Game theoretic analysis of a three-stage interconnected forward and reverse supply chain

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Received: 5 June 2021 / Accepted: 13 August 2021 / Published online: 8 September 2021
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Abstract
The dynamic economic scenario of today ensures that industrial and environmental policies that contribute to greener supply chain are incorporated. This paper considers an interconnected three-stage forward and reverse supply chain, which provides green products to a green conscious market. The procurement of raw materials is responsible for the first stage of the supply chain; the second manufacturing/remanufacturing process; and the third stage of marketing the products to the consumer. There is one supplier, one manufacturer, and one retailer in the forward supply chain. New raw materials are used in this supply chain, and new products are manufactured and sold. There is also a market for remanufactured products, and in this market, the same retailer also sells remanufactured products. There is one collector, one remanufacturer, and one retailer in the reverse supply chain. From consumers, the collector collects used products, processes and sells the remanufacturable ones to the remanufacturer. If the raw materials supplied by the collector are not adequate to satisfy the demand, the remanufacturer purchases the remainder from the seller. Both the manufacturer and the remanufacturer use green manufacturing processes. Two models, namely centralized model and decentralized model, are formulated. A numerical example is taken to illustrate the two model and perform sensitivity analysis.

Keywords Supply chain management · Green supply chain · Green sensitive · Remanufacturing · Game theory

1 Introduction

Nowaday, humanity is under a wonderful danger due to speedy environmental deterioration, global warming, population density, rapid urbanization and additionally usage of environmental resources. So, many industries are giving a great deal significance to sustainability for the sake of existing and even for future generations. There are a number of
approaches reachable for sustainable improvement in exercise; one of the most effective seems to be green purchasing (Dubey et al., 2013; Kumar et al., 2012). In order to save resources and reduce the environmental impact, green purchasing has become highly essential (Luthra et al., 2016). Sekem is a well-known company which has a major emphasis on sustainability, and Sekem has emerged as a leading producer of organic foods in Egypt. From 2006 to 2011, the company experienced annual growth of 14%. Therefore, the integration of environmental and industrial strategies is now becoming extremely relevant. Although staying competitive in the business, companies also have to implement such plans to minimize negative impacts on the environment. However, if environmental conservation precautions are not taken nowadays, the cost of atmospheric emissions would, throughout the near future, rise even more, even contributing to an unlivable climate. In order to raise demand for green goods and services, this recognition has also impacted customers. As a result, companies are focusing hard to overcome the possible impacts of their products or services. There are many companies that introduce greening practices in their supply chains, Walmart, Dell, Toyota, Honda, Nestle, Coca-Cola, etc., for example (Bag et al., 2017). The above examples demonstrate that in both real life and literature, management of green supply chain (GSCM) is receiving a lot of attention nowadays. In multiple phases of forward and reverse supply chain management, GSCM combines environmental consideration, such as product design, supplier selection and partnership (Van Hoof & Thiell, 2014), manufacturing and remanufacturing processes, forward and reverse logistics (Liu et al., 2020) and management of end-of-life products (Srivastava, 2007). Even with that environmental point of view, economic and social dimensions are often taken into account, and at various levels of supply chain management, it seeks to combine all three aspects (Hart, 1997).

There have been few studies that look at forward as well as reverse supply chains with various connectivity. As far as we know it’s the first analysis of an interconnected forward as well as reverse green supply chain comprising three stages, five participants, and five links. The responses to the relevant research questions are analyzed throughout this study. When does the greening level of a product in a supply chain increase or decrease? Which system possesses the maximum level of product greening? Which system seems to be more affordable to customers? Which system seems to have the maximum demand for the product? When does the product’s demand increase or decrease? Which system significantly increases the amount of profit for the entire supply chain? How is the supply chain able to succeed both economically and ecologically?

The research issue in this work is based on green procurement strategies both forward and reverse and is motivated by such examples already given. Companies in a supply chain work collaboratively with the other participants to focus on making the supply chains more environment friendly and sustainable. A centralised supply chain is one that has an only one decision maker that keeps trying to optimise the entire structure. However, when every individual with this same supply chain wants to consider optimising its own framework, the system is said to be decentralized. In most instances, the members in a supply chain possess competing objectives. As a result, when a member only possesses her strategy, she will respond individually rather than the optimal for the entire supply chain. This study analyzes an interconnected three-stage supply chain forward as well as reverse, which delivers green products to the market of green awareness. The procurement of raw materials is liable for the first stage of the supply chain; the second stage of manufacturing/remanufacturing; as well as the third stage of the marketing of products to the consumer. There is one supplier, one manufacturer, and one retailer in the forward supply chain. New raw materials are used in this supply chain, and new goods are manufactured and sold.
There has also been a demand for remanufactured products and even the same retailer also sells remanufactured products on this market. There is one collector, one remanufacturer, and one retailer in the reverse supply chain. From consumers, the collector stores the products used, processes and delivers to the remanufacturer the re-manufacturable items. Even when the raw materials provided by the collector are not adequate to satisfy the requirement, the remanufacturer purchases the remainder from the seller. Green manufacturing techniques are used by both the manufacturer and the remanufacturer.

The remaining part of this paper is as follows: Three distinct sources of research paper are reviewed in Sect. 2. In Sect. 3, two models, namely centralized model and decentralized model, are formulated, and the solutions obtained from the two models (centralized and decentralized) are compared. In Sect. 4, a numerical example is taken to illustrate the two models and perform sensitivity analysis. Conclusion and recommendations for future research are presented in Sect. 5.

2 Literature review

In this segment, we are going to review the various literature in consideration of three distinct sources of research paper: reverse supply chain, Stackelberg game, and sustainable development in supply chain.

2.1 The literature on reverse supply chain

Throughout the last three decades, supply chain management (SCM) has gained considerable interest from both corporate and academic research. Almost all of the SCM paper focuses upon on forward movement and conversion of products from manufacturers to end users. However, there has not been much interest in the reverse transport of resources from customers to upstream manufacturers. Reverse flow management is an extension to conventional supply chains of reused goods or services that either return to or being discarded by reprocessing companies. Management of the reverse supply chain (RSCM) is identified as the proper management of the sequence of processes needed to acquire and distribute goods from a customer or to regain value. Prahinski and Kocabasoglu (2006) have shown managers can enhance productivity improvements, customer support, negotiation of contracts, product development, post market sales volume with after service through proper planning of the RSC. In recent decades, significant attention has been given to return of products, reuse of products and obsolete products’ re-distribution due to growing environmental concern. Reverse logistics (RL) relating to the distribution operations regarding product returns have gained significant attention and many industries use it to support their customers as a marketing weapon and therefore can deliver significant revenues. Sasikumar and Kannan (2008) have analysed the RL studies and recommended a classification primarily focused on the issues with reverse distribution. The outcome of this study offers a deeper idea of RL and identifies several new approaches for advanced visualization research. de la Fuente et al. (2008) have investigated supply chain management as incorporated in organizations that operate with forward and reverse logistics simultaneously. They have investigated the processes involved, indicating that the IMSCM (Integrated Model for Supply Chain Management) integrated model requires new reverse logistics processes. Mokhtar et al. (2019) have analysed the role of supply chain management styles in the outcome measures of suppliers relevant to reverse processes. In addition, the mediating position in this relationship is explored by two governance
structures (i.e. trust and legal-legitimate power). The results of this studies are explained using systemic equation designing from 190 Malaysian manufacturing companies. The article demonstrated that change and transactional leadership are relevant and constructive contributors to the reverse higher level of performance of suppliers; trust and power initiate these interactions dramatically. Doan et al. (2019) have shown that in developing countries, product flow amounts to reverse supply chain (RSC) facilities, and also other parameters, are uncertain and vague. In this research, a fuzzy approach to address all ambiguous parameters to help electronics enterprises set up more powerful RSCs. In order, to make it more systematic, risk factors are implemented into the model. Gorji et al. (2020) have considered a supply chain including an end-of-life vehicles (ELV) take-back centre, an inspection centre, and a repair centre. Three decision variables seem to be the purchase price of the ELVs, the sales price of the repaired vehicle and the reconstructed level of vehicle. The influence of government subsidies on stability values of its choice variables of the centres in the supply chain of the ELV was evaluated in different scenarios using the game theory technique. Taleizadeh and Sadeghi (2019) have considered two accumulating reverse supply chains which contains one retailer and one manufacturer. One of these chains aims to accelerate the collection process and achieve greater market share through the use of direct and conventional networks. One of the others only uses the conventional channel. To obtain the optimal channels rewards, they have applied three game theory structures, and using numerical analysis, the outcomes of the decision variables as well as the profit function of its participants are compared throughout all three structures.

2.2 The literature on Stackelberg game

Dastidar (2004) put together all the traditional results of Stackelberg quantity games in a homogeneous product party system with concave demand and purely convex costs. They include some new results with Stackelberg price games and compare the equilibrium structure of the quantity games with the price games. Yang and Zhou (2006) have considered the price and quantity preferences of a two-stage system with a manufacturer who provides two successful retailers with a single commodity. They have assumed a Stackelberg structure, wherein the manufacturer functioning as a leader specifies its wholesale price to every retailers and retailers operating like followers individually set their selling prices and corresponding inventory levels under the price structure of the manufacturer. Almehdawe and Mantin (2010) have constructed a model in which the inventory system is controlled by the Stackelberg game under two scenarios. In this system, the supply chain is composed of a single manufacturer and many retailers. They implement the conventional approach in which the manufacturer is the dominant participant in the supply chain, followed by retailers. Include a game-theoretical model consists of three stages (Sheu & Chen, 2012) have analysed the impact of governmental financial interference on rivalry in sustainable development. For government and chain member decisions, they formulated Nash equilibrium solutions. Huang et al. (2013) discuss optimal closed-loop supply chain (CLSC) strategies for two separate recycling streams. In the forward supply chain, the manufacturer sells products via the retailer and the retailer as well as a third party compete to collect used products in the reverse supply chain. With the approach of game theory, they interpret the efficiency of the supply chain function of pricing choices and recycling methods in both of the decentralized and centralized cases. Li and Chen (2018) have considered a retailer’s Stackelberg supply chain and developed a model with the approach of game theory. By introducing a comparatively overall market demand feature, Chaab and
Rasti-Barzoki (2016) analyse supply chain management through cooperative advertising and pricing. They have explained four potential game scenarios, consisting of the nash, retailer Stackelberg, manufacturer Stackelberg, and cooperative games.

