Chern-Simons $AdS_5$ supergravity in a Randall-Sundrum background

Richard S. Garavuso* and Francesco Toppan†
Centro Brasileiro de Pesquisas Físicas
Rio de Janeiro, RJ 22290-180, Brazil

Abstract

Chern-Simons AdS supergravity theories are gauge theories for the super-AdS group. These theories possess a fermionic symmetry which differs from standard supersymmetry. In this paper, we study five-dimensional Chern-Simons AdS supergravity in a Randall-Sundrum scenario with two Minkowski 3-branes. After making modifications to the $D = 5$ Chern-Simons AdS supergravity action and fermionic symmetry transformations, we obtain a $\mathbb{Z}_2$-invariant total action $S = \tilde{S}_{\text{bulk}} + S_{\text{brane}}$ and fermionic transformations $\tilde{\delta}$. While $\tilde{\delta} \tilde{S}_{\text{bulk}} = 0$, the fermionic symmetry is broken by $S_{\text{brane}}$. Our total action reduces to the original Randall-Sundrum model when $\tilde{S}_{\text{bulk}}$ is restricted to its gravitational sector. We solve the Killing spinor equations for a bosonic configuration with vanishing $su(N)$ and $u(1)$ gauge fields.

*garavuso@cbpf.br
†toppan@cbpf.br
1 Introduction

Chern-Simons AdS supergravity [1, 2, 3] theories can be constructed only in odd spacetime dimensions. As the name implies, they are gauge theories for supersymmetric extensions of the AdS group [1]. They have a fiber bundle structure and hence are potentially renormalizable [2]. The dynamical fields form a single adS superalgebra-valued connection and hence the supersymmetry algebra closes automatically off-shell without requiring auxiliary fields [3]. The Lagrangian in dimension $D = 2n - 1$ is a Chern-Simons $(2n - 1)$-form for the super-adS connection and is a polynomial of order $n$ in the corresponding curvature. Unlike standard supergravity theories, there can be a mismatch between the number of bosonic and fermionic degrees of freedom [1]. For this reason, the ‘supersymmetry’ of Chern-Simons AdS supergravity theories is perhaps better referred to as a fermionic symmetry.

$D = 11, N = 1$ Chern-Simons AdS supergravity may correspond to an off-shell supergravity limit of M-theory [2–3]. It has expected features of M-theory which are not shared by $D = 11$ Cremmer-Julia-Scherk (CJS) supergravity [5]. These features include an $osp(32|1)$ superalgebra [6] and higher powers of curvature [7]. Hořava-Witten theory [8] is obtained from CJS supergravity by compactifying on an $S^1/Z_2$ orbifold and requiring gauge and gravitational anomalies to cancel. This theory gives the low energy, strongly coupled limit of the heterotic $E_8 \times E_8$ string theory. In light of the above discussion, it would be interesting to reformulate Hořava-Witten theory with $D = 11, N = 1$ Chern-Simons AdS supergravity.

Reformulating Hořava-Witten theory as described above may prove to be difficult. It is simpler to compactify the five-dimensional version of Chern-Simons AdS supergravity on an $S^1/Z_2$ orbifold and ignore anomaly cancellation issues. Canonical sectors of $D = 5$ Chern-Simons AdS supergravity have been investigated in locally AdS$_5$ backgrounds possessing a spatial boundary with topology $S^1 \times S^1 \times S^1$ located at infinity [9]. In this paper, as a preamble to reformulating Hořava-Witten theory, we will study $D = 5$ Chern-Simons AdS supergravity in a Randall-Sundrum background with two Minkowski

---

1 The AdS group in dimension $D \geq 2$ is $SO(D - 1, 2)$. The corresponding super-AdS groups are given in [3]. For $D = 5$ and $D = 11$, the super-AdS groups are respectively $SU(2, 2|N)$ and $OSp(32|N)$.

2 For example, in $D = 5$ Chern-Simons AdS supergravity [1], the number of bosonic degrees of freedom $(N^2 + 15)$ is equal to the number of fermionic degrees of freedom $(8N)$ only for $N = 3$ and $N = 5$.
3-branes [10]. We choose coordinates $x^\mu = (x^\bar{\mu}, x^5)$ to parameterize the five-dimensional spacetime manifold. In terms of these coordinates, the background metric takes the form

$$g_{\mu\nu}dx^\mu dx^\nu = a^2(x^5)\eta^{(4)}_{\bar{\mu}\bar{\nu}}dx^{\bar{\mu}}dx^{\bar{\nu}} + (dx^5)^2,$$

(1.1)

where $\eta^{(4)}_{\bar{\mu}\bar{\nu}} = \text{diag}(-1, 1, 1, 1)$, $a(x^5) \equiv \exp(-|x^5|/\ell)$ is the warp factor, and $\ell$ is the $AdS_5$ curvature radius. The coordinate $x^5$ parameterizes an $S^1/\mathbb{Z}_2$ orbifold, where the circle $S^1$ has radius $\rho$ and $\mathbb{Z}_2$ acts as $x^5 \rightarrow -x^5$. We choose the range $-\pi \rho \leq x^5 \leq \pi \rho$ with the endpoints identified as $x^5 \sim x^5 + 2\pi \rho$. The Minkowski 3-branes are located at the $\mathbb{Z}_2$ fixed points $x^5 = 0$ and $x^5 = \pi \rho$. These 3-branes have corresponding tensions $T(0)$ and $T(\pi \rho)$ and may support $(3+1)$-dimensional field theories.

