Phenomenological description of space-time foam

Giovanni AMELINO-CAMELIA

Dipartimento di Fisica, Universitá “La Sapienza”, P.le Moro 2, I-00185 Roma, Italy

ABSTRACT

The expectation that it should not be possible to gain experimental insight on the structure of space-time at Planckian distance scales has been recently challenged by several studies. With respect to space-time fluctuations, one of the conjectured features of quantum-gravity foam, the experiments that have the best sensitivity are the ones which were originally devised for searches of the classical-physics phenomenon of gravity waves. In experiments searching for classical gravity waves the presence of space-time fluctuations would introduce a source of noise just like the ordinary (non-gravitational) quantum properties of the photons composing the laser beam used in interferometry introduce a source of noise. Earlier studies of the noise induced by quantum properties of space-time have shown that certain simple pictures of fluctuations of space-time occurring genuinely at the Planck scale would lead to an observably large effect. Experimentalists would benefit from the guidance of detailed description of this noise, but quantum-gravity theories are not yet developed to the point of allowing such detailed analysis of physical processes. I propose a new phenomenological approach to the description of foam-induced noise.
1 Introduction and summary

Work done in the last two decades [1, 2] and particularly over the last few years [3, 4, 5, 6, 7, 8] has corrected an old misconception that it would not be possible to gain experimental insight on “quantum gravity” (the sought theory of the interplay between general relativity and quantum mechanics) and on the structure of space-time at Planckian distances. The pessimistic expectations for “quantum-gravity phenomenology” that are described in traditional quantum-gravity reviews basically rely on two simple observations. First one observes that the interplay of general relativity and quantum mechanics can be the dominant element in the analysis of a physical context only if this context involves strong gravitational forces and short distances. This is a condition that was realized in the early stages of evolution of the Universe, but we would not be able to realize similar conditions experimentally. The second observation is based on the fact that in the contexts that we can study experimentally, which only involve length scales much larger than the Planck length

\[ L_p = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-33}\text{cm}, \]

all effects induced by quantum gravity would be very small. In fact, since \( L_p \) is proportional to both the gravitational constant, \( G \), and the Planck constant, \( \hbar \), we expect that the magnitude of these effects should be set by some power of the ratio between the Planck length and a characteristic length scale of the process under investigation.

Even without any detailed analysis of the interplay between general relativity and quantum mechanics it should be clear that these two observations cannot be sufficient for justifying the radical assumption that there is no hope for quantum-gravity phenomenology. In fact, for example, similar arguments would apply also to grandunified theories of the electroweak and strong interactions, which predict large new effects for collision processes involving particles with wavelengths of the order of \( 10^{-30}\text{cm} \) (which, of course, are not available to us), but predict only very small effects for the type of processes we can access experimentally. Yet experimentalists did manage to devise experiments with very good sensitivity to grandunification predictions. The probability of proton decay in grandunification is extremely small, since it is governed by the fourth power of the ratio between the mass of the proton and the grandunification scale, but, in spite of such a strong suppression, experimentalists are managing to set significant bounds on proton decay by keeping under observation a large number of protons (so that the experiments measure the probability that one among many protons decays).

As the analogy with proton decay in grandunification suggests, quantum-gravity phenomenology is not impossible, but it is of course extremely hard. Opportunities to test experimentally the nature of the interplay between general relativity and quantum mechanics remain extremely rare, but we have now a handful of proposals [1, 2, 3, 4, 5, 6, 7, 8] which represent a significant step forward with respect to the expectations of not many years ago.

\[ \text{In spite of the fact that I am somewhat responsible for the fact that the community refers to this field as “quantum-gravity phenomenology”, I now believe that this is a rather inappropriate terminology. Especially for a phenomenological (hopefully data-driven) programme it is not proper to make any a priori dogmatic assumptions about the outcome of the studies to be conducted, and the name quantum-gravity phenomenology could erroneously suggest that we have experimental evidence indicating that a relatively straightforward quantization of Einstein’s gravity is realized in Nature (while, as it is well known, there is no evidence of this type). A better name for the field would have been “Planck-length phenomenology” reflecting the fact that its objective is the investigation of the structure of space-time at Planckian distance scales. We also do not have any robust experimental evidence of new physics occuring at those distance scales, but the question “what is the structure of space-time at Planckian distances?” is fully meaningful physically: it is in a sense operatively well defined. (Moreover, the conceptual arguments suggesting that new physical theories are required in the Planck-length regime come from several complementary lines of analysis and appear to be robust.)} \]
In this paper I focus on one of these opportunities for quantum-gravity phenomenology: the study of quantum space-time fluctuations using modern interferometers [6, 11, 12, 13]. My main objective here is the proposal of a new phenomenological approach to the description of quantum-gravity-induced interferometric noise. This approach was recently sketched out in a short non-technical paper [15], and here I intend to elaborate on the proposal, also discussing some of the technical points omitted in Ref. [15].

