Majorana Phase in Minimal $S_3$ Invariant Extension of the Standard Model

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Abstract

The leptonic sector in a recently proposed minimal extension of the standard model, in which the permutation symmetry $S_3$ is assumed to be an exact flavor symmetry at the weak scale, is revisited. We find that $S_3$ with an additional $Z_N$ symmetry allows CP violating phases in the neutrino mixing. The leptonic sector contains six real parameters with two independent phases to describe charged lepton and neutrino masses and the neutrino mixing. The model predicts: an inverted spectrum of neutrino mass, $\tan \theta_{23} = 1 + O(m_e^2/m_{\mu}^2)$ and $\sin \theta_{13} = m_e/\sqrt{2}m_{\mu} + O(m_\alpha m_{\mu}/m_{\tau}^2) \simeq 0.0034$. Neutrino mass as well as the effective Majorana mass $<m_{ee}>$ in the neutrinoless double-$\beta$ decay can be expressed in a closed form as a function of $\phi_{\nu}, \Delta m_{21}^2, \Delta m_{23}^2$ and $\tan \theta_{12}$, where $\phi_{\nu}$ is one of the independent phases. The model also predicts $<m_{ee}> \geq (0.036 - 0.066)$ eV.

PACS numbers: 11.30.Hv, 12.15.Ff, 14.60.Pq

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The Yukawa sector of the standard model (SM), which is responsible for the generation of the mass of leptons and quarks, and their mixing, has too many redundant parameters. This not only weakens the predictivity of the SM, but also makes ambiguous how to go beyond the SM. An exact flavor symmetry could reduce this redundancy, thereby giving useful hints about how to unify the flavor structure of the SM.

Recently, a minimal $S_3$ invariant extension of the SM was suggested in [1], while assuming that the Higgs, quark and lepton including the right-handed neutrino fields belong to the three-dimensional reducible representation of the permutation group $S_3$. This smallest nonabelian symmetry based on $S_3$ is only spontaneously broken, because the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ is spontaneously broken. It was found in [1] that this flavor symmetry is consistent with experiments, and that in the leptonic sector an additional discrete symmetry $Z_2$ can be introduced. It was argued there that due the additional discrete $Z_2$ symmetry the neutrino mixing matrix $V_{\text{MNS}}$ cannot contain any CP violating phase $\delta$. We now believe this is incorrect, and we would like to re-investigate the leptonic sector of the model in this letter.

We will find that it is possible to introduce two independent CP violating phases in the neutrino mixing even with an additional $Z_N$ symmetry in the leptonic sector. The permutation symmetry $S_3$ with $Z_N$ allows three real mass parameters for the charged lepton mass matrix, and three real parameters and two phases for the neutrino mass matrix. The model predicts: an inverted spectrum of neutrino mass, $\tan^2 \theta_{23} = 1 + O(m_e^2/m_{\mu}^2)$ and $\sin^2 \theta_{13} = m_e/m_\mu \sqrt{2} + O(m_e m_\mu/m_\tau^2)$. Neutrino mass as well as the effective Majorana mass $m_{\nu e}$ in the neutrinoless double-$\beta$ decay can be expressed in a closed form as a function of $\phi_\nu, \Delta m_{23}^2, \Delta m_{12}^2$ and $\tan \theta_{12}$, where $\phi_\nu$ is one of the independent phases. We find that the minimum of $m_{\nu e}$ as well as $m_{\nu e}$ occurs at $\phi_\nu = 0$, which is approximately $\Delta m_{23}^2 / \sin 2 \theta_{12}$.

