ABSTRACT

Surveys are an important tool for many areas of social science research, but privacy concerns can complicate the collection and analysis of survey data. Differentially private analyses of survey data can address these concerns, but at the cost of accuracy—especially for high-dimensional statistics. We present a novel privacy mechanism, the Tabular-DDP Mechanism, designed for high-dimensional statistics with incomplete correlation. The Tabular-DDP Mechanism satisfies dependent differential privacy, a variant of Pufferfish privacy; it works by building a causal model of the sensitive data, then calibrating noise to the level of correlation between statistics. An empirical evaluation on survey data shows that the Tabular-DDP Mechanism can significantly improve accuracy over the Laplace mechanism.

1 INTRODUCTION

Survey data remains an important part of research in many different areas, including political science [17]. Many survey questions are about political or otherwise personal beliefs or intentions, and individuals will rightfully be concerned if their responses may be made public. This concern even has the potential to reduce participation, which may bias the survey results. To address this problem, survey researchers typically keep their datasets secret in order to protect the privacy of respondents, and take additional steps to protect privacy when revealing aggregate results. These practices make it difficult to share survey data with other researchers, and in spite of the steps taken to protect privacy, respondents often remain concerned about the privacy of their responses.

Differential privacy [4, 5] is a strong formal definition of individual privacy, and it has been previously applied to survey data to protect the privacy of respondents [7]. Differential privacy works by adding noise to results destined for public release. Releasing more results requires adding more noise, because of the potential for correlation between results to reveal more information about a respondent than any single result does on its own. In differential privacy, this principle is called sequential composition.

For survey researchers, sequential composition means that the error in the differentially private statistics they release increases with the number of statistics. For summary statistics about per-question responses, the error can grow large for long surveys with many questions.

We propose a novel mechanism for releasing differentially private statistics, the Tabular-DDP Mechanism, that can significantly improve error for releases of multiple statistics—including summary statistics about survey results. The key insight of the Tabular-DDP Mechanism is that a single respondent’s answers to different survey questions are not necessarily 100% correlated, so the amount of noise required to use sequential composition is larger than necessary.

The Tabular-DDP Mechanism works by building an approximate causal model of the distribution underlying the collected survey data, then using the model to estimate correlations between statistics in the desired data release. The mechanism leverages incomplete correlations (and independence) to reduce the amount of noise required, based on a relaxed privacy definition called dependent differential privacy [12].

In this paper, we formalize the Tabular-DDP Mechanism and prove that it satisfies dependent differential privacy. Then, we apply the Tabular-DDP Mechanism to real-world survey data from the American National Election Studies (ANES). We conduct an empirical evaluation of the accuracy of the Tabular-DDP Mechanism; the results suggest that the Tabular-DDP Mechanism can improve accuracy for summary statistics for this kind of survey data by several times in comparison to the standard Laplace mechanism (with sequential composition).

Contributions. We make the following contributions:

- We initiate the study of optimal mechanisms for differentially private summary statistics for survey results, based on the insight that responses are not completely correlated
- We define the Tabular-DDP Mechanism, a novel dependent differential privacy mechanism designed for incompletely-correlated high-dimensional statistics
- We evaluate the Tabular-DDP Mechanism experimentally using real survey data to demonstrate its accuracy benefit

2 BACKGROUND

2.1 Survey Data

The motivating use case for our work is privacy in survey data. Such data is collected by posing survey questions like the examples in Figure 1 to individuals, and aggregating and analyzing the responses. To protect privacy, the responses themselves are typically kept secret; even summary statistics about the responses are often not released publicly, because they could potentially reveal information about individual respondents.

The standard approach for protecting privacy in survey data is de-identification: the removal of personally identifiable information (PII) like names and phone numbers [2, 14] before sharing the data. However, de-identification approaches do not always fully protect privacy: they are frequently subject to re-identification attacks [8], which recover the removed
ANES 2006 Survey. The survey has a total of 72 questions, then releasing the results of both mechanisms satisfies the database, so the privacy guarantee obtained in practice. The standard approach [5] is to assume that each individual contributes exactly one row to the database, so the distance between two databases is equal to the number of rows on which they differ. When one individual may contribute multiple rows, a different distance metric must be used to ensure privacy.

