Umklapp scattering in transport through a 1D wire of finite length.

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Suppression of electron current $\Delta I$ through a 1D channel of length $L$ connecting two Fermi liquid reservoirs is studied taking into account the Umklapp interaction induced by a periodic potential. This interaction opens band gaps at the integer fillings and Hubbard gaps $2m$ at some rational fillings in the infinite wire: $L \to \infty$. In the perturbative regime where $m \ll v_c/L$ ($v_c$ : charge velocity), and for small deviations $\delta n$ of the electron density from its commensurate values $-\Delta I/V$ can diverge with some exponent as voltage or temperature $V,T$ decreases above $E_c = \max(v_c/L, v_c\delta n)$, while it goes to zero below $E_c$. This results in a non-monotonous behavior of the conductance. In the case when the Umklapp interaction creates a large Mott-Hubbard gap $2m \gg T$, inside the wire, the transport is suppressed near half-filling everywhere inside the gap except for an exponentially small region of $V,T < T_L \exp(-2m/T_L)$.

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I. INTRODUCTION

Recent developments in the nano-fabrication technique have made the 1D interacting electron systems an experimental reality, and its quantum transport properties have been the subject of extensive studies both experimentally [1-3] and theoretically [4-16]. In realistic experimental set-ups, the quantum wire is attached to two-dimensional regions called reservoirs or leads. A metallic phase of the infinite wire is known as a Tomonaga-Luttinger liquid [4]. To describe 1D transport phenomena in the realistic configuration a model was recently formulated of the inhomogeneous Tomonaga-Luttinger liquid (ITTL) [4]. It recovers the conductance $G = 2e^2/h$ observed experimentally even in the presence of the electron-electron interaction in the wire [6] (below we use the units, where $\hbar = e = 1$), although the previous calculations on an infinitely long wire [5] predicted the renormalized conductance (see [1] for the further development). Calculation of the conductance suppressed by a weak random impurity potential in this model [10] had agreed with both the previous theoretical prediction [8] and an experiment [4].

In this paper we consider effect of opening a spectral gap in the wire on the 1D transport. Theory [11] predicts that in 1D besides band gaps produced by a periodical potential in the wire at the integer fillings, a repulsive interaction between electrons opens Mott-Hubbard gaps at some rational fillings. Furthermore recently Tarucha et al. [12] succeeded in introducing the 1D periodic potential with a periodicity of order 40nm into the wire 2$\mu$m in length and 50nm wide. This induces Umklapp scattering. The electron density $n$ can be continuously controlled by the gate voltage, and one can satisfy the half-filling condition within an accessible value of $n$. If this condition is satisfied the system will becomes a 1D (doped) Mott insulator with the Mott-Hubbard gap $m$ for the infinite length wire. Then it will offer an ide-
voltage, bias voltage, and temperature.

In the next section IV, motivated by a recent discussion [19,20] that in a non-perturbative case of the large Mott-Hubbard gap $2m \gg v_c/L \equiv T_L$ the conductance is strongly suppressed at any low energy $(\hbar \nu)$ inside the gap similar to the band gap case, we consider transport through a 1D Mott-Hubbard insulator of a finite length $L$ beyond perturbative approach. Our presentation follows [21] where we used a special value of the low energy constant of the interaction to map the problem onto the exactly solvable models. We find current vs. voltage at high temperature $T > \max(m,T_L)$ and at low energy $T,V < T_L$. The result shows that for the strong interaction creating a large Mott-Hubbard gap $2m \gg T_L$ inside the wire, the transport is suppressed near half-filling everywhere inside the gap except for an exponentially small region of $V,T < T_L \exp(-2m/T_L)$

**II. MODEL**

Our model can be derived following [3] from a 1 channel electron Hamiltonian

$$\mathcal{H} = \int dx \left\{ \sum_\sigma \psi_\sigma^\dagger(x)(-\frac{\partial^2}{2m^*} - E_F)\psi_\sigma(x) + \varphi(x)\rho^2(x) + [V_{\text{imp}}(x) + V_{\text{period}}(x)]\rho(x) \right\}$$

with the periodic potential $V_{\text{period}}(x)$ (period $a$) assumed to be weak enough to justify the perturbative consideration of the Umklapp backscattering. The Fermi momentum $k_F$ and the Fermi energy $E_F$ is determined by the filling factor $\nu$ as $\nu = k_F a/\pi$ and $E_F \approx v_F k_F$. In Eq. (1) the function $\varphi(x) = \text{const} \times \theta(x)\theta(L-x)$ switches on the electron-electron interaction inside the wire confined in $0 < x < L$. This interaction is assumed to be local, as the close metallic gate used in experiments to form the wire inevitably screens the long range Coulomb. Contribution of the random impurity potential $V_{\text{imp}}(x)\rho(x)$ to the conductance has been considered in [3,6], some results of which we will use below. Following Haldane’s generalized bosonization procedure [14] to account for the nonlinear dispersion one has to write the fermionic fields as $\psi_\sigma(x) = \sqrt{k_F/(2\pi)} \sum \exp(i(n+1)(k_F x + \phi_\sigma(x)/2) + i\theta_\sigma(x)/2)$ and the electron density fluctuations as $\rho(x) = \sum \rho_\sigma(x)$, $\rho_\sigma(x) = [\theta_\sigma \phi_\sigma(x) + 2k_F]/(2\pi) \sum \exp(in(k_F x + \phi_\sigma(x)/2))$ where summation runs over even $n$ and $\phi_\sigma, \theta_\sigma$ are mutually conjugated bosonic fields $[\phi_\sigma(x), \theta_\sigma(y)] = i2\pi \text{sgn}(x-y)$.

