Dirac dark matter in a radiative neutrino model

Hiroshi Okada\textsuperscript{1,2,\ast} and Yutaro Shoji\textsuperscript{3,†}

\textsuperscript{1}Asia Pacific Center for Theoretical Physics (APCTP) - Headquarters San 31, Hyojadong, Nam-gu, Pohang 790-784, Korea
\textsuperscript{2}Department of Physics, Pohang University of Science and Technology, Pohang 37673, Republic of Korea
\textsuperscript{3}Kobayashi-Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, Nagoya, Aichi 464-8602, Japan

(Dated: March 26, 2020)

Abstract

We propose a simple radiative neutrino mass scenario with a Dirac dark matter candidate, which is minimally realized by a $Z_3$ symmetry. We introduce two Dirac neutrinos and two inert doublets. We demonstrate that the model has a large allowed region that satisfies the constraints from neutrino oscillation data, lepton flavor violations, direct detection of dark matter and dark matter relic density. We also propose an efficient parameterization of the neutrino Yukawa couplings, which reproduces a desired active neutrino mass matrix.

Keywords:
I. INTRODUCTION

It is very important to understand the nature of neutrinos such as their oscillations, mass ordering, CP phases, Dirac or Majorana, and any interactions related to neutrinos since they would have a clue to derive new physics. For example, the leptonic CP phases could explain the Baryon Asymmetry of Universe (BAU) through leptogenesis [1]. It contrasts with the CP phase in the quark sector, which is too small to explain the observed BAU. Another example is the neutrinoless double beta decay, which would tell us whether neutrino is Dirac or Majorana, although the measurement of this process is challenging.

Neutrino would have a connection to the idea of dark matter (DM), which occupies $\sim 20\%$ of the energy density of the Universe [2]. Although we have not discovered DM yet, there are several suggestions from experiments. For instance, direct detection searches tell us an upper bound on the scattering cross section of DM and nucleus. The most stringent bound is put by XENON1T [3] and is $\sigma_{SI} \lesssim 4.1 \times 10^{-47}$ cm$^2$ at 30 GeV of DM mass for the spin independent interaction, which is strong enough to exclude many models in this mass range. Meanwhile, indirect detection searches provide very attractive signatures although there are rather big uncertainties from astrophysical background. Several interesting excesses in the electron-positron flux of the cosmic rays are found by experiments such as AMS-02 [4-9], DAMPE [10], and CALET [11, 12]. One of the interpretations for them is to create electron-positron pair via DM annihilation or decay. For example, let us consider a fermionic DM candidate that annihilates into an electron-positron pair since it will be natural if DM has similar nature as that of the neutrinos. In this case, we need a rather large cross section to explain these experimental results compared with the canonical cross section of $\sim 10^{-9}$ GeV$^{-2}$ for the relic density. Such enhancement can be obtained by Sommerfeld [13] or Breit-Wigner [14, 15] and these enhancement mechanisms require s-wave dominant annihilation. Thus, Dirac DM is favored from this viewpoint.

In this paper, we propose a simple model to realize the neutrino mass matrix and a Dirac DM in the framework of the radiative seesaw mechanism at the one-loop level. We show an allowed region that satisfies the constraints from neutrino oscillation data, lepton flavor violations (LFVs), direct detection searches and correct relic density of DM. Since the radiative seesaw scenario has been discussed in this context for a long time, many authors have come up with various kinds of ideas. Here, we list representative references at one-
TABLE I: The charge assignments of the relevant particles under $SU(2)_L \times U(1)_Y \times Z_3$, where $\omega \equiv e^{2\pi i/3}$. We assume $N$ is Dirac and has two families and $H_1$ and $H_2$ do not develop VEVs.

|               | Fermions | Bosons |
|---------------|----------|--------|
| Fields        | $L_L$    | $e_R$  | $N$  | $H$ | $H_1$ | $H_2$ |
| $SU(2)_L$     | 2        | 1      | 1    | 2   | 2     | 2     |
| $U(1)_Y$      | $-\frac{1}{2}$ | $-1$  | 0    | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $Z_3$         | 1        | 1      | $\omega$ | 1   | $\omega$ | $\omega^2$ |

In ref. [22], this idea is proposed as one of the possibilities.  

