1E 1207.4-5209: a low-mass bare strange star?

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ABSTRACT

Both rotation- and accretion-powered low-mass bare strange stars are studied, the astrophysical appearances of which are especially focused. It is suggested that low-mass bare strange stars, with weaker ferromagnetic fields than that of normal pulsars, could result from accretion-induced collapses (AIC) of white dwarfs. According to its peculiar timing behavior, we propose that the radio-quiet object, 1E 1207.4-5209, could be a low-mass bare strange star with polar surface magnetic field $\sim 6 \times 10^{10}$ G and a few kilometers in radius. The low-mass bare strange star idea is helpful to distinguish neutron and strange stars, and is testable by imaging pulsar-like stars with the future Constellation-X telescope.

Key words: dense matter — pulsars: individual (1E 1207.4-5209) — pulsars: general — stars: neutron — elementary particles

1 INTRODUCTION

Astrophysics offers an alternative channel for us to explore the fundamental laws in the nature, and the study of quark stars obeys exactly this spirit. It is a clear goal for part of laboratory physicists to find quark-gluon-plasma (or quark matter) in order to research into the problem of the elemental color interaction, whereas to detect astrophysical quark matter may be a shortcut. However the peculiar nature of quark surface has not been noted until recently (Alcock, Farhi & Olinto 1986; Bombaci 1997; Shapiro & Teukolsky 1983). As the strange star model (Li et al. 1999) may work well to understand the various observations of pulsar-like stars, including glitches and free-precessions (Zhou et al. 2004), there are at least three natural motivations to study such stars with low-masses.

(1) The formation of low-mass strange stars is a direct consequence of the presumption that pulsar-like stars are actually quark stars rather than neutron stars. One can not rule out this possibility (but neglected unfortunately) now from either first principles or astrophysical observations. This paper is trying to catch astrophysicist’s attention to the investigation relevant.

(2) “Low-mass” may helpful for identifying strange stars. Bare strange stars can be very low-massive with small radii, while normal neutron stars can not. It is well known that the masses and radii of neutron and strange stars with almost the maximum mass are similar (Glendenning 2000; Weber 1999; Madsen 1999), since no convincing work, neither in theory from first principles nor in observation, has confirmed Baade-Zwicky’s original idea that supernovae produce neutron stars. Therefore, the question of detecting astrophysical quark matter is changed to: how to identify a quark star?

One kind of frequently discussed quark stars are those with strangeness, namely strange stars, which are very likely to exist. There are a few ways (e.g., of cooling behaviors, mass-radius relations, etc.) proposed, by which neutron and strange stars could be distinguished. However the peculiar nature of quark surface has not been noted until 1998 (Usov 2002; Xu 2003). As the strange star model (Xu 2003b) may work well to understand the various observations of pulsar-like stars, including glitches and free-precessions (Zhou et al. 2004), there are at least three natural motivations to study such stars with low-masses.

(1) $R = 1.04 \times 10^6 B_{60}^{-1/3} (M/M_\odot)^{1/3}$ cm, and $R = (4.8, 2.3, 1.0)$ km for $M = (10^{-1}, 10^{-2}, 10^{-3})M_\odot$, if the bag constant $B = 60$ MeV/fm$^3$. But the radii of neutron stars with $\sim 0.5M_\odot$ are generally greater than 10 km, and the minimum mass of a stable neutron star is $\sim 10^{-1}M_\odot$, with radius $R \sim 160$ km (about two orders larger than that of low-mass bare strange stars with similar masses) (Shapiro & Teukolsky 1983). It is consequently possible that we can distinguish neutron and strange stars by direct mea-

This is the reason that one generally believes that neutron and strange stars can not be distinguished by measuring only their masses or radii, but tries to compare the observation-determined mass-radius relations with the theoretical ones in order to identify strange stars.
measurements of the radii\(^2\) of low-mass pulsar-like stars by X-ray satellites. Fortunately, low-mass neutron stars have been cared recently (Carriere, Horowitz & Piekarewicz 2003), the radii of which may correlate with that of \(^{208}\text{Pb}\) because of the stellar central densities to be near the nuclear-matter saturation density. It is then worth studying low-mass strange stars in order to obtain crucial evidence for quark stars.

(3) The conventional method to estimate the polar magnetic fields of radio pulsars has to be modified if no certain reason forces us to rule out the possibility that low-mass bare strange stars could exist in the universe (see §3.1 for details).

Additionally, the identification of a low-mass strange star, with mass \(\lesssim 0.1\,\text{M}_\odot\), may also tell us whether the star is bare, since the radius of a bare strange star is much smaller than that of a crusted one in the low-mass limit (Bombaci, Parenti & Vidaña 2004; Xu 2003a).

The layout of the rest of this paper is as follows. First, a phenomenological view of strange quark matter is introduced in §2. After a study of the general nature of rotation-powered as well as accretion-driven low-mass bare strange stars in §3, we focus our attention on the central compact object 1E 1207.4-5209 in §4, with some investigations of other potential candidates in §5. Although the major points of the paper are to propose candidates of low-mass quark stars according to astrophysical observations, an effort of probing into the origin of such stars is tried in §6. Finally, conclusions and discussions are presented in §7.

2 QUARK MATTER PHENOMENOLOGY

Strange quark stars are composed of quark matter with almost equal numbers of \(u, d,\) and \(s\) quarks. There are actually two different kinds of quark matter to be investigated in lab-physics and in astrophysics, which appear in two regions in the quantum chromo-dynamics (QCD) phase diagram (Fig.1). Quark matter in lab-physics and in the early universe is temperature-dominated (temperature \(T \gg 0\), baryon chemical potential \(\mu_B \gg 0\)), while that in quark stars or as cosmic rays is density dominated \((T \sim 0, \mu_B \gg 0)\). Previously, Monte Carlo simulations of lattice QCD (LQCD) were only applicable for cases with \(\mu_B = 0\). Only recent attempts are tried at \(\mu_B \neq 0\) (quark stars or nuggets) in LQCD. We have then to rely on phenomenological models to speculate on the properties of density-dominated quark matter.

In different locations of the diagram (Fig.1), besides that the interaction strength between quarks and gluons is weak or strong, the vacuum would have different features and is thus classified into two types: the perturbative-QCD (pQCD) vacuum and nonperturbative-QCD (QCD) vacuum. The coupling is weak in the former, but is strong in the later. Quark-antiquark (and gluons) condensations occur in QCD vacuum (i.e., the expected value of \(\langle \bar{q} q \rangle \neq 0\), but not in pQCD vacuum. The chiral symmetry is spontaneously broken in case the vacuum is changed from pQCD to QCD vacuums, and quarks become then massive constituent ones. LQCD calculations (Kogut et al. 1991) show that the value of \(\langle \bar{q} q \rangle\) increases when the color coupling becomes strong (i.e., as the temperature or the baryon density decrease). Therefore, we note that the quark de-confinement and the chiral symmetry restoration may not take place at a same time.

Considerable theoretical efforts have been made in past years to explore the QCD phase diagram. When \(T\) or \(\mu_B\) are extremely high, there should be quark-gluon plasma (QGP) phase because of the asymptotic freedom, and vacuum is of pQCD. However, in a relatively lower energy limit, especially in the density-dominated region, the vacuum is phase-converted to QCD one but the quarks could be still deconfined. It is a hot point to investigate the possibility that real quarks may also be condensed (i.e., \(\langle \bar{q} q \rangle \neq 0\)) simultaneously when \(\langle \bar{q} q \rangle \neq 0\), the so-called color-superconducting (CSC) phases (for recent reviews, see, e.g., Ren (2004), Rischke (2004)). Actually two CSC phases are currently discussed. One corresponds to Cooper pairing among the two flavors of quarks \((u\) and \(d)\) only, the two-flavor color super-conductivity (2SC) phase, in case that \(s\) quark is too massive to participate. Another one occurs at higher \(\mu_B\) in which \(s\) quarks are relatively less massive and are thus involved in Cooper pairing, the color-flavor locked (CFL) phase.

