Natural realization of a large extra dimension in 5D supersymmetric theory

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An exponentially large extra dimension can be naturally realized by the Casimir energy and the gaugino condensation in 5D supersymmetric theory. The model does not require any hierarchies among the 5D parameters. The key ingredient is an additional modulus other than the radion, which generically exists in 5D supergravity. SUSY is broken at the vacuum, which can be regarded as the Scherk–Schwarz SUSY breaking. We also analyze the mass spectrum and discuss some phenomenological aspects.

Subject Index B11, B12, B13, B41

1. Introduction

Models with extra dimensions are intriguing candidates for the new physics, and have been investigated in a vast number of articles. Such models were originally proposed in order to explain the large hierarchy between the electroweak scale $M_{\text{EM}}$ and the Planck scale $M_{\text{Pl}}$. In the warped spacetime \cite{1}, for example, $M_{\text{EM}}$ emerges only from the five-dimensional (5D) parameters that are roughly of the same order as $M_{\text{Pl}}$. In the flat spacetime, however, the smallness of $M_{\text{EM}}/M_{\text{Pl}}$ is just translated into largeness of the size of the extra dimensions $L_{ED}$ compared to the Planck length $M_{\text{Pl}}^{-1}$ \cite{2, 3, 4}. We should note that $L_{ED}$ is not a parameter of a theory but a quantity dynamically determined by some moduli stabilization mechanism. In contrast to models in the warped geometry \cite{5}, most of the known models on the flat spacetime need to admit some hierarchies among the fundamental parameters in order to realize the tiny ratio $M_{\text{EM}}/M_{\text{Pl}}$.

In our previous work \cite{6}, we have shown that an exponentially large extra dimension can be obtained without introducing any hierarchies among the model parameters in the context of 5D supergravity. The moduli is stabilized by the Casimir energy and a superpotential term induced by the gaugino condensation. The purpose of this paper is to investigate this scenario further in more general setup, and understand the essential structure of the model by comparing a model with a similar setup in Ref. \cite{7} that does not generate a large extra dimension. We also discuss the phenomenological aspects of the model by evaluating the mass spectrum of the moduli and the superparticles. The latter spectrum largely depends on whether the gauge and matter fields are in the bulk or on the boundary.

The paper is organized as follows. In Sect. 2, we provide a compact review of the stabilization of the extra dimension by the Casimir effect in a simple setup. In Sect. 3, we extend the model in
Sect. 2 by introducing an additional modulus, and show that an exponentially large extra dimension is naturally realized. In Sect. 4, the mass spectrum of our model is discussed in various cases according to where the gauge and matter fields live. Section 5 is devoted to the summary. In Appendix A, we show the consistency of our formula for the effective potential with the previous works [7, 8] by noting the equivalence of supersymmetry (SUSY) breaking by a boundary constant superpotential and by the Scherk–Schwarz twisted boundary condition [9].

2. Radion stabilization by Casimir energy

In this section, we provide a brief review of the radius stabilization by the Casimir energy [7, 10] in the superfield formalism.

We consider a 5D flat spacetime compactified on $S^1/Z_2$ whose fundamental region is denoted as $0 \leq y \leq L$. The physical field content consists of hypermultiplets $Q_a$, where the index $a$ specifies a representation of the gauge group and the generation; vector multiplets $V_G$ ($G = SU(3)_C, SU(2)_L, U(1)_Y, \ldots$); and the gravitational multiplet. These multiplets can be expressed in terms of $N = 1$ superfields [11–13] as

$$1_Q = (Q_a, Q^*_a)$$

and

$$1_V = (V_G, \Sigma_1 G),$$

where $V_G$ is an $N = 1$ vector superfield and the others are chiral superfields. The matter superfields $Q_a$ and the gauge superfields $V_G$ in the SUSY standard model can be identified with the zero-modes of $Q_a$ and $V_G$ whose $Z_2$-parities are even at both boundaries. In addition to these, there may exist antiperiodic fields that have opposite $Z_2$-parities at $y = 0$ and $y = L$. They do not have zero-modes, but contribute to the Casimir energy. We introduce $n_P^a$ periodic and $n_A^a$ antiperiodic hypermultiplets $Q_a$, and $n_P^V$ periodic and $n_A^V$ antiperiodic vector multiplets.

In the 4D effective theory, there also appears a chiral superfield $T$ that comes from the gravitational multiplet, which is called the radion superfield. Its lowest component $T|_0$ is identified as

$$T|_0 \equiv \tau + i \rho = L_{ED} |\varphi_C| - i \rho,$$

where $L_{ED} = \int_0^L dy e^y$ is the size of the extra dimension, $\rho$ is the Wilson line phase for the graviphoton field along the extra dimension, and $\varphi_C$ is the compensator scalar, which will be fixed by the superconformal gauge-fixing condition explained later.

The 5D bulk Lagrangian is expressed in terms of the $N = 1$ superfields [12, 13]. In addition, we introduce the following boundary superpotential terms localized at $y = 0$:

$$W^{(0)} = W_0 + W_{\text{yukawa}}(Q),$$

where $W_0$ is a constant and $W_{\text{yukawa}}$ contains the Yukawa couplings. We will not introduce the Kähler potentials nor the gauge kinetic functions at the boundaries until Sect. 4.

The 4D effective Lagrangian is expressed as

$$L = - \left[ \int d^2 \theta \sum_G \frac{f_G}{2} \text{tr} \left( W_G^2 \right) + \text{h.c.} \right] + \int d^4 \theta \ |\varphi_C|^2 \Omega + \left[ \int d^2 \theta \ \varphi_C^3 W + \text{h.c.} \right],$$

where $W_0$ denotes SUSY with four supercharges in this paper.

We have dropped the gravitational fluctuation modes. Their dependence on the superspace is provided in Ref. [14].

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where $\phi_C$ is the chiral compensator superfield whose lowest component is $\varphi_C$, and $W^G_{\frac{c}{\Omega}}$ is the field strength superfield for $V_G$. The holomorphic functions $f_G$ and $W$ are the gauge kinetic functions and the superpotential, which are derived as [15]

$$f_G(T) = C_GT, \quad W(Q, T) = W_0 + W_{\text{yukawa}}(Q),$$

(4)

where $C_G$ is a constant. Notice that there is no bulk contribution to $W$ because $N = 2$ SUSY in the bulk prohibits it. The real function $\Omega$ is related to the Kähler potential $K$ by $\Omega = -3e^{-K/3}$.

