1/m_Q order contributions to B → πlν decay in HQEFT

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Abstract

Contributions to B → πlν decay from 1/m_Q order corrections are analyzed in the heavy quark effective field theory (HQEFT) of QCD. Transition wave functions of 1/m_Q order are calculated through light cone sum rule (LCSR) within the HQEFT framework. The results are compared with those from other approaches.

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I. INTRODUCTION

Heavy to light semileptonic decay $B \rightarrow \pi l \nu$ has attracted much interest due to its special role in determining the important Cabibbo-Kabayashi-Maskawa (CKM) matrix element $|V_{ub}|$. A lot of work concerning this decay may be found in the literatures [1–3]. Since $B$ meson consists of a heavy quark and a light quark, it is natural to explore $B$ decays with considering the heavy quark symmetry. This has been proved to be powerful in relating various heavy hadron processes, and distinguishing between long and short distance dynamics reliably.

The leading order wave functions of the decay $B \rightarrow \pi l \nu$ have been calculated in Ref. [4] by using light cone sum rules within the heavy quark effective field theory (HQEFT) [5–9]. In this paper, we present the next to leading order study for $B \rightarrow \pi l \nu$ decay in HQEFT. In section II we formulate this decay up to $1/m_Q$ order in HQEFT. In section III the $1/m_Q$ order wave functions are calculated by using light cone sum rule method in HQEFT framework. Section IV devotes to the numerical analysis and relevant discussions in comparison with other approaches. A short summary is outlined in section V.

II. $B \rightarrow \pi L \nu$ DECAY IN HQEFT

The semileptonic decay $B \rightarrow \pi l \nu$ is determined by the matrix element

$$\langle \pi(p')|\bar{u}\gamma^\mu b|B(p_B)\rangle = 2 f_+(q^2) p_\mu + (f_+(q^2) + f_-(q^2)) q_\mu$$

$$= f_+(q^2) \left[ p_B^\mu + p^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

with

$$f_0(q^2) = \frac{q^2}{m_B^2 - m_\pi^2} f_-(q^2) + f_+(q^2),$$

where $q = p_B - p$ is the momentum carried by the lepton pair.

In HQEFT, the heavy quark field $Q$ is decomposed into two parts $Q^+$ and $Q^-$, which are formally corresponding to the two solutions of the Dirac equation. For the free particle case, they are known to be the quark and antiquark fields respectively. In order to shift out the large component of heavy quark momentum and also to make the leading order Lagrangian explicitly $m_Q$ independent, an effective heavy quark field can be conveniently defined as

$$Q^+_v = e^{i\hbar m_Q v^\mu x^\mu v} \hat{Q}^+_v = e^{i\hbar m_Q v^\mu x^\mu (1 + \hat{v})/2} P_+ Q^+$$

with $v$ being an arbitrary four-velocity satisfying $v^2 = 1$, and $P_+ \equiv (1 + \hat{v})/2$ being a projection operator. Under this definition, $Q^+_v$ is actually the “large component” of heavy quark field. Similarly, one can define the “large component” of heavy antiquark field. This becomes transparent if one considers a free quark field in the frame $v = (1, 0, 0, 0)$, where one has in the momentum space

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Here $u_s$ and $v_s$ are the four-component spinors, while $\varphi_s$ and $\chi_s$ are the two-component Pauli spinor fields that annihilate a heavy quark and create a heavy antiquark, respectively. A detailed discussion on the “large” and “small” components of the heavy quark and antiquark fields can be found in Refs. [5,6].

Integrating out the small component of quark field as well as the antiquark field, the effective Lagrangian can be represented in terms of $Q^+_v$ as

$$L_{eff}^{(+)} = \bar{Q}^+_v iv \cdot DQ^+_v + \frac{1}{m_Q} \bar{Q}_v^+(i\not{D})^2Q^+_v + \mathcal{O}(\frac{1}{m_Q^2}).$$  \hspace{1cm} (2.5)$$

At the same time, a heavy-light quark current $\bar{q}\Gamma Q$ with $\Gamma$ being arbitrary Dirac matrices is expanded into

$$\bar{q}\Gamma Q \rightarrow e^{-im_Q v \cdot x} \left\{ \bar{q}\Gamma Q^+_v + \frac{1}{2m_Q} \bar{q}\Gamma \frac{1}{i\not{D}}(i\not{D})^2Q^+_v + \mathcal{O}(\frac{1}{m_Q^2}) \right\},$$ \hspace{1cm} (2.6)$$

where $q$ denotes an arbitrary light quark. In Eqs.(2.5) and (2.6) the operators $i\not{D}$ and $i\not{D}_\perp$ are defined by

$$i\not{D}_\parallel = i\not{v} \cdot D, \quad i\not{D}_\perp = i\not{D} - i\not{D}_\parallel.$$

For detailed derivation of HQEFT, the interested readers are referred to Refs. [5,6].

