Pareto-optimal fronts to diminish lane-changing impact in mixed traffic

Yang Li  
Ph.D., Candidate  
Key Laboratory of Road and Traffic Engineering of Ministry of Education,  
Tongji University, China,  
4800 Cao’an Road, Shanghai, 201804,  
E-mail: cc960719@tongji.edu.cn

Linbo Li (corresponding author)  
Ph.D., Associate Professor  
Key Laboratory of Road and Traffic Engineering of Ministry of Education,  
Tongji University, China,  
4800 Cao’an Road, Shanghai, 201804  
E-mail: llinbo@tongji.edu.cn

Daiheng Ni  
Ph.D., Professor  
Civil and Environmental Engineering,  
University of Massachusetts Amherst, Massachusetts 01003, USA  
E-mail: ni@engin.umass.edu

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Abstract—In order to minimize the impact of LC (lane-changing) maneuver, this research proposes a novel LC algorithm in mixed traffic. The LC maneuver is decomposed into two stages: one is from the decision point to the execution point (finding a suitable gap), and the other is from the execution point to the end point (performing the LC maneuver). Thereafter, a multiobjective optimization problem integrating these two stages is posed, in which the comfort, efficiency and safety of the LC vehicle and the surrounding vehicles are simultaneously considered. Through introducing the NSGA-II (Non-dominated Sorting Genetic Algorithm), the pareto-optimal front of this problem is obtained. The nearest point of the frontier to the origin is used as the final solution. Through the micro-level analysis of the operating status of each vehicle, macro-level analysis of the traffic flow state within the LC area, and the sensitivity analysis of pareto-optimal fronts, we verify the performance of our proposed algorithm. Our results demonstrate that compared with the existing algorithm, our algorithm provides the optimal execution point and trajectory with the least impact on surrounding vehicles. The operation status of the traffic flow within the LC area has been significantly improved. This research could provide valuable insights into autonomous driving technology.

Index Terms—Lane-changing (LC) behavior, Lane-changing impact, Pareto-optimal frontier, Mixed traffic flow, Longitudinal control model.
1 INTRODUCTION

RECENTLY, considerable attention has been paid to the study of AV (autonomous vehicle) technology, and research results have suggested that the advent of AVs could effectively improve traffic efficiency, enhance traffic safety, mitigate or reduce the impact of track shock wave (smooth vehicle trajectories) [1, 2]. It is foreseeable that high level AVs will soon appear in daily life, and will co-exist with HVs (human-driving vehicles) in the near future. One of the indispensable components of AV technology is the LC (lane-changing) maneuver research, which is a very challenging undertaking that requires exploration of solution spaces to achieve competing goals of safety, mobility, and environmental factors [2]. Along with CF (car-following) behavior, LC is also an indispensable component of traffic flow theories [3], which describes the lateral movement of the vehicles from the current lane to the target lane while proceeding forward. The LC behavior can be mainly divided into mandatory and discretionary. The LC behavior due to diversion, merging or avoiding collision with obstacles can be viewed as mandatory, since the drivers are forced to complete LC within a certain time and specific location. LC resulting from the driver’s need for higher speed or more comfortable driving space could be considered as discretionary. For the convenience of discussion, we present the typical LC scenario in Fig. 1. In the most complex scenario, the LC behavior will involve interaction with up to four vehicles. Therefore, under certain circumstances, LC behavior has more impairment to the traffic flow than CF behavior, for example, reducing efficiency, increasing risk, enlarging emissions, etc.

![LC schematic](image)

Fig. 1 A typical LC schematic where five vehicles are simultaneously involved

Generally speaking, existing research on lateral maneuver could be roughly divided into: modeling the decision-making process of LC [4, 5], the impact of LC on surroundings [6-9], the duration of LC [10-12], and motion planning of LC [4, 13-18]. The research on LCD (LC decision-making) model mainly addresses the question of whether perform LC or LK (lane keeping). Over the past decades, a considerable effort has been devoted on this topic. A large number of existing studies have been conducted in this direction, which are mainly based on the most famous Gipps-type [19], Utility Theory [5], Game Theory [20], Deep Learning [13], etc. For readers interested in these models, see Zheng [7] for a review. The studies on LCI (LC impact) mainly estimate, model and alleviate the impact of LC maneuver on surroundings [1, 8, 9, 21]. The research on LC duration mainly answers the question about the distribution and affecting factors of LC duration [10]. Extensively studies demonstrate that the LC duration roughly ranges from 1s to 16s, and various factors might influence LC duration, including but not limited to traffic density, vehicle type, driver characteristics, the direction of LC, etc.
The research on LC motion planning could be divided into the LTP (LC trajectory planning) part and LTT (LC trajectory tracking) part. When the vehicle has determined to perform LC, the vehicle would gradually turn the steering wheel and drive towards the target lane. More specifically, the LTP algorithm calculates a designed LC trajectory in advance, and the LTT controls the vehicle to drive along this trajectory until it arrives the center-line in the target lane. This research mainly revolves around two issues: (1) How to obtain a safe, comfort and efficient LC trajectory equation? (2) How to control the vehicle to accurately track the planned trajectory? Over the past decades, considerable efforts have been made on this direction. The research on LTP algorithm could be divided into analytical method [15-17], artificial potential field method [4, 18], and data-driven method [13, 14].

The analytical method predetermines the trajectory form equation. The problem of obtaining the corresponding equation coefficient is solved through transferring this into an optimization problem, which takes the safety, comfort and efficiency of the vehicle into account [15-17]. Up to now, the most commonly-used mathematical equations are quintic polynomial equation, cubic polynomial equation, sine(cosine) curve equation, trapezoidal curve equation, Bezier curve equation, etc. Based on the V2V communication, a dynamic automated LC algorithm was proposed in [15], which consists of LTP and LTT algorithm. Bai, Shen, Wei and Feng [22] established the rectangular collision boundary to analyze the possible collision points. Yang, Zheng, Wen, Jin and Ran [23] proposed a dynamic LTP algorithm, which consists of the trajectory decision, trajectory generation, and starting-point determination module. Huang, Ji, Peng and Hu [24] incorporated the personalized driving style into the quintic polynomial function so as to meet driver’s personalized LC needs. Luo, Yang, Xu, Qin and Li [25] absorbed the cooperative safety spacing model into a multi-vehicle cooperative automated LTP algorithm. Chen, Hu and Wang [17] synchronously fused the cooperative safety spacing model and the prediction of leading vehicle into the LTP algorithm. Lim, Lee, Sunwoo and Jo [26] proposed a hybrid LC trajectory planning method with the strength of sampling and optimization methods.

