The economical 3-3-1 model revisited

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We show that the economical 3-3-1 model poses a very high new physics scale of 1000 TeV order due to the FCNC constraint, and its new implication for neutrino masses, inflation, leptogenesis, and superheavy dark matter is recognized. Alternatively, we modify the model by rearranging the third quark generation differently from the first two as well as changing the scalar sector, and it now predicts a consistent new physics at TeV scale unlike the previous case, possibly fully probed at the current colliders. Due to the different minimal particle contents, the models manifestly accommodate dark matter and neutrino masses with characteristic production mechanisms. The large FCNCs which come from the ordinary and exotic quark mixings can be avoided due to the approximate $B - L$ symmetry.

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I. INTRODUCTION

There have been up to now certain experimental evidences for the physics beyond the standard model prediction. The most important issues of which must include neutrino oscillations, baryon-number asymmetry, dark matter, and inflation. The traditional proposals such as supersymmetry, extradimension, and grand unification solve only some of the questions separately, and obviously they obey several theoretical and experimental issues. In this work, we will show that the model based on the gauge symmetry $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) may be an intriguing choice for the new physics due to its ability to solve the underlined problems integrally.

The weak isospin group $SU(3)_L$ that is directly extended from $SU(2)_L$ of the standard model is well-established as it is able to determine the number of generations to match that of colors as observed by the $[SU(3)_L]^3$ anomaly cancelation. However, the electric charge $Q$ neither commutes nor closes algebraically with $SU(3)_L$. Hence, a new Abelian group is deduced as a result to close those symmetries by the gauge group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, where $Q = T_3 + \beta T_8 + X$ with $T_n$ ($n = 1, 2, 3, ..., 8$) and $X$ indicating to the $SU(3)_L$ and $U(1)_X$ charges, respectively (cf. [4]). And, the color group has also been involved for completeness. The nontrivial commutations for $Q$ are $[Q, T_1 \pm iT_2] = \pm (T_1 \pm iT_2)$, $[Q, T_4 \pm iT_5] = \mp q(T_4 \pm iT_5)$, and $[Q, T_6 \pm iT_7] = \mp (1 + q)(T_6 \pm iT_7)$, where $q = -(1 + \sqrt{3} \beta)/2$ will define the electric charges for the new particles in representations. Note that $\beta$ is arbitrary on the theoretical ground, independent of all anomalies.

We have actually been interested in some calculable 3-3-1 model that contains a minimal content of fermions and scalars. The first one was the economical 3-3-1 model extracted from the 3-3-1 model with right-handed neutrinos, where $\beta = -1/\sqrt{3}$. The new physics implications as well as supersymmetric extension have been extensively investigated in [9–22]. The second one was first introduced as the reduced 3-3-1 model deduced from the minimal 3-3-1 model, where $\beta = -\sqrt{3}$. However, this version was encountered with the problems of the $\rho$-parameter and FCNC constraints as well as the Landau pole limit (cf. [24]). The realistic theory for the second approach overcoming such issues was finally achieved, called the simple 3-3-1 model, with the phenomenological aspects extensively studied in [26, 27]. Although not presenting a low Landau pole as the reduced 3-3-1 model, the economical 3-3-1 model matters to the other bounds similarly, showing the corresponding interesting consequences to be examined.

We will reconsider the new physics scale of the economical 3-3-1 model due to the FCNC

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1 The introduction of the embedding coefficient $\beta$ was given early in 3, 6.
constraint. We show that there are such two distinct versions of it: (i) the first one that is just original model works surprisingly beyond $O(1000) \text{ TeV}$, which provides seesaw mechanism, inflation, leptogenesis, and superheavy dark matter, by contrast to the previous interpretations \cite{7–21}; and (ii) the second one with the fermion and scalar content appropriately recast works beyond $O(1) \text{ TeV}$, showing fully-testable new-physics phenomena such as neutrino mass mechanism, new fermions, new Higgs and gauge bosons, and WIMP at the current colliders.

For this aim, we first consider the 3-3-1 model with arbitrary $\beta$ and extract the bound for the 3-3-1 breaking scale from FCNCs, which depends only on the arrangement of quark representations, as presented in Sec. III. Turning to the model under investigation, two folds for it, i.e. the economical 3-3-1 model, are derived, and the corresponding consequences are discussed, as given in Sec. III. Note that these economical 3-3-1 models are not limited by a Landau pole since this pole is actually higher than the Planck scale \cite{28}. We finally conclude this work in Sec. IV.

II. FCNCS

The 3-3-1 model with arbitrary $\beta$ is given by the electric charge operator as mentioned

$$Q = T_3 + \beta T_8 + X.$$  

(1)

Supposing that the first quark generation transforms differently from the last two under $SU(3)_L$, the fermion content which is anomaly free is achieved as

$$\psi_{aL} = \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ k_{aL} \end{pmatrix} \sim \left(1, 3, -\frac{1+q}{3} \right), \quad (2)$$

$$Q_{1L} = \begin{pmatrix} u_{1L} \\ d_{1L} \\ j_{1L} \end{pmatrix} \sim \left(3, 3, \frac{1+q}{3} \right), \quad (3)$$

$$Q_{aL} = \begin{pmatrix} d_{aL} \\ -u_{aL} \\ j_{aL} \end{pmatrix} \sim \left(3, 3^*, -\frac{q}{3} \right), \quad (4)$$

$$e_{aR} \sim (1, 1, -1), \quad k_{aR} \sim (1, 1, q), \quad \nu_{aR} \sim (1, 1, 0), \quad (5)$$

$$u_{aR} \sim \left(3, 1, \frac{2}{3} \right), \quad d_{aR} \sim \left(3, 1, -\frac{1}{3} \right), \quad (6)$$

$$j_{1R} \sim \left(3, 1, \frac{2}{3} + q \right), \quad j_{aR} \sim \left(3, 1, -\frac{1}{3} - q \right), \quad (7)$$
where \( a = 1, 2, 3 \) and \( \alpha = 2, 3 \) are generation indices, and the electric charges of the new particles are related to the basic electric charge, \( q = -(1 + \sqrt{3})/2 \), by \( Q(k_a) = q \), \( Q(j_1) = q + 2/3 \), and \( Q(j_\alpha) = -q - 1/3 \), as aforementioned. The numbers in parentheses denote representations with respect to the 3-3-1 groups. \( \nu_{aR} \) are sterile which may be imposed or not. This similarly applies for \( k_{aR} \) if \( q = 0 \). Two minimal 3-3-1 versions have been studied, where the singlets \( \nu_{aR} \) and \( k_{aR} \) are omitted, while \( k_{aL} \) are replaced by \( (e_{aR})^c \) or \( (N_{aR})^c \), respectively \( [2, 3] \). Here \( N_{aR} \) are some neutral fermions like \( \nu_{aR} \). The above ingredient does not apply for quarks since \( SU(3)_C \), \( SU(3)_L \), and spacetime symmetry commute. By contrast, if the second or third quark generation is arranged differently from the two others, \( \alpha \) takes values, \( \alpha = 1, 3 \) or \( \alpha = 1, 2 \), respectively.

