Another Model with Interacting Composites

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Abstract

We show that we can construct a model in 3 + 1 dimensions where it is necessary that composite vector particles take place in physical processes as incoming and outgoing particles. Cross-section of the processes in which only the constituent spinors take place goes to zero. While the spinor-spinor scattering goes to zero, the scattering of composites gives nontrivial results.

1 Introduction

To find a nontrivial field theoretical model is one of the outstanding problems in theoretical high energy physics. The perturbatively nontrivial \( \phi^4 \) theory in four dimensions was shown to go to a free theory as the cut-off is lifted [1, 2]. The Nambu-Jona-Lasinio model [3], which is non-renormalizable perturbatively, was also shown to go to a trivial model [4, 5]. There are claims that the gauged Nambu-Jona-Lasinio model [6, 7] may give a non-trivial theory for a non realistically large number of flavors [8, 9]. All these examples show that the search for non-trivial models may be an interesting endeavor.

On the other hand, there are two-dimensional models with infinite number of local and non-local charges [10, 11]. These models were shown to give scattering matrices without particle production [12, 13] indicating a behavior not expected from fully interacting theories. When the symmetry present in these models is broken, these models give rise to particle production in perturbative calculations [14].

Another class of models are those with zero scattering amplitudes for the constituent fields, even though their Lagrangians seem to have non-zero interaction terms. These interactions arise from constraints. An example of them is taking a product of the constituent field equal to a power of the auxiliary field [15, 16, 17], i.e. forming composites of the constituent fields. The interaction of the constituent field with the composite field is defined. The Lagrangian contains the kinetic part of only the constituent spinor fields and the kinetic...
terms for the composites are formed as a result of vacuum polarizations of the composite field due to its interaction with the constituent field. Such models simulate the models with only spinors.

The first work on models with only spinors goes back to the work of Heisenberg\cite{Heisenberg}. Gursey proposed his model as a substitute for the Heisenberg model in fifties \cite{Gursey}. This spinor model is important since it is conformally invariant classically and has classical solutions \cite{19} which may be interpreted as instantons and merons \cite{21}, similar to the solutions of pure Yang-Mills theories in four dimensions. This original model can be generalized to include vector, pseudovector and pseudoscalar interactions.

A while ago, one of us, M.H., with collaborators, claimed to have written a polynomial Lagrangian equivalent to the one given by Gursey\cite{15,16,17}. Now we know that the models studied are only naively equivalent to these spinor models.

He, with collaborators, has also shown that\cite{15,16,17,22} in some of these models, the interaction of the spinor field with other spinors goes to zero. The model is not only asymptotically free, it is actually free, meaning that the coupling constant goes to zero as the cut-off is removed. We could not find physical processes\cite{22} which give results indicative of an interacting theory.

There is one more aspect of these models we missed in our work in the eighties\cite{15,16,17,22}. A further study shows that although the constituent fields do not give finite scattering amplitudes with each other, two composites can scatter from each other and may give rise to composite particle creation with finite results. These fact makes these models, which we had discarded as “trivial”, interesting once more. M.H., with a collaborator, has already studied a model of this class with scalar composites \cite{23}. In this work we will study another form of these models. We will first show why the constraint model is not totally equivalent to the spinor model with non polynomial interaction. Then we will repeat the calculations of given in reference \cite{16}. Higher order calculations and the calculation of processes where the composites take place will be studied in the next section. Since the two models are similar, there will be some repetition of the work in \cite{23}. We will end with remarks where this model is coupled to a constituent scalar field. This is a model where the scalar field can be considered as coupled to the spinor field via the gauge field, with the difference from the usual case that here the gauge field is the induced composite field.

