Numerical Simulation of Granular Column Collapse with Fractal Particle Size Distribution Using Discrete Element Method

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Abstract. This study conducts numerical simulations of the granular column collapse with Fractal Particle Size Distributions (FPSDs) via the Discrete Element Method (DEM) and investigated kinetic behaviours of dry granular flows. The aim of this paper is to explore the effects of the fractal dimension of FPSD on the kinetics of dry granular flows. When the fractal dimension of the flows consisting of granular materials increases, the horizontal particle translational velocities become greater and the mobility improves, whereas the particle rotational velocities decrease. Meanwhile, the change in the potential energy increases, and the particle kinetic energy in the rotational form reduces; thus, the particle kinetic energy in the translational form increases. The reducing particle rotational movement may be related to the reducing particle shearing behaviours because only the contact shearing can affect particle rotational motion. In conclusion, a larger fractal dimension of FPSD of a dry granular flow leads to a longer spreading distance and a smaller rotational velocity.

Keywords: Granular column collapse, particle behaviours, flow mobility, flow dynamics.

1. Introduction

1.1. Background
Complicated flow dynamics of granular media, e.g., rock avalanches, debris flows, landslides, and grains in the pharmaceutical and food industries, are typically investigated using the granular column collapse on a transversal wall. This model is effective and has been widely used by many researchers. Figure 1 illustrates the granular column collapse movement model. By moving the wall immediately in the longitudinal orientation or slowly in the transversal orientation, the collapse or quasi-static movement of granular column can be modelled.

1.2. Related Work
Many investigations have been performed via the granular column collapse model on a transversal wall. Geometric configuration parameters own the power-law function relationship with the initial geometric parameters [1-4]. The energy evolution during the collapse was obviously influenced by the aspect ratio [5]. How the factors exert effects on the dynamics of granular columns, e.g. the granular column initial porosity [6], mono-dispersed particle size [7], particle shape [8], particle friction...
coefficient [9], particle restitution coefficient [10], particle stiffness [11] and the fluid situation [12], have also been examined in detail. Deep insights into the movement mechanisms of the granular column collapse have been obtained.

Although there are many studies about the granular column collapse mechanisms, the investigations on mechanisms of how the FPSD has effects on the granular column collapse model are limited. The existing studies focused on the granular column movement with mono-dispersed [13], bi-dispersed [14], tri-dispersed [15-16] and poly-dispersed mixtures with non-fractal size distributions [17]. However, Crosta et al. [18] found that the rock blocks size distribution in natural rock avalanche deposits is fractal from field measurements. Lai et al. [19] have explored the mechanisms of how FPSD influences the runout distance and the kinetics of different sized grains. However, how FPSD affects particle motion and energy conversion in translational and rotational forms.

We analyzed the collapse processes of the granular columns with varying fractal dimensions of FPSD through DEM. The details about our DEM simulations were then given. We attempted to use the evolutions of particle motion and energy conversion in translational and rotational forms to explore how the flow mobility changes. The motive is to understand the reason why the flows consisting of granular media can own tremendous mobility.

![Diagram of granular column collapse movement model.](image)

**Figure 1.** Diagram of granular column collapse movement model.

2. DEM Numerical Model Setup

2.1. The Meaning of FPSD

We employed the method employed by the references [20-21] to generate the numerical samples with FPSDs. The mathematical meaning of FPSD is that the number, \( N(d) \), of \( d \)-sized particles is \( 2^D \) times the \( 2d \)-sized number. The parameter \( D \) indicates the fractal dimension. Then, we can use the following equation to represent the flows with FPSD.

\[
N(d) = N_0(d/d_0)^D
\]

where \( N_0 \) is the \( d_0 \)-sized particle number. On the other hand, the greater value of \( D \) means that the flow owns the larger proportion of small sized particles. If the Particle Size Distributions (PSD) of the flow is fractal, it means that the PSD is self-similar from different scales. The other uniform, multi- and poly-dispersed types of distributions don’t own this kind of character.

2.2. Granular Column Collapse Processes Simulations with Different FPSDs Using DEM

The DEM has been commonly employed to model the movement process of the flows consisting of granular media [22]. The model in DEM consists of grains and walls. The number of the grains and walls is finite. It is based on a Lagrangian theory where particles are discrete and their movement trajectories are recorded according to the solutions of the Newton's motion equations. The DEM descriptions in detail can be seen in Itasca [23] and Kermani et al. [24] and hence are not presented herein.
The numerical samples with $D = 1.0, 1.5, 2.5$ and $3.5$ are built according to Lai et al. [19] using DEM code. First, we utilized the disks as the particles and their thicknesses are unit. We employed six kinds of particles with varying sizes, which are small-sized particles with 0.2 and 0.4 m radii, medium-sized particles with 0.6 and 0.8 m radii, and large-sized particles with 1.0 and 1.2 m radii, respectively. Small-sized, medium-sized and large-sized particles are in blue, red and yellow, respectively. The reason why these sizes are defined is that they can describe the common rock grains in nature according to Crosta et al. [18] and the acceptable computational efficiency of DEM.

We obtained the particles with specified sizes in DEM via the equation (1). Figure 1 gives the FPSDs of the numerical samples. Then, a rectangular reservoir is built. The length of this reservoir is 40.0 m and its depth is 80.0 m, respectively. The particles with different sizes are randomly generated there. The numbers of particles are specified according to Figure 2. A linear contact model with the viscous damping in the longitudinal and transversal orientations and the slip determination represents the collision and shearing behaviours among the particles. Finally, the particles fall to accumulate stably in the rectangular under the applied gravity. All the simulation parameters are used from the reference of Lai et al. [19], which are shown in Table 1. Figure 3 illustrates the setup of the samples with varying $D$.

