The parity-violating nucleon-nucleon force in the $1/N_c$ expansion

Daniel R. Phillips,1 Daris Samart,2,3 and Carlos Schat4

1Institute of Nuclear and Particle Physics and Department of Physics and Astronomy, Ohio University, Athens, Ohio 45701, USA
2Department of Applied Physics, Faculty of Sciences and Liberal Arts, Rajamangala University of Technology Isan, Nakhon Ratchasima, 30000, Thailand
3Thailand Center of Excellence in Physics (ThEP), Commission on Higher Education, Bangkok 10400, Thailand
4Departamento de Física, FCEyN, Universidad de Buenos Aires and IFIBA, CONICET, Ciudad Universitaria, Pab. 1, (1428) Buenos Aires, Argentina

Several experimental investigations have observed parity violation in nuclear systems—a consequence of the weak force between quarks. We apply the $1/N_c$ expansion of QCD to the P-violating T-conserving component of the nucleon-nucleon (NN) potential. We show there are two leading-order operators, both of which affect $p\bar{p}$ scattering at order $N_c$. We find an additional four operators at order $N_c^0 \sin^2 \theta_W$ and six at $O(1/N_c)$. Pion exchange in the PV NN force is suppressed by $1/N_c$ and $\sin^2 \theta_W$, providing a quantitative explanation for its non-observation up to this time. The large-$N_c$ hierarchy of other PV NN force mechanisms is consistent with estimates of the couplings in phenomenological models. The PV observed in $p\bar{p}$ scattering data is compatible with natural values for the strong and weak coupling constants: there is no evidence of fine tuning.

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Protons are composite systems that interact with one another via all Standard Model forces. The strong-, nuclear- and electromagnetic forces play the most prominent role in proton-proton ($pp$) scattering, and myriad $pp$ data have been collected over the last seventy years. There are also parity-violating (PV) $pp$ interactions, which manifest the presence of weak interactions between the quarks inside each proton. Measurements of this effect are far fewer, but observations of longitudinal beam asymmetries $\sim 10^{-7}$ at Bonn [1], PSI [2], and TRIUMF [3] demonstrate that PV nucleon-nucleon (NN) forces exist. PV in NN systems is also probed via an asymmetry in the reaction $n\bar{p} \rightarrow d\gamma$ [4], measurement of which is ongoing at the Fundamental Nuclear Physics Beamline at the Spallation Neutron Source [5]. And ab initio calculations of few-nucleon systems allow us to take models of the PV NN force and predict what will occur, e.g., in $^3$He($n,p)^3$H [6], soon to be measured at the SNS FNPB [7]. Nuclear parity violation is also observed in larger systems (e.g., the radiative decay of the first excited state of $^{19}$F), but there theoretical uncertainties in the relationship between the observable and the model of the PV NN force are harder to quantify. Much work has gone into constraining the PV NN force from a variety of nuclear experiments, see Refs. [8, 9] for recent reviews.

The prevailing paradigm in such analyses is based on single-meson exchange between nucleons, most commonly in the framework developed by Desplanques, Donoghue, and Holstein (DDH) [10]. The quantum numbers of the exchanged mesons determine the operator structures that contribute, while operator coefficients involve products of strong and weak meson-nucleon-nucleon coupling constants. In this paper we show that Standard Model couplings and the $1/N_c$ expansion of QCD predict the operators, and the sizes of the associated coefficients, which appear in the PV NN potential.

An alternative framework—suitable for studying PV at very low energies—that systematizes pioneering studies [11, 12] has recently emerged [13–15], but has, as yet, been applied to far fewer experiments. The extension of chiral perturbation theory to few-nucleon systems, $\chi$EFT (see Ref. [10] for a recent review) has also been invoked. In $\chi$EFT the one-pion-exchange piece of the PV NN force dominates, with all other effects suppressed by two orders in the chiral expansion. $\chi$EFT calculations of PV few-nucleon observables have been reported in Refs. [17, 20].

It seems natural to expect one-pion exchange dominates the PV NN interaction, since it gives the long-distance parity-conserving potential, and drives many of the properties of light nuclei. But, thus far, experimental data show no evidence for pion exchange in the PV NN force; only upper bounds on its impact on observables have been obtained. We will show that the smallness of the PV NN operator associated with one-pion exchange is a consequence of the large-$N_c$ expansion.

