Fermi Edge Singularity in Quantum Hall Systems far from Equilibrium

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In this paper we study the non-equilibrium one dimensional physics with the example of quantum Hall edge channel at integer filling factor Coulomb interacting with an artificial impurity. Electrons in an integer quantum Hall system effectively behave as free fermions, thus the interaction with a charged impurity normally leads to the orthogonality catastrophe and the Fermi edge singularity. Unlike in 3D materials where the Fermi edge singularity is commonly observed, the artificial impurity allows for a controllable interaction that can be made strong, leading to strong correlations and large scattering phases, resulting in resonant suppression of coherence on the channel, which can be detected by embedding the channel into an electronic interferometer and measuring the dip of the visibility of the interference pattern. However, in equilibrium the transition rates at the impurity satisfy the detailed balance equation and therefore the shape of the visibility dip is trivial. Thus we consider a more interesting regime where the transitions are induced by the non-equilibrium partitioning noise created in the interferometer itself by the beam splitter. The best method to describe such a system is the non-equilibrium bosonization technique which reduces the problem to the calculation of the full counting statistics of the currents after a tunneling contact. The full counting statistics is calculated analytically in the Markovian limit in the regimes of weak tunneling or weak backscattering, and supplemented by the direct numerical calculation of the electron correlation functions for intermediate transparencies. The main result is that the visibility is determined by both the Fermi edge singularity and the non-equilibrium physics via the Fermi edge singularity exponent and the transparency of the noise emitting QPC. The non-equilibrium effects are also manifested in the asymmetry of the visibility dip and in the non-trivial dependence of the dip position on the transparency.

INTRODUCTION

The Fermi liquid theory breaks down for one dimensional electronic systems due to the increased role of interactions and the formation of gapless collective modes, which puts the unique 1D physics apart from the other dimensionalities. Such systems have not been well studied until recently, and it is still little known about the role of disorder in one dimensional systems at the mesoscopic scales, where the dephasing and energy equilibration processes takes place. However, the progress in the semiconductor experimental techniques has made it possible to fabricate nanoscale systems suitable for studies of disorder physics.

The simplest and well controllable system for this purpose is a one dimensional quantum Hall edge channel interacting with a single localized impurity with a fluctuating charge, i.e., a quantum dot. The natural quantity to investigate in such system is the correlation function of the edge excitations. It can be studied experimentally by investigating in such system is the correlation function of the edge excitations. It can be studied experimentally by introducing interferometers utilizing one dimensional chiral quantum Hall edge states in place of optical beams and use two quantum point contacts (QPC) as beam splitters. This type of interferometers have been extensively studied recently from both experimental \[2,5\] and theoretical \[6,14\] sides.

First experimental and theoretical steps \[13,17\] in the investigation of controlled dephasing of Mach-Zehnder interferometer by a quantum dot strongly coupled to one
of its arms have been done very recently. The setup, schematically pictured in Fig. 1 consists of an equal arms MZ interferometer an $\nu = 2$, with an artificial impurity – a quantum dot with the level energy $\varepsilon_0$. Coulomb interacting with one arm, and several additional channels with Coulomb and tunnel coupling to the dot. When one of the incoming channels is biased with the voltage $\Delta \mu$, the left partitioning quantum point contact with transparency $T$ creates a non-equilibrium distributions in the arms of the interferometer. To concentrate on the effects of interaction with the impurity, the system must be kept in the low-energy limit where the relevant energies $\varepsilon_0, \Delta \mu$ are much smaller than the inverse time of flight through the interferometer. Then the internal dephasing in the channels can be neglected and the loss of visibility can only be caused by the interaction with the impurity. We will assume this requirement throughout the paper.

The principal energy scales determining the operating regime of such a system are: the environment temperature $\beta^{-1}$, the quantum $|\gamma_{L,R}|^2 \nu_F$ and classical $T \Delta \mu$ level broadening widths due to tunneling and transitions. In the essentially quantum case, when the quantum level broadening $|\gamma_{L,R}|^2 \nu_F$ is the dominant energy scale, the interferometer is found close to its ground state, and the effect of interaction consist in phase shift of the correlation function on the upper arm, as a non-trivial function of the level energy $\varepsilon_0$ which is determined by the virtual transitions at the dot. However, in the ground state this phase shift results only in a nudge of the interference pattern, but does not cause any loss of visibility.

