Measuring the transmission of a quantum dot using Aharonov-Bohm Interferometers

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The conductance $G$ through a \textit{closed} Aharonov-Bohm mesoscopic solid-state interferometer (which conserves the electron current), with a quantum dot (QD) on one of the paths, depends only on $\cos \phi$, where $\Phi = \hbar c \phi / e$ is the magnetic flux through the ring. The absence of a phase shift in the $\phi-$dependence led to the conclusion that closed interferometers do not yield the phase of the “intrinsic” transmission amplitude $t_D = |t_D| e^{i \alpha}$ through the QD, and led to studies of \textit{open} interferometers. Here we show that (a) for single channel leads, $\alpha$ can be deduced from $|t_D|$, with no need for interferometry; (b) the explicit dependence of $G(\phi)$ on $\cos \phi$ (in the closed case) allows a determination of both $|t_D|$ and $\alpha$; (c) in the open case, results depend on the details of the opening, but optimization of these details can yield the two-slit conditions which relate the measured phase shift to $\alpha$.

I. INTRODUCTION

Recent advances in nanoscience raised much interest in quantum dots (QDs), which represent artificial atoms with experimentally controllable properties \cite{1}. The quantum nature of the QD is reflected by resonant tunneling through it, as measured when the QD is connected via metallic leads to electron reservoirs. The measured conductance $G$ shows peaks whenever the Fermi energy of the electrons crosses a resonance on the QD. Experimentally, the energies of these resonances are varied by controlling the plunger gate voltage on the QD, $V$. Quantum mechanically, the information on the tunneling is contained in the complex transmission amplitude, $t_D = \sqrt{T_D e^{i \alpha}}$. It is thus of great interest to measure both the magnitude $T_D$ and the phase $\alpha$, and study their dependence on $V$. Although the former can be deduced from measuring $G$, via the Landauer formula \cite{2}, $G = \frac{2 e^2}{h} T$, experimental information on the latter has only become accessible since 1995 \cite{3}, using the Aharonov-Bohm (AB) interferometer \cite{4}.

In the AB interferometer, an incoming electronic waveguide is split into two branches, which join again into the outgoing waveguide. Aharonov and Bohm \cite{5} predicted that a magnetic flux $\Phi$ through the ring would add a difference $\phi = e \Phi / h c$ between the phases of the wave functions in the two branches of the ring, yielding a periodic dependence of the overall transmission $T$ on $\phi$. Placing a QD on one of the branches, as in Fig. 1a, and using the other path as a “reference path”, with a transmission amplitude $t_B$, one expects $T$ also to depend on the “\textit{intrinsic}” amplitude $t_D$. In the two-slit limit, one has $T = |t_D e^{i \phi} + t_B|^2 = A + B \cos(\phi + \beta)$, with $\beta = \alpha + \kappa$, where the reference phase $\kappa$ is independent of the QD parameters, and thus set at zero. However, for the “closed” two-terminal geometry, as shown in Fig. 1a and used by Yacoby \textit{et al.} \cite{6}, the two-slit expectation that $\beta = \alpha$ was clearly wrong: Unitarity (conservation of current) and time reversal symmetry imply the Onsager relations \cite{7}, which state that $G(\phi) = G(-\phi)$, and therefore $\beta$ \textit{must} be equal to zero or $\pi$. Indeed, a fit of the experimental data \cite{4} to the above two-slit formula, with $B > 0$, gives a phase shift $\beta$ which “jumps” from 0 to $\pi$ near each resonance of the QD, and then exhibits an a priori unexpected abrupt “phase lapse” back to 0, between every pair of resonances.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1}
\caption{Model for the AB interferometer: (a) Closed two-terminal case, (b) schematic picture of the six-terminal open interferometer, (c) model for the open interferometer.}
\end{figure}
Aiming to measure a non-trivial AB phase shift $\beta$ then led to experiments with six-terminal “open” interferometers (Fig. 1b) [10], where the additional four “leaky” channels lead to absorbing reservoirs. These interferometers break the Onsager symmetry, and yield a non-trivial phase shift $\beta$ which increases gradually from zero to $\pi$ through each resonance, and then jumps back to zero between resonances. Much of the early literature assumed that the measured $\beta$ is equal to the “intrinsic” $\alpha$. Recently [11] we showed that this assumption is not necessarily always valid: the detailed $V$-dependence of $\beta$ depends on the strength of the coupling to the additional terminals. Thus, we posed the challenge of finding clear criteria as to when the measured $\beta$ equals $\alpha$.

