Research Article

Analysis of Asymmetric Piecewise Linear Stochastic Resonance Signal Processing Model Based on Genetic Algorithm

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1. Introduction

When the nonlinear system interacts with the signal to be measured and the nonsignal to be measured (noise), resonance will occur. At this time, when the noise intensity increases, the noise will not inundate the signal but will increase the signal-to-noise ratio, and the noise energy will transfer to the signal. This shows that stochastic resonance is a powerful method to extract a weak signal from strong noise \cite{1,2}. However, a large number of studies have shown that the classical SR theory has obvious detection advantages only in the case of small parameters, i.e., adiabatic approximation. In engineering application, most target signals are not small parameter signals, which greatly limits the application in engineering. In recent years, a series of achievements have been made in the resonance phenomenon of large parameter signals, such as the use of secondary sampling, single sideband modulation, frequency-domain information exchange, and other methods, which can make large parameter signals produce resonance phenomenon \cite{3}. In reference \cite{4}, it is known that the piecewise linear model has better performance than the classical model, and in reference \cite{5}, the asymmetric model can obtain better performance than the symmetric model by adjusting the asymmetric factor. Zhang et al. applied a stochastic resonance system for bearing fault detection \cite{6,7}. Zhang et al. found that the performance of the system is also affected by the system parameters \cite{8}, and the selection of parameters will directly affect the quality of the system. Zhang and He obtained better system parameters through an adaptive genetic algorithm and applied them to bearing fault detection \cite{9}.

At present, some scholars are committed to exploring the effect of asymmetry on system performance \cite{2,4}. Some scholars have devoted themselves to the study of piecewise linear systems. Both of them can achieve better system performance \cite{10}. However, there is no research on the combination of them to obtain better performance. Based on this, an asymmetric linear piecewise bistable model is proposed. The analytical expression and signal-to-noise ratio of the model are derived and compared with the symmetrical...
bistable piecewise linear stochastic resonance system and the continuous bistable system. At the same time, in order to obtain better performance in the application of bearing fault detection, a genetic algorithm is used to obtain better system parameters. In the second part of the paper, the concrete formula derivation is given. It provides a theoretical basis for the simulation and experiment. In the third part, numerical simulation is carried out to verify the correctness of the formula derivation. In the fourth part, the genetic algorithm simulation is carried out to verify the correctness of the simulation and experiment. In the third part, numerical detection, a genetic algorithm is used to obtain better system parameters and are all greater than 0. Under the static condition, the system has two potential wells and one barrier. The bottom of the two wells is \( x_m = k_2 \) and \( x_n = -r k_2 \), respectively, and the barrier height is \( \Delta U = c \).

Among them, \( r \) is asymmetric factor, \( k_1, k_2 \) and \( k_3 \) are system parameters and are all greater than 0. Under the static condition, the system has two potential wells and one barrier. The bottom of the two wells is \( x_m = k_2 \) and \( x_n = -r k_2 \), respectively, and the barrier height is \( \Delta U = c \).

When \( r = 1 \) is a symmetric piecewise linear model, other cases are asymmetric linear systems. It can be seen from Figure 1 that the width of a potential well varies with the asymmetric factor.

2.2 System Response and Signal-to-Noise Ratio. The steady state of the system is \( +x_m, -x_n \) and \( r x_m = x_n \). Let \( W_+(t) \) be the probability of time \( t \) system in bistability \( +x_m, -x_n \) and define \( n_+(t) \) as the probability of transition from steady state \( +x_m, -x_n \) at time \( t \). Because of the asymmetry of the two potential wells, the transition probability of the two potential wells is not the same, that is, \( n_+(t) \neq n_-(t) \). When the asymmetry factor is \( r = 1 \), the equation holds. According to the adiabatic approximation theory [2], the escape rate of the asymmetric linear bistable system is as follows:

\[
\lambda_+ = \frac{1}{D} \int_{-r k_3}^{r k_3} e^{-\left(1/D\right)\left((k_3/r k_3)x\right)\left(x + r k_3\right)} dx, \quad \lambda_- = \frac{1}{D} \int_{-r k_3}^{r k_3} e^{-\left(1/D\right)\left((k_3/r k_3)x\right)\left(x + r k_3\right)} dx
\]

where \( D \) is the noise intensity, \( r \) is asymmetric factor, \( k_1, k_2 \) and \( k_3 \) are system parameters. The transition rate \( n_+(t) \) is generally considered to have an exponential form. Under the action of periodic signal \( s(t) = A \cos(\omega_0 t) \), it is expanded by Taylor series, as shown in the following equation:

\[
n_+ = \lambda_+ \left[ 1 + \frac{A x_m}{D} \cos(\omega_0 t) + \frac{1}{2} \left( \frac{A x_m}{D} \right)^2 \cos^2(\omega_0 t) + \cdots \right],
\]

\[
n_- = \lambda_- \left[ 1 - \frac{A x_m}{D} \cos(\omega_0 t) + \frac{1}{2} \left( \frac{A x_m}{D} \right)^2 \cos^2(\omega_0 t) + \cdots \right].
\]