Data-driven approaches usually refer to methods employing machine learning or deep learning algorithms, which aims to extract LC dynamics from massive trajectory data [13, 14]. Zhang, Sun, Qi and Sun [14] employed the LSTM neural network to model the CF and LC behavior simultaneously. A hybrid retraining constrained training method is further established to improve the model. Xie, Fang, Jia and He [13] fused the DBN and LSTM together to model the process of LC decision and LC execution. Results demonstrate that the proposed model could better reproduce the behavior of LC. Artificial potential field method regards the various elements of driving environment, such as road edges, static obstacles, and moving obstacles as a potential energy field [4, 18]. The vehicle tries to find a trajectory with the lowest total potential field. Hang, Lv, Huang, Cai, Hu and Xing [4] combined the potential field with the MPC model to acquire the optimal path for the vehicle. Zheng, Zeng, Yang, Li and Zhan [18] established an effective LTP algorithm based on the quartic Bézier curve, where the potential field functions are employed to evaluate the real-time collision risk. After obtaining the final form of the planned trajectory, the vehicle would track this curve under the controller. This may involve the determination of the vehicle model (for example, the kinematic model, the kinetics model, etc.), and the controller (for example, the PID method, MPC method, etc.). Moreover, many scholars consider the influence of the uncertainty of the external system on the controller, and conduct the robustness analysis of the control system.

From the aforementioned literature review, it can be seen that a wealth of results has been yielded in terms of LC maneuver research. Nevertheless, existing research often overlooks the critically important need for AV to complete LC maneuver safely while minimizing its impact on surrounding traffic flow. Consequently, existing LC algorithms might indeed be optimal for the LC vehicle, but might not be optimal for other vehicles within the LC area. More specifically, existing LTP algorithms only take the comfort, safety, and efficiency of the LC vehicle in the optimization objective function, while overlooking the cost of surroundings, especially the vehicles behind the target lane. Admittedly, we are also aware that in some cases, the impact of LC behavior on the traffic flow within the LC area is unavoidable, so how to minimize this impact, or rather to smooth out the traffic...
shock wave, is of great interest and yet to be solved.

Therefore, this paper aims to propose a novel LC algorithm that could achieve local-optimum within the LC region. And it is interesting and also crucial research topic about how to simultaneously consider the LC vehicle and surrounding vehicles in obtaining a fair automatic LC algorithm. On the other hand, with the continuous development of autonomous driving technology, it gradually becomes possible for AV to actively dissipate or alleviate traffic shock wave from the source as much as possible. This may transform traffic control from conventional passive and global means to active and individualized control, and this research could also be seen as a preliminary groundwork to apply the application of this concept.

Under our algorithm, not only the safe completion of LC maneuver is ensured, but also the impact of LC is diminished through obtain the optimal execution point and the LC trajectory equation. The “optimal” here means that the overall cost within the LC area is minimized, rather than the single LC vehicle. A multiobjective optimization function that integrates the above two parts is constructed. In order to solve this problem, the NSGA (Non-dominated sorting genetic algorithm) -II algorithm [27] is introduced, through which the pareto-optimal frontier of is acquired. The main contributions of this paper are fourfold: (a) a multiobjective optimization objective that simultaneously integrates the cost of the LC vehicle and surrounding vehicles is formulated. The comfort, efficiency and safety are all concurrently considered in this objective. (b) the pareto-optimal solutions and frontier of the above optimization problem is obtained through introducing the NSGA-II algorithm. (c) our algorithm is established in mixed traffic, which might have indispensable research implications to promote the study of LC algorithms in mixed traffic. (d) a comprehensive numerical simulation is conducted to verify the effectiveness of our proposed algorithm. Micro-level (trajectory of each vehicle), macro-level (flow, space-mean speed and density), and sensitivity analysis methods are introduced.

The paper is structured as follows: Section II presents the research problem and scopes. Section IV presents the mathematical model of our proposed algorithm. Section IV presents the introduction of the pareto-optimal frontier and NSGA. Section V presents the Simulation experiment design and result analysis. Section VI presents the conclusion of this paper.
2 RESEARCH PROBLEM

The behaviors of individual drivers are often random, and they often take actions to maximize their own interests at all times in real traffic environment. These actions may generally be best for individuals, but not for traffic flow in their vicinity. These self-optimized actions usually result in low throughput, poor driver comfort, or even traffic accidents. Traffic management agencies typically passively alleviate traffic congestion through Ramp Metering [28], Variable Speed Control [29], etc. Although these measures could address traffic shock waves and congestion that are recurrent in nature, they are unable to tackle issues that are spontaneous such as those caused by LC maneuvers. With the continuous deepening of the application of intelligent driving environment, the advent of self-driving vehicles provides a new means of solving this dilemma. The estimation of traffic state has been improved from the analysis based on historical data to real-time monitoring data. The V2X technology could organically connects drivers, vehicles, roads and other transportation participation elements to realize information collection, information exchange and control command execution (as shown in Fig. 2). It becomes possible for AVs to actively dissipate or mitigate traffic congestion caused by LC maneuver from the source as much as possible. This may transform traffic control from conventional passive and global means to active and individualized control. It is in this context that this paper is presented. An ideal hierarchical control architecture schematic to realize this idea is given in Fig. 2. This architecture meets the need of real-time and large-scale calculation of vehicle-road interactive information in traffic control. The strategic level collects the information of all road sections, gives and distributes the optimization strategy at the overall level of the road network. The tactical level completes the fusion and process of multi-source data in its jurisdiction, and coordinates the driving of AVs in its own jurisdiction. The operational level refers to the real-time actions and states of AVs at each moment.

Traffic management agencies tend to expect the LC maneuver to have the least impact on surroundings. Specifically, they want to minimize the cost of efficiency, comfort and safety within the area. As for the LC vehicles, they are more likely to maximize their own interests, and often ignore the cost of neighboring vehicles in their strategy as we elaborated in the Introduction. Succinctly, the former tends to consider the problem from the perspective of the system as a whole, while the latter tends to start from self-interest. We assume that the algorithm could not only be distributed by the RSU to the LC vehicle, but the LC vehicle could also generate this algorithm itself (it is bound to be equipped with sensors that could obtain real-time trajectory information of surroundings). And our algorithm could also be directly translated into the algorithm commonly used in existing studies as well, by simply setting the weighting factor of the cost of neighboring vehicles to zero. Thus, our algorithm could achieve both self-optimum and local-optimum. The major reason for showing this architecture here is to elaborate the general context in which this research is carried out. To facilitate the illustration of the subsequent study, we illustrate the constructed algorithm by standing in the perspective of RSU. Fig. 3 presents the specific LC scenarios to which the algorithm applies. Typically, the LC maneuver could be divided into two stages. One is from the decision to the execution point (stage 1). And other stage is the execution process of LC (stage 2).
Stage 1: after the AV has made the LC decision (at time $t_0$), it needs to search for a suitable gap on the target lane. If the gap distance is acceptable, AV will perform the execution of LC at time $t_{start}$ (start to steer the wheel). Seen from the surface, the AV is still in the process of CF, but in fact, the AV is making preparation (finding a suitable gap). Therefore, this stage could be viewed as internal part of LC maneuver. It is worth noting that there are also situations where the vehicle directly executes LC after making the decision ($t_0 = t_{start}$).

