Observations indicate that our universe is in a phase of accelerated expansion. Some mysterious dark energy seems to drive this acceleration. Revealing its true nature will likely entail a breakthrough in fundamental physics. One explanation is Einstein’s cosmological constant. It describes observations well, but is plagued by an enormous fine tuning problem: Quantum Field Theory generically yields a $10^{120}$ times larger value. Scalar field dark energy cosmologies addressing this issue have been under investigation for more than a decade. Currently, observations only provide bounds on the evolution of such an (effective) scalar field. In addition, it may well be that the field evolution closely mimics that of a cosmological constant in the late universe. For years to come, astrophysical and cosmological tests may not be able to settle the issue and perhaps may never be. Tabletop experiments cannot be used to measure the vacuum energy and hence provide no clue to the true nature of dark energy. Likewise, direct detection of a scalar dark energy field is next to impossible if the interaction strength of the field is at the gravitational level and no detectable violation of the equivalence principle is induced. Using the cooling of an ordinary black body provides no solution either: even though the cooling rate is proportional to the number of degrees of freedom, scalar dark energy couples too weakly to reach thermal equilibrium and radiate.

Here, we propose a measurement with the potential to exclude a light scalar field as well as generic models of modified gravity as dark energy candidates. Our mechanism uses the spectrum radiated by black holes and can equally well be applied to determine the number of light degrees of freedom. We obtain the grey body factors for massive scalar particles and calculate the total emissivity. While the Large Hadron Collider (LHC) may not get to the desired accuracy, the measurement is within reach of next generation colliders.

This spectrum is given by

$$T_H = \frac{n + 1}{4 \pi T_H} = \frac{n + 1}{4 \sqrt{\pi}} M_*^{n+1} \sqrt{\frac{M_*}{M_H}} \left( \frac{n + 2}{8 \Gamma \left( \frac{n + 2}{2} \right)} \right).$$

FIG. 1: Total emissivity of a scalar particle as a function of particle mass $m$ over temperature $T_H$ of a black hole with mass $M_H \gg m$. The total emissivity is obtained by integrating Equation over energy $\omega$ and using grey body factors for massive particles (see Equation). We have normalized such that a massless scalar corresponds to unity both for $n = 1$ dashed (blue) line and $n = 7$ dashed-dotted (red) line. The normalization for $n = 7$ (and hence the un-scaled emissivity) is $\sim 136$ times larger than that for $n = 1$. For comparison we show the result for a perfect black body (solid black line).
that leave undetected (such as neutrinos), we can\(^1\) sum up and compare to the known mass of the black hole. A scalar dark energy field will be excluded if there is no energy missing.

Unfortunately, astrophysical black holes in four dimensional space-time have temperatures \(T_H \sim 62M_\odot/M_B nK\), that are too low to emit detectable radiation. This and the (luckily) inconveniently large distance to the next black hole make a measurement prohibitively difficult.

The way out are black holes with temperatures \(0.1\text{eV} \lesssim T_H \lesssim 100\text{GeV}\). Here, the lower limit ensures sufficient emission, while the upper bound keeps the effect in an energy range where our understanding of existing particle species is good. If the accelerator energy is comparable to the fundamental Planck mass \(M_\star\) in higher dimensional theories \([22]\), such black holes will be produced \([23,24,25]\). In theories with extra dimensions, all standard model particles except for gravitons are confined to a four dimensional membrane (simply called ‘brane’) embedded in a higher dimensional ‘bulk’ space-time. As the size \(l \lesssim 1\text{mm}\) of these extra dimensions may be much larger than the typical scale \(l_P \sim 1/M_P \sim 10^{-32}\text{mm}\) of our four dimensional theory, \(M_P \sim M_\star^{2+n/m}\) can substantially exceed \(M_\star\). For \(n \geq 5\) higher dimensional Planck masses as low as \(M_\star \sim \text{TeV}\) are allowed by current constraints \([21]\). For lower dimensions the constraints are stronger. Roughly speaking, a black hole forms when some energy \(E \sim M_\star\) is concentrated within a radius \(r_H\). Hence the production cross section is \(\sigma \sim \pi r_H^2\), where \(r_H\) is inferred from the center of mass energy plugged into Equation (1). Due to the smaller fundamental Planck scale \(M_\star\), higher dimensional black holes are larger (and cooler) than their four dimensional counterparts. This substantially increases the cross section and black holes may be formed at reasonable rate at LHC \([26,27,28]\) or by interactions of cosmic rays with our atmosphere \([24,30,31,32]\).