In the present paper we review three aspects of this problem. In Sec. 2 we show that in some cases one has $T_D = \gamma_0^2 \sin^2 \alpha$, where $\gamma_0$ is a constant measuring the asymmetry of the QD. In these cases, $\alpha$ is determined by $T_D$, and there is no need to build special interferometers to measure $\alpha$. We also show that in many cases the detailed flux-dependence of the measured conductance of the closed interferometer contains information which allows the deduction of both $T_D$ and $\alpha$, eliminating the need to open the interferometer. In Sec. 3 we then review recent work [12] in which we showed that an opening like that shown in Fig. 1c, with “forks” of “lossy” channels on each segment of the AB ring, can be tuned so that one reproduces the two-slit conditions. In those cases, one indeed has $\beta = \alpha$. However, this tuning requires some optimization of the parameters, and cannot be guaranteed for an arbitrary open interferometer.

II. CLOSED AB INTERFEROMETER

A. Model with one QD resonance

Our model is shown in Fig. 2: We start with an isolated quantum dot, with a single localized level (of single particle energy $\epsilon_D$, representing the gate voltage $V$) and on-site Hubbard interaction $U$, and with a one-dimensional tight-binding chain, with sites at integer coordinates, with zero on-site energies and with nearest-neighbor (nn) hopping matrix elements equal to $-J$. We next connect the dot to the sites $\pm 1$ on the chain, via matrix elements $-J_L$ to the left, and $-J_R$ to the right, and also modify the lower branch of the resulting triangle: site 0 becomes the “reference” site, with on-site energy $\epsilon_0$, with no interactions and with nn hopping energies $-I$ (replacing the original $-J$). Experimentally, the reference site can represent a simple point contact, tunnel junction, etc. The triangle so formed contains an Aharonov-Bohm phase, $\phi = \phi_L + \phi_R$, where (for simplicity), the phase $\phi_L$ ($\phi_R$) is attached to the bond with the hopping matrix element $J_L$ ($J_R$) (we choose $J$, $I$, $J_L$ and $J_R$ to be real). Hence, the Hamiltonian of the system reads

$$
\mathcal{H} = \mathcal{H}_D + \sum_{k \sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \epsilon_0 \sum_{\sigma} c_{0\sigma}^\dagger c_{0\sigma} \\
+ (I/J - 1) \sum_k \epsilon_k \left( c_{k\sigma}^\dagger c_{0\sigma} + c_{0\sigma}^\dagger c_{k\sigma} \right) / \sqrt{N} \\
+ \sum_{k \sigma} \left( V_L d_{sL}^\dagger c_{k\sigma} + V_R d_{sR}^\dagger c_{k\sigma} \right),
$$

(1)

where the operator $c_{k\sigma}^\dagger$ creates single particle eigenstates (with spin $\sigma$) on the unperturbed chain (with $I = J, J_L = J_R = 0$), with eigenenergy $\epsilon_k = -2J \cos k$, while $c_{0\sigma} = \sum_k c_{k\sigma} / \sqrt{N}$, and $V_L = -(J_L e^{i\phi_L} - iK + J_R e^{-i\phi_R}) / \sqrt{N}$. The operators on the dot are denoted by $d_{sL}$ and $d_{sR}^\dagger$, and they anti-commute with the band operators $c_{k\sigma}, c_{k\sigma}^\dagger$. The dot Hamiltonian is

$$
\mathcal{H}_D = \epsilon_D \sum_{\sigma} d_{\sigma}^\dagger d_{\sigma} + \frac{1}{2} U \sum_{\sigma} n_{\sigma}\sigma n_{\sigma}. 
$$

(2)

with $n_{\sigma}\sigma = d_{\sigma}^\dagger d_{\sigma}$, and $\sigma$ denotes the spin opposite to $\sigma$.

The Hamiltonian (1) is a simple generalization of that used by Ng and Lee [13], to which we added the reference path.

