The analysis of problem based learning implementation and its influence to the students generalization thinking skills on solving r-dynamic vertex coloring

L O Harjito\textsuperscript{1,2}, Dafik\textsuperscript{1,2}, A I Kristiana\textsuperscript{1,3}, I N Maylisa\textsuperscript{1,2} and Y Wangguway\textsuperscript{1,2}
\textsuperscript{1}CEREBEL University of Jember, Indonesia
\textsuperscript{2}Department of Mathematics Education Post Graduate University of Jember, Indonesia
\textsuperscript{3}Department of Mathematics Education University of Jember, Indonesia

Email: lelitaoktafianti@gmail.com

Abstract. In this study, we aim to analyze the generalization thinking skills of students on solving r-dynamic vertex coloring by applying the Problem Based Learning (PBL). This research used a mixed-method, which combines qualitative methods and quantitative methods. The research subjects consisted of two classes, the first class is the control class of 30 students and the second class is an experimental class of 32 students. The instruments used in this research were pre-test, post-test, observation sheet, and interview sheet. Testing the homogeneity of both classes using the results of the pretest were held before starting the research. Homogeneity test results showed the significance probability value was 0.424> 0.05, so the difference in the average of the two classes was not significant. This shows that the two classes are homogeneous or have the same ability. After testing the homogeneity, we also tested normality by using post-test data. It shows that the independent sample test on the post-test data of the control class and the experimental class is obtained that the score of significance probability is 0.00 <0.05 so that it is significant which means that there is a difference between the control class and the experimental class. While students' generalization thinking skills showed that 56% were in the very high category, 23% were in the high category, 13% were in the average category, and 8% were in a low category. The results of the study indicate that there is an influence of the application of the Problem Based Learning (PBL) in improving students' generalization thinking skills in on solving of r-dynamic vertex coloring.

1. Introduction
Mathematics can be used to solve problems both in the field of mathematics itself and in daily life's problem. To improve the quality of education, especially in the field of mathematics, educators are required to always improve themselves both in the knowledge of mathematics and the management of teaching and learning as well as the source of teaching materials that will be given to students.

One of the ability that must be possessed in the process of learning mathematics is reasoning skill. Improving students' reasoning abilities can be done by developing their generalization skills. Generalization skills are a process of concluding the problem in the general way to simplify the problem. This skill can help students in solving problems especially in the field of discrete mathematics. Generalization is a process of formulating a conjecture and note the order [1].

According to Mason [2] there are 4 indicators in generalization skills, namely: (a) perception of generality, this indicator shows the ability of students when agreed to paired or patterned. Students can
also solve or overcome patterns. They know what can be overcome by using rules or patterns, (b) expression of generality, in this indicator students can use the results of determining patterns to determine structure / data. In this indicator, students can also choose rules or patterns, (c) symbolic expression of generality, these students have succeeded in making general rules and patterns and can formulate numerical or verbal publications, and (d) manipulation of generality, in this indicator students have been able to use the results of generalizations to solve problems and have solved problem patterns.

In this research based on the generalization thinking skills that have been explained, we develop the skills to think generalizations based on solving r-dynamic vertex coloring. Indicators and sub-indicators of generalization thinking skills can be seen on Table 1

| Indicator                          | Sub-indicator                                                                 |
|------------------------------------|-------------------------------------------------------------------------------|
| Perseption of generality           | a. Student is can to recognize a graph                                        |
|                                    | b. Student is can to give notation to graph                                   |
| Expression of generality           | a. Student is can to calculate the cardinality of a graph                     |
|                                    | b. Student is can to formulate the generality of the cardinality calculation results of a graph |
| Symbolic expression of generality  | a. Student is can to label the point of a graph                               |
| Manipulation of generality         | a. Student is can to find the chromatic number of a graph                     |

The possibility of generalization thinking must be conveyed to students with the help of learning models. The learning model used is PBL. PBL learning model is a learning strategy that uses real-world problems as a context for students to learn about critical thinking and problem-solving skills, as well as to gain knowledge and concepts from the material being studied. PBL is a learning model that uses irregular open questions and invites students to be active in learning [3]. PBL has been used in a variety of subjects to solve problems and think critically in authentic learning [4].

