Numerical Investigation of Nanofluid Laminar Forced Convective Heat Transfer inside an Equilateral Triangular Tube

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Abstract. In this article distilled water and CuO particles with volume fraction of 1%, 2% and 4% are studied numerically. The steady state flow regime is considered laminar with Reynolds number of 100 and nanoparticles diameters (dp) are set in the range of 20 nm and 80 nm. The hydraulic diameter and the length of equilateral triangular channel are 8 mm and 1000 mm respectively. The problem is solved using finite volume method with constant heat flux for two sides and constant temperature for one side. Convective heat transfer coefficient, Nusselt number and convective heat transfer coefficient distribution on walls are investigated in details. The fluid flow is supposed to be one phase flow. It can be observed that nanofluid leads to a remarkable enhancement on heat transfer coefficient pressure loss through the channel. The computations reveal that the size of nanoparticles has no significant influence on heat transfer properties. Besides, the study shows a good agreement between current results and experimental data in the literatures.

1. Introduction
The thermal industrial equipment has experienced unprecedented improvements in enhancing heat transfer capabilities and this has resulted in new challenges for thermal sciences especially with the advancement of nanotechnology. Nanotechnology as well as nanoparticles have been used in a wide variety of industries. In recent years, nanofluids have become popular in heat exchanger installations for achieving higher heat transfer rates. Nanoparticles are suspended in a base fluid and it is called nanofluid [1]. Nanofluids are completely different from microfluids and they have distinctive features in comparison to conventional solid-liquid mixtures in which small sized particles in the range of mm or μm of metals and non-metals are dispersed [1]. Knowledge about nanofluids using in heat transfer enhancement mechanism is still in its primary. A new class of polymer nanofluids, drag-reducing nanofluids, aim at enhanced heat transfer as well as flow friction reduction have been studied by Phelan et al [2] where a wide range of active self-assembly mechanisms for nanoscale structures has been investigated in details.

Khaled et al [3] studied the laminar heat transfer regime in a channel with and without nanoparticles. They found an obvious enhancement for the case where nanoparticles have been added to the base fluid. Heris et al [4] investigated a numerical method for laminar heat transfer in a channel...
with constant heat flux on all walls. Their results clearly showed that addition of nanoparticles to the base fluid produces remarkable enhancement of heat transfer. Nanoparticle concentration can increase heat transfer coefficients and decreasing their size can enhance heat transfer coefficients as well. Wen et al [5] focused on the study of Al₂O₃-water nanofluid in the laminar flow regime with constant heat flux on walls of a coppery pipe. Their results showed that the increase in Reynolds number and nanoparticle size with 1.6% volume fraction caused 47% enhancement of Nusselt number. Several different numerical methods including Eulerian, one-phase and two-phase-mix methods were used by Lotfi et al [6] for exploring the effect of a vast variety of volume fractions of nanoparticles on heat transfer parameters. Ahmad et al [7] studied heat transfer in three nanofluids which nanoparticles Al₂O₃, CuO and SiO₂ were suspended in the base fluid of ethylene glycol with nanoparticles concentrations ranged from 1 to 6% in a channel with triangular cross section numerically. Flow regime was considered laminar with Reynolds number of 100-800. Their results disclosed 50% rise of Nusselt number when Reynolds number changed from 100 to 800. In an experimental research carried out by Kim et al [8], the heat transfer of both laminar and turbulent nanofluid flow regimes in a channel with constant heat flux on walls was studied. They found that Al₂O₃-water nanofluid with volume fraction of 3%, caused 8% rise in conduction and 20% increase in convective heat transfer coefficients.

In summary, the heat transfer of none-circular cross section channels is generally less than that of circular ones [9]. In the present work, the finite volume method has been used for simulation of water and CuO nanofluid with volume fraction of 0%, 1%, 2% and 4%. The steady state laminar flow regime with Reynolds number of 100 and nanoparticles diameters of 20 nm and 80 nm are considered. The convective and diffusive terms in Navier-Stokes equations have been simulated by a second order upwind method in all simulations meanwhile the SIMPLE procedure has been employed for the velocity and pressure relation. The fully developed x-velocity and uniform temperature \(T_0=300\) K is assumed at the tube inlet meanwhile the fully developed conditions are assumed at the tube outlet. The stationary wall conditions and uniform heat flux are imposed on the solid walls of tubes. The geometry of equilateral triangular cross section tube with hydraulic diameter of 8 mm and length of 1000 mm can be observed in Figure 1.