![Model for the closed interferometer.](image)

B. Transmission

For simplicity, we discuss only zero temperature, so that $\epsilon_0$ is set equal to the Fermi energy in the leads. The $2 \times 2$ scattering matrix is easily related to the matrix of retarded single-particle Green functions, $G_{kk'}^\sigma(\omega)$, evaluated on the energy shell, $\omega = \epsilon_k$. [13] We use the equation-of-motion method to express $G_{kk'}^\sigma(\omega)$ and $G_{dd'}^\sigma(\omega)$ in terms of the Green function on the dot, $G_{dd''}^\sigma(\omega)$ [14]. The equation-of-motion for the latter has the form

$$
(\omega - \epsilon_d)G_{dd''}^\sigma(\omega) = 1 + \sum_k V_k G_{kd''}^\sigma(\omega) + U \Gamma_{dd'd''}^\sigma(\omega).
$$

(3)

The last term on the RHS represents the effect of the interactions, with $\Gamma_{dd'd''}^\sigma(\omega)$ being the temporal Fourier
transform of $\Gamma_{\text{dd},d}(t) = -i\mathcal{O}(t)\{[d^\dagger_d(t)d_d(t), d^\dagger_d(t)]\}$. The second term on the RHS of Eq. (3) can be written as $\sum_k V_k G_{\text{dd},k}^\sigma(\omega) = -A(\omega)G_{\text{dd},k}^\sigma(\omega)$, implying that $-A(\omega)$ contains the full contribution of the non-interacting Hamiltonian to the self-energy on the dot. Thus we write

$$G_{\text{dd},k}^\sigma(\omega)^{-1} = \omega - \epsilon_D - \Sigma_{\text{int}}^\sigma(\omega) + A(\omega),$$

where $\Sigma_{\text{int}}^\sigma(\omega)$ represents the self energy due to the interactions, which vanishes when $U = 0$. At $\omega = \epsilon_k$ we have

$$A(\epsilon_k) = e^{i|k|J_F^2 + J_B^2}[1 - \frac{t_B e^{i|k|}}{2t_B |k|}(1 + \gamma_D \cos \phi)] = \frac{j_F^2 + J_B^2}{\epsilon_k + |k|} + \sin |k|(Z + iY),$$

where the $\phi$-dependent quantities $Y$ and $Z$ are defined in this equation, while $\gamma_D = 2Jt_F/(J_F^2 + J_B^2)$ is the asymmetry factor for the dot, and

$$t_B = -i \sin \delta_B e^{i\delta_B} = 2t_B \sin |k|/(J + 2V_B e^{i|k|})$$

is the transmission amplitude of the “background” path (when $J_F = J_R = 0$, with the effective hopping energy $V_B = I^2/(\epsilon_k - \epsilon_0)$).

The other equations of motion [13] then yield the other Green functions, and we end up with

$$t^\sigma = t_B G_{\text{dd},k}^\sigma(\epsilon_k) \{\frac{1}{V_B} e^{i\phi} + G_{\text{dd},k}^\sigma(\epsilon_k)^{-1} - A(\epsilon_k)\}$$

$$= t_B G_{\text{dd},k}^\sigma(\epsilon_k) \{\frac{1}{V_B} e^{i\phi} + \epsilon_k - \epsilon_D - \Sigma_{\text{int}}^\sigma(\epsilon_k)\}. \quad (7)$$

Equation (5) is one of our main results. It expresses $t^\sigma$ in terms of the fully dressed single particle QD Green function, which depends on both paths of the interferometer and therefore also on the AB phase $\phi$. The remaining discussion aims to see if one can extract information on the “intrinsic” QD transmission from measurements of $T^\sigma = |t^\sigma|^2$.

The Onsager relations require that the conductance, and therefore also $T^\sigma = |t^\sigma|^2$, must be an even function of $\phi$. It is clear from Eq. (5) that this holds only if

$$\Im[G_{\text{dd},k}^\sigma(\epsilon_k)^{-1} - A(\epsilon_k)]$$

$$= \Im[\epsilon_k - \epsilon_D - \Sigma_{\text{int}}^\sigma(\epsilon_k)] \equiv 0. \quad (8)$$

Indeed, we found the same condition to follow from the unitarity of the scattering matrix. The same sort of relation appears for the single particle scattering, in connection with the Friedel sum rule [10]. Equation (8) implies that the interaction self-energy $\Sigma_{\text{int}}^\sigma(\epsilon_k)$ is real, and is an even function of $\phi$. It also implies that $\Im[G_{\text{dd},k}^\sigma(\epsilon_k)]^{-1}$ is fully determined by the non-interacting self-energy $\Im A(\epsilon_k)$.