Before we will come to our main purpose of the letter, let us briefly summarize the basic ingredient of the $S_3$ invariant SM of [1]. The quark, lepton and Higgs fields are denoted by $Q^T = (u_L, d_L), u_R, d_R, L^T = (\nu_L, e_L), e_R, \nu_R, H$. Each of them forms a reducible representation $1 + 2$ of $S_3$. The doublets carry capital indices $I, J$ which run from 1 to 2, and the singlets are denoted by $Q_3, u_{3R}, u_{3R}, L_3, e_{3R}, \nu_{3R}, H_3$. The most general renormalizable Yukawa interactions are given by

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu},$$

where

$$\mathcal{L}_{Y_D} = -\overline{Q} \sum_{i=1}^{3} Y^d_{H_i} H_i d_R + h.c.,$$

$$\mathcal{L}_{Y_U} = -\overline{Q} (i\sigma_2) \sum_{i=1}^{3} Y^u_{H_i} H_i u_R + h.c.,$$

$$\mathcal{L}_{Y_E} = -\overline{L} \sum_{i=1}^{3} Y^e_{H_i} H_i e_R + h.c.,$$

1. A partial list for permutation symmetries is [2]–[17]. See for instance [8] for a review. The basic idea of [1] is similar to that of [2, 5, 6].

2. See for instance [18] for recent reviews on CP violation in the leptonic sector.

3. Similar but different predictions are obtained from different types of discrete symmetry [10]–[15]. See also [16] and [17].
\[ \mathcal{L}_{Y_i} = -\mathcal{T}(i\sigma_2) \sum_{i=1}^{3} Y_{H_i}^\nu H_i \nu_R + h.c., \]

and the Yukawa coupling matrices are given by

\[
Y_{H_1}^k = \begin{pmatrix} 0 & Y_2^k & Y_5^k \\ Y_2^k & 0 & 0 \\ Y_4^k & 0 & 0 \end{pmatrix}, \quad Y_{H_2}^k = \begin{pmatrix} Y_2^k & 0 & 0 \\ 0 & -Y_2^k & Y_5^k \\ 0 & Y_4^k & 0 \end{pmatrix}, \tag{2}
\]

\[
Y_{H_3}^k = \begin{pmatrix} Y_1^k & 0 & 0 \\ 0 & Y_1^k & 0 \\ 0 & 0 & Y_3^k \end{pmatrix}, \quad k = d, u, l, \nu. \tag{3}
\]

Further, the Majorana mass terms for the right-handed neutrinos is given by

\[ \mathcal{L}_M = -M_1 \nu_{IR}^T C \nu_{IR} - M_3 \nu_{3R}^T C \nu_{3R}, \tag{4} \]

where \( C \) is the charge conjugation matrix.\(^4\)

Pakvasa and Sugawara\(^2\) analyzed the Higgs potential. The potential they analyzed has not only an abelian discrete symmetry (which we will use for selection rules of the Yukawa couplings), but also a permutation symmetry \( S_2: H_1 \leftrightarrow H_2 \), which is not a subgroup of the flavor group \( S_3 \) of the model. We assume throughout this letter that the vacuum can respect this accidental symmetry of the Higgs potential, and

\[ < H_1 > = < H_2 > \tag{5} \]

is satisfied. \([< H_1 > = -< H_2 > \] would yield the same physics.] Then the Yukawa interactions \( [1] \) yield the mass matrices of the general form

\[ M = \begin{pmatrix} m_1 + m_2 & m_2 & m_3 \\ m_2 & m_1 - m_2 & m_3 \\ m_4 & m_4 & m_3 \end{pmatrix}. \tag{6} \]

The Majorana mass for \( \nu_L \) can be obtained from the see-saw mechanism\(^{20}\), and the corresponding mass matrix is given by \( M_\nu = M_{\nu D} \bar{M}^{-1} (M_{\nu D})^T \), where \( \bar{M} = \text{diag}(M_1, M_1, M_3) \). The mass matrices are diagonalized by the unitary matrices \( U' \)’s as

\[ U_{d(u,e)}^T M_d(u,e) U_{d(u,e)} \equiv U_\nu^T M_\nu U_\nu. \]

The diagonal masses can be complex, and so the physical masses are their absolute values, which we denote by \( m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, m_e, m_\mu, m_\tau, \) etc.

