Staircase temperature profiles and plasma transport self-organisation in a minimum kinetic model of turbulence based on the trapped ion mode instability

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Abstract. A minimum model of plasma turbulence in a kinetic framework is presented. It is based on trapped ion turbulence, gyro and bounce averaged, and implemented in the versatile and efficient code TERESA. Zonal flow - streamer interplay are readily shown to be key players that govern the confinement properties of the model. The parameter space of the model is explored with brute force numerics. A generic result is either a streamer dominated pattern with large transport, or a staircase temperature profile with very marked corrugations and quenched transport. A case with off-axis heating is found to exhibit quasiperiodic relaxation events relevant to investigate dynamical turbulence self-organisation.

1. Introduction

Although the issue of confinement performance of burning plasmas could be considered as solved, at least empirically, the JET results with the ITER like wall have eroded the performance capability of this device [1]. The possibility of stepping up the fusion gain on JET with respect to the 1997 DT campaign [2] is clearly questioned by the latter results. Theoretical investigation of confinement performance then appears to be required to help in clarifying the situation and drawing appropriate research lines to master plasma performance with this new wall environment. Of particular importance is the issue of transport barriers, their onset, their width and their confinement properties [3]. In major gyrokinetic simulations such transport barriers have been triggered by external plasma polarisation [4, 5]. However, self-generated barriers have only been observed with micro-barriers leading to corrugations in the temperature gradient [6]. Such a regulation of turbulence can lead to key regimes for more favourable confinement performance with either short lived distributed corrugations that govern an enhanced confinement time, or the seed to the transition towards stronger barriers, and consequently improved confinement regimes. In this paper we analyse the onset and evolution of such corrugations with a reduced kinetic model of turbulence.

State of the art simulations show complex confinement properties combining all scales of the system and in particular on going self-organisation and interplay between different turbulence scales [7, 8, 9]. However, such models are very demanding to run and analyse. We have
thus developed a simple kinetic code, the TERESA code [10], modelling trapped ion turbulence [11, 12, 13, 14, 15, 16, 17]. This model is the minimum kinetic model of plasma turbulence. It is derived in the framework of Hamiltonian dynamics and action-angle variables [17], and benefits from a versatile and efficient parallelisation implementation in the code TERESA [10].

First simulations have led to staircase temperature profiles and thus strong corrugations of the temperature gradient. This signature of self-organisation of plasma transport is reminiscent of that obtained with the GYSELA 5D kinetic code [6] and the TOKAM2D fluid code [18]. TERESA simulations thus offer the possibility to address the key elements of turbulence self-organisation whilst retaining the efficiency of toy models like TOKAM2D [19]. Furthermore, it addresses kinetic features that are considered mandatory [20] but still require appropriate investigation to assess their actual effect. The paper is built as follows. The model and numerical settings are presented in Section 2. In the case where the plasma is driven out of equilibrium by a contact with two thermal baths, the long-time behaviour is observed to settle in either a streamer or in a zonal flow dominated regime, Section 3. In Section 4, a more systematic exploration of the space of control parameters shows that this feature is rather stiff. With off-axis heating, therefore a flux-driven case, quasi-periodic relaxation of the transport barriers have been observed, hence with time dependent self-organisation properties. Finally, Section 5 closes the paper.

2. A reduced model for trapped ion dynamics

Compared to the gyrokinetic case that is based on averaging out the fast gyrophase [21], the bounce average reduction consists of averaging out another fast phase related to the bounce motion. This model is thus restricted to trapped ion turbulence. The kinetic model is then reduced from the original 6D phase space \((r, \theta, \varphi, v_r, v_\theta, v_\varphi)\) in toroidal geometry, with given magnetic equilibrium, to 4D \((\alpha, \psi, \kappa, E)\) [11, 12, 13, 14, 15, 16], with two invariants, \(\kappa \in [0, 1]\) the trapping parameter, and the normalised particle energy \(E \in [0, +\infty]\) [17]. Given the safety factor \(q\), the variable \(\alpha = \varphi - q\theta\) labels a trapped orbit and is comparable to the toroidal angle \(\varphi\) at fixed poloidal angle \(\theta\). It is the conjugate coordinate [17] of the normalised poloidal magnetic flux \(\psi \in [0, 1]\) that also labels the radial position of a trapped orbit. Note that \(\psi\) is normalised by the radial extent of the box, according to the convention used in the experiments with \(\psi = 0\) on the magnetic axis and \(\psi = 1\) at the outer boundary. Similarly, \(\alpha\) is defined according to the \(\varphi\) variation \(\alpha \in [0, 2\pi]\) when the reference poloidal angle is set at \(\theta = 0\) (midplane low field side)

