Spectral transformation-based technique for reducing effect of limited pre-correlation bandwidth in the GNSS receiver filter in presence of noise and multipath

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Abstract: This paper proposes an efficient scheme to reduce the pre-correlation bandwidth effect in the global navigation satellite system (GNSS) receiver filtering process. It is mainly based on the application of a spectral transformation to the satellite-emitted signal that effectively reduces its band. At the receiver's end, this operation causes the spreading of noise over a much wider band than that used by the radio frequency stage. Consequently, the resulting auto-correlation function in the acquisition process acquires properties that enhance considerably the performance of the receiver in the presence of the multipath and noise disturbing phenomena. The simulation results demonstrate that the proposed method is a plausible solution for both multipath and noise problems in the GNSS applications for any limited value of the pre-correlation bandwidth in the receiver filter.

Keywords: precorrelation bandwidth, spectral transformation, multipath, noise.

DOI: 10.23919/JSEE.2020.000003

1. Introduction

Multi-path (MP) signal propagation is widely identified as a major source of the error in the global navigation satellite system (GNSS), including the global positioning system (GPS) [1], global navigation satellite system (GLONASS) [2], Galileo [3] and many others.

The noise and MP signals seriously degrade the positioning accuracy of the GNSS receivers since they cause code-tracking errors in the delay locked loop (DLL), where a set of correlators combination is used to achieve the tracking process [4].

The most conventional structure of code tracking is based on a feedback delay estimation known as the early-minus-late (EML) technique. Unfortunately, the classical EML is unable to cope efficiently with the MP and noise drawbacks [5]. As a result, a number of improved EML-based techniques are introduced to mitigate the impact of these two problems. One class of such improved schemes, known as narrow EML (nEML) or narrow correlator (NC), is based on the idea of reducing the spacing between early and late correlators [6]. Another class, relying on the DLL variant, uses more than two correlators in the tracking loop [6] for the so-called double-delta (ΔΔ) GNSS receivers. The ΔΔ technique, in the case of older generation applications with a medium-to-long MP delay and a good carrier-to-noise ratio (C/N0), offers a better MP rejection [7]. A few well-known particular cases of the ΔΔ technique are the high-resolution correlator (HRC) [6], the strobe correlator (SC) [8], the pulse aperture correlator (PAC) [9] and the modified correlator reference waveform [10,11].

Nevertheless, these techniques need to be adapted to the new generation (NG) binary offset carrier (BOC) modulated signals. Such an adaptation requires the knowledge of mathematical models of different functions characterizing GNSS signals such as the auto-correlation function (ACF), the power spectral density (PSD), the discriminator function (DF) and the MP error envelopes (MEE). For this reason, the analytical models of the ACF, DF and MEE for the coherent EML (C-EML) DLL configuration in sine-BOC receivers were established firstly in [12,13]. Meanwhile, the non C-EML (NC-EML) analytical models were also proposed in [14]. Afterwards, the theoretical models of the ACFs and the PSDs for all BOC and modified BOC (MBOC) signals, based on their statistical pro-
Recently, Zitouni et al. [16] improved the work presented in [14,15] by proposing the analytical models of the ACF, DF and MEE for both BOC\(\cos(\alpha, \beta)\) and MBOC modulated signals in the C-EML and NC-EML tracking configurations.

Based on all previous models, and in order to adapt the conventional structures to the new variants of GNSS signals, several NG tracking techniques were proposed in the literature, such as the multiple gate delay (MGD) correlator model [7] whose parameters are the number of early and late gates and the weighting factors. These parameters, which are used to be combined in the discriminator, can be optimized according to the MP profile as illustrated in [7].

Another NG tracking configuration, which is closely related to the \(\Delta \Delta\) technique, is the tracker Early1/Early2 (E1/E2) [6,17]. In this latter structure, the main goal is to find a tracking point on the correlation function (CF) which is not distorted by the MP. In [6], the tracker E1/E2 showed some improvement in performance, compared to the \(\Delta \Delta\) technique, but only for very short MP delays in the case of GPS L1 coarse/acquisition (C/A) signals.

One more NG technique called the early-late slope (ELS), also known as the MP elimination technique (MET), was proposed in [18]. The simulation results of the comparative study in [6], using the MEE performance criterion, showed that the ELS was exceeded by the HRC for binary phase shift keying (BPSK) and sine-BOC modulated signals.

The recent NG approach, called a-posteriori MP estimation (APME), was proposed in [19] for estimating the MP. It reposes on the a-posteriori estimation of the MP tracking errors, which is made independently with an MP estimator based on the correlation values derived from the phase and the correlator delays.

In [20], a completely different scheme was used to solve the MP problem in GNSS receivers. The proposed technique, called tracking error compensator (TrEC), uses the invariant properties of the MP in the received CF to provide significant performance advantages over the nEML for narrowband GPS receivers [20,21].

