Power spectral density analysis of relative phase jitter in a twin-soliton molecule

Haochen Tian*, Defeng Zou, Youjian Song and Minglie Hu

Ultrafast Ultrafast Laser Laboratory, Key Laboratory of Opto-electronic Information Science and Technology of Ministry of Education, School of Precision Instruments and Opto-electronics Engineering, Tianjin University, Tianjin 300072, China
*Corresponding author: haochentian@tju.edu.cn

Abstract: Investigation on the relative phase evolution between two bounded optical solitons is essential for its potential applications in development of larger telecommunication capacity of optical fiber transmission lines, resolution improvement in advancing ultrafast characterization approaches and all-optical information storage. In this paper we characterized relative phase jitter power spectral density (PSD) of a soliton molecule pair generated from a passively mode-locked Er:fiber laser (DOI: 10.1364/CLEO_SI.2019.SW3H.6). Through tracking fast shifts of one certain spectral interference fringe, the relative phase noise PSD is obtained by balanced detection. The estimated measurement resolution is at $10^{-13}$ rad$^2$/Hz level and the integrated phase noise from 10 MHz to 100 Hz is only 3.5 mrad. The estimated relative linewidth is far below 1 mHz. Comparison between phase noise PSD and intensity noise PSD indicates that AM-PM conversion plays an important role in relative phase jitter dynamics between the two solitons. It should be pointed out that, the calibration of PSD at high Fourier frequency is not rigid, due to the reason that interpolating process is not a strict calibration method and thus may cause error on the whole noise spectrum. This would bring uncertain to the determination of measurement resolution and integrated phase noise. Despite of this, our spectral interference fringe tracking technique is still attractive for its simplicity and shows potential in ultra-high resolution in phase noise measurement.

1. Introduction

Mode-locked lasers have long been applied as practical platforms for the study of complex dissipative nonlinear dynamics widespread in nature. Twin-soliton molecule, which consists of two strongly bounded optical solitons from a mode-locked laser, has attracted special attention due
to the central role on the investigation of dynamic attraction behaviors in dissipative systems [1]. In particular, study on the relative phase evolution [2-5] between two optical solitons is essential for its potential applications in development of larger telecommunication capacity of optical fiber transmission lines, resolution improvement in advancing ultrafast characterization approaches and all-optical information storage [6].

In-depth investigation into the soliton interaction dynamics based on various dissipative systems requires advanced probing methods. Routinely, observation of relative phase evolution in soliton molecule relies on time-stretch dispersive Fourier-transform (TS-DFT) technique [7]. This technique permits single shot optical spectrum measurement by employing chromatic dispersion-stretched optical pulse to map the broadband spectrum from temporal waveform. Polar diagrams depicting relative phase evolution information have been resolved by this technique in a variety of ultrafast and transient soliton pairs with different soliton separations [8]. Recent years, dynamics of distorted and undistorted soliton molecules [9], dissipative optical soliton molecules [10], optical soliton molecular complexes [11], harmonic mode-locking states [12] and other variety of compound pulsation process [13] have been systematically studied implementing TS-DFT as a real-time probe. Despite TS-DFT technique provides real-time probing method to observe versatile phase variations in round-trip soliton pair, its capacity of fulfilling high-resolution phase noise measurement is restrained by the finite sampling rate of digital oscilloscope.

In this work, we push resolution of phase measurement to a higher level. The relative phase jitter within a twin-soliton molecule has been monitored with high speed and sub-mrad precision by tracking spectral shifts of an optical spectral interference fringe. The relative phase noise power spectral density (PSD) has been characterized up to 10 MHz Fourier frequency with estimated measurement resolution of $10^{-13}$ rad$^2$/Hz. The integrated phase noise from 10 MHz to 100 Hz is only 3.5 mrad and the estimated relative linewidth is far below 1 mHz. Comparison between phase noise PSD and intensity noise PSD indicates that AM-PM conversion plays an important role in relative phase jitter dynamics between the two solitons. It should be pointed out that, the calibration of PSD at high Fourier frequency is not rigid, due to the reason that interpolating process is not a strict calibration method and may cause several dB error on the whole PSD. This would bring uncertain to the determination of measurement resolution and integrated phase noise. Despite of this, our spectral interference fringe tracking technique is still attractive for its simplicity and
shows superior potential in ultra-high resolution in phase noise measurement. Our noise spectrum analysis provides in-depth insight into ultrafast soliton molecular dynamics, which sets essential fundaments to physical mechanisms in nonlinear optical phenomena, larger optical communications capacity and high-resolution spectroscopy.