Under the assumption of adiabatic approximation, the probability equation of the model can be established according to (2) and (3):

\[
\frac{d W_+(t)}{d t} = -\lambda_- W_-(t) + \lambda_+ W_+(t),
\]

\[
\frac{d W_-(t)}{d t} = -\lambda_+ W_+(t) + \lambda_- W_-(t).
\]

The equations of solution (4) can be obtained as follows:

\[
W_-(t) = \frac{1}{2} \left[ \frac{1}{\lambda_-} + e^{-\lambda_- t} \right],
\]

\[
W_+(t) = \frac{1}{2} \left[ \frac{1}{\lambda_+} - e^{-\lambda_+ t} \right].
\]

The probability distribution function obtained from formula (5) is shown in the following equation:
Select the system parameter $k$, in reference [5], and the piecewise linear model has been compared with the continuous bistable system, in the process of noise intensity from small to large, the system is always larger than the other two systems. It can be seen that the higher SNR can be obtained for the system, but in the process of noise intensity from small to large, the system has the same phenomenon, but in the process of noise intensity from small to large, the system is always larger than the other two systems.

The output power spectrum can be written as follows:

$$S_N(\omega) = \left[ 1 - \left( \frac{Ax_m^2}{2D} \right) \left( \frac{\lambda_* + \lambda_*}{2} \right) \right] \frac{\lambda_* + \lambda_*}{(\lambda_* + \lambda_*)^2 + \omega_0^2).}$$

The output power spectrum is obtained by Fourier transform of the autocorrelation function, as follows:

$$S(\omega) = \frac{\pi}{2} x^2(D)[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + S_N(\omega).$$

Then SNR is as follows:

$$SNR = \frac{\int_{0}^{\infty} S(\omega) - S_N(\omega)d\omega}{S_N(\omega)}$$

$$= \frac{(1 + r^2)\pi \lambda^2 x_m^2(\lambda_* + \lambda_*)}{4D^2 \left[ 1 - (1/2)(Ax_m/D)^2(\lambda_* + \lambda_*)^2/(\lambda_* + \lambda_*)^2 + \omega_0^2) \right]}$$

For the system, $x_m = b$ and obtained from formula (2):

$$\lambda_* = \frac{k_2}{(k_1 - k_2)k_2} e^{-(k_1/D)},$$

$$\lambda_* = \frac{k_2}{(k_1 - k_2)k_2} e^{-(k_1/D)}.$$
Asymmetric piecewise linear model is better. Therefore, only asymmetric and symmetric piecewise linear models are compared. According to formula (1), the system model can be written as follows:

$$\dot{x}(t) = -\frac{dU(x)}{dx} + H(t),$$  \hspace{1cm} (16)

where $H(t) = A \cos(2\pi ft) + \varepsilon(t)$, $\varepsilon(t)$ for noise. Its mean value is zero, and autocorrelation function can be expressed as $\langle \varepsilon(t) \varepsilon(t + \tau) \rangle = 2D\delta(t - \tau)$. $D$ is the noise intensity, $\tau$ is the delay time, $f$ is the input signal frequency, and $A$ is its amplitude. The fourth-order Runge Kutta of (1) is simulated [4], and the resonance behavior of $r = 1$ and $r \neq 1$ is compared and explored.

Figure 3 is the input signal frequency $f = 0.01$, sampling frequency, and amplitude noise intensity $fs = 5$, $A = 0.1$, $D = 4$, system parameter $k_1 = 2, k_2 = 1, k_3 = 0.25$. Figures 3(b) and 3(d) show the amplitude frequency characteristics of the input signal and the output signal, respectively. It can be seen that the spectrum energy concentrates on the low-frequency component. While Figure 4 is under the condition that other conditions remain unchanged, making the asymmetric factor $r = 1.5$, it can be seen from Figure 4(a) that the reduction effect is better than Figure 3(c), and from the frequency domain, it can be seen that the energy concentrated on the input signal frequency is higher.

