We present a study of the cross sections \( \sigma(J/\psi X \rightarrow D^{(*)} \bar{D}^{(*)}) \) based on the calculation of the effective tri- and four-linear couplings \( J/\psi(X)D^{(*)} \bar{D}^{(*)} \) within a constituent quark model. In particular, the details of the calculation of the four-linear couplings \( J/\psi XD^{(*)} \bar{D}^{(*)} \) are given. The results obtained have been used in a recent analysis of \( J/\psi \) absorption by the hot hadron gas formed in peripheral heavy-ion collisions at SPS energies.

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I. INTRODUCTION

The problem of computing \( J/\psi \) strong couplings to \( \pi, \rho \) and other pseudo-scalar and vector particles has its own interest because it opens the way to the calculation of cross sections of the kind (see Fig. 1):

\[
\sigma \left( J/\psi \{\pi, \rho, \ldots\} \rightarrow D^{(*)} \bar{D}^{(*)} \right).
\] (1)

Such cross sections are the basic ingredients to estimate the hadronic absorption background of \( J/\psi \) in heavy-ion collisions, as it is thoroughly discussed in Refs. [1, 2]. The description of processes like those in Eq. (1) is a hard task because no experimental test can be performed and moreover they are not amenable to first principles calculations, so that one has to resort to build models and make approximations to describe their dynamics.

The dissociation process of the \( J/\psi \) by hadrons has been considered in several approaches, but the predicted cross sections show very different energy dependence and magnitude near threshold. Anyway, using different approaches, one consistently finds non negligible cross section values (at least comparable with the nuclear one \( \sigma_{J/\psi}^{N} \), \( N \) = nucleon) especially for the reactions with \( \pi \)'s and \( \rho \)'s, the most studied cases; for a review see for instance Ref. [3]. This is certainly a clear indication that the picture of \( J/\psi \) absorption by nuclear matter, as an antagonist mechanism to the plasma suppression, is incomplete as long as interactions with the hadronic gas formed in nucleus-nucleus collisions are not considered.

The problem of calculating the \( J/\psi \) dissociation by pseudo-scalar and vector mesons has been addressed in Refs. [1, 2] within the Constituent-Quark-Meson model (CQM), originally devised to compute exclusive heavy-light meson decays and tested on a quite large number of such processes [4]. The basic calculations refer to \( \pi \) and \( \rho \) contributions. The couplings to other mesons have been obtained under the hypothesis of flavour/octet symmetry.

The typical effective Feynman diagrams contributing to the \( J/\psi \) dissociation are depicted in Fig. 1. The tri-linear couplings \( \rho D^{(*)} \bar{D}^{(*)} \) have been calculated in Ref. [2] and the \( J/\psi D^{(*)} \bar{D}^{(*)} \) couplings have been recently discussed in Ref. [5] (we report these results at the end of Sect. II), where also four-line ar couplings involving pions have been derived. The aim of the present note is to explain the method used and the results obtained in evaluating the four-linear couplings of the kind \( J/\psi \phi D^{(*)} \bar{D}^{(*)} \) (see Fig. 1, third diagram), since these are not calculated elsewhere within the CQM framework. The numerical values of the \( J/\psi \phi D^{(*)} \bar{D}^{(*)} \) couplings are also given. For completeness we report also the expressions for the tri-linear couplings discussed in Refs. [1, 2]. In the end, we present the cross section predictions, based on the
FIG. 1: Tree level effective Feynman diagrams for the $J/\psi \rho \rightarrow H \bar{H}$ reaction, $H$ being $D^{(*)}$, with $D^{(*)} = D$ or $D^*$. complete set of contributing diagrams, for the processes $J/\psi \rho \rightarrow D^{(*)} \bar{D}^{(*)}$ and $J/\psi \Phi \rightarrow D_s^{(*)} \bar{D}_s^{(*)}$, together with an estimate of the associated theoretical uncertainties.

II. THE MODEL

CQM is based on an effective Lagrangian which incorporates the heavy quark spin-flavor symmetries and the chiral symmetry in the light sector. In particular, it contains effective vertexes between an heavy meson and its constituent quarks (see the vertexes in the r.h.s. of Fig. 2) whose emergence has been shown to occur when applying bosonization techniques to Nambu–Jona-Lasinio (NJL) interaction terms of heavy and light quark fields \[^8\]. On this basis we believe that CQM can be considered as a quite reasonable approach to the computation of $J/\psi$ strong couplings to be compared to the various methods available in the literature, often based on $SU(4)$ symmetry \[^9\].

In Fig. 2 we show the typical diagrammatic equation to be solved in order to obtain $g_4(g_3)$, four(tri)-linear couplings, in the various cases at hand: on the l.h.s. it is represented the effective four-linear coupling to be used in the cross section calculation (to obtain one of the relevant tri-linear couplings we could discard either the $J/\psi$ or the $\rho$); the effective interaction at the meson level (l.h.s.) is modeled as an interaction at the quark-meson level (r.h.s. of Fig. 2).

The $J/\psi$ is introduced using a Vector Meson Dominance (VMD) Ansatz: in the effective loop on the r.h.s. of Fig. 2 we have a vector current insertion on the heavy quark line $c$ while on the l.h.s. the $J/\psi$ is assumed to dominate the tower of $1^-, \bar{c}c$ states mixing with the vector current (for more details see \[^8\]). Similarly, vector particles coupled to the light quark component of the heavy mesons $\rho, \omega$, when $q = (u,d)$, or $K^*, \Phi$, when one or both light quarks involved are $q = s$, are also taken into account using VMD arguments.

The pion and other pseudo-scalar fields have a derivative coupling to the light quarks of the Georgi-Manohar kind \[^10\].

In this paper we will mainly focus on the reaction:

$$J/\psi \rho \rightarrow D^{(*)} \bar{D}^{(*)}$$

(2)

and in particular on the four-linear coupling $J/\psi \rho H \bar{H}$ (third graph) in Fig. 1.

