Factorization for Hard Exclusive Electroproduction

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This talk summarized the proof of hard-scattering factorization for hard exclusive electroproduction processes: deeply virtual Compton scattering and exclusive meson production.

1 Introduction

The processes discussed in this talk were $ep \rightarrow eM p$, $ep \rightarrow e\gamma p$, and $ep \rightarrow e\mu^+\mu^- p$, all in the deep-inelastic region where $Q$ is large. The factorization theorems that I will discuss provide a sound basis for the phenomenology of these processes, which was discussed extensively at this workshop. They express the amplitudes for the processes in terms of operator matrix elements and perturbatively calculable coefficient functions.

As always, the fundamental problem is that we do not know how to solve QCD exactly, and we must appeal to approximations. In high energy processes, such as we consider here, large ranges of scales and of rapidities are important. The factorization theorems demonstrate that enormous simplifications occur in suitable asymptotic limits, and then enable calculations to be done in perturbation theory, where asymptotic freedom implies that hard scattering coefficients etc. can be usefully approximated by low-order Feynman graph calculations.

My presentation of the arguments tends to be graphical, since I find this is the easiest way to exhibit their structure. However they correspond to quantitative mathematical proofs. The principles are quite general; the processes under discussion turn out to provide the simplest situations for explaining how to prove factorization theorems. Although the proofs are apparently based on Feynman graph arguments, some of the key elements are actually non-perturbative, as I will emphasize.

2 The simplest case: muon-pair production

The simplest case is the deep-inelastic production of high-mass muon pairs: $ep \rightarrow e\mu^+\mu^- p$, for which the kinematic variables are defined in Fig. 1. The asymptotics can be treated by the simplest version of the light-cone expansion

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However, I will explain methods that generalize to more complicated processes, such as meson production. Other relevant references are [4, 5].

From the momenta defined in Fig. 1, we define a set of 4 scalar variables to specify the kinematics of the process by: $Q = \sqrt{-q^2}$, $\alpha = r^2/Q^2$, $x = Q^2/2p\cdot q$, and $t = (p - p')^2$. We consider the process at large $Q$, with $x$, $\alpha$ and $t$ fixed. Then the leading-power part of the amplitude is given by regions symbolized in Fig. 2. There the lines in the upper bubble, $H$, have large virtualities, of order $Q^2$, and the lines in the lower bubble are approximately collinear to the proton and have relatively low virtualities. Power-counting arguments [3, 4] show that only two lines connect the two bubbles, except that extra longitudinally polarized gluon lines can join the two bubbles; these can be eliminated by a gauge transformation.

The methods for the power-counting can be reduced [3] to dimensional analysis for the finite-order graphs used for calculating $H$, together with an analysis of the behavior of the meson and proton subgraphs under boosts from the rest frames of the hadrons. Thus the methods are actually non-perturbative as regards the parts of the amplitude that are fundamentally non-perturbative.

An expansion of $H$ in powers of small parameters (relative to $Q$) gives the factorization theorem. This expansion is conveniently performed with the aid of light-front coordinates ($V^\pm = (V^0 \mp V^z)/\sqrt{2}$), so that

\[
\begin{align*}
(p^+, p^-, p_T) &= (p^+, \text{ small}, 0_T), \\
q^\mu &= (-xp^+, Q^2/2xp^+, 0_T), \\
r^\mu &= (\alpha xp^+, Q^2/2xp^+, 0_T) + \text{ small}, \\
p'^\mu &= (p^+(1 - x - \alpha x), \text{ small}, \text{ small}).
\end{align*}
\]

The fractional momenta of the lines joining the two bubbles in Fig. 2 are $x_1$ and $x_2$, and they obey $x_1 - x_2 = x(1 + \alpha)$. The small variables in Eq. (1) are small compared with $Q$, and the variables $x$ and $\alpha$ have also gotten redefined.

Figure 1: Deeply virtual muon-pair production.
Figure 2: Leading regions for deeply virtual muon-pair production.
by a fraction of order $M^2/Q^2$ to give the formulae for $q^\mu$ and $r^\mu$.

A factorization theorem results after one sums over all possibilities for the graphs and after one neglects relatively small external momentum components for $H$:

$$A = \int d\xi \sum_i H_i(Q^2, x/\xi, \alpha) f_{i/p}(\xi, \xi - x(1 + \alpha)) + \text{non-leading power.} \quad (2)$$

where the momentum fractions have been rewritten as $x_1 = \xi$ and $x_2 = \xi - x(1 + \alpha)$. The sum is over the flavors of the partons joining the two bubbles.