Since quantum-gravity phenomenology is still a relatively young research subject, I start, in the next Section, with a brief review of the status of this field. I describe the main proposals, with emphasis on one of these proposals which has recently enjoyed some success, in the sense that it provided the first ever instance of experimental data that appear to be a manifestation of Planck-scale physics. While this exciting development is not directly connected with the type of space-time-foam studies which are the main focus of this Article, it appeared appropriate to discuss it in some detail since it shows that the development of quantum-gravity phenomenology has already encountered a non-empty set (one example) of “success stories”, at least in the sense that the analysis of certain present-day experimental paradoxes does appear to invite considerations of the type that is natural in quantum-gravity phenomenology. This in turn might provide encouragement to colleagues involved in experiments relevant for my present proposal of space-time-foam studies: having found already one example in another quantum-gravity phenomenology research line, one can look with more optimism at the possibility of encountering a second example of “quantum-gravity phenomenology success story”, and perhaps this second example will emerge as we pursue the space-time-foam experimental programme here outlined.

In Section 3 I start developing my characterization of space-time foam based on distance noise. I give an operative definition of space-time foam and show that the sensitivity of modern interferometers could be sufficient for detecting distance fluctuations occurring genuinely at the Planck scale. I illustrate my point by showing that the sensitivity achieved by modern interferometers would, for example, be sufficient for detecting the distance-noise level that corresponds to quantum fluctuations of the length of the arms of the interferometer (of a few kilometers!) that are only of Planck-length magnitude and occur with a frequency of one per Planck time.

In Section 4 I observe that (having established in Section 3 that modern interferometers are potentially relevant for quantum-gravity studies) it would now be necessary for the various theoretical approaches to the quantum-gravity problem to provide to the experimentalists detailed models of the noise to be expected in interferometers, but I also argue that the development of these theoretical approaches is still too preliminary for such “physical” predictions. As a way to by-pass this limitation of quantum-gravity theories, I then propose a new phenomenological approach that describes directly quantum-gravity-induced distance noise. My task is partly facilitated by the fact that in order to guide interferometric studies of foam it is only necessary to estimate a relatively simple (single-variable) function: the power spectrum of the distance noise [16, 17]. The quantum-gravity-induced strain noise should depend only on its variable argument, the frequency $f$ at which observations are made, on the Planck length, on the speed-of-light constant $c$ ($c \approx 3 \cdot 10^8 m/s$), and, perhaps, on a length scale characterizing the properties of the apparatus with respect to quantum gravity. I find that within this conceptual framework there is a compellingly-simple candidate for a foam-induced “white noise” (noise with constant, $f$-independent, power spectrum). More generally, I show that upon adopting a given qualitative description of quantum distance fluctuations (e.g., “white noise”, “$f^{-1}$ noise”, “random-walk $f^{-2}$ noise”....) one already has a lot of information on distance noise, because dimensional analysis constrains very strongly the structure of the noise spectrum. This leads me to consider a few natural candidates for a phenomenological description of quantum-gravity-induced distance noise, and, surprisingly, I find that over the next few years experiments will put under scrutiny quite a few of these candidates.
In Section 5 I discuss some urgent theory issues which could benefit the development of the phenomenological approach here proposed. The main point made in Section 4 is that the information on distance noise needed by experimentalists is such that with only a few qualitative guidelines from theory one can develop rather powerful phenomenological models. However, the guidelines we can extract presently from quantum-gravity theories are very limited. In Section 5 I describe a sort of “wish list” describing the type of guidelines that my new phenomenological approach most urgently needs from theory. One important bit of qualitative information concerns the structure of the limiting procedure, which of course must be present in quantum gravity, by which a quantum space-time approaches its corresponding classical space-time. The outlook of model-building for distance noise would be affected strongly by results indicating, for example, that this classical-limit procedure must involve in some way the ratio of the wavelengths of the particles versus the Planck length (as one would expect in pictures in which the same space-time appears to be “quantum” to particles of very short wavelength and appears to be classical to particles of very large wavelength, which basically average over the short-distance structure of space-time). Another important theory issue concerns the role of energy considerations in quantum gravity. While energy considerations are rather elementary when the analysis is supported by a fixed background space-time, the fact that quantum gravity cannot a priori rely on a background space-time renders energy considerations much more subtle [18]. If however theoretical analyses eventually develop that capability of providing some (even partial) information on the role of energy considerations in quantum gravity, the phenomenological approach I am proposing would acquire a very powerful tool for discriminating between different distance-noise models.