This paper is organized as follows: In Section 2, we construct a $\mathbb{Z}_2$-invariant bulk theory. This bulk theory is obtained by making modifications to the $D=5$ Chern-Simons AdS supergravity action and fermionic symmetry transformations which allow consistent orbifold conditions to be imposed. The variation of the resulting bulk action $S_{\text{bulk}}$ under the resulting fermionic transformations $\delta_\epsilon$ vanishes everywhere except at the $\mathbb{Z}_2$ fixed points. We calculate $\delta_\epsilon S_{\text{bulk}}$ in Section 3. In Section 4, we modify $S_{\text{bulk}}$ and $\delta_\epsilon$ to obtain a modified $\mathbb{Z}_2$-invariant bulk theory. The modified bulk action $\tilde{S}_{\text{bulk}}$ is invariant under the modified fermionic transformations $\tilde{\delta}_\epsilon$. In Section 5, we complete our model by adding the brane action $S_{\text{brane}}$. We show in Section 6 that our total action

$$S = \tilde{S}_{\text{bulk}} + S_{\text{brane}}$$

(1.2)

reduces to the original Randall-Sundrum model [10] when $\tilde{S}_{\text{bulk}}$ is restricted to its gravitational sector. In Section 7, we solve the Killing spinor equations for a purely bosonic configuration with vanishing $su(N)$ and $u(1)$ gauge fields. Our concluding remarks are given in Section 8. Finally, in the Appendix, we work out the fünfbein, spin connection, curvature 2-form components, Ricci tensor, and Ricci scalar for our metric (1.1).

---

3 We use indices $\mu, \nu, \ldots = 0, 1, 2, 3, 5$ for local spacetime and $a, b, \ldots = 0, 1, 2, 3, 5$ for tangent spacetime. The corresponding metrics, $g_{\mu\nu}$ and $\eta_{ab} = \text{diag}(-1, 1, 1, 1, 1)$, are related by $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$, where $e_\mu^a$ is the fünfbein. Barred indices $\bar{\mu}, \bar{\nu}, \ldots = 0, 1, 2, 3$, and $\bar{a}, \bar{b}, \ldots = 0, 1, 2, 3$ denote the four-dimensional counterparts of $\mu, \nu, \ldots$ and $a, b, \ldots$, respectively.
2 \( Z_2 \)-invariant bulk theory

In this section, we construct a \( Z_2 \)-invariant bulk theory. The bulk theory is obtained by making modifications to the \( D = 5 \) Chern-Simons AdS supergravity \[1\] action and fermionic symmetry transformations which allow consistent orbifold conditions to be imposed.

The field content of \( D = 5 \) Chern-Simons AdS supergravity is the fünfbein \( e_{\mu}^a \), the spin connection \( \omega_{\mu}^{ab} \), the \( su(N) \) gauge connection \( A_\mu = A_i^\mu \tau_i \), the \( u(1) \) gauge connection \( B_\mu \), and \( N \) complex gravitini \( \psi_{\mu r} \) which transform as Dirac spinors in a vector representation of \( su(N) \). These fields form a connection for the adS superalgebra \( su(2,2|N) \). The action and fermionic symmetry transformations are given in \[9\] in terms of the \( AdS_5 \) curvature radius \( \ell \). The only free parameter in the action is a dimensionless constant \( k \). To allow consistent \( Z_2 \) orbifold conditions to be imposed, we make the following modifications:

1. Rescale the \( su(N) \) and \( u(1) \) gauge connections:
   \[ A \rightarrow g_A A, \quad B \rightarrow g_B B. \]

2. Replace \( g_A, g_B, \ell^{-1}, \) and \( k \) by the \( Z_2 \)-odd expressions\[\[^4\]
   \[ G_A \equiv g_A \text{sgn}(x^5), \quad G_B \equiv g_B \text{sgn}(x^5), \quad L^{-1} \equiv \ell^{-1} \text{sgn}(x^5), \quad K \equiv k \text{sgn}(x^5). \]

In this manner, we obtain the bulk action

\[ S_{\text{bulk}} = S_{\text{grav}} + S_{su(N)} + S_{u(1)} + S_{\text{ferm}}, \] \[2.1\]

\[^4\] We use indices \( i, j, \ldots = 1, \ldots, N^2 - 1 \) to label the \( N \times N \)-dimensional \( su(N) \) generators \( \tau_i \). The indices \( r, s, \ldots = 1, \ldots, N \) label vector representations of \( su(N) \). We will use the notation \( A_r^i \equiv A_i^r \tau_i^r \). Spinor indices \( \alpha, \beta, \ldots \) will sometimes be suppressed.

\[^5\] The signum function \( \text{sgn}(x^5) \) is +1 for \( 0 < x^5 < \pi \rho \) and −1 for \( -\pi \rho < x^5 < 0 \). It obeys \( \text{sgn}^2(x^5) = 1 \) and \( \partial_5 \text{sgn}(x^5) = 2[\delta(x^5) - \delta(x^5 - \pi \rho)] \).
where
\[ S_{\text{grav}} = \frac{1}{k^2} K \varepsilon_{abcde} \left( \frac{1}{4} R_{ab} R_{cd} e^e + \frac{2}{3L} R_{ab} e^e d e + \frac{1}{3L} e^a e^b e^c e^d \right), \]
\[ S_{su(N)} = \int i K \, \text{str} \left( G_A^3 A F^2 - \frac{1}{2} G_A^4 A^3 F + \frac{1}{10} G_A^5 A^5 \right), \]
\[ S_{u(1)} = \int K \left[ - \left( \frac{1}{3^2} - \frac{1}{N^2} \right) G_B^3 B(dB)^2 + \frac{3}{4L^2} \left( T^a T_a - \frac{L^2}{2} R^{ab} R_{ab} \right) \right. \]
\[ \left. - R_{ab} e_a e_b G_B B - \frac{3}{8} G_A^2 G_B F_s F_r B \right], \]
\[ S_{\text{ferm}} = \int \frac{3}{2L} K \left( \bar{\psi}^r \mathcal{R}^a_{\beta} \nabla \psi^a r_{\beta} + \bar{\psi}^a \mathcal{F}^r_{a} \nabla \psi^a r_{r} \right) + \text{c.c.}, \] (2.2)
and the transformations
\[ \delta e^a = - \frac{1}{2} \left( \bar{\psi}^r \Gamma^a_{\beta} \epsilon_r - \bar{\epsilon}^r \Gamma^a_{\beta} \psi_r \right), \quad \delta \epsilon^{ab} = \frac{1}{4} \left( \bar{\psi}^r \Gamma^{ab} \epsilon_r - \bar{\epsilon}^r \Gamma^{ab} \psi_r \right), \]
\[ \delta \bar{\psi}^r = - \nabla \epsilon_r, \quad \delta \bar{\epsilon}^r = - \nabla \psi_r, \]
\[ \delta e^a = i \left( \bar{\psi}^r \epsilon_s - \bar{\epsilon}^r \psi_s \right), \quad \delta B = i \left( \bar{\psi}^r \epsilon_r - \bar{\epsilon}^r \psi_r \right). \] (2.3)