With such a reduction, the new particles are not local but extended in space, the extent being the Larmor radius in the \(\alpha\)-direction and the so-called banana width in the \(\psi\)-direction [17, 22]. Regarding the trapping parameter, \(\kappa \to 0\) for highly trapped particles and \(\kappa \to 1\) for barely trapped particles. It is interesting to note that averaging out the last angle \(\alpha\) would then yield a fully integrable system, hence without transport. A large part of the interest of the model is therefore that it is a minimum model for turbulent transport with a kinetic description.

The two fast angle averaging procedures yield a low pass filter in time, by construction, and in space via Finite Size Effects due to the spatial extent of the averaged trajectories. The gyrophase average introduces the Larmor radius, here \(\rho_0\), as characteristic scale for the FSE in both the radial and poloidal directions, while the bounce average introduces the banana width \(\delta_0\) for the FSE in the radial direction. Removing the two fast angle dependences, restricts the physics of the model to low frequency turbulence comparable to the drift frequency of the trapped orbit, \(\Omega_d E\). This frequency depends on both invariants, \(E\) as made explicit here, and \(\kappa\). For the sake of simplicity the latter can be ignored, which is the case for deeply trapped particles corresponding to harmonic oscillations, so that the model is effectively reduced to 3D.
The characteristic frequency is then $\Omega_d/\Omega_0 \propto q_0\rho_s^2/A_0$ where $\rho_s = \rho_0/r_0$, $r_0$ being a reference minor radius, $q_0 = q(r_0)$ the reference value of the safety factor, and $A_0$ the reference aspect ratio $A_0 = r_0/R_0$ and $\Omega_0$ the reference ion cyclotron frequency.

Given the description in terms of action-angle variables, the Vlasov equation is readily recovered given the Hamiltonian $\mathcal{H}$ describing the dynamics of the trapped orbits [17], namely:

$$\mathcal{H} = E(1 + \Omega_d \psi) + \Phi$$  \hspace{1cm} (1)

Here $\Phi$ is the fast angle averaged of the actual electric potential $\phi$. It comprises therefore a gyroaverage and a bounce average, hence $\Phi = \mathcal{J}\phi$, where $\mathcal{J}$ is the fast angle averaging operator. For numerical efficiency, the FSE governed by this averaging process is taken into account by a Pade approximation, hence:

$$\mathcal{F} = \mathcal{J} \mathcal{F} = \left(1 - \frac{E\delta^2}{4} \frac{\partial^2}{\partial \psi \partial \psi}\right)^{-1} \left(1 - \frac{E\rho_0^2}{4} \frac{\partial^2}{\partial \alpha \partial \alpha}\right)^{-1} \mathcal{F}$$  \hspace{1cm} (2)

The code then evolves the trapped orbit distribution function $\mathcal{F}(\alpha, \psi, \kappa, E)$ given the Vlasov equation with possible volumetric source term $S$, and diffusion transport $D$ in buffer regions, Fig.(1):

$$\frac{\partial}{\partial t} \mathcal{F}(\alpha, \psi, \kappa, E) - \left[\Phi, \mathcal{F}(\alpha, \psi, \kappa, E)\right] + E \Omega_d \frac{\partial}{\partial \alpha} \mathcal{F}(\alpha, \psi, \kappa, E) = D(\mathcal{F}) + S(\mathcal{F})$$  \hspace{1cm} (3)

