Pseudogap state near a quantum critical point

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In the standard picture of a quantum phase transition, a single quantum critical point separates the phases at zero temperature. Here we show that the two-dimensional case is considerably more complex. Instead of the single point separating the antiferromagnet from the normal metal, we have discovered a broad region between these two phases where the magnetic order is destroyed but certain areas of the Fermi surface are closed by a large gap. This gap reflects the formation of a quantum state characterized by a superposition of \textit{d}-wave superconductivity and a quadrupole density wave, which builds a checkerboard pattern with a period incommensurate with that of the original spin-density wave. At moderate temperatures both orders coexist over comparatively large distances but thermal fluctuations destroy the long-range order. Below a critical temperature the fluctuations are less essential and superconductivity becomes stable. This phenomenon may help to explain the origin of the mysterious pseudogap state and of the high-temperature transition into the superconducting state in the cuprates. In particular, we show that spectroscopic probes on the oxygen and copper sites reveal checkerboard order.

Quantum criticality has become one of the most fascinating subjects in theoretical physics for the past two decades. At zero temperature, when the system reaches a certain threshold point, quantum fluctuations are so strong that the metallic state is broken. Electrons do not behave as stable quasi-particles anymore but become heavy and short-lived, hence affecting profoundly the properties of those materials in both thermodynamics and transport. Quantum critical points (QCPs) are among the strongest disturbances that can be exerted on the metallic state; as such they are top candidates to explain the mysterious behaviour of high-temperature superconducting cuprates\textsuperscript{1-3}, heavy fermions\textsuperscript{4,5} or doped ferromagnets\textsuperscript{6}.

The beginning of the study of the quantum critical metals dates back to the mid-1970s\textsuperscript{7,8}. The main issue in dealing with quantum criticality in metals is that the fermions around the Fermi surface create dangerous massless modes, which makes the physical properties singular.

An important step in the study of the quantum criticality was the formulation of a spin-fermion model designed to picture two-dimensional metals near the antiferromagnetic\textsuperscript{9,10} and ferromagnetic\textsuperscript{11} instability. In this model, low-energy fermions interact with a collective spin-fluctuation mode $\phi$ that is driven into criticality near the magnetic transition. The authors of refs 9,10 developed an Eliashberg-like technique similar to the one well known in the context of strongly coupled superconductivity and basically reproduced the results of the Hertz theory$^9$.

However, as recently unveiled\textsuperscript{12-15}, this ansatz is incomplete because such a theory misses some important contributions in both perturbation theory and renormalization group, thus invalidating the whole preceding picture.

Model and method of calculations

Here, we revisit the issue of quantum antiferromagnet–normal-metal transitions in two-dimensional models of itinerant electrons and demonstrate, within a slightly modified version of the spin-fermion model of refs 9,10, that the coupling of the bosonic spin mode to the electrons generates at the QCP a pseudogap in the spectrum corresponding to an order completely different from the original spin-density wave (SDW). This new state may be understood as a superposition of $d$-wave superconductivity and a quadrupole-density wave (QDW), and its emergence around the QCP constitutes an unexpected outcome of our theory. Highly relevant for the high-$T_c$ cuprates, the spin-fermion model lives on the Cu sites in the CuO$_2$ lattice. When the oxygen sites are incorporated, the quadrupole order induces a modulated charge order of the four O atoms surrounding the same Cu atom (Fig. 1a) that, in turn, leads to an energy modulation on Cu atoms. Altogether, the modulation forms a checkerboard structure.

Interestingly, our formalism shows a certain analogy with the theory of Anderson localization by disorder\textsuperscript{16} and in both cases the low-energy physics is finally captured in terms of a nonlinear $\sigma$-model. We sketch the derivation here and refer the reader to the Supplementary Information for details.

The physics of electrons interacting through critical bosonic modes is described\textsuperscript{17,18} by the Lagrangian $L = L_\phi + L_\psi$ with

$$L_\phi = \psi^\dagger [i\tau \cdot (-i\nabla) + \lambda \phi \sigma] \psi$$

$$L_\psi = \frac{1}{2} \phi D^{-1} \phi + \frac{g}{2} (\phi^2)^2$$

Herein, $L_\psi$ is the Lagrangian of electrons with spectrum $\varepsilon(p)$ that propagate in the fluctuating field $\phi$ representing the bosonic spin excitations modelled by the quantum critical Lagrangian $L_\phi$.

