A NOTE ON INFLATIONARY STRING COSMOLOGY

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Abstract

Cosmological solutions are obtained by continuation of black D-brane solutions into the region between the horizons. It is investigated whether one can find exponential expansion when probing the cosmology with D-branes. A unique configuration exhibiting exponential expansion is discussed.

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1 Introduction

String (or M-) theory is believed to describe physics at the Planck scale. The only experimental window into this region is the early universe. Therefore it is very interesting to derive implications for the evolution of the universe from string theory. In [1-16] we give a list of some references dealing with this question. A prominent stringy version of a cosmological model is the pre-big-bang (PBB) scenario [2]. There, two solutions connected by scale factor duality are used to describe an inflationary and a Friedmann-Robertson-Walker (FRW) phase of the universe. One of the main problems in the PBB scenario is to describe the smooth phase transition from the inflationary to the FRW phase. This is known as the graceful exit problem [3]. A related problem is that typically cosmological solutions of low energy effective string theories run into singularities. Methods of how to avoid those singularities are for example discussed in [4, 5, 6, 10]. Furthermore, there are recent indications that the PBB scenario also requires an exponentially large world radius at its onset [16].

In this note we are going to investigate whether we can find solutions exhibiting an inflationary phase with exponentially growing world radius. We will deal with solutions obtained in the spirit of [7], i.e. by continuing a black D-brane solution into the region between inner and outer horizon. Black D-brane solutions minimize the type II low energy effective action,

$$S = \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left[ R + 4 (\partial \phi)^2 \right] - \frac{2}{(8 - p)!} F^2 \right\}.$$  \hspace{1cm} (1)

$F$ is a RR-form field strength. An object extended along $p$ spatial dimensions couples electrically to a $p + 1$ form gauge potential, and thus corresponds to non-zero $p + 2$ form field strength. Here, we will discuss magnetically charged objects and hence will have non-zero $8 - p$ form field strength. In [13] it has been observed that for an $(n, m)$ five-brane of type IIB theory the presence of RR charges increases the acceleration of the expansion near the big bang singularity. Therefore, there is some hope that one can find exponential expansion from D-brane backgrounds. In section two we will scan all the black D-brane backgrounds (continued to the region between the horizons) for exponentially growing world radii near the singularity at the inner horizon. The scan includes T-dual solutions (or, in other words D-branes whose transverse space has compact directions), and the possibility of probing the universe with several kinds of D-branes. We will find that there is exactly one option to obtain exponential inflation within the given framework. This is a T-dualized D5-brane solution probed by D0-branes. (After T-duality the D5-brane becomes a D4-brane)

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2 Additional references dealing with string cosmology in general and the PBB scenario in particular can be found at Maurizio Gasperini’s homepage: http://carmen.to.infn.it/gasperin/.
and therefore probing with D0-branes makes sense. In section three we will investigate the inflationary model and find that it expands in all spatial directions and is therefore a truly ten dimensional solution. We comment on our results in a concluding section.

2 Scan for inflationary string cosmology

The strategy is as follows. We take a black $p$-brane solution and ‘go’ in the region between the horizons. In the region between the inner and the outer horizon, time becomes space-like and the radial distance becomes time-like. Thus the solution (depending on the radial distance) will become a time dependent cosmological solution. Typically, the solutions obtained in this way possess a big bang singularity when time is at the inner horizon and a big crunch singularity when it reaches the outer horizon (or vice versa). Our aim is to investigate whether we can find frames with an exponential expansion in the vicinity of the inner horizon. In such frames the universe would exhibit inflationary expansion close to the big bang singularity. Especially we would like to find solutions for which three spatial directions exponentially expand whereas the others contract or at least do not expand.

Before discussing the general black D-brane solution we describe what we mean by brane frame. A brane frame is defined such that the world volume action of the corresponding D-brane starts off with the ‘canonical’ Nambu-Goto term,

$$\int d^{p+1}x \sqrt{g_{\text{brane}}} \equiv \int d^{p+1}x e^{-\phi+\phi_0} \sqrt{g_{\text{string}}},$$  

where in our convention $e^{\phi_0}$ is the string coupling and the index $i$ on the metrics refers to the fact that they are induced. From the above we obtain for the metric components

$$(g_{\text{brane}})^{\mu\nu} = e^{-2\phi+2\phi_0/(p+1)} (g_{\text{string}})^{\mu\nu}. \tag{3}$$

When we probe space-time with a D-p-brane we will measure the corresponding brane-frame metric.