Originally suggested by ’t Hooft [21], this technique notes that QCD has a “hidden”, perhaps small, parameter in $1/N_c$. Multiple simplifications of QCD occur in the limit of a large number of degrees of freedom. In particular, the expansion in powers of $1/N_c$ provides insight into hadronic matrix elements. This approach to the nonperturbative regime of QCD has proven very useful in the study of baryons: for reviews, see Refs. [22, 23]. In the context of nuclear forces the $1/N_c$ expansion was used to study the NN potential in Refs. [24, 25]. These works analyzed the NN potential for momenta of order $N_c^0$, i.e., $p \sim \Lambda_{QCD}$, and found that it is an expansion in $1/N_c^2$. Furthermore, the $1/N_c^2 \approx 1/10$ hierarchy be-
tween the different contributions to the NN potential is roughly borne out in the Nijm93 [26] NN potential. In Ref. [27] this analysis was extended to the 3N potential; here we tackle the parity-violating component of the NN force. Some of our results have previously been obtained within the chiral soliton model [28, 31], or from consistency relations for PV pion-nucleon scattering [31]. But a model-independent derivation of all pertinent scalings and comparison with experimental data and phenomenological potentials appears here for the first time.

We begin with the Standard Model operators that govern PV between quarks, since the fact that sin^2θ_W ≈ 0.23 [32] is key to the hierarchy of PV NN operators. First, we recall the effective Lagrangian for the PV four-quark operators involving u and d quarks [33]:

\[ \mathcal{L}_{4q}^{\text{eff}} = - \frac{G_F}{\sqrt{2}} \cos^2 \theta_C \sum_{a=1,2} \left( V^a_\mu - A^a_\mu \right)^2 \]

\[ - \frac{G_F}{\sqrt{2}} \cos 2\theta_W V^3_\mu + A^3_\mu - 2 \sin^2 \theta_W I^1_\mu \]

\[ = \sqrt{2} G_F \left\{ \cos^2 \theta_C \sum_{a = 1,2} V^a I^{a3} + \cos 2\theta_W V^3 A^{a3} \right\} - 2 \sin^2 \theta_W I^1 A^{a3} \]  

(1)

where we kept only the parity-violating terms. Here \( G_F = 1.16 \times 10^{-5} \) GeV^2, \( \cos^2 \theta_C = 0.946 \) and we dropped the Cabibbo suppressed terms. The vector, axial and isoscalar currents are given by \( V^\mu_a = \frac{i}{2} q \gamma^\mu \tau^a q \), \( A^\mu_a = \frac{i}{2} q \gamma^\mu \gamma^5 \tau^a q \) and \( I^1_\mu = \frac{1}{2} q \gamma_\mu q \) respectively. Importantly, the factor \( \sin^2 \theta_W \) accompanies the product of isoscalar and axial currents, which is the only source of \( \Delta I = 1 \) operators. In contrast, \( \Delta I = 0 \) and \( \Delta I = 2 \) operators have pre-factors of \( O(1) \). Running from the \( Z^0 \)-mass down to the strong scale \( \sim 1 \text{ GeV} \) does not significantly modify this hierarchy [34, 35].

Our task now is to estimate the size of the NN matrix elements of quark operators in Eq. (1). We do this by employing the Hartree expansion for the nuclear Hamiltonian in the large-\( N_c \) limit [27, 36]:

\[ H = N_c \sum_{s,t,u} U_{stu} \left( \frac{S}{N_c} \right)^s \left( \frac{I}{N_c} \right)^t \left( \frac{G}{N_c} \right)^u . \]

(2)

The explicit factors of \( 1/N_c \) ensure that an m-body interaction scales generically as \( 1/N_c^{m-1} \), as mandated by large-\( N_c \) QCD counting [37]. The coefficients are \( O(1) \) functions of the momenta. From now on we take a quark basis representation for the operators:

\[ S^i = q^i (q^i/2) q, \quad I^a = q^i (q^i/2) q, \quad G^{ia} = q^i (q^i/2) q . \]