After substitution of these expressions into (1) and introduction of the charge and spin bosonic fields as $\phi_{c,s} = (\phi_\Sigma \pm \phi_L)/\sqrt{2}$ the Hamiltonian takes its bose-form $\mathcal{H} = \mathcal{H}_D + \mathcal{H}_{bs}$. Here the free electron movement modified by the forward scattering interaction is described by [4,5]

$$\mathcal{H}_D = \int dx \sum_{b=c,s} v_b(x) \left\{ \frac{1}{g_b(x)} \left( \frac{\partial_x \phi_b(x)}{\sqrt{4\pi}} \right)^2 + g_b(x) \left( \frac{\partial_x \theta_b(x)}{\sqrt{4\pi}} \right)^2 \right\}$$

with $g_c(x) = g$ for $x \in [0,L]$ ( $g$ is less than 1 for the repulsive interaction and it will be assumed below ), $g_c(x) = 1$, otherwise and $v_c(x) = v_F/g_c(x)$. The constants in the spin channel $g_s = 1, v_s = v_F$ are fixed by $SU(2)$ symmetry. Keeping only the most slowly decaying terms among others with the same transferred momentum one could write the backscattering interaction as

$$\mathcal{H}_{bs} = \frac{E_F^2}{v_F} \int_0^L dx \sum_{\text{even } m > 0} U_m \cos(2k_{mF}mx + \frac{m\phi_s(x)}{\sqrt{2}}) + \sum_{\text{odd } m > 0} U_m \cos(\frac{\phi_s(x)}{\sqrt{2}})\cos(2k_{mF}mx + \frac{m\phi_s(x)}{\sqrt{2}})$$

most singular is the $m = 1$ term of the second sum responsible for opening the band gap in the infinite wire at $\nu = \text{integer}$. The dimensionless coefficients $U_m$ originate from $\varphi(x)$.

In the spinless case we should put $\phi_s = \phi_c$ in (3) and change the backscattering interaction to

$$\mathcal{H}_{bs} = \frac{E_F^2}{v_F} \int_0^L dx \sum_{m > 0} U_m \cos(2k_{mF}mx + m\phi_c(x))$$

shows out in transport properties of the wire filled with the non-interacting electrons. The model is equivalent to a Dirac equation with the mass switched on inside the wire:
with $m = \pi E_F U_1/2$ in the spin case of Eq. (3) and $m = \pi E_F U_1$ in the spinless case of Eq. (1). Here $\psi_{R(L)}$ describes the right (left) chiral electron and $k_F$ is counted from $\pi l/a$ at $\nu \simeq l$. The transport is determined by the transmittance $D$ which is a function of the electron energy $\varepsilon$ counted from the one of the middle of the gap $\nu_F \pi l/a$:

$$D(\varepsilon) = \left[1 + m^2 \sin^2(\sqrt{\varepsilon^2 - m^2 t_L})/\varepsilon^2 - m^2\right]^{-1}$$

(6)

where the analytical extension is assumed at $\varepsilon < m$ and $t_L = L/\nu_F$. In particular, the linear bias conductance $G$ is equal to

$$G(T, \mu_0) = \frac{1}{2\pi T} \int d\varepsilon \frac{D(\varepsilon)}{1 + \cosh(\varepsilon - \mu_0)/T}$$

(7)

for spin electrons where $\mu_0 = k_F \nu_F$ gives a deviation of the chemical potential of the wire from that of the $\nu = l$ filling. The conductance has two regimes of behavior.

1. **High temperatures $T > T_L$, $m$** - The asymptotics to (6) can be written as

$$G = \frac{1}{\pi} \left(1 - \frac{m}{2T} \frac{B(m/T_L)}{1 + \cosh(\mu_0/T)}\right)$$

(8)

after noticing that $D$ is a quickly oscillating function against the slowly varying temperature factor. The coefficient $B$ changes slowly from 0 to $\simeq \pi$ and is defined below in Eq. (8). The expression (9) shows that at the high temperature the band gap produces a smooth well in the conductance vs. chemical potential which becomes more narrow and deeper as $T$ decreases.

2. **Low temperatures $T < T_L$.** The conductance is about $D(\mu_0)/\pi$. It is suppressed in the middle of the gap $G(0) = \left[\pi(1 + (mt_L^2))\right]^{-1}$ and approach its maximum $1/\pi$ away from the gap oscillating with period $\pi l/a$. In the limit $mt_L \ll 1$ this interference structure is perfectly periodical, as

$$\Delta G_1 = -(mt_L)^2 \sin^2((k_F - \pi l/a)L) / ((k_F - \pi l/a)L)^2$$

(9)

Therefore the conductance has $L/a$ humps in between two neighbor band gaps. We expect this being correct at any finite $m$ from comparison with a tight binding model. In our model it holds on asymptotically if $\pi \nu_c/(am) \gg 1$.

Then the model works well.