Andini, et al [5] states that there are several characteristics of the PBL model, namely: (a) asking questions or problems, (b) focusing on knowledge, (c) actual investigations, (d) students’ work in the form of artifacts and exhibits, and (e) collaboration. The PBL syntax has 5 phases, namely: (1) directing students to the problem, (2) organizing students to learn, (3) helping independent and group investigations, (4) developing and presenting artifacts and exhibits, and (5) analyze and evaluate the problem-solving process.

Learning by using PBL aims to develop social skills or collaborative skills of students through discussion activities. These skills are in the form of collaboration, interpersonal, and active roles in group success that can be assessed through observation activities. Learning will focus on discrete modeling of r-dynamic vertex coloring material. this material is from graph theory material which will later become a parameter of students’ generalization thinking skills. Based on this explanation, the researcher wants to analyze the application of PBL and its effect on the generalization skills of students by solving r-dynamic vertex coloring problems. The purpose of this study was to determine the effect of the PBL on student generalization skills in the study of r-dynamic vertex coloring, where r-dynamic vertex coloring was a problem.

**Definition 1.** $G = (V, E)$ is a simple graph. The vertex set and the edge set $G$ are given as $V(G)$ and $E(G)$, respectively. Therefore, the graphs $\Delta(G)$ represent the maximum degrees and $\delta(G)$ represent the minimum degrees. For each $v \in V(G)$, $d(v)$ represent the degree of $v$, $N(v)$ represent the relative set of $v$, and $c(v)$ represent the color of $v$. Move the corresponding code from $G$ by $k$ color is the destination of $c: V(G) \rightarrow \{1, 2, ..., k\}$ with this character: if $u, v \in V(G)$ and $u, v \in E(G)$ are adjacent, then $c(u)$ and $c(v)$ are different. r-dynamic coloring of graph $G$, introduced by Montgomery
exactly $k$ implementation $G$ coloring of graph for each of $v$ expected only $\min\{r, d(v)\}$ different color. The chromatic number of $r$-dynamic, $\chi_r(G)$ is the minimum $k$ so the graph $G$ has $r$-dynamic $k$ colors. Useful research for our research, submitted by Montgomery [6]. Let the maximum degree of graph $G$. It holds the coloring of $r$ on graph $G$ is bigger and equal to minimum between the $r$ and the maximum degree on the graph $G$ and then to add 1. The $r$-dynamic chromatic number has been inquired by several authors, it can be seen in [7, 8, 9, 10, 11, 12, 13, 14, 15].

2. Research methods
This type of research used in this study is a mixed method. The mixed method is a research method that combines qualitative and quantitative methods. With this combination, it can complement the weaknesses in qualitative and quantitative methods. In this study, quantitative data is the main data obtained from tests arranged based on indicators of student generalization skills in solving $r$-dynamic vertex coloring problems in graphs. While qualitative data is supporting data obtained from the results of student interviews to illustrate a portrait of students' thinking phases. The subjects of this study were students in semester 3 of the Mathematics Education study program, FKIP University of Jember. This research was conducted with 32 experimental class students and 30 control class students. Then both classes were given a pretest and posttest that had been arranged based on generalization skills indicators.

The research design used is in the form of non equivalent control group design. Both groups were given different treatments. The control class is applied conventional learning methods and in the experimental class the PBL is applied.

| Class    | Pretest | Treatment | Postest |
|----------|---------|-----------|---------|
| Experiment N=32 | R₁ | PBL       | R₂ |
| Control N=30   | R₃     | Conventional | R₄ |

2.1 Population
The population in this study is the third semester students of the Mathematics Education study program, FKIP Jember University. Samples were randomly selected in 2 classes consisting of 32 experimental classes and 30 student control classes. The experimental class is given the implementation of PBL, while the control class is given the implementation of the conventional learning model.

