The cosmological implications of a fundamental length: a DSR-inspired de Sitter spacetime

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Abstract. We study a de Sitter model in the framework of a deformed special relativity (DSR) inspired structure. The effects of this framework appear as the existence of a fundamental length which influences the behavior of the scale factor. We show that such a deformation can be used either to control the unbounded growth of the scale factor in the present accelerating phase or to account for the inflationary era in the early evolution of the universe.

Keywords: inflation, quantum gravity phenomenology, physics of the early universe
The cosmological implications of a fundamental length

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1. Introduction

It is generally believed that the existence of a fundamental length is a natural feature in all theories that endeavor to answer the old and interesting question of how to quantize gravity [1,2]. In string theory, considered as an alternative to the unified theory of interactions, the fundamental length plays a crucial role [3]. Also, the fundamental length appears in loop quantum gravity [4] as a promising alternative to quantum gravity, in the form of discrete spectra of the area and volume operators [5]. This fundamental length is introduced by hand in some effective theories [6]–[8], where it is shown that it can be recovered as the limit of full quantum gravity [6,9,10]. Therefore, the motivation behind the construction of these effective models is to study the effects of such a fundamental length in simple scenarios which are amenable to exact solutions. A common approach to introduce a fundamental length is to modify or deform the algebraic structure of the phase-space, which may be done in various manners, a few examples of which can be found in [11]–[13].

To study the effects of the existence of a fundamental length on the behavior of the universe, one can construct a model based on the noncommutative structure of DSR [6], which is described by what is known as the $\kappa$-deformation [14]. The concept of $\kappa$-deformation is introduced and studied in [15,16]. The $\kappa$-Minkowski space [16,17] arises from the $\kappa$-Poincare algebra [15] such that the ordinary brackets between coordinates are replaced by

$$\{x_0, x_i\} = \frac{1}{\kappa} x_i,$$  

where $\{,\}$ represents the Poisson bracket and $\kappa$ is the deformation (noncommutativity) parameter, which has the dimension of mass $\kappa = \epsilon \ell^{-1}$ for $c = \hbar = 1$, where $\epsilon = \pm 1$ [18] such that $\kappa$ and $\ell$ can be interpreted as dimensional parameters corresponding to a fundamental energy and length, respectively. This fundamental length can be identified with an invariant minimum length, e.g., the Planck length. In what follows, we restrict ourselves to the $\epsilon = 1$ sector. In the next section we will study the phase-space structure of
a de Sitter spacetime. The DSR-inspired deformation in de Sitter spacetimes is introduced in section 3. Sections 4 and 5 deal with the interpretation of the results.

2. The de Sitter spacetime

Let us start by briefly examining a simple de Sitter model

\[ ds^2 = -N^2(t) \, dt^2 + \frac{a^2(t)}{(1 + (k/4)r)^2} (dx^2 + dy^2 + dz^2), \]  

(2)

where \( N(t) \) is the lapse function and \( k = -1, 0, +1 \) represents an open, flat or closed universe, respectively. The Einstein–Hilbert Lagrangian with cosmological constant \( \Lambda \) becomes

\[ L = \sqrt{-g} \left( R[g] - 2\Lambda \right) = -6N^{-1}a\dot{a}^2 + 6kNa - 2\Lambda Na^3, \]  

(3)

where \( R[g] \) is the Ricci scalar and in the second line the total derivative term has been ignored. The corresponding Hamiltonian, up to a sign, becomes

\[ \mathcal{H}_0 = \frac{1}{24}Na^{-1}\dot{p}_a^2 + 6kNa - 2\Lambda Na^3. \]  

(4)

Here, we note that since the momentum conjugate to \( N(t) \), \( \pi = \partial L/\partial \dot{N} \), vanishes, the term \( \lambda \pi \), where \( \lambda \) is a Lagrange multiplier, must be added as a constraint to Hamiltonian (4), so that the Dirac Hamiltonian is

\[ \mathcal{H} = \frac{1}{24}Na^{-1}\dot{p}_a^2 + 6kNa - 2\Lambda Na^3 + \lambda \pi. \]  

