Spin-electron-acoustic waves and solitons in high-density degenerate relativistic plasmas

Pavel A. Andreev

Department of General Physics, Faculty of physics, Lomonosov Moscow State University, Moscow, Russian Federation, 119991.
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The spin-electron-acoustic waves (sometimes called the spin-plasmons) can be found in degenerate electron gas if the spin-up electrons and spin down electrons move relatively each other. Here, we suggest relativistic hydrodynamics with the separate spin evolution which allows us to study linear and nonlinear spin-electron-acoustic waves, including the spin-electron-acoustic solitons. Presented hydrodynamic model is the corresponding generalization of the relativistic hydrodynamic model with the average reverse gamma factor evolution which consists of the equations for evolution of the following functions the partial concentrations (for spin-up electrons and spin down electrons), the partial velocity fields, the partial average reverse relativistic gamma factors, and the partial flux of the reverse relativistic gamma factors. We find that the relativistic effects decreases the phase velocity of spin-electron-acoustic waves. Numerical analysis of the changes of spectra of Langmuir wave, spin-electron-acoustic wave, and ion-acoustic wave under the change of the spin polarization of electrons is presented. It is demonstrated that spectra of Langmuir wave and spin-electron-acoustic wave getting closer to each other in the relativistic limit. Spin dependence of the amplitude and width of the relativistic spin-electron-acoustic soliton is demonstrated as well. Reformation of the bright soliton of potential of the electric field into the dark soliton under the influence of the relativistic effects is found.

Keywords: relativistic plasmas, hydrodynamics, degenerate electrons, spin polarization, separate spin evolution.

I. INTRODUCTION

The spin-electron-acoustic waves (SEAWs) in degenerate plasmas are suggested in 2015-2016 for the three-dimensional, two-dimensional, near surface geometries of electron-ion, electron-positron, and electron-positron-ion plasmas [1], [2], [3], [4], [5]. It it found together with the original model called the separate spin evolution quantum hydrodynamic (SSE-QHD) [1], which is found from the Pauli equation being combination of two equations. Therefore, the spin-up electrons and spin-down electrons are considered as two different fluids (species or subsystems). A number of spincaused effects are demonstrated in the SSE-QHD equations including the Euler and continuity equations which are discussed in Refs. [6], [7], [8], [9], [10], [11]. The separate spin evolution kinetic is developed and used as well [12], [13], [14], [15]. Study of the separate spin evolution is part of the field of quantum plasmas developed in the large number of works including [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], where the complete quantum hydrodynamic model derivation based on the trace of the microscopic motion of quantum particles [16], [17], [22], [32], [33], [34]. However, the SEAWs are related to relatively simple quasi-classic manifestation of the spin effects in the Euler equation. It is the difference of the partial pressures entering the Euler equations spin-up and spin-down electrons. It opens up a possibility for the partial generalization of the SSE-QHD, i.e. the generalization of the quasi-classic limit of the SSE-QHD. This kind of partial generalization is made in this paper. Here, we present the quasi-classic relativistic SSE-QHD. This model based on the transition of basic features quasi-classic SSE-QHD up on to classic relativistic hydrodynamic model with the average reverse gamma factor evolution, which is recently developed for the degenerate electron gas [35]. The degenerate relativistic hydrodynamic model with the average reverse gamma factor evolution [35] is the result of generalization of classic relativistic hydrodynamic model with the average reverse gamma factor evolution of the relativistically hot plasmas [36], [37], [38]. This model is applied to analysis of spectra of Langmuir, and electromagnetic waves in magnetized relativistic plasmas including the low frequency dynamics governed by ions [39], [40], [41]. The derivation of the relativistic hydrodynamic model is based on the original method developed in Refs. [42], [43], [44], where no probabilistic or statistical assumption is made. While the derivation is completely based on deterministic classical mechanics.

In order to describe the SSE in the nonrelativistic regime we present the equations for SSE-QHD following Ref. [1]

$$\partial_t n^{\uparrow}/\downarrow + \nabla (n^{\uparrow}/\downarrow \mathbf{v}^{\uparrow}/\downarrow) = \pm \frac{\mu}{\hbar} (S_x B_y - S_y B_x),$$

(1)

where $n^{\uparrow}$ ($n^{\downarrow}$) is the concentration of electrons located in the spin-up (spin-down) state, $\mathbf{v}^{\uparrow}$ ($\mathbf{v}^{\downarrow}$) is the velocity field of electrons located spin-up (spin-down). The right-hand side of the continuity equation contains functions $S_x$ and $S_y$, which are projections of the spin density vector, $\mu$ is the magnetic moment. The concentration of all electrons

*Electronic address: andreepa@physics.msu.ru*
The transverse projections of spin density evolve in accordance with the following pair of equations

\[ \partial_t S_{\uparrow/\downarrow} + \frac{1}{2} \nabla \left[ S_{\uparrow/\downarrow} \left( \mathbf{v}_\uparrow + \mathbf{v}_\downarrow \right) \right] \]

\[ - \frac{\hbar}{4m} \nabla \left[ \left| \mathbf{S} \times \mathbf{e}_z \right| \right]_{\uparrow/\downarrow} \left( \frac{\nabla n_\uparrow}{n_\uparrow} - \frac{\nabla n_\downarrow}{n_\downarrow} \right) = T_{x/y}, \quad (2) \]

where we have no spin-up and spin-down indexes for the spin density functions, and the following expressions for the spin torque are applied

\[ T_x = \frac{2\mu}{m^2} (B_x S_y - B_y (n_u - n_d)) \]

\[ T_y = \frac{2\mu}{m} (B_x (n_u - n_d) - B_z S_x) \]

Finally, we include the pair of vector equations for the partial velocity fields

\[ \begin{align*}
\left( \partial_t + \frac{\mathbf{v}_\uparrow + \mathbf{v}_\downarrow}{2} \nabla \right) \mathbf{v}_\uparrow/\downarrow - \frac{\hbar^2}{4m} n_{\uparrow/\downarrow} \nabla \left( \frac{n_{\uparrow/\downarrow}}{n_{\uparrow/\downarrow}} - \frac{\left( \nabla n_{\uparrow/\downarrow} \right)^2}{2n_{\uparrow/\downarrow}^2} \right) \\
+ \nabla p_{\uparrow/\downarrow} = q_n n_{\uparrow/\downarrow} \left( \mathbf{E} + \frac{1}{c} [\mathbf{v}_\uparrow/\downarrow, \mathbf{B}] \right) \\
\end{align*} \]

\[ + m \frac{\mu}{\hbar} (J_{(x)} B_y - J_{(y)} B_x) - m \mathbf{v}_\uparrow/\downarrow \cdot \frac{\mu}{\hbar} (S_x B_y - S_y B_x) \]

\[ + \frac{\mu}{2} (S_x \nabla B_x + S_y \nabla B_y) \pm \mu n_{\uparrow/\downarrow} \nabla B_z, \quad (3) \]

where

\[ J_{(x)} = \frac{1}{2} (\mathbf{v}_\uparrow + \mathbf{v}_\downarrow) S_{x/y} - \frac{\hbar}{4m} \left( \frac{\nabla n_\uparrow}{n_\uparrow} - \frac{\nabla n_\downarrow}{n_\downarrow} \right) [S_y \times \mathbf{e}_z]_{x/y} \quad (4) \]

is the spin current.