It is now convenient to rewrite $G_{\text{dd},k}^\sigma(\epsilon_k)$ in terms of its phase, $\delta$. Writing $[G_{\text{dd},k}^\sigma(\epsilon_k) \sin |k|(J_F^2 + J_B^2)/J]^{-1} = (1 + Y)(i - \cot \delta)$, we find

$$T^\sigma = |t^\sigma|^2 = \frac{\sin^2 \phi}{(1 + Y)^2} [\tilde{\gamma}_D^2 \sin^2 \phi + (\tilde{\gamma}_D \cos \phi - \sqrt{TB}((1 + Y) \cot \delta + \cot |k| + Z)]^2, \quad (9)$$

with $\tilde{\gamma}_D = \gamma_D J/|J + 2V_B e^{i|k|}| = \gamma_D |\sin(\delta_D + |k|)/|k||$ and $TB = |t_B|^2$. Interestingly, the second term in the square brackets is of the Fano form [17]. At $\phi = 0$, it reflects the possibility for a complete destructive interference, with $T^\sigma = 0$. A similar expression for $T^\sigma$ was derived by Hofstetter et al [18], but their approximations ignore the explicit dependence of some parameters (e.g. $Y$) on $\phi$.

When one cuts off the reference path, $V_B = 0$, Eq. (4) reduces to the “intrinsic” QD transmission amplitude, $t_D^\sigma = -i\gamma_D \sin \alpha e^{\alpha}$, with

$$-\cot \alpha = \cot |k| + \frac{\epsilon_D - \Sigma_{\text{int}}^\sigma(\epsilon_k)}{\sin |k|(J_F^2 + J_B^2)/J}, \quad (10)$$

where $\Sigma_{\text{int}}^\sigma(\epsilon_k) = \Sigma_{\text{int}}^\sigma(\epsilon_k)/|V_B|=0$. It is interesting to note that for our single-channel (one-dimensional) leads, we have $TB = \gamma_D^2 \sin^2 \alpha$ (as already noted by Ng and Lee [13]). Since $\gamma_D$ does not depend on the energy $\epsilon_k$ or on the gate voltage $V = \epsilon_D$, it follows that a measurement of $T_D$ immediately also yields the phase $\alpha$, via $\sin \alpha = \sqrt{T_D}/\max(T_D)!$ In many cases, the measurement of $T_D$ already yields $\alpha$, eliminating the need to perform complicated interferometer measurements. It would be very interesting to test this, for cases where $\alpha$ is measured independently (e.g. with an open interferometer, see below).

The phase $\delta$ of $G_{\text{dd},k}^\sigma(\epsilon_k)$ is now given by

$$\cot \delta = (\cot \alpha - Z + X)/(1 + Y), \quad (11)$$

where $X = (\Sigma_{\text{int}}^\sigma(\epsilon_k) - \Sigma_{\text{int}}^\sigma(\epsilon_k))/|\sin |k|(J_F^2 + J_B^2)/J|$ contains only the effects of the reference path on the interaction self-energy of the dot. One way to proceed is to calculate $X$, e.g. using a perturbative expansion in $V_B$ or using a full solution of the interacting case. Even without calculating $X$, we expect it to be an even function of $\phi$. Although $\alpha$ does not depend on $\phi$, $\delta$ usually depends on $\phi$ (via $X$, $Y$ and $Z$).

Equations (5),(6),(9) represent our second main result: they relate the $\phi$-dependence of the measured $T^\sigma$ with the QD parameters. We are not aware of earlier discussions which separate between the roles played by $\alpha$ and $\delta$.

