Learning Simpler Language Models with the Differential State Framework

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Abstract

Learning useful information across long time lags is a critical and difficult problem for temporal neural models in tasks such as language modeling. Existing architectures that address the issue are often complex and costly to train. The Differential State Framework (DSF) is a simple and high-performing design that unifies previously introduced gated neural models. DSF models maintain longer-term memory by learning to interpolate between a fast-changing data-driven representation and a slowly changing, implicitly stable state. This requires hardly any more parameters than a classical, simple recurrent network. Within the DSF framework, a new architecture is presented, the Delta-RNN. In language modeling at the word and character levels, the Delta-RNN outperforms popular complex architectures, such as the Long Short-Term Memory (LSTM) and the Gated Recurrent Unit (GRU), and, when regularized, performs comparably to several state-of-the-art baselines. At the subword level, the Delta-RNN’s performance is comparable to that of complex gated architectures.

1 Introduction

Recurrent neural networks are increasingly popular models for sequential data. The simple recurrent neural network (RNN) architecture [Elman, 1990] is, however, not suitable for capturing longer-distance dependencies. Architectures that address this shortcoming include the Long Short-Term Memory (LSTM, Hochreiter and Schmidhuber 1997a), the Gated Recurrent Unit (GRU, Chung et al. 2014, 2015), and the structurally constrained recurrent network (SCRN, Mikolov et al. 2014). While these can capture
some longer-term patterns (20 to 50 words), their structural complexity makes it difficult to understand what is going on inside. One exception is the SCRN architecture, which is by design simple to understand. It shows that the memory acquired by complex LSTM models on language tasks does correlate strongly with simple weighted bags-of-words. This demystifies the abilities of the LSTM model to a degree: while some authors have suggested that the LSTM understands the language and even the thoughts being expressed in sentences (Choudhury, 2015), it is arguable whether this could be said about a model that performs equally well and is based on representations that are essentially equivalent to a bag of words.

One property of recurrent architectures that allows for the formation of longer-term memory is the self-connectedness of the basic units: this is most explicitly shown in the SCRN model, where one hidden layer contains neurons that do not have other recurrent connections except to themselves. Still, this architecture has several drawbacks: one has to choose the size of the fully connected and self-connected recurrent layers, and the model is not capable of modeling non-linearities in the longer-term memory component.

In this work, we aim to increase representational efficiency, i.e., the ratio of performance to acquired parameters. We simplify the model architecture further and develop several variants under the Differential State Framework, where the hidden layer state of the next time step is a function of its current state and the delta change computed by the model. We do not present the Differential State Framework as a model of human memory for language. However, we point out its conceptual origins in Surprisal Theory (Boston et al., 2008; Hale, 2001; Levy, 2008), which posits that the human language processor develops complex expectations of future words, phrases, and syntactic choices, and that these expectations and deviations from them (surprisal) guide language processing, e.g., in reading comprehension. How complex the models are (in the human language processor) that form the expectation is an open question. The cognitive literature has approached this with existing parsing algorithms, probabilistic context-free grammars, or n-gram language models. We take a connectionist perspective. The Differential State Framework proposes to not just generatively develop expectations and compare them with actual state changes caused by observing new input; it explicitly maintains gates as a form of high-level error correction and interpolation. An instantiation, the Delta-RNN, will be evaluated as a language model, and we will not attempt to simulate human performance such as in situations with garden-path sentences that need to be reanalyzed because of costly initial mis-analysis.

2 The Differential State Framework and the Delta-RNN

In this section, we will describe the proposed Differential State Framework (DSF) as well as several concrete implementations one can derive from it.
2.1 General Framework

The most general formulation of the architectures that fall under DSF distinguishes two forms of the hidden state. The first is a fast state, which is generally a function of the data at the current time-step and a filtration (or summary function of past states). The second is a slow state, or data-independent state. This concept can be specifically viewed as a composition of two general functions, formally defined as follows:

\[ h_t = q_\Theta(x_t, M_{t-1}) = f_\psi[g_\Theta(x_t, M_{t-1}), M_{t-1}] \]  

where \( \Theta = \{\theta, \psi\} \) are the parameters of the state-machine and \( M_{t-1} \) is the previous latent information the model is conditioned on. In the case of most gated architectures, \( M_{t-1} = h_{t-1} \), but in some others, as in the SCRN or the LSTM, \( M_{t-1} = \{h_{t-1}, c_{t-1}\} \) or could even include information such as de-coupled memory, and in general will be updated as symbols are iteratively processed. We define \( g_\Theta(\cdot) \) to be any, possibly complicated, function that maps the previous hidden state and the currently encountered data point (e.g. a word, subword, or character token) to a real-valued vector of fixed dimensions using parameters \( \theta \). \( f_\psi(\cdot) \), on the other hand, is defined to be the outer function that uses parameters \( \psi \) to integrate the fast-state, as calculated by \( g_\Theta(\cdot) \), and the slowly-moving, currently un-transformed state \( h_{t-1} \). In the sub-sections that follow, we will describe simple formulations of these two core functions and, later in Section 3, we will show how currently popular architectures, like the LSTM and various simplifications, are instantiations of this framework. The specific structure of Equation [1] was chosen to highlight that we hypothesize the reason behind the success of gated neural architectures is largely because they have been treating the next-step prediction tasks, like language modeling, as an interaction between two functions. One inner function focuses on integrating observed samples with a current filtration to create a new data-dependent hidden representation (or state “proposal”) while an outer function focuses on computing the difference, or “delta”, between the impression of the sub-sequence observed so far (i.e., \( h_{t-1} \)) with the newly formed impression. For example, as a sentence is iteratively processed, there might not be much new information (or “suprasal”) in a token’s mapped hidden representation (especially if it is a frequently encountered token), thus requiring less change to the iteratively inferred global representation of the sentence. However, encountering a new or rare token (especially an unexpected one) might bias the outer function to allow the newly formed hidden impression to more

\[ ^1 \text{c}_t \text{ refers to the “cell-state” as in (Hochreiter and Schmidhuber, 1997b).} \]

\[ ^2 \text{One way to extract a “sentence representation” from a temporal neural language model would be to simply to take the last hidden state calculated upon reaching a symbol such as punctuation (e.g., period or exclamation point). This is sometimes referred to as encoding variable-length sentences or paragraphs to a real-valued vector of fixed dimensionality.} \]
strongly influence the overall impression of the sentence, which will be useful when predicting what token/symbol will come next. In Section 5 we will present a small demonstration using one of the trained word-models to illustrate the intuition just described.

