Ground-state cooling of mechanical resonators by hot thermal light

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We propose a scheme to cool a mechanical resonator to its quantum ground-state, which is interacting with a cavity mode via the optomechanical coupling. As opposed to standard laser cooling schemes where coherence renders the state of the resonator to its ground-state, here we use incoherent thermal light to achieve the same aim. We show that simultaneous cooling of two degenerate or near-degenerate mechanical resonators is possible in our scheme, which is otherwise a challenging goal to achieve in optomechanics. The generalization of this method to the simultaneous cooling of multiple resonators is straightforward. The underlying physical mechanism of cooling is explained by revealing a direct connection between the laser sideband cooling and “cooling by heating” in a standard optomechanical setting.

I. INTRODUCTION

The ground-state cooling of mechanical resonators is crucial for various applications, such as gravitational wave detection [1], ultrasensitive precision measurements [2], and for the investigation of the quantum behavior of mechanical systems [3], where only relevant motion is due to zero-point fluctuations. For the quantum information processing applications of optomechanical systems, mechanical resonators need to be close to their quantum ground-state [4, 5]. To achieve this goal, different methods have been proposed and realized such as resolved sideband cooling [6–13], feedback-assisted cooling [14–19], and backaction cooling [20, 21]. In a standard approach, thermal energy is removed from a single mode of mechanical resonator to bring its state near to the ground-state. In the recent past, optomechanical systems having multiple mechanical resonators caught much attention [22–26], finding various applications such as heat management [27, 28] and mesoscopic entanglement [29]. However, ground-state cooling of degenerate or near-degenerate multiple mechanical resonators is quite a challenging task to accomplish in optomechanics. This is because the existence of dark modes in multiple resonators case, which decouple from the common cavity mode and hinders the simultaneous ground-state cooling of multiple resonators [18, 30–32]. There are few recent proposals [33, 34] to overcome this difficulty, however, these demand phase-dependent coherent interaction between the mechanical resonators.

A complementary approach, namely “cooling by heating”, exploits incoherent driving of the quantum system to cool it down [35, 36]. As opposed to laser cooling, in this method, incoherent thermal drive removes energy from the quantum system only to dump it in another bath at a lower temperature. For optomechanical setting, this has been proposed for two optical modes coupled to a single mechanical resonator [35]. This scheme requires not only three-body interaction but also it can not cool down the mechanical resonator near to its ground-state for the typical optomechanical system parameters.

In this work, we propose a scheme based on cooling by heating mechanism, and it requires only two-body interaction to cool a single mechanical resonator to its ground-state. In addition, simultaneous ground-state cooling of multiple degenerate or near-degenerate mechanical resonators is also possible, which is otherwise challenging to accomplish. A key to realize this method is bath spectral filtering (BSF) [37–44], in which unwanted frequencies are filtered out from the system-bath coupling spectra. The BSF has previously been shown to enhance performance of different thermal functions [43, 44]. Finally, we reveal a direct connection between the laser sideband cooling and cooling by heating in the optomechanical setting, which also explains the underlying physical mechanism of cooling in our scheme.

This paper is organized as follows. In Sec. II, we present our basic model and outline the derivation of the master equation. In Sec. III, we discuss the results of single and multiple resonators cooling. We present the conclusions of our work in Sec. IV.

II. DESCRIPTION OF THE MODEL

We consider a multimode optomechanical system (Fig. 1), which consists of a single optical mode interacting with two mechanical resonators. The optical mode is coupled to two thermal baths at temperatures $T_c$ and $T_h$, and each mechanical resonator of frequency $\omega_i$ is in contact with an independent thermal bath at temperature $T_{hi}$, $i = 1, 2$. The Hamiltonian of the isolated optomechanical system is given by (we take $\hbar = 1$)

$$\hat{H}_s = \omega_\alpha \hat{a}^\dagger \hat{a} + \sum_{i=1}^2 \left[ \omega_i \hat{b}_i^\dagger \hat{b}_i - g_i \hat{a}^\dagger \hat{a} (\hat{b}_i + \hat{b}_i^\dagger) \right],$$