(5)

The equations of motion with respect to the above Hamiltonian become

\[ \dot{a} = \{ a, \mathcal{H} \} = \frac{1}{12}Na^{-1}p_a, \]  

(6)

\[ \dot{p}_a = \{ p_a, \mathcal{H} \} = \frac{1}{24}Na^{-2}\dot{p}_a^2 - 6kN + 6\Lambda Na^2, \]  

(7)

\[ \dot{N} = \{ N, \mathcal{H} \} = \lambda, \]  

(8)

\[ \dot{\pi} = \{ \pi, \mathcal{H} \} = -\frac{1}{24}a^{-1}\dot{p}_a^2 - 6ka + 2\Lambda a^3. \]  

(9)

Note that to satisfy the constraint \( \pi = 0 \) at all times the secondary constraint \( \dot{\pi} = 0 \) must also be satisfied. A simple calculation leads to

\[ \dot{a} = \sqrt{\frac{1}{3}\Lambda a^2 - k}, \]  

(10)

where we have fixed the gage by taking \( N = 1 \); that is, we work in the comoving gauge. Note that the other equations will be satisfied automatically if the above equation is satisfied. The solutions for non-vanishing \( \Lambda \) become

\[ a(t) = \frac{C_1}{4\Lambda} \left\{ \begin{array}{l} C_1^2 e^{\sqrt{(\Lambda/3)t}} + 24k e^{-\sqrt{(\Lambda/3)t}} \vspace{0.2cm} \\ C_1^2 e^{-\sqrt{(\Lambda/3)t}} + 24k e^{\sqrt{(\Lambda/3)t}} \end{array} \right\}, \]  

(11)

where \( C_1 \) is the integration constant. For vanishing \( \Lambda \) we have

\[ a(t) = C_2 \pm i\sqrt{k}t, \]  

(12)
where \( C_2 \) is the integration constant, and the solutions become meaningful only for \( k = 0 \) and \(-1\). In all the possible solutions the scale factor has a growing behavior without any limit, save for the trivial cases \( \Lambda = 0 \) and \( k = 0 \). This means that the scale factor goes to infinity for large \( t \), either exponentially or linearly for non-vanishing and vanishing \( \Lambda \), respectively.

3. The DSR-inspired phase-space

It has long been argued that a deformation in phase-space can be seen as an alternative path to quantization. This argument is based on the Wigner quasi-distribution function and Weyl correspondence between quantum-mechanical operators in Hilbert space and ordinary \( c \)-number functions in phase-space; see for example [19] and the references therein. The deformation in the usual phase-space structure is introduced by Moyal brackets which are based on the Moyal product [11]. However, to introduce such deformations, it is more convenient to work with Poisson brackets rather than Moyal brackets.

From a cosmological point of view, models are built in a minisuper-(phase)-space. It is therefore safe to say that studying such a space in the presence of the deformations mentioned above can be interpreted as studying the quantum effects on cosmological solutions. One should note that in gravity, and consequently in cosmology, the effects of quantization are woven into the existence of a fundamental length [1], as mentioned in the introduction. The question then arises as to what form of deformations in phase-space is appropriate for studying quantum effects in a cosmological model. The modified structure of the geometry, that is, the noncommutative geometry [8], has become the basis from which similar modifications in phase-space have been inspired. In this approach, the fields and their conjugate momenta play the role of the coordinate basis in noncommutative geometry [20]. Of course, in doing so an effective model is constructed whose validity depends on its power of prediction. For example, if in a model field theory the fields are taken as noncommutative, as has been done in [20], the resulting effective theory predicts the same Lorentz violation as a field theory in which the coordinates are considered as noncommutative [21]. Over the years, a large number of works on noncommutative fields [11] have been inspired by noncommutative geometry model theories [8]. As a further example, it is well known that string theory can be used to suggest a modification in the bracket structure of coordinates, also known as the generalized uncertainty principle [22], which is used to modify the phase-space structure [12]. In this paper we will examine a new kind of modification in phase-space inspired by relation (1), much the same as has been done in [11]–[13]. In what follows we introduce noncommutativity based on \( \kappa \)-Minkowskian space and study its consequences on the solutions discussed in the previous section.