In the absence of other moduli than $T$, the $T$-dependent part of $\Omega$ is given by\(^3\)

$$\Omega = -3\text{Re} T - \frac{\xi(\text{Re} T)}{(\text{Re} T)^2} + \cdots,$$

(5)

where the ellipsis denotes terms independent of $T$, and

$$\xi(\tau) \equiv \frac{\zeta(3)}{32\pi^2} \left\{ \sum_a n^A_a Z_P(c_a \tau) - \frac{3}{4} \sum_a n^A_a Z_A(c_a \tau) - n^\dagger_P + \frac{3}{4} n^\dagger_A - 2 \right\},$$

$$Z_P(x) \equiv -\frac{4}{\zeta(3)} \int_{|x|}^\infty d\lambda \frac{\ln \left( 1 - e^{-2\lambda} \right)}{\lambda},$$

$$Z_A(x) \equiv \frac{16}{3\zeta(3)} \int_{|x|}^\infty d\lambda \frac{\lambda}{\lambda + x} \ln \left( 1 + \frac{\lambda - x}{\lambda + x} e^{-2\lambda} \right).$$

(6)

The first term in (5) is the well-known radion Kähler potential at tree level. It has the no-scale structure, and does not induce the potential for the radion. This structure is broken by the second term in (5), which is the one-loop correction. A constant $c_a$ in the arguments of $Z_{P,A}$ is defined as $c_a \equiv \frac{M_a}{M_5}$, where $M_a$ is a bulk mass for $Q_a$ and $M_5$ is the 5D Planck mass,\(^4\) and $\zeta(s)$ is Riemann’s zeta function. Functions $Z_P(x)$ and $Z_A(x)$ are normalized so that $Z_P(0) = Z_A(0) = 1$. The bulk mass $M_a$ for $Q_a$ (or $c_a$) controls the wave function profile along the extra dimension, and the flip $c_a \to -c_a$ changes the boundary toward which the wave function localizes. This is the reason why $Z_P(x)$ is symmetric while $Z_A(x)$ is not under $x \to -x$. Since both functions exponentially decrease in the region $|x| > 1$, only modes spread over the bulk contribute to $\Omega$ in the one-loop diagrams. There are no brane-to-brane contributions to $\Omega$ [17,18], which correspond to the third term in (4.1) of Ref. [6], because we introduce the matter-dependent superpotential only at $y = 0$.

As shown in Ref. [7], the extra dimension is stabilized by this setup (see Appendix A). For later convenience, however, we introduce a non-Abelian gauge sector in the bulk, in which the gaugino condensation occurs. Since the gauge kinetic function is proportional to $T$, the following superpotential term is induced:

$$W = Ae^{-aT} + \cdots,$$

(7)

where $A$ is a complex constant and $a$ is a real constant of $O(4\pi^2)$.

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\(^3\) The general formula for the one-loop correction to $\Omega$ is calculated in Ref. [16], where only periodic fields are considered. The contribution of the antiperiodic fields is obtained from the formula by taking a limit of infinite brane mass terms at one of the boundaries.

\(^4\) In supergravity theory (SUGRA), every mass parameter is introduced by gauging some isometry. The bulk mass $M_a$ is associated with the gauging of the $U(1)$ symmetry that rotates the phases of $Q_a$ and $Q_a^\dagger$ with opposite charges. The ratio $c_a$ is the gauge coupling constant for this gauging.
Then the effective potential $V_{\text{eff}}$ is calculated as

$$V_{\text{eff}} = |\phi_C|^4 \left\{ \frac{|W_0|^2 F(\tau)}{\tau^4} + \frac{4a |W_0A| e^{-a\tau} \cos(a\rho - \vartheta)}{\tau^4} \right\} + \cdots, \quad (8)$$

where $\vartheta \equiv \text{arg}(\bar{W}_0A)$, and

$$F(\tau) \equiv \frac{6\xi(\tau) - 4\tau^2 \xi'(\tau) + \tau^2 \xi''(\tau)}{\tau^4}. \quad (9)$$

We have dropped higher-order terms in the loop factor $\frac{\zeta(3)}{32\pi^2}$ or $e^{-a\tau}$ and $Q_a$-dependent terms. In order to obtain the ordinary Poincaré SUGRA, we have to impose the superconformal gauge-fixing conditions. According to the action formula in Refs. [19,20], the gravitational part of (3) is

$$L_{\text{grav}} = \sqrt{-g} |\phi_C|^2 \Omega_0 |\mathcal{R} + \cdots, \quad (10)$$

where $g \equiv \det(g_{\mu\nu})$, $\mathcal{R}$ is the Ricci scalar, and $\Omega_0$ is the lowest component of $\Omega$. Thus, the condition to obtain the canonically normalized Einstein term is

$$\phi_C = \left( -\frac{3}{\Omega_0} \right)^{1/2} \tau^{-\frac{3}{2}} + \cdots, \quad (11)$$

where we have used (5) at the second equality, and the ellipsis denotes terms suppressed by $\frac{\zeta(3)}{32\pi^2}$ and $Q_a$-dependent terms. Throughout this paper, we take the unit of the 4D Planck mass, i.e., $M_{\text{Pl}} = 1$. In this gauge, we find from (1) that

$$\tau = L_{\text{ED}}^{2/3} + \cdots. \quad (12)$$

From (8) with (11), the minimization conditions for $V_{\text{eff}}$ are

$$-6F(\tau) + \tau F'(\tau) + 4a(a\tau + 2) \left| \frac{A}{W_0} \right| \tau^4 e^{-a\tau} = 0,$$

$$\cos(a\rho - \vartheta) = -1. \quad (13)$$

When the 5D masses for $Q_a$ are zero (i.e., $c_a = 0$), $\xi(\tau)$ becomes constant and the first equation is reduced to

$$(a\tau + 2)\tau^4 e^{-a\tau} = \frac{3\xi}{2a} \left| \frac{W_0}{A} \right|.$$

This does not have a solution in the region $\tau \gg 1$ unless $|W_0/A|$ is exponentially small.\(^6\)

In Ref. [7], the extra dimension is stabilized by the $\tau$-dependence of $F(\tau)$ in the absence of the gaugino condensation ($A = 0$). In this case, the $O(\text{TeV})$ Kaluza–Klein (KK) scale is obtained by assuming the bulk hypermultiplet mass $M_H$ is also of $O(\text{TeV})$. In any case, we have to admit a large hierarchy among the fundamental scales of the 5D theory—see (A15) in Appendix A. This stems from the assumption that there is only one modulus, i.e., the radion. Hence we will extend the model in the next section so that an additional modulus appears in the effective theory.

