Interpolating Points on a Non-Uniform Grid using a Mixture of Gaussians

Ivan Skorokhodov  
KAUST  
Thuwal, Saudi Arabia  
iskorokhodov@gmail.com

Abstract

In this work, we propose an approach to perform non-uniform image interpolation based on a Gaussian Mixture Model. Traditional image interpolation methods, like nearest neighbor, bilinear, Hamming, Lanczos, etc. assume that the coordinates you want to interpolate from, are positioned on a uniform grid. However, it is not always the case in practice and we develop an interpolation method that is able to generate an image from arbitrarily positioned pixel values. We do this by representing each known pixel as a 2D normal distribution and considering each output image pixel as a sample from the mixture of all the known ones. Apart from the ability to reconstruct an image from arbitrarily positioned set of pixels, this also allows us to differentiate through the interpolation procedure, which might be helpful for downstream applications. Our optimized CUDA kernel and the source code to reproduce the benchmarks is located at https://github.com/universome/non-uniform-interpolation.

1. Introduction

Imagine that we have access to some image in a functional form. I.e. the image is represented as a function $f: p \mapsto c$ which takes a pixel coordinate $p = (x, y) \in \mathbb{R}^2$ as an input and produces its corresponding RGB value $c = (r, g, b) \in \mathbb{R}^3$. Such representations arise, for example, in differentiable rendering pipelines [9, 3] or implicit representations of images [5, 7, 6, 1].

Now imagine, that we want to generate a high-resolution raster image from this functional representation $f(p)$. This means that we need to evaluate $f(p)$ in every coordinate location of $H \times W$ grid to generate a $H \times W$ sized image. If evaluating $f(p)$ is costly then it is a tedious procedure. What can we do?

One approach would be to speed up the inference for $f(p)$. Another one is to generate a low-resolution version of an image and then upsample it with one of the existing methods. Such upsampling methods assume that the points you are trying to upsample from, are positioned on a uniform grid, i.e. they have a fixed equal horizontal and vertical spacing between each other, as depicted on figure 1a. However, in practice there sometimes occur situations when your points are positioned on a non-uniform grid, like on image 1b, limiting the applicability of the existing tools.

To alleviate the issue, we propose a novel interpolation method that makes it possible to reconstruct an image from a subset of points that are arbitrarily scattered across the image. We achieve this by representing each known color $c^i$ at location $(x^i, y^i)$ as a 2D normal distribution $\mathcal{N}(\mu^i, \sigma^2 I)$ for $\mu^i = (x^i, y^i)$ and some predefined variance $\sigma^2$.

To summarize, our contributions are the following:

- We propose a novel interpolation technique which is based on representing the known points as a GMM model and inferring the value for the unknown ones as an expectation.

- We develop an optimized CUDA kernel for both the forward and backward passes of the proposed interpolation procedure.

- We conduct the experiments on ImageNet dataset and show that our proposed interpolation technique outperforms in several scenarios 6 other standard interpolation methods based on the reconstruction quality.

2. Method

Our interpolation method treats each known point $(x^i, y^i)$ with color $c^i$ as a 2D gaussian distribution with mean $\mu^i = (x^i, y^i)$ and some diagonal covariance matrix $\sigma^2 I$ for some fixed hyperparameter $\sigma$. To compute the point value in some unknown pixel coordinate $q = (x, y)$ we evaluate its expected color value as:

$$c(q) \triangleq \mathbb{E}_{p(c|q)}[c] = \sum_{i=1}^{N} c(p^{(i)}) \cdot \frac{\mathcal{N}(q|\mu^i, \sigma^2 I)}{Z_q}$$  \hspace{1cm} (1)
Figure 1: Example of (a) uniform and (b) non-uniform interpolation. Blue points are “known” points and red points are “unknown” points, i.e. points we want to compute the value in. Existing interpolation methods assume a uniform grid, but it is not always true in practice.

where $Z_q$ is the normalizing factor for the point computed as:

$$Z_q = \sum_{i=1}^{N} \mathcal{N}(q | \mu^{(i)}, \sigma^2 I)$$

(2)

To speed up the procedure, we consider only those known points, that are close enough to the query one. The gaussian densities are considered to be weights by which the known points influence the resulted color of the unknown one. We illustrate this on Figure 2.

We interpolate each color channel independently.

3. Experiments

3.1. Validating the correctness of the computations

Since writing CUDA kernels is very error-prone, especially for the backward pass, one needs to ensure that all the computations are correct. For this, we implemented a (very) slow python version using Pytorch automatic differentiation framework [4]. After that, we performed the forward pass on the same input for both the python version and our optimized CUDA kernel. Comparing that the results of the both procedures are equal, confirms that the implemented computations are correct.

3.2. Testing the reconstruction quality

The first set of experiments we conduct is to test the reconstruction quality of the proposed interpolation method. For this, we take 1000 images from ImageNet dataset [2] — one image per class — then downsample them to a specified factor and then upsample with one of the methods. We test against 6 standard interpolation techniques that are shipped into PIL image library [8]: nearest neighbour, box, bilinear, bicubic, Hamming and Lanczos. The results are presented on Figure 3. As one can see, our method is competitive for small downsampling factors and outperforms the existing methods when the downsampling factor increases.
3.3. Optimizing the points locations

Since our procedure permits the optimization of points positions, it is a natural idea to minimize the reconstruction quality with gradient descent. Concretely, $\mu^{(1)}, ..., \mu^{(N)}$ become learnable parameters that are being optimized using the derivatives compute with respect to them.

We take an image of a room, define how many points we allow ourselves to have, randomly sample the points on an image using the uniform distribution and then optimize their locations.

The results are presented on Figure 4. As one can see, it is more important to select a proper variance value than optimizing the coordinates. We hypothesize that the model is being stuck in a local minimum. On Figure 5, we illustrate that the model is very reluctant to updating its coordinates positions.

4. Conclusion

In this work, we proposed an interpolation technique based on the gaussian mixture model which is able to reconstruct an image from arbitrary positioned points. We developed an optimized CUDA kernel for both the forward procedure and the corresponding backward pass. We benchmarked it against 6 existing interpolation techniques and showed that it outperforms them in terms of the reconstruction quality for a broad range of setups on ImageNet dataset. Investigating why the model is not amenable to the optimization is a fruitful future research direction.

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