On topological bias of discrete sources in the gas of wormholes

A.A. Kirillov, E.P. Savelova, G.D. Shamshutdinova
Branch of Uljanovsk State University in Dimitrovgrad,
Dimitrova str 4., Dimitrovgrad, 433507, Russia

Abstract
The model of space in the form of a static gas of wormholes is considered. It is shown that the scattering on such a gas gives rise to the formation of a specific diffuse halo around every discrete source. Properties of the halo are determined by the distribution of wormholes in space and the halo has to be correlated with the distribution of dark matter. This allows to explain the absence of dark matter in intergalactic gas clouds. Numerical estimates for parameters of the gas of wormholes are also obtained.

1 Introduction
As it was shown recently all the variety of dark matter phenomena can be prescribed to the locally non-trivial topological structure of space or equivalently to the existence of the specific topological bias for sources (e.g., see Ref. [1]). The simplest model of such a space is given by a static gas of wormholes which was shown to give rise to the scale dependent renormalization of intensities of pure gravitational sources, i.e., to the origin of “dark matter halo” around any point-like gravitating mass [2]. In the present paper we show that together with the effect pointed above out the gas of wormholes gives rise to an analogous renormalization of all cosmic discrete sources of radiation. By other words any discrete source turns out to be surrounded with a diffuse halo. Moreover, by virtue of their common origin, such a halo should strongly correlate with the dark matter halo. The basic results of the present paper are the expressions (9) and (17) for the halo and numerical estimates for the astrophysical parameters of the gas of wormholes (mean density, the characteristic scale of the throat, relative brightness of the halo). When observing galaxies such a diffuse halo has a low surface brightness and is usually considered as a cosmic background which has different from radiation of the galaxy origin. We recall that usually, the observed diffuse halos in galaxies are attributed to reflection from dust, and the general diffuse component is assumed to originate from very fade and remote galaxies. This leads to an essential overestimation of the mas-to-luminosity ratio.
in galaxies $M/\ell$ and, therefore, this is usually interpreted as the presence of dark matter. As we show in the present paper such a ratio should be bigger $M/\ell \gg 1$ in smaller objects (e.g. dwarfs galaxies), while in larger objects (e.g., cluster size plasma clouds), due to their huge size (more than or of the order of the characteristic scale of the halo), radiation from the halo sums always up with radiation from the hot cloud itself and the parameter $M/\ell \sim 1$, e.g., dark matter is always absent [3].

We recall that nontrivial topological structure (or the topological bias) was to form during quantum stage of the evolution of the Universe, when space-time topology underwent fluctuations and the spacetime itself had the foam-like structure [4]. During the expansion the Universe cools down, quantum gravity processes stop and the topological structure tempers. There are no convincing theoretical arguments of why such a foam-like structure of space should decay upon the quantum period. Moreover, the presence of a considerable portion of dark energy (i.e. of an effective cosmological constant) in the present Universe and in the past, on the inflationary stage [5], may be considered as the very basic indication of a nontrivial topological structure of space [6].

Indeed, dark energy violates the weak energy condition $\varepsilon + 3p > 0$. Save speculative theories (or pure phenomenological models [5]), there is no matter which meets such a property. However in the presence of a non-trivial topology, vacuum polarization effects are known to give rise quite naturally to such a form of matter [7]. By other words, up to date the only rigorous way to introduce dark energy is to consider the vacuum polarization effects on manifolds of a non-trivial topological structure. It is necessary to point out that stability of wormholes requires the presence of matter violating the weak energy condition. In particular, wormholes are known to be possibly supported by the vacuum polarization induced by the wormholes themselves (e.g., see Ref. [8]).

2 The topological bias of sources

Consider a unite discrete source of radiation and the problem of scattering on the gas of wormholes. For the sake of simplicity we consider the case of a static gas, i.e., we assume that wormholes do not move in space. In this case the scattering is not accompanied with the frequency shift. We point also out that our results can easily be generalized to the case of the expanding Universe.