In these expressions, \( \Gamma^a \) are the Dirac matrices, \( \Gamma^{ab} \equiv \frac{1}{2} \left( \Gamma^a \Gamma^b - \Gamma^b \Gamma^a \right) \), \( R^{ab} = d\omega^{ab} + \omega^{ac} \omega_c^b \) is the curvature 2-form, \( T^a = d e^a + \omega^a \sigma^b \) is the torsion 2-form, \( F = dA + G_A A^2 = F^i \tau_i \) is the \( su(N) \) curvature,
\[ \mathcal{R}^a_{\beta} \equiv \frac{1}{2L} T^a(\Gamma_a)_{\beta} + \frac{1}{4} (R_{ab} + \frac{1}{2L} e^a e^b) (\Gamma_{ab})_{\beta} + i \frac{1}{4} G_B d B \delta^a_{\beta} - \frac{1}{2} \bar{\psi}^a \bar{\psi}_s, \]
\[ \mathcal{F}^r_s \equiv F^r_s + \frac{1}{N} G_B d B \delta^r_s - \frac{1}{2} \bar{\psi}^r \bar{\psi}_s, \] (2.4)
\( \text{str} \) is a symmetrized trace satisfying \( \text{str}(\tau_i \tau_j \tau_k) \equiv \frac{1}{2L} \text{tr} \{ \tau_i \tau_j \tau_k \} \), \( \nabla \) is the \( adS_5 \times su(N) \times u(1) \) covariant derivative, and
\[ \nabla \psi_r \equiv (d + \frac{1}{4} \omega_{ab} \Gamma_{ab} + \frac{1}{2L} e^a \Gamma_a) \psi_r - G_A A^s_r \psi_s + i \left( \frac{1}{4} - \frac{1}{N} \right) G_B B \psi_r, \]
\[ \nabla \epsilon_r \equiv (d + \frac{1}{4} \omega_{ab} \Gamma_{ab} + \frac{1}{2L} e^a \Gamma_a) \epsilon_r - G_A A_s^r \epsilon_s + i \left( \frac{1}{4} - \frac{1}{N} \right) G_B B \epsilon_r. \] (2.5)

Note that the results in the Appendix can be used to show that the torsion vanishes for our metric.

We impose the following orbifold conditions:

---

6 We choose a chiral basis for the 4 \( \times \) 4 Dirac matrices
\[ \Gamma^a = \left( \Gamma^a, \Gamma^5 \right) = \left( \begin{array} {cc} 0 & -i \sigma^a \\ -i \bar{\sigma}^a & 0 \end{array} \right), \left( \begin{array} {cc} -1 & 0 \\ 0 & 1 \end{array} \right), \] where \( \sigma^a = (1, \bar{\sigma}) \) and \( \bar{\sigma}^a = (1, -\sigma) \). These matrices satisfy \( \text{tr} (\Gamma_a \Gamma_b \Gamma_c \Gamma_d \Gamma_e) = -4i \varepsilon_{abcde} \), where \( \varepsilon_{abcde} \) is the Levi-Civita tensor and \( \varepsilon^{01235} = 1 \).
1. **Periodicity on** $S^1$. The fields and the fermionic parameters $\epsilon_r$, denoted generically by $\phi$, are required to be periodic on the circle $S^1$. That is,

$$\phi(x^\mu, x^5) = \phi(x^\mu, x^5 + 2\pi \rho). \tag{2.6}$$

2. $\mathbb{Z}_2$ **parity assignments**. The bosonic field components

$$\Phi = e^\mu_\bar{a}, e^5_{\dot{5}}, A^i_{\bar{5}}, B_5, \quad \Theta = e^5_{\bar{5}}, e^\mu_{\dot{5}}, A^i_{\mu}, B_{\dot{5}}$$

are chosen to satisfy

$$\Phi(x^\mu, x^5) = +\Phi(x^\mu, -x^5), \quad \Theta(x^\mu, x^5) = -\Theta(x^\mu, -x^5). \tag{2.7}$$

That is, the $\Phi$ components are $\mathbb{Z}_2$-even and the $\Theta$ components are $\mathbb{Z}_2$-odd. For the gravitini, we require

$$\Gamma^{\dot{5}} \psi_{\bar{r}(\mu, x^5)} = \psi_{\dot{r}(\mu, -x^5)}, \quad \Gamma^{\dot{5}} \psi_{5r}(\mu, x^5) = -\psi_{5r}(\mu, -x^5). \tag{2.8}$$

Finally, the fermionic parameters $\epsilon_r$ are required to satisfy

$$\Gamma^{\dot{5}} \epsilon_r(x^\mu, x^5) = +\epsilon_r(x^\mu, -x^5). \tag{2.9}$$

These conditions imply that the $\mathbb{Z}_2$-odd quantities vanish at the orbifold fixed points. It is straightforward to check that $S_{\text{bulk}}$ is $\mathbb{Z}_2$-even and that the transformations (2.3) are consistent with the $\mathbb{Z}_2$ parity assignments.

