An Improved Volumetric Metric for Quantum Computers via more Representative Quantum Circuit Shapes.

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In this work, we propose a generalization of the current most widely used quantum computing hardware metric known as the quantum volume [1, 2]. The quantum volume specifies a family of random test circuits defined such that the logical circuit depth is equal to the total number of qubits used in the computation. However, such square circuit shapes do not directly relate to many specific applications for which one may wish to use a quantum computer. Based on surveying available resource estimates for known quantum algorithms, we generalize the quantum volume to a handful of representative circuit shapes, which we call Quantum Volumetric Classes, based on the scaling behavior of the logical circuit depth (time) with the problem size (qubit number).

As a technology, quantum computing is in its infancy but developing rapidly. In the near term, noisy and intermediate-scale quantum (NISQ) systems may become useful for specific niche applications [3]. In the long term, with the development of fault-tolerant (FT) systems, this technology is expected be extremely disruptive and transformative. Clear metrics to evaluate this technology are crucial for evaluating and comparing performance of various quantum devices and platforms in the near term, as well as creating quantitative tools to better anticipate more long-term disruptions of this technology.

An ideal metric has several key features, which we list in Table 1 below. First, we want such a metric to be defined in such a way that it is universal across potential quantum computers. This means we want a metric that is defined at the logical computational level, independent of the underlying physical platform. In addition, we want a metric that is universal across maturity levels. We do not just want a metric for NISQ devices, but one that applies equally to both NISQ and more mature FT systems that utilize error correction.

| Feature of an ideal metric |
|-----------------------------|
| 1. Universal and platform independent |
| 2. Applicable to both near (NISQ) and long term (FT) systems |
| 3. Simple enough to be useful and understandable to non-experts |
| 4. Representative of the computational power needed to execute quantum algorithms |

Table 1: Features of an ideal metric

We also require the metric to be simple and understandable enough to be useful, including to non-experts. There will inevitably be trade-offs between precision and simplicity. However, one of the key
reasons for high-level metrics is to provide some level of guidance to non-experts, whether they are end users, application specialists, leadership within a company, or a government organization. Such potential stakeholders need quantitative tools to make strategic decisions or to track the technology over time.

Finally, the ideal metric needs to represent the computational power in a straightforward way, i.e., a bigger number represents a more powerful device. These values should also be tied closely to applications, making it possible to more quickly determine if a device can run some specific application. Conversely, having a metric closely aligned with the application space allows specific applications to be framed directly in terms of this framework. This makes any such system far more useful to potential end users.

The rest of this paper is laid out as follows. Section 1 gives a detailed description of the quantum volume metric, including its shortcomings. In particular, there are two fundamental issues. First, the quantum volume does not obviously relate to many of the applications that one may wish to use a quantum computer for in terms of resources needed to perform a computation. The second is the fact that the quantum volume does not explicitly take into account quantum error correction, a feature that will be necessary for this technology to mature beyond the NISQ era that exists today. This is especially important in light of the rapid progress towards implementing error correction below the fault-tolerant threshold [4–11].

1 Quantum Volume

One of the most widespread high-level metrics currently in use is the quantum volume, originally introduced by IBM in 2017 [1, 2]. The quantum volume is meant to capture the ability of a quantum computing device to prepare any state from a random sampling of the state space of a set of qubits. A quantum computer with $n$ qubits is represented by a $2^n$-dimensional Hilbert space. However, if the qubits are poorly controlled or subject to excess noise, the device cannot effectively sample the full state space of the $n$ qubits.

The ability to randomly access any part of the $2^n$-dimensional state space of $n$ qubits, or equivalently the ability to randomly scramble an initial state, requires a gate depth of $\mathcal{O}(n)$ [12]. For this reason, the quantum volume of a device was defined as the largest square circuit that the device can implement. Mathematically this can be written as [2]

$$\log_2(V_Q) = \max_{n < n_{\text{max}}} \left( \min \left[ n, d \right] \right),$$

where $n$ is the subset of the $n_{\text{max}}$ qubits available used in the circuit and $d$ is the circuit depth. Each of the $d$ layers of the algorithm consists of a random permutation of the qubits followed by the pairwise application of a random 2-qubit SU(4) matrix (see Figure 1). The qubit permutation may consist of sets of swap gates to move qubit states around, or a simple logical relabeling of the qubits without applying any additional gates to the extent the qubit layout and connectivity allows.

Figure 1: A circuit diagram for benchmarking the quantum volume. The circuit consists of $d$ layers of a random permutation of the qubits (represented by $\pi$) followed by random two-qubit SU(4) gates.

In order to determine the quantum volume for a specific hardware platform, a definition of suc-
cess is needed for a given test circuit. The original quantum volume benchmark uses the heavy output criteria where so-called “heavy outputs” are measured with a probability $> 2/3$ [2, 13]. A downside of this criteria is that probability outputs must be computed in advance, a problem that is generally computationally hard classically. However, it is possible to construct benchmark circuits that scale efficiently enough to be useful for large-scale quantum circuits [14].

