Introduction. — The doubly charmed baryon \( \Xi_{cc}^{++} \) has received much attention since its discovery \([1]\). It unlocked a new type of hadron spectrum, the doubly heavy baryons. Different from light or singly heavy baryons, doubly heavy baryons resemble a 'double-star' system with a light 'planet' surrounded, providing a new hadronic structure to study strong interactions. The \( \Xi_{cc}^{++} \) discovery also motivated many theoretical studies on doubly heavy tetraquarks, which would help us probe the nature of exotic four quark states or structures, e.g. cusps or true resonances. Recently, the observation of the first doubly heavy tetraquark state \( T_{cc}^{++} \) was reported by the LHCb collaboration \([2,3]\). Other than doubly charmed baryons, a different kind of doubly heavy baryons, the beauty-charmed baryons containing one bottom quark and one charm quark should also exist. Although similar to doubly charmed baryons, the beauty-charmed baryons have different features and thus unique research values. Compared to the charm-charm system, the bottom-charm system is expected to have a smaller size, behaving more like a point-like particle. Moreover, the beauty-charmed baryons involve more energy scales, the bottom mass, the charm mass and the nonperturbative QCD scale \( \Lambda_{QCD} \), and therefore implicating more affluent physics.

Because of their importance, experimentalists have been putting efforts to find beauty-charmed baryons. For example, the exclusive channels \( \Xi_{bc}^{0} \to D^{0} p K^{-} \) \([4]\) \( \Xi_{bc}^{+} \to \Xi_{cc}^{++} \pi^{-} \) \([5,6]\) were used to search for \( \Xi_{bc} \), but no signals were observed. Obviously, compared to \( \Xi_{cc}^{++} \), fewer \( \Xi_{bc} \)'s are produced at the LHC, which makes \( \Xi_{bc} \) searches more difficult. More importantly, the main difficulty in such normal searches for \( \Xi_{bc} \) is induced by a much smaller reconstruction efficiency, because a bottom typically decays with fractions of \( O(10^{-3}) \) even to the most abundant exclusive final states \([6,7]\). To overcome this difficulty, we propose to search \( \Xi_{bc} \) via an inclusive decay channel \( \Xi_{bc} \to \Xi_{cc}^{++} + X \), where \( X \) stands for all possible particles.

Using \( \Xi_{bc} \to \Xi_{cc}^{++} + X \) to search for \( \Xi_{bc} \) has several advantages. Firstly, it has a much larger branching ratio than any exclusive decay channels. Secondly, the detection efficiency is high because only \( \Xi_{cc}^{++} \) needs to be reconstructed. Lastly but most importantly, because the weakly decaying \( \Xi_{bc} \) has a relatively long lifetime which typically form a sub-millimeter displaced secondary decaying vertex, the \( \Xi_{cc}^{++} \)'s decaying from \( \Xi_{bc} \)'s do not point back to the proton-proton collision vertices. This feature can clearly distinguish the signal events from the main
background, strongly produced $\Xi^{+}_{bc}$'s. To clarify this point, a diagrammatic sketch is displayed in FIG. 1.

In the following, we also analyze the feasibility of this inclusive approach for $\Xi_{bc}$ search. Based on the heavy diquark effective theory [8–10], we calculate that the $\Xi_{bc} \to \Xi^{+}_{bc} + X$ branching ratio is about 3%. Combining it with the $\Xi_{bc}$ production rate and a careful analysis of the $\Xi^{+}_{bc}$ detection efficiency, we find that hundreds of signal events are expected at the LHCb Run3, corresponding to an integrated luminosity of 23 fb$^{-1}$ by 2024. Consequently, the proposed inclusive approach for $\Xi_{bc}$ search is feasible and also timely for the LHCb.

