Imprints of Schwinger Effect on Primordial Spectra

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We study the Schwinger effect during inflation and its imprints on the primordial power spectrum and bispectrum. The produced charged particles by Schwinger effect during inflation can leave a unique angular dependence on the primordial spectra. If such features are observed in future observations, they will be the smoking gun for the existence of the primordial electric field.

I. INTRODUCTION

Schwinger effect is a fascinating effect in quantum field theory [1]. A pair of charged particles are produced in the vacuum, when the external electric force are strong enough. If this effect is observed, it will help our theoretical understanding of quantum field theory. But so far it has not been observed. The main obstacle is that we need $E \sim 1.3 \times 10^{13} \text{V/m}$ [2]. This made us to consider Schwinger effect in astrophysical and cosmological context [3–5]. In this paper, we focus on searching for the observational signature of Schwinger effect in inflation.

The existence of a large enough electric field during inflation is conventionally considered theoretically challenging. This is due to the fact that radiation typically drops with the scale factor as $a^{-4}$. During inflation, the electric field and magnetic field are quickly diluted away with the rapid expansion of the universe. However, we do observe a large scale magnetic field of order micro Gauss on 10 kpc scale [6–8] and $10^{-16}$ Gauss even on Mpc scale expected in cosmic voids [9–11]. These large scale coherent magnetic fields can hardly be explained without a primordial origin. A natural setting to generate the large scale coherent primordial magnetic field is inflation [12]. However, due to the conformal invariance, the magnetic field also drops as $a^{-4}$. Lots of efforts have been made to generate the primordial magnetic field during inflation by breaking the conformal invariance [13–27]. By far, the best model we know of is still not sufficient to generate the required amount of primordial magnetic field to explain the large scale magnetic field today. One encounters the problem of either a backreaction of the electric field or a strong coupling regime at very early times [28]. This suggests that a background magnetic field should be continuously generated during inflation to counter the effect that it is diluted away quickly. Similarly, we would expect that the same mechanism may be used to generate the electric field to compensate for the fact that the electric field is also diluted away. Such examples do exist, and they are mainly obtained by the breaking of the conformal symmetry of the gauge fields. For example, in [29, 30], a dilatonic coupling between the inflaton and the gauge field in the action of the type $f(\phi)^2 FF$ can generate a constant electric field with energy density not changing with respect to the expansion of the universe. It is shown in [31] that this constant electric field is even an attractor solution in the context of anisotropic inflation.

In this work, we investigate the consequence that the electric field may bring us, instead of focusing on the details of how to generate a constant electromagnetic field in inflation. Also we do not consider the backreaction to the geometry or the constant electric field. We assume that there is a constant electric field with an unchanged energy density in a physical volume during inflation. In short, we focus on the signatures produced. Our setup follows the same line as the seminal works of the recently developed Schwinger effect in 2D [32, 33] and 4D [34, 35] de Sitter space. There are also other related works about Schwinger effect in 2D [36] and 4D [37–39] with slightly different setups. Unlike the flat space case, strong electric field is not needed in inflation to produce super light particles. Charged super light particles will be mainly produced gravitationally during inflation with weak electric field. This phenomenon is known as “hyperconductivity”.

One way to observe the Schwinger effect during inflation is to measure the properties of charged fields produced. If the charged fields are coupled to the inflaton, they will decay to inflatons during inflation, thus leaving signatures on the primordial power spectrum and bispectrum. The idea stems from the so-called quasi-single field inflation [40–42], or cosmological collider physics [43], which states that if there exist some massive fields of mass $m \sim H$, they can leave imprints on the squeezed limit of non-Gaussianities. Interestingly, we found that if there exist a constant electric field during inflation, the Schwinger effect will cause an angular dependence on the primordial power spectrum. This

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angular dependence is different from the other mechanisms that produce the angular dependence on the primordial power spectrum. For example, Bianchi universes such as anisotropic inflation generated by a vector field background [44–48], Galileons [49] or higher-order of curvature terms [50–52] can produce an angular dependence $\cos^2 \theta$ (See [53, 54] for reviews and many more related works therein); inflation with a massive spin-1 field can produce the angular dependence $P_1(\cos \theta)$. Moreover, since the magnitude of non-Gaussianities is directly proportional to the number of particles produced during inflation, the bispectrum has an angular dependence as more charged particles are produced in the direction parallel with the direction of the electric field.

This paper is organized as follows, in Section II, we introduce the model we are considering. In Section III, we derive the geodesic equation of a charged scalar particle. In Section IV, we give the primordial power spectrum. In Section V, we give the bispectrum. In Section VI, we give the result of loop corrections to the bispectrum. We give a conclusion in Section VII.

II. MODEL

We consider QED coupled to a pair of charged scalar $\sigma$ and $\sigma^*$ in four dimensional de Sitter space.

$$S_\sigma = \int d^4x \sqrt{-g} \left[ -g^{\mu\nu}D_\mu \sigma^* D_\nu \sigma - m^2 \sigma^* \sigma - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right],$$  \hspace{1cm} (1)

where $D_\mu \equiv \partial_\mu - ieA_\mu$ is the covariant derivative. $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic tensor. The FRW metric is

$$ds^2 = a^2(\tau) (-d\tau^2 + dx^2),$$

where $\tau$ is the conformal time. We neglect the backreaction of the produced charged scalar particles on the electric field and the FRW background in this work. We consider a constant electric force in the $z$ direction,

$$A_\mu = \frac{E}{H^2 \tau} \delta^z_\mu, \quad E = \text{const}.$$ \hspace{1cm} (3)

Thus the equation of motion for the $\sigma$ field is

$$\sigma'' + 2\frac{a'}{a} \sigma' - \partial_z \partial_z \sigma - 2ieA_z \partial_z \sigma + e^2 A_z^2 \sigma + a^2 m^2 \sigma = 0.$$ \hspace{1cm} (4)

We quantize the $\sigma$ field in the following way

$$\sigma_k = v_k a_k + v_k^* b_k^\dagger,$$ \hspace{1cm} (5)

$$\sigma_k^* = v_k^* a_k^\dagger + v_k b_k,$$ \hspace{1cm} (6)

where $a_k$ and $b_k^\dagger$ are annihilation and creation operators of the positively charged scalar particle, and $b_k$ and $a_k^\dagger$ are annihilation and creation operators of the negatively charged scalar particle. They satisfy the commutation relations

$$[a_k, a_p^\dagger] = [b_k, b_p^\dagger] = (2\pi)^3 \delta^{(3)}(k - p),$$ \hspace{1cm} (7)

$$[a_k, a_p] = [b_k, b_p] = [a_k, b_p] = [a_k, b_p^\dagger] = \cdots = 0.$$ \hspace{1cm} (8)

We introduce the variables

$$z \equiv 2ki\tau, \quad \kappa \equiv -i \frac{k_z eE}{k H^2}, \quad \mu^2 \equiv \frac{9}{4} - \frac{e^2 E^2}{4 H^4} - m^2 \frac{2}{H^2},$$ \hspace{1cm} (9)

where $\kappa$ is imaginary. The real part of the parameter $\kappa$ characterizes the magnitude of the electric field projected to the direction of the trajectory of the negative charged particle. In this work, we focus on the parameter regime where $e^2 E^2/H^4 + m^2/H^2 > 9/4$, thus $\mu$ is imaginary. Our work can be easily generalized to the $e^2 E^2/H^4 + m^2/H^2 < 9/4$ case. The real part of the parameter $\mu$ can be understood as the effective mass of a charged particle in de Sitter space in Hubble units with correction $9/4$ coming from the curved space time and $e^2 E^2/H^2$ from the electric field. The mode function satisfies the equation

$$\frac{d^2}{dz^2} (av_k) + \left\{ \frac{1}{z^2} \left( \frac{1}{4} - \mu^2 \right) + \frac{\kappa}{z} - \frac{1}{4} \right\} (av_k) = 0.$$ \hspace{1cm} (10)
There are two solutions, which are given by the Whittaker functions \( W_{\kappa, \mu}(z) \) and \( M_{\kappa, \mu}(z) \). Since in the sub-horizon limit \( |z| \to \infty \), the solution must approach to the Minkowski solution, we obtain the mode function

\[
a_{\nu_k} = e^{i\kappa z/2 \sqrt{2k}} W_{\kappa, \mu}(z) ,
\]

In the late time limit, the mode function behaves as

\[
a_{\nu_k} = \frac{e^{-|\mu| z/2}}{2 \sqrt{k|\mu|}} \left\{ \alpha_k M_{\kappa, \mu}(z) + \beta_k (M_{\kappa, \mu}(z))^* \right\} .
\]

The coefficients \( \alpha_k \) and \( \beta_k \) satisfies the normalization condition

\[
|\alpha_k|^2 - |\beta_k|^2 = 1 .
\]

The Bogoliubov coefficients can be obtained as

\[
\alpha_k = (2|\mu|)^{1/2} e^{i(\kappa + |\mu|) z/2} \frac{\Gamma(-2\mu)}{\Gamma(\frac{1}{2} - \mu - \kappa)} , \quad \beta_k = -i(2|\mu|)^{1/2} e^{i(\kappa - |\mu|) z/2} \frac{\Gamma(2\mu)}{\Gamma(\frac{1}{2} + \mu - \kappa)} .
\]