### III. NARROW GAP: PERTURBATIVE APPROACH

In this section we assume that the dimensionless coefficients $U_m$ in Eq. (3) are small enough to justify perturbative calculation of the current. The variation of the current due to the backscattering is given by :

$$\Delta I = -i/(2\sqrt{2\pi}) \int dx [\delta \rho_s(x), \mathcal{H}] = \sqrt{2} \int dx [\delta \phi_s(x)] \mathcal{H}_{bs}.$$  

At finite voltage $V$ applied symmetrically to neglect the momentum transfer variation, the average of $\Delta I$ decomposes into sum of the different backscattering mechanism contributions $<\Delta I_m>$ in the lowest order. The even $m$ terms involving only $\phi_c$ field are equal to

$$<\Delta I_m> = -\frac{m}{4} \frac{U_m E_F^2}{\nu_F} \int_{-\infty}^{\infty} dt \int_0^{L} dx_1 dx_2 <e^{im\phi_c(x_1,t)/\sqrt{2}} e^{-im\phi_c(x_2,0)/\sqrt{2}} >$$

(10)

The current operator $\Delta m$ has a high energy scaling dimension $m^2 g/2$ and a free electron ($g = 1$) behavior at low energy. We will see below that the integral (11) scales at low energy with $(m^2 - 1)$ exponent and with $(m^2 g - 2)$ exponent at high energy. The most singular behavior is due to Umklapp backscattering at $m = 2$ with the threshold voltage $V = E_{thr} = 2k_F \nu_c$ going to zero at the half filling. We assume $V, E_{thr} \geq 0$ below. Less singular correction with $m = 4$ could become relevant at the one and three quarters fillings and so on. Expressions for the odd $m$ terms include additionally a spin field correlator $<e^{i\phi_c(x_1,t)/\sqrt{2}} e^{-i\phi_c(x_2,0)/\sqrt{2}}>$ under the integrals in (11). The high energy dimension of $\Delta I_m$ in this case is $m^2 g/2 + 1/2$. The most singular behavior occurs to the $m = 3$ term at the one and two thirds fillings. It has two threshold energies $E_{thr,c,s} = 2k_F \nu_{c,s}$ for $\nu_c \neq \nu_s$. Neglecting a change of the TLL compressibility produced by the Umklapp scattering we can relate the threshold energy to a deviation of the chemical potential of the wire $\mu$ at $V = 0$ from that of the rational filling. Since $\Delta \rho_c = 2k_F/\pi$ and $\partial \mu/\partial \rho_c = \pi \nu_c/(2g)$ we gather $E_{thr,c} = 2g \mu$. However, in an experiment it is the average of the electrochemical potentials of the leads but not $\mu$ that is known. The latter is proportional to $\mu$ with the coefficient $[1 + 2ge^2/(\pi e \hbar c_g)]$ if the gate voltage is fixed (22). This coefficient is about 1 if the density $c_g$ of the capacitance between the wire and the screening gate is large.

Correlator of the charge field exponents $e^{i\phi_c(x,t)}$,
evolution of which is specified by $\mathcal{H}_0$, could be compiled from the correlators of the uniform TL liquid $K(x,t) = K(x,t,g,v_c)$. (10) follows way 

\begin{equation}
< e^{i\phi_n(x,t)} e^{-i\phi_n(y,0)} > = K(x - y, t) \prod_{n=1}^{\infty} \left( \frac{K(2nL,0)}{K(2nL \pm |x-y|, t)} \right)^{-\tau^{2n}} 
\times \prod_{n=0}^{\infty} \left( \frac{K(2(nL + x),0)K(2(nL + y),0)}{K^2(2nL + x + y, t)} \right)^{-\tau^{2n+1/2}} 
\times \prod_{n=1}^{\infty} \left( \frac{K(2(nL - x),0)K(2(nL - y),0)}{K^2(2nL - x - y, t)} \right)^{-\tau^{2n-1/2}} \tag{11}
\end{equation}

Here $\beta$ is inverse temperature $1/T$ and $\alpha = 1/E_F$ is the ultraviolet cut-off. This complicated form comes about through a multiple scattering at the points of point $x = 0, L$. As a result of the scattering the correlator $\phi_n(x, t) \phi_n(y, 0)$ becomes an infinite sum of the uniform correlators taken along the different paths connecting points $x$ and $y$ and undergoing reflections from the boundaries at $x = 0, L$. Each reflection brings additional factor $r = (1 - g)/(1 + g)$. The similar correlator $< e^{i\phi_n(x,t)} e^{-i\phi_n(y,0)} >$ for spin field is $K(x - y, t, 1, v_c)$. Below we analyze the current corrections \[1\] for high $(T > 1/t_L = T_L)$ and low $(T \ll T_L)$ temperatures, respectively.

1. High temperatures $T > T_L$ - The uniform correlator $K(x, t)$ goes down exponentially if distance between the points $|x| > T_L$. Therefore only paths with length less than $\beta$ contribute to the correlator \[1\]. This means that the high temperature form of the correlator \[1\] reduces to the first multiplier $K(x - y, t) > 0$ to a factor $(1 + O(exp(-T_L/\beta))$. Neglecting $O(T_L, T)$ quantity we can extend integration over $x_1 - x_2$ in \[1\] from $-\infty$ to $+\infty$. Then calculation of the $m = 2$ contribution reduces to finding Fourier transformation $F_{2g}(q, \varepsilon)$ of the correlator $K^2(x, t, g, v_c)$:

\begin{equation}
< \Delta I_2 > = \frac{1}{4} \left( \frac{U_2 E_F^2}{g} \right)^2 t_L \sum_{\pm} \mp F_{2g}(2E_{thr}, \pm2V) = -2 \left( \frac{2^{2(g-1)U_2^2}}{(2g)^2} \frac{E_F^2}{T_L} \left( \frac{\pi T}{E_F} \right)^{4g-2} \sinh \left( \frac{V}{T} \right) \right) \prod_{\pm} \Gamma \left( g \pm i \frac{E_{thr}}{2\pi T} \right) \tag{12}
\end{equation}

