Threshold Cross Section Measurements

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Abstract. Accurate measurements of particles masses, couplings and widths are possible by measuring production cross sections near threshold. We discuss the prospects for performing such measurements at a high luminosity muon collider.

INTRODUCTION

A muon collider is particularly well suited to the threshold measurement because the spread in energy of the beam is very small [1]. Pair production of W-bosons, \( t\bar{t} \) production and the Bjorken process \( \mu^+\mu^- \rightarrow ZH \) have been considered as possible places to study thresholds at a muon collider [2–4]. Threshold production of chargino pairs at a muon collider offers a possible way of accurately measuring the chargino mass [5].

We assume here that the muon collider has a relatively modest beam energy spread of \( R = 0.1\% \), where \( R \) is the rms spread of the energy of a muon beam. We assume that 100 fb\(^{-1}\) integrated luminosity is available and that this amount of luminosity could be accumulated at the relevant energies for the measuring the threshold cross sections; high luminosity is essential if the threshold measurements are to prove interesting.

\( M_W \) MEASUREMENT AT THE \( \mu^+\mu^- \rightarrow W^+W^- \) THRESHOLD

The threshold cross section is most sensitive to \( M_W \) just above \( \sqrt{s} = 2M_W \), but a tradeoff exists between maximizing the signal rate and the sensitivity of the cross section to \( M_W \). Detailed analysis [6] shows that if the background level is small

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and systematic uncertainties in efficiencies are not important, then the optimal
measurement of $M_W$ is obtained by collecting data at a single energy

$$\sqrt{s} \sim 2M_W + 0.5 \text{ GeV} \sim 161 \text{ GeV},$$

where the threshold cross section is sharply rising.

At a muon collider with high luminosity, systematic errors arising from uncer-
tainties in the background level and the detection/triggering efficiencies will be domi-
nant unless some of the luminosity is devoted to measuring the level of the back-
ground (which automatically includes somewhat similar efficiencies) at an energy
below the $W^+W^-$ threshold. Then, assuming that efficiencies for the background
and $W^+W^-$ signal are sufficiently well understood that systematic uncertainties
effectively cancel in the ratio of the above-threshold to the below-threshold rates,
a very accurate $M_W$ determination becomes possible.

We analyzed [2] the possible precision obtainable for the $W$ mass via just two
measurements: one at center of mass energy $\sqrt{s} = 161$ GeV, just above threshold,
and one at $\sqrt{s} = 150$ GeV. The optimal $M_W$ measurement is obtained by expending
about two-thirds of the luminosity at $\sqrt{s} = 161$ GeV and one-third at $\sqrt{s} =
150$ GeV. Combining the three modes, an overall precision of $\Delta M_W = 6$ MeV
should be achievable with 100 fb$^{-1}$ integrated luminosity.

HIGGS BOSON MEASUREMENT AT THE $\mu^+\mu^- \rightarrow Zh$ THRESHOLD

The SM Higgs boson is easily discovered in the Bjorken Higgs-strahlung pro-
cess [7] $\ell^+\ell^- \rightarrow Zh$ running the machine well above threshold, e.g. at $\sqrt{s} =
500$ GeV. For $m_h \lesssim 2M_W$ the dominant Higgs boson decay is to $b\bar{b}$ and most back-
grounds can be eliminated by $b$-tagging. A very accurate determination of $m_h$
could then obtained by measuring the threshold cross section of $Zh$ production, which
rises rapidly as shown in Fig. 1(a) since the threshold behavior is $S$-wave.

The sensitivity to the SM Higgs boson mass is maximized by a single measu-
rement of the cross section at $\sqrt{s} = M_Z + m_h + 0.5 \text{ GeV}$, just above the real particle
threshold provided that the normalization of the measured $Zh$ cross section as a
function of $\sqrt{s}$ can be precisely predicted, including efficiencies and systematic ef-
fects. We employed $b$-tagging and cuts in order to reduce the background to a
very low level. These cuts and other systematic uncertainties are discussed in more
detail in Ref. [3]. The background is very much smaller than the signal unless
$m_h$ is close to $M_Z$. The electroweak radiative corrections to the cross section are
estimated to be less than 1% for $m_H \sim 100$ GeV [8], and the measurement of the
cross section described here is at the 2% level. We found a precision of the SM
Higgs mass determination to within 45 MeV for $m_h = 100$ GeV may be achievable
at a muon collider. More generally the precision ranges from 20-100 MeV for
$m_h < 150$ GeV.
FIGURE 1. The cross section vs. \( \sqrt{s} \) for (a) the process \( \mu^+\mu^- \to Z^*h \to f\bar{f}h \) for a range of Higgs masses, and for (b) \( \mu^+\mu^- \to \tilde{\chi}^+\tilde{\chi}^- \) for various sneutrino masses and \( m_{\tilde{\chi}^\pm} = 103.7 \) GeV.