For the $B \rightarrow \pi$ weak transition matrix element, it receives $1/m_Q$ order contributions not only from the expansion of effective current (2.6) but also from the insertion of the effective Lagrangian (2.5). As a result, the heavy quark expansion (HQE) of the transition matrix element can be simply written as [6]

$$\langle \pi|\bar{u}\gamma^\mu b|B\rangle = \sqrt{\frac{m_B}{\Lambda_B}} \left\{ \langle \pi|\bar{u}\gamma^\mu Q^+_v|M_v\rangle + \frac{1}{2m_Q} \langle \pi|\bar{u}\gamma^\mu \frac{1}{iv \cdot D}P_+(D^2_\perp) \right\} + \frac{i}{2} \sigma_{\alpha\beta}F^{\alpha\beta}Q^+_v|M_v\rangle + \mathcal{O}(1/m_Q^2) \right\},$$ \hspace{1cm} (2.8)$$

where $\Lambda_B$ is defined by the mass difference of $B$ meson and bottom quark, $\Lambda_B = m_B - m_b$, and $F^{\alpha\beta}$ is the gluon field strength tensor. $|M_v\rangle$ is an effective heavy meson state defined in HQEFT to manifest the spin-flavor symmetry,

$$\langle M_v|\bar{Q}^+_v\gamma^\mu Q^+_v|M_v\rangle = 2\Lambda v^\mu$$ \hspace{1cm} (2.9)$$

with the binding energy $\bar{\Lambda} \equiv \lim_{m_Q \rightarrow \infty} \Lambda_M$ being heavy flavor independent.

Note that in Eqs.(2.6) and (2.8) we have used the operator $1/(iv \cdot D)$ to effectively represent the contraction of effective heavy quark fields (or say heavy quark propagator) [6,9]. In calculating Green functions in the next section, we will treat this operator as contraction.
of $Q_i^+$ and $Q_i^–$. It is seen in Eq.(2.8) that the $1/m_Q$ order corrections to $B \to \pi$ transition are simply attributed to one kinematic operator and one chromomagnetic operator.

Based on heavy quark spin-flavor symmetry, we parameterize the matrix elements in HQEFT as

$$\langle \pi(p)|\bar{u} \Gamma Q_v^+|M_v\rangle = -Tr[\pi(v,p)\Gamma \mathcal{M}_v],$$

$$\langle \pi(p)|\bar{u} \Gamma P_+ \frac{p}{iv \cdot D} D \perp Q_v^+|M_v\rangle = -Tr[\pi_1(v,p)\Gamma \mathcal{M}_v],$$

$$\langle \pi(p)|\bar{u} \Gamma P_+ i \frac{p}{iv \cdot D} 2 \sigma_{\alpha \beta} F_{\alpha \beta} Q_v^+|M_v\rangle = -Tr[\pi_1^{\alpha \beta}(v,p)\Gamma P_+ \frac{i}{2} \sigma_{\alpha \beta} \mathcal{M}_v],$$

where $\mathcal{M}_v$ is the flavor independent spin wave function for pseudoscalar heavy mesons,

$$\mathcal{M}_v = -\sqrt{A} P_+ \gamma^5.$$

The functions $\pi(v,p)$, $\pi_1(v,p)$ and $\pi_1^{\alpha \beta}(v,p)$ can be generally decomposed into

$$\pi(v,p) = \gamma^5 [A(v \cdot p) + \hat{p} B(v \cdot p)],$$

$$\pi_1(v,p) = \gamma^5 [f_a(v \cdot p) + \hat{p} f_b(v \cdot p)],$$

$$\pi_1^{\alpha \beta}(v,p) = \gamma^5 [(\hat{p}^\alpha \gamma^\beta - \hat{p}^\beta \gamma^\alpha)(s_1(v \cdot p) + \hat{p} s_2(v \cdot p)) + i \sigma^{\alpha \beta}(s_3(v \cdot p) + \hat{p} s_4(v \cdot p))].$$

with $\hat{p}^\mu = p^\mu/(v \cdot p)$. The Lorentz scalar functions $A$, $B$, $f_a$, $f_b$ and $s_i(i = 1, 2, 3, 4)$ are independent of the heavy quark mass. Their scale dependence is suppressed in these formulae. In Eqs.(2.13)-(2.15) we use for convenience $v \cdot p$ as the variable of wave functions, which is related to the momentum transfer $q^2$ by

$$\xi \equiv v \cdot p = \frac{m_B^2 + m_\pi^2 - q^2}{2m_B}.$$

Now after trace calculation Eq.(2.8) yields

$$\langle \pi(p)|\bar{u} \gamma^\mu b|B(p_B)\rangle = 2\sqrt{\frac{m_B A}{\Lambda_B}} \left\{v^\mu \left( A + \frac{1}{2m_Q} A_1 \right) + \hat{p}^\mu \left( B + \frac{1}{2m_Q} B_1 \right) + \mathcal{O}\left(\frac{1}{m_Q^2}\right) \right\}$$

