A Mixed Multiscale FEM for the Eddy-Current Problem With \( T, \Phi-\Phi \) in Laminated Conducting Media

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A novel mixed multiscale finite-element method for the eddy-current problem is presented to avoid the necessity of modeling each laminate of the core of electrical devices. The method is based on a current vector potential \( T \) and a reduced magnetic scalar potential (RMSP) \( \Phi \) and copes with the 3-D problems. The edge effect is considered. Material properties are assumed to be linear. Hence, the method is developed for the frequency domain. External currents are represented by the Biot–Savart field serving as excitation. The planes of symmetry are exploited. Numerical simulations are presented, showing excellent accuracy at minimal computational costs.

Index Terms—Biot–Savart field, current vector potential (CVP), eddy-current problem (ECP), laminated media, mixed multiscale finite-element method (MMSFEM), reduced magnetic scalar potential (RMSP).

I. INTRODUCTION

An accurate prediction of the eddy-current distribution in the laminated iron cores of electric devices is a challenging task in the design process. Modeling of each laminate requires many finite elements, leading to extremely large equation systems. The computational costs to solve these systems are prohibitively high.

The solution obtained by prescribing a current vector potential (CVP) \( T \) having a single component normal to the lamination [1] or using an anisotropic electric conductivity [2] has to be corrected in a post-processing step to consider the eddy currents due to the main magnetic flux. These approaches are questionable in the context of nonlinear material properties. Multiscale finite-element methods (MSFEMs) provide the solution in one step taking account of both magnetic stray flux perpendicular to the lamination and main magnetic flux parallel to the lamination.

The capacity of the MSFEMs is well known [3]. An MSFEM in 3-D for eddy currents in laminated iron cores based on the magnetic vector potential \( A \) has been presented in [4] recently. A CVP \( T \) [5] with a reduced magnetic scalar potential (RMSP) \( \Phi \) can also be used for an eddy-current problem (ECP) [6], [7]. The \( T, \Phi-\Phi \) formulation is popular to simulate the eddy currents, for instance, in the core of transformers.

Often 2-D/1-D-methods are used to simulate the electrical machines. These methods are based on the assumption that the influence of the magnetic stray fields in the end region of the electrical machines can be neglected. This means that each laminate is exposed to the same electromagnetic field, and therefore, a simulation of a single laminate suffices [8], [9]. Such problems are solved elegantly with 2-D/1-D-MSFEMs [10]. However, this kind of methods is not in the scope of this article.

II. \( T, \Phi-\Phi \) FORMULATION

The \( T, \Phi-\Phi \) formulation is introduced extremely shortened, and the associated boundary value problem along with the weak form for eddy currents is summarized in the following.

A. \( T, \Phi-\Phi \) Formulation

Considering Ampere’s law \( \text{curl} \ H = J + J_0 \) with an impressed current density \( J_0 \), the magnetic-field strength

\[
H = T + T_{BS} - \text{grad} \Phi
\]  

(1)
in the conducting domain $\Omega_c$ can be represented by a CVP $T$ and an RMSP $\phi$, where $J_0$ is replaced by its Biot–Savart field $T_{BS}$. The potentials $T$ and $\phi$ describe the quasi-static magnetic field in $\Omega_c$. The static magnetic field in the non-conducting domain $\Omega_0$ can be written as

$$H = T_{BS} - \nabla \phi.$$

(2)

### B. Boundary Value Problem With $T$, $\phi$–$\Phi$

The ECP to be solved in this article is sketched in Fig. 1. It consists of laminates $\Omega_c$ enclosed by air $\Omega_0$, $\Omega = \Omega_c \cup \Omega_0$. The laminated domain $\Omega_m$ used by the MMSFEM consists of the conducting laminates $\Omega_c$ and the insulation layers in between.

Thus, the following boundary value problem for the $T$, $\phi$–$\Phi$ formulation is obtained [7].

