Integrated approach methodology for evaluating the feed rate of mechanical disk hopper-feeding devices

E V Pantyukhina
Tula state University, 92, Lenin Ave., Tula, 300041, Russia
E-mail: elen-davidova@mail.ru

Abstract. The paper considers the methodology of the proposed integrated approach to mathematical modeling of the actual feed rate of various designs of mechanical disk pocket hopper-feeding devices. The approach allows us to estimate the actual feed rate of the device for various types of axisymmetric parts of the shape of rotation bodies with sufficient accuracy for practice at the design stage. The mathematical model of feed rate takes into account the probability of gripping axisymmetric parts of the shape of rotation bodies with different geometric parameters, as well as the design and kinematic parameters of feeding devices.

1. Introduction
Hopper-feeding devices of mechanical type are widely used for feeding with small-sized piece parts of the shape of rotation bodies into process machines and lines.

The difficulty of reliably estimating actual feed rate when designing such devices is as follows. Their work is based on the probabilistic principle of gripping one part from the total mass. Therefore, it is impossible to determine the number of correctly oriented parts issued from the device per unit of time in some cases without conducting experimental studies.

Development of mathematical models of actual feed rate of mechanical hopper-feeding devices at the stage of their design is necessary. It will allow to get values of actual feed rate of the device with sufficient accuracy for practice. The obtained mathematical model will allow to design an operable device and when using parametric synthesis, the hopper-feeding device will ensure reliable delivery of parts to process machines with the required feed rate.

In practice, there are several historically developed approaches to modeling the feed rate of hopper-feeding devices [1]. Most of the works of Soviet and foreign scientists are based on experimental research and the laws of classical mechanics [2-45]. They allow you to get only the areas of boundary values of kinematic parameters and empirical feed rate models of specific designs of hopper-feeding devices with certain design parameters for certain parts with specific parameters. The first work that suggested using probability theory to describe the process of gripping a part from the total mass was that a part falling on the surface of a rotating disk, if there are obstacles from other parts and the speed of the gripping organs, should take a favorable position for its grip. The disadvantage of this approach, which prevents its use for a mechanical hopper-feeding device, is an incorrect description of the probability that takes into account the impact on the process of gripping the part of the speed of movement of the gripping organs.

To assess the actual feed rate of mechanical hopper-feeding devices at the stage of their design, a comprehensive approach is proposed based on the results of known works and consisting in a combination of methods of classical mechanics and probability theory. The models of actual feed rate obtained using an integrated approach take into account the probabilistic nature of the process of gripping a part from the total mass, the influence of the speed and design parameters of the gripping bodies, as well as the geometric parameters of the parts on the process of gripping them.

The proposed integrated approach was applied to known and several improved designs of mechanical disk hopper-feeding devices. For the first time the proposed approach was used in
modeling the feed rate of a hopper-feeding device with radial sockets and an annular orientator for hollow parts of the shape of rotation bodies with a conical end face and implicit asymmetry at the ends [5]. Later, the integrated approach was tested on designs of hopper-feeding devices with radial [6] and tangential [6] profile sockets for similar parts, as well as with teeth and an annular orientator for continuous parts with a conical and spherical end face [1] and hollow parts with a spherical end [8]. The designs have an obliquely positioned rotating disk and are designed for rod parts. In [9], a mathematical model of the feed rate of a hopper-feeding device with a vertical rotating disk for solid flat parts is described. The correctness of mathematical feed rate models obtained using the integrated approach was confirmed by the results of experimental studies. This indicates that this approach can be used to model the feed rate of various designs of mechanical hopper-feeding devices.

The range of manufactured products is constantly increasing; their shape and size are changing. This requires the development of new designs of hopper-feeding devices for each piece component of the manufactured product. The mathematical models given in the articles describe the actual feed rate of only specific structures. Therefore, it is advisable to present a unified methodology for an integrated approach to modeling the actual feed rate of various designs of mechanical hopper-feeding devices for axisymmetric parts of different shapes.

This article discusses the methodology of the proposed theoretical integrated approach to the mathematical description and evaluation of the feed rate of various designs of mechanical disk pocket hopper-feeding devices for axisymmetric parts in the form of rotation bodies.

