The minimal scenario of leptogenesis

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\textbf{Abstract.} We review the main features and results of thermal leptogenesis within the type I seesaw mechanism, the minimal extension of the Standard Model explaining neutrino masses and mixing. After presenting the simplest approach, the vanilla scenario, we discuss various important developments of recent years, such as the inclusion of lepton and heavy neutrino flavour effects, a description beyond a hierarchical heavy neutrino mass spectrum and an improved kinetic description within the density matrix and the closed-time-path formalisms. We also discuss how leptogenesis can ultimately represent an important phenomenological tool to test the seesaw mechanism and the underlying model of new physics.

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1. Introduction: the double side of leptogenesis

A successful model of baryogenesis cannot be realized within the Standard Model (SM) and therefore the observed matter–antimatter asymmetry of the Universe can be regarded as evidence for new physics beyond the SM.

The discovery of neutrino masses and mixing in neutrino oscillation experiments in 1998 [1] showed directly for the first time, in particle physics experiments, that the SM is indeed incomplete, since in the limit of an infinite cutoff it strictly predicts that neutrinos are massless and therefore cannot oscillate.

This discovery has greatly increased the interest in leptogenesis [2, 3], a model of baryogenesis that is a cosmological consequence of the most popular way to extend the SM in order to explain why neutrinos are massive but at the same time much lighter than all the other fermions: the seesaw mechanism [4]. As a matter of fact, leptogenesis realizes a highly non-trivial link between two completely independent experimental observations: the absence of primordial antimatter in the observable Universe and the observation that neutrinos mix and (therefore) have masses. In fact, leptogenesis has a naturally built-in double-sided

4 For some earlier reviews on leptogenesis, see [3].
nature. On the one hand, it describes a very early stage in the history of the Universe characterized by temperatures \(T_{\text{lepton}} \gtrsim 100\text{ GeV}\), much higher than those probed by Big Bang nucleosynthesis, \(T_{\text{BBN}} \sim 0.1-1\text{ MeV}\); on the other hand, it complements low-energy neutrino experiments providing a completely independent phenomenological tool to test models of new physics embedding the seesaw mechanism.

In this paper, we review the main features and results of leptogenesis. Let us give a brief outline of the paper. In section 2 we present the status of low-energy neutrino experiments measuring neutrino masses and mixing parameters, and we introduce the seesaw mechanism, which provides an elegant framework to explain them. In section 3 we discuss the vanilla leptogenesis scenario. In section 4 we show the importance of accounting for flavour effects for a correct calculation of the final asymmetry. In section 5 we discuss the density matrix formalism which properly takes into account decoherence effects, which are crucial for describing the transition from a one-flavoured regime to a fully flavoured regime. In section 6 we relax the assumption of hierarchical right-handed (RH) neutrino mass spectrum, and discuss how the asymmetry can be calculated in the degenerate limit. In section 7 we discuss different ways of improving the kinetic description beyond the density matrix formalism. In section 8 we discuss the other effects (thermal corrections, spectator processes and scatterings) that have been considered and that can give in some cases important corrections. In section 9 we show how leptogenesis represents an important guideline to test the models of new physics. Finally, in section 10 we conclude the paper, outlining the prospects of testing leptogenesis in the future.

2. Neutrino masses and mixing

Neutrino oscillation experiments have established two fundamental properties of neutrinos. The first one is that neutrinos mix. This means that the neutrino weak eigenfields \(\nu_\alpha (\alpha = e, \mu, \tau)\) do not coincide with the neutrino mass eigenfields \(\nu_i (i = 1, 2, 3)\) but are obtained applying to them a unitary transformation described by the \((3 \times 3)\) leptonic mixing matrix \(U\),

\[
\nu_\alpha = \sum_i U_{\alpha i} \nu_i . \tag{1}
\]

The leptonic mixing matrix is usually parameterized in terms of six physical parameters, three mixing angles, \(\theta_{12}, \theta_{13}\) and \(\theta_{23}\), and three phases, two Majorana phases, \(\rho\) and \(\sigma\), and one Dirac phase \(\delta\),

\[
U = \begin{pmatrix}
c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\
s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & c_{23} s_{13} \\
s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -s_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & c_{23} c_{13}
\end{pmatrix}
\text{diag}(e^{i\rho}, 1, e^{i\sigma}) , \tag{2}
\]

where \(s_{ij} \equiv \sin \theta_{ij}\) and \(c_{ij} \equiv \cos \theta_{ij}\). A global analysis [5] of all existing neutrino data, prior to the results of a non-vanishing \(\theta_{13}\) from short baseline reactors, gives \(\theta_{12} = 34^{\circ} \pm 1^{\circ}\) for the solar mixing angle, \(\theta_{23} = 40.4^{\circ} \pm 1.6^{\circ}\) for the atmospheric mixing angle and \(\theta_{13} = 9.0^{\circ} \pm 1.3^{\circ}\) for the reactor mixing angle, where the latter is mainly dominated by the evidence for a non-vanishing \(\theta_{13}\) found by the T2K experiment [6] that confirmed previous hints [7]. Recently, the Daya Bay and Reno short-baseline reactor neutrino experiments, respectively, found \(\theta_{13} = 8.8^{\circ} \pm 0.8^{\circ} \pm 0.3^{\circ}\) [8] and \(9.8^{\circ} \pm 0.6^{\circ} \pm 0.85^{\circ}\) [9], confirming, at more than \(5\sigma\), the previous results.

The second important property established by neutrino oscillation experiments is that neutrinos are massive. More specifically, defining the three neutrino masses in a way that
Figure 1. Neutrino masses \(m_i\) versus the lightest neutrino mass \(m_1\). The three upper bounds (at 95% CL) discussed in the body text from absolute neutrino mass scale phenomenologies are also indicated.

\[ m_1 \leq m_2 \leq m_3, \]

neutrino oscillation experiments measure two mass-squared differences that we can indicate with \(\Delta m^2_{\text{atm}}\) and \(\Delta m^2_{\text{sol}}\) since, historically, the first one was first measured in atmospheric neutrino experiments and the second one in solar neutrino experiments. Two options are currently allowed by previous experiments. A first option is ‘normal ordering’ (NO) and in this case

\[ m_2^2 - m_1^2 = \Delta m^2_{\text{atm}} \quad \text{and} \quad m_3^2 - m_2^2 = \Delta m^2_{\text{sol}}, \]

whereas a second option is represented by ‘inverted ordering’ (IO) and in this case

\[ m_3^2 - m_2^2 = \Delta m^2_{\text{sol}} \quad \text{and} \quad m_2^2 - m_1^2 = \Delta m^2_{\text{atm}}. \]

It is convenient to introduce the atmospheric neutrino mass scale \(m_{\text{atm}} \equiv \sqrt{\Delta m^2_{\text{atm}} + \Delta m^2_{\text{sol}}} = (0.049 \pm 0.001)\) eV and the solar neutrino mass scale \(m_{\text{sol}} \equiv \sqrt{\Delta m^2_{\text{sol}}} = (0.0087 \pm 0.0001)\) eV [5].

The measurements of \(m_{\text{atm}}\) and \(m_{\text{sol}}\) are not sufficient to fix all three neutrino masses. If we express them in terms of the lightest neutrino mass \(m_1\), we can see from figure 1 that while \(m_2 \geq m_{\text{sol}}\) and \(m_3 \geq m_{\text{atm}}\), the lightest neutrino mass can be arbitrarily small, implying that the lightest neutrino could even be massless.

The lower limits for \(m_2\) and \(m_3\) are saturated when \(m_1 \ll m_{\text{sol}}\). In this case one has hierarchical neutrino models, either normal and in this case \(m_2 \simeq m_{\text{sol}}\) and \(m_3 \simeq m_{\text{atm}}\), or inverted, and in this case \(m_2 \simeq \sqrt{m^2_{\text{atm}} - m^2_{\text{sol}}} \simeq m_3 \simeq m_{\text{atm}}.\) On the other hand, for \(m_1 \gg m_{\text{atm}}\) one obtains the limit of quasi-degenerate neutrinos when all three masses can be arbitrarily close to each other.

However, the lightest neutrino mass is upper bounded by absolute neutrino mass scale experiments. Tritium beta decay experiments [10] place an upper bound on the effective electron neutrino mass \(m_{\nu_e} \lesssim 2\) eV (95% CL) that translates into the same upper bound on \(m_1\). This is
derived from model-independent kinematic considerations that apply independently of whether neutrinos have a Dirac or Majorana nature.

Neutrinoless double beta decay (0νββ) experiments place a more stringent upper bound on the effective 0νββ Majorana neutrino mass, \( m_{ee} \lesssim 0.34–0.78 \) eV (95% CL) as obtained by the CUORICINO experiment [11].\(^5\) It translates into the following upper bound on \( m_1 \) [12]:

\[
m_1 \lesssim m_{ee}/(\cos 2\theta_{12} \cos^2 \theta_{13} - \sin^2 \theta_{13}) \lesssim 3.45 \ m_{ee} \lesssim (1.2–2.7) \text{ eV \ (95\% CL)}.
\]

Here, the wide range is due to theoretical uncertainties in the calculation of the involved nuclear matrix elements. However, this upper bound applies only if neutrinos are of Majorana nature, which is the relevant case for us, since the seesaw mechanism predicts Majorana neutrinos.

From cosmological observations, within the ΛCDM model, one obtains a very stringent upper bound on the sum of the neutrino masses [15]

\[
\sum_i m_i \lesssim 0.58 \text{ eV \ (95\% CL)}.
\]

This translates into the most stringent upper bound we currently have on the lightest neutrino mass \( m_1 \lesssim 0.19 \) eV (95% CL), an upper bound that almost excludes quasi-degenerate neutrino models.\(^6\)

A minimal extension of the SM, able to explain not only why neutrinos are massive but also why they are much lighter than all the other massive fermions, is represented by the seesaw mechanism [4]. In the minimal type I version, one adds RH neutrinos \( N_{iR} \) to the SM Lagrangian with Yukawa couplings \( h \) and a Majorana mass term that violates lepton number

\[
\mathcal{L} = \mathcal{L}_{SM} + i \bar{N}_{iR} \gamma_\mu \partial^\mu N_{iR} - \bar{\ell}_a \lambda_{a i} N_{iR} \Phi - \frac{1}{2} M_i \bar{N}_{iR}^c N_{iR} + h.c.
\]

For definiteness, we will consider the case of three RH neutrinos \((i = 1, 2, 3)\). This is the most attractive case corresponding to having one RH neutrino for each generation, such as for example nicely predicted by \( SO(10) \) grand-unified models. Note, however, that all current data from low-energy neutrino experiments are also consistent with a more minimal model with only two RH neutrinos that will be discussed in section 9.

After spontaneous symmetry breaking, a Dirac mass term \( m_D = v \ H \) is generated by the Higgs vev \( v \). In the seesaw limit, \( M \gg m_D \), the spectrum of neutrino masses splits into a light set given by the eigenvalues \( m_1 < m_2 < m_3 \) of the neutrino mass matrix

\[
m_\nu = -m_D \frac{1}{M} m_D^T
\]

and into a heavy set \( M_1 < M_2 < M_3 \) coinciding to a good approximation with the eigenvalues of the Majorana mass matrix corresponding to eigenstates \( N_i \simeq N_{iR} + N_{iR}^c \). The symmetric neutrino mass matrix \( m_\nu \) is diagonalized by a unitary matrix \( U \),

\[
D_m = \text{diag}(m_1, m_2, m_3) = -U_\nu^T m_\nu U_\nu^*.
\]

In a basis where the charged lepton mass matrix is diagonal, \( U_\nu \) coincides with the leptonic mixing matrix \( U \). In this way, the lightness of ordinary neutrinos is explained as just an algebraic

\(^5\) At 90% CL the CUORICINO result is \( m_{ee} \lesssim 0.27–0.57 \) eV. For comparison, the bound from the Heidelberg–Moscow experiment is \( m_{ee} \lesssim 0.21–0.53 \) eV (90% CL) [13], whereas the EXO-200 experiment has recently found the upper bound \( m_{ee} \lesssim 0.14–0.38 \) eV (90% CL) [14].