with

$$A_1 = f_a + 2s_1 - 2s_2 \frac{m_\pi^2}{\xi^2} - 3s_3,$$

$$B_1 = f_b - 2s_1 + 2s_2 - 3s_4,$$

where $p^2 = m_\pi^2$ is used. Eqs.(2.1) and (2.16) yield relations between the form factors $f_\pm$ and the universal wave functions,

$$f_\pm(q^2) = \frac{\Lambda}{m_B \Lambda_B} \left\{ \left[ A(\xi) + \frac{1}{2m_Q} A_1(\xi) \right] \pm \left[ B(\xi) + \frac{1}{2m_Q} B_1(\xi) \right] \frac{m_B}{\xi} + \mathcal{O}\left(\frac{1}{m_Q^2}\right) \right\}.$$
\[ L_{\text{eff|HQET}} = \bar{Q}_v^i v \cdot DQ_v^+ + \frac{1}{2m_Q} \bar{Q}_v^i (i D_{\perp})^2 Q_v^+ + \mathcal{O}\left(\frac{1}{m_Q^2}\right), \]

and the heavy-light current expansion (at tree level)

\[ \bar{q} \Gamma Q^+ \rightarrow \bar{q} \Gamma Q_v^+ + \frac{1}{2m_Q} \bar{q} i D_{\perp} Q_v^+ + \mathcal{O}\left(\frac{1}{m_Q^2}\right). \]

In some references the notation \( h_v \) is used for the field variable \( Q_v^+ \) here.

For the heavy quark expansion of the transition matrix element in HQET, the \( 1/m_Q \) order corrections from the current expansion (2.21) and those from the insertion of the Lagrangian (2.20) cannot be attributed to the same set of operators. In particular, there are both operators with even and odd powers of \( i D_{\perp} \) which contribute at \( 1/m_Q \) order. Therefore the corrections from these two sources have to be characterized by different sets of wave functions and considered separately.

In HQEFT, however, there are only operators with even power of \( i D_{\perp} \) appearing in the effective Lagrangian (2.5) and current (2.6). The terms with odd powers of \( i D_{\perp} \) are canceled due to the inclusion of the antiquark contributions. Therefore the \( 1/m_Q \) order corrections to transition matrix elements from the two sources can be attributed to the same set of operators, and the final formulation in HQEFT is simple [6]. In this paper, we need only calculate the two composite functions \( A_1 \) and \( B_1 \).

### III. LIGHT CONE SUM RULES IN HQEFT

To calculate the \( 1/m_Q \) order wave functions \( A_1 \) and \( B_1 \), we consider the two-point correlation function

\[ F^\mu = i \int d^4x e^{i(q-m_Qv) \cdot x} \langle \pi(p) | T \{ \bar{u}(x) \gamma^\mu \frac{1}{i v \cdot D} P_+ (D_{\perp}^2 + i \frac{1}{2} \sigma_{\alpha\beta} F^{\alpha\beta}) Q_v^+(x), \bar{Q}_v^+(0) i \gamma^5 d(0) \} | 0 \rangle \]

where \( \bar{Q}_v^+(0) i \gamma^5 d(0) \) is the interpolating current for \( B \) meson. Inserting between the two currents in Eq.(3.1) a complete set of intermediate states with the \( B \) meson quantum number, one gets

\[ 2i F A_1 v^\mu + B_1 \hat{p}^\mu + \int_{s_0}^\infty ds \rho_a(v \cdot p, s) v^\mu + \rho_b(v \cdot p, s) \hat{p}^\mu \]

\[ \frac{s - \omega}{s - \omega} + \text{subtraction} \]  

with \( \omega \equiv 2v \cdot k \), where \( k = p_B - m_Qv \) is the residual momentum. \( F \) is the decay constant of \( B \) meson at the leading order of \( 1/m_Q \) expansion, defined by [8]

\[ \langle 0 | q \Gamma Q_v^+ | B_v \rangle = \frac{F}{2} Tr[\Gamma M_v]. \]