The qualitative feature of \( |\alpha_k|^2 \), \( |\alpha_k||\beta_k| \), \( |\beta_k|^2 \) are plotted in FIG. 1. We set \( m = 3H/2 \). We see that as \( E \) increases, both \( |\alpha_k|^2 \) and \( |\beta_k|^2 \) first decrease exponentially and then eventually approach to a constant value, with \( |\alpha_k|^2 \) approaching to 2 and \( |\beta_k|^2 \) approaching to 1. The number of charged particles being produced with charge \( e \) and wave number \( k \) per comoving three volume \( \int d^3k/(2\pi)^3 \) is

\[
n_k = |\beta_k|^2 = \frac{e^{2i\kappa z} + e^{-2|\mu| z}}{2 \sinh(2|\mu| z)} .
\]

The particle production rate can be also be calculated by the instanton method. In the classical limit \( |\mu| \pm i\kappa \gg 1 \),

\[
n_k \approx e^{-S_0} = e^{-2\pi(|\mu|\pm i\kappa)} = e^{-S_+} = e^{-2\pi \frac{\mu}{mH}(\sqrt{1 + l^2} \pm \frac{l}{2H})} ,
\]

where \( l = eE/\tilde{m}H \) characterizes the relative magnitude of the electric field and mass. \( \tilde{m}^2 = m^2 - 9H^2/4 \) is the effective mass of a neutral particle in de Sitter space. \( S_+ \) is the action corresponding to the action of the process that the charged particles are produced but moving to the direction that increases the electric potential energy of itself, whereas \( S_- \) corresponds to the action of the process that the charged particles are produced and moving to the direction that decreases the electric potential energy. In the terminology of the instanton method, \( S_+ \) corresponds to upward tunneling and \( S_- \) corresponds to the downward tunneling. It is always the \( S_- \) that gives the dominant contribution.
There are two interesting limits that we can discuss this problem quite intuitively. The first limit is the weak electric field limit, where \( l \ll 1 \). The classical actions \( S_{\pm} \) in (16) can be approximated as

\[
S_{\pm} = 2\pi \left( \frac{\tilde{m}}{H} \pm \frac{k_z eE}{k H^2} \right).
\]  

(17)

The first term can be understood as the usual Boltzman factor coming from the production of neutral massive particles of effective mass \( \tilde{m} \). From the point of view of a geodesic observer, de Sitter space is associated with a thermal bath with the Hawking temperature \( T = \frac{H}{2\pi} \). The second term is understood as the chemical potential from the electric field. This chemical potential can assist the production of charged particles along the direction of decreasing potential. Since in this limit, the electric field is very weak, the dominant contribution comes from the first term. Hence, although the particles are charged, they are mainly produced gravitationally due to the expansion of the universe. In \([55–57]\), other examples of the chemical potential is also discussed in the context of fermion production in inflation.

The other limit is the large electric field limit, where \( l \gg 1 \). In this limit, the electric field is so strong that the modes do not feel the curvature of the spacetime. The classical actions \( S_{\pm} \) can be approximated as

\[
S_+ = 2\pi \frac{eE}{H^2} \left( 1 + \frac{k_z}{k} \right) \quad S_- = 2\pi \left( \frac{eE}{H^2} \left( 1 - \frac{k_z}{k} \right) + \frac{\tilde{m}^2}{2eE} \right).
\]  

(18)

If we consider the charged particle pairs moving along the \( z \) direction, the second term dominates. As we can see, it also reproduces the flat spacetime result.

In order to study the time scale of mass production of charged particles, it is useful to consider the WKB approximation of solution (11).

\[
\alpha v_k = \frac{1}{\sqrt{2|w_k|}} \exp \left\{ -i \int d\tau |w_k| \right\},
\]  

(19)

where \( w_k \) is the effective frequency given by

\[
w_k^2 = (k_x^2 + eA_x)^2 + k_y^2 + k^2 + a^2m^2 - \frac{a''}{a} = \frac{1}{\tau^2} \left( \frac{e^2E^2}{H^4} + \frac{m^2}{H^2} - 2 \right) + \frac{2k_z eE}{\tau H^2} + k^2, \quad k \equiv (k_x^2 + k_y^2 + k_z^2)^{1/2}.
\]  

(20)

Then the adiabatic parameter is evaluated as

\[
\left| \frac{\dot{w}_k}{w_k} \right| = \left| \frac{H^3(e^2E^2 + eEH^2k_z\tau - 2H^4 + H^2m^2)}{(e^2E^2 + 2eEH^2k_z\tau - 2H^4 + H^2m^2 + k^2H^4\tau^2)^{3/2}} \right|.
\]  

(21)

It is around the time

\[
\tau \sim -\frac{1}{k} \left( |\mu|^2 + \frac{1}{4} \right)^{1/4},
\]  

(22)

that the quantity \( \dot{w}_k/w_k^2 \) approaches its maximum. This means that most particles are produced at this time scale.

Now we observe the production of the particle via the Schwinger effect during inflation. One may think about the mechanism in the context of quasi-single field inflation. The charged particles can decay into the primordial curvature perturbations, thus leaving an imprint on the primordial power spectrum and bispectrum. \( \zeta \) is the primordial curvature perturbation. The second order action of the primordial curvature perturbation can be written down following the procedure in \([58, 59]\)

\[
S_\zeta = M_p^2 \int dt \frac{q^d}{(2\pi)^d} \epsilon (a^3 \zeta^2 - k^2 a \zeta^2).
\]  

(23)

Quantizing it in the following way

\[
\zeta_k = u_k c_k + u_k^* c_k^\dagger,
\]  

(24)

where \( c_k^\dagger, c_k \) are the creation and annihilation operators satisfying the usual commutation relations

\[
[c_k, c_{k'}^\dagger] = (2\pi)^3 \delta^{(3)}(k - p).
\]  

(25)
The mode function satisfies the following equation of motion

$$\ddot{\xi} + (3 + \eta)H \dot{\xi} + \frac{k^2}{a^2} \xi = 0.$$  \hfill (26)

To the lowest order in slow roll parameter, the solution is

$$u_k(\tau) = \frac{H}{2\sqrt{\epsilon M_p}} \frac{1}{k_3/2} (1 + ik\tau) e^{-ik\tau},$$  \hfill (27)

We consider the following coupling between the primordial curvature perturbation and the positive charged scalar fields

$$L_{\sigma\zeta'} = c_2 \int d^3 x d\tau a^3 \sigma \zeta', \quad L_{\sigma'\zeta'} = c_3 \int d^3 x d\tau a^2 \sigma' \zeta'.$$  \hfill (28)

The coupling between the inflaton and the negative charged scalar fields are

$$L_{\sigma'\zeta'} = c_2^* \int d^3 x d\tau a^3 \sigma^* \zeta', \quad L_{\sigma\zeta'} = c_3^* \int d^3 x d\tau a^2 \sigma \zeta'.$$  \hfill (29)

and the coupling between the primordial curvature perturbation and the positive and negative charged scalar fields are

$$L_{\sigma'\sigma^*} = c_2' \int d^3 x d\tau a^3 \sigma^* \zeta', \quad L_{\sigma \sigma^*} = c_3' \int d^3 x d\tau a^2 \sigma \sigma^* \zeta'.$$  \hfill (30)

where $c_2, c_3, c_2', c_3'$ and $c_3'$ are some constants coming from the background of the $\sigma$ or $\sigma^*$ field. Here (28) and (29) do not conserve the charge of $\sigma$. They correspond to cases where the phase symmetry of $\sigma$ is broken, for example, in an Abelian Higgs model (see [60] for discussion of a similar case). In this case, tree level contribution dominates correction to the spectra. Equation (30) corresponds to the case where charge is conserved. In this case, loop diagrams has to be computed. One may worry that these background values contribute to the mass of the gauge field and it won’t be able to support a long range force. But now we are considering very small coefficients. In order to calculate the primordial spectrum, we used the Schwinger-Keldysh formalism (For the application in quasi-single field inflation, see [61]). Now we derive the four types of free propagators for both the curvature perturbation and the massive charged scalar fields. For the curvature perturbation sector, the generating functional can be written as

$$Z_0[J_+, J_-] = \int D\zeta_+ D\zeta_- \exp \left[ i \int_{\tau_0}^{\tau_f} d\tau d^3 x \left( L_0[\zeta_+] + L_0[\zeta_-] + J_+ \zeta_+ - J_- \zeta_- \right) \right].$$  \hfill (31)

The four propagators can be generated using

$$-i\Delta_{ab}(\tau_1, x_1; \tau_2, x_2) = \frac{\delta}{i\alpha J_a(\tau_1, x_1) J_b(\tau_2, x_2)} \left. Z_0[J_+, J_-] \right|_{J_+ = 0}, \quad a, b = \pm.$$  \hfill (32)

Fourier transforming it into momentum space gives

$$G_{ab}(k, \tau_1, \tau_2) = -i \int d^3 x e^{-ik\cdot x} \Delta_{ab}(\tau_1, x; \tau_2, 0).$$  \hfill (33)