After the description of a “wish list for theory” given in Section 5, in Section 6 I outline a sort of “wish list for experiments”, emphasizing some key points that experimentalists planning to contribute to this research programme should take into account.

Finally Section 7 is devoted to some closing remarks.

2 Status of quantum-gravity phenomenology

As mentioned, we finally have some, although still very few, research lines attempting to gain experimental information on the interplay between quantum mechanics and general relativity. Let us denominate “quantum gravity” the still unknown, theory that describes this interplay. There are clearly two qualitatively different features that one can expect in quantum gravity: (i) the presence of new particles (e.g. the graviton) (ii) the presence of completely new phenomena [18] not describable in terms of the propagation of a particle in a background space-time, related with the non-classicality of space-time. For short I’ll refer to (i) as “particle-like effects” or “new particles propagating in a classical background space-time” and to (ii) as “non-particle-like effects” or simply “genuinely quantum space-time”.

Of course the most exciting class of quantum-gravity phenomena are the ones involving “non-particle-like effects”, the ones related to the emergence of profoundly non-classical (possibly quantum) properties of space-time, such as noncommutativity, discreteness, topology fluctuations.... General arguments [18, 19, 20] as well as specific proposals based on space-time discreteness or noncommutativity [21, 22, 23, 24, 25] provide motivation for the exploration of this “non-particle” possibility. I start this short review from those experiments which might provide opportunities to uncover this profoundly new realm of physics.

For what concerns the development of a corresponding phenomenological program the most promising opportunities for the exploration of the possibility of genuinely quantum space-times come from contexts in which there is no (classical) curvature. If the interplay between general relativity and quantum mechanics requires that at a fundamental level space-time is not classical, then this should in particular be true for the space-times that are
perceived by our low-energy probes as flat and classical. A space-time that appears to be Minkowski when probed by low-energy probes, would be perceived as “quasi-Minkowski” [11, 20] (some appropriate “quantum deformation” of Minkowski space-time) by probes of higher energies. [An example of such space-times has been discussed in Ref. [23] and references therein.]

One of the effects that could characterize a quasi-Minkowski space-time are quantum fluctuations of distances. These are the focus of the present Article and will be discussed in detail in the following Sections. Modern interferometers will be shown to provide exciting opportunities to search for these distance fluctuations, extending the preliminary indications of previous works on this subject [4, 6, 11, 12, 13, 14].

Another important property of quasi-Minkowski space-times is that the symmetries that characterize a deformation of Minkowski space-time are of course a deformed version (deformed algebras) of the symmetries of Minkowski space-time. This is extremely clear in the case of certain noncommutative space-times [22, 23] whose symmetries are properly described by Hopf algebras (e.g. the $\kappa$-Poincaré Hopf algebra, which reproduces the ordinary Poincaré algebra only in the low-energy limit). The realization that quantum gravity might lead to deformed symmetries has led to renewed interest in certain symmetry tests, as indirect tests of the short-distance structure of space-time.

In particle physics the symmetries of Minkowski space-time lead to the emergence of CPT symmetry. CPT tests have been discussed in relation with quantum gravity for more than 15 years [2, 26, 27, 28, 29, 30]. The present upper limits on violations of CPT symmetry (see, e.g., Ref. [27]) have reached a level which is significant for the study of the structure of space-time at Planckian distances. This is basically due to the fact that limits on the neutral-kaon “$\delta M_{K^0}/M_{K^0}$”, one of the CPT-violation parameters that can be introduced in the analysis of the neutral-kaon system, have reached the level $\delta M_{K^0}/M_{K^0} < 10^{-19} \sim L_p M_{K^0}$.