\(^5\)Note that $\tau = L_{\text{ED}} + \cdots$ in the unit of $M_s$ since $M_s = L_{\text{ED}}^{-1/3}$.

\(^6\)A solution to (13) is generically a (SUSY-breaking) anti de Sitter (AdS) vacuum. So we need to uplift the vacuum energy to achieve the 4D Minkowski space by introducing an additional sector. Such an uplifting sector affects the vacuum solution, but this does not improve the situation drastically.
3. Realization of the large extra dimension

In this section, we extend the model in the previous section, and construct a model in which the extra dimension is stabilized at an exponentially large size without introducing any hierarchical parameters in 5D theory.

Notice that the vacuum expectation value (VEV) of the radion \( \tau \) determines both the size of the extra dimension \( L_{\text{ED}} \) and the gauge coupling constant of the condensation sector. The former is relevant to the volume suppression of the Casimir energy \( \tau^{-6} \), and the latter is to the exponential factor in the second term in (8).\(^7\) In a case where additional moduli appear in the 4D effective theory, the above two quantities can be controlled by different moduli separately. Such additional moduli originate from 5D vector multiplets whose 4D vector components are \( Z_2 \)-odd at both boundaries. They commonly exist if the 5D theory is an effective theory of a higher-dimensional theory, such as 10D superstring theory.

In the following, we consider the case where there is one additional modulus other than the radion. These moduli generically mix with each other, so it is convenient to treat them on an equal footing by denoting them as \( T^I \) (\( I = 1, 2 \)). The mixing is described by the cubic polynomial called the norm function \( \mathcal{N}(\tau) \):

\[
\mathcal{N}(\tau) = \tilde{C}_0(\tau^1)^3 + 3\tilde{C}_1(\tau^1)^2\tau^2 + 3\tilde{C}_2\tau^1(\tau^2)^2 + \tilde{C}_3(\tau^2)^3,
\]

where \( \tilde{C}_{0,1,2,3} \) are real constants, and \( \tau^I \equiv \text{Re} T^I \). This corresponds to the prepotential of 4D \( N = 2 \) SUSY theory. Then (12) is extended to

\[
\mathcal{N}(\tau) = L_{\text{ED}}^2 + \cdots.
\]

Hence we are interested in a situation where \( \mathcal{N}(\langle \tau \rangle) \gg 1 \). It is convenient to rotate the moduli fields \( (T^1, T^2) \) to a new basis \( (T_b, T_s) \) so that the norm function takes the form

\[
\mathcal{N}(\tau) = \tau_b^3 + 3C_1\tau_s^2\tau_b + 3C_2^2\tau_b\tau_s^2 + C_3\tau_s^3,
\]

where \( C_1 \) and \( C_3 \) are real constants.\(^8\) Note that this redefinition is always possible. We will look for a vacuum where \( \langle \tau_b \rangle \gg \langle \tau_s \rangle = \mathcal{O}(1) \). In such a vacuum, \( \tau_b \) is almost identified with the radion.

Before discussing the moduli stabilization, we comment on the 4D coupling constants in the effective theory. The gauge coupling constants \( g_G \) are read off from the gauge kinetic functions as

\[
g_G = f_G(\langle \tau \rangle)^{-\frac{1}{2}} = (C_G^b(\tau_b) + C_G^s(\tau_s))^{-\frac{1}{2}},
\]

where \( C_G^b \) and \( C_G^s \) are real constants. Thus we assume that all the gauge kinetic functions depend only on \( T_b \), i.e., \( C_G^b = 0 \).\(^9\) Otherwise, the 4D gauge couplings become too small by the large VEV of \( \tau_b \).

\(^7\) The radion also determines the wave function profile of \( Q_u \).

\(^8\) The model discussed in our previous work \[6\] is the case of \( C_1 = 0 \) and \( C_3 < 0 \).

\(^9\) In this case, the 5D gauge coupling constants \( g_G^{(S)} = g_G L_{\text{ED}}^{1/2} \) are exponentially larger than \( M_s^{1/2} = L_{\text{ED}}^{1/6} \). This is not a problem because \( g_G^{(S)} \) are not fundamental parameters in 5D SUGRA, but are essentially just VEVs of the moduli. There is no reason to think that \( g_G^{(S)} = \mathcal{O}(M_s^{-1/2}) \) is natural.
The Yukawa coupling constants $y_{abc}$ are read off from the matter part of $\Omega$ [21–23],

$$\Omega_{\text{matter}} = 2N^{1/3}(\text{Re} \, T) \sum_{a} Y_{c,a}(\text{Re} \, T) |Q_{a}|^2 + \cdots,$$

where

$$Y_{c}(\text{Re} \, T) \equiv \frac{1 - e^{-2c \cdot \text{Re} \, T}}{2c \cdot \text{Re} \, T}.$$  \hspace{1cm} (20)

After the canonical normalization of $Q_{a}$, we obtain

$$y_{abc} = \frac{\lambda_{abc}}{\sqrt{8N(\tau) Y_{c,a}(\tau) Y_{c,b}(\tau) Y_{c,c}(\tau)}}.$$  \hspace{1cm} (21)

where $\lambda_{abc}$ are the coupling constants in the boundary superpotential term $W_{\text{yukawa}}(Q)$. Since $N(\langle \tau \rangle)$ is exponentially large in our case, the standard model matter fields $Q_{a}$ must be charged for the radion multiplet $\nabla_{b}$ with positive charges, i.e., $c_{b} > 0$, in order to obtain the realistic values of the Yukawa couplings.\footnote{We cannot explain the fermion mass hierarchy by the wave function localization in this case. We need some mechanism that generates the hierarchical structure of $\lambda_{abc}$.} Namely, the wave functions of $Q_{a}$ strongly localize toward $\gamma = 0$ where $W_{\text{yukawa}}(Q)$ exists. Such localized modes do not contribute to the Casimir energy as mentioned in the previous section.