The problem of the modification of the Newton’s law in the presence of the gas of wormholes (i.e., origin of the topological bias) was solved recently in Ref. [2]. It turns out that in the case of radiation we can also speak of the topological bias of sources. We shall be interested in the behavior of the bias on scales $L \gg a$, where $a$ is a characteristic size of a wormhole throat and, therefore, in such an approximation the throat looks like a point object. Our aim is to find the Green function to the wave equation

$$\left(k^2 + \nabla^2\right) G(r, r_0) = 4\pi\delta(r - r_0)$$

in the presence of the wormhole. Recall that that equation describes the distri-
bution of radiation on the frequency \( \omega = kc \) (i.e., any component \( E, H \) or the vector potential \( A \)) produced by a unite stationary source \( j \sim e^{-i\omega \delta(r - r_0)} \). Indeed, in the Lorentz gauge \( \partial_i A^i = 0 \) the Maxwell equations take the form

\[
\frac{\partial^2}{\partial x^i \partial x^i} A^i = \frac{4\pi}{c} j^i, \tag{2}
\]

where \( j^i \) is 4–current. Using now the linear relation between the strength tensor and the vector potential \( F_{ik} = \partial_i A_k - \partial_k A_i \) we see that both \( E^i = F^0_i \) and \( H^i = \frac{1}{2} \delta^{ij} F_{jk} \) obey the above equation with the obvious replacement \( j^i \to J^i = \partial_i j_k - \partial_k j_i \). Note that in the case of an arbitrary source the vector potential (and respectively field strengths) can be expressed via the Green function as follows

\[
A^k = \frac{1}{c} \int G(r, r_0) j^k(r_0) d^3r_0. \tag{3}
\]

In the case of the flat space the (retarded) Green function is known to have the standard form \( G_0(R) = e^{ikR}/R \) (where \( R = |r - r_0| \)). Due to the conformal invariance of the Maxwell equations, the same function can be used for the class of conformally flat metrics.

The simplest wormhole can be constructed as follows. Consider two spheres \( S_{\pm} \) of the radius \( a \) and at the distance \( d = |R_{+} - R_{-}| \) between their centers. The interior of the spheres is removed and the surfaces of the spheres are glued together. Such spheres \( S_{\pm} \) can be considered as conjugated mirrors, so that while the incident signal falls on one mirror the reflected signal outgoes from the conjugated mirror. Thus, every wormhole is determined with a set of parameters \( a, R_{\pm}, \) and \( U \), where \( a \) is the radius of the throat, \( R_{\pm} \) stands for positions of centers of spheres (i.e. of throats), and \( U \) stands for the rotation matrix which defines the gluing procedure for the surfaces of the spheres. In the approximation used below the dependence on \( U \) disappears and will not be accounted for.

The exact solution of the scattering problem in the case of a unique wormhole is rather tedious and will be presented elsewhere. For astrophysical needs it is sufficient to consider diffraction effects in the geometrical optics limit \( ka \gg 1 \). According to the Huygens principle the scattering on a wormhole can be prescribed to the presence of secondary sources on throats which can be accounted for by additional terms

\[
G(R) = G_0(R) + u^+_R - u^+_A + u^-_R - u^-_A, \tag{4}
\]

where terms \( u^\pm_{A,R} \) describe absorption and reflection by throats \( S_{\pm} \) respectively. Every such term can be described by the surface integral (e.g., see the standard book [9])

\[
u^\pm_A (r, r_0) = \frac{k}{2\pi i} \int_{S^R_{\pm}} G_0(r', r_0) \frac{e^{ikR'}}{R'} df_{n} \tag{5}
\]

where \( R' = |r' - r| \), and \( S^R_{\pm} \) denotes the lighted and dark sides of throats.
respectively. If we neglect the throat size then the integration gives the square $\pi a^2$. Then we find terms which describe reflection and absorption of the signal and correspond to secondary sources placed on the throats

$$u_\pm = \frac{k}{2\pi i} \pi a^2 G_0 (R_\pm - r_0) G_0 (r - R_\pm),$$

$$u_\mp = \frac{k}{2\pi i} \pi a^2 G_0 (R_\mp - r_0) G_0 (r - R_\mp),$$

which are spherical waves from the two additional sources at the positions $\vec{R}_\pm$.

