Scaling of Multi-Tension Cosmic Superstring Networks

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Abstract

Brane inflation in superstring theory ends when branes collide, initiating the hot big bang. Cosmic superstrings are produced during the brane collision. The cosmic superstrings produced in a D3-brane-antibrane inflationary scenario have a spectrum: \((p,q)\) bound states of \(p\) fundamental (F) strings and \(q\) D-strings, where \(p\) and \(q\) are coprime. By extending the velocity-dependent one-scale network evolution equations for abelian Higgs cosmic strings to allow a spectrum of string tensions, we construct a coupled (infinite) set of equations for strings that interact through binding and self-interactions. We apply this model to a network of \((p,q)\) superstrings. Our numerical solutions show that \((p,q)\) networks rapidly approach a stable scaling solution. We also extract the relative densities of each string type from our solutions. Typically, only a small number of the lowest tension states are populated substantially once scaling is reached. The model we study also has an interesting new feature: the energy released in \((p,q)\) string binding is by itself adequate to allow the network to reach scaling. This result suggests that the scaling solution is robust. To demonstrate that this result is not trivial, we show that choosing a different form for string interactions can lead to network frustration.

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I. INTRODUCTION

The cosmic microwave background (CMB) data from WMAP \cite{1,2} strongly supports the inflationary universe \cite{3} as the origin of the hot big bang. Recently, the brane world scenario suggested by superstring theory was proposed, where the standard model of the strong and electroweak interactions are open string (brane) modes while the graviton and the radions (which govern the size/shape of the bulk) are closed string (bulk) modes. Consider a generic brane world scenario that closely describes our universe today (i.e., a KKLT-like vacuum \cite{4}). If we take an extra brane-anti-brane pair in the early universe their brane tensions provide the cosmological constant that drives inflation. There is an attractive force between a brane and an anti-brane so they tend to move towards each other while inflation is taking place. Thus, brane inflation is a natural feature of the brane world \cite{5,6,7}; in brane inflation, the inflaton is an open string mode identified with the interbrane separation. Inflation ends when the D3-D3-brane pair collides and annihilates, releasing energy that starts the hot big bang. Note that the inflaton field no longer exists after the annihilation of the D3-D3-brane pair. Towards the end of inflation, the D3-D3-brane collision produces D1-branes – or cosmic superstrings – but neither monopoles nor domain walls \cite{8}.

We can estimate the cosmic string tension $\mu$ using the density perturbation magnitude in the CMBR data from COBE \cite{1}. In the simplest realistic scenario, namely, the KKLMMT D3-D3-brane inflationary scenario \cite{7}, one finds that \cite{9}

$$5 \times 10^{-7} \geq G\mu \geq 4 \times 10^{-10}$$

(1)

where $G$ is the Newton’s constant. The upper bound comes from WMAP data and other data \cite{10,11}, while the lower values require some fine-tuning in the model. These predictions are not sensitive to different warping schemes because the normalization to COBE is always performed after the warp effect is taken into account. In this scenario, the density perturbation responsible for structure formation is dominated by the inflaton, with cosmic strings playing a secondary role.

The evolution of networks of cosmic strings is a well studied problem \cite{12,13}. After the initial production of cosmic strings, the strings interact among themselves. When two cosmic strings intersect, they reconnect or intercommute. When a cosmic string intersects itself, a closed string loop is broken off. Such a loop will oscillate quasi-periodically and gradually lose energy via gravitational radiation. Its eventual decay transfers the cosmic
string energy to gravitational waves. A higher (lower) string density leads to a higher (lower) interaction rate so, not surprisingly, cosmic string networks evolve towards scaling solutions. A consequence of scaling is that the physics of simple, abelian Higgs networks is essentially dictated by a single parameter, the dimensionless string tension $G\mu$. This scaling feature can be seen by considering the evolution of the number density in a one-scale model, where the scaling solution emerges as an attractive fixed point.

There are many different hybrid inflationary models in which one can construct a variety of cosmic string-producing scenarios. What is different in brane inflation is that the string network that is produced has a large spectrum of possible string tensions $[14, 15, 16]$. For the D3-$\overline{D3}$-brane inflationary scenario, one expects a spectrum of $(p, q)$ string bound states $[15, 16]$, where the tension of a particular bound state $(p, q)$ is given by

$$G\mu_{(p,q)} = G\mu\sqrt{p^2g_s^2 + q^2}. \quad (2)$$

where $g_s$ is the superstring coupling. For the string to be stable, $p$ and $q$ must be coprime, i.e., $p$ and $q$ have no common factors greater than 1. Written this way, $(1, 0)$ corresponds to the fundamental F-string while $(0, 1)$ corresponds to the $D1$-brane, or D-string, so $\mu$ is the tension of the $(0, 1)$ superstring. We can see the emergence of such a spectrum in several ways. Consider the following simple picture: the gauge group at the end of inflation just before brane-antibrane annihilation is

$$U(1)_D \times U(1)_D = U(1)_+ \times U(1)_-,$$

where the open string (complex) tachyon field stretching between the branes couples only to $U(1)_-$. The usual operation of the Higgs mechanism generates abelian vortices following the spontaneous symmetry breaking of $U(1)_-$. These are the D-strings. Since no free $U(1)$ gauge symmetry remains after the annihilation, it is believed that the $U(1)_+$ symmetry becomes confining, yielding confining fluxes that may be identified as fundamental closed strings, or F-strings $[17]$. The production of D-strings may be estimated via the Kibble mechanism. Most of the decay products are expected to be very massive non-relativistic closed strings, which are expected to decay to gravitons, standard model particles and other light modes. We expect some of the massive closed strings to be extended. These are the F-strings. In a cosmological setting, their production is likely, again, to be dictated by the Kibble mechanism. The production of D- and F-strings are not independent, so we expect some initial spectrum of $(p, q)$ strings to be produced.
Since the interactions between \((p, q)\) strings is not simple \([18]\), one expects that \((p, q)\) network evolution might be quite involved. It is not obvious, \textit{a priori}, that the network can even approach scaling. For example, it could oscillate (i.e., the density of any specific \((p, q)\)-type could oscillate indefinitely), approach scaling only asymptotically, or simply frustrate. One way to address this problem would be to do a full numerical simulation of a \((p, q)\) string network. However, this is a highly non-trivial problem. String network evolution is a complex physical process; accurately modeling the build-up of small-scale string structure is computationally demanding, even in the context of abelian Higgs models, which have only one type of vortex. A radically simpler alternative would be simply to generalize the one-scale string network model due to Kibble \([19]\) to the case of \((p, q)\) string evolution. Recall that the scaling of the cosmic string network appears as a stable fixed point in this one-scale model. However, previous researchers have found it useful to include more of the network physics than the original one-scale model allowed. In particular, the velocity-dependent one-scale (VOS) model developed by Martins and Shellard for the abelian Higgs case \([20]\) provides a very convenient and reliable method for calculating the large-scale quantitative properties of string networks in many contexts, including a cosmological setting. This model performs exceptionally well when tested against high resolution numerical simulations of string networks \([13]\). It allows one to see analytically how scaling emerges, and to calculate reliably a small number of macroscopic quantities useful for cosmological applications. We take this model as the starting point for our own model building. We recognize that there are a number of other analytic approaches to the string evolution \([21]\). Some may characterize the details of small-scale stringy structures more accurately, but they also require more phenomenological input parameters which can only be obtained from simulations. Since there is, as yet, no simulation of a cosmic superstring network, and since we are chiefly interested in the overall properties of such a string network, we choose the simplest possible “analytic” model that highlights the most important physical effects.