At the same time, in deep Euclidean region the correlator (3.1) can be calculated in effective field theory. The result can be written as an integral over a theoretic spectral density,

\[ \int_0^\infty ds \rho_a^{th}(\xi, s) v^\mu + \rho_b^{th}(\xi, s) \hat{p}^\mu \]

\[ \frac{s - \omega}{s - \omega} + \text{subtraction}. \]
In the sum rule analysis, one assumes the quark-hadron duality, and equals the hadronic representation (3.2) and the theoretic one (3.4), which provides a sum rule equation. Furthermore, in order to enhance the importance of the ground state contribution, to suppress higher order nonperturbative contributions and also to remove the subtraction, a Borel transformation

$$
\hat{D}^{(\omega)}_T \equiv \lim_{-\omega, n \to \infty} \frac{(-\omega)^{n+1}}{n!} \left( \frac{d}{d\omega} \right)^n
$$

should be performed to both sides of the sum rule equation. With using the formulae

$$
\hat{D}^{(\omega)}_T \frac{1}{s - \omega} = e^{-s/T}, \quad \hat{D}^{(\omega)}_T e^{\omega} = \delta(\lambda - \frac{1}{T}),
$$

one gets

$$
2iF(A_1 v^\mu + B_1 \tilde{B}^\mu)e^{-2\lambda_B/T} = \int_0^{s_0} ds e^{-s/T} \left[ \rho_a(\xi, s)v^\mu + \rho_b(\xi, s)\tilde{B}^\mu \right]
$$

with

$$
\rho_a(\xi, s)v^\mu + \rho_b(\xi, s)\tilde{B}^\mu = \hat{D}^{(-1/T)}_T \hat{D}^{(\omega)}_T F^\mu(\xi, \omega).
$$

The operator $1/(iv \cdot D)$ in Eq.(3.1) may be effectively regarded as a heavy quark propagator [6,9]. Namely, in the following we conveniently calculate the correlator (3.1) via the three-point one:

$$
i \int d^4x e^{i(q-mQ)\cdot x} \langle \pi(p)| \left\{ \bar{u}(x)\gamma^\mu Q_\nu(x), i^3 \int d^4y \bar{Q}_\nu(y) \left( D^2_\perp + \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta} \right) Q_\nu^+(y), \int d^4x e^{i(q-mQ)\cdot x} \left\{ \bar{u}(x)\gamma^\mu \gamma^5 d(0)|0 \right\} \right\} \rangle.
$$

In calculating the three-point function (3.8), we represent the nonperturbative contributions embeded in the hadronic matrix element in terms of light cone wave functions up to twist 4. Among them are the two-particle distribution functions defined by [1,2,10]

$$
\langle \pi(p)| \bar{u}(x)\gamma^\mu \gamma^5 d(0)|0 \rangle = -ip^\mu f_\pi \int_0^1 du e^{iup\cdot x} \left[ \varphi_\pi(u) + x^2 g_1(u) \right]
$$

$$+ f_\pi(x^\mu - \frac{x^2 p^\mu}{x \cdot p}) \int_0^1 du e^{iup\cdot x} g_2(u),

\langle \pi(p)| \bar{u}(x)\gamma^5 d(0)|0 \rangle = \frac{f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du e^{iup\cdot x} \varphi_\pi(u),

\langle \pi(p)| \bar{u}(x)\sigma_{\mu\nu} \gamma^5 d(0)|0 \rangle = ip_{\mu x_{\nu} - p_{\nu} x_{\mu}} m_\pi^2
\frac{f_\pi m_\pi^2}{6(m_u + m_d)} \int_0^1 du e^{iup\cdot x} \varphi_\sigma(u),
$$

where $\varphi_\pi$ is the leading twist 2 distribution amplitude. $\varphi_\pi$ and $\varphi_\sigma$ are twist 3 distribution amplitudes, while $g_1$ and $g_2$ are of twist 4. Furthermore, we would also include three-particle distribution functions [10–12],

\[\text{6}\]
Using Eqs. (3.5), (3.9) and (3.10), we finally get

\[
\langle \pi(p)|\bar{u}(x)\gamma^5F_{\alpha\beta}(\omega x)d(0)|0\rangle = -f_{3\pi}[(p_{\alpha}p_{\mu}g_{\beta\nu} - p_{\beta}p_{\mu}g_{\alpha\nu}) - (p_{\alpha}p_{\nu}g_{\beta\mu} - p_{\beta}p_{\nu}g_{\alpha\mu})] \int D\alpha_i \varphi_{3\pi}(\alpha_i) e^{i(\alpha_1 + \omega_\alpha_3)p \cdot x},
\]

\[
\langle \pi(p)|\bar{u}(x)\gamma^5F_{\alpha\beta}(\omega x)d(0)|0\rangle = if_{\pi}[p_{\beta}(g_{\alpha\mu} - x_{\alpha}p_{\mu}/p \cdot x) - p_{\alpha}(g_{\beta\mu} - x_{\beta}p_{\mu}/p \cdot x)] \times \int D\alpha_i \varphi_{\perp}(\alpha_i)e^{i(\alpha_1 + \omega_\alpha_3)p \cdot x} + if_{\pi}p_{\mu}/p \cdot x(p_{\alpha}x_{\beta} - p_{\beta}x_{\alpha}) \times \int D\alpha_i \varphi_{\parallel}(\alpha_i)e^{i(\alpha_1 + \omega_\alpha_3)p \cdot x},
\]

