.

In this section, we derive a proper nonlinear extension of the classical Klein Gordon equation which has no tachyonic solutions for one particle as well for multiple particles.

3.1 The single particle

Polar decomposition of the Klein Gordon field $\Psi(x^\mu) = R(x^\mu) \exp(iS(x^\mu)/\hbar)$ gives the following pair of equations:

$$\partial_\mu (R^2 \partial_\mu S) = 0$$
$$\partial_\mu S \partial_\mu S = m^2 c^2 (1 + Q_{rel}) \equiv \mathcal{M}^2 c^2$$

4When we ignore internal degrees of freedom or demand that the latter trivially factorize. This so called absolute recognizability constraint is to say the least very questionable and not satisfied in most practical situations.

5The treatment of the $U(1)$ factor is entirely similar to the mathematics behind the Bohm Aharonov effect. In the latter however, the topological defect is generated by the fact that the vector potential - which dictates the coupling between the particle's momentum and the electromagnetic field - cannot be continuously defined on a simply connected domain in space. As such, this effect has a dynamical reason while the spin statistics theorem is entirely kinematical.

6Perhaps there should be attraction on very tiny scales.

7Something which I strongly disagree with.
where \( Q_{rel} = \frac{\hbar^2}{m c^2} \frac{\partial^2}{\partial t^2} - \Delta \) is the dimensionless relativistic quantum potential and \( \partial^\mu \partial^\nu = \frac{1}{c^2} \partial^2 - \Delta \) is the d'Alembertian. Now, there exist solutions of the KG equation for which \( Q_{rel} < -1 \) so that the relativistic current defined by
\[
\frac{dx^\mu(\lambda)}{d\lambda} = \frac{\partial^\mu S}{m}
\]
gets spacelike as is illustrated in the next example.

**Example 1**

For ease of notation, we shall work in the so called natural units \( \hbar = c = 1 \) and examine the problem of one relativistic particle moving in two dimensional Minkowski space time. The family of normalized wave packages we are interested in is given by:
\[
\phi_\epsilon(x, t) = \frac{1}{\sqrt{\epsilon \sqrt{1 + A^2}}} \left( \int_{-\epsilon/2}^{\epsilon/2} \exp(-i(\omega t - kx))dk + A \int_{s-\epsilon/2}^{s+\epsilon/2} \exp(-i(\omega t - kx))dk \right)
\]
where \( \omega^2 - k^2 = m^2 \) and \( A, s \) are real nonzero constants. For any \( L \) we can take \( \epsilon \) sufficiently small such that for all \( (x, t) \in [-L, L]^2 \) we have:
\[
\phi_\epsilon(x, t) \sim \frac{\sqrt{\epsilon}}{\sqrt{1 + A^2}} (\exp(-imt) + A \exp(-i(\omega t - sx)))
\]
To calculate the particle’s paths, note that with some slight abuse of notation \( S = -i \ln(\phi_\epsilon) \). Then, \( E = -\partial_t S = \frac{\epsilon}{(1 + A^2)\epsilon^2} \left( m + A^2 \omega + A(m + \omega) \cos((m - \omega)t - sx) \right) \) and the momentum \( P = \frac{\epsilon}{\epsilon^2} A(s \omega + A(m + \omega) \cos((m - \omega)t - sx)) \). Now the ratio \( \frac{P}{E} \) equals:
\[
\frac{P}{E} = \frac{As(A + \cos((m - \omega)t - sx))}{(m + A^2 \omega + A(m + \omega) \cos((m - \omega)t - sx))}
\]
For \( \cos((m - \omega)t - sx) = -1 \) the above expression reduces to:
\[
\frac{P}{E} = \frac{As(A - 1)}{m + A^2 \omega - A(m + \omega)}.
\]
For \( A = \frac{2}{m^2} \), one obtains \( \frac{P}{E} = \frac{2s}{\omega(2 - m^2)} \). Obviously, for \( 0 < m < 2 \) and \( \omega > \frac{2\sqrt{m}}{\sqrt{1 + m}} \) we have \( \frac{P}{E} > 1 \) which is what we wanted to show.

