On pattern formation in ferrocolloid convection

A Bozhko¹, G Putin¹, T Tynjälä²,³, M Dabagh Meshin², P Jalali²

¹Department of Physics, Perm State University, Bukirev Str. 15, 614990 Perm Russia
²Department of Energy and Environmental Technology, Lappeenranta University of Technology, 53851 Lappeenranta, Finland
tynjala@lut.fi

Abstract. Experimental studies and numerical simulations of stability of buoyancy-driven flows in a ferrocolloid for the cases of horizontal and inclined vertical orientation of a thin cylindrical cavity are performed. The influence of a homogeneous longitudinal magnetic field on convective instability and spatio-temporal patterns were also investigated. In the case of ferrocolloids the gradients of magnetic permeability may arise due to both temperature and particle concentration gradients. The particle mass flux in a classical form is summarized from the translation diffusion coefficient and the thermal diffusion ratio. However, the explanation for the observed self-oscillation regimes in magnetic fluid for the cavities of sufficiently large thickness is conditioned by the competition of density variations originating from the fluid thermal expansion and barometric sedimentation. The results prove that a uniform longitudinal magnetic field allows to control the stability and the shape of secondary convection motions at inclined orientation of layer. In a ferrocolloid the repeated transients involving localized roll convection and pure shear flow took place. Under action of uniform longitudinal magnetic field orientated perpendicular to flux velocity of shear motion on such long-wave transients can lead to complicated types of chaotic localized states or solitary vortices.

1. Introduction

Experiments and numerical simulations were conducted in order to study convection patterns in a layer of ferrocolloid heated from below and subjected to uniform magnetic field. The ferrocolloid contains single domain magnetite particles suspended in kerosene carrier liquid and has the susceptibility thousand times higher then natural media. In the case of ferrocolloids the gradients of magnetic permeability may arise due to both temperature and particle concentration gradients. The particle mass flux in a classical form is summarized from the translation diffusion coefficient and the thermal diffusion ratio [1]. However, the explanation for the observed self-oscillation regimes in magnetic fluid for the cavities of sufficiently large thickness (several to tens of millimeters) is conditioned by the competition of density variations originating from the fluid thermal expansion and barometric sedimentation. As known, thermally driven shear flow in an inclined layer draws up convection rolls in the direction of inclination. Furthermore, in an inclined layer the direction of

³ Author to whom any correspondence should be addressed.
gravity sedimentation is not aligned parallel to fluid thermal expansion, thermodiffusion and magnetodiffusion, which introduces more complexity to the problem by breaking the symmetry and leading to three dimensional flow field.

2. Problem description

2.1. Experimental setup

Geometry of the studied problem is shown in figure 1. Magnetic fluid is held in a cylindrical cavity heated from bottom and cooled from top. Experiments were performed with a kerosene-based magnetic fluid. Properties of the studied fluid are shown in table 1. In this study the alignment angle $\theta$ of the plate is varied from horizontal $\theta = 0^\circ$ to $25^\circ$. The thickness of the layer $h = 3.5 \text{ mm}$ and diameter $D = 75 \text{ mm}$.

![Figure 1. Geometry of the studied problem.](image)

**Table 1.** Properties of the studied ferrocolloid. Fluid consists of magnetite particles suspended in a kerosene carrier.

| Property                              | Symbol | Value       |
|--------------------------------------|--------|-------------|
| Diameter of particles                | $d_p$  | 11 nm       |
| Density of kerosene                  | $\rho_c$ | 800 kg/m$^3$ |
| Density of magnetite                 | $\rho_p$ | 5000 kg/m$^3$ |
| Density of mixture                   | $\rho_m$ | 1250 kg/m$^3$ |
| Kinematic viscosity of mixture       | $\nu_m$ | $6.4 \times 10^{-6} \text{ m/s}$ |
| Conductivity                         | $\lambda_m$ | 0.22 W/mK   |
| Thermal conductivity                 | $\alpha_m$ | $5.0 \times 10^8 \text{ m/s}^2$ |
| Diffusion coefficient                | $D$    | $2.0 \times 10^{-11} \text{ m}^2/\text{s}$ |
| Coefficient of thermal expansion     | $\beta$ | $0.8 \times 10^{-3} \text{ 1/K}$ |
| Prandtl number                       | $Pr = \nu/\alpha$ | 128         |
| Lewis number                         | $Le = D/\alpha$ | $4 \times 10^4$ |
| Schmidt number                       | $Sc = \nu/D$ | $3 \times 10^3$ |
| Saturation magnetization             | $M_s$  | 48 kA/m     |
| Vacuum permeability                  | $\mu_0$ | $4\pi \times 10^{-7} \text{ H/m}$ |
| Particle magnetic moment             | $m_p$  | $2.5 \times 10^{-19} \text{ Am}^2$ |

In the experimental device [2] the magnetic fluid is confined between copper and transparent heat exchangers from below and above, respectively. The circular sidewall of the layer was made of plexiglas. The patterns were visualized by the liquid crystal sheet, which undergoes its entire color change at temperature interval from 24°C to 27°C. The temperature difference $\Delta T$ across the layer was
measured with the help of thermocouples in order to determine the critical temperature difference for the onset of convection. Homogeneous magnetic field was generated by a Helmholtz coil arrangement.

