Ranging Optimization and Research on Indoor 3D Positioning Algorithm Based on UWB

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Abstract. Nowadays, the indoor space of people is more and more vast and complex, so the service of positioning and guidance in parking lots, shopping malls, and airport is increasingly needed. Thus, especially indoor 3D positioning, IPS (Indoor Positioning System) has the chance to develop rapidly. This paper proposed UWB as the indoor 3D positioning technology and developed a method to improve TWR (tow-way-ranging) algorithm to enhance the precision of indoor 3D positioning. Furthermore, a 3D positioning algorithm based on TOF ranging with the LSE (Least Squares Estimation) method is introduced. Lastly, the simulation results in Matlab are showed to prove that this algorithm can achieve a relatively accurate 3D positioning precision, which is meaningful and practical for many applications.

1. Introduction

Ultra wideband (UWB) is a wireless communication technology, which utilizes very short pulses with the duration of less than 1 ns. The short duration of the UWB waveform enables the system to provide high data rate communication as well as accurate positioning [1]. Thus, UWB is widely used for indoor precise positioning with the rapid development of LSB (Location Based Services) and WSN (Wireless Sensor Network). Nevertheless, currently, most positioning algorithms are based on tow-dimensional plane, but with the expansion of positioning space, so that we need to prove 2D node positioning algorithm to get 3D algorithm.

There are so many wireless positioning algorithms, such as AOA (Angle of Arrival) [2], RSSI (Received Signal Strength Indication) [3], TDOA (Time Difference of Arrival) [4], TOA (Time of Arrival) [5] and TOF (Time of Fight), the last one is a widely used location algorithm, because it does not require any clock synchronization and offset the clock reference of nodes by the interaction of two signals back and forth. Therefore, using the TOF for positioning calculations simplifies the complexity of the positioning system to some extent. In 3D space, there needs four known anchor nodes and one target tag node which can mobile in the area, and then the target node sends the data to the positioning server that calculates the relative location of the tag.

In this paper, the aim is to introduce an improved ranging method and a 3D indoor positioning algorithm. The main method for ranging is the TOF based on TWR, and lastly, the Least Squares Estimation (LSE) is used for getting the coordinates of the target node. The research can be used in indoor 3D positioning and is also meaningful for further development of the positioning methods.
2. Program Framework and Algorithm Design

2.1. Ranging Algorithm based on Time of Fight
The distance between anchor and tag in the system usually is calculated by a method called two-way ranging algorithm[6], which is the one of classical ranging methods and do not require clock synchronization like TOA or TDOA, while offsetting the reference difference of the respective clock by interacting two signals back and forth. The algorithm execution process is as follows in figure 1.

The tag sends a Poll message to the anchor and notes the send time, $T_{sp}$ and it listens for the Response message, and if no response arrives after some period the tag will time out and send the poll again.

The anchor listens for a Poll message addressed to it. When the anchor receives a poll it notes the receive time $T_{rp}$ and sends a Response message back to the tag, noting its send time $T_{sr}$.

When the tag receives the Response message it notes the receive time $T_{rr}$ and sets the future send time of the Final response message $T_{sf}$, it embeds this time in the message before initiating the delayed sending of the Final message to the anchor. The anchor receiving this Final response message (at $T_{rf}$) now has enough information to work out the range.

So, we can get first round-trip time $T_{round1}=T_{rr}-T_{sp}$, first turn around time $T_{reply1}=T_{sr}-T_{rp}$, second round trip time $T_{round2}=T_{rf}-T_{sr}$ and second turn around time $T_{reply2}=T_{sf}-T_{rr}$. Lastly, one-way fight time TOF is:

$$ TOF = \frac{(T_{round1}-T_{reply1}+T_{round2}-T_{reply2})}{4} \quad (2.1) $$

$$ = \frac{(T_{rr}-T_{sp})-(T_{sr}-T_{rp})+(T_{rf}-T_{sr})-(T_{sf}-T_{rr})}{4} \quad (2.2) $$

Thus, the distance of the two nodes is (C is the speed of light):

$$ D = TOF \times C \quad (2.3) $$

![Figure 1. TOF Flow Chart](image)
2.2. Indoor 3D Positioning Algorithm

Based on the distance using the TOF between a tag and multiple known anchors, the spherical positioning technique is an effective method in the case of ideal error-free ranging estimation. Here, we assume that the coordinates of the three anchors in 3D space are \((x_1, y_1, z_1)\), \((x_2, y_2, z_2)\), \((x_3, y_3, z_3)\), and the coordinate of tag is \((x, y, z)\), three spheres intersect at one point, as shown in Figure 2.