In this section, we review our scenario. We introduce two families of neutral Dirac fermions, $N$, to reproduce the neutrino data and two inert doublets, $H_1 = (\eta^+_1, \eta^0_1)^T$ and $H_2 = (\eta^+_2, \eta^0_2)^T$. The masses for $N_a$ is denoted as $M_a$ and those for $\eta^0_a$ and $\eta^+_a$ are denoted as $m_{\eta^0_a}$ and $m_{\eta^+_a}$ with $a \in \{1, 2\}$, respectively. We denote the SM Higgs boson as $H$ and its vacuum expectation value (VEV) as $v_H/\sqrt{2} \approx 174$ GeV. In order to assure the Dirac nature of $N$ and the inert feature of $H_1$ and $H_2$, we impose a discrete Abelian symmetry, $Z_3$. Under this rather simple framework, the neutrino mass is induced at the one-loop level via the new particles and the lightest particle among them can become a DM candidate. We assume the DM is the lighter $N$, which we denote as $N_1$, and its stability is assured by $Z_3$.  

1 We summarize the relevant field contents and their charge assignments in Tab. I.
The Lagrangian for the lepton sector is given by

\[-\mathcal{L}_L = y_\ell \bar{L}_i H e_{R_i} + y_{\nu a_i} \bar{L}_i \tilde{H}_1 N_{R_a} + y_{\nu a_i} \tilde{N}_{L_a} \tilde{H}_2 L_{L_j}^c + M_a \tilde{N}_{L_a} N_{R_a} + \text{c.c.}, \tag{II.1}\]

where \(i, j \in \{1, 2, 3\}\), \(a, b \in \{1, 2\}\) and \(\tilde{H}_{1,2} \equiv i \sigma_2 H_{1,2}^*\) with \(i \sigma_2\) being the complete anti-symmetric matrix. We assume that \(y_\ell\) and \(M\) are diagonal; we consider the mass eigen-bases for the charged-leptons and the neutral Dirac fermions.

Meanwhile, the relevant part of the Higgs potential is given by

\[V = \lambda_0 (H^\dagger H_1) (H^\dagger H_2) + \text{c.c.}, \tag{II.2}\]

where \(\lambda_0\) plays an important rule in generating nonzero neutrino masses by connecting \(H_1\) and \(H_2\) appropriately. We presume \(\lambda_0\) to be small enough so that we can identify \(\eta_1^0\) and \(\eta_2^0\) as mass eigenstates. In addition, we presume the neutral component and the charged component of an inert doublet have degenerated masses, i.e. \(m_{\eta_1^0} = m_{\eta_1^+} \equiv m_{\eta_1}\) and \(m_{\eta_2^0} = m_{\eta_2^+} \equiv m_{\eta_2}\). This condition is appropriate since oblique parameters \([23, 24]\) are in favor of these degeneracy.

### A. Active neutrino mass

The dominant contribution to the active neutrino mass matrix comes from the one-loop diagrams involving the Dirac neutral fermions and the neutral inert Higgs bosons. At the leading order in \(\lambda_0\), the active neutrino mass matrix is given by \([25]\)

\[m_{\nu_{ij}} = y_{\nu a_i} R_{b b} y_{N_{L_a b}} + y_{N_{L_i b}}^T R_{b b} y_{N_{R_{b j}}}^T, \tag{II.3}\]

where the dimensionful parameter, \(R\), is defined as

\[R_{bb} = \frac{\lambda_0^* v_H^2 M_b}{32 \pi^2} F(M_b^2, m_{\eta_1^0}^2, m_{\eta_2^0}^2), \tag{II.4}\]

\[F(a, b, c) = -\frac{(b - c)a \ln a + (c - a)b \ln b + (a - b)c \ln c}{(a - b)(b - c)(c - a)}. \tag{II.5}\]

The neutrino mass matrix is diagonalized by a unitary matrix, \(U_\nu\), as

\[U_\nu^T m_\nu U_\nu = \text{diag}(m_1, m_2, m_3) \equiv D_\nu. \tag{II.6}\]
We regard $U_{\nu}$ as the PMNS matrix, which we denote as $U_{\text{PMNS}}$ [20], because the charged-lepton mass matrix is assumed to be diagonal. We assume that $D_{\nu}$ does not have zero eigenvalues in the following discussion\textsuperscript{2}.

