1. Introduction

Spontaneous parametric down-conversion (SPDC) is a simple high-rate and high-fidelity source of entangled quantum states. Entangled polarization qubits are ideal systems for demonstration of quantum teleportation, quantum dense coding, quantum key distribution etc. But for further improvement of the listed techniques, it is necessary to increase the system dimensionality and turn to entangled qudits, which have much higher degrees of entanglement [1–6].

For this purpose one can exploit frequency [7], temporal [8–12], and spatial SPDC modes [13–17], including orbital angular momentum [18, 19] modes. Frequency and temporal modes do not allow one to manipulate and register them without postselection [20]. This can be overcome by using spatial modes and integrated optics [21].

Separated Schmidt modes in the angular spectrum of biphotons

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Abstract

We prepared qudits based on angular multimode biphoton states by modulating the pump angular spectrum. The modes were prepared in the Schmidt basis and their intensity distributions did not overlap in space. This allows one to get rid of filtering losses while addressing single modes and to realize a single-shot qudit readout.

Keywords: spontaneous parametric down-conversion, angular spectrum, qudit, Schmidt decomposition

((Some figures may appear in colour only in the online journal)
Figure 1. (a) Schematic illustration of the double-Gaussian TPA [27] and (b) the modes $f_{ij}(k)$ obtained after slits are placed in the far field into the signal and idler beams.

\[
F(k_{xx}, k_{sx}) = \exp \left( -\frac{(k_{xx}+k_{sx})^2}{2\sigma_1^2} \right) \times \exp \left( -\frac{(k_{xx}-k_{sx})^2}{2\sigma_2^2} \right)
\]

where $\sigma_1$ reflects the pump angular divergence and $\sigma_2$ is determined by phase-matching relations (figure 1(a)). For type-I collinear and noncollinear frequency-degenerate SPDC, $\sigma^2_1$ and $K$ take the values $\sigma^2_1 = \sqrt{4K\sigma_1/\gamma_1}$, $K = 0$ [26] and $\sigma^2_1 = \sqrt{4\sigma_2/(\gamma_2 \gamma_1)}$, $K = 2\pi \sqrt{2}\sigma_2/\gamma_1\lambda_p = k_\parallel \sin(\theta_0)$ [24, 25], respectively, where $n_0$ and $n_p$ denote the signal and pump refractive indices, $\lambda_p$ is the pump wavelength, and $\theta_0$ is the angle between the pump and signal beams at the degenerate wavelength. $\gamma_1 = 0.249$ [27] and $\gamma_2 = 0.195$ [24] are the parameters which come from the approximations $\sin(x^2) \approx \exp(-\gamma_1 x^2)$ and $\sin(x) \approx \exp(-\gamma_2 x^2)$.

The TPA of two entangled qudits, each having $d$ modes, has the form of a single sum:

\[
|\Psi\rangle = \sum_{m=1}^{d} c_m |\psi_m^{(i)}\rangle |\psi_m^{(j)}\rangle,
\]

where $\{\psi_m\}$ denotes a set of eigenvectors of the reduced density matrices for the signal and idler photons and $|c_m|^2$ is the probability of registering the state in the $m$th mode. To extract entangled modes in signal and idler channels one often places slits in front of the detector (figure 1(b)). The thinner the slits, the closer the state to a single-mode one, but, at the same time, the greater the losses and the portion of uncorrelated photons in each mode.

A more useful approach realized here is to choose qudit modes to coincide with Schmidt modes [26]. To achieve this we should perform decomposition of the TPA in the form of a single sum of factorized terms instead of double integration (1):

\[
F(k_s, k_i) = \sum_{m=1}^{d} c_m f_{ij}^{(m)}(k_s) f_{ji}^{(m)}(k_i),
\]

with the profiles of the Schmidt modes $f^{(m)}_{ij}(k_{ij})$ non-overlapping in the $k$ space.

For a double-Gaussian TPA, the Schmidt modes can be chosen in the Hermite–Gauss or Laguerre–Gauss basis [28] and will spatially overlap (figure 2). Projective measurements in this case require a spatial light modulator (SLM) for each mode combined with a spatial filter (e.g. single-mode fiber), which is a source of considerable losses and does not allow a single-shot qudit readout. To overcome this one has to use complicated mode-sorting schemes based on phase modulation [29–31] which also lead to a drop in efficiency. Non-overlapping Schmidt modes can help to eliminate this disadvantage (figure 3(b)). In the same way as demonstrated in the frequency domain [7], it can be realized for spatial modes in the near or far field (figure 4).