Typically, the scalar content includes

\[
\eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^g \end{pmatrix} \sim \left( 1, 3, \frac{q - 1}{3} \right),
\]

(8)

\[
\rho = \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^{q+1} \end{pmatrix} \sim \left( 1, 3, \frac{q + 2}{3} \right),
\]

(9)

\[
\chi = \begin{pmatrix} \chi_1^{-q} \\ \chi_2^{-q-1} \\ \chi_3^0 \end{pmatrix} \sim \left( 1, 3, -\frac{2q + 1}{3} \right).
\]

(10)

The scalar \( \chi \) develops VEV, \( \langle \chi \rangle = \frac{1}{\sqrt{2}}(0 0 w) \), breaking the 3-3-1 symmetry as well as generating the new particle masses. So, it should be presented. The scalar \( \eta \) or \( \rho \), which have \( \langle \eta \rangle = \frac{1}{\sqrt{2}}(u 0 0) \) and \( \langle \rho \rangle = \frac{1}{\sqrt{2}}(0 v 0) \), will break the standard model symmetry and give mass for the ordinary particles. Note that the other scalar components if electrically neutral can have VEV, but strongly constrained (cf. \[29\], for instance). The minimal 3-3-1 model and the 3-3-1 model with right-handed neutrinos work with the above three scalar triplets, and even they impose several scalar multiplets, e.g. sextets, additionally. However, the simple 3-3-1 model and the economical 3-3-1 model work with only two scalar triplets, \( (\chi, \eta) \) and \( (\chi, \rho) \), respectively.

The quark generations are not universal under the \( SU(3)_L \otimes U(1)_X \) symmetry, therefore there could be FCNCs. The neutral current takes the form

\[
\mathcal{L} \supset i \gamma^\mu D_\mu F \supset -g F \gamma^\mu [T_3 A_3 + T_8 A_8 + t_X (Q - T_3 - \beta T_8) B_\mu] F,
\]

(11)

where \( F \) runs over all fermion multiplets, and note that the covariant derivative is \( D_\mu = \partial_\mu + ig s_t G_\mu + ig T_n A_\mu + ig X B_\mu \), with the last three terms containing gauge coupling, generators,
and gauge bosons for the 3-3-1 groups, respectively. We have also used \( X = Q - T_3 - \beta T_8 \) and \( t_X = g_X/g = t_W/\sqrt{1 - \beta^2 t_W^2} \). It is clear that the ordinary leptons and the new fermions do not flavor change, because the corresponding flavor groups that possibly mix such as \( \{ \nu_{aL} \}, \{ e_{aL} \}, \{ e_{aR} \}, \{ k_{aL} \}, \{ k_{aR} \}, \{ j_{aL} \}, \) and \( \{ j_{aR} \} \) are each identical under the gauge charges. Additionally, the terms of \( T_3 \) and \( Q \) do not flavor change, because all the mentioned flavor groups including those of the ordinary quarks are identical under these charges, respectively. Thus, the FCNCs only couple the ordinary quarks to \( T_8 \), arising in part from

\[
\mathcal{L} \supset -g q_L \gamma^\mu T_{8L} q_L (A_{8\mu} - \beta t_X B_\mu),
\]

where we denote either \( q = (u_1, u_2, u_3) \) for up quarks or \( q = (d_1, d_2, d_3) \) for down quarks, and \( T_{8L} = \frac{1}{2\sqrt{3}} \text{diag}(1, -1, -1) \) summarizes \( T_8 \) values for \( q_{1L}, q_{2L}, \) and \( q_{3L} \), respectively.

In the mass basis, we have \( q_{L,R} = V_{qL,qR} q'_{L,R} \), where either \( q' = (u,c,t) \) or \( q' = (d,s,b) \), and \( V_{qL,qR} \) are quark mixing matrices that diagonalize the corresponding quark mass matrices such that \( V_{aL}^\dagger M_a V_{aR} = \text{diag}(m_u, m_c, m_t) \) and \( V_{dL}^\dagger M_d V_{dR} = \text{diag}(m_d, m_s, m_b) \). The CKM matrix takes the form, \( V_{CKM} = V_{aL}^\dagger V_{dL} \). Using \( A_{8\mu} - \beta t_X B_\mu = (1/\sqrt{1 - \beta^2 t_W^2}) Z'_\mu \), it follows

\[
\mathcal{L} \supset -g \frac{q'_L \gamma^\mu (V_{qL}^\dagger T_{8L} V_{qL}) q'_L Z'_\mu}{\sqrt{1 - \beta^2 t_W^2}},
\]

\[
\supset -g \frac{q'_L \gamma^\mu q'_j L(V_{qL}^\dagger L ij(V_{qL}^\dagger L) j Z'_\mu}{\sqrt{3(1 - \beta^2 t_W^2)}}.
\]

which causes FCNCs for \( i \neq j \), where \( i, j = 1, 2, 3 \) indicate to respective physical quark states. \( Z'_\mu \) might mix with \( Z \) and \( V = A_4 \) or \( V = A_6 \) for \( q = 0 \) or \( q = -1 \), respectively. The contribution of \( V \) to the FCNCs is negligible. We write \( Z' = -s_\varphi Z_1 + c_\varphi Z_2 \), where \( Z_{1,2} \) are two physical neutral gauge bosons with masses

\[
m_{Z_1}^2 \simeq \frac{g^2}{4c_W^2} (u^2 + v^2), \quad m_{Z_2}^2 \simeq \frac{g^2 u^2}{3(1 - \beta^2 t_W^2)},
\]

and the \( Z-Z' \) mixing angle is

\[
t_{2\varphi} \simeq \frac{\sqrt[3]{3(1 - \beta^2 t_W^2)}}{2c_W u^2} \left[ (1 + \sqrt{3} \beta t_W^2) u^2 - (1 - \sqrt{3} \beta t_W^2) v^2 \right].
\]

\[For the 3-3-1 models without exotic charges (\( q = 0 \) or \( -1 \)), there might be mixings between ordinary quarks and exotic quarks, which have different weak isospins and lead to large FCNCs associated with \( Z \) boson, independent of the generation nonuniversality. This effect might be more dangerous than the nonuniversal \( Z' \) couplings and is only suppressed if such mixing is small, compared to the ordinary quark mixings, as shown below. Alternatively, the FCNCs may be also associated with neutral scalars as discussed in [31, 32].
Substituting $Z'$ into the above FCNCs and integrating $Z_{1,2}$ out, we obtain the effective Lagrangian describing meson mixings,

$$\mathcal{L}_{\text{FCNC}}^{\text{eff}} = \frac{g^2}{3(1 - \beta^2 t_W^2)} (\bar{q} \gamma^\mu q_1')^2 [(V^*_q)_{1i}(V_q)_{ij}]\left(\frac{s^2_\rho}{m^2_{Z_1}} + \frac{c^2_\rho}{m^2_{Z_2}}\right).$$