2.2 Instruments
The instruments used in this study were pretest, posttest, observation, and interview. On the grading scale in the pretest and post-test interval 0-100 was used. Meanwhile, the observation and interview sheets are used in the rating scale or interval 0-4 that has been validated by experts. In this study consists of three stages in the research design, namely a preliminary study (qualitative research), analysis of student generalization skills and the application of PBL (quantitative research), portrait phase (qualitative research). A description of the research procedure is illustrated in the following chart.
2.3 Task
Student generalization skills are measured based on indicators that have been converted into test instruments. One of the test instruments used is to discuss r-dynamic vertex coloring. For each \( v \in V(G), d(v) \) represent the degree of \( v \), \( N(v) \) represent the relative set of \( V \), and \( c(v) \) represent the color of \( v \). Move the corresponding code from \( G \) by \( k \) color is the destination of \( c: V(G) \rightarrow \{1,2,\ldots,k\} \) with this character: if \( u,v \in V(G) \) and \( u,v \in E(G) \) are adjacent, then \( c(u) \) and \( c(v) \) are different. r-
dynamic coloring of graph $G$, exactly $k$ implementation $G$ coloring of graph for each of $v$ expected only $\min\{r, d(v)\}$ different color. The chromatic number of $r$-dynamic, $\chi_r(G)$ is the minimum $k$ so the graph $G$ has $r$-dynamic $k$ colors. Useful research for our research. Let the maximum degree of graph $G$. It holds the coloring of $r$ on graph $G$ is bigger and equal to minimum between the $r$ and the maximum degree on the graph $G$ and then to add 1. The Graph used is the $L_{on}(2,1)$ lobster graph. The following are the stages of completion of the $r$-dynamic vertex coloring on $L_{on}(2,1)$.

Giving notation to vertices on lobster graph $L_{on}(2,1)$

The $r$-dynamic vertex coloring for $r = 1$

$$|c(N(a_1))| \geq \min\{r, d(a_1)\}$$

$$1 \geq \{1,4\}$$

Giving color on $a_1$, with color 1 and each neighborhood's vertex must have a different color. So that it meet the color requirement of $r = 1$.

The $r$-dynamic vertex coloring for $r = 2$

$$|c(N(a_1))| \geq \min\{r, d(a_1)\}$$

$$2 \geq \{2,4\}$$

Giving color on $a_1$, with color 1 and each two neighborhood's vertices must have a two different colors. So that it meet the color requirement of $r = 2$.

The $r$-dynamic vertex coloring for $r = 3$

$$|c(N(a_1))| \geq \min\{r, d(a_1)\}$$

$$3 \geq \{3,4\}$$

Giving color on $a_1$, with color 1 and each three neighborhood's vertices must have a three different colors. So that it meet the color requirement of $r = 3$. 
The r-dynamic vertex coloring for \( r = k \)

\[
|c(N(a_1))| \geq \min\{r, d(a_2)\}
\]

\[
4 \geq \{4,4\}
\]

Giving color on \( a_1 \), with color 2 and each four neighborhood's vertices must have a four different colors. So that it meet the color requirement of \( r = k \).

**Figure 2.** Example of r-dynamic vertex coloring of lobster graph \( Lo_n(2,1) \)

The presentation of r-dynamic vertex coloring is the purpose of this task. Where, each vertex must have a different color so we get the color of each neighborhood vertices are different. The independent sample t-test is the method to test the research hypothesis formulated with a significance level of 5% or 0.05

\( H_0 = \) Generalization thinking skills of students using PBL is less than or equal to generalization thinking skills of students not using PBL.

\( H_1 = \) Generalization thinking skills of students using PBL is greater than to generalization thinking skills of students not using PBL.

Information:
- If \( p_{value} < 0.05 \) then reject \( H_0 \) and accept \( H_1 \)
- If \( p_{value} \geq 0.05 \) then accept \( H_0 \) and reject \( H_1 \)

### 2.4 Data collection and data analysis

In this research, quantitative data analysis uses t-test. The qualitative data uses interviews, observations, and uses ordinal data. In addition, qualitative and quantitative data were also analyzed using descriptive and inferential statistics. Descriptive statistical data obtained from the average value, standard deviation and frequency. Whereas inferential statistical data that calculate with PBL uses the normality test, homogeneity test, and independent test between the control class and the experimental class. Independent samples were used to compare second grade with a significant value of 0.05.

