Difficulties Distinguishing Dark Energy from Modified Gravity via Redshift Distortions

Fergus Simpson and John A. Peacock

1SUPA, Institute for Astronomy, University of Edinburgh, Royal Observatory, Blackford Hill, Edinburgh EH9 3HJ

(Dated: March 2, 2010)

The bulk motion of galaxies induced by the growth of cosmic structure offers a rare opportunity to test the validity of general relativity across cosmological scales. However, modified gravity can be degenerate in its effect with the unknown values of cosmological parameters. More seriously, even the ‘observed’ value of the RSD (redshift-space distortions) used to measure the fluctuation growth rate depends on the assumed cosmological parameters (the Alcock-Paczynski effect). We give a full analysis of these issues, showing how to combine RSD with BAO (baryon acoustic oscillations) and CMB (Cosmic Microwave Background) data, in order to obtain joint constraints on deviations from general relativity and on the equation of state of dark energy whilst allowing for factors such as non-zero curvature. In particular we note that the evolution of \( \Omega_m(z) \), along with the Alcock-Paczynski effect, produces a degeneracy between the equation of state \( w \) and the modified growth parameter \( \gamma \). Typically, the total marginalized error on either of these parameters will be larger by a factor \( \approx 2 \) compared to the conditional error where one or other is held fixed. We argue that future missions should be judged by their Figure of Merit as defined in the \( w_p - \gamma \) plane, and note that the inclusion of spatial curvature can degrade this value by an order of magnitude.

I. INTRODUCTION

Models of dark energy leave a characteristic signature embedded in both the cosmic expansion and structure formation histories. Recent observational progress has been made with the former, due to its relative ease of measurement, leading to a measurement of the dark energy equation of state parameter \( w \equiv P/\rho c^2 \) with better than 10% precision [1, 2]. This work is geometrical, and so probes dark energy only through its influence on the evolving expansion rate of the Universe. It is thus possible that dark energy may be an illusion, indicating the need to revise general relativity and thus also the Friedmann equation. In either case, the phenomenological dark energy term may well differ from a cosmological constant \( (w = -1) \), and may change its equation of state with redshift. These possible degrees of freedom need to be allowed for before we can claim any evidence for a deviation from general relativity. This paper thus considers how we can make simultaneous measurements of the properties of dark energy and of modified gravity.

A number of probes are capable of measuring \( w \) via its influence on the redshift-distance relation. This measurement alone is effectively completely degenerate with a modification of gravity on the scale of the Hubble radius. But for many models, the Mpc scales of galaxy clustering may be affected in a different way; the growth rate of density fluctuations has thus emerged as a key means of breaking this degeneracy between gravity and dark energy [3, 4]. It is rather more difficult to study the growth rate, due to uncertainty in the behaviour of galaxy bias, but there are currently two promising avenues available for future exploration. Weak gravitational lensing provides a direct measurement of the dark matter distribution, and its evolution with redshift. It can also probe broader aspects of modified gravity, particularly the balance between perturbations to the time and space parts of the metric [5, 6]. The focus of the present work will be the alternative technique, known as redshift-space distortions (RSD), which exploit the relationship between the large-scale coherent velocities of galaxies and the growth rate of perturbations.

In real space, we expect the clustering of galaxies to be statistically isotropic. However, in redshift space the line-of-sight component of a galaxy’s peculiar velocity breaks this symmetry. Inside a virialized cluster of galaxies, the orbital velocity dispersion scatters galaxy redshifts, creating the ‘Fingers of God’, and thereby erasing spatial information on small scales. Across larger scales, galaxies coherently fall out of voids and into overdense regions, considerably amplifying the power in redshift space. These two effects are often treated independently, although a more complex model is required to attain a higher degree of precision [7]. For the present purpose, the large-scale effect is the aspect of interest, since continuity relates coherent peculiar velocities directly to the growth rate of density fluctuations.

Observations to date have led to estimates of the growth rate at various redshifts, although not yet at a useful level of precision [4, 8–10]. Future surveys are likely to cover orders of magnitude larger volumes, thereby delivering the precision needed to discriminate interesting models of modified gravity. But we shall see that, when approaching this target, it may no longer be appropriate to make the simplifying assumptions adopted to date.