To introduce noncommutativity we start from

\[
\{ N'(t), a'(t) \} = \ell a'(t),
\]

where one can interpret \( N(t) \) and \( a(t) \), appearing as the coefficients of \( dt \) and \( d\vec{x} \), in the same manner as \( x_0 \) and \( x_i \), respectively. For this reason we name this kind of phase-space the \( \kappa \)-Minkowskian (minisuper) phase-space. For the primed variables then,
Hamiltonian (4) becomes
\[ H'_0 = \frac{1}{24} N' a^{-1} p_a^2 + 6 k N a - 2 \Lambda N a^3, \]  
(14)
where the ordinary Poisson brackets are satisfied except for (13). Following [23], we introduce the new variables
\[ N'(t) = N(t) - \ell a(t) p_a(t), \]
\[ a'(t) = a(t). \]  
(15)

It can be easily checked that the above variables will satisfy (13) if the unprimed variables satisfy the ordinary Poisson brackets. The term \(-\ell a(t) p_a(t)\) may be looked upon as a direct consequence of a phase-space deformation of relation (13) which, as has been suggested, could originate from string theory, noncommutative geometry and so on; see [13, 19, 24]. With the above transformations, Hamiltonian (14) changes to
\[ H^{nc}_0 = \frac{1}{24} N a^{-1} p_a^2 + 6 k N a - 2 \Lambda N a^3 - \frac{1}{24} \ell p_a^3 - 6 \ell k a^2 p_a + 2 \ell \Lambda a^4 p_a. \]  
(16)

Clearly, the momentum \(\pi\) conjugate to \(N(t)\) does not appear in (16), i.e. it should be taken as the primary constraint. It can be checked by using Legendre transformations that the conjugate momentum corresponding to \(N(t)\), that is, \(\pi = \partial \mathcal{L} / \partial \dot{N}\), vanishes. Therefore, the term \(\lambda \pi\) must be added to Hamiltonian (16), so that we find
\[ H^{nc} = \frac{1}{24} N a^{-1} p_a^2 + 6 k N a - 2 \Lambda N a^3 - \frac{1}{24} \ell p_a^3 - 6 \ell k a^2 p_a + 2 \ell \Lambda a^4 p_a + \lambda \pi. \]  
(17)

The equations of motion with respect to Hamiltonian (17) are
\[ \dot{a} = \{a, H^{nc}\} = \frac{1}{12} N a^{-1} p_a - \frac{1}{8} \ell p_a^2 - 6 \ell k a^2 + 2 \ell \Lambda a^4, \]  
(18)
\[ \dot{p}_a = \{p_a, H^{nc}\} = \frac{1}{24} N a^{-2} p_a^2 - 6 k N + 6 \Lambda N a^2 + 12 \ell k a p_a - 8 \ell \Lambda a^3 p_a, \]  
(19)
\[ \dot{N} = \{N, H^{nc}\} = \lambda, \]  
(20)
\[ \dot{\pi} = \{\pi, H^{nc}\} = -\frac{1}{24} a^{-1} p_a^2 - 6 k a + 2 \Lambda a^3. \]  
(21)

Again we restrict ourselves to the comoving gauge for which \(N = 1\). Combining the first and last equations we find
\[ \dot{a} + 12 \ell a^2 \left(\frac{1}{3} \Lambda a^2 - k\right) = \sqrt{\frac{1}{3} \Lambda a^2 - k}, \]  
(22)
which is compatible with other equations. Note that this equation reduces to the commutative case (10) when \(\ell \to 0\). The analytic solutions for this equation exist only for the special cases \(\Lambda = 0\) or \(k = 0\). For \(\Lambda = 0\), the solution of equation (22) is complex:
\[ a(t) = \frac{(1 + i) \tan[(1 + i) \sqrt{6 \ell k^{3/4}} (t + C_3)]}{2 \sqrt{6 \ell k^{1/4}}}, \]  
(23)

