D3-brane intersecting with dyonic BIon

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Abstract

The Born–Infeld theory of a D3-brane intersecting with (p, q) strings is reconsidered. From the assumption that the electromagnetic fields are those of a dyon, and using the kappa invariance of the action, the explicit scalar field and its charge are derived. Considering perturbations orthogonal to both branes, the $SL(2, Z)$-invariant S-matrix is obtained. Owing to the selfduality of the brane the latter can be evaluated explicitly in both high and low energy regions.

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1 Introduction

The Dp-brane Born–Infeld action has BPS-saturating solutions which have a spacetime interpretation as intersections with other branes \[1, 2\]. These 1/2 supersymmetric solutions appear as spikes on the world–volume with uniform energy per unit length and their infinite energy is associated with an infinite string (or brane) of fixed tension. The sources of these BIonic solutions on a brane appear as the endpoints of strings outside the brane. The fundamental string attached to the brane is described by the electric point charge solution. In the case of the D3-brane there are dyonic BIons corresponding to \((p,q)\) strings of infinite length that end on the D3-brane.

The perturbations propagating via the attached fundamental string reveal specific nonlinearities of the BI action. The equation describing these fluctuations was shown to be convertible into a modified Mathieu equation \[3, 4, 5\] which allows an explicit and exact expression for the S-matrix to be obtained as a function of the frequency of the fluctuations. In the following we generalise these considerations to the case of the full \((p,q)\) strings in the background and show that the S-matrix is a function of the product of the \(SL(2,\mathbb{Z})\)-invariant tension of the \((p,q)\) strings and the frequency of the fluctuation. It turns out that as the frequency increases, the S-matrix goes over from perfectly reflecting to perfectly absorbing; these considerations require a knowledge of the S-matrix in both the low-frequency and the high-frequency regions. The reflection-absorption behaviour obtained is in full compliance with Pochinski’s picture of D-branes as boundaries for open strings.

The basic procedure to obtain such type of solutions is to consider the BPS limit \[6, 7, 8\]. In this way finite length strings stretched between two D3-branes together with the string junctions were obtained in \[8\]. One of the D3-branes was considered as a IIB supergravity solution and taken as a background for another D3-brane. Static BIonic solutions of this type have a finite energy proportional to the distance between the branes with the \((p,q)\) string tension as coefficient. They preserve 1/2 of the supersymmetries, while string junctions preserve 1/4 of the supersymmetries. A further interesting development was achieved in \[9, 10\], where the ’t Hooft–Polyakov monopole of Yang–Mills theory is analysed in 3+1 dimensions. The Higgs scalar was interpreted as a transverse scalar for a two D3-brane configuration connected by the monopole solution. It was found that part of the D-string flux lives on the D3-brane, while another part passes through the space between the branes.
The D3-brane carries Fundamental and Dirichlet string charges \([11,12]\). In the BPS limit the brane becomes tensionless so that its energy is entirely due to the electromagnetic field and is the sum of the electric and magnetic charges \([13,14,15]\). Then the D3-brane can be interpreted as a blown up \((p,q)\) string which is the three-dimensional analog of supertubes \([16]\).

It is unclear whether the BPS bound can be obtained from examination of the complicated Hamiltonian under consideration. However, there is an alternative approach which we pursue here. In Section 2 we present the action, Hamiltonian and equations of motion. In Section 3 we first accept that electric intensity and magnetic induction are those created by the dyon and next find the explicit expression for the scalar field using the supersymmetry argument. The solution obtained satisfies the equations of motion and preserves half of the supersymmetry. In Section 4 the equation of small fluctuations is derived and converted into a modified Mathieu equation. Investigation of the resulting S-matrix then leads to the conclusions referred to above.

### 2 Equations of motion

The D3-brane action with constant dilaton and vanishing Kalb–Ramond fields is given by

\[
I = -T_3 \int d^4 \xi \sqrt{-\det (g_{\alpha \beta} + 2\pi F_{\alpha \beta})},
\]

where \(g_{\alpha \beta}\) is the induced metric and \(F_{\alpha \beta}\) is the electromagnetic field strength.