In this expression the Poisson bracket is defined by $[\Phi, \mathcal{F}] = \partial_\psi \Phi \partial_\alpha \mathcal{F} - \partial_\alpha \Phi \partial_\psi \mathcal{F}$, which also readily takes the form of the divergence of a flux, $[\Phi, \mathcal{F}] = \partial_\alpha \left(\partial_\psi (\Phi) \mathcal{F}\right) + \partial_\psi \left(-\partial_\alpha (\Phi) \mathcal{F}\right)$. The initial condition is defined as an $\alpha$-dependent perturbation to an initial equilibrium distribution function:

$$\mathcal{F}(t = 0) = \mathcal{F}_{eq}(\psi, \kappa, E)(1 + \varepsilon \zeta(\alpha, \psi))$$  \hspace{1cm} (4)

where $\mathcal{F}_{eq}(\psi, \kappa, E)$ is an equilibrium solution at $\Phi = 0$, since it does not depend on $\alpha$ and where $\varepsilon$ is a small parameter that sets the magnitude of the initial perturbation $\zeta(\alpha, \psi)$, a random function, of the equilibrium solution. Unless specified, $\mathcal{F}_{eq}$ is a non-shifted Maxwellian with density and temperature depending on $\psi$ in such a way that this steady state is unstable. In particular, the temperature ranges from $T_1$ at $\psi = 1$, $\mathcal{F}_{eq}(\psi = 1, \kappa, E) = F_1$, to $T_0$ at $\psi = 0$, $\mathcal{F}_{eq}(\psi = 0, \kappa, E) = F_0$, with constant temperature e-folding length $L_T$. Conversely the density profile is usually flat. In the 3D version of TERESA, no dependence on $\kappa$ is taken into account. The perturbation yields a modification of the density of trapped orbits:

$$\delta_n(\alpha, \psi) = \varepsilon \int_{0}^{1} J_\kappa d\kappa \int_{0}^{+\infty} J_E dE \mathcal{F}_{eq}(\psi, \kappa, E) \zeta(\alpha, \psi)$$  \hspace{1cm} (5)

where $J_\kappa$ and $J_E$ are the Jacobians associated to the change of coordinates.

Boundary conditions of eq.(3) are Dirichlet on the cold side $\mathcal{F}(\psi = 1) = F_1$ and Dirichlet on the hot side $\mathcal{F}(\psi = 0) = F_0$ for the Thermal Bath (TB) driven case or Neumann conditions $\partial_\psi \mathcal{F}(\psi = 0) = 0$ for the Flux Driven (FD) case [23]. The diffusion operator $D(\mathcal{F}) = \partial_\psi (D(\psi) \partial_\psi \mathcal{F})$ with $D(\psi)$ different from zero only close to $\psi$ boundaries, Fig.(1), is a term which locally changes the nature of transport, converting turbulent convection into diffusive transport. In these buffer regions the Trapped Ion Modes are stable (large damping) so
that a constant electric potential can be imposed at the boundaries. Furthermore, these buffer regions ensure a thermal contact with the thermal bath conditions with a heat flux:

$$Q_D = -D(\psi)\partial_\psi \int_0^{1^-} J_\kappa d\kappa \int_0^{+\infty} J_E dE \mathcal{F}$$

that depends on the radial gradient of the distribution function. Conversely, in the FD case this flux is vanishing at $\psi = 0$ due to the symmetry boundary condition.

