Abstract

We perform a joint fit to HERA measurements of the unpolarized deep inelastic scattering structure functions $F_2(x,Q^2)$ and $F_L(x,Q^2)$ over a very large kinematical range in the photon virtuality $Q^2 \leq 400$ GeV$^2$ and Bjorken variable $x < 0.01$. This is done by considering a $U(1)$ vector gauge field minimally coupled to the graviton Regge trajectory in improved holographic QCD. We find good agreement with the data from HERA achieving a $\chi^2$ of 1.4 without removing presumed outliers existing in data. If we consider only data with $Q^2 \leq 10$ GeV$^2$ the quality of the fit improves to a $\chi^2$ of 1.3. An approximate relation between the structure functions, valid at NLO QCD, is used to estimate the parton distribution function of the gluon inside the proton. Our PDFs are in agreement with the NLO results presented by CTEQ and NNPDF collaborations.

1. Introduction

The finding that the Pomeron is dual to the graviton Regge trajectory [1] ignited the application of the gauge/gravity duality in the analysis of QCD processes dominated by Pomeron exchange. This observation has been studied in a multitude of diffractive processes, like low-$x$ deep inelastic scattering (DIS), deeply virtual Compton scattering, vector meson production, double diffractive Higgs production, central production of mesons, as well as other inclusive processes [2-39]. These studies showed that holographic QCD is a valuable tool to study physical processes where the partonic structure is dominated by a gluon rich medium.

*emails: arturj.c.amorimdesousa@gmail.com, miguelc@fc.up.pt
Currently the proton is seen as a collection of partons, each carrying a fraction $x$ of the longitudinal momentum. The probability density of finding a given parton carrying a momentum fraction $x$ at a squared energy scale $Q^2 = -q^2$ is known as a Parton Distribution Function (PDF). The precise knowledge of the PDFs is vital to test predictions of the Standard Model and beyond Standard Model models in the LHC. However, these objects are nonperturbative and cannot be derived from first principles in QCD. Instead the functional dependence of the PDFs on $x$ is parameterised at some high enough scale $Q^2 = Q^2_0$ where nonperturbative effects are not important. These input distributions can then be evaluated at another scale $Q^2$ using the DGLAP equations, provided the formalism of perturbative QCD is adequate. The input parameters are then fixed by data from different experiments. Despite the successes of this procedure, perturbative QCD techniques, like the BFKL pomeron [40, 41, 42], breakdown in the low $x$ kinematical regime where the gluons dominate.

In this paper we focus on unpolarized DIS at low Bjorken $x$, extending the previous work [33]. The basic idea is to construct the holographic Regge theory for the glueball exchange associated with the Pomeron trajectory. In DIS the Pomeron couples to the quark bilinear electromagnetic current $J^\mu = \bar{\psi}\gamma^\mu\psi$, which is described holographically by the interaction between a bulk $U(1)$ vector gauge field and the graviton Regge trajectory. Here we shall extend the analysis for the structure function $F_2$ of [33] by including HERA data for the structure function $F_L$. We shall perform a chi squared fit of our model to $F_2$ and $F_L$ HERA data presented in [43, 44]. The fit uses 313 data points, covering the very large kinematical range of $x < 10^{-2}$ and $Q^2 \leq 400 \text{ GeV}^2$ for $F_2$ and $Q^2 \leq 45 \text{ GeV}^2$ for $F_L$, where $Q^2$ is the photon virtuality. We have found a $\chi^2$ of 1.4. For a reduced set of data points with $Q^2 \leq 10 \text{ GeV}^2$ we found a $\chi^2$ of 1.3.

The gluon PDF can be extracted from both structure functions $F_2$ and $F_L$. This was done in the context of holographic QCD in [45] using the BPST kernel for pomeron exchange and a hard-wall in $AdS_5$ to add confinement effects. The results presented in [45] were obtained using a $\chi^2$ fit with data for $Q^2 \leq 10 \text{ GeV}^2$ (also obtaining $\chi^2 = 1.3$). Since we have a good description of both $F_2$ and $F_L$ in a wider photon virtuality range, we use the same method to compute the holographic gluon PDF. We compare our results with the NLO results of the CTEQ and NNPDF PDF sets, finding good agreement throughout the larger kinematical region.

This letter is organized as follows. We first review how to obtain ex-
pressions for the DIS structure functions \( F_2(x, Q^2) \) and \( F_L(x, Q^2) \) in generic AdS/QCD models. Later we focus on the improved holographic QCD model of \cite{46, 47, 48} and we fit our model for Pomeron exchange to low unpolarized \( x \) DIS data. Then we extract the gluon PDF from our holographic computation of \( F_2 \) and \( F_L \).

2. Holographic computation of \( F_2 \) and \( F_L \) structure function

In this section we review how to compute the structure functions \( F_2 \) and \( F_L \) holographically. Details can be found in \cite{33} and hence we will just present the main formulas that allow us to derive expressions for \( F_2 \) and \( F_L \) in terms of holographic quantities.

Using the optical theorem the structure functions can be related to the amplitude for forward Compton scattering

\[
A^{FC}(q, P) = i(2\pi)^4 \delta^4 \left( \sum_i k_i \right) \left[ n_T^2 \bar{F}_1(x, Q^2) + 2x Q^2 (n_T \cdot P)^2 \bar{F}_2(x, Q^2) \right],
\]

where \( q \) is the momentum of the incoming photon, \( P \) is the momentum of the incoming hadron and \( n_T = n_T(q) \) is the transverse projection of the virtual photon polarization \( n^\mu \). The DIS structure functions are extracted from the forward Compton amplitude through

\[
F_i(x, Q^2) = 2\pi \text{Im}\bar{F}_i(x, Q^2), \quad (i = 1, 2)
\]

and \( F_L = F_2 - 2xF_1 \).