In this equation, there is implicitly the usual DGLAP scale-dependence for the hard scattering and for the skewed parton densities. To get a valid perturbative calculation of the hard scattering coefficient $H_i$ one should choose the renormalization and factorization scale $\mu_{\overline{MS}}$ to be of the order of a typical scale of transverse momentum in $H$, i.e., of order $Q$. The hard scattering can be expanded in Feynman graphs for muon production from a quark or gluon target, but the higher order terms have subtractions. The subtractions have two effects: they prevent double counting of contributions to the amplitude from different orders of perturbation theory, and they cancel the collinear divergences in the unsubtracted graphs.

The skewed parton densities are matrix elements of suitable non-local gauge-invariant operators, whose details are in Refs.\cite{2,3,4}. Taking into account the dependence on the polarization of the proton state, Ji \cite{5} decomposed this parton density into scalar form factors. There are a number of different conventions on the kinematic variables and the normalization for the parton densities, but the details are not important for my purposes.

It is important that there exists an operator definition of the parton densities: (a) it permits a precise definition of the non-perturbative quantities involved, and (b) it gives a definite link to non-perturbative physics. Positivity constraints have been derived for the parton densities in Ref.\cite{7}.

Since at fixed $Q$ and $t$, both the amplitude and the skewed parton density depend on two scalar variables, one can in principle unfold the skewed parton density from the amplitude, up to the usual issue of flavor dependence. This is unlike the case of deeply virtual Compton scattering or meson production, where the amplitude depends on one variable, and the unfolding is much problematic (pace Freund \cite{8}).

Finally, conformal symmetry, exactly valid at lowest order, gives great simplification in the calculation of the DGLAP evolution kernels for skewed parton distributions \cite{9}. 

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3 Deeply virtual Compton Scattering

Deeply virtual Compton scattering \((\gamma^* + p \rightarrow \gamma p)\), is intermediate between the case of high-mass muon-pair production that was treated in the previous section and deeply inelastic meson production to be treated in the next section, so I will not cover it explicitly. See Refs. [4–6].

4 Meson production

The case of meson production \(e + p \rightarrow e + M + p\), needs a generalization of the methods used for muon pair production. No longer does a simple light-cone expansion work, and one must use the methods summarized in Sec. 2. The reason for this is that there are two hadrons in the final state and therefore the factorization theorem not only separates the short-distance part of the scattering from the long-distance part, but also it gives separate factors for each of the two hadrons. The original discussions were for vector mesons only \((\rho, \omega, \text{etc.})\), but Ref. [3] showed that the production of other mesons, like the \(\pi\), also obey a factorization theorem.

4.1 Momentum regions

![Figure 3: The two kinds of leading region for meson production. In (a), the soft subgraph is optional. In both graphs, there may be extra gluons connecting the subgraphs A and B to S and H.](image)

The usual power-counting techniques tell us that the leading power contributions to the amplitude come from regions symbolized by Fig. 3, which has groups of lines that are: collinear to the target proton, collinear to the meson, soft, and hard. The possible soft lines joining the two collinear subgraphs are a new feature of this process compared with muon-pair production. Roughly, one
can characterize the sizes of the momenta in the various subgraphs associated with the leading regions by:

\[ H : (O(Q), O(Q), O(Q)) \]
proton : \( (O(Q), O(M^2/Q), O(M)) \)
meson : \( (O(M^2/Q), O(Q), O(M)) \)
soft : small compared with \( Q \).

There are two kinds of potentially leading region:

- Fig. 3(a), where two lines connect each of the meson and proton subgraphs to the hard scattering, and only gluons connect the soft subgraph to the meson and proton subgraphs.

- Fig. 3(b), where one quark connects each of the meson and proton subgraphs to the hard scattering, and where one (anti)quark plus any number of gluons connect the soft subgraph to the meson and proton subgraphs.

Relative to the first kind of region, the second kind gets an enhancement because of the lower number of external lines of the hard scattering and it gets a suppression because the external lines of the soft subgraph are quarks. Which kind of region actually is most important depends on the polarization of the meson. It turns out [3, 10], by a quite non-trivial argument, that the leading-most power only occurs for longitudinally polarized virtual photons making either pseudo-scalar mesons or longitudinally polarized vector mesons, and that only regions of type (a) contribute.

