EFFECTIVE NUCLEON-NUCLEON INTERACTION AND
FERMI LIQUID THEORY

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We present two novel relations between the quasiparticle interaction in nuclear
matter and the unique low momentum nucleon-nucleon interaction in vacuum.
These relations provide two independent constraints on the Fermi liquid parameters
of nuclear matter. Moreover, the new constraints define two combinations of Fermi
liquid parameters, which are invariant under the renormalization group flow in the
particle-hole channels. Using empirical values for the spin-independent Fermi liquid
parameters, we are able to compute the major spin-dependent ones by imposing
the new constraints as well as the Pauli principle sum rules.

The work we present at this conference was motivated by the results of Bogner
et al., who have constructed a low momentum nucleon-nucleon interaction \( V_{\text{low } k} \)
using traditional nuclear effective interaction methods. Starting from a realistic
nucleon-nucleon interaction, such as the Paris, Bonn-A, and Argonne potentials
or a chiral effective field theory model, they integrate out relative momenta larger
than a cutoff \( \Lambda \) in the sense of the renormalization group (RG). The hard momenta
renormalize \( V_{\text{low } k} \), such that the original low momentum half-on-shell \( T \)
matrix, i.e. phase shifts and low momentum components of the scattering wave functions,
as well as bound state properties remain unchanged.

Diagrammatically \( V_{\text{low } k} \) sums all ladders with bare potential vertices and inter-
mediate momenta greater than the cutoff. Subsequently, the energy dependence of
the ladder sum is removed by adding the folded diagram corrections. For the case
of the nonrelativistic two-nucleon problem, the nuclear model space effective inter-
action methods are equivalent to solving a RG flow equation obtained by requiring
\( d T(k', k; k^2)/d \Lambda = 0 \) with the various bare potentials as large \( \Lambda \) initial conditions.
The results of the RG decimation to \( V_{\text{low } k} \) are shown in Fig. (1). For \( \Lambda \ll 2 \) fm
all model potentials flow to a unique low momentum interaction \( V_{\text{low } k} \), which to a
good approximation and for reasonable values of the cutoff is merely shifted by a
constant compared to the bare potentials. This constant shift in momentum space,
corresponding to a smeared delta function in coordinate space, is of such strength
that it removes the experimentally undetermined core from the bare models. For
\( \Lambda > m_\pi \), \( V_{\text{low } k} \) can be projected on pion exchange terms, which determine the
low momentum potential shape, plus contact terms, which account for the short
range part of the interaction. In this setup, the non-pionic contributions of the bare
potentials flow to “fixed point” values.

In accordance with the ideas of effective field theories, the results are insensitive
to the cutoff, provided there is a separation of scales. The insensitivity can best
be seen in the plateau of the RG flow of \( V_{\text{low } k}(0,0; \Lambda) \), Fig. (1). The lowest order
approximation to $V_{\text{low } k}(0, 0)$ is given by the effective energy dependent potential $V_{\text{eff}}(0, 0; p^2 = 0)$ at zero energy. For small $\Lambda$, the RG equation for energy dependent effective potentials of Birse et al.\textsuperscript{4} is solved by $V_{\text{eff}}(0, 0; p^2 = 0) = (1/a - 2\Lambda/\pi)^{-1}$. This is plotted as solid line in Fig. (1) and agrees well with $V_{\text{low } k}$.

In this work we will take $V_{\text{low } k}$ as dynamical input to Fermi liquid theory. This results in an intriguing relation between the effective interactions in vacuum and in medium. It is even more so, as the role of the local repulsive core in the bare interactions is taken over by non-local parts, which in turn contribute to the non-locality of the quasiparticle interaction in Fermi liquid theory. For $\Lambda \sim k_F$ the energy independent $V_{\text{low } k}$ includes the effects of the repulsive core, is generally smooth, and can therefore be used as a $G$ matrix. This is approximate from a Brueckner theory standpoint, since self-energy insertions and the dependence on the center of mass momentum are ignored. However, it is expected that the effects of the self-energy insertions are small. The novel relations have as input the s-wave matrix elements $V_{\text{low } k}(0, 0)$ at $\Lambda = k_F$, i.e. in the weakly $\Lambda$ dependent region. Similar energy independent effective potentials are in agreement with $V_{\text{low } k}$ up to a few percent.\textsuperscript{5, 6} As the Fermi liquid parameters are fixed points under the RG flow towards the Fermi surface, the constraints relate $V_{\text{low } k}$ to these fixed points. At low temperatures, strongly interacting normal Fermi systems can be described by weakly interacting quasiparticles and quasiholes. As in any effective theory, the quasiparticle interaction is only restricted by the symmetries. For nuclear matter these are rotations in space, spin, and isospin, and the quasiparticle interaction $\mathcal{F}$ in units of the density of states at the Fermi surface is given by

