THAP: A Matlab Toolkit for Learning with Hawkes Processes

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Abstract

As a powerful tool of asynchronous event sequence analysis, point processes have been studied for a long time and achieved numerous successes in different fields. Among various point process models, Hawkes process and its variants attract many researchers in statistics and computer science these years because they capture the self- and mutually-triggering patterns between different events in complicated sequences explicitly and quantitatively and are broadly applicable to many practical problems. In this paper, we describe an open-source toolkit implementing many learning algorithms and analysis tools for Hawkes process model and its variants. Our toolkit systematically summarizes recent state-of-the-art algorithms as well as most classic algorithms of Hawkes processes, which is beneficial for both academical education and research. Source code can be downloaded from https://github.com/HongtengXu/Hawkes-Process-Toolkit.

Keywords: Hawkes processes, learning algorithms, Granger causality, clustering structure

1. Introduction

Real-world interactions among multiple entities are often recorded as event sequences, such as user behaviors in social networks, earthquakes in different locations, and diseases and their complications. The entities or event types in these sequences often exhibit complicated self- and mutually-triggering patterns — historical events are likely to have influences on the happenings of current and future events, and the historical events at different time stamps have different impacts. Modeling these event sequences and analyzing the triggering patterns behind them are classical problems in statistics and computer science, which can be solved based on point process models and their learning algorithms.

As a special kind of point processes, Hawkes process model (Hawkes 1971) attracts a lot of researchers and has been widely used in many fields because it can represent the triggering patterns explicitly and quantitatively. Additionally, Hawkes process is very flexible, which has many variants and can be extended and connected with existing machine learning models. Its typical applications include, but not limited to, financial analysis, bioinformatics, social network analysis and control, and crowd behavior modeling. Because of these properties and broad applications, most existing toolkits of point processes are actually developed focusing on Hawkes processes.

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However, although many new models and learning algorithms of Hawkes processes have been proposed for these years, the development of existing Hawkes processes’ toolkits lags behind. On the one hand, they concentrate on implementing traditional algorithms rather than the rapidly evolving state-of-the-art. On the other hand, it is difficult to have a fair and comprehensive comparison for modern algorithms because they are implemented over different sources.

Focusing on modeling and learning Hawkes process and its variants, we describe a new toolkit THAP (Toolkit for HAwkes Processes) in this paper, implementing a wide variety of learning and analysis algorithms for Hawkes processes. THAP offers a Matlab-based implementation of modern state-of-the-art learning and analysis algorithms and provides two real-world date sets (the IPTV data (Luo et al., 2014, 2015) and the Linkedin data (Xu et al., 2017)). It has an ability to compare different algorithms using a variety of evaluation metrics, and thus, may clarify which algorithms perform better under what circumstance. The Matlab-based implementation makes it have some benefits for academical education and research — students can understand the basic concepts of Hawkes processes and the details of the corresponding learning algorithms and accelerate their research in their initial phases. The open-source nature of THAP makes it easy for third parties to contribute additional implementations, and the modules of THAP are extendable to develop more complicated models and functions, e.g. Wasserstein learning (Xiao et al., 2017) and recurrent neural networks (Du et al., 2016).

2. Implementation

THAP is a multi-platform Matlab software (R2016a or higher version required). It is compatible with MS Windows, Linux, and Mac OS. The toolkit consists of five main components, as shown in Fig. 1: Data: Import real-world data (i.e., csv files), convert them to Matlab’s format (i.e., mat files), and implement data preprocessing like sampling, stitching, and thinning. Simulation: Implement three simulation methods to generate synthetic data, including the branch clustering method (Hawkes and Oakes, 1974; Møller and Rasmussen, 2006), Ogata’s modified thinning method (Ogata, 1981), and the fast thinning method for the Hawkes process with exponential impact functions (Dassios and Zhao, 2013). Model: Define Hawkes process model and its variants and implement their learning algorithms. Analysis: Achieve the Granger causality analysis and the clustering analysis of event sequences. Visualization: Visualize data, models, and learning results.

