How can GeoGebra support physics education in upper-secondary school—a review

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Abstract
Recently, GeoGebra, a mathematics education software, has entered the scene of physics education; however, research on how the software can be used to support teaching and learning physics is limited and scattered. The aim of this article is to present a review of the current literature on how GeoGebra can be used to support physics education in upper-secondary schools. The general conclusion that comes from these studies is that GeoGebra is a user-friendly software that can be operated intuitively by teachers and students. It provides an environment in which the underlying mathematical structures are always at hand, enabling users to see connections between physical phenomena and their formal representations. In addition, teachers with or without programming skills can use the software to design custom-made computer simulations and augment real experiments with virtual objects. Our intention is to help teachers who would like to start using GeoGebra or to broaden the use of the software in physics education.

Keywords: physics education, GeoGebra, review

1. Introduction
Educational researchers have long explored how teaching and learning physics can be supported by the use of computers. A variety of educational software, including simulations, has been developed in recent decades, and many of them have been continuously refined and supported [1]. One such software, GeoGebra, is an open-source mathematics software which has recently received increasing interest in physics education [2]. Compared to prebuilt learning environments, such as PhET Interactive Simulations (https://phet.colorado.edu), GeoGebra enables teachers and students without in-depth programming
knowledge to create their own computer simulations [2–5]. While PhET simulations are based on educational research, less work has been done to inform the use of GeoGebra in physics education.

In order to contribute to teachers’ acquaintance with recent development in digital technology [6], the aim of this study is to present a review of the current literature on how GeoGebra can be used to support physics education in upper-secondary school. Our intention is to provide a collection of research-based educational materials and to discuss features of GeoGebra that have been acknowledged as useful tools for teaching and learning physics.

2. Background to GeoGebra

2.1. Basic facts

GeoGebra is an educational software developed by Austrian Markus Hohenwarter in 2001/2002 [7]. Although GeoGebra was originally designed for mathematics education in secondary schools, it now has users in both higher and lower mathematics and science education.

GeoGebra is an open-source software, freely available from www.geogebra.org. It works on many operating systems such as Windows, MacOS, Linux, iOS, Android and also from a web browser. GeoGebra is multilingual (more than 70 languages) in its menus and its commands.

The full version of the software was GeoGebra Classic. Other GeoGebra applications, such as the Graphing Calculator and 3D Calculator, offer a limited number of features.

There is a large community of GeoGebra users, many of which upload educational materials, such as simulations, to the software website (www.geogebra.org/materials).

2.2. How does it work?

The default GeoGebra Classic view consists of an Algebra Window, a Graphics Window and a Tool Bar, but any other view can be added if necessary. Mathematical objects (e.g. points, vectors, segments, lines, 2D and 3D objects, or diagrams) can be constructed either in the Algebra Window or in the Graphics Window and are displayed automatically in both of them. These representations are connected dynamically so that users can go back and forth between them and automatically generate responses on their actions. For example, in figure 1, two points (A and B) and a line (f) through these points are represented geometrically in the Graphics Window, while the points’ coordinates and the line’s equation are displayed in the Algebra Window. When the user changes the $x$ coordinate of A in the Algebra Window, the position of A and the position of the line will simultaneously change in the Graphics Window, as well as the line’s equation in the Algebra Window.

Images and text can easily be inserted into the Graphics Window, where the user can determine the length, area, or angle of an object. Another useful tool is the slider, which is a graphical representation of a free number or free angle. The slider enables users to dynamically vary the value of a parameter with a cursor. For example, these features can be used to determine the equation of the function describing the trajectory of a basketball (figure 2). Because of the dynamic link between algebraic and graphic representations, it is possible, by dragging the sliders, to examine how different values of the parameters $a$, $h$ and $k$ affect the corresponding path of the basketball.
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The 3D features in GeoGebra enable users to draw 3D graphs and create 3D geometric constructions. With augmented reality (AR) enabled, users can place mathematical objects on any surface and walk around them. GeoGebra can also be used to create simulations without extensions such as Flash or Java. Users do not need any programming skills to create simulations in GeoGebra, as long as they understand the mathematics behind potential simulations.