\(^6\) For a more general discussion of neutrino mass bounds, in particular on theoretical assumptions and uncertainties, see, for example, [12, 16, 17].

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by-product. If the largest eigenvalue in the Dirac neutrino mass matrix is assumed to be of the order of the electroweak scale, as for the other massive fermions, then the atmospheric neutrino mass scale $m_{\text{atm}}$ can be naturally reproduced for $M_3 \sim 10^{14-10^{15}}$ GeV, very close to the grand-unified scale. This is the minimal version of the seesaw mechanism, often indicated as the type I seesaw mechanism.

The seesaw formula \((8)\) can be recast as an orthogonality condition for a matrix $\Omega$ that in a basis where simultaneously the charged lepton mass matrix and the Majorana mass matrix are diagonal provides a useful parametrization of the neutrino Dirac mass matrix \[\text{18}\]

$$m_D = U D_m^{1/2} \Omega D_M^{1/2},$$

where $D_M \equiv \text{diag}(M_1, M_2, M_3)$. The orthogonal matrix contains six independent parameters and, together with the neutrino masses, determines the properties of the heavy RH neutrinos, the three lifetimes and the three total CP asymmetries. In this way, one can easily see that the total number of additional parameters introduced by the seesaw Lagrangian is 18 in the case of three RH neutrinos. The orthogonal parametrization is quite useful since, given a model that specifies $m_D$ and the three RH neutrino masses $M_i$, one can easily impose both the experimental information on the nine low-energy neutrino parameters (six contained in $U$ and the three $m_i$) and, as we will see, also the requirement of successful leptogenesis.

3. The simplest scenario: vanilla leptogenesis

Leptogenesis belongs to a class of models of baryogenesis where the asymmetry is generated from the out-of-equilibrium decays of very heavy particles. Interestingly, this is the same class to which the first model proposed by Sakharov belongs. This class of models became very popular with the advent of Grand Unified Theories (GUT) that provided a well-defined and motivated framework. In GUT baryogenesis models, the very heavy decaying particles generating the asymmetry are the same new gauge bosons predicted by the GUTs. However, the final asymmetry depends on too many untestable parameters, so that imposing successful baryogenesis does not lead to compelling experimental predictions. This lack of predictability is made even stronger considering that the decaying particles are too heavy to be produced thermally and one has, therefore, to invoke a non-thermal production mechanism of the gauge bosons. This is because while the mass of the gauge bosons is about the GUT scale, $M_X \sim 10^{15-10^{16}}$ GeV, the reheating temperature at the end of inflation $T_{\text{reh}}$ cannot be higher than $\sim 10^{15}$ GeV from cosmic microwave background (CMB) observations, where the reheating temperature is the value of the temperature at the beginning of the radiation-dominated regime after inflation \[\text{19}\].

The minimal (and original) version of leptogenesis, which we discuss in this review, is based on the type I seesaw mechanism \[\text{2}\], where the asymmetry is produced by the three heavy RH neutrinos predicted by the seesaw mechanism and whose masses can have values orders of magnitude below the upper bound on the $T_{\text{reh}} \lesssim 10^{15}$ GeV from inflation and CMB observations. We will call ‘minimal leptogenesis scenarios’ those scenarios based on a type I seesaw mechanism and on thermal production of the RH neutrinos, implying that $T_{\text{reh}}$ cannot be too much below (at least) the lightest RH neutrino mass $M_1$. At these high temperatures, RH neutrinos can be produced by Yukawa interactions of leptons and Higgs bosons in the thermal

\[\text{7}\]

For a description of leptogenesis from RH neutrino oscillations, see \[\text{20}\] in this focus issue. For leptogenesis scenarios based on models beyond the type I seesaw mechanism, see \[\text{21}\] also in this focus issue.
bath. A solution of the Boltzmann equations shows that, more exactly, \( T_{\text{reh}} \) can be even up to ten times lower than \( M_1 \), the exact value depending on the strength of the coupling \([22]\).

After their production, RH neutrinos decay either into lepton and Higgs doublets \( N_i \rightarrow \ell_i + \Phi \) (when leptogenesis takes place, the electroweak symmetry is not broken) with a decay rate \( \Gamma \) or into anti-lepton and conjugate Higgs doublets \( N_i \rightarrow \bar{\ell}_i + \Phi^\dagger \) with a decay rate \( \bar{\Gamma} \). Both inverse processes and decays violate lepton number (\( \Delta L = 1 \)) and in general \( CP \) as well. It is also important to note that they also violate the \( B - L \) asymmetry (\( \Delta (B - L) = 1 \)). At temperature \( T \gg 100 \text{ GeV} \), non-perturbative SM processes called sphalerons are in equilibrium \([23]\). They violate both lepton and baryon number while they still conserve \( B - L \). Hence the lepton asymmetry produced in the elementary processes is reprocessed in such a way that \( \sim 1/3 \) of the \( B - L \) asymmetry is in the form of a baryon number, while \( \sim -2/3 \) of the \( B - L \) asymmetry is in the form of a lepton number \([24]\). Therefore, two of the Sakharov conditions are satisfied. The third Sakharov condition, departure from thermal equilibrium is also satisfied since a fraction of the decays occur out of equilibrium, implying that part of the asymmetry survives the washout from inverse processes.

In the most general approach, even within minimal leptogenesis, the asymmetry depends on all 18 seesaw parameters and the calculation itself presents different technical difficulties. However, there is a simplified scenario \([2, 22, 25–30]\) that grasps most of the main features of leptogenesis and is able to highlight important connections with the low-energy neutrino parameters in an approximate way. We will refer to this scenario as ‘vanilla leptogenesis’. Let us discuss the main assumptions and approximations.

The first assumption is the one-flavour approximation, where the flavour composition of the leptons produced by (or producing) the RH neutrinos has no influence on the final value of the asymmetry and can therefore be neglected. It also assumes that leptons produced by different RH neutrinos can be treated, for all practical purposes, as having the same flavour (i.e. \( \ell_1 = \ell_2 = \ell_3 = \ell \)). This is equivalent to saying that what counts is just the asymmetry between the total number of leptons and the total number of anti-leptons irrespective of whether these leptons are electron, muon or tauon doublets, and irrespective of the RH neutrino that generated it. Under this assumption, the final \( B - L \) asymmetry is the sum of two contributions

\[
N_{B-L}^f = N_{B-L}^{\text{pre-ex.f}} + N_{B-L}^{\text{lep.f}}. \tag{11}
\]

The first term on the right-hand side, \( N_{B-L}^{\text{pre-ex.f}} \), is the residual value of a possible pre-existing asymmetry generated by some external mechanism prior to the onset of leptogenesis. It has to be regarded as a possible external contribution whose initial value would be set up by some independent source of leptogenesis. Therefore, if leptogenesis is responsible for the observed asymmetry, this term has to be negligible.

The second term is the genuine leptogenesis contribution to the final asymmetry and is given by the sum of the three contributions from each RH neutrino species,

\[
N_{B-L}^{\text{lep.f}} = \sum_i \varepsilon_i \kappa_{i}^f(K_1, K_2, K_3). \tag{12}
\]

Each contribution is the product of the total \( CP \) asymmetry \( \varepsilon_i \) times the final value of the efficiency factor, \( \kappa_{i}^f(K_1, K_2, K_3) \), which depends on the decay parameters \( K_i \), defined as \( K_i \equiv \Gamma_{D,i}(T = 0)/H(T = M_i) \), where \( H \) is the Hubble expansion rate and the total decay rates are \( \Gamma_{D,i} \equiv \Gamma_i + \bar{\Gamma}_i \). The total \( CP \) asymmetries are defined as

\[
\varepsilon_i \equiv -\frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i}, \tag{13}
\]
and a perturbative calculation from the interference of tree level with one-loop self-energy and vertex diagrams gives [31–33]

\[ \varepsilon_i = \frac{3}{16\pi} \sum_{j \neq i} \text{Im} \left[ \frac{(h^+ h)^2}{(h^+ h)_{ii}} \right] \frac{\xi(x_j/x_i)}{\sqrt{x_j/x_i}}, \]

having introduced

\[ \xi(x) = \frac{2}{3} x \left[ (1 + x) \ln \left( \frac{1 + x}{x} \right) - \frac{2 - x}{1 - x} \right], \]

and defined \( x_j \equiv M_j^2/M_1^2 \). The efficiency factors are computed by solving a simple set of Boltzmann equations integrated over the momenta (rate equations)

\[ \frac{dN_{N_i}}{dz} = -D_i (N_{N_i} - N_{N_i}^{\text{eq}}), \quad i = 1, 2, 3, \]

\[ \frac{dN_{B-L}}{dz} = \sum_{i=1}^{3} \varepsilon_i D_i (N_{N_i} - N_{N_i}^{\text{eq}}) - N_{B-L} \left[ \Delta W(z) + \sum_i W_i^{\text{ID}}(z) \right], \]

where \( z \equiv M_1/T \) and where we denote as \( N_X \) any particle number or asymmetry \( X \) calculated in a portion of co-moving volume containing one heavy neutrino in ultra-relativistic thermal equilibrium, so that \( N_{N_i}^{\text{eq}}(T \gg M_i) = 1 \). With this convention, the predicted baryon-to-photon ratio \( \eta_B \) is related to the final value of the final \( B - L \) asymmetry by the equation

\[ \eta_B = a_{\text{sph}} \frac{N_{B-L}^{\text{rec}}}{N_{\gamma}^{\text{rec}}} \simeq 0.96 \times 10^{-2} N_{B-L}^{\text{rec}}, \]

where \( N_{\gamma}^{\text{rec}} \simeq 37 \) and \( a_{\text{sph}} = 28/79 \). Defining \( z_i \equiv z \sqrt{x_i} \), the decay factors are defined as

\[ D_i \equiv \frac{\Gamma_{D,i}}{H z} = K_i x_i \frac{1}{\gamma_i}, \]

where the thermally averaged dilation factors \( \langle 1/\gamma \rangle \) are given by the ratio \( K_1(z)/K_2(z) \) of the modified Bessel functions. After a proper subtraction of the resonant contribution from \( \Delta L = 2 \) processes in order to avoid double counting [29, 34], the inverse decay washout terms are simply given by

\[ W_i^{\text{ID}}(z) = \frac{1}{4} K_i \sqrt{x_i} K_1(z_i) z_i^3. \]

The washout term \( \Delta W(z) \) is the non-resonant \( \Delta L = 2 \) processes contribution. It gives a non-negligible effect only at \( z \gg 1 \) and in this case it can be approximated as [27]

\[ \Delta W(z) \simeq \frac{\omega}{z^2} \left( \frac{M_1}{10^{10} \text{ GeV}} \right) \left( \frac{m_i^2}{eV^2} \right), \]

where \( \omega \simeq 0.186 \) and \( m_i^2 \equiv m_1^2 + m_2^2 + m_3^2 \). The efficiency factors can be simply calculated analytically as

\[ \kappa_i^\xi(K_i) = -\int_{z_{\text{in}}}^{\infty} dz' \frac{dN_{N_i}}{dz'} e^{-\int_{z'}^{\infty} dz'' \left[ \Delta W(z'') + \sum_i W_i^{\text{ID}}(z''; K_i) \right]}. \]