2. Principle

The principle of the phase noise measurement is shown in Fig. 1. The main idea of this approach is borrowed from a recent carrier-envelope phase noise measurement [14]. Optical spectrum of soliton molecule pair has obvious interferometric fringes due to the similar optical spectra and low intrinsic phase noise between two optical solitons. The fluctuations of relative phase between two optical solitons correspond to the shift of interferometric fringes. To this end, one could filter out two narrow wavelengths symmetrically on the center of rising edge and falling edge in one certain interferometric fringe, as shown in the grey-shaded part in Fig. 1. One could use a balanced photodetector to detect these two wavelengths separately. If the intensity of two selected wavelengths are equal, the resulted subtraction of two photodiodes’ current is zero. However, when the relative phase between two solitons drifts, the interferometric fringes shifts, leading to an intensity difference between two selected wavelengths. Therefore, the output voltage from balanced photodetector would deviate from zero voltage. This error voltage is proportional to the relative phase change. In this way, the relative phase change in soliton molecule pair is convert to change in output of photodiode. We ran a simple simulate on the discrimination slope. Through implementing two same pulses with 35-nm FWHM spectral bandwidth and 1.2-ps pulse separation, which is coordinate with the pulse pair under test in our experiment, the discrimination slope is
quite linear with relative phase change from -0.24 \pi to 0.24 \pi, as shown in the inset in Fig. 1. Moreover, balanced detection would suppress the intensity fluctuation of the optical spectrum itself which leads to a high-resolution, intensity-drift-free phase noise measurement.

3. Experimental setup and result

![Diagram of experimental setup](image)

Fig. 2. Experiment setup. FBG, fiber Bragg grating; Fiber Atten: tunable fiber attenuator. (i) scheme of PSD measurement at low Fourier frequency (ii) scheme of PSD measurement at high Fourier frequency

The experiment setup for relative phase noise PSD measurement of soliton molecules is shown in Fig. 2(a). The soliton molecule under test is generated from a nonlinear amplified loop mirror (NALM) based Er:fiber mode-locked laser. At single-pulse operation condition, the laser outputs a pulse train with 20 mW average power. The pulse duration is 120 fs. Increase of pump power and slightly tune of wave plate would lead to stable soliton molecule from the laser due to the soliton energy quantization effect. The output optical spectrum is shown in Fig. 3(a). The low frequency and high frequency phase jitter PSD have been characterized separately. For low-frequency PSD measurement, we use an optical spectrum analyzer to record a series of output spectra over 30 minutes, as shown in Fig. 2(i). The recorded evolutive optical spectra is shown in Fig. 3(b). The relative phase change can be retrieved from the wavelength shift of interference fringes. The PSD at low frequency (< 100 mHz) is obtained by Fourier transform, as shown in Fig. 3(a) curve (i). For high frequency phase jitter PSD characterization, two reflective FBGs with
center wavelengths of $\lambda_1=1553$ nm and $\lambda_2=1556$ nm filter out two narrow wavelengths belong to the same interference fringe, as shown in Fig. 1(b). The two wavelengths are tuned to similar average power by fiber attenuators and directed into separate detectors of a balanced photodiode detector (Thorlabs PDB420C). The relative phase change in soliton molecule pair will result in a proportional differential voltage while balanced photodetection effectively eliminates common-mode intensity fluctuations as illustrated in the principle section. A fast Fourier transform (FFT) analyzer and a radio frequency (RF) spectrum analyzer are implied to characterize the relative phase noise spectrum of the soliton molecule, as shown in Fig. 3(a) curve (ii). The high frequency PSD (curve ii) has been calibrated by interpolating curve (i) towards high frequency. The estimated measurement noise floor is at $10^{-13}$ rad$^2$/Hz level. The integrated phase noise from 10 MHz to 100 Hz is only 3.5 mrad.