Decay rate. — In the feasibility analysis of this approach, the inclusive $\Xi_{bc} \to \Xi^{+}_{bc} + X$ decay rate was calculated in the following several steps. Benefitting from the heavy quark symmetry and heavy diquark symmetry, each step of the calculation is basically model independent. Firstly, under the heavy (d)quark symmetry, it can be demonstrated that the leading contribution to the inclusive $\Xi_{bc} \to \Xi^{+}_{bc} + X$ decay is from $X_{bc} \to X_{cc} + fJ$, where $X_{QQ'}$ stands for a heavy diquark constituted by the heavy $Q$ and $Q'$ quarks and $fJ$ can be any possible quark or lepton pairs. Subsequently, the unknown $X_{bc} \to X_{cc}$ diquark transition current was evaluated by matching the $b \to c$ transition. Afterwards, the decay rate of $\Xi_{bc} \to \Xi^{+}_{bc} + X$ was numerically calculated, with possible theoretical uncertainties taken into account.

By this paragraph, we demonstrate the validation of treating the two heavy quarks $QQ'$ as a point-like heavy diquark in a doubly heavy baryon. To start with, the two heavy quarks $QQ'$ form a color anti-triplet and have an attractive potential. As illustrated by [11][13], the dynamics in the two heavy quark system involves three different energy scales, the heavy quark mass $m_Q$, the 3-momentum $m_Qv$ and the kinetic energy $m_Qv^2$, with $v$ being the heavy quark velocity in the baryon rest frame. Therefore, the distance between the two heavy quarks is estimated as $r_{QQ} \sim 1/(m_Qv)$, while the spatial size of the light quark is approximately $r_{Qq} \sim 1/A_{QCD}$. According to [13], $v$ is as small as $v \propto \alpha_s(m_Q)$ if $m_Q$ is heavy enough. For doubly heavy baryons, it can be deduced that $v_Q \ll 1$ and $m_Qv_Q^2 \ll m_Qv^2 \ll m_Q$. Practically, numerical calculations confirm this hierarchy by giving $v_Q^2 \approx 0.3$ for charm and $v_Q^2 \approx 0.1$ for bottom, respectively [15]. It indicates that $m_{bb}v_b^2 \approx m_{cc}v_c^2 \approx A_{QCD} \approx 350 – 500$ MeV, while $m_{bb}v_b \approx 1.5$ GeV and $m_{cc}v_c \approx 800$ MeV. Therefore, $r_{bb}/r_{Qq} \approx A_{QCD}/(m_{bb}v_b) \ll 1$ is perfectly satisfied and $r_{cc}/r_{Qq} \approx A_{QCD}/(m_{cc}v_c)$ is also suppressed.

In conclusion, in a doubly heavy baryon the two heavy quarks can be treated as a point-like diquark compared to the baryon size. This greatly simplifies the structure of a doubly heavy baryon to a bound state of a heavy diquark and a light quark, analogous to a heavy meson constituted by a heavy quark and a light quark.

Benefitting from the simple structure, the inclusive decay of a doubly heavy baryon can be formulated in the same way with that of a heavy meson. It is well known that making use of the operator product expansion within the heavy quark effective theory, the inclusive decay rate of a heavy meson can be systematically expanded by inverse powers of the heavy quark mass (see e.g. [10]). The leading term is just the corresponding decay rate of a free heavy quark. Following the same procedure, within the heavy diquark effective theory [8–10], the inclusive $\Xi_{bc} \to X_{cc}$ decay rate can be expanded by the inverse power of the diquark mass $M_X$, with the leading-power contribution given by the free diquark decay rate,

$$\Gamma(\Xi_{bc} \to X_{cc}) = \sum_{f,f'} \Gamma(X_{bc} \to X_{cc}f\bar{f}') + O\left(\frac{1}{M_X}\right).$$

Practically, the fermion pairs $\bar{f}f'$ include $\bar{v}_l\ell^-$ ($\ell = e, \mu, \tau$) and $\bar{u}d, \bar{u}s, \bar{c}d, \bar{c}s$.