The four types of propagators are given as the following

$$G_{++}(k, \tau_1, \tau_2) = \theta(\tau_1 - \tau_2) u_k(\tau_1) u_k(\tau_2)^* + \theta(\tau_2 - \tau_1) u_k(\tau_1)^* u_k(\tau_2)$$
$$G_{+-}(k, \tau_1, \tau_2) = u_k(\tau_1)^* u_k(\tau_2)$$
$$G_{-+}(k, \tau_1, \tau_2) = u_k(\tau_1) u_k(\tau_2)^*$$
$$G_{--}(k, \tau_1, \tau_2) = \theta(\tau_1 - \tau_2) u_k(\tau_1)^* u_k(\tau_2) + \theta(\tau_2 - \tau_1) u_k(\tau_1) u_k(\tau_2)^*.$$  \hfill (34)

For charged massive scalar pairs, we need to introduce two more sources $J_{\pm}^*$ and $J_{\pm}^*$ to source the complex conjugate of the $\sigma$ field

$$Z_0[J_+, J_-, J_+^*, J_-^*]$$
$$\equiv \int D\sigma_+ D\sigma_- D\sigma_+^* D\sigma_-^* \exp \left[ i \int_{\tau_0}^{\tau_f} d\tau d^3 x \left( L_0[\sigma_+, \sigma_+]^* + L_0[\sigma_-^*, \sigma_-^*] + J_+ \sigma_+ - J_- \sigma_- + J_{\pm}^* \sigma_{\pm}^* - J_{\pm}^* \sigma_{\pm} \right) \right].$$  \hfill (35)
The four propagators can be generated using
\[-i \Delta_{ab}(\tau_1, x_1; \tau_2, x_2) = \left. \frac{\delta}{i \alpha \delta J_a(\tau_1, x_1)} \frac{\delta}{i \beta \delta J_b^*(\tau_2, x_2)} Z_0[J_+, J_-, J_+^*, J_-^*] \right|_{J_\pm=0, J_\pm^*=0}, \quad a, b = \pm. \tag{36}\]

Then we have
\[
\begin{bmatrix}
D_{++}(k, \tau; k', \tau') \\
D_{+-}(k, \tau; k', \tau') \\
D_{-+}(k, \tau; k', \tau') \\
D_{--}(k, \tau; k', \tau')
\end{bmatrix}
= i \begin{bmatrix}
\langle T \sigma_k(\tau) \sigma_k^*(\tau') \rangle \\
\langle \sigma_k^*(\tau) \sigma_k^*(\tau') \rangle \\
\langle \sigma_k(\tau) \sigma_k(\tau') \rangle \\
\langle T \sigma_k^*(\tau) \sigma_k^*(\tau') \rangle
\end{bmatrix}.	ag{37}\]

The four types of propagators in the Schwinger-Keldysh formalism are
\[
\begin{align*}
D_{++}(k, \tau_1, \tau_2) &= \theta(\tau_1 - \tau_2) v_k(\tau_1)v_k(\tau_2)^* + \theta(\tau_2 - \tau_1)v_k(\tau_1)^*v_k(\tau_2) \\
D_{+-}(k, \tau_1, \tau_2) &= v_k(\tau_1)v_k(\tau_2) \\
D_{-+}(k, \tau_1, \tau_2) &= v_k(\tau_1)v_k(\tau_2)^* \\
D_{--}(k, \tau_1, \tau_2) &= \theta(\tau_1 - \tau_2)v_k(\tau_1)^*v_k(\tau_2) + \theta(\tau_2 - \tau_1)v_k(\tau_1)v_k(\tau_2)^*. \tag{38}\end{align*}
\]

III. THE GEODESIC EQUATION

To understand the charged particles motion in inflation, we solve the geodesic equation as an intuitive understanding. Following from our metric in (2), the following geodesic equation for a massive charged particle can be written down following the standard procedure (see textbooks [62, 63]).

\[
\frac{d^2 x^\mu}{ds^2} = -\Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} + \frac{e}{m} F^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} + g^\alpha\beta \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = -1,	ag{39}\]

where the connection is
\[
\Gamma^\lambda_{\alpha\beta} = \frac{1}{2} g^{\lambda\gamma} \left( \frac{\partial g_{\gamma\alpha}}{\partial x^\beta} + \frac{\partial g_{\gamma\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\gamma} \right). \tag{40}\]

Here, we would like to observe the change in the physical velocity of the massive charged particle. We would like to solve the following geodesic equation:

\[
\frac{d^2 x(t)}{dt^2} = -2 \frac{\dot{a}}{a} \frac{dx(t)}{dt} + \frac{e}{m} a^{-1} E. \tag{41}\]

The initial condition we would choose is \(x'(t_0) = 0\), which means that the particles are produced at zero velocity. This is the choice following from the instanton method. The solution to this equation subjected to the initial condition is

\[
x(t) = \frac{E e}{m H^2} e^{-Ht} \left( \cosh(H(t - t_0)) - 1 \right). \tag{42}\]

The velocity of the particle is

\[
x'(t) = \frac{E e}{m H} \left( e^{-Ht} - e^{Ht_0 - 2Ht} \right). \tag{43}\]

At first, the force from electric field dominates over the Hubble friction, thus the particle starts to accelerate. Later, due to the Hubble friction, the particle begins to decelerate. The maximum velocity occurs in the subhorizon.

IV. POWER SPECTRUM

In this section, we study the power spectrum of our model. It can be evaluated using the Schwinger-Keldysh formalism [61]

\[
\langle \zeta(\kappa_t^0_k \zeta(-k) \rangle = |c|^2 \sum_{a, b = \pm} \int_{-\infty}^{0} \int_{-\infty}^{0} \frac{d\tau_1}{(-H\tau_1)^3} \frac{d\tau_2}{(-H\tau_1)^3} \partial_{\tau_1} G_{a+}(k, \tau_1, 0) D_{ab}(k, \tau_1, \tau_2) \partial_{\tau_2} G_{b+}(k, \tau_2, 0) + (\kappa \rightarrow -\kappa), \tag{44}\]
The indefinite integral can be integrated directly, which yields

$$\langle \zeta_k \zeta_{k'} \rangle^{(1)} = 2|c_2|^2 e^{i\pi \kappa} \frac{32k^3 M_p^2 c^2}{\varepsilon^2} \int_0^\infty dx_1 e^{i \pi x_1 W_{\kappa,\mu}(-2ix_1)} x_1^2 + (\kappa \rightarrow -\kappa)$$  \hspace{1cm} (45)

The integral can be evaluated as

$$I = \int dx \frac{e^{ix} W_{\kappa,\mu}(-2ix)}{x} = G_{2,1}^{2,1} \left( \frac{1}{2+i, \mu + 1, 0 - \kappa} \right).$$  \hspace{1cm} (46)

where $G$ is the Meijer function defined through the Gamma function in the following way

$$G_{m,n}^{p,q} \left( x \left| \begin{array}{c} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{array} \right. \right) = \frac{1}{2\pi i} \int \frac{d\gamma}{\gamma} \prod_{j=1}^n \Gamma(b_j - s) \prod_{j=n+1}^p \Gamma(1 - a_j + s) \frac{x^s ds}{\Gamma(1 - \kappa)}.  \hspace{1cm} (47)

At $x = 0$, the integral $I$ gives 0. At $x \rightarrow \infty$, it gives

$$\frac{\Gamma(\frac{1}{2} - \mu)\Gamma(\frac{1}{2} + \mu)}{\Gamma(1 - \kappa)}.  \hspace{1cm} (48)

The second contributions is

$$\langle \zeta_k \zeta_{-k} \rangle^{(2)} = -4|c_2|^2 e^{i\pi \kappa} \Re \left[ \int_0^\infty dx_2 e^{-ix_2 W_{\kappa,\mu}(-2ix_2)} \int_0^{x_2} dx_1 e^{ix_1 W_{\kappa,\mu}(-2ix_1)} \right] + (\kappa \rightarrow -\kappa).  \hspace{1cm} (49)

This integral is difficult to evaluate, thus we integrate it numerically. However, in the limit of large mass and small electric field, an analytical result is possible by integrating out the charged massive scalar fields. We present the results in Appendix B1.

The power spectrum $P_k$ is obtained as

$$\langle \zeta_k \zeta_{-k} \rangle' = \langle \zeta_k \zeta_{-k} \rangle^{(1)} + \langle \zeta_k \zeta_{-k} \rangle^{(2)} = \frac{2\pi^2}{k^3} P_k(k).  \hspace{1cm} (50)$$
From here and the following, when making the plot, we set $c_2 = c_3 = M_{\text{pl}} = \epsilon = H = 1$. We plot the angular dependence of the power spectrum in FIG. 3. The produced charged particles can leave non trivial angular dependence on the power spectrum. The power spectrum grows exponentially as the quantity $k_z/k$ increases. This signature is understood as the production of virtual particles increases exponentially when the momentum of the positive charged massive scalar particle is aligned with the electric field $E$ whereas the momentum of the negative charged massive scalar particle is opposite to the direction of the electric field $E$. This signature is a unique signature which cannot be generated by other mechanisms to the knowledge we know of.