Restoring dimensions in the above example makes one aware that, since \( A \) has to be dimensionless, the factor 2 must have dimension of \( 1/s \). Hence one needs an invariant length, that is the Planck length \( l_p \). Recall that in standard units, \( l_p \sim 1.6 \times 10^{-35} \), \( \hbar \sim 6.6 \times 10^{-34} \) and \( c \sim 3 \times 10^8 \). Putting \( A = \frac{2\alpha c}{l_p} \), where \( \alpha \) is a dimensionless numerical constant, leads to
\[
\frac{Pc}{E} = \frac{2c^2 l_p^{-1} \alpha \hbar}{\omega(2c l_p^{-1} \alpha - mc^2)}
\]
and we may safely assume that $mc^2 < cl^{-1}_p \hbar \alpha$ or $0 < m < \frac{cl^{-1}_p \hbar \alpha}{e} \sim (1, 4) \alpha.10^{-7}$ kilo. In order for the above expression to exceed one, it is necessary that:

$$\omega > \sqrt{\frac{4mc^4l^{-2}_p \alpha^2}{(4cl^{-1}_p \hbar \alpha - mc^2)}}$$

For $\alpha = 10^{-22}$ and $m = 10^{-30}$ which is about the electron mass, the above estimate reduces to $\omega > 10^{21}$ Hertz, which is in the gamma ray frequency spectrum with wave vector $s$ larger than about $10^{13}$. All these numbers fall well within the reach of conventional space time physics.

This result is making a consistent Bohmian interpretation of the relativistic KG equation impossible. However, there is no good reason why the KG equation should be the physically correct one, i.e. it might very well be that the equation requires non linear corrections. This is not strange at all, since it is well known that the superposition principle in -say- an environmental decoherence interpretation of QM is responsible for the measurement problem\(^8\). The latter problem is solved in a Bohmian interpretation and therefore one can take the point of view that a correct relativistic equation should be Poincaré invariant, have a probability interpretation (scaling invariance with respect to $R$) and no acausal Bohmian trajectories. The most obvious ansatz which satisfies all these requirements (and does not affect the classical limit) is given by $\mathcal{M}^2 = m^2 \exp(Q_{rel})$ instead of $m^2(1+Q_{rel})$. Actually, the latter formula is suggested by the following series of arguments\(^9\). Equation (8) can be rewritten as

$$\frac{dx^\mu(\tau)}{d\tau} = \frac{\partial^\mu S(x^\nu(\tau))}{M}$$

where the parameter $\tau$ yields the eigentime of the congruence. Direct calculation yields:

$$\mathcal{M}(x^\gamma(\tau)) \frac{d^2x^\mu(\tau)}{d\tau^2} = \left(c^2 \eta^\mu\nu - \frac{dx^\mu(\tau)}{d\tau} \frac{dx^\nu(\tau)}{d\tau}\right) \partial_\nu \mathcal{M}(x^\gamma(\tau)). \quad (9)$$

Now, we study the non-relativistic limit of the above equation. That is: $\tau \equiv t$, where $t$ is the time coordinate associated to a freely falling congruence of observers, $(x^\mu(t)) = (ct, x^\alpha(t))$ with $|\dot{x}^\alpha| \ll c$. The geodesic equation \(^9\) is decomposed as follows:

$$\mathcal{O} \left(\frac{\dot{x}^\alpha(t)}{c}\right)^2 = c^2 \dot{x}^\alpha \partial_\alpha \mathcal{M}$$

$$\mathcal{M} \ddot{x}^\alpha = c^2 \partial^\alpha \mathcal{M} + \dot{x}^\beta \dot{x}^\beta \partial_\beta \mathcal{M} - \dot{x}^\alpha \partial_\alpha \mathcal{M}$$

\(^8\)In this particular approach, one has the so called pointer basis problem as well as the problem of Poincaré recurrence times.