Before showing how to compute \cite{11} holographically let us introduce the kinematics. We use light-cone coordinates \((+, -, \perp)\), with the flat space metric given by \( ds^2 = -dx^+dx^- + dx^2 \), where \( x_\perp \in \mathbb{R}^2 \) is a vector in impact parameter space. We take for the large \( s \) kinematics of \( 12 \rightarrow 34 \) scattering the following

\[
k_1 = \left( \sqrt{s}, -\frac{Q^2}{\sqrt{s}}, 0 \right), \quad k_2 = \left( \frac{M^2}{\sqrt{s}}, \sqrt{s}, 0 \right),
\]

where \( k_1 \) is the incoming photon momenta and \( k_2 \) is the incoming proton momentum with mass \( M \). For the forward Compton scattering amplitude the momentum transfer \( q_\perp = 0 \) so that the outgoing photon has \( k_3 = -k_1 \).
Figure 1: Tree level Witten diagram representing spin $J$ exchange in $12 \rightarrow 34$ scattering. The $n_1$ and $n_3$ labels denote the incoming/outgoing photon polarizations, for forward scattering $n_1 = n_3$.

and the outgoing proton $k_4 = -k_2$. The incoming and outgoing photon polarizations are the same. The possible polarization vectors are

$$n(\lambda) = \begin{cases} (0, 0, \epsilon_\lambda), & \lambda = 1, 2, \\ (\sqrt{s}/Q, Q/\sqrt{s}, 0), & \lambda = 3, \end{cases}$$

where $\epsilon_\lambda$ is just the usual transverse polarization vector.

In the framework of AdS/QCD the above scattering amplitude can be computed with the Witten diagram shown in figure 1. The upper part of the diagram is related to the incoming and outgoing virtual photons, whereas the bottom part to the proton target. We are interested in the Regge limit where the amplitude is dominated by the exchange of the graviton Regge trajectory, which includes fields of even spin $J$. We also need to define our holographic external states. Among other fields, the holographic dual of QCD will have a dilaton field and a five-dimensional metric, which in the vacuum will have the form

$$ds^2 = e^{2A(z)} \left[ dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu \right], \quad \Phi = \Phi(z),$$

for some unknown functions $A(z)$ and $\Phi(z)$. We shall use greek indices in the boundary, with flat metric $\eta_{\mu\nu}$. We will work in the string frame.

In DIS the external photon is a source for the conserved $U(1)$ current $\bar{\psi} \gamma^\mu \psi$, where the quark field $\psi$ is associated to the open string sector. The five dimensional dual of this current is a massless $U(1)$ gauge field $A$. We shall assume that this field is made out of open strings and that is minimally
coupled to the metric, with the following action

\[ S_A = -\frac{1}{4} \int d^5X \sqrt{-g} e^{-\Phi} F_{ab} F^{ab} \]  

(6)

where \( F = dA \) and we use the notation \( X^a = (z, x^a) \) for five-dimensional points. We will fix the gauge of the \( U(1) \) bulk field to be \( D_a A^a = 0 \), which gives \( A_z = 0 \) and \( \partial_\mu A^\mu = 0 \). In this gauge, the solution to the equation of motion \( \nabla_a (e^{-\Phi} F^{ab}) = 0 \) is given by

\[ A_\mu(X; k, \lambda) = n_\mu(\lambda) f_k(z) e^{i k \cdot x}, \]  

(7)

where \( f_k(z) \) solves the differential equation

\[ \left[ -Q^2 + e^{\Phi} \partial_z (e^{A - \Phi} \partial_z) \right] f_Q(z) = 0. \]  

(8)

The momentum \( k \) and the polarisation vector \( n(\lambda) \), given in (4), satisfy

\[ k^2 = Q^2, \quad n_z = 0, \quad k \cdot n = 0. \]  

(9)

The UV boundary condition \( f(0) = 1 \) gives the non-normalizable solution, since the off-shell photon acts as a source for the quark bilinear current \( \bar{\psi} \gamma^\mu \psi \).

For the proton target we consider a scalar field \( \Upsilon \) that represents an unpolarised proton described by a normalizable mode of the form

\[ \Upsilon(X; p) = v_m(z) e^{ip \cdot x}, \]  

(10)

where \( p \) is the momentum and \( m^2 = -p^2 \). As explained in detail in [33], the specific details of the function \( v_m(z) \) will not be important because it will appear in an integral that can be absorbed in the coupling between the pomeron and the proton.

