Synchronised neutrino oscillations from self interaction and associated applications

Yvonne Y. Y. Wong

Department of Physics and Astronomy, University of Delaware, Newark, DE 19716

Abstract. A recent revival of interest in synchronised oscillations due to neutrino–neutrino forward scattering in dense gases has led to two interesting applications with notable outcomes: (i) cosmological bounds on neutrino–antineutrino asymmetries are improved owing to flavour equilibration prior to the onset of big bang nucleosynthesis, and (ii) a neutron-rich environment required for $r$-process nucleosynthesis is shown to be always maintained in a supernova hot bubble irrespective of flavour oscillations, contrary to results from previous studies. I present in this talk a pedagogical review of these works.

1. THE NEUTRINO–NEUTRINO FORWARD SCATTERING SAGA

It is well known that neutrino oscillations in a medium are affected by the presence of other particles. A familiar example is oscillations in the sun: Extra interaction channels available exclusively to the electron neutrino induce for it an excess “effective” mass through coherent forward scattering on the ambient electrons [1].

Depending on the electron number density, this excess serves as a refractive index for the oscillating neutrino system, modifies the oscillation frequency and amplitude from their vacuum values, and is responsible for such phenomenon as the Mikheyev–Smirnov–Wolfenstein (MSW) effect [3] and, with it, the large mixing angle (LMA) solution to the solar neutrino problem [4].

In certain astrophysical and cosmological settings, neutrinos form a gas so dense that forward scattering of neutrinos on the neutrinos themselves may constitute a significant source of refraction [5, 6]. Furthermore, such scattering differs markedly from scattering on other fermions, since in the former instance it is possible for the “beam” and “background” neutrinos to exchange flavour via the neutral current. The net outcome is a set of “on-” and “off-diagonal” refractive indices that are dependent on the state of every other neutrino in the gas, thereby rendering the evolution of the whole ensemble highly nonlinear.

In the very earliest works incorporating the refractive index from neutrino–neutrino forward scattering, the off-diagonal refractive indices were erroneously left out. This was rectified by Pantaleone in 1992 [7]. Shortly after, the implications of these “self in-

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1 This statement assumes that $\nu_e$ oscillates into $\nu_\mu$ and/or $\nu_\tau$. If a sterile neutrino is involved, then neutral current scattering of $\nu_e$ on the surrounding electrons and nucleons will also contribute to the excess [2]. Note that we shall not be considering sterile neutrinos in this talk.
teractions” for a multi-momentum gas were investigated extensively by Samuel [8], who discovered in his numerical studies that when a certain critical density is reached, the gas begins to exhibit self-maintained coherence (a.k.a. synchronisation) when decoherence is a priori expected. This interesting effect went unexplained for almost a decade, and was recently reexamined by Pastor, Raffelt, and Semikoz [9], who succeeded in giving it a clean and physical interpretation on which I shall elaborate in this talk.

The result of Ref. [9] has since been extended and applied to the only two environments in which self interactions are expected to play a dominant role: the early universe, and a core collapse supernova. In the first setting, synchronised oscillations are investigated in the context of relic neutrino–antineutrino asymmetries in the epoch preceding big bang nucleosynthesis (BBN). Independent studies by several groups have demonstrated that if the neutrino oscillation parameters are indeed those inferred from the solar and atmospheric neutrino data, then the asymmetries in the $\nu_\mu$ and the $\nu_\tau$ sectors must be equilibrated with that in the $\nu_e$ sector prior to BBN, and be subject to the same stringent constraints on the $\nu_e$ asymmetry derived from uncertainties in the primordial $^4$He abundance [10, 11, 12] (see also Ref. [13]). In the second case, self interactions modify in an unexpected way the flavour evolution of both neutrinos and antineutrinos in the hot bubble region above a nascent neutron star a few seconds after core bounce [14]. This impinges directly on the success or otherwise of $r$-process nucleosynthesis, which is believed to take place in this setting.

In this talk, I shall present a pedagogical review of these works, starting with the interpretation of synchronised oscillations in Sec. 1, and then moving on to the applications in Secs. 2 and 3. The material used here is drawn largely from Refs. [9], [10], [11], [12], and [14]. If you the reader happen to the author of one of these works, please pardon me for quoting you almost verbatim in some places.

1.1. Equations of motion

1.1.1. Density matrix formalism

We begin with a simple two-flavour system. In order to understand the synchronisation phenomenon, it is convenient to work with bilinears of the neutrino wave function (and hence density matrices), since, as we shall see, the neutrino self interaction potential appears also in this form.

For a group of neutrinos with momentum $p$, the density matrix at time $t$ in the flavour basis is defined as

$$
\rho(p,t) \equiv \begin{pmatrix}
\rho_{\alpha\alpha} & \rho_{\alpha\beta} \\
\rho_{\beta\alpha} & \rho_{\beta\beta}
\end{pmatrix} = \frac{1}{N_0(p)} \sum_i |\psi_i(p,t)\rangle \langle \psi_i(p,t)|
$$

$$
= \frac{1}{N_0(p)} \sum_i \left( a_i^*(p,t)b_i(p,t) b_i^*(p,t) |a_i(p,t)|^2 \right), \quad (1)
$$
where the subscripts $\alpha$ and $\beta$ denote two different neutrino flavours, $N_0(p)$ is some time-independent normalisation factor, and

$$|\psi_i(p,t)\rangle = a_i(p,t)|\nu_{\alpha}\rangle + b_i(p,t)|\nu_{\beta}\rangle$$  \hspace{1cm} (2)$$
is the wave function of the $i$th neutrino in the group. The functions $|a_i(p,t)|^2$ and $|b_i(p,t)|^2$ are probabilities in the one-particle formulation. Thus the sums

$$N_{\nu_{\alpha}} = \sum_i |a_i(p,t)|^2 = \rho_{\alpha\alpha}N_0(p), \quad N_{\nu_{\beta}} = \sum_i |b_i(p,t)|^2 = \rho_{\beta\beta}N_0(p)$$  \hspace{1cm} (3)$$
in the diagonal of $\rho(p,t)$ represent respectively the occupation numbers of $\nu_{\alpha}$ and $\nu_{\beta}$ at a given momentum, while the off-diagonal $\rho_{\alpha\beta}$ and $\rho_{\beta\alpha}$ are intangible quantities containing the system’s phase information.