where $\omega_\alpha$ and $\omega_i$ are the frequencies of the optical and mechanical resonators, respectively. The bosonic annihilation (creation) operators of the optical and mechanical modes are given by $\hat{a}$ ($\hat{a}^\dagger$) and $\hat{b}_i$ ($\hat{b}_i^\dagger$), respectively. The $g_i$ terms represent the single-photon optomechanical coupling between the optical mode and mechanical resonators. This multimode optomechanical model can be realized via a photonic crystal optomechanical system [45] or circuit electromechanical system [46, 47]. The free Hamiltonians of the independent ther-
mal baths is given by
\[ \hat{H}_{SB} = \sum_k g_{k,a}(a + a^\dagger) \otimes (\hat{a}_{k,a} + \hat{a}_{k,a}^\dagger) + \sum_{k,i} g_{k,i}(\hat{b}_i + \hat{b}_i^\dagger) \otimes (\hat{a}_{k,i} + \hat{a}_{k,i}^\dagger), \] (3)

where, \( \alpha = H, C \), and the first term describes the interaction of hot and cold baths with the optical mode, and the second term represents the interaction between the mechanical resonator \( \hat{b}_i \) and its bath \( B_i \).

To derive the master equation, we diagonalize the Hamiltonian of the isolated optomechanical system by a unitary transformation
\[ \hat{S} = e^{-\sum_i \xi_i a^\dagger a (\hat{b}_i^\dagger - \hat{b}_i)}, \] (4)

where \( \xi_i = g_i / \omega_i \). The diagonalized Hamiltonian reads as
\[ \hat{H}_s = \omega_a \hat{a}^\dagger \hat{a} + \sum_i [\omega_i \hat{b}_i^\dagger \hat{b}_i - g_i^2 / \omega_i (\hat{a}^\dagger \hat{a})^2], \] (5)

and the transformed operators are given by
\[ \hat{a} = \hat{S} \hat{a} \hat{S}^\dagger = \hat{a} e^{-\sum_i \xi_i (\hat{b}_i^\dagger - \hat{b}_i)}, \] (6)
\[ \hat{b}_i = \hat{S} \hat{b}_i \hat{S}^\dagger = \hat{b}_i - \xi_i \hat{a}^\dagger \hat{a}. \] (7)

The eigenenergies of the isolated optomechanical system are expressed as
\[ E_{n_a,m_i} = n_a \omega_a + m_i \omega_i - n_a^2 g_i^2 / \omega_i, \] (8)

where \( m_i \) is the number of phonons in the mechanical resonator of frequency \( \omega_i \), and \( n_a \) is the number of photons in the cavity. The master equation in the interaction picture evaluate to
\[ \hat{\rho} = \hat{L}_H + \hat{L}_C + \hat{L}_i, \] (9)

where the interaction of the baths with the system is described by Liouville superoperators \( \hat{L}_x \), which are given by [43, 44, 48]
\[ \hat{L}_x = G_x(\omega_x) \hat{D}[\hat{a}] + G_x(\omega_1 - \omega_x) \hat{D}[\hat{a}^\dagger] + \sum_i \xi_i^2 \left[ G_x(\omega_-) \hat{D}[\hat{a}^\dagger \hat{b}_i] + G_x(\omega_-) \hat{D}[\hat{a} \hat{b}_i^\dagger] \right], \] (10)

and ignored all higher order terms \( O(\xi^3) \).

### III. RESULTS

In this section, we present the results for the ground-state cooling of the multiple mechanical resonators in an optomechanical system. To explain the underlying physical mechanism, first, we will consider a single mechanical resonator.
This simple model helps us to make a direct connection between the “cooling by heating” with laser sideband cooling obvious in the optomechanical system.

For the numerical simulations, we have considered parameters of a typical optomechanical system [49]: \( \omega_i = 2 \pi \times 10 \) MHz, \( g_i = 2 \pi \times 100 \) kHz, \( \kappa_{i} = 2 \pi \times 100 \) Hz, \( \kappa_{s} = 2 \pi \times 10^6 \) MHz. Initial average phonons in the mechanical resonator is \( \bar{n}_e(0) \sim 2 \times 10^3 \), and the corresponding temperature becomes \( T \sim 1.5 \) K. We take the temperature of the cold bath \( T_c \sim nK \), while the temperature of the hot bath is larger by many order of magnitude, i.e., \( 4 \times 10^4 \lesssim T_h \lesssim 4 \times 10^7 \) K. In the results, we have scaled all these parameters with the optical frequency \( \omega_a \).