In the sub-sections to follow, we will describe the ways we chose to formulate $g_\theta(\cdot)$ and $f_\psi(\cdot)$ in the experiments of our paper. The process we followed for developing the concrete implementations of $g_\theta(\cdot)$ and $f_\psi(\cdot)$ involved starting from the simplest possible form using the fewest (if any) possible parameters to compose each function and testing it in preliminary experiments to verify its usefulness.

It is important to note that Equation 1 is still general enough to allow for future design of more clever or efficient functions that might improve the performance and long-term memory capabilities of the framework. More importantly, one might view the parameters $\psi$ that $f_\psi(\cdot)$ uses as possibly encapsulating structures that can be used to store explicit memory-vectors, as is the case in stacked-based RNNs (Das et al., 1992; Joulin and Mikolov, 2015) or linked-list-based RNNs (Joulin and Mikolov, 2015).

2.2 Forms of the Outer Function

Keeping $g_\theta(\cdot)$ as general as possible, here we will describe several ways one could design $f_\psi(\cdot)$, the function meant to decide how new and old hidden representations will be combined at each time step. We will strive to introduce as few additional parameters as necessary and experimental results will confirm the effectiveness of our simple designs.

One form that $f_\psi(\cdot)$ could take is a simple weighted summation, as follows:

$$h_t = f_\psi\left[ g_\theta(x_t, h_{t-1}), h_{t-1} \right] = \Phi(\gamma g_\theta(x_t, h_{t-1}) + \beta h_{t-1})$$

where $\Phi(\cdot)$ is an element-wise activation applied to the final summation and $\gamma$ and $\beta$ are bias vectors meant to weight the fast and slow states respectively. In Equation 2 if $\gamma = \beta = 1$, no additional parameters have been introduced making the outer function simply a rigid summation operator followed by a non-linearity. However, one will notice that $h_{t-1}$ is transmitted across a set of fixed identity connections in addition to being transformed by $g_\theta(\cdot)$.

While $\gamma$ and $\beta$ could be chosen to be hyper-parameters and tuned externally (as sort of per-dimension scalar multipliers), it might prove to be more effective to allow the model to learn these coefficients. If we introduce a vector of parameters $r$, we can choose the fast and slow weights to be $\gamma = (1 - r)$ and $\beta = (r)$, facilitating simple interpolation. Adding these negligibly few additional parameters to compose an interpolation mechanism yields the state-model:

$$h_t = \Phi((1 - r) \otimes g_\theta(x_t, h_{t-1}) + r \otimes h_{t-1}).$$
Note that we define $\otimes$ to be the Hadamard product. Incorporating this interpolation mechanism can be interpreted as giving the Differential State Framework model a flexible mechanism for mixing various dimensions of its longer-term memory with its more localized memory. Interpolation, especially through a simple gating mechanism, can be an effective way to allow the model to learn how to turn on/off latent dimensions, potentially yielding improved generalization performance, as was empirically shown by Serban et al. (2016).

Beyond fixing $r$ to some vector of pre-initialized values, there are two simple ways to parametrize $r$:

$$r = 1/(1 + \exp(-b_r)),$$  \hspace{1cm} (4) \\
$$r = 1/(1 + \exp(-[Wx_t + b_r]))$$  \hspace{1cm} (5)

where both forms only introduce an additional set of learnable bias parameters, however Equation 5 allows the data at time step $t$ to interact with the gate and thus takes into account additional information from the input distribution when mixing stable and local states together. Unlike Serban et al. (2016), we constrain the rates to lie in the range $[0, 1]$ by using the logistic link function, $\sigma(v) = 1/(1 + \exp(-v))$, which will transform the biases into rates much like the rates of the SCRN. We crucially choose to share $W$ in this particular mechanism for two reasons: 1) we avoid adding yet another matrix of input to hidden parameters and, much to our advantage, reuse the computation of the linear pre-activation term $Wx_t$, and 2) additionally coupling the data pre-activation to the gating mechanism will serve as further regularization of the input-to-hidden parameters (by restricting the amount of learnable parameters, much as in classical autoencoders). Two error signals, $\frac{\partial r}{\partial W}$ and $\frac{\partial z_t}{\partial W}$, now take part in the calculation of the partial derivative $\frac{\partial L(y_t, x_{t+1})}{\partial W}$ ($y_t$ is the output of the model at $t$).

Figure 1 depicts the architecture using the simple late-integration mechanism.

### 2.3 Forms of the Inner Function – Instantiating the Delta-RNN

When a concrete form of the inner function $g_\theta(\cdot)$ is chosen, we can fully specify the Differential State Framework. We will also show, in Section 3, how many other commonly-used RNN architectures can, in fact, be treated as special cases of this general framework defined under Equation 1.

Starting from Equation 2 if we fix $\gamma = 1$ and $\beta = 0$, we can recover the classical Elman RNN, where $g_\theta(x_t, h_{t-1})$ is a linear combination of the projection of the current data point and the projection of the previous hidden state, followed by a non-linearity $\phi(\cdot)$.

However, if we also set $\beta = 1$, we obtain a naive way to compute a delta change of states. Specifically, the simple-RNN’s hidden state, where $\Phi(v) = v$ (the identity
function), is:

\[
\begin{align*}
    h_t &= \gamma \otimes g_\theta(x_t, h_{t-1}) + \beta \otimes h_{t-1} \\
    h_t &= 1 \otimes \phi(Vh_{t-1} + Wx_t + b) + 0 \otimes h_{t-1} \\
    h_t &= \phi(Vh_{t-1} + Wx_t + b)
\end{align*}
\]

(6)

where \(h_t\) is the hidden layer state at time \(t\), \(x_t\) is the input vector, and \(\theta = \{W, V\}\) contains the weight matrices. In contrast, the simple Delta-RNN, where instead \(\phi(v) = v\), we have:

\[
    h_t = \Phi(Vh_{t-1} + Wx_t + b + h_{t-1}).
\]

(7)

Thus, the state can be implicitly stable, assuming \(W\) and \(V\) are initialized with small values and \(\phi(\cdot)\) allows this by being partially linear. For example we can choose \(\phi(\cdot)\) to be the linear rectifier (or initialize the model so to start out in the linear regime of the hyperbolic tangent). In this case, the simple Delta-RNN does not need to learn anything to maintain the state constant over time.

