Abstract. The paper presents the approximate formulas for calculating the deflection angle and the misalignment of the electron beam from the optical axis of the electron gun caused by the action of magnetic fields during the electron beam welding. Mathematical model of the effect of magnetic field induced by thermoelectric currents on the electron beam position in the process of electron beam welding of dissimilar materials is presented. The method of monitoring of the misalignment of the scanning electron beam and its mathematical model are proposed. Monitoring of the misalignment of the scanning electron beam is based on the processing of the signal of the collimated X-ray sensor directed to the optical axis of the electron gun by synchronous detection method. The method of compensation of the effect of magnetic fields by passing through the welded seam the currents which compensate thermoelectric currents is considered.

Introduction

One of the main causes of deflection of an electron beam during electron beam welding (EBW) is the effect of magnetic fields. These fields can be caused by the magnetization of the equipment, the residual magnetization of welded products, the action of various electromagnetic devices, thermoelectric currents produced by temperature gradients in some dissimilar materials [1]. Different ways of reducing the effect of magnetic fields are used. There are demagnetization of products, shielding of electron beam, and the compensation of magnetic field in the welding zone. Demagnetization of products can significantly reduce the noise level, but it may be re-magnetized. In addition, demagnetization of large parts is a labor-consuming and expensive process. Shielding of the electron beam by magnetic shield is the protection from external fields in the space of the shield location. This shield degrades possibility to monitor the process of EBW. Therefore, the compensation of magnetic field in the zone of magnetic field action is most appropriate [1].
Assessing the impact of the magnetic field

To assess the effect of magnetic field on the trajectory of the electron beam the formulas for calculating the deflection of the electron beam from the axis of the electron gun should be derived. Also, determination of the angle of the beam trajectory is an interesting, because it characterizes deflection of the weld seam from the plane of the joint.

The action of magnetic fields in the space between the electron gun and the work piece during EBW can be described with sufficient accuracy by the equations [2]:

\[
\psi = \frac{e}{mV} \int_0^z B(z)dz, \quad X = \frac{e}{mV} \int_0^z \int_0^z B(z)dzdz,
\]

where \(\psi\) – angle of inclination of the trajectory of the electron beam relative to the optical axis of the electron gun \(z\); \(X\) – displacement of the electron beam from the joint; \(e\) – electron charge; \(m\) – mass of the electron; \(V\) – electron speed, which depends on the accelerating voltage; \(B(z)\) – projection of induction of the magnetic field in the plane of the joint as a function of the coordinate \(z\).

The origin of coordinates \((z=0)\) is the center of the focusing system.

The error of calculations according to formulas (1) for most cases occurring in practice doesn’t exceed 2.5 %.

A mathematical model of magnetic fields induced by thermoelectric currents

When welding dissimilar materials the deflection of the electron beam from the joint can be caused by magnetic field of thermoelectric currents. According to Ohm's law, the current \(I\) in a closed circuit is directly proportional to the electromotive force and inversely proportional to the total resistance \(R\) of the entire circuit. Value of arising thermoelectric power \(E\) depends on the material of welded products and temperatures of hot \((T_1)\) and cold \((T_2)\) contacts. In a small range of the temperature the thermoelectric power can be considered as proportional to the difference of the temperature.

When welding two dissimilar materials 1 and 2 the total resistance of the entire circuit is the sum of the resistances \(R_1\) and \(R_2\) of these materials. Resistance of the conductor constant cross-section depends on the properties of the material of the conductor, its length and cross-section.

Thus, the thermoelectric current is determined by the formula

\[
I = \frac{E}{R} = \frac{\alpha_{12}S(T_2 - T_1)}{\rho_1 + \rho_2 l_R},
\]

where \(\alpha_{12}\) – thermoelectric power coefficient; \(\rho_1, \rho_2\) – resistivities of welded materials; \(l_R\) – length of the conductor; \(S\) – cross-sectional area of the conductor; \(T_1\) – temperature at the center of the weld where the circuit of thermoelectric currents is closed; \(T_2\) – melting temperature.

For determining the temperature \(T_1\) the equation of the temperature distribution in the infinite plate heated by a fast-moving line source without the heat transfer from the surface can be used [3]:

\[
T(x, y) = k_{12}T_{max} \sqrt{\frac{v}{y}} \exp \left( \frac{x^2y}{4ay} \right),
\]

where \(k_{12}\) – coefficient characterizing the thermophysical properties of welded materials;
$T_{\text{max}}$ – maximum temperature (evaporation temperature of welded materials); $v$ – welding speed; $a$ – coefficient of the temperature conductivity; $x$ – coordinate across the joint; $y$ – coordinate along the joint.

