A reassessment of the systematic gravitational error in the LARES mission

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Abstract

In this letter we reexamine the evaluation of the error due to the even zonal harmonics of the geopotential in some proposed tests of relativistic gravitomagnetism with existing and proposed laser–ranged LAGEOS–like satellites in the gravitational field of the Earth. It is particularly important because the error due to the even zonal harmonics of the geopotential is one of the major sources of systematic errors in this kind of measurements. A conservative, although maybe pessimistic, approach is followed by using the diagonal part only of the covariance matrix of the EGM96 Earth’s gravity model up to degree $l = 20$. It turns out that, within this context and according to the present level of knowledge of the terrestrial gravitational field, the best choice would be the use of a recently proposed combination which involves the nodes $\Omega$ of LAGEOS, LAGEOS II and LARES and the perigees $\omega$ of LAGEOS II and LARES. Indeed, it turns out that the unavoidable orbital injection errors in the inclination of LARES would induce a gravitational error which turns out to be insensitive both to the even zonal harmonics of degree higher than $l = 20$ and to the correlation among them.
1 Introduction

One of the most intriguing predictions of the General Theory of Relativity, in its linearized weak-field and slow-motion approximation, is the so-called frame-dragging or Lense-Thirring effect (Lense and Thirring 1918). It can be thought of as a consequence of a gravitational coupling between the proper angular momentum $J$ of a central body of mass $M$, which acts as source of the gravitational field, and the angular momentum $s$ of a particle freely orbiting it. It turns out that the spin $s$ undergoes a tiny precessional motion (Schiff 1960). The most famous experiment devoted to the measurement of such a gravitomagnetic effect is the Stanford University GP–B mission (Everitt et al. 2001) which is scheduled to fly in April 2003.

If we consider the whole orbit of a test particle in its geodesic motion around $M$ as a sort of giant gyroscope, its orbital angular momentum $l$ undergoes the Lense-Thirring precession, so that the longitude of the ascending node $\Omega$ and the argument of pericenter $\omega$ of the orbit of the test particle (Sterne 1960) are affected by small secular precessions\(^1\) $\dot{\Omega}_{LT}$, $\dot{\omega}_{LT}$ (Lense and Thirring 1918; Ciufolini and Wheeler 1995; Iorio 2001).

1.1 The LAGEOS-LAGEOS II Lense-Thirring experiment

Up to now, the only attempts to detect them in the gravitational field of the Earth are due to Ciufolini and coworkers (Ciufolini et al. 1998; Ciufolini 2000; 2002) who analysed the laser data of the existing geodetic SLR (Satellite Laser Ranging) satellites LAGEOS and LAGEOS II over time spans of some years. The observable is a suitable combination of the orbital residuals of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II according to an idea exposed in (Ciufolini 1996). The relativistic signal is a linear trend with a slope of almost 60.2 milliarcseconds per year (mas yr\(^{-1}\) in the following). The claimed total accuracy is of the order of 20% – 30%. The main sources of systematical errors in such kind of measurements are the unavoidable aliasing effect due to the mismodelling in the classical secular precessions induced by the even zonal coefficients of the multipolar expansion of the Earth’s gravitational field (Kaula 1966) and the non-gravitational perturbations affecting especially the perigee of LAGEOS II. It turns out that the mismodelled classical precessions due to the first two even