We evolve $\rho(p,t)$ by taking its commutator with the Hamiltonian $H(p,t)$ in flavour space,

$$\partial_t \rho(p,t) = -i[H(p,t),\rho(p,t)].$$  \hspace{1cm} (4)$$
As an example, the Hamiltonian governing $\nu_e \leftrightarrow \nu_{\mu,\tau}$ oscillations in the sun takes the form

$$H(p,t) = \frac{\delta m^2}{4p} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} A(p,t) & 0 \\ 0 & 0 \end{pmatrix},$$  \hspace{1cm} (5)$$
in which the first term describes vacuum oscillations, and the second term with matter potential $A(p,t) = \sqrt{2}G_F n_{e^-}(t)$, where $n_{e^-}(t)$ denotes the solar electron number density, is responsible for refractive effects.$^2$

For our purposes, it is useful to recast the equation of motion (4) into Bloch form by first parameterising $\rho(p,t)$ and $H(p,t)$ in terms of the functions $(P_0, \mathbf{P})$ and $(V_0, \mathbf{V})$:

$$\rho(p,t) = \frac{1}{2}[P_0 + \mathbf{P} \cdot \mathbf{\sigma}] = \frac{1}{2} \begin{pmatrix} P_0 + P_z & P_x - iP_y \\ P_x + iP_y & P_0 - P_z \end{pmatrix},$$

$$H(p,t) = \frac{1}{2}[V_0 + \mathbf{V} \cdot \mathbf{\sigma}] = \frac{1}{2} \begin{pmatrix} V_0 + V_z & V_x - iV_y \\ V_x + iV_y & V_0 - V_z \end{pmatrix}.$$  \hspace{1cm} (6)$$
Here, $\mathbf{\sigma} = \sigma_x \mathbf{x} + \sigma_y \mathbf{y} + \sigma_z \mathbf{z}$ are the Pauli matrices. The quantities $\mathbf{P} = P_x \mathbf{x} + P_y \mathbf{y} + P_z \mathbf{z}$ and $\mathbf{V} = V_x \mathbf{x} + V_y \mathbf{y} + V_z \mathbf{z}$ may be regarded, respectively, as a three-dimensional “spin polarisation” and a “magnetic field” vector, whose time and momentum dependences are implicit. The equation of motion (4) transforms into

$$\partial_t \mathbf{P} = \mathbf{V} \times \mathbf{P}, \quad \partial_t P_0 = 0,$$  \hspace{1cm} (7)$$
such that the dynamics of the oscillating system is analogous to a spin vector predisposed to precessing around a magnetic field $\mathbf{V}$ at a rate $|\mathbf{V}|$, with

$$P_z = \frac{N_{\nu_{\alpha}}(p,t) - N_{\nu_{\beta}}(p,t)}{N_0(p)}$$  \hspace{1cm} (8)$$

$^2$ Rigorous derivations of the equation of motion and various interaction potentials can be found in Refs. [15] and [16]. See also Ref. [17].
parameterising the excess of $\nu_\alpha$ over $\nu_\beta$, and conservation of probability demands that the function

$$P_0 = \frac{N_{\nu_\alpha}(p,t) + N_{\nu_\beta}(p,t)}{N_0(p)}$$

remain constant with time.

Observe in Eq. (7) that $V_0$ is redundant for flavour oscillations, since it contributes only an unobservable overall phase to the neutrino wave function. Some typical examples of the field vector $V$ are (i)

$$V = \frac{\delta m^2}{2p} B = \frac{\delta m^2}{2p} (\sin 2\theta x - \cos 2\theta z),$$

for oscillations in vacuum, where, pictorially, the unit vector $B$ forms an angle $2\theta$ with the negative $z$-axis and $\delta m^2/2p$ is the precession frequency, and (ii)

$$V = \frac{\delta m^2}{2p} B + A(p,t) z = \frac{\delta m^2}{2p} \sin 2\theta x + [A(p,t) - \frac{\delta m^2}{2p} \cos 2\theta] z,$$

for oscillations in a medium, with the matter potential $A(p,t)$ entering in the $z$-direction. As mentioned before, the presence of $A(p,t)$ generally results in the modification of the oscillation frequency and amplitude. In the spin precession picture, this is equivalent to defining a field vector $B_{\text{eff}}$

$$\frac{\delta m^2}{2p} B + A(p,t) z = \frac{\delta m_{\text{eff}}^2}{2p} (\sin 2\theta_{\text{eff}} x - \cos 2\theta_{\text{eff}} z) \equiv \frac{\delta m_{\text{eff}}^2}{2p} B_{\text{eff}},$$

which makes an angle $2\theta_{\text{eff}}$ with the negative $z$-axis, and $P$ precesses around it at a rate $\delta m_{\text{eff}}^2/2p$, where

$$\sin 2\theta_{\text{eff}} = \frac{\sin 2\theta}{\sqrt{(2pA(p,t)/\delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta}},$$

$$\delta m_{\text{eff}}^2 = \delta m^2 \sqrt{(2pA(p,t)/\delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta}.$$

If $|A(p,t)|$ is much larger than $|\delta m^2/2p|$, the former will force the total field vector to align with the $z$-axis. The effective mixing angle $\theta_{\text{eff}}$ approaches 0 or $\pi/2$, thereby suppressing flavour oscillations. On the other hand, if the condition

$$A(p,t) = \frac{\delta m^2}{2p} \cos 2\theta$$

happens to hold, then the vector $V$ points in the $x$-direction, forming a right angle with the $z$-axis which corresponds to maximal mixing, $\theta_{\text{eff}} = \pi/4$. This is the so-called resonance condition.

Finally, observe that the field vector $B_{\text{eff}}$ differs for each momentum mode. Thus the spin vectors of an ensemble of neutrinos possessing a spectrum of momenta do not in general precess in the same direction, or at the same rate.
1.1.2. *Add some neutrino–neutrino forward scattering*

Suppose now we have a spatially homogeneous and isotropic neutrino gas of volume $V$. Contribution from neutrino–neutrino forward scattering to the total field vector $V$ appears in the form

$$\frac{\sqrt{2}G_F}{V} J,$$

where

$$J = \sum_k P_k N_0(p_k)$$

is the total polarisation vector, and the subscript $k$ labels the $k$th momentum mode. Thus the simplest equation of motion one can write down to describe flavour oscillations in the $k$th momentum mode of a self-interacting neutrino gas will consist of a term from vacuum oscillations, and another from self interactions:

$$\partial_t P_k = \frac{\delta m^2}{2p_k} B \times P_k + \frac{\sqrt{2}G_F}{V} J \times P_k.$$

Note that Eq. (17) is valid only for the case of active–active flavour oscillations. For an active–sterile system, all interaction potentials must couple exclusively to the active neutrino, and their contributions to the total field vector $V$ in the Bloch formulation always point in the $\pm z$-direction.