We also plot the dependence of the power spectrum on electric field strength in FIG. 4 and the dependence of the power spectrum on the mass of massive field in FIG. 5.

![FIG. 3](image1.png)

*FIG. 3:* At different electric field strength, the power spectrum has angular dependence. We set the mass of massive field is $3H/2$. When the orientation is much close to field’s orientation, the power spectrum is much larger. At the orientation perpendicular to the field’s orientation, although the electric field is strong, Schwinger effect is weak and the effective mass is large enough to suppress the power spectrum, hence, the power spectrum can be small with strong electric field. At the orientation parallel to the field’s orientation, the Schwinger effect is strong enough to get a large power spectrum.

![FIG. 4](image2.png)

*FIG. 4:* We set $k_z/k = 1$ and the mass of the massive field to be $3H/2, 9H/2, 15H/2, 21H/2, 27H/2, 33H/2, 39H/2$ respectively. The power spectrum increases exponentially with respect to the electric field strength.
where 

\[ \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle = c_2 c_3^* \sum_{a,b=\pm} \int_0^0 \int_0^0 \frac{d\tau_1}{(-H\tau_1)^3} \frac{d\tau_2}{(-H\tau_2)^3} \partial_{\tau_1} G_{a+}(k_3, \tau_1, 0) D_{ab}(k_3, \tau_1, \tau_2) \partial_{\tau_2} G_{b+}(k_2, \tau_2, 0) \]

+ (\kappa \rightarrow -\kappa, c_2 \rightarrow c_2^*, c_3 \rightarrow c_3) . \tag{51}

There are three contributions

\[ \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle = \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle^{(1)} + \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle^{(2)} + \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle^{(3)} + 5 \text{ Permutations} + (\kappa \rightarrow -\kappa, c_2 \rightarrow c_2^*, c_3 \rightarrow c_3) . \tag{52} \]

where \( \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle^{(1)} \), \( \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle^{(2)} \) and \( \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle^{(3)} \) are computed as

\[ \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle^{(1)} = -2c_2 c_3^* \frac{H^3 e^{i\pi \kappa}}{128 e^3 k_1 k_2 k_3 M_{pl}^4} \int_0^\infty x_1^e \left( \begin{array}{c} x_1 \end{array} \right) \int_0^\infty x_2 x_3 e^{-\frac{i x_1 + k_3 x_2}{8} - \frac{i k_1 x_3}{8}} W_{-\kappa, -\mu}(2ix_1) . \]

\[ \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle^{(2)} = 2c_2 c_3^* \frac{H^3 e^{i\pi \kappa}}{128 e^3 k_1 k_2 k_3 M_{pl}^4} \left[ \int_0^\infty x_1 x_2 e^{-i \frac{k_3 x_1}{8} - \frac{x_1 + k_3 x_2}{8}} W_{-\kappa, -\mu}(2ix_1) \right] . \]

\[ \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle^{(3)} = 2c_2 c_3^* \frac{H^3 e^{i\pi \kappa}}{128 e^3 k_1 k_2 k_3 M_{pl}^4} \left[ \int_0^\infty x_1 x_2 e^{-i \frac{x_2}{8} - \frac{k_3 x_1}{8}} W_{-\kappa, -\mu}(2ix_1) \right] . \]

We can define the bispectrum in the form of dimensionless shape function \( S(k_1, k_2, k_3) \) \[57\], defined as,

\[ \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle \equiv (2\pi)^4 S(k_1, k_2, k_3) \frac{1}{(k_1 k_2 k_3)^2} P^{(0)}_{\zeta} , \tag{53} \]

where \( P^{(0)}_{\zeta} = \frac{1}{2\pi^2 M_{pl}^2} \frac{H^2}{\epsilon} \) is the power spectrum for the curvature perturbation without the correction coming from massive fields.

The bispectrum can be evaluated using numerical integration. We plot the angular dependence of the amplified bispectrum shape function \( (k_1/k_3) \times S(k_1, k_2, k_3) \) as a function of \( k_3 / k_3 \) in FIG. 7. The bispectrum increases exponentially with increasing absolute value of \( k_3 / k_3 \). When \( k_3 / k_3 \) is positive, the main contribution comes from the
FIG. 6: This figure shows how we measure the non-Gaussianity. The z direction denotes the direction of the electric field. When we measure the non-Gaussianity, we should fix the ratio of the long wavelength momentum and the short wavelength momentum \( k_1/k_3 \). In the meanwhile, we measure the angular dependence of the non-Gaussianity for different \( k_{3z}/k_3 \). \( k_{3z} \) is the magnitude of the long wavelength momentum projected onto the z direction.

![](image1)

FIG. 7: This figure shows the amplified shape function \( S(k_1,k_2,k_3) \times k_1/k_3 \) as a function of the angular dependence of the soft momentum \( k_{3z}/k_3 \). In making this figure, we set \( m = 3H/2 \) and \( k_1/k_3 = 100 \). When orientation is much close to field’s orientation, the amplitude of the bispectrum is much larger. At the orientation perpendicular to the field’s orientation, although the electric field is strong, Schwinger effect is weak and the effective mass is large enough to suppress the bispectrum. At the orientation parallel to the field’s orientation, the Schwinger effect is strong enough to generate a bispectrum of large amplitude.

We plot the clock signals of the squeeze limit of the bispectrum with fixed effective mass \( \mu \) in FIG. 8 and with fixed mass \( m \) in FIG. 9. In both cases, we see that the clock signal is less obvious when the strength of the electric field increases. In the case of fixing the effective mass \( \mu \) case, the absolute value of the clock signal increases with increasing electric field strength due to the enhanced particle production rate. However, the relative amplitude between the clock signal and the contribution of the non-oscillating part coming from some local process decreases. This is because the contribution of the non-oscillating part increases faster than the clock signal when the electric field strength increases. For the fixed mass \( m \) case, when the electric field strength increases, the amplitude of the clock signal relative to the non-oscillating part decreases more dramatically compared with the fixed effective mass case. This is because the electric field strength contributes to the effective mass of the charged massive scalar particles. The energy needed to produce a particle increases accordingly. The combination of these two effect causes the amplitude of the clock signal

positive charged particles whereas when \( k_{3z}/k_3 \) is negative, the main contribution comes from the negative charged particles.
relative to the non-oscillating part barely observable starting from $\kappa = 4i$. The analytical expression of the clock

\[\frac{k_1}{k_3} \times S(k_1, k_2, k_3)\]

FIG. 8: The figure shows the clock signal from the bispectrum. Here we set the momentum to be in the z-direction and hence we set $k_{3z}/k_3 = 1$. We can see the suppression of the clock signal as the electric field increases. The frequency of the clock signal remains unchanged as the change in velocity of the produced particles occurs in the subhorizon and remains stationary during horizon crossing. Hence, no change in frequency of the oscillatory signal would be seen. However, the amplitude of the oscillatory signal would increase as the magnitude of the electric field strength increase.

signal in the large mass and small electric field strength can also be obtained by standard procedure. The derivation is shown explicitly in Appendix C. We are interested in the squeezed limit where $k_1 \sim k_2 \gg k_3$. In this limit, the shape function is

\[S(k_1, k_3) = \frac{1}{H M^2_{pl}} \text{Re} \left[2^{\mu-9/2} c_2 c_3^* f(\mu, \kappa) \left(\frac{k_1}{k_3}\right)^{-1/2-\mu} \right] + (\kappa \to -\kappa, c_2 \to c_2^*, c_3^* \to c_3), \tag{54}\]

with the prefactor given by

\[f(\mu, \kappa) \equiv e^{i\pi \kappa} \frac{\Gamma(-2\mu)^2 \Gamma \left(\frac{1}{2} + \mu\right) \Gamma \left(\frac{5}{2} + \mu\right)}{\Gamma \left(\frac{1}{2} - \mu - \kappa\right) \Gamma \left(\frac{1}{2} + \mu - \kappa\right)} (1 + \sin (\pi \mu)). \tag{55}\]

We can see that in the large mass limit, all the $\Gamma$ functions contribute a factor of $e^{-2\pi |\mu|}$ and $\sin(\pi \mu)$ would give a contribution of $e^{\pi |\mu|}$. Hence, the Boltzmann suppression factor $e^{-\pi |\mu|}$ is recovered in this limit. At the end of this section, we would like to compare several mechanisms that can generate large clock signals even if the mass of the $\sigma$ field are large. There are a few categories of mechanisms listed as the following.
FIG. 9: The figure shows the clock signal from the bispectrum for $m = 4H^2$. Here we set $k_3/k_3 = 1$ again. We can see the suppression of the clock signal as the electric field increases.

- The presence of a new scale. In [64], non-adiabatic production of very heavy fields is studied. The signatures of this model can be large due to the existence of another scale $\phi$ with $\phi$ as the inflaton.

- Finite temperature effect. In [65], the clock signal of the quasi-single field inflation is studied in the context of warm inflation. The particle production rate can be unsuppressed when the effective mass of the particle is changed due to the finite temperature effect.

- The presence of chemical potential. In [55–57], the effect of the chemical potential is studied. The chemical potential can assist the production of the massive particles during inflation thus leaving a less suppressed clock signal. The mechanism we studied here also belongs to this category. However, our studies shows that although it is promising to generate a larger clock signal, one may worry that the contribution from the non-oscillating part will also increase.