Preliminary experimentation with this simple form (Equation 7) often yielded unsatisfactory performance. This further motivated the development of the simple interpolation mechanism presented in Equation 3. However, depending on how one chooses the non-linearities, \(\phi(\cdot)\) and \(\Phi(\cdot)\), one can create different types of interpolation. Using an Elman RNN for \(g_\theta(x_t, h_{t-1})\) as in Equation 6, substituting into Equation 3 can create what we propose as the “late-integration” state model:

\[
\begin{align*}
    z_t &= g_\theta(x_t, h_{t-1}) \\
    &= \phi(Vh_{t-1} + Wx_t + b), \text{ and,} \\
    h_t &= \Phi((1 - r) \otimes z_t + r \otimes h_{t-1}).
\end{align*}
\]

(8)

where \(\Phi(\cdot)\) could be any choice of activation function, including the identity function. This form of interpolation allows for a more direct error propagation pathway since gradient information, once transmitted through the interpolation gate, has two pathways: through the non-linearity of the local state (through \(g_\theta(x_t, h_{t-1})\)) and the pathway composed of implicit identity connections. \(\text{3}\)

When using a simple Elman RNN, we have essentially described a first-order Delta-RNN. However, historically, second-order recurrent neural architectures have been shown to be powerful models in tasks such as grammatical inference (Giles et al., 1991) and noisy time-series prediction (Giles et al., 2001) as well as incredibly useful in rule-extraction when treated as finite-state automata (Giles et al., 1992; Goudreau et al., 1994).

3Late-integration might remind the reader of the phrase “late fusion”, as in the context of Wang and Cho (2015). However, Wang and Cho was focused on merging the information from an external bag-of-words context vector with the standard cell state of the LSTM.
Very recently, Wu et al. (2016) showed that the gating effect between the state-driven component and data-driven components of a layer’s pre-activations facilitated better propagation of gradient signals as opposed to the usual linear combination. A second-order version of \( g_\theta(x_t, h_{t-1}) \) would be highly desirable, not only because it further mitigates the vanishing gradient problem that plagues back-propagation through time (used in calculating parameter gradients of neural architectures), but because the form introduces negligibly few additional parameters. We do note that the second-order form we use, like in Wu et al. (2016), is a rank-1 matrix approximation of the actual tensor used in Giles et al. (1992); Goudreau et al. (1994).

We can take the late-integration model, Equation 9, and replace, similar to Giles et al. (1991), \( z_t \) with:

\[
    z_t = \phi (V h_{t-1} \otimes W x_t + b) \tag{10}
\]

or a more general form (Wu et al. 2016):

\[
    d_1^1 = \alpha \otimes V_d h_{t-1} \otimes W x_t \\
    d_1^2 = \beta_1 \otimes V_d h_{t-1} + \beta_2 \otimes W x_t \\
    z_t = \phi (d_1^1 + d_1^2 + b), \tag{11}
\]

where we note that \( z_t \) can be a function of any arbitrary incoming set of information signals that are gated by the last known state. The Delta-RNN will ultimately combine this data-driven signal \( z_t \) with its slow-moving state. More importantly, observe that even in the most general form (Equation 11), only a few further bias vector parameters, \( \alpha, \beta_1, \) and \( \beta_2 \) are required.

Assuming a single hidden layer language model, with \( H \) hidden units and \( V \) input units (where \( V \) corresponds to the cardinality of the symbol dictionary), a full late-integration Delta-RNN that employs a second-order \( g_\theta(x_t, h_{t-1}) \) (Equation 11), has
only \((H \ast H) + 2(H \ast V) + 5H + V\) parameters\(^4\), which is only slightly larger than a classical RNN with only \((H \ast H) + 2(H \ast V) + H + V\) parameters. This stands in stark contrast to the sheer number of parameters required to train commonly-used complex architectures such as the LSTM (with peephole connections), with \((4(H \ast H) + 8(H \ast V) + 4H + V)\) parameters, and the GRU, with \((3(H \ast H) + 4(H \ast V) + 3H + V)\) parameters.

### 2.4 Regularizing the Delta-RNN

Regularization is often important when training large, over-parametrized models. To control for overfitting, approaches range from structural modifications to impositions of priors over parameters (Neal, 2012). Commonly employed modern approaches include drop-out (Srivastava et al., 2014) and variations (Gal and Ghahramani, 2016) or mechanisms to control for internal covariate drift, such as Batch Normalization (Ioffe and Szegedy, 2015) for large feedforward architectures. In this paper, we investigate the effect that drop-out will have on the Delta-RNN’s performance.\(^5\)

To introduce simple (non-recurrent) drop-out to the framework, our preliminary experiments uncovered that drop-out was most effective when applied to the inner function \(g(\cdot)\) as opposed to the outer function’s computed delta-state. For the full Delta-RNN, under drop-out probability \(p_{\text{drop}}\), this would lead to the following modification:

\[
h_t = \Phi((1 - r) \otimes \text{DROP}(g_\Theta(x_t, h_{t-1}), p_{\text{drop}}) + r \otimes h_{t-1}). \tag{12}
\]

\(\text{DROP}(x, p_{\text{drop}}) = x \otimes (\sim B(1, p_{\text{drop}}))\) is the drop-out operator which masks its input argument with a binary vector sampled from \(H\) independent Bernoulli distributions.

### 2.5 Learning under the Delta-RNN

Let \(w_1, \ldots, w_N\) be a variable-length sequence of \(N\) symbols (such as words that would compose a sentence). In general, the distribution over the variables follows the graphical model:

\[
P_\Theta(w_1, \ldots, w_T) = \prod_{t=1}^{T} P_\Theta(w_t|w_{<t}), \tag{13}
\]

where \(\Theta = \{\psi, \theta\} = \{V, W, R, b, b_r, \alpha, \beta_1, \beta_2\}\) are the model parameters (of a full Delta-RNN).

\(^4\)\(H\) counts the hidden bias, the full interpolation mechanism \(r_t\) (Equation 5), and the second-order biases, \(\{\alpha, \beta_1, \beta_2\}\).

