A study of $\pi-$ and $\rho-$ mesons with a non-perturbative approach

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Abstract. In this poster, we give a summary of work in progress regarding the study of meson’s PDAs. We explore the consequences of a momentum-independent interaction as an ad-hoc tool to rebuild the pion distribution amplitude (PDA). The PDA is obtained through its moments using the Schwinger-Dyson formalism within QCD in the rainbow-ladder approximation.

1. Introduction
We can say that and mesons are the simplest bound-states to study in QCD. This can be achieved by describing light-quark confinement and dynamical chiral symmetry breaking (DCSB), and admits a symmetry-preserving truncation scheme. QCD’s Dyson-Schwinger equations (DSEs) provide a picture of these mesons in hadron physics. We assume that $u/d$ mesons are produced by a vector-vector current-current interaction that is mediated by a momentum-independent boson propagator, i.e., by the symmetry preserving regularisation of a contact interaction.

2. Pion Distribution Amplitude
Consider $S$ the dressed-quark propagator, which is obtained from the gap equation:

$$S^{-1}(p) = i \gamma \cdot p + m + \int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^{\mu}}{2} \gamma_{\nu} S(q) \frac{\lambda^{\nu}}{2} \Gamma_{\nu}(q,p)$$

(1)
where \( m \) is the Lagrangian current-quark mass, \( D_{\mu\nu} \) is the vector-boson propagator, and \( \Gamma_\nu \) is the quark vector-boson vertex which contains the non-perturbative information.

In order to build a stock of material that can be used to identify unambiguous signals in experiment for the pointwise behavior of interaction between light-quarks, their mass-functions and other quantities, we use a description through a contact interaction. We thus define

\[
g^2 D_{\mu\nu}(p - q) = \frac{1}{m_{G}^2} \delta_{\mu\nu} \delta
\]

where \( m_{G} \) is a gluon mass-scale, and proceed by embedding this interaction in a rainbow-ladder truncation of the DSEs, which implies \( \Gamma_\nu(k, p) = \gamma_\nu \). This leads to the quark propagator equation:

\[
S^{-1}(p) = i\gamma \cdot p + M
\]

where \( M \) is momentum independent and is defined as:

\[
M = m + \frac{4}{3} \frac{1}{m_{G}^2} \int \frac{d^4q}{(2\pi)^4} \gamma_\mu S(q) \gamma_\mu
\]

The regularisation procedure used is

\[
\frac{1}{s + M^2} = \int_0^\infty d\tau e^{-\tau(s+M^2)} \to \int_{\tau_{ir}}^{\tau_{uv}} d\tau e^{-\tau(s+M^2)}
\]

where \( \tau_{uv,ir} \) are, respectively, infrared and ultraviolet regulators. A nonzero value of \( \tau_{ir} = 1/\Lambda_{ir} \) implements confinement by ensuring the absence of quark production thresholds. \( \tau_{ir} = 1/\Lambda_{ir} \) plays a dynamical role and sets the scale of all dimensioned quantities.

The homogeneous Bethe-Salpeter Equation (BSE) for the pseudoscalar meson for this interaction is

\[
\Gamma_\pi(P) = \frac{4}{3} \frac{1}{m_{G}^2} \int \frac{d^4q}{(2\pi)^4} \gamma_\mu \chi_\pi(q_+, q_-) \gamma_\mu
\]

where \( q_+ = q + P \) and \( q = q \) are the gluons relative momentum. With a symmetry-preserving regularisation of the interaction, the Bethe-Salpeter amplitude cannot depend on relative momentum. We use

\[
\Gamma_\pi(P) = \gamma_5[iE_\pi(P) + \frac{1}{M} \gamma \cdot PF_\pi(P)]
\]

We want to build the pion distribution amplitude (PDA) from a moments calculation of it. The PDA is

\[
f_\pi \phi_\pi(x) = Z_2 N_c \int \frac{d^4q}{(2\pi)^4} \delta(n \cdot q - x n \cdot P) \ tr[\gamma_5 \gamma_\cdot n S(q_+)\Gamma_\pi(q; P)S(q_-)]
\]

The PDAs moments are

\[
\langle x^m \rangle = \int_0^1 dx \ x^m \phi_\pi(x)
\]

So, we need to solve

\[
f_\pi \langle x^m \rangle = Z_2 N_c \int \frac{d^4q}{(2\pi)^4} \ (n \cdot q)^m \ (n \cdot P)^{m+1} \ tr[\gamma_5 \gamma_\cdot n S(q_+)\Gamma_\pi(q; P)S(q_-)]
\]

where \( f_\pi \) is the pions leptonic decay constant define as the zero-moment of the PDA.
2.1. Results