\[
\langle \pi(p)|\bar{u}(x)\gamma^5\tilde{F}_{\alpha\beta}(\omega x)d(0)|0\rangle = -f_{\pi}[p_{\beta}(g_{\alpha\mu} - x_{\alpha}p_{\mu}/p \cdot x) - p_{\alpha}(g_{\beta\mu} - x_{\beta}p_{\mu}/p \cdot x)] \times \int D\alpha_i \varphi_{\perp}(\alpha_i)e^{i(\alpha_1 + \omega_\alpha_3)p \cdot x} - f_{\pi}p_{\mu}/p \cdot x(p_{\alpha}x_{\beta} - p_{\beta}x_{\alpha}) \times \int D\alpha_i \varphi_{\parallel}(\alpha_i)e^{i(\alpha_1 + \omega_\alpha_3)p \cdot x}, \tag{3.10}
\]

where \(D\alpha_i = d\alpha_1d\alpha_2d\alpha_3\delta(1 - \alpha_1 - \alpha_2 - \alpha_3)\) and \(\tilde{F}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\rho\sigma}F^{\rho\sigma}\). \(\varphi_{3\pi}\) is a distribution amplitude of twist 3, while \(\varphi_{\perp}, \varphi_{\parallel}, \tilde{\varphi}_{\perp}\) and \(\tilde{\varphi}_{\parallel}\) are of twist 4.

After contracting the effective heavy quark fields \(Q_+^a(x_1)\bar{Q}_-^b(x_2)\) into propagator \(P_+\int_0^\infty dt\delta(x_1 - x_2 - vt)\), the correlation function (3.8) becomes

\[
i\int d^4x\int d^4y\int_0^\infty dl\int_0^\infty dte^{i(q - m_0v) \cdot x}\langle \pi(p)|\bar{u}(x)\gamma^\mu P_+\delta(x - y - vl)\times\left[\partial_y^2 - v_\alpha v_\beta \partial_y^{\alpha\beta} - \partial_y^{\alpha\beta}A_\alpha(y) - A_\alpha(y)\partial_y^{\alpha\beta} + v_\alpha v_\beta \partial_y^{\alpha\beta}A^\beta(y) + v_\alpha v_\beta A^\alpha(y)\partial_y^{\beta} + \frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}(y)\right]P_+\delta(y - vt)\gamma^5d(0)|0\rangle, \tag{3.11}
\]

with \(\partial_y^n \equiv \partial/(\partial y_n)\), which includes only terms related to the two-particle and three-particle distribution functions (3.9) and (3.10), while other terms are not considered in this paper.

Then one can represent the matrix elements by the distribution functions defined in Eqs. (3.9) and (3.10). In doing this we choose to work in the fixed-point gauge \(x^\mu A_\mu(x) = 0\), where one has

\[A_\mu(x) = x^\nu \int_0^1 d\omega\omega F_{\nu\mu}(\omega x).\]

Using Eqs. (3.5), (3.9) and (3.10), we finally get

\[
\rho_\alpha(\xi, s) = \frac{i}{12s^3}f_\pi \left[36\mu_\pi(u - 1)\xi^2 \varphi_\sigma + 18\mu_\pi(u - 1)^2\xi^2 \varphi_\sigma' - 6\mu_\pi m_\pi^2 \xi \varphi_\sigma 
+ \mu_\pi(u - 1)\xi(10\xi^2 - 2m_\pi^2)\varphi_\sigma' + 3\mu_\pi(u - 1)^2\xi^2 \varphi_\sigma'' + 36m_\pi^2 g_2 
-(u - 1)(60\xi^2 - 12m_\pi^2)g_2' - 18(u - 1)^2\xi^2 g_2''\right]_{u = 1 - \frac{1}{2s}} 
+ \frac{i}{2s^3}f_\pi \int_0^1 d\alpha_3 \frac{1}{\alpha_3} \left[2\xi^2(\varphi_\perp - \varphi_\parallel) + m_\pi^2(\varphi_\perp + \varphi_\parallel)\right]_{\alpha_2 = 1 - \alpha_3 - \alpha_1 = 1 - \frac{1}{2s}}, \tag{3.12}
\]

\[
\rho_\beta(\xi, s) = \frac{i}{12s^3}f_\pi \left[36(u - 1)\xi^2 \varphi_\sigma + 18(u - 1)^2\xi^2 \varphi_\sigma' + 2\mu_\pi(\xi^2 + 3m_\pi^2)\varphi_\sigma 
- \mu_\pi(u - 1)\xi(10\xi^2 - 2m_\pi^2)\varphi_\sigma' - 3\mu_\pi(u - 1)^2\xi^2 \varphi_\sigma'' + 36(\xi^2 + 3m_\pi^2)g_1'\right],
\]
where $\tilde{\xi}^2 \equiv \xi^2 - m_B^2$, and for arbitrary function $f(x, y)$ we use the symbols $[f(x, y)]_{x=x_0} \equiv f(x_0)$ and $[f(x, y)]_{x=x_2} \equiv f(x_1, y) - f(x_2, y)$. For the detailed techniques used in evaluating Eqs.(3.12)-(3.13), we refer to Ref. [4].