**Figure 1. Geometry of problem**

### 2. Governing Equations and Numerical Simulation Approach

The governing equations in single phase model are similar to those of the base liquid equations. The effective conservation equations including continuity, momentum and energy for steady state flow regime are as follow (All following symbols, subscripts and Greek letters are tabed in nomenclature section at the end of the paper.):

\[
\nabla (\rho \nu) = 0
\]

(1)
\[ \nabla \cdot (\rho v v) = -\nabla p + \nabla \cdot (\mu \nabla v) \]  
(2)

\[ \nabla \cdot (\rho c_p v T) = \nabla \cdot (k \nabla T) \]  
(3)

where thermo physical properties including \( \mu, \rho, k, c_p \) in before equations should be replaced with effective values \( \mu_{\text{eff}}, \rho_{\text{eff}}, k_{\text{eff}}, c_{p,\text{eff}} \) respectively which are listed as below [10,11]:

\[ \mu_{\text{eff}} = (123 \emptyset^2 + 7.3 \emptyset + 1) \mu_f \]  
(4)

\[ \rho_{\text{eff}} = (1-\emptyset) \rho_f + \emptyset \rho_p \]  
(5)

\[ k_{\text{eff}} = \left\{ 1 + 64.7 \emptyset^{0.7460} \left( \frac{d_f}{d_p} \right)^{0.3690} \left( \frac{k_f}{k_f} \right)^{0.7476} \right\}^{0.9955} \text{Pr}^{1.2321} k_f \]  
(6)

\[ c_{p,\text{eff}} = (1-\emptyset) c_{p,f} + \emptyset c_{p,p} \]  
(7)

Prandtl and Reynolds numbers are described as \( \text{Pr} = \frac{\mu}{\rho_f \alpha_f} \) and \( \text{Re} = \frac{\rho_f \beta_c T_3}{3 \pi \mu L_{bf}} \) respectively.

In above equation, \( L_{bf}=0.17 \text{nm} \) and \( \beta_c=1.3807\text{E}-23 \) are the mean free path of water and Boltzmann constant for entire tested temperature range of water respectively. Furthermore, thermal diffusivity and dynamic viscosity are calculated by \( \alpha_f = k_f / \rho_f c_{p,f} \) and \( \mu = a 10^{b(T-c)} \), \( a = 2.414 \times 10^{-5} \), \( b=247 \) and \( c=140 \), [11].

Diameter of water molecule is calculated by \( d_f = 0.1 \left( \frac{6 M}{\pi N} \right)^{1/3} \) where \( M = 18 \text{ g}r/\text{mol} \)

Thermo physical properties of water and nanoparticles are listed in Table 1.

| Thermo physical properties | Water | CuO |
|---------------------------|------|-----|
| \( \rho \) (kg/m\(^3\))    | 997  | 6300 |
| \( C_p \) (J/kgK)           | 4181.7 | 537 |
| \( \mu \) (Pa.s)            | 0.00089 | -  |
| \( k \) (W/mK)              | 0.6069 | 17.65 |

Constant heat flux and constant temperature of 320 K are considered on two sides of channel and horizontal side respectively. The average convective heat transfer coefficient and Nusselt number for constant temperature plate are defined as follow:

\[ \bar{h} = \frac{mc_p (T_{m,o}-T_{m,i})}{A_s \Delta T_{lm}} \]  
(8)

\[ \bar{N}U = \frac{kD_h}{k_{\text{eff}} \Delta T_{lm}} \]  
(9)

where \( \Delta T_{lm} \) means LMTD and it is defined by the below relation:

\[ \Delta T_{lm} = \frac{(T_2-T_{m,o})-(T_2-T_{m,i})}{\ln[(T_2-T_{m,o})/(T_2-T_{m,i})]} \]  
(10)

To find a proper as well as independent grid for solving the problem of heat transfer in a three dimensional channel, several different grid distributions have been evaluated and the average temperature of outlet water is computed for three different grid sizes as in Table 2.

| Grid size(nm) | Node numbers   | \( T \) (K) |
|---------------|----------------|-------------|
| 1             | 14 \( \times \) 12 \( \times \) 1000 | 306.012     |
| 0.8           | 17 \( \times \) 15 \( \times \) 1000 | 305.543     |
| 0.6           | 23 \( \times \) 20 \( \times \) 1000 | 305.490     |
It can be seen that the average temperature of water in outlet flow for the grid size of 1 nm has not adequate accuracy, however, the grid size of 0.8 nm and 0.6 nm are much accurate and are very close to each other. Therefore, the grid size of 0.8 nm is chosen in order to have less calculation time.

3. Numerical Results

Forced convective heat transfer in a channel with equilateral triangular cross section and CuO-water nanofluid for several volume fractions and two nanoparticle sizes of 20 nm and 80 nm has been investigated extensively. With introducing a heat flux ratio that shows heat flux relation between two sides of channel (\( r_\text{h}=q_2/q_1 \)). The horizontal side is considered with constant temperature of 320 K. As it can be seen in Figures 2 and 3 convective heat transfer coefficient and average Nusselt number versus heat flux ratio for different volume fractions are grown linearly. Furthermore, the rise of heat flux ratio leads to growth of average convective heat transfer coefficient and average Nusselt number (See Figures 2 and 3). This work is done for three different heat flux ratio of 0, 0.5 and 1. The Reynolds number for all cases is equal to \( \text{Re}=100 \) and average convective heat transfer coefficient and average Nusselt number are obtained for horizontal side of the tube.

