Complexity for Dynamical Anisotropic Sphere in $f(G, T)$ Gravity

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Abstract

This paper is devoted to the formulation of a complexity factor for dynamical anisotropic sphere in the framework of $f(G, T)$ gravity, where $G$ is the Gauss-Bonnet invariant and $T$ is the trace of energy-momentum tensor. Inhomogeneous energy density, anisotropic pressure, heat dissipation and modified terms create complexity within the self-gravitating system. We evaluate the structure scalars by orthogonal splitting of the Riemann tensor to evaluate a complexity factor which incorporates all the fundamental properties of the system. Moreover, we examine the dynamics of the sphere by assuming homologous mode as the simplest pattern of evolution. We also discuss dissipative as well as non-dissipative scenarios corresponding to homologous and complexity free conditions. Finally, we establish a criterion under which the complexity free condition remains stable throughout the process of evolution. We conclude that the presence of dark source terms of $f(G, T)$ gravity increase the system’s complexity.

Keywords: $f(G, T)$ gravity; Self-gravitating systems; Complexity factor
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1 Introduction

The universe includes small as well as large scale structures ranging from small planets to galaxies that contain trillions of stars. These cosmic systems influence the evolution of cosmos and lay the foundation of cosmological study. In this regard, Einstein presented general theory of relativity (GR) to study the dynamical relationship among matter, gravity, space and time. This theory opened new avenues in the field of cosmology and astrophysics by assuming that the universe is static, i.e., it is neither contracting nor expanding. Later in 1929, Edwin Hubble proposed that the universe is in a phase of accelerated expansion due to the presence of some mysterious force known as dark energy. Different astrophysical phenomena such as Supernovae Ia and cosmic microwave background radiation [1] confirm the accelerated expansion of the universe. Dark matter is another mysterious component of the universe whose presence is indicated by the dynamics of galaxies in cluster and rotation curves of spiral galaxies. Due to the problems of cosmic coincidence and fine tuning related to the cosmological constant in GR, the current cosmological accelerated expansion cannot be adequately discussed in this theory. Therefore, modified theories are considered to explain these mysterious components. For this purpose, the Einstein-Hilbert (EH) action is modified either by replacing or adding the generic scalar invariants instead of curvature scalar ($R$).

In this regard, Nojiri and Odintsov [2] put forward $f(G)$ gravity by adding the generic Guass-Bonnet (GB) term in EH action. They discussed the transition from deceleration to acceleration phase via the Guass-Bonnet invariant. This invariant is composed of Ricci scalar, Ricci tensor ($R_{\alpha\beta}$) and Riemann tensor ($R_{\alpha\beta\mu\nu}$) as $G = R^2 - 4R^{\alpha\beta}R_{\alpha\beta} + R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu}$. Different cosmological and astrophysical phenomena have been investigated in this theory. Bamba et al. [3] determined the stability of a bouncing cosmos by reconstructing $f(G)$ gravity. They also studied the early-time bounce as well as late-time acceleration of the cosmos through a scale factor involving a combination of two exponential functions. Abbas et al. [4] employed the Krori-Barua metric along with power-law model to analyze the formation of compact stars in Guass-Bonnet gravity and investigated their viable and stable behavior. Shamir and Saeed [5] examined the power as well as exponential law solutions for static spacetime by considering a viable $f(G)$ gravity model. Sharif and Saba [6] examined the viability and stability of anisotropic spherical object by using decoupling technique with suitable $f(G)$ gravity models. Sharif and
Ramzan [7] adopted embedding class-1 technique to inspect different physical aspects of anisotropic structures.

Recently, Sharif and Ikram [8] introduced a modification of $f(G)$ with non-minimal coupling by including the trace of energy-momentum tensor $(T)$ in the EH action which led to $f(G, T)$ gravity. They established the energy conditions for FRW universe by reconstructing de Sitter and power-law models. It was noted that an extra force compelled the massive particles to follow a non-geodesic trajectory for non-zero pressure. Furthermore, they [9] analyzed stability of the Einstein universe by using the perturbation scheme. Hossienkhani et al. [10] derived the energy conditions for anisotropic universe filled with perfect fluid and found that weak and null energy conditions are fulfilled for a certain range of parameters. Moreover, they observed that an increment in anisotropy led to the increasing behavior of weak energy condition. Sharif and Gul [22] determined the collapse rate of spherical object filled with perfect fluid through dynamical equations in the context of $f(G, T)$ gravity. Yousaf [11] determined the structure scalars and examined their effects on Weyl tensor, shear scalar and Raychaudhuri equations in the same theory.

Massive objects such as galaxies, stars and their clusters compose most of the visible universe. The prominent characteristics (pressure, energy density, heat flow, etc.) of the cosmic objects are interrelated which contribute to the complex nature of these stellar configurations. Thus, a factor mathematically relating the aforementioned factors through a single relation aids in the study of stellar configurations. There have been several attempts in various scientific fields to accurately explain complexity. Nevertheless, an interpretation of complexity which is suitable for all fields has not been achieved. López-Ruiz et al. [12] were among the first to characterize complexity in the form of entropy (measure of disorderness) and information. Initially, ideal gas and perfect crystal were taken into account to define complexity. The entropy of a perfect crystal is zero due to symmetric arrangement of atoms. On the other hand, random motion of particles within ideal gas generate maximum entropy. Moreover, a small portion of perfect crystal contains characteristics of the entire structure. Thus, its probability distribution depends on the perfect symmetry of all accessible states whereas ideal gas has maximum information for a brief part as all the accessible states have same probability. However, both setups are assigned zero complexity in physics. It means that the measure of complexity of a system in terms of entropy and information has not proven to be helpful.
The concept of complexity was extended to disequilibrium to check how different probabilistic states vary from the equiprobable distribution of the system [13]. Moreover, the complexity of perfect crystal as well as ideal gas reduces to zero under this definition. This definition has also been employed to calculate complexity of some compact cosmic objects by substituting energy density in place of probability distribution [14]. However, this definition is not sufficient to measure the complexity as it does not include the effects of other state determinants such as temperature, heat flux, pressure, etc. Recently, Herrera [15] resolved this issue by identifying complexity in terms of anisotropic pressure, inhomogeneous energy density and active gravitational mass for a static self-gravitating matter distribution. Four structure scalars were generated through the orthogonal splitting of the Riemann tensor. The structure scalar encompassing the effects of all state determinants was termed as the complexity factor. Herrera [16] extended the definition of complexity to non-static dissipative fluid and also discussed two patterns of evolution. This definition of complexity has also been applied to axially symmetric spacetime [17].

Using Herrera’s technique, Sharif and Butt [18] determined the complexity factor for a static matter distribution in cylindrical spacetime. They also studied the influence of electromagnetic field on static spherical [19] as well as cylindrical [20] structures and observed an increment in complexity in the presence of charge. The concept of complexity has also been analyzed in the framework of \( f(G) \) and \( f(G, T) \) theories. Sharif et al. [21] determined the complexity of static anisotropic spherical objects in \( f(G) \) gravity which led to more complex system. Complexity factors have been derived for different symmetries and matter distributions in the background of other modified theories as well [23]. Recently, Yousaf et al. [24] checked the behavior of complexity factor in the context of \( f(G, T) \) gravity for charged as well as uncharged static sphere and concluded that complexity increases in both scenarios. It is interesting to mention here that the definition of complexity given in [15] is not associated to entropy and information, instead it is based on the premise that the simplest system is characterized by the isotropic pressure in homogeneous matter distribution.

In this paper, we evaluate the complexity factor for a non-static dissipative spherical distribution and discuss the evolution patterns in the framework of \( f(G, T) \) gravity. We have used the metric signatures \((-,-,+,+\)) and relativistic units, i.e., \( G = c = 1 \). The paper is organized as follows. In section 2, the field equations and essential characteristics of the matter...
source describing the dynamics of the system are evaluated. In section 3, we decompose the Riemann tensor to obtain scalar functions. Two patterns of evolution (homologous and homogeneous) are discussed in section 4. In section 5, kinematical and dynamical quantities are derived to discuss possible solutions for both non-dissipative and dissipative scenarios. The deviation of self-gravitating object from zero complexity condition is checked in section 6. In section 7, we summarize the obtained results.