We define the bare propagator $D$ of the spin-wave boson mode entering equation (2) through its Fourier transform

$$D^{-1}(\omega, \mathbf{q}) = \frac{\omega^2/n^2 + (\mathbf{q} - \mathbf{Q})^2}{\omega^2/n^2 + a}$$

where $n$ is the spin-wave velocity, $a$ is a mass characterizing the distance to the QCP (at the QCP, $a = 0$, whereas $a > 0$ on the metallic side), and $\mathbf{Q}$ is the ordering wave vector in the SDW phase. Pursuing the application of the spin-fermion model to cuprates, it is usually assumed that the spectrum $\varepsilon(p)$ in equation (1) leads to a Fermi surface of the shape represented in Fig. 1b. To facilitate a controlled theoretical analysis (see below), we instead consider in the following a slightly deformed Fermi surface as in Fig. 1c. The points on the Fermi surface connected by the vector $\mathbf{Q}$ are the hotspots in the model and, close to criticality, the most interesting physics is formed.

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in their vicinity. Figure 1b,c illustrates that there are eight hotspots on the Fermi surface. It is implied that the coupling constant $\lambda$ is small, $\lambda^2 \ll \nu p_0$, where $\nu$ is the Fermi velocity and $p_0$ is the radius of curvature at the hotspots. The quartic $\phi^4$-term in $L_0$ is usually neglected for $\lambda \gtrsim 0$.

We keep the theory under control by assuming that the Fermi velocities $v_{1,2}$ of two hotspots connected by vector $Q$ in Fig. 1c are close to being parallel to each other,

$$\delta \ll 1 \tag{4}$$

with the angle $\delta$ defined in the inset of Fig. 1c. In the more realistic situation of Fig. 1b, we expect qualitatively similar results, although the theory would not formally be justified.

The Landau damping modifies the form of $D^{-1}(\omega, q)$, equation (3), adding to the latter the term $\gamma |\omega|$ with $\gamma = (2\lambda^2/(\pi\nu^2\sin \alpha))$ and $\nu = \nu_{1,2}$. In the limit (4), the Landau damping is strong, leading to a weak-coupling limit of our theory, and dominates over the $\omega^2$-term in the bare propagator $D^{-1}$.

### Mean-field equations and pseudogap state

In the mean-field approximation, the fermion–fermion interaction mediated by the boson propagator $D$ leads to a superposition of particle–particle and particle–hole pairings,

$$\Sigma^{pp}_{\text{p}}(\omega, q) = \Sigma^{ph}_{\text{p}}(\omega, q) + \Sigma^{ph}_{\text{h}}(\omega, q) \tag{5}$$

with the momentum $p$ located at hotspots opposite to each other on the Fermi surface. In equation (5), $\Sigma^{pp}_{\text{p}}$ is the Pauli matrix for the electron spin and $\Sigma^{ph}_{\text{p}}$ is the amplitude of the particle–particle (particle–hole) pairing. The pairings of the type in equation (5) are purely singlet and thus do not lead to any spin order. The signs of coefficients $\Sigma^{pp/ph}_{\text{p}}$ at neighbouring hotspots on each connected piece of the Fermi surface are opposite, indicating a $d$-wave-like structure of the gap in the electron spectrum. Therefore, neither local charge- nor current-density modulations arise. The particle–hole correlations in equation (5) have been discussed in ref. 14 and classified by its authors as corresponding to modulated correlations in a valence bond solid. In that work, the authors have concluded that the superconducting correlations were stronger owing to curvature effects while, at the same time, they have already noticed an emerging SU(2) symmetry in the linearized spin-fermion model. It is our central finding that the particle–hole and superconducting particle–particle pairings have to be considered on equal footing even for a finite curvature of the Fermi surface, and that they together form a composite SU(2) order parameter.

Let us have a closer look at the analytical form of the mean-field pseudogap in the spin-fermion model. At the end of this section, we discuss its implications for the CuO$_2$ lattice in the high-$T_c$ cuprates. The general solution $O$ of the mean-field equations for the order parameter at a given hotspot may be represented in the form $O(\epsilon) = b(\epsilon)u$, with $u$ being an arbitrary SU(2) unitary matrix, $u^*u = 1$, $\det u = 1$ and $b(\epsilon)$ a real positive function of the fermionic Matsubara frequency $\epsilon$. After a rescaling to dimensionless quantities, $\epsilon \rightarrow \epsilon \Gamma, b \rightarrow b \Gamma$, and $T \rightarrow \tilde{T} \Gamma$ with the characteristic energy $\Gamma = (3\lambda^2/8\pi\nu^2\sin \alpha)$, we obtain at criticality ($\alpha = 0$) a set of remarkably universal self-consistency equations that are independent of the parameters of the model,