2.3 The literature on sustainable development

Green supply chain management has been recognized as an important management style for reducing environmental problems. Using an interpretive structural modelling (ISM) system, Diabat and Govindan (2011) have considered a model of the drivers influencing the effectiveness of green supply chain management. Awan et al. (2018) have investigated the influence of ambidextrous power structure on the connection for both contract management and relational management in terms of social sustainable development. According to this research, ambidextrous decision making and contractual management practices meet the demands of industrial companies while also serving as a way of enhancing sustainable development. Focused on literature and consultations from the GSM with industry leaders, the numerous generators of green supply chain management (GSCM) are described. Li et al. (2016) have considered a dual-channel supply chain in which the manufacturer produces green goods that become eco-friendly. Both for centralized and decentralized scenarios, while using Stackelberg control strategy, they have investigated the pricing and greening strategies for the chain members. In the development of effective supply chain networks, Nagurney and Nagurney (2010) have constructed a comprehensive analysis and research model. They have considered a company involved in evaluating the capacities of its various supply chain operations, i.e. the manufacturing, storage and transport of the commodity to the locations of demand. Golroudbary et al. (2019) have demonstrated that logistic control is a crucial issue for industrial companies. This study demonstrates how to integrate two simulation methodologies to construct a structure in new ways, resulting in a system which is effective for logistics services. The research design provides decision-makers with such an alternative method of managing logistics. The natural ecosystem has a subdiscipline called industrial ecology. The goals of industrial ecology are to reconstruct its industrial ecosystem in terms of controlling and developing straight to closed-loop commercial manufacturing and consumption processes. Awan (2020) has shown that industrial ecology highlights the importance of an idea generation approach in decision-making, considering resource utilisation in approaches that indicate systemic limitations. The suggested research could help to clarify the procedure of promoting and reinforcing industrial ecology in business firms, as well as the processes in which enterprises in systems can realise social, environmental and financial advantages. The company is considered to have been a decision-maker with multiple requirements who not only seeks to minimize the overall costs associated with design/construction and service, but also to minimize the generated emissions. The reduction of carbon emissions is a hot issue for environmental protection, and the implementation of limits is considered an appropriate way to improve the environment. Sustainable products indicate, as per practical world standards, that the manufacturing processes encourage the impact on carbon emissions and actually respond to consumer demand. Dong et al. (2016) have studied sustainable investment in sustainable goods with awareness of pollution control for decentralized and centralized supply chains. Using the innovative valence theory (VT) approach, Dhir et al. (2021) have examined how customers consider their choices for recycling e-waste. The research results have significant importance for decision-makers, governments, and academics interested in understanding more about consumer attitudes toward e-waste.
recycling. Within this dual supply chain with three echelons comprising one collector, one recycler, and one manufacturer, Jafari et al. (2017) have considered the waste recycling process. The game-theoretical models are developed among the members under its different power structures. Then, the solutions of equilibrium are derived, and different leadership perspectives are identified. They have shown that the manufacturer gets a larger advantage when the collector and recycler have similar decision-making powers than when they make decisions with individuals who have different powers. In the case of environmental contamination, prevention seems preferable to cure. Awan et al. (2019) have explained that industrial enterprises should consider zero-pollution choice criteria and environmentally friendly material handling as significant marketing tactics. Making the supply chain of a greener product, Ghosh et al. (2021) have formulated an organized structure on the advanced payment policy of the manufacturer and the retailer’s trade credit facility. They have proposed a model that enhances the sales effort of the retailer, the wholesale price required by the manufacturer, the green procurement level, and also the retailers’ selling price.

3 Model formulation

In this paper, we consider a three-stage interconnected supply chain in both directions (forward and reverse), wherein green products, both new as well as remanufactured are shipped to the marketplace. Figure 1 represents the three-stage interconnected forward and reverse supply chain. We have used some notation throughout the paper which are given in the Table 1. In the forward flow, the supply chain is composed of a single raw material supplier, a single manufacturer, and a single retailer. At a unit price of $c_1$, new raw materials are purchased and processed by the supplier and sold at such unit price of $s_1$ to the manufacturer. The manufacturer produces new product using green manufacturing process with a greening level of $\theta_1$, where $\theta_1 \geq 0$. The manufacturer takes an increasing quadratic function $I\theta^2_2$ as the green investment cost, where the green investment parameter is $I$ and $I > 0$. Compared with green investment costs, all other production costs are
ignored. At such wholesale price per unit of $w_1$, the manufacturer sells new products to the retailer; then, the retailer sells the new products to the customers at a price of $p_1$ per unit.

In the reverse supply chain, the retailer also sells remanufactured products to the same market. In the reverse flow, the supply chain is composed of a single collector, a single remanufacturer, and a single retailer. From customers, used products are collected by the collector and processed at a unit cost $c_2$ and it is sold at such a price per unit $s_2$ to the remanufacturer. The probability to meet the demand of the raw materials of the remanufacturer supplied by the collector is $\beta$, where $0 < \beta \leq 1$. If the collector is unable to satisfy the remanufacturer’s requirement, the remanufacturer purchases the remainder from the seller at such price per unit of $s_1$. Then, the probability of buying raw materials from seller by the remanufacturer is $1 - \beta$. The remanufacturer produces product using green manufacturing process with a greening level of $\theta_2$, where $\theta_2 \geq 0$. The remanufacturer takes an increasing quadratic function $I\theta_2^2$ as the green investment cost, where $I$ indicates the parameter for green investment and $I > 0$. Compared with green investment costs, all other remanufacturing expenditures are ignored. At a wholesale price of $w_2$ per unit, the retailer purchases products from the remanufacturer; the remanufactured products are sold to the consumer at a price of $p_2$ per unit by the retailer.

### Table 1 Model notation

| Notations | Definition |
|-----------|------------|
| Parameters | |
| $a$ | Total market potential |
| $b$ | Price sensitivity of consumer demand |
| $d_1$ | Sensitivity of consumers to new product’s greening level |
| $d_2$ | Sensitivity of consumers to remanufactured product’s greening level |
| $c_1$ | The supplier’s unit cost |
| $c_2$ | The collector’s unit cost |
| $I$ | Green investment parameter |
| $\beta$ | Probability of receiving adequate raw materials from the collector to satisfy the demand of the remanufacturer |
| Decision variables | |
| $D_1$ | Demand in the market for new products |
| $D_2$ | Demand in the market for remanufactured products |
| $p_1$ | The retailer’s unit selling price for new products |
| $p_2$ | The retailer’s unit selling price for remanufactured products |
| $s_1$ | The supplier’s unit selling price |
| $s_2$ | The collector’s unit selling price |
| $w_1$ | Manufacturer’s unit wholesale price |
| $w_2$ | Remanufacturer’s unit wholesale price |
| $\theta_1$ | Manufacturer’s greening level |
| $\theta_2$ | Remanufacturer’s greening level |
| $Q$ | Total sales volume |
| $\pi_R$ | Profits of the supply chain member, where $R$ denotes retailer; $M$ denotes manufacturer; $RM$ denotes remanufacturer; $S$ denotes supplier; and $C$ denotes collector |
| $\pi_{SC}$ | Profit of the entire supply chain |
Market demand takes the following functional form:

\[ D_j = a - bp_j + d_j \theta_j, \quad j = 1, 2. \]  

(1)

where \( a \) (\( > 0 \)) is the total market potential, \( b \) (\( > 0 \)) is the price sensitivity of consumer demand. The consumer sensitivity to the greening level of new products is \( d_1 \), and the consumer sensitivity to the greening level of re-manufactured products is \( d_2 \) and \( d_j > 0 \) for \( j = 1, 2 \).

Some of assumptions in this paper are listed below:

1. It is expected that the reverse supply chain will have more positive impact than the forward supply chain on the environment. It is also believed that consumers are more susceptible than new products to the greening level of re-manufactured products, i.e. \( d_2 > d_1 \).

2. \( c_2 \leq c_1 \), i.e. the unit cost of the collector, is considered to be less than or equal to the seller’s unit cost.

3. To avoid the trivialities, the following assumptions are made throughout the paper.

   (i) \( 2bl - d_1^2 \geq 0 \) to ensure that \( p_1 > 0 \)
   
   (ii) \( 2bl - d_2^2 \geq 0 \) to ensure that \( p_2 > 0 \)
   
   (iii) \( a \geq bc_1 \) to ensure that \( \theta_1 \geq 0, \theta_2 \geq 0 \)

3.1 The centralized model

Let us consider maximizing the profit supply chain model with the retailer (R), the manufacturer (M), the remanufacturer (RM), the supplier (S), and the collector (C). Decision variables are the retailer’s unit sale rates for new and remanufactured products; and the greening levels of the manufacturer as well as remanufacturer. The supply chain members’ profit functions are as follows.

\[ \pi_R(p_1, p_2) = (p_1 - w_1)D_1 + (p_2 - w_2)D_2 \]  

(2)

\[ \pi_M(\theta_1, w_1) = (w_1 - s_1)D_1 - I\theta_1^2 \]  

(3)

\[ \pi_{RM}(\theta_2, w_2) = (w_2 - \beta s_2 - (1 - \beta)s_1)D_2 - I\theta_2^2 \]  

(4)

\[ \pi_S(s_1) = (s_1 - c_1)(D_1 + (1 - \beta)D_2) \]  

(5)

\[ \pi_C(s_2) = (s_2 - c_2)\beta D_2 \]  

(6)

