Teleportation of the one-qubit state with environment-disturbed recovery operations

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Abstract. We study standard protocol \( P_0 \) for teleporting the one-qubit state with both the transmission process of the two qubits constitute the quantum channel and the recovery operations performed by Bob disturbed by the decohering environment. The results revealed that Bob’s imperfect operations do not eliminate the possibility of nonclassical teleportation fidelity provided he shares an ideal channel state with Alice, while the transmission process is constrained by a critical time \( t_0 \), longer than which will result in failure of \( P_0 \) if the two qubits are corrupted by the decohering environment. Moreover, we found that under the condition of the same decoherence rate \( \gamma \), the teleportation protocol is significantly more fragile when it is executed under the influence of the noisy environment than those under the influence of the dissipative and dephasing environments.

PACS. 03.67.Hk Quantum communication – 03.65.Yz Decoherence; open systems; quantum statistical methods – 75.10.Jm Quantized spin models

1 Introduction

Quantum teleportation [1], the disembodied transport of a quantum state based on the nonlocal properties of an entangled state resource, has been demonstrated to be one of the most peculiar and fascinating aspects of quantum information theory. Together with the help of local operations and classical communication (LOCC), it allows sending the quantum information from a sender, conventionally named Alice, to a distant receiver Bob, with fidelity (see Section 2) better than that achievable via classical communication alone, with the cost of destroying the original state. Due to the important role it played in quantum information theory, a lot of theoretical works [2,3,4,5] have been devoted to it in recent years. Experimental realization of quantum teleportation has also been successfully demonstrated with photonic qubits [6,7] and atomic qubits [8,9].

The practical implementation of quantum teleportation begins with the preparation of a pair of entangled qubits which are shared by two parties, Alice and Bob. This step establishes a quantum link between them (see Figure 1). Alice receives a state to be teleported and performs the Bell state measurement on her two qubits, and then communicates classically the measurement result (two bits of classical information) to Bob, who uses it to perform recovery operations on his qubit, thus completing the process of teleportation. The perfect implementation of this quantum protocol requires the sharing of the maximally entangled channel state and complete coherent control over a system’s quantum state. In real circumstances, however, decoherence due to the inevitable interaction of the system with the surrounding environment makes it very difficult to prepare the maximally entangled channel states [10,11,12,13], and the amount of entanglement may be further reduced when the two qubits being distributed to Alice and Bob because during the transmission process, the qubits may also be exposed to decohering environment. For this reason, a number of schemes using non-maximally entangled state as resource have been proposed [2,3,4]. These works reveal several interesting aspects of quantum entanglement in terms of their teleportation capacity. Particularly, it is shown in reference [3] that standard teleportation with an arbitrary entangled mixed state resource is equivalent to a generalized depolarizing channel with probabilities given by the maximally entangled components of the resource.

Although teleportation with system decoherence have been studied by many authors in recent years [14,15,16,17,18], we noted that the effects of imperfect operations (e.g., the Bell state measurement performed by Alice and the recovery operations performed by Bob) on quantum teleportation has seldom been considered. However, a through understanding of this problem is obviously vital for the achievement of high efficient and long distance quantum communication, for in the presence of a decohering environment, it is difficult to execute the complete coherent control of a quantum state. Indeed, several recent works [14,18] have demonstrated that the noisy operations may have significant influence on reducing fidelity...
of the expected outcomes. Stimulated by this observation, in the present paper we would like to reexamine the standard teleportation protocol $P_0$ with the addition of environment-disturbed recovery operations. Although this issue is somewhat similar to that discussed in reference [15], we concentrated on, however, different mechanisms of decoherence (see Section 2 for more detail), and thus one may expects that it will include new features characteristic of the considered system here. Our results revealed that in order to execute the quantum teleportation protocol with fidelity better than the classical communication alone will do, the rotation rate $\omega$ of Bob’s recovery operations must be larger than a critical value. Moreover, if the channel state is prepared maximally and distributed to Alice and Bob without decoherence, then Bob’s imperfect recovery operations do not rule out the possibility of nonclassical teleportation of a quantum state. For the decohered channel states, however, the transmission time $t_0$ (see Figure 1) of the two qubits must be shorter than a critical value to ensure success of the standard teleportation protocol.

