The parameters identification of the automatic control system with the controller

K M Bobobekov, G V Troshina, A A Voevoda
Novosibirsk State Technical University, 20, Karla Marksa ave., Novosibirsk, 630073, Russia
E-mail: troshina@corp.nstu.ru

Abstract. The active identification problem of an unstable object is considered. The technique is illustrated on the examples of parameters identification for the first and second order unstable object corresponding to an inverted pendulum, entering an automatic control system with controllers and the example of the first-order unstable object with the controller which parameters are estimated with the use of test sinusoidal signal with a changeable frequency. In the work the use of the specially picked up test signals is offered which significantly simplifies the identification problem in comparison with passive methods. The identification is carried out according to the nomograms built as a result of the processes modeling in the automatic control system at the variation of the object and controller parameters and the test signals impact on system.

1. Introduction
In the devices of the automatic control, in robotics and the mechanical engineering such situation when the object parameters change eventually takes place are quite often and fine tuning of the control units parameters (controllers) is required [1, 2]. The optimal control problems for the dynamic system identification are provided in works [3, 4]. The active identification problem of the unstable object parameters causes special difficulties and isn't solvable without the controllers introduction providing the system stability that results in the need of automatic control system [5, 6]. It is possible to note that the Kalman filter is used extensively for state vector estimation [7, 8]. The problem of the adaptive robust output regulation are considered in works [9]. The need of the interval observers design for linear systems is proved in works [10, 11]. The system block diagram represents the consecutive connection of the controller and the object captured by the negative feedback [12, 13]. Typical examples in robotics are unstable objects control which parameters are required to be defined or identified [14]. The periodic signals like a sinusoid, meander or their combination is offered to use as the test signals [15, 16].

2. Control object
The controller transfer function \( w_{c}(s) = x(s)/y(s) \) is calculated on the basis of the known object transfer function \( w_{ob}(s) = n(s)/d(s) \), where \( \deg(x(s)) = \deg(y(s)) = n - 1 \), \( \deg(n(s)) \leq \deg(d(s)) = n \). Here \( \deg(\cdot) \) designates the corresponding polynomial degree. It is supposed that the object and the controller are captured by the negative feedback. The synthesis polynomial method is used for the controller transfer function calculation. This method requires to calculate the transfer function of the
opened system \( w_{op}(s) = w_{r}(s)w_{ob}(s) \), the transfer function of the closed system
\[ w_{op}(s) = w_{r}(s)w_{ob}(s)(1 + w_{r}(s)w_{ob}(s))^{-1} \]
and the characteristic polynomial of the closed system
\[ c(s) = d(s)y(s) + n(s)x(s) \].
Further the following designations are accepted: \( v \) – a task, \( u \) – the input signal, \( y \) – the object output signal. The statistical methods or the standard signals, for example, \( l(t) \) or \( \delta(t) \) are used for the active identification methods. The alternative approach consists in the usage of a test signal.

Now the input signal on the object planning methods at which the identification problem optimal solution is reached are actively developed. The different signals of a step-like signal (Heaviside function), a sinusoid, a meander and their combinations are used as a test signal at the active identification of the object or controller parameters. For convenience in this work for the object or controller parameters identification a signal like sinusoid and meander is used.

For the object or controller parameters estimation the nomograms which are built in advance are used. The nomograms are built when periodic signals are given on the system input, and at the system exit the output signal amplitudes in the fixed timepoints \( t \) are measured, and these values are marked on axes \( X \) and \( Y \).

3. Experimental

Example 1: the first order object. The \( k \) and \( \tau \) parameters active identification problem is considered for the first order unstable object

\[ w_{ob}(s) = k/(1 - \tau s) \]  \hspace{1cm} (1)

For the purpose of the system stability ensuring the PI-regulator is used

\[ w_{r}(s) = (\alpha + \beta s)/s \],  \hspace{1cm} (2)

where the controller parameters \( \alpha \) and \( \beta \) are calculated by the polynomial method from the condition that the system poles are equal \( 0.5 \pm 0.5i \). The controller parameters have the following values \( \alpha = -0.25 \) and \( \beta = -1 \) for the basic values of the object parameters \( k = 2 \) and \( \tau = 1 \).

It was defined that at the object parameters variation within \( k \in [1.5; 2.5] \) and \( \tau \in [0.5; 1.5] \) the system is steady, transient comes to the end approximately for 5–13 sec and reregulation is about 50-100%. On the system the periodic signal like a meander with the amplitude \( \pm 1 \) and the period \( T = 2\pi \) sec is given. We consider that necessary information about the object parameters can be obtained as the result of the transient measurement in two points [14]. The points corresponding to timepoints of 14 sec and 15 sec are chosen. At these timepoints - the output signals values significantly depend on the object parameters variation, therefore they are used in the identification procedure.

We build the nomogram in which the first measurement results corresponding to the timepoint of 14 sec are marked on axis \( X \) and the value corresponding to the measurement of 15 sec are marked on axis \( Y \). The object parameters change as follows \( k = \{1.5, 1.63, 1.75, 1.88, 2, 2.13, 2.25, 2.38, 2.5\} \), \( \tau = \{0.5, 0.63, 0.75, 0.88, 1, 1.13, 1.25, 1.38, 1.5\} \).

