OPTIMIZATION OF CAPITAL STRUCTURE IN REAL ESTATE ENTERPRISES

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ABSTRACT. On the basis of capital structure theory and the option pricing model, the revenues and costs of debts are quantified. Combining with the financing characteristics of real estate enterprises, a mathematical model in consideration of the effect of interest-free debt was established in this paper to determine the optimal capital structure of real estate enterprises, and then a simulation analysis was conducted. The results indicated that the interest-bearing debt interest rate, the tax rate, the risk-free interest rate and the proportion of interest-bearing debt are all positively correlated with the optimal debt ratio of real estate enterprises, the annual average growth rate of housing price and the annual volatility of enterprise assets are negatively correlated with that, and as the debt maturity increases, the optimal debt ratio of real estate enterprises will decrease.

1. Introduction. The fund sources and constitution of real estate enterprises have some particularities. According to statistics, in 2011, Chinese real estate enterprise-owned funds accounted for only 30%, with the rest mainly from bank loans, corporate bonds and presale. This special fund constitution of real estate enterprises has drawn considerable attention to their capital structure issues. Theoretically, based on the trade-off between revenues and costs of debts, there lies a capital structure that can maximize the enterprise value - the optimal capital structure. Study on optimal capital structure is significant for enterprises to implement financing strategies.

Different from the real capital structure, the optimal capital structure must be calculated indirectly with appropriate methods, other than being obtained directly from the corporate financial statements. Currently, typical methods to determine the optimal capital structure can be categorized into econometric analysis [7, 8, 13] and numerical simulation analysis [1, 3, 5, 6, 11, 12, 15]. Wherein, the econometric analysis, represented by the partial adjustment model, is applied to set the optimal capital structure as the linear function of firm characteristics and macroeconomic variables. A dynamic adjustment equation is introduced to establish the relationship between the optimal capital structure and the real capital structure, and then the optimal capital structure could be indirectly calculated by regression analysis. The method simplifies the complex mechanism among variables to the
linear relationship, simple but requiring high-quality data. What’s more, the conclusions are derived from historical data and thus cannot reflect the dynamical relationship between the economic variables and the optimal capital structure. The numerical simulation analysis is applied mainly to establish a mathematical model, analyze the effects of debts on tax shield revenue, bankruptcy costs, agency costs and investment demands, determine the optimal capital structure and dynamically analyze the optimal capital structure utilizing numerical simulation technology. In contrast to the econometric analysis model, this method can reveal the optimal capital structure formation mechanisms and reflect the effects of various economic factors on the optimal capital structure dynamically without historical data limitation. In this connection, this paper will construct a mathematical model based on the capital structure theory and make a simulation analysis on the optimal capital structure of real estate enterprises.

Compared to the enterprises in other industries, real estate enterprises have large amounts of interest-free debts from real estate presale. Existing literature shows that there is a great difference between the influences of interest-free debts and interest-bearing debts in capital operation [10]. Real estate presale has no interest and no corresponding tax shield revenue. Similar to interest-bearing debts, real estate presale without interest will also bring bankruptcy risk to the enterprises. In addition, real estate enterprises are also likely to bear the losses caused by rising housing prices after the presales. However, the existing studies on the optimal capital structure usually ignore the effect of interest-free debts, and thus are not suitable for real estate companies. In view of this, it is for the first time that this paper, by expanding the existing researches, establishes a mathematical model in consideration of the effects of interest-free debts, determines an optimal capital structure of real estate enterprises, and conducts a simulation analysis. In consideration of the particularities of real estate enterprises, research on the optimal capital structure contributes to enriching the theoretical models of related fields and provides a basis for real estate enterprises’ financing decisions.

2. Capital structure, optimal capital structure and determinants.

2.1. Capital structure and optimal capital structure. Capital structure refers to the sources and proportions for all kinds of capitals, usually denoted as the ratio of debt to total assets. In order to reflect the debt created during the financing progress, financing debt ratio (the ratio of financing debt to total assets, hereinafter referred to as debt ratio), is taken as a representative indicator for capital structure in this paper.

Optimal capital structure refers to such a capital structure where the marginal debt revenue equals to the marginal debt cost [14]. As an ideal capital structure, the optimal capital structure can enable companies to maximize corporate values and to minimize operating costs.

Enterprises have to bear the corresponding debt cost at the same time they obtain the debt revenue. With the increasing corporate debts, both the revenue and the cost are increasing, but the growth rate of the former is lower than that of the latter. When the debt ratio is at a low level, the marginal revenue is greater than the marginal cost, thus the enterprise value will increase as the debt increases. However, with the debt ratio increasing, the marginal debt cost will inch closer to the marginal debt revenue gradually. When the debt ratio reaches a certain point, the marginal debt revenue will equal to the marginal debt cost. At this point, the
trade-off value from debt (that is, the debt revenue minus the debt cost, hereinafter referred to as trade-off value) can come to its maximum, which means that debts have been optimally utilized and the debt ratio is the optimal capital structure of the enterprise.