1.2. *Physical interpretation of synchronised oscillations*

1.2.1. *Vacuum-like oscillations*

In the pioneering work of Samuel [8], Eq. (17) was solved numerically for a neutrino gas with a distribution of momenta. To his surprise, the oscillating multi-momentum gas exhibited synchronised, monochromatic behaviours when the self interaction potential was allowed to dominate over the vacuum oscillation rate:

$$\frac{\sqrt{2}G_F}{V} |J| \gg \left| \frac{\delta m^2}{2p_k} \right|.$$

A series of papers by Samuel and co-workers on the same subject emerged in the years following this initial discovery [18, 19, 20], extending the results of Ref. [8] to include antineutrinos and a non-neutrino background in the context of the early universe. Exact solutions to the equations of motion were found [19], and the stability of the synchronised state analysed [20]. Nonetheless, a clean, physical explanation for the synchronisation phenomenon had continued to elude the players. This interesting effect was elucidated recently by Pastor, Raffelt, and Semikoz in Ref. [9], whose argument goes as follows.
When the neutrino gas is sufficiently dense and the condition (18) satisfied, the equation of motion (17) for a single momentum mode becomes dominated on the right hand side by the self interaction potential:

$$\partial_t P_k \simeq \frac{\sqrt{2G_F}}{V} J \times P_k. \quad (19)$$

This implies an approximate solution

$$P_k \simeq (P^i_k \cdot \hat{J}) \hat{J} + \text{precessions around } J, \quad (20)$$

where the superscript “i” denotes initial. In plain English, all polarisation vectors $P_k$ across the momentum distribution are individually first and foremost drawn towards the giant total polarisation vector $J$, and precess around it at a rapid rate of $\sim \sqrt{2G_F}|J|/V$. If the evolution of $J$ proceeds at a rate much slower than this precession frequency, then all $P_k$’s will remain pinned to $J$, which, as we shall see, forms the basis of synchronisation.

The dynamics of $J$ is governed by an equation of motion constructed from summing the single mode equation (17) over all momentum modes,

$$\partial_t J = B \times \sum_k \frac{\delta m^2}{2p_k} P_k N_0(p_k). \quad (21)$$

Substituting the approximate solution (20) and averaging the oscillations to zero over the momentum spread, we immediately see that the evolution of $J$ is non-dissipative,

$$\partial_t J \simeq \left\langle \frac{\delta m^2}{2p} \right\rangle B \times J, \quad (22)$$

with

$$\left\langle \frac{\delta m^2}{2p} \right\rangle \equiv \frac{\sum_k \frac{\delta m^2}{2p_k} (P^i_k \cdot \hat{J}) N_0(p_k)}{|\sum_k P_k N_0(p_k)|}. \quad (23)$$

Since the slowly-moving part of every polarisation vector $P_k$ follows to first approximation the dynamics of $J$, an apparently monochromatic behaviour emerges within the multi-momentum neutrino gas, and $\left\langle \frac{\delta m^2}{2p} \right\rangle$ is the synchronised oscillation frequency with an explicit dependence on the initial conditions.

### 1.2.2. Add a non-neutrino background

Granted that neutrino–neutrino forward scattering continues to constitute the dominant interaction potential, it is easy to generalise the previous picture to include a non-neutrino background $A(p,t)$. The equation of motion is essentially the same as Eq. (17), save for the replacement

$$\frac{\delta m^2}{2p_k} B \rightarrow \frac{\delta m^2}{2p_k} B + A(p_k,t)z, \quad (24)$$
and the analogue of the “dense gas” condition (18) is

$$\underbrace{\frac{\sqrt{2}G_F}{V}}_{\text{J}} \gg |\frac{\delta m^2}{2p_k}|, |A(p_k,t)|.$$ (25)

When this condition is satisfied, all polarisation vectors \(P_k\) become pinned to \(J\) as per the approximate solution (20), and the evolution of \(J\) is equally well described by Eq. (22) upon replacing

$$\langle \frac{\delta m^2}{2p} \rangle B \to \langle \frac{\delta m^2}{2p} \rangle B + \langle A(p,t) \rangle z,$$ (26)

where

$$\langle A(p,t) \rangle \equiv \frac{\sum_k A(p_k,t)(P_k \cdot \hat{J})N_0(p_k)}{|\sum_k P_k N_0(p_k)|}.$$ (27)

represents an average background.

Perhaps the most interesting aspect of this result is that a strong self interaction potential not only forces all modes in a multi-momentum neutrino gas to oscillate at the same frequency, but also with the same amplitude, regardless of the individual modes’ response to the non-neutrino background. Previously, we saw that electrons in the solar interior alter the mixing angle and the oscillation frequency for \(\nu_e \leftrightarrow \nu_\mu, \nu_\tau\) oscillations in a momentum-dependent fashion. In the present case, however, the effective field vector for \(J\) in Eq. (26) necessarily implies a common mixing angle

$$\sin^2 \theta_{\text{synch}} = \frac{\sin^2 \theta}{\sqrt{\langle A(p,t) \rangle / \langle \frac{\delta m^2}{2p} \rangle - \cos^2 \theta}^2 + \sin^2 2\theta},$$ (28)

and a common oscillation frequency

$$\omega_{\text{synch}} = \left\langle \frac{\delta m^2}{2p} \right\rangle \sqrt{\left(\langle A(p,t) \rangle / \langle \frac{\delta m^2}{2p} \rangle - \cos^2 \theta\right)^2 + \sin^2 2\theta},$$ (29)

for all momentum modes. A corollary of Eqs. (28) and (29) is that if the “average” resonance condition

$$\langle A(p) \rangle = \left\langle \frac{\delta m^2}{2p} \right\rangle \cos^2 \theta$$ (30)

is met, then it is necessarily met by all momenta at the same time or at the same place, and any MSW resonant conversion that is incurred must proceed with the same adiabaticity across the spectrum [11, 12].

### 1.2.3. Add some antineutrinos

In a realistic system such as the early universe, it is inevitable that neutrinos and antineutrinos will coexist, in which case it is necessary to consider also neutrinos scattering
on antineutrinos and vice versa. To this end, we first define the density matrix for antineutrinos of momentum \( p \) in flavour space, and the corresponding polarisation vector \( \vec{P} \):

\[
\bar{\rho}(p) = \begin{pmatrix}
\bar{\rho}_{\alpha\alpha} & \bar{\rho}_{\alpha\beta} \\
\bar{\rho}_{\alpha\beta} & \bar{\rho}_{\beta\beta}
\end{pmatrix} = \frac{1}{2}[\bar{P}_0 + \bar{P} \cdot \sigma].
\]

(31)

To first order in \( G_F \), the equations of motion are [16]

\[
\begin{align*}
\partial_t \bar{P}_k &= +\frac{\delta m^2}{2p_k} \mathbf{B} \times \bar{P}_k + (A_{CP^+} + A_{CP^-}) \mathbf{z} \times \bar{P}_k + \frac{\sqrt{2} G_F}{V} (\mathbf{J} - \bar{\mathbf{J}}) \times \bar{P}_k, \\
\partial_t \bar{P}_q &= -\frac{\delta m^2}{2p_q} \mathbf{B} \times \bar{P}_q - (A_{CP^+} - A_{CP^-}) \mathbf{z} \times \bar{P}_q + \frac{\sqrt{2} G_F}{V} (\mathbf{J} - \bar{\mathbf{J}}) \times \bar{P}_q,
\end{align*}
\]

(32)

where \( \bar{\mathbf{J}} \) is the antineutrino analogue of \( \mathbf{J} \), and we have segregated the background interaction potentials into a \( CP \) symmetric \( (A_{CP^+}) \) and a \( CP \) asymmetric \( (A_{CP^-}) \) part, with implicit momentum and time dependences. A \( CP \) asymmetric potential, such as that arising from a local four-fermion coupling, carries opposite signs in the interaction Hamiltonian for neutrinos and antineutrinos, while \( A_{CP^+} \) is identical for both \( \nu \) and \( \bar{\nu} \), and comes usually from higher order propagator effects [5, 16].