One of the most promising advanced MP mitigation techniques is the MP estimating DLL (MEDLL), implemented for GPS receivers [22]. The MEDLL is considered as a significant progress in the attenuation of MP effects in the GNSS receiver [22,23]. It uses several correlators to determine accurately the ambiguity created by the MP in the CF. According to [23], the MEDLL provides higher MP attenuation performance than nEML [24]. Its principle consists of estimating all the parameters of line of sight (LOS) and MP signals such as delays, amplitudes and phases.

Then, the MP signals are eliminated, leaving the LOS path to be tracked alone.

However, at this point, it is worth to notice that all above mentioned methods are seriously and simultaneously affected by the two contradictory issues caused by noise and receiver pre-correlation bandwidth (P-BW). In fact, in practice, the receiving filter P-BW is usually limited to minimize the noise and interferences effects [25].

This P-BW must be sufficiently wide to accommodate most of the relevant ACF’s frequency components, and thus avoid the distortion of the magnitude and phase responses of the ACF (or the resulting DF). Moreover, the P-BW not only determines the sharpness of the filter magnitude response, but also quantifies the high frequency noise level introduced by the filter itself [26].

Actually, it is well-known that the shape of the ACF is a critical factor that dominates the tracking performance of the DLL discriminator design in terms of accuracy and stability [27]. The change in the parameters of the precorrelation filter will influence the discriminator performance. In fact, the choice of the infinite P-BW represents the ideal case and provides a sharper ACF, also increases the noise power at the same time, which disturbs the measurements and causes bias towards the localization process. In contrast, a limited P-BW deforms the correlation peak and makes it rounded, causing a measurement offset [28].

In this paper, we propose an efficient method to reduce the noise and MP effects on GNSS applications. It is mainly based on the application of a spectral transformation to the satellite-emitted signal that effectively reduces its band and makes it insensitive to the receiving filter P-BW limitation. This paper is organized as follows: firstly, a description of the GNSS signal in the presence of the MP and noise is given followed by the presentation of the MP and the receiving filter P-BW limitation effects. After that, the principle of our proposed method is presented. Then, based on simulation tests, a comparative study among the proposed method and the classical ones is carried out to experiment the robustness in the presence of the MP and noise. Finally, we end up with a conclusion.

2. GNSS signal in MP and noise environments

In presence of the noise, the received signal can be expressed as follows:

\[
s(t) = \sum_{i=1}^{M} \sqrt{P_i(t)} D_i(t - \tau_i) C_i(t - \tau_i) + W(t)
\]

where \(P_i(t)\) is the instantaneous power of the \(i\)th satellite received signal; \(M\) is the number of visible satellites;
$D_i(t)$ is the $i$th satellite signal navigation data; $C_i(t)$ is the pseudo random noise (PRN) code and subcarrier corresponding to the $i$th satellite; $\tau_i$ is the $i$th received signal delay; $f_{fi}$ is the $i$th received signal carrier frequency; $f_{Di}$ is the $i$th received signal Doppler frequency shift; $\varphi_i$ is the $i$th received signal carrier phase; $W(t)$ is the additive white Gaussian noise (AWGN).

At the pre-correlation filter output (after passing through the intermediate frequency stage), a signal received from only one GNSS satellite and affected by MP signals, can be given as follows:

$$s_r(t) = \text{Re}\left( \sum_{l=0}^{N} a_l C_{fi}(t - \tau_l) D_{fi}(t - \tau_l) \right) \exp[j2\pi(f_{IF} + f_{Di})t + \varphi_l] + n(t)$$  \hspace{1cm} (2)

where $N$ is the number of MP components; $\tau_i$ is the LOS or MP signal delay; $\varphi_i$ is the LOS or MP signal phase; $f_{IF}$ is the intermediate frequency (IF); $f_{Di}$ is the Doppler frequency; $a_i$ is the LOS or MP signal coefficient amplitude; $n(t)$ is the narrow band noise; $C_{fi}(t)$ is the filtered PRN code and subcarrier; $D_{fi}(t)$ is the filtered navigation data.

The most important characteristics of each MP component are given as follows [29–31]:

(i) Its path is generally longer than that of the LOS.
(ii) Its power is lower than that of the LOS.

In the following, it assumes that there is only one reflected component that presents a phase difference of 0° or 180° with the LOS; both values correspond to the maximum error that the GNSS receiver can reach.

### 3. GNSS receiver major limitations

In GNSS receivers, the propagation delay estimation requires synchronization between the received and the locally generated signals. This is accomplished by using a correlation process as follows:

$$R(\tau) = \int_{0}^{T} s_r(t)s_L(t + \tau)dt$$  \hspace{1cm} (3)

where $\tau$ is the phase shift of the locally generated code; $s_r(t)$ is the received signal envelope; $s_L(t)$ is the locally generated code and subcarrier; $T$ is the integration time.