3. Research design
For the review, we decided to search for and select scientific articles and conference proceedings on the use of GeoGebra in physics education published in English using a methodological approach based on the work of Petticrew and Roberts [8]. We restricted our search to papers published between 2011 and 2021. The research studies were identified using the following two-step procedure.
(a) We ran the first search in the following databases: Education Resources Information Centre (ERIC), Scopus, and Web of Science, using the combination of two keywords: GeoGebra AND ‘physics education’. From this combination, we had an outcome of 17 articles from 2011 to 2021. We defined various inclusion and exclusion criteria to assist in the identification of appropriate studies (table 1). Through this procedure, we identified 13 papers for this review, that is, studies on how GeoGebra can be used in upper secondary school physics.
(b) We extended our search to Google Scholar using the same time interval and the same combination of keywords. From this database, we had an outcome of more than 500 articles. Using the same inclusion and exclusion criteria, an additional 21 papers were selected. In total, the review comprises 34 papers.

4. Results and discussion
4.1. Current literature on the use of GeoGebra to support teaching of a specific physics content
All the papers selected for this review are presented in table 2, where they are classified according to their physics content. In addition, the papers that offer step-by-step instructions on how to create the provided material are marked with an asterisk. These instructions can be valuable for teachers who wish to re-create or modify the educational material provided.
Table 1. Inclusion and exclusion criteria of the literature review process.

| Inclusion criteria                                                                 | Exclusion criteria                                                                 |
|-----------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| • Physics content appropriate for upper-secondary school                           | • Papers on GeoGebra not related to physics education                                |
| • Qualitative, quantitative and mixed studies of students’ use of GeoGebra         | • Theoretical and philosophical publications, that do not consider the use of GeoGebra in physics courses |
| • Experience from GeoGebra-based educational material in classroom learning, online learning or mixed learning | • Papers that do not provide information about where to find or how to re-create the used educational material |
| • Experience from GeoGebra-based educational material in training of physics teachers |                                                                                     |
| • English language                                                                |                                                                                     |
| • Scientific articles or conference proceedings                                    |                                                                                     |
| • Articles from 2011 to 2021                                                      |                                                                                     |

Table 2. Scientific papers from 2011 to 2021 on how GeoGebra can be used to support physics education.

| Content                                      | Article                                                                 |
|----------------------------------------------|------------------------------------------------------------------------|
| MECHANICS (projectile motion, free fall, inclined plane, oscillatory motion, pendulum, circular motion, collision, friction) | [4, 9–13], [14–17], [18–23], [24] |
| WAVES (mechanical waves, Young’s double slit experiment)                               | [25, 26] |
| GEOMETRICAL OPTICS (mirrors, reflection, refraction)                                     | [5], [13, 23, 27], [28, 29], [30–32] |
| ELECTRO-MAGNETISM (electric charge in a uniform magnetic field)                           | [23, 29] |
| ASTRONOMY (planetary motion)                                                             | [13], [14], [33–35], [36] |
| QUANTUM PHYSICS                                                                         | [3, 37–39] |

Two characteristics can be observed in the selected papers: (a) most papers are developmental papers; they contain detailed descriptions of educational material specifically designed for teaching a specific physics content, but little information about the outcome of the use of the material in teaching, and (b) the number of empirical research studies is limited. Within the first category of papers, the provided educational material is designed with the defined purpose of exemplifying how various GeoGebra features can be used to support teaching of a specific physics content. The texts mainly describe how to create simulations, how to involve students in the process of designing simulations, or how real experiments can be augmented by using GeoGebra.

4.2. Descriptions of how to use GeoGebra in physics education

In this section, we provide an overview of papers that describe how GeoGebra simulations can be designed for use in physics education, how the software can be used to augment recordings of physical experiments, and how students can be engaged in modelling physical phenomena with GeoGebra.

4.2.1. Designing simulations of physical phenomena. Based on his experience with the software, Walsh [4] offers guidelines on how to create a simulation of projectile motion (figure 3), and how to upload it on GeoGebra’s website.
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Figure 3. GeoGebra simulation of simple projectile motion retrieved from [4], and available at www.geogebra.org/u/tomwalsh, with the permission of the American Association of Physics Teachers. Available at doi.org/10.1119/1.4981047.

Similar with Walsh, Kolár [13] describes the basic applications of GeoGebra. He introduces yet another handy feature of GeoGebra, namely tracing, where a point leaves coloured dots on the Graphics Window as it is changing its position in time. Kolár also presents applets that he designed using GeoGebra, and he gives suggestions about how the applets can be adopted in physics class. These applets can be found at ggbm.at/WcgpxH2e.

