Comparative theoretical study of the resonance frequencies of the longitudinal-shear and torsion modes of the magnetoelectric effect in a two-layer magnetostrictive-piezoelectric structure Metglas / GaAs

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Abstract. The article is devoted to a comparative theoretical study of the frequencies of the electromagnetic resonance (EMR) of the magnetoelectric (ME) effect in the magnetostrictive-piezosemiconductor structure Metglas / GaAs of the longitudinal-shear and torsional modes. It is found that the resonance frequencies for the torsional mode are approximately 2 times higher than the corresponding frequencies for the longitudinal-shear mode. Therefore, it is quite possible to observe the torsional mode of the ME effect against the background of the longitudinal-shear mode, since the resonance frequencies are well distinguishable. The results obtained can find application in the construction of new ME devices.

1. Introduction
As is known [1], the ME effect in a two-phase layered composite consists in the induction of an electric voltage in the piezophase during its magnetization (direct effect) and, conversely, in a change in the magnetic state of the magnetophase when an electric voltage is connected to the piezophase (reverse effect). It was theoretically and experimentally established [2] that the ME effect significantly increases when passing from the low-frequency range to the region of electromechanical resonance (EMR). This is due to the enhancement of mechanical deformations that transfer excitations between the phases of the composite. The results of studying the ME effect in the field of various EMR modes are known: planar, bending [3] and thickness-shear [4]. Excitation of various EMR modes allows covering a wide frequency range, which is important in the design of various resonant ME devices: sensors, gyrators, harvesters, etc. [5]. Recently, interest has arisen in the study of the torsional mode and the torsional one of the ME effect in a YIG / GaAs layered structure has been studied [6]. Also in [7], the delta-E effect was investigated in the torsional and bending modes in the layered ME structure FeCoSiB / AlN on a polysilicon substrate. Depending on the method of fixing a two-layer ME composite, both the longitudinal-shear and torsional modes of the ME effect can be excited in it. Both modes are associated with shear deformations; therefore, it is of great interest to consider the EMR frequencies of both modes in order to identify them. In this article, it is proposed to consider the ME effect in the region of the longitudinal-shear and torsional modes in a layered Metglas / GaAs composite.
2. Longitudinal-shear mode
Let us consider a magnetolectric composite in the form of a thin narrow plate. Axis 1 is directed along the length of the plate, axis 3 is perpendicular to the plane of the sample.

![Figure 1. Magnetolectric composite. 1 - piezoelectric phase, 2 - magnetostrictive phase, 3 - electrodes.](image)

Let us consider small longitudinal-shear mechanical vibrations in the composite along its width under the influence of a small external alternating magnetic field \( h_2(t) \) in the presence of a constant magnetic field \( H_0 \). The ac field strength is directed along the 2 (y) axis, the constant magnetizing field is directed along the 1 (x) axis. The time dependence of the alternating magnetic field is harmonic.

\[
h_2(t) = h_2 e^{i\omega t}.
\]

Material equation for a piezoelectric layer

\[ p S_6 = d_{36} E_3 + p s_{66} \nu T_6. \]

Let us express the necessary shear component of the stress tensor in a piezoelectric

\[ p T_6 = \frac{1}{p s_{66}} p S_6 - \frac{d_{36}}{p s_{66}} E_3. \]

The required shear component of the stress tensor of the magnetostrictive phase

\[ m T_6 = m G \left( m S_6 - q_{26} h_2 \right) = m G m S_6 - q_{26} h_2. \]

where

\[ q_{26} = m G q_{26}. \]

With longitudinal-shear vibrations

\[ m S_6 = p S_6 = \frac{\partial U}{\partial y} + \frac{\partial U}{\partial x} = \frac{\partial U}{\partial y}. \]

Shear component of the stress tensor of the composite
\[ T_6 = \rho \nu T_6 + \rho \nu T_6 = c_{66} S_6 - \nu \bar{q}_{26} h_2 - \rho \nu \frac{d_{36}}{s_{66}} E_3, \]  
(7)

where the volume fractions of the piezoelectric and magnetostrictive phases

\[ \rho \nu = \frac{\rho t}{\rho t + m t}, \]
\[ m \nu = \frac{m t}{\rho t + m t}, \]  
(8)

effective shear stiffness of the composite

\[ c_{66} = \frac{\rho \nu}{s_{66}} + m \nu m G. \]  
(9)

effective composite density

\[ \rho = \rho \nu \rho + m \nu m \rho. \]  
(10)