In the Thermal Bath driven case, the control parameter is the temperature difference $\Delta T$ between the two boundaries. The heat flux is not constrained and characterises the confinement regime. In the Flux Driven case, the mean heat flux is imposed and the temperature profile then characterises the confinement regime. The volumetric source term $S(\mathcal{F}) = S_0 S_\psi(\psi)L_1(E)e^{-E}$ is a heating operator that transfers sub-thermal particles into supra-thermal particles. Using the first Laguerre polynomial $L_1(E)$ in $E$, energy is injected without particles [22, 23]. At each time step, the magnitude of the source $S_0$ is monitored to avoid generating negative values of the distribution function. This corresponds to some form of saturation of the heating efficiency. Two $\psi$-profiles $S_\psi(\psi)$ of the source term have been used in FD simulations, Fig.(1): central heating with a gaussian shape centred on $\psi = 0$ in the core buffer region, and off-axis heating with a gaussian shape shifted radially out of the core buffer region. The initial condition of the distribution function is an $\alpha$-independent function (hence an equilibrium function) with given exponential temperature profile, therefore constant gradient length $L_T$, perturbed by a random function $\zeta(\alpha, \psi)$ of amplitude $\varepsilon$.

The Maxwell-Gauss equation, in the quasineutral asymptotic limit, is used to compute the electric potential. The finite size effects yield the non-isotropic operator $\Delta_{\text{FSE}}$ [17].

$$C_e(\phi - \lambda \langle \phi \rangle_\alpha) - C_i \Delta_{\text{FSE}} \phi = \frac{1}{\sqrt{\pi n_{eq}}} \int_0^{1^-} J_\kappa d\kappa \int_0^{+\infty} J_E dE \ J \mathcal{F} - 1 \quad (7)$$

$$\Delta_{\text{FSE}} = \rho_0^2 \frac{\partial^2}{\partial \alpha^2} + 6 \rho_0^2 \frac{\partial^2}{\partial \psi^2} \quad (8)$$

$C_e$ and $C_i$ are constant depending on $T_i$ and $T_e$ the ion and electron temperatures as well as on the fraction of trapped particles. The standard adiabatic electron response takes the form

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Profiles of the diffusion operator (red), core heating within the core buffer region (blue) and off-axis heating outside the buffer region (green).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Cancelled modes of the adiabatic response: 5D with gyro-averaging only the $(n = 0, m = 0)$ mode is removed (red point) and 4D with gyro and bounce averages where one cannot separate the poloidal modes at given toroidal mode, red plus orange points for $n = 0$.}
\end{figure}
\[ \delta n_e/n_0 = e(\phi - \langle \phi \rangle_{\theta,\varphi})/T_e \]

where the average is a magnetic flux surface average, hence on both the poloidal angle \( \theta \) and the toroidal angle \( \varphi \). This enforces that \( \langle \delta n_e \rangle_{\theta,\varphi} = 0 \). In Fourier space, it corresponds to \( \tilde{n}_{m,n} = 0 \) for \( (m,n) = (0,0) \), Fig.(2). Given the electric potential, the only average available after the reduction is \( \langle \phi \rangle_{\alpha} \). For the sake of simplicity, let us assume that the \( \alpha \)-average corresponds to the \( \varphi \)-average. The electron response in eq.(7) is then such that \( \langle \delta n_e \rangle_{\varphi} \approx C_e(1 - \lambda) \langle \delta \phi \rangle_{\varphi} \). Hence for \( \lambda = 1 \), \( \langle \delta n_e \rangle_{\varphi} = 0 \). Such a constraint is much stronger than the actual one for adiabatic electrons. In Fourier space it corresponds to \( \tilde{n}_{m,n} = 0 \) for \( n = 0, \forall m \), Fig.(2). Conversely, the other limit \( \lambda = 0 \) removes altogether the cancellation of the electron density fluctuation averaged on a magnetic surface. Since the model does not allow one to properly compute the adiabatic electron response, we introduce \( \lambda \) as a free parameter, \( 0 \leq \lambda \leq 1 \), and assume the form chosen in eq.(7). Boundary conditions for the quasineutrality equation are set with \( \Phi(\psi = 0) = 0 \) which cancels the radial component of the electric drift velocity, \( \partial_\alpha \Phi(\psi = 1) = 0 \) and \( \partial_\alpha \Phi(\psi = 1) = 0 \).

The numerical implementation of TERESA 4D is similar to that of GYSELA [24]. It is a semi-Lagrangian code using spectral method on \( \alpha \) [10]. Several convergence studies have been performed to ensure momentum and energy conservation. The parallelisation scheme is based on MPI and OpenMP with high efficiency from 10 to 10k-cores. A recent upgrade of TERESA 4D allows one to perform simulations using several ion species and electrons [25].