2 $f(G, T)$ Gravity and Physical Variables

The action integral for this theory is given by

$$S_{f(G, T)} = \frac{1}{2k^2} \int \sqrt{-g}d^4x [f(G, T) + R] + \int \sqrt{-g}d^4x L_m,$$

where $L_m$, $g$ and $k^2$ denote the Lagrangian density, determinant of the metric tensor $(g_{\alpha\beta})$ and coupling constant, respectively. To proceed we consider $L_m = P$. The Lagrangian density and the energy-momentum tensor are related as

$$T_{\alpha\beta} = g_{\alpha\beta} L_m - \frac{2\partial L_m}{\partial g^{\alpha\beta}}.$$  

The field equations are derived by varying Eq.(1) with respect to $g_{\alpha\beta}$ as

$$G_{\alpha\beta} = 8\pi T_{\alpha\beta} - (\Theta_{\alpha\beta} + T_{\alpha\beta}) f_T(G, T) + \frac{1}{2} g_{\alpha\beta} f(G, T) + (4R_{\mu\beta}R^\mu_{\alpha} - 2RR_{\alpha\beta}) + 4R_{\mu\nu}R_{\alpha\beta\nu} - 2R_{\mu\nu}R_{\alpha\beta\nu} + (4R_{\alpha\beta} + 4g_{\alpha\beta} R^{\mu\nu} \nabla_\mu \nabla_\nu)_f(T(G, T)) + (\Theta_{\alpha\beta} + T_{\alpha\beta}) \nabla^\gamma \ln f_T(G, T) + \nabla_\alpha \Theta_{\alpha\beta} \times f_G(G, T),$$

where $\nabla^2 = \Box = \nabla^l \nabla_l$ and $G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}$ demonstrate the d’Alembert operator and the Einstein tensor, respectively. Moreover, $\Theta_{\alpha\beta} = -2T_{\alpha\beta} + P g_{\alpha\beta}$ and $f(G, T)$ is an arbitrary function of $G$ and $T$ whose partial derivatives with respect to $G$ and $T$ are expressed by $f_G(G, T)$ and $f_T(G, T)$, respectively. The covariant divergence of Eq.(3) turns out to be

$$\nabla^\alpha T_{\alpha\beta} = \frac{f_T(G, T)}{k^2 - f_T(G, T)} \left[ -\frac{1}{2} g_{\alpha\beta} \nabla^\alpha T + (\Theta_{\alpha\beta} + T_{\alpha\beta}) \nabla^\alpha (\ln f_T(G, T)) + \nabla^\alpha \Theta_{\alpha\beta} \right].$$
Equation (3) can be rewritten in the following form

\[ G_{\alpha\beta} = 8\pi T^{(\text{tot})}_{\alpha\beta} = 8\pi (T^{(M)}_{\alpha\beta} + T^{(GT)}_{\alpha\beta}), \]  

where the correction terms of \( f(G, T) \) are represented as

\[ T^{(GT)}_{\alpha\beta} = \frac{1}{8\pi} \left[ \left( \mu P + V_\alpha V_\beta + \Pi_{\alpha\beta} + q(V_\alpha \chi_\beta + \chi_\alpha V_\beta) \right) f_T(G, T) \right. \]

\[ \left. + \left( 4R_{\mu\beta} R_\alpha^\mu + 4R_{\mu\nu} R_{\alpha\beta}^{\mu\nu} - 2RR_{\alpha\beta} - 2R_{\alpha}^{\mu\nu} R_{\beta\mu\nu} \right) f_G(G, T) \right) \]

\[ + \left( 4g_{\alpha\beta} R_{\mu\nu} \nabla_\mu \nabla_\nu - 4R_{\alpha\mu\beta\nu} \nabla^\mu \nabla^\nu - 4R_{\alpha}^{\mu\nu} \nabla_\mu \nabla_\nu - 2g_{\alpha\beta} R \nabla^2 \right) f_T(G, T) \]

\[ \left. + 2R \nabla_\alpha \nabla_\beta - 4R_\alpha^{\mu} \nabla_\mu \nabla_\beta + 4R_{\alpha\beta} \nabla^2 \right) f_G(G, T) \]  

where the energy-momentum tensor describing the anisotropic and dissipative matter configuration is given as

\[ T^{(M)}_{\alpha\beta} = \mu V_\alpha V_\beta + Ph_{\alpha\beta} + \Pi_{\alpha\beta} + q(V_\alpha \chi_\beta + \chi_\alpha V_\beta). \]

Here, the quantities \( V^\alpha = (A^{-1}, 0, 0, 0) \), \( q^\alpha = (0, qB^{-1}, 0, 0) \) and \( \chi^\alpha = (0, B^{-1}, 0, 0) \) are four velocity, heat flux and radial four-vector, respectively. Moreover,

\[ \Pi_{\alpha\beta} = \Pi \left( \chi_\alpha \chi_\beta - \frac{h_{\alpha\beta}}{3} \right), \quad \Pi = P_r - P_\perp, \]

\[ P = \frac{P_r + 2P_\perp}{3}, \quad h_{\alpha\beta} = g_{\alpha\beta} + V_\alpha V_\beta, \]

\[ \chi^\alpha \chi_\alpha = 1, \quad \chi^\alpha V_\alpha = 0, \quad V^\alpha q_\alpha = 0, \quad V^\alpha V_\alpha = -1. \]

The geometry of the non-static dissipative sphere enclosed by a hypersurface \( \Sigma \) is represented by the line element

\[ ds^2 = -A^2 dt^2 + B^2 dr^2 + C^2 d\theta^2 + C^2 \sin^2 \theta d\phi^2, \]

where the metric coefficients \( A, B \) and \( C \) are functions of \( t \) and \( r \). The modified field equations in this framework are evaluated as

\[ 8\pi (A^2 \mu + T^{(GT)}_{00}) = -\frac{A^2}{B^2} \left[ \frac{2C''}{C} + \frac{C'^2}{C^2} - \frac{B^2}{C^2} - \frac{2C'B'}{CB} \right] + \frac{\dot{C}(\frac{2B'}{B} + \frac{\dot{B}}{C})}{C}, \]

\[ 8\pi (-qAB + T^{(GT)}_{01}) = \frac{2A'C'}{AC} + \frac{2C'B^2}{CB} - \frac{2C''}{C}, \]

\[ 8\pi (-qAB + T^{(GT)}_{01}) = \frac{2A'C'}{AC} + \frac{2C'B^2}{CB} - \frac{2C''}{C}, \]
\[ 8\pi(B^2 P_r + T_{11}^{(GT)}) = \frac{-B^2}{A^2} \left[ \frac{2\dot{C}}{C} - \frac{\dot{C}(2\dot{A} - \ddot{C})}{A} \right] - \frac{B^2}{C^2} + \left( C'^2 + \frac{2A'C'}{AC} \right), \]

(14)

\[ 8\pi(C^2 P_\perp + T_{22}^{(GT)}) = \frac{8\pi T_{33}^{(tot)}}{\sin^2 \theta} = \frac{-C^2}{A^2} \left[ \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} - \frac{\dot{A}}{A} \frac{\dot{C}}{C} + \frac{\dot{B}}{B} \right] + \frac{C^2}{B^2} \left[ \left( \frac{A'}{A} - \frac{B'}{B} \right) \frac{C'}{C} - \frac{A'B'}{AB} + \frac{A''}{A} + \frac{C''}{C} \right], \]

(15)

where dot and prime stand for differentiation with respect to \( t \) and \( r \), respectively. The terms \( T_{00}^{(GT)} \), \( T_{01}^{(GT)} \), \( T_{11}^{(GT)} \) and \( T_{22}^{(GT)} \) include the modified terms whose values are given in Eqs.(A1)-(A4) of Appendix A. The Bianchi identity has only two non-zero components which are written as

\[ -\frac{1}{A} \left[ \dot{\mu} + A^2 \left( \frac{T^{00(GT)}}{A^4} \right) - \left( \mu + \frac{T^{00(GT)}}{A^2} + P_r + \frac{T^{11(GT)}}{B^2} \right) \frac{\dot{B}}{B} + 2(\mu) \right. \]

\[ \times \left. \frac{T^{00(GT)}}{A^2} + P_\perp \frac{T^{22(GT)}}{C^2} \right) \frac{\dot{C}}{C} \right] - \frac{1}{B} \left( q' - AB \left( \frac{T^{01(GT)}}{A^2B^2} \right)' + 2(q) \right. \]

\[ \left. - \frac{T^{01(GT)}}{AB^2} \left( \frac{AC}'{AC} \right) = Z_1 + \frac{2\dot{A}T^{00(GT)}}{A^4} - \frac{A'T^{01(GT)}}{A^2B^2} - \frac{B'T^{01(GT)}}{AB^3}, \right. \]

(16)

\[ \frac{1}{A} \left[ \dot{q} - AB \left( \frac{T^{01(GT)}}{A^2B^2} \right) - \left( 2q - \frac{2T^{01(GT)}}{AB} \right) \frac{\dot{B}}{B} + \left( -2q - \frac{2T^{01(GT)}}{AB} \right) \frac{\dot{C}}{C} \right] \]

\[ + \frac{1}{B} \left[ P_r + B^2 \left( \frac{T^{11(GT)}}{B^4} \right)' + \left( \mu + \frac{T^{00(GT)}}{A^2} + P_r + \frac{T^{11(GT)}}{B^2} \right) \frac{A'}{A} + 2P_r \right. \]

\[ \left. + \frac{2T^{11(GT)}}{B^2} - 2P_\perp \frac{2T^{22(GT)}}{C^2} \right) \frac{C'}{C} \right] = Z_2 + \frac{\dot{A}T^{01(GT)}}{A^3B} - \frac{2B'T^{11(GT)}}{B^4}, \]

(17)

where \( Z_1 \) and \( Z_2 \) consist of the additional curvature terms given in Eqs.(A5) and (A6) of Appendix A. It is noted that the presence of extra force in \( f(G, T) \) gravity results in the non-conservation of the energy-momentum tensor.