(16)

The $Z_1$ contribution is negligible too, since

$$\frac{s^2_\rho/m^2_{Z_1}}{c^2_\rho/m^2_{Z_2}} \sim \frac{(1 + \sqrt{3} t_W^2) u^2 - (1 - \sqrt{3} t_W^2) v^2)^2}{4(1 + \sqrt{3} t_W^2) w^2} < \left(\frac{1 + \sqrt{3} t_W}{2}\right)^2 \frac{v_\rho^2}{w^2},$$

(17)

which is suppressed due to $v_\rho \ll w$, where we have used $|\beta| < 1/t_W$ derived from the photon field normalization and gauge coupling matching, $s_W = e/g = t_X/\sqrt{1 + (1 - \beta^2) t_X^2}$, as partly mentioned, and $v_\rho^2 = u^2 + v^2 = (246 \text{ GeV})^2$ identified from the $W$ boson mass. It is easily proved that the $\rho$-parameter deviation from the standard model value due to $Z$-$Z'$ mixing is $\Delta \rho = \rho - 1 \simeq (s^2_\rho/m^2_{Z_1})/(c^2_\rho/m^2_{Z_2})$, which again implies the nonsignificant contribution of $Z_1$ due to $\Delta \rho < 0.0006$ from the global fit [1]. Therefore, only $Z_2$ governs the FCNCs, leading to

$$\mathcal{L}_{\text{FCNC}}^{\text{eff}} \simeq \frac{1}{w^2} (\bar{q} \gamma^\mu q_1')^2 [(V^*_q)_{1i}(V_q)_{ij}]^2,$$

(18)

which is independent of $\beta$ and the Landau pole if presented for large $|\beta|$, which is a new observation of this work and in agreement with a partial conclusion in [25].

In both economical 3-3-1 models discussed below, the ordinary $(u_a, d_a)$ and exotic $(U, D_a)$ quarks correspondingly representing in the same triplet/antitriplet with the same electric charge might mix. Hence, the mixing matrices are now redefined as $(u_1 u_2 u_3 U)^T_{L,R} = V_{uL,aR}(u c t T)^T_{L,R}$ and $(d_1 d_2 d_3 D_2 D_3)^T_{L,R} = V_{dL,dR}(d s b B')^T_{L,R}$, such that the $4 \times 4$ mass matrix of up-type quarks $(u_a, U)$ and the $5 \times 5$ mass matrix of down-type quarks $(d_a, D_a)$ are diagonalized [34]. The FCNC Lagrangian as coupled to $Z'$ is now changed to

$$- \frac{g}{\sqrt{3(1 - \beta^2 t_W^2)}} \bar{q} \gamma^\mu q_1' [V^\dagger_q V_q]_{ij} Z'_{\mu},$$

(19)

where we denote $[V^\dagger_{uL} V_{uL}]_{ij} \equiv (V^*_u)_{i1}(V_{uL})_{1j} - \frac{1}{2}(V^*_u)_{i4}(V_{uL})_{4j}$ for up-type quarks and $[V^\dagger_{dL} V_{dL}]_{ij} \equiv (V^*_d)_{i1}(V_{dL})_{1j} + \frac{1}{2}(V^*_d)_{i4}(V_{dL})_{4j} + \frac{3}{2}(V^*_d)_{5i}(V_{dL})_{5j}$ for down-type quarks. The corresponding effective Lagrangian is achieved as

$$\frac{1}{w^2} (\bar{q} \gamma^\mu q_1')^2 [V^\dagger_q V_q]^2_{ij}.$$  

(20)

As mentioned in the above footnote, the ordinary and exotic quark mixings also lead to the FCNCs associated with $Z$, obtained by the Lagrangian,

$$\pm \frac{g}{2 c_W} \bar{q} \gamma^\mu q_1' (V^*_q)_{i1}(V_q)_{ij} Z_{\mu},$$

(21)
where “+” and \( I = 4 \) are applied for \( V_u \), whereas “−” and \( I = 4, 5 \) are applied for \( V_d \). Integrating \( Z \) out, the corresponding effective Lagrangian is

\[
\frac{1}{v^2_w}(q^\dagger_{iL} \gamma^\mu q_{jL})^2[(V_{qL}^*)_{ii}(V_{qL})_{lj}]^2,
\]

which would spoil the standard model prediction for the neutral meson mass differences if the mixing of the ordinary and exotic quarks was compatible to the ordinary quark mixing. For instance, the \( K^0-\bar{K}^0 \) mixing bounds \( |(V_{qL}^*)_{i1}(V_{dL})_{lj}| \lesssim 10^{-5} \), which is much smaller than the smallest CKM matrix element. To avoid the large FCNCs, we assume

\[
|(V_{qL}^*)_{i1}(V_{qL})_{lj}| \ll |(V_{qL}^*)_{i1}(V_{qL})_{1j}|,
\]

so that (22) is insignificant, and (20) is thus reduced to (18). The above inequality is also valid when 1’s are replaced by \( \alpha = 2, 3 \), due to the unitarity condition, \((V_{qL}^\dagger V_{qL})_{ij} = 0\). Furthermore, the \( B-L \) conservation demands that the exotic and ordinary quark mixings vanish \([29, 35]\). Hence, the suppressions like (23) are naturally preserved by an approximate \( B-L \) symmetry, as interpreted in \([4, 25, 36]\). Lastly, there may exist tree-level FCNCs induced by new non-Hermitian gauge bosons \( X^{0,0*} \) that couple \( u_1 \) with \( U \), and \( d_\alpha \) with \( D_\alpha \), given by the Lagrangian,

\[
\mathcal{L} \supset -\frac{g}{\sqrt{2}}(\bar{u}_{1L} \gamma^\mu U_{1L} - \bar{D}_{\alpha L} \gamma^\mu d_{\alpha L})X_\mu^0 + H.c.
\]