The mentioned experiment by Weisz at al. have focused on the investigation of the thermal equilibrium case $\beta^{-1} \gg T \Delta \mu, |\gamma_{L,R}|^2 \nu_F$, where the fluctuations of the charge at the impurity are activated by the finite temperature of the sample. The dot is found then in the classical mixture with the Boltzmann occupations, and its influence on the interferometer consists of a fixed phase shift in one arm, and thus a shift of interference pattern, when the dot is occupied. Since the time resolution is insufficient to see each interference pattern separately, the observed visibility

$$V e^{i \Delta \phi_{AB}} = P_+ + P_+ e^{i 2 \pi \eta_D}$$  \hspace{1cm} (1)

is an averaging of the shifted patterns with the corresponding occupations, yielding the result $V e^{i \Delta \phi_{AB}} = (1 + e^{-\beta \varepsilon_0} e^{i 2 \pi \eta_D})/(1 + e^{-\beta \varepsilon_0})$ \cite{10}, where $2 \pi \eta_D$ is the scattering phase by the impurity on the upper arm. In the limit of strong interaction and symmetric coupling $\eta_D = 0.5$, a complete loss of visibility is observed, accompanied, respectively, by the $\pi$-valued jump of the phase shift of the Aharonov-Bohm oscillations when the energy level of the dot crosses the Fermi level. We must, however, argue that the thermal regime lacks particularly remarkable features, and suggest that much more interesting effects that may give important insights onto the underlying physics can be accessed in extended experiments at zero temperature $\beta^{-1} \rightarrow 0$ in the non-equilibrium regime. The visibility is then found in the same way \cite{1}, but the transitions in this case are induced by the non-equilibrium noise. The rates of these transition, which are the central subject of the present paper, are determined by the two cooperating factors: the non-equilibrium excitations in the channel coming from the partitioning QPC, and the backaction of the quantum fluctuations of the Fermi sea due to the perturbation by the event of tunneling itself, which leads to the so called orthogonality catastrophe known as the classical problem of the Fermi edge singularity (FES). This problem appears and has been studied in physics of many different kinds of systems \cite{18,22}.

The backaction effects are not small due to the strong Coulomb coupling, reflected in the large scattering phase (e.g., $2 \pi \eta_D \sim \pi$) on the channel passing by the charged impurity. This strong coupling requires a non-perturbative treatment of both Coulomb interactions and of the tunneling at the partitioning QPC that creates the non-equilibrium state at the edge. Indeed, the interactions strengths enter both into the resulting level width, and into the power-law exponent of the level energy dependence of the transition rates. The energy dependence of the rates is equivalent to the FES absorption spectrum of the Fermi sea with a deep level impurity.

The theoretical description of the non-equilibrium noise induced fluctuations in 1D systems is not trivial because the conventional bosonization technique could not be utilised in this situation. Instead, the non-equilibrium bosonization approach that has been recently developed in Ref. \cite{9,24,25} allows the description of such a system. In this approach the tunnelling currents through the quantum dot, which allows one to compare the value obtained from two different quantities measured independently. Interestingly, we find only one asymmetric visibility dip with the position reflecting the level energy of the dot crosses the Fermi level. We must, however, argue that the thermal regime lacks particularly remarkable features, and suggest that much more interesting effects that may give important insights onto the underlying physics can be accessed in extended experiments at zero temperature $\beta^{-1} \rightarrow 0$ in the non-equilibrium regime. The visibility is then found in the same way \cite{1}, but the transitions in this case are induced by the non-equilibrium noise. The rates of these transition, which are the central subject of the present paper, are determined by the two cooperating factors: the non-equilibrium excitations in the channel coming from the partitioning QPC, and the backaction of the quantum fluctuations of the Fermi sea due to the perturbation by the event of tunneling itself, which leads to the so called orthogonality catastrophe known as the classical problem of the Fermi edge singularity (FES). This problem appears and has been studied in physics of many different kinds of systems \cite{18,22}.

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verified experimentally and possible modifications of the experimental setup are also discussed.

