Cosmological interplay between general relativity and particle physics

Michael Mazia

Andronikashvili Institute of Physics, 6 Tamarashvili St., Tbilisi 0177, Georgia
Faculty of Physics and Mathematics, Chavchavadze State University, 32 Chavchavadze Ave., Tbilisi 0179, Georgia

We clearly formulate and study further a conjecture of effective field theory interaction with gravity in the cosmological context. The conjecture stems from the fact that the melding of quantum theory and gravity typically indicates the presence of an inherent UV cutoff. Taking note of the physical origin of this UV cutoff, that the background metric fluctuations does not allow QFT to operate with a better precision than the background space resolution, we conjecture that the converse statement might also be true. That is, an effective field theory could not perceive the background space with a better precision than it is allowed by its intrinsic UV scale. Some of the subtleties and cosmological implications of this conjecture are explored.

PACS numbers: 04.60.-m, 95.36.+x, 98.80.-k

Introduction

A fairly generic fact in describing of nature is that the low energy behavior of a system is largely independent on the details of what is going on in higher energy scales. For instance, the actual physical theory based on quantized fields is an effective description applicable at some energy scale. In cosmology we have lots of particle physics involved at various energy scales. In this regard it is important to look carefully at the coupling of gravity with the effective field theory in the cosmological context. The effective field theory description usually includes characteristic UV energy scale Λ. This fact immediately underlines that there is a certain amount of frame dependence in any effective field theory description. For our consideration it is cosmological frame, that is, a preferred frame in which the Cosmic Microwave Background Radiation is spatially isotropic. For our consideration it is cosmological frame, that is, a preferred frame in which the Cosmic Microwave Background Radiation is spatially isotropic. For our consideration it is cosmological frame, that is, a preferred frame in which the Cosmic Microwave Background Radiation is spatially isotropic.

In early cosmology, for instance estimating the horizon volume \( S \) with UV cutoff \( \Lambda \), one arrives at the relationship between energy density of the background metric fluctuations takes the form

\[
\rho_{\text{vacuum}} \sim \frac{\Lambda^3(t)}{t} .
\]  

To illuminate this conjecture further let us notice that well-known quantum gravity arguments indicate the existence of a natural UV cutoff in nature. Such UV cutoff even when it is not set immediately by the Planck mass \( (m_P \sim 10^{19}\text{GeV}) \) is very high at the particle physics scale. Let us sketch one of the discussions of this kind. For an effective quantum field theory in a box of size \( l \) with UV cutoff \( \Lambda \) the entropy \( S_{QFT} \) scales as,

\[
S_{QFT} \sim l^3 \Lambda^3 .
\]

That is, the effective quantum field theory counts the degrees of freedom simply as the number of cells \( \Lambda^{-3} \) in the box \( l^3 \). Imposing black hole entropy bound \( S_{QFT} \lesssim S_{BH} \sim (l/l_P)^2 \) one arrives at the relationship between UV and IR cutoffs

\[
\Lambda \lesssim \frac{1}{l_P^{3/2} l^{1/3}} .
\]  

Albeit in Eq. (2) \( \Lambda \ll m_P \) whenever \( l^{1/3} \gg l_P^{1/3} \), still it is very high from the standpoint of particle physics we have in early cosmology. For instance estimating the horizon distance for radiation dominated epoch

\[
a(t) \propto t^{1/2} \quad , \quad l = a(t) \int_0^t \frac{d\xi}{a(\xi)} = 2t ,
\]
from Eq. (2) one finds that at EW phase transition when the universe was about \( \sim 10^{-12} \text{ sec} \), old \( \Lambda \sim 10^9 \text{ GeV} \). Clearly the EW theory that represents an effective field theory description of this phase transition is characterized by the EW scale \( \sim 10^2 \text{ GeV} \) and does not care about gravity as the dimensionless gravitational coupling \( G_N E^2 \) at this energy scale \( (E \sim 10^2 \text{ GeV}) \) is negligibly small. In general particle physics describing the cosmological plasma does not care about gravity as long as its temperature is much less than Planck energy \( T \ll m_p \). Before passing to the subject of our discussion let us recall some of the lore concerning QFT vacuum energy.