where \(C_3\) is an integration constant. Real solutions are only obtained for \(k = 0\) or \(-1\), that is
\[ a(t) = C_4, \quad k = 0, \]  
(24)
\[ a(t) = \frac{\tanh[2 \sqrt{3} \ell (t + C_5)]}{2 \sqrt{3} \ell}, \quad k = -1, \]  
(25)
where $C_4$ and $C_5$ are constants. The scale factor for $k = 0$ and $\Lambda = 0$, represented by equation (24), is a constant, similar to the commutative case. The solution for $k = -1$ is interesting since its behavior at late times, $t \to \infty$, is completely different from that of the commutative case. Here, the scale factor becomes constant, in contrast to the commutative case given by equation (12), where the scale factor grows linearly.

For the observationally interesting case $k = 0$, the scale factor can be written as

$$a(t) = 3^{1/6} \left( \frac{e^{\sqrt{3} \Lambda t}}{C_6 + 12\ell \sqrt{\Lambda} e^{\sqrt{3} \Lambda t}} \right)^{1/3},$$

where $C_6$ is a constant of integration. This solution reduces to the commutative one given by equation (11) for $\ell \to 0$. Note that the above solution for $t \to \infty$ becomes constant:

$$a(t \to \infty) = \left( \frac{\sqrt{3}}{12\ell \sqrt{\Lambda}} \right)^{1/3},$$

which is different from that of the commutative solution given by equation (11). The general solution for the scale factor given by equation (22) cannot be obtained analytically. This equation may however be solved numerically, showing that these solutions follow the general behavior of the special solutions, namely, the scale factor becomes constant for $t \to \infty$, in contrast to the commutative case. This behavior can be seen in figure 2 for different values of $k$.

The scale factor calculated in (26) and its behavior are the main results of the present work. To describe these results we may interpret $\Lambda$ both as a cosmological constant for the present accelerating and the early inflationary phases of the universe. In what follows we study the implications of the fundamental length on the scale factor given by relation (26) in these two distinct scenarios.

4. The present accelerating phase

The first scenario can be realized by noting that the main ingredients in these calculations have been the introduction of the above deformation to the ordinary Poisson brackets, which resulted in a damping exponential behavior in the evolution of the universe. However, in contrast to usual scenarios where $\ell = 0$, the scale factor asymptotically approaches a constant value at late times $t \to \infty$. It is clear from the plots in figure 1 that for $\ell = 0$ the curve is concave in all regions. For $\ell \neq 0$ the curve is concave at first but becomes convex at late times. This means that the deceleration parameter is negative at the beginning and becomes positive later. Note that the above results are a direct effect of the existence of a non-vanishing fundamental length $\ell$ (noncommutativity parameter). It is interesting to note that in studying the phenomenological aspects of quantum gravity one has to go to high energies to zoom in on very small distances where we expect the effects of quantum gravity to become dominant. However, the results

1 For the uninteresting case $\Lambda = 0$ and $k = -1$ the scale factor (25) is convex in all regions.

2 It appears that this result is in contradiction with the observation that the deceleration parameter is positive at early times and becomes negative for late times, corresponding to a matter-radiation dominated regime and a $\Lambda$ dominated phase, respectively [25]. Here, our model is restricted to the $\Lambda$ dominated phase and shows that the existence of $\ell$ can change the acceleration to a deceleration phase.
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Figure 1. The solid line shows the scale factor for non-vanishing $\ell$ ($\ell = 0.0001$) and the dashed line that for vanishing $\ell$. The initial condition is $a(0) = 1$ with $\Lambda = 6$, $k = 0$ for both cases.

Figure 2. The behavior of the scale factor for $\Lambda = 6$, $\ell = 0.0001$, $a(0) = 1$ and $k = +1$, left, $k = -1$, right. Note that we have rescaled the horizontal axes such that $t' = 100t$.

presented here point to an alternative possibility for the observation of the fundamental length in that it is directly related to the possibility of the observation of a constant scale factor. The price to be paid, however, is that one has to wait, possibly, for an incredibly long time. This could be interpreted as the duality of time and energy, namely, either spending large amounts of energy to penetrate small distances in a short span of time or using little energy but facing the possibility of having to endure the passage of a long interval of time. In this scenario, the cosmos can be taken as a large laboratory in which the study of cosmological models can pave the way for a full understanding of quantum gravity. Otherwise, ‘an accelerator powerful enough to study ... Planckian objects would have to be as large as the entire galaxy’ [27].