In this manner we see that the scattering on wormholes can be prescribed to additional sources, i.e., to the bias of the point source in (1) of the form

$$\delta (r - r_0) \rightarrow \delta (r - r_0) + b (r, r_0),$$

where $b (r, r_0)$ is the bias function. Indeed, in this case the Green function remains formally the same as in the flat space $G_0 (R)$, while the scattering on the topology is described by the bias of sources $J (r) \rightarrow J (r) + \int b (r, r') J (r') d^3 r'$. Thus, in the case of the static gas of wormholes the bias function takes the form

$$b (r, \omega) = \frac{\omega}{2\pi ic} \sum_m \pi a^2 \left( \frac{e^{ikR_m}}{R_m} - \frac{e^{ikr_0}}{r_0} \right) \left[ \delta (\vec{r} - \vec{R}_m) - \delta (\vec{r} - \vec{R}_m') \right],$$

where for the sake of simplicity we set $r_0 = 0$, and the index $m$ enumerates different wormholes.

Thus the presence of the gas of wormholes leads to the origin of a specific radiating halo around every point source. Due to the randomness of phases in multipliers in (9) such a halo has incoherent (or diffuse) nature.

Using the density of distribution of wormholes $F (R_-, R_+, a)$ and transforming sums in integrals the above equation can be cast to the form

$$b (r, \omega) = \frac{\omega}{2\pi ic} n \int \left( \frac{e^{ikR}}{R} - \frac{e^{ikr}}{r} \right) [g (R, r) + g (r, R)] d^3 R$$

where $g = \frac{1}{a^2} \int a^2 F (R_-, R_+, a) da$ and $n$ denotes the mean density of wormholes in space. In the case of a homogeneous distribution of wormholes the function $g$ depends only on $d = |R_+ - R_-|$. Then the bias function takes the simplest form for the Fourier transforms

$$b (k, \omega) = \frac{\omega}{2\pi c} n \frac{4\pi (g (k) - g (0))}{k^2 - \frac{\omega}{c^2} (\omega + io)^2},$$

where $g (k) = (2\pi)^{-3/2} \int g (r) e^{-ikr} d^3 r$ is the Fourier transform for the function $g (\vec{d})$.

\footnote{We note that the lighted side of a throat $S_\pm$ is turned by the matrix $U^{\pm 1}$ with respect to the dark side, so that in general the union $S^R \cup S^A \neq S$.}
We note that since we are working in the geometric optics limit, the Lorentz invariance and the standard dispersion relations for photons are not violated. Nevertheless, the violations surely take place at the scales, where the geometric optics does not work. Namely, at sufficiently large distances where topological defects start to show up (e.g., dark matter starts to show up at \( \lambda = 2\pi/k \gtrsim c/\omega \) of the order of a few Kpc \([1]\) which is much more, than any wavelength \( \lambda \sim c/\omega \) of a photon detected).

The violation of the Lorentz invariance has intensively been discussed in the literature \([10]\). All the existing estimates coincide by the order of magnitude and concern only of extremely small scales \([11,12]\). In particular in a recent paper \([12]\) it was established the well-known (e.g., see also Refs. \([11,13]\)) very strong limit for the "first" correction to the standard dispersion relation. Namely, if \( \omega^2 = k^2 (1 + kl_1 + k^2 l_2^2 + ...) \), then \( l_1 < L_{Pl} \), where \( L_{Pl} \) is the plankian length.

We point out that such a correction corresponds to the decomposition over the small parameter \( kl \ll 1 \), where \( l \) has the sense of a characteristic scale connected to wormholes \(^2\), which leads to a very strong limit on the existence of microscopic wormholes (i.e., we admit only the existence of wormholes at scales \( l_1 < L_{Pl} \), see also analogous estimates in Refs. \([11,13]\)).

However, (we hope that this should not be a surprise for readers) the same restriction can be used to estimate parameters of wormholes having the astrophysical meaning (scales). Indeed, in the presence of wormholes of astronomical scales (\( L \sim \) a few Kpc), the small parameter is already the ratio of the photon wavelength to the characteristic scale of a wormhole, which is the inverse parameter \( 1/(kL) \). Therefore, in the dispersion relation the first non-trivial corrections should have the form \( \omega^2 = k^2 \left( 1 + \frac{1}{kl_1} + \frac{1}{k^2 l_2^2} + ... \right) \). Then, if we use the above restrictions on the absolute value of the existing correction to the dispersion relation (e.g., \( k \sim 10^6 \) cm\(^{-1}\) and \( kl_1 < 10^{-27} \)), then we find that the characteristic scales of astrophysically significant wormholes should obey the "restrictions" \( L_1 > (1/5) Kpc \). Those are just the scales at which dark matter effects start to display themselves and according to our interpretation of dark matter effects \([1]\) the density of wormholes should achieve the value \( nL_1^3 \sim 1 \).