\(^9\)I have no conclusive proof here.
The second equation has only a good non relativistic limit provided that $\partial_\beta M \sim M c^2$ which implies that the first equation is identically satisfied. Because of the former equation, it is reasonable to think that $\partial_t M \sim M c^2$ and therefore

$$\mathcal{M} \ddot{x}^\alpha = c^2 \partial^\alpha \mathcal{M}.$$ 

Recalling the non-relativistic equation implies:

$$m \ddot{x}^\alpha = -\partial_\alpha \left( m c^2 \ln \left( \frac{M}{\mu} \right) \right) = -\partial_\alpha Q_{cl}$$

where $\mu$ is any mass scale. This latter implies that $\mathcal{M} = m \exp \left( \frac{Q_{cl}}{mc^2} \right)$ which suggests that the relativistic mass satisfies $\mathcal{M} = m \exp \left( \frac{Q}{c^2} \right)$. An important aspect is that equation (9) is the geodesic equation associated to the space time metric $g_{\mu \nu} = M^2 m^2 \eta_{\mu \nu}$, a remark which seems to go back to de Broglie. Since in the Bohmian interpretation, a point particle is localized somewhere within the wave package, it seems unnatural to attribute a mass field $\mathcal{M}$ to it. Moreover, the fact that equation (8) is equivalent to

$$g^{\mu \nu} \partial_\mu S \partial_\nu S = m^2 c^2$$

strongly suggests that one really should see the quantum effects on the motion of the relativistic particle as being given by the above conformal transformation of the space time metric. Consequently, the continuity equation should be modified\(^\text{10}\) to

$$\nabla_\nu (R^2 \nabla^\nu S) = 0$$

where $\nabla$ is the metric connection associated to $g_{\mu \nu}$ and, moreover

$$\frac{dx^\mu (\tau)}{d\tau} = \frac{\nabla^\mu S}{m}.$$ 

The latter considerations bring us to the anticipated conclusion that one can attribute a natural space time geometrical meaning to the non local quantal guidance mechanism which takes away its mystery. That is: the wave character of the particle deforms space time geometry in such a way that the geodesic pattern of point particles equals the extremely chaotic congruence of curves in Bohm’s theory. The only remaining uncertainty being the initial position of the particle.

It is unfortunately not well known that the one particle Klein Gordon theory as enunciated above has a consistent interpretation\(^\text{3}\). The theory is time reversal invariant which allows for particles to travel the same space time curves but in opposite time directions. It is the choice of time of the observer which determines the notion of particle and antiparticle. Obviously, this can give rise to negative probabilities but this is not a problem as explained in\(^\text{3}\).

\(^\text{10}\)A simple calculation reveals that $\nabla_\mu (R^2 \nabla^\mu S) = \frac{2}{D-1} \partial_\mu (R^2 \partial^\mu S) + \frac{2}{D-1} (D - 1) R^2 \partial_\mu \mathcal{M} \partial^\mu S$. Therefore the continuity equation changes for $D > 1$. 

11
3.2 Multiple particles

In this section, I approach the objections in section (2) from a relativistic point of view and show that the standard formalism is incomplete. The same comment applies anyway to any approach aiming to solve the Unitary/Reduction problem in quantum mechanics, without addressing the issue of local coordinate (active Lorentz) invariance of the quantum mechanical equations in a curved (flat) background properly. For simplicity, I present the equations in the $D + 1$ dimensional special relativistic case; the extension to a curved background being straightforward. In relativity, configuration space is $R^{N(D+1)}$ (!) and it is therefore natural to demand the wave function $\Psi$ to depend on the $N$ tuple of space time coordinates (unlike what is done in quantum field theory where a split of space and time is made from the start), i.e. $\Psi(x_{\mu}^1, \ldots, x_{\mu}^N)$. Then, the $N$ particle relativistic equations in Minkowski would look as follows:

$$\sum_{j=1}^{N} \frac{1}{m_j} \partial_{\mu} S \partial^\mu_j S \quad = \quad \sum_{j=1}^{N} \frac{\hbar^2}{m_j} \frac{\partial_{\mu} \partial^\mu R}{R} + \sum_{j=1}^{N} \frac{m_j c^2}{R^2} \quad (10)$$