The composite BOC is shorten as CBOC, and time-multiplexed BOC is shorten as TMBOC. In Fig. 1, we illustrate the normalized ACFs of BOCsin(1,1), CBOC(6,1,11,1)+, TMBOC(6,1,4/33) and BOCcos(1,1) modulated signals for the infinite P-BW [32]. As a result, the composite ACF and its corresponding S-curve are distorted, causing a bias in the zero-crossing point as shown in Fig. 3 for infinite and 4 MHz P-BWs, respectively.
4. Principle of the proposed method

4.1 Signal processing at the emitter

The following processing steps are performed on the pilot GNSS signal $x_p(t)$ (without any data message) before the transmission:

(i) The spectrum $X_p(f)$ of $x_p(t)$ is limited to a bandwidth $B$ by using a transmission filter.

(ii) As shown in Fig. 4 and Fig. 5, the limited bandwidth $B$ of $X_p(f)$, at the output of the transmission filter, is divided theoretically, using a uniform N-channel bank filter, into $N$ adjacent sub-bands $B_i$ ($i = 1, \ldots, N$) of equal bandwidths $B_i = f_N = B/N$.

(iii) The discrete Fourier transform (DFT) $X_i(K\nu_e)$ of each separately filtered $B_i$ signal ($i = 1, \ldots, N$) is now calculated and the obtained DFT coefficients are multiplied by the hadamard coefficients $C_i(K, \alpha)$ of values 1 and $-1$ ($K$ characterises the coefficient index and $\alpha$ is the number of repetition of each coefficient) to obtain the new weighted spectrums $X_{p_i}(K\nu_e)$ of the sub-bands $B_i$ signals ($i = 1, \ldots, N$), whose analytical models and corresponding frequency bands, for $\alpha = 1$ ($C_i(K, 1) = C_i(K)$), are given in Fig. 6. $\nu_e$ is the sampling period in the spectral domain.

In Fig. 6, we colour the different sub-bands to show the effect of weights in the spectral domain. In reality, the weighting operation causes the masking of the frequency response of each $B_i$ band filter ($i = 1, \ldots, N$).
Using Fig. 6, (5) and (6), the analytical models of stored and translated to the same carrier frequency are given in Fig. 7.

\[
X_p(Kv) = X_p(Kv_p)P_{s_{mp}}(Kv - f_p/2)C_i(K)
\]

\[
X_p(Kv) = X_p(Kv_p)P_{s_{mp}}(Kv - (N-1)f_p/2)C_2(K)
\]

\[
\vdots
\]

\[
X_p(Kv) = X_p(Kv_p)P_{s_{mp}}((N-1/2)f_p)C_n(K)
\]

Fig. 6 Weighted spectrums and their frequency bands

For a given case \( m \), the masked frequency response of the \( B_m \) band filter is given as follows:

\[
P_{w_m}(Kv_e) = P_{s_{mp}}(Kv_e - \left( m - \frac{1}{2} \right) f_N)C_m(K).
\] (5)

The corresponding \( X_{p_m}(Kv_e) \) spectrum is obtained as follows:

\[
X_{p_m}(Kv_e) = X_p(Kv_e)P_{w_m}(Kv_e).
\] (6)

(iv) The spectrums of the analog signals \( X_p(i) \) \((i = 1, \ldots, N)\) corresponding to each band \( B_i \), are then restored and translated to the same carrier frequency \( f_p \) to get \( G_i(f) \) spectrums, all centred on \( f_p \), and given by

\[
G_i(f) = X_p(f + \left( i - \frac{1}{2} \right) f_N - f_p) = X_p(f + \left( i - \frac{1}{2} \right) f_N - f_p)P_{w_i}(f + \left( i - \frac{1}{2} \right) f_N - f_p).
\] (7)

Using Fig. 6, (5) and (6), the analytical models of \( G_i(f) \) \((i = 1, \ldots, N)\) and their corresponding frequency bands are given in Fig. 7.

\[
G_1(f) = X_p(f + f_p/2 - f_p)P_{w_1}(f + f_p/2 - f_p)
\]

\[
G_2(f) = X_p(f + f_p/2 - f_p)P_{w_2}(f + f_p/2 - f_p)
\]

\[
\vdots
\]

\[
G_N(f) = X_p(f + (N-1)f_p/2 - f_p)P_{w_N}(f + (N-1)f_p/2 - f_p)
\]

Fig. 7 Modulation of the resulted weighted \( B_i \) signals spectrums

(v) Finally, the spectrums of all modulated \( B_i \) signals are summed up to obtain the overall narrowband emitted signal of bandwidth \( B/N \) given by

\[
S_{em}(f) = \sum_{i=1}^{N} G_i(f) = S_c(f - f_p)
\] (8)

where \( S_c(f) \) is the spectrum of the signal without the carrier.