There is a group of equations:

\[
\begin{align*}
(x_1-x)^2 + (y_1-y)^2 + (z_1-z)^2 &= R_1^2 \\
(x_2-x)^2 + (y_2-y)^2 + (z_2-z)^2 &= R_2^2 \\
(x_3-x)^2 + (y_3-y)^2 + (z_3-z)^2 &= R_3^2
\end{align*}
\]

Figure 2. 3D spherical location method.

In fact, indoor positioning systems usually use more anchors to improve the positioning accuracy, so there will be over-determined equations to solve it commonly using the LSE [7]. Thus, it is assumed that \(X=[x, y, z]^T\) is the coordinate vector of the tag and \(X_n=[x_n, y_n, z_n]^T\) is the vector of the nth anchor coordinates, so that the distance \(D_n\) between tag and nth anchors is:

\[
D_n = \|X_n - X\| = \left( (x_n-x)^2 + (y_n-y)^2 + (z_n-z)^2 \right)^{1/2}
\]

Since \(D_n\) has been obtained by UWB ranging and \(X\) is known, the calculation of the tag coordinate is to solve the following three-dimensional nonlinear equations:

\[
\begin{align*}
\left( (x_1-x)^2 + (y_1-y)^2 + (z_1-z)^2 \right)^{1/2} &= D_1 \\
\left( (x_2-x)^2 + (y_2-y)^2 + (z_2-z)^2 \right)^{1/2} &= D_2 \\
\ldots & \ldots \\
\left( (x_n-x)^2 + (y_n-y)^2 + (z_n-z)^2 \right)^{1/2} &= D_n
\end{align*}
\]

In the above formulas, \(n\) represents the number of anchor nodes, and satisfies the condition \(n\geq 4\). When the number of anchors is exactly four, that is, \(n=4\), the system of equations can just find the exact solution. However, in order to improve the positioning accuracy, the number of anchors is usually more than four \((n>4)\). In the case, the equation is an overdetermined nonlinear equation group. It generally does not have an exact solution, so it needs to be solved by using simple LSE.
3. Improvement of Algorithm

3.1. Ranging Optimization

In section 2.1, equation (2.2), we can find that Tsp, Ts, Tsf is the RMARKER of each transmission frame (the moment when the first pulse of the data frame leaves the transmitting antenna), which contains the delay TX_ANTD from the transmitting circuit to the transmitting antenna. While Tr, Trp, Ttf is the RMARKER of each reception frame (the moment when the first pulse of the data frame reaches the receiving antenna), which includes delay LDE_RXANTD from receiving antenna to receiving circuit.

Thus, the accuracy of the TX_ANTD and LDE_RXANTD values affect the TOF, which have a further effect on the measurement distance between the two nodes. The adjustment method is as follows:

1. Letting the transmitting and receiving antenna be at an LOS standard distance, such as 5m.
2. Selecting empirical value DWT_PRF_64M_RFDLY(515.6f) or DWT_PRF_16M_RFDLY(515.0f) according to pulse PRF (Pulse Repetition Frequency).
3. Performing send and receive experiments again, comparing the distance and standard distance calculated according to formula (1) above. Then adjusting the antenna delay value until the calculated distance is equal to the standard, so that we can obtain accurate antenna delay value.