2.2. Determinants of optimal capital structure. Optimal capital structure is a trade-off outcome between the debt revenue and the debt cost. Therefore, both of them are the key determinants affecting the optimal capital structure.

Debt revenue\(^2\) generally refers to the tax shield revenue, i.e., the hidden benefit that is brought to the enterprise because the interest on debt is outside tax levy scope. The tax shield revenue is equal to the summary of the present discounted value of the annual tax-free income during the debt maturity.

Debt cost\(^3\) mainly refers to the bankruptcy cost arose from debt. Bankruptcy cost is the cost incurred when enterprises cope with financial crisis, and can be divided into direct cost and indirect cost. Expenses on legal proceedings and liquidation when enterprises apply for bankruptcy due to insolvency belong to direct bankruptcy cost; the resulting costs arose from excessive debts that lead to the company solvency being questioned and affect management behaviors on financing, procurement and sales belong to indirect bankruptcy cost. As far as bankruptcy enterprises are concerned, bankruptcy cost means the direct bankruptcy costs; while for most of the normal enterprises, bankruptcy cost refers to the indirect bankruptcy cost. With respect to the normal enterprises, this study focuses mainly on indirect bankruptcy cost. However, the existing studies show that indirect cost is difficult to measure accurately \([2, 9]\). Therefore, it is necessary to choose an appropriate method or theory to measure indirect bankruptcy cost more accurately.

Inspired by the research conducted by Zhang and Xiao \([17]\), this paper quantifies indirect bankruptcy cost by using debt guarantee fee. It is assumed that the guarantor provides full guarantees for the enterprise debt in order to ensure that the creditor can get the book value of the repayment of principal and interest of debts to schedule. In this case, the management behaviors such as financing, procurement and sales will not be affected by the solvency of the enterprises. Namely, full guarantee eliminates the risk or cost of enterprise bankruptcy. In order to obtain a full guarantee, the debtor must pay a guarantee fee. If the guarantee fee is greater than indirect bankruptcy cost, the debtor is unwilling to pay, and if the guarantee fee is less than indirect bankruptcy cost, the guarantor is unwilling to bear. Therefore, the fair full guarantee fee should be equal to the indirect bankruptcy cost of the enterprise. Thus, the measure of indirect bankruptcy cost is converted into determining the full guarantee fee. In the sense of option, full guarantee is equivalent to the guarantor’s providing the debtor with a European put option that takes company assets as the subject matter and debt book value as the exercise price \([16]\). The value of the option is just the guarantee fee that the debtor is willing to pay.

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\(^2\)Debt revenue especially refers to the benefit from debt, not including the investment revenue from debt capital. The reason is that for given investment plan, the investment revenue from debt is equal to that from equity. Namely, the investment revenue from debt capital has no influence on the optimal capital structure.

\(^3\)Debt cost specially refers to the cost from debt, not including the cost for using debt (interest). The reason is that according to statistics, there is little difference between the cost for using debt and equity. To simplify the analysis, it is reasonable to assume that the two are equal. Namely, the cost for using debt will not affect the optimal capital structure.
and also the indirect bankruptcy cost of the enterprise. In summary, the option pricing model can be used to determine the indirect bankruptcy cost.

3. Mathematical model of optimal capital structure in real estate enterprises. According to the above analysis, the optimal capital structure is the debt ratio that can maximize the trade-off value. Trade-off value equals to debt revenue minus debt cost. Therefore, debt revenue and debt cost become key factors of the optimal capital structure model. Based on the existing research, and combined with the characteristics of capital structure in real estate enterprises, a mathematical model consisting of debt revenue and debt cost was established, and an objective function of trade-off value to determine the optimal capital structure was conducted in this paper.

3.1. Debt revenue. Financing debts for real estate enterprises consist primarily of interest-bearing debts (bank loans, corporate bonds, etc.) and interest-free debts (real estate presale). Among them, the revenue that interest-bearing debts bring to the enterprise is mainly the tax shield revenue, while the interest-free debt revenue can be regarded as a kind of implicit revenue due to non-payment of interest.

