Longitudinal Equivalent Flexural Stiffness of Rectangular Pipe-jacking Tunnel in Integrated Pipe Gallery

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Abstract. The rectangular pipe-jacking tunnel (RPJT) has a high section utilization ratio and is one of the primary forms of urban integrated pipe gallery (IPG) at present. This kind of tunnel will produce uneven longitudinal displacement under adjacent construction. Longitudinal bending stiffness is the crucial parameter to analyse its longitudinal stress characteristics. Combined with the structural elements of RPJT, the calculation models of the longitudinal bending stiffness of RPJT with round corners was put forward. The influence of longitudinal bolt number, ring width, and segment thickness on the neutral axis position and the longitudinal equivalent stiffness (LES) efficiency was further analysed. The influence of bending moment on the opening angle of the inter-ring joint was also analysed. The results show that increasing the number of longitudinal bolts, width of segment ring, and thickness of segment are beneficial to enhancing the LES efficiency and promoting the upward shift of the neutral axis. In addition, an increase of bending moment leads to the increase in the opening of the inter-ring joint, which is not conducive to structure safety. The research results can provide theoretical support for the structural design and longitudinal safety analysis of RPJT.

1. Introduction

With the rapid increase of urban construction, the IPG has become an essential infrastructure for comprehensively solving the intensive treatment of urban lifeline pipelines. RPJT has gradually become the construction form's preference in the urban IPG project due to its advantages such as higher section utilization ratio and lower underground space occupancy rate. Many cities have adopted RPJT to build urban IPG projects, such as the Subway Tozai Line in Kyoto, Japan [1], the subway line 3 project in Ningbo, China [2], the underground passage project of Shanghai Hongqiao Linkong Business Park, China [3].

A pipe-jacking tunnel is a complex tubular structure composed of segments connected by bolts. In order to facilitate analysis, the longitudinal deformation of pipe-jacking tunnel is usually simplified as one-dimensional problem. The theoretical research related to longitudinal deformation of tunnels includes mainly the longitudinal beam-spring model [4] and the longitudinal equivalent continuous
model [5]. The coefficients of the longitudinal beam-spring model need to be determined based on experiments, resulting in certain restrictions in the model’s application. The longitudinal equivalent continuous model regards the tunnel as a uniform continuous beam with the same stiffness and structural characteristics by using the equivalent stiffness in the longitudinal direction. This model had been combined with Euler-Bernoulli beam theory to calculate the stress and displacement of the tunnel [6].

When the longitudinal equivalent continuous model is used to simplify the calculation, the equivalent flexural stiffness of the pipe-jacking tunnel is important as it will directly affect the accuracy of the tunnel structure's stress and deformation analysis. Many studies on the bending stiffness of circular shield tunnels have been conducted. For example, considering the deflection and shear deformation of the beam simultaneously, Huang et al. [7] simplified the tunnel to be a homogeneous Timoshenko beam. They used the equivalent shear stress to consider the joints' effect, thereby obtaining a new longitudinal bending stiffness model. Assuming that the tunnel lining includes bolts, chain links and rubber washers, all of which are linear elastic structures, and that the bolts only work in tension, while the rubber washers only work in compression, Yu et al. [8] obtained the analytical solution of the longitudinal bending stiffness of the segmented lining. Wang et al. [9] proposed a four-stage analysis model for determining the bending stiffness of joints by considering the influence of bending moment and axial stress. Li et al. [10] modified the traditional equivalent continuous model by considering the impact of the longitudinal axial stress and evaluated the efficiency of longitudinal bending stiffness. However, RPJT, as a relatively new structural form is rarely studied on the mechanical properties of its longitudinal structure.

Based on the theoretical research on the longitudinal equivalent flexural stiffness of the traditional circular shield tunnel, this paper proposes a model calculating the longitudinal flexural stiffness of the RPJT. Taking the rounded RPJT as an example, the effects of the number of longitudinal bolts, ring width and thickness on the position of the neutral axis and the longitudinal equivalent flexural stiffness of the RPJT, and the influence of the bending moment on the opening angle of ring joint are further investigated.

2. Theoretical derivation of longitudinal equivalent flexural stiffness

To protect the tunnel structure and reduce the resistance of the jacking process, rounded corners are generally designed at the four corners of the RPJT. To improve the integrity between the tunnels, bolt connections are used between the pipe rings.