\(^5\)In preliminary experiments, we also investigated incorporating layer normalization (Ba et al., 2016) into the Delta-RNN architecture, the details of which may be found in the Appendix. We did not find observe noticeable gains using layer normalization over drop-out, and thus only report the results of drop-out in this paper.
No matter how the hidden state $h_t$ is calculated, in this paper, it will ultimately be fed into a maximum-entropy classifier defined as:

$$P(w, h_t) = P_{\Theta}(w|h_t) = \frac{\exp(w^T R h_t)}{\sum_{w'} \exp(w'^T R h_t)},$$

(14)

To learn parameters for any of our models, we optimize with respect to the sequence negative log likelihood:

$$L = -\sum_{i=1}^{N} \sum_{t=1}^{T} \log P_{\Theta}(w_t|h),$$

(15)

Model parameters, $\Theta = \{\theta, \psi\}$, of the Delta-RNN are learned under an empirical risk minimization framework. We employ back-propagation of errors (or rather, reverse-mode automatic differentiation with respect to this negative log likelihood objective function) to calculate gradients and update the parameters using the method of steepest gradient descent. For all experiments conducted in this paper, we found that the ADAM adaptive learning rate scheme (Kingma and Ba, 2014) (followed by a Polyak average (Polyak and Juditsky, 1992) for the subword experiments) yielded the most consistent and near-optimal performance. We therefore use this set-up for optimization of parameters for all models (including baselines), unless otherwise mentioned. For all experiments, we unroll computation graphs $T$ steps in time (where $T$ varies across experiments/tasks), and, in order to approximate full back-propagation through time, we carry over the last hidden from the previous mini-batch (within a full sequence). More importantly, we found that by furthermore using the derivative of the loss with respect to the last hidden state, we can improve the approximation and thus perform one step of iterative inference to update the last hidden state carried over. We ultimately used this proposed improved approximation for the sub-word models (since in those experiments we could directly train all baseline and proposed models in a controlled, identical fashion to ensure fair comparison).

For all Delta-RNNs experimented with in this paper, the output activation of the inner function $g(\cdot)$ was chosen to be the hyperbolic tangent. The output activation of the outer function $f(\cdot)$ was set to be the identity for the word and character benchmark experiments and the hyperbolic tangent for the subword experiments (these decisions were made based on preliminary experimentation on sub-sets of the final training data). The exact configuration of the implementation we used in this paper involved using the late-integration form, either the un-regularized (Equation 9) or the drop-out regularized (Equation 12) variant, for the outer function and Equation 11.

We compare our proposed models against a wide variety of un-regularized baselines, as well several state-of-the-art regularized baselines for the benchmark experiments.

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6 Note that the bias term has been omitted for clarity.

7 We searched the step-size $\lambda$ over the values $\{0.05, 0.1, 0.15\}$ for all experiments in this paper.
These baselines include the LSTM, GRU, and SCRN as well as computationally more efficient formulations of each, such as the MGU. The goal is to see if our proposed Delta-RNN is a suitable replacement for complex gated architectures and can capture longer term patterns in sequential text data.

3 Related Work: Recovering Previous Models

A contribution of this work is that our general framework, presented in Section 2.1, offers a way to unify previous proposals for gated neural architectures (especially for use in next-step prediction tasks like language modeling) and explore directions of improvement. Since we will ultimately compare our proposed Delta-RNN of Section 2.3 to these architectures, we will next present how to derive several key architectures from our general form, such as the Gated Recurrent Unit and the Long Short Term Memory. More importantly, we will introduce them in the same notation / design as the Delta-RNN and highlight the differences between previous work and our own through the lens of $f_\psi(\cdot)$ and $g_\theta(x_t, M_{t-1})$.

Simple models, largely based on the original Elman RNN (Elman, 1990), have often been shown to perform quite well in language modeling tasks (Mikolov et al., 2010, 2011). The Structurally Constrained Recurrent Network (SCRN, Mikolov et al. 2014), an important predecessor and inspiration for this work, showed that one fruitful path to learning longer-term dependencies was to impose a hard constraint on how quickly the values of hidden units could change, yielding more “stable” long-term memory. The SCRN itself is very similar to a combination of the RNN architectures of (Jordan, 1990; Mozer, 1993). The key element of its design is the constraint that part of recurrent weight matrix must stay close to the identity, a constraint that is also satisfied by the Delta-RNN. These identity connections (and corresponding context units that use them) allow for improved information travel over many time-steps and can even be viewed as an exponential trace memory (Mozer, 1993). Residual Networks, though feed-forward in nature, also share a similar motivation (He et al., 2016). Unlike the SCRN, the proposed Delta-RNN does not require a separation of the slow and fast moving units, but instead models this slower time-scale through implicitly stable states.

The Long Short Term Memory (LSTM, Hochreiter and Schmidhuber 1997a) is arguably the currently most popular and often-used gated neural architecture, especially in the domain of Natural Language Processing. Starting from our general form, Equation 1, we can see how the LSTM can be deconstructed, where setting $c_t = g_\theta(x_t, M_{t-1})$,
yields:

\[
\begin{align*}
    h_t &= f_{\psi}[g_{\theta}(x_t, M_{t-1}), M_{t-1}] \\
    h_t &= r_t \otimes \Phi(c_t), \quad \text{where}, \quad (16) \\
    r_t &= \sigma(W_r x_t + V_r h_{t-1} + U_r c_t + b_r) \quad \text{(17)}
\end{align*}
\]

where \( M_{t-1} = \{h_{t-1}, c_{t-1}\} \), noting that \( c_{t-1} \) is the cell-state designed to act as the constant error carousel in mitigating the problem of vanishing gradients when using back-propagation through time. A great deal of recent work has attempted to improve the training of the LSTM, often by increasing its complexity, such as through the introduction of so-called “peephole connections” \((Gers \text{ and Schmidhuber, 2000})\). To compute \( c_t = g_{\theta}(x_t, M_{t-1}) \), using peephole connections, we use the following set of equations:

\[
\begin{align*}
    c_t &= f_t \otimes c_{t-1} + i_t \otimes z_t, \quad \text{where}, \\
    z_t &= \Phi(W_z x_t + V_z h_{t-1} + b_z), \\
    i_t &= \sigma(W_i x_t + V_i h_{t-1} + U_i c_{t-1} + b_i), \\
    f_t &= \sigma(W_f x_t + V_f h_{t-1} + U_f c_{t-1} + b_f).
\end{align*}
\]