In CQM, as in Heavy Quark Effective Theory (HQET) \[^11\], the heavy super-field $H$ describes the charmed states $D$ and $D^*$, respectively associated to the annihilation operators $P_5$, $P^\mu$. $H$ is written in the following way:

$$H(v) = \frac{1 + \gamma_5}{2} (P - P_5\gamma_5)$$

(3)

where $v$ is the four-velocity of the heavy meson; the limit of very large heavy quark mass is understood. The heavy quark propagator is:

$$\frac{1 + \gamma_5}{2} \frac{i}{v \cdot k}$$

(4)

where $k$ is the residual momentum defined by the equation $p^\mu_Q = m_Qv^\mu + k^\mu$ and related to the interaction of the light degrees of freedom with the heavy quark ($k \sim O(\Lambda_{QCD})$).

The $\rho$ is described by the interpolating field $\rho^\mu$ \[^7\] and its kinetic term in the effective Lagrangian is built out by the tensor field strength:

$$F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + [\rho_\mu, \rho_\nu]$$

(5)

In the approach followed by \[^7\], $\rho^\mu$ is defined by $\rho^\mu = im_\rho/f_\rho \hat{\rho}^\mu$ where $\hat{\rho}$ is a $3 \times 3$ hermitian traceless matrix analogous to the $3 \times 3 \pi$ matrix of the pseudo-scalar octet.
In the following we will use the Feynman rules defined in [4]. The interaction terms relevant to this calculation are:

\[ -\bar{q} \, \not{H} \, Q_v \, \text{h.c.}, \]  

which describes the vertex light quark \((q)\), heavy quark \((Q_v)\), heavy meson \((H)\), and

\[ \bar{q} \left( \frac{m^2}{f} \gamma^\mu \gamma^\nu \right) q, \]  

describing the vertex light quark, light quark, \( \rho \). Here the decay constant \( f_\rho \) is defined by

\[ \langle 0 | V_{\mu} | \rho(q, \epsilon) \rangle = i f_\rho \epsilon^\mu, \]  

where \( f_\rho = 0.152 \text{ GeV}^2 \). We use a similar definition for the decay constant of \( J/\psi \) but we factor out the mass \( m_j \) in the latter case, thus obtaining \( f_J = 0.405 \text{ GeV} \).

Once established the form of the effective vertexes occurring in the loop diagram in the r.h.s. of Fig. 2 one has just to compute it using some regularization; we will adopt the Schwinger proper time.

CQM does not include any confining potential and an infrared cutoff \( \mu \) is needed to prevent low integration momenta to access the energy region where confinement should be at work. The kinematic condition for the free dissociation of \( H \) in \( m_Q \) and \( m_q \) is given by:

\[ m_H > m_Q + m_q \]  

with \( p_H = m_H \, v \approx m_Q \, v + k \). It follows that

\[ k \cdot v > m_q. \]  

In the hadron rest frame we have \( k^0 > m_q \) and we can therefore require:

\[ \mu \approx m_q. \]  

The value of the constituent light quark mass in the model at hand is given by a gap equation [4]:

\[ m_q - m_0 - 8 G I_1(m_q^2) = 0, \]  

where \( G = 5.25 \text{ GeV}^{-2} \), \( m_0 \) is the current mass and the \( I_1 \) integral is defined in the Appendix. As a consistency check, putting a zero current mass for the \( u, d \) species we get a constituent mass of 300 MeV, while for a strange current mass of \( m_0 = 131 \text{ MeV} \) we obtain a strange constituent mass of 500 MeV using \( \mu = 300, 500 \text{ MeV} \) respectively in the calculation of \( I_1 \).

The residual momentum has an upper limit given by the chiral symmetry braking scale \( \Lambda_\chi \simeq 4\pi f_\pi \) which we adopt as a UV cutoff [4].

Then the momenta running in the loop are limited by two cutoff’s: \( \mu \) and \( \Lambda \). These two cutoff’s are implemented by the Schwinger regularization on the light propagator as follows:

\[ \int \frac{d^4l}{(l^2 - m_q^2)} \rightarrow \int d^4l \int_{1/\Lambda^2}^{1/\mu^2} ds \, e^{-s(l^2 + m_q^2)}. \]
The diagrammatic equation in Fig. 2 states that the effective vertex $J/\psi \rho H \bar{H}$ is given by:

$$(-1)\sqrt{Z_H m_H Z_{H'} m_{H'}} \times$$

$$\times N_c \int \frac{d^4l}{(2\pi)^4} \text{Tr} \left[ ( -i H(\nu') \frac{i}{v' \cdot l + \Delta} \frac{m^2}{f_{J \rho} m_J} \frac{i}{v' \cdot l + \Delta} ( -i H(\nu) \frac{i}{\nu - m} \frac{m^2}{f_{\rho} m_\rho} \frac{i}{\nu + \nu - m} ) \right].$$

(14)

$H$ and $\bar{H}'$ represent the heavy-light external meson fields labeled by their four-velocities $v, v'$ while the $\sqrt{Z_H m_H Z_{H'} m_{H'}}$ coupling factor of heavy mesons to quarks is calculated in [4]. The parameter $\Delta$ appearing in the heavy propagator is defined by:

$$\Delta = M_H - m_Q,$$

(15)
i.e., the mass of the heavy-light meson minus the mass of the heavy quark contained in it. $\Delta$ is the main free parameter of the model. It varies in the range $\Delta = 0.3, 0.4, 0.5$ GeV for $u, d$ light quarks and $0.5, 0.6, 0.7$ GeV for strange quarks [12]. Varying $\Delta$ gives an handle to estimate the theoretical error. $m$ is the constituent mass of light quarks as defined above.

### III. THE CALCULATION

The $\rho$ is coupled to the light quarks by VMD, $\epsilon$ being its polarization and $q$ its 4-momentum. The $J/\psi$, having polarization $\eta$, is also coupled via VMD, but to the heavy quarks ($\eta$ appears in the trace between the two heavy quark propagators, while $\epsilon$ appears between the two light quark propagators). In front of this expression we have the fermion loop factor.