A problem with the quantum volume metric is the reliance on defining quantum volume as an exponential, e.g. $2^n$, rather than just quoting the integer $n$ itself. This means that large-scale powerful systems will have exponentially large values due to the fact that incremental improvements lead to large differences in the quantum volume. Therefore, the quantum volume fails to be a fair representation of computational power (item four in Table 1). For this reason, the logarithm of the quantum volume is generally quoted rather than the volume itself. In this spirit, we also adopt this convention when considering a generalized metric.

A second problem with the quantum volume is that as defined the quantum volume does not explicitly specify how quantum error correction should be handled when determining the metric value of a specific device, i.e., whether a quantum volume test circuit is defined at the physical or logical level. As such, the quantum volume is only applicable to the NISQ devices that currently exist which do not utilize error correction. Therefore, this metric fails the universality condition (item two) of Table 1. Given our emphasis on trying to relate computational power to known algorithms (e.g., by looking for representative circuit shapes based on known algorithms), we choose to define our metric at the logical level. By this we mean that circuit shapes that define our metric are defined in terms of logical resources, and the actual physical implementation of this circuit incorporates as much (or as little) quantum correction overhead as needed to successfully implement the circuit in question. Ideally, one would also like to have an idea of how the computational resources defined at the logical level scale with the physical features (e.g., qubit number and physical error rates), however, this type of resource estimation is beyond the scope of this current work.

A final problem that we see with the quantum volume is that a square $n \times n$ circuit represents a minimum necessary circuit depth needed for many potential quantum algorithms of interest (see section 2). This means that such a circuit represents at best the first step in a full computation. In fact, many quantum algorithms have significantly greater circuit depth than qubit number. That makes the quantum volume a poor representative as a stand-in for the capability of the device (item four in Table 1).

A generalization of the metric that instead considers the qubit number and circuit depth as separate independent resources has been proposed [15]. Although more robust, this generalization comes at the cost of much greater complexity, as a quantum computer is no longer represented by a single quantitative value, but rather an entire family of successful circuit shapes and sizes. Therefore, this full generalization although more descriptive and thus more useful as a generalized framework for benchmarking [16], fails our simplicity criteria as an ideal metric by itself (item three in Table 1).

In order to retain the advantages of this generalized framework [15] we look at a restricted subset of possible circuit shapes. In this way we can balance the simplicity of a few numbers representing our metric, while not restricting ourselves to the original square circuit shape. To this end, a survey of resource requirements for known quantum algorithms was performed in order to determine typical circuit shapes to use in defining a generalized metric. The results of this survey and the proposed circuit shapes are discussed in section 2.

## 2 Identifying Volumetric Shapes

Quantum volume provides meaningful insight about a quantum computer’s ability to implement algorithms which require a gate depth that scales no faster than linearly in relation to number of qubits. In reality, however, several algorithms appear to require a gate depth that scales faster than linearly in relation to number of qubits. Thus, quantum volume has limited utility as a universal metric for comparing the performance of quantum
computers.

In order to look for more useful quantum circuit shapes, we performed a survey of known quantum algorithms with available resource estimates. Data collection included algorithms designed for both NISQ and FT devices. After reviewing over 225 research papers, we found 58 [17–61] algorithms with resource estimates that could be used to approximate the scaling of gate depth in relation to the number of qubits. Many of the remaining research papers that were not used were due to a lack of clearly defined circuit shape, e.g., by only specifying resources as exponential or not.

Of the 58 algorithms considered, 27 algorithms were designed with NISQ computers in mind [17–34], while 31 remain applicable only for larger or more universal fault-tolerant quantum computers [35–61]. A breakdown of the number of algorithms by application area is given in Table 2.

2.1 Assumptions During Data Collection

Data collection required a number of assumptions. First, several research papers provided resource estimates that could be used to approximate the scaling of gate depth, but did not clearly or explicitly specify the scaling of qubits. In these cases, qubits were assumed to scale linearly.

Second, the estimated scaling of gate depth for many of the algorithms depended on other variables in addition to simple qubit count. For instance, on the condition number of a specific matrix or function that is being computed, or the desired error tolerance in a quantum simulation. In these situations, these additional variables were treated as constants with the assumption that their exact values would be determined when applied to a specific problem.

Finally, we found that some of the resource estimation papers provided specific estimates for the scaling of gate depth, while others provided estimates for the scaling of similar (but different) variables such as gate count, operations, runtime, or time complexity (See Table 3 for a breakdown of the algorithms by depth estimation type). Estimates for the scaling of gate depth, runtime, and time complexity typically account for the ability to implement quantum gates in parallel whereas estimates for the scaling of gate count and operations typically do not. Therefore, estimates associated with gate count or operations were considered over-estimations that needed to be accounted for when analyzing the data.