$$\tilde{b}(\epsilon) = T \sum \frac{\cos \Theta(\epsilon')}{\sqrt{Q(\epsilon - \epsilon')}} = \epsilon + T \sum \frac{\sin \Theta(\epsilon')}{\sqrt{Q(\epsilon - \epsilon')}}$$

$$\Omega(\tilde{\omega}) = 2\pi T \sum \sin^2 \left( \frac{\Theta(\epsilon + \tilde{\omega}) - \Theta(\epsilon)}{2} \right) \tag{6}$$

In these equations, $\sin \Theta(\epsilon) = f(\epsilon)[f(\epsilon)^2 + f(\epsilon)]^{1/2}$. The functions $b(\epsilon)$ and $f(\epsilon)$ are by construction even, $b(\epsilon) = b(-\epsilon)$, and odd, $f(\epsilon) = -f(-\epsilon)$, respectively, and $\tilde{\omega}$ is a rescaled bosonic Matsubara frequency. The function $f(\epsilon)$ replaces the frequency term $\epsilon$ in the bare fermion propagator. We note that similar equations have been written previously$^{17}$ for studying the superconducting instability.

A quick glance at equations (6) reveals the trivial solution $b(\epsilon) = 0$, leading to $\Omega(\tilde{\omega}) = |\tilde{\omega}|$ and $f(\epsilon) = \text{sign}(\epsilon)|\epsilon|/|\epsilon| + 2/\pi\sqrt{|\epsilon|}$. This solution corresponds to the one-loop self-energy corrections$^{8,10,18}$ to the bosonic and fermionic propagators. Here, of greater interest is the existence of a non-trivial and so far unanticipated energy-dependent solution $b(\epsilon)$. It can be computed numerically and its dependence on $\epsilon$ and $T$ is shown in Fig. 2a. We have checked that the free energy corresponding to the non-trivial solution is lower than the one for $b(\epsilon) = 0$ (see Supplementary Information).

The characteristic value of $b(\epsilon)$ is of order $\Gamma$, implying that it scales linearly with the interaction constant $\lambda^2$. This is in sharp contrast with the exponentially small values of the gap encountered in conventional superconductors$^{19}$ and, therefore, one can expect at a QCP much higher values of the gap.

Equations (6) have been obtained by linearizing the electron spectrum near the hotspots but their solution formally does not depend on the position on the Fermi surface. In fact, the order parameter $O$ depends not only on the frequency $\epsilon$ but also on the distance from the hotspots, decaying at momenta of order $\gamma/v$. The characteristic length of the arc of the Fermi surface under the gap, however, should be calculated taking into account the curvature of the Fermi surface. As a result, the gap is large only in the vicinity of the hotspots. Schematically, $b(0, p)$ on the Fermi surface is depicted in Fig. 2b. At the same time, increasing the effective coupling constant—which corresponds to the more realistic Fermi surface in Fig. 1b with the angle $\delta$ inevitably of order of unity—leads

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**Figure 1 | Real-space CuO$_2$ plane, Brillouin zone and Fermi surface for the spin-fermion model.** a. Structure of Cu (3$d_{x^2-y^2}$) and O (2$p_x/2p_y$) orbitals in the CuO$_2$ plane. Partial charges at the O atoms can induce an effective elementary quadrupole at a site in the Cu lattice. b. Brillouin zone for the square Cu lattice after integrating out the O atoms. c. Brillouin zone in our weak-coupling model at a small angle $\delta$ between the Fermi velocities $v_1$ and $v_2$. The index $l$ labels the four pairs of hotspots. In both $\textbf{b}, \textbf{c}$, electrons at hotspots connected by the vector $Q$ interact through the critical bosonic mode. The vectors $Q_{1/2}$ and the given linear combinations $Q_{l}$ modulate the amplitudes of particle–hole pairings.

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to spreading the region with a large gap over an essential part of the Fermi surface\(^2\). In this situation two hotspots located across the boundary of the Brillouin zone can effectively merge, such that the gap \( b(0, p) \) reaches eventually the maximum value at the antinodal points (see Fig. 2c). This effect can be enforced by inhomogeneities and, as a result, lead to a \( d \)-wave-like dependence on the position on the Fermi surface.