(3)

These 15 operators generate an \( SU(4) \) algebra. We wish to take their matrix elements in the \( |NN \rangle \) piece of the hadronic Hilbert space, \( S, I, G \) in Eq. (2) can have any nucleon index; we denote by \( O_n \) the nucleon \( (n = 1, 2) \) on which they act. Products of operators acting on the same nucleon are reduced to a single operator using relations and reduction rules for the powers of \( S, I, G \) discussed in Refs. [27, 30]. The contributions to the PV NN force that result after such reduction can be straightforwardly ordered according to \( 1/N_c \) counting, since matrix elements of \( S \) and \( I \) between nucleon states are \( O(1) \), while matrix elements of \( G \) are \( O(N_c) \). The mass of the nucleon, \( m_N \), scales with \( N_c \), and so tends to infinity in the large-\( N_c \) limit. This implies that any large-\( N_c \) NN potential is (modulo exchange diagrams) local: it is a function of the relative co-ordinate \( r \); or, equivalently, in momentum space, depends solely on the difference of final- and initial-state relative momenta, \( p_- \equiv p' - p \). The combination \( p_+ \equiv p' + p \) can appear only via relativistic corrections, and so its occurrence is always suppressed by a factor of \( 1/N_c \). Both \( p_- \) and \( p_+ \) are parity odd, with \( p_- \) (\( p_+ \)) being even (odd) under time reversal.

We now use these momentum operators to counterbalance the spin-flavor structures obtained after using the reduction rules in Eq. (2). We do this such that we obtain a Hamiltonian that is a rotational scalar, time-reversal even, and parity odd. As to its isospin transformation properties, we have already seen that \( \Delta I = 0, 1, 2 \) operators arise in the Standard Model. At the hadronic level the leading-in-\( N_c \) operators are:

\[ U^N_{PV} = N_c \left( U^*_{PV} (p_-^2) \right] \begin{pmatrix} \\{ p_+ \cdot (\sigma_1 \times \sigma_2) \tau_1 \cdot \tau_2 \end{pmatrix} \right) \]

\[ + U^5_{PV} (p_-^2) \left[ \begin{pmatrix} p_+ \cdot (\sigma_1 \times \sigma_2) \tau_1 \cdot \tau_2 \end{pmatrix} \right], \]

(4)

where \( \{ \ldots \}_2 \) denotes a symmetric and traceless rank-two tensor. These mediate \( \Delta I = 0, 2 \) transitions. Since \( p_- \) is \( O(1) \) an arbitrary function of \( p_-^2 \) can appear as a pre-factor without changing the \( N_c \) order of any contribution. The four \( O(N_c^0 \sin^2 \theta_W) \) operators—all \( \Delta I = 1 \)—are:

\[ U^N_{PV} = N_c \left( U^*_{PV} (p_-^2) \right] \begin{pmatrix} \\{ p_+ \cdot (\sigma_1 \times \sigma_2) \tau_1 \cdot \tau_2 \end{pmatrix} \right) \]

\[ + U^5_{PV} (p_-^2) \left[ \begin{pmatrix} p_+ \cdot (\sigma_1 \times \sigma_2) \tau_1 \cdot \tau_2 \end{pmatrix} \right]. \]

(5)

At \( O(1/N_c) \) there are a number of additional \( \Delta I = 0, 2 \) operators that arise:

\[ U^N_{PV} = N_c^{-1} \left( U^*_{PV} (p_-^2) \right] \begin{pmatrix} \\{ p_+ \cdot (\sigma_1 \times \sigma_2) \end{pmatrix} \right) \]

\[ + U^8_{PV} (p_-^2) \left[ \begin{pmatrix} p_+ \cdot (\sigma_1 \times \sigma_2) \tau_1 \cdot \tau_2 \end{pmatrix} \right] \]

\[ + U^{10}_{PV} (p_-^2) \left[ \begin{pmatrix} p_+ \cdot (\sigma_1 \times \sigma_2) \tau_1 \cdot \tau_2 \end{pmatrix} \right] \]

(6)
while at $\mathcal{O}(1/N_c^2)$ corrections to the coefficient functions in Eq. (5) and additional $\Delta I = 1$ operators appear. Note that the $1/N_c$ expansion says very little about the coefficient functions $U^1_p - U^1_D$ and $U^1_{D, p}$; the only constraint on them is that they should be $\mathcal{O}(1)$.