To construct a D3-brane intersecting with \((p,q)\) strings we keep the electric field \(\mathbf{E}\), the magnetic induction \(\mathbf{B}\) (both with the factor \(2\pi\) absorbed) and a transverse scalar field \(\Phi\) in the ninth direction. The freedom of the world-volume diffeomorphism invariance is fixed by choosing the static gauge for which the world-volume coordinates \(x^\alpha (\alpha = 0, 1, 2, 3)\) are equated with the first four spacetime coordinates of the flat target space. The action reduces to

\[
I = T_3 \int \mathcal{L} \, d^4 x,
\]

where the Lagrangian density \(\mathcal{L}\) is

\[
\mathcal{L} = -\sqrt{1 - E^2 + \nabla \Phi^2 + B^2 + (B \cdot \nabla \Phi)^2 - (E \cdot B)^2 - |E \times \nabla \Phi|^2}.
\]
The conjugate momentum $\Pi$ associated with the electromagnetic potential is given by

$$\Pi = \frac{\partial L}{\partial E} = -\frac{1}{L}[(1 + \nabla \Phi^2)E + (E \cdot B)B - (E \cdot \nabla \Phi)\nabla \Phi].$$

This nonlinear relation can be reversed to give the electric field $E$ in terms of canonical momentum $\Pi$, magnetic induction $B$ and scalar field $\Phi$,

$$E = \frac{v \Pi - w B + [v (\Pi \cdot \nabla \Phi) - w (B \cdot \nabla \Phi)]\nabla \Phi}{\sqrt{(1 + \nabla \Phi^2)(wv - w^2)}},$$

where the quantities $u, v, w$ are defined as

$$u \equiv 1 + \Pi^2 + \nabla \Phi^2 + (\Pi \cdot \nabla \Phi)^2,$$
$$v \equiv 1 + B^2 + \nabla \Phi^2 + (B \cdot \nabla \Phi)^2,$$
$$w \equiv \Pi \cdot B + (\Pi \cdot \nabla \Phi)(B \cdot \nabla \Phi).$$

After performing the Legendre transform

$$\mathcal{H} = E \cdot \Pi - L,$$

the Hamiltonian density $\mathcal{H}$ appears as

$$\mathcal{H} = \sqrt{uv - w^2} \frac{1}{1 + \nabla \Phi^2}.$$

The Hamiltonian (8) is invariant under the electric–magnetic duality rotation through an arbitrary angle $\theta$

$$\Pi \to \Pi \cos \theta + B \sin \theta,$$
$$B \to B \cos \theta - \Pi \sin \theta.$$

The invariance (9) takes solutions of equations of motion into solutions and permits to construct a set of S–dual solutions.

The equations of motion are conveniently presented by introducing the magnetic field strength $H$,

$$H = -\frac{\partial L}{\partial B} = -\frac{1}{L}[B - (B \cdot E)E + (B \cdot \nabla \Phi)\nabla \Phi],$$

4
and the spatial part \( \Phi \) of the four-momentum of the scalar field

\[
\Phi = -\frac{\partial L}{\partial \nabla \Phi} = -\frac{1}{\mathcal{L}} \left[ (1 - \mathbf{E}^2) \nabla \Phi + (\mathbf{E} \cdot \nabla \Phi) \mathbf{E} + (\mathbf{B} \cdot \nabla \Phi) \mathbf{B} \right].
\] (11)

The Euler–Lagrange static equations are

\[
\nabla \cdot \Pi = 0, \quad \text{(12a)}
\]

\[
\nabla \times \mathbf{H} = 0, \quad \text{(12b)}
\]

\[
\nabla \cdot \Phi = 0, \quad \text{(12c)}
\]

supplemented by the corresponding Bianchi identities

\[
\nabla \times \mathbf{E} = 0, \quad \text{(13a)}
\]

\[
\nabla \cdot \mathbf{B} = 0. \quad \text{(13b)}
\]

The terms containing the time derivative of the scalar field \( \dot{\Phi} \) have been omitted in the Lagrangian (3) for simplicity. Considering that

\[
\left. \frac{\partial L}{\partial \dot{\Phi}} \right|_{\dot{\Phi}=0} = 0,
\]

these terms give no contribution to the Hamiltonian (8), nor to the equations of motion (12) in the static case.