In the calculation of $\Gamma(X_{bc} \to X_{cc}f\bar{f}')$ induced by the weak interaction vertices such as $\bar{c}c'\mu P_L\bar{f}f'\gamma_{\mu}P_Lf'$, the $f'\bar{f}$ part can be factorized out at the leading order of $\alpha_s$, and the nontrivial part is the remaining diquark current $\langle X_{cc}^{i} | \bar{c}c'\mu P_Lb | X_{bc}^{i'} \rangle$, where $i, i'$ are color indices. The S-wave diquark $X_{bc}$ might be either a scalar or axial-vector, but as implicated by studies of beauty-charmed baryon spectroscopy [17–21], an axial-vector $X_{bc}$ state is dominant in the $\Xi_{bc}$ baryon. On the other hand, $X_{cc}$ can only be an axial-vector due to the flavor and spin symmetries. The calculation of the diquark current is performed in two different kinematic regions, the large recoil region and the small recoil region. In the former, the perturbative calculation is applicable because typically a hard gluon is required to exchange between the spectator quark and the weak interacting quarks, as displaced in FIG. 2. In practice, we adopt the non-relativistic QCD (NRQCD) factorization for this calculation as the $B_c \to \eta_c, J/\psi$ form factor studies in [22, 23]. In the small recoil region, the soft overlap contribution is dominant and the perturbative QCD expansion is not convergent. However, the heavy quark symmetry determines the form of the diquark current in this region. For the intermediate region, we use the simplified $z$-series expansion [24] to perform the extrapolation.

In the small recoil region, the diquark currents can be obtained by taking the heavy quark limit, and the results for the vector and axial-vector weak currents read

$$\langle X_{cc}^{i} (v, \epsilon) | \bar{c}c'\mu b | X_{bc}^{i'} (v', \epsilon') \rangle = \delta_{il} \sqrt{2M_{cc}M_{bc}} \left[ -a_0 \epsilon^* \cdot \epsilon' v'^{\mu} - a_1 \epsilon^* \cdot \epsilon' v^{\mu} + a_2 \epsilon^* \cdot \epsilon' v'^{\mu} + a_3 \epsilon \cdot \epsilon' v^{\mu} \right],$$

$$\langle X_{cc}^{i} (v, \epsilon) | \bar{c}c'\gamma_{5}\gamma_{l} b | X_{bc}^{i'} (v', \epsilon') \rangle = \delta_{il} \sqrt{2M_{cc}M_{bc}} \left[ -ib_0 \epsilon^* \epsilon' v'^{\mu} - ib_1 \epsilon^* \epsilon' v^{\mu} \right].$$

(2)
with
\[
a_{0,1,2,3}(q_{\text{max}}^2) = b_{0,1}(q_{\text{max}}^2) = 1.
\] (3)

The \( v^{(i)} \), \( \epsilon^{(i)} \) and \( M_{cc(bc)} \) are the 4-velocity, polarization vector and mass of \( \chi_{cc(bc)} \). It can be derived in the following way. Due to the heavy quark symmetry, the ground state \( QQ' \) diquark can be represented by a Lorentz bilinear field
\[
D_v^{QQ}(x) = \frac{1 + \gamma^\mu X_\mu(x)}{2} + i\gamma_5 S(x)C ,
\] (4)

where the axial-vector field \( X_\mu(x) \) annihilate an axial-vector diquark with a polarization vector \( \epsilon_\mu \) and the scalar field \( S(x) \) annihilate a scalar diquark. All the color indices are hidden for convenience. Its Lorentz transformation property is \( D_v(x) \to D_v'(x) = D(\Lambda)D_\epsilon(\Lambda^{-1}x)D(\Lambda)^T \), with \( \Lambda \) and \( D(\Lambda) \) the Lorentz transformation matrices for Lorentz vectors and Dirac spinors, respectively. The the conjugate field of \( D_v \) can be introduced as \( \bar{D}_v = \gamma^j D_v^{jQ} \). We can first match the quark transition currents to the corresponding diquark transition currents. The heavy quark spin symmetries and the Lorentz covariance require that the leading contribution must take the form
\[
e_{TB} = \text{tr} \left[ X^T \bar{D}_v^{cQ} \Gamma D_v^{bQ} \right].
\] (5)