The quasi-static magnetic field in the conducting domain $\Omega_c$

$$\nabla \times (\rho \nabla \times T) + j_0 \mu T - j_0 \mu \nabla \phi = - \nabla (\rho \nabla \times T_{BS}) - j_0 \mu T_{BS}$$

(3)

$$j_0 \nabla \cdot (\mu T - \nabla \phi) = - j_0 \nabla \cdot (\mu T_{BS}) \quad \text{in} \quad \Omega_c$$

(4)

$$\nabla \cdot (\mu (T - \nabla \phi) \cdot n) = - \mu T_{BS} \cdot n \quad \text{on} \quad \Gamma_E$$

(5)

$$T \times n = - T_{BS} \times n \quad \text{(12)} \quad \text{on} \quad \Gamma_E$$

for all $(v_h, v_h) \in V_{h0}$, where $U_{h0}$ and $V_{h0}$ are the finite-element subspaces of $H(\nabla \times, \Omega_c)$ and $H^1(\Omega_c)$, respectively.

### C. Weak Form With $T$, $\phi$–$\Phi$

The weak form for the finite-element method is as follows.

Find $(T_h, \phi_h) \in V_{h0} := \{(T_h, \phi_h): T_h \in U_{h0}, \phi_h \in V_{h0} \text{ and } T_h \times n = -T_{BS} \times n \quad \text{on} \quad \Gamma_E, \quad \phi_h = \phi_0 \quad \text{on} \quad \Gamma_{hH}, \quad \phi_h = \phi_0 \quad \text{on} \quad \Gamma_{hH} \cup \Gamma_{H0})$, such that

$$\int_{\Omega_c} \rho \nabla \times T_h \cdot \nabla \times v_h \, d\Omega + j_0 \int_{\Omega_c} \mu (T_h - \nabla \phi_h) \cdot v_h \, d\Omega$$

$$\quad = - \int_{\Omega_c} \rho \nabla \times T_{BS}h \cdot \nabla \times v_h \, d\Omega - j_0 \int_{\Omega_c} \mu T_{BS}h \cdot v_h \, d\Omega$$

(13)

$$\quad = j_0 \int_{\Omega} \mu \nabla \phi_h \cdot \nabla v_h \, d\Omega - j_0 \int_{\Omega} \mu T_h \cdot \nabla v_h \, d\Omega$$

(14)

$$\quad = j_0 \int_{\Omega} \mu T_{BS} \cdot \nabla v_h \, d\Omega - j_0 \int_{\Gamma} b_h v_h \, d\Gamma$$

for all $(v_h, v_h) \in V_{h0}$, where $U_{h0}$ and $V_{h0}$ are the finite-element subspaces of $H(\nabla \times, \Omega_c)$ and $H^1(\Omega_c)$, respectively.

### III. MIXED MULTISCALE FINITE-ELEMENT METHOD

#### A. Mixed Multiscale Approach for $T$, $\phi$–$\Phi$

Multiscale approaches are based on the fact that the problem can be observed on the large scale with the overall dimensions of the laminated core, on the one hand, and, on the other, on the small scale with the very small thickness of the laminates $d$ and the width of the insulation layer $d_0$ in between (see Fig. 3). Thus, it would be obvious to assume the mixed multiscale approach

$$\begin{align*}
\bar{T} &= T_0 + \phi_2 T_2 \\
\bar{\phi} &= \phi_0 + \phi_2 \phi_2
\end{align*}$$

(15)

(16)

where the mean values $T_0$ and $\phi_0$ consider the large-scale variation of the solution and $T_2$ and $\phi_2$ with the even periodic micro-shape function $\phi_2$ (see Fig. 2), the highly oscillating variation of the solution on the small scale.

Therefore, the magnetic field represented in (1) and (2) could now be written as

$$\bar{H} = T_0 + T_2 \phi_2 + T_{BS} \nabla \phi_0 - \nabla (\phi_2 \phi_2) \quad \text{in} \quad \Omega_m$$

(17)

$$\bar{H} = T_{BS} \nabla \phi_0 \quad \text{in} \quad \Omega_0.$$
has been used in this article. Thus, the magnetic field in (17) and (18), respectively, can be written for the MMSFEM

$$\tilde{H} = T_{BS} + T_2 \phi_0 - \text{grad} \Phi_0 \quad \text{in} \quad \Omega_m$$

(21)

and

$$\tilde{H} = T_{BS} - \text{grad} \phi_0 \quad \text{in} \quad \Omega_0.$$  

(22)

B. Weak Form for $\tilde{T}$, $\tilde{\Phi} - \Phi$

The weak form for the MMSFEM reads as follows:

Find $(T_{2h}, \Phi_0h) \in V_{Dh} := \{(T_{2h}, \Phi_0h) : T_{2h} \in \mathcal{U}_h, \Phi_0h \in \mathcal{V}_h \text{ and } T_{2h} \times n = 0 \text{ on } \Gamma_{m0} \setminus \Gamma_T \cup \Gamma_H, \Phi_0h = \Phi_0 (= 0) \text{ on } \Gamma_H \cup \Gamma_{H_0}\}$, such that

$$\int_{\Omega} \rho \text{curl}(T_{2h} \phi_2) \cdot \text{curl}(v_{2h} \phi_2) d\Omega + j\omega \int_{\Omega} \mu(T_{2h} \phi_2 - \text{grad} \Phi_0h) \cdot v_{2h} \phi_2 d\Omega = - \int_{\Omega} \rho \text{curl} T_{BS} \cdot \text{curl}(v_{2h} \phi_2) d\Omega - j\omega \int_{\Omega} \mu T_{BS}h \cdot v_{2h} \phi_2 d\Omega$$

$$j\omega \int_{\Omega} \mu \text{grad} \Phi_0h \cdot \text{grad} v_{0h} d\Omega - j\omega \int_{\Omega} \mu T_{2h} \phi_2 \cdot \text{grad} v_{0h} d\Omega$$

$$= j\omega \int_{\Omega} \mu T_{BS}h \cdot \text{grad} v_{0h} d\Omega - j\omega \int_{\Gamma_T} b_h v_{0h} d\Gamma$$

(23)

for all $(v_{2h}, v_{0h}) \in V_{0h}$, where $\mathcal{U}_h \subset H(\text{curl}, \Omega_m)$, $\mathcal{V}_h \subset H^1(\Omega)$, and $\phi_2 \in H^1_{\text{per}}(\Omega_m)$ have been selected.

The interface between $\Omega_m$ and $\Omega_0$ is denoted by $\Gamma_{m0}$ and $\Gamma_T$ is the part of $\Gamma_{m0}$ which represents the smooth surface of the laminated core, compared with Figs. 1 and 3. The arising coefficients in (23) and (24) have been averaged, as demonstrated in [3]. Since the micro-shape function $\phi_2$ is the even function shown in Fig. 2 with respect to the center of the laminate, the highly oscillating second term on the RHS in (23) does not vanish.

IV. Numerical Simulations

A. Numerical Problems

Problems consisting of different numbers of laminates have been investigated, compared with Fig. 3. The thickness of the laminates and an unfavorable width of the insulation layers have been selected with $d = 0.45$ mm and $d_0 = 0.05$ mm. A conductivity of $\sigma = 2 \times 10^6$ S/m and a relative permeability of $\mu_r = 1000$ have been assumed. For the sake of simplicity, the filamentary current was selected to deal easily with the linear forms on the right-hand sides of the weak form in (23) and (24). The rectangular filamentary current generates the Biot–Savart field $T_{BS}$, which serves as a source field. Each straight segment of the rectangular loops has been split into four sub-segments. Gauss integration with three integration points was used to approximate

$$T_{BS} = \frac{\mu_0 I}{4\pi} \int_s ds \times \frac{r_{SF}}{||r_{SF}||^3}$$

(25)

where $I$ is the peak value of the current selected with 10 A, $r_{SF}$ the source-field-point vector, and $s$ the integration path.
V. CONCLUSION

The presented MMSFEM is an attractive alternative to the MSFEM based on a magnetic vector potential $\mathbf{A}$ [4]. Contrary to the MSFEM with $\mathbf{A}$, considering the edge effect does not require an additional unknown. The edge effect is simply realized by prescribing the homogenous tangential Dirichlet boundary conditions of $T_2$.

The novel MMSFEM requires only as many unknowns as a brute force simulation using $T, \Phi, -\Phi$ with an anisotropic conductivity in the first step, except that the MMSFEM provides the eddy currents due to the main magnetic field including the edge effect. In general, the savings of the MSFEM compared with RSA and RST grow clearly with the problem size measured by the number of laminates $N_L$.

ACKNOWLEDGMENT

This work was supported by the Austrian Science Fund (FWF) under Project P 27028.

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