Following the existing research idea that the time value of capital is measured in the form of continuous compounding with risk-free interest rate \( r \), we assume the interest-bearing debt is \( x_i \), the interest-bearing debt interest rate is denoted as \( i \), and then the annual interest that should be paid for the interest-bearing debt is \( x_i \times i \). According to the income tax levy provisions, interest is out of the scope of tax levy, which means that the annual tax shield revenue from interest-bearing debt is \( x_i \times i \times r_t \), where \( r_t \) is the tax rate. By discounting the tax shield revenue of the first year in continuous compounding (risk-free interest rate \( r \)), we can get the present value of the tax shield revenue of the first year as \( x_i \times i \times r_t \times e^{-r} \). Assuming the interest-bearing debt maturity is \( T \), the present value of the tax shield revenue at year \( T \) is \( x_i \times i \times r_t \times e^{-rT} \). By summarizing the present value of the tax shield revenue of each year, we get the present value \( Y_i \) of the tax shield revenue from the interest-bearing debt (debt maturity is \( T \)) as:

\[
Y_i = x_i \times i \times r_t \times \frac{e^{-r}(1 - e^{-rT})}{1 - e^{-r}} \tag{1}
\]

Real estate presale is a kind of interest-free debt for real estate enterprises. Assume that the interest-free debt is \( x_n \), and the corresponding annual debt revenue is \( x_n \times i \). Further, we suppose that both the interest-bearing debt and the interest-free debt will expire simultaneously at year \( T \). Similar to that of the interest-bearing debt, the present value \( Y_n \) of the interest-free debt revenue is shown as:

\[
Y_n = x_n \times i \times \frac{e^{-r}(1 - e^{-rT})}{1 - e^{-r}} \tag{2}
\]

By summarizing \( Y_i \) and \( Y_n \), we obtain the real estate enterprises financing debt revenue \( Y \):

\[
Y = (x_i \times i \times r_t + x_n \times i) \times \frac{e^{-r}(1 - e^{-rT})}{1 - e^{-r}} \tag{3}
\]

The main purpose of real estate enterprise financing is to conduct real estate development. When the real estate flows into the market, the enterprise will repay bank loans and corporate bonds, and meanwhile deliver the real estate to the buyers as schedules (which can be regarded as paying off the real estate presale). Therefore, it is reasonable to suppose that both the interest-bearing liability and the interest-free liability of the real state enterprises will expire simultaneously.
3.2. **Debt cost.** Real estate enterprises debts include interest-bearing debts and interest-free debts, both being accompanied with corresponding bankruptcy costs. In addition, interest-free debts that consist mainly of real estate presale also bear implicit costs resulted from rising housing prices, which is one of the important features of the real estate enterprises that are different from other enterprises. In this paper, the real estate enterprises bankruptcy costs and the implicit costs caused by interest-free debts will be discussed respectively.

As mentioned, the bankruptcy cost can be quantified according to the European put option pricing model. On the basis research conducted by [4], for a real estate enterprise, assuming that the interest-bearing debt is $x_i$, the interest-free debt is $x_n$, and the total assets is $S$, then the bankruptcy cost $C_B$ can be obtained as:

$$C_B = (x_i + x_n)N(-d_2) - SN(-d_1)$$  \hspace{1cm} (4)

In (4), $N()$ is the standard normal distribution function, $d_1$ and $d_2$ are respectively defined as follows:

$$d_1 = \frac{\ln[S/(x_i + x_n)] + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$ \hspace{1cm} (5)

$$d_2 = \frac{\ln[S/(x_i + x_n)] + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$ \hspace{1cm} (6)

In (5) and (6), $r$ is the risk-free interest rate, and $\sigma$ denotes the annual volatility of enterprise assets, and $T$ represents the debt maturity.

Formula (4) determines the debt bankruptcy cost. However, the interest-free debt that real estate enterprises acquired from presale will have to cover the implicit cost caused by rising housing prices at the same time it bears the bankruptcy cost. Real estate presale is a part of the purchasing funds that the buyer paid to the real estate enterprise in advance. This part of funds will bring hidden revenue to the enterprise on one hand, but on the other hand, the enterprise will lose the price gap as a result of the increasing housing price. Assuming the average annual growth rate of the housing price is $\delta$, the implicit cost $C_P$ brought by the increasing housing price is presented in formula (7).

$$C_P = x_n[(1 + \delta)^T - 1]e^{-rT}$$ \hspace{1cm} (7)

By summarizing the bankruptcy cost $C_B$ and the hidden cost $C_P$ caused by rising housing price, we obtain the real estate enterprises debt cost $C$ as:

$$C = (x_i + x_n)N(-d_2) - SN(-d_1) + x_n[(1 + \delta)^T - 1]e^{-rT}$$ \hspace{1cm} (8)

3.3. **Optimal capital structure.** On the basis of debt revenue and debt cost, the optimal capital structure for real estate enterprise can be determined with an objective function for net trade-off value. In this paper, the net trade-off value is defined to be the difference between the revenue and cost of debt. When the capital structure comes to optimal level, the net trade-off value is maximized. Therefore, the aim of optimization of capital structure is to maximize the net trade-off value $V$. The optimization problem is:

$$\max \ V = [x_i \times i \times r_t + x_n \times i] \times \frac{e^{-r(1 - e^{-rT})}}{1 - e^{-r}} - (x_i + x_n)N(-d_2)$$

$$+ SN(-d_1) - x_n[(1 + \delta)^T - 1]e^{-rT}$$ \hspace{1cm} (9)