Within each isospin sector the expansion is thus in $1/N_c^2$, as for the strong NN and NNN force. Since $\Delta I = 1$ operators are suppressed by $\sin^2 \theta_W$ and $1/N_c$, the two operators in Eq. (4) give the entire PV NN force up to corrections that are formally of relative order $1/N_c^2$, $\sin^2 \theta_W/N_c$. Below we will argue, though, that the numerical suppression is not quite the $\approx 10\%$ this implies.

We now compare our result to the dominant paradigm for PV NN potentials, that of DDH. The relevant expressions can be found in Eqs. (4) and (5). By computing the one-meson-exchange diagrams from the weak and strong meson-nucleon Hamiltonians given there the DDH potential is obtained as a set of operators, each of which is multiplied by one strong and one weak meson-nucleon-nucleon coupling. Up to $\mathcal{O}(N_c sin^2 \theta_W)$ only one spin-flavor structure is produced by the $1/N_c$ analysis that does not appear in the DDH potential, it is the operator multiplied by the coefficient function $U^1_{D, p}$ and it corresponds to a tensor constructed out of $L$ and $p_-$ coupled to the rank-two spin tensor. Such a structure is not generated straightforwardly in the meson-exchange picture.

The rest of the DDH structures can each be matched to one structure in the LO or $\mathcal{O}(N_c sin^2 \theta_W), \mathcal{O}(1/N_c)$ potentials. DDH made a prediction for the strength of these operators based on standard values for the strong meson-nucleon-nucleon couplings and estimates of the “best values” for the weak couplings. In what follows we will use these weak-coupling estimates as our point of comparison. For a discussion of the pitfalls of this approach see Ref. [31]. In order to extract values for the weak couplings from our $1/N_c$ analysis, we recall the $N_c$-scaling rule of the strong couplings from Ref. [28]:

$$g_\omega \sim \sqrt{N_c}, \quad g_\rho \sim \frac{1}{\sqrt{N_c}}, \quad \xi_V \sim N_c, \quad \xi_S \sim \frac{1}{N_c}. \quad (7)$$

For the pion we count its coupling as $g_{\pi NN} \sim \sqrt{N_c}$. This, together with the Goldberger-Treiman relation, means that the pseudoscalar $\pi NN$ coupling which appears in the DDH potential, $g_{\pi NN} = \frac{m_N}{\Lambda_\chi} g_{\pi NN}$, scales as $N_c^{3/2}$.

In a similar vein, we replaced DDH’s parameters $\chi_{V, S}$ by $m_N \xi_{V, S}/\Lambda_\chi$ and $h^0_p$ by $m_N h^0_p/\Lambda_\chi$, so that there are no spurious factors of $N_c$ appearing in the coefficients of operators via the nucleon mass. Here $\Lambda_\chi$ is $\approx 1$ GeV is a strong-interactions mass scale that suppresses higher dimensional operators and is independent of $N_c$.

Matching the spin-isospin structures between the $1/N_c$ expansion of the PV NN potential in $1/N_c$ and the DDH potential, we then extract the $N_c$ and $\sin^2 \theta_W$ scalings of the weak couplings in DDH potential as:

$$h^0_p \sim \sqrt{N_c}, \quad h^2_p \sim \sqrt{N_c}, \quad \frac{h^1_p}{\sin^2 \theta_W} \lesssim \sqrt{N_c}, \quad \frac{h^1_\omega}{\sin^2 \theta_W} \sim \sqrt{N_c}, \quad \frac{h^1_\omega}{\sin^2 \theta_W} \lesssim \frac{1}{\sqrt{N_c}}, \quad \frac{h^0_\omega}{\sin^2 \theta_W} \sim \frac{1}{\sqrt{N_c}}. \quad (8)$$