The general expression for \( X \) with the correct parity and time-reversal properties is \( X = X_0 + X_1 + X_2 + X_3 \), where the coefficients \( X_i \) are functions of \( w = v \cdot v' \). The property \( \gamma \bar{D}_v = D_v \) together with \( v \cdot \epsilon = v' \cdot \epsilon' = 0 \) simplifies the current \( e_{TB} \) by replacing \( X^T \) by a scalar function \( \xi(w) \). Evaluating the trace with a proper normalization gives the matrix elements
\[
\frac{1}{\sqrt{2M_{bc}M_{cc}}} \left( \chi_{cc}(v, \epsilon) | \bar{c} \gamma_j b \right) | \chi_{bc}(v', \epsilon') \rangle
\]
\[
= \xi(w) \left[ - \epsilon^* \cdot v \epsilon' v' - \epsilon^* \cdot v' \epsilon v + \epsilon^* \cdot v' \epsilon v + v \cdot \epsilon^{*} v' \right],
\] (6)

\[
\frac{1}{\sqrt{2M_{bc}M_{cc}}} \left( \chi_{cc}(v, \epsilon) | \bar{c} \gamma_j b \right) | \chi_{bc}(v', \epsilon') \rangle
\]
\[
= \xi(w) \left[ -i \epsilon^* \cdot v' \epsilon' - i \epsilon^* \cdot v \epsilon \right],
\] (6)

where the symmetry factor \( \sqrt{2} \) is induced by the identical c quarks in \( \chi_{cc} \), and the mass square roots are divided to make \( \xi(w) \) dimensionless. Replacing \( \chi_{cc}(v, \epsilon) \to \chi_{bc}(v', \epsilon') \) and \( b \to \bar{b} \), the above vector-current expression leads to \( \langle \chi_{bc}(v', \epsilon') | \bar{b} \gamma_j b | \chi_{bc}(v', \epsilon') \rangle / M_{bc} = 2 \xi(1) v^\mu = 2 v^\mu \) and similarly \( \langle \chi_{cc}(v, \epsilon) | \bar{c} \gamma_j b | \chi_{cc}(v, \epsilon) \rangle / (2M_{cc}) = 2 \xi(1) v^\mu = 2 v^\mu \). Both of them indicate that \( \xi(w = 1) = 1 \) with \( w = 1 \) corresponding to the minimal recoil, leading to the final result in [2] and [3].

In the large recoil region, the diquark currents are induced by the exchange of hard gluons. At the leading order with one hard gluon exchanging, the corresponding Feynman diagrams are shown by Fig. 2. The gluon line near the current vertex is the hard one and leads to the large recoil, while the dashed lines denote a number of soft gluon exchanging which can be absorbed into the initial and final diquark wave functions. The velocities of the initial- and final-state diquarks are \( v' \) and \( v \), respectively, and \( k' \) and \( k \) are the relative momenta between two heavy quarks.

**FIG. 2:** Feynman diagrams of \( \chi_{bc} \to \chi_{cc} \) diquark transition induced by the \( V \to A \) current. The double lines denote the heavy quarks. The gluon line close to the current vertex denotes a hard gluon. The dashed lines denote a number of soft gluons which can be absorbed into the initial and final diquark wave functions. The velocities of the initial- and final-state diquarks are \( v' \) and \( v \), respectively, and \( k' \) and \( k \) are the relative momenta between two heavy quarks.

\[
a_{ij}[\chi_{bc}(v') \to \chi_{cc}(v)] = \sum_{jk} \frac{\xi_{jk}^{\chi_{bc}}(\mu)}{m_c^2 - m_{bc}^2} \times \langle 0 | K_{ij}(\mu) \rangle \chi_{bc}(v') \rangle \langle \chi_{cc}(v) | K_{ij}(\mu) \rangle 0 \),
\] (7)

where the \( K^{(\mu)}(\mu) \) are all the possible independent biquark bilinear combinations of two component operators which can be power-counted by the relative velocity \( v^{(i)} \), while \( \xi_{ij}^{\chi_{bc}}(\mu) \) are the short-distance Wilson coefficients which can be calculated order by order in series of the strong coupling constant \( \alpha_s \). At the leading order, we have the following result
\[
a_{2,3}(q^2) = \frac{\alpha_s}{2(1-w)2\sqrt{w}} \frac{N_c+1}{N_c} m_c R_{bc}(0) R_{cc}(0) ,
\]
\[
a_{0}(q^2) = \frac{b_0(q^2)}{\xi_{1}a_{2,3}(q^2)},
\]
\[
a_{1}(q^2) = \frac{b_1(q^2)}{\xi_{1}a_{2,3}(q^2)} ,
\] (8)