2 Basic formalism

During the teleportation process, the unavoidable interaction of an open quantum system with its surrounding environment is an important source of decoherence [19]. In order to describe such process, a master equation approach can be used. Under the assumption of Markovian and Born approximations and after performing the partial trace over the environmental degrees of freedom, the reduced density operator $\rho$ of the open quantum system evolves according to a general master equation in the Lindblad form [19,20]

$$\frac{d\rho}{dt} = -i[H, \rho] + \frac{\gamma}{2} \sum_{k,i} (2\mathcal{L}_{k,i}\rho\mathcal{L}_{k,i}^\dagger - \mathcal{L}_{k,i}^\dagger\mathcal{L}_{k,i}\rho - \rho\mathcal{L}_{k,i}^\dagger\mathcal{L}_{k,i}),$$

(1)

where $H$ denotes the Hamiltonian of the system, and $\gamma$ is the phenomenological parameter that describes the coupling strengths of the qubits with their respective environment. The generators of decoherence here are defined in terms of the raising and lowering operators $\sigma^{\pm} = (\sigma^x \pm i\sigma^y)/2$ ($\sigma^z$ with $n = 0, 1, 2, 3$ signify the $2 \times 2$ identity matrix and the three Pauli spin operators) as $\mathcal{L}_k = \sigma_k^x$ for the dissipative environment, $\mathcal{L}_{k,1} = \sigma_k^z$ and $\mathcal{L}_{k,2} = \sigma_k^y$ for the noisy environment, and $\mathcal{L}_{k} = \sigma_k^x \sigma_k^z$ for the dephasing environment [19]. Moreover, we have assumed that during the decoherence process each qubit of the open system interacts only, and independently, with its own environment. This assumption is reasonable provided the constituents composing the quantum system are separated by distances large enough [19].

In the present work, we explore standard teleportation protocol of the one-qubit state when it is executed in the presence of dissipative, noisy and dephasing environments [19]. For simplicity, we consider throughout this paper the situation in which Alice’s Bell state measurement is perfect, while the decoherence only takes place during the establishment of the channel state (e.g., a third party prepares the maximally entangled Bell state at time $t = 0$, and then sends one qubit to Alice and another one to Bob after a time interval $t_0$. During the transmission process, the two qubits may be exposed to the decohering environment, and thus degrades entanglement between them) as well as Bob performs the recovery operations (see Figure 1 for an illustration of this process).

For the ideal situation (i.e., no decoherence), if Alice and Bob share one of the maximally entangled Bell state $|\Psi^0\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$, then according to the definition of the standard teleportation protocol $P_0$ as stated by Bennett et al. [11], the joint state composed of the state to be teleported and the quantum channel can be expressed as

$$\rho_{123} = \frac{1}{4} \sum_{m=0}^{3} \Pi_{12}^{m}(\sigma^m \rho_\alpha \sigma^m),$$

(2)

where $\rho_\alpha = |\varphi_\alpha\rangle\langle\varphi_\alpha|$, and $|\varphi_\alpha\rangle$ is the unknown one-qubit state Alice seeks to teleport to Bob, which can be represented on a Bloch sphere as $|\varphi_\alpha\rangle = \cos(\theta/2)|0\rangle + e^{i\phi} \sin(\theta/2)|1\rangle$, where $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$ are the polar and azimuthal angles, respectively. Moreover, $\Pi_{12}^m = |\Psi^m\rangle\langle\Psi^m|$, $\Psi^m$ denote the Bell state measurements performed by Alice, with $|\Psi^{0,3}\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$ and $|\Psi^{1,2}\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$ being the four Bell states. It follows immediately from equation (2) that if Alice’s measurement outcome is $m$, then Bob’s unitary operation to recover $|\varphi_\alpha\rangle$ will be $\sigma^m$. For general cases, the channel state $\rho^{(\alpha)}$ established between Alice and Bob at an arbitrary time $t_0$ will be mixed due to the unavoidable interaction of the two-qubit system with the decohering environment, and thus severely undermines the feasibility of
entanglement as a resource for teleportation. The explicit forms of $\rho^{(a)}$ can be obtained by solving the appropriate master equation (1) with $\rho^{(a)}(0) = |\Psi^0\rangle\langle\Psi^0|$ as the initial condition. Here $a = p, di, no$ or $de$ indicates the case that the channel state is protected perfectly, or corrupted by the dissipative, noisy or dephasing environment. If Alice’s measurement has outcome $m$, she tells this measurement result to Bob in a classical way, then the output state (teleported via the state $\rho^{(a)}$) after Bob’s recovery operations conditioned on the two bits of classical information received from Alice is given by
\[
\mathcal{E}_m^{(\beta)}[\rho^{(a)}] = \frac{1}{P_m^{(a)} R_m^{(\beta)}}(1 - \sum_{p=1}^{3} \{\rho_m \otimes \sigma^{(a)}\}(\rho_m \otimes \rho^{(a)})),
\]  
where $P_m^{(a)} = \text{tr}_{1,2,3}[(\Pi_m^{(a)} \otimes \sigma_3^{(a)})(\rho_m \otimes \rho^{(a)})]$ is the probability for Alice to get the measurement outcome $m$. $\sigma_3^{(a)}$ is the $2 \times 2$ identity matrix acting on qubit $3$. $R_m^{(\beta)}$ is a trace-preserving quantum operation carried out by Bob for the purpose of accomplishing the teleportation process, where $\beta = di, no$ or $de$ indicates if Bob’s operation is infected with the dissipative, noisy or dephasing environment. The explicit form of $R_m^{(\beta)}[\rho]$ can be derived from equation (1) with the system Hamiltonian $\hat{H} = \hat{H}_m = -\omega \sigma^m$. From which one can see that for a given decoherence forms of the average fidelity with Bob’s recovery operation of the relations $\mathcal{F}^{(\beta)}[\rho^{(a)}]$ are given by
\[
\mathcal{F}^{(\beta)}[\rho^{(a)}] = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta \sum_{m=0}^{3} P_m^{(a)} R_m^{(\beta)}[\rho^{(a)}],
\]  
where $4\pi$ is the solid angle.

