The Cosmological Slingshot Scenario in details

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We generalize the Cosmological Slingshot Scenario for a Slingshot brane moving in a Klebanov-Strassler throat. We show that the horizon and isotropy problems of standard cosmology are avoided, while the flatness problem is acceptably alleviated. Regarding the primordial perturbations, we identify their vacuum state and elucidate the evolution from the quantum to the classical regimes. Also, we calculate their exact power spectrum showing its compatibility with current data. We discuss the bouncing solution from a four dimensional point of view. In this framework the radial and angular motion of the Slingshot brane are described by two scalar fields. We show that the bouncing solution for the scale factor in String frame is mapped into a monotonically increasing (in conformal time) solution in the Einstein frame. We finally discuss about the regularity of the geometry in Einstein frame.

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I. INTRODUCTION

The Cosmological Slingshot Scenario, shortly the “Slingshot”, is a proposal for the cosmic early-time evolution in the String Theory context. According to that, our Universe is a D3-brane moving in a String Theory background of the form $M^4 \times K^6$. $M^4$ is a “warped” Minkowskian space-time and $K^6$ is a compact Calabi-Yau (CY) space. The latter includes a “throat” sourced by a stack of a large number ($N$) of other D3 branes. The Slingshot is characterized by a non-trivial orbital motion of the Universe in the compact space around the stack of D3-branes. If back-reactions due to the brane motion can be neglected (probe brane approximation), a brane observer measures a 3 + 1 dimensional induced metric in terms of the brane embedding. This metric defines a cosmological brane evolution commonly called Mirage Cosmology.

The early-time evolution (i.e. well before nucleosynthesis) corresponds to the motion of the D3-brane deep into the throat (Slingshot era) moving towards the hat of the compact space. The late-time cosmology starts when the D3-brane reaches the hat of the CY. There, the probe brane approximation breaks down and local gravity à la Randall-Sundrum dominates the cosmological evolution. Since $N$ is taken to be large, close to the stack, the probe brane approximation of can be used. Under this approximation, the Slingshot brane observer experiences, at early-times, a non-singular bouncing cosmology in the String frame. In Einstein frame on the other hand, the induced cosmology is monotonically increasing (in conformal time). Moreover, assuming that in the past infinity the brane starts in the hat of the Calabi-Yau without modifying its regular structure, no past singularities can ever be present in the both frames, as we shall argue in the paper. This scenario is then a realization of the emergent Universe idea, although in a technically very different way. Additionally, as we shall show later on, the brane induced cosmology naturally avoids the Standard Cosmology problems (i.e. horizon, isotropy and flatness).

This model can be included in a more general class of bouncing/cyclic models that have successfully tried to address some of the Standard Cosmology problems, such that ekpyrotic models, phantom based cyclic models and emergent cyclic models. For a review on bouncing cosmologies see.
The plan of the paper is as follows. In section II we show how the standard cosmological problems are solved when the slingshot brane is moving in a Klebanov-Strassler throat. In section III we calculate the primordial perturbation spectrum and the corresponding spectral index. In section IV we write the effective 4D theory, which reproduces in the string frame the mirage solution, we discuss the Slingshot in the Einstein frame and we finally conclude in section V.

II. THE COSMOLOGICAL SLINGSHOT SCENARIO IN A KLEBANOV-STRASSLER THROAT

To make the discussion concrete, we shall consider a probe D3-brane moving in the Klebanov-Strassler (KS) throat of a Calabi-Yau (CY) compact manifold. The KS geometry is obtained by putting together a stack of $N$ D3-branes and $M$ fractional branes at the apex of a conifold. Then, the conical singularity is deformed by blowing up a 3-sphere at the tip. We restrict the probe brane motion to a region far from the tip of the KS geometry. In this case, the KS geometry can be well approximated by the Klebanov-Tseytlin (KT) metric.

In the KS background, the minkowskian coordinates of the string frame the mirage solution, we discuss the Slingshot in the Einstein frame and we finally conclude in section V.

The dynamics of a probe brane is governed by the Dirac-Born-Infeld action with a Wess-Zumino term (that takes into account the coupling to the bulk Ramond-Ramond five-form with the brane charge). The supergravity approximation is valid as long as the curvature radius of the solution is large compared to the string length $l_s$. String perturbation theory on the other hand requires all other fields on the brane are switched off and matter is created later. The sign of the Wess-Zumino term has been chosen to represent a slow (adiabatic) brane motion ($\mathbf{h}\mathbf{r}\mathbf{'} \ll 2$) denotes a derivative with respect to $r$.

In (5) we used the symmetries of the background to write $r^2 = r^2 + r^2 \Omega_5^2$, where $\Omega_5$ parameterizes one of the $\mathbb{T}_{1,1}$ angles.

The equations of motion of the probe brane follow from varying the action with respect to $r, \Omega_5$

$$r'' - r\Omega_5^2 = 0, \quad (r^2 \Omega_5')' = 0. \quad (6)$$

First integrals of this system are provided by

$$r'^2 + \frac{J^2}{r^2} = 2U, \quad r^2 \Omega_5' = J. \quad (7)$$

In (3) we take into account that all other fields on the brane are switched off and matter is created later. The sign of the Wess-Zumino term has been chosen to represent a D3-brane in the mostly plus convention for the metric, and $T_3 = 1/(2\pi)^3 g_s l_s^4$ is the tension of the probe. The probe brane is extended parallel to the stack of D3s, so that it looks like a point particle moving in transverse space (for inhomogeneous embedding see [17]). In the static gauge we identify the minkowskian coordinates of $ds^2_{\text{Mink}}$ in eq. (1) with the brane world-volume coordinates. The resulting induced metric is

$$ds^2 = h^{-1/2} \left[-(1-hr'^2)\, d\eta^2 + d\vec{x} \cdot d\vec{x}\right], \quad (4)$$

where a prime (') denotes a derivative with respect to $\eta$ and $\vec{r}$ indicates the position of the brane in the bulk. In the case of slow (adiabatic) brane motion ($hr'^2 \ll 1$) the brane action turns out to be

$$S = \frac{T_3}{2} \int d^4 x \left(r'^2 + r^2 \Omega_5^2\right). \quad (5)$$

In $S$ we used the symmetries of the background to write $r'^2 = r'^2 + r^2 \Omega_5^2$, where $\Omega_5$ parameterizes one of the $\mathbb{T}_{1,1}$ angles.

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First integrals of this system are provided by

$$r'^2 + \frac{J^2}{r^2} = 2U, \quad r^2 \Omega_5' = J. \quad (7)$$
and eq.(6) is then solved by the bouncing solution

\[ r = \sqrt{2U \eta^2 + \frac{J^2}{2U}}, \quad \Omega_5 = \arctan \left( \frac{2U}{J} \eta \right). \]

A constant of integration has been fixed by requiring that at \( \eta = 0 \) the probe is at the turning point \( r_{\text{min}} = J/\sqrt{2U} \).

One can easily find that the non-relativistic approximation is accurate whenever \( J^4 \gg 8U^3L^4 \). In this approximation, the induced metric on the brane reads

\[ ds_i^2 = h^{-1/2} ds_{\text{Mink}}^2. \]

An observer on the brane will therefore experience a Friedman-Robertson-Walker metric with scale factor

\[ a(\eta) = h^{-1/4} = r/(L \ln^{1/4}(r/r_s)), \]

where \( \eta \) is the observer conformal time. Since \( r(\eta) \) has a turning point, it is easy to see that the same happens to \( a(\eta) \), generating a nonsingular bouncing cosmology.