One can easily see its behavior making use of the following asymptotics:

\begin{equation}
< \Delta I_2 > \sim \left( \frac{U_2}{g} \right)^2 \frac{E_F^2}{T_L} \sinh \left( \frac{V}{T} \right) \left( \frac{E_{thr}^2 - V^2}{E_F^2} \left( \frac{T}{E_F} \right)^{4g-2} \right) \tag{13}
\end{equation}

FIG. 1. Schematic voltage dependence of the high temperature current corrections produced by the $m = 2$ Umklapp interaction $\Delta I_2$. Solid lines, $E_{thr} = 0$; dashed lines, $E_{thr} \gg T$.

These asymptotics show that the threshold singular-
changing: $4g \to m^2g, 4k_{2F} \pm 2V \to m(2k_{mF} \pm V)$. In the case of the spinless electrons: $4g \to 2m^2g, 4k_{2F} \pm 2V \to m(2k_{mF} \pm V)$ with arbitrary integer $m$. In particular, we find again $T^{-1}$-dependence for decrease of the conductance produced by the band gap opening in the spectrum of free electrons. The edge singularity is characterized by a half of the scaling dimension for $\Delta I_m$ since only one chiral component of the field $\phi_{c}$ contributes.

As to the odd $m$ terms, the two threshold energies $E_{\text{thr},c,s} = 2k_{mF}v_c,s$ become distinguishable if their difference exceeds $T$. The leading high-temperature current correction reads as:

$$<\Delta I_m> = \frac{m}{4} \left( \frac{U_m E_{c}^{\mp}}{2\pi g} \right)^2 t_L v_s \sum_{\pm} \int \int dq \frac{F_n}{t_L} (mE_{\text{thr}} - qv_s, \pm mV - \varepsilon) F_{m+x}^{\mp} (qv_c, \varepsilon)$$  \hspace{1cm} (14)

Threshold energies could be observed at the integer filling factor where opening the band gap eventually makes electrons non-interacting.

2. Low temperature, $T,V << T_L$ - With lowering temperature we should expect that above current correction dependencies will be modulated by a $\pi T_L$ quasiperiodic interference structure [3][4] and also a new low energy scaling behavior of the current correction operators will appear at $V,T < T_L$. The dominant contribution to the integral of [4] comes from long times $t \gg t_L$. One can neglect the spacious dependence compared with large time asymptotics:

$$<e^{i\phi_c(x,t)} e^{-i\phi_c(y,0)}> = e^{\gamma(T_L/T)} (\frac{\alpha}{t_L})^{2g} (\frac{\pi t_L/\beta}{\sinh(\pi(t - i\alpha)/\beta)} \sinh(\pi(-t + i\alpha)/\beta))^{1-z} (\frac{\sqrt{xy(L-x)(L-y)}}{E^2})^{2rg}$$  \hspace{1cm} (15)

where $z(T_L/T) = \nu \beta/(T_L \pi)$ and $\gamma(T_L/T)$ approach the constant $\gamma(\infty)$ on the order of $1$ as $\ln(T_L/T) z(T_L/T)$. Our asymptotic analysis following in essential Maslov’s paper [10] shows that the low energy exponents approach their free electron values as $\exp[T_L \ln \nu/(T \pi)]$. The effect accounts for prolongation of the paths due to the finite reflection. In particular, it determines the coefficient $c(g)$ of the $T^2$ corrections to the non-universal zero temperature value of the conductance variation due to impurities: $\Delta G_{\text{imp}} \propto -(L/l)(T/E_F)^{\gamma-1}(1-c(g)(T/T_L)^g)$ in a universal way [3]. After substitution of [3] into Eq. [10], the current suppression produced by the even $m$ terms of the interaction becomes equal to:

$$<\Delta I_m> = -\frac{m^2g^{2(1-z)}e^{mg/2}}{\Gamma(m^2(1-z))} \left( \frac{U_m}{g} \right)^2 t_L \pi \frac{2g}{a} (2mk_{mF}L) T_L \left( \frac{\pi T}{T_L} \right)^{m^2-1} \left( \frac{T_L}{E_F} \right)^{m^2(g-1)} f_{m,x} (V/T)$$  \hspace{1cm} (16)

where function $f_{x}(x) = \sinh(x) \prod_{\pm} \Gamma(\alpha + ix/\pi)$ characterizes the $V-T$ cross over. It approaches $\Gamma^2(a)x(1 - (\ln \Gamma(\alpha))^2(x/\pi)^2)$ at $x \ll 1$ and $\pi(x/\pi)2a_1$ at $x \gg 1$. Function $R$ specifies the $k_F - 1/L$ crossover as:

$$R_{2g}(x) = \frac{\pi T^2 (1 + 2rg) J_{1/2 + 2rg}(x/2)}{x^{1+2rg}} \simeq \Gamma^2(1 + 2rg) \left\{ \begin{array}{ll} \frac{\pi}{4(1 + 4rg)} \Gamma^2(3/2 + 2rg), & x \ll 1 \\ 4 \sin^2(x/2 - \pi rg) x^{-2 - 4rg}, & x \gg 1 \end{array} \right\}$$  \hspace{1cm} (17)

