Weak Antilocalization and Conductance Fluctuation in a Sub-micrometer-sized Wire of Epitaxial Bi$_2$Se$_3$

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In this paper, we address the phase coherent transport in a sub-micrometer-sized Hall bar made of epitaxial Bi$_2$Se$_3$ thin film by probing the weak antilocalization (WAL) and the magnetoresistance fluctuation below 22 K. The WAL effect is well described by the Hikami-Larkin-Nagaoka model, where the temperature dependence of the coherence length indicates that electron conduction occurs quasi-one-dimensionally in the narrow Hall bar. The temperature-dependent magnetoresistance fluctuation is analyzed in terms of the universal conductance fluctuation, which gives a coherence length consistent with that derived from the WAL effect.

I. INTRODUCTION

A topological insulator (TI) is a new phase of matter that was theoretically predicted and was later realized in two-dimensional and three-dimensional materials. This new phase originates from the strong spin-orbit interaction to cause band inversion. As a result, TI has a band gap in the bulk, while there exist gapless states on the surface. Two-dimensional TI has one-dimensional states at the edge and three-dimensional TI (3DTI) has two-dimensional states at the surface. Electrons in these states satisfy the linear dispersion relation and their spins are polarized. In particular, in 3DTI, electrons in the surface states behave as Dirac electrons. These gapless surface states are expected to be robustly protected against the time-reversal-invariant perturbation, and therefore, it is expected that they can be applied to spintronics and quantum computing.

In order to address and make the best use of this protection as well as the electronic properties as Dirac electrons, it is necessary to experimentally investigate TI from the viewpoint of quantum transport. Some quantum transport phenomena in 3DTI have already been observed; several groups reported the weak antilocalization (WAL) effect in 3DTI crystals, nanoribbons, and thin films. Considerable fluctuation in the magnetoresistance, similar to the universal conductance fluctuation (UCF), was also reported for bulk crystal. Nevertheless, only a few studies have simultaneously addressed and analyzed both the WAL effect and the magnetoresistance fluctuation.

Bi$_2$Se$_3$ is one of the representative 3DTIs with a single Dirac cone; it was predicted and established by angle-resolved photoemission spectroscopy (ARPES) in 2009. Because the bulk band gap of Bi$_2$Se$_3$ is relatively large compared to that of other 3DTI materials, it has attracted attention from the viewpoint of 3DTI and the Landau levels of surface states, and its thickness dependence has been reported. However, two technical difficulties were encountered in these studies. First, the synthesized Bi$_2$Se$_3$ material is often naturally doped by electrons because of Se vacancies, and therefore, the bulk states could contribute to the transport properties of the Bi$_2$Se$_3$ sample, which prevents the pure surface state contribution from being determined. Second, the surface of Bi$_2$Se$_3$ is sensitive to water and oxidation, and therefore, the electron mobility in this state is usually small.

Herein, we report the WAL effect and magnetoresistanc-
istance fluctuation in a sub-micrometer-sized Hall bar (wire) sample fabricated on an epitaxial Bi$_2$Se$_3$ thin film. We use Bi$_2$Se$_3$ thin film to minimize the bulk contribution and to address the mesoscopic coherence by using the WAL effect and the magnetoresistance fluctuation at low temperatures. We show that the WAL effect is well described by the Hikami-Larkin-Nagaoka model and that the temperature dependence of the coherence length indicates that electron conduction occurs quasi-one-dimensionally in the narrow Hall bar. The temperature-dependent magnetoresistance fluctuation is analyzed in terms of UCF, which gives a coherence length consistent with that derived from the WAL effect.

II. EXPERIMENT

We grew a 20-nm-thick Bi$_2$Se$_3$ thin film by molecular beam epitaxy (MBE) on a sapphire substrate, as described previously. This thickness is equivalent to 20 quintuple layers. The ARPES measurement clarified the electron band structure in the momentum-energy space under the Fermi energy, as shown in Fig. 1(a). The Fermi energy is located 150 meV above the Dirac point and in the bulk conduction band. This is due to the natural doping by Se vacancies. Therefore, the carriers of both surface states and the bulk conduction band contribute to the transport phenomena of the thin films.