In [11] we review the process of determining the growth rate from redshift distortions, before constructing a
Fisher matrix. Our results are presented in §IV while in §IV we consider the implications for the proposed dark energy Figure of Merit.

II. SIGNATURES OF MODIFIED GRAVITY

For any theory of gravity that could play the role of dark energy, there is little reason to believe the formation of structure on large scales would match that of General Relativity. A simple phenomenological model to quantify such a deviation has been suggested by Linder [3] (see also Wang & Steinhardt [11]), parameterising the growth of linear density perturbations as

$$f(z) = \frac{d \ln \delta}{d \ln a} \simeq \Omega_m(z),$$

where δ is the fractional density fluctuation and a is the scale factor; this approximation typically holds to a precision of $\sim 0.1\%$. This explicitly restricts the value of $f(z)$ to unity at high redshift, but this is unlikely to be problematic given that dark energy only appears to be of cosmological significance at low redshift. An exception to this would require a rather contrived functional form of $w(z)$, one that maintains a significant amount of dark energy at high redshift yet not sufficient to modify our observation of the CMB. Observational limits on these `early dark energy’ models have been studied in [12].

In the context of ΛCDM, γ takes the value of 0.55. Galaxy surveys are sensitive to this parameter via the Kaiser effect [13], which in its simplest form is given by

$$P(k, k_\perp) = P(k) \left(1 + \beta \mu^2\right)^2,$$

where $\mu = k_\parallel / |k|$, the parameter $\beta(z)$ is defined as

$$\beta(z) = \frac{f(z)}{b(z)},$$

and where we must restrict ourselves to the large-scale linear regime of scale-independent bias, or be prepared to model non-linear redshift distortions. Given that our application of (2) will extend beyond General Relativity, it is important to ensure that the validity of this formalism remains intact. In arriving at the above equations we have implicitly assumed a continuity relation linking the velocity and density fields. This relies upon the conservation of comoving matter, and as such should hold under all metric theories of gravity.

Whilst the linear bias $b(z)$ is not directly observable, it can be inferred either from the bispectrum or from the amplitude of galaxy clustering – where on large scales $\xi_{gg} = b^2 \xi_{mm}$ and for a given cosmology $\xi_{mm}(z)$ is known from the CMB. Any claimed deviation from $\gamma = 0.55$ will inevitably be met with great scepticism unless the methodology is highly robust, and the bispectrum is unlikely to match this requirement, given that it is inherently a nonlinear quantity. We thus consider the argument from clustering amplitude: $b = \sigma_{gal}(z)/\sigma_S(z)$, where $\sigma_{gal}$ denotes the observed fractional rms in galaxy number density. A measurement of $\beta$ from redshift-space distortions thus yields $f(z)\sigma_S(z)$ if we assume that $\sigma_{gal}$ can be measured with negligible uncertainty.

The matter fluctuation $\sigma_S(z)$ is of course not an observable, but it can be inferred for a given choice of cosmological parameters by taking the CMB as a reference point. This effectively allows us to deduce $f(z)$ in terms of observed fluctuations and known growth laws:

$$f(z) = \Omega_m(z) = \beta(z) \frac{\sigma_{gal}(z)}{\sigma_S(z_{CMB})} \frac{G(z_{CMB})}{G(z)};$$

where the universal linear growth function is $\delta \propto G(z)$. This argument neglects the weak dependence of the last-scattering redshift on the cosmological parameters; a more precise version is given below, following equation [10].

A. Alcock-Paczynski

Provided the distance-redshift relation is well known, the values of $f \sigma_S(z)$ and $\sigma_S(z_{CMB})$ may be extracted from redshift distortions and the CMB respectively. Note however that this assumes perfect knowledge of the background expansion history, $H(z)$, allowing us to map the true observables (angles and redshifts) onto $k$ space. Strictly speaking, the $f \sigma_S$ term is thus not directly observable; the actual measurement inevitably incorporates corrections from the Alcock-Paczynski effect [14].