Here we demonstrate the preparation of Gaussian-shaped spatially separated Schmidt modes in the far field. Our approach is based on the TPA dependence on the transverse pump beam shape (2), which has been realized previously for frequency modes [7]. In the three-mode SPDC state demonstrated in our work, each mode is generated from the corresponding Gaussian pump beam, coming to the crystal at a slightly different angle (figures 3 and 4(b)). Note that the divergence of each pump beam should be adjusted in such a way that each maximum of the TPA shows no wavevector correlations and can be therefore treated as a single term in the Schmidt decomposition (5) (figure 3). The required divergence depends on the phase-matching conditions for a given crystal. In a recent work [32] the authors used a similar approach of...
pump beam modulation to achieve the right pump profile on the crystal surface. In contrast to the current work they placed slits in the near field of the pump beam (figure 4(a)) and did not exploit the Schmidt decomposition framework. Because of that the width of the slits was about three times the value determined from the expression below (3).

The idea of pump spatial profile modulation has been used as well in experiments investigating the orbital angular momentum degrees of freedom of the SPDC signal [33–37].

2. Theoretical description

To prove the principle we chose the type-I degenerate noncollinear SPDC generation regime characterized by the transverse wavevector $K$ and the far-field biphoton distribution to be a set of $M$ separated Gaussian peaks along the $x$ axis (perpendicular to the pump wavevector; see figure 5):

$$F(k_x, k_y) = \frac{1}{}\sqrt{2\pi \sigma_x \sigma_y} \exp\left(-\frac{(k_x-k_0)^2}{2\sigma_x^2}\right) \exp\left(-\frac{(k_y-k_0)^2}{2\sigma_y^2}\right) \exp\left(-\frac{(k_x+k_y-K_z)^2}{2\sigma_z^2}\right) \exp\left(-\frac{(k_x-k_y-K_z)^2}{2\sigma_z^2}\right).$$

(6)

where $k(m) = (M - 1 - 2m)k_0/2$, with $k_0$ denoting the distance between neighboring peaks (figure 3(a)). If $\sigma_x = \sigma_y$, (6) becomes factorized and acquires the form of the Schmidt decomposition.

The distribution of the pump field along the $x$ coordinate on the crystal surface is the product of a periodic function, which defines the distance between neighboring biphoton Schmidt modes in the far field, and a Gaussian envelope, which defines the angular size of each mode:

$$E_p(x) \propto e^{-x^2/2\sigma_x^2} \sum_{m=0}^{M-1} \cos(xk(m)).$$

(7)

Consequently, to obtain a set (6) of separated Schmidt modes in the angular spectrum of SPDC we should choose $\sigma_x$ so that without the pump cosine modulation a single-mode regime in the SPDC spectrum is achieved.

3. Experiment

Our setup is shown in figure 5. Two-photon light was generated in the noncollinear degenerate regime (the angle between the pump and signal beams at the degenerate wavelength outside the crystal was equal to $10^\circ$) in a 3 mm thick BBO crystal from a cw diode laser pump with a wavelength of 405 ± 0.01 nm. To achieve the desired pump distribution we used an SLM (Holoeye Pluto-VIS) and prepared holograms according to paper [38]. The phase encrypted into the hologram (figure 7) had the form of a blazed diffraction grating (six pixels of SLM per period) with a three-peak Gaussian envelope defining pump amplitude modulation in the far field (6). The pump beam was expanded before the SLM and its first-order diffracted part was focused on the crystal by lenses L2, L3, and L4, and the zero-order diffracted part was filtered out. The pump power on the crystal was 0.5 mW. The SPDC signal was registered in the far field by spatial filters. Each filter consisted of a slit placed in the focal plane of a collimating lens (L5/L6), with a focal distance of 100 mm, and followed by a lens of a 2.1 mm focal distance and a multimode fiber. The filters were scanned in the focal planes of the lenses (L5/L6).

