An experiment for the measurement of the bound-beta decay of the free neutron

W. Schott\textsuperscript{1}, G. Dollinger\textsuperscript{2}, T. Faestermann\textsuperscript{1}, J. Friedrich\textsuperscript{1}, F. J. Hartmann\textsuperscript{1}, R. Hertenberger\textsuperscript{3}, N. Kaiser\textsuperscript{1}, A. R. Müller\textsuperscript{1}, S. Paul\textsuperscript{1}, and A. Ulrich\textsuperscript{1}

\textsuperscript{1} Physik-Department, Technische Universität München, D-85748 Garching, Germany
\textsuperscript{2} Institut für Angewandte Physik und Messtechnik, Universität der Bundeswehr München, D-85577 Neubiberg, Germany
\textsuperscript{3} Sektion Physik der Ludwig-Maximilians-Universität München, D-85748 Garching, Germany

Received: date / Revised version: date

Abstract. The hyperfine-state population of hydrogen after the bound-beta decay of the neutron directly yields the neutrino left-handedness or a possible right-handed admixture and possible small scalar and tensor contributions to the weak force. Using the through-going beam tube of a high-flux reactor, a background free hydrogen rate of ca. $3 \text{s}^{-1}$ can be obtained. The detection of the neutral hydrogen atoms and the analysis of the hyperfine states is accomplished by Lamb shift source type quenching and subsequent ionization. The constraints on the neutrino helicity and the scalar and tensor coupling constants of weak interaction can be improved by a factor of ten.

PACS. 1 3.30.Ce, 14.20.Dh

1 Introduction

The neutron decay is for many years subject of intense studies, as it reveals detailed information about the structure of the weak interaction. However, the studies have so far only addressed the main decay channel (classical neutron three-body $\beta$-decay), where decay rates and decay asymmetries have been determined with great precision \cite{1}. Symmetries of the interaction are accessible by the precise measurement of momentum spectra of the decay products and/or their correlation with the neutron spin alignment. In addition, experiments are planned, where the polarization of the final-state particles (electron or proton) is measured. However, there is a very elegant method to measure very precisely the relative spin alignments of the daughter products and their correlation. Using the two-body neutron $\beta$-decay into a hydrogen atom (H) and an electron antineutrino ($\bar{\nu}$)

$$n \rightarrow H + \bar{\nu}$$ (1)

one can investigate the hyperfine population of the emerging hydrogen atom. The challenge lies in the very small branching ratio $BR = 4 \cdot 10^{-6}$ of the total neutron $\beta$-decay rate \cite{2}, which is the result of the small phase space for the efficient coalescence necessary for H-formation. We expect only states with zero angular momentum in the hydrogen atom to be populated; these are mainly the 1s and the metastable 2s states with 83.2\% and 10.4\% probability, respectively. This is because states of angular momentum $l > 0$ have a negligible expectation value in the center of the potential. The residual 6.4\% $ns$ states with $n > 2$ decay within nanoseconds, mainly into the 1s state.

Using standard V-A theory the possible spin configurations emerging from this bound-$\beta$ decay and the resulting hyperfine states of the emitted hydrogen atom are given in table 1.

According to \cite{4} the population probabilities $W_i$ of the various configurations $i$ can be deduced to be

$$W_1 = \frac{(\chi - 1)^2}{2(\chi^2 + 3)}, \quad \chi = (1 + g_S)/(\lambda - 2g_T),$$ (2)

$$W_2 = \frac{2}{\chi^2 + 3}, \quad \chi = (1 + g_S)/(\lambda - 2g_T),$$ (3)

$$W_3 = \frac{(\chi + 1)^2}{2(\chi^2 + 3)}, \quad \chi = (1 + g_S)/(\lambda - 2g_T),$$ (4)

depending only on one variable $\chi = (1 + g_S)/(\lambda - 2g_T)$, with $\sum_{i=1}^{3} W_i = 1$. $\lambda$ is the ratio

$$\lambda = g_A/g_V = -1.2695 \pm 0.0029$$

$g_A, g_V, g_S, g_T$ are the axial, vector, scalar and tensor coupling constants, respectively. Thus, by means of $W_i$ only a combination of $g_S$ and $g_T$ can be measured. $g_S$ is obtained from $W_i$ only if $g_T$ is known from somewhere else and vice versa.

The V-A contribution to the emission of H in one of its hyperfine spin states with $F = 1$ and $m_F = 1$ (config. 3 of table 1) is suppressed by about two orders of magnitude (cf. eq. 4). Therefore, by measuring the population of
Table 1. Spin projections \(i\) in the neutron bound-\(\beta\) decay. As a convention, the \(H\) moves to the right, the \(\bar{\nu}\) to the left. Fe and GT mean Fermi and Gamov-Teller transition, respectively. \(W_i\) are the populations according to pure V-A interaction (cf. eqs. [2, 41] of ref. [2]. \(\chi = g_\nu / g_\lambda\), \(F\) the total spin (with hyperfine interaction) and \(m_F\) the \(F\) projection, \(|m_{SM}\rangle\) the Paschen-Back state, where \(m_S\) and \(m_I\) denote the \(e^-\) and \(p\) spin quantum numbers (+ means +1/2, \(i\), e. spin points to the right in the magnetic quantization field direction)

| configuration \(i\) | \(\bar{\nu}\) | \(n\) | \(p\) | \(e^-\) | trans. | \(W_i(\%)\) |
|----------------------|-------------|--------|------|--------|--------|---------|
| 1                    | \(\leftrightarrow\) | + | + | Fe/GT | 44.14 ± 0.05 |
| 2                    | \(\leftrightarrow\) | + | - | Fe/GT | 55.24 ± 0.04 |
| 3                    | \(\leftrightarrow\) | + | - | Fe/GT | 622.2 ± 0.11 |
| 4                    | \(\leftrightarrow\) | + | + | Fe/GT | 0.0 |
| 2'                   | \(\leftrightarrow\) | - | + | Fe/GT | 0.0 |
| 1'                   | \(\leftrightarrow\) | - | - | GT | 0.0 |