It would be certainly desirable to classify, in a similar way as in \([21, 22]\), all possible mass matrices that are consistent with an additional discrete abelian symmetry and with experimental data. We, however, leave this program to feature work. Here we simply adopt the result of \([1]\) that

\[ Y_{1e} = Y_{3e} = Y_{1\nu} = Y_{5\nu} = 0, \tag{7} \]

\(^4\)Supersymmetrization of the present model has been proposed in \([9]\).
and consequently
\[ m_1^e = m_3^e = m_5^e = m_5^e = 0 \]  

follows from a $Z_2$ symmetry. We emphasize that there are a number of different charge assignments of $Z_N$ that can yield \[ 7 \].\footnote{We do not consider $U(1)$ to avoid the appearance of a (nearly) massless particle.} Provided that the charge of $H_3$, $Q(H_3)$, is different from $Q(H_{1,2})$, only the conditions
\[
Q(L_3) = Q(L_{1,2}) + Q(e_{3R}) + Q(H_{1,2}) = Q(e_{1,2R}) + Q(H_{1,2})
\]
\[
= Q(\nu_{1,2R}) - Q(H_{1,2}) - Q(H_3)
\]

modulo $N$ should be satisfied to forbid $Y_1^e, Y_3^e, Y_1^e$ and $Y_3^e$. Unfortunately, none of the abelian discrete symmetries above is a symmetry in the quark sector. Note that if $Z_N$ is chiral, it is broken by QCD anyway ($S_3$ is not broken by QCD, because it is not a chiral symmetry.) The symmetry violating effect of the quark sector appears first at the two-loop level in the leptonic sector, so that the violation of $Z_N$ in the leptonic sector may be assumed to be negligibly small. Therefore, we throughout neglect that violating effect \[ 6 \].

To proceed with our discussion, we calculate the unitary matrix $U_{eL}$ from
\[
U_{eL}^\dagger M_e M_e^\dagger U_{eL} = \text{diag}(m_1^2, m_\mu^2, m_\tau^2),
\]

where
\[
M_e M_e^\dagger = \begin{pmatrix}
2(m_2^e)^2 + (m_5^e)^2 & (m_5^e)^2 & 2m_5^e m_4^e \\
(m_5^e)^2 & 2(m_2^e)^2 + (m_5^e)^2 & 0 \\
2m_5^e m_4^e & 0 & 2(m_4^e)^2
\end{pmatrix},
\]

and all the mass parameters appearing in \ref{11} are real. We find that $U_{eL}$ can be approximately written as \[ 9 \]
\[
U_{eL} \simeq \begin{pmatrix}
-\frac{y}{2} \left(1 + \frac{1}{x^2}\right) & -\frac{1}{\sqrt{2}} \left(1 - \frac{y^2}{4} + \frac{y^2}{2x^2}\right) & \frac{1}{\sqrt{2}} \\
\frac{y}{2} \left(1 - \frac{1}{x^2}\right) & \frac{1}{\sqrt{2}} \left(1 - \frac{y^2}{4} - \frac{y^2}{2x^2}\right) & \frac{1}{\sqrt{2}} \\
1 - \frac{y^2}{4} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \frac{y}{x^2}
\end{pmatrix},
\]

where $x = m_5^e/m_2^e \simeq m_\tau/m_\mu$ and $y = m_4^e/m_2^e \simeq \sqrt{2} m_e/m_\mu$.

The Majorana masses of the right-handed neutrinos, $M_1$ and $M_3$ in \[ 11 \] which may be complex, can be absorbed by a redefinition of $m_2^e, m_4^e$ and $m_3^e$, and we may therefore assume that $M_1$ and $M_3$ are real. After rescaling of $m_2^e, m_4^e$ and $m_3^e$ as
\[
(m_2^e) \to \rho_2^e = (m_2^e)/M_1^{1/2}, \quad (m_4^e) \to \rho_4^e = (m_4^e)/M_1^{1/2}, \quad (m_3^e) \to \rho_3^e = (m_3^e)/M_3^{1/2},
\]

\[ 13 \]
we obtain

\[
M_\nu = M_\nu^T M^{-1}(M_\nu^T)^T = \left( \begin{array}{ccc}
2(\rho_2^\nu)^2 & 0 & 2\rho_2^\nu\rho_4^\nu \\
0 & 2(\rho_2^\nu)^2 & 0 \\
2\rho_2^\nu\rho_4^\nu & 0 & 2(\rho_4^\nu)^2 + (\rho_3^\nu)^2
\end{array} \right).
\] (14)

