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Sensitivity to dark-energy candidates by searching for four-wave mixing of high-intensity lasers in the vacuum

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On the four-wave mixing yield in the quasi-parallel system (QPS) evaluated in our paper [1], the following corrections are provided from left (before corrections) to right (after corrections):

\begin{equation}
\mathcal{L}(z) = \frac{I(N_c, N_i)}{\pi w^2(z)} = \frac{I(N_c, N_i)}{\pi w_0^2} \frac{z_R^2}{z^2 + z_R^2} \rightarrow \mathcal{L}(z) = \theta_r^2 \frac{I(N_c, N_i)}{\pi w_0^2} \frac{z_R^2}{z^2 + z_R^2} = \theta_r^2 \mathcal{L}(z),
\end{equation}

\begin{equation}
\mathcal{L} = b \int_0^f f^{-1} \mathcal{L}(z) dz = \frac{I(N_c, N_i)}{c\tau \lambda} \tan^{-1} \left( \frac{f}{z_R} \right) \rightarrow \mathcal{L} = b \int_0^f f^{-1} \mathcal{L}(z) dz = \theta_r^2 \mathcal{L}(z),
\end{equation}

\begin{equation}
\gamma = K_0(\lambda, \tau) K_2(f, d) K_3(\vec{u}, \vec{v}) \mathcal{F} \left( \frac{g}{M} \right)^2 C_{mb} N_c^2 N_i \rightarrow \gamma = K_0(\lambda, \tau) K_2(f, d) K_3(\vec{u}, \vec{v}) \mathcal{F} \left( \frac{g}{M} \right)^2 C_{mb} N_c^2 N_i \theta_r^2
\end{equation}

\begin{equation}
\frac{g}{M} = m^{-1} \sqrt{\frac{\gamma}{K_0 K_2 F_5 C_{mb} N_c^2 N_i}} \rightarrow \frac{g}{M} = \frac{2 \omega}{m^2} \sqrt{\frac{\gamma}{K_0 K_2 F_5 C_{mb} N_c^2 N_i}}
\end{equation}

\begin{equation}
\frac{g}{M} = m^{-1} \sqrt{\frac{\gamma}{K_0 K_2 F_5 C_{mb} N_c^2 N_i (\tau_c / \tau_i)}} \rightarrow \frac{g}{M} = \frac{2 \omega}{m^2} \sqrt{\frac{\gamma}{K_0 K_2 F_5 C_{mb} N_c^2 N_i (\tau_c / \tau_i)}}
\end{equation}

for Eqs. (32), (33), (45), (49), and (50) of Ref. [1], respectively.

The source of these corrections originates in Eq. (32) of Ref. [1], the left-hand side of Eq. (1) above, where we wrongly applied the luminosity factor in the center-of-mass system (CMS) to that in the QPS, as pointed out in Ref. [2]. In order to clarify this point, let us partially repeat the statements made in the erratum to our previous paper [3]. The relation between a signal yield $\gamma$, the interaction
cross section $\sigma$, and the time-integrated luminosity factor $\mathcal{L}$ is as follows:

$$\mathcal{Y} = \mathcal{L}\sigma,$$

where

$$\mathcal{L} \equiv K \int n_1 n_2 dV dt$$

with the numbers of particles per bunch volume $V$, $N_i \equiv \int n_i dV$ for two colliding beams $i = 1, 2$. The $K$-factor [4], which also appears in Möller’s Lorentz-invariant factor [5], is identified as the relative velocity of the incoming particles’ velocities $\vec{v}_1$ and $\vec{v}_2$ with the velocity of light $c$,

$$K \equiv \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - \left(\frac{\vec{v}_1 \times \vec{v}_2}{c^2}\right)^2} = 2c \sin^2 \vartheta,$$

where the incident angle $\vartheta$ as illustrated in Fig. 1 of Ref. [3] is used for the second equation. We then consider the time-integrated luminosity factor for the case of a pulse crossing in CMS, where two pulsed beams with the transverse cross section $A$ and the pulse length $L$ collide in a head-on geometry. With $dV = AL$, $dt = L^2/c$, $n_i = N_i/(AL)$, and $K = 2c$, Eq. (7) gives

$$\mathcal{L} = \frac{K N_1 N_2}{2c A} = \frac{N_1 N_2}{A}.$$

We can confirm that the luminosity factor included in the left of Eq. (1) certainly corresponds to the expression in CMS, if we identify $A = \pi w_0^2$ with the laser beam waist $w_0$. We can thus identify $K/(2c)$ as the correction factor with respect to the luminosity factor defined in the CMS in order for us to apply the luminosity viewpoint to a non-head-on collision.

Therefore, we need to multiply $\sin^2 \vartheta_r \sim \vartheta_r^2$ ($c = 1$ in natural units) in Eq. (1) above, because we expect that effectively only $\vartheta \sim \vartheta_r$ contributes to both the $K$-factor and the cross section via the narrow Breit–Wigner resonance function. Accordingly, the other equations above must also be corrected. The sensitivity to weak coupling is then modified by this $K$-factor correction. Because $\vartheta_r$ satisfies the resonance condition $\vartheta_r \sim m/(2\omega)$, Eqs. (4) and (5) result in different mass dependences from $m^{-1}$ to $m^{-2}$.

We have also provided a more direct formulation based on Bjorken–Drell’s parametrization [6,7] in Appendix A of Ref. [8] without recourse to the luminosity–cross section viewpoint, which results in the equivalent modifications above. We note that in this appendix, however, we have further introduced a more accurate acceptance factor for the inducing effect by extending from the plane wave approximation to the Gaussian beam approximation, in order to require that a final-state photon wave vector of the spontaneous process coincides with a wave vector of the inducing laser field. This extension changes the $\vartheta$ dependence on the four-wave mixing yield from $\vartheta$ in Eq. (36) via Eqs. (34) and (35) of Ref. [1] to $\vartheta^2$ with $\vartheta \sim \vartheta_r$ on this acceptance factor. This detail can be found in Eqs. (52) through (54) in Appendix A of Ref. [8], which is applied to the first experimental result. Hence, the overall mass dependence of the accessible coupling strength is eventually modified from $m^{-1}$ to $m^{-5/2}$. Therefore, the slope of the expected sensitivity curve in Fig. 4 of Ref. [1] becomes much steeper, which indicates that the sensitivity to lower mass regions relatively diminishes.

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