Since there are a sufficient number of parameters, there exist parameterizations that always reproduce the neutrino oscillation data\textsuperscript{3}. In the following, we show one of such parameterizations.

We define

\begin{align}
    x &= U_{\text{PMNS}}^* \sqrt{D_{\nu}}(w_1 + iw_2), \\
y &= U_{\text{PMNS}}^* \sqrt{D_{\nu}}w_3, \\
z &= U_{\text{PMNS}}^* \frac{1}{\sqrt{D_{\nu}}}(w_1 - iw_2),
\end{align}

where the elements of $\sqrt{D_{\nu}}$ are the square roots of the elements of $D_{\nu}$, $1/\sqrt{D_{\nu}}$ is its inverse and $w_i$'s are arbitrary real vectors satisfying

\[ w_i \cdot w_j = \delta_{ij}. \] (II.10)

Here, we assume $z_2 \neq 0$. Then, we construct an orthonormal basis as

\begin{align}
    \hat{x} &= \frac{x}{|x|}, \\
    \hat{y} &= \frac{y - \frac{x^\dagger y}{|y|^2} x}{\sqrt{|y - \frac{x^\dagger y}{|y|^2} x|}}, \\
    \hat{z} &= \frac{z}{|z|}.
\end{align}

Using them, the Yukawa couplings for the Dirac neutral fermions are constructed as

\begin{align}
    y_{N_R} &= \left( \hat{x} \quad \hat{y} \right) Y, \\
y_{N_L} &= \frac{1}{2} R^{-1} Y^{-1} \left( \hat{x}^\dagger \quad \hat{y}^\dagger \right) \left[ U_{\text{PMNS}}^* D_{\nu} U_{\text{PMNS}}^\dagger + A \right],
\end{align}

\textsuperscript{2} One can assume a tiny eigenvalue if needed.

\textsuperscript{3} The rank of the mass matrix for $N$ has nothing to do with the rank of the active neutrino mass matrix since $y_{N_R} \neq y_{N_L}^T$.  

5
where $Y$ is an arbitrary full rank $2 \times 2$ matrix and $A$ is an anti-symmetric matrix satisfying

$$A_{12} = - \frac{1}{z_2^*} \left[ A_{13} z_3^* - (U_{PMNS}^* D_{\nu} U_{PMNS}^\dagger z_3^*)_1 \right]. \quad (II.16)$$

$$A_{23} = - \frac{1}{z_2^*} \left[ A_{13} z_1^* + (U_{PMNS}^* D_{\nu} U_{PMNS}^\dagger z_3^*)_3 \right]. \quad (II.17)$$

Then, one can easily check that

$$\hat{z}^\dagger \left[ U_{PMNS}^* D_{\nu} U_{PMNS}^\dagger + A \right] = 0, \quad (II.18)$$

and

$$y_{\nu R}^T R y_{\nu L} + y_{\nu L}^T R y_{\nu R}^T = U_{PMNS}^* D_{\nu} U_{PMNS}^\dagger. \quad (II.19)$$

We take $A_{13}, Y$ and $w_i$'s as the input parameters instead of $y_{\nu R}$ and $y_{\nu L}$.

We take into account the perturbativity conditions, $|y_{\nu R_{ib}}| < \sqrt{4\pi}$ and $|y_{\nu L_{bi}}| < \sqrt{4\pi}$, and the cosmological constraint on the sum of the neutrino masses, $\sum_i m_i \lesssim 0.146$ eV for the normal hierarchy and $\sum_i m_i \lesssim 0.172$ eV for the inverted hierarchy [27–29].