2.2 Categorization of Circuit Shapes

Data analysis first required identifying the circuit shapes that align with each algorithm’s specific scaling of gate depth. We considered each algorithm’s circuit shape, $S$, to be of the form

$$ S = n \times d(n), $$

where the circuit depth $d(n)$ is explicitly some function of the number of required qubits $n$. The scaling of circuit depth for all algorithms fell into one of the following forms, arranged from slower-growing to faster-growing:

$$ d(n) \sim \begin{cases} 
O(1) \\
O(\text{polylog}(n)) \\
O(\sqrt{n} \text{ polylog}(n)) \\
O(n \text{ polylog}(n)) \\
O(n) \\
O(n \text{ polylog}(n)) \\
O(n^2) \\
O(n^2 \text{ polylog}(n)) \\
O(n^3) \\
O(n^3 \text{ log}(n)) \\
or \\
O(n^5) 
\end{cases}. $$

Ignoring polylog($n$) factors in Equation 3, we filtered the possible circuit shapes into the following “Quantum Volumetric Classes” listed in Figure 2. Most resource estimates for scaling are rough order of magnitude estimates and ignore things such as overheads which scale at least of order log($n$) [62]. Therefore, we end up with a more reasonable number of categories with minimal loss of informational value.
| Application Area                        | Quantum Computing Era |
|----------------------------------------|------------------------|
|                                        | NISQ | FT | All |
| Machine Learning                       | 11   | 13 | 24  |
| Optimization                           | 9    | 5  | 14  |
| Many-Body Physics/Chemistry            | 9    | 7  | 16  |
| Quantum Data Hiding                    | 0    | 6  | 6   |
| Numerical Solvers                      | 0    | 3  | 3   |
| Other                                  | 0    | 2  | 2   |

Table 2: Breakdown of the algorithms with known resource estimates considered in this paper [17–61]. We note that the total count of all algorithms exceeds 58 due to the fact that some algorithms apply to both machine learning and optimization.

Figure 2: The Quantum Volumetric Classes used in this work, as well as the initial assumptions about which circuit depths from Equation 3 fall within each class.
Table 3: Count of how many algorithms fall within each category of gate depth estimation types.

| Circuit Depth Estimate Type | Count |
|-----------------------------|-------|
| Gate Depth                  | 14    |
| Gate Count/Operations       | 19    |
| Runtime/Time Complexity/etc.| 25    |

Mathematically, these classes can be written as

\[
\begin{align*}
QV-1 &= \max_{n < n_{\text{max}}} \left( \min \left[ n, d \right] \right) \\
QV-2 &= \max_{n < n_{\text{max}}} \left( \min \left[ n, d^{1/2} \right] \right) \\
QV-3 &= \max_{n < n_{\text{max}}} \left( \min \left[ n, d^{1/3} \right] \right) \\
QV-4 &= \max_{n < n_{\text{max}}} \left( \min \left[ n, d^{1/4} \right] \right),
\end{align*}
\]

where the variables are defined just as they were in Equation 1. It should be noted that we do not follow the exponential convention from the original quantum volume for our classes. In particular, although QV-1 contains the exact same information as the original quantum volume \(V_Q\), these two are not equal. Instead the relationship between the two is given as

\[QV-1 \equiv \log_2(V_Q).\]  \(\text{(5)}\)

This should not present an issue as it is already common to quote \(\log_2(V_Q)\) rather than \(V_Q\) anyways.

As an example of how these classes work, a given quantum computer may score a value of \(n\) for the QV-1 metric. This corresponds to the determination that the device is able to implement square circuits up to a size \(S = n \times n\), and is equivalent in this case to a quantum volume of \(\log_2(Q_V)\). Likewise a device with a QV-2 score of \(n\) means the device can successfully implement circuits whose depth grows quadratically with qubit number up to a circuit shape of \(S = n \times n^2\). In general, a value of \(n\) for the QV-\(k\)th volumetric metric corresponds to the circuit of shape \(S = n \times n^k\).

The following subsections walk through the data analysis process using these classes of quantum volume metrics. First we will present initial results without any adjustments to the data. Then, section 3.2 will highlight adjustments made to the data based on (1) an assumption that algorithms will require additional gate depth to account for gate overhead and (2) the lack of parallelism in resource estimates for the scaling of gate count and operations. The final subsection then presents final results.

3 Results

The following subsections walk through the data analysis process using these classes of quantum volume metrics. In section 3.1, we will present initial results without any adjustments to the data. Then, section 3.2 will highlight adjustments made to the data based on (1) an assumption that algorithms will require additional gate depth to account for gate overhead and (2) the lack of parallelism in resource estimates for the scaling of gate count and operations. Section 3.3 then presents final results.

