Uncertainty Sets For Wind Power Generation

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Abstract—As penetration of wind power generation increases, system operators must account for its stochastic nature in a reliable and cost-efficient manner. These conflicting objectives can be traded-off by accounting for the variability and uncertainty of wind power generation. This letter presents a new methodology to estimate uncertainty sets for parameters of probability distributions that capture wind generation uncertainty and variability.

I. INTRODUCTION

Wind power generation (WPG) introduces variability and uncertainty in power system operations [1]. As defined in [1], variability of WPG is the random fluctuation of wind speed caused by physical processes in the atmosphere while uncertainty of WPG results from wind forecast errors [1]. To account for variability and uncertainty, approaches to robust unit commitment (RUC) [2], chance constrained optimal power flow (CC-OPF) [3] and distributionally robust chance constrained [1]. To account for variability and uncertainty, approaches of wind speed caused by physical processes in the atmosphere depend on a rigorous methodology to derive the parameters of probability distributions that capture wind generation uncertainty and variability.

Our analysis is inspired by [3] and [4]. The CC-OPF formulation in [3] considers Gaussian-distributed deviations of WPG with precisely known mean and variance. In an extension, [3] and [4] consider a distributionally robust CC-OPF where the parameters of the Gaussian deviations (both mean and variance) fall within uncertainty sets. The uncertainty set for the mean WPG represents the forecast error while the uncertainty set for the variance continues to represent the variability. Here, we present a methodology to derive these uncertainty sets from historical data. We note that while the underlying distributions are Gaussian, the extension to uncertainty sets results in a non-Gaussian representation of the WPG that better fits historical data.

II. METHODOLOGY

1) Data: Two sources of data are used—historical wind speed measurements at 5-minute resolution from the Goodnoe meteorological station in the Bonneville Power Authority (BPA) power system [6] and one-hour resolution wind speed forecasts produced by the NOAA Rapid Refresh numerical weather prediction model [7] for the same location. The historical measurements and forecasts are detrended by using data from the same calendar season (December–February) and the same period of the day (00:00–04:00 AM).

2) Wind Speed Variability: For any hour-long interval indexed by $t$, wind speed $w_t$ can be written as $w_t = \mu_t + \epsilon_t$ [3], where $\mu_t$ and $\epsilon_t$ are the hourly-average wind speed and intra-hour, zero-mean variability around $\mu_t$ [1], respectively. Hour-long windows of the Goodnoe data [6] are binned by their hourly average $\mu^*$ to generate empirical conditional probability density functions (pdf), $f^E(w_t|\mu^*)$, as in Fig. [1a]. Gaussian distributions are fit to these empirical distributions yielding an estimated $\sigma^*$ for each $\mu^*$. Figure [1b] shows the estimated $\sigma^*$ scales linearly with $\mu^*$ for $\mu^* \in [0, 25]$ m/s—the typical operating range of wind turbines. This linear scaling is consistent with velocity distributions for high Reynolds number atmospheric flows [3]. If there is no error in the hourly-averaged wind speed forecast, i.e. $\mu_t = \mu^*_t$, the wind speed variability is estimated using the linear mapping of $\sigma^*(\mu^*)$ shown in Fig. [1b]. The resulting distribution is parametrized as the Gaussian (normal) distribution, i.e. $N(\mu_t^*, \sigma^*(\mu_t^*)^2)$, and can be used for constructing uncertainty sets for wind speed variability in the RUC [2] and CC-OPF [3].

3) Wind Speed Uncertainty and Variability: Wind forecast errors cause $\mu_t \neq \mu^*_t$. The wind forecast error $\epsilon_t$ is calculated from the NOAA [7] and Goodnoe [6] data as $\epsilon_t(\Delta T) = \mu_t - \mu^*_t(\Delta T)$, where $\mu^*_t(\Delta T)$ is the forecast for hour $t$ made $\Delta T$ hours in advance. The empirical distribution $f^E(\epsilon_t; \Delta T)$ for $\Delta T = 1$ hour is shown in Fig. [2a]. We propose to represent $f^E(\cdot)$ with a generalized Gaussian distribution, $f^G(\cdot)$:

$$f^G(\epsilon_t; \Delta T, \mu^-, \mu^+) = \int_{\mu^-}^{\mu^+} d\mu \mathcal{N}[\epsilon_t; \mu, \sigma^*(\mu)],$$

where $\sigma^*(\mu)$ is the fit from Fig. [1b] and the dependence of $\mu^+$ and $\mu^-$ on $\Delta T$ is suppressed. Note that $f^G(\cdot)$ is no longer a Gaussian. Instead, it is a linear superposition of Gaussian distributions over a range of means $[\mu^-, \mu^+]$ that represent