Therefore, the entire supply chain’s (SC) profit function is

\[ \pi_{SC}(p_1, p_2) = (p_1 - c_1)D_1 + (p_2 - \beta c_2 - (1 - \beta)c_1)D_2 - I(\theta_1^2 - \theta_2^2) \]  

(7)

For maximizing the profit function of the overall supply chain given in Eq. (7), the unique global optimal solution is are as follows.

\[ p_1^* = \frac{2al + c_1(2bl - d_1^2)}{4bl - d_1^2} \]  

(8)
The optimal values of demands for the new products and remanufactured products are as follows.

\[ D_1^* = \frac{2bl(a - bc_1)}{4bl - d_1^2} \] (12)

\[ D_2^* = \frac{2bl(a - \beta bc_2 - (1 - \beta)bc_1)}{4bl - d_2^2} \] (13)

The optimal profit of the entire supply chain is given by,

\[ \pi_{SC}^* = \frac{1}{2} \left[ \frac{(a - bc_1)^2}{4bl - d_1^2} + \frac{(a - \beta bc_2 - (1 - \beta)bc_1)^2}{4bl - d_2^2} \right] \] (14)

**Proof** See 5.1 “Appendix 1”.

It is easy to understand from the solution that the manufacturer’s (remanufacturer’s) greening level and the retailer’s unit selling price for new (remanufactured) products increase, while customer sensitivity to the greening level of new as well as remanufactured products increases. Moreover, the demand for new (remanufactured) products increases, and the entire supply chain obtains greater profit as consumers become more receptive to new (remanufactured) product’s greening levels. Contrarily, if the price sensitivity of consumer demand and the green investment parameter increase, then the retailer’s selling price per unit for new (remanufactured) products, the demands of both products, the manufacturer’s (remanufacturer’s) greening level and the overall supply chain profit decrease. It turns out that the remanufacturer decides a higher greening level as the sufficiency of the collector to satisfy the demand of the remanufacturer rises, and the customers can purchase the remanufactured products for a cheaper price. As a consequence, the demand for remanufactured products increases and the entire supply chain obtains more profit.

### 3.2 The decentralized model

Each supply chain member tries to maximize their own profit in the decentralized model. Equations (2)–(6) are the profit functions of the retailer, the manufacturer, the remanufacturer, the supplier and the collector, respectively. Considering game theoretical approach, in the first stage, the supplier determines the selling prices of their unit to the manufacturer and the collector determines the selling prices of their unit to the remanufacturer at the same time, i.e. with the expectation of manufacturer’s action the supplier determines \( s_1 \) and collector determines \( s_2 \) in expectation of remanufacturer’s action. In the
second step, with the expectation of retailer’s actions the manufacturer and the remanufacturer determine their unit wholesale prices and greening levels simultaneously. The manufacturer determines greening levels and unit wholesale prices $\theta_1$ and $w_1$; the remanufacturer determines greening levels and unit wholesale prices $\theta_2$ and $w_2$. In the final step, the retailer decides the unit sale prices for new and remanufactured products, $p_1$ and $p_2$, respectively.

Using backward induction method, we have solved this model. After computing hessian matrix, we get retailer profit function as specified in Eq. (2) is strictly concave. We get the retailer’s best response functions by using first order conditions with respect to $p_1$ and $p_2$ simultaneously.

\[ p_1^d(\theta_1, w_1) = \frac{a + d_1 \theta_1 + bw_1}{2b} \]  
\[ p_2^d(\theta_2, w_2) = \frac{a + d_2 \theta_2 + bw_2}{2b} \]  
where superscript stands for the decentralised case.

Then, we substitute these values into the profit functions of the manufacturer and remanufacturer, which are given in Eqs. (3) and (4), respectively.

By computing hessian matrix we get, manufacturer’s profit function is strictly concave. Using first order conditions with respect to $\theta_1$ and $w_1$ simultaneously, we get the best response functions of the manufacturer as follows:

\[ \theta_1^d(s_1) = \frac{d_1(a - bs_1)}{8bI - d_1^2} \]  
\[ w_1^d(s_1) = \frac{4aI + s_1(4bI - d_1^2)}{8bI - d_1^2} \]  
Similarly, remanufacturer’s profit function is strictly concave. Using first order conditions with respect to $\theta_2$ and $w_2$ simultaneously, we get the best response functions of the remanufacturer as follows:

\[ \theta_2^d(s_1, s_2) = \frac{d_2(a - \beta bs_2 - (1 - \beta)bs_1)}{8bI - d_2^2} \]  
\[ w_2^d(s_1, s_2) = \frac{4aI + (\beta s_2 + (1 - \beta)s_1)(4bI - d_2^2)}{8bI - d_2^2} \]  
In the final step, we put the Eqs. (15)–(20) into the seller’s profit function given in Eq. (5) and Eqs. (16), (19), (20) into the collector’s profit function given in Eq. (6).

Similarly, we get seller’s and collector’s profit functions are concave. Using first order condition with respect to $s_1$ of Eq. (5) and first order condition with respect to $s_2$ of Eq. (6) simultaneously, we get

\[ s_1^d = \frac{2(8bI - d_2^2)(a + bc_1) + (8bI - d_1^2)(1 - \beta)(a - \beta bc_2 + 2(1 - \beta)bc_1)}{b(4(8bI - d_2^2) + 3(8bI - d_1^2)(1 - \beta)^2)} \]
\[
s^d_2 = \frac{a + \beta bc_2 - (1 - \beta) \left[ \frac{2(8bl - d^d_2)(a + bc_1) + (8bl - d^d_2)(1 - \beta)(a - \beta bc_2 + 2(1 - \beta)bc_1)}{4(8bl - d^d_2) + 3(8bl - d^d_2)(1 - \beta)^2} \right]}{2\beta b}
\] (22)

For the retailer, the manufacturer, the remanufacturer, the seller, and the collector, the unique global optimal solutions for decentralized models are as follows. The supplier and collector decide \( s^d_1 \) and \( s^d_2 \) according to Eqs. (20) and (21). After that the manufacturer determines \( \theta^d_1(s_1) \) and \( w^d_1(s_1) \) and the remanufacturer determines \( \theta^d_2(s_1, s_2) \) and \( w^d_2(s_1, s_2) \) according to Eqs. (17)–(20). In the end, retailer decides \( p^d_1(\theta_1, w_1) \) and \( p^d_2(\theta_2, w_2) \) according to Eqs. (15) and (16). The optimal values of demands for the new products and remanufactured products as follows.

\[
D^d_1 = \frac{2bl(a - bs^d_1)}{8bl - d^d_1}
\] (23)

\[
D^d_2 = \frac{2bl(a - b(\beta s^d_2 + (1 - \beta)s^d_1))}{8bl - d^d_2}
\] (24)

The optimal profit of the overall supply chain is

\[
\pi^d_{sc} = \frac{1}{2} \left( \frac{(a - bs_1)\psi_1}{(8bl - d^d_1)^2} + \frac{(a - \beta bs_2 - (1 - \beta)bs_1)\psi_2}{(8bl - d^d_2)^2} \right)
\] (25)

where

\[
\psi_1 = a(12bl - d^d_1) - 2bc_1(8bl - d^d_1)^2 + b(4bl - d^d_1)s_1
\]

\[
\psi_2 = a(12bl - d^d_2) - 2bc_1(8bl - d^d_2)(1 - \beta)
\]

\[
+ b(4bl - d^d_2)(s_1(1 - \beta) - (2c_2 - s_2)\beta) - 8b^2\beta c_2
\]

\textbf{Proof} \ See 5.1 “Appendix 2”.

In the forward supply chain, it can be noticed that \( c_1 \leq s^d_1 \leq w^d_1 \leq p^d_1 \). In a similar way in the reverse supply chain \( c^d_2 \leq s^d_2 \) and \( \beta s^d_2 + (1 - \beta)s^d_1 \leq w^d_2 \leq p^d_2 \), where \( \beta s^d_2 + (1 - \beta)s^d_1 \) is the purchasing price of the remanufacturer. \( \square \)

### 3.3 Comparison between the solutions of the centralized model and decentralized model

After obtaining centralized and decentralized solutions, comparing it gives the following observations:

1. The greening level of the manufacturer in the centralized solution is greater than or equal to the greening level in the decentralized solution, i.e. \( \theta^c_1 \geq \theta^d_1 \)

   \textbf{Proof}: \( c_1 \leq s^d_1 \Rightarrow a - bc_1 \geq a - bs^d_1 \) (Since \( b > 0 \))

   Since, \( 4bl - d^d_1 < 8bl - d^d_1 \) and \( d^d_1 > 0 \)

   Therefore, \( \frac{d_1(a - bc_1)}{4bl - d^d_1} \geq \frac{d_1(a - bs^d_1)}{8bl - d^d_1} \) i.e. \( \theta^c_1 \geq \theta^d_1 \)

2. The greening level of the remanufacturer in the centralized solution is greater than or equal to the greening level in the decentralized solution, i.e. \( \theta^c_2 \geq \theta^d_2 \)

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Proof: \( c_1 \leq s_1^d \text{ and } c_2 \leq s_2^d \Rightarrow a - \beta bc_2 - (1 - \beta)bc_1 \geq a - \beta bs_2^d - (1 - \beta)bs_1^d \) (Since \( b > 0 \))

Since, \( 4bl - d_2^2 < 8bl - d_2^2 \) and \( d_2 > 0 \)

Therefore, \( \frac{d_2 \{a - bc_2(1 - \beta) - bc_1\}}{4bl - d_2^2} \geq \frac{d_2 \{a - bs_2^d(1 - \beta) - bs_1^d\}}{8bl - d_2^2} \) i.e. \( \theta_2^* \geq \theta_2^d \)

3. The unit selling price in the decentralized solution for the retailer’s new products is greater than or equal to the unit selling price in the centralized solution, i.e. \( p_1^d \geq p_1^* \)

Proof: \( p_1^* = \frac{2al + c_1 (2bl - d_1^2)}{4bl - d_1^2} \)

