Role of multichannel \( \pi\pi \) scattering in decays of bottomia

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The effect of isoscalar S-wave multichannel \( \pi\pi \rightarrow \pi\pi, K\overline{K}, \eta\eta \) scattering is considered in the analysis of decay data of the \( \Upsilon \)-mesons. We show that when allowing for the final state interaction contribution to the decays \( \Upsilon(mS) \rightarrow \Upsilon(nS)\pi\pi \) \((m > n, m = 2, 3, n = 1, 2)\) in our model-independent approach, we can explain the two-pion energetic spectra of these \( \Upsilon \) transitions including the two-humped shape of the di-pion mass distribution in \( \Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi \) as the coupled-channel effect. It is shown also that the considered bottomia decay data do not offer new insights into the nature of the \( f_0 \) mesons, which were not already deduced in our previous analyses of pseudoscalar meson scattering data.

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I. INTRODUCTION

A comprehensive study of the properties of scalar mesons is important for the most profound topics concerning the QCD vacuum, because both sectors affect each other due to possible “direct” transitions between them. The problem of a unique structure interpretation of the scalar mesons is far away from being solved completely [1]. For example, applying our model-independent method in the three-channel analyses of processes \( \pi\pi \rightarrow \pi\pi, K\overline{K}, \eta\eta \) we have obtained parameters for the scalar mesons \( f_0(500) \) and \( f_0(1500) \), which considerably differ from results of analyses which utilize other methods — mainly those based on dispersion relations and Breit–Wigner approaches (see detailed discussion in Ref. [2]).

The decays of heavy quarkonia into a pair of pseudoscalar mesons and a spectator are a good laboratory for studying the \( f_0 \) mesons if the pseudoscalar meson pair is produced in an S-wave. This occurs, e.g., in the specific decays of charmonia and bottomia: \( J/\psi \rightarrow \phi(\pi\pi, K\overline{K}), \psi(2S) \rightarrow J/\psi \pi\pi \) and \( \Upsilon(2S) \rightarrow \Upsilon(1S)\pi\pi, \Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi, \Upsilon(3S) \rightarrow \Upsilon(2S)\pi\pi \).

In recent papers [3,4] we have already presented a combined analysis of the isoscalar S-wave processes \( \pi\pi \rightarrow \pi\pi, K\overline{K}, \eta\eta \). The analysis was performed in our model-independent approach based on analyticity, unitarity, on the use of the uniformization procedure, and on the inclusion of charmonium decay processes \( J/\psi \rightarrow \phi(\pi\pi, K\overline{K}) \), \( \psi(2S) \rightarrow J/\psi \pi\pi \). Taking into account the data on charmonium decays helped to narrow down the \( f_0(500) \) solution to the one with the larger width. Other resonance parameters were practically not changed after using the charmonium data. At this stage it is worth performing a combined analysis including data on decays of the \( \Upsilon \)-meson family: \( \Upsilon(2S) \rightarrow \Upsilon(1S)\pi\pi, \Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi \) and \( \Upsilon(3S) \rightarrow \Upsilon(2S)\pi\pi \). These decays have been studied intensively using various approaches (see, e.g., Ref. [5] and the references therein).

Note that here, except for a possible confirmation and specification of the scalar resonance parameters, there is the problem of explaining the two-humped shape of the di-pion mass distribution in the decay \( \Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi \). This distribution might be the result of the destructive interference of the relevant contributions to the decay \( \Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi \). However, in this scenario the phase space cuts off possible contributions, which might interfere destructively with the \( \pi\pi \) scattering contribution giving the specific shape of the di-pion spectrum.

After the experimental evidence for the two-humped shape of the di-pion spectrum Lipkin and Tuan [6] suggested that the decay \( \Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi \) proceeds as follows:
\[ \Upsilon(3S) \rightarrow B\overline{B} \rightarrow B\overline{B} \pi \rightarrow BB\pi \rightarrow \Upsilon(1S)\pi\pi. \]
In the heavy-quarkonium limit, when neglecting the recoil of the final quarkonium state, they obtained a transition amplitude containing a term proportional to \( p_1 \cdot p_2 \propto \cos\theta_{12} \), where \( \theta_{12} \) is the angle between the pion three-momenta \( p_1 \) and \( p_2 \), multiplied by some function of the kinematical invariants. If the latter was a constant, then the angular distribution \( d\Gamma/d\cos\theta_{12} \propto \cos^2\theta_{12} \) (and \( d\Gamma/dM_{\pi\pi} \)) would have the two-humped shape. However, this scenario was not tested numerically by fitting to data. It is possible that this effect is negligible due to the small coupling of \( \Upsilon \) to the b-flavored sector.

