Diagnosis of an actuated seat in a constrained space via Takagi-Sugeno sliding mode observers

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Abstract. This paper deals with actuator faults detection and isolation for an actuated seat described by Takagi-Sugeno multiple models. The goal is to ensure the comfort and the security of the users in simulator applications. Sliding mode observers based on T-S models are designed to estimate the system state vector. Residuals are generated by the comparison of measured and estimated outputs. In this work, a multi-observers technique is used. It consists of the construction of many observers such that each observer must be robust to noises and to other uncertainties but sensitive to one actuator fault. Simultaneous faults occurring on the actuated seat can be detected and isolated using this method. Simulation results are given to show the effectiveness of this approach.

1. Introduction

Nowadays, multimedia space simulators (flight simulator, automotive, video games ...) overgrown the field of new technologies. Users spend several hours a day on their chairs in front of the screens. For simulators, the user is in general installed on an articulated mobile platform equipped with an instrumented seat, a joystick and one or more wide-screen. For example, to be an efficient racing simulator, the user has to feel every sensation as in real situations, such as acceleration, braking or jumps.

In this study, which is part of an industrial project \footnote{Confidentiality has been contracted with the industrial company, therefore, its name and the studied application can not be provided in this paper. All the parts of this contract has read this paper and agreed to its publication.}, we are interested in the diagnostic of the seat for such applications. Indeed, the seat is instrumented in a confined space and it has to ensure the comfort and the security of the users. Such systems integrate actuators, sensors and embedded intelligent systems (control, optimization, supervision, ...) whose aim is to increase their performances. This context is motivating the present diagnostic study with application on an actuated seat which can be seen as a robot with tree structure. Due to high reliability demands of such systems, recent supervision and fault diagnosis concepts are of particular importance \cite{16}. There are three main approaches to deal with fault diagnosis \cite{15}, \cite{12}, \cite{19}, \cite{10}, \cite{2}:

- Signal analysis-based approach,
- Knowledge-based approach,
Model-based approach.

Model-based approaches to fault diagnosis in dynamic processes have received considerable attention since the beginning of the 70’s, both in the research context and in the domain of application on real processes [16], [9], [14], [11]. These model-based fault diagnosis systems are structured on two levels:

- Residual generation: Its purpose is to generate a residual signal which is sensible to a fault.
- Decision making: The residuals are examined for the likelihood of a fault and a decision rule is then applied to determine if some fault appears.

Most of model-based fault diagnosis methods are based on linear system models. For nonlinear dynamic systems, the fault diagnosis problem has been traditionally analyzed in two steps: First, the model is linearized around an operating point, and second, some techniques are applied to generate signal-residual, e.g. Kalman filter, observers, parity relations, parameter estimation [12]. Among nonlinear observers, T-S fuzzy model-based approaches have become popular, because such models provide universal approximations of nonlinear systems. Therefore in the past few decades, T-S fuzzy models have been the subject of many theoretical studies (e.g. [24] [22]) and applications (e.g. [13], [20], [21]). The major interest of such modelling approaches is that they allow extending some linear observer or controller design methodologies to nonlinear systems. In this work, we consider the model-based approach and the technique used in [1], in order to deal with the fault diagnosis problem. The strategy is based on obtaining a Takagi-Sugeno (T-S) fuzzy model and then using sliding mode observers to estimate the system state vector; the residual signal is then generated by the comparison of the measured and the estimated outputs. Observers based diagnosis is a technique that has been much discussed in the literature [17], [4], [25], [10], [3]. Without the occurrence of a faults, the residuals are close to zero while they deviate significantly from zero upon the occurrence of a faults on the system. The detection of the faults is generally quite easy; however, their localization is more delicate. Therefore, one frequently uses multi-observers to generate the residuals, whose are analysed through logical decision rules to find the location of the faults.

The paper is organized as follows: In section 2, the dynamical model of the seat and the corresponding Takagi-Sugeno fuzzy model are described. In section 3, we design a fuzzy sliding mode observer for the actuator fault detection and isolation. In section 4, simulation results show the effectiveness of the approach.

2. Description and modelling of the actuated seat

In this paper we will consider the diagnostic of an empty actuated seat, depicted in figure 1. That is to say, without considering the interaction with the user, which can be considered as modelling uncertainties and external disturbances. Therefore, in this preliminary study, one assumes that the robustness of the following designed observers will handle such uncertainties. The actuated seat contains three actuated joints $r_1$, $\theta_2$, $\theta_4$ and one unactuated joint $\theta_3$, whose significations are given in Table 1 ($r_1$ is a prismatic joint, $\theta_i$ ($j = 2,3,4$) are rotary joints).