**Vacuum energy in view of particle physics**

Particle physics contributes to the vacuum energy in two different ways [4]. First, we have the divergent Nullpunktsenergie characteristic of generic quantum field theories. Second, according to the SM, and even more so in its unified extensions, what we commonly regard as empty space is full of condensates. On the quite general grounds, as long as QFT respects Lorentz invariance, one infers that the vacuum energy mimics the cosmological constant [5], to wit: the vacuum energy density defined as a

\[
\langle 0 | T_{\mu\nu} | 0 \rangle = (0|T_{00}|0) \eta_{\mu\nu} . \tag{3}
\]

**Nullpunktsenergie** - The vacuum energy density defined as a Nullpunktsenergie appears to be infinite. However, the infinity arises from the contribution of modes with very small wavelengths and for we do not know what actually might happen at such scales it is reasonable to introduce a cutoff and hope that a more complete theory will eventually provide a physical justification for doing so. Before going on let us make a brief comment on the regularization of Nullpunktsenergie. In presence of UV cutoff it is customary to set the energy density coming from Nullpunktsenergie as \( \sim \Lambda^4 \) (one can do so simply on the dimensional grounds). However, one should care the Eq. (3) to be satisfied [6]. Regularizations of the Nullpunktsenergie which respect the Lorentz symmetry of the underlying theory disfavor its quartic dependence on the UV scale, but rather it appears to depend quadratically on the UV scale, \( \sim m^2 \Lambda^2 \), where \( m \) is the mass scale of theory [6]. This point has attracted little attention hitherto for many authors still follow the old customary. Returning to the main stream of reasoning let us notice that in QFT the energy-momentum operator \( T_{\mu\nu} \) (and correspondingly the source of gravity \( (0|T_{\mu\nu}|0) \)) is not uniquely defined because of operator ordering. In the framework of QFT we are usually subtracting this (divergent) Nullpunktenergie which is equivalent to the normal operator ordering in \( T_{\mu\nu} \). Or equivalently in the path integral approach one observes that the equations of motion for matter fields are invariant under the shift of the matter Lagrangian by a constant that results in a new energy-momentum tensor

\[
T_{\mu\nu} \rightarrow T_{\mu\nu} + \text{const.} \eta_{\mu\nu} .
\]

Usually in the framework of QFT the vacuum energy \( H(0) = E_0|0\rangle \) is treated as an unphysical quantity that may be set arbitrarily\(^1\). Let us notice that in curved space-time one can still renormalize the Nullpunktenergie and set it completely arbitrary like in the case of Minkowski background [8].

**Vacuum condensates** - The origin of masses is a fundamental problem in particle physics. Conventionally, for generating masses to the particles a self-interacting scalar field is introduced in the SM that acquires a non-vanishing vacuum expectation value and breaks the electroweak symmetry down to the electromagnetic one. The classical, zero-temperature Higgs potential is of the form

\[
V_{\text{cl}}(\phi) = \text{const.} - \mu^2 \phi^2 + g \phi^4 , \tag{4}
\]

with \( \mu^2, g > 0 \) (usually \( g \) is small enough as to ensure the perturbative regime of the theory). The minimum of \( V_{\text{cl}} \) breaks the \( Z_2 \) symmetry as \( \phi \) acquires a non-vanishing vacuum expectation value \( \langle \phi \rangle = \mu / \sqrt{2g} \).

At high temperature there is an additional contribution to the scalar mass. Ignoring the gauge and fermion sectors, the effective potential is approximately of the form \( V_{\text{eff}}(\phi, T) \approx \text{const.} + \left( -\mu^2 + \frac{g T^2}{2} \right) \phi^2 + g \phi^4 . \) (5)

Diagrammatically, the thermal mass term in Eq. (5) arises from the quadratically divergent loop of Fig. 1, where

\(^1\) For a crystal the Nullpunktenergie represents the vibration energy of crystal molecules at a zero temperature that manifests itself even at a finite temperature and has therefore quite definite physical meaning, see for instance very readable popular book by Kaganov [7].
the UV divergence is cut off at momenta of order $\sim T$. This quadratically divergent diagram evidently provides a contribution of the order $\sim gT^2$ for temperatures large compared to the scalar mass $\mu$. The addition of the thermal mass term above is responsible for symmetry restoration at high temperature. Namely for $T \gtrsim \mu/\sqrt{g}$, the effective potential has a positive coefficient for $\phi^2$ and then the minimum occurs at $\langle \phi \rangle = 0$. If we adjust the constant in Eq. (5) to get the zero effective potential for $T < \mu/\sqrt{g}$, then we have to put up with a very large cosmological constant before the electroweak phase transition, $V_{\text{eff}}(\phi = 0) \propto \mu^4/g$. Nevertheless, as it was observed in \cite{11}, this huge cosmological constant is negligible compared to the radiation energy present at that time, $T \sim \mu/\sqrt{g} \Rightarrow \rho_{\text{rad}} \sim \mu^4/g^2$. Thus, we see that the vacuum energy density is suppressed by a large factor $1/g$ compared to the radiation energy density and can be safely ignored notwithstanding its large value.