### 3 Calculation of $\delta \epsilon S_{\text{bulk}}$

The $D = 5$ Chern-Simons AdS supergravity action is invariant (up to a boundary term) under its fermionic symmetry transformations. In Section 2 we modified this action and its fermionic transformations to obtain a $\mathbb{Z}_2$-invariant bulk theory. Due to the signum functions introduced by the modifications, $\delta \epsilon S_{\text{bulk}}$ contains terms which have no counterpart in the unmodified theory. More specifically, the extra terms arise from $\partial_5$ acting on
the signum functions to yield delta functions. Such ‘delta function’ contributions to $\delta_c S_{\text{bulk}}$ can potentially spoil the fermionic symmetry only at the $\mathbb{Z}_2$ fixed points. Thus, $S_{\text{bulk}}$ is invariant under its fermionic transformations everywhere except perhaps at the $\mathbb{Z}_2$ fixed points. In this section, we will calculate $\delta_c S_{\text{bulk}}$.

For our metric and $\mathbb{Z}_2$ parity assignments, the uncanceled variation $\delta_c S_{\text{bulk}}$ arises from the variation of the 4-Fermi terms. The 4-Fermi terms are

$$S_{\psi^4} = \frac{3i}{4} \int K \left( \bar{\psi}_r^\mu \psi_\alpha \bar{\psi}_s^\alpha \nabla \psi_r^\mu + \bar{\psi}_s^\mu \psi_r^\alpha \nabla \psi_s^\alpha \right) + \text{c.c.}$$

Let us now compute $\delta_c S_{\text{bulk}}$ by applying $\delta_c$ to (3.1) and dropping all terms which contribute no delta functions. For our metric and $\mathbb{Z}_2$ parity assignments, we can drop all but

1. The $\partial_\mu$ part of $\nabla_\mu$.
2. The $-\partial_\mu \epsilon_r$ part of $\delta_c = -\nabla_\mu \epsilon_r$.

The only contributing terms are thus contained in the expression

$$Q \equiv -\frac{3i}{2} \frac{1}{4!} \int d^5 x K \left\{ \varepsilon^{\mu_5 \rho_5 \lambda_5} (\partial_5 \epsilon_5 \psi_{\nu_5}) (\bar{\psi}_r^\mu \partial_\sigma \psi_{\lambda r}) + \varepsilon^{\mu_5 \rho_5 \lambda_5} (\bar{\psi}_r^\mu \partial_5 \psi_{\lambda r}) (\partial_5 \epsilon_5 \partial_\sigma \psi_{\lambda r}) \right. $$

$$+ \varepsilon^{\mu_5 \rho_5 \lambda_5} \left[ (\partial_\mu \epsilon_5 \psi_{\nu_5}) (\bar{\psi}_r^\mu \partial_5 \psi_{\lambda r}) + (\bar{\psi}_r^\mu \partial_5 \epsilon_5) (\bar{\psi}_r^\mu \partial_5 \psi_{\lambda r}) \right. $$

$$+ \left( \bar{\psi}_r^\mu \psi_{\nu_5} \right) (\partial_5 \epsilon_5 \partial_5 \psi_{\lambda r}) + (\bar{\psi}_r^\mu \psi_{\nu_5} \partial_5 \epsilon_5) \left. \right\} + \text{c.c.}$$

(3.2)
More specifically, the delta function terms contained in $Q$ are obtained by integrating by parts and keeping only the terms in which $\partial_5$ acts on $K$. Thus,

$$
\delta_\epsilon S_{\text{bulk}} = \frac{3i}{2} \int d^5x \, \partial_5 K \left\{ \varepsilon^{5\rho\delta\lambda} \left( \bar{\psi}_\rho \partial_5 \psi_{\lambda r} \right) + \varepsilon^{5\mu\delta\lambda} \left( \bar{\psi}_\mu \partial_5 \psi_{\lambda r} \right) + \varepsilon^{5\mu\delta\lambda} \left( \bar{\psi}_\mu \partial_5 \psi_{\lambda r} \right) \right\} + c.c.,
$$

where

$$\partial_5 K = 2k \left[ \delta(x^5) - \delta(x^5 - \pi \rho) \right].$$

4 Modified $\mathbb{Z}_2$-invariant bulk theory

The result (3.3) for $\delta_\epsilon S_{\text{bulk}}$ demonstrates that $S_{\text{bulk}}$ is not invariant under the fermionic transformations $\delta_\epsilon$. In this section, we will modify $S_{\text{bulk}}$ and $\delta_\epsilon$ by replacing the $adS_5 \times su(N) \times u(1)$ covariant derivative $\nabla$ with $\tilde{\nabla}$, where

$$
\tilde{\nabla}_\sigma \psi_{\lambda r} \equiv \nabla_\sigma \psi_{\lambda r} + 2\delta_5^5 \delta_\lambda^\lambda \left[ \delta(x^5) - \delta(x^5 - \pi \rho) \right] \text{sgn}(x^5) \Gamma_5 \psi_{\lambda r},
$$

$$
\tilde{\nabla}_\sigma \epsilon_r \equiv \nabla_\sigma \epsilon_r + 2\delta_5^5 \left[ \delta(x^5) - \delta(x^5 - \pi \rho) \right] \text{sgn}(x^5) \Gamma_5 \epsilon_r.
$$