We deposited a 30-nm-thick amorphous Se layer to prevent Bi$_2$Se$_3$ from being destroyed by water or oxidation. The thin film was fabricated into a narrow 260-nm-wide wire-shaped Hall bar by using electron beam lithography (EBL) and Ar ion milling. Ti (5 nm) and Au (100 nm) were then deposited as electrodes. Figure 1(b) shows a scanning electron microscope (SEM) image of the Hall bar sample. A schematic of the sample geometry is shown in Fig. 1(c). We measured the magnetic-field dependence of the longitudinal resistance and the Hall resistance at 11 different temperatures between 2 and 22 K. All the experiments were carried out by using the standard lock-in technique. We measured two devices fabricated from the same Bi$_2$Se$_3$ film and obtained consistent results as reported below.

III. RESULTS AND DISCUSSIONS

A. Basic parameters

First, we present the basic electronic properties of our device. Figure 2(a) shows the Hall resistance as a function of magnetic field between -7 and 7 T at 2 K. The carriers are electrons, the density of which is $n_e = 6.2 \times 10^{13}/\text{cm}^2$. The electron mobility, $\mu = l/(weR_n)$, is $807 \text{ cm}^2/\text{Vs}$, and the mean free path of electrons, $l_m = h\mu \sqrt{2m_e/e} = 105 \text{ nm}$, are obtained at 2 K based on the longitudinal resistance $R$ at 0 T and the length, $l = 2.0 \mu\text{m}$, and width, $w = 260 \text{ nm}$, of the Hall bar. The carrier density and mobility are shown as a function of temperature in Fig. 2(b). Both show little dependence on temperature, indicating that our sample is in the metallic regime.

When the number of electron bands that contribute to the carrier transport is more than one, the Hall resistance should have nonlinear components as a function of the magnetic field. Actually, such a nonlinearity was reported for the TI devices already. In our case, however, the applied magnetic field ($\lesssim 7$ T) was not high enough to observe the expected nonlinearity to prove that both the surface and bulk states contribute to transport. Thus, the present data obtained in the transport measurement are not sufficient to affirm that the surface states play a role in the conduction, while the ARPES result in Fig. 1(a) clearly implies that there are both surface states and bulk conduction states at the Fermi level. Based on this fact, it is believed that electrical current flows through both the surface states and the bulk conduction states. We will discuss this point later based on our experimental findings.

Two important features are noted in the longitudinal resistance as a function of the magnetic field (Fig. 2(c)). One is a dip structure around 0 T, which originates from the WAL effect. The other is the resistance fluctuation occurring over a wide magnetic field range, which be-
FIG. 3. Magnetoconductance around 0 mT is shown. Solid curves indicate the experimental results and dashed curves indicate the results of the fitting by the HLN model. The y-axis shows the data obtained at 2 K; the other data are incrementally shifted downward by 0.12$e^2/h$.

comes prominent at lower temperatures and is thus similar to UCF. These two features are the main topics of this paper, as discussed in the following.

B. Weak antilocalization

We first consider the WAL effect. Without spin-orbit interaction, quantum interference between the time-reversal pair of electron waves scattered by impurities is destructive and results in electron localization. This localization gives rise to the conductance correction to reduce the conductance around a zero magnetic field. This quantum interference occurs at the scale of the coherence length. If there is strong spin-orbit interaction, on the contrary, the correction enhances conductance, which is known as the WAL effect. Actually, as seen in Fig. 2(c), the dip structure in the resistance becomes more remarkable with decreasing temperatures. The data shown in Fig. 3 are the conductance around zero magnetic fields [converted from the data in Fig. 2(c)]. The enhancement in the peak structure as the temperature decreases is characteristic of the WAL effect.

We first assume that the electrons in the present device behave as in a two-dimensional system and analyze the magnetoconductance with two-dimensional WAL models. In the case of a two-dimensional system, when a magnetic field is applied perpendicularly to the two-dimensional plane, the conductance correction gradually vanishes with an increase in the magnetic field because of the breaking of the time-reversal symmetry. Therefore, the effect of the quantum interference weakens. Conventionally, this reduction in conductance can be explained by the Hikami-Larkin-Nagaoka (HLN) model, where the spin relaxation is governed by the Elliot-Yafet mechanism. In this model, the conductance correction $\delta G_{\text{WAL}}(B)$ is given by

$$\delta G_{\text{WAL}}(B) \equiv G(B) - G(0) = \alpha \frac{e^2}{2\pi^2 \hbar} \left[ \psi(B\phi_B) + \frac{B\phi_B}{B} - \ln\left(\frac{B\phi_B}{B}\right) \right]. \quad (1)$$

$G(B)$ is the conductance as a function of magnetic field, $B$. $\psi$ represents the digamma function. $B_\phi = \hbar/4eI_\phi^2$ is a magnetic field characterized by coherence length $l_\phi$. In the case of a system with strong spin-orbit interaction, the prefactor $\alpha$ is equal to $-0.5$ and this model explains the conductance correction of the WAL effect.