The Gated Recurrent Unit (GRU, Chung et al. 2014, 2015) can be viewed as one of the more successful attempts to simplify the LSTM. We see that \( f_{\psi}(\cdot) \) and \( g_{\theta}(\cdot) \) are still quite complex, requiring many intermediate computations to reach an output. In the case of the outer mixing function, \( f_{\psi}(\cdot) \), we see that:

\[
\begin{align*}
    h_t &= \Phi(\gamma [g_{\theta}(x_t, h_{t-1}) + \beta h_{t-1}]) \\
    \gamma &= r_t \quad \text{and} \quad \beta = (1 - r_t), \quad \text{where}, \quad (18) \\
    r_t &= \sigma(V_r h_{t-1} + W_r x_t + b_r) \quad (19)
\end{align*}
\]

noting that the state gate \( r_t \) is also a function of the RNN’s previous hidden state and introduces parameters specialized for \( r \). In contrast, the Delta-RNN does not use an extra set of input-to-hidden weights, and more directly, the pre-activation of the input projection can be reused for the interpolation gate. The inner function of the GRU, \( g_{\theta}(x_t, h_{t-1}) \), is defined as:

\[
\begin{align*}
    g_{\theta}(x_t, h_{t-1}) &= \phi(V_h (q_t \otimes h_{t-1}) + W_h x_t + b_h) \\
    q_t &= \sigma(V_q h_{t-1} + W_q x_t + b_q)
\end{align*}
\]

where \( \phi(\cdot) \) is generally set to be the hyperbolic tangent activation function. A mutated architecture (MUT, Jozefowicz et al. 2015) was an attempt to simplify the GRU somewhat, as, much like the Delta-RNN, its interpolation mechanism is not a function of the previous hidden state but is still largely as parameter-heavy as the GRU, only shedding a single extra parameter matrix, especially since its interpolation mechanism retains
a specialized parameter matrix to transform the data. The Delta-RNN, on the other hand, shares this with its primary calculation of the data’s pre-activation values. The Minimally Gated Unit (MGU, Zhou et al. 2016) is yet a further attempt to reduce the complexity of the GRU by merging its reset and update gates into a single forget gate, essentially using the same outer function under the GRU defined in Equation [19], but simplifying the inner function \( g_\theta(x_t, h_{t-1}) \) to be quite close to the Elman-RNN but conditioned on the forget gate as follows:

\[
g_\theta(x_t, h_{t-1}) = \phi(V_r(r_t \otimes h_{t-1}) + W_h x_t + b_h).
\]

While the MGU certainly does reduce the number of parameters, viewing it from the perspective of our general Delta-RNN framework, one can see that it still largely uses a \( g_\theta(x_t, h_{t-1}) \) that is rather limited (only the capabilities of the Elman-RNN). The most effective version of our Delta-RNN emerged from the insight that a more powerful \( g_\theta(x_t, h_{t-1}) \) could be obtained by (approximately) increasing its order, which requires a few more bias parameters, and nesting it within a non-linear interpolation mechanism that will compute the delta-states. Our framework is general enough to also allow designers to incorporate functions that augment the general state-engine with an external memory to create architectures that can exploit the strengths of models with decoupled memory architectures (Weston et al., 2014; Sukhbaatar et al., 2015; Graves et al., 2016) or data-structures that serve as memory (Sun et al., 1998; Joulin and Mikolov, 2015).

A final related, but important, strand of work uses depth (i.e., number of processing layers) to directly model various time-scales, as emulated in models such as the hierarchical/multi-resolutional recurrent neural network (HM-RNN) (Chung et al., 2016). Since the Delta-RNN is designed to allow its interpolation gate \( r \) to be driven by the data, it is possible that the model might already be learning how to make use of boundary information (word boundaries at the character/sub-word level, sentence boundaries as marked by punctuation at the word-level). The HM-RNN, however, more directly attacks this problem by modifying an LSTM to learn how to manipulate its states when certain types of symbols are encountered. (This is different from models like the Clockwork RNN that require explicit boundary information (Koutnik et al., 2014).) One way to take advantage of the ideas behind the HM-RNN would be to manipulate the Differential State Framework to incorporate the explicit modeling of time-scales through layer depth (each layer is responsible for modeling a different time-scale). Furthermore, it would be worth investigating how the HM-RNN’s performance would change when built from modifying a Delta-RNN instead of an LSTM.

### 4 Experimental Results

Language modeling is an incredibly important next-step prediction task, with applications in downstream applications in speech recognition, parsing, and information...
retrieval. As such, we will focus this paper on experiments on this task domain to
gauge the efficacy of our Delta-RNN framework, noting that the Delta-RNN framework
might prove useful in, for instance, machine translation (Bahdanau et al., 2014) or light
chunking (Turian et al., 2009). Beyond improving language modeling performance, the
sentence (and document) representations iteratively inferred by our architectures might
also prove useful in composing higher-level representations of text corpora, a subject
we will investigate in future work.

4.1 Datasets

4.1 The Penn Treebank Corpus

The Penn Treebank corpus (Marcus et al., 1993) is often used to benchmark both word
and character-level models via perplexity or bits-per-character, and thus we start here.8
The corpus contains 42,068 sentences (971,657 tokens, average token-length of about
4.727 characters) of varying length (the range is from 3 to 84 tokens, at the word-level).

4.2 The IMDB Corpus

The large sentiment analysis corpus (Maas et al., 2011) is often used to benchmark al-
gorithms for predicting the positive or negative tonality of documents. However, we opt
to use this large corpus (training consists of 149,714 documents, 1,875,523 sentences,
40,765,697 tokens, average token-length is about 3.4291415 characters) to evaluate our
proposed Delta-RNN as a (subword) language model. The IMDB data-set serves as a
case when the context extends beyond the sentence-level in the form of actual docu-
ments.

4.2 Word & Character-Level Benchmark

The first set of experiments allow us to examine our proposed Delta-RNN models
against reported state-of-the-art models. These reported measures have been on tradi-
tional word and character-level language modeling tasks—we measure the per-symbol
perplexity of models. For the word-level models, we calculate the per-word perplexity
(PPL) using the measure $PPL = \exp \left[ -\frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \log P_{\theta}(w_t|h) \right]$. For the
character-level models, we report the standard bits-per-character (BPC), which can be
calculated from the log likelihood using the formula: $BPC = -1/(N \log(2)) \sum_{i=1}^{N} \sum_{t=1}^{T} \log P_{\theta}(w_t|h)$.