The model is completed by smoothly pasting this Mirage era to a local gravity driven late evolution when the brane reaches the top of the CY and gravity becomes localized \( \text{"a la Randall-Sundrum} \). There, the standard late time evolution of the observed Universe, is supposed to be well reproduced by the brane dynamics. This assumption involves a transition from a mirage dominated era with a moving brane without any matter, into a local gravity dominated era with a static brane and matter fields excited on it. This transition has to be understood as an analogous of the reheating process in standard inflationary models. It entails a dynamical mechanism under which the kinetic energy of the brane is passed to matter fields and the brane motion is stabilized by acquiring a mass for the radion and the angular scalar fields. The exact description of this dynamics as well as the robustness of our predictions for physical observable is an open point of the model which is left for future research.

### A. The Standard Cosmological Problems

In [1] we showed how the standard cosmological problems are solved for the simpler version of the Slingshot scenario, in which the probe brane moves in an \( AdS_5 \) throat. In what follows, we will complete the proof, extending it to the KS case. An important ingredient in our argument is that the scale factor for a brane moving in a KS throat (10) can be rewritten as a conformal re-scaling of the corresponding scale factor for a brane moving in \( AdS_5 \), namely \( a_{\text{AdS}} = r/L \). Indeed, we can write (10) as

\[ a = \Omega_{KS}(a_{\text{AdS}}) a_{\text{AdS}} \quad \text{with} \quad \Omega_{KS}(a_{\text{AdS}}) = \log^{-1/4}(a_{\text{AdS}}L/r_s). \]

Note that the above scaling should not be considered as a change of frame but just as a relation between two different Friedman-Robertson-Walker scale factors (for example the Lagrangian will not be re-scaled as one would do for a conformal transformation). It should be kept in mind that our approximations are valid whenever \( a_{\text{AdS}} \gg r_s/L \).

Under such a re-scaling, the Hubble constant changes as

\[ H = \left( 1 + a_{\text{AdS}} \frac{d\ln \Omega_{KS}}{da_{\text{AdS}}} \right) H_{\text{AdS}}. \]

\( H_{\text{AdS}} \) is the mirage Hubble constant of a brane moving in an \( AdS_5 \) throat [1], namely

\[ H_{\text{AdS}}^2 = -\frac{1}{L^2} \left[ \frac{\kappa}{a_{\text{AdS}}^3} - \frac{2U}{a_{\text{AdS}}^4} + \frac{(J/L)^2}{a_{\text{AdS}}^6} + \frac{\kappa(J/L)^2 - U^2}{a_{\text{AdS}}^8} \right]. \]

We are now ready to study how standard cosmological problems are solved in the Slingshot scenario.

#### A. Horizon

As explained in [1], in the KS throat the probe brane experiences a bounce in the String frame. This immediately ensures that horizon problem is solved. To check this explicitly, we write the comoving horizon as

\[ \Delta \eta = \int_{\eta_i}^{\eta_f} d\eta \]
4

FIG. 1: The function $\rho_{\text{shear}}$ in the String frame (solid line) and Einstein frame (dashed line) as a function of $r/r_s$. The vertical axes has been plotted up to a proportionality constant that can be adjusted in order to have small shear today. We note that, since our Universe brane moves in regions well to the right of the horizontal axes $r \gg r_s$, shear never dominates.

where $\eta_i$ is the smallest conformal time. To solve horizon problem it is required that $\Delta \eta > H_0^{-1}$. Since we have $\eta_i \to -\infty$ due to the bounce, this condition is trivially satisfied.

B. Isotropy. In the $AdS$ case, mirage matter contributes to Hubble equation with a term $\rho \sim a^{-8}_{AdS}$ [1]. This term dominates over the shear $\rho_{\text{shear}} \sim a^{-6}_{AdS}$ at early times, avoiding the chaotic behavior [22]. To check whether this is true in the KS case, we should verify that the corresponding mirage contribution dominates over the shear. The form of this contribution can be read from [12]. On the other hand, the $\rho_{\text{shear}}$ term will still be given as $a^{-6}$ in the KS throat. To compare them we can write the quotient as

$$\sqrt{\frac{\rho_{\text{shear}}}{\rho}} \propto \left(1 + a_{AdS} \frac{d \ln \Omega_{KS}}{d a_{AdS}}\right)^{-1} a_{AdS} \frac{\Omega_{KS}}{\Omega_{AdS}}.$$  (15)

The proportionality constant in (15) parameterize the anisotropic perturbations in the pre-bounce era.

It is simple to check that (15) is an increasing function of $a_{AdS}$ in the region $a_{AdS} \gtrsim 1.57 r_s/L$ (figure 1). As we assumed that the Slingshot brane never approaches the tip of the KS throat, this condition is automatically satisfied. Therefore, $\rho_{\text{shear}}/\rho$ decreases very rapidly close to the bouncing point in the pre-bounce era solving the isotropy problem.

C. Flatness. The curvature contribution to the Hubble equation [47] can be disregarded if the quantity $|\Omega_{\text{Total}} - 1| = 1/a^2H^2$ passes through a minimum where it satisfies the phenomenological constraint

$$|\Omega_{\text{Total}} - 1|_{\text{min}} < 10^{-8},$$  (16)

where the value $10^{-8}$ represents the measured curvature during the BBN.

In the $AdS$ case, this condition results in a restriction to a two dimensional region of parameter space. In this sense flatness problem might be alleviated in the Slingshot scenario.

For the KS case we have, after conformal re-scaling

$$|\Omega_{\text{Total}} - 1| = \frac{f^2}{a^2_{AdS}H^2_{AdS}}, \quad f = \frac{4 \ln(a_{AdS}L/r_s)}{4 \ln(a_{AdS}L/r_s) - 1}.$$  (17)

The KT approximation is valid for $r_{\text{min}} \gg r_s$; to fix ideas we will use $r_{\text{min}} > 10^2 r_s$. In this region we have $f = O(1)$ and decreasing in $a_{AdS}$. Consequently, the flatness problem in the KS space might, in good approximation, be alleviated by the same choice of parameters used in the $AdS$ case.
III. PRIMORDIAL PERTURBATIONS

Before calculating the power spectrum of density perturbations, let us recall a lesson from inflation. In inflation the primordial perturbations are produced by quantum fluctuations of a scalar field, the inflaton. These fluctuations are codified into the two point correlation function of the inflaton in its vacuum state (the Bunch-Davis vacuum), which also sets the initial conditions. However, these fluctuations are over-damped by the expansion of the Universe at super-horizon scales. At these scales then, the quantum state becomes characterized by a large occupation number and the system collapses into a classical state. This classical state represent a random (gaussian) spectrum of perturbations with variance given by matching the classical correlations with the quantum correlations at the quantum to classical transition point [23] (see [24] for other cases).