Beyond the Standard Model the cross section generally depends on the \( ZZh \) coupling (\( g_{ZZh} \)) and the total Higgs width (\( \Gamma_H \)) in addition to \( m_h \). In order to simultaneously determine these three quantities, measurements could be made at the three c.m. energies \( \sqrt{s} = m_h + M_Z + 20 \) GeV, \( \sqrt{s} = m_h + M_Z + 0.5 \) GeV, and \( \sqrt{s} = m_h + M_Z - 2 \) GeV. With a three-parameter fit to \( m_h, g_{ZZh}^2 B(h \to b\bar{b}) \) and \( \Gamma_H \), the attainable error in \( m_h \) is about 110 MeV at the 1\( \sigma \) level for a 100 GeV Higgs. Measurements that would simultaneously determine \( m_h, \sigma(Zh)B(h \to b\bar{b}) \) and \( \Gamma_H \) could be done at a level of accuracy that could distinguish a Standard Model Higgs boson from its many possible (e.g. supersymmetric) extensions [3].

TOP-QUARK MASS MEASUREMENT AT THE \( \mu^+\mu^- \to t\bar{t} \) THRESHOLD

The top-quark threshold cross section is calculable since the large top-quark mass puts one in the perturbative regime of QCD [9]. One can perform a scan of the threshold curve by devoting to 10 fb\(^{-1} \) integrated luminosity to measuring the cross section at each of ten energies in 1 GeV intervals. Then the top-quark mass can be determined to within \( \Delta m_t \sim 70 \) MeV, provided systematics and theoretical uncertainties are under control. Considerable progress has been made recently in the theoretical calculations of the some NNLO corrections to the threshold cross section [10]. The remaining theoretical uncertainties [11] in the threshold cross section are still fairly large and make it difficult to fully exploit the large luminosity for determining say the strong coupling \( \alpha_s \) or a light Higgs boson mass (and the top quark Yukawa coupling) from the size of the cross section. Furthermore there is theoretical ambiguity in the mass definition of the top quark. The theoretical ambiguity in relating quark pole mass to other definitions of the top quark mass (that might be relevant as input to radiative correction calculations) is of order
\( \Lambda_{QCD}, \text{i.e., or a few hundred MeV} \) [12]. So it is not clear that an extraction of the top-quark mass better than this is useful, at least at the present time.

**CHARGINO SIGNAL AND BACKGROUND**

The mass of the lighter chargino in the minimal supersymmetric standard model (MSSM) can be determined accurately by measuring the cross section\(^2\) for

\[
\mu^+\mu^- \rightarrow \chi^+\chi^-
\]

near the threshold [5]. The precision that can be obtained in the chargino mass depends substantially on the mass of the chargino mass itself: the heavier the chargino the smaller the production cross section. The cross section also depends on the mass of the sneutrino which appears in the \( t \)-channel since this contribution interferes destructively with the \( s \)-channel graphs. The cross section is displayed in Fig. 1(b) for several values of the sneutrino mass. If the lightest chargino is gaugino-dominated, then changing the parameters of the chargino mass matrix essentially changes the mass but not the chargino couplings significantly. The width of the lightest chargino is usually less than a few MeV, and often substantially less when two-body decays are kinematically impossible. Therefore one can envision a measurement of the cross section that depends on just two parameters: the chargino mass \( m_{\tilde{\chi}^\pm} \) and the sneutrino mass \( m_{\tilde{\nu}} \).

As in the other threshold measurements, the statistical precision on the chargino mass is maximized just above \( 2m_{\tilde{\chi}^\pm} \). A simultaneous measurement of the chargino and sneutrino masses requires a sampling of the cross section at at least two points. It turns out to be advantageous for the chargino mass measurement to choose this higher energy measurement at a point where the chargino cross section is not flat.