![Fig. 3. (a) Spectrum of soliton molecule. Inset shows two selected wavelengths. (b) Recorded output spectra over 30 min.](image)

**It should be pointed out that, the calibration of curve (ii) can’t be regarded to be 100% strict and rigid.** Interpolating process is not a strict calibration method and may cause several dB error on the whole PSD in high Fourier frequency, bringing uncertain to the determination of measurement resolution and integrated phase noise. To solve this problem, we tried following another two methods to calibrate. Due to the reason that the relative phase ($\Delta\phi$) of soliton pair is quite sensitive to the pumping power and cavity dispersion. The first method is to add pump modulation to the pump diode of the fiber laser. A sinusoidal pump modulation was implied on the laser pump diode in order to change $\Delta\phi$ artificially. The interferometric spectrum was supposed
to shift ‘one fringe’ (which refers that peaks turn into gaps while gaps turn into peaks),
corresponding to over $\pi$ change in $\Delta \varphi$. Accordingly, the error signal would reach its minimum ($\Delta \varphi = -1/2 \pi$) and maximum ($\Delta \varphi = 1/2 \pi$) on oscilloscope successively. In this way, calibration from error voltage to phase could be fulfilled using the linear range of sinusoidal discrimination signal. Unfortunately, when we were modulating the pump power, the soliton molecule mode-locking state always disappeared before reaching $\pi$ change of $\Delta \varphi$. Secondly we made an attempt in cavity dispersion change. It was hard to alter cavity dispersion continuously for an Er:fiber laser without an intra-cavity grating pair or prism pair. To this end, we did this in a Ti:sapphire mode-locked laser with an intra-cavity a prism pair, which emits soliton molecule pair as well. But again, the soliton molecule mode-locking state couldn’t be held on before reaching $\pi$ change of $\Delta \varphi$. As so much statement above, the rigid calibration still remains a technical difficulty to us. We have to admit that, to some extent, our phase noise measurement is a rough estimation and more strict calibration method needs to be explored in the future.

Fig. 4. (a) Phase noise spectral density of soliton molecule. Relative intensity noise of soliton molecule pair is shown in the inset. (b) Frequency noise spectral density of the soliton molecule.

However the general trend of PSD is still convincing. It can be seen from the relative phase noise PSD that, for the Fourier frequency < 1 kHz, the spectrum shows a $1/f^2$ slope characteristic. This slope indicates a random walk nature of the relative phase between the two pulses. For the frequency beyond 1 kHz, the spectrum is dominated by white phase noise and starts to roll off at >1 MHz, meaning that the phases of the two solitons are only bounded at >1 MHz bandwidth. The spikes at ~ kHz arise from acoustic vibrations. We also measure the relative intensity noise (RIN)
of the soliton molecule, as shown in Fig. 3(d). The RIN spectrum has the similar acoustic spikes at ~ \( \text{kHz} \) and rolling off features at > 1 MHz. This indicates that AM-PM conversion plays an important role in relative phase jitter dynamics between the two solitons. Linewidth estimation is realized after converting the phase noise spectrum to frequency noise spectrum, as shown in Fig. 2(c). It can be seen from the \( \beta \)-separation line that, the relative linewidth of soliton molecule pair is far below 1 mHz [15]. Assuming \( 1/f^2 \) slope along lower frequency, the relative linewidth is estimated to be at 1 \( \mu \text{Hz} \) level.

4. Conclusion and outlook

In conclusion, we characterized the relative phase PSD for a temporal soliton molecular for the first time with high resolution. The relative phase noise PSD has been characterized up to 10 MHz Fourier frequency with estimated measurement resolution of \( 10^{-13} \, \text{rad}^2/\text{Hz} \). The integrated phase noise from 10 MHz to 100 Hz is only 3.5 mrad and the relative linewidth is far below 1 mHz. Comparison between phase noise PSD and intensity noise PSD reveals that AM-PM conversion plays an important role in relative phase jitter dynamics between the two solitons. Despite that the calibration of PSD at high Fourier frequency is not rigid, our spectral interference fringe tracking technique still shows superior potential in ultra-high resolution in phase noise measurement. Compared with well-established TS-DFT technique, our phase noise measurement method gets rid of long-term drift of refractive index from kilometer-long fiber [16]. In the meanwhile, our method pushes measurement resolution to ~40 dB higher than TS-DFT technique, where the resolution is mainly limited by finite sampling rate [17].

![Fig. 5. Summary of resolution in representative phase noise measurements](image-url)
On the other side, compared with other carrier-envelope phase and comb-mode phase noise measurements in mode-locked lasers and CW lasers, the estimated resolution of our method exceeds the best performance of phase noise measurement discussed in literature [14, 18-24], as summarized in Fig. 5. This study provides a new perspective towards the soliton molecular dynamics and may contribute to the development of more rigorous models of complex nonlinear systems. Moreover, spectrum analysis of noise dynamics is motivated for both shedding new light on the detailed nature of fundamental physics towards the ultrafast nonlinear optics and extending potential applications of larger telecommunication in optical fiber transmission lines. real-time spectroscopy, etc.

