Numerical simulations of Modified Newtonian Dynamics

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Abstract. The ΛCDM standard cosmological model is strongly supported by multiple lines of evidence, particularly from observations at large scales such as the CMB and large scale structure. There are some indications, however, of problems at smaller scales. An alternative to the CDM approach is to modify the gravitational force, as exemplified by the MOdified Newtonian Dynamics (MOND) idea. While evidence suggests MOND cannot account for dynamics at all scales without dark matter, it has been successful at galactic scales. Due to the complexity of the theory, however, most tests of MOND have extended no further than using a simple scaling relation to determine rotation curves or velocity dispersions. Therefore, to test the concept more thoroughly we require numerical simulations. We discuss the development and testing of a new N-body solver, using two distinct formulations of MOND, that is incorporated into the RAMSES code. The theory of MOND as a modification of Newtonian gravity is briefly summarised. We then show how it is implemented in the code, providing an example of an idealised test case and future applications.

1. Introduction

According to the standard model of cosmology a cold dark matter component comprises around 27% of the energy budget of the Universe. Observational support for this picture includes CMB observations [1], large-scale structure, gravitational lensing and cluster dynamics (see e.g. [2] or [3] for a more recent review and references). At small scales, however, there remains several unresolved problems, such as the missing satellites problem, the cusp/core issue and the predicted existence of satellite halos that are “too big to fail” (see e.g. [4] for a recent review of these issues). Recent observations [5, 6] have also pointed to the existence of large thin rotating disks of satellite galaxies around both the Milky Way and Andromeda, which may be difficult to incorporate into the hierarchically formation mechanism of the standard ΛCDM cosmological model.

An alternative approach, that may alleviate these problems, is to treat the dark matter phenomenology as indicative of a modification of gravity in the very weak field regime. One well-known example of this idea is the MOdified Newtonian Dynamics (MOND) first proposed in [7], in which the standard Newtonian gravitational force is enhanced for accelerations below an empirically determined value of $a_0 \sim 1.2 \times 10^{-10}$ m/s². Applied to galaxy rotation curves this simple idea has been remarkably effective [8]. The proposal has been developed into a full relativistic theory, which is necessary for the construction of a cosmological model, in [9, 10, 11]
among others (see [12] and references therein), although so far none of these theories is known to pass all cosmological tests. Furthermore, it is known that additional unseen matter is required in MOND at cluster scales and beyond [13].

Nevertheless, the success of the MOND approach at galactic scales suggests it is worthwhile to investigate the possible consequences to galaxy dynamics of such a modification of gravity. To investigate such questions we require numerical simulations using a MOND gravitational solver. Several numerical studies of MOND have been undertaken, in particular the pioneering work of [14]. In [15] a MOND N-body code was used to investigate the stability of MOND disks, as well as the behaviour of galaxy mergers. A modification of the cosmological N-body code AMIGA along similar lines is developed in [16]. A publically available code known as NMODY [17] has been available for some time, and has been used to investigate various aspects of stellar dynamics in MOND, such as galaxy mergers and dynamical friction [18, 19]. This code works on a fixed spherical grid, however, making it cumbersome to perform simulations of systems that exhibit little symmetry.

As we wish to study systems covering a wide range of length scales (as necessary to consider satellite galaxies orbiting around hosts, or galaxies falling into clusters, for example) and to include gas physics, we have chosen to modify the powerful N-body/hydrodynamics adaptive mesh refinement code RAMSES [20]. Here we will briefly describe the code itself, which we call “RayMOND,” and we will demonstrate the performance of the code using an idealised test. In the course of developing our code a similar code was developed in [21], although our code uses two formulations of MOND, rather than just one, and a somewhat different numerical treatment. A more thorough description of the code and testing may be found in [22].

