Potential models and lattice correlators for quarkonia at finite temperature

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We update our recent calculation of quarkonium Euclidean correlators at finite temperatures in a potential model by including the effect of zero modes in the lattice spectral functions. These contributions cure most of the previously observed discrepancies with lattice calculations, supporting the use of potential models at finite temperature as an important tool to complement lattice studies.

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Quarkonia at finite temperature are an important tool for the study of quark-gluon plasma formation in heavy ion collisions (see, e.g., Ref. [1]). Many efforts have been devoted to determine the dissociation temperatures of $QQ$ states in the deconfined medium, using either lattice calculations of quarkonium spectral functions [2, 3, 4, 5, 6, 7, 8] or non-relativistic calculations based upon some effective (screened) potential [9, 10, 11, 12, 13, 14, 15, 16].

Lattice studies are directly based on quantum chromodynamics and should provide, in principle, a definite answer to the problem. However, in lattice studies the spectral functions have to be extracted — using rather limited sets of data — from the Euclidean (imaginary time) correlators, which are directly measured on the lattice. This fact, together with the intrinsic technical difficulties of lattice calculations, somehow limits the reliability of the results obtained so far, and also their scope, which in fact is essentially limited to the mass of the ground state in each $QQ$ channel.

Potential models, on the other hand, provide a simple and intuitive framework for the study of quarkonium properties at finite temperature, allowing one to calculate quantities that are beyond the present possibilities for lattice studies. The main problem of the latter approach is the determination of the effective potential: although at zero temperature the use of effective potentials and their connection to the underlying field theory is well established, at finite $T$ the issue is still open [17, 18, 19].

Calculations of the $cc$ and $bb$ dissociation temperatures, using different potential models based upon the lattice free and internal energies, have found on the whole a reasonable agreement with the results from the lattice studies [11, 12, 13, 15]. On the other hand, calculations of Euclidean correlators using a variety of potential models were not able to reproduce the temperature dependence of the lattice correlators [14, 15, 20].

In Ref. [15] it was pointed out that a precise quantitative agreement with the lattice correlators should not be expected, mainly because lattice spectral functions are strongly affected in the continuum region by artifacts due to the finite lattice spacing. Actually, while some degree of qualitative agreement had been found for the $S$-wave correlators, it was somehow disturbing the finding that for the $P$-wave correlators the temperature dependence of the potential model was even qualitatively different from the lattice one. In Ref. [15] this different behavior has been shown to be due to a sizable amount of strength at low energy that was present in the $P$-wave lattice spectral functions, but not in the potential model estimate. This sizable amount of spectral strength — which is present below the $QQ$ threshold even when no bound states are supported — strictly speaking goes beyond a potential model description. Hence, at very low energy the effective potential approach has to be supplemented by different mechanisms.

Recently, in Ref. [21] it has been shown that the lattice calculations of meson correlators at finite temperature contain a constant contribution, due to the presence of zero modes in the spectral functions. While the presence of a zero mode in the vector channel had already been discussed in the literature, in the $P$-wave channels it had generally been overlooked.

In the following, we want to show that the inclusion of the zero mode contributions in the calculations of Ref. [15] can reconcile potential model and lattice correlators. These contributions have also been considered in Ref. [10] and we shall comment below on the results obtained there. An alternative explanation of the potential model vs lattice discrepancy has been proposed in Ref. [13] and, again, we defer at the end our comments.

Here we briefly outline the model developed in Refs. [12, 15], to which we refer the reader for all the details. The object we are interested in is the Euclidean time correlator at finite temperature $T$, defined as the thermal expectation value of a hadronic current-current correlation function in Euclidean time $\tau$ for a given mesonic channel $H$ (see, e.g., Ref. [22], Chap. 7):

$$ G_H(\tau, T) = \langle j_H(\tau)j_H^\dagger(0) \rangle, $$

where $j_H = \bar{q}_H q$ and $\Gamma_H = 1, \gamma_5, \gamma_{\mu}, \gamma_{\mu}\gamma_5$. The four vertex operators $\Gamma_H$ correspond, respectively, to the scalar, pseudoscalar, vector and axial-vector mesonic channels, which in turn, at zero temperature, correspond to the $\chi_{c0}(\chi_{b0})$, $\eta_c(\eta_b)$, $J/\Psi$, and $\chi_{c1}(\chi_{b1})$ quarko-
nium states for the $c\bar{c}$ ($bb$) system, respectively. In some lattice studies only the spatial components in the vector and axial-vector channels are considered ($\Gamma_H = \gamma_i, \gamma_i \gamma_5$). Moreover, we shall restrict ourselves, as in most lattice calculations, to the case of spatial momentum $p = 0$.