Inserting \(M_\star = 1\text{TeV}\) and \(M_H = 10\text{TeV}\) into Equation (1) we find temperature in the range \(55\text{GeV} \lesssim T_H \lesssim 580\text{GeV}\) for \(n = 1 - 7\). As seen in Figure (1) a particle can only contribute efficiently to the evaporation for as long as its mass \(m \lesssim T_H\). So for our purpose, temperatures \(T_H \lesssim 580\text{GeV}\) are quite agréable, in particular since future colliders may improve our knowledge of particles up to \(\sim \text{TeV}\). More massive black holes are still preferable since they are cooler, emit more particles and are less subject to quantum gravity effects.

The emission rate for one species is described by \([32]\)

\[
\dot{E}^{(s)}(\omega) = \sum_j \sigma^{(s)}_{j,\nu}(\omega, r_H) \frac{\omega}{\exp(\frac{\omega}{T_H})} \pm 1 \ (2\pi)^{n+3}, \quad (2)
\]

Here \(s\) and \(j\) are spin and angular momentum of the emitted particle, \(\omega = \sqrt{m^2 + k^2}\) is the energy and \(\sigma\) is the grey body factor \([33,34]\). In the case of a black body, \(\sigma\) is the area of the emitter. For black holes, it is a function of the frequency of the emitted particle which depends on the state of the black hole and in particular on the particles mass and angular momentum. Essentially, \(\sigma\) incorporates that a particle emitted at the horizon may be reflected back into the black hole due to the non-trivial interaction with the black hole. We have extended the calculation of \([33]\) to incorporate scalar particles of mass \(m\). For these, the Klein-Gordon equation in the induced black hole metric becomes

\[
\frac{d^2 R(r)}{dr^2} = -\left(\frac{2}{r} + \frac{d\ln[h(r)]}{dr}\right) \frac{dR(r)}{dr} + R(r) \left(\frac{m^2}{h} - \frac{\omega^2}{h^2} + \frac{\lambda}{hr^2}\right), \quad (3)
\]

which is to be compared to Equation (3.3) in \([33]\). We have numerically integrated \([3\) to obtain the transmission coefficients and grey body factors along the lines of \([32]\). For the massless case presented in \([32]\), our results are in perfect agreement (i.e. we reproduced Figure 1 of \([33]\)).

Although it seems like a complication the dependence of the grey body factors on the properties of the black hole is quite useful. In particular, it can be used to determine the number of extra dimensions \([33]\) via the ratio of the energies emitted into particles with different spin, i.e. scalars, gauge bosons and fermions.

Standard model particles and the dark energy scalar live on the brane (of course, the dark energy scalar might also live in the bulk). For these, one sets \(n = 0\) in the integration measure of Equation (2), whereas bulk scalars and gravitons command the full \(4+n\) dimensional phase space. This does not lead to a drastic enhancement of radiation into the bulk \([23,33]\). Indeed, the emitted energy per degree of freedom for bulk fields is comparable to those on the brane. There are, however \((n+3)(n+2)/2-1\) graviton polarization states which for \(n = 7\) yields a substantial number of 44 states (see also Figure 2).

The higher dimensional Planck mass can be determined from the production cross section of gravitons in collisions where no black hole is formed \([36]\). As the grey body factors depend on the number of extra dimensions, we can furthermore infer \(n\) from the relative abundances \([32]\) of particles with different spin. Measuring the spectrum of particles emitted and using Equations (1) and (2) one can infer the radius, temperature and mass of the black hole.

We define the effectiveness \(n(x)\) of some degree of freedom \(x\) by comparing the emission rate into channel \(x\) to the emission into one massless scalar

\[
n(x)(M_H) = \int_m^{M_H} d\omega \dot{E}(\omega) / \int_m^{M_H/2} d\omega \dot{E}(m.s.)(\omega). \quad (4)
\]

\(^1\) This is only simple in a Gedankenexperiment. In reality the task might be a challenge to even the finest experimental physicists.
Here, the cut-off $\Lambda = \min(M_H[1 + m^2/M_H^2]/2, M_H)$ limits the energy of emitted particles and is due to energy-momentum conservation and finite black hole mass and $\Lambda \geq m$ is understood. Please note that $M_H$ decreases steadily during evaporation. The number of effective degrees of freedom is then given by $n_{\text{eff}}(M_H) = \sum_\omega n(\omega)(M_H)$. It is not directly observable, as experiments lack resolution to connect particles to their corresponding emission times. What we can observe is the integral over the evaporation process, where the energy deposited into one massless scalar is

$$E^{(\text{m.s.})}(M_H^{\text{init}}) = \int_0^{M_H^{\text{init}}} \frac{dM}{n_{\text{eff}}(M)}.$$  \hspace{1cm} (5)

Inverting this relation and normalizing to $M_H^{\text{init}}$, the integrated number of degrees of freedom $\bar{n}_{\text{eff}}(M_H^{\text{init}}) = M_H^{\text{init}}/E^{(\text{m.s.})}(M_H^{\text{init}})$ follows. In contrast to $n_{\text{eff}}$, we can measure $\bar{n}_{\text{eff}}$ from the total energy $E(\omega)$ deposited into a known species $\omega$ using $E^{(\text{m.s.})} = E(\omega)/\int_0^{M_H^{\text{init}}} n(\omega)(M) \ dM$ from Equation (4).