Reference:

[1] P. Grelu, and N. Akhmediev. "Dissipative solitons for mode-locked lasers." Nature photonics 6.2 (2012): 84.

[2] N. N. Akhmediev, A. Ankiewicz, and J. M. Soto-Crespo. "Multisoliton solutions of the complex Ginzburg-Landau equation." Physical review letters 79.21 (1997): 4047.

[3] V. V. Afanasjev, B. A. Malomed, and P. L. Chu. "Stability of bound states of pulses in the Ginzburg-Landau equations." Physical Review E 56.5 (1997): 6020.

[4] B. A. Malomed. "Bound solitons in the nonlinear Schrödinger–Ginzburg-Landau equation." Physical Review A 44.10 (1991): 6954.

[5] A. Zavyalov, et al. "Discrete family of dissipative soliton pairs in mode-locked fiber lasers." Physical Review A 79.5 (2009): 053841.

[6] M. Pang, et al. "All-optical bit storage in a fibre laser by optomechanically bound states of solitons." Nature Photonics 10.7 (2016): 454.

[7] K. Goda, and B. Jalali. "Dispersive Fourier transformation for fast continuous single-shot measurements." Nature Photonics 7.2 (2013): 102.

[8] G. Herink, et al. "Real-time spectral interferometry probes the internal dynamics of femtosecond soliton molecules." Science 356.6333 (2017): 50-54.

[9] J. Igbonacho, et al. "Dynamics of distorted and undistorted soliton molecules in a mode-locked fiber laser." Physical Review A 99.6 (2019): 063824.

[10] J. Peng and H. Zeng. "Build - Up of Dissipative Optical Soliton Molecules via Diverse Soliton Interactions." Laser & Photonics Reviews 12.8 (2018): 1800009.

[11] Z. Q. Wang, et al. "Optical soliton molecular complexes in a passively mode-locked fibre laser." Nature communications 10.1 (2019): 1-11.
[12] X. Liu and M. Pang. "Revealing the Buildup Dynamics of Harmonic Mode-Locking States in Ultrafast Lasers." Laser & Photonics Reviews 13.9 (2019): 1800333.

[13] Z. Wang, et al. "Self-organized compound pattern and pulsation of dissipative solitons in a passively mode-locked fiber laser." Optics letters 43.3 (2018): 478-481.

[14] A. Liehl, et al. "Ultrabroadband out-of-loop characterization of the carrier-envelope phase noise of an offset-free Er: fiber frequency comb." Optics letters 42.10 (2017): 2050-2053.

[15] G. Di Domenico, S. Schilt, and P. Thomann. "Simple approach to the relation between laser frequency noise and laser line shape." Applied optics 49.25 (2010): 4801-4807.

[16] K. Goda and B. Jalali. "Dispersive Fourier transformation for fast continuous single-shot measurements." Nature Photonics 7.2 (2013): 102.

[17] A. Mahjoubfar, et al. "Time stretch and its applications." Nature Photonics 11.6 (2017): 341.

[18] S. Koke, et al. "Direct frequency comb synthesis with arbitrary offset and shot-noise-limited phase noise." Nature Photonics 4.7 (2010): 462.

[19] T. D. Shoji, et al. "Ultra-low-noise monolithic mode-locked solid-state laser." Optica 3.9 (2016): 995-998.

[20] R. Liao, et al. "Active f-to-2f interferometer for record-low jitter carrier-envelope phase locking." Optics letters 44.4 (2019): 1060-1063.

[21] L. C. Sinclair, et al. "Invited Article: A compact optically coherent fiber frequency comb." Review of scientific instruments 86.8 (2015): 081301.

[22] N. Kuse, et al. "All polarization-maintaining Er fiber-based optical frequency combs with nonlinear amplifying loop mirror." Optics express 24.3 (2016): 3095-3102.

[23] C. Li, et al. "All-optical frequency and intensity noise suppression of single-frequency fiber laser." Optics letters 40.9 (2015): 1964-1967.

[24] N. Coluccelli, et al. "Frequency-noise measurements of optical frequency combs by multiple fringe-side discriminator." Scientific reports 5 (2015): 16338.