To achieve differential privacy, we can add noise as prescribed by one of several basic mechanisms. The two most commonly-used mechanisms are the Laplace mechanism, which ensures pure \(\epsilon\)-differential privacy, and the Gaussian mechanism, which ensures \((\epsilon, \delta)\)-differential privacy. In both cases, the scale of the noise is determined by the query’s sensitivity, which measures the influence of a single individual’s data on the query’s output. The \(L1\) sensitivity of a function \(f : \mathcal{D} \rightarrow \mathbb{R}^k\) is defined as follows, where \(d\) is a distance metric on databases:

\[
\Delta_1 f = \max_{D, D'} \| f(D) - f(D') \|_1
\]

The \(L2\) sensitivity \(\Delta_2 f\) is defined the same way, but with the \(L2\) norm instead of the \(L1\) norm.

**Theorem 1 (The Laplace Mechanism).** Given a numeric query \(f : \mathcal{D} \rightarrow \mathbb{R}^k\), the Laplace mechanism adds to the query answer \(f(D)\) with a vector \((\eta_1, \ldots, \eta_k)\), where \(\eta_i\) are i.i.d. random variables drawn from the Laplace distribution centred at 0 with scale \(b = \Delta_1 f / \epsilon\), denoted by \(\text{Lap}(b)\). The Laplace mechanism preserves \((\epsilon, 0)\)-differential privacy.

### 2.3 Dependent Differential Privacy

Sometimes, correlations may exist between individuals that allow an adversary to make inferences about one individual based on the data of another. Consider, for example, a dataset of GPS locations that includes members of a chess club. If the chess club meets at 3pm on Thursdays, then the locations of the club’s members at that time will be highly correlated with one another! The adversary may be able to learn the most popular location of chess club members during the meeting time, and then infer, based on their belief about correlations in the data, that an individual chess club member is highly likely to have been at the popular location. In this case, the correlation in the data enabled the inference: absent the knowledge that chess club members are likely to be in the same location during the meeting time, the adversary would not be able to make the inference.

Importantly, differential privacy does not promise to prevent this inference. Arguably, it is not a privacy violation at all. However, in some cases such inferences are highly likely to reveal information that may prove harmful, so a significant body of work has investigated ways of refining the definition of differential privacy to account for this risk [10–13, 16, 20].

The most important for our setting is dependent differential privacy, due to Liu et al. [12]. Dependent differential privacy can be seen as a strengthening of differential privacy, which reduces to differential privacy when no correlations are present in the data. Dependent differential privacy is defined as follows:

**Definition 2 (Dependent Neighboring Databases).** Two databases \(D(L, \mathcal{R})\) and \(D'(L, \mathcal{R})\) are dependent neighboring databases if the modification of a tuple value in database \(D(L, \mathcal{R})\) causes a change in at most \(L - 1\) other tuple values in \(D'(L, \mathcal{R})\) due to the probabilistic dependence relationship \(\mathcal{R}\) between the data tuples.

**Definition 3 (Dependent Differential Privacy).** A randomized mechanism \(M\) satisfies \((\epsilon, \delta)\)-dependent differential privacy if for all pairs of dependent neighboring databases \(D(L, \mathcal{R})\) and \(D'(L, \mathcal{R})\) such that 

1. First, how much do you think people can change the kind of person they are?
   - Completely
   - A lot
   - A moderate amount
   - A little
   - Not at all

2. If you wanted to defend an opinion of yours, how successfully do you think you could do that?
   - Extremely successfully
   - Very successfully
   - Moderately successfully
   - Slightly successfully
   - Not successfully at all

Figure 1: Example questions and responses from the ANES 2006 Survey. The survey has a total of 72 questions.
and $D'(L, R)$ and all possible sets of outcomes $S$:

$$\Pr[M(D(L, R)) \in S] \leq e^\delta \Pr[M(D'(L, R)) \in S] + \delta$$

This definition is designed to capture inferences made on the dependence relationship $R$ while preserving important properties of differential privacy. Like differential privacy, dependent differential privacy is compositional and closed under post-processing.

Liu et al. [12] propose a definition of dependent sensitivity that allows the use of the Laplace mechanism to satisfy dependent differential privacy. Dependent sensitivity is larger when significant correlations in the data could enable inferences like our earlier example, and is equal to $L1$ sensitivity when no correlations exist.

