Nested Sparse Approximation: Structured Estimation of V2V Channels Using Geometry-Based Stochastic Channel Model

Sajjad Beygi Student Member, IEEE, Urbashi Mitra Fellow, IEEE, and Erik G. Ström* Senior Member, IEEE

Abstract

Future intelligent transportation systems promise increased safety and energy efficiency. Realization of such systems will require vehicle-to-vehicle (V2V) communication technology. High fidelity V2V communication is, in turn, dependent on accurate V2V channel estimation. Such V2V channels have characteristics differing from classical cellular communication channels. Herein, geometry-based stochastic modeling is employed to develop a V2V channel model. The resultant model exhibits significant structure; specifically, the V2V channel is characterized by three distinct regions within the delay-Doppler plane. Each region has a unique combination of specular reflections and diffuse components resulting in a particular element-wise and group-wise sparsity. This joint sparsity structure is exploited to develop a novel channel estimation algorithm. A general machinery is provided to solve the jointly element/group sparse channel (signal) estimation problem using proximity operators of a broad class of regularizers. The alternating direction method of multipliers using the proximity operator is adapted to optimize the mixed objective function. Key properties of the proposed objective functions are proven which ensure that the optimal solution is found by the new algorithm. The effects of pulse shape leakage are explicitly characterized and compensated resulting in measurably improved performance. Simulation results show that the new algorithm can achieve up to a 10 dB gain over previously proposed methods.

This research was funded in part by one or all of these grants: ONR N00014-09-1-0700, AFOSR FA9550-12-1-0215, DOT CA-26-7084-00, NSF CCF-1117896, NSF CNS-1213128, NSF CCF-1410009, NSF CPS-1446901, Barbro Osher Pro Suecia Foundation, Ericsson’s Research Foundation POSTFFT-13:038, and Adlerbert Research Foundation.

School of Electrical Engineering, University of Southern California *Dept. of Signals and Systems, Chalmers University of Technology Emails: {beygihar, ubli}@usc.edu, erik.strom@chalmers.se
I. INTRODUCTION

Vehicle-to-vehicle (V2V) communication is central to future intelligent transportation systems, which will enable efficient and safer transportation with reduced fuel consumption [1]. In general, V2V communication is anticipated to be short ranges with transmission ranges varying from a few meters to a few kilometers between two mobile vehicles on a road.

A big challenge of realizing V2V communication is the inherent fast channel variations (faster than in cellular [2], [3]) due to the mobility of both the transmitter and receiver. Since channel state information can improve communication performance, a fast algorithm to accurately estimate V2V channels is of interest. Furthermore, V2V channels are highly dependent on the geometry of road and the local physical environment [1], [4].

A popular estimation strategy for fast time-varying channels is to apply Wiener filtering [5], [6]. Recently, [2], [5] have proposed an adaptive Wiener filter to estimate V2V channels using subspace selection. The main drawback of Wiener-filtering is that the knowledge of the scattering function is required [6]; however, the scattering function is not typically known at the receiver. Often, a flat spectrum in the delay-Doppler domain is assumed, which introduces performance degradation due to the mismatch with respect to the true scattering function [5].

In this work, we develop a V2V channel model in the delay-Doppler domain, using geometry-based stochastic channel modeling [1], [4], [7]. Our analysis reveals the special structure of the V2V channel components in the delay-Doppler domain. We show that the delay-Doppler representation of the channel exhibits three key regions; within these regions, the channel is a mixture of specular reflections and diffuse components. While the specular contributions appear all over the delay-Doppler plane sparsely, the diffuse contributions are concentrated in specific regions of the delay-Doppler plane. In our prior work [8], [9], a Hybrid Sparse/Diffuse (HSD) model was presented for a mixture of sparse and Gaussian diffuse components, which we have applied to estimate the V2V channel [10]. This approach requires information about the V2V channel such as the sparsity level, and power delay profile (PDP) of the diffuse and sparse components [8]. Another approach for time-varying frequency-selective channel estimation is via compressed sensing (CS) or sparse approximation based on an $l_1$-norm regularization [11]-[13]. These algorithms perform well for channels with a small number of scatterers or, clusters of scatterers. For V2V channels, diffuse contributions from reflections along the roadside will degrade the performance of
CS methods that only consider element-wise sparsity [5], [12].

In this work, to exploit the particular structure of the V2V channel, we adapt recent work in 2D sparse signal estimation [14], [15], to design a novel joint element- and group-wise sparsity estimator to estimate the 2D time-varying V2V channel using received data. Our proposed method provides a general machinery to solve the joint sparse structured estimation problem with a broad class of regularizers that promote sparsity. We show that our proposed algorithm covers both well-known convex and non-convex regularizers such as smoothly clipped absolute deviation (SCAD) regularizers [17], and the minimax concave penalty (MCP) [18] that were proposed for element-wise sparsity estimation. We have also presented a general way to design a proper regularizer function for joint sparsity problem in our previous work [16].

Recent algorithm for hierarchical sparsity (sparse groups with sparsity within the groups) [19–21] also consider a mixture of penalty functions (group-wise and element-wise). Of particular note is [20] where a similar nested solution is determined, also in combination with alternating direction method of multipliers (ADMM) as we do herein. Our modeling assumption can be viewed as a generalization of their assumptions which results in the need for different methods for proving the optimization of the nested structures. In particular, [20] examines a particular proximity operator for which the original regularizing function is never specified. This proximity operator is built by generalizing the structure of the proximity operator for the $l_p$ norm. In contrast, we begin with a general class of regularizing functions, we specify the properties needed for such functions (allowing for both convex and non-convex functions); thus, our proof methods just rely on the properties induced by these assumptions. Furthermore, our results are also applicable to the problem of hierarchical sparsity [19–21]. We observe that the results in [19–21] cannot be applied to non-convex regularizer functions such as SCAD and MCP due to their concavity and a nonlinear dependence on the regularization parameter.

To find the optimal solution of our joint sparsity objective function, we take advantage of the alternating direction method of multipliers [23], which is a very flexible and efficient tool, for optimization problems whose objective functions are the combination of multiple terms. Furthermore, we use the proximity operator [24] to show that the estimation can be done by using simple thresholding operations in each

---

1Note given a particular regularization function, there is a unique proximity operator, but for a given proximity operator there may exist more than one regularizer function.
We also address the channel leakage effect due to finite block length and bandwidth for channel estimation in the delay-Doppler plane. In [12], the basis expansion for the scattering function is optimized to compensate for the leakage which can degrade performance. The resulting expansion in [12] is computationally expensive. Herein, we take an alternative view and show that with the proper sampling resolution in time and frequency, we can explicitly derive the leakage pattern and robustify the channel estimator with this knowledge at the receiver to improve the sparsity, compensate for leakage, and maintain modest algorithm complexity. Our overall approach can lead to a performance gain of up to 10 dB over previously proposed algorithms.

The main contributions of this work are as follows:

1) A general framework for joint sparsity estimation problem is proposed, which covers a broad class of regularizers including convex and non-convex functions. Furthermore, we show that the solution for joint sparse estimation problem is computed by applying the element-wise and group-wise structure in a nested fashion using simple thresholding operations.

2) We provide a simple model for V2V channel in the delay-Doppler plane, using geometry-based stochastic channel modeling. We characterize the three key regions in the delay-Doppler domain with respect to the presence of sparse specular and diffuse components.

3) The leakage pattern is explicitly computed and a compensation procedure proposed.

4) A low complexity joint element- and group-wise sparsity V2V channel estimation algorithm is proposed exploiting the aforementioned channel model and optimization result.

The rest of this paper is organized as follows. In Section II, we review some definitions from variational analysis and present our key optimization result for a joint sparse and group sparse signal estimation. In Section III, the system model for V2V communications is presented. In Section IV, the geometry-based V2V channel model is developed. The observation model and leakage effect are computed in Section V. In Section VI, the channel estimation algorithm for the time-varying V2V channel model using joint sparsity structure is presented. In Section VII, we provide simulation results and compare the performance of the estimators. Finally, Section VIII concludes the paper. Furthermore, we present our proofs, region specifier algorithm, and analysis of our proximal ADMM in the Appendices.

Notation: We denote a scalar by $x$, a column vector by $x$, and its $i$-th element with $x[i]$. Similarly, we
denote a matrix by \( X \) and its \((i, j)\)-th element by \( X[i, j] \). The transpose of \( X \) is given by \( X^T \) and its conjugate transpose by \( X^H \). A diagonal matrix with elements \( x \) is written as \( \text{diag}\{x\} \) and the identity matrix as \( I \). The set of real numbers by \( \mathbb{R} \), and the set of complex numbers by \( \mathbb{C} \). The element-wise (Schur) product is denoted by \( \odot \).

II. JOINTLY SPARSE SIGNAL ESTIMATION: OPTIMIZATION RESULT

In this section, we propose a unified framework using proximity operators to solve the optimization problem imposed by a jointly sparse signal estimation problem. Then, we apply this machinery to estimate the V2V channel, exploiting the group- and element-wise sparsity structure discovered in Section IV.

A. Proximity Operator

We start with the definition of a proximity operator from variational analysis \[25\].

**Definition 1.** Let \( \phi(a; \lambda) \) be a continuous real-valued function of \( a \in \mathbb{R}^N \), the proximity operator \( P_{\lambda, \phi}(b) \) is defined as

\[
P_{\lambda, \phi}(b) := \arg\min_{a \in \mathbb{R}^N} \left\{ \frac{1}{2} \| b - a \|_2^2 + \phi(a; \lambda) \right\},
\]

where \( b \in \mathbb{R}^N \) and \( \lambda > 0 \).

**Remark 1.** If \( \phi(.) \) is a separable function, i.e., \( \phi(a; \lambda) = \sum_{i=1}^{N} f(a[i]; \lambda) \). Then, \( [P_{\lambda, \phi}(a)]_i = P_{\lambda, f}(a[i]) \).

**Remark 2.** If the objective function \( J(a) = \frac{1}{2} \| b - a \|_2^2 + \phi(a; \lambda) \) is a strictly convex function, the proximity operator of \( \phi(a; \lambda) \) admits a unique solution.

**Remark 3.** Furthermore, \( P_{\lambda, \phi}(b) \) is characterized by the inclusion

\[
\forall (a^*, b), \ a^* = P_{\lambda, \phi}(b) \iff a^* - b \in \partial \phi(a^*; \lambda),
\]

where \( \partial \phi(.) \) is the sub-gradient of the function \( \phi \) \[25\].

\(^2\)A jointly sparse signal in this paper, is a signal that has both element-wise and group-wise sparsity.
Note that $\phi$ does not need to be a convex or differentiable function to satisfy the conditions noted in Remarks 2 and 3. Proximity operators have a very natural interpretation in terms of denoising. Consider the problem of estimating a vector $\mathbf{a} \in \mathbb{R}^N$ from an observation $\mathbf{b} \in \mathbb{R}^N$, namely $\mathbf{b} = \mathbf{a} + \mathbf{n}$ where $\mathbf{n}$ is additive white Gaussian noise. If we consider the regularization function $\phi(\mathbf{a}; \lambda)$ as the prior information about the vector $\mathbf{a}$, then $P_{\lambda, \phi}(\mathbf{b})$ can be interpreted as a maximum a posteriori (MAP) estimate of the vector $\mathbf{a}$.