THE FORMALISM

We concentrate on the case of non-equilibrium noise and backaction induced transitions. In the low-energy limit there is no intrinsic loss of coherence along the channels, and the interference pattern produced after the right beam splitter is determined completely by the phase shift in the upper arm. This allows us to separate the description of the relevant physics that is taking place in the region around the quantum dot, from the physics of the interferometer and the source of the non-equilibrium excitations. The tunneling in the beam splitter QPCs is therefore taken into account non-perturbatively through the boundary conditions for the edge excitations discussed in the next section. While the most suitable way to describe the edge channels, the quantum dot, and the Coulomb and tunnel couplings is to proceed straight from the bosonized Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_d + \mathcal{H}_i + \mathcal{H}_t,$$  
(2)

where the Hamiltonian of the edge channels is

$$\mathcal{H}_0 = \int \frac{dxdy}{8\pi^2} \sum_{ab} \partial_x \phi_a(x)V_{ab}(x,y)\partial_y \phi_b(y),$$  
(3)

and $V_{ab}(x,y)$ is the Coulomb interaction of the charge densities $\rho(x) = \partial_x \phi_a(x)/2\pi$ in different points of the channel. The bosonic fields $\phi$ satisfy the commutation relations $[\partial_x \phi_a(x), \phi_b(y)] = 2i\pi \delta_{ab}(x-y)$. The indexes $a, b$ enumerating the fields are taking the values $L, R, U, D$, according to the location of the corresponding channel to the left, right, up, or down from the quantum dot. The quantum dot Hamiltonian is $\mathcal{H}_d = \varepsilon_0 d^d d$, where $\varepsilon_0$ is the bare level energy determined by the applied gate voltage, which is the controllable parameter of the system. The Coulomb interaction between the quantum dot charge $d^d d$ and the charge density on the channels with the potential $U_a(x)$ is represented by

$$\mathcal{H}_i = d^d d \int \frac{dx}{2\pi} \sum_a U_a(x)\partial_x \phi_a(x).$$  
(4)

The tunneling Hamiltonian reads $\mathcal{H}_t = d^d \sum_a \tau_a e^{i\phi_a(0)} + h.c.$, with the tunneling amplitudes $\tau_a$ from the corresponding channel $a = L, R$, and $x = 0$ is chosen at the location of the quantum dot.

Since we are interested in the noise activated transitions, when the quantum level broadening is smaller than the noise temperature $|\tau_a|^2 \nu_F \ll T\Delta\mu$, where $\nu_F$ is the Fermi density of states, the tunneling $\mathcal{H}_t$ can be taken into account perturbatively. The remaining part of the Hamiltonian can be diagonalized with the help of the transformation $\mathcal{H} = e^{iS} \mathcal{H} e^{-iS}$, where

$$S = d^d d \int dx \sum_a \sigma_a(x)\phi_a(x).$$  
(5)

The functions $\sigma_a(x)$ are chosen such that the interaction part $\mathcal{H}_i$ is cancelled after the transformation, the requirement that is expressed by the integral equation

$$U_a(x) = -\int dx' \sum_b V_{ab}(x,x')\sigma_b(x').$$  
(6)

On the other hand, the Coulomb potential along the channel $a$, created by the impurity charge and the arbitrary charge density distribution $\rho_b(x)$ on the channel $b$, would be

$$\varphi_a(x) = U_a(x)d^d d + \int dx' \sum_b V_{ab}(x,x')\rho_b(x'),$$  
(7)

from where it follows that the solution $\sigma_a(x)$ of Eq. (5) is nothing but the charge density $\rho_b(x) = d^d d \delta_{ab}(x)$, accumulated on the ground channels $\varphi_b(x) = 0$, screening the charge present on the quantum dot. These densities are, naturally, localized in the interaction region around the quantum dot at $x = 0$.

The total charges can be expressed in terms of the zero-frequency Fourier components of the potentials $\eta_a \equiv -\int dx \sigma_a(x) = \sum_b V_{ab}^{-1} U_b|_{k=0}$. Given that no other metallic objects are located in the proximity of the quantum dot, the electro-neutrality principle implies that the total charge accumulated on the channels is equal to the dot charge $\sum_s \eta_a = 1$. Since the charge on the channel is directly related to the additional phase $\Delta \phi_a = 2\pi \eta_a$ acquired by a fermion passing by the dot, the latter equation repeats the statement of the Friedel sum rule [21].