The four acceleration, expansion scalar and shear tensor of the fluid are...
respectively, given as
\[
a_1 = \frac{A'}{A}, \quad a = \sqrt{a^\alpha a_\alpha} = \frac{A'}{AB},
\]
(18)
\[
\Theta = \left( \frac{2 \dot{C}}{C} + \frac{\dot{B}}{B} \right) \frac{1}{A},
\]
(19)
\[
\sigma_{11} = \frac{2}{3} B^2 \sigma, \quad \sigma_{22} = \frac{\sigma_{33}}{\sin^2 \theta} = -\frac{1}{3} C^2 \sigma,
\]
(20)
\[
\sigma^{\alpha\beta} \sigma_{\alpha\beta} = \frac{2}{3} \sigma^2, \quad \sigma = \left( \frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right) \frac{1}{A}.
\]
(21)

In order to examine the impact of expansion and shear on the matter distribution, we express Eq.(13) as
\[
4\pi \left( qB - \frac{T_{01}^{(GT)}}{A} \right) = \frac{1}{3} \left( \Theta - \sigma \right)' - \sigma \frac{C''}{C} = \frac{C''}{B} \left[ \frac{1}{3} D_C \left( \Theta - \sigma \right) - \frac{\sigma}{C} \right],
\]
(22)
where \( D_C = \frac{1}{C} \frac{\partial}{\partial r} \) is the proper radial derivative. The mass of a spherical object is defined by Misner and Sharp [25] as
\[
m = \frac{1}{2} C^3 R_{232}^3 = \left[ 1 - \left( \frac{C''}{B} \right)^2 + \left( \frac{\dot{C}}{A} \right)^2 \right] \frac{C}{2}.
\]
(23)

We introduce the proper time derivative \( (D_T = \frac{1}{A} \frac{\partial}{\partial t}) \) to analyze the dynamics of self-gravitating objects. The velocity \( (U) \) of the collapsing matter, defined in terms of areal radius \( (C) \), is negative in case of collapse as
\[
U = D_T C < 0.
\]
(24)
The relation between velocity and mass of the sphere is given as
\[
E \equiv \frac{C'}{B} = \left( 1 - \frac{2m}{C} + U^2 \right)^{\frac{1}{2}}.
\]
(25)
The proper derivatives (time and radial) of the mass function become
\[
D_T m = -4\pi \left[ \left( P_r + \frac{T_{11}^{(GT)}}{B^2} \right) U + \left( q - \frac{T_{01}^{(GT)}}{AB} \right) E \right] C^2,
\]
(26)
\[ D_C m = 4\pi \left[ \left( \mu + \frac{T_{00}^{(GT)}}{A^2} \right) + \left( q - \frac{T_{01}^{(GT)}}{AB} \right) \frac{U}{E} \right] C^2, \quad (27) \]

which lead to
\[
\frac{3m}{C^3} = 4\pi \left( \mu + \frac{T_{00}^{(GT)}}{A^2} \right) - \frac{4\pi}{C^3} \int_0^r C^3 \left[ D_C \left( \mu + \frac{T_{00}^{(GT)}}{A^2} \right) \right. \\
- 3 \left( q - \frac{T_{01}^{(GT)}}{AB} \right) \frac{U}{CE} \left. \right] C' dr. \quad (28)
\]

The Weyl tensor \( (C^\lambda_{\alpha\beta\mu}) \) measures the stretch in a self-gravitating object due to varying gravitational field of another body. This tensor can be decomposed into magnetic and electric parts. However, for a spherical system the magnetic part disappears while electric part is given as
\[
E_{\alpha\beta} = C_{\alpha\mu\nu} V^\mu V^\nu = \varepsilon (\chi_\alpha \chi_\beta - \frac{h_{\alpha\beta}}{3}), \quad (29)
\]
where
\[
\varepsilon = \frac{1}{2A^2} \left[ \frac{\dot{C}}{C} - \frac{\dot{B}}{B} \right] - \left( \frac{\dot{C}}{C} + \frac{\dot{A}}{A} \right) \left( \frac{\dot{C}}{C} - \frac{\dot{B}}{B} \right) + \left( \frac{C'}{C} - \frac{A'}{A} \right) \left( \frac{C'}{C} + \frac{B'}{B} \right) \right]. \quad (30)
\]

The effect of the tidal force on the mass of the fluid distribution is gauged from the following relation
\[
\frac{3m}{C^3} = 4\pi \left[ \left( \mu + \frac{T_{00}^{(GT)}}{A^2} \right) - \Pi^{(tot)} \right] - \varepsilon, \quad (31)
\]
where \( \Pi^{(tot)} = \Pi + \Pi^{(GT)} \) and \( \Pi^{(GT)} = \left( \frac{T_{11}^{(GT)}}{B^2} - \frac{T_{22}^{(GT)}}{C^2} \right) \). The hypersurface separates the spacetime into two regions, i.e., inner and outer regions. The Darmois junction conditions must be satisfied to prevent any discontinuity at the boundary. The Vaidya spacetime describes the outer region of a dissipative sphere as
\[
ds^2 = - \left( 1 - \frac{2M(v)}{r} \right) dt^2 - 2dr dv + \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) r^2, \quad (32)\]
where $M(v)$ and $v$ demonstrate the total mass and retarded time, respectively. The matching of both regions at the boundary surface is ensured if the following conditions are fulfilled

$$ (m(t, r))_{\Sigma} = (M(v))_{\Sigma}, $$

$$ 2 \left( \frac{\dot{C}'}{C} - \frac{C'B}{CB} - \frac{\dot{C}A'}{CA} \right)_{\Sigma} = \left( -\frac{\left( \frac{2\dot{C}}{C} - \frac{2\dot{A}}{A} - \frac{\dot{C}}{C} \right) B}{A} \right. $$

$$ + \left. \frac{\left( -\frac{B^2}{C^2} + \frac{(2A')'}{A} + \frac{C'}{C} \frac{C'}{C} \right) A}{B} \right)_{\Sigma}, $$

$$ \left( q - \frac{T^{(GT)}}{AB} \right)_{\Sigma} = \left( P_r + \frac{T^{(GT)}}{B^2} \right)_{\Sigma}, $$

where the subscript $\Sigma$ shows that the values are analyzed at the boundary. In order to discuss the dissipative spherical geometry along with the conformal flatness condition, Herrera et al. [26] proposed some analytical solutions to the Einstein equations. The obtained solutions were matched with the Vaidya spacetime, and the consequences of relaxational effects on temperature and evolution of the system have been observed. Tewari [27] computed the exact solutions of field equations corresponding to shear-free, dissipative spherical object. At the boundary, the acquired solutions were compared to the Vaidya metric and it was observed that after applying some constraints, the solutions representing the static fluid evolved to radiating collapse. Vertogradov [28] utilized the general Vaidya metric to discuss the gravitational collapse by employing an equation of state and found that it would either be a black hole or a naked singularity.

### 3 Structure Scalars

In order to determine the complexity of the system, we split the Riemann tensor in terms of structure scalars by following Herrera’s technique [29]. The disintegrated form of the Riemann tensor in terms of Ricci scalar, Ricci and Weyl tensors reads

$$ R^\rho_{\alpha\beta\mu} = C^\rho_{\alpha\beta\mu} + \frac{1}{2} R_{\alpha\mu} \delta^\rho_\beta + \frac{1}{2} R^\rho_{\beta\mu} g_{\alpha\beta} - \frac{1}{2} R_{\alpha\beta} \delta^\rho_\mu - \frac{1}{2} R^\rho_\mu g_{\alpha\beta} - \frac{1}{6} R \left( \delta^\rho_\beta g_{\alpha\mu} - g_{\alpha\beta} \delta^\rho_\mu \right), $$

(36)
which is rewritten as

\[ R^{\alpha\gamma}_{\beta\delta} = C^{\alpha\gamma}_{\beta\delta} + 16\pi T^{(tot)\mu}_{\beta \delta} + 8\pi T^{(tot)} \left( \frac{1}{3} \delta_{[\beta \delta]}^{\alpha \gamma} - \delta_{[\beta \delta]}^{[\alpha \gamma]} \right). \]  \hfill (37)