**Definition 4 (Dependent Sensitivity [12]).** The dependent sensitivity of a query $Q$ with $L1$ sensitivity $\Delta Q$ is:

$$DS_Q = \sum_{i \in C_i} p_{ij} \Delta Q$$

Where $p_{ij}$ represents the dependence coefficient between records $i$ and $j$.

## 3 PRIVACY FOR SURVEY DATA

Differential privacy assumes complete correlation between the attributes of a single individual, and so releasing statistics about multiple columns of a tabular dataset requires the use of sequential composition. The key insight of our approach is the observation that complete correlation often does not exist between attributes, so the use of sequential composition provides very loose upper bounds on the actual privacy loss for these statistics.

We propose the use of a dependent differential privacy mechanism for releasing statistics about multiple attributes in tabular data, including survey data. Under valid assumptions about the distribution of the underlying data, dependent differential privacy provides strong privacy protection for participants in the dataset—but with less noise required.

The primary challenges lie in modeling correlations between columns and in efficiently calculating the dependent sensitivity of queries over the data based on these models.

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### 3.1 Example: Monty Hall

As a simple example of our setting, consider the Monty Hall problem. The problem describes a game involving a contestant, a host (Monty Hall), and three doors. One door contains a goat, one contains a prize, and one is empty; the contestant’s goal is to choose the door with the prize. The game proceeds in three steps:

1. The contestant chooses a door (the “First Selection”).
2. Monty opens a door that is neither the “First Selection” nor the door with the prize (revealing either the goat or nothing at all).
3. The contestant is given the opportunity to change their selection to the other non-open door, or keep their first selection.
4. The contestant’s final selection is opened. If the door contains the prize, the contestant wins.

The Bayesian network corresponding to the Monty Hall problem appears in Figure 3. This problem is famous for being counterintuitive—we assume that the event of Monty opening one of the doors does not affect the probability that the contestant has made the right choice, but in fact it does! This effect is encoded in the Bayesian network: which door Monty opens depends on both the location of the prize and the contestant’s first selection.

Imagine we have collected observations of Monty Hall games, as in Figure 2, and we would like to release statistics about these games under differential privacy. We can release histograms for all three attributes summarizing the game outcomes, and add Laplace noise with scale $\frac{1}{\epsilon}$ to each one. By the sequential proposition property of differential privacy, the total privacy cost is $3\epsilon$. Note that it is not possible to use parallel composition in this case, because adding or removing a whole row of data changes the results of all three histograms.

### 3.2 Modeling Correlations

Calculating dependent sensitivity requires the ability to evaluate the probability that an attribute takes a particular value given the values of the other attributes in the same row. We model these correlations using a Bayesian network, in a similar way to previous work [12, 16].

A Bayesian network is a graphical model (directed acyclic graph) that represents conditional dependencies between variables. In our setting, each column of the dataset is represented by a variable in the Bayesian network (i.e. a node in the graph) and the conditional probability table associated with each edge in the graph encodes the conditional dependencies between column values.
3.4 Privacy Considerations

The approach we have outlined raises several important concerns about the real-world privacy we can expect from the guarantee. First, Pufferfish privacy and its variants (including dependent differential privacy) represent weaker guarantees than \( \epsilon \)-differential privacy; in the context of survey data, the weakening of the guarantee is similar to the difference between node- and edge-level privacy in graphs [9]. In our setting:

- \( \epsilon \)-differential privacy protects the presence or absence of an individual in the survey results.
- \( \epsilon \)-dependent differential privacy protects the presence or absence of an answer to a survey question in the survey results.

The difference between these guarantees is significant, and our weaker guarantee may not be applicable in some cases. In cases where survey answers may be sensitive, but participation in the survey is not, the dependent differential privacy guarantee may be appropriate, and enable better utility in the results.

Second, learning a Bayesian network from the sensitive data presents two additional concerns: (1) the model’s structure may reveal properties of the underlying distribution (e.g. enabling attribute inference), and (2) the model’s structure may reveal properties of individual records in the data (enabling inferences about individuals). In our setting, (1) is not a major concern, since the underlying distribution of responses is what we would like to learn.