The trace computation in (14) will introduce a number of scalar combinations of the momenta and polarizations of the external particles that we will list in the Appendix. Each of these combinations will be weighted by a scalar integral which amounts to a numerical factor: what we call the coupling. Actually, as we will see, such scalar integrals depend on the energy of the $\rho$. In general the expressions obtained for the four-linear couplings appear to be quite complicated functions of $E_\rho$ if compared, e.g., to those obtained when studying only $J/\psi$ interactions with pions [4]. It is therefore difficult to write down general polar expressions for the $E_\rho$ behaviour. On the other hand we have in mind to use these results to compute cross sections $\sigma_{J/\psi}$ and thermal averages $\langle \rho \cdot \sigma \rangle_T$ in a hadron gas at a temperature $T \approx 170$ MeV where the Boltzmann factor is presumably more effective than any polar form factor in damping the high energy tails.

Using the Feynman trick the fermion loop of the above equation (14) becomes:

$$\frac{m^2_J m^2_\rho}{f_{J \rho} m_J m_\rho} \sqrt{Z_H m_H Z_{H'} m_{H'}} \int_0^1 dx \frac{\partial}{\partial m^2(x)} N_c \int \frac{d^4l}{(2\pi)^4} \text{Tr} \left[ \bar{H}' \frac{\eta}{ \nu - \eta x + m} \eta \frac{\eta}{ \nu - \eta x + m} \right] \frac{1}{(l^2 - \tilde{m}^2) (v \cdot l + \delta) (v' \cdot l + \delta')},$$

(16)
in which we have defined:

$$\tilde{m}^2 = m^2 + x m^2_\rho (x - 1),$$

(17)
and

$$\delta = \Delta - x q \cdot v = \Delta - x E_\rho,$$

$$\delta' = \Delta - x q \cdot v' = \Delta - x \omega E_\rho,$$

(18)
(19)
where $E_\rho$ is the energy of the incident $\rho$ and $\omega = v \cdot v'$ ($v' = \omega v$). The cross section computation is performed in the frame where $J/\psi$ is at rest. $\omega$, in this frame, is related to the meson masses by:

$$\omega = \frac{m^2_{J/\psi} + m^2_\rho - m^2_H - m^2_{H'}}{2m_H m_{H'}} + 2 E_\rho m_{J/\psi} 2 m_H m_{H'}.$$  

(20)

By kinematic considerations the energy threshold of the reactions (2) for $D \bar{D}$ and $D^* \bar{D}$ channels is $E_\rho \approx 0.77$ GeV whereas for $D^* \bar{D}^*$ channel is $E_\rho \approx 0.96$ GeV, with $\omega \approx 1$. We consider $\rho$ particles with energies in the range between 0.77 and 1 GeV where the two final state mesons are almost at rest.

All the couplings that we can extract by direct computation can be written in terms of 7 basic expressions which we call: $L_5, A, B, C, D, E, F$. The latter are linear combinations of the $I_i, L_i$ integrals listed in the Appendix and are defined by:

$$\frac{\partial}{\partial m^2} N_c \int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2 - \tilde{m}^2) (v \cdot l + \delta) (v' \cdot l + \delta')} = L_5$$

(21)
\[
\frac{\partial}{\partial \hat{m}^2} iN_c \int \frac{d^4 l}{(2\pi)^4} \frac{l_\mu}{(l^2 - \hat{m}^2)} \frac{l_{\mu'} l_{\nu}}{(l' \cdot l + \delta) (l' \cdot l' + \delta')} = A v_\mu + B v'_\mu \tag{22}
\]

\[
\frac{\partial}{\partial \hat{m}^2} iN_c \int \frac{d^4 l}{(2\pi)^4} \frac{l_\mu l_{\nu}}{(l^2 - \hat{m}^2)} \frac{l_{\mu'} l_{\nu'}}{(l' \cdot l + \delta) (l' \cdot l' + \delta')} = C g_{\mu \nu} + D v_\mu v_\nu + E v'_\mu v'_\nu + F (v_\mu v'_\nu + v'_\mu v_\nu). \tag{23}
\]

The final expression of the loop integral can then be reduced to a sum of terms of the general form:

\[
\sum_S S(H, H') C \int_0^1 dx \; g_4^{(S)}(x, E_\rho) \tag{24}
\]

where \(S(H, H')\) represent the scalar combinations of momenta and polarizations of \(H\) and \(H'\) occurring in the calculation; \(g_4^{(S)}\), are the corresponding couplings. Here \(C\) is given by:

\[
C = \frac{m_3^2}{f_J m_J} \frac{m_2^2}{f_\rho} \sqrt{Z_H m_H Z_{H'} m_{H'}}, \tag{25}
\]

with \(f_J = 0.405 \text{ GeV}\). Our central values for the couplings are obtained for \(\Delta = 0.4 \text{ GeV}\) (and \(Z_H = 2.36 \text{ GeV}^{-1}\))

while for \(m_H\) we use the experimental value for the \(D(\pi)\) mass.