The correlators of Eq. (11) are related to the corresponding spectral functions through an integral transform,

$$G_H(\tau, T) = \int_0^\infty d\omega \sigma_H(\omega, T) K(\tau, \omega, T), \quad (2)$$

which is regulated by the temperature kernel

$$K(\tau, \omega, T) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh[\omega/2T]}. \quad (3)$$

The temperature dependence of the correlators is usually studied by introducing the ratio between the correlation function $G_H(\tau, T)$ at some temperature $T$ and the so-called reconstructed correlator,

$$G_H^{rec}(\tau, T_r) = \int_0^\infty d\omega \sigma_H(\omega, T_r) K(\tau, \omega, T), \quad (4)$$

calculated using the kernel at the temperature $T$ and the spectral function at some reference temperature $T_r$. This procedure should eliminate the trivial temperature dependence due to the kernel and differences from one in the ratio should then be ascribed to the temperature dependence of the spectral function.

In the potential model of Ref. [15], the spectral function has been expressed as

$$\sigma_H(\omega, T) = \sum_n F_{H,n}^2 \delta(\omega - M_n) + \theta(\omega - s_0) F_{H,\omega-s_0}^2, \quad (5)$$

where in the right hand side the sum over $n$ runs over the bound states of mass $M_n$ and the last term represents the continuum contribution, $s_0$ being the continuum threshold; $F_{H,n}$ and $F_{H,c}$ are the couplings associated to the discrete and continuum states, respectively, and they can be expressed in terms of the wave function at the origin for the $S$-states (pseudoscalar and vector channels),

$$F_{PS}^2 = \frac{N_c}{2\pi} |R(0)|^2 \quad \text{and} \quad F_{V}^2 = \frac{3N_c}{2\pi} |R(0)|^2, \quad (6a)$$

and in terms of the first derivative of the wave function at the origin for the $P$-states (scalar and axial-vector channels),

$$F_3^S = \frac{9N_c}{2\pi m^2} |R'(0)|^2 \quad \text{and} \quad F_3^A = \frac{9N_c}{\pi m^2} |R'(0)|^2, \quad (6b)$$

$N_c$ being the number of colors and $m$ the quark mass. Since the integration over the energy in Eq. (2) can reach very high excitation energies, the wave functions have been renormalized to account for relativistic kinematical effects, that is, the spectral functions have the correct asymptotic dependence ($\sim \omega^2$).

The potential that we employ to generate the $Q\bar{Q}$ wave functions [12, 15] is given by a linear combination [11] of the color-singlet free and internal energies obtained on the lattice from the Polyakov loop correlation functions [21, 23, 27]. The only partially free parameter one has in this approach is the bare heavy quark mass [30]: for instance, using the physical mass for the quark $c$, the $J/\psi$ dissociation temperature occurs below $1.5T_c$, whereas with slightly heavier quarks it can be brought above $1.5T_c$ [15].

When addressing the Euclidean correlators, one should also account for the presence of zero mode contributions to the spectral functions at finite temperature. There is no zero mode in the pseudoscalar channel, whereas the vector channel has been discussed in Refs. [21, 28]. Recently, is has been pointed out that these modes provide an important contribution also in the lattice scalar and axial-vector correlators [21]. In the non-interacting case, they yield a term proportional to $\delta(\omega)$ in the spectral function and a $\tau$-independent term in the Euclidean correlator:

$$\sigma_H^{(0)} = \chi_H(T) \delta(\omega) \quad (7a)$$

$$G_H^{0} = T \chi(H). \quad (7b)$$

In the interacting theory, one may assume the delta function to be smeared to a Lorentzian, with a width related to the heavy quark diffusion constant [28].

Here, for simplicity, we employ the free expressions of Eq. (7), where all the susceptibilities $\chi_H$ can be expressed in terms of a scalar piece,

$$\chi_S(T) = -4N_c \int \frac{dk}{(2\pi)^3} \frac{m^2}{\omega_k^2} \frac{\partial n_F}{\partial \omega_k}, \quad (8a)$$

and a charge piece,

$$\chi_0(T) = -4N_c \int \frac{dk}{(2\pi)^3} \frac{\partial n_F}{\partial \omega_k}, \quad (8b)$$

as

$$\begin{align*}
\text{pseudoscalar} & : \chi_{PS} = 0 \\
\text{scalar} & : \chi_S \\
\text{vector} & : \chi_V = -\chi_S \\
\text{axial} & : \chi_A = 3\chi_S \\
\text{axial} & : \chi_A = \chi_0 + 2\chi_S.
\end{align*} \quad (9)$$

In Eq. (8), $\omega_k = \sqrt{k^2 + m^2}$ and $n_F = 1/\exp(\omega_k/T) + 1$.

The zero mode contributions to the spectral function arise from processes in which the meson current, rather than inducing transitions resulting in the creation-annihilation of a $Q\bar{Q}$ pair, is absorbed by a heavy (anti-)quark of the thermal bath, which is in turn scattered with a slightly modified momentum. The detailed balance between this process and its inverse results in the factors $\partial n_F/\partial \omega_k$ in Eq. (8).
In the vector channel the strong enhancement of the ratio at large \( \tau \) is also well described, owing again to the zero mode term in the spectral function. On the whole, the data of Ref. \[8\] are described semi-quantitatively: we stress again, as thoroughly discussed in Ref. \[15\], that an accurate description of the lattice data should not be expected, due to the discretization effects in the lattice calculations.