However, another possibility can not be ruled out: \(\langle \bar{q} q \rangle = 0\) while \(\langle \bar{q} q \rangle \neq 0\). When \(T\) is not high, along the reverse direction of the \(\mu_B\) axis, the value of \(\langle \bar{q} q \rangle\) increases, and the color coupling between quarks and gluons becomes stronger and stronger. The much strong coupling may favor the formation of \(n\)-quark clusters \((n: \text{the number of quarks in a cluster})\) in the case (Xu 2003a). Such quark clusters could be very likely in an analogy of \(\alpha\) clusters moving in nuclei, which are well known in nuclear physics. Recent experimental evidence for multi-quark \((n > 3)\) hadrons may increase the possibility of quark clustering. The clusters are localized\(^3\) to become classical (rather than quantum) par-

\(^2\) Also the very distinguishable mass-radius \((M - R)\) relations of neutron and quark stars are helpful. The study of Lane-Emden equation with \(n = 2/3\) (corresponding to the state of non-relativistic neutron gas with low masses) results in \(M \propto R^{-3}\), whereas, for low-mass quark stars, \(M \propto R^3\), due to the color confinement of quark matter.

\(^3\) We apply “local” to refer that “quark wavefunctions do almost not overlap”. In this sense, localized clusters can still move from place to place when \(T\) is high, but could be solidified at low \(T\).
ticles when the thermal de Broglie wavelength of clusters \( \lambda \sim h/\sqrt{3 m k T} < l \sim [3 n/(4 \pi f n_{0})]^{1/3} \) \((m: \text{the mass of clusters, } l: \text{the mean cluster distance, } n_{0}: \text{the baryon number density, } f: \text{quark flavor number}),\) assuming no interaction is between the clusters. Calculation based on this inequality for \( f = 3 \) shows that cluster localization still exists even in temperature \( T \sim 1 \text{ MeV} \) if \( n \sim 10^{2} \). In addition, the interaction in-between, which is neglected in the inequality, would also favor this localization.

In case of negligible interaction, quark clusters would become a quantum system in case of low temperature. However, the interaction is certainly not weak since the vacuum is of QCD \((\langle q \bar{q} \rangle \neq 0)\), now, a competition between condensation and solidification appears then, just like the case of laboratory low-temperature physics. Quark matter would be solidified as long as the interaction energy between neighboring clusters is much larger than that of the kinetic thermal energy. This is why only helium, of all the elements, shows superfluid phenomenon though other noble elements have similar weak strength of interaction due to filled crusts of electrons. The essential reason for the occurrence of CSC is that there is an attractive interaction between two quarks at the Fermi surface. But, as discussed, much strong interaction may result in the quark clustering and thus a solid state of quark matter. In conclusion, a new phase with \( \langle q \bar{q} \rangle \neq 0 \) but \( \langle q \bar{q} \rangle = 0 \) is suggested to be inserted in the QCD phase diagram (Fig.1), which could exist in quark stars.

Astrophysics may teach us about the nature of density-dominated quark matter in case of those theoretical uncertainties. There are at least two astrophysical implications of the solid quark matter state proposed.

1. Quark stars in a solid state can be used to explain naturally the observational discrepancy between glitches and the solid quark matter state proposed.

2. Ferromagnetism may occur in a solid quark matter. In this case, the magnetic field derived from \( P \) and \( \dot{P} \) is

\[
B = 6.4 	imes 10^{19} \sqrt{P \dot{P}} \text{ G},
\]

if “typical” values \( I = 10^{45} \text{ g cm}^{2} \) and \( R = 10^{6} \text{ cm} \) are assumed. Note: the field is only half the value in Eq. (4) if one simply suggests \( \mu = BR^{3} \) (Manchester & Taylor 1977).

However, neutron star’s \( I \) and \( R \) change significantly for different equations of state, or for different masses even for a certain equation of state (Lattimer & Prakash 2001). This means that the “typical” values may actually be not approximately constants. The inconsistency becomes more serious if pulsar-like stars are in fact strange quark stars since such a quark star could be as small as a few hundreds of baryons (strangelets).

Let’s compute \( P \) and \( \dot{P} \) for a quark star with certain mass \( M \) and radius \( R \). First we approximate the moment of inertia to be \( I \approx 2 M R^{2}/5 \) (i.e., a star with uniform density). This approximation is allowed for low-mass strange stars (Alcock, Farhi & Olinto 1986). In this case, the magnetic field derived from \( P \) and \( \dot{P} \) is

\[
\begin{align*}
\mu &= a M^{\alpha} + b R^{\beta},
\end{align*}
\]

where \( \{a, \alpha; b, \beta\} \) is a parametric set. If the magnetized momentum per unit volume is a constant \( \mu_{c} \), one has \( \{a = 0, \alpha = 0; b = 4 \mu_{c}/3, \beta = 3\} \). In case the magnetized momentum per unit mass is a constant \( \mu_{m} \), one has then

\[
\mu_{m} = \frac{4 \mu_{c} R^{3}}{3M},
\]

for different equations of state, or for different masses even for a certain equation of state (Lattimer & Prakash 2001).
described by a simplified version of the MIT bag model, in which are listed in Table 1. The mass-radius relations for four cases possible are investigated, the calculational results when 

\[ P = \frac{20\pi^2 (a\mu^3 + bR^3)^2}{3c^5 M R^2} \]

This equation shows that a pulsar with certain initial parameters (\( M \) and \( R \)) evolves along constant \( \dot{P} \). Integrating Eq. (7) above, one obtains the pulsar age \( T \),

\[ T = \frac{P^2 - P_0^2}{2\dot{P}} \]

where \( P_0 \) is the initial spin period. The age \( T = T_c \equiv P/(2\dot{P}) \) when \( P_0 < P \).

The mass of a pulsar is detectable dynamically if it is in a binary system, but precise mass estimates are only allowed by the measurement of relativistic orbital effects. It is therefore to determine the model parameters (\( \mu_m \) and \( \mu_v \)) in Eq. (4) by pair neutron star systems (Thorsett & Chakrabarty 1999; Lyne et al. 2004). Four cases possible are investigated, the calculation results of which are listed in Table 1. The mass-radius relations for the calculation are in the regime of strange quark matter described by a simplified version of the MIT bag model, in which the relation between pressure \( P \) and the density \( \rho \) is given by

\[ P = \frac{1}{3}(\rho - 4B) \]

where the bag constant \( B \) is chosen to be 60 MeV/fm\(^3\) (low limit) and 110 MeV/fm\(^3\) (up limit), respectively. It is assumed that the mass and radius of a star do not change significantly during quark-clustering and solidification as the star cools.

The values of \( \mu_m \) and \( \mu_v \) are grouped into two classes in Table 1. One class has higher \( \mu_m \) or \( \mu_v \) for normal pulsars (B2303 & J0737B only), but the other (of millisecond pulsars) has lower \( \mu_m \) or \( \mu_v \). We just average the values of B2303 & J0737B for indications of normal pulsars; Model 1: \( \mu_m = 4.97 \times 10^{-4} \) G-cm\(^3\)-g\(^{-1}\), Model 2: \( \mu_m = 3.88 \times 10^{-4} \) G-cm\(^3\)-g\(^{-1}\), Model 3: \( \mu_v = 2.57 \times 10^{11} \) G, and Model 4: \( \mu_v = 4.21 \times 10^{11} \) G.

Figure 2. The \( \dot{P} \) values based on Eq. (4). Model 1: \( \mu_m = 4.97 \times 10^{-4} \) G-cm\(^3\)-g\(^{-1}\), Model 2: \( \mu_m = 3.88 \times 10^{-4} \) G-cm\(^3\)-g\(^{-1}\), Model 3: \( \mu_v = 2.57 \times 10^{11} \) G, and Model 4: \( \mu_v = 4.21 \times 10^{11} \) G.