The transformed tunneling Hamiltonian then reads:

$$\tilde{\mathcal{H}}_t = \sum_b \tau_b d^d d e^{i\int dx \sum_a \sigma_a(x)\varphi_a(x)} e^{i\phi_b(0)} + h.c.$$.  
(8)

In the low-energy limit considered here the wavelength of the excitations $u/\varepsilon$ is larger than the length $L$ of the interferometer, therefore the fields $\varphi_a(x)$ are almost constant in the region of the interaction, and can be approximated $\varphi_a(x) \approx \varphi_a(0)$ by their value at $x = 0$ in (8), which then becomes

$$\tilde{\mathcal{H}}_t = \sum_b \tau_b d^d d e^{i\phi_b(0)} - i \sum_a \eta_a \varphi_a(0) + h.c.$$  
(9)

The level energy in the dot Hamiltonian $\tilde{\mathcal{H}}_d = \varepsilon_0 d^d d$ is renormalized by the static self-interaction of the accumulated charge density:

$$\varepsilon_0 = \varepsilon_0 + \sum_a \int dx U_a(x)\sigma_a(x).$$  
(10)

The physical quantities, such as the visibility, will be investigated as functions of this parameter $\varepsilon_0$ throughout the rest of the paper.
TRANSITION RATES AND THE NON-EQUILIBRIUM BOSONIZATION

The rates of the tunneling transitions between the quantum dot and the side channels are found from the Golden rule expression

$$\Gamma_{\pm} = \int dt \left( \langle -| \hat{H}_t(0) \hat{H}_t | + \rangle \right)$$

(11)

where $\langle -|$ and $\langle +|$ are the states with the quantum dot empty and occupied, correspondingly, of the whole system unperturbed by the tunneling.

Using the prescription $\phi_a(x, t) = -2\pi Q_a(t)$ of the non-equilibrium bosonization [4], the fields $\phi_a(x, t)$ are expressed in terms of the total charges $Q_a(t)$ that have passed through a certain cross-section, such as right after the partitioning QPC for $\phi_a(x_L, t)$ on the biased channel, and the corresponding cross-sections for the other channels at ground state. We reiterate that although in general the interaction between the channels on the same edge split the boson excitations into the charged and dipole modes that scatter near the quantum dot, the mixing of these modes can be neglected in the low-energy limit, since it takes place on the length much larger than the size of the interferometer. Moreover, even for an interacting chiral system, the local correlation functions are not affected by these intra-edge interactions [3].

Substituting the tunneling Hamiltonian [4] into (11) and using the time translation invariance, the transition rates read

$$\Gamma_{b,\pm} = |\eta_b|^2 \int dt \ e^{i\epsilon_0 t} \left\langle \exp \left\{ \pm 2\pi i \left[ Q_b(0) - \sum_a \eta_a Q_a(0) \right] \right\} \times \exp \left\{ \mp 2\pi i \left[ Q_b(\pm t) - \sum_a \eta_a Q_a(\pm t) \right] \right\} \right\rangle$$

(12)

and can be further rewritten through the following important ingredient of the theory: the full counting statistics of the excitations on the biased edge channel

$$\chi(\lambda, t) = \langle e^{i\lambda Q(t)} e^{-i\lambda Q(0)} \rangle,$$

(13)

which is defined for $t > 0$ and continued for negative times by $\chi(\lambda, -t) = \chi^*(\lambda, t)$. The rates therefore read

$$\Gamma_{b,+} = |\eta_b|^2 \int dt \ e^{-i\epsilon_0 t} \chi_b(2\pi(1 - \eta_b), t) \prod_{a \neq b} \chi_a(-2\pi \eta_a, t)$$

(14a)

$$\Gamma_{b,-} = |\eta_b|^2 \int dt \ e^{i\epsilon_0 t} \chi_b(-2\pi(1 - \eta_b), t) \prod_{a \neq b} \chi_a(2\pi \eta_a, t).$$

(14b)

Here $\chi_{a \neq b}(\lambda) \propto (it+0)^{-1/2} \frac{\Delta \mu}{\pi}$ are the equilibrium correlation functions on the unbiased channels. The imaginary shift of the time is needed to keep track of the complex branches for arbitrary $\lambda$. The dimensional proportionality constant is non-universal and will be omitted from here onwards. For the biased arm of the interferometer, the generating function is non-trivial. It can be calculated analytically or numerically in different approximations and limits, which is discussed in the two following sections.