Over 100 epochs, word-level models with mini-batches of 64 (padded) sequences.
(Early stopping with a lookahead of 10 was used.) Gradients were clipped using a sim-
ple magnitude-based scheme (Pascanu et al., 2013), with the magnitude threshold set to

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8To be directly comparable with previously reported results, we make use of the specific pre-processed
train/valid/test splits found at http://www.fit.vutbr.cz/~imikolov/rnnlm/.
| Penn Treebank: Word Models | PPL |
|---------------------------|-----|
| N-Gram (Mikolov et al., 2014) | 141 |
| NLM (Mikolov, 2012) | 140.2 |
| N-Gram+cache (Mikolov et al., 2014) | 125 |
| RNN (Gulcehre et al., 2016) | 129 |
| RNN (Mikolov, 2012) | 124.7 |
| LSTM (Mikolov et al., 2014) | 115 |
| SCRNN (Mikolov et al., 2014) | 115 |
| LSTM (Sundermeyer, 2016) | 107 |
| MI-RNN (Wu et al., our impl.) | 109.2 |
| Delta-RNN (present work) | 100.324 |
| Delta-RNN, dynamic #1 (present work) | 93.296 |
| Delta-RNN, dynamic #2 (present work) | 90.301 |

| Penn Treebank: Character Models | BPC |
|---------------------------------|-----|
| N-discount N-gram (Mikolov et al., 2012) | 1.48 |
| RNN+stabilization (Krueger et al., 2016) | 1.48 |
| linear MI-RNN (Wu et al., 2016) | 1.48 |
| Clockwork RNN (Koutnik et al., 2014) | 1.46 |
| RNN (Mikolov et al., 2012) | 1.42 |
| GRU (Jernite et al., 2016) | 1.42 |
| HF-MRNN (Mikolov et al., 2012) | 1.41 |
| MI-RNN (Wu et al., 2016) | 1.39 |
| Max-Ent N-gram (Mikolov et al., 2012) | 1.37 |
| LSTM (Krueger et al., 2016) | 1.356 |
| Delta-RNN (present work) | 1.347 |
| Delta-RNN, dynamic #1 (present work) | 1.331 |
| Delta-RNN, dynamic #2 (present work) | 1.326 |

|                              |     |
|------------------------------|-----|
| LSTM-norm stabilizer (Krueger et al., 2016) | 1.352 |
| LSTM-weight noise (Krueger et al., 2016) | 1.344 |
| LSTM-stochastic depth (Krueger et al., 2016) | 1.343 |
| LSTM-recurrent drop (Krueger et al., 2016) | 1.286 |
| RBN (Cooijmans et al., 2016) | 1.32 |
| LSTM-zone out (Krueger et al., 2016) | 1.252 |
| H-LSTM + LN (Ha et al., 2016) | 1.25 |
| TARDIS (Gulcehre et al., 2017) | 1.25 |
| 3-HM-LSTM + LN (Chung et al., 2016) | 1.24 |
| Delta-RNN-drop, static (present work) | 1.251 |
| Delta-RNN-drop, dynamic #1 (present work) | 1.247 |
| Delta-RNN-drop, dynamic #2 (present work) | 1.245 |

Table 1: Test-set results on the Penn Treebank word-level and character-level language modeling tasks. Note that “impl.” means implementation.

5. A simple grid-search was performed to tune the learning rate, $\lambda = \{0.002, 0.001, 0.0005, 0.0002\}$, as well as the size of the hidden layer $H = \{500, 1000, 1500\}$. Parameters (non-biases) were initialized from zero-mean Gaussian distributions with variance tuned,
\[ \sigma = \{0.1, 0.01, 0.005, 0.001\} \]

The character-level models, on the other hand, were updated using mini-batches of 64 samples over 100 epochs. (Early stopping with a lookahead of 10 was used.) The parameter initializations and grid-search for the learning rate and hidden layer size were the same as for the word models, with the exception of the hidden layer size, which was searched over \( H = \{500, 1000, 1500, 2000\} \).

A simple learning rate decay schedule was employed: if the validation loss did not decrease after a single epoch, the learning rate was halved (unless a lower bound on the value had been reached). When drop-out was applied to the Delta-RNN (\textit{Delta-RNN-drop}, we set the probability of dropping a unit to \( p_{\text{drop}} = 0.15 \) for the character-level models and \( p_{\text{drop}} = 0.5 \) for the word level models. We present the results for the un-regularized and regularized versions of the models. For all of the Delta-RNNs, we furthermore experiment with two variations of dynamic evaluation, which facilitates fair comparison to compression algorithms, inspired by the improvements observed in (Mikolov, 2012). \textit{Delta-RNN-drop, dynamic #1} refers to simply updating the model sample-by-sample after each evaluation, where in this case, we update parameters using simple stochastic gradient descent (Mikolov, 2012), with a step-size \( \lambda = 0.005 \). We develop a second variation of dynamic evaluation, \textit{Delta-RNN-drop, dynamic #2}, where we allow the model to first iterate (and update) once over the validation set and then finally the test-set, completely allowing the model to “compress” the Penn Treebank corpus. These two schemes are used for both the word and character-level benchmarks. It is important to stress the BPC and PPL measures reported for the dynamic models follow a strict “test-then-train” online paradigm, meaning that each next-step prediction is made before updating model parameters.

The standard vocabulary for the word-level models contains 10K unique words (including an unknown token for out-of-vocabulary symbols and an end-of-sequence token) and the standard vocabulary for the character-level models includes 49 unique characters (including a symbol for spaces). Results for the word-level models are reported in Table and results for the character-level models are reported in Table.

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9 We also experimented with other initializations, most notably the identity matrix for the recurrent weight parameters as in (Le et al., 2015). We found that this initialization often worsened performance.

For the activation functions of the first-order models, we experimented with the linear rectifier, the parametrized linear rectifier, and even our own proposed parametrized smoothed linear rectifier, but found such activations lead to less-than-satisfactory results. The results of this inquiry is documented in the code that will accompany the paper.

10 Note that \( H = 2000 \) would yield nearly 4 million parameters, which was our upper bound on total number of parameters allowed for experiments in order to be commensurable with the work of Wu et al. (2016), which actually used \( H = 2048 \) for all Penn Treebank models.

11 We use a special “null” token (or zero-vector) to mark the start of a sequence.
4.3 Sub-word Language Modeling

We chose to measure the negative log likelihood of the various architectures in the task of **subword modeling**. Subwords are particularly appealing not only in that the input distribution is of lower dimensionality but, as evidenced by the positive results of Mikolov et al. (2012), sub-word/character hybrid language models improve over the performance of pure character-level models. Sub-word models also enjoy the advantage held by character-level models when it comes to handling out-of-vocabulary words, avoiding the need for an “unknown” token. Research in psycholinguistics has long suggested that even human infants are sensitive to word boundaries at an early stage (e.g., Aslin et al. 1998), and that morphologically complex words enjoy dedicated processing mechanisms (Baayen and Schreuder, 2006). Subword-level language models may approximate such an architecture. Consistency in subword formation is critical in order to obtain meaningful results (Mikolov et al., 2012). Thus, we design our sub-word algorithm to partition a word according to the following scheme:

1. Split on vowels (using a predefined list)
2. Link/merge each vowel with a consonant to the immediate right if applicable
3. Merge straggling single characters to subwords on the immediate right unless a subword of shorter character length is to the left.