The chargino decay mode is \( \tilde{\chi}^\pm \rightarrow \tilde{\chi}^0 f f' \) provided the chargino is lighter than the muon sneutrino. The cross section is reduced near threshold, so the cuts to reduce backgrounds need to be reoptimized. The backgrounds to chargino pair-production have been investigated in Refs. [16,17] where the signal efficiencies have been obtained for the various final states when the center-of-mass energy is \( \sqrt{s} = 500 \text{ GeV} \). The primary background is \( W \) pair production which is very large, but can be effectively eliminated because the \( W \)'s are produced in the very-forward direction. However, if the energy is reduced so that the collider is operating in the chargino threshold region, then the effectiveness of these cuts might be reduced (the signal events might be expected to be more spherical as well). Therefore the efficiencies were reinvestigated for the threshold measurement.

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\(^2\) The measurement of the chargino mass via the threshold cross section has been considered previously for electron-positron machines in Ref. [13,14]. We consider the measurement at a muon collider with high luminosity, carefully taking into account the beam effects and reoptimizing cuts to eliminate the background in the threshold region.

\(^3\) The overall normalization of the cross section could also depend on radiative corrections which could be substantial in some cases [15].
TABLE 1. Precision of mass measurements assuming 100 fb$^{-1}$ luminosity. The ranges considered for the Higgs and
chargino masses are also shown.

| Particle | Mass Measurement (MeV) | Mass Range (GeV) |
|----------|------------------------|-----------------|
| $W$      | 6                      | –               |
| $t$      | 70                     | –               |
| $h$      | 20-150                 | 50-200          |
| $\tilde{\chi}^+$ | 30-200     | 100-200*         |

$^a m_\tilde{\nu} > 300$ GeV

A further advantage of the threshold measurement is that the chargino mass measurement is somewhat isolated from its subsequent decays. Distributions in the final state observables, say e.g. $E_{jj}$ from the decay $\tilde{\chi}^\pm \rightarrow \tilde{\chi}^0 jj$ [16], depend on the neutralino mass. The cross section for chargino pair production, on the other hand, is independent of the final state particles, and only the branching fractions and detector efficiencies for the various final states impact this measurement (as indicated above, if $m_{\tilde{\chi}^\pm} - m_{\tilde{\chi}^0} > M_W$ the branching fractions of chargino decay is given essentially in terms of the $W$ branching fractions).

The chargino production cross section decreases with increasing chargino mass. Therefore the precision with which the mass can be measured is better at smaller values of the mass with precisions of as small as 30 MeV possible for $m_{\tilde{\chi}^\pm} = 100$ GeV. For $m_{\tilde{\chi}^\pm} = 200$ GeV the chargino mass can be determined to 100 (200) Mev for $m_\tilde{\nu} = 500$ (300) GeV. The sneutrino mass can be measured to about 6 GeV accuracy for $m_\tilde{\nu} = 300$ GeV and to about 20 GeV accuracy for $m_\tilde{\nu} = 500$ GeV. This provides an indirect method of measuring the sneutrino mass (the sneutrino might be too heavy to produce directly).

CONCLUSION

A muon collider would provide an opportunity for precision mass measurements in the respective threshold regions$^4$. The precisions that can be obtained for particle masses is shown in Table 1 assuming an integrated luminosity of 100 fb$^{-1}$. The precisions for the Higgs and chargino measurements are correlated with the (as of yet unknown) mass, so the ranges we considered are shown as well. To utilize the highest precision measurements achievable at the statistical level, theoretical uncertainties and other systematics need to be under control in all cases. The muon sneutrino mass can also be simultaneously measured to a few GeV if it is less than 500 GeV in the process $\mu^+\mu^- \rightarrow \chi^+\chi^-$.  

$^4$ The most recent TESLA design envisions a beam energy spread of $R = 0.2\%$ [18] while the NLC design expects a beam energy spread of $R = 1.0\%$. A high energy $e^+e^-$ collider in the large VLHC tunnel would have a beam spread of $\sigma_E = 0.26$ GeV [19] which should give numbers precisions comparable to those considered here.
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