In this paper we adapt the velocity-dependent one-scale model to describe a multi-tension cosmic string network that includes both string self-intersection and string-string binding interactions. For a multi-tension network, the string density evolution equation generalizes to a set of (infinitely many) coupled equations. We then specify the string interaction terms in our particular multi-tension, \((p, q)\) string model and solve this set of equations numerically. Fortunately, we find that the \((p, q)\) string network, with stringy interactions turned on,
rapidly approaches scaling. This fast convergence and rapid decrease of string densities with increasing tension allows us to truncate the set of equations at low, computationally tractable values of \((p, q)\). To show that this scaling result is not somehow built into the model we have constructed, we demonstrate that the same set of evolution equations with a different interaction term can lead to a frustrated network.

Because of the various approximations we use, our results are limited to overall macroscopic network features, which are nonetheless those features that are needed for cosmological applications. For instance, we find for the \((p, q)\) superstring network:

- The \((p, q)\) string network approaches a scaling network rapidly. The final scaling solution is independent of the initial densities of the various types of strings. The fractional density in strings, for \(F \neq 0\), is given by
  \[
  \Omega_{cs} = 8\pi G\mu_{(0,1)}\Gamma \\
  \Gamma \simeq \begin{cases} 
  20/(0.55 P + F) & g_s = 1.0 \\
  15/(0.53 P + F) & g_s = 0.5
  \end{cases} 
  \tag{3}
  \]
  where \(P\) measures the probability of self-interaction and \(F\) measures the overall probability of interaction among different types of strings. For the \((p, q)\) cosmic superstring network, we do not know the value of \(F\), though we expect \(P \lesssim F \lesssim 1\). It is interesting to note that scaling is achieved even if we turn off the string self-interaction, i.e., when \(P = 0\). For the abelian Higgs model we have \(\Gamma_{U(1)} \simeq 20\) and \(P \simeq 0.28\).

Using this value for \(P\) and taking \(F = P\), we find that, for \(g_s = 1\), \(\Gamma \simeq 46\); for \(F = 1\), \(P = 0.28\), we have \(\Gamma \simeq 17\). Thus, the total density of the \((p, q)\) cosmic superstring network is comparable to standard cosmic strings. Differentiating between the two kinds of networks based on their string densities will require more detailed modeling.

- The relative number density of each type of string is roughly given by
  \[
  N_{(p,q)} \sim \mu_{(p,q)}^{-\gamma} \\
  6 < \gamma \lesssim 10.
  \tag{4}
  \]
  The fall-off is power-like, not exponential. The rapid convergence of the coupled set of equations is brought about by this rapid fall-off. The power law is most accurate for high values of \(p\) and \(q\); the spectrum tends to be somewhat flatter for the first few string types. Indeed, we find that when scaling is reached, the relative numbers of \((0,1)\), \((1,0)\), and \((1,\pm 1)\) strings are comparable and far larger than the population of
the remaining \((p, q)\) states with \(p, |q| > 1\). In the case of \(F = P = 0.28\), \(g_s = 1.0\), we find \(N_{(p,q)} \propto \mu_{(p,q)}^{-7.5}\).

- The adapted multi-tension velocity-dependent one-scale model (MTVOS) that we describe in §II can be used for many different kinds of multi-tension networks by a simple change of the inter-string interactions term.

Clearly, this analysis can be improved in a variety of ways. We shall comment on some of them. However, we are confident that the rapid approach to scaling and the fast power drop-off in the densities are generic features of cosmic superstring networks.

In the past, the evolution of cosmic strings in models more complicated than the abelian Higgs model have been considered. For example, Pen and Spergel studied the evolution of \(S_N\) strings by simulating a network of \(S_3\) and \(S_8\) strings \[22\]. The \(S_N\) symmetry enforces identical tension and number densities among the \(N\) string types. In terms of the model we construct, the set of \(N\) coupled evolution equations collapses to a single equation. McGraw also modeled non-abelian \(S_3\) strings with two different tensions \[23\]. The evolution of networks of \(Z_N\) strings connected to monopoles, which have some qualitative similarities to the networks we consider, has been studied in Ref. \[24\]. We believe that a network of \((p, q)\) strings is the first network type that truly requires a set of coupled equations. The formalism may be adapted for other non-trivial string network.

In Section 2, we adapt the velocity-dependent one-scale model to a model for the evolution of cosmic strings that have a spectrum of tensions. In Section 3, we specialize the model to the \((p, q)\) superstring network by defining our string interaction term. In Section 4 we present our numerical results and in Section 5 we briefly discuss some observational implications of these networks.

II. THE MULTI-TENSION VELOCITY-DEPENDENT ONE SCALE MODEL

Consider a set of different types of cosmic strings \(\{\alpha\}\) with tensions \(\{\mu_{\alpha}\}\). Let the number of cosmic strings of type \(\alpha\) per unit area be \(n_{\alpha}\). Suppose that all of the cosmic strings may be characterized by a single length scale \(L\) and a single average velocity \(v\), and that cosmic strings of type \(\alpha\) can evolve either by interaction mediated loop formation or by binding to cosmic strings of other types \(\beta \neq \alpha\). The following model is motivated by the model of
Martins and Shellard [20], but has been altered somewhat to accommodate the new string physics we introduce.