Next, to compute the Witten diagram of figure 1, we need to consider the interaction between the external scattering states and a spin \( J \) field in the graviton Regge trajectory. The higher spin fields come from the closed string sector, while the external fields come from the open sector. Their coupling is done by extending the minimal coupling between the graviton and the external states. This issue has been discussed in detail in [33], thus we will just write the final result. We start by decomposing the spin \( J \) field \( h_{a_1...a_J} \) in \( SO(1,3) \) irreducible representations. Then, in the Regge limit, we are only interested in the TT components of this field \( h_{a_1...a_J} \), with \( \partial^\nu h_{\nu a_2...a_J} = 0 \).
and $h_{\nu\alpha_1\ldots\alpha_J} = 0$. The coupling between the $U(1)$ gauge field and the TT components of the spin $J$ field has the form

$$\kappa_J \int d^5X \sqrt{-g} e^{-\Phi} h_{\alpha_1\ldots\alpha_J} F^{\alpha_1\alpha} \partial^{\alpha_2} \ldots \partial^{\alpha_{J-1}} F^{\alpha_J}. \quad (11)$$

For the scalar field $\Upsilon$ we have

$$\bar{\kappa}_J \int d^5X \sqrt{-g} e^{-\Phi} h_{\alpha_1\ldots\alpha_J} \Upsilon \partial^{\alpha_1} \ldots \partial^{\alpha_J} \Upsilon. \quad (12)$$

Using the ingredients we have just introduced the contribution to the forward Compton scattering amplitude due to spin $J$ exchange can be computed. Then one needs to sum over the fields with of spin $J = 2, 4, \ldots$ in the graviton Regge trajectory. This sum can be converted into an integral in the complex $J$-plane through a Sommerfeld-Watson transform. From the resulting expression and using (2) one obtains for the DIS structure functions $F_1$ and $F_2$ [33]

$$xF_i(x, Q^2) = 4\pi Q^2 \int dz d\bar{z} P^{(i)}_{13}(Q^2, z) P_{24}(P^2, \bar{z}) \text{Im}[\chi(s, t = 0, z, \bar{z})], \quad (13)$$

where

$$P^{(1)}_{13}(Q^2, z) = e^{A(z) - \Phi(z)} f_Q^2, \quad (14)$$

$$P^{(2)}_{13}(Q^2, z) = e^{A(z) - \Phi(z)} \left[ f_Q^2 + \frac{1}{Q^2} (\partial_z f_Q)^2 \right], \quad (15)$$

$$P_{24}(P^2, \bar{z}) = e^{3A(\bar{z}) - \Phi(\bar{z})} \Upsilon^2(P^2, \bar{z}), \quad (16)$$

and

$$\chi(s, t, z, \bar{z}) = \frac{\pi}{4} \int \frac{dJ}{2\pi i} \frac{S(z, \bar{z})^{J-1} \left(1 - (-1)^J\right) k_J \bar{k}_J}{\sin(\pi J)} G_J(z, \bar{z}, t), \quad (17)$$

with $S(z, \bar{z}) = se^{-A(z) - A(\bar{z})}$. The eikonal phase $\chi(s, t, z, \bar{z})$ results from the $+ \cdots +, - \cdots -$ component of the spin J propagator $G_J(z, \bar{z}, t)$, which admits a spectral representation

$$G_J(z, \bar{z}, t) = e^{B(z) + B(\bar{z})} \sum_n \frac{\psi_n(J, z) \psi_n^*(J, \bar{z})}{t_n(J) - t}, \quad (18)$$
where $\psi_n(J,z)$ are the normalizable modes associated to the spin $J$ fields, that is they describe massive spin $J$ glueballs. The function $B(z)$ depends on the particular holographic QCD model, for the model here considered $B = \Phi - A/2$.

The next step is to assume that the $J$-plane integral can be deformed from the poles at even $J$, to the poles $J = j_n(t)$ defined by $t_n(J) = t$. The scattering domain of negative $t$ contains these poles along the real axis for $J < 2$. After this step the expressions for $F_1$ and $F_2$ become

$$2x F_1(x, Q^2) = \sum_n g_n x^{1-j_n(0)} Q^{2 j_n(0)} \tilde{P}_{13}^{(1,n)}(Q^2),$$

$$F_2(x, Q^2) = \sum_n g_n x^{1-j_n(0)} Q^{2 j_n(0)} \tilde{P}_{13}^{(2,n)}(Q^2),$$

where

$$\tilde{P}_{13}^{(1,n)} = \int dz f_Q^2 e^{(2-j_n) A(z) + B(z) - \Phi(z)} \psi_n(j_n, z),$$

$$\tilde{P}_{13}^{(2,n)} = \int dz \left[ f_Q^2 + \frac{1}{Q^2} (\partial_z f_Q)^2 \right] e^{(2-j_n) A(z) + B(z) - \Phi(z)} \psi_n(j_n, z),$$

with $j_n$ evaluated at $t = 0$. The constants $g_n$, which involve the AdS local couplings and an integral over the proton wavefunction, will be used as fitting constants of the model.

The above discussion is applicable to any holographic model of QCD. We shall consider the improved holographic QCD model introduced in [46, 47, 48]. Solving the model such that the spectrum of the scalar and tensor glueballs is reproduced fixes the background fields $A(z)$ and $\Phi(z)$, which give an approximate dual description of the QCD vacuum. We may then compute the non-normalizable modes for any $Q^2$ by solving numerically the equation (8) with the UV boundary condition $f_Q(0) = 1$.