Table 1. Pulsars and the model parameters derived. Models numbered 1, 2, 3, and 4 are for \{\( \mu_m = \alpha = 1; b = 0, \beta = 0; B = 60\) MeV/fm\(^3\)}; \{\( \mu_m = \alpha = 1; b = 0, \beta = 0; B = 110\) MeV/fm\(^3\)}; \{\( \mu = 0, \alpha = 0, b = 4\pi\mu_v / 3, \beta = 3; B = 60\) MeV/fm\(^3\)}; and \{\( \mu = 0, \alpha = 0, b = 4\pi\mu_v / 3, \beta = 3; B = 110\) MeV/fm\(^3\)}

\[
\begin{array}{cccccccc}
\text{Pulsars} & P (\text{ms}) & \dot{P} (\text{s}/\text{ms}) & M (M_\odot) & \mu_m (\text{model 1}) & \mu_m (\text{model 2}) & \mu_v (\text{model 3}) & \mu_v (\text{model 4}) \\
J1518a & 40.94 & 2.73E-20 & 1.56 & 4.27E-7 & - & 2.31E+8 & - \\
B1534a & 37.90 & 2.42E-18 & 1.34 & 4.00E-6 & 3.10E-6 & 2.10E+9 & 3.52E+9 \\
B1913a & 59.03 & 8.63E-18 & 1.44 & 9.25E-6 & 6.89E-6 & 4.98E+9 & 8.99E+9 \\
B2127a & 30.53 & 4.99E-18 & 1.35 & 5.15E-6 & 3.97E-6 & 2.72E+9 & 4.57E+9 \\
B2303 & 1066 & 5.69E-16 & 1.30 & 2.38E-4 & 1.71E+11 & 2.80E+11 & \\
J0737 & 22.70 & 1.74E-18 & 1.34 & 2.62E-6 & 2.03E+9 & 2.31E+9 & \\
J0737B & 2773 & 8.85E-15 & 1.25 & 6.45E-4 & 4.33E+11 & 5.61E+11 &
\end{array}
\]
of bare strange stars can be well approximated by (Alcock, Farhi & Olinto 1986), because of Eq. (9).

\[ M = \frac{4}{3} \pi R^3 (4\bar{B}), \]  

(10)

where \( \bar{B} = (60 \sim 110) \text{ MeV}/\text{fm}^3 \), i.e., \((1.07 \sim 1.96) \times 10^{14} \text{ g/cm}^2 \). Combining Eq. (11) and Eq. (10), one comes to

\[ \dot{P} \bar{P} = \frac{320 \pi^3 \mu_m^2}{9 c^3} \bar{B} R, \]

(11)

for models 1 and 2, and

\[ \dot{P} \bar{P} = \frac{20 \pi^3 \mu_m^2}{9 c^3} R, \]

(12)

for models 3 and 4 in the low-mass limit.

Lines of constant potential drops in the \( P - \dot{P} \) diagram.

The potential drop in the open-field line region is essential for pulsar magnetospheric activity. We adopt only Model 1 in the following indications. From Eq. (7), (8), and (10), one has

\[ B = \frac{32 \pi}{3} \mu_m. \]

(13)

This shows that the polar fields of homogeneously magnetized quark stars, with certain \( \mu_m \), of different low masses are approximately the same. The potential drop between the center and the edge of a polar cap is (Ruderman & Sutherland 1975)

\[ \phi = \frac{2 \pi^2 R^3 \bar{B} \dot{P}}{c^2}. \]

(14)

In case of approximately constant \( \mu_m \), Eq. (14) can be conveniently expressed as, from Eq. (13),

\[ \phi = \frac{64 \pi^3}{3 c^2} \mu_m R^3 \dot{P}^2. \]

(15)

From Eq. (11) and (15), one has

\[ \dot{P} \bar{P}^3 = \frac{2.026 \times 10^6 c^2}{\bar{B}^3 \mu_m \phi}. \]

(16)

However, if the variance of pulsar masses (or radii) is smaller than that of \( \mu_m \), it is better to express potential drop as, from Eq. (7) and Eq. (14),

\[ \phi = \frac{2 \pi}{5c} \sqrt{16 \pi \bar{B} R^2 \dot{P} P^{-3}}, \]

(17)

where Eq. (10) is included. The lines of constant \( \phi \) are drawn in Fig.3, based on both Eq. (10) (dashed lines labelled a, b, and c, with a slop of \(-1/3\)) and Eq. (17) (dash-dotted lines labelled d and e, with a slop of 3). The parameters for these lines are as following: a: \( \phi = 10^{12} \text{ V}, \mu_m = 10^{-6} \text{ G cm}^3 \text{ g}^{-1}; \) b: \( \phi = 10^{10} \text{ V}, \mu_m = 10^{-6} \text{ G cm}^3 \text{ g}^{-1}; \) c: \( \phi = 10^{12} \text{ V}, \mu_m = 10^{-7} \text{ G cm}^3 \text{ g}^{-1}; \) d: \( \phi = 10^{12} \text{ V}, R = 10 \text{ km}; \) and e: \( \phi = 10^{11} \text{ V}, R = 1 \text{ km}. \)

Pair production mechanisms are essential for pulsar radio emission. A pulsar is called to be “death” if the pair production condition can not be satisfied. A general review of understanding radio pulsar death lines can be found in (Zhang 2003). Although a real deathline depends upon the dynamics of detail pair and photon production, the deathline can also be conventionally taken as a line of constant potential drop \( \phi \). It is found in Fig.3 that the slope of constant \( \phi \) is \(-1/3\) (or 3) if the scattering distribution of pulsar points in \( P - \dot{P} \) diagram is due to different masses (or polar field \( B \)) but with constant \( \mu_m \) (or mass or radius). The deathline slope may be expected to be between \(-1/3\) and 3 if the distributions of mass and polar field are combined.

### 3.2 Accretion-dominated spindown

The physical process of accretion onto rotating pulsar-like stars with strong magnetic fields is very complex but is essential to know the astrophysical appearance of the stars (e.g., the variation of X-ray flux, the evolutionary tracks, etc.,) which is still not be understood well enough (Lipunov 1992). Nevertheless, it is possible and useful to describe the accretion semi-quantitatively.

For an accretion scenario in which the effect of kinematic energy of accreted matter at infinite distance is negligible (such as the case of supernova fall-back accretion), three typical radii are involved. The radius of light cylinder of a spinning star with period \( P \) is

\[ r = \frac{cP}{2\pi} = 4.8 \times 10^9 P \text{ cm}. \]

(18)

If all of the accretion material is beyond the cylinder, the star and the accretion matter could evolve independently. The magnetospheric radius, defined by equating the kinematic energy density of free-fall particles with the magnetic one \( B^2/(8\pi) \), is

\[ r_m = \left( \frac{B^2 R^6}{M \sqrt{2G M}} \right)^{2/7}, \]

(19)

where \( M \) is the accretion rate. In the low-mass limit of bare strange stars, considering the mass-radius relation of Eq. (10), Eq. (19) becomes

\[ r_m = \left( \frac{4}{3\pi c^2} \right)^{1/7} \bar{B}^{-1/7} R^{24/7} \dot{P}^{3/7} M^{-2/7}, \]

(20)
where the bag constant \( \hat{B} = \hat{B}_{60} \times 60 \text{ MeV/fm}^3 \). If a star is homogeneously magnetized per unit mass (i.e., in Models 1 and 2), according to Eq. (16), one has from Eq. (20)

\[
\begin{align*}
\rho_m & = \left( \frac{8\pi^3}{27} \right)^{1/3} \hat{B}^{1/3} \mu_0^{-1/3} R_0^{2/3} \hat{M}^{-2/3} \\
& = 4.9 \times 10^8 \hat{B}_{60}^{1/3} \mu_0^{-1/3} R_0^{2/3} \hat{M}^{-2/3}.
\end{align*}
\]

Due to the strong magnetic fields around a spinning star, matter is forced to co-rotate, and both gravitational and centrifugal forces work. At the so-called corotating radius \( r_c \), these two forces are balanced,

\[
\begin{align*}
r_c & = \left( \frac{GM}{4\pi^2} \right)^{1/3} \rho V^2/3 = 1.2 \times 10^{-3} \hat{M}^{1/3} P^{2/3}.
\end{align*}
\]

In the low-mass limit, one has from Eq. (10) and Eq. (22)

\[
\begin{align*}
r_c & = \left( \frac{4G}{3\pi} \right)^{1/3} \hat{B}^{1/3} R P^{2/3} \\
& = 145 \hat{B}_{60}^{1/3} R P^{2/3}.
\end{align*}
\]

In another case, in which the kinematic energy at infinity is not zero (i.e., ISM or stellar wind accretion), besides those three radii, an additional one is the accretion radius \( r_a \), at which the total energy (kinematic and gravitational forces) is zero,

\[
r_a = 2GMV_\infty^{-2},
\]

where \( V_\infty \) is the relative velocity of the star to the surrounding media. The motion of matter only at a radius \( r < r_a \) could be affected by gravity, and the mass capture rate is then

\[
\begin{align*}
\dot{M}_c & = \pi r_a^2 \rho V_\infty = 4\pi G^2 \hat{M}^2 \rho V_\infty^{-3},
\end{align*}
\]

where \( \rho \) is the density of diffusion material.

