Few-Body Physics: (Some) Recent Developments

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Abstract. Understanding hadron structure and dynamics based on quantum chromodynamics is one of the central challenges of contemporary nuclear physics. In this paper, some recent efforts towards this goal within the framework of effective field theory are reviewed. The main focus is on the theory of nuclear forces and few-nucleon systems.

1. Introduction

The nuclear force has been at the heart of nuclear physics during the past eight decades. Traditionally, one of the major goals of nuclear physics is to understand the properties of nuclei and nuclear matter in terms of the interaction between the nucleons which requires the solution of the nuclear many-body problem. Few-body systems play a special role since the small number of particles allows for an accurate numerical solution of the Schrödinger equation without introducing approximations unavoidable when dealing with more complex systems such as heavy nuclei. Vastly increased computational resources coupled with algorithm developments and new ideas on optimizing the efficiency of calculations by eliminating some redundant (at low energy) properties of the nucleon-nucleon potentials (such as e.g. the short-range repulsive core) have allowed for considerable progress in making heavier nuclei accessible to microscopic ab initio calculations, see [1] for recent review articles.

The second major challenge of modern nuclear physics is the quantitative understanding of hadronic interactions from Quantum Chromodynamics. Utilizing a formulation of QCD on a discrete four-dimensional space-time lattice, the so-called lattice QCD, allows one to evaluate Green functions numerically using Monte Carlo techniques. This method has proved to be very successful e.g. in the description of hadron spectra [2]. On the other hand, lattice QCD simulations of few-baryon observables are much more challenging and currently far from reaching the level of accuracy of simulations in the single-hadron sector. Another interesting development was initiated recently by Aoki et al. who use lattice QCD to extract nuclear potentials, see [3] for a recent review article. Chiral perturbation theory is another systematic approach to low-energy hadron dynamics which exploits the symmetries and symmetry-breaking pattern of QCD. In particular, QCD with two flavors of the $u$- and $d$-quarks and, to a less extent, with three flavors of the $u$-, $d$- and $s$-quarks exhibits an approximate chiral symmetry which is spontaneously broken down to its vector subgroup. This symmetry/symmetry-breaking pattern manifest themselves in the hadron spectrum leading, in particular, to a natural explanation of the very small (compared to other hadrons) observed masses of pions which are identified with the corresponding Goldstone bosons. Moreover, the Goldstone boson nature of the pions implies that they interact weakly at low energy. These two features are at the heart of chiral perturbation theory, the low-energy
2. Nuclear forces from chiral EFT

Nuclear forces can be derived from the most general effective chiral Lagrangian for pions and nucleons by eliminating the pionic degrees of freedom. This can be achieved employing various techniques and making use of the chiral expansion, i.e. perturbative expansion in powers of $Q/\Lambda_{\chi}$ with $Q$ being the soft scale related to external three-momenta of the nucleons and the pion mass and $\Lambda_{\chi}$ the pertinent hard scale of the order of the $\rho$-meson mass. Presently, the two-nucleon force is worked out and employed in few- and many-body calculations up to next-to-next-to-next-to-leading order (N$^3$LO) [8,9]. As visualized in Fig. 1, it involves the contributions from $1\pi$, $2\pi$, and $3\pi$-exchange which are parameter-free since the corresponding low-energy constants (LECs) are known from $\pi N$ scattering as well as short-range contact interactions with up to four derivatives whose strengths have been adjusted to NN low-energy data. Relativistic
Figure 2. Neutron-proton differential cross section (left panel) and analyzing power (right panel) at $E_{\text{lab}} = 50$ MeV based on the chiral N$^3$LO NN potential [9], Nijm I,II and Reid 93 potentials [11], CD-Bonn 2000 [12], covariant spectator theory of Ref. [13] in comparison with the data and the results of Nijmegen PWA [14]. References to data can be found in [9].

and isospin-violating corrections have also been studied in great detail within the framework of chiral EFT, see [6, 7] and references therein. For a reasonably chosen values of the cutoff in the Lippmann-Schwinger equation, the resulting values of the LECs can be well understood in terms of resonance saturation [10]. Both available chiral N$^3$LO NN potentials allow for an accurate description of neutron-proton phase shifts and observables up to laboratory energy of about 200 MeV, see Fig. 2 for two representative examples.

A still open issue in the two-nucleon sector is related to the consistency of the Weinberg power counting for short-range operators. The meaning of the non-perturbative renormalization of the Schrödinger equation in the context of chiral EFT and the implications on the power counting are currently under discussion, see [15–20] and references therein for a sample of different views on this issue.