where \( \xi_1 \equiv m_b/M_{bc}, \xi_2 \equiv m_c/M_{cc}, \) and the number of colors \( N_c = 3 \). The wave functions at the origin of the diquarks are defined through the nonperturbative matrix.
\begin{align}
\varepsilon_{ijk} \langle 0 | \psi_{c i}^T i \sigma_j \sigma_k \psi_{b j} | \Psi_b^c (\vec{r}) \rangle &= N_c \frac{R_{bc}(0)}{\sqrt{4\pi}} \varepsilon, \\
\varepsilon_{ijk} \langle X_{cc}^0 (\vec{r}) | \psi_{c i}^T i \sigma_j \sigma_k \psi_{c j} | 0 \rangle &= N_c \frac{R_{cc}^0(0)}{\sqrt{4\pi}} \varepsilon^n. 
\end{align}

In principle, both NLO \( \alpha_s \) corrections and subleading power corrections to the diquark transition amplitudes can be calculated as the \( B_c \to J/\psi \) transition in \cite{22, 23}, where the NLO correction K factors of 1.15 – 1.30 were obtained. We leave these calculations to future works. To obtain the numerical result, it requires the input of the diquark wave functions at the origin \( \psi_{bc,cc}(0) \). They were obtained by solving the nonrelativistic Shrödinger’s equations, with the nonrelativistic potential \( V(\vec{r}) = -\frac{2}{3} \frac{\alpha_s(q^2)}{r} + \frac{\alpha_s(q^2)}{c_{i r} r + 1} + \sigma \tau \) with \( \mu_{th} = 2.16 \text{ GeV} \), \( \sigma = 0.21 \text{ GeV}^2 \), \( c_1 = 1.948 \text{ GeV} \), \( c_2 = 15.782 \text{ GeV} \), \( c_3 = 9.580 \text{ GeV} \), which were fitted from the lattice calculation \cite{26, 27}. The quark masses take values of \( m_c = 1.392(11) \text{ GeV} \) and \( m_b = 4.749(18) \text{ GeV} \). The ground-state solutions of the Shrödinger’s equations give \( R_{bc}(0) = (0.66 \pm 0.06) \text{ GeV}^{3/2} \) and \( R_{cc}(0) = (0.87 \pm 0.09) \text{ GeV}^{3/2} \). The uncertainties were estimated from the difference between the central values and the results obtained with modeling the diquark potentials to be the Cornell potentials \cite{28}.

To extrapolate the diquark current in the whole range from the above results in the small and large recoil regions, a simplified z-series expansion \cite{24} was adopted with the formulation

\[ f(q^2) = \frac{f(0)}{1 - q^2/m_b^2} \left[ 1 + b \zeta(q^2) + \alpha \zeta(q^2)^2 \right], \tag{10} \]

for \( a_0 = b_0 \), \( a_1 = b_1 \) and \( a_2 = a_3 \), where \( \zeta(q^2) = \zeta(q^2) - \zeta(0) \), \( \zeta(q^2) = \zeta(q^2) = \left( \sqrt{1 - q^2} - \sqrt{1 + q^2} \right) / \left( \sqrt{1 - q^2} + \sqrt{1 + q^2} \right) \), \( t_b = (m_c + m_b)^2 \), \( t_q = t_q (1 - \sqrt{t_q}) \), and \( t_0 = t_0 (1 - \sqrt{t_0}) / \sqrt{m_b} \) and the free parameters \( b \) and \( c \) are to be determined. Fitting the formula to the numerics of \cite{3} and \cite{3}, the central values of the parameters were extracted as: \( b = -58.6, c = 238.2 \) for \( a_2 \); \( b = -52.9, c = 1898.2 \) for \( a_0 \); \( b = -57.0, c = 388.0 \) for \( a_1 \). The corresponding uncertainties are plotted in FIG. 3. Considering the uncertainties of the diquark wave functions at origin, the \( \pm 1 \sigma \) uncertainties for \( a_0,1,2 \) have been obtained, also plotted in FIG. 3.