We will show that the modified bulk action

$$
\tilde{S}_{\text{bulk}} \equiv S_{\text{bulk}}(\nabla \to \tilde{\nabla}) \equiv S_{\text{bulk}} + \Delta S_{\text{bulk}}
$$

is invariant under the modified transformations

$$
\tilde{\delta}_\epsilon \equiv \delta_\epsilon(\nabla \to \tilde{\nabla}) \equiv \delta_\epsilon + \Delta \delta_\epsilon.
$$

That is, we will show that

$$
\tilde{\delta}_\epsilon \tilde{S}_{\text{bulk}} = \delta_\epsilon S_{\text{bulk}} + (\Delta \delta_\epsilon) S_{\text{bulk}} + \tilde{\delta}_\epsilon (\Delta S_{\text{bulk}})
$$

vanishes. It is straightforward to check that $\tilde{S}_{\text{bulk}}$ is $\mathbb{Z}_2$-invariant and the transformations $\tilde{\delta}_\epsilon$ are consistent with our $\mathbb{Z}_2$ parity assignments.
We begin by computing \((\Delta \delta_\epsilon) S_{\text{bulk}}\). For our metric and \(\mathbb{Z}_2\) parity assignments, the only part of \(S_{\text{bulk}}\) which is not invariant under \(\Delta \delta_\epsilon\) is \(S_{\psi^4}\) (given by (3.1)). Note that

\[
(\Delta \delta_\epsilon) \psi_{\lambda r} = -2\delta_\epsilon^5 \left[ \delta(x^5) - \delta(x^5 - \pi \rho) \right] \text{sgn}(x^5) \Gamma \xi \epsilon_r. \tag{4.5}
\]

Thus, after using \(K \text{sgn}(x^5) = k, (2.9),\) and (3.1), we obtain

\[
(\Delta \delta_\epsilon) S_{\text{bulk}} = -\frac{3i}{24} \int d^5x \partial_\xi K \left[ \varepsilon^{5\rho\beta\xi} \left( \tilde{\psi}_\rho^s \partial_\beta \psi_{\lambda r} \right) \right.
\]
\[
+ \varepsilon^{\mu\beta\xi\lambda} \left( \tilde{\psi}_\mu^s \partial_\beta \psi_{\lambda r} \right) + \varepsilon^{\mu\beta\xi\lambda} \left( \tilde{\psi}_\mu^s \partial_\beta \bar{\psi}_{\lambda r} \right)
\]
\[
+ \varepsilon^{\mu\beta\xi\lambda} \left( \tilde{\psi}_\mu^s \partial_\beta \bar{\psi}_{\lambda r} \right) \] + c.c. \tag{4.6}
\]

Now, let us compute \(\tilde{\delta}_\epsilon (\Delta S_{\text{bulk}})\). For our metric and \(\mathbb{Z}_2\) parity assignments, the only part of \(S_{\text{bulk}}\) which is changed by the replacement \(\nabla \rightarrow \tilde{\nabla}\) is \(S_{\psi^4}\). After using \(K \text{sgn}(x^5) = k, (2.8),\) and (3.1), we obtain

\[
\Delta S_{\text{bulk}} = \frac{3i}{24} \int d^5x \partial_\xi K \varepsilon^{\mu\beta\xi\lambda} \left( \tilde{\psi}_\mu^s \partial_\beta \psi_{\lambda r} \right) + c.c. \tag{4.7}
\]

Applying \(\tilde{\delta}_\epsilon\) to (4.7) yields

\[
\tilde{\delta}_\epsilon (\Delta S_{\text{bulk}}) = -\frac{3i}{24} \int d^5x \partial_\xi K \varepsilon^{\mu\beta\xi\lambda} \left[ \partial_\mu \bar{\psi}_{\mu s} \psi_{\lambda r} + \partial_\beta \bar{\psi}_{\lambda r} \psi_{\mu s} \right. \left. + \left( \tilde{\psi}_\mu^s \partial_\lambda \psi_{\beta r} \right) \right] + c.c. \tag{4.8}
\]

Using the results (3.3), (4.6), and (4.8) in (4.1) yields

\[
\tilde{\delta}_\epsilon \tilde{S}_{\text{bulk}} = 0. \tag{4.9}
\]

5 Brane action

To complete our model, we add the brane action \(S_{\text{brane}}\). In the absence of particle excitations, the brane action consists of brane tensions. That is,

\[
S_{\text{brane}} = -\int d^5x e^{(4)} \left[ T^{(0)} \delta(x^5) + T^{(\pi \rho)} \delta(x^5 - \pi \rho) \right] + \text{excitations}, \tag{5.1}
\]

where \(e^{(4)} \equiv \det(e_{\mu}^{\ a})\). As discussed in Section 2, \(\mathbb{Z}_2\)-odd quantities vanish at the \(\mathbb{Z}_2\) fixed points. Thus, it is clear that \(S_{\text{brane}}\) is \(\mathbb{Z}_2\)-even. Further discussion of 3-brane actions can be found in [11].
6 Connection with original RS model

In this section, we will show that our total action \( S = \tilde{S}_{\text{bulk}} + S_{\text{brane}} \) reduces to the original Randall-Sundrum model [10] when \( \tilde{S}_{\text{bulk}} \) is restricted to its gravitational sector.