3. Nucleon-deuteron scattering and light nuclei

While two-nucleon potentials are sufficient to describe the bulk of nuclear observables, three- (3NF) and, in principle, also more-nucleon forces have to be taken into account to achieve a quantitative description of few-nucleon observables. In fact, already 70 years ago Primakoff and Holstein pointed out that “... replacement of field interactions by two-body action-at-a-distance potentials is a poor approximation in nuclear physics” [21]. The emergence of two- and, more generally, many-nucleon forces follows inevitably from QCD due to residual color interaction between the elementary constituents of the colorless nucleons. Three- and more-nucleon forces also appear naturally in chiral EFT, see Fig. 1. Moreover, chiral power counting provides a qualitative explanation of the observed hierarchy of nuclear forces $V_{2N} \gg V_{3N} \gg V_{4N}$... with three-/four-nucleon forces starting to contribute at next-to-next-to-leading/next-to-next-to-leading order in the chiral expansion. The first non-vanishing contributions to the 3NF at next-to-next-to-leading order (N$^3$LO) result from $2\pi$-exchange, $1\pi$-exchange with 2N contact interaction and the purely short-range 3N contact term [22, 23]. While the $2\pi$-exchange contribution is parameter-free, the two other topologies depend on the unknown LECs $D$ and $E$ which can be determined e.g. from the $^3$H binding energy, the nd doublet scattering length [23], the $^4$He binding energy [24], the properties of light nuclei [25] or triton beta decay [26]. In addition, the $D$-term also figures importantly in pion production [27] and photo-production...
The results for three-nucleon scattering observables and spectra of light nuclei are available up to N\textsuperscript{2}LO. In particular, applications of chiral EFT to nucleon-deuteron scattering show a reasonable agreement between the theory and the data at low energy [6, 7, 29], see Fig. 3 for some examples. The well-known A\textsubscript{y} puzzle related to the underprediction of the nucleon vector analyzing power in low-energy elastic Nd scattering is not resolved at N\textsuperscript{2}LO. More precisely, the theoretical uncertainty at this order is comparable to the deviation from the data when using modern phenomenological two- and three-nucleon force models. Calculations based on N\textsuperscript{3}LO chiral NN potential of Ref. [8] in a combination with N\textsuperscript{2}LO 3NF show, however, a similar deviations from the data as in the case of phenomenological potentials [33]. A complete N\textsuperscript{3}LO analysis including the corresponding 3NFs is not yet available. Another puzzle is observed in the so-called symmetric space-star (SST) and the related constant relative-energy (SCRE) configurations in the deuteron breakup reaction. In these configurations the angles between the outgoing nucleons in the center-of-mass system (CMS) are 120°. The SCRE geometry is characterized by the angle \( \alpha \) between the beam axis and the plane in the CMS spanned by the outgoing nucleons. A particular case of \( \alpha = 90° \) corresponds to the SST geometry. As shown in the right panel of Fig. 3, there are large deviations between the theory and the data for the cross section measured in Köln [30] for \( \alpha = 56° \). The discrepancy turns out to increase with increasing values of \( \alpha \). The included 3NFs have little effect on the cross section while the effect of the Coulomb interaction is significant but removes only a part of the discrepancy.

The spectra of light nuclei have also been explored within chiral EFT. In particular, the \( \alpha \)-particle binding binding energy is found to be \( B = 24.4 \ldots 28.8 \) MeV (\( B = 27.8 \ldots 29.6 \) MeV) at next-to-leading order (N\textsuperscript{2}LO) [6] to be compared with the experimental value \( B_{\text{exp}} = 28.3 \) MeV. The properties of S-shell and P-shell nuclei with \( A \leq 13 \) have been studied within the no-core-shell-model framework, see Ref. [34] for a recent review article. The inclusion of the chiral three-nucleon force is shown to yield an improved description of the data compared to the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Differential cross section (in mb/sr) for elastic \textit{N}d scattering at 10 MeV (left panel) and 65 MeV (middle panel) at next-to-leading order (light-shaded bands) and N\textsuperscript{2}LO (dark-shaded bands). References to data can be found in [23]. Right panel shows \textit{pd} breakup cross section (in mb MeV\(^{-1}\) sr\(^{-2}\)) along the kinematical locus \( S \) at 19 MeV nucleon energy in the SCRE configuration with \( \alpha = 56° \) [30]. Dashed and dashed-dotted lines are results based on the CD Bonn 2000 2NF [12] combined with the TM99 3NF [31] and the coupled channel calculation including the explicit \( \Delta \) and the Coulomb interaction [32], respectively.}
\end{figure}
calculations based only on the two-nucleon forces.