### B. Lepton flavor violations (LFVs)

Due to the newly introduced couplings, $y_{\nu R}$ and $y_{\nu L}$, LFVs arise at the one-loop level. The branching ratios for $\ell_i \rightarrow \ell_j \gamma$ are calculated as

$$\frac{BR(\ell_i \rightarrow \ell_j \gamma)}{BR(\ell_i \rightarrow \ell_j \nu_j \nu_j)} = \frac{3\alpha_{em}}{16\pi G_F^2} \left| y_{\nu R_{ia}} y_{\nu R_{a}}^* G(M^2_{a}, m_{\nu_{i1}}^2) + y_{\nu L_{aj}} y_{\nu L_{ai}}^* G(M^2_{a}, m_{\nu_{j1}}^2) \right|^2, \quad (II.20)$$

$$G(m_a^2, m_b^2) = \frac{2m_a^6 + 3m_a^4m_b^2 - 6m_a^2m_b^4 + m_b^6 + 12m_a^4m_b^2 \ln(m_b/m_a)}{12(m_a^2 - m_b^2)^4}, \quad (II.21)$$

where $G_F \approx 1.17 \times 10^{-5}[\text{GeV}]^{-2}$ is the Fermi constant and $\alpha_{em} \approx 1/129$ is the fine structure constant. The current experimental upper bounds are found as [30, 31]

$$BR(\mu \rightarrow e\gamma) \leq 4.2 \times 10^{-13}, \quad BR(\tau \rightarrow \mu\gamma) \leq 4.4 \times 10^{-8}, \quad BR(\tau \rightarrow e\gamma) \leq 3.3 \times 10^{-8}. \quad (II.22)$$

The same couplings contribute to the muon anomalous magnetic moment as well, but its sign is always negative, which is against the current observation results [32–35]. Thus, we do not consider it in this paper.
III. DARK MATTER

In our model, the lightest particle among the Dirac fermions and the inert doublets becomes a DM candidate. In this paper, we assume the lighter Dirac fermion, \( \chi \equiv N_1 \), is the DM and denote its mass as \( m_\chi \equiv M_1 \). We also assume that the masses of the other \( Z_3 \) charged particles are not so close to the DM mass and that there is no initial asymmetry between the DM density and the anti-DM density.

We consider the case where the DM relic density is explained by the freeze-out mechanism. The relevant annihilation channels of the DM are \( \chi \chi \to \nu \nu, \ell \bar{\ell} \), which occur at the tree level.

In order to estimate the relic density, we solve the Boltzmann equation, which is given by

\[
\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma|v_\chi| \rangle [n_\chi^2 - (n_{\chi}^{\text{eq}})^2],
\]

where \( H \) is the Hubble parameter, \( n_\chi \) is the number density of the DM (not including the anti-DM), \( n_{\chi}^{\text{eq}} \) is the equilibrium number density and \( \langle \sigma|v_\chi| \rangle \) is the thermally averaged cross section, which is given in Appendix [A].

Then, we compare it with the observed relic density at 2\( \sigma \), which is given by [2]

\[
\Omega_\chi h^2 = 0.1199 \pm 0.0054.
\]

Next, we discuss the direct detection searches for the DM. We consider only the spin independent (SI) processes since the spin-dependent ones do not give a stringent constraint. The DM-Higgs coupling is induced at one-loop level through \( y_{N_R} \) and \( y_{N_L} \). In order to estimate the scattering cross section, we write down the relevant interactions as

\[
-L = \kappa_1^0 v_H h |\eta_1^0|^2 + \kappa_1^+ v_H h |\eta_1^+|^2 + \kappa_2^0 v_H h |\eta_2^0|^2 + \kappa_2^+ v_H h |\eta_2^+|^2,
\]

where \( h \) is the SM Higgs boson. The effective couplings of the DM to the quarks are calculated as

\[
-L_{\text{eff}} = C_{St} m_q \left( \chi \frac{-i \partial_\mu \gamma^\mu}{m_\chi} \chi \right)(\bar{q}q),
\]

\[
C_{St} = \frac{1}{32\pi^2} \frac{1}{m_h^2 m_\chi} \left[ |y_{N_{R_{a1}}}|^2 \left( \kappa_1^0 I(m_{\eta_1^0}^2, m_\chi^2) + \kappa_1^+ I(m_{\eta_1^+}^2, m_\chi^2) \right) \\
+ |y_{N_{L_{a1}}}|^2 \left( \kappa_2^0 I(m_{\eta_2^0}^2, m_\chi^2) + \kappa_2^+ I(m_{\eta_2^+}^2, m_\chi^2) \right) \right],
\]

\[
I(a, b) = 1 - \frac{a - b}{b} \ln \frac{a}{a - b},
\]

on page 7.
where $a > b$. Then, the SI cross section is given by

$$\sigma_{SI}^p = \frac{m_p^2}{\pi} \left( \frac{m_\chi m_p}{m_\chi + m_p} \right)^2 |C_{SI}|^2 f_p^2,$$