ANALYTIC SOLUTION IN THE MARKOVIAN LIMIT

In the Markovian, long time limit of the generating function can be found as a classical probability game result, known as the Levitov-Lee-Lesovik formula [27]:

$$\log \chi_a(\lambda, t) = -\frac{\lambda^2}{4\pi^2} \log (it + 0) + \frac{\Delta \mu t}{2\pi} \log (R + T e^{i\lambda}).$$

(15)

It is applicable when the main contribution to the integrals in Eqs. (13) comes from the long times $t \sim 1/\epsilon_0 \gg \Delta \mu^{-1}$, which implies small transparency $T \sim 1$ or small reflection $1 - T \ll 1$, and small detunings $\epsilon_0 \ll \Delta \mu$. One thus arrives to the expressions

$$\Gamma_{b,+}(\epsilon_0) \propto \text{Re} \int_0^{\infty} dt \ e^{-i\epsilon_0 t} \frac{\Delta \mu t}{(it + 0)^{1 - \alpha_b}},$$

(16a)

$$\Gamma_{b,-}(\epsilon_0) \propto \text{Re} \int_0^{\infty} dt \ e^{i\epsilon_0 t} \frac{\Delta \mu t}{(it + 0)^{1 - \alpha_b}},$$

(16b)

where the exponent

$$\alpha_b = 1 - \left[ (1 - \eta_b)^2 + \sum_{a \neq b} \eta_a^2 \right] = 2\eta_b - \sum_a \eta_a^2$$

(17)

turns out to be the conventional FES exponent with the accumulated charges $\eta_a$ that are, as previously mentioned, directly related to the scattering phases on the impurity.

By switching to the dimensionless time $\Delta \mu t$ one obtains the simple result for the transition rates

$$\Gamma_{b,+}(\epsilon_0) \propto \text{sign}(\alpha_b) \text{Im} \left[ \frac{-1}{(\epsilon_0 + i\gamma)^\alpha_b} \right]$$

(18a)

$$\Gamma_{b,-}(\epsilon_0) \propto \text{sign}(\alpha_b) \text{Im} \left[ \frac{e^{i\pi \alpha_b}}{(\epsilon_0 + i\gamma)^\alpha_b} \right]$$

(18b)

with the characteristic Lorentzian-like line shape with the power-law behavior of the FES absorption rates (see the dashed lines on Figs. 2 and 3).

The singularity is smeared by the real part $\text{Re} \gamma$ of

$$\gamma = -\frac{\Delta \mu}{2\pi} \log \left[ 1 + T(e^{i2\pi \eta_0} - 1) \right],$$

(19)
while its imaginary part $\text{Im} \gamma$ also contributes to the shift of the level energy $\varepsilon_0$. For $\eta_0 = 1/2$ the argument of the logarithm is real, therefore this shift is zero for the transparencies $T < 0.5$ and $-\Delta \mu/2$ for $T > 0.5$, thus the positions of the peaks of $\Gamma_\pm(T)$ are at $\varepsilon_0 = 0$ and $\varepsilon_0 = -\Delta \mu/2$. This corresponds to the phase transition studied in Ref. [9] and discussed further below in the context of the impurity energy level shift (see Fig. 7). In the equilibrium limit $\gamma = 0$ an imaginary shift of the singularity at $\varepsilon_0 = 0$ has to be kept for proper choice of the complex branch. And in the free-fermion case $\alpha = 0$ the rates are reduced to the corresponding step functions.

In the exponents $\alpha$, the positive term $2\eta_0$ represents the so-called “Mahan contribution” [18], and the negative term $-\sum_\alpha \eta_0^2$ is the “Anderson contribution” [19].

Note that depending on the distribution of the couplings and, correspondingly, the screening charges between the channels surrounding the quantum dot, $\alpha_0$ can be either smaller or larger than zero, therefore changing qualitatively the energy dependence of the transition rates. Interaction with the same channel on which the tunneling occurs increases $\alpha$ and favours the low energy transitions, therefore leading to the more singular character of the transition rates line shape as in Fig. [2] for $\alpha = 0.5$; while interaction with all the other channels decreases $\alpha$ and favours the higher energy transition, as shown on the Fig. 3 for $\alpha = -0.5$. Due to the device geometry (see Fig. 1 and complete screening of the dot in the experiment [15] at filling factor $\nu = 2$, the Coulomb interaction of the dot with the side channels is much weaker than with the horizontal channels which leads to $\alpha \sim -0.5$. However, by switching from weak tunneling to weak backscattering of the inner channels it should be possible to shift the balance and achieve positive $\alpha$ up to $\alpha = 0.5$ while also maintaining the phase shift of $\pi$, or even up to $\alpha = 1$ though sacrificing the strength $\eta_D$ of the coupling with the interfering channel.

**NUMERICS**

Due to the limited applicability of the Markovian approximation, it is especially interesting to complement the analytic results by the numerical data obtained in Ref. [28] using the Lemanicus BG/Q supercomputer [29]. The data is valid for the intermediate transparencies $0 < T < 1$ and for the shorter times $\Delta \mu t \sim 1$.