### A. Single mechanical resonator

In the case of a single mechanical resonator, the index \( i \) can be dropped in Eq. (9). We are interested in the dynamics of the mechanical resonator, therefore, upon taking trace over the optical mode, Eq. (9) takes the form

\[
\frac{d \tilde{\rho}}{dt} = (A^- + A^+_\text{th}) \tilde{D}[\hat{b}_1] + (A^+ + A^+_\text{th}) \tilde{D}[\hat{b}^+_1],
\]

where \( A^- (A^+_\text{th}) \) is downward transition rate, and \( A^+ (A^+_\text{th}) \) is upward transition rate in the mechanical resonator due to the optical (mechanical) baths, and these are defined as

\[
A^- := \xi_1^2 \sum_{a=H,C} (G_a(\omega_+)|\bar{n}_a + 1\rangle + G_a(-\omega_-)|\bar{n}_a + 1\rangle),
\]

\[
A^+ := \xi_1^2 \sum_{a=H,C} (G_a(\omega_-)|\bar{n}_a\rangle + G_a(-\omega_+)|\bar{n}_a + 1\rangle),
\]

\[
A^+_\text{th} := G_1(\omega_1), \quad A^+_\text{th} := G_1(-\omega_1).
\]

Note that we use \( A^\pm \) for the transition rates to compare our results with the sideband cooling in the atomic and optomechanical systems [49].

In the rest of the work, we consider one dimensional Ohmic spectral densities of all the baths, given by [38–40]

\[
G_x(\omega) = \begin{cases} 
\kappa_x(\omega)|1 + \bar{n}_x(\omega)\rangle \langle 1 + \bar{n}_x(\omega)| & \omega > 0, \\
\kappa_x(|\omega|)|\bar{n}_x(|\omega|)\rangle \langle |\omega| |\bar{n}_x(|\omega|)| & \omega < 0,
\end{cases}
\]

where \( \kappa_x \) is the system-bath coupling strength, and \( \bar{n}_x(\omega) = 1/\left[ e^{\omega/T_x} - 1 \right] \) is the mean number of quanta in the respective bath. For the cooling of mechanical resonator, bath spectral filtering is applied only to optical baths, which yields [37, 44]

\[
\tilde{G}_a := \frac{1}{\pi} \left[ \left( (\omega - (\omega(n + 1/2) + \Delta_a(\omega)) \right)^2 + (\pi G_a(\omega))^2 \right]^{-1},
\]

where \( \Delta_a(\omega) \) is the Lamb-shift induced by the thermal bath.

**Cooling:** In a typical optomechanical system, resolved-sideband (\( \omega_1 > \kappa_{h,c} \)) cooling can be induced if the input laser light of frequency \( \omega_L \) is resonant with the lower sideband \( \omega_1 = \omega_- = \omega_a - \omega_L \) [12, 49]. In this case, the Stokes process is suppressed and anti-Stokes scattering dominates, as shown in Fig. 2(a). As long as the upper sideband \( \omega_+ = \omega_a + \omega_L \) is far off-resonant, the input laser can drive the mechanical resonator near to its ground-state. Compared with the standard optomechanical setup, we don’t have input laser drive contribution in the Hamiltonian given in Eq. (1). Instead, the cavity is driven by the hot and cold thermal baths, however, both lower and upper phonon sidebands appear as a...
result of these thermal drives. By analogy with laser sideband cooling, we may want to drive the cavity at the lower sideband and suppress the upper sideband. This can be achieved by using bath spectral filtering [37–44].

