Mathematical methods to evaluate ecological stability of industrial park

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Abstract. A range of theories and methods were developed for improving productivity in every industrial activity without damaging the quality of the environment. Each industrial activity must be carried out within the stable region of ecological carrying capacity. A quality of the environment can be achieved by maintaining the ecological stability of the environment. This paper presents mathematical methods to evaluate ecological stability of oleo chemical based industrial park. The stability occurs when there is ecological balance on every part of the perturbation variables that affect the stability of ecological indicators. Measuring and calculating equilibrium constant, predicting equilibrium constant, and the effect of perturbation on equilibrium, and phase distribution equilibria are also discussed.

1. Introduction
Stability is a central notion in several academic disciplines, but the concept has been almost exclusively subject specific. There are many concepts of stability known. Some of them are constancy, persistence, resilience, elasticity, amplitude, cyclical stability, trajectory stability, global stability, local stability, and alternate stable states [1]. Among those many concepts, 3 basic concept of stability known in general are: constancy, robustness and, resilience. However, stability relates to transitions between states. Robustness can be shown as a limiting case of resilience, and neither constancy nor resilience can be defined in terms of other. Hence, there are two basic concepts of stability, both of which are used in both the social and the natural sciences.

Ecologists have become more and more concerned with providing a precise notion of ecological stability over the last few decades, especially given the continual interest in the “balance of nature,” and the “diversity/complexity-stability hypothesis”—the hypothesis that as the diversity or complexity of a community increases so does the stability of the community.

Some theories believes that ecological stability depends on how ecological systems respond to perturbation. Perturbations may induce, for instance, changes in environmental parameters that modify the strength and qualitative nature of interspecific and intraspecific community interactions. Tolerance and resilience of community depend upon how community respond to perturbations that affect model variables and parameters.

It seems that stability has many different meanings and hence ecological theories are imprecise and inapplicable to environmental policy. Moreover, if stability has many different meanings, then theories that employ stability concepts will lead to different conservation strategies. It appears then that theoretical ecology, more specifically complexity-stability hypotheses, cannot provide aid to
conservation biologists. An appropriate definition of ecological stability therefore requires a concept representing how communities respond to specific types of structural change on industrial parks.

Ecological stability in this paper refers to the ability of an ecosystem to resist changes in the presence of perturbations. Viewed locally, this definition is entirely equivalent to the sensitivity based concept of stability. Within a (local) neighborhood of the equilibrium state, it is assumed throughout the following development that the functional interdependence among the various compartments remains essentially the same, and that choice is the dominant factor affecting the ecosystem response to perturbations.

2. Methods and Approaches

2.1 Ecological Stability

Like many scientific concepts, fully adequate definitions of some ecological concepts have not yet been formulated. Ecological stability is one such concept. Ecologists have proposed several incompatible definitions of ecological stability. Proposed definitions of it are not fully satisfactory and seem incompatible. Mathematical ecologists commonly define it as Lyapunov stability, named after the Russian mathematician who first precisely defined the concept to describe the apparently stable equilibrium behavior of the solar system [2]. His definition has found widespread application outside this context and is frequently used to analyze mathematical models of biological communities [3]. Ecological stability can refer to types of stability in a continuum ranging from resilience (returning quickly to a previous state) to constantly to persistence.

Lyapunov was the first to consider the modifications necessary in nonlinear systems to the linear theory of stability based on linearizing near a point of equilibrium. The first method developed the solution in a series which was then proved convergent within limits. Then, the so-called "Second Method of Lyapunov" was found to be applicable to the stability of aerospace guidance systems which typically contain strong nonlinearities not treatable by other methods. The second method, which is almost universally used nowadays, makes use of a Lyapunov function V(x) which has an analogy to the potential function of classical dynamics.

Within ecology, however, parameters representing external factors affecting species and their interactions are much more likely to change than in systems studied within classical mechanics because they are regularly altered by real-world perturbations. Enrichment of an ecosystem (increasing the available resource) may cause instability leading to collapse of the system in finite time. May used this definition, for instance, in his influential analysis of relationships between the stability and complexity of such models [4]. The definition has some clear advantages. Unlike other definitions, it integrates ecological stability into a thoroughly studied mathematical theory that has proved fruitful in many sciences, especially physics. It also seems to formalize the intuition that ecological stability depends on community response to perturbation. Stability consists of local stability and global stability. Local stability indicates that a system is stable over small short-lived disturbances, while global stability indicates a system highly resistant to change in species composition and/or food web dynamics.