The key modules of THAP are modeling and analysis modules. In particular, Hawkes processes can be categorized into parametric models and nonparametric ones. The parametric models include the Hawkes processes with predefined impact functions, e.g., exponential impact functions.
and Gaussian impact functions. The nonparametric models include the Hawkes processes with arbitrary impact functions, and those impact functions can be represented by a set of basis functions or discretized as a set of sample points with fixed time lags. According to the representation of impact function, different learning algorithms are applied. For the Hawkes processes with continuous impact functions (i.e., those represented by predefined functions or basis), we can apply maximum likelihood estimation (MLE) directly to estimate the parameters of the models. For the Hawkes processes with discretized impact functions, we can (a) treat event sequences as time series and apply the least-squares (LS) method (Eichler et al., 2016), or (b) combine MLE with a solver of ordinary differential equations (ODE) (Zhou et al., 2013) to estimate the parameters of the models.

Additionally, THAP provides us with three cutting-edge analysis tools. The first is Granger causality analysis. For multi-dimensional Hawkes processes, the self-and mutually-triggering patterns between different event types can be represented by a Granger causality graph. THAP combines MLE with various regularizers, e.g., sparse, group-sparse, and low-rank regularizers, and learns the adjacent matrix of the Granger causality graph (i.e., the infectivity matrix of event types) robustly. The second is clustering analysis. THAP contains two methods to cluster the event sequences generated by different Hawkes processes: (a) learning a mixture model of Hawkes processes (Xu and Zha, 2017); (b) implementing a distance metric for marked point processes (Iwayama et al., 2017). The third is dynamical analysis of event sequence. THAP implements the time-varying Hawkes process (TVHP) model in (Xu et al., 2017) and captures the change of infectivity matrix over time.

In summary, we visualize some typical functions achieved by THAP in Fig. 2. Specifically, using THAP, we can (a) visualize event sequences and their intensity functions; (b) simulate event sequences by different simulators and compare their runtime; (c) learn Hawkes processes by different algorithms and visualize learned impact functions; (d) calculate estimation errors of parameters; (e) calculate log-likelihood of data obtained by different algorithms; (f) learn the Granger causality graph of event types (e.g., the infectivity between TV program categories); (g) learn the dynamics of infectivity matrix (e.g., the infectivity between companies for employees at different ages); and (h) learn clustering structures of event sequences and distances between them.
Table 1: Models and algorithms of Hawkes processes in different toolkits.

| Model       | Type                  | Parametric | Nonparametric |
|-------------|-----------------------|------------|---------------|
|             | Impact function       |            |               |
|             | Exponential           | Nonparametric |
| Simulato    | Branch clustering     | ![star]     | ![star]       |
|             | (Fast) Thinning       | ![star]     | ![star]       |
| Learning    | MLE(Regularizer)      | ![star]     | ![star]       |
|             | MLE + ODE             | ![star]     | ![star]       |
|             | Least-squares         | ![star]     | ![star]       |
| Analysis    | Granger causality     | ![star]     |               |
|             | Clustering (Mixture model) | ![star] | ![star]       |
|             | Clustering (Distance metric) | ![star] | ![star]       |
|             | Longtime dynamics (TVHP) | ![star]   | ![star]       |

⋆ = THAP, ♠ = R-hawkes, ■ = pyhawkes, ♣ = PtPack, ♠ = tick.

3. Related work

Table 1 summarizes the implemented features in other open-source point process toolkits and compares them to those in THAP. The functions implemented by different toolkits are labeled by different symbols. THAP covers most of functions of other toolkits and contains many new functions. Specifically, the R-based library R-hawkes just contains a single estimation algorithm of traditional Hawkes processes (Da Fonseca and Zaatour 2014). The Python-based library pyhawkes only implements its contributors’ published algorithms (Linderman and Adams, 2014, 2015). The C++ library PtPack includes some traditional and advanced learning techniques of point processes. It is not very user-friendly because it does not have Python or Matlab interfaces. Recently, a new C++ library tick is developed with a Python interface (Bacry et al., 2017), which includes most PtPack’s functions and further improves their performance.

4. Summary

THAP contributes to point process research community by (a) providing an easy and fair comparison among most existing models and learning algorithms of Hawkes processes, (b) supporting advanced analysis tools which have not been available for other libraries, and (c) filling the blank of point process’s education and research with a Matlab-based toolkit. In the future, we plan to add extensions to go beyond existing Hawkes process models.

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1. https://cran.r-project.org/web/packages/hawkes/hawkes.pdf
2. https://github.com/slinderman/pyhawkes
3. https://github.com/dunan/MultiVariatePointProcess
4. https://github.com/X-DataInitiative/tick
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