Towards robust coupled field induced Josephson junctions

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Abstract

The concept of coupled robust field induced Josephson junctions placed in complex electromagnetic environments is presented. The methodology of modeling of structures and possible implementations is introduced. The presented scheme is expected to implement and describe both classical and quantum computer with use of Josephson junction and artificial evolution. In such case one can obtain unexpected new topologies of circuits that can contribute in enhancement of known and used circuit schemes.

1. Research motivation in relation to superconducting electronics

The Josephson effect [1] allows to use unique properties of superconducting macroscopic quantum effect. Great progress was achieved in theoretical understanding of this phenomena and in implementation of Josephson junctions in various physical systems as in superconductors, superfluids, Bose-Einstein condensate or polaritons. From perspective of current development of electronics the most promising is the usage of low temperature s-wave superconductors although highly correlated materials and majorana fermions are also important topics of fundamental research. Likharev proposed the usage of fluxons [2] as representation of logical bit 1 and until now it seems to be the most optimal implementation of classical information in superconductor that gives the base for development of Rapid Single Quantum Flux (RSFQ) electronics. The greatest integration of Josephson junctions was achieved indeed in RSFQ electronics [3], [4]. The development of complex superconducting qubit circuits [5] is also promising but not so dynamical as it is the case of RSFQ electronics. In implementation of Josephson junction circuits most common is the use of standard topology of superconductor and insulators or non-superconducting materials which makes design and implementation more easy to be achieved from technological and simulation point of view. This results in assigning concept of simple washboard potential to particular Josephson junctions of rather simple topology as presented in [5]. Presence of simple form of washboard potentials simplifies description of the system from computational point of view. Nevertheless the simplicity of washboard potential does not exploit many physical phenomena that could take place and which might be technically useful. The purpose of this work is to draw and motivate new research direction that can take into account broad class of washboard potentials with various complexities and topologies. This shall have its relevance in implementation of both classical and quantum computer in superconductor which requires use of system with many Josephson junctions. It should also have its relevance in studying various properties of condensates from fundamental point of view and also is related to science of complexity. Thus it is highly interdisciplinary research as it is shown in Fig.1.

In the next section the definition of field induced Josephson junction will be generalized to various classes of physical systems. Section 3 gives brief mathematical description will be stated. Finally the possible experimental setups and schemes will be drawn and possible future directions of research will be briefly discussed.

2. Field Induced Josephson junctions (FIJJs) and their generalizations

The first Josephson junction system was obtained for superconductor-insulator-superconductor with thin insulator barrier [1]. In such case one of the most important parameters is the probability of tunneling from one superconductor into another for wavepackets of certain energy. Later it was shown that broad class of constrictions (narrowings) in superconductor can generate the effect similar to that observed in tunneling Josephson junction. In such a way, weak-link Josephson junctions were found as the system with higher transmission probability than the case of tunneling structure. However one should not only limit in usage of physical constrains encoded in physical lattice [11] (and hence boundary conditions) to obtain Josephson effect. One can use the external magnetic or electric fields in order to modulate magnitude of superconducting order parameter (SCOP). First idea of usage of magnetic field in modulating the superconducting order parameter was coming from Clinton [6] although the concept of topological defect in superconducting order parameter is old. Clinton has
placed the ferromagnetic strip on the top of superconducting strip separated with insulating barrier. In such case the fringe field coming from ferromagnet was used to diminish superconducting order parameter locally. Further idea of similar kind was developed by L.Gomez and A.Maeda \cite{11} and relied on direct evaporating of ferromagnetic material on the top of superconducting strip. In such case one deals with diffusion of spin polarized particles from ferromagnet into superconductor and with penetration of magnetic field into superconductor. Various possible effects can occur as induction of triplet phase in s-wave superconductor. Due to many possible physical phenomena and complex mathematical picture in two or three dimensions as pointed in \cite{12} it is rather recommended to study at first one dimensional superconducting strip polarized with external source magnetic field as it is depicted in four subfigures of Fig.\ref{f1:11} situation Fig.\ref{f1:11} is nearly equivalent to Fig.\ref{f1:11} and case of Fig.\ref{f1:11} can be approximated by Fig.\ref{f1:11} in the limit of long normal strip). It could be equivalent to Clinton/Johnson system of ferromagnet placed on the superconducting with presence of insulating barrier that blocks spin polarized electron diffusion from ferromagnet into superconductor. We assume that thickness of superconductor is up to two superconducting coherence lengths so we can neglect the variation of superconducting order parameter across superconductor and assume that it is constant. In first approximation we will neglect the occurrence of quantum phase slips \cite{13} in superconducting nanowire.