We introduce the tensors \( Y_{\alpha\beta} \) and \( X_{\alpha\beta} \), respectively defined as

\[ Y_{\alpha\beta} = R_{\alpha\gamma\beta\delta} V^\gamma V^\delta, \]  \hfill (38)

\[ X_{\alpha\beta} = *R^{*}_{\alpha\gamma\beta\delta} V^\gamma V^\delta = \frac{1}{2} \eta_{\alpha\gamma}^{\mu\nu} R_{\epsilon\mu\beta\delta} V^\gamma V^\delta, \]  \hfill (39)

where \( \eta_{\alpha\gamma}^{\mu\nu} \) denotes the Levi-Civita symbol and \( R_{\alpha\beta\gamma\delta} = \frac{1}{2} \eta_{\epsilon\mu\gamma\delta} R^{\epsilon\mu}_{\alpha\beta} \). In electrodynamics, the tensor \( Y_{\alpha\beta} \) specifies the electric component of the Riemann tensor whereas the tensor \( X_{\alpha\beta} \) has no analogy. Dual provides a simple constitutive relation for free space, with vacuum having its own meaningful electric and magnetic fields. The tensors \( Y_{\alpha\beta} \) and \( X_{\alpha\beta} \) will later be expressed in terms of matter variables such as energy density \( \mu \), anisotropic pressure \( \Pi \) and heat flux \( q \) as these tensors are defined in the form of Riemann tensor. These tensors are written in the combination of their trace-free \( (Y_{TF}, X_{TF}) \) and trace parts \( (Y_T, X_T) \) as

\[ Y_{\alpha\beta} = \frac{h_{\alpha\beta} Y_T}{3} + (\chi_{\alpha\beta} - \frac{h_{\alpha\beta}}{3}) Y_{TF}, \]  \hfill (40)

\[ X_{\alpha\beta} = \frac{h_{\alpha\beta} X_T}{3} + (\chi_{\alpha\beta} - \frac{h_{\alpha\beta}}{3}) X_{TF}. \]  \hfill (41)

The structure scalars corresponding to the current setup are obtained as

\[ Y_T = 4\pi (\mu + 3P_r - 2\Pi) + \frac{(\mu + P) f_T}{2} + M^{(GT)}, \]  \hfill (42)

\[ Y_{TF} = \varepsilon - 4\pi \Pi + \frac{\Pi f_T}{2} + L^{(GT)}, \]  \hfill (43)

\[ X_T = 8\pi \mu + Q^{(GT)}, \]  \hfill (44)

\[ X_{TF} = -\varepsilon - 4\pi \Pi + \frac{\Pi f_T}{2}, \]  \hfill (45)

where \( L^{(GT)} = \frac{j_{(GT)}^\alpha}{s_{\alpha\beta} \frac{1}{s_{\alpha\beta}}} \). The terms \( M^{(GT)} \), \( j_{\alpha\beta}^{(GT)} \) and \( Q^{(GT)} \) are provided in Appendix B. The scalars \( X_T \) and \( Y_T \) are responsible for the energy density and local anisotropic pressure, respectively. Employing Eqs. (30) and (43), the scalar function \( Y_{TF} \) is expressed as

\[ Y_{TF} = -4\pi \Pi - 4\pi \Pi^{(GT)} - L^{(GT)} + \frac{4\pi}{C^3} \int C^3 D_C \left( \mu + \frac{T_{00}^{(GT)}}{A^2} \right). \]
\[
-3 \left( q - \frac{T_{01}^{(GT)}}{AB} \right) \frac{U}{CE} \right] C'dr,
\]

where \( \Pi, \mu \) and \( q \) represent the anisotropic pressure, energy density and heat flux, respectively. The above equation shows that \( Y_{TF} \) involves the contribution from inhomogeneous energy density, heat flux and anisotropic pressure. The scalar \( X_{TF} \) measures the inhomogeneity within the system in \( f(G,T) \) gravity as

\[
X_{TF} = 4\pi \Pi^{(GT)} - \frac{4\pi}{C^3} \int C^3 \left[ D_C \left( \mu + \frac{T_{00}^{(GT)}}{A^2} \right) - 3 \left( q - \frac{T_{01}^{(GT)}}{AB} \right) \frac{U}{CE} \right] C'dr.
\]

4 Evolution Modes

Different interrelated physical factors such as pressure and energy density play an important role in the complex nature of the cosmic structure. The scalar \( Y_{TF} \) involves energy density inhomogeneity, heat flux and pressure anisotropy along with \( f(G,T) \) corrections. Therefore, we identify \( Y_{TF} \) as the complexity factor for the fluid distribution in a non-static system. The condition \( Y_{TF} = 0 \) corresponds to complexity free system. As fluid evolves with time, we need to determine its pattern of evolution. For this purpose, we discuss two modes of evolution, i.e., homologous evolution and homogeneous expansion in the subsequent sections and choose the simplest mode to minimize complexity.

4.1 Homologous Evolution

The word homologous means self-similar or having the same pattern. Therefore, homologous collapse refers to a linear relation between velocity and the radial distance, i.e., matter is pulled to the core at the same rate throughout the collapse of astrophysical objects. The object undergoing homologous collapse emits strong gravitational radiation as compared to the object whose core collapses first. We rewrite Eq.[22] as

\[
D_C \left( \frac{U}{C} \right) = \frac{4\pi}{E} \left( q - \frac{T_{01}^{(GT)}}{AB} \right) + \frac{\sigma}{C},
\]
which after integration produces

\[ U = h(t)C + C \int_0^r \left[ \frac{4\pi}{E} \left( q - \frac{T_{01}^{(GT)}}{AB} \right) + \frac{\sigma}{C} \right] C' dr, \tag{49} \]

where \( h(t) \) is an integration constant whose evaluation at the boundary yields

\[ U = \frac{U_\Sigma}{C_\Sigma} C - C \int_r^{r_{\Sigma}} \left[ \frac{4\pi}{E} \left( q - \frac{T_{01}^{(GT)}}{AB} \right) + \frac{\sigma}{C} \right] C' dr. \tag{50} \]

Heat dissipation and shear scalar cause the fluid to deviate from homologous mode. Therefore, if the terms in the integral cancel each other then \( U \sim C \) which is the required condition for homologous evolution \(^{[30]}\) which provides \( U = h(t)C \) with \( h(t) = \frac{U_\Sigma}{C_\Sigma} \). The homologous condition in \( f(G,T) \) gravity is evaluated as

\[ \frac{4\pi B}{C'} \left( q - \frac{T_{01}^{(GT)}}{AB} \right) + \frac{\sigma}{C} = 0. \tag{51} \]

### 4.2 Homogeneous Expansion

The condition for homogeneous expansion reads \( \Theta' = 0 \). In other words, the rate of collapse or expansion of the cosmic objects is homogeneous if it does not depend upon \( r \). Applying this condition on Eq.(22) leads to

\[ 4\pi \left( q - \frac{T_{01}^{(GT)}}{AB} \right) = -\frac{C'}{B} \left[ \frac{1}{3} D_C(\sigma) + \frac{\sigma}{C} \right]. \tag{52} \]

Comparing the above equation with (51) yields

\[ D_C(\sigma) = 0. \tag{53} \]

Moreover, the regularity conditions at the middle yield \( \sigma = 0 \). Thus, Eq.(52) takes the form

\[ q = \frac{T_{01}^{(GT)}}{AB}, \tag{54} \]

which implies that the fluid is dissipative. However, in the context of GR, the homogeneous pattern of evolution yields a shear-free and non-dissipative fluid distribution.
5 Kinematical and Dynamical Variables

We select the simplest mode of evolution by investigating the behavior of different physical variables. For sake of simplifying the calculations, we assume $C$ as a separable function of $r$ and $t$. The homologous condition along with Eq.(22) leads to

\[(\Theta - \sigma)' = \left(\frac{3\dot{C}}{AC}\right)' = 0.\]  

(55)

This implies that the fluid is geodesic as $A' = 0 (a = 0)$. We take $A = 1$ without any loss of generality. Conversely, the geodesic condition provides

\[\Theta - \sigma = \frac{3\dot{C}}{C}.\]  

(56)

The successive derivatives of the above equation closer to the core lead to the homologous condition. Thus, the geodesic condition implies a homologous fluid and vice-versa. In GR, the fluid distribution is shear-free when $q = 0$. However, in $f(G, T)$ theory the shear scalar for $q = 0$ is evaluated as

\[\sigma = 4\pi \frac{T_{01}^{(GT)}C}{C'}.\]  

(57)

If a non-dissipative fluid collapses homogeneously, we obtain $T_{01}^{(GT)} = 0$ while Eq.(52) yields the shear scalar as

\[\sigma = \frac{12\pi}{C^3} \int \frac{C^3T_{01}^{(GT)}}{A}dr + \frac{b(t)}{C^3},\]  

(58)

where arbitrary integration function $(b(t))$ must be zero at the center $(C = 0)$. Thus, homogeneous expansion infers homologous evolution for the non-dissipative case as $T_{01}^{(GT)} = 0 \Rightarrow \sigma = 0 \Rightarrow U \sim C$. As homogeneous expansion implies homologous evolution, this shows that the homologous evolution is the simplest mode. When a fluid undergoes homologous evolution, the relation between mass and velocity during collapse becomes

\[D_TU = -\frac{m}{C^2} - 4\pi \left(P_t + \frac{T_{11}^{(GT)}}{B^2}\right)C.\]  