Let us consider the equation of motion for deformations

\[ \rho \frac{\partial^2 U_x}{\partial t^2} = \frac{\partial T_6}{\partial y}. \]  
(11)

Let us substitute (7) into (11) and get

\[ -\rho \omega^2 U_x = c_{66} \frac{\partial^2 U_x}{\partial y^2}. \]  
(12)

Solution to equation (12)

\[ U_x = A \cos (k y) + B \sin (k y), \]  
(13)

where

\[ k = \sqrt{\frac{\rho}{c_{66}}}. \]  
(14)

Then

\[ S_6 = \frac{\partial U_x}{\partial y} = (B \cos (k y) - A \sin (k y)) k, \]  
(15)

\[ T_6 = c_{66} (B \cos (k y) - A \sin (k y)) k - \nu \bar{q}_{26} h_2 - \rho \nu \frac{d_{36}}{s_{66}} E_3. \]  
(16)

To find A, B, we use the boundary conditions for a free sample

\[ T_6 \bigg|_{y = \frac{b}{2}} = 0, \]
\[ T_6 \bigg|_{y = \frac{b}{2}} = 0, \]  
(17)

where b is composite width.
Let us substitute (16) in (17)

\[
\begin{align*}
\epsilon_{66} \left( B \cos(\eta) + A \sin(\eta) \right) k - m \sqrt{2} h_2 - \nu \frac{d_{36}}{p_{s66}} E_3 &= 0 \\
\epsilon_{66} \left( B \cos(\eta) - A \sin(\eta) \right) k - m \sqrt{2} h_2 - \nu \frac{d_{36}}{p_{s66}} E_3 &= 0,
\end{align*}
\]

where

\[
\eta = \frac{kb}{2}.
\]

We get

\[
A = 0
\]

\[
B = \frac{m \sqrt{2} s_{66} h_2 + \nu d_{36} E_3}{p_{s66} c_{66} k \cos(\eta)}.
\]

Let us find the third component of the electric displacement vector

\[
D_3 = \varepsilon_0 E_3 + d_{36} T_0 = \left[ \varepsilon_0 - \frac{d_{36}^2}{p_{s66}} \right] E_3 + \frac{d_{36}}{p_{s66}} S_6.
\]

The third component of the electric field strength vector is found from the condition of equality to zero of the electric induction flux through the upper surface of the composite

\[
\int_{-\frac{b}{2}}^{\frac{b}{2}} D_3 dy = 0.
\]

Let us substitute (21) into (22) and get

\[
\left[ \varepsilon_0 - \frac{d_{36}^2}{p_{s66}} \right] E_3 b + 2d_{36} \frac{B \sin(\eta)}{p_{s66}} = 0.
\]

Let us substitute (20) into (23) and get

\[
\left[ \varepsilon_0 - \frac{d_{36}^2}{p_{s66}} \right] E_3 b + 2d_{36} \frac{\sin(\eta) m \sqrt{2} s_{66} h_2 + \nu d_{36} E_3}{p_{s66} c_{66} k \cos(\eta)} = 0.
\]

From this expression we find \( E_3 \)

\[
E_3 = -\frac{m \sqrt{2} d_{36} s_{66} \tan(\eta)}{\varepsilon_0 \frac{p_{s66} c_{66} \eta}{d_{36}^2} + \frac{\nu \tan(\eta)}{d_{36}} - \frac{c_{66} p_{s66} \eta}{d_{36}^2}} h_2.
\]

Since there is an electric field only in the piezoelectric phase, the voltage drop

\[
U = E_3 \rho t.
\]

Average electric field strength in the composite
\[ E = \frac{U}{m + p} = \rho V E. \]  \hspace{1cm} (27)

Then the ME-coefficient for voltage

\[ \alpha = \frac{E}{h^2} = -\frac{m V \nu q_{26} d_{36} p s_{66} \tan(\eta)}{\varepsilon_0 p s_{66} c_{66} \eta + d_{36} \left[ \rho V \tan(\eta) - c_{66} p s_{66} \eta \right]} . \]  \hspace{1cm} (28)

