Mass Spectrum of a Meson Nonet is Linear

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Abstract

It is argued that the mass spectrum of a meson nonet is linear, consistent with the standard Gell-Mann–Okubo mass formula and leading to an extra Gell-Mann–Okubo mass relation for the masses of the isoscalar states. This relation is shown to hold with an accuracy of up to \(\sim 3\%\) for all well-established nonets. It also suggests a new \(q\bar{q}\) assignment for the scalar meson nonet.

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The hadronic mass spectrum is an essential ingredient in theoretical investigations of the physics of strong interactions. It is well known that the correct thermodynamic description of hot hadronic matter requires consideration of higher mass excited states, the resonances, whose contribution becomes essential at temperatures $\sim O(100 \text{ MeV})$ [1, 2]. The method for taking into account these resonances was suggested by Belenky and Landau [3] as considering unstable particles on an equal footing with the stable ones in the thermodynamic quantities; e.g., the formulas for the pressure and energy density in a resonance gas read

$$p = \sum_i p_i = \sum_i g_i \frac{m_i^2 T^2}{2\pi^2} K_2 \left( \frac{m_i}{T} \right),$$  \hspace{1cm} (1)

$$\rho = \sum_i \rho_i, \quad \rho_i = T \frac{dp_i}{dT} - p_i,$$  \hspace{1cm} (2)

where $g_i$ are the corresponding degeneracies ($J$ and $I$ are spin and isospin, respectively),

$$g_i = \frac{\pi^4}{90} \times \begin{cases} (2J_i + 1)(2I_i + 1) & \text{for non-strange mesons} \\ 4(2J_i + 1) & \text{for strange (K) mesons} \\ 2(2J_i + 1)(2I_i + 1) \times 7/8 & \text{for baryons} \end{cases}$$

These expressions may be rewritten with the help of a resonance spectrum,

$$p = \int_{m_1}^{m_2} dm \tau(m) p(m), \quad p(m) \equiv \frac{m^2 T^2}{2\pi^2} K_2 \left( \frac{m}{T} \right),$$  \hspace{1cm} (3)

$$\rho = \int_{m_1}^{m_2} dm \tau(m) \rho(m), \quad \rho(m) \equiv T \frac{dp(m)}{dT} - p(m),$$  \hspace{1cm} (4)

normalized as

$$\int_{m_1}^{m_2} dm \tau(m) = \sum_i g_i,$$  \hspace{1cm} (5)

where $m_1$ and $m_2$ are the masses of the lightest and heaviest species, respectively, entering the formulas (1),(2).

\footnote{For simplicity, we neglect the chemical potential and approximate the particle statistics by the Maxwell-Boltzmann one.}
In both the statistical bootstrap model \[4, 5\] and the dual resonance model \[6\], a resonance spectrum takes on the form
\[
\tau(m) \sim m^a \, e^{m/T_0},
\]
where \(a\) and \(T_0\) are constants. The treatment of a hadronic resonance gas by means of the spectrum (6) leads to a singularity in the thermodynamic functions at \(T = T_0\) \[4, 5\] and, in particular, to an infinite number of the effective degrees of freedom in the hadron phase, thus hindering a transition to the quark-gluon phase. Moreover, as shown by Fowler and Weiner \[7\], an exponential mass spectrum of the form (6) is incompatible with the existence of the quark-gluon phase: in order that a phase transition from the hadron phase to the quark-gluon phase be possible, the hadronic spectrum cannot grow with \(m\) faster than a power.

