Multipartite Dark Matter

Qing-Hong Cao, Ernest Ma, and José Udaka
Department of Physics and Astronomy,
University of California, Riverside, CA 92521

C.-P. Yuan
Department of Physics and Astronomy,
Michigan State University, East Lansing, MI 48824

Dark matter (comprising a quarter of the Universe) is usually assumed to be due to one and only one weakly interacting particle which is neutral and absolutely stable. We consider the possibility that there are several coexisting dark-matter particles, and explore in some detail the generic case where there are two. We discuss how the second dark-matter particle may relax the severe constraints on the parameter space of the Minimal Supersymmetric Standard Model (MSSM), as well as other verifiable predictions in both direct and indirect search experiments.
Dark matter (DM) is at the heart of any study regarding the interface between particle physics, astrophysics, and cosmology. Its relic abundance has now been measured with precision. Combining the results of the WMAP Collaboration and the Sloan Digital Sky Survey, \( \Omega_{CDM} h^2 = 0.110 \pm 0.013 \) (2\( \sigma \)) [1], where \( \Omega_{CDM} \) is the DM energy density normalized by the critical density of the Universe and \( h = 0.71 \pm 0.05 \) (2\( \sigma \)) is the scaled Hubble parameter. Many dark-matter candidates have been suggested in various models beyond the Standard Model (SM) of particle physics, but a nearly universal implicit assumption is that one and only one such candidate (1DM) is needed and its properties are constrained accordingly. This is of course not a fundamental principle and the possibility of multipartite dark matter should not be ignored. In this Letter we study its impact on the conventional picture of 1DM physics, such as that of supersymmetry, using a simple generic scenario of two dark-matter candidates (2DM), one a fermion singlet (neutralino) and the other a scalar singlet. Our conclusions are broadly applicable to any 2DM model.

**Model**

The simplest way to have at least two DM candidates is to append the SM with the exactly conserved discrete symmetry \( \mathbb{Z}_2 \times \mathbb{Z}_2' \). As pointed out in Ref. [2], this may be realized naturally in the framework of \( N = 2 \) supersymmetry. Alternatively, if the SM is extended to include an exactly conserved \( \mathbb{Z}_2 \) symmetry without supersymmetry, then the supersymmetric version of this extension will have \( \mathbb{Z}_2 \times \mathbb{Z}_2' \), as in Refs. [3] and [4]. To explore generically the impact of such a scenario, we first observe that the details of the specific model are mostly irrelevant, as far as the relic abundance, and the direct and indirect detection of dark matter are concerned, except for the masses of the two DM candidates and their interactions with the SM particles and with each other. This is because the relevant processes are either elastic scattering at almost zero momentum transfer or annihilation at rest.

Specifically we add two new fields which are singlets under the SM gauge group: a new fermion \( \chi \) and a new scalar \( S \). Under \( \mathbb{Z}_2 \times \mathbb{Z}_2' \), \( \chi \sim (-, +) \) and \( S \sim (+, -) \), whereas all SM particles are \( (+, +) \). This means that \( \langle S \rangle = 0 \) is required. In a complete theory such as that of Ref. [3], there may also be \( (-, -) \) particles. For simplicity we assume that all such particles are heavy enough to decay into \( \chi \) and \( S \). If not, we would then have to consider three coexisting DM candidates.
The Lagrangian of our generic 2DM model is given by

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{DM}^\chi + \mathcal{L}_{DM}^S + \mathcal{L}_{int},$$

(1)

where $\mathcal{L}_{SM}$ denotes the usual SM Lagrangian, and

$$\mathcal{L}_{DM}^\chi = i\bar{\chi} \partial \chi - m_1 \bar{\chi} \chi,$$

$$\mathcal{L}_{DM}^S = \frac{1}{2} \partial_\mu S \partial^\mu S + \frac{1}{2} m_2 S^2 + \frac{1}{4} \lambda_1 S^4,$$

$$\mathcal{L}_{int} = \frac{1}{2} \lambda_2 H^\dagger H SS + \frac{\lambda_3}{\Lambda} H^\dagger H \bar{\chi} \chi + \frac{\lambda_4}{2\Lambda} \bar{\chi} \chi SS,$$