### 3. Research finding

The study was conducted in the experimental class and the control class using qualitative methods to determine student generalization skills. The research was carried out after carrying out the validity and reliability tests of the research instruments. Then the experimental class and the control class were given a pretest to find out the initial abilities of the students' generalization skills.

After conducting a pretest in the control class and the experimental class, learning in the experimental class will be carried out using the PBL learning model, while the control class uses the conventional learning model. Then the data will be obtained which will be analyzed using the SPSS application. Following are the results of data analysis using SPSS and Excel applications.

#### 3.1 Instrument validation

Before showing results, it is necessary to test the reliability and validity of the posttest instrument. The following table shows the reliability and validity of the results.
Based on table 2, it can be seen that the $r_{count}$ value of problem 1 is 0.688, problem 2 is 0.756, problem 3 is 0.512, problem 4 is 0.500, and problem 5 is 0.642. All items produce the value $r_{count} > r_{table}$ (0.3610) with $df = N-2 = 30-2 = 28$, so all items are valid.

Table 4. The Question Reliability Test Results

| Cronbach's Alpha | N of Items |
|------------------|------------|
| .592             | 5          |

Based on table 4, it can be seen that the overall reliability value is 0.592 and $r_{table}$ with the terms of significance 5% $df = N-2 = 30-2 = 28$, $r_{table} = 0.3610$, therefore $r_{count} > r_{table}$. This concludes that the instrument items are reliable.

3.2 Results

Initial research was conducted on 30 students in the control class to determine the level of student generalization skills. Thirty students were tested with a pretest (diagram 1), in the control class it was found that 7% were in the very high category, 13% were in the high category, 23% were in the average category, and 57% were in the low category in student generalization skills.
Diagram 1. The Distribution of Pretest of Student Generalization Skills in the Control Class

![Pie chart showing the distribution of pretest results in the control class.]

The study was conducted on 32 students in the experimental class to determine the level of student generalization skills. 32 students were tested with a pretest (diagram 2), in the experimental class it was found that 7% were in the very high category, 15% were in the high category, 30% were in the average category, and 48% were in the low category in student generalization skills.

Diagram 2. The Distribution of Pretest of Student Generalization Skills in Experimental Class

![Pie chart showing the distribution of pretest results in the experimental class.]

The next step is to analyze the data obtained from the pretest and posttest using SPSS. Data analysis that was carried out was quantitative method. Statistical tests conducted in second grade are normality test, homogeneity test, and independent test. The first step in the analysis using SPSS is the homogeneity of the second class to ask whether the ability of the second class is the same or not. Based on table 5, the homogeneity test gets sig results. 0.424. This is significant if it is greater than 0.05 (based on mean = 0.424 > 0.05), so that the pretest data variance of the control and experimental class is homogeneous.

Table 5 The Pretest Homogeneity Test Results of Control and Experiment Class

| Levene Statistic | df1 | df2 | Sig.   |
|------------------|-----|-----|--------|
| .648             | 1   | 60  | .424   |

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After that will be followed by an analysis of normality test. This test is conducted to determine whether the distribution of pretest data in the control class and experimental class is normally distributed or not. Data distribution will be said to be significant if the value is greater or equal to 0.05. Based on table 5 shows the significant value of the experimental class is 0.085 ≥0.05 and the control class is 0.053 ≥0.05. So this means that the data from the control class and the experimental class are normally distributed.

| Table 6. Pretest Normality Test Results for the Control and Experiment Class |
|--------------------------------------------------|
| Group    | Kolmogorov-Smirnov<sup>a</sup> | Df | Sig. | Shapiro-Wilk |
| Pretest  |                      |    |      |             |
| Control class | .159 | 30 | .053 | .962 | 30 | .341 |
| Experimental class | .145 | 32 | .085 | .965 | 32 | .374 |