The right-hand side of the continuity equations is related to the spin flip. The second and third group of terms on the right-hand side of the Euler equation (they are simultaneously proportional to the magnetic moment \( \mu \) and magnetic field \( \mathbf{B} \) with no operators acting on the magnetic moment) are also related to the spin flip. This effect is not involved in the quasi-classic relativistic model presented below. Same is true for the force field acting from nonuniform magnetic field on the spins. Moreover, the spin precession is neglected below as well.

It would be better to create background of our model being based on the Dirac equation. However, the quantum hydrodynamic model derived for single electron from the Dirac equation is rather complex (see Ref. 45 for the modern description of this model). It covers a number of physical effects. The Dirac equation is developed for the single-electron dynamics \( \gamma^\mu (i\hbar \partial_\mu - q A_\mu / c) \psi - mc \psi = 0 \) which describes the evolution of four component wave bi-spinor function \( \psi \), where we also have \( \gamma^\mu \) are the four by four Dirac matrices, \( A^\mu \) is the electromagnetic four-potential, and \( \mu = 0, 1, 2, 3 \).

The spin-electron-acoustic waves are theoretically demonstrated in degenerate plasmas after derivation of the SSE-QHD equations [1] and the derivation of kinetic equations with separate spin evolution [12], however the waves of the same nature called the spin-plasmons are suggested in the condensed matter physics for the two-dimensional electron gas and electron in graphene [46], [47], [48], [49].

It is also essential to mention that developed here model of relativistic degenerate electron gas is novel look on the structure of the relativistic hydrodynamic equations which are earlier developed for classical [50], [51], [52], [53], [54], [55], [56], [57], [58], [59], [60], [61], and quantum plasmas [29], [45], [62], [63], [64], [65].

This paper is organized as follows. In Sec. II the relativistic hydrodynamic model with the average reverse gamma factor evolution is adopted for the separate spin evolution in systems of degenerate partially spin polarized electrons. In Sec. III single fluid reduction of the relativistic separate spin evolution hydrodynamics. In Sec. IV the linear and nonlinear analysis of the small amplitude spin-electron-acoustic waves is given. In Sec. V a brief summary of obtained results is presented.

II. TWO FLUID RELATIVISTIC HYDRODYNAMIC MODEL WITH THE AVERAGE REVERSE GAMMA FACTOR EVOLUTION

In this paper we present the separate spin evolution relativistic hydrodynamics as the two fluid hydrodynamic model with the average reverse gamma factor evolution. This model is based on generalization of models developed in Refs. [1], [32], [33], [34].

We start presentation of the suggested separate spin evolution model with the continuity equation

\[ \partial_t n_s + \nabla \cdot (n_s \mathbf{v}_s) = 0, \quad (5) \]

which has rather traditional form, it is also corresponds to Refs. [32], [33], [34]. However, we consider the separate spin evolution, so we should present comparison with the separate spin evolution quantum hydrodynamics obtained from the Pauli equation [1], [2]. It shows that the continuity equations for the partial concentrations \( n_\uparrow \) and \( n_\downarrow \) have non zero right-hand side. The spin polarization of electrons changes under action of the magnetic field. It corresponds to the change of difference of partial concentrations \( n_\uparrow \) and \( n_\downarrow \) and corresponding change of each of them caused by the torque created by magnetic field and acting on the magnetic moments of electrons. More exactly, we have the z-projection of the torque while the z-direction is the chosen direction and we consider the spin-projections relatively this direction. The z-projection of the torque appears to be nonlinear, so we have no restrictions at the analysis of the linear effects. The nonlinear analysis of the longitudinal perturbations has also zero contribution of the torque. Therefore, the model under...
Greek indexes are deposited for the four-vector notations.

The second equation of evolution is the Euler equation is given which is the velocity field evolution equation

\[ n_s \partial_t v_s + n_s (v_s \cdot \nabla) v_s + \frac{1}{m_s} \nabla \tilde{p}_s \]

\[ = \frac{q_s}{m_s} \left( \Gamma_s - \frac{\tilde{t}_s}{c^2} \right) E + \frac{q_s}{m_s c} \left[ (\Gamma_s v_s + t_s) \times B \right] \]

\[ - \frac{q_s}{m_s c^2} \left( \Gamma_s v_s (v_s \cdot E) + v_s (t_s \cdot E) + t_s (v_s \cdot E) \right), \quad (6) \]

where \( m_s \) and \( q_s \) are the mass and charge of particle of \( s \) species, \( c \) is the speed of light, tensor \( p_s^{ab} = \tilde{p}_s^{\delta \delta} \) is the flux of the velocities for electrons with fixed spin projection, tensor \( t_s^{ab} = \tilde{t}_s^{\delta \delta} \) is the flux of the average reverse gamma-factor for spin-\( s \) electrons, \( \delta \) is the three-dimensional Kronecker symbol. Moreover, we work in the Minkovskii space, hence the metric tensor has diagonal form with the following sings \( g^{\alpha \beta} = \{ -1, +1, +1, +1 \} \). We mostly use the three dimensional notations, therefore, we can change covariant and contravariant indexes for the three-vector indexes: \( v_s^a = v_{sa} \). The Latin indexes like \( a, b, c \) etc describe the three-vectors, while the Greek indexes are deposited for the four-vector notations. We also have Latin index \( s \) which refers to the species or subspecies of electrons with different spin projections. However, the indexes related to coordinates are chosen from the beginning of the alphabet, while other indexes are chosen in accordance with their physical meaning. The model under presentation and this Euler equation includes no effects related to change of the spin projections of electrons on the chosen direction.