$$\sum_{j=1}^{N} \partial_{\mu} \left( \frac{R^2 \partial^\mu_j S}{m_j} \right) \quad = \quad 0 \quad (11)$$

where $\partial_{\mu}$ denotes the partial derivative to the $\mu$'th coordinate of the $j$'th particle. The latter equation is again the usual continuity equation in configuration space. Both equations being second order hyperbolic partial differential equations in $R$ and $S$. In the above equations, each particle is allowed to have its own independent Lorentz coordinate system and unless we fix a gauge by specifying the relative boost and rotation factors between the different coordinate systems, the above system leaves $2(N-1)$ variables undetermined$^{11}$. The point is that from a relativistic perspective, one should not know these relative Lorentz transformations in order to find a unique solution. This becomes very clear when one imagines the $N$ particles to be initially spatially separated meaning that the initial $N$ particle wave function is the product of $N$ one particle wave functions whose supports are a distance $> \epsilon$ apart in the Riemannian metric determined by the initial hypersurface. In this setup, it is completely irrelevant to even wonder about the relative Lorentz transformations since in case interactions are present, they are mediated by gauge fields and therefore the coupling to other particles is of a local nature. This argument becomes even more compelling in the context of true local coordinate transformations in a fixed curved background space time where, in case transition functions would exist, nothing is

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$^{11}$Even in case $R_0, S_0$ and the first time derivatives would be equal to a product, respectively a sum of one particle initial conditions (expressed in the different local Lorentz coordinate systems), there would not exist a unique solution. In the specific case of a two particle system, this is reflected in the existence of a free mass parameter. Obviously, for entangled (in the standard meaning of the word) initial conditions, the situation is far worse.
said about that part of the coordinate patches which have no overlap anyway\textsuperscript{12}.

In quantum field theory, a very special form of the above discussed gauge fixing is accomplished by setting the relative boost and rotation factors to zero, i.e. a globally consistent coordinate system is chosen. In order to avoid all possible confusion, let me stress that this kind of gauge freedom should not be mixed up with the so called local Lorentz invariance which lives on the tangent bundle; a symmetry the above equations obviously posses also. Moreover, the active Lorentz invariance of QFT (independence of the observer) is not the same as the above coordinate symmetry, since the relative boost and rotation factors remain null. In the latter context, it is well known that the necessity of a \textit{globally well defined} observer for a \textit{fundamental} equation of physics to be written down becomes a highly problematic matter, in particular in the standard interpretation of quantum gravity where the status of the observer him/herself gets jeopardized. For all the above reasons, it seems very natural and necessary to me that the laws of physics are observer independent and that the well known problems that observers bring along are indeed problems (of global geometrical or topological nature) of the observers themselves and not of fundamental physics.

Since standard quantum mechanics is shown to be incomplete, we need to add $2(N-1)$ hyperbolic equations to the above system. Taken into account the results of section (2) and the discussion about the positivity of $\mathcal{M}^2$ in section (3.1), the extension to the relativistic case becomes obvious and the result reads:

$$
\left( \partial_{\mu} S_{\gamma}(x^\nu_j) + \frac{\epsilon_j}{c} \sum_i A_{\mu i}(x^\nu_j) \right) \left( \partial^\mu S_{\gamma}(x^\nu_j) + \frac{\epsilon_j}{c} \sum_i A^\mu_i(x^\nu_j) \right) = \mathcal{M}^2_j(x^\nu_j)c^2
$$

$$
\partial_{\mu} \left( \frac{R^2_j(x^\nu_j) \left( \partial^\mu S_{\gamma}(x^\nu_j) + \frac{\epsilon_j}{c} \sum_i A^\mu_i(x^\nu_j) \right)}{m_j} \right) = 0
$$

$$
\frac{dx^\mu(\tau_j)}{d\tau_j} - \left( \partial^\mu S_{\gamma}(x^\nu_j) + \frac{\epsilon_j}{c} \sum_i A^\mu_i(x^\nu_j) \right) = 4\pi c^{-1} J^\mu_j(x^\nu_j)
$$

where $\mathcal{M}^2_j(x^\nu_j) = m_j^2 \exp \left( \frac{\beta_j}{m_j c^2} \frac{\partial_{\mu} S_j(x^\nu_j)}{R^2_j(x^\nu_j)} \right)$ and

$$
J^\mu_j(x^\nu_j) = \frac{\epsilon_j R^2_j(x^\nu_j) \left( \partial_{\mu} S_j(x^\nu_j) + \frac{\epsilon_j}{c} \sum_i A_{\mu i}(x^\nu_j) \right)}{m_j}
$$