If standard model particles and gravitational polarization states can account for $n_{\text{eff}}$, a scalar dark energy field will be ruled out. The same is true for bulk and weakly interacting brane particles with masses $\lesssim T^{\text{init}}$. This would leave us with a cosmological constant as the only explanation for the acceleration of our Universe. Please note that long distance modifications such as $[37]$ would also be ruled out as they are equivalent to scalar field models with light scalars $[35, 36, 40]$.

If, on the other hand, we find missing energy which cannot be accounted for then possible candidates must have $m \lesssim T^{\text{init}}$. Distinguishing between bulk and brane fields would then require a high precision measurement making use of the slightly different emission rates.

Standard model particles contribute roughly one hundred degrees of freedom. In addition, we have $(n + 3)(n + 2)/2 - 1$ gravitational modes. Assuming that the latter radiate approximately like a scalar field, we see that $\bar{n}_{\text{eff}}$ needs to be determined to better than 0.5% (see also Figure 2). A recent study of possible black hole decays at LHC $[26]$ predicts an accuracy for the measurement of the total energy emitted into known particles of $\sim 30\%$ for a 5TeV and $\sim 15\%$ for a 8TeV black hole ($M_* = 1\text{TeV}$).

This is not yet sufficient for our measurement – but only by two orders of magnitude. Future colliders will probe higher and higher energies and produce black holes with ever increasing mass. As more massive black holes are cooler, they emit a smaller variety of particles with considerably better statistics. Hence, the measurement proposed is within reach of next generation colliders.

Beside experimental challenges there remain theoretical problems to be solved. First, the emission of gravity modes into the bulk must be better understood. Moreover, the data for Figures 1 and 2 was inferred for a $7\text{ TeV}$ model, because graviton polarizations contribute 44 states. While the initial temperature for $n = 7$ in the mass range depicted is sufficient to radiate all known degrees of freedom, the initial temperature $T^{\text{ini}} \sim 1.7\text{ GeV}$ for $n = 1$ and $M^{\text{ini}} = 10^4\text{TeV}$ is too low to efficiently radiate top and bottom quarks, the Higgs scalar as well as W and Z bosons. As the black hole evaporates, the mass drops leading to an increase in $T_{\text{H}}$ and hence to an increase in $n_{\text{eff}}$ until black hole masses $M_{\text{H}} \sim 1\text{TeV}$ are reached. At even smaller masses, $n_{\text{eff}}$ drops as more and more particles approach the kinematically allowed cut off $\Lambda = \min(M_H[1 + m^2/M_H^2]/2, M_H)$.

![FIG. 2: Degrees of freedom $n_{\text{eff}}(M_H)$ as a function of black hole mass for $n = 7$ (solid upper [black] line), $n = 3$ (solid middle [red] line) and $n = 1$ (solid lower [blue] line). The dashed lines below each solid line are the corresponding integrated degrees of freedom $\bar{n}_{\text{eff}}(M^{\text{ini}}_H)$. Experimentally, one cannot resolve $n_{\text{eff}}$, but rather measures $\bar{n}_{\text{eff}}$. Overall, there are more degrees of freedom for the $n = 7$ model, because graviton polarizations contribute 44 states. While the initial temperature for $n = 7$ in the mass range depicted is sufficient to radiate all known degrees of freedom, the initial temperature $T^{\text{ini}} \sim 1.7\text{ GeV}$ for $n = 1$ and $M^{\text{ini}} = 10^4\text{TeV}$ is too low to efficiently radiate top and bottom quarks, the Higgs scalar as well as W and Z bosons. As the black hole evaporates, the mass drops leading to an increase in $T_{\text{H}}$ and hence to an increase in $n_{\text{eff}}$ until black hole masses $M_{\text{H}} \sim 1\text{TeV}$ are reached. At even smaller masses, $n_{\text{eff}}$ drops as more and more particles approach the kinematically allowed cut off $\Lambda = \min(M_H[1 + m^2/M_H^2]/2, M_H)$.]