Transport properties of structures can be determined when we combine both Ginzburg-Landau and Bogoliubov-de Gennes formalism. It should be pointed that we should specify the asymptotic states at first so SCOP and vector potential would be constant. In general case the polarizing cables topology and distribution of polarizing current is variational problem when we try to approximate Fig.\ref{f1:11} by \ref{f1:11} or \ref{f1:11} by \ref{f1:11}. In general case we have two classes of polarizing cables generating magnetic field that polarizes superconductor (closed loops with \( A_x\) and \( A_z\) vector potential components and open infinite cable generating \( A_z\) vector potential) and we assume that they are not affected by superconductor. The Bogoliubov-de Gennes (BdGe) wavepacket propagating via superconductor is affected by the presence of non-uniform vector potential and hence is partly scattered when it propagates from \( x = -\infty \) to \( x = +\infty \). The shape of polarizing loops is not fixed. Quite similar situation could occur when we have continuously deformed superconducting wire (quasi-one dimensionality is preserved and this allows the usage of non-linear Ordinary Differential Equations (ODE) instead of non-liner Partial Differential Equations(PDE) ) as it is the case of Fig.\ref{f1:11}. In similar fashion we can deform the polarizing cables as well which can result in amorphic robust but field induced Josephson junction depicted in Fig.\ref{f1:11}. Continuing this type of reasoning we can obtain two (or more) coupling in inductive way FIJJs as it is depicted in Fig.\ref{f1:11}. Most general scheme is given by Fig.\ref{f1:11} when one can deal with net of coupled robust field induced Josephson junctions embedded in network of polarizing cables of complex topology.

3. **Mathematical modeling of robust FIJJs**

Let us concentrate on the case depicted in Fig.\ref{f1:11} when we place long ferromagnetic strip on a thin superconductor and separate both systems by insulator. In such case we need to consider the scattering problem on non-uniform vector potential generated by ferromagnetic strip placed on the top of superconducting strip so in general case we have three non-zero components of vector potential. So particles moving in superconductor do not have many degrees of freedom since its y and z coordinates are fixed. Therefore one obtains the following Schrodinger Hamiltonian \( \hat{\mathcal{H}} = \frac{\hbar}{2m} \nabla^2 - \frac{\hbar^2}{2V} A_x(x) \) and \( \hat{\mathcal{H}}_p = (-\frac{\hbar^2}{2} A_x(x), \hat{p}_y = (-\frac{\hbar^2}{2} A_z(x)) \) that can be rewritten

\[
\mathcal{H} = \frac{\hbar}{2m} \left( \left( \frac{\hbar^2}{2} A_x(x) \right)^2 + \left( -\frac{\hbar^2}{2} A_z(x) \right)^2 + \left( -\frac{\hbar^2}{2} A_x(x) \right)^2 \right)
\]

and place into Bogoliubov-de Gennes (BdGe) equations of the form:

\[
\begin{bmatrix}
H(x) & \Delta(x) \\
\Delta^*(x) & -H^*(x)
\end{bmatrix}
\begin{bmatrix}
u_n(x) \\
\nu_n^*(x)
\end{bmatrix}
= E_n
\begin{bmatrix}
u_n(x) \\
\nu_n^*(x)
\end{bmatrix}
\]

where \( \nu_n(x) \) and \( \nu_n^*(x) \) are wavefunctions of electron and hole and \( \Delta \) is superconducting order parameter and \( |\psi\rangle \) is fermion wavefunction. Self-consistency for vector potential as a function of electric current distribution in the system requires that \( \Delta(x) \propto \int dx' f(x') \rho(x - x') \) has to be fulfilled. Simple distribution of \( A_z \) vector potential component is the case \( \rho_z(x,y=0) \propto \rho_z(x,y=0) \) when one uses the one polarizing cable with \( z \) non-zero current component \( I_z \) and constant \( a_0 \). One of the constants of motion is the conservation of electric charge that introduces many constrains in quasi-one-dimensional case. Since both Bogoliubov-de Gennes model and Ginzburg-Landau (GL) can be derived from BCS theory it is easier and justified to start from GL. In Ginzburg-Landau model conservation of electric charge essentially means that \( \text{constant} = \int -c_1 A_x(x,y=0)|\psi(x,y=0)|^2 \) with constant \( c_1 \) and SCOP \( \psi \) that refers to physical situation from Fig.\ref{f1:11} and Fig.\ref{f1:11}. Having the distribution of vector potential from GL we can place into BdGe equations as initial condition for computations and obtain new superconducting order parameter and new
vector potential after certain number of iterations. From computational point of view usage of BdGe is equivalent to eigenvalue problem for eigenenergy and eigenstate of given Bogoliubov-de Gennes matrix from equation 1. In general case we shall consider time-dependent case and Zeeman splitting in equation 1.