The effective theory has the superpotential

$$W = W_{0} + A e^{-aT_{s}} + W_{\text{yukawa}}(Q_{a}),$$  \hspace{1cm} (22)

where $W_{0}$ and $A$ are complex constants and $a = O(4\pi^{2})$ is a real constant. Notice that the second term induced by the gaugino condensation is independent of $T_{b}$ since we have assumed that the gauge kinetic functions only depend on $T_{s}$.\footnote{We can relax this assumption so that there may be a gaugino condensation term that depends on $T_{b}$, but such a term is negligible when $\langle \tau_{b} \rangle \gg O(1)$ and does not affect the following discussion.}

Now $\Omega$ in (5) is modified as

$$\Omega = -3N^{1/3}(\text{Re} \, T) - \frac{\xi(\text{Re} \, T)}{N^{2/3}(\text{Re} \, T)} + \cdots.$$  \hspace{1cm} (23)

Here, $c_{a} \tau$ in the definition of $\xi(\tau)$ in (6) is now understood as $c_{a} \cdot \tau \equiv c_{a}^{b} \tau_{b} + c_{a}^{s} \tau_{s}$. Let us consider a case where all the standard model fields are in the bulk and no other zero-modes exist, i.e., $n_{P}^{V} = 12$ and $\sum_{a} n_{a}^{P} = 52$. We should note that $Z_{P}(c_{a} \tau) \simeq 0$ because the matter fields strongly localize toward the boundary. By the same reason, we can neglect contributions of antiperiodic hypermultiplets that are charged for $\nabla_{b}$. Therefore, $\xi(\tau)$ is expressed as

$$\xi(\tau) = \frac{3\zeta(3)}{128\pi^{2}} \left\{ n_{V}^{A} - \sum_{a} n_{a}^{A} Z_{A}(c_{a} \tau_{s}) - 20 \right\}.$$  \hspace{1cm} (24)

Notice that there is an additional contribution $-\zeta(3)/32\pi^{2}$ compared to (6), which comes from the additional modulus multiplet. In the following, we focus on the case $n_{a}^{A} = 0$ to simplify the discussion. Then $\xi(\tau)$ becomes a constant $\xi_{0} \equiv 3\zeta(3)(n_{V}^{A} - 20)/128\pi^{2}$. The effective potential for
the moduli is now calculated as
\begin{align}
V_{\text{eff}} &= |\varphi_c|^4 e^{K/3} \left\{ |W|^2 \left( K_I K_I^J K_J - 3 \right) + \left( k^{IJ} K_I \tilde{W}_J + \text{h.c.} \right) + k^{IJ} W_I \tilde{W}_J \right\} \\
&= \frac{6\xi_0 |W_0|^2}{\tau_b^6} + \frac{4a \tau_s |W_0| A e^{-2\alpha \tau_s}}{\tau_b^5} \cos(\alpha \tau_s - \vartheta) + \frac{2a^2 |A|^2 e^{-2\alpha \tau_s}}{3(C_1^3 - C_3)} + \cdots,
\end{align}

where \( \vartheta = \arg(W_0 A) \), and the ellipsis denotes terms suppressed by \( 1/\tau_b \). We have used at the second equality that
\[ |\varphi_c|^2 = -\frac{3}{\Omega} = \frac{1}{\tau_b} + O(\tau_b^{-2}). \]

The third term in (25), which was dropped in (8), is now crucial to stabilize \( \tau_s \).

From the minimization condition for \( V_{\text{eff}} \), we obtain
\begin{align}
\tau_b^3 &= \frac{3\xi_0}{a \tau_s} \left| \frac{W_0}{A} \right| e^{a \tau_s}, \\
2(a \tau_s - 1) - \frac{\xi_0(2a \tau_s + 1)}{(C_1^3 - C_3)\tau_s^3} &= 0, \\
\cos(\alpha \tau_s - \vartheta) &= -1.
\end{align}

Since \( a = O(4\pi^2) \gg O(1) \), the second equation is solved as
\[ \tau_s = \left( \frac{\xi_0}{C_1^3 - C_3} \right)^{1/3} + O(a^{-1}). \]

If this value is of \( O(1) \), we obtain an exponentially large extra dimension. For example, \( L_{\text{ED}} \asymp 10^{15}, 10^7, 10^3 \) for \( (n_A^4, |W_0/A|, a, C_1^3 - C_3) = (50, 1, 8\pi^2, 0.1), (40, 1, 8\pi^2, 0.5), (40, 1, 4\pi^2, 0.5) \), respectively.

We have assumed that the gauge kinetic function of the condensation sector \( f_C(T) \) is independent of \( T_b \). Even if this is not satisfied, we can always redefine the moduli so that \( f_C(T) = \bar{T}_s \). However, this redefinition breaks the structure of the norm function in (17). In such a case, the third term in (25) is suppressed by \( \tau_b \) instead of \( \tau_s \), and thus is negligible. Then we need the \( \tau_s \)-dependence of \( \xi(\tau) \) in (24) by considering the case \( n_A^4 \neq 0 \), in order for \( \tau_s \) to be stabilized at an \( O(1) \) value. We should also note that a vacuum with an exponentially large \( \langle \tau_b \rangle \) exists even if \( W \) has other gaugino condensation terms that depend on \( T_b \) because such terms are highly suppressed around the vacuum.

4. Mass spectrum

4.1. Uplifting and moduli masses

The solution to (27) is an AdS vacuum with a negative cosmological constant:
\[ V_{\text{eff}}(\langle \tau \rangle) = -\frac{18\xi_0 |W_0|^2}{2a \langle \tau_s \rangle + 1} < 0. \]

In order to achieve the 4D Minkowski spacetime, we cancel this with a nonvanishing F-term of a chiral superfield \( X \) in 4D effective theory, which is tuned as
\[ \left| F_X \right|^2 = K^{-1} \frac{1}{XX} |V_{\text{eff}}(\langle \tau \rangle)|, \]
where $K_{X\bar{X}}$ is the $(X, \bar{X})$-component of the Kähler metric. Since the negative cosmological constant (29) is exponentially small, the effects of the uplifting are tiny. However, as we will show in Sect. 4.3, it provides nonnegligible contributions to the superparticle masses in some cases.