\( p_1^d = \frac{a + d_1 \theta_1^d + bn_1^d}{2b} \) where \( \theta_1^d = \frac{d_1 (a - bs_1^d)}{8bl - d_1^2} \) and \( w_1^d = \frac{4a + s_1^d (4bl - d_1^2)}{8bl - d_1^2} \)

Therefore,

\[
p_1^d = \frac{a + d_1 \frac{d_1 (a - bs_1^d)}{8bl - d_1^2} + b \frac{4a + s_1^d (4bl - d_1^2)}{8bl - d_1^2}}{2b} = \frac{6a + s_1^d (2bl - d_1^2)}{8bl - d_1^2}
\]

Now,

\[
p_1^d - p_1^* = \frac{6a + s_1^d (2bl - d_1^2)}{8bl - d_1^2} - \frac{2al + c_1 (2bl - d_1^2)}{4bl - d_1^2}
\]

\[
= \frac{(2bl - d_1^2) [4I(a - bc_1) + (s_1^d - c_1) (4bl - d_1^2)]}{(8bl - d_1^2) (4bl - d_1^2)}
\]

As \( 2bl - d_1^2 \geq 0, a \geq bc_1 \) and \( s_1^d \geq c_1 \)

Therefore, \( p_1^d - p_1^* \geq 0, \) i.e. \( p_1^d \geq p_1^* \)

4. The unit selling price in the decentralized solution for the retailer’s remanufactured products is greater than or equal to the unit selling price in the centralized solution, i.e. \( p_2^d \geq p_2^* \)

Proof: \( p_2^* = \frac{2al + (bc_2 + (1 - \beta)c_1) (2bl - d_2^2)}{4bl - d_2^2} \)

\( p_2^d = \frac{a + d_1 \theta_2^d + bn_2^d}{2b} \) where, \( \theta_2^d = \frac{d_2 (a - bs_2^d(1 - \beta)bc_1)}{8bl - d_2^2} \) and \( w_2^d = \frac{4a + (bs_2^d + (1 - \beta)s_1^d) (4bl - d_2^2)}{8bl - d_2^2} \)

Therefore,

\[
p_2^d = \frac{a + \frac{d_2 (a - bs_2^d(1 - \beta)bc_1)}{8bl - d_2^2} + b \frac{4a + (bs_2^d + (1 - \beta)s_1^d) (4bl - d_2^2)}{8bl - d_2^2}}{2b} = \frac{6a + (bs_2^d + (1 - \beta)s_1^d) (2bl - d_2^2)}{8bl - d_2^2}
\]

Now,

\[
p_2^d - p_2^* = \frac{6a + (bs_2^d + (1 - \beta)s_1^d) (2bl - d_2^2)}{8bl - d_2^2} - \frac{2al + (bc_2 + (1 - \beta)c_1) (2bl - d_2^2)}{4bl - d_2^2}
\]

\[
= \frac{(2bl - d_2^2) [4I((a - bc_1) + \beta(c_1 - c_2)) + (\beta s_1^d - c_2) + (1 - \beta)(s_1^d - c_1) (4bl - d_2^2)]}{(8bl - d_2^2) (4bl - d_2^2)}
\]

As \( 2bl - d_2^2 \geq 0, a \geq bc_1, c_2 \geq c_2, s_2^d \geq c_2 \) and \( s_1^d \geq c_1 \)

Therefore, \( p_2^d - p_2^* \geq 0 \) i.e. \( p_2^d \geq p_2^* \)

5. In the centralized solution, new product demand is greater than or equal to demand in the decentralized solution, i.e. \( D_1^* \geq D_1^d \)

\( \text{Springer} \)
Proof: $c_1 \leq s_1^d \Rightarrow a - bc_1 \geq a - bs_1^d$ (Since $b > 0$)

Since $4bI - d_1^2 < 8bI - d_2^2$, $b > 0, I > 0$

Therefore, $\frac{2bI(a - bc_1)}{4bI - d_1^2} \geq \frac{2bI(a - bc_1^d)}{8bI - d_2^2}$, i.e. $D_1^e \geq D_1^d$

6. In the centralized solution, remanufactured product demand is greater than or equal to demand in the decentralized solution, i.e. $D_2^c \geq D_2^d$

Proof: $D_2^c = \frac{2bI(a - \beta bc_2 - (1 - \beta)bc_1)}{4bI - d_2^2}$ and $D_2^d = \frac{2bI(a - \beta bs_2 - (1 - \beta)bs_1^d)}{8bI - d_2^2}$

Now,

$$D_2^c - D_2^d = \frac{2bI(a - \beta bc_2 - (1 - \beta)bc_1) - 2bI(a - \beta bs_2 - (1 - \beta)bs_1^d)}{8bI - d_2^2} = \frac{2b^2I[4I((a - bc_1) + \beta(c_1 - c_2)) + (\beta(s_2^d - c_2) + (1 - \beta)(s_1^d - c_1))(4bI - d_2^2)]}{(8bI - d_2^2)(4bI - d_2^2)}$$

As $2bI - d_2^2 \geq 0$, $a \geq bc_1, c \geq c_2$, $s_2^d \geq c_2$ and $s_1^d \geq c_1$

Therefore, $D_2^c - D_2^d \geq 0$, i.e. $D_2^c \geq D_2^d$

7. In the centralized solution, the profit of the entire supply chain is greater than or equal to the profit in the decentralized solution, i.e. $\pi_{SC}^* \geq \pi_{SC}^d$

Proof:

$$\pi_{SC}^* = I \left[ \frac{(a - bc_1)^2}{4bI - d_1^2} + \frac{(a - \beta bc_2 - (1 - \beta)bc_1)^2}{4bI - d_2^2} \right]$$

$$\pi_{SC}^d = I \left[ \frac{(a - bs_1)\psi_1}{(8bI - d_1^2)^2} + \frac{(a - \beta bs_2 - (1 - \beta)bs_1^d)\psi_2}{(8bI - d_2^2)^2} \right]$$

where

$$\psi_1 = a(12bI - d_1^2) - 2bc_1(8bI - d_1^2) + b(4bI - d_1^2)s_1$$

$$\psi_2 = a(12bI - d_2^2) - 2bc_1(8bI - d_2^2)(1 - \beta) + b((4bI - d_2^2)(s_1(1 - \beta) - (2c_2 - s_2)\beta)) - 8b^2I\beta c_2$$

Now,
\[ \pi^*_{\text{SC}} - \pi^d_{\text{SC}} = I \left( \frac{(a - b c_1)^2}{4bl - d_z^2} + \frac{(a - b) c_2 - (1 - \beta) b c_1)^2}{4bl - d_z^2} \right) \]

\[ - I \left( \frac{(a - b c_1)(a)(2bl - d_z^2) - 2bc_1(8bl - d_z^2) + b(4bl - d_z^2) (s_1)}{(8bl - d_z^2)^2} \right) \]

\[ + \frac{(a - b) c_2 - (1 - \beta) b c_1}{(8bl - d_z^2)^2} \times \frac{(a)(2bl - d_z^2) - 2bc_1(8bl - d_z^2)}{(8bl - d_z^2)^2} \]

\[ (1 - \beta) + b((4bl - d_z^2)(s_1(1 - \beta) - (2c_2 - s_2)\beta)) - b^2fCfc_2 \]

\[ = - (a - b c_1)^2 d_z^2 + \frac{2b(a - b c_1)f}{(4bl - d_z^2)^2} \left( -c_1 + \frac{c_2 - 2a - b c_1}{d_z^2} \right) \]

\[ + d_z^2 f(a - b c_1)^2 \left( \frac{8bl - d_z^2}{d_z^2} + \frac{2b(a - b c_1)(-1 + \beta - b c_1)^2}{(4bl - d_z^2)^2} \right) \]

\[ + d_z^2 f(a - b c_1)^2 \left( \frac{8bl - d_z^2}{d_z^2} + \frac{2b(a + b c_1)(-1 + \beta - b c_1)^2}{(4bl - d_z^2)^2} \right) \]

\[ + \frac{d_z^2 f(a - b c_1)^2}{(4bl - d_z^2)^2} \left( \frac{8bl - d_z^2}{d_z^2} + \frac{2b(a + b c_1)(-1 + \beta - b c_1)^2}{(4bl - d_z^2)^2} \right) \]

\[ = I \left( \frac{(a - b c_1)^2}{(4bl - d_z^2)^2} + \frac{4b(a - b c_1)^2}{(4bl - d_z^2)^2} \right) \]

\[ + \frac{2b(a - b c_1)(a)(6sl + c_1(d_2 - 8bl) - d_z^2 + 2bl_1)}{(d_2 - 8bl)^2} \]

\[ + \frac{2b(a + b c_1)(a)(-1 + \beta - b c_1)^2}{(d_2 - 8bl)^2} \]

\[ + \frac{2b(a + b c_1)(a)(-1 + \beta - b c_1)^2}{(d_2 - 8bl)^2} \]

\[ + \frac{2b(a + b c_1)(a)(-1 + \beta - b c_1)^2}{(d_2 - 8bl)^2} \]

\[ + \frac{(2b(a + b c_1)(a)(-1 + \beta - b c_1)^2}{(d_2 - 8bl)^2} \]

\[ + \frac{(2b(a + b c_1)(a)(-1 + \beta - b c_1)^2}{(d_2 - 8bl)^2} \]

\[ + \frac{(2b(a + b c_1)(a)(-1 + \beta - b c_1)^2}{(d_2 - 8bl)^2} \]

Since, \(4bl - d_z^2 > 0\) and \(4bl - d_z^2 > 0\)

Therefore, \(\pi^*_{\text{SC}} - \pi^d_{\text{SC}} \geq 0\), i.e. \(\pi^*_{\text{SC}} \geq \pi^d_{\text{SC}}\)
4 Results and discussion