Then proceed with data analysis using quantitative statistics to find differences in learning outcomes with the PBL. Data analysis using SPSS applications with pretest learning outcomes data.

| Table 7. The Average Test Results for Pretest of Control and Experiment Class |
|--------------------------------------------------|
| Group    | N     | Mean   | Std. Deviation | Std. Error Mean |
| Pretest  |       |        |                |                 |
| Control class | 30 | 59.8333 | 8.17938 | 1.49334 |
| Experimental class | 32 | 62.8750 | 8.42328 | 1.48904 |

Based on Table 7 the results of statistical tests show the average of each control class group is 59.83 and the experimental class is 62.87 where the average control class is lower than the experimental class. Then proceed with independent t test in both classes. Samples are said to be significant if the Sig. (2-tailed) is greater than 0.05.

| Table 8. The Independent Pretest Test Results for the Control and Experiment Class |
|--------------------------------------------------|
| Group    | Levene's Test for Equality of Variances | t-test for Equality of Means | 95% Confidence Interval of the Difference |
| Pretest  | Equal variances assumed |                      |                                                         |
|          | F      | Sig.  | T    | df  | Sig. (2-tailed) | Mean Difference | Std. Error Difference | Lower | Upper |
|          | .648   | .424  | -1.441 | 60  | .155 | -3.04167 | 2.11090 | -7.26409 | 1.18076 |
|          | Equal variances not assumed |                      |                                                         |
|          | -1.442 | 59.921 | .154  | -3.04167 | 2.10887 | -7.26014 | 1.17681 |

Based on Table 8, the Sig. (2-tailed) is 0.155 > 0.05, so Ho is accepted, which means there is no significant difference (real) in the pretest mean values of the control class and the experimental class. The research then continued with implementing learning using conventional learning models in the
control class and PBL in the experimental class, then continued with the posttest at the end of learning. The study was conducted on 30 students in the control class to determine the level of generalization skills after learning. Thirty subjects were tested by posttest (diagram 3), in the control class it was found that 31% were in the very high category, 25% were in the high category, 27% were in the average category, and 17% were in the low category in student generalization skills.

Diagram 3. The Distribution of Posttest of Student Generalization Skills in the Control Class

Furthermore, research was conducted on 32 students in the experimental class to determine the level of generalization skills of students after applying the PBL. 32 students were tested by posttest (diagram 4), in the experimental class it was found that 56% were in the very high category, 23% were in the high category, 13% were in the average category, and 8% were in the low category in the student generalization skills.

Diagram 4. The Distribution of Posttest of Student Generalization Skills in Experimental Class

After the learning process in the control class and the experimental class, the data obtained from the post-test results were analyzed using the normality test, homogeneity test, and independent test. The first step in the analysis using SPSS is the homogeneity test of the two classes to find out whether the abilities of the two classes are the same or not. Based on table 9, homogeneity tests get sig results.
0.096. This is significant if it is greater than 0.05 (based on mean = 0.096> 0.05), so the variance of pretest data from the control and experimental classes is homogeneous.

**Table 9. Homogeneity Test Results of the Control and Experiment Class**

| Levene Statistic | df1 | df2 | Sig. |
|------------------|-----|-----|------|
| 2.856            | 1   | 60  | .096 |

After that will be followed by an analysis of normality test. This test is conducted to determine whether the posttest data distribution in the control class and experimental class is normally distributed or not. Data distribution will be said to be significant if the value is greater or equal to 0.05. Based on table 10 shows the significant value of the experimental class is 0.200 ≥0.05 and the control class is 0.200 ≥0.05. So this means that the data from the control class and the experimental class are normally distributed.

**Table 10. Test Results for Postt Normality Control and Experiment Class**

| Group            | Kolmogorov-Smirnov<sup>a</sup> | Shapiro-Wilk          |
|------------------|---------------------------------|-----------------------|
|                  | Statistic | Df | Sig. | Statistic | Df | Sig. |
| Posttest         |           |    |      |           |    |      |
| Control class    | .070      | 30 | .200<sup>+</sup> | .957 | 30 | .266 |
| Experimental class | .120 | 32 | .200<sup>+</sup> | .952 | 32 | .162 |

The next step is data analysis using quantitative statistics to find differences in learning outcomes with the PBL. Data analysis using SPSS applications with posttest learning outcome data.