---

2 There is one subtlety concerning the still unknown nature of neutrinos. Dirac neutrinos will effectively contribute twice as much degrees of freedom as Majorana neutrinos. Turned around this might also give us a hint about the true nature of neutrinos. Nevertheless, it is quite likely that the nature of neutrinos can be inferred from other experiments like, e.g., ones to detect the neutrinoless double $\beta$ decay.

Conclusions: We have shown how missing energy in the decay of higher dimensional black holes produced at colliders may be used to discern the number of light par-
particles/fields. In particular, a scalar dark energy field can be excluded provided all energy radiated away from the black hole is accounted for by known particles and graviton polarization states. Counting light degrees of freedom could answer additional questions. It might, for example, reveal the Majorana/Dirac nature of neutrinos. The proposed measurement is challenging for experimentalists and necessitates a better understanding of black holes produced at colliders. Yet, it may be the one and only way to rule out a light scalar field or modified gravity as dark energy candidates.

Acknowledgments

We would like to thank R. R. Caldwell, A. Ringwald and C. Wetterich for discussions.

[1] A. G. Riess et al. [Supernova Search Team Collaboration], Astrophys. J. 607, 665 (2004)
[2] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148, 175 (2003)
[3] M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D 69 (2004) 103501
[4] Ya. B. Zeldovich, Pis'ma Zh. Eksp. Teor. Fiz. 6 (1967) 883 [JETP Lett. 6 (1967) 316].
[5] C. Wetterich, Nucl. Phys. B 302, 668 (1988)
[6] B. Ratra and P. J. Peebles, Phys. Rev. D 37, 3406 (1988)
[7] R. R. Caldwell, Phys. Lett. B 545, 23 (2002).
[8] A. Upadhye, M. Ishak and P. J. Steinhardt, astro-ph/0411803
[9] R. R. Caldwell and M. Doran, Phys. Rev. D 69, 103517 (2004).
[10] C. Kolda and D. H. Lyth, Phys. Lett. B 458 (1999) 197.
[11] M. Doran and J. Jaeckel, Phys. Rev. D 66 (2002) 043519.
[12] J. Kratochvil, A. Linde, E. V. Linder and M. Shmakova, JCAP 0407 (2004) 001.
[13] M. Doran in: M.A. Chown, New Scientist 183 2455 (2004) 11.
[14] P. Jetzer and N. Straumann, astro-ph/0411034
[15] C. Wetterich, Nucl. Phys. B 302 (1988) 645.
[16] H. B. Sandvik, J. D. Barrow and J. Magueijo, Phys. Rev. Lett. 88, 031302 (2002).
[17] T. Damour, F. Piazza and G. Veneziano, Phys. Rev. D 66 (2002) 046007.
[18] D. Parkinson, B. A. Bassett and J. D. Barrow, Phys. Lett. B 578 (2004) 235.
[19] M. Doran, astro-ph/0411606
[20] S. W. Hawking, Commun. Math. Phys. 43 (1975) 199.
[21] P. Kanti, hep-ph/0402168 and references therein.
[22] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429 (1998) 263.
[23] P. C. Argyres, S. Dimopoulos and J. March-Russell, Phys. Lett. B 441 (1998) 96.
[24] T. Banks and W. Fischler, arXiv:hep-th/9906038
[25] R. Emparan, G. T. Horowitz and R. C. Myers, Phys. Rev. Lett. 85 (2000) 499.
[26] S. B. Giddings and S. Thomas, Phys. Rev. D 65 (2002) 056010.
[27] S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. 87 (2001) 161602.
[28] C. M. Harris, M. J. Palmer, M. A. Parker, P. Richardson, A. Sabetfakhri and B. R. Webber, hep-ph/0411022.
[29] J. L. Feng and A. D. Shapere, Phys. Rev. Lett. 88 (2002) 021303.
[30] R. Emparan, M. Masip and R. Rattazzi, Phys. Rev. D 65 (2002) 064023.
[31] L. Anchordoqui and H. Goldberg, Phys. Rev. D 65 (2002) 047502.
[32] A. Ringwald and H. Tu, Phys. Lett. B 525 (2002) 135.
[33] C. M. Harris and P. Kanti, JHEP 0310 (2003) 014.
[34] P. Kanti and J. March-Russell, Phys. Rev. D 66 (2002) 024023.
[35] D. Ida, K. y. Oda and S. C. Park, Phys. Rev. D 67 (2003) 064025 [Erratum-ibid. D 69 (2004) 049901].
[36] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B 544 (1999) 3.
[37] S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, Phys. Rev. D 70 (2004) 043528.
[38] Teyssandier P and Tourrenc P 1983 J. Math. Phys. 24 2793
[39] D. Wands, Class. Quant. Grav. 11 (1994) 269.
[40] T. Chiba, Phys. Lett. B 575 (2003) 1.