### B. Optimality of the Nesting of Proximity Operators

We consider the estimation of the vector $\mathbf{a}$ from $\mathbf{b}$ as noted above. Furthermore, suppose that the vector $\mathbf{a}$ is a jointly sparse vector. The desired optimization problem is as follows

\[
\hat{\mathbf{a}} = \arg\min_{\mathbf{a} \in \mathbb{R}^N} \left\{ \frac{1}{2} \| \mathbf{b} - \mathbf{a} \|_2^2 + \phi_g(\mathbf{a}, \lambda_g) + \phi_e(\mathbf{a}; \lambda_e) \right\},
\]

where $\phi_g(\mathbf{a}; \lambda_g)$ is a regularization term to induce group sparsity and $\phi_e(\mathbf{a}; \lambda_e)$ is a term to induce the element-wise sparsity. In general, the weighting parameters, $\lambda_g > 0, \lambda_e > 0$, can be selected from a given range via cross-validation, by varying one of the parameters and keeping the others fixed. To define the group sparsity regularizer, we consider a general definition for a group of elements from vector $\mathbf{a}$.

**Definition 2.** The $i$th group vector is defined as $\mathbf{a}_i = \Gamma_i \mathbf{a} \in \mathbb{R}^N$, where $\Gamma_i$ is a square matrix with entries 0 or 1 such that in each column or row there is at most one 1, and also for $i \neq j$ (two distinct groups), we have $\Gamma_i^T \Gamma_j = 0$. Furthermore, we have $\mathbf{a} = \sum_{i=1}^{N_g} \mathbf{a}_i$, where $N_g$ is the number of groups in vector $\mathbf{a}$.

We further consider penalty functions $\phi_g(\mathbf{a}; \lambda_g)$ and $\phi_e(\mathbf{a}; \lambda_e)$ of the form:

\[
\phi_g(\mathbf{a}; \lambda_g) = \sum_{j=1}^{N_g} f_g (\| \mathbf{a}_j \|_2; \lambda_g), \quad \text{and}
\]

\[
\phi_e(\mathbf{a}; \lambda_e) = \sum_{i=1}^N f_e (\mathbf{a}[i]; \lambda_e),
\]

where $f_g : \mathbb{R} \to \mathbb{R}$ and $f_e : \mathbb{R} \to \mathbb{R}$ are continuous functions to promote sparsity on groups and elements, respectively, $N$ is the length of vector $\mathbf{a}$, and $N_g$ is the number of groups in vector $\mathbf{a}$. Our goal here is to derive the solution of the optimization problem in (1) using the proximity operators of the functions $f_e$.\[\text{December 10, 2014 DRAFT}\]
and \( f_g \). We state the conditions imposed on the regularizers, in terms of the univariate functions \( f_g(x; \lambda) \) and \( f_e(x; \lambda) \) to promote sparsity and also to control the stability of the solution of the optimization problem in (1).

**Assumption I:** For \( k \in \{ e, g \} \)

i. \( f_k \) is a non-decreasing function of \( x \) for \( x \geq 0 \); \( f_k(0; \lambda) = 0 \); and \( f_k(x; 0) = 0 \).

ii. \( f_k \) is differentiable except at \( x = 0 \).

iii. For \( \forall z \in \partial f_k(0; \lambda) \), then \( |z| \leq \lambda \), where \( \partial f_k(0; \lambda) \) is the sub-differential of \( f_k \) at zero.

iv. There exists a \( \mu \leq \frac{1}{2} \) such that the function \( f_k(x; \lambda) + \mu x^2 \), is convex.

v. \( f_g \) is a **homogeneous** function, i.e., \( f_g(\alpha x; \alpha \lambda) = \alpha^2 f_g(x; \lambda) \) for \( \forall \alpha > 0 \).

vi. \( f_e \) is a **scale invariant** function, i.e., \( f_e(\alpha x; \lambda) = f_e(x; \alpha \lambda) = \alpha f_e(x; \lambda) \) for \( \forall \alpha > 0 \).

It can be observed that conditions (i), (iv), (v), and (vi) ensure the existence of the minimizer of the optimization problem in Eq. (1), and they induce norm properties on the regularizer function. Assumption (ii) promotes sparsity, (iii) controls the stability of the solution in Eq. (1), finally assumption (iv) enables the inclusion of many non-convex functions in the optimization problem. Note that the scale invariant property of \( f_e \) implies that \( f_e \) also satisfies (v) (is a homogeneous function).

Many pairs of regularizer functions satisfy Assumption I. For instance, the \( l_1 \)-norm, namely \( f_g(x; \lambda_g) = |x| \) and \( f_e(x; \lambda_e) = \lambda_e |x| \), satisfy Assumption I (see Appendix A). We note that two recently popularized non-convex functions, SCAD and MCP regularizers, also satisfy Assumption I. It is worth pointing out that SCAD and MCP are more effective in promoting sparsity than the \( l_p \) norms. The SCAD regularizer [17], is given by

\[
 f_g(x; \lambda) = \begin{cases} 
 \lambda |x| & \text{for } |x| \leq \lambda \\
 -\frac{x^2 - 2\mu_S \lambda |x| + \lambda^2}{2(\mu_S - 1)} & \text{for } \lambda < |x| \leq \mu_S \lambda, \\
 \frac{(\mu_S + 1)\lambda^2}{2} & \text{for } |x| > \mu_S \lambda
\end{cases}
\]  

(2)

where \( \mu_S > 2 \) is a fixed parameter, and the MCP regularizer [18], is

\[
 f_g(x; \lambda) = \text{sign}(x) \int_0^{\frac{|x|}{\lambda}} \left( \lambda - \frac{z}{\mu_M} \right)_+ \, dz,
\]  

(3)

where \( (x)_+ = \max(0, x) \) and \( \mu_M > 0 \) is a fixed parameter. In Appendix A we show that Assumption I is met for SCAD and MCP. The following theorem is our main technical result that presents the solution
of the optimization problem in (1) based on the proximity operators of the functions $f_g$ and $f_e$.

**Theorem 1.** Consider functions $f_e$ and $f_g$ that satisfy Assumption I. The optimization problem in (1) can be decoupled as follows

$$
\hat{a}_i = \arg\min_{a_i \in \mathbb{R}^N} \left\{ \frac{1}{2} \|b_i - a_i\|_2^2 + g(a_i; \lambda_g) + E(a_i; \lambda_e) \right\},
$$

where index $i = 1 \ldots N_g$ denotes the group number, $g(a_i; \lambda_g) = f_g(\|a_i\|_2; \lambda_g)$, and $E(a_i; \lambda_e) = \sum_j f_e(a_i[j]; \lambda_e)$.

$$
\hat{a}_i = P_{\lambda_g, g}(P_{\lambda_e, E}(b_i)),
$$

where $P_{\lambda_g, g}$ and $P_{\lambda_e, E}$ are the proximity operators of $E$ and $g$, respectively (see Definition 1) and can be written as

$$
P_{\lambda_g, g}(b) = \frac{P_{\lambda_g, f_g}(\|b\|_2)}{\|b\|_2} b,
$$

$$
[P_{\lambda_e, E}(b)]_j = P_{\lambda_e, f_e}(b[j]).
$$

The proof is provided in Appendix B. This result states that within a group, joint sparsity is achieved by first applying the element proximity operator and then the group proximity operator. We observe the $f_g$ and $f_e$ can be chosen structurally different, the resultant complexity is modest, and we can use our result with any optimization algorithm based on proximity operators, e.g., ADMM [23], proximal gradient methods [25], proximal splitting methods [24], and so on.

**C. Proximity Operator of Sparse-Inducing Regularizers**

In this section, we compute closed form expressions for the proximity operators of the sparsity inducing regularizers introduced in Section II-B using Definition 1 and Remarks 1 to 3. All of the aforementioned regularizers satisfy the condition noted in Remark 2 due to property (iv) in Assumption I. Using Definition 1 we can compute the proximity operators of the $l_p$, SCAD, and MCP regularizers. The proximity operator for the $l_p$-norm is given by, $P_{\lambda, f_p}(x) = \text{sign}(x) \max\{0, \lambda u\}$, where $u^{p-1} + \frac{u}{p} = \frac{|x|}{\lambda_p}$. For $p = 1$, i.e., $f_e(x; \lambda) = \lambda |x|$, the resulting operator is often called *soft-thresholding* (see e.g. [21]),

$$
P_{\lambda, f_e}(x) = \text{sign}(x) \max\{0, |x| - \lambda\}.
$$
The closed form solution of the proximity operator for the SCAD regularizer [17] can be written as,

\[
P_{\lambda, f_s}(x) = \begin{cases} 
0, & \text{if } |x| \leq \lambda \\
-x \cdot \text{sign}(x) \lambda, & \text{if } \lambda \leq |x| \leq 2\lambda \\
\frac{x - \text{sign}(x) \lambda \cdot \mu_S}{1 - \mu_S}, & \text{if } 2\lambda < |x| \leq \mu_S \lambda \\
x & \text{if } |x| > \mu_S \lambda
\end{cases}
\]

and finally the proximity operator for the MCP regularizer [18] is

\[
P_{\lambda, f_s}(x) = \begin{cases} 
0, & \text{if } |x| \leq \lambda \\
\frac{x - \text{sign}(x) \lambda}{1 - \mu_M}, & \text{if } \lambda < |x| \leq \mu_M \lambda \\
x & \text{if } |x| > \mu_M \lambda
\end{cases}
\]

In Fig. 1 MCP and SCAD proximity operators are depicted for \( \lambda = 1 \) and three different values of the parameters \( \mu_S \) and \( \mu_M \). It is clear that when \( \mu_S \) and \( \mu_M \) are large, both SCAD and MCP operators behave like the soft-thresholding operator (for \( x \) smaller than \( \mu_S \lambda \) and \( \mu_M \lambda \), respectively).