This simple partitioning scheme was designed to ensure that no subword was shorter than two characters in length. Future work will entail designing a more realistic subword partitioning algorithm. Subwords below a certain frequency were discarded, and combined with 26 single characters to create the final dictionary. For Penn Treebank, this yields a vocabulary of 2405 symbols was created (2,378 subwords + 26 characters + 1 end-token). For the IMDB corpus, after replacing all emoticons and special non-word symbols with special tokens, we obtain a dictionary of 1926 symbols (1899 subwords + 26 single characters + 1 end-token). Results for all sub-word models are reported in Table 2.

| PTB-SW   | Performance | IMDB-SW   | Performance |
|----------|-------------|-----------|-------------|
|          | # Params    | NLL       | # Params    | NLL         |
| **RNN**  | 1,272,464  | 1.8939    | **RNN**    | 499,176    | 2.1691     |
| **SCRN** | 1,268,604  | 1.8420    | **SCRN**   | 496,196    | 2.2370     |
| **MGU**  | 1,278,692  | 1.8694    | **MGU**    | 495,444    | 2.1312     |
| **MI-RNN** | 1,267,904 | 1.8441    | **MI-RNN** | 495,446    | 2.1741     |
| **GRU**  | 1,272,404  | 1.8251    | **GRU**    | 499,374    | 2.1551     |
| **LSTM** | 1,274,804  | 1.8412    | **LSTM**   | 503,664    | 2.2080     |
| **Delta-RNN** | 1,268,154 | 1.8260    | **Delta-RNN** | 495,570    | 2.1333     |

Table 2: Test-set negative log likelihoods while holding number of parameters approximately constant. Subword modeling tasks on Penn Treebank and IMDB.
Specifically, we test our implementations of the LSTM\textsuperscript{12} (with peephole connections as described in Graves\textsuperscript{2013}), the GRU, the MGU, the SCRN, as well as a classical Elman network, of both 1st and 2nd-order (Giles et al., 1991; Wu et al., 2016).\textsuperscript{13} Subword models were trained in a similar fashion as the character-level models, updated (every 50 steps) using mini-batches of 20 samples but over 30 epochs. Learning rates were tuned in the same fashion as the word-level models, and the same parameter initialization schemes were explored. The notable difference between this experiment and the previous ones is that we fix the number of parameters for each model to be equivalent to that of an LSTM with 100 hidden units for PTB and 50 hidden units for IMDB. This ensures a controlled, fair comparison across models and allows us to evaluate if the Delta-RNN can learn similarly to models with more complicated processing elements (an LSTM cell versus a GRU cell versus a Delta-RNN unit). Furthermore, this allows us to measure parameter efficiency, where we can focus on the value of actual specific cell-types (for example, allowing us to compare the value of a much more complex LSTM memory unit versus a simple Delta-RNN cell) when the number of parameters is held roughly constant. We are currently running larger versions of the models depicted in the table above to determine if the results hold at scale.

5 Discussion

With respect to the word and character-level benchmarks, we see that the Delta-RNN outperforms all previous, un-regularized models, and performs comparably to regularized state-of-the-art. As documented in Table\textsuperscript{2}, we further trained a second-order, word-level RNN (MI-RNN) to complete the comparison, and remark that the second-order connections appear to be quite useful in general, outperforming the SCRN and coming close to that of the LSTM. This extends the results of Wu et al. (2016) to the word-level. However, the Delta-RNN, which also makes use of second-order units within its inner function, ultimately offers the best performance and performs better than the LSTM in all experiments. In both Penn Treebank and IMDB subword language modeling experiments, the Delta-RNN is competitive with complex architectures such as the GRU and the MGU. In both cases, the Delta-RNN nearly reaches the same performance as the best performing baseline model in either data-set (i.e., it nearly reaches the same performance as the GRU on Penn Treebank and the MGU on IMDB). Surprisingly, on IMDB, a simple Elman network is quite performant, even outperforming the MI-RNN. We argue that this might be the result of constraining all neural architectures to only a small number of parameters for such a large data-set, a constraint we intend to relax in future work.

\textsuperscript{12}We experimented with initializing the forget gate biases of all LSTMs with values searched over \{1, 2, 3\} since previous work has shown this can improve model performance.

\textsuperscript{13}We will publicly release code to build and train the architectures in this paper upon publication.
future work.

The Delta-RNN is far more efficient than a complex LSTM and certainly a memory-augmented network like TARDIS [Gulcehre et al., 2017]. Moreover, it appears to learn how to make appropriate use of its interpolation mechanism to decide how and when to update its hidden state in the presence of new data. Given our derivations in Section 3, one could argue that nearly all previously proposed gated neural architectures are essentially trying do the same thing under the Differential State Framework. The key advantage offered by the Delta-RNN is that this functionality is offered directly and cheaply (in terms of required parameters).

It is important to contrast these (un-regularized) results with those that use some form of regularization. Zaremba et al. (2014) reported that a single LSTM (for word-level Penn Treebank) can reach a PPL of $\sim 80$, but this was achieved via dropout regularization (Srivastava et al., 2014). There is a strong relationship between using dropout and training an ensemble of models. Thus, one can argue that a single model trained with dropout actually is not a single model, but an implicit ensemble (see also Srivastava et al., 2014). An ensemble of twenty simple RNNs and cache models did previously reach PPL as low as 72, while a single RNN model gives only 124 (Mikolov, 2012). Zaremba et al. (2014) trained an ensemble of 38 LSTMs regularized with dropout, each with 100x times more parameters than the RNNs used by Mikolov, achieving PPL 68. This is arguably a small improvement over 72, and seems to strengthen our claim that dropout is an implicit model ensemble and thus should not be used when one wants to report the performance of a single model. However, the Delta-RNN is amenable to regularization, including drop-out. As our results show, when simple drop-out is applied, the Delta-RNN can reach much lower perplexities, even similar to the state-of-the-art with much larger models, especially when dynamic evaluation is permitted. This even extends to very complex architectures, such as the recently proposed TARDIS, which is a memory-augmented network (and when dynamic evaluation is used, the simple Delta-RNN can outperform this complex model). Though we investigate the utility of simple drop-out in this paper, our comparative results suggest that more sophisticated variants, such as variational drop-out (Gal and Ghahramani, 2016), could yield yet further improvement in performance.