However, concern (2) is an issue in our setting. It is possible that learning the Bayesian network from the data could reveal information specific to individuals—though in large datasets, this information is likely to be minimal. To alleviate this issue, a differentially private learning algorithm could be used [19].

An additional concern is that the learning process could produce a model that does not actually match the underlying distribution—either because the learning process fails to learn the correct model, or because the data does not represent the underlying distribution very well. In this case—as in other applications of Pufferfish privacy—unexpected privacy failures could occur due to the mismatch between expected and actual correlations in the data.

All of these concerns represent limitations of our approach, and are important areas for future improvement.

4 DEPENDENT SENSITIVITY FOR TABULAR DATA

This section describes the Tabular-DDP Mechanism, formalized in Algorithm 1, which adapts the dependent sensitivity approach of Liu et al. [12] to the setting of multi-attribute tabular data.

Transforming the data. We adopt the definition of dependent sensitivity from Liu et al., as defined earlier. To scale noise to dependent sensitivity, we need the data to be represented in the form \( X = \{X_1, \ldots, X_n\} \), where we assume that the attributes of each \( X_i \) may be completely dependent on one another, and there may additionally be correlations between two tuples \( X_i \) and \( X_j \).

To fit these assumptions, we transform the tabular representation of our data table into a single-column table, as shown in Figure 2, by concatenating the columns. After this transformation, each tuple has only a single attribute, and the domain of that attribute is the product of the table’s original attributes.
The transformed data fits the assumptions of dependent sensitivity. In the new representation, which has only a single column, the Bayesian network in Figure 3 encodes correlations between rows rather than columns, as expected for dependent sensitivity.

**Calibrating noise to dependent sensitivity.** With the transformed data, it is possible to apply the mechanisms of Liu et al. directly:

1. Transform the tabular data to a single-column representation.
2. Add Laplace noise to the results of querying the transformed data, scaled to the dependent sensitivity of the query.

Next, we introduce a slight modification to the mechanism that avoids the need for explicit transformation of the data.

**The Tabular-DDP Mechanism.** The Tabular-DDP Mechanism, defined in Algorithm 1, simulates the process described above, and scales the additive Laplace noise to the effective dependent sensitivity of applying a query to multiple attributes of a tabular dataset in parallel.

First, the mechanism splits the dataset into chunks column-wise (line 1), to make the modeling task computationally tractable. Next, for each chunk, the mechanism learns a Bayesian network encoding the causal relationships in the data (line 3). The LearnNetwork function refers to an off-the-shelf tool for learning the network and returning a representation containing the conditional probability table, as described earlier (Section 3.2). The larger the number of columns \( k \) in each chunk, the more computationally challenging this task is. Then, the mechanism computes the effective dependent sensitivity by summing the dependence coefficients for all attributes in the table (line 5). Here, the mechanism uses the conditional probability table in the learned Bayesian network to calculate the probability ratio:

\[
\frac{\Pr[X_j = d_j | X_i = d_i]}{\Pr[X_j = d_j | X_i = d_i]}
\]

Finally, the mechanism releases the result of running the query \( Q \) and adding Laplace noise scaled to the dependent sensitivity (line 8). Like the process defined above, the Tabular-DDP Mechanism satisfies \( \epsilon \)-dependent differential privacy, as long as the learned Bayesian network accurately represents the underlying data distribution. For each column-wise chunk of the dataset, the mechanism satisfies \( \frac{k}{\epsilon} \) \( \epsilon \)-dependent differential privacy, for a total privacy cost bounded by \( \epsilon \)-dependent differential privacy by sequential composition.

**Privacy.** To prove privacy for the Tabular-DDP Mechanism, we will view the dataset implicitly in the single-column representation described above (with correlations between tuples, rather than columns) and leverage the privacy result of Liu et al. [12]:

**Lemma 2 (Liu et al. [12, Theorem 8]).** The dependent sensitivity for publishing any query \( Q \) over a dependent (correlated) dataset is

\[
DS^Q = \max_i DS_i^Q
\]

Here, \( i \) refers to a tuple index, and \( DS_j^Q = \sum_i \rho_{i,j} \Delta Q_j \) is the dependent sensitivity for the \( i \)th tuple. If we can show that the Tabular-DDP Mechanism correctly calculates \( DS^Q \) and adds Laplace noise scaled to that sensitivity, then it follows that the Tabular-DDP Mechanism satisfies dependent differential privacy.