To synthesize the relevant observers, dynamical model of the seat is required. It is given as the following second order mechanical system:

$$J(q(t)) \ddot{q}(t) + D(q(t), \dot{q}(t))\dot{q}(t) + G(q(t)) = u(t)$$

where $q(t) = [r_1 \theta_2 \theta_4]^T$ is the vector of generalized coordinates, $J(q(t))$ is the inertia matrix (symmetric and positive definite), $D(q(t), \dot{q}(t))$ is the matrix of Coriolis and centrifugal forces, $G(q(t))$ is the vector of gravitation and $u(t)$ is the vector of the generalized forces produced by
Figure 1. Different joints of the seat.

Table 1. Joints significations

|   |                                                                 |
|---|------------------------------------------------------------------|
| r<sub>1</sub> | Tracking position expressed in m                                 |
| θ<sub>2</sub> | Angle of the seating expressed in rad                           |
| θ<sub>3</sub> | Angle of the legrest expressed in rad                           |
| θ<sub>4</sub> | Angle of the seat-back expressed in rad                          |

the actuators. The control input \( u \) is assumed to be given by some known feedback controllers. For lightening the mathematical expressions, the time \( t \) will be committed in the sequel when there is no ambiguity. The matrices of the model, obtained from the Lagrange formalism [18], are detailed as follows:

\[
J(q) = \begin{bmatrix}
M & (\ast) & (\ast) \\
m_3 L_3 \sin(\beta - \theta_{23}) & J_3 & 0 \\
-m_4 L_6 \sin(\beta - \theta_{24}) & 0 & J_4
\end{bmatrix}
\]

\[
D(q, \dot{q}) = \begin{bmatrix}
0 & -m_4 L_3 \cos(\beta - \theta_{23}) \dot{\theta}_3 & m_6 L_6 \cos(\beta - \theta_{24}) \dot{\theta}_4 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
G(q) = \begin{bmatrix}
-M g \cos \beta \\
-(g m_3 L_3^2) \sin(\theta_{23}) \\
-(g m_4 L_4^2) \sin(\theta_{24})
\end{bmatrix}
\]

where \( \theta_{23} = \theta_2 + \theta_3; \ \theta_{24} = \theta_2 + \theta_4; M \) is the total mass of the seat; \( m_3 \) and \( m_4 \) are respectively the masses of the footrest and the headrest of the seat; \( L_3 \) and \( L_4 \) are respectively the lengths of the footrest and the headrest of the seat; \( \beta \) is the angle between the seat tracking axis and the horizontal plane; \( g \) is the gravitational acceleration.

Note that \( \theta_2 \) is an unactuated joint, which is not considered as a generalized coordinate since it is geometrically linked to the prismatic joint \( r_1 \) by means of a guide (constrained regarding to the mechanical design of the seat). Therefore, the nonlinear relation between \( \theta_2 \) and \( r_1 \) has
been approximated from experimental measurements (see Figure 2) as a third order polynomial given by:

\[ \theta_2 = ar_1^3 + br_1^2 + cr_1 + e \]  

(2)

where \( a = -36.54, b = 1.079, c = 1.49 \) and \( e = 0.1 \).

2.1. State space dynamical model of the seat and Takagi-Sugeno modelling

Let us consider the state vector \( x = [q \dot{q}]^T \) and the control input \( u = [u_1u_2u_3]^T \), the model (1) can be rewritten as the following nonlinear state space model:

\[
\begin{align*}
\dot{x}(t) &= A(q, \dot{q})x(t) + B(q)u(t) + Fd(t) \\
y(t) &= Cx(t)
\end{align*}
\]  

(3)

with

\[
A(q, \dot{q}) = \begin{bmatrix}
0_3 & I_3 \\
-J(q)^{-1}\bar{G}(q) & -J^{-1}(q)D(q, \dot{q})
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0_3 \\
-J(q)^{-1}
\end{bmatrix}
\]

where \( \bar{G}(q) \in \mathbb{R}^{3 \times 3} \) is defined such that \( G(q) = \bar{G}(q)q \); \( F \) and \( C \) are constant matrices of appropriate dimensions and \( d(t) \) is an unknown input vector containing external disturbance or faults in the system.

Let us notice that all the nonlinear terms included in the matrices of the state space model (3) are bounded (since \( r_1, \theta_3 \) and \( \theta_4 \) are physically bounded and chosen such that they do not cross zero) and given by: \( f_{J_1} = \sin(\beta - \theta_{23}), f_{J_2} = \sin(\beta - \theta_{24}), \bar{f}_{G_1} = \frac{1}{r_1}, \bar{f}_{G_2} = \frac{\sin(\theta_{24})}{\theta_4}, f_{G_3} = \frac{\sin(\theta_{24})}{\theta_4}, f_{D_1} = \cos(\beta - \theta_{23})\dot{\theta}_3 \) and \( f_{D_2} = \cos(\beta - \theta_{24})\dot{\theta}_4 \). Therefore, a Takagi-Sugeno model...
(4), which exactly matches the nonlinear model (3), is obtained by applying the well-known sector nonlinearity approach [23].

\[
\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) (A_i x(t) + B_i u(t) + F_d(t))
\]

where \( z(t) \) is the premise vector assumed to depend only on the state variables, \( r \) is the number of fuzzy sets for the right-hand side of the system (4), respectively; \( h_i(z(t)) \geq 0 \) are the membership functions that satisfy the convex sum property: \( \sum_{i=1}^{r} h_i(z(t)) = 1 \).