Equation (3) describes the temperature of an area which located behind the welding source. Temperature at the center of the weld is determined by the formula

$$T = k_{12}T_{\text{max}}\sqrt{\frac{v}{y}}.$$  

Assume that the conductor with the thermoelectric current is an ellipse, as shown in Fig. 1. The length of the ellipse can be calculated from the approximate formula

$$I_R = \pi(a_y + a_x),$$

where $a_y = \frac{-y - y_w}{2}$ – semi-major axis of the ellipse; $y_w$ – distance from the optical axis of the electron gun to the conductor with current (the length of the welding cavity along the axis $y$); $a_x$ – semi-minor axis of the ellipse.

\[ \text{Figure 1. Schematic representation of the model.} \]

Substitution of formulas 4 and 5 into expression 2, differentiation of the resulting expression with respect to $y$, equating of the derivative to zero allow to define coordinate $y_{\text{max}}$ of closing the circuit of the maximal thermoelectric current $I_0$. Semi-minor axis of the ellipse should be set.

Values of circulating thermoelectric currents are defined by the expression

$$I_n = \alpha_{12}\delta b_x\left( k_{12}T_{\text{max}}\sqrt{\frac{v}{y_w + nb_y}} - k_{12}T_{\text{max}}\sqrt{\frac{v}{y_{\text{max}} - nb_y}} \right) \rho_1 + \rho_2 \cdot \pi \cdot \left( -\frac{y_{\text{max}} - y_w}{2} + a_x - 2nb_y \right).$$
where \( b_x \) – width of the conductor with current; \( \delta \) – thickness of the product; \( n=0, 1, 2, 3, \ldots \).

Contours with thermoelectric currents are approximated by rectangles of length \((-y_{\text{max}} - y - 2nb_y)\) and width \((2b_x - 2nb_y)\), where \( n=0, 1, 2, 3, \ldots \). Components of magnetic field directed along the welding line, have the greatest impact on deflection of the electron beam from the joint. These magnetic fields are induced by line segments of current which are perpendicular to the weld.

According to the law of Biot-Savart-Laplace in the case when \( \alpha_2=180-\alpha_1, \cos \alpha_1-\cos \alpha_2=2\cos \alpha_1 \)

the induction of magnetic field of the straight line segment of current having a finite length is described by the equation [4]

\[
B = \frac{\mu_0 I}{2\pi r} \cos \alpha_1 = \frac{\mu_0 I}{2\pi r} \cdot \frac{b_x}{\sqrt{b_x^2 + r^2}},
\]

where \( b_x \) – half the length of the straight line segment of current; \( r \) – distance from the straight line segment of current to the point under consideration.

During the welding process conductors with a current are removed from the axis of the electron gun to distances \((y_w+nb_y)\) and \((-y_{\text{max}} - nb_y)\), distance from the point under consideration \( M \) to the surface of the welded parts is \((l - z)\), where \( l \) – distance from the surface of the welded parts to the electron gun, as shown in Fig. 1. Distance from the straight line segment of current to the point under consideration is determined by formula

\[
r = \sqrt{(y_w + nb_y)^2 + (l - z)^2}.
\]

Thus, the equation (7) can be written as

\[
B_n^+(z) = \frac{\mu_0 I_n}{2\pi \sqrt{(y_w + nb_y)^2 + (l - z)^2}} \cdot \frac{b_x - nb_y}{\sqrt{(b_x - nb_y)^2 + (y_w + nb_y)^2 + (l - z)^2}},
\]

\[
B_n^-(z) = \frac{\mu_0 I_n}{2\pi \sqrt{(-y_{\text{max}} - nb_y)^2 + (l - z)^2}} \cdot \frac{b_x - nb_y}{\sqrt{(b_x - nb_y)^2 + (-y_{\text{max}} - nb_y)^2 + (l - z)^2}}.
\]

Indices \( \leftrightarrow \) and \( \rightarrow \) show that vectors of magnetic induction are directed oppositely to each other.

Using expressions (6), (9) and (10), total deflection can be calculated according to equation (1).

For comparative assessment of calculated and experimental data the materials and their characteristics considered in paper [5] are used. Properties of welded materials are presented in Table 1. The results of calculations produced for welding stainless steel SS 304 and low-carbon steel showed that at an accelerating voltage of 60 kV, a welding speed of 40 m/h, the product thickness of 2.5 cm deflection of the weld from the joint at the depth of 6 mm is 0.6 mm. The calculation results agree well with experimental data obtained in the work [5].

### Table 1. Properties of alloys

| Alloy               | \( T_2 \) (K) | \( a \) \( (\text{m}^2/\text{s}) \times 10^3 \) | \( \rho \) \( (\text{Ohm}\cdot\text{m}) \times 10^{-7} \) |
|--------------------|--------------|---------------------------------------------|---------------------------------------------|
| Stainless steel SS 304 | 1700        | 0.45                                        | 6.667                                       |
| Low-carbon steel   | 1783        | 1.28                                        | 1.136                                       |
**System of control and compensation of deflection caused by the action of magnetic field**

In the interaction of the electron beam with the processed material electrons lose their energy as a result of braking. This process is accompanied by the excitation of the X-ray radiation, localized in a place where the electron beam interacts with the material.