Due to the centrifugal inhibition, since the radius of matter nearest to the star could be \( r_m \), massive accretion onto stellar surface is impossible when \( r_m > r_c \). This is the so-called supersonic propeller spindown phase. A star spins down to the equilibrium period \( P_{eq} \), defined by \( r_m = r_c \). In the low-mass limit, one has, from Eq. (20) and Eq. (22),

\[
\begin{align*}
P_{eq} & = 0.72 G^{-5/7} \hat{B}^{-5/7} \hat{B}^{6/7} R^{3/7} \hat{M}^{-3/7},
\end{align*}
\]

or assuming a homogenous magnetic momentum per unit mass, from Eq. (21) and Eq. (22),

\[
\begin{align*}
P_{eq} & = 15G^{-5/7} \hat{B}^{1/7} \hat{B}^{6/7} \mu_0^{-1/7} R^{3/7} \hat{M}^{-3/7}.
\end{align*}
\]

However, accretion with rate \( \dot{M} \) onto the stellar surface is not possible, although the centrifugal barrier is not effective when \( P > P_{eq} \), until the star spins down to a so-called break period \( \dot{P}_{br} \) [Davies, Fabian & Pringle 1979 Davies & Pringle 1981 [Khanlari et al. 2006]]

\[
\begin{align*}
P_{br} & = 60^{16/21} M_1^{5/7} \dot{M}_1^{-4/21} s \\
& = 360 \hat{B}_{60}^{-4/21} B_1^{16/21} M_1^{-5/7} R_6^{-12/7} s \\
& = 4.9 \hat{B}_{60}^{16/21} B_1^{16/21} M_1^{-5/7} R_6^{-12/7} s,
\end{align*}
\]

in the low-mass limit, where Eq. (3), Eq. (10), and Eq. (13) have been included, and the convention \( Q = 10^9 \hat{Q}_9 \) has been adopted. Pulsars with periods between \( P_{eq} \) and \( P_{br} \) do still spin down. This phase is called as subsonic propeller. Only a negligible amount of accretion matter can penetrate into the magnetosphere (onto the stellar surface) during both the supersonic and subsonic propeller phases (Khanlari et al. 2006), and the expected accretion luminosity is thus very low.

How to determine quantitatively the spindown rate when a pulsar is in those two propeller phases? No certain answer known hitherto for the propeller torques, even not to be certain about the accretion configuration (disk or sphere). Nonetheless, if the spinup effect of matter accreted onto stellar surface is neglected, the spindown rate can be estimated according to the conservation laws of angular-momentum and/or rotational-energy [Davies, Fabian & Pringle 1979]. The escape velocity at radius \( r \) is \( \sqrt{2GM/r} \). Approximating the stellar angular-momentum loss rate \( -2\pi\dot{P}/P^2 \) to that of accretion material near \( r_m \) (based on the angular-momentum conservation), we have

\[
\dot{P}_I = \frac{\pi}{2} \hat{B}^{1/2} I^{-1} M\hat{r}_m^{1/2} P^2 \]

\[
\dot{P}_E = \frac{G}{5\pi} M^{-1}\hat{r}_m^{-1} P^3 \]

4 THE CASE OF 1E 1207.4-5209

The radio-quiet central compact object in the supernova remnant PKS 1209-51/52, 1E 1207.4-5209, is a unique pulsar-like star which is worth noting since we known much information about it: the rotating period \( P = 0.424 \) s, the cyclotron energy \( E_{cyc} = 0.7 \) keV [Bignami et al. 2003], the age \( T \approx 7 \) kys estimated from the remnant, with an uncertainty of a factor of 3 [Roger et al. 2003], the timing properties [Zavlin et al. 2004], and the thermal X-ray spectrum of long-time observations [De Luca et al. 2004]. The distance to the remnant is \( d \approx 1.3 \sim 3.9 \) kpc, the X-ray flux in a range of 0.4-8 keV is \( 2.3 \times 10^{12} \text{ erg cm}^{-2} \text{ s}^{-1} \), and the corresponding X-ray luminosity is then \( L = (0.47 \sim 4.2) \times 10^{33} \text{ erg/s} \) [Pavlov et al. 2003]. However, the more we observe, the knottier astrophysicists can have a model to understand its nature.

Two issues are addressed at first. One is about its absorption lines. The lines at 0.7, 1.4, and 2.1 keV (and possibly 2.8 keV) are identified, which are phase-dependent [Mereghetti et al. 2002]. These imply a cyclotron-origin of the features [Bignami et al. 2003 Xu et al. 2003], although this possibility was considered to be unlikely when discovered by Chandra [Saulat et al. 2002]. However, there are still two questions relevant to this issue.

1) Where does the absorption form (near the stellar surface or in the magnetosphere)? An \( e^\pm \) plasma surrounding a magnetized neutron star, maintained by the cyclotron-resonance process, was suggested to prevent a direct detection of the stellar surface in X-ray band, the existence of which seems to explain the age dependence of the effective radiating area [Ruderman 2003]. The cyclotron lines would thus form at a height where resonant scattering occurs. In the regime, all neutron stars with high B-fields should present cyclotron absorption in their thermal X-ray spectra; but this conflicts with the observations. In addition, the physics of the plasma is still not well studied, and its density and stability are not sure. An alternative and intuitive suggestion is that the line forming region is near the stellar surface. In this case, we may need bare quark surface, with an electron layer of density \( \sim 10^{32} \text{ cm}^{-3} \) and thickness of a few thousands of fermis, in order to explain those absorption
dips. Due to the degeneracy of electrons, only electrons near the surface of fermi-sea can be exited to higher levels, the number of which is energy-dependent. For instance, the number of electrons which resonantly scatter photons with energy \( \sim 1.4 \) keV could be about the double of that with \( \sim 0.7 \) keV. The number of electrons, which are responsible to cyclotron-resonant of photons with higher energy, is therefore larger although absorption cross-section is smaller. Another factor, which may also favor more electrons to absorb higher energy photons, could be of radiative transfer process (e.g., a certain layer might be optically thick at \( \sim 0.7 \) keV, but optically thin at \( \sim 2.1 \) eV), but a detail consideration on this is necessary in the future. We conclude then that it is reliable to assume a surface origin for the cyclotron-resonant lines.

(2) Is the cyclotron resonant in terms of electrons or protons? The fundamental electron cyclotron resonant lies at \( \Delta E_e = 11.6B_{12}\sqrt{1-r_s/R} \) keV, while the proton one at \( \Delta E_p = 6.3B_{12}\sqrt{1-r_s/R} \) eV, where \( r_s \equiv 2GM/c^2 \) is the Schwartzchild radius. For a star with 10\(^7\) cm and 1\(M_\odot\), the factor \( \sqrt{1-r_s/R} = 0.84 \). In case of lower mass strange stars, \( r_s/R \sim R^2 \) is smaller, and the factor is closer to 1. We thus just approximate the factor to be 1, the field is then \( B = 6 \times 10^{10} \) G in terms of electrons, \( B = 10^{14} \) G of protons. If the lines are proton-originated, there is still two scenarios. One is that the multipole fields have strength \( B_m \sim 10^{14} \) G, but the global dipole field \( B_d \sim 3 \times 10^{12} \) G is much smaller in order to reconcile the spindown rate expected from Eq. (2). This means that the stellar surface is full of flux loops with typical length \( l_{loop} \), which can be estimated to be [Thompson & Duncan 1993].

\[
l_{loop} \sim \frac{B_p}{B_m} R \sim 10^5 R_6 \text{ cm.} \tag{31}
\]

The maximum release energy due to magnetic reconnection could be \( \sim (B_{12}/8\pi)\Theta_{loop} \sim 10^{12} \) erg. If the dynamical instability takes place at a short timescale of \( \sim 1 \) s, as observed in soft \( \gamma\)-ray repeaters, bursts with \( \sim 10^{12} \) erg/m\(^3\) might have been detected in 1E 1207.4-5209; but we do not. Another scenario is that the dipole field of 1E 1207.4-5209 is \( \sim 10^{14} \) G, but the accreted material onto the stellar surface contributes a positive angular momentum. The magnetodipole radiation should spin down the object at a rate \( P = 2 \times 10^{-10} \) s/s, based on Eq. (1), which is much larger than observed \( \sim 10^{-14} \) s/s. This discrepancy might be circumvented if accreted matter contributes a positive momentum. Yet, this can only be possible when \( P > P_{br} \), which results in an accretion rate, according to Eq. (22), \( \dot{M} > 0.9 \times 10^{15} \) g/s and an X-ray luminosity \( L_x > 10^{35} \) erg/s, to be much larger than observed \( L \sim 10^{33} \) erg/s, for typical neutron star parameters. In both pictures, however, a strange thing is: why does not this star with magnetar-field show magnetar-activity (e.g., the much higher luminosity \( \sim 10^{34-35} \) erg/s of persistent X-ray emission, observed in anomalous X-ray pulsars and soft \( \gamma\)-ray repeaters)? In addition, there is still no idea to answer why the feature strength is similar at \( \sim 0.7 \) and \( \sim 1.4 \) keV (and even the appearance of a line at \( \sim 2.1 \) keV), due to the high mass-energy of protons (\( \sim 10^3 \) times that of electrons), as discussed in the case of SGR 1806-20 [Xu et al. 2003]. We therefore tend to suggest an electron-cyclotron-origin of the absorption features.