3 Teleportation with environment-disturbed recovery operations

To begin with, we first explore the special case that the channel state is prepared maximally and distributed perfectly between Alice and Bob. For this case, we always have $\rho^{(p)} = |\Psi^0\rangle\langle\Psi^0|$, combination of this with equations (3) and (4) one can derive exactly the complete analytical forms of the average fidelity with Bob’s recovery operation corrupted by the dissipative, noisy or dephasing environment. Their explicit expressions are as follows
\[
\mathcal{F}^{(di)}[\rho^{(p)}] = \frac{1}{12} \left( e^{-2\gamma t} + 2\alpha_1 + 2\alpha_2 + \alpha_3 \right) + \frac{1}{6} e^{-\gamma t} \sin^2 \omega t, \\
\mathcal{F}^{(no)}[\rho^{(p)}] = \frac{7}{12} + \frac{1}{6} (\beta_2 - \beta_1) + \frac{1}{12} e^{-2\gamma t} + \frac{1}{6} e^{-\gamma t} \sin^2 \omega t, \\
\mathcal{F}^{(de)}[\rho^{(p)}] = \frac{2}{3} + \frac{1}{6} (\mu_2 - \mu_1) + \frac{1}{6} e^{-\gamma t} \sin^2 \omega t,
\]  
where the corresponding parameters $\alpha_i$ ($i = 1, 2, 3$), $\beta_j$ and $\mu_j$ $(j = 1, 2)$ are given by
\[
\alpha_1 = \frac{4u(\gamma^2 + \omega^2) \cosh(ut) - 3\gamma(\gamma^2 + 5\omega^2) \sinh(ut)}{8u},
\]
\[
\alpha_2 = \left( \frac{\gamma \sinh(ut) - 4u \cosh(ut)}{8u} \right),
\]
\[
\alpha_3 = \left( \frac{\gamma \sinh(ut) + 4u \cosh(ut)}{8u} \right),
\]
\[
\beta_1 = \frac{2v - \gamma \sinh(ut) - 4u \cosh(ut)}{4v},
\]
\[
\beta_2 = \frac{2v - \gamma \sinh(ut) + 4u \cosh(ut)}{4v},
\]
\[
\mu_1 = \frac{4u + \gamma \sinh(ut) + 4u \cosh(ut)}{8u},
\]
\[
\mu_2 = \frac{4u - \gamma \sinh(ut) - 4u \cosh(ut)}{8u},
\]  
with
\[
u = \frac{1}{2} \sqrt{\gamma^2 - 16\omega^2}, \quad v = \frac{1}{2} \sqrt{\gamma^2 - 4\omega^2}.
\]  