By replacing \( G_i(f) \) with the expression given in (7), we obtain

\[
S_{em}(f) = \sum_{i=1}^{N} X_p(f + \left( i - \frac{1}{2} \right) f_N - f_p)P_{w_i}(f + \left( i - \frac{1}{2} \right) f_N - f_p).
\] (9)

\[
S_c(f) = \sum_{i=1}^{N} X_p(f + \left( i - \frac{1}{2} \right) f_N)
\]

\[
P_{w_i}(f + \left( i - \frac{1}{2} \right) f_N - f_p)\]

\[
(10)
\]

where \( f \in \left[ f_p - \frac{f_N}{2}, f_p + \frac{f_N}{2} \right] \).

4.2 Signal processing at the receiver

In the transmission channel, it is assumed that the transmitted signal \( s_{em}(t) \) is affected by a simple delay, a Doppler frequency and an AWGN. Thus, after the receiver preprocessing and the intermediate frequency conversion, the complex envelope of the received signal, in absence of the MP, can be written from (2) as follows:

\[
s_r(t) = s_e(t - \tau_0)\exp(i(2\pi(f_{IF} + f_D)t + \varphi_0)) + n(t)
\] (11)

where \( s_e(t) \) is the emitted baseband signal that represents the inverse Fourier transform of \( S_c(f) \); \( \tau_0 \) is the signal propagation delay.

To recover the received baseband pilot signal, the operations performed at the emitter level are reversed at the end of the receiver where the following processing steps are performed.

Step 1 Acquire the received signal

At the end of the receiver, after removing the IF frequency, the demodulated received signal \( s_{rd}(t) \), can be given from (11) as follows:

\[
s_{rd}(t) = s_e(t - \tau_0)\exp(i(2\pi f_D t + \varphi_0)) + n_d(t)
\] (12)

where \( n_d(t) \) is the demodulated narrow band noise.
The Fourier transform of (12), which represents the spectrum of the demodulated received signal, is given as follows:

\[ S_{rd}(f) = S_{c}(f - f_D) \exp(-j(2\pi f \tau_0 - \phi_0)) + N(f) \]  

(13)

where \( N(f) \) is the PSD of the narrow band noise \( n_d(t) \).

Substituting (10) into (13) gives

\[ S_{rd}(f) = \left( \sum_{i=1}^{N} X_p \left( f + \left( i - \frac{1}{2} \right) f_N - f_D \right) \right) \cdot P_{w_i} \left( f + \left( i - \frac{1}{2} \right) f_N - f_D \right) \cdot \exp(-j(2\pi f \tau_0 - \phi_0)) + N(f). \]  

(14)

The DFT \( S_{rd}(K\nu_e) \) of \( s_{rd}(t) \) can be given from (14) as follows:

\[ S_{rd}(f) = \left( \sum_{i=1}^{N} X_p \left( K\nu_e + \left( i - \frac{1}{2} \right) f_N - f_D \right) \right) \cdot P_{w_i} \left( K\nu_e + \left( i - \frac{1}{2} \right) f_N - f_D \right) \cdot \exp(-j(2\pi K\nu_e \tau_0 - \phi_0)) + N(K\nu_e). \]  

(15)

**Step 2** Recover the received \( B_i \) signals

The received and unweighted baseband \( B_i \) signals \( i = 1, \ldots, N \), noted by \( Y_{B_i}(K\nu_e) \), are recovered by multiplying the shifted version of \( S_{rd}(K\nu_e) \) with the same coefficients of the Hadamard sequence \( C_i \). Here, the shift value is equal to \( \left[-\left( i - \frac{1}{2} \right) f_N + \tilde{f}_D \right] \) (\( \tilde{f}_D \) is the estimated Doppler value).

Mathematically, for the \( B_m \) signal general case, this can be given as follows:

\[ Y_{B_m}(K\nu_e) = S_{rd}(K\nu_e) \ast \delta \left( K\nu_e - \left( m - \frac{1}{2} \right) f_N + \tilde{f}_D \right) C_m(K) \]  

(16)

where \( \delta(K\nu_e) \) is the unit impulse (Dirac).

