Backgammon is Hard

R. Teal Witter

NYU Tandon, Brooklyn NY 11201, USA rtealwitter@nyu.edu

Abstract. We study the computational complexity of the popular board game backgammon. We show that deciding whether a player can win from a given board configuration is NP-Hard, PSPACE-Hard, and EXPTIME-Hard under different settings of known and unknown opponents’ strategies and dice rolls. Our work answers an open question posed by Erik Demaine in 2001. In particular, for the real life setting where the opponent’s strategy and dice rolls are unknown, we prove that determining whether a player can win is EXPTIME-Hard. Interestingly, it is not clear what complexity class strictly contains each problem we consider because backgammon games can theoretically continue indefinitely as a result of the capture rule.

Keywords: Computational complexity · Games

1 Introduction

Backgammon is a popular board game played by two players. Each player has 15 pieces that lie on 24 points evenly spaced on a board. The pieces move in opposing directions according to the rolls of two dice. A player wins if they are the first to move all of their pieces to their home and then off the board.

The quantitative study of backgammon began in the early 1970’s and algorithms for the game progressed quickly. By 1979, a computer program had beat the World Backgammon Champion 7 to 1 [2]. This event marked the first time a computer program bested a reigning human player in a recognized intellectual activity. Since then, advances in backgammon programs continue especially through the use of neural networks [11,17,18].

On the theoretical side, backgammon has been studied from a probabilistic perspective as a continuous process and random walk [10,19]. However, the computational complexity of backgammon remains an open problem two decades after it was first posed [5]. One possible explanation (given in online resources) is that the generalization of backgammon is unclear.

From a complexity standpoint, backgammon stands in glaring contrast to many other popular games. Researchers have established the complexity of numerous games including those listed in Table 1 [4] but we are not aware of any work on the complexity of backgammon.

In this paper, we study the computational complexity of backgammon. In order to discuss the complexity of the game, we propose a natural generalization of backgammon. Inevitably, though, we have to make arbitrary choices such as
Table 1. A selection of popular games and computational complexity results.

| Game       | Complexity Class |
|------------|------------------|
| Tic-Tac-Toe| PSPACE-Complete  |
| Checkers   | EXPTIME-Complete |
| Chess      | EXPTIME-Complete |
| Bejeweled  | NP-Hard          |
| Go         | EXPTIME-Complete |
| Hanabi     | NP-Hard          |
| Mario Kart | PSPACE-Complete  |

the number or size of dice in the generalized game. Nonetheless, we make every effort to structure our reductions so that they apply to as many generalizations as possible.

There are two main technical issues that make backgammon particularly challenging to analyze. The first is the difficulty in forcing a player into a specific move. All backgammon pieces follow the same rules of movement and there are at least 15 unique combinations of dice rolls (possibly more for different generalizations) per turn. For other games, this problem has been solved by more complicated reductions and extensive reasoning about why a player has to follow specified moves [4]. In our work, we frame the backgammon problem from the perspective of a single player and use the opponent and dice rolls to force the player into predetermined moves.

The second challenge is that the backgammon board is one-dimensional. Most other games with computational complexity results have at least two dimensions of play which creates more structure in the reductions [8]. We avoid using multiple dimensions by carefully picking Boolean satisfiability problems to reduce from.

We show that deciding whether a player can win is NP-Hard, PSPACE-Hard, and EXPTIME-Hard for different settings of known or unknown dice rolls and opponent strategies. In particular, in the setting most similar to the way backgammon is actually played where the opponent’s strategy and dice rolls are unknown, we show that deciding whether a player can win is EXPTIME-Hard. Our work answers an open problem posed by Demaine in 2001 [5].