Similarly, the form of the power spectrum $P(k)$ presented in [2], commonly used in the analysis of redshift distortions [13][4], implicitly assumes knowledge of the distance-redshift relation. In reality the difference in the true functions $D_A(z), H(z)$ may differ from our adopted fiducial values $\hat{D}_A(z), \hat{H}(z)$, leading to the two scaling factors

$$f_\perp = D_A(z)/\hat{D}_A(z),$$

$$f_\parallel = H(z)/\hat{H}(z),$$

which generate apparent wavenumbers $k_\perp = f_\perp k_\parallel$ and $k_\parallel = f_\parallel k_\parallel$, where the prime denotes the coordinate system derived from the assumed cosmology. Thus the assumed cosmology changes the inferred value of $\beta$ [10], and will also alter the value of $\sigma_{gal}$ deduced from the data.

Guzzo et al. [4] propose an iterative method to converge on the correct cosmology. While this proves useful for independently determining $\Omega_m$ alone, it is unlikely to succeed when extending the parameter set to include $w_0$, its evolution $w_a \equiv -dw/da$, and the global curvature $\Omega_k$. 
B. The apparent power spectrum

Our chosen parameter set $p_i$ consists of

$$[w_0, w_a, \Omega_L, \Omega_k, \Omega_m h^2, \Omega_b h^2, n_s, A_s, \beta, \gamma, \sigma_p].$$

The main cosmological parameters are taken to have fiducial values as derived from WMAP5 [1]. An assumption about the redshift dependence of bias is required, and we take $b(z) = 0.6(1 + z)$. Combined with $\gamma = 0.55$, this determines the value of $\beta$ for a given redshift. Finally, $\sigma_p$ denotes the one-dimensional rms pairwise velocity dispersion, which we take to be $300$ kms$^{-1}$ or $3 h^{-1}$ Mpc when converted to length units.

Why include both $w$ and $\gamma$ as free parameters? A more limited parameter set may lead us to misinterpret a simple dark energy fluid as a sign of modified gravity: quantities such as $[w_0, w_a, \Omega_k]$ all have an impact upon our inferred value of $\gamma$ by modifying the function $\Omega_m(z)$.

Our model for the fiducial galaxy bias is simply parameterized as $b(z) = 0.6(1 + z)$. Within a given redshift bin, it is assumed that there is negligible scale or redshift evolution, although in reality we may typically expect a change of $\sim 10\%$ across a bin of width $0.2$. We do not treat the bias as a nuisance parameter to be marginalized over, since its perturbed value is given exactly for a given set of parameters:

$$b = \frac{\Omega_m(z)}{\beta}.$$  

Adjustment of these parameters would thus in effect also change the amplitude of the apparent power spectrum $P'_\text{gal}(k')$. We have experimented with an alternative parameter set, in which $\beta$ is replaced by $b$, and find that our overall results are unchanged, as required.

The isotropic real-space matter power spectrum, $P(k)$, is generated following the HALOFIT [17] prescription, and we extend (2) to incorporate a simple model of the nonlinear redshift distortions

$$P(k, k_{\perp}) = P(k)\left(1 + \beta \mu^2\right)^2 D(k\mu\sigma_p).$$

where $D$ is a Lorentzian given by

$$D(k\mu\sigma_p) = \frac{1}{1 + (k\mu\sigma_p)^2 / 2}.$$  

The apparent power spectrum $P'_\text{gal}(k')$ is recast in the form below, as outlined in (A8) from Ballinger et al. [10] (see also Matsubara & Suto [13]):

$$P'_\text{gal}(k') = \frac{1}{f_{\parallel}^2 F_{\parallel}} b^2 P_m \left(\frac{k'}{f_{\parallel}} \sqrt{1 + \mu^2 \left(1 - \frac{1}{F_{\parallel}^2} - 1\right)} \right) \times \left[1 + \mu^2 \left(1 - \frac{1}{F_{\parallel}^2} - 1\right)\right]^{-2} \times \left[1 + \mu^2 \left(\beta + 1 - \frac{1}{F_{\parallel}^2} - 1\right)\right] D\left(k'\mu\sigma_p\right),$$

where $\mu = k_{\parallel} / |k|$ and $F \equiv f_{\parallel} / f_{\perp}$.