\[
\supset -\frac{g}{\sqrt{2}}[\bar{u}^l_{iL} \gamma^\mu u_{jL}^l (V_{uL}^*)_{i1}(V_{uL})_{4j} - \bar{d}^l_{\alpha L} \gamma^\mu d_{\alpha L}^l (V_{dL}^*)_{i1}(V_{dL})_{\alpha j}]X_\mu^0 + H.c.,
\]

where \( I = 2 + \alpha \). This yields the effective Lagrangian,

\[
\frac{1}{w^2} \left\{ (\bar{u}^l_{iL} \gamma^\mu u_{jL}^l)^2[(V_{uL}^*)_{i1}(V_{uL})_{4j}]^2 + (\bar{d}^l_{\alpha L} \gamma^\mu d_{\alpha L}^l)^2[(V_{dL}^*)_{i1}(V_{dL})_{\alpha j}]^2 \right\},
\]

where we have used \( m_X^2 = g^2/4(u^2 + w^2) \approx g^2w^2/4 \). The \( X \) contributions to FCNCs as (25) are radically smaller than those of \( Z' \) in (18) due to the conditions (23). In summary, for any 3-3-1 model the FCNCs due to \( Z' \) in (18) would dominate, which will be taken into account.

Without loss of generality, by alignment in the up quark sector, i.e. \( V_{uL} = 1 \), the CKM matrix is just \( V_{\text{CKM}} = V_{dL} \). The \( K^0-\bar{K}^0 \) mixing yields a bound \([1, 37]\),

\[
\frac{1}{w^2}|(V_{dL}^*)_{i1}(V_{dL})_{lj}|^2 < \frac{1}{(10^4 \text{ TeV})^2}.
\]

The CKM factor is \( |(V_{dL}^*)_{i1}(V_{dL})_{lj}| \approx 0.22 \) \([1]\), which implies

\[
w > 2.2 \times 10^3 \text{ TeV}.
\]
\[(V_{dL}^*)_{21}(V_{dL})_{22} \simeq 0.22,\] which presents the same bound for \(w\) as the previous case. Furthermore, putting the third quark generation differently from the first two, the CKM factor is now smaller, \[(V_{dL}^*)_{31}(V_{dL})_{32} \simeq 3.5 \times 10^{-4},\] which yields

\[w > 3.5 \text{ TeV}.\] (28)

Let us stress again that the bounds achieved in (27) and (28) are independent of \(\beta\), applying for every 3-3-1 model with appropriate fermion content, i.e. quark arrangement, which is a new investigation of this work, in agreement with the special cases in [4, 24].

We can similarly study for the \(B^0_s-\bar{B}^0_s\) mixing where \((i,j) = (2,3)\), i.e. \[\frac{1}{w^2} \left| (V_{dL}^*)_{12}(V_{dL})_{13} \right|^2 < 1/(100 \text{ TeV})^2\] [1, 37] for nonuniversal first quark generation, and so forth for other cases. With the aid of the CKM factors in [1], if the second or third quark generation is different from the two others, it gives a bound \(w > 4 \text{ TeV}\). Otherwise, when the first quark generation is different, it gives a negligible contribution to the \(B\) meson mixing. We see that the \(B\) mixing effect does not discriminate the second and third quark generations, unlike the case of the kaon mixing. The \(B\) mixing gives the bound in agreement with the \(K\) mixing when the third generation is different. However, it gives a negligible contribution to the \(B\) mixing when the kaon mixing bound is applied to the model with nonuniversal first or second quark generation.

It is noteworthy that the bound (27) applies for both the original economical 3-3-1 model [7, 8] and the reduced 3-3-1 model [23], where the first quark generation is nonuniversal. The latter is ruled out as it is limited by a low Landau pole, \(w \lesssim 5 \text{ TeV}\) [25, 28], and additionally, it is encountered with a large \(\rho\)-parameter [24]. The former presents a new physics beyond 1000 TeV order, and of course the previous predictions for the model at TeV are useless [4, 21]. On the other hand, the bound (28) is valid for both the minimal 3-3-1 model (including the simple 3-3-1 model as well) and the 3-3-1 model with right-handed neutrinos, where the third quark generation is nonuniversal, as often studied. We will also introduce a new economical 3-3-1 model working at the TeV scale, avoiding the large bound (27).

Let us remind the reader that the detailed outcomes of the FCNCs (18) using the neutral meson mass differences are worth studying, but the overall bounds as obtained above would be expected (see, for instance, [31, 32]). In other words, it is sufficient for the purpose of this work as to classify and interpret the new directions of the economical 3-3-1 models, to be discussed below.
III. TWO SCENARIOS FOR THE ECONOMICAL 3-3-1 MODEL

An economical 3-3-1 model is defined as to work with the minimal fermion and scalar content that includes $\nu_{aR}$ in lepton triplets and contains only two scalar triplets, either $(\chi, \rho)$ or $(\chi, \eta)$. Such theory has electric charge operator $Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X$. As a result of the above analysis, there are two distinct economical 3-3-1 models. The first one has particle content as the original economical 3-3-1 model (i.e., possessing nonuniversal first quark generation and $\chi, \rho$), but the 3-3-1 breaking scale is beyond 1000 TeV order, called type-I economical 3-3-1 model. By contrast, the second one has nonuniversal third quark generation and works with $\chi, \eta$, which implies a TeV 3-3-1 breaking scale, called type-II economical 3-3-1 model.

A. Type-I economical 3-3-1 model

The fermion and scalar content is given as [8, 10]

$$\psi_{aL} = \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ \nu^c_{aR} \end{pmatrix} \sim \begin{pmatrix} 1 \\ 3 \\ -\frac{1}{3} \end{pmatrix}, \quad e_{aR} \sim (1, 1, -1), \quad (29)$$

$$Q_{1L} = \begin{pmatrix} u_{1L} \\ d_{1L} \\ U_L \end{pmatrix} \sim \begin{pmatrix} 3 \\ 3 \\ \frac{1}{3} \end{pmatrix}, \quad Q_{aL} = \begin{pmatrix} d_{aL} \\ -u_{aL} \end{pmatrix} \sim (3, 3^*, 0), \quad (30)$$

$$u_{aR}, U_R \sim \begin{pmatrix} 3, 1 \\ \frac{2}{3} \end{pmatrix}, \quad d_{aR}, D_{aR} \sim \begin{pmatrix} 3, 1 \\ -\frac{1}{3} \end{pmatrix}, \quad (31)$$

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^1 \\ \chi_3^0 \end{pmatrix} \sim \begin{pmatrix} \frac{1}{\sqrt{2}} u' + G_X^0 \\ G_Y^- \frac{1}{\sqrt{2}} (w + H_1 + iG_Z') \end{pmatrix} \sim \begin{pmatrix} 1 \\ 3 \\ -\frac{1}{3} \end{pmatrix}, \quad (32)$$

$$\rho = \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^+ \end{pmatrix} \sim \begin{pmatrix} G_W^+ \\ \frac{1}{\sqrt{2}} (v + H + iG_Z) \end{pmatrix} \sim \begin{pmatrix} 1 \\ 3 \\ \frac{2}{3} \end{pmatrix}. \quad (33)$$

Recall that $\alpha = 2, 3$, and $U, D$ have ordinary electric charges like $u, d$, respectively.