The approach is based on the calculation of the full

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{The transition rates $\Gamma_-$ and $\Gamma_+$ in arbitrary units as functions of the level energy $\varepsilon_0/\Delta \mu$, for $\eta_0 = 0.5$ and $\alpha = 0.5$ at different transparencies $T = 0.15$ (blue) and 0.85 (red). Analytic results in the Markovian limit are presented by the dashed lines and the numerical data by the solid lines. As expected for $\eta = 0.5$, the positions of the corresponding peaks for $T \to 0$ and $T \to 1$ are located near $\varepsilon_0 = 0$ and $\varepsilon_0 = -\Delta \mu/2$. Notice the more specific particle-hole symmetry $\Gamma_+(T) \leftrightarrow \Gamma_-(1 - T)$ demonstrated by the numerical results, see also Eq. (21), while in the Markovian limit all the four rate profiles are identical. While the analytic curves maintain the consistent power-law tails on both sides, the quantum non-equilibrium effects are represented by the characteristic suppression of the tails of the numerical curves at $\varepsilon = -\Delta \mu$ and $\varepsilon = \Delta \mu/2$ (see the insets) that are discussed below.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{The transition rates $\Gamma_+$ for the negative exponent $\alpha = -0.5$, in the same units, scale, and the coloring and dashing conventions same as on Fig. (2). Unlike in the $\alpha = 0.5$ case, the non-equilibrium effects are less pronounced for the intermediate transparencies, while the numerics provide less precision in the asymptotic regions. Nevertheless one can still identify the tail of $\Gamma_+(T = 0.15)$ reaching until $\varepsilon_0 = \Delta \mu/2$ (solid red lines).}
\end{figure}
counting statistics generator of currents at the left QPC [25, 26], expressed in terms of a free fermion determinant (see Refs. 27 and 30):
\[ (e^{i\Delta Q(t)} - e^{-i\Delta Q(0)}) = \det[1 - f(\varepsilon) + \exp(i\lambda P(t) \otimes S(\varepsilon))f(\varepsilon)], \]
where \( f(\varepsilon) \) is the electron distribution function, \( P(t) \) is the projector on the time interval \([0, t]\), and \( S(\varepsilon) \) is the scattering matrix of the QPC. Such determinant can be then evaluated numerically [28]. The numerical evaluation has been implemented with \( N = 1000 \) electrons. Although the data is only available for one special value of the phase shift of \( 2\pi \eta_0 = \pi \), this is the particular value relevant for the existing experiment. The correlation functions are then utilized in (14).

A distinct feature revealed in the numerics is the consequence of the particle-hole symmetry reflected in the correspondence
\[ \Gamma_+(T, \varepsilon_0) = \Gamma_-(1 - T, -\varepsilon_0 - \Delta \mu/2) \]  
while the opposite direction rates for the same transparency \( \Gamma_\pm(T, \varepsilon_0) \) are clearly distinct. In this respect the Markovian limit result has a degeneracy, since all the rates there \( \Gamma_\pm(T), \Gamma_\pm(1 - T) \) have identical energy profiles.

Though the algorithms are increasingly unstable in the limits of small \( T \ll 1 \) or large \( 1 - T \ll 1 \) transparencies, and the direct comparison with the analytics is not possible, one still sees a very adequate correspondence. For \( T = 0.15 \) and for \( T = 0.85 \) the quantitative agreement is still apparent for the peaks and the non-vanishing tails of the transition rates. The quantum non-equilibrium effects, presumably related to the second step in the initial non-equilibrium distribution function, although not captured by the Markovian limit calculation, are manifested by the sharp decline of \( \Gamma_+(T > 1/2) \) at \( \varepsilon_0 \sim \Delta \mu/2 \) and, in accordance with the symmetry [21], of \( \Gamma_-(T < 1/2) \) at \( \varepsilon_0 \sim -\Delta \mu \) (see the insets in Fig. 2). This effect is further emphasized in the strong suppression of the dot occupation probability and the visibility (Fig. 4) at the same values of \( \varepsilon_0 \), since they are composed of the ratios of the transition rates.