After the canonical normalization, we obtain the mass matrix for the moduli. The radion $\tau_h$ and the nongeometric modulus $\tau_s$ generically have a mixing and their mass-squared matrix is

$$M^2_\tau \equiv \begin{pmatrix} \sqrt{2} K_1/\sqrt{K_2} & 0 \\ 0 & \sqrt{2} K_2/\sqrt{K_1} \end{pmatrix} U_K \begin{pmatrix} \frac{\partial^2 V_{\text{eff}}}{\partial \tau_h^2} & \frac{\partial^2 V_{\text{eff}}}{\partial \tau_h \partial \tau_s} \\ \frac{\partial^2 V_{\text{eff}}}{\partial \tau_h \partial \tau_s} & \frac{\partial^2 V_{\text{eff}}}{\partial \tau_s^2} \end{pmatrix} U_K^{-1} \begin{pmatrix} \sqrt{2} K_1/\sqrt{K_2} & 0 \\ 0 & \sqrt{2} K_2/\sqrt{K_1} \end{pmatrix}, \tag{31}$$

where $K_1, K_2,$ and $U_K$ are the eigenvalues and the diagonalizing matrix of the Kähler metric, and given by

$$K_1 = \frac{3(1 + C_1^2)}{4\tau_b^2} + \cdots, \quad K_2 = \frac{3(C_1^3 - C_3)\tau_s}{2(1 + C_1^2)\tau_b^3} + \cdots,$$

$$U_K = \frac{1}{\sqrt{1 + C_1^2}} \begin{pmatrix} 1 - C_1^2 \delta & C_1(1 + \delta) \\ -C_1(1 + \delta) & 1 - C_1^2 \delta \end{pmatrix} + \cdots, \quad \delta \equiv \frac{2(C_1^3 - C_3)\tau_s}{(1 + C_1^2)^2\tau_b}, \tag{32}$$

where the ellipses are terms suppressed by $\tau_s/\tau_b$, and (31) is evaluated at the vacuum. By diagonalizing (31), we obtain the moduli masses as follows:

$$m_{\tau_h} \simeq \begin{cases} \frac{12 \sqrt{6} s_0 \langle \tau_s \rangle |W_0|}{\sqrt{a} C_1} & (C_1 \neq 0) \\ \frac{12a \sqrt{6} s_0 |W_0|}{\sqrt{a} \langle \tau_s \rangle} & (C_1 = 0) \end{cases}, \quad m_{\rho_h} \simeq 0,$$

$$m_{\tau_s} \simeq m_{\rho_s} \simeq \begin{cases} \frac{4 C_1 a \langle \tau_s \rangle |W_0|}{\langle \tau_b^{3/2} \rangle} & (C_1 \neq 0) \\ \frac{4a \langle \tau_s \rangle |W_0|}{\langle \tau_b^{3/2} \rangle} & (C_1 = 0) \end{cases} \tag{33}$$

where $\tau_1$ ($\tau_h$) is the lighter (heavier) mass eigenstate. When $C_1 = 0$, $\tau_1, \tau_h$ are almost reduced to $\tau_b, \tau_s$ up to the normalization factors.

4.2. SUSY-breaking F-terms

The vacuum (27) breaks SUSY because of the constant superpotential $W_0$, which is equivalent to the Scherk–Schwarz twisted boundary condition (see Appendix A). The gravitino mass is then

$$m_{3/2} = \langle e^{K/2} W \rangle \simeq \frac{|W_0|}{\langle \tau_b^{3/2} \rangle} \simeq \frac{|W_0|}{L_{\text{ED}}} \tag{34}.$$ 

The moduli F-terms are estimated from the equations of motion as

$$F_{\tau_b}^{\tau_b} = \frac{W_0}{\langle \tau_b^{3/2} \rangle} \left[ 1 + \mathcal{O}(\langle \tau_b^{-1} \rangle) \right] \simeq m_{3/2}.$$ 

$$F_{\tau_s}^{\tau_s} = \frac{W_0}{a \langle \tau_s^{3/2} \rangle} \left[ \frac{\xi_0 + 2(C_1^3 - C_3)\langle \tau_s^3 \rangle}{2(C_1^3 - C_3)\langle \tau_s^3 \rangle} \left[ 1 + \mathcal{O}(\langle \tau_b^{-1} \rangle) \right] \right] \simeq \frac{m_{3/2}}{a \langle \tau_s \rangle}. \tag{35}$$

The compensator F-term $F^{\phi_C}$ is negligible as a result of the (approximate) no-scale structure.
The uplifting superfield \( X \) can originate from either a bulk hypermultiplet or a brane-localized chiral multiplet. Since the Kähler potential in each case is given by

\[
\Omega = \begin{cases} 
-N^{1/3}(\text{Re } T) \left\{ 3 - 2Y_{cX}(\text{Re } T) |X|^2 + \cdots \right\}, & \text{(Bulk origin)} \\
-3N^{1/3}(\text{Re } T) + h_X |X|^2 + \cdots, & \text{(Brane origin)} 
\end{cases}
\]

(36)

where \( Y_c(\tau) \) is defined in (20), \( (c_X^b, c_X^s) \) are charges of \( X \) for \( (\mathbb{V}_b, \mathbb{V}_s) \), and \( h_X \) is a real constant, the Kähler metric is

\[
K_{X\bar{X}} = -3 \left( \frac{\Omega_{X\bar{X}}}{\Omega} - \frac{|\Omega_X|^2}{\Omega^2} \right) \sim \begin{cases} 
2Y_{cX}(\tau), & \text{(Bulk origin)} \\
\frac{h_X}{N^{1/3}(\tau)}, & \text{(Brane origin)} 
\end{cases}
\]

(37)

where we have assumed that \( |X| \ll 1 \). Hence the F-term of \( X \) is estimated from (29) and (30) as

\[
|F^X| = \begin{cases} 
\frac{3m_{3/2}}{(\tau_b^{3/2})^{1/2}} \sqrt{\frac{\xi_0}{(2a(\tau_b) + 1)Y_{cX}(\langle \tau \rangle)}}, & \text{(Bulk origin)} \\
\frac{3m_{3/2}}{(\tau_b)} \sqrt{\frac{2\xi_0}{(2a(\tau_b) + 1)h_X}}, & \text{(Brane origin)} 
\end{cases}
\]

(38)

When \( X \) lives in the bulk, \( F^X \) is negligible for \( c_X \cdot \langle \tau \rangle < 0 \), and it grows up to the same order as that in the case of \( X \) on the the boundary for \( c_X \cdot \langle \tau \rangle > 0 \). Thus we assume that \( c_X \cdot \langle \tau \rangle = 0 \), i.e., \( Y_{cX}(\tau) = 1 \) in the following. In this case, \( X \) also contributes to (24) and \( \xi_0 \) in (25) is modified as \( \xi_0 = \zeta(3)(3n_0^4 - 56)/128\pi^2 \).