In Table II we list the explicit expressions of the couplings. We call them \(g_4 = \{g, h, f\}\), respectively related to the four linear couplings \(J/\psi D \bar{D}, J/\psi D^* \bar{D}, J/\psi D^* \bar{D}^*\). All these expressions have to multiplied by \(C\) and integrated over \(x\); the numerical values in Table I are then:

\[
C \int_0^1 dx \; g_4(x, E_\rho). \tag{26}
\]

These are typically complicated functions of \(E_\rho\) and it is not as easy as in \[\text{Eq. (24)}\] to determine an explicit polar form factor dependency common to all of them. On the other hand we have in mind to adopt these couplings to compute cross sections of the kind \(\sigma_{J/\psi \rho \rightarrow \text{open charm}}\), and use this information to compute thermal averages in the form:

\[
\langle \rho \cdot \sigma_{J/\psi \rho \rightarrow \text{open charm}} \rangle_T = \frac{N}{2\pi^2} \int_{E_0}^{\infty} \frac{dE}{E^2} \frac{p E \sigma(E)}{e^{E/kT} - 1}, \tag{27}
\]

where \(E_0\) is the energy threshold needed to open the reaction channel and the Bose distribution is used to describe an ideal gas of mesons. \(\rho\) in the l.h.s of Eq. \[\text{Eq. (27)}\] is the number density of particles in the gas. The Boltzmann factor \(\exp (-E/T)\) will be at work as an exponential form factor cutting high energy tails faster than any polar one. We could therefore avoid any arbitrary Ansatz on form factors at the interaction vertexes. We limit ourself to study the dependency of our couplings on \(E_\rho\) in the range of energy where we reasonably think to have \(\rho\) mesons in the hadron gas excited by a peripheral heavy-ion collision. Estimating the \(J/\psi\) absorption background to the suppression signal in heavy-ion collisions amounts to compute the attenuation lengths (inverse of the thermal averages in Eq. \[\text{Eq. (27)}\]) in a hot gas, \(T \approx 170 \text{ MeV}\), populated by \(\pi, K, \eta, \rho, \omega, \ldots\) mesons. We could therefore expect to have \(E_\rho \approx 770 \div 1000 \text{ MeV}\).

The loop that must be calculated in the case in which we substitute a \(\Phi\) particle to the \(\rho\) is the same as in Fig. \[\text{Fig. 2}\] but with \(q,q' = s \; (m_\rho = 500 \text{ MeV}, \mu \sim m_\rho)\), and with super-fields \(H_s\) in place of \(H\). The reaction in this case is

\[
J/\psi \; \Phi \rightarrow D_s^{(*)} \bar{D}_s^{(*)}. \tag{28}
\]

The structure of the coupling of \(\Phi\) to light quark current is identical to the one of the \(\rho\), and all the above equations are valid also in this case with the substitutions: \(m_\rho \rightarrow m_\Phi, f_\rho \rightarrow f_\Phi, H \rightarrow H_s\). Numerically we have used \(f_\Phi = 0.249 \text{ GeV}^2\), while for \(m_{H_s}\) we have used the experimental value for \(D_s^{(*)}\). The numerical values are reported in Table II.

We conclude this section by writing the tri-linear couplings \(\rho HH\); they are computable within the same framework by simply not including the \(J/\psi\) in the diagrammatic equation of Fig. \[\text{Fig. 2}\].

The vertex \(\rho D^{(*)} \bar{D}^{(*)}\) is described by two constants \(g_3 = \beta, \lambda\) and the effective Lagrangian describing this interaction can be written as \[\text{Eq. (28)}\]:

\[
\mathcal{L}_{HH \rho} = -i \beta \text{Tr}[\bar{H} H] \; v \cdot \rho + i \lambda \text{Tr}[\bar{H} \sigma_{\mu \nu} H] F_{\mu \nu}, \tag{29}
\]
\( J/\psi \bar{D} \bar{D} \) & \( X = \rho \) & \( X = \Phi \) \\
\( g_1 \) & \( \frac{2}{m_D^2} Ax \) & \( 4 \pm 2 \) & \( 1.5 \pm 0.5 \) & \( \text{GeV}^{-4} \) \\
\( g_2 \) & \( \frac{1}{m_D^2} B(x-1) \) & \( -2.3 \pm 1.0 \) & \( -1.1 \pm 0.2 \) & \( \text{GeV}^{-4} \) \\
\( g_3 \) & \( \frac{1}{m_D^2} (A + B + 2x(A\omega - 1) - mL_5) \) & \( 27 \pm 4 \) & \( 13 \pm 1 \) & \( \text{GeV}^{-2} \) \\
\( g_4 \) & \( \frac{2}{m_D^2} (D + F - Am) \) & \( -9 \pm 3 \) & \( -7 \pm 1 \) & \( \text{GeV}^{-2} \) \\
\( g_5 \) & \( \frac{1}{m_D^2} ((m^2 + m^2_{\omega}x(1-x))L_5 - 2Am - 2C + D - E + 2F(1-\omega)) \) & \( -8 \pm 3 \) & \( -7 \pm 1 \) & \( \text{GeV}^{-2} \) \\
\( g_6 \) & \( \frac{1}{m_D^2} (A - B + 2B(x - \omega x + \omega) - mL_5) \) & \( 25 \pm 4 \) & \( 12 \pm 1 \) & \( \text{GeV}^{-2} \) \\
\( g_7 \) & \( \frac{1}{m_D^2} ((m^2 + m^2_{\omega}x(1-x))L_5 - 2Bm - 2C - D + E + 2F(1-\omega)) \) & \( -6 \pm 2 \) & \( -5 \pm 1 \) & \( \text{GeV}^{-2} \) \\
\( g_8 \) & \( \frac{1}{m_D^2} (E + F - Bm) \) & \( -7 \pm 2 \) & \( -5 \pm 1 \) & \( \text{GeV}^{-2} \) \\
\( g_9 \) & \( ((m^2 + m^2_{\omega}x(1-x))L_5 - 2C - D - E - 2F\omega)(1-\omega) \) & \( -0.4 \pm 0.4 \) & \( -0.4 \pm 0.2 \) & \\