**Step 3** Derive the \( B_1 \) signal expression

From (16), \( Y_{B_1}(K\nu_e) \) can be given as follows:

\[ Y_{B_1}(K\nu_e) = S_{rd}(K\nu_e) \ast \delta \left( K\nu_e - \frac{3f_N}{2} + \tilde{f}_D \right) C_1(K). \]  

(17)

Then, by using (15), (17) becomes

\[ Y_{B_1}(K\nu_e) = \left( \sum_{i=1}^{N} \left( X_p(K\nu_e + (i - 1)f_N + \tilde{f}_D) \right) \right) \cdot P_{w_i} \left( K\nu_e + (i - 1)f_N + \tilde{f}_D \right) \cdot \exp(-j(2\pi (K\nu_e - \frac{3f_N}{2} + \tilde{f}_D) \tau_0 - \phi_0)) + N(K\nu_e - \frac{3f_N}{2} + \tilde{f}_D) C_1(K). \]  

(18)

During the acquisition process, if the \( f_D \) frequency is ideally estimated, the term \( (\tilde{f}_D - f_D) \) becomes null and thus the development of (18) gives

\[ Y_{B_1}(K\nu_e) = X_p(K\nu_e) P_{w_1}(K\nu_e) C_1(K). \]  

(19)

where

\[ B_1(K\nu_e) = \left( \sum_{i=2}^{N} \left( X_p(K\nu_e + (i - 1)f_N) \right) \right) \cdot P_{w_i} \left( K\nu_e + (i - 1)f_N \right) \cdot \exp(-j(2\pi (K\nu_e - \frac{3f_N}{2} + \tilde{f}_D) \tau_0 - \phi_0)) + N(K\nu_e - \frac{3f_N}{2} + \tilde{f}_D) C_1(K). \]  

(20)

**Step 4** Derive the \( B_2 \) signal expression

By the same reasoning as previously, we find

\[ Y_{B_2}(K\nu_e) = S_{rd}(K\nu_e) \ast \delta \left( K\nu_e - \frac{3f_N}{2} + \tilde{f}_D \right) C_2(K). \]  

(21)

The development of (21) gives

\[ Y_{B_2}(K\nu_e) = \left( \sum_{i=1}^{N} X_p(K\nu_e + (i - 2)f_N) P_{w_i}(K\nu_e + (i - 2)f_N) \right) \cdot \exp(-j(2\pi (K\nu_e - \frac{3f_N}{2} + \tilde{f}_D) \tau_0 - \phi_0)) + N(K\nu_e - \frac{3f_N}{2} + \tilde{f}_D) C_2(K). \]  

(22)

Similarly, for \( \tilde{f}_D - f_D = 0 \), we have

\[ Y_{B_2}(K\nu_e) = X_p(K\nu_e) P_{w_2}(K\nu_e) C_2(K). \]  

(23)
where

\[
B_2(K\nu_c) = \left( \sum_{i=2,i\neq 2}^{N} X_p(K\nu_c + (i-2)f_N) \cdot P_{w_i}(K\nu_c + (i-2)f_N) \cdot \exp\left( -j \left( 2\pi \left( K\nu_c - \left( m - \frac{1}{2} \right) f_N + \tilde{f}_D \right) \tau_0 - \varphi_0 \right) \right) \right.
\]

\[
+ N(K\nu_c - \frac{3f_N}{2} + \tilde{f}_D) C_2(K).
\]

**Step 5** Derive the resulting recovered signal expression

In the same way, by using (15) and (16), the general analytical model of any recovered \(B_n\) signal spectrum can now be derived as follows:

\[
Y_{B_n}(K\nu_c) = X_p(K\nu_c) P_{w_n}(K\nu_c) C_n(K).
\]

\[
\exp \left( -j \left( 2\pi \left( K\nu_c - \left( m - \frac{1}{2} \right) f_N + \tilde{f}_D \right) \tau_0 - \varphi_0 \right) \right) + B_n(K\nu_c)
\]

\[
(25)
\]

where

\[
B_n(K\nu_c) = \left( \sum_{i=2,i\neq m}^{N} X_p(K\nu_c - (m-i)f_N) \cdot P_{w_i}(K\nu_c - (m-i)f_N) \cdot \exp \left( -j \left( 2\pi \left( K\nu_c - \left( m - \frac{1}{2} \right) f_N + \tilde{f}_D \right) \tau_0 - \varphi_0 \right) \right) \right.
\]

\[
+ N\left( K\nu_c - \left( m - \frac{1}{2} \right) f_N + \tilde{f}_D \right) C_n(K).
\]

(26)

Each resulted spectrum is then filtered by using the same prototype required for the analysis part (Emitter). The resulting signal spectrum is therefore the sum of all \(Y_{B_n}(K\nu_c)\) given as follows:

\[
Y(K\nu_c) = \sum_{i=1}^{N} Y_{B_i}(K\nu_c).
\]

(27)

The development of (27), using (25) and (26), gives

\[
Y(K\nu_c) = \sum_{i=1}^{N} \left( X_p(K\nu_c) P_{w_i}(K\nu_c) C_i(K) \cdot \exp \left( -j \left( 2\pi \left( K\nu_c + \left( 1 - \frac{1}{2} \right) f_N + \tilde{f}_D \right) \tau_0 - \varphi_0 \right) \right) \right) + \sum_{i=1}^{N} B_i(K\nu_c).
\]

(28)

By using (5), we have

\[
P_{w_i}(K\nu_c) C_i(K) = P_{fN} \left( K\nu_c - \left( i - \frac{1}{2} \right) f_N \right)
\]

\[
C_i(K) C_i(K) = P_{fN} \left( K\nu_c - \left( i - \frac{1}{2} \right) f_N \right)
\]

(29)

where \(C_i(K) C_i(K) = 1 (i = 1, \ldots, N)\).