5. The early inflationary phase

The second scenario stems from the observation of the general belief that the effects of a fundamental length or, equivalently in this model, the noncommutative structure, can be observed in the early universe. It would therefore be interesting and natural to discuss
the results of the present work at the early stages of the evolution of the universe. We will show below that this simple model may be useful in describing the early inflationary phase of the universe. Let us then suppose that this phase begins at an initial time $t_i$ and ends at a final time $t_R$, also known as the reheating time [28,29]. During such a phase the problem of e-folding, that is, the order of magnitude of the size of the universe with which the scale factor expands during inflation and the length of this period, which should be finite, must be addressed whenever an inflationary model is discussed [30]. The value of the e-folding according to the present data is at least 60, which means that the scale factor of the inflationary phase must satisfy the relation $60 \sim \log(a(t_R)/a(t_i))$. From equation (26), it is clear that at early stages, since $\ell \ll 1$, the scale factor behaves exponentially and $\Lambda$ is viewed as the cosmological constant of the inflationary era. As time passes, the behavior of the scale factor would only depend on $\ell$, and clearly it would no longer follow the exponential growth that it once had. This means that the presence of $\ell$ could provide a natural mechanism with which to exit from the inflationary phase. To address the e-folding problem we note that one may introduce a relation between the inflationary cosmological constant $\Lambda$ and the fundamental length (noncommutativity parameter) $\ell$ using the above definition for e-folding, that is, $\ell \sim e^{-180\sqrt{3}/(12\sqrt{\Lambda})}$, which would clearly address the 60 e-folding problem [30]. We may now see that such a relatively simple model can address the above issues in inflationary models. Note that the initial time can be chosen as zero [28], as has been done in the above calculations.

It would be appropriate here to point out that there are other DSR-inspired theories for the inflationary era in the evolution of the universe [31]. These theories essentially amount to an adaptation of the old varying speed of light idea to the more compelling logics of the DSR setup; for a comprehensive review see [32] and the references therein. In these models, the assumption of a varying speed of light is invoked to address the problems of the standard cosmology without resorting to the ubiquitous scalar field invariably referred to as the inflaton. The well-known problems of the standard cosmology, namely the flatness, cosmological constant, homogeneity and isotropy, have all a direct solution in such DSR-inspired models [31]. An important feature of these models is the functional form of $c(t)$ which, amongst other properties, should be able to account for the rapid expansion of the universe during the inflationary era. To this end, $c(t)$ is so constructed as to predict an almost $10^{30}$-fold increase in the speed of light during inflation over its present value. In contrast, the model presented in this work has a deformation parameter, $\ell$, which makes a ‘graceful exit’ from the inflationary era possible. The inflationary era itself is produced by the existence of a cosmological constant, $\Lambda$. In summary, in our model the existence of a deformation, inspired by DSR, controls the rate of inflation. It has also been shown [33] that in a DSR-inspired Friedmann–Robertson–Walker model the scale factor shows an exponential (inflationary) behavior just after dust-domination as a consequence of the fundamental length. This latter model is more relevant to other DSR-inspired works [31,32] since in these models the inflationary behavior appears just after the radiation-dominated era.

6. Conclusions

More often than not, it is the case that the study of effective theories can shed light on the blurred corners of the corresponding full theory. The same is true in describing the
quantum effects on cosmology since the full theory is immensely difficult to handle [34]. The DSR can be interpreted as one such effective theory. Therefore, the study of cosmological models within the framework of such effective theories could pave the way for a more profound understanding. In the present study, we have introduced a fundamental length by employing an effective theory, namely a DSR-inspired model. In doing so we have chosen a simple cosmological model, namely a de Sitter spacetime. Here, the introduction of a fundamental length causes additional terms to appear in Hamiltonian (17) as compared to (5). As has been mentioned in [13, 19, 24], these extra terms can be interpreted as the effects of high energy corrections of a full theory, e.g., string theory. Our results can also be used to address the problems of interest in cosmology such as the inflation or present accelerating phase, discussed above. It is therefore reasonable to assume that the modifications introduced in this work, based on relation (13), are relevant in model theories dealing with the problems mentioned above.