In order to simplify the calculations under the premise of not affecting the results of the analysis, let $x_i + x_n = x$, $x_i = kx$, $x_n = (1 - k)x$, $0 \leq k \leq 1$, where $k$ is the
proportion of interest-bearing debt (the ratio of interest-bearing debt to the whole financing debt). Thus, optimization problem (9) can be changed into:

$$\max \ V(x) = [kx \times i \times r_1 + (1 - k)x \times i] \times \frac{e^{-r}(1 - e^{-rT})}{1 - e^{-r}} - xN(-d_2)$$
$$+ SN(-d_1) - (1 - k)x[(1 + \delta)^T - 1]e^{-rT} \tag{10}$$

Problem (10) is an unconstrained extremum problem, where the total financing debt $x$ is the decision variable and equal to the sum of interest-bearing debt and interest-free debt. In problem (10), there is no constraint on decision variable $x$, which is applicable to an ideal capital market. In an ideal market without financing constraints, the corporate financing behavior just depends on the tradeoff between debt revenue and debt cost. In the real economy, however, financing constraints exist everywhere. For instance, in credit markets, in order to control the risk of funds, financial institutions such as banks usually limit the amount of loans to enterprises; in stock markets, equity financing will be influenced by a variety of financing policies. Thus, under such a realistic condition, the determination of optimal capital structure is a constrained extremum problem, where the variation of $x$ is restricted to a certain degree. As a reflection of debt financing, $x$ is essentially limited by debt financing constraints. With that in mind, the constraints of problem (10) can be set up based on debt financing constraints.

In the case of interest-bearing debt, with the assessment on corporate operating conditions (such as risk, profitability and cash flow, etc.), banks and other financial institutions usually give enterprises appropriate authorization on loan amount, which can be regarded as the ceiling of interest-bearing debt. Similarly, due to the limited number of presale houses, interest-free debt also has upper boundary. To sum up, there is an upper threshold for debt financing $x$. In this paper, the threshold is defined as a specific proportion of total assets, $pS$, where $p$ is the proportion and $S$ denotes total assets. The parameter $p$ reflects the degree of debt financing constraints. The smaller the value of $p$ is, the greater the debt financing constraint is. In addition to the upper limit, there is also a lower threshold for $x$. Under any circumstance, $x$ cannot be negative. Hence, 0 is a rational lower threshold for debt financing $x$. Based on the above analysis, the constraints of problem (10) can be set as $0 \leq x \leq pS$.

So far, the optimization of capital structure can be formulated as a constrained nonlinear programming problem.

$$\begin{align*}
\max \ V(x) &= [kx \times i \times r_1 + (1 - k)x \times i] \times \frac{e^{-r}(1 - e^{-rT})}{1 - e^{-r}} - xN(-d_2) \\
&+ SN(-d_1) - (1 - k)x[(1 + \delta)^T - 1]e^{-rT} \\
\text{s.t.} \quad 0 \leq x \leq pS
\end{align*} \tag{11}$$

Kuhn-Tucker condition is employed to resolve the above nonlinear programming. Its process includes the following 3 steps: first, find the derivative of $V(x)$ with respect to $x$; second, establish the Kuhn-Tucker condition with the constraints $0 \leq x \leq pS$; finally, utilize numerical simulation method for solving.

First, find the derivative of $V(x)$ with respect to $x$. In problem (10), both $xN(-d_2)$ and $S \times N(-d_1)$ are compound functions of $x$. Let $g_1(x) = xN(-d_2)$, $g_2(x) = SN(-d_1)$, and respectively take the derivatives of $g_1(x)$ and $g_2(x)$.

$$g_1'(x) = N(-d_2) + x \times N'(-d_2) \times (-d_2)' \tag{12}$$
\[ g^2(x) = S \times N'(-d_1) \times (-d_1)' \quad (13) \]

Both (12) and (13) include derivative function of standard normal distribution function. The derivative function of distribution function is density function. Namely, \( N'(-d_1) = f(-d_1), \) \( N'(-d_2) = f(-d_2). \) \( f() \) is the density function of standard normal distribution. The derivative functions of \(-d_1\) and \(-d_2\) can be calculated with formulas (5) and (6), respectively.

\[ (-d_1)' = (-d_2)' = -\frac{1}{\sigma\sqrt{T}} \times \frac{x}{s} \times (-\frac{S}{x^2}) = \frac{1}{\sigma\sqrt{T}} \times \frac{1}{x} \quad (14) \]

