Power counting in baryon chiral perturbation theory including vector mesons

Thomas Fuchs,\textsuperscript{1} Matthias R. Schindler,\textsuperscript{1} Jambul Gegelia,\textsuperscript{1,2,*} and Stefan Scherer\textsuperscript{1}
\textsuperscript{1}Institut für Kernphysik, Johannes Gutenberg-Universität, D-55099 Mainz, Germany
\textsuperscript{2}High Energy Physics Institute, Tbilisi State University, University St. 9, 380086 Tbilisi, Georgia

(Dated: August 1, 2003)

Abstract

It is demonstrated that using a suitable renormalization condition one obtains a consistent power counting in manifestly Lorentz-invariant baryon chiral perturbation theory including vector mesons as explicit degrees of freedom.

PACS numbers: 11.10.Gh, 12.39.Fe.

\*Alexander von Humboldt Research Fellow
I. INTRODUCTION

Because of their phenomenological importance, vector mesons were included in low-energy chiral Lagrangians already at an early stage [1, 2, 3]. Usually they were treated—within some approximation—as gauge bosons of local chiral symmetry. Refs. [4] and [5] contain reviews of these approaches including one of a “hidden” local chiral symmetry (see also Ref. [6]). Details of the Lagrangian describing the interaction of vector mesons with mesons and baryons in a chirally invariant way can be found in Refs. [7, 8, 9]. Different formulations of vector meson effective field theories were shown to be equivalent in Ref. [10]. However, to the best of our knowledge the incorporation of (axial) vector mesons into (baryon) chiral perturbation theory [11, 12, 13] remains an important open problem [14] as long as a systematic power counting is not available. In the present paper we show how the approach formulated recently in Ref. [15] is capable to consistently include (axial) vector mesons in the manifestly Lorentz-invariant formulation of the effective field theory of the strong interactions.

The basic idea of Ref. [15] (see also Refs. [16, 17]) can be summarized as follows. If one uses the modified minimal subtraction scheme of (baryon) chiral perturbation theory (\(\tilde{\text{MS}}\)) [13], then the diagrams with an arbitrary number of loops contribute to lower-order calculations. As mentioned in Ref. [13], these contributions lead to a renormalization of the low-energy constants, i.e., they can be absorbed into a redefinition of these constants. The renormalized coupling constants of our extended on-mass-shell (EOMS) scheme correspond to a re-summation of the (infinite) series of loop corrections. The coupling constants and fields of the \(\tilde{\text{MS}}\) scheme are expressed in terms of EOMS quantities. Expanding \(\tilde{\text{MS}}\) quantities in terms of a power series of EOMS-renormalized couplings generates counterterms which precisely cancel the contributions of multi-loop diagrams to lower-order calculations. The EOMS scheme thus leads to a consistent power counting in baryon chiral perturbation theory. The inclusion of (axial) vector mesons into this scheme does not introduce any new complications as long as they appear only as internal lines in Feynman diagrams involving soft external pions and nucleons with small three momenta. We will show this below by means of three select examples.

II. EFFECTIVE LAGRANGIAN AND POWER COUNTING

In our discussion we will make use of the effective Lagrangian (including vector mesons) in the form given by Weinberg [3],

\[
\mathcal{L} = \frac{1}{2} \partial_\mu \pi^a \partial^{\mu} \pi^a - \frac{M^2}{2} \pi^a \pi^a + \bar{\Psi} (i\gamma^\mu \partial_\mu - m) \Psi - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} M^2 A_\mu^a A^{a\mu} \\
+ g e^{abc} \pi^b \partial^{\mu} A_\mu^c + g \bar{\Psi} \gamma^\mu \tau^a \frac{g_A}{2F} \Psi A_\mu^a - \frac{\partial A}{2F} \bar{\Psi} \gamma^\mu \gamma^\nu \tau^a \Psi \partial^{\mu} \pi^a + \mathcal{L}_1,
\]