The 3-3-1 breaking scale is bounded by $w > 2200$ TeV. Since $\chi_1^0$ has $L = 2 \neq 0$, its VEV, $u'$, which breaks this charge should be much smaller than the weak scale, $u' \ll v$. Because of $u' \neq 0$ there mix in the gauge boson sectors, the charged $W-Y$ and the neutral $Z-Z'-A_4$, in addition to the ordinary $Z-Z'$ mixing. Diagonalizing these sectors we get physical eigenstates and masses similarly
Consequently, from the $W$ boson mass, $m_W^2 = g^2v^2/4$, we determine the weak scale $v \simeq 246$ GeV, as usual. The mixings in both gauge boson sectors shift the tree-level $\rho$-parameter from the standard model prediction by $\Delta \rho = \rho - 1 = \frac{m_W^2}{c_W^2 m_Z^2} - 1 \simeq 3u'^2/v^2$, implying $|u'| < 3.5$ GeV due to the global fit $\Delta \rho < 0.0006$ [1]. Therefore, the mixings between exotic and ordinary quarks are proportional to $u' / w \sim 10^{-6}$ which does not affect the FCNCs due to the $Z$ exchange as well as the non-unitarity of ordinary quark mixing matrices as remarked before [29].

Note that all the new particles including Higgs bosons $H_{1,2}$, gauge bosons $Z', Y, X$, and exotic quarks $U, D$ gain the masses proportional to the $w$ scale, which are quite heavy as expected. The ordinary particles get consistent masses after the electroweak symmetry breaking, expect for the followings. Because of the minimal scalar content, there are 3 light quarks possessing vanishing tree-level masses. However, they can obtain appropriate masses induced by radiative corrections or effective interactions since the Peccei-Quinn symmetry is completely broken [11, 17].

At the tree-level, the neutrinos have Dirac masses, one zero and two degenerate, which are unacceptable. But up to the five dimensional interactions, the relevant Yukawa Lagrangian is

$$\mathcal{L}_{\text{Yukawa}} \supset h_\nu^{\mu \nu} \bar{\nu}_a L \psi_b L \rho + \frac{h^{\nu \nu}}{\Lambda} (\bar{\psi}_a L \psi_b L) (\chi X)^* + H.c.,$$

where $\Lambda$ is a cut-off scale which can be taken as $\Lambda \sim w$. Therefore, the observed neutrinos ($\sim \nu_L$) gain Majorana masses via a seesaw mechanism, evaluated to be

$$m_\nu \sim h^{\nu \nu} (h^{\nu \nu})^{-1} (h^{\nu \nu})^T \frac{v^2}{w},$$

which naturally fits the data since $w$ is large as 2200 TeV, e.g. taking $m_\nu \sim 0.1$ eV and $h^{\nu \nu} \sim 1$ yields $h^{\nu \nu} \sim 10^{-4}$ which looks like the charged lepton Yukawa couplings. It is to be noted that the above neutrino mass generation scheme may be radiatively induced [12].

The scalar field that breaks $SU(3)_L \otimes U(1)_X$ down to $SU(2)_L \otimes U(1)_Y$ is $\chi_3^0 = \frac{1}{\sqrt{2}} (w + H_1 + i G_{Z'})$, which provides the masses for the new particles as well as setting the seesaw scale, as mentioned. Further, the imaginary part is the Goldstone boson of $Z'$, while the real part includes a new neutral Higgs boson living in the $w$ scale. In the early universe, the real field $\Phi = \sqrt{2} R(\chi_3^0)$ can be interpreted as an inflaton field involving (in time) toward the potential minimum $\Phi_{\text{min}} = w$, driving the cosmic inflation. Let us consider its potential when the inflation scale is either not too high but significantly larger than $w$ or close to the Planck scale.

For the first case, the inflation potential is radiatively contributed by the gauge bosons, fermions, and scalars which takes the form (up to the leading-log approximation) [38],

$$V(\Phi) = \frac{\lambda}{4} (\Phi^2 - w^2)^2 + \frac{a}{64 \pi^2} \Phi^4 \ln \frac{\Phi^2}{w^2} + V_0,$$
where the renormalization scale has been fixed at \( w \), and
\[
a \simeq \frac{13 + 4t_W^4}{48(3 - t_W^2)^2} g^4 - \frac{1}{2} (h_U^4 + h_{D_2}^4 + h_{D_3}^4) + 9\lambda^2 + \frac{1}{4} \lambda' \lambda'.
\]
(37)

Here, \( g_X = g t_W / \sqrt{1 - t_W^2 / 3} \) is used; \( h_{U,D_\alpha} \) denote the Yukawa couplings of inflaton with exotic quarks \( U, D_\alpha \); and \( \lambda, \lambda' \) correspond to the self-inflaton and Higgs-inflaton quartic couplings, respectively. This potential yields an appropriate local minimum given that \( a/\lambda \gg -63.165 \). Additionally, since \( w \) is radically smaller than the Planck scale, the inflation potential is governed by the quartic and log terms. The number of e-folds will be chosen \( N \gtrsim 40 \) so that the inflation scale is correspondingly higher than the expected 2200 TeV value. The CMB measurements yield a constraint on the curvature perturbation which leads to \( \lambda \lesssim 10^{-12} \) [1]. Further, the spectral index \( n_s \), the tensor-to-scalar ratio \( r \), and the running index \( \alpha \) can be evaluated as functions of \( a' \equiv a/\lambda \) and fitting to the data [1]. Then we obtain \( a' \sim -10 \), and thus \( g \sim h_{U,D_\alpha} \sim \sqrt{\lambda} \ll \sim 10^{-2} \times 10^{75} \), which contradicts the electroweak data \( g \sim 0.5 \). Conversely, this regime of the potential is not flat to reproduce a suitable inflation scenario.