All that is left is the equation of motion for the spin $J$ fields that are dual to the twist two operators, whose exchange gives the dominant contribution in DIS at low $x$. This equation is then analytically continued in $J$, in order to do the Sommerfeld-Watson transform in Regge theory. As described in detail in [33] the normalisable modes of the spin $J$ field $\psi_n(z)$ solve a Schrödinger problem

$$\left( -\frac{d^2}{dz^2} + U_J(z) \right) \psi_n(z) = t_n \psi_n(z),$$

7
where
\[
U_J(z) = \frac{3}{2} \left( \ddot{A} - \frac{2}{3} \ddot{\Phi} \right) + \frac{9}{4} \left( \dot{A} - \frac{2}{3} \dot{\Phi} \right)^2 + (J - 2)e^{-2A} \left[ \frac{2}{l_s^2} \left( 1 + \frac{d}{\sqrt{\lambda}} \right) + \frac{J + 2}{\lambda^{4/3}} + e^{2A} \left( a\ddot{\Phi} + b \left( \dot{A} - \dot{\Phi}^2 \right) + c\dot{\Phi}^2 \right) \right].
\] (24)

The first line in this equation represents the potential for the graviton and the remaining terms deform the graviton potential. This potential is analytically continued in \( J \) in such a way that the value of the intercept \( J = j_n \) is obtained when the \( n \)-th eigenvalue satisfies \( t_n(J) = 0 \). We will use a Chebyshev algorithm with 1000 points to compute the eigenvalues \( t_n \) and the eigenfunctions \( \psi_n \). From the low energy effective string theory perspective, \( l_s \) is related to the string tension; \( d \) is related to the anomalous dimension curve of the twist 2 operators, or it can also be thought as encoding the information of how the masses of the closed strings excitations are corrected in a slightly curved background; the constants \( a \), \( b \) and \( c \) encode the first order derivative expansion in effective field theory. All these constants will be adjusted from fitting \( F_2 \) and \( F_L \) data.

Finally, the DIS structure functions \( F_2 \) and \( F_L \) can be written in Regge theory in the following form
\[
F_i(x,Q^2) = \sum_n f_{i,n}(Q^2) x^{1-j_n},
\] (25)

where
\[
f_{2,n}(Q^2) = g_n Q^{2j_n} \int dz e^{-(j_n-\frac{3}{2})A} \left( f_Q^2 \frac{\dot{f}_Q^2}{Q^2} \right) \psi_n, \quad (26)
\]
\[
f_{L,n}(Q^2) = g_n Q^{2j_n} \int dz e^{-(j_n-\frac{3}{2})A} \frac{\dot{f}_Q^2}{Q^2} \psi_n. \quad (27)
\]

The constants \( g_n \) will be fixed by a joint fit to \( F_2 \) and \( F_L \), thus the details of the proton holographic wave function are not important in the fit.

3. Data analysis and results

With the previously described setup we proceed to find the best values for the potential parameters \( l_s, a, b, c \) and \( d \), as well as for the coupling values
Figure 2: Structure function $F_2(Q^2, x)$. Shown are the experimental points vs prediction with a fit of \( \chi^2_{d.o.f} = 1.4 \). Each line corresponds to a given \( Q^2 \) (GeV\(^2\)) as indicated.

$g_n$ that better fit the data. We use the first four Reggeons, which are enough to reproduce the non-trivial $x$ behaviour that can be recasted in terms of an effective $Q$ dependent exponent as $\sigma = f(Q^2) x^{-\epsilon(Q^2)}$. Adding several trajectories explains the so-called hard-pomeron behaviour for large $Q$ and the soft-pomeron behaviour for smaller $Q$, as discussed in [33]. We look, as usual, for the best set of parameter values such that the sum of the weighted difference squared between experimental data and model predicted values is minimum, using as weight the inverse of the experimental uncertainty. The sum involves both data from HERA measurements of $F_2$ and $F_L$. $F_2$ data contributes with 249 experimental points with $x < 10^{-2}$ and $Q^2 \leq 400$ GeV\(^2\), while $F_L$ measurements include 64 points with $x < 10^{-2}$ and $Q^2 \leq 45$ GeV\(^2\).

Because there is no known relation between the potential parameters and its spectrum we can not compute efficiently the gradient of the function to be optimized. We use then the implementation of the Nelder-Mead (NM) algorithm in C++ presented in the book Numerical Recipes. This algorithm requires only function calls in order to find the minimum of a function. In our case we have an optimisation problem with 9 free parameters which is non-convex in general. We are then in the presence of a non-trivial optimisation problem because when the NM algorithm terminates we do not know if the minimum found is a local or global minimum. To mitigate this problem we started the algorithm with different initial guesses in the parameter space and took as the global minimum the lowest value found for $\chi^2$. 
Table 1: Values of the parameters for the best fit found. All parameters are dimensionless except for $l_s = L$, whose inverse numerical value is expressed in GeV units. As an output we also show the intercept of the first four Pomeron trajectories.

| parameter | value | coupling | value | Intercept |
|-----------|-------|----------|-------|-----------|
| $l_s^{-1}$ | 6.491 | $g_0$    | 0.166 | $j_0 = 1.17$ |
| a         | -4.567| $g_1$    | 0.088 | $j_1 = 1.08$ |
| b         | 1.485 | $g_2$    | 0.245 | $j_2 = 0.975$ |
| c         | 0.653 | $g_3$    | -1.551| $j_3 = 0.913$ |
| d         | -0.113|          |       |            |

The best fit we found had a $\chi^2$ around 1.4 and the best fit parameters are displayed in table 1. In this table we also show the value of the intercept for the four Pomerons here considered. The first two trajectories are precisely compatible with QCD values for the hard and soft Pomerons. Our best fit results for $F_2(x, Q^2)$ are presented in figure 2 and for $F_L(x, Q^2)$ in figure 3. Overall we see that there is an improvement in comparison with the results of [33] where the best fit to $F_2(x, Q^2)$ gave a $\chi^2 = 1.7$.

