The effect of boundary conditions on mixing of 2D Potts models at discontinuous phase transitions.

Summary: We study Swendsen-Wang dynamics for the critical $q$-state Potts model on the square lattice. For $q = 2, 3, 4$, where the phase transition is continuous, the mixing time $t_{\text{mix}}$ is expected to obey a universal power-law independent of the boundary conditions. On the other hand, for large $q$, where the phase transition is discontinuous, the authors recently showed that $t_{\text{mix}}$ is highly sensitive to boundary conditions: $t_{\text{mix}} \geq \exp(cn)$ on an $n \times n$ box with periodic boundary, yet under free or monochromatic boundary conditions, $t_{\text{mix}} \leq \exp(n^{o(1)})$.

In this work we classify this effect under boundary conditions that interpolate between these two (torus vs. free/monochromatic). Specifically, if one of the $q$ colors is red, mixed boundary conditions such as red-free-red-free on the 4 sides of the box induce $t_{\text{mix}} \geq \exp(cn)$, yet Dobrushin boundary conditions such as red-red-free-free, as well as red-periodic-red-periodic, induce sub-exponential mixing.

MSC:
60K35 Interacting random processes; statistical mechanics type models; percolation theory
82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics
82B27 Critical phenomena in equilibrium statistical mechanics
82C20 Dynamic lattice systems (kinetic Ising, etc.) and systems on graphs in time-dependent statistical mechanics

Keywords:
Potts model; Swendsen-Wang; mixing times; surface tension; boundary conditions
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