Stage 2: the other is from the execution point to the end point. The AV begins to turn the steering wheel at time $t_{start}$. Then, the vehicle gradually drives along the planned trajectory until it arrives the target lane at time $t_{end}$. This stage could be viewed as the external part of LC maneuver, since we could see from the surface that the AV is performing LC.

In fact, the impact of LC maneuver on surroundings mainly exists in the execution point (stage 1), and the execution process (stage 2). Assume that the vehicles on the target lane maintains a stable CF distance and speed. First, we illustrate the impact of the stage 2 of LC on the surroundings. Suppose the execution point of LC is fixed, the LTP algorithm is actually selecting one of the many curve equations that has the least cost (e.g., comfort, efficiency, and safety) as shown in the bottom right of Fig. 3. Each sampling point on the curve has a corresponding position, speed and acceleration, all of which would affect the response of the vehicles behind the target lane in real time (assuming they are under the control of CF model), resulting in a corresponding cost.
for these vehicles as well. Thus, the total loss within the LC area may not be minimal when the loss of the \( AV_{lc} \) is minimal. At this stage, we need to answer whether there exists a curve that could minimize the cost of both at the same time. Second, when the trajectory equation is fixed, the total cost within the LC area might be very different under different execution point. It is even possible that the impact of the execution point is greater than the impact of the LC trajectory equation. At the same time, it is worth noting that it might not be appropriate to research these two stages separately, since they might not be optimal at the same time. It is interesting and also crucial research topic about how to simultaneously combine these two stages in obtaining a fair automatic LC algorithm. Therefore, a novel LC algorithm that simultaneously integrates the above two stages is established. The driving environment considered is the mixed traffic environment. Under our proposed algorithm, \( AV_{lc} \) could not only find the optimal starting point, but also the optimal LC trajectory. The total cost is composed of the \( AV_{lc} \) and the mixed vehicles behind the target lane, where comfort, efficiency and safety are simultaneously considered.

To facilitate the subsequent discussion, it is necessary to explain the scope and the assumption of this paper. (a) only LC behavior and CF behavior on the mainline of expressway are considered in this paper, and the established algorithm is more suitable for the mandatory LC scenarios. The AVs are with SAE Level 4/5 automation that could drive fully-autonomously. The AVs could obtain the real-time status of surroundings (either through its own sensors, or through RSU). (b) The strategic level controller has given the range of the coordination zone to the RSU (Road Side Unit) in the tactical level. And the tactical level distributes the optimal execution point and trajectory information to the \( AV_{lc} \). (c) the \( AV_{lc} \) has already determined to change lane, and the time before \( t_n \) is outside the scope of this paper. Out of special reasons, the \( AV_{lc} \) has to perform LC. This may be due to the lower speed or the occurrence of traffic accident of the preceding vehicle on the current lane. This leads to a poor driving experience of \( AV_{lc} \), which causes the \( AV_{lc} \) to perform LC.

3 Mathematical model

This section presents the mathematical model of the proposed algorithm, which contains the CF model, LC model and the multiobjective optimization objective function. Since our algorithm involves the longitudinal and lateral movement of AVs and HVs, to facilitate the research, the LCM (Longitudinal Control Model) [3, 30] is introduced to characterize the longitudinal motions of HVs. The reason why we choose this model is to the unified perspective casted on the existing microscopic traffic flow models [3]. In addition, this model considers the reaction time as well as the aggressive type of the driver. The IDM (Intelligent Driver Model) [31] is employed to model the longitudinal motions of AVs. This model could better reproduce the CF behavior of AVs, and has been widely used as an autonomous CF model in existing research [32, 33]. As for the LC model, the trajectory planning and tracking model are adopted. The time-based quintic polynomial function is employed to model the longitudinal and lateral LC trajectory, since it has high reliability and flexibility. A multiobjective optimization function is formulated, where the cost of the \( AV_{lc} \) and the mixed vehicles are simultaneously considered. Finally, the vehicle kinematics model, vehicle error model, and the MPC controller are integrated together to track the planned trajectory.

3.1 Car-following model

3.1.1 LCM model

The LCM model is derived through focusing the forces for the vehicle in the longitudinal direction of Field Theory. Field Theory represents everything in the environment (highways and vehicles) as a field perceived by the subject driver whose mission is to achieve his or her goals by navigating through the overall field. The formula is given below:
\[ a_{LCM}(t + \tau_i) = A_1[1 - \frac{v_{i}(t)}{v_{desire}} - e^{-v_{i}(t)\xi_i(t)}] \]  
(1)

\[ s_{ij}^{(i)}(t) = \frac{v_{i}^2(t)}{2b_i} - \frac{v_{j}^2(t)}{2B_j} + v_{i}(t)\tau_i + l_j \]  
(2)

\[ \dot{a}_{LCM}(t + \tau_i) = A_2[-\frac{a_{i}(t)}{v_{desire}} - \left(\frac{s_{ij}^{(i)}(t)}{s_{ij}^{(i)}(t)}\right)\frac{v_{ij}(t)}{s_{ij}^{(i)}(t)}e^{-v_{i}(t)\xi_i(t)}] \]  
(3)

\[ (s_{ij}^{(i)})' = \frac{a_{i}(t)}{b_i} - \frac{a_{i}(t)}{B_j} + a_{i}(t)\tau_i \]  
(4)

Where \( i \) denotes the follower vehicle, \( j \) denotes the leader vehicle, \( l_j \) denotes the vehicle length, \( \tau_i \) denotes the reaction time, \( s_{ij}(t) \) denotes the spacing between vehicle \( i \) and \( j \), \( v_{desire} \) denotes the desired speed, \( A_i \) denotes the maximum acceleration, \( s_{ij}^* \) denotes the desired safe spacing of driver \( i \), \( b_i \) denotes the maximum deceleration that the driver can confidently apply in an emergency, \( B_j \) denotes the driver’s estimation of the leader vehicle’s comfortable deceleration in an emergency brake.