With respect to previously published papers on Trapped Ion turbulence in the kinetic collisionless regime, the present work is based on several novel aspects. The code has been completely rewritten to meet the high standard required by massively parallel computing, as indicated above. As in [22, 23] the system is flux driven and thus departs significantly from the initial work [17]. However, a major difference has been introduced. Indeed, the volumetric source term implemented in the code is an operator taking subthermal particles and changing them into suprathermal particles at constant particle number. The possibility of maintaining a constant heating power then relies on the transport properties of the system that must replenish the subthermal particle population in the heating volume via transport mechanisms. When the latter are too weak, the heating operator generates negative values of the distribution function. This issue was identified in [22, 23] and taken into account by assuming that the code only addressed part of the trapped population, as would a \( \delta f \) code. In fact, this working assumption is not appropriate. We have therefore introduced a saturation mechanism in the heating efficiency as the number of available subthermal particles tends to zero. This prevents negative values of the distribution function at the cost of a time dependent heating power. This more complex but correct approach to the heating mechanism is used to address the critical issue of self-generated transport barriers [6].

Regarding the physics at hand, it is interesting to note that this model is in fact a kinetic model of the Rayleigh-Bénard instability [26] in the 2D approach. Indeed the equations of the latter, in a fluid approach, combine a vorticity equation for the stream function and a heat transport equation. The coupling between the two equations is governed by the fluid convection of heat on the one hand and on the g-term, a buoyancy force due to the fluid expansion with temperature on the other hand. At lowest order this force is proportional to gravity and the temperature variation. The Trapped Ion turbulence model can be changed by computing the evolution equation of the integral of the distribution function \( \mathcal{F} \) using the Vlasov equation (3). Then using the quasineutrality condition, eq.(7), one obtains a vorticity equation for the electric potential. In this equation the drift term proportional to \( \Omega_d E \) plays the same role as the g-term in the fluid vorticity equation. One can then draw an equivalence between the stream function and the electric potential. Finally, the Vlasov equation is similar to the heat transport equation of neutral fluids, the distribution function replacing the temperature variation. In the
latter, the differences stem from the molecular collision closure of the fluid equation that yields
the heat diffusion term. This term competes with heat convection and leads to a threshold for
the Rayleigh-Bénard instability. Regarding the vorticity equation, one finds another difference
which corresponds to the electron response to the electric potential fluctuations. This term
introduces a volumetric loss term. When this term is not null it will tend to stabilise the larger
modes and modify the inverse cascade properties.

3. Thermal Bath regimes: Streamers versus Zonal Flows
Zonal Flows [27] and Streamers [28] are two different kinds of structures present in the electric
potential that govern large scale transport. Zonal Flows are aligned in the \( \alpha \)-direction and thus
convect in the direction perpendicularly to the radial one. They do not yield radial transport but
generate shear [27]. Conversely streamers are aligned in the \( \psi \) direction that directly connect the
source to the sink regions of the plasma [28]. The control parameter \( \lambda \) [22] governs the transition
from zonal flow dominated transport, \( \lambda = 1 \), to streamer dominated transport, \( \lambda = 0 \), see Fig.(3).

Figure 3. Large scale patterns of the temperature for different values of \( \lambda \). Left hand side:
streamer regime with \( \lambda = 0 \). Right hand side zonal flow regime with \( \lambda = 1 \).