All the phases in (14), except for one, can be absorbed. Without loss of generality, we may assume that \(\rho_3^\nu\) is complex. We find that \(M_\nu\) can be diagonalized as

\[
U_\nu^T M_\nu U_\nu = \left( \begin{array}{ccc}
m_{\nu_1} e^{i\phi_1} & 0 & 0 \\
0 & m_{\nu_2} e^{i\phi_2} & 0 \\
0 & 0 & m_{\nu_3}
\end{array} \right),
\] (15)

where

\[
U_\nu = \left( \begin{array}{ccc}
-s_{12} & c_{12} e^{i\phi_\nu} & 0 \\
0 & 0 & 1 \\
c_{12} e^{-i\phi_\nu} & s_{12} & 0
\end{array} \right),
\] (16)

\[
m_{\nu_3} \sin \phi_\nu = m_{\nu_2} \sin \phi_2 = m_{\nu_1} \sin \phi_1,
\] (17)

and \(c_{12} = \cos \theta_{12}\) and \(s_{12} = \sin \theta_{12}\). The mixing angle is given by

\[
\tan^2 \theta_{12} = \frac{(m_{\nu_2}^2 - m_{\nu_1}^2 \sin^2 \phi_\nu)^{1/2} - m_{\nu_1} \cos \phi_\nu}{(m_{\nu_2}^2 - m_{\nu_1}^2 \sin^2 \phi_\nu)^{1/2} + m_{\nu_1} \cos \phi_\nu},
\] (18)

from which we find

\[
\frac{m_{\nu_2}^2}{\Delta m_{23}^2} = \frac{(1 + 2t_{12}^2 + t_{12}^4 - r t_{12}^4)^2}{4t_{12}^2(1 + t_{12}^2)(1 + t_{12}^4 - r t_{12}^4) \cos^2 \phi_\nu} - \tan^2 \phi_\nu
\] (19)

\[
\approx \frac{1}{\sin^2 2\theta_{12} \cos^2 \phi_\nu} - \tan^2 \phi_\nu \text{ for } |r| << 1,
\] (20)

where \(t_{12} = \tan \theta_{12}, r = \Delta m_{21}^2 / \Delta m_{23}^2\). As in [1], we find that only an inverted mass spectrum

\[
m_{\nu_3} < m_{\nu_1}, m_{\nu_2}
\] (21)

is consistent with the experimental constraint \(|\Delta m_{21}^2| < |\Delta m_{23}^2|\) in the present model. To see this, we first derive

\[
m_{\nu_1} \cos \phi_1 - m_{\nu_3} \cos \phi_\nu = -2\rho_2^\nu \rho_4^\nu A_1
\] (22)

\[
m_{\nu_2} \cos \phi_2 - m_{\nu_3} \cos \phi_\nu = 2\rho_2^\nu \rho_4^\nu A_2
\] (23)

where

\[
A_1 = \sin 2\theta_{12} + \cos^2 \theta_{12} / \tan 2\theta_{12}, \quad A_2 = \sin 2\theta_{12} - \sin^2 \theta_{12} / \tan 2\theta_{12}.
\] (24)

Then we use the fact that if \(A_1\) is positive (negative), then \(A_2\) is always positive (negative). Suppose that \(2\rho_2^\nu \rho_4^\nu A_2\) is positive, which implies that \(m_{\nu_2} \cos \phi_2 > m_{\nu_3} \cos \phi_\nu\) and \(m_{\nu_1} \cos \phi_1 < m_{\nu_3} \cos \phi_\nu\). In this case, eq. (17) can be satisfied, only if \(m_{\nu_2} > m_{\nu_3}\) or \(m_{\nu_1} > m_{\nu_3}\). Similarly, if
Figure 1: $m_{\nu_2}$ versus $\sin \theta_{12}$ for $\Delta m_{21}^2 = 6.9 \times 10^{-5} \text{ eV}^2$, $\Delta m_{23}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ and $\sin \phi_\nu = 0$ (solid), 0.6 (dotted) and 0.96 (dot-dashed).