In Ref. [7] Moxhay suggested that the two-humped
shape is a result of the interference between two parts of the decay amplitude. The first part, in which the \( \pi \pi \) final state interaction is allowed for, is related to a mechanism which acts as well in the decays of excited quarkonia states \( \psi(2S) \rightarrow J/\psi \pi \pi \) and \( \Upsilon(2S) \rightarrow \Upsilon(1S)\pi \pi \) and which, obviously, should also occur in the process \( \Upsilon(3S) \rightarrow \Upsilon(1S)\pi \pi \). The second part is responsible for the Lipkin–Tuan mechanism. However, nothing remains from the latter term because the author says that this part does not dominate the amplitude and “the other tensor structures conspire to give a distribution in \( M_{\pi \pi} \) that is more or less flat” – it is constant. It seems that the approach of Ref. [8] resembles the above one. The authors simply supposed that a pion pair is formed in the \( \Upsilon(3S) \) decay both as a result of rescattering and direct production. One can, however, believe that the latter is not reasonable because the pions interact strongly.

In the present paper we show that the indicated effect of destructive interference can be achieved by taking into account our previous conclusions on the wide resonances [4, 9], without any further assumptions.

II. MULTICHANNEL \( \pi \pi \) SCATTERING IN THE DECAYS OF BOTTOMIA

When carrying out our combined analysis, data for the processes \( \pi \pi \rightarrow \pi \pi, K \overline{K}, \eta \eta \) were taken from many sources (see the corresponding references in [4]). For the \( J/\psi \rightarrow \phi \pi \pi \), \( \phi K \overline{K} \) decays data were taken from the Mark III, DM2 and BES II collaborations; for \( \Upsilon \) also in [4]). For \( \Upsilon(2S) \rightarrow \Upsilon(1S)\pi \pi \) the pairs of pseudoscalar mesons in the final state have a large dependence of di-pion spectra (Fig. 2, upper panel) are the result of a destructive interference between the \( \pi \pi \) decay both as a result of rescattering and direct production and “the other tensor structures conspire to give a distribution in \( M_{\pi \pi} \) that is more or less flat” – it is constant. It seems that the approach of Ref. [8] resembles the above one. The authors simply supposed that a pion pair is formed in the \( \Upsilon(3S) \) decay both as a result of rescattering and direct production. One can, however, believe that the latter is not reasonable because the pions interact strongly.

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\( F_n(s) = (\rho_{n0} + \rho_{n1} s) T_{11} + (\omega_{n0} + \omega_{n1} s) T_{21}, \)

where \( \rho_i = \sqrt{1-s_i/s} \) and \( s_i \) is the reaction threshold. The \( S \)-matrix elements are parametrized on the uniformization plane of the \( \pi \pi \) scattering amplitude by poles and zeros which represent resonances. The uniformization plane is obtained by a conformal map of the \( S \)-sheeted Riemann surface, on which the three-channel \( S \) matrix is determined, onto the plane. In the uniformizing variable used we have neglected the \( \pi \pi \)-threshold branch point and allowed for the \( K \overline{K} \)- and \( \eta \eta \)-threshold branch points and left-hand branch point at \( s = 0 \) related to the crossed channels. The background is introduced to the amplitudes in a natural way: on the threshold of each important channel there appears generally speaking a complex phase shift. It is important that we have obtained practically zero background of the \( \pi \pi \) scattering in the scalar-isoscalar channel. It confirms well our representation of resonances.

The expressions for the decay \( \Upsilon(2S) \rightarrow \Upsilon(1S)\pi \pi \)

\[
N|F|^2 \sqrt{(s-s_1)\lambda(m^2_{\Upsilon(2S)},s,m^2_{\Upsilon(1S)})},
\]

where \( \lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz \) is the Källén function, and the analogue relations for \( \Upsilon(3S) \rightarrow \Upsilon(1S)\pi \pi \) give the dimeson mass distributions. \( N \) (normalization to experiment) is as follows: for \( \Upsilon(2S) \rightarrow \Upsilon(1S)\pi^+\pi^- \), 4.3439 for ARGUS, 2.1776 for CLEO(94), 1.2011 for CUSB; for \( \Upsilon(2S) \rightarrow \Upsilon(1S)\pi^0\pi^0 \), 0.0788 for Crystal Ball(85); for \( \Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^- \) and \( \pi^0\pi^0 \), 0.5096 and 0.2235 for CLEO(97), and for \( \Upsilon(3S) \rightarrow \Upsilon(2S)(\pi^+\pi^- \) and \( \pi^0\pi^0 \), 7.7397 and 3.8587 for CLEO(94), respectively. The parameters of the coupling functions of the decay particles \( \Upsilon(2S) \) and \( \Upsilon(3S) \) to channel \( i \), obtained in the analysis, are for \( \Upsilon(2S) \rightarrow \Upsilon(1S)\pi^+\pi^- \), \( (\rho_{10},\rho_{11},\omega_{10},\omega_{11}) = (0.4050, 47.0963, 1.3352,-21.4343) \), \( (\rho_{20},\rho_{21},\omega_{20},\omega_{21}) = (1.0827, -2.7546, 0.8615, 0.6600) \), \( (\rho_{30},\rho_{31},\omega_{30},\omega_{31}) = (7.3875, -2.5598, 0.0, 0.0) \).