Table 2. \(W_i(\%)\) for various \(g_S\) and \(g_T\).

| config. \(i\) | \(g_S = 0\) | \(g_S = 0.1\) | \(g_S = 0\) | \(g_T = 0\) | \(g_T = 0.02\) |
|--------------|-------------|-------------|-------------|-------------|-------------|
| 1            | 44.14       | 46.44       | 43.40       | 45.14       | 44.70       |
| 2            | 55.24       | 53.32       | 55.82       | 55.96       | 56.00       |
| 3            | 622.2       | 238.3       | 780.0       | 780.0       | 780.0       |
| 4            | 0.0         | 0.0         | 0.0         | 0.0         | 0.0         |

this configuration in relation to the configurations 1 and 2 (both \(m_F = 0\)), the existence of \(S\), \(P\), \(T\) weak interaction variants can be verified. While the states \(F = 1, m_F = ±1\) (configurations 3 and 4) are pure eigenstates of the Hamiltonian describing the \(p\) \(- e^-\) interaction in an external magnetic field \(B_1\), linear combinations of the configurations 1 and 2 states with coefficients depending on \(B_1\) yield the other two eigenstates. Therefore, the configurations 3 and 4 are separated in any case from configurations 1 and 2, whereas a separation of configurations 1 and 2 is only possible, if the decays occur within a magnetic field \(B_1\) (cf. below).

The sensitivity of \(W_i\) with respect to \(g_S\) and \(g_T\) is shown in table 2 [3], where the \(W_i\) are calculated assuming small admixtures of \(g_S\) and \(g_T\).

There are three configurations, 4, 1', and 2', which correspond to a scenario with right-handed neutrinos. However, only configuration 4 is unique and leads to a HFS state not populated by any other transition. The negative helicity \(\bar{\nu}\) configurations 1' and 2' would contribute to configurations 1 and 2, respectively.

A possible small contribution of negative helicity to the \(\bar{\nu}\) would manifest itself by a non-zero value of \(W_4\) in table 1. The population of configuration 4, predicted by a left-right symmetric V+A model, is given by [3].

Wertext:

\[
W_4 = \frac{(x + \lambda y)^2}{2(1 + 3\lambda^2 + x^2 + 3\lambda^2 y^2)},
\]

with \(x = \eta - \zeta\) and \(y = \eta + \zeta\), where \(\eta\) is the mass ratio squared of the two intermediate charged vector bosons and \(\zeta\) the boson mass eigenstate mixing angle, \(\eta\) and \(\zeta\) being \(\eta < 0.036\) [5] and |\(\zeta| < 0.03\) (C.L. 90%), respectively.

In this model the antineutrino helicity \(H_\bar{\nu}\) becomes

\[
H_\bar{\nu} = \frac{1 + 3\lambda^2 - x^2 - 3\lambda^2 y^2}{1 + 3\lambda^2 + x^2 + 3\lambda^2 y^2}.
\]

The neutron decay is then mediated by right-handed currents, the details of which are absorbed in \(\eta\) and \(\zeta\). The best data for neutrino helicities come from \(\tau\) and \(\mu\) decay spectra and the extraction of the Michel parameters. Typical accuracies are in the order of 15% [7]. Similar accuracies can be achieved using the B-coefficient in neutron decay.

A certain background to the configurations 1-4, 1', 2' comes from \(s\) states with larger principal quantum number \(n\), also originally populated. These \(ns\) states with \(n > 2\) subsequently decay into the 1s and 2s states by spontaneous emission of photons, where the spin quantum number \(m_S\) of the \(e^-\) is changed. If only the 2s state is used for the spin analysis, the 4s and higher-state population yield a 2s background slowly converging with \(n\); as an example \(W(4s \rightarrow 2s) = 3.07 \cdot 10^{-4}\) and \(W(5s \rightarrow 2s) = 2.18 \cdot 10^{-4}\) (cf. Appendix). From the sum \(W(4s \rightarrow 2s) + W(5s \rightarrow 2s) = 5.25 \cdot 10^{-4}\) a fraction of 44.2% (that is \(2.32 \cdot 10^{-4}\)) would contribute to configuration 4 as background, and 55.2% of the sum (2.90 \(\cdot 10^{-4}\)) would show up in configuration 3. The latter background constitutes already 47% of the resulting \(W_3\) rate, which contributes 6.2 \(\cdot 10^{-4}\) to the 2s population.

In order to improve the present \(g_S\), \(g_T\) and \(H_\bar{\nu}\) accuracies (cf. below), the background due to optical \(m_S\) changing transitions from \(ns\) states with \(n > 2\) into the 2s-state analyzed must be eliminated, e.g. by ionizing these \(ns\) H-atoms using a laser prior to their decay.