![Figure 2](image1.png)

**Figure 2.** Effect of heat flux ratio on average convective heat transfer coefficient for diameter of nanoparticles (a) 20 nm and (b) 80 nm

According to Figures 2 and 3, it can be found that the average convective heat transfer coefficient increases versus varying of nanoparticle diameter from 80 nm to 20 nm. For instance, average convective heat transfer coefficient of nanofluid with 4% volume fraction, heat flux ratio of 1 and nanoparticle size of 20 nm and 80 nm rise 11.6% and 10% respectively in comparison with distilled water. Furthermore, average convective heat transfer coefficient of nanofluid with 4% volume fraction, heat flux ratio of 0 and nanoparticle size of 20 nm and 80 nm increase 18% and 16% respectively in comparison with the distilled water. A decline in nanoparticle size from 80 nm to 20 nm diminishes average Nusselt number. For example, average Nusselt number of nanofluid with 4% volume fraction, heat flux ratio of 1 and nanoparticle size of 20 nm and 80 nm records 1.64% and 4.1% increases respectively in comparison to the distilled water. As an another example average Nusselt number of nanofluid with 4% volume fraction, heat flux ratio of 0 and nanoparticle size of 20 nm and 80 nm show 7.8% and 9.69% growths respectively in comparison to water. Totally decreasing nanoparticle size leads to increasing average convective heat transfer coefficient and decreasing average Nusselt number. So decreasing nanoparticle size does not lead to much more heat transfer. For a constant nanoparticle size, by decreasing heat flux ratio from 1 to 0, average Nusselt number increases. So unbalancing heat fluxes on walls leads to a better convective heat transfer and Nusselt number as well.
Figure 3. Effect of heat flux ratio on average Nusselt number for diameter of nanoparticles (a) 20 nm and (b) 80 nm

On the other hand, Figures 4 (a) and (b) show local convective heat transfer coefficient and Nusselt number on hot plate for heat flux ratio of 1 and nanoparticle size of 80 nm along the channel respectively. The plotted local convective heat transfer coefficient in Figure 4(a) for three volume fractions of nanofluid depicts the much more differences at channel inlet in comparison to that of outlet and becomes less with moving along the channel.

Figure 4. Local (a) convective heat transfer coefficient and (b) Nusselt number for $r_q=1$, $d_p=80$ nm

4. Conclusion

In this paper, the influence of CuO nanoparticle on flow structure and heat transfer pattern has been carried out comprehensively. As a result of grid independency study, a fine as well as accurate grid size has been selected for achieving reliable global parameters of fluid flow and heat transfer. Average convective heat transfer coefficient goes up versus heat flux ratio and nanoparticle volume fraction meanwhile the computations reveal that the size of nanoparticles has not a significant influence on heat transfer properties. Moreover, average convective heat transfer and Nusselt number along the channel become less in comparison with that of incoming flow due to heated wall as well as decline of thermal boundary layer. Furthermore, in the presence of nanoparticles, a significant rise on heat transfer coefficient and pressure loss through the channel is observed where the maximum magnitude of velocity is computed at the centre of channel cross section.
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Nomenclature

| Symbol | Description                          |
|--------|--------------------------------------|
| $C_p$  | Specific heat [J/kgK]                |
| $d$    | Diameter [m]                         |
| $d_p$  | Nanoparticle diameter [m]            |
| $D_h$  | Hydraulic diameter of the triangular tube [m] |
| $h$    | Convective heat transfer coefficient [W/mK] |
| $k$    | Conduction heat transfer coefficient [W/mK] |
| $L$    | Length [m]                           |
| $N$    | Avogadro’s number [$= 6.022 \times 10^{23}$] |
| $N_U$  | Nusselt number                       |
| $p$    | Pressure [Pa]                        |
| $Pr$   | Prandtl number                       |
| $q'$   | Heat flux [W/m²]                     |
| $Re$   | Reynolds number                      |
| $T$    | Temperature [K]                      |
| $T_m$  | Fluid mean temperature [K]           |
| $T_s$  | Solid wall temperature [K]           |
| $q_o$  | Heat flux ratio                      |
| $x$    | Cartesian coordinate axis [m]        |
| $y$    | Cartesian coordinate axis [m]        |
| $z$    | Cartesian coordinate axis [m]        |
| $\alpha$ | Thermal diffusivity [m²/s]         |
| $\mu$  | Dynamic viscosity [kg/ms]            |
| $\nu$  | Viscosity [m²/s]                     |
| $\rho$ | Density [kg/m³]                      |
| $\phi$ | Volume fraction [%]                  |

Greek Letters

| Symbol | Description                          |
|--------|--------------------------------------|

Subscript

| Subscript | Description |
|-----------|-------------|
| ave       | Average     |
| $bf$      | Mean free path |
| eff       | Effective   |
| $f$       | Fluid       |
| $i$       | Inlet flow  |
| $lm$      | Logarithm mean temperature difference |
| $m$       | Mean        |
| $o$       | Outlet flow |
| $p$       | Nanoparticle |
| $s$       | Solid wall  |

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