Another issue is about its radius detected. In principle, one can obtain a radius of a distant object by detecting its spectrum (fitting the spectrum gives out a temperature \( T \) if a Plankian spectrum approximation is good enough) and flux \( F \), through \( F = \sigma T^4 \times R^2/d^2 \) (\( R \) is radiation radius, not the coordinate radius \( R_{coord} \) in the Schwartzchild metric. \( R = R_{coord}/\sqrt{1-r_s/R_{coord}} \) if the spectrum is Plankian), if the distance \( d \) is measured by other methods (e.g., parallax). However, we are not sure if the thermal spectrum of 1E 1207.4-5209 is really Plankian, and we do not know the distance neither. Nonetheless, since the spectrum depends also on the ISM absorption (the neutral hydrogen density is supposed to be known), one may fit the spectrum by free parameters \( T, d, \) and \( R \). Note that radii determined in this way are highly uncertain. A radius of \( \sim 1 \) km was obtained in single-blackbody models by ROSAT [Mereghetti, Bignami & Caraveo 1996] and ASCA [Yasisht et al. 1997] observations, whereas a 10 km-radius was suggested in an atmosphere model of light-elements by [Zavlin, Pavlov & Trümper 1998]. An XMM-Newton observation yields recently two-blackbody radii fitted: \( R = 0.8 \) km and 4.6 km for hotter and cooler components, respectively [De Luca et al. 2004]. The possibility, that 1E 1207.4-5209 may have a radius to be much smaller than 10 km of conventional neutron stars, is thus not ruled out.

Now we turn to a low-mass bare strange star model for 1E 1207.4-5209. Besides the absorption features, the most outstanding nature, that makes the pulsar be puzzling enough, should be its timing behavior: it does not spin down stably but seems to spin up occasionally. Furthermore, two or more probability peak-frequencies are identified during each of the five observations [Zavlin et al. 2004]. Pulsar glitches and Doppler shift in a binary system were proposed for the “spinup” [Zavlin et al. 2004] [De Luca et al. 2004], but the multi-frequency distributions are still not well understood. An alternative model proposed here is that the pulsar is a low-mass bare strange star which is at a critical point of its subsonic propeller phase, \( P > P_{br} \).

Steady accretion onto a magnetized and spinning star is possible only if \( P > P_{br} \), when magnetohydrodynamic (e.g., Rayleigh-Taylor and Kelvin-Helmholtz) instabilities occur at the magnetospheric boundary [Arons & Lea 1976, Elsner & Lamb 1981]. Before the onset of the instabilities, accretion plasma can only penetrate into the magnetosphere by diffusion, with a rate much smaller than \( \dot{M} \) [Kulsanov 2003]. Propeller torque may spin down a star to a period of \( P \geq P_{br} \). At this period, steady accretion does not occur until the density just outside the boundary increases to a critical density of \( \rho_{crit} \sim 7\rho_m \) when the Rayleigh-Taylor instability happens [Elsner & Lamb 1984], where \( \rho_m \sim M/(4\pi r_{in}^3\sqrt{2GM/r_{in}}) \). The star should then be spun up by steady accretion torque\(^5\). However, increasing the spin-frequency may dissatisfy the necessary condition for steady accretion, \( P > P_{br} \). The star will then return back

\(^5\) In case of wind-fed accretion in binary systems, the star will spin up (or down) if the accreted material has a positive (negative) angular momentum. Whereas, in case of fallback disk accretion or ISM-fed accretion, the momentum of accreted matter should be positive, and the steady-accretion-induced torque leads the star to spin up. We are not considering accretion in a binary system here.
to a subsonic propeller phase when the decreasing period is low enough, and it spins down again. We could therefore expect an erratic timing behavior when a pulsar is at this critical phase, \( P \sim P_{\text{br}} \). The object 1E 1207.4-5209 could be an ideal laboratory for us to study the detailed physics in such an accretion stage.

For 1E 1207.4-5209, setting \( P_{\text{br}} = 0.424 \, \text{s}, B_{12} = 0.06 \), one has \( \mu_{\text{br}} = 9.1 \times 10^{-6} B_{110}^{-1} \text{G-cm}^3/\text{g}^{-2} \) from Eq. (18), which is close to that values of millisecond pulsars (Table 1). The accretion rate reads, from Eq. (25),

\[
M = 10^{14} B_{60}^{-4/15} R_{5}^{2/5} \, \text{g/s},
\]

and the magnetospheric radius is then, from Eq. (20),

\[
r_m = 2.5 \times 10^{5} B_{60}^{-1/15} R_{5}^{2/5} \, \text{cm}.
\]

As for the instantaneous spin-down rate in the subsonic propeller phase, we apply Eq. (24) and Eq. (50) for estimation, according to the momentum and energy conservations, respectively. One has then,

\[
\left\{ \begin{array}{l}
\dot{P}_s = 1.9 \times 10^{-12} B_{60}^{-4/5} R_{5}^{4/5} \\
\dot{P}_k = 1.3 \times 10^{-13} B_{60}^{-1/5} R_{5}^{-2/5}.
\end{array} \right.
\]

These results imply that the instantaneous period increase could be one or two orders larger than the averaged one (\( \sim 10^{-14} \, \text{s/s} \)) if \( R_5 \sim 1 \). A precision measurement of instantaneous \( P_{\text{inst}} \) may tell us the actual radius. In fact, possible spinup has been noted in the XMM-Newton observations of August 2002, with an instantaneous period decrease rate \( \dot{P}_{\text{inst}} = -(3 \sim 6) \times 10^{-14} \), whereas spindown in Chandra observations of June 2003, with \( \dot{P}_{\text{inst}} \sim 2 \times 10^{-13} \) to be much larger than the averaged one (Zavlin et al. 2001). The conjectured radius is then likely to be \( \sim 1 \, \text{km} \), based on this instantaneous rate and the spindown rule of Eq. (50), and 1E 1207.4-5209 could be a low-mass bare strange star (\( M \sim 10^{-3} M_\odot \)). Such a high accretion rate (\( \sim 10^{14} \, \text{g/s} \)) can certainly not of the capture by the star from interstellar medium matter, but could be due to a fallback flow, since the age is relatively young (\( \sim 7 \, \text{kys} \)). The age discrepancy between the SNR age and that\(^6\) of \( P/(2P) \) is not surprising since the pulsar is not dominantly rotation-powered. It depends on the detail physics of magnetospheric boundary interactions to solve the age problem for this pulsar.

The accretion X-ray luminosity, for steady accretion with rate of \( M \), could be \( \sim G M M/R \sim 10^{32} \, \text{erg/s} \), which is comparable with the observed \( L \sim 10^{33} \, \text{erg/s} \). This hints that the X-ray radiation of 1E 1207.4-5209 could be powered by both of cooling and accretion. Note that, from Eq. (2), the energy-loss rate due to magnetodipole radiation from such a low-mass star is only \( \sim 10^{24} \, \text{erg/s} \), to be much smaller than the X-ray luminosity \( L \). Therefore, such pulsar-like stars have negligible magnetospheric activities, and can be observed only if they are near and young enough.

The models proposed, including this strange-star model and that of (Zavlin et al. 2001), can be tested by future timing observations. The Doppler shift model should be ruled out if future pulses are not arrive at the time expected in the model. If random spin nature can be confirmed as a general nature in more precise observations, we may tend to suggest \( P \sim P_{\text{br}} \) for 1E 1207.4-5209, since glitching pulsars usually spin stably except during glitches.

### 5 OTHER CANDIDATES OF LOW-MASS STRANGE STARS

A first candidate pulsar of low-mass strange stars was suggested in 2001 (Xu et al. 2001), the fastest rotating millisecond pulsar PSR 1937+21 (\( P = 1.558 \times 10^{-3} \, \text{s}, P = 1.051 \times 10^{-19} \, \text{s/s} \)). In order to explain its polarization behavior of radio pulses and the integrated profile (pulse widths of main-pulse and inter-pulse, and the separation between them), this pulsar is supposed to have mass < 0.2\( M_\odot \) and radius < 1 km. The polar magnetic field is \( 8.2 \times 10^8 \, \text{G} \) based on Eq. (1) for conventional neutron stars, but could be \( 2.2 \times 10^5 B_{60}^{-1/2} \) G if the radius \( R = 1 \, \text{km} \) and Eq. (2) and Eq. (19) are applied.