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2.2 Stability of Industrial Parks

A range of theories and methods is developed for improving productivity in every industrial activity without damaging the quality of the environment. A quality of the environment can be achieved by
maintaining the ecological stability of the environment. Each industrial activity must be carried out within the stable region of ecological carrying capacity. The “stability” of a community is thus characterized in one of the following ways:

a. Stable: a system is stable just in case all the variables return to their initial equilibrium values following a perturbation.

b. Resilience: how fast the variables return to their equilibrium following a perturbation.

c. Persistence: how long the value of a variable lasts before it is changes to a new value.

d. Resistance: the degree to which a variable is changed following a perturbation.

e. Variability: the degree to which a variable varies over time.

A community is equilibrium stable if and only if all of the species abundances return to their stable equilibrium after a perturbation. In the context of industrial park, community is all entities, plants or activities involved in the symbiotic chain within the park and the society living in the neighborhood. A community’s resilience stability is determined by how fast the variable of the interest returns to its pre-perturbed stable equilibrium.

2.3 Ecological Indicators

Ecological Indicator is a measure, or a collection of measures, that describes the condition of an ecosystem or one of its critical components. Ecological indicators are used to communicate information about ecosystems and the impact human activity has on ecosystems to groups such as the public or government policy makers. According to the Community Environmental Council, good indicators should:

• reflect something basic and fundamental to the long-term economic, social or environmental health of a community over generations.

• be understood and accepted by the community as a valid sign of sustainability or symptom of distress

• have interest and appeal for use by local media in monitoring, reporting and analysing general trends toward or away from sustainable community practices; and

• be statistically and practically measurable in a geographical area, preferably comparable to other cities/communities, and yield valid data [5].

The basic principles of developing indicators are: use existing data, re-evaluate underlying assumptions, integrate long-term focus with short-term change, relate indicators to individual and vested stakeholders, identify the direction of sustainability, present indicators as a whole system and determine linkages. It is also important to use a simple and easy to understand format for presenting data so that decision makers or other stakeholders can base on the existing data to seek further information that addresses issues of primary concerns in the community.

There are the number options for formulating a complex definition of ecological stability. Adopting ecological stability defined as the ability of an ecosystem to resist changes in the presence of perturbations, in the context of stability on industrial parks, perturbations consists of social, economic, environmental and political influence on the management of industrial park.

Assume $X_1 = $ social perturbation ; $X_2 = $ economic perturbation ; $X_3 = $ environmental perturbation, and $X_4 = $ political perturbation. All vectors are confined within some closed arbitrary boundary.
3. Results and Discussions

3.1 Predicting Equilibrium Constant

By adopting Rutledge’s concepts about ecological stability, to develop an index for the stability a model diagram can be developed to describe the dependence on time for each perturbation component. All the compartment model diagram has a dependence on time. Hence, each main component is represented at two arbitrary times $t_1$ and $t_2$.

Let $Q_i$ be the initial conditions of the industrial park at time $t_1$. $P_j$ is the conditions of the industrial parks at time $t_2$, $f_{ij}$ is the percentage of the total perturbation flow through the $i^{th}$ component that passes to the $j^{th}$ component between times $t_1$ and $t_2$.

The $Q_i$ and $P_i$ refer to component of perturbation $X_i$ occurs at different times with any difference in these components and subcomponents therein accounted by $f_{ij}$. The relationship between these variables is provided by the equation:

$$
\sum_{i=1}^{4} P(X_i) = 1
$$

Each variables can either be independently affects the stability industrial park, or have simple causal relationship or dependence among each vectors as well as sub vectors.

Figure1. Diagram of main components from the original conditions to perturbed conditions
Perturbation flow in an industrial park ecosystem is a function of time. It can occur either in a pathways between entities or in a resources point itself affected by internal or external perturbation. The variables $X_i$ can be defined to be of discrete or continue in nature which represent perturbation flows over some arbitrary time period.

Let $a_k$ be the passage of a given increment of perturbation through the $k^{th}$ component at time $t_1$ and the $b_j$ represent the passage of a given increment of perturbation through the $j^{th}$ component at time $t_2$. The diversity of the ecosystem in terms of its throughput is given by:

$$D = -\sum_{i=1}^{d} P(a_k) \log P(a_k)$$  \hspace{1cm} (3)

Figure 2. Example of 3-symbiosis entities where perturbation can occur.