Now we will scan systematically all D-q-brane backgrounds for the possibility of inflationary phases in some p-brane frame. For that we use the general form of the black brane solution in [17]. From their solution we obtain a cosmological solution by continuation into the region between the two horizons. We change the notation of [17] according to

$$t \rightarrow y, \quad r \rightarrow t, \quad r_\pm \rightarrow t_\pm, \tag{4}$$

3The procedure of going behind the outer horizon should be merely understood as an easy way of obtaining cosmological solutions from known black brane solutions. Nevertheless we will keep the terminology of inner and outer horizons for convenience. The world-volume of the D-brane has Euclidean signature.
and chose \( t \) to be in the interval between \( t^- \) and \( t^+ \), \((t^+ > t^-)\). For a D-q-brane we find for

the metric (in the string frame)

\[
\begin{align*}
\text{ds}^2 &= \left(\frac{(t^+)}{(t^-)}\right)^{7-q} dy^2 - \frac{dt^2}{\sqrt{1 - \left(\frac{(t^+)}{(t^-)}\right)^{7-q}}} \\
&\quad + t^2 \left(1 - \left(\frac{t^-}{t^+}\right)^{\frac{7-q}{2}}\right)^{\frac{7-q}{2}} d\Omega_{8-q}^2 + \sqrt{1 - \left(\frac{t^-}{t^+}\right)^{7-q}} dx^i dx^i,
\end{align*}
\]

(5)

where the sum over \( i = 1, \ldots, q \) is understood. The dilaton is (for convenience we put \( \phi_0 = 0 \), it can be reintroduced by noting that constant shifts in \( \phi \) are moduli of the low energy effective theory)

\[ e^{-2\phi} = \left(1 - \left(\frac{t^-}{t^+}\right)^{7-q}\right)^{\frac{3-q}{2}}. \]

(6)

For a magnetically charged q-brane one has an \( 8-q \) form field strength,

\[ F = Q\epsilon_{8-q} \]

(7)

where \( \epsilon_{8-q} \) is the unit volume form of a \( 8-q \) sphere and the value for \( Q \) is

\[ Q = \frac{1}{2} \sqrt{(7-q)^2 (t^+ t^-)^{7-q}}. \]

(8)

Now we consider the region near the inner horizon, \( t = t^- + \varepsilon \),

\[ 1 - \left(\frac{t^-}{t^+}\right)^{7-q} = (7-q)\frac{\varepsilon}{t^-} + O(\varepsilon^2). \]

(9)

In this region the metric is approximated by

\[
\begin{align*}
\text{ds}^2 &\approx \left(\frac{(t^+)}{(t^-)}\right)^{7-q} \sqrt{\frac{t^-}{(7-q)\varepsilon}} dy^2 - \frac{t^2 \frac{1}{2} \left(\frac{t^+}{t^-}\right)^{\frac{7-q}{2}}} {\sqrt{\left(\frac{t^+}{t^-}\right)^{7-q} - 1} \left(\frac{(7-q)}{\varepsilon}\right)^{\frac{1}{2} \left(\frac{7-q}{2}\right)}} \\
&\quad + t^2 \left(\frac{(7-q)}{\varepsilon}\right)^{\frac{1}{2} \left(\frac{7-q}{2}\right)} d\Omega_{8-q}^2 + \sqrt{(7-q) \frac{\varepsilon}{t^-}} dx^i dx^i.
\end{align*}
\]

(10)

After compactifying the \( x^i \) and \( y \) directions (spanning the Euclidean world-volume of the brane) on circles, T-duality can be performed along the \( x^i \) directions or along the \( y \) direction. The \( g_{tt} \) component of the metric is not affected by these T-dualities and will always behave like

\[ g_{tt} \sim \varepsilon^{-\frac{1}{2} \left(\frac{7-q}{2}\right)}. \]

(11)

\[ ^4 \text{For } q = 3 \text{ one replaces } F \text{ by } F + *F \text{ in order to have a self-dual field strength} \].