In Section 2, we introduce the relevant rules of backgammon and generalize it from a finite board to a board of arbitrary dimension. In Section 3, we prove that deciding whether a player can win even when all future dice rolls and the opponent’s strategy are known is NP-Hard. In Section 4, we prove that deciding whether a player can win when dice rolls are known and the opponent’s strategy is unknown is PSPACE-Hard. Finally in Section 5, we prove that deciding whether a player can win when dice rolls and the opponent’s strategy are unknown is EXPTIME-Hard.
2 Backgammon and its Generalization

We begin by describing the rules of backgammon relevant to our reductions. When played in practice, the backgammon board consists of 24 points where 12 points lie on Player 1’s side and 12 points lie on Player 2’s side. However, without modifying the structure of the game, we will think of the board as a line of 24 points where Player 1’s home consists of the rightmost six points and Player 2’s home consists of the leftmost six points. Figure 1 shows the relationship between the regular board and our equivalent model. Player 1 moves pieces right according to dice rolls while Player 2 moves pieces left. The goal is to move all of one’s pieces home and then off the board.

Players move their pieces by taking turns rolling dice. On their roll, a player may move one or more pieces ‘forward’ (right for Player 1 and left for Player 2) by the numbers on the dice provided that the new points are not blocked. A point is blocked if the opponent has at least two pieces on it. The turn ends when either the player has moved their pieces or all moves are blocked. Note that a player must always use as many dice rolls as possible and if the same number appears on two dice then the roll ‘doubles’ so a player now has four moves (rather than two) of the number.

If only one of a player’s pieces is on a point, the opponent may capture it by moving a piece to the point. The captured piece is moved off the board and must be rolled in from the opponent’s home before any other move may be made. This sets back the piece and can prove particularly disadvantageous if all of the points in the opponent’s home are blocked.
The obvious way to generalize the backgammon board used in practice is to concatenate multiple boards together, keeping the top right as Player 1’s home and the bottom right as Player 2’s home. In the line interpretation, we can equivalently view this procedure as adding more points between the respective homes. The rules we described above naturally extend. We formalize this generalization in Definition 1.

**Definition 1 (Generalized Backgammon).** Let \( m \) be a positive integer given as input. We define constants \( h \geq 6 \), and \( d \geq 2 \) and \( s \geq 6 \) where the lower bounds originate from traditional backgammon. Then a generalized backgammon instance consists of \( n \) points on a line with the leftmost \( h \) and rightmost \( h \) marked as each player’s home and \( d \) dice each with \( s \) sides. We require the number of pieces \( p \) to be polynomial in \( m \) and specify our choice of \( p \) in the reductions.

In our proofs, we fix the constants without loss of generality by modifying our reductions. We assume \( h = 6 \) home points by blocking additional points in the opponent’s home. We also assume \( d = 2 \) dice and \( s = 6 \) sides by rolling blocked pieces for the player and using dummy moves for the opponent.

3 NP-Hardness

In this section, we show that determining whether a player can win against a known opponent’s strategy and known dice rolls (KSKR) is NP-Hard. We begin with formal definitions of Backgammon KSKR and the NP-Complete problem we reduce from.

**Definition 2 (Backgammon KSKR).** The input is a configuration on a generalized backgammon board, a complete description of the opponent’s strategy, and all future dice rolls both for the player and opponent. The problem is to determine whether a player can win the backgammon game from the backgammon board against the opponent’s strategy and with the specified dice rolls.

We do not require that the configuration is easily reachable from the start state. However, one can imagine that given sufficient time and collaboration between two players, any configuration is reachable using the capture rule.

An opponent’s strategy is known if the player knows the moves the opponent will make from all possible positions in the resulting game. Notice that such a description can be very large. However, in our reduction, we limit the number of possible positions by forcing the player to make specific moves and predetermining the dice rolls. Therefore the reduction stays polynomial in the size of the 3SAT instance. We formalize this intuition in Lemma 1.

**Definition 3 (3SAT).** The input is a Boolean expression in Conjunctive Normal Form (CNF) where each clause has at most three variables. The problem is to determine whether a satisfying assignment to the CNF exists.
Given any 3SAT instance, we construct a backgammon board configuration, an opponent strategy, and dice rolls so that the solution to Backgammon KSKR yields the solution to 3SAT. Since 3SAT is NP-Complete \[9\], Backgammon KSKR must be NP-Hard. We state the result formally below.