\(^1\)In the original paper by Lense and Thirring the longitude of the pericenter $\varpi = \Omega + \omega$ is used instead of $\omega$. 
zonal harmonics of the geopotential $J_2$ and $J_4$ are the most insidious source of error for the \textit{Lense–Thirring} measurement with LAGEOS and LAGEOS II. The combination of (Ciufolini 1996) is insensitive just to $J_2$ and $J_4$. According to the full covariance matrix of the EGM96 gravity model (Lemoine \textit{et al} 1998), the error due to the remaining uncancelled even zonal harmonics amounts to almost 13%. A reliable evaluation of the impact of the geopotential is a particularly subtle and important topic. Indeed, it is based on the use of the covariance matrix of the even zonal harmonics of the geopotential of some terrestrial gravity models like EGM96. As pointed out in (Ries \textit{et al} 1998), in obtaining the solution of EGM96 and of other previous gravity models a multidecadal observational time span has been used and many seasonal, stochastic and secular variations, which is known that they affect the geopotential, have not been accounted for. Then, according to the remarks of (Ries \textit{et al} 1998), nothing would assure that during any particular relatively short time span as that used in the LAGEOS–LAGEOS II \textit{Lense–Thirring} experiment the correlation among the geopotential coefficients would be just that of the EGM96 covariance matrix. For example, it seems that the 13\% favorable estimate is based on a particular correlation between $J_6$ and $J_8$ (Ries \textit{et al} 1998) which do affect the observable by Ciufolini. If, with a more conservative, although maybe pessimistic, approach, we use the diagonal part only of the covariance matrix of EGM96 up to degree $l = 20$ the error due to the even zonal harmonics of the geopotential for the LAGEOS–LAGEOS II \textit{Lense–Thirring} experiment amounts to 46.6\% (Iorio 2002a). Moreover, it turns out to be insensitive to the even zonal harmonics of degree higher than $l = 20$ due to the high altitude of the LAGEOS satellites. Indeed, the classical precessions of the node and the perigee depend on $(\frac{R}{a})^l a^{-\frac{3}{2}}$, where $R$ is the Earth’s radius and $a$ is the satellite’s semimajor axis.

\textbf{1.2 The LARES project}

The originally proposed LARES mission (Ciufolini 1986; 1998) consists of the launch of a LAGEOS–type satellite–the LARES–with the same orbit of LAGEOS except for the inclination $i$ of its orbit, which should be supplementary to that of LAGEOS, and the eccentricity $e$, which should be one order of magnitude larger. In Table 1 the orbital parameters of the existing and proposed LAGEOS–type satellites are quoted. The choice of the particular value of the inclination for the LARES is motivated by the fact that in this way, by using as observable the
Table 1: Orbital parameters of LAGEOS, LAGEOS II, LARES and POLARES and their Lense–Thirring precessions.

| Orbital parameter | LAGEOS | LAGEOS II | LARES | POLARES |
|-------------------|--------|-----------|-------|---------|
| a (semi major axis (km)) | 12,270 | 12,163 | 12,270 | 8,378 |
| e (eccentricity) | 0.0045 | 0.014 | 0.04 | 0.04 |
| i (inclination (deg)) | 110 | 52.65 | 70 | 90 |
| \(\dot{\Omega}_{LT}\) (mas yr\(^{-1}\)) | 31 | 31.5 | 31 | 96.9 |
| \(\dot{\omega}_{LT}\) (mas yr\(^{-1}\)) | 31.6 | -57 | -31.6 | 0 |

sum of the nodes of the LAGEOS and the LARES, it should be possible to cancel out exactly all the contributions of the even zonal harmonics of the geopotential, which depends on \(\cos i\), and add up the Lense–Thirring precessions which, instead, are independent of \(i\).

Of course, it would not be possible to obtain practically two orbital planes exactly 180 deg apart due to the unavoidable orbital injection errors. It turns out that all depends on the last stadium of the rocket used. According to conservative estimates, if a solid propellant is used for it an error in inclination of the order of 1 deg is to be expected, while if a liquid propellant, which is more expensive, is used the error should amount to 0.5-0.6 deg (Anselmo, private communication 2002). In Figure 1, page 4314 of (Iorio et al 2002) the impact of such source of error on the originally proposed LAGEOS–LARES mission has been shown. It should be noted that the simple sum of the nodes of the LAGEOS and the LARES is affected by all the even zonal harmonics of the geopotential. Then, when the impact of the departures of the inclination of LARES from its nominal values on the error due to the even zonal harmonics of the geopotential has to be calculated, the role of all the correlations among the even zonal harmonics should be important. If we decide to take into account the remarks of (Ries et al 1998), the results obtained in (Iorio et al 2002) might be considered optimistic in the sense that they are based on an extrapolation of the validity of the full covariance matrix of EGM96 up to degree \(l = 20\) to arbitrary future time spans.