To drive the cavity at lower sideband with the hot thermal bath, the filtered hot bath couples only to a transition frequency at \( \omega_- \) (lower sideband), and the coupling frequency at upper sideband \( \omega_+ \) is filtered (Fig. 1(c)). To complete the refrigeration process, the cold bath couples to a transition frequency at \( \omega_a \). The coupling spectra of the hot and cold baths are filtered and well-separated (Fig. 1(c)), and satisfy

\[
\tilde{g}_C(\omega_a) \gg \tilde{g}_H(\omega_a), \\
\tilde{g}_C(\omega_-) \ll \tilde{g}_H(\omega_-).
\]

For the choice of this bath spectral filtering, the master equation given in Eq. (9) modifies to

\[
\begin{align*}
\dot{\tilde{\rho}}_C &= \tilde{g}_C(\omega_a) \tilde{D}[a] + \tilde{g}_C(-\omega_a) \tilde{D}[\dagger a^\dagger], \\
\dot{\tilde{\rho}}_H &= \tilde{\zeta}_1^2 (\tilde{g}_H(\omega_-) \tilde{D}[\dagger b^\dagger b] + \tilde{g}_H(-\omega_-) \tilde{D}[a^\dagger b^\dagger]), \\
\dot{\tilde{\rho}}_1 &= \Gamma_1 (\omega_1) \tilde{D}[b^\dagger] + \tilde{g}_1 \tilde{D}[\dagger b^\dagger],
\end{align*}
\]

we note that \( \dot{\tilde{\rho}}_1 \) remains the same. The optical mode reaches to almost a thermal steady-state at temperature \( T_c \) with corrections of the order \( O(\tilde{\zeta}_2^2) \) [40, 41, 50], and contribution from the mechanical bath. In our results, we ignore the correction term of the order \( \tilde{\zeta}_2^2 \) from the steady-state mean phonon number \( \langle \tilde{n}_a \rangle_{ss} \), which then takes the form

\[
\langle \tilde{n}_a \rangle_{ss} \approx \frac{\tilde{g}_C(-\omega_a) + \tilde{g}_1(-\omega_1)}{\tilde{g}_C(\omega_a) + \tilde{g}_C(-\omega_a) + \tilde{g}_1(\omega_1) + \tilde{g}_1(-\omega_1)}.
\]

The comparison between the numerical simulation of the master equation given in Eq. (20), and our approximate results is given in Appendix A (Fig 5). From Eq. (20), one can obtain the mean number of phonons in the mechanical resonator

\[
\frac{d\langle \tilde{n}_1 \rangle}{dt} = (A^+_c + A^+_m) \langle \tilde{n}_1 + 1 \rangle - (A^-_c + A^-_m) \langle \tilde{n}_1 \rangle,
\]

where \( A^+_c \) (\( A^-_c \)) represents upward (downward) transition rate due to the optical filtered baths (Fig. 1(c)), and given by

\[
A^-_c := \tilde{\zeta}_1^2 \tilde{g}_H(\omega_-) \langle \tilde{n}_a + 1 \rangle, \quad A^+_c := \tilde{\zeta}_2^2 \tilde{g}_H(\omega_-) \langle \tilde{n}_a \rangle.
\]

The steady-state average phonon number is

\[
\langle \tilde{n}_1 \rangle_{ss} = \frac{A^+_c + \kappa_1 \tilde{n}_1}{\Gamma_c + \kappa_1},
\]

here \( \Gamma_c = A^-_c - A^+_c \) is the net optical baths damping rate. This equation of phonon-number is similar to the steady-state phonon number in a standard optomechanical system for laser sideband cooling [7], which is given by \( \langle \tilde{n}_1 \rangle_{ss} = (A^+ + \kappa_1 \tilde{n}_1)/\Gamma_{opt} + \kappa_1) \). \( \Gamma_{opt} = A^- - A^+ \) is the net optomechanical damping rate, further, \( A^+ \) and \( A^- \) represent the rates of Stokes (heating) and anti-Stokes (cooling) processes, respectively. For the resolved-sideband ground-state laser cooling, (i) upper sideband should be far off-resonant, (ii) the optical damping \( \Gamma_{opt} \) must be much greater than the mechanical damping \( \kappa_1 \), and (iii) the initial phonon number \( \tilde{n}_1(0) \) must be much smaller than the quality factor of the mechanical resonator \( \tilde{n}_1 \ll \omega_1/\kappa_1 \) [7]. Similar to the resolved-sideband cooling, in our scheme ground-state cooling is possible provided, (i) upper sideband (\( \omega_+ \)) from the hot bath spectrum is filtered, (ii) for sufficiently low cold bath temperature, the thermal dissipation due to the optical baths is greater than the mechanical bath coupling strength, i.e., \( \Gamma_c \gg \kappa_1 \). (iii) The initial phonon number in the mechanical resonator is much smaller than its quality factor \( \tilde{n}_1 \ll \omega_1/\kappa_1 \), in addition, we note that these conditions are stated for \( T_h > T_c > T_c \). For the bath spectra shown in Fig. 1(c), and considered system parameters (Fig. 3), \( \Gamma_c \) is larger than \( \kappa_1 \) by three order of magnitude. The other conditions mentioned here also satisfy with in the considered system parameters. To investigate the steady-state cooling performance of our scheme, we plot the average number of phonons \( \langle \tilde{n}_1 \rangle_{ss} \) in the mechanical resonator as a function of the hot bath temperature \( T_h \) in Fig. 3. The results indicate that ground-state cooling is possible for the reasonable hot bath temperature.

