Excitation energy of superdeformed bands in Relativistic Mean Field Theory

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Abstract

Constrained Relativistic Mean Field (RMF) calculations have been carried out to estimate excitation energies relative to the ground state for superdeformed bands in the mass regions $A \sim 190$ and $A \sim 150$. It is shown that RMF theory is able to successfully reproduce the recently measured superdeformed minima in Hg and Pb nuclei.

Superdeformation has become the recent years one of the most interesting topics of nuclear structure studies. After the well known area in the vicinity of the mass number $A \sim 150$ a second region with $A \sim 190$ has been discovered, where an impressive number of results has been obtained. However, despite the rather large amount of experimental information on superdeformed bands, there are still a number of very interesting properties, which have not yet been measured. A characteristic example is the excitation energy between ground and superdeformed bands. The excitation energy and the well depth of the superdeformed minimum are amongst the most important factors which affect the decay of the superdeformed bands to the ground state.

Very recently discrete $\gamma$ rays directly connecting states of a superdeformed band in $^{194}$Hg to the yrast states have been discovered \cite{1}. This has made it possible to determine accurately the excitation energy of the levels of the superdeformed band above the yrast line. Extrapolating to zero angular momentum the superdeformed minimum was found to be 6.017 MeV above the ground state. Similar measurements at about the same time have also been reported for the superdeformed band of $^{194}$Pb nucleus \cite{2}. The excitation energy between the SD band and the ground state was estimated to be 4.471(6) MeV.
There are several non-relativistic theoretical predictions for the excitation energy relative to the ground state. Such as Hartree-Fock calculations with density dependent Skyrme \cite{4} or Gogny interactions \cite{5} as well as calculations using the Strutinsky method built on a Woods-Saxon average potential \cite{6}. In this letter we report and discuss the predictions of the RMF theory for the excitation energies relative to the ground state of the SD bands of $^{194}$Hg and $^{194}$Pb. In addition predictions for nuclei in the rare earth mass region are also provided.

Relativistic Mean Field (RMF) \cite{7} theory has recently gained considerable success in describing various facets of nuclear structure properties. With a very limited number of parameters, RMF theory is able to give a quantitative description of ground state properties of spherical and deformed nuclei \cite{8, 9} at and away from the stability line \cite{10, 11, 12}. Moreover good agreement with experimental data has been found recently for collective excitations such as giant resonances \cite{13} and for twin bands in rotating superdeformed nuclei \cite{14, 15}. Afansajev et al, have recently, and with great success carried out a systematic investigation of the entire A~$\sim$140 to A~$\sim$150 mass region \cite{17}.

The starting point of RMF theory is a standard Lagrangian density \cite{8}
\begin{equation}
\mathcal{L} = \bar{\psi} \left( \gamma (i\partial - g_\omega \omega - g_\rho \vec{\rho} \vec{\tau} - eA) - m - g_\sigma \sigma \right) \psi \\
+ \frac{1}{2} \left( \partial \sigma \right)^2 - U(\sigma) - \frac{1}{4} \eta_{\mu \nu} \eta^{\mu \nu} + \frac{1}{2} m^2 \omega^2 \\
- \frac{1}{4} \vec{R}_{\mu \nu} \vec{R}^{\mu \nu} + \frac{1}{2} m^2 \rho^2 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu}
\end{equation}
which contains nucleons $\psi$ with mass $m$, $\sigma$-, $\omega$-, $\rho$-mesons, the electromagnetic field and non-linear self-interactions of the $\sigma$-field,
\begin{equation}
U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4.
\end{equation}

In the present work we have carried out constrained RMF calculations by imposing a quadratic constraint $H'$ \cite{18}, for the quadrupole moment using a mean field Hamiltonian $H_{RMF}$ and minimizing instead of $H_{RMF}$ the expectation value of $H_{RMF} + H'$ where
\begin{equation}
H' = \frac{c}{2} (\langle Q \rangle - q)^2
\end{equation}
and $q$ is the actual value of the quadrupole moment. This constraint enables us to trace the energy surface as function of the quadrupole moment.

For our calculations we have adopted the frequently used parameter set NL1 \cite{19, 20}, along with the recently proposed parametrization NL3 \cite{21} using a new version of the “axially deformed” code \cite{22}. The values of the two parameter sets are listed in Table 1. In the calculations pairing has been included within the BCS formalism. The experimental odd-even mass differences were used to determine for the ground state gap parameters $\Delta_p$ (for
protons) and $\Delta_n$ (for neutrons). These values were then used to determine the corresponding monopole pairing constants $G_p$ and $G_n$ using the energy cut-off of $82A^{-1/3}$ MeV. In order to treat properly the pairing correlations on going to the superdeformed minimum the corresponding gaps were not kept constant but calculated using these strength parameters for each value of the constraint.