(III.7)

where $f_p = 0.326$ [36]. Since the newly introduced couplings, $\kappa_0^1$, $\kappa_1^1$, $\kappa_2^0$ and $\kappa_2^+,$ are not related to the neutrino masses, the LFVs or the DM relic density, we cannot get a rigid prediction for the cross section. Instead, we consider the maximum value of the cross section with

$$|\kappa_1^0|, |\kappa_1^+|, |\kappa_2^0|, |\kappa_2^+| < \kappa_{\text{max}}.$$  

(III.8)

The maximum value is evaluated as

$$\sigma_{SI}^p < \sigma_{SI}^{p,\text{max}} = \kappa_{\text{max}}^2 \sigma_{SI}^p |\kappa_0^0\rightarrow1, \kappa_1^+\rightarrow1, \kappa_2^0\rightarrow1, \kappa_2^+\rightarrow1|.$$  

(III.9)

To discuss the maximum detectability of this model, we compare $\sigma_{SI}^{p,\text{max}}$ with the current experimental constraints such as that of XENON1T [3], which provides the most stringent constraint. 

A. Numerical analysis 

Here, we perform a numerical analysis to find allowed regions, where the relic density of DM, the neutrino oscillation data and the LFV constraints are satisfied. First of all, we fix the ranges for input parameters as

$$|\lambda_0| < 4\pi, \quad 0 \leq \alpha_{2,3} < 360^\circ,$$

(III.10)

$$m_0 < m_0^{\text{max}}, \quad 200 \text{ GeV} < M_1 < M_2 < 10 \text{ TeV},$$

(III.11)

$$M_1 < m_{\eta_{1,2}}, \quad 1 \text{ TeV} < m_{\eta_{1,2}} < 10 \text{ TeV},$$

(III.12)

$$|Y_{ab}| < \sqrt{4\pi}, \quad |A_{13}| < 10 \text{ eV},$$

(III.13)

where $m_0$ is the (non-vanishing) lightest neutrino mass and $\alpha_{2,3}$ are the Majorana phases. Here, $m_0^{\text{max}}$ is the maximum value of the lightest neutrino mass to satisfy the cosmological constraint on the sum of the neutrino mass. We have $m_0^{\text{max}} = 0.0403$ eV for the normal hierarchy and $m_0^{\text{max}} = 0.0419$ eV for the inverted hierarchy. For the neutrino oscillation parameters, we use the central values given by [37]

$$\theta_{12} = 33.82^\circ, \quad \theta_{23} = 49.7^\circ, \quad \theta_{13} = 8.61^\circ, \quad \delta_{\text{CP}} = 217^\circ,$$

(III.14)

$$\Delta m_{21}^2 = 7.39 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{31}^2 = 2.525 \times 10^{-3} \text{ eV}^2,$$

(III.15)
for the normal hierarchy and

\[ \theta_{12} = 33.82^\circ, \quad \theta_{23} = 49.7^\circ, \quad \theta_{13} = 8.65^\circ, \quad \delta_{\text{CP}} = 280^\circ, \quad (III.16) \]
\[ \Delta m_{21}^2 = 7.39 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{32}^2 = -2.512 \times 10^{-3} \text{ eV}^2, \quad (III.17) \]

for the inverted hierarchy. Here, \( \Delta m_{ij}^2 = m_{\nu_i}^2 - m_{\nu_j}^2 \).