In addition, problems dealing with repetitive numerical computation and graphical 2D representation can be solved in GeoGebra. For example, Hasek [36] shows how to simulate the motion of a planet around the Sun using Feynman’s method of numerical analysis (figure 4) which is made possible due to simultaneous interconnection between algebraic, geometric and numeric representations.

The possibility of connecting geometric and numerical representations enabled Hruby and Vesenka [11] to design a GeoGebra applet to represent the one-dimensional momentum conservation of two colliding objects. The momentum is represented both as a vector and as the area of a rectangle whose width is the mass of the object and whose height is the velocity of the object (figure 5). The total momentum, represented as the vector sum of momenta, shows that the total momentum remains constant but is ambiguous about the velocity changes of the objects. The area representation shows that the velocities differ depending on collision type, providing a better conceptual understanding of the conservation of momentum in terms of final speeds and directions [11].

4.2.2. Augmenting real experiments with representations of physical concepts. Teichrew and Erb [23] present how pictures or video recordings of physical experiments can be augmented by virtual objects, such as force arrows and light rays, constructed with GeoGebra. If a computer or smartphone has a camera, such virtual objects can
Figure 4. GeoGebra simulation of the motion of a planet around the Sun using Feynman’s method of numerical analysis retrieved from [36]. CC BY 4.0.

Figure 5. GeoGebra simulation of momentum conservation of two colliding objects retrieved from [11], available at www.geogebra.org/m/M6NSNRBk. CC BY 4.0.
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Figure 6. Motion of an object on an inclined plane augmented by a dynamic model constructed in GeoGebra, available at www.geogebra.org/m/pafx6xfu#material/qhb4yeht. Created with GeoGebra.

be opened with GeoGebra 3D Calculator simply by pressing the tool’s AR button. For example, the motion of an object on an inclined plane can be augmented by a dynamic GeoGebra model of the resulting force (figure 6). The model displays the resulting force as the vector sum of the gravitational force and the normal force. The mass of the cart and the angle of inclination can be modified to correspond to the real setup (figure 7).

4.2.3. Involving students in the process of modelling physical phenomena. Some of the studies describe how students can be engaged in the construction of GeoGebra simulations. In a study conducted by Marciuc et al [15] ninth-grade students were encouraged to create several simulations of uniform accelerated motion, such as motion on an inclined plane (figure 8). The commands for creating these representations (table 3) are also included in the paper and can be used as inspiration for developing activities where students are involved in the process of modelling physical phenomena in a quantitative way.

Similarly, in the context of teaching geometrical optics to students without prior knowledge of trigonometry, Santana et al [32] created supervised activities where students were instructed to construct geometrical objects (lines or segments) in specified ways to create and analyse ray diagrams for image formation in lenses, mirrors and prisms (figure 9). A detailed description of the activities supplied in this article can be used to develop similar materials.

Involving students in the process of modelling physical phenomena might be a starting point for discussions about the limitations of computer models. This is similar to what has been suggested in exercises with Algodoo [40], a software in which students can create simulations by using simple drawing tools such as boxes, circles, polygons, ropes, or chains. However, in comparison with Algodoo, where the underlying mathematical architecture is less accessible, GeoGebra explicitly shows the physical and mathematical modelling. The dynamic connection between the two construction forms might strengthen students’ understanding of the role of mathematical modelling in physics.
Figure 7. GeoGebra model of the resulting force constructed obtained by adding the force arrows of gravitational force and normal force, available at www.geogebra.org/m/pafx6xfu#material/qhb4yeht. Created with GeoGebra.

Figure 8. GeoGebra simulation of the sliding motion on an inclined plane retrieved from [15], with the permission of the ADL ROMANIA.
Table 3. GeoGebra commands for modelling of sliding motion on an inclined plane retrieved from [15].

| Commands | Results |
|----------|---------|
| $\alpha = 25^\circ$; $v_0 = 1.6; g = 9.8; L = 7; t = 0; O = (0,0)$ | Defining system parameters |
| $a = g \cdot \sin(\alpha)$ | Defining acceleration |
| $d = V_0 \cdot t + \frac{1}{2} \cdot a \cdot t^2$ | The displacement at time $t$ |
| $t_{nf} = \frac{(-V_0 + \sqrt{2aL + V_0^2})}{a}$ | The time required to cover the distance $L$ |
| $M = ((L - d) \cdot \cos(\alpha),(L - d) \cdot \sin(\alpha))$ | The position of the body at time $t$ |

Figure 9. GeoGebra representation of the image formation through two thin biconvex lenses retrieved from [32]. © IOP Publishing Ltd. All rights reserved.