Let us now turn our attention to the mechanism proposed by [25]. A perturbation of wavelength $\lambda$ smaller than the typical quantum scale (say $l_c$) of a given system, is in its pure quantum state (vacuum). However, in an expanding background, the wavelength of a perturbation grows in time ($\lambda \propto a$). In this case whenever $\lambda \sim l_c$, or in other words, as soon as the perturbation becomes macroscopic, wavelengths bigger than the horizon scale collapse into a classical random state, with the same mechanism discussed before. In the proposal of [25], the relevant fluctuations are so continuously “created” at “super-horizon” scales. Thus, a coherent (gaussian) spectrum of classical perturbations is produced with variance given by matching the classical correlations with the quantum correlations at the quantum to classical transition point [48].

In the Slingshot the mechanism of [25] is used, however the region in which the perturbations are frozen is not parameterized by the horizon but by the centrifugal barrier, as we shall discuss later on.

Before working out the perturbation spectrum, a clarification must be added. In the original proposal of [25] the perturbation was produced by the same radiation which sets the Cosmic Microwave Background. However, as pointed out by [28], the perturbation coming out from the horizon today, should have been born when the curvature sourced by the energy density of radiation was much bigger than the Plank scale, which makes the mechanism unreliable. In the Slingshot this problem is avoided. There, perturbations are in fact created by brane fluctuations in a regime in which the supergravity approximation is still valid. Moreover, the “quantum gravity” scale in this context, i.e. the string length, is much smaller than the wavelengths of the relevant observable brane fluctuations. For this reason, no extra quantum gravity effect participate to the primordial perturbation spectrum. Finally, also note that the general result that bouncing cosmologies produce a blue spectrum in a Bunch-Davis vacuum (see [27] for this result in mirage models) is avoided if the vacuum of [25] is used.

Having set the scenario, let us make more explicit the calculation of the primordial power spectrum. In the mechanism introduced, we can say that classical modes are created at the time $\eta$, when the proper wavelength of the corresponding quantum mode reaches the value

$$a(\eta_c)/k \equiv a_*/k = l_c.$$  \hspace{1cm} (18)

In the Slingshot scenario, a suitable value for the collapse length $l_c$ compatible with all the observational constraints is $l_c = l_s g_s^{-\gamma}$ where $\gamma > 1/3$ [1]. Even if this choice is not essential for the model, let us point out that is compatible with a flat spectrum only in a background with a constant $g_s$, like KS or $AdS_5 \times S_5$.

We start by perturbing the embedding of the probe brane by writing $r = r(\eta) + \delta r(\eta, \vec{x})$ and $\Omega_5 = \Omega_5(\eta) + \delta \Omega(\eta, \vec{x})$. The action (3) can be expanded to quadratic order in $\delta$’s and their derivatives, getting in Fourier space

$$S = T_3 \frac{3}{2} \sum_k \int d\eta \left( \delta r_k^2 + r^2 \delta \Omega_k^2 - \left( k^2 - \frac{J^2}{r^4} \right) \delta r_k^2 - r^2 k^2 \delta \Omega_k^2 + \frac{4J}{r} \delta \Omega_k^3 \delta r_k \right).$$  \hspace{1cm} (19)

In what follows, we will find convenient to use the Bardeen potential [29] $\delta \Phi_k = \delta r_k/r$ instead of $\delta r_k$. With this change of variable we get

$$S = T_3 \sum_k \int d\eta \left( \frac{r^2}{2} \left( \delta \Phi_k^2 + \delta \Omega_k^2 - k^2 (\delta \Phi_k^2 + \delta \Omega_k^2) \right) + J \delta \Omega_k \delta \Phi_k - J \delta \Omega_k \delta \Phi_k' \right).$$  \hspace{1cm} (20)

The resulting generalized momenta are

$$\Pi_{\delta \Phi_k} = T_3 \left( r^2 \delta \Phi' - J \delta \Omega \right),$$  \hspace{1cm} (21)

$$\Pi_{\delta \Omega_k} = T_3 \left( r^2 \delta \Omega' + J \delta \Phi \right).$$  \hspace{1cm} (22)

Then the Hamiltonian obtained form the above Lagrangian is

$$H = \frac{1}{2T_3 r^2} \left( (\Pi_{\delta \Phi_k} + J \delta \Omega_k)^2 + (\Pi_{\delta \Omega_k} - J \delta \Phi_k)^2 \right) + \frac{1}{2} k^2 J^2 \left( \delta \Phi_k^2 + \delta \Omega_k^2 \right),$$  \hspace{1cm} (23)
which is positive definite and has a well defined ground state so that a quantum mechanical description of this system is possible.

A. Normalized operators

The equations of motion for the fluctuations are derived from the action \[20\] and are written as

\[
\frac{d}{d\eta} \left( r^2 \delta \Omega_k' + 2J \delta \Phi_k \right) + r^2 k^2 \delta \Omega_k = 0 ,
\]

\[
\frac{d}{d\eta} \left( r^2 \delta \Phi_k' - 2J \delta \Omega_k \right) + r^2 k^2 \delta \Phi_k = 0 ,
\]

The normalized solution for these equations are

\[
\delta \Phi_k = u_1 a_1 + u_2 a_2 + c.c. ,
\]

\[
\delta \Omega_k = v_1 a_1 + v_2 a_2 + c.c. ,
\]

where

\[
u_1 = \sqrt{\frac{U}{kT_3}} \eta \frac{J}{r^2} e^{-i k \eta}, \quad u_2 = \sqrt{\frac{1}{U k T_3}} \frac{J}{2r^2} e^{-i k \eta}
\]

\[
v_1 = u_2 = \sqrt{\frac{1}{U k T_3}} \frac{J}{2r^2} e^{-i k \eta}, \quad v_2 = -u_1 = -\sqrt{\frac{U}{k T_3}} \eta \frac{J}{r^2} e^{-i k \eta} .
\]

They satisfy the commutation rules

\[
[\delta \Phi_k, \Pi_{\delta \Phi_{k'}}] = i \delta_{k,k'}, \quad [\delta \Phi_k, \Pi_{\delta \Omega_{k'}}] = 0 , \quad [\delta \Omega_k, \Pi_{\delta \Phi_{k'}}] = 0 , \quad [\Pi_{\delta \Phi_k}, \Pi_{\delta \Omega_{k'}}] = 0 ,
\]

provided that the operators \(a_i, a_i^\dagger\) are standard annihilation and creation operators, \(i.e.,\)

\[
[a_i, a_j^\dagger] = \delta_{ij}, \quad [a_i, a_j] = 0 .
\]

We are interested in the correlation of the Bardeen potential \(\delta \Phi\) at the time of creation \(\eta_*\). It is straightforward to check that

\[
\langle \delta \Phi_{k'} \delta \Phi_{k} \rangle = \delta_{k,k'} \frac{1}{2k T_3 r_*^2} ,
\]

where \(r_* = r(\eta_*)\).

