Challenges of Nuclear Shadowing in DIS

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Abstract

Nuclear shadowing in DIS at moderately small $x$ is suppressed by the nuclear formfactor and depends on the effective mass of a hadronic fluctuation of the virtual photon. We propose a solution to the problems (i) of how to combine a definite transverse size of the fluctuation with a definite effective mass, and (ii) of how to include the nuclear formfactor in the higher multiple scattering terms. Comparison of the numerical results with known approximations shows a substantial difference.
1. Introduction

Shadowing in deep-inelastic scattering (DIS) off nuclei is a hot topic for the last two decades. In the infinite momentum frame of the nucleus it can be interpreted as a result of parton fusion leading to a diminishing parton density at low Bjorken $x$ \[1\] - \[4\]. A more intuitive picture arises in the rest frame of the nucleus where the same phenomenon looks like nuclear shadowing of hadronic fluctuations of the virtual photon \[5\] - \[12\]. To crystallize the problem and its solution we restrict ourselves in this paper to only quark-antiquark fluctuations of the photon, neglecting those higher Fock components which contain gluons and $q\bar{q}$ pairs from the sea. The lifetime of the $q\bar{q}$ fluctuation (called coherence time) is given by

$$t_c = \frac{2\nu}{Q^2 + M^2} \quad (1)$$

where $\nu$ is the photon energy, $Q^2$ its virtuality and $M$ is the effective mass of the $q\bar{q}$ pair.

Provided that the coherence time is much longer than the nuclear radius, $l_c \gg R_A$, the total cross section on a nucleus reads \[13\],

$$\sigma_{\text{tot}}^\gamma A(x, Q^2) = 2 \int \! \! \! d^2b \int \! \! \! d^2r \, G_{\gamma^*}(Q^2, r) \left\{ 1 - \exp\left[ -\frac{1}{2} \sigma(r) T(b) \right] \right\} \equiv 2 \int \! \! \! d^2b \left\{ 1 - \left\langle \exp\left[ -\frac{1}{2} \sigma(r) T(b) \right] \right\rangle \right\} \quad (2)$$

Here $G_{\gamma^*}(Q^2, r)$ characterizes the probability for the photon to develop a $q\bar{q}$ fluctuation with transverse separation $r$. The condition $t_c \gg R_A$ insures that the $r$ does not vary during propagation through the nucleus (Lorentz time dilation). Then the $q\bar{q}$ pair with a definite transverse separation is an eigenstate of the interaction with the eigenvalue of the total cross section $\sigma(r)$. Therefore, one can apply the eikonal expression \(2\) for the interaction with the nucleus. The nuclear thickness function $T(b) = \int_{-\infty}^{\infty} \! \! \! dz \, \rho_A(b, z)$ is the integral of nuclear density over longitudinal coordinate $z$ and depends on the impact parameter $b$.

The color dipole cross section $\sigma(r)$ introduced in \[13\] vanishes like $r^2$ at small $r \to 0$ due to color screening. This is the heart of the phenomenon called nowadays color transparency \[14, 13, 15\]. For this reason nuclear shadowing in \(2\) is dominated by large size fluctuations corresponding to highly asymmetric sharing of the longitudinal momentum carried by the
and \( \bar{q} \). This leads to \( Q^2 \) scaling of shadowing.

Note that the averaging of the whole exponential in (2) makes this expression different from the Glauber eikonal approximation where \( \sigma(r) \) is averaged in the exponent. The difference is known as Gribov’s inelastic corrections [17]. In the case of DIS the Glauber approximation does not make sense, and the whole cross section is due to the inelastic shadowing.

For the other case, \( t_c \sim R_A \), one has to take into account the variation of \( r \) during the propagation of the \( q \bar{q} \) fluctuation through the nucleus. At present this can only be done for the double scattering term [12] in the expansion of the exponential in (2),

\[
\frac{\sigma_{\gamma^*A}^{\gamma^*N}}{\sigma_{\gamma^*N}^{\gamma^*N}} \approx 1 - \frac{1}{4} \frac{\langle \sigma^2(r) \rangle}{\langle \sigma(r) \rangle} \langle T \rangle \int d^2b F_{A}^2(q, b) + \ldots ,
\]