In our previous work \[8\] we considered a model for a transition from a phase of strongly interacting hadron constituents, described by a manifestly covariant relativistic statistical mechanics which turned out to be a reliable framework in the description of realistic physical systems \[9\], to the hadron phase described by a resonance spectrum, Eqs. (3),(4). An example of such a transition may be a relativistic high temperature Bose-Einstein condensation studied by the authors in ref. \[10\], which corresponds, in the way suggested by Haber and Weldon \[11\], to spontaneous flavor symmetry breakdown, \(SU(3)_F \rightarrow SU(2)_I \times U(1)_Y\), upon which hadronic multiplets are formed, with the masses obeying the Gell-Mann–Okubo formulas \[12\]
\[
m^\ell = a + bY + c \left[ \frac{Y^2}{4} - I(I + 1) \right],
\]
here \(I\) and \(Y\) are the isospin and hypercharge, respectively, \(\ell\) is 2 for mesons and 1 for baryons, and \(a, b, c\) are independent of \(I\) and \(Y\) but, in general, depend on \((p, q)\), where \((p, q)\) is any irreducible representation of \(SU(3)\). Then only the assumption on the overall degeneracy being conserved during the transition is required to lead to the unique form of a resonance spectrum in the hadron phase:
\[
\tau(m) = Cm, \quad C = \text{const}.
\]
Zhirov and Shuryak \[13\] have found the same result on phenomenological grounds. As shown in ref. \[13\], the spectrum (8), used in the formulas (3),(4) (with the upper limit of integration infinity), leads to the equation of state \(p, \rho \sim T^6, p = \rho/5\), called by Shuryak the “realistic” equation of state for hot hadronic matter \[1\], which has some experimental support. Zhirov and Shuryak \[13\] have calculated the velocity of sound, \(c_s^2 \equiv dp/d\rho = c_s^2(T)\), with \(p\) and \(\rho\) defined in Eqs. (1),(2), and found that \(c_s^2(T)\) at first increases with \(T\) very quickly and then saturates at the value of \(c_s^2 \simeq 1/3\) if only the pions are taken into account, and at \(c_s^2 \simeq 1/5\) if resonances up to \(M \sim 1.7\) GeV are included.
We have checked the coincidence of the results given by the linear spectrum (8) with those obtained directly from Eq. (1) for the actual hadronic species with the corresponding degeneracies, for all well-established multiplets, the mesons:

1 \( ^3S_1 \) \( J^{PC} = 1^{--} \) nonet, \( \rho(770), \omega(783), \phi(1020), K^*(892) \)

1 \( ^1P_1 \) \( J^{PC} = 1^{+-} \) nonet, \( b_1(1235), h_1(1170), h_1(1380), K_1(1270) \)

1 \( ^3P_1 \) \( J^{PC} = 1^{++} \) nonet, \( a_1(1260), f_1(1285), f_1(1510), K_1(1400) \)

1 \( ^3P_2 \) \( J^{PC} = 2^{+-} \) nonet, \( a_2(1320), f_2(1270), f'_2(1525), K^*_2(1430) \)

1 \( ^3D_3 \) \( J^{PC} = 3^{--} \) nonet, \( \rho_3(1690), \omega_3(1670), \phi_3(1850), K^*_3(1780) \)

the baryons:

\( J^P = \frac{1}{2}^+ \) octet, \( N(939), \Lambda(1116), \Sigma(1190), \Xi(1320) \)

\( J^P = \frac{3}{2}^+ \) decuplet, \( \Delta(1232), \Sigma(1385), \Xi(1530), \Omega(1672) \)

\( J^P = \frac{1}{2}^- \) nonet, \( N(1520), \Lambda(1690), \Sigma(1670), \Xi(1820), \Lambda(1520) \)

\( J^P = \frac{5}{2}^+ \) octet, \( N(1680), \Lambda(1820), \Sigma(1915), \Xi(2030) \),

and found it excellent \[8\]. Shown are typical figures of ref. \[8\] in which the results given by both, Eq. (1), and Eq. (3) with a linear spectrum, are compared \[8\]. Thus, the theoretical implication that a linear spectrum is the actual spectrum in the description of individual hadronic multiplets, is consistent with experiment as well. In our recent paper \[14\] we have applied a linear spectrum to the problem of establishing the theoretical implication that a linear spectrum is the actual spectrum in the description of individual hadronic multiplets, is consistent with experiment as well. In our recent paper \[14\] we have applied a linear spectrum to the problem of establishing the correct \( q\bar{q} \) assignment for the problematic meson nonets, like the scalar, axial-vector and tensor ones, and separating out non-\( q\bar{q} \) mesons.

The easiest way to see that a linear spectrum corresponds to the actual spectrum of a meson nonet is as follows\[4\]. Let us calculate the average mass squared for a spin-\( s \) nonet:

\[
\langle m^2 \rangle_g \equiv \frac{\sum_i g_i m_i^2}{\sum_i g_i} = \frac{3m_1^2 + 4m_{1/2}^2 + m_{0'}^2 + m_{0''}^2}{9},
\]

where \( m_1, m_{1/2}, m_0, m_{0'} \) are the masses of isovector, isospinor, and two isoscalar states, respectively, and the spin degeneracy, \( 2s + 1 \), cancels out. In general, the isoscalar states\[4\] \( \omega_{0'} \) and \( \omega_{0''} \) are the octet \( \omega_8 \) and singlet \( \omega_0 \) mixed states because of \( SU(3) \) breaking,