C. Possible Measurements

We now discuss a few limits in which a measurement of $T^\sigma$, Eq. (11), can yield information on the “intrinsic” phase $\alpha$ and thus on the full “intrinsic” transmission $t_D$. First, consider a relatively open dot, with small barriers at its contacts with the leads. In such circumstances, the
electron wave function spreads over the leads, and it has only a small amplitude on the dot itself, thus reducing the effects of interactions. This, and the related better screening [19] imply that $|X| \ll |Z|$. In this limit, Eq. (11) (with $X = 0$) gives the detailed dependence of $\delta$ on $\phi$. Equation (11) then has the form

$$
\mathcal{T} \equiv |t|^2 = A \left| \sum_{\nu} \frac{e^{i \phi + K}}{1 + \frac{2K}{1 + 2i\nu} \cos \phi + \frac{1}{2} \nu^2 \cos^2 \phi} \right|^2
$$

(12)

with coefficients which depend on $\gamma_D, \delta_B, |k|$ and $\alpha$. For the closed interferometer, $K$ is real. The $\cos \phi$ in the denominator, which results from the self energy in the dot Green function, is due to interference within the ring, between clockwise and counterclockwise motions of the electron. In the limit of small $V_B$ one has $z \propto t_D$. Fitting data to Eq. (12) would confirm that one is in this non-interacting limit, and would yield the $V$-dependence of the “intrinsic” phase $\alpha$ (as well as the $V$-independent parameters $\gamma_D, \delta_B$ and $|k|$). Note that a fit to the explicit $\phi$-dependence in Eq. (12) is much preferred over a fit to a harmonic expansion of the form $\mathcal{T} = \sum a_n \cos n \phi$. Figure 3 shows an example of the $V$- and $\phi$-dependence of $\mathcal{T}$ for this limit. The denominator in Eq. (12), which contains the information on $\alpha$ via $z$ (and thus determines the higher harmonics in $\mathcal{T}$), is mostly visible near the resonance at $V = 0$. Far away from the resonance this denominator is small, and $\mathcal{T}$ can be approximated by $A + B \cos(\phi + \beta)$, where $\beta$ is zero or $\pi$ below or above the point $V = 0$.

Second, note that all the $\phi$-dependent functions $X, Y$ and $Z$ become small when $V_B$ (and therefore $\delta_B \sim V_B/J$) is small. Except very close to the QD resonance, where $\alpha = \pi/2$, we conclude that $\delta = \alpha + O(\delta_B)$. Thus, at least away from resonances one can study the transmission for several values of $V_B$ (which can be varied via the point contact voltage $e_0$), estimate $\delta(V)$ (as some average over $\phi$) for each $V_B$ and extrapolate $V_B$ to zero to obtain $\alpha$, without explicit knowledge of $X(\phi)$.

The situation becomes even more interesting in the Kondo regime, when $\alpha$ and/or $\delta$ remain close to $\pi/2$ over a wide range of $V$. Hofstetter et al. [18] assumed that $\delta = \pi/2$, and deduced the $\phi$-dependence of $\mathcal{T}^\sigma$. As seen from Eq. (11), one cannot have $\delta = \pi/2$ without neglecting the $\phi$-dependence of $X, Y$ and $Z$. Alternatively, if the Kondo condition implies that $\alpha = \pi/2$ without neglecting the $\phi$-dependence of $X, Y$ and $Z$. For larger $V_B$, a resonance on the QD (i.e. $\alpha = \pi/2$) may result in significant deviation of $\delta$ from $\pi/2$. The question whether this deviation relates to the variety of plateau values observed by Ji et al. [11] deserves further study.

In any case, the above discussion demonstrates that although a measurement of the transmission of the closed AB interferometer does not yield a non-trivial AB phase shift $\beta$, the data still contain much information on the properties of the “intrinsic” QD.

III. OPEN AB INTERFEROMETER

A. Model for multi-resonances

We next discuss the conditions for obtaining the equality $\beta = \alpha$ in an open AB interferometer. In principle, we could repeat the above discussion for the open case; as seen below, this simply amounts to replacing each “lossy” channel by a complex self-energy at its “base”, and then proceeding as in the closed case. However, for the present purpose it suffices to discuss an effectively non-interacting case. [12] Obviously, if one cannot achieve this aim in that case then there will be problems also in the more complicated interacting cases. We thus restrict this discussion to an approximate treatment of the Coulomb blockade (and not the Kondo) region, and treat the interaction term in the Hartree approximation, replacing $n_{dL} n_{R}$ by $(n_d \sigma) n_{dR}$. We further simplify this by replacing $(n_d \sigma)$ by a constant which increases for consecutive resonances. Thus, we replace the single QD by a set of smaller dots, each containing a single resonant state, with energy $\epsilon_D = E_D(n) \equiv V + U(n - 1) \equiv V + U(25)$. Each such state is connected to its left and right nearest neighbors (nn’s) on the leads (denoted by $L$ and $R$) via
bonds with hopping amplitudes \{-J_L(n), -J_R(n), n = 1, ..., N\} (see Fig. 4). The problem now reduces to a simple tight-binding model, and for the “intrinsic” QD we find