2.2. Simulation model

In the simulation model [3] the magnetic fluid was treated as a two-phase mixture of magnetic particles in a carrier phase. The mixture model uses a single phase approximation and can be considered as an intermediate between the single phase approximation and full set of equations governing the dynamics of multiphase flow [3]. This kind of mixture model or drift-flux model as they called it was first suggested by Zuber and Findlay [4]. In the model the conservation equations (1) – (3) for mass, momentum and energy are solved for the mixture phase and in addition the mass conservation equation is solved separately for the dispersed phase (4).

\[
\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m u_m) = 0 \tag{1}
\]

\[
\frac{\partial}{\partial t} (\rho_m u_m) + \nabla \cdot (\rho_m u_m u_m) = -\nabla \rho_m + \eta_m \nabla^2 u_m - \nabla \left( \alpha_p \rho_p u_{Mp} + \alpha_c \rho_c u_{Mc} \right) + \rho_m g + \frac{\mu_0 m_p}{V_p} L(\xi) \nabla H \tag{2}
\]

\[
\frac{\partial}{\partial t} (\rho_m c_{mT}) + \nabla \cdot \left[ \left( \alpha_p \rho_p u_{Mp} + \alpha_c \rho_c u_{Mc} \right) + \rho_m g \right] = \nabla \cdot (\lambda_m \nabla T) \tag{3}
\]

\[
\frac{\partial}{\partial t} \left( \alpha_p \rho_p \right) + \nabla \cdot \left[ \rho_p \alpha_p \left( u_m + \frac{\alpha_c \rho_c}{\rho_m} (u_p - u_c) \right) \right] = 0 \tag{4}
\]

Subscripts \( m, p \) and \( c \) in above equations stands for mixture, particle and carrier phase, respectively. Mixture properties used in the simulations are calculated as volume fraction weighted averages of component values. Eg. the mixture density is calculated form equation (5).

\[
\rho_m = \alpha_p \rho_p + \alpha_c \rho_c \tag{5}
\]

Similarly for the mixture velocity,

\[
u_m = \frac{1}{\rho_m} \left( \alpha_p \rho_p u_p + \alpha_c \rho_c u_c \right) . \tag{6}
\]

In momentum equation (2) \( u_{Mp} = (u_p - u_m) \) and \( u_{Mc} = (u_c - u_m) \) are the diffusion velocities of particles and carrier phases, respectively, \( m_p \) is the particle magnetic moment, \( \mu_0 \) vacuum permeability, \( L(\xi) = \coth(\xi) - 1/\xi \) is the Langevin function, where Langevin parameter \( \xi = \mu_0 m_p H / (k_B T) \) is the ratio of magnetic and thermal energies.

Magnetic field inside the simulation domain was calculated from Maxwell equations written for electrically non-conducting fluid as

\[
\nabla \cdot B = 0 \quad \nabla \times H = 0 \tag{7}
\]
Introducing magnetic scalar potential defined as \( \nabla \phi_m = -H \) and using the definition of magnetic induction \( B = \mu_0 (H + M) \), together with magnetic equation of state (8) linearized about the applied magnetic field \( H_0 \), as first suggested by Finlayson [5]

\[
M = M_0 + \frac{\partial M}{\partial H} (H - H_0) + \frac{\partial M}{\partial T} (T - T_0) + \frac{\partial M}{\partial \alpha_p} (\alpha_p - \alpha_{p,0}),
\]

the flux function for magnetic scalar potential can be written as

\[
\nabla \cdot \left[ \left( 1 + \frac{\partial M}{\partial H} \right) \nabla \phi_m \right] = \nabla \cdot \left[ \frac{\partial M}{\partial T} (T - T_0) + \frac{\partial M}{\partial \alpha_p} \nabla (\alpha_p - \alpha_{p,0}) \right]
\]

The slip of the magnetic particles relative to carrier fluid was estimated balancing the drag to magnetic and gravitational body forces affecting the particles, equation (10),

\[
u_s = \frac{m_p \bar{L}(\xi)}{3 \pi \mu_c d_p} \nabla H + \frac{d_p^2 \rho_p - \rho_s}{18 \pi \mu_c} g
\]