It brings out an interference structure in the conductance versus the chemical potential at low energy. This structure coincides with the one of Eq. (3) at $m = 1$. At larger $m$, however, the oscillations are more frequent. In particular, there can be the unchanged $L/a$ number of maximums of the conductance in between its neighbor minimums at the half-filling and the integer filling.
The odd $m$ terms of the spin electron current will meet Eq. [4] after substitution $m^2 + 1$ instead of $m^2$ into the powers and the index of the $f$ function in this equation. In the spinless case we have to substitute $2m^2$ there and into the index of $R$. Combining the above results we can outline a temperature dependence of the conductance correction produced by the $m = 2$ Umklapp interaction (Fig. 4). For spin electrons its magnitude increases/decreases following $(E_F/T_L)(T/E_F)^{4g-3}$ as $T$ going down above $T_L$ and follows $(T/T_L)^2(T/L/E_F)^{4g-4}$, if $E_{thr} < T_L$; otherwise, the correction starts to decrease exponentially $\exp(-E_{thr}/T)$ below $E_{thr}$ and keeps on decreasing like $(T/T_L)^2(T/L/E_F)^{4g-4}(T/E_{thr})^{-2+g2g}\sin^2(2k_2gL - \pi g\gamma)$ below $T_L$. The $T > T_L$ dependence is similar to that of the conductivity of infinite wire found by Giamarchi [5]. Similar dependence with $T$ replaced by $V$ could be predicted for the zero temperature differential conductance $dG_2(V)$ at $V < T_L$.

In summary under perturbative condition we have described a hierarchy of the threshold features produced by the Umklapp backsscatterings at the rational values of the occupation number inside the 1D channel connecting two Fermi liquid reservoirs. In the differential conductance (its derivative) vs. the chemical potential /threshold energy at a finite voltage, the threshold structure is an asymmetric peak of width $\max(T, T_L)$ located at the crossover $E_{thr} \approx V$ as the chemical potential moves away from the rational filling. These peaks are produced by any repulsive interaction at the half-filling of spin electrons in the wire. In the conductance vs. temperature, we predicted a maximum below $E_{thr}$ due to crossover from the Umklapp backscattering to the impurity suppression and an asymmetric minimum at $E_{thr}$ if the interaction is strong enough. However, if the interaction is weak so that $g > 3/4$ the suppression of the conductance even at the half-filling may be difficult for observation while $m/T_L \ll 1$.

IV. 1D MOTT-HUBBARD INSULATOR: NON-PERTURBATIVE RESULTS.

In this section, to clear up the difference between the Mott-Hubbard insulator and the band gap one, we map the problem at low energies and at high temperature onto the exactly solvable models making use of a free fermion value of the constant $g$ of the forward scattering inside the wire. The results are shown in Figs. 3 and 4. At low energies when $T,V < T_L$ (temperature; $V$: voltage), we have found that a new energy scale $T_T \propto T_L \exp[-2\sqrt{m^2 - \mu^2/T_L}]$ appears in the system if the chemical potential $\mu = E_{thr}/(2g)$ of the wire is small enough: $\sqrt{m^2 - \mu^2/T_L} \ll 1$. Below $T_T$ the conductance is not suppressed and the current increases linearly. Above this energy the current saturates and the conductance goes down as $T_T/T$ reaching small values $\approx \exp[-2\sqrt{m^2 - \mu^2/T_L}]$ at $T \approx T_L$. At high temperature $T \gg T_L, m$ we confirmed the asymptotical behavior of the conductance: $G = (1 - \cst \gamma (1 + \cosh \gamma)^{-1})/\pi$ for a Mott-Hubbard insulator which has been found in the previous section in the perturbative regime of a small gap [18]. A brief physical explanation to these results follows. At low energies $T < T_L$ and $\mu \ll m$ the charge field is quantized inside the wire at its values related to the degenerate sin-Gordon vacua. Rare low energy excitations tunnel through the wire with the amplitude $\propto \exp[-m/T_L]$ as (anti)solitons switching the quantized value of the field. The whole process of tunneling, however, includes transformation of the reservoir electron into the sin-Gordon quasiparticles and back. This transformation results in a non-trivial scaling dimension of the tunneling operator equal to 1/2 for the Mott-Hubbard insulator connected to the Fermi liquid reservoirs independently of any parameters. In the case of the band insulator, this dimension is marginal (= 1): the transformation is trivial and does not introduce additional energy dependence. The infrared relevance of the tunneling with the 1/2 dimension brings out above resonance at zero energy. Meanwhile, the exponentially small tunneling amplitude specifies the narrow width of this resonance equal to the crossover energy. Increase of $|\mu|$ favors tunneling of the quasiparticles of the same sort and ultimately produces their finite density in the wire. Then the interaction between these quasiparticles described with the two-particle $S$-matrices dependent on $g$ emerges. At low momenta the $S$-matrix for the quasiparticles of the same sort is inevitably free fermion like, as at $g = 1/2$. It manifests in the renormalization group (RG) flow derived from the Bethe ansatz solution for the massive phase [20] of the sin-Gordon model and in the exponent calculated for the Tomonaga Luttinger liquid (TLL) phase at low
Increase of $T$, on the other hand, is expected to entail, first, a thermally activated behavior of the conductance $\propto \exp[-2m/T]$ at $T_L < T < m$ \cite{24} and then a power law dependence at $m < T$. Since the effective value of $g$, in general, scales with energy, the $1/T$ dependence we found for $g = 1/2$ may vary at higher energies $T \gg m$ depending on the high energy value of $g$.