Synchronisation occurs when the condition

\[
\frac{\sqrt{2} G_F}{V} |\mathbf{J} - \bar{\mathbf{J}}| \gg \left| \frac{\delta m^2}{2p_k} \right|, |A_{CP^+}|, |A_{CP^-}|
\]

(33)

holds, and the vector \( \mathbf{I} \equiv \mathbf{J} - \bar{\mathbf{J}} \) now assumes the role played by the total polarisation vector \( \mathbf{J} \) in the previous neutrino-only case. All \( \bar{P}_k \) and \( \bar{P}_q \) become pinned to \( \mathbf{I} \),

\[
\begin{align*}
\bar{P}_k &\approx (\bar{P}_k \cdot \hat{\mathbf{I}}) \hat{\mathbf{I}} + \text{precessions around } \mathbf{I}, \\
\bar{P}_q &\approx (\bar{P}_q \cdot \hat{\mathbf{I}}) \hat{\mathbf{I}} + \text{precessions around } \mathbf{I},
\end{align*}
\]

(34)

and the corresponding evolution equation for the compound system is

\[
\partial_t \mathbf{I} \approx \left[ \left\langle \frac{\delta m^2}{2p} \right\rangle \bar{\mathbf{B}} + \left( \langle A_{CP^+} \rangle + \langle A_{CP^-} \rangle \right) \mathbf{z} \times \mathbf{I} \right],
\]

(35)

where

\[
\begin{align*}
\langle \frac{\delta m^2}{2p} \rangle &= \frac{\sum_k \frac{\delta m^2}{2p_k} (\bar{P}_k^i \cdot \hat{\mathbf{I}}) N_0(p_k) + \sum_q \frac{\delta m^2}{2p_q} (\bar{P}_q^i \cdot \hat{\mathbf{I}}) N_0(p_q)}{|\sum_k \bar{P}_k N_0(p_k) - \sum_q \bar{P}_q N_0(p_q)|}, \\
\langle A_{CP^+} \rangle &= \frac{\sum_k A_{CP^+} (\bar{P}_k^i \cdot \hat{\mathbf{I}}) N_0(p_k) + \sum_q A_{CP^+} (\bar{P}_q^i \cdot \hat{\mathbf{I}}) N_0(p_q)}{|\sum_k \bar{P}_k N_0(p_k) - \sum_q \bar{P}_q N_0(p_q)|}, \\
\langle A_{CP^-} \rangle &= \frac{\sum_k A_{CP^-} (\bar{P}_k^i \cdot \hat{\mathbf{I}}) N_0(p_k) - \sum_q A_{CP^-} (\bar{P}_q^i \cdot \hat{\mathbf{I}}) N_0(p_q)}{|\sum_k \bar{P}_k N_0(p_k) - \sum_q \bar{P}_q N_0(p_q)|}.
\end{align*}
\]

(36)
Observe that the present formulation is invalid if
\[ \sum_k P_k N_0(p_k) \neq \sum_q \bar{P}_q N_0(p_q), \tag{37} \]
although the case of identical \( \nu \) and \( \bar{\nu} \) spectra can lead to some curious effects [9]. It is interesting to note that both neutrinos and antineutrinos are synchronised and evolve in an identical manner, regardless of the presence of a background medium that distinguishes between the \( CP \) partners [11, 14]. This has an interesting implication for \( r \)-process nucleosynthesis in a supernova hot bubble, to be discussed later.

This is as much background material as we shall need. Let us now turn to the applications.

2. APPLICATION 1: EQUILIBRATION OF RELIC NEUTRINO–ANTINEUTRINO ASYMMETRIES

One of the many open questions in cosmology is the possibility of admitting a large relic neutrino–antineutrino asymmetry,
\[ L_{\nu\alpha} \equiv \frac{n_{\nu\alpha} - n_{\bar{\nu}\alpha}}{n_\gamma}, \tag{38} \]
where \( n_\psi = \int N_\psi p^2 dp \) denotes the number density of the particle species \( \psi \). At temperatures \( T > 2 \) MeV, thermal and chemical equilibria are generally believed to hold for an ensemble of neutrinos and antineutrinos of some active flavour \( \alpha \). The distribution functions \( N_{\nu\alpha} \) and \( N_{\bar{\nu}\alpha} \) assume the Fermi–Dirac form,
\[ N_{\nu\alpha} = N_{\text{eq}}(\xi_{\nu\alpha}), \quad N_{\bar{\nu}\alpha} = N_{\text{eq}}(\xi_{\bar{\nu}\alpha}), \tag{39} \]
where
\[ N_{\text{eq}}(\xi) = \frac{1}{2\pi^2} \frac{p^2 dp}{1 + e^{p/T - \xi}}, \tag{40} \]
and \( \xi_{\bar{\nu}\alpha} = -\xi_{\nu\alpha} \), such that the asymmetry in \( \nu\alpha \) and \( \bar{\nu}\alpha \) may be alternatively expressed in terms of the species’ chemical potential \( \xi_{\nu\alpha} \),
\[ L_{\nu\alpha} = \frac{1}{n_\gamma} \int (N_{\nu\alpha} - N_{\bar{\nu}\alpha}) = \frac{1}{12 \zeta(3)} \left( \frac{\pi^2}{\Sigma_{\nu\alpha}} + \xi_{\nu\alpha}^3 \right), \tag{41} \]
with \( n_\gamma = 2\zeta(3) T^3 / \pi^2 \), and \( \zeta \) is the Riemann zeta function.\(^3\)

\(^3\) The terms neutrino–antineutrino asymmetry, neutrino chemical potential, and neutrino degeneracy parameter will be used interchangeably throughout this talk.
As of now, there are no direct observations of the cosmic neutrino background. Thus the existence or otherwise of any sizeable $\xi$ can only be established indirectly from the study of the cosmic microwave background radiation (CMBR) anisotropy spectrum, and from requiring consistency with the observationally highly successful theory of BBN. In the former case, we note that the contribution by one neutrino species ($\nu_\alpha$ and $\bar{\nu}_\alpha$) to the total energy density in radiation in the universe depends immediately on the species’ chemical potential:

$$\rho_{\nu_\alpha\bar{\nu}_\alpha} = T^4 \frac{7\pi^2}{120} \left[ 1 + \frac{30}{7} \left( \frac{\xi_{\nu_\alpha}}{\pi} \right)^2 + \frac{15}{7} \left( \frac{\xi_{\bar{\nu}_\alpha}}{\pi} \right)^4 \right].$$  \hspace{1cm} (42)

An increase in the energy density in radiation tends to delay the epoch of matter–radiation equality, which is known to magnify the amplitude of the first acoustic peak in the CMBR angular power spectrum, and shift all other peaks to higher multipoles [21]. Currently, CMBR limits $\xi$ to below 3, applicable to all neutrino flavours [22].