**Table 11. Average Test Results of Posttest Control and Experiment Class**

| Group         | N  | Mean   | Std. Deviation | Std. Error Mean |
|---------------|----|--------|----------------|-----------------|
| Posttest      |    |        |                |                 |
| Kelas Kontrol| 30 | 74.500 | 8.80341        | 1.60728         |
| Control class | 32 | 85.625 | 6.76685        | 1.19622         |

The posttest average test results in table 11 show that the average of each control class group is 74.50 and the experimental class is 85.62 where the average control class is lower than the experimental class. Then proceed with independent t test in both classes. Samples are said to be significant if the Sig. (2-tailed) is greater than 0.05.

**Table 12. Independent Posttest Test Results of Control and Experiment Class**

| Levene's Test for Equality of Variances | t-test for Equality of Means |
|----------------------------------------|-----------------------------|
|                                        | F   | Sig. | t  | Df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | Lower  | Upper |
| Posttest Equal variances assumed     | 2.856 | .096 | -5.600 | 60 | .000 | -11.125 | 1.98674 | -15.099 | -7.1509 |
| Equal variances not assumed          | -5.553 | 54.408 | .000 | -11.125 | 2.00357 | -15.141 | -7.10878 |
The posttest independent test results in the control class and the experimental class in table 1 show that the Sig. (2-tailed) is 0.000 <0.05, then Ho is rejected, which means there is a significant (real) difference in the posttest average value of the control class and the experimental class.

The next analysis is the analysis of the distribution of observations from the PBL conducted in the experimental class with 32 students. Based on diagram 5, it was found that 59% of students were very active in PBL learning, 23% of students were active in PBL learning, 12% of students were quite active in PBL learning, 5% of students were not active in PBL learning, and 1% of students were very active in learning PBL. So the PBL can give effect to work in solving r-dynamic vertex coloring problems.

**Diagram 5** The Observation of Result Distribution of All Subject in the Experimental Class

| Distribution of Student Activities in Implementation PBL |
|----------------------------------------------------------|
| Indicators                                              |
| Orient students to problem                              |
| Organizing students to learn                             |
| Guiding individual and group investigations              |
| Develop and present the work                             |
| Analyze and evaluate the problem solving process         |
| Indicators                                              |
| Very active                                            | 27 | 22 | 22 | 17 | 13 |
| Active                                                 | 2  | 5  | 6  | 5  | 8  |
| Hesitate                                               | 2  | 4  | 2  | 7  | 6  |
| Inactive                                               | 1  | 1  | 2  | 2  | 4  |
| Very inactive                                          | 0  | 0  | 0  | 1  | 1  |

3.3 Phase portrait

The phase portrait was taken to draw a picture of the students' thinking skills process. 3 subjects were selected from the experimental class and the control class according to the post est results. Interviews were conducted on subjects that have been selected to determine student generalization skills in completing r-dynamic vertex coloring.

The analysis of the work was done to know the process of students 'completion worksheets in producing the final results, to strengthen the solution of these students' worksheets, the results of the worksheets related to the interview and the data of observation.

After students solve problems related to r-dynamic vertex coloring, researchers conduct interviews for mind mapping in solving problems about r-dynamic vertex coloring. This interview aims to find out the student's thinking framework when solving the r-dynamic vertex coloring problem.
Subject 1 interview results:

Researcher : After reading this issue, what did you understand? Or what is the first thing that comes to mind?:

Student : In this problem ordered to find chromatic numbers or minimum color graph cycle with \( n = 6 \).

Researcher : Can you finish it? And do you know what this graph is?

Student : Yes I know this is a cycle graph with \( n = 6 \).

Researcher : Then are you able to give the graph notation?

Student : Yes, I give a notation \( x_1, x_2, \ldots, x_6 \) on the graph.

Researcher : Can you calculate the cardinality too?