2. MOND formulations
Several theories that reproduce the MOND phenomenology in the weak field limit have been proposed over the years. Here we focus on two specific examples, referred to as the AQUAL formulation [23] and the QUMOND formulation [24]. The modified Poisson equation of the AQUAL formulation arises from an quadratic gravitational Lagrangian of the form

$$\mathcal{L}_G = -\frac{1}{8\pi G} a_0^2 F \left( \frac{\nabla \phi^2}{a_0^2} \right)$$

for which the associated modified Poisson equation is

$$\nabla \cdot \left[ \mu \left( \frac{\nabla \phi}{a_0} \right) \nabla \phi \right] = 4\pi G \rho,$$

where $\mu(x) = dF(z)/dz$, $z = x^2$, $\rho$ is the density distribution, $\phi$ is the gravitational potential and $a_0$ is the MOND acceleration scale. The function $\mu(x)$ is the interpolation function, common to all current MOND formulations, that interpolates between the Newtonian regime in the presence of large accelerations, to the deep MOND regime when the accelerations are small. Therefore the form of $\mu(x)$ is only constrained at the limits:

$$\mu(x) \rightarrow 1, \quad x \gg 1$$

$$\mu(x) \rightarrow x, \quad x \ll 1.$$  

Eq. 2 is clearly highly non-linear, requiring the application of particular numerical algorithms, as will be discussed below.

An alternative formulation, more amenable to a numerical treatment, is the quasi-linear QUMOND formulation. In this case the gravitational Lagrangian involves an auxiliary potential
and may be written as

\[ \mathcal{L}_G = -\frac{1}{8\pi G} \left[ 2\nabla \phi \cdot \nabla \phi^N - a_0^2 Q \left( \frac{|\nabla \phi^N|^2}{a_0^2} \right) \right]. \tag{4} \]

A variation of the action involving this Lagrangian leads to the following equations for the \( \phi \) and \( \phi^N \) fields:

\[ \nabla^2 \phi^N = 4\pi G \rho \]
\[ \nabla^2 \phi = \nabla \cdot \left[ \nu \left( \frac{|\nabla \phi^N|}{a_0} \right) \nabla \phi^N \right] \tag{5} \]

where \( \nu(y) = \frac{dQ(z)}{dz}, \ y = z^2 \), and the field \( \phi^N \) satisfies the standard Newtonian Poisson equation. The function \( \nu(y) \) again performs an interpolation between Newtonian and MONDian regimes, depending on the accelerations in the system. The limiting behaviour in this formulation is

\[ \nu(y) \rightarrow 1, \quad y \gg 1 \]
\[ \nu(y) \rightarrow y^{-1/2}, \quad y \ll 1. \tag{6} \]

This formulation is more straightforward numerically: one need only solve two standard linear Poisson equations, albeit with an unusual source term for the second equation. This source term, as well as including the contribution from standard matter, also contains a component of so-called “phantom dark matter,” which is responsible for the MOND phenomenology in this formulation. Unlike the non-baryonic cold dark matter of the standard cosmology, however, the distribution of this component is fully determined by the normal “baryonic” matter of the galaxy.

2.1. Relativistic extensions

To be a complete theory of modified gravity MOND clearly requires a relativistic completion. Although we will not use the relativistic extensions of MOND in our work, it is nevertheless worth pointing out that several proposals for such a theory now exist, most of them reducing in the weak field limit to the AQUAL formulation discussed above. Almost all of these theories (except the recently proposed metric-only theory of [25]) propose to extend the degrees of freedom in the gravitational sector beyond those of a standard massless spin-2 gravitational field. Perhaps the most well-studied example is the TeVeS model of [9], which involves a metric, a vector field and a scalar field. Many more details regarding this theory may be found in the review of modified gravity theories of [26] and references therein. TeVeS appears to have difficulty reproducing certain cosmological observables, particularly CMB data.

There is also a relativistic completion of the QUMOND formulation which adds additional degrees of freedom in the form of a second metric, and so lies in the class of bimetric theories of gravity [11]. While somewhat less well-studied than TeVeS, early indications seem promising, assuming that the theory does not exhibit the pathologies that are known to afflict many other multi-metric theories, such as inconsistencies and ghosts.