The corrections to the 3NF at N^3LO are currently being worked out and implemented in the scattering calculations [35, 36]. An important step along these lines was done in Ref. [37] where a new method for partial wave decomposition of the 3NF is suggested. In addition to the 3NF corrections, four-nucleon force (4NF) starts to contribute at N^3LO in the chiral expansion. The parameter-free expressions for the leading 4NF can be found in Ref. [38]. The expectation values of the individual 4NF contributions obtained in the pioneering study of Ref. [39, 40] are typically of the order of few hundreds of keV, which is in line with estimations based on dimensional arguments.

4. Nuclear lattice simulations

As an alternative to lattice QCD, the properties of few- and many-nucleon systems can be simulated on the lattice based on chiral EFT. By using hadronic degrees of freedom it is possible to probe large volumes and greater numbers of nucleons than in lattice QCD. This strategy has been followed by several groups, see e.g. Refs. [41–43] and a review article [44]. Clearly, the method is only applicable at low energies where chiral EFT is expected to converge. The effective Lagrangian for pions and nucleons treated as point-like particles is formulated on a spacetime lattice and the path integral is evaluated by Monte Carlo sampling. The main object in such calculations is the correlation function for \( A \) nucleons in the Euclidean space-time defined by

\[
Z_A(t) = \langle \Psi_A \exp(-tH) | \Psi_A \rangle .
\]

Here, the states \( |\Psi_A\rangle \) refer to the Slater determinants for \( A \) free nucleons, \( H \) is the Hamiltonian of the system and \( t \) the Euclidean time. The ground state energy of the \( A \)-nucleon system can be derived from the asymptotic behavior of the correlation function for large \( t \),

\[
E_A^0 = -\lim_{t \to \infty} \frac{d}{dt} \ln Z_A(t).
\]

Expectation value of a normal ordered operator \( \mathcal{O} \) can be derived in a similar way by

\[
\langle \Psi_A^0 | \mathcal{O} | \Psi_A^0 \rangle = \lim_{t \to \infty} \frac{Z_A^\mathcal{O}(t)}{Z_A(t)}, \quad Z_A^\mathcal{O}(t) = \langle \Psi_A | \exp(-tH/2) \mathcal{O} \exp(-tH/2) | \Psi_A \rangle,
\]

where the states \( |\Psi_A^0\rangle \) denote the ground states of \( A \)-nucleons system. The correlation functions \( Z_A(t) \) and \( Z_A^\mathcal{O}(t) \) can be calculated on the lattice utilizing the Hubbard-Stratonovich transformation in order to get rid of terms quartic in the nucleon fields (at the expense of introducing interactions with auxiliary fields) and using standard Monte Carlo techniques, see [43, 44] for more details.

The above sketched program is carried out for \( A = 2, 3, 4 \) in [43] at leading order (LO) in chiral EFT incorporating the physics of instantaneous one-pion exchange and the LO 2N contact interactions. The corresponding LECs are determined from the deuteron binding energy and the \( ^1S_0 \) scattering length using Lüscher’s formula [45]. To avoid a multi-particle clustering instability at coarse lattice spacing, a Gaussian smearing of contact interactions is introduced. The resulting triton (\( \alpha \)-particle) binding energy agrees with the experimental value to within 5% (25%). These studies were extended to next-to-next-to-leading order (NLO) in Ref. [47] and N^2LO in [48], where the corresponding 3NFs were taken into account. The LECs accompanying the subleading NN contact interactions were determined by fitting the low-energy Nijmegen phase shifts using the spherical wall method [46]. Recently, we computed the ground state energies of \(^3\)H, \(^3\)He, \(^4\)He, \(^6\)Li and \(^{12}\)C at N^2LO including the Coulomb interaction and the leading isospin-breaking effects [49,50], see Fig. 4. Notice that in these simulations we have also taken into account the leading short-range four-nucleon interaction (which contributes at a higher
order) whose strength is adjusted to reproduce the α-particle binding energy. The calculated ground state energy of $^6$Li, $-32.9(9)$ MeV is in a good agreement with the experimental value of $-32.0$ MeV. A slight overbinding for the ground state energy of $^{12}$C ($-99(2)$ MeV to be compared with the experimental value of $-92.2$ MeV) might be due to the finite lattice size (we used a periodic box of a length 13.8 fm). This issue is under investigation. For more details the reader is referred to [50]. Last but not least, the results for the ground state energy of dilute neutron matter at NLO can be found in Ref. [51].