where \(\pi^a\) and \(A_\mu^a\) are isotriplets of pion and \(\rho\) meson fields with masses \(M\) and \(M_\rho\), respectively, and \(\Psi\) is an isodoublet of nucleon fields with mass \(m\). The constants \(F\) and \(g_A\) denote the chiral limit of the pion decay constant and the axial-vector coupling constant, respectively. Moreover, we use the universal \(\rho\) coupling, i.e., \(g = g_{\rho\pi\pi} = g_{\rho NN}\). The field strengths are defined as \(F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g e^{abc} A_\mu^b A_\nu^c\) and \(\mathcal{L}_1\) contains an infinite number of terms.
Our renormalization scheme is devised so that, after renormalization, a given diagram has a so-called chiral power \(D\) which is determined by applying the following power counting rules. Let \(Q\) collectively stand for the pion mass, a (small) external four momentum of a pion or a (small) external three-momentum of a nucleon. The pion nucleon interaction counts as \(O(Q)\), the \(\rho\) nucleon interaction as \(O(Q^0)\), the \(\pi\pi\rho\) interaction as \(O(Q)\), and the \(\rho\) self-interaction terms as \(O(Q^0)\), respectively. The integration of a loop counts as \(O(Q^n)\) (in \(n\) dimensions). Finally, we count the \(\rho\) meson propagator as \(O(Q^0)\), the nucleon propagator as \(O(Q^0)\), and the pion propagator as \(O(Q^{-2})\), respectively.

III. APPLICATIONS

Let us now consider three characteristic diagrams illustrating our approach. We start with the vector-meson loop contribution to the nucleon self-energy (see Fig. 1):

\[
- i\Sigma_1(\not{p}) = -\frac{g^2}{4} \int \frac{d^nk}{(2\pi)^n} \gamma_\nu \gamma^\alpha \frac{1}{\not{k} - \not{p} - m + i\epsilon} \gamma_\mu \gamma^\alpha \frac{g^{\mu\nu} - k^\mu k^\nu}{k^2 - M_\rho^2 + i\epsilon}.
\]

According to the above power counting \(\Sigma_1\) is assigned the chiral order \(Q^{n-1}\). Calculating the expression of Eq. (2) we obtain:

\[
\Sigma_1 = -\frac{3g^2}{8M_\rho^2 p^2} [AI_\rho(0) + BI_N(0) + CI_\rho N(0, -p)],
\]

where

\[
A = \not{p} \left[ p^2 - m^2 + (2 - n)M_\rho^2 \right],
\]

\[
B = \not{p} \left[ p^2 + m^2 - (2 - n)M_\rho^2 \right] - 2p^2 m,
\]

\[
C = \not{p} \left[ (3 - n)M_\rho^2 (p^2 + m^2) - (2 - n)M_\rho^4 - (p^2 - m^2)^2 \right] + 2(n - 1)M_\rho^2 p^2 m,
\]

and

\[
I_\rho(0) = i \int \frac{d^nk}{(2\pi)^n} \frac{1}{k^2 - M_\rho^2 + i\epsilon},
\]

\[
I_N(0) = i \int \frac{d^nk}{(2\pi)^n} \frac{1}{k^2 - m^2 + i\epsilon},
\]

\[
I_\rho N(0, -p) = i \int \frac{d^nk}{(2\pi)^n} \frac{1}{[k^2 - M_\rho^2 + i\epsilon] [(k - p)^2 - m^2 + i\epsilon]}.
\]

Before renormalization, the expression of Eq. (3) for \(\Sigma_1\) is of order \(Q^0\), i.e., the unrenormalized diagram clearly violates the power counting rules.

To renormalize \(\Sigma_1\) we first apply the \(\tilde{\text{MS}}\) scheme (modified minimal subtraction scheme of chiral perturbation theory) \(^{12, 13}\). To obtain the final renormalized expression we perform additional finite subtractions. For this purpose we expand the coefficients and integrals in Eq. (3) in powers of \(p^2 - m^2\) and \(\not{p} - m\), both counting as \(O(Q)\). The integrals

\(^{1}\) Fermion loops are integrated out, their contributions being included in low-energy constants.
\( I_\rho(0) \) and \( I_N(0) \) do not depend on \( p^2 \); \( I_{\rho N}(0,-p) \) is an analytic function of \( p^2 \) in the vicinity of the point \( p^2 = m^2 \). Hence the expansion of Eq. (4) in powers of \( p^2 - m^2 \) contains only integer powers. The final renormalized expression for \( \Sigma_1 \) is obtained by subtracting from Eq. (3) the terms proportional to \( (p^2 - m^2)^0 \), \( (p^2 - m^2)^1 \), \( (p^2 - m^2)(\not{p} - m) \), and \( (p^2 - m^2)^2 \). As a result of this subtraction we obtain that the renormalized self energy \( \Sigma_1^R \) starts as \( (p^2 - m^2)^3 \) in agreement with power counting as \( n \to 4 \). All counterterms corresponding to the above subtractions are generated by expanding bare quantities of the Lagrangian in terms of renormalized ones.