We also performed a fit for $Q^2 \leq 10$ GeV$^2$, in the region where holography is better suited. For this fit we obtained a $\chi^2$ around 1.3 with the best fit parameters in table 2. The fit improves only slightly in comparison with the previous one, at the expense of covering a much smaller kinematical region. The latter fit is of the same quality as that obtained in [45] for the same reduced set of data points. That work considered the BPST kernel for pomeron exchange and a hard-wall in $AdS_5$ to add confinement effects.

Table 2: Values of the parameters for the best fit found for $Q^2 \leq 10$ GeV$^2$ and intercept of the four Pomeron trajectories.

| parameter | value     | couplings | value     | Intercept |
|-----------|-----------|-----------|-----------|-----------|
| $l_s^{-1}$ | 6.53642   | $g_0$     | 0.173979  | $j_0 = 1.17$ |
| a         | -4.36898  | $g_1$     | 0.119573  | $j_1 = 1.08$ |
| b         | 1.43068   | $g_2$     | 0.229406  | $j_2 = 0.967$ |
| c         | 0.627532  | $g_3$     | -1.65699  | $j_3 = 0.899$ |
| d         | -0.116385 |           |           |            |
Figure 3: Predicted structure function $F_L(Q^2, x)$ vs experimental data with a $\chi^2_{d.o.f} = 1.4$. Each line corresponds to a given $Q^2$(GeV$^2$) as indicated.
4. Gluon Parton distribution functions

The structure functions \( F_2 \) and \( F_L \) of the proton can be written in terms of the proton’s PDFs. Their physical meaning is the probability density for finding a parton with a certain longitudinal momentum fraction \( x \) at resolution scale \( Q^2 \). In the naive quark parton model \( F_2 = \sum_i e_i^2 x q_i(x) \), where the phenomena of Bjorken scaling is predicted, i.e. \( F_2 \) depends only on \( x \) and not \( Q^2 \). The parton model also predicts that

\[
F_L(x, Q^2) = F_2(x, Q^2) - 2xF_1(x, Q^2) = 0, \tag{28}
\]

which is known as the Callan-Gross relationship and is satisfied if the partons inside the proton have spin-\( \frac{1}{2} \). These relations follow from considering only the QED diagram \( \gamma^*\text{-parton} \) and assuming that the partons have zero transverse momentum. At NLO QCD gluon radiation and \( g \rightarrow q\bar{q} \) processes give rise to \( \ln Q^2 \) scaling violations and to partons with non-zero transverse momentum. Hence the Callan-Gross relation is no longer true, and \( F_L \) can be related to \( F_2 \) and the gluon PDF \( g(x, Q^2) \) through [49]

\[
F_L(x, Q^2) = \frac{\alpha_s(Q^2)}{4\pi} \left( \frac{16}{3} I_F + 8\bar{e}^2 I_G \right), \tag{29}
\]

where

\[
I_F = \int_x^1 \frac{dy}{y} \left( \frac{x}{y} \right)^2 F_2(y, Q^2), \tag{30}
\]

\[
I_G = \int_x^1 \frac{dy}{y} \left( \frac{x}{y} \right)^2 \left( 1 - \frac{x}{y} \right) G(y, Q^2), \tag{31}
\]

and where \( G(y, Q^2) = yg(y, Q^2) \), \( \alpha_s \) is the QCD coupling and \( \bar{e}^2 \) is the sum of the squares of the electric charges of the active quark flavours. The number of active flavours \( n_f \) changes with the scale at which we want to evaluate the PDF. In this work we will assume that \( n_f = 3 \) for \( Q^2 \leq m_c \), \( n_f = 4 \) for \( m_c < Q^2 \leq m_b \) and \( n_f = 5 \) otherwise. \( m_c \) and \( m_b \) are the masses of the charm and bottom quarks, respectively. We wish to express the gluon PDF \( g(x, Q^2) \) in terms of the structure functions \( F_2 \) and \( F_L \). This can be done by computing the derivative with respect to \( x \) of \( \text{[29]} \) and using the definition
of the integrals $I_F$ and $I_G$. A straightforward computation yields\footnote{The approximate formula $\bar{e}^2 g(x, Q^2) = \frac{5.9 \pi}{2 \alpha_s(Q^2)} F_L(0.4x, Q^2) - \frac{5.9}{4} F_2(0.8x, Q^2)$ was derived in [50], however we shall use \cite{52} since it follows from the NLO expression \cite{29} without further approximations.}

$$\bar{e}^2 g(x, Q^2) = -2 + \frac{2x}{3} \frac{\partial}{\partial x} F_2(x, Q^2) + \frac{\pi}{\alpha_s(Q^2)} \left( 3 - 2x \frac{\partial}{\partial x} + \frac{x^2}{2} \frac{\partial^2}{\partial x^2} \right) F_L(x, Q^2). \quad (32)$$

We use the NLO result \cite{32} to estimate the function $G(x, Q^2)$ at small $x$ from the holographic values for $F_2$ and $F_L$ computed using the best fit parameters in table \cite{1}. We shall compare our gluon PDFs with the CT18NNLO and NNPDF collaborations \cite{51, 52}, also at NLO. For the coupling constant $\alpha_s(Q^2)$ in \cite{32} we used the holographic value (see \cite{53} for a plot of this function). Since the functions $F_2$ and $F_L$ do not reproduce the data in the range $10^{-2} < x < 1$, and in particular have the wrong asymptotics for $x \to 1$, we do not expect to be able to reproduce $G(x, Q^2)$ in the transition region $x \sim 10^{-2}$ where there are other contributions to the structure functions from quarks. To match the correct asymptotic values in this transition region, we added a constant function $f(Q^2)$ to $G(x, Q^2)$ such that we match the value of $G(x, Q^2)$ given by the average of the other collaborations at the specific value $x = 10^{-2}$. Our main goal is to assess whether or not we can predict the correct low $x$ evolution of the gluon PDFs. Our results are presented in figure \cite{4}. It clear that we are able to reproduce the correct behaviour within the other collaborations allowed regions. Of course these results should be taken as a simple qualitative indication, since we are using a NLO expression for the gluon PDFs together with the holographic structure functions and coupling constant.