### 3 D3-brane intersecting with \((p, q)\) string

By \( SL(2, \mathbb{Z}) \) invariance, both Fundamental and Dirichlet strings can end on the self–dual D3–brane, the former ending on electric charges and the latter on magnetic charges. The endpoint of a \((p, q)\) string will appear in the world–volume as a dyon. This idea together with supersymmetry arguments allows to construct a D3–brane intersecting with a \((p, q)\) string.
3.1 Supersymmetric solution

Pointlike electric and magnetic charges at the same point are sources of an electromagnetic field with spherical symmetry and components

\[ E = e_0 \frac{r}{r^3}, \quad B = b_0 \frac{r}{r^3}, \]

where \( e_0 \) and \( b_0 \) are constants. The preserved part of the supersymmetry of the IIB Minkowski vacuum is defined by the condition

\[ \Gamma \varepsilon = \varepsilon, \]

where \( \Gamma \) is the projection operator appearing in the \( \kappa \)-symmetry transformations. In the case under consideration it is given by \[17, 18\]

\[ \Gamma = \frac{1}{L}(\Gamma_I \otimes I - \Gamma_J \otimes J), \]

where

\[ \Gamma_I = \Gamma_{0123} - (\nabla \Phi \cdot \Gamma)\Gamma_{01239} + E \cdot B, \]

\[ \Gamma_J = (E \cdot \Gamma)\Gamma_{123} - ((E \times \nabla \Phi) \cdot \Gamma)\Gamma_9 + (B \cdot \nabla \Phi)\Gamma_{09} - (B \cdot \Gamma)\Gamma_0, \]

and

\[ I = i\sigma_2, \quad J = \sigma_1. \]

The 10 dimensional matrices \( \Gamma_\alpha \) act on the Weyl–Majorana index while the matrices \( I, J \) act on the \( SO(2) \) index of the spinor. The matrices \( \Gamma_I, \Gamma_J \) have the properties

\[ [\Gamma_I, \Gamma_J] = 0, \quad \Gamma_J^2 - \Gamma_I^2 = L^2 \mathbb{1}. \]

We denote the \( SO(2) \) components of the spinor \( \varepsilon \) by \( \varepsilon_1 \) and \( \varepsilon_2 \) and set

\[ \varepsilon_1 = \Gamma_{123}\varepsilon, \quad \varepsilon_2 = -\Gamma_0\varepsilon. \]

Equation (15), omitting the vanishing term \( E \times B \), splits into two equations

\[ -\frac{1}{L}[(1 + B^2)\Gamma_{123} - (B - \Gamma_{123}\Gamma + \Gamma_0 E \times \Gamma) \cdot S] \varepsilon = \Gamma_{123}\varepsilon, \quad (23a) \]

\[ -\frac{1}{L}[(1 + B^2)\Gamma_0 - \Gamma_{0123}(B + \Gamma_{123}\Gamma - \Gamma_0 E \times \Gamma) \cdot S] \varepsilon = \Gamma_0\varepsilon, \quad (23b) \]
where
\[ S = \nabla \Phi \Gamma_9 - \mathbf{E} \Gamma_0 + \mathbf{B} \Gamma_{123}. \] (24)

The pair of equations (23) is equivalent to
\[ L = -(1 + \mathbf{B}^2), \] (25a)
\[ S \epsilon = 0. \] (25b)