$$t_D = \frac{S_{LR} 2i \sin k}{(S_{LL} + e^{-ik})(S_{RR} + e^{-ik}) - |S_{LR}|^2},$$  \hspace{1cm} (13)$$

where $$S_{XY} = \sum_n J_X(n)J_Y(n)^*/[\epsilon_k - E_R(n)]/J$$, $$X, Y = L, R$$.

![FIG. 4. Model for a QD with four resonances (to replace the single dot in Fig. 2).](image)

Figure 5 shows typical results for the “intrinsic” transmission $$T_D$$ and phase $$\alpha$$, where the zero of $$\alpha$$ is set at its ($$k$$-dependent) value at large negative $$V$$. Here and below, we choose $$k = \pi/2$$, so that $$\epsilon_k = 0$$ and the resonances of the transmission, where $$T_D = 1$$, occur exactly when $$E_R(n) = \epsilon_k = 0$$, i.e. when $$V = -U(n - 1)$$. We also use the simple symmetric case, $$J_L(n) = J_R(n) \equiv J$$, and measure all energies in units of $$J$$. Interestingly, this model reproduces the behavior apparently observed by Schuster et al. [5]: $$\alpha$$ grows smoothly from 0 to $$\pi$$ as $$\epsilon_k$$ crosses $$E_R(n)$$, and exhibits a sharp “phase lapse” from $$\pi$$ to 0 between neighboring resonances, at points where $$T_D = 0$$. These latter points, associated with zeroes of $$S_{LR}$$, represent Fano-like destructive interference between the states on the QD [17,20,21]. These zeroes in $$T$$ are missed, and the corresponding “phase lapses” are superfluously smeared, when one replaces the exact Eq. (13) by a sum of Breit-Wigner resonances [22].

**B. Model for the open AB interferometer**

Placing the above QD model on one branch of the closed AB interferometer, one can easily solve for $$T$$. In the absence of interactions, we again find the form (12), which is thus also valid for many resonances. Near each resonance, results are similar to those shown in Fig. 3.

As expected, Eq. (12) is still an even function of $$\phi$$, with no phase shift $$\beta$$ (except for the apparent jumps between zero and $$\pi$$).

Before we discuss the open interferometer, it is useful to understand the criteria for having the two-slit situation. A crucial condition for having $$t = t_D e^{i\phi} + t_B$$ is that the electron go through each branch only once. Equivalently, there should be no reflections from the “forks”, which connect the ring with the external leads, back into the ring [11]. We achieve this by the construction shown in Fig. 1c: each of the four “lossy” channels in Fig. 1b is now replaced by a “comb” of $$M$$ channels. An analysis of each such “comb” shows that (a) the transmission $$T$$ and reflection $$R$$ through the “comb” is only weakly dependent on the energy $$\epsilon_k$$ near the band center $$k = \pi/2$$ (Fig. 6a) and (b) the transmission $$T$$ decreases and the reflection $$R$$ increases with the coupling of each “lossy” channel in the “comb” to its “base”, $$J_x$$ (Fig. 6b) and
with the number \( M \). Note that \( T + R < 1 \), due to the loss into the “teeth” of the “comb”. Ideally, one would like to have \( T, R \ll 1 \), so that in practice the electron crosses each “comb” only once, and is not reflected from the “comb” back into the QD. Given Fig. 6b, we expect to have optimal two-slit conditions for intermediate values of \( J_x \), e.g. \( J_x \sim 0.9J \) for \( M = 6 \).