By repeating this procedure many times one finds superconducting order parameter and vector potential in self-consistent way when results from previous iteration step are nearly the same as the results from current iteration step from numerical point of view. Because of vector potential self-consistency across the whole system we obtain coupled BdGe equations that describe propagation of wavepackets in each designated superconducting wire. In our considerations we assume that distribution of current is unchanged and fixed in polarizing cables generating magnetic field that affect and bias cables propagating BdGe wavepackets. One can use also superconductors for biasing cables and in such case quasi-one dimensional GL equations can describe this situation together with BdGe equations related to cables when one considers the field induced Josephson junctions. Obviously superconducting polarizing cables should have critical current much higher than critical current of cables described by BdGe wavepackets.

4. Future experiments and outlook

Future experiments can be designed based on the concept presented in Fig.3d. Instead of conducting very large number experiments referring to each possible case of polarizing and BdGe cable configuration one can built the system where topology of network can be controlled in electrical way. It will limit certain number of configurations and point main Cartesian directions. In such case we have to consider two lattices (BdGe cables + polarizing cables) in close proximity and such that they are embedded one into each other. One lattice will generate complex pattern of magnetic field and this will be the network of cable polarizes and magnetic field entangler (MFE). Another network will be the network of thin superconductors propagating BdGe wavepackets that we name simply BdGe cables.

First approach is presented in Fig.4 when one uses the network of normal cables as cable polarizes controlled by semiconductor N-P-N or P-N-P transistors. In case of BdGe cables we need to use superconducting version of transistor that is Josephson junction with controlled channel by external voltage as presented by [9]. Such approach is very difficult in implementation since dissipative current flowing via polarizing cables will generate heat that will affect network of superconducting cables even in case of intensive cooling. What is more voltages involved in transistor operation are 3 orders of magnitude higher than voltages necessary to destroy superconducting state and bring it to normal state what requires excellent insulators. The necessity of maintenance of high electric insulation and cooling makes such architecture rather unrealistic in real implementation.

It is rather desirable to use only superconducting cables as cable polarizes. The control of their topology can be obtained by destroying locally the superconducting state with usage of another cable with high magnetic field as it is depicted in Fig.4 and in more detailed in 2 dimensions in Fig.4a,4b,4d and in 3 dimensions in Fig.4c. In such case we need 4 networks of cables and this suggests the usage both of low and high temperature superconductors. Two of them can be used as the cables ‘cutting’ other cables locally by use of generated magnetic field. The magnetic field cutting scheme is depicted in Fig.4. Since ‘cutting cable’ need to have higher critical current or \( T \) than cable being cut it is recommendable to use 4 types of superconductors with different critical temperatures.

Two networks doing cutting can be interpreted as the signalling networks and they will conduct the electric current of the highest intensity. In order to minimize the magnetic field coming from this current (that is parasitic magnetic field) one needs to use signalling cables in such way that cables with opposite current flow will be kept at minimum distance in accordance to principle that dipole generates smaller value and quicker decaying field than the case of single monopole (concept also known broadly among electric engineers as twisted cables generating minimal magnetic field). What is more one should use two loops of cutting cables with opposite flow of current as it is pointed in left Fig.4 that minimizes the leakage of magnetic flux. In more detailed the proposed scheme of topology of signalling cables in relation to polarizing and BdGe cables is depicted in Fig.4 (in two dimensions) and it can be generalized quite easily to 3 dimensions as it is presented in Fig.5. In general case we can engineer network of N inputs/outputs with BdGe wavepackets and M closed superconducting cables propagating BdGe wavepackets (blue cable from Fig.3d) that are subjected to the network of \( N_1 \) polarizing cables of arbitrary topology. In such case one expects to obtain the network of coupling field induced Josephson junctions (shown in Fig.2a) that have continuously tuned electric parameters (for example shape of washboard potential, critical Josephson current etc.). This should bring the possibility of creating network of Josephson junctions with wide class of complex washboard potentials. Consequently this should widen class of possible schemes of implementations of classical and quantum computer in superconducting environments. What is more presented structure in Fig.3d and having experimental implementation as subfigures of Fig.4 and 5 can be used as physical system with continuously tuned microscopic
scattering matrix controlled by outside macroscopic signals (current fed to magnetic field polarizers and cutting cables). The bigger is 2 or 3 dimensional network the bigger is the signalling box collecting all signalling cables that is schematically depicted in left Fig 4 and in Fig 4a.