(59)
Using the definition of $Y_{TF}$ in the above expression yields

$$
\frac{3D_T U}{C} = Y_{TF} - L^{(GT)} + 4\pi \Pi^{(GT)} - 4\pi \left[ \mu + \frac{T_{00}^{(GT)}}{A^2} \right] + 3 \left( P_r + \frac{T_{11}^{(GT)}}{B^2} \right) - 2\Pi,
$$

which leads to

$$
\frac{\ddot{C}}{C} - \frac{\ddot{B}}{B} = Y_{TF} - L^{(GT)} + 4\pi \Pi^{(GT)}. \tag{60}
$$

Thus, the matter distribution is free from complexity when $\frac{\ddot{C}}{C} - \frac{\ddot{B}}{B} + L^{(GT)} - 4\pi \Pi^{(GT)} = 0$. Integration of Eq. (61) for the case $Y_{TF} = 0$ produces

$$
B = C_1(t) \left[ g_1(r) \int \frac{1}{C_1(t)^2} e^{f(L^{(GT)} - 4\pi \Pi^{(GT)})} \frac{C_1(t)}{C_1(t)} dt + g_2(r) \right], \tag{62}
$$

where $g_1(r)$ and $g_2(r)$ are integration functions. The above equation can be conveniently written as

$$
B = C_1(t) C_2'(r) \left[ \tilde{g}_1(r) \int \frac{1}{C_1(t)^2} e^{f(L^{(GT)} - 4\pi \Pi^{(GT)})} \frac{C_1(t)}{C_1(t)} dt + \tilde{g}_2(r) \right] = Z C', \tag{63}
$$

where $Z = \tilde{g}_1(r) \int \frac{1}{C_1(t)^2} e^{f(L^{(GT)} - 4\pi \Pi^{(GT)})} \frac{C_1(t)}{C_1(t)} dt + \tilde{g}_2(r)$, $\tilde{g}_1(r) = \frac{g_1(r)}{C_1'(r)}$ and $\tilde{g}_2(r) = \frac{g_2(r)}{C_2'(r)}$.

We now propose possible solutions for non-dissipative as well as dissipative fluids satisfying the homologous and vanishing complexity conditions. As the considered system is non-static and $f(G, T)$ field equations are highly non-linear, it is convenient to choose a linear $f(G, T)$ model of the form $f(G, T) = h_1(G) + h_2(T)$, where $h_1(G) = \xi G^n$ and $h_2(T) = \kappa T$. Further, $\xi$ and $\kappa$ are real numbers while $n > 0$. For the present work, we select $\xi = n = 1$. One of the extension of modified Gauss-Bonnet gravity is $f(G, T)$ gravity. The term GB alone is a total derivative and can be integrated out leaving no modified results. However, in $f(G, T)$ theory, it encompasses the effects of both GB invariant and $T$ which does not correspond to the total derivative and therefore cannot be integrated out in the action as a surface term. The model mentioned in the manuscript is not the same as GR because it includes the contribution from the GB term as well as $T$. Hence, the model $\xi G + \kappa T$ cannot be considered as GR version. In order to inspect the complexity of self-gravitating objects, some other $f(G, T)$ models can also be utilized such as
\[ f(G, T) = G^2 + 2T \]
\[ f(G, T) = f(G) + \lambda T. \]

At the current stage, the system contains two unknowns: \( B(t, r) \) and \( C(t, r) \) which can be determined via the homologous and vanishing complexity conditions. For the case of non-dissipative fluid, these conditions are computed as

\[
\frac{1}{(\kappa + 8\pi)B^3C^2} \left[ B \left( \dot{B}C'\ddot{C} - 2C \left( \dot{B}C' + 5\dot{B}\ddot{C}' \right) \right) + 10\ddot{B}\dot{B}CC' \right. \\
+ \left. 2B^2\dot{C}' \left( C - 4\ddot{C} \right) \right] = 0, \quad (64)
\]

\[
\frac{1}{BC} \left[ -2\dot{B}B^3\dot{C} \left( C - 4\ddot{C} \right) - C' B \left( -32\dot{B}\ddot{C}'^2 + \left( 3\dot{B} + \dot{B}^2 \right) C' \right. \\
+ \left. 2B'C' + 4\dot{C}'^2 \ddot{B}^2 - 2B^2 \left( -CC'' + 2\ddot{C} + 8\dot{C}'^2 + C'^2 \right) \right] \\
+ \left. 2B^4 \left( \dot{C}^2 + 1 \right) \right] = 0. \quad (65)
\]

In dissipative case, the obtained vanishing complexity condition is the same as (65) while the homologous condition is given as

\[
q = \frac{1}{16\pi^2(\kappa + 1)B^5C^4} \left[ C' \left( \dot{B}C - B\dot{C} \right) \left( B \left( C \dot{B}C' + 40\pi\dot{C}'^2 \dot{B} \right) \right. \\
- \left. 4\pi\ddot{C}\dot{B}C' \right) + \left. 4\pi\dot{B}\ddot{B}CC' + B^2 \left( -C'' \right) \left( C - 32\pi\ddot{C} \right) \right]. \quad (66)
\]

The homologous condition and vanishing complexity condition for the non-dissipative and dissipative cases in the context of GR are as follows

\[
\frac{1}{4\pi B^2C^2} \left( \dot{B}C' - \ddot{B}C' \right) = 0, \quad (67)
\]

\[
\frac{B^2 \left( \dot{C}^2 + 1 \right)}{C} + C'' = \frac{B'C'}{B} + B\dot{B}\dot{C} + \frac{C'^2}{C}, \quad (68)
\]

\[
q = \frac{1}{16\pi^2 B^4C^3} \left( C'' \left( \dot{B}C' - \ddot{B}C' \right) \left( B\ddot{C}' - \dot{B}C' \right) \right). \quad (69)
\]

In order to compare the complex nature of self-gravitating fluids in \( f(G, T) \) gravity, we have shown the behavior of \( B, C \) and \( q \) in the non-static scenario...
Figure 1: Plots of metric potentials and heat flux versus $r$ and $t$ in $f(G, T)$ framework.
in Figure 1. In the upper left graph, the value of $B$ is slightly reduced by increasing the value of $t$ while the increment in $r$ makes $B$ maximum. The graph of $C$ (upper right) indicates that its value gradually decreases with increasing $t$ and remains at its maximum throughout the $r$ range. The graphical analysis of $q$ illustrates that it has more dissipation at the center as compared to GR with respect to $t$ and it goes on increasing at the boundary. Radially, it is maximum at the center and then begins to decrease for larger values of $r$. Figure 2 exhibits the functioning of $B$, $C$ and $q$ in the context of GR. The metric coefficient $B$ linearly increases with $t$ whereas its value monotonically increases towards the boundary with $r$. The graphic representation of $C$ depicts the similar behaviour as in $f(G,T)$ with respect to $t$ whereas it first increases and then shifts to the decreasing behavior with $r$. The lower graph in Figure 2 interprets that heat flux is minimum at the center and becomes maximum at the larger values of $t$, while it shows the similar pattern as in $f(G,T)$ for the radial trend.

6 Stability of Zero Complexity Condition

Following the procedure in [29], the evolution equation is attained in terms of complexity factor by using Eqs.(16), (43) and (45)

\[
-4\pi \left( \mu + T^{00(GT)} + P_r + \frac{T^{11(GT)}}{B^2} \right) \sigma - \frac{4\pi}{B} \left[ q' - B\left( \frac{T^{00(GT)}}{B^2} \right)' \right] \\
- (q - \frac{T^{01(GT)}}{B^2}) C' - \dot{Y}_{TF} - \dot{L}^{(GT)} - 8\pi \Pi - 4\pi \dot{\Pi}^{(GT)} - 4\pi \left( T^{00(GT)} \right)' \\
-3\frac{\dot{C}}{C} \left( Y_{TF} - L^{(GT)} \right) - 4\pi Z_1 + \frac{4\pi T^{00(GT)} B'}{B^2} - \frac{12\pi T^{00(GT)} \dot{C}}{C} \\
+ \left( 12\pi T^{(GT)}_{00} - 12\pi \Pi^{(GT)} + \frac{12\pi T^{(GT)}_{11}}{B^2} \right) \frac{\dot{C}}{C} - 16\pi \Pi^{(tot)} \dot{C} = 0. \tag{70}
\]

At the initial time $t = 0$, the substitution $Y_{TF} = \Pi^{(tot)} = q = \sigma = 0$ for the non-dissipative case yields

\[
-\dot{Y}_{TF} + 4\pi \left( \frac{T^{00(GT)}}{B^2} \right)' - 4\pi \left( T^{00(GT)} \right)' - \dot{L}^{(GT)} - 8\pi \Pi - 4\pi \dot{\Pi}^{(GT)} + 3L^{(GT)} \\
\times \frac{\dot{C}}{C} - 4\pi Z_1 + \frac{4\pi T^{00(GT)} B'}{B^2} - \frac{12\pi T^{00(GT)} \dot{C}}{C} + \left( 12\pi T^{00(GT)} + \frac{12\pi T^{11(GT)}}{B^2} \right)
\]
Figure 2: Plots of metric potentials and heat flux versus $r$ and $t$ in GR framework.