The identification algorithm is as follows. The system with the test signal like meander is started. The measurements are performed for timepoints of 14 sec and 15 sec. For the error reduction it is possible to make several measurements, for example, in a period. The average values are marked on the nomogram (Figure 1). According to the nomogram the object parameters are defined by the interpolation method on the next curves.
Figure 1. The nomogram for the parameters determination of the first order unstable object when on the system input the signal like a meander is given.

Example of the object parameters determination. We will show how the nomogram is used. For an experiment we take $k = 1.7$ and $\tau = 0.7$. On the system the signal like a meander with a single amplitude and the period $T = 2\pi$ sec is given. Let's say that the object parameters are unknown. At the system exit the amplitudes are measured at $t_1 = 14$ sec and $t_2 = 15$ sec and we receive the following values $A_1 = 2.18$ and $A_2 = 2.66$. The value $A_1$ is marked on an axis $X$ and the value $A_2$ is marked on an axis $Y$. The point $M$ is represented on the nomogram as a result (Figure 1). For the parameters determination $k$ and $\tau$ we will draw the auxiliary lines passing through a point $M$ and supplementing the families of lines corresponding to the families $k$ and $\tau$. It gives us a chance to determine the object parameters $k = 1.73$ and $\tau = 0.73$ by the interpolation method on two families of the lines corresponding to changes $k$ and $\tau$ in the set limits. Really, $k = (1.8 - 1.65) / 2 + 1.65 \approx 1.73$, $\tau = (0.8 - 0.65) / 2 + 0.65 \approx 0.73$. Thus, the determination problem of the unstable object parameters is solved.

Example 2: the second order object (the inverted pendulum). We will consider the active identification problem of the unstable object parameters like the inverted pendulum $-k/(s^2 - b)$ included in automatic control system (Figure 2) [15]. For the astaticism the integrator which conditionally belongs to the object is entered into the controller

$$W_{ob}(s) = \frac{n(s)}{d(s)} = -\frac{k}{s^2 - bs}.$$  \tag{3}

At the basic parameters values $k = 1$, $b = 5$ the controller is defined by a polynomial method, such that the characteristic polynomial of the closed system was the set look, for example, $(\tau s + 1)^5$. Thus, all poles of the closed system are equal $-\tau^{-1}$.

Figure 2. The block diagram of the automatic control system with the second order unstable object.
Further we believe $\tau = 0.2$. The transfer function for the second order controller is chosen according to the technique of the polynomial method:

$$W_c(s) = \frac{x(s)}{y(s)} = \frac{x_2s^2 + x_1s + x_0}{y_2s^2 + y_1s + y_0}. \quad (4)$$

Further the characteristic polynom of the closed system $c(s) = d(s)y(s) + n(s)x(s)$ is written out and the polynoms $d(s), n(s), y(s), x(s)$ from the equations (3), (4) are substituted. The members in the left part are grouped on degrees $s$:

$$y_2s^5 + y_1s^4 + (-by_2 + y_0)s^3 + (-by_1 - kx_2)s^2 + (-by_0 - kx_1)s - kx_0 = (\tau s + 1)^2. \quad (5)$$

Opening the brackets and equating the coefficients which are in identical degrees at $s$, the equations system is received which can be written down as a matrix $Ax = b$ where $x = (y_2 \ y_1 \ y_0 \ x_2 \ x_1 \ x_0)^T$, $b = (0.0003 \ 0.008 \ 0.0815 \ -0.44 \ -1.4075 \ -1)^T$.

The Nyquist hodograph family of the closed system at the object parameters variation $k = \{0.8, 1.2, 1.4\}, \ b = \{4, 4.5, 5, 5.5\}$ is built for the frequency determination of the test signal (Figure 3). It has allowed to choose the first frequency of the test signal equal $\omega = 2.5$ (the corresponding points are connected by the line), and the second frequency - $\omega = 10$. Two test sinusoidal signals are given in turn in this example. It is supposed that during the work of the control system the object parameters can be in limits $k \in [0.8; 1.4], \ b \in [4; 6]$.

**Example of the object parameters determination.** We will show how it is necessary to use the nomograms represented in the Figure 4 and Figure 5. Let's say that the object parameters are $k = 0.9$ and $b = 4.3$. On the system input the sinusoidal signal is given by the frequency $\omega = 10$ and the single amplitude. In the result the amplitude at the object exit is defined and it is equal 0.67. The parameter $k = 0.9$ is determined by the nomogram represented in the Figure 4. After that on the system input the sinusoidal signal $\omega = 2.5$ is given and at the system exit the output signal amplitude is measured and it is equal 2.37. The parameter $b = 4.32$ is determined by the Figure 5 for the schedule $k = 0.9$. Thus, the estimation task of the unstable object parameters like the inverted pendulum is carried out with insignificant mistakes.
Example 3: the controller parameters identification. The active identification problem of the PI-controller parameters presented in the discrete form is considered:

\[ W(z) = ((\beta + 0.125\alpha)z + (0.125\alpha - \beta))/(z - 1), \]  

what corresponds to the PI-controller equation in a continuous case \( W(s) = (\beta s + \alpha)/s \), where the controller parameters \( \alpha \) and \( \beta \) are identified. This controller provides the system stability with the first order unstable object

\[ W_{obj}(z) = -k(0.125z + 0.125)/(\tau - 0.125)z - (\tau - 0.125), \]  

what corresponds to the object in the continuous representation \( W_{obj}(s) = -k/(1 - \tau) \) included in automatic control system. Here \( k = 2 \) and \( \tau = 1 \).