**Theorem 3.** If the learned Bayesian network \((V, E, P)\) accurately represents the underlying distribution of the dataset \( D \), then the Tabular-DDP mechanism (Algorithm 1) satisfies \( \epsilon \)-dependent differential privacy.

**Proof.** We show that Algorithm 1 satisfies \( \frac{k\epsilon}{n} \)-dependent differential privacy for each chunk of columns. By sequential composition, if there are at most \( \frac{n}{k} \) chunks, then the mechanism has a total privacy cost of \( \epsilon \)-dependent differential privacy. For each chunk of columns, we have the following for the sensitivity calculated by Algorithm 1, leveraging the fact that our counting queries have sensitivity \( \Delta Q_j = 1\):

\[
DS = \sum_{i,j} \rho_{i,j} \\
\geq \max_j \sum_i \rho_{i,j} \Delta Q_j \\
= DS^Q
\]

By Lemma 2, noise scaled to \( DS^Q \) will satisfy \( \epsilon \)-dependent differential privacy. Algorithm 1 adds Laplace noise scaled to:

\[
\frac{nDS}{k\epsilon}
\]

which satisfies \( \frac{k\epsilon}{n} \)-dependent differential privacy, as required.

\(\square\)
Utility. The same accuracy bounds proven by Liu et al. [12] also apply to the Tabular-DDP Mechanism. These results bound the error for any individual column of the statistics returned by the mechanism.

Definition 5 ((\(\alpha, \beta\))-accuracy [5, 12]). A randomization algorithm \(A\) satisfies (\(\alpha, \beta\))-accuracy for a query function \(Q\) if:

\[
\Pr_D[|A(D) - Q(D)| > \alpha] \leq \beta
\]

Lemma 4. The Tabular-DDP Mechanism provides (\(\alpha, \beta\))-accuracy for each column of the dataset \(D\), for \(\beta = \exp\left(-\frac{\alpha^2}{4DS}\right)\).

Proof. Follows directly from Liu et al. [12], Theorem 10.

In addition, we can extend the utility bounds from Liu et al. [12] to bound \(L_1\) error for Tabular-DDP Mechanism. We leverage a result on the sum of Laplace samples from Chan et al. [12] to bound each \(L_1\) accuracy in the same way as (\(\alpha, \beta\)) accuracy, but using the \(L_1\) error.

Lemma 5 (Sum of independent Laplace samples ([1, Lemma 2.8])). Suppose \(y_i\)'s are independent random variables, where each \(y_i\) has Laplace distribution \(Lap(b_i)\). Suppose \(Y := \sum_i y_i\), and \(b_M := \max_i b_i\). Let \(\nu \geq \sqrt{\sum_i b_i^2}\) and \(0 < \lambda < \frac{\sqrt{2\nu^2}}{b_M}\). Then:

\[
\Pr[Y > \lambda] \leq \exp\left(-\frac{\lambda^2}{8\nu^2}\right)
\]

Definition 6 (\(L_1\)-accuracy). A randomization algorithm \(A\) satisfies \(L_1(\alpha, \beta)\)-accuracy for a query function \(Q\) if:

\[
\Pr_D[\max_i|A(D) - Q(D)|_1 > \alpha] \leq \beta
\]

Theorem 6. The Tabular-DDP Mechanism provides \(L_1(\alpha, \beta)\)-accuracy for \(\beta = \exp\left(-\frac{\sqrt{2\nu^2}}{4DS}\right)\).

To prove the accuracy bound, we consider that the \(L_1\) error introduced by the mechanism is a result only of the noise samples added to each result in line 8 of Algorithm 1 (i.e. \(|A(D) - Q(D)|_1\) is exactly equal to the sum of the noise samples added by the mechanism). Each of these noise samples is conditionally independent from the others, so Lemma 5 applies, and gives an upper bound on the \(L_1\) error resulting from the noise.