These two conditions (25a) and (25b) are compatible and give rise to the preservation of half supersymmetry. Indeed, from Eqs. (25b) and (14) it follows that
\[ \nabla \Phi = \mathbf{G} \frac{\mathbf{r}}{r^2}, \] (26)
where \( \mathbf{G} \) is a constant. Equation (25b) thus assumes the form
\[ \hat{M} \epsilon = 0, \] (27)
where
\[ \hat{M} = \mathbf{G} \mathbb{1} + e_0 \Gamma^{09} - b_0 \Gamma^{1239}. \] (28)

The matrix \( \hat{M} \) satisfies the relation
\[ \hat{M}^2 = (e_0^2 + b_0^2 - G^2) \mathbb{1} + 2G \hat{M}, \] (29)
therefore its eigenvalues must satisfy the same relation (29). Thus Eq. (27) has nontrivial solutions only when
\[ G = \pm \sqrt{e_0^2 + b_0^2}. \] (30)

Owing to condition (30) the eigenvalues of the matrix \( \hat{M} \) are either 0 or 2\( G \). On the other hand the trace of the matrix \( \hat{M} \) is 32\( G \), consequently it has sixteen 0 and sixteen 2\( G \) eigenvalues. Eqs. (14) (26) (30) together also change to the identity the condition (25a). As a consequence the solution preserves half of the supersymmetries.
3.2 The \((p, q)\) string

The supersymmetric configuration \([14, 26, 30]\) simplifies the expressions for the conjugate momentum \(\Pi\), magnetic intensity \(H\) and conjugate scalar field \(\Phi\) resulting in

\[
\Pi = E, \quad H = B, \quad \Phi = \nabla \Phi. \tag{31}
\]

Equations (31) manifest that the supersymmetry requirements imposed on solution are:

i) the electromagnetic field must be a solution of the linear electrodynamic equations,

ii) the scalar field must be a harmonic function,

iii) the scalar charge \(G\) expressed in terms of electric and magnetic charges has an \(SL(2, \mathbb{Z})\) symmetry.

These prescriptions can be used to construct multiple \((p, q)\) strings ending at arbitrary locations on a brane. It is clear that the solution also satisfies the Euler–Lagrange equations (12) and the Bianchi identities (13), and is a BPS state.

Charge quantisation \([19]\)

\[
\int \star \left( \frac{\delta I}{\delta F} \right) = p, \quad \frac{1}{2\pi} \int F = q, \tag{32}
\]

where \(\star\) means Hodge duality operation, connects the constants \(e_0\) and \(b_0\) with the electric \(p\) and magnetic \(q\) charges respectively,

\[
e_0 = \pi g_s p, \quad b_0 = \pi q. \tag{33}
\]

The above relations allow to express the energy of the configuration in terms of charges. The Hamiltonian density \(\mathcal{H}\) of the static solution is equal to

\[
\mathcal{H} = 1 + \nabla \Phi^2. \tag{34}
\]

This gives for the energy \(\mathcal{E}\)

\[
\mathcal{E} = T_3 \int dV + \sqrt{(p T_F)^2 + (q T_D)^2} \int d \Phi(r), \tag{35}
\]

where \(T_F\) and \(T_D\) are Fundamental and Dirichlet string tensions, respectively. The first term is the energy of the flat brane with no background. The second term has a natural interpretation as a \((p, q)\) string attached to the brane and running off to infinity. The total energy is the sum of these two terms which is the typical feature of BPS states.
4 Orthogonal perturbations

We obtain the equation of small fluctuations $\eta$ along an additional spatial direction perpendicular to the brane and string by evaluating the appropriately extended Born–Infeld Lagrangian at the BPS background. The Lagrangian density then becomes

$$\mathcal{L} = -\sqrt{(1 + B^2_c)^2 + (1 + B^2_c)[(\nabla \eta)^2 - (1 + \nabla \Phi_c^2)(\partial_0 \eta)^2]}$$  \hspace{1cm} (36)$$

with

$$\nabla \Phi_c = \pm G \frac{r}{r^3}, \quad G = \sqrt{(\pi g_s p)^2 + (\pi q)^2}.$$  \hspace{1cm} (37)$$

The $SL(2, \mathbb{Z})$-invariance of $G$, essentially the tension of the $(p, q)$-string, has been shown, for instance, in ref. [20].