Transport through the finite length wire under a constant voltage $V$ between the left and right leads could be described in the inhomogeneous Tomonaga-Luttinger liquid model (TLL) with the Lagrangian \cite{22,23}: \[ \int dx \{ \sum_b \mathcal{L}_b(x, \phi_b, \partial_t \phi_b) + \mathcal{L}_{bs}(x, Vt, \phi_c, \phi_s) \} \] associated to the Hamiltonian \cite{23}. The bosonic fields $\phi_b(x, t)$, $b = c, s$ relate to the deviations of the charge and spin densities from their average values as following: $\rho_b(x, t) = (\partial_x \phi_b(x, t))/((\sqrt{2} \pi))$, respectively. The first part of the Lagrangian describes a free electron motion modified by the forward scattering interaction. The second part of the Lagrangian introduces backscattering inside the wire. Only its term corresponding to the Umklapp process of four Fermi momenta transfer is important near half-filling. This term does not involve the spin field. Therefore, our consideration will be restricted to the charge field only. For the clean wire this field is characterized by the Lagrangian:

\[ \int dx \mathcal{L}_c = \int dx \left( \frac{\nu_v(x)}{2g(x)} \left\{ \frac{1}{v_c^2} \left( \frac{\partial_t \phi_c(t, x)}{\sqrt{4\pi}} \right)^2 - \left( \frac{\partial_x \phi_c(t, x)}{\sqrt{4\pi}} \right)^2 \right\} - \frac{E_F^2 U}{v_F} \phi_c(x) \cos(4k_F x + 2Vt + \sqrt{2} \phi_c(t, x)) \right) \] (18)

where $\phi_c(x) = \theta(x)\theta(L-x)$ specifies a one channel wire of the length $L$ adiabatically attached to the leads $x > L, x < 0$ and $\nu_F(E_F)$ denotes the Fermi velocity(energy) in the channel. The parameter $4k_F$ varies the chemical potential $\mu$ of the wire from its zero value at half-filling. In a real experiment as we discussed before this chemical potential is linearly changed by variation of the electrochemical potential of the screening gate or the average of the reservoir potentials. Outside the Hubbard gap this parameter coincides with the momentum transferred by the backscattering: four Fermi momenta minus a vector of the reciprocal lattice, and relates the present results to the ones \cite{23} of the previous section. We assume $\mu \geq 0$ below. The constant of the forward scattering varies from $g_c(x) = g$ inside the wire ($x \in [0, L]$) to $g_c(x) = g_\infty = 1$ inside the leads, and the Umklapp scattering of the strength $U$ is introduced inside the wire. The charge velocity $v_c(x)$ changes from $v_F$ outside the wire to a some constant $v_c$ inside it. In the absence of the Umklapp scattering, $v_c \simeq v_F/g$ and $0 < g < 1$ is determined by the forward scattering amplitude of the bare short range interaction between electrons. Approaching the half-filling put the Umklapp scattering on. It entails an essential renormalization of the low energy value of $g$, which flows to its free fermion value $g = 1/2$ in the massive phase \cite{24} ($\mu < m$) where the coefficient of the cos-term scales to $\simeq m^2$ and on approaching this phase \cite{24} $|\mu| > m$. This value of $g$ will be assumed below. The zero frequency current through the wire equals $I = V/\pi + <\hat{I}_{bs}>$, where the backscattering current \cite{24} is $\hat{I}_{bs} = -2E_F^2 U/v_F \int_0^L dx \sin(4k_F x + 2Vt + \sqrt{2} \phi_c(x))$. It will be shown later that $2\pi E_F U$ is a half gap $m$, opened by the backscattering \cite{1} in the charge mode spectrum inside the wire.

1. High temperatures $T > T_L$, $m$ - The average backscattering current $<\hat{I}_{bs}> = \int D\phi \hat{I}_{bs} \exp\{i \int dt \int dx (\mathcal{L}_c + \mathcal{L}_{bs}) \}$ can be written as a formal infinite series in $U$. Each term of it is an integral product of the free bosonic correlators $\langle \exp\{i\sqrt{2}\phi(x, t)\}\exp\{-i\sqrt{2}\phi(y, 0)\} \rangle$. Such a correlator approaches its uniform TLL expression when $x, L - x, y, L - y \gg v_c/T$. Substitution of this form into the above series allowed us \cite{18} to find $L$-proportional part of the backscattering current neglecting the boundary contribution in the perturbative case. However, the problem is not perturbative, in general, due to a finite gap $2m$ creation. Therefore, application of the uniform correlator will give us a part of the backscattering current $\propto \min(L, v_c/m)$ with the relative error $O(\max(T_L, m)/T)$, which is of the order of ratio of the border piece $\sim v_c/T$ to the essential part of the "bulk" one. This relates to the high-temperature asymptotics of the whole current.