The gravitational sector of \( \tilde{S}_{\text{bulk}} \) is \( S_{\text{grav}} \), given by the first equation of (2.2). \( S_{\text{grav}} \) consists of three terms:

1. The ‘Gauss-Bonnet’ term \( \int \frac{1}{8} K \varepsilon_{abce} R^{ab} R^{cd} e^{e} / L \).
2. The ‘Einstein-Hilbert’ term \( \int \frac{1}{8} \cdot \frac{2}{3} K \varepsilon_{abce} e^{c} e^{d} e^{e} / L^{3} \).
3. The ‘cosmological constant’ term \( \int \frac{1}{8} \cdot \frac{1}{5} K \varepsilon_{abce} e^{a} e^{b} e^{c} e^{d} e^{e} / L^{5} \).

For our metric, the first term can be expressed as a linear combination of the other two. Summing the three terms yields an effective Einstein-Hilbert term and an effective cosmological constant term. To demonstrate this explicitly, let us evaluate \( S_{\text{grav}} \) for our metric. Using the results in the Appendix, we obtain

\[\varepsilon_{abce} R^{ab} R^{cd} e^{e} = d^{5} x e \left( -\frac{120}{\ell^{4}} + \frac{192}{\ell^{3}} \left[ \delta(x^{5}) - \delta(x^{5} - \pi \rho) \right] \right),\]

\[\varepsilon_{abce} e^{c} e^{d} e^{e} = d^{5} x e (-6 R),\]

\[\varepsilon_{abce} e^{a} e^{b} e^{c} e^{d} e^{e} = d^{5} x e (-5!),\]

where \( e \equiv \det(e_{\mu}^{a}) \). Thus,

\[S_{\text{grav}} = \int d^{5} x \frac{1}{8} \left\{ \frac{k}{\ell} \left( -\frac{120}{\ell^{4}} + \frac{192}{\ell^{3}} \left[ \delta(x^{5}) - \delta(x^{5} - \pi \rho) \right] \right) \right.\]

\[+ \frac{2k}{3\ell^{3}} (-6 R) + \frac{k}{5\ell^{5}} (-5!) \}\]

\[= \int d^{5} x \frac{k}{\ell^{3}} \left\{ -\frac{15}{\ell^{2}} + \frac{24}{\ell} \left[ \delta(x^{5}) - \delta(x^{5} - \pi \rho) \right] - \frac{1}{2} R - \frac{3}{\ell^{2}} \right\}\]

\[= \int d^{5} x \frac{k}{\ell^{3}} \left\{ \frac{3}{2} \left( -\frac{20}{\ell^{2}} + \frac{16}{\ell} \left[ \delta(x^{5}) - \delta(x^{5} - \pi \rho) \right] \right) - \frac{1}{2} R + \frac{12}{\ell^{2}} \right\}\]

\[= \int d^{5} x \frac{k}{\ell^{3}} \left( R + \frac{12}{\ell^{2}} \right)\]

\[= \int d^{5} x \left( 2M^{3} R - \Lambda \right),\]

(6.2)
where $M$ is the five-dimensional gravitational mass scale, $\Lambda$ is the bulk cosmological constant, and

$$M^3 = \frac{k}{2\ell^3}, \quad \Lambda = -\frac{24M^3}{\ell^2}. \quad (6.3)$$

Combining the result (6.2) with (5.1), we obtain the action of the original Randall-Sundrum model. It is shown in [10] that the five-dimensional vacuum Einstein’s equations for this system,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{1}{4M^3} \left\{ g_{\mu\nu}\Lambda + e^{(4)}_\mu\delta^\rho_\nu g_{\rho\sigma} \left[ T^{(0)}(\delta(x^5)) + T^{(\pi\rho)}(\delta(x^5 - \pi\rho)) \right] \right\},$$

are solved by our metric provided that the relations

$$T^{(0)} = -T^{(\pi\rho)} = 24M^3/\ell, \quad \Lambda = -24M^3/\ell^2 \quad (6.4)$$

are satisfied.

### 7 Killing spinors

In this section, we will solve the Killing spinor equations for a purely bosonic configuration with vanishing $su(N)$ and $u(1)$ gauge fields. In this case, the Killing spinor equations reduce to

$$0 = \delta_\epsilon \psi_\mu = -\partial_\mu \epsilon_r - \frac{1}{2} \frac{a'}{a} \Gamma_\mu (\Gamma_5 - 1) \epsilon_r,$$

$$0 = \delta_\epsilon \psi_5 = -\partial_5 \epsilon_r + \frac{1}{2} \frac{a'}{a} \Gamma_5 \epsilon_r - 2 \left[ \delta(x^5) - \delta(x^5 - \pi\rho) \right] \text{sgn}(x^5) \Gamma_5 \epsilon_r. \quad (7.1)$$

To solve these equations, split $\epsilon_r$ into $\mathbb{Z}_2$-even ($\epsilon_r^+$) and $\mathbb{Z}_2$-odd ($\epsilon_r^-$) pieces:

$$\epsilon_r = \epsilon_r^+ + \epsilon_r^-,$$

where

$$\epsilon_r^\pm \equiv \frac{1}{2} (\epsilon_r \pm \Gamma_5 \epsilon_r) = \pm \Gamma_5 \epsilon_r^\pm. \quad (7.3)$$