In the sequel, first we model a V2V communication system. Then, we show that V2V channel representation in the delay-Doppler domain has both element-wise and group-wise sparsity structures. Finally, we apply our key optimization result derived in this section to estimate the V2V channel using an ADMM algorithm.
III. COMMUNICATION SYSTEM MODEL

We will consider communication between two vehicles as shown in Fig. 2. The transmitted signal \( s(t) \) is generated by the modulation of the transmitted pilot sequence \( s[n] \) onto the transmit pulse \( p_t(t) \) as,

\[
s(t) = \sum_{n=-\infty}^{+\infty} s[n] p_t(t - nT_s),
\]

where \( T_s \) is the sampling period. Note that this signal model is quite general, and encompasses OFDM signals as well as single-carrier signals. The signal \( s(t) \) is transmitted over a linear, time-varying, V2V channel. The received signal \( y(t) \) can be written as,

\[
y(t) = \int_{-\infty}^{+\infty} h(t, \tau) s(t - \tau) d\tau + z(t).
\]

Here, \( h(t, \tau) \) is the channel’s time-varying impulse response, and \( z(t) \) is a complex white Gaussian noise. At the receiver, \( y(t) \) is converted into a discrete-time signal using an anti-aliasing filter \( p_r(t) \). That is,

\[
y[n] = \int_{-\infty}^{+\infty} y(t) p_r(nT_s - t) dt.
\]

The relationship between the discrete-time signal \( s[n] \) and received signal \( y[n] \), using Eqs. 4–6, can be written as,

\[
y[n] = \sum_{m=-\infty}^{+\infty} h_l[n, m] s[n - m] + z[n],
\]

where \( h_l[n, m] \) is the discrete time-delay representation of the observed channel, which is related to the continuous-time channel impulse response \( h(t, \tau) \) as follows,

\[
h_l[n, m] = \int_{-\infty}^{+\infty} h(t + nT_s, \tau) p_t(t - \tau + mT_s) p_r(-t) dtd\tau.
\]

With some loss of generality, we assume that \( p_r(t) \) has a root-Nyquist spectrum with respect to the sample duration \( T_s \), which implies that \( z[n] \) is a sequence of \( i.i.d \) circularly symmetric complex Gaussian random variables with variance \( \sigma^2_z \), and that \( h_l[n, m] \) is causal with maximum delay \( M - 1 \), \( i.e., h_l[n, m] = 0 \)
for $m \geq M$ and $m < 0$. We can then write

$$y[n] = \sum_{m=0}^{M-1} \left( \sum_{k=-K}^{K} H_l[k, m] e^{j \frac{2\pi nk}{2K+1}} \right) s[n - m] + z[n], \quad (8)$$

for $n = 0, 1, ..., N_r - 1$,

where $2K + 1 \geq N_r$, and

$$H_l[k, m] = \frac{1}{2K + 1} \sum_{n=0}^{N_r-1} h_l[n, m] e^{-j \frac{2\pi nk}{2K+1}}, \text{ for } |k| \leq K, \quad (9)$$

is the discrete delay-Doppler, spreading function of the channel.

**IV. GEOMETRY-BASED V2V CHANNEL MODELING**

In this section, we adopt the V2V geometry-based stochastic channel model from [7] and analyze the structure such an model imposes on the delay-Doppler spreading function. The V2V channel model considers four types of multipath components (MPCs): (i) the effective line-of-sight (LOS) component, which may contain the ground reflections, (ii) discrete components generated from reflections of discrete mobile scatterers (MD), e.g., other vehicles, (iii) discrete components reflected from discrete static scatterers (SD) such as bridges, large traffic signs, etc., and (iv) diffuse components (DI). Thus, the V2V channel impulse response can be written as

$$h(t, \tau) = h_{LOS}(t, \tau) + \sum_{i=1}^{N_{MD}} h_{MD,i}(t, \tau)$$

$$+ \sum_{i=1}^{N_{SD}} h_{SD,i}(t, \tau) + \sum_{i=1}^{N_{DI}} h_{DI,i}(t, \tau), \quad (10)$$

where $N_{MD}$ denotes the number of discrete mobile scatterers, $N_{SD}$ is the number of discrete static scatterers and $N_{DI}$ is the number of diffuse scatterers, respectively. Typically, $N_{DI}$ is much larger than $N_{SD}$ and $N_{MD}$ [7]. In the above representation, the multipath components can be modeled as

$$h_i(t, \tau) = \eta_i \delta(\tau - \tau_i)e^{-j2\pi\nu_i t},$$

where $\eta_i$ is the complex channel gain, $\tau_i$ is the delay, and $\nu_i$ is the Doppler shift associated with path $i$ and $\delta(t)$ is the Dirac delta function. The spatial distribution of the scatterers and the statistical properties of the complex channel gains are specified in [7] for rural and highway environments. It is shown that
the channel power delay profile is exponential. Further details about the spatial evolution of the gains can be found in [7], [27]. In geometry-based stochastic channel modeling, point scatterers are randomly distributed in a geometry according to a specified distribution. The position and speed of the scatterers, transmitter, and receiver determine the delay-Doppler parameters for each MPC, which in turn, together with the transmitter and receive pulse shapes, determine $H_l[k,m]$.

We next determine the delay and Doppler contributions of an ensemble of point scatterers of type (i)-(iv) for the road geometry depicted in Fig. 2. If vehicles are assumed to travel parallel with the $x$-axis, the overall Doppler shift for the path from the transmitter (at position TX) via the point scatterer (at position P) to the receiver (at position RX) can be written as [7]

$$\nu(\theta_t, \theta_r) = \frac{1}{\lambda_{\nu}} [(v_T - v_P) \cos \theta_t + (v_R - v_P) \cos \theta_r], \quad (11)$$

where $\lambda_{\nu}$ is the wavelength, $v_T$, $v_P$, and $v_R$ are the speed of the transmitter, scatterer, and receiver, respectively, and $\theta_t$ and $\theta_r$ is the angle of departure and arrival, respectively. The path delay is

$$\tau = \frac{d_1 + d_2}{c_0}, \quad (12)$$

where $c_0$ is the propagation speed, $d_1$ is the distance from TX to P, and $d_2$ is the distance from P to RX. The path parameters $\theta_t$, $\theta_r$, $d_1$, and $d_2$ are easily computed from TX, P, and RX. The delay and Doppler parameters of each component (i)-(iv) can now be specified by Eqs. (11) and (12).

**LOS:** If it exists, the most significant component of the V2V channel is the line of sight (LOS) component, which will have delay and Doppler parameters $\tau_0 = \frac{d_0}{c_0}$ and $\nu_0 = \frac{1}{\lambda_{\nu}} (v_T - v_R) \cos(\theta)$, where $d_0$ is the distance from TX and RX and $\theta$ is the angle between the $x$-axis (i.e., the moving direction) and the line passing through TX and RX.

**Diffuse Scatterers:** The diffuse (DI) scatterers are static ($v_P = 0$) and uniformly distributed in the shadowed regions in Fig. 2. Suppose we place a static scatterer at the coordinates $(x, y)$. The delay-Doppler pair, $(\tau(x, y), \nu(x, y))$, for the corresponding MPC can be calculated from Eqs. (11) and (12). If we fix $y = y_0$ and vary $x$ from $-\infty$ to $+\infty$, the delay-Doppler pair will trace out a U-shaped curve in the delay-Doppler plane, as depicted in Fig. 3.

Repeating this procedure for the permissible $y$-coordinates for the DI scatterers, $|y_0| \in [D/2, d + D/2]$, will result in a family of curves that are confined to a U-shaped region, see Fig. 4. Hence, the DI
scatterers will result in multipath components with delay-Doppler pairs inside this region. The maximum and minimum Doppler of the region is easily found from Eq. (11). In fact, it follows from Eq. (11) that the Doppler parameter of an MPC due to a static scatterer will be confined to the symmetric interval $[-\nu_S, \nu_S]$, where $\nu_S = \frac{1}{\lambda_v}(v_T + v_R)$.

**Static Discrete Scatterers:** The static discrete (SD) scatterers can appear outside the shadowed regions in Fig. 2. In fact, the $y$-coordinates of the SD scatterers are drawn from a Gaussian mixture consisting of two Gaussian pdfs with the same standard deviation $\sigma_{y,SD}$ and means $y_{1,SD}$ and $y_{2,SD}$.

The delay-Doppler pair for an MPC due to an SD scatterer can therefore appear also outside the U-shaped region in Fig. 4. However, since the SD scatterers are static, the Doppler parameter is in the interval $[-\nu_S, \nu_S]$, i.e., the same interval as for the diffuse scatterers.

**Mobile Discrete Scatterers:** We assume that no vehicle travels with an absolute speed exceeding $v_{\text{max}}$. It then follows from Eq. (11) that the Doppler due to a mobile discrete (MD) scatterer is in the interval $[-\nu_{\text{max}}, \nu_{\text{max}}]$, where $\nu_{\text{max}} = \frac{4v_{\text{max}}}{\lambda_v}$. For example, in Fig. 4 the Doppler shift $\nu_p$ is due to an MD scatterer (vehicle) that travels in the oncoming lane ($v_p < 0$).

Based on the analysis above, we can conclude that the delay-Doppler parameters for the multipath...
components can be divided into three regions,

\[ R_1 \triangleq \{(\tau, \nu) \in \mathbb{R}^2 : \tau \in (\tau_0, \tau_0 + \Delta \tau), \nu \in (-\nu_S, \nu_S)\} \]

\[ R_2 \triangleq \{(\tau, \nu) \in \mathbb{R}^2 : \tau \in [\tau_0 + \Delta \tau, \tau_{\text{max}}], |\nu| \in [\nu_S - \Delta \nu, \nu_S]\} \]

\[ R_3 \triangleq \{(\tau, \nu) \in \mathbb{R}^2 : \tau \in [\tau_0, \tau_{\text{max}}], |\nu| \leq \nu_{\text{max}}\} \setminus (R_1 \cup R_2), \]

where \( \tau_{\text{max}} - \tau_0 \) is the maximum significant excess delay for the V2V channel. Here, \( \Delta \tau \) and \( \Delta \nu \) are chosen such that the contributions from all diffuse scatterers are confined to \( R_1 \cup R_2 \). The exact choice of \( \Delta \tau \) is somewhat arbitrary. In this paper, we consider a thresholding rule to compare the noise level and channel components, which results in a particular choice of \( \Delta \tau \); the method is specified in Appendix \( \square \).

However, regardless of method, once \( \Delta \tau \) is chosen, we can compute the height \( \Delta \nu \) of the two strips that make up \( R_2 \). This can be done by placing an ellipsoid with its foci at the transmitter and receiver such that the path from the transmitter to the receiver via any point on the ellipsoid has propagation delay \( \tau_0 + \Delta \tau \). By computing the associated Doppler along the part of ellipsoid that is in the diffuse region (i.e., in the strips just outside the highway, see Fig. 2), we can determine the smallest absolute Doppler value among them as \( \nu' \) and calculate \( \Delta \nu \) as \( \Delta \nu = \nu_S - \nu' \). In Appendix \( \square \) we present a data driven approach to approximate \( \Delta \nu \). Note that the regions gather channel components with similar behavior. Region \( R_1 \),
contains the LOS, ground reflections, and (strong) discrete and diffuse components due scatterers near the transmitter and receiver. In Region $R_2$, the delay-Doppler contribution of static discrete and diffuse scatterers from farther locations appear. Region $R_3$ contains contributions from moving discrete and static diffuse scatters only.

**Remark 4.** In Fig. 4, we see that there are sparse contributions from the SD and MD scatterers in all regions $R_1$ to $R_3$. However, clusters of DI components are confined to $R_1 \cup R_2$. This structure of the DI components can be captured by a group sparsity regularizer. In Sec. [VI] we propose a method based on joint element-wise and group-wise sparsity and Theorem 1 of Section [II-B] to estimate the discrete delay-Doppler representation of V2V channel exploiting this structure.