The low mass may actually favor a high spin frequency \( \Omega \) during the birth of a bare strange star. The defined Kepler frequency of such stars could be approximately a constant,

\[
\Omega_k = \sqrt{\frac{GM}{R^3}} = 1.1 \times 10^4 B_{60}^{1/2} \, \text{s}^{-1},
\]

with a prefactor of \( \sim 0.65 \) at most for \( M \sim M_\odot \) and \( R \sim 10^6 \, \text{cm} \) (Glendenning 2000). The initial rotation periods of strange stars are limited by the gravitational radiation due to \( r \)-mode instability (Madsen 1998). At this early stage, the stars are very hot, with temperatures of a few 10 MeV, and we may expect a fluid state of quark matter, without color superconductivity. The critical \( \Omega \) satisfies the equation

\[
\frac{1}{\tau_{\text{gw}}} + \frac{1}{\tau_{\text{sw}}} + \frac{1}{\tau_{\text{bv}}} = 0,
\]

where the growth timescale for the instability (the negative sigh indicates that the model is unstable) is estimated to be,

\[
\tau_{\text{gw}} = -3.85 \times 10^{21} \Omega^{-5/9} M^{-4/9} R^{-4},
\]

and \( \tau_{\text{sw}} \) and \( \tau_{\text{bw}} \) are the dissipation timescales due to shear and bulk viscosities, respectively,

\[
\left\{ \begin{array}{l}
\tau_{\text{sw}} = 1.85 \times 10^{-9} \alpha_s^{5/3} M^{-5/9} R^{11/3} T^{5/3}, \\
\tau_{\text{bw}} = 5.75 \times 10^{-2} M_{100}^{1/3} \Omega^{2} R^{2} T^{-2},
\end{array} \right.
\]

and \( \alpha_s \) the coupling constant of strong interaction, \( T \) the temperature, \( m_{100} \) the strange quark mass in 100 MeV.

The calculated results, based on Eq. (36), are shown in Fig. 4. It is found that low-mass bare strange stars can rotate very fast, even faster than the Kepler frequency (Note: the surface matter is not broken at super-Kepler frequency, due to the self-bounding of quark matter), and one would then not be surprising that the fastest rotating pulsar could be a low-mass bare strange star. However, though it needs advanced technique of data collection and analysis to detect sub-millisecond radio pulsar, we have not find one yet (Edwards, van Straten & Bailes 2001; Han et al. 2004). This negative result could be due to: (1) The dynamical process does not result in a sub-millisecond rotorator; (2) No magnetospheric activity exists for very low-mass strange stars whose \( P \) is very small, since the potential drop is not high enough to trigger pair production (see §3.1). In the later case, a nearby sub-millisecond radio pulsar could be found.

\footnote{See Eq. (2), for the case of \( P_0 \ll 0.424 \, \text{s}. \)
by X-ray observations since a hotspot, powered by rotation and/or accretion, may form on the stellar surface.

RX J1856.5-3754 is another candidate. RX J1856.5-3754 could be low-massive, based on the X-ray spectrum, but the main puzzle is the origin of its optical radiation, the intensity of which is about 7 times that extrapolated from the Rayleigh-Jeans law of X-ray spectrum (Burwitz et al. 2003). If RX J1856.5-3754 is a spinning magnetized star, its magnetosphere could be surrounded by a spherically quasi-static atmosphere, in which the plasma temperature is of the order of the free-fall temperature (Lipunov 1992; Ikhsanov 2003).

\[
T_{\text{ff}} = \frac{G M \rho_0}{k r_m},
\]

where \(\rho_0\) is the proton mass and \(k\) the Boltzmann constant. The dissipation of stellar rotation energy, as well as the gravitational energy of accreted matter, may heat the envelope, which could be responsible for the UV-optical emission. The soft-component-fitted parameters could thus be of this quasi-static envelope, with temperature < 33 eV and radius > 17 km. Assuming \(T_{\text{ff}} < 33\) eV and \(r_m > 17\) km, one has low limit\(^7\) of the stellar mass \(M > 4 \times 10^{-7} M_\odot\), or radius \(R > 0.1\) km. While the hard-component-fitted stellar radius is \(R \approx 4.4\) km (Burwitz et al. 2003). One can also infer an accretion rate \(M \sim 4 \times 10^{10}\) g/s, from Eq. (21) for \(\mu_m = 10^{-6}\) G cm\(^{-2}\) g\(^{-1}\). The X-ray luminosity due to accretion is then \(> 3 \times 10^{36}\) erg/s. This model for soft component could be tested by more observations in sub-mm bands, besides in optical and UV bands, since the quasi-static atmosphere could also be effective in radiating infrared photons if it is dusty.

Strange quark matter with mass \(\ll M_\odot\) could be ejected by a massive strange star (\(\sim M_\odot\)) during its birth or by collision of two strange stars, and such low-mass matter may explain a few astrophysical phenomena (Xu & Wu 2003): the planets around pulsars could be quark matter with mass \(\sim 10^{23-28}\) g, while very low mass strange quark matter (called as strangelets) with baryon numbers of \(\sim 10^9\) may be the nature of ultra-high energy cosmic rays beyond the GZK cutoff. The bursts of soft \(\gamma\)-ray repeaters could be due to either the starquake-induced magnetic reconnection or the collision between a strange planet and solar-mass bare strange star. The collision chance would be not low if both objects form during a same supernova, or in a same binary system. Some of the transient unidentified EGRET sources (Wallace et al. 2000) may represent such collision events, the gravitational energy release of which is

\[
E_k \sim \frac{G M_\odot m_B}{(R_B^3 + R_a^3)^{1/3}} = 2 \times 10^{52} \frac{R_a^3 R_B^3}{(R_B^3 + R_a^3)^{1/3}} \text{ ergs},
\]

where “\(a\)” and “\(b\)” denotes two objects of strange quark matter. The released energy could be \(\sim 10^{45}\) erg if \(R_a \sim 10^5\) cm and \(R_b \sim 10^4\) cm. This strong (color) interaction may result in photon emission by various hadron process (e.g., hadron annihilation), with energy \(\gtrsim 100\) MeV (the EGRET telescope covers an energy range from 30 MeV to over 20 GeV), and could be another way to produce strangelets.

Merging quark stars, rather than neutron stars (Erich et al. 1989), may result in cosmic \(\gamma\)-ray bursts observed (GRBs), which could help to eliminate the baryon load problem. The released energy is \(\sim 10^{53}\) ergs during the collision of two quark stars with \(\sim 10^6\) cm. The residual body should be expected to rapidly rotate, and such a high spin frequency may result in a beaming pattern of emission. A fireball with low-baryonic contamination in this color interaction event may favor the emission of photons and neutrinos with high energy. Alternatively, rapid rotating quark stars being residual via hypernovae could also be possible as the central engines. Only non-baryonic particles (e.g., photons, neutrinos, and \(e^\pm\)) can massively radiated from the quark surfaces, since the stellar temperature is initially very high (\(> 10\) MeV). A fireball forms then, and the Usov mechanism (Usov 1995) of pair production in strong electric field near quark surface may play an important role at that time. The rotation may cause the very irregular lightcurves.

6 THE ORIGIN OF LOW-MASS STRANGE STARS

This is a real problem which is difficult to answer with certainty now. Several scenarios and arguments relevant are suggested in this sections, though, in this paper, we focus on proposing low-mass strange star candidates and trying to attract one’s attention to such stars neglected previously.

1. The origin of millisecond pulsars. An open debate on this issue took place in 1996 (Bhattacharya et al. 1996). Millisecond pulsars are recycled ones in low-mass X-ray binaries, the magnetic fields of which decay (by, e.g., enhanced Ohmic dissipation, diamagnetic screening effect, etc.) during accretion process, according to the standard model. After years of searching for coherent millisecond X-ray pulsations, five accretion-driven millisecond pulsars have been discovered since 1998 (Wijnands 2004), which are important to test the model. Besides the old problems (e.g., birthrate, millisecond pulsars with planets, etc.), new puzzling issues are raised in the standard arguments (Rappaport, Frejageau & Spruit 2004).
However, the puzzles may disappear if low-mass bare strange stars, with rapidly spinning, result directly from accretion-induced collapse (AIC) of white dwarfs, but could be covered by normal matter if they had high-accretion phases in the later evolutions (while the core collapses may produce only normal pulsars with mass $\sim M_0$ and radius $\sim 10^8$ cm). Normal neutron stars created by AIC had been investigated with great efforts (Fryer et al. 1999). A deflagration wave, separating nuclear matter and quark matter, should form inside white dwarfs, and propagate outwards, if quark stars are born via AIC. Though the possibility of forming a massive strange star with $\sim M_0$ by AIC might not be ruled out, the reason that AIC produces low-mass strange star might be simple: The density and temperature in an accreting white dwarf may not be so high that the detonation flame reaches near the stellar surface. A low-mass quark star could form if the detonation surface is far below the stellar surface.