2 An additional symmetry

We start with the pure spinor Lagrangian

\[ L = \bar{\psi} \left( i \partial - i e g \partial g^{-1} - m \right) \psi + \alpha \left[ \bar{\psi} \gamma_{\mu} \psi \right] \left[ \bar{\psi} \gamma_{\mu} \psi \right]^{2/3} + s \left[ \bar{\psi} \gamma_{\mu} \gamma_{\nu} \psi - G_{\mu} G^{\nu} \right]. \]  

(1)

This is the vector form of the Gursey model\cite{19}. Here only the spinor fields have their kinetic part. The $g$ field is used to construct a pure gauge term, just
to restore the local gauge symmetry, when the spinor field is transformed. We introduce an auxiliary field $G^\mu$ and an anti ghost field $\bar{w}_\mu$ into the Lagrangian and also add a term using $s$ as a symmetry operator to give us the constraint and Faddeev-Popov terms. The symmetry $s$ transforms the fields as

\[ s\bar{\omega}^\mu = \lambda^\mu, \quad s\lambda^\mu = 0, \quad sG^\mu = \omega^\mu \]
\[ s\omega^\mu = 0, \quad s\psi = s\bar{\psi} = 0 \quad s^2 = 0 \]

The $g$ field also transforms trivially.

Our Lagrangian should be invariant under this operation. Performing the $s$ operation, we rewrite our Lagrangian as

\[ L = \bar{\psi} \left( i\partial - ieg\gamma^g - m \right) \psi + \alpha \left[ \bar{\psi}\gamma^\mu \psi \right]^2 + \lambda^\mu \left( \bar{\psi}\gamma^\mu \psi - G^\mu G^\mu \right) + \bar{w}^\mu \left( G^\mu G^\mu + 2w^\lambda G^\lambda \right) \]

(3)

Since the spinor field vanishes under the $s$ operation we do not get any contribution from this part. Acting with $s$ on the total Lagrangian gives

\[ sL = -\lambda^\mu w^\mu G^2 - 2\lambda^\mu G^\mu w^\lambda G^\lambda + \lambda^\mu w^\mu G^2 + 2\lambda^\mu G^\mu w^\lambda G^\lambda \]
\[ + 2\bar{w}^\mu w^\mu G^2 - 2\bar{w}^\mu w^\lambda G^\lambda + 2\lambda^\mu G^\mu w^\lambda w^\lambda = 0 \]

(4)

This result validates our assertion.

In a paper written by M.H., with collaborators, [17] it has been asserted that the model given by the above Lagrangian is equivalent with the model described by the Lagrangian given below.

\[ \hat{L} = \bar{\psi} \left( i\partial - ieg\gamma^g - m + eG^\mu \gamma^\mu + e\lambda^\mu \gamma^\mu - m \right) \psi - e^4 \lambda^\mu G^\mu \left( G^\lambda G^\lambda \right) + \text{ghost terms} \]

(5)

Although the new model has the correct relation between the auxiliary field $G^\mu$ and the spinor field, we see that replacing the fractional spinor interaction by a spinor vector coupling may not be allowed. The pure spinor term with fractional powers have the symmetry described by the $s$ operation, whereas the latter term does not. From this point on we will study the properties of the second model. We will view the Gursey model as a motivation, a pure spinor model which is only naively equivalent to the latter model.

We will find that although the $G^\mu$ field does not have a kinetic term in the original Lagrangian, the one-loop corrections will generate them, making this composite field behave as a dynamical entity. In the literature there are similar models with differential operators in the interaction Lagrangian [24]. We will not take such terms in our model.
3 The Model

We start with the polynomial Lagrangian, where the fractional interaction of eq.(1) is replaced by the product of fields and a constraint term.

\[ L = \bar{\psi} \left( i\slashed{\partial} - i\gamma^\mu \partial_\mu - \gamma^\mu \partial_\mu + m \right) \psi - e^4 \lambda_\mu G^\mu \left( G_\gamma G^\gamma \right) \]  

(6)

In this expression only the spinor fields have the kinetic part. \( \lambda_\mu \) and \( G_\mu \) are two auxiliary vector fields. If we write the Euler-Lagrange equations for the \( \lambda_\mu \) and \( G_\mu \) fields, the equations of motion give

\[
\lambda_\mu (\bar{\psi} \gamma^\mu \psi - e^3 G^\mu G^2) = 0, \\
\bar{\psi} \gamma^\mu \psi - e^3 (\lambda^\mu G^\kappa G_\kappa + 2 \lambda^\kappa G_\kappa G^\mu) = 0. 
\]

(7)

Since we have a constraint Lagrangian, we have to perform the constraint analysis à la Dirac [25].