The equation of evolution of the averaged reverse relativistic gamma factor for electrons with fixed spin projection in absence of the spin-flip effects has the following form

\[ \partial_t \Gamma_s + \nabla (\Gamma_s v_s + t_s) \]

\[ = - \frac{q_s}{m_s c^2} n_s (v_s \cdot E) \left( 1 - \frac{1}{c^2} \left( v_s^2 + 5 \tilde{p}_s \right) \right). \quad (7) \]

Function \( \Gamma_s \) is the hydrodynamic Gamma function considered here for electrons with particular spin projection.

The model also includes the equation of evolution for the current of the reverse relativistic gamma factor (the hydrodynamic Theta function) presented for electrons with fixed spin projection:

\[ (\partial_t + v_s \cdot \nabla) t_s^a + \nabla t_s^a + (t_s \cdot \nabla) v_s^a + t_s^a (\nabla \cdot v_s) \]

\[ + \Gamma_s (\partial_t + v_s \cdot \nabla) v_s^a = \frac{q_s}{m_s} n_s E^a \left[ 1 - \frac{v_s^2}{c^2} - 3 \tilde{p}_s \right] \]

The hydrodynamic equations \( (5), (6), (7) \) and \( (8) \) are obtained in the mean-field approximation (the self-consistent field approximation). The fourth rank tensor \( M^{abcd} \) existing in equation \( (8) \) is presented via its partial trace \( M^{abcc} = M^{abc} \). This tensor is constructed via the Kronecker symbols \( M^{abcd} = (M_{st} / 3)(\delta^{ab} \delta^{cd} + \delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc}) \). So we have the following elements of this tensor: \( M^{xxxx} = M^{yyyy} = M^{zzzz} = M_{st} \) and \( M^{xyxy} = M^{zzxx} = M_{0} / 3 \). Otherwise the element of tensor \( M^{abcd} \) is equal to zero. Required partial trace \( M^{abc} \) has the following form \( M^{abc} = (5 M_{st} / 3) \delta^{ab} \).

The equations of electromagnetic field have the traditional form presented in the three-dimensional notations \( \nabla \cdot B = 0 \),

\[ \nabla \times E = - \frac{1}{c} \partial_t B, \quad (9) \]

\[ \nabla \cdot E = 4 \pi (q_e n_{e\uparrow} + q_e n_{e\downarrow} + q_i n_i), \quad (10) \]

and

\[ \nabla \times B = \frac{1}{c} (\partial_t E + 4 \pi q_e n_{e\uparrow} v_{e\uparrow} + 4 \pi q_e n_{e\downarrow} v_{e\downarrow} + 4 \pi q_i n_i v_i), \quad (11) \]

where the ions exist as the motionless background.

A. Equations of state for spin-up and spin-down electrons

Equations \( (5)-(8) \) are derived for the relativistically hot plasmas \( (36) \). Next, it is adopted for the degenerate electron gas \( (37) \). Our goal here is further generalization of the hydrodynamic model with the average reverse gamma factor evolution to consider SEAW \( (1), (3) \), so we need to include the separate spin evolution to this model. We consider the degenerate electron gas of high concentration, hence we find the Fermi velocity \( v_{FS} = p_{FS} / \sqrt{1 + p_{FS}^2 / m_s^2 c^2} \), where \( p_{FS} = (6 \pi^2 n_s)^{1/3} \).

For the analysis of the high density relativistic degenerate electron gas subspecies with the chosen spin projection we derive the equations of state for functions \( p_s^{ab}, t_s^{ab} \), and \( M^{abcd} \).

Degenerate electrons with the fixed spin projection are described within the zero-temperature limit of the Fermi-Dirac distribution, which is given by the Fermi step distribution

\[ f_{s0} = \begin{cases} \frac{1}{(2 \pi)^{3/2}} & \text{for } p \leq p_{FS} \\ 0 & \text{for } p \geq p_{FS} \end{cases} \quad (12) \]
The concentration has well-known form in terms of the distribution function
\[ n_s = \int f_s \delta d^3 p. \]  
(13)

where \( p = m_s \sqrt{1 - v^2 / c^2} \). The flux of the current of particles, which has the following representation in form of the distribution function
\[ \tilde{p}_s^{ab} = \int v^a v^b f_s \delta d^3 p. \]  
(14)

We substitute distribution function (12) go calculate the equation of state \( p_s^{ab} = \tilde{p}_s \delta^{ab} \) with
\[ \tilde{p}_s = \frac{m^3 e^5}{6\pi^2\hbar^3} \left[ \frac{1}{3}\xi_s^3 - \xi_s + \arctan \xi_s \right]. \]  
(15)

where \( \xi_s \equiv p_{Fs}/mc \). The flux of the current of the average reverse gamma factor can also be presented via the distribution function
\[ \tilde{t}_s^{ab} = \int \left( \frac{v^a v^b}{\gamma} \right) f_s \delta d^3 p. \]  
(16)

We obtain \( t_s^{ab} = \tilde{t}_s \delta^{ab} \), where
\[ \tilde{t}_s = \frac{m^3 e^5}{12\pi^2\hbar^3} \left[ \xi_s \sqrt{\xi_s^2 + 1} + \frac{2\xi_s}{\sqrt{\xi_s^2 + 1}} - 3\text{Arsinh} \xi_s \right], \]  
(17)

with \( \text{Arsinh} \xi = \ln | \xi + \sqrt{\xi^2 + 1} | \), and \( \text{sinh}(\text{Arsinh} \xi) = \xi \). The fourth rank tensor \( M_s^{abcd} \) is also calculated
\[ M_s^{abcd} = \int v^a v^b v^c v^d f_s \delta d^3 p, \]  
(18)

to get the following expression \( M_s^{abcd} = (M_{s0}/3)(\delta^{ab} \delta^{cd} + \delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc}) \), where
\[ M_{s0} = \frac{m^3 e^7}{60\pi^2\hbar^3} \left[ 2\xi_s (\xi_s^2 - 6) - \frac{3\xi_s}{\xi_s^2 + 1} + 15\arctan \xi_s \right]. \]  
(19)

We also need to find the equilibrium expression for the partial average reverse gamma factor \( \Gamma_{0s} \) for the spin-

\[ \Gamma_s = \int \frac{1}{\gamma} f_s \delta d^3 p, \]  
(20)

where we find
\[ \Gamma_{0s} = \frac{m^3 e^3}{4\pi^2\hbar^3} \left[ \xi_s \sqrt{\xi_s^2 + 1} - \text{Arsinh} \xi_s \right]. \]  
(21)

These expressions leads to the closed set of hydrodynamic equations, which are applied below to the longitudinal



III. SINGLE FLUID REDUCTION OF SEPARATE SPIN EVOLUTION RELATIVISTIC HYDRODYNAMIC MODEL WITH THE AVERAGE REVERSE GAMMA FACTOR EVOLUTION

Spin polarized system of electrons can be described as the single fluid \([17], [18], [25]\), where the spin density exists among other well-known hydrodynamic functions like the concentration and the velocity field. Method of transition from the separate spin evolution quantum hydrodynamics to the single fluid quantum hydrodynamics of electrons is described in Ref. [2] for the nonrelativistic regime. Here we apply this method for the relativistic separate spin evolution quasi-classic hydrodynamics described above.