- Non-trivial sound speed. The non-trivial sound speed of the massive field is studied in [66]. The magnitude of the clock signal can also be larger than expected when the ratio of sound speed of the massive field and the inflaton is less than one. In [67, 68], the non-trivial sound speed of the inflaton is studied. It is shown in [68] that when the sound speed of the inflaton is close to zero, there will also be a change in the suppression factor of the clock signal.
VI. LOOP CORRECTION TO BISPECTRUM

In this section, we investigate loop corrections coming from the extra massive fields to the primordial non-Gaussianities. The technique of dealing loop correction in quasi-single field inflation can be found in [43, 69–72]. The non-oscillatory part of the diagram is usually UV divergent and we need a systematic way of regularization and renormalization following [73–76]. Luckily, the clock signal is free from UV divergence and we can evaluate it easily.

Using the Schwinger-Keldysh formalism, the bispectrum corresponding FIG. 10 can be obtained as

\[ \langle \zeta_{k_1}, \zeta_{k_2}, \zeta_{k_3} \rangle' = c_2^2 c_3^3 \sum_{a,b=\pm} \int_{-\infty}^{0} d\tau_1 \int_{-\infty}^{0} d\tau_2 (-H\tau_1)^{3/2} (-H\tau_2)^{1/2} \partial_{\tau_1} G_{a+}(k_3, \tau_1, 0) \int \frac{d^3 q}{(2\pi)^3} D_{ab}(\mathbf{p}, \tau_1, \tau_2) D_{ba}(\mathbf{q}, \tau_2, \tau_1) \times \partial_{\tau_2} G_{b+}(k_1, \tau_2, 0) \partial_{\tau_2} G_{b+}(k_2, \tau_2, 0) + (\kappa \rightarrow -\kappa) . \]

where \( \mathbf{p} \) and \( \mathbf{q} \) are the loop momentum that satisfies the constraint \( \mathbf{p} + \mathbf{q} = \mathbf{k}_3 \). After evaluation, we get

\[ \langle \zeta_{k_1}, \zeta_{k_2}, \zeta_{k_3} \rangle' = c_2^2 c_3^3 \Re \left[ g(\mu, \kappa) \frac{2^{1-8\mu} H^5}{k_1 k_2 k_3^4 M_{pl}^6 e^{3}} \left( \frac{k_2}{k_3} \right)^{2\mu} \right] + (\kappa \rightarrow -\kappa) , \]

with the prefactor given by

\[ g(\mu, \kappa) = e^{2i\pi \kappa} \frac{\Gamma(2-2\mu)\Gamma(4-2\mu)\Gamma(-2\mu)^4}{\Gamma(1/2-\mu-\kappa)^2\Gamma(1/2+\mu-\kappa)^2} (\sin(\pi \mu))^2 \]

and with the definition of the shape function \( S(k_1, k_2, k_3) \) in (53) and taking the limit \( k_1 = k_2 \gg k_3 \), we can obtain the expression for the shape function

\[ S(k_1, k_1, k_3) = c_2^2 c_3^3 \Re \left[ g(\mu, \kappa) \frac{2^{1-8\mu} H^5}{M_{pl}^6 e^{3}} \left( \frac{2k_1}{k_3} \right)^{2\mu-2} \right] + (\kappa \rightarrow -\kappa) . \]

We can see that from the loop diagram, the massless curvature modes resonates with two pairs of massive fields and generate two sets of clock signals. The final clock signal would be contributed from the interference of these two clock signals and hence has a doubled frequency of the frequency of the tree level diagram. The total Boltzmann suppression factor is of \( e^{-2\pi\mu} \) where all the \( \Gamma \) functions contribute \( e^{-4\pi\mu} \) in total and the \( \sin \) functions contribute a factor of \( e^{2\pi\mu} \). The suppression for the loop correction is the square of the tree-level case due to the excitation of the two massive fields in the loop diagram.

VII. CONCLUSION AND OUTLOOK

In this work, we consider the imprints of the Schwinger effect on the primordial power spectrum and bispectrum. Both the power spectrum and bispectrum obtained an angular dependence due to the fact that the electric field can assist the production of charged massive particles by adding a chemical potential to them. This angular dependence differs from other models that can too cause an angular dependence on the primordial power spectrum and bispectrum.
As a result, the production rate aligned or opposite to the direction of the charged particles gets enhanced. On the other hand, the production rate perpendicular to the direction of the electric field is suppressed due to the contribution of the electric field strength to the effective mass of the charged scalar particles.

There are many interesting possibilities to explore. We list a few of them and hope to address some of these possibilities in the future.

- The influence of the primordial magnetic field on the power spectrum and bispectrum in the context of quasi-single field inflation. The pair production of the charged scalar field in the presence of a constant electric and magnetic field is studied in [77]. We would like to couple the charged particles with the inflaton and see what kind of signature these particles would imprint on the primordial power spectrum and bispectrum of the curvature perturbations. These signatures on the power spectrum and bispectrum will provide supporting evidences to the existence of the primordial magnetic field.

- The backreaction of the produced particles on the primordial electric and magnetic fields as the particles being produced will weaken the primodial electric and magnetic field. This effect is studied in (1+1)D in [78] and general dimension in [79, 80]. We would like to estimate the actual magnitude of the clock signal generated when taking into account the backreaction effect.

- The signature from other fields produced by Schwinger effect during inflation. Schwinger effect not only produces charged scalar particles, but charged fermions too [81]. The production rate is very similar. However, the production of fermions may lead to other types signatures on the power spectrum and bispectrum.

- Other sources like SU(2) gauge fields [82, 83]. In this case, the production rate is suppressed as the interaction strength increases. However, some signal of cosmological collider type can be generated by the spin-2 field which is required by this type of model.

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Appendix A: Instanton Method

In this section, we use the instanton method to derive the Euclidean action $S_E$ in four dimensional de Sitter space. We followed the approach of [84, 85].

We have the Euclidean action

$$S_E = m \int_\Sigma ds + e \int_\Sigma A_\mu dx^\mu - \frac{1}{4} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu} + \int d^4x \partial_\mu (\sqrt{-g} F^{\mu\nu} A_\nu) . \quad (A1)$$

Then the variation of action with respect to $A_\mu$ yields the Maxwell equation

$$\partial_\mu (\sqrt{-g} F^{\mu\nu} A_\nu) = -e \int ds \delta^4(x - x(s)) \frac{dx^{\nu}(s)}{ds} . \quad (A2)$$

We expand the derivative in the last term of Euclidean action and use the Maxwell equation

$$S_E = m \int_\Sigma ds + \frac{1}{4} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu} . \quad (A3)$$

We consider the case where the existence of electric field causes two types of vacuum and the electric field in the true vacuum can be eliminated from the Gauss law

$$E_i = E_0 - \frac{k_z}{k} e . \quad (A4)$$
Then we use $F_{\mu\nu}F^{\mu\nu} = 2E^2$, we have

$$S_E = m \int_\Sigma ds - \left( \frac{k_z e E_0}{k} - \frac{E_0^2}{2} - \frac{k_z^2 e^2}{k^2} \right) \int_V \hat{\epsilon}. \quad (A5)$$

Here, $V$ is the volume of the inside region and $\hat{\epsilon} = \sqrt{|g|} dx^\mu \wedge dx^\nu$. We can also simplify the Euclidean action by dropping the constant term $\frac{1}{4} \int d^4x \sqrt{g} E_0^2$ and the small term $\frac{1}{4} \int d^4x \sqrt{g} k_z^2 e^2$. Then we can obtain the Euclidean action.

$$S_E = m \int_\Sigma ds - \frac{k_z e E_0}{k} \int_V \hat{\epsilon}. \quad (A6)$$

According to the calculation of the surface and the volume of N-spherical worldsheets, we can obtain the Euclidean action.

$$S_E = m \frac{2\pi^{\frac{N+1}{2}}}{\Gamma\left(\frac{N+1}{2}\right)} R_0^N - \frac{k_z e E_0}{k} \frac{2\pi^{\frac{N+1}{2}}}{\Gamma\left(\frac{N+1}{2}\right)} H^{-(N+1)} \int_0^{\theta_0} \sin^N \theta d\theta . \quad (A7)$$

Here, $R_0$ is the radius of N-spherical worldsheet and $\theta_0$ is the polar angle on the d-sphere of radius $R_0$. We then extremize Euclidean action with respect to $\theta_0$

$$\frac{dS_E}{d\theta_0} = 0 . \quad (A8)$$

Then

$$\tan \theta_0 = NH \frac{m}{e E_0} , \quad (A9)$$

the radius of the Euclidean worldsheet would be given by

$$R_0 = \frac{N m}{(N^2 H^2 m^2 + e^2 E_0^2)^{1/2}} . \quad (A10)$$

When we consider the particle production, the dimension of the worldsheet is 1, and we would obtain the following Euclidean action for the instantons

$$S_E = 2\pi H^{-2} \left[ (m^2 H^2 + e^2 E_0^2)^{1/2} - \frac{k_z e E_0}{k} \right] . \quad (A11)$$

Appendix B: Large Mass Limit

In this section, we derive the analytical expression for the power spectrum and bispectrum in the large mass and weak electric field limit. In this limit, the charged massive scalar fields can be integrated out, leaving an effective general single field inflation with sound speed correction.