Figure 5 is a comparison of the average signal-to-noise ratio gain [10] of the asymmetric piecewise linear model and the symmetric piecewise linear model under the same other conditions as Figure 4. It can be seen from the figure that the average signal-to-noise ratio gain of the asymmetric model is always higher than that of the symmetric model with the increase of noise intensity, which is consistent with the conclusion of the formula derivation. Figure 5 shows that the performance of the asymmetric system is not superior to that of the symmetric model because the selection of asymmetric factors is not optimal. Figure 6 is a three-dimensional graph of noise intensity $D$, asymmetric factor $r$ an average signal-to-noise ratio gain $MSNRI$ under the same conditions as Figure 5. It can be seen that $r = 2$ is not the optimal case. As you can see, the performance improvement is not very high. This is because the asymmetric coefficient is not very suitable. It is because of this that the adaptive algorithm mentioned below is needed to get the appropriate system parameters.

### 4. Adaptive Genetic Algorithm

The model proposed in this paper has four parameters, namely $k_1, k_2, k_3, r$ and the dimension of the genetic algorithm is set as four dimensions [11–13]. The key of the algorithm is to transform the solution into the chromosome needed in a genetic algorithm. There are a variety of "chromosome" transformation methods, which can be divided into real number coding (parameter optimization problem) and integer coding (shortest path problem) according to requirements. The genetic algorithm has a wide range of applications. Usually, some changes will be made according to actual needs. In this study, the range of parameter optimization is determined by theoretical analysis, that is, the range of initial gene in genetic algorithm. By replacing the signal-to-noise ratio with fitness function, the purpose of improvement is achieved. The genetic algorithm can be well applied to the stochastic resonance system. The specific process is shown in Figure 7. Specific steps of the proposed algorithm:

- The fitness function of the paper is the output signal-to-noise ratio. Signal-to-noise ratio (SNR) is a common measure in signal processing research. The higher the SNR, the better the fitness. Compared with correlation and average signal-to-noise ratio, the genetic algorithm has lower time complexity and little difference in effect.
- Initialize population size and iteration: Random selection of individuals to build an initial population.
- The fitness function constructed in step (1) is used to calculate the fitness of all individuals. And keep the best individuals to the next generation.
- The individual genes in the population were crossed, and when the crossing process met the variation conditions, the cross was executed (5). The crossover operator of the following formula is used:

$$X'_1 = \lambda_1 X_1 + (1 - \lambda_2 X_2),$$
$$X'_2 = \lambda_1 X_2 + (1 - \lambda_2 X_1).$$  \hspace{1cm} (17)

- Variation, that is to say, the offspring produced genes that the parents did not have. The construction of the mutation operator is as follows:

$$X' = X + \Delta.$$  \hspace{1cm} (18)

Generate offspring and replace any random individual in the offspring with the optimal solution individual in step (3). According to whether the termination condition reaches the maximum number of iterations, the algorithm’s branch flow is determined.
Figure 3: Time-domain and frequency-domain graphics of input and output signals.

Figure 4: Time-domain and frequency-domain graphics of input and output signals.

Figure 5: Average SNR gain.
Adding optimization parameters to the genetic algorithm will not exponentially increase the time complexity like an ordinary iterative algorithm. The time complexity of the iterative algorithm is $O(N^n)$, $N$ is the number of iterations, and $n$ is the number of parameters. The time complexity of GA is $O(N \times n^n)$. $N$ is the genetic algebra, $n$ is the number of parameters.
5. Bearing Fault Detection

5.1. Fault Characteristics Extraction of the Bearing Type 6205-2RS JEM SKF

5.1.1. Characteristic Frequency. The bearing model is 6205-2RS JEM SKF deep groove ball bearing. The main parameters are shown in Table 1. Because the condition of a small parameter is not satisfied, the resonance phenomenon is produced by using the method of second sampling.

| Inner diameter (cm) | Outer ring diameter (cm) | Ball straight (cm) | Thickness (cm) | Stanza (cm) | Number of ball bearings (one) |
|---------------------|--------------------------|-------------------|----------------|-------------|-------------------------------|
| 2.5001              | 5.1999                   | 0.7940            | 1.5001         | 3.904       | 9                             |

Sampling frequency $f_s = 12000$ Hz, number of sampling points $N = 10000$. Here, $5$ Hz is selected as the second sampling frequency. The calculation formula of the characteristic frequency is as follows:

$$f_{BPFI} = \frac{n_r f_r}{2} \left(1 + \frac{D_1}{D_2} \cos \alpha\right),$$

$$f_{BPFO} = \frac{n_r f_r}{2} \left(1 - \frac{D_1}{D_2} \cos \alpha\right).$$

(19)

$n_r$ represents the number of rolling elements, $D_1$ represents the diameter, $D_2$ represents the bearing, the rotation frequency is $f_r$, the contact angle is $\alpha$. $f_{BPFI}$ and $f_{BPFO}$ are the characteristic frequencies of the inner and outer rings of the bearing, respectively. Substituting the data in the table into equation (19), it is known that the characteristic frequency of the outer ring fault is 107.28 Hz, and that of the inner ring fault is 162.11 Hz.