## V. OBSERVATION MODEL AND LEAKAGE EFFECT

In this section, we show how pulse shaping and a finite-length training sequence can be taken into account when formulating a linear observation model of the V2V channel. We assume that $p_t(t)$ and $p_r(t)$ are causal with support $[0, T_{\text{supp}})$. The contribution to the received signal from one of the $1 + N_{MD} + N_{SD} + N_{DI}$ terms in (10) is of the form

$$s(t) * h_i(t, \tau) = \sum_{l=-\infty}^{\infty} s[l] \eta_i e^{j2\pi \nu_i l} p_t(t - lT_s - \tau_i)$$

$$\approx \sum_{l=-\infty}^{\infty} s[l] \eta_i e^{j2\pi \nu_i (lT_s + \tau_i)} p_t(t - lT_s - \tau_i),$$

where the approximation is valid if we make the (reasonable) assumption that $\nu_i T_{\text{supp}} \ll 1$, and * denotes convolution. If we let $p(t) = p_t(t) * p_r(t)$, we can write the contribution after filtering and sampling as

$$y_i[n] = s(t) * h_i(t, \tau) * p_r(t)|_{t=nT} = \sum_{l=-\infty}^{\infty} s[l] \eta_i e^{j2\pi \nu_i (lT_s + \tau_i)} p((n - l)T_s - \tau_i)$$

$$= \sum_{l=-\infty}^{\infty} s[n - m] \eta_i e^{j2\pi \nu_i ((n - m)T_s + \tau_i)} p(mT_s - \tau_i),$$

and identify $h_i[n, m] = \eta_i e^{j2\pi \nu_i ((n - m)T_s + \tau_i)} p(mT_s - \tau_i)$. Suppose we have access to $h_i[n, m]$ for $n = 0, 1, \ldots, N_r - 1$ and let $\omega = \exp(j2\pi/(2K + 1))$. The $(2K + 1)$-point DFT of $h_i[n, m]$, where we choose
(2K + 1) ≥ Nr, is

\[ H_{l,i}[k, m] = \frac{1}{2K + 1} \sum_{n=0}^{N_r-1} h_i[n, m] \omega^{-nk} \quad k \in K \]

\[ = \eta_i e^{-j2\pi \nu_i(mT_s - \tau_i)} p(mT_s - \tau_i) w(k, \nu_i), \]

where \( K = \{0, \pm 1, \pm 2, \ldots, \pm K\} \) and \( w(k, x) \) is the \((2K + 1)\)-point DFT of a discrete-time complex exponential with frequency \( x \) and truncated to \( N_r \) samples

\[
w(k, x) = \begin{cases} 
\frac{N_r}{2K+1}, & x = k/(2K + 1) \\
\frac{e^{-j\pi((2K+1)x)} \sin(\pi(\frac{k}{2K+1}-x))}{\sin(2\pi(\frac{k}{2K+1}-x))}, & \text{otherwise}
\end{cases}
\]

We note that the leakage in the delay and Doppler plane is due to the non-zero support of \( p \) and \( w \). The leakage with respect to Doppler decreases with the observation length, \( N_r \), and the leakage with respect to delay decreases with the bandwidth of the transmitted signal. For simplicity, and for the purpose of deriving a relative low-complexity method to compensate for the leakage, we assume that we can write \( \nu_i T_s = k_i/(2K + 1) \) and \( \tau_i = m_i T_s \), where \( k_i \) and \( m_i \) are integers for all \( i \). We can then write

\[ H_{l,i}[k, m] = \eta_i g[k, m, k_i, m_i], \quad k \in K, m \in M \]
where $\mathcal{M} = \{0, 1, \ldots, M - 1\}$ and

$$g[k, m, k', m'] = \omega^{-k'(m - m')}w(k - k', 0)p((m - m')T_s).$$

Due to the linearity of the discrete Fourier transform, we can conclude that the channel with leakage is given by

$$H_l[k, m] = \sum_i H_{l,i}[k, m] = \sum_i \eta_i g[k, m, k_i, m_i],$$

where the summation is over the LOS component and all the $N_{MD} + N_{SD} + N_{DI}$ scatterers. The channel without leakage is

$$H[k, m] = \sum_i \eta_i \delta[k - k_i] \delta[m - m_i],$$

where $\delta[n]$ is the Kronecker delta function. The channels in (13) and (14) can be represented for $k \in \mathcal{K}$ and $m \in \mathcal{M}$ by the vectors $x_l \in \mathbb{C}^N$ and $x \in \mathbb{C}^N$, respectively, where $N = |\mathcal{K}||\mathcal{M}| = (2K + 1)M$, as

$$x_l = \text{vec} \begin{bmatrix} H_l[-K, 0] & \cdots & H_l[-K, M - 1] \\ \vdots & \cdots & \vdots \\ H_l[K, 0] & \cdots & H_l[K, M - 1] \end{bmatrix}$$

$$x = \text{vec} \begin{bmatrix} H[-K, 0] & H[-K, M - 1] \\ \vdots & \vdots \\ H[K, 0] & \cdots & H[K, M - 1] \end{bmatrix}. \quad (15)$$

The relationship between $x_l$ and $x$ can be written as

$$x_l = Gx,$$

where $\text{vec}(A)$ is the vector formed by stacking the columns of $A$ on the top of each other.
where $G \in \mathbb{C}^{N \times N}$ is defined as

$$G = \begin{bmatrix}
\text{vec}(G_0) & \text{vec}(G_1) & \cdots & \text{vec}(G_{N-1})
\end{bmatrix} \quad (17)$$

$$G_j = \begin{bmatrix}
g[-K,0,k',m'] & \cdots & g[-K,M-1,k',m'] \\
\vdots & \ddots & \vdots \\
g[K,0,k',m'] & \cdots & g[K,M-1,k',m']
\end{bmatrix},$$

where the one-to-one correspondence between $j = 0, 1, \ldots, (2K + 1)M - 1$ and $(k', m') \in \mathcal{K} \times \mathcal{M}$ is given by $j = m'(2K + 1) + k' + K$.

The structure for $G$ is a direct consequence of how we vectorize $H_l[k, m]$ in (15). If we consider a different way of vectorizing $H_l[k, m]$, then the leakage matrix $G$ needs to be recomputed accordingly, by appropriate permutation of the columns and rows of leakage matrix defined in (17). As $K$, $M$, and the pulse shape are known, $G$ is completely determined in (16). Thus, we can utilize the relationship in (16) to compensate for leakage.

Consider that the source vehicle transmits a sequence of $N_r + M - 1$ pilots, $s[n]$, for $n = -(M - 1), -(M - 2), \ldots, N_r - 1$, over the channel. We collect the $N_r$ received samples in a column vector

$$y = [y[0], y[1], \ldots, y[N_r - 1]]^\top.$$

Using (8), we have the following matrix representation:

$$y = Sx_l + z, \quad (18)$$

where $z \sim \mathcal{CN}(0, \sigma^2 z I_{N_r})$ is a Gaussian noise vector, and $S$ is a $N_r \times M(2K + 1)$ block data matrix of the form

$$S = [S_0, \ldots, S_{M-1}],$$

where each block $S_m \in \mathbb{C}^{N_r \times (2K + 1)}$ is of the form

$$S_m = \text{diag} \{s[-m], \ldots, s[N_r - m - 1]\} \Omega,$$

for $m = 0, 1, \ldots, M - 1$, and $\Omega \in \mathbb{C}^{N_r \times (2K + 1)}$ is a Vandermonde matrix, $\Omega[i, j] = \omega^{i(j-K)}$, where
\( i = 0, 1 \ldots, N_r - 1 \) and \( j = 0, 1, \ldots, 2K \). Finally, by combining (18) and (16) we have
\[
y = Sx_l + z = Ax + z,
\]
where \( A = SG \).

### VI. CHANNEL ESTIMATION

Based on our analysis in Section IV, the components in the vector \( x \) have both element- and group-wise sparsity. Thus, we consider regularizations to exploit the jointly sparse structure of the V2V channel as follows
\[
\hat{x} = \arg\min_{x \in \mathbb{C}^N} \left\{ \frac{1}{2} \| y - Ax \|_2^2 + \phi_g(|x|; \lambda_g) + \phi_e(|x|; \lambda_e) \right\}, \quad (P_0)
\]
where \(|x| = [|x[1]|, \ldots, |x[N]|]^T \) with \( N = M(2K + 1) \) and \(|x[i]| = \sqrt{\text{Re}(x[i])^2 + \text{Im}(x[i])^2} \). Here the regularization functions are
\[
\phi_g(|x|; \lambda_g) = \sum_{j=1}^{N_g} f_g(\|x_j\|_2; \lambda_g),
\]
\[
\phi_e(|x|; \lambda_e) = \sum_{i=1}^{N} f_e(|x[i]|; \lambda_e).
\]

Next, we develop a proximal alternating direction method of multipliers (ADMM) algorithm to solve problem \( P_0 \). Problem \( P_0 \) can be rewritten using an auxiliary variable \( w \) as follows,
\[
\min_{x, w \in \mathbb{C}^N} \frac{1}{2} \| y - Ax \|_2^2 + \phi_g(|w|; \lambda_g) + \phi_e(|w|; \lambda_e)
\]
\[
\text{s.t. } w = x \tag{19}
\]

For the optimization problem in (19), ADMM consists of the following iterations,

- **Initialize**: \( \rho \neq 0, \lambda_{pg} = \frac{\lambda}{\rho}, \lambda_{pc} = \frac{\lambda}{\rho}, \theta^0 = w^0 = 0, A_0 = (\rho^2 I + A^H A)^{-1}, \) and \( x_0 = A_0 A^H y \).

- **Update-\( x \):**
\[
x^{n+1} = \rho^2 A_0 (w^n - \theta^n_{\rho}) + x_0,
\]
where \( \theta^n_{\rho} = \theta^n_{\rho^2} \).
• **Update-w:** for $i = 1, 2, \ldots, N_g$,

$$
\mathbf{w}_i^{n+1} = \arg\min_{\mathbf{w}_i} \frac{1}{2} \left\| \mathbf{x}_i^{n+1} + \theta_{\rho i} - \mathbf{w}_i \right\|^2_2 + g(\left| \mathbf{w}_i \right|; \lambda_{pg}) + E(\left| \mathbf{w}_i \right|; \lambda_{pe})
$$

(20)

where $\mathbf{x}_i = \Gamma_i \mathbf{x}$, $\mathbf{w}_i = \Gamma_i \mathbf{w}$, $\theta_{\rho i} = \Gamma_i \theta_{\rho}$, and

$$
E(\left| \mathbf{w}_i \right|; \lambda_{pe}) = \sum_j f_e \left( \frac{\left| w_i[j] \right|}{\rho}; \lambda_{pe} \right),
$$

(21)

$$
g(\left| \mathbf{w}_i \right|; \lambda_{pg}) = f_g \left( \frac{\| \mathbf{w}_i \|_2}{\rho}; \lambda_{pg} \right).
$$

(22)

• **Update-dual variable-$\theta$:**

$$
\theta_{\rho}^{n+1} = \theta_{\rho}^n + (\mathbf{x}_i^{n+1} - \mathbf{w}_i^{n+1}).
$$

The details of this derivation are provided in Appendix D. Note that both $A_0$ and $x_0$ are known and can be computed in advance. In the update-w step, the index $i$ denotes the group number, thus, this step can be done in parallel for all groups. Since the vectors in optimization problem for updating $\mathbf{w}$ in (20) are complex vectors, Theorem 1 in Section II-B cannot directly be applied to find a closed-form solution for this optimization problem. However, in order to apply Theorem 1, we introduce the following notation and lemma. The vector $\mathbf{w} \in \mathbb{C}^n$ can be written as $\mathbf{w} = |\mathbf{w}| \odot \text{Phase}(\mathbf{w})$, where the $n$th element of $\text{Phase}(\mathbf{w})$ is $\exp(j \text{Ang}(\mathbf{w}[n]))$, and $\text{Ang}(\mathbf{w}[n])$ is the angle of $\mathbf{w}[n]$ in polar form, i.e., $\mathbf{w}[n] = |\mathbf{w}[n]| \exp(j \text{Ang}(\mathbf{w}[n]))$, and $\odot$ is component-wise multiplication (Schur product).