As the second example we consider the vertex diagram of Fig. 2. Applying Feynman rules, the corresponding expression reads

\[
V^a = \frac{ig^2 g_A}{2F} \gamma^a \int \frac{d^n k}{(2\pi)^n} \frac{1}{(\not{q} - \not{k})\gamma_5} \frac{g^{\mu\nu} - \frac{k^\mu k^\nu}{M^2}}{k^2 - M^2 + i\epsilon} \frac{(2q - k)^\nu}{(q - k)^2 - M^2 + i\epsilon}. \tag{5}
\]

Power counting suggests that this diagram is of order \( Q^3 \). For the sake of simplicity, we only consider Eq. (5) evaluated between on-mass-shell nucleons \( [p^2 = m^2 = (p + q)^2] \), \(3\)

\[
\bar{u}(p + q)V^a u(p) = g^2 g_A \frac{m}{F} \{ A I_{\rho \pi}(0,-q) + B [I_{\rho \pi}(0,-q) - I_{\rho \pi}(0,0)] + C J(112|n + 2) + D J(121|n + 2) + E I_{\rho \pi N}(0,-q,p) + F I_\rho(0) \} \bar{u}(p + q)\gamma_5\tau^a u(p), \tag{6}
\]

where the coefficients \( A, \ldots, F \) are given by

\[
A = -\frac{1}{2} \left( 5 + \frac{q^2 - M^2}{M^2} \right),
B = \frac{M_{\rho}^2 - M^2}{2q^2} \left( 1 + \frac{q^2 - M^2}{M^2} \right),
C = 8\pi(2m^2 - q^2),
D = -8\pi q^2,\]
\[
E = -2q^2, \quad F = \frac{1}{M^2},
\]

and the integrals read

\[
I_{\rho \pi}(0,p) = i \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - M^2 + i\epsilon)[(k + p)^2 - M^2 + i\epsilon]},
I_{\rho \pi N}(0,-q,p) = i \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - M^2 + i\epsilon)[(k - q)^2 - M^2 + i\epsilon][(k + p)^2 - M^2 + i\epsilon]}. \tag{7}
\]

Moreover, we have introduced the auxiliary integral

\[
J(abc|d) = i \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - M^2 + i\epsilon)^a[(k - q)^2 - M^2 + i\epsilon]^b(k^2 + 2p \cdot k + i\epsilon)^c}. \tag{8}
\]

\(^2\) We do not quote the specific values of the subtraction constants, because they are not relevant for our discussion.

\(^3\) Recall that \( \bar{u}(p + q)\gamma_5 u(p) \) counts as \( \mathcal{O}(Q) \).
which, with shifted space-time dimension \( n + 2 \), contributes to Eq. (6) as a result of reducing tensor integrals to scalar ones. An explicit calculation yields

\[
I_{\rho\sigma}(0, -q) = -\frac{1}{(4\pi)^{\frac{n}{2}}} \left( \frac{M^2}{M_{\rho}^2} \right)^{\frac{n}{2} - 2} \sum_{l=0}^{\infty} \frac{1}{l!} \left( \frac{q^2}{M_{\rho}^2} \right)^l \frac{\Gamma(2 - \frac{n}{2} + l) \Gamma(1 + l) \Gamma(1 + l)}{\Gamma(2 + 2l)}
\]

\[
\times F\left(1 + l, 2 - \frac{n}{2} + l; 2 + 2l; 1 - \frac{M^2}{M_{\rho}^2}\right)
\]

\[
= -\frac{1}{(4\pi)^{\frac{n}{2}}} \left( \frac{M^2}{M_{\rho}^2} \right)^{\frac{n}{2} - 2} \Gamma(2 - \frac{n}{2}) - \frac{1}{16\pi^2} \frac{1}{8\pi^2 M_{\rho}^2} \ln \left( \frac{M}{M_{\rho}} \right) - \frac{q^2}{32\pi^2 M_{\rho}^2}
\]

\[+ \mathcal{O}(Q^4), \quad (9)\]

where \( F(a, b; c; z) \) is the hypergeometric function \[19\] and \( Q \) stands for either \( q \) or \( M \).