\[
\omega_{0'} = \omega_8 \cos \theta_M - \omega_0 \sin \theta_M,
\]

\[
\omega_{0''} = \omega_8 \sin \theta_M + \omega_0 \cos \theta_M,
\]

where \( \theta_M \) is a mixing angle. Assuming that the matrix element of the Hamiltonian between the states yields a mass squared, i.e., \( m_{0'}^2 = \langle \omega_{0'} | H | \omega_{0'} \rangle \) etc., one obtains

\[2\] Instead of a direct comparison of Eqs. (1) and (3), we compared the expressions \( p/p_{SB} \) for both cases, where \( p_{SB} \equiv \sum_i g_i \pi^2/90 T^4 \), i.e., \( p_{SB} \) is the pressure in an ultrarelativistic gas with \( g = \sum_i g_i \) degrees of freedom.

\[3\] For a baryon multiplet, it is more difficult to show that the mass spectrum is linear, since the Gell-Mann–Okubo formulas are linear in mass for baryons. More detailed discussion is given in \[8\].

\[4\] The \( \omega_{0'} \) is a mostly octet isoscalar.
from the above relations [5],
\begin{align}
m_0^2 &= m_8^2 \cos^2 \theta_M + m_0^2 \sin^2 \theta_M - 2m_{0B}^2 \sin \theta_M \cos \theta_M, \\
m_0^{2\nu} &= m_8^2 \sin^2 \theta_M + m_0^2 \cos^2 \theta_M + 2m_{0B}^2 \sin \theta_M \cos \theta_M.
\end{align}

Since $\omega_0'$ and $\omega_0''$ are orthogonal, one has further
\begin{equation}
m_0^{2\nu} = 0 = (m_8^2 - m_0^2) \sin \theta_M \cos \theta_M + m_{0B}^2 (\cos^2 \theta_M - \sin^2 \theta_M).
\end{equation}

Eliminating $m_0$ and $m_{0B}$ from (10)-(12) yields
\begin{equation}
\tan^2 \theta_M = \frac{m_8^2 - m_0'^2}{m_0^{2\nu} - m_8^2}.
\end{equation}

It also follows from (10),(11) that, independent of $\theta_M$, $m_0^2 + m_0^{2\nu} = m_8^2 + m_0^2$, and therefore, Eq. (9) may be rewritten as
\begin{equation}
\langle m^2 \rangle_9 = \frac{3m_1^2 + 4m_{1/2}^2 + m_8^2 + m_0^2}{9}.
\end{equation}

For the octet, (3 $m_1$, 4 $m_{1/2}$, 1 $m_8$), the Gell-Mann–Okubo formula (as follows from (7)) is
\begin{equation}
4m_{1/2}^2 = 3m_8^2 + m_1^2.
\end{equation}

Therefore, the average mass squared for the octet is
\begin{equation}
\langle m^2 \rangle_8 = \frac{3m_1^2 + 4m_{1/2}^2 + m_8^2}{8} = \frac{m_1^2 + m_8^2}{2},
\end{equation}

where Eq. (15) was used. In the exact $SU(3)$ limit where the $u$, $d$ and $s$ quarks have equal masses, all the squared masses of the nonet states are equal as well. Since in this limit all the squared masses of the octet states are equal to the average mass squared of the octet [4], Eq. (16), the mass of the singlet should have the same value.

5In a manifestly covariant theory, this holds since a total mass squared is rigorously conserved. In the standard framework, for pseudoscalar mesons, this is easily seen by using the lowest order relations [10] $m_1^2 \equiv m_u^2 = 2mB$, $m_{1/2}^2 \equiv m_s^2 = (m + m_s)B$, where $m = (m_u + m_d)/2$, and $B$ is related to the quark condensate. Therefore, it follows from (15),(16) that $m_8^2 = 2/3 (2m_u + m)B$, $\langle m^2 \rangle_8 = 2/3 (m_u + 2m)B = 2/3 (m_u + m_d + m_s)B$. In the exact $SU(3)$ limit, $m_u = m_d = m_s = m$, and hence $m_1^2 = m_{1/2}^2 = m_8^2 = \langle m^2 \rangle_8 = 2mB$. For higher mass mesons, since the states with equal isospin (and alternating parity) lie on linear Regge trajectories, one may expect the relations of the form ($c = C/B$) $m_1^2 = 2mB + C = (2m + c)B$, $m_{1/2}^2 = (m + m_s)B + C = (m + m_s + c)B$, $m_8^2 = 2/3 (2m_u + m)B + C = 2/3 (2m_u + m + 3/2 c)B$, consistent with the Gell-Mann–Okubo formula (15), leading to $m_1^2 = m_{1/2}^2 = m_8^2 = \langle m^2 \rangle_8 = 2mB + C$ in the $SU(3)$ limit $m_u = m_d = m_s = m$. For vector mesons, such a relation was obtained by Balázs in the flux-tube fragmentation approach to a low-mass hadronic spectrum [17], $m_{v}^2 = m_{v}^2 + 1/2 \alpha^\prime$, with $\alpha^\prime$ being a universal Regge slope, in good agreement with the experiment.

