Fermionic Ising glasses with BCS pairing interaction in the presence of a transverse field

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Abstract

In the present work we have analyzed a fermionic infinite-ranged Ising spin glass with a local BCS coupling in the presence of transverse field. This model has been obtained by tracing out the conduction electrons degrees of freedom in a superconducting alloy. The transverse field $\Gamma$ is applied in the resulting effective model. The problem is formulated in the path integral formalism where the spins operators are represented by bilinear combination of Grassmann fields. The problem can be solved by combining previous approaches used to study a fermionic Heisenberg spin glass and a Ising spin glass in a transverse field. The results are shown in a phase diagram $T/J$ versus $\Gamma/J$ ($J$ is the standard deviation of the random coupling $J_{ij}$ for several values of $g$ (the strength of the pairing interaction). For small $g$, the line transition $T_c(\Gamma)$ between the normal paramagnetic phase and the spin glass phase decreases when increases $\Gamma$, until it reaches a quantum critical point. For increasing $g$, a PAIR phase (where there is formation of local pairs) has been found which disappears when is close to $\Gamma_c$ showing that the transverse field tends to inhibited the PAIR phase.

Key words: Quantum spin glass, Transverse field, Lattice theory and statistics
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Spin glass-like phase has been found in several physical systems such as cuprate superconductors [1], heavy fermions [2] and conventional superconductors [3] doped with magnetic impurities like $\text{Gd}_x\text{Th}_{1-x}\text{Ru}_2$. Recent theoretical studies have been trying to model the equivalent phase transition problem, at finite temperatures, between a spin glass phase and a BCS pairing among localized fermions of opposite spins (see Ref. [4] and references therein). Nevertheless, disorder and frustration nearby a quantum critical point (QCP) can be a source of non-trivial effects. In fact, it has been shown that there is a deviation of a Fermi-liquid behavior near $T = 0$ in the transition between a metallic paramagnetic and a metallic spin glass [5].

Quite recently, a quantum Ising spin glass in a transverse field $\Gamma$ has been investigated [6]. The non-commutativity of quantum mechanical spins operators has been treated within the framework of path integral formalism where both spins operators $\hat{S}^z_i$ and $\hat{S}^x_i$ are represented by bilinear combination of Grassmann fields. Both static approximation and the replica symmetry “ansatz” have been used. The results show the freezing temperature $T_f(\Gamma)$ decreases (when $\Gamma$ increases) toward a QCP.

In the present work, we studied the competition between a spin glass ordering and BCS pair formation in a presence of a mechanism, which can lead to a quantum phase transition. The formalism used in the present approach is a combination of those introduced in Ref. [4] and [6]. This has been done using a Hamiltonian with a fermionic Ising spin glass and a BCS pairing interaction in a real space [4] with a transverse magnetic field $\Gamma$. Therefore, the Hamiltonian is given by:

$$\hat{H} = -\sum_{ij} (J_{ij} \hat{S}^z_i \hat{S}^z_j + \frac{g}{N} \hat{c}^\dagger_i \hat{c}^\dagger_j \hat{c}_j \hat{c}_i) - 2\Gamma \sum_j \hat{S}^x_j.$$  \hspace{1cm} (1)

The spins operators $\hat{S}^z_i$ and $\hat{S}^x_j$ are represented as Ref.
The coupling $J_{ij}$ is infinite ranged with Gaussian distribution, zero mean and variance $\langle J_{ij}^2 \rangle = J^2/2$.

The Grand Canonical potential can be obtained using the replica trick [4]. The action in the replicated partition function has three components $A = A_{BCS} + A_{SG} + A_{\Gamma}$. In the first one, the $|\psi \rangle$ order parameter can be introduced by a Hubbard-Stratonovich transformation following closed the approach of Ref. [4] which corresponds to a long range order where there is double occupation of the sites (PAIR phase). The spin glass order parameter $q_{\alpha \beta}$ is also introduced in a similar way, but in the quantum case the diagonal component of the spin glass order parameter is no longer constrained to unity. Therefore, the resulting $A_{BCS}$ can be written in terms of Nambu matrices (see Ref. [4]) and the resulting spin part of the action $A_{SG} + A_{\Gamma}$ in terms of Spinhos matrices (see Ref. [6]). In order to solve the functional integral over the Grassmann fields, the elements of Spinors and Nambu matrices have been mixed (see Ref. [4], Eq. (20)-(22)). Further details will be given elsewhere [7]. Therefore, the Grand Canonical potential can be found for the half-filling case as

$$\frac{\Omega}{\beta} = A - \int_{-\infty}^{\infty} Dz \ln[\cosh \beta g|\eta|] + \int_{-\infty}^{\infty} Du \cosh \beta |h|, \quad (2)$$

where $A = (\beta J)^2 \tilde{\chi}(\chi + 2q)/2 + \beta g|\eta|^2$, $h = J\sqrt{\theta^2 + (\Gamma/J)^2}$ and $\theta = \sqrt{2\tilde{\chi}u + \sqrt{2g}}$. In both equation $Dz = dz \exp(-z^2/2)/\sqrt{2\pi}$ and $\beta = 1/T$ ($T$ is the temperature). In $A$, $q$ is the non-diagonal spin glass order parameter, $\tilde{\chi} = \chi/\beta$, where $\chi$ is the local susceptibility related with the non-diagonal spin glass order parameter.

Phases diagrams can be obtained in $T/J - g/J$ ($g$ is the pairing strength) and $T/J - \Gamma/J$ spaces which are shown in Fig. 1 and Fig. 2, respectively. In Fig. 1, three phases can be identified for $\Gamma = 0$: a normal-paramagnetic phase (NORMAL) at high temperature and small $g$, a spin glass phase (SG) at low temperature and enhancing $g$ one gets a phase transition at $g = g_c(T)$ to a PAIR phase. When $\Gamma$ is non-null, it can be seen that the line transition $g_c(T)$ is displaced showing that the existence of the PAIR phase requires greater values of $g$, at the same time the freezing temperature $T_f$ decreases towards zero. These changes with increasing $\Gamma$ can be better seen in Fig. 2, for $g = 0$ there is just a transition line between the NORMAL and the SG phases with a QCP at $\Gamma_c = 2\sqrt{2}J$ as already found in Ref. [6]. If $g$ is turned on, which energetically favors the double occupation, the PAIR phase disappears when $\Gamma$ is increased. The sequence Fig 2.b-Fig 2.d shows that it is necessary even greater values of $g$ in order to PAIR phase starts to occupy a larger region than the SG phase. The QCP remains the same as $g = 0$ and the replica symmetric solution is unstable in the whole SG phase.

To conclude, our results suggest that the competition between the SG and the PAIR phases is strongly affected by the spin flipping mechanism ($\Gamma$) which tends to inhibited the pair formation in the sites, particularly near to the QCP.

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