First we have the spinor-Dirac constraints. The auxiliary fields give us the extra constraints

\[
\Sigma^\mu = \delta L / \delta (\partial_0 \lambda_\mu) \approx 0, \\
\Omega^\mu = \delta L / \delta (\partial_0 G_\mu) \approx 0
\]

(8)

To write the canonical hamiltonian we use the relation

\[ H_c = p_i \dot{q}_i - L \]

(9)

which gives

\[
H_c = \bar{\psi} \left( i\gamma^\lambda \partial_\lambda + e\slashed{\partial}g^{-1} - eG_\mu \gamma^\mu - e\lambda_\mu \gamma^\mu + m \right) \psi + e^4 \lambda_\mu G^\mu \left( G_\lambda G^\lambda \right) 
\]

(10)

The constraints are added to the canonical expression to give

\[
H_p = \bar{\psi} \left( i\gamma^\lambda \partial_\lambda + e\slashed{\partial}g^{-1} - eG_\mu \gamma^\mu - e\lambda_\mu \gamma^\mu + m \right) \psi + e^4 \lambda_\mu G^\mu \left( G_\lambda G^\lambda \right) \\
\quad + a\pi + (\pi - i\bar{\psi}\gamma_0) b + c_\mu \Sigma^\mu + d_\mu \Omega^\mu 
\]

(11)

Here \( a, b, c_\mu, d_\mu \) are Lagrange multipliers. The condition that the constraints should not change in time dictates that the Poisson brackets of the constraints with the Hamiltonian must vanish.

\[ \{ \theta_i, H_p \} = 0 \]

(12)

\( a \) and \( b \) are evaluated by these relations.
The rest of the Poisson parentheses give us further constraints.

\[
\kappa^\mu = \overline{\psi} \gamma^\mu \psi - G^\nu G^\mu \nu \\
\zeta^\mu = G^\nu G_\nu G^\mu - \overline{\psi} \gamma^\mu \psi - 2(\lambda_\mu G^\nu) G^\mu
\] (13)

When the Poisson parentheses of these secondary constraints are taken with the Hamiltonian, we get new relations which evaluate \( c_\mu \) and \( d_\mu \). Thus the system is closed and we do not get new constraints.

Now we study the different classes our constraints may belong. To be a first class constraint that constraint should have vanishing Poisson parentheses with every other constraint, i.e. for every \( i \) and \( j \),

\[ \{\theta_i, \theta_j\} \equiv 0. \] (14)

If a single parenthesis is different from zero, we get second class constraints. Our constraints turn out to second class.

We form the Faddeev Popov determinant by taking the determinant of the matrix formed by second class constraints.

\[
\Delta_F = |\operatorname{det}\{\theta_i, \theta_j\}|^{1/2} = g_{\mu\nu} G^2 + 2G^\mu G_\mu
\] (15)

We can write this result in the partition function formalism.

\[
Z = \int D\pi D\chi \delta(\theta_i) \Delta_F \exp \left( -i \int (\chi \pi - H_c) \right)
\] (16)

Here \( \chi \) is the generic notation for all the fields and \( \pi \) represents all the momenta. If we integrate over all the momenta, we get

\[
Z = \int D\psi D\chi D\lambda D\pi \exp \left( i \int L_{\text{eff}} dx^4 \right)
\] (17)

By using ghost fields, the effective lagrangian can be written as

\[
L_{\text{eff}} = \overline{\psi} \left( i\partial - ieg\gamma^\mu - e[G + \chi] - m \right) \psi - e^4 \lambda_\mu G^\mu G^2 - L_{\text{ghost}}
\] (18)

where \( L_{\text{ghost}} = \overline{\psi} (G^2 + 2w_\lambda G^\lambda) w_\mu \). If we take an integral over the spinor fields the effective action is written as

\[
S_{\text{eff}} = Tr \ln \left[ i\partial - e(ig\gamma^\mu - G^\mu_\lambda \gamma^\lambda - \lambda_\mu \gamma^\mu) + m \right] \\
- \int dx^4 (e^4 \lambda_\mu G^\mu (G_\lambda G^\lambda) + L_{\text{ghost}})
\] (19)