3. Modifications to RAMSES for the MOND calculation

The technical details of the RAMSES code may be found in [20] and the details of the modified code RAyMOND may be found in [22]. Here we will summarise the changes necessary to implement both the AQUAL and QUMOND formulations.

The non-linearity of the AQUAL modified Poisson equation limits the possible numerical schemes that may be practically utilised in an N-body solver. In particular, we cannot use direct force summation methods such as those used in tree codes. Nor can we use the fast Fourier transform method that is used to solve the Poisson equation in Fourier space. Instead
we must use an iterative solver, which is readily adaptable to non-linear equations such as Eq. 2 above.

With this in mind, RAMSES is a good candidate for modification to MOND gravity, as it uses an iterative Gauss-Siedel solver accelerated with a multigrid scheme (see e.g. [27] for details of multigrid techniques) to solve the Newtonian Poisson equation. Both the numerical stencil for the iterative solver and the multigrid acceleration algorithm must be modified for the non-linear AQUAL formulation. We give a very brief description of these changes here, more details may be found in [22].

3.1. The modified Poisson solver

3.1.1. AQUAL The Gauss-Siedel solver in RAMSES uses a standard 7-point stencil to update the value of $\phi$ at the central grid point by the average of $\phi$ on the six neighbouring grid points. This stencil is shown for comparison in Fig. 1.

We will follow the procedure of [14], [15] and [16] by modifying the Gauss-Siedel solver used in RAMSES to use an extended stencil of points around each grid point in the calculation of $\phi$, as shown in Fig 2. The updated value of $\phi$ at each grid point is now given by

Figure 1. 6-point stencil used in the Gauss-Siedel iterative solver of standard RAMSES.

Figure 2. 18-point stencil used for the MOND solver. The squares indicate the locations where the $\mu$ function is evaluated.
\[
\phi_{i,j,k} = (\mu_{i+1/2,j,k} \phi_{i+1,j,k} + \mu_{i-1/2,j,k} \phi_{i-1,j,k} + 
\mu_{i,j+1/2,k} \phi_{i,j+1,k} + \mu_{i,j-1/2,k} \phi_{i,j-1,k} + 
\mu_{i,j,k+1/2} \phi_{i,j,k+1} + \mu_{i,j,k-1/2} \phi_{i,j,k-1} - 
4\pi \rho_i \rho_{i,j,k}) / \sum N \mu_N.
\]

The subscripts on the \( \mu \) interpolation function denote that this is to be evaluated midway between the grid points, as indicated by the squares in Fig. 2, and the denominator is the sum over \( \mu \) at all 6 neighbouring midway points. The evaluation of \( \mu \) requires a finite-difference calculation of the gradient of \( \phi \) at the neighbouring grid points, and this requires the extended stencil shown in Fig. 2. As an example, the gradients of \( \phi \) at the \( i + 1/2, j, k \) grid point (used in the evaluation of \( \mu \) at that location) are calculated as follows:

\[
(\nabla \phi)_{i+1/2,j,k} = \frac{\phi_{i+1,j,k} - \phi_{i,j,k}}{dx}
\]

\[
(\nabla \phi)_{i+1/2,j,k} = \frac{\phi_{i+1,j+1,k} + \phi_{i+1,j+1,k} - \phi_{i-1,j+1,k} - \phi_{i+1,j-1,k}}{4dx}
\]

\[
(\nabla \phi)_{i+1/2,j,k} = \frac{\phi_{i,j,k+1} + \phi_{i+1,j,k+1} - \phi_{i,j,k-1} - \phi_{i+1,j,k-1}}{4dx}
\]

Similar expressions apply for the 5 other midway points at which \( \mu \) is evaluated. Clearly the dependence of \( \mu \) on \( \phi \) leads to a \( \phi \) dependence in the coefficients used in the Gauss-Siedel solver and thus a non-linear behaviour. We can easily see from Eq. 7 that we recover the Newtonian version of the Gauss-Siedel solver when \( \mu = 1 \), as is the case whenever all accelerations in the system are well above the MOND scale.

