Simultaneous suppression of forward and backward light scattering by high-index nanoparticles based on Kerker-like effects

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Abstract. The ability of all-dielectric nanostructures to perform exotic photonics effects is with superior efficiency compared to their metallic counterparts. Free from joules losses, high-index dielectrics support comparable excitation of electric and magnetic resonances and pave a way to advanced technologies of light energy manipulation. One of the most important effects is directive light scattering provided by the Kerker and anti-Kerker effects giving the potential to realize Huygens source of light, transparent metasurfaces, router nanoantennas etc. Here we study an effect where most of the scattered power is redirected to the side directions rather than to the forward and/or backward directions. This kind of scattering on isotropic scatterer requires at least the presence of the first two orders of multipoles to enable simultaneous forward and back-scattering suppressions. Electric dipole Fano resonance profile and quadrupoles off-resonance characteristics provide the required phase and amplitude conditions to obtain such an optical signature. We find the individual scatterers sustain the transverse scattering conditions when assembled into a metasurface so exhibit invisibility effect. We investigate this phenomenon analytically and numerically in the visible and microwave domains and provide the proof-of-the-concept experiment in the gigahertz frequency and showing very good agreement with the theoretical predictions.

1. Introduction

Optical properties of high-refractive-index dielectric nanoparticles are attracting a great scientific interest nowadays [1, 2, 3]. These subwavelength scatterers can support the excitation of electric and magnetic multipolar resonances which allow the control over the electric and magnetic components of a light by changing the nanoparticles size, geometry, and material [4, 5, 6]. Desirable overlapping of certain multipole resonances of dielectric nanoparticles can be employed for the configuration of the scattered radiation. This, in its turn, could be useful for different applications, including nanoantennas [2], metadevices [7], multifunctional metasurfaces [8] etc. For example, directional light scattering due to the resonant Kerker effect in dielectric nanodiscs is used for the realization of Huygens nanoantennae and fully transparent metasurfaces [8].
2. Transverse Scatterer

Starting from the Cartesian multipole decomposition of a field scattered by an arbitrary particle being much smaller, than the wavelength. The surrounding medium is a free space with relative permittivity $\varepsilon_d = 1$. The incident wave is assumed to be linearly polarized along the $z$-axis: $E_{\text{inc}} = E_0 e^{ikz} \hat{x}$, where $k = |k|$ is the wavenumber and $\hat{x}$ is a unit vector along the $x$ axis. The scattered light is defined as a summation of multipoles with the following form (up to the quadrupole terms) [9],

$$E_{\text{sca}}(n) \cong \frac{k^2 e^{ikr}}{4\pi r\varepsilon_0} \left( [n \times [p \times n]] + \frac{1}{c} [m \times n] + \frac{ik}{6} [n \times (\hat{Q} \cdot n)] + \frac{ik}{2c} [n \times (\hat{M} \cdot n)] \right), \quad (1)$$

where the unit vector $n$ is in the direction of scattering vector $r$, $\varepsilon_0$ is the vacuum electric permittivity, $c$ is the light speed in the vacuum; $k$ is the wavenumbers in vacuum, correspondingly. $m$ is the magnetic dipole moment (MD) of a particle; $p$ is the total electric dipole moment (TED); $\hat{Q}$, $\hat{M}$ and $\hat{O}$ are the electric quadrupole moment tensor (EQ), the
magnetic quadrupole moment tensor (MQ), respectively. Note that these tensors are symmetric and traceless and in tensor notation e.g. $\hat{Q}$ is equal to $Q_{\alpha\beta}$, where subscript indices denote components (e.g $\alpha = x, y, z$) [11]. To derive the conditions of the simultaneous suppression for forward and backward scattering [10], we insert the unit vector $\mathbf{n} = [0, 0, \pm 1]$ into Eq.1 and obtain

$$
c_{p_x} = m_y; \quad cQ_{xz} = 3M_{yz}; \quad \Re(p_x) = -\Re\left(\frac{kM_{yz}}{2ic}\right); \quad \Im(p_x) = \Im\left(\frac{kM_{yz}}{2ic}\right).
$$