or in hadronic representation [18],

\[
\frac{\sigma_{\gamma^*A}^{\gamma^*N}}{\sigma_{\gamma^*N}^{\gamma^*N}} \approx 1 - \frac{1}{4\pi} \frac{\langle T \rangle}{\langle \sigma(r) \rangle} \int d^2b \int dM^2 \frac{d\sigma(\gamma^*N \rightarrow XN)}{dM^2 dt} \Big|_{t=0} F_{A}^2(q, b) + \ldots ,
\]

where the mean nuclear thickness and the formfactor read,

\[
\langle T \rangle = \frac{1}{A} \int d^2b T^2(b) ,
\]

\[
F_{A}(q, b) = \frac{1}{\langle T \rangle} \int_{-\infty}^{\infty} dz \rho_A(b, z) e^{iqz} ,
\]

with longitudinal momentum transfer \( q = 1/t_c \) given by (1). In the case of (3) the uncertain fluctuation mass is fixed at \( M^2 = Q^2, \quad q = 2m_Nx \). Two expressions (3) and (4) are related since the integrated forward diffractive dissociation cross section \( \gamma^*N \rightarrow XN \) equals to \( \langle \sigma^2 \rangle /16\pi \).

There are two problems remaining which are under discussion:

- How the nuclear formfactor can be included in the higher order scattering terms which are of great importance for heavy nuclei? For instance, the shadowing term in (3), (4) for lead is of the order of one at low \( x \), so the need of the higher order terms is obvious.
Even for the double scattering term in (3) it is still unclear which argument should enter the formfactor. Indeed, the effective mass of the $q\bar{q}$ fluctuation needed for the coherence time in (4) cannot be defined in the quark representation with a definite $q\bar{q}$ separation. On the other hand, Eq. (4) exhibits an explicit dependence on $M_X$ and the longitudinal momentum transfer is known. However, unknown in this case is the absorptive cross section of the intermediate state $X$.

We suggest a solution of both problems in the next section. The goal of this paper is restricted to the study of the difference between the predictions of the correct quantum-mechanical treatment of nuclear shadowing and known approximations. We do it on an example of the valence $q\bar{q}$ part of the photon and neglect the higher Fock components containing gluons and sea quarks, which may be important if to compare with data especially at very low $x$. Nuclear anti-shadowing effect is omitted as well, since we believe it is beyond the shadowing dynamics (e.g. bound nucleon swelling). Numerical results and a comparison with the standard approach are presented in section 3.

2. The Green function of a $q\bar{q}$ pair in nuclear medium

We start with the generalizing of eq. (2) for the case $l_c \leq R_A$,

$$\sigma_{tot}^{\gamma^*} (x, Q^2) = \int d^2 b \int_0^1 d\alpha \sigma_{tot}^{\gamma^*} (x, Q^2; b, \alpha) , \quad (7)$$

where

$$\sigma_{tot}^{\gamma^*} (x, Q^2; b, \alpha) = T(b) \int d^2 r |\Psi_{\gamma^*} (\vec{r}, \alpha)|^2 \sigma(r)$$

$$- \quad 2 \text{Re} \int_{-\infty}^{\infty} dz_1 \rho_A(b, z_1) \int_{z_1}^{\infty} dz_2 \rho_A(b, z_2) A(z_1, z_2, \alpha) . \quad (8)$$

The first term in r.h.s. of (8) corresponds to the second, lowest order in $\sigma(r)T(b)$, term in expansion of the exponential in (3). The shadowing terms are contained in the second term in (8). $\Psi_{\gamma^*} (\vec{r}, \alpha)$ is the (non-normalized) wave function of the $q\bar{q}$ fluctuation of the virtual photon, where $\alpha$ is the fraction of the light-cone momentum of the photon carried
by the quark. An explicit expression of transverse and longitudinally polarized photons can be found in \[19, 9\].

The function $A(z_1, z_2, \alpha)$ in (8) reads,

$$A(z_1, z_2, \alpha) = \frac{1}{4} \int d^2r_1 d^2r_2 \Psi_{\gamma^*}(\vec{r}_2, \alpha) W(\vec{r}_2, z_2; \vec{r}_1, z_1) \Psi_{\gamma^*}(\vec{r}_1, \alpha) \sigma(r_2) \sigma(r_1) e^{i q_{\text{min}} (z_2 - z_1)},$$

(9)

with

$$q_{\text{min}} = \frac{Q^2 \alpha (1 - \alpha) + m_q^2}{2 \nu \alpha (1 - \alpha)}.$$  

(10)

This expression was first suggested in unpublished paper \[20\].