5.1.2. Outer Ring Fault Detection. Figure 8 shows the input signal of bearing fault to be detected and the output signal of the symmetrical piecewise linear system. It can be seen that the fault signal is completely covered by other high-frequency noises. After passing the SR system, the fault signal is detected. The parameters of this check are all found through the adaptive parameter optimization of the genetic algorithm. The population number of genetic algorithms is 200, the genetic algebra is 200, the crossover probability is set to 0.4, and the mutation probability is set to 0.2. (In the actual natural environment, the crossover probability is much greater than the mutation probability, but in order to jump out of the local optimum, the mutation probability is set to be larger here.)

Figure 9 is an output signal through an asymmetrical piecewise linear system. Compared with Figure 8, it can be seen that the accuracy of amplitude value and inspection at fault frequency is higher than that of the symmetrical segmented system. The time-domain waveforms of the stochastic resonance system are compared. It can be seen that the burr of time-domain waveform of an asymmetric system is obviously less than that of a symmetric system. It is
Figure 9: Time and frequency-domain diagrams of the output of outer circle fault detection system \((a = 1.00619166102941, b = 0.11, r = 1.6, c = 0.28425683835532)\).

Figure 10: Time-domain and frequency-domain diagrams of the input and output of inner-loop fault detection system \((a = 1.26382755905825, b = 1.226069, r = 1, c = 0.253128458753468)\).

Figure 11: Time and frequency-domain diagrams of the output of outer circle fault detection system \((a = 0.938548936195951, b = 0.11, c = 2.11048612862940, r = 1.6)\).
consistent with higher amplitude at fault frequency of frequency-domain waveform.

5.1.3. Inner Ring Fault Detection. In order to prove that there is still a lot of room to improve the performance of the system, the number of population and the number of iterations are doubled when a genetic algorithm is used for parameter optimization. Figure 10 shows the signal to be detected for the inner ring fault and the output signal after the piecewise linear system model. It can be found that the fault signal has been detected, but after a longer time of parameter optimization, the system performance has not been significantly improved.

Figure 11 shows the output waveform of the signal to be detected through the asymmetric piecewise linear model. It can be found that the performance is greatly improved after the time of parameter optimization is increased. It is the

Figure 12: The entity diagram of the ID-25/30 test bench.

Figure 13: Signal to be detected.
same as the previous conclusion and proves the superiority of the system.

5.2. Fault Diagnosis of ID-25/30 Bearing Health Test Bench. In order to further verify the practicability of the system, it is applied to another group of experimental platform for verification. The physical figure of the ID-25/30 bearing health test bench is shown in Figure 12. The calculation method of the characteristic frequency signal of bearing fault is the same as the previous one. The frequency of the inner ring and outer ring can be obtained from the formula. The inner raceway frequency of the bearing can be calculated theoretically to be $f_{\text{inner}} = 117.14$ Hz.

Figure 13 shows the fault signal to be tested collected by the platform. It can be seen that the fault model is completely
submerged by noise and cannot be detected. Figure 14 is the output graph of the piecewise linear system. It can be seen that the fault signal is detected. This part of the parameters is also optimized by genetic algorithm, which proves the practical value of the system again. Figure 15 shows the output of the asymmetrical piecewise linear system. Figure 15 has a higher amplitude and the frequency doubling has been checked, indicating that the detection effect performance is very good.

6. Conclusion
An unsymmetrical piecewise linear system has good efficiency. This paper discusses the performance of the system from three aspects: formula derivation, numerical simulation, and engineering application. The results show that it has a good performance both in theory and in practice, which is consistent with the formula derivation results. Of course, the performance of a system is closely related to its parameters, and it is difficult to find out the coordination between parameters. Therefore, a genetic algorithm is proposed to find better parameters.

The asymmetry mentioned in this paper is the asymmetry of the width of the potential well and the steady state is assumed to be two steady states, which is not explored in the asymmetry of the depth of the potential well and the multisteady state, so we will continue to study in the following work.

Data Availability
Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Conflicts of Interest
The authors declare that there are no conflicts of interest.

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