A Comparison Between Two Shape Parameters Estimators for (Burr-XII) Distribution

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Received 16/6/2019, Accepted 29/12/2019, Published 8/9/2020

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Abstract

This paper deals with defining Burr-XII, and how to obtain its p.d.f., and CDF, since this distribution is one of failure distribution which is compound distribution from two failure models which are Gamma model and weibull model. Some equipment may have many important parts and the probability distributions representing which may be of different types, so found that Burr by its different compound formulas is the best model to be studied, and estimated its parameter to compute the mean time to failure rate. Here Burr-XII rather than other models is consider because it is used to model a wide variety of phenomena including crop prices, household income, option market price distributions, risk and travel time. It has two shape-parameters (α, r) and one scale parameter (λ) which is considered known. So, this paper defines the p.d.f. and CDF and derives its Moments formula about origin, and also derive the Moments estimators of two shapes parameters (α, r) in addition to maximum likelihood estimators as well as percentile estimators, the scale parameter (λ) is not estimated (as it is considered known). The comparison between three methods is done through simulation procedure taking different sample size (n=30, 60, 90) and different sets of initial values for (α, r, λ). It is observed that the moment estimators \( \hat{\alpha}_{mom} \) and \( \hat{\alpha}_{mom} \) are the best estimator with percentage (46%), (42%) respectively compared with other estimators.

Key word: Burr-XII failure model, Maximum likelihood estimator, Moments estimator, Percentile estimator.

Introduction

Twelve different methods of cumulative distribution functions are presented by Burr on the data of the lifetime modeling or the data of the survival (1). It is worthy to mention that there are two types from these twelve methods mentioned above which are considered as the most important methods due to their application in the study of biological, industrial, reliability and life testing, and several industrial and economic experiments, these types are Burr Type XII and Burr Type X (2).

The Burr type XII distribution became a vital research area for many authors and many studies. Evans and Ragab. 1983 (3) present a Bayes that estimates the shape parameter (α) and the reliability function based on type-II censored samples. Saracoglu et al. 2013 (4) progressive type-II right censored samples are used to obtain the maximum likelihood, weighted least squares ,ordinary least squares, and best linear unbiased estimators for the shape parameter α. According to Abuzaid 2015 (5), middle-censoring is considered as a modern general scheme of censoring and studying the analysis of middle-censored data with Burr-XII distribution which is considered one of the most popular and flexible distributions for modeling stochastic events and lifetime for many products. Nasser and et al.2016(6) introduced an adaptive type-II progressive hybrid censoring scheme that is used to obtain the maximum likelihood and Bayesian estimation for the unknown parameters of the Burr type XII distribution and Bayes estimates of the unknown parameters

The objective of this paper is to estimate two parameters (α, r),where scale parameter (λ) is
considered known of the Burr XII distribution by the three different types of estimators Moments, Maximum likelihood as well as percentile estimators. The paper is presented as follows: Section 2, gives an introduction about (Burr-XII), finding this p.d.f. and its cumulative CDF, and discuss the Moments, Maximum likelihood as well as percentile estimators for the two parameters (α, r) ,the scale parameter (λ) is not estimated (considered known). Section 3 focuses on the results and compares between three methods through simulation procedure. Section 4 covers some conclusions from the results.

Theoretical Aspect

The compound p.d.f. of Burr-XII distribution can be obtained by compounding (p.d.f. of Gamma)distribution with (p.d.f. of weibull) distribution. The formula for the probability density function of the Weibull distribution is

\[ f(y) = \beta \alpha y^{\alpha-1} e^{-\beta y^\alpha}, \ y > 0, \ \alpha, \beta > 0 \ldots (1) \]

where, \( \alpha \) is the shape parameter and \( \beta \) is the scale parameter and, the formula for the probability density function of the gamma distribution is

\[ f(\beta) = \frac{\lambda^r}{\Gamma(r)} \beta^{r - 1} e^{-\lambda \beta}, \ \beta > 0, \ \lambda > 0 \ldots (2) \]

where, \( r \) is the shape parameter and \( \lambda \) is the scale parameter since \( y \) be r.v ~ weibull(\( \alpha, \beta \)) and one of its parameter \( \beta \) be r.v ~ Gamma(\( r, \lambda \)) then , \( y \) has a compound density function is (7)

\[ f(y) = \int f(y|\beta).f(\beta) \ d\beta \]