We assume that the length scale evolves via the equation

\[ \dot{L} = HL + c_1v, \]  

(5)

where the loop parameter \( c_1 \leq 1 \) is a dimensionless factor and \( H \) is the Hubble parameter. We take the equation of motion for the velocity to have the Martins-Shellard form

\[ \dot{v} = (1 - v^2) \left( -2Hv + \frac{c_2}{L} \right); \]  

(6)

where the “momentum parameter” \( c_2 \) is a second constant. This term is the acceleration due to the curvature of the strings. In the absence of expansion, these two equations imply

\[ \gamma = (1 - v^2)^{-1/2} = (L/L_0)^{c_2/c_1}, \]

and with \( c_1 = c_2 = 0 \), but with expansion retained, they imply

\[ \gamma v a^2 = \text{constant}, \]

\( L/a = \text{constant}, \)

and so, \( \gamma v L^2 = \text{constant}. \)

Thus, the “self-acceleration” due to string curvature and expansion have opposite effects: self-acceleration increases string velocity, whereas expansion dilutes it. This suggests that the two effects can cancel one another. We can see how this comes about by rewriting Eq. (5) as

\[ \frac{d(HL)}{dt} = H(HL + c_1v) + \frac{\dot{H}}{H}HL = H(HL) \left[ \frac{c_1v}{HL} - \left( \frac{1 + 3w}{2} \right) \right] \]  

(7)

and combining it with the velocity equation; we then find that there is a quasi-steady solution

\[ v = HL \left( \frac{1 + 3w}{2c_1} \right) = \frac{c_2}{2HL}, \]  

(8)

which implies

\[ HL = \sqrt{\frac{c_2c_1}{1 + 3w}}, \quad v = \frac{1}{2} \sqrt{\frac{c_2(1 + 3w)}{c_1}}. \]  

(9)

In this solution, both \( HL \) and \( v \) are constants that differ in the radiation \( (w = 1/3) \) and matter \( (w = 0) \) eras. Clearly, there is no quasi-steady solution for \( w \leq -1/3; \) thus, quasi-steady solutions only exist in the radiation and matter eras. We require that in both eras, \( v \leq 1, \) a condition that is more restrictive in the radiation era, where it demands that \( c_2/2c_1 \leq 1. \) In practice, we choose \( c_1 \) and \( c_2 \) such that the scaling values of \( HL \) and \( v \) match the values given in [20], where similar constants \( \tilde{c} \) and \( k \) are chosen to line up with full network simulations. The translation between our constants and theirs is simple: The translation between our constants and theirs is simple: in the radiation era, our \( c_2 = k, \)
while our $c_1 = (k + \bar{c})/2$; in the matter era, $c_2 = (3/4)k$ and $c_1 = (3/8)(k + \bar{c})$. For example, in the radiation era they find $HL = 0.1375$ and $v = 0.655$, which for us fixes $c_1 = 0.21$ and $c_2 = 0.18$.

Next, add to these equations an equation of energy conservation, at first in the absence of interactions between strings of different types. Let the cosmic string energy density be

$$\rho = \frac{n\mu}{\sqrt{1 - v^2}}$$

(10)

where $\mu$ is the mass per unit length of a string, and $n$ is the mean string number density. In the absence of interactions, assume that

$$\dot{\rho} = -2H\rho(1 + v^2)$$

(11)

Differentiating the expression, Eq. (10) for the string energy density and using Eq. (11) and Eq. (6), the equation for $\dot{v}$, implies

$$\dot{n} = -\left(2H + \frac{c_2v}{L}\right)n.$$ (12)

where we see that the first term comes from cosmological expansion. The second term can be interpreted in two different ways. For straight strings, it would reflect a net expansion in the velocity field orthogonal to the strings. For kinky strings, we should interpret $n$ as the characteristic number of intersections per unit area on average for a two dimensional surface intersecting the network. As the strings straighten out (the source term in $\dot{v}$) the number of intersections will fall, at a rate that is $\sim nv/L$ characteristically. We may interpret this to mean that as string kinks straighten out, the number of intersections of strings with any two dimensional cut through three dimensional space will decrease; it is also the straightening of kinks that raises $v$. Note that when the string velocity approaches its asymptotic value $v = c_2/2HL$, the energy equation becomes

$$\dot{\rho} = -\left(2H + \frac{c_2v}{L}\right)\rho = -H\left[2 + \frac{c_2(1 + 3w)}{2c_1}\right]\rho,$$ (13)

i.e. it assumes exactly the same form as the equation for the number density of strings. This is sensible because the energy per unit length per string $\mu\gamma \rightarrow$ constant asymptotically.

Next, let us consider what happens when we allow interactions between strings. Recall that we assume a single characteristic length scale, $L$, for all types of strings; however, unlike the single-$\mu$ case, this length need not be related directly to string density. We
further assume that the different types of cosmic strings interact by binding, first at a point and then zipping up to form a new cosmic string with the same length as the original two, which enforces equal lengths for all different kinds of strings. The zipping up takes a time $L$, but let us suppose that $L$ is small enough compared to the Hubble length that we can regard the zipping up as instantaneous; $HL \sim 0.1$ is good enough for our purposes. Let the equation for $n_\alpha$ be

$$
\dot{n}_\alpha + 2Hn_\alpha = -\frac{c_2n_\alpha v}{L} - Pn_\alpha^2 vL + FvL \left[ \frac{1}{2} \sum_{\beta,\gamma} P_{\alpha\beta\gamma} n_\beta n_\gamma (1 + \delta_{\beta\gamma}) - \sum_{\beta,\gamma} P_{\beta\gamma\alpha} n_\gamma n_\alpha (1 + \delta_{\gamma\alpha}) \right],
$$

(14)

where the first term on the RHS arises from the breaking off of loops from individual undulating strings of type $\alpha$, the second term arises from breaking off of loops after the collision of two strings of type $\alpha$, and the third term arises from the zipping up of two strings of different types that collide and bind.

As an aside, we note that the term proportional to $P$ could equally well be written as a term under the summation, proportional to $P_{0\alpha\alpha}$. We have pulled this term out of the sum to make the contrast between self interaction and interaction between strings of different types clear. But a few comments on this term are in order:

- The term proportional to $P$ or $P_{0\alpha\alpha}$ is the usual term that drives networks without multiple string types to scaling.
- By writing $P$ instead of $P_{0\alpha\alpha}$, we are assuming that the self-interaction rate does not depend upon $\alpha$.
- If we take $P_{\alpha\beta\gamma} = 0$ for $\beta \neq \gamma$, then a self-interaction term of this form drives all string species which have a sufficient initial density to the same final scaling number density.
- After we have taken $P$ out of the sum, we either restrict the sum to $\beta \neq \gamma$ or assume $P_{0\alpha\alpha} = 0$.

We assume that our constants $c_2$, $P$, and $F$ are identical for cosmic strings of all types. We define $F$ as a measure of the overall probability that two strings of different types interact at all. We have assumed that the interaction of strings of two different types can only result in zipping up, if anything – there are no reconnections and no breaking off of loops directly associated with such interactions.
In the third term, $P_{\alpha\beta\gamma}$ is the probability of forming a string of type $\alpha$ when strings of types $\beta$ and $\gamma$ collide, whenever the strings interact at all. The factor
\[
\frac{1}{2} (1 + \delta_{\beta\gamma})
\]
is introduced so that we do not double count the production of strings of type $\alpha$ when strings of different types $\beta$ and $\gamma$ collide; i.e. since we do not restrict the sum, symmetry on $\beta \leftrightarrow \gamma$ implies we have two identical source terms from $\beta + \gamma \rightarrow \alpha$; the factor
\[
1 + \delta_{\gamma\alpha}
\]
in the loss rate arises because in each $\alpha - \alpha$ collision we lose two long strings.