Secondly, an asymmetry in the electron neutrino sector impinges directly on the $\beta$-processes

$$n + \nu_e \leftrightarrow p + e^- , \quad p + \bar{\nu}_e \leftrightarrow n + e^+$$  \hspace{1cm} (43)

that are responsible for setting the neutron-to-proton ratio prior to BBN ($T \sim 1$ MeV)—a positive $\xi_{\nu_e}$ tends to lower $n_n/n_p$ while a negative $\xi_{\nu_e}$ raises it. Since the primordial $^4$He mass fraction $Y_p$ depends exclusively on the value of $n_n/n_p$ at the weak freeze-out, the measurement of $Y_p$ can be used to limit the allowed values of $\xi_{\nu_e}$, which cover the range $-0.01 < \xi_{\nu_e} < 0.07$ at present [10, 23]. Furthermore, a nonzero $\xi$ in any neutrino flavour tends to accelerate the expansion of the universe through its contribution to the energy density a la Eq. (42). This has the effect of raising the freeze-out temperature for the processes of Eq. (43) and therefore also the instantaneous neutron-to-proton ratio, affecting again the value of $Y_p$. An interesting nucleosynthesis scenario that allows for large neutrino asymmetries while simultaneously preserving the standard primordial $^4$He yield is known as degenerate BBN (DBBN), whereby an increased $n_n/n_p$ due to a large $\xi_{\nu_\mu}$ and/or $\xi_{\nu_\tau}$ is compensated for by a sizeable positive $\xi_{\nu_e}$ [23]. Several recent combined analyses of (D)BBN and CMBR have generated the constraints

$$-0.01 < \xi_{\nu_e} < 0.22 , \quad |\xi_{\nu_\mu,\nu_\tau}| < 2.6 ,$$  \hspace{1cm} (44)

assuming no neutrino oscillations [24, 25].

### 2.1. Neutrino oscillations?

Naturally, one is curious to find out how the numbers in Eq. (44) hold up in the presence of oscillations. To date, there are three pieces of evidence for neutrino oscillations.

(i) Electron neutrinos originating from nuclear reactions in the sun have been observed to transform largely into $\nu_\mu$ and/or $\nu_\tau$ [26]. Currently, these oscillation parameters all provide acceptable explanations for the $\nu_e$ deficit [4]:

LMA \quad $\delta m^2 \approx 5 \times 10^{-5}$ eV$^2$ \quad $\sin 2\theta \approx 0.8$, 

LOW $\delta m^2 \simeq 10^{-7} \text{eV}^2$ $\sin 2\theta \simeq \text{large}$,
Vacuum $\delta m^2 \simeq 10^{-10} \text{eV}^2$ $\sin 2\theta \simeq \text{large}$,
SMA $\delta m^2 \simeq 7 \times 10^{-6} \text{eV}^2$ $\sin 2\theta \simeq 0.05$. (45)

The LMA solution, however, is unanimously the most favoured. (ii) The disappearance of atmospheric $\nu_\mu$ and $\bar{\nu}_\mu$ is generally attributed to oscillations into $\nu_\tau$ and $\bar{\nu}_\tau$, with oscillation parameters $\delta m^2 \simeq 3 \times 10^{-3} \text{eV}^2$ and $\sin 2\theta \simeq 1$ [27]. (iii) The Liquid Scintillator Neutrino Detector (LSND) experiment has reported evidence for $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ oscillations with $\delta m^2 > 1 \text{eV}^2$ and $\sin 2\theta \simeq 0.03 \rightarrow 0.1$ [28]. Since it is impossible to accommodate all three signals with only three neutrinos, the most common practice is to either introduce a fourth sterile neutrino (which in fact does not provide a very good fit to the data [29]), or ignore the LSND result. We shall be considering the latter option for simplicity.

In a rudimentary study by Lunardini and Smirnov [13], it was suggested that three flavour oscillations in a near-bimaximal mixing framework (i.e., if LMA holds) necessarily lead to flavour equilibrium prior to BBN (see also an earlier work by Savage, Malaney, and Fuller [30]). This proposal was later examined in detail in the numerical studies of Dolgov et al. [10], and then confirmed by analytical means in a paper by myself [11], and in another independent work by Abazajian, Beacom, and Bell [12]. The common conclusion of these works is that constraints from BBN on the $\nu_e$ asymmetry must now apply also to asymmetries in the $\nu_\mu$ and the $\nu_\tau$ sectors, which constitutes a significant improvement over the DBBN/CMBR bounds quoted in Eq. (44).

A rigorous three flavour analytical study is at present beyond our means. Fortunately, the hierarchical nature of the mass splittings allows us to understand the full equilibration mechanism in terms of two separate equilibrating transitions—the first between $L_{\nu_\mu}$ and $L_{\nu_\tau}$, governed by the atmospheric neutrino oscillation parameters, and the second between $L_{\nu_e}$ and $L_{\nu_x}$, where $\nu_x$ denotes some linear combination of $\nu_\mu$ and $\nu_\tau$, and the $\nu_e \leftrightarrow \nu_x$ transformation proceeds with the parameters of Eq. (45). I shall outline the mechanics of these transformations.

### 2.2. Equilibration of $L_{\nu_e}$ with $L_{\nu_x}$

Suppose there is some pre-existing asymmetry in either or both of the $\nu_e$ and the $\nu_x$ sectors. The one-body reduced density matrices for neutrinos and antineutrinos are as defined previously. These are again parameterised in terms of two polarisation vectors $\vec{P}$ and $\vec{\bar{P}}$, and for the present application, we choose the normalisation factor $N_0(p)$ to be $N_{\text{eq}}(0)$ as defined in Eq. (40). The evolution of the neutrino–antineutrino ensemble at temperatures $T \sim \mathcal{O}(1) \rightarrow \mathcal{O}(10)$ MeV is governed by Eq. (32), where the volume $V$ accompanying the self interaction term is already absorbed into the present definition of $\delta$.\footnote{In principle, if the mixing angle commonly known as $\theta_{13}$ is nonzero, a third equilibrating transition is also present. I shall not talk about this possibility here. The interested reader is referred to the original works for details.}
\( N_0(p) \), and the background potentials are

\[ A_{CP^+} = -\frac{8\sqrt{2}G_F p}{3m_W^2} E_{ee}, \quad A_{CP^-} = \sqrt{2}G_F n_\gamma \eta, \]  

(46)

in which \( E_{ee} \) is the electron–positron energy density, \( m_W \) is the mass of the \( W \) boson, and \( \eta \) is the electron–positron asymmetry. By universal charge neutrality, \( \eta \) must be of the order of the baryon asymmetry so that \( A_{CP^-} \) is negligible in comparison with \( A_{CP^+} \).