---

7 $M$ is related to the four-dimensional gravitational mass scale $M_{(4)} = 2.43 \times 10^{18}$ GeV by $M^2_{(4)} = M^3 \int_{-\pi\rho}^{\pi\rho} dx^5 a^2(x^5) = M^3 \ell [1 - \exp(-2\pi\rho/\ell)]$. The effective mass scales on the 3-branes at $x^5 = 0$ and $x^5 = \pi\rho$ are respectively $M_{(4)}$ and $M_{(4)} e^{-\pi\rho/\ell}$. If the Standard Model fields live on the 3-brane at $x^5 = \pi\rho$, then $M_{(4)} e^{-\pi\rho/\ell}$ can be associated with the electroweak scale.
We obtain the following system of equations:

\[
\begin{align*}
\partial_\mu \epsilon^+ &= - \left( a'/a \right) \Gamma_\mu \Gamma_5 \epsilon^-,
\partial_\mu \epsilon^- &= 0,
\partial_5 \epsilon^+ &= + \frac{1}{2} \left( a'/a \right) \epsilon^+ - 2 \left[ \delta(x^5) - \delta(x^5 - \pi \rho) \right] \text{sgn}(x^5) \epsilon^+,
\partial_5 \epsilon^- &= - \frac{1}{2} \left( a'/a \right) \epsilon^- + 2 \left[ \delta(x^5) - \delta(x^5 - \pi \rho) \right] \text{sgn}(x^5) \epsilon^-.
\end{align*}
\] (7.4)

These equations are solved by

\[
\begin{align*}
\epsilon^+ &= a^{1/2} \left[ - \left( a'/a^2 \right) x^\mu \Gamma_\mu \Gamma_5 \text{sgn}(x^5) \chi^-(0) + \chi^{+}(0) \right]
= a^{1/2} \left[ (1/\ell) x^\mu a_{\mu} \Gamma_\mu \Gamma_5 \chi^-(0) + \chi^{+}(0) \right],
\epsilon^- &= a^{-1/2} \text{sgn}(x^5) \chi^-(0),
\end{align*}
\] (7.5)

where \( \chi^{+}(0) \) and \( \chi^{-}(0) \) are constant (projected) spinors.\footnote{Thus, our solution for the Killing spinors is}

\[
\epsilon_r = a^{1/2} \chi^{+}(0) + a^{-1/2} \text{sgn}(x^5) \left( 1 - \frac{a'}{a} x^\mu \Gamma_\mu \Gamma_5 \right) \chi^-(0).
\] (7.6)

\section{Conclusion}

We have constructed a Randall-Sundrum scenario from \( D=5 \) Chern-Simons AdS supergravity. Our total action \( S = \tilde{S}_{\text{bulk}} + S_{\text{brane}} \) is \( \mathbb{Z}_2 \)-invariant. \( \tilde{S}_{\text{bulk}} \) is invariant under the fermionic transformations \( \tilde{\delta}_\epsilon \). However,

\[
\tilde{\delta}_\epsilon S_{\text{brane}} = - \int d^5 x \tilde{\delta}_\epsilon e^{(4)} \left[ T(0) \delta(x^5) + T(\pi \rho) \delta(x^5 - \pi \rho) \right] + \cdots,
\] (8.1)

where

\[
\tilde{\delta}_\epsilon e^{(4)} = e^{(4)} \left[ - \frac{1}{2} \left( \bar{\psi}_{\mu} \Gamma^\mu \epsilon_r - \bar{\epsilon}^\nu \Gamma^\nu \psi_{\mu} \right) \right].
\] (8.2)

Thus, the fermionic symmetry is broken by \( S_{\text{brane}} \). Nevertheless, the Killing spinors of Section 7 are globally defined.
Our model reduces to the original Randall-Sundrum model \cite{10} when $\tilde{S}_{\text{bulk}}$ is restricted to its gravitational sector. The original Randall-Sundrum model involves the fine-tuning relations

$$
T^{(0)} = -T^{(\pi \rho)} = 24M^3/\ell, \quad \Lambda = -24M^3/\ell^2.
$$

Randall-Sundrum scenarios constructed from standard $D = 5$ supergravity theories yield these relations (up to an overall normalization factor) as a consequence of local supersymmetry (some examples are given in \cite{12}). In our case, the relation $\Lambda = -24M^3/\ell^2$ follows from our metric choice. We do not obtain the relations $T^{(0)} = -T^{(\pi \rho)} = 24M^3/\ell$ as a consequence of local fermionic symmetry. These are fine-tuning relations in our model.

**A Appendix**

In this Appendix, we work out the fünfbein, spin connection, curvature 2-form components, Ricci tensor, and Ricci scalar for our metric (1.1). For the fünfbein, we obtain

$$
e_{\bar{\mu}}^\bar{a} = a \delta_{\bar{\mu}}^\bar{a}, \quad e_{\bar{a}} \bar{\mu} = e_\mu^b \eta_{\bar{a}b}, \quad e^{\bar{\mu} \bar{a}} = g^{\bar{\mu} \bar{\rho}} e_\rho^\bar{a},$$

$$
e_{\bar{a}}^\mu = a^{-1} \delta_{\bar{a}}^\mu, \quad e_{\mu \bar{a}} = e_\mu^b g_{\bar{a} \bar{b}}, \quad e^{\bar{a} \bar{\mu}} = \eta^{\bar{a} \bar{b}} e_\bar{b}^\mu,$$

$$e_5^\bar{5} = e_{\bar{5}\bar{5}} = e^{\bar{5}\bar{5}} = 1, \quad e_5^\bar{5} = e_{\bar{5}\bar{5}} = e^{\bar{5}\bar{5}} = 1. \quad (A.1)
$$

Our conventions for the spin connection, curvature 2-form components, Ricci tensor, and Ricci scalar are respectively

$$
\omega_{\mu \nu}^{ab} = \frac{1}{2} e^{\nu a} (\partial_\mu e_\nu^b - \partial_\nu e_\mu^b) - \frac{1}{2} e_{\nu b} (\partial_\mu e_\nu^a - \partial_\nu e_{\mu}^a) - \frac{1}{2} e_\nu^{a\bar{b}} e^{ab} (\partial_{\bar{\rho}} e_{\sigma \bar{c}} - \partial_{\sigma} e_{\rho \bar{c}}) e_{\mu \bar{c}},$$

$$R_{\mu \nu}^{ab} = \partial_\mu \omega_{\nu}^{ab} - \partial_\nu \omega_{\mu}^{ab} + \omega_{\mu \sigma}^{ac} \omega_{\nu \bar{c}}^b - \omega_{\nu \sigma}^{ac} \omega_{\mu \bar{c}}^b,$$

$$R_{\nu \sigma} = R_{\mu \nu}^{ab} e_\mu^a e_\sigma^b, \quad R = e_\mu^a e_\nu^b R_{\mu \nu}^{ab}.$$  