What is the lesson to be learned from the Differential State Framework? First, and foremost, we can obtain strong performance in language modeling with a simpler, more efficient (in terms of number of parameters), and thus faster, architecture. Second, the Delta-RNN is designed from the interpretation that the computation of the next hidden state is the result of a composition of two functions. One inner function decides how to “propose” a new hidden state while the outer function decides how to use this

\[14\text{At greater computational cost, a somewhat lower perplexity for an LSTM may be attainable, such as the perplexity of 107 reported by Sundermeyer (2016) (see Table 1). However, this requires many more training epochs and precludes batch training.}\]
Figure 2: L1 norm of deltas between consecutive states of model trained on Penn Treebank plotted over words of example sentences. A simple polynomial trend-line (dashed red) was fit to the bar heights in order to illustrate the informative “bumps” of each sample sentence. The main observation is that the norm is, in general, lower for low-information content words, such as the article “the”, and higher for informative words, such as “government”.
new proposal in updating the previously calculated state. The data-driven interpolation mechanism is used by the model to decide how much impact the newly proposed state has in updating what is likely to be a slowly changing representation. The SCRN, which could be viewed as the predecessor to the Delta-RNN framework, was designed with the idea that some constrained units could serve as a sort of cache meant to capture longer-term dependencies. Like the SCRN, the Delta-RNN is designed to help mitigate the problem of vanishing gradients, and through the interpolation mechanism, has multiple pathways through which the gradient might be carried, boosting the error signal’s longevity down the propagation path through time. However, the SCRN combines the slow-moving and fast-changing hidden states through a simple summation and thus cannot model non-linear interactions between its shorter and longer term memories, furthermore requiring tuning of the sizes of these separated layers. On the other hand, the Delta-RNN, which does not require special tuning of an additional hidden layer, can non-linearly combine the two types of states in a data-dependent fashion, possibly allowing the model to exploit boundary information from text, which is quite powerful in the case of documents. The key intuition is that the gating mechanism only allows the state proposal to affect the maintained memory state only if the currently observed data-point carries any useful information. This warrants a comparison, albeit indirect, to Surprisal Theory. This “surprisal” proves useful in iteratively forming a sentence impression that will help to better predict the words that come later.

With respect to the last point made, we briefly examine the evolution of a trained Delta-RNN’s hidden state across several sample sentences. The first two sentences are hand-created (constrained to use only the vocabulary of Penn Treebank) while the last one is sampled from the Penn Treebank training split. Since the Delta-RNN iteratively processes symbols of an ordered sequence, we measure the L1 norm across consecutive pairs of hidden states. We report the (min-max) normalized L1 scores in Figure 2 and observe that, in accordance with our intuition, we can see that the L1 norm is lower for high-frequency words (indicating a smaller delta) such as “the” or “of” or “is”, which are words generally less informative about the general subject of a sentence/document. As this qualitative demonstration illustrates, the Delta-RNN appears to learn what to do with its internal state in the presence of symbols of variable information content.

\[ L_{1 \text{score}} = \frac{(L1(h_{t-1}, h_t) - min(L1_{seq}))(max(L1_{seq}) - min(L1_{seq}))}{(max(L1_{seq}) - min(L1_{seq}))}. \]
6 Conclusions

We present the Differential State Framework, which affords us a useful perspective on viewing computation in recurrent neural networks. Instead of recomputing the whole state from scratch at every time step, the Delta-RNN only learns how to update the current state. This seems to be better suited for many types of problems, especially those that involve longer term patterns where part of the recurrent network’s state should be constant most of the time. Comparison to the currently widely popular LSTM and GRU architectures shows that the Delta-RNN can achieve similar or better performance on language modeling tasks, while being conceptually much simpler and with far less parameters. Comparison to the Structurally Constrained Recurrent Network (SCRN), which shares many of the main ideas and motivation, shows better accuracy and a simpler model architecture (since, in the SCRN, tuning the sizes of two separate hidden layers is required, and this model cannot learn non-linear interactions within its longer memory). Future work includes larger-scale language modeling experiments to test the efficacy of the Delta-RNN framework as well as architectural variants that employ decoupled memory. Since the Delta-RNN can also be stacked just as any other neural architecture, we intend to investigate if depth (in terms of hidden layers) might prove useful on larger-scale data-sets. In addition, we intend to explore how useful the Delta-RNN might be in other tasks that the architectures such as the LSTM currently hold state-of-the-art performance in. Finally, it would be useful to explore if Delta-RNN’s simpler, faster design can speed up the performance of grander architectures, such as the Differentiable Neural Computer (Graves et al., 2016) (composed of multiple LSTM modules).

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Appendix A: Layer Normalized Delta-RNNs

In this appendix, we describe how layer normalization would be applied to a Delta-RNN. Though our preliminary experiments did not uncover that layer normalization gave much improvement over drop-out, this was only observed on the Penn Treebank benchmark. Future work will investigate the benefits of layer normalization over drop-out (as well as model-ensembling) on larger-scale benchmarks.

A simple RNN requires the layer normalization to be applied after calculating the full linear pre-activation (a sum of the filtration and the projected data point). On the other hand, a Delta-RNN requires further care (like the GRU) to ensure the correct components are normalized without damaging the favorable properties inherent to the model’s multiplicative gating. If layer normalization is applied to the pre-activations of the late-integration Delta-RNN proposed in this paper, the update equations become:

\[
\begin{align*}
  d_r^{rec} &= LN(V_d h_{t-1}), \quad d_l^{dat} = LN(W x_t) \\
  z_t &= \phi_{hid}(d_r^{rec} \otimes d_l^{dat} + d_l^{rec} + d_l^{dat}), \text{ and,} \\
  h_t &= \Phi((1 - r) \otimes z_t + r \otimes h_{t-1}), \text{ and,} \\
  r &= 1/(1 + \exp(-[d_l^{dat} + b_r])).
\end{align*}
\]

Note that the additional bias parameters introduced in the original update equations are now omitted. This can be done since the layer normalization operation applied will now perform the work of shifting and scaling. Since the Delta-RNN takes advantage of parameter-sharing, it notably requires substantially fewer layer normalizations than a more complex model (such as the GRU) would. A standard GRU would require nine layer normalizations while the Delta-RNN simply requires two.