By expanding $\mathcal{L}$ and retaining the lowest order terms we obtain the equation

$$-(1 + \nabla \Phi_c^2) \frac{d^2 \eta}{dt^2} + \triangle \eta = 0,$$  \hspace{1cm} (38)$$

where $\triangle$ is the 3-dimensional Laplacian. Different from previous considerations this is the equation for the case with the full $(p, q)$ string in the background.

In spherical coordinates and with

$$\eta = e^{i\omega t} r^{1/2} Y_{lm}(\theta, \phi) \psi(r), \quad r = G^{1/2} e^z, \quad h^2 = G \omega^2, \quad a = l + \frac{1}{2},$$  \hspace{1cm} (39)$$

the equation becomes the modified Mathieu equation

$$\frac{d^2 \psi}{dz^2} + [2h^2 \cosh 2z - a^2] \psi = 0.$$  \hspace{1cm} (40)$$

For the present case of the D3-brane in Born–Infeld theory the complete solution of the equation with derivation of the S-matrix has been obtained in ref. [5]. Although the considerations there utilized expansions for large $\omega^2$, i.e. $h^2$, the resulting S-matrix expressed in terms of modified Mathieu functions of the first kind, $M^{(1)}_{\nu}(z, h^2)$, with Floquet exponent $\nu$, was shown to have the unique property to be identical with that obtained for small $h^2$ derived in considerations of the supergravity background of an extremal D3-brane in refs. [3], [4] except for the difference in parameters. Thus the case
of the D3-brane is a rare case permitting an exact solution. The explicit and exact S-matrix element for the lth partial wave is [5]

$$S_l = \frac{\sin \pi \gamma}{\sin \pi (\gamma + \nu)} e^{i\pi(l+1/2)}, \quad e^{i\pi \gamma} = \frac{M_{1,\nu}^{(1)}(0, h^2)}{M_{1,\nu}^{(1)}(0, h^2)}.$$  \hfill (41)

The Floquet exponent has to be calculated separately in the cases of $h^2$ small and $h^2$ large. Expansions in ascending powers of $h^2$ have a finite radius of convergence; expansions in descending powers (valid in the complementary domain) are asymptotic.

### 4.1 Low energy

In the low energy, small $h^2$, case and S-waves

$$\nu = a + \frac{h^4}{4a(1 - a^2)} + O(h^6), \quad a = \frac{1}{2}. \hfill (42)$$

In ref. [4] it was shown that

$$e^{i\pi \gamma} = \frac{\nu!(\nu - 1)! \sin \pi \nu}{\pi (h/2)^{2\nu}} [1 + \frac{4\nu}{(\nu^2 - 1)^2} (\frac{h^2}{2})^4 + \cdots] \hfill (43)$$

implying here

$$e^{i\pi \gamma} \approx \frac{1}{h} [1 + O(h^4)], \hfill (44)$$

i.e. $\gamma$ imaginary. We define the amplitudes of the incident wave, and reflected and transmitted waves respectively as

$$A_i = 2i \sin \pi (\gamma + \nu), \quad A_r = 2i \sin \pi \gamma, \quad A_t = 2i \sin \pi \nu. \hfill (45)$$

The transmitted wave at large negative $z$ represents particles falling into the D3-brane. In the present case we obtain

$$\left| \frac{A_r}{A_i} \right| = |\tan \pi \gamma| = 1 - O(h^2), \quad \left| \frac{A_t}{A_i} \right| = \frac{1}{|\cos \pi \gamma|} = 2h - O(h^3). \hfill (46)$$