The cooling mechanism can be explained by direct analogy with the laser sideband cooling shown in Fig. 2. By increasing the temperature \( T_h \) of the hot bath, the number of photons of “correct frequency” in the cavity to induce the red (lower) sideband transition increases. In a refrigeration process, the hot bath induces a transition of frequency \( \omega_- \) at the cost of destroying a single phonon (Fig. 2). Then the energetic (blue-shifted) photon created in previous transition is dumped in to the cold bath to complete the refrigeration process.

![FIG. 3. (Color online) The steady-state mean phonon number \( \langle \tilde{n}_1 \rangle_{ss} \) as a function of the hot bath temperature \( T_h \).]
that, this explanation is only valid for the filtered baths spectra shown in Fig. 1(c). For baths spectra including upper phonon sideband or spectrally overlapping baths, this explanation is invalid.

*Heating*: if we select the optical baths spectra of the form shown in Fig. 1(d), and they follow

\[
\begin{align*}
\tilde{G}_C(\omega_a) & \gg \tilde{G}_H(\omega_b), \\
\tilde{G}_C(\omega+) & \ll \tilde{G}_H(\omega+),
\end{align*}
\]

then the master equation given in Eq. (9) simplifies to

\[
\begin{align*}
\hat{L}_C &= \tilde{G}_C(\omega_a)\hat{D}[\hat{a}] + \tilde{G}_C(-\omega_a)\hat{D}[^{\dagger}\hat{a}], \\
\hat{L}_H &= \tilde{G}_H(\omega+)\hat{D}[\hat{a}\hat{b}_1] + \tilde{G}_H(-\omega+)\hat{D}[^{\dagger}\hat{a}\hat{b}_1^\dagger)],
\end{align*}
\]

and \(\hat{L}_1\) remains the same. The energy transitions induced by these dissipators \((\hat{D}[^{\dagger}\hat{a}\hat{b}^\dagger], \hat{D}[\hat{a}\hat{b}])\) lead to heating of the mechanical resonator as shown in Fig. 2. \(\Gamma_h = A_h^- - A_h^+\) is the net optical thermal dissipation rate with the downward and upward transition rates

\[
A_h^- := \tilde{\kappa}_1^2 \tilde{G}_H(\omega_+)\langle \hat{n}_a \rangle, \quad A_h^+ := \tilde{\kappa}_1^2 \tilde{G}_H(-\omega+)\langle \hat{n}_a + 1 \rangle.
\]

The dynamical evolution of the initial phonon number is given by

\[
\langle \hat{n}_i(t) \rangle = \frac{A_h^+ + \kappa_i \hat{n}_i}{\Gamma_h + \kappa_i} + \hat{n}_i e^{-t(\Gamma_h + \kappa_i)},
\]

in this case, \(\Gamma_h\) is negative, consequently, the mechanical resonator heats up and the system becomes unstable in the long-time limit.