5. Conclusion

In this letter we extended the previous work of \cite{33}, that considered the improved holographic QCD model to study DIS at low $x$, to the case of unpolarized DIS. The $\chi^2$ quality of our fit improves on this previous work from $\chi^2 = 1.7$ to $\chi^2 = 1.4$. This is due to the fact that, in addition to the 249 data points from the structure function $F_2$, we have an extra of 64 data points
from the structure function $F_L$, which can also be described holographically. These results are obtained in a very large kinematical window of $x < 10^{-2}$ and $Q^2 \leq 400$ GeV$^2$ for $F_2$, and $Q^2 \leq 45$ GeV$^2$ for $F_L$. Reducing the data set to $Q^2 \leq 10$ GeV$^2$ we found a fit with $\chi^2 = 1.3$, of the same quality as in the recent work \cite{45}.

Using the NLO relation (32) we were able to reproduce the low $x$ evolution of gluonic PDFs using as input the holographic functions $F_2$, $F_L$ and $\alpha_s(Q^2)$. In the region of larger $Q^2$ there is more tension in matching to the PDFs of the other collaborations, as can be seen in the $Q^2 = 100$ GeV$^2$ plot of figure 4. According to equation (32), in this region it is essential to have a good description of $F_L$ because it is divided by $\alpha_s$ which is small for high values of $Q^2$ due to asymptotic freedom. Since we are performing a $\chi^2$ fit to the data, the fitting process favours a good description of $F_2$ because the uncertainties are lower than those of $F_L$ as compared with the value measured.
The uncertainties of $F_L$ are of the same size as the measured value. Thus, better measurements of $F_L$ might help our model give a better description of the gluon PDFs. Moreover, for high values of $Q^2$ it is important to include heavy quarks in global QCD fits. Nowadays PDF groups use variable flavour schemes in order to produce high quality results. To holography this means that the holographic dual must contain quark flavour degrees of freedom if the model ought to be successful at computing them. The IHQCD model we used as our QCD vacuum exhibits the properties of large $N_c$ Yang-Mills, including the running of the coupling constant. Following the ideas of [54], it would be very interesting to include flavour degrees of freedom in this holographic QCD model and to test if the quality of our fits generically improve, mainly for high $Q^2$. Including quarks is actually necessary because it is believed that the third and fourth dominant Regge trajectory actually come from the mesonic sector, instead of the glueball sector.

Another problem would be to determine holographically the gluonic PDFs, without making reference to perturbative QCD definitions. Holography may actually be the right set up for a non-perturbative definition. In fact, gluonic PDFs can be defined using a Wilson loop operator (see for example [55]). Thus, it would be very interesting to use the duality between Wilson loops and strings in the dual geometry to explore this problem.

For future work we also wish to extend the range of process involving pomeron exchange that can be described holographically. In a companion paper [56] we show how to reproduce low $x$ data for $\gamma^*\gamma \to X$, $\gamma^*p \to X$ and $pp \to X$ processes using the parameters that describe Pomeron exchange fixed in this letter (and given by table 1).

6. Acknowledgments

This research received funding from the Simons Foundation grants 488637 (Simons collaboration on the Non-perturbative bootstrap) and from the grant CERN/FIS-PAR/0019/2017. Centro de Física do Porto is partially funded by Fundação para a Ciência e a Tecnologia (FCT) under the grant UID-04650-FCUP. AA is funded by FCT under the IDPASC doctorate programme with the fellowship PD/BD/114158/2016.

References

[1] R. C. Brower, J. Polchinski, M. J. Strassler and C. I. Tan, “The Pomeron and gauge/string duality,” JHEP 12 (2007), 005 [arXiv:hep-th/0603115].
[2] C. A. Ballon Bayona, H. Boschi-Filho, N. R. F. Braga, Deep inelastic scattering from gauge string duality in the soft wall model, JHEP 03 (2008), 064 [arXiv:hep-th/0711.0221]

[3] Y. Hatta, E. Iancu, A. H. Mueller, Deep inelastic scattering at strong coupling from gauge/string duality: the saturation line, JHEP 01 (2008), 026. [arXiv:hep-th/0710.2148]

[4] L. Cornalba, M. S. Costa, Saturation in deep inelastic scattering from the AdS/CFT correspondence, PRD 78 (9) (2008) 096010 [arXiv:hep-ph/0804.1562]

[5] B. Pire, C. Roiesnel, L. Szymanowski, S. Wallon, On AdS/QCD correspondence and the partonic picture of deep inelastic scattering, Physics Letters B 670 (1) (2008) 84–90 [arXiv:hep-ph/0805.4346]