Let us monitor the state of the plasma by the \( \alpha \)-averaged heat flux \( < Q_D >_\alpha \), eq.(6), the
zonal flow amplitude measured by \( ZF = \int_0^1 |\partial_\psi < \Phi >_\alpha | d\psi \) and the zonal flow shear magnitude
\( ZF' \) defined by \( ZF' = \int_0^1 |\partial^2_\psi < \Phi >_\alpha | d\psi \). One then finds that the simulation evolution in the
TB case is organised in 5 characteristic steps, Fig.(4):

- a) Linear phase. Streamer structures develop according to the most unstable mode \( k_\alpha \). The
heat flux increases exponentially.
- b) Nonlinear saturation phase. The heat flux reaches its maximum value just before the
development of zonal flows, increase of \( ZF \).
- c) Relaxation phase. Fast dynamics, apparently chaotic, take place with strong zonal flow
and streamer interplay. This step is the most demanding from a numerical point of view
because of the fast dynamics and sharp gradients in both \( (\alpha, \psi) \) directions.
- d) Bifurcation phase. As the fast dynamics slow down, the system evolves either towards a
zonal flow or a streamer dominated case. In some simulations, a slowly decaying streamer
regime governs a long transient before switching finally towards a zonal flow dominated
regime.
- e) final streamer dominated phase or e') final zonal flow dominated phase. In the latter,
the heat flux is quenched while in the former state the heat flux is strong. In both cases
the variation of \( < Q_D >_\alpha \), \( ZF \) and \( ZF' \) are small compared to the prior phases. For the
streamer dominated state some reorganisation in the number of vortices can take place.
Figure 4. Phase portraits in the planes heat flux, Zonal Flow ($<Q_D>_{\alpha},ZF$) and Zonal Flow, Zonal Flow shear ($ZF,ZF'$) of the 5 chronological steps of typical simulations. Blue and red: $\lambda = 0$, green and cyan: $\lambda = 1$. Blue and green: large temperature difference $\Delta T$, red and cyan: small temperature difference $\Delta T$.

While the transition from the linear state a) to the nonlinear quasi-steady state e) is chaotic, it is possible that some memory effects take place. This aspect is related to the streamer pattern and the $k_\alpha$ selection which stems both from the initial noise, which defines the spectrum of injected $k_\alpha$-modes, and the $k_\alpha$-dependent growth rate during the linear phase. After that phase, the energy cost to reorganise the $k_\alpha$ pattern can be too large to take place, so that different final states can be reached for otherwise comparable initial conditions.

4. Phase portrait of the TERESA simulation domain

4.1. The numerical investigation

The equations, eqs. (3, 7), initial condition, eq. (5) and boundary conditions determine the parameter space associated to the TERESA toy system. This must be completed by the finite size effects that appear in eq. (8) and the averaging operators $\mathcal{J}$, including the relation $\Phi = \mathcal{J}\phi$. The reduction process that is followed to obtain the TERESA system has led us to introduce several arbitrary control parameters such as $\lambda$, that can play a role in the behaviour of the system and lead to differences with respect to the reference case discussed in Section 3.

We aim at describing the properties of the non-linear state, in particular possible interplay between zonal flow pattern and streamer pattern [29], hence between state e) and state e'). The exploration of the impact of 7 control parameters is based on a brute force analysis with some 650 simulations using 85000 CPU.h. For each selected control parameter, between 3 and 10 values distributed in geometric progressions have been tested over specified ranges, see table 1 and table 2. Scans in 1D and 2D have been performed in order to recover a 7D reconstruction of the pattern behaviour. Such an exploration is now possible given TERESA numerical and computational optimisation and the increasing performance of supercomputers.

From a numerical point of view, a wide range of values, table 3, have been used to ensure that numerical error on the energy conservation of the distribution function would not exceed 1%. The number of points in the $(\alpha,\psi)$ space, is $N_\alpha$ and $N_\psi$ respectively. Several runs with different resolution have been performed to check the numerical convergence. A similar check has been performed for the time step $\Delta t$ and regarding the energy $E$, depending on the maximum value $E_{max}$ and the number of energy values $N_E$, which determine the precision of the computation.
of the trapped orbit density in eq.(7). In the present analysis the trapping parameter is kept constant.

| Parameters | $N_a$ | $N_\psi$ | $N_\kappa$ | $N_E$ | $E_{max}$ | $\Delta t$ |
|------------|-------|----------|------------|-------|-----------|----------|
| Ref case   | 257   | 129      | 1          | 48    | 25        | $[10^{-5}:10^{-2}]$ |
| Interval   | $[129:4097]$ | $[129:257]$ | $[1]$     | $[48:144]$ | $[25:50]$ | $[10^{-8}:10^{-2}]$ |

Table 3. Range of values used in the numerical tests, and retained values of the parameters that control the numerical precision of TERESEA.