6It is also seen from the relations of a previous footnote: since the total mass squared of a nonet is proportional to the total mass of quarks the nonet members are made of, $\sum_i m_i^2 = (12m + 6m_s)B + 9C$, it follows from the above expressions for $m_1^2$, $m_{1/2}^2$ and $m_8^2$ that $m_0^2 = 2/3 (2m + m_s)B + C = \langle m^2 \rangle_8 = \langle m^2 \rangle_9$. 

5, 6
i.e.,

$$m_0^2 = \frac{m_1^2 + m_8^2}{2}$$  \hspace{1cm} (17)

With Eq. (15), it then follows from (17) that

$$m_0^2 + m_8^2 = 2m_{1/2}^2,$$

which reduces, through $m_0^2 + m_8^2 = m_{0\ell}^2 + m_{0\nu}^2$, to

$$m_{0\ell}^2 + m_{0\nu}^2 = 2m_{1/2}^2,$$  \hspace{1cm} (18)

which is an extra Gell-Mann–Okubo mass relation for a nonet. We will check this relation below, and show that with the experimentally available meson masses, the relative error in the values on the l.h.s. and r.h.s. of Eq. (18) does not exceed 3% for all well-established nonets (except for the pseudoscalar nonet for which Eq. (18) does not hold, perhaps because the $\eta_0$ develops a large dynamical mass due to axial $U(1)$ symmetry breakdown before it mixes with the $\eta_8$ to form the physical $\eta$ and $\eta'$ states). For a singlet-octet mixing close to “ideal” one, $\tan \theta_M \simeq 1/\sqrt{2}$; it then follows from (13) that

$$2m_{0\ell}^2 + m_{0\nu}^2 \simeq 3m_8^2,$$

which reduces, through (15),(18), to

$$m_{0\nu}^2 \simeq m_1.$$  \hspace{1cm} (19)

Now it follows clearly why the ground states of all well-established nonets\(^7\) (except for the pseudoscalar one) are almost mass degenerate pairs, like $(\rho, \omega)$.\(^8\) In the close-to-ideal mixing case, Eq. (18) may be rewritten, with the help of (19), as

$$m_1^2 + m_{0\nu}^2 \simeq 2m_{1/2}^2.$$  \hspace{1cm} (20)

This relation for pseudoscalar and vector mesons with the ground states being the mass degenerate pairs $(\pi, \eta_0)$ and $(\rho, \omega)$, respectively, was previously obtained by Balázs and Nicolescu using the dual-topological-unitarization approach to the confinement region of hadronic physics (Eq. (21) of ref. [18]). With (16) and (17), Eq. (10) finally reduces to

$$\langle m^2 \rangle_9 = \frac{m_1^2 + m_8^2}{2},$$  \hspace{1cm} (21)

which, of course, coincides with both, $\langle m^2 \rangle_8$ in (16) and $m_0^2$ in (17), which is the property of the $SU(3)$ limit (or the conservation of a total mass squared in a manifestly covariant theory).

\(^7\)This is also true for $q\bar{q}$ assignment of the scalar meson nonet suggested by the authors in ref. [14].

\(^8\)It follows from the relations of footnote 5 that, in the close-to-ideal mixing case, $m_{0\ell}^2 \simeq 2m_sB + C$ and $m_{0\nu}^2 \simeq 2mB + C = m_1^2$. 

6
For the actual mass spectrum of the nonet, the average mass squared \( \langle m^2 \rangle_9 \) may be represented in the form\( ^9 \)

\[
\langle m^2 \rangle_9 = \frac{\int_{m_1}^{m_8} dm \, \tau(m) \, m^2}{\int_{m_1}^{m_8} dm \, \tau(m)} ,
\]

and one sees that the only choice for \( \tau(m) \) leading to the relation (21) is \( \tau(m) = C m, \) \( C = \text{const}. \)

Indeed, in this case

\[
\langle m^2 \rangle_9 = \frac{\int_{m_1}^{m_8} dm \, m^3}{\int_{m_1}^{m_8} dm \, m} = \frac{(m_8^4 - m_1^4)/4}{(m_8^2 - m_1^2)/2} = \frac{m_1^2 + m_8^2}{2} ,
\]

in agreement with (21).