$$\times \frac{\dot{C}}{C} = 0.$$ 

Inserting the above condition in the derivative of Eq.(46) at $t = 0$ leads to

$$\frac{4\pi}{C^3} \int_0^r [(\mu + \frac{T^{(GT)}}{A^2})'] = +4\pi \left( \frac{T^{(GT)}}{B^2} \right)' - 4\pi \left( T^{(GT)} \right) - \frac{12\pi T^{(GT)}}{C} \hat{C}$$

$$+ \frac{4\pi T^{(GT)} B'}{B^2} - 4\pi Z_1 + 3L^{(GT)} \frac{\dot{C}}{C} + (12\pi T^{(GT)} + \frac{12\pi T^{(GT)}}{B^2}) \frac{\dot{C}}{C}.$$  (71)

Moreover, differentiation of Eq.(70) with respect to time along with the condition $Y_{TF} = \sigma = 0$ is expressed as

$$-\ddot{Y}_{TF} + \left[ 4\pi \left( \frac{T^{(GT)}}{B^2} \right) \right]' - 4\pi \left( T^{(GT)} \right)' - \tilde{L}^{(GT)} - 4\pi \tilde{Z}_1 + \tilde{L}^{(GT)}$$

$$-4\pi \tilde{H}^{(GT)} - 12\pi \left( \frac{T^{(GT)}}{C} \right) - \frac{3\dot{C}}{C} + \frac{1}{4\pi} \left( \frac{T^{(GT)}}{B^2} \right)' + 4\pi \left( T^{(GT)} \right).$$
\begin{align*}
+4\pi Z_1 - 4\pi \left( \frac{B'T^{00(GT)}}{B^2} \right) + 4\pi \dot{\Pi}^{(GT)} + 12\pi \left( \frac{T^{00(GT)}\dot{C}}{C} \right) - 3L^{(GT)}\frac{\dot{C}}{C} \\
- \left( 12\pi T^{00(GT)}_{00} + \frac{12\pi T^{11(GT)}}{B^2} \right) \frac{\dot{C}}{C} + 8\pi \dot{\Pi} \right] + 3 \left( \frac{L^{(GT)}\dot{C}}{C} \right) - \frac{16\pi \Pi^{(tot)}\dot{C}}{C} \\
- 8\pi \dot{\Pi} + \left[ (12\pi T^{00(GT)}_{00} + \frac{12\pi T^{11(GT)}}{B^2}) \frac{\dot{C}}{C} \right] + 4\pi \left( \frac{B'T^{00(GT)}}{B^2} \right) = 0. \quad (72)
\end{align*}

The higher derivatives of Eq.(46) along with (72) show that the inhomogeneous energy density, anisotropic pressure and dark source terms lead to the instability of vanishing complexity condition. The state with zero complexity is achieved and maintained in the scenario of isotropic pressure, homogeneous energy density with no modified terms. Furthermore, the presence of heat flux also influences the stability of the zero complexity factor in the dissipative mode. In the present manuscript, the vanishing complexity factor condition \(Y_{TF} = L^{(GT)} - 4\pi \Pi^{(GT)}\) is determined which comes out in the form of matter variables and extra curvature terms. With the help of experimental data of any compact objects regarding mass and radius, this condition can be found helpful to check the stability and viability of that particular object. Different techniques have been employed to analyze the stability and viability of various astrophysical objects [33].

7 Conclusions

The focus of this paper is to determine the complexity of a non-static sphere in the background of \(f(G,T)\) gravity. For this purpose, we have assumed an inhomogeneous sphere with heat flux and anisotropic pressure. We have adopted Herrera’s technique to split the Riemann tensor into four scalars which specify the structure of self-gravitating objects. We have chosen \(Y_{TF}\) as the complexity factor based on the following reasons.

1. In the static case, it has already served as an appropriate measure of complexity [15].

2. It is the only factor which incorporates the energy density inhomogeneity, anisotropic pressure, heat dissipation and correction terms.
We have inspected two possible modes of evolution namely homologous and homogeneous patterns. Choosing homologous mode as the simplest pattern of evolution, we have derived the solutions for dissipative and non-dissipative cases by applying the condition of zero complexity. We have interpreted the numerical solutions through graphical analysis in GR as well as $f(G, T)$ scenarios to distinguish the behavior of complexity between them. We have also discussed the factors which cause the system to deviate from vanishing complexity condition during the evolution process.

The structure scalars in $f(G, T)$ theory include the contribution from dark source terms and potential functions. Consequently, the presence of higher order curvature terms produce complexity in the dynamical sphere. Moreover, the geodesic nature of the fluid evolving homologously (in dissipative and non-dissipative scenarios) leads to the choice of homologous pattern as the simplest mode of evolution. We have devised the homologous condition in this theory which includes the modified terms. Further, the condition $Y_{TF} = 0$ was achieved in GR when $\mu = \Pi = q = 0$ whereas, in $f(G, T)$ theory, an extra constraint $L^{(GT)} - 4\pi \Pi^{(GT)} = 0$ along with the aforementioned conditions provide a complexity free system. Moreover, the shear tensor does not vanish for the non-dissipative case in contrast to GR. The modified homologous and vanishing complexity conditions have been used to evaluate the possible solutions for non-dissipative and dissipative models. Isotropic pressure and homogeneous energy density lead to the stability of vanishing complexity condition in the absence of correction terms and heat flux. We conclude from Eq. (43) that the $f(G, T)$ system is more complex as compared to its GR analog. In the formalism of $f(G, T)$ gravity, the complexity of self-gravitating system for the static case has been evaluated [24] and the core equation (vanishing complexity condition) of the complexity work is recovered from the non-static system. It is worthwhile to mention here that our all results reduce to those obtained in GR when $f(G, T) = 0$.

### Appendix A

The modified terms in $f(G, T)$ gravity appearing in the field equations are

$$T_{00}^{(GT)} = \frac{1}{8\pi} \left[ (\mu + P) A^2 f_T - \frac{A^2}{2} f + \left( \frac{8B\dot{C}\ddot{C}}{A^2 BC^2} - \frac{16A'\dot{C}'\dddot{C}'}{AB^2 C^2} + \frac{8AC'\dot{A}'\dddot{C}'}{B^4 C^2} \right) \right]$$
\[
\begin{align*}
&\quad + \frac{4\dot{B} + 8A'\dot{B}'\dot{C}'C'}{BC^2} + \frac{8\dot{A}'\dot{C}'\dot{C}''}{AB^3C^2} - \frac{8\dot{A}'\dot{B}'C'\dot{C}'}{AB^2C^2} - \frac{8A'\dot{B}'\dot{C}''}{A^3BC^2} - \frac{12\dot{A}'\dot{B}'\dot{C}'}{AC^2} + \frac{4\dot{C}'^2A'}{BC^2} \\
&\quad \times \frac{B'}{B^3} - \frac{12A\dot{A}'\dot{B}'C'^2}{B^3C^2} + \frac{4C''\dot{A}'\dot{B}}{AB^3C^2} + \frac{4\dot{A}'\dot{A}'\dot{C}''}{A^2BC^2} + \frac{4\dot{C}'^2\dot{B}}{B^2C^2} + \frac{8\dot{C}'^2}{B^2C^2} \\
&\quad + \frac{8\dot{B}'A\dot{C}'\dot{C}'}{B^3C^2} - \frac{16\dot{B}'\dot{C}'\dot{C}''}{C^2B^3} - \frac{4\dot{A}'\dot{B}}{AC^2B} - \frac{8\dot{A}'\dot{C}''}{B^2AC^2} + \frac{8A'^2\dot{C}''}{B^2A^2C^2} + \frac{8\dot{C}'\dot{C}''}{C^2B^4} \\
&\quad - \frac{8\dot{C}'\dot{C}''}{C^2B^2} - \frac{4\dot{A}'\dot{A}'\dot{C}''}{B^3C^2} - \frac{4\dot{C}'^2\dot{B}}{B^3C^2} \left( \dot{f}_G + \left( \frac{8\dot{C}'\dot{C}''}{B^2C^2} - \frac{8B\dot{C}'\dot{C}''}{B^3C^2} \right) \right) + \frac{8\dot{B}'\dot{C}'\dot{C}'}{B^2C^2} - \frac{4\dot{B}'\dot{C}'\dot{C}''}{B^3C^2} - \frac{4\dot{A}'\dot{B}' \dot{C}''}{B^3C^2} \\
&\quad + \frac{12A'^2\dot{B}'\dot{C}'\dot{C}''}{B^3C^2} - \frac{8A'^2\dot{C}'\dot{C}''}{B^3C^2} \left( \dot{f}_G + \left( \frac{4\dot{C}^2}{B^2C^2} + \frac{4A'^2}{B^3C^2} - \frac{4A'^2\dot{C}'^2}{B^3C^2} \right) \right) \\
&\quad \times f''_G.
\end{align*}
\]

\[T_{01}^{(GT)} = \frac{1}{8\pi} \left[-qAB f_T + \left( \frac{10A'\dot{B}'\dot{C}'B}{AB^3C} - \frac{10\dot{A}'\dot{B}'\dot{C}'A'}{A^4BC} - \frac{8\dot{A}'\dot{B}'\dot{C}'A'}{A^3BC^2} - \frac{8A'}{AB^3} \right) \right. \]

