Production Bottleneck Prediction of Multi-Variety and Small Batch Production Workshop

Shuo Lin^{1a*}, Junhui Liu^{1b*}, Shishi Li^{1c}

1 Faculty of Information and Control Engineering, Shenyang Jianzhu University, Shenyang, Liaoning, 110168, China

^{a*} farewell_lin@163.com, ^{b*} fearless_liu2019@163.com, ^{c} 1512985353@qq.com

Abstract—Aiming at the bottleneck dynamic drift problem of production line, a dynamic bottleneck prediction method based on Markov chain was proposed to predict the bottleneck before the bottleneck appeared and predict the bottleneck change state. First, Markov state probability transition matrix is determined by analyzing the past data. Second, the current operation state of the process is brought into the Markov chain to calculate the probability of each process bottleneck at the next moment. Finally, the bottleneck index model is used to accurately locate the bottleneck location, and then get the exact bottleneck process.

1. The introduction

In recent years, people have more and more personalized requirements for products, and the previous production mode of mass assembly line workshop has been quietly changed to multi-varieties and small batch production mode. The transformation of this model makes the mode of production also changed accordingly. Due to the different production processes of different products, the production processes required and the production time of each link are also different. These problems lead to the uncertainty of the production process, which leads to production bottlenecks in the production process, and makes the improvement of production efficiency face huge challenges.

Moreover, due to product customization and personalized production demand, the production mode of multi-varieties and small-batch production are promoted. As a result, factories can not produce as many goods as they used to. In this production mode, the production can only be carried out according to the quantity of orders to avoid losses. Therefore, this leads to unpredictable changes in demand and production capacity, which greatly increases the probability of bottleneck generation and drift, posing a great test to production efficiency and production speed [1]. Therefore, in order to improve production efficiency and ensure the balance between supply capacity and demand capacity, we can start from reducing the emergence of bottlenecks. At present, the main bottleneck identification methods can be divided into three categories: identification method based on external performance characteristics of bottleneck (such as equipment blocking and starvation time, equipment load, equipment utilization rate.), and bottleneck identification method based on scheduling vision (multiple optimization of production scheduling, the equipment with the longest average active time after each optimization is the potential bottleneck equipment. The bottleneck process that becomes the most possible bottleneck device after multiple optimization) and the bottleneck identification method based on capacity load ratio [2].

Among the above three methods of bottleneck identification, the method to determine the bottleneck based on the external performance characteristics of the bottleneck looks more intuitive and clear, but in
the actual production, there will be several different index data close to each other. In this case, it is difficult to distinguish where the bottleneck is generated by external characteristics. The bottleneck identification method based on scheduling vision is only based on the impact of production scheduling on production bottlenecks, and it needs a lot of simulation and experiments, so it has limitations. The bottleneck identification method based on capacity load ratio solves the shortcomings of the previous two methods, but it cannot accurately predict the location of the main bottleneck when there are multiple bottleneck processes in the system.

At the same time, the above three methods mainly calculate the average bottleneck of the system, and lack the real-time analysis of the dynamic drift process of the system bottleneck, which has various limitations in the small batch production of multiple varieties with high real-time requirements, so the Markov chain is introduced here to make real-time prediction\cite{3}.

2. Bottleneck prediction method

2.1 Introduction and application of Markov chain

Markov chains are random processes with Markov properties in discrete exponential sets and state spaces in probability theory and mathematical statistics. A Markov chain is a set of discrete random variables with Markov properties. That is, for the set of random variables $X=\{X_n: n > 0\}$ in probability space $(\omega, F, P)$ with one-dimensional countable set as exponential set, if the values of random variables are in the countable set $X=s_i, s_i \in S$, and the conditional probability of random variable satisfies the following relation:

$$P(X_{t+1}|X_t, \ldots, X_1) = P(X_{t+1}|X_t)$$

Then X is called Markov chain, countable set $S \subset \mathbb{Z}$ is called state space, and the value of Markov chain in state space is called state. According to the definition of Markov chain, the state of a process conforming to Markov property at time $t+1$ is only related to its state at time $T$, and has nothing to do with its state at other time\cite{4}.