There are two classes of sensitive tests of certain types of deformations of Lorentz invariance that could be induced by non-trivial structure of space-time at Planckian distances. One class of studies is based on the fact that observations of the gamma rays we receive from distant astrophysical sources allow to establish that there is no anomalous effect (within the achieved experimental accuracy) leading to relative delays between the times of arrival of simultaneously emitted photons. The fact that these tests can be significant for quantum gravity follows from the fact that the time-delay sensitivity $\Delta T$ of the relevant experiments is remarkably small as compared to the overall duration $T$ of the journey that the relevant particles make from their far away astrophysical emission point to the Earth: $\Delta T/T \sim 10^{-21} \sim L_p E$, where $E$ is the energy of the particle. A second class of tests of deformations of Lorentz invariance is based on the fact that a deformation of Lorentz symmetries would of course affect our estimates of the threshold energies required for certain particle-production processes (those thresholds are basically kinematical). Recent experimental results concerning this second class of Lorentz-symmetry tests have led to some excitement. In two energy regimes, photons around 10 TeV and cosmic rays around $10^{20}$ eV, certain puzzling data admit interpretation as a manifestation of a departure from ordinary Lorentz invariance [4, 8, 34, 35]. As I shall emphasize again in Section 6, the way in which these exciting results on “threshold anomalies” [8] have emerged can provide encouragement for other quantum-gravity-phenomenology studies.

These four experimental programmes, the space-time-foam studies proposed in Ref. [4], the CPT tests proposed in Ref. [5], the time-of-flight Lorentz-invariance tests proposed in Ref. [3] and the threshold-energy Lorentz-invariance tests described in Ref. [8] (extending preliminary observations reported in Refs. [6, 34, 35]) are all we have at present as opportunities to explore experimentally effects associated with genuinely quantum space-times. The objective of quantum-gravity phenomenology, as defined earlier in this Article, is however even more general: one is hoping to gain insight on all aspects of the interplay between
general relativity and quantum mechanics is of interest. In this perspective there are at least two more classes of observations which should be mentioned in this review Section. The proposal put forward in Ref. [3] concerns possible tests (again using interferometers, but in a way that is different from the one discussed in the present Article) of the residual traces of some strong quantum-gravity effects (specifically string-theory effects [37]) which might have occurred in the early Universe. The quantum-gravity effects considered in Ref. [3] are not of the type here defined as “effects due to a genuinely quantum space-time”, since these effects basically amount to the introduction of new particles (the gravitons) in a classical background space-time.

Some information on the behaviour of quantum mechanics in presence of strong (but classical) gravitational fields has been obtained in studies of the quantum phases induced by large gravitational fields [1, 4]. These experiments explore a very special regime of quantum gravity, the one in which the space-time aspects of the problem can be analyzed within classical physics. All aspects of space-time are treated classically (one does not even introduce some new particles, e.g. the gravitons, with space-time degrees of freedom) but these experiments have provided [4] some insight on the role that the Equivalence Principle should have in quantum gravity.

3 Planck-scale distance fluctuations could be detected by modern interferometers

Having briefly reviewed, in the preceding Section, the overall status of quantum-gravity phenomenology, I now focus my attention on the role that interferometers (and other types of detectors, such as resonant bars) could have in the study of quantum-gravity-induced distance fluctuations.

A prediction of nearly all approaches to the unification of general relativity and quantum mechanics is that at very short distances the sharp classical concept of space-time should give way to a somewhat “fuzzy” (or “foamy”) picture (see, e.g., Refs. [38, 39, 40]), but these new concepts are usually only discussed at a rather formal level. If we are to test this prediction we must define space-time fuzziness in physically meaningful (operative) terms. Interferometers are the best tools for monitoring the distance between test masses, and I propose as operative definition of the distance fluctuations that could be induced by quantum gravity one which is expressed directly in terms of strain noise in interferometers [4]. In achieving their remarkable accuracy modern interferometers must deal with several classical-physics strain noise sources (e.g., thermal and seismic effects induce fluctuations in the relative positions of the test masses). Importantly, strain noise sources associated with effects of ordinary quantum mechanics are also significant for modern interferometers: the combined minimization of photon shot noise and radiation pressure noise leads to a noise source which originates from ordinary quantum mechanics [16]. The operative definition of fuzzy distance which I advocate characterizes the corresponding quantum-gravity effects as an additional source of strain noise. A theory in which the concept of distance is fundamentally fuzzy in this operative sense would be such that the read-out of an interferometer would still be

Since modern interferometers were planned to look for classical gravity waves (gravity waves are their sought “signal”), it is reasonable to denominate as “noise” all test-mass-distance fluctuations that are not due to gravity waves. I choose to adopt this terminology which reflects the original objectives of modern interferometers, even though this terminology is somewhat awkward for the type of studies I am proposing in which interferometers would be used for searches of quantum-gravity-induced distance fluctuations (and therefore in these studies quantum-gravity-induced distance fluctuations would play the role of “signal”).
noisy (because of quantum-gravity effects) even in the idealized limit in which all classical-physics and ordinary-quantum-mechanics noise sources are completely eliminated. Just like the quantum properties of the non-gravitational degrees of freedom of the apparatus induce noise (e.g. the mentioned combination of photon shot noise and radiation pressure noise) it is of course plausible that noise be induced by the quantum properties of the gravitational degrees of freedom of the apparatus (e.g. the distances between the test masses).