Thus in the zero energy limit, i.e. $h \to 0$, one has total reflection. The corresponding fluctuation transverse to brane and string is

$$\eta \sim r^{1/2} \psi = r^{1/2} \sinh(z/2) + O(h^2) = r^{1/2} \left[ \sqrt{\frac{r}{G^{1/2}}} - \sqrt{G^{1/2}/r} \right] + O(h^2) \approx \frac{1}{2G^{1/4}}(r - G^{1/2}) + O(h^2) \hfill (47)$$
in lowest order of $h^2$ and is seen to satisfy in the case of the fundamental open string and in the weak-coupling limit, in which the dynamics can be studied using string perturbation theory, and $h^2 \sim g_s \omega^2$, the Dirichlet boundary condition $\eta = 0$ at $r = G^{1/2} \to 0$ with $g_s \to 0$. In this case the frequency $\omega$ can be large compared with the string mass scale, i.e. $\omega \leq m_s/\sqrt{g_s}$.

### 4.2 High energy

In the high energy, large $h^2$ case, the Floquet exponent is a complicated function of $h$ as shown in ref. [5]. However, we can obtain the limits from formulae derived there. Thus for large $h$ one obtains with $a^2 \simeq -2h^2 + 2hq$ the large-$h^2$ expression

$$|S_i| \simeq \left| \frac{A_r}{A_i} \right| \simeq \frac{2\pi (16h)^q}{e^{8h} \Gamma(\frac{q+1}{2})^2} \to 0$$

and

$$\left| \frac{A_t}{A_i} \right| \simeq |e^{-i\pi(q+1)/2}| = 1.$$  

Hence in the high energy limit the S-matrix yields total absorption. These results confirm the expectations of ref. [1] obtained there on the basis of a simplified model.

Finally we observe that the study of the convergent small $h^2$ expansions shows that these are valid within a radius of convergence of some number (depending on $a$) $\times h^2$. Beyond this the asymptotic expansions take over. Thus there is a critical value of $h^2 \sim$ some number determining a critical energy $\omega_{\text{crit}}$, i.e.

$$\omega_{\text{crit}} \sim \frac{1}{\sqrt{G}} = \frac{1}{[(\pi g_s p)^2 + (\pi q)^2]^{1/4}}.$$  

In the case of the pure fundamental or p-string, $\omega_{\text{crit}} \sim 1/\sqrt{g_s}$ which becomes large in the weak-coupling limit $g_s \to 0$.

Considering the fluctuation $\eta$, one obtains as the leading factor

$$\eta \simeq r^{1/2} \exp[\pm \sqrt{2h^2 \cosh 2z + 2h^2}] (1 + O(1/h^2))$$

which again reflects the $z \to -z$ inversion symmetry.
5 Conclusion

In the above we considered the D3-brane intersecting with the \((p, q)\) string. T-duality permits the construction of the Dp-brane intersecting with the electric flux carrying the D(p-2)-brane. The method applied above does not allow to find an analytic expression for the S-matrix in this case. To understand this we first note that the potential acts as a sink. The waves can escape to infinity or to the origin. In the case of the selfdual D3-brane these interior and outer regions are equivalent to the effect that Eq. (40) is invariant under the interchange \(r \leftrightarrow 1/r\). This symmetry played an important role in deriving the analytic expression and disappears in considering higher or lower dimensional branes. However, the following simplification is available: one can arrange the electric and magnetic fluxes in the attached D(p-2)-brane in such a way that the brane becomes tensionless. This will be a higher dimensional supertube and its energy is an additive quantity—the sum of the electric and magnetic charges in appropriate units. Comparing the potential in Eq. (38) and the Hamiltonian of the static solution in (34) one observes that these coincide. We have not checked whether such a relation holds for other branes also, but certainly the supertube (or its higher dimensional analog) in the background has to give a simplified potential.

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