Where, \( f(y|\beta) \) is a conditional density function depending on the parameter \( \beta \)

\[ f(y|\beta, r, \lambda) = \int \frac{\beta^r}{\Gamma(r)} \beta^{r - 1} e^{-\lambda \beta} y^{\alpha-1} e^{-\beta y^\alpha} \ d\beta = \frac{\alpha \lambda^r}{\Gamma(r)} \int_0^{\infty} \beta^{r - 1} e^{-\beta(\lambda + y^\alpha)} \ d\beta = \frac{\alpha \lambda^r}{\Gamma(r)} y^{\alpha-1} \int_0^{\infty} \beta^{r} e^{-\beta(\lambda + y^\alpha)} \ d\beta \]

After some steps,

\[ f(y | \alpha, \lambda, r) = \frac{\alpha r}{\lambda} y^{\alpha-1} \left[ 1 + \left( \frac{y^\alpha}{\lambda} \right) \right]^{-r-1}, \ y > 0 \]

the p.d.f. in equation (4) is (Burr-XII) distribution with (\( \lambda \)) is scale parameter and (\( r, \alpha \)) are shape parameters.

Also, the C.D.F of (Burr-XII) corresponding to p.d.f. in equation (4) is given in equation (5):

\[ F(y | \alpha, r, \lambda) = 1 - \left[ 1 + \frac{y^\alpha}{\lambda} \right]^{-r}, \ y > 0 \]

Moments derivation

The (\( mn \)) moments formula about origin is

\[ \mu_{mn} = E(Y^m) = \int_0^\infty y^m f(y) dy \]

Applying formula

\[ \beta(a, b) = \int_0^{\infty} \frac{z^{a-1}}{(1+z)^a+b} \ dz \]

Assume \( z = \frac{y^\alpha}{\lambda} \)

\[ dy = \frac{1}{\alpha} (z\lambda)^{a-1} \lambda dz \]

\[ \mu_m = \int_0^{\infty} ((z\lambda)^{a})^{m+a-1} \frac{r^{m+1}}{\alpha} [1 + z]^{-r-1} \frac{\lambda}{\alpha} (z\lambda)^{a-1} \ dz \]

Then:

\[ E(Y^m) = \lambda \alpha \beta \left( \frac{m}{\alpha} + 1, r - \frac{m}{\alpha} \right) \]

Then

\[ \text{Mean}= E(Y) = \frac{1}{\alpha} \lambda \alpha \beta \left( \frac{1}{\alpha} + 1, r - \frac{1}{\alpha} \right) \]

and \( E(Y^2) = \frac{2}{\alpha} \lambda \alpha \beta \left( \frac{2}{\alpha} + 1, r - \frac{2}{\alpha} \right) \)

this gives

\[ \text{variance} = S^2 = \frac{2}{\alpha} \lambda \alpha \beta \left( \frac{2}{\alpha} + 1, r - \frac{2}{\alpha} \right) - \left[ \frac{1}{\alpha} \lambda \alpha \beta \left( \frac{1}{\alpha} + 1, r - \frac{1}{\alpha} \right) \right]^2 \]

And from equations:

\[ E(Y^2) = \frac{\sum y_i^2}{n} \]

\[ E(Y) = \frac{\sum y_i}{n} \]

According to given values of \( \lambda \) equation (9) can be solved to obtain \( \hat{\alpha}_{MOM} \) and \( \hat{\beta}_{MOM} \)

Maximum Likelihood Estimator

Maximum likelihood estimation (MLE) is a procedure of finding the value of one or more
parameters of a statistical model given observations, by finding the parameter values that maximize the likelihood of making the observations give the parameters. The maximum likelihood estimator is widely used in practice largely because of its conceptual simplicity.