The second assumption is that the RH neutrino mass spectrum is hierarchical with \( M_2 \gtrsim 3M_1 \).
Under these two assumptions (i.e. the unflavoured description plus the hierarchical RH neutrino mass spectrum) and barring a particular case, which we will discuss later on, the final asymmetry is typically produced dominantly by the lightest RH neutrino out-of-equilibrium decays, in such a way that the sum in equation (11) can be approximated by the first term \((i = 1)\), explicitly

\[
N^i_{B-L} \simeq \varepsilon_1 \kappa^i_1 (K_1),
\]

so that an ‘\(N_1\)-dominated scenario’ is realized. This happens either because \(|\varepsilon_{2,3}| \ll |\varepsilon_1|\) or because the asymmetry initially produced by the \(N_{2,3}\) decays is afterwards washed out by the lightest RH neutrino inverse processes such that \(\kappa^j_{2,3}(K_{2,3}) \ll \kappa^1_i (K_1)\). Indeed, if we denote as \(N^{(2,3)}_{B-L}(T \gtrsim M_i)\) the contribution to the \(N_{B-L}\) asymmetry from the two heavier RH neutrinos prior to the lightest RH neutrino washout, the final values are given simply by

\[
N^{(2,3),f}_{B-L} = N^{(2,3)}_{B-L}(T \gtrsim M_i) \ e^{-\frac{2\pi}{\kappa_1}} \kappa_i.
\]

The same exponential washout factor also suppresses the residual value of a possible pre-existing asymmetry. Therefore, it is sufficient to impose the strong washout condition \(K_1 \gg 1\) for the pre-existing asymmetry and for the contribution from heavier RH neutrinos to be negligible. The decay parameters can be expressed in terms of the seesaw parameters as

\[
K_i = \frac{(h^ih)_{ii} v^2}{M_i m_*},
\]

where \(m_* \simeq 1.1 \times 10^{-3} \text{ eV}\) is the equilibrium neutrino mass [26, 28]. It is therefore quite straightforward that, barring cancellations in the seesaw formula, one typically has \(K_1 \sim K_{\text{sol}} - K_{\text{atm}} \simeq 10-50 \gg 1\), where \(K_{\text{sol(atm)}} \equiv m_{\text{sol(atm)}}/m_*\). The same condition also guarantees that the final asymmetry is independent of the initial \(N_1\) abundance. It is then quite suggestive that the measured values of \(m_{\text{sol}}\) and \(m_{\text{atm}}\) have just the right values to produce a washout that is strong enough to guarantee independence of the initial conditions but still not strong enough to prevent successful leptogenesis [28]. This leptogenesis conspiracy between experimental results and theoretical predictions is the main reason for the success of leptogenesis during the last few years.

There is a particular case when \(K_1 \gg 1\) does not hold, and in this case the final asymmetry can be dominated by the contribution from the next-to-lightest RH neutrinos [35]. For this case one still has \(K_2 \gg 1\), so that the independence of the initial conditions holds nonetheless. For the time being, as an additional third assumption, we will not consider this particular case.

If one excludes cancellations among the different terms contributing to the neutrino masses in the seesaw formula, one obtains the Davidson–Ibarra upper bound on the lightest RH neutrino \(CP\) asymmetry [36]

\[
\varepsilon_1 \leq \varepsilon_1^{\text{max}} \simeq 10^{-6} \ M_1 \frac{m_{\text{atm}}}{10^{10} \text{ GeV}} \frac{m_{\text{atm}}}{m_1 + m_3}.
\]

Imposing \(\eta^\text{max}_{\text{IB}} \simeq 0.01\ \varepsilon_1^{\text{max}} \kappa_1 \sim \eta^\text{CMB}_{\text{IB}}\), one obtains the allowed region in the plane \((m_1, M_1)\) shown in the left panel of figure 2. One can notice the existence of an upper bound on the light neutrino masses \(m_1 \lesssim 0.12 \text{ eV}\) [28, 37], incompatible with quasi-degenerate neutrino mass models, and a lower bound on \(M_1 \gtrsim 3 \times 10^9 \text{ GeV}\) [28, 36, 37], implying a lower bound on the reheat temperature \(T_{\text{reh}} \gtrsim 10^9 \text{ GeV}\) [22].

An important feature of vanilla leptogenesis is that the final asymmetry does not directly depend on the parameters of leptonic mixing matrix \(U\). That implies that one cannot establish
a direct model-independent connection. In particular, the discovery of \(CP\) violation in neutrino mixing would not be a smoking gun for leptogenesis and, vice versa, a non-discovery would not rule out leptogenesis. However, within more restricted scenarios, for example imposing some conditions on the neutrino Dirac mass matrix, links can emerge. In section 9 we will discuss in detail the interesting case of \(SO(10)\)-inspired models.

Over the last few years, different directions beyond the vanilla leptogenesis scenario have been explored, usually aiming at evading the above-mentioned bounds. For example, it was noticed that the Davidson–Ibarra bound equation (26) is, strictly speaking, evaded by an extra term contribution \(\Delta \varepsilon_1\) to the total \(CP\) asymmetry [30, 38]. However, this extra term vanishes if \(M_3 \simeq M_2\) and is suppressed as \(\Delta \varepsilon_1 \propto (M_1/M_2)^2\). It can significantly relax the bounds on neutrino masses only when \(|\Omega_{ij}|^2 \gtrsim (M_2/M_1)^2\), implying cancellations with a certain degree of fine-tuning in the seesaw formula for the neutrino masses. For example, in usual models with \(|\Omega_{ij}|^2 \lesssim 10\), this extra contribution can be safely neglected for \(M_2 \gtrsim 10M_1\).

The development that proved to have the most important impact on the final asymmetry, compared to a calculation within the vanilla scenario, is certainly the inclusion of flavour effects. For this reason, we discuss them in detail in the next section.

4. The importance of flavour

The inclusion of flavour effects provides the most significant modification in the calculation of the final asymmetry compared to the vanilla scenario. Two kinds of flavour effects are neglected in the vanilla scenario: heavy neutrino flavour effects, how heavier RH neutrinos...
can contribute to the final asymmetry, and lepton flavour effects, how the flavour composition of the lepton quantum states produced in the RH neutrino decays affects the calculation of the final asymmetry. We first discuss the two effects separately and then show how their interplay can have very interesting consequences.

4.1. Heavy neutrino flavour effects

In the vanilla scenario the contribution to the final asymmetry from the heavier RH neutrinos is negligible because either the $CP$ asymmetries are suppressed in the hierarchical limit compared to $\varepsilon_1^{\text{max}}$ (cf equation (26)) and/or because, even assuming that a sizeable asymmetry (compared to the observed one) is produced at $T \sim M_{2,3}$, it is later on washed out by the lightest RH neutrino inverse processes.

However, as we anticipated, there is a particular case when, even neglecting the lepton flavour composition and assuming a hierarchical heavy neutrino mass spectrum, the contribution to the final asymmetry from next-to-lightest RH neutrino decays can be dominant and explain the observed asymmetry [35]. This case corresponds to a particular choice of the orthogonal matrix such that $N_1$ is so weakly coupled (corresponding to $K_1 \ll 1$) that its washout can be neglected. For the same choice of parameters, the $N_2$ total $CP$ asymmetry $\varepsilon_2$ is unsuppressed if $M_3 \lesssim 10^{15}$ GeV. In this case an $N_2$-dominated scenario is realized. Note that in this case the existence of a third (heavier) RH neutrino species is crucial in order to have a sizeable $\varepsilon_2$.

The contribution from the two heavier RH neutrino species is also important in the quasi-degenerate limit when $\delta_i \equiv (M_i - M_1)/M_1 \ll 1$, $i = 2, 3$. In this case the $CP$ asymmetries $\varepsilon_{2,3}$ are not suppressed, and the washout from the lighter RH neutrino species is moderate, with no exponential prefactor [39, 40].

4.2. Lepton flavour effects

The importance and generality of flavour effects in leptogenesis is fully highlighted in [41, 42]. Their role was first discussed in [26] and included in specific scenarios in [43–45].

For the time being, let us continue to assume that the final asymmetry is dominantly produced from the decays of the lightest RH neutrinos $N_1$, neglecting the contribution from the decays of the heavier RH neutrinos $N_2$ and $N_3$. If $M_1 \gg 10^{12}$ GeV, the flavour composition of the quantum states of the leptons produced from $N_1$ decays has no influence on the final asymmetry and a one-flavour regime holds [26, 41, 42]. This is because the lepton quantum states evolve coherently between the production from a $N_1$-decay and a subsequent inverse decay with a Higgs boson. In this way, the lepton flavour composition does not play any role.

However, if $10^{12} \gtrsim M_1 \gtrsim 10^9$ GeV, during the relevant period of generation of the asymmetry, the produced lepton quantum states will, on average, have an interaction with RH tauons before undergoing the subsequent inverse decay. In this way the tauon component of the lepton quantum states is measured by the thermal bath and the coherent evolution breaks down [26, 41, 42]. Therefore, at the subsequent inverse decays, the lepton quantum states are an incoherent mixture of a tauon component and of a (still coherent) superposition of an electron and a muon component that we can indicate with $\tau^\perp$.

The fraction of asymmetry stored in each flavour component is not proportional, in general, to the branching ratio of that component. This implies that the two flavour asymmetries, the tauon and the $\tau^\perp$ components, evolve differently and have to be calculated separately. In this
way, the resulting final asymmetry can considerably differ from the result in the one-flavour regime. This can indeed be approximated by the expression

\[ N_{B-L}^\ell \simeq 2 \varepsilon_1 \kappa(K_1) + \frac{\Delta p_{1\tau}}{2} \left[ \kappa(K_{1\tau^-}) - \kappa(K_{1\tau}) \right], \quad (27) \]

where \( K_{1\alpha} \equiv p_{1\alpha}^0 \), the \( p_{1\alpha}^0 \)'s (\( \alpha = \tau, \tau^- \)) are the tree level probabilities that the leptons \( \ell_1 \) and the anti-leptons \( \bar{\ell}_1 \) produced in the decays of the \( N_1 \)'s are in the flavour \( \alpha \), while \( \Delta p_{1\tau} \) is the difference between the probability of finding \( \ell_1 \) in the flavour \( \tau \) and that of finding \( \bar{\ell}_1 \) in the flavour \( \bar{\tau} \). If we compare this expression with the one-flavour regime result equation (23), one can see that if \( \Delta p_{1\tau} = 0 \) (or if \( K_{1\tau} = K_{1\tau^-} \)), then the final symmetry is enhanced only by a factor of 2. However, in general, leptons and anti-leptons have different flavour compositions \([26, 42]\) and in this case \( \Delta p_{1\tau} \neq 0 \), and the final asymmetry can be much higher than in the one-flavour regime. The most extreme case is when the total \( B - L \) number is conserved (i.e. \( \varepsilon_1 = 0 \)), and still the second term in equation (27) can be non-vanishing \([42]\) and even explain the observed asymmetry \([50]\).

If \( M_\ell \lesssim 10^9 \) GeV, even the coherence of the \( \tau^- \) component is broken by the muon interactions between decays and inverse decays and a full three-flavour regime applies. In the intermediate regimes, a density matrix formalism is necessary to properly describe decoherence \([41, 46-48]\).