**Theorem 1.** Backgammon KSKR is NP-Hard.

**Proof.** Our proof consists of a reduction from 3SAT to Backgammon KSKR. Assume we are given an arbitrary 3SAT instance with \(n\) variables and \(k\) clauses. First, we force Player 1 (black) to choose either \(x_i\) or \(\neg x_i\) for every \(i \in [n]\). Then, we propagate their choice into the appropriate clauses in the Boolean expression from the 3SAT instance. Finally, we reach a board configuration where Player 1 wins if and only if they have chosen an assignment of bits that satisfies the Boolean expression.

We compartmentalize the process into “gadgets.” Each gadget simulates the behavior of a part of the 3SAT problem: There is an assignment gadget for every variable that forces Player 1 to choose either \(x_i\) or \(\neg x_i\) for every \(i \in [n]\). There is a clause gadget for every clause that records whether the assignment Player 1 chose satisfies \(c_j\) for every \(j \in [k]\).

Player 1 wins if and only if their assignment satisfies all clauses. We ensure this by putting a single black piece in each clause. Player 1 satisfies the clause by protecting their piece. Once the assignment has been propagated to the clauses, Player 2 (white) captures any unprotected piece. If even a single clause is unsatisfied (i.e. a single piece is open), Player 2 traps the piece and moves all the white pieces home before Player 1 can make a single additional move. We block Player 2’s home and use the rule that a captured piece must be rolled in the board before any other move can be made.

If, on the other hand, Player 1 satisfies every clause then Player 1 will win since we will feed rolls with larger numbers to Player 1 and smaller numbers to Player 2. Player 1 will then beat Player 2 given their material advantage in the number of pieces on the board.

We now describe the variable and clause gadgets in Figure 2. In order to simplify the concepts, we explain the gadgets in the context of their function in the reduction rather than providing a complicated, technical definition. There are \(n\) variable gadgets followed by \(k\) clause gadgets arrayed in increasing order of index from left to right.

For each \(x_i\), we repeat the following process: There are initially two white pieces each on point 4 and point 16 (the top of Figure 2). We move these pieces to 1 and 13 respectively while feeding Player 1 blocked rolls e.g. one. We then give Player 1 a two and a three. The only moves they can make are from 2 to 4 to 7 or from 14 to 16 to 19. This choice corresponds to setting \(x_i\). Without loss of generality, Player 1 chooses \(\pi_i\) and Player 2 blocks 4 from 6 and 7 from 9. We feed Player 1 rolls of two and three until all the \(\pi_i\) pieces are on 19; the number of these pieces is exactly the number of times \(\pi_i\) appears in clauses. We give Player 1 enough rolls to move all the pieces corresponding to either \(x_i\) or \(\neg x_i\). We then move to the next variable.
Once all the variables have been set, we move down the variable gadgets from \( x_n \) to \( x_1 \) propagating the choice of \( x_i \) to the appropriate clauses. We use Player 2’s pieces to block 1 and 13 for all variable gadgets with lower indices so only the pieces in gadget \( x_i \) can move. (Variable gadgets with higher indices have already been emptied to clauses.) For each \( x_i \), we move the pieces on 19 through the variable gadgets \( x_{i+1}, \ldots, x_n \) with rolls of sixes. Once we reach the clause gadgets, we move one piece at a time with sixes until we reach a clause that contains \( \overline{x_i} \). Once it reaches its clause, we give the piece a two to protect the open piece.

We use a similar set of rolls for the \( x_i \) pieces on 7. Since the rolls have to be deterministic, we give the rolls for both the \( x_i \) and \( \overline{x_i} \) before moving on to the \( x_{i-1} \) variable gadget. The rolls for whichever of \( x_i \) and \( \overline{x_i} \) Player 1 did not choose simply cannot be used.

Notice that every roll we give Player 1 can be played by exactly one piece (except when Player 1 sets \( x_i \)). While Player 1 receives rolls, we give Player 2 ‘dummy’ rolls of one and two to be used on a stack of pieces near Player 2’s home.