2 Experiment

Figure 1 depicts the suggested setup to perform a neutron bound-\(\beta\) decay experiment. Hydrogen atoms from this decay have a kinetic energy of \(T_H = 326.5\) eV, which corresponds to a velocity \(v \approx 0.83 \cdot 10^{-3}\). They are produced in the high neutron flux from an intense neutron source and extracted from a through-going beam pipe. In the following we will assume a high-flux neutron beam from a reactor. Different possibilities for such a neutron source will be discussed below.

The proposed setup of observing the emerging hydrogen atom parallel to a magnetic field has the virtue that we automatically define a helicity axis which coincides with the quantization axis for the various HFS states. On the opposite side the neutrino helicity axis follows from angular momentum conservation. With this, the spin alignment of the original neutron is reconstructed. We thus do not
need to operate with polarized neutrons (see table 1). At the same time we can reverse the direction of the quantization axis by operating the system with reversed $B$ fields. This will become important when discussing the spin analysis of the emerging hydrogen by means of Paschen-Back splitting in a strong field.

The $B$ field in the decay volume is also necessary for a second reason, which at the same time defines its minimum strength. In order to preserve the magnetic quantum numbers of electron and proton, $m_S$ and $m_I$, (also in the case of total spin $F = 0$), the Zeeman coupling to the external fields must be larger than the HFS coupling within the hydrogen atom. The latter one corresponds to an internal magnetic field of 507 Gauss (63.4 Gauss) for the $1s(2s)$ levels, also called the critical field $B_c$ [8]. Thus the field in the decay volume should exceed the value of 63.4 Gauss, since the $2s$ level splitting is used.

During their passage in the beam tube the hydrogen atoms move to the right, the hyperfine spin states $F = 1, m_F = 0, \pm 1$ or $|m_Sm_I\rangle$ states are kept in a longitudinal magnetic quantization field $B_1$, which extends from the neutron-decay volume to the analyzing station. For separating the configuration 1 and 2 states, $B_1$ should be at least as large as the critical field $B_c$ of the 1s or 2s level ($B_c$ is the magnetic field of the electron at the position of the proton, which causes the hyperfine splitting (fig. 3)). Also from this point of view, using only the 2s hydrogen atoms, $B_1$ must be $B_1 \geq 63.4$ Gauss.

### 2.1 Spin analysis and detection of 326.5 eV H-atoms

The hyperfine analyzing part starts with a transverse magnetic field $B_2 = 10$ Gauss. Within the $B_2$ field region of 5.4 mm extension the $2s, F = 1, m_F = 0, \pm 1$ states are rotated adiabatically by $\pi/2$ into the same states, however, with the quantization axis being perpendicular to the direction of flight and parallel to the magnetic field $B_3$ with $B_3 = 575$ Gauss, immediately following the $B_2$ field region.

Figure 4 depicts the Breit-Rabi diagram of the $2s_{1/2}$ hyperfine states in a magnetic field, also showing the corresponding $2p_{1/2}$ states [8].

The $\alpha$ and $\beta$ states are related to the $|m_Sm_I\rangle$ states as

$$|\alpha11\rangle = |++\rangle$$

$$|\alpha10\rangle = \cos \theta |+-\rangle + \sin \theta |-+\rangle$$

$$|\beta1-1\rangle = |-\rangle$$

$$|\beta00\rangle = \sin \theta |+-\rangle - \cos \theta |-+\rangle,$$

where $\theta$ is given by $\tan 2\theta = B_c/B$ with the quantization field $B$ and the critical field $B_c$.

If there exists a magnetic field $B_1$ in the through-going beam tube at the point of neutron decay, the $|\alpha10\rangle$ and $|\beta00\rangle$ population of the hydrogen atoms from bound-$\beta$ decay is given by

$$N_{\alpha10} = N_1 \cos^2 \theta + N_2 \sin^2 \theta$$

and

$$N_{\beta00} = N_1 \sin^2 \theta + N_2 \cos^2 \theta$$

with $N_1$ and $N_2$ being the configuration 1($|+-\rangle$) and 2($|+-\rangle$) state population from table 1 respectively. Thus, $|\alpha10\rangle$ and $|\beta00\rangle$ are not equally populated if the bound-$\beta$ decays occur in a magnetic field $B_1 \neq 0$ (and, thus, $\theta \neq \pi/4$). This population is conserved up to the region of spin analysis (with $B_3$ field) if no level transitions are induced.

Owing to Stark mixing in an electric field superimposed transversely to $B_3$, the states $\beta(1,-1)$ and $\beta(0,0)$, originally metastable, can mix with the $e(1,1)$ and $e(1,0)$ states at a $B_3$ field of 575 Gauss. Using $E_1 = 4.3$ V/cm,
the lifetime $\tau_\beta$ of the $\beta$ states is thus strongly reduced ($v \cdot \tau_\beta = 1.1 \text{ cm}$) as compared to the lifetime $\tau_\alpha$ of the $\alpha$ states ($v \cdot \tau_\alpha = 1.8 \cdot 10^3 \text{ cm}$) [9]. Thus, in the $E_1 \times B_3$ field the $2s_{1/2}, F = 1, m_F = -1$ and the $2s_{1/2}, F = 0$ states are quenched immediately, decaying back into the respective $1s_{1/2}$ ground state, whereas the two $2s_{1/2}$ states with $F = 1, m_F = 1$ and $F = 1, m_F = 0$ survive.