By defining $N_\alpha = a^2 n_\alpha$, we can rewrite Eq. (14) as
\[
\dot{N}_\alpha = -\frac{c_2 N_\alpha v}{L} - \frac{PN_\alpha^2 vL}{a^2} + \frac{FvL}{a^2} \left[ \frac{1}{2} \sum_{\beta,\gamma} P_{\alpha\beta\gamma} N_\beta N_\gamma (1 + \delta_{\beta\gamma}) - \sum_{\beta,\gamma} P_{\beta\gamma\alpha} N_\gamma N_\alpha (1 + \delta_{\gamma\alpha}) \right],
\]
and if we define conformal time by $d\eta = v dt/a$ and introduce the comoving string length, $L = a \ell$, then we find
\[
N'_\alpha = -\frac{c_2 N_\alpha}{\ell} - PN_\alpha^2 \ell + F \ell \left[ \frac{1}{2} \sum_{\beta,\gamma} P_{\alpha\beta\gamma} N_\beta N_\gamma (1 + \delta_{\beta\gamma}) - \sum_{\beta,\gamma} P_{\beta\gamma\alpha} N_\gamma N_\alpha (1 + \delta_{\gamma\alpha}) \right],
\]
with a prime denoting differentiation with respect to conformal time. In terms of $N_\alpha$, we find that $\rho_\alpha = \mu_\alpha N_\alpha / a^2 \sqrt{1 - v^2}$.

The remaining two equations are those for $\ell$ and $v$. Substituting $L = \ell a$ into Eq. (5) gives
\[
\dot{\ell} = \frac{c_1 v}{a} \quad \Rightarrow \quad \ell = \ell(0) + c_1 \eta.
\]
When we change the independent variable from $t \rightarrow \eta$, Eq. (6) becomes
\[
v' = \frac{(1 - v^2)}{v} \left( -2Hav + \frac{c_2}{\ell} \right).
\]
To complete the set of equations, we need to find $a(\eta)$; we use
\[
H = \frac{d(\ln a)}{dt} = \frac{d(\ln a)}{d\eta} \frac{d\eta}{dt} = \frac{v}{a} \frac{d(\ln a)}{d\eta}
\]
to get
\[
\frac{da}{d\eta} = \frac{Ha^2}{v}.
\]
In the radiation dominated era, which is of greatest interest to us practically, \( Ha^2 \) is approximately constant. Thus, in the quasi-steady state, the scale factor grows linearly with \( \eta \).

Eq. (14) may yield a steady state solution of the form \( N_\alpha = f_\alpha/\ell^2 = f_\alpha/[(\ell(0) + c_1\eta)]^2 \) provided that

\[
-2c_1f_\alpha = -(c_2f_\alpha + Pf_\alpha^2) + F \left[ \frac{1}{2} \sum_{\beta,\gamma} P_{\alpha\beta\gamma} f_\beta f_\gamma (1 + \delta_{\beta\gamma}) - \sum_{\beta,\gamma} P_{\beta\gamma\alpha} f_\gamma f_\alpha (1 + \delta_{\gamma\alpha}) \right] \tag{21}
\]

has a nontrivial solution.

It is instructive to consider what we get when \( F = 0 \). In that case, Eq. (21) has two solutions, \( f_\alpha = 0 \), which is not relevant, and

\[
f_\alpha = \frac{2c_1 - c_2}{P} \tag{22}
\]

which is physically realizable only if \( 2c_1 > c_2 \). Let us assume that this is so. At sufficiently late times, we will therefore find that

\[
\rho_\alpha = \frac{\mu_\alpha (2c_1 - c_2)}{Pc_1^2v^2\sqrt{1 - v^2}} \tag{23}
\]

where we have assumed that \( c_1\eta \gg \ell(0) \), and that \( v \) relaxes to its asymptotic value. In this limit, we find that \( a \approx Ha^2\eta/v \) in the radiation era, and therefore \( \eta \approx va/Ha^2 = v/Ha; \) use this in Eq. (23) to find

\[
\rho_\alpha \approx \frac{\mu_\alpha H^2 (2c_1 - c_2)}{Pc_1^2v^2\sqrt{1 - v^2}} \implies \Omega_\alpha = \frac{8\pi G\rho_\alpha}{3H^2} \approx \frac{8\pi G\mu_\alpha (2c_1 - c_2)}{3Pc_1^2v^2\sqrt{1 - v^2}}. \tag{24}
\]

Because this version of the network equations assumes common \( L \) and \( v \) for all string types, when \( F = 0 \) we expect to find \( \Omega_\alpha/\mu_\alpha \) independent of \( \alpha \), assuming nonzero initial populations. (Remember that \( \Omega_\alpha \propto f_\alpha = 0 \) is also a solution.) Notice that Eq. (22) implies \( \Omega_\alpha \propto (2c_1 - c_2)/P \), and for small self-interaction probability, this is the expected \( P^{-1} \) scaling. Moreover, nonzero \( P \) is essential for time-independent \( \Omega_\alpha \) to arise in networks with only interactions among strings of the same type. Also, since this is the limit in which our model reduces to the abelian Higgs case, we can use prior simulation results to fix the value of our parameter \( P \). Numerical studies of the radiation dominated era have \( \Gamma = \Omega_{cs}/(8\pi G\mu) \approx 20 \), which for us implies \( P = 0.28 \), taking \( c_1 = 0.21, c_2 = 0.18 \), and \( v = 0.655 \). Because of this we take \( P = 0.28 \) as the fiducial value for \( P \) in our numerical solutions.
It is also instructive to consider what happens if $P = F = 0$. In this case, Eq. (14) has an exact solution

$$N_\alpha = \frac{N_\alpha(0)\ell(0)c_2/c_1}{[\ell(0) + c_1\eta]c_2/c_1} \rightarrow N_\alpha(0) \left[ \frac{\ell(0)}{c_1\eta} \right]^{c_2/c_1}$$

(Eq. (25) is a special case of the general solution that can be found for $F = 0$, and the results of this and the previous paragraph follow from appropriate limiting cases of this solution.). From this result, we find that

$$\Omega_\alpha \approx \frac{8\pi G\mu_\alpha N_\alpha(0)\ell(0)c_2/c_1}{3(H\alpha)^2-c_2/c_1\sqrt{1-v^2}}$$

In the radiation dominated era, when $H\alpha \propto a^{-1}$, we find that $\Omega_\alpha \propto a^{2-c_2/c_1}$, which either rises or falls depending on the sign of $2 - c_2/c_1$.