The HLN model has already been applied to explain the results of the WAL effect in Bi$_2$Se$_3$ samples. Following these studies, we analyzed the experimental results of magnetoconductance using the HLN model and estimated the coherence length at each temperature. To estimate the prefactor and coherence length, we fitted the magnetoconductance with Eq. (1), as shown in Fig. 3. Equation (1) can be used to describe the conductance correction when the magnetic field is less than $B_m = \hbar/2eI_m^2$. $B_m$ may be derived from the mean free length $l_m$.
path of our sample to be \( \sim 120 \text{ mT} \) so that we can use the conductance peak within \( \pm 100 \text{ mT} \) for the fitting. The prefactor \( \alpha \) is approximately \(-0.3\) at 2 K, which is of the same order as in the symplectic case \( (\alpha = -0.5) \). This means that carrier electrons are strongly affected by strong spin-orbit interaction. The obtained coherence lengths as a function of temperature are shown in Fig. 4.

The coherence length increases from 0.2 \( \mu \text{m} \) to around 0.5 \( \mu \text{m} \) as the temperature decreases from 22 to 2 K. The observed behavior of the coherence length is consistent with that reported for thin films of Bi\(_2\)Se\(_3\).\(^{13,14}\)

From the temperature dependence of the coherence length, the dimensionality of the system is known. Theoretically, the coherence length is proportional to \( T^{-1/2} \) for the two-dimensional system and \( T^{-1/3} \) for the one-dimensional system.\(^{15}\) Figure 4 shows that the coherence length is proportional to \( T^{-0.37} \) above \( \sim 4 \text{ K} \), which is very close to the exponent expected for the one-dimensional system. Remarkably, as shown in Fig. 4, \( l_\phi > l_0 \) at \( T < 15 \text{ K} \), which is consistent with the fact that electrons move quasi-one-dimensionally. Although the above analysis is made based on the two-dimensional case, as a consistency check, the one-dimensional model is used to analyze the observed WAL effect.\(^{16}\) We have confirmed that the coherence lengths based on this one-dimensional model are very consistent with what has been derived from the conventional HLN model.

C. Magnetoresistance fluctuation

As shown in Fig. 2(c), the magnetoresistance fluctuates as the magnetic field changes. We observe that this fluctuation is perfectly reproducible at a fixed temperature and that it becomes prominent as the temperature lowers. Now we show that this can be attributed to UCF, a typical mesoscopic effect. The conductance of mesoscopic samples differs from each other, and if phase coherence is maintained over the entire sample, the conductance fluctuation \( \delta G \) is nearly equal to a universal value, \( e^2/h \) at 0 \( T \).\(^{17,18}\) We calculated the correlation function \( F(\Delta B) \) of the components of the conductance fluctuation \( \delta G(B) \) by extracting the smoothed conductance from the original conductance data between 1 and 7 T, as shown in Fig. 5(a). The fluctuations \( \delta G(B) \) in Fig. 5(a) are reproducible as a function of magnetic field at different temperatures, while their amplitude decreases as the temperature increases. This supports that the conductance fluctuation is intrinsic to the device transport. The correlation function is defined by \( F(\Delta B) = \langle [\delta G(B + \Delta B) - \delta G(B)] \rangle \), where \( \langle \rangle \) expresses the ensemble average.