A satisfactory combined description of all considered processes is obtained with a total \( \chi^2/\text{ndf} = 640.302/(564 - 70) \approx 1.30 \) for the \( \pi \pi \) scattering, \( \chi^2/\text{ndf} \approx 1.15 \) for \( \pi \pi \rightarrow K \overline{K}, \chi^2/\text{ndf} \approx 1.65 \); for \( \pi \pi \rightarrow \eta \eta \), \( \chi^2/\text{ndf} \approx 0.87 \); for decays \( J/\psi \rightarrow \phi(\pi^+\pi^- \) and \( \pi^0\pi^0 \), \( \chi^2/\text{ndf} \approx 1.21 \); for \( \psi(2S) \rightarrow J/\psi(\pi^+\pi^- \) and \( \pi^0\pi^0 \), \( \chi^2/\text{ndf} \approx 2.43 \); for \( \psi(2S) \rightarrow \Upsilon(1S)\pi^+\pi^- \) and \( \pi^0\pi^0 \), \( \chi^2/\text{ndf} \approx 1.01 \); for \( \psi(3S) \rightarrow \Upsilon(1S)\pi^+\pi^- \) and \( \pi^0\pi^0 \), \( \chi^2/\text{ndf} \approx 0.97 \); for \( \psi(3S) \rightarrow \Upsilon(2S)(\pi^+\pi^- \) and \( \pi^0\pi^0 \), \( \chi^2/\text{ndf} \approx 0.54 \).

In Figs. 1 and 2 we show the fits to the experimental data on above indicated bottomia decays in the combined analysis with the processes \( \pi \pi \rightarrow \pi \pi, K \overline{K}, \eta \eta \) and the decays \( J/\psi \rightarrow \phi \pi \pi \), \( \phi K \overline{K} \). The dips in the energy dependence of di-pion spectra (Fig. 2, upper panel) are the result of a destructive interference between the \( \pi \pi \) scattering and \( K \overline{K} \rightarrow \pi \pi \) contributions to the final states of the decays \( \Upsilon(3S) \rightarrow \Upsilon(1S)(\pi^+\pi^- \) and \( \pi^0\pi^0 \).

The description of the processes \( \pi \pi \rightarrow \pi \pi, K \overline{K}, \eta \eta \) and charmonia decays and the resulting resonance parameters practically did not change when compared to the case without bottomia decays. The description of the re-
spective data and the resonance parameters can be found in Refs. [3, 4].

III. SUMMARY

The combined analysis was performed for data on isoscalar S-wave processes $\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta$ and on the decays of heavy quarkonia $J/\psi \rightarrow \phi(\pi\pi, K\bar{K})$, $\psi(2S) \rightarrow J/\psi \pi\pi$, $Y(2S) \rightarrow Y(1S)\pi\pi$, $Y(3S) \rightarrow Y(1S)\pi\pi$ and $Y(3S) \rightarrow Y(2S)\pi\pi$ from the ARGUS, Crystal Ball, CLEO, CUSB, DM2, Mark II, Mark III, and BES II collaborations. It was shown that in the final states of the bottomia decays the contribution of the coupled processes, e.g., $K\bar{K} \rightarrow \pi\pi$, is important even if these processes are energetically forbidden. This is in accordance with our previous conclusions on wide resonances [4, 9, 17]: when a wide resonance cannot decay into a channel, which opens above its pole mass and which is strongly coupled [e.g. the $f_0(500)$ and the $K\bar{K}$ channel], one should consider this resonance as a multi-channel state. E.g., on the basis of this consideration the new and natural mechanism of destructive interference in the decay $Y(3S) \rightarrow Y(1S)\pi\pi$ is indicated, which provides the two-humped shape of the di-pion mass distribution (Fig. 2).

The results of the analysis confirm all of our earlier conclusions on the scalar mesons [4]. Hence the considered bottomia decay data do not offer new insights into the nature of the scalar mesons, which were not already deduced in previous analyses of pseudoscalar meson scattering data.

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FIG. 1: Decay $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$.

FIG. 2: Decays $\Upsilon(3S) \to \Upsilon(nS)\pi\pi$, $n = 1, 2$. 