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The other metric components show generically some power-like behavior with respect to \( \varepsilon \). In order to get exponential expansion (or contraction) in a proper time frame the time component of the metric should go like \( \varepsilon^{-2} \). To find inflation for a \( p \)-brane probe we need thus a dilaton behaving like (the tilde on \( \tilde{\phi} \) indicates that we allow for T-duality transformations)

\[
e^{-2\tilde{\phi}} \sim \varepsilon^{\left(-\frac{3}{2} + \frac{5-q}{7-q}\right)(p+1)},
\]

where \( p + 1 \) is the world-volume dimension of the probe. Using the T-duality relation

\[
\tilde{\phi} = \phi + \frac{1}{4} \log \frac{\tilde{g}}{g}
\]

we observe that performing T-duality with respect to \( n \) of the \( x^i \) \( (n = 0, \ldots, q) \) and \( m \) times with respect to \( y \) \( (m = 0, 1) \) will lead to the following expression

\[
e^{-2\tilde{\phi}} \sim \varepsilon^{\frac{3q-2}{2} + \frac{5-q}{7-q}}.
\]

Comparing (12) with (14) we arrive at the condition

\[
\frac{3 - q}{2} + \frac{n}{2} - \frac{m}{2} = (p + 1) \left( -\frac{3}{2} + \frac{5 - q}{7 - q} \right).
\]

Within the allowed parameter regions there is only one solution to (15), namely \( q = 5, p = 0, m = 1, n = 0 \).

So, we have to start with the D5-brane background. By T-dualizing the \( y \) direction this will turn into a D4-brane (with world-volume along \( x^i, i = 1, \ldots, 5 \)). Probing this D4-brane background with D0-branes will result in measuring exponentially growing scale factors near the singularity at the inner horizon. (Later we will see that in order to get expansion and not contraction we have to reverse the time direction. Then the singularity at the inner horizon corresponds to a final singularity which will occur in the infinite future of a proper time.) It is quite surprising that we find exactly one possibility to obtain inflation within the given framework. Fortunately, the background and the probes are both objects of type IIA theory.

### 3 Inflationary string cosmology from the D5-brane

In the previous section we gave general arguments that starting with a (continued) D5-brane solution, performing T-duality along the \( y \) direction and moving to the D0-brane frame will...
give inflationary cosmology near the singularity at \( t = t_- \). Here, we will repeat the previous discussion for the D5-brane and analyze the result. We start with the solution

\[
 ds^2 = \frac{((t_+/t)^2 - 1)}{\sqrt{1 - (t_-/t)^2}} dy^2 - \frac{dt^2}{((t_+/t)^2 - 1)} \left( 1 - (t_-/t)^2 \right) \]

\[
 + t^2 \left( 1 - (t_-/t)^2 \right)^{\frac{1}{2}} d\Omega_3^2 + \sqrt{1 - (t_-/t)^2} dx^i dx^j, 
\]

\[ (17) \]

\[
 e^{-2\phi} = \left( 1 - \left( \frac{t_-}{t} \right)^2 \right)^{-1}. 
\]

The three-form RR-field strength is

\[
 F = Q \epsilon_3 
\]

where \( \epsilon_3 \) is the unit volume form of a three-sphere and

\[
 Q = t_+ t_- . 
\]

T-dualizing along the \( y \) direction yields

\[
 ds^2 = \frac{1}{((t_+/t)^2 - 1)} \sqrt{1 - (t_-/t)^2} dy^2 - \frac{dt^2}{((t_+/t)^2 - 1)} \left( 1 - (t_-/t)^2 \right) \]

\[
 + t^2 \left( 1 - (t_-/t)^2 \right)^{\frac{1}{2}} d\Omega_3^2 + \sqrt{1 - (t_-/t)^2} dx^i dx^j, 
\]

\[ (21) \]

\[
 e^{-2\phi} = \left( 1 - \left( \frac{t_-}{t} \right)^2 \right)^{-\frac{1}{2}} \left( \left( \frac{t_+}{t} \right)^2 - 1 \right). 
\]

and a four form field strength (whose gauge field couples magnetically to the D4-brane),

\[
 F \sim \epsilon_3 \wedge dy. 
\]