At this point we redefine our fields as
\[ A_\mu = -ig\partial_\mu g^{-1} + G_\mu + \lambda_\mu \]
\[ F_\mu = \lambda_\mu - G_\mu \]
\[ J_\mu = G_\mu + \lambda_\mu + 2g\partial_\mu g^{-1}. \quad (20) \]

Using the inverse transformations
\[ ig\partial_\mu g^{-1} = \frac{2J_\mu - 2A_\mu}{6} \]
\[ G_\mu = \frac{2A_\mu - 3F_\mu + J_\mu}{6} \]
\[ \lambda_\mu = \frac{2A_\mu + 3F_\mu + J_\mu}{6} \quad (21) \]

we write the effective action as
\[ S_{\text{eff}} = \text{Tr} \ln (i\partial + eA + m) + \int dx^4 \left[ e^4 \left( A_\mu A^\mu A_\lambda A^\lambda \right) + \text{other terms} \right]. \quad (22) \]

Here we see that only the \( A_\mu \) field would appear quadratically upon expanding this action. All the other fields are raised to third or higher powers. We first take the first derivatives of the effective action with respect to the fields and set these expressions equal to zero to kill the tadpoles. The vacuum expectation values of all the vector fields are zero, also as dictated by Lorentz invariance. We then take second derivatives to calculate the propagators, using the zero values wherever these fields appear. For the \( A_\mu \) field we get
\[ \frac{\partial^2 S_{\text{eff}}}{\partial A_\mu \partial A_\nu} |_{F_\mu = 0} = 0, \quad \frac{\partial^2 S_{\text{eff}}}{\partial J_\mu \partial A_\nu} = 0 \]
\[ \frac{\partial^2 S_{\text{eff}}}{\partial J_\mu \partial J_\nu} |_{J_\mu = 0} = 0, \quad \frac{\partial^2 S_{\text{eff}}}{\partial F_\mu \partial A_\nu} = 0 \quad (24) \]

Thus all the fields, except \( A_\mu \) decouple from the model. This is also true for the ghost fields. We can write the last expression as
\[ S'_{\text{eff}} = \text{Tr} \ln (i\partial + eA + m) + \int dx^4 e^4 \left( A_\mu A^\mu A_\lambda A^\lambda \right) \quad (25) \]
At this point we expand the logarithm term to write the vector field kinetic term, being the usual vector field expression. We will use the usual massless vector field propagator for the $A_\mu$ field assuming we can invert this expression, if necessary by introducing a gauge fixing term into the lagrangian.

At this point we want to bring a new interpretation to the old work. We will interpret the infinities coming from the $A_\mu$ propagator as wave function renormalization. Assuming this expression can be inverted, the propagator for this field may be written as $\frac{c\gamma_\mu}{p^2}$ in the Feynman gauge. We will use this expression to perform calculations in higher orders.

4 Spinor Propagator

In this section we calculate the above results in higher orders. To justify our result that no mass is generated for the fermion we may study the Bethe-Salpeter equation obeyed for this propagator. The Dyson-Schwinger equation for the spinor propagator is written as

$$iA\phi + B = i\phi + 4\pi\epsilon\gamma^\mu \int \frac{d^4q}{(iA\gamma + B)(p - q)^2}\gamma^\mu.$$  \hspace{1cm} (26)

Here $iA\phi + B$ is the dressed fermion propagator. We use the one loop result for the scalar propagator. After rationalizing the denominator, we can take the trace of this expression over the $\gamma$ matrices to give us

$$B = 16\pi\epsilon \int d^4q \frac{B}{(A^2q^2 + B^2)(p - q)^2}.$$ \hspace{1cm} (27)

The angular integral on the right hand side can be performed to give

$$B = 16\pi\epsilon \left[ \int_0^{p^2} dq^2 \frac{q^2B}{p^2(A^2q^2 + B^2)} + \int_{p^2}^{\infty} dq^2 \frac{B}{(A^2q^2 + B^2)} \right].$$ \hspace{1cm} (28)

If we differentiate this expression with respect to $p^2$ on both sides, we get

$$\frac{dB}{dp^2} = -16\pi\epsilon \left[ \int_0^{p^2} dq^2 \frac{q^2B}{(p^2)^2(A^2q^2 + B^2)} \right].$$ \hspace{1cm} (29)

This integral is clearly finite. We get zero for the right hand side as $\epsilon$ goes to zero. Since mass is equal to $m$ in the free case we get this constant equal to $m$. This choice satisfies eq. (26).