Again, one notices that the distributional charge and mass densities are smeared out by the amplitude $R^2_j$ and $\mathcal{M}_j$ respectively, which is different from the well

\textsuperscript{12}Note that coordinate transformations in a curved background have nothing to do with diffeomorphism invariance as is often mistakingly quoted. The correct behavior under coordinate transformations is a property of tensors while diffeomorphism invariance is a dynamical property of spacetime.
known result in quantum field theory where the dressed mass and charge are still parameters and not fields. A discussion about the geometrization of the relativistic multi particle system will appear elsewhere.

In this section, we have shown that the principle of local coordinate invariance forces one to extend standard quantum mechanics. In the latter extension, each particle has its own wave package and interaction is mediated causally through (electromagnetic) gauge fields. We used the modification of the standard KG equation proposed in [1], meaning that the superposition principle is abandoned. At this point, the reader may wonder how it will be possible to describe entanglement between, say, two spin \(\frac{1}{2}\) particles as in the Einstein Podolsky Rosen experiments since, in the standard description, this requires use of states of the form \(\frac{1}{\sqrt{2}} (|\uparrow\rangle - |\downarrow\rangle)\). Now one can take two points of view: either the above state has no space time meaning and belongs to a preordained Clifford bundle with the canonically induced connection, or spin has a space time interpretation. Let me discuss the former point of view first: the problem is that a change of spinor bundle generically gives rise to different physics which implies that the Dirac picture is a subjective one, that is the left (right) up and down states do not correspond to a physical reality of the left (right) particle, but to the outcome of a so called spin (in reality a position) measurement on it.\(^{13}\) Therefore, from an objective point of view, it is entirely meaningless to speak about a delocalized state of total spin zero. The same holds for the notion of angular momentum since its very definition requires a global preferred coordinate system, something which has generically no canonical meaning. Of course, one might dismiss these objections just on practical grounds but I am afraid that such attitude is against any serious attempt of unification and I shall grasp this opportunity to take a strict relativistic point of view\(^{14}\). The only relativistically sane explanation is that correlations in the outcome of measurements are the consequence of interactions between the wave packages (and perhaps both measurement apparatus) in their common causal past (assuming that both particles have the same time orientation!) and no singlet state should be needed to describe that. Since we are aiming for an objective world view and rejected measurement as it stands in the Copenhagen interpretation, the standard explanation cannot be maintained here. Therefore, the reason for the correct amount of correlations in experiments must be of an entirely different physical nature. Hence, we are forced to take the position that a particle simply has a definite spin vector which obeys a deterministic equation of motion and that, in case of the EPR setup, it are merely the initial spin vectors which have opposite directions\(^{15}\). As mentioned before, this cannot be realized within the

\(^{13}\)This raises again the question about the meaning of a spin observable whose existence is not, in any sense, proven to be necessary. Likewise, one can question the need for a momentum observable which is a (infinitely) delocalized concept and therefore highly unphysical.

\(^{14}\)In case the multi particle Schroedinger equation were more criticized before, this attitude would perhaps not be a rarity now.

\(^{15}\)That is, a hidden variable model is needed. Something which has always been maintained by Einstein.
context of conventional Dirac theory and another set of equations will be necessary. Let me also note that there is no conflict with the so called Kochen Specker theorems: the fact that a particle has classical properties does not imply that these can be determined simultaneously through experiments, just as momentum and position of a particle cannot be simultaneously measured in classical physics\(^\text{16}\). However, I am contradicting some interpretations of experiments concerning Bell’s theorem \(^\text{3}\) (as did de Broglie and more recently ’t Hooft\(^\text{4}\)) and there exists a vast literature about possible loopholes in the relevant experiments which still allow for the possibility of a local hidden variable theory. However, this is not the place to enter into this subject in detail. The reader will most likely feel uncomfortable with my urge for reinstating classical concepts, but let me point out that I merely came to these conclusions by respecting well known relativistic principles. Moreover, the possible states which can occur in nature are narrowed down immensely due to the exclusion of delocalized multi particle entangled states which I believe, is good news.