We can briefly say that lepton flavour effects induce three major consequences that are all encoded in expression (27) valid in the two-fully flavoured regime. (i) The washout can be considerably lower than in the unflavoured regime \([41, 42]\), since one can have that the asymmetry is dominantly produced in a flavour \( \alpha \) with \( K_{1\alpha} \ll K_1 \). (ii) The leptonic mixing matrix enters directly the calculation of the final asymmetry; more specifically and in particular the low-energy phases contribute as a second source of \( CP \) violation in the flavoured \( CP \) asymmetries \([42, 49-51]\). As an interesting phenomenological consequence, the same source of \( CP \) violation that could take place in neutrino oscillations could be sufficient to explain the observed asymmetry \([50, 51]\), although under quite stringent conditions on the RH neutrino mass spectrum and with an exact determination requiring a density matrix calculation \([52]\). Note that this is a particular case realizing the above-mentioned scenario with \( \varepsilon_1 = 0 \) \([42]\). This problem becomes particularly interesting in the light of the recent discovery of a non-vanishing \( \theta_{13} \) angle \([6-9]\), a necessary condition to have \( CP \) violation in neutrino oscillations. In particular, a calculation of the lower bound on \( \theta_{13} \) necessarily requires the use of density matrix equations. (iii) The flavoured \( CP \) asymmetries, given by

\[ \varepsilon_{ia} = \frac{3}{16\pi (h^\dagger h)_{ii}} \sum_{j \neq i} \left\{ \text{Im} \left[ h_{ai}^* h_{aj} (h^\dagger h)_{ij} \right] \frac{\xi(x_j/x_i)}{\sqrt{x_j/x_i}} + \frac{2}{3(x_j/x_i - 1)} \text{Im} \left[ h_{ai}^* h_{aj} (h^\dagger h)_{ij} \right] \right\}, \quad (28) \]

contain a second term that conserves the total lepton number (it cancels exactly when summing over flavour in the total \( CP \) asymmetry and it, therefore, contributes to \( \Delta p_{1\tau} \) in equation (27) but not to \( \varepsilon_1 \)), and therefore the upper bound in equation (26) does not strictly apply to the flavoured \( CP \) asymmetries. As a consequence, allowing for a mild cancellation in the neutrino mass matrix, corresponding to \( |\Omega_{ij}| \sim 1 \), and also for a mild RH neutrino mass hierarchy \((M_2/M_1 \sim 10)\), the lower bound on \( T_{\text{reh}} \) can be relaxed by about one order of magnitude, down to \( 10^8 \) GeV \([30]\), as shown in the right panel of figure 2. However, for many models, such as sequential dominated models \([53]\), these cancellations do not occur, and lepton flavour effects cannot relax the lower bound on \( T_{\text{reh}} \) \([50]\). One known exception is given by the inverse seesaw
model [54] which naturally explains large cancellations in the neutrino mass matrix and leads to large CP asymmetries thanks to an underlying lepton number symmetry. This leads to the relaxation of the lower bound on \( T_{\text{reh}} \) by up to three orders of magnitude [55, 56] (see a more detailed discussion in the next section).

Whether or not the upper bound \( m_s \lesssim 0.1 \text{ eV} \) on neutrino masses found in the vanilla scenario still holds in a flavoured \( N_1 \)-dominated scenario, is relaxed or even completely evaporates is a controversial topic. It still surely holds in the one-flavour regime for \( M_1 \gtrsim 10^{12} \text{ GeV} \) but it certainly does not hold in the two-fully flavoured regime [30, 41, 57]. However, it was found that the two-fully flavoured regime is not respected at large values \( m_s \gtrsim 0.1 \text{ eV} \) [46]. In [47] it was found that it holds up to \( m_s \sim 2 \text{ eV} \), implying in any case an upper bound much above current experimental bounds and, therefore, uninteresting. In [30], including the information from low-energy neutrino data and accounting for the Higgs asymmetry, it was found again that it holds only up to \( m_s \gtrsim 0.1 \text{ eV} \) and in this case a density matrix approach would be required for a conclusion.

4.3. The interplay between lepton and heavy neutrino flavour effects

As we have seen, when lepton flavour effects are neglected, the possibility that the next-to-lightest RH neutrino decays contribute to the final asymmetry relies on a special case realizing the \( N_2 \)-dominated scenario [35]. On the other hand, when lepton flavour effects are taken into account, the contribution from heavier RH neutrinos cannot be neglected in a much more general situation. Even the contribution from the heaviest RH neutrinos can be sizeable (i.e. explain the observed asymmetry) and has to be taken into account in general.

As a result, the calculation of the final asymmetry becomes much more involved. Assuming hierarchical mass patterns and that the RH neutrino processes occur in one of the three different fully flavoured regimes, one has to consider ten different mass patterns, shown in figure 2, that require specific multi-stage sets of classical Boltzmann equations for the calculation of the final asymmetry.

4.3.1. The (flavoured) \( N_2 \)-dominated scenario. Among these ten RH neutrino mass patterns, for those three where the \( N_1 \) washout occurs in the three-fully flavoured regime, for \( M_1 \ll 10^9 \text{ GeV} \), the final asymmetry has necessarily to be produced by \( N_2 \) either in the one or in the two fully flavoured regime (for \( M_2 \gg 10^9 \text{ GeV} \)). They have some particularly attractive features and realize a ‘flavoured \( N_2 \)-dominated scenario’ [45] (it corresponds to the fifth, sixth and seventh mass patterns in figure 3).

While in the unflavoured approximation the lightest RH neutrino washout yields a global exponential washout factor, when lepton flavour effects are taken into account, the asymmetry produced by the heavier RH neutrinos, at the \( N_1 \) washout, gets distributed into an incoherent mixture of charged lepton flavour eigenstates [45]. It turns out that the \( N_1 \) washout in one of the three flavours is negligible, corresponding to having at least one \( K_{1\alpha} \lesssim 1 \), in quite a wide region of the parameter space [30]. In this way, accounting for flavour effects, the region of applicability of the \( N_2 \)-dominated scenario enlarges considerably, since it is not necessary that \( N_1 \) fully decouples but it is sufficient that it decouples just in a specific lepton flavour [45]. The unflavoured \( N_2 \)-dominated scenario is recovered in the limit where all three \( K_{1\alpha} \lesssim 1 \) and either \( M_2 \gtrsim 10^{12} \text{ GeV} \) or \( 10^{12} \text{ GeV} \gtrsim M_2 \gtrsim 10^9 \text{ GeV} \) and \( K_2 \lesssim 1 \).
In recent years, it has also been realized that, accounting for the Higgs and for the quark asymmetries, the dynamics of the flavour asymmetries couple and the lightest RH neutrino washout in a particular flavour can be circumvented even when $N_1$ is strongly coupled in that flavour [59]. Another interesting effect arising in the $N_2$-dominated scenario is **phantom leptogenesis**. This is a pure quantum-mechanical effect that, for example, allows parts of the electron and of the muon asymmetries, the phantom terms, to undergo a weaker washout at the production than the total asymmetry. It has recently been shown that phantom terms associated with an RH neutrino species $N_i$ with $M_i \gg 10^9$ GeV are present not just in the $N_2$-dominated scenario [60]. However, it should be noted that phantom terms produced by the lightest RH neutrinos cancel each other and thus do not contribute to the final asymmetry, although they can induce flavoured asymmetries much larger than the total asymmetry, something potentially relevant in active-sterile neutrino oscillations [61].

### 4.3.2. Heavy neutrino flavour projection

Even assuming a strong RH neutrino mass hierarchy, coupled $N_1$ in all lepton flavours ($K_{1\alpha} \gg 1$ for any $\alpha$) and $M_1 \gtrsim 10^{12}$ GeV (it corresponds to the first panel in figure 2), the asymmetry produced by the heavier RH neutrino decays at $T \sim M_i$, in particular by the $N_2$ decays, can be large enough to explain the observed asymmetry by avoiding most of the washout from the lightest RH neutrino processes. This is because, in general, there is an ‘orthogonal’ component that escapes the $N_1$ washout [26, 62], while the remaining ‘parallel’ component undergoes the usual exponential washout. For a mild mass hierarchy, $\delta_3 \lesssim 10$, even the asymmetry produced by the $N_3$ decays can be large enough to explain the observed asymmetry and escape the $N_1$ and $N_2$ washout. Heavy neutrino flavour projection is also occurring when $10^{12} \gtrsim M_1 \gtrsim 10^9$ GeV, in this case in the $e-\mu$ plane.

When the effect of heavy neutrino flavour projection is taken into account jointly with an additional contribution to the flavoured $CP$ asymmetries $\varepsilon_{2\alpha}$ that is not suppressed when...
the heaviest RH neutrino mass $M_3 \gtrsim 10^{15}$ GeV, this can lead to the possibility of a dominant contribution from the next-to-lightest RH neutrinos even in an effective two RH neutrino model that can be regarded as a limit case in a (more appealing) three RH neutrino case with $M_3 \gtrsim 10^{15}$ GeV [59].

4.3.3. The problem of the initial conditions in flavoured leptogenesis. As we have seen, in the vanilla scenario the unflavoured assumption reduces the problem of the dependence on the initial conditions to simply imposing the strong washout condition $K_1 \gg 1$. In other words, there is a full equivalence between strong washout and independence of the initial conditions.

When (lepton and heavy neutrino) flavour effects are considered, the situation is much more involved, and for example, imposing strong washout conditions on all flavoured asymmetries ($K_{i\alpha}$) is not enough to guarantee independence of the initial conditions. Perhaps the most striking consequence is that in a traditional $N_1$-dominated scenario there is no condition that can guarantee independence of the initial conditions. The only possibility to have independence of the initial conditions is represented by a tauon $N_2$-dominated scenario [58], i.e. a scenario where the asymmetry is dominantly produced from the next-to-lightest RH neutrinos, and therefore $M_2 \gg 10^9$ GeV, in the tauon flavour. The condition $M_1 \ll 10^9$ GeV is also important to have projection on the orthonormal three-lepton flavour basis before the lightest RH neutrino washout [62].

5. Density matrix formalism

As explained in the last section, leptogenesis is sensitive to the temperature range in which asymmetry is produced. This determines whether the lepton quantum states produced in RH neutrino decays remain coherent or undergo decoherence and get projected in the flavour space before scattering in inverse processes. Moreover, since the lepton states produced in heavy neutrino decays differ, in general, from the lepton flavour eigenstates, lepton flavour oscillations can also, in principle, arise in a similar way as neutrino oscillations happen in vacuum or in a medium. In order to treat the problem of flavour oscillations and partial loss of decoherence in a consistent way, one has to extend the classical Boltzmann framework to account for these intrinsically quantum effects. The formalism of the density matrix is appropriate for this purpose [63]. The density matrix for leptons with momentum $\mathbf{p}$ is defined as

$$\rho_\ell (\mathbf{p}) = \begin{pmatrix} \langle a_\alpha^\dagger (\mathbf{p}) a_\alpha (\mathbf{p}) \rangle & \langle a_\beta^\dagger (\mathbf{p}) a_\alpha (\mathbf{p}) \rangle \\ \langle a_\alpha^\dagger (\mathbf{p}) a_\beta (\mathbf{p}) \rangle & \langle a_\beta^\dagger (\mathbf{p}) a_\beta (\mathbf{p}) \rangle \end{pmatrix},$$

(29)

with $a$ ($a^\dagger$) denoting the annihilation (creation) operator, i.e. it is the expectation value (to be understood in statistical terms) of the generalized number operator. For anti-leptons one has analogously a density matrix $\rho_\bar{\ell} (\mathbf{p})$. The diagonal elements of the density matrix contain nothing else than the occupation numbers of the two flavoured leptons, and the off-diagonal elements encode flavour correlations.