4.3. Empirical research on GeoGebra in physics teaching

As mentioned, there is little empirical research on the use of GeoGebra in physics education. However, several studies on a teaching sequence supported by GeoGebra simulations have been conducted by Malgieri, Onorato, and de Ambrosis [3, 37–39]. A collection of GeoGebra simulations was developed to assist students in learning quantum physics at a basic level based on Feynman’s sum over paths’ approach. The software was chosen because it offers the possibility of creating highly interactive simulations that are easy to use and modify [37]. The simulations were used in both classroom activities and for homework, displaying elements of physical experiments. For example, the simulation of the two-slit experiment with point-like slits (figure 10) displays in the left window the physical setup and the detection probability of the photon (black curve). In the right window, the simulation displays the vectors (phasors) associated with the two paths in the same colour with which the paths are represented in the left window (red and green), making the formal representations physically intuitive. The resultant amplitude, which is the sum of the two vectors associated with the paths, is shown in black.

Relevant physical parameters, such as the position of the detector, light wavelength, and the source detector can be varied using sliders and checkboxes. The teaching sequence has been tested and revised in the context of training courses for pre-service and in-service high school teachers, and for the use of students in their final year of high school.

The studies show that by using this teaching sequence, students have improved their understanding of several conceptual issues and their ability to use the ‘sum over path’ method for problem solving as well as their ability to express themselves using an expert-like language. Regarding the collection of simulations and the software used for designing it, the researchers consider that GeoGebra is a valuable supporting software as it ‘makes the mathematical models behind the simulations completely transparent and easily accessible to the user, and avoids producing the impression that complex and exotic algorithms are at work’ [3].

As part of our research, we conducted a study that focused on upper-secondary school students’ interaction with a freely available GeoGebra simulation of friction [22]. The simulation is a dynamic representation of a hand pulling a block over a surface. The hand holds a dynamometer which shows the value of the pulling force. The force–time graph that appears simultaneously displays small ripples as opposed to the idealised linear relationships often provided in textbooks (figure 11).

A class of 19 first-year upper-secondary school students (16 years old) specialising in natural sciences in Sweden took part in a 1 h lesson
following written instructions developed around the GeoGebra simulation [22]. Students collaborated in pairs, and video cameras were placed behind three pairs to record students’ voices and gestures and their computer screens. The results showed that the students quickly understood how the simulation works. Taking advantage of the flexibility of the tool, students could investigate the effect of varying parameters by dragging sliders and by pausing or rerunning the simulation. They observed that the mass and surface of the material, but not the base area, influenced the frictional force. The opportunity to observe how the force–time graph was rendered as the
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hand pulled the block and specific features of the provided GeoGebra simulation of friction, such as the ripply graph, encouraged the students to come up with their own representations in the form of enactments, gestures, and drawings, particularly in order to explain microscopic aspects of frictional force. Overall, we found the simulation, in conjunction with the provided work sheet, to be a valuable tool in upper-secondary school teaching of friction by enabling students to cooperate and to describe their understanding using scientifically appropriate language.

5. Concluding remarks

The current literature on how GeoGebra can be used to support upper-secondary school physics education is comprised primarily of descriptions of educational material, mostly simulations. These studies acknowledge GeoGebra’s features as valuable for physics education. For example, the simultaneous interconnection between algebraic, geometric, and numeric representations allows for the possibility of analysing inserted videos and pictures of real experiments (figure 2), giving students the opportunity to obtain an intuition for the formal representation of physical phenomena. The ability to augment photos and videos of real experiments with virtual objects, such as arrows (figure 6), makes GeoGebra a suitable tool for making formal representations of physical phenomena and concepts, such as vectors, more experienceable and intuitive. Using GeoGebra, teachers can create custom-made computer simulations or modify existing simulations that are freely available on the software’s website. This provides teachers with the opportunity to choose the desired approach to teaching and learning physics. In addition, students can build an understanding of the role of computer modelling in physics by being involved in the process of creating computer simulations of physical phenomena. In addition, the empirical studies selected in our review show that GeoGebra is a user-friendly software that can be operated intuitively by students. It provides an environment where the underlying mathematical structures are always at hand, while enabling users to connect physical phenomena to formal representations of them, as well as informal representations such as drawings, gestures, and enactment.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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All figures in this paper are used with kind permission from the authors of the reviewed studies.

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