\( J/\psi \bar{D}^* \bar{D} \) & \( X = \rho \) & \( X = \Phi \) \\
\( h_1 \) & \( (mL_5 + (A - B)x)(\omega - 1) \) & \( 1 \pm 2 \) & \( 0.1 \pm 0.6 \) & \( \text{GeV}^{-1} \) \\
\( h_2 \) & \( B(x-1) \) & \( -9 \pm 4 \) & \( -5 \pm 1 \) & \( \text{GeV}^{-1} \) \\
\( h_3 \) & \( mL_5 - Bx \) & \( -6 \pm 12 \) & \( -6 \pm 3 \) & \( \text{GeV}^{-1} \) \\
\( h_4 \) & \( mL_5 + A(x - 1) - B \) & \( -35 \pm 15 \) & \( -20 \pm 4 \) & \( \text{GeV}^{-1} \) \\
\( h_5 \) & \( A \) & \( 35 \pm 11 \) & \( 15.7 \pm 3 \) & \( \text{GeV}^{-1} \) \\
\( h_6 \) & \( Ax \) & \( 15 \pm 8 \) & \( 6 \pm 2 \) & \( \text{GeV}^{-1} \) \\
\( h_7 \) & \( B - mL_5 \) & \( 16 \pm 16 \) & \( 11 \pm 4 \) & \( \text{GeV}^{-1} \) \\
\( h_8 \) & \( (m^2 + m^2_{\omega}x(1-x))L_5 - 2C - D - E - 2F\omega \) & \( 1.3 \pm 2 \) & \( 1.1 \pm 0.8 \) & \\
\( h_9 \) & \( D + F - mA \) & \( -19 \pm 7 \) & \( -17 \pm 3 \) & \\
\( h_{10} \) & \( E + F - mB \) & \( -15 \pm 6 \) & \( -13 \pm 2 \) & \\

\( J/\psi \bar{D}^* \bar{D} \) & \( X = \rho \) & \( X = \Phi \) \\
\( f_1 \) & \( \frac{1}{m_D^2} (A + B - mL_5) \) & \( 25.5 \pm 0.6 \) & \( 13.6 \pm 0.1 \) & \( \text{GeV}^{-2} \) \\
\( f_2 \) & \( \frac{1}{m_D^2} (B - mL_5 + A(2\omega x - 2x + 1)) \) & \( 26 \pm 1 \) & \( 13.86 \pm 0.08 \) & \( \text{GeV}^{-2} \) \\
\( f_3 \) & \( \frac{1}{m_D^2} Ax \) & \( 4 \pm 2 \) & \( 1.5 \pm 0.5 \) & \( \text{GeV}^{-4} \) \\
\( f_4 \) & \( \frac{1}{m_D^2} (A - mL_5 + B(-2\omega x + 2x + 2\omega - 1)) \) & \( 26.0 \pm 0.5 \) & \( 13.8 \pm 1 \) & \( \text{GeV}^{-2} \) \\
\( f_5 \) & \( \frac{2}{m_D^2} B(x-1) \) & \( -2.2 \pm 0.7 \) & \( -1.2 \pm 0.1 \) & \( \text{GeV}^{-4} \) \\
\( f_6 \) & \( (m^2L_5 + 2C + D - E + 2F\omega + m^2_{\omega}(x^2 - x)L_5)(1-\omega) \) & \( 0.03 \pm 0.1 \) & \( 0.03 \pm 0.03 \) & \\
\( f_7 \) & \( \frac{1}{m_D^2} (m^2L_5 + 2C + D + E + 2F\omega + m^2_{\omega}(x^2 - x)L_5) \) & \( -0.1 \pm 2 \) & \( -0.2 \pm 0.2 \) & \( \text{GeV}^{-2} \) \\
\( f_8 \) & \( \frac{m^2}{m_D^2} (D + F - mA) \) & \( -8 \pm 2 \) & \( -8.2 \pm 0.3 \) & \( \text{GeV}^{-2} \) \\
\( f_9 \) & \( \frac{m^2}{m_D^2} (m^2L_5 + 2mA + 2C - D + E + 2F(\omega - 1) + m^2_{\omega}(x^2 - x)L_5) \) & \( 8.3 \pm 2 \) & \( 8.0 \pm 0.1 \) & \( \text{GeV}^{-2} \) \\
\( f_{10} \) & \( \frac{m^2}{m_D^2} (m^2L_5 + 2mB + 2C + D - E + 2F(\omega - 1) + m^2_{\omega}(x^2 - x)L_5) \) & \( 7.3 \pm 0.8 \) & \( 6.3 \pm 0.2 \) & \( \text{GeV}^{-2} \) \\
\( f_{11} \) & \( \frac{m^2}{m_D^2} (E + F - mB) \) & \( -7 \pm 1 \) & \( -6.5 \pm 0.5 \) & \\

TABLE I: The couplings \( g_4 = \{ g_i, h_i, f_i \} \) expressed as linear combinations of the basic scalar integrals listed in the Appendix. The numerical values are given by \( C \int_0^1 dx g_4 \) : the mean values are estimated by setting \( \Delta = 0.4 \) GeV \((\Delta = 0.6 \) GeV\) and varying the energy of \( \rho (\Phi) \) \n
\( \mathcal{F}^{\mu \nu} \), have been defined above. The numerical values are:

\[
\beta = -0.98
\]
\[
\lambda = +0.42 \text{ GeV}^{-1};
\]

where the field \( \rho^\mu \) and the tensor \( \mathcal{F}^{\mu \nu} \) have been defined above. The numerical values are:

\[
\beta = -0.98
\]
\[
\lambda = +0.42 \text{ GeV}^{-1};
\]
analogously $\beta$ and $\lambda$ for the $\mathcal{L}_{HH\Phi}$ are

\begin{align}
\beta &= -0.48 \\
\lambda &= +0.14 \text{ GeV}^{-1}.
\end{align}

As for the couplings $J/\psi D^{(*)} \bar{D}^{(*)}$, they have been extensively discussed in [6]. Here we just report the main results. Observe that

$$\mathcal{L}_{J/\psi HH} = ig_{J/\psi HH} \text{Tr}[\bar{H} \gamma_\mu H] J^\mu,$$

where $H$ can be any of the pairs $D D^*$ or $D_s D_s^*$ (neglecting $SU(3)$ breaking effects). As a consequence of the spin symmetry of the HQET we find:

$$g_{J/\psi D^* D^*} = g_{J/\psi DD}, \quad g_{J/\psi DD^*} = \frac{g_{J/\psi DD}}{m_D}.$$  