3.1 Initial Results

| Class | Circuit Depth | Counts |
|-------|---------------|--------|
| QV-1  | \(O(n)\)      | 16 (28%) |
|       | \(O(\log(n))\) | 8 (14%) |
|       | \(O(\log^2(n))\) | 6 (10%) |
|       | \(O(1)\)       | 2 (3%)  |
|       | \(O(\sqrt{n} \cdot \text{polylog}(n))\) | 1 (2%) |
| QV-2  | \(O(n^2)\)    | 10 (17%) |
|       | \(O(n \cdot \text{polylog}(n))\) | 2 (3%) |
| QV-3  | \(O(n^3)\)    | 8 (14%) |
|       | \(O(n^2 \cdot \text{polylog}(n))\) | 1 (2%) |
| QV-4  | \(O(n^4)\)    | 2 (3%)  |
| QV-5  | \(O(n^5)\)    | 2 (3%)  |

Table 4: Number of algorithms by circuit depth scaling and grouped by QV class.

Our initial sorting of algorithms into QV classes is shown in Figure 2. The counts for the number of algorithms for each of the given depth scalings of Equation 3 is given in Table 4, together with the initial sorting by QV class. Under this initial grouping, 33 (57%) algorithms fall into QV-1, 12 (21%) into QV-2, 9 (16%) into QV-3, and 2 (3%) into either QV-4 or QV-5. Therefore the original quantum volume, represented by our QV-1 class, accounts for only 57% of the quantum algorithms surveyed. If we add the additional categories of
QV-2, QV-3, and QV-4 to the original quantum volume metric, we would increase coverage to 93% of algorithms surveyed.

When analyzing the initial data by application area we notice a few trends (see Figure 3). First, machine learning and optimization algorithms primarily seem to align with QV-1. Many-body physics and chemistry algorithms align evenly across QV-1 and the combination of QV-2 and QV-3. Quantum data hiding algorithms align with QV-1 for watermarking and QV-2/QV-3 for steganography. Numerical solvers (i.e., Shor’s algorithm) align with QV-3 or QV-4.

3.2 Adjustments to Results

After an initial analysis, the categorization of algorithms into their respective QV classes was adjusted to account for the following assumptions. First, it is anticipated that implementing algorithms on quantum computers will require gate overhead due to the physical limitations of qubit connectivity within the physical devices. Qubits will likely not be able to directly interact with all other qubits; therefore, additional quantum gates will likely be needed to make the necessary connections. This gate overhead is expected to be at least of order \( \log(n) \) [62].

Keeping this overhead in mind, the initial data presented in section 3.1 is likely an underestimation of actual gate depth requirements. To account for this, algorithms with circuit depth estimates scaling exactly with the shape of their respective QV metrics are pushing the upper limits of the metric, and they should therefore move to the next higher QV metric to leave room for potential gate overhead. To adjust for this, algorithms with circuit depths \( \mathcal{O}(n) \) were moved from QV-1 to QV-2, \( \mathcal{O}(n^2) \) were moved into QV-3, and \( \mathcal{O}(n^3) \) were considered part of QV-4. This change is represented graphically in Figure 4.

![Figure 3: Initial breakdown of application areas by QV class.](image)

A second consideration was the fact that the resource estimates of the various algorithms were not all of the same type (see Table 3). In particular, resource estimations that rely on assumptions of total gate or operation counts do not take into account parallelisation of the actual algorithm. Since these cases already represent overestimations of gate depth, we did not move any such algorithm to a higher QV class. Counts for how many algorithms were moved to a higher QV classes is shown in Table 5.

3.3 Final Results

After adjusting the data as described in section 3.2 we arrived at the final results, presented in Table 6 below. Under this adjusted grouping, the utility of QV-1 is reduced, as it now only accounts for 21 (36%) of the algorithms, while QV-2 saw a significant boost to 18 (31%) algorithms. QV-3 increased slightly to 11 (19%), while QV-4 now has 6 (10%) of all algorithms surveyed. QV-5 remained at 2 (3%). Therefore we see, a set of metrics consisting of QV-1, QV-2, QV-3, and QV-4 should cover a vast majority of quantum algorithms (91%).

We again analyzed the updated data by application area, which we represent graphically in Figure 5 Machine learning and optimization algo-
Table 6: Number of algorithms by circuit depth scaling and grouped by QV class based on the adjustments described in section 3.2.

| Class | Circuit Depth | Counts |
|-------|---------------|--------|
| QV-1  | $O(n)$        | 4 (7%) |
|       | $O(\log(n))$ | 8 (14%)|
|       | $O(\log^2(n))$ | 6 (10%)|
|       | $O(1)$        | 2 (3%) |
|       | $O(\sqrt{n} \log(n))$ | 1 (2%)|
| QV-2  | $O(n)$        | 12 (21%)|
|       | $O(n^2)$      | 4 (7%) |
|       | $O(n \log(n))$ | 2 (3%)|
| QV-3  | $O(n^2)$      | 6 (10%)|
|       | $O(n^3)$      | 4 (7%) |
|       | $O(n^2 \log(n))$ | 1 (2%)|
| QV-4  | $O(n^3)$      | 4 (7%) |
|       | $O(n^4 \log(n))$ | 2 (3%)|
| QV-5  | $O(n^n)$      | 2 (3%) |

Figure 4: Adjustments to the categorization of algorithms into QV classes as originally represented in Figure 2 for different scaling behaviors for the circuit depths.