We now wish to check the formula (18) for all well-established meson nonets (we indicate the actual particle masses, as given by the recent Particle Data Group \( ^{19} \), and take \( m_\pi = 1/3 \) \( (2m_\pi + m_\pi^0) \), \( m_K = 1/2 \) \( (m_K^+ + m_K^0) \) etc.):

1) \( ^1S_0 J^{PC} = 0^{-+}, \pi(138), \eta(547), \eta'(958), K(495) \). In the assumption of no mixing of the \( \eta_8 \) and \( \eta_0 \) states, it follows from (15),(17) that

\[
m_\eta^2 = m_{\eta_8}^2 = 1/3 \left( 4m_K^2 - m_\pi^2 \right) \approx 4/3 \, m_K^2 ,
\]

\[
m_{\eta'}^2 = m_{\eta_0}^2 = 1/3 \left( 2m_K^2 + m_\pi^2 \right) \approx 2/3 \, m_K^2 ,
\]

so that\( ^{10} \)

\[
m_\eta \approx \sqrt{4/3} \, m_K \approx 566 \, \text{MeV} ,
\]

in fair agreement with the experiment, but

\[
m_{\eta'} \approx \sqrt{2/3} \, m_K \approx 400 \, \text{MeV} ,
\]

in strong disagreement with the experiment. The reason for the invalidity of Eq. (18) for the pseudoscalar nonet is, probably, a large dynamical mass of the \( \eta_0 \) due to axial \( U(1) \) symmetry breakdown developed before it mixes with the \( \eta_8 \) to form the physical \( \eta \) and \( \eta' \) states.

2) \( ^1S_1 J^{PC} = 1^{-+}, \rho(769), \omega(783), \phi(1019), K^*(894) \). In this case one has 1.65 GeV\(^2 \) on the l.h.s. of Eq. (18) vs. 1.60 GeV\(^2 \) on the r.h.s.

3) \( ^1P_1 J^{PC} = 1^{++}, b_1(1231), h_1(1170), h_1(1380), K_1(1273) \). Now one has 3.27 GeV\(^2 \) on the l.h.s. of (18) vs. 3.24 GeV\(^2 \) on the r.h.s.

4) \( ^3P_1 J^{PC} = 1^{++}, a_1(1230), f_1(1282), f_1(1512), K_1(1402) \). In this case the both values on the different sides of Eq. (18) are equal to 3.93 GeV\(^2 \).

5) \( ^3P_2 J^{PC} = 2^{++}, a_2(1318), f_2(1275), f_2'(1525), K^*_2(1425) \). Now one has 3.95 GeV\(^2 \) on the l.h.s. of (18) vs. 4.06 GeV\(^2 \) on the r.h.s.

\(^9\)Since \( m_s > m, m_1 < m_{1/2} < m_8 \), as seen in the relations of footnote 5. Moreover, \( m_1 < m_0 < m_8 \), and therefore, the range of integration in Eq. (22) is \( (m_1, m_8) \).

\(^{10}\)These relations for \( m_\eta \) and \( m_{\eta'} \) are also contained in ref. [1], p. 20.
6) $1^{3}D_{3} J^{PC} = 3^{--}, \rho_{3}(1680), \omega_{3}(1668), \phi_{3}(1854), K^{*}_{3}(1770)$. In this case one has 6.23 GeV$^{2}$ on the l.h.s. of Eq. (18) vs. 6.27 GeV$^{2}$ on the r.h.s.

Thus, Eq. (18) holds with an accuracy of up to $\sim$3% for the vector and tensor meson nonets and with a higher accuracy for other nonets; for the axial-vector nonet it is exact.

7) Even for the $2^{3}P_{2} J^{PC} = 2^{++}$ nonet for which there is no isovector candidate at present, and two of the three remaining states need experimental confirmation, $f_{2}(1811), f_{2}(2011), K^{*}_{2}(1975)$, one has 7.32 GeV$^{2}$ on the l.h.s. of (18) vs. 7.80 GeV$^{2}$ on the r.h.s., with a still satisfactory accuracy of $\sim$6%.