The slits for spatial filtering had sizes of $0.2 \times 4$ mm in one channel and $0.4 \times 10$ mm in another (the smaller size was along the direction of the pump modulation). For frequency filtering we placed in front of the detectors bandpass filters with a full width at half-maximum (FWHM) of 10 nm and a central wavelength of 810 nm. The fibers transmitted the coupled light to PerkinElmer single-photon detectors based on avalanche photodiodes.

4. Results and discussion

At the first stage we adjusted the pump beam size in order to fulfill the equation $\sigma_x = \sigma_y^\prime$ (figure 3(b)) so that a single Gaussian pump beam would produce spatially single-mode biphotons. This condition was satisfied by using the pump with an FWHM of $(250 \pm 2) \mu$m. The results are shown in figure 6. Blue dots correspond to the single-detector counting rate, and red ones show coincidences between the two detectors one of which was collecting all radiation in the signal channel and the second was scanning along the modulation axis in the idler channel. The ratio of the widths of these two curves is very close to the Schmidt number of the state [39]. Thus, we can conclude that the state is nearly single-mode.

Next, we modulated the angular distribution of the pump field according to (7). The corresponding hologram sent to the
SLM is shown in figure 7. As a result, we achieved spatially multimode SPDC generation with $M = 3$. The control parameters were $k_0$ (inversely proportional to the period of the pump modulation at the crystal) and $\sigma_k$ (inversely proportional to the width of the envelope $\delta x$). The distribution of the pump intensity on a CCD camera placed at the position of the crystal is shown in figure 8.

Because the pump beam incident on the SLM had a Gaussian intensity distribution, the central peak in the resulting SPDC spectrum was brighter than the sidebands. To reflect this fact we inserted a parameter $\alpha = 0.63$ into equation (7) and took it into account in all theoretical calculations:

$$E_p(x) \propto (1/2 + \alpha \cos(2k_0x)) \exp \left(-\frac{x^2}{\sigma_k^2} \right).$$

Using the cross-section of the CCD image, we estimated the parameters of the pump distribution: the field envelope FWHM was equal to $(246 \pm 2) \mu m$ and $k_0 = (0.168 \pm 0.002) \mu m^{-1}$. Note that the resolution of the camera (the size of each pixel was $4.4 \mu m$) was not good enough to properly resolve the peaks but still allowed us to estimate the period and the width of the envelope.

The measured and calculated angular intensity distributions for SPDC are shown in figure 9. Blue lines show the single count rate spectra obtained by scanning a slit in the signal beam. Red, brown and magenta are joint two-photon count rate spectra obtained by scanning a slit in the signal beam after placing another slit in the idler beam at three different positions of maxima in the single count rate distribution. Solid lines in panel (b) are calculated for the parameters of the experiment and dashed ones reflect the case of a weak pump focusing in both the $x$ and $y$ directions and narrow slits of sizes of $0.2 \times 0.5 \text{mm}$ in both directions.

The noticeable broadening of the experimental distributions compared to the theoretical ones is due to the large height of the slits, which results in the integration of the spectrum intensity in the $x$ direction (compare with the dashed lines in figure 9(b) corresponding to the theoretical distributions with very narrow slits). Another reason for the broadening was the effect of pump beam walkoff, which can be overcome by modulating the pump beam shape not only in $y$ but in both the $x$ and $y$ directions. The use of frequency filters of smaller widths will also result in a narrower SPDC distribution. We explain the difference in conditional and unconditional experimental spectra as a result of misalignment of the slit positions.

Note that the geometry of the experiment does not lead to considerable broadening due to the anisotropy of the pump refractive index, an effect investigated in [40].

In our approach the purity of the SPDC distributions produced by each of the three interfering Gaussian pump beams can theoretically reach almost 1 as it is equal to the inverse of the Schmidt number [41], with cross-correlations less
Figure 10. Intensity correlations calculated for each pair of modes for the parameters of the experiment.

than $10^{-41}$ (see figure 10). This value was calculated as a normalized square of an overlapping area for the theoretical angular distributions of each pair of modes.

5. Conclusion

Here we have demonstrated a simple method of achieving separated Schmidt modes in the angular spectrum of SPDC. The method is based on spatially modulating the pump intensity distribution on a nonlinear crystal. Another advantage of our approach is that an arbitrary shape of the SPDC angular distribution can be achieved due to the use of a spatial light modulator at the stage of pump preparation.

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