It is useful to observe that any of the regions involved can be interpreted in terms of a space-time picture, Fig. 4. The incoming and outgoing hadrons move in almost light-like directions, and the hard scattering is localized close to the intersection of the world-lines of the external hadrons. What will be important for our treatment of the soft subgraph in the next section is that the lines collinear to the meson are created at the hard scattering. This implies that the soft interactions happen in the final state, and not in the initial state.
### 4.2 Analyticity and causality

Consider a region of the form of Fig. 3(b). The line coupling the hard scattering to the final-state meson has a factor

\[
\frac{1}{(V - k)^2 - m^2 + i\epsilon} \simeq \frac{1}{V^2 - 2V - k^+ - k_T^2 - 2V_T \cdot \mathbf{k}_T - m^2 + i\epsilon}, \quad (4)
\]

where the approximation is valid to the leading power of \(Q\) given that the integration momentum \(k\) is soft. Now there are two cases to consider for the relative sizes of the components of \(k^\mu\):

(i) The longitudinal components are small: \(k^+ k^- \ll k_T^2\). This is the region that is conventionally associated with rescattering corrections, i.e., small angle scattering of low mass objects.

(ii) The longitudinal and transverse components are comparable: \(k^+ k^- \sim k_T^2\); this is the conventional soft region.

In the first case, the only significant dependence on \(k^+\) is in the propagator \(4\). Hence the \(k^+\) integration can be deformed into the lower half-plane. Then the propagator \(4\) is far off-shell, so that in effect conventional rescattering interactions do not exist. This derivation implements the statement that, as in Fig. 4, the meson only has final-state interactions and that there is no causal connection between the oppositely moving final-state particles. The second case has some line(s) forced off-shell, by order \(QM\), where \(M\) is a typical hadron scale.

Given that case (i) is not important, a further approximation to \(4\) is valid whenever \(k\) is soft:

\[
\frac{1}{(V - k)^2 - m^2 + i\epsilon} \simeq \frac{1}{V^2 - 2V - k^+ - m^2 + i\epsilon} \simeq \frac{1}{-2V - k^+ + i\epsilon}, \quad (5)
\]

which only depends on \(k^+\), and not on \(k_T\). The second half of the approximation depends on the non-perturbative statement that soft lines have virtualities of at least of the order of a hadron mass. The same approximation applies to all the soft lines coupling to the meson subgraph in Fig. 3. The resulting power counting shows \(\delta\) that of the regions in Fig. 3 only (a) is leading: the soft lines connecting to the meson are all gluons and they have a polarization that permits certain Ward identity arguments to be used — see Sec. 4.4. Only the first part of the approximation is needed for the Ward identity arguments; these give the factorization theorem.

The above arguments are quite general. As stated earlier, the results of the power counting encompass relevant non-perturbative effects. In addition,
the analyticity arguments go beyond perturbation theory; they are similar in style to the arguments of DeTar, Ellis and Landshoff [11], and of Landshoff and Polkinghorne [12]. So we should expect that the proof is valid beyond perturbation theory, and, in particular, as regards final-state interactions.

4.3 “Endpoint” contribution

The soft contributions can also be viewed as the endpoint contributions of the situations where lines joining the proton to the hard scattering have small momentum fractions. Hence the treatment of the soft contributions has some notable implications for the analyticity properties of the hard-scattering coefficients — see the papers of Radyushkin [4] and of Collins and Freund [6] for details.

4.4 Actual leading contributions

We have now found that the actual leading-power contributions with a non-trivial soft subgraph have the form of Fig. 2(a), where the only soft lines joining to the meson subgraph are gluons, and where the soft momenta put lines collinear to the meson off-shell by order $QM$. Effectively the soft gluons are on the same footing as the extra collinear longitudinal gluons joining $A$ to $H$.

A Ward identity argument [3] valid to the leading power shows that all effects of these gluons either cancel or are effectively in the skewed parton distributions. This implements the statement that the “color of the meson is zero”. The final result is as if the soft subgraph $S$ is not present, and we obtain a factorization theorem of the form:

$$\text{Amplitude} = H \otimes \text{skewed pdf} \otimes \text{meson distribution amplitude} + \text{non-leading power.}$$

(6)

As usual, the hard scattering $H$ is perturbatively calculable in powers of $\alpha_s(Q)$, the skewed parton distribution is the same as in other processes, and is computed at a scale $Q$ (or of that order), while the distribution amplitude of the meson is the same as in other exclusive processes and is computed at a scale $Q$.

At small $x$ (e.g., for $\rho$ production at the HERA collider) additional steps are used to discuss the Regge limit and to relate the skewed parton density to the ordinary gluon density [13].

For $\pi$ production the skewed parton densities have a relation to the ordinary polarized parton density, since the operators are the same, and there are some interesting selection rules.
5 Summary

The full proof of hard-scattering factorization provides a sound basis for the phenomenology of the processes discussed, since it follows from the QCD Hamiltonian. There are many non-perturbative elements in the proof, so that it goes beyond mere perturbation theory. This includes definite operator definitions of the parton densities.

Part of the theorem is that amplitudes involving transverse polarization are power suppressed. More work is needed here to prove an effective generalized factorization theorem for this case.

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