Once all variables have been set and the choices propagated to the clauses, we give Player 2 a one to capture any unprotected pieces. If Player 2 captures the unprotected piece, Player 2’s home is blocked so Player 1 cannot make any additional moves until all of Player 2’s pieces are in their home. At this point, Player 2 easily wins. Otherwise, none of Player 1’s pieces are captured and the game becomes a race to the finish. We give Player 2 low rolls and Player 1 high rolls so Player 1 quickly advances and wins.
Since Player 1 wins if and only if they find a satisfying assignment, determining whether Player 1 can win determines whether a satisfying assignment to the 3SAT instance exists. Then, with lemma 1 Backgammon KSKR reduces from 3SAT in polynomial space and time so Backgammon KSKR is NP-Hard.

Lemma 1. The reduction from 3SAT to Backgammon KSKR is polynomial in space and time complexity with respect to the number of variables and the number of clauses in the 3SAT instance.

Proof. The length of the board is linear with respect to the variables and clauses plus some constant buffer on either end. Player 1 has at most twice the number of clauses for each variable while Player 2 has at most a constant number of pieces per variable and clause. Only one piece is captured per reduction so the number of moves is at most the product of the length of the board and the number of pieces.

While it is potentially exponential with respect to the input, the description of the dice rolls and opponent’s move may be stored in polynomial space due to their simplicity. In the assignment stage, the rolls and opponent moves are the same for each variable gadget and can be stored in constant space plus a pointer for the current index. In the propagation stage, the rolls and opponent moves are almost the same for each variable gadget and clause gadget except that the number of rolls necessary to move between the variable and clause gadgets varies. However, we can store the number of rolls by the index in addition to constant space for the rules. In the end game, Player 1 and Player 2 both move with doubles if able and the rolls are repeated until one player wins.

4 PSPACE-Hardness

In this section, we show that determining whether a player can win against an unknown opponent’s strategy and known dice rolls (USKR) is PSPACE-Hard. We begin with formal definitions of Backgammon USKR and the PSPACE-Complete problem we reduce from.

Definition 4 (Backgammon USKR). The input is a configuration on a generalized backgammon board, an opponent’s strategy which is unknown to the player, and known dice rolls. The problem is to determine whether a player can win the backgammon game from the backgammon board against the opponent’s unknown strategy and with the specified dice rolls.

An opponent’s strategy is unknown if the player does not know what the opponent will play given a possible position and dice rolls in the resulting game. The opponent’s strategy is not necessarily deterministic; it can be adaptive or stochastic.

Definition 5 (Gpos [15]). The input is a positive CNF formula (without negations) on which two players will play a game. Player 1 and Player 2 alternate setting exactly one variable of their choosing. Once it has been set, a variable
may not be set again. Player 1 wins if and only if the formula evaluates to True after all variables have been set. The problem is to determine whether Player 1 can win.

Given any $G_{pos}$ instance, we construct a backgammon board configuration, an unknown opponent’s strategy, and known dice rolls so that the solution to Backgammon USKR yields the solution to $G_{pos}$. Since $G_{pos}$ is PSPACE-Complete [15], Backgammon USKR must be PSPACE-Hard. We state the result formally below.

**Theorem 2.** Backgammon USKR is PSPACE-Hard.

**Proof.** The reduction from $G_{pos}$ to Backgammon USKR closely follows the reduction from 3SAT to Backgammon KSKR so we primarily focus on the differences. Assume we are given an arbitrary $G_{pos}$ instance with $n$ variables and $k$ clauses. The key observation is that, since the CNF is positive, Player 1 will always set variables to True while Player 2 will always set variables to False. We can therefore equivalently think about the game as Player 1 moving variables to a True position while Player 2 blocks variables from becoming True. Once all the variables have been set, we propagate the choices to the clause gadgets as we did in the 3SAT reduction.

The winning conditions also remain the same. Player 1 wins if and only if they are able to cover the open piece in each clause. We require the opponent’s strategy to be unknown so they can adversarially set variables.