The numerical values are given by:

$$g_{J/\psi DD} = 8.0 \pm 0.5$$

$$g_{J/\psi DD^*} = 4.05 \pm 0.25 \text{ GeV}^{-1}$$

$$g_{J/\psi D^* D^*} = 8.0 \pm 0.5.$$

In Fig. 3 we report the cross section curves as functions of the $\sqrt{s}$ of the process for the three final states under consideration ($DD, DD^*, D^* D^*$). This calculation has been made by using the tri- and four-linear couplings quoted above, assuming their validity in the energy range $\sqrt{s} \approx 3.8 \div 4.5$, and computing the tree level diagrams for the process at hand (for a sketch of the diagrams involved see [1]). The dashed curves define the uncertainties bands obtained by varying $\Delta$ and $E_\rho$, as discussed in Tab I.

![FIG. 3: The cross sections of the processes $J/\psi \rho \rightarrow D^{(*)} \bar{D}^{(*)}$ and $J/\psi \Phi \rightarrow D_s^{(*)} \bar{D}_s^{(*)}$ on the left and on the right panel respectively. The dashed curves define the uncertainty bands obtained by varying $\Delta$ and $E_\rho$, as discussed above. Some of the reactions, the one initiated by $\rho$ giving $D \bar{D}$ in the final state and those initiated by $\phi$ giving $D_s \bar{D}_s$ and $D_s^* \bar{D}_s$ (or $D_s^* \bar{D}_s^*$, a sum of the two is taken) show the typical “exothermic” peak for zero $\rho(\phi)$ momentum. The remaining reactions show the usual threshold behaviour (endothermic).]

IV. SUMMARY

We have presented the calculation method of the effective couplings $J/\psi(X)D^{(*)} \bar{D}^{(*)}$, with $X = \rho, \Phi$, within the CQM model. The resulting cross section predictions, together with an estimate of the associated theoretical uncertainties, have been presented as functions of $\sqrt{s}$, showing values of the same order as the cross sections for $J/\psi \pi \rightarrow D^{(*)} \bar{D}^{(*)}$. This, given also the higher spin multiplicity of the $\rho$ meson with respect to pions, demonstrates the importance of the $\rho$ contribution to the $J/\psi$ absorption in the hot hadron gas, formed in peripheral heavy-ion collisions at SPS energy, as discussed thoroughly in Ref. [2]. Aiming at calculating thermal averages with $T \approx 170$ MeV, we didn’t discuss in the present paper the introduction of any arbitrary form factors since the exponential statistical weight acts as a cut off in the high energy tail.
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Appendix

In this Appendix are listed the $I_i$ and $L_i$ integrals occurring in the calculation and their linear combinations $A, B, ..., F$. These integrals have been computed adopting the proper time Schwinger regularization prescription, with cut-off $\mu = 0.3$ GeV ($0.5$ GeV when is present a strange quark), $\Lambda = 1.25$ GeV. In the following $N_c = 3$.

$$I_1 = iN_c \int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2 - \tilde{m}^2)}$$
$$= \frac{N_c}{16\pi^2} \tilde{m}^2 \Gamma \left(-1, \frac{\tilde{m}^2}{\lambda^2}, \frac{\tilde{m}^2}{\mu^2} \right)$$

$$I_3(\delta) = -iN_c \int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2 - \tilde{m}^2)} (v \cdot l + \delta)$$
$$= \frac{N_c}{16\pi^{3/2}} \int_{1/\Lambda^2}^{1/\mu^2} ds \ \frac{e^{-s(\tilde{m}^2 - \delta^2)}}{s^{3/2}} \left(1 + \text{Erf} (\delta \sqrt{s}) \right)$$

$$I_5(\delta, \delta', \omega) = iN_c \int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2 - \tilde{m}^2)} (v \cdot l + \delta) (v' \cdot l + \delta')$$
$$= \int_0^1 dy \ \frac{1}{1 + 2y^2(1 - \omega) + 2y(\omega - 1)} \times$$
$$\left[ \frac{6}{16\pi^{3/2}} \int_{1/\Lambda^2}^{1/\mu^2} ds \ \sigma e^{-s(\tilde{m}^2 - \sigma^2)} s^{-1/2} \left(1 + \text{Erf} (\sigma \sqrt{s}) \right) \right. \right.$$
$$\left. + \frac{6}{16\pi^2} \int_{1/\Lambda^2}^{1/\mu^2} ds \ e^{-s\sigma^2} s^{-1} \right],$$

in the last expression we have defined

$$\sigma \equiv \sigma(\delta, \delta', y, \omega) = \frac{\delta (1 - y) + \delta' y}{\sqrt{1 + 2(\omega - 1)y + 2(1 - \omega)y^2}}$$

In the previous equations $\tilde{m}^2$, $\delta$ and $\delta'$ are given by

$$\tilde{m}^2 = m^2 + xm^2_p (x - 1)$$

$$\delta = \Delta - x q \cdot v = \Delta - x E_\rho$$

$$\delta' = \Delta - x q \cdot v \omega = \Delta - x \omega E_\rho$$

with $m = 0.3$ GeV the constituent mass for light quark $u, d$. The expression of $\omega = v \cdot v'$ in the rest frame of $J/\psi$ is

$$\omega = \frac{m_{J/\psi}^2 + m_p^2 - m_H^2 - m_{H'}^2 + 2E_\rho m_{J/\psi}}{2m_H m_{H'}}$$