8) Let us now dwell upon a problematic nonet, the scalar meson one, whose currently adopted $q\bar{q}$ assignment is $^{[13]} a_{0}(982), f_{0}(1300), f_{0}(980), K^{*}_{0}(1429)$. It is seen that for this assignment Eq. (18) does not hold (980 < 1300 < 1429). As we have shown, this relation should hold, independent of a mixing angle (i.e., even for a “non-ideal” nonet), and there is no apparent physical reason for the mass shifts of the scalar nonet members, like that of the $\eta_{0}$, which would lead to the invalidity of Eq. (18). Therefore, we conclude that the currently adopted $q\bar{q}$ assignment of the scalar meson nonet is incorrect.

There are five established isoscalars with $J^{PC} = 0^{++}$, the $f_{0}(980), f_{0}(1300), f_{0}(1370), f_{0}(1525)$ and $f_{0}(1590)$. In the quark model, one expects two $1^{3}P_{0}$ states and one $2^{3}P_{0}$ ($u\bar{u} + d\bar{d}$)-like state below 1.8 GeV. Therefore, at least two of the five cannot find a place in the quark model. Many broad $\pi\pi$ elastic resonances claimed in the past (like the $\sigma(700), \epsilon(1200), f_{0}(1400)$) were collected by the recent Particle Data Group under one entry, the $f_{0}(1300)$. Similarly, all the broad $\pi\pi$ inelastic $S$-wave resonance claims were collected under one entry, the $f_{0}(1370)$, although they could be the $f_{0}(1300)$ provided the inelasticity of the latter is in fact larger than is presently believed $^{[20]}$. Although it is currently adopted as a member of the nonet, there exists an interpretation of the $f_{0}(980)$ as a $K\bar{K}$ molecule $^{[21]}$ since it [and the $a_{0}(980)$] lies just below the $K\bar{K}$ threshold which is 992 MeV $^{[22]}$. If the $f_{0}(980)$ is not the $1^{3}P_{0}$ $s\bar{s}$ state, the latter should be found near 1500 MeV with partial decay widths close to the flavor symmetry predictions for an ideal nonet $^{[23]}$. The weak signal as 1515 MeV claimed by the LASS group $^{[21]}$ does not have the expected large width. In this case, the $f_{0}(1525)$ (or $f_{0}(1520)$) could be a candidate for the $1^{3}P_{0}$ $s\bar{s}$ state $^{[24]}$. This $f_{0}(1525)$ has been identified as $K\bar{K}$ $S$-wave intensity peaking at the mass of the $f_{2}^{0}(1525)$ and having a comparable width $^{[25], [27]}$. The $f_{0}(1520)$ (as well as $f_{0}(1370)$) has been recently observed by the Crystal Barrel Collaboration in a simultaneous fit to the $\bar{p}p \rightarrow 3\pi^{0}$ and $\bar{p}p \rightarrow \eta\pi^{0}$ data $^{[28]}$. The both, $f_{0}(1525)$ and $f_{0}(1520)$ are adopted by the recent Particle Data Group as one entry, the $f_{0}(1525)$. The $f_{0}(1590)$ has been seen in $\pi^{-}p$ reactions at 38 GeV/c $^{[29], [30]}$. It has a peculiar decay pattern for

$$\pi^{0}\pi^{0} : K\bar{K} : \eta \bar{\eta} : \eta' \bar{\eta}' : 4\pi^{0} = < 0.3 : < 0.6 : 1 : 2.7 : 0.8, \tag{11}$$

These are the $f_{2}(1810)$ and $K^{*}_{2}(1980)$. 

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which could favor a gluonium interpretation [31]. Another possibility is that it is a large deuteron-like \((\omega\omega - \rho\rho)/\sqrt{2}\) bound state ("deuson") [32].

The established isovector with \(J^{PC} = 1^{++}\) is the \(a_0(980)\). Its mass, \((982 \pm 2)\) MeV, is low, compared to its isovector partners, like the \(a_1(1260)\), \(a_2(1320)\) and \(b_1(1235)\). Its apparent width (as measured in its \(\eta\pi\) decay mode), \((54 \pm 10)\) MeV, is small, compared to its partners (which have 100 MeV and more). Moreover, neither the relative coupling of the \(a_0(980)\) to \(\eta\pi\) and \(K\bar{K}\), nor its width to \(\gamma\gamma\), are known well enough to draw firm conclusions on its nature (\(q\bar{q}2q\bar{q}\) state, \(K\bar{K}\) molecule, etc.). If the \(a_0(980)\) is not the \(3P_0\) state, the latter should be observed near 1300 MeV, with partial decay width as expected from flavor symmetry. The candidate \(a_0(1320)\) identified by GAMS as intensity peaking at the mass of the \(a_2(1320)\) and having a comparable width [33], needs experimental confirmation.