One of the two remaining $\alpha$ states can be selected by the spin filter method using the simultaneous interaction of the $\alpha$, $\beta$ and $e$ levels at $B_3$ [8]. Owing to the static transverse electric field $E_1$ (fig. 1), $\beta$ and $e$ are mixed. Then, we apply an rf field of 1.609 GHz, causing the $\alpha$ states to interact with the coupled $\beta - e$ levels. At $B_3 \approx 538$ Gauss the $\alpha(1,0)$ will be quenched and the $\alpha(1,1)$ state will be selected, whereas with the same frequency at $B_3 \approx 605$ Gauss the $\alpha(1,0)$ will remain, and the $\alpha(1,1)$ will be removed. If the quantizing field $B_1$ is reversed, $\alpha(1,1)$ will be replaced by $\beta(1,-1)$ and $\alpha(1,0)$ by $\beta(0,0)$, respectively. The transmission curve for a single spin state is 0.4 Gauss wide because of the width of the perturbed $\beta$ state [10]. Therefore, for a perfect separation of the spin states, $B_3$ must be homogeneous to $\pm 2$ Gauss. The rf field can be produced within a box-shaped cavity in the TE101 mode, yielding $B_{rf}$ field lines perpendicular to $B_3$ (fig. 1) surrounding the cavity center $y$ axis corresponding to an rf electric field in the $B_3$ direction and, thus, causing $\alpha - \beta$ transitions. Because of the $\beta - e$ mixing required at the position of $B_{rf}$, the static $E_1$ field must be implemented within the cavity.

As discussed in the introduction, the detection of a population of configuration 4 from table 1, which feeds the state $\beta(1,-1)$, is of eminent importance. The spin filter must thus be operated such that only the $| - - \rangle$ configuration survives, requiring a full quenching of the $\alpha$-states. This can be achieved by operating the experiment with reversed fields. The magnetic level splitting will thus be reversed in sign, where $\beta(1,-1)$ becomes $\alpha(1,1)$ and $\beta(0,0)$ becomes $\alpha(1,0)$, respectively (however, we will keep the
sired configuration with subsequent quenching. We will be left with the de-
states. We are left with the two mixing will cause a depopulation of the corresponding \(2s\) field. When passing the fields of 538 and 605 Gauss, Stark depopulation of the two nomenclature defined in the Rabi diagram of fig. 3). A going beam tube
Thermal neutron flux along the FRMII SR6 through-
false effects. Some of them are listed below.
- inefficient depopulation of the unwanted HFS states.
- atomic cascades from \(ns\) states (with \(n > 2\)) which may feed \(2s\) via an intermediate \(p\)-state in an uncontrolled way. See the Appendix for details.
- radiative effects: additional soft photons may distort our arguments on angular momentum conservation and lead to a population of \(np\) states (with \(n > 2\)). This can be calculated in the framework of radiative corrections.

\(\chi\) can be obtained by measuring the ratios \(v_{\alpha\beta} = N_{\alpha10}/N_{\beta00}\) or \(v_{\alpha\alpha} = N_{\alpha11}/N_{\alpha10}\).

Downstream of the cavity and the \(B_3\) field the remaining state-selected H-atoms (e. g., \(2S_1/2, F = 1, m_F = 1\)) are ionized by, e. g., a \(\lambda_1 = 364\) nm laser beam (fig. 1). Alternatively, the ionization may be driven by an optical two-step process, using, e. g., a \(2s \rightarrow 3p\) transition (\(\lambda = 656.28\) nm) and subsequent ionization by a high-power laser (\(\lambda = 816.33\) nm) or an incoherent light source of that wavelength. The produced protons are accelerated by an electrostatic field \(E_2\) to about 20 keV energy and fo-
cussed to a small spot on the beam axis, from where they are deflected by 90° by an analyzing magnetic field and transported by point to point imaging onto a CsI(Tl) scintillator. Using a photomultiplier to detect the light output the protons can be detected with high efficiency as demonstrated in a test experiment, where 18.5 keV protons have been measured (fig. 1). A box-shaped magnetic field \(B_4\) is used for the bending, which provides radial and, owing to non-zero edge angles, also axial focussing of the proton beam.

One interesting addition would be an intense \(\lambda_2 = 243\) nm laser beam crossing the beam line at an angle \(\delta = 100\) mrad for exciting the H-atoms from \(1s\) to \(2s\) by means of two-\(\lambda_2\) photon absorption (fig. 2). Although not considered in the present context, this pumping station would allow to use about 50% of all hydrogen atoms produced in the subsequent hyperfine analysis, as compared to the 10.4% H-atoms from direct \(2s\) population, the basis for all following rate discussions. However, no high-power Lyman-\(\alpha\) laser is presently available.

In order to suppress the configuration 1–4 background due to \(ns\) states with \(n > 2\), the corresponding H-atoms can be ionized by a \(\lambda = 816.33\) nm laser with resonators positioned at both ends of the experiments straight section, i.e. at the left end of the through-going beam tube (fig. 4) and behind the \(E_2\) focus.

### 2.2 Possible event rates

In the following we will evaluate possible event rates. We will assume the set-up, as depicted in fig. 1, to be installed at the new Munich high-flux reactor FRMII. Extrapolations to other neutron sources will be derived at the end.