Thus, by using (29), (28) becomes

\[
Y(K\nu_c) = \sum_{i=1}^{N} X_p(K\nu_c) P_{fN} \left( K\nu_c + \left( \frac{1}{2} - i \right) f_N + \tilde{f}_D \right) \tau_0 - \varphi_0 \right) \right) + \sum_{i=1}^{N} B_i(K\nu_c).
\]

(30)

Equation (30) can be simplified as follows:

\[
Y(K\nu_c) = \exp(j\varphi_0) \cdot \exp(-j2\pi f_N \tau_0) \cdot \exp \left( -j2\pi f_N \tau_0 \right) \cdot \exp \left( -j2\pi f_N \tau_0 \right)
\]

\[
\sum_{i=1}^{N} P_{fN} \left( K\nu_c - \left( i - \frac{1}{2} \right) f_N \right) \cdot \exp(j2\pi f_N \tau_0) + \sum_{i=1}^{N} B_i(K\nu_c).
\]

(31)

The discrete time domain \(y(nT_e)\) of the received signal, after the calculation of the inverse DFT of \(Y(K\nu_c)\), may be given as follows:

\[
y(nT_e) = \exp(j\varphi_0) \cdot \exp(-j2\pi f_N \tau_0) \cdot \exp \left( -j2\pi f_N \tau_0 \right) \cdot \exp \left( -j2\pi f_N \tau_0 \right)
\]

\[
\left[ x_p(nT_e - \tau_0) p_{\text{bank}}(nT_e) \right] + \sum_{i=1}^{N} B_i(nT_e)
\]

where

\[
p_{\text{bank}}(nT_e) = p(nT_e) \cdot \exp(-j\pi nT_e f_N).
\]

(32)

According to (33), \(p_{\text{bank}}(nT_e)\) is a function of the prototype filter impulse response \(p(nT_e)\). \(p(nT_e)\) represents the inverse DFT of \(P_{fN}(K\nu_c)\). In order to achieve the perfect reconstruction, suitable filters and parameter \(N\) must be chosen.

The received signal as given by (32), can be rewritten as follows:

\[
y(nT_e) = \exp(j\varphi_{\text{comp}}) x_{\text{rec}}(nT_e - \tau_0) + \eta(nT_e)
\]

(34)
where
\[ x_{rec}(nT_e - \tau_0) = x_p(nT_e - \tau_0) \ast p_{bank}(nT_e) \]  
\[ \varphi_{\text{comp}} = \varphi_0 - 2\pi \hat{f}_D \tau_0 - 2\pi \frac{f_N}{2} \tau_0 \]  
\[ \eta(nT_e) \text{ represents the noise and the interferences that is given by} \]
\[ \eta(nT_e) = \sum_{i=1}^{N} B_i(nT_e). \]  

At the receiver, a local reference replica of the transmitted signal is generated and filtered by a filter bank similar to that used in the transmitter. During the acquisition process, this replica must be shifted until it is aligned with the received signal. In the tracking process, the composite phase \( \tilde{\varphi}_{\text{comp}} \) and the delay \( \tilde{\tau}_0 \) parameters are estimated. The local reference replica can be written as follows:
\[ y_L(nT_e) = \exp(j \tilde{\varphi}_{\text{comp}}) \cdot x_{B_i}(nT_e - \tilde{\tau}_0) \]  
where
\[ x_{B_i}(nT_e - \tilde{\tau}_0) = x_p(nT_e - \tilde{\tau}_0) \ast p_{\text{bank}}(nT_e). \]  

Using (3), the result of the correlation of the received signal with \( y_L(nT_e) \), after all calculation done, can be given as follows:
\[ R_{yy_L}(\tau) = R_{x_{B_i}x_{B_i}}(\tau - \tilde{\tau}_0) + R_{\eta x_{B_i}}(\tau) \]  
where \( R_{\eta x_{B_i}}(\tau) \) is the correlation between the noise together with the interference and the locally generated code. Note here that, since the interference is multiplied for the first time by the locally generated code which will be spread.