Student : Yes, I can calculate and determine the cardinality formula. I can also formulate general form in cardinality calculation.

Researcher : Can you give labels or colors for \( r = 1 \) and \( r = 2 \) on the graph?

Student : Yes, I can give colors for each \( r = 1 \) and \( r = 2 \).

Researcher : Can you find the chromatic numbers?

Student : Yes I can, by looking back at the general requirements of \( r \)-dynamic vertex coloring.

Figure 3. The result of student work on very high level

Figure 4. Phase portrait of a student work on high level

Figure 4 shows the process of student generalization skills in solving \( r \)-dynamic vertex coloring problems in graphs. In step 1a to stage 1b, student 3 is able to recognize a circle graph with \( n = 6 \) and is able to represent the graph. Then student 2 proceeds to stages 2a and 2b. Student 3 has been able to calculate and determine the cardinality formula of the graph. It also has been able to formulate the cardinality formula into general calculations. The next step to 3a, namely student 3 has been able to label points or coloration for \( r = 1 \) and \( r = 2 \). Then finally to stage 4a, student 3 is able to calculate and find the chromatic number of the cycle graph.
Subject 2 interview results:

Researcher: After reading this issue, what did you understand? Or what is the first thing that comes to mind?

Student: In this problem ordered to find chromatic numbers or the minimum color of the cycle graph.

Researcher: Can you finish it? And do you know this graph, what's its name?

Student: Yes I know this is a cycle graph with \( n \) = 6.

Researcher: Then are you able to give the graph notation?

Student: I give a notation on the graph.

Researcher: Can you calculate the cardinality too?

Student: Yes, I can calculate and determine the cardinality formula. I can also formulate general form in cardinality calculation.

Researcher: Can you give labels or colors for \( r = 1 \) and \( r = 2 \) on the graph?

Student: You can, but it's a bit difficult so I have to look again at the initial graph while still notation.

Researcher: Can you find the chromatic numbers?

Student: I can't yet ma'am, just stop until the labeling of the points.

Figure 5. The result of student work on high level

Figure 6. Phase portrait of a student work on high level

Figure 6 shows the process of student generalization skills in solving r-dynamic vertex coloring problems in graphs. In step 1a to stage 1b, student 3 is able to recognize a circle graph with \( n = 6 \) and is able to represent the graph. Then student 2 proceeds to stages 2a and 2b. Student 3 has been able to
calculate and determine the cardinality formula of the graph. It also has been able to formulate the cardinality formula into general calculations. The next step to 3a, namely student 3 has been able to give a point label or coloring for $r = 1$ and $r = 2$ but still have a little difficulty so they have to look again at the initial graph when it is given a notation only. Student 3 only stops at stage 3a, not yet able to continue to 4a. This means student 3 did not find the chromatic number.

Subject 3 interview results:

Researcher : After reading this issue, what did you understand? Or what is the first thing that comes to mind?
Student : On this issue ordered to find said chromatic.
Researcher : Can you finish it? And do you know this graph, what's its name?
Student : Yes I can, but a little confused. This is a cycle graph with $n = 6$.
Researcher : Then are you able to give the graph notation?
Student : Yes, I can give it a notation $x_1, x_2, x_3, x_4, x_5, x_6$
Researcher : Can you calculate the cardinality too?
Student : Yes I can calculate and determine the cardinality formula. But it was a little difficult to formulate it in calculations, so I tried again to the stage of determining cardinality.
Researcher : What other difficulties are you experiencing?
Student : I also can not label or color on the graph. So I haven't been able to find the chromatic numbers yet.