1. Power Spectrum

The large mass limit of the power spectrum of the quasi-single field model with neutral scalar particles is obtained in [86–94]. In the large mass limit and the absence of electric field, the power spectrum scales as $1/|\mu|^2$, where $\mu$ is defined in (9) with electric field $E$ set to be zero. Here we follow the same idea and derive the power spectrum in the large mass and small electric field limit by integrating out the charged massive scalar fields.

From (49), we know that (45) is exponentially suppressed in the large mass limit. Since using the effective field theory approach, we are only able to capture this part of the contribution, we do not need to consider it.

Now we start to evaluate the contribution coming from (49). We expand the Whittaker function in the limit $x \to 0$,

$$W_{-\kappa,-\mu}(2ix_1) = \frac{(2ix_1)^{1/2-\mu} \Gamma(2\mu)}{\Gamma(\kappa + \mu + \frac{1}{2})} + \ldots . \quad (B1)$$
We expand the Whittaker function in the limit

\[ \int_{x_1}^{\infty} dx_2 e^{-ix_2} \frac{(2i)^{1/2-\mu}\Gamma(2\mu)x_2^{-1/2-\mu}}{\Gamma(\kappa + \mu + \frac{1}{2})} \sim -e^{-ix_1} \frac{(2i)^{1/2-\mu}\Gamma(2\mu)x_1^{1/2-\mu}}{\Gamma(\kappa + \mu + \frac{1}{2})} (\frac{1}{2} - \mu). \]  

(B2)

Inserting it into the second integral

\[ \int_{0}^{\infty} dx_1 e^{-ix_1} \frac{(2i)^{1/2+\mu}\Gamma(-2\mu)x_1^{-1/2+\mu}}{\Gamma(-\kappa + \mu + \frac{1}{2})} \left( -e^{-ix_1} \frac{(2i)^{1/2-\mu}\Gamma(2\mu)x_1^{1/2-\mu}}{\Gamma(\kappa + \mu + \frac{1}{2})} \right) = \int_{0}^{\infty} dx_1 e^{-2ix_1} \frac{2(-1)^{-\mu}\cos(\pi(\kappa + \mu))\csc(2\pi\mu)}{\mu(-1 + 2\mu)} e^{-i(-1)^{-\mu}\cos(\pi(\kappa + \mu))\csc(2\pi\mu)} \mu(-1 + 2\mu). \]  

(B3)

The two point function of the primordial curvature perturbations would be given by

\[ \langle \zeta_k \zeta_{-k} \rangle' = -4|c_2|^2 \frac{e^{i\pi\kappa}}{32k^3 M_{pl}^4 \epsilon^2} \text{Re} \left[ \frac{i(-1)^{-\mu}\cos(\pi(\kappa + \mu))\csc(2\pi\mu)}{\mu(-1 + 2\mu)} \right] + (\kappa \rightarrow -\kappa). \]  

(B4)

Since this method automatically removes the contribution that is exponentially suppressed in the large \( \mu \) limit, we don’t need to consider the non-time ordered integral. The power spectrum can be written as

\[ P_\zeta = -4 \frac{|c_2|^2}{2\pi^2} \frac{e^{i\pi\kappa}}{32 M_{pl}^4 \epsilon^2} \left[ \frac{i(-1)^{-\mu}\cos(\pi(\kappa + \mu))\csc(2\pi\mu)}{\mu(-1 + 2\mu)} \right] + (\kappa \rightarrow -\kappa). \]  

(B5)

Using the technique very similar to [94], we obtained the expression which is suitable for all \( c_2 < |\mu|^2 \).

\[ P_\zeta = -4 \frac{1}{2\pi^2} \frac{1}{32 M_{pl}^4 \epsilon^2} \sqrt{1 + 2 \frac{e^{i\pi\kappa}|c_2|^2 i(-1)^{-\mu}\cos(\pi(\kappa + \mu))\csc(2\pi\mu)}{\mu(-1 + 2\mu)}} + (\kappa \rightarrow -\kappa), \]  

(B6)

2. Bispectrum

The large mass limit of the bispectrum oin the quasi-single field model with neutral scalar particles is obtained in [95]. After integrating out the massive field, an equilateral non-Gaussianity is obtained. This part contains no clock signal, however, it is the dominant contribution in the large mass limit since it is only suppressed by \( 1/|\mu|^2 \) whereas the clock signal is supressed by \( \exp(-\pi|\mu|) \) in the large mass limit. In this section, we would like to derive the bispectrum by integrating out the charged massive scalar particles.

First, we calculate the second term of the bispectrum, we can write it as

\[ \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle^{(2)} = 2c_2 c_3^* \frac{H^3 e^{i\pi\kappa}}{128 \epsilon^4 k_1 k_2 k_3 M_{pl}^4} \text{Re} \left[ \int_{x_1}^{\infty} dx_1 e^{-ix_1} W_{\kappa,\mu}(-2ix_1) \int_{x_1}^{\infty} dx_2 x_2 e^{-i(k_1+k_2) x_2} W_{-\kappa,\mu}(2ix_2) \right] \]

\[ + (\kappa \rightarrow -\kappa, c_2 \rightarrow c_2^*, c_3 \rightarrow c_3^*), \]  

(B7)

We expand the Whittaker function in the limit \( x \to 0 \) following (B1), the first layer of integral is evaluated to

\[ \int_{x_1}^{\infty} dx_2 x_2 e^{-i(k_1+k_2) x_2} \frac{(2i x_2)^{1/2-\mu}\Gamma(2\mu)}{\Gamma(\kappa + \mu + \frac{1}{2})} \sim -(2i)^{\frac{1}{2}-\mu} \frac{\Gamma(2\mu)}{\Gamma(\kappa + \mu + \frac{1}{2})} (\frac{1}{2} - \mu) e^{-i(k_1+k_2) x_1}. \]  

(B8)

Inserting it into the second integral

\[ \int_{0}^{\infty} dx_1 \frac{e^{-ix_1} (-2ix_1)^{1/2+\mu}\Gamma(-2\mu)}{\Gamma(-\kappa + \mu + \frac{1}{2})} (-1)(2i)^{\frac{1}{2}-\mu} \frac{\Gamma(2\mu)}{\Gamma(\kappa + \mu + \frac{1}{2})} (\frac{1}{2} - \mu) e^{-i(k_1+k_2) x_1} \]

\[ = (2i)(\frac{1}{2})^{\frac{1}{2}+\mu} \frac{\Gamma(2\mu)}{\Gamma(\kappa + \mu + \frac{1}{2})} (-1)(2i)^{\frac{1}{2}-\mu} \frac{\Gamma(2\mu)}{\Gamma(\kappa + \mu + \frac{1}{2})} (\frac{1}{2} - \mu) e^{-i(k_1+k_2) x_1} \]

\[ = 2(\frac{1}{2})^{\frac{1}{2}+\mu} \frac{\csc(2\pi\mu) \cos(\pi(\kappa + \mu))}{\mu(\mu - \frac{1}{2})} \left( \frac{1}{k_1 + k_2 + k_3} \right)^3. \]  

(B9)
Hence, the second term of the bispectrum in the large mass limit can be expressed as
\[
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle^{(2)} = 2c_2c_3^\xi \frac{H^3 e^{i\pi k}}{128c^4 M_p^4 k_1 k_2 k_3 (k_1 + k_2 + k_3)^3} \text{Re}[2(-1)^{1+\mu} \csc(2\pi \mu) \cos(\pi(k + \mu)) / \mu(\mu - \frac{1}{2})] \\
+ (\kappa \rightarrow -\kappa, c_2 \rightarrow c_2^*, c_3 \rightarrow c_3).
\] (B10)

We can get the third term of the bispectrum in the large mass limit in the similar way.
\[
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle^{(3)} = 2c_2c_3^\xi \frac{H^3 e^{i\pi k}}{128c^4 M_p^4 k_1 k_2 k_3 (k_1 + k_2 + k_3)^3} \text{Re}[2(-1)^{1-\mu} \csc(2\pi \mu) \cos(\pi(k + \mu)) / \mu(\mu - \frac{1}{2})] \\
+ (\kappa \rightarrow -\kappa, c_2 \rightarrow c_2^*, c_3 \rightarrow c_3).
\] (B11)