where we have incorporated the fact that, since they arise from the $\mu^0 A^0$ product in the effective four-quark Lagrangian, the $\Delta I = 1$ weak meson-nucleon-nucleon couplings must all include a factor of $\sin^2 \theta_W$. The bounds on the scalings of $h^0_p, h^1_p, h^1_\omega$ follow from requiring that the large-$N_c$ scaling is not violated, while the scalings of $h^0_\omega, h^1_\omega$ are needed in order that the $U$'s in Eqs. (4) scale as $\mathcal{O}(N_c), \mathcal{O}(N_c^0)$). Some of these results in Eq. (8) were previously derived in nucleon soliton models [28–31]. And Ref. [31] computed the large-$N_c$ scaling of $h^0_\omega$, but did not account for its $\sin^2 \theta_W$ suppression. This is the first time all weak-coupling scalings have been computed in a rigorous $1/N_c$ analysis.

The two operators in Eq. (4) give the entire leading-order PV NN potential. When written in terms of the DDH couplings they are proportional to $g_{\rho} h^0_p, \chi_V / m_N$. Taking the product of all scalings in this expression verifies that the potential in Eq. (4) is $\mathcal{O}(N_c)$, but also shows that one of the factors of $N_c$ is associated with the factor of $m_N/\Lambda_\chi \sim 1$ that (implicitly) occurs in the DDH coupling $\chi_V$. This effectively demotes the leading-order piece of the PV NN potential to a numerical size typical of a $\mathcal{O}(N_c)$ contribution. We therefore conclude that the two operators in Eq. (4) determine the parity-violating NN force up to $\approx 30\%$ corrections.

Although the DDH ranges are large, the preferred values fall in the relatively narrow bands predicted by the $1/N_c$ hierarchy, except for $h^0_p$ and $h^1_\omega$, see Fig. 1. Notably, the two leading-order operators are associated with the largest couplings, $h^0_p, h^0_\omega$. The DDH best value for the coupling $h^1_\omega$ is also within 30% of the natural value once the $\sin^2 \theta_W$ suppression is taken into account. The $\sin^2 \theta_W/N_c$ suppressed couplings include $h^1_\omega$. This is in contrast to DDH, who have a $h^1_\omega$ glaringly larger than the large-$N_c$ prediction. A much smaller $h^1_\omega$ is found in soliton models [28, 29], and appears to be borne out by experiment (see, e.g., Fig. 3 in Ref. [4]). Lastly, Ref. [39] used the quark model to argue that the coupling $h^1_p$ was small, and as a consequence it has been neglected in many subsequent analyses. In contrast, large-$N_c$ gives no reason that this coupling is any less important than, say, $h^1_\omega$; both generate the operator structure $(\sigma_1 + \sigma_2)(\tau_1 \times \tau_2)^z$ in Eq. (5) and consistency with the $N_c$ scaling of $U^1_p$ only requires that at least one of $h^1_p, h^1_\omega$ saturates the bounds given in Eq. (5).

None of this, though, is a comparison at the level of observables. As already alluded to, there are many problems with the extraction of weak meson-nucleon-nucleon couplings from data; Ref. [4] emphasized that the extracted weak coupling constants depend on the strong
coupling constants used. For that reason, constraining
the products of weak and strong couplings from experi-
ment may be a better choice. Therefore we conclude
our discussion by considering the dominant combina-
tions \( g_\rho h_\rho^0, g_\rho h_\rho^1, g_\omega h_\omega^0 \sim \mathcal{O}(1) \) and \( g_\omega h_\omega^1 \sim \mathcal{O}(N_c \sin^2 \theta_W) \) ask-
ing what they predict for experiments. All four con-
tribute to proton-proton scattering. The \( p\bar{p} \) asymmetry
has been measured at 15 \[1\], 45 \[2\], and 221 MeV \[3\].
In the main, the first two experiments constrain the PV-
induced mixing between \( \Sigma_0^- \) and \( \Xi_0^- \) waves, while the
third constrains the mixing between \( \Xi_0^+ \) and \( \Xi_2^- \) waves.
In plane-wave Born approximation the information that can
be parameterized by the quantities \( A_{SP} \) and \( A_{PD} \) \[\[4, 10\]\],
the pertinent combinations of coupling constants that
govern these mixings. In the DDH approach they are:
\[
A_{SP} \equiv g_\rho h_\rho^{pp}(2 + \chi_V) + g_\omega h_\omega^{pp}(2 + \chi_S),
A_{PD} \equiv g_\rho h_\rho^{pp} \chi_V + g_\omega h_\omega^{pp} \chi_S,
\]
for the strong couplings \( \{g_\omega, g_\rho, \xi_8, \xi_V\} \). This produces
the red point in Fig. 2. All nine couplings should be as-
signed a 30% error, due to omitted terms in the \( 1/N_c \)
expansion. Uncorrelated variation over this range pro-
duces the blue shaded area in the figure. The yellow
shaded area is the result found from solely varying the
five weak couplings. The prediction for \( A_{SP} \) and \( A_{PD} \)
from large-\( N_c \) and naturalness is thus consistent with the
constraints extracted by Carlson et al. \[10\] within the
combined theoretical and experimental uncertainties. It
shows no evidence of fine tuning. The black dot is ob-
tained with DDH “best values” for the weak and strong
couplings. For these observables those values are consist-
ent with large-\( N_c \) and naturalness, but such consistency
will not occur in observables where \( h_\rho^1 \) contributes.