Note that the network equation, Eq. (14), may also be used to describe entanglement. In that case, instead of setting $\alpha = (p_\alpha, q_\alpha)$, as we shall do to describe $(p, q)$ networks, we simply let $\alpha = p_\alpha$. Moreover, we set $P_{\alpha\beta\gamma} = \delta_{\alpha-(\beta+\gamma)}$; taking $c_2 = P = 0$, the network equations are

$$N'_\alpha = F\ell \left[ \frac{1}{2} \sum_{\beta, \gamma} \delta_{\alpha-(\beta+\gamma)} N_\beta N_\gamma (1 + \delta_{\beta\gamma}) - N_\alpha \sum_\gamma N_\gamma (1 + \delta_{\gamma\alpha}) \right].$$

These equations cannot lead to a scaling solution because there is a conservation law, basically conservation of energy, that restricts the evolution of the system. Multiply Eq. (27) by $\alpha$ and sum over $\alpha$; the result is

$$\left( \sum_\alpha \alpha N_\alpha \right)' = F\ell \left[ \frac{1}{2} \sum_{\alpha, \beta \neq \gamma} \alpha \delta_{\alpha-(\beta+\gamma)} N_\beta N_\gamma - \sum_{\alpha \neq \gamma} \alpha N_\alpha N_\gamma 
+ \sum_{\alpha, \beta} \alpha \delta_{\alpha-2\beta} N_\beta^2 - 2 \sum_{\alpha} \alpha N_\alpha^2 \right]$$

$$= F\ell \left[ \frac{1}{2} \sum_{\beta \neq \gamma} (\beta + \gamma) N_\beta N_\gamma - \sum_{\alpha \neq \gamma} \alpha N_\alpha N_\gamma + 2 \sum_\beta \beta N_\beta^2 - 2 \sum_{\alpha} \alpha N_\alpha^2 \right]$$

$$= F\ell \left[ \sum_{\beta \neq \gamma} \beta N_\beta N_\gamma - \sum_{\alpha \neq \gamma} \alpha N_\alpha N_\gamma + 2 \sum_\beta \beta N_\beta^2 - 2 \sum_{\alpha} \alpha N_\alpha^2 \right] = 0,$$

(28)
where the next to last line was obtained by relabelling $\gamma \rightarrow \beta$ in one of the two sums over $\beta \neq \gamma$. Thus, here we have an example where scaling is not achieved. The network evolves toward ever larger values of $\mu$, but its overall comoving energy density does not decline.

Thus, neither the existence nor the nature of a scaling solution for a particular multi-tension network is obvious. If each type of string evolves independent of all other types, scaling will be achieved eventually for all types present originally, with $\Omega_{\alpha} \propto \mu_{\alpha}/P$. Turning on the interactions between string types will populate the different tensions, and, once produced, their self-interactions and energy-losing binding interactions will propel them toward scaling solutions. The final spectrum of string tensions may be broad or narrow, depending on the efficiency with which the reaction terms operate. The reaction terms themselves may promote scaling even if there are no self-interactions, but the fact that entanglement can be described by a reaction network with particular choices of interaction probabilities shows that there are certainly circumstances in which scaling cannot arise solely from the reactions among strings of different types. Note, finally, that we have assumed very little about the nature of the multi-tension network that is described by these equations beyond the assumption that string interactions lead either to loop formation or the formation of other kinds of strings through some sort of binding. Thus these equations may easily be adapted for any particular multi-tension string network model simply by determining the form of $P_{\alpha\beta\gamma}$ for that model; the particular $(p, q)$ network that we consider below is only one example of the sort of network these equations can describe.

III. F- AND D-STRING NETWORK

To specialize the preceding network to the $(p, q)$ strings of [16, 18], we define $P_{\alpha\beta\gamma}$ – taking $\alpha = (p, q)$, $\beta = (k, l)$, and $\gamma = (m, n)$ – and motivate the overall interaction probability, $F$. For this investigation, we make a first and very crude approximation: we assume that the probability of two strings of different types interacting is a single, universal constant, rather than a function of of $\alpha, \beta, \gamma$ or the relative velocity of the strings. By discarding all these complexities, we retain only a kinematically determined branching ratio (see both Fig. 1 and the discussion in the text below); in future work, we may attempt to retain more of the physics contained in $F$ to obtain more realistic results. Before we can write down this branching ratio, we shall state the relevant properties of $(p, q)$ strings, since these determine
the form of $P_{\alpha,\beta\gamma}$:

- Strings with positive and negative values of $p$ and $q$ are generically allowed; the sign of $p$ or $q$ indicates the direction of the string's charge. Because of a reflection symmetry, we can always choose the orientation of the string such that $p \geq 0$. For $p = 0, q > 0$; for $p > 0, q \in \mathbb{Z}$.

- Strings with $q = 0$ are only stable for $p = 1$. It is probable that $(0, q)$ strings are marginally bound; operationally, we assume that the non-zero momentum transfers in the string collisions that accompany binding unbind these states. An interaction that formally would create an $(N, 0)$ or $(0, N)$ string thus, in fact, creates $N (1, 0)$ or $(0, 1)$ strings.

- We assume that two strings of different types interact with probability $F$. If two strings of different types do interact, there are two possible products, or bound states, that they can form: a $(p, q)$ string interacting with a $(p', q')$ string can form either a $(p + p', q + q')$ string or a $(p - p', q - q')$ string, where we always take $p > p'$. As stated above, if either of these product bound states has a resulting $p''$ and $q''$ that are not coprime, then what is actually formed is a set of strings with stable, lower $p, q$ values.

- In agreement with our comments above, we assume $P_{0aa} = 0$ or, equivalently, restrict our summations to $\beta \neq \gamma$.

- For bound states of strings to be stable, $p$ and $q$ must be coprime. If a bound state of a string is formed with $p$ and $q$ not coprime, then the new state is, in reality, a collection of lower-tension $p$ and $q$ strings that are coprime: i.e., a string which nominally has $p = Nk, q = Nl$ is actually a set of $N (k, l)$ strings. We may view this as the “decay” of a $(Nk, Nl)$ string:

  - comes about because the $N (k, l)$ strings that compose this “state” are BPS with respect to each other; that is to say, they have no mutual binding interactions. Any possible marginal binding may be ignored since the strings are moving with relativistic speeds.

  - has no energy cost – the resulting collection of strings always has lower energy than the strings that bound to form them; there is no actual $(Nk, Nl)$ bound
state – a collection of stable \((k, l)\) strings is formed immediately following the interaction.

- can untie itself through reconnection events if the collection of \((k, l)\) strings that are created in the interaction are tangled or tied up immediately after their creation.

![Diagram of string intersection](image)

**FIG. 1:** A schematic view of a string intersection. The intersection angle, \(\theta\), determines whether the additive \(-(p + p', q + q')\) or subtractive \(-(p - p', q - q')\) binding occurs.