Note that, the above described T-S model contains 64 rules, therefore the matrices constituting the local subsystems and the membership functions are not detailed here for space reason. The reader may refer to [23] for more details on how to apply the well-known sector nonlinearity approach. Nevertheless, to illustrate the effectiveness of proposed T-S modelling approach, simulations of the seat have been performed in Matlab/Simulink to show that the T-S fuzzy model of the seat represents perfectly the nonlinear model, see Figures 3, 4 and 5.

\[\text{Figure 3. Comparison of the nonlinear model of the seat and its T-S model: tracking and seating position.} \quad \text{T-S Model, Nonlinear model}\]
3. Synthesis of the sliding mode observer

The goal is now to design convenient observers dedicated to the diagnosis of the seat actuators faults. From the model-based approach, it is necessary to build the mathematical model of the
system that is to be observed. In the general case of nonlinear dynamic systems, the design of observers is not an easy task. Therefore, in this work we consider the Takagi-Sugeno fuzzy model described in (4) and derived from (3).

It is assumed that the T-S fuzzy model (4) is locally observable and our goal is to design an observer such that \( x(t) - \hat{x}(t) \to 0 \) as \( t \to \infty \), where \( \hat{x}(t) \) denotes the estimated state vector. In this work, one considers an adaptation of the fuzzy sliding mode observer design methodology proposed in [8] to the T-S model of the actuated seat. This method uses a number of local linear time-invariant observers; each local model represented by (4) is associated with a local observer given as follows:

\[
\begin{align*}
\dot{x}(t) &= A_i x(t) + B_i u(t) + F_i d(t) + L_i (y(t) - \hat{y}(t)) + \phi_i(t) \\
\dot{\hat{y}}(t) &= C_i \hat{x}(t)
\end{align*}
\]

(5)

where \( L_i \) is the observer gain and \( \phi_i \) is the discontinuous vector of sliding mode which will be defined later.

By the use of the Parallel Distributed Compensation (PDC) technique [9], the total state estimation is the combination of local observer outputs.

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{r} h_i(z(t)) \left\{ A_i \dot{x}(t) + B_i u(t) + F d(t) + L_i (y(t) - \hat{y}(t)) + \phi_i(t) \right\} \\
\dot{\hat{y}}(t) &= \sum_{i=1}^{r} h_i(z(t)) C_i \dot{x}(t)
\end{align*}
\]

(6)

To analyze the convergence of this observer, the state estimation error \( e(t) = x(t) - \hat{x}(t) \) dynamics is examined:

\[
\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t)
\]

(7)

Using (4) and (6), one can find:

\[
\dot{e}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \left[ A_{ij} e(t) + F_i d(t) - \phi_i(t) \right]
\]

(8)

where \( A_{ij} = A_i - L_i C_j \).

One chooses the following Lyapunov function:

\[
v(t) = e^T(t) P e(t)
\]

(9)

Deriving:

\[
v(t) = e^T(t) P \dot{e}(t) + \dot{e}^T(t) P e(t)
\]

(10)

Using (8), one finds:

\[
\dot{v} = e^T(t) P \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \left( \bar{A}_{ij} e + F_i d - \phi_i \right) + \\
\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \left( \bar{A}_{ij} e + F_i d - \phi_i \right)^T P e
\]

(11)

\[
= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \left\{ e^T(P \bar{A}_{ij} + \bar{A}_{ij}^T P) e + 2 e^T P F_i d - 2 e^T P \phi_i \right\}
\]

Then:

\[
\dot{v}(t) \leq -\gamma \| e(t) \|^2 + \sum_{i=1}^{r} \left\{ 2 \| e^T(t) P \| \| F_i d(t) \| - 2 e^T(t) P \phi_i(t) \right\}
\]