The neutron decay volume within the beam pipe SR6 of the FRMII has a length \(l = 2 \cdot s_z\), with \(s_z = 4.7\) m and a radius \(r_S = 0.0715\) m. \(s_z\) is the distance between center and far end of SR6. The expected H-rate \(\dot{N}_H\) at one exit of this beam pipe, at a distance \(s_z = 4.7\) m from its center (fig. 1), is given by

\[
\dot{N}_H = BR \int (\Phi(z) \Omega_S(z) \, dz) / 4\pi \cdot 1/\tau_n \cdot 1/v_n \tag{13}
\]

where \(\tau_n = 886\) s is the neutron lifetime, \(\Phi(z)\) the thermal neutron flux along the SR6 z axis (fig. 5) \(11\) \(12\). \(V\) the SR6 volume, \(\Omega_S = A_S/(z-z_S)^2 = \pi r_S^2/(z-z_S)^2\) the solid angle of the hydrogen spectrometer, with \(A_S = 0.016\) m² being the SR6 cross section, \(v_n = 2.2 \cdot 10^3\) m/s is the average thermal-neutron velocity. Thus, the integral can be written as

\[
(1/4\pi) \int \Phi(z) \Omega_S(z) \, dz =
= (1/4\pi) \int_{-z_S}^{z_S} \Phi(z) \cdot A_S^2/(z-z_S)^2 \cdot dz =
= r_S^4 \cdot \pi/4 \cdot \int_{-z_S}^{z_S} \Phi(z)/(z-z_S)^2 \cdot dz. \tag{14}
\]

\(\dot{N}_H = 3 s^{-1}\) is obtained using the thermal neutron flux distribution of refs. \(11\) \(12\) (fig. 5). According to eqs. \(13\) and \(14\) the neutron density \(N_n/V\), i. e. the number of neutrons \(N_n\) in the observed SR6 volume \(V\), is given by

\[N_n/V = \int_0^{s_z} (\Phi(z) \, dz) / (v_n s_z) = 2.4 \cdot 10^8 \text{ cm}^{-3},\]
being about $3 \cdot 10^4$ times larger than the ultra-cold neutron (UCN) density in the recently proposed UCN sources, e.g. ref. [13].

### 2.3 Expected experimental constraints on $g_s$, $g_T$ and $H_\nu$

A small $g_s$ or $g_T$ contribution including the sign may be measured via the population probability $W_3$. In [14] a 1σ confidence level upper limit for the absolute value of $g_s$ is quoted to be $g_s \leq 6 \cdot 10^{-2}$. In order to obtain for $g_T = 0$ an assumed value for $g_s = 6 \cdot 10^{-2}$ with the same accuracy using bound-β decay, the statistical error $(\delta W_3)_{\text{stat}}$ must be 1σ, where $\sigma$ is the standard deviation. The difference

$$\Delta W_3 = \left[ W_3(g_s = 6 \cdot 10^{-2}, g_T = 0) - W_3(g_s = 0, g_T = 0) \right] = 2.54 \cdot 10^{-3}$$

(15)

can be written as $\Delta W_3 = (\delta W_3)_{\text{stat}} + 2 \left( \delta W_3 \right)_\lambda$, where $(\delta W_3)_{\text{stat}}$ is the statistical error, and $(\delta W_3)_\lambda$, the uncertainty due to the error on $\lambda$. Using $d\lambda = 2.9 \cdot 10^{-3}$ [1], we obtain $(\delta W_3)_\lambda = (\delta W_3/d\lambda)d\lambda = -1.1 \cdot 10^{-4}$. Hence, for measuring $W_3(\lambda = 6 \cdot 10^{-2}, g_T = 0) = 3.68 \cdot 10^{-3}$ with 68% confidence, $(\delta W_3)_{\text{stat}} = 2.32 \cdot 10^{-3}$ corresponds to 1σ. $W_3$ can be written as $W_3 = N_3/N$, with $N = \sum_i N_i$ the total number of counts and $N_i$ the individual number of counts for configuration $i$, respectively. $\sigma$ is then $\sigma \approx \sqrt{N_3/N}$ yielding $N = W_3/\sigma^2 = 684$. The measuring time required is $t = N/N_i$, with $N = N_i$ of all hydrogen atoms from bound-beta decay moving along the beam tube were considered, $t$ becomes about 228s. As only the $2s_{1/2}$ states can be used (see the discussion above) with $N = 0.1 N_i$, the necessary time to confirm the present $g_s = 6 \cdot 10^{-2}$ upper limit is 2280s.

The statistical error for $g_s$ can be written as

$$\frac{(\delta g_s)_{\text{stat}}}{(\delta W_3)_{\text{stat}}} = \frac{(\partial g_s}{(\partial W_3)}_{g_s=6 \cdot 10^{-2}, g_T=0} \cdot (\delta W_3)_{\text{stat}}$$

$$= \lambda(\chi^2 + 3)^2 \sqrt{W_3}/N.$$  

(16)

With $(\delta g_s)_{\text{stat}} = 6 \cdot 10^{-3}$ ($\chi \approx 1/\lambda$, $W_3 = 3.683 \cdot 10^{-3}$), $N = 4.4 \cdot 10^4$ results, which corresponds to 40 h measuring time, when only the $2s_{1/2}$ state is used. This reduces the present $g_s$ upper limit by a factor of ten.