B. Classical Solution and Matching

We will consider the transition point of the quantum to the classical description in the region in which \(k \ll J/r^2\) (this was called the frozen region in \[1\]). Here in fact the oscillations of the classical solutions of the equations of motion are drastically damped and therefore the system can be considered classical, as it happens in inflation \[23\]. In this limit, we can discard the \(k^2\) term in the equations \[24,25\] and the resulting equations of motion read

\[
\frac{d}{d\eta} \left( r^2 \delta \Omega_k' + 2J \delta \Phi_k \right) = 0 , \quad \frac{d}{d\eta} \left( r^2 \delta \Phi_k' - 2J \delta \Omega_k \right) = 0 .
\]

The real solutions of these equations are

\[
\delta \Phi_k = \frac{C_k}{2J} + A_k \sin (2 \theta + \phi_k) ,
\]

\[
\delta \Omega = -\frac{D_k}{2J} + A_k \cos (2 \theta + \phi_k) ,
\]
where \( \phi_k, C_k, D_k, A_k \) are constants, \( \theta = \Omega_5(\eta) - \Omega_5(\eta_*) \) and
\[
C_k = r^2 \delta \Omega'_k + 2J \delta \Phi_k ,
D_k = r^2 \delta \Phi'_k - 2J \delta \Omega_k .
\]

With these definitions, it is easy to invert for \( A_k \) to obtain
\[
A_k = \frac{r^2}{2J} \left[ \delta \Phi'_k \cos (2\theta + \phi_k) - \delta \Omega'_k \sin (2\theta + \phi_k) \right] .
\]

We now consider initial conditions arising from the matching of the classical to the quantum system at the time \( \eta = \eta_* \). Therefore \( C_k, D_k, A_k \) will be taken as Gaussian stochastic variables with correlations \( \langle \cdots \rangle_c \) matching the quantum correlators \( \langle \cdots \rangle \) at \( \eta = \eta_* \). The constants \( \phi_k \) will instead be related to the fact that the quantum system picks up only positive frequencies in the vacuum state.

Using the quantum solutions described above at the matching point \( \eta = \eta_* \), we then get in the limit \( \eta \gg \eta_\text{asym} \).

The matching of
\[
\langle \delta \Phi_k \delta \Phi_{k'} \rangle_c = \langle \hat{\delta \Phi}_k \hat{\delta \Phi}_{k'} \rangle ,
\]
requires \( \phi_k = \pi/2 \); this is the selection of positive frequencies. So we are left for each mode with
\[
\delta \Phi_k \equiv \frac{C_k}{2J} + A_k \cos 2\theta .
\]

In general correlators depend on time through \( \theta \). However in the frozen region, the oscillation rapidly stabilizes in time. We will consider this asymptotic region \( (2U \eta_\text{asym} / J > 2\pi) \) to be well before the nucleosynthesis. At this time then
\[
\delta \Phi_k = \frac{C_k}{2J} - A_k \cos (2\Omega_5) = \frac{C_k}{2J} + A_k (1 - \frac{r_{\text{min}}^2}{r_*^2}) .
\]

Using the initial conditions found above we then get in the limit \( k \ll J/r_*^2 < J/r_{\text{min}}^2 \)
\[
\langle \delta \Phi_k \delta \Phi_{k'} \rangle_c \bigg|_{\eta > \eta_\text{asym}} \sim \frac{\delta_{k,k'}}{2kT r_*^2} \left[ 1 - \left( \frac{r_{\text{min}}}{r_*} \right)^2 \right] ,
\]
so the power spectrum of temperature fluctuations is
\[
P(k) \sim \frac{1}{2kT r_*^2} \left[ 1 - \left( \frac{r_{\text{min}}}{r_*} \right)^2 \right] ,
\]

A consistency condition for the production of the perturbation is that \( r_{\text{min}} < r_* \). So we see that in the limit \( r_{\text{min}} \ll r_* \) we obtain the power spectrum introduced in [1]. Note that \( r_* = r_*(k) \) as follows from eq. (18).

C. Spectral index

Since we assumed that perturbations are created when the physical wavelength reaches a fixed value \( l_c \), we have from eq. (18), \( k l_c = a_* \). In the Klebanov-Tseytlin (KT) metric this means
\[
r_* = r_* \exp \left( -\frac{1}{4} W_{-1}(-\zeta) \right) ,
\]
where \( \zeta = 4 r_s^4/L^4 l_s^4 k^4 \leq e^{-1} \) and \( W_{-1}(x) \) is the negative branch of Lambert’s W-function \([1]\). Then the power spectrum \([43]\) is explicitly written as

\[
P(k) = \frac{1}{2 T_3 k r_s^2} e^\frac{1}{2} W_{-1}(-\zeta) \left( 1 - \left( \frac{r_{\text{min}}}{r_s} \right)^2 e^\frac{1}{2} W_{-1}(-\zeta) \right),
\]

whereas, the scalar spectral index \( n_s = d \ln k^3 P(k)/d \ln k \), turns out to be

\[
n_s = 1 + \frac{2}{1 + W_{-1}(-\zeta)} \left( 1 - \frac{W_{-1}(-\zeta)}{1 - r_s^2/r_{\text{min}}^2 e^{-\frac{1}{2} W_{-1}(-\zeta)}} \right),
\]

The first term in the parenthesis in \([46]\) above is the redshift due to the KT metric reported in \([1]\). The second term in the parenthesis, gives a new correction coming form the time evolution of the correlation function. In the case in which \( \zeta \ll 1 \), we can use the expansion of the Lambert W function for small argument \( W(-\zeta) \approx \ln(\zeta) + \cdots \) so to get

\[
P(k) \approx \frac{1}{k^3 L^2 T_3} \left( 1 - \left( \frac{r_{\text{min}}}{r_s} \right)^2 \sqrt{\zeta} \right),
\]

and

\[
n_s \approx 1 + \frac{2}{\ln(\zeta)} - \frac{2 \sqrt{\zeta}}{\sqrt{\zeta - r_s^2/r_{\text{min}}^2}}.
\]

Since \( \ln(\zeta) < 0 \) for small \( \zeta \), the first correction on \( n_s \) is negative. On the other hand, \( r_s^2/r_{\text{min}}^2 \) and the overall sign of the correction has to be evaluated taking into account the joint contribution of both terms in \([43]\). After some manipulations we find that the correction is red whenever

\[
\sqrt{\zeta} > r_s^2/r_{\text{min}}^2,
\]

from which we immediately see that long wavelengths are red-shifted.

On the other hand, if the last term is positive, then \( \sqrt{\zeta} < r_s^2/r_{\text{min}}^2 \) and the overall sign of the correction has to be evaluated taking into account the joint contribution of both terms in \([43]\). After some manipulations we find that the correction is red whenever

\[
\sqrt{\zeta} \left( 1 - 2 \log(\zeta) \right) < \frac{r_s^2}{r_{\text{min}}^2},
\]

from which we conclude that short wavelengths are also red-shifted, and there is an intermediate range of wavelengths that is blue-shifted.

The various parameters appearing in the formulas for the power spectrum and the spectral index may be partially fixed by using the above expressions at a pivot wavelength \( \lambda_p = \zeta_p^{1/4} \lambda_0 \) where \( \lambda_0 = a_0 L T / \sqrt{2} r_s \). Then we can write

\[
P(k_p) k_p^3 \approx \frac{1}{L^2 T_3} \left( 1 - \left( \frac{r_{\text{min}}}{r_s} \right)^2 \sqrt{\zeta_p} \right) = 10^{-10},
\]

and

\[
n_s \approx 1 + \frac{2}{\log(\zeta_p)} - \frac{2 \sqrt{\zeta_p}}{\sqrt{\zeta_p - r_s^2/r_{\text{min}}^2}} = .95,
\]

which gives two constraints for the three unknowns \( r_{\text{min}}/r_s, L^2 T_3 \) and \( \zeta_0 \).