In the $I_1$ integral the gamma-function is

$$\Gamma (\alpha, x_0, x_1) = \int_{x_0}^{x_1} dt \ e^{-t} e^{t^{-1}}$$

while the error function is

$$\text{Erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z dx \ e^{-x^2}.$$
The $L_i$ integrals are defined in the following way:

$$L_i = \frac{\partial}{\partial \tilde{m}^2} I_i$$

and they are

$$L_1 = \frac{N_c}{16\pi^2} \int \frac{d^4l}{(2\pi)^4} \left[ \Gamma \left( -1, \frac{\tilde{m}^2}{\Lambda^2}, \mu^2 \right) + \tilde{m}^2 \frac{\partial}{\partial \tilde{m}^2} \Gamma \left( -1, \frac{\tilde{m}^2}{\Lambda^2}, \mu^2 \right) \right]$$

$$L_3(\delta) = -\frac{\partial}{\partial \tilde{m}^2} \frac{N_c}{16\pi^{3/2}} \int \frac{d^4l}{(2\pi)^4} \left[ \Gamma \left( -1, \frac{\tilde{m}^2}{\Lambda^2}, \mu^2 \right) - s^{-1/2} \left( 1 + \text{Erf} \left( \frac{s}{\sqrt{2}} \right) \right) \right]$$

$$L_5(\delta, \delta', \omega) = \frac{6}{16\pi^{3/2}} \int \frac{d^4l}{(2\pi)^4} \left[ \Gamma \left( -1, \frac{\tilde{m}^2}{\Lambda^2}, \mu^2 \right) - s^{-1/2} \left( 1 + \text{Erf} \left( \frac{s}{\sqrt{2}} \right) \right) \right]$$

The $A, B, C, \ldots$ functions are all function of $\delta, \delta'$ and $\omega$:

$$A = \frac{L_3(\delta') + \delta L_3(\delta) - (L_3(\delta') + \delta L_3(\delta) \omega)}{\omega^2 - 1}$$

$$B = \frac{L_3(\delta) + \delta' L_5(\delta, \delta', \omega) - (L_3(\delta) + \delta' L_5(\delta, \delta', \omega) \omega)}{\omega^2 - 1}$$

$$C = \frac{1}{2(\omega^2 - 1)} \left[ L_5(\delta, \delta', \omega) \delta^2 + (L_3(\delta') - (L_3(\delta') + 2\delta L_5(\delta, \delta', \omega)) \omega \delta') + \delta(L_3(\delta') + \delta L_5(\delta, \delta', \omega)) \right]$$

$$D = \frac{1}{2(\omega^2 - 1)^2} \left[ 2(L_1 + \delta L_3(\delta)) \omega^3 + (I_5(\delta, \delta', \omega) + 2\delta L_5(\delta, \delta', \omega)) \omega \delta') + \delta(L_3(\delta') + \delta L_5(\delta, \delta', \omega)) \right]$$

$$E = \frac{1}{2(\omega^2 - 1)^2} \left[ 2(L_1 + \delta' L_3(\delta')) \omega^3 + (I_5(\delta, \delta', \omega) + 2\delta L_5(\delta, \delta', \omega)) \delta^2 \right.\left. + \delta(L_3(\delta') + \delta L_5(\delta, \delta', \omega)) \delta^2 \right]$$

The determined couplings weight the scalar combinations of external particle momenta and polarizations in the combinations obtained by the loop computation in Eq. (46). We list below the scalar combinations $S_1 (D^{(i)} D^{(i)})$ together with the couplings they are weighted by, and with the effective Lagrangian interactions $\mathcal{L}$ one can write down for them. These terms describe the four-linear diagrams in Fig. 1; for the actual cross section computation one has to add also the $t$ and $u$ channels.
\[ S(DD) \]

\[
\begin{align*}
q \cdot p_2 \eta \cdot p_1 \epsilon \cdot p_1 & \quad -g_1 \quad -J_\alpha \partial_\mu \rho_\nu \partial^\mu D \partial^\nu \bar{D} \\
q \cdot p_2 \eta \cdot p_1 \epsilon \cdot p_1 & \quad g_2 \quad J_\alpha \partial_\mu \rho_\nu \partial^\mu \bar{D} \partial^\nu D \\
q \cdot \rho_1 \epsilon \cdot p_2 \eta \cdot p_1 & \quad g_1 \quad -J_\alpha \partial_\mu \rho_\nu \partial^\mu \bar{D} \partial^\nu D \\
q \cdot \rho_1 \epsilon \cdot p_2 \eta \cdot p_1 & \quad -g_2 \quad J_\alpha \partial_\mu \rho_\nu \partial^\mu D \partial^\nu \bar{D} \\
\eta \cdot q \epsilon \cdot p_1 & \quad g_3 \quad -\partial_\mu \rho_\nu \bar{J}^\mu \partial^\nu D \\
\eta \cdot p_1 \epsilon \cdot \eta & \quad g_4 \quad -J_\mu \rho_\nu \partial^\mu \bar{D} \partial^\nu D \\
\eta \cdot p_2 \epsilon \cdot \eta & \quad g_5 \quad J \cdot \partial \bar{D} \rho \cdot \partial D \\
q \cdot \eta \epsilon \cdot p_2 & \quad -g_6 \quad \partial_\mu \rho_\nu \bar{J}^\mu \partial^\nu \bar{D} \\
q \cdot p_2 \epsilon \cdot \eta & \quad g_6 \quad \partial_\mu \rho_\nu \bar{J}^\mu \partial^\nu \bar{D} \\
\epsilon \cdot \rho_2 \eta \cdot p_1 & \quad g_7 \quad J \cdot \partial \bar{D} \rho \cdot \partial \bar{D} \\
\epsilon \cdot p_2 \eta \cdot p_2 & \quad g_8 \quad -J_\mu \rho_\nu \partial^\mu \bar{D} \partial^\nu \bar{D} \\
\eta \cdot \epsilon & \quad g_9 \quad J \cdot \rho D \bar{D}
\end{align*}
\]

\[ S(DD^*) \]