![Figure 2. The FPSDs of the samples in DEM.](image)

### Table 1. DEM simulation parameters.

| Parameter                              | Value               |
|----------------------------------------|---------------------|
| Sample length $L_0$                    | 40.0 m              |
| Sample height $H_0$                    | 67.0 m              |
| Density of particles $\rho$            | 2500 kg / m$^3$     |
| Stiffness of particles and walls       | $k_n = k_t = 10^{10} N / m$ |
| Friction of particles $\mu_{\text{particle}}$ | 0.6                |
| Friction of the left and right wall sides $\mu_{\text{lateral}}$ | 0                  |
| Friction of the bottom wall side $\mu_{\text{bottom}}$ | 0.6               |
| Gravity $g$                            | 9.81 m/s$^2$        |
| Damping $\beta_n$ in the normal orientation | 0.12              |
| Damping $\beta_t$ in the transversal orientation | 0.12             |
3. DEM Simulation Results

3.1. Motion Kinematics

The right walls of the initial granular columns with varying $D$ are deleted to trigger the collapse. The friction of the walls is set to the same value of the particles. Due to the friction, the main granular flow becomes still and the velocities in the translational and rotational forms are close to zero gradually. Figure 3 shows the accumulation shapes for the flows with the four varying $D$. The velocities of the grains in the frontal part of the granular flow are larger than the others, thus making them move away from the main body as shown in the granular flow with $D = 1.0$. The same trend can be seen for the other three samples. The particles that accumulate away from the main body are deleted because only the accumulation shapes of the main bodies of the flows are focused in our study.

The longitudinal and transversal orientations are built as $x$ and $y$ axes, individually. Then, the translational average values of $v_x$ and $v_y$ in the longitudinal and transversal orientations, and the average values of $\bar{\omega}$ in the rotational form are defined:

$$v_x = \frac{1}{n} \sum_{i=1}^{n} v'_x / n$$
$$v_y = \frac{1}{n} \sum_{i=1}^{n} v'_y / n$$
$$\bar{\omega} = \frac{1}{n} \sum_{i=1}^{n} \omega_i / n$$
$$t^* = t / \sqrt{H_0 / g}$$

The motion velocity distributions of $v_x$ and $v_y$ for the samples with $D = 1.0$ and 3.5 during the movement are given in Figure 4. The definition of $t^*$ is chosen according to the literature of Jing et al. [11]. According to Figure 4, $v_x$ shows a greater peak than $v_y$, and the peaks of $v_x$ and $v_y$ occur at different values of $t^*$. Besides, $v_x$ increases significantly as $D$ increases from 1.0 to 3.5, whereas $v_y$ increases only slightly, and $\bar{\omega}$ decreases obviously.
3.2. Energy Conversion

We use the normalized energy $E^t$, $E^r$ and $E^p$ in the translational, rotational and potential forms to compare the effects of $D$. The energies in the translational, rotational and potential forms are divided by the initial potential energy $E^p_0 = \sum_{i=1}^{n} m^i g h^i_0$, where $m^i$ is the particle $i$ mass, and $h^i_0$ is the initial vertical distance from the bottom wall.

Figure 5 gives the evolutions of $E^t$, $E^r$ and $E^p$ and the average values are given in parentheses. Compared with the values of $E^r$ and $E^p$ of the flows with $D = 1.0$ and 3.5 are much greater. The evolutions of $E^t$ for the flows $D = 1.0$ and 3.5 are almost the same until $t^* = 1.0$. This is in accordance with the time when the differences appear in $\bar{v}_x$ and $\bar{v}_y$. It is shown that $E^r$ reduces and $E^t$ enlarges with the increasing $D$.

Figure 5 also gives the evolution of $E^p$ during the movement. The values of $E^p$ at $t^* = 6.0$ are shown in brackets. It can be seen that the evolutions of $E^p$ for the flows $D = 1.0$ and 3.5 are almost the same until $t^* = 1.0$ and then the difference occurs with the continuation of the movement. According to the values in brackets, $E^p$ enlarges as $D$ increases. This means that more energy in potential forms is transformed into the energy in translational forms and the energy dissipated by particle friction and collision behaviours.

Figure 5. The normalized energy distributions of $E^t$, $E^r$ and $E^p$ in the translational, rotational and potential forms of the samples with $D = 1.0$ and 3.5.
4. Conclusions
The granular flow is a type of flow including rock avalanches, debris flows, landslides in nature and grains in the pharmaceutical and food industries [25-27]. Based on DEM, we performed the granular column collapse simulations with different $D$ and analysed the influences of $D$ on the velocities and energy in the translational and rotational forms. The FPSD affects the flow mobility significantly. As the fractal dimension $D$ becomes greater, the spreading distance increases. When $D$ becomes greater, the runout distance becomes greater which represents the increasing mobility. During the process of the entire movement, the translational velocity in the longitudinal orientation increases at the beginning and then the translational velocity in the transversal orientation becomes much greater at the following spreading and final stages. The velocity in the rotational form reduces obviously. This is directly responsible for the increasing runout distance of the flows with the increasing $D$.

More energy in potential forms is transformed into the energy in translational forms and the energy dissipated by particle friction and collision behaviours with the increasing $D$. Therefore, the particle velocities in the translational form increase. However, the particle velocities in the rotational form reduce. Only the particle shearing behaviours can exert the effects on the particle movement in the rotational motion. Therefore, we reckon that the decreasing particle shearing behaviours may be responsible for the decreasing particle rotational motion.

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