Finally, with the numerical results for the diquark currents, the inclusive \( \Xi_{bc} \to X_{cc} \) decay rate \cite{1} can be calculated. The leading power free diquark decay rates were calculated by phase space integration over the amplitude square. Summing over the contributions from all possible channels with \( f f' = \mu \bar{\nu}, \tau \bar{\nu}, \bar{u} d, u s, c d, c s \), the numerical result for the inclusive decay rate reads

\[ \Gamma(\Xi_{bc} \to X_{cc}) = (1.9 \pm 0.1 \pm 0.3 \pm 0.4) \times 10^{-13} \text{ GeV}, \tag{11} \]

where the uncertainties in order are from the quark mass variation, the model dependence of the diquark wave functions at the origin, and the scale dependence, respectively. In additional, one would expect more uncertainties induced by unknown power corrections. The most dominant source should be \( \nu^2 \) corrections, which is expected to potentially modify the result by \( \sim 30 \% \). The decay rate translates to the branching ratio as

\[ \mathcal{B}(\Xi_{bc} \to X_{cc}) \approx 6 \% \cdot \frac{\tau_{\Xi_{bc}}}{200 \text{ fs}}, \tag{12} \]

where the \( \Xi_{bc} \) lifetimes have been evaluated in \cite{29} as 93 fs < \( \tau(\Xi^0_{bc}) \) < 118 fs and 409 fs < \( \tau(\Xi^{+}_{bc}) \) < 607 fs.

**Signal and background.** — Based on the \( \Xi_{bc} \to X_{cc} \) branching ratio calculated above, with also the information of \( \Xi_{bc} \) production and \( \Xi^{+}_{cc} \) detection efficiency, the number of signal events containing a displaced \( \Xi^{+}_{cc} \) can be estimated.

The \( X_{cc} \) is a hadronic state containing doubly charmed hadrons, including ground-state baryons like \( \Xi^{++}_{cc} \) and \( \Omega^{++}_{cc} \) as well as tetraquarks \( T_{cc} \), and also excited states, which will eventually decay strongly (or electromagnetically) into their ground-state doubly charmed hadrons. As the fragmentation rates to stranged baryons and tetraquarks are much smaller than the rates to non-stranged baryons, the \( \Xi_{cc} \) and \( T_{cc} \) fragmentations are neglected in the following discussion for convenience. With this approximation, all the inclusive \( \Xi_{bc} \to X_{cc} \) decays produce signals with a displaced \( \Xi_{cc} \), half \( \Xi^{++}_{cc} \) and half \( \Xi^{+}_{cc} \) in the limit of isospin symmetry, i.e., \( \mathcal{B}(\Xi_{bc} \to X_{cc}) \approx 2 \mathcal{B}(\Xi_{bc} \to \Xi^{+}_{cc} \to X) \).

The \( \Xi_{bc} \) production cross section at the LHC has been theoretically studied in \cite{20, 31}. To reduce potential systematic theoretical uncertainties, we adopted, instead of the result for the cross section itself, the cross section ratio \( \sigma(\Xi_{bc}) / \sigma(\Xi_{cc}) \approx 40 \% \) evaluated by \cite{31}. The signal is nothing but a displaced \( \Xi^{+}_{cc} \), so its detection efficiency is expected to be identical to that of a normal \( \Xi^{+}_{bc} \), \( e(\Xi^{+}_{bc}) \).
Based on these inputs, the expected number of signal events $N_s$ can be written as

$$N_s = N_p(\Xi_{bc}) \cdot B(\Xi_{bc} \rightarrow \Xi^{++} + X) \cdot \epsilon(\Xi^{++})$$

$$= 2N_o(\Xi^{++}) \cdot \frac{\sigma(\Xi_{bc})}{\sigma(\Xi_{cc})} \cdot B(\Xi_{bc} \rightarrow \Xi^{++} + X) \cdot \epsilon(\Xi^{++})$$

where $N_{p,o}$ represents the number of produced and detected particles and the factor of 2 is due to the equal production of $\Xi^+_{bc}$ and $\Xi^0_{bc}$ induced by the isospin symmetry. Quantitatively, it is expected that the LHCb Run3 (23 fb$^{-1}$) will collect about seven thousand $\Xi^{++}$’s through the $\Lambda^+_c K^- \pi^+ \pi^+$ reconstruction [32]. If combining the $\Xi^{++}_c$ channel [33], the number will increase to $N_o(\Xi^{++}) \approx 10^4$. Combining the inclusive decay branching ratio [12] and the $\Xi_{bc}$ production information, it finally arrives at the number of expected signal events at the LHCb Run3,

