Zero refraction in natural materials and the mechanism of metal superlens

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**Abstract** – We found that a single negative material has the characteristic of zero refraction in near field, and point out that the mechanism of a metal superlens to image with a resolution exceeding the diffraction limitation is different from that of a perfect lens made of a double negative material. The principle of metal superlens is disclosed. Our numerical results based on the zero-refraction characteristic is well in agreement with the experimental results. This work brings new understanding about the single negative materials and will lead to the applications of the superlens in the right direction.

All electromagnetic materials can be sorted into four classes according to their permittivity \( \epsilon \) and permeability \( \mu \), i.e. the double positive (DPS, \( \epsilon > 0, \mu > 0 \)), electric negative (ENG, \( \epsilon < 0, \mu > 0 \)), double negative (DNG, \( \epsilon < 0, \mu < 0 \)) [1], and magnetic negative (MNG, \( \epsilon > 0, \mu < 0 \)) materials, just as shown in fig. 1. It has been realized that the first (DPS) and third (DNG) quadrants correspond to positive and negative refractions [1–13], respectively. In the second (ENG) and fourth (MNG) quadrants, media are opaque. On the \( \epsilon \) and \( \mu \) axes, as \( n = \sqrt{\epsilon / \mu} = 0 \), the corresponding materials show the zero-refraction characteristic. The zero-refraction material is expected to have great potential in applications, but it seems that the zero-refraction materials have never been found in Nature. However, the above conclusion is valid in traditional optics. In subwavelength optics, the near-field characteristic of a single negative (SNG) material, which may be either an ENG or a MNG material, has not been clarified. In 2000, Pendry [14] predicted that the superlens made of a SNG slab could replace the perfect lens made of a DNG slab to challenge the diffraction limitation [14,15]. Following the idea, Zhang et al. experimentally obtained the sub-diffraction–limited image with one-fourth of the illumination wavelength by using a silver film as superlens [16]. After that, based on the superlens properties, the magnifying optical superlens consisting of a curved periodic stack of Ag and a dielectric was realized [17,18]. As the Ag film is an ENG material, easier to be fabricated and less expensive than the DNG materials, it is called the “poor man’s superlens” [19]. These results paved the way for the applications of SNG materials in nanoscale optical imaging and ultrasmall optoelectronic devices. For instance, the optical lithography technology of micro electric circuits will be proceeded to nanometer size to raise the storage density [20–22].

Nevertheless, the refraction in the second and fourth quadrants in the near field has to be elucidated. Although a DNG slab can focus light from a point source by negative refraction, just as displayed in the third quadrant of fig. 1, one cannot assume the case as a matter of course in ENG and MNG materials, because the refraction characteristics in the second and fourth quadrants are not clear. We find that the SNG materials correspond to zero refraction. This discovery is applied to disclose the principle of the metal superlens, which is quite different from that of a perfect lens proposed by Pendry [14,15]. The resolution of the metal superlens used by Zhang et al. [16] is numerically calculated based on zero refraction, and their experimental results are well explained.

First of all, let us consider an isolated ideal SNG slab with dielectric constant \( \epsilon < 0 \) and thickness \( d \). A Cartesian coordinate is set with the \( z \)-axis perpendicular to the slab and the origin is at the incident surface. In an \( S \)-polarized

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and transmitted waves can be written as
\[ \hat{E}_{1S}(x, z) = \exp(i k_x x + i k_{1z} z - i \omega t) \hat{y}. \]  
Here the incident amplitude is assumed to be 1. Then at the two sides of the incident surface the reflected and transmitted waves can be written as
\[ \hat{E}_{1S}^r(x, z) = r_{12} \exp(i k_{1x} x - i k_{1z} z - i \omega t) \hat{y}, \]  
and
\[ \hat{E}_{2S}(x, z) = t_{12} \exp(i k_{2x} x - |k_{2z}| z - i \omega t) \hat{y}, \]  
respectively, where
\[ |k_{2z}| = \sqrt{(\omega / c)^2 |\epsilon_2| + k_x^2} \]  
and
\[ k_{1z} = \sqrt{(\omega / c)^2 - k_x^2}. \]  