Appendix C: Analytical Expression for the Clock Signal

In this section, we compute the analytical expression for the clock signal following the standard approach developed in [43]. In order to calculate the clock signal of primordial bispectrum analytically, we first simplify the propagators $D_{++}, D_{+-}, D_{-+},$ and $D_{--}$ in the following way. Using the late time behavior (12) and focusing on the non-local terms
\[
v_k(\tau_1)v_k^*(\tau_2) = \frac{e^{-\pi |\mu|}}{a(\tau_1)a(\tau_2)4k|\mu|} \left( |\alpha|^2 M_{\kappa,\mu}(2k\tau_1)(M_{\kappa,\mu}(2k\tau_2))^* + \alpha^* \beta M_{\kappa,\mu}(2k\tau_1)M_{\kappa,\mu}(2k\tau_2) \\
+ \alpha^* \beta (M_{\kappa,\mu}(2k\tau_1)) (M_{\kappa,\mu}(2k\tau_2))^* + |\beta|^2 (M_{\kappa,\mu}(2k\tau_1))^* M_{\kappa,\mu}(2k\tau_2) \right),
\] (C1)
\[
v_k(\tau_1)v_k^*(\tau_2) = \frac{e^{-\pi |\mu|}}{a(\tau_1)a(\tau_2)4k|\mu|} \left( |\alpha|^2 (M_{\kappa,\mu}(2k\tau_1))^* M_{\kappa,\mu}(2k\tau_2) + \alpha^* \beta M_{\kappa,\mu}(2k\tau_1)M_{\kappa,\mu}(2k\tau_2) \\
+ \alpha^* \beta (M_{\kappa,\mu}(2k\tau_1))^* (M_{\kappa,\mu}(2k\tau_2))^* + |\beta|^2 M_{\kappa,\mu}(2k\tau_1)(M_{\kappa,\mu}(2k\tau_2))^* \right),
\] (C2)

from where we know that only $|\alpha|^2$ and $|\beta|^2$ terms are different. However, the $|\alpha|^2$ and $|\beta|^2$ terms are local, and hence, do not contribute to the clock signal. In order to understand the bispectrum in the squeeze limit ($\alpha^* \beta$ and $\alpha^* \beta$), the four types of propagators $D_{++}, D_{+-}, D_{-+},$ and $D_{--}$, defined in (38) becomes identical.
\[
D(k, \tau_1, \tau_2) = D_{++}(k, \tau_1, \tau_2) = D_{+-}(k, \tau_1, \tau_2) = D_{-+}(k, \tau_1, \tau_2) = D_{--}(k, \tau_1, \tau_2)
\]
\[
= \frac{e^{-\pi |\mu|}}{a(\tau_1)a(\tau_2)4k|\mu|} \left( \alpha^* \beta M_{\kappa,\mu}(2k\tau_1)M_{\kappa,\mu}(2k\tau_2) + \alpha^* \beta (M_{\kappa,\mu}(2k\tau_1))^* (M_{\kappa,\mu}(2k\tau_2))^* \right).
\] (C3)

The squeezed limit bispectrum can be calculated for all orders in the $(k_1/k_3)$ expansion. However, for simplicity, we focus on the first order in $(k_1/k_3)$ expansion, where the Whittacker M is expanded as
\[
M_{\kappa,\mu}(z) = z^{\mu+1/2} + O(z^{\mu+3/2}).
\] (C4)

Taking the effective mass to be $e^2 E^2/H^4 + m^2/H^2 > 9/4$. The charged massive scalar propagator becomes
\[
D(k, \tau_1, \tau_2) = D_{++}(k, \tau_1, \tau_2) = D_{+-}(k, \tau_1, \tau_2) = D_{-+}(k, \tau_1, \tau_2) = D_{--}(k, \tau_1, \tau_2)
\]
\[
= \frac{e^{-\pi |\mu|}}{a(\tau_1)a(\tau_2)4k|\mu|} \left( \alpha^* \beta (-4k^2 \tau_1 \tau_2)^{-\mu+1/2} + c.c \right).
\] (C5)

The bispectrum is further simplified to
\[
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = 2c_2c_3^\xi \text{Re} \left[ \int_{-\infty}^{0} \int_{-\infty}^{0} \frac{d\tau_1}{(-H \tau_1)^3} \frac{d\tau_2}{(-H \tau_2)^3} [\partial_{\tau_1} G_{++}(k_3, \tau_1, 0) - \partial_{\tau_1} G_{-+}(k_3, \tau_1, 0)] D(k_3, \tau_1, \tau_2) \\
\partial_{\tau_2} G_{++}(k_1, \tau_2, 0) \partial_{\tau_2} G_{++}(k_2, \tau_2, 0) \right] + (\kappa \rightarrow -\kappa, c_2 \rightarrow c_2^*, c_3 \rightarrow c_3).
\] (C6)
After evaluation, the bispectrum becomes
\[
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle' = c_2 c_3^* \Re \left[ f(\mu, \kappa) \frac{2^{2\mu} H^3}{16k_1 k_2 k_3^{3/2} k_{12}^{5/2} M_{\text{pl}}^3} e^{3} \left( \frac{k_1 + k_2}{k_3} \right)^{-\mu} \right] + (\kappa \to -\kappa, c_2 \to c_2^*, c_3 \to c_3) \,.
\]
where the prefactor is given by
\[
f(\mu, \kappa) \equiv e^{i\pi \kappa} \frac{\Gamma(-2\mu)^2 \Gamma\left(\frac{1}{2} + \mu\right) \Gamma\left(\frac{3}{2} + \mu\right)}{\Gamma\left(\frac{1}{2} - \mu - \kappa\right) \Gamma\left(\frac{3}{2} + \mu - \kappa\right)} \left(1 + \sin (\pi \mu)\right) .
\] (C7)

So the shape function is
\[
S(k_1, k_2, k_3) = \frac{c_2 c_3^*}{HM_{\text{pl}}^2} \Re \left[ f(\mu, \kappa) \frac{2^{2\mu-2} k_1 k_2 k_3^{1/2}}{k_{12}^{5/2}} \left( \frac{k_1 + k_2}{k_3} \right)^{-\mu} \right] + (\kappa \to -\kappa, c_2 \to c_2^*, c_3 \to c_3) .
\] (C8)

[1] J. S. Schwinger, “On gauge invariance and vacuum polarization,” Phys. Rev. 82, 664 (1951).
[2] A. Di Piazza, C. Muller, K. Z. Hatsagortsyan and C. H. Keitel, “Extremely high-intensity laser interactions with fundamental quantum systems,” Rev. Mod. Phys. 84, 1177 (2012) [arXiv:1111.3886 [hep-ph]].
[3] R. Ruffini, G. Vereshchagin and S. S. Xue, “Electron-positron pairs in physics and astrophysics: from heavy nuclei to black holes,” Phys. Rept. 487, 1 (2010) [arXiv:0910.0974 [astro-ph.HE]].
[4] W. Tangarife, K. Tobioka, L. Ubaldi and T. Volansky, “Dynamics of Relaxed Inflation,” JHEP 1802, 084 (2018) [arXiv:1706.03072 [hep-ph]].
[5] W. Tangarife, K. Tobioka, L. Ubaldi and T. Volansky, “Relaxed Inflation,” arXiv:1706.00438 [hep-ph].
[6] T. E. Clarke, P. P. Kronberg and H. Boehringer, “A New radio - X-ray probe of galaxy cluster magnetic fields,” Astrophys. J. 727, L4 (2011) [arXiv:1009.1782 [astro-ph.HE]].
[7] F. Govoni and L. Feretti, “Magnetic field in clusters of galaxies,” Int. J. Mod. Phys. D 13, 1549 (2004) [astro-ph/0410182].
[8] C. Vogt and T. A. Ensslin, “A Bayesian view on Faraday rotation maps - Seeing the magnetic power spectra in galaxy clusters,” Astron. Astrophys. 434, 67 (2005) [astro-ph/0501211].
[9] K. Dolag, M. Kachelriess, S. Ostapchenko and R. Tomas, “Lower limit on the strength and filling factor of extragalactic magnetic fields,” Astrophys. J. 727, L4 (2011) [arXiv:1009.1782 [astro-ph.HE]].
[10] A. Neronov and I. Vovk, “Evidence for strong extragalactic magnetic fields from Fermi observations of TeV blazars,” Science 328, 73 (2010) [arXiv:1006.3504 [astro-ph.HE]].
[11] F. Tavecchio, G. Ghisellini, L. Foschini, G. Bonnoli, G. Ghirlanda and P. Coppi, “The intergalactic magnetic field constrained by Fermi/LAT observations of the TeV blazar 1ES 0229+200,” Mon. Not. Roy. Astron. Soc. 434, 1348 (2013) [arXiv:1312.5815 [astro-ph.CO]].
[12] M. S. Turner and L. M. Widrow, “Inflation Produced, Large Scale Magnetic Fields,” Phys. Rev. D 37, 2743 (1988).
[13] B. Ratra, “Cosmological seed’’ magnetic field from inflation,’’ Astrophys. J. 391, L1 (1992).
[14] A. Dolgov, “Breaking of conformal invariance and electromagnetic field generation in the universe,’’ Phys. Rev. D 48, 2499 (1993) [hep-ph/9301280].
[15] M. Gasperini, M. Giovannini and G. Veneziano, “Primordial magnetic fields from string cosmology,’’ Phys. Rev. Lett. 75, 3796 (1995) [hep-th/9504083].
[16] M. Giovannini, “Primordial magnetic fields from inflation,’’ Phys. Rev. D 62, 123505 (2000) [hep-ph/0007163].
[17] K. Bamba and J. Yokoyama, “Large scale magnetic fields from inflation in dilaton electromagnetism,’’ Phys. Rev. D 69, 043507 (2004) [astro-ph/0310824].
[18] K. Bamba and M. Sasaki, “Large-scale magnetic fields in the inflationary universe,’’ JCAP 0702, 030 (2007) [astro-ph/0611701].
[19] M. Giovannini and K. E. Kunze, “Magnetized CMB observables: A Dedicated numerical approach,’’ Phys. Rev. D 77, 063003 (2008) [arXiv:0712.3483 [astro-ph]].
[20] J. Martin and J. Yokoyama, “Generation of Large-Scale Magnetic Fields in Single-Field Inflation,’’ JCAP 0801, 025 (2008) [arXiv:0711.4307 [astro-ph]].
[21] K. Subramanian, “Magnetic fields in the early universe,’’ Astron. Nachr. 331, 110 (2010) [arXiv:0911.4771 [astro-ph.CO]].
[22] A. Kandus, K. E. Kunze and C. G. Tsagas, “Primordial magnetogenesis,’’ Phys. Rept. 505, 1 (2011) [arXiv:1007.3891 [astro-ph.CO]].
[23] R. Durrer, “Cosmic Magnetic Fields and the CMB,’’ New Astron. Rev. 51, 275 (2007) [astro-ph/0609216].
[24] K. Attuejeet, I. Pahwa, T. R. Seshadri and K. Subramanian, “Cosmological Magnetogenesis From Extra-dimensional Gauss Bonnet Gravity,’’ Phys. Rev. D 89, no. 6, 063002 (2014) [arXiv:1312.5815 [astro-ph.CO]].
[25] T. Fujita, R. Namba, Y. Tada, N. Takeda and H. Tashiro, “Consistent generation of magnetic fields in axion inflation models,’’ JCAP 1505, no. 05, 054 (2015) [arXiv:1503.05802 [astro-ph.CO]].
1. X. Chen, Y. Wang and Z. Z. Xianyu, “Schwinger-Keldysh Diagrammatics for Primordial Perturbations,” JCAP 1712, no. 12, 006 (2017) [arXiv:1703.10166 [hep-th]].