In the presence of a single reflected specular component, the received signal can be expressed as follows:
\[ s_r(t) = s_c(t - \tau_0) \exp(j(2\pi(f_1f + f_D)t + \varphi_0) + a_1 s_c(t - \tau_1) \exp(j(2\pi(f_1f + f_D)t + \varphi_1)) + n(t). \]  

Following the same steps as above, in presence of the MP, the CF becomes
\[ R_{yy_L}(\tau) = R_{x_{B_i}x_{B_i}}(\tau - \tilde{\tau}_0) + a_1 R_{x_{B_i}x_{B_i}}(\tau - \tilde{\tau}_1) + R_{\eta x_{B_i}}(\tau) \]  
where
\[ a_{1r} = a_1 \cos \varphi_1 \]  
where \( \varphi_1 \) is the phase shift due to the MP and \( a_1 \) is the MP amplitude.

Fig. 8 shows the form of the ACF with \( N = 10 \) sub-bands decomposition. In Fig. 8, the ACF of the proposed method coincides with that of the traditional one.

5. Complexity of the proposed method

As shown in previous sections, the proposed scheme employs, in addition to the existing components of the traditional emitter/receiver, the following blocks:

(i) An analysis/synthesis filter bank block, which consists of a set of filters, requires a number of multipliers and adders to generate the DFT coefficients corresponding to each sub-band. Note here that the number is directly related to the complexity and desired performance.

(ii) A weighting operation block deals with the weighting of the coefficients resulting from the first block. When the Hadamard matrix needs to be used in the transmitter or receiver, it can be stored in read-only memory (ROM) and the values of the matrix can be called up whenever required. Thus, the Hadamard weighting operation can be implemented by virtually ruling out the time consuming and tiresome multiplication operation. Even if a need of multiplication arises, the DFT coefficient being multiplied with the Hadamard coefficient will be the coefficient itself or the coefficient itself with a negative sign before as the kernels of the Hadamard matrix are only “+1 s” and “−1 s”.

(iii) A digital modulator block, which consists of a discrete element, allows coefficients corresponding to each band to be delayed by a number of samples and to be centred accordingly to a central frequency.

The complexity is defined as the number of complex multiplications and additions. This number depends on the filter topology, the fast Fourier transform (FFT) algorithm used for enhancing the performance of the DFT and other operations related to the modulation and shift processes. It can be seen that there is no complicated computation involved in the proposed scheme. Additionally, the proposed
scheme produces a much better performance in relation to the noise and MP and is robust to the problem of limiting the band of the precorrelation filter in the receiver. Therefore, the proposed scheme is more efficient although its computation time is slightly greater than those of the conventional ones.

6. Simulation results

In order to test the performance of the proposed method, four scenarios are adopted. In the first scenario, the MP effect is tested by using the MEE criterion, while in the second and the third ones the noise effect is studied by using the root mean square error (RMSE) and the DLL tracking error standard deviation (STD) criteria, respectively. In the last scenario, we compare the proposed scheme to the other four schemes especially NC, HRC and strobe double phase estimator (SDPE) for its both variants (SDPE G1, SDPE G2). The RMSE criterion is used to illustrate the effectiveness of the proposed scheme. In the first scenario, the MEE is calculated for different values of the receiver P-BW. The simulation parameters are summarized in Table 1.

### Table 1: Simulation parameters used for the MEE test

| Received signal | P-BW | MP relative delay | MP relative amplitude | MP relative phase |
|----------------|------|-------------------|-----------------------|------------------|
| BPSK(1), CBOC, BOCsin(1,1), BOCos(1,1) | From 4 MHz to 12 MHz with a step equal to 2 MHz | 0.5 chip with respect to the LOS delay | 0.5 with respect to the LOS amplitude | 0° with respect to the LOS phase |

The results of the comparison between the proposed method and the classical one in terms of MEE as a function of the P-BW are shown in Fig. 9 for different received signals.

![Graphs showing MEE as a function of the P-BW](image)

Fig. 9 MEE as a function of the P-BW (classical and proposed schemes for BPSK (1), CBOC, BOCsin(1,1), and BOCos(1,1) modulated signals)

It shows the efficiency of the proposed method compared to the classical one for the considered signals. In fact, for 4 MHz P-BW the differences between the MEE values of the proposed method and those of the classical one are approximately 11 m, 35 m, 4.5 m and 7.5 m respectively, for BPSK(1), CBOC, BOCsin(1,1), and BOCos(1,1) modulated signals. Besides, when the P-BW increases, the MEE differences values in all four cases decrease to reach zero. Our method presents an almost constant error that proves its non-sensitivity to the P-BW limitation in the receiver.

In the second scenario, the code tracking RMSE versus the relative MP delay is used to realize the comparison between the classical and the proposed reception schemes for four different values of the P-BW. The simulation parameters are shown in Table 2, and the results are depicted in Fig. 10 – Fig. 14, respectively, for BPSK(1), CBOC, BOCsin(1,1), BOCos(1,1) and BPSK(10) modulated signals.