The superposition of partial concentrations \( n_s \) gives the full concentration of electrons: \( n = n_\uparrow + n_\downarrow \). The flux of electrons is the superposition of partial fluxes, so it gives us relation between velocities: \( v = \frac{n_\uparrow u_\uparrow + n_\downarrow u_\downarrow}{n_\uparrow + n_\downarrow} \). Superposition of the velocity evolution equations \([6]\) shows the structure of the full flux of particle current \( \Pi_{ab} \) constructed as the sum of spin-s fluxes of particle current \( \Pi_{sa} = n_\uparrow v_\uparrow v_s + p_{sa}; \Pi_{sb} = \Pi_{ab} + \Pi_{db}. \) It contains the contribution of flux on the difference of partial velocity fields: \( \Pi_{ab} = n_\uparrow v_\uparrow v_s + p_{sa} + \frac{m^3 e^5}{n_\uparrow + n_\downarrow} (v^a_\uparrow - v^a_\downarrow)(v^b_\uparrow - v^b_\downarrow) \). The last term gives transition of center of mass of electrons at the relative motion of the spin-up and spin-down electrons. Hence, if the difference of velocity fields for the spin-up and spin-down electrons is large we cannot make transition to the single fluid dynamics. However, if this difference is small in compare with the velocity field \( v \) we can consider the single fluid hydrodynamics of the spin polarized degenerate electrons.

The average reverse gamma factor is the sum of the partial functions: \( \Gamma = \Gamma_\uparrow + \Gamma_\downarrow \). While the flux of average reverse gamma factor has additional contribution related to different partial velocity fields \( t = t_\uparrow + t_\downarrow + (n_\uparrow \Gamma_\uparrow - n_\downarrow \Gamma_\downarrow)(v^a_\uparrow - v^a_\downarrow) \).

The effects of nonzero spin polarization in the single fluid can be described by the following parameter \( \eta = \frac{n_\uparrow - n_\downarrow}{(n_\uparrow + n_\downarrow)} \). We use to present the equations of state for the high-rank tensors entering the single fluid relativistic hydrodynamics of the spin-polarized degenerate electrons.

The flux of current of particles is found for the single fluid regime using expression \([13]\)
\[ \tilde{p}_\uparrow + \tilde{p}_\downarrow = \frac{m^3 e^5}{6\pi^2\hbar^3} \left[ \frac{2}{3}\xi^3 - \left( (1 + \eta)^{1/3} + (1 - \eta)^{1/3} \right) \xi \right. \]
\[ + \arctan (1 + \eta)^{1/3} \xi + \arctan (1 - \eta)^{1/3} \xi \],
(22)

where the first term on the right hand-side \( \xi^3 \) has no spin dependent coefficient except coefficient 2 appearing from superposition of \( (1 + \eta) \) and \( (1 - \eta) \), and \( \xi \equiv p_{Fe}/mc \), with the Fermi momentum \( p_{Fe} = (3\pi^2 n_0)^{1/3} \).
The single fluid flux of the current of the average reverse gamma factor at the nonzero spin polarization appears as the superposition of partial functions given by equation (23)

\[ \tilde{\mathbf{i}} = \tilde{\mathbf{i}}_\uparrow + \tilde{\mathbf{i}}_\downarrow = \frac{m^3 c^5}{12 \pi^2 \hbar^4} \times \]

\[ \left[ \frac{2(1 + \eta)^{1/3} \xi}{\sqrt{(1 + \eta)^{2/3} \xi^2 + 1}} + \frac{2(1 - \eta)^{1/3} \xi}{\sqrt{(1 - \eta)^{2/3} \xi^2 + 1}} ight. \]

\[ + (1 + \eta)^{1/3} \xi \sqrt{(1 + \eta)^{2/3} \xi^2 + 1} \]

\[ + (1 - \eta)^{1/3} \xi \sqrt{(1 - \eta)^{2/3} \xi^2 + 1} \]

\[ -3 \text{Arsh}(1 + \eta)^{1/3} \xi - 3 \text{Arsh}(1 - \eta)^{1/3} \xi \].

Equation (23) is the generalization of the expression found in Ref. [3] up to the account of nonzero spin polarization.

Same generalization is made for the function \( M_0 \):

\[ M_0 = M_{\uparrow 0} + M_{\downarrow 0} = \frac{m^3 c^7}{60 \pi^2 \hbar^4} \times \]

\[ \left[ 15 \text{arctan}(1 + \eta)^{1/3} \xi + 15 \text{arctan}(1 - \eta)^{1/3} \xi \right. \]

\[ + 2(1 + \eta)^{1/3} \xi [(1 + \eta)^{2/3} \xi^2 - 6] \]

\[ + 2(1 - \eta)^{1/3} \xi [(1 - \eta)^{2/3} \xi^2 - 6] \]

\[ - \frac{3(1 + \eta)^{1/3} \xi}{(1 + \eta)^{2/3} \xi^2 + 1} \]

\[ - \frac{3(1 - \eta)^{1/3} \xi}{(1 - \eta)^{2/3} \xi^2 + 1} \].

Finally, we find expression for the equilibrium average reverse gamma factor at the nonzero spin polarization

\[ \Gamma_0 = \Gamma_{\uparrow 0} + \Gamma_{\downarrow 0} = \frac{m^3 c^3}{4 \pi^2 \hbar^3} \times \]

\[ \left[ (1 + \eta)^{1/3} \xi \sqrt{(1 + \eta)^{2/3} \xi^2 + 1} \right. \]

\[ + (1 - \eta)^{1/3} \xi \sqrt{(1 - \eta)^{2/3} \xi^2 + 1} \]

\[ - \text{Arsh}(1 + \eta)^{1/3} \xi + \text{Arsh}(1 - \eta)^{1/3} \xi \].

Results presented in this section allows to partially consider the effects of the spin polarization in terms of single fluid hydrodynamic model. However, the spin-electron-acoustic discussed below can not appear in the single fluid model of electrons.