The F-terms of the other chiral superfields are negligible. Therefore the dominant source of SUSY breaking is \( F^{T_b} \).

### 4.3. Superparticle masses

In this subsection, we estimate the mass spectrum of the superparticles in three cases classified according to where the gauge and the matter fields live.

#### 4.3.1. Gauge and matter fields in the bulk

First we discuss the case that both the gauge and the chiral matter fields live in the bulk. In this case, the gaugino masses \( M_G \) (\( G = SU(3)_C, SU(2)_L, U(1)_Y \)) are calculated as

\[
M_G = \langle F^I \partial_I \ln(\text{Re } f_G(T)) \rangle \simeq \frac{F_{T_i}^{T_i}}{2\tau_b} \simeq \frac{m_{3/2}}{a(\tau_b)}. 
\]

(39)

Notice that there is no contribution from \( F^{T_b} \) since \( f_G(T) \) does not depend on \( T_b \) by assumption. The soft scalar masses of \( Q_a \) are

\[
m_{Q_a}^2 = -F^I \tilde{F}^J \partial_I \partial_J \ln \left( \partial_{Q_a} \partial_{Q_a} \Omega \right) 
\]

\[
\simeq m_{3/2}^2 \left\{ 1 - (c_a \cdot \langle \tau \rangle)^2 \mathcal{Y}_a(c_a \cdot \langle \tau \rangle) \right\},
\]

(40)

where we have used (19), and

\[
\mathcal{Y}(x) \equiv \frac{1 + e^{4x} - 2e^{2x}(1 + 2x^2)}{(1 - e^{2x})^2 x^2}
\]

(41)

is an even and monotonically decreasing function of \( |x| \) and \( \mathcal{Y}(0) = 1/3 \). Recall that the quark and lepton superfields are strongly localized toward \( y = 0 \), i.e., \( c_a \cdot \langle \tau \rangle \gg 1 \), to achieve the observed
fermion masses. Thus these masses become much smaller than $m_{3/2}$ since $\lim_{x \to \infty} x^2 \gamma(x) = 1$. This can be understood because such fields are almost regarded as the boundary-localized fields, which do not have couplings with the moduli at tree level. Thus we have to consider the next-leading contributions, and obtain

$$m^2_{Qa} \simeq \frac{2c^3 \langle \tau_s \rangle}{c^0 \langle \tau_b \rangle} m^2_{3/2} \ll m^2_{3/2}. \quad (42)$$

Although the scalar masses are much smaller than the gaugino masses at the compactification scale $m_{KK} = \pi/L_{ED}$ in this case, the former become comparable to the latter at low energies by the renormalization group effect if $m_{KK}$ is high enough, for example $m_{KK} = \mathcal{O}(10^{16} \text{ GeV})$. This situation is similar to the gaugino mediation [24], and we obtain the flavor universal soft masses in such a case.

4.3.2. Gauge and matter fields on the boundary. Next we consider the case that both the gauge fields and the chiral matter multiplets live on the boundary $y = 0$. Since the brane-localized fields do not couple with the moduli at tree level, the contribution from the moduli F-terms does not exist. Here we assume the following gauge kinetic function $f^{(0)}_G$ localized at $y = 0$:

$$f^{(0)}_G(X) = k_{0G} + k_{1G} X, \quad (43)$$

where $k_{0G}$ and $k_{1G}$ are $\mathcal{O}(1)$ constants.\(^{12}\) Then the gaugino masses $M_G$ are expressed as

$$M_G = \left| F^T \partial_1 \ln f^{(0)}_G \right| = g^2_G k_{1G} \left| F^X \right| = \begin{cases} \mathcal{O} \left( \frac{g^2_G}{\langle \tau_b \rangle} \frac{\xi_0}{a \langle \tau_s \rangle} \right) m^3_{3/2} \quad (X: \text{Bulk origin}) \\ \mathcal{O} \left( \frac{g^2_G}{\langle \tau_b \rangle} \frac{\xi_0}{a \langle \tau_s \rangle} \right) m^3_{3/2} \quad (X: \text{Brane origin}) \end{cases} \quad (44)$$

Here the gauge coupling constants are given by $g^2_G = (\text{Re} \langle f^{(0)}_G \rangle)^{-1}$.

As for the chiral matter multiplets, we assume the brane-localized Kähler potential $\Omega^{(0)}$ to have the form

$$\Omega^{(0)} = \sum_a \left( h_a |q_a|^2 - \kappa_{aX} |q_a|^2 |X|^2 \right), \quad (45)$$

where $h_a$ and $\kappa_{aX}$ are positive $\mathcal{O}(1)$ constants, and $q_a$ are brane-localized chiral superfields. Then the effective Kähler potential is calculated as

$$\Omega = \Omega^{(0)} - \frac{\zeta(3)}{8\pi^2 \mathcal{N} \text{Re} T} \Omega^{(0)} + \cdots \quad (46)$$

Therefore, the soft scalar masses for $q_a$ are computed as

$$m^2_{q_a} = -F^T \bar{F}^j \partial_j \partial_j \ln (\partial_{q_a} \delta_{q_a} \Omega) = \begin{cases} \mathcal{O} \left( \frac{m^3_{3/2}}{(a \langle \tau_s \rangle^2 b)} \right) \quad (X: \text{Bulk origin}) \\ \mathcal{O} \left( \frac{m^3_{3/2}}{(a \langle \tau_s \rangle^2 b)} \right) \quad (X: \text{Brane origin}) \end{cases} \quad (47)$$

The first term in the second line is the contribution of $F^T b$.

\(^{12}\) Since the R-symmetry is explicitly broken by the constant superpotential $W_0$, there is no reason to forbid the second term.
Hence all the superparticles have masses of the same order of the magnitude, which can be set to be $\mathcal{O}(\text{TeV})$. In this case, $L_{\text{ED}} = \mathcal{O}(10^7)$ and $m_{3/2} = 10^{11} \text{ GeV}$ when $X$ originates from the bulk field, and $L_{\text{ED}} = \mathcal{O}(10^8)$ and $m_{3/2} = 10^{10} \text{ GeV}$ for $X$ from the brane field.