$-2\rho_2^i\rho_4^i A_1$ is positive, then $m_{\nu_2} > m_{\nu_3}$ or $m_{\nu_1} > m_{\nu_3}$ has to be satisfied. Therefore, $m_{\nu_3}$ cannot be the largest among $m_{\nu_i}$’s.\footnote{Of course, $m_{\nu_1} > m_{\nu_3} > m_{\nu_2}$ or $m_{\nu_2} > m_{\nu_3} > m_{\nu_1}$ is mathematically allowed, but is excluded by experiments.}

In fig. 1 we plot $m_{\nu_2}$ versus $\sin \theta_{12}$ for $\Delta m_{21}^2 = 6.9 \times 10^{-5} \text{ eV}^2$, $\Delta m_{23}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ (best-fit values reported in \cite{25,26,27}) and $\sin \phi_\nu = 0$ (solid), 0.6 (dotted) and 0.96 (dashed). The $\sin \phi_\nu$ dependence of $m_{\nu_2}$ is shown in fig. 2 for $\tan \theta_{12} = 0.68$ and the same values of $\Delta m_{21}^2$ and $\Delta m_{23}^2$ as in fig. 1. As we see from (20) and also from fig. 2, $m_{\nu_2}$ assumes at $\sin \phi_\nu = 0$ its minimal value

$$m_{\nu_2,\text{min}} \simeq \sqrt{\Delta m_{23}^2 / \sin 2\theta_{12}} = (0.036 - 0.066) \text{ eV},$$

where we have used $\Delta m_{23}^2 = (1.3 - 3.0) \times 10^{-3} \text{ eV}^2$ and $\sin 2\theta_{12} = 0.83 - 1.0$ \cite{26,28}.

Now the product $U_{e\ell}^T P U_\nu$ with $P = \text{diag.}(1,1,\exp i \arg(Y_{4\nu}))$ defines a neutrino mixing matrix, which we bring by an appropriate phase transformation to the popular form

$$V_{\text{MNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}. \quad (26)$$
We find:

\[
s_{13} = \frac{1}{\sqrt{2}} \frac{m_e}{m_{\mu}} + O(m_e m_{\mu} / m_{\tau}^2) \simeq 0.0034,
\]
\[
t_{23} = \frac{s_{23}}{c_{23}} = 1 - \frac{1}{2} \left( \frac{m_e}{m_{\mu}} \right)^2 + O(m_e^2 / m_{\tau}^2),
\]
\[
\delta = \text{arg}(Y_4^\nu) - \phi_{\nu},
\]
\[
\sin 2\alpha = \sin(\phi_1 - \phi_2)
\]
\[
= \pm \frac{m_{\nu_3} \sin \phi_{\nu}}{m_{\nu_1} m_{\nu_2}} \left( \sqrt{m_{\nu_2}^2 - m_{\nu_3}^2 \sin^2 \phi_{\nu}} + \sqrt{m_{\nu_1}^2 - m_{\nu_3}^2 \sin^2 \phi_{\nu}} \right),
\]
\[
\simeq \pm 2 \sin \phi_{\nu} (m_{\nu_3} / m_{\nu_2}) \sqrt{1 - (m_{\nu_3} / m_{\nu_2})^2 \sin^2 \phi_{\nu}},
\]
\[
\sin 2\beta = \sin(\phi_1 - \phi_2)
\]
\[
= \pm \frac{\sin \phi_{\nu}}{m_{\nu_1}} \left( m_{\nu_1} \sqrt{1 - \sin^2 \phi_{\nu}} + \sqrt{m_{\nu_1}^2 - m_{\nu_3}^2 \sin^2 \phi_{\nu}} \right),
\]

for \( \phi_1 + \phi_2 \sim \pm \pi \), where \( \phi_1, \phi_2 \) and \( \phi_{\nu} \) are defined in (15). Since \( \sin^2 2\theta_{13} \simeq 4.6 \times 10^{-5} \), future oscillation experiments such as J-Park experiment [28] can easily exclude the model. In fig. 3 we plot \( \sin 2\alpha \) (solid) and \( \sin 2\beta \) (dotted) as a function of \( \sin \phi_{\nu} \). As we can see, \( \sin 2\alpha \) reaches its maximal value 1 at \( \sin \phi_{\nu} \simeq 0.94 \). Similarly, the maximal value of \( \sin 2\beta \), which is about 1,
sin ϕ