\[\times \frac{B'C'^2}{C^2} - \frac{10\dot{A}'\dot{B}'\dot{C}'C'}{A^3B^2C} - \frac{8A'^2\dot{C}'\dot{C}'}{A^2B^2C^2} + \frac{10A'^2\dot{B}'\dot{C}'}{A^2B^3C} + \frac{\dot{B}'\dot{C}'\dot{C}'}{A^2B^2C^2} + \frac{10\dot{B}'\dot{B}}{A^2B^2} \]

\[\times \frac{A'B'C'^2\dot{C}'}{A^3BC} - \frac{10\dot{A}'\dot{A}'\dot{C}'\dot{C}'}{A^4BC^2} - \frac{10\dot{A}'\dot{A}'\dot{C}'\dot{C}'}{A^3BC^2} + \frac{\dot{A}'\dot{B}'\dot{C}'\dot{C}'}{A^3BC} + \frac{8\dot{A}'\dot{C}'\dot{C}'}{A^3BC} + \frac{8\dot{A}'\dot{C}'\dot{C}'}{A^2B^2} - 10 \]

\[\frac{AB^3C'^2}{A^3C'^2} - \frac{A'^2\dot{C}'\dot{C}'}{A^3C'^2} + \frac{A'^2\dot{C}'\dot{C}'}{A^3C'^2} - \frac{A'^2\dot{C}'\dot{C}'}{A^3C'^2} - \frac{A'^2\dot{C}'\dot{C}'}{A^2B'^2} \]

\[\frac{8\dot{C}'\dot{C}''}{A^2C'^3} \left( f_G + \left( -\frac{8\dot{A}'\dot{A}'\dot{C}'}{A^4B} - \frac{4\dot{A}'\dot{A}'\dot{C}'}{A^2C} + \frac{8\dot{C}'\dot{C}''}{A^2B^2} + \frac{8A'^2\dot{B}'}{2B^3C^2} + \frac{12\dot{B}'\dot{C}'\dot{C}'}{A^3B^2} \right) \dot{f}_G \right) \left( \frac{8\dot{A}'\dot{C}'\dot{C}'}{A^2B^2C^2} + \frac{8A'\dot{B}'\dot{B}'}{A^3B^3} - \frac{4\dot{B}'\dot{C}'\dot{C}'}{A^2B^2C^2} \right) \dot{f}_G + (4 \]

\[\times \frac{\dot{C}'^2}{A^2C'^2} + \frac{4\dot{C}'^2}{B^2C^2} + \frac{8A''}{AB^2} - \frac{8B}{A^2B} - \frac{8A''}{AB^2} - \frac{8A'B'}{AB^3} + \frac{8\dot{B}'}{A^3B} \right) f_G', \tag{A2}\]
\[ T^{(GT)}_{11} = \frac{1}{8\pi} \left[ \frac{2}{3} \Pi B^2 f_T + \frac{B^2}{2} f \left( \frac{32 \dot{B} C' \dot{C}''}{A^2 B C^2} - \frac{8 A'C'' C''}{A B^2 C^2} - \frac{8 B \ddot{B} \ddot{C}}{A^2 C^2} - 8 \right) \times \right. \\
 \left. \frac{B' \dddot{C} C'}{B C^2 A^2} + \frac{\dddot{B} C' C'}{A^2 B C^2} - \frac{4 A'' C' C'}{A B^2 C^2} - \frac{16 \dot{C} C'}{A^2 C^2} - \frac{8 \dot{A} \dot{C} C''}{A^3 C^2} + \frac{32 A' \dot{C} C'}{A^3 C^2} \right. \\
 \left. - 4 B \dddot{B} \dddot{C} + \frac{8 \dddot{C} C'' C''}{A^2 B C^2} - \frac{4 B \dddot{B} \dddot{A}}{A^2 C^2} + \frac{4 A''}{A^2 C^2} - \frac{8 \dot{A} \dot{C} C''}{A^3 C^2} - 24 \right. \\
 \left. \frac{A' \dot{B} \dot{C} C'}{A^3 B C^2} + \frac{8 \dot{A} \dot{C} B' C'}{A^3 B C^2} - \frac{4 B B'}{A^2 C^2} + \frac{4 A''}{A^2 C^2} - \frac{16 A'' C'}{A^4 C^2} + \frac{12 A' B' C'^2}{A^3 B C^2} \right. \\
 \left. - \frac{4 A' B' \dddot{C} C'}{A^3 B C^2} + \frac{4 \dddot{A} \dddot{B} C'^2}{A^3 B C^2} \right) \dot{f}_G \left( \frac{8 B^2 \dddot{C} C'' C''}{A^4 C^2} - \frac{12 B^2 \dddot{A} \dddot{C}}{A^5 C^2} + \frac{4 \dddot{A} \dddot{C} C'^2}{A^5 C^2} \right) \dot{f}_G \left( \frac{8 \dddot{C}}{A^3} \right) \\
 \left. \times \frac{A C''}{C^2} - \frac{4 \dddot{A} \dddot{C} C'^2}{A^3 C^2} - \frac{4 A'}{A C^2} + \frac{12 C'' A'}{A B^2 C^2} - \frac{\dddot{C} C''}{A^2 C^2} \right] \right] \\
 \left( A^3 \right) \\
 \] \\
 \[ T^{(GT)}_{22} = \frac{1}{8\pi} \left[ \frac{2}{3} \Pi f_T + \frac{C^2}{2} f \left( \frac{-4 \dot{C} C}{A^2 B^2} + \frac{4 A'}{A B^2} - \frac{4 \dot{B}}{A^2 B} + \frac{8 \dot{A} \dot{C} B' C''}{A^3 B^3} - 4 \right) \times \right. \\
 \left. \frac{A' \dot{B} \dot{C} C'}{A^3 B^2} + \frac{4 \dddot{B} C'^2}{A^2 B^3} - \frac{8 \dddot{C} C'' C''}{A^3 B^2} + \frac{8 \dddot{A} \dddot{C} C''}{A^3 B^2} + \frac{8 \dddot{C} C C''}{A^3 B^2} - \frac{4 \dddot{B} C'^2}{A^2 B^4} \right) \dot{f}_G + \\
 \left. + \left( \frac{4 \dddot{B} C C}{A B} - \frac{12 \dddot{A} \dddot{B} C C}{A^3 B^3} - \frac{4 \dddot{A} \dddot{B} C C''}{A^3 B^3} + \frac{8 \dddot{A} \dddot{C} C' B}{A^3 B^3} + \frac{4 \dddot{A} \dddot{B} \dot{C} C}{A^3 B^3} - 4 \right) \right] \\
 \left. \times \frac{A'' \dot{C} C}{B^2} + \frac{4 \dddot{B} \dddot{C} C}{A B} - \frac{12 A' C'' C''}{A^3 B^3} + \frac{4 \dddot{A} \dddot{C} C''}{A^3 B^3} + \frac{12 \dddot{C} \dddot{C} A^2}{A^4 B^2} \right) \dot{f}_G + \left( 8 \right) \\
 \left. \times \frac{A' \dddot{B} \dddot{C} C}{A^3 B^3} + \frac{4 \dddot{B} C'C' C'}{A^3 B^3} + \frac{4 \dddot{A} \dddot{B} \dddot{C} C'}{A^3 B^3} - \frac{12 A' \dddot{B} \dddot{C} C'}{A^3 B^3} - \frac{4 \dddot{B} \dddot{C} \dddot{C} C'}{B^3 A^3} + \frac{4}{B^4} \right) \\
 \left. \times \frac{A'' \dddot{C} C'}{A} + \frac{8 \dddot{B} \dddot{C} C'}{A^2 B^3} - \frac{8 \dddot{B} \dddot{C} C'}{B^3 A^2} - \frac{12 \dddot{B} C\ddot{C} C'}{A^3 B^2} + \frac{4 \dddot{A'} \dddot{C} C''}{A^3 B^2} + \frac{12 A^2}{A^2} \right] \\
 \]}
\[
\times \frac{CC'}{B^3} \right) f_G' + \left( \frac{4\dot{A}^2CC}{A^3B^2} + \frac{4\dot{A}'C'C}{AB^4} - \frac{4\dot{C}C}{A^2B^2} \right) f_G'' + \left( \frac{12CC'}{A^2B^2} - \frac{12C}{A^2} \right)
\]
\[
\times \frac{\dot{B}C'}{B^3} - \frac{12A'C'C}{A^3B^3} \right) f_G' + \left( \frac{4\dot{B}^2CC}{A^4B} + \frac{4B'C'C}{A^2B^3} - \frac{4CC''}{A^2B^2} \right) f_G'.
\]