Substituting (14) into (12) and (13), we can get \( g^1(x) \) and \( g^2(x) \):

\[ g^1(x) = N(-d_2) + \frac{f(-d_2)}{\sigma\sqrt{T}} \]
\[ g^2(x) = S \times \frac{f(-d_1)}{x} \times \frac{1}{\sigma\sqrt{T}} \quad (16) \]

So far, the derivative function of \( V(x) \) can be expressed as:

\[ V'(x) = [k \times i \times r_t + (1 - k) \times i] \times \frac{e^{-r(1 - e^{-rT})}}{1 - e^{-r}} - N(-d_2) - \frac{f(-d_2)}{\sigma\sqrt{T}} + \]
\[ S \times \frac{f(-d_1)}{\sigma\sqrt{T}} \times \frac{1}{x} - (1 - k)[(1 + \delta)^T - 1]e^{-rT} \quad (17) \]

Second, list the Kuhn-Tucker condition of problem (11).

Problem (11) can be changed into:

\[
\begin{cases}
\min \quad \bar{V}(x) = -[kx \times i \times r_t + (1 - k)x \times i] \times \frac{e^{-r(1 - e^{-rT})}}{1 - e^{-r}} \\
\quad + xN(-d_2) - S \times N(-d_1) + (1 - k)x[(1 + \delta)^T - 1]e^{-rT} \\
\quad + s \times \frac{f(-d_1)}{\sigma\sqrt{T}} \times \frac{1}{x} + (1 - k)[(1 + \delta)^T - 1]e^{-rT} - \mu^*_1 + \mu^*_2 = 0 \\
\quad s.t. \quad h_1(x) = x - 0 \geq 0 \\
\quad h_2(x) = pS - x \geq 0 \\
\end{cases} \quad (18) \]

The Kuhn-Tucker condition of (18) is as follows:

\[
\begin{cases}
\quad -[k \times i \times r_t + (1 - k) \times i] \times \frac{e^{-r(1 - e^{-rT})}}{1 - e^{-r}} + N(-d_2) + \frac{f(-d_2)}{\sigma\sqrt{T}} - \]
\[ \quad S \times \frac{f(-d_1)}{\sigma\sqrt{T}} \times \frac{1}{x} + (1 - k)[(1 + \delta)^T - 1]e^{-rT} - \mu^*_1 + \mu^*_2 = 0 \\
\quad \mu^*_1(x^* - 0) = 0 \\
\quad \mu^*_2(pS - x^*) = 0 \\
\quad \mu^*_1 \geq 0 \\
\quad \mu^*_2 \geq 0 \quad (19) \]

By sorting equations (19), the optimal financing debt \( x^* \) can be calculated. However, due to its nonlinear property, it is impossible to get analytical solution. In this case, we can only get numerical solution.

The optimal capital structure \( CS^* \) is:

\[ CS^* = \frac{x^*}{S} \quad (20) \]

By substituting \( x^* \) into (11), the maximum net trade-off value \( V^* \) can be calculated.
4. Simulation of optimal capital structure model in real estate enterprises. By quantifying the debt revenue and the debt cost, we derive the objective function, and further determine the optimal capital structure through optimization. In this paper, the values of the relevant parameters are set in consideration of the real economic data, and the optimal capital structure model is simulated to determine the optimal capital structure of the real estate enterprises under given conditions. At last, the impact of each parameter on the optimal capital structure and the maximum net trade-off value are analyzed.

4.1. Model parameter settings and initial simulation. The model parameters should be set based on the real economic conditions before simulation analysis. Among them, the initial value of the interest-bearing debt interest rate \( i \) is 6.38%, i.e., the average interest rate of 2012 mid-and-long term bank loans; the tax rate \( r_t \) is 25%, i.e., the Chinese tax rate; the risk-free interest rate \( r \) is 3.25%, i.e., the one-year bank deposit rate of 2012; the debt maturity \( T \) is 3.0 years, according to the real estate project development cycle; the annual volatility of enterprise assets \( \sigma \) is 30%, that is calculated from the market value of 91 listed real estate enterprises of A-stock market in 2000-2012; the annual housing price growth rate \( \delta \) is 6%, that is calculated with the 2012 housing sale price index; the proportion of interest-bearing debt \( k \) and the total assets \( S \) are respectively 59% and 13.4 billion yuan, i.e., the average derived from the annual reports of 91 listed real estate enterprises of A-stock market in 2012. In accordance with the provisions of the People’s Bank of China, real estate enterprises’ own capital ratio shall not be less than 35%. Meanwhile, taking into account other possible influences, the financing constraints \( p \) is set as 0.6 in this paper. Model parameter settings are shown in Table 1.

