Base parameters that can be identified from measured values and a control scheme for planar Torque-unit manipulator

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Abstract. A base parameter set which is defined to be a minimum set of inertial parameters that can determine dynamic model uniquely is shown for Torque-unit Manipulator (TUM) which has been proposed as a new design concept of manipulator. The elements of the base parameter set are also the parameters that can be identified independently from motion data and input data, i.e., measured values. The base parameters play very important role for dynamic modelling of manipulators such as model based control and simulation of the manipulator motion. The base parameters are given completely in closed form of linear combinations of the link inertial parameters. Also, a control strategy for all state variables of TUM is given by using the base parameters.

1. Introduction

“Torque unit Manipulator (TUM)” is a new design concept of (space) manipulator[1]. Figure 1 shows a schematic picture of TUM. TUM has an open-loop kinematic chain as the traditional types of manipulators, however the differences are as follows.

(i) Each joint of TUM is free joint which is 1 degree-of-freedom.

(ii) Each link has a “torque unit” on an arbitrary position. The “torque unit” applies torque to the link on the position. The “torque unit” can be made of a rotary actuator such as DC-servo motor and a disc which is connected to the rotating shaft of the actuator.

Packing the actuator and the disc into one “unit”, we can make a torque-unit and attach it on an arbitrary position of each like of a kinematic chain whose joints are all free. (See Fig.2) Thereby, we would be able to construct a Torque-unit Manipulator very easily. The TUM would have merits, for example,

• Easy maintenance since all joints are all free,
Possibility of construction of redundant actuator system since we could attach a few torque-unit on each link.

The merits would give TUM high-reliability. The high-reliability is very important if we utilize manipulators in remote and hazardous environments such as the space. TUM is an interesting sample of Multibody Systems.

![Figure 1. TUM; A Design Concept of Manipulator](image1.png)

![Figure 2. Torque-unit](image2.png)

### 2. Torque-unit Manipulator

Fig.1 shows a \( N \) degree-of-freedom planar Torque-unit manipulator which we treat in this paper. We don’t take the effect of gravity into account. All joint axes and the rotational axes of the discs are parallel. Then, we attach a coordinate system \((o_i; x_i, y_i, z_i)\) on link \( i \). \( x_i \) is taken along the longitudinal direction of link \( i \), \( z_i \) is taken along the join axis. Thereby we can define the joint variable of joint \( i \). Let \( \theta_i \) denote the joint angle that is measured from \( x_{i-1} \)-axis to \( x_i \)-axis about \( z_i \)-axis. Let \( \theta = [\theta_1 \ \theta_2 \ \ldots \ \theta_N]^T \), where superscript \( T \) denotes transposition. For link \( i \), let \( l_i \) denote the length from \( o_i \) to \( o_{i+1} \), \( m_i \) denote the mass, \( I_i \) denote the inertia moment around \( z_i \), and \( r_i \) the distance from \( o_i \) to the center of mass of link \( i \), respectively. We assume that the center of mass of link \( i \) is located on the line segment that connects \( o_i \) and \( o_{i+1} \). Then, we attach a coordinate system \((d_{di}; x_{di}, y_{di}, z_{di})\) to the disc \( i \) such that we take \( z_{di} \) in alignment with the rotational axis of the disc \( i \) and \( o_{di} \) at the center of mass of the disc and we set \( o_{di} \) on the line that passes \( o_i \) and \( o_{i+1} \). The rotational angle of disc \( i \) is denoted by \( \varphi_i \) which is the angle between \( x_i \) and \( x_{di} \) and let \( \varphi = [\varphi_1 \ \varphi_2 \ \ldots \ \varphi_N]^T \). For disc \( i \), let \( l_{di} \) denote the length form \( o_i \) to \( o_{di} \), \( m_{di} \) the mass, and \( I_{di} \) the moment around \( o_{di} \), respectively. Here, \( m_i \), \( m_{di} \), \( l_i \), \( l_{di} \) will be called “link inertial parameters”. Any liner combination of the link inertial parameters is defined to be an “inertial parameter”.

### 3. Dynamic Model for the TUM

In this section we derive the dynamic model for the TUM. The dynamic model of the TUM can be described as a system of two equations:

\[
\begin{align*}
H_{11}(\theta)\ddot{\theta} + H_{12}(\theta)\dot{\varphi} + H_{11}(\theta)\dot{\theta} - \frac{\partial}{\partial \theta} \left\{ \frac{1}{2} \dot{\theta}^T H_{11}(\theta) \ddot{\theta} \right\} &= 0 \\
H_{12}^T \ddot{\varphi} + H_{22}(\varphi)\dot{\varphi} &= \tau
\end{align*}
\]

where \( \tau_{disc} = [\tau_{disc_1}, \tau_{disc_2}, \ldots, \tau_{disc_N}] \) denotes the vector of applied torques to the discs and they are the input to the TUM. \( H_{11}(\theta) \) is the inertial matrix when we assume that all the discs are frozen, \( H_{12} \) is upper triangular matrix whose elements are \( I_{di} \) and \( H_{22} \) is diagonal matrix whose elements are also \( I_{di} \).