**Lemma 1.** For any $\mathbf{c} \in \mathbb{C}^N$

$$
\arg\min_{\mathbf{z} \in \mathbb{C}^N} \| \mathbf{c} - \mathbf{z} \|_2^2 = \text{Phase}(\mathbf{c}) \odot \arg\min_{|\mathbf{w}| \in \mathbb{R}^n} \| \mathbf{c} - |\mathbf{w}| \|_2^2.
$$

The proof is provided in Appendix E. Since the two last terms in (20) are independent of the phase of $\mathbf{w}_i$, we use Lemma 1 to write the $i$th group problem in (20) as

$$
\mathbf{w}_i^{n+1} = \text{Phase} \left( \mathbf{x}_i^{n+1} + \theta_{\rho i} \right) \odot \left( \arg\min_{|\mathbf{w}_i|} \frac{1}{2} \left\| \mathbf{x}_i^{n+1} + \theta_{\rho i} - |\mathbf{w}_i| \right\|^2_2 + g(\left| \mathbf{w}_i \right|; \lambda_{pg}) + E(\left| \mathbf{w}_i \right|; \lambda_{pe}) \right)
$$

(23)
Now, the vectors in the optimization problem in (23) involve only real vectors, therefore the solution for this optimization problem can be directly computed using Theorem 1. We determine a closed form solution for the update-\(w\) step in Corollary 1 below, using the proximity operators of the univariate functions \(f_e\) and \(f_g\). This update rule is a direct consequence of Theorem 1.

**Corollary 1.** The second step, update-\(w\), can be performed as follows

\[
    w^{n+1}_i = P_{\lambda_g g} \left( P_{\lambda_e E} \left( |x^{n+1}_i + \theta^n_{\rho_i}| \right) \right) \odot \text{Phase} \left( x^{n+1}_i + \theta^n_{\rho_i} \right),
\]

where \(E\) and \(g\) are defined in Equations (21) and (22).

Based on Corollary 1, the update-\(w\) step only depends on the proximity operators of the regularizer functions \(f_e\) and \(f_g\). In summary, the proposed algorithm to estimate the channel vector \(x\) from the received data vector \(y\) can be written as:

**PROPOSED V2V CHANNEL ESTIMATION**

**Input:** \(y\), \(A\), \(\lambda_g\), \(\lambda_e\), \(\rho\).

**Initialize:** \(w^0 = \theta^0 = 0\).

**Pre-computation:**

\[
    A_0 = \left( \rho^2 I + A^H A \right)^{-1},
\]

\[
    x^0 = A_0 A^H y.
\]

**For** \(n = 0\) to END

\[
    x^{n+1} = \rho^2 A_0 \left( w^n - \theta^n_{\rho} \right) + x^0
\]

\[
    w^{n+1}_i = P_{\lambda_g g} \left( P_{\lambda_e E} \left( |x^{n+1}_i + \theta^n_{\rho_i}| \right) \right) \odot \text{Phase} \left( x^{n+1}_i + \theta^n_{\rho_i} \right)
\]

for \(i = 1, 2, \ldots, N_g\)

\[
    \theta^{n+1}_{\rho} = \theta^n_{\rho} + (x^{n+1} - w^{n+1})
\]

**End**

**Output:** Vector \(x\).

The proposed algorithm for channel estimation clearly incurs modest complexity.
VII. Numerical Results

In this section, we demonstrate the performance gains that can be achieved with our proposed, structured, estimation using both convex and non-convex sparsity-inducing regularizers, relative to prior methods such as Wiener filtering [5], the Hybrid Sparse/Diffuse (HSD) estimator [8–10], and the $l_1$-based compressive estimator [11], [12].

To simulate the channel, we consider a geometry with length of 1 km around the transmitter-receiver pair, road width $D = 50$ m, and the width of the diffuse strip around the road $d = 25$ m, as Fig. 2. The locations of the transmitter and receiver are chosen in this geometry with distance 100 m to 200 m from each other. The speed of the transmit and receive vehicles are chosen randomly from the interval [60, 160] (km/h), the speed limits for a highway. It is assumed that the number of MD scatterers $N_{MD} = 10$, and their speed are also chosen randomly from the interval [60, 160] (km/h); we have $N_{SD} = 10$ and $N_{DI} = 1000$, SD and DI scatterers, respectively [7]. Using these parameters, we compute the delay and Doppler values for each scatterer. The statistical parameter values for different scatterers are selected as in Table I of [7], which are determined from experimental measurements. The scatterer amplitudes were randomly drawn from zero mean, complex Gaussian distributions with three different power delay profiles for the LOS and mobile discrete (MD) scatterers, static discrete scatterers, and diffuse scatterers. We assume that the mean power of the static scatterers is 10 dB less than the mean power of the LOS and MD scatterers, and the mean power of the diffuse scatterers also is 20 dB less than the mean power of the LOS and MD scatterers [7]. Furthermore, we have considered $f_c = 5.8$ GHz, $T_s = 5$ ns, $N_r = 4096$, $K = 2048$, and $M = 512$. The pilot samples are drawn from a zero-mean, unit variance Gaussian distribution. The interpolation/anti-aliasing filters $p_l(t) = p_r(t)$ are root-raised-cosine filters with roll-off factor 0.25. The required regularization parameters were found by trial and error. We consider the total number of groups to be $N_g = 2048$ with a common size. These value of penalty parameters $\lambda_e$ and $\lambda_g$ were found by trial and error. Performance is measured by the normalized mean square error (NMSE), normalized by the mean energy of the channel coefficients. The NMSE is defined as $\text{NMSE} = \frac{E\{\|\hat{x} - x\|^2_2/\|x\|^2_2\}}{E\{\|z\|^2_2\}}$, where $\hat{x}$ is the estimated channel vector and SNR defined as $\text{SNR} = \frac{E\{\|y - z\|^2_2\}}{E\{\|z\|^2_2\}}$.

Fig. 5 depicts the MSE using our proposed estimator, our previously proposed hybrid sparse/diffuse (HSD) estimator [8–10], and the Wiener based estimator of [5]. The HSD estimator is adapted for
V2V channels [8]-[10]; the HSD estimator considers the channel components as a summation of sparse and diffuse components, i.e., $\mathbf{x} = \mathbf{x}_s + \mathbf{x}_d$. Sparse components, $\mathbf{x}_s$, are modeled by the element-wise product of an unknown deterministic amplitude, $\mathbf{a}_s$, and a random Bernoulli vector, $\mathbf{b}_s$, $\mathbf{x}_s = \mathbf{a}_s \odot \mathbf{b}_s$. Furthermore, the diffuse components are assumed to follow a Gaussian distribution with exponential profile, $\mathbf{x}_d \sim \mathcal{N}(\mathbf{0}, \Sigma_d)$ where $\Sigma_d$ is diagonal and is the Kronecker product of covariance matrices of the channel component vectors with common Doppler values [10]. The profile parameters are retrieved using the expectation-maximization algorithm [8].

The HSD model estimation procedure can be simply stated as: first, the location of sparse components, i.e., the Bernoulli vector, are determined as $\hat{\mathbf{b}}_s[k] = 1 \left( |x_{LS}[k]|^2 \geq \gamma \left( (\Sigma_e[k,k])^{-1} + \Sigma_d[k,k] \right) \right)$, where $1(.)$ is the indicator function, $x_{LS} = (\rho^2 \mathbf{I} + \mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{y}$ is the regularized LS estimation, $\Sigma_e$ is the covariance matrix of the noise vector after LS estimation and $\gamma$ is a known parameter [10]. Then, the sparse components are computed as $\hat{\mathbf{x}}_s = x_{LS} \odot \hat{\mathbf{b}}_s$, and finally the diffuse components can be estimated as $\hat{\mathbf{x}}_d = \Sigma_d (\Sigma_d + \Sigma_e^{-1})^{-1} (x_{LS} - \hat{\mathbf{x}}_s)$ [8], [10].

The Wiener based estimator [5] uses the Wiener filter structure to estimate the channel as $\hat{\mathbf{x}} = \mathbf{R}_x \mathbf{A}^H (\mathbf{A} \mathbf{R}_x \mathbf{A}^H + \sigma_z^2 \mathbf{I})^{-1} \mathbf{y}$ with $\mathbf{R}_x = \mathbb{E}\{\mathbf{x}\mathbf{x}^H\}$, which is not known at the receiver side, but is approximated by assuming a delay-Doppler scattering function prototype with flat spectrum in a 2D region as in [6]. The maximum path delay and Doppler in the support of scattering function are considered.
as $\tau_{\text{max}} = 1.5\mu s$ and $\nu_{\text{max}} = 860$ Hz, respectively. As we will see, this idealized scattering function assumption results in degraded performance.

It is clear from Fig. 5 that there is performance enhancement when we consider the joint structural information of the channel in the delay and Doppler domain. For our proposed method, we have considered different types of regularizers. In Fig. 5, **Structured-Soft** corresponds to our proposed structured estimator with a soft-thresholding regularizer, i.e., $f_g(x; \lambda_g) = \lambda_g|x|$ and $f_e(x; \lambda_e) = \lambda_e|x|$; **Structured-SCAD** corresponds to the case where $f_g(x; \lambda_g)$ is the SCAD regularizer function with $\mu_S = 3$ in Equation (2) and $f_e(x; \lambda_e) = \lambda_e|x|$; and **Structured-MCP** corresponds to the case where $f_g(x; \lambda_g)$ is the MCP regularizer function with $\mu_M = 2$ in Equation (3) and $f_e(x; \lambda_e) = \lambda_e|x|$, respectively. The penalty parameters for the simulations have been considered as $\lambda_e \in [0.25, 1.5]$ and $\lambda_g \in [5, 8]$. Fig. 5 shows that the non-convex regularizers improve estimation quality by about 3 dB at low SNR and 5 dB at high SNR values with the same computational complexity compared to the convex soft-thresholding regularizer.

There is also a significant improvement in SNR due to the exploitation of the V2V channel structure in the delay-Doppler domain. For instance, to achieve $\text{MSE} = -20$ dB, the performance curve related to the the structured estimator shows a 6dB improvement in SNR compared to that for the HSD estimator, and 15 dB improvement in SNR compared to that for the Wiener Filter estimator.