[6] J. L. Albacete, Y. V. Kovchegov, A. Taliotis, DIS on a large nucleus in AdS/CFT, JHEP 07 (2008), 074 [arXiv:hep-th/0806.1484]

[7] Y. Hatta, Relating $e^+e^-$ annihilation to high energy scattering at weak and strong coupling, JHEP 11 (2008), 057 [arXiv:hep-ph/0810.0889]

[8] R. C. Brower, M. Djuric, C.-I. Tan, Saturation and Confinement: Analyticity, Unitarity and AdS/CFT Correspondence, [arXiv:hep-th/0812.1299]

[9] E. Levin, J. Miller, B. Z. Kopeliovich, I. Schmidt, Glauber-Gribov approach for DIS on nuclei in N = 4 SYM, JHEP 02 (2009), 048 [arXiv:hep-ph/0811.3586]

[10] R. Brower, M. Djuric, C.-I. Tan, Elastic and Diffractive Scattering after AdS/CFT, [arXiv:hep-th/0911.3463]

[11] J.-H. Gao, B.-W. Xiao, Polarized deep inelastic and elastic scattering from gauge/string duality, PRD 80 (1) (2009) 015025 [arXiv:hep-ph/0904.2870]

[12] Y. Hatta, T. Ueda, B.-W. Xiao, Polarized DIS in $\mathcal{N} = 4$ SYM: where is spin at strong coupling?, JHEP 08 (2009), 007 [arXiv:hep-ph/0905.2493]
[13] Y. V. Kovchegov, Z. Lu, A. H. Rezaeian, Comparing AdS/CFT calculations to HERA $F_2$ data, PRD 80 (7) (2009) 074023 [arXiv:hep-ph/0906.4197]

[14] E. Avsar, E. Iancu, L. McLerran, D. N. Triantafyllopoulos, Shockwaves and deep inelastic scattering within the gauge/gravity duality, JHEP 11 (2009), 105 [arXiv:hep-th/0907.4604]

[15] S. K. Domokos, J. A. Harvey, N. Mann, Pomeron contribution to pp and $p\bar{p}$ scattering in AdS/QCD, PRD 80 (12) (2009) 126015. [arXiv:hep-ph/0907.1084]

[16] L. Cornalba, M. S. Costa, J. Penedones, Deep inelastic scattering in conformal QCD, JHEP 3 (2010), 1–65. [arXiv:hep-th/0911.0043]

[17] F. Dominguez, Particle production in DIS off a shockwave in AdS, JHEP 9 (2010), 7. [arXiv:hep-th/0912.1641]

[18] L. Cornalba, M. S. Costa, J. Penedones, AdS Black Disk Model for Small-x Deep Inelastic Scattering, PRL 105 (7) (2010) 072003. [arXiv:hep-ph/1001.1157]

[19] M. A. Betemps, V. P. Gonçalves, J. T. de Santana Amaral, Diffractive deep inelastic scattering in an AdS/CFT inspired model: A phenomenological study, PRD 81 (9) (2010) 094012. [arXiv:hep-ph/1001.3548]

[20] J.-H. Gao, Z.-G. Mou, Polarized deep inelastic scattering off the neutron from gauge/string duality, PRD 81 (9) (2010) 096006. [arXiv:hep-ph/1003.3066]

[21] Y. V. Kovchegov, R-current dis on a shock wave: Beyond the eikonal approximation, PRD 82 (5) (2010) 054011. [arXiv:hep-ph/1005.0374]

[22] E. Levin, I. Potashnikova, Inelastic processes in DIS and $\mathcal{N} = 4$ SYM, JHEP 8 (2010) 112. [arXiv:hep-ph/1007.0306]

[23] S. K. Domokos, J. A. Harvey, N. Mann, Setting the scale of the $pp$ and $p\bar{p}$ total cross sections using AdS/QCD, PRD 82 (10) (2010) 106007. [arXiv:hep-th/1008.2963]
[24] R. C. Brower, M. Djuric, I. Sarcevic, C.-I. Tan, String-gauge dual description of deep inelastic scattering at small-x, JHEP 11 (2010) 1–26. [arXiv:hep-ph/1007.2259]

[25] M. S. Costa, M. Djuric, Deeply virtual Compton scattering from gauge/gravity duality, PRD 86 (1) (2012) 016009. [arXiv:hep-th/1201.1307]

[26] R. C. Brower, M. Djuric, C.-I. Tan, Diffractive Higgs Production by AdS Pomeron Fusion, JHEP 09 (2012) 097. [arXiv:hep-ph/1202.4953]

[27] A. Stoffers, I. Zahed, Holographic Pomeron: Saturation and DIS, PRD 87, 075023 (2013). [arXiv:hep-ph/1205.3223]

[28] M. S. Costa, M. Djuric, N. Evans, Vector meson production at low x from gauge/gravity duality, JHEP 9 (2013) 1–18. [arXiv:hep-ph/1307.0009]

[29] N. Anderson, S. K. Domokos, J. A. Harvey, N. Mann, Central production of $\eta$ and $\eta'$ via double Pomeron exchange in the Sakai-Sugimoto model, PRD 90 (8) (2014) 086010. [arXiv:hep-ph/1406.7010]

[30] E. Koile, N. Kovensky, M. Schvellinger, Hadron structure functions at small x from string theory, JHEP 5 (2015) 001. [arXiv:hep-th/1412.6509]