Numerical simulations were conducted using commercial finite volume simulation code, Fluent, with user defined functions added for the magnetic terms. Second-order upwind scheme was used for continuity, momentum and energy equations, whereas the first order scheme was used for the calculation of magnetic potential. Pressure values at the faces are evaluated using staggered control volume around each face, so called PRESTO scheme [6], [7]. As an iteration procedure to solve conservation equations, segregated semi-implicit method for pressure linked equations (SIMPLE) [7], [8], was used. Constant temperature boundary conditions were applied for bottom and top plate of the cylinder and sidewalls were assumed perfectly insulated. To create constant magnetic field strength \( H \) in \( x \)-direction, boundary condition \( \phi_m = -H x \), was applied for the magnetic scalar potential at the sidewalls.

3. Results and discussion

Experimental investigations revealed three clearly distinguishable regions based on inclination angle, magnetic field strength and temperature difference. Figure 2 (a) shows the case where convection rolls are aligned with the applied field. It has been shown [9] that uniform longitudinal magnetic field causes only re-orientation of rolls but does not influence on convective instability in horizontal fluid layer. Figure 2 (b) presents moderately inclined layer in the absence of magnetic field. In this case the convection rolls are aligned with the direction of shear flow. Figure 2 (c) shows the case of pure shear flow observed for large inclination angles and/or small temperature differences.

![Figure 2](image-url)

**Figure 2.** Schematic figures and liquid crystal photos of convective patterns. (a) Horizontal cavity where the convection rolls are aligned with the applied field, (b) inclined layer and weak (or absent) magnetic field, where the orientation of convection rolls is determined by the shear flow, and (c) pure shear flow for large inclination angles and/or small temperature differences.
In this study the focus was in the area of moderate inclination angles \( \theta \leq 25^\circ \) and weak magnetic field strengths. In this region, intermediate between figure 2 (a) and (b), neither effect of magnetic field nor inclination angle is dominant. Interaction between two competing mechanisms leads to complicated patterns and chaotic localized states and solitary vortices were observed in the experiments. Figure 3 shows one example of localized pulses observed experimentally. In this case the inclination angle \( \theta = 25^\circ \), the temperature difference is about two times critical temperature difference for the onset of Rayleigh convection for horizontal cavity, and strength of applied magnetic field \( H = 1 \) kA/m.

**Figure 3.** Experimentally observed localized pulses close to intersection of different stability areas. Time interval between the snapshots is 2 minutes, inclination angle \( \theta = 25^\circ \), \( \Delta T/\Delta T_{cr} \approx 2 \) and \( H = 1 \) kA/m. Here the critical temperature difference for the onset of Rayleigh convection for horizontal cavity \( \Delta T_{cr} = 5.1 \) K.

Figure 4 presents the time advancement of convective patterns for the case \( \theta = 15^\circ \), \( \Delta T/\Delta T_{cr} = 2.4 \) and \( H = 0 \). Slow drift of longitudinal rolls in direction perpendicular to their axes towards the lateral sides can be observed. The basic unicellular motion draws up convection rolls in the direction of inclination. In particular, the drift arises due to the appearance and the evolution of defects. In figure 4 the structural defect displaces and moves along the rolls so-called dislocation of climbing. Then, it is pinned by a neighboring roll pairs, and a new defect begins to travel perpendicular to the rolls - dislocation of gliding. The identical defect motion was observed in numerical simulations (figure 5). Temperature drop from cool (black) to warm (white) liquid is approximately 3 K. Each light (dark) strip in photograph corresponds to the same handedness of two neighboring rolls.

**Figure 4.** Defect motion in the absence of field at inclination angle \( \theta = 15^\circ \) at \( \Delta T/\Delta T_{cr} = 2.4 \).
Figure 5. Simulated defect motion at $\theta = 15^\circ$, $\Delta T/\Delta T_{cr} \approx 2$, $H = 0$.

In contrast to the case of horizontal cavity, where the longitudinal magnetic field has no effect to the onset of convection in a tilted layer the horizontal longitudinal magnetic field extinguishes the convection perturbations along the field direction and stabilizes Rayleigh flows. Attenuation of convective motion is presented in figures 6 and 7, where shown are experimental results and simulated temperature contours at the cylinder midplane after turning on the horizontal longitudinal magnetic field.

Figure 6 Breaking of convection rolls as the strength of magnetic field is increased from 0 to 1.5kA/m at $\theta = 15^\circ$, $\Delta T/\Delta T_{cr} \approx 2$.

Figure 7. Stabilization of Rayleigh convection in a tilted layer $\theta = 15^\circ$, $\Delta T/\Delta T_{cr} \approx 2$. At time $t = 0$ s horizontal magnetic field $H = 8$ kA/m was turned on. Shown are the temperature contours at cylinder midplane for times $t = 5$, 20 and 35 s.