Now, let \( y_1, y_2, \ldots, y_n \) be a r.s. from p.d.f. in equation (4), then:

\[
L = \prod_{i=1}^{n} f(y_i, \alpha, \lambda, r) = \alpha^n \lambda^{-n} r^n \prod_{i=1}^{n} \left( \frac{y_i}{\lambda} \right)^{\alpha} \left[ 1 + \frac{y_i}{\lambda} \right]^{-r-1}
\]

\log L = n \log r + n \log \alpha - n \log \lambda + (\alpha - 1) \sum_{i=1}^{n} \log y_i

- (r + 1) \sum_{i=1}^{n} \log \left[ 1 + \frac{y_i}{\lambda} \right] = 0

\[
\hat{\alpha}_{MLE} = \frac{n}{\bar{y}} = \frac{n}{\sum_{i=1}^{n} \frac{y_i^\alpha}{\lambda} \log y_i - \sum_{i=1}^{n} \log y_i}
\]

\[
\hat{\lambda}_{MLE} = \frac{\sum_{i=1}^{n} y_i^\alpha \log y_i}{(r+1) \sum_{i=1}^{n} \frac{y_i^\alpha}{\lambda} \log y_i - \sum_{i=1}^{n} \log y_i}
\]

\[
\hat{\lambda}_{MLE} = \left\{ \begin{array}{l}
\frac{\sum_{i=1}^{n} y_i^\alpha \log y_i}{(r+1) \sum_{i=1}^{n} \frac{y_i^\alpha}{\lambda} \log y_i - \sum_{i=1}^{n} \log y_i} \\
1 - \frac{\sum_{i=1}^{n} \frac{y_i^\alpha}{\lambda} \log y_i}{\sum_{i=1}^{n} \frac{y_i^\alpha}{\lambda} \log y_i} \end{array} \right.
\]

(10)

(11)

Since the scale parameter \((\lambda)\) known is considered so \(\frac{\delta \log L}{\delta \lambda}\) do not find and \((\hat{\lambda})\) may be found from mean time to failure according to given values \(\alpha\), \(r\).

**Percentile estimators**

The estimation by this method is obtained from minimizing the total sum squares of difference between cumulative distribution function and its non-Parametric estimator:

\[
\hat{F}(y_i|\alpha, r, \lambda) = \frac{i}{n+1},
\]

\[
T = \sum_{i=1}^{n} \left[ F(y_i|\alpha, r, \lambda) - \hat{F}(y_i|\alpha, r, \lambda) \right]^2
\]

\[
T = \sum_{i=1}^{n} \left[ 1 - \left( \frac{y_i^\alpha}{\lambda} \right)^r - \frac{i}{n+1} \right]^2
\]

(12)

Here the scale parameter \((\lambda)\) is constant and estimating \((\alpha, r)\) by moments and maximum likelihood method and percentile method

Now the percentile estimator obtained is:

\[
\frac{\partial T}{\partial \alpha} = 2 \sum_{i=1}^{n} \left[ 1 - \left( \frac{y_i^\alpha}{\lambda} \right)^r - \frac{i}{n+1} \right] - \frac{1}{\lambda} \log y_i
\]

\[
\frac{\partial T}{\partial \alpha} = 2 \sum_{i=1}^{n} \left[ 1 - \left( \frac{y_i^\alpha}{\lambda} \right)^r - \frac{i}{n+1} \right] \left( \frac{y_i^\alpha}{\lambda} \right) \log(\lambda)
\]

(13)

Solved numerically to obtain \(\hat{\sigma}_{PEC}\).