### B. Multiple mechanical resonators

Here, we consider two mechanical resonators couple to a single optical cavity, as shown in Fig. 1, however, the extension of our scheme to \(N\) mechanical resonators is straightforward. For the case of the hot and cold baths spectra considered in Fig. 1(c), the reduced master equation of the two mechanical resonators simplifies to

\[
\frac{d\hat{\rho}}{dt} = (A_i^- + A_{th_i}^-)\hat{D}[\hat{b}^\dagger] + (A_i^+ + A_{th_i}^+)\hat{D}[^{\dagger}\hat{b}],
\]

and the transitions rates due to the optical baths have the form

\[
\begin{align*}
A_i^- & := \tilde{\kappa}_i^2 \tilde{G}_H(-\omega_{ij})\langle \hat{n}_a \rangle + 1, \\
A_i^+ & := \tilde{\kappa}_i^2 \tilde{G}_H(\omega_{ij})\langle \hat{n}_a \rangle,
\end{align*}
\]

\(\omega_{ij} = \omega_a - \omega_i\). At steady-state the average phonon number in each resonator is given by

\[
\langle \hat{n}_i \rangle_{ss} = \frac{A_i^+ + \kappa_i \hat{n}_i}{\Gamma_i + \kappa_i},
\]

here \(\Gamma_i = A_i^- - A_i^+\). This shows that the steady-state mean phonon number of each resonator depends on the relaxation rates of both resonators with their baths. The simultaneous cooling of both degenerate and non-degenerate multiple resonators is possible provided these relaxation rates are not very high. On contrary, in optomechanical sideband cooling, existence of the dark modes hinder the simultaneous multiple resonators cooling [31]. In addition, for degenerate or nearly degenerate resonators, one can not cool down either of the mechanical resonators [18, 30]. In our scheme, Eq. (29) shows that, simultaneous cooling of both degenerate and non-degenerate resonators is possible. This is shown in Fig. 4, in which we plot the steady-state phonon number \(\langle \hat{n}_i \rangle_{ss}\) of each resonator as a function of the hot bath temperature \(T_h\). For the identical resonators (Fig. 4(a)) coupled to thermal baths of the same temperature, the cooling curves are identical. However, for the non-degenerate case (Fig. 4(b)), the steady-state phonons in the second resonator is greater than the first resonator. The lesser cooling of the second resonator is because of the choice of the hot bath spectrum (Fig. 1(c)), which peaks at \(\omega_2 - \omega_1\). The second resonator frequency \(\omega_2\) is below this peak, due to which it is weakly driven as compared to the resonator one, consequently, \(\tilde{\Gamma}_2 < \tilde{\Gamma}_1\). The effect of optomechanical coupling strength \(g_i\) is shown in Fig. 4(c), the weakly coupled second resonator have smaller optical thermal dissipation \((\tilde{\Gamma}_2 < \tilde{\Gamma}_1)\), which results in lesser cooling. The mechanical resonator with larger system-bath coupling cools lesser as shown in Fig. 4(d).

Extension of our scheme to \(N\) independent mechanical resonators coupled to a single cavity mode is straightforward. Because Eqs. (28) and (29) remain valid for arbitrary number of independent resonators. Another key feature of our method is that it is not only limited to optomechanics, but can also be realized in different platforms provided the target system to be cooled interacts dispersive with the working medium through the Hamiltonian of the form \(\hat{H}_{int} = g N_o X_m\). Where \(X_m\) is observable of the target system, and \(N_o = \eta \hat{H}_{wm}\) with \(\eta\) a positive constant, such an interaction yields a master equa-
n

10

10

⟨

states. The use of small Hilbert space dimensions for the optical use truncated Hilbert space with 7 photon states and 70 phonon master equation given in Eq. (20). In the numerical simulation, we ter than the solar temperatures ∼ are most useful for cooling when they are comparable or hot-

table to remove the unwanted transition frequencies from the system-bath coupling spectra. Remarkably, the steady-

temperature for the cavity field could be to use micro-

maser scheme of heating [53], where the pump atoms can be prepared in quantum coherent superpositions [54, 55]. Instead of a beam of atoms, one can also consider using a single atom making repeated interactions with the cavity field. By this way, one can cool multiple mechanical resonators using even a single non-thermal atom carrying quantum information [56].

Our scheme of cooling establishes a direction connection between sideband cooling and cooling by heating methods proposed previously [35, 36]. These results are relevant to the miniaturization of quantum devices based on the novel multimode optomechanical systems [28, 57].