In summary, we have studied a noncommutative model theory in which noncommutativity was inspired by invoking $\kappa$-deformation in phase-space. The noncommutativity parameter appearing in our model was interpreted as a fundamental length. Starting from a de Sitter universe, the effects of noncommutativity are to predict either a ‘graceful exit’ from the inflationary phase or the emergence from a late time accelerating phase. These results can be interpreted as a phenomenological feature for the existence of such a length and indirectly for the existence of a quantum theory of gravity.

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References

[1] Garay L J, Quantum gravity and minimum length, 1995 Int. J. Mod. Phys. A 10 145 [SPIRES] [gr-qc/9403008]
[2] Rovelli C, Notes for a brief history of quantum gravity, 2000 Preprint gr-qc/00060613
[3] Green M, Schwarz J H and Witten E, 1987 Superstring Theory vol 1 and 2 (Cambridge: Cambridge University Press)
Pochinski J, 1998 String Theory vol 1 and 2 (Cambridge: Cambridge University Press)
[4] Rovelli C, 2004 Quantum Gravity (Cambridge: Cambridge University Press)
[5] Rovelli C and Smolin L, Discreteness of area and volume in quantum gravity, 1995 Nucl. Phys. B 442 593 [SPIRES] [gr-qc/9411005]
Rovelli C and Smolin L, 1995 Nucl. Phys. B 456 753 (erratum)
Ashtekar A and Lewandowski J, Quantum theory of gravity I: area operators, 1997 Class. Quantum Grav. 14 A55 [SPIRES] [gr-qc/9602046]
[6] Amelino-Camelia G, Doubly-special relativity: first results and key open problems, 2002 Int. J. Mod. Phys. D 11 1643 [SPIRES] [gr-qc/0210063]
Kowalski-Glikman J, Introduction to doubly special relativity, 2005 Lect. Notes Phys. 669 131 [hep-th/0405273]
[7] Amelino-Camelia G, Relativity in space–times with short-distance structure governed by an observer-independent (Planckian) length scale, 2002 Int. J. Mod. Phys. D 11 35 [SPIRES] [gr-qc/0012051]

3 This interpretation is consistent with the one introduced at the beginning of section 3 in that the extra terms can be interpreted as quantum effects.
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Amelino-Camelia G, Testable scenario for relativity with minimum-length, 2001 Phys. Lett. B 510 255 [SPIRES] [hep-th/0012238]

Magueijo J and Smolin L, Lorentz invariance with an invariant energy scale, 2002 Phys. Rev. Lett. 88 190403 [SPIRES] [hep-th/0112090]

Magueijo J and Smolin L, Generalized Lorentz invariance with an invariant energy scale, 2003 Phys. Rev. D 67 044017 [SPIRES] [gr-qc/0207085]

[8] Connes A, 1994 Noncommutative Geometry (New York: Academic)

Connes A, A short survey of noncommutative geometry, 2000 J. Math. Phys. 41 3832 [SPIRES] [hep-th/0003006]

Majid S, Meaning of noncommutative geometry and the Planck-scale quantum group, 2000 Lect. Notes Phys. 541 227 [hep-th/0006166]

[9] Amelino-Camelia G, Smolin L and Starodubtsev A, Quantum symmetry, the cosmological constant and Planck scale phenomenology, 2004 Class. Quantum Grav. 21 3095 [SPIRES] [hep-th/0306134]

[10] Seiberg N and Witten E, String theory and noncommutative geometry, 1999 J. High Energy Phys. JHEP09(1999)032 [SPIRES] [hep-th/9908142]

[11] Garcia-Compean H, Obregon O and Ramirez C, Noncommutative quantum cosmology, 2002 Phys. Rev. Lett. 88 161301 [SPIRES] [hep-th/0107250]