Figure 3: sin 2α (solid) and sin 2β (dotted) versus sin φν for tan θ12 = 0.68, Δm21 = 6.9 × 10^{-5} eV^2 and Δm23 = 2.3 × 10^{-3} eV^2 in the case of φ1 + φ2 ~ π.

occurs at sin φν ≃ 0.85. We then consider the effective Majorana mass

\[ < m_{ee} > = | \sum_{i=1}^{3} m_{\nu_i} V_{ei}^2 | \simeq | m_{\nu_1} c_{12}^2 + m_{\nu_2} s_{12}^2 \exp i2\alpha |, \]  

which can be measured in neutrinoless double β decay experiments. (α is given in (29).)

In fig. 4 we plot < m_{ee} > as a function of sin φν. As we can see from fig. 4, the effective Majorana mass stays at about its minimal value < m_{ee} >_{min} for a wide range of sin φν. Since < m_{ee} >_{min} is approximately equal to m_{\nu_2,min} (which is given in (25)), it is consistent with recent experiments [29, 30] and is within an accessible range of future experiments [31]. An experimental verification of (20), (21) and (27)–(33) would strongly indicate the existence of the smallest nonabelian symmetry based on the permutation group S3 along with an abelian discrete symmetry Z_N at the electroweak scale, where Z_N is only an approximate symmetry of the whole theory, but the effect of its violation is of two-loop order in the leptonic sector.

S3 is obviously a possible answer to the question why there exist three generations of leptons...
Figure 4: The effective Majorana mass $\langle m_{ee} \rangle$ as a function of $\sin \phi_\nu$ with $\sin^2 \theta_{12} = 0.3$ and $\Delta m^2_{21} = 6.9 \times 10^{-5} \text{eV}^2$. The dashed, solid and dot-dashed lines stand for $\Delta m^2_{23} = 1.4, 2.3$ and $3.0 \times 10^{-3} \text{eV}^2$, respectively. The $\Delta m^2_{21}$ dependence is very small.
and quarks. \( S_3 \), of course, cannot explain the hierarchy of the fermion mass spectrum, but \( S_3 \) with \( Z_N \) in the leptonic sector can relate the mass spectrum and mixing in this sector, making testable predictions, which have been re-investigated in the present letter. Therefore, \( S_3 \) solves partially the flavor problem of the SM. Since there are three \( SU(2)_L \) doublet Higgs fields in the model, there exist FCNC processes at the tree level. In [1] the magnitude of various tree level FCNC amplitudes have been estimated, and it has been found that they are sufficiently suppressed. The suppression follows from the smallness of the corresponding Yukawa couplings, where \( S_3 \) plays an important role for that smallness. However, we find that \( \Delta m_K \), the difference of the mass of \( K_L \) and \( K_S \), exceeds the experimental value, unless the mixing of the Higgs fields is fine tuned. This problem is currently under investigation, and we will report the result elsewhere.

It is straightforward to keep the discrete flavor symmetries, \( S_3 \) in the hadronic sector and \( S_3 \times Z_N \) in the leptonic sector, in a supersymmetric extension of the standard model [9]. The supersymmetric flavor problem has been investigated there, and it has been explicitly found that thanks to the flavor symmetries the dangerous FCNC and CP violating processes, that originate from soft supersymmetry breaking terms, are sufficiently suppressed, in a similar manner as it was found in [32].

Acknowledgments
This work is supported by the Grants-in-Aid for Scientific Research from the Japan Society for the Promotion of Science (No. 13135210).

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