The similar argument can be used to show that $A$ is the dressed spinor propagator is a constant. We multiply eq. (26) by $\phi$ and then take the trace over the spinor indices. We end up with

$$p^2A = p^2 - 8\pi\epsilon \left[ \int_0^{p^2} dq^2 \left( \frac{(g^4)A}{p^2(A^2q^2 + B^2)} + \int_{p^2}^{\infty} dq^2 \frac{p^2A}{(A^2q^2 + B^2)} \right) \right].$$ \hspace{1cm} (30)
We divide both sides by $p^2$ and differentiate with respect to $p^2$. The end result

$$\frac{dA}{dp^2} = 16\pi\epsilon \int_0^{p^2} dq^2 \left( \frac{(q^4)A}{(p^2)^3(A^2q^2 + B^2)} \right). \quad (31)$$

shows that $A$ is a constant as $\epsilon$ goes to zero. Since the integral is finite, it equals unity for the free case, we take $A = 1$.

### 5 Other Processes

In this section we will try to analyze the contribution of the higher order diagrams to our basic terms, i.e. to the three and four point functions. We will use only the contributions of the ladder diagrams, anticipating the result we would have if we had an inner symmetry index $N$ to justify an $\frac{1}{N}$ expansion.

Our model is not a gauge invariant model. The gauge invariance, which may be present in the original model via the $g$ term, is broken by the new quartic term for the $A_\mu$ fields. If this term were absent, we could fix the gauge, say by imposing the Lorentz gauge, we would bring new, this time propagating ghost fields. As it can be shown easily, the additional contribution of these to the vector field propagator would again be of the transverse type. For the vector field propagator, we assume that we can invert the inverse propagator by, if necessary adding the necessary term by hand, as an additional constraint. We note that the $\epsilon$, which is in the denominator in the inverse propagator, is brought to the denominator in the propagator.

We will first study the four spinor diagrams. We do not have four spinor coupling in our Lagrangian. We need vector particles coupling to spinors to obtain this expression, which necessitates the use of vector propagators as internal lines. Since each vector propagator contains an $\epsilon$ contribution, the four spinor process goes to zero as $\epsilon$. When we go beyond tree diagrams, we need at least two vector propagators to end up with two spinors, which means extra powers of $\epsilon$. This is true for all the higher order processes.

We can justify our claims also by writing the Bethe-Salpeter equations for this process. The Bethe-Salpeter equation for the four spinor interaction reads as

$$G^{(2)}(p, q; P) = G^{(2)}_0(p, q; P) + \frac{1}{(2\pi)^8} \int d^4p' d^4q' G^{(2)}_0(p, p'; P) K(p', q' ; P) G^{(2)}(q', q; P). \quad (32)$$

Here $G^{(2)}_0(p, q; P)$ is two non-crossing spinor lines, $G^{(2)}(p, q; P)$ is the proper four point function, $K$ is the Bethe-Salpeter kernel.

The four spinor kernel in this expression is at least to the first power in $\epsilon$. We can use the quenched ladder approximation [26], where the kernel is separated to a vector contribution which is limited to bare propagator only, connected to the proper kernel with two spinor legs. The proper kernel is of order $\epsilon$. 

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The contribution of a vector and two spinor propagators is finite, recalling that the vector propagator has an extra $\epsilon$. The total result is of order $\epsilon$ for the $\bar{\psi}\psi\bar{\psi}\psi$ vertex. This result shows that the spinor-spinor scattering process is vanishes as $\epsilon$ goes to zero. This result will be important when we study the $<\bar{\psi}\psi A_\mu>$ vertex.

We note that the $<\bar{\psi}\psi A_\mu>$ vertex is finite, hence does not need any infinite regularization. The first correction to the vertex is the one loop diagram. Here we have two spinor and one vector propagators. The divergence coming from the loop momentum integration is cancelled by the $\epsilon$ factor coming from the vector propagator. Naively the higher diagrams do not change this result, since each momentum integration is accompanied by an $\epsilon$ term.