Let us consider an $N_1$-dominated scenario for simplicity. Moreover, let us consider a momentum integrated description, introducing the matrix of lepton number densities $N_\ell / R^3 = g_\ell \int \frac{d^3 p}{(2\pi)^3} \rho_\ell (\mathbf{p})$, where $R$ is the scale factor. Within the density matrix formalism,
the asymmetry can be calculated from the following density matrix equation in the flavour space \( \tau - \tau' \) [41, 47, 48, 60, 64]:

\[
\frac{dN_{a\bar{b}}^{B-L}}{dz} = \varepsilon_{a\bar{b}}^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - \frac{1}{2} W_1 \left\{ \mathcal{P}^{0(1)} \right\}_{a\bar{b}} + \frac{1}{H z} \left[ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} , N_{\tau+\bar{\tau}} \right]_{a\bar{b}} \\
- \frac{\text{Im}(\Lambda)_{a\bar{b}}}{H z} \left[ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} , N_{\tau+\bar{\tau}} \right]_{a\bar{b}},
\]

where we defined \( N_{\tau+\bar{\tau}} \equiv N_{a\bar{b}}^{\tau} + N_{a\bar{b}}^{\bar{\tau}} \) and where \( \mathcal{P}^{0(1)} \) is a matrix projecting the lepton quantum states along the flavour ‘1’ of a lepton \( \ell_1 \) produced from the decay of an RH neutrino \( N_1 \). The \( CP \) asymmetry matrix is a straightforward generalization of equation (28) [48, 49, 60]. The real and imaginary parts of the tau–lepton self-energy are, respectively, given by [65, 66]

\[
\text{Re}(\Lambda) = \frac{f^2}{64} T \quad \text{and} \quad \text{Im}(\Lambda) = 8 \times 10^{-3} f_t^2 T,
\]

where \( f_t \) is the tauon Yukawa coupling. The commutator structure in the third term on the rhs of equation (30) accounts for oscillations in the flavour space driven by the real part of the self-energy, and the double commutator accounts for damping of the off-diagonal terms driven by the imaginary part of the self-energy.

In order to close the system of equations, we also need an equation for the matrix \( N_{\tau+\bar{\tau}} \), which is given by

\[
\frac{dN_{a\bar{b}}^{\tau+\bar{\tau}}}{dz} = -\frac{\text{Re}(\Lambda)_{a\bar{b}}}{H z} (\sigma_2)_{a\bar{b}} N_{a\bar{b}}^{B-L} - S_k (N_{a\bar{b}}^{\tau+\bar{\tau}} - 2 N_{a\bar{b}}^{eq} \delta_{a\bar{b}}),
\]

where \( S_k \equiv \Gamma_k / (H z) \) accounts for gauge interactions. As shown in [48], this term has the effect of damping the flavour oscillations. This can be understood by noting that gauge interactions force \( N_{a\bar{b}}^{\tau+\bar{\tau}} \equiv 2 N_{a\bar{b}}^{eq} \delta_{a\bar{b}} \), which in turn renders the oscillatory term equation (30) negligible.

Equation (30) should be solved in any of the intermediate regimes where lepton states are partially coherent. Actually, the range of relevance of the density matrix equation might be more important than previously believed. Indeed, it was found in [48] that in order to recover the unflavoured regime, one should have masses well above \( 10^{13} \) GeV.

As already discussed, the contribution from heavier RH neutrinos cannot be neglected in general. Therefore, the density matrix equation (30) should be extended to account for such effects; this was done in [60], where a general equation was presented, valid for any RH neutrino decays and any temperature range. Flavour projection effects as well as phantom terms are readily taken into account in this framework.

6. Limit of quasi-degenerate heavy neutrinos

If \( \delta_2 \ll 1 \), the \( CP \) asymmetries \( \varepsilon_{1,2} \) get resonantly enhanced as \( \varepsilon_{1,2} \propto 1/\delta_2 \) [31–33]. If, more stringently, \( \delta_2 \lesssim 10^{-2} \), then \( \eta_B \propto 1/\delta_2 \) and the degenerate limit is obtained [40]. In this limit the lower bounds on \( M_1 \) and on \( T_{\text{reh}} \) get relaxed proportionally to \( \delta_2 \) and at the resonance they completely disappear [39, 67]. The upper bound on \( m_1 \) also disappears in this extreme case. In a more realistic case where the degeneracy of the RH neutrino masses is comparable to the degeneracy of the light neutrino masses, as typically occurring in models with flavour symmetries, the upper bound \( m_1 \lesssim 0.1 \) eV obtained in the hierarchical case imposing the validity

\[
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Boltzmann equations gets relaxed to \( m_1 \lesssim 0.4 \text{ eV} \) [30, 38] in the case of \( M_3 \gg M_2 \), while it is basically unchanged if \( M_3 = M_1 \) [30]. The difference is due to the fact that the relaxation is to be ascribed mainly to the extra term in the \( CP \) asymmetry mentioned at the end of section 3. This extra term, subdominant in the hierarchical case, can become dominant (if \( M_3 \gg M_2 \)) in the quasi-degenerate case \( (\delta_2 \equiv (M_2 - M_1)/M_1 \ll 1) \) and grows with the absolute neutrino mass scale instead of being suppressed as the usual term [38]. On the other hand, this extra term vanishes exactly when \( M_2 = M_1 \), a more reasonable assumption for \( \delta_2 \ll 1 \).

In the full three-flavour regime, the contributions from all quasi-degenerate RH neutrinos should be taken into account and in this case the final asymmetry can be calculated as

\[
N_{B-L}^f = \sum_{i,\alpha} \varepsilon_{i\alpha} \kappa \left( \sum_j K_{j\alpha} \right). \tag{33}
\]

Note that, for each lepton flavour \( \alpha \), the washout in the degenerate limit is described by the sum of the flavoured decay parameters for each RH neutrino species.

The simplest way to obtain a quasi-degenerate RH neutrino mass spectrum is to postulate the existence of a slightly broken lepton number symmetry in the lepton sector [68–70]. Assuming the existence of only two RH neutrinos at first for simplicity, it is possible to write down Yukawa couplings and a Majorana mass term in a way that conserves lepton number. In the ‘flavour’ basis, which we denote by a prime, one RH neutrino can be assigned lepton number +1, and the other can be assigned −1, so that the seesaw mass matrix from equation (7) takes the form

\[
M^\nu = \begin{pmatrix}
0 & h'_{\alpha_1} v^2 & 0 \\
h'_{\alpha_1} v^2 & 0 & M \\
0 & M & 0
\end{pmatrix}, \tag{34}
\]

which conserves lepton number. Rotating to the RH neutrino mass basis, one finds that

\[
M^\nu = \begin{pmatrix}
0 & h'_{\alpha_1} v^2 & i h'_{\alpha_1} v^2 \\
h'_{\alpha_1} v^2 & M & 0 \\
0 & M & 0
\end{pmatrix} . \tag{35}
\]

It can be seen that the two RH neutrinos are exactly degenerate in this limit, and one can easily show that the neutrino mass matrix \( m_\nu \) in equation (8) vanishes identically. In other words, having non-zero neutrino masses requires a small breaking of the lepton number symmetry, which automatically splits the two RH neutrinos into a quasi-Dirac fermion pair.

There are different ways to implement the breaking of the lepton number symmetry, thus generating non-zero neutrino masses. For instance, in the above two-RH neutrino model, we can write

\[
M^\nu = \begin{pmatrix}
0 & h'_{\alpha_1} v^2 & \epsilon_{\alpha} v^2 \\
h'_{\alpha_1} v^2 & \mu_1 & M \\
\epsilon_{\alpha} v^2 & M & \mu_2
\end{pmatrix}, \tag{36}
\]

which implies that the full light neutrino mass matrix is given by [71, 72]

\[
m^\nu \simeq v^2 \left( \frac{1}{M} h^T + h' \frac{1}{M} \epsilon^T \right) - v^2 \left( h' \frac{1}{M} \mu_2 \frac{1}{M} h^T \right), \tag{37}
\]
proportional to the breaking parameters $\epsilon$ and $\mu_2$, as expected. If the lepton number symmetry is broken as in the first term in equation (37), it is referred to as ‘linear’ [73]; if the second term is at work, it is referred to as the inverse/double seesaw mechanism [54]. In the following, we will refer to these models, for simplicity, as ‘inverse seesaw models’. No matter how the lepton number symmetry is broken, the bottom line is that these models fall in the category of low-scale seesaw models, where the size of the Yukawa couplings is not necessarily suppressed if the RH neutrino mass scale is lowered to the electroweak scale. This can lead to interesting non-unitarity effects in neutrino oscillation experiments [74, 75], as well as observable lepton flavour violating rates in experiments looking for $\mu \to e\gamma$, $\tau \to \mu(e)\gamma$, $\mu \to eee$ or $\mu \to e$ conversion in nuclei [44]. Note that, within the orthogonal parameterization equation (10), the inverse seesaw model with two RH neutrinos is obtained in the limit $|\Omega| \to \infty$ [76].

The simple model in equation (34) can be trivially extended to have a third massive RH neutrino of mass still conserving lepton number. It would have zero lepton number and be decoupled from leptons. Without further assumptions, the other RH neutrino mass scale is independent of $M$, and therefore it can be much lower or much higher. Leptogenesis in the context of these low-scale seesaw models was intensively studied in recent years. It was found in [44] that it is possible to have at the same time successful leptogenesis and low-energy observable effects (beyond standard neutrino oscillation phenomenology) such as, for example, charged lepton flavour violation processes. This is possible in a model with three RH neutrinos and with the help of very large flavour effects (hence the name ‘resonant $\tau$-leptogenesis’).

More recently, this possibility was examined in the context of a two-RH neutrino model, and it was found that, in the limit $M_2 - M_1 \gg \Gamma_{1,2}$, leptogenesis does not allow large enough Yukawa couplings to have non-trivial consequences at low energies [76]. This conclusion was re-examined in [72] relaxing the requirement on the mass splitting, and allowing for more extreme quasi-degeneracies. In the limit $M_2 - M_1 \ll \Gamma_{1,2}$, it was argued that the decay parameter $K$ should be replaced by an effective decay parameter $K_{eff} \propto K_\alpha (M_2 - M_1)^2 / \Gamma_i^2$ which depends explicitly on the small breaking of the lepton number symmetry. As a matter of fact, it is expected that the washout of lepton number vanishes in the limit of lepton number conservation, and in [72] it was rigorously derived from the negative interference between the two RH neutrinos exchanged in the $\Delta L = 2$ process $\ell \Phi \to \bar{\ell} \Phi^\dagger$.

A more controversial issue is the behaviour of the $CP$ asymmetry parameter in the limit $M_2 - M_1 \ll \Gamma_{1,2}$, which is directly related to the form of the regulator for the RH neutrino propagator in the self-energy diagram. The reason is that the location of the pole for the RH neutrino determines the maximum enhancement of the $CP$ asymmetry. Following Blanchet et al [72], Anisimov et al [77] use

$$
\epsilon_{ia} \simeq \frac{1}{8\pi(h^i h)_{ii}} \sum_{j \neq i} \left\{ \text{Im} \left[ h_{ai}^* h_{aj} (h^i h)_{ij} \right] + \text{Im} \left[ h_{ai} h_{aj} (h^i h)_{ij} \right] \right\} \frac{M_j^2 - M_i^2}{(M_j^2 - M_i^2)^2 + (M_1 \Gamma_i - M_2 \Gamma_j)^2}
$$

(38)

with the regulator given by the difference $M_i \Gamma_i - M_j \Gamma_j$, whereas Pilaftsis [44] finds the regulator $M_1 \Gamma_1$. Within the inverse seesaw model considered (with two RH neutrinos) the decay rates $\Gamma_1$ and $\Gamma_2$ are predicted to be equal in the lepton number conserving limit. Therefore, a regulator $M_1 \Gamma_1 - M_2 \Gamma_2$ allows for a much larger enhancement of the $CP$ asymmetry than $M_1 \Gamma_1$. 