2. Formulation of the problem

Parts of different shapes and sizes are widely distributed in various industries. The axisymmetric parts of the shape of the bodies of rotation are divided into three main groups depending on the ratio \( z \) of the main dimensions (length and diameter) and the possibility of their feeding by hopper-feeding devices. The first group includes core parts with \( z > 1.5 \). The second group consists of equal-sized and similar parts with a range of values of \( 0.8 < z < 1.5 \). The third group includes flat parts with \( z < 0.8 \).

Sketches indicating the geometric parameters of axisymmetric parts of the shape of rotation bodies of various shapes are shown in Figure 1. They may refer to each of the three groups described, depending on the ratio of their overall dimensions \( z \).

![Figure 1. Sketches of axisymmetric parts of the shape of rotation bodies with conical (a), spherical (b), step (b) end, in the form of a cap (d): \( l \) – the length of the part; \( d_1, l_1 \) – the diameter and the length of the cylindrical end of the part; \( x_c \) – the distance from the cylindrical end of the part to its center of mass; \( d_2 \) – the diameter of the conical end of the part; \( r \) – the radius of the spherical end of the part.](image)

Various designs of mechanical disk hopper-feeding devices can be used for automatic feeding of such parts. The main distinguishing feature of feeding devices is the implementation of processes for gripping and orienting parts in them.

For rod parts with obvious asymmetry, mechanical disk hopper-feeding devices with tangentially positioned gripping organs (pockets) are most common due to the high probability gripping of parts. Depending on the shape parts, these devices can provide one of two ways of orientation. Active orientation is most useful to achieve higher device feed rate. In this case, the parts are gripped by the pocket in two positions. The incorrectly gripped parts are then reoriented to the desired position using
special orientators. Passive orientation is used for parts that are difficult to reorient to the desired position due to the lack of orientation keys. In this case, incorrectly gripped parts are ejected from the pocket when the disk is rotated at the top of the device [7].

The radial arrangement of the pockets reduces the probability of gripping parts by 1.5-2 times than in tangential ones. It is justified when implementing active orientation of parts by shape. In this case, the parts are in a horizontal position relative to the rotating disk. The use of passive orientation in such structures in most cases is due to the absence of obvious asymmetry in the shape of the parts. In this case, undirected parts fall out of the pocket due to their own weight or the coordinates of their center of mass.

Improved hopper-feeding devices with radial [1, 5] and tangential [7] pockets implement passive orientation of rod parts with implicit asymmetry of shape and absence of displacement of mass center. The principle of operation of the developed devices is based on precisely constructed geometry of the gripping organ, which simultaneously orients the parts.

Mechanical disk hopper-feeding devices with vertically arranged rotating disk are used for equally sized and flat parts [9]. Passive orientation of parts is realized in most of them. The active orientation of the cap-shaped, equidistant and flat parts may be provided outside the device using jets of air separating the parts into two streams.

The feed rate of the device depends largely on the grip process and the way the parts are oriented. Therefore, in mathematical modeling of actual feed rate using a complex approach, it is necessary to know the physical basis of implementation of processes of gripping and orientation of parts and structural features of gripping and orienting organs of hopper-feeding devices.

3. Theoretical part
Feed rate $F$ [parts / min] of mechanical disk hopper-feeding devices in accordance with the integrated approach is determined by the formula

$$F = k n E_{max} \left[1 - \left(\frac{n}{n_{max}}\right)^4\right],$$

where $k$ is the number of pockets; $n$ - the rotational frequency of the disk [rpm]; $E_{max}$ - the maximum efficiency or probability of gripping parts $E$; $n_{max}$ - the maximum value of the rotational frequency of the disk which it is impossible to grip the part with a pocket.

Thus, the main task in modeling the actual feed rate of devices using an integrated approach is to determine the probability of gripping parts ($E_{max}$) and the conditions under which gripping a part is impossible ($n_{max}$).

The probability of gripping a part $E_{max}$ is determined by finding the part on the rotating disk before gripping in the desired position (probability $p_1$) and the absence of interference from other parts (probability $p_2$) according to the formula

$$E_{max} = p_1 p_2.$$  

The part must be in the desired position on the rotating disk before being gripped. Each part on the surface of a rotating disk can take as many positions as the surfaces limit it. For example, a part with a conical end has three surfaces, each of which can fall on the surface of a rotating disk-bases I and II with different diameters and a cylindrical surface III.