To establish this result more firmly we write the Dyson-Schwinger equation for this vertex. Here we use the result of the four spinor diagram. Three point function has the vector particle going to two spinors which go into the four spinor kernel. The loop integration brings a divergence which is cancelled by the $\epsilon$ coming from the kernel. The end result is the finite renormalization of the three point vertex.

We see that we do not need an infinite renormalization for the four $A_\mu$ vertex. The first correction to the tree diagram is the box diagram with four spinor propagators. This diagram in spinor electrodynamics is known to be finite [27, 28, 29]; hence the coupling constant for this process does not run. The finite contribution of this diagram just renormalizes the coupling constant by a finite amount. There are no infinities for this function coming from higher orders. The two loop diagrams contains an $A_\mu$ propagator, making this diagram finite. The three-loop diagram contains eight spinor and two vector propagators, which are linear in $\epsilon$. Higher orders also do not give infinite contributions; so, the sole coupling constant of the model does not need infinite renormalization.

6 Conclusion

As a result of the arguments in the earlier sections, we can construct a model where the composites can scatter from each other, whereas the process whose sole result is the scattering of the constituent spinor fields from each other vanish as the cut-off is removed. The scattering of the composite fields from each other will be a finite expression. There is also a tree-diagram process where the spinor scatters from a composite particle, a Compton-like scattering, with a finite cross-section. This diagram can be written in the other channel, which can be interpreted as spinor production out of vector particles. There are also processes where a single spinor scatters with a vector composite and creating additional vector particles in the tree approximation. Here we have to exclude any internal vector lines as a rule, which makes us use tree diagrams only. There may be insertions to vacuum fluctuations of a spinor field, but these just add finite contributions to the propagator, since the presence of vector propagators make these loop diagrams at most finite. Whenever the composites do not take part as incoming or outgoing particles, the cross section goes to zero.
The creation of composite particles, out of two incoming composites is finite, if the outcome is an even number of composites. The creation of an odd number of composites is forbidden by the Furry theorem in the one loop calculation. Any loop diagram which creates spinor particles as a result of the interaction of two vector composites vanishes as the cut-off is removed. The lowest of these, which creates two spinors, are the triangle diagram made out of spinors, or a box diagram, made out of three spinors and one vector particle. The first expression is zero and the amplitude vanishes due to the presence of the vector propagator in the latter case. The diagram which involves the production of four spinors involves two vector propagators, giving also zero cross-section, since it is proportional to a power of $\epsilon$.

We can also have scattering processes where two vector particles go to an even number of vector particles. In the one loop approximation all these diagrams give finite results, like the case in the standard electrodynamics. Since going to an odd number of vector particles is forbidden by the Furry theorem, we can also argue that vector $\phi$ particles can go to an even number of vector particles only. This assertion is easily checked by diagrammatic analysis.

As a result of our calculations we find a model which is trivial for the constituent spinor fields, aside from Compton scattering in first order only, whereas finite results are obtained for the scattering of the composite vector particles.

The processes where a single spinor particle giving scattering with a composite particle and giving rise to additional vector particles are allowed. Here we have to exclude any internal vector lines as a rule, which makes us stick to tree diagrams only. There may be insertions to a vacuum fluctuations of a spinor which are not zero, but these contributions are finite.

An addition to the model is to add a constituent complex scalar field to the model, doing just the complementary thing to the work of Bardeen et al. Since we already have a composite vector field, all we can add is a scalar field which has its kinetic term in the lagrangian. Work on this model is going on. We just want to make the remark that it does not seem to be feasible to generate the gauge interaction of the scalar field, the $<\phi\partial_{\mu}\phi A^\mu>$ term, using the triangular spinor diagram, due to the Furry theorem. A triangular loop with one $\gamma^\mu$ insertion will be zero. A way out may be introducing a pseudovector composite particle instead of a vector one in the first place. Another way out would be to give internal structure to fields, say $U(n)$ symmetry.

The $<A_{\mu}A^\mu\phi^2>$ can be generated by the spinor box diagram, thus making contact between the scalar and the vector particles in the theory, although such an interaction does not exist in the original lagrangian. Such a term may break the gauge invariance, though. We still hope a sense can be made out of this model which may be complementary to the gauged version of the scalar case. Our model is a toy model. We could not find a physical system that is effectively described by it.

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