Fuzzy Topological Systems

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Abstract. Dialectica categories are a very versatile categorical model of linear logic. They have been used to model many seemingly different systems, for example Petri nets and the Lambek calculus. In this note, we expand our previous work on fuzzy Petri nets to deal with fuzzy topological systems. One basic idea is to use as the dualizing object in the Dialectica categories construction, the unit real interval $I = [0, 1]$, which has all the properties of a lineale. The second basic idea is to generalize Vickers’s notion of a topological system.

1 Introduction

Fuzzy set theory and fuzzy logic have been invented by Lotfi ali Asker Zadeh. Zadeh’s theory started from a generalization of the set concept and of the notion of a truth value (for an overview, for example, see [1]). In fuzzy set theory, an element of a fuzzy subset belongs to it to a degree, which is usually a number between 0 and 1. For example, if we have a fuzzy subset of white colors, then all the gray-scale colors are white to a certain degree and, thus, belong to this set with a degree. The following definition by Zadeh himself explains what fuzzy logic is:

Definition 1. Fuzzy logic is a precise system of reasoning, deduction and computation in which the objects of discourse and analysis are associated with information which is, or is allowed to be, imprecise, uncertain, incomplete, unreliable, partially true or partially possible.

Categories, invented by Samuel Eilenberg and Saunders Mac Lane, form a very high-level abstract mathematical theory that unifies all branches of mathematics. Category theory plays a central role in modern mathematics and theoretical computer science, and, in addition, it is used in mathematical physics, in software engineering, etc. Categories have been used to model and study logical systems. In particular, the Dialectica categories of de Paiva [2] are a collection of related categorical models of linear logic [3]. These categories have been used

$^3$ The definition was posted to the bisc-group mailing list on 22/11/2008.