Where the event $a_k$ is defined as the passage of a given increment of perturbation through the $k^{th}$ component and $P(Q_k)$ is the probability that event $a_k$ occurred. The diversity is a function of time, since the perturbation flow in an ecosystem is a function of time. Hence, the time dependent nature is obtained by defining the appropriate events of perturbation occurrence as functions of time is the logarithm of the ratio of a posteriori to a priori probabilities.

$$I(a_k;b_j) = \log \frac{P(a_k / b_j)}{P(a_k)}$$ \hspace{1cm} (4)

Uncertainty as measured by equation (4) is equivalent to the uncertainty resolved about the occurrence of perturbation event $b_j$ by the occurrence of perturbation event $a_k$ (Gallagher, 1986) and is given by:

$$I(a_k;b_j) = \log \frac{f_{kj}}{P_j}$$ \hspace{1cm} (5)

Since the complexity of the symbiotic chain reflects the opportunities for choice of pathways, a measure of choice is an appropriate index for symbiotic chain and hence for ecological stability.

### 3.2 The Effect of Perturbation on Equilibrium

The most common kind of equilibrium problem will be encountered starting with an arbitrary amount of perturbation on each component.

Observing individual concentrations or partial perturbation directly may be not always be practical, however. If one of the components is perturbed, the extent to which it is affected may serve as an index of its ecological stability. As perturbation occurrence is a function of time, equilibrium will
dinamically change depend on time as well. Continuous perturbation may lead to the occurrence of phase distribution equilibrium.

For every perturbation passing through a component in an ecosystem, a probability assignment can be made to its destination or source. Given a specific perturbation has passed through the $k^{th}$ component, $P(Q_j/P_k)$ is the probability that the increment of perturbation will affect or taken up by the $j^{th}$ component, $P(Q_j/P_k)$ is the probability that the perturbation passed from the $k^{th}$ component to the $j^{th}$ component. The occurrence of perturbation $b_j$ changes the probability of the occurrence of perturbation $a_k$ from the a proiri probability, $P(Q_k)$ to the a posteriori probability $P(Q_k/P_j)$. A quantitative measure of the uncertainty about the occurrence of perturbation events

### 3.3 Phase distribution equilibrium

Phase distribution equilibrium occurs when the perturbation event occurs continuously. Continuous perturbation may occur by temperature, energy flow, and chemical reactions. Equilibrium change dinamically continuous. In such case, equilibrium constant can is calculated on each defined phase. Phase can either be time period, or symbiotical phase.

![Figure 3](image)

**Figure 3.** A control is added to get equilibrium reestablished

If control is added while the system is at equilibrium, the system must respond to counteract the control. The system must consume the control and produce products until a new equilibrium is established.

For future research, ecological stability can also be added to the parameter lists to evaluate environmental performance of Industrial Park. Ecological stability might affect the environmental performance of a system, e.g. Industrial Park, Oleochemical based Industrial Park, Sustainable Housing, Tourism Region. Environmental Performance of the Industrial Park consists of Operational Performance and Environmental Performance [6]. Hence, the ecological stability could be included in the parameters lists to evaluate environmental performance of the oleo chemical based industrial park. How significant this stability affects the performance of a system, depends on the type of stability applied to the system, and the which components and/or sub components of the performance parameters which has direct related path to the stability within the system.

### 4. Conclusion

A measure of ecological stability for industrial parks can be developed based on choice of pathways for symbiotic structure. The concept of choice is developed from a qualitative concept from information theory. Relationships among ecological stability, diversity and complexity consistent with observed behavior during succession arise naturally in the development of the stability index. Theoretical community ecology can provide a much needed resource even when it does not give definitive answers about what to do in particular cases but only explores possibilities.

There are a variety of stability concepts and ecologists have begun to systematically explore and use them to remove various confusions concerning the complexity-stability hypotheses.
Theoretical community ecology is important for environmental policy. A variety of precise stability concepts exist that allow theoretical ecologists to probe complexity stability hypotheses and provide important considerations for policy. The ecological stability could be included in the indicator lists to evaluate environmental performance of the industrial park.

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