For the second case, the interaction of inflaton to gravity via a non-minimal coupling \( \xi \) may be important,
\[
S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} (m_P^2 + \xi \Phi^2) R + \frac{1}{2} \partial^\mu \Phi \partial_\mu \Phi - \frac{\lambda}{4} (\Phi^2 - w^2)^2 \right],
\]
(38)
where \( R \) is the scalar curvature and \( m_P = (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18} \) GeV is the reduced Planck mass. We assume \( \xi \gtrsim 1 \) and the action can be rewritten in the Einstein frame as [39]
\[
S = \int d^4 x \sqrt{-\hat{g}} \left[ \frac{1}{2} m_P^2 \hat{R} + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - U(\phi) \right],
\]
(39)
where the inflation potential is related to the canonically-normalized inflaton field \( \phi \) as
\[
U(\phi) = \frac{\lambda m_P^4}{4\xi^2} \left( 1 + e^{-\sqrt{\frac{2}{3} \phi} m_P} \right)^{-2}.
\]
(40)
That said, the inflation potential is flat due to the large field values, \( \phi \gg m_P \) or \( \Phi \gg m_P / \sqrt{\xi} \), and it successfully fits the data if \( \xi \sim 10^4 \sqrt{\lambda} \), in agreement to [39]. In this case, the number of e-folds set is about 60. Since \( \lambda = m_{H_1}^2 / (2w^2) \) can be small for a \( H_1 \) mass of few TeVs, the unitarity condition \( \xi \lesssim O(10) \) is recognized, and the inflation begins from the Planck regime \( \Phi \sim M_P \). The reheating happens when the inflaton decays into the exotic quarks or new gauge bosons. Considering the first case, it yields \( T_R \sim h_{U,D_\alpha} (w/1000 \text{ TeV})^{1/2} \times 10^{11} \text{ GeV} \sim 10^{11} \text{ GeV} \).

Since the right-handed neutrinos do not directly couple to the inflaton, they could only be produced from the thermal bath of radiations. The CP asymmetry decays of these right-handed
neutrinos into a heavy charged Higgs boson and a charged lepton $\nu_R \rightarrow H^\pm e^\mp$ due to the Yukawa couplings $h_{ab} \bar{\psi}_a L \bar{u}_b R + H.c.$ can generate the expected baryon asymmetry via a leptogenesis mechanism similarly to the standard technique provided that $m_{\nu_R} \gtrsim m_{H_2}$ [40]. However, it differs from the standard prediction due to the fact that the channels $\nu_R \rightarrow G^\pm e^\mp$ via the couplings $h'_{ab} \bar{\psi}_a L \bar{u}_b \rho + H.c.$ are negligible as suppressed by $h' \ll h$ and $m_W \ll m_{H_2}$. Additionally, like the neutral field $H_1$, the finding of the charged field $H_2$ with some mass in the TeV regime can mark the existence of this baryon-asymmetry production scheme.

Let us emphasis that the economical 3-3-1 model has a natural room for dark matter as basic scalars filling up the model [25, 36]. As studied in [36], the dark matter candidate might be resided in an inert scalar triplet $\eta$, as a replication of $\chi$ and odd under a $Z_2$ symmetry. We may have another inert scalar triplet for dark matter as a replication of $\rho$. However, in this model, the candidate has a mass proportional to the 3-3-1 scale of 1000 TeV order. Therefore, if its mass is in or beyond this scale, it cannot be generated as thermal relics as of [36], since otherwise it overcloes the universe due to the unitarity constraint [41]. Interestingly, this superheavy dark matter can be generated in the early universe by some mechanisms such as gravitational and thermal productions, associated with the discussed inflation and reheating, analogous to [42]. By contrast, the thermal generations may be interpreted as in [36] if the inert field masses are at TeV scale.

Hence, by the realization of a high 3-3-1 breaking scale, the 3-3-1 model might integrally explain the neutrino masses and the cosmological issues, comparable to the other theories [4, 24, 29, 35, 42–45]. Note that the usual 3-3-1 models do not reveal the inflation and associated superheavy dark matter. A detailed investigation of all the issues for this kind of the model is out of the scope of the present work, which should be published elsewhere [46].

B. Type-II economical 3-3-1 model

A low bound for the 3-3-1 breaking scale is only if the third quark generation is discriminative. For this case, the scalar that breaks the electroweak symmetry should be $\eta$ instead of $\rho$, in order to generate the top quark mass consistently (by contrast, with the scalar content as the previous model retained, the top quark has vanishing tree-level mass and it is impossible to be induced by radiative corrections or effective interactions, cf. [25], for instance). Thus, the fermion and scalar
content is appropriately derived as

$$\psi_{aL} = \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ \nu_{aR}^c \end{pmatrix} \sim \begin{pmatrix} 1, 3, -\frac{1}{3} \end{pmatrix}, \quad e_{aR} \sim (1, 1, -1),$$

$$Q_{aL} = \begin{pmatrix} d_{aL} \\ -u_{aL} \\ D_{aL} \end{pmatrix} \sim (3, 3^*, 0), \quad Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \end{pmatrix} \sim \begin{pmatrix} 3, 3, \frac{1}{3} \end{pmatrix},$$

$$u_{aR}, U_R \sim \begin{pmatrix} 3, 1, \frac{2}{3} \end{pmatrix}, \quad d_{aR}, D_{aR} \sim \begin{pmatrix} 3, 1, -\frac{1}{3} \end{pmatrix},$$

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^{-1} \\ \chi_3^0 \end{pmatrix} \approx \begin{pmatrix} \frac{1}{\sqrt{2}} w' + c \zeta G_X^0 - s \zeta H_2^0 \\ G_Y^s \\ \frac{1}{\sqrt{2}} (w + H_1 + i G_Z') \end{pmatrix} \sim \begin{pmatrix} 1, 3, -\frac{1}{3} \end{pmatrix},$$

$$\eta = \begin{pmatrix} \eta_1^{0} \\ \eta_2^{-1} \\ \eta_3^{0} \end{pmatrix} \approx \begin{pmatrix} \frac{1}{\sqrt{2}} (u + H + i G_Z) \\ G_W^s \\ \frac{1}{\sqrt{2}} w' + s \zeta G_X^0 + c \zeta H_2^0 \end{pmatrix} \sim \begin{pmatrix} 1, 3, -\frac{1}{3} \end{pmatrix},$$

where note that $\alpha = 1, 2$ and $t_\xi = u'/w'$ as well as the physical scalar spectrum as explicitly displayed can be obtained from the following scalar potential. Recall also the FCNC bounds:

$w > 3.5$ TeV for the $K$ mixing and $w > 4$ TeV for the $B_s$ mixing.