The controller parameters \( \alpha = -0.5 \) and \( \beta = -1.5 \) are calculated by the polynomial method. The transfer function for the closed system at the basic values of the controller parameters is defined as follows

\[ W_{cl}(z) = \frac{0.331z^2 - 0.2577}{z^2 - 1.533z + 0.6065}. \]  

The equation (8) corresponds to the transfer function for the closed system in continuous time

\[ W_{cl}(s) = \frac{3s^2 + 1.5}{s^2 + 2s + 1.5}. \]  

The transient in the discrete closed system (8) is given in Figure 6. The analysis of the Nyquist hodograph family for the closed system (Figure 7) built for the values number of the controller parameters is carried out. The test signal
frequency is chosen and it is equal \( \omega = 1.4 \text{ sec} \) that corresponds to the signal with the period \( T = 4.49 \text{ sec} \). The points on the hodograph are allocated with the bold lines corresponding \( \omega = 1.4 \text{ sec} \) for five families at \( \beta = \{-2; -1.75; -1.5; -1.25; -1\} \). In these points hodographs are the most sensitive to the controller parameters variation. Thus, it is possible to carry out the active identification with the chosen frequency. In Figure 7 the hodograph family allocated \( \alpha^* \) and \( \beta^* \) corresponds \( \alpha^* \in [-0.25; -1.25] \) and \( \beta^* = -1 \).

The amplitude choice of the test signal depends on the noise level generally. In this task it is supposed that noises are insignificant and therefore the amplitude is chosen to equal unit. The transient comes to the end approximately in 13 sec (Figure 8).

**Figure 7.** The Nyquist hodograph family for closed systems at \( \alpha = [-0.25; -1.25] \) and \( \beta = [-1; -2] \)

**Figure 8.** The transient in the system (Eq. 8) when giving the signal like a sinusoidal signal
The necessary information about the controller parameters, as appears from Figure 8, can be received as the result of the output signal amplitude measurement and the output signal phase measurement.

4. Conclusion
The use of the active identification technique for the parameters specification of the control system (controllers) is shown in this article. The main idea of the considered technique consists in need of giving periodic signals like sinusoid, meanders or their combination. The measurements of the amplitude and the phase are carried out at the system exit. The simulation results show the identification process for the control system parameters as a result of the test signals action on the automatic control system. This identification technique can be generalized also for more difficult devices used in mechanical engineering where there is a possibility of periodic testing and the parameters fine tuning for the control systems. For the input signal optimization it is necessary to use the Fischer information matrix which allows to estimate dispersions and to conduct the researches in local area.

References
[1] Chen C T 1999 Linear system theory and design (New York Oxford: Oxford University Press)
[2] Ljung L 1987 System identification: theory for the user (Prentice Hall, Englewood Cliffs)
[3] Astrom K J 1970 Introduction to stochastic control theory (New York: Academic Press)
[4] Sage A P, Melse J L 1972 Estimation Theory with Application to Communication and Control (New York: McGraw-Hill)
[5] Fedorov V V 1972 Theory of Optimal Experiments theory (New York: Academic Press)
[6] Mehra R K 1974 IEEE Trans. Aut. Contr. 19(6) 753–768
[7] Julier S J, Uhlmann J K, Durrant-whyte H F 2000 IEEE Trans. Aut. Contr. 5(3) 477–482
[8] Crassidis J L 2006 IEEE Trans. on Aerospace and Electronic System 42(2) 750–756
[9] Zhang Z, Serrani A 2009 IEEE Trans. Aut. Contr. 54(1) 266–278
[10] Cacace F, Germani A, Manes C, Setola R 2012 IEEE Trans. Aut. Contr. 57(1) 119–134
[11] Efimov D, Perruquetti W, Raissi T, Zolghadri A. 2013 IEEE Trans. Aut. Contr. 58(12) 3218–3224
[12] Antsaklis P J, Michel A N 1997 Linear systems (New York: McGraw-Hill)
[13] Doyle J C, Francis B, Tannenbaum A 1990 Feedback control theory (New York: Mscmillan)
[14] Troshina G V, Voevoda A A, Bobobekov K M 2016 11 Int. Forum on Strategic Technology (IFOST 2016) 594–596
[15] Troshina G V, Voevoda A A, Bobobekov K M 2016 13 Int. Conf. on Actual Problems of Electronic Instrument Engineering (APEIE 2016) 1 180–182
[16] Troshina G V, Voevoda A A, Bobobekov K M 2017 18 Int. Conf. on Young Specialists on Micro/Nanotechnologies and Electron Devices (EDM 2017) 138–141