The use of the planetary differential in the mechanisms of energy recovery

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Abstract. The paper assesses the possibility of using planetary differential variators to change the gear ratio of the kinematic control chain of a flywheel accumulator. The possibility of using a differential mechanism as a variator for the flywheel battery drive is related to the calculation of the required power for the transmission ratio control. The equation of the power in the engagement of central wheels with the satellite was derived as a ratio of the power ratio. The obtained ratios of speeds and capacities of the differential mechanism allowed us to prove that the flywheel accumulator can be used without being included in the control system, by using the property of self-leveling the kinetic energy of the links of the planetary differential variator.

Recently, a new direction has appeared in the design of hoisting-and-transport vehicles. This direction related to the improvement of hybrid power plants. The goal is to reduce energy consumption due to the recovery of braking energy. As a rule, braking energy recovery occurs in two ways:

a) We can use an electric drive, energy recovery produces with an electric battery. This direction has a particular advantage for use in electric vehicles. The weak point in the development of this direction is an electric battery – it has a limited cycle of discharging and charging. Although its use in electric vehicles is natural (when it recharges once a day), its use in a cycle of acceleration-braking movement is problematic.

b) Even in electric vehicles, attempts to use flywheel batteries were made [1-2] to operate in the acceleration-braking cycles. Their success was promoted by the insignificant amount of pumped energy in one cycle and the high service life of mechanical energy recovery devices. However, the complexity of devices for organizing the recovery of braking energy by using mechanical devices is quite high. The successful development of the design of automatic transmissions allows to use the experience of their application for the recovery of braking energy. In the design of automatic transmissions planetary differentials are widely used.

Despite significant progress in this area of the automotive industry, there has not been developed a “classical” scheme of a hybrid power plant and no full theoretical generalization has been made to select possible schemes of mechanisms for specific conditions.

The classification of schemes of three-link planetary mechanisms, which is convenient for practical use, was developed by Professor A.F. Krainev. In the developed schemes, the axis of any of the wheels can be taken as a fixed axis. The most convenient for use are coaxial schemes. For us, the most interesting are the schemes with a variable gear ratio. Fundamentally changing the gear ratio of the planetary mechanism is possible in several ways:
A change in its structure by braking this mechanism or braking one of the wheels. This method allows you to step change the range of the gear ratio, but does not allow you to change the gear ratio. This changes the internal gear ratio of the interacting gears when one of the elements of the planetary gear stops.

A smooth change in the gear ratio of gear mechanisms is possible due to the introduction of hydraulic and electric machines with variable torque. Parallel and serial connection of several mechanisms is possible, the formation of closed gears composed of several mechanisms with two degrees of freedom is promising for this method of changing the gear ratio.

It is advisable to smoothly change the gear ratio of the planetary gear mechanism by combining it with a continuously variable transmission based on a friction or belt variator. In this case, their connection with the planetary gear mechanism with two degrees of freedom is especially often used. In some cases, gearboxes with three or four degrees of freedom are also used.

The main issue that needs to be solved when creating an efficient flywheel energy accumulator - is the development of an energy recovery management system. It is necessary to solve this problem from the standpoint of matching the kinematic characteristics of the planetary differential with the requirements of the control system for the recovery of braking energy.

**Figure 1.** Scheme of single-row: 1, 3 - central wheels; 2 - satellite; 4 – drove.

**Figure 2.** Scheme of double-row flat differential: 1, 3 - central wheels; 2 - satellite; 4 – drove; 5 - output gear.

Figure 1 shows a diagram of a single-row flat planetary differential with a plan of speeds of its links. Figure 2 shows a two-row differential circuit.

The goal of the differential kinematics study is to develop recommendations for choosing the optimal kinematic parameters of the differential planetary variator. Minimize of the power required to control the differential, which connects the flywheel to the hybrid powertrain transmission [3-6].

The main parameter that determines the properties of the planetary differential is the gear ratio. In the general case of differential with \( W = 2 \) the so-called internal gear ratios can be determined. However, in practice, only two are usually used - between the input and output links: the ratio of the rotational speeds of the large and small central wheels when the carrier is stopped \( \omega_4 = 0 \).

\[
U_{13}^4 = \frac{\omega_1}{\omega_3},
\]

where 1 is the index of the small central wheel (solar)
3 - index of a large central wheel (epicycle)
4 - carrier index
We pose the problem of determining the power ratio as a combination of kinematic and power parameters. It is necessary to derive a mathematical model - the dependence of the sign and power modules in the control and controlled channels of the planetary variator. On such parameters as the direct ratio of the angular velocities in the channel under consideration, the number of teeth and the type of gearing.

Let us consider the solution of this problem by the example of a single-row planetary differential mechanism with the number of degrees of freedom $W = 2$ (figure 1).

The ratio of the speeds of the solar (1) and corona wheels (3) can be obtained using the method of reverse movement.

\[
\frac{\omega_1 - \omega_4}{\omega_3 - \omega_4} = \frac{z_2}{z_1} \quad \text{for external gearing}
\]

\[
\frac{\omega_2 - \omega_4}{\omega_3 - \omega_4} = \frac{z_3}{z_2} \quad \text{for internal gearing},
\]

where $\omega$ – angular velocity of the link, $z$ – number of wheel teeth.