Then the electric field is written as
\[ \hat{E}_{1S}^i(x, z) = F(x, z) \exp(i k_x x + i k_{1z} z - i \omega t) \hat{y}, \]  
where \( F(x, z) \) is the rectangle function. On the incident surface, \( z = 0 \), \( F(x, z = 0) \) it is expressed as
\[
F(x) = \begin{cases} 
1/\sqrt{b/\cos \theta}, & |x| \leq b/(2 \cos \theta), \\
0, & |x| > b/(2 \cos \theta).
\end{cases}
\]  
Therefore, the electric-field distribution at the incident surface (at time \( t = 0 \)) becomes
\[ \hat{E}_{1S}^i(x, z) = F(x) \exp(i k_x x) \hat{y}. \]  
This electric field can be expanded by plane waves, and each plane-wave component corresponds to one transmission field described by eq. (3). The total field in the slab should be the integral of these transmission fields, so we write
\[
\hat{E}_{2S}^i(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{dk'_x}{k_x} \int_{-b/(2 \cos \theta)}^{b/(2 \cos \theta)} dz' t_{12} \times \exp[i (k_x - k'_x) x'] \exp(i k_{2x} x - |k_{2z}| z') \hat{y}.
\]  
Now we give numerical examples to show how the transmitted thin beam propagates in the slab from the incident to output surfaces. In general, \( t_{12} \leq 1 \), but for the present purpose, we may take \( t_{12} = 1 \). The metal is Ag and the slab thickness is set as 0.035 \( \mu m \). The incident wavelength is \( \lambda_0 = 0.365 \mu m \), and accordingly the permittivity of Ag is \( \epsilon = -2.4012 + i0.2488 \) [15]. The beam width is \( b = 10 \lambda_0 = 3.65 \mu m \). In this case, the material is nonideal, but the imaginary part can be neglected since it is less than the real part by one order of magnitude, i.e., the metal film is approximated by an ideal ENG material. The calculation results for the cases of \( \theta = 0 \) and \( \theta = 30^\circ \) are displayed in fig. 2. Figure 2(a) shows the case of normal incident \( \theta = 0 \). The profile of the electric field at the incident surface is exactly rectangular. When the wave reaches the exit surface, the profile is still a rectangle with the width unchanged, although the intensity is weakened and the edges become slightly blurred. As for the case of inclined incidence \( \theta = 30^\circ \) (see fig. 2(b)), the electric field shows the oscillation distribution on both incident and output surfaces, with less amplitudes on the latter. Thus, we achieve the knowledge that the profile of the electric field is copied from the incident plane to the exit plane without any shift along the \( z \)-direction, but only the intensity decreases. In other words, when a thin light beam strikes on the surface of an ENG slab, it will transmit along the direction perpendicular to the surface. The refractive angle is always zero independently of the incident angle.

It can be verified in the same way that the zero refraction will also occur when light goes through the surface of an MNG material. We outline the general conclusion concerning refraction with materials as follows: the refractive angle is positive in a double positive (DPS)
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![Electric field distributions (arb. units)](image1)

Fig. 2: (Colour on-line) The profiles of the electric field at incident surface (dashed curves) and at output surface (solid curves) of a thin Ag slab when the incident angle is (a) $\theta = 0$ and (b) $\theta = 30^\circ$.

![Sketch of the superlens set in the photolithography experiment](image2)

Fig. 3: (Colour on-line) The sketch of the superlens set in the photolithography experiment.

Material (positive refraction), negative in a DNG material (negative refraction) and zero in a SNG material, as depicted in Fig. 1. This conclusion is valid for both S- and P-polarized waves.

As an application example of our discovery, we propose the superlens theory based on the zero-refraction characteristic. The installation utilized in the experiment [16] is drawn in Fig. 3. An Ag slab is sandwiched by two dielectrics with permittivities $\epsilon_1$ and $\epsilon_3$, respectively. For convenience, we label the regions of $\epsilon_1$, $\epsilon_2$, and $\epsilon_3$ by characteristics I, II, and III, respectively. A Cartesian coordinate is set with the $z$-axis perpendicular to the metal film. The interfaces I/II and II/III are the incident and output surfaces of the metal slab.

Before doing numerical calculations, let us consider the working principle of the superlens based on the zero-refraction theory. The red arrow in Fig. 3 indicates a light line from the mask slit, which is zero-refracted by the metal slab and then arrives at the surface of the $\epsilon_3$ material. Without the metal slab, the light would go along the dashed arrow and reach the surface of the $\epsilon_3$ material at the point farther from the mask slit. Obviously the metal slab can beam the light to form a narrower image of the mask slit, which is termed as the beaming effect.