### 4. A Base Parameter Set

We show a base parameter set of the dynamic model for the TUM.
4.1. Definition of Base Parameters

The dynamic model of TUM (1) can be determined if and only if each element of \( H_{11}(\theta) \) is determined as a function of \( \theta \). Then, we can describe each element of \( H_{11}(\theta) \) in the following form:

\[
\sum_{v=1}^{T} p_v f_v(\theta)
\]

(2)

where \( p_v \) are inertial parameters and \( f_v \) are a polynomial of trigonometric functions of \( \theta \). (\( f_v \) is allowed to be a constant function.) The from (2) will be called a function of \( \theta \) generated by \( p_1, p_2, \ldots, p_T \). These forms include the values of the kinematic parameters. Then, assuming that the values of kinematic parameters are known, if we give values to all the link inertial parameters, we can determine all the elements of \( H_{11} \) as a function of \( \theta \) and hence the dynamic model (1). To investigate the problem, following definitions and properties have been given in [2].

**Definition 1**

An inertial parameter \( p \) is called a fundamental parameter if any two choices of values to all the link inertial parameters, which give different values to \( p \), never determine the same dynamic model.

**Definition 2**

A set \( F \) of inertial parameters is said to generate the dynamic model if the same dynamic model are always determined by any choices of values to all the link inertial parameters as long as they give the same value to each inertial parameter in \( F \).

**Definition 3**

A set \( F \) of linearly independent fundamental parameters that generates the dynamic model is called a base parameter set and each fundamental parameter in \( F \) is called a base parameter.

**Property 1**

The set of all the fundamental parameters constitutes a linear vector space, and a base parameter set from a base set of the linear vector space.

**Property 2**

A base parameter set is a minimum set of inertial parameters that can generate the dynamic model.

A fundamental parameter corresponds to an identifiable parameter from motion data and input torque data which are measured values using sensors that are equipped on manipulator. It is impossible to estimate all the link inertial parameter values from the measured values. Hence, the concept of base parameter set is very important from the view of measurement.

4.2 Base Parameter set of planer TUM

We give a base parameter set of planer TUM show in Fig.1.

**Proposition**

The following inertial parameters constitute a base parameter set of the dynamic model (1).

\[
R_i = m_i r_i + M_{i+1} l_i + m_{di} l_{di},
\]

(3)

\[
J_i = I_i + M_{i+1} l_i^2 + m_{di} l_{di}^2,
\]

(4)

\[
I_{di},
\]

(5)

for \( 1 \leq i \leq N \), where,

\[
M_i = \sum_{j=i}^{N} (m_j + m_{dj}).
\]

(6)

We omit the proof.

4.3 Identification of the base parameter values of planer TUM

We can rewrite the equations of motion (1) as:
\[
\begin{pmatrix}
H_{11}(\theta) & H_{12}(\theta) \\
H_{21}(\theta) & H_{22}(\theta)
\end{pmatrix}
\begin{pmatrix}
\dot{\theta} \\
\dot{\phi}
\end{pmatrix}
+ \left( H_{11}(\theta)\dot{\theta} - \frac{\partial}{\partial \theta} \frac{1}{2} \dot{\theta} \cdot H_{11}(\theta)\dot{\theta} \right)

= Y(\ddot{\theta}, \dot{\theta}, \theta, \dot{\phi}, \phi)p = \begin{pmatrix}
0 \\
\tau_{\text{disc}}
\end{pmatrix}
\]

where \(Y(\cdot)\) is a regressor matrix and \(p\) represents the vector of base parameters which are the linear independent parameters and can determine the equations of motion uniquely. Then the matrix \(Y(\cdot)\) turns to be full rank, hence, we can estimate \(p\) by the least-squares method with motion data and input torque data.

5. A control scheme of the planer TUM

We have shown the position controllability of the links for general type of N d.o.f TUM by using a conventional control law for manipulators[1]. However there arises a problem. When we use such conventional control laws, we only focus on the behavior of links and ignore the behavior of the discs in torque-units. Hence, we only ensure that positions of the links approach desired positions and that the angular velocities of the discs never go infinity even asymptotically. The angular velocities of the discs usually do not approach zero and result to be constant when the links approach desired position. The residual angular velocity might give bad influence to the actuator. The problem arose because we ignored the behavior of the disc. Then, taking the all variables to represent the state of TUM into account, i.e., taking the behavior of the discs into account, we have found that TUM is a system with non-holonomic constraints: a Non-holonomic system[3]. Hence the angular velocity of each disc at each time depends on the trajectory of the links: how the links moved from the beginning. To remove the residual angular velocities of the discs, we have given a control strategy which consists of two phases as follows[4]:

- **Phase 1** Controlling the positions of the links to desired position
- **Phase 2** Controlling the residual angular velocities of the discs to zero by planning trajectory of the links

In planning the trajectory in Phase2 we need the exact values of the base parameters of the TUM. They can be identified in Phase1.

6. Conclusion

Torque-unit Manipulator (TUM) is a design concept of (space) manipulator. We have shown a base parameter set for planer TUM and described the identification of them in the paper. If we control the links of TUM with conventional control laws, there remain residual angular velocities of the discs in the torque unit. By taking the advantage of the feature in the dynamics of TUM, we have given a control scheme. The control scheme consists of two phases and we need exact values of base parameter for the control scheme.

References

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