We emphasise that the bias parameter here is the ‘true’ bias, and we do not need to define an ‘apparent’ value: the rationale for this equation is that the amplitude of apparent galaxy number density fluctuations is unchanged by the Alcock-Paczynski transformation, and only the direction of wavevectors is altered.

Seo & Eisenstein [19] present an equivalent calculation to the above, which involves evaluating the distortions first before applying the transformation, thereby simplifying the form to

$$P'_\text{gal}(k') = \frac{1}{f_{\parallel}^2 f_{\perp}^2} b^2 P_m(k)\left(1 + \beta \mu^2\right)^2 D(k\mu\sigma_p).$$

This should give rise to equivalent results, but we prefer the approach of (10) in which the Alcock-Paczynski corrections are exhibited explicitly. This has several advantages: it makes it clear that there is a potentially strong degeneracy between $\beta$ and $F$, as discussed by Ballinger et al. [10], and it allows us to show directly the impact of the geometrical corrections on the RSD signal, as discussed below.

The size of the parameter space can be reduced by one if we focus on $w_p$, the value of $w$ at the pivot redshift. In the usual linear evolution model, this is $w(a) = w_p + w_a(a_p - a)$, where the pivot era $a_p$ is chosen so that errors in $w_p$ and $w_a$ are uncorrelated. In principle $w_a$ is an important parameter, since detection of $w_a \neq 0$ would disprove the cosmological constant hypothesis. But in practice it is rather poorly measured, and the present analysis is not greatly changed if we marginalize over it.
C. Constructing likelihood contours

As usual, we predict parameter uncertainties using the Fisher-matrix formalism. The Fisher matrix (expectation of the Hessian matrix of 2nd derivatives of \( \ln \mathcal{L} \)) is constructed by numerical integration of the following expression \[20\], up to a cut-off at \( k_c = 0.3 \, h \, \text{Mpc}^{-1} \).

\[
F_{ij} = \frac{1}{4\pi^2} \int_0^{k_c} \int_0^{k_c} \left( \frac{\partial \ln P'}{\partial p_i} \right) \left( \frac{\partial \ln P'}{\partial p_j} \right) V_{\text{eff}}(k) k'_i dk'_i dk'_j, \tag{13}
\]

Here, the effective volume of the survey compensates for the shot noise, as defined by \[21\]:

\[
V_{\text{eff}} \equiv V_0 \left( \frac{n_P}{1 + n_P} \right)^2. \tag{14}
\]

We emphasise that this is an integration over the full apparent power spectrum defined above; in this way, the Alcock-Paczynski effects are fully included.

The galaxy power spectrum alone will not yield well-defined cosmological conclusions. In order to include constraints from the CMB, we add the Planck Fisher matrix defined by the DETF\[29\]. As usual in such work, the DETF Fisher matrix uses a different parameter set from our preferred choice, and so the matrix has to be subject to a coordinate transformation, using the Jacobian matrix between one parameter set and another. The final Fisher matrices used in this analysis can be found at \url{www.roe.ac.uk/~frgs/wgamma.html}.

III. RESULTS

Once we are in possession of a full Fisher matrix, marginalization can be performed in the usual analytic manner, in order to isolate the constraints on the parameters of interest. The resulting confidence contours are shown in Figures \[14\]. Here we illustrate a survey that should be feasible \((N = 10^6 \, \text{redshifts})\). The striking aspect of these plots, which is the main result of our paper, is that there is a strong degeneracy between \( \gamma \) and \( \omega \), in the sense that less negative \( \omega \) requires a smaller value of \( \gamma \); the slope of this degeneracy depends on redshift, but is always in this sense.

Huterer & Linder \[22\] highlighted the bias induced in \( \omega \) by neglecting \( \gamma \). Here we would stress that the reverse may also be true, a deviation in general relativity may be erroneously inferred by neglecting a deviation from \( \omega = -1 \). To state the issue more simply: the conditional errors in \( \omega \) and \( \gamma \) may seriously underpredict the total error in either parameter when marginalizing over the unknown value of the other. In the examples we have shown, this almost doubles the error.