The total Lagrangian is $\mathcal{L} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Yukawa}} - \mathcal{V}_{\text{scalar}}$, where

$$\mathcal{L}_{\text{kinetic}} = \sum_F \bar{F} i \gamma^\mu D_\mu F + \sum_S (D^\mu S)^\dagger (D_\mu S) - \frac{1}{4} G^\mu\nu G_{\mu\nu} - \frac{1}{4} A^\mu\nu A_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu},$$

where $F, S$ run over fermion and scalar multiplets, respectively. $G_{\mu\nu}, A_{\mu\nu},$ and $B_{\mu\nu}$ are field strength tensors corresponding to the 3-3-1 groups, and $D_\mu$ is covariant derivative as previously supplied. The Yukawa Lagrangian and scalar potential are

$$\mathcal{L}_{\text{Yukawa}} = h_{33}^U \bar{Q}_{3L} \chi U_R + h_{3\alpha}^D \bar{Q}_{\alpha L} \chi^* D_{\beta R} + h_{3a}^\eta \bar{Q}_{3L} \eta u_{aR} + h_{ao}^d \bar{Q}_{\alpha L} \eta^* d_{aR}$$

$$+ h_{3a}^u \bar{Q}_{3L} \eta u_{aR} + h_{ao}^d \bar{Q}_{\alpha L} \chi^* d_{aR} + h_{3a}^{\eta U} \bar{Q}_{3L} \eta U_R + h_{oa}^{\eta D} \bar{Q}_{\alpha L} \eta^* D_{\beta R} + H.c.,$$

$$V_{\text{scalar}} = \mu_1^2 \eta^\dagger \eta + \mu_2^2 \chi^\dagger \chi + \lambda_1 (\eta^\dagger \eta)^2 + \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 (\eta^\dagger \eta)(\chi^\dagger \chi) + \lambda_4 (\eta^\dagger \chi)(\chi^\dagger \eta)$$

$$+ \left[ \mu_3^2 \eta^\dagger \chi + \lambda_5 (\eta^\dagger \chi)^2 + (\lambda_6^0 \eta^\dagger \eta + \lambda_7^0 \chi^\dagger \chi) \eta^\dagger \chi + H.c. \right].$$

By the criteria in [4, 35] with $L(\nu_R) = 1$ and $B(\nu_R) = 0$, the baryon minus lepton number $B - L$ neither commutes nor closes algebraically with $SU(3)_L$. Indeed, we see that the minimal interactions of the model (unprimed couplings) conserve a new Abelian symmetry, $U(1)_N$, that
close those symmetries by \( B - L = - \frac{4}{\sqrt{3}} T_8 + N \). The \( N \)-charges for multiplets are obtained as
\[
N(\psi_{aL}, Q_{3L}, Q_{aL}, e_{aR}, u_{aR}, d_{aR}, U_R, D_{aR}, \eta, \chi) = -1/3, 1, -1/3, -1, 1/3, 1/3, 7/3, -5/3, 2/3, -4/3,
\]
while the nontrivial \( B - L \) charges for new particles are \([B - L](U, D, \eta_3^0, \chi_1^0, \chi_2^0, X^0, Y^-) = 7/3, -5/3, 2, -2, -2\), respectively. Here, the fields \( X \) and \( Y \) are non-Hermitian gauge bosons correspondingly coupled to \( T_{4,5} \) and \( T_{0,7} \). It is clear that the nonminimal Yukawa couplings that are primed violate \( B - L \) by two units, while the nonminimal scalar couplings and mass-parameters as primed violate this charge by one or two units. Additionally, \( \eta_3^0 \) and \( \chi_3^0 \) have \( B - L \neq 0 \), thus their VEVs \( u', w' \) break \( B - L \) unlike \( u, w \) that conserve this charge. For consistency with the standard model, the violating parameters such as couplings and VEVs should be much smaller than the corresponding conserved ones, \( u' \ll u, w' \ll w, h' \ll h, \chi' \ll \lambda, \) etc.

It is easily checked that the leptons and three ordinary quarks have vanishing tree-level masses. Furthermore, the Lagrangian automatically contains the Peccei-Quinn like symmetries, similarly to the original economical 3-3-1 model \[17\]. Such massless particles can get appropriate masses when the Peccei-Quinn like symmetries are completely broken via radiative corrections or effective interactions \[17\]. Let us impose the latter, which is up to five dimensions, given by
\[
\mathcal{L}_{\text{Yukawa}} = \frac{1}{\Lambda} (Q_{3L} \eta^* \chi^*) (h^u_{aR} d_{aR} + h^D_{aR} D_{aR}) + \frac{1}{\Lambda} (\bar{Q}_{aL} \eta \chi) (h^u_{aR} u_{aR} + h^D_{aR} U_R)
+ \frac{1}{\Lambda} h^e_{ab} \bar{\psi}_{aL} \eta^* \chi^* e_{bR} + \frac{1}{\Lambda} (h^c_{ab} \bar{\psi}_{bL} (f^\nu_{ab} \eta \eta + g^\nu_{ab} \chi \chi + h^\nu_{ab} \eta \chi^*) + H.c.,
\]
where as usual the unprimed couplings conserve \( B - L \), while the primed couplings stand for the violating ones. The cutoff scale \( \Lambda \) can be taken in the same order as \( w \). Specially, \( f^\nu_{ab} \) and \( g^\nu_{ab} \) are symmetric, while \( h^\nu_{ab} \) is general, in flavor indices.