The new born low-mass bare strange stars could rotate very fast, even to be super-Keplerian (Fig.4), and would spin down if the accretion rates are not very high. Unless the mass is smaller than $(0.1 \sim 0.3)M_0$, the thickness of crust could be negligible (Xu 2003a), and the maximum X-ray luminosity at the break period $[M = 2.3 \times 10^{16} P_{br-3}^{7/5} P_{br-6}^{12/15} \mu_{m-6}^{16/15} R_5^{12/5}]$ g/s from Eq. (41) is approximately,

$$L_x = \frac{G M \dot{M}}{R} \sim 2.8 \times 10^{34} P_{br-3}^{-7/5} P_{br-6}^{9/5} \mu_{m-6}^{16/15} R_5^{22/5} \text{erg/s}.$$ (41)

A strange star with low mass may explain the low time-averaged accretion luminosity in bursting millisecond X-ray pulsars since $L_x \propto R^{-4}$, if it is assumed that bursting X-ray pulsars are in a critical stage of $P \sim P_{br}$. It is worth noting that, in this stage, the real accretion-luminosity could be much lower than $L_x$ presented in Eq. (41) because only part of the accretion matter with rate $\dot{M}$ could bombard directly the stellar surfaces if $P < P_{br}$.

It is generally suggested that, during an iron-core collapse supernova, the gravitation-released energy $E_g \sim 10^{54}$ erg is almost in the form of neutrinos, $\sim 10^{-3}$ of which is transformed into the kinetic energy of the outgoing shock and $\sim 10^{-4}$ of which contributes to the photon radiation. However, this idea is not so successful in modern supernova simulations (Baras et al. 2003; Lieb 2004), since the neutrino luminosity could not be large enough for a successful explosion even in the models with the inclusion of convection below the neutrinosphere (2D-calculations). We note that the bare quark surfaces may be essential for successful explosions of both types of iron-core collapse and AIC. The reason is that, because of the color binding, the photon luminosity of a quark surface is not limited by the Eddington limit. It is possible that the prompt reverse shock could be revived by photons, rather than neutrinos. A hot quark surface, with temperature $T \sim 10^{11}$ K, of a newborn strange star$^{10}$ will radiate photons at a rate of

$$E_\nu > 4 \pi R^2 \sigma T^4 \sim 7 \times 10^{50} R_{5}^{22/5} \text{erg/s},$$

while the Thomson-scattering-induced Eddington luminosity is only

$$L_{Edd} = \frac{64 \pi^2 c G M_0}{3 \pi T} BR^3 \sim 10^{35} B_{60} R_5^{2} \text{erg/s}.$$ (43)

This means that the photon emissivity may play an important role in both types of supernova explosions (i.e., for the birth of solar-mass as well as low-mass bare strange stars).

Strange stars born via this way are certainly bare, since any normal matter can not survive from the strong photon bursts. A high fall-back accretion may not be possible due to massive ejecta, rapid rotation, and strong magnetic fields, and such stars could keep to be bare as long as the accretion rates are not very high, since accreted matter with low rates could penetrate the Coulomb barrier (Xu 2004). As a white dwarf collapses to a state with nuclear or supranuclear densities, strange quark matter seeds may help to trigger the transition from normal matter to quark matter. However, the seeds may not be necessary, since the transition could occur automatically at that high density.

In the core collapse case, the total photon energy could be much larger than the energy ($\sim 10^{-2} E_g \sim 10^{51}$ erg), with which the out envelope should be expelled, since the time scale for a proto-strange star with $T \sim 10^{11}$ is usually more than $1$ s. In the AIC case, the bounding energy of a progenitor white dwarf with mass $\sim M_0$ and radius $\sim 10^9$ cm is $E_{boun} \sim 3 \times 10^{50}$ erg, and a minimum mass $\sim 3 \times 10^{-3} M_0$ of bare strange stars would release an amount of $E_{boun}$ if each baryon contributes about 50 MeV after conversion from hadron matter to strange quark matter. In this sense, such explosions to produce low-mass strange stars are powered by the phase-transition energy, rather than the gravitational energy. Certainly, the low-limit can be smaller if the progenitor white dwarf is less massive. It is then not surprising that AIC may produce low-mass strange stars as long as a strange quark phase conversion$^{11}$ can occur in the center of a white dwarf with much high temperature and density. The mass of the residual bare quark stars may depend on the details of combustion of nuclear matter into strange quark matter, especially on the last detonation surface where the phase-transition can not occur anymore. Certainly, this surface would be determined by various microphysics (e.g., how much energy per baryon is released during the phase conversion?). In case the kick energy is approximately the same, only solar-mass millisecond pulsars can survive in binaries since low-mass pulsars may be ejected by the kick.

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8 The crust thickness could be much smaller than previous calculation for static and cold cases due to a high penetration rate during hot bursts.

9 This is estimated by only gravitational energy release. The temperature of low-mass bare strange stars could also be $> 10^{11}$ K, since each baryon would release $10 \sim 100$ MeV during the quark phase transition.

10 The thermal conductivity of quark matter is much larger than that of normal matter in proto-neutron star crusts. The surface temperature of proto-neutron stars can not be so high, otherwise a significant amount of stellar matter should be expelled as wind (the neutron star mass might then be very small).

11 The critical condition may depends on various parameters, e.g., the chemical composition, the accretion rate, the thermal history, the stellar rotation, etc.
Recently, it is suggested by Popov (2004), that low-mass compact objects can only form by fragmentation of rapidly rotating protoneutron stars, and that such objects should have large kick velocities. It is worth noting that AIC-produced low mass compact stars might not have large kick velocities, which may serve as a possible test of the models of formation mechanisms. On one hand, if the kick energy \( E_{\text{kick}} \sim MV^2 \) (\( V \) the kick velocity) is dominantly part of the gravitational energy \( \sim GM^2/R \), then one comes to \( V \sim \sqrt{M} \). On the other hand, if \( E_{\text{kick}} \) is mainly part of phase-transition energy \( \alpha M \), then low-mass stars could have similar \( V \) as that of strange stars with \( \sim M_\odot \). Fragmentation might hardly occur for quark stars due to the color-confinement.

Additionally, AIC-created pulsar-like stars may help to explain various astrophysical phenomena, the recent work including the kick velocities of millisecond pulsars (Tauris & Bailes 1996), the r-process nucleosynthesis of heavy (baryon number \( A > 130 \)) nuclei [Fryer et al. 1999] Qian & Wasserburg 2003, the ultra-high-energy protons accelerated in the pulsar magnetospheres [de Gouveia Dal Pino & Lazarian 2001], as well as the numerical calculations of millisecond pulsar formation in binary systems [Tauris et al. 2000]. Nonetheless, besides the high-mass stars, it is interesting to determine whether an LI-star (a low- and intermediate-mass star, with mass \( \sim 2 \leq M/M_\odot \leq \sim 8 \)) can die via a violent event (e.g., supernova) and maybe produce a low-mass strange star after unstable nuclear explosion, by detail calculations and/or observations.

2. The origin of pulsar magnetic fields. The different values of \( \mu_\alpha \) (or \( \mu_\sigma \)) of normal pulsars and millisecond pulsars (Table 1) may result from their dissimilar physical processes at birth (iron-core collapse or AIC). The magnetic momentum per unit baryon, \( \mu_\sigma \), could be dependent on the number, \( n \), of quarks of each cluster. It is suggestive that \( \mu_\sigma \) of pulsars created by core-collapse is bigger than that by AIC according to Table 1. A nucleon has a magnetic momentum of about nuclear magneton, \( \mu_N = 5 \times 10^{-24} \text{erg/G, and the corresponding value } \mu_\sigma \sim \mu_N/m_\sigma \sim 3 \text{G-cm}^{-3}\text{g}^{-1} \), which is much larger than that observed in pulsars. This hints that the quark clusters in solid quark stars may have a magnetic momentum per baryon to be \( \sim 10^{-4(4-6)} \) orders weaker than that of nucleons.