In Fig. 1, we show the scatter plots on the \((m_\chi, \Omega_\chi h^2)\)-plane and on the \((m_\chi, \sigma_{\text{SI}}^{\text{max}}/\kappa_{\text{max}}^2)\)-plane. The upper panels are for the normal hierarchy and the lower panels are for the inverted hierarchy. The blue points reproduce the active neutrino mass matrix and satisfy the perturbativity constraints, the LFV constraints and \(\Omega_\chi h^2 < 0.2\). The yellow points also explain the DM relic density within 2\(\sigma\).

We find a large number of points that satisfy all the constraints we have discussed. With \(\kappa_{\text{max}} = 1\), the spin-independent cross section is below the current constraints of the direct detection experiments. However, it can still become larger than the cosmic neutrino background, which is around \(10^{-48} - 10^{-47} \text{ cm}^2\) for \(1 \text{ TeV} < m_\chi < 10 \text{ TeV}\).

**IV. CONCLUSIONS AND DISCUSSIONS**

We have studied a simple neutrino mass scenario at the one-loop level, which also contains a Dirac DM candidate. The model is minimally realized by two Dirac neutrinos and two inert doublets with a \(Z_3\) symmetry. In formulating the neutrino sector, we have developed an efficient method to parameterize the neutrino Yukawa couplings. We have shown an allowed region that satisfies the constraints from neutrino oscillation data, lepton flavor violations, direct detection of dark matter and dark matter relic density. We have found that the allowed region covers almost whole the range of DM mass we have assumed, \(0.2 - 10 \text{ TeV}\). With an enhancement mechanism of the cross section, it may also explain the excesses in indirect detection searches, whose DM range is \(O(100 \text{ GeV})\) to \(O(1 \text{ TeV})\) depending on the excesses. Considering that they require flavor specific final state, *i.e.* into an electron positron pair, it could lead to a verifiable scenario by introducing flavor dependent gauged \(U(1)\) symmetries. Although we did not give a detailed analysis on the indirect searches, it would be worthwhile to mention this issue since the Dirac nature of DM might be a hint of the enhancement of the cross section.
FIG. 1: Scatter plots of the allowed points for the normal hierarchy (upper) and for the inverted hierarchy (lower). The blue points reproduce the active neutrino mass matrix and satisfy the perturbativity constraints, the LFV constraints and $\Omega_\chi h^2 < 0.2$. The yellow points also explain the DM relic density within $2\sigma$.

Acknowledgments

This research was supported by an appointment to the JRG Program at the APCTP through the Science and Technology Promotion Fund and Lottery Fund of the Korean Government. This was also supported by the Korean Local Governments - Gyeongsangbuk-do Province and Pohang City (H.O.). H.O. is sincerely grateful for the KIAS member. Y.S. is supported by Grant-in-Aid for Scientific research from the Ministry of Education, Science, Sports, and Culture (MEXT), Japan, No. 16H06492.

Appendix A: Dark matter cross section

In this appendix, we give the thermally averaged cross sections for the DM, which is used for the calculation of the thermal relic density.
For the $\chi \bar{\chi} \rightarrow 2\nu$ processes, we have

$$\langle \sigma_{\chi \bar{\chi} \rightarrow 2\nu} | v_\chi \rangle = \sum_{a,b} \left( \frac{1}{n_{\chi}^{eq}} \right)^2 \frac{T}{2048 \pi^3} \int_0^\infty ds \, K_1 \left( \frac{\sqrt{s}}{T} \right) \sqrt{s - 4m_\chi^2} \times \left\{ |y_{N_{R_{a1}}} y_{N_{R_{b1}}}^*|^2 W_1(m_{\eta_1^0}^2, s) \right. + |y_{N_{L_{1a}}} y_{N_{L_{1b}}} y_{N_{L_{1a}}}^*|^2 W_1(m_{\eta_2^0}^2, s) \right. - \left( y_{N_{R_{a1}}} y_{N_{R_{b1}}}^* y_{N_{L_{1b}}} y_{N_{L_{1a}}}^* + c.c. \right) \frac{m_\chi^2}{s} W_2(m_{\eta_1^0}^2, m_{\eta_2^0}^2, s) \right\}. \quad (A.1)$$