Figure 5: Adjusted breakdown of application areas by QV class.

4 Conclusions

Quantum computing is a rapidly developing technology that promises to be broadly disruptive across many fields. This necessitates the development of better metrics to enable potential end
users to evaluate and compare hardware platforms for their desired applications, as well as to track the development of the technology over time. The generalized quantum volume classes presented in this work (see Equation 4) provide a generalization to the original quantum volume benchmark that is applicable to a much broader range of potential applications, while not becoming too complex to be useful to the potential end users for these applications.

Adoption of these metrics by the community would help in a number of ways. These metrics more explicitly tie the power of quantum computing to specific use cases, enhancing the utility to end-users. Focusing on developing metrics from the end-user perspective will also help companies, governments, and other entities to make more informed decisions concerning this field. Adoption of these metrics or similar frameworks will help guide the field by quantifying how success and improvement is measured. Having a consistent framework should also help minimize some of the assumptions specified in section 2.

In the coming years we can expect to see more powerful quantum computing systems with both more qubits and better fidelities. Soon, we will even see systems whose error rates fall below error correction thresholds, opening the door to further progress through the application of quantum error correction to these systems. The original quantum volume does not specify how to account for error correction, i.e., whether the specified square circuit shape applies at the logical or physical level. In order for a metric to be sufficiently simple and useful to end users who may not be experts in quantum computing, the metric should be defined at the logical level to best align with the application space. However, it would also be useful to be able to connect the user space metric, such as that presented here, with estimates of system level performance. Such an accounting is beyond the scope of this work, but is forthcoming in a future paper.

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References

[1] Nikolaj Moll, Panagiotis Barkoutsos, Lev S. Bishop, Jerry M. Chow, Andrew Cross, Daniel J. Egger, Stefan Filipp, Andreas Fuhrer, Jay M. Gambetta, Marc Ganzhorn, Abhinav Kandala, Antonio Mezzacapo, Peter Müller, Walter Riess, Gian Salis, John Smolin, Ivano Tavernelli, and Kristan Temme. Quantum optimization using variational algorithms on near-term quantum devices. Quantum Science and Technology, 3(3):030503, 2018. DOI: 10.1088/2058-9565/aab822.
[2] Andrew W. Cross, Lev S. Bishop, Sarah
Sheldon, Paul D. Nation, and Jay M. Gambetta. Validating quantum computers using randomized model circuits. *Physical Review A*, 100(3):032328, 2019. DOI: 10.1103/PhysRevA.100.032328.

[3] John Preskill. Quantum Computing in the NISQ era and beyond. *Quantum*, 2:79, 2018. DOI: 10.22331/q-2018-08-06-79.

[4] Robin Harper and Steven T. Flammia. Fault-Tolerant Logical Gates in the IBM Quantum Experience. *Physical Review Letters*, 122(8):080504, 2019. DOI: 10.1103/PhysRevLett.122.080504.

[5] Google Quantum AI, Zijun Chen, Kevin J. Satzinger, Juan Atalaya, Alexander N. Korotkov, Andrew Dunsworth, Daniel Sank, Chris Quintana, Matt McEwen, Rami Barends, Paul V. Klimov, Sabrina Hong, Cody Jones, Andre Petukhov, Dvir Kafri, Sean Demura, Brian Burkett, Craig Gidney, Austin G. Fowler, Alexandru Paler, Harald Putterman, Igor Aleiner, Frank Arute, Kunal Arya, Ryan Babbush, Joseph C. Bardin, Andreas Bengtsson, Alexandre Bourassa, Michael Broughton, Bob B. Buckley, David A. Buell, Nicholas Bushnell, Benjamin Chiaro, Roberto Collins, William Courtney, Alan R. Derk, Daniel Eppens, Catherine Erickson, Edward Farhi, Brooks Foxen, Marissa Giustina, Ami Greene, Jonathan A. Gross, Matthew P. Harrigan, Sean D. Harrington, Jeremy Hilton, Alan Ho, Trent Huang, William J. Huggins, L. B. Ioffe, Sergey V. Isakov, Evan Jeffrey, Zhang Jiang, Kostyantyn Kechedzhi, Seon Kim, Alexei Kitaev, Fedor Kostritsa, David Landhuis, Pavel Laptev, Erik Lucero, Orion Martin, Jarrod R. McClean, Trevor McCourt, Xiao Mi, Kevin C. Miao, Masoud Mohseni, Shirin Montazeri, Wojciech Mruczkiewicz, Josh Mutus, Ofer Naaman, Matthew Needley, Charles Neill, Michael Newman, Murphy Yuezheng Niu, Thomas E. O’Brien, Alex Opremcak, Eric Ostby, Bálint Pató, Nicholas Redd, Pedram Roushan, Nicholas C. Rubin, Vladimir Shvarts, Doug Strain, Marco Szalay, Matthew D. Trevithick, Benjamin Villalonga, Theodore White, Z. Jamie Yao, Ping Yeh, Juhwan Yoo, Adam Zalcman, Hartmut Neven, Sergio Boixo, Vadim Smelyanskiy, Yu Chen, Anthony Megram, and Julian Kelly. Exponential suppression of bit or phase errors with cyclic error correction. *Nature*, 595(7867):383–387, 2021. DOI: 10.1038/s41586-021-03588-y.