A. Effect of Alcock-Paczynski

One of the main differences between this and most earlier works is the inclusion of the full distortion of the power spectrum, as given by \[10\]. This includes both the BAO and RSD information, although these have previously been discussed as separate effects. To clarify this, consider \(10\) again. The first term,

\[
P_{\text{gal}}'(k') = \frac{1}{f_{\perp}^2 f_{\parallel}^2} b^2 P_m \left( \frac{k'}{f_{\perp}^2} \sqrt{1 + \mu^2 \left( \frac{1}{F^2} - 1 \right)} \right), \tag{15}
\]

accounts for BAO (plus the information in the overall curvature of the power spectrum, which we will not attempt to separate out). This appears different, but has the same content as the standard approach, which is to compute \( P_{\text{gal}}'(k') \) and hence the acoustic scale using one geometry only, but then to argue that the scale for other geometries should change \( \propto D_V \equiv [(1 + z)^2 D_A^2 c z / H^2]^{1/3} \). This approximate scaling applies only in the absence of RSD, however; since these are always present whether or not the analysis focuses on BAO only, the full analysis is to be preferred.

RSD have frequently been discussed in isolation, using the Kaiser formula. One might have thus been tempted to approach a combined BAO+RSD analysis by adopting an incorrect model that treats BAO as above, together with RSD without the Alcock-Paczynski corrections:

\[
P_{\text{gal}}'(k') = \frac{1}{f_{\perp}^2 f_{\parallel}^2} b^2 P_m \left( k' \sqrt{1 + \mu^2 \left( \frac{1}{F^2} - 1 \right)} \right) \times [1 + \beta \mu^2] D \left( \frac{k'}{\sigma_p} \right). \tag{16}
\]

It is instructive to compare this form with the correct power spectrum, in order to demonstrate the impact of the Alcock-Paczynski corrections. This modification results in the dashed contours shown in Figure \[1\] which illustrates how neglecting the geometric distortion of \( P'(k) \) leads to erroneously small conditional errors on both \( \omega \) and \( \gamma \). Furthermore, the lack of anisotropic amplification reduces our capability of distinguishing between deviations in \( \omega \) and \( \gamma \), thereby increasing the (negative) covariance between the two parameters.

In earlier work, both Sapone & Amendola \[23\] and Stril et al. \[24\] address this issue, although here our analysis extends to include parameters such as \( \Omega_k \) and \( \sigma_p \), of which we find the former provides a substantial impact. Wang \[25\] explored the potential for galaxy redshift survey to measure the linear growth \( f(z) \), including the Alcock-Paczynski effect. Yet this parameterisation conceals uncertainties in \( H(z) \), so here we focus on purely growth-dependent term, \( \gamma \).

We note in passing that it is not so straightforward to achieve the converse of a BAO “wiggles-only” analysis and cleanly isolate the RSD signal alone. For instance, if
FIG. 1: Left: Joint constraints on modified gravity and dark energy from a combination of the Cosmic Microwave Background and Large Scale Structure. The solid contours represent the 1- and 2-σ constraints for a $10^{-3}h^{-3}\text{Gpc}^3$ redshift survey at $z = 1$, with $\bar{n} = 10^{-4}h^3\text{Mpc}^{-3}$, and combined with the DETF Planck Fisher matrix. If we had chosen to neglect the Alcock-Paczynski effect from the redshift distortions, but leaving the BAO information intact as discussed in the text, we would arrive at the dashed contours. In both cases, a weak prior is applied to $\sigma_p$, on the basis it may be well measured on scales much smaller than those considered here. Right: Varying the redshift bin of the survey from $z = 0.5, 1, 2$, shown as dashed, solid and dotted respectively.

we were to null the isotropic component

$$\bar{P}(k_{\parallel}, k_{\perp}) = \frac{P(k_{\parallel}, k_{\perp})}{\int P(k)\,dk}, \quad (17)$$

this removes an essential component of the signal we require, namely the amplitude of the power spectrum itself.