Figure 7. The result of student work on average level

Figure 8. Phase potrait of a student work on average level

Figure 8 shows the process of student generalization skills in solving r-dynamic vertex coloring problems in graphs. In step 1a to stage 1b, student 2 is able to recognize a circle graph with $n = 6$ and is able to represent the graph. Then student 2 proceeds to stages 2a and 2b. Student 2 has been able to calculate and determine the cardinality formula of the graph. It also has been able to formulate the cardinality formula into general calculations. But student 2 is still unable to label or color, so it has not been able to find the chromatic numbers.
Subject 4 interview results:
Researcher : After reading this issue, what did you understand? Or what is the first thing that comes to mind?
Student : Initially I did not understand. But after reading the instructions on the problem, I understood. The problem that I have to solve is finding chromatic numbers in the graph.
Researcher : Can you finish it? And do you know this graph, what's its name?
Student : Yes I can, but a little confused. This is a cycle graph.
Researcher : Then are you able to give the graph notation?
Student : Yes, I'll give a notation starting from $x_1, x_2, x_3, x_4, x_5, x_6$
Researcher : Can you calculate the cardinality too?
Student : Yes, but there are difficulties
Researcher : What difficulties are you experiencing?
Student : I can only determine the final cardinality, I can't find a way to find it. Because by chance the graph is easy so the final cardinality is easy
Researcher : Are there any more difficulties besides that?
Student : I also can not label or color on the graph. So I haven't been able to find the chromatic numbers yet.

Figure 9. The result of student work on low level

Figure 10 shows the process of student generalization skills in solving r-dynamic vertex coloring problems in graphs. In step 1a to stage 1b, student 1 is able to recognize a circle graph and is able to symbolize the graph. But student 1 does not go to stage 2a, but jumps from 1b to 2b. Student 1 also cannot label or color the graph, and cannot find the chromatic number.

4. Discussion
This study was conducted aiming to determine the effect of generalization skills in the implementation of PBL. Generalization Skills have 4 indicators, namely (a) perception of generality, this indicator shows the ability of students when agreed to paired or patterned. Students can also solve or overcome
patterns. They know what can be overcome by using rules or patterns, (b) expression of generality, in this indicator students can use the results of determining patterns to determine structure / data. In this indicator, students can also choose rules or patterns, (c) symbolic expression of generality, these students have succeeded in making general rules and patterns and can formulate numerical or verbal publications, and (d) manipulation of generality, in this indicator students have been able to use the results of generalizations to solve problems and have solved problem patterns. The subjects used were 30 students from the control class and 32 students in the experimental class. The implementation of the problem-based learning method is done in the experimental class. It is known that from the results of data analysis in the experimental class there are good effects in conducting learning methods problem-based learning. There is also an effect of generalization skills and the effect is significant because students are more effective in the learning process.

The results of the independent sample t-test on the pre-test questions in the control and experimental class were obtained by Sig. (2-tailed) is 0.155 and 0.155 > 0.05 in the condition if Sig (2-tailed) > 0.05, then there is no significant difference between the learning outcomes of the control class and the experimental class. While in the post-test results, independent tests were obtained in the control class and the experimental class with Sig. (2-tailed) is 0.00 <0.05 so that it gets a significant value and can prove a second class that is different in terms of student achievement tests after applying PBL.

While students' generalization thinking skills showed that 56% were in the very high category, 23% were in the high category, 13% were in the average category, and 8% were in a low category. This study uses a PBL to see its effect on student generalization skills so that students can effectively carry out the learning process. PBL affects the student's learning process. For this research, it is expected that the results of the implementation of PBL can improve deductive reasoning, discipline, and effective student learning.

This results is closed to the result of Dini, et.al [16] showing the awareness of the impact of generalization thinking skills on academic achievement. This shows that when we can improve students' generalization thinking skills, it will imply an increase in student academic achievement. Furthermore, this result is closed to the of Septory, et.al [14] show that PBL is applied in the study of r-dynamic vertex coloring and can improve combinatorial thinking skills.

5. Conclusion
Research that has been done shows that the application of the PBL has a significant influence on the generalization skills of students in the experimental class. Students in the experimental class showed positive generalization skills compared to the control class. The results showed that an increase in student learning outcomes and generalization skills were seen from the post-test. We can claim that PBL implementation has a significant effect on students' generalization skills.

The portrait of the student phase is an illustration that explains the flow of students' generalization skills in completing r-dynamic vertex coloring studies. It can be known the difference students’ way of thinking in the control class has a relatively lower flow than the experimental class.

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