3.2. 3D Positioning Algorithm Optimization

Simple LSE algorithm is usually affected by the ranging error. In addition, the least square method is used to solve the problem by subtracting the nth equation from the n1 equation and the precision of the solution is affected by the error of the nth equation. In this section, we propose a joint positioning algorithm that combines the LSE algorithm and the Taylor [8] series expansion method. This iterative algorithm is suitable for various channel environments, but it requires that the coordinate estimate of the initial position of the positioning is close to the actual position coordinate value, so as to guarantee the convergence and accuracy of the algorithm. The algorithm execution process is as follows:

First, defining function f_i(x, y, z):

\[ f_i(x, y, z) = \left( (x_{i+1} - x)^2 + (y_{i+1} - y)^2 + (z_{i+1} - z)^2 \right)^{1/2} - \left( (x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2 \right)^{1/2} \] (3.1)

And \( d_i \) is the estimated distance difference, and \( \epsilon_i \) is the covariance matrix of the estimated distance difference error. Assuming that the initial value of \((x, y, z)\) is \((x_0, y_0, z_0)\), then \( x = x_0 + \delta_x, y = y_0 + \delta_y, z = z_0 + \delta_z \) can be obtained, and \( \delta_x, \delta_y, \delta_z \) are the positioning estimation error.

Expanding the function \( f_i(x, y, z) \) with Taylor series, and retaining the first two terms:

\[ f_i\hat{v} = f_i(x, y, z) + \frac{\partial f_i}{\partial x} \delta_x + \frac{\partial f_i}{\partial y} \delta_y + \frac{\partial f_i}{\partial z} \delta_z = d_i + \epsilon_i \] (3.3)

Then,

\[ f_i\hat{v} = f_i(x, y, z) \]

\[ \begin{align*}
\alpha_{i,1} & = \frac{\partial f_i}{\partial x} \bigg|_{x=x_0, y=y_0, z=z_0} = \frac{x_{i+1} - x_1}{d_i} + \frac{x_{i+1} - x_0}{d_{i+1}} \\
\alpha_{i,2} & = \frac{\partial f_i}{\partial y} \bigg|_{x=x_0, y=y_0, z=z_0} = \frac{y_{i+1} - y_1}{d_i} + \frac{y_{i+1} - y_0}{d_{i+1}} \\
\alpha_{i,3} & = \frac{\partial f_i}{\partial z} \bigg|_{x=x_0, y=y_0, z=z_0} = \frac{z_{i+1} - z_1}{d_i} + \frac{z_{i+1} - z_0}{d_{i+1}}
\end{align*} \]

The weighted least squares estimate of equation (3.3) is expressed as:

\[ A \delta = D + e \] (3.4)
Finally, rewriting formula (3.4) as:

\[ \delta = [A^T A]^{-1} A^T D \]  

(3.5)

According to the formula (3.5), \( \delta \) is calculated from the initial estimated position \( (x_v, y_v, z_v) \), and updating \( (x_v, y_v, z_v) \) continuously according to the following formula (3.6) until the required precision is reached or the predetermined number of iterations is reached. Finally, the last \( (x_v, y_v, z_v) \) is the positioning coordinate of the target node.

\[ (x_v, y_v, z_v) \rightarrow (x_v+\delta_x, y_v+\delta_y, z_v+\delta_z) \]  

(3.6)

The result of the least-squares method is used as the initial value, and then the Taylor series is iterated until the final convergence or accuracy is achieved. The final result is the desired position coordinate.

4. Simulation Results and Analysis

In order to verify the above inference, we have tested the LSE-Taylor algorithm with the improved ranging method in Matlab, figure 3 shows the four known anchor nodes that are separately located at \( (0,0,0), (0,5,0), (5,0,0), (0,0,5) \) and the size of space is 5m×5m×5m.

In addition, although we have conducted several experiments, we only show the three representative results in Fig.3. Thus, we can find that the location of the tag is very close to the true location, which has a relatively accurate precision in LOS.

![Figure 3. Location Simulation](image)

So, the precision of ranging method directly improves the positioning and the development of the location further enhances the accuracy.

5. Conclusion

In this paper, we first proposed a 3D location algorithm based on TOF with the LSE. Then, according to analysis, ranging error has a great influence on LSE algorithm, so that we improved the ranging and positioning algorithm. Lastly, using LSE-Taylor joint positioning algorithm to achieve more...
accurate positioning which could be useful for some location scenes and positive effect on further researches.

6. References
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