4.3.3. Gauge fields in the bulk and matter fields on the boundary. Finally we consider the case that the gauge fields live in the bulk while the matter fields are localized on the boundary $y = 0$. In this case, the gaugino masses are obtained by (39), and the soft scalar masses by (47). Thus the situation is similar to the case in Sect. 4.3.1, and the flavor universal soft masses are obtained if $m_{\text{KK}}$ is high enough.

In either case discussed above, the higgsino mass can be obtained by adding the Giudice–Masiero terms \cite{25} to $\Omega^{(0)}$,

$$\Omega^{(0)}_{\text{GM}} = \eta H_u H_d + \text{h.c.}, \quad (48)$$

where $H_u$ and $H_d$ are the up- and the down-type Higgs superfields, and $\eta$ is a constant. Note that these terms cannot be introduced in the bulk because of the $N = 2$ SUSY structure.

Because the analysis in this section is performed in 4D effective theory, all masses in the above expressions must be laid below $m_{\text{KK}}$. This condition is satisfied if $|W_0| < (a \langle \tau_s \rangle)^{-1}$. However, we should note that our mechanism for the realization of a large extra dimension still works even when $|W_0| = \mathcal{O}(1)$, although the expressions of $m_{\tau_h}$ and $m_{\rho_s}$ in (33) have to be modified.

4.4. Comment on cosmology

Before closing this section, we provide some comments on cosmology based on our model. Notice that the lighter modulus $\tau_l$ and the axionic component $\rho$ are much lighter than the MSSM superparticles in all cases discussed in the previous subsection. Such light particles may cause some cosmological problems.

In the F-term inflation models, $\tau_l$ is not stabilized during inflation if the Hubble scale at that time $H_{\text{inf}}$ is larger than $m_{3/2}$. This so-called overshooting problem also occurs in the models in Refs. \cite{26–28}, and some solutions to it have been proposed in Refs. \cite{29–31}. Besides, the radion $\tau_b$ generically takes a different value from the present minimum during inflation, and starts to oscillate after it ends. Such oscillation dominates the energy density of the universe, and its decay ruins successful big bang nucleosynthesis. Low-scale inflation may be a way out of these problems. In the MSSM inflation model \cite{32}, for example, $H_{\text{inf}} = \mathcal{O}(0.1 \text{ GeV})$ and the correction to the moduli potential during inflation is small. So the above problems do not occur. However, some fine-tunings among the model parameters are generically required to realize low-scale inflation.

The axionic component $\rho$ remains massless and thus contributes to the dark radiation. It is pointed out in Ref. \cite{33} that $\rho$ produced from the decay of $\tau_l$ leads to too much dark radiation which contradicts the observation, even if $m_{\tau_l} \gtrsim \mathcal{O}(10 \text{ TeV})$. One of the solutions suggested there is to increase the partial width of the $\tau_l$ decay into the standard model particles such as the Higgs bosons or the gauge bosons. This can be achieved in our model by increasing the coupling constant $\eta$ in (48), for example.

5. Summary

We have explicitly shown that an exponentially large extra dimension can be naturally realized by the Casimir energy and the gaugino condensation in 5D supergravity on $S^1/Z_2$. The key ingredient is the nongeometric moduli, which are generically present in 5D supergravity. The relevant modulus to the Casimir energy is the geometric modulus, i.e., the radion. However, there is no reason that
the same modulus also determines the gauge coupling constant of the condensation sector. When the relevant modulus to that sector is different from the radion, the moduli potential has a minimum at an exponentially large VEV for the radion even if no hierarchies among the 5D parameters are assumed. Therefore we can dynamically obtain the TeV-scale KK scale only from the Planck-scale parameters.

The potential does not exist at tree level due to the no-scale structure of the Kähler potential. The one-loop correction breaks the structure and generates the potential for the moduli. The situation is similar to the LARGE volume scenario in string theory [27,28], but our mechanism does not need any stringy effects and works within the field theory. Besides, we can explicitly calculate the spectrum and effective couplings because 5D supergravity is much more tractable than string theory.

SUSY is broken at the vacuum. This is essentially the Scherk–Schwarz SUSY breaking, which is equivalent to the constant superpotential on the boundary [8,34]. The spectrum of the superparticles depends on whether they are in the bulk or on the boundary. If we do not assume any hierarchies among the 5D parameters, an $O(\text{TeV})$ KK scale is allowed only in the case that all the standard model fields are localized on the boundary. In the other cases, the gauginos become much heavier than the sfermions at the KK scale $m_{KK}$, and the spectrum becomes similar to that of the gaugino mediation when $m_{KK} = O(10^{16} \text{ GeV})$. We also provided some comments on cosmology.

Finally, we comment on an assumption we made. The vector sector of 5D SUGRA is specified by the cubic polynomial called the norm function. We have assumed that it does not mix the moduli multiplet $\mathbb{V}_b$ whose scalar component is $\tau_b$ and the gauge multiplets $\mathbb{V}_G$, i.e., $C^b_G = 0$ in (18). Notice that our 5D theory should be regarded as an effective theory of a more fundamental theory, e.g. a string theory, because it is nonrenormalizable. Such fundamental theories often have further extra dimensions compactified on a small manifold. Thus $C^b_G = 0$ can be easily realized if $T_b$ is geometrically separated from $V_G$ in a higher-dimensional compact space. In that case, however, the matter fields cannot live in the bulk because they are not charged for $\mathbb{V}_b$ and the observed fermion masses cannot be obtained [see (21)].