### 3.1.2 IDM model

The IDM (Intelligent driver model), the most popular model today, assumes that each driver has a desired headway, speed and headway, and that the model consists of acceleration in free-flow conditions and deceleration to avoid collision with the vehicle in front. The IDM model takes the following forms:

\[ a_{IDM}(t) = A_1[1 - \frac{v_{i}(t)}{v_{desire}} - (\frac{s_{ij}^*(t)}{s_{ij}(t) - l_j})^2] \]  
(5)

\[ s_{ij}^{(i)}(t) = s_{ij}^{(jam)} + s_{ij}^{(i)} \sqrt{\frac{v_{i}(t)}{v_{desire}}} + v_{i}(t)T + \frac{v_{i}(t)\Delta v_{ij}(t)}{2\sqrt{A_i^2}} \]  
(6)

\[ \dot{a}_{IDM}(t) = A_2[-\delta \left(\frac{v_{i}(t)}{v_{desire}}\right)^{\delta-1} - \frac{2s_{ij}^*(t)(s_{ij}^*(t))'}{(s_{ij}(t) - l_j)^2}] \]  
(7)

\[ (s_{ij}^*(t))' = \frac{s_{ij}^{(i)}}{2\sqrt{v_{i}(t)v_{desire}}} + \dot{a}_{IDM}(t)T - \frac{a_{IDM}(t)^2}{2\sqrt{A_i^2a_i^2}} \]  
(8)

Where \( \delta \) is the acceleration coefficient, \( s_{ij}^{(jam)} \) is the distance between blocked vehicles, \( s_{ij}^{(i)} \) is the stopping distance, \( T \) is the safe headway, \( a_{i}^{conf} \) is the comfortable deceleration.

### 3.2 Lane-changing model

#### 3.2.1 Trajectory planning model

The time-based quintic polynomial function is introduced to model the longitudinal and lateral trajectory of \( AV_{LC} \). The longitudinal and lateral trajectory with respect to time is defined as below:

\[
x_{LC}(t) = a_0 + at + a_1t^2 + a_2t^3 + a_3t^4 + a_4t^5
\]

\[
y_{LC}(t) = b_0 + bt + b_2t^2 + b_3t^3 + b_4t^4 + b_5t^5
\]

\[
\theta_{LC}(t) = a \tan \left( \frac{y_{LC}(t)}{x_{LC}(t)} \right)
\]  
(9)
Where $x_{LC}(t), y_{LC}(t), \theta_{LC}(t)$ denote the longitudinal position, lateral position and course angle. $a_i, i=0,1,...5$ and $b_j, j=0,1,...5$ are the corresponding coefficients.

Assuming time $t$, it is reasonable to assume that the velocity and acceleration of $AV_{LC}$ are desired to be zero at the start and the end position in the longitudinal direction. Therefore, we could derive the following equations constrains.

\[
\begin{align*}
    x_{LC}(t_{\text{start}}) &= x_{\text{start}}, \dot{x}_{LC}(t_{\text{start}}) = v_{LC}^{\text{start}}, x_{LC}(t_{\text{end}}) = a_{LC}^{\text{end}} \\
    x_{LC}(t_{\text{end}}) &= x_{\text{end}}, \dot{x}_{LC}(t_{\text{end}}) = v_{LC}^{\text{end}}, \ddot{x}_{LC}(t_{\text{end}}) = a_{LC}^{\text{end}} \\
    y_{LC}(t_{\text{start}}) &= 0, \dot{y}_{LC}(t_{\text{start}}) = 0, \ddot{y}_{LC}(t_{\text{start}}) = 0 \\
    y_{LC}(t_{\text{end}}) &= D_0, \dot{y}_{LC}(t_{\text{end}}) = 0, \ddot{y}_{LC}(t_{\text{end}}) = 0
\end{align*}
\]  

(10)  

(11)

Where $D_0$ denotes the lane width, $v_{LC}^{\text{start}}$ and $v_{LC}^{\text{end}}$ denote the initial and final speed in the longitudinal direction, $a_{LC}^{\text{start}}$ and $a_{LC}^{\text{end}}$ denote the initial and final acceleration in the longitudinal direction. $x_{\text{start}}, x_{\text{end}}$ denotes the initial and final longitudinal position of the $AV_{LC}$.

### 3.2.2 Trajectory tracking model

Vehicle kinematics model considering the longitudinal, lateral, and yaw motions is introduced. According to the kinematic constraints of the front and rear axles, we could derive the following equations:

\[
\dot{\psi} = [\dot{x}, \dot{y}, \dot{\phi}]^T = [\cos \phi, \sin \phi, \frac{1}{l}] \cdot \tan \delta \cdot v,
\]

(12)

Where $\delta, l, v$ denotes the front wheel steering angle, wheel base, and the speed at the center of the rear axle. In order to facilitate the design of MPC (Model Predictive Controller), we expand this model by Taylor series at the reference trajectory point $(x_r, y_r)$, and the vehicle error model is formulated as:

\[
\dot{\tilde{\psi}}(k+1) = \tilde{A} \cdot \tilde{\psi}(k) + \tilde{B} \cdot \tilde{\mu}(k), (k = 1, 2, 3, ...)
\]

(13)

Where $\tilde{A}$ and $\tilde{B}$ are rational functions of $\theta$.

\[
\tilde{A} = \begin{bmatrix} 1 & 0 & -v \sin \phi \cdot T \\ 0 & 1 & v \cos \phi \cdot T \\ 0 & 0 & 1 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} \cos \phi \cdot T & 0 \\ \sin \phi \cdot T & 0 \\ \tan \delta \cdot T & \frac{v \cdot T}{l \cos \delta} \end{bmatrix}
\]

(14)

Where $T$ denotes the sampling time, $\tilde{\psi} = \psi - \psi_r$ denotes the difference with the reference state, $\tilde{\mu} = \mu - \mu_r$ denotes the difference with the reference input, $\tilde{\psi}_r = [x_r, y_r, \phi_r]^T$, $\mu_r = [a_r, \delta_r]^T$.

The design ideas of the MPC controller mainly include: the current state should converge to the reference value as soon as possible, and the control input should as small as possible. Therefore, the deviation of the system state quantity and the control quantity need to be optimized. The objective function has the following form:

\[
I(kq) = \sum_{i=1}^{N_p} \left[ q(y_{r}(k+i)) - \eta,(k+i) \right]_{\gamma}^2 + \sum_{j=1}^{N_c} \left[ \Delta u(k+j) \right]_{\rho}^2
\]

(15)

Where the first item on the right side describes the rapidity of tracking control system, and the right side describes the stationarity of the tracking control system. $N_p$ is the prediction horizon, $N_c$ is the control horizon, and $\rho$ is the weight.
coefficient. $\varepsilon$ is the relaxation factor, which could directly limit the control increment, avoid the sudden change of control quantity, and prevent the situation that there is no feasible solution in the optimization process.

3.3 Multiobjective Optimization problem

3.3.1 Formulation of the cost function

Suppose at time $t_0$, there are $m$ mixed vehicles behind the target lane (vehicles are marked from 1 to $m$ from near to far).