[6] Alexander Erhard, Hendrik Poulsen Nautrup, Michael Meth, Lukas Postler, Roman Stricker, Martin Stadler, Vlad Negnevitsky, Martin Ringbauer, Philipp Schindler, Hans J. Briegel, Rainer Blatt, Nicolai Friis, and Thomas Monz. Entangling logical qubits with lattice surgery. *Nature*, 589(7841):220–224, 2021. DOI: 10.1038/s41586-021-03079-6.

[7] Caterina Vigliar, Stefano Paesani, Yunhong Ding, Jeremy C. Adcock, Jianwei Wang, Sam Morley-Short, Davide Bacco, Leif K. Oxenløwe, Mark G. Thompson, John G. Rarity, and Anthony Laing. Error-protected qubits in a silicon photonic chip. *Nature Physics*, pages 1–7, 2021. DOI: 10.1038/s41567-021-01333-w.

[8] Laird Egan, Dripto M. Debroy, Crystal Noel, Andrew Risinger, Daiwei Zhu, Debopriyo Biswas, Michael Newman, Muyuan Li, Kenneth R. Brown, Marko Cetina, and Christopher Monroe. Fault-tolerant control of an error-corrected qubit. *Nature*, pages 1–6, 2021. DOI: 10.1038/s41586-021-03928-y.

[9] C. Ryan-Anderson, J. G. Bohnet, K. Lee, D. Gresh, A. Hankin, J. P. Gaebler, D. Francois, A. Chernoguzov, D. Lucchetti, N. C. Brown, T. M. Gatterman, S. K. Halit, K. Gilmore, J. A. Gerber, B. Neyenhuis, D. Hayes, and R. P. Stutz. Realization of Real-Time Fault-Tolerant Quantum Error Correction. *Physical Review X*, 11(4):041058, 2021. DOI: 10.1103/PhysRevX.11.041058.

[10] Lukas Postler, Sascha Heußen, Ivan Pogorelov, Manuel Rispler, Thomas Feldker, Michael Meth, Christian D. Marciniak, Roman Stricker, Martin Ringbauer, Rainer Blatt, Philipp Schindler, Markus Müller, and Thomas Monz. Demonstration of fault-tolerant universal quantum gate operations. *Nature*, 605(7911):675–680, 2022. DOI: 10.1038/s41586-022-04721-1.

[11] Youwei Zhao, Yangsen Ye, He-Liang Huang, Yiming Zhang, Dachao Wu, Huijie Guan, Qinglei Zhu, Zulin Wei, Tan He, Sirui Cao,
Fusheng Chen, Tung-Hsun Chung, Hui Deng, Daojin Fan, Ming Gong, Cheng Guo, Shaojun Guo, Lianchen Han, Na Li, Shaowei Li, Yuan Li, Futian Liang, Jin Lin, Haoran Qian, Hao Rong, Hong Sun, Lihua Sun, Shiyu Wang, Yulin Wu, Yu Xu, Chong Ying, Jiale Yu, Chen Zha, Kaili Zhang, Yong-Heng Huo, Chao-Yang Lu, Cheng-Zhi Peng, Xiaobo Zhu, and Jian-Wei Pan. Realizing an Error-Correcting Surface Code with Superconducting Qubits. arXiv:2112.13505 [quant-ph], 2021. DOI: 10.48550/arXiv.2112.13505.

[12] Yasuhiro Sekino and L. Susskind. Fast scramblers. Journal of High Energy Physics, 2008(10):065–065, 2008. DOI: 10.1088/1126-6708/2008/10/065.

[13] Scott Aaronson and Lijie Chen. Complexity-Theoretic Foundations of Quantum Supremacy Experiments. In 32nd Computational Complexity Conference (CCC 2017), volume 79 of Leibniz International Proceedings in Informatics (LIPIcs), pages 22:1–22:67, 2017. DOI: 10.4230/LIPICS.CCC.2017.22.

[14] Timothy Proctor, Kenneth Rudinger, Kevin Young, Erik Nielsen, and Robin Blume-Kohout. Measuring the capabilities of quantum computers. Nature Physics, pages 1–5, 2021. DOI: 10.1038/s41567-021-01409-7.

[15] Robin Blume-Kohout and Kevin C. Young. A volumetric framework for quantum computer benchmarks. Quantum, 4:362, 2020. DOI: 10.22331/q-2020-11-15-362.