Again, we find that the unrenormalized diagram violates the power counting. We renormalize Eq. (6) by first subtracting all ultraviolet divergences using the \( \overline{\text{MS}} \) scheme which amounts to dropping all terms proportional to \( \bar{\lambda} \) [see Eqs. (10) and (12) below].

To determine the additional finite subtractions, we expand all coefficients and integrals in powers of \( M^2 \) and \( q^2 \). The integrals contain non-analytic parts which are proportional to non-integer powers of \( M^2 \) and/or \( q^2 \) for non-integer \( n \). These non-analytic contributions separately satisfy the power counting. We only expand the analytic parts and obtain the final renormalized expression of the diagram by subtracting all terms of the above expansion which are of order \( Q^2 \) or less.

The terms which need to be subtracted read

\[
\tilde{u}(p + q)V^{a}_{\text{subtr}} u(p) = g^2 g_{\Lambda} \frac{m}{2F} \left\{ \frac{5(M_{\rho}^2)^{\frac{n}{2} - 2}}{(4\pi)^{\frac{n}{2}}} \Gamma(2 - \frac{n}{2}) + \frac{5}{16\pi^2} \frac{1}{32\pi^2} \right. \]

\[
+ 32\pi m^2 J_0(112|n + 2) + \frac{2}{M_{\rho}^2} I_{\rho}(0) - 2 \frac{q^2 - M^2}{M_{\rho}^2} \bar{\lambda} \bigg\} \tilde{u}(p + q)\gamma_5 \pi^a u(p), \quad (10)\]

where

\[
J_0(abc|d) = i \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2 - M_{\rho}^2 + i\epsilon)^a(k^2 + i\epsilon)^b(k^2 + 2p\cdot k + i\epsilon)^c}, \quad (11)\]

and

\[
\bar{\lambda} = \frac{m^{n-4}}{16\pi^2} \left\{ \frac{1}{n - 4} - \frac{1}{2} [\ln(4\pi) + \Gamma'(1) + 1] \right\}. \quad (12)\]

Taking into account that

\[
J(112|n + 2) - J_0(112|n + 2) \sim Q^2,
\]

it is now straightforward to check that the difference of Eq. (6) and Eq. (10) satisfies the power counting, i.e., is of order \( Q^3 \) as \( n \to 4 \).

As a final example, we discuss the one-loop diagram of the pion self energy given in Fig. 3. The corresponding expression reads

\[
- i \Sigma^{ab}_\pi(p) = - i \Sigma_\pi(p)\delta^{ab} = - g^2 \epsilon^{acde} \epsilon^{bcd} \int \frac{d^nk}{(2\pi)^n} \frac{(2p + k)^\mu(2p + k)^\nu}{[(k + p)^2 - M^2 + i\epsilon][k^2 - M_{\rho}^2 + i\epsilon]}, \quad (13)\]
from which we obtain
\[
\Sigma_\pi(p) = -2g^2 \left\{ \left(1 + \frac{p^2 - M^2}{M_\rho^2}\right) I_\rho(0) + \left[ 2M^2 + 2p^2 - M_\rho^2 - \frac{(p^2 - M^2)^2}{M_\rho^2} \right] I_\rho\pi(0, p) - I_\pi(0) \right\},
\]
where
\[
I_\pi(0) = - \frac{M^2 (M_\rho^2)^{\frac{n}{2} - 2} \Gamma(2 - n/2)}{(4\pi)^{\frac{n}{2}}} - \frac{M^2}{16\pi^2} + \frac{M^2}{8\pi^2} \ln \left( \frac{M}{M_\rho} \right),
\]
and \(I_\rho\pi(0, p)\) is given in Eq. (9).