(2)

where $H$ is the SM Higgs doublet. After the electroweak symmetry is spontaneously broken, $H = (v + h)/\sqrt{2}$ with $v = 246$ GeV and the masses of $\chi$ and $S$ are given by $m_\chi = m_1 - \lambda_3 v^2/2\Lambda$ and $m_2^2 = m_2^2 + \lambda_2 v^2/2$. The various effective interaction terms, relevant to our discussion, are

$$\mathcal{L}_{h\chi\chi} = g_\chi h \bar{\chi} \chi,$$

$$\mathcal{L}_{hh\chi\chi} = \frac{g_\chi}{2v} hh \bar{\chi} \chi,$$

$$\mathcal{L}_{hhSS} = \frac{1}{2} g_S h SS,$$

$$\mathcal{L}_{\chi\chi SS} = \frac{g_\chi S}{v} \bar{\chi} \chi SS,$$

(3)

where we have introduced the dimensionless couplings $g_\chi = \lambda_3 v/\Lambda$, $g_S = \lambda_2$, and $g_\chi S = \lambda_4 v/2\Lambda$. Note that this is not meant to be an effective theory in powers of $1/\Lambda$ for all processes. It is applicable only to DM-nucleus elastic scattering (with almost zero momentum transfer) and DM annihilation at rest.

As an example of how the effective couplings of Eq. (3) may be generated in a complete model, let us consider Ref. [3], where a second pair of scalar superfields $(\eta_1^0, \eta_1^-)$ and $(\eta_2^0, \eta_2^-)$ are added, which are odd under a new $\mathbb{Z}_2$, whereas the usual $(\phi_1^0, \phi_1^-)$ and $(\phi_2^0, \phi_2^-)$ of the MSSM are even. Together with the conventional $R$ parity, we then have an exactly conserved $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry. Consider now the interaction

$$\frac{1}{2} g_\gamma \tilde{B}(\eta_1^0 \eta_1^- - \eta_2^0 \eta_2^-) + \frac{1}{2} g_\gamma \tilde{B}(\phi_1^0 \phi_1^- - \phi_2^0 \phi_2^-),$$

(4)

where $\tilde{B}$ is the $U(1)_Y$ gaugino. We may thus identify the $\chi$ of our generic model with $\tilde{B}$, and $S(h)$ with a linear combination of the real parts of $\eta_{1,2}^0(\phi_{1,2}^0)$. The effective $\bar{\chi} \chi SS$ and $hh \bar{\chi} \chi$...
interactions are then generated from the exchange of the $\tilde{\eta}$ and $\tilde{\phi}$ higgsinos, respectively. Assuming the masses of these higgsinos to be comparable to $m_\chi$ and $m_S$, the effective couplings $g_\chi$ and $g_\chi S$ are not necessarily very much suppressed. This allows us to consider three characteristic scenarios, as depicted in Fig. 1. For definiteness, we consider $m_\chi > m_S$ in this analysis, but our conclusions are mostly the same if we switch them around.

![Diagram showing possible annihilation scenarios in the 2DM model](image)

**FIG. 1:** Possible annihilation scenarios in the 2DM model where the (red) arrow line denotes the DM annihilation.

Scenario A [Fig. 1(a)]: $g_\chi, g_S \neq 0$ but $g_\chi S = 0$; both $\chi$ and $S$ can annihilate into SM particles but they do not interact with each other. Scenario B [Fig. 1(b)]: $g_\chi = 0$ but $g_S, g_\chi S \neq 0$; $\chi$ can only annihilate into $S$, after which $S$ will annihilate into SM particles. All of $\chi$’s information is hidden behind $S$, hence it will be superdark and cannot be observed directly. It may be revealed nevertheless if apparent discrepancies occur among different experiments. Scenario C [Fig. 1(c)]: $g_\chi, g_S, g_\chi S \neq 0$; $\chi$ can annihilate into both $S$ and SM particles, after which $S$ will annihilate into SM particles. The special case of $g_\chi S \gg g_\chi, g_S$ is of particular interest. Here $\chi \bar{\chi}$ will annihilate predominantly into $SS$, resulting in a much smaller $\chi$ relic abundance, thereby relaxing the constraints on its parameter space, which may be identified with that of the MSSM. Scenarios A and B are of course just two special limits of C, but they have qualitatively different predictions on the direct and indirect search experiments of dark matter, as shown below.

**Observational Constraints**  If two DM candidates coexist, the usual observational constraints also apply, but with modifications.