The cross section of the RPJT with four round corners is shown in Figure 1, where $A$ and $B$ are the half-length and half-width, respectively, of the straight part of the outer contour, while $a$ and $b$ are the half-length and half-width, respectively, of the straight part of the inner contour; $R$ is the radius of the arc of the outer contour, while $r$ is the radius of the arc at the corner of the dashed line. In addition, $c$
and $\varphi$ defines the position of the neutral axis, $ds$ is any differential unit on the rectangular segment ring ($ds_{12}$ represents any differential unit between 1 and 2), $d\alpha$ is the angle of the straight part $ds$ relative to the point $o$, $dy$ is the angle of the arc part $ds$ relative to the center of the circle, $x$ is the distance between $ds$ and the neutral axis ($x_{12}$ indicates the distance from $ds$ to the neutral axis between 1 and 2), $t$ is the thickness of the sheet ring. To make the arc and the straight line connect smoothly, the RPJT-like fillet is usually designed as a quarter arc, which is the more commonly used design method for the rounded RPJT.

Excluding the compression deformation of the lining ring and hoop joint, it can be seen from the in Figure 1 that the differential unit satisfies the following conditions:

i. below the neutral axis (compression area)

$$ds_{12} = (b + r) \left(1 + \tan^2 \alpha \right) d\alpha, \quad ds_{23} = rd\gamma, \quad ds_{34} = (a + r) \left[1 + \tan^2 \left(\frac{\pi}{2} - \alpha \right) \right] d\alpha$$

$$x_{12} = b + r - c, \quad x_{23} = r \cos \gamma + b - c, \quad x_{34} = (a + r) \tan \left(\frac{\pi}{2} - \alpha \right) - c$$

(1)

ii. above the neutral axis (tension area)

$$ds_{45} = (a + r) \left[1 + \tan^2 \left(\frac{\pi}{2} - \alpha \right) \right] d\alpha, \quad ds_{56} = rd\gamma, \quad ds_{67} = (b + r) \left(1 + \tan^2 \alpha \right) d\alpha$$

$$x_{45} = (a + r) \tan \left(\frac{\pi}{2} - \alpha \right) + c, \quad x_{56} = r \cos \gamma + b + c, \quad x_{67} = b + r + c$$

(2)

According to the geometric conditions of the triangle formed by the neutral axis and the origin, we obtain $c(a + r)^{-1} = \tan \varphi$, and $\varphi$ can be obtained.

Image:

**Figure 3.** The stress deformation diagram of the loop unit with rounded corners

The length $l_x$ between the center lines of two adjacent lining segments is taken as a calculation unit. Therefore, the relationship between the unit rotation angle $\theta$ and the length $l_x$ can be shown in Figure 2 where the following assumptions are made:

1. The cross section of the pipe-jacking tunnel is continuous and uniform, which means that the joint between rings is ignored.

2. The deformation of the tunnel cross-section is proportional to the distance from the neutral axis.

3. In the tension zone of the tunnel, both the pipe section concrete and the connecting bolts are subject to tensile stresses, while in the compression zone, only the pipe section concrete is subject to compressive stress and is always in an elastic state, as shown in Figure 3.

4. The bolts are continuously and evenly distributed along the circumferential direction and their average linear stiffness $k_t$ [5] is $k_t = nE_bA_b(4a + 4b + 2\pi r)^{-1}l_b$. And $n$ is the total number of the longitudinal bolts, $E_b$ is the Young’s modulus of the bolt, $l_b$ denote the length of the bolt.

5. As $l_b$ is so small compared with the tunnel diameter, the position of the neutral axis and the stress distribution in the segment ring are considered to be unchanged along the pipe-jacking tunnel axis.

According to the deformation coordination conditions in Figure 3, we obtain
where $\varepsilon_i$ and $\varepsilon_c$ are the tensile and compressive strains, respectively, of the piece joint, respectively; $\delta_j$ is the joint opening farthest from the neutral axis.

Integrating and summing the stress on different areas of the tunnel section, and introducing the opening displacement of the ring seam in the tension zone, force balance expression is obtained

$$\frac{2E_i\varepsilon_c}{B + R - c} \int_0^{a_r} \frac{a_r}{D} x_1 t^2 s_1 ds_1 + \frac{2E_i\varepsilon_c}{B + R - c} \int_0^{a_r} x_2 t^2 ds_2 + \frac{2E_i\varepsilon_c}{B + R - c} \int_0^{a_r} x_3 t^2 ds_3 + \frac{2E_i\varepsilon_c}{B + R - c} \int_0^{a_r} x_4 t^2 ds_4$$

$$= \frac{2k_3\delta_j}{B + R - c} \int_0^{a_r} x_6 t^2 ds_6 + \frac{2k_3\delta_j}{B + R - c} \int_0^{a_r} x_7 t^2 ds_7 + \frac{2k_3\delta_j}{B + R - c} \int_0^{a_r} x_8 t^2 ds_8 + \frac{2k_3\delta_j}{B + R - c} \int_0^{a_r} x_9 t^2 ds_9$$