The joint optimization object contains the $AV_{LC}$ and the $m$ mixed vehicles, and the joint optimization objective function contains the comfort, efficiency and safety part. The jerk variable is introduced to quantify the comfort of each vehicle; the difference between the current and desired speed is to employed to characterize efficiency [34]; the relative speed and relative distance with preceding vehicle is used to quantify driving risk [4]. The general formulas of these three cost take the following form:

$$J^\text{comfort}_i(t) = |\dot{a}_i(t)|$$ (16)

$$J^\text{efficiency}_i(t) = |v_i(t) - v_0|$$ (17)

$$J^\text{safety}_i(t) = \lambda^\text{safety} (\Delta v_i(t))^2 + 1/\left[ (s_i(t))^2 + v_{\text{small}} \right]$$ (18)

Where $J^\text{comfort}_i(t)$, $J^\text{efficiency}_i(t)$, $J^\text{safety}_i(t)$ denotes the comfort, efficiency and safety cost of vehicle $i$ at time $t$. $\lambda^\text{safety}$ equals to 1 when $\Delta v_i(t) \geq 0$, and equals to 0 when $\Delta v_i(t) < 0$. $v_{\text{small}}$ is a small value to avoid zero dominator. When calculating the above these three items, the following two points need to be noted. (1) if vehicle $i$ is a HV, the $\dot{a}_i(t)$ adopts the acceleration of the LCM model, and the $v_0$ equals to the corresponding parameter $v_{\text{desire}}$. (2) if the vehicle $i$ is an AV, the $\dot{a}_i(t)$ adopts the acceleration of the IDM model in the longitudinal direction, and adopts the acceleration of the planned trajectory in the lateral direction.

Ideally, if the speed of the target lane traffic is the same as the desired speed, the construction of Equation 17 could also be understood from the perspective of traffic shock wave. Let point B on the flow-density curve be the condition represented by the LC vehicle, and point A be the condition represented by the following vehicles in the target lane. As long as point B is on the right of point A, the shock wave will be formed eventually, and the speed of the shock wave is denoted by the slope of the chord, $U_{AB}$. If point B moves toward point A along the curve, the impact of shock wave reduces, and diminishes when B coincides with A.

![Fig. 4 Illustration of the calculation of the traffic shock wave using the LWR model [35]](image-url)
The total cost of the $AV_{LC}$ takes the following form:

$$J_{LC} = \omega_{\text{comfort}} \frac{\sum_{t=t_0}^{t_{end}} J_{\text{LCcomfort}}(t)}{N_{\text{comfort}}} + \omega_{\text{efficiency}} \frac{\sum_{t=t_0}^{t_{end}} J_{\text{LCefficiency}}(t)}{N_{\text{efficiency}}}$$

$$+ \omega_{\text{safety}} \frac{\sum_{t=t_0}^{t_{end}} J_{\text{LCsafety}}(t)}{N_{\text{safety}}}$$

(19)

Where $J_{LC}$ denotes the total cost of the $AV_{LC}$. During $t_0$ to $t_{start}$, the $AV_{LC}$ is controlled under the IDM model, and during $t_{start}$ to $t_{end}$, the $AV_{LC}$ tracks the planned LC trajectory under MPC controller. $N_{\text{comfort}}$, $N_{\text{efficiency}}$, $N_{\text{safety}}$ are the normalized values of the corresponding terms in the objective function (to make the units consistent). $\omega_{\text{comfort}}$, $\omega_{\text{efficiency}}$, $\omega_{\text{safety}}$ denote the weight coefficient of comfort, efficiency and safety.

The total cost of the $m$ mixed vehicles behind the target lane takes the following form:

$$J_{TF} = \omega_{\text{comfort}} \frac{\sum_{i=1}^{m} \sum_{t=t_0}^{t_{start}} \omega_i J_{i\text{comfort}}(t)}{N_{\text{comfort}}} + \omega_{\text{efficiency}} \frac{\sum_{i=1}^{m} \sum_{t=t_0}^{t_{start}} \omega_i J_{i\text{efficiency}}(t)}{N_{\text{efficiency}}}$$

$$+ \omega_{\text{safety}} \frac{\sum_{i=1}^{m} \sum_{t=t_0}^{t_{start}} \omega_i J_{i\text{safety}}(t)}{N_{\text{safety}}}$$

(20)

Where $J_{TF}$ denotes the total cost of the $m$ mixed vehicles (TF denotes target following). $\omega_i$ denote the weight coefficient of the vehicle $i$. For vehicle with close distance and high relative speed difference, the more they are affected by the LC behavior, the larger its weight coefficient in the objective function. The $\omega_i$ takes the following form:

$$\omega_i = \sigma_i / \sum_{k=1}^{m} \sigma_k$$

(21)

$$\sigma_i = |\Delta v_{i-AV_{LC}}(t_0)| / \sqrt{\Delta x_{i-AV_{LC}}(t_0)}$$

(22)

Where $\Delta x_{i-AV_{LC}}(t_0)$ denotes the initial distance between vehicle $i$ and $AV_{LC}$ at time $t_0$, $\Delta v_{i-AV_{LC}}(t_0)$ denotes the initial speed difference between vehicle $i$ and $AV_{LC}$ at time $t_0$.

### 3.3.2 Constrains of the problem

The $AV_{LC}$ needs to meet the speed, stability, comfort, safety constrains during the execution of LC. The speed of $AV_{LC}$ should not exceed the maximum speed but should be greater than the minimum speed. The acceleration and jerk of AV should within the reasonable range. In addition, the LC duration ($t_{end} - t_{start}$) and the longitudinal moving distance of LC ($x_{end} - x_{start}$) should also within a reasonable range. Existing research demonstrate that the LC duration roughly ranges from 1s to 16s [10, 36]. Therefore, we specify the size of the longest and shortest LCD. The $AV_{LC}$ has to avoid excessive occupation of the surrounding road resources, since the LC maneuver occupies two lanes at the same time.

$$v_{\text{max}} \leq \sqrt{v(t)^2 + \dot{v}(t)^2} \leq v_{\text{max}}$$

(23)
\[
\begin{align*}
    a_{\text{min}} & \leq a(t) \leq a_{\text{max}} \\
    a_{\text{min}} & \leq a(t) \leq a_{\text{max}} \\
    j_{\text{min}} & \leq j(t) \leq j_{\text{max}} \\
    j_{\text{min}} & \leq j(t) \leq j_{\text{max}} \\
    0 & \leq t_{\text{start}} - t_0 \\
    t_{\text{LC, min}} & \leq t_{\text{end}} - t_{\text{start}} \leq t_{\text{LC, max}} \\
    x_{\text{LC, min}} & \leq x_{\text{end}} - x_{\text{start}} \leq x_{\text{LC, max}}
\end{align*}
\] (24)

Where \( v_{\text{min}}, v_{\text{max}}, a_{\text{min}}, a_{\text{max}}, j_{\text{min}}, j_{\text{max}}, t_{\text{LC, min}}, t_{\text{LC, max}}, x_{\text{LC, min}}, x_{\text{LC, max}} \) represent the minimum and maximum speed limit, acceleration, jerk, LC duration, longitudinal moving distance.

The AV\(_{LC}\) must not collide with surrounding vehicles at any time. The definition of the collision boundary area is shown below. The \( l_a, l_b, C_a, C_b \) are defined as vehicle length, vehicle width, ellipse long radius and ellipse short radius respectively.