![FIG. 3. Schematic linear bias conductance $G$ vs. temperature near half-filling $\mu < T_L$; curve 1 for the weak interaction $m < T_L$, curve 2 for the strong one.](image-url)

Calculation of above series with the uniform TLL correlator is equivalent to expanding the value $g = 1/2$ into the leads. Following Luther and Emery \cite{28} we map this bosonic Lagrangian onto the free massive fermion one \cite{19,20} with the density of Lagrangian
Here $\psi_{R(L)}$ is right (left) chiral fermion field. The fermionized backscattering current $I_{bs} = 2mi \int_0^L dx \exp[i(4k_F x + 2V t)] \psi_R^+(x,t) \psi_R(x,t) - h.c.$ is the doubled backscattering current for the fermions under doubled voltage. To find its average we just need to know the fermionic reflection coefficient $R$ as a function of dimensionless energy $\omega = \varepsilon/m$:

$$R(\omega) = \frac{\sin^2(\sqrt{\omega^2 - 1}t_L)}{(\omega^2 - 1) + \sin^2(\sqrt{\omega^2 - 1}t_L)}$$

(20)

where $t_L \equiv m/T_L$ denotes the dimensionless traversal time. The analytical continuation is assumed for $|\omega| < 1$. Since the chemical potential for the right/left chiral fermions is $\mu \pm V$, respectively, the total current can be expressed as

$$I = \frac{V}{\pi} - \frac{m \sinh(V/T)}{\pi} \times \int d\omega \frac{R(\omega)}{\cosh((m\omega - \mu)/T) + \cosh(V/T)}$$

(21)

Only the leading term in $\max(m, T_L)/T$ of the right hand side of (21) is meaningful. Extracting it, we find the high-temperature asymptotics as following

$$I = \frac{V}{\pi} - \frac{m \sinh(V/T) B(mt_L)}{\pi \cosh(\mu/T) + \cosh(V/T)}$$

(22)

$$B(x) = \int d\omega \frac{\sin^2(\sqrt{\omega^2 - 1} x)}{(\omega^2 - 1) + \sin^2(\sqrt{\omega^2 - 1} x)}$$

where function $B(x)$ increases as $\pi x$ at small $x > 0$ and approaches the constant $\approx \pi$ at $x \gg 1$. Accuracy of this calculation of (22) may be written as a factor $1 + O(\max(m, T_L)/\max(T, V))$ to (11) if $|\mu| \ll \max(T, V)$ or as $1 + O[(\max(m, T_L)/\max(T, V)](\max(T, V)/\mu)^2 e^{\mu|\mu|/T}$, otherwise. The high-temperature conductance (Fig.3)

$$\int dx \mathcal{L}_t = \int dx \frac{v_F}{2} \left\{ \frac{1}{v_F} \left( \frac{\partial_t \phi_+(t, x)}{\sqrt{4\pi}} \right)^2 - \left( \frac{\partial_t \phi_-(t, x)}{\sqrt{4\pi}} \right)^2 \right\} - \frac{Y T_L u}{\pi v_F} \cos(2V t + \sqrt{2} \phi_+(0, t))$$

(24)

where we rescaled $\phi$ back and introduced a new energy cut-off parameter $Y T_L$ with dimensionless constant $Y$ which will be specified later. Parameter $u$ is related to the weak reflection coefficient as $u^2 = v_F^2 R(\mu/m)$. For the strong backscattering the tunneling Hamiltonian approach may be applied (31). It was associated (7) to the dual representation using the field $\theta$ mutually conjugated to $\phi$ : $[\theta_\sigma(x), \phi_\sigma(y)] = i2\pi sgn(x - y)$. The appropriate Lagrangian reads

$$\int dx \mathcal{L}_t = \int dx \frac{v_F}{2} \left\{ \frac{1}{v_F} \left( \frac{\partial_t \theta_+(t, x)}{\sqrt{4\pi}} \right)^2 - \left( \frac{\partial_t \theta_-(t, x)}{\sqrt{4\pi}} \right)^2 \right\} - \frac{Y T_L u'}{\pi v_F} \cos(V t + \theta_+(t, 0)/\sqrt{2})$$

(25)
with \( u'^2 = v^2(1 - R(\mu/m)) \) proportional to the free massive fermion transmission and the voltage multiplied by \( g \) factor \( \beta \). Both these Lagrangian are, indeed, equivalent \( \beta \) if interaction dependent relation between \( u \) and \( u' \) is met \( \beta \). The above model \( \beta \) or \( \beta \) characterizes the point scatterer of any backscattering strength at low energy \( \beta \). Although, the exact relation between \( u \) or \( u' \) and the bare parameters of the scatterer remains unknown. Our problem is dually symmetrical to that of Kane and Fisher: suppression of the direct current in their problem equals suppression of the backscattering one in our case. This correspondence allows us to re-write their solution \( \beta \) as follows:

\[
I = \frac{T_x}{\pi} \Im \psi(\frac{1}{2} + \frac{T_x + iV}{\pi T})
\]

\[
< e^{i\phi_{\beta}(x,t)}e^{-i\phi_{\beta}(y,0)} > \text{ const} \left( \alpha T_L \right)^{2\beta} \left( \frac{(\pi T/T_L)^2}{\sinh(\pi(T - i\alpha)T)\sinh(\pi(-t + i\alpha)T)} \right) F(x)F(y)
\]

where \( \alpha = 1/E_F \) and \( F \) was simplified in the previous section as:

\[
F(x) = \frac{L}{(L - x)}/\prod_{m=1}^{\infty} (m \pm x/L)^{2m+1} \approx \text{const}' |x(L-x)/L|^2/\text{const}' = e^{1/\beta}.
\]