![Fig. 3. Reduction from $G_{pos}$: variable gadgets. Player 1 sets $x_i$ to True while Player 2 has already set $x_{i+1}$ to False.](image-url)

We now describe the variable gadgets in Figure 3. In the 3SAT reduction, we needed a stack of $x_i$ pieces and a stack of $\bar{x}_i$ pieces since Player 1 could set $x_i$ to True or False. Here, since the CNF is positive, Player 1 will only set variables to True and so only require an $x_i$ stack.

At the beginning of Player 1’s turn, all unset variables are blocked by two pieces on point 4. Then, while Player 1 receives dummy rolls of one, Player 2 moves all the blocking pieces to 1. Next, Player 1 receives a roll of two and
three and must choose which unset variable to set to True. Once the variable is chosen, Player 2 blocks all other unset variables by moving two pieces from 6 to 4 while Player 1 again receives dummy rolls. The remaining pieces for the chosen variable are then moved from 2 to 4 to 7.

Player 2’s turn is more simple. They choose which variable to set to False and do so by blocking 16 from 18. For the remainder of Player 1’s turns, the blocking pieces on 16 will not be moved.

After all the variables have been set to True or False, we move the pieces set to True to the clauses they appear in. We again move from $x_n$ to $x_1$, removing the blocking pieces on 13 and then 1 as we go. The process and clause gadgets are the same as in the 3SAT reduction.

Player 1 wins in Backgammon USKR if and only if the positive CNF instance in $G_{pos}$ is True after alternating setting variables with Player 2. Therefore the solution to Backgammon USKR yields an answer to $G_{pos}$ and, by Lemma 2, $G_{pos}$ reduces in polynomial space to Backgammon USKR.

**Lemma 2.** The reduction from $G_{pos}$ to Backgammon USKR is polynomial in space complexity with respect to the number of clauses and variables in $G_{pos}$.

**Proof.** As in the reduction from 3SAT to Backgammon KSKR, the size of the board is linear in the number of clauses and variables plus some constant buffer. By similar arguments, the size of the description is polynomial because it depends only on the stage of the reduction and the index of the current variable and clause gadgets.

### 5 EXPTIME-Hardness

In this section, we show that determining whether a player can win against an unknown opponent strategy and unknown dice rolls (USUR) is EXPTIME-Hard. We begin with formal definitions of Backgammon USUR and the EXPTIME-Complete problem we reduce from.

**Definition 6 (Backgammon USUR).** The input is a configuration on a generalized backgammon board. The opponent’s strategy and dice rolls are unknown to the player. The problem is to determine whether a player can win the backgammon game from the backgammon configuration against the unknown strategy and dice rolls.

**Definition 7 ($G_6$ [16]).** The input is a CNF formula on sets of variables $X$ and $Y$ and an initial assignment of the variables. Player 1 and Player 2 alternate changing at most one variable. Player 1 may only change variables in $X$ while Player 2 may only change variables in $Y$. Player 1 wins if the formula ever becomes true. The problem is to determine whether Player 1 can win.

Given any $G_6$ instance, we construct a backgammon board configuration and exhibit an opponent’s strategy and dice rolls such that the solution to Backgammon USUR yields the solution to $G_6$. Since $G_6$ is EXPTIME-Complete [16], Backgammon USUR must be EXPTIME-Hard.
**Theorem 3.** Backgammon USUR is EXPTIME-Hard.

**Proof.** The proof consists of a reduction from $G_6$ to Backgammon USUR. Assume we are given a CNF formula with $n_x$ variables $X$, $n_y$ variables $Y$, $k$ clauses, and an initial assignment to $X$ and $Y$. First, Player 1 and Player 2 take turns changing variables in their respective sets $X$ and $Y$. Then, once Player 1 gives the signal, the game progresses to a board state where Player 1 wins if and only if the CNF formula is True on the current assignment.

Player 1 changes variable $x_i$ by moving a signal piece corresponding to $x_i$. Then, with Player 2’s help, we feed Player 1 dice rolls that update the clause gadgets that contain $x_i$. We require that the dice rolls adaptively respond to Player 1 and that Player 2 can adversarially set variables in $Y$.