From equation (3) and equation (4), the tensile and compressive strains of the joint are obtained

$$\varepsilon_c = \left( \frac{B + R - c}{l_s} \right) \theta, \quad \varepsilon_i = \frac{\theta K_j (B + R + c)}{l_s K_j + E_s (4a + 4b + 2\pi r)}$$

After substituting equation (5) into equation (4), the conditional equation that the neutral axis satisfies is obtained $q c^2 + nc + s = 0$.

$$n = \left[ -2(a - b) - \frac{1}{2} \pi r \right] p, \quad s = \left[ b^2 + 2a(b + r) + \frac{1}{2} b \pi r + \sqrt{r^2} \right] q$$

where

$$p = \frac{1}{l_s} \left( \frac{K_j}{4a + 4b + 2\pi r} E_s + K_j l_s \right), \quad q = \frac{1}{l_s} \left( \frac{K_j}{4a + 4b + 2\pi r} E_s + K_j l_s \right)$$

From the moment balance condition, we obtain

$$\frac{2E_i\varepsilon_c}{B + R - c} \int_0^{a_r} \frac{a_r}{D} x_1 t^2 s_1 ds_1 + \frac{2E_i\varepsilon_c}{B + R - c} \int_0^{a_r} x_2 t^2 ds_2 + \frac{2E_i\varepsilon_c}{B + R - c} \int_0^{a_r} x_3 t^2 ds_3 + \frac{2E_i\varepsilon_c}{B + R - c} \int_0^{a_r} x_4 t^2 ds_4 + \frac{2E_i\varepsilon_i}{B + R + c} \int_0^{a_r} x_6 t^2 ds_6 + \frac{2E_i\varepsilon_i}{B + R + c} \int_0^{a_r} x_7 t^2 ds_7 + \frac{2E_i\varepsilon_i}{B + R + c} \int_0^{a_r} x_8 t^2 ds_8 + \frac{2E_i\varepsilon_i}{B + R + c} \int_0^{a_r} x_9 t^2 ds_9 = M$$

Combining equation (6) and equation (5) leads to the unit rotation angle, $\theta$,

$$\theta = \frac{M l_s}{E_i t R_1 + R_2 l_s}$$

where

$$R_1 = 2 a (b - c + r)^3 + \frac{1}{2} (b - c)^3 r \pi + 2 \sqrt{2} (b - c) r^2 + \frac{(2 + \pi)}{4} r^3 + \frac{2 (b - c)^3}{3}$$

$$R_2 = \left[ 2 a (b + c + r)^3 + \frac{1}{2} (b + c)^3 r \pi + 2 \sqrt{2} (b + c) r^2 + \frac{(2 + \pi)}{4} r^3 + \frac{2 (b + c)^3}{3} \right] R_3,$$

$$R_3 = \frac{p - q}{2}$$

The available LES is

$$(EI)_{eq} = E_i t (R_1 + R_2 l_s)$$

The LES efficiency is defined as the ratio of the equivalent bending stiffness to the bending stiffness of a homogeneous tunnel [11]
where $\eta$ reflects the change of the equivalent flexural stiffness compared to the flexural stiffness of the original segment, which is of great significance in the pipe-jacking tunnel design.

3. Results and analysis

3.1. Case studies

The IPG project located between Zhongyi Road and Hexie Avenue in Wuhan, China, is constructed along the west side of the Huangxiao River, with an overall approach from the north to south, starting from Xingye Road in the south and crossing Jinqiao Avenue with a length of 139.3 m. The integrated pipe gallery and low-level box culvert crossing the Rail Line 3 adopt RPJT construction, and all other sections adopt support excavation construction.