2. Weinberg, Steven. Gravitation and cosmology: principles and applications of the general theory of relativity. Vol. 1. New York: Wiley, 1972.

3. S. M. Carroll, San Francisco, USA: Addisson-Wesley (2004) 513 p

4. R. Faurer, M. Mirbabayi, L. Senatore and E. Silverstein, “Productive Interactions: heavy particles and non-Gaussianity,” JCAP 1710, no. 10, 058 (2017) [arXiv:1606.00513 [hep-th]].

5. X. Tong, Y. Wang and S. Zhou, “Unsuppressed primordial standard clocks in warm quasi-single field inflation,” JCAP 1806, no. 06, 013 (2018) [arXiv:1801.05688 [hep-th]].

6. X. Chen, W. Z. Chua, Y. Guo, Y. Wang, Z. Z. Xianyu and T. Xie, “Quantum Standard Clocks in the Primordial Trispectrum,” JCAP 1805, no. 05, 049 (2018) [arXiv:1803.04412 [hep-th]].

7. T. Nomm, Y. Yamaguchi and D. Yokoyama, “Effective field theory approach to quasi-single field inflation and effects of heavy fields,” JHEP 1306, 051 (2013) [arXiv:1211.1624 [hep-th]].

8. H. Lee, D. Baumann and G. L. Pimentel, “Non-Gaussianity as a Particle Detector,” JHEP 1612, 040 (2016) [arXiv:1607.03735 [hep-th]].

9. X. Chen, Y. Wang and Z. Z. Xianyu, “Loop Corrections to Standard Model Fields in Inflation,” JHEP 1608, 051 (2016) [arXiv:1604.07841 [hep-th]].

10. X. Chen, Y. Wang and Z. Z. Xianyu, “Standard Model Background of the Cosmological Collider,” Phys. Rev. Lett. 118, no. 26, 261302 (2017) [arXiv:1610.06597 [hep-th]].

11. X. Chen, Y. Wang and Z. Z. Xianyu, “Standard Model Mass Spectrum in Inflationary Universe,” JHEP 1704, 058 (2017) [arXiv:1612.08122 [hep-th]].

12. Y. P. Wu and J. Yokoyama, “Loop corrections to primordial fluctuations from inflationary phase transitions,” JCAP 1805, no. 05, 009 (2018) [arXiv:1704.05920 [hep-th]].

13. S. Weinberg, “Quantum contributions to cosmological correlations,” Phys. Rev. D 72, 043514 (2005) [hep-th/0506236].

14. L. Senatore and M. Zaldarriaga, “On Loops in Inflation,” JHEP 1012, 008 (2010) [arXiv:0912.2734 [hep-th]].

15. L. Senatore and M. Zaldarriaga, “On Loops in Inflation II: IR Effects in Single Clock Inflation,” JHEP 1301, 109 (2013) [arXiv:1203.6354 [hep-th]].

16. G. L. Pimentel, L. Senatore and M. Zaldarriaga, “On Loops in Inflation III: Time Independence of zeta in Single Clock Inflation,” JHEP 1207, 166 (2012) [arXiv:1203.6651 [hep-th]].

17. E. Bavarsad, S. P. Kim, C. Stahl and S. S. Xue, “Effect of a magnetic field on Schwinger mechanism in de Sitter spacetime,” Phys. Rev. D 97, no. 2, 025017 (2018) [arXiv:1707.03975 [hep-th]].

18. C. Stahl and S. S. Xue, “Schwinger effect and backreaction in de Sitter spacetime,” Phys. Lett. B 760, 288 (2016) [arXiv:1603.07166 [hep-th]].

19. E. Bavarsad, C. Stahl and S. S. Xue, “Scalar current of created pairs by Schwinger mechanism in de Sitter spacetime,” Phys. Rev. D 94, no. 10, 104011 (2016) [arXiv:1602.06556 [hep-th]].

20. O. O. Sobol, E. V. Gorbar, M. Kamarpour and S. I. Vlkhinskii, “Influence of backreaction of electric fields and Schwinger effect on inflationary magnetogenesis,” Phys. Rev. D 98, 063534 (2018) [arXiv:1807.09851 [hep-ph]].

21. T. Hayashinaka, T. Fujita and J. Yokoyama, “Fermionic Schwinger effect and induced current in de Sitter space,” JCAP 1607, no. 07, 010 (2016) [arXiv:1604.04165 [hep-th]].

22. K. D. Lozanov, A. Maleknejad and E. Komatsu, “Schwinger Effect by an SU(2) Gauge Field during Inflation,” arXiv:1805.09318 [hep-th].

23. A. Maleknejad and E. Komatsu, “Production and Backreaction of Spin-2 Particles of SU(2) Gauge Field during Inflation,” arXiv:1808.09076 [hep-ph].

24. J. Garriga, “Pair production by an electric field in (1+1)-dimensional de Sitter space,” Phys. Rev. D 49, 6343 (1994).

25. J. Garriga, “Nucleation rates in flat and curved space,” Phys. Rev. D 49, 6327 (1994) [hep-ph/9308280].

26. A. J. Tolley and M. Wyman, “The Gelaton Scenario: Equilateral non-Gaussianity from multi-field dynamics,” Phys. Rev. D 81, 043502 (2010) [arXiv:0910.1853 [hep-th]].

27. A. Achucarro, J. O. Gong, S. Hardeman, G. A. Palma and S. P. Patil, “Mass hierarchies and non-decoupling in multi-scalar field dynamics,” Phys. Rev. D 84, 043502 (2011) [arXiv:1005.3848 [hep-th]].

28. A. Achucarro, J. O. Gong, S. Hardeman, G. A. Palma and S. P. Patil, “Effective theories of single field inflation when heavy fields matter,” JHEP 1205, 066 (2012) [arXiv:1201.6342 [hep-th]].

29. X. Chen and Y. Wang, “Quasi-Single Field Inflation with Large Mass,” JCAP 1209, 021 (2012) [arXiv:1205.0160 [hep-th]].

30. S. Pi and M. Sasaki, “Curvature Perturbation Spectrum in Two-field Inflation with a Turning Trajectory,” JCAP 1210, 051 (2012) [arXiv:1205.0161 [hep-th]].

31. R. Gwyn, G. A. Palma, M. Sakellariadou and S. Sypsas, “Effective field theory of weakly coupled inflationary models,” JCAP 1304, 004 (2013) [arXiv:1210.3020 [hep-th]].

32. H. An, M. McAneny, A. K. Ridgway and M. B. Wise, “Quasi Single Field Inflation in the non-perturbative regime,” JHEP 1806, 105 (2018) [arXiv:1706.09971 [hep-ph]].

33. X. Tong, Y. Wang and S. Zhou, “On the Effective Field Theory for Quasi-Single Field Inflation,” JCAP 1711, no. 11, 045 (2017) [arXiv:1708.01709 [astro-ph.CO]].

34. A. V. Iyer, S. Pi, Y. Wang, Z. Wang and S. Zhou, “Strongly Coupled Quasi-Single Field Inflation,” JCAP 1801, no. 04, 041 (2018) [arXiv:1710.03654 [hep-th]].

35. J. O. Gong, S. Pi and M. Sasaki, “Equilateral non-Gaussianity from heavy fields,” JCAP 1311, 043 (2013) [arXiv:1306.3691 [hep-th]].