![Collision area diagram](image)

**Fig. 5** The boundary of the collision area of the LC vehicle

Taking the starting point as the original coordinates, suppose at time \( t \), let \( P_{AV_{LC}}(t) = (x_{AV_{LC}}(t), y_{AV_{LC}}(t)) \) denotes the center position of the vehicle AV\(_{LC}\). The real-time boundary of collision area of AV\(_{LC}\) is defined as \( G_{AV_{LC}}(x, y) \).

\[
\begin{align*}
    M^2 / C_a^2 + N^2 / C_b^2 = 1 \\
    M = (x - x_{AV_{LC}}(t)) \cos \theta_{AV_{LC}} - (y - y_{AV_{LC}}(t)) \sin \theta_{AV_{LC}} \\
    N = (x - x_{AV_{LC}}(t)) \sin \theta_{AV_{LC}} + (y - y_{AV_{LC}}(t)) \cos \theta_{AV_{LC}}
\end{align*}
\] (26)

It is worth noting that the four corners of the smallest circumscribed rectangle of the vehicle outline should fall on the ellipse or within the ellipse. The real-time minimum distance between two collision boundaries could be obtained through the Lagrangian solution algorithm [34].

### 4 Solution method

#### 4.1 Pareto-optimal solutions

To diminish the impact of LC behavior on surroundings, we need to minimize the cost function \( J_{TF} \) and \( J_{LC} \) at the same time. This problem could be converted into multiobjective optimization problem, where we could only find a set of acceptable solutions as much as possible [37]. Since the multiobjective model does not possess a universal optimal solution, it is more important to find good compromises, or trade-offs, rather than a single solution. When the value of \( J_{LC} \) reaches the minimum, the value of \( J_{TF} \) may has unsatisfactory outcome. After the pioneering work of [38], the concept of Pareto-optimality has been widely utilized to characterize a solution of multiobjective optimization problem. Let \( \Gamma \) be the set of all feasible solutions for our
formulated cost function of $J_{TF}$ and $J_{LC}$ under our proposed framework. $x_A$, $x_B$, $x_C$, $x_D$, $x_E$ denote five different feasible solutions within the hypothetical feasibility region as shown in Fig. 6.

If the value of the objective function corresponding to solution A is better than the value of the objective function corresponding to solution B, it could be called solution A strongly Pareto dominance solution B (for example, solution $x_E$ is better than solution $x_A$ and solution $x_E$). If an objective function value corresponding to solution A is better than an objective function value corresponding to solution B, but another objective function value corresponding to solution A is worse than an objective function value corresponding to solution B, then solution A is indistinguishable from solution B, also known as solution A can Pareto dominate solution B (for example the relation between $x_D$ and $x_E$). For solution A, if no other solution can be better than solution A in the variable space $\Gamma$ (both objective function values are better than the function value corresponding to A), then solution A is the pareto-optimal solution (for example the solution $x_A$ and $x_B$). The solid dots in Fig. 6 are all pareto-optimal solutions. All pareto-optimal solution constitute the pareto-optimal solution set, and these solutions are mapped by the objective function to form the pareto-optimal frontier of the problem.

![Fig. 6 The illustration of Pareto-optimal solutions](image)

4.2 NSGA-II algorithm

Over the past decades, multiobjective GA (Genetic Algorithm) [27, 39-42] , as one kind of evolutionary algorithm, has been extensively employed to solve the multiobjective optimization problem. Various kinds of multiobjective GA has been proposed, such as vector evaluated GA [39], non-dominated sorting GA [40], random weight GA [41], adaptive weight GA[42], non-dominated sorting GA II [27], etc. Among these algorithms, the most famous is the NSGA-II algorithm, which has been widely applied in the field of transportation[43-46]. Compared with the first version of NSGA, NSGA-II algorithm has lower computational complexity. Crowding degree and crowding degree comparison operators are introduced to maintain the diversity of populations. The elite strategy is adopted to expand the sampling space and improve the population level rapidly [27]. Therefore, this algorithm will be adopted in this study to solve the optimization problem we have constructed.

The basic process of the NSGA-II algorithm is presented in Fig. 7. Three main steps are summarized below. Step 1: The initial population of size is randomly generated, and the first generation of offspring population is obtained by the three basic operations of selection, crossover and mutation of genetic algorithm after non-dominated sorting. Step 2: From the second generation, the
parent population is merged with the offspring population to perform fast non-dominated sorting, and the crowding degree is calculated for the individuals in each non-dominated layer. Based on the non-dominance relationship and the crowding degree of the individuals, the appropriate individuals are selected to form the new parent population. Step 3: A new population of children is generated by the basic operations of the genetic algorithm, and so on until the end of the program is satisfied. For more details about this algorithm, please refer to [27].

![Diagram of the NSGA-II algorithm](image)

**Fig. 7 Basic process of the NSGA-II algorithm**

### 4.3 Testing environment

All the simulation is programmed in Python, and run on an i9-9700CK 3.6GHz processor, with 16G RAM and RTX 2070. The NSGA-II algorithm in pymoo package is employed to solve the multiobjective problem. This package is developed under the supervision of Kalyanmoy Deb [27]. In order to improve the speed of the algorithm solution, we define the magnitude of change for each variable each time. We set the minimum change unit of each variable to 0.1, since too many decimal places only increase the complexity of the solution, and do not make a fundamental difference to the overall LC maneuver (for example, the LC duration takes the values of 5.11 and 5.12, or the final speed takes the values of 24.51 and 24.52). This could be realized in the pymoo package (the Discrete Variable Problem in Customization).

### 5 Simulation experiment design and results analysis

#### 5.1 Scenario and parameters settings

The simulation step $\Delta t$ is set as 0.1s. The normalized values of the corresponding cost terms in the objective function are set as: $N_{\text{comfort}} = 8m/s^3$, $N_{\text{efficiency}} = 25m/s$, $N_{\text{speed}} = 0.5s^{-1}$. The parameters of the LCM model are from the literature [47], and the
parameters of the IDM model are from the literature [48]. Other parameters are set as follows: \( l_o = 5m, l_p = 2m, C_a = 2.5m, C_s = 1m, v_{\text{max}} = 30m/s, v_{\text{min}} = 5m/s, a_{\text{max}} = 8m/s^2, a_{\text{min}} = -8m/s^2, j_{\text{max}} = 8m/s^3, j_{\text{min}} = -8m/s^3, D_o = 3.5m \) [34]. The simulation duration is set to 500s, with a total of 20 mixed vehicles on the target lane. After a period of warm-up simulation time, the vehicles on the target lane all maintain a steady CF distance and speed. Suppose at moment 300s, the longitudinal position of \( AV_{LC} \) is between the 10th and the 11th vehicle. At this moment, two vehicles 100m ahead of suddenly collide and both vehicles stop abruptly.