Finally we move to the D0-brane frame by replacing \( ds^2 \rightarrow ds_0^2 = e^{-2\phi} ds^2 \),

\[
 ds_0^2 = \frac{1}{1 - (t_+/t)^2} dy^2 - \frac{dt^2}{(1 - (t_+/t)^2)^2} 
\]

\[
 + \frac{t^2 \left( (t_+/t)^2 - 1 \right)}{1 - (t_-/t)^2} d\Omega_3^2 + \frac{\left( t_+/t \right)^2 - 1}{1 - (t_-/t)^2} dx^i dx^j. 
\]

\[ (24) \]
In order to discuss the characteristics of (24) we transform to the proper time coordinate defined via
\[ d\tau = \pm \frac{dt}{1 - \left(\frac{t}{\tau}\right)^2}, \] which is solved by
\[ \pm \tau = t + \frac{t_-}{2} \log \frac{t - t_-}{t + t_-} + \text{constant}. \] Unfortunately we cannot obtain an analytic expression for \( t \) as a function of \( \tau \) and therefore we will just give a qualitative discussion. We reverse the time direction by choosing the lower sign in (26) and take the initial condition such that
\[ t(\tau_0) = t_+. \] Now the big bang is at \( t = t_+ \) where the scale factor in front of \( dx^i dx^i \) and \( d\Omega_3^2 \) vanishes. Near the initial value \( \tau_0 \), \( t \) depends linearly on \( \tau \). As we approach \( t_- \) all the scale factors grow exponentially with \( \tau \) and diverge at \( t(\tau = \infty) = t_- \). To illustrate this we give the approximative metric near \( t = t_- \). In that region we can solve for \( t(\tau) \) by
\[ t(\tau) \approx t_- + ct_- e^{-\frac{\tau}{t_-}}, \] with \( c \) being some integration constant depending on the initial condition (27). Then the metric becomes
\[ ds^2 \approx -d\tau^2 + \frac{e^{\frac{2\pi}{2c}}}{2c} \left\{ dy^2 + \left( \frac{t_+}{t_-} \right)^2 - 1 \right\} \left[ t_-^2 d\Omega_3^2 + dx^i dx^i \right]. \] The dilaton behaves like
\[ e^{-2\phi} \approx \left( \frac{t_+}{t_-} \right)^2 - 1 \right\} (2c)^{-\frac{3}{2}} e^\frac{3\pi}{t_-}, \] and large \( \tau \) is seen to correspond to weak coupling. However, for large \( \tau \) the curvature increases and \( \alpha' \) corrections will become important. (Note that we are dealing with non-BPS states.) To summarize, we have found a cosmological solution which after some power-like expansion enters an inflationary phase with exponential expansion, when probed with D0-branes. For large proper time the curvature will blow up and \( \alpha' \) corrections will be important. In the presented solution all the nine spatial directions become exponentially large.

4 Conclusions

In this note we used the fact that a black D-brane corresponds to a cosmological solution when continued to the region between the inner and the outer horizon. From these cosmological solutions new solutions can be generated via T-duality. Then we searched this class
of cosmological solutions for a metric describing exponential expansion when probed with D-branes. Surprisingly we found exactly one solution. This is a D5-brane T-dualized to a D4-brane. When this background is probed with D0-branes, the D0-brane like observer will see a universe which enters an exponential expansion after some time. Since all nine space directions expand exponentially the universe is truly ten dimensional.

One may be tempted to use the presented solution in some kind of a double-pre big bang scenario in order to achieve an exponentially large world radius at the onset of the usual pre-big-bang scenario. However, first of all there will be problems because one has to introduce a second graceful exit from the exponential expansion to the inflationary expansion of the PBB scenario. This may be merely a technical question. But even if one was able to solve for the graceful exit one would have just reversed the problem raised in [16]. Now, we would need an exponential contraction in order to get the non-observable six space dimensions down to string scale. Before interpreting this result as a no-go theorem for phenomenologically interesting string cosmology one should note that we only considered a special class of cosmological string vacua. Extending the scan for exponential expansion to more general solutions one might be more lucky in finding some model of phenomenological relevance. As a step towards that direction one may consider for example intersecting black branes [19] as a starting point.

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