**IV. NUMERICAL ANALYSIS**

With the scale dependence written explicitly, the distribution amplitudes read [2,10–13]

\[\varphi_\pi(u, \mu) = 6u(1-u)\left\{1 + \frac{3}{2}a(\mu)[5(2u-1)^2 - 1] + \frac{15}{8}a_4(\mu)[21(2u-1)^4 - 14(2u-1)^2 + 1]\right\},\]

\[\varphi_\rho(u, \mu) = 1 + \frac{1}{2}B_2(\mu)[3(2u-1)^2 - 1] + \frac{1}{8}B_4(\mu)[35(2u-1)^4 - 30(2u-1)^2 + 3],\]

\[\varphi_\sigma(u, \mu) = 6u(1-u)\left\{1 + \frac{3}{2}C_2(\mu)[5(2u-1)^2 - 1] + \frac{15}{8}C_4(\mu)[21(2u-1)^4 - 14(2u-1)^2 + 1]\right\},\]

\[g_1(u, \mu) = \frac{5}{2}\delta(\mu)u^2(1-u)^2 + \frac{1}{2}\epsilon(\mu)\delta^2(\mu)[u(1-u)(2+13u(1-u)) + 10u^3\log u(2 - 3u + \frac{6}{5}u^2) + 10(1-u)^3\log((1-u)(2 - 3(1-u)) + \frac{6}{5}(1-u)^2)],\]

\[g_2(u, \mu) = \frac{10}{3}\delta^2(\mu)(1-u)(2u-1),\]

\[\varphi_{3\pi}(\alpha_1, \mu) = 360\alpha_1\alpha_2\alpha_3^2[1 + \frac{1}{2}\omega_{1,0}(\mu)(7\alpha_3 - 3) + \omega_{2,0}(\mu)(2 - 4\alpha_1\alpha_2 - 8\alpha_3 + 8\alpha_3^2) + \omega_{1,1}(\mu)(3\alpha_1\alpha_2 - 2\alpha_3 + 3\alpha_3^2)],\]

\[\varphi_{\perp}(\alpha_1, \mu) = 30\delta^2(\mu)(\alpha_1 - \alpha_2)\alpha_3^2[\frac{1}{3} + \epsilon(\mu)(1 - 2\alpha_3)],\]

\[\varphi_{||}(\alpha_1, \mu) = 120\delta^2(\mu)\epsilon(\mu)(\alpha_1 - \alpha_2)\alpha_1\alpha_2\alpha_3,\]

\[\varphi_{\perp}(\alpha_1, \mu) = 30\delta^2(\mu)\alpha_3^2(1 - \alpha_3)[\frac{1}{3} + \epsilon(\mu)(1 - 2\alpha_3)],\]

\[\varphi_{||}(\alpha_1, \mu) = -120\delta^2(\mu)\alpha_1\alpha_2\alpha_3\frac{1}{3} + \epsilon(\mu)(1 - 3\alpha_3)].\]

The asymptotic form of these functions and the renormalization scale dependence are given by perturbative QCD [11,14]. In the current study, $\mu$ should be a typical scale of $B \to \pi l \nu$ decay. We set $\mu$ to the typical virtuality of the bottom quark,
\[ \mu_b = \sqrt{m_B^2 - m_B^0} \approx 2.4 \text{GeV}, \]  

(4.2)

which is the same as that used in Refs. [2,10]. Accordingly, the parameters appearing in the distribution amplitudes (4.1) are taken as [2,10]

\[
\begin{align*}
    a_2(\mu_b) &= 0.35, \quad a_4(\mu_b) = 0.18, \quad B_2(\mu_b) = 0.29, \quad B_4(\mu_b) = 0.58, \\
    C_2(\mu_b) &= 0.059, \quad C_4(\mu_b) = 0.034, \quad \delta^2(\mu_b) = 0.17 \text{GeV}^2, \quad \varepsilon(\mu_b) = 0.36.
\end{align*}
\]

(4.3)

Other meson quantities needed in the sum rule study have been studied via sum rules and other approaches. Here we use \( \mu_\pi = 2.02 \text{GeV}, \ f_\pi = 0.132 \text{GeV} \) [2,10], \( \Lambda_B = 0.53 \text{GeV} \) and \( F = 0.30 \text{GeV}^{3/2} \) [8].