In order to understand the outcomes, this section provides a numerical analysis. Sensitivity analysis is carried out for the profit functions of the members of the green supply chain, product’s greening level, unit selling price and market demand. The initial parameters are adjusted as follows: \( a = 200, b = 2.5, c_1 = 10, c_2 = 8, I = 40, \beta = 0.90, d_1 = 2 \) and \( d_2 = 3 \). The values of the initial parameters are changed for sensitivity analysis in such a way that all the assumptions are fulfilled in the paper. Table 2 represents the change in the level of greening and retailer selling prices w.r.t. % change of \( a, b, c_1, c_2, I \) and \( \beta \). Table 3

| Parameter | % Change | \( \theta_1 \) | \( \theta_2 \) | \( p_1 \) | \( p_2 \) |
|-----------|----------|----------------|----------------|----------------|----------------|
| \( a \)   | -20      | 1.130430, 0.255732 | 1.75318, 0.377122 | 39.1304, 58.3739 | 38.8544, 58.591 |
|           | -10      | 1.30435, 0.295334  | 2.00726, 0.431436 | 43.3043, 65.5026 | 43.2102, 65.812 |
|           | 0        | 1.47826, 0.334937  | 2.26134, 0.485749 | 47.4783, 72.6314 | 47.5659, 73.033 |
|           | 10       | 1.65217, 0.37454   | 2.51543, 0.540062 | 51.6522, 79.7601 | 51.9216, 80.254 |
|           | 20       | 1.82609, 0.414143  | 2.76951, 0.594376 | 55.8261, 86.8889 | 56.2773, 87.475 |
| \( b \)   | -20      | 2.51811, 0.563935  | 3.36855, 0.708655 | 64.5595, 100.655 | 66.5003, 101.719 |
|           | -10      | 2.19165, 0.492712  | 3.01408, 0.434697 | 43.5591, 66.1082 | 43.327, 66.3818  |
|           | 0        | 1.47826, 0.334937  | 2.26134, 0.485749 | 47.4783, 72.6314 | 47.5659, 73.033 |
|           | 10       | 1.31496, 0.298403  | 2.01408, 0.434697 | 43.5591, 66.1082 | 43.327, 66.3818  |
|           | 20       | 1.17986, 0.268057  | 1.81103, 0.392453 | 40.3165, 60.6801 | 39.8462, 60.8546 |
| \( c_1 \) | -20      | 1.53043, 0.34783   | 2.27659, 0.487592 | 46.3304, 72.3478 | 47.3472, 73.0065 |
|           | -10      | 1.50435, 0.341383  | 2.26897, 0.48667  | 46.9043, 72.4896 | 47.4566, 73.0198 |
|           | 0        | 1.47826, 0.334937  | 2.26134, 0.485749 | 47.4783, 72.6314 | 47.5659, 73.033 |
|           | 10       | 1.45217, 0.328491  | 2.25372, 0.484828 | 48.0522, 72.7732 | 47.6752, 73.0462 |
|           | 20       | 1.42609, 0.322045  | 2.2461, 0.483906  | 48.6261, 72.915  | 47.7845, 73.0594 |
| \( c_2 \) | -20      | 1.47826, 0.334263  | 2.302, 0.495383   | 47.4783, 72.6462 | 46.9828, 72.8948 |
|           | -10      | 1.47826, 0.3346    | 2.28167, 0.490566 | 47.4783, 72.6388 | 47.2743, 72.9639 |
|           | 0        | 1.47826, 0.334937  | 2.26134, 0.485749 | 47.4783, 72.6314 | 47.5659, 73.033 |
|           | 10       | 1.47826, 0.335274  | 2.24102, 0.480932 | 47.4783, 72.624  | 47.8574, 73.1021 |
|           | 20       | 1.47826, 0.335612  | 2.22069, 0.476115 | 47.4783, 72.615  | 48.149, 73.1712  |
| \( I \)   | -20      | 1.86813, 0.420735  | 2.89095, 0.613693 | 47.8681, 72.7634 | 48.4473, 73.302 |
|           | -10      | 1.65049, 0.372965  | 2.53768, 0.542276 | 47.6505, 72.6899 | 47.9527, 73.1518 |
|           | 0        | 1.47826, 0.334937  | 2.26134, 0.485749 | 47.4783, 72.6314 | 47.5659, 73.033 |
|           | 10       | 1.33858, 0.303946  | 2.03928, 0.439894 | 47.3386, 72.5837 | 47.255, 72.9366  |
|           | 20       | 1.22302, 0.278204  | 1.85693, 0.40195  | 47.223, 72.5441  | 46.9997, 72.8568 |
| \( \beta \) | -20      | 1.47826, 0.331612  | 2.24102, 0.435613 | 47.4783, 72.7045 | 47.8574, 73.7521 |
|           | -10      | 1.47826, 0.331352  | 2.25118, 0.460391 | 47.4783, 72.7103 | 47.7117, 73.3967 |
|           | 0        | 1.47826, 0.334937  | 2.26134, 0.485749 | 47.4783, 72.6314 | 47.5659, 73.033 |
|           | 10       | 1.47826, 0.342599  | 2.27151, 0.511047 | 47.4783, 72.4628 | 47.4201, 72.6701 |
|           | 20       | 1.47826, 0.354352  | 2.28167, 0.535585 | 47.4783, 72.2042 | 47.2743, 72.3182 |
represents the change of greening levels and retailer selling prices w.r.t. % change of $d_1$ and $d_2$. The change of unit selling prices of manufacturer, remanufacturer, seller, collector and market demand for both new and remanufactured products w.r.t. % change of $a, b, c_1, c_2, I$ and $\beta$ are presented in Table 4. In Table 5, the change of unit selling prices of manufacturer, remanufacturer, seller, collector and market demand for both new and remanufactured products w.r.t. % change of $d_1$ and $d_2$ are presented. The change of different profit functions w.r.t. % change of $d_1$ and $d_2$ are presented. In Table 6, the change of different profit functions w.r.t. % change of $d_1$ and $d_2$ are presented. It’s in Figs. 2, 3, 4, 5, we investigate the influence of consumer sensitivity on the level of greening of new (remanufactured) products, the green investment parameter and the probability of receiving adequate raw materials from the collector for the profit of the entire supply chain. The findings indicate that in centralized as well as decentralized models, entire profit increases as customers have become more sensitive to the level of greening, entire profit increases as the probability that the collector will meet the demand of the remanufacturer increases, entire profit decreases as green investment wages increase. The importance of increasing profits for the centralized system seems to be effective, and the supply chain gains more profit in the centralized system. It’s in Fig. 6, we investigate the effect of sensitivity of the consumer to the green investment parameter on the manufacturer’s (remanufacturer’s) greening level. The findings indicate that in both centralized and decentralised systems, if the consumer demand price sensitivity and green investment parameter increase, the greening level of the manufacturer (remanufacturer) decreases. In addition, it is noticed that in both centralized and decentralized systems, remanufacturer determines a higher greening level than the manufacturer. In Fig. 7, the effect of sensitivity of the consumer to the green investment parameter on the manufacturer’s greening level is analysed. In Fig. 8, the effect of sensitivity of the consumer to the greening level of remanufactured products and green investment parameters on the remanufacturer’s greening level is analysed. In both cases, we observed that as consumer sensitivity to the greening level of new (remanufactured) products increases so does the manufacturer’s (remanufacturer’s) greening level and while the green investment parameter increases, the manufacturer’s (remanufacturer’s) greening level decreases. In all the above cases, the importance of increasing the greening level of manufacturer’s

| Para. | $\theta_1$ | $\theta_2$ | $p_1$ | $p_2$ |
|-------|------------|------------|-------|-------|
|       | $C$ | $D$ | $C$ | $D$ | $C$ | $D$ | $C$ | $D$ |
| $d_1$ | 20 | 1.16438 | 0.265755 | 2.26134 | 0.485721 | 46.9315 | 72.4526 | 47.5659 | 73.0334 |
|       | 10 | 1.31953 | 0.300136 | 2.26134 | 0.485734 | 47.1876 | 72.5366 | 47.5659 | 73.0332 |
|       | 0 | 1.47826 | 0.334937 | 2.26134 | 0.485749 | 47.4783 | 72.6314 | 47.5659 | 73.033 |
|       | 10 | 1.64107 | 0.370213 | 2.26134 | 0.485766 | 47.8052 | 72.7371 | 47.5659 | 73.0327 |
|       | 20 | 1.80851 | 0.406017 | 2.26134 | 0.485784 | 48.1702 | 72.8541 | 47.5659 | 73.0325 |
| $d_2$ | 20 | 1.47826 | 0.335329 | 1.75295 | 0.382778 | 47.4783 | 72.6228 | 46.3633 | 72.655 |
|       | 10 | 1.47826 | 0.335146 | 2.00139 | 0.433693 | 47.4783 | 72.6268 | 46.9218 | 72.8321 |
|       | 0 | 1.47826 | 0.334937 | 2.26134 | 0.485749 | 47.4783 | 72.6314 | 47.5659 | 73.033 |
|       | 10 | 1.47826 | 0.334703 | 2.53482 | 0.539107 | 47.4783 | 72.6365 | 48.3036 | 73.2588 |
|       | 20 | 1.47826 | 0.334442 | 2.82412 | 0.593939 | 47.4783 | 72.6423 | 49.1445 | 73.5108 |
Table 4 Change of unit selling prices and market demand w.r.t. % change of $a, b, \ldots$