Markov chain has four properties, namely irreducibility, recurrence, periodicity and ergodicity. Irreducibility means that random variables can be transformed in any state during the evolution of Markov chain, and conversely, it is called reducibility. If the Markov chain reaches a state and can return to the state repeatedly in the process of continuous transfer, then the Markov chain is recurrent, and on the contrary, it is called transient. The state of a Markov chain is periodic if it returns a state only if it is a multiple of an integer greater than 1. Therefore, starting from state $I$, you can only return to state $I$ after an integer multiple of $k$, which is the maximum of all integers that satisfy this condition. If $k = 1$, state $I$ is not periodic, if $k > 1$, state $I$ is periodic. If a state of a Markov chain is normally recurrent and aperiodic, the state is ergodic. If a Markov chain is irreducible and a certain state is ergodic, then all states of the Markov chain are ergodic and are called ergodic.

According to the definition of Markov chain, we can know that Markov chain forms a probability matrix according to the past data, and then puts the current state into the probability matrix for calculation, so as to obtain the state probability of the next moment, and then make probability prediction. The advantage of Markov chain prediction is that the state at the next moment is only related to the current state, so it can be judged as accurately as possible\cite{5}.

In the production line with many varieties and small quantities, it is impossible to store a certain product in large quantities because of the variety and small demand. Therefore, in the case of some sudden orders, it is easy to produce production bottlenecks, which will lead to failure to complete the project on time. Therefore, we can introduce the concept of Markov chain to obtain the probability matrix of state transition of the bottleneck, and then predict the possible production links of the bottleneck at the next moment. Thus, the bottleneck process can be predicted and the work efficiency can be improved. However, since the prediction of Markov chain is only a probability prediction, the result can only be said that there is a large probability of bottleneck problems in the next process, so the concept of bottleneck index can be introduced to calculate the bottleneck degree of each process to get more accurate location of the bottleneck\cite{6}.
2.2. Bottleneck index

Every process has the possibility of being called a bottleneck, so every process has a bottleneck degree. The concept of bottleneck index can be introduced to measure the size of the bottleneck degree and then determine the exact location of the bottleneck. Bottleneck index is an index to measure the degree of process bottleneck. The larger the bottleneck index is, the more likely the process will become a bottleneck.

First of all, we need to know the concept of comprehensive bottleneck degree. Comprehensive bottleneck degree refers to the ability of manufacturing unit to become a bottleneck under the joint influence of external reasons (such as order change, raw material supply, etc.) and inherent factors (such as processing equipment problems, operators, etc.). There are great limitations in describing the comprehensive bottleneck degree qualitatively. In many cases, this method can not provide support for the effective prediction and analysis of the bottleneck drift, and provide a basis for the effective control of the production process. Therefore, we need to start from the most fundamental cause of bottleneck shift, that is, the matching degree of demand capacity and supply capacity, establish a measurement index system and specific measurement method, in order to achieve quantitative description of comprehensive bottleneck degree. Because the supply capacity of the production line cannot meet the demand, the production capacity leads to bottlenecks. Therefore, it is the most accurate to find the bottleneck from the perspective of supply and demand balance [7].

There are three aspects to consider when establishing metrics. First of all, we need to consider the comparison of the combined bottlenecks of the same manufacturing unit. In this way, it is convenient for us to analyze the significance of the change, and then analyze the change speed and change direction of the bottleneck dynamic attribute of the manufacturing unit. Secondly, we need to consider the comparison of integrated bottleneck degree of different manufacturing units to ensure simple and easy analysis. Finally, we need to consider the comprehensiveness and precision of the indicators, and take into account as many factors as possible that can affect the bottleneck. Therefore, based on the difference of time and quality measurement unit, the bottleneck index $I_{BN}$ is constructed by taking time, quality and cost as parameters from two aspects of processing capacity and processing demand. Among them, the bottleneck index mathematical model of the processing unit is shown below:

$$I_{BN,i} = \omega_t f_q (\sigma, \mu) + \omega_c C_a / C_0 + \omega_l \left( \frac{l_i}{T_i - T_i(t)} + \frac{1}{T_i - F_i(t) - \Delta p_i(t)} \sum_{j=1}^{N_i} q_{ij} p_{ij} E_{ij} \right)$$  （2）