The quantity we are interested in is the difference between the two asymmetries

\[
L_{\nu_e} - L_{\nu_x} = \frac{1}{n_\gamma} \int \left[ (N_{\nu_e} - N_{\bar{\nu}_e}) - (N_{\nu_x} - N_{\bar{\nu}_x}) \right] = \frac{1}{n_\gamma} \int (P_z - \bar{P}_z) N_{\text{eq}}(0)
\]

\[
= \frac{1}{n_\gamma} (J_z - \bar{J}_z).
\]

(47)

Hence we would like to track the evolution of the vector \( I \equiv (J - \bar{J})/n_\gamma \) (note the new definition), and flavour equilibrium necessarily implies \( I_z = 0 \).

The next question is, are all the modes synchronised so that we can apply results from the previous sections? The answer comes from figuring out the conditions under which Eq. (33) holds true, and for the temperature range of interest, this amounts to requiring only that the disparity between the initial asymmetries be larger than, say, \( 10^{-5} \) in magnitude [11]. Then, assuming that all neutrinos and antineutrinos are initially in thermal and chemical equilibrium, i.e.,

\[
P_i \cdot \hat{I}_i \approx \frac{N_{\text{eq}}(\xi_{\nu_e}) - N_{\text{eq}}(\xi_{\nu_x})}{N_{\text{eq}}(0)}, \quad \bar{P}_i \cdot \hat{I}_i \approx \frac{N_{\text{eq}}(-\xi_{\bar{\nu}_e}) - N_{\text{eq}}(-\xi_{\bar{\nu}_x})}{N_{\text{eq}}(0)},
\]

(48)

the general compound evolution equation (35) can be rewritten explicitly for this application as

\[
\partial_t I \approx \frac{3}{2} \left( \frac{\tilde{y} (\xi_{\nu_e}^2 - \xi_{\nu_x}^2)}{\pi^2 (\xi_{\nu_e}^2 - \xi_{\nu_x}^2) + (\xi_{\bar{\nu}_e}^2 - \xi_{\bar{\nu}_x}^2)} \right) \left( \frac{\Delta m^2}{2pB} - \frac{8\sqrt{2}G_F p}{3m_W^2} E_{ee} \right) \times I,
\]

(49)

where

\[
\tilde{y} = \frac{\tilde{p}}{T} \equiv \sqrt{\pi^2 + \frac{1}{2} \left( \xi_{\nu_e}^2 + \xi_{\nu_x}^2 \right)}
\]

(50)

represents some average momentum.

Equation (49) has a straightforward interpretation. Consider the terms inside the parentheses. These are identically the vacuum and electron–positron background terms that one would find in a single momentum evolution equation, and control the ensemble’s synchronised mixing angle,

\[
\sin 2\theta_{\text{synch}} = \frac{\sin 2\theta}{\sqrt{(2pA_{CP^+}/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta}} \bigg|_{p=\bar{p}}
\]

(51)
evaluated for $A_{CP} = -8\sqrt{2}G_F\tilde{p}E_{ee}/3m_W^2$ at $p = \tilde{p}$. The factor multiplying the bracketed terms, which we shall label as $\kappa$,
\[
\kappa = \frac{3}{2} \frac{\tilde{\gamma} \left( \xi_{\nu_e}^2 - \xi_{\nu_s}^2 \right)}{\pi^2 (\xi_{\nu_e} - \xi_{\nu_s}) + (\xi_{\nu_s}^3 - \xi_{\nu_s}^3)},
\]
does not play a role in determining the effective mixing angle. It does, however, influence directly the oscillation frequency,
\[
\frac{\delta m^2_{\text{synch}}}{2\tilde{p}} = \kappa \delta m^2 \sqrt{(2pA_{CP}/\delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta} \bigg|_{p=\tilde{p}},
\]
For instance, $\delta m^2_{\text{synch}}$ vanishes for $\xi_{\nu_e} = -\xi_{\nu_s}$, and oscillations are switched off completely.

An approximate solution to Eq. (49) can be obtained in the adiabatic limit. Specifically, the expression for the variable $I_z$ as a function of time is
\[
I_z \equiv L_{\nu_e} - L_{\nu_s} \simeq \left( \cos 2\theta_c \cos 2\theta_i^c + \sin 2\theta_c \sin 2\theta_i^c \cos \int^t_{t_i} \frac{\Delta m^2_{\text{eff}}}{2\tilde{p}} dt' \right) I_{\nu_e}^i,
\]
assuming the validity of the adiabatic condition
\[
\gamma \equiv \left| \frac{V_z \partial_t V_x - V_x \partial_t V_z}{\kappa (V_x^2 + V_z^2)^{3/2}} \right|_{p=\tilde{p}} < 1,
\]
where $V_x = (\delta m^2/2p) \sin 2\theta$, and $V_z = A_{CP} - (\delta m^2/2p) \cos 2\theta$, both evaluated for $p = \tilde{p}$. Equation (54) predicts for maximal vacuum mixing an MSW-like effect, transforming $I_z$ from $I_{\nu_e}^i$ to 0 (plus some small amplitude oscillations) when vacuum oscillations overcome refractive matter effects. The temperature at which this equilibrating transition takes place can be established roughly by solving
\[
\frac{8\sqrt{2}G_F\tilde{p}}{3m_W^2} \simeq \frac{\left| \delta m^2 \right|}{2\tilde{p}}.
\]
For initial chemical potentials satisfying the constraints (44), the equilibration temperature turns out to be $\simeq 2.6$ MeV for the LMA solution. In the cases of the LOW and the Vacuum mass splittings, the temperatures are $\simeq 0.9$ MeV and $\simeq 0.3$ MeV respectively. Evidently, only the LMA equilibrating transition can take place well ahead of BBN.$^5$

For the oscillation parameters of the SMA solution, $I_z$ remains close to its initial value even after the “transition” at $T \simeq 1.9$ MeV, since both the vacuum and the background

---

$^5$ The real LMA solution encompasses a range of mixing parameters that are merely large, but not maximal [4]. Thus only a partial equilibrium between the asymmetries can be achieved from synchronised oscillations alone. However, collisions with the background medium (i.e., momentum-changing non-forward scattering) are inevitable, and these are expected to drive the equilibration to a more complete state.
terms are predominantly in the negative $z$-direction for $\delta m^2 > 0$, and the usual MSW resonance condition cannot be satisfied.