The structure of the tails of the transition rates has a similar origin as the dynamic Coulomb blockade effect. On the example of the \( \Gamma_- \) rate at the partitioning transparency \( T \ll 1 \) (see the solid blue line in the lower panel of Fig. 2), the electronic excitations on the arm of the interferometer repel the electron on the dot thus assisting the energetically unfavorable tunneling transition from the dot into the side channel, and therefore enhancing the tail at \( \varepsilon_0 < 0 \), compared to the Markovian limit result (dashed line). However, further than the broadening width \( \text{Re}(\gamma) \) from the resonance the effects become perturbative in transparency \( T \), and the tunneling is enabled by the single particle excitations with the energy \( \Delta \mu \), so the tail is eventually suppressed at \( \varepsilon_0 = -\Delta \mu \). Conversely, the transitions onto the dot are disadvantaged by the non-equilibrium excitations at low transparency, leading to the rapid decay of the \( \Gamma_+ \) rate at \( \varepsilon_0 > 0 \) (see the short tail of solid blue line in the upper panel of Fig. 2). At high transparency the same happens with the hole excitations in place of the electrons, according to the symmetry [21].

**RESULTS AND DISCUSSION**

With the transition rates and the stationary occupation probabilities one can immediately construct the re-
The resulting visibility profile \( \eta \), which for \( \eta_0 = 1/2 \) is

\[
V e^{i \Delta \phi_{AB}} = 1 - \frac{2 \text{Im} [ \varepsilon_0 + i \gamma]^{-\alpha}}{\text{Im} [ (1 + e^{i \pi (1 - \alpha)}) (\varepsilon_0 + i \gamma)^{-\alpha} ]}
\]

(22)

shown on Fig. 4. The visibility dip exhibits a crossover between the two asymptotes: \( V \sim 2 |\varepsilon_0|/\pi \theta_1 \) at \( \varepsilon_0 \ll T \Delta \mu \) and \( V \sim 1 - 2 \theta_2/\pi \varepsilon_0 \) at \( T \Delta \mu \ll \varepsilon_0 \ll \Delta \mu \) where the effective temperatures are, correspondingly,

\[
\theta_1 = \text{Re} \gamma \frac{2}{\pi \alpha} \tan \frac{\pi \alpha}{2},
\]

(23a)

\[
\theta_2 = \text{Re} \gamma \frac{\pi \alpha}{\sin(\pi \alpha)}.
\]

(23b)

Both these temperatures become equal to the “free fermion” noise temperature \( \text{Re} \gamma \) in the weak coupling limit \( \alpha = 0 \), when the transition rates become regular Lorentzians \( 31 \). Here \( \alpha \) has been left as the free parameter which can in principle vary due to the remaining couplings, even at fixed phase shift \( \pi \).

The power-law factor \( (\varepsilon_0 + i \gamma)_{-\alpha} \) enters homogeneously to the transition rates \( 15 \), and is thus cancelled in the visibility. Moreover, one can demonstrate that for the Markovian limit there is a symmetry \( V_M(\alpha) = V_M(-\alpha) \), even though the transition rates profiles, Fig. 2 for \( \alpha = 0.5 \), and Fig. 3 for \( \alpha = -0.5 \), are unmistakably distinct. This observation hints that the visibility itself is not the most fully characteristic quantity. We therefore suggest that a small bias \( \delta \mu \) is applied to one of the tunneling contacts, and the linear response tunneling current \( I_\beta = P_0 \Gamma_\beta^+ - P_1 \Gamma_\beta^- \) through the quantum dot is also measured

\[
I_L = \frac{\Gamma_{L+} \Gamma_{R-} - \Gamma_{R+} \Gamma_{L-}}{\Gamma_{R+} + \Gamma_{R-} + \Gamma_{L+} + \Gamma_{L-}}.
\]

(24)

Assuming symmetric tunnel couplings \( \tau_L = \tau_R = \tau \) and charge screening \( \alpha_L = \alpha_R = \alpha \), the resulting differential conductance \( G_L = \partial I_L/\partial \delta \mu \) then retains the signature of the FES power law in the Lorentzian-like peak

\[
G \propto \left| \frac{\varepsilon_0^{\alpha - 1}}{(\varepsilon_0 + i \gamma)^{\alpha + 2}} \text{Re}(\gamma) \right|^{\alpha} \cos \left[ \alpha \left( \frac{\pi}{2} - \text{arg}(\varepsilon_0 + i \gamma) \right) \right]
\]

(25)

with the power-law \( |\varepsilon_0|^{\alpha + 2} \) tails, the resonance width \( \text{Re} \gamma = -\Delta \mu / 2 \pi \log |R + Te^{i 2 \pi \eta_0}| \) due to the non-equilibrium noise, and the suppression of the tails at \( |\varepsilon_0| = \Delta \mu \). Because of this suppression, however, the power law only spans the limited segment of \( T \Delta \mu < \varepsilon_0 < \Delta \mu \) that can prove problematic to get a good fit at moderately small transparencies \( T \).