(2)
The first term of Eq.2 represents in-phase dipoles, and similarly the second term indicates in-phase quadrupoles. The remaining terms show the coherent dipoles has to be in $\pi$-phase relation to the coherent quadrupoles in order to nearly suppress forward and back-scattering simultaneously. Note, the imaginary parts of the multipoles (equivalent to polarizabilities) must be always positive values. [9].

In fig. 1(a,b), we study the aforementioned conditions for Si nanocube with edge size equal to 250 nm at the invisible spectrum. At the particular wavelength $\lambda = 788$ nm (black arrows), one can see that the dipoles are in-phase with the nearly equal real and imaginary parts indicating the Kerker effect. At the same time, the quadrupoles, have comparable values, and are in phase as well. The real parts of the dipoles are equal to the quadrupoles real parts but with opposite polarity. The corresponding scattering patterns are shown in Fig.1(c). Hence, there is the almost complete scattering suppression in the forward and backward directions. However, the suppression of the forward scattering is not complete owing to the optical theorem [9].

Due to scaling properties of Maxwell equations, the transverse scattering effect of high-index particles can be obtained in different electromagnetic spectrum. Also, because spheres and cubes have a similar symmetry, we performed an experiment at the microwave frequency for ceramic sphere in order to prove the concept. In Fig.1(d) the calculated and the measured transverse scattering pattern are in a good agreement.

3. Metasurface and reflection
We consider here square and periodic lattice of Si cubic particles with side 250 nm arranged in the $xy$-plane. The incident plane wave is $E_z$ polarized and propagates normally along the $z$-direction. Figure 2(a) illustrates the reflection resonances for different lattice spacing 400 nm, 500 nm and 600 nm. In this figure, increasing the spacing further widens the zero-reflection region around the transverse scattering wavelength 788 nm. This is because, the spectral point of Rayleigh

Figure 2. Reflection spectra calculated for periodic square lattice of silicon nanocubes with side 250nm. (a) For different spacing in free space. (b) The evolution of the reflection as the refractive index of the introduced substrate increases from ref= 1 to ref= 1.3 and ref= 1.5 with fixed lattice spacing $p = 400nm$. 

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anomaly shifted closer to the region of reflection suppression; and as a result, coupling between particles becomes stronger and the effective resonances are shifted [11]. However, multipoles resonances shifting is not uniform as we can see from fig.2(a).

Figure 2(b) shows the reflection evolution for several substrate refractive indices 1, 1.3, and 1.5, while the lattice spacing is fixed to be 400 nm. Note, that at the point of zero reflection we have now another physical mechanism destructive interference of the partial waves reflected by the substrate and metasurfaces, so called, antireflection [12, 13, 14, 15, 16]. The metasurface now is not invisible, and reflects a wave with totally the same amplitude, as the substrate, but in antiphase. It is obvious, increasing the refractive index will lead to a more pronounced spectral separation of these two points corresponding to the aforementioned different optical phenomena.

4. Conclusion
In summary, we have carried out the conditions for the formation of transverse scattering pattern for high-index nanoparticles of different shapes. This unusual behavior could take place when the in-phase coherent electric and magnetic dipoles is out of phase to its counterpart quadrupoles and all have comparable values; which ordinarily occurs in off-resonance scattering regions of the spectrum. Experimental measurements proved the concept for the ceramic sphere situated in a free space and good agreement between the theoretical and experimental scattering pattern is obtained. Finally, to extend the discussion to 2D nanostructures, we considered regular metasurfaces consisting of the nanoparticles with the transverse scattering patterns. We showed the metasurface obtain near-zero reflection independently on lattice spacing and the substrate.

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