Below Figure 2 shows the dependence of the ME-voltage coefficient on the frequency of the alternating magnetic field. To take into account losses in the calculation it is necessary

\[ \omega = 2\pi \left( 1 + \frac{1}{2Q} \right) f \]  where Q is the Q-factor of resonance. In the calculation, the following material parameters of the initial components were used: for Metglas: \( m \rho = 7180 \text{ kg/m}^3 \), \( m G = 3.85 \cdot 10^{10} \text{ Pa} \), \( q_{26} = 1.0 \cdot 10^{-9} \text{ m/A} \), \( m t = 58 \text{ \mu m} \); for gallium arsenide (GaAs): \( p \rho = 5320 \text{ kg/m}^3 \), \( p G = 5.94 \cdot 10^{10} \text{ Pa} \), \( \varepsilon = 12.9 \), \( d_{36} = -2.69 \cdot 10^{-12} \text{ m/V} \), \( p t = 6.25 \cdot 10^{-4} \text{ m} \).

Sample length \( l = 2 \cdot 20^{-2} \text{ m} \), width \( b = 5 \cdot 10^{-3} \text{ m} \). Q-factor of resonance \( Q = 300 \).

\[ \text{Figure 2. Voltage magnetoelectric coefficient for Metglas / GaAs for the longitudinal-shear mode.} \]

The fundamental frequency of the EMR for the longitudinal-shear mode is 325 kHz, the second harmonic is 975 kHz.

3. Torsional mode
The X axis will be drawn along the length of the sample in the corresponding plane of symmetry of the sample, the Y axis - along the axis of rotation of the composite beam during torsional vibrations in the direction of the sample width (see Figure 3).
Figure 3. The position of the interface between the piezoelectric and magnetostrictive phases in a two-layer composite.

The constant bias magnetic field $H_0$ is directed along the X axis, the alternating magnetic field $h_2$ is directed along the Y axis.

Total composite thickness.

$$t = ^pt + ^mt.$$  \hspace{1cm} (29)

Shear components of the strain tensor

$$S_4 = x \frac{\partial \theta}{\partial y}$$

$$S_6 = -z \frac{\partial \theta}{\partial y}.$$ \hspace{1cm} (30)

where $\theta$ is the twist angle.

Material equations for a piezoelectric

$$S_4 = \frac{1}{\rho G} ^pT_4$$

$$S_6 = \frac{1}{\rho G} \left( ^pT_6 + d_{36} ^pE_3 \right).$$ \hspace{1cm} (31)

From (31) we find the tangent components of the stress tensor of the piezoelectric

$$^pT_4 = ^pGS_4 = ^pGx \frac{\partial \theta}{\partial y},$$

$$^pT_6 = ^pG \left( S_6 - d_{36} ^pE_3 \right) = -^pGz \frac{\partial \theta}{\partial y} - d_{36} ^pG^pE_3.$$ \hspace{1cm} (32)

Material equations for a ferromagnet

$$S_4 = \frac{1}{mG} ^mT_4$$

$$S_6 = \frac{1}{mG} ^mT_6 + q_{26} h_2.$$ \hspace{1cm} (34)

From (34) the tangent components of the stress tensor of the magnetostrictive phase
\[ m T_4 = m G S_4 = m G x \frac{\partial \theta}{\partial y}, \]
\[ m T_6 = m G \left( S_6 - q_{26} h_2 \right) = -m G z \frac{\partial \theta}{\partial y} - \overline{q}_{26} h_2, \]
\[ \text{(35)} \]

where
\[ \overline{q}_{26} = m G q_{26}. \]
\[ \text{(36)} \]

Electrical displacement in piezoelectric
\[ D_3 = d_{36} ^p T_6 + \varepsilon e_0 ^p E_3 = -d_{36} ^p G z \frac{\partial \theta}{\partial y} + \left( \varepsilon e_0 - \varepsilon G d_{36}^2 \right) ^p E_3. \]
\[ \text{(37)} \]