| Parameters       | Definitions                                      | Units | Values |
|------------------|--------------------------------------------------|-------|--------|
| \( i \)          | Interest-bearing debt interest rate              | –     | 6.38%  |
| \( r_t \)        | Tax rate                                         | –     | 25%    |
| \( r \)          | Risk-free interest rate                          | –     | 3.25%  |
| \( T \)          | Debt maturity                                    | Year  | 3.0    |
| \( \sigma \)     | Annual volatility of enterprise assets           | –     | 30%    |
| \( \delta \)     | Annual housing price growth rate                 | –     | 6%     |
| \( k \)          | The proportion of interest-bearing debt          | –     | 59%    |
| \( S \)          | Total assets                                     | Billion | 13.4 |
| \( p \)          | Financing constraints                            | –     | 0.6    |

With the parameters in Table 1, we can get 3 solutions of the nonlinear equations (19): \( X_1 = 0; \ X_2 = 4.422; \ X_3 = 8.040 \). Substituting \( X_1 - X_3 \) into problem (11), the net trade-off values can be calculated as \( NY_1 = 0; \ NY_2 = 0.107; \ NY_3 = -0.201 \). By comparison, the optimal amount of debt is \( X_2 \), and the optimal debt ratio \( CS^* \) is \( \frac{4.422}{13.4} = 0.33 \). The initial simulation results of the optimal capital structure model are shown in Table 2.

As shown in Table 2, with respect to a real estate enterprise with total assets of 13.4 billion yuan, given market environment (the relevant parameters are shown in Table 1), and given 3-year financing cycle, the optimal debt ratio is 0.33. That is, the ideal amount of financing debt is 4.422 billion yuan, of which loans, bonds and
other interest-bearing debts reach 2.609 billion yuan, while presale is 1.813 billion. At this point, the enterprise obtains the debt revenue of 0.445 billion yuan, and at the same time bears the debt cost of 0.338 billion yuan, and thus has a maximum net trade-off value of 0.107 billion yuan. The simulation results are normal compared to the statistics of the 91 listed real estate companies, showing that the model is adequate in reflecting the real market conditions and also valuable for the follow-up studies.

4.2. Simulation analysis of optimal capital structure model. By reviewing the optimal capital structure model, it could be found that \( i \) and \( r_t \) only have influences on debt revenue, while \( \delta \) and \( S \) just act on the debt cost. Based on the simple analysis of formulas (3), (8), (10) and (17), we can find that both \( i \) and \( r_t \) are positively correlated with the optimal debt ratio and the maximum net trade-off value, while \( \delta \) is negatively correlated with both, \( S \) has no relationship with the optimal debt ratio, but is positively correlated with the maximum net trade-off value. However, as far as the remaining 5 parameters \( r, T, \sigma, k \) and \( p \) in Table 1 are concerned, it is difficult to determine their impacts on the optimal capital structure and the maximum net trade-off value by simple observation. Therefore, the method of simulation analysis is utilized in this paper.

The process of implementing the simulation analysis is as follows. First, the variation range of the parameters is determined based on the realistic data. Second, the variation range of each parameter is equally divided into 100 intervals with 101 points. Finally, the corresponding optimal capital structures and net trade-off values can be calculated by introducing the parameters of these points into equations (19).

(1) Risk-free interest rate \( r \)

In order to analyze the impacts of risk-free interest rate \( r \) on the optimal debt ratio and the maximum net trade-off value, we let \( r \) be a value in the range of [2%, 11%], referring to the deposits interest rate during 1996 to 2012. The curves of the optimal debt ratio and the maximum net trade-off value are shown in Figure 1.

Figure 1 shows that:

1) The optimal debt ratio and the maximum net trade-off value are positively correlated with the risk-free interest rate, which means that as the time value of capital increases, the real estate enterprises are more inclined to acquiring capital through debt financing, and thus the net debt revenue also increases. The reason might be that corporate debt financing occupies the time value of money. A risk-free interest rate increase means an addition to the time value of money. As far as borrowers are concerned, debt financing becomes more profitable. Therefore, the optimal debt ratio and the maximum net trade-off value increase with risk-free interest rate.

2) When the risk-free interest rate changes from 2% to 11%, the ranges of the optimal debt ratio and the maximum net trade-off value (billion) are approximately

| Optimal debt ratio | Financing debt (billion) | Interest-bearing debt (billion) | Interest-free debt (billion) | Debt revenue (billion) | Debt cost (billion) | Maximum net trade-off value (billion) |
|--------------------|--------------------------|-------------------------------|-----------------------------|------------------------|-------------------|-------------------------------------|
| 0.33               | 4.422                    | 2.609                         | 1.813                       | 0.445                  | 0.338             | 0.107                               |

Table 2. Initial model simulation results of optimal capital structure model
Figure 1. Impacts of risk-free interest rate on optimal capital structure and maximum net trade-off value

[0.328, 0.378] and [0.105, 0.126], and the growth rates are respectively 15.2% and 20.0%.