The SU(2) matrix \( u \) reflects the degeneracy of the order parameter and may be parametrized as

\[
 u = \left( \begin{array}{cc} \Delta_+ & \Delta_- \\ -\Delta_+^* & -\Delta_-^* \end{array} \right) \quad \text{with} \quad |\Delta_+|^2 + |\Delta_-|^2 = 1
\] (7)

The complex numbers \( \Delta_+ \) and \( \Delta_- \) should be interpreted as order parameters for the superconducting and particle–hole order, respectively. In contrast to the conventional superconductivity, we are confronted with both particle–particle and particle–hole pairs at the same time (see Fig. 3a,b).

Studying the symmetries of this order within the spin-fermion model, we find that the \( d \)-wave structure does not lead to local charge- or current-density modulations. However, the electron–hole pairing breaks the rotational symmetry of the electron gas, giving rise to finite modulated quadrupole density

\[
 D_{xy}(r) \propto |\Delta_-| \sin(Q_x r - \varphi_x) \cos(Q_y r - \varphi_y)
\] (8)

with \( \varphi_x \) and \( \varphi_y \) denoting phases. This formula describes a spatial oscillation of the off-diagonal elements of the quadrupole tensor with the wave vectors \( Q_x = (Q_1 + Q_2)/2 \) and \( Q_y = (Q_1 - Q_2)/2 \), where \( Q_1 \) and \( Q_2 \) connect two hotspots at \( \pm p \) (Fig. 1b,c). Note that the vectors \( Q_{\pm} \) are considerably smaller than the SDW wave vector \( Q \) and that the resulting chequerboard structure is incommensurate with the original lattice. This new type of particle–hole order is referred to here as a QDW.

In the CuO\(_2\) lattice, the QDW corresponds to a rather unique charge-density-wave order on the oxygen O (2p) orbitals. In contrast, the average charge is the same on all Cu (3d\(_{x^2-y^2}\)) orbitals; however, we predict a modulation with the same wave vectors \( Q_{\pm} \).

The transition energies between Cu (3d\(_{x^2-y^2}\)) and Cu (2p) states, an effect due to Coulomb interaction between Cu (2p) electrons and charge modulation on the four neighboring O atoms. Indeed, the local electron (hole) density \( \rho_Q(r, a_0) \) on an O atom located between the two neighboring Cu sites at \( r \) and \( r + a_0 \) may be reduced to the bond correlation function \( \langle \psi^*_Q \psi_{r+a_0} \rangle \). In the presence of the pseudogap, we derive in the Supplementary Information that

\[
 \rho_Q(r, a_0) \propto \pm |\Delta_+| \sin(Q_x r - a_0/2 - \varphi_x) \cos(Q_y r - a_0/2 - \varphi_y)
\] (9)

The overall sign in this formula is + or − depending on whether \( a_0 \) is directed along an O(2p) or an O(2p) orbital, respectively. The elementary quadrupole plaquette corresponding to equation (9) is represented in Fig. 1a. Thus, the order obtained within the spin-fermion model reflects a charge modulation on the oxygen sites with the same wave vectors \( Q_{\pm} \) as in equation (8). In Fig. 4 illustrating the resulting chequerboard structure on the CuO\(_2\) lattice, the vectors \( R_{\pm} = 2\pi Q_{\pm} / |Q_{\pm}| \) reciprocal to \( Q_{\pm} \) are drawn. Remarkably, although the vectors \( Q_{\pm} \) are close to those connecting the antinodes (see Fig. 1(b)) we did not assume any nesting in the model under consideration. Instead, the emergence of the particle–hole QDW order is a consequence of the proximity to the QCP.

The QDW is a pure charge order with neither spin correlations nor orbital currents involved. It is different from the stripe structures proposed in refs 21, 22 involving both the charge and spin orders and from the \( d \)-density wave pairing of ref. 23 containing correlated charge and current modulations.