While the percentile estimator of \(r\) is obtained from:

\[
\frac{\partial T}{\partial r} = 2 \sum_{i=1}^{n} \left[ 1 - \left( \frac{y_i^\alpha}{\lambda} \right)^r - \frac{i}{n+1} \right] \left( \frac{y_i^\alpha}{\lambda} \right) \log(1 + \frac{y_i^\alpha}{\lambda}) - \left( 1 + \frac{y_i^\alpha}{\lambda} \right) \log y_i
\]

\[
\frac{\partial T}{\partial r} = 0 \rightarrow \left[ \sum_{i=1}^{n} \left[ 1 - \left( \frac{y_i^\alpha}{\lambda} \right)^r - \frac{i}{n+1} \right] \right] - \left( \frac{y_i^\alpha}{\lambda} \right) \log(1 + \frac{y_i^\alpha}{\lambda}) = 0
\]

(14)
Solved numerically to $\hat{\eta}_{pec}$

From solving $\frac{\partial r}{\partial a} = 0$, $\frac{\partial r}{\partial \eta} = 0$ by Newton – Raphson techniques, $\hat{\alpha}_{pec}$ and $\hat{\eta}_{pec}$ are obtained while ($\lambda$ is constant).

**Simulation Procedure**

In this section, Monte Carlo simulation results have been conducted to examine and compare the performance of three Methods (moment , maximum likelihood and Percentile) for the unknown shape parameters ($\alpha$, $r$) considering scale parameter $\lambda$ constant respecting to their MSE values with different cases and different sample sizes $n=30,60,90$. For a given values of ($r$, $\alpha$, $\lambda$), generated a random sample, say $y$ as Burr-XII distribution through the adoption inverse transformation method

$$y_i = \lambda \frac{1}{\alpha} \left[ (1 - U_i)^\frac{1}{r} - 1 \right]^\frac{1}{\alpha}$$

The results of the simulation study are summarized and tabulated in Table 1, Table 2 and 3 of the three estimators for all sample sizes and ($r$, $\alpha$, $\lambda$) values.
Table 1: Comparing estimators of (α) by three Methods