We next present results for the open interferometer shown in Fig. 1c. The algebra is similar to that for the closed case: each “tooth” of the “combs” can be eliminated from the equations, at the cost of adding a self energy on the site of its “base”, equal to \( J_x^2 e^{ik} / J \). The result is again of the form of the first expression in Eq. (12), except that now \( K \) becomes complex and therefore the numerator in the second expression contains \( \cos(\phi + \gamma) \) instead of \( \cos \phi \). To demonstrate qualitative results, we chose four identical “combs”, with \( M = 6 \) “teeth” on each and with the same hopping matrix elements; all the hopping energies were set to \(-J\), except for the bonds connecting each “tooth” to its base, denoted \(-J_x\). Figure 7 shows exact results for \( A, B, C \) and \( \beta \) in fits of \( T \) to the form

\[
T = A + B \cos(\phi + \beta) + C \cos(2\phi + \gamma) + \ldots ,
\]

FIG. 6. Transmission (thick line) and reflection (thin line) through a “comb”, (a) versus \( k \) at \( J_x = .7J \) and (b) versus \( J_x \) (in units of \( J \)) at \( k = \pi/2 \). The number on each frame gives the number of “teeth”, \( M \).

FIG. 7. \( A, B, C \) and \( \beta \) for transmission through (a) the closed AB ring (\( J_x = 0 \)), and for the open interferometer with (b) \( J_x = .15J \), (c) \( J_x = .9J \) and (d) \( 1.5J \). The thin line in the lowest frames shows the exact intrinsic phase \( \alpha \) (from Fig. 5). The QD parameters are the same as in Fig. 5.
with the conventions $B, C > 0$, as used in the analysis of experiments. Note the decreasing magnitude of the amplitudes as $J_x$ increases, due to the large losses to the lossy channels. Note also that for $J_x = 0$ the results reproduce those for the closed interferometer, with $\beta$ jumping discontinuously between 0 and $\pi$. As $J_x$ increases, the “data” for $\beta$ become smoother, and they approach the “intrinsic” values of $\alpha$ for intermediate values of $J_x \sim 0.9J$. However, as $J_x$ increases further, $\beta$ “retracts” towards a more steep variation near each resonance. Although the electron crosses each “comb” only once, due to the small values of the comb transmission $T$, it is reflected several times from the “combs” back towards the QD, due to the increasing “comb” reflection. Therefore, the electron visits the QD several times, and the final AB transmission does not reflect the correct desired $t_D$.

Finally, we allow also a lossy channel connected directly to the dot. As seen in Fig. 8, this eliminates the Fano zeroes of $B$ and causes a “smearing” of the sharp Fano “phase lapses” in $\beta$. Technically, the losses from the QD introduce complex self-energies on the dot, which move the zeroes of $T$ away from the real energy axis. Interestingly, Fig. 8 resembles the experiments of Schuster et al. [5].

IV. CONCLUSIONS

Basically, we have made three explicit predictions:

- For single channel leads, the QD transmission and its phase are related via $T_D = \gamma_B^2 \sin^2 \alpha$. In such cases, the measurement of $T_D$ also yields $\alpha$.
- Measurements of the transmission $T$ in a closed interferometer contain much information on both the magnitude $T_D$ and the phase $\alpha$ of the “bare” QD.
- Open interferometers do not usually obey the two-slit criteria. Therefore, the phase $\beta$ measured via a fit to Eq. (4) will usually not yield the intrinsic QD phase $\alpha$. However, optimization of the losses can achieve the two-slit conditions, and yield $\beta = \alpha$.

In principle, the configuration of Fig. 1c allows a full study of all the cases discussed here: setting the voltage on the reference site $\epsilon_0 \to \pm \infty$ sends $V_B$ to zero, and yields the “intrinsic” transmission through the QD, $T_D$. Setting the gate voltage on the QD $V = \epsilon_D \to \pm \infty$ yields the reference transmission $T_B = \sin^2 \delta_B$. Setting the coupling to the “lossy” channels ($J_x$) to zero, by some manipulations of the relevant gates, yields the “closed” case. In this case, variation of $\epsilon_0$ allows variation of $V_B$, and extrapolation of $\delta$ to $\alpha$. Finally, varying $J_x$ allows optimization of the two-slit condition, yielding another measurement of the “intrinsic” phase $\alpha$. We hope that this review will stimulate the buildup of such flexible experimental systems.

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