Alternatively, spectroscopy of the quantum dot can be performed by filling or draining the dot with a finite bias in one lead at highly asymmetric tunnel coupling (see, for example, recent experiment \( 32 \)), and measuring directly the transition rates in one of the contacts. In this case the
exponent can be accessed from the regular non-vanishing tail, and the characteristic shape of the decaying tails should also be available for observation.

The last notable feature is the dependence of the value \( \varepsilon^*_\eta \) of the parameter \( \varepsilon_\eta \) at which the visibility is maximally reduced, on the transparency of the partitioning QPC \( T \). When this QPC if fully transparent, the arm of the interferometer acts as an additional gate with the voltage \( \Delta \mu \), which for electrostatic reasons rises the energy level of the quantum dot by \( \eta_\eta \Delta \mu \). The position of the resonance therefore occurs at lower value of the original parameter \( \varepsilon^*_\eta = -\eta_\eta \Delta \mu \). This corresponds to the shift obtained from the Markovian approximation as the imaginary part \( \text{Im} \gamma \) of [19], although the approximation is applicable for small or large transparency and is not valid for the intermediate values. One can also consider the Gaussian approximation, relevant in case where the higher cumulants in the correlation function [19] are suppressed for some reason. This results in a linear drift of the level energy on the QPC transparency (red dashed line on Fig. 7). The numerical result for \( \alpha = 0.5 \) agrees asymptotically with the Markovian limit, while for \( \alpha = -0.5 \) it exhibits a dependence somewhat intermediate between the Markovian and the Gaussian limits. The actual profile that can be measured in the experiment can, therefore, tell which regime is rather taking place, and to what extent the higher cumulants are contributing to the statistics of the fluctuations.

FIG. 7: The position \( \varepsilon^*_\alpha / \Delta \mu \) of the visibility dip center as a function of the QPC transparency. The numerical results for \( \alpha = 0.5 \) (blue) and \( \alpha = -0.5 \) (red) presented with solid lines. Blue dashed line is for the Markovian limit, although it is applicable only asymptotically for \( T \to 0 \) and \( T \to 1 \); red dashes are for the Gaussian case with suppression of the higher cumulants. The numerics for \( \alpha = 0.5 \) clearly demonstrate an intermediate situation between the Markovian and Gaussian cases. For \( \alpha = -0.5 \) the numerics resemble more the Gaussian result. Note that the visible oscillations of the numerical curves are not physical but an artifacts of the discretization of the possible position of the dip minimum.

CONCLUSION

A quantum Hall edge channel, embedded in an electronic interferometer, with controllable Coulomb interaction with an artificial impurity is a very convenient system for investigation of the interaction effects and the Fermi edge singularity manifestations in one-dimensional electronic systems. In the low-energy limit when there is no intrinsic loss of coherence on the edge channels, the visibility of the interference pattern is only suppressed due to the averaging of the phase of scattering on the fluctuating impurity charge [1]. First experimental efforts studying such a system have been done recently [15], however, concentrating on the thermal equilibrium case, where the observed visibility is trivial because of the thermal occupation of the impurity. We extend the discussion onto the case when the transitions at the impurity are induced by the non-equilibrium partitioning noise created in the interferometer, and the backaction of the Fermi sea perturbations due to tunneling. Both factors are strong and are taken into account non-perturbatively. The non-equilibrium bosonization technique [4] is the framework of choice, that allows us to express all the electron correlation functions in terms of the full counting statistics [20] of the current passing through the beam splitter. Analytical expression for the FCS [15] is used in the Markovian limit for weak tunneling or backscattering in the partitioning QPC, and for the intermediate transparencies the FCS is computed numerically. The visibility profile (Fig. 4) is found non trivially depending on both the Fermi edge singularity exponent [17] and the parameters of the non-equilibrium noise, such as the partitioning QPC transparency. The non-equilibrium effects are also manifested in the asymmetry of the visibility dip and the non-trivial dependence of the dip position on the transparency (Fig. 7). Considering all the predicted features, we can strongly recommend further experimental investigations with this kind of setups in non-equilibrium regime.

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