| n   | λ   | α  | r   | $\hat{\alpha}_{mom}$ | $\hat{\alpha}_{mle}$ | $\hat{\alpha}_{pec}$ |
|-----|-----|----|-----|---------------------|---------------------|---------------------|
| 30  | 1.5 | 2  | 2.5 | 2.09633             | 2.29803             | 2.06934             |
|     | 2   | 4  | 2.5 | 2.66050             | 2.59070             | 2.77532             |
|     | 4   | 2.5| 2.53821          | 2.57762             | 2.63645             |
|     | 4   | 4  | 2.48650           | 2.53870             | 2.61693             |
| 30  | 0.8 | 2  | 2.5 | 2.53786             | 2.54610             | 2.61684             |
|     | 2   | 4  | 2.41586           | 2.00670             | 2.73040             |
|     | 4   | 2.5| 2.57764           | 2.57860             | 2.66050             |
|     | 4   | 4  | 2.82110           | 2.57782             | 2.77630             |
| 60  | 1.5 | 2  | 2.5 | 2.02647             | 2.04130             | 2.06060             |
|     | 2   | 4  | 2.56560           | 2.59700             | 2.56070             |
|     | 4   | 2.5| 2.52050           | 2.56760             | 2.48750             |
|     | 4   | 4  | 2.41329           | 2.44560             | 2.53802             |
| 60  | 0.8 | 2  | 2.5 | 2.52906             | 2.51862             | 2.45910             |
|     | 2   | 4  | 2.37015           | 2.00577             | 2.63621             |
|     | 4   | 2.5| 2.56604           | 2.56064             | 2.63620             |
|     | 4   | 4  | 2.76311           | 2.5741              | 2.57736             |
| 90  | 1.5 | 2  | 2.5 | 2.01950             | 2.01408             | 2.01298             |
|     | 2   | 4  | 2.54633           | 2.47230             | 2.55616             |
|     | 4   | 2.5| 2.51176           | 2.56607             | 2.46361             |
|     | 4   | 4  | 2.40770           | 2.40380             | 2.40120             |
| 90  | 0.8 | 2  | 2.5 | 2.36678             | 2.34351             | 2.39020             |
|     | 2   | 4  | 2.35776           | 2.00701             | 2.62878             |
|     | 4   | 2.5| 2.51362           | 2.40059             | 2.62861             |
|     | 4   | 4  | 2.75085           | 2.29360             | 2.31820             |
| 30  | 1.5 | 2  | 2.5 | 2.36450             | 2.28670             | 2.57030             |
|     | 2   | 4  | 2.28790           | 2.62201             | 2.55310             |
|     | 4   | 2.5| 2.57630           | 2.65630             | 2.63020             |
|     | 4   | 4  | 2.63190           | 2.66450             | 2.77430             |
| 30  | 0.8 | 2  | 2.5 | 2.63020             | 2.28780             | 2.48670             |
|     | 2   | 4  | 2.59130           | 2.59135             | 2.59630             |
|     | 4   | 2.5| 2.47060           | 2.47050             | 2.66390             |
|     | 4   | 4  | 2.57310           | 2.47056             | 2.33060             |
| 60  | 1.5 | 2  | 2.5 | 2.30280             | 1.98767             | 1.99560             |
|     | 2   | 4  | 1.99760           | 1.98720             | 1.88500             |
|     | 4   | 2.5| 1.63560           | 1.64670             | 1.89320             |
|     | 4   | 4  | 1.77320           | 1.63020             | 1.94900             |
| 60  | 0.8 | 2  | 2.5 | 2.29850             | 2.25699             | 2.25389             |
|     | 2   | 4  | 2.39670           | 2.57749             | 2.56637             |
|     | 4   | 2.5| 2.35776           | 2.46887             | 2.65778             |
|     | 4   | 4  | 2.46356           | 2.46671             | 2.18940             |
| 90  | 1.5 | 2  | 2.5 | 1.77030             | 1.86210             | 1.74610             |
|     | 2   | 4  | 1.77310           | 1.55820             | 1.87994             |
|     | 4   | 2.5| 1.62641           | 1.637521            | 1.88100             |
|     | 4   | 4  | 1.76791           | 1.62631             | 1.55420             |
| 90  | 0.8 | 2  | 2.5 | 2.28886             | 2.25577             | 2.25238             |
|     | 2   | 4  | 2.6605           | 2.5590              | 2.4853              |
|     | 4   | 2.5| 2.34786           | 2.26161             | 2.26163             |
|     | 4   | 4  | 2.45773           | 2.15582             | 2.15542             |
### Table (2): MSE for $\hat{\alpha}$