Assuming a finite detection efficiency $\epsilon$, these times scale with the factor $1/\epsilon$. Since $\epsilon \geq 1$ seems to be realistic, the $g_s$ upper limit could be lowered by a factor of ten within a few weeks of measuring time.

According to eqs. [5] and [6] with $\zeta = 0$, i.e., $x = y = \eta$, $W_4$ and $H_\nu$ are approximately given by [6]

$$W_4 \approx \frac{\eta^2(1 + \lambda)^2}{2(1 + 3\lambda^2)}.$$  

(17)

$$H_\nu \approx 1 - 2\eta^2 = 1 - \frac{4(1 + 3\lambda^2)}{(1 + \lambda)^2} \cdot W_4.$$  

The statistical error of $H_\nu$ becomes

$$\frac{(\delta H_\nu)_{\text{stat}}}{(\delta W_3)_{\text{stat}}} = \frac{4(1 + 3\lambda^2)}{(1 + \lambda)^2} \cdot \sqrt{W_4}/N.$$  

(18)

Assuming $\eta = 0.036$, we obtain $W_4 = 8.1 \cdot 10^{-6}$ and $H_\nu = 0.997$. This yields $N = 8.3 \cdot 10^3$ for $(\delta H_\nu)_{\text{stat}} = 1 \cdot 10^{-2}$, i.e. 8 h measuring time using only the $2s_{1/2}$ state.

### 2.4 Observables for $\chi$

$\chi$ is given by the ratios $v_{\alpha\beta}$ or $v_{\alpha\alpha}$, defined in section 2.1 according to eqs. [2] to [4] and [10] to [12],

$$v_{\alpha\beta} = \frac{(\chi - 1)^2 \cos^2 \theta + 4 \sin^2 \theta}{(\chi - 1)^2 \sin^2 \theta + 4 \cos^2 \theta},$$

(19)

$$v_{\alpha\alpha} = \frac{(\chi + 1)^2}{(\chi - 1)^2 \cos^2 \theta + 4 \sin^2 \theta},$$

(20)

with $\chi$ either

$$\chi = 1 \pm 2 \sqrt{\sin^2 \theta - v_{\alpha\beta} \cos^2 \theta \over v_{\alpha\beta} \sin^2 \theta - \cos^2 \theta},$$

(21)

or

$$\chi = {-(1 + v_{\alpha\alpha} \cos^2 \theta) \pm 2 \sqrt{v_{\alpha\alpha}(1 - v_{\alpha\alpha} \sin^2 \theta \cos^2 \theta) \over 1 - v_{\alpha\alpha} \cos^2 \theta}}.$$  

(22)

If $g_s$ and $g_T$ are small, the `-` sign holds in eq. [21] and the `+' sign holds in eq. [22].

### 2.5 Necessary power of the lasers

The ionization of $2s$ hydrogen atoms can be obtained in a two-step process. The excitation from the $2s$ to the $3p$ state by a $\lambda_1 = 656.28$ nm laser beam (fig. 1) is selective to the moving H atoms from bound-β decay because of the Doppler shift. The consecutive ionization can be achieved by an intense incoherent 816.33nm light source. The necessary power $Q_1$ in the $\lambda_1$ laser resonator is given by

$$Q_1 = j_1 E_1 \delta X \delta Y.$$  

(23)

$j_1$ is the photon current density, $E_1 = 1.889 eV$ the photon energy, $\delta X = 0.3$ mm and $\delta Y = 0.1$ m the $\lambda_1$ laser-beam waist width in $x$ and $y$ direction, respectively. The requested photon current $j_1$ can be written as

$$j_1 = \frac{-\ln(1 - P_1)}{\sigma_1 \Delta t},$$

(24)

where $P_1(2s \rightarrow 3p)$ is the desired $2s \rightarrow 3p$ excitation probability, $\sigma_1$ the $2s \rightarrow 3p$ photon-absorption cross section.
and $\Delta t$ the exposure time, $\Delta t = \delta X/(v \delta) = 12$ ns. $v$ is the neutron-decay hydrogen-atom velocity and $\delta = 100$ mrad the $\lambda_1$ laser resonator inclination with respect to the beam axis $z$. $\sigma_1$ is given by

$$
\sigma_1 = \frac{\lambda^3 A_{ik} \lambda}{8 \pi c} = 5.08 \cdot 10^{-16} \text{m}^2,
$$

with the Einstein transition coefficient $A_{ik}$

$$
A_{ik} = \frac{6.67 \cdot 10^{-13} \cdot f_{ik}}{(\lambda(\text{mm})^2)}
$$

where the oscillator strength $f_{ik} = 0.641$ for the $2s \rightarrow 3p$ transition. The $\lambda_1$ laser resonator quality $\lambda_1/d\lambda_1$ is matched to the relative width $d\lambda_1/\lambda_1$ which is necessary because of the Doppler broadening $\Delta \lambda$ caused by the thermal motion of the decaying neutron, $d\lambda/\lambda = v_n/c = 0.73 \cdot 10^{-5}$. The Doppler shift $\Delta \lambda$ renders a high selectivity against thermally moving $H$ with $\Delta \lambda/\lambda = 0.83 \cdot 10^{-3}$. A laser power $Q_1 = 2.4$ W is needed for $P_1 = 0.8$.

