Modelling of distribution of circular beams of Airy in parabolic fiber

E O Monin\textsuperscript{1} and S G Volotovskiy\textsuperscript{2}

\textsuperscript{1}Samara National Research University, Moskovskoe shosse 34a, Samara, Russia, 443086
\textsuperscript{2}Image Processing Systems Institute - Branch of the Federal Scientific Research Centre “Crystallography and Photonics” of Russian Academy of Sciences, Molodogvardeyskaya str. 151, Samara, Russia, 443001

e-mail: monin23.23@gmail.com, sv@smr.ru

Abstract. Recently, the attention of researchers has been turned to various beams possessing the property of autofocusing, among which are circular beams of Airy, beams of Pierce, hypergeometric and other beams. The sharp autofocusing property inherent in the above beams is very useful in optical manipulation, useful for multiphoton polymerization, is used for nonlinear effects and for polarization transformations. A classic focusing element is a lens that has a quadratic dependence on the radius. The diffractive version of the parabolic lens has radial lines, condensing to the edge of the optical element as a linear chirp. This structure can be obtained by "twirling" the Airy function along the circumference with displacement and scaling. The study of the behavior of various types of self-focusing laser beams in parabolic environments extends the spectrum of optical signals used for telecommunications. In particular, a fractional Fourier transform is used to describe fibers with a parabolic refractive index. In this paper we consider circular Airy beams, which have a radial dependence. Modelling of the passage of these beams through an optical fiber with a parabolic change in the refractive index was performed on the basis of the use of a fractional Fourier transform.

1. Introduction

The attention of researchers has recently been drawn to various beams with the property of autofocusing [1-9], among which are classical, circular and generalized Airy beams [6, 7, 9-16], Pierce's beams [17-19], hypergeometric [8, 20-23] and other beams [24-26]. The sharp autofocusing property inherent in such beams is claimed for optical manipulation [27-29], useful for multiphoton polymerization [30], is used for nonlinear effects [31] and for polarization transformations [32, 33].

The Airy functions are infinitely extended and do not have finite energy, so their physical implementation requires truncation. In [11], finite-energy Airy beams were considered, representing the product of the classical Airy mode and the exponential function. Although multiplying by an exponential [11] or Gaussian [12] function makes it possible to easily form such beams using a spatial light modulator, in both cases the generated beams actually cease to be non-diffractive, although they approximately retain their appearance to a certain distance.

In [13] another way of truncating the infinite Airy mode was considered - with the help of a rectangular aperture truncating the function in the positive part of the argument, with its decrease practically to zero, and in the negative part - up to the \( n \)-th zero. In [13], the degree of divergence of three types of truncated Airy beams was compared: exponential, Gaussian and bounded by a
diaphragm - and it was numerically shown that in the latter case the oscillating structure of the beam and the clearly marked intensity maximum persists much longer than in the first two.

In this paper, a numerical study is made of the behavior of circular Airy beams bounded by a circular aperture (similar to [13]) in an optical fiber with a parabolic change in the refractive index based on the use of a fractional Fourier transform [34-36].

2. Circular Airy beams

In this paper we consider the circular bundles of Airy. They are radially symmetric beams whose intensity along the radius is described by the Airy version [10]:

\[ \text{Ai}(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \exp \left[ i \left( \frac{t^3}{3} - xt \right) \right] dt \]  

(1)

In [14], Airy vortex beams are considered, the radial component of which is expressed in terms of the truncated Airy functions truncated in the nth root or extremum [13]. During propagation, such beams retain a pronounced ring structure, similar to the Laguerre-Gauss modes, although the ratio of the radiuses of the rings varies. We note that the type of Airy vortex functions considered in [14] differs substantially from the circular Airy functions proposed in [1-3], which are characterized by sharp autofocusing. Thus, the choice of the point of rotation in the formation of circular beams from one-dimensional distributions substantially changes the properties of the beam.

After the "twisting" of the function (1) in the manner of [1-3], the imaginary and real part of the complex distribution of the obtained function was divided. The imaginary part is a periodic undamped function, and in the zone of interest to us, it is substantially less than the real component. So in the future we will only consider it. We will also consider only those values that fall into a certain radius, since the simulation will take place in a circular waveguide:

\[ R_{Ai}(r, \rho, \omega) = \text{Re} \left\{ \text{Ai} \left[ \omega (r - \rho) \right] \cdot \text{circ} \left( \frac{r}{R_{\text{max}}} \right) \right\} \]  

(2)

The parameters \( \rho \) and \( \omega \) are responsible for the radius of the bounding aperture and the scale increase, respectively.

Figure 1 shows the distribution obtained with the help of the obtained formula (2). The following values were used in the simulation: \( \rho = 1, \omega = 6 \).

![Figure 1. Circular Airy beam.](image)

Also in this paper we consider vortex circular Airy beams with an orbital angular momentum. The propagation of such beams in free space was investigated in [4, 37, 38]. The azimuthally modulated circular Airy beams, which can be regarded as a superposition of vortex circular Airy beams, were proposed and investigated in [39].