\[
\begin{align*}
i\varepsilon_{\alpha\beta\gamma\delta} q^\alpha \partial^\gamma \eta^\epsilon \eta^\delta & \quad h_1 \quad h_1 & \quad -i\varepsilon_{\alpha\beta\gamma\delta} \partial^\alpha \rho^\beta J^\gamma D^\delta \partial^\epsilon \bar{D} \\
i\varepsilon_{\alpha\beta\gamma\delta} q^\alpha \partial^\gamma \eta^\epsilon \eta^\delta & \quad 1 \quad \frac{1}{m_D} \quad h_2 \quad \frac{1}{m_D} \quad h_2 & \quad -i\varepsilon_{\alpha\beta\gamma\delta} \partial^\alpha \rho^\beta \bar{J}^\gamma D^* \partial^\epsilon \bar{D} \\
i\varepsilon_{\alpha\beta\gamma\delta} q^\alpha \partial^\gamma \eta^\epsilon \eta^\delta & \quad 1 \quad \frac{1}{m_D} \quad h_2 \quad \frac{1}{m_D} \quad h_2 & \quad -i\varepsilon_{\alpha\beta\gamma\delta} \partial^\alpha \rho^\beta \bar{J}^\gamma \partial^\epsilon D \partial^\delta \bar{D} \\
i\varepsilon_{\alpha\beta\gamma\delta} q^\alpha \partial^\gamma \eta^\epsilon \eta^\delta & \quad 1 \quad \frac{1}{m_D} \quad h_2 \quad \frac{1}{m_D} \quad h_2 & \quad -i\varepsilon_{\alpha\beta\gamma\delta} \partial^\alpha \rho^\beta \partial^\epsilon \bar{D}^* \partial^\delta \bar{D} \\
i\varepsilon_{\alpha\beta\gamma\delta} q^\alpha \partial^\gamma \eta^\epsilon \eta^\delta & \quad 1 \quad \frac{1}{m_D} \quad h_2 \quad \frac{1}{m_D} \quad h_2 & \quad -i\varepsilon_{\alpha\beta\gamma\delta} \partial^\alpha \rho^\beta \partial^\epsilon \bar{D}^* \partial^\delta \bar{D} \\
i\varepsilon_{\alpha\beta\gamma\delta} q^\alpha \partial^\gamma \eta^\epsilon \eta^\delta & \quad 1 \quad \frac{1}{m_D} \quad h_2 \quad \frac{1}{m_D} \quad h_2 & \quad -i\varepsilon_{\alpha\beta\gamma\delta} \partial^\alpha \rho^\beta \partial^\epsilon \bar{D}^* \partial^\delta \bar{D} \\
i\varepsilon_{\alpha\beta\gamma\delta} q^\alpha \partial^\gamma \eta^\epsilon \eta^\delta & \quad 1 \quad \frac{1}{m_D} \quad h_2 \quad \frac{1}{m_D} \quad h_2 & \quad -i\varepsilon_{\alpha\beta\gamma\delta} \partial^\alpha \rho^\beta \partial^\epsilon \bar{D}^* \partial^\delta \bar{D} \\
i\varepsilon_{\alpha\beta\gamma\delta} q^\alpha \partial^\gamma \eta^\epsilon \eta^\delta & \quad 1 \quad \frac{1}{m_D} \quad h_2 \quad \frac{1}{m_D} \quad h_2 & \quad -i\varepsilon_{\alpha\beta\gamma\delta} \partial^\alpha \rho^\beta \partial^\epsilon \bar{D}^* \partial^\delta \bar{D} \\
i\varepsilon_{\alpha\beta\gamma\delta} q^\alpha \partial^\gamma \eta^\epsilon \eta^\delta & \quad 1 \quad \frac{1}{m_D} \quad h_2 \quad \frac{1}{m_D} \quad h_2 & \quad -i\varepsilon_{\alpha\beta\gamma\delta} \partial^\alpha \rho^\beta \partial^\epsilon \bar{D}^* \partial^\delta \bar{D} \\
\end{align*}
\]
\[
S(D^*D^*) = f(L)
\]

\[
\begin{align*}
q \cdot \eta & \cdot \eta_1 \eta_2 \cdot p_1 & f_1 & -\partial_\mu \rho_\alpha J^\mu \partial_\alpha D^\nu D^\alpha \\
q \cdot \eta & \cdot \eta_2 \eta_1 \cdot p_2 & f_2 & -\partial_\mu \rho_\alpha J^\mu \partial_\alpha D^\nu D^\alpha \\
q \cdot \eta & \cdot p_1 \eta_1 \eta_2 \cdot p_2 & f_3 & -\partial_\mu \rho_\alpha J^\mu \partial_\alpha D^\nu D^\alpha \\
q \cdot \eta & \cdot \eta_2 \eta_1 \cdot p_1 & f_4 & -\partial_\mu \rho_\alpha J^\mu \partial_\alpha D^\nu D^\alpha \\
q \cdot \eta & \cdot p_1 \eta_1 \cdot p_2 & f_5 & -\partial_\mu \rho_\alpha J^\mu \partial_\alpha D^\nu D^\alpha \\
q \cdot \eta & \cdot \eta_2 \eta_1 \cdot p_2 & f_6 & -\partial_\mu \rho_\alpha J^\mu \partial_\alpha D^\nu D^\alpha \\
q \cdot \eta & \cdot p_1 \eta_1 \cdot \eta_2 & f_7 & -\partial_\mu \rho_\alpha J^\mu \partial_\alpha D^\nu D^\alpha \\
q \cdot \eta & \cdot \eta_2 \eta_1 \cdot \eta_2 & f_8 & -\partial_\mu \rho_\alpha J^\mu \partial_\alpha D^\nu D^\alpha \\
q \cdot \eta & \cdot p_1 \eta_1 \cdot p_2 & f_9 & -\partial_\mu \rho_\alpha J^\mu \partial_\alpha D^\nu D^\alpha \\
q \cdot \eta & \cdot \eta_2 \eta_1 \cdot p_2 & f_{10} & -\partial_\mu \rho_\alpha J^\mu \partial_\alpha D^\nu D^\alpha \\
\end{align*}
\]
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