Let us express from (37) \(^p E_3\)
\[ ^p E_3 = h_{36} z \frac{\partial \theta}{\partial y} + \beta_{33}^z D_3, \]
\[ \text{(38)} \]

where
\[ h_{36} = \frac{d_{36} ^p G}{\varepsilon e_0 - \varepsilon G d_{36}^2}, \]
\[ \beta_{33}^z = \frac{1}{\varepsilon e_0 - \varepsilon G d_{36}^2}, \]
\[ \text{(39)} \]

and substitute in (33). As a result, we get
\[ ^p T_6 = -^p G^D z \frac{\partial \theta}{\partial y} - h_{36} D_3, \]
\[ \text{(40)} \]

where
\[ ^p G^D = \frac{\varepsilon e_0 ^p G}{\varepsilon e_0 - \varepsilon G d_{36}^2}. \]
\[ \text{(41)} \]

Torque
\[ M = \int_{-l/2}^{l/2} dx \int_{z_0}^{z_2} \left( x^p T_4 - z^p T_6 \right) dz + \int_{-l/2}^{l/2} dx \int_{z_0}^{z_2} \left( x^m T_4 - z^m T_6 \right) dz = \]
\[ = \int_{-l/2}^{l/2} dx \int_{z_0}^{z_2} \left( x^p G x \frac{\partial \theta}{\partial y} - z \left( -^p G^D z \frac{\partial \theta}{\partial y} - h_{36} D_3 \right) \right) dz + \]
\[ + \int_{-l/2}^{l/2} dx \int_{z_0}^{z_2} \left( x^m G z \frac{\partial \theta}{\partial y} - z \left( -m G z \frac{\partial \theta}{\partial y} - \overline{q}_{26} h_2 \right) \right) dz = K_0 \frac{\partial \theta}{\partial y} + l^p r^2 \langle h_{36} \rangle D_3 + l^m r^2 \langle q_{26} \rangle h_2 \]
\[ \text{(42)} \]

where
\[ K_0 = \rho K + m K \]
\[ \rho K = \frac{1}{3} \rho G \left( z_0^3 - (z_0 - \rho t)^3 \right) l + \frac{1}{12} \mu G \rho t l^3, \quad (43) \]
\[ m K = \mu K^\mu I \]

where the polar moment of the ferromagnet
\[ m I = \frac{1}{3} \left( (z_0 + m t)^3 - z_0^3 \right) l + \frac{1}{12} m t l^3, \quad (44) \]
\[ \langle h_{36} \rangle = \frac{1}{\rho t^2} \int_{z_0 - \rho t}^{z_0} z h_{36} dz = \frac{h_{36} \left( 2z_0 - \rho t \right)}{2 \rho t} \]
\[ \langle q_{26} \rangle = \frac{1}{m t^2} \int_{z_0}^{z_0 + m t} z q_{26} dz = \frac{q_{26} \left( 2z_0 + m t \right)}{2 m t} \quad (45) \]

Since the cross-section of the composite is a very narrow rectangle, then with a more accurate calculation of the torque taking into account the Saint-Venant hypothesis [8], it turns out that the total polar moment of the shear stiffness coefficients
\[ K = 4K_0. \quad (46) \]

Let us find the voltage across the piezoelectric
\[ U = \int_{z_0 - \rho t}^{z_0} \rho E_3 dz = \int_{z_0 - \rho t}^{z_0} \left( h_{36} \frac{\partial \theta}{\partial y} + \beta_{33}^5 D_3 \right) dz = \rho t^2 \langle h_{36} \rangle \frac{\partial \theta}{\partial y} + \rho t \langle \beta_{33}^5 \rangle D_3, \quad (47) \]

where
\[ \langle \beta_{33}^5 \rangle = \frac{1}{\rho t} \int_{z_0 - \rho t}^{z_0} \beta_{33}^5 dz = \beta_{33}^5. \quad (48) \]

From (47), we express the electrical displacement in the piezoelectric
\[ D_3 = \frac{U}{\rho t \langle \beta_{33}^5 \rangle} \left( \frac{\partial \theta}{\partial y} \right) \quad (49) \]

And substitute in (42)
\[ M = -l t^3 \langle G \rangle \frac{\partial \theta}{\partial y} - l \rho t \langle h_{36} \rangle \frac{\partial \theta}{\partial y} U + l m t^2 \langle q_{26} \rangle h_2, \quad (50) \]

where
\[ \langle G \rangle = \frac{1}{l t^3} \left( K - \frac{l \rho t \langle h_{36} \rangle^2}{\langle \beta_{33}^5 \rangle} \right) \quad (51) \]