The second (shadowing) term in (8) is illustrated in fig. 1. At the point $z_1$ the photon

diffractively produces the $q\bar{q}$ pair ($\gamma^* N \rightarrow q\bar{q} N$) with transverse separation $\vec{r}_1$. The pair propagates through the nucleus along arbitrarily curved trajectories (should be summed over) and arrives at the point $z_2$ with a separation $\vec{r}_2$. The initial and the final separations are controlled by the distribution amplitude $\Psi_{\gamma^*}(\vec{r})$. While passing the nucleus the $q\bar{q}$ pair interacts with bound nucleons via the cross section $\sigma(r)$ which depends on the local separation $\vec{r}$. The function $W(\vec{r}_2, z_2; \vec{r}_1, z_1)$ describing the propagation of the pair from $z_1$ to
$z_2$ also includes that part of the phase shift between the initial and the final photons, which is due to transverse motion of the quarks, while longitudinal motion is already included in (10) via the exponential.

Thus, Eq. (8) does not suffer from either of the two problems of the approximations (3) - (4). The longitudinal momentum transfer is known and all the multiple interactions are included.

The propagation function $W(\vec{r}_2, z_2; \vec{r}_1, z_1)$ in (9) satisfies the equation

$$i \frac{\partial W(\vec{r}_2, z_2; \vec{r}_1, z_1)}{\partial z_2} = -\frac{\Delta(r_2)}{2\nu\alpha(1-\alpha)} W(\vec{r}_2, z_2; \vec{r}_1, z_1)$$

$$- \frac{i}{2} \sigma(r_2) \rho_A(b, z_2) W(\vec{r}_2, z_2; \vec{r}_1, z_1),$$

with the boundary condition $W(\vec{r}_2, z_1; \vec{r}_1, z_1) = \delta(\vec{r}_2 - \vec{r}_1)$. The Laplacian $\Delta(r_2)$ acts on the coordinate $\vec{r}_2$. The full derivation of (11) will be given elsewhere. Here we only notice that it looks natural like Schrödinger equation with the kinetic term $\Delta/[2\nu\alpha(1-\alpha)]$ which takes care of the varying effective mass of the $q\bar{q}$ pair and provides a proper phase shift, and $z_2$ plays the role of the time. The imaginary part of the optical potential describes the absorptive process.

In the “frozen” limit $\nu \to \infty$ the kinetic term in (11) can be neglected and

$$W(\vec{r}_2, z_2; \vec{r}_1, z_1) = \delta(\vec{r}_2 - \vec{r}_1) \exp \left[ -\frac{1}{2} \sigma(r_2) \int_{z_1}^{z_2} dz \rho_A(b, z) \right].$$

(12)

When this expression is substituted into (8) - (9) and with $q_{\text{min}} \to 0$ one arrives at result (2) with $G_{\gamma^*}(Q^2, r) = \int_0^1 d\alpha |\Psi_{\gamma^*}(\vec{r}, \alpha)|^2$.

We can also recover the approximation (3) - (4) if one neglects the absorption of the $q\bar{q}$ pair in the medium. Then $W$ becomes the Green function of a free motion,

$$W(\vec{r}_2, z_2; \vec{r}_1, z_1)|_{\sigma \to 0} = \frac{1}{2\pi} \int d^2k \exp \left[ i\vec{k}(\vec{r}_2 - \vec{r}_1) + \frac{i k^2(z_2 - z_1)}{2\nu\alpha(1-\alpha)} \right],$$

(13)

where $\vec{k}$ is the transverse momentum of the quark.

With this expression the shadowing term in (8) reproduces the second term in (4). Indeed, the amplitude of the photon diffractive dissociation in the plane wave approximation
reads,

$$f_{dd}(k) = \frac{1}{2} \int d^2 r \, \Psi^* (\vec{r}, \alpha) \, \sigma(r) \, e^{i\vec{k}\vec{r}}.$$  \hfill (14)

Therefore, (9) can be represented as,

$$A(z_1, z_2, \alpha) = \frac{1}{2\pi} \int d^2 k \, \left| f_{dd}(k) \right|^2 \exp \left[ \left( z_2 - z_1 \right) \frac{Q^2 \alpha (1 - \alpha) + m_2^2 + k^2}{2\nu \alpha (1 - \alpha)} \right]$$  \hfill (15)

Taking into account that $M_X^2 = (m_q^2 + k^2)/\alpha (1 - \alpha)$ is the effective mass squared of the $q\bar{q}$ pair and substituting (15) to (8) we arrive at eq. (4).