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implying that leptogenesis is compatible with observable lepton flavour violation rates [72]. The precise value of the $CP$ asymmetry in the regime $M_2 - M_1 \ll \Gamma_{1,2}$, which is especially relevant in inverse seesaw models, is currently still an open issue, whose resolution presumably lies beyond the classical Boltzmann approach (see the next section).

Note that the Weinberg–Nanopoulos [78] requirement, that at least two couplings should violate lepton (or baryon) number to have a generation of asymmetry, is satisfied with both regulators in the inverse seesaw model. Indeed, when four of the five lepton-number-violating couplings in equation (36) are turned off, the $CP$ asymmetry vanishes with both regulators, as it should [72]. However, the limit of all couplings taken simultaneously to zero is not well behaved for the regulator in equation (38), as noted in [79].

A different scenario was considered in [55] but still within the inverse seesaw framework. There, a third almost decoupled RH neutrino was added, with a mass $M_1 \ll M$. Lepton number violation was included in the coupling of the lightest RH neutrino, such that it can decay, producing a lepton asymmetry. Therefore, in this case, the quasi-degenerate pair of RH neutrinos is not responsible for the generation of asymmetry. Nonetheless, their large couplings to leptons (leading for instance to non-unitarity effects in neutrino oscillations) imply that the flavoured $CP$ asymmetry, more precisely the lepton number conserving part in equation (28), can be large even for TeV-scale RH neutrino masses. However, this scenario of ‘non-unitarity driven leptogenesis’ has intrinsically large lepton flavour violating interactions that lead to flavour equilibration [80]. Using the cross-sections for flavour violating interactions obtained in [44], one finds that the asymmetry cannot be generated in the right amount if $M_1 \lesssim 10^8$ GeV [55]. However, these cross-sections were recently found to be significantly more suppressed than in [44], leading to an interesting relaxation of the bounds down to $M_1 \gtrsim 10^6$ GeV [56].

It is worth mentioning that leptogenesis was investigated in the context of the inverse seesaw model also when lepton number is exactly conserved [81]. In this case, the observed baryon asymmetry can be generated if leptogenesis occurs during the electroweak phase transition, when the sphaleron rate progressively goes out of equilibrium. The lepton flavour asymmetries, generated exclusively thanks to flavour effects (again the second term in equation (28)), are then converted into a baryon asymmetry before total washout.

Another example where leptogenesis occurs in the resonant regime is radiative leptogenesis [82, 83]. There, RH neutrinos are assumed to be exactly degenerate at some high scale (for instance, the GUT scale), and small RH neutrino mass splittings are generated by running the seesaw parameters.

In these low-scale seesaw models with TeV-scale RH neutrinos, it is natural to wonder whether some interesting signatures could be observed at collider experiments such as the LHC. Unfortunately, it seems that within the simplest type-I seesaw, the prospects are rather dim [70, 84]. The main problem is that the mixing of RH neutrinos with light neutrinos has strong upper limits from rare (lepton-flavour-violating) decays [74], which prevents an important production of RH neutrinos at the LHC. However, in extended models such as type II seesaw, type III seesaw, left–right symmetric models or simply an extra $U(1)_{B-L}$, the prospects are much more encouraging (see [21] in this issue).

7. Improved kinetic description

We have already discussed the density matrix formalism, which goes beyond the traditional kinetic treatment with Boltzmann (rate) equations. This section is devoted to other kinetic effects
that are important both for an estimation of the theoretical uncertainties in the calculation and for a better conceptual understanding of the minimal leptogenesis framework.

7.1. Momentum dependence

Within the vanilla scenario, the final asymmetry is computed by solving classical Boltzmann equations for the RH neutrino and lepton number densities, the so-called rate equations. These are obtained from the Boltzmann equations for the distribution function integrating over momenta with some approximations (see below). One can then wonder what is the theoretical error introduced by this integrated description.

Given a particle species \( X \), the number density is obtained by integrating the distribution function over momentum

\[
n_X = \frac{g_X}{(2\pi)^3} \int d^3p_X f_X,
\]

where \( g_X \) is the number of degrees of freedom of particle \( X \). For leptogenesis with decays and inverse decays, the system of Boltzmann equations (one for the RH neutrino and one for lepton number) in the expanding Friedmann–Robertson–Walker Universe is given by

\[
\frac{\partial f_N}{\partial t} - |p_N| H \frac{\partial f_N}{\partial |p_N|} = C_D[f_N],
\]

\[
\frac{\partial f_\ell - \bar{\ell}}{\partial t} - |p_\ell| H \frac{\partial f_\ell - \bar{\ell}}{\partial |p_\ell|} = C_D[f_\ell - \bar{\ell}],
\]

where the collision integrals on the right-hand side are defined as

\[
C[f_A, A \leftrightarrow B C] = \frac{1}{2E_A} \int \frac{d^3p_B}{2E_B(2\pi)^3} \frac{d^3p_C}{2E_C(2\pi)^3} (2\pi)^4 \delta^4(p_A - p_B - p_C) \times \left[ f_B f_C (1 - f_A) |M(B C \rightarrow A)|^2 - f_A (1 - f_B)(1 - f_C) |M(A \rightarrow B C)|^2 \right].
\]

Note that the double-counting problem is solved here in the same way as for the integrated Boltzmann equations, by consistently including the resonant part of the \( \Delta \mathbf{L} = 2 \) scatterings. In order to recover the usual Boltzmann equations (16) and (17), one has to introduce three approximations: (i) kinetic equilibrium for the RH neutrinos, which can be expressed as \( f_N / f_N^{eq} = n_N / n_N^{eq} \); (ii) Maxwell–Boltzmann distributions for RH neutrinos, leptons and Higgs fields; and (iii) neglect Pauli blocking and Bose enhancement factors.

RH neutrinos are only coupled to the thermal bath via their Yukawa couplings. It is therefore clear that in the weak washout regime, \( K_1 \ll 1 \), the assumption of kinetic equilibrium is not a very good one, and indeed, it was found that the lepton asymmetry computed with the above equations can differ by up to 50% in the weak washout regime compared to the usual treatment with integrated equations [85–87]. However, as expected, in the strong washout regime, the above-mentioned approximations are very good and the integrated rate equations can be used safely.

7.2. Non-equilibrium formalism

Leptogenesis is an intrinsic non-equilibrium problem. One of Sakharov’s conditions is indeed that a departure from thermal equilibrium is necessary to produce the baryon asymmetry.
Therefore, it does not come as a surprise that recently a huge effort [48, 88–95] was made to understand leptogenesis within non-equilibrium quantum field theory, also known as the closed-time path (CTP) or Keldysh–Schwinger formalism. This more rigorous, although far more complex, approach has the advantage of taking into account quantum effects that are completely missed by the usual approach, such as memory effects and off-shell effects. Moreover, it allows the straightforward inclusion of flavour oscillations and decoherence [48], it has the advantage of being able to consistently account for finite-density corrections and it has no double-counting problem.

Concretely, in the non-equilibrium framework, one needs to find the equations of motion for the two-point correlation functions (i.e. the propagators or Green’s functions) of RH neutrinos and leptons from the general Schwinger–Dyson equation on the CTP,

\[ S^{-1}_\ell (x, y) = S^{-1}_0 (x, y) - \Sigma_\ell (x, y), \]  

\[ S^{-1} (x, y) = S^{-1}_0 (x, y) - \Sigma_N (x, y), \]  

which are obtained from the variational principle on the effective action, a functional of the full propagators \( \Delta_\phi, S_\ell \) and \( S \) for the Higgs, lepton and RH neutrino, respectively. In the above equations, the subscript 0 denotes the free propagators, and \( \Sigma_\ell \) and \( \Sigma_N \) are the self-energies for the leptons and RH neutrinos, respectively. Note that the self-energies are themselves functions of the propagators. For instance, the one-loop lepton self-energy depends on the RH neutrino and Higgs propagators:

\[ \Sigma_\ell^{\alpha \beta} (x, y) = - h_{\alpha \gamma} h^{\gamma \beta}_j P_R S^{ij}_\ell (x, y) P_L \Delta_\phi (y, x). \]  

It usually proves convenient to decompose any two-point function \( D(x, y) \) into a spectral, \( D_\rho \), and a statistical component, \( D_F \):

\[ D(x, y) = D_F (x, y) - \frac{i}{2} \text{sgn}(x^0 - y^0) D_\rho (x, y). \]  

Convolting the Schwinger–Dyson equations with the full propagator, we finally arrive at a system of two coupled integro-differential equations, the so-called Kadanoff–Baym equations:

\[ i \not\!\partial_z S^{\alpha \beta}_F (x, y) = \int_0^{x^0} dz \Sigma_\rho^{\alpha \gamma}_\ell (x, z) S^{\gamma \beta}_F (z, y) - \int_0^{y^0} dz \Sigma_\rho^{\alpha \gamma}_\ell (x, z) S^{\gamma \beta}_F (z, y), \]  

\[ i \not\!\partial_z S^{\alpha \beta}_\rho (x, y) = \int_{y^0}^{x^0} dz \Sigma_\rho^{\alpha \gamma}_\ell (x, z) S^{\gamma \beta}_\rho (z, y). \]  

The corresponding equations for the RH neutrino two-point function are obtained by changing \( \not\!\partial_x \rightarrow \not\!\partial_x - M \) and \( S^{\alpha \beta} \rightarrow S^{ij} \), where \( \alpha, \beta = e, \mu, \tau \) are lepton flavours and \( i, j = 1, 2, 3 \) are RH neutrino flavours. It can be noted that the Kadanoff–Baym equations contain an integration over the entire history of the system, a ‘memory’ integral, which encodes all previous interactions with momentum and spin correlations. An attempt at studying memory effects in the context of leptogenesis [89] found that large effects could arise in the resonant limit in the weak washout regime. In order to recover a Markovian description of the system, characterized by uncorrelated initial states at every time step, one has to perform a gradient expansion (for an alternative
approach, see [91]), relying on the fact that the microscopic timescale \( t_{\text{mic}} \sim 1/M_i \) is much smaller than the macroscopic timescales \( t_{\text{mac}} \sim 1/\Gamma_i, 1/H \). This is also known as the molecular chaos approximation.