In (Iorio et al 2002) an alternative observable based on the combination of the residuals of the nodes of LAGEOS, LAGEOS II and LARES and the perigee of LAGEOS II and LARES has been proposed. It would allow to cancel out the first four even zonal harmonics \(J_2, J_4, J_6, J_8\).
so that the error due to the even zonal harmonics of the geopotential would be rather insensitive to the orbital injection errors in the LARES inclination and would amount to 0.02% only.

In (Iorio 2002b) the recent proposal of inserting the LARES in a low-altitude polar orbit (Lucchesi and Paolozzi 2001), so to obtain the so called POLARES, and to analyze only its node has been critically analyzed from the point of view of the impact of the orbital injection errors in the POLARES inclination.

1.3 Motivation of the present work

The conclusions obtained in (Iorio et al 2002; Iorio 2002b) are based on the assumption of the validity of the EGM96 full covariance matrix in arbitrary future time spans during which, instead, it might happen that the correlations between the even zonal harmonics will be different. This problem is particularly relevant for those observables which are sensitive to the full range of the even zonal harmonics of the geopotential like the originally proposed node-only LAGEOS–LARES combination and the node-only POLARES observable.

Consequently, we wish to reanalyze such issues in a more conservative, although pessimistic, approach by using the diagonal part only of the covariance matrix of EGM96. We expect that the error due to the even zonal harmonics of the geopotential of the new proposed observable based on the use of the orbital elements of LAGEOS, LAGEOS II and LARES, which is $J_2 - J_8$-free, should be relatively insensitive to the correlation among the remaining even zonal harmonics, contrary to the sum of the nodes of the LAGEOS and the LARES and the node of the POLARES. If it will be so, the reliability of the modified version of the LARES project will be enforced and posed on a more firm basis.

2 The LARES mission

2.1 The originally proposed LARES scenario

As pointed out in (Iorio et al 2002), the impact of the unavoidable orbital injection errors in the LARES inclination on the error due to the even zonal harmonics of the geopotential is of crucial importance for the originally proposed LAGEOS–LARES observable, especially if the LARES satellite will be finally launched with a relatively cheap rocket of not too high quality due to
budget restrictions. Figure 1 of page 4314 in (Iorio et al. 2002) has been obtained by considering the root–sum–square error due to the full covariance matrix of EGM96, up to degree \( l = 20 \), as a function of the inclination of LARES. Since there are no even zonal harmonics cancelled out by the sum of the nodes of LAGEOS and LARES, the role of the correlation among all the various geopotential's harmonics in the assessment of the error due to the even zonal harmonics of the geopotential should not be neglected. Then, the estimates of (Iorio et al. 2002) might reveal to be rather optimistic. A more conservative approach consists of repeating the analysis by using the diagonal part only of the covariance matrix of EGM96. The results are summarized in Figure 1. It can be noticed that, in this pessimistic but perhaps more realistic approach, the

![Figure 1: Influence of the injection errors in the LARES inclination on the error of the originally proposed LAGEOS–LARES nodal observable due to the even zonal harmonics of the geopotential according to the diagonal part only of the covariance matrix of EGM96 up to degree \( l = 20 \).](image)

error due to the even zonal harmonics of the geopotential is of the order of 4%–4.5%, contrary to the 1%–1.4% of (Iorio et al. 2002). Note also that, even for the nominal values of Table 1 for the LARES orbit, the error due to the even zonal harmonics of the geopotential would amount
to almost 1%, contrary to 0.3% of (Iorio et al 2002). This further confirms that, according to the present knowledge of the terrestrial gravitational field, the implementation of the originally proposed LAGEOS–LARES observable would pose some problems in term of accuracy. Of course, the situation should greatly improve when the new data for the geopotential from the CHAMP and GRACE missions will be available. It should also be considered that, in the case of the node–only LAGEOS–LARES configuration, the error due to the even zonal harmonics of the geopotential would represent the most relevant part of the systematic error because the non–gravitational perturbations acting on the nodes of the LAGEOS–like satellites are far less relevant.