The Higgs mechanism by means of which the standard models particles (gauge bosons, leptons and quarks) acquire the mass has little to do with the origin of mass of the observable world, it is necessary to clarify the origin of nucleon mass. The nucleon consists of $u$ and $d$ quarks. But the masses of $u$ and $d$ quarks are very small compared to the mass of nucleon, $m_u + m_d \simeq 10\text{MeV}$ and their contribution to the nucleon mass is about 2 percent. It can be shown that even in the formal limit $m_u, m_d \to 0$, the nucleon mass remains practically unaltered. The nucleon mass arises due to spontaneous violation of chiral symmetry in QCD and can be expressed through the chiral symmetry violating vacuum condensates \cite{12}.

In addition there are neutrino masses connected with the energy scale beyond the SM. This energy scale ranges from $\sim \text{TeV}$ to the GUT scale $\sim 10^{16}\text{GeV}$ with respect to the different scenarios. The underlaying physics still awaits better understanding, but in any case this energy scale does not appear to be of great importance for what follows.

To summarize, the vacuum energy density coming from the SM particle physics can be made consistent with the observed value of dark energy density by adding a suitable counter term to $\langle T_{\mu\nu} \rangle$

$$\langle T_{\mu\nu} \rangle \to \langle T_{\mu\nu} \rangle + \text{const.} \eta_{\mu\nu},$$

but certainly it does not explain anything concerning the origin of dark energy. It merely tells us that the SM particle physics does not contradict the observed value of dark energy density.

### Effective field theory

The modern view is to regard our fundamental theories, the standard model of particle physics and general relativity, as a low energy effective theories. The renormalizability technically corresponds to the possibility of sending the energy cutoff $\Lambda$ of a system to infinity (while keeping all the physical quantities finite). Physically this means that the theory can be extrapolated to infinitely small distances without encountering new microscopic structures. However, we have no good reason to suspect that the effects of our present theory are the whole story at the highest energies. Happily enough, we do not need to know what is going on at all scales at once in order to figure out how nature works at a particular scale. Effective field theory allows us to make predictions at low energies without making unwarranted assumptions about what is going on at high energies. Various important energy scales of particle physics come into play in cosmology \cite{13}. Let us briefly sketch the interplay between particle physics and cosmology in the early universe. The melting of particle physics and cosmology leads to the expectation that cosmic plasma in the early universe underwent a few phase transitions related to the grand unified symmetry breaking\(^\text{3}\), to the electroweak symmetry breaking, during which SM particles acquire the mass, and chiral symmetry breaking of strong interaction. Then the evolution of the early universe proceeds according to known high energy physics. During the early evolution of the universe the temperature and density of the cosmic plasma are very high and respectively collisions are exceedingly frequent to keep different species of particles in thermal equilibrium with each other. As the universe expands and cools, the reaction rates begin to lag behind the expansion rate and various particle species drop out of equilibrium, or as it is said freeze out/decouple. Being direct experimental signature, it is important to study the relic abundance of decoupled species. When the temperature of the universe drops down to $T \sim 1\text{MeV}$ the weak interactions become frozen, neutrinos decouple from the rest of matter and shortly thereafter neutrons and protons cease to inter-convert. Soon thereafter, as the temperature drops somewhat below the nuclear binding energy $\sim 1\text{MeV}$,

---

\(^2\) The idea of high temperature electroweak symmetry restoration was originally put forward by Kirzhnits on the bases of similarity between Higgs mechanism and the Ginzburg-Landau theory of superconductivity \cite{10}.