IV. SPIN-ELECTRON-ACOUSTIC WAVES IN THE RELATIVISTIC MAGNETIZED PARTIALLY SPIN POLARIZED PLASMAS

A. Linearized equations for relativistic separate spin evolution hydrodynamics

We focus on degenerate electron-ion plasmas, where both components are degenerate. Moreover, the concentrations of both components \( n_{oe} = n_o \) are equal. Nevertheless, the concentration is large enough, so the Fermi velocity \( v_F \) getting close to the speed of light \( c \).

All species are assumed to be macroscopically motionless in the equilibrium state \( v_{0\uparrow} = v_{0\downarrow} = v_0 = 0 \). The macroscopic equilibrium electric field is equal to zero either. Wave propagate parallel to the constant uniform external magnetic field, so effects except the spin polarization are caused by the magnetic field. The spin polarization manifests itself in the equilibrium in the difference of the partial concentrations of electrons \( n_{0\uparrow} \neq n_{0\downarrow} \).

We consider small amplitude plane wave perturbations of the equilibrium state. Two linearized continuity equation are found for spin-up and spin-down electrons

\[ \partial_t \delta n_s + n_o \partial_x \delta v_{xs} = 0, \tag{26} \]

where \( s = \uparrow, \downarrow \), while ions are assumed to be motionless, so we do not present equations of motion for ions. Below we include the motion of ions. To include the motion of ions we assume that parameter \( s \) has three values \( \uparrow \) for the spin-up electrons, \( \downarrow \) for the spin-down electrons, and \( i \) for ions. So, we do not need to repeat the set of linearized hydrodynamic equations for the second regimes of mobile ions.

The linearized equations for the evolution of the partial velocity fields appear from equation (6)

\[ n_o \partial_t \delta v_{xs} + \delta \delta n_s = \frac{q_s}{m_s} \Gamma_{0s} \delta E_x - \frac{q_s}{m_s c^2} \tilde{E}_0 \delta E_x, \tag{27} \]

where parameters \( \Gamma_{0s} \), \( \delta \delta n_s \), and \( \tilde{E}_0 \) appear from equations of state presented above (21), (15), and (17), correspondingly.

The third and fourth pairs of equations in the set of relativistic linearized equations are obtained for evolution of \( \delta \Gamma \) and \( \delta t \). They appear from equations (7) and (8). They have the following form

\[ \partial_t \delta \Gamma_s + \Gamma_0 \delta \partial_x \delta v_{xs} + \partial_x \delta t_{xs} = 0, \tag{28} \]

and

\[ \partial_t \delta t_{xs} + \partial_x \delta \tilde{t}_s - \frac{\Gamma_{0s}}{n_{os}} \partial_x \delta \tilde{p}_s + \frac{q_s \Gamma_{0s}^2}{m_s n_{os}} \delta E_x = \frac{q_s}{m_s} n_{0s} \delta E_x - \frac{5q_s}{m_s c^2} \tilde{E}_0 \delta E_x + \frac{10q_s}{3m_s c^2} M_{0s} \delta E_x, \tag{29} \]

where \( M_{0s}^{Zee} = (5/3) M_{0s} \). However, equations (25) and (29) do not give any contribution in the spectra of longitudinal waves propagating parallel to the external magnetic field.
FIG. 1: The spectra for the Langmuir wave (dashed blue line), and SEAW (continuous green line) are demonstrated via plotting of the dimensionless square of frequency $\xi = \omega/\omega_{Le}$ as the function of the dimensionless wave vector $\kappa = kc/\sqrt{3}\omega_{Le}$. Figures are made for chosen spin polarization $\eta = 0.1$, but for different concentrations of electrons. The concentrations are presented via dimensionless parameter $a = (3\pi^2 n_0e^2)\bar{\hbar}/m_ec$.

All figures of spectra contain the ion-acoustic wave spectrum. However, it is almost invisible red dotted line near zero on this scale. Horizontal line $\xi = 1$ corresponds to $\omega = \omega_{Le}$.

We need to consider one of the Maxwell equations for divergence of electric field since we study the longitudinal waves propagating parallel to the external magnetic field. The linearized Poisson equation has the well-known form

$$\partial_x \delta E_x = 4\pi (q_e \delta n_e + q_i \delta n_i).$$

(30)

FIG. 2: The spectra for the Langmuir wave, SEAW, and ion-acoustic wave (red dotted line) are demonstrated in the ultra-relativistic limit $a = 100$ at $\eta = 0.1$. The upper figure shows the spectra on the large scale. The lower figure shows the small frequency part of the spectra.

B. Spectra of the Langmuir waves, spin-electron-acoustic waves, and ion-acoustic waves

If we consider the relativistic Langmuir waves in the zero-spin-polarization single fluid model of degenerate electrons we find [35]

$$\omega^2 = \frac{\omega_{Le}^2}{\gamma_{Fe}^2} + \frac{1}{3} v_{Fe}^2 k_z^2,$$

(31)

where $\Gamma_0 = \frac{v_F^2}{c^2} = \frac{1}{\gamma_{Fe}^2}$, and $v_{Fe}^2 = c^2 \frac{p_{Fe}^2}{m_F e c^2} = \frac{p_{Fe}^2}{m_{Fe} c^2}$, with $p_{Fe} = (3\pi^2 n_0e)^{1/3}\hbar$, and $\gamma_{Fe} = 1/\sqrt{1 - v_{Fe}^2/c^2} = \sqrt{1 + p_{Fe}^2/m_{Fe}^2}$ is the standard relativistic gamma factor considered for the Fermi velocity.

Equations (29)-(30) allows us to find the dispersion equation for the high-frequency longitudinal waves

$$1 = \frac{\omega_L^2}{\gamma_F^2 \omega^2 - u_{ps}^2 k_z^2} + \frac{\omega_{L1}^2}{\gamma_{F1}^2 \omega^2 - u_{ps}^2 k_z^2},$$

(32)

where the characteristic velocities $u_{ps}$ have simple expressions via the partial Fermi velocities $u_{ps}^2 = v_{ps}^2, 1/3$.

For the intermediate frequency regime $v_{Fe}^2 k_z^2/3 \gg \omega^2 \gg v_{F1}\kappa_z^2/3$ considered in the long-wavelength limit $\kappa_z \to 0$ equation (32) simplifies to the spectrum of the
FIG. 3: The spectra for the Langmuir wave, and SEAW are presented for chosen spin polarization \( \eta = 0.9 \).