A second deciding factor on the efficacy of flavour equilibration is the adiabaticity of the transition from matter-suppressed to vacuum oscillations. Unlike that encountered in, for instance, solar neutrino analyses, the adiabaticity parameter $\gamma$ of Eq. (55) is strongly dependent on the initial conditions. Take for concreteness the case of $\xi_{\nu_e}^i = 0$. The adiabatic condition $\gamma < 1$ always holds for LMA if $|\xi_{\nu_x}^i| > 0.01$, but can be badly violated at the transition point for the LOW $\delta m^2$ unless $|\xi_{\nu_x}^i| > 0.1$ [11]. In the case of maximal mixing, violation of adiabaticity at the transition point generally results in large amplitude “post-transition” oscillations about the equilibrium point at an angular frequency roughly equal to $\delta m^2_{\text{synch}}/2\tilde{p}$. Naturally, this is quite a separate phenomenon from true equilibrium. On the other hand, an adiabaticity parameter that evaluates to infinity at all times (because, for example, $\kappa \to 0$) signifies that there is no transition at all. From the perspective of equilibrating two vastly different asymmetries, the requirement of $|\xi_{\nu_x}^i| > 0.01$ (assuming $\xi_{\nu_e}^i = 0$) in the LMA case for a smooth transition is, by definition, not a major concern.

### 2.3. Equilibration of $L_{\nu_\mu}$ and $L_{\nu_\tau}$, and new constraints on neutrino–antineutrino asymmetries

Super-Kamiokande’s atmospheric neutrino data indicate maximal mixing between $\nu_\mu$ and $\nu_\tau$, with a mass splitting that is some two orders of magnitude larger than the maximum acceptable solar $\delta m^2$. This means, in general terms, that equilibration of the asymmetries in the $\nu_\mu$ and the $\nu_\tau$ sectors will occur in a manner similar to that outlined before, but at a higher temperature, and subject to a background potential proportional to the muon–antimuon energy density $E_{\mu\mu}$. A comparison of the magnitudes of the vacuum and the background potential terms returns an equilibration temperature of $\approx 12$ MeV for $\delta m^2 = 3 \times 10^{-3}$ eV$^2$ and $p = \tilde{p} \approx \pi/T$, and complete equilibrium between $L_{\nu_\mu}$ and $L_{\nu_\tau}$ can always be achieved because of maximal mixing and collision-induced flavour relaxation not discussed here [10, 11, 12].

Thus, summing up the findings, we conclude that all active neutrino–antineutrino asymmetries or chemical potentials must come to an equal value before the onset of BBN, if the flavour oscillation parameters are those indicated by the atmospheric neutrino data and by the solar LMA solution. The stringent constraint imposed by the primordial $^4$He abundance on $\xi_{\nu_e}$,

$$\xi_{\nu_e} \simeq \xi_{\nu_\mu} \simeq \xi_{\nu_\tau} < 0.07,$$

now applies to all three neutrino flavours.
3. APPLICATION 2: R-PROCESS NUCLEOSYNTHESIS IN A SUPERNOVA HOT BUBBLE

Heavy nuclei with mass number \( A > 70 \) are predominantly produced by slow and rapid neutron capture—the \( s \)- and the \( r \)-processes. The most plausible site for \( r \)-process nucleosynthesis suggested so far is the hot bubble between a protoneutron star and the escaping shock wave in a core collapse supernova a few seconds after core bounce. A key condition for this process to be successful, however, is that the environment must be rich in neutrons (relative to protons), and neutrinos released from the cooling of the nascent neutron star play an important role in establishing this condition.

Just as in the early universe, the neutron-to-proton ratio is fixed by the \( \beta \)-processes in Eq. (43). Assuming charge neutrality, it is common to express this ratio in terms of the electron fraction \( Y_e \):

\[
\frac{n_n}{n_p} = \frac{1}{Y_e - 1}.
\]

Clearly, \( Y_e < 0.5 \) will satisfy the minimum requirement for neutron predominance, but a successful \( r \)-process may call for \( Y_e < 0.45 \). Because neutrinos are far more abundant than electrons and positrons, \( Y_e \) is governed largely by the neutrino spectra and fluxes. Near the weak freeze-out radius (\( \sim 30 \rightarrow 35 \) km), this is well approximated by [31]

\[
Y_e \simeq \left( 1 + \frac{L_{\nu_e} \bar{\epsilon}}{L_{\bar{\nu}_e} \tilde{\epsilon}} \right)^{-1},
\]

where \( L_{\nu_e} \) and \( L_{\bar{\nu}_e} \) are the electron neutrino and antineutrino luminosities (typically of order \( 10^{51} \) erg s\(^{-1} \)), \( \epsilon \equiv \langle E^2_{\nu_e} \rangle / \langle E_{\nu_e} \rangle \), and \( \bar{\epsilon} \equiv \langle E^2_{\bar{\nu}_e} \rangle / \langle E_{\bar{\nu}_e} \rangle \). Neutrinos streaming out of the neutrino sphere at a radius of \( \sim 10 \) km generally possess spectra of Fermi–Dirac form, with mean energies

\[
\langle E_{\nu_e} \rangle \simeq 11 \text{ MeV}, \quad \langle E_{\bar{\nu}_e} \rangle \simeq 16 \text{ MeV}, \quad \langle E_{\nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau} \rangle \simeq 25 \text{ MeV}.
\]

These numbers yield for \( Y_e \) a value of \( \sim 0.41 \), allowing for a successful \( r \)-process, in the absence of neutrino oscillations.

Contrastingly, if flavour oscillations between \( \nu_e \) and \( \nu_\mu, \tau \) occur outside the neutrino sphere but still within the weak freeze-out radius, then the two dissimilar neutrino spectra will be exchanged to various degrees depending on the oscillation parameters, and \( Y_e \) becomes modified as a generic consequence [31]. Since a large, positive electron–positron asymmetry is always present in the background medium, and the interaction Hamiltonian dominated by the \( CP \) asymmetric potential \( \sqrt{2} G_F (n_{e^-} - n_{e^+}) \), conventional wisdom tells us that the MSW resonance condition cannot be satisfied simultaneously by both neutrinos and antineutrinos. If the former meets the condition, the more energetic \( \nu_\mu \)’s will be transformed across the resonance into \( \nu_e \)’s, while the \( \bar{\nu}_e \) energy spectrum remains virtually unaffected. This then leads to an over-production of electrons, pushes \( Y_e \) to above 0.5, and consequently ruins the \( r \)-process. The mass splitting required for such oscillations to be effective is \( \delta m^2 > 1 \text{eV}^2 \), which overlaps with the region of parameter space needed to explain the LSND result [28].
This situation changes drastically when neutrino–neutrino forward scattering is also taken into account. Specifically, for the nominal neutrino luminosity $L_\nu = 10^{51}$ erg, the electron fraction always stays below 0.5, such that the minimum requirement for a neutron-rich environment can always be satisfied, irrespective of neutrino oscillations. This interesting result can be easily understood in terms of synchronisation [14].