In the above equation, $\omega_t$, $\omega_q$, $\omega_c$ are respectively the weights of production time, product quality and production cost in the bottleneck degree, $\omega_t + \omega_q + \omega_c = 1$; $l_i$, $T_i$ are respectively the production load and planned available production time of manufacturing unit $i$; $F_i(t)$ is the variation of production capacity of manufacturing unit $i$ caused by changes in actual production conditions; $p_{ij}$, $E_{ij}$, $q_{ij}$ respectively, rejection rate, unit processing time and processing quantity of product $J$ in manufacturing unit $i$; $\Delta p_i(t)$ is the repair time of unqualified product $J$ in manufacturing unit $i$; $N_i$, $f_q (\sigma, \mu)$ is the influence function of the number of products processed in manufacturing cell $I$ and the quality assurance capability of manufacturing cell $I$ on the comprehensive bottleneck degree. $\sigma$ and $\mu$ are the standard deviation and mean of quality characteristics, respectively. $C_0$, $C_a$ respectively are the target cost and actual cost of the manufacturing unit.

We can calculate the size of the process bottleneck index through the above formula, and then compare the size of the bottleneck index to determine which process the bottleneck is generated. Thus, the bottleneck process can be mobilized to reduce the probability of bottleneck and improve the efficiency of work[9].

3. Case analysis

There are five processes in a multi variety and small batch production line, they are process 1, process 2, process 3, process 4 and process 5 respectively, five process could become a bottleneck in the process of production process, through to the five processes the number of bottlenecks can get after finishing the five process bottleneck transition probability between each other. The bottleneck transfer probability of these five processes is shown in the following table:
Table 1 Bottleneck transfer probability table

| Bottleneck 2 | Process 1 | Process 2 | Process 3 | Process 4 | Process 5 |
|--------------|-----------|-----------|-----------|-----------|-----------|
| Process 1   | 0.05      | 0.5       | 0.2       | 0.15      | 0.1       |
| Process 2   | 0.2       | 0.1       | 0.15      | 0.2       | 0.35      |
| Process 3   | 0.15      | 0.2       | 0.05      | 0.2       | 0.4       |
| Process 4r  | 0.4       | 0.1       | 0.2       | 0.15      | 0.15      |
| Process 5   | 0.5       | 0.05      | 0.15      | 0.1       | 0.2       |

In the table, the vertical column is the procedure number of the bottleneck occurring for the first time, and the horizontal column represents the bottleneck serial number after the bottleneck is transferred. The data in the table are the probability of bottleneck transition. From the data in the table, we can get the probability of the bottleneck shifting from one process to another. With these available data, we can predict the shift of bottlenecks.

The state transition probability among the five processes of this production line is shown in the table above, from the table we can get the bottleneck transition probability matrix, according to the principle of Markov chain prediction, we construct a state matrix with the current state of each process, and then multiply the state matrix with the state transition probability matrix to get the probability of bottleneck occurrence after the transition at the next moment. After obtaining the probability of bottleneck in each process, the process with high probability of bottleneck can be monitored, and the possibility of bottleneck can be reduced by adjusting production tasks.

We can assume that the current running state of the production line is (1,0,0,0,0), that is, process 1 is the bottleneck process under the current condition, and the prediction result of the probability of bottleneck transfer by Markov chain is (0.05, 0.5, 0.2, 0.15, 0.1), that is, the next step will become a new bottleneck process with the greatest probability. At this point we need to redeploy process 2, this can reduce the probability that it will become a bottleneck.

After several iterations of the prediction of bottleneck probability, we can find that the final probability tends to be stable, that is, with the continuous transfer of the state, the bottleneck state constantly changes and finally stabilizes near a state probability. It can be found in this production line that the bottleneck probability of each process after 30 iterations is [0.25, 0.21, 0.15, 0.16, 0.23]. The specific state transfer process is shown in Figure 1. We can see it clearly in the picture, after several iterations, the state transition probability approaches steady state.