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Appendix A. Equivalence to Scherk–Schwarz SUSY breaking

Here we comment on an interpretation of the result in Sect. 2 from the viewpoint of the Scherk–Schwarz SUSY breaking, and compare (8) with the result in Ref. [7]. For this purpose, we consider the case where no gaugino condensation term exists ($A = 0$) and there are $n_{H1}$ massless hypermultiplets and $n_{H2}$ hypermultiplets with a common bulk mass $M_H$, and no antiperiodic fields (i.e., $n^A_a = n^\phi_a = 0$). Then (8) becomes

\[
V_{\text{eff}} = -\frac{6|\phi_C|^4|W_0|^2}{\tau^4} \left\{ \xi_1 - \frac{\xi_2}{12} F(c_H \tau) \right\} + \cdots \\
= -\frac{3|\phi_C|^2|F|^2}{2\tau^4} \left\{ \xi_1 - \frac{\xi_2}{12} F(c_H \tau) \right\} + \cdots , \tag{A1}
\]

13 This is not a fine-tuning because there is no nonzero value that $C^b_G$ must be tuned. It can take any values as long as it is exponentially suppressed.
where the ellipsis denotes higher-order terms in $\xi_1$ or $\xi_2$ and $Q_a$-dependent terms, and

$$
\xi_1 \equiv \frac{(n_V - n_{H1} + 2)\xi(3)}{32\pi^2}, \quad \xi_2 \equiv \frac{n_{H2}}{8\pi^2},
$$

$$
F(x) \equiv \xi(3) \left\{ 3 Z_P(x) - 2x Z'_P(x) + \frac{x^2}{2} Z''_P(x) \right\}. \quad (A2)
$$

At the second equality, we have used that

$$
P^T = \frac{2\phi^2_W}{\varphi_C} + \ldots. \quad (A3)
$$

It is well known that SUSY breaking by $F^T$ is equivalent to the Scherk–Schwarz SUSY breaking as shown in Refs. [8,34,36,37]. In the latter mechanism, SUSY is broken by the twisted boundary condition

$$
\Phi(x, y + 2L) = e^{-2\pi i \tilde{\omega} \tilde{\sigma}} \Phi(x, y), \quad (A4)
$$

where $2L$ is the circumference of $S^1$ and $\Phi$ denotes a $SU(2)_R$-doublet field, i.e., the gravitino, the gaugino, or the hyperscalar. From the consistency with the orbifold projection, the twist vector must be $\tilde{\omega} = (\omega_1, \omega_2, 0)$. We can always go to the periodic field basis by redefining fields as $\Phi \rightarrow e^{i \tilde{\omega} \tilde{\sigma} f(y)} \Phi$, where a function $f(y)$ satisfies $f(y + 2L) = f(y) + 2\pi$. Then the radion F-term $F^T$ is shifted by [13,34]:

$$
F^T \rightarrow F^T + 2\pi (\omega_2 - i \omega_1) |\varphi_C|. \quad (A5)
$$

Conversely, the nonzero value of $F^T$ in (A3) can be translated into the $SU(2)_R$ twist$^{14}$ with

$$
\tilde{\omega} = \frac{1}{2\pi |\varphi_C|} \left( -\text{Im} F^T, \text{Re} F^T, 0 \right), \quad (A6)
$$

by the field redefinition $\Phi \rightarrow e^{-i \tilde{\omega} \tilde{\sigma} f(y)} \Phi$.\textsuperscript{15}

Using (A6) and (11), the potential (A1) is rewritten as

$$
V_{\text{eff}} \simeq \frac{6\pi^2 |\tilde{\omega}|^2}{\tau^6} \left\{ \frac{\xi_1}{12} F(c_H \tau) \right\} + \ldots. \quad (A7)
$$

Now we assume that $c_H \tau \gtrsim 2$. Then, since

$$
Z_P(x) \simeq e^{-2x} \frac{\xi(3)}{\xi(3)} (1 + 2x), \quad (A8)
$$

for $x \gtrsim 1$, the function $F(x)$ is approximated as

$$
F(x) \simeq e^{-2x} \left( 3 + 6x + 6x^2 + 4x^3 \right). \quad (A9)
$$

We find that (A7) and (A9) agree with (3.6) in Ref. [7] under the following identification:

$$
\tau \leftrightarrow (\pi L)^{2/3} \phi^{1/3},
$$

$$
c_H = M_H L_{ED}^{1/3} \leftrightarrow M(L\pi)^{1/3},
$$

$$
(n_{H1}, n_{H2}, n_V) \leftrightarrow (N_h, N_H, N_V),
$$

$$
|\tilde{\omega}| \leftrightarrow \omega. \quad (A10)
$$

\textsuperscript{14}This translation is possible only in the flat spacetime. In the warped spacetime, $SU(2)_R$-twisted boundary conditions lead to an inconsistency [34,35].

\textsuperscript{15}In particular, we can cancel the boundary constant superpotential $W_0$ by choosing the function $f(y)$ as a step function [34].
The quantities on the right-hand side are the ones in Ref. [7]. We have used that $M_5 = L_{\text{ED}}^{-1/3}$ in the unit of $M_{\text{Pl}}$. In this comparison, we assumed that $\omega \ll 1$ and use the formula

$$\sum_{k=1}^{\infty} \frac{\sin^2(\pi \omega k)}{k^5} = \pi^2 \zeta(3) \omega^2 + O(\omega^4). \quad (A11)$$

From the relation (12), we find that $V_{\text{eff}}$ scales as $L_{\text{ED}}^{-4}$, which is peculiar to the Casimir energy. In Ref. [7], the (negative) bulk cosmological constant and the brane tensions are introduced as counterterms to absorb the nonvanishing vacuum energy and ensure the 4D Minkowski spacetime. In the context of 5D SUGRA, the introduction of the 5D cosmological constant requires gauging some isometry by the graviphoton. As pointed out in Ref. [34], this gauging is inconsistent with the Scherk–Schwarz twisted boundary condition (A4). Thus we do not introduce such counterterms here. Instead we assume the uplifting sector that consists of a chiral superfield $X$ originating from a brane-localized chiral multiplet to achieve a vanishing 4D cosmological constant.

When

$$\frac{1}{3} \leq \frac{\xi_2}{12 \xi_1} = \frac{n_{H_2}^2}{3(n_V - n_{H_1} + 2) \zeta(3)} \lesssim O(1), \quad (A12)$$

the potential (A7) has a minimum at $c_H \tau = O(1)$. Namely, the KK scale is

$$m_{\text{KK}} \equiv \frac{\pi}{L_{\text{ED}}} = O(1) \times M_H, \quad (A13)$$

and all the superparticles in the bulk have common masses,

$$m_{SB} = \frac{2\pi |\vec{\omega}|}{L_{\text{ED}}} = \frac{2 |W_0|}{L_{\text{ED}}}, \quad (A14)$$

where we have used that $|\vec{\omega}| = |W_0| / \pi$. Thus, to obtain TeV-scale superparticles, we have to assume a large hierarchy among the fundamental scales of the 5D theory $M_5, M_H$, and $|W_0|^{1/3}$, since

$$O(|W_0| M_H) = O\left(|W_0| M_5^3\right) = O\left(10^{-15}\right). \quad (A15)$$

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