The position of the interface between the piezoelectric and magnetostrictive phases relative to the axis of rotation of the composite beam \( z_0 \) is determined from the condition of the minimum effective
shear modulus of the sample \( \langle G \rangle \)

\[
z_0 = \frac{\nu G D \beta_3^5}{2 \left( m G^m t + \nu G D p t - h_3 p t \right)}.
\] (52)

Equation of torsional vibrations

\[
J \frac{\partial^2 \theta}{\partial t^2} = \frac{\partial M}{\partial y},
\] (53)

where the moment of inertia of the sample per unit width

\[
J = \nu \rho l^3 I + m \rho m l^3.
\] (54)

where the polar moment of the piezoelectric

\[
\nu I = \frac{1}{3} \left( z_0^3 - \left( z_0 - \nu t \right)^3 \right) l + \frac{1}{12} \nu t l^3.
\] (55)

Let us substitute (50) into (53)

\[
J \frac{\partial^2 \theta}{\partial t^2} = -lt^3 \langle G \rangle \frac{\partial^2 \theta}{\partial y^2}.
\] (56)

The dependence of the twist angle on time is harmonic \( \theta \sim e^{i\omega t} \); therefore, we have

\[
\frac{\partial^2 \theta}{\partial y^2} + k^2 \theta = 0,
\] (57)

where the wavenumber is

\[
k = \omega \sqrt{\frac{J}{lt^3 \langle G \rangle}}.
\] (58)

General solution of the equation of motion (57)

\[
\theta = A \cos (ky) + B \sin (ky).
\] (59)

Open circuit condition

\[
\int_{-b/2}^{b/2} D_y dy = 0.
\] (60)

We integrate (47) over \( y \)

\[
Ub = \nu t^2 \langle h_{36} \rangle \theta_{y}^{b/2}_{-b/2} = 2 \nu t^2 \langle h_{36} \rangle B \sin \eta.
\] (61)

where
Boundary conditions for a free sample

\[
M\left(\frac{b}{2}\right) = 0
\]
\[
M\left(-\frac{b}{2}\right) = 0
\]

Combining boundary conditions (63) with (61), we obtain a linear system of three inhomogeneous algebraic equations for three unknowns \(A, B, U\)

\[
-klt^3 \left\langle G \right\rangle (B \cos \eta - A \sin \eta) - \frac{l^p t \left\langle h_{36} \right\rangle}{\beta_{33}^S} U + l^n t^2 \left\langle q_{26} \right\rangle h_2 = 0
\]

\[
-klt^3 \left\langle G \right\rangle (B \cos \eta + A \sin \eta) - \frac{l^p t \left\langle h_{36} \right\rangle}{\beta_{33}^S} U + l^n t^2 \left\langle q_{26} \right\rangle h_2 = 0.
\]

Solving this system, we find \(U\) and then the ME voltage coefficient

\[
\alpha_x = \frac{2^n t^2 m^2 \left\langle h_{36} \right\rangle \left\langle q_{26} \right\rangle \left\langle \beta_{33}^S \right\rangle \tan \eta}{t \left(klt^3 \left\langle G \right\rangle \left\langle \beta_{33}^S \right\rangle + 2 \left\langle h_{36} \right\rangle^2 n t^3 \tan \eta \right)}.
\]

Figure 4 shows the dependence of the ME voltage coefficient on the frequency of the alternating magnetic field \(f\), kHz. To take into account losses in the calculation it is necessary

\[
\omega = 2\pi \left(1 + \frac{1}{2Q} \right) f.
\]

The calculation uses the same material parameters.

**Figure 4.** Voltage magnetoelectric coefficient for Metglas / GaAs in torsion mode.
The fundamental frequency of the EMR for the torsional mode is 649 kHz, the second harmonic is 1948 kHz.

4. Conclusion
The calculation results show that the resonance frequencies of the EMR of the torsional mode of the ME effect in the magnetostrictive-piezosemiconductor structure Metglas / GaAs are approximately 2 times higher than the corresponding resonance frequencies of the EMR of longitudinal-shear vibrations propagating along the width of the composite ME sample. The ME effect in the magnetostrictive-piezosemiconductor structure Metglas / GaAs in the torsional mode can be observed experimentally, since the resonance frequencies of the EMR are well distinguishable against the background of the resonance frequencies of the EMR of the longitudinal-shear mode. The results obtained can find application in the construction of new ME devices.

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