To make a prediction for \( A_{SP} \) and \( A_{PD} \) we take
\( G_F f_\pi \Delta \chi \sim 1.0 \times 10^{-6} \) (with \( f_\pi = 92.4 \text{ MeV} \sim \sqrt{N_c} \)
as the naturalness estimate for the leading-order weak
couplings \( h_\rho^{0,2} \). Assuming a natural value for \( g_\rho \approx 4\pi \) \[42\],
we determine other couplings from these by the \( N_c, \sin^2 \theta_W \) scalings of Eqs. \(7\) and \(8\). This gives
\( \{-1.0, -0.077, -1.0, -0.33, -0.23\} \times 10^{-6} \) for the weak
couplings \( \{h_\rho^0, h_\rho^1, h_\omega^0, h_\omega^1\} \) and \( \{12.4, 0.0, -0.33, 3.0\} \)
for the strong couplings \( \{g_\omega, g_\rho, \xi_8, \xi_V\} \). This produces
the red point in Fig. 2. All nine couplings should be as-
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will not occur in observables where \( h_\rho^1 \) contributes.

The \( 1/N_c \) expansion for hadronic matrix elements,
superimposed on suppressions by factors of \( \sin^2 \theta_W \) pre-
dicted by the Standard Model, provides a new bench-
mark for parity-violating NN couplings. This approach
not only estimates the couplings, it also gives plausible
ranges for them based on \( 1/N_c \) scaling. The results for
the only non-zero measurements of parity-violating ef-
certs in the NN system are consistent with the experi-
mental data. It also naturally predicts a small \( h_\rho^1 \): \( |h_\rho^1| \lesssim (0.8 \pm 0.3) \times 10^{-7} \). This is consistent with the bound ob-
tained from \( ^{18}\text{F} \) experiments \[43, 48\], \( |h_\rho^1| \lesssim 1.3 \times 10^{-7} \). It is also consistent with the first lattice calculation of \( h_\rho^1 \) \[49\]. The \( 1/N_c \) expansion thus explains the other-
wise puzzling failure of pion effects to yet manifest them-
selves in hadronic parity-violation experiments. Finally,

![FIG. 1. The hierarchy of weak couplings for the DDH best values \[40\].](image1)

![FIG. 2. The ellipse gives the 90% C.L. constraint from experiment \[1, 3\] on the combinations of couplings \( A_{SP} \) and \( A_{PD} \) (in units of \( 10^{-7} \)) via the analysis of Refs. \[8, 40\]. The black point corresponds to the DDH best value and the red point is obtained from naturalness and our large-\( N_c \) analysis. The blue region is then found by varying the weak and strong couplings by 30% around their natural values. The smaller yellow region is obtained by only varying the weak couplings by a 30%, keeping the strong couplings fixed.](image2)
at $O(N^0 \sin^2 \theta_W)$, involving three momenta coupled to a rank-two tensor, should be included in analyses that examine the subleading piece of the NN force generated by the weak interaction.

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