| n   | $\lambda$ | $\alpha$ | $r$ | $\hat{\alpha}_{mom}$ | $\hat{\alpha}_{mle}$ | $\hat{\alpha}_{pec}$ | Best   |
|-----|-----------|----------|-----|-----------------------|-----------------------|-----------------------|--------|
| 30  | 1.5       | 2        | 2.5 | 0.3828                | 0.20844               | 0.18389               | PEC    |
|     |           | 2        | 2.5 | 0.38047               | 0.21518               | 0.39860               | MLE    |
|     |           | 2        | 2.5 | 0.32414               | 0.21664               | 0.33450               | MOM    |
|     |           | 4        | 2.5 | 0.30340               | 0.09630               | 0.20537               | MLE    |
|     | 0.8       | 2        | 2.5 | 0.33802               | 0.82013               | 0.64490               | MOM    |
|     |           | 2        | 2.5 | 0.33425               | 0.44630               | 0.59317               | MOM    |
|     |           | 2        | 2.5 | 0.27560               | 0.45020               | 0.55680               | MOM    |
|     |           | 4        | 2.5 | 0.15340               | 0.13040               | 0.48790               | MLE    |
| 60  | 1.5       | 2        | 2.5 | 0.15047               | 0.08091               | 0.06052               | PEC    |
|     |           | 2        | 2.5 | 0.35007               | 0.09560               | 0.05530               | PEC    |
|     |           | 4        | 2.5 | 0.20844               | 0.08332               | 0.05082               | PEC    |
|     |           | 4        | 2.5 | 0.211580              | 0.06654               | 0.05091               | PEC    |
|     | 0.8       | 2        | 2.5 | 0.08984               | 0.07981               | 0.05002               | PEC    |
|     |           | 2        | 2.5 | 0.07076               | 0.04457               | 0.05031               | MLE    |
|     |           | 4        | 2.5 | 0.04117               | 0.03950               | 0.05006               | MLE    |
|     |           | 4        | 2.5 | 0.03534               | 0.03267               | 0.05001               | MLE    |
| 90  | 1.5       | 2        | 2.5 | 0.01039               | 0.03626               | 0.06043               | MOM    |
|     |           | 2        | 2.5 | 0.00625               | 0.03024               | 0.04898               | MOM    |
|     |           | 4        | 2.5 | 0.00234               | 0.01937               | 0.04670               | MOM    |
|     |           | 4        | 2.5 | 0.00122               | 0.00971               | 0.04117               | MOM    |
|     | 0.8       | 2        | 2.5 | 0.00139               | 0.00380               | 0.00366               | MOM    |
|     |           | 2        | 2.5 | 0.00128               | 0.00170               | 0.00347               | MOM    |
|     |           | 4        | 2.5 | 0.00962               | 0.00781               | 0.00583               | PEC    |
|     |           | 4        | 2.5 | 0.00127               | 0.00976               | 0.00970               | MOM    |

### Table (3): MSE of $\hat{r}$

| n   | $\lambda$ | $\alpha$ | $r$ | $\hat{r}_{mom}$ | $\hat{r}_{mle}$ | $\hat{r}_{pec}$ | Best   |
|-----|-----------|----------|-----|----------------|----------------|----------------|--------|
| 30  | 1.5       | 2        | 2.5 | 0.00641        | 0.00218        | 0.00360        | MLE    |
|     |           | 2        | 2.5 | 0.00128        | 0.01276        | 0.03290        | MOM    |
|     |           | 4        | 2.5 | 0.00656        | 0.00624        | 0.00223        | PEC    |
|     |           | 4        | 2.5 | 0.00312        | 0.00209        | 0.00241        | MOM    |
|     | 0.8       | 2        | 2.5 | 0.000322       | 0.000309       | 0.00110        | PEC    |
|     |           | 2        | 2.5 | 0.00139        | 0.00151        | 0.00130        | PEC    |
|     |           | 4        | 2.5 | 0.00102        | 0.00135        | 0.00145        | MOM    |
|     |           | 4        | 2.5 | 0.00125        | 0.00277        | 0.00672        | MOM    |
| 60  | 1.5       | 2        | 2.5 | 0.00141        | 0.00203        | 0.00263        | MOM    |
|     |           | 2        | 2.5 | 0.00127        | 0.00330        | 0.00329        | MOM    |
|     |           | 4        | 2.5 | 0.00136        | 0.00224        | 0.00225        | MOM    |
|     |           | 4        | 2.5 | 0.00129        | 0.00126        | 0.00189        | MLE    |
|     | 0.8       | 2        | 2.5 | 0.00305        | 0.00102        | 0.00104        | MLE    |
|     |           | 2        | 2.5 | 0.00138        | 0.00125        | 0.00127        | MLE    |
|     |           | 4        | 2.5 | 0.00101        | 0.00100        | 0.00142        | MLE    |
|     |           | 4        | 2.5 | 0.00123        | 0.00241        | 0.00589        | MOM    |
| 90  | 1.5       | 2        | 2.5 | 0.00102        | 0.00202        | 0.00243        | MOM    |
|     |           | 2        | 2.5 | 0.00126        | 0.00122        | 0.00125        | MLE    |
|     |           | 4        | 2.5 | 0.00125        | 0.00223        | 0.00224        | MOM    |
|     |           | 4        | 2.5 | 0.00121        | 0.00124        | 0.00123        | MOM    |
|     | 0.8       | 2        | 2.5 | 0.00135        | 0.00101        | 0.00103        | MLE    |
|     |           | 2        | 2.5 | 0.00112        | 0.00115        | 0.00107        | PEC    |
|     |           | 4        | 2.5 | 0.00031        | 0.00092        | 0.00022        | PEC    |
|     |           | 4        | 2.5 | 0.00122        | 0.00235        | 0.00360        | MOM    |
Conclusion