Since $\mathcal{F}^{(\beta)}[\rho^{(p)}] (\beta = di, no, de)$ is a function of the rotation rate $\omega$, decoherence rate $\gamma$ as well as the time interval $t$ during which Bob performs the recovery operations, there exists a critical time $t_c^{(\beta)}[\rho^{(p)}]$ at which the average teleportation fidelity attains its maximum, denoted by $\mathcal{F}^{(\beta)}[\rho^{(p)}]_{\text{max}}$. This maximum is reached whenever $t$ satisfies the relations $\partial F^{(\beta)}[\rho^{(p)}]/\partial t = 0$ and $\partial^2 F^{(\beta)}[\rho^{(p)}]/\partial t^2 < 0$. Complete forms of the above two nonlinear equations can be derived straightforwardly from equations (5), (6) and (7), however, since the expressions of $\mathcal{F}^{(\beta)}[\rho^{(p)}]$ are so complicated, it is very difficult to obtain analytical solutions of them, thus we resort to numerical methods. The corresponding results are plotted in Figure 2(a) and Figure 3(a), from which one can see that for a given decoherence rate $\gamma$, the critical time $t_c^{(\beta)}[\rho^{(p)}]$ decreases monotonously with increasing $\omega$ and we always have $t_c^{(de)}[\rho^{(p)}] > t_c^{(di)}[\rho^{(p)}] > t_c^{(no)}[\rho^{(p)}]$, while the maximum average fidelity $\mathcal{F}^{(\beta)}[\rho^{(p)}]_{\text{max}}$
increases with increasing \( \omega \) or decreasing \( \ell_{c}^{(d)}[\rho^{(p)}] \) and approaches to its asymptotic value when \( \omega \) is infinitely large (when \( \gamma = 0.1 \) and \( \omega = 200 \), we can obtain \( F_{\max}^{(d)}[\rho^{(p)}] \approx 0.92163 \), \( F_{\max}^{(no)}[\rho^{(p)}] \approx 0.92138 \) and \( F_{\max}^{(de)}[\rho^{(p)}] \approx 0.92177 \), and the differences between them becomes smaller and smaller with increasing \( \omega \)). However, in contrast to those in reference [18] (i.e., the channel is perfect while Bob’s recovery operations are corrupted by intrinsic, bit-flip or bit-phase-flip noise) where the maximum of the average teleportation fidelity is always larger than the classical limiting value of 2/3 [21], there exists a critical rotation rate \( \omega_{c}^{(d)}[\rho^{(p)}] \) smaller than which the standard teleportation protocol \( P_{0} \) will fail to achieve nonclassical fidelity. \( \omega_{c}^{(d)}[\rho^{(p)}] \) increases monotonously with increasing value of \( \gamma \), and when \( \gamma = 0.1 \) we have \( \omega_{c}^{(di)}[\rho^{(p)}] \approx 0.11192 \), \( \omega_{c}^{(no)}[\rho^{(p)}] \approx 0.12999 \) and \( \omega_{c}^{(de)}[\rho^{(p)}] \approx 0.03829 \). Moreover, for any fixed decoherence rate \( \gamma \), we always have the relations \( F^{(di)}[\rho^{(p)}] > F^{(d)}[\rho^{(p)}] > F^{(no)}[\rho^{(p)}] \) associated with the maximum average teleportation fidelity, which indicates that the devastating effects of the noisy environment is more severe than those of the dissipative and the dephasing environments.