SEAWs

\[
\omega^2 = \frac{n_{0\uparrow} \gamma_{F\uparrow} \nu_{F\uparrow}^2 k_z^2 / 3}{n_{0\downarrow} \gamma_{F\downarrow}}
\]

or this expression can be represented in order to explicitly show the dependencies on the spin polarization and concentration of all electrons

\[
\omega^2 = \frac{1 - \eta}{(1 + \eta)^{1/3}} \frac{(3\pi^2)^{2/3} \hbar^2 n_{0\uparrow}^{2/3} \nu_{F\uparrow}^2 k_z^2}{3m_e^2 \sqrt{1 + (1 + \eta)^{2/3} (3\pi^2)^{2/3} n_{0\uparrow}^{2/3} \nu_{F\uparrow}^2 k_z^2 / m_e^2}},
\]

where we include \( \gamma_{F\uparrow} \approx 1 \) since \( c \sim \nu_{F\uparrow} \gg \nu_{F\downarrow} \). Presented assumptions also require \( 1 - \eta \ll 1 \), hence we can apply \( 1 + \eta \approx 2 \).

Numerical solution of equation (32) shows that conditions \( \nu_{F\uparrow}^2 k_z^2 / 3 \gg \omega^2 \gg \nu_{F\downarrow}^2 k_z^2 / 3 \) cannot be satisfied with the frequency \( \omega \) given by equations (33) or (34). Hence, we conclude that equations (33) or (34) give a rough analytical illustration of spectrum. Real frequency of SEAW is considerably higher than given by equations (33) or (34).

Approximate analysis of the SEAWs presented above is made as the analytical illustration of relativistic effects in the spectrum of the SEAWs. Complete analysis is made numerically with the account of the motion of ions. Therefore, the dispersion equation has the following form:

\[
1 = \frac{1}{\gamma_{F\uparrow} \omega^2 - u_{p\uparrow}^2 k_z^2}
+ \frac{1}{\gamma_{F\downarrow} \omega^2 - u_{p\downarrow}^2 k_z^2}
+ \frac{1}{\gamma_{Fi} \omega^2 - u_{pi}^2 k_z^2},
\]

where the three wave spectrum is presented, it includes the Langmuir wave, SEAW, and ion-acoustic wave.

Numerical analysis of dispersion equation (35) is presented in Figs. (1), (2), (3), and (4). We consider two regimes of the relatively large spin polarizations \( \eta = 0.1 \) and \( \eta = 0.9 \) to give distinctive illustration of the spin effects. Fig. (1) shows spectra for three different concentrations at the fixed spin polarization \( \eta = 0.1 \). We apply the dimensionless parameter \( a = (3\pi^2 n_{0e})^{1/3} \hbar / m_e c \)
which characterize the role of large concentration and the role of the relativistic effects. Value of $a$ above 1 corresponds to the noticeable relativistic effects. Concentrations are chosen to show the spectra in the weakly relativistic regime $a = 0.1$ (the upper figure in Fig. (1)), the considerable relativistic effects $a = 1$ (the middle figure in Fig. (1)), the strong relativistic effects $a = 10$ (the lower figure in Fig. (1)). In the weakly relativistic regime $a = 0.1$ we find that the SEAW is a low frequency wave, but its frequency is well above the frequency of the ion-acoustic wave. The Langmuir wave in this regime has the well-known non-relativistic form, but in small $k$ limit its frequency becomes smaller then $\omega_{Le}$. It is the manifestation of small relativistic effects. Increase of concentration leads to the increase of the Fermi velocity, hence the phase velocities of waves increase either. We see this effect in the middle and lower figures in Fig. (2). Moreover, the increase of the relativistic effects gives the decrease of minimal frequency of the Langmuir wave. Same effect is found for the relativistically hot plasmas (36). Further increase of concentration leads to the ultra relativistic regime demonstrated in Fig. (2). Simultaneous increase of the relativistic effects and phase velocity of waves leads to rapprochement of the dispersion dependencies of the Langmuir wave and SEAW. However, on the smaller scale their frequencies have considerable difference (see the lower figure in Fig. (2)). Larger spin effects $\gamma = 0.9$ leads to larger differences of frequencies of the Langmuir wave and SEAW (see Figs. (3) and Fig. (4)). The phase velocity of waves decreases under influence of the relativistic effects at the fixed concentration. However, we change the concentration during the numerical analysis. Therefore, the increase of phase velocity at the increase of concentration hide this effect in figures.

C. Small amplitude spin-electron-acoustic soliton

In this section we consider the nonlinear small amplitude dynamics of the spin-electron-acoustic waves, which leads to the spin-electron-acoustic solitons in the high-density low-temperature electron-ion plasmas. Technically this analysis shows similarity to the model of the ion-acoustic solitons described in Ref. [35]. However, the final equations give different physical picture due to the account of the separate spin evolution of electrons.

In order to study the SEAWs we introduce the following scaling (36)

$$\xi = \varepsilon^{1/2}(z - Ut), \quad \tau = \varepsilon^{3/2}t,$$

where parameter the parameter $\tau$ is the slow time, while faster dependence on time $t$ is presented via the parameter $\xi$.

Next, we use the following expansions of the hydrodynamic functions on the small parameter $\varepsilon$:

$$n_s = n_{0s} + \varepsilon n_{1s} + \varepsilon^2 n_{2s}, \quad (37)$$

$\eta = 0.1$

$a = 0.02$ - red line;

$a = 0.1$ - green dashed line;

$a = 1$ - blue dotted line.

$\eta = 0.1$

$a = 1$ - red line;

$a = 1.2$ - green dashed line;

$a = 1.5$ - blue dotted line.

FIG. 5: The form of the scalar potential of the electric field in the spin-electron-acoustic soliton is demonstrated for different concentrations presented via parameter $a = (3\pi^2 n_{0s})^{1/3} \hbar/m_e c$ at fixed spin polarization $\eta = 0.1$. The dimensionless parameters defining the profile of soliton have following definitions $\Phi = \varepsilon \varphi_1 / \bar{u}_{Fe}$ and $\Sigma = (m_e \omega_{Le} \sigma / p_{Fe}) (\sqrt{m_e \bar{u} / p_{Fe}})$. This is represented by the equation:

$$v_{sz} = 0 + \varepsilon v_{1sz} + \varepsilon^2 v_{2sz}, \quad (38)$$

$$\Gamma_s = \Gamma_{0s} + \varepsilon \Gamma_{1s} + \varepsilon^2 \Gamma_{2s}, \quad (39)$$

$$t_{sz} = 0 + \varepsilon t_{1sz} + \varepsilon^2 t_{2sz}, \quad (40)$$

and

$$\phi = 0 + \varepsilon \phi_1 + \varepsilon^3 \phi_2, \quad (41)$$

where subindex $s$ corresponds to $s = i$ for ions, $s = \uparrow$ for the spin-up electrons, $s = \downarrow$ for the spin-down electrons, function $\Gamma_{0s}$ is given by the equilibrium equation of state (21), and $\phi$ is the potential of the electric field $E = -\nabla \phi$.