Another effect is that for the radiation wave, the transmissivity will monotonously decrease with the increase of the tangential component of the wave vector. This characteristic acts as an angular filter to weaken the light with a larger incident angle so as to concentrate the energy at the central region. This effect was in fact first obtained by Pendry [14] when he deduced the total amplitude transmissivity of a DNG slab, and later discussed in ref. [16]. Here we name it as angular filter effect.

Having analyzed the physical mechanism of the superlens, we give the numerical calculation of the image profile. The model coincident to the photolithography experiment in ref. [16] is sketched in Fig. 3. The electric-field distributions of $S$ and $P$ waves on the image material surface (interface II/III) are

$$E_S(x, z) = \int_{-a/2}^{a/2} H_0 \left[ \frac{2\pi \sqrt{\epsilon_1}}{\lambda_0} \sqrt{(x-x_1)^2 + d_1^2} \right] \times T_S(x-x_1) dx_1,$$

and

$$E_P(x, z) = \int_{-a/2}^{a/2} H_0 \left[ \frac{2\pi \sqrt{\epsilon_1}}{\lambda_0} \sqrt{(x-x_1)^2 + d_1^2} \right] \times T_P(x-x_1) \sqrt{\epsilon_1/\epsilon_3} dx_1,$$
Fig. 4: (Colour on-line) The energy distributions on the surface of the image material (right side of the interface II/III in fig. 3).

respectively. Here \( H_0 \) is the zeroth-order Hankel function, and \( T_S, T_P \) the amplitude transmissivities of the metal slab for \( S \) and \( P \) waves, respectively. Similarly to the way used in ref. [13], we obtain the transmissivities of the metal slab sandwiched by dielectrics I and III:

\[
T_{S,P} = \frac{t_{S,P}^{12} t_{S,P}^{23} \exp(ik_{2z}d_2)}{1 - r_{S,P}^{21} r_{S,P}^{23} \exp(2ik_{2z}d_2)}.
\] (13)

The interface transmissivities and reflectivities are in turn expressed as

\[
r_{i,j}^{S} = \frac{k_{iz} - k_{jz}}{k_{iz} + k_{jz}},
\] (14)

\[
t_{i,j}^{S} = \frac{2k_{iz}}{k_{iz} + k_{jz}},
\] (15)

\[
r_{i,j}^{P} = \frac{k_{iz}/\epsilon_i - k_{jz}/\epsilon_j}{k_{iz}/\epsilon_i + k_{jz}/\epsilon_j},
\] (16)

\[
t_{i,j}^{P} = \frac{2k_{iz}/\epsilon_i}{k_{iz}/\epsilon_i + k_{jz}/\epsilon_j},
\] (17)

where \( i = 1, 2 \) and \( j = i + 1 \). In the calculations, \( k_{iz} \) is related to \( k_{iz} = \sqrt{\epsilon_i(\omega/c_0) \sin \arctan((x - x_i)/d_2)} \), by the common electromagnetism relationship just as, for instance, eq. (4). In the photolithography experiment, light will cut a notch on the image material. For a linear image material, its notch depth is proportional to the striking light energy, and the energy distribution is proportional to the square of the amplitude of the total electric field: \( |E|^2 = |E_S|^2 + |E_P|^2 \).

The parameters are according to the experiments [16] taken as follows: the mask slit width is \( a = 40 \) nm; the incident wavelength \( \lambda_0 = 365 \) nm; \( \epsilon_1 = 2.3013 + i 0.0014 \) for PMMA; \( \epsilon_2 = -2.4012 + i 0.2488 \) for the silver superlens; \( \epsilon_3 = 1.5170 + i 0.00046 \) for the image material. On the image material, an exposed notch with the half-maximum width (HMW) of \( w_1 = 89 \) nm was observed. When the superlens was replaced by a PMMA with thickness 35 nm, the HMW became \( w_2 = 321 \) nm. This shows that the superlens has narrowed the HMW from 321 to 89 nm. It is appropriate to define a ratio \( \eta = w_2/w_1 \), named as narrowing factor, to describe the narrowing effect. In the experiment, this ratio is \( \eta_e = 321/89 \approx 3.6 \).