4.2. Changing the response of the adiabatic electrons

Figure 5. Density, temperature and pressure of trapped orbits for different values of $\lambda$.

In Section 3, the two extreme values of $\lambda$ are used, namely 0 and 1. Extending this analysis to intermediate values allows one to modify the step like pattern of the profiles. The trapped orbit density and pressure are computed as the zero and first energy moment of the distribution function $F$, the ratio of these moments defining the temperature, see Fig.(5). First, one can observe non-monotonic profiles with temperature inversion which underlines the very efficient insulation property of the zonal flow shear layers, red curve for $\lambda = 0.2025$. A small variation of $\lambda$ governs a strong change of the profile, see $\lambda = 0.2$, which exhibits a monotonic temperature...
profile, Fig.(5). Furthermore, one also finds a variation in the number of steps, essentially leading to a doubling of the number steps as $\lambda$ increases. Odd numbers of steps have not been observed: this is likely governed by symmetry and momentum conservation. The control parameter $C_i$ and $C_e$ eq.(7) can play a similar role as the parameter $\lambda$ or modify the finite size effects that are discussed in the following section. However, their impact is not addressed in the present study.

4.3. Finite Size Effects
The number of degrees of freedom is determined by $\rho_0$ and $\delta_0$. The first one determines the number of structures in the $\alpha$ direction while the second constrains the $\psi$ direction. Comparing TERESA simulations to more standard gyrokinetic simulations one can state that $\delta_0$ is equivalent to the standard $\rho_*$ parameter while $\delta_0/\rho_0$ is a measure of shaping properties, in particular the elongation [30]. Small values of $\rho_0$ and $\delta_0$ yield a large number of structures within the simulation box. Note that the radial size of the simulation region is used to normalise $\psi$ so that: $L_\alpha \times L_\psi = [0 : 2\pi] \times [0 : 1]$.

![Figure 6. Temperature of trapped orbits for $2\pi/\rho_0 = 20500$ during phase e'), factor 10 zoom on the right hand side.](image)

Since $\rho_0$ controls the number of modes in the $\alpha$-direction, it determines the number of streamers Fig.(6) and consequently the heat flux from one thermal bath to the other. However the threshold value for switching from streamers to zonal flows does not appear to depend on $\rho_0$, Fig.(7 (top row)). The parameter $\delta_0$ modifies the zonal flow pattern. It yields a comparable effect as the parameter $\lambda$ on the transition between streamer and zonal flow dominated states, Fig.(7(bottom row)). This result is in line with the fact that both parameters play similar role regarding the zonal flow, mode 0, in the quasi-neutrality equation.

4.4. Heat source effect on transport
As readily expected for the present Rayleigh-Bénard setting, the temperature difference $\Delta T$ governs the instability threshold. Otherwise it appears to essentially change the steps height, and consequently the gradients with standard consequences on the numerical robustness of the simulation. Similarly, $\Omega_d$ is a key parameter in determining the critical temperature difference $\Delta T_c \propto \Omega_d/(1 + \mathcal{O}(\delta_0^2) + \mathcal{O}(\rho_0^2))$. This is readily expected since, the energy normalisation, and consequently the temperature, are equivalent control parameters in the Vlasov equation.