Consider now the situation in which the transmission process of the two qubits constitute the quantum channel are infected with the dissipative environment (see Figure 1) under an interval of time, say \( t_{0} \). Then the initial maximally entangled Bell state \( |\Psi^{0}\rangle \) will be destroyed, and by solving the appropriate master equation (1) with \( |\Psi^{0}\rangle \) as the initial condition and the generators of decoherence given by \( \mathcal{L}_{k} = \sigma_{k}^{-} \), one can obtain the analytical expressions of the nonzero elements of \( \rho^{(d)}(t_{0}) \) explicitly as \( \rho_{11}^{(d)}(t_{0}) = e^{-\gamma t_{0}}/2 \), \( \rho_{14}^{(d)}(t_{0}) = e^{-\gamma t_{0}}/2 \), \( \rho_{22}^{(d)}(t_{0}) = (e^{-\gamma t_{0}} - e^{-2\gamma t_{0}})/2 \), and \( \rho_{44}^{(d)}(t_{0}) = 1 - e^{-\gamma t_{0}} + e^{-2\gamma t_{0}}/2 \), whose concurrence (a measure of pairwise entanglement introduced by Wootters [22]) \( C(t_{0}) = e^{-2\gamma t_{0}} \) decays exponentially with increasing time \( t_{0} \), which forecasts the possible depression of the teleportation fidelity of the expected outcomes. By solving again the master equation (1) with \( \rho^{(d)}(t_{0}) \) as the initial state and \( \mathcal{H}_{m} = -\omega m^{2}/2 \) as the Hamiltonian of the system and then combining the corresponding solutions with equations (3) and (4), one can derive the explicit forms of the average fidelity \( F^{(d)}[\rho^{(d)}] \) with Bob’s recovery operations disturbed by the dissipa-
tive, noisy and dephasing environments, as
\[
F^{(di)}[\rho^{(di)}] = \frac{1}{2} + \frac{1}{12} (2e^{-2\gamma t} - e^{-\gamma t}) \\
\times (e^{-\gamma t} - \alpha_1 + \alpha_3) + \frac{1}{6} \alpha_3 \\
+ \frac{1}{6} e^{-\gamma t} \left( \alpha_2 - \alpha_3 + e^{-\gamma t/2} \sin^2 \omega t \right),
\]
\[
F^{(no)}[\rho^{(di)}] = \frac{2}{3} + \frac{1}{6} (e^{-2\gamma t} - e^{-\gamma t}) (e^{-2\gamma t} - 2\beta_1 + 1) \\
- \frac{1}{6} \beta_1 + \frac{1}{6} e^{-\gamma t} \left( \beta_2 + e^{-\gamma t/2} \sin^2 \omega t \right),
\]
\[
F^{(de)}[\rho^{(di)}] = \frac{2}{3} + \frac{1}{6} (e^{-2\gamma t} - e^{-\gamma t}) (1 - \mu_1) - \frac{1}{6} \mu_1 \\
+ \frac{1}{6} e^{-\gamma t} \left( \mu_2 + e^{-\gamma t/2} \sin^2 \omega t \right),
\]
(8)
where the corresponding parameters \(\alpha_i\) (\(i = 1, 2, 3\)), \(\beta_j\) and \(\mu_j\) (\(j = 1, 2\)) appeared in the above equations are completely the same as those expressed in equation (6). Since their expressions are still so complicated, we resort to numerical methods again. The critical time \(t^{(\gamma)}_c[\rho^{(di)}(t_0)]\) at which the average fidelity attains its maximum versus the rotation rate \(\omega\) are displayed in Figure 2(b), while the maximum of \(F^{(\gamma)}[\rho^{(di)}(t_0)]\), denoted by \(F^{(\gamma)}_{max}[\rho^{(di)}(t_0)]\), versus \(\omega\) are displayed in Figure 3(b), both with the decoherence rate and the transmission time given by \(\gamma = 0.1\) and \(t_0 = 2\), respectively. From Figure 2(b) one can see that although the dissipative environment disentangling the two qubits involved in the quantum channel exponentially, the \(\omega\) dependence of the critical time \(t^{(\gamma)}_c[\rho^{(di)}(t_0)]\) displays nearly the same behaviors as those of \(t^{(\gamma)}_c[\rho^{(p)}]\). In general, we have \(t^{(\gamma)}_c[\rho^{(di)}(t_0)] > t^{(\gamma)}_c[\rho^{(p)}]\), \(t^{(\gamma)}_c[\rho^{(no)}(t_0)] > t^{(\gamma)}_c[\rho^{(p)}]\) and \(t^{(\gamma)}_c[\rho^{(de)}(t_0)] < t^{(\gamma)}_c[\rho^{(p)}]\), however, the differences between them are very small. Particularly, in the large \(\omega\) region this difference can even be neglected (cf. the curves displayed in Figures 2(a) and 2(b)). When considering the maximum average fidelity \(F^{(\gamma)}_{max}[\rho^{(di)}(t_0)]\), as can be seen from Figure 3(b), it also increases with increasing \(\omega\), and approaches to its asymptotic value which is smaller than that of Alice and Bob share the ideal channel state \(|\Psi^0\rangle\) in the limit of \(\omega \to \infty\) (when \(\gamma = 0.1\), \(t_0 = 2\) and \(\omega = 200\), one can obtain \(F^{(\gamma)}_{max}[\rho^{(di)}(t_0)] \approx 0.82629\), \(F^{(no)}_{max}[\rho^{(di)}(t_0)] \approx 0.82609\) and \(F^{(de)}_{max}[\rho^{(di)}(t_0)] \approx 0.82638\)).

The depression of the average fidelity can be attributed to the exponential decay of the entanglement of the channel state during the transmission time \(t_0\).

For any fixed decoherence rate \(\gamma\), equation (8) also gives the constraint on a critical rotation rate \(\omega_c^{(\gamma)}[\rho^{(di)}]\) smaller than which the standard teleportation protocol will fail to yield an average fidelity better than classically possible. \(\omega_c^{(\gamma)}[\rho^{(di)}]\) increases with increasing value of \(\gamma\) (when \(\gamma = 0.1\) and \(t_0 = 2\), we can obtain \(\omega_c^{(\gamma)}[\rho^{(di)}] \approx 0.14719\), \(\omega_c^{(\gamma)}[\rho^{(p)}] \approx 0.31763\) and \(\omega_c^{(\gamma)}[\rho^{(no)}] \approx 0.06229\).