| Para. | $w_1$ | $w_2$ | $s_1$ | $s_2$ | $D_1$ | $D_2$ |
|-------|-------|-------|-------|-------|-------|-------|
|       | %     | C     | D     | C     | D     | C     | D     | C     | D     |
| $a$   | 20    | 52.2363 | 52.126 | 39.9612 | 39.0048 | 67.8261 | 15.3439 | 75.1361 | 16.1624 |
|       | 10    | 58.4146 | 58.4159 | 44.2386 | 43.4702 | 78.26.9 | 17.7201 | 86.0254 | 18.4901 |
|       | 0     | 64.5929 | 64.7058 | 48.5159 | 47.9355 | 88.6957 | 20.0962 | 96.9147 | 20.8178 |
|       | 10    | 70.7712 | 70.9958 | 52.7933 | 52.4008 | 99.1304 | 22.4724 | 107.804 | 23.1455 |
|       | 20    | 76.9494 | 77.2857 | 57.0706 | 56.8662 | 109.565 | 24.8486 | 118.693 | 25.4732 |
| $b$   | 20    | 99.6934 | 100.754 | 72.6245 | 73.0469 | 96.6953 | 21.6551 | 106.557 | 22.2336 |
|       | 10    | 88.8296 | 89.5702 | 65.1794 | 65.297 | 94.6791 | 21.2852 | 103.86 | 21.8671 |
|       | 0     | 64.5929 | 64.7058 | 48.5159 | 47.9355 | 88.6957 | 20.0962 | 96.9147 | 20.8178 |
|       | 10    | 58.9465 | 58.9299 | 44.6232 | 43.8766 | 86.7874 | 19.6946 | 94.9493 | 20.4929 |
|       | 20    | 54.2467 | 54.1269 | 41.38 | 40.4942 | 84.9496 | 19.3001 | 93.1386 | 20.1833 |
| $c_1$ | 20    | 63.9998 | 64.6478 | 47.304 | 48.087 | 91.8261 | 20.8698 | 97.5681 | 20.8968 |
|       | 10    | 64.2964 | 64.6768 | 47.91 | 48.0113 | 90.2609 | 20.483 | 97.2414 | 20.8573 |
|       | 0     | 64.5929 | 64.7058 | 48.5159 | 47.9355 | 88.6957 | 20.0962 | 96.9147 | 20.8178 |
|       | 10    | 64.8894 | 64.7349 | 49.1219 | 47.8598 | 87.1304 | 19.7095 | 96.588 | 20.7783 |
|       | 20    | 65.1859 | 64.7639 | 49.7278 | 47.784 | 85.5652 | 19.3227 | 96.2613 | 20.7388 |
| $c_2$ | 20    | 64.6239 | 64.4025 | 48.5793 | 47.1276 | 88.6957 | 20.0558 | 98.657 | 21.2307 |
|       | 10    | 64.6084 | 64.5542 | 48.5476 | 47.5315 | 88.6957 | 20.076 | 97.7858 | 21.0243 |
|       | 0     | 64.5929 | 64.7058 | 48.5159 | 47.9355 | 88.6957 | 20.0962 | 96.9147 | 20.8178 |
|       | 10    | 64.5774 | 64.8575 | 48.4842 | 48.3395 | 88.6957 | 20.1165 | 96.0436 | 20.6114 |
|       | 20    | 64.5619 | 65.0092 | 48.4525 | 48.7434 | 88.6957 | 20.1367 | 95.1724 | 20.4049 |
| Para. | %   | \( w_1 \) | \( w_2 \) | \( s_1 \) | \( s_2 \) | \( D_1 \) | \( D_2 \) |
|-------|-----|--------|--------|--------|--------|--------|--------|
|       | C   | D      | C   | D    | C   | D    | C   | D    |
| \( I \) | –20 | –64.6853 | –64.8856 | –48.529 | –47.9339 | 89.6703 | 20.1953 | 99.1183 | 21.0409 |
|       | –10 | –64.6338 | –64.7853 | –48.5217 | –47.9348 | 89.1262 | 20.1401 | 97.8819 | 20.9164 |
|       | 0   | –64.5929 | –64.7058 | –48.5159 | –47.9355 | 88.6957 | 20.0962 | 96.9147 | 20.8178 |
|       | 10  | –64.5595 | –64.6414 | –48.5112 | –47.9361 | 88.3465 | 20.0604 | 96.1375 | 20.7379 |
|       | 20  | –64.5319 | –64.5881 | –48.5073 | –47.9366 | 88.0576 | 20.0307 | 95.4993 | 20.6717 |
| \( \beta \) | –20 | –64.7458 | –66.2844 | –48.8284 | –52.767 | 88.6957 | 19.8967 | 96.0436 | 18.6691 |
|       | –10 | –64.7578 | –65.5043 | –48.8529 | –50.0564 | 88.6957 | 19.8811 | 96.4791 | 19.7311 |
|       | 0   | –64.5929 | –64.7058 | –48.5159 | –47.9355 | 88.6957 | 20.0962 | 96.9147 | 20.8178 |
|       | 10  | –64.2404 | –63.9093 | –47.7957 | –46.1957 | 88.6957 | 20.5559 | 97.3503 | 21.902 |
|       | 20  | –63.6998 | –63.1367 | –46.6909 | –44.6939 | 88.6957 | 21.2611 | 97.7858 | 22.9536 |
Table 5  Change of unit selling prices and market demand w.r.t. % change of $d_1, d_2$

| Para. | $w_1$ | $w_2$ | $s_1$ | $s_2$ | $D_1$ | $D_2$ |
|-------|-------|-------|-------|-------|-------|-------|
|       | C     | D     | C     | D     | C     | D     | C     | D     | C     | D     | C     | D     |
| $d_1$ |       |       |       |       |       |       |       |       |       |       |       |       |
| -20   | -     | 64.4799 | -     | 64.7067 | -     | 48.5346 | -     | 47.9332 | 87.3288 | 19.9316 | 96.9147 | 20.8166 |
| -10   | -     | 64.533  | -     | 64.7063 | -     | 48.5258 | -     | 47.9343 | 87.969  | 20.009  | 96.9147 | 20.8172 |
| 0     | -     | 64.5929 | -     | 64.7058 | -     | 48.5159 | -     | 47.9355 | 88.6957 | 20.0962 | 96.9147 | 20.8178 |
| 10    | -     | 64.6597 | -     | 64.7053 | -     | 48.505  | -     | 47.9369 | 89.5129 | 20.1934 | 96.9147 | 20.8185 |
| 20    | -     | 64.7337 | -     | 64.7047 | -     | 48.4931 | -     | 47.9384 | 90.42555| 20.3009 | 96.9147 | 20.8193 |
| $d_2$ |       |       |       |       |       |       |       |       |       |       |       |       |
| -20   | -     | 64.5749 | -     | 64.4527 | -     | 48.479  | -     | 47.9401 | 88.6957 | 20.1198 | 93.9083 | 20.5059 |
| -10   | -     | 64.5833 | -     | 64.5713 | -     | 48.4963 | -     | 47.938  | 88.6957 | 20.1087 | 95.3044 | 20.652 |
| 0     | -     | 64.5929 | -     | 64.7058 | -     | 48.5159 | -     | 47.9355 | 88.6957 | 20.0962 | 96.9147 | 20.8178 |
| 10    | -     | 64.6037 | -     | 64.8571 | -     | 48.5379 | -     | 47.9328 | 88.6957 | 20.0822 | 98.7599 | 21.0042 |
| 20    | -     | 64.6157 | -     | 65.0259 | -     | 48.5625 | -     | 47.9297 | 88.6957 | 20.0665 | 100.861 | 21.2121 |
Table 6 Change of different profit functions w.r.t. % change of \(a, b, \ldots\)

| Para. | %  | \(\pi_{SC}\) |  | \(\pi_R\) |  | \(\pi_M\) |  | \(\pi_{RM}\) |  | \(\pi_S\) |  | \(\pi_C\) |  |
|-------|----|-------------|---|-------------|---|-------------|---|-------------|---|-------------|---|-------------|---|
|       | C  | D           |   | C           | D |            |   | C           | D |            |   | C           | D |
| \(a\) | -20| 3837.24     | 1503.84 | - | 198.663 | - | 184.424 | - | 200.445 | - | 519.418 | - | 400.89 |
|       | -10| 5066.23     | 1985.83 | - | 262.354 | - | 245.967 | - | 262.339 | - | 690.488 | - | 524.678 |
|       | 0  | 6465.82     | 2534.76 | - | 334.896 | - | 316.356 | - | 332.548 | - | 885.868 | - | 665.096 |
|       | 10 | 8035.99     | 3150.65 | - | 416.29  | - | 395.59  | - | 411.072 | - | 1105.56 | - | 822.145 |
|       | 20 | 9776.77     | 3833.5  | - | 506.535 | - | 483.67  | - | 497.912 | - | 1349.56 | - | 995.824 |
| \(b\) | -20| 11654.3     | 4487.03 | - | 602.047 | - | 567.099 | - | 578.49  | - | 1582.41 | - | 1156.98 |
|       | -10| 10004.9     | 3874.19 | - | 517.348 | - | 488.832 | - | 501.167 | - | 1364.51 | - | 1002.33 |
|       | 0  | 6465.82     | 2534.76 | - | 334.896 | - | 316.356 | - | 332.548 | - | 885.868 | - | 665.096 |
|       | 10 | 5670.1      | 2228.97 | - | 293.758 | - | 276.749 | - | 294.086 | - | 776.208 | - | 588.173 |
|       | 20 | 5016.77     | 1976.61 | - | 259.953 | - | 244.018 | - | 262.336 | - | 685.633 | - | 524.671 |
| \(c_1\) | -20| 6729.12     | 2639.75 | - | 348.889 | - | 341.179 | - | 335.076 | - | 944.453 | - | 670.151 |
|       | -10| 6596.49     | 2586.84 | - | 341.832 | - | 328.65  | - | 333.811 | - | 914.926 | - | 667.621 |
|       | 0  | 6465.82     | 2534.76 | - | 334.896 | - | 316.356 | - | 332.548 | - | 885.868 | - | 665.096 |
|       | 10 | 6337.1      | 2483.52 | - | 328.081 | - | 304.296 | - | 331.288 | - | 857.279 | - | 662.576 |
|       | 20 | 6210.34     | 2433.11 | - | 321.386 | - | 292.47  | - | 330.03  | - | 829.159 | - | 660.06 |
| \(c_2\) | -20| 6590.98     | 2582.83 | - | 341.191 | - | 315.083 | - | 345.87  | - | 888.947 | - | 691.741 |
|       | -10| 6528.12     | 2558.68 | - | 338.026 | - | 315.719 | - | 339.176 | - | 887.407 | - | 678.353 |
|       | 0  | 6465.82     | 2534.76 | - | 334.896 | - | 316.356 | - | 332.548 | - | 885.868 | - | 665.096 |
|       | 10 | 6404.07     | 2511.08 | - | 331.8   | - | 316.993 | - | 325.985 | - | 884.331 | - | 651.97 |
|       | 20 | 6342.88     | 2487.63 | - | 328.739 | - | 317.631 | - | 319.487 | - | 882.795 | - | 638.975 |
| Para. | %  | $\pi_{SC}$ | $\pi_R$ | $\pi_M$ | $\pi_{RM}$ | $\pi_S$ | $\pi_C$ |
|-------|----|------------|--------|--------|-----------|--------|--------|
|       |    | C  | D  | C  | D  | C  | D  | C  | D  | C  | D  | C  | D  | C  | D  | C  | D  | C  | D  | C  | D  | C  | D  | C  | D  | C  | D  | C  | D  |
| $I$   | -20| 6577.4 | 2557.74 | - | 340.227 | - | 317.782 | - | 336.098 | - | 891.435 | - | 672.196 | |
|       | -10| 6514.89 | 2544.92 | - | 337.248 | - | 316.988 | - | 334.116 | - | 888.333 | - | 668.233 | |
|       | 0  | 6465.82 | 2534.76 | - | 334.896 | - | 316.356 | - | 332.548 | - | 885.868 | - | 665.096 | |
|       | 10 | 6426.27 | 2526.52 | - | 332.992 | - | 315.839 | - | 331.276 | - | 883.863 | - | 662.552 | |
|       | 20 | 6393.73 | 2519.7 | - | 331.419 | - | 315.41 | - | 330.223 | - | 882.2 | - | 660.446 | |
| $\beta$ | -20| 6404.07 | 2390.49 | - | 297.766 | - | 310.106 | - | 267.443 | - | 980.286 | - | 534.887 | |
|       | -10| 6434.87 | 2455.93 | - | 313.829 | - | 309.619 | - | 298.734 | - | 936.278 | - | 597.468 | |
|       | 0  | 6465.82 | 2534.76 | - | 334.896 | - | 316.356 | - | 332.548 | - | 885.868 | - | 665.096 | |
|       | 10 | 6496.9 | 2626.05 | - | 360.898 | - | 330.995 | - | 368.088 | - | 829.894 | - | 736.175 | |
|       | 20 | 6528.12 | 2727.93 | - | 391.562 | - | 354.095 | - | 404.285 | - | 769.419 | - | 808.569 | |
Table 7 Change of different profit functions w.r.t. % change of $d_1$, $d_2$