The fermion mass matrices can be derived when substituting the VEVs of the scalars into the Lagrangians \[17\] and \[19\]. Using the conditions that the violating parameters are radically smaller than the corresponding conserved ones, the charged leptons obtain masses, \([M_e]_{ab} \simeq h^e_{ab} u \frac{w}{\Lambda} \), where \( w \sim \Lambda \) and \( u = 246 \text{ GeV} \) as imposed from the \( W \) boson mass, which can fit the data as the standard model. Since \( u \ll w \), we have seesaw mechanism for the neutrino masses. Indeed, the right-handed neutrinos achieve large Majorana masses, \([M_R]_{ab} \simeq -g^\nu_{ab} \frac{w^2}{\Lambda} \). The left-handed neutrinos gain small Majorana masses, \([M_L]_{ab} \simeq -f^\nu_{ab} \frac{w^2}{\Lambda} \). The neutrino Dirac masses take the form, \([M_D]_{ab} \simeq -h^\nu_{ab} \frac{w^2}{2\Lambda} \). Thus, the observed neutrinos \( (\sim \nu_L) \) obtain small masses,
\[
M_\nu \simeq M_L - M_D^T M_R^{-1} M_D \simeq - \frac{w^2}{\Lambda} \left[ f^\nu - \frac{1}{4} (h^\nu)^T (g^\nu)^{-1} h^\nu \right].
\]
The new observation is that the neutrinos get masses when both the Peccei-Quinn and \( B - L \) symmetries are broken. The strength of the symmetry breakings is set by the primed couplings of
the effective interactions, commonly called $h'$, thus $M_\nu \sim \frac{u^2}{\Lambda} h'$. Note that for the 3-3-1 model, if $B - L$ is conserved, it must be a gauged charge, and thus the effective interactions must absent themselves [4, 24, 29, 35, 42–44]. Therefore, $h'$ measures the approximate $B - L$ symmetry as well as the nonunitarity of the 3-3-1 model, as imprinted from the 3-3-1-1 model. The $h'$ strength can be obtained by integrating $U(1)_N$ gauge boson out in the 3-3-1-1 model, which matches $h'/\Lambda = g_N/\Lambda_N$. Further, we have $h' \sim \Lambda/\Lambda_N \sim 10^{-11}$, where $\Lambda_N \sim 10^{14}$ GeV is just inflation scale and $g_N \sim 1$ [42, 43]. This implies $M_\nu \sim 0.1$ eV as desirable. Alternatively, comparing $M_\nu/M_e \sim \frac{w h'}{u h'}$ with $u/w \sim 0.1$ and $M_\nu/M_e \sim 10^{-6}$, it yields $h'/h \sim 10^{-5}$, and thus the breaking strength $h'$ is suitably smaller than the electron Yukawa coupling, in agreement to [25].

The mixings of the exotic and ordinary quarks are proportional to $u'/u$, $w'/w$, and $h'/h$—the ratios of the $B - L$ violating parameters over the corresponding normal ones [34]. Again, the VEVs $u'$, $w'$ and couplings $h'$ should be small, $u' \ll u$, $w' \ll w$, $h' \ll h$, in order to suppress the dangerous FCNCs coming from $Z$ boson exchange due to the ordinary and exotic quark mixings. Generalizing the result in [29], we obtain $u'/u \sim w'/w \sim h'/h \sim \sqrt{|(V_{dL})_{11}(V_{dL})_{12}|} \lesssim 3.16 \times 10^{-3}$, where $(V_{dL})_{ij}$ is the element that connects the corresponding exotic and ordinary quarks in the mixing matrix. It yields $u' \lesssim 0.77$ GeV due to $u = 246$ GeV, and $w' \lesssim 3.16, 15.8$, and 31.6 GeV for $w = 1, 5$, and 10 TeV, respectively, as well as $h'$ is more suppressed, similarly to the ones for the neutrino masses. In practice, the VEVs $u'$, $w'$ break $B - L$ (i.e., the lepton number), and they are suppressed to be small by the corresponding lepton-number violating scalar-potential. From the potential, we have roundly $u' \sim \lambda_{12}^2 u$ and $w' \sim \lambda_7^2 w$. Thus, $u'$ and $w'$ should be small since its absence, i.e. $\lambda_{12}^2 = 0$, enhances the 3-3-1-1 gauge symmetry.

Following the approach in [25, 36], the model can provide realistic dark matter. The inert triplet $\rho$, which is odd under a $Z_2$ symmetry and analogous to the one in the 3-3-1 model with right-handed neutrinos, if introduced cannot be dark matter since the candidate $\rho_2^0 = \frac{1}{\sqrt{2}}(H + iA)$ yields degenerate masses for $H$ and $A$, which implies a large direct detection cross-section via $Z$ exchange, already ruled out by the experiment [47]. However, an inert triplet as replication of $\eta$ or $\chi$, called $\zeta = (\zeta_1^0, \zeta_2^0, \zeta_3^0)$, provides a consistent candidate as the combination of either real or imaginary parts of $\zeta_1^0, \zeta_2^0, \zeta_3^0$. The inert scalar sextet responsible for dark matter can be also interpreted, similarly to the simple 3-3-1 model in [25].

In summary, the 3-3-1 model with right-handed neutrinos has a nontrivial vacuum for $u' \neq 0$ and $w' \neq 0$, and this yields the appropriate new-physics consequences as obtained. Interestingly, the type II economical 3-3-1 model is a minimal realization of this vacuum, while explicitly indicating to dark matter. See [48] for other interpretations. Note that the previous studies [3] only consider
the vacuum with \( u' = w' = 0 \), and thus the above consequences were not recognized, although they include more than two scalar triplets.

IV. CONCLUSION

As a fundamental element, the 3-3-1 model presents the FCNCs via \( Z' \) boson due to nonuniversal fermion generations under the gauge symmetry. We have proved that the FCNCs describing meson mixings are independent of the embedding of electric charge operator as well as the potential Landau pole. Applying for the \( K \) and \( B_s \) mixings, we obtain the new physics scale \( w > 2200 \text{ TeV} \) if the first or second fermion generation is discriminative, and \( w > 3.5 \text{ TeV} \) for the \( K \) system or \( w > 4 \text{ TeV} \) for the \( B_s \) system if the third fermion generation is discriminative.

Due to the above constraint, the original economical 3-3-1 model works in a large energy regime, yielding the seesaw mechanism, inflation, leptogenesis, and superheavy dark matter integrally. The 3-3-1 breaking field, \( \chi^0_3 \), is important to set the seesaw scale originating from inflation scale and define inflaton. Dark matter is a hidden/inert field, a replication of \( \chi \), called \( \eta \), or of \( \rho \), called \( \rho' \), which might be created in the early universe by nonthermal processes/mechanisms associated with the inflation and reheating. The imprints at TeV scale of the inflation and leptogenesis mechanisms are the new Higgs fields \( H_{1,2} \) which may be verified at the LHC.

Alternatively, we have introduced a new economical 3-3-1 model, where the third fermion generation is rearranged differently from the first two and the scalar content includes \( \eta, \chi \). This model works naturally at the TeV scale. The lepton number breaking/violating parameters are suppressed, \( u' \ll u, w' \ll w, h' \ll h \), and \( \lambda' \ll \lambda \). The strength of the lepton number breaking might have a source from the 3-3-1-1 breaking to be actually small, responsible for the neutrino masses. It is shown that a hidden scalar field \( \zeta \) as a replication of \( \eta \) or \( \chi \) can provide WIMPs as thermal relics. However, \( \rho \) if included as an inert scalar cannot be dark matter.

Let us stress that the discrimination of fermion generations as recognized at 1000 TeV order is surprisingly close to the WIMP mass bound. Although the 3-3-1 model does not directly solve this coincidence, it provides both scenarios for dark matter as nonthermal and thermal relics. Therefore, these two economical 3-3-1 models would predict and connect the particle physics to the cosmological issues with rich phenomenologies, attracting much attention [46].
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