In order to simulate the behaviour of the system from Fig 3d, it seems to be sufficient to use quasi-one dimensional BdGe wavepacket for signals propagating wavepackets and to use quasi-one dimensional GL model for other cables. The self-consistency relation for vector potential as generated by all current sources shall be included in the calculations as well as self-consistency in superconducting order parameter in cables propagating BdGe wavepackets. Therefore proposed scheme can be investigated further both in theoretical and in experimental way and is of interdisciplinary character as pointed by Fig 1. It seems to be fruitful to use concept of fitness function that is known from evolutionary algorithms (as for example in antenna design [12]) and science of complexity and use it to design new scheme of both classical and quantum circuits. The fitness function should take into account the correctness of logical operations and their vulnerability to error against external noise. One can conduct the artificial evolution referring to Fig 3d. in numerical simulation (artificial gene is expressed by topology of BdGe and polarizing cables) or can conduct evolution in real experiment as it is described in subfigures of Fig 3, 4a and 4c. The presented scheme is expected to describe the implementation of both classical and quantum computer basing on Josephson junctions. One expects the emergence of new schemes that are not accounted or considered as well as pointing the already existing schemes. Such approach is already known in the context of programmable matter [13] and suggested evolution would be example of it. The obtained new schemes can be optimized further with use of well-known methodologies. What is more the provided scheme can also provide the platform for experimental studies of BEC condensate interacting with superconducting environment as it is depicted in Fig 2b. In order to control the interaction of BdGe cables with superconducting cables in proper way one shall use very low \( T_c \) superconductors in BdGe cables. What is more one shall use helium 3 instead of helium 4. The natural place for BEC condensate would be space in proximity to helium 3. In order to describe the system in mathematical way one needs usage of coupled Gross-Pitaevskii equations (describing BEC condensate) together with coupled BdGe and GL equations. In such case the basic source of concepts can be taken from [14]. However they need to be readjusted for the class of systems described here. Particular expectation for the case of superconductors interacting with BEC condensate would be the emergence of robust Josephson junctions in BEC condensate as well as complex network of vortices. This issue is the subject of future research.

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Fig. 2 Concept of Field Induced Josephson junction in basic configurations (a or b) with approximation schemes (c or d).

(a) Physical system 1.

(b) Physical system 2.

(c) Scheme 1-approximation of (a)

(d) Scheme 2-approximation of (b)

(e) Deformed superconducting cable in 2D among many ferromagnets.

(f) Deformed superconducting and polarizing cables in 3D.

(g) Two robust FIJJs interacting in inductive way in the field of polarizing cables.

(h) Robust FIJJs (rFIJJs) interacting in complex electromagnetic environment-most general case with optional existence of BEC condensate.

Fig. 3 Stages of generalization of concept of FIJJ (Field Induced Josephson junction).
Fig. 4 Prototype of magnetic field entangler (MFE) embedded in superconducting cable environment and connected to current sources (CS). Each node of MFE can be controlled by 6 semiconductor transistors so one can promote certain directions of current flow and hence control topology of MFE. Same concept but basing on 6 Josephson junction transistors [9] can be assigned to BdGe cable nodes.

(a) 2 dim BdGe cables and polarizing cable lattice

(b) 2 dim BdGe cable lattice + polarizing cable lattice with cutting cables and signalling boxes

(c) 3 dim BdGe + polarizing cable lattice + signalling cables and signalling boxes.

(d) Detailed view of 2 dim unit cell of BdGe and polarizing cables with signalling cables and box.

Fig. 5 Detailed scheme of 2 [Fig.4a, 4b, 4d] and 3 [Fig.4c] dimensional BdGe cables with polarizing cable networks that are controlled by two signalling networks (cutting cables).
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