![Fig. 1: a) $f^E(w_t|\mu^*)$ and their Gaussian best fits for a few $\mu^*$. b) $\sigma$ from a) versus the average wind speed $\mu^*$ and its linear fit $\sigma^*(\mu^*)$.](image-url)
the uncertainty in the wind speed forecast. The best fit $\mu^-$ and $\mu^+$ are computed by solving the optimization problem:

$$\arg\min_{\mu^-, \mu^+} \int [f^G(e_t; \Delta T, \mu^-, \mu^+) - f^E(e_t; \Delta T)]^2 \, de_t,$$

(2)

which minimizes the mean square difference between $f^E(\cdot)$ and $f^G(\cdot)$ and provides a better fit to the historical data than a single Gaussian distribution, as shown in Fig. 2(b). The range $[\mu^+_t, \mu^-_t]$, where $\mu^-_t = \mu^- + \mu^-$ and $\mu^+_t = \mu^+ + \mu^+$, can be interpreted as the bounds of the uncertainty set for the mean wind speed. Fig. 2(b) displays $\mu^-_t$ and $\mu^+_t$ for $\mu_t = 10$ m/s for different $\Delta T$. Using $\sigma^*(\mu^*)$ from Fig. 1(b), we compute the bounds on $\sigma^*$ as $\sigma^*(\mu^+)$ and $\sigma^*(\mu^-)$, which are shown in Fig. 1(b). The ranges $[\mu^+_t, \mu^-_t]$ and $[\sigma^*(\mu^+_t), \sigma^*(\mu^-_t)]$, if converted to wind generation as explained below, can be used in a distributionally robust CC-OPF.

4) Wind Power Uncertainty and Variability: We illustrate the conversion to wind power using the single wind turbine power curve $p(\mu)$ shown in Fig. 3(a). This procedure can be generalized to multiple turbines by using an aggregated wind power curve. The conversion of the range $[\mu^+_t, \mu^-_t]$ is given by:

$$[\mu^+_t, \mu^-_t] \rightarrow [p(\mu^+_t), p(\mu^-_t)].$$

(3)

Figure 3(b) shows $p(\mu^-_t)$ and $p(\mu^+_t)$ corresponding to $\mu^-_t$ and $\mu^+_t$, respectively, from Fig. 2(b). In Fig. 2(b), the growth of $p(\mu^+_t)$ at larger $\Delta T$ is eventually clipped by $p(w)$ as $\mu^+_t$ enters Region III of the turbine curve. If the entire range $[\mu^+_t, \mu^-_t]$ is in Region III, then range $[p(\mu^-_t), p(\mu^+_t)]$ collapses to zero width around the maximum output of the turbine curve.

The conversion of the range $[\sigma^*(\mu^+_t), \sigma^*(\mu^-_t)]$ is shaped by the formulation of the distributionally robust CC-OPF in [4]. Note that $\sigma_p$ can be obtained $\forall p(\mu_t) \in [p(\mu^-_t), p(\mu^+_t)]$ using the relation $\sigma^*(\mu)$ from Fig. 1(b) and the slope $s$ of the turbine curve as $\sigma_p = s(\mu_t) \cdot \sigma^*(\mu_t)$. However, using the wind turbine power curve from Fig. 3(b) results in difficult nonconvexity in distributionally robust formulations (e.g., Eq. 27 in [4]). To avoid this nonconvexity, we assume that the range on the mean value in (Eq. 3) and range on the standard deviations are independent. Therefore, the range on the standard deviations is given by:

$$\sigma^-_p = \min_{\mu \in [\mu^-_t, \mu^+_t]} s(\mu)\sigma^*(\mu)$$

(4)

$$\sigma^+_p = \max_{\mu \in [\mu^-_t, \mu^+_t]} s(\mu)\sigma^*(\mu)$$

(5)

Figure 3(c) shows the conversion to the robust interval on the standard deviation of WPG for the data in Fig. 2(c). b) Ranges for the on the mean WPG (i.e. forecast error) computed using Eq. 3 for the data in Fig. 2(b). c) Ranges on WPG standard deviation (variability) computed using Eqs. 4 and 5 for the same data as in b).

5) Conclusion: We have presented a data-driven method to develop robust intervals for distribution parameters, which preserves the physical relationship between instantaneous and hour-average wind speed and are suitable for uncertainty sets in the RUC and distributionally robust CC-OPF.

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