IV. CONCLUSIONS

We proposed a scheme to implement ground-state cooling of a single mechanical resonator in the standard optomechanical setting. As opposed to laser cooling, we used hot thermal light to cool down the mechanical resonator. We showed that it is possible to cool multiple degenerate or near-degenerate mechanical resonators to their ground-state using our method. To realize our proposed scheme, we used bath spectrum filtering to remove the unwanted transition frequencies from the system-bath coupling spectra. Remarkably, the steady-state mean phonon number given in Eq. (23) shares similarities with the average phonon number obtained in laser sideband cooling. Consequently, ground-state cooling conditions in our scheme are also similar to the those given in standard resolved-sideband cooling. Temperatures for the cavity field are most useful for cooling when they are comparable or hotter than the solar temperatures ∼ 5000 K. Intriguingly, the mechanical components would not equilibrate with such a high temperature, which would melt them, but thermalize to a much colder temperature. An appealing possibility to get high temperatures for the cavity field could be to use micro-

Appendix A: Dynamics of the system

For the optomechanical system shown in Fig. 1(a), if we consider a single mechanical resonator of frequency ω₁, and bath spectrum filtering of Fig. 1(c), the master equation can be written as [58]

\[ \dot{\mathcal{L}}_C = \gamma_C (\hat{D}[\hat{a}] + e^{-\beta_e \omega} \hat{D}[\hat{a}^\dagger]) \]

\[ \dot{\mathcal{L}}_H = \gamma_H \hat{c}^2 (\hat{D}[\hat{b}^\dagger] + e^{-\beta_h \omega} \hat{D}[\hat{b}^\dagger]) \]

\[ \dot{\mathcal{L}}_1 = \gamma_1 (\hat{D}[\hat{b}] + e^{-\beta_1 \omega} \hat{D}[\hat{b}^\dagger]) \]

\[ \dot{\mathcal{L}}_2 = \gamma_2 (\hat{D}[\hat{b}^\dagger] + e^{-\beta_2 \omega} \hat{D}[\hat{b}]) \]

\[ \dot{n}_a = -\gamma_a (1 - e^{-\beta_e \omega}) \hat{n}_a + \gamma_C e^{-\beta_e \omega} \hat{n}_a \]

\[ \dot{n}_1 = -\gamma_1 (1 - e^{-\beta_1 \omega}) \hat{n}_1 + \gamma_1 e^{-\beta_1 \omega} \hat{n}_1 \]

\[ \dot{n}_2 = -\gamma_2 (1 - e^{-\beta_2 \omega}) \hat{n}_2 + \gamma_2 e^{-\beta_2 \omega} \hat{n}_2 \]

here γ₁, γ₄, and γ₆ are relaxation rates which depend on the specific model of the cold, hot and mechanical baths, respectively. In addition, β_e, β_h, and β₁ are inverse temperatures of the cold, hot and mechanical baths, respectively. In the limit β_h → 0, the rate equations for the mean number of photons and phonons read

\[ \frac{d}{dt} \langle \hat{n}_a \rangle = -\gamma_a (1 - e^{-\beta_e \omega}) \langle \hat{n}_a \rangle + \gamma_C e^{-\beta_e \omega} \langle \hat{n}_a \rangle \]

\[ \frac{d}{dt} \langle \hat{n}_1 \rangle = -\gamma_1 (1 - e^{-\beta_1 \omega}) \langle \hat{n}_1 \rangle + \gamma_1 e^{-\beta_1 \omega} \langle \hat{n}_1 \rangle \]

\[ \frac{d}{dt} \langle \hat{n}_2 \rangle = -\gamma_2 (1 - e^{-\beta_2 \omega}) \langle \hat{n}_2 \rangle + \gamma_2 e^{-\beta_2 \omega} \langle \hat{n}_2 \rangle \]

and at steady-state the mean number of quanta in the mechanical resonator take the form

\[ \langle \hat{n}_1 \rangle_{ss} = \frac{\gamma_C e^{-\beta_e \omega} + \gamma_1 e^{-\beta_1 \omega}}{\gamma_C e^{-\beta_e \omega} - \gamma_1 e^{-\beta_1 \omega} + \gamma_C + \gamma_1} \]

For one-dimensional Ohmic spectral densities of the baths, Eq. (A6) simplifies to

\[ \langle \hat{n}_1 \rangle_{ss} = \frac{\omega_a K_a \hat{n}_a + \omega_1 K_1 \hat{n}_1}{\omega_a K_a + \omega_1 K_1} \]

which shows that ground-state cooling is possible for the conditions stated in Sec. III.A.
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