Not all positions that the part takes on the surface of the disk will be favorable for grip. Some of them are easy to grip. Some require a u-turn at a certain angle to grip them. In some positions, grip is not possible at all. For example, in a hopper-feeding device with radial pockets, it is favorable for the grip to find the part on its cylindrical surface III with further rotation towards the pocket by the base II. In the toothed hopper-feeding device, in addition to the position III, the pocket can also grip the part when it is in a vertical position on the base II. The positions that are favorable for grip are determined by the shape of the part and the design features of the device. If you know the physical basis for implementing grip processes in the device, these positions are easily detected.
The larger and smaller diameters, the center of mass coordinate, and the length of the cylindrical end of the part were expressed through the length of the part using coefficients \(a\), \(b\), \(c\), and \(f\) in the expressions \(d_1 = al\), \(d_2 = cl\), \(x_c = bl\), \(l_1 = fl\). This simplifies probability calculations.

The graphs determine the probabilities of the part being located on each of its bases on the rotating disc depending on the coefficients \(a\), \(b\) and \(c\) (Figure 2).

![Graphs for determining the probabilities of finding a part on bases I (a) and II (b).](image)

**Figure 2.** Graphs for determining the probabilities of finding a part on bases I (a) and II (b).

The probability of finding a part on a cylindrical surface III is determined by the formula

\[
p_{\text{III}} = 1 - (p_I + p_{II}),
\]

(3)

If only positions I or II are favorable for grip and rotation of the part to the pocket is not required, then the probability \(p_I\) will be equal to the probability \(p_I\) or \(p_{II}\) depending on which of the bases is favorable for grip.

If the part is gripped by the pocket only from a horizontal position relative to the disk surface and its rotation is required for grip, then the probability of finding the part on the rotating disk before grip in the desired position is determined by the expression

\[
p_I = 1 - (1 - p_{III} p_{12\text{max}})^3 (1 - p_{III} p_{12\text{min}}) \frac{\pi(R - d_l)R^2}{2l},
\]

(4)

where \(R\) is the radius of the location of the gripping organs; \(p_{12}\) – the probability of making a turn, represented in practical calculations by the minimum \(p_{12\text{min}}\) and maximum \(p_{12\text{max}}\) values. In this case, the lowest probability is that even a part that deviates from the pocket by a certain angle will be correctly gripped. The highest probability will be in the spot where there are no other parts.

The probability \(p_{12\text{min}}\) is determined using the expression presented in [1, 4-8], or using a graph based on the coefficients \(a\) and \(f\) known for the part (Figure 3).

The highest probability \(p_{12\text{max}}\) is determined by the central angle \(\delta\) drawn from the center of mass of the part to the base that grips the part, according to the expression

\[
p_{12\text{max}} = \frac{1}{2\pi} \left( \delta - 2\arcsin \frac{\mu}{\alpha} \right),
\]

(5)

where \(\alpha\) is the angle of inclination of the surface on which the parts are located in the hopper-feeding device; \(\mu\) – the coefficient of friction of parts on the surface of the rotating disk.

The table 1 contains formulas that can be used to determine the angle \(\delta\) depending on the surface that the part sinks into the pocket, and the coefficients \(a\), \(b\), and \(c\).
Figure 3. Graph for determining the probability \( P_{12_{\text{min}}} \).

Table 1. Formulas for determining the central angle included in the expression (4).

| Sinking of the base I | Sinking of the base II | Sinking of the cylindrical surface III |
|-----------------------|-----------------------|----------------------------------------|
| \( \delta_1 = 2 \arctan \frac{a}{2b} \) | \( \delta_{\text{II}} = 2 \arctan \frac{c}{2(1-b)} \) | \( \delta_{\text{III}} = \arctan \frac{2b}{a} + \arctan \frac{2(1-b)}{a} \) |

In some devices, the pocket can grip a part from several positions of the part (for example, base I or II without a u-turn and a cylindrical surface III with a u-turn). Then the probability \( p_1 \) will be equal to the sum of one of these probabilities (\( p_1 \) or \( p_{\text{II}} \) depending on which side of the part is possible to grip) and the probability of turning the cylindrical surface III, defined by the expression (4).