Thus, an attractive choice for the \(q\bar{q}\) scalar meson nonet could be the \(a_0(1320)\), \(K^*_0(1430)\), \(f_0(1300)\) (or \(f_0(1370)\)) and \(f_0(1525)\) (or \(f_0(1520)\)). This choice would leave out the \(a_0(980)\) and \(f_0(980)\) which could be then interpreted in terms of four-quark or \(K\bar{K}\) molecule states, and one may then speculate that the \(f_0(1590)\) is a glueball, or, at least, a state rich in glue. For the \(q\bar{q}\) assignment suggested above, one has 4.00 – 4.20 GeV\(^2\) on the l.h.s. of Eq. (18) (4.00 corresponds to the assignment which includes \(f_0(1300)\) and \(f_0(1520)\) while 4.20 to that with \(f_0(1370)\) and \(f_0(1525)\)) vs. 4.08 GeV\(^2\) on the r.h.s., and we conclude that for this assignment, the formula (18) holds, with a high accuracy, as should be the case for a genuine meson nonet.

9) Similar analysis for the \(1^3D_1\) \(J^{PC} = 1^{--}\) and \(2^3S_1\) \(J^{PC} = 1^{--}\) nonets will be given in a separate publication [34].

Evidently, one may choose an opposite way, viz., starting from a linear spectrum as the actual spectrum of a nonet, to derive the Gell-Mann–Okubo mass formula. To this end, one should first calculate the average mass squared, Eq. (21). Then one has to place 9 nonet states in the interval \((m_1, m_8)\) in a way that preserves the average mass squared. As we already know, the isoscalar singlet mass squared should coincide with the average mass squared; for the remaining 8 states one would have the relation (16) which would in turn reduce to the Gell-Mann–Okubo formula (15). One sees that the assumption on a linear mass spectrum turns out to be a good alternative to the group theoretical mechanism of symmetry breaking, for the derivation of the Gell-Mann–Okubo type relations, which may be rather difficult technical task for a higher symmetry group.

The method of the derivation of the Gell-Mann–Okubo mass relations described above may be easily generalized to the case of four or more flavors. In our recent paper [35], by applying this method to an \(SU(4)\) hexadecuplet, we have derived the corresponding Gell-Mann–Okubo mass formula and found it to be in good agreement with the experimentally established masses of the charmed mesons.
Acknowledgements

One of us (L.B.) wish to thank E.V. Shuryak for very valuable discussions on hadronic resonance spectrum.

References

[1] E.V. Shuryak, *The QCD Vacuum, Hadrons and the Superdense Matter*, (World Scientific, Singapore, 1988)

[2] C.R. Alcock, G.M. Fuller and G.J. Mathews, Astrophys. J. 320 (1987) 439
H. Bebie, P. Gerber, J.L. Goity and H. Leutwyler, Nucl. Phys. B 378 (1992) 95

[3] S.Z. Belenky and L.D. Landau, Sov. Phys. Uspekhi 56 (1955) 309; Nuovo Cim. Suppl. 3 (1956) 15

[4] R. Hagedorn, Nuovo Cim. 35 (1965) 216; Nuovo Cim. Suppl. 6 (1968) 169, 311

[5] S. Frautschi, Phys. Rev. D 3 (1971) 2821

[6] S. Fubini and G. Veneziano, Nuovo Cim. A 64 (1969) 811

[7] G.N. Fowler and R.M. Weiner, Phys. Lett. B 89 (1980) 394

[8] L. Burakovsky, L.P. Horwitz and W.C. Schieve, Hadronic Resonance Spectrum: Power vs. Exponential Law. Experimental Evidence; to be published

[9] L.P. Horwitz and C. Piron, Helv. Phys. Acta 46 (1973) 316
L.P. Horwitz, W.C. Schieve and C. Piron, Ann. Phys. (N.Y.) 137 (1981) 306
L.P. Horwitz, S. Shashoua and W.C. Schieve, Physica A 161 (1989) 300
L. Burakovsky, Manifestly Covariant Relativistic Statistical Mechanics as a Framework for Description of Realistic Physical Systems, Ph.D. thesis (Tel-Aviv University, 1995), unpublished; L. Burakovsky, L.P. Horwitz and W.C. Schieve, Mass - Proper Time Uncertainty Relation in a Manifestly Covariant Relativistic Statistical Mechanics, to be published