Proof. Set \(\lambda = \frac{2\nu\sqrt{2\nu^2}}{DS}\) and \(\nu = \sqrt{n\frac{DS}{\alpha^2}}\). By Lemma 5, we have:

\[
\Pr_D[\max_i|A(D) - Q(D)|_1 > \alpha] \leq \exp\left(-\frac{\alpha^2}{8\nu^2}\right) = \exp\left(-\frac{\alpha^2}{8\nu^2}\right) = \exp\left(-\frac{2\nu^2}{4DS}\right) = \exp\left(-\frac{\sqrt{2\nu^2}}{4DS}\right)
\]

Thus the accuracy of Tabular-DDP Mechanism is independent of the dimensionality of the statistic being released, except as encoded in the dependent sensitivity.

Limitations. Our approach has several important limitations. First, as discussed in Section 3.4, the privacy guarantee is strictly weaker than standard \(\epsilon\)-differential privacy, and additional unexpected privacy failures could occur if the learned Bayesian networks do not actually correspond to the underlying population distribution. Second, the Tabular-DDP Mechanism is based on Laplace noise, and uses \(L_1\) sensitivity; for high-dimensional data, if \((\epsilon, \delta)\)-differential privacy is sufficient, the Gaussian mechanism with \(L_2\) sensitivity may produce better accuracy. We hope to extend the Tabular-DDP Mechanism to Gaussian noise with \(L_2\) sensitivity in future work.

5 EVALUATION

Our empirical evaluation seeks to answer two questions:

1. Q1: Accuracy. How does the accuracy of the Tabular-DDP Mechanism compare to the Laplace mechanism?
2. Q2: Scalability. How does the size and dimensionality of the dataset impact the running time of the Tabular-DDP Mechanism?

To answer the first question, we evaluated the accuracy of the Tabular-DDP Mechanism for computing summary statistics for 9 survey datasets released by the American National Election Studies. The results suggest that Tabular-DDP Mechanism can significantly increase accuracy over the Laplace mechanism for these real-world datasets. To answer the second question, we measured running time for each component of Tabular-DDP Mechanism; the results suggest that Tabular-DDP Mechanism scales to realistic datasets, and that the primary scalability challenge comes from learning the Bayesian network from the data.

Datasets. Our datasets were drawn primarily from the American National Election Studies (ANES) database. Each dataset included columns that corresponded to the answers for questions in the metadata datasheet. Questions that were multiple choice were frequently designed by indexed characters (for example, in the dataset ANES 2011, multiple choice question responses are represented as sequential columns ("C3c1", "C3c2", where each possible answer is indicated by a number and choice, or otherwise, for that column "-l. Inapplicable, legitimate skip").), and if there was branch logic for indexed questions, the numeric values were sentinel values.

Methodology. We compared Tabular-DDP Mechanism to the Laplace mechanism, which provides \(\epsilon\)-differential privacy and assumes that attributes in each individual record may be completely correlated with one another. To simulate the computation of summary statistics for each survey, we ran a histogram query on each column of the survey results (i.e. we queried the count of each response category for each question of the survey).

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5.1 Experiment 1: Accuracy

Experiment Setup. Our first experiment examines the accuracy of the Tabular-DDP Mechanism by comparing it to the standard Laplace mechanism. We ran 100 trials for each experiment, and report $L^2$ error. We used $\varepsilon \in \{0.1, 1, 10\}$ for both mechanisms.

Results. The results for $\varepsilon = 0.1$ appear in Figure 5. Additional results for other values of $\varepsilon$ appear in Figure 9 in the Appendix, and are consistent with these. We set $k = 10$ (i.e. 10 columns per “chunk” of the dataset, so that each Bayesian network covers 10 columns). The results show that the Tabular-DDP Mechanism consistently outperforms the Laplace mechanism in terms of accuracy at a given level of privacy.

Figure 7 shows accuracy results for various values of the chunk size $k$. The results suggest that the accuracy advantage of the Tabular-DDP Mechanism over the Laplace mechanism increases as $k$ increases; when $k = 5$, for example, the accuracy advantage of the Tabular-DDP Mechanism is fairly small, and it is much larger when $k = 15$. These results match our expectations about the Tabular-DDP Mechanism: as $k$ increases, the Tabular-DDP Mechanism takes better advantage of the partiality of correlations between attributes.