Our simulation scenarios are randomly generated to ensure that the algorithm applies to all possible traffic conditions. The following scenarios is one of them with penetration rate as 50\%, the AV and HV alternatively appear in the queue. The initial speed of the \( AV_{LC} \) is 20m/s, the distance to the 11th vehicle to 20m. We would perform further sensitivity analysis for these parameters in the next subsection. The \( AV_{LC} \) has to perform LC between the gap between the 10th and the 11th vehicle. And it is difficult for the \( AV_{LC} \) to perform LC in the next gap since several vehicles following him closely behind.

\[ \text{Fig. 8 The illustration of the testing scenario} \]

5.2 Performance of our proposed algorithm

Fig. 9 presents the pareto-optimal frontier obtained by our algorithm under this scenario. With the origin as the center, a circle is drawn with the closest distance of the frontier from the origin (the radius is 36.81), and the intersection of the circle and the frontier is taken as the final optimal solution. Since the construction of the cost function for \( AV_{LC} \) and the mixed vehicles in Section 3 is the same, thus, we could take such a intersection point, and this similar approach has also been adopted in [49]. It could be seen under this solution, the cost of \( AV_{LC} \) is about 14.11, the cost of mixed vehicles is 34.00, and the total cost is about 48.11. It is noteworthy that when the loss of \( AV_{LC} \) is small, the loss of mixed vehicles is extremely large, for example, the leftmost scatter point, when the loss of \( AV_{LC} \) is less than 3, the loss of mixed vehicles is all greater than 45, and the total loss is all greater than the loss corresponding to the intersection we take, which verifies the importance of this study conducted.

Subplot (b), (c), (d) in Fig. 9 presents the longitudinal trajectories (position, speed and headway) of all vehicles. The red curve represents the \( AV_{LC} \), the blue curve represents the AVs, and the black curve represents the HVs. It can be seen that after a period of simulation warm-up time, the 20 mixed vehicles maintain a constant CF headway and speed. The time headway of AVs is lower than that of the HVs (about 2.4s for AV and 3.2s for HV). When the \( AV_{LC} \) is inserted into the 10th and 11th vehicle, its LC behavior significantly affected the mixed vehicles behind, which all made corresponding deceleration. The vehicle speed gradually decreased from the steady following speed of 25m/s.
Fig. 10 presents a more detailed microscopic view of the movement and cost of each vehicle. We show the longitudinal and lateral trajectory of $AV_{LC}$, the longitudinal trajectories of three vehicles behind, and the cost of each vehicle at each moment in Subplot (a) and Subplot (b). At the moment of 300s, the $AV_{LC}$ has decided to change lane, but he do not choose to steer the wheel at this moment, but until the moment of 300.6s. And the $AV_{LC}$ finally reached the centerline of the target lane at 306.6s. Within 300s to 300.6s, $AV_{LC}$ still travels according to the IDM model, and three vehicles behind the target lane also travel according to the steady CF speed. The speed of $AV_{LC}$ is 20m/s at the moment of 300s. The speed of $AV_{LC}$ is 20.5m/s, and the acceleration is about 0.81m/s$^2$ when time is 300.6s.

Within 300s to 300.6s, $AV_{LC}$ follows the trajectory equation given by our algorithm, and the 11th vehicle, 12th vehicle, 13th vehicle gradually decelerate. At the moment of 306.6s, the final longitudinal speed of $AV_{LC}$ arrives at 26m/s, and the longitudinal acceleration reaches at 0 (these values are all solved by our proposed algorithm). At the same time, it is worth noting that the speed and acceleration of the $AV_{LC}$ are all continuous everywhere during the period from stage 1 (from 300s~300.6s) to stage 2 (from 300.6s to 306.6s). Subplot (c) and Subplot (d) also present the lateral trajectory information of the $AV_{LC}$. The lateral trajectory of $AV_{LC}$ smoothly and gradually changes from 0m to 3.5m, and the lateral speed slowly increases from 0m to 1.1m/s and gradually decreases to 0. The four subplots at the bottom of Fig. 10 presents the total loss, comfort loss, efficiency loss and safety loss for these four vehicles for each moment. At moment 300.6s, the instantaneous comfort loss of the 11th vehicle exceeds 20, while its safety loss is 18.12, resulting in a total loss of nearly 25 at that moment, causing it to be the one vehicle most affected by the LC behavior.
5.3 Comparison with the existing algorithm

In the previous section, we analyze the pareto-optimal frontier, and we can intuitively see that when $A_{LCV}$ takes the leftmost solutions, the cost of $A_{LCV}$ could indeed take a smaller value, but the corresponding cost of the mixed vehicles is larger at this point, leading to an overall loss that is not optimal. Since we could directly analyze from the pareto-optimal frontier that our algorithm does improve the local traffic flow to minimize its total loss, we introduce a macro-level analysis approach in this subsection to compare and analyze the impact of these two solutions on traffic flow.

In order to facilitate the presentation of our macro-level analysis, we increase the traffic flow in the target lane to 200 vehicles, with $A_{LCV}$ changing lanes between the 100th and 101th vehicles. At the same time, we increase the simulation step size to 6000s, and the $A_{LCV}$ decides to change lane at 4000s, keeping all the remaining parameters the same as in 5.1 (we only increase the simulation time and traffic flow size).

Since the speed of $A_{LCV}$ during LC is not fixed, it is not appropriate for us to estimate the traffic shock wave using the steady-state macroscopic model. Nevertheless, we could employ the basic method to explore the flow, space-mean speed, and density [50] through defining the rectangular area within the LC area. This method takes the following three forms:

\[ q_{flow} = \frac{d(A)}{|A|} \]
\[ V_{space-mean} = \frac{d(A)}{t(A)} \]
\[ k_{density} = \frac{t(A)}{|A|} \]

Where $|A|$ denotes the area of rectangle, $d(A)$ and $t(A)$ denotes the total distance and time travelled of all vehicles within this area.

Fig. 11 presents the speed time-space heatmap of vehicles within the LC area. The abscissa represents the simulation time, the ordinate represents the longitudinal position, and each curve represents the trajectory of the vehicle. The red line in subplot (a) represents the LC vehicle. Through the speed time-heatmap in subplot (b), we could obviously find that, with the insertion of the LC vehicle, the speed of the vehicles behind the target lane all made a corresponding deceleration. The closer the color is to blue,
the higher the speed of vehicles. The closer the color is to red, the lower the speed of vehicles. The blue area represents the speed around 22.5~25m/s, the red area represents the speed around 20~22.5 m/s. The color within the LC area gradually shifts from dark blue to light blue, where the color of the LC vehicle and its immediate rear line both show red.