Next, in Fig. 6 we consider the effect of promoting group-sparsity in the performance of V2V channel

![Fig. 6. Effect of group sparsity in channel estimation - Comparison between only element-wise sparsity ($\lambda_g = 0$) (LASSO method/$l_1$-approximation), only group-wise sparsity ($\lambda_e = 0$), and joint element- and group-sparsity algorithms such as Joint-Soft, Joint-SCAD, and Joint-SCAD-2.](image-url)
estimation using the sparse regularization approach. We show that the group-sparsity inducing term improves the channel estimation performance. In order to apply our approach to estimate the channel, we have considered different assumptions by which to determine the estimators. In the first scenario, we design estimators under the assumption that the entire channel exhibits only *element-wise* sparsity, i.e., $\lambda_g = 0$. In the second scenario, we design estimators under the assumption that the entire channel exhibits only *group-wise* sparsity, i.e., $\lambda_e = 0$. In the third scenario, we design algorithm for both joint element- and group-wise sparsity; we use a soft-thresholding, $l_1$ regularizer to estimate the channel (Joint-Soft). In the Joint-SCAD scenario, we assume that the channel follows both the element- and group-wise sparsity but we use the SCAD regularizer for group sparsity regularization to estimate the channel. In the last curve (Joint-SCAD-2), we consider the joint sparsity structure with SCAD regularizer to estimate the channel but with two different grouping sizes for the regions $R_1 \cup R_2$ and $R_3$, based on the algorithm proposed in Appendix C. The ratio of the number of groups for regions $R_1 \cup R_2$ and $R_3$, is $\frac{N_{gd}}{N_{gs}} \approx 0.02$.

In Fig. 6, we see that there is significant improvement (6 dB to 10 dB) of joint-sparsity estimation compared to only element-wise estimation over the plotted SNR range. We see that considering different group sizes for key regions can improve the quality of estimation up to 4 dB for high SNR.

Finally, in Fig. 7 we consider the effect of leakage compensation (Section V) on the performance of
sparse estimator of V2V channels, such as the HSD estimator and our proposed joint sparsity estimator using SCAD regularizer function for group sparsity. From the results in Fig. 7, we observe that the uncompensated leakage effect reduces performance severely, more than 7 dB, particularly at higher SNR, due to the channel mismatch introduced by the channel leakage.

VIII. CONCLUSIONS

We provide a comprehensive analysis of V2V channels in the delay-Doppler domain using geometry-based stochastic channel modeling. Our modeling reveals that the V2V channel model has three key regions, and these regions exhibit different sparse/hybrid structures which can be exploited to improve channel estimation in this domain. Using this structure, we have proposed a joint element- and group-wise sparse approximation method using general regularization functions. We prove that for the needed optimization, the solution result in a nested estimation of the channel vector based on the group and element wise penalty functions. Our proposed method exploits proximity operators and the alternating direction method of multipliers, resulting in a modest complexity approach with excellent performance. We characterized the leakage effect on the sparsity of the channel and robustified the channel estimator by explicitly compensating for pulse shape leakage at the receiver using the leakage matrix. Simulation results reveal that exploiting the joint sparsity structure with non-convex regularizers yields a 5 dB improvement in SNR compared to our previous state of the art HSD estimator in low SNR. Furthermore, we showed that structured estimators yield 10 dB to 15 dB improvement in SNR compared to unstructured approaches at high SNR.

APPENDIX A

SPARSITY INDUCING REGULARIZERS

In this section, we show that the regularizer functions summarized in Section II-B satisfy Assumptions I. We note that showing the assumptions for the convex regularizers is straightforward and thus omitted; we focus on the non-convex regularizers which will be used to induce group sparsity.
• **SCAD regularizer** [17]: This penalty takes the form

\[
fg(x; \lambda) = \begin{cases} 
\lambda|x|, & \text{for } |x| \leq \lambda \\
\frac{-x^2 - 2\mu S\lambda|x| + \lambda^2}{2(\mu S - 1)} & \text{for } \lambda < |x| \leq \mu S \lambda \\
\frac{(\mu S + 1]\lambda^2}{2} & \text{for } |x| > \mu S \lambda
\end{cases}
\]

where \(\mu_S > 2\) is a fixed parameter. This penalty function is non-decreasing, \(fg(0; \lambda) = fg(x; 0) = 0\) and it is clear that \(fg(\alpha x; \alpha \lambda) = \alpha^2 fg(x; \lambda)\) for \(\forall \alpha > 0\). The derivative of the SCAD penalty function for \(x \neq 0\) is given by

\[
fg'(x; \lambda) = \text{sign}(x) \left( \frac{(\mu S \lambda - |x|)}{\mu_S - 1} + \mathbb{I}\{|x| > \lambda\} + \lambda \mathbb{I}\{|x| \leq \lambda\} \right)
\]

where \(\mathbb{I}(.)\) is the indicator function, and any point in interval \([-\lambda, +\lambda]\) is a valid subgradient at \(x = 0\), so condition (iv) is satisfied. For \(\mu = \frac{1}{\mu S - 1}\) the function \(fg(x; \lambda) + \mu x^2\) is also convex. Thus, Assumption I holds for the SCAD penalty function with \(\mu_S \geq 3\).

• **MCP regularizer** [18]: This penalty takes the form

\[
fg(x; \lambda) = \text{sign}(x) \int_0^{|x|} \left( \lambda - \frac{z}{\mu M} \right)_+ dz
\]

where \(\mu_M > 0\) is a fixed parameter. This penalty function is non-decreasing for \(x \geq 0\) and \(fg(0; \lambda) = fg(x; 0) = 0\). Also, \(fg(\alpha x; \alpha \lambda) = \alpha^2 fg(x; \lambda)\) for \(\forall \alpha > 0\). The derivative of the MCP penalty function for \(x \neq 0\) is given by

\[
fg'(x; \lambda) = \text{sign}(x) \left( \lambda - \frac{|x|}{\mu M} \right)_+
\]

and any point in \([-\lambda, +\lambda]\) is a valid subgradient at \(x = 0\). For \(\mu = \frac{1}{\mu M}\) the function \(fg(x; \lambda) + \mu x^2\) is also convex. Thus, Assumption I holds for the MCP penalty function with \(\mu_M \geq 2\).

**APPENDIX B**

**PROOF OF THEOREM**

We first prove two lemmas, needed for the proof of Theorem[1] and Corollary[1].

**Lemma 2.** Consider that \(g(a; \lambda) = f\left(\frac{||a||_2}{\rho}; \lambda\right)\), where \(f(x; \lambda)\) is a non-decreasing function of \(x\).
Furthermore, \( f(x; \lambda) \) is a homogeneous function, i.e., \( f(\alpha x; \alpha \lambda) = \alpha^2 f(x; \lambda) \). Then,

i) \( P_{\lambda,g}(a) = \gamma a \), where \( \gamma \in [0, 1] \).

ii) \( \gamma = \begin{cases} \frac{1}{\|a\|_2} P_{\rho \lambda, \frac{\rho}{\rho^2}}(\|a\|_2) & \text{if } \|a\|_2 > 0 \\ 0 & \text{if } \|a\|_2 = 0 \end{cases} \)

**Proof:** i). For every \( z \), we can write \( z = a^\perp + \gamma a \), where \( a^\perp \perp a \) and \( \gamma \in \mathbb{R} \). Therefore, based on the proximity operator definition we have,

\[
P_{\lambda,g}(a) = \arg\min_z \left\{ \frac{1}{2} \|a - z\|_2^2 + g(z; \lambda) \right\} = \arg\min_{z=a^\perp + \gamma a} \left\{ \frac{1}{2} \|a - \gamma a - a^\perp\|_2^2 + g(\gamma a + a^\perp; \lambda) \right\}
\]

Since \( \|\gamma a + a^\perp\|_2 \geq \max\{\|\gamma\|_2, \|a^\perp\|_2\} \) and \( f \) is a non-decreasing function, we have

\[
g(\gamma a + a^\perp; \lambda) \geq g(\gamma a; \lambda).
\]

Therefore, \( a^\perp = 0 \) in the optimization problem (24) and we can rewrite it as,

\[
P_{\lambda,g}(a) = \arg\min_{\gamma} \left\{ \frac{1}{2} (\gamma - 1)^2 \|a\|_2^2 + g(\gamma a; \lambda) \right\}.
\]

The two terms in the objective function in (25) are increasing when \( \gamma \) increases from \( \gamma = 1 \) or decreases from \( \gamma = 0 \). Hence, the minimizer lies in the interval \([0, 1]\). Therefore, we have \( P_{\lambda,g}(a) = \gamma a \), where \( \gamma \in [0, 1] \).

ii). Let \( t = \gamma \|a\|_2 \), then the optimization problem in (25) for \( \|a\|_2 > 0 \), can be written as,

\[
P_{\lambda,g}(a) = \frac{a}{\|a\|_2} \arg\min_{t \in [0, \|a\|_2]} \left\{ \frac{1}{2} (\|a\|_2 - t)^2 + f\left(\frac{t}{\rho}; \lambda\right) \right\} = \frac{a}{\|a\|_2} \arg\min_{t \in [0, \|a\|_2]} \left\{ \frac{1}{2} (\|a\|_2 - t)^2 + \frac{1}{\rho^2} f\left(t; \rho \lambda\right) \right\} = \frac{a}{\|a\|_2} P_{\rho \lambda, \frac{\rho}{\rho^2}}(\|a\|_2).
\]

Equality (a) is due to the homogeneity of function \( f \), i.e., \( f\left(\frac{t}{\rho}; \lambda\right) = f\left(\frac{t}{\rho}; \frac{\rho \lambda}{\rho}\right) = \frac{1}{\rho^2} f\left(t; \rho \lambda\right) \). Equality (b) is due to the definition of the proximal operator. Thus,

\[
\gamma = \frac{1}{\|a\|_2} P_{\rho \lambda, \frac{\rho}{\rho^2}}(\|a\|_2),
\]
Lemma 3. If the function \( f(x; \lambda) \) is homogenous i.e., \( f(\alpha x; \alpha \lambda) = \alpha^2 f(x; \lambda) \) for all \( \alpha > 0 \), then

\[
P_{\alpha \lambda, f}(\alpha b) = \alpha P_{\lambda, f}(b)
\]

for \( \forall b \in \mathbb{R} \) and \( \lambda > 0 \).

Proof: By definition of the proximity operator, we have

\[
P_{\alpha \lambda, f}(\alpha b) = \arg\min_x \left\{ \frac{1}{2} (\alpha b - x)^2 + f(x; \alpha \lambda) \right\}
\]

Consider \( x = \alpha z \), and using the homogenous properties of \( f \), we have

\[
P_{\alpha \lambda, f}(\alpha b) = \alpha \arg\min_z \left\{ \frac{\alpha^2}{2} (b - z)^2 + f(z; \lambda) \right\} = \alpha P_{\lambda, f}(b).
\]

Proof of Theorem \([\text{I}]\): Since the regularizer functions \( \phi_e \) and \( \phi_g \) are separable, it is easy to show that the solution of optimization problem in Equation \([\text{I}]\) can be computed in parallel for all the groups as,

\[
\hat{a}_i = \arg\min_{a_i \in \mathbb{R}^N} \left\{ \frac{1}{2} \|b_i - a_i\|^2 + g(a_i; \lambda_g) + E(a_i; \lambda_e) \right\}
\]

for \( i = 1, \ldots, N_g \),

where \( g(a_i; \lambda_g) = f_g (\|a_i\|_2; \lambda_g) \) and \( E(a_i; \lambda_e) = \sum_j f_e (a_i[j]; \lambda_e) \). For the sake of simplicity in notation of the proof for Theorem \([\text{I}]\) and Corollary \([\text{I}]\) we drop the group index and we consider

\[
\hat{a} = \arg\min_a \left\{ \frac{1}{2} \|b - a\|^2 + g(a; \lambda_{pg}) + E(a; \lambda_{pe}) \right\}
\]

where \( g(a; \lambda_{pg}) = f_g (\frac{\|a\|_2}{\rho}; \lambda_{pg}) \) and \( E(a; \lambda_{pe}) = \sum_j f_e (\frac{a[j]}{\rho}; \lambda_{pe}) \). Here \( \lambda_{pg} = \frac{\lambda_g}{\rho} \) and \( \lambda_{pe} = \frac{\lambda_e}{\rho} \).