![Fig. 4. Reduction from $G_6$: variable gadget (top) and clause gadget (bottom).](image)

We next describe the gadgets in Figure 4. The variable gadgets consist of stacks of pieces corresponding to $x_i$ and $\overline{x_i}$ for $i \in [n_x]$. On their turn, Player 1 changes a variable by using their six to move the appropriate piece. For example, Player 1 can change $x_i$ to False by moving a piece on point 7 to 13 as shown at the top of Figure 4. If $x_i$ is already False, Player 1 has effectively skipped their turn (which is an acceptable move in $G_6$).

Once Player 1 changes a variable, Player 2 and the dice rolls work together to update the appropriate clauses. The key insight of a clause is that it is True if at least one variable in the clause is True in either $X$ or $Y$. We represent this on the backgammon board as shown at the bottom of Figure 4. Point 8 is empty or contains Player 1’s pieces if at least one of the $X$ variables in the clause is True and 10 is empty if at least one of the $Y$ variables in the clause is True. Therefore
Player 1 can progress a piece from 5 to 13 on rolls three and five if and only if either a variable in \( X \) or \( Y \) in the clause is True.

We update the clause when Player 1 sets a variable in one of two ways: If the variable is True in the clause, we move two pieces from 1 to 5 to 8. If the variable is False in the clause, we move two pieces from 8 to 13 to 17. If 8 becomes empty, we move two pieces from a white stack to 8 in order to block Player 1 from unfairly using it to bypass the clause. By using blocking pieces on 5 and 13 we ensure the correct clause is modified.

We update the clause when Player 2 sets a variable in one of two ways: If the variable is True in the clause and all other \( Y \) variables in the clause are True, we move two pieces from 10 to another white stack. If the variable is False in the clause and all other \( Y \) variables in the clause are True, we move two pieces from another white stack to 10. In all other cases, 10 should remain in its current ‘open’ or ‘closed’ position.

Notice that the process of changing variables could continue indefinitely. We make sure that we do not run out of pieces by using the capture rule. If Player 1 needs more pieces in the variable or clause gadgets, we feed them rolls to move excess pieces through the board towards their home where Player 2 will capture them one by one. We perform an analogous process if Player 2 needs more pieces.

The variable changing process ends when, instead of moving a variable piece, Player 1 moves a specified signal at the end of the variable gadgets. Then Player 1 will receive enough six and 4-3-4-5-4 rolls to move all of their pieces home while Player 2 makes slow progress. If all of the clauses are True, Player 1 can successfully get all their pieces home and win the game. Otherwise, they will be blocked at a False clause and Player 2 will continue their slow progress until all white pieces except for those in the False clause remain. We will then feed small rolls to Player 1 and large rolls to Player 2 so Player 2 can capitalize on their advantage and win.

We have therefore simulated the \( G_6 \) instance and Player 1 can win Backgammon USUR if and only if they can win the corresponding \( G_6 \) game. As before, the reduction space is polynomial in the \( G_6 \) input size since there are a constant number of pieces and points for every clause and variable.

Notice that Backgammon USUR is not obviously EXPTIME-Complete because the game can progress indefinitely as a result of the capture rule.

6 Conclusion

We show that deciding whether a player can win a backgammon game under different settings of known or unknown opponent strategies and dice rolls is NP-Hard, PSPACE-Hard, and EXPTIME-Hard. It would seem that our results show backgammon is hard even when it is a one-player game. However, in the settings for our PSPACE-Hardness and EXPTIME-Hardness results, the second player is hidden in the unknown nature of the opponent’s strategy and dice rolls.

Despite the popularity of backgammon and academic interest in the computational complexity of games, to the best of our knowledge our work is the
first to address the complexity of backgammon. One possible explanation is the apparent ambiguity in generalizing backgammon. We contend, however, that the backgammon generalization we use is as natural as those for other games like checkers or chess. Another explanation is the difficulty in forcing backgammon moves as needed for a reduction.

Interestingly, it is not clear that the problems we consider are in EXPTIME because backgammon games can theoretically continue indefinitely. One natural follow up question to our work is what complexity class contains these backgammon problems.

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