With these quantities, the variation of \( 1/m_Q \) order wave functions with respect to the Borel parameter \( T \) is shown in Fig.1, where \( v \cdot p = 2.64 \text{ GeV} \) is fixed. Appropriate values of the threshold \( s_0 \) should be determined by the stability of the wave functions. Due to the general criterion that both the higher nonperturbative corrections and the contributions from excited and continuum states should not exceed 30\%, we focus on the region around \( T = 1.5 \text{GeV} \). In this region, as shown in Fig.1, there is no corresponding value of threshold which makes the curves of wave functions very flat. We choose \( s_0 \approx 1.8 \text{GeV} \) and \( s_0 \approx 0.8 \text{GeV} \) respectively, where the wave functions \( A_1 \) and \( B_1 \) become relatively stable, though the stability here is not as optimistic as that for the leading order wave functions \( A \) and \( B \) [4].

Using the thresholds given above, the wave functions are obtained straightforwardly from Eqs.(3.6), (3.12) and (3.13). And the form factors \( f_+ (q^2) \) and \( f_0 (q^2) \) can be easily evaluated from the relations (2.2) and (2.19). However, it is known that the light cone expansion and the sum rule method would break down as \( q^2 \) approaches near \( m_B^2 \) [2]. At large \( q^2 \) (or very small \( y \)), the results directly evaluated from the sum rule (3.6) turn out to be not trustworthy. In order to probe the dynamics of the whole kinematically allowed region, we have to extrapolate the sum rule results in the small \( q^2 \) region to large \( q^2 \) region in an appropriate way. For this purpose, for \( f_+ (q^2) \) we use the sum rule results at small momentum transfer, whereas at large \( q^2 \) we assume the single pole approximation [2]

\[ f_+ (q^2) = \frac{f_{B^*} g_{B^* B\pi}}{2 m_{B^*} (1 - q^2/m_{B^*}^2)}, \]

(4.4)

where \( m_{B^*} = 5.325 \text{GeV}, \ f_{B^*} \sim 0.16 \text{GeV} \) and \( g_{B^* B\pi} \sim 29 \) [2] would be used. Furthermore, it is convenient to parametrize the form factors as

\[ F(q^2) = \frac{F(0)}{1 - a_F q^2/m_B^2 + b_F q^4/m_B^4}, \]

(4.5)

where \( F \) can be \( f_+ \) and \( f_0 \), respectively. We fit the parameters \( a_{f_+} \) and \( b_{f_+} \) by using the sum rules and Eq.(4.4), i.e., requiring Eq.(4.5) approach the sum rule results at \( q^2 < 15 \text{GeV}^2 \) but be compatible with Eq.(4.4) at \( q^2 > 15 \text{GeV}^2 \). \( f_0 \) is not important in extracting \( |V_{ub}| \), because when the lepton masses are neglected, the \( B \to \pi l \nu \) decay width depends only on \( f_+ \). So we use the low \( q^2 \) (\( q^2 < 15 \text{GeV}^2 \)) region results from LCSR to fix the relevant parameters for \( f_0 \) in Eq.(4.5). Since \( f_0 \) is nearly constant in \( q^2 \) [15], one may expect this parameterization yield reasonable approximation for \( f_0 \) in the whole kinematical region.

In this way we get the fitted parameters in table 1. Then the variation of \( f_+ \) and \( f_0 \) with respect to the momentum transfer \( q^2 \) are plotted in Fig.2. In table 2 we present values of wave functions at some kinematical points of momentum transfer.
Table 1. Results of LCSR calculations up to leading (LO) and next leading order (NLO) in HQEFT. The leading order results are obtained in Ref. [4].

|        | \(F(0)\) | \(a_F\) | \(b_F\) | Order |
|--------|----------|--------|--------|-------|
| \(f_+\) | 0.35     | 1.31   | 0.35   | LO    |
|         | 0.38     | 1.19   | 0.25   | NLO   |
| \(f_0\) | 0.35     | 0.60   | −0.19  | LO    |
|         | 0.38     | 0.71   | 0.41   | NLO   |

Table 2. Values of the form factors \(f_+\) and \(f_0\) at some kinematical points.

| \(q^2(\text{GeV}^2)\) | 0  | 6  | 12 | 18 | 24 | \(\text{HQET-NLO-LCSR}\) |
|------------------------|----|----|----|----|----|--------------------------|
| \(f_+\)                |    |    |    |    |    | HQEFT-NLO-LCSR [4]       |
| 0.38                   | 0.51 | 0.72 | 1.16 | 2.50 | HQEFT-LO-LCSR             |
| 0.35                   | 0.48 | 0.71 | 1.18 | 2.70 | HQET-NLO-LCSR [16]        |
| 0.42                   | 0.54 | 0.77 |        |     | HQET-LO-LCSR [16]         |
| 0.36                   | 0.47 | 0.68 |        |     |                           |
| 0.28                   |    |    |        |     | QCD-LCSR [3]              |
| 0.27                   |    |    |        |     | QCD-LCSR [2]              |
| 0.24                   |    |    |        |     | QCD-SR [15]               |
| \(f_0\)                |    |    |        |     |     | HQEFT-LO-LCSR [17]        |
| 0.38                   | 0.44 | 0.50 | 0.54 | 0.55 | HQEFT-NLO-LCSR            |
| 0.35                   | 0.41 | 0.50 | 0.66 | 1.02 |                           |