### 3. Numerical results

We calculate nuclear shadowing for calcium and lead from the above displayed equations. As was mentioned in the Introduction, only the valence $q\bar{q}$-part of the photon is taken into account, but the higher Fock components containing gluons and sea quarks are neglected, as well as the effect of anti-shadowing. Therefore, we do not compare our results with data, but only to the standard approach (3) - (4).

We do the same calculations again, using the free Green function (13). This makes it possible to disentangle between the influence of higher scattering terms and the formfactor.

We approximate the cross section by the dipole form $\sigma(r) = C r^2$, $C \approx 3$, which is a good approximation at $r > 0.2 - 0.3$ fm [21]. However, we calculated the proton structure function $F_2(x, Q^2)$ perturbatively (we fixed the quark masses at $m_q = 0.3$ GeV, $m_s = 0.45$ GeV and $m_c = 1.5$ GeV) what leads to an additional logarithmic $r$-dependence at small $r$. This is important since results in the double-log $Q^2$ dependence of $F_2$. Nuclear shadowing, however, is dominated by soft fluctuations with large separation [12], therefore, the dipole form of the cross section is sufficiently accurate.

We use a uniform density for all nuclei, $\rho_A = 0.16$ fm$^{-3}$, what is sufficient for our purpose, comparison with the standard approach calculated under the same assumption.

Within these approximations it is possible to solve (11) analytically. The solution is the harmonic oscillator Green function with a complex frequency [21],

$$W(\vec{r}_2, z_2; \vec{r}_1, z_1) = \frac{a}{2 \pi \sinh (\omega \Delta z)} \exp \left\{ -\frac{a}{2} \left[ \left( \frac{\vec{r}_2^2 + \vec{r}_1^2}{\sinh (\omega \Delta z)} - \frac{2\vec{r}_2 \cdot \vec{r}_1}{\sinh (\omega \Delta z)} \right) \right] \right\}.$$

(16)
where

\[ \Delta z = z_2 - z_1 \]
\[ \omega^2 = i \frac{C \rho_A}{\nu \alpha (1 - \alpha)} \]
\[ a^2 = -i C \rho A \nu \alpha (1 - \alpha). \]  

This formal solution properly accounts for all multiple scatterings and finite lifetime of hadronic fluctuations of the photon, as well as for fluctuations of the transverse separation of the $q\bar{q}$ pair.

The results of calculations are shown in fig. 2. The dashed curves show predictions of (3) which we call standard approach. The mean values of $\sigma^2$ and $\sigma$ are calculated using the same $q\bar{q}$ distribution functions [19, 9] as in (9) and the intermediate state mass is fixed at $M^2 = Q^2$. At low $x < 0.01$ shadowing saturates because $q = 2m_N x \ll 1/R_A$. The thin solid curve also corresponds to a double scattering approximation, i.e. absorption (the second term in (11)) is omitted. However, the formfactor is treated properly, i.e. the kinetic term in (11) taking into account the relative transverse motion of the $q\bar{q}$ pair, correctly reproduces the phase shift. The difference between the curves is substantial. The thin solid curve does not show saturation even at $x = 0.001$.

The next step is to do the full calculations and study importance of the higher order rescattering terms in (11). The results are shown by the thick solid curves. Higher order scattering brings another substantial deviation (especially for lead) from the standard approach. At very low $x$ the curves saturate at the level given by (2).

4. Conclusions and outlook

We suggest a solution for the problem of nuclear shadowing in DIS with correct quantum-mechanical treatment of multiple interaction of the virtual photon fluctuations and of the nuclear formfactor. We perform numerical calculations for $q\bar{q}$ fluctuations of the photon and find a significant difference with known approximations. Realistic calculations to be compared with data on nuclear shadowing should incorporate the higher Fock components which include gluons. The same path integral technique can be applied in this case. The
$x$-dependence of the dipole cross section $\sigma(r, x)$ (correlated with $\vec{r}$) should be taken into account. One should also include the effect of anti-shadowing, although it is only a few percent. A realistic form for the nuclear density should be used (this can be done replacing

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Figure 2: Nuclear shadowing for calcium and lead. The dotted curve is calculated in the standard approach (3). The thin solid curve corresponds to the double scattering approximation with the free Green function, (13), and the thick solid curve shows the full calculation, (14).
\[ \rho_A(b, z) \] by a multistep function like in [21]). We are going to settle these problems in a forthcoming paper.

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