One can see that substitution of this asymptotics in the whole formal series for the backscattering current discussed above implies transformation of the Lagrangian \( \beta \) into the one of \( \beta \) with the coefficient for the cos-term:

\[
e^{1/\beta}(1+r)J_{r+1/2}(2E_{th}t_L)/\sqrt{\pi(2E_{th}t_L)^{r+1/2}}
\]

instead of \( 2\pi \). and another energy cut-off \( T_L \). This model would be equivalent to that we constructed before in the weak perturbative regime if we can meet

\[
\frac{Y}{\sin(\mu t_L)} = \sqrt{\pi(1+r)}J_{r+1/2}(2\mu t_L)^{-r-1/2}
\]

At zero \( r \) it exactly specifies \( Y \) as a constant on the order of 1. However, if \( r \neq 0 \) (\( r = 1/3 \) for \( y = 1/2 \) we assumed), \( Y \) increases \( \propto (\mu t_L)^r \) at large \( \mu t_L \). Moreover, there is a mismatching between the oscillating structures of \( J_{r+1/2}(2\mu t_L) \) and \( \sin(\mu t_L) \) which cannot be naturally accounted for by a smooth variation of the energy cut-off, but sooner by a small deviation of the traversal time \( t_L \) in the free electron reflection coefficient \( \beta \) from its bare value \( L/v \) as \( \mu \) changes. Such behavior results from penetration of the interaction inside the wire. It is described by the finite reflection coefficient \( r \) in the inhomogeneous TLL model. The phenomenon is described more important for \( \mu \gg \max(T_L, m) \) when the electron propagation through the wire is not suppressed. In the opposite regime of small \( |\mu|/T_L \ll 1 \) no interference structure is expected and \( Y \) remains constant. Finally, under this choice \( \beta \) of \( Y \) one can see that \( T_L \gg T_L \). Therefore, the solution \( \beta \) coincides with the perturbative result \( \beta \) that is \( I = -V/\pi \propto -V^3 \) and \( G - 1/\pi \propto -T^2 \).

\[
I = \frac{2e^{-\gamma}}{\sqrt{\pi(1+r)}J_{r+1/2}(2\mu t_L)^{-r-1/2}}
\]

\[
G = \frac{T_x}{\pi T} \psi(\frac{1}{2} + \frac{T_x}{\pi T})
\]

where \( \psi \) denotes the digamma function and satisfies:

\[
\psi'(1/2) = \frac{1}{2}, \quad \psi'(x) \propto 1/x, \quad x \to \infty,
\]

and a new energy scale \( T_x \) \( \beta \) varies from \( T_x = YT_L/\sqrt{\alpha} \) at the weak backscattering \( \beta \) to \( T_x = YT_L(1 - R)/\pi \) at the strong one \( \beta \).

Let us, first, compare this result with the perturbative one \( \beta \) of the previous section. The latter was derived making use of the long-time asymptotics for the correlator \( \beta \):

\[
\text{FIG. 4. Schematic current } I \text{ vs. voltage } V \text{ near half-filling } \mu \ll T_L: \text{ curve 1 is zero temperature dependence, curve 2 is the high temperature } T \gg T_L \text{ one, the dashed lines are the low voltage } I = V/\pi \text{ and high voltage asymptotics.}
\]

Turning to the case \( m/T_L \geq 1 \) we cannot use the perturbative expression \( \beta \) anymore: The perturbative series is not convergent due to a finite gap creation. Then above non-perturbative consideration is necessary. Application of the solution \( \beta \) in this case reveals a quite remarkable property of low energy transport through the Mott-Hubbard insulator. There is an exponentially small value of \( T_x = (1 - R(\mu/m))YT_L/\pi \propto T_L \exp(-2m/T_L) \) for \( \mu \ll m \). Hence, the zero temperature current \( I \) (Fig.4) is not suppressed for the voltage less than \( T_x \) and saturates at \( T_x/2 \) value when \( T_x < V < T_L \). Similarly, the conductance (Fig.5) displays a small decrease \( \propto T^2 \) below its zero temperature value \( 1/\pi \) with increase of \( T \) for \( T < T_x \) and approaches its exponentially small asymptotics \( G = \frac{T_x}{4\pi} \exp(-2m/T_L)T_L/T \) above \( T_x < T < T_L \). As \( |\mu| \) increases, the reflection coefficient
$R(\mu/m)$ on the Fermi level goes down and $T_L$ exceeds $T_g$, finally, approaching its weak backscattering value $2T_g = YT_L/\sqrt{R(\mu/m)}$, where the perturbative consideration is applicable.

In summary, we studied transport through a 1D Mott-Hubbard insulator beyond perturbative approach. Assuming that $g = 1/2$ near the half-filling in agreement with the Bethe ansatz solutions we mapped the problem onto the exactly solvable models and found current vs. voltage at high temperature $T > \max(m, T_L)$ and at low energy $T, V < T_L$. The solution of these models shows, in particular, that the high-temperature transport through the Mott-Hubbard insulator is similar to the one through the band gap insulator at $g = 1/2$. At low energies, however, there is always a regime where the transport remains non-suppressed in the absence of the impurity backscattering. For the strong interaction resulting in the opening of the large Mott-Hubbard gap, the transport through the wire is suppressed near the half-filling almost everywhere inside the gap except for an exponentially small low energy region $V, T < T_L \exp(-m/T_L)$.

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