![Fig. 4. (Color online) Critical rotation rate \(\omega^{(\gamma)}[\rho^{(no)}]\) versus the transmission time \(t_0\) with the decoherence rate given by \(\gamma = 0.1\) and Bob's recovery operations corrupted by the dissipative (denoted by the black circles with \(t_0 \in [0.15, 13.8]\)), noisy (denoted by the red circles with \(t_0 \in [0.1, 3.43]\)) and dephasing (denoted by the blue circles with \(t_0 \in [0.15, 11.85]\)) environment. Here the black, red and blue lines show the corresponding data fitting results with \(t_0 \in [0.15, 7.85]\) (a), \(t_0 \in [0.1, 2.98]\) (b), and \(t_0 \in [0.15, 11.85]\) (c). Moreover, the insets in (a) and (b) show behaviors of \(\omega^{(\gamma)}[\rho^{(no)}]\) in the small regions of \(t_0\), where the labels of the horizontal and vertical axes are omitted for compaction of the plots.](image-url)
Thus it is reasonable to conjecture that when the transmission time \( t_0 \) becomes longer than \([\ln(\sqrt{2} + 1)]/2\gamma\), the teleportation protocol will fail to achieve nonclassical fidelity. In fact, the classical transmission time for \( F_{\text{max}}^{(3)}(\rho^{(no)}(t_0)) \) is much smaller than \([\ln(\sqrt{2} + 1)]/2\gamma\) (see the following text). To see this more clearly, we solve again the master equation (1) with \( \rho^{(no)}(t_0) \) as the initial state and \( \dot{H}_m = -\omega m^2/2 \) as the system Hamiltonian, and by combination of the corresponding solutions with equations (3) and (4), we obtain

\[
F^{(\text{dec})}(\rho^{(no)}(t_0)) = \frac{1}{2} \left[ 1 + \frac{1}{12} e^{-4\gamma t_0} (e^{-\gamma t} - 2\beta_1 + 1) + \frac{1}{6} e^{-2\gamma t} \left( \beta_2 + e^{-\gamma t} \sin^2 \frac{\omega t}{2} \right) \right] .
\]

The maximum of the average fidelity \( F^{(3)}(\rho^{(no)}(t_0)) \) is achieved at the critical time \( t^{(\text{dec})}_{\gamma}(\rho^{(no)}(t_0)) \), whose \( \omega \) dependence is plotted in Figure 2(c) with the decoherence rate and the transmission time given by \( \gamma = 0.1 \) and \( t_0 = 2 \), while the \( \omega \) dependence of \( F^{(3)}(\rho^{(no)}(t_0)) \) is plotted in Figure 3(c), still with \( \gamma = 0.1 \) and \( t_0 = 2 \). From these two figures one can observe that they display very similar behaviors as those with the transmission process of the two qubits disturbed by the dissipative environment, i.e., \( t^{(\text{dec})}_{\gamma}(\rho^{(no)}(t_0)) \) decreases while \( F^{(3)}(\rho^{(no)}(t_0)) \) increases with increasing value of \( \omega \). But now the magnitudes of \( F_{\text{max}}^{(3)}(\rho^{(no)}(t_0)) \) are further depressed (for the case of \( \gamma = 0.1 \), \( t_0 = 2 \) and \( \omega = 200 \), we have \( F_{\text{max}}^{(3)}(\rho^{(no)}(t_0)) \approx 0.74651, F_{\text{max}}^{(4)}(\rho^{(no)}(t_0)) \approx 0.74637 \) and \( F_{\text{dec}}^{(\text{max})}(\rho^{(no)}(t_0)) \approx 0.74658 \)). This phenomenon may be caused by the competition between the two generators \( \gamma_{k=1} = \sigma_k \) and \( \gamma_{k=2} = \sigma_k^2 \) of decoherence. Also we found that the critical rotation rate \( \omega_{\phi}^{(3)}(\rho^{(no)}(t_0)) \) after which \( F_{\text{max}}^{(3)}(\rho^{(no)}(t_0)) \) exceeds the classical limiting value \( 2/3 \) is enhanced. For instance, we have \( \omega_{\phi}^{(3)}(\rho^{(no)}(t_0)) \approx 0.27823, \omega_{\phi}^{(4)}(\rho^{(no)}(t_0)) \approx 0.55646 \) and \( \omega_{\phi}^{(\text{dec})}(\rho^{(no)}(t_0)) \approx 0.127941 \) if \( \gamma = 0.1 \) and \( t_0 = 2 \). Moreover, the maximum average teleportation fidelity cannot exceed \( 2/3 \) when the transmission time \( t_0 \) is larger than a critical value \( t_{0,c}(\gamma = 0.1) \), which is approximately \( t_{0,c} \approx 0.3549 \). When \( t_0 < t_{0,c} \), from Figure 4(b) one can observe that \( \omega_{\phi}^{(3)}(\rho^{(no)}) \) can also be fitted very well as \( \omega_{\phi}^{(3)}(\rho^{(no)}) = a \omega + b + c \omega^d \) (with the coefficients \( a, \beta, c \) and \( d \) given in Table 1) in the small \( t_0 \) region, while in the large \( t_0 \) region, the divergence of the corresponding data fitting results from the real critical rotation rate also increases with increasing \( t_0 \).
For the case that the transmission process of the two qubits from the EPR source to Alice and Bob is infected with the dephasing environment, we obtain the nonzero elements of $\rho^{(de)}(t_0)$ as $\rho^{(de)}_{1,44}(t_0) = 1/2$ and $\rho^{(de)}_{4,14}(t_0) = e^{-\gamma t_0}/2$, which yields the concurrence $[22] C(t_0) = e^{-\gamma t_0}$. $C(t_0)$ also decays exponentially with increasing $t_0$, with however, the decay rate smaller than that under the influence of the dissipative environment, thus it is natural to expect an enhancement of the average teleportation fidelity. By solving the master equation (1) with $\rho^{(de)}(t_0)$ as the initial state and $\hat{H}_m = -\omega m^2/2$ as the system Hamiltonian, and then inserting the corresponding solutions into equations (3) and (4), we obtain $F^{(\beta)}[\rho^{(de)}]$ with Bob's recovery operations executed in the presence of the dissipative, noisy and dephasing environments, as