B. Redshift evolution

As we progress towards higher redshifts $\Omega_m(z)$ approaches unity, and so for a fixed fractional error on $\beta$ we arrive at a larger error $\delta\gamma$, as given by

$$\delta\gamma = \frac{1}{\ln(\Omega_m(z))} \frac{\delta f}{f}. \quad (18)$$

This effect is illustrated in the right panel of Figure 1. A tilting in the degeneracy direction is also induced.

C. Curvature

Non-zero curvature not only contributes to the Alcock-Paczynski squashing, but invokes further uncertainty in $\Omega_m(z)$, which in turn enlarges the uncertainty in $\gamma$. Figure 2 demonstrates the dangers associated with assuming a flat cosmology, where even a modest deviation from flatness $\Omega_k = 0.01$ can be seen to generate a significant bias in the estimation of both $w$ and $\gamma$.

D. Errors on $\beta$

Two fitting functions have recently been proposed to predict the precision with which $\beta$ may be measured. Guzzo et al. [4] utilised the correlation function in real-space, and found the error on $\beta$ was well described by

$$\frac{\delta\beta}{\beta} = \frac{50}{\sqrt{0.5\bar{n}^{0.44}}}, \quad (19)$$

while White et al. [20] considered the analysis in Fourier space, noting

$$\frac{\delta\beta}{\beta} = b^{-1} \left[ \beta^2 F_{bb}^{-1} - 2\beta F_{bf}^{-1} + F_{ff}^{-1} \right]^{1/2}, \quad (20)$$

which exhibits the same scaling with volume, but is rather more pessimistic at high number densities.

When fixing the background cosmology, our findings are consistent with White et al.

E. Degeneracy direction

To establish the expected direction of degeneracy in the $w - \gamma$ plane, we consider the partial derivatives of the relevant parameters

$$\frac{\partial\gamma}{\partial w} = -\frac{\partial \ln f}{\partial \ln w} + \frac{\partial \ln g}{\partial \gamma}, \quad (21)$$

where $g \equiv \sigma_8(z)/\sigma_8(z_{\text{CMB}})$. This predicted degeneracy gradients are plotted as dotted lines in Figure 4 and
FIG. 2: The same survey specifications as in Fig 1 but now assuming a flat universe. The dashed contours illustrate the bias induced by an actual value of $\Omega_k = -0.01$.

FIG. 3: The substantial error in $\gamma$ that can arise from a relatively small uncertainty in $\beta$. The dataset is the same as Fig 1 and the dashed line is for a fixed $w = -1$.

can be seen to closely align with the redshift distortion contours for a constant equation of state. The dataset matches that of Figure 1.

A simple qualitative interpretation of the slope direction is that a more positive value of $w(z)$ generates a lower $\Omega_m(z)$, which in turn requires a lower value of $\gamma$ in order to maintain the same value of $f$.

IV. FIGURE OF MERIT

In recent work by the Joint Dark Energy Mission Figure of Merit Science Working Group [26], the relative merits of future surveys are quantified by separately considering the errors on the dark energy equation of state (in the form of eigenmodes) and the modified growth index $\Delta \gamma$. However as we have seen, these quantities can clearly exhibit significant covariance. To compensate for this, a simple prescription could be adopted in terms of the marginalised Fisher elements, analogous to that previously used for $w_0$ and $w_a$.

$\text{FoM} = \sqrt{F_{ww} F_{\gamma\gamma} - F_{w\gamma}^2}$.  \hfill (22)

Note we have omitted the factor of approximately $6\pi$ which would reduce this to the area within the 95% confidence contours. Some examples of this FoM are presented in Table I for a selection of survey volumes and redshifts.

We also stress that the quantity $\gamma$ is just as likely to exhibit redshift variation as $w$, which raises the question: where are we measuring its value? The functional form is such that $z < 2$ is strongly preferred, since at higher redshifts $\Omega_m \sim 1$ and $\gamma$ is unable to exert much influence. This reflects our prior that regions of low $\Omega_\Lambda$ are less likely to demonstrate unusual activity in the growth rate. Dark energy is only known to exist as a low-redshift phenomenon, and as such unearthing the growth rate in this era presents a most enticing prospect. This issue is elaborated in a companion paper [27].