[10] L. Burakovsky, L.P. Horwitz and W.C. Schieve, A New Relativistic High Temperature Bose-Einstein Condensation; Phys. Rev. D, in press

[11] H.A. Haber and H.E. Weldon, Phys. Rev. D 25 (1982) 502 (1994) 4725

[12] S. Okubo, Prog. Theor. Phys. 27 (1962) 949, 28 (1962) 24
M. Gell-Mann and Y. Ne'eman, *The Eightfold Way*, (Benjamin, N.Y., 1964)

[13] O.V. Zhirov and E.V. Shuryak, Sov. J. Nucl. Phys. 21 (1975) 443
[14] L. Burakovsky and L.P. Horwitz, Hadronic Resonance Spectrum May Help in Resolution of Meson Nonet Enigmas; to be published

[15] D.H. Perkins, *Introduction to High Energy Physics*, 3rd ed., (Addison Wesley, Reading, Ma, 1987), section 5.6

[16] M. Gell-Mann, R.J. Oakes and B. Renner, Phys. Rev. 175 (1968) 2195

[17] L.A.P. Balázs, Phys. Lett. B 120 (1983) 426

[18] L.A.P. Balázs and B. Nicolescu, Phys. Rev. D 28 (1983) 2818

[19] Particle Data Group, Phys. Rev. D 50 (1994) 1173

[20] Particle Data Group, Phys. Rev. D 50 (1994) 1478

[21] J. Weinstein and N. Isgur, Phys. Rev. D 41 (1990) 2236

[22] S. Flatte, Phys. Lett. B 63 (1976) 224

[23] N.A. Törnqvist, Nucl. Phys. B (Proc. Suppl.) 21 (1991) 196

[24] D. Leith and B. Ratcliff, in Proceedings of the 3rd International Conference on Hadron Spectroscopy, “Hadron 89”, Ajaccio, France, 1989; ed. F. Binon *et al.*, Editions Frontieres (Gif-sur-Yvette) C29 (1990) 3, 15

[25] L. Montanet, Nucl. Phys. B (Proc. Suppl.) 39B,C (1995) 281

[26] D. Aston *et al.*, Nucl. Phys. B 301 (1988) 525

[27] M. Baubillier *et al.*, Z. Phys. C 17 (1983) 309

[28] V.V. Anisovich *et al.*, Phys. Lett. B 323 (1994) 233 (1982) 2446

[29] F. Binon *et al.*, Sov. J. Nucl. Phys. 38 (1983) 561; Nuovo Cim. A 78 (1983) 313, *ibid.* 80 (1984) 363

[30] D. Alde *et al.*, Phys. Lett. B 198 (1987) 286; Z. Phys. C 36 (1987) 603

[31] S.S. Gershtein, A.K. Likhodey and Yu.D. Prokoshkin, Yad. Fiz. 39 (1984) 251

[32] N.A. Törnqvist, Phys. Rev. Lett. 67 (1991) 556 (1986) 485 Rev. Lett. (1990) 569

[33] M. Boutemeur *et al.*, ref. [24], p. 119

[34] L. Burakovsky and L.P. Horwitz, Comments on the $K^*(1410)$ Meson, in preparation
[35] L. Burakovsky and L.P. Horwitz, Gell-Mann–Okubo Mass Formula for $SU(4)$ Meson Hexadecuplet, to be published
FIGURE CAPTIONS

Fig. 1. Temperature dependence of the ratio \( p/p_{SB} \) as calculated from: a) Eq. (1), b) Eq. (3) with a linear spectrum, for the \( 1^1P_1 \ J^{PC} = 1^{+-} \) meson nonet, \( b_1(1235), h_1(1170), h_1(1380), K_1(1270) \).

Fig. 2. The same as Fig. 1 for the \( 1^3P_2 \ J^{PC} = 2^{++} \) meson nonet, \( a_2(1320), f_2(1270), f_2'(1525), K_2^*(1430) \).

Fig. 3. The same as Fig. 1 for the \( J^P = \frac{1}{2}^+ \) baryon octet, \( N(939), \Lambda(1116), \Sigma(1193), \Xi(1318) \).

Fig. 4. The same as Fig. 1 for the \( J^P = \frac{3}{2}^- \) baryon nonet, \( N(1520), \Lambda(1690), \Sigma(1670), \Xi(1820), \Lambda(1520) \). replaced by assignment