[16] Thomas Lubinski, Sonika Johri, Paul Varosy, Jeremiah Coleman, Luning Zhao, Jason Neece, Charles H. Baldwin, Karl Mayer, and Timothy Proctor. Application-Oriented Performance Benchmarks for Quantum Computing. arXiv:2110.03137 [quant-ph], 2021. DOI: 10.48550/arXiv.2110.03137.

[17] Joonho Lee, William J. Huggins, Martin Head-Gordon, and K. Birgitta Whaley. Generalized Unitary Coupled Cluster Wave functions for Quantum Computation. Journal of Chemical Theory and Computation, 15(1):311–324, 2019. DOI: 10.1021/acs.jctc.8b01004.

[18] Robert M. Parrish, Edward G. Hohenstein, Peter L. McMahon, and Todd J. Martinez. Quantum Computation of Electronic Transitions using a Variational Quantum Eigensolver. Physical Review Letters, 122(23):230401, 2019. DOI: 10.1103/PhysRevLett.122.230401.

[19] Zidu Liu, L.-M. Duan, and Dong-Ling Deng. Solving Quantum Master Equations with Deep Quantum Neural Networks. arXiv:2008.05488 [cond-mat, physics:quant-ph], 2020. DOI: 10.48550/arXiv.2008.05488.

[20] Zixuan Hu, Rongxin Xia, and Sabre Kais. A quantum algorithm for evolving open quantum dynamics on quantum computing devices. Scientific Reports, 10(1):3301, 2020. DOI: 10.1038/s41598-020-60321-x.

[21] Wen Wei Ho and Timothy H. Hsieh. Efficient variational simulation of non-trivial quantum states. SciPost Physics, 6(3):029, 2019. DOI: 10.21468/SciPostPhys.6.3.029.

[22] Wen Wei Ho, Cheryne Jonay, and Timothy H. Hsieh. Ultrafast Variational Simulation of Non-trivial Quantum States with Long Range Interactions. Physical Review A, 99(5):052332, 2019. DOI: 10.1103/PhysRevA.99.052332.

[23] Seth Lloyd, Maria Schuld, Aroosa Ijaz, Josh Izaac, and Nathan Killoran. Quantum embeddings for machine learning. arXiv:2001.03622 [quant-ph], 2020. DOI: 10.48550/arXiv.2001.03622.

[24] Adrián Pérez-Salinas, Alba Cervera-Lierta, Elies Gil-Fuster, and José I. Latorre. Data re-uploading for a universal quantum classifier. Quantum, 4:226, 2020. DOI: 10.22331/q-2020-02-06-226.

[25] Jonathan Romero and Alan Aspuru-Guzik. Variational Quantum Generators: Generative Adversarial Quantum Machine Learning for Continuous Distributions. Advanced Quantum Technologies, 4(1):2000003, 2021. DOI: 10.1002/qute.202000003.

[26] Samuel Yen-Chi Chen, Chao-Han Huck Yang, Jun Qi, Pin-Yu Chen, Xiaoli Ma, and Hsi-Sheng Goan. Variational Quantum Circuits for Deep Reinforcement Learning. IEEE Access, 8:141007–141024, 2020. DOI: 10.1109/ACCESS.2020.3010470.

[27] Owen Lockwood and Mei Si. Reinforcement Learning with Quantum Variational Circuit. Proceedings of the AAAI Conference on Artifi-
Pierre Dupuy de la Grand’rive and Jean-Francois Hullo. Knapsack Problem variants of QAOA for battery revenue optimization. arXiv:1908.02210 [quant-ph], 2019. DOI: 10.48550/arXiv.1908.02210.

Jeremy Cook, Stephan Eidenbenz, and Andreas Bärtschi. The Quantum Alternating Operator Ansatz on Maximum k-Vertex Cover. 2020 IEEE International Conference on Quantum Computing and Engineering (QCE), pages 83–92, 2020. DOI: 10.1109/QCE49297.2020.00021.

Zhihui Wang, Nicholas C. Rubin, Jason M. Dominy, and Eleanor G. Rieffel. XY-mixers: Analytical and numerical results for QAOA. Physical Review A, 101(1):012320, 2020. DOI: 10.1103/PhysRevA.101.012320.

Shouvanik Chakrabarti, Andrew M. Childs, Tongyang Li, and Xiaodi Wu. Quantum algorithms and lower bounds for convex optimization. Quantum, 4:221, 2020. DOI: 10.22331/q-2020-01-13-221.

Gavin E. Crooks. Performance of the Quantum Approximate Optimization Algorithm on the Maximum Cut Problem. arXiv:1811.08419 [quant-ph], 2018. DOI: 10.48550/arXiv.1811.08419.

Martin Roetteler, Michael Naehrig, Krysta M. Svore, and Kristin Lauter. Quantum resource estimates for computing elliptic curve discrete logarithms. arXiv:1706.06752 [quant-ph], 2017. DOI: 10.48550/arXiv.1706.06752.