Vortex circular beams are described by the following expression:

\[ \Psi_{Ai,l}(x, y, \rho, \omega) = R_{Ai} \left( \left(x^2 + y^2 \right)^{\frac{l}{2}}, \rho, \omega \right) \cdot (x + iy) \]  

(3)

Figure 2 shows beams with different vortex order \( l \).
3. Fractional Fourier Transform

It is often the case that an operation originally defined for integer orders can be generalized to fractional or even complex orders in a meaningful and useful way. A basic example is the power operation. The $a$th power of $x$, denoted by $x^a$, might be defined as the number obtained by multiplying unity $a$ times with $x$. Thus $x^4 = x \cdot x \cdot x \cdot x$. This definition makes sense only when $a$ is an integer. However, it is an elementary fact that the definition of the power of a number can be meaningfully and consistently extended to real and complex values of $a$. Likewise, the original definition of the derivative of a function makes sense only for integral orders; that is, we can speak of the first or the second derivative and so on. However, it is possible to extend the definition of the derivative to noninteger orders by the use of an elementary property of Fourier transforms [40].

A fractional Fourier transform [34] is used to describe the propagation of laser radiation through an optical fiber with a parabolic refractive index (Fig. 3). For a bounded fiber, laser modes with distributions limited in the object and spectral regions can be found [35, 41].

\[
F(u,v,z) = -\frac{ik}{2\pi f \sin \tau} \exp \left\{ \frac{ik \cos \tau (u^2 + v^2)}{2f \sin \tau} \right\} \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) \exp \left\{ \frac{ik \cos \tau (x^2 + y^2)}{2f \sin \tau} - \frac{ik(xu + yv)}{f \sin \tau} \right\} dx dy
\]

where $\tau = \frac{z}{f} \cdot \frac{\pi}{2}$. 

Figure 2. Vortex circular Airy beams: $l = 1$ (a), $l = 4$ (b), $l = 7$ (c).

Figure 3. Waveguide Profile.
Note that expression (4) has singularities (division by zero) in the case when the value \( z = 2nf \), \( n \) is an integer. Also, a high error is observed near these points. To solve this problem, consider the entire path of the beam propagation and divide this path into 4 parts, as shown in Fig. 4.

![Figure 4. Full period.](image)

The least influence of the error in the calculation will be on the interval \([1/2f, 3/2f]\). Then in order to simulate the longitudinal propagation using formula (4) on the interval \([f, 2f]\), it is necessary to calculate the transverse distribution in the plane \( z = 1/2f \) and perform a fractional Fourier transform for it for the distance of the smallest error. Thus, we avoid division by zero at the point \( z = 2f \). Similarly, the remaining segments are calculated. However, in order to calculate the first interval \([0, f]\), we need to find the distribution in the plane \( z = -1/2f \). It turns out that you need to know what was before our input beam. Then remember that the fractional Fourier transform is a periodic function along the \( z \) axis with a period equal to \( 4f \). From this it follows that to calculate the first segment one can consider a beam in the plane \( z = -1/2f + 4f = 7/4f \).

4. Modelling and results
The simulation results using expression (4) and the algorithm described in the previous section are shown in Figures 5-9.

Figures 5-8 show longitudinal views on the period \([0, 4f]\), \( f = 1000 \) mm, the input beam size is \( 5 \times 5 \) mm. As can be seen, the self-focusing of the beam (the maximum concentration of the intensity on the optical axis) at a distance \( z = 660 \) mm takes place up to the focal plane (\( z = f \)). A similar effect for one-dimensional fractional Airy beams was noted in [9]. It should be noted that the self-focusing distance does not depend on the order of the optical vortex.

![Figure 5. Longitudinal view of the vortex circular Airy beam with \( l = 0 \).](image)

![Figure 6. Longitudinal view of the vortex circular Airy beam with \( l = 1 \).](image)

The transverse distribution of the beams in the plane of self-focusing with \( z = 660 \) mm is shown in Figure 9. At this point, the maximum energy concentration occurred during propagation in a parabolic
optical fiber. With the growth of the degree of the vortex, the internal energy ring expands. Also, the outer rings become more pronounced.

**Figure 7.** Longitudinal view of the vortex circular Airy beam with \( l = 4 \).

**Figure 8.** Longitudinal view of the vortex circular Airy beam with \( l = 7 \).

**Figure 9.** Transverse view for the vortex circular Airy beams: \( l = 0 \) (a), \( l = 1 \) (b), \( l = 4 \) (c), \( l = 7 \) (d).

### 5. Conclusion

We numerically investigate the behavior of vortex circular Airy beams bounded by a circular aperture in an optical fiber with a parabolic change in the refractive index. For the simulation, a fractional Fourier transform was used, based on an algorithm that allows the field to be calculated correctly at the singular points (when dividing by zero) and near them. To solve this problem, the algorithm
provides for a half-focal distance offset. The simulation results show that the self-focusing distance of the circular Airy beams does not depend on the order of the optical vortex.

6. References
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