The large-section RPJT technology used in the IPG project is less applied in China due to technical requirements. It faces many challenges such as RPJT structural design, rectangular shield design, manufacturing, rectangular shield construction comprehensive technology, and organization and management. Therefore, the longitudinal equivalent bending stiffness of the tunnel was studied in this research. The size of the exterior contour of the rectangular section of the tunnel is $8.300 \times 5.250$ m, the radius of the rounded corner is 0.5 m, while the interior contour size is $7.100 \times 4.050$ m and the radius of the rounded corner is 0.15 m. The bolt is 375 mm away from the exterior contour and 225 mm from the interior contour, and 45 8.8 grade M30 bolts are used longitudinally. The main parameters of the joint and lining are shown in Tables 1 and 2, respectively.

| Table 1. Parameters of the tunnel longitudinal joint. |
|-----------------------------------------------------|
| Number of bolts ($n$) | Diameter ($d$/mm) | Length ($l_h$/mm) | Young’s modulus ($E_s$/GPa) |
|-----------------------------------------------------|
| 45 | 30 | 450 | 206 |

| Table 2. The setting of tunnel lining parameters. |
|------------------------------------------------|
| Outer half-length ($A$/m) | Inner half-width ($B$/m) | Half-width of bolt rectangle ($a$/m) | Half-length of bolt rectangle ($b$/m) | Bolt fillet radius ($r$/m) | Thickness of the segment ($t$/m) | Ring width ($l_s$/m) | Elastic modulus ($E_s$/GPa) |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 3.650 | 2.125 | 0.5 | 3.494 | 1.969 | 0.281 | 0.60 | 1.2 | 34.5 |

| Table 3. Results for Rectangular Tunnels |
|----------------------------------------|
| Elastic stiffness coefficient ($K_i$) | Moment of inertia ($I_e$/m$^3$) | Neutral axis ($c$/m) | LES efficiency ($\eta$) |
|----------------------------------------|
| 14.5613 | 60.7634 | 2.1125 | 0.0400 |

After substituting the data in Table 2 into equations (1) to (9), the results shown in Table 3 are obtained.

3.2. The influence of the number of longitudinal bolts on the LES efficiency

Based on the above project, combined with RPJT segment structure parameters of Huangxiao River, the number of longitudinal bolts is changed from 35 to 55, and the other segment parameters remain unchanged. The neutral axis position and the LES are calculated separately, as shown in Figure 4.

It can be seen from Figure 4 that with the increase of the number of longitudinal bolts, $n$, the distance between the neutral axis and the midline, $c$, decreases linearly. When the number of bolts increases from 35 to 55, the neutral axis moves up by approximately 0.06 m. The compression zone gradually increases and the tension zone gradually decreases, which makes the RPJT segment more...
become uniform in terms of stresses, thus tending to prevent the cracking caused by the tension of the RPJT segment and facilitating the waterproofing of the component.

When the number of longitudinal bolts, \(n\), increases by one, the corresponding LES efficiency, \(\eta\), increases by about 0.08\%, because the primary function of the bolts is to resist the tensile stress caused by the bending moment \(M\). With the increase of longitudinal bolts, the connection between the rings is strengthened, and the average line stiffness at the interface of the tubes is increased, which enhances the integrity of the shield tunnel structure and leads to the increase of the longitudinal equivalent flexural stiffness of the tunnel. Therefore, in tunnel design, the LES of the tunnel can be increased by appropriately increasing the number of longitudinal ring joint bolts.

It should be noted that as the number of bolts increases, the number of bolt holes on the same segment also increases. Too many bolt holes will reduce the strength of a single segment and the transverse load-bearing performance of the segment, which poses threat to the tunnel structure's safety. Therefore, multiple factors should be considered comprehensively when considering the number of bolts.

![Figure 4. Number of the longitudinal bolt number vs. LES efficiency and position of the neutral axis.](image)

![Figure 5. Ring width vs. LES efficiency and position of the neutral axis.](image)

3.3. The influence of ring width on the LES efficiency

Combined with the IPG project of Huangxiao River, the number of longitudinal bolts is taken as 45. The ring width of the tunnel segment varies from 0.8 m to 1.6 m, and other structural parameters remain unchanged. The position of the neutral axis and the LES efficiency are calculated separately, as shown in Figure 5.

Figure 5 shows the effect of the ring width on the LES efficiency and the position of the neutral axis. As the ring width of the tunnel segment, \(l_0\), increases, the distance between the neutral axis and the centerline \((c)\) decreases linearly. When the ring width increases from 0.8 m to 1.6 m, the neutral axis moves upwards by about 0.09 m. The compression zone becomes smaller, and the stress in the segment is more uniform, which is beneficial to the overall stability of the tunnel. At the same time, the increase in the tension zone area will reduce the opening of the joint between the segment rings, which can effectively prevent water from penetrating through the joint.