Another simple way to discuss this operative definition of distance fuzziness is the following. Let us assume that we have established experimentally the exact dependence on all relevant physical observables of the total noise present in an interferometer. The resulting strain noise spectrum will include terms that are independent of both the Planck constant $\hbar$ and the gravitational constant $G$, terms that depend either on $\hbar$ or on $G$, and there could also be terms that depend on both $\hbar$ and $G$. This last class of contributions to noise, depending on both $\hbar$ and $G$ (possibly on the particular combination of $\hbar$ and $G$ given by the Planck length $L_p$), is here being defined as the quantum-gravity contribution to noise.

This operative definition of quantum-gravity-induced distance noise immediately confronts us with a potentially serious challenge, which is the central challenge of all quantum-gravity-phenomenology research lines: if indeed quantum-gravity effects are proportional to (some power of) the Planck length $L_p$, the smallness of $L_p$ will automatically lead to very small effects. However, modern interferometers have a truly remarkable sensitivity to distance fluctuations and it is actually not difficult to realize that this sensitivity is potentially significant for the detection of fluctuations occurring genuinely at the Planck scale. In order to support this observation with a simple intuitive argument let us consider the possibility that the distances $L$ between the test masses of an interferometer be affected by Planck-length fluctuations of random-walk type occurring at a rate of one per Planck time ($\sim 10^{-44}s$). It is easy to show that such fluctuations would induce strain noise with power spectrum given by $L_p c L^{-2} f^{-2}$. For $f \sim 100Hz$ and $L \sim 1Km$ (as for some modern interferometers) this corresponds to strain noise at the level $10^{-37}Hz^{-1}$, well within the reach of the sensitivity of modern interferometers.

Fluctuations genuinely at the Planck scale (the simple scheme I used to illustrate my point involves Planck-length fluctuations occurring at a rate of one per Planck time) can lead to an effect that, while being very small in absolute terms, is large enough for testing with modern interferometers. This originates from the fact that random-walk fluctuations do not fully average out. They have zero mean (in this sense they do average out) but the associated standard deviation grows with the time of observation (with the random-walk-characteristic $\sqrt{t}$ dependence which translates into the $f^{-2}$ dependence of the power spectrum). A reasonable scale to characterize the time of observation in interferometry is provided by $f^{-1}$ which, for $f \sim 100Hz$, is much larger than the Planck time. $[(100Hz)^{-1}/10^{-44}s \sim 10^{40}$ and therefore over a time of order $(100Hz)^{-1}$ the standard deviation can become much greater than the Planck length.]

4 Phenomenological description of space-time foam and the sensitivity of planned experiments

The example of random-walk fluctuations is quite interesting since various quantum-gravity scenarios have random-walk elements. However, the random-walk case was here analyzed only as an example in which the classical space-time picture breaks down on distance scales of order $L_p \sim 10^{-35}m$, but the nature of this breaking is such that an interferometer working at a few hundred $Hz$ is sensitive to a collective effect of a very large number of
minute fluctuations. It may well be that the fluctuations induced by quantum gravity are not of random-walk type, but it appears that interferometers (and possibly resonant-bar detectors) should have significant sensitivity to various scenarios in which the fluctuations average out only in the sense of the mean and not in the sense of the standard deviation.

Having established that it is not preposterous to hope that modern interferometers might have the capability to detect quantum-gravity-induced distance fluctuations, it is now important for theorists to provide to the experimentalists a detailed description of the fluctuations. Unfortunately, the scarcity of experimental information on the quantum-gravity realm has not yet allowed a proper “selection process”, so there are a large number of quantum-gravity candidates. Moreover, even the two approaches whose mathematical/logical consistency has been already explored in some depth, the one based on “critical superstrings” \[43, 44\] and the one based on “canonical/loop quantum gravity” \[45, 46, 47\], have not yet matured a satisfactory understanding of their physical implications, such as the properties of space-time foam. In the few phenomenological programmes investigating other quantum properties of space-time \[4, 24, 27, 28, 5\] the difficulties