\(^3\) Most likely the universe did not undergo the GUT phase transition. The point is that the inflation energy scale, $E_{\text{inflation}}$, is bounded from above by (non) observation of tensor fluctuations of the cosmic microwave background radiation (relict gravitational wave background) \cite{13}, with the current limit being $E_{\text{inflation}} \lesssim 10^{16}\text{GeV}$ \cite{14}. 


namely around $T \sim 100$ keV, Big-Bang nucleosynthesis begins. The neutrons combine with protons into light nuclei, mostly helium-4, but also deuterium, helium-3, lithium-7 and others. These elements remain in the universe, so that their primordial abundance is measurable today. Big-Bang nucleosynthesis ended when the universe was about $t \sim 200$ sec old. Much later occurred hydrogen recombination at $T \sim 0.2$ eV, $t \sim 3 \times 10^8$ yrs, after that the cosmic plasma became electrically neutral and CMBR decoupled from the rest of matter. Now turning to our discussion, the question is to get good understanding of $\Lambda(t)$ that enters the Eq.(1). We said nothing about the dark matter. If all the matter were baryonic, by noticing that gravity is the dominant force for large scale structure of the universe, one could find it well motivated to take $\Lambda(t) = \Lambda_{QCD}$ after the QCD phase transition as this energy scale is responsible for generating (most of) the mass of the baryonic matter. That is, in this case one could find it convincing that the effective field theory appropriate for large scale structure of the universe should be that one describing the origin of the nucleon mass. As it was mentioned in the previous section, the mass of nucleon arises due to chiral symmetry breaking of strong interaction the characteristic energy scale for which is set by the $\Lambda_{QCD}$ [12]. Indeed, by substituting $\Lambda_{QCD} \simeq 170$ MeV and $t_0 \simeq 10^{60} t_P$ in Eq.(1), one gets pretty good value for observed dark energy density. Such dark energy decays linearly with time and thereby can not spoil the successes of early cosmology. Assuming that this energy component dominates presently

$$H^2 = \frac{8\pi}{3m_P^2} \rho_{\text{vacuum}} ,$$

the equation of state can be simply estimated by using energy-momentum conservation

$$p = -\frac{\dot{\rho}}{3H} - \rho ,$$

giving

$$\rho_{\text{vacuum}} \simeq \sqrt{\frac{m_P^2 \Lambda_{QCD}^2}{24\pi t^3}} - \frac{\Lambda_{QCD}^3}{t} . \quad (6)$$

The second term in Eq.(4) becomes dominant for

$$t \gtrsim \frac{m_P^2}{24\pi \Lambda_{QCD}^2} \simeq 10^{58} t_P .$$

So, this dark energy exhibits a negative pressure just recently. This scenario was proposed in [2]. Let us notice that linearly decaying dark energy has been proposed earlier in [16] on the bases of Dirac’s large number hypothesis.

Now, let us summarize the main lines of our discussion and take note of the fact that the matter content of the universe is dominated by the dark matter.

Conclusion

As is well known the melding of quantum theory and gravity is marked with an intrinsic UV cutoff. Namely, the combination of relativistic and quantum effects implies that the conventional notion of distance breaks down the latest at the Planck scale. Physically the emergence of this UV cutoff is understood as a result of background metric fluctuations. The coupling of gravity with QFT implements this UV cutoff into QFT. While the concrete realization of this picture may involve many specifics, see for instance well known example of this kind implemented through the generalized uncertainty relations [17], physically it is this fluctuation term that renders the UV cutoff of QFT bounded from above. That is, UV scale of QFT can not be greater that the resolution of the background space. The question that immediately occurs is, is not the converse statement true? That is, can the effective field theory describing the processes with spatial resolution $\Lambda^{-1}$ see the background space to a better accuracy than $\Lambda^{-1}$? This is the conjecture put forward in [2]. This question is not only interesting in its own right; it could also cast new light on dark energy. Two important questions that immediately occur are as follows. First, effective field theory is characterized with some energy scale, thereby reflecting appreciable amount of frame dependance. In our discussion we assume the cosmological frame defined by the Cosmic Microwave Background Radiation. Second question is to get good understanding of $\Lambda(t)$ in the cosmological context. The dominant force for large scale structure of the universe is gravity. Presently the matter outweighs the radiation by a wide margin. The most natural assumption would be that the effective field theory appropriate for large scale structure of the universe should be that one describing the origin of the mass of matter. The mass of the visible matter we see around us and are part of comes overwhelmingly from nucleons. Operating simply by the visible matter one would find it motivated to hold $\Lambda(t) \simeq \Lambda_{QCD}$ after the QCD phase transition. But visible matter contributes to the energy budget of the universe about 5 percent while dark matter is about 25 percent. Albeit they are of the same order, matter in the universe is more dark than visible. Therefore, from our discussion one finds it reasonable to insert in Eq.(1) the energy scale appropriate to the dark matter (rather then visible matter)