The construction is this section is remarkably similar to the so called u-wave description in the double wave theory of de Broglie\(^\text{4}\) as I found out later. He also recognized some of the afore mentioned problems with the standard formalism but did not want to give up the multi particle Schroedinger equation. Instead, he forced himself to think of the \(\Psi\) wave as an entirely fictitious object, living on configuration space cross one (!) time, which had only a probabilistic meaning. I can only imagine that the reason for his obstinate recognition of the Schroedinger wave must have been rooted in a long tradition of classical Newtonian mechanics\(^\text{17}\) which was only questioned by then for about 10 years and which is still today the prevalent view on physical reality. This claim is supported by the fact that de Broglie frequently speaks about three dimensional physical space, indicating the presumption of a globally preferred notion of time.

4 Conclusions

In this paper, I constructed a fully consistent relativistic quantum theory of particles in which each particle has its own wave function which lives in space time. It remains to be seen how the theory could account for particle-anti particle pair creation, this could in principle happen in case the phase factors \(S_j\) get singular. At least, our theory has the potentiality to make such process explicit or at least indicate in which direction one should extend it. This is in sharp contrast to quantum field theory where this process happens instantaneously and is put in by hand through the specific form of the Lagrangian density. Moreover, the theory is observer independent implying that particles have an objective existence. Consequently: the distinction particle - antiparticle and particle energy

\(^{16}\)Even classically, a measurement of position is going to influence the momentum of the particle. Therefore, in my opinion, the Kochen Specker theorems deal with the wrong question.

\(^{17}\)Not to speak about the success of its application! An argument which, even more today, is supposed to kill all resistance.
are just observer dependent concepts which can be introduced later. Moreover, we have deduced by pretty airtight arguments that a (deterministic) hidden variable model for spin is necessary, something Einstein has maintained for most of his life. The latter implies that somehow the derivation of the Bell inequalities must be incomplete and I believe that a careful reexamination is a necessary and extremely worthwhile enterprise, taken into account the possible reward. If this turns out to be successful, then quantum mechanics is compatible with the laws of general relativity and the objective of a universal field theory suddenly seems a lot more realistic. In case any imaginable attempt to give a hidden variable explanation for the experimental outcome would fail, then and only then would I see myself obliged to resign from one of relativity’s beautiful principles.

Regarding this universal field theory, I see the program being developed in two steps. The first step consists in ignoring particle spin for the moment, keeping the description of point particles and writing down a geometrical action principle which contains gravitation, electromagnetism and point matter. I believe we are prepared to take that step now and I have commented a few times on how this could be done. A second, and much more difficult step, consists in probing the geometrical structure of an elementary particle. I am entirely convinced of the idea that elementary particles are just bound states of an interacting ensemble of a fundamental substance of space time\(^\text{18}\) and that quantum numbers such as particle spin should be encoded into this geometrical form. Therefore, the contemporary bundle descriptions of external quantum numbers (how elegant they may be) are in my opinion merely subjective pictures of a much deeper physical reality. Insight into the latter subject would also throw more light upon the how of the particle scattering processes.

Let me end by recalling the historical happening of 1949, where on the occasion of Einstein’s 70’th birthday quantum physicists such as Born, Pauli and Heitler, members of the orthodox school, shamelessly expressed in their articles their disappointment at seeing Einstein persist in a negative attitude towards the purely probabilistic approach to Wave Mechanics. Perhaps people should be more careful in making claims about issues which are not settled yet, especially towards the great scientist who was the main actor in both revolutions.

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\(^{18}\)This in contrast to string theory where elementary particles correspond to vibration modes of a string which lives in a background space time.
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