| Mean Redshift | Volume $(h^{-3}\text{Gpc}^3)$ |
|---------------|-------------------------------|
| 0.5           | 16.8 3.8 0.6 0.3              |
| 1             | 158.0 35.4 5.8 2.6            |
| 1.5           | 329.5 55.1 25.2              |
| 2             |                               |

TABLE I: The Figure of Merit, as defined by (22), for various permutations of volume and redshift, for the case of a single redshift bin and with a number density $\bar{n} = 10^{-3}h^3\text{Mpc}^{-4}$.
By relaxing the common assumption of a fixed background cosmology, we have highlighted some of the difficulties encountered when attempting to study gravity via the bulk motion of galaxies. Rather than a pure probe of structure, redshift distortions also comprise a geometric component. This enters at the stage of converting the true observables, angles and redshifts, into distances and Fourier modes. Furthermore, when determining the growth index $\gamma$ it is essential that its corresponding radix $\Omega_m(z)$ is well determined. With these two factors in mind, it appears unlikely that the galaxy power spectrum alone could provide conclusive evidence against General Relativity.

To converge on the true underlying cosmology, iterating over a value for $\Omega_m$ has proved adequate for current data. However with the greater degrees of freedom required to test relativity $(w_0, w_a, \Omega_k)$, the available volume of parameter space appears too great. Fortunately future data will inevitably be accompanied by improved measurements of the baryon acoustic oscillations. Ironically the squashing effect that empowers the BAO is the very same Alcock-Paczynski effect that confounds the redshift distortions.

One concern in the formalism may be the assumption of scale-independence for both the growth and bias. More physically motivated forms of modified gravity, such as $f(R)$ models, lead to rather different scale-dependent growth factors. However, as highlighted in [28], such models also generate very prominent deviations on intermediate scales, which would become more immediately apparent.

Nevertheless, neglect of these issues is more likely to lead to a bias in the results of analyses that assume scale-independent effects, rather than changing their statistical precision. In this work, we have concentrated on the latter aspect, and our main conclusion is that the parameters $\gamma$ and $w_p$ will generally be strongly anti-correlated. We therefore suggest that a natural Figure of Merit for future experiments in fundamental cosmology should be the reciprocal of the area of the error contour in the $\gamma-w_p$ plane.

Acknowledgements
We thank Luigi Guzzo for many helpful comments on an earlier draft of this paper, and also Thomas Kitching and Will Percival for several productive discussions. FS was supported by an STFC Rolling Grant.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Volume $(h^{-3}\text{Gpc}^3)$ & 1 & 1.5 & 2 & \hline
1 & 175.8 & 62.4 & 25.3 & 12.0 \hline
10 & 1122.3 & 405.2 & 166.6 & 80.8 \hline
100 & 1514.1 & 606.9 & 311.2 & \hline
\hline
\end{tabular}
\caption{The same Figure of Merit as in Table I but now under the assumption of a flat universe.}
\end{table}