In Fig. 1 we show the potential energy surface for the nucleus $^{194}$Hg as a function of the quadrupole parameter $q$ in Eq. (3), calculated with the parameter sets NL3 and NL1. It is seen that for both Lagrangian parametrizations the RMF theory predicts an oblate shape for the ground state of the nucleus $^{194}$Hg. This is in accordance with experiment [23]. The calculations predict in addition a stationary point at prolate deformation. In an axially symmetric calculation it looks like a local minimum, but including triaxial deformations it corresponds probably to a saddle point connected to the oblate minimum by a valley in the $\beta$-$\gamma$ plane, which avoids the maximum at $\beta = 0$. For this nucleus the excitation energy $E_x$ of the superdeformed minimum with respect to the oblate ground state is 5.62 MeV for the parameter set NL1. It is close to the measured value (6.02 MeV) [1]. The parameter set NL3 does even better giving a value of 6 MeV in excellent agreement with the experiment. The depth of the potential well of the isomeric superdeformed state is about 2.66 MeV for NL1 and 1.25 MeV for NL3.

We have also performed similar calculations for the nucleus $^{192}$Hg. These are shown in Fig. 2. For this nucleus the excitation energy $E_x$ is 3.93 MeV for NL1 while 4.46 MeV for NL3. The depths of the isomeric wells were found to be 2.70 MeV and 1.17 MeV respectively.

It is seen from Figs. 1 and 2 that going from $^{194}$Hg to $^{192}$Hg the excitation energy $E_x$ as well as the depth of the superdeformed minimum are decreased for both parametrizations. For NL1 the relative decrease is 30 % and 1.5 % respectively while for NL3 it is 26 % and 6.5 %. Moreover, it is also seen that for the two Hg nuclei NL3 predicts higher excitation energies and shallower potential wells as compared with NL1. An interesting feature which is common for both parametrizations is the fact that the product of the well depth and width of the superdeformed barrier decreases by about 20% on going from $^{192}$Hg to $^{194}$Hg. This quantity is relevant for the tunneling probability between the superdeformed and the other minima and therefore for the lifetime in the superdeformed well. Similar observations have also been made in experimental investigations [24]. It should be noted at this point that the parameter set NL3 gives values for the ground state binding energies which are in excellent agreement with the empirical values. Specifically NL3 predicts for $^{192}$Hg 1519.87 MeV and for $^{194}$Hg 1535.90 MeV the experimental values being 1519.20 MeV and 1535.50 MeV respectively. The corresponding predictions of NL1 are 1524.99 and 1540.31 MeV respectively.

Non-relativistic Density Dependent Hartree-Fock calculations predict val-
ues for the excitation energies $E_x$ of $^{194}$Hg, which deviate somewhat from the experimental values. For Skyrme forces one finds \[4\] 5.0 MeV, while the Gogny’s force \[3\] predicts 6.9 MeV. Finally, using a Woods-Saxon potential \[2\] a value of 4.6 MeV was obtained. We can conclude that the predictions of RMF theory are in better agreement with experiment than other theoretical studies.

Our calculations stayed strictly in the mean field approximation. There were no corrections of spurious rotational contributions subtracted. Those corrections would influence in some way or another the energy of all the stationary points in the energy surface. However only the differences between the minima would be of importance. To calculate these contributions in an appropriate way is a very difficult task, which definitely goes beyond the present state of the art.

We next, carried out calculations for the nucleus $^{194}$Pb using the NL3 force. The potential energy landscape is shown in Fig. 3 (left side). RMF theory predicts for the excitation energy $E_x$ a value 4.53 MeV, which is very close to the experimentally measured value 4.471(6) MeV. The depth of the isomeric well was found to be 2.2 MeV. The Hartree-Fock calculations with a Skyrme interaction give an estimate of 4.86 MeV, which is also in good agreement with the measured value. The Woods-Saxon potential gives a lower value of 3.8 MeV. In contrast to the Hg-nuclei we observe here a rather sharp maximum between the ground state minimum at zero deformation and the superdeformed minimum. However, this does not mean pairing correlations vanish at this saddle point. As expected we observe a local increase of pairing in going over the saddle because of the increasing level density at this point.

Recently its was observed \[25\] that the superdeformed band of $^{194}$Pb is populated at lower spin values than that of $^{192}$Hg and de-excites towards normally deformed states at a spin value lower than that of $^{192}$Hg. This fact suggests that the barrier separating the superdeformed and normal deformed minima in the potential energy surface is higher in $^{194}$Pb than in $^{192}$Hg. This agrees with the predictions of the RMF theory. Our calculations using the force NL3 for these two isotonic nuclei (N=90) show that the height of the barrier of the nucleus $^{194}$Pb is about 1 MeV higher than $^{192}$Hg and also the excitation energy $E_x$ of $^{194}$Pb is slightly higher than that of $^{192}$Hg.

Superdeformed nuclei in the rare earth region are of great interest and there are projects for the measurement of the excitation energies $E_x$ \[26\]. We have therefore, also performed calculations for some even-even rare earth nuclei which exhibit superdeformed bands. In Table 2 we list the predictions of RMF theory for the excitation energy $E_x$. It is seen for Gd and Dy isotopes the $E_x$ values increase with increase of the mass number. However, the $E_x$ energies of $^{146}$Gd and $^{148}$Dy nuclei appear to be larger than their neighboring counterparts. This could attributed to the magic character of these nuclei.
which are expected to have a deeper minimum. Moreover, it is observed that the isotonic nuclei, $^{142}$Sm, $^{144}$Gd, (N=80), $^{148}$Gd, $^{150}$Dy (N=84), $^{150}$Gd, $^{152}$Dy (N=86) have rather similar excitation energies $E_x$.