For the $\chi \bar{\chi} \rightarrow \ell\bar{\ell}$ processes, we have

$$\langle \sigma_{\chi \bar{\chi} \rightarrow \ell\bar{\ell}} | v_\chi \rangle = \sum_{a,b} \left( \frac{1}{n_{\chi}^{eq}} \right)^2 \frac{T}{2048 \pi^3} \int_0^\infty ds \, K_1 \left( \frac{\sqrt{s}}{T} \right) \sqrt{s - 4m_\chi^2} \times \left\{ |y_{N_{R_{a1}}} y_{N_{R_{b1}}}^*|^2 W_1(m_{\eta_1^0}^2, s) \right. + |y_{N_{L_{1a}}} y_{N_{L_{1b}}} y_{N_{L_{1a}}}^*|^2 W_1(m_{\eta_2^0}^2, s) \right. - \left( y_{N_{R_{a1}}} y_{N_{R_{b1}}}^* y_{N_{L_{1b}}} y_{N_{L_{1a}}}^* + c.c. \right) \frac{m_\chi^2}{s} W_2(m_{\eta_1^0}^2, m_{\eta_2^0}^2, s) \right\}. \quad (A.2)$$

Here, $K_1(x)$ is the modified Bessel function of the second kind and

$$W_1(m_1^2, s) = 1 + \frac{(a - 1)^2}{a^2 - \beta^2} - \frac{a - 1}{\beta} \ln \frac{a + \beta}{a - \beta}, \quad (A.3)$$

$$W_2(m_1^2, m_2^2, s) = \frac{2}{\beta(a + b)} \ln \frac{(a + \beta)(b + \beta)}{(a - \beta)(b - \beta)}, \quad (A.4)$$

with

$$a = \frac{2(m_1^2 - m_\chi^2)}{s}, \quad (A.5)$$

$$b = \frac{2(m_2^2 - m_\chi^2)}{s}, \quad (A.6)$$

$$\beta = \sqrt{1 - \frac{4m_\chi^2}{s}}, \quad (A.7)$$

for $a > 1$ and $b > 1$.

[1] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45.

[2] P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 571, A16 (2014) doi:10.1051/0004-6361/201321591 [arXiv:1303.5076 [astro-ph.CO]].
[3] E. Aprile et al. [XENON Collaboration], Phys. Rev. Lett. 121, no. 11, 111302 (2018) doi:10.1103/PhysRevLett.121.111302 [arXiv:1805.12562 [astro-ph.CO]].

[4] M. Aguilar et al. [AMS Collaboration], Phys. Rev. Lett. 110, 141102 (2013). doi:10.1103/PhysRevLett.110.141102

[5] L. Accardo et al. [AMS Collaboration], Phys. Rev. Lett. 113, 121101 (2014). doi:10.1103/PhysRevLett.113.121101

[6] M. Aguilar et al. [AMS Collaboration], Phys. Rev. Lett. 113, 121102 (2014). doi:10.1103/PhysRevLett.113.121102

[7] M. Aguilar et al. [AMS Collaboration], Phys. Rev. Lett. 113, 221102 (2014). doi:10.1103/PhysRevLett.113.221102

[8] M. Aguilar et al. [AMS Collaboration], Phys. Rev. Lett. 122, 101101 (2019). doi:10.1103/PhysRevLett.122.101101

[9] M. Aguilar et al. [AMS Collaboration], Phys. Rev. Lett. 122, no. 4, 041102 (2019). doi:10.1103/PhysRevLett.122.041102

[10] G. Ambrosi et al. [DAMPE Collaboration], Nature 552 63 (2017) [arXiv:1711.10981 [astro-ph.HE]].

[11] O. Adriani et al. [CALET Collaboration], Phys. Rev. Lett. 119, no. 18, 181101 (2017) doi:10.1103/PhysRevLett.119.181101 [arXiv:1712.01711 [astro-ph.HE]].

[12] O. Adriani et al. [CALET Collaboration], Phys. Rev. Lett. 120, no. 26, 261102 (2018) doi:10.1103/PhysRevLett.120.261102 [arXiv:1806.09728 [astro-ph.HE]].

[13] A. Sommerfeld, Ann. Phys. 403, 257 (1931).