Other constraints come from the requirement that the relevant primordial perturbations related to the today’s observational scales were born frozen. These cross-constraints have been studied in \([1]\) in the case in which \( r_{\text{min}} \ll r_s \). The space of parameter is obviously larger in the more general case in which \( r_{\text{min}} < r_s \). However, for what concern the analysis in this paper we will be content in using the constraints of \([1]\).

The last comment we need to add regards possible Trans-Planckian contributions to the primordial spectrum of perturbations. As we have already stressed before, these contributions are not present in our model as the dynamics is always controlled by the low energy regime of String Theory, i.e. the supergravity approximation, and hence quantum stringy corrections are suppressed. This is due to the fact that the brane is always slowly moving and that \( r_{\text{min}} > l_s \), i.e. the brane is never too close to the stack. The semiclassical treatment of the brane fluctuation is therefore the dominant effect.
IV. FOUR DIMENSIONAL POINT OF VIEW

The 4D effective theory for warped compactifications of IIB supergravity with (static) D-branes has been derived by a perturbative approach in [31], and by a gradient expansion method in [30]. In this last approach, the universal Kähler modulus $\rho(x)$ is obtained by writing the metric as

$$ds^2 = h^{-\frac{2}{3}}(x,y)\tilde{g}_{\mu\nu}(x,y)dx^\mu dx^\nu + \tilde{h}^2(x,y)\gamma_{ab}(y)dy^a dy^b,$$

with $h(x,y) = h(y) + h_0(x)$, and identifying

$$\rho(x) = iH(x) = ih_0(x) + \frac{i}{V_6} \int d^6y\sqrt{-\gamma} h(y).$$

Here, $V_6$ is the volume of the (un-warped) compact manifold. The perturbative potential for $\rho$ has been also obtained in [30] and it depends on the curvature of the transverse metric $\gamma_{ab}$, the localized negatively charged objects like anti-branes and the 3-form fluxes. The effective four dimensional action is found to be [30]

$$S = \frac{1}{2\bar{k}^2} \int d^4x \sqrt{-\tilde{g}} \left[ HR[\tilde{g}] - 2V(H) \right],$$

where $\bar{k}^2 = k_{10}^2/V_6$ and $k_{10}^2$ is the ten dimensional Newtonian constant.

We consider the background to be stabilized on the minimum of the potential of the universal Kähler modulus $V = 0$ (by non-perturbative corrections or some other yet unknown mechanism) and we focalize on simplest case of an $AdS_5 \times S_5$ throat sourced by a stack of $N$ D3-branes, this can be easily generalized to the KS case. In this case, the background close to the stack of D3-branes has the solution found by [38] where $h_0(x) = 0$, $H \sim 3L^4/r_{max}^4$ and $r_{max}$ is a cut-off radius for which the approximate solution of [38] is no longer valid. Recalling that for this solution $L^4 = 4\pi l_s^4 N g_s$ and plugging it into the action (55), we get an overall $N$ factor that can be reabsorbed by re-scaling the metric $\tilde{g}_{\mu\nu} = N\tilde{g}_{\mu\nu}$ (and defining $k^2 = r_{max}^4 \bar{k}^2/12\pi l_s^4 g_s$).

We are now ready to add to the action (55) the contribution of a moving brane. A single brane slowly moving on a background sourced by a stack of others $N$ D3-branes will obviously produce gravity back-reactions sourced by the kinetic DBI energy of the brane. In order to keep these back-reactions small we need to assume that we are in the adiabatic limit $|h(y)g^{\mu\nu}\gamma_{ab}\partial_\mu y^a \partial_\nu y^b| \ll 1$. The following effective action at order $1/N$ and in the slow velocity limit, i.e. at leading order on the tension expansion in local curvature units, is thus obtained

$$S_{eff} = \int d^4x \sqrt{-g} \left[ \frac{1}{2\bar{k}^2} \frac{T_3 L^2}{12N} \Phi^2 \right] R - \frac{T_3 L^2}{2N} ((\nabla \Phi)^2 + \Phi^2(\nabla \Omega)^2).$$

Here $\Omega$ and $\Phi = r/L$ parameterize respectively the angular and the linear motion of the moving D3 brane on the warped CY. We have also included the conformal coupling to the four dimensional Ricci scalar of [33], which, consistently, is of order $1/N$. The second term in (56) is nothing else than the DBI action [30] in the adiabatic limit. In the expansion leading to (56) the ratio $|h(y)g^{\mu\nu}\gamma_{ab}\partial_\mu y^a \partial_\nu y^b|/N$ has been considered to be at second order in the spirit of the Mirage approximation of [40].

This action has been derived assuming a negligible backreaction on the CY geometry. This assumption should be valid whenever the brane is deep inside the throat or, in other words, whenever the value of the effective scalar field $\Phi$ (the radion) is small. As soon as the brane exits the throat, i.e. for a large $\Phi$, the radion motion would acquire an effective (large) mass due to the backreaction on the compact embedding. This in principle should stabilize $\Phi$ at late time. Together with that, another mechanism enters into the stabilization of the effective String frame Newtonian constant. At late time $\Phi$ is supposed to couple to brane matter fields so to re-heat the Universe. From the ten dimensional point of view this re-heating can be understood as the dissipation of the kinetic energy of the brane into world-volume excitations. As explained in the introduction, the details of this late time dynamics are however out of the scope of the present paper and are left to future work.

A. String frame

The effective action (56) does not however describe any physically relevant frame. In fact, the metric $g_{\mu\nu}$ is not related to any physical choice of units. Units might be fixed either in terms of particle masses (String frame) or of the four dimensional Planck length (Einstein frame). In the String frame all particles masses are constant in time as the Standard Model is supposed to live on the Slingshot brane. In the Einstein frame instead the gravitational coupling
is constant in time. Although both the Einstein and String frames are of physical relevance, only the String frame has a geometrical meaning from the ten dimensional perspective. To make connection to what has been described in the previous sections we will start discussing this case.

The induced metric in the slow velocity limit can be written as
\[ g_{\mu\nu}(x) = h^{-\frac{1}{2}}(y(x)) \left( g_{\mu\nu}(x) + h(y(x)) \gamma_{ab}(y(x)) \partial_{\mu}y^{a} \partial_{\nu}y^{b} \right) \simeq h^{-\frac{1}{2}}(y(x)) g_{\mu\nu}(x), \]

where \( \partial_{\mu}y^{a} \) describe the brane embedding. In the simplest \( AdS_{5} \times S_{5} \) case \( h^{-1/2} = r^{2}/L^{2} = \Phi^{2} \). The effective action on the String frame is then
\[ S_{\text{brane}} = \int d^{4}x \sqrt{-g} \left[ \left( \frac{1}{2k^{2}L^{4}} - \frac{T_{3}L^{2}}{12N} \right) R[g] + \frac{3}{k^{2}L^{6}}(\nabla\Phi)^{2} - \frac{T_{3}L^{2}}{2N}(\nabla\Omega)^{2} \right] . \]