(12)
In order to have $\dot{v}(t) < 0$, we propose that the discontinuous term has the following form:

$$
\phi_i(t) = k_i \text{sign} \left( e^T(t)P \right) = k_i \frac{e^T(t)P}{\| e^T(t)P \|} \quad (13)
$$

where $k_i > 0$ is constant and $P > 0$ so that the following Lyapunov inequality is satisfied:

$$
P \tilde{A}_{ij} + \tilde{A}_{ij}^T P < 0 \quad (14)
$$

From this, $\lim_{t \to \infty} e(t) = 0$ if (12) and (13) are respected, therefore if:

$$
2 \| e^T(t)P \| \sum h_i(z(t)) \{ \| F_i d(t) \| - k_i \} < 0 \quad (15)
$$

then one obtains the following condition:

$$
k_i > \| F_i d(t) \| \quad (16)
$$

Furthermore, it should be mentioned that an observer with converging state estimation is considered as a stable observer. Nevertheless, its stability may be checked, for validation purpose, from a well-known bounded real lemma (stability conditions) [23]. These conditions come from the inequalities (14), which can be efficiently solved via convex optimization algorithms within the linear matrix inequalities (LMI) framework [7]. Moreover, from the solution of such LMIs, the observer gains $L_i$ can be obtained as in [1]. Hence, if a positive definite matrix $P$ exists, the sliding mode fuzzy observer (6) is defined.

Finally, once the state or output is estimated, the residual signals are generated by the comparison of the measured and estimated output.

$$
r(t) = y(t) - \hat{y}(t) \quad (17)
$$

Therefore, analysing the residuals, one may detect and isolate actuator faults. Indeed, to detect and isolate the actuator faults, a set of three observers is used such that each observer is sensitive to one actuator fault [5], [6]. So the $i^{th}$ observer will have as inputs $y$ and $\bar{u}_i$ which is the vector of inputs without the $i^{th}$ input $u_i$. This structure is presented in Figure 6 for the actuator fault detection and isolation. In the considered seat, a set of three observers for each actuator is sufficient. Each observer generates one residual which is sensitive to only one actuator fault.

**Figure 6.** Multi-observers for actuator fault detection and isolation.
4. Simulation results

Simulations trials have been performed in the absence of faults, thresholds can be set on the basis of the maximum absolute values of the residuals. If some faults occur, they will cause deviations from zero of some residuals much greater than the differences caused by uncertainties and measurement noises. Thresholds should be chosen correctly in view to reduce the false alarms and the non detections. In our case, we assume that uncertainties and measurements noises do not have a great influence on the residuals. For our simulations, thresholds are chosen as $+/−0.5$ meters for the residual $res_1$, related to the prismatic joint $r_1$, and $+/−0.5$ radians for the residuals $res_2$ and $res_3$, respectively related to the rotary joints $θ_3$ and $θ_4$.

To show the effectiveness of the approach, some scenarios for some actuator faults are simulated using Matlab/Simulink. Figure 7 shows that the first residual $res_1$ is sensitive to the prismatic joint actuator fault where the other residuals are not sensitive. Figure 8 shows that the second residual $res_2$ is sensitive to the seat legrest rotary joint actuator fault while the others are not sensitive to this fault. Residuals $res_3$, $res_4$ and $res_5$ are sensitive to the seat footrest, the seat-back and the seat headrest actuator faults respectively. Figure 9 shows that we can detect the seat-back and the tracking actuator faults simultaneously.

![Figure 7. Detection of the tracking actuator fault.](image)

Figure 7. Detection of the tracking actuator fault.

--- thresholds, --- residuals

Figure 7 shows the sensitivity of the first residual to the actuator of the first joint. This fault consists of blocking of the actuator several times at the instants 5s, 12.5s, 15s and 18.2s respectively. Similar fault is simulated for the actuator of the rotary joint 2 and detected by the second residual as it is shown by the Figure 8. Finally, simultaneous faults are simulated for the first and the third actuator and detected by the two residuals $res_1$ and $res_3$ as is shown in figure 9.
5. Conclusion

In this paper an analysis for fuzzy sliding mode observer based FDI applied to a Takagi-Sugeno fuzzy model of an actuated seat has been presented. A multi-observers approach is used to generate residuals that allow us to detect and to isolate actuator faults. In the actuated seat, considered as a robot with tree structure, it is important to detect the faulty actuator and then to reconfigure the control law. The effectiveness of the approach allows avoiding false alarms and non detections. Because of decentralization of actuators, we notice that each residual is sensitive to only one actuator fault. These residuals are presented as fault signature. Five observers are
used and each of them is dedicated to only one actuator. Therefore, five residuals are obtained and each of them is sensitive to only one actuator fault. Simulation results show that each residual generated by the sliding mode observers is sensitive to only one actuator fault. All real systems have uncertain characteristics, and cannot be modelled perfectly. Therefore, one has considered some bounded modelling uncertainties. In the simulation results we can observe that robustness is achieved under parametric variations of the system. Residual only depend of external inputs or fault in the system. Therefore, the sliding mode technique provides a way to find a possible solution to this problem.

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