In order to reduce the background from the population of higher $ns$ states in the neutron decay, a minimum power for the ionization laser with $\lambda = 816.33$ nm of $Q_1 = 216.7$ W is required, as may be derived using eqs. (23) to (26). Considering $W_4$, the efficiency for the removal has to be $P_1 = 1 - 8.1 \cdot 10^{-7}$, thus closer to one by one order of magnitude than the value of $W_4 = 8.1 \cdot 10^{-6}$ to be measured (cf. subsection 23). We assume $d\lambda/\lambda = 0.73 \cdot 10^{-5}$, the oscillator strength $f \approx 1$ for the transition $n \geq 3$ into the continuum, the lifetime of the 3s state $\Delta t = \tau(3s) = 158.4$ ns, the photon energy $E_1 = 1.512$ eV and the waist widths $\delta X = \delta Y = 0.1$ m.

At present, we have only considered $H$ atoms in the 2s state. A much improved count rate could be obtained when also the 1s hydrogen atoms could be used which make up 83% of the resulting hydrogen final states. However, this requires a pumping of all 1s to 2s states of the hydrogen atoms before entering the spin selector. This can in principle be achieved by means of a UV laser beam crossing the hydrogen path under a small angle $\delta$ (cf. fig. 1). The 1s $\rightarrow$ 2s transition requires a two-photon process. We estimated $P(1s \rightarrow 2s)$ from the ionization probability $P(1s \rightarrow p)$, which has been calculated for the three-photon process by [17].

The photon density for the two-photon absorption must be large, requiring a small $\lambda_2$ laser beam waist $\delta X$ (fig. 1). $\delta X$ is limited by the uncertainty relation, constraining the $\lambda_2$ laser beam angular width within the resonator to be $\delta \Theta \geq \lambda_2/(2\pi \delta X)$, yielding $\delta \Theta \geq 0.13$ mrad for $\delta X = 0.3$ mm and the resonator length $(2\delta X/\delta \Theta)$ to be less than 4.6 m. The passage time $\Delta t$ of the H atoms through the $\lambda_2$ laser resonator inclined by $\delta = 100$ mrad with respect to the beam $z$ axis is 12 ns.

The necessary power density $I_2$ for the 1s to 2s transition $\lambda_1$ laser resonator may be estimated from the calculated ionization probability $P(1s \rightarrow p) = 6.5 \cdot 10^{-4}$ for a $\lambda_2$ laser pulse with $\Delta t = 10$ fs duration and $I_2 = 5 \cdot 10^{12}$ W/cm$^2$ [17]. Assuming $P(1s \rightarrow p) = (C_1 \cdot I_2 \cdot \Delta t)^3$, $C_1 = 1.73 \cdot 10^{-4} \text{kg results.} P_2 = P(1s \rightarrow 2s)$ should be given as $P_2 = (C_1 \cdot I_2 \cdot \Delta t)^2$, yielding $I_2 = 1.66 \cdot 10^7$ W/cm$^2$ for $P_2 = 0.1$. The high power is at the present technical limit for a continuous beam covering an area of $\delta X \cdot \delta Y = 0.3$ mm$\cdot$100 mm = 30 mm$^2$.

### 2.6 Background suppression

The fast hydrogen atoms from the bound-$\beta$ decay of the neutron, emitted to the right-hand side of fig. 1, are observed through collimators outside the through-going beam pipe. The collimators have to be placed such that the wall of the inner tube is not visible by the detectors, because backscattered neutralized protons from normal $\beta$ decays would produce a huge background. The setup also minimizes the neutron flux at the end of the beam tube. The protons emitted to the left are deflected by electric or magnetic fields such that they do not hit the back end of the beam tube, where they could be backscattered and neutralized producing a background flux of H atoms in the analyzing direction. High-vacuum conditions ($\leq 10^{-6}$ hPa) are also recommended to avoid neutralization of protons from three-body neutron decay and to avoid scattering of the H atoms from bound-$\beta$ decay. The background, i.e. the probability $P_3$ for neutron or proton scattering at the rest-gas atoms in the beam-tube vacuum should be less than 1%. $P_3$ is given by $P_3 = \sigma_3 n_3 \Delta z \leq 10^{-2}$, where $\sigma_3 = 10^{-16}$ cm$^2$ is a typical scattering cross section, $n_3$ the atom density of the rest gas and $\Delta z = 10$ m the vacuum chamber length, yielding $n_3 \leq 10^{11}$ cm$^{-3}$: the beam line vacuum should be better than $4.1 \cdot 10^{-4}$ Pa.

### 3 Conclusion

In this manuscript we outline an experiment to determine the scalar and tensor contribution to the weak interaction including their signs as well as the constraints on the left-handedness of the neutrino with ten times higher precision than the present value. Currently, the upper limit on $g_S$ is $|g_S| \leq 6 \cdot 10^{-2}$ (C.L. 68%) [13], whereas $|g_T/g_A| \leq 0.09$ (C.L. 95%) [15]. From the measurements of neutron $\beta$-decay asymmetry coefficients upper limits for the left handed and right handed boson mass squared and the absolute value of the boson mixing angle are determined to be $\eta < 0.036$ [5] and $|\zeta| < 0.03$ (C.L. 90%) [6], respectively. In the proposed experiment the upper limits of $g_{S}$ or $g_{T}$ and $\eta$ or $\zeta$ can be reduced by a factor of ten.