The last part of this study is based on two series of simulations exploring the FD approach for on axis and off-axis heating, see Fig.(1). For on-axis heating, the diffusion buffer region ensures the contact between the source and the turbulent plasma and preserves the symmetry in the $\alpha$-direction. This heating leads to the zonal flow dominated regime. The transport is quenched and no steady state regime can be achieved. Similar observations have been made in ELM-free H-modes [31, 32, 33]. In the streamer dominated regime, with off-axis heating, standard large scale convection can be achieved. However, as in the ELM-free experiments, stable conditions
Figure 7. Heat flux $< Q_D >_\alpha$ (left column) and $ZF$ (right column) depending on $\lambda$ and $\rho_0$ (top row) or $\delta_0$ (bottom row). Colour and radius of circles are proportional to the magnitude of $< Q_D >_\alpha$ or $ZF$ respectively. Black stars locate simulations that have been analysed.

are only achieved with a power step-down procedure [33].

Figure 8. Pressure of trapped particle orbits during a barrier relaxation: before (top left), relaxation onset (top right), relaxation (bottom left) and barrier restoration (bottom right).

Off-axis heating is observed to have less efficient coupling to the plasma. Indeed, on-axis
heating is combined to the strong diffusion in the core buffer region that replenishes efficiently the
distribution function, hence preventing negative values. In the off-axis case, turbulent transport
is too weak to prevent the self regulation of the magnitude of the heating source to come into
play. The coupling of the heating source is then switched-off to avoid generating negative values
in the distribution function.

The interesting point with these TERESA simulations is to understand why the corrugation
results are particularly stiff, generally preventing steady state regimes in Flux Driven
simulations. Indeed, the corrugation dynamics observed in GYSELA [9] do not lead to such
strong barriers [3]. Furthermore, regarding simulations with barriers, secondary instabilities
occur leading to Barrier Relaxation Modes, as in GYSELA kinetic simulations [5, 4] and in
edge fluid simulations [34, 35]. The case with off-axis heating is still under investigation to
understand what process unlocks the transport barrier, leading to quasi-periodic relaxation
events, Fig.(8). Although the overall heat transport remains small compared to the streamer
regime, these relaxation events contribute to sufficient energy transfer to reach steady state
regimes. As observed in the fluid simulations, a Dupree like mechanism, with competition
between the growth of instabilities and large damping processes [35] can delay the relaxation
mechanism so that a weak energy source is mandatory to allow the perturbation to grow and
trigger the relaxation before too large gradients lock the barrier. This mechanism would then
be reminiscent of the power step-down experiments [33]. However, further analysis is required
to identify the actual mechanism at hand.

5. Discussion and Conclusion

The turbulence governed by trapped ion instabilities provides a minimum kinetic model when
averaging out two fast angles, that of the gyro-motion and that of the bounce-motion. In the
limit of deeply trapped ions, when the trapping parameter is nearly constant, one can further
reduce the model to 3D: 2D space conjugate variables and the energy –setting the trapping
parameter to a single value. This interchange instability model is akin to a kinetic Rayleigh
Bénard model. It provides a toy model, reduced to 3D, which is implemented in the TERESA
code.

In the reduction procedure, several free parameters are introduced in the model leading to a
large parameter space where turbulence can be investigated. The zonal flow streamer interaction
is addressed by using the control parameter that determines the response of the adiabatic elec-
trons. One then finds that the self-organisation of the system leads either to streamer dominated
solutions, with large heat flux, and thus low confinement, or to zonal flow dominated solutions,
where the radial transport is locked. In the latter limit, staircase temperature profiles develop
with marked corrugations. Tuning the control parameter allows one to change the number of
steps in the temperature profiles and even to achieve temperature reversal.

A brute force exploration of the parameter space of the model has allowed one to draw a
first series of phase portraits of the system. On the overall, the system is observed to be rather
stiff, switching between streamer and zonal flow states, sometimes after very long transients,
but then remaining in steady state. Quasiperiodic relaxation cycles between these two states,
as characterised by predator-prey models, is most often not observed. It appears that off-axis
heating seems to allow the onset of a secondary instability and subsequent relaxation events.
Further analysis of the model is under-way to complete the description of the parameter space.
This reduced model, with versatile and efficient numerics, is a powerful tool to investigate the
physics at play in more relevant and complete models of plasma turbulence. Furthermore, it
provides a means to investigate in great detail the kinetic features, an aspect that is most often
overlooked.
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