5. Chiral EFT with $\Delta(1232)$

All theoretical developments described so far are based on the chiral EFT formulated in terms of the asymptotically observed ground state fields, the pions and nucleons chirally coupled to external sources. The excitation of baryon and meson resonances is encoded in LECs of the pion-nucleon interaction beyond leading order. Such implicit treatment of the $\Delta(1232)$, the close-by resonance with the excitation energy of only $m_\Delta - m_N = 293$ MeV leads to unnaturally large values of certain LECs which can, potentially, spoil the convergence of the chiral expansion. Indeed, the chiral expansion of the two-nucleon force exhibits a somewhat unnatural convergence pattern. For example, by far the most important $2\pi$-exchange contribution arises at $N^2$LO [52] from formally subleading diagrams. The corresponding attractive central potential turns out to be an order of magnitude stronger than the (nominally) dominant $2\pi$-exchange contributions at NLO. This unnatural convergence pattern is clearly reflected in two-nucleon scattering observables such as e.g. peripheral phase shifts. As visualized in Fig. 5, the $N^2$LO corrections are much larger than the NLO ones which in the case of the $^3F_4$ phase shift are negligibly small. The origin of the convergence issue can be traced back to the large values of the LECs $c_{3,4}$ accompanying the subleading $\pi\pi N N$ vertices (filled circles in Fig. 1) which are well understood in terms of resonance saturation [53] and are largely driven by the $\Delta(1232)$. One can, therefore, argue that the explicit inclusion of the $\Delta$ will allow to resum a certain class of important contributions and improve the convergence as compared to the delta-less theory, provided a proper power counting scheme such as the small-scale expansion (SSE) [54] is employed, see also [55]. The SSE is a phenomenological extension of chiral perturbation theory in which the small expansion parameter includes external momenta, pion masses and the nucleon-delta

1 Here and in what follows, the error bars are one standard deviation estimates which include both Monte Carlo statistical errors and uncertainties due to extrapolation at large Euclidean time.
mass splitting, i.e. the delta-nucleon mass splitting is treated as \( m_\Delta - m_N \sim M_\pi \) rather than \( m_\Delta - m_N \sim \Lambda_\chi \gg M_\pi \). The improved convergence of the \( \Delta \)-full chiral EFT has been confirmed for \( \pi N \) scattering [56] and in the first few-nucleon studies [57–61]. In particular, the values of the LECs \( c_{3,4} \) obtained from a fit to threshold coefficients of \( \pi N \) scattering at order \( \mathcal{O}(Q^2) \) are of a natural size in the \( \Delta \)-full theory, \( c_3 = -0.8 \text{ GeV}^{-1} \) and \( c_4 = 1.3 \text{ GeV}^{-1} \), and considerably reduced compared to the ones in the \( \Delta \)-less approach: \( c_3 = -3.9 \text{ GeV}^{-1} \) and \( c_4 = 2.9 \text{ GeV}^{-1} \). In addition, the major part of the subleading 2\( \pi \)-exchange contribution is shifted to a lower order (NLO) in the \( \Delta \)-full theory. These two features indeed result in a much better convergence pattern in chiral EFT with explicit \( \Delta \)'s as exemplified for the case of the \( ^3F_3 \) and \( ^3F_4 \) phase shifts in Fig. 5. Notice that since the N\( ^3 \)LO 3NF and 4NF diagrams in the \( \Delta \)-less theory only involve leading pion-nucleon interactions, one expects them to miss sizable contributions due to the \( \Delta \)-isobar similar to e.g. the leading 3\( \pi \)-exchange 2NF [62], see [63] for a recent discussion.

6. Summary and outlook
To summarize, chiral EFT provides an accurate description of low-energy 2N data at N\( ^3 \)LO. The results for >2N systems are presently available at N\( ^2 \)LO. While most of the calculated 3N observables are in a reasonable agreement with the data, the theoretical uncertainty at intermediate energies becomes significant at this order. Extension of these studies to N\( ^3 \)LO is underway. I also discussed recent progress in lattice simulations of few- and many nucleon systems based on chiral EFT. These studies demonstrate that lattice EFT is a promising tool for a quantitative description of light nuclei and nuclear matter. Chiral EFT with explicit \( \Delta \)-isobar degrees of freedom is another important frontier area of research in the field.

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