(i) *Relic abundance:* Since both DM candidates contribute to the relic abundance, they
must add up to account for the current observation:

\[ \Omega_\chi h^2 + \Omega_S h^2 = \Omega_{CDM} h^2 = 0.110 \pm 0.013. \] (5)

It is well-known that the relic density of each DM species is approximately given by \( \Omega_i h^2 \approx (0.1 \, \text{pb})/\langle \sigma v \rangle_i \), where \( \langle \sigma v \rangle_i \) is the thermally averaged product of its annihilation cross section with its velocity. Using Eq. (5), we then obtain

\[ \frac{\langle \sigma v \rangle_\chi \langle \sigma v \rangle_S}{\langle \sigma v \rangle_\chi + \langle \sigma v \rangle_S} \equiv \langle \sigma v \rangle_0 \sim \text{pb}. \] (6)

(ii) Halo density profile: For simplicity, we assume the two DM candidates to have the same density profile and use that given by Navarro, Frenck and White (NFW) \[5\] in our analysis. (It is of course straightforward to extend our results to other density profiles.) In the 2DM model, the dark-matter mass density profile of the galactic halo is thus given by

\[ \rho(r) = \frac{\epsilon_\chi \rho_0}{(r/r_c) (1 + r/r_c)^2} + \frac{\epsilon_S \rho_0}{(r/r_c) (1 + r/r_c)^2}, \] (7)

where \( r_c = 20.0 \, \text{kpc} \) and \( \rho_0 \) is adjusted to reproduce the local halo density at the Earth position. Here, \( \epsilon_i \) represents the fraction of the mass density of the \( i \)th dark matter in our local dark-matter halo as well as in the Universe, i.e.

\[ \epsilon_i = \frac{\rho_i}{\rho_0} \approx \frac{\Omega_i h^2}{\Omega_{CDM} h^2}, \] (8)

where \( \rho_i \) is the local density of the \( i \)th DM and \( \sum_i \epsilon_i = 1 \). For our 2DM model, we obtain

\[ \epsilon_\chi = \frac{\langle \sigma v \rangle_0}{\langle \sigma v \rangle_\chi}, \quad \epsilon_S = \frac{\langle \sigma v \rangle_0}{\langle \sigma v \rangle_S}. \] (9)

(iii) Direct search: Assuming that DM is the dominant component of the halo of our galaxy, it is expected that a certain number of these weakly interacting massive particles (WIMPs) will cross the Earth at a reasonable rate and be detected by measuring the energy deposited in a low-background detector through the scattering of a WIMP with a nucleus of the detector. So far most experimental limits of this direct detection are given in terms of the cross section per nucleon under the 1DM hypothesis. The event rate per unit time per nucleon is given by

\[ R \approx \sum_i n_i \langle \sigma \rangle_i = \sum_i \frac{\rho_i}{m_i} \langle \sigma \rangle_i, \] (10)
where $n_i$ is the local number density of the $i$th DM and $\langle \sigma \rangle_i$ is the $i$th DM-nucleon elastic scattering cross section which is averaged over the relative DM velocity with respect to the detector. The measured experimental rate in the 1DM case is given by $R_{\text{exp}} \approx \rho_0 \sigma_0 / m_0$ where $\sigma_0$ denotes the “zero-momentum-transfer” cross section of DM-nucleon scattering and $m_0$ is the DM mass. The current direct-search limit implies $R < R_{\text{exp}}$, i.e.

$$\frac{\epsilon_s}{m_s} \sigma_{sN} + \frac{\epsilon_N}{m_N} \sigma_{sN} < \frac{\sigma_0}{m_0},$$

where $\sigma_{sN}$ denotes the scattering cross section of $s(N)$ with a nucleon $N$. Although the experimental sensitivities and limits are often described in terms of the dark-matter elastic scattering with a single nucleon, one should keep in mind that nuclear form factors may need to be taken into account.

In Scenario B, there is no scattering of $\chi$ with the nucleon, hence the limit in Eq. (11) becomes $\sigma_{sN} < \sigma_0 / \epsilon_s$, i.e. bounds from direct detection become weaker in this case. On the other hand, if dark matter is observed in direct-detection experiments, the DM-nucleon cross section may be underestimated by a factor of $1/\epsilon_s$.

(iv) Indirect gamma-ray search: The relic dark matter may collect and become gravitationally bound to the center of the galaxy, the center of the Sun and the center of the Earth. If this happens, then a variety of indirect dark-matter detection opportunities arise. In particular, the measurement of secondary particles coming from dark-matter annihilation in the halo of the galaxy will help to decipher the nature of dark matter. Efforts to detect the annihilation products of dark-matter particles in the form of gamma rays, antimatter and neutrinos are collectively known as indirect detection. Of these, the observation through gamma rays is the simplest and most robust. The diffusion gamma-ray spectrum is given by

$$\frac{d\Phi}{dE_\gamma} = \epsilon_s^2 \frac{d\Phi_s}{dE_\gamma} + \epsilon_N^2 \frac{d\Phi_X}{dE_\gamma},$$