(2) Debt maturity $T$
To examine the impacts of debt maturity $T$ on the optimal capital structure and the maximum net trade-off value, by letting $T$ vary in the range of [1, 10], we can get the curves of the optimal debt ratio and the maximum net trade-off value as shown in Figure 2.

As seen from Figure 2:
1) With the debt maturity increasing, the optimal debt ratio will decrease gradually. The reason is that with the extension of the liability period, companies may have accumulated a lot of long-term debt. In such companies, their solvencies may be in serious doubt. For the demand for risk management, banks will limit the amount of loans to such companies, reducing their optimal debt ratios.

2) The maximum net trade-off value will increase first and then decrease with its maximum near the point $T = 5$. That is, “$T = 5$” is the critical point: to its left, the maximum net trade-off value will increase as the debt maturity increases, while to its right, the situation is just the opposite. The maximum net trade-off value reaches its maximum when the debt maturity is just 5 years. With debt maturity increasing, on one hand, the debt revenue will increase as shown in formula (3); on the other hand, the cumulative long-term debt may also lead to an increase in bankruptcy cost. A small increase in debt maturity will provide the benefit to the enterprise greater than the cost of bankruptcy. At this point, the maximum net trade-off value is positively correlated with debt maturity. However, when the debt maturity exceeds a certain threshold, the growth of debt revenue can’t offset the increase in bankruptcy cost, and the maximum net trade-off value will decline as debt maturity increases.
3) When the debt maturity changes in the range of \([1, 5]\), the range of the optimal debt ratio and the maximum net trade-off value (billion) are approximately \([0.26, 0.48]\) and \([0.060, 0.120]\), and the change rates are respectively 84.6% and 100%, which means that in this range, the maximum net trade-off value is more sensitive to the debt maturity compared to the optimal debt ratio. However, in \([5, 10]\), the variation ranges of both are respectively \([0.18, 0.26]\) and \([0.010, 0.120]\), and the change rates are respectively 44.4% and 20.0%, which means that when the debt maturity is beyond 5 years, compared to the maximum net trade-off value, the optimal debt ratio is more sensitive to the debt maturity.

(3) Annual volatility of enterprise assets \(\sigma\)

In order to investigate the impacts of the annual volatility of enterprise assets \(\sigma\) on the optimal debt ratio and the maximum net trade-off value, we let \(\sigma\) vary in the range of \([0.1, 1.0]\). The curves of the optimal debt ratio and the maximum net trade-off value are shown in Figure 3.

It is shown in Figure 3 that:

1) With the annual volatility of enterprise assets less than 0.15, the optimal debt ratio is 0.6. It indicates that under financing constraints \((p=0.6)\), if \(\sigma \leq 0.15\), the optimal debt level depends on the upper limit of financing capability. If \(\sigma > 0.15\), the optimal debt ratio and the maximum net trade-off value are negatively correlated with the annual volatility of enterprise assets, and the curve slope has an obvious mutation near “0.4”. That is, “\(\sigma = 0.4\)” is the critical point. In the financial sense, enterprises with greater asset volatility usually face greater operational risk, and then greater bankruptcy cost. In other words, with other conditions unchanged, the optimal debt ratio and the maximum net trade-off value will decrease as the annual volatility of enterprise assets increase.
2) Both the optimal debt ratio and the maximum net trade-off value have faster rates of change in the left of the critical point and slower rates of change in the right of it.

(4) The proportion of interest-bearing debt \( k \)

In order to measure the influences of the proportion of interest-bearing debt \( k \) on the optimal debt ratio and the maximum net trade-off value, by letting \( k \) vary in the range of \([0, 1.0]\), we can get the curves of the optimal debt ratio and the maximum net trade-off value as shown in Figure 4.

Maximum net trade-off value

Figure 4 indicates that:

1) As the proportion of interest-bearing debt increases, both the optimal debt ratio and the maximum net trade-off value will increase gradually. Given the parameters in this paper, the marginal net trade-off value from interest-free debt is less than that from interest-bearing debt. The difference between the two marginal net trade-off values can be calculated with \( \Delta = i \times (1 - r_t) \times e^{-r(1-e^{-rT})} - [(1 + \delta)^T - 1]e^{-rT} \). With the parameters in Table 1, we can get \( \Delta \) as -0.04. Thus it can be seen that given market environment, interest-bearing debt will be more beneficial for enterprises than interest-free debt. Therefore, both the optimal debt ratio and the maximum net trade-off value are positively correlated with the proportion of interest-bearing debt.