$$\rho_{\text{vacuum}} \simeq \frac{\Lambda_{DM}^3}{t} .$$

The question of energy scale $\Lambda_{DM}$ appropriate to an effective field theory describing the origin of mass(es) of dark matter particle(s) is not yet understood well. However, if the presented conjecture really works, the black hole energy bound on Eq.(1) for present value of horizon distance $l_0 \sim 10^{60} t_P$, that is to require $l_0 \gtrsim \frac{\Lambda_{DM}^3}{t} \rho_{\text{vacuum}}$, puts an upper limit on $\Lambda$ to be of the order of $\sim 1$ GeV. That is, in this case $\Lambda_{DM}$ could not be far above the
\[ \Lambda_{QCD} \]. For the moment we can say nothing definitely about \( \Lambda_{DM} \), but certainly it would be nice this energy scale to be not very far from \( \Lambda_{QCD} \).

This work was supported by the CRDF/GRDF and the Georgian President Fellowship for Young Scientists.

[1] N. Sasakura, Prog. Theor. Phys. 102 (1999) 169, hep-th/9903146; M. Maziashvili, Int. J. Mod. Phys. D16 (2007) 1531, gr-qc/0612110.
[2] M. Maziashvili, Phys. Lett. B663 (2008) 7, arXiv: 0712.3756 [hep-ph].
[3] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Rev. Lett. 82 (1999) 4971, hep-th/9803132.
[4] S. Weinberg, Rev. Mod. Phys. 61 (1989) 1.
[5] Ya. B. Zeldovich, Pisma Zh. Eksp. Teor. Fiz. 6 (1967) 883; Usp. Fiz. Nauk 95 (1968) 209.
[6] E. Kh. Akhmedov, hep-th/0204048; G. Ossola and A. Sirin, Eur. Phys. J. C31 (2003) 165, hep-ph/0305050.
[7] M. I. Kaganov, *Electrons, Phonons, Magnons*, (Moscow, Nauka, 1979).
[8] L. H. Ford, gr-qc/9707062.
[9] P. Arnold and O. Espinosa, Phys. Rev. D47 (1993) 3546, Erratum-ibid. D50 (1994) 6662, hep-ph/9212235.
[10] D. A. Kirzhnits, Pisma Zh. Eksp. Teor. Fiz. 15 (1972) 745; D. A. Kirzhnits, Uspeh Fiz. Nauk, 125 (1978) 169.
[11] S. Bludman and M. Ruderman, Phys. Rev. Lett. 38 (1977) 255.
[12] B. L. Ioffe, Nucl. Phys. B188 (1981) 317, Erratum - Nucl. Phys. B191 (1981) 591; Usp. Fiz. Nauk, 171 (2001) 1273, hep-ph/0104017; Prog. Part. Nucl. Phys. 56 (2006) 232, hep-ph/0502148.

Usp. Fiz. Nauk, 176 (2006) 1103, hep-ph/0601250.
[13] G. Steigman, Ann. Rev. Nucl. Part. Sci. 29 (1979) 313; Ann. Rev. Nucl. Part. Sci.57 (2007) 463, arXiv: 0712.1100 [astro-ph]; A. D. Dolgov and Ya. B. Zeldovich, Usp. Fiz. Nauk, 130 (1980) 559; Rev. Mod. Phys. 53 (1981) 1; D. N. Schramm and M. S. Turner, Rev. Mod. Phys. 70 (1998) 303, astro-ph/9706069; J. R. Ellis, astro-ph/0305038.
[14] V. Rubakov, M. Sazhin and A. Veryaskin, Phys. Lett. B115 (1982) 189; R. Fabbri and M. Pollock, Phys. Lett. B125 (1983) 445; L. Abbott and M. Wise, Nucl. Phys. B244 (1984) 541; A. Starobinsky, Sov. Astron. Lett. 11 (1985) 133.
[15] D. Spergel et al., Astrophys. J. Suppl. 170 (2007) 377, astro-ph/0603449.
[16] A. Zee, in *High Energy Physics: in Honor of P. A. M. Dirac in his Eighties Year* (Eds. S. L. Mintz and A. Perlmutter, Plenum Press, New York, 1985); Mod. Phys. Lett. A19 (2004) 983, hep-th/0403064.
[17] A. Kempf and G. Mangano, Phys. Rev. D55 (1997) 7009, hep-th/9612084.