$$F^{(d)}[\rho^{(de)}] = \frac{1}{2} + \frac{1}{12} \left( e^{-\gamma t} - \alpha_1 + \alpha_3 \right) + \frac{1}{6} e^{-\gamma t_0} \left( \alpha_2 + e^{-\gamma t} \sin^2 \frac{\omega t}{2} \right),$$

$$F^{(n)}[\rho^{(de)}] = \frac{7}{12} + \frac{1}{12} e^{-2\gamma t} - \frac{1}{6} \beta_1 + \frac{1}{6} e^{-\gamma t_0} \left( \beta_2 + e^{-\gamma t} \sin^2 \frac{\omega t}{2} \right),$$

$$F^{(e)}[\rho^{(de)}] = \frac{2}{3} - \frac{1}{6} \mu_1 + e^{-\gamma t_0} \left( \mu_2 + e^{-\gamma t} \sin^2 \frac{\omega t}{2} \right).$$

(11)

Still there exists a critical time $t_{c}^{(\beta)}[\rho^{(de)}(t_0)]$ at which the average fidelity $F^{(\beta)}[\rho^{(de)}]$ attains its maximum value. $t_{c}^{(\beta)}[\rho^{(de)}(t_0)]$ and $F^{(\beta)}_{\max}[\rho^{(de)}(t_0)]$ versus the rotation rate $\omega$ are shown in Figure 2(d) and Figure 3(d), where the decoherence rate and the transmission time are chosen to be $\gamma = 0.1$ and $t_0 = 2$, respectively. These two figures display very similar behaviors with those of the former cases. But now the magnitudes of $F^{(\beta)}_{\max}[\rho^{(de)}(t_0)]$ are slightly enhanced (e.g., when $\gamma = 0.1$, $t_0 = 2$ and $\omega = 200$, we have $F^{(d)}_{\max}[\rho^{(de)}(t_0)] \approx 0.87496$, $F^{(n)}_{\max}[\rho^{(de)}(t_0)] \approx 0.87474$ and $F^{(e)}_{\max}[\rho^{(de)}(t_0)] \approx 0.87510$) compared with that of the transmission process being disturbed by the noisy environment. Also our numerical results demonstrated that in order for $F^{(\beta)}_{\max}[\rho^{(de)}(t_0)]$ to exceed 2/3, the transmission time must be shorter than a critical value $t_{0,c} \approx 12.194$ when $\gamma = 0.1$. In the region of $t_0 < t_{0,c}$, the critical rotation rate $\omega_{c}^{(\beta)}[\rho^{(de)}]$ after which $F^{(\beta)}_{\max}[\rho^{(de)}(t_0)]$ exceeds 2/3 exhibits an exponential increase with increasing $t_0$ (we have $\omega_{c}^{(d)}[\rho^{(de)}] \approx 0.13194$, $\omega_{c}^{(n)}[\rho^{(de)}] \approx 0.26389$ and $\omega_{c}^{(e)}[\rho^{(de)}] \approx 0.04273$ when $\gamma = 0.1$ and $t_0 = 2$), and as can be seen from Figure 4(c), it can also be fitted very well as $\omega_{c}^{(\beta)}[\rho^{(de)}] = \alpha e^{\beta t_0} + \gamma e^{\delta t_0}$, where the coefficients $a$, $c$, $\gamma$ and $\delta$ are displayed in Table 1.