[14] M. Ibe, H. Murayama and T. T. Yanagida, Phys. Rev. D 79, 095009 (2009) doi:10.1103/PhysRevD.79.095009 [arXiv:0812.0072 [hep-ph]].

[15] W. L. Guo and Y. L. Wu, Phys. Rev. D 79, 055012 (2009) doi:10.1103/PhysRevD.79.055012 [arXiv:0901.1450 [hep-ph]].

[16] E. Ma, Phys. Rev. D 73, 077301 (2006) doi:10.1103/PhysRevD.73.077301 [hep-ph/0601225].

[17] S. Kanemura, T. Nabeshima and H. Sugiyama, Phys. Rev. D 85, 033004 (2012) doi:10.1103/PhysRevD.85.033004 [arXiv:1111.0599 [hep-ph]].

[18] Y. Kajiyama, H. Okada and K. Yagyu, Nucl. Phys. B 874, 198 (2013) doi:10.1016/j.nuclphysb.2013.05.020 [arXiv:1303.3463 [hep-ph]].

[19] L. M. Krauss, S. Nasri and M. Trodden, Phys. Rev. D 67, 085002 (2003)
doi:10.1103/PhysRevD.67.085002 [hep-ph/0210389].

[20] M. Aoki, S. Kanemura and O. Seto, Phys. Rev. Lett. 102, 051805 (2009) arXiv:0807.0361.

[21] M. Gustafsson, J. M. No and M. A. Rivera, Phys. Rev. Lett. 110, 211802 (2013) arXiv:1212.4806 [hep-ph].

[22] E. Ma, arXiv:1912.11950 [hep-ph].

[23] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65, 964 (1990). doi:10.1103/PhysRevLett.65.964

[24] M. E. Peskin and T. Takeuchi, Phys. Rev. D 46, 381 (1992). doi:10.1103/PhysRevD.46.381

[25] H. Okada and Y. Orikasa, Phys. Rev. D 94, no. 5, 055002 (2016) doi:10.1103/PhysRevD.94.055002 [arXiv:1512.06687 [hep-ph]].

[26] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28, 870 (1962). doi:10.1143/PTP.28.870

[27] S. Roy Choudhury and S. Hannestad, arXiv:1907.12598 [astro-ph.CO].

[28] N. Aghanim et al. [Planck Collaboration], arXiv:1807.06209 [astro-ph.CO].

[29] S. Vagnozzi, E. Giusarma, O. Mena, K. Freese, M. Gerbino, S. Ho and M. Lattanzi, Phys. Rev. D 96, no. 12, 123503 (2017) doi:10.1103/PhysRevD.96.123503 [arXiv:1701.08172 [astro-ph.CO]].

[30] A. M. Baldini et al. [MEG Collaboration], Eur. Phys. J. C 76, no. 8, 434 (2016) doi:10.1140/epjc/s10052-016-4271-x [arXiv:1605.05081 [hep-ex]].

[31] J. Adam et al. [MEG Collaboration], Phys. Rev. Lett. 110, 201801 (2013) doi:10.1103/PhysRevLett.110.201801 [arXiv:1303.0754 [hep-ex]].

[32] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura and T. Teubner, J. Phys. G 38, 085003 (2011) [arXiv:1105.3149 [hep-ph]].

[33] A. Keshavarzi, D. Nomura and T. Teubner, Phys. Rev. D 97, no. 11, 114025 (2018) arXiv:1802.02995 [hep-ph].

[34] J. Grange et al. (Muon g-2) (2015), 1501.06858.

[35] H. Inuma (J-PARC muon g-2/EDM), J. Phys. Conf. Ser. 295, 012032 (2011).

[36] Y. Mambrini, Phys. Rev. D 84, 115017 (2011) doi:10.1103/PhysRevD.84.115017 [arXiv:1108.0671 [hep-ph]].

[37] I. Esteban, M. C. Gonzalez-Garcia, A. Hernandez-Cabezudo, M. Maltoni and T. Schwetz, JHEP 1901, 106 (2019) doi:10.1007/JHEP01(2019)106 [arXiv:1811.05487 [hep-ph]].