In the same table we show predictions of RMF theory for the ground state (g.s) binding energy (BE) together with the corresponding empirical values (BE$_{exp}$), taken from the most recent compilation of Audi and Wapstra [27]. It is seen that RMF theory with the effective force NL3 is able to reproduce the experimental values with very high accuracy, the disagreement being less than 0.1 %. The excellent predictions for the g.s binding energies give us confidence for the correctness of our estimate for the excitation energy $E_x$. Finally, we show in Fig. 3 (right side) as an illustration the energy curve of the nucleus $^{146}$Gd calculated with the parameter set NL3. The excitation energy $E_x$ is 10.63 MeV and the depth of the superdeformed minimum is 0.66 MeV.

In summary, we have performed constrained RMF calculations using the effective forces NL1 and NL3 for several nuclei in the A $\sim$ 190 and A $\sim$ 150 regions to estimate the excitation energy of the superdeformed minimum relative to the ground state. It has been shown that RMF theory is able to reproduce the recently reported values for the excitation energies of $^{194}$Hg and $^{194}$Pb nuclei with a higher precision than other non-relativistic calculations. Moreover predictions for the heights of the superdeformed barrier in the A $\sim$ 190 mass region have also been given. The calculated values agree with scenarios suggested by experimental finding. Finally, RMF theory predictions have been also made for the excitation energy $E_x$ of the well known superdeformed nuclei in the rare earth mass region, which should also be useful for comparisons with future experiments.

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Table 1: The parameter sets of the effective interactions NL1 and NL3 of the Lagrangian of the RMF theory used in the present work. The masses are given in (MeV) and the coupling constant $g_2$ in (fm$^{-1}$)

|     | $M$   | $m_\sigma$ | $m_\omega$ | $m_\rho$ | $g_\sigma$ | $g_\omega$ | $g_\rho$ | $g_2$   | $g_3$   |
|-----|-------|-------------|-------------|-----------|-------------|-------------|-----------|---------|---------|
| NL1 | 938   | 492.250     | 795.359     | 763.0     | 10.138      | 13.285      | 4.976     | -12.172 | -36.265 |
| NL3 | 939   | 508.194     | 782.501     | 763.0     | 10.217      | 12.868      | 4.474     | -10.431 | -28.885 |

Table 2: The predictions of the RMF theory for the excitation energy $E_x$ relative to the ground state of some Sm, Gd and Dy isotopes using the effective force NL3. Also shown are the calculated and empirical values of the (g.s) binding energies.

| Nucleus | BE   | BE_{exp} | $E_x$ |
|---------|------|----------|-------|
| $^{142}$Sm | 1177.02 | 1176.62 | 6.80 |
| $^{144}$Gd | 1184.62 | 1184.12 | 6.45 |
| $^{146}$Gd | 1205.43 | 1204.44 | 10.63 |
| $^{148}$Gd | 1220.91 | 1220.77 | 7.82 |
| $^{150}$Gd | 1236.85 | 1236.40 | 8.00 |
| $^{148}$Dy | 1211.47 | 1210.75 | 10.32 |
| $^{150}$Dy | 1228.35 | 1228.39 | 7.98 |
| $^{152}$Dy | 1245.69 | 1245.33 | 8.32 |
| $^{154}$Dy | 1262.94 | 1261.75 | 9.90 |

Figure Captions

**Fig. 1** Potential energy surface as a function of the quadrupole parameter $q$ for the nucleus $^{194}$Hg calculated using two different parameter sets NL3 (upper part) and NL1 (lower part).

**Fig. 2** Potential energy surface as a function of the quadrupole parameter $q$ for the nucleus $^{192}$Hg calculated with two different parameter sets NL3 (upper part) and NL1 (lower part).

**Fig. 3** Potential energy surface as a function of the quadrupole parameter $q$ for the nuclei $^{194}$Pb (l.h.s) and $^{146}$Gd (r.h.s.)
Fig. 1

$^{194}$Hg $E_x = 6.0$ MeV $V = 1.25$ MeV

$^{194}$Hg $E_x = 5.62$ MeV $V = 2.66$ MeV

NL3

NL1
Fig. 2

\[ ^{192}\text{Hg} \quad E_x = 4.46 \text{ MeV} \quad V = 1.17 \text{ MeV} \]

\[ ^{192}\text{Hg} \quad E_x = 3.93 \text{ MeV} \quad V = 2.70 \text{ MeV} \]
\(^{194}\)Pb \(E_x = 4.53\) MeV \(V = 2.2\) MeV

\(^{146}\)Gd \(E_x = 10.63\) MeV \(V = 0.66\) MeV

Fig. 3