Note that for \( \rho = 1 \), we have the claim in Theorem \([\text{I}]\).

Assume \( v = P_{\lambda_{pe}, E}(b) \) and \( u = P_{\lambda_{pg}, g}(v) \).

Based on above definitions, to prove the claim of Theorem \([\text{I}]\) we need to show that \( a = u \) is the minimizer of \( J(a) = \frac{1}{2} \|b - a\|^2 + g(a; \lambda_{pg}) + E(a; \lambda_{pe}) \). To prove this claim, we consider two cases: I: \( u \neq 0 \), and II: \( u = 0 \).

Case (I): \( u \neq 0 \). Since \( u = P_{\lambda_{pg}, g}(v) \) and \( g(a; \lambda_{pg}) = f_g (\frac{\|a\|_2}{\rho}; \lambda_{pg}) \), and \( f_g \) is a homogenous non-decreasing function, by Lemma \([\text{II}]\) we have \( u = \gamma v \), for some \( \gamma \in (0, 1] \). Furthermore, \( u \) should satisfy
30

the first order optimality condition for the objective function in

\[ u = \arg \min_a \left\{ \frac{1}{2} \| v - a \|_2^2 + g(a; \lambda_{\rho g}) \right\}, \]

namely

\[ 0 \in u - v + \partial g(u; \lambda_{\rho g}). \quad (26) \]

Using the definition of the proximity operator and Remark 1, we have \[ [P_{\lambda_{\rho e}, E}(b)]_i = P_{\lambda_{\rho e}, f_e} \left( \frac{b[i]}{\rho} \right). \]

Since \( f_e \) is a homogeneous function, using Lemma 3, we have

\[ P_{\gamma \lambda_{\rho e}, f_e} \left( \frac{b[i]}{\rho} \right) = \gamma P_{\lambda_{\rho e}, f_e} \left( \frac{b[i]}{\rho} \right) \]
equivalently,

\[ P_{\gamma \lambda_{\rho e}, E}(\gamma b) = \arg \min_a \left\{ \frac{1}{2} \| \gamma b - a \|_2^2 + E(a; \gamma \lambda_{\rho e}) \right\} \]

\[ = \gamma \arg \min_z \left\{ \frac{1}{2} \| b - z \|_2^2 + E(z; \lambda_{\rho e}) \right\} \]

\[ = \gamma P_{\lambda_{\rho e}, E}(b) = \gamma v = u \]

and by the first order optimality condition (of \( u \)) for the objective function in (27), we have

\[ 0 \in u - \gamma b + \partial E(u; \gamma \lambda_{\rho e}). \]

Since \( \gamma \neq 0 \), above Equation can be rewritten as

\[ 0 \in v - b + \frac{1}{\gamma} \partial E(u; \gamma \lambda_{\rho e}). \quad (28) \]

Since \( E(u; \lambda_{\rho e}) = \sum_j f_e \left( \frac{u[j]}{\rho}; \lambda_{\rho e} \right) \), applying scale invariant property of function \( f_e \), i.e., \( f_e \left( \frac{u[j]}{\rho}; \gamma \lambda_{\rho e} \right) = \gamma f_e \left( \frac{u[j]}{\rho}; \lambda_{\rho e} \right) \), we have \( \frac{1}{\gamma} \partial E(u; \gamma \lambda_{\rho e}) = \partial E(u; \lambda_{\rho e}) \). Therefore, we can rewrite (28) as

\[ 0 \in v - b + \partial E(u; \lambda_{\rho e}). \quad (29) \]

Summing Equations (26) and (29), we have

\[ 0 \in u - b + \partial g(u; \lambda_{\rho g}) + \partial E(u; \lambda_{\rho e}). \]

which is the first order optimality of \( u \) for the objective function \( J(a) \).
Case (II): \( u = 0 \). Here, we need to show the first-order optimality conditions for \( u = 0 \) for the objective function \( J(a) \), i.e.,

\[
0 \in \{ u - b + \partial g(u; \lambda_{pg}) + \partial E(u; \lambda_{pe}) \} |_{u=0} = \partial g(0; \lambda_{pg}) + \partial E(0; \lambda_{pe}) - b
\]

This is equivalent to showing the existence of a \( \chi_1 \in [-1, +1]^N \), (equivalent to the term \( \partial E(0; \lambda_{pe}) \)), where \( \chi_2 \) with \( \| \chi_2 \|_2 \leq \frac{1}{\rho} \) (equivalent to the term \( \partial g(0; \lambda_{pg}) \)) such that \( b = \lambda_{pe} \chi_1 + \lambda_{pe} \chi_2 \), due to property (iii) in Assumption I. By definition of the proximity operator we have

\[
u = P_{\lambda_{pg}}g(v) = \arg\min_{z} \left\{ \frac{1}{2} \| v - z \|_2^2 + g(z; \lambda_{pg}) \right\} = \arg\min_{z} \left\{ \frac{1}{2} \| v - z \|_2^2 + f_g \left( \frac{\| z \|_2}{\rho}; \lambda_{pg} \right) \right\}.
\]

Using the first optimality condition of \( u = 0 \) for the objective function in (30), we have

\[
0 \in -v + \partial f_g(0; \lambda_{pg}) \frac{\partial(\|0\|)}{\rho}.
\]

Since \( \partial(\|0\|) = \{ x \in \mathbb{R}^N, \| x \|_2 \leq 1 \} \) and \( \| z \| \leq \lambda_{pg} \) for all \( z \in \partial f_g(0; \lambda_{pg}) \) (using property (iii) in Assumption I), using Equation (31) we have \( \| v \|_2 \leq \frac{\lambda_{pg}}{\rho} \). Furthermore, since \( v = P_{\lambda_{pe}, E}(b) \), the first-order optimality condition implies that \( 0 \in \partial E(v; \lambda_{pe}) + v - b \). Thus for \( \chi_1 \in \partial E(v; \lambda_{pe}) \) and \( \chi_2 = \frac{v}{\lambda_{pe}} \), we have \( b = \lambda_{pe} \chi_1 + \lambda_{pe} \chi_2 \) and proof is completed.

**APPENDIX C**

**REGION AND GROUP SPECIFICATION**

Here, we propose a heuristic method to find the regions \( R_1 \), \( R_2 \), and \( R_3 \), depicted in Fig. 8 and introduced in Section [**V**]. To describe the regions, we need to compute the value of \( k_S \), \( \Delta k \), and \( \Delta m \), i.e., the discrete Doppler and delay parameters that corresponds to \( \nu_S \), \( \Delta \nu \), and \( \Delta \tau \), respectively. To estimate the \( \Delta m \) and \( \Delta k \), we use a regularized least-squares estimate of \( x \) given by \( x_{LS} = A_0 y = A_0(Ax + z) \approx x + e \) where \( e = A_0 z \) and \( A_0 = (A^H A + \rho^2 I)^{-1} A^H \) and \( \rho \) is a small real value. Based on relationship in the Eq. [15], we can write \( x_{LS} = \text{vec} \{ H_{LS} \} \), where \( H_{LS} \) is an estimate of the discrete delay-Doppler spreading function. Let us define the function \( E_d(m) = \sum_{i=-K}^{+K} |H[i, m]|^2 \) for
Fig. 8. Discrete representation of the regions $R_1$, $R_2$, and $R_3$.

$1 \leq m \leq M$. This function represent the energy profile of channel components in delay direction. Then,

$$\Delta m = \min_m \{m | \mathcal{E}_d (m) \leq T_1 \},$$

where $T_1 = \alpha_d \mathcal{E}_d (0)$. Here $0 < \alpha_d < 1$ is a tuning parameter. For a highway environment [7], based on our numerical analysis, $\alpha_d \approx 0.4$ is a reasonable choice.

After computing $\Delta m$, to compute the values of $\Delta k$ and $k_s$ as labeled in Fig.8, due to symmetry of channel diffuse components around zero Doppler value, we define functions $\mathcal{E}_\nu (k) = \sum_{i=\Delta m}^{M} |H[k, i]|^2 + |H[-k, i]|^2$ for $0 \leq k \leq K$. This function represents the energy profile of channel components in Doppler direction. Let us define $k_0 = \max_k \mathcal{E}_\nu (k)$. Then, we can estimate $\Delta k$ and $k_s$ using following equations,

$$k_s - \Delta k = \max_k \{k | \mathcal{E}_\nu (k) < T_2, 0 \leq k < k_0 \},$$

$$k_s = \min_k \{k | \mathcal{E}_\nu (k) < T_2, k_0 < k \leq K \},$$

where $T_\nu = \alpha_\nu \mathcal{E}_\nu (k_0)$ with $0 < \alpha_\nu < 1$. For highway environment, based on our numerical analysis, $\alpha_\nu \approx 0.6$ is a good choice.

Given estimates of the parameters that define regions $R_1$, $R_2$, and $R_3$, we can determine two different group sizes for regions $R_1 \cup R_2$ and $R_3$. Let $g_d$ be the group size for the channel components in region $R_1 \cup R_2$ and $g_s$ is the group size for the channel components in region $R_3$. Consider that the total number
of groups is $N_g$ then we have $N_g = N_{gd} + N_{gs}$ where $N_{gd} = \frac{|R_1 \cup R_2|}{g_d}$ is the total number of groups in $R_1 \cup R_2$ and $N_{gs} = \frac{|R_3|}{g_s}$ is the total number of groups in $R_3$, respectively. To specify the group sizes, we assume that $\frac{N_{gd}}{N_{gs}} \leq \epsilon$ where $\epsilon \ll 1$, then both $g_s$ and $g_d$ can be evaluated for a given total number of groups. We empirically observed that performance is robust to the selection of the parameters, i.e., modest changes in parameters do not affect performance significantly. Furthermore, note that finding the optimal group sizes are beyond the scope of this work, thus, we rely on the choice of such parameters that make the number of groups in region $R_1 \cup R_2$ smaller relative to the total number of groups.

Appendix D

Proximal ADMM Iteration Development

For the optimization problem given in (19), we form the augmented Lagrangian

$$L_\rho (x, w, \theta) = \frac{1}{2} \| y - Ax \|^2_2 + \phi_g(|w|; \lambda_g) + \phi_e(|w|; \lambda_e) + \langle \theta, x - w \rangle + \frac{\rho^2}{2} \| x - w \|^2_2,$$

where $\theta$ is the dual variable, $\rho \neq 0$ is the augmented Lagrangian parameter, and $\langle a, b \rangle = \text{Re}(b^H a)$. Thus, ADMM consists of the iterations:

- **update-x:** $x^{n+1} = \arg\min_x \frac{1}{2} \| y - Ax \|^2_2 + \langle \theta^n, x - w^n \rangle + \frac{\rho^2}{2} \| x - w^n \|^2_2$.
- **update-w:** $w^{n+1} = \arg\min_w \phi_g(|w|; \lambda_g) + \phi_e(|w|; \lambda_e) + \langle \theta^n, x^{n+1} - w \rangle + \frac{\rho^2}{2} \| x^{n+1} - w \|^2_2$.
- **update-dual variable:** $\theta^{n+1} = \theta^n + \rho^2 (x^{n+1} - w^{n+1})$.