| Para. | %  | $\pi_{SC}$ | $\pi_R$ | $\pi_M$ | $\pi_{RM}$ | $\pi_S$ | $\pi_C$ |
|-------|----|------------|--------|--------|------------|--------|--------|
|       |    | C  | D  | C  | D  | C  | D  | C  | D  | C  | D  | C  | D  |
| $d_1$ | -20 | 6419.34 | 252.365 | - | 332.24 | - | 313.578 | - | 332.509 | - | 880.299 | - | 665.018 |
|       | -10 | 6441.11 | 252.87 | - | 333.487 | - | 314.884 | - | 332.528 | - | 882.918 | - | 665.055 |
|       | 0   | 6465.82 | 253.76 | - | 334.896 | - | 316.356 | - | 332.548 | - | 885.868 | - | 665.096 |
|       | 10  | 6493.6 | 254.34 | - | 336.474 | - | 317.996 | - | 332.571 | - | 889.157 | - | 665.142 |
|       | 20  | 6524.63 | 254.82 | - | 338.228 | - | 319.809 | - | 332.596 | - | 892.793 | - | 665.191 |
| $d_2$ | -20 | 6358.79 | 251.35 | - | 330.119 | - | 317.097 | - | 327.604 | - | 885.557 | - | 655.208 |
|       | -10 | 6408.49 | 252.35 | - | 332.347 | - | 316.75 | - | 329.92 | - | 884.639 | - | 659.84 |
|       | 0   | 6465.82 | 253.47 | - | 334.896 | - | 316.356 | - | 332.548 | - | 885.868 | - | 665.096 |
|       | 10  | 6531.47 | 254.77 | - | 337.787 | - | 315.913 | - | 335.502 | - | 887.251 | - | 671.003 |
|       | 20  | 6606.31 | 256.15 | - | 341.047 | - | 315.42 | - | 338.797 | - | 888.795 | - | 677.595 |
Fig. 2  Entire supply chain (SC) profit in the centralized model and decentralized model

Fig. 3  Entire supply chain (SC) profit in the centralized model and decentralized model

Fig. 4  Entire supply chain (SC) profit in the centralized model and decentralized model

Fig. 5  Entire supply chain (SC) profit in the centralized model and decentralized model
(remanufacturer’s) for the centralized system is significant and the manufacturer (remanufacturer) takes into account higher levels of greening in the centralized system. Figure 9 shows that the effect of sensitivity of the consumer to the greening level of new (remanufactured) products on the manufacturer’s (remanufacturer’s) greening level in the decentralised system. Results indicate that the greening level of manufacturer (remanufacturer) is proportional to the sensitivity of the consumer to the greening level of new (remanufactured) products. The effect of the probability that the collector meets the demand of the remanufacturer and green invest parameter on the manufacturer’s greening level is analysed in Fig. 10. Results indicate that the greening level of remanufacturer increases if the probability of receiving adequate raw materials from the collector increases and decreases if the green investment parameter increases. And also, the remanufacturer determines higher greening levels in the centralized system. Figures 11 and 12 show how consumer sensitivity to the level of greening on new (remanufactured) products and green investment parameters impact the retailer’s unit selling price for new (remanufactured)
products. It seems that when customers are more concerned about the greening level of new (remanufactured) products, the retailer’s unit selling price for new (remanufactured) products increases. And as a result of rising green investment parameter, the retailer’s unit selling price for new (remanufactured) products increases. Figure 13 shows how the probability of receiving adequate raw materials from the collector and green investment parameter impact the retailer’s unit selling price for new (remanufactured) products. The analysis showed that if the probability of the collector meeting the demand of the remanufacturer increases, the retailer can set a lower unit retail price for remanufactured products. In all of the above cases, the retail price per unit for new as well as remanufactured products in the centralized scenario is relatively low. In Fig. 14, we investigate the influence of sensitivity of customer on the level of greening of new and remanufactured products to the total sales volume of the supply chain. Results indicate that the total sales volume of the supply chain is proportional to the sensitivity of the consumer to the
greening level of new (remanufactured) products. Figure 15 shows how the probability of the collector meeting the demand of the remanufacturer and green investment parameter impact the total sales volume of the supply chain. The analysis revealed that if the possibility of the collector to fulfill the needs of the remanufacturer increases, then the total supply chain sales volume increases, and if the green investment parameter increases, the total supply chain sales volume decreases. In both cases, the sales volume is relatively high in the centralized system. We consider forward and reverse supply chains which are interconnected in three stages, where both new as well as remanufactured products are shipped to the marketplace. The manufacturer and the remanufacturer both invest and decide their greening levels in green manufacturing processes. Each supply chain member tries to maximize their own profit in the decentralized model, and lower greening levels are
decided by both the manufacturer and the remanufacturer than the optimum solution, i.e. the solution in centralized scenario. Therefore, both products’ demand is lower than the optimal demand; as a result, less profit is obtained by the supply chain than the optimum. The production of eco-friendly products or services is becoming ever more relevant because of deteriorating environmental conditions. The customer is also being impacted by such a consciousness. On the other hand, for eco-friendly products, green-conscious customers have to pay higher. Therefore, governments should offer certain incentives to green companies to lower selling prices to raise demand for environmentally friendly products with a more sustainable environment. From this viewpoint, when the results in this study are evaluated, it has been demonstrated that as customers become more conscious of the environmental impact of new and remanufactured products, greening levels are increased by the manufacturer and the remanufacturer; the demands for both kinds of green products are therefore going to increase; and also the entire supply chain gains more. As demand rises, more green products are sold, which already enhances the collector’s ability to purchase used products from the market. The remanufacturer decides a higher greening standard, while the collector’s sufficiency increases. The demand for remanufactured products and the profit of the entire supply chain are therefore going to increase. As a result, the sustainable green approach throughout the supply chain provokes others to and it also contributes to a greener and more sustainable and therefore more financial gains.

5 Conclusion

The purpose of this investigation is to examine how the decisions of the supply chain leader affects social sustainable development. As per this research, having one decision controller in a supply chain can contribute to enhancing sustainability objectives. Improving product greening levels, lowering product retail prices, and increasing entire supply chain profit are generally considered significant objectives for a sustainable supply chain. Members of the supply chain should have to follow the centralized model in order to achieve these objectives. Certain policies should be implemented by the government to ensure that supply chain members must follow the centralised model. As a consequence, the supply chain has been both economically and ecologically able to succeed.

5.1 Practical implications, limitations and future research

This new study has important practical applications for all supply chain decision-makers. Supply chain decision-makers should utilize these research outcomes to properly understand the advantages of such a centralized structure. The results of the study demonstrate
how to maximise the profit of the entire supply chain. This study indicates how to increase products’ greening levels and how consumers can get green products at lower prices without comprising the overall profit of the supply chain representatives. Using the findings of this study, supply chain members can promote an even more sustainable environment.

The current study has certain limitations, even after its significance and usefulness. For example, product delivery time is not taken into account; thus, an improved approach to evaluate the influence of product delivery time would be beneficial. The ideas discussed in this study can be developed in a number of different ways. It might be useful to include a competition between supply chain members, in a future research. Throughout this supply chain, the impact of the retailer’s strategic inventory also can be investigated. Within different game systems, such as Stackelberg games with various leaders, this supply chain can be analysed. Including probabilistic demand functions will become another very possible future research suggestion in such a topic.