7 CONCLUSIONS AND DISCUSSIONS

General properties of both rotation- and accretion-powered low-mass bare strange stars are presented. It is suggested that normal pulsars with \( \sim M_\odot \) masses are produced after core-collapse supernova explosions, whereas millisecond pulsars with \( \sim (0.1-1)M_\odot \) (and even lower) masses could be the remains of accretion-induced collapses (AIC) of massive white dwarfs. These different channels to form pulsars may result in two types of ferromagnetic fields: weaker for AIC (\( \mu_\sigma \sim 10^{-6} \text{G-cm}^{-3}\text{g}^{-1} \)) while stronger for core-collapse (\( \mu_\sigma \sim 10^{-4} \text{G-cm}^{-3}\text{g}^{-1} \)). We note that the low-mass quark stars involved in this paper are also with small radii, which may be distinguished from low-mass quark stars with large radii (Alaverdyans, Harutyunyan & Vartanyan 2001).

Some potential astrophysical appearances relevant to low-mass bare strange stars are also addressed. We suggest that the radio-quiet central compact object, IE 1207.4-5209, is a low-mass bare strange star with polar surface magnetic field \( \sim 6 \times 10^{10} \text{G} \) and likely a few kilometers in radius, and it is now at a critical point of subsonic propeller phase, \( P \sim P_{\text{tr}} \), in order to understand its timing behavior. A newborn low-mass strange star could rotate very fast, even with a super-Kepler frequency. The radius of the dim thermal object, RX J1856.5-3754, is \( R > 0.1 \text{ km} \) if its soft UV-optical component radiates from a spherically quasi-static atmosphere around. It is proposed that some of the transient unidentified EGRET sources may result from the collisions of two low-mass strange stars. It is worth noting, in our sense, that the so called Massive Compact Halo Objects, discovered through gravitational microlensing (Alcock et al. 1993), could also be probably low-mass quark stars formed by evolved stars, rather than quark nuggets born during the QCD phase transition of the early Universe (Banerjee et al. 2003).

The mass of strange quark matter could be as low as of a few hundreds of baryons (strangelets). Strangelets can be evaporated through bare surfaces of new-born quark stars, or produced during collision of two (low-mass) quark stars. Strangelets with \( \sim 10^{-9} \) baryons could be detected as ultra-high energy cosmic rays (Xu & Wu 2003). In this sense the discrepancy between the observational fluxes (Anchordoqui et al. 2004) of AGASA and of HiRes/Fly’s Eye might be explained, since solid strangelets at initially low temperature should be heated enough to ionize the atmosphere and would result thus in low radiation of fluorescence in the later detector.

Can we confirm the small radius of a low-mass bare strange star by a direct observation of future advanced space telescopes? This work might be done by the next generation Constellation X-ray telescope (to be launched in 2009-2010), which covers an energy band of (0.25-100) keV. The radii, \( R \), of neutron stars are generally greater than 10 km (\( R \sim 0.1M_\odot \) mass neutron stars is \( \sim 160 \text{ km} \)). If pulsars are neutron stars, their surfaces should be imaged by the Constellation-X with much high space resolution, as long as the separation between the four satellites is greater than \( \sim 10^5 \text{pc} \). Can we confirm the small radius of a low-mass bare strange star? (Manchester 1992), although the inferred polar fields range...
about 4 orders, based on Eq. (11). However, their fields range only \( \sim 2 \) orders when the ingredient of mass-changing is included, according to Table 1 and Eq. (14), if pulsars are actually strange stars. Why has no millisecond pulsar with characteristic age \( T_c < 10^9 \) years been found (or why are most millisecond pulsars so old)? The answer could be that the initial periods, \( P_0 \), of millisecond pulsars spread over a wide range of \( \sim 1 \) ms (or even smaller) to \( \sim 50 \) ms, so that \( P \) is not much larger than \( P_0 \) [it is thus not reasonable to estimate age by \( T_c = P/(2P_0) \)]. Such a distribution of \( P_0 \) may be relevant to their birth processes of AIC.

Can a core-collapse supernova also produce a low-mass bare strange star? This possibility could not be ruled out in principle. Likely astrophysical hints could be that the thermal X-ray emission and rotation power of such a star should be lower than expected previously. Additionally, the cooling history of a low-mass strange star should be significantly different from that of solar-mass ones. Observationally, two isolated “low-field” weak radio pulsars could be low-mass normal pulsars (Lorimer et al. 2003): PSR J0609+2130 \((P = 55.7 \text{ ms}, \ P = 3.1 \times 10^{-10})\) and PSR J2235+1506 \((P = 57.9 \text{ ms}, \ P = 1.7 \times 10^{-10})\). The polar fields inferred from Eq. (15) of low-mass bare strange stars could be higher than that from Eq. (11) for rotation powered pulsars, since one has

\[
R_c = 2.9 \times 10^{15} B_{\infty} B_{2}^{2} P \dot{P},
\]

(44) from Eq. (5) and Eq. (10). We can therefore obtain radii of rotation-powered pulsars in case that electron cyclotron absorptions from their surfaces are detected by advanced X-ray spectrometry. If these two pulsars have polar fields \( B = 5 \times 10^{15} \) G, the radii of PSR J0609+2130 and PSR J2235+1506 are thus \( 2 \) km and \( 1 \) km, respectively. The very weak radio luminosity might be due to a very small rotation energy \( \Omega^2/2 \propto R^3 \) and a small potential drop \( \phi \) in Eq. (11).

In this sense, many low-mass bare strange stars may be not detectable in radio band.

It is sincerely proposed to search low-mass bare strange stars, especially with masses of \( \sim \left(10^{-1} - 10^{-2}\right) M_\odot\), by reprocessing the timing data of radio pulsars. The systemic long-term variation of timing residual of the millisecond pulsar \( \text{B1937+21}\) might uncover a companion star with mass \( \sim 10^{-2} M_\odot\) (Gong 2003). The companion masses of pulsar/white-dwarf binaries are estimated to be a few \( 0.1 M_\odot\) (Thorsett & Chakrabarty 1999). All these companions real white dwarfs (or part of them to be just low-mass strange stars)? Only part of the companions (a few percent) of pulsar/white-dwarf systems have been optically detected. A further study on this issue is then surely necessary.

Finally, it is very necessary and essential to probe strange quark stars through various observations of millisecond-pulsar’s environments (planets, accretion disks), in order to distinguish these two scenarios on millisecond-pulsar’s nature: to be (A) recycled or (B) supernova-originated. In case (A), planets and residual accretion disks could be around such pulsars, but possible mid- or far-infrared emission is still not been detected (Lazio & Fischer 2001; Löhmer, Wolczan & Wielebinski 2004). Another point to be not natural in case (A) is: it is observed that planets orbiting main-sequence stars can only form around stars with high metallicities, but the planet captured by PSR B1620-26 is in a low-metallicity environment (the globular cluster M4). In addition, the formation of pulsar planets is still a matter of debate (e.g., Miller & Hamilton 2001). In case (B), however, observations relevant could be well understood, since a pulsar (with possibly low mass) and its planet(s) may born together during a supernova (see the discussion at the end of §4 and Xu & Wu 2003), and no infrared emission can be detected if no significant supernova-fall-back disk exists. If the first scenario is right, infrared radiations from both the disks and the planets could be detectable by the Spitzer Space Telescope and by the present SCUBA-1 or future -2 detectors of JCMT 15-m ground telescope. But if the later is true, only negative results can be concluded. Surely, these are exciting and interesting subjects to be proposed when these advanced telescopes operate. Besides the radio-loud pulsars, it is also valuable to detect sub-mm emission from radio-quiet pulsar-like compact objects discovered in high-energy X-ray bands, in order to find hints of quark stars. In the model presented, compact center objects (CCOs, e.g., 1E 1207.4-5209) and dim thermal Neutron stars (DTNs, e.g., RX J1856.5-3754) might have sub-mm radiation from the cold material around the stars; but no sub-mm emission is possible if no accretion occurs there. It is then very interesting to test and constrain the models through observations of CCOs and DTNs in sub-mm wavelengths.

In conclusion, if pulsar-like stars are strange quark stars, part of them should consequently be of low-masses unless one can convince us that no astrophysical process results in the formation of low-mass quark stars. Since they are also X-ray emitters, we may expect that some of them could have been detected by Chandra or XMM-Newton (e.g., the Chandra \( \sim 1 \) Ms X-ray survey).

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12 Comet-like planets with density of \( \sim 1 \text{ g/cm}^3 \) are much larger and more grained than that strange planets with \( \sim 10^{15} \text{g/cm}^3 \), and contribute consequently more infrared emission.