From here you can get a kinematic relationship of the angular velocities of the links of the planetary mechanism with $W = 2$.

\[
\frac{\omega_1 - \omega_4}{\omega_3 - \omega_4} = \frac{z_3}{z_2} \quad \text{or}
\]

\[
\omega_1 = \omega_4 \left( 1 + \frac{z_3}{z_1} \right) - \omega_3 \frac{z_3}{z_1},
\]

where the expression of the internal gear ratio of a single-row planetary gear $W = 1$ with the differential axis with the wheel stopped 3 is known $\omega_3 = 0$.

\[
U_{14}^3 = \frac{\omega_1}{\omega_4} = 1 + \frac{z_3}{z_1} > 0
\]

and the internal gear ratio of the differential when the carrier is stopped 4, $\omega_4 = 0$

\[
U_{13}^4 = \frac{\omega_1}{\omega_3} = -\frac{z_3}{z_1}
\]

From this we get for the mechanism with $W = 2$ the link speeds of the links 1, 3 and 4

\[
\frac{\omega_1}{\omega_4} = U_{14}^3 + U_{13}^4 \frac{\omega_1}{\omega_3}
\]

The resulting expression allows you to determine the change in the speed of rotation of the flywheel, for example, by connecting it with the gear wheel 3 depending on the change in speed of links 1 and 4 associated with the transmission of the machine and the engine.

This connection is clearly visible after the construction of velocity triangles (figure 1). In the coordinate system “point radius - linear velocity”, the angular velocity of the link is proportional to the tangent of the angle of inclination of the beam originating from the origin, which is shown at figure 1.
The rays representing the laws of velocity distribution intersect at points corresponding to the radii of the poles of engagement.

The dependencies in figure 1 are shown for positive values \( \omega_1 \) and \( \omega_4 \) at the same time when \( \omega_3 \) has a negative direction.

The fulfillment of this condition of a different direction of speeds is not necessary, but it is intentionally chosen due to the fact that the minimum loss of braking energy of the machine corresponds to the condition of ensuring a constant value of the kinematic energy of the transmission of the machine and the flywheel battery. The fulfillment of this condition requires the presence of the opposite change in the speeds of the differential links.

The resulting expression allows you to relate to each other the speeds of the input and output links of the differential (for example, carrier 4 and the central wheels) to analyze the required power to control the gear ratio.

Another property of the planetary differential is its ability to work in gear mode. There are several ways to achieve this. As an example, consider a gearbox, making one of the moving parts of the differential stationary. We will stop the large central wheel 3 (epicycle) of the single-row differential, the carrier will be assigned as the driven link, and the solar central wheel will be the driving link of the gearbox. Then the gear ratios of the gearbox will be determined by the following relationship:

\[
U_{13}^4 + U_{14}^3 = 1
\]

That is, the sum of the gear ratios for different stopped links is equal to one. Using the property of reversibility, you can organize a reverse gear. Choosing a large central wheel (epicyclic of a single-row differential) by the driven link and stopping (braking) the carrier, we make the solar (small central) wheel - the driving link. Then the gear ratio of the gearbox will be equal to the internal gear ratio of the planetary gear set, that is, it will be negative.

Thus, having determined the internal gear ratio for a particular planetary differential, and it is constant, and having a relationship linking the angular velocities of the three main links of the planetary differential, it is possible to determine the properties of this mechanism [7-10].

In automatic transmissions, the following planetary differential lock property is often used. If the angular velocities of the two differential links are equal, then the angular velocity of the third link will be equal to the angular velocity of these two links. That is, all links in this case will rotate as one. A similar result can be obtained in other cases, when \( \omega_1 = \omega_2 \) and \( \omega_2 = \omega_3 \). Thus, if you install a locking clutch between any two links of the differential, then when it is turned on, the differential will be locked and its gear ratio will be 1.

Thus, planetary differentials, different in purpose, device and characteristics, are used in gearboxes to obtain compact coaxial structures with a variable gear ratio, in gearboxes by convenient control by means of brakes and friction clutches. In addition to gearboxes, this principle has been widely used in automobile differentials, as well as summing kinematic schemes of metal-cutting machines. In modern gearboxes, several planetary differentials can be used to get a wider range of gear ratios. Many automatic transmissions work on this principle. Smoothly changing gear ratios can be obtained in automatic transmissions with a belt variator. Comparison of the characteristics of gearboxes with a stepwise and smooth change in the gear ratio can be done using figure 3a,b.

Figure 3a shows the kinematic characteristic of a speed gearbox connecting the speeds of the input \( \omega_1 \) and output \( \omega_2 \) shafts. Acceleration in the transmission ends when the maximum input shaft speed \( \omega_1_{\text{max}} \) associated with the engine is reached.
After gear shifting, a new acceleration (increase in the speed $\omega_2$ of the output shaft) begins with the minimum engine speed. The process of accelerating the car with gear shifting ends with the achievement of the maximum speed of the transmission $\omega_2$ and the machine. In the selected coordinate system $\omega_1$ and $\omega_2$, the gear ratio is constant and corresponds to the slope of the beam passing through the origin [11-12]. Figure 3b additionally shows the law of change in the gear ratio of a continuously variable gearbox, which is similar to the law of change in the gear ratio with a constant kinetic energy of a machine with a flywheel energy accumulator. On this principle, a flywheel control method is proposed by connecting it to an automatic transmission using a planetary differential with two degrees of mobility.

The transfer of kinematic energy from the transmission of the machine to the flywheel is possible by increasing the gear ratio of the variator. At transmission speed $\omega_2 = 0$ this number tends to infinity. Since no variator can provide an infinitely large gear ratio, it becomes necessary to choose its highest value. It determines the values of the kinetic energy loss of the machine at the end of the energy recovery process when the minimum transmission speed $\omega_2_{\text{min}}$ is reached. This choice determines the value of the recovery efficiency. Thus, the end of the braking process of the machine can occur without energy recovery, that is, with small losses, determined by the limitation of the gear ratio of the variator.

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