The extended stencil requires further modification of the RAMSES code as we now need \( \phi \) values at diagonal neighbours, such as \( \phi_{i+1,j+1,k} \). The data structure of RAMSES that allows access to the grid values only stores the six nearest neighbour points shown in Fig. 1. Rather than substantially revise this data structure to incorporate the diagonal grid points as “neighbours,” we choose to use the existing linked list architecture to extract the points we require by finding the neighbours of neighbours.

Finally, the non-linear AQUAL formulation of MOND requires that we also modify the multigrid algorithm, in addition to modifying the Gauss-Siedel solver. Several approaches are possible: we choose the Full Approximation Storage scheme. In this scheme the full non-linear equation is solved on all coarse levels, rather than simply the correction to the fine grid solution, as in the standard linear algorithm. More details regarding this scheme may be found in [27].

3.1.2. QUMOND

The QUMOND version of the code does not require any modifications to the Gauss-Siedel solver, as this formulation involves solving the standard Poisson equation, albeit twice, and with a modified density field the second time. The source term of Eq. 5 may be rewritten in terms of an additional density component, as discussed earlier:

\[
\nabla^2 \phi = 4\pi G(\rho + \rho')
\]

where \( \rho \) is the physical density of the system and \( \rho' \) is given by

\[
\rho' = \frac{1}{4\pi G} \nabla \cdot \left( \nu \left( \frac{1}{a_0} \frac{\nabla \phi^N}{a_0} \right) \nabla \phi^N \right).
\]

The calculation of \( \rho' \) makes use of the extended stencil described earlier and the Newtonian potential \( \phi^N \) is determined from a standard pass through the RAMSES multigrid algorithm. Note that the interpolation function used here is related to \( \nu(y) \) by \( \nu = \nu + 1 \).
The principal changes necessary for this scheme are additional arrays to store the standard and phantom density components, and the Newtonian and MONDian potentials. The execution of the code through a timestep is also modified in order to solve two Poisson equations rather than just one.

4. Code test - spiral galaxy model

We show an example test of the RAyMOND code using a galaxy model comprising a disk and a bulge component. We use the initial conditions code GalactICs [28] to generate a Newtonian disk and bulge inside a dark matter halo. We run a simulation of this model as a control run using the normal Newtonian RAMSES code. For the MOND runs, we simply remove the dark matter halo particles from the initial conditions and run the disk and bulge particles only using RAyMOND. Fortuitously, this seems to result in a disk with a very similar rotation curve: it appears that in this case the Newtonian dark matter halo corresponds closely with the MONDian “phantom dark matter” halo determined by the stellar disk and bulge, but this may not generally be true.

The parameters in the disk, bulge and halo are those of the MW-A model in [28]. The disk is modelled as an exponential disk, with mass \(4.2 \times 10^{10} M_\odot\), scale radius 4.5 kpc, and scale height 0.43 kpc. The bulge is a King model, with mass \(2.1 \times 10^{10} M_\odot\) and the halo uses a lowered Evans model with mass \(2.7 \times 10^{11} M_\odot\) (see [28] for the full set of parameters used to initialise the model). We use 60000 particles in the disk, 30000 in the halo and 10000 in the bulge.

Snapshots of the Newtonian run viewed face-on and edge-on are shown in Fig. 3. Although not clear in the plots, the disk forms weak spiral structure soon after the simulation starts, but is clearly globally stable, as expected due to the presence of the dark matter halo. There is some disk thickening over time, as is standard in numerical disk simulations. The AQUAL run snapshots are shown in Fig. 4. The disk forms more pronounced spiral structure than in the Newtonian case, but again exhibits no global instability. A bar instability does develop after around 3 Gyr of evolution. The disk thickening is considerably less pronounced in this case, although the bar becomes clearly visible by 4 Gyr.