Equations of state (15)-(19) for functions $p_s$, $t_s$, and $M_s$ are applied to get representations of these functions via $n_{0s}$, $n_{1s}$, $n_{2s}$, and $n_{2s}$. For the flux of current of particles we obtain:

$$\dot{\bar{p}}_s \approx \bar{p}_{0s} + \varepsilon \bar{u}_{ps}^2 n_{1s} + \varepsilon^2 \bar{u}_{ps}^2 n_{2s} + \varepsilon^3 \frac{\bar{v}_{ps}^2}{\gamma \bar{v}_{p}^2} n_{0s}.$$  

$\Sigma$ 1234567890

-1.0 0.0 1.0 2.0

-0.1 0.0 0.1 0.2 0.3 0.4 0.5

-0.08 0.06 0.04 0.02

\(Φ\)

\(Σ\)

\(η=0.1\)

\(a=0.02\) - red line;

\(a=0.1\) - green dashed line;

\(a=1\) - blue dotted line.
where $u_{ps}^2 = v_{ps}^2/3$. Second, we present the expansion of the partial fluxes of the reverse relativistic gamma factors

$$\tilde{\eta}_s \approx \tilde{\eta}_0 + \varepsilon \frac{v_{ps}^2}{3 \gamma_{ps}} n_{1s}. \quad (43)$$

We need it up to the first order on $\varepsilon$. We do not need to consider the expansion for function $M_{s0}$, since we need their equilibrium expressions only.

The continuity equations for partial concentrations of electrons and for the concentration of ions considered in the first (lowest) order of the expansion is

$$n_{0s} \partial_\xi v_{sz1} = U \partial_\xi n_{s1}, \quad (44)$$

while in the second order of expansion we obtain

$$\partial_\tau n_{s1} - U \partial_\xi n_{s2} + \partial_\xi (n_{s0} v_{sz2} + n_{s1} v_{sz1}) = 0. \quad (45)$$

During expansion of equations we obtain coefficients in front of $\varepsilon^{3/2}$ and $\varepsilon^{5/2}$ for the first and second orders of expansion, correspondingly.

We can integrate equation (44). We use the boundary condition, where the perturbation caused by soliton goes to zero at infinite distance from its center $v_{sz1} \to 0$ and $n_{s1} \to 0$ at $\xi \to \pm \infty$. Therefore, we obtain

$$n_{0s} v_{sz1} = U n_{s1}. \quad (46)$$

As the second equation, we consider the expansion of the Poisson equation

$$n_{\uparrow 1} + n_{\downarrow 1} - n_{i1} = 0, \quad (47)$$

and

$$-\partial_\xi^2 \varphi_1 = 4\pi q_s (n_{\uparrow 2} + n_{\downarrow 2} + q_in_{i2}). \quad (48)$$

Equation found in the first order (47) does not contain the explicit contribution of the potential of the electric field. The Poisson equation found in the second order (48) contains the first order expansion of the potential of the electric field $\varphi_1$.

The influence of the electric field on the dynamic of electrons and ions can be found from the Euler equation, which is presented in the first and second orders of expansion:

$$-Un_{0s} \partial_\xi v_{sz1} + u_{ps}^2 \partial_\xi n_{s1} = -\frac{q_s n_{s0}}{m_s} \frac{1}{\gamma_{ps}} \partial_\xi \varphi_1. \quad (49)$$
The first order Euler equation (49) can be integrated. It gives additional relation between \( v_{s+1}, n_{s+1} \), and \( \varphi_1 \).

The second order Euler equation (50) gives relation between \( v_{s+2}, n_{s+2}, v_{s+1}, \) and \( \varphi_1 \), but it also includes the first order perturbation of the relativistic hydrodynamic gamma function \( \Gamma_{s+1} \). We include equation (7) in the lowest order of expansion

\[
-U \partial_t \Gamma_{s+1} + \Gamma_{0s} \partial_t v_{s+1} + \partial_t s_{s+1} = 0.
\]

Equation (51) shows that we need to include equation for the first order perturbation of the flux of the relativistic hydrodynamic gamma function \( t_{s+1} \):

\[
U \partial_t t_{s+1} - u_{s+1}^2 \partial_t n_{s+1} + \frac{\Gamma_{0s}}{n_{0s}} u_{sp}^2 \partial_t n_{s+2} + \frac{q_s}{m_s} \Gamma_{0s} \frac{1}{\gamma_{Fs}} \partial_t \varphi_1
\]

\[
= \frac{q_s}{m_s} n_{0s} \left( 1 - \frac{5 u_{sp}^2}{c^2} + \frac{10 u_{sp}^4}{3 c^4} \right) \partial_t \varphi_1,
\]

where \( u_{sp}^4 \equiv M_{0s}/n_{0s} \).

In the first order on \( \varepsilon \) we obtain the expressions for the perturbation of concentration of each species as the function of the potential of the electric field

\[
n_{s+1} = -\frac{q_s}{m_s} \frac{n_{0s}}{\gamma_{Fs} \gamma_{Fp}} \frac{u_{sp}^4}{U^2 - u_{sp}^2} \varphi_1.
\]

We substitute expression (53) obtained for each species in the Poisson equation (17). It gives us equation for the velocity of perturbation \( U \):

\[
\frac{1}{\gamma_{Fp} U^2 - u_{sp}^2} + \frac{1}{\gamma_{Fp} U^2 - u_{sp}^2} + \frac{m_e}{\gamma_{Fp} U^2 - u_{sp}^2} = 0.
\]

Equation (54) can have solutions under condition that denominators of different terms have different sings, for instance \( w_{sp}^4 < U^2 < u_{sp}^2 \). The right-hand side of equation (55) is equal to zero. This condition exclude the Langmuir wave solution. This equation gives two solutions. One corresponds to the SEAW and the second solution corresponds to the ion-acoustic wave.