For the metric (1.1), the nonzero quantities here are

$$\omega_{\mu}^{\bar{a} \bar{a}} = -\omega_{\bar{a}}^\mu = a' \delta_{\mu}^\bar{a} = -e_{\mu}^{\bar{a}} / \ell, \quad (A.2)$$

$$R_{\bar{a} \bar{b}} = -a'^2 (\delta_{\bar{a}}^\mu \delta_{\bar{b}}^\nu - \delta_{\bar{b}}^\mu \delta_{\bar{a}}^\nu) = -(e_{\mu}^{\bar{a}} e_\nu^b - e_{\mu}^{\bar{b}} e_\nu^a) / \ell^2,$$

$$R_{\bar{5} \bar{5}} = a' \delta_{\mu}^\bar{a} = e_{\mu}^{\bar{a}} \left\{ 1 / \ell^2 - 2 [\delta(x^5) - \delta(x^5 - \pi \rho)] / \ell \right\}, \quad (A.3)$$

13
\[
R_{\mu\nu} = -(aa'' + 3a'^2)\eta_{\mu\nu} = -\left\{4/\ell^2 - 2[\delta(x^5) - \delta(x^5 - \pi \rho)]/\ell\right\} g_{\mu\nu},
\]
\[
R_{55} = -4a^{-1}a'' = -\left\{4/\ell^2 - 8[\delta(x^5) - \delta(x^5 - \pi \rho)]/\ell\right\}, \tag{A.4}
\]
\[
R = -8a^{-1}a'' - 12a^{-2}a'^2 = -20/\ell^2 + 16[\delta(x^5) - \delta(x^5 - \pi \rho)]/\ell, \tag{A.5}
\]
and those related to (A.3) by \( R_{\mu\nu}^{ab} = -R_{\nu\mu}^{ab} = -R_{\mu\nu}^{ba} \). The prime symbol \( \prime \) denotes partial differentiation with respect to \( x^5 \).

**Acknowledgements**

This work is supported by CNPq Edital Universal and FAPERJ.

F.T. acknowledges discussions with J. Zanelli about Reference [9].

**References**

[1] A.H. Chamseddine, ‘Topological gravity and supergravity in various dimensions’, *Nucl. Phys.* **B346** (1990) 213.

[2] R. Troncoso and J. Zanelli, ‘New gauge supergravity in seven and eleven dimensions’, *Phys. Rev.* **D58** (1998) 101703 [hep-th/9710180].

[3] R. Troncoso and J. Zanelli, ‘Gauge supergravities for all odd dimensions’, *Int. J. Theor. Phys.* **38** (1999) 1181 [hep-th/9807029].

[4] R. Troncoso and J. Zanelli, ‘Chern-Simons supergravities with off-shell local superalgebras’, in ‘Black Holes and Structure of the Universe’, C. Teitelboim and J. Zanelli, (eds.), World Scientific, Singapore, 1999 [hep-th/9902003].

[5] E. Cremmer, B. Julia and J. Scherk, ‘Supergravity theory in 11 dimensions’, *Phys. Lett.* **B76** (1978) 409.

[6] P.K. Townsend, ‘p-Brane democracy’, published in PASCOS/Hopkins 1995:0271-286 (QCD161:J55:1995) [hep-th/9507048].

[7] M. Green and P. Vanhove, ‘D instantons, strings and M-theory’ *Phys. Lett.* **B408** (1997) 122 [hep-th/9704145].
[8] P. Hořava and E. Witten, ‘Heterotic and Type I string dynamics from eleven dimensions’, *Nucl. Phys.* **B460** (1996) 506 [hep-th/9510209];

P. Hořava and E. Witten, ‘Eleven-dimensional supergravity on a manifold with boundary’, *Nucl. Phys.* **B475** (1996) 94 [hep-th/9603142].

[9] O. Mišković, R. Troncoso and J. Zanelli, ‘Dynamics and BPS states of $AdS_5$ supergravity with a Gauss-Bonnet term’ *Phys. Lett.* **B637** (2006) 317 [hep-th/0603183].

[10] L. Randall and R. Sundrum, ‘A large mass hierarchy from a small extra dimension’, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[11] R. Sundrum, ‘Effective field theory for a three-brane universe’, *Phys. Rev.* **D59** (1999) 085009 [hep-ph/9805471].

[12] R. Altendorfer, J. Bagger and D. Nemeschansky, ‘Supersymmetric Randall-Sundrum scenario’, *Phys. Rev.* **D63** (2001) 125025 [hep-th/0003117];

A. Falkowski, Z. Lalak and S. Pokorski, ‘Supersymmetrizing branes with bulk in five-dimensional supergravity’, *Phys. Lett.* **B491** (2000) 172 [hep-th/0004093];

E. Bergshoeff, R. Kallosh and A. Van Proeyen, ‘Supersymmetry in singular spaces’, *JHEP* **0010** (2000) 033 [hep-th/0007044];

R.S. Garavuso, ‘Randall-Sundrum scenario from $D=5$, $\mathcal{N}=2$ gauged Yang-Mills/Einstein/tensor supergravity’, *Nucl. Phys.* **B750** (2006) 73 [hep-th/0605002].