Lopez-Dominguez J C, Obregon O, Ramirez C and Sabido M, Towards noncommutative quantum black holes, 2006 Phys. Rev. D 74 084024 [SPIRES] [hep-th/0607002]

Barbosa G D and Pinto-Neto N, Noncommutative geometry and cosmology, 2004 Phys. Rev. D 70 103512 [SPIRES] [hep-th/04071112]

Khosravi N, Jalalzadeh S and Sepangi H R, Non-commutative multi-dimensional cosmology, 2006 J. High Energy Phys. JHEP01(2006)134 [SPIRES] [hep-th/0601116]

Vakili B and Sepangi H R, Generalized uncertainty principle in Bianchi type I quantum cosmology, 2007 Phys. Lett. B 651 79 [SPIRES] [hep-th/0706.0273] [gr-qc]

Battisti M V and Montani G, The big-bang singularity in the framework of a generalized uncertainty principle, 2007 Phys. Lett. B 656 96 [SPIRES] [gr-qc/0703025]

Hassan S F and Sloth M S, Trans-Planckian effects in inflationary cosmology and the modified uncertainty principle, 2003 Nucl. Phys. B 674 434 [SPIRES] [hep-th/0204110]

[13] Khosravi N, Sepangi H R and Sheikh-Jabbari M M, Stabilization of compactification volume in a bicrossproduct structure of a κ-Poincare algebra and non-commutative geometry, 2004 Phys. Lett. B 617 102 [SPIRES] [hep-th/0404011]

Kowalski-Glikman J and Nowak S, Doubly special relativity theories as different bases of the Planck-scale quantum group, 2000 Phys. Lett. B 539 126 [SPIRES] [hep-th/0203040]

Kowalski-Glikman J, Doubly special quantum and statistical mechanics from quantum κ-Poincare algebra, 2002 Phys. Lett. A 299 454 [SPIRES] [hep-th/0111110]

Lukierski J, Nowicki A, Ruegg H and Tolstoy V N, q-deformation of Poincare algebra, 1991 Phys. Lett. B 268 331 [SPIRES]

Lukierski J, Nowicki A and Ruegg H, New quantum Poincare algebra and κ-deformed field theory, 1992 Phys. Lett. B 293 344 [SPIRES]

Majid S and Ruegg H, Bicrossproduct structure of κ-Poincare group and non-commutative geometry, 1994 Phys. Lett. B 334 348 [SPIRES] [hep-th/9405107]

Freidel L, Kowalski-Glikman J and Nowak S, From noncommutative κ-Minkowski to Minkowski space–time, 2007 Phys. Lett. B 648 70 [SPIRES] [hep-th/0612170]

Bruno N R, Amelino-Camelia G and Kowalski-Glikman J, Deformed boost transformations that saturate at the Planck scale, 2001 Phys. Lett. B 522 133 [SPIRES] [hep-th/0107039]

Zachos C K, Fairlie D B and Curtright T L (ed), 2005 Quantum Mechanics in Phase Space (Singapore: World Scientific)

Carmona J M, Cortés J L and Gamboa J and Méndez F, Quantum theory of noncommutative fields, 2003 J. High Energy Phys. JHEP03(2003)058 [SPIRES] [hep-th/0301248]

Carmona J M, Cortés J L, Gamboa J and Méndez F, Noncommutativity in field space and Lorentz invariance violation, 2003 Phys. Lett. B 565 222 [SPIRES] [hep-th/0207158]

[21] Carroll S M, Harvey J A, Kostelecky V A, Lane C D and Okamoto T, Noncommutative field theory and Lorentz violation, 2001 Phys. Rev. Lett. 87 141601 [SPIRES] [hep-th/0105082]

Carlson C E, Carone C D and Lebed R F, Bounding noncommutative QCD, 2001 Phys. Lett. B 518 201 [SPIRES] [hep-ph/0107291]

Carlson C E, Carone C D and Lebed R F, Supersymmetric noncommutative QED and Lorentz violation, 2002 Phys. Lett. B 549 337 [SPIRES] [hep-ph/0209077]
The cosmological implications of a fundamental length