2) When the proportion of interest-bearing debt varies in the range of \([0, 1.0]\), the ranges of the optimal debt ratio and the maximum net trade-off value (billion) are approximately \([0.25, 0.36]\) and \([0.020, 0.180]\), and the growth rates are respectively
44.0% and 800%, indicating that the maximum net trade-off value is much more sensitive to the proportion of interest-bearing debt.

(5) Financing constraints $p$

In order to analyze the impacts of financing constraints $p$ on the optimal debt ratio and the maximum net trade-off value, we let $p$ be a value in the range of $[0, 1]$. The curves of the optimal debt ratio and the maximum net trade-off value are shown in Figure 5.

Figure 5 shows that: With $p$ less than 0.33, the optimal debt ratio is equal to $p$, and the maximum net trade-off value is positively correlated with $p$. It indicates that when $p \leq 0.33$, the financing constraint determines the optimal debt ratio. When $p > 0.33$, the optimal debt ratio and the maximum net trade-off value have nothing to do with $p$, which means that loose constraints have no effect on the optimal financing decision.

5. Conclusion. By combining with the financing debt characteristics of real estate enterprises and analyzing respective revenues and costs of the interest-bearing debt and the interest-free debt, this paper established a mathematical model of the optimal capital structure and conducted numerical simulation to draw the following conclusions:

1) According to the mathematical model, the interest-bearing debt interest rate and the tax rate are positively correlated with the optimal debt ratio and the maximum net trade-off value of the real estate enterprises; annual housing price growth rate is negatively correlated with both; the total assets has no relationship with the optimal debt ratio, but is positively correlated with the maximum net trade-off value.
2) As seen from the simulation results, the risk-free interest rate and the proportion of interest-bearing debt are positively correlated with the optimal debt ratio and the maximum net trade-off value for the real estate enterprises; the annual volatility of enterprise assets is negatively correlated with both; as the debt maturity increases, the optimal debt ratio of real estate enterprises will decrease, while the maximum net trade-off value will increase at first and then decrease.

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REFERENCES

[1] R. Anderson and A. Carverhill, Corporate liquidity and capital structure, *Review of financial studies*, 25 (2012), 797–837.

[2] G. Andrade and S. N. Kaplan, How costly is financial (not economic) distress? Evidence from highly leveraged transactions that became distressed, *The Journal of Finance*, 53 (1998), 1443–1493.

[3] E. Barucci and L. Del Viva, Dynamic capital structure and the contingent capital option, *Annals of Finance*, 9 (2013), 337–364.

[4] F. Black and M. Scholes, The pricing of options and corporate liabilities, *The journal of political economy*, 81 (1973), 637–654.

[5] N. Chen and S. G. Kou, Credit spreads, optimal capital structure, and implied volatility with endogenous default and jump risk, *Mathematical Finance*, 19 (2009), 343–378.

[6] Y. Chu, Optimal capital structure, bargaining, and the supplier market structure, *Journal of Financial Economics*, 106 (2012), 411–426.

[7] D. O. Cook and T. Tang, Macroeconomic conditions and capital structure adjustment speed, *Journal of Corporate Finance*, 16 (2010), 73–87.

[8] M. J. Flannery and K. P. Rangan, Partial adjustment toward target capital structures, *Journal of Financial Economics*, 79 (2006), 469–506.
[9] R. A. Haugen, and L. W. Senbet, Bankruptcy and agency costs: Their significance to the theory of optimal capital structure, Journal of Financial and Quantitative Analysis, 23 (1988), 27–38.

[10] W. Jin, W. Zhang, B. Zhou and X. Xiong, Dynamic capital structure of the real estate companies in China, The theory and practice of finance and economics, 168 (2010), 67–71. (in Chinese)

[11] D. C. Mauer and S. Sarkar, Real options, agency conflicts, and optimal capital structure, Journal of Banking & Finance, 29 (2005), 1405–1428.

[12] E. Morellec, B. Nikolov and N. Schürhoff, Corporate governance and capital structure dynamics, The Journal of Finance, 67 (2012), 803–848.

[13] Ö. Öztekin and M. J. Flannery, Institutional determinants of capital structure adjustment speeds, Journal of Financial Economics, 103 (2012), 88–112.

[14] A. A. Robichek and S. C. Myers, Problems in the theory of optimal capital structure, Journal of Financial and Quantitative Analysis, 1 (1966), 1–35.

[15] B. Yang, Dynamic capital structure with heterogeneous beliefs and market timing, Journal of Corporate Finance, 22 (2013), 254–277.

[16] Z. Zhang, The value of debt guarantee, Research on financial and economic issues, 187 (1999), 22–27. (in Chinese)

[17] Z. Zhang and S. Xiao, Tax shield, bankruptcy cost and the optimal capital structure, Accounting Research, (2009), 47–55. (in Chinese)

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