In the pseudoscalar channel, where no constant contribution to the correlators is present, the ratio evaluated on the lattice tends (also in the quenched case) to stay around one up to some temperature (interpreted as the dissociation one) and then it starts decreasing; in our calculation, on the other hand, the ratio is monotonically decreasing with \( \tau \) at all the temperatures. A remnant of this behavior is also visible in the vector channel, where the ratio drops slightly below one, before jumping up.

Indeed, the binding due to the potential extracted from the lattice free energies gets softer with increasing \( T \) and this is reflected into the decrease of the square wave function at the origin \[15\]. On the other hand, the lattice spectral functions generally display ground state peaks of nearly constant strength up to the dissociation temperature. Note that when in the potential model the bound state energy is going to zero, a strong resonance-like \[31\] contribution appears in the continuum (see the spectral functions displayed in Ref. \[15\]). Although it might be possible that on the lattice this contribution has been mistakenly identified as a bound state, in our calculations it still has a strength gradually decreasing with \( T \).

Note, anyway, that the trend predicted in the potential model is qualitatively correct, since the quenching of the pseudoscalar ratio gets stronger with increasing \( T \). Similar results are also obtained for the quenched calculations of Refs. \[4, 7\] and we do not display them here.

In Ref. \[13\] an alternative explanation for the flatness, up to the dissociation temperature, of the \( G/G_{\text{rec}} \) ratio in the pseudoscalar channel has been proposed: in that potential model no continuum contribution is present when there are \( S \)-wave bound states; thus, assuming the existence of just one bound state, one should have

\[
\frac{G_{\text{PS}}(\tau,T)}{G_{\text{PSA}}(\tau,T,T_r)} = \left| \mathcal{R}_{\text{PS}}(0,T) \right|^2 \cdot K(\tau,M(T),T)/K(\tau,M(T_r),T) \quad \text{(note that this expression in general is not normalized to one at } \tau = 0) \].

However, in Ref. \[13\] the ratio between the wave functions at the origin has been neglected: it has a strong temperature dependence and it is precisely this term that generates the quenching with increasing \( T \) of the ratio in the pseudoscalar channel. Since the mass of the bound states turns out to be quite stable, the remaining factor, \( K(\tau,M(T),T)/K(\tau,M(T_r),T) \), is nearly constant and yields the flat behavior of the lattice calculations, which thus here appears just as an artifact of the approximation.

A potential model calculation including the zero modes has also been done in Ref. \[16\], using a hybrid model...
where the spectral functions are calculated by solving a Schrödinger equation with a screened potential at low energies and employing the perturbative expressions at large energies. In spite of the different models, also this approach yields very good results for the ratio $G/G_{rec}$ of the $P$-wave correlators, indicating that this ratio is essentially dominated by the zero mode contribution. On the other hand, the results of Ref. [15] in the pseudoscalar channel are rather different from ours: in the $c\bar{c}$ system at $T$ slightly above $T_c$, the authors of Ref. [15] get a rather flat ratio, compatible with the lattice data; when $T$ grows the lattice ratios get increasingly quenched, while their calculations show a moderate enhancement (cf. Fig. 11 of Ref. [16]). According to the authors of Ref. [16] the discrepancy should be acried to relativistic effects.

It should be noted that in Ref. [16] the finite $T$ correlators are compared to the $T = 0$ one and in the latter an ad hoc factor is introduced to account for radiative corrections: in the pseudoscalar channel this factor yields a 100% correction to the low energy part of the spectral function. Apparently, no radiative corrections have been included at finite $T$.

To summarize, we have updated a recent calculation [15] of quarkonium Euclidean correlators at finite $T$, by including the zero mode contributions that are present in the lattice spectral functions. The model developed in Refs. [12, 15] is based upon an effective potential extracted from $Q\bar{Q}$ free and internal energies measured on the lattice. Good agreement with the lattice results has been found for the $P$-wave correlators (scalar and axial-vector channels). Also the vector channel appears to be well described.

The pseudoscalar correlators, on the other hand, show the same $T$-dependence as the ones calculated on the lattice, but not the same $\tau$-dependence below the dissociation temperature. This discrepancy has already been discussed in Ref. [13] as being related to the fact that the strength of the bound states (and of the zero-energy resonances) in the potential model is gradually decreasing with $T$, whereas the peak associated to the ground state in the lattice measurements seems to keep a constant strength up to dissociation. Hopefully, more accurate lattice calculations should shed some light on this issue in the near future.

We would like to stress that these results have been obtained without any adjustment of any parameter, the effective potential being fixed by independent lattice measurements of the $Q\bar{Q}$ free energies.

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