Now we make some discussion about the physical implications of the phenomena displayed in Figures 2 and 3. Since decoherence occurs in all real physical entities, it is essential to reduce the gap between the experimentally achieved coherence lifetimes and those required by the theory for quantum teleportation to become practical. For spin qubits the coherence times in general are very short (e.g., coherence time of about 3.0 $\mu$s has been reported for quantum dot electron spins [24], and coherence time of about 30 $\mu$s has been demonstrated experimentally in irradiated malonic acid crystals at temperature 50 K [25]), thus for the teleportation protocol to remain effective, the rotation operation to the target qubit must be performed on a timescale in which the coherence of the spin is preserved. The curves in Figures 2 and 3 show evidently that the quality of quantum teleportation can be improved significantly by increasing the rotation rate $\omega$ and thus the resultant gate operation times can be shortened, this indicates that the large rotation rate to the target spin is beneficial to quantum teleportation.

Finally, we would like to make a comparison of the robustness of the standard teleportation protocol $P_0$ when it is executed with different (perfect or decohering) quantum channels as well as different environment-disturbed recovery operations. First, for fixed $\alpha = p, d, n, o$ or $de$ (i.e., the case with the same quantum channels but different imperfect recovery operations), we always have $F^{(de)}_{\max}[\rho^{(\alpha)}(t_0)] > F^{(di)}_{\max}[\rho^{(\alpha)}(t_0)] > F^{(no)}_{\max}[\rho^{(\alpha)}(t_0)]$. This relation reveals that under the condition of the same decoherence rate $\gamma$, the destructive effects of the noisy environment on Bob’s recovery operation is more severe than that of the dissipative or the dephasing environment, thus it is reasonable to obtain the relation $\omega_{c}^{(de)}[\rho^{(\alpha)}(t_0)] < \omega_{c}^{(di)}[\rho^{(\alpha)}(t_0)] < \omega_{c}^{(no)}[\rho^{(\alpha)}(t_0)]$ for the critical rotation rate. Second, for any fixed $\beta = d, n, n$ or $de$ (i.e., the case with different quantum channels but the same recovery operations), we always have the following two relations: $F^{(d)}_{\max}[\rho^{(p)}] > F^{(d)}_{\max}[\rho^{(de)}(t_0)] > F^{(d)}_{\max}[\rho^{(di)}(t_0)] > F^{(d)}_{\max}[\rho^{(no)}(t_0)]$ for the maximum average teleportation fidelity, and $\omega_{c}^{(d)}[\rho^{(p)}] < \omega_{c}^{(d)}[\rho^{(de)}(t_0)] < \omega_{c}^{(d)}[\rho^{(di)}(t_0)] < \omega_{c}^{(d)}[\rho^{(no)}(t_0)]$ for the critical rotation rate. These demonstrate again that the devastating effects imposed by the noisy environment on the standard teleportation protocol $P_0$ is the most serious one compared with the other cases.

4 Summary and discussion

In summary, we have studied standard teleportation process of an arbitrary one-qubit state with both the transmission process of the two qubits constitute the quantum channel and the recovery operations performed by Bob disturbed by the dissipative, noisy and dephasing environments. Through detailed analyzation of the average fidelities with different situations, we demonstrated that provided Alice and Bob share an ideal channel state, Bob’s environment-disturbed recovery operations do not eliminate the possibility for teleporting the one-qubit state with nonclassical fidelity. When the transmission process of the two qubits is corrupted by decohering environment, however, there exists a constraint on the transmission time $t_0$, denoted by $t_{0,c}$, after which the standard teleportation
protocol $P_0$ will fail to attain an average fidelity better than classically possible.

We have also compared the robustness of the teleportation protocol $P_0$ with different (perfect or decohered) quantum channels and different imperfect recovery operations, and obtained the following two general relations for the average fidelities: (i) $F_{\max}[\rho^{(\alpha)}(t_0)] > F_{\max}[\rho^{(\alpha)}(t_0)]$ > $F_{\max}[\rho^{(\alpha)}(t_0)]$, and (ii) $F_{\max}[\rho^{(\beta)}] > F_{\max}[\rho^{(\beta)}(t_0)]$ > $F_{\max}[\rho^{(\beta)}(t_0)]$. We have therefore revealed that under the condition of the same decoherence rate $\gamma$, the standard teleportation protocol $P_0$ is significantly more fragile under the influence of the noisy environment than those under the influence of the dissipative and the dephasing environments.

Besides decoherence processes considered in this work, Alice’s Bell state measurement may also be disturbed by the decohering environments [14]. Even though not shown here it will result in a further depression of the average teleportation fidelity. Thus in spite of the existence of certain special conditions under which the teleportation fidelity may be enhanced to some extent by the local environment [26,27], finding ways to stabilize the entangled channel state and to minimize, delay or even eliminate the devastating effects of the decohering environment is still an challenging task in the physical realization of the teleportation protocol. Moreover, understanding the influence of surrounding environments on an open quantum system such as the mechanisms of decoherence and the dynamics of entanglement is also both of fundamental interest in quantum foundation issues and of practical importance in quantum information theory.

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