From the action (58) we can finally find the effective Slingshot equations at zeroth order in the expansion. Note that to get zeroth order back-reactions to the metric we need to consistently solve at first order the scalar field equations. At first order in \( 1/N \) expansion we have
\[ G_{\alpha\beta} = -2\frac{\nabla_{\alpha}\nabla_{\beta}\Phi}{\Phi} + 2\frac{\Box \Phi}{\Phi} g_{\alpha\beta} - 3\frac{(\nabla\Phi)^{2}}{\Phi^{2}} g_{\alpha\beta} + \frac{T_{3}L^{2}k^{2}}{N} \left[ \Phi^{2}\nabla_{\alpha}\Omega_{\beta}\Omega - \frac{\Phi^{2}}{2}(\nabla\Omega)^{2} g_{\alpha\beta} - \frac{\Phi}{3}\nabla_{\alpha}\nabla_{\beta}\Phi + \frac{\Phi}{3} \nabla_{\alpha} \nabla_{\beta} \Phi - \frac{1}{2}(\nabla\Phi)^{2} g_{\alpha\beta} \right] . \]

By varying the action (58) with respect to the scalar fields \( \Phi \) and \( \Omega \) and by substituting the Ricci scalar obtained by the Einstein equations (59), the scalar field equations turn out to be, at leading order,
\[ \Box \Omega = 0 , \]
\[ \Box \Phi - 2\frac{(\nabla\Phi)^{2}}{\Phi} = \Phi(\nabla\Omega)^{2} . \]

B. Cosmology

We now consider the theory (59) in a Friedman-Robertson-Walker (FRW) background. Here we have
\[ ds^{2} = -dt^{2} + a^{2}(t)d\vec{x} \cdot d\vec{x} , \]
and all fields are time dependent.

The equation for \( \Omega \) reads
\[ \frac{d}{dt} \left( a^{3}\Omega \right) = 0 \quad \Rightarrow \quad \Omega = \frac{J}{a^{3}} , \]
where \( J \) is a constant of integration. Using this solution in the equation for \( \Phi \) we find that
\[ \frac{\ddot{\Phi}}{\Phi} + 3\frac{\dot{\Phi}}{a} \frac{\dot{\Phi}}{\Phi} - 2\frac{\dot{\Phi}^{2}}{\Phi^{2}} = \frac{J^{2}}{a^{6}} . \]

Moreover, from the \( G_{tt} \) component of (59), we get, at leading order in \( 1/N \), the Friedman equation
\[ \frac{\dot{a}^{2}}{a^{2}} = \frac{\dot{\Phi}}{\Phi} \left( 2\frac{\dot{a}}{a} - \frac{\dot{\Phi}}{\Phi} \right) , \]
which is solved by the mirage solution \( \dot{a}/a = \dot{\Phi}/\Phi \) or in other words by \( a = C\Phi \). The constant \( C \) can be fixed to \( C = 1 \) by appropriate rescaling of the coordinates. The scalar equation (64) ensures instead the bouncing behavior
\[ \frac{\dot{\Phi}}{\Phi} = \frac{J^{2}}{a^{6}} \frac{\dot{\Phi}^{2}}{\Phi^{2}} , \quad \Rightarrow \quad \dot{\Phi}^{2} = \frac{2\Phi}{L^{2}\Phi^{2}} - \frac{J^{2}}{\Phi^{4}} , \]
where \( U \) is another constant of integration and the bounce occurs at \( \Phi^{2} = J^{2}/2UL^{2} \). The effective theory therefore reproduces the bouncing solution of [41] to leading order in the back-reaction parameters \( |h(y)g^{\mu\nu}\gamma_{ab}\partial_{\mu}y^{a}\partial_{\nu}y^{b}| \) and \( 1/N \), which define the mirage approximation of [40].

First order corrections are certainly interesting and even non-negligible when the brane leaves the CY throat far from the bouncing point. However, the zeroth order solution describes very well the motion of the Slingshot brane deep into the throat of the CY. This is exactly the framework in which the early time Slingshot is based. We leave for future research the investigation of higher order corrections to the mirage approximation.
C. Energy conditions

The Slingshot solution is ultimately related to the violation of the energy conditions. To verify this, let us define the effective energy condition $T_{\alpha\beta}$ as the right-hand side of eq. (59), which is, to leading order

$$T_{\alpha\beta} = -2 \frac{\nabla_\alpha \nabla_\beta \Phi}{\Phi} + 2 \frac{\Box \Phi}{\Phi^2} g_{\alpha\beta} - 3 \left( \frac{\nabla \Phi}{\Phi} \right)^2 g_{\alpha\beta}. \quad (67)$$

In a FRW background we may define the effective energy density $\rho = -T_{t\bar{t}}$ and the effective pressure $p_{\delta \bar{t}} = T_{\bar{t}t}$. By using the field equations (64-65) we get

$$\rho + 3p = 6H^2 \left( 1 - \frac{\dot{\Omega}^2}{H^2} \right), \quad (68)$$

and

$$\rho + p = 4H^2 \left( 1 - \frac{\dot{\Omega}^2}{2H^2} \right). \quad (69)$$

When the centrifugal barrier double the Hubble expansion, or in other words, when $\dot{\Omega}^2 > 2H^2$, both strong ($\rho + 3p \geq 0$) and weak ($\rho + p \geq 0$) energy conditions are violated and the bounce occurs. Strong energy conditions alone are however violated as soon as the centrifugal barrier overtake the Hubble expansion, i.e. for $\dot{\Omega} > H$, this generates a short period of acceleration. The number of e-foldings ($N$) of cosmological acceleration, can then be easily bounded by

$$N = \int H dt < \int \dot{\Omega} dt < 2 \pi \simeq 6. \quad (70)$$

A more precise numerical calculation shows that the number of e-foldings is actually bounded by $2 \ [1, 21]$. On the explicit solution it is easy to check that the energy conditions are both violated for

$$a^2 < \frac{3J^2L^2}{4U} . \quad (71)$$

From a more geometrical perspective, brane curvatures violate energy conditions via the projection of bulk curvatures onto the brane, as discussed in [1].

D. Perturbations

In section III we considered the primordial spectrum of perturbation by explicitly neglecting the local gravity contribution (mirage approximation). One might ask whether this is appropriate. Unfortunately, to fully answer this question a non-linear effective four dimensional theory, in the expansion parameter $1/N$, must be developed. Nevertheless, a convincing positive answer can be obtained by just looking at the first order effective theory (59). We will use the short notation $T_{\alpha\beta}$ for the right hand side of (59). Standard arguments [34], shows that the local gravity contribution can be neglected if the Bardeen potential $\Psi$ satisfies

$$\Psi \ll \frac{\delta T_{t\bar{t}}}{p + \rho} , \quad (72)$$

whenever $\Psi$ is in the frozen region. In our model, all perturbations are frozen at their value $a_\ast \propto k$. For small $k$s (i.e. for super-horizon scales) the Bardeen potential $\Psi$ is then well approximated with [34]

$$3a_\ast^2H^2\Psi \simeq \frac{\delta T_{t\bar{t}}}{2} a_\ast^2 , \quad (73)$$

where $\delta T_{t\bar{t}}$ is the perturbation of the energy momentum tensor $T_{t\bar{t}}$. The condition for which local gravity can be neglected in the calculation of the Bardeen potential is therefore

$$\frac{1}{3} \ll \frac{1}{2} \frac{1}{1 - \frac{a_\ast^2}{a_\ast^2}} , \quad (74)$$
where we have used \( \Omega_0 \). The inequality (74), is then always satisfied for large scale modes. This analysis therefore shows that local gravity can be safely neglected in the calculation of the scalar primordial perturbations related with the CMB.