In fig. 4 our calculated energy distributions at the right side of the II/III interface are displayed. The distributions are proportional to the depths of the exposure notch. The solid and dashed curves correspond to the result with and without the superlens, respectively. The corresponding HMWs are 72 and 246 nm, respectively, both being narrower than those in the experiment by about 20 percent. We believe that the experimental wider HMWs resulted from the factors such as the roughness of the interface between the superlens and dielectrics, the mask slit edges and so on. Nevertheless, our calculated narrowing factor is \( \eta_t \approx 3.4 \), quite close to the experimental result.

If the superlens is removed from the experimental set while the PMMA thickness is kept unchanged, the image material will be separated by the 40 nm PMMA from the mask, the corresponding theoretical distribution of the exposure depth is expressed by the dotted line in fig. 4. Its HMW, being 144 nm, is narrower and the peak is higher than the dashed curve. Since the image material is closer to the mask slit when the metal slab is absent, the exposure distance becomes shorter, so that the light beam is more concentrated. When the superlens is inserted between the
image material and PMMA, the exposure distance will be extended. The extension does not cause the light diffusion due to zero refraction, which copies the wave from the incident plane to the exit plane. However, the copy effect should keep the HMW at the original width of 144 nm; why does the HMW become 72 nm? This is due to the angular filter effect that whittles down the light with larger incident angle, which causes the exposure notch to be narrowed to the solid curve in fig. 4.

In summary, this paper makes the following contributions: i) in the near field, the ENG and MNG materials have the zero-refraction characteristic. This finding filled the cognition gap about the natural materials sorted by their electromagnetic properties. The zero refraction was realized only in metamaterials, such as photonic crystals, but never in natural materials before; ii) the principle of the superlens made of an ENG or MNG material is disclosed, which enhances the image resolution by beaming and filtering effects, instead of the focusing effect in the perfect lens; iii) a method is proposed to calculate the resolution of the superlens, and our numerical results are in good agreement with experiment.

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REFERENCES

[1] Veselago V. G., Sov. Phys. Usp., 10 (1968) 509.
[2] Smith D. R., Padilla W. J., Vier D. C., Nemat-Nasser S. C. and Schultz S., Phys. Rev. Lett., 84 (2000) 4184.
[3] Smith D. R. and Kroll N., Phys. Rev. Lett., 85 (2000) 2933.
[4] Shelby R. A., Smith D. R. and Schultz S., Science, 292 (2001) 77.
[5] Cubukcu E., Aydin K., Ozbay E., Fotiopoulos C. A. and Soukoulis C. M., Nature (London), 423 (2003) 604.
[6] Parimi P. V., Lu W. T., Vodo P. and Sridhar S., Nature, 426 (2003) 404.
[7] Ward D. W., Nelson K. A. and Webb K. J., New J. Phys., 7 (2005) 213.
[8] Aydin K., Bulu I. and Ozbay E., New J. Phys., 8 (2006) 221.
[9] Lezec H. J., Dionne J. A. and Atwater H. A., Science, 316 (2007) 430.
[10] Yao J., Liu Z., Liu Y., Wang Y., Sun C., Guy B., Stacy Angelica M. and Zhang X., Science, 321 (2008) 930.
[11] Chen J., Wang Y., Jia B. et al., Nat. Photon., 5 (2011) 239.
[12] Burgos Stanley P., de Waele Rene, Polman Albert and Atwater Harry A., Nat. Mater., 9 (2010) 407.
[13] Zhang Y., Zhang X., Mei T. and Fiddy M., Opt. Express, 18 (2010) 12213.
[14] Pendry J. B., Phys. Rev. Lett., 85 (2000) 3966.
[15] Wee W. H. and Pendry J. B., Phys. Rev. Lett., 106 (2011) 165503.
[16] Fang N., Lee H., Sun C. and Zhang X., Science, 308 (2005) 534.
[17] Liu Z., Lee H., Xiong Y., Sun C. and Zhang X., Science, 315 (2007) 1686.
[18] Zhang X. and Liu Z., Nat. Mater., 7 (2008) 435.
[19] Pendry B. J., Metamaterials and Negative Refraction, http://www.cmth.ph.ic.ac.uk/phononics/.
[20] Ozbay E., Science, 311 (2006) 189.
[21] Soukoulis C. M., Linden S. and Wegener M., Science, 315 (2007) 47.
[22] Engheta N., Science, 317 (2007) 1698.