$$\mathcal{F} = \sum_l \left( F_l + F'_l \tau \cdot \tau' + G_l \sigma \cdot \sigma' + G'_l \tau \cdot \tau' \sigma \cdot \sigma' \right) P_l(\cos \theta) + \text{tensor} + \mathcal{O}(A^{-1/3}),$$

where $\theta$ denotes the angle between the quasiparticles $p$ and $p'$ on the Fermi surface. The Fermi liquid parameters of this effective theory are determined by experiment. On a microscopic level, the quasiparticle interaction includes all diagrams that are quasiparticle-quasihole irreducible in the zero sound channel.
In order to satisfy the Pauli principle, one needs an integral equation, which generates an infinite set of diagrams for the quasiparticle interaction. This is achieved by the induced interaction of Babu and Brown.\textsuperscript{7,8} The induced interaction in combination with the antisymmetrized bare interaction as driving term generates the complete particle-hole parquet, i.e. all fermion planar diagrams without the particle-particle channel.\textsuperscript{9} This is the minimum set of diagrams mandated by the Pauli principle. The induced interaction incorporates the response of the system to the presence of the quasiparticle. We approximate the driving term, which in principle includes all particle-hole irreducible diagrams, e.g. particle-particle ladders and non-planar diagrams, by $z^2 V_{\text{low } k}$ with a density dependent cutoff $\Lambda = k_F$. Given the driving term, the induced interaction is exact for $p = p'$. Solving the integral equations for the scattering amplitude and the induced interaction in this limit simultaneously leads to the two constraints, in units where the nucleon mass $m = 1$

\begin{equation}
\sum_l \left\{ 2F_l - \frac{F_l}{1 + F_l/(2l+1)} + 2F'_l - \frac{F'_l}{1 + F'_l/(2l+1)} - 3(2G_l - \frac{G_l}{1 + G_l/(2l+1)}) \right\} = z^2 \frac{16k_F(1+F_l/3)}{\pi} V_{\text{low } k}(0,0; \Lambda = k_F, l 0 S_0) \tag{2}
\end{equation}

\begin{equation}
\sum_l \left\{ 2F_l - \frac{F_l}{1 + F_l/(2l+1)} - 3(2F'_l - \frac{F'_l}{1 + F'_l/(2l+1)}) + 2G_l - \frac{G_l}{1 + G_l/(2l+1)} \right\} = z^2 \frac{16k_F(1+F'_l/3)}{\pi} V_{\text{low } k}(0,0; \Lambda = k_F, l 0 S_0) \tag{3}
\end{equation}

In analogy to Fermi liquid theory, where one separates the quasiparticle contribution of the full Green’s function from the multi-pair background, one can separate the soft modes of the quasiparticle-quasihole propagators from the hard ones. We show that in this setup and neglecting the flow in the BCS channel, the two constraints as well as the two Pauli principle sum rules are RG invariant under the flow to the Fermi surface.\textsuperscript{1} The induced interaction permits the calculation of the beta function for the quasiparticle scattering amplitude and interaction to all orders in the limit $p \to p'$ and thus considerably improves one-loop approximations thereof. To lowest order, our RG equations agree with the perturbative one-loop flow of Dupuis.\textsuperscript{10}

Are these constraints consistent with phenomenological values for the Fermi liquid parameters? For this purpose, we approximate the Legendre series with its $l = 0, 1$ terms. We use phenomenological values for the spin-independent parameters and then compute the spin dependent ones imposing the Pauli principle sum rules as well as the constraints. From the incompressibility, the effective mass, the symmetry energy, the anomalous orbital gyromagnetic ratio, and a self-consistent calculation of the nucleon spectral function, one extracts

\begin{equation}
F_0 = -0.27 \quad F_1 = -0.85 \quad F'_0 = 0.71 \quad F'_1 = 0.14 \quad z = 0.8. \tag{4}
\end{equation}

In Fig. (2) we show the solution for the spin-dependent Fermi liquid parameters without tensor interactions. We thus find

\begin{equation}
G_0 = 0.15 \pm 0.3 \quad G_1 = 0.45 \pm 0.3 \quad G'_0 = 1.0 \pm 0.2 \quad G'_1 = 0 \pm 0.2. \tag{5}
\end{equation}
The value of $G_0'$ is to be compared to the experimental constraint from the Gamow-Teller resonance, after correcting for the effects of the Delta-hole polarization $G_0' = 1.0$. Tensor interactions are included easily into the constraints. A simple argument however shows that the tensor parameters have to be treated self-consistently within the induced interaction. The one-bubble recoupling of $G_0'$ to tensor interactions reduces the tensor parameters significantly, in agreement with the results of Dickhoff et al.\textsuperscript{11}

A solution of the flow equations, as presented by Metzner for the Hubbard model at this conference, would provide the scattering amplitude also for non-forward scattering, which is of high interest for the calculation of superfluid gaps and transport processes, e.g. in neutron star interiors.

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