Determining the probability \( p_2 \) that there is no interference from other parts has been implemented in several ways. In the first case, the probability \( p_2 \) was determined using a simplified formula [1, 4-6]. In the second case, the probability \( p_2 \) was determined on the basis of complex calculations with the determination of the areas of all surfaces of the part and the partial coefficients of linear braking [7]. It was shown that for solid parts and parts that have an internal cavity that does not lead to coupling of parts, the probability \( p_2 \) values in both cases are completely the same. Therefore, it is advisable to use a simplified formula for such parts

\[
p_2 = 1 - \frac{\arctan \mu_0}{\pi} \left( \frac{0.9a + 1.4}{a + 2} \right),
\] (6)

where \( \mu_0 \) is the coefficient of friction between parts.

For parts with an internal cavity that facilitates their inter-coupling with each other, the probability \( p_2 \) is determined by the expression (7)

\[
p_2 = 1 - \frac{\sum_{i=1}^{\gamma_i}}{\gamma_i},
\] (7)

where \( \gamma_i \) is the number of surfaces of the part; \( X_i \) and \( \gamma_i \) the partial coefficient of linear braking and its weight, determined respectively by the formulas

\[
X_i = \frac{\xi_i}{\pi} = \frac{1}{\sqrt{2\pi}} \sqrt{\xi_i^2 + \gamma_i^2}, \quad \gamma_i = \frac{F_1 + F_2}{2\pi \sum_{i=1}^{F_i}}.
\] (8)
where $\xi_{xi}, \xi_{yi}$ are the greatest angles of rotation of two adjacent sides of parts without separating them; $F_1$ and $F_2$ the area of adjacent surfaces of parts; $\sum F_i$ - the area of all surfaces of the part.

After determining the probabilities $p_1$ and $p_2$ using the formula (2), calculate the probability $E_{max}$ of gripping the part.

Determining the speed of rotation of the disk $n_{max}$ at which it is impossible to grip the part is based on the condition that the part collides with the elements of the gripping organs. As a result of a collision, the item is thrown out of the pocket. The expression for the definition $n_{max}$ will depend on the design of the gripping organ and how the part gets into the pocket.

The frequency of rotation of the disk of hopper-feeding devices is determined by the known formula

$$n_{max} = \frac{30\nu_{max}}{\pi R},$$

where $\nu_{max}$ is the maximum value of the circumferential speed of the pockets, at which it is impossible to grip the part.

Expressions to define $\nu_{max}$ of many mechanical disk pocket hopper-feeding devices depending on the method of sinking the part into the pocket have been derived from result of many years of research. The results are shown in Figure 4. They allow you to determine $\nu_{max}$ depending on the diameter $d_1$ and the method of sinking the part into the pocket of the hopper-feeding device:

– falling of a part sliding on a fixed surface into a moving pocket (Figure 4, a, curve 1), for example, in the construction of a vertical hopper-feeding device [9];
– rolling from the surface of the protrusions on the rotating disk into its radial pockets with the thickness of the protrusion equal $d_1$ (Figure 4, a, curve 2.1), 0.75$d_1$ (Figure 4, a, curve 2.2), 0.5$d_1$ (Figure 4, a, curve 2.3), for example, in the construction of a hopper-feeding device with teeth [1];
– rolling the part from a fixed surface into moving tangential pockets at (Figure 4, b, curve 3.1) and (Figure 4, b, curve 3.2) for parts that, for example, have a device design with tangential profile pockets [7];
– by rolling the part from the surface of the rotating disk into its tangential pockets (Figure 4, b, curve 4), for example, in the design of a device with tangential rectangular pockets [10];
– by rolling the part from the surface of the rotating disk into its radial pockets (Figure 4, b, curve 5), for example, in the design of a device with radial sockets and an annular orientator [5].

An expression for determining the actual feed rate of mechanical disk hopper-feeding devices was obtained after substituting the obtained coefficients $E_{max}$ and $n_{max}$ in the formula (1).
4. Discussion of results

Consider the simulation of actual feed rate using an example of a mechanical disc hopper-feeding device with tangentially arranged rectangular pockets and a tipping device. The pockets provide a grip of the part. The tipping device is located in the upper part of the hopper-feeding device and provides active orientation of parts in the form of a cap [1].