![Graphs showing RMSE as a function of the P-BW](image)

**Fig. 10** RMSE of the classical and proposed schemes of BPSK (1) modulated signals for different P-BWs

### Table 2: Simulation parameters used for the RMSE test

| Received signal | P-BW | SNR/dB | MP relative delay | MP relative amplitude | MP relative phase |
|----------------|------|--------|-------------------|-----------------------|------------------|
| BPSK(1), CBOC, BOCsin(1,1), BOCos(1,1) | 4 MHz, 8 MHz, 24 MHz and 36 MHz | –35 | From 0 to 450 m with respect to the LOS delay | 0.5 with respect to the LOS amplitude | 0° with respect to the LOS phase |
| BPSK(10) | 24 MHz and 36 MHz | | | |
Fig. 11 RMSE of the classical and proposed schemes of CBOC modulated signals for different P-BWs

As illustrated in Figs. 10 – 14, for all considered P-BWs, the proposed method has a lower RMSE than that of the classical one for all MP delay values, which shows its robustness in relation to the noise and MP.

The third scenario is realized to compare the STDs of the proposed method with those of the classical one. The simulations are done for BPSK (1), CBOC, BOCsin(1,1), BOCcos(1,1) and BPSK (10) modulated signals with different values of the P-BW in each signal case. The simulation parameters are shown in Table 3, and the results of the STD versus the signal to noise ratio (SNR), for all aforementioned signals, are shown in Figs. 15 – 19, respectively.

Table 3 Simulation parameters used for the STD criterion test

| Received signal | P-BW | SNR      |
|-----------------|------|----------|
| BPSK(1), CBOC,  | 4 MHz, 8 MHz | From −40 dB to −25 dB |
| BOCsin(1,1), BOCcos(1,1) | 24 MHz and 36 MHz |   |
| BPSK(10)        | 24 MHz and 36 MHz |   |

Fig. 12 RMSE of the classical and proposed schemes of BOCsin(1,1) modulated signals for different P-BWs

Fig. 13 RMSE of the classical and proposed schemes of BOCcos(1,1) modulated signals for different P-BWs

Fig. 14 RMSE of the classical and proposed schemes of BPSK (10) modulated signals for different P-BWs

Fig. 15 STD of the classical and proposed schemes of BPSK(1) modulated signals for different P-BWs
As shown in Figs. 15 – 19, for all five signals, the proposed method presents lower STDs values than those of the classical one over the entire SNR variation range and for all considered P-BW values, which shows the robustness of the proposed method vis-à-vis the noise.

In the last scenario, the RMSE values versus SNR on one hand and versus MP delay on the other hand are estimated to make a comparison between the proposed method and the other four methods, especially NC, HRC, SDPE G1 and SDPE G2. The simulation parameters are shown in Table 4 and Table 5. The results are shown in Fig. 20 and Fig. 21.

| Table 4 | Simulation parameters used for RMSE versus SNR |
|---------|------------------------------------------------|
| Scheme  | P-BW/ MHz | MP delay | MP relative amplitude | SNR |
| Proposed method, NC, HRC, SDPE G1, SDPE G2 | 24 | 0.5 chip | 0.5 with respect to the LOS | From –40 dB |

| Table 5 | Simulation parameters used for RMSE versus MP delay |
|---------|------------------------------------------------------|
| Scheme  | P-BW/ MHz | MP delay | MP relative amplitude | MP relative phase |
| Proposed method, NC, HRC, SDPE G1, SDPE G2 | 24 | From 0 to 450 m | 0.5 with respect to the LOS | 0° with respect to the LOS |
Fig. 20 RMSE versus SNR (comparison between the performance of the proposed scheme and those of NC, HRC, SDPE G1 and SDPE G2)

Fig. 21 RMSE versus relative MP delay (comparison between the performance of the proposed scheme and those of NC, HRC, SDPE G1 and SDPE G2)

As shown in Fig. 20 and Table 4, the performance for RMSEs versus SNR which varies from – 40 dB to – 22 dB. The small RMSE for the proposed method through the whole SNR range confirms the applicability of the proposed scheme. As shown in Table 5, the simulation is realized in presence of one MP signal in phase with LOS, the delay varies from 0 m to 450 m with respect to the LOS. With an SNR equal to – 30 dB, the result is shown in Fig. 21. As illustrated in Fig. 21, for all values of MP delay, the proposed method presents a low RMSE with regard to what we observe for all the other methods which confirms its best performance.

7. Conclusions

In this paper, an efficient method for the reduction of the signal pre-correlation filtering effect in GNSS applications is proposed. The proposed method is based on the realization of a spectral transformation that effectively reduces the band of the emitted signal. Such a processing causes, at the end of the receiver, noise spreading over a much larger bandwidth, allowing a better resistance to the MP and noise phenomena. The simulation results demonstrate effectively that the proposed method promotes an overall enhancement in the performance of the GNSS receivers and presents a credible solution for both MP and noise problems for any limitation of the P-BW in the receiver filtering process.

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