As noted before, when neutrino–neutrino forward scattering constitutes the dominant interaction potential, it synchronises both neutrinos and antineutrinos, regardless of the nature of the non-neutrino background medium. Consequently, if the neutrinos were to encounter an MSW resonance, the antineutrinos would also be dragged along for the ride, and any spectral swap that accompanies the transitions would occur with identical efficiency. The net effect is that the post-resonance $\nu_e$ and $\bar{\nu}_e$ will now have identical spectra, having taken over those which once belonged to $\nu_\mu,\tau$ and $\bar{\nu}_\mu,\tau$ respectively. Assuming that $L_{\nu_e} \simeq L_{\bar{\nu}_e}$, Eq. (59) always gives $Y_e \simeq 0.5$ for the electron fraction.

4. CONCLUSIONS

I have reported in this talk the findings of a series of recent works devoted to explaining and applying the phenomenon of synchronised flavour oscillations due to neutrino self interaction, in the early universe and in a core collapse supernova.

For the “explanation” part, I presented the original interpretation of Pastor, Raffelt, and Semikoz [9], as well as extensions to it introduced by myself [11], and separately, by Abazajian, Beacom, and Bell [12].

Application-wise, the analyses of Dolgov et al., and Refs. [11] and [12] demonstrate that neutrino self interactions in the epoch immediately prior to BBN are sufficiently intense to synchronise the multi-momentum flavour oscillations, which, together with the large mixing angles indicated by present neutrino data, allows for the efficient equilibration of all pre-existing neutrino–antineutrino asymmetries. I have reviewed these works in considerable detail, and relayed the unanimous conclusion that stringent BBN bounds on the electron neutrino–antineutrino asymmetry must now apply also to asymmetries in the $\nu_\mu$ and the $\nu_\tau$ sectors.

Lastly, I spoke briefly about a possible upset of the neutron-rich environment and thus the success of $r$-process nucleosynthesis in a supernova hot bubble by “conventional” neutrino oscillations. I proceeded to report the new results of Pastor and Raffelt [14], which show that, if neutrino self interactions are properly taken into account, synchronised oscillations will always prevent the neutron-to-proton ratio from dropping below the minimum value required for neutron predominance.

And on this note, I rest my case.

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REFERENCES

1. L. Wolfenstein, Phys. Rev. D 17, 2369 (1978); 20, 2634 (1979).
2. V. Barger, N. Deshpande, P. B. Pal, R. J. L. Phillips, and K. Whisnant, Phys. Rev. D 43, R1759 (1991).
3. S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. 42, 1441 (1985) [Sov. J. Nucl. Phys. 42, 913 (1985)]; Nuovo Cimento C 9, 17 (1986).
4. See, for example, M. C. González-García, M. Maltoni, C. Peña-Garay, and J. W. F. Valle, Phys. Rev. D 63, 033005 (2001).
5. D. Nötzold and G. Raffelt, Nucl. Phys. B307, 924 (1988).
6. G. M. Fuller, R. W. Mayle, J. R. Wilson, and D. N. Schramm, Astrophys. J. 322, 795 (1987).
7. J. Pantaleone, Phys. Lett. B 287, 128 (1992); Phys. Rev. D 46, 510 (1992).
8. S. Samuel, Phys. Rev. D 48, 1462 (1993).
9. S. Pastor, G. G. Raffelt, D. V. Semikoz, Phys. Rev. D 65, 053011 (2002).
10. A. D. Dolgov, S. H. Hansen, S. Pastor, S. T. Petcov, G. G. Raffelt, and D. V. Semikoz, Nucl. Phys. B 632, 363 (2002).
11. Y. Y. Wong, Phys. Rev. D 66, 025015 (2002).
12. K. N. Abazajian, J. F. Beacom, and N. F. Bell, Phys. Rev. D 66, 013008 (2002).
13. C. Lunardini and A. Yu. Smirnov, Phys. Rev. D 64, 073006 (2001).
14. S. Pastor and G. Raffelt, Phys. Rev. Lett. 89, 191101 (2002).
15. B. H. J. McKellar and M. J. Thomson, Phys. Rev. D 49, 2710 (1994).
16. G. Sigl and G. Raffelt, Nucl. Phys. B406, 423 (1993).
17. G. Raffelt, G. Sigl, L. Stodolsky, Phys. Rev. Lett. 70, 2363 (1993).
18. V. A. Kostelecký, J. Pantaleone, and S. Samuel, Phys. Lett. B 315, 46 (1993); V. A. Kostelecký and S. Samuel, ibid. 318, 127 (1993); 385, 159 (1996); Phys. Rev. D 49, 1740 (1994); 52, 3184 (1995).
19. V. A. Kostelecký and S. Samuel, Phys. Rev. D 52, 621 (1995); S. Samuel, ibid. 53, 5382 (1996).
20. J. Pantaleone, Phys. Rev. D 58, 073002 (1998).
21. J. Lesgourgues and S. Pastor, Phys. Rev. D 60, 103521 (1999).
22. S. Hannestad, Phys. Rev. Lett. 85, 4253 (2000).
23. H. Kang and G. Steigman, Nucl. Phys. B372, 494 (1992); S. Esposito, G. Mangano, G. Miele, and O. Pisanti, J. High Energy Phys. 09, 038 (2000); S. Esposito, G. Mangano, A. Melchiorri, G. Miele, and O. Pisanti, Phys. Rev. D 63, 043004 (2001).
24. S. H. Hansen, G. Mangano, A. Melchiorri, G. Miele, and O. Pisanti, Phys. Rev. D 65, 023511 (2001).
25. J. P. Kneller, R. J. Scherrer, G. Steigman, and T. P. Walker, Phys. Rev. D 64, 123506 (2001).
26. Homestake collaboration, B. T. Cleveland et al., Astrophys. J. 495, 505 (1998); GALLEX collaboration, W. Hampel et al., Phys. Lett. B 447, 127 (1999); SAGE collaboration, J. N. Abdurashitov et al., Phys. Rev. C 60, 055801 (1999); Super-Kamiokande collaboration, S. Fukada et al., Phys. Rev. Lett. 86, 5651 (2001); 86, 5656 (2001); GNO collaboration, M. Altman et al., Phys. Lett. B 490, 16 (2000); SNO collaboration, Q. R. Ahmad et al., Phys. Rev. Lett. 87, 071301 (2001).
27. Super-Kamiokande collaboration, S. Fukada et al., Phys. Rev. Lett. 85, 3999 (2000); 81, 1562 (1998).
28. LSND collaboration, A. Aguilar et al., Phys. Rev. D 64, 112007 (2001).
29. See, for example, M. Maltoni, T. Schwetz, and J. W. F. Valle, Phys. Rev. D 65, 093004 (2002); see also, R. Foot, hep-ph/0210393.
30. M. J. Savage, R. A. Malaney, and G. M. Fuller, Astrophys. J. 368, 1 (1991).
31. Y.-Z. Qian, G. M. Fuller, G. J. Mathews, R. W. Mayle, J. R. Wilson, and S. E. Woosley, Phys. Rev. Lett. 71, 1965 (1993).