In leptogenesis, one needs to compute the evolution of the lepton number density. The latter is given by the average expectation value of the zeroth component of the lepton number current, given by

\[
j_{\text{Lap}}^\mu(x) = -\text{tr} [\gamma^\mu S_{\text{Lap}}(x, x)].
\]

One then obtains for the lepton number density

\[
n_{\text{Lap}}(t) = i \int \frac{d^3p}{(2\pi)^3} \int_0^{t_f} dt' \int_0^{t_f} dt'' \text{tr} \left[ \Sigma_{\ell p, \ell' p, \ell'' p}(t', t'') S_{\ell F p}(t'', t') - \Sigma_{\ell p, \ell' p, \ell'' p}(t', t'') S_{\ell p, \ell p}(t'', t') \right],
\]

after switching to momentum space. From this master equation, one then needs to input the equilibrium expression for the lepton and Higgs propagators, as well as the non-equilibrium Majorana neutrino propagator. In order to obtain a Boltzmann-like equation, two further simplifications are required. First, the quasi-particle ansatz, also known as the on-shell approximation, which states

\[
D_\rho(X, p) = 2\pi \delta(p^0) \delta(p^2 - m^2),
\]

where \( X \equiv (x + y)/2 \) is the central coordinate. Then, one can express the statistical propagator for the occupation number using the so-called Kadanoff–Baym ansatz,

\[
D_F(X, p) = \left[ f(X, p) + \frac{1}{2} \right] D_\rho(X, p),
\]

which is chosen such that, in equilibrium, the correct fluctuation dissipation relation between \( D_F^{\text{eq}}(p) \) and \( D_\rho^{\text{eq}}(p) \) is automatically obtained (see [90] for more details). With these approximations, one arrives at a Boltzmann equation which includes finite-density effects. This equation would be similar to the kinetic equation (41), with however the important new property

\[
|\mathcal{M}(N \rightarrow \ell \Phi)|^2 = |\mathcal{M}(\ell \Phi \rightarrow N)|^2 = \frac{\mathcal{M}_0(N \leftrightarrow \ell \Phi)}{1 + \varepsilon(p, T)},
\]

where subscript ‘0’ denotes tree level, and [92]

\[
\varepsilon(p, T) = \varepsilon \times \left( 1 + \int \frac{d\Omega}{4\pi} \left[ f_0^{\text{eq}}(E_1) - f_\ell^{\text{eq}}(E_2) \right] \right),
\]

where \( \varepsilon \) is the \( CP \) asymmetry defined in equation (14), and \( E_{1,2} = \frac{1}{2} (M^2 + p^2)^{\frac{1}{2}} \pm p \cos \theta \). Note that the above property in equation (53) explicitly avoids the double-counting problem, which plagues the momentum-dependent description of the last subsection (as well as the usual integrated one). As can be seen, the \( CP \) asymmetry parameter now includes finite-density effects via a dependence on the Higgs and lepton distribution functions. This result agrees with that found using thermal field theory in the real-time formalism, when the right convention is used [92], as well as in the imaginary time formalism as obtained recently [96]. It, however, disagrees with an earlier work based on thermal field theory in the real-time formalism, where a term quadratic in the distribution functions was found [29].

One should then wonder whether the non-equilibrium formalism described above leads to substantial differences in the leptogenesis predictions for the baryon asymmetry. It turns out that the modifications are very important (up to an order of magnitude) in the regime
where \( M/T \ll 1 \), but tend to vanish in the non-relativistic regime \( M/T \gg 1 \). In other words, major changes are expected mainly in the weak washout regime for a vanishing initial RH neutrino abundance, because in this case the asymmetry is produced in both regimes: the initial asymmetry is produced with the wrong sign at early times and is compensated for by the right sign asymmetry at later times. This was confirmed in [93], where it was found that the sign of the final asymmetry could even get changed by finite-density effects. In all other cases, corrections are quantitatively small, in particular in the strong washout regime, \( K \gg 1 \), where the asymmetry is produced exclusively in the non-relativistic regime [92, 93].

7.2.1. Resonant limit. It is interesting to examine whether the above quantum kinetic formalism can provide some insight into the \( CP \) asymmetry produced in the extreme quasi-degenerate limit, \( |M_1 - M_2| \ll M_{1,2} \). This formalism includes off-shell effects, as well as the proper inclusion of coherent transitions \( N_i \rightarrow N_j \), both of which should be important when \( |M_1 - M_2| \sim \Gamma_{1,2} \). This problem has attracted some attention in recent years [94, 95]. In [94] it was found that the enhancement of the \( CP \) asymmetry found within the Boltzmann formalism

\[
R_{\text{BE}} = \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1 \Gamma_1 - M_2 \Gamma_2)^2} \tag{55}
\]

differs from the Kadanoff–Baym (KB) result specifically because of a contribution from coherent RH neutrino oscillations. It is indeed found that [94]

\[
R_{\text{KB}} = \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1 \Gamma_1 + M_2 \Gamma_2)^2}, \tag{56}
\]

which prevents any additional enhancement when \( \Gamma_1 \sim \Gamma_2 \), as predicted by the inverse seesaw model. In such a case, the new region in the parameter space of successful leptogenesis explored in [72] is not available.

7.2.2. Adding flavour. We saw in section 5 how the first kind of ‘quantum’ effects were included, namely flavour oscillations and decoherence, by switching from classical Boltzmann equations for lepton number densities to evolution equations for the density matrix. For the lightest RH neutrino and two relevant lepton flavours, we presented equation (30). The CTP formalism can also account for a flavour matrix structure, through the lepton propagators and self-energies. Reassuringly, the structure of equation (30) was also found within this formalism [48].

8. Other corrections

8.1. Thermal effects

Leptogenesis occurs in the very hot thermal bath of the early Universe. Leptons and Higgs fields have very fast interactions with the thermal bath due to their gauge couplings. This gives them an effective mass which is proportional to temperature [65]. This effective mass allows processes that were otherwise kinematically forbidden to occur, such as \( CP \)-violating Higgs decays to RH neutrinos, when \( T \gg M_i \). Thermal effects have therefore a direct impact on leptogenesis, and
the first study to try and quantify them [29] employed a real-time formalism and hard thermal loop resummation. It was found that the CP violating parameter has a strong temperature dependence, and that in the weak washout regime, there could be important differences with the zero-temperature treatment. On the other hand, in the strong washout regime, $K_i \gg 1$, the usual results from a vacuum calculation are recovered with good accuracy. Recently, a new study came out using the imaginary time formalism [96], and the CP asymmetry parameter was found to differ from [29], and to agree with more recent attempts with non-equilibrium quantum field theory (see the previous section).

8.2. Spectator processes

Chemical equilibrium holds among SM particles in the early Universe above the electroweak phase transition thanks to gauge interactions. On the other hand, Yukawa interactions only force some new conditions (between left- and right-handed fermions) when the temperature is high enough, depending on the size of the Yukawa coupling. If one imposes additionally hypercharge neutrality and the effect of electroweak and strong sphaleron equilibrium, one arrives at a set of relations among the chemical potentials (or among the asymmetries) of leptons, Higgs and baryons [97]. These processes, although not directly involved in the leptogenesis process, hence the name ‘spectator processes’ [98], have an indirect effect on the final asymmetry through a modified washout. The first effect is the inclusion of the Higgs asymmetry as a new contribution to the washout. The second effect is that the asymmetry originally produced in leptons of a particular flavour gets redistributed into the other flavours following precise relations [26]. This means that the set of Boltzmann equations to solve for the different lepton flavours are now coupled to each other via a flavour coupling matrix.

Within the $N_1$-dominated scenario, the overall effect of spectator processes is usually subdominant compared to flavour effects, but can change the final result by as much as 40% depending on the temperature at which the asymmetry is produced [57, 97]. Supersymmetry has new degrees of freedom and new constraints can be derived. A detailed study was performed recently in [99] and the overall effect was found to be again of the order of one. Within the $N_2$-dominated scenario, however, potentially much bigger effects are possible [59].

8.3. Scattering processes

It can be shown that leptogenesis is well described in the strong washout regime, $K \gg 1$, by just decays and inverse decays. The reason is that in this regime the asymmetry is produced at relatively late times, at $T \ll M$, with no dependence on the dynamics happening at $T \gg M$ when scattering processes are important. However, they should be included in the weak washout regime for initial vanishing RH neutrino abundance. For instance, $\Delta L = 1$ Higgs-mediated scatterings involving top quarks, such as $\ell N \leftrightarrow Q_3 \bar{t}$, contribute to the washout and CP asymmetry [49], and their inclusion is crucial to have a correct estimation of the final asymmetry in the weak washout regime [100]. Scatterings involving gauge bosons, such as $\ell N \leftrightarrow \Phi A^*$, have also been included, especially their contribution to the $CP$ asymmetric source term [101]. It was found that factorization of the $CP$ asymmetry from decays and scatterings involving top quarks does not happen with scatterings involving gauge bosons. In particular, there is a new source of lepton-number-conserving $CP$ asymmetry.
9. Testing new physics with leptogenesis

The seesaw mechanism with three RH neutrinos extends the SM by introducing 18 new parameters. On the other hand, low-energy neutrino experiments can only potentially test the nine parameters in the low-energy neutrino mass matrix $m_\nu$. Nine high-energy parameters, those characterizing the properties of the three RH neutrinos, the three masses and the six parameters encoded in the seesaw orthogonal matrix of equation (10), basically fixing, together with the light neutrino masses, the three lifetimes and the three total CP asymmetries, are not tested by low-energy neutrino experiments. Quite interestingly, the requirement of successful leptogenesis,

$$\eta_B(m_\nu, \Omega, M) = \eta_{CMB}^B,$$

(57)

provides an additional constraint on a combination of both low-energy neutrino parameters and high-energy neutrino parameters. However, just one additional constraint would not seem sufficient to over-constrain the parameter space leading to testable predictions. In spite of this observation, as we have seen, in the vanilla leptogenesis scenario one can derive an upper bound on the neutrino masses. The reason is that, within this scenario, the dependence of $\eta_B$ on the six parameters related to the properties of the two heavier RH neutrinos cancels out. In this way the asymmetry depends on a reduced subset of high-energy parameters (just three instead of nine). At the same time, the final asymmetry gets strongly suppressed when the absolute neutrino mass scale is larger than the atmospheric neutrino mass scale. For all these reasons, by maximizing the final asymmetry over the high-energy parameters and by imposing successful leptogenesis, an upper bound on the neutrino masses is found.

When flavour effects are considered, the vanilla leptogenesis scenario holds only under very special conditions, as we have seen. In general, the final asymmetry depends also on the parameters in the leptonic mixing matrix. Therefore, accounting for flavour effects, one could naively hope to derive definite predictions on the leptonic mixing matrix as well, in addition to the upper bound on the absolute neutrino mass scale. However, the situation is quite different when flavour effects are taken into account. This is because the final asymmetry depends, in general, also on the six parameters related to describing the two heavier RH neutrino properties and that were cancelling out in the calculation of the final asymmetry in the vanilla scenario, and this goes at the expense of predictability.

For this reason, in a general scenario with three RH neutrinos and flavour effects included, it is not possible to derive any prediction on low-energy neutrino parameters. As we have discussed, whether the upper bound $m_i \lesssim 0.1$ eV on neutrino masses still holds or not is an open issue and a precise value seems to depend on a precise account of many different subtle effects and in particular it necessarily requires a density matrix formalism.

In order to gain predictive power, two possibilities have been explored in past years.

A first possibility is to consider non-minimal scenarios giving rise to additional phenomenological constraints. For example, as discussed in section 6, the inverse seesaw model, which technically can be still regarded as part of the minimal-type I seesaw model, allows for non-trivial phenomenologies at low energy beyond standard neutrino oscillations, such as non-unitarity effects and observable lepton flavour violation. An experimental observation of any of these signatures would be of great value to reduce the freedom in the choice of seesaw parameters. In recent years, during the Large Hadron Collider era, the possibility has also been intensively explored that, within a non-minimal version of the seesaw mechanism, one can have
successful low scale leptogenesis together with collider signatures. It has also been noted that in the supersymmetric version of the seesaw, the branching ratios of lepton-flavour-violating processes or electric dipole moments are typically enhanced and hence the existing experimental bounds further constrain the seesaw parameter space [102, 103].

A second possibility is to search for a reasonable scenario where the final asymmetry depends only on a reduced set of independent parameters over-constrained by the successful leptogenesis condition, as with the vanilla scenario. From this point of view, the account of flavour effects has opened very interesting new opportunities or even re-opened old attempts that fail within a strict unflavoured scenario. Let us briefly discuss some of the main ideas that have been proposed within this second possibility, the first one being covered elsewhere in this issue [21].