We've obtained the following expression for the PDAs moments:

\[
f_\pi(x^m) = \frac{1}{4\pi^2} Z_2 N_c \frac{1}{M} \int_0^1 d\alpha (-\alpha)^m \left[ E_\pi M^2 C_1(M^2) + \frac{1}{2} F_\pi \left( (M^2 - 4M^2)C_1(M^2) + C(M^2) \right) \right]
\]

(11)

where \( M^2 = \alpha(1 - \alpha)P^2 + M^2 \) and the constants

\[
C_1(M^2; \tau_{uv}^2, \tau_{ir}^2) = \Gamma(0; M^2 \tau_{uv}^2) - \Gamma(0; M^2 \tau_{ir}^2)
\]

\[
C(M^2; \tau_{uv}^2, \tau_{ir}^2) = M^2 [\Gamma(-1; M^2 \tau_{uv}^2) - \Gamma(-1; M^2 \tau_{ir}^2)]
\]

(12)

The \( f_\pi \), define as \( m = 0 \), is

\[
f_\pi = \frac{1}{4\pi^2} Z_2 N_c \frac{1}{M} \int_0^1 d\alpha \left[ E_\pi M^2 C_1(M^2) + \frac{1}{2} F_\pi \left( (M^2 - 4M^2)C_1(M^2) + C(M^2) \right) \right]
\]

(13)

This means \( \langle x^0 \rangle = 1 \), i.e. PDAs moments describe a point-like particle.

Figure 1. Results for the Pions moments and the point-like behavior of the PDA.

3. Pion Distribution Amplitude

We want to build a distribution amplitude for the \( \rho \)-meson by getting a procedure analogous to the PDAs moments calculation. In this case, we need to change the Bethe-Salpeter amplitude
in order to have a description of a vector rather than a pseudoscalar meson. The vector BSA takes the form

\[ \Gamma_\mu^\nu(P) = \gamma^\nu \gamma^\mu E_\rho(P) \] (14)

with

\[ P^\mu \gamma^\mu_\mu = 0 \quad \gamma^L_\mu + \gamma^T_\mu = \gamma_\mu \] (15)

The first attempt for a \( \rho \) distribution amplitude (\( \rho \)DA) from a moment calculation is

\[ f_\rho \phi_\rho(x) = \frac{1}{3} Z_2 N_c \int \frac{d^4 q}{(2\pi)^4} \delta(n \cdot q - x n \cdot P) tr[\gamma_\mu S(q_+ \Gamma_\mu^\rho(q; P) S(q_-))] \] (16)

The moments are given by

\[ f_\rho \langle x^m_L \rangle = \frac{1}{3} Z_2 N_c \int \frac{d^4 q}{(2\pi)^4} \frac{(n \cdot q)^m}{(n \cdot P)^{m+1}} tr[\gamma_\mu S(q_+ \Gamma_\mu^\rho(q; P) S(q_-))] \] (17)

\[ f_\rho \langle x^m_T \rangle = \frac{1}{3} Z_2 N_c \int \frac{d^4 q}{(2\pi)^4} \frac{(n \cdot q)^m}{(n \cdot P)^{m+1}} tr[\sigma_\mu n_\nu S(q_+ \Gamma_\mu^\rho(q; P) S(q_-))] \] (18)

\[ f_\rho \langle x'^m \rangle = -\frac{i}{3} Z_2 N_c \int \frac{d^4 q}{(2\pi)^4} \frac{(n \cdot q)^m}{(n \cdot P)^{m+1}} tr[n \cdot \gamma_\mu S(q_+ \Gamma_\mu^\rho(q; P) S(q_-))] \] (19)

where \( f_\rho \) is the \( \rho \)'s leptonic decay constant define as the zero-moment of the \( \rho \)DA. The test moments \( \langle x^m_L \rangle \) and \( \langle x^m_T \rangle \) are a longitudinal-like and a transverse-like, respectively, forms inspired from [4]. On the other hand, test moment \( \langle x'^m \rangle \) it’s a fit to the point-like form that we expect.

The results of these test expressions are presented in the following graph.
4. Summary
Weve obtained expressions for the DAs moments using the contact interaction. This represent an easy way to describe the behavior of the \( \pi \)- and \( \rho \)-mesons as point-like particles. Its important to note that weve only tested expressions that still need to be explored and fixed for a better description. It is also necessary to find a way to reconstruct \( \rho \)DA using the moments for the complete picture.

References
[1] Pion form factor from a contact interaction. \textit{Phys. Rev.} C 81 065202
[2] pi- and rho-mesons, and their diquark partners, from a contact interaction. \textit{Phys. Rev.} C 83 065206
[3] Pion and kaon valence-quark parton distribution functions. \textit{Phys. Rev.} C 83 062201
[4] The rho- meson light-cone distribution amplitudes of leading twist revisited. \textit{Phys. Rev.} D 54 2182
[5] Bethe-Salpeter study of vector meson masses and decay constants. \textit{Phys. Rev.} C 60 055214