The main parameters parts: $l = 0.03\ m$; $d_1 = 0.015\ m$; $d_2 = 0.01\ m$; $l_1 = 0.026\ m$; $x_c = 0.022\ m$. Radius of the rotating disk along the pocket axis $R = 0.2\ m$, number of pockets $k = 45$. The coefficient of friction of parts on the elements of the device design $\mu = 0.2$ and among themselves $\mu_0 = 0.25$. Angle of inclination of the disk surface $\alpha = 45^\circ$.

Calculate the coefficients $a$, $b$, $f$, and $c$: $a = 1/2$, $c = 1/3$, $f = 7/8$, $b = 2/3$.

Part in the form of a cap has three surfaces (see Figure 1, g): is the base I with an open portion, a closed base II and the cylindrical surface III. To grip a part with a pocket, it must end up on a rotating disk on its cylindrical surface III. Therefore the probability $p_{1l} = p_{III}$. From the graphs, we determine that $p_1 = 0.03$ (see Figure 2, a), $p_{II} = 0.05$ (see Figure 2, b). Then $p_{1l} = p_{III} = 1 - (p_1 + p_{II}) = 0.92$.

In the future, the part should turn to some angle described by the probability $p_{12}$. Using the graph, we determine its minimum value $p_{12\text{min}} = 0.04$ (see Figure 3). To determine the maximum probability $p_{12\text{max}}$, we calculate the angle of the part's sinking into the pocket by its cylindrical surface III (table 1). We get $\delta_{III} = 2.14\rad$ and calculate $p_{12\text{max}} = 0.276$ its using expression (5). Then we define $p_1 = 0.947$ by the formula (4).

The probability $p_1 = 0.947$ of no interference from other parts is determined by the expression (6), we get $p_2 = 0.942$.

After determining the probabilities $p_1$ and $p_2$ using the formula (2) we calculate the probability of gripping the part, we get $E_{\text{max}} = 0.892$.

Since in the considered device, the part is sunk into the pocket by rolling the part from the surface of the rotating disk into its tangential pockets, we determine the $\nu_{\text{max}} = 0.485\ m/s$ (Figure 4, b, curve 4). Then, using the formula (9), we calculate $n_{\text{max}} = 23\ rpm$.

We substitute the coefficients values $E_{\text{max}}$ and $n_{\text{max}}$ in formula (1) and obtain an expression to estimate the actual feed rate of a mechanical disk hopper-feeding device with tangential rectangular pockets depending on the frequency of rotation of the disk.

The results of computer simulation are shown in Figure 5 in the form of graphs of dependencies of actual feed rate and the device grip coefficient on the disk rotation frequency $n$ in the range of its change from 0 to 15 rpm.

![Figure 5](image-url)

**Figure 5.** Graphs of the dependence of feed rate and grip coefficient on the frequency of rotation of the disk of a mechanical disk hopper-feeding device with tangential rectangular pockets.

The graphs show that when feeding rod parts with a spherical end face in the shape of a cap, the maximum feed rate of a mechanical disk hopper-feeding device with tangential rectangular pockets
reaches \( F = 500 \text{ parts/min} \) at a disk rotation frequency of 16 rpm (see Figure 5). The recommended disk rotation frequency is 10-20\% less than the disk rotation frequency at which feed rate is maximum. This is due to strong fluctuations and instability of the device, which dramatically reduces the reliability of its operation.

The problem of modeling the feed rate of mechanical disk hopper-feeding devices can be approached from the opposite side. For example, after making sure from the graphs that the device feed rate and, accordingly, the required probabilities do not take sufficient values to ensure the required performance of technological machines and lines, it is advisable to make structural changes to the device (for example, increase the angle of the rotating disk, adjust the pocket size). This will improve the feed rate of the device and achieve the desired results.

5. Conclusions
The integrated approach methodology is the result of many years of research. It takes into account the related effect on the feed rate of the hopper-feeding device of its main kinematic and structural parameters, geometric and physical-mechanical parameters of the parts. Physical laws of the process of probabilistic grip of parts by moving gripping organs of the device are the basis of the mathematical model of actual feed rate built using an integrated approach.

The proposed method for evaluating the actual feed rate of various designs of mechanical disk hopper-feeding devices allows you to determine the feed rate of devices for various axisymmetric parts of the shape of rotation bodies with sufficient accuracy for practice at the design stage and greatly simplifies engineering calculations.

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