5.2 Experiment 2: Scalability

Experiment Setup. Our second experiment measures running time of Tabular-DDP Mechanism to determine whether or not it can scale to realistic datasets. We instrumented our implementation to separately measure the running time of (1) learning the Bayesian network from the data, (2) calculating the dependent sensitivity, and (3) generating the noise samples themselves. We ran Tabular-DDP Mechanism on the same datasets and recorded the running time of each component; we performed 5 trials and report the average running time of each component. We set $k = 10$ (i.e. 10 columns per “chunk” of the dataset, so that each Bayesian network covers 10 columns).

Results. The results appear in Figure 8, and suggest that Tabular-DDP Mechanism is capable of scaling to realistic...
datasets like the ANES surveys we considered. The running time for Tabular-DDP Mechanism in this experiment is dominated by the time to calculate dependent sensitivity based on the Bayesian network associated with the target columns. Running time was higher for surveys with more questions (e.g. the ANES 2012 and 2019 surveys, which had more columns than other datasets). For all of the datasets we considered, when $k = 10$, Tabular-DDP Mechanism was able to compute summary statistics for all columns in about 10 seconds or less.

For small values of $k$, the running time is dominated by the time taken to calculate dependent sensitivity. However, as $k$ increases, the model learning time quickly dominates the total time, due to the fundamental scalability challenges of learning models over many attributes. The running time for the ANES 2020 survey increases 10x—from about 10 seconds to over 100 seconds—when $k$ increases from 10 to 15.

5.3 Discussion

Based on the results of our experiments, we answer the original research questions as follows. (1): for the survey data we studied, the accuracy of the Tabular-DDP Mechanism improves on the Laplace mechanism—when $k \geq 10$, the improvement is often 2x or more. (2): the Tabular-DDP Mechanism is slower than the Laplace mechanism, but for $k \leq 10$, it scales easily to realistic survey datasets with hundreds of columns and thousands of responses.

Our experimental results clearly demonstrate the tradeoff between running time and accuracy in the Tabular-DDP Mechanism: accuracy increases with larger values of $k$, but running time also increases (exponentially!). Fortunately, the results suggest that significant accuracy gains can be achieved with small enough values of $k$ that running time is reasonable. More scalable approaches for learning Bayesian networks may allow increasing $k$ further, and thus improving accuracy even more.

6 RELATED WORK

A significant amount of previous work has considered the privacy implications of correlations within sensitive data. The most general framework for formalizing privacy while taking correlations into account is Pufferfish privacy [16], introduced earlier. The Pufferfish framework allows specifying any model of correlations in the underlying population as a probability distribution over possible datasets. Dependent differential privacy [12] can be defined as a particular variant of Pufferfish privacy. Our work builds on these definitions, providing a new mechanism that satisfies dependent differential privacy (and thus, Pufferfish privacy).

Many different mechanisms have been proposed for Pufferfish privacy; most are designed for a specific purpose where the correlations in the underlying data are known ahead of time to the analyst and have a specific structure. Many of these consider temporal correlations—multiple data records contributed by the same individual over time—and model these correlations using Markov chains. Solutions have been proposed for social media settings [16], smart meter data [10, 13], and web browsing data [11]. In contrast to these approaches, the Tabular-DDP Mechanism is designed to learn a general model of the underlying correlations from the data itself.

Recent work by Zhang et al. [20] proposes Pufferfish mechanisms for attribute privacy. This work uses similar techniques to ours, but has a different privacy goal: attribute privacy aims to prevent population-level inferences about attributes of the dataset (for example, the distribution of race and gender in the original dataset). Our work, in contrast, aims to prevent inferences about individuals.

Previous work has explored the application of differential privacy to protect privacy in survey data [3, 6, 7]. This work has focused on ensuring statistical validity and avoiding bias in the inferences made using differentially private statistics. Previous work in this area has applied well-known differential privacy mechanisms like the Laplace mechanism.

7 CONCLUSION

We have presented the Tabular-DDP Mechanism, a novel dependent differential privacy mechanism that can improve accuracy over the standard Laplace mechanism for high-dimensional
statistics that are not completely correlated. We have shown how to apply the Tabular-DDP Mechanism to protect privacy in summary statistics for survey data; our experimental results show a significant improvement in accuracy compared to the standard Laplace mechanism in that setting.

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APPENDIX

See Figure 9 for additional experimental results.
Figure 9: Accuracy results: comparison between the Tabular-DDP mechanism and the Laplace mechanism for $\epsilon \in \{0.1, 1, 10\}$.