The relation of $\gamma(G \odot H)$ and $\gamma(G \odot H)$

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Abstract. Every graph $G$ with vertex $V$ and edge $E$ usually referred to as $G = (V, E)$ in this research is a simple, undirected, and non-trivial graph. A set of vertex $D$ of graph $G = (V, E)$ is called Dominating Set if every vertex from $u \in V(G) - D$ is adjacent to a vertex $v \in D$. In this paper, we analyze the relation of domination number of corona product graphs $\gamma(G \odot H)$ and edge corona product graphs $\gamma(G \diamond H)$. The difference between corona product and edge corona product is located on each vertex of graph $G$, while at edge corona product, copy of $H$ graph is located on each edge of graph $G$. The difference in the number of vertices and edges between corona product and edge corona product is $p(G \odot H) + m = p(G \diamond H)$ and $q(G \odot H) = q(G \diamond H) - m(n - 2)$. The result of this research is $\gamma(G) \leq \gamma(G \odot H) \leq \gamma(G \diamond H)$. There is a relation between all domination numbers that have been generated.

1. Introduction

Every graph $G$ with vertex $V$ and edge $E$ usually referred to as $G = (V, E)$ in this research is a simple, undirected, and non-trivial graph. Graph $G$ is the pair of $(V(G), E(G))$, $V(G) = \{v_1, v_2, \ldots, v_n\}$ is finite and not empty set $E(G) = \{e_1, e_2, \ldots, e_n\}$ is a set of irregular pairs $\{v_1, v_2\}$ from $v_1, v_2 \in V(G)$ and it’s edges are defined by $[7, 3]$.

The domination number was first introduced by Haynes in 1998 denoted by $\gamma(G)$. Domination number is the minimum cardinality of a dominating set which is the development of previous studies. The value of the domination number is always $\gamma(G) \subseteq V(G)$. Regarding the upper bound of the domination number is the number of vertices of the graph. When at least one vertex is needed for the dominating set of the graph, then $1 \leq \gamma(G) \leq n$ for each graph having the order $n$. Let $D \subset V$, if each vertex of graph $G$ without the dominating set of graph $G$ is adjacent to at least one vertex of the dominating set of graph $G$, then $D$ is said to be a dominating set of graph $G$ $[8]$. Dominating sets is not only within one distance, but there are also distances of two, three and so on until $k$-th distance. The difference is at the of reach, where at the dominating set with one distance, the vertex that becomes the dominating set reaches another vertex that is spaced one from that dominator, while at the dominating set is distance two, three, four and so on, the vertex that dominating set reach another vertex with a distance of at least two to the $k$-th distance from the vertex that serves as the dominating set. But in this research only focus on dominating sets with a distance of one.

Researches related to dominating sets is growing quite rapidly, including $[6]$, $[4]$, $[5]$, $[11]$, $[12]$ $[13]$, $[14]$, $[1]$ and many more. All of researches only examined the domination number of special...
graph and its operation. In 2011, Carmelito et. al [2] examined Domination in the Corona of Graphs, and determined Corollary as follows:

**Corollary 1.** Let $G$ graph with order $n$ and $H$ graph with order $m$ be a connected graph. We have $\gamma(G \circ H) = n$.

The theorem regarding the upper and lower bounds of the dominating set to be used in this study is as follows:

**Theorem 1.** [9]Let $G$ be any graph,

\[ \left\lfloor \frac{p}{1+\Delta(G)} \right\rfloor \leq \gamma(G) \leq p - \Delta(G) \]

2. Result

In this research, we determined the relation of Domination Number of corona graphs $\gamma(G \circ H)$ and edge corona graphs $\gamma(G \circ H)$. This study examined the domination number $\gamma(G)$ of edge corona of graph $G \circ H$. The definition of edge corona product of graph $(G \circ H)$ is introduced by [10]. The difference between corona product and edge corona product is located on the placement. If on corona product, copy of $H$ graph is located at each vertex of graph $G$, while at edge corona product, copy of $H$ graph is located on each edge of graph $G$. The difference in the number of vertices and edges between corona product and edge corona product is $p(G \circ H) + m = p(G \circ H)$ and $q(G \circ H) = q(G \circ H) - m(n-2)$.

**Lemma 1.** Let $G$ and $H$ with order $n$ and $m$, respectively be a connected graph. The dominating set of edge corona product of graph $(G \circ H)$ is located in $G$.