Our analysis of the second order approximation leads to the nonlinear equation for the electric potential

\[
\partial^2_t \varphi_1 + \sum_{s=\uparrow,\downarrow,i} \frac{U \omega_{ip}^2}{\gamma_{Fs} \gamma_{Fp}} \partial_t \varphi_1 + \sum_{s=\uparrow,\downarrow,i} \frac{q_s^2 \omega_{ip}^2}{m_s \gamma_{Fs} \gamma_{Fp}} \left[ \frac{\Gamma_{0s}}{n_{0s}} \left( 1 - \frac{u_{sp}^2}{U^2} \right) + \frac{1}{U^2} \left( \frac{\gamma_{Fp} \omega_{ip}^2}{3 \gamma_{Fs} \gamma_{Fp} U^2} - \frac{u_{sp}^2 \omega_{ip}^2}{c^2} \right) \right] \varphi_1 \partial_t \varphi_1
\]

\[
+ \sum_{s=\uparrow,\downarrow,i} \frac{q_s}{m_s} \frac{\omega_{ip}^2}{U^2 - u_{sp}^2} \left( 1 - \frac{\sum_{s=\uparrow,\downarrow,i} \frac{\omega_{ip}^2}{m_s \gamma_{Fs} \gamma_{Fp}} \left[ \frac{\Gamma_{0s}}{n_{0s}} \left( 1 - \frac{u_{sp}^2}{U^2} \right) + \frac{1}{U^2} \left( \frac{\gamma_{Fp} \omega_{ip}^2}{3 \gamma_{Fs} \gamma_{Fp} U^2} - \frac{u_{sp}^2 \omega_{ip}^2}{c^2} \right) \right] \varphi_1 \partial_t \varphi_1 + \frac{10 u_{sp}^4}{3 c^4} \varphi_1 \partial_t \varphi_1 = 0.
\]
Here we have Korteweg-de Vries (KdV) equation in terms of electrostatic potential. In schematic form it can be rewritten as \( \partial_t^2 \varphi_1 + D \partial_x \varphi_1 + (P/2) \partial_x^6 \varphi_1^2 = 0 \), where we partially follow notations of Ref. [6]. Hence, parameter \( D \) corresponds to the coefficient in the second term of equation \(55\), while \( P \) corresponds to the superposition of coefficients of the third, fourth and fifth terms. To get the soliton solution of the Korteweg-de Vries equation \(55\), we present novel variable \( \sigma = \xi - \tilde{u} \tau \), where \( \tilde{u} \) is the soliton propagation velocity. Consequently, we find

\[
\varphi_1 = \frac{3D \tilde{u} }{P} \frac{1}{\cosh^2(\sqrt{D \tilde{u} \sigma}/2)}. \tag{56}
\]

Let us illustrate main features of the spin-electron-acoustic solitons at the numerical analysis presented on Figs. \(5\), \(6\), \(7\), \(8\). Presentation of results is made for two spin polarization regimes similarly to analysis of spectra of small amplitude waves. Fig. \(5\) shows the existence of the ”bright” soliton, the soliton of increased potential of the electric field. The upper figure in Fig. \(6\) shows that the increase of concentration from \( a = 0.02 \) up to \( a = 0.1 \) (both regimes corresponds to the small relativistic corrections) we have increase of amplitude and width of the spin-electron-acoustic soliton. Further increase of concentration up to \( a = 1 \) (the relativistic regime) leads to decrease of the amplitude and increase of the width.

The lower figure in Fig. \(6\) demonstrates further growth of the concentration, where the increase of the amplitude with the simultaneous increase of width is found.

The upper figure in Fig. \(6\) presents the following modification of the soliton profile during the increase of concentration at the fixed spin polarization \( \eta = 0.1 \). Small increase of concentration from \( a = 1.5 \) to \( a = 1.57 \) demonstrates considerable increase of amplitude from \( \Phi = 0.08 \) to \( \Phi = 0.55 \). Further small increase of concentration to \( a = 1.59 \) leads to dramatic change of the soliton profile, so we have dark soliton (area of negative potential of the electric field) instead of the bright soliton. While module of amplitude shows no considerable changes (there is small increase). Following small increase of concentration to \( a = 1.6 \) shows considerable decrease of the module of amplitude and width of soliton. Continuing the increase of concentration up to values \( a = 1.7 \), \( a = 2 \), \( a = 3 \) we find monotonic decrease of amplitude and small decrease of width (see the lower figure in Fig. \(6\)). Figs. \(7\) and \(8\) show same tendency for another spin polarization \( \eta = 0.9 \), but the critical point, where the sign of the amplitude of soliton is changed. The change happens at \( a = 1.18 \). So, the critical concentration becomes smaller at larger spin polarization.

V. CONCLUSION

In order to consider spin-electron-acoustic waves and spin-electron-acoustic solitons propagating parallel to the external magnetic field in the relativistic degenerate partially spin polarized electron gas, we have presented a generalization of the relativistic hydrodynamic model with the average reverse gamma factor evolution. Suggested model has been based on the following background. The relativistic hydrodynamics with is originally developed for the hot plasmas with temperature of electrons \( T_e \) of order of the rest energy \( T_e \sim m_e c^2 \). However, this model has a restriction from the area of large concentrations: \( T_e \gg T_{Fe} = (3n_e^2/m_e)^{1/3}/2m_e \). Modification of hydrodynamic model with the average reverse gamma factor evolution for the high-density degenerate relativistic electrons \( T_{Fe} \sim m_e c^2 \gg T_e \) is developed in literature as well in the regime of electrons with the zero spin polarization. The area of applications of the single fluid model of spin polarized electrons has been discussed.

Degenerate electrons is a quantum system. Proper justification of this model should be based on quantum dynamics of particles. However, major plasma effects are based on the electromagnetic interaction and form of distribution of particles in the momentum space, which can be captured in the quasi-classic limit, as it has been done in this paper. Other quantum effects like the quantum Bohm potential or dynamic of the transverse projections of the spin density have been neglected.

The dispersion equation for the spin-electron-acoustic wave is considered in two regimes:
1) immobile ions;
2) separate spin evolution of electrons with the account of the motion of ions.

The first regime shows that the large spin polarization regime leads to the relatively small phase velocity of the spin-electron-acoustic waves, so the influence of ions is essential. Similar conclusion is correct in the nonrelativistic regime. However, the relativistic effects considerably decreases the phase velocity of the spin-electron-acoustic waves. Numerical analysis of spectra of the Langmuir, spin-electron-acoustic and ion-acoustic waves have been presented. We stress our attention on the dependence of spectra on the spin polarization of electrons. The spin polarization of ions is neglected. The properties of spin-electron-acoustic soliton has been studied.

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VII. DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study, which is a purely theoretical one.
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