Deriving a closed form expressions for update-x is straightforward,

$$x^{n+1} = \rho^2 A_0 (w^n - \theta^n_\rho) + x_0,$$

where $\theta^n_\rho = \frac{\theta^n}{\rho}$, $A_0 = (\rho^2 I + A^H A)^{-1}$ and $x_0 = A_0 A^H y$. If we pull the linear terms into the quadratic ones in the objective function of update-w and ignoring additive terms, independent of $w$, then we can express this step as

$$w^{n+1} = \arg\min_w \left\{ \frac{1}{2} \| x^{n+1} + \theta^n_\rho - w \|^2_2 + \frac{1}{\rho^2} (\phi_g(|w|; \lambda_g) + \phi_e(|w|; \lambda_e)) \right\}$$

$$= \arg\min_w \sum_{i=1}^{N_g} \left\{ \frac{1}{2} \| x_i^{n+1} + \theta^n_\rho - w_i \|^2_2 + f_g \left( \frac{\| w_i \|_2}{\rho}; \lambda_g; \frac{\lambda_e}{\rho} \right) + \sum_j f_e \left( \frac{\| w_i[j] \|_2}{\rho}; \lambda_e; \frac{\lambda_e}{\rho} \right) \right\}$$
where $x_i = \Gamma_i x$, $w_i = \Gamma_i w$, and $\theta_{pi} = \Gamma_i \theta_p$. Thus, we can perform the update-w step in parallel for all groups,

$$w_i^{n+1} = \arg\min_{w} \left\{ \frac{1}{2} \| x_i^{n+1} + \theta_{pi} - w_i \|^2 + f_g \left( \frac{\|w_i\|_2}{\rho}; \frac{\lambda_g}{\rho} \right) + \sum_j f_e \left( \frac{|w_i[j]|}{\rho}; \frac{\lambda_e}{\rho} \right) \right\}$$

Here, for simplicity in representation, we define $\lambda_{pe} = \frac{\lambda_e}{\rho}$ and $\lambda_{pg} = \frac{\lambda_g}{\rho}$. In addition, we define

$$E(|w_i|; \lambda_{pe}) = \sum_j f_e \left( \frac{|w_i[j]|}{\rho}; \frac{\lambda_{pe}}{\rho} \right),$$

$$g(|w_i|; \lambda_{pg}) = f_g \left( \frac{\|w_i\|_2}{\rho}; \frac{\lambda_{pg}}{\rho} \right).$$

Thus, we have

$$w_i^{n+1} = \arg\min_{w} \left\{ \frac{1}{2} \| x_i^{n+1} + \theta_{pi} - w_i \|^2 + g(|w_i|; \lambda_{pg}) + E(|w_i|; \lambda_{pe}) \right\}$$

To guarantee convergence to the optimal solution in $(P_0)$, the overall objective function, i.e., $\frac{1}{2} \| y - Ax \|^2_2 + \phi_g(|x|; \lambda_g) + \phi_e(|x|; \lambda_e)$, should be a convex function $[23]$. Note that since the first term in the objective function, i.e., the quadratic penalty function, is convex, for any functions $f_e$ and $f_g$ that satisfies condition (iv) in Assumption I, the overall objective function is convex as well. Thus, ADMM yields convergence for all choices of the convex and non-convex functions given in Section II-B.

**APPENDIX E**

**PROOF OF LEMMA 1**

The function $\|c - w\|^2_2 = \|c\|^2_2 + \|w\|^2_2 - 2\text{Re}\{c^H w\} = \|c\|^2_2 + \|w\|^2_2 - 2\text{Re}\{c^H w\}$ is minimized, with respect to phase of $w$, when $\text{Re}\{c^H w\}$ is maximized. Now,

$$\text{Re}\{c^H w\} = \sum_{n=1}^{N} |c[n]| |w[n]| \cos(\text{Ang}(c[n]) - \text{Ang}(w[n]))$$

$$\leq \sum_{n=1}^{N} |c[n]| |w[n]| = |c| |w|$$
with equality if and only if \( \text{Phase}(w) = \text{Phase}(c) \), which in turn implies that \( \|c - w\|_2^2 = \|c - |w|\|_2^2 \).

Hence,

\[
\arg\min_{\|w\|\circ \text{Phase}(w) \in \mathbb{C}^N} \|c - w\|_2^2 = \arg\min_{\|w\|\circ \text{Phase}(c) \in \mathbb{C}^N} \|c - |w|\|_2^2 = \text{Phase}(c) \odot \arg\min_{|w| \in \mathbb{R}^N} \|c - |w|\|_2^2
\]

and the lemma follows.

REFERENCES

[1] D.W. Matolak, “V2V communication channels: State of knowledge, new results, and whats next,” Commun. Tech. for Veh. Springer, pp. 1–21, 2013.

[2] L. Bernadó, T. Zemen, F. Tufvesson, A. F. Molisch, and C.F. Mecklenbräuker, “Delay and doppler spreads of non-stationary vehicular channels for safety relevant scenarios,” IEEE Trans. Veh. Technol., vol. 63, pp. 82–93, 2014.

[3] J. Nuckelt, M. Schack, and T. Kürner, “Deterministic and stochastic channel models implemented in a physical layer simulator for car-to-x communications,” J. Adv. in Radio Science, vol. 9, no. 12, pp. 165–171, 2011.

[4] A.F. Molisch, F. Tufvesson, J. Karedal, and C.F. Mecklenbräuker, “A survey on vehicle-to-vehicle propagation channels,” IEEE Trans. Wireless Commun., vol. 16, no. 6, pp. 12–22, 2009.

[5] T. Zemen and A.F. Molisch, “Adaptive reduced-rank estimation of nonstationary time-variant channels using subspace selection,” IEEE Trans. Veh. Technol., vol. 61, no. 9, pp. 4042–4056, 2012.

[6] P. Bello, “Characterization of randomly time-variant linear channels,” IEEE Trans. Commun. Systems, vol. 11, no. 4, pp. 360–393, 1963.

[7] J. Karedal, F. Tufvesson, N. Czink, A. Paier, C. Dumard, T. Zemen, C.F. Mecklenbräuker, and A.F. Molisch, “A geometry-based stochastic MIMO model for vehicle-to-vehicle communications,” IEEE Trans. Wireless Commun., vol. 8, no. 7, pp. 3646–3657, 2009.

[8] N. Michelusi, U. Mitra, A.F. Molisch, and M. Zorzi, “UWB sparse/diffuse channels, part I: Channel models and Bayesian estimators,” IEEE Trans. Signal Process., vol. 60, no. 10, pp. 5307–5319, 2012.

[9] ——, “UWB sparse/diffuse channels, part II: Estimator analysis and practical channels,” IEEE Trans. Signal Process., vol. 60, no. 10, pp. 5320–5333, 2012.

[10] S. Beygi, E.G. Ström, and U. Mitra, “Geometry-based stochastic modeling and estimation of vehicle to vehicle channels,” in Proceedings IEEE Int. Conf. Acoustics, Speech and Signal Process., pp. 4289-4293, 2014.

[11] W.U. Bajwa, J. Haupt, A. M. Sayeed, and R. Nowak, “Compressed channel sensing: A new approach to estimating sparse multipath channels,” Proceedings of the IEEE, vol. 98, no. 6, pp. 1058–1076, 2010.

[12] G. Tauböck, F. Hlawatsch, D. Eiwen, and H. Rauhut, “Compressive estimation of doubly selective channels in multicarrier systems: Leakage effects and sparsity-enhancing processing,” IEEE J. Sel. Topics Signal Process., vol. 4, no. 2, pp. 255–271, 2010.

[13] C. Carbonelli, S. Vedantam, and U. Mitra, “Sparse channel estimation with zero tap detection,” IEEE Trans. Wireless Commun., vol. 6, no. 5, pp. 1743–1763, 2007.
[14] M. Stojnic, F. Parvaresh, and B. Hassibi, “On the reconstruction of block-sparse signals with an optimal number of measurements,” IEEE Trans. Signal Process., vol. 57, no. 8, pp. 3075–3085, 2009.

[15] Y.C. Eldar and M. Mishali, “Robust recovery of signals from a structured union of subspaces,” IEEE Trans. Info. Theory, vol. 55, no. 11, pp. 5302–5316, 2009.

[16] S. Beygi, E.G. Ström, and U. Mitra, “Structured sparse approximation via generalized regularizers: with application to V2V channel estimation,” IEEE Global Communication Conference, (Globecom), pp. 1–6, Austin, Tx, Dec. 2014.

[17] J. Fan and R. Li, “Variable selection via nonconcave penalized likelihood and its oracle properties,” J. Amer. Statistical Assoc., vol. 96, no. 456, pp. 1348–1360, 2001.

[18] C.-H. Zhang, “Nearly unbiased variable selection under minimax concave penalty,” The Annals of Statistics, pp. 894–942, 2010.

[19] P. Sprechmann, I. Ramirez, G. Sapiro, and Y. Eldar, “C-HiLasso: A collaborative hierarchical sparse modeling framework,” IEEE Trans. Signal Process., vol. 59, no. 9, pp. 4183–4198, Sept 2011.

[20] R. Chartrand and B. Wohlberg, “A nonconvex ADMM algorithm for group sparsity with sparse groups,” in Proceedings IEEE Int. Conf. Acoustics, Speech and Signal Process., pp. 6009–6013, 2013.

[21] J. Friedman, T. Hastie, and R. Tibshirani, “A note on the group lasso and a sparse group lasso,” Technical report, Department of Statistics, Stanford University, 2010.

[22] F. Bach, R. Jenatton, J. Mairal, and G. Obozinski, “Optimization with sparsity-inducing penalties,” Foundations and Trends® in Machine Learning, vol. 4, no. 1, pp. 1–106, 2012.

[23] J. Eckstein, “Augmented lagrangian and alternating direction methods for convex optimization: A tutorial and some illustrative computational results,” RUTCOR Research Reports, vol. 32, 2012.

[24] P. L. Combettes and J.-C. Pesquet, “Proximal splitting methods in signal processing,” in Fixed-point algorithms for inverse problems in science and engineering. Springer, 2011, pp. 185–212.

[25] R.T. Rockafellar and R.J.-B. Wets, Variational analysis. Berlin: Springer-Verlag, 2004, vol. 317.

[26] R. Griponval, “Should penalized least squares regression be interpreted as maximum a posteriori estimation?” IEEE Trans. Signal Process., vol. 59, no. 5, pp. 2405–2410, 2011.

[27] A. Borhani and M. Patzold, “Correlation and spectral properties of vehicle-to-vehicle channels in the presence of moving scatterers,” IEEE Trans. Veh. Technol., vol. 62, pp. 4228–4239, 2013.