We assume that when the current running state of the production line is (0,1,0,0,0), that is, step 2 in the current state is the bottleneck process, and the bottleneck probability multiplied by state matrix and probability transition matrix is [0.2, 0.1, 0.15, 0.2, 0.35], and the bottleneck probability at the next moment will be step 5. After thirty iterations of this state, we can find that when the initial bottleneck state is step 2, the final steady-state probability is [0.25, 0.21, 0.15, 0.16, 0.23]. The specific state transfer process is shown in Figure 2. It is also clear from this figure that the state transition probability come to a steady state after several iterations. We can see that although the initial states are different, both initial states end up with the same steady-state probability.
Figure. 1 Bottleneck state transition probability diagram from the beginning of the process

It can be found from these two changes of the bottleneck transition probability that, when the state transition probability matrix is determined, no matter what the initial bottleneck state of each process is, the bottleneck probability of the steady state will be the same. Therefore, the running state of the process will only have a certain influence on the probability of the next bottleneck process. However, the average probability of the bottleneck in each process has nothing to do with the current state, so in the absence of the current bottleneck state, we can still rely on the balance state bottleneck probability of the production line to judge the probability of the bottleneck in each process.

Figure. 2 Bottleneck state transition probability diagram from step 2

That is, after a certain number of iterations from an initial state, the probability of each process becoming a bottleneck tends to be stable. At this point, the probability of each process appearing a bottleneck can be regarded as the probability of each process appearing a bottleneck without considering the initial state. According to the final steady-state probability, it can be seen that in this paper, process one has the greatest probability of becoming the bottleneck of the production line. Therefore, the first process needs to be adjusted to a certain extent. In the production operation stage, it is necessary to focus on process one control, reduce the influence of bottleneck process of normal production. And then improve work efficiency, more quickly to complete the production of products [9].
4. Conclusion
This paper analyzes the bottleneck probability of more and more multi-variety and small-batch production lines recently, because the demand for single type of products produced by this production line is small, so it is impossible to overstock a large number of products, and the production demand is prone to big changes, and temporary changes or orders are easy to be added. Therefore, compared with the previous large-scale assembly line production, production bottlenecks are more likely to occur. This paper introduces the concept of Markov chain and combines it with the prediction of production bottleneck. With the feature that the state transfer in Markov chain is only related to the state at the previous moment, we can get the probability of bottleneck transfer more simply. Then accurately and quickly find the location of the bottleneck and processing. The bottleneck and its drift can be dealt with more properly. Then we can ensure the balance between supply capacity and demand capacity, and ensure the timely completion of orders.

Acknowledgments
This work was supported by the Liaoning provincial key R & D plan project (No.2020jh2/10100039), Project of Liaoning Province Education Department of China (No. LJKZ0583) and Shenyang Municipal Science and Technology Project of China (No. Z18-5-015).

References
[1] Sun J.Y, Wang Y.H.( 2017) Research on the Bottleneck Resolution of multi-product job shop [J].Science and Technology Innovation and Application, (19):93+95.
[2] Liu Z, Jiang Z.Q, Gong BG. (2014) Dynamic prediction method of multi-bottleneck in manufacturing shop based on two bottleneck degrees[J].China mechanical engineering, 25(14):1910-1915+1921.
[3] Li L, Xia J, Li L, Gui L.H. (2020) Study on scheduling of Automotive electronics Production shop under the influence of bottleneck Process [J].Modern Manufacturing Engineering, (03):26-32.
[4] Chen Y.S, Zhou L.F. (2020) Forecasting model of vehicle ownership based on exponential smoothing method and markov chain [J]. Journal of changchun institute of technology (natural science),21(01):124-128.
[5] Liu Y, Wang Y.H, Qian W.Y. (2015)Dynamic conflict analysis model based on markov chain [J]. Chinese journal of management science, 23(S1):325-332.
[6] Liu Z, Tang J, Fei Z.M . (2012)Close-loop prediction method of logistics bottleneck based on bottleneck polymorphism [J]. Computer Integrated Manufacturing Systems, 18(11):2554-2561.
[7] Jayakumar K P A T V, Saravanan R . (2014)Genetic architecture and bottleneck analyses of Salem Black goat breed based on microsatellite markers [J]. Veterinary World, 7(9):733-737.
[8] Li X.J, Sun W.L, Yuan Y.P, Li H.H.(2016) Research on multi-bottleneck identification method of Job-shop network in disturbed environment [J]. Journal of xi 'an jiaotong university ,50(12):64-72+78.
[9] LEE HOCHANG, SEO Dong-won. (2016)Performance Evaluation of WIP-controlledline Production Systems with Constant Processing Times [J]. Computers & Industrial Engineering, 94 (5) : 138-146.