where $d\Phi_i / dE_\gamma$ ($i = \chi, S$) is the differential gamma-ray flux along a direction that forms an angle $\psi$ with respect to the direction of the galactic center:

$$\frac{d\Phi_i}{dE_\gamma} = \frac{dN_\gamma}{dE_\gamma} \langle \sigma v \rangle_i \frac{1}{4\pi m_i^2} \int \left[ \frac{\rho_0}{(r/r_c) (1 + r/r_c)^2} \right]^2 dl.$$
The integral is performed along the line of sight. All annihilation channels of the \( i \)th DM are summed, and \( dN_\gamma/dE_\gamma \) is the differential gamma spectrum per annihilation coming from the decay of annihilation products.

Consider the special case of \( m_\chi = m_S = m_0 \). After some simple algebra, one can show that

\[
\frac{d\Phi}{dE_\gamma} \simeq \frac{dN_\gamma}{dE_\gamma} \frac{1}{4\pi m_0^2} \int \frac{\rho_0}{(r/r_c)(1 + r/r_c)^2} dl, \tag{14}
\]

where we have used the fact that \( dN_\gamma/dE_\gamma \) is almost the same for most of the final states. The integrated flux of the 2DM model is of the same order as that of the 1DM model.

\( (v) \) Collider search: Since \( \Omega h^2 \propto 1/\langle \sigma v \rangle \), the requirement of the correct relic density (\( \Omega_{\text{CDM}} h^2 \sim 0.1 \)) implies that DM annihilation was efficient in the early Universe. It also suggests efficient annihilation now, implying large indirect detection rates, as well as efficient scattering now, implying large direct detection rates. The sum rule, cf. Eq. (5), means that the DM annihilation of each individual candidate has to be more efficient than that of the 1DM case. Hence larger cross sections of DM production are expected at the collider. The smaller the fraction \( \epsilon_i \), the easier is the detection. In our simplistic case where the two DM candidates interact only with the SM Higgs boson, the vector-boson-fusion process \( qq \to qqVV \to qqh \), with the subsequent decay \( h \to \chi\bar{\chi}/SS \), provides the most promising collider signature of the model [6] when \( m_h > m_\chi/S \).

\textbf{2DM Implications} We first study the cosmological implications of either \( \chi \) or \( S \) as the sole source of dark matter. In Fig. 2(a) and (b) we present the correlations between the effective coupling and the DM mass [7], which is derived from WMAP data. The black-solid (red-dashed, blue-dotted) curve denotes \( \Omega_i h^2 \simeq 0.1 \) (0.05, 0.01), respectively. In the region below the black-solid curve the dark matter is overproduced. Fig. 2(c) shows the spin-independent cross section of DM-nucleon scattering for \( \chi \) (black-solid) and \( S \) (black-dashed). Current CDMS limit and projected sensitivity of CDMS2007 [8] are also plotted. Using Eq. (11), we then derive a realistic bound on the 2DM model. For a large range of the DM mass, \( \sigma_0/m_i \) is almost a constant, e.g. \( \sigma_0/m_i \simeq 2 \times 10^{-9} \text{pb/GeV} \) (2 \( \times 10^{-10} \text{pb/GeV} \) for the current CDMS data (projected CDMS2007 sensitivity). In Fig. 2(d) we present the allowed parameter space of Scenario A in the 2DM model in the \((m_S, m_\chi)\) plane for
FIG. 2: (a) and (b) show the correlations between the coupling and the mass of a single DM candidate as determined by the WMAP data: (a) for $\chi$ and (b) for $S$. (c) shows the spin-independent cross sections of DM-nucleon scattering in the 1DM model together with the CDMS limit and future projected sensitivities of CDMS2007. We choose $m_h = 200$ GeV throughout in this work. (d) shows the allowed ($m_\chi, m_S$) parameter space of Scenario A in the 2DM model.

$\epsilon_\chi = \epsilon_S = 0.5$. In Scenario B, the limits only depend on $S$ and the bounds become weaker.