To renormalize the pion self energy we first apply the \(\overline{\text{MS}}\) scheme. The additional finite subtraction counterterms are obtained by expanding the coefficients and the analytic parts of the integrals in Eq. (14) and identifying those terms which are of lower order than suggested by power counting, i.e., order \(Q^4\). Note that the last term in Eq. (14), which is non-analytic in \(M\), will give a contribution to Eq. (14) which, if taken separately, violates the power counting. It cannot be removed by counterterms in the Lagrangian, but exactly cancels with an analogous contribution coming from the \(I_\rho\pi(0, p)\) term in Eq. (14). We arrive at the following renormalized expression:
\[
\Sigma^R_\pi(p) = -2g^2 \left\{ \left(2M^2 + 2p^2 - M_\rho^2\right) I_\rho(0, p) + \frac{1}{16\pi^2} + \frac{(M_\rho^2)^{\frac{n}{2} - 2} \Gamma\left(2 - \frac{n}{2}\right)}{(4\pi)^{n/2}} \right\}
\]
\[- \frac{(p^2 - M^2)^2}{M_\rho^2} \left[ I_\rho\pi(0, p) - 2\lambda \right] - \frac{p^2}{32\pi^2} - \frac{M^2}{8\pi^2} \ln \left( \frac{M}{M_\rho} \right) \right\}. \tag{16}
\]
Using Eq. (9) we see that Eq. (16) satisfies the power counting, i.e., is of order \(Q^4\).

Two- (and multi-) loop diagrams have a more complicated structure, but the outcome remains the same. Those terms which are non-analytic in small expansion parameters satisfy the systematic power counting after subtracting the one-loop-order sub-diagrams. The contributions which violate the power counting are analytic in small expansion parameters and are subtracted by a finite number of local counterterms in the Lagrangian.

**IV. SUMMARY AND CONCLUSIONS**

We have demonstrated that the inclusion of explicit degrees of freedom corresponding to (axial) vector particles in manifestly Lorentz-invariant baryon chiral perturbation theory does not violate the power counting if a suitable renormalization condition is used. As an important test of our method it is now necessary to apply a full calculation to physical processes such as, e.g., the determination of the nucleon electromagnetic form factors \[20\], where a one-loop calculation in ordinary baryon chiral perturbation theory does not show a satisfactory agreement with data beyond very small values of \(Q^2 \approx 0.1 \text{ GeV}^2\) \[21, 22\].
Acknowledgments

The work of T.F. and S.S. was supported by the Deutsche Forschungsgemeinschaft (SFB 443). J.G. acknowledges the support of the Alexander von Humboldt Foundation.

[1] J. S. Schwinger, Phys. Lett. B 24, 473 (1967).
[2] J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967).
[3] S. Weinberg, Phys. Rev. 166, 1568 (1968).
[4] M. Bando, T. Kugo, and K. Yamawaki, Phys. Rept. 164, 217 (1988).
[5] U.-G. Meißner, Phys. Rept. 161, 213 (1988).
[6] M. Harada and K. Yamawaki, Phys. Rept. 381, 1 (2003).
[7] G. Ecker, J. Gasser, H. Leutwyler, A. Pich, and E. de Rafael, Phys. Lett. B 223, 425 (1989).
[8] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B 321, 311 (1989).
[9] B. Borasoy and U.-G. Meißner, Int. J. Mod. Phys. A 11, 5183 (1996).
[10] M. C. Birse, Z. Phys. A 355, 231 (1996).
[11] S. Weinberg, Physica A 96, 327 (1979).
[12] J. Gasser and H. Leutwyler, Annals Phys. 158, 142 (1984).
[13] J. Gasser, M. E. Sainio, and A. Svarc, Nucl. Phys. B307, 779 (1988).
[14] U.-G. Meißner, PiN Newslett. 16, 1 (2002).
[15] T. Fuchs, J. Gegelia, G. Japaridze, and S. Scherer, [arXiv:hep-ph/0302117] to appear in Phys. Rev. D.
[16] J. Gegelia and G. Japaridze, Phys. Rev. D 60, 114038 (1999).
[17] J. Gegelia, G. Japaridze, and X. Q. Wang, [arXiv:hep-ph/9910260]
[18] S. Weinberg, Nucl. Phys. B363, 3 (1991).
[19] M. Abramowitz and I. A. Stegun (Eds.), Handbook of Mathematical Functions (Dover, New York, 1972).
[20] M. R. Schindler, J. Gegelia, and S. Scherer, in preparation.
[21] B. Kubis and U. G. Meißner, Nucl. Phys. A679, 698 (2001).
[22] T. Fuchs, J. Gegelia, and S. Scherer, [arXiv:nucl-th/0305070]
FIG. 1: One-loop contribution to the nucleon self energy due to $\rho$ meson dressing.

FIG. 2: One-loop contribution to the $\pi NN$ vertex including an internal $\rho$ meson.

FIG. 3: Pion self energy diagram with $\rho$ meson dressing.