[31] E. Koile, N. Kovensky, M. Schvellinger, Deep inelastic scattering cross sections form the gauge/gravity duality, JHEP 12 (2015) 009. [arXiv:hep-th/1507.07942]

[32] D. Jorrin, N. Kovensky, M. Schvellinger, Deep inelastic scattering off scalar mesons in the $1/N$ expansion from the D3D7-brane system, JHEP 3 (2016) 003. [arXiv:hep-th/1609.01202]

[33] A. Ballon-Bayona, R. Carcasses Quevedo, M. S. Costa, Unity of pomerons from gauge/string duality, JHEP 08 (2017) 085 [arXiv:hep-ph/1704.08280]

[34] R. Nally, T. G. Raben, C.-I. Tan, Inclusive Production Through AdS/CFT, JHEP 11 (2017) 075. [arXiv:hep-ph/1702.05502]

[35] N. Kovensky, G. Michalski, M. Schvellinger, Deep inelastic scattering from polarized spin-1/2 hadrons at low x from string theory, JHEP 10 (2018), 084 [arXiv:hep-th/1807.11540]
[36] C. H. Lee, H. Y. Ryu, I. Zahed, Diffractive Vector Photoproduction using Holographic QCD, PRD 98, 056006 (2018). [arXiv:hep-ph/1804.09300]

[37] A. Amorim, R. Carcassés Quevedo and M. S. Costa, “Nonminimal coupling contribution to DIS at low x in Holographic QCD,” PRD 98 (2018) no.2, 026016. [arXiv:hep-ph/1804.07778]

[38] N. Kovensky, G. Michalski, M. Schvellinger, 1/N corrections to $F_1$ and $F_2$ structure functions of vector mesons from holography, PRD 99, 046005 (2019) [arXiv:hep-th/1809.10515]

[39] K. A. Mamo, I. Zahed, Diffractive photoproduction of $J/ψ$ and $ϒ$ using holographic QCD: gravitational form factors and GPD of gluons in the proton, PRD 101, 086003 (2020). [arXiv:hep-ph/1910.04707]

[40] V. S. Fadin, E. A. Kuraev, L. N. Lipatov, On the Pomeranchuk Singularity in Asymptotically Free Theories, Phys. Lett. B60 (1975) 50–52.

[41] E. A. Kuraev, L. N. Lipatov, V. S. Fadin, The Pomeranchuk Singularity in Nonabelian Gauge Theories, Sov. Phys. JETP 45 (1977) 199–204, [Zh. Eksp. Teor. Fiz.72,377(1977)].

[42] I. I. Balitsky, L. N. Lipatov, The Pomeranchuk Singularity in Quantum Chromodynamics, Sov. J. Nucl. Phys. 28 (1978) 822–829, [Yad. Fiz.28,1597(1978)].

[43] F. D. Aaron, et al., Combined Measurement and QCD Analysis of the Inclusive $e^{±}p$ Scattering Cross Sections at HERA, JHEP 01 (2010) 109 [arXiv:hep-ex/0911.0884]

[44] F. D. Aaron, et al., Measurement of the Inclusive $e^{±}p$ Scattering Cross Section at High Inelasticity $y$ and of the Structure Function $F_L$, Eur. Phys. J. C71 (2011) 1579. [arXiv:hep-ex/1012.4355]

[45] A. Watanabe, T. Sawada, M. Huang, Extraction of gluon distributions from structure functions at small x in holographic QCD (2019). [arXiv:hep-ph/1910.10008]

[46] U. Gürsoy, E. Kiritsis, Exploring improved holographic theories for QCD: part I, JHEP 02 (2008) 032. [arXiv:hep-th/0707.1324]
[47] U. Gürsoy, E. Kiritsis, F. Nitti, Exploring improved holographic theories for QCD: part II, JHEP 02 (2008) 019. [arXiv:hep-th/0707.1349]

[48] U. Gürsoy, E. Kiritsis, L. Mazzanti, G. Michalogiorgakis, F. Nitti, Improved Holographic QCD [arXiv:hep-th/1006.5461]

[49] G. Altarelli, G. Martinelli, Transverse Momentum of Jets in Electroproduction from Quantum Chromodynamics, Phys. Lett. 76B (1978) 89–94.

[50] A. M. Cooper-Sarkar, G. Ingelman, K. Long, R. Roberts, D. Saxon, Measurement of the Longitudinal Structure Function and the Small X Gluon Density of the Proton, Z. Phys. C 39 (1988) 281.

[51] T.-J. Hou, et al., New CTEQ global analysis of quantum chromodynamics with high-precision data from the LHC [arXiv:hep-ph/1912.10053]

[52] R. D. Ball, et al., Parton distributions from high-precision collider data, Eur. Phys. J. C 77 (10) (2017) 663. [arXiv:hep-ph/1706.00428]

[53] A. Ballon-Bayona, R. Carcasses Quevedo, M. S. Costa, M. Djuric, Soft Pomeron in Holographic QCD, PRD 93 (2016) 035005. [arXiv:hep-ph/1508.00008]

[54] M. Jarvinen, E. Kiritsis, Holographic Models for QCD in the Veneziano Limit, JHEP 03 (2012) 002. [arXiv:hep-th/1112.1261]

[55] J. C. Collins, D. E. Soper, Parton Distribution and Decay Functions, Nucl. Phys. B194 (1982) 445-492.

[56] A. Amorim, M. Costa, $\gamma^*\gamma$, $\gamma^*p$ and $pp$ scattering at low $x$ in IHQCD, work in progress.