Fluctuations and phase transition to superconducting state. The degeneracy with respect to rotations of the matrix \( u \) can lead to gapless excitations destroying the long-range order. We study the contributions of these excitations to thermodynamics by deriving the proper nonlinear \( \sigma \)-model in a way developed in the localization theory\(^8\). Fluctuations are most dangerous at finite temperatures when the dimension is reduced from 2 + 1 to effectively 2. The derivation of the effective free-energy functional \( F[u] \) describing the fluctuations yields

\[
 F[u] = \frac{1}{T} \int \text{tr} \left[ \nabla u^\dagger \nabla u + \kappa^2 u^\dagger u \tau_3 u \tau_3 \right] dR
\] (10)

In the pseudogap state \( T \sim T_c^* \), whereas \( 1/T \) vanishes at the mean-field transition temperature \( T_c^* \sim T_c \). Further, \( \kappa \sim \hbar^2 v/m \), with \( m \)
being the effective electron mass, and $R = [x / \sin(\delta/2), y / \cos(\delta/2)]$, where $x$ and $y$ are coordinates according to Fig. 1c. The coupling constant $t$ does not contain $\sin \delta$ and is small for all temperatures below but not too close to $T^*$. In the second term of equation (10), $\tau_3$ is the third Pauli matrix in the space of the matrix $u$ (equation (7)). Its presence is due to the finite curvature of the Fermi surface and breaks the symmetry between the superconducting and QDW orders, favouring superconductivity with a large gap $\sim \Gamma$. It is well known that fluctuations in the two-dimensional nonlinear $\sigma$-model at finite $t$ and $\kappa = 0$ produce logarithmically divergent contributions destroying the long-range order, in our case both superconducting and QDW orders. At finite curvature, $\kappa$ is finite and serves as an infrared cutoff so that below a critical temperature $T_c$, the superconducting order can be stabilized. In the one-loop approximation, a renormalization group analysis $^{24}$ yields for the effective coupling constants $t(\kappa_0)$ and $\kappa(\kappa_0)$

$$t(\kappa) = t_0 \left(1 - \frac{3\kappa_0}{16\kappa_0 \ln \frac{\Gamma}{\kappa_0}}\right)^{-1} \quad (11)$$

$$\kappa^2(\kappa_0) = \kappa_0^2 \left(1 - \frac{3\kappa_0}{16\kappa_0 \ln \frac{\Gamma}{\kappa_0}}\right) \quad (12)$$

where $t_0$ and $\kappa_0$ are the bare values written below equation (10) and $\Gamma$ plays the role of the upper energy cutoff in the $\sigma$-model.

Formally, equations (11) and (12) are applicable as long as $t(\kappa) \ll 1$ where superconductivity is stabilized. On increasing the temperature, the bare coupling $t$ and, hence, $t(\kappa)$ grows, whereas $\kappa^2$ decays. These tendencies imply that fluctuations of the order parameter $u$ become strong, while the anisotropy between the superconducting and QDW components vanishes. This should lead at $t(\kappa) \sim 1$ to a phase transition into a disordered (pseudogap) phase where no long-range order exists anymore but the pseudogap $b(\kappa)$ is still finite and remains large up to considerably high temperatures. The pseudogap state should exhibit combined properties of $d$-wave superconductivity and QDW, although sharp dependencies are necessarily smeared by thermal fluctuations. An estimate for the transition temperature $T_c$ between the superconducting and pseudogap phase is provided by the equation $t(\kappa) = 1$. At the same time, smearing by fluctuations prevents a sharp transition at $T^*$. In contrast to Bardeen–Cooper–Schrieffer superconductivity, $T_c$ is related to the gap value quite non-trivially.

**Phase diagram**

We summarize our findings with the help of the phase diagram in Fig. 3c. The red dashed line denotes the antiferromagnet–normal metal phase transition in the absence of interaction between the spin-wave modes and electron spins. Here, we have studied the region to its right. We identify the pseudogap region and represent the crossover to the normal metal state by the blue line. The green superconducting region is a part of a broader pseudogap region. Moving to the right by increasing the bosonic mass $a$ in equation (3) makes the spin-fermion interaction less singular and the effects of curvature of the Fermi surface more pronounced. As a result, QDW pairing is suppressed but one still obtains $d$-wave superconductivity. The transition temperature $T_c$ first grows because the fluctuations are suppressed but then decays because the interaction becomes less singular. At large $a$, the temperatures $T^*$ and $T_c$ merge.

Although our results are applicable only to the right of the dashed line ($a > 0$), the pseudogap should remain finite for a while when crossing the dashed line to the left ($a < 0$) because it would cost a large energy for it to vanish. Then, we expect a region at $a < 0$ with a still finite pseudogap that extends until the phase transition (crossover) to the antiferromagnetic state sets in. In this region, three phases may compete with each other. We do not know yet the details of how the superconducting state converts into the antiferromagnetic one and therefore have marked this region by a question mark in Fig. 3c.

The phase diagram represented in Fig. 3c looks similar to that of the cuprates (see, for example, refs 1, 2). The large values of the pseudogap can clearly be understood from the linear dependence of the pseudogap on the effective electron–electron interaction obtained in the present work.