1. $\hat{r}_{mom}$ is the best estimator with percentage (46%) and $\hat{r}_{mle}$ also with percentage (33%) while $\hat{r}_{pec}$ is dominated with (21%).
2. For $\alpha$ (shape parameter), $\tilde{\alpha}_{mom}$ is dominated with percentage 42% and $\tilde{\alpha}_{mle}$ (7/24) = 29%, while $\tilde{\alpha}_{pec}$ is best percentage 29%.
3. The compound (Burr-XII) model is important for estimating time to failure of distribution that represents the time of failure for compound model of many parts especially for big system and equipment.

Estimate the expected mean time to failure of this compound distribution from applying $E(x) = r \lambda \text{Beta } \left( \frac{1}{\alpha}+1, r-\frac{1}{\alpha} \right)$ when $\text{Beta } (a,b)= \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ after applying the best estimators of $(r, \alpha)$ (λ is known) and also estimating $E(x^2)$, where these are necessary to obtain estimated (variance), which is important for finding confidence internal of estimators.

Author’s declaration:
- Conflicts of Interest: None.
- I hereby confirm that all the Figures and Tables in the manuscript are mine. Besides, the Figures and images, which are not mine, have been given the permission for re-publication attached with the manuscript.
- The author has signed an animal welfare statement.
- Ethical Clearance: The project was approved by the local ethical committee in University of Mustansiriyah.

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(Burr-XII)

مقارنة بين المقدرات لمعملي الشكل من توزيع

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قسم الرياضيات, كلية العلوم, الجامعة المستنصرية, العراق

المتبرع

يتناول هذا البحث تعريف Burr-XII و كيفية الحصول على دالة كثافة الاحتمال و دالة التوزيع له, نظراً لأنه أحد توزيعات الفشل والذي يركز من نموذجين للفشل كما نموذج كاما ونموذج ويبل ويل المعدات تحتوي على العديد من الإجزاء الهامة وقد يكون هناك أزواج مختلفة من توزيع بور الاحتمالية الذي يمثلها , بواسطة صيغة المركبة المختلفة لذلك وجدنا أنه هو أفضل نموذج للدراسة. و تقييم المعادلة لحساب متوسط الزمن لمعدل الفشل والتي تعتبرها معلومة.

بادأ من النماذج الأخرى وهذا التوزيع له معلم لي للشكل ومعملي قبض واحدة لذلك في هذا البحث عرفنا دالة الكثافة ودالة التوزيع ونظام المعادلات حول نقطة الصل, وكذلك أنشأنا مقدرات العزوم والأمثلة الأعظم كذلك التقدير المئوي لمعملي النماذجдерاج عند محاكاة النماذج المتعادلة ومعمليات مثالية من الفترات الزمنية. لوحظ أن مقدرات العزوم $\hat{r}_{mom}$ and $\tilde{\alpha}_{mom}$ تم المقارنة بين طرق الثلاثة من خلال المحاكاة باخذ حوسم عينة مختلفة ومجموعات مختلفة من الفترات الزمنية. لوحظ أن مقدرات العزوم $r_{mom}$ and $r_{mle}$ 42% and 46% على التوالي مقارنة مع مقدرات الأخرى $r_{pec}$.

الكلمات المفتاحية: نموذج فشل Burre-XII, مقدر العزوم. مقدر الأمكان الأعظم, المقدر المئوي.