When the ring width of the tunnel segment, \(l_0\), increases from 0.8 m to 1.6 m, the LES efficiency, \(\eta\), increases linearly from 0.0272 to 0.0524. When the ring width of shield tunnel segment increases, the number of longitudinal rings decreases in the same length of shield tunnel, which makes the shield tunnel more inclined to a continuous integral structure and leads to the enhancement of LES in the tunnel. In addition, reduction in the number of ring joints reduces the possibility of water leakage between the pipe segments, which is good for safe construction and use of the pipe gallery. Therefore, in the tunnel design process, the overall bending resistance of the shield tunnel can be increased by increasing the ring width of the tunnel segment.
However, the increase of the ring width will increase the jack stroke, construction difficulty, and transportation difficulty of the shield machine simultaneously, and the construction cost and time will also increase accordingly [12]. Therefore, the ring width value needs to be designed reasonably in the actual project, where 1~1.6 m wide of the pipe-jacking tunnel segment is generally adopted.

3.4. The influence of the segment’s thickness on the LES efficiency

The thickness of the segment is one of the primary parameters that affects the shape of the rectangular cross-section. It is assumed that the distance ratio between the bolt position and the inner and outer contour of the rectangle remains constant and the fillet radius of the inner and outer contour remains the same. When the overall size of the outer contour of the tunnel (8.300×5.250m) remains unchanged, $a$ and $b$ will also change with $t$, satisfying

$$a = \frac{8.3 - 375}{2 \times 600} t - r, \quad b = \frac{5.25 - 375}{2 \times 600} t - r$$

(10)

Combined with the IPG project of the Huangxiao River, the number of longitudinal bolts is 45, the ring width is 1.2 m, and the thickness of the segment varies from 0.4 m to 0.8 m. The position of the neutral axis and the LES efficiency of the RPJT are calculated, as shown in Figure 6.

Figure 6 shows that as the thickness of the segment ($t$) increases from 0.4 m to 0.8 m, the distance between the neutral axis and the center line ($c$) decreases by 0.2426 m linearly. The compression zone of the RPJT segment increases while the tension zone decreases, resulting that the overall stress area is uniform, which is beneficial to the waterproofing of the RPJT segment and the overall stability of the tunnel.

When the tunnel segment thickness ($t$) is doubled from 0.4 m, the LES efficiency ($\eta$) linearly increases from 0.0392 to 0.0408. The reason for this phenomenon is due to the fact that the space occupied by the entire tunnel remains the same when the size of the rectangular outer contour and the fillet radius remains the same. In other words, with unchanged overall stress of the tunnel, when the segment’s thickness increases, the stress-bearing area at the cross-section increases, and the pressure shared by each segment of the segment decreases, making the tunnel safer.

However, the increase of the thickness of the pipe will also increase the cost of materials and construction, and when occupying the same underground space, the usable area of the tunnel is reduced, which violates the original intention of optimizing the space of RPJT, and should be considered comprehensively in the design.

![Figure 6. Effect of segment thickness on equivalent stiffness and neutral shafts](image)

![Figure 7. Relationship between bending moment and maximum joint opening](image)

3.5. The influence of bending moment on the maximum joint opening of the tunnel
In this section, the IPG project of Huangxiao River is still used as an example. The number of longitudinal bolts is 45 and the ring width is 1.2 m. Under different bending moments, the joint opening farthest from the neutral axis is calculated, as shown in Figure 7.

From Figure 7 when the bending moment $M$ increases, the maximum joint opening at the farthest point from the neutral axis increases linearly. The reason for this is due to the fact that the bending moment does not affect the position of the neutral axis and the LES efficiency. The area of the tension zone remains unchanged. When the sectional bending moment increases, the longitudinal bending of the tunnel structure increases, as well as the bending deformation, unit rotation angle $\theta$ and opening of the annular joint.

4. Conclusion

In this paper, by simplifying the actual RPJT, the longitudinal equivalent continuity model of the rounded RPJT is established, and the corresponding equivalent bending stiffness is derived. Then taking the integrated pipe gallery (IPG) project of Huangxiao River as a reference, the effect of different parameters on the position of the neutral axis and the longitudinal equivalent stiffness (LES) efficiency is analysed. The following conclusions are obtained:

1) When the number of longitudinal joint bolts, ring width and thickness of the RPJT segments increases, the neutral axis will move upwards and the longitudinal equivalent bending stiffness of RPJT increases, which is beneficial to the waterproofing of the segment and the overall stability of the tunnel.

2) Increase in the bending moment of the tunnel will not affect the position of the neutral axis and the longitudinal equivalent bending stiffness, but increase the segment unit's corner, resulting in the increase of the opening of ring joint.

3) Any excessive increase or decrease of any parameter will have an impact on the economic effect of the project. Various factors should be considered comprehensively when designing the tunnel.

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