As we have shown in section [IV.B] when local gravity can be neglected, the effective action (68) faithfully reproduce the mirage cosmology results. We can define an effective matter content and pressure by making use of Hubble equation \( \dot{a}^2/a^2 = (8\pi G/3)\rho_{\text{eff}} \) and Raychaudhuri equation \( \dot{a}/a = -(4\pi G/3)(\rho_{\text{eff}} + 3p_{\text{eff}}) \). Since far form the bounce the angular momentum contribution is subdominant, equation (64) imply
\[
\frac{\dot{a}}{a} + \frac{\dot{a}^2}{a^2} = \frac{4\pi G}{3} (\rho_{\text{eff}} - 3p_{\text{eff}}) \simeq 0.
\] (75)
or in other words \( p_{\text{eff}} = 1/3\rho_{\text{eff}} \). This is the well known result that mirage matter behaves like radiation [2]. The Hollands-Wald result [24] of an almost flat power-spectrum can then be qualitatively used from the effective theory point of view. Note however that here perturbations are created whenever the angular momentum plays a determinant role. Nevertheless, the angular momentum signatures on the later time evolution of the perturbations are expected to “decay” very quickly, as shown before in the Mirage analysis.

E. Einstein frame

The Slingshot was originally developed in the so called String frame [1]. In the String frame, particle masses are defined by the oscillations of the fundamental string, and therefore are constant in time. However, a brane observer experiences an induced time varying gravitational coupling until, at late times, the brane leaves the throat and ends into the CY. In [33], it was claimed that a physical description of a gravitational system may only be performed in a frame in which the induced gravitational coupling is constant, i.e. in the so called Einstein frame. However, as it is clearly explained in [36], the only requirement of the constancy of the newtonian coupling \( G_1 \) is meaningless. In local gravity for example, a physical meaningful quantity is in fact not the Newton constant but rather the gravitational force. In particular, the gravitational attraction \( F \) between two bodies of masses, let us say, \( m_1 \) and \( m_2 \), is \( F \propto G_4(t)m_1m_2 \). The change to the Einstein frame in which \( G_4 = \text{const.} \), does not make the gravitational force time-independent, as in this case masses are \( m_i = m_i(t) \) [30]. The same conclusion can be obtained by considering cosmological observations that involve the redshift. The redshift is indeed defined as \( z = \omega_1/\omega_2 - 1 \) where \( \omega_1 \) and \( \omega_2 \) are respectively the frequencies emitted and observed of some atomic transition. In this physical system \( \omega_1 \propto m_e/a(t) \), where \( m_e \) is the electron mass setting the units. By a conformal transformation \( g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu} \), we also have \( m \rightarrow \Omega m \) and therefore the redshift is un-modified. Finally, we also wish to quickly comment on the power spectrum of primordial perturbations. In String frame the perturbed metric is \( ds^2_{\text{eff}} = a^2(1 + 2\Psi)dt^2 + ... \) where \( \Psi \) is the Bardeen potential. By re-scaling to a conformal frame such that \( ds^2_0 = (a\Omega)^2(1 + 2\Psi)dt^2 + ... \), we still obtain the same Bardeen potential \( \Psi \). The power spectrum of primordial perturbations (a physical quantity) is therefore unchanged by the change of conformal frames. To conclude, one can then in general show that any conformal frame is physically equivalent [37].

It seems much more economical and natural to attribute the time dependence of the gravitational force to a corresponding time variation of the gravitational coupling rather than to the masses of the string excited states. Only during nucleosynthesis the question of whether the gravitational coupling is constant or not is well posed and very strict constraints do apply. Therefore, the time variation of the four dimensional newtonian constant in the Slingshot era can never be ruled out by observations.

During the Slingshot era, our Universe is a D3-brane moving in a throat of a CY space. When the throat is embedded in a compactification scheme, it should be cut off at some \( r_{UV} \) and glued back to the rest of the compactification space [38]. On the other hand, the conical singularity, at the tip of the cone, is smoothed out by effectively cutting off the throat at \( r_{IR} \) and appropriately deforming the space so that a smooth geometry is obtained. In the case in which the throat is taken to be \( AdS_5 \times S_5 \) as in [1], and neglecting brane velocities contributions, this construction strongly resembles the RS1 scenario [39] (see [40] and for a 10D tentative construction see [30]), where the vicinity of \( r_{IR} \) and \( r_{UV} \) correspond to the IR and UV branes, respectively (see fig.1). The correct analysis for the induced Einstein theory on a third brane in the RS1 background has been performed in [41]. The result for a probe brane is
\[
8\pi G_4 = \frac{2\kappa_5^2}{L} \left[ e^{2d_{UV}/L} - e^{-2d_{IR}/L} \right]^{-1},
\] (76)
where \( d_{IR} \) and \( d_{UV} \) are the proper distances from the probe brane to, respectively, the IR and UV boundary and \( \kappa_5 \) is the higher dimensional gravitational coupling. One can easily see that, once the probe brane reaches the UV
FIG. 2: The geometry of the KS throat glued to a CY. The tip of the throat is deformed at \( r = r_{\text{min}} \) corresponding to an IR cutoff of the dual theory whereas the "far" UV region of the throat is glued at \( r = r_{\text{max}} \) to a CY space. The probe brane is wondering in the IR vicinity. This is similar (in the zero angular momentum case) to the RS1 setup with a probe brane, where the tip corresponds to the IR brane and the far region to the UV one.

boundary, the newtonian constant stabilizes to a constant value. In an exact \( AdS_5 \) geometry we have

\[
d_{IR} = \int_{r_{IR}}^{r} \frac{L}{r'} dr' = L \ln \left( \frac{r}{r_{IR}} \right),
\]

and

\[
d_{UV} = \int_{r}^{r_{UV}} \frac{L}{r'} dr' = L \ln \left( \frac{r_{UV}}{r} \right),
\]

where \( r \) is the probe brane position at fixed bulk time. Thus we get that

\[
8\pi G_4 = \frac{2\kappa_5^2}{L} \left( \frac{r_{UV}^2}{r^2} - \frac{r_{IR}^2}{r^2} \right)^{-1}.
\]

As the probe is moving in the vicinity of the IR region \( r > r_{IR} \) and far from the UV, \( r < r_{UV} \), we may write (as already noted in [42])

\[
G_4 \approx G_N \left( \frac{r}{r_{UV}} \right)^2
\]

where \( 8\pi G_N \approx 2\kappa_5^2/L \) is the Newton constant on the UV brane. Whenever instead the brane reaches the UV boundary, the local gravitational coupling on the moving brane stabilizes to the measured Newtonian constant. The string theory system is slightly more involved as one needs to consider the conformal coupling between the local Ricci scalar and the brane position introduced before. This is due to the fact that the bulk is not sourced by a cosmological constant, as in RS1 case, but by a Ramond-Ramond five form. Nevertheless, we can use this toy model for a qualitative description of what we would expect in the late time evolution of the Sling shot brane.

Let us consider two particles living on the brane. As the extra-dimensions are compact, the Newtonian force between them behaves four-dimensionally, i.e. \( F \propto G_4/r^2 \), at distances \( r \gg \left( \ell_s/G_4 \right)^{1/6} \). When the probe brane reaches the UV region after the bounce, \( G_4 \) stabilizes to the measured newtonian constant and therefore local 4D gravity dominates at low energies. Viceversa, as the brane approaches the bouncing point, \( G_4 \) becomes smaller and smaller and therefore the scale for which gravity looks four-dimensional, become larger and larger. Eventually, gravity would look four-dimensional only above cosmological distances in the vicinity of the bouncing point. In other words there, the 10D gravity dominates over the local gravitational attraction up to very large scales and the probe brane approximation of [2] is justified. In this case, local 4D gravity cannot be used to describe gravitational attraction up to cosmological scales.