$$N_s \approx 240 \cdot \left(\frac{N_o(\Xi^{++})}{10^4}\right) \cdot \frac{\sigma(\Xi_{bc}/\sigma(\Xi_{cc}))}{40\%} \cdot \frac{\Gamma(\Xi_{bc} \rightarrow X_{cc})}{1.9 \times 10^{-15}\text{GeV}} \cdot \tau(\Xi^{++}_{bc}) \cdot \frac{\epsilon(\Xi^{++})}{400\text{fs}}$$

It was proposed by [31] to search for $\Xi_{bb}$ via the inclusive $\Xi_{bb} \rightarrow B_c^- + X$ decay channel, and the expected number of signal events was estimated by [35]. Compared to [35], the final state detection efficiency is very carefully handled from available $\Xi^{++}_{bc}$ measurements, and the number in [14] is very reliable.

If we take into account the isospin breaking effects, the situation will be even better. Firstly, the $\Xi_{bc}$ baryon has the $u$ quark as the spectator, which enhances its decay branching ratio to $\Xi^{++}_{bc}$ compared to $\Xi^0_{bc}$. As a result, most of the displaced $\Xi^{++}_{bc}$’s should decay from $\Xi^0_{bc}$. Simultaneously, $\Xi^{0}_{bc}$ is expected to have a longer lifetime than $\Xi^0_{bc}$ [29], indicating its secondary decaying vertex is farther away from the collision point. This makes it easier to reconstruct at the LHC.

As for the background, a displaced $\Xi^{++}_{cc}$ is also possibly produced from $B_c^+ \rightarrow B_c^{++} + X$. However, such background is negligible because the $B_c^+ \rightarrow B_c^{++} + X$ branching ratio is expected to be tiny owing to very strict phase space suppression. First, because of the baryon number conservation, $B_c$ decays into at least two baryons if one $\Xi^{++}_{cc}$ is required. Then, the dominant quark-level transition for such decays is $b \rightarrow c \bar{s} s$, so the least massive final state is $\Xi^{++}_{cc} \Xi^0$. Checking their masses, only 0.18 GeV is left for the phase space. Considering a similar decay channel $B \rightarrow \Lambda^+ + \Xi$ with about 0.5 GeV phase space having a branching ratio of $O(10^{-3})$ [29], the $B_c^+ \rightarrow B_c^{++} \Xi^0$ branching ratio is expected to be smaller than $O(10^{-3})$. It also allows decay processes with some other final states such as $\Xi^{++}_{cc} \Xi^0 \gamma$, $\Xi^{++}_{cc} \Xi^0 \pi$, $\Xi^{++}_{cc} \Xi^0$, $\Xi^{++}_{cc} \Xi^0$, all of them are expected to have similar or even smaller branching ratios. As the production cross sections of $B_c$ and $\Xi_{bc}$ are of the same order, the number of the displaced $\Xi^{++}_{cc}$‘s produced via $B_c$ decays is smaller than that of the signal by at least one to two orders of magnitude, and hence this only background source can be safely neglected.

**Conclusion.** — We have proposed that the inclusive $\Xi_{bc} \rightarrow \Xi^{++}_{bc} + X$ decay channel can be used to search for the $\Xi_{bc}$ baryons, with a very clean and simple signal — a displaced $\Xi^{++}_{cc}$. Making use of effective theories of QCD, we have calculated its branching ratio at the leading order to be about 3%, while the radiative corrections and power corrections are left for future studies. Based on the result for the $\Xi_{bc} \rightarrow \Xi^{++}_{bc} + X$ branching ratio, the $\Xi_{bc}$ production rate, and the $\Xi^{++}_{bc}$ detection efficiency extracted from previous experiments, it was estimated that the LHCb Run3 will collect about 240 such signal events. The only possible background, the $B_c^+ \rightarrow B_c^{++} + X$ decay, has been demonstrated to be negligible. In conclusion, the inclusive $\Xi_{bc} \rightarrow \Xi^{++}_{bc} + X$ decay may serve as the discovery channel for $\Xi_{bc}$.

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