We also point out to the fact that at galaxy scales the Lorentz invariance violates also due to relativistic gravitational effects (i.e., due to general relativity). By other words to essentially improve the above restrictions on the dispersion relations violation is hardly possible.

As it was pointed out above the reason of the absence of the Lorentz invariance violation, e.g., in the optic range, is very simple. For rather short wave-length the forming halo secondary sources (cf. expression \( (9) \)) have random phases and, therefore, do not contribute to the amplitude of a signal. By other words the halo carries the diffuse character. In such a case it is more correct to consider the expression for the energy flux which comes to the point \( r \) which for the diffuse field becomes an additive quantity.

\(^2\)Recall that wormholes relate to the three basic parameters. Those are the density of wormholes \( n^{-1/3} \), the mean throat size \( a \), and the mean distance between throats \( d = |R_+ - R_-| \).
3 The diffuse halo

Consider now the renormalization of the intensity of radiation. By virtue of the diffuse character of the current the intensity of radiation is determined by the square of the current, e.g.,

\[ I(R)I^*(R') = |I(R)|^2 \delta(R - R'), \]  

where the averaging out should be thought as either over the period of the field \( T = 2\pi/\omega \), or over the random phases. When we do not account for the scattering on topology, i.e., in the ordinary flat space, the intensity of radiation \( W = E^2 + H^2 / 8\pi \) is determined by the intensity of the current \( |I|^2 \) as

\[ W(r) \sim \int |I(r')|^2 d^3r' \]  

and, in particular, for a point source \( |I(r)|^2 = |I_0|^2 \delta(r - r_0) \) the intensity of the energy flow is

\[ W(r) \sim \int |I(r')|^2 d^3r' \]  

The scattering on wormholes leads to the replacement \( G_0 \rightarrow G \) which effectively can be described as the origin of the bias \( (8), (9) \) or as the origin of an additional halo which has the property pointed out to be delta-correlated and which leads to a renormalization of the current intensity \( |I_0|^2 \rightarrow |\tilde{I}(r)|^2 \). We shall use the Green function, i.e., we consider the sum

\[ G(r)G^*(r) = |G(r)|^2 = |G_0(r)|^2 + \sum_{s=A, R, p=\pm} |u_p|^2 \]  

which gives

\[ |G(r)|^2 = \frac{1}{r^2} + \frac{\omega^2}{4\pi^2 c^2} \sum_m \pi^2 a_m^4 \left( \frac{1}{|R_-|^2} + \frac{1}{|R_+|^2} \right) \left( \frac{1}{|R_- - r|^2} + \frac{1}{|R_+ - r|^2} \right). \]  

In the above expression due to the randomness of phases the intersection terms are omitted. Then the above expression defines the bias for the intensity of a unite source in the form

\[ |G(r)|^2 = \frac{1}{r^2} + \int \frac{\tilde{b}^2(R)}{|R - r|^4} d^3r \]  

where

\[ \tilde{b}^2(R) = \frac{\omega^2 n}{c^2} \int \left( \frac{1}{R^2} + \frac{1}{X^2} \right) (\tilde{g}(R, X) + \tilde{g}(X, R)) d^3X \]  

and \( \tilde{g}(R, X) = \frac{1}{n} \int a^4 F(R_-, R_+, a) da \). For an isotropic distribution of wormholes \( \tilde{g} = \tilde{g}(|R_+ - R_-|) \) and therefore we find the bias function in the form

\[ \tilde{b}^2(R) = \frac{k^2}{2} n \int \left( \frac{1}{R^2} + \frac{1}{|X + R|^2} \right) \tilde{g}(X) d^3X. \]  

In this manner the relation between the intensity of an actual \( |I_0(r)|^2 \) and the apparent \( |I(r)|^2 \) currents (or, which is equivalent, between the actual \( \ell_0 \) and
observed $\ell$ luminosity) is determined by the distribution of wormholes in space
and is given by the expression

$$|\tilde{I}|^2(r) = |I_0|^2(r) + \int \tilde{b}^2(r-r')|I_0|^2(r')d^3r.$$  \hspace{1cm} (18)