Whether the interaction of two strings forms a \((p + p', q + q')\) bound state or a \((p - p', q - q')\) bound state is determined by a simple consideration of force balance: if the angle between the interacting strings is small enough, then the heavier, additive bound state is formed because the two interacting strings’ tensions can balance the tension of the heavier bound state; if the angle is greater than some critical angle of force balance, then the lighter, subtractive bound state is formed. The critical angle which determines which binding occurs is given by

\[
\cos \theta_{\text{crit}}^{klmn} = \frac{e_{kl} \cdot e_{mn}}{|e_{kl}| |e_{mn}|} \quad e_{mn} = ([m - Cn] g_s, n)
\]

where \(C\) is the RR scalar. If we assume a stochastic distribution of string orientations, then the strings’ interaction angle should have a flat distribution in \(\cos \theta\), that is, that each value of the cosine between -1 and 1 should be equally likely. If we assume this, and remember that the directionality of the \(F\) and \(D\) charge must be taken into account – i.e., \(\theta = \pi/4\) is not equivalent to \(\theta = 3\pi/4\) – then the probability of forming the additive bound state is simply the fraction of cosine-space with \(\theta\) less than the angle of force balance; the subtractive
bound state is formed otherwise. We can write this as:

\[ P_{\alpha \beta \gamma}^\pm = \frac{1}{2} \left( 1 \mp \frac{(k+Cl)(m+Cn)g_s^2 + ln}{[(k+Cl)^2g_s^2 + l^2]^{1/2}[(m+Cn)^2g_s^2 + n^2]^{1/2}} \right), \]  

(29)

where \( P^+ \) indicates \( \alpha = (p, q) = (k+m, l+n) \) and \( P^- \) indicates \( \alpha = (p, q) = (k-m, l-n) \). This form captures the kinematic branching ratio, but as stated leaves out an important process: the creation of non-coprime \( (p, q) \) strings, which are in reality collections of two or more coprime strings. To take this into account, we must slightly modify the way in which we insert \( P_{\alpha \beta \gamma} \) into our equations: we take

\[ P_{(Nk, Nl)\,(p, q)}(p', q') = NP_{(k, l)\,(p, q)}(p', q'). \]

The inclusion of this process is extremely important. This break-up of non-coprime strings is a nonreversible process that is fundamentally dissipative – it helps to keep the average tension of the network low both by limiting the pathways by which high-tension bound states can be reached and by providing a mechanism through which a single interaction can destroy a high-tension bound state and replace it with a collection of low-tension strings. Thus, in summary,

- Two strings of different types \( \alpha \) and \( \beta \) interact with probability \( F \).
- When these strings interact, either a subtractive or additive bound state is formed, with probabilities \( P^- \) and \( P^+ \) given by Eq. (29). The two interacting strings are annihilated in the production of the new bound string state.
- When the bound state \( \gamma \) is stable, one such string is produced.
- When the nominal bound state \( \gamma \) is unstable, it immediately forms \( N \) lower-tension stable strings, which helps keep the tension dependence of string density spectrum steep.

In all our numerical runs we have assumed \( C = 0 \).

Some possible physical effects that our model neglects include:

- Velocity dependence of the interaction probabilities due to variation in the binding energy of the resulting bound states: the increase binding energy that holds very high tension states together is small – the energy gained by binding decreases greatly for
high-$\mu$ states. Thus we expect that the momentum transferred in even moderate-velocity interactions that involve these lightly-bound states may lead them to unbind spontaneously.

- This velocity dependence may lead to an effective cut-off in $\mu$, irrespective of the interaction dynamics.

- We might not expect strings of widely varying tensions all to have the same velocities and characteristic length scale in reality, contrary to what our model assumes.

- We have decoupled the evolution of the r.m.s. velocity and length scale of our network from the network’s $P$ and $F$ dependent interactions, though one would generally expect these interactions to be relevant to determining the network parameters.

IV. NETWORK RESULTS

The equations given in §II require numerical solution. For all numerical results, we work in the radiation-dominated era, assume (for convenience) that the RR scalar, $C = 0$, and fix our constants $c_1 = 0.21$ and $c_2 = 0.18$ to match (these choices are made so that, at scaling, we have $HL = 0.1375$ and $v = 0.655$). Furthermore we have done each run twice, with two different values of the superstring coupling, $g_s = 0.5$ and 1.0. We were less certain about how to initialize the cosmic string network; cosmic string creation immediately after brane inflation is not understood completely. Fortunately, scaling has proven to be quite robust to a wide variety initial conditions; for an illustration of this, see Fig. 2. On energetic grounds, we believe that, in general, networks will be formed with primarily the lowest-lying states populated; thus, for our calculations we chose initially to populate only the $(1, 0)$ and $(0, 1)$ states, and those with equal number densities. Final scaling results are always insensitive to these choices; at worst, very different initial conditions can alter the rate at which the network approaches the scaling regime. We similarly find that any initial choices for network velocity, $v$, and length scale, $L$, quickly approach their analytically-predicted scaling values. To integrate our equations numerically, another choice we had to make was how many $(p, q)$ states to allow. After testing networks of many different sizes, we found that our results showed a steep, power law dependence of number density on tension in all cases. The relative densities of the low-lying tension states, furthermore, were not changed when
more high-tension states were included. Thus we were able to obtain accurate results from a relatively small network: for the runs we show here, we have taken $p \in [0, 5]$, $q \in [-5, 5]$, though the way in which we solve the equations allows nominally higher-valued, temporary $(p, q)$ states to “form” if the values of $p$ and $q$ are non-coprime, but only if the decay products of the unstable $(p, q)$ state are a collection of stable $(k, l)$ strings with $k, |l| \leq 5$. Finally, we take the scale factor of the universe $a = 1$ at network initialization.

![Comparison among three different sets of network initial conditions, all taking $F = 1$, $P = 0$. The higher three lines represent the evolution of the overall density in cosmic strings (summed over all string states), $\tilde{\Omega}_{cs} \equiv \Omega_{cs}/((8/3)\pi G \mu(0,1))$, with scale factor, $a$. The lower three lines represent the evolution of the density in $(0, 1)$, or $D_-$, strings, $\tilde{\Omega}_{cs}^{(0,1)} \equiv \Omega_{cs}^{(0,1)}/((8/3)\pi G \mu(0,1))$, with $a$. Our standard initial conditions, equal initial populations of $(1, 0)$ and $(0, 1)$ strings and $HL = 1$, are shown by the dashed lines. The solid line represents the results from a network run with a short initial length scale ($10^{-2}$ of our usual choice) and with over half of the initial string $(p, q)$ states in our network equally populated. Finally, the dotted line shows the results for a very large initial population of strings – $\tilde{\Omega}_{cs} \sim 1000$ – equally spread over half of the tension states included in our network, with our usual choice for the initial network length scale. For all the runs shown here we have set the superstring coupling, $g_s = 1.0$.](image)

An interesting new result from this network model is that string networks with no loop
creation – those with $P = P_{0\alpha} = 0$ – still exhibit cosmologically acceptable scaling; enough energy is lost through string binding and binding-mediated annihilation to keep the comoving network number densities $N_\alpha \eta^2$ constant, regardless of initial conditions, after an initial relaxation period following network formation. Because of this, these networks are very robust; though there are regions of parameter space where the network never truly reaches scaling, in all reasonable cases (where we keep $F \neq 0, P \lesssim F$) we never find cosmologically-disastrous solutions where cosmic strings come to dominate the energy density of the Universe.