The ability to control the deflection of the electron beam from the optical axis of the gun by the X-ray radiation from the zone of braking of electrons is based on the fact that the intensity of X-ray radiation depends from the surface area through which the radiation flux passes. Deflection of the beam from the axis of the electron gun caused by the action of magnetic fields, leads to a decrease of share of X-ray radiation passing through the crystal area of the sensor. Consequently, the intensity of X-ray radiation $I_d$ measured by the sensor is reduced.

X-ray sensor includes X-ray detector and collimator, which is configured in the form of a blind with the slit oriented to the optical axis of the gun, as shown in Fig. 4.

For monitoring the beam deflection the technological scanning, which improves the quality of the welded joint is used.

A mathematical model of the X-ray sensor as an element having the frequency spectrum of the output signal at presence of the periodic signal of scanning of the electron beam is expressed by the relation [6]

\[
I_d(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt),
\]

where coefficients of the series are given by the expressions:

\[
a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} I_d(t) dt,
\]

\[
a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} I_d(t) \cos nt dt,
\]

\[
b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} I_d(t) \sin nt dt.
\]

Here, the intensity of X-ray radiation from the surface of the work piece is determined by expression

\[
I_d = kk_1 U_0^2 Z I_e \frac{1}{\sigma_x \sqrt{2\pi}} \times \int_{-\infty}^{\infty} f_d(x) \exp \left( -\frac{(x - \varepsilon_0 - \varepsilon_m \sin \omega t)^2}{2\sigma_x^2} \right) dx,
\]

where $f_d(x)$ – function of the view area of the collimator of X-ray sensor; \( k = 1.5 \times 10^{-9} \text{ V}^{-1} \) – proportionality coefficient; \( k_1 \) – coefficient considering the orientation of the sensor in the space; \( I_e \) – electron beam current; \( Z \) – atomic number of the material of welded product; \( U_0 \) – accelerating voltage; \( h \) – width of the slit of the collimator of X-ray sensor; \( l_1 \) – length of the collimation channel; \( l_2 \) – distance from the collimator to the workpiece surface; \( \sigma_x \) – root mean square deflections of electrons from the axis of the beam along the \( x \) axis; \( \varepsilon_0 \) – deflection of the beam from the optical axis of the electron gun; \( \varepsilon_m \) – amplitude of the beam
scanning across the joint; \( \varepsilon_x \) – the position of the beam axis along the \( x \) axis.

Dependencies of odd and even harmonics from the beam deflection from the axis of the gun are shown in Fig. 2 and 3. The calculation of harmonics was produced by the formulas (13) and (14).

![Figure 2](image1.png)

**Figure 2.** Dependence \( b_n(\varepsilon_0) \) at \( \varepsilon_m=1 \).

![Figure 3](image2.png)

**Figure 3.** Dependence \( a_n(\varepsilon_0) \) at \( \varepsilon_m=1 \).

According to the developed mathematical model of X-ray sensor the first harmonic of frequency of scanning allows to determine the beam deflection from the axis of the electron gun. The amplitude of the second harmonic is proportional to the sensitivity of the
measurement transducer. It can be used to stabilize the amplification factor of the compensation system and focusing the beam.

In welding of dissimilar materials magnetic field \( B(z) \) is induced by thermoelectric currents, which are distributed in welded products.

Functional block diagram of system of automatic compensation of effect of magnetic field of thermoelectric currents is shown in Fig. 4.

![Functional block diagram of system of automatic compensation of effect of magnetic field of thermoelectric currents.](image)

**Figure 4.** Functional block diagram of system of automatic compensation of effect of magnetic field of thermoelectric currents.

This system includes an electron gun 1, a focusing system 2, source of the current of focusing 3, deflecting coils 4 and 5, a generator 7 of scanning of the electron beam across the
joint and along it, a block 16 aiming the electron beam at the joint, electric drive 15 moving the electron gun, the current sources 17 and 18, X-ray sensor 8, selective amplifier 9 tuned to the frequency of scanning of the electron beam across the joint, demodulator 10, the amplifier 13 with controlled amplification factor, the integrator 14, selective amplifier 11 of the second harmonic, and a rectifier 12.

Current sources 17 and 18 contain current leads which are located on the top and bottom surfaces of a welded product 6 and symmetrical to the weld in the zone of maximum temperature.

The method of compensation of the effect of magnetic fields caused by currents of thermopower includes monitoring the deflection of the electron beam from the optical axis of the electron gun by X-ray radiation from the processing zone and input into the welded product compensating currents directed oppositely thermoelectric currents. Compensating currents eliminate the beam deflection. Value of compensating currents varies depending on deflection of the electron beam from the optical axis of the electron gun.

**Conclusions**

The conclusions drawn are the following:

1. The considered mathematical model allows to calculate deflection of the electron beam and angle of slope of the trajectory depending on the distribution of magnetic induction in the space between the electron gun and the work piece.

2. Modeling the impact of thermoelectric currents on deflection of the beam when welding dissimilar materials agrees well with the experimental data.

3. Application of method of synchronous detection of the signal of X-ray sensor of the beam deflection allows to automate the process of compensation the impact of magnetic fields in EBW.

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