\begin{thebibliography}{99}
\bibitem{1} E. Komatsu, J. Dunkley, M. R. Nolta, C. L. Bennett, B. Gold, G. Hinshaw, N. Jarosik, D. Larson, M. Limon, L. Page, et al., Astrophys. J. Supp. \textbf{180}, 330 (2009), 0803.0547.
\bibitem{2} W. J. Percival, B. A. Reid, D. J. Eisenstein, N. A. Bahcall, T. Budavari, M. Fukugita, J. E. Gunn, Z. Ivezic, G. R. Knapp, R. G. Kron, et al., ArXiv e-prints (2009), 0907.1660.
\bibitem{3} E. V. Linder, Phys. Rev. D \textbf{72}, 043529 (2005), arXiv:astro-ph/0507263.
\bibitem{4} L. Guzzo, M. Pierleoni, B. Meneux, E. Branchini, O. Le Fèvre, C. Marinoni, B. Garilli, J. Blaizot, G. De Lucia, A. Pollo, et al., Nature (London) \textbf{451}, 541 (2008), 0802.1944.
\bibitem{5} R. Bean, ArXiv e-prints (2009), 0909.3853.
\bibitem{6} B. Jain and P. Zhang, Phys. Rev. D \textbf{78}, 063503 (2008), 0709.2375.
\bibitem{7} R. Scoccimarro, Phys. Rev. D \textbf{70}, 083007 (2004), arXiv:astro-ph/0407214.
\bibitem{8} F. Hoyle, P. J. Outram, T. Shanks, B. J. Boyle, S. M. Croom, and R. J. Smith, Mon.Not.Roy.As.Soc. \textbf{332}, 311 (2002), arXiv:astro-ph/0107348.
\bibitem{9} J. da Ângela, P. J. Outram, and T. Shanks, Mon.Not.Roy.As.Soc. \textbf{361}, 879 (2005), arXiv:astro-ph/0505469.
\bibitem{10} N. P. Ross, J. da Ângela, T. Shanks, D. A. Wake, R. D. Cannon, A. C. Edge, R. C. Nichol, P. J. Outram, M. Colless, W. J. Couch, et al., Mon.Not.Roy.As.Soc. \textbf{381}, 573 (2007), arXiv:astro-ph/0612400.
\bibitem{11} L. Wang and P. J. Steinhardt, Astrophys. J. \textbf{508}, 483 (1998), arXiv:astro-ph/9804015.
\bibitem{12} M. Doran and G. Robbers, Journal of Cosmology and Astro-Particle Physics \textbf{6}, 26 (2006), arXiv:astro-ph/0601544.
\bibitem{13} N. Kaiser, Mon.Not.Roy.As.Soc. \textbf{227}, 1 (1987).
\bibitem{14} C. Alcock and B. Paczynski, Nature (London) \textbf{281}, 358 (1979).
\bibitem{15} J. A. Peacock, S. Cole, P. Norberg, C. M. Baugh, J. Blund-Hawthorn, T. Bridges, R. D. Cannon, M. Colless, C. Collins, W. Couch, et al., Nature (London) \textbf{410}, 169 (2001), arXiv:astro-ph/0103143.
\bibitem{16} W. E. Ballinger, J. A. Peacock, and A. F. Heavens, Mon.Not.Roy.As.Soc. \textbf{282}, 877 (1996), arXiv:astro-ph/9605017.
\bibitem{17} R. E. Smith, J. A. Peacock, A. Jenkins, S. D. M. White, C. S. Frenk, F. R. Pearce, P. A. Thomas, G. Efstathiou, and H. M. P. Couchman, Mon.Not.Roy.As.Soc. \textbf{341}, 1311 (2003), arXiv:astro-ph/0207664.
\bibitem{18} T. Matsubara and Y. Suto, Astrophys. J. Lett. \textbf{470}, L1+ (1996), arXiv:astro-ph/9604142.
\bibitem{19} H.-J. Seo and D. J. Eisenstein, Astrophys. J. \textbf{598}, 720 (2003).
\bibitem{20} M. White, Y.-S. Song, and W. J. Percival, Mon.Not.Roy.As.Soc. pp. 925+- (2009), 0810.1518.
\bibitem{21} M. Tegmark, A. J. S. Hamilton, M. A. Strauss, M. S.
Vogeley, and A. S. Szalay, Astrophys. J. 499, 555 (1998), arXiv:astro-ph/9708020.

[22] D. Huterer and E. V. Linder, Phys. Rev. D 75, 023519 (2007), arXiv:astro-ph/0608681.

[23] D. Sapone and L. Amendola, ArXiv e-prints (2007), 0709.2792.

[24] A. Stril, R. N. Cahn, and E. V. Linder, ArXiv e-prints (2009), 0910.1833.

[25] Y. Wang, Journal of Cosmology and Astro-Particle Physics 5, 21 (2008), 0710.3885.

[26] A. Albrecht, L. Amendola, G. Bernstein, D. Clowe, D. Eisenstein, L. Guzzo, C. Hirata, D. Huterer, R. Kishner, E. Kolb, et al., ArXiv e-prints (2009), 0901.0721.

[27] F. Simpson, ArXiv Astrophysics e-prints (2009), arXiv:0910.3836.

[28] W. Hu and I. Sawicki, Phys. Rev. D 76, 104043 (2007), 0708.1190.

[29] http://www.physics.ucdavis.edu/DETFast/DETFast.zip