Although the effective theory discussed above is strictly valid only in the RS1 context, it actually captures the essential fact that the gravitational coupling scales like \( a^{-2} \) close to the IR boundary, as pointed out by [33]. Let us indeed consider the String frame effective theory [35], which is again valid close to the bouncing point. The Einstein frame can be found by re-scaling the metric by a conformal factor

\[
\chi^2 = \frac{1}{\Phi^2} \left( 1 - \frac{T_3 L^2 k^2 \Phi^2}{6N} \right).
\]
The scale factor in Einstein frame is therefore

\[ a_E = \frac{a}{\Phi} \sqrt{1 - \frac{T_3 L^2 k^2 \Phi^2}{6 N}} = 1 + \mathcal{O}\left(\frac{T_3 L^2 k^2 \Phi^2}{6 N}\right), \quad (82) \]

where in the last equality we used the solutions of the equations (59-61), at leading order. In Einstein frame, the spacetime is therefore not evolving at zeroth order [49]. However, as already stressed before, effective masses vary as \( m_{\text{eff}} = \Phi^{-1} m \), where \( m \) are the bare (String frame) particle masses. The physical observable are therefore not only non-trivial in the Einstein frame, as conversely claimed by [35], but are physically equivalent to the physical observable in the String frame [37].

To get an insight of what the next to leading order could be, we refer the reader to the exact solution of eqs. (59-61) found in [43]

\[ a = \frac{1}{L} \sqrt{\frac{J^2}{2U} + 2U \eta^2}, \quad \Omega' = \frac{1}{L^2 a^2}, \quad \Phi = \frac{a}{1 + k \sqrt{\frac{U T_3}{3 N} \eta}}. \quad (83) \]

Note that in this solution there is a past singularity at negative \( \eta \) in the field \( \Phi \). However the effective theory (58) is valid for \( \Phi^2 \ll \frac{T_3 L^2 k^2}{6 N} \), i.e. the effective theory cannot be trusted when the brane sits in the hat of the CY. Since from the ten dimensional point of view the hat of the Calabi-Yau is a regular compact geometry, everything behaves regularly there and the value of \( \Phi \) is bounded from above by constant \( \Phi_{\text{max}} \) determined by the details of the compactification. In conclusion, the field singularity of solution (83) is only an apparent singularity and if an exact solution could be found this should be regular in the past infinity and future. To properly show this assertion however the full back-reacting String solution must be performed, this is beyond the scope of this paper.

In terms of (83), the Einstein frame scale factor reads

\[ a_E = 1 + k \sqrt{\frac{U T_3}{3 N} \eta} + \mathcal{O}\left(\frac{T_3 L^2 k^2 \Phi^2}{6 N}\right), \quad (84) \]

Note that the apparent past singularity on \( \Phi \) gets mutated into a scale factor singularity, however again the same discussions on its regularity apply. The exact solution for the Einstein frame scale factor should then smoothly approach in the past infinity a constant which is fixed by the scale of compactification. This resembles the emergent Universe idea [4]. From the above considerations it is then clear that, even in the Einstein frame, no singularities should appear whenever the Slingshot brane wanders in the throat of the CY, which is the case studied in this paper.

An interesting question is whether the spectrum of primordial perturbations analyzed in the probe brane approximation can be reproduced from a four dimensional point of view. To achieve this goal, a more detailed effective four dimensional theory than the one used here to describe the cosmological background, is needed. However, as this study is far beyond the scope of the present paper, we leave it for future work.

V. CONCLUSIONS

The Cosmological Slingshot Scenario, aspires to provide a new paradigm for the early time cosmology. In there, the observable universe is a \( D3 \)-brane moving in a warped throat of a CY solution of the IIB supergravity. The inflationary era is replaced by a period of Mirage Cosmology [2], where local gravity effects are neglected. At later times instead, when the probe brane is approaching the base of the CY throat, local gravity becomes important and completely takes over the dynamics.

In the Slingshot, the probe brane trajectory has a non-vanishing angular momentum in the KS throat region of a CY space. The presence of the angular momentum provides a turning point on the brane orbit at a finite distance to the tip of the throat. From the point of view of an observer living in the brane, the turning point prevents an initial singularity and gives rise to a bouncing cosmology in String frame, without passing through a quantum gravity regime. In Einstein frame on the other hand the scale factor starts as a finite constant in the past corresponding to the Slingshot brane sitting at the hat of the Calabi-Yau. In this context, by generalizing the results of [1] to a KS throat, we showed that the problems of standard cosmology (horizon and isotropy) are naturally avoided and the flatness problem is acceptably alleviated.

Concerning the calculation of the quantum primordial perturbations, we have here identified their vacuum state and elucidated the matching conditions from the quantum to the classical regime of the perturbation. Using this detailed analysis, we have found the exact power spectrum of primordial perturbations showing its compatibility with latest WMAP data [44] (see [45] for the exact match of the Slingshot primordial power spectrum with the full set of
WMAP data). A comment on a criticism raised by [33] should be added here. As pointed out by [28] the scenario of [29] is unreliable. In [29] the primordial perturbations are created by the same radiation that fills the Universe today. However, if this is the case, the perturbation coming out from the horizon today would have been born when the curvature scale was much bigger than the Plank scale. Fortunately, in the Slingshot this problem is avoided as perturbations are created by brane fluctuations in a regime in which the supergravity approximation is still valid.

In the second part of our paper we have studied the Slingshot cosmology from a four dimensional perspective. In this context, we showed that the results found in [1], obtained by using the mirage approach of [40], are reproduced. Interestingly, we also clarified that strong and weak energy conditions are violated in the effective theory approach during the bouncing.

In the Slingshot frame (or String frame), particle masses are fixed and Newton constant $G_4$ is time-dependent. This time variation might seem odd. However, no observational constraints on the variation of $G_4$ are applicable to the pre-nucleosynthesis era. The Slingshot, that aims for an early time description of the Universe, is therefore observationally safe. Nevertheless, a frame where $G_4$ is constant might be more familiar for some [33]. This frame, at leading order in the tension expansion with respect local curvatures, is easily found from the action [68] by a conformal transformation. The same conformal transformation that fix in time the Newtonian constant, makes however particle masses time varying. It is then easy to note that the physical observable are frame-independent, as already elucidated by Dicke’s original paper [46] (see [37] for a modern view).

Finally, one may ask of whether cosmological singularities might occur in the Einstein frame. Here we have showed that during the early time cosmology, there are no singularities in any frame. Although the effective theory for the whole brane trajectory is not known, and thus, definite statements cannot be made, one might speculate that if no singularities are formed during the brane motion deep into the throat, future singularities, in the hat of the CY, are unlikely to appear. The physical system reaches high energies only when the brane is close to the stack. Therefore, from the “mirage” perspective, there cannot possibly be any future cosmological singularities [50]. As Physics is frame independent, the same conclusion should be valid in Einstein frame as well. Nevertheless, a quantitative exploration of this point, involving the transition from the mirage to the local era, should be done in future research.

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