**Proof.** The edge corona graph $G \circ H$ is a connected graph with vertex set $V(G \circ H) = V(G) \cup \{x_b,c; b = 1..E(G); c = 1..V(H)\}$ and edge set $E(G \circ H) = E(G) \cup E(H) \cup E(G_b)V(H_b)$. The vertex and edge cardinality of $(G \circ H)$ are $|V(G \circ H)| = p(G) + q(G)p(H)$, $|E(G \circ H)| = q(G) + q(H) + q(G)p(H)$.

Based on the definition of edge corona graph, every graph $H_b$ connected with edge $e_b$ of graph $G$ so every vertex in graph $H_b$ connected with $e_b = u, v$ then we only need vertex $u$ or $v$ in graph $G$ to dominated vertices in graph $H_b$. So it is proven that the dominating set of $\gamma(G \circ H)$ is located in $G$.

**Lemma 2.** Let $G$ and $H$ with order $n$ and $m$, respectively be a connected graph; we have $\gamma(G) \leq \gamma(G \circ H)$

**Proof.** Base on Lemma 1 and definition of edge corona graph, then the dominator is at least that is $\gamma(G)$ and can be more. So it is proven that $\gamma(G) \leq \gamma(G \circ H)$.

**Lemma 3.** Let $G$ and $H$ with order $n$ and $m$, respectively be a connected graph; we have $\gamma(G \circ H) \leq \gamma(G \circ H)$

**Proof.** Based on definition of edge corona graph, then graph $H$ only needs one vertex on edge of graph $G$ to dominated graph $H$. While according to [2] on the corona graph requires every vertex on graph $G$ to become a dominator. So it prove that the number of dominators on corona graph is bigger than the edge corona graph, denoted by $\gamma(G \circ H) \leq \gamma(G \circ H)$.

**Corollary 1.** Let $G$ and $H$ with order $n$ and $m$, respectively be a connected graph; we have $\gamma(G) \leq \gamma(G \circ H) \leq \gamma(G \circ H)$

From Lemma 2 and Lemma 3, we can write $\gamma(G) \leq \gamma(G \circ H) \leq \gamma(G \circ H)$. 

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Theorem 1. If \( G \) is graph of edge corona of path graph \( P_n \) and path graph \( P_m \) for every positive integer \( n, m \geq 3 \), then \( \gamma(P_n \circ P_m) = \lfloor \frac{n}{2} \rfloor \).

Proof. The vertex set of \((P_n \circ P_m)\) is \( V(P_n \circ P_m) = \{x_b; b = 1..n\} \cup \{x_b,c; b = 1..n; c = 1..m\} \) and the edge set of \((P_n \circ P_m)\) is \( E(P_n \circ P_m) = \{x_bx_{b+1}; b = 1..n-1\} \cup \{x_bx_{b,c}; b = 1..n-1; c = 1..m\} \cup \{x_{b,b-1,c}; b = 2..m; c = 1..m\} \cup \{x_{b,cx_{b,c+1}}; b = 1..n-1; c = 1..m-1\} \). The vertex and edge cardinality of graph are \( |V(P_n \circ P_m)| = n + (n-1)m \), \( |E(P_n \circ P_m)| = 3m(n-1) \) respectively. Thus, \( \Delta(P_n \circ P_m) = \Delta(G) + 2m = 2m + 2 \) and \( \delta(P_n \circ P_m) = \delta(H) + 2 = 3 \).

Base on Lemma 1 the dominator located in \( P_n \), base on Theorem 1 \( \lfloor \frac{n}{3} \rfloor \leq \gamma(P_n) \leq n \). Choose \( D = \{x_b; b \equiv 2 \mod 3 \) and \( x_n \) while \( n \equiv 1 \mod 3 \) \) as the dominating set of \( P_n \circ P_m \) for \( n, m \geq 3 \), thus \( |D| = \lfloor \frac{n}{3} \rfloor \). Therefore \( \gamma(P_n \circ P_m) = \lfloor \frac{n}{2} \rfloor \).

Figure 1 is dominating set of \( P_3 \circ P_3 \).

Corollary 2. If \( G \) is graph of edge corona of path graph \( P_n \) and \( H \) order \( m \) for every positive integer \( n, m \geq 3 \), then \( \gamma(P_n \circ H) = \lfloor \frac{n}{2} \rfloor \).