A promising way to tell if there are two coexisting DM candidates, assuming that they are very different in mass, is through indirect gamma-ray observations. The overlap of the two distributions might change the line shape of the gamma-ray distribution which
FIG. 3: Predicted gamma-ray spectra in the 2DM model for $\epsilon_x = \epsilon_S = 0.5$. The predicted gamma flux is from a $\Delta \Omega = 10^{-3}$ srad region around the direction of the galactic center, assuming the NFW halo profile (with a boost factor as indicated in the figure). For comparison we also show the scaled gamma-ray distribution in the 1DM case. EGRET and HESS observations are also shown here for comparison.

is distinguishable from that of the 1DM case. In Fig. 3 this is illustrated by showing the predicted fluxes from a $\Delta \Omega = 10^{-3}$ srad region around the direction of the galactic center together with the existing EGRET [9] and HESS [10] observations in the same sky direction. We adopt the NFW density profile for the DM in our galaxy ($\bar{J} \times \Delta \Omega \sim 1$ for $\Delta \Omega = 10^{-3}$ srad) and allow the flux to be scaled by a “boost factor”. For demonstration we choose $\epsilon_X = \epsilon_S = 0.5$, $m_X = 260$ GeV, and $m_S = 60$ GeV. Clearly, the resulting gamma-ray flux distribution from the overlap of 2DM distribution is significantly different from that of the 1DM model, which can be probed by the GLAST experiment [11]. The gamma-ray spectra can also be used to distinguish Scenario A from B of the 2DM model. Unfortunately, it is difficult to observe a shape change if $m_X - m_S$ is small. On the other hand, they may be discriminated at the Large Hadron Collider (LHC) because in Scenario A, both $\chi$ and $S$ are produced; whereas in Scenario B, only $S$ is. A discrepancy between relic abundance and
LHC production may reveal Scenario B.

Consider the special case \((g_{\chi S} \gg g_{\chi}, g_S)\) of Scenario C, where the new annihilation channel \(\chi \bar{\chi} \rightarrow SS\) opens. This case is very interesting because it has a crucial impact on the conventional supersymmetric DM model. For example, the lightest neutralino is a well-motivated dark-matter candidate, but its relic abundance is typically too large, or equivalently, its annihilation rate is too small. The WMAP data thus impose very tight constraints on the parameter space of the MSSM. But those constraints can be relaxed if there exists an additional DM candidate which opens up a new annihilation channel for the neutralino. For illustration, we choose \(g_{\chi S} = (0.5, 0.55, 0.6)\) and \(m_S = 120\) GeV with \(\epsilon_{\chi}(\epsilon_S) = 0.9(0.1)\) in the 2DM model. Using Fig. 2(b), we then fix \(g_S = 0.114\). The (black) dotted curve in Fig. 4 denotes \(\Omega_{\chi} h^2 = 0.1\) in the 1DM model and the region below it will exceed the relic abundance. After including the new annihilation channel \(\chi \bar{\chi} \rightarrow SS\), more of the parameter space is reclaimed. Increasing \(g_{\chi S}\) will open up even more parameter space.

**Conclusion** In this Letter we presented a simple generic model of two coexisting dark-
matter candidates. We discussed its three characteristic annihilation scenarios and its impact on the observational constraints of dark matter. We note that the cosmic gamma-ray observation is a good probe for confirming the 2DM model. We also demonstrate that with a second dark-matter candidate, the usual severe constraints on the parameter space of the MSSM can be relaxed. More detailed studies of this new idea of multipartite dark matter are forthcoming.

Acknowledgements This work is supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837 and the U. S. National Science Foundation under award PHY-0555545.

[1] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 170, 377 (2007).
[2] C. Boehm, P. Fayet and J. Silk, Phys. Rev. D 69, 101302 (2004).
[3] E. Ma, Annales Fond. Broglie 31, 285 (2006).
[4] T. Hur, H. S. Lee and S. Nasri, arXiv:0710.2653 [hep-ph].
[5] J. F. Navarro, C. S. Frenk and S. D. M. White, Astrophys. J. 462, 563 (1996).
[6] Q. H. Cao, E. Ma and G. Rajasekaran, Phys. Rev. D 76, 095011 (2007).
[7] Y. G. Kim and K. Y. Lee, Phys. Rev. D 75, 115012 (2007); J. McDonald, Phys. Rev. D 50, 3637 (1994).
[8] R. Abusaidi et al. [CDMS Collaboration], Phys. Rev. Lett. 84, 5699 (2000); D. Abrams et al. [CDMS Collaboration], Phys. Rev. D 66, 122003 (2002).
[9] H. A. Mayer-Hasselwander et al., Astron. Astrophys. 335 (1998) 161.
[10] F. Aharonian et al. [H.E.S.S. Collaboration], Phys. Rev. Lett. 97, 221102 (2006) [Erratum-ibid. 97, 249901 (2006)].
[11] G. Zaharijas and D. Hooper, Phys. Rev. D 73, 103501 (2006); M. Gustafsson, E. Lundstrom, L. Bergstrom and J. Edsjo, Phys. Rev. Lett. 99, 041301 (2007).