Neglecting lepton mass, the differential decay width of \(B \to \pi l \nu\) is given by

\[
\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} (E_\pi^2 - m_\pi^2)^{3/2} [f_+(q^2)]^2
\]

with \(E_\pi = (m_B^2 + m_\pi^2 - q^2)/(2m_B)\). Now using the branching ratio \(\text{Br}(B^0 \to \pi^- l^+ \nu_l) = (1.8 \pm 0.6) \times 10^{-4}\) and the lifetime \(\tau_{B^0} = 1.542 \pm 0.016 \text{ ps}\) [18], one gets from Eq.(4.6)

\[
|V_{ub}| = (3.2 \pm 0.5 \pm 0.2) \times 10^{-3},
\]

where the first (second) error corresponds to the experimental (theoretical) uncertainty. Here the theoretical uncertainty is mainly considered from the threshold effects. Besides this, theoretical uncertainty may arise also from using the single pole approximation to fit the form factors at small recoil. To take this into account effectively, if we let the couplings in Eq.(4.4) change in the regions \(f_{B^*} = 0.16 \pm 0.03\text{ GeV}\) and \(g_{B^*B\pi} = 29 \pm 3\) [2], we get a more conservative value for \(|V_{ub}|\):

\[
|V_{ub}| = (3.2 \pm 0.5 \pm 0.4) \times 10^{-3}.
\]

Eq.(4.7) and (4.8) can be compared with the average obtained by CLEO [18],

\[
|V_{ub}| = (3.25^{+0.25}_{-0.32} \pm 0.55) \times 10^{-3},
\]

where the uncertainties are statistical and experimental ones.
It is also interesting to compare our results with the relations predicted by the large energy effective theory (LEET) \[19\]:

\[
f_+(q^2) = \frac{m_B^2 + m_a^2}{m_B^2 + m_a^2 - q^2} f_0(q^2).
\] (4.10)

In Fig. 3 we see that the form factor ratio \(f_+/f_0\) derived from our calculations in HQEFT is compatible with that predicted by the LEET relation (4.10). They agree well with each other at regions not very far from the maximum recoil point. As a comparison, both numerical results of the \(1/m\) order contribution in this manuscript and that in Ref. \[16\] are not large, but the former turns out to be a little smaller than the latter. This may be attributed to the fact that we have considered the antiquark contributions and also the input values for the involved parameters are slightly different.

V. SUMMARY

We have studied the \(1/m_Q\) order corrections to \(B \to \pi l \nu\) decay in HQEFT, while most formulae and discussions here can also be generally applied to other heavy-to-light pseudoscalar meson decays. In HQEFT, \(1/m_Q\) order corrections from the effective current and from effective Lagrangian are given by the same operator forms. Consequently the independent wave functions are reduced in HQEFT in comparison with the ones in HQET. These \(1/m_Q\) order contributions have been calculated using light cone sum rule with considering two-particle and three-particle distribution amplitudes up to twist 4. Numerically, the \(1/m_Q\) order wave functions give only about 10% correction to the transition form factor \(f_+\). This correction indicates a slightly smaller value of the CKM matrix element \(|V_{ub}|\). As to the form factor ratio \(f_+/f_0\), good agreement is observed between the sum rules in HQEFT and the LEET at large recoil region.

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Fig. 1. Variation of $1/m_Q$ order wave functions $A_1$ ((a)) and $B_1$ ((b)) with respect to the Borel parameter $T$ at the momentum transfer $q^2 = 0\text{GeV}^2$ (or $\xi = v \cdot p = 2.64\text{GeV}$). The dashed, solid and dotted curves in (a) correspond to the thresholds $s_0 = 1.5$, 1.8 and 2.1 GeV respectively, while those in (b) are derived at $s_0 = 0.3$, 0.8 and 1.1 GeV respectively.

Fig. 2. Form factors $f_+ ((a))$ and $f_0 ((b))$ obtained from light cone sum rules in HQEFT. The dashed curves are the leading order results in HQEFT [4,17], while the solid curves are the results with including $1/m_Q$ order correction.
Fig. 3. Comparison of the ratio $f_+(q^2)/f_0(q^2)$. The dashed curve is obtained from the LEET relation (4.10), and the dotted curve is from HQEFT sum rule with including only the leading order wave functions [4,17], while the solid curve is the result of this paper, which includes also the $1/m_Q$ order contributions.