The average rotational velocities as a function of radius in the disk are shown in Fig. 5. The rotation curve at the beginning of the simulation (identical in both the Newtonian and AQUAL runs) is shown as a black line, the Newtonian rotation curve after 4 Gyr is shown as a dashed blue line, while the AQUAL rotation curve is shown as a solid green line. In the Newtonian case, the flat rotation curve is evident due to the presence of the dark matter halo, and it remains stable throughout the evolution. The AQUAL rotation curve remains effectively flat at large radii, entirely due to the MOND enhancement of the gravitational accelerations in the disk, as
Figure 4. AQUAL disk and bulge (no dark matter halo). These are subsets of 20000 particles.

Figure 5. Mean rotational (azimuthal) velocity of the Newtonian disk and bulge and the AQUAL disk and bulge at the beginning and end of their respective runs, in $r$-bins of width 200 pc. The black line denotes the initial conditions of the simulations, the blue dashed line shows the Newtonian rotation curve after approximately 4 Gyr, while the green solid line shows the AQUAL rotation curve after the same time.

there is no dark matter halo in this simulation. The MOND curve is slightly lower than that of the Newtonian system after 4 Gyr, and the existence of the bar is evident in the dip in the rotation curve at small radii.

This test demonstrates that our code can evolve a rotationally-supported system over a significant timescale, and that our code may be used in the study of secular processes in MOND disks.

5. Case study - collisional ring galaxies
We will now discuss an explicit example of a study for which the RAyMOND code is being used: a collisional ring galaxy system. In these systems a spiral galaxy suffers a collision through the disk by a smaller, elliptical, companion. The resulting perturbation of the disk causes a density wave to propagate through the galaxy, producing a striking ring structure in the disk. The famous Cartwheel Galaxy is the paradigmatic example of such an object (for a review of collisional ring galaxies see [29]). Numerical simulations, such as that of [30], allow us to gain considerable insight into the formation and evolution of these systems. An example simulation in MOND gravity is shown in Figs. 6 and 7. We can see the ring structure resulting from the
collision clearly in the stellar particles comprising the disk, as well as the dragging of stars out of the disk by the elliptical collider. Similar structures are visible in observed collisional ring systems in the gaseous material as well, which may also be simulated with the RAyMOND code.

These systems provide an opportunity to examine one key difference between MONDian dynamics and Newtonian dynamics in the presence of dark matter: the differing amounts of dynamical friction. Normally, the presence of large dark matter haloes leads to dynamical friction, where the gravitational interaction between a galaxy (and its associated dark matter halo) and the particles comprising a larger host halo gives rise to a friction effect, slowing the motion of the galaxy [31]. In MOND, the effect of dynamical friction is only due to the baryonic material (i.e. the stars and gas) when the two galaxies undergo a collision. While the enhanced gravity of MOND will increase the effectiveness of dynamical friction during the collision, there is essentially no effect after the collision. In the ΛCDM context the presence of the dark matter haloes leads to longer dynamical friction timescales, and potentially more slowing of the galactic motion.

Thus galaxy collisions proceed differently in MOND and ΛCDM. In a collisional ring galaxy, observations of the ring properties can provide constraints on the timescales of the collision, and thus we may find significant differences in the modelling of these systems.

6. Conclusions

We have developed a modification of the N-body/hydrodynamics code RAMSES that enables high resolution simulations using MOND gravity. The code utilises two formulations of MOND, known as AQUAL and QUMOND, allowing a direct comparison between them and the investigation of possible observational consequences from the different manifestations of MONDian behaviour. The execution time of the code has also been demonstrated to be as fast as may be expected, considering the complexity of the MOND calculation. Furthermore, using idealised tests we have confirmed that the RAyMOND code is able to evolve dispersion-supported and rotation-supported stellar systems in a stable manner in a MOND gravitational potential.

There are, of course, many possible applications of our code at galactic scales, including galaxy interactions, the behaviour of dwarf satellites, star cluster dynamics in the Milky-Way and so on. A particular application of the code to the study of collisional ring galaxy systems was discussed. Further studies of the effects of MOND gravity on galaxy dynamics in various environments will be the subject of future studies.
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