2.2 The modified LARES scenario

The combination of orbital residuals including the nodes of LAGEOS, LAGEOS II and LARES and the perigees of LAGEOS II and LARES of eq. (9) in (Iorio et al 2002) seems to be a better choice. Indeed, also in this pessimistic approach the error due to the even zonal harmonics of the geopotential turns out to be very small and insensitive to the orbital injection errors in the inclination of the LARES satellite, as shown in Figure 2. From it the error due to the even zonal harmonics of the geopotential remains almost fixed at the level of 0.1%, while in Figure 3 of page 4317 in (Iorio et al 2002) it is of the order of 0.02%–0.04%. As it could be expected, since the first four even zonal harmonics are cancelled out by the combined residuals, the impact of the correlation among the remaining ones is relatively not relevant.

Notice also that the part of the systematic error due to the non–gravitational perturbations, as evaluated in (Iorio et al 2002) over a time span of 7 years, amounts just to 0.3%. This is very important because this means that the combined residuals approach would yield a more accurate measurement of the Lense–Thirring effect than the simple sum of the nodes of the LAGEOS and LARES satellites, even with the conservative evaluations of the error due to the even zonal harmonics of the geopotential presented here. Indeed, for, say, $\delta i_{\text{LARES}}^{\text{inj}} \sim 1$ deg, the total systematic error of the combination of residuals, according to Figure 2, would amount to 0.33%, while for the sum of the nodes it would be of the order of 4.5%, even if the impact of the non–gravitational perturbations on the nodes is very small and is considered negligible.

Also in this case the role which will be played by the results of the CHAMP and GRACE
missions will be of decisive importance.

3 The POLARES scenario

The approach followed in this letter for the error due to the even zonal harmonics of the geopotential clearly shows in Figure 3 that the option of inserting the LARES satellite in a polar, low-altitude orbit should be considered rather impracticable. Indeed, the effects of the orbital injection errors in the inclination of POLARES would be greatly enhanced by its low altitude of 2000 km only at a level of 100%–150%. It should also be considered that the results of Figure 3, as those of Figure 1 of page L178 in (Iorio 2002b), are optimistic because they have been obtained by neglecting the contributions of the classical nodal precessions of the even zonal harmonics of degree higher than \( l = 20 \) which, contrary to the LAGEOS satellites, would not be negligible in this case due to the low altitude of the POLARES. Moreover, it is
not probable that the improvements due to the CHAMP and GRACE missions will reduce the error due to the even zonal harmonics of the geopotential to a level comparable to the other proposed configurations, which, in turn, will benefit from the new Earth gravity models.

4 Conclusions

In this letter we have followed a more conservative and realistic approach in evaluating the impact of the orbital injection errors of LARES and POLARES on the error due to the even zonal harmonics of the geopotential of some proposed observables aimed to the measurement of the Lense–Thirring effect with LAGEOS–like SLR satellites. In particular, we have used, in a root–sum–square fashion, the diagonal part only of the covariance matrix of the even zonal harmonic coefficients of the EGM96 Earth’s gravity model up to degree $l = 20$.

With regard to the LARES project, it turns out that the recently proposed residuals com-
bination involving the LAGEOS, LAGEOS II and LARES satellites, which cancels out the first four even zonal harmonics, should yield not only a more accurate measurement than the simple sum of the nodes of LAGEOS and LARES, but also more reliable because it would be less dependent on the correlation between the remaining even zonal harmonics of higher degree.

The use of POLARES, according to the present level of knowledge of the terrestrial gravitational field and to the results presented here, should be considered unpracticable. It is probable that, even with the new gravity models from CHAMP and GRACE, such a proposed configuration would not be competitive with the other multisatellite observables which, in turn, would benefit of the new data of the gravitational field.

The new gravity models from the CHAMP and GRACE missions should yield great benefits for a more confident and reliable assessment of the error due to the even zonal harmonics of the geopotential in all the examined missions. However, also in this case the originally proposed LARES orbital configuration, in conjunction with the data from LAGEOS and LAGEOS II, should be the best choice.

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