Numerical simulations were conducted with parameters close to those, with which in the experiments the localized states, such as shown in figure 3, were observed. Figure 8 shows simulation results for inclination angle $\theta = 25^\circ$, $\Delta T/\Delta T_{cr} \approx 2$ and $H = 1.5$ kA/m. Simulations could not reproduce
attenuation of convective motion and appearance of solitary convection rolls seen in the experiments and shown in figures 3 and 9.

**Figure 8** Simulated time advancement of convection patterns for inclination angle $\theta = 25^\circ$, $\Delta T/\Delta T_c \approx 2$ and $H = 1.5$ kA/m. Time between the snapshots is 30 s.

(a) (b) (c)

**Figure 9** Experimentally observed solitary vortices at $\theta = 25^\circ$, $\Delta T/\Delta T_c \approx 2$ and $H = 8$ kA/m. Time between (a) and (b) is 5 minutes and between (b) and (c) 1 minute.

4. Conclusions

When the magnetic field is small enough the hydrodynamic orientation mechanism in the inclined layer predominates over the demagnetizing one, and the axis of convection rolls are lined up along the shear flow, i.e. perpendicular to the imposed magnetic field. On the other hand at strong magnetic fields and not large inclination angles, the demagnetizing effect results in a horizontal orientation of the convection rolls. Among the various wave regimes which take place in the ferrocolloid convection, one should particularly note the chaotic localized states. The shape of these states depends on values of control parameters $\Delta T$, $\theta$, and magnetic field $H$. The typical patterns are presented in figures 3 and 9. Under transition from hydrodynamic to magnetic mechanisms of convection rolls orientation a comparable contribution of both mechanisms leads to formation of different types of chaotic localized states or pulses. Usually, these pulses appear and die at irregular locations and times, have unique forms, and vary irregularly in dimension. Even in the absence of magnetic field the strong amplitude modulation of convection rolls can lead to attenuation of roll motion partly or in the entire cell due to large concentration inhomogeneities of colloidal particles, stratified by gravity.

Numerical simulations for the horizontal plate and for inclined plate in the absence of magnetic field reproduced expected flow patterns. For inclined plate the simulations could not reproduce experimentally observed localized states and solitary vortices. Comparison of experiments and simulations was done only qualitatively since quantitative comparison is difficult due to uncertainty of fluid properties. Indeed, experimentally observed chaotic states should not appear in homogeneous and stable ferrocolloids. Concentration inhomogenities within the fluid are likely to be the cause for observed phenomena. Size of the suspended particles and particle clusters is critical parameter for the
development of the density variations within the fluid. For example viscosity of ferrocolloid in magnetic field essentially depends on size of magnetic particles. On the other hand viscosity is key parameter in Grashof number $Gr = \beta \Delta T gh^3 / \nu$, a key dimensionless parameters determining the onset of convection. Viscosity is also present in characteristic time $\tau = h^2 / \nu$, for reaching the quasi-steady state in the system [10]. Moreover the particle size is crucial when terminal velocity and effect of Brownian motion are calculated. To gain better understanding from this complex system, more information about the mass transfer phenomena and cluster formation in ferrocolloids in the presence of magnetic field should be obtained. To this end, experiments in the weightlessness could produce information, where the effect of uncontrollable gravity sedimentation could be minimized.

Acknowledgments

The research described in this publication was made possible in part by Russian Foundation for Basic Research under grant 04-01-00586, Finnish Academy grant 110852 and Award No. PE-009-0 CRDF.

References

[1] Blums E 1995 *J. Magn. Magn. Mat.* 149 111-15
[2] Bozhko A A and Putin G F 2003 *Magnetohydrodynamics* 39 147-68
[3] Tynjälä T 2005 *Theoretical and Numerical Study of Thermomagnetic Convection in Magnetic Fluids (D. Sc. Thesis, Series Acta Universitatis Lappeenrantaensis 220)* (Lappeenranta: Lappeenranta University of Technology)
[4] Zuber N and Findlay J A 1965 *J. Heat Trans. – T. ASME* 87 453-68.
[5] Finlayson B A 1970 *J. Fluid Mech.* 40 753-67
[6] Fluent Inc. 2005 *Fluent 6.2 Users Guide* (Lebanon: Fluent Inc.)
[7] Patankar S V 1980 *Numerical Heat Transfer and Fluid Flow* (Washington: Hemisphere Publishing Co.)
[8] Patankar S V and Spalding D B 1972 *Int. J. Heat Mass Tran.* 15 1787-806
[9] Schwab L and Stierstadt K 1987 *J. Magn. Magn. Mat.* 65 315-16
[10] Odenbach S 1995 *Adv. Space. Res.* 16 99-104