First, we estimated the coherence length from the obtained correlation function. In the UCF theory, the conductance fluctuation \( \delta G(B) \) correlates with \( \delta G(B + B_\text{c}) \) when \( B < B_\text{c} \), where \( B_\text{c} \) is characterized by the coherence length. As the coherence length derived from the WAL effect analyzed using the HLN model shows the temperature dependence expected for a one-dimensional system, we assumed that the present sample is one-dimensional and analyzed the conductance fluctuation as one-dimensional UCF. \( B_\text{c} \), defined by \( F(B_\text{c}) = F(0)/2 \), is expressed as \( B_\text{c} = 0.95h/ewl_\phi \) in a one-dimensional system that is in the diffusive regime with \( l_\phi > l_T \). Here, \( l_T \) is a thermal diffusion length.\(^{39}\) As shown in Fig. 5(b), the coherence length derived based on UCF is satisfactorily consistent with that derived from the WAL effect. These coherence lengths are proportional to \( T^{-0.28} \), which is consistent with the quasi-one-dimensional case. The observed quasi-one-dimensionality is also consistent with the results from the WAL effect using the HLN model.

Second, we investigated the temperature-dependent behavior of the root mean square of the conductance fluctuation defined by \( \text{rms}(\delta G) = F(0)^{1/2} \). According to the UCF theory,\(^{20}\) for the one-dimensional system, if \( l > l_\phi \) holds, \( \text{rms}(\delta G) \) reflects the size of the sample, which causes \( \text{rms}(\delta G) \propto (l_\phi/l_T)^{3/2} \). In addition, if \( l_\phi > l_T \) holds, \( \text{rms}(\delta G) \) is affected more greatly by the finite temperature, which yields \( \text{rms}(\delta G) \propto l_T/l_\phi \). Therefore, \( \text{rms}(\delta G) \propto l_T^{1/2}/l_\phi^{2/3} \) is expected. As already discussed, \( l_\phi \propto T^{-1/3} \) and \( l_T \propto T^{-1/2} \); therefore, \( \text{rms}(\delta G) \propto T^{-2/3} \).
In order to confirm this, we investigated the temperature dependence of rms(δG). Figure 5(b) shows rms(δG) as a function of temperature. The temperature dependence of rms(δG) was fitted by $T^{-0.69}$ except for the data at 2 K, which is consistent with the prediction of the UCF theory, $T^{-2/3}$. This suggests that the observed fluctuation originates from UCF in a quasi-one-dimensional system.

Recently, a similar magnetoresistance fluctuation signal was reported by another group, who proposed that the observed fluctuation should not be categorized as conventional UCF. They argue that the fluctuation reflects the hybridization between the surface state and the bulk state confined at surface. A possible reason for the difference between their interpretation and ours may arise from the difference in the thickness of the Bi$_2$Se$_3$ sample used; we use an ultra-thin film whereas they use a macroscopic crystal. In the thin film, the bulk state confined to the surface may decrease, and therefore, the effect of hybridization is likely to be reduced. Although we currently do not know the definite reason for the difference between the two results, it should be noted that the present analysis based on UCF is as expected theoretically and is also consistent with the WAL effect.

D. Contribution from the surface states

Although we have reported the quantum transport phenomena of Bi$_2$Se$_3$ thin film, within the experimental results presented here, it is not easy to clarify to what extent the electronic properties as TI materials are relevant. On the other hand, even in our experiment, there are a few signatures that the properties as TI materials play some roles in the quantum transport phenomena. One signature is that the surface states and the bulk conduction states are at the Fermi level as seen in the ARPES result of Fig. 1(a). This may suggest that electrons flow through the surface states and the bulk conduction states. The second one is based on the theoretical report that a fitting parameter $\alpha$ of the HLN model can be nearly equal to $-0.3$ if the surface states cause the WAL effect and the bulk conduction states give rise to the weak localization effect. This is consistent with our observation [see Fig. 4]. The above discussion supports that the measured quantum transport phenomena is originated from not only the bulk conduction states but also the surface states. Further experimental efforts, such as the electrical depletion of the bulk states or the carrier doping are necessary to systematically detect phenomena originated only from the Dirac electrons in the surface states of topological insulators.

IV. CONCLUSION

In summary, in the longitudinal resistance of a Bi$_2$Se$_3$ narrow Hall bar sample, a dip near 0 T and a fluctuation as a sweep of field were observed. This dip originates from the WAL effect, and we derived the coherence length by using the HLN model. By analyzing the temperature dependence of the coherence lengths, it is confirmed that conduction electrons move in a quasi-one-dimensional system. The coherence length calculated from the fluctuation on the assumption of UCF is consistent with that calculated from the WAL effect based on the HLN model. The conductance fluctuation as a function of temperature is consistent with the prediction of the UCF theory.

V. ACKNOWLEDGMENT

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