[22] Kempf A, Mangano G and Mann R B, *Hilbert space representation of the minimal length uncertainty relation*, 1995 Phys. Rev. D **52** 1108 [SPIRES] [hep-th/9412167]

[23] Chaichian M, Sheikh-Jabbari M M and Tureanu A, *Hydrogen atom spectrum and the Lamb shift in noncommutative QED*, 2001 Phys. Rev. Lett. **86** 2716 [SPIRES] [hep-th/0010175]

[24] Nascimento J R, Petrov A Yu and Ribeiro R F, *Noncommutative fields in three dimensions and mass generation*, 2007 Europhys. Lett. **77** 51001 [SPIRES] [hep-th/0601077]

[25] Holz D E, *An accelerated history of the universe*, 2006 AIP Conf. Proc. (July 2006) vol 842 (New York: AIP) p 741

Cheng T-P, 2005 *Relativity, Gravitation and Cosmology* (Oxford Master Series in Physics no. 11) (Oxford: Oxford University Press)

Perkins D, 2005 *Particle Astrophysics* (Oxford Master Series in Physics no. 10) (Oxford: Oxford University Press)

[26] Amelino-Camelia G and Piran T, *Planck-scale deformation of Lorentz symmetry as a solution to the UHECR and the TeV-γ paradoxes*, 2001 Phys. Rev. D **64** 036005 [SPIRES] [astro-ph/0008107]

Amelino-Camelia G, *Quantum-gravity phenomenology: status and prospects*, 2002 Mod. Phys. Lett. A **17** 899 [SPIRES] [gr-qc/0204051]

Amelino-Camelia G, 2005 *Introduction to Quantum-Gravity Phenomenology* (Lecture Notes in Physics vol 669) ed G Amelino-Camelia and J Kowalski-Glikman (Berlin: Springer) [gr-qc/0412136]

Jacobsen T, Liberati S, Mattingly D and Stecker F W, *New limits on Planck scale Lorentz violation in QED*, 2004 Phys. Rev. Lett. **93** 021101 [SPIRES] [astro-ph/0309681]

Smolin L, *Falsifiable predictions from semiclassical quantum gravity*, 2006 Nucl. Phys. B **742** 142 [SPIRES] [hep-th/0501091]

Hossenfelder S, *Interpretation of quantum field theories with a minimal length scale*, 2006 Phys. Rev. D **73** 105013 [SPIRES] [hep-th/0603032]

[27] Susskind L, 2005 *The Cosmic Landscape: String Theory and the Illusion of Intelligent Design* (New York: Little, Brown)

[28] Brandenberger R H, *Inflationary cosmology: progress and problems*, 1999 Preprint hep-ph/9910410

[29] Linde A D, *A new inflationary universe scenario: a possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems*, 1982 Phys. Lett. B **108** 389 [SPIRES]

[30] Liddle A R, *An introduction to cosmological inflation*, 1998 Proc. ICTP Summer School in High-Energy Physics [astro-ph/9901124]

Guth A H, *Inflation and eternal inflation*, 2000 Phys. Rep. **333** 555 [SPIRES] [astro-ph/0002156]

[31] Albrecht A and Magueijo J, *A time varying speed of light as a solution to cosmological puzzles*, 1999 Phys. Rev. D **59** 043516 [SPIRES] [astro-ph/9811018]

Alexander S, Brandenberger R and Magueijo J, *Non-commutative inflation*, 2003 Phys. Rev. D **67** 081301 [SPIRES] [hep-th/0108190]

[32] Magueijo J, *New varying speed of light theories*, 2003 Rep. Prog. Phys. **66** 2025 [astro-ph/0305457]

[33] Khosravi N and Sepangi H R, *A fundamental length as a candidate for dark energy: a DSR inspired FRW spacetime*, 2008 Preprint 0802.0767 [gr-qc]

[34] Bojowald M, Hernandez H and Skirzewski A, *Effective equations for isotropic quantum cosmology including matter*, 2007 Phys. Rev. D **76** 063511 [SPIRES] [0706.1057] [gr-qc]