**Comparison with experiments on high-$T_c$ cuprates**

The number of experimental works on cuprates is huge and we shall therefore restrict ourselves to the discussion of the most direct evidence from recent experiments to confirm our theoretical predictions. Whereas the existence of the pseudogap at temperatures considerably exceeding the superconducting transition temperature $T_c$ was discovered long ago in NMR experiments $^{25}$, followed by a variety of experimental observations including most transport and thermodynamic probes $^{2}$, more detailed information about the structure of the pseudogap has come from angle-resolved photoemission spectroscopy measurements $^{26, 27}$. They show that the pseudogap has a $d$-wave-like form and competes with superconductivity. As we have argued, the maximum of the pseudogap is indeed expected to move to the antinodal points, thus acquiring $d$-wave symmetry.

A very specific prediction from our analysis is the formation of the QDW order described by equation (9) and represented in Fig. 4. The wave vectors $Q_{x}$ and $Q_{y}$ of this modulated order are close to the vector connecting the antinodes.

Using scanning tunnelling microscopy (STM), the authors of ref. 28 have come to the conclusion that charge-modulated chequerboard patterns exist in several high-$T_c$ compounds and their wave vectors are close to the vectors connecting the antinodes. On the other hand, a quadrupole-like order on O atoms has been discussed in a different STM experiment $^{29}$ on BSCCO compounds. Periodic structures have been observed with STM also in ref. 30.

Competition between superconductivity and charge modulation with and without magnetic field was studied in a very recent experiment on YBa$_2$Cu$_3$O$_{6.67}$ compounds using high-energy X-ray diffraction $^{31}$. As in ref. 28, vectors close to those connecting the antinodes determined the periods of the charge chequerboard

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**Figure 4 | Chequerboard structure.** The quadrupole density amplitude (normalized to values between $\pm 1$, colour scale) is represented in real space. It is incommensurate with the square Cu lattice of the compound. The marked vectors are $R_{\pm} = 2nQ_{x}/Q_{y}^2$ (Fig. 1b). A similar picture for the valence bond solid correlations can be found in ref. 14.
structure. The charge structure was reported to develop above the superconducting transition temperature $T_c$. Below $T_c$, this structure appeared at magnetic fields suppressing the superconductivity, which is in perfect agreement with our theory.

Resonant soft X-ray scattering with photon energies near the copper $L_3$ absorption edge was used in ref. 32 for studying YBa$_2$Cu$_3$O$_{6+y}$ with hole concentrations of 0.09 to 0.13 per planar Cu atom. Again, a charge modulation has been uncovered with approximately the same wave vector. Remarkably, the intensity of the charge-density-wave signal is maximal at $T_c$. This effect is well reflected in our theory: above $T_c$, fluctuations destroy the charge correlations, whereas below $T_c$ the superconductivity suppresses them.

More detailed information about the modulation has been obtained in a similar experiment in ref. 33. The period of the superstructure corresponds to those of ref. 32, yet the peaks in the X-ray scattering were attributed to a spatial modulation of energies of the intra-atomic Cu (2p)–Cu (3d$_{x^2-y^2}$) transition, which is in agreement with our theory.

A magnetic-field-induced charge order in YBa$_2$Cu$_3$O$_y$ without any spin has been identified in ref. 34 in NMR measurements. Although the authors interpreted their results in terms of a stripe order, the data do not exclude a chequerboard one. The fact that the charge modulation appears under a magnetic field destroying the superconductivity agrees with our picture of the composite order parameter $O$.

Finally, a very recent measurement of the sound velocity in underdoped YBa$_2$Cu$_3$O$_y$ in a magnetic field$^{35}$ has indicated the existence of a two-dimensional charge order in magnetic fields exceeding the critical field $B_{c2} \approx 18$ T. This is the same behaviour as in the previous experiments and the results clearly agree with our theory.

**Outlook**

Solving the spin-fermion model we have shown that the antiferromagnet–normal metal quantum phase transition in two dimensions is a complex phenomenon belonging to a unique universality class different from those known in the theory of phase transitions. Our most remarkable finding is the emergence of a pseudogap state at and near the critical line. It is characterized by an SU(2) matrix order parameter that physically corresponds to a superposition of superconducting and QDW orders.

A good agreement of our findings with experimental results and their controlled derivation suggest that the theory presented here may be a good candidate for the explanation of phenomena related to high-temperature superconductivity.

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