Appendix 1

The Hessian matrix of the objective function \( \pi_{SC}(p_1, p_2, \theta_1, \theta_2) \) of the centralized model given in Eq. (7) is calculated as

\[
H(p_1, p_2, \theta_1, \theta_2) = \begin{bmatrix}
\frac{\partial^2 \pi_{SC}}{\partial p_1^2} & \frac{\partial^2 \pi_{SC}}{\partial p_1 \partial p_2} & \frac{\partial^2 \pi_{SC}}{\partial p_1 \partial \theta_1} & \frac{\partial^2 \pi_{SC}}{\partial p_1 \partial \theta_2} \\
\frac{\partial^2 \pi_{SC}}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi_{SC}}{\partial p_2^2} & \frac{\partial^2 \pi_{SC}}{\partial p_2 \partial \theta_1} & \frac{\partial^2 \pi_{SC}}{\partial p_2 \partial \theta_2} \\
\frac{\partial^2 \pi_{SC}}{\partial \theta_1 \partial p_1} & \frac{\partial^2 \pi_{SC}}{\partial \theta_1 \partial p_2} & \frac{\partial^2 \pi_{SC}}{\partial \theta_1^2} & \frac{\partial^2 \pi_{SC}}{\partial \theta_1 \partial \theta_2} \\
\frac{\partial^2 \pi_{SC}}{\partial \theta_2 \partial p_1} & \frac{\partial^2 \pi_{SC}}{\partial \theta_2 \partial p_2} & \frac{\partial^2 \pi_{SC}}{\partial \theta_2 \partial \theta_1} & \frac{\partial^2 \pi_{SC}}{\partial \theta_2^2}
\end{bmatrix}
\]

The leading principal minors \( \Delta_k \) of \( H(p_1, p_2, \theta_1, \theta_2) \) of order \( k \) are given by

\[
\Delta_1 = -2b, \Delta_2 = 4b^2, \Delta_3 = -2b(4bI - d_1^2), \Delta_4 = (4bI - d_1^2)(4bI - d_2^2)
\]

From assumptions 3(i) and 3(ii), we get \( 4bI - d_1^2 > 0 \) and \( 4bI - d_2^2 > 0 \). Therefore, \((-1)^k \Delta_k > 0\) for all leading principle minors of \( H(p_1, p_2, \theta_1, \theta_2) \). Thus, \( H(p_1, p_2, \theta_1, \theta_2) \) is negative definite and \( \pi_{SC}(p_1, p_2, \theta_1, \theta_2) \) is strictly concave. Using the first order conditions of Eq. (7) simultaneously, we get the unique global optimal solution to the centralized model as given in Eqs. (8)–(11). Putting the Eqs. (7)–(10) into Eq. (1), we get the optimal values of demands for the new and remanufactured products as given in Eqs. (12)–(13). Finally, putting the Eqs. (8)–(10) into Eq. (7), we get the optimal profit of the entire supply chain as given in Eq. (14).
Appendix 2

In decentralized model, first we consider retailer’s profit function. The Hessian matrix of the objective function \( \pi_R(p_1, p_2) \) given in Eq. (2) is calculated as

\[
H(p_1, p_2) = \begin{bmatrix}
\frac{\partial^2 \pi_R}{\partial p_1^2} & \frac{\partial^2 \pi_R}{\partial p_1 \partial p_2} \\
\frac{\partial^2 \pi_R}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi_R}{\partial p_2^2}
\end{bmatrix} = \begin{bmatrix}
-2b & 0 \\
0 & -2b
\end{bmatrix}
\]

The leading principal minors \( \Delta_k \) of \( H(p_1, p_2) \) of order \( k \) are given by,

\[
\Delta_1 = -2b, \Delta_2 = 4b^2
\]

Therefore, \((-1)^k \Delta_k > 0\) for all leading principle minors of \( H(p_1, p_2) \). Thus, \( H(p_1, p_2) \) is negative definite and \( \pi_R(p_1, p_2) \) is strictly concave. Using the first order conditions of Eq. (2) with respect to \( p_1 \) and \( p_2 \), we get the retailer’s optimum response functions as given in Eqs. (15)–(16).

Next, we consider the manufacturer’s and the remanufacturer’s profit functions. We first substitute the retailer’s optimum response function given in Eq. (15) into the manufacturer’s profit function given in Eq. (3). The Hessian matrix of the objective function \( \pi_M(\theta_1, \omega_1) \) given in Eq. (3) is calculated as

\[
H(\theta_1, \omega_1) = \begin{bmatrix}
\frac{\partial^2 \pi_M}{\partial \theta_1^2} & \frac{\partial^2 \pi_M}{\partial \theta_1 \partial \omega_1} \\
\frac{\partial^2 \pi_M}{\partial \omega_1 \partial \theta_1} & \frac{\partial^2 \pi_M}{\partial \omega_1^2}
\end{bmatrix} = \begin{bmatrix}
-2I & d_1 \\
\frac{d_1}{2} & -b
\end{bmatrix}
\]

The leading principal minors \( \Delta_k \) of \( H(\theta_1, \omega_1) \) of order \( k \) are given by,

\[
\Delta_1 = -2I, \Delta_2 = 2bI - \frac{d_1^2}{4}
\]

From assumptions 3(i), we get \( 2bI - \frac{d_1^2}{4} > 0 \). Therefore, \((-1)^k \Delta_k > 0\) for all leading principle minors of \( H(\theta_1, \omega_1) \). Thus, \( H(\theta_1, \omega_1) \) is negative definite and \( \pi_M(\theta_1, \omega_1) \) is strictly concave. Using the first order conditions of Eq. (3) with respect to \( \theta_1 \) and \( \omega_1 \), we get the manufacturer’s optimum response functions as given in Eqs. (17)–(18).

In a similar way, we substitute the retailer’s optimum response function given in Eq. (16) into the remanufacturer’s profit function given in Eq. (4). The Hessian matrix of the objective function \( \pi_{RM}(\theta_2, \omega_2) \) given in Eq. (4) is calculated as

\[
H(\theta_2, \omega_2) = \begin{bmatrix}
\frac{\partial^2 \pi_{RM}}{\partial \theta_2^2} & \frac{\partial^2 \pi_{RM}}{\partial \theta_2 \partial \omega_2} \\
\frac{\partial^2 \pi_{RM}}{\partial \omega_2 \partial \theta_2} & \frac{\partial^2 \pi_{RM}}{\partial \omega_2^2}
\end{bmatrix} = \begin{bmatrix}
-2I & d_2 \\
\frac{d_2}{2} & -b
\end{bmatrix}
\]

The leading principal minors \( \Delta_k \) of \( H(\theta_2, \omega_2) \) of order \( k \) are given by,

\[
\Delta_1 = -2I, \Delta_2 = 2bI - \frac{d_2^2}{4}
\]
From assumptions 3(ii), we get $2bI - d_2^2 > 0$. Therefore, $(-1)^k\Delta_k > 0$ for all leading principle minors of $H(\theta_2, w_2)$. Thus, $H(\theta_2, w_2)$ is negative definite and $\pi_{RM}(\theta_2, w_2)$ is strictly concave. Using the first order conditions of Eq. (4) with respect to $\theta_2$ and $w_2$, we get the remanufacturer’s optimum response functions as given in Eqs. (19)–(20).

In the last step, we consider the supplier’s and the collector’s profit functions. We first substitute the optimum response functions of the retailer, manufacturer, remanufacturer given in Eqs. (15)–(20) into the supplier’s profit function given in Eq. (5). Then, the second order derivative of the supplier’s objective function $\pi_S(s_1)$ given in Eq. (5) with respective to $s_1$ is calculated as

$$\frac{\partial^2 \pi_S}{\partial s_1^2} = -\frac{4b^2I}{8bl - d_1^2} - \frac{4b^2I(1 - \beta)^2}{8bl - d_2^2}$$

From assumptions 3(i) and 3(ii), we get $8bl - d_1^2 > 0$ and $8bl - d_2^2 > 0$. And since $I > 0$, it can be seen that $-\frac{4b^2I}{8bl - d_1^2} - \frac{4b^2I(1 - \beta)^2}{8bl - d_2^2} < 0$. Therefore, $\frac{\partial^2 \pi_S}{\partial s_1^2} < 0$, i.e. $\pi_S(s_1)$ is a strictly concave function.

In a similar way, we first substitute the optimum response functions of the retailer, remanufacturer given in Eqs. (16), (19) and (20) into the collector’s profit function given in Eq. (6). Then, the second order derivative of the collector’s objective function $\pi_C(s_2)$ given in Eq. (5) with respective to $s_2$ is calculated as

$$\frac{\partial^2 \pi_C}{\partial s_2^2} = -\frac{4b^2I\beta^2}{8bl - d_2^2}$$

From assumptions 3(ii), we get $8bl - d_2^2 > 0$. And since $I > 0$, it can be seen that $-\frac{4b^2I\beta^2}{8bl - d_2^2} < 0$. Therefore, $\frac{\partial^2 \pi_C}{\partial s_2^2} < 0$, i.e. $\pi_C(s_2)$ is a strictly concave function.

Finally, simultaneously solving the first order condition of Eq. (5) with respect to $s_1$ and first order condition of Eq. (6) with respect to $s_2$ gives the optimal unit selling prices of the supplier and the collector as given in Eqs. (21) and (22). Putting the Eqs. (15)–(22) into Eq. (1), we get the optimal values of demands for the new and remanufactured products as given in Eqs. (23)–(24). Finally, putting the Eqs. (15)–(22) into Eq. (7), we get the optimal profit of the overall supply chain as given in Eq. (25).

**Authorship contributions** Prof.(Dr.) Dipak Kumar Jana has formulated the models. Mr. Manojit Das has solved the models and compared the solutions of the models and drawn the figures. Dr. Shariful Alam has done the Numerical part. This paper is written by Mr. Manojit Das.

**Declarations**

**Conflict of interest** There have been no conflicts of interest declared by the authors.

**Ethical approval** Any of the authors’ investigations with human participants or living creatures are not included in this article.

**Informed consent** Each individual participant in the research has given their informed consent.
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