4 Estimates

Consider now some simplest estimates. In order to find estimates for the
renormalization of the surface brightness of a source we consider the case when
all wormholes have the same value for $d = |\vec{R}_- - \vec{R}_+| = r_0$. In this case we can
take $\tilde{g}(X) = \frac{r_0}{4\pi r_0^3}\delta(X - r_0)$ and for a point source we get the bias in the form

$$\tilde{b}^2(R) = \frac{k^2}{2n a R^2} \left(1 + \frac{R}{2r_0} \ln \left|\frac{R + r_0}{R - r_0}\right|\right).$$  \hspace{1cm} (19)

We note that the characteristic behavior $\tilde{b}^2 \sim 1/R^2$ of the halo density is the
specific attribute of the point-like structure of a source. In the case of actual
sources such a halo acquires the cored character $\tilde{b}^2 \sim \tilde{b}^2(l) \sim \text{const}$, where $l$
corresponds to the linear size of the source.

To get the estimate to the number density of wormholes is rather straight-
forward. First wormholes appear at scales when dark matter effects start to
display themselves, i.e., at scales of the order $L \sim (1 \div 5)Kpc$, which gives in
that range the number density

$$n \sim (3 \div 0.024) \times 10^{-66}\text{cm}^{-3}.$$  \hspace{1cm} (20)

The characteristic size of throats can be estimated as follows \[2, 6\]. As it was
pointed out above in the case of a homogeneous distribution of wormholes the value of $a$
defines the amount of dark energy in the Universe. Indeed, consider
a single wormhole in the flat (Minkowsky) space. Then the metric can be taken
in the form (e.g., see Ref. \[2\])

$$ds^2 = dt^2 - f^2(r)(dr^2 + r^2 \sin^2 \vartheta d\varphi^2 + r^2 d\vartheta^2),$$  \hspace{1cm} (21)

where $f(r) = 1 + \theta(a - r)(\frac{r^2}{a^2} - 1)$ and $\theta(x)$ is the step function. We can replace
$f(r)$ with any smooth function, this however will not change the subsequent
estimates. Both regions $r > a$ and $r < a$ represent portions of the ordinary
flat Minkowsky space and therefore the curvature is $R^k_i = 0$. However on the
boundary $r = a$ it has the singularity which defines the scalar curvature as $R = -8\pi G T = \frac{2}{a} \delta(r - a)$ where $T$ stands for the trace of the stress energy tensor
which one has to add to the Einstein equations to support such a wormhole. It
is clear that such a source violates the weak energy condition and, therefore,
it reproduces the form of dark energy (i.e., $T = \varepsilon + 3p < 0$). If the density
of such sources (and respectively the density of wormholes) is sufficiently high,
then this results in an acceleration of the scale factor for the Friedmann space as \( t^\alpha \) with \( \alpha = \frac{2\varepsilon}{3(\varepsilon+p)} > 1 \), i.e., see Refs. [5].

Every wormhole gives contribution \( \int T r^2 dr \sim \bar{a} \) to the dark energy, while the dark energy density is \( \varepsilon_{DE} \sim (8\pi G)^{-1} n \bar{a} \). Since the density of dark energy has the order \( \varepsilon_{DE} \sim 0.75\varepsilon_0 \), where \( \varepsilon_0 \) is the critical density, then we immediately find the estimate \( \bar{a} \sim (1 \div 125) \times 10^{-3} R_\odot \), where \( R_\odot \) is the Solar radius. Now by means of use of the expression (19) we find the estimate for the relative brightness of the halo

\[
\ell/\ell_0 \sim k^2/2m^4l \sim 4l/R_\odot \left( \frac{k}{k_0} \right)^2 [1 \div 2 \times 10^6] \times 10^{-14}. \tag{22}
\]

Here \( l \) is the linear size of the source around which the diffuse halo forms, and \( k_0 \) defines the wavelength \( \lambda_{max} \) which corresponds to the temperature \( T_\odot = 6 \times 10^3K \). It is clear that the relative brightness of the halo is small \( \ell/\ell_0 \ll 1 \) and it reaches the order of the unity only for sufficiently extended objects of the characteristic size \( [0.5 \times 10^{-6} \div 1] \times 10^{14} R_\odot \). We also point out that outside the radiating region the halo brightness decays according to (19) as \( \sim 1/R^2 \).

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