In order to further analyze the LC impact, we extract the trajectories within the blue box as shown in subplot (c). Tab. 1 presents comparison of the time and distance travelled of each vehicle within the LC area under our proposed algorithm and the existing algorithm. According to the method in [50], the flow, space-mean speed, and density could be estimated. $|A|$ equals to $L \times T = 500 \times 15 = 7500 \text{m} \cdot \text{s}$. In total, there are 14 vehicles in the region, from the 94th to 106th vehicle. It can be seen that the distance and time travelled of the previous vehicles including the 100th itself are the same under the two different algorithms. And with the insertion of $AV_{LC}$, the trajectories of the vehicles after the 100th have a significant difference. Since the duration of LC is not very long, this difference is not very huge, but under our algorithm, the 101th, 102th and 103th vehicles travel almost 3m, 4m and 2m more in this area. This results in an improvement in space-mean speed and traffic flow, with an overall speed increase of 0.7m/s and a flow increase of 5.88veh/h. It is shown that the solution we take does improve the operation of the regional traffic flow compared to the solution on the leftmost side of the frontier.

![Fig. 11 Longitudinal speed time-space heatmap of the vehicles within the LC area](image)

**Tab. 1 Time and distance travelled of each vehicle within the LC area, and the corresponding macroscopic traffic flow index**

|                  | The existing algorithm | Our proposed algorithm |
|------------------|------------------------|------------------------|
|                  | Distance travelled(m)  | Time travelled(s)      | Distance travelled(m)  | Time travelled(s)      |
| 94th vehicle     | 5.00                   | 0.30                   | 5.00                   | 0.30                   |
| 95th vehicle     | 85.00                  | 3.50                   | 85.00                  | 3.50                   |
| 96th vehicle     | 140.00                 | 5.70                   | 140.00                 | 5.70                   |
| 97th vehicle     | 220.00                 | 8.90                   | 220.00                 | 8.90                   |
| 98th vehicle     | 275.00                 | 11.10                  | 275.00                 | 11.10                  |
| 99th vehicle     | 355.00                 | 14.30                  | 355.00                 | 14.30                  |
| 100th vehicle    | 375.00                 | 15.10                  | 375.00                 | 15.10                  |
| $AV_{LC}$        | 368.67                 | 15.10                  | 370.77                 | 15.10                  |
| 101th vehicle    | 343.41                 | 15.10                  | 346.37                 | 15.10                  |
| 102th vehicle    | 305.26                 | 13.10                  | 309.79                 | 13.20                  |
| 103th vehicle    | 236.84                 | 10.00                  | 238.31                 | 10.00                  |
| 104th vehicle    | 184.95                 | 7.70                   | 185.74                 | 7.70                   |
| 105th vehicle    | 110.57                 | 4.60                   | 110.89                 | 4.60                   |
| 106th vehicle    | 56.98                  | 2.40                   | 57.07                  | 2.40                   |
| **Total**        | **3061.68**            | **126.90**             | **3073.93**            | **127.00**             |
Flow (veh/h) & 1469.61 & 1475.49 \\ Speed (m/s) & 24.13 & 24.20 

5.4 Sensitivity analysis of the parameters

Fig. 12 shows the sensitivity analysis of the parameters, including the initial speed, initial distance with the 11th vehicle, and the penetration ratio of AVs. As the initial speed rises from 18m/s to 25m/s, the frontier gradually shifts down. For the $AV_{LC}$, the distribution of the cost values is approximately the same, and the main difference is that the cost of the rear vehicle is gradually decreasing. Similarly, as the initial distance increases from 40m to 100m, the frontier gradually moves downward and the overall cost gradually decreases. When the penetration ratio changes from 0% to 100%, the cost value of the frontier rises by nearly 10. This may be due to the fact that the AVs have a shorter CF headway, and when $AV_{LC}$ is inserted, the efficiency cost for them is much greater than that for HVs, thus causing the frontier to rise.

5.5 Discussion

The central objective of this paper is to minimize the impact of LC behavior on traffic flow. To achieve this goal, we construct cost functions for the LC vehicle and surrounding vehicles, including efficiency, comfort, and safety, respectively. Since the multiobjective model does not possess a universal optimal solution, it is more important to find good compromises, or trade-offs, rather than a single solution. Therefore, we introduce the NSGA-II algorithm [27] in the concept of Pareto-optimality [38] to solve our formulated multi-objective optimization problem. After obtaining the pareto-optimal frontier, we take the origin as the center and draw a circle to take the intersection point on the tangent line with the frontier as the final optimal solution (the magnitudes are the same).

With the pareto-optima frontier, we can intuitively see the difference and superiority of our solution to those employed in existing studies. Although the cost functions we construct may differ, the existing studies follow the idea of solving the LC strategies that minimize itself cost. These strategies all correspond to the leftmost point of the surface. It can be seen that the LC vehicle can indeed have a very low cost when it takes the leftmost strategy, but the value is not optimal for others, and the overall cost is also not optimal. Also, it is worth noting that when a set of parameters is set. It is possible to draw a line along the surface that is wirelessly close to the frontier, but at the same time parallel to the X-axis. The intersection of this line with the Y-axis could be identified as “baseline impact”. It is understood that whatever LC strategy the vehicle adopts at that point, it causes at least that...
value of cost to the surrounding vehicles. The magnitude of this loss may be influenced to a large extent by the initial vehicle speed, as well as the distance to the first rear vehicle on the target lane. Subsequent research could dig deeper at this point.

Through the analysis of the driving status of each vehicle at the micro level and the analysis of the impact on traffic flow at the macro level, we verify the effectiveness and superiority of our proposed algorithm. Compared with existing algorithms, our proposed algorithm could indeed improve the state of traffic flow operation in the region to a certain extent. Undoubtedly, many aspects of this paper need further research. How to simultaneously manipulate the LC vehicle and the surrounding AVs to abate the impact of LC behavior at the control level. The optimization objective can be borrowed from the cost function defined in this study, how to design the controller to obtain the optimal acceleration for each AV at each moment to achieve the same research objective as this study. On the other hand, this study distinguishes between the concepts of LC decision point and LC execution point. We assume that the vehicle has completed the process of LC decision, but has not yet found the optimal LC execution point. The subsequent study can consider the fusion of three processes, one is the process from normal driving to LC decision generation, the second is from the decision point to the execution point, and the third is from the execution point to the end point (as shown in Fig. 3), so as to further minimize the impact of LC behavior on surroundings.

6 Conclusion

In order to minimize the impact of LC on surroundings, this paper proposes a novel LC algorithm. We parse the LC maneuver into two stages as shown in Fig. 3. The core of our proposed algorithm is the formulation of the multiobjective optimization problem and the adoption of the NSGA-II algorithm to obtain the pareto-optimal frontier. Through comprehensive numerical simulation, we have verified the validity and excellence of the model. With our algorithm, the impact of LC behavior on the surrounding traffic flow is diminished as much as possible. This is reflected in the reduction of the total cost of surrounding vehicles and the increase of traffic flow within the investigated region. We hope this paper could shed some light on the research of the automatic LC algorithms.

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