The issue is addressed by measuring the spin correlation between the proton and the electron. They are defined with respect to their direction of flight (measuring direction) by selecting different hyperfine states from the bound-$\beta$ decay of the neutron. Although the experimental requirements are challenging, we show a pathway to perform such an experiment at the through-going beam tube of a high-flux reactor like the FRMII. Slightly better conditions do exist at the corresponding beam tube of the high-flux reactor in Grenoble, and the usable neutron density is about four times higher [19]. Next generations of spallation neutron sources like the SNS may be even better suited, if a through-going beam tube is provided,
since higher neutron densities are expected and the pulsed neutron beam makes the detection of hydrogen atoms and the background suppression easier.

4 Appendix

The most important contribution from higher excited states of hydrogen to the 2s hyperfine states are given by the contribution to $W(4s \rightarrow 2s)$ changing $m_S$. This may be written as

$$W(4s \rightarrow 2s) = 2 \cdot W(4s) \cdot W(4s \rightarrow 3p) \cdot W(3p \rightarrow 2s) \cdot W(\Delta j = 0) \cdot W(\Delta j = \pm 1) = 3.07 \cdot 10^{-4}$$

where $W(4s) = 1.3\%$ is the original 4s population.

$$W(4s \rightarrow 3p) = A_{4s3p}/(A_{4s3p} + A_{4s2p}) = 0.416 \text{ and } W(3p \rightarrow 2s) = A_{3p2s}/(A_{3p2s} + A_{3p1s}) = 0.118$$

is the 4s $\rightarrow$ 3p and 3p $\rightarrow$ 2s transition probability, respectively. $A_{4s3p}$ is the corresponding 4s $\rightarrow$ 3p Einstein transition coefficient. The factor two arises from the interchangeability of the time-ordered transitions with $\Delta j = 0$ and $\Delta j = \pm 1$. Assuming the two transition probabilities for the $e^-$ spin quantum number $m_S$ ($m_S$ changing with $\Delta j = 0$ and $m_S$ non-changing with $\Delta j = \pm 1$) to be equal, where $j = l + s$ is the spin orbit coupling realized, the probability for $\Delta j = 0$ and $\Delta j = \pm 1$ photons of the $p \rightarrow s$ transition is given by the multiplicities of the Zeeman-split $p$ and $s$ levels. They are $W(\Delta j = 0) = 2/5$ and $W(\Delta j = \pm 1) = 3/5$.

Correspondingly, the $m_S$ changing background contribution from the 5s state is given by

$$W(5s \rightarrow 2s) = 2 \cdot W(\Delta j = 0) \cdot W(\Delta j = \pm 1) \cdot W(5s) \cdot W(5s \rightarrow 4p) \cdot W(4p \rightarrow 2s) \cdot W(5s \rightarrow 3p) \cdot W(3p \rightarrow 2s) = 2.18 \cdot 10^{-4}$$

with $W(5s) = 0.7\%$,

$$W(5s \rightarrow 4p) = A_{5s4p}/(A_{5s4p} + A_{5s3p} + A_{5s2p}),$$

$$W(4p \rightarrow 2s) = A_{4p2s}/(A_{4p2s} + A_{4p3s} + A_{4p1s}),$$

$$W(5s \rightarrow 3p) = A_{5s3p}/(A_{5s3p} + A_{5s4p} + A_{5s2p}),$$

$$W(3p \rightarrow 2s) = A_{3p2s}/(A_{3p2s} + A_{3p1s}).$$

5 Acknowledgement

We are especially grateful to Prof. J. Byrne for carefully reading the manuscript and many valuable comments. We also want to thank Dr. A. Röhrmoser for calculating the neutron and gamma flux in the FRMII SR6 beam tube.

References

1. S. Eidelman et al., Phys. Lett. B 592, 1 (2004) and 2005 partial update for edition 2006.
2. L. L. Nemenov, Sov. J. Nucl. Phys. 31, 115 (1980).
3. L. L. Nemenov and A. A. Ovchinnikova, Sov. J. Nucl. Phys. 31, 659 (1980).
4. J. Byrne, Eur. Phys. Lett. 56, 633 (2001).
5. A. Gaponenko et al., Phys. Rev. D 71, 071101 (2005).
6. J.R. Musser et al., Phys. Rev. Lett. 94, 101805 (2005).
7. SLD Collaboration, K. Abe et al., Phys. Rev. Lett. 78, 4691 (1997).
8. W. Haeberli, Ann. Rev. Nucl. Sci. 17, 373 (1967), and references quoted therein.
9. W. E. Lamb and R. C. Retherford, Phys. Rev. 79, 549 (1950).
10. F.-J. Eckle, Diploma thesis, Sektion Physik der Universität München, 1982.
11. W. Gaubatz, Technische Universität München (TUM), private communication.
12. I. Altarev et al., internal report, TUM, 2000.
13. U. Trinks et al., Nucl. Instr. and Meth. 440, 666 (2000).
14. E. G. Adelberger et al., Phys. Rev. Lett. 83, 1299 (1999).
15. W. L. Wiese et al., in National Standard Reference Data Series, National Bureau of Standards 4, Category 3 - Atomic and Molecular Properties, 1966.
16. K. Omidvar and A. M. McAllister, Phys. Rev. A 51, 1063 (1995).
17. M. Roth et al., HMI Annual Report 2001, p. 20.
18. A. I. Boothroyd, J. Markey, and P. Vogel, Phys. Rev. C 29, 603 (1984).
19. E. G. Kessler et al., Nucl. Instr. and Meth. A457, 187 (2001).