9.1. The two-right-handed neutrino model

A phenomenological possibility that has attracted great attention is the two-RH neutrino model [104], where the third RH neutrino is either absent or effectively decoupled in the seesaw formula. This necessarily happens when \( M_3 \gg 10^{14} \text{ GeV} \), implying that the lightest left-handed neutrino mass \( m_1 \) has to vanish. It can be shown that the number of parameters decreases from 18 to 11 in this case. In particular, the orthogonal matrix is parameterized in terms of just one complex angle.

In leptogenesis the two-RH neutrino model has been traditionally considered as a sort of benchmark case for the \( N_1 \)-dominated scenario, where the final asymmetry is dominated by the contribution from the lightest RH neutrinos [40, 49, 105]. However, recently, as we anticipated already, it was shown that there are some regions in the one-complex angle parameter space that are \( N_2 \)-dominated [106] and that correspond to the so-called light sequential dominated models [53].

It should be mentioned that even though the number of parameters is highly reduced, in a general two-RH neutrino model it is still not possible to make predictions on the low-energy neutrino parameters. To this extent, one should further reduce the parameter space, for example assuming texture zeros in the neutrino Dirac mass matrix.

9.2. \( SO(10) \)-inspired models

In order to gain predictive power, one can impose conditions within some model of new physics embedding the seesaw mechanism. An interesting example is represented by the ‘\( SO(10) \)-inspired leptogenesis scenario’ [107–109], where \( SO(10) \)-inspired conditions are imposed on the neutrino Dirac mass matrix \( m_D \). In the basis where the charged lepton mass matrix and the Majorana mass matrix are diagonal, in the bi-unitary parametrization, one has \( m_D \equiv V^\dagger_L D_{mD} U_R \),

where \( D_{mD} \equiv \text{diag}(\lambda_1, \lambda_2, \lambda_3) \) is the diagonalized neutrino Dirac mass matrix and the mixing angles in \( V_L \) are of the order of the mixing angles in the CKM matrix \( V_{\text{CKM}} \). The \( U_R \) and three \( M_i \) can then be calculated from \( V_L, U \) and \( m_D \), since the seesaw formula equation (8) directly leads to the Takagi factorization of \( M^{-1} \equiv D_{mD}^{-1} V_L U D_m U^T V_L^T D_{mD}^{-1} \), or explicitly \( M^{-1} = U_R D_M^{-1} U_R^T \).

In this way, the RH neutrino masses and the matrix \( U_R \) are expressed in terms of the low-energy neutrino parameters, of the eigenvalues \( \lambda_i \) and of the parameters in \( V_L \). Typically, one obtains a very hierarchical spectrum \( M_1 \sim 10^5 \text{ GeV} \) and \( M_2 \sim 10^{11} \text{ GeV} \); the asymmetry
Figure 4. Constraints on some of the low-energy neutrino parameters in the $SO(10)$-inspired scenario for normal ordering and $I < V_L < V_{CKM}$ [111]. The yellow, green and red points correspond, respectively, to $\alpha_2 = 5, 4, 1$.

produced from the lightest RH neutrino decays is by far unable to explain the observed asymmetry [108].

However, when the $N_2$-produced asymmetry is taken into account, successful ($N_2$-dominated) leptogenesis can be attained [110]. In this case, imposing the leptogenesis bound and considering that the final asymmetry does not depend on $\lambda_1$ and on $\lambda_3$, one obtains constraints on all low-energy neutrino parameters, which have some dependence on the parameter $\lambda_2$ typically parameterized in terms of $\alpha_2 \equiv \lambda_2 / m_c$, where $m_c$ is the charm quark mass. Some examples of the constraints on the low-energy neutrino parameters are shown in figure 4. They have been obtained scanning over the $2\sigma$ ranges of the allowed values of the low-energy parameters and over the parameters in $V_L$ assumed to be $I < V_L < V_{CKM}$ and for three values of $\alpha_2 = 5, 4, 1$. It is particularly interesting that when the independence of the initial conditions is imposed, negative values of $J_{CP}$ seem to be favoured [112], establishing a connection between the sign of $J_{CP}$ and of the matter–antimatter asymmetry.

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A supersymmetric version of this scenario including the renormalization group evolution of all the relevant couplings was also studied previously in [113] and that including a type II contribution to the seesaw mechanism from a triplet Higgs in left–right symmetric models in [114].

Together with strongly hierarchical RH neutrino mass patterns, there exist also ‘level crossing’ regions where at least two RH neutrino masses are arbitrarily close to each other, in such a way that the lightest RH neutrino mass is uplifted and CP asymmetries are resonantly enhanced to some level. Also in this case, imposing the successful leptogenesis condition, one then obtains conditions, although quite fine-tuned ones, on the low-energy neutrino parameters [109].

9.3. Discrete flavour symmetries

An account of heavy neutrino flavour effects is also important when leptogenesis is embedded within theories that try to explain tribimaximal mixing for the leptonic mixing matrix via flavour symmetries. It has been shown in particular that if the symmetry is unbroken, then the CP asymmetries of the RH neutrinos would exactly vanish. On the other hand, when the symmetry is broken, for the naturally expected values of the symmetry breaking parameters, the observed matter–antimatter asymmetry can be successfully reproduced [115–118]. It is interesting that in a minimal picture based on an $A^4$ symmetry, one has an RH neutrino mass spectrum with $10^{15}$ GeV $\gtrsim M_1 \gtrsim M_2 \gtrsim M_1 \gg 10^{12}$ GeV. One has therefore that all the asymmetry is produced in the unflavoured regime and that the mass spectrum is only mildly hierarchical (it has actually the same kind of hierarchy as light neutrinos). At the same time, the small symmetry breaking imposes a quasi-orthogonality of the three-lepton quantum states produced in the RH neutrino decays. Under these conditions the washout of the asymmetry produced by one RH neutrino species from the inverse decays of a lighter RH neutrino species is essentially negligible. The final asymmetry then receives a non-negligible contribution from the decays of all three RH neutrino species.

9.4. Supersymmetric models

Within a supersymmetric vanilla framework, the final asymmetry is only slightly modified compared to the non-supersymmetric calculation [119]. However, supersymmetry introduces a conceptually important issue: the stringent lower bound on the reheat temperature, $T_{\text{reh}} \gtrsim 10^9$ GeV, is typically marginally compatible with an upper bound from the avoidance of the gravitino problem $T_{\text{reh}} \lesssim 10^{6–10}$ GeV, with the exact value depending on the parameters of the model [120–122]. It is quite remarkable that the solution of such an issue inspired intensive research activity on supersymmetric models able to reconcile minimal leptogenesis and the gravitino problem. Of course, on the leptogenesis side, some of the discussed extensions beyond the vanilla scenario that relax the RH neutrino mass bounds also relax the $T_{\text{reh}}$ lower bound. However, note that in the $N_2$-dominated scenario, while the lower bound on $M_1$ simply disappears, there is still a lower bound on $T_{\text{reh}}$ that is even more stringent, $T_{\text{reh}} \gtrsim 6 \times 10^9$ GeV [35].

As already mentioned, with flavour effects one has the possibility to relax the lower bound on $T_{\text{reh}}$ if a mild hierarchy in the RH neutrino masses is allowed together with a mild cancellation in the seesaw formula [30]. However, for most models, such as sequential
dominated models [53], this solution does not work. A major modification introduced by supersymmetry is that the critical value of the mass of the decaying RH neutrinos setting the transition from an unflavoured regime to a two-flavour regime and from a two-flavour regime to a three-flavour regime is enhanced by a factor $\tan^2 \beta$ [41, 123]. This has practical relevance in the calculation of the asymmetry within supersymmetric models and it is quite interesting that leptogenesis becomes sensitive to such a relevant supersymmetric parameter. Recently, a refined analysis, mainly discussing how the asymmetry is distributed among all particle species, has shown different subtle effects in the calculation of the final asymmetry within supersymmetric models finding corrections below $O(1)$ [99].

10. Future prospects for testing leptogenesis

In 2011, two important experimental results were announced that, if confirmed, can be interpreted as positive for future tests of leptogenesis.

The first result is the discovery of a non-vanishing $\theta_{13} \simeq 9^\circ$ (cf section 2) confirmed now by various experiments (both long-baseline and reactor). An important consequence of the measurement of such a ‘large’ $\theta_{13}$ is the encouraging prospects for discovery of the neutrino mass ordering (either normal or inverted) in current or near-future neutrino oscillation experiments such as T2K and NOνA. For instance, if the ordering is found to be inverted in these experiments, we should expect a signal in $0\nu\beta\beta$ experiments in the next decade. If it is not found, the Majorana nature of neutrinos, and therefore the seesaw mechanism, would be ruled out.

Moreover, such a large value of $\theta_{13}$ opens the possibility of a measurement of the neutrino oscillation CP-violating invariant $J_{CP} \propto \sin \theta_{13} \sin \delta$ during the next few years. This would have some direct model-independent consequences. If a non-vanishing and close-to-maximal value is found ($|\sin \delta| \sim 1$), even the small contribution to the final asymmetry uniquely stemming from a Dirac phase could be sufficient to reproduce the observed final asymmetry. More generally, the presence of CP violation at low energies would certainly support the presence of CP violation at high energies as well, since given a generic theoretical model predicting the neutrino Dirac mass matrix $m_D$, these are, in general, both present.

A more practical relevance of such a measurement of $J_{CP}$, even if in the end it indicates a vanishing value within the experimental error, is that it will provide an additional constraint on specific models embedding the seesaw, satisfying successful leptogenesis and able to make predictions on the low-energy neutrino parameters. In this way, the expected improvements in low-energy neutrino experiments, made easier by a non-vanishing $\theta_{13}$, will test the models more and more stringently.

The second important experimental result of 2011 is the hint of the existence of the Higgs boson reported by the ATLAS and CMS collaborations. There are at least two reasons why this hint can also be interpreted as positive for leptogenesis: firstly, because the whole leptogenesis mechanism relies on the Yukawa coupling between Higgs, lepton and RH neutrino; secondly, because the measurement of the Higgs boson mass could in future open opportunities for additional phenomenological information to be imposed on leptogenesis scenarios, for example relying on the requirement of SM electroweak vacuum stability [124]. In this respect, it is interesting to note that the current value of 125 GeV is compatible with reheating temperatures as high as $10^{15}$ GeV as needed by thermal leptogenesis, and is also compatible with the
requirement that the Yukawa couplings of the RH neutrinos do not destabilize the Higgs potential when RH neutrino masses $M_i$, assumed to be quasi-degenerate, are smaller than $10^{14}$ GeV.

What are the other possible future experimental developments that could further support the idea of leptogenesis? Improved information from absolute neutrino mass scale experiments, both on the sum of the neutrino masses from cosmology and on $m_{ee}$ from $0\nu\beta\beta$ experiments, could be crucial. For instance, if cosmology provides a measurement of the neutrino mass $0.01 \lesssim m_1 \lesssim 0.2$ eV in the next few years, as it is reasonable to expect, then a positive signal in $0\nu\beta\beta$ experiments must be found; otherwise Majorana neutrinos and the seesaw mechanism will be disfavoured. On the other hand, in the case of a positive signal in $0\nu\beta\beta$ experiments, we will be able to say that minimal leptogenesis with hierarchical heavy neutrino masses works in an optimal neutrino mass window $10^{-3} \lesssim m_1 \lesssim 0.1$ eV [28, 30, 58], where independence of the initial conditions is more easily obtained and where we know that successful leptogenesis can be safely obtained from existing calculations using Boltzmann equations. Moreover, a determination of the allowed region in the plane $m_{ee} - \sum m_i$ could provide an additional test of specific leptogenesis scenarios, such as, for example, $SO(10)$-inspired scenarios.

In conclusion, we are living in an exciting time where new experimental information is coming, which is providing and will continue to provide crucial tests for new physics models, in particular leptogenesis, in the near future.

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