Craig Gidney and Martin Ekerå. How to factor 2048 bit RSA integers in 8 hours using 20 million noisy qubits. Quantum, 5:433, 2021. DOI: 10.22331/q-2021-04-15-433.
[46] Patrick Rebentrost, Masoud Mohseni, and Seth Lloyd. Quantum Support Vector Machine for Big Data Classification. *Physical Review Letters*, 113(13):130503, 2014. DOI: 10.1103/PhysRevLett.113.130503.

[47] Maria Schuld, Ilya Sinayskiy, and Francesco Petruccione. Prediction by linear regression on a quantum computer. *Physical Review A*, 94(2):022342, 2016. DOI: 10.1103/PhysRevA.94.022342.

[48] Dominic W. Berry, Andrew M. Childs, and Robin Kothari. Hamiltonian Simulation with Nearly Optimal Dependence on all Parameters. In *2015 IEEE 56th Annual Symposium on Foundations of Computer Science*, pages 792–809, 2015. DOI: 10.1109/FOCS.2015.54.

[49] Ivan Kassal, Stephen P. Jordan, Peter J. Love, Masoud Mohseni, and Alán Aspuru-Guzik. Polynomial-time quantum algorithm for the simulation of chemical dynamics. *Proceedings of the National Academy of Sciences*, 105(48):18681–18686, 2008. DOI: 10.1073/pnas.0808245105.

[50] L.-A. Wu, M. S. Byrd, and D. A. Lidar. Polynomial-Time Simulation of Pairing Models on a Quantum Computer. *Physical Review Letters*, 89(5):057904, 2002. DOI: 10.1103/PhysRevLett.89.057904.

[51] François Fillion-Gourdeau, Steve MacLean, and Raymond Laflamme. Algorithm for the solution of the Dirac equation on digital quantum computers. *Physical Review A*, 95(4):042343, 2017. DOI: 10.1103/PhysRevA.95.042343.

[52] Tim Byrnes and Yoshihisa Yamamoto. Simulating lattice gauge theories on a quantum computer. *Physical Review A*, 73(2):022328, 2006. DOI: 10.1103/PhysRevA.73.022328.

[53] Yan Huang, Zhaofeng Su, Fangguo Zhang, Yong Ding, and Rong Cheng. Quantum algorithm for solving hyperelliptic curve discrete logarithm problem. *Quantum Information Processing*, 19(2):62, 2020. DOI: 10.1007/s11128-019-2562-5.

[54] Shengbin Wang, Zhimin Wang, Wendong Li, Lixin Fan, Zhiqiang Wei, and Yongjian Gu. Quantum fast Poisson solver: The algorithm and complete and modular circuit design. *Quantum Information Processing*, 19(6):170, 2020. DOI: 10.1007/s11128-020-02669-7.

[55] Chao-Hua Yu, Fei Gao, Chenghuan Liu, Du Huynh, Mark Reynolds, and Jingbo Wang. Quantum algorithm for visual tracking. *Physical Review A*, 99(2):022301, 2019. DOI: 10.1103/PhysRevA.99.022301.

[56] Chao-Hua Yu, Fei Gao, and Qiao-Yan Wen. An Improved Quantum Algorithm for Ridge Regression. *IEEE Transactions on Knowledge and Data Engineering*, 33(3):858–866, 2021. DOI: 10.1109/TKDE.2019.2937491.

[57] Jia Luo, Ri-Gui Zhou, GaoFeng Luo, Yao-Chong Li, and GuangZhong Liu. Traceable Quantum Steganography Scheme Based on Pixel Value Differencing. *Scientific Reports*, 9(1):15134, 2019. DOI: 10.1038/s41598-019-15198-8.

[58] Shahrokhi Heidari, Mohammad Rasoul Pourarian, Reza Gheibi, Mosayeb Naseri, and Monireh Housshmand. Quantum red–green–blue image steganography. *International Journal of Quantum Information*, 15(05):1750039, 2017. DOI: 10.1142/S0219749917500393.

[59] S. Miyake and K. Nakamae. A quantum watermarking scheme using simple and small-scale quantum circuits. *Quantum Information Processing*, 15(5):1849–1864, 2016. DOI: 10.1007/s11128-016-1260-9.

[60] Xianhua Song, Shen Wang, Ahmed A. Abd El-Latif, and Xiamu Niu. Dynamic watermarking scheme for quantum images based on Hadamard transform. *Multimedia Systems*, 20(4):379–388, 2014. DOI: 10.1007/s00530-014-0355-3.

[61] Seth Lloyd and Christian Weedbrook. Quantum Generative Adversarial Learning. *Physical Review Letters*, 121(4):040502, 2018. DOI: 10.1103/PhysRevLett.121.040502.

[62] Steven Herbert. On the depth overhead incurred when running quantum algorithms on near-term quantum computers with limited qubit connectivity. *arXiv:1805.12570 [quant-ph]*, 2020. DOI: 10.48550/arXiv.1805.12570.