The expressions \( Z_1 \) and \( Z_2 \) come out to be
\[
Z_1 = \frac{f_T}{k^2 - f_T} \left[ \left( \frac{\mu}{A} + \frac{T^{00(GT)}}{A^3} \right) \right] (\ln f_T)' + 2 \left( \frac{q}{B} - \frac{T^{01(GT)}}{AB^2} \right) + \frac{1}{2A}
\]
\[
\times (\mu + 3P)' + \left( \frac{q}{B} - \frac{T^{01(GT)}}{AB^2} \right) (\ln f_T)' + 2 \left( \frac{2\mu}{A} + \frac{2T^{00(GT)}}{A^3} + \frac{P}{A} \right) \right],
\]
\[
Z_2 = \frac{f_T}{k^2 - f_T} \left[ \left( -\frac{P_r}{B} + \frac{T^{11(GT)}}{B^3} + \frac{P}{B} \right) (\ln f_T)' + 2 \left( -\frac{q}{A} + \frac{T^{01(GT)}}{A^2B} \right) - \frac{1}{2}
\]
\[
\times (\ln f_T)' \right].
\]

**Appendix B**

The correction terms in the structure scalars are evaluated as
\[
M^{(GT)} = 2 \left[ R_{\mu\beta} R_{\alpha}^\mu f_G + R_{\mu\nu} R_{\mu\beta\nu\alpha} f_G - \frac{1}{2} R R_{\alpha\beta} f_G - \frac{1}{2} R_{\beta\mu\nu} R_{\alpha}^\mu f_G \right]
\]
\[
+ \frac{1}{2} R \nabla_\alpha \nabla_\beta f_G + R_{\alpha\beta} \Box f_G - R_{\alpha}^\mu \nabla_\beta \nabla_\mu f_G - R_{\beta}^\mu \nabla_\alpha \nabla_\mu f_G
\]
\[
- \ R_{\beta\mu\nu} \nabla^\mu \nabla^\nu f_G \right] g^{\alpha\beta} + 2 \left[ -R_{\mu\beta} R_{\alpha}^\mu f_G - R_{\mu\nu} R_{\mu\beta\alpha} f_G + \frac{1}{2} R R_{\alpha\delta} f_G \right]
\]
\[
+ \frac{1}{2} R_{\beta\mu\nu} R_{\alpha}^\mu f_G - R_{\alpha\beta} \Box f_G + R_{\beta\mu\nu} \nabla^\mu \nabla^\nu f_G + R_{\alpha}^\mu \nabla_\delta \nabla_\mu f_G
\]
\[
- \frac{1}{2} R \nabla_\alpha \nabla_\beta f_G + R_{\delta}^\mu \nabla_\alpha \nabla_\mu f_G \right] V_\beta V_\delta g^{\alpha\beta} + 2 \left[ -R_{\mu\beta} R^{\mu\gamma} f_G \right]
\]
\[
+ \frac{1}{2} R_{\beta\mu\nu} R^{\mu\nu\gamma} f_G - \frac{1}{2} R R_{\beta}^\gamma f_G R_{\mu\beta\nu} f_G + R^{\mu\nu} \nabla_\beta \nabla_\mu f_G
\]
\[
- \frac{1}{2} R \nabla_\gamma \nabla_\beta f_G + R_{\beta\mu\nu} \nabla^\mu \nabla^\nu f_G - R_{\beta}^\gamma \Box f_G + R_{\beta}^\gamma \nabla_\gamma \nabla_\mu f_G \right] V_\alpha V_\gamma g^{\alpha\beta}
\]

24
\[ J^{(GT)}_{(\alpha \beta)} = [2 R_{\mu d} R^\mu_{\nu} f_{G} + 2 R^{\mu \nu} R^{\gamma}_{\mu \nu \rho \sigma} f_{G} - R R_{\epsilon d} f_{G} - R_{\delta \mu \nu m} R^{\mu n} f_{G} + 2 R_{\epsilon d} \Box f_{G} + R \nabla_{\epsilon} \nabla_{\sigma} f_{G} - 2 R^{\mu}_{\epsilon d} \nabla_{\delta} \nabla_{\epsilon} f_{G} - 2 R^{\mu \nu} \nabla_{\mu} \nabla_{\nu} f_{G} - 2 R^{\mu \nu} \nabla_{\mu} \nabla_{\nu} f_{G} - 8 \nabla^{\mu} \nabla_{\mu} f_{G} - 6 R \Box f_{G}, \]

\[ + 2 \left[ R_{\mu \delta} R^{\mu \gamma} f_{G} + R^{\mu \nu} R_{\mu \epsilon \nu \rho \delta} f_{G} - \frac{1}{2} R_{\delta \mu \nu m} R^{\mu n} f_{G} + \frac{1}{2} R \nabla^{\gamma} \nabla_{\delta} f_{G} \right] \]

\[ \times g_{\alpha \beta} V_{\gamma} V^{\delta} g_{\alpha \beta} - 2 R^{\mu \nu} R_{\mu \nu \rho \sigma} f_{G} + 4 R^{\mu \nu} R^{\rho}_{\mu \nu \sigma} f_{G} - 2 R^{\mu \nu} \nabla_{\mu} \nabla_{\nu} f_{G} - 2 R^{\mu \nu} \nabla_{\mu} \nabla_{\nu} f_{G} - 4 R^{\mu \nu} \nabla_{\mu} \nabla_{\nu} f_{G} - 4 R^{\mu \nu} \nabla_{\mu} \nabla_{\nu} f_{G} - \frac{1}{2} f + 12 R^{\mu \nu} \nabla_{\mu} \nabla_{\nu} f_{G} - 6 R \Box f_{G}, \]
\[Q^{(GT)} = \left[ \frac{1}{2} R_{\mu
u} R^{\mu\nu}_G + \frac{1}{2} R^{\mu\nu} R_{\mu\nu} f_G - \frac{1}{4} R R^p \delta_{fg} - \frac{1}{4} R_{\mu\nu\rho} R^{\mu\nu\rho}_G \right. \\
+ \left. \frac{1}{2} R^p R^p \delta_{fg} - \frac{1}{4} R_{\mu\nu} \nabla^\mu \nabla^\nu f_G - \frac{1}{4} R^p \nabla^\mu \nabla^\nu f_G - \frac{1}{2} R^p \nabla^\mu \nabla^\nu f_G \right. \\
- \frac{1}{2} R_{\mu\nu} \nabla^\mu \nabla^\nu f_G \right] g^{\alpha\beta} \varepsilon_{\delta\beta} \varepsilon_{\gamma}^{\alpha} + \left[ -\frac{1}{2} R_{\mu\delta} R^{\mu\gamma}_G - \frac{1}{2} R^{\mu\nu} R^{\mu\nu}_G \right. \\
+ \frac{1}{4} R R^p \delta_{fg} - \frac{1}{4} R_{\delta\mu\nu} R^{\mu\nu\gamma}_G - \frac{1}{2} R^p R^p \delta_{fg} - \frac{1}{4} R \nabla^\mu \nabla^\nu f_G \\
+ \frac{1}{4} R_{\delta\mu\nu} R^{\mu\nu\gamma}_G f_G + \frac{1}{2} R^p R^p \nabla^\mu \nabla^\nu f_G + \frac{1}{4} R^p R^p \nabla^\mu \nabla^\nu f_G \right] g^{\alpha\beta} \varepsilon_{\delta\beta} \varepsilon_{\gamma}^{\alpha} \\
- \left. \frac{1}{2} R_{\mu\delta} R^{\mu\gamma}_G - \frac{1}{2} R^{\mu\nu} R^{\mu\nu}_G + \frac{1}{4} R R^p \delta_{fg} + \frac{1}{4} R_{\delta\mu\nu} R^{\mu\nu\gamma}_G f_G \right. \\
+ \frac{1}{2} R^p R^p \delta_{fg} - \frac{1}{4} R_{\delta\mu\nu} R^{\mu\nu\gamma}_G f_G + \frac{1}{2} R^p R^p \delta_{fg} + \frac{1}{4} R \nabla^\mu \nabla^\nu f_G \\
- \frac{1}{4} R^{\mu\nu} \nabla^\beta \nabla^\gamma f_G - \frac{1}{2} R^p \nabla^\mu \nabla^\nu f_G - \frac{1}{2} R^p \nabla^\mu \nabla^\nu f_G \right] g^{\alpha\beta} \varepsilon_{\delta\beta} \varepsilon_{\gamma}^{\alpha} \\
- 12R^{\mu\nu} \nabla^\mu \nabla^\nu f_G + 6R \nabla^\mu f_G + [(\mu + P) f_T + 4R^{\mu\nu} R_{\mu\nu} f_G \\
+ 4R^{\mu\nu} R_{\mu\nu} \nabla^\mu \nabla^\nu f_G - 2R^2 f_G - 2R^2 \nabla^\mu \nabla^\nu f_G - 4R \nabla^\mu f_G \\
+ 16R^{\mu\nu} \nabla^\mu \nabla^\nu f_G - 4R^{\mu\nu} \nabla^\mu \nabla^\nu f_G - 4R^2 \nabla^\mu \nabla^\nu f_G \\
- 4R^m_{\mu\nu} \nabla^\mu \nabla^\nu f_G \right] + \frac{1}{2} f.
\]

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