There are three regimes of interest for these solutions:

1. For $F \gg P$: The network will be dominated by $F$, $D$, and $(1, \pm 1)$ strings. Higher tension states are present, but maximally suppressed (these networks have the steepest spectra).

2. For $F \to 0$: All string tensions that are present initially eventually reach the same scaling density. If there are a great many string states, this can cause a catastrophe, since the formation of loops tends to drive all types of strings to the same value of $\Omega_{cs}/\mu_\alpha$.

3. For $P \sim F$: The interactions terms will populate the higher $(p, q)$’s, and the $P$ terms will flatten the spectrum somewhat because of its tendency to equally populate all levels. The larger $P$ is, the more quickly this happens. In Figs. 3, 4, 5 & 6 a variety of combinations are shown. For larger values of $P$, the “final” scaling state is not an exact scaling solution, but one that continues to evolve slowly to late times. Some features that appear to generic in this regime include

- $\Omega_{cs}/((8/3)\pi G \mu_{(0,1)}) = 60/(F + 0.55P)$ for $g_s = 1.0$, $46/(F + 0.53P)$ for $g_s = 0.5$.
  This formula is only valid for $F \neq 0$.

- $a_{\text{scaling}} \sim 1000$, so scaling is achieved at $T_{\text{scaling}} \sim 10^{-3}T_{\text{reheat}}$, where $T_{\text{reheat}}$ is the temperature to which the universe reheats at the end of inflation. Since $T_{\text{reheat}} \sim M_s$, the string scale, $T_{\text{scaling}} \sim 10^{-3}M_s \gg \text{TeV}$, so scaling is reached long before the electroweak phase transition.

- Scaling results are insensitive to initial conditions unless $F = 0$

- Steep final spectra: $N_\alpha^{\text{final}} \propto \mu_\alpha^{-n}$, $6 < n \lesssim 10$

There are several aspects of these results that require further discussion.
A. Scaling

- The overall properties of the networks are fairly insensitive both to initial conditions and to particular parameter choices; i.e., $\Omega_{cs}$ never grows fast enough ever to come to dominate the universe.

- However, the final state of the string network depends upon the relationship between $F$ and $P$. When $P \sim F$, the network does not quickly reach a true scaling solution. Instead, it continues to evolve to late times (see Figs. 3 and 5). Because of the efficient energy loss from both binding and loop formation, the network’s overall density grows very slowly – e.g., $(d \log \Omega / d \log a) \sim 0.07$ for $a \sim 10^4$; $\sim 0.01$ for $a \sim 10^5$, for the case of $P = F = 0.28$; note that $(d \log \Omega / d \log a) = 0$ defines entry into the scaling regime. This late growth comes about because of the continuing competition between loop formation, which wants equally to populate all string states, and binding interactions, which tend to destroy high-tension bound states. This late evolution is not dangerous cosmologically.

- In agreement with our understanding of the late evolution of $P \sim F$ networks, such networks tend to develop somewhat flatter final spectra, as their high-tension bound states tend to be more populated than those in $F \gg P$ networks. Their spectra still exhibit very steep power law behavior, however (see Figs. 4 and 6).

- The scale factor at which the network enters the scaling regime is somewhat dependent upon initial conditions: networks with more states initially populated tend to take slightly longer to reach scaling, though the greater frequency of interactions caused by such initial conditions means that these networks tend to be less dense throughout their evolution than networks that are formed with only low-lying string states (see Fig. 2); networks that begin with much smaller initial $L$, on the other hand, can take a good bit longer to reach scaling. Recall also that the binding interactions of high-tension states will very often lead to non-coprime combinations, which leads such networks quickly to develop the same kinds of steep spectra that are seen when less democratic initial conditions are used.
B. Low-F Catastrophe

- An aspect of superstring networks that has been as-yet unappreciated is that their ability to populate arbitrarily high tensions through the formation of bound states can lead to a cosmological catastrophe if such states cannot be made to decay. In traditional network evolution, which is what our equations reduce to when $F \rightarrow 0$, even very small initial populations of each possible string state will each eventually reach the same final scaling density. When this happens, $\Omega_{cs} = \sum_{\alpha} \Omega_{cs}^{\alpha} \propto \sum_{\alpha} N_{\alpha} \mu_{\alpha}$ can become huge, and thus disastrous, even if the energy density in each individual state is small.

C. Final Spectra

- The fact that our numerical solutions have found a very strong dependence of string number density on tension – with $N_{\alpha} \propto \mu_{\alpha}^{-n}$, and $6 < n \lesssim 10$ – is another important aspect of these networks. If the spectrum were flat, or nearly so, a scenario very much like the low-F catastrophe outlined above would ensue: since the effect of many string states is additive, and since there are many more possible states at higher tensions, such a flat-spectrum network would be ruled out immediately by cosmological considerations.

- Computationally, the steepness of the spectra greatly eases our task. Numerical tests showed that the addition of many high-tension states with low number densities scarcely affected any of the results. We were thus able to limit ourselves to small networks, with $p_{\text{max}} = |q_{\text{max}}| = 5$.

- Careful study of Figs. 4 and 5 shows that the spectra plotted there are not strict power laws. The spectra are, in fact, somewhat flatter for very low tensions, where the number of possible states is small. Thus, in all cases there are proportionally more $F$, $D$, and $(1,1)$ strings than anything else; particularly when $F \gg P$, these states will dominate the cosmic string network. The relative populations of these low-lying states are tabulated in Tab. I.

- The effect of varying the superstring coupling, $g_s$, is to vary the tension of the $F$
strings relative to the D strings, with this variation propagating up the ladder of bound states. Here, we have taken only two representative values of $g_s$: 0.5 and 1.0. Reducing $g_s$ affected the network as our $N \propto \mu$ results suggest: those states which were previously degenerate (equally populated)– $(1,0)$ or $[(2,1) + (2,-1)]$ relative to $(0,1)$ or $[(1,2) + (1,-2)]$, for instance – had their degeneracy lifted, with the lighter state’s number density increasing. The precise amount of increase was dependent upon the values of $F$ and $P$. Again, see Tab. I.

- We note in passing that (unphysical) networks with $P = 0$ and with the loop term $\propto c_2$ removed from Eq. (14) also go to scaling.