Graph of \((P_n \circ H)\) have vertex set \( V(P_n \circ H) = \{x_b; b = 1..n\} \cup \{x_{b,c}; b = 1..n; c = 1..m\} \). The vertex and edge cardinality of graph \((P_n \circ H)\) are \( |V(P_n \circ H)| = p(G) + p(H)q(G), |E(P_n \circ H)| = q(G) + q(H)q(G) + 2p(H)q(G) \) respectively. Thus, \( \Delta(P_n \circ H) = \Delta(G) + 2p(H) \) and \( \delta(P_n \circ H) = \delta(H) + 2 \). Base on Lemma 1 the dominator located in \( P_n \) and base on Theorem 1 \( \lfloor \frac{n}{3} \rfloor \leq \lfloor \frac{n}{2} \rfloor \) \( \leq n \).

Theorem 2. If \( G \) is graph of edge corona of star graph \( S_n \) and path graph \( P_m \) for every positive integer \( n, m \geq 3 \), then \( \gamma(S_n \circ P_m) = 1 \).

Proof. The vertex set of \((S_n \circ P_m)\) is \( V(S_n \circ P_m) = \{A, x_b; b = 1..n\} \cup \{x_{b,c}; b = 1..n; c = 1..m\} \) and edge set of \((S_n \circ P_m)\) is \( E(S_n \circ P_m) = \{Ax_b; b = 1..n\} \cup \{Ax_{b,c}; b = 1..n; c = 1..m\} \cup \{x_{b,cx_{b,c+1}}; b = 1..n; c = 1..m-1\} \). The vertex and edge cardinality of graph are \( |V(S_n \circ P_m)| = mn + n + 1, |E(S_n \circ P_m)| = 3mn \) respectively. Thus, \( \Delta(S_n \circ P_m) = \Delta(G) + nm = n + nm \) and \( \delta(S_n \circ P_m) = \delta(H) + 2 = 3 \).

Base on Lemma 1 the dominator located in \( S_n \), base on Theorem 1 \( 1 \leq \gamma(S_n) \leq 1 \). Select \( D = \{A\} \) as the dominating set of \( S_n \circ P_m \) for \( n, m \geq 3 \), thus \( |D| = 1 \). Therefore \( \gamma(S_n \circ P_m) = 1 \). Figure 2 is dominating set of \( S_5 \circ P_3 \).

Corollary 3. If \( G \) is graph of edge corona of star graph \( S_n \) and \( H \) order \( m \) for every positive integer \( n, m \geq 3 \), then \( \gamma(S_n \circ H) = 1 \).
Graph \((S_n \circ H)\) have vertex set \(V(S_n \circ H) = \{A\} \cup \{x_b; b = 1..n\} \cup \{x_{b,c}; b = 1..n; c = 1..m\}\). The vertex and edge cardinality of graph \((S_n \circ H)\) are \(|V(S_n \circ H)| = p(G) + p(H)q(G), |E(S_n \circ H)| = q(G) + q(G)q(H) + 2p(H)q(G)\) respectively. Thus, \(\Delta(S_n \circ H) = \Delta(G) + p(H)q(G)\) and \(\delta(S_n \circ H) = \delta(H) + 2\). Base on Lemma 1 the dominator located in \(S_n\) and base on Theorem 1 \(1 \leq 1 \leq n + 1\).

**Theorem 3.** If \(G\) is graph of edge corona of cycle graph \(C_n\) and path graph \(P_m\) for every positive integer \(n, m \geq 3\), then \(\gamma(C_n \circ P_m) = \lceil \frac{n}{2} \rceil\).

**Proof.** The vertex set of \((C_n \circ P_m)\) graph is \(V(C_n \circ P_m) = \{x_b; b = 1..n\} \cup \{x_{b,c}; b = 1..n; c = 1..m\}\) and edge set of \((C_n \circ P_m)\) graph is \(E(C_n \circ P_m) = \{x_b x_b+1; b = 1..n-1\} \cup \{x x_1\} \cup \{x_b x_{b,c}; b = 1..n; c = 1..m\}\) \(\cup \{x_{b,c} x_{b,c+1}; b = 1..n; c = 1..m-1\}\). The vertex and edge cardinality of graph are \(|V(C_n \circ P_m)| = n(m + 1), |E(C_n \circ P_m)| = 3nm\) respectively. Thus, \(\Delta(C_n \circ P_m) = \Delta(G) + 2m = 2m + 2\) and \(\delta(C_n \circ P_m) = \delta(H) + 2 = 3\).

Base on Lemma 1 the dominator located in \(C_n\), base on Theorem 1 \(\lceil \frac{n^2}{3} \rceil \leq \gamma(C_n) \leq n\). Select \(D = \{x_b; b \equiv 2 \mod 3\}\) and \(x_n\) while \(n \equiv 1 \mod 3\) as the dominating set of \(C_n \circ P_m\) for \(n, m \geq 3\), thus \(|D| = \lceil \frac{n^2}{3} \rceil\), we can see while \(c\); \(i \equiv 0 \mod 3\) then \(p(H_b; b \equiv 0 \mod 3)\) not dominated. Base on Corollary 1 we select the other vertex, Select \(D = \{x_b; b\) is odd \} as the dominating set of \(C_n \circ P_m\) for \(n, m \geq 3\), thus \(|D| = \lceil \frac{n^2}{2} \rceil\). Therefore \(\gamma(C_n \circ P_m) = \lceil \frac{n^2}{2} \rceil\). \(\Box\)

This Theorem also applies to \(C_n \circ H\). Figure 3 is dominating set of \(S_5 \circ P_3\).

**Corollary 4.** If \(G\) is graph of edge corona of cycle graph \(C_n\) and \(H\) order \(m\) for every positive integer \(n, m \geq 3\), then \(\gamma(C_n \circ H) = \lceil \frac{n}{2} \rceil\).

Graph \((C_n \circ H)\) have vertex set \(V(C_n \circ H) = \{x_b; b = 1..n-1\} \cup \{x_n x_1\} \cup \{x_{b,c}; b = 1..n; c = 1..m\}\). The vertex and edge cardinality of graph \((C_n \circ H)\) are \(|V(C_n \circ H)| = p(G) + p(H)q(G), |E(C_n \circ H)| = q(G) + q(G)q(H) + 2p(H)q(G)\) respectively. Thus, \(\Delta(C_n \circ H) = \Delta(G) + 2p(H)\) and \(\delta(C_n \circ H) = \delta(H) + 2\). Base on Lemma 1 the dominator located in \(C_n\) and base on Theorem 1 \(\lceil \frac{n^2}{3} \rceil \leq \lceil \frac{n}{2} \rceil \leq n\).
Theorem 4. If $G$ is graph of edge corona of wheel graph $W_n$ and path graph $P_m$ for every positive integer $n,m \geq 3$, then $\gamma(W_n \diamond P_m) = \lceil \frac{n}{2} \rceil + 1$.

Proof. The vertex set of $(W_n \diamond P_m)$ graph is $V(W_n \diamond P_m) = \{A, x_b; b = 1..n\} \cup \{x_{b,c}, y_{b,c}; b = 1..n; c = 1..m\}$ and edge set of $(W_n \diamond P_m)$ graph is $E(W_n \diamond P_m) = \{Ax_b; b = 1..n\} \cup \{x_{b,x_{b+1}}; b = 1..n\} \cup \{y_{b,y_{b+1}}; b = 1..n; c = 1..m\} \cup \{x_{b,x_{b-1}}; b = 1..n; c = 1..m\} \cup \{x_{b,x_{b+1}}; b = 1..n; c = 1..m\} \cup \{y_{b,y_{b+1}}; b = 1..n; c = 1..m\} \cup \{x_{n,x_1}; c = 1..m\} \cup \{x_{1,x_{n-1}}; c = 1..m\}$. The vertex and edge cardinality of graph are $|V(W_n \diamond P_m)| = n(m + 1), |E(W_n \diamond P_m)| = 3nm$ respectively. Thus, $\Delta(W_n \diamond P_m) = \Delta(G) + nm = n + nm$ and $\delta(W_n \diamond P_m) = \delta(H) + 2 = 3$.

Based on Lemma 1 the dominator located in $W_n$, base on Theorem 1 $1 \leq \gamma(W_n) \leq 1$. Choose $D = \{A\}$ as the dominating set of $W_n \diamond P_m$ for $n,m \geq 3$, thus $|D| = 1$, we can see the edge of cycle graph do not dominate $p(H_b; b = x_{b,x_{b+1}} \cup x_{b,x_1})$. Base on Corollary 1 so we select the other vertex, select $D = \{A, x_b; b \text{ is odd}\}$ as the dominating set of $W_n \diamond P_m$ for $n,m \geq 3$, thus $|D| = \lfloor \frac{n}{2} \rfloor + 1$. Therefore $\gamma(W_n \diamond P_m) = \lfloor \frac{n}{2} \rfloor + 1$. 

Figure 4 is dominating set of $W_5 \diamond P_3$.

Corollary 5. If $G$ is graph of edge corona of wheel graph $W_n$ and $H$ order $m$ for every positive integer $n,m \geq 3$, then $\gamma(W_n \diamond H) = \lfloor \frac{n}{2} \rfloor + 1$. Because wheel graph formed from cycle graph and star graph, then it is influenced.
Figure 4. Dominating Set of $W_5 \odot P_3$

$1..m \cup \{y_{b,c}; b = 1..n; c = 1..m\}$. The vertex and edge cardinality of graph $(W_n \odot H)$ are $|V(W_n \odot H)| = p(G) + p(H)2q(G), |E(W_n \odot H)| = q(G) + q(G)q(H) + 2p(H)q(G)$ respectively. Thus, $\Delta(W_n \odot H) = \Delta(G) + 2p(H)q(G)$ and $\delta(W_n \odot H) = \delta(H) + 2$. Base on Lemma 1 the dominator located in $W_n$ and base on Theorem 1 $1 \leq \lceil \frac{n}{2} \rceil + 1 \leq n + 1$.

**Theorem 5.** If $G$ is graph of edge corona of complete graph $K_n$ and path graph $P_m$ for every positive integer $n,m \geq 3$, then $\gamma(K_n \odot P_m) = \lceil \frac{n}{2} \rceil$.

**Proof.** The vertex set of $(K_n \odot P_m)$ graph is $V(K_n \odot P_m) = \{x_b; b = 1..n\} \cup \{x_{b,c}; b = 1..n; c = 1..m\}$. The vertex and edge cardinality of graph are $|V(K_n \odot P_m)| = n(m + 1), |E(K_n \odot P_m)| = 2nm + (m - 1)n + \frac{n(n - 1)}{2}$ respectively. Thus, $\Delta(K_n \odot P_m) = \Delta(G) + 2m = 2m + n - 1$ and $\delta(K_n \odot P_m) = \delta(H) + 2 = 3$.

Base on Lemma 1 the dominator located in $C_n$, base on Theorem 1 $\lceil \frac{n}{3} \rceil \leq \gamma(C_n) \leq n - 2$. Select $D = \{x_b; b \equiv 2 \mod 3 \text{ and } x_n \text{ while } n \equiv 1 \mod 3\}$ as the dominating set of $C_n \odot P_m$ for $n,m \geq 3$, thus $|D| = \lceil \frac{n}{3} \rceil$, we can see while $e_b; b \equiv 0 \mod 3$ then $p(H_b; b \equiv 0 \mod 3)$ not dominated. Base on Corollary 1 we select the other vertex, Select $D = \{x_b; b \text{ is odd }\}$ as the dominating set of $C_n \odot P_m$ for $n,m \geq 3$, thus $|D| = \lceil \frac{n}{2} \rceil$. Therefore $\gamma(C_n \odot P_m) = \lceil \frac{n}{2} \rceil$.

This Theorem also applies to $K_n \odot H$.

**Corollary 6.** If $G$ is graph of edge corona of complete graph $K_n$ and $H$ order $m$ for every
positive integer $n, m \geq 3$, then $\gamma(K_n \odot H) = \lceil \frac{n}{2} \rceil$.

Graph $(K_n \odot H)$ have vertex set $V(K_n \odot H) = \{x_b; b = 1..n\} \cup \{x_{b,c}; b = 1..n; c = 1..m\}$. The vertex and edge cardinality of graph $(K_n \odot H)$ are $|V(K_n \odot H)| = p(G) + p(H)q(G)$, $|E(K_n \odot H)| = q(G) + q(G)q(H) + 2p(H)q(G)$ respectively. Thus, $\Delta(K_n \odot H) = \Delta(G) + 2p(G)$ and $\delta(K_n \odot H) = \delta(H) + 2$. Base on Lemma 1 the dominator located in $K_n$ and base on Theorem 1 $1 \leq \lceil \frac{n}{2} \rceil \leq n$.

3. Concluding Remarks
In this research, we have analyze the relation of domination number of corona graph $(G \odot H)$ and edge corona graph $(G \odot H)$. Based on Carmelito, the domination number of corona graph is $\gamma(G \odot H) = n$ and based on this paper the domination number of edge corona graph is in between $\gamma(G)$ and $\gamma(G \odot H)$. The open problem of this research is:

Open Problem 1. Let $G$ be a connected graph, analyze the relation of domination number with the other.

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