- In all our networks, the lowest tension states dominate the network energy density. We expect this to be a feature of any multi-tension networks that interact via binding, even if the spectrum of possible bound states is much more complicated than the one we have considered.

V. OBSERVATIONAL CONSEQUENCES

Several aspects of $(p,q)$ networks are potentially observationally distinct from regular cosmic string networks. The most obvious difference is that these networks feature a spec-
FIG. 3: The bottom panel shows the evolution of $\tilde{\Omega}_{cs} = \Omega_{cs}/((8/3)\pi G \mu_{0,1})$ for various parameter values, taking the string coupling $g_s = 1.0$. The top panel shows the rate of change in the comoving number density $N_{\eta^2}$; in the scaling regime, $d \log N_{\eta^2}/d \log \eta = 0$. 

trum of string tensions. We suggest a few possible observational signatures that could allow one to distinguish a $(p, q)$ network from a standard, abelian Higgs network:

- Previous studies of cosmic string lensing probability have been based on results from standard, abelian Higgs network models. In such networks, $\Gamma = \Omega_{cs}/(8\pi G \mu) \approx 20$. In principle, our model allows values for $\Gamma$ both less than and greater than the abelian Higgs value. However, we expect the extra-dimensional nature of superstrings to reduce their interaction rates, which leads to higher values of $\Gamma$ and $\Omega_{cs}$. If future lensing surveys find a rate of cosmic string lensing substantially higher than that predicted by a abelian Higgs network, that rate could both be a signature of a cosmic superstring network as well as an observational constraint on the parameters of such a network.

- If the overall densities in cosmic superstring networks are generally higher than those in abelian Higgs models, then observational bounds on cosmic string tension (e.g. [11]) that depend on overall network properties will need to be reinterpreted. We expect that
FIG. 4: The final scaling-era spectra for a variety of parameter combinations, taking the string coupling $g_s = 1.0$, with $N_{(0,1)}$ normalized to unity and the other number densities altered accordingly.

The net effect will be to tighten such bounds, though how much the bounds will change is difficult to predict since observational bounds depend on many aspects of string networks (e.g., string substructure, or “wiggliness”), while the detailed properties of multi-tension networks have not yet been fully fleshed out.

- The Y-shaped junction of two strings in the act of binding is a good signature of the non-trivial properties of cosmic superstrings. Such a junction, if present, could be detected by cosmic string lensing, or by observation of the Kaiser-Stebbins effect, where a temperature difference is seen in the cosmic microwave background radiation due to a string-induced Doppler shift \[30\]. In the latter case, we would expect to see a different temperature in each of the 3 patches of sky. The relativistic motion of the binding strings could also be an indicator of a binding event: the cosmic string lensing angle is enhanced by a factor of $\gamma$ for moving strings \[31\] (depending on string orientation), which is moderate for usual network motions ($\sim 1.3$, in the radiation-dominated era). The strings motions near a binding site, however, are very relativistic, though over a very small spatial region, and thus would exhibit exaggerated lensing.
FIG. 5: The bottom panel shows the evolution of $\tilde{\Omega}_{cs} = \Omega_{cs}/((8/3)\pi G\mu_{(0,1)})$ for various parameter values, taking the string coupling $g_s = 0.5$. The top panel shows the rate of change in the comoving number density $N\eta^2$; in the scaling regime, $d \log N\eta^2 / d \log \eta = 0$.

near the binding site. Random variation among string velocities within the network should also lead to the existence of some individual fast-moving strings whose lensing will also be enhanced.

- A recent analysis [32] of the direct detectability of cosmic string-generated temperature anisotropies in the data from the upcoming Planck satellite suggests that relatively high tension or fast moving strings within cosmic superstring networks would be marginally within the Planck range of detectability (they estimate that strings with $G\mu \approx 6 \times 10^{-6}$, $v = 1/\sqrt{2}$ would be directly detectable by Planck; for a multi-tension network with a fiducial tension $G\mu \sim 5 \times 10^{-7}$, only strings with a combined $\beta\gamma$ and high-($p, q$) tension enhancement of $\sim 10$ would be seen).

- Direct observation of more than one cosmic string tension from observational techniques, such as gravity wave bursts [33] or gravitational lensing, that are sensitive to a particular string’s tension would be a definite prediction of this kind of network. In the case of lensing, however, the velocity, orientation, and string substructure depen-
FIG. 6: The final scaling-era spectra for a variety of parameter combinations, taking the string coupling $g_s = 0.5$, with $N_{(0,1)}$ normalized to unity and the other number densities altered accordingly.

...dences of the lensing angle may overwhelm this effect for the most probable lensing strings (since over 90% of the strings in our network are $F$, $D$, and $(1, \pm 1)$ strings, whose tensions are all of the same approximate magnitude). For random string orientations, the string lensing angle can vary by as much as a factor of six because of velocity dependent effects, though we expect typical velocity dependent variation of only a factor of two or so (these variations arise because the string lensing angle is proportional both to the sine of the orientation angle of the cosmic string relative to the line of sight as well as $\gamma(1 + \hat{n} \cdot v)$, where $v$ is the string velocity and $\hat{n}$ is a unit vector in the direction between the observer and the string $[31]$). Thus, there are two ways that lensing measurements could indicate the existence of a multi-tension network. The most dramatic would be a single very large ($\gtrsim 10$) variation between two observed lensing angles. Just as compelling, however, would be if a large number of lensing measurements were made with a typical variation among events that is greater than one would expect to arise, statistically, from random string orientations and string velocity directions. In any event, accurate follow-up observations of the Kaiser-
Stebbins effect, where the string’s relative velocity enters the equations differently (in a cross product rather than a dot product) could perhaps allow us to disentangle to some degree the string’s velocity and intrinsic tension. Another possible avenue for discriminating between variation due to string orientation and velocity and intrinsic string tension would be if a series of lensing events were observed along a single long string within a small patch of the sky. Since we expect strings to be curved, it could be possible to observe the same cosmic string at several different orientations. This could allow us to extract the string’s actual tension through a statistical analysis of the events’ lensing angles.

- Another signature would be a mismatch between a particular string tension measured directly – from lensing, perhaps – and a $G\mu$ measurement coming from a technique like CMB fluctuations or pulsar timing analysis that is only sensitive to the averaged network as a whole; however, the effects of string substructure (i.e., string wiggliness), which alter string-generated CMB spectra, could mask more subtle expressions of this effect. If limits on string substructure (see [11]) improve, then a direct detection via lensing of a string with a tension that is several times larger than what CMB limits would lead us to expect for a single-tension network would be a strong indication of the existence of a multi-tension network.

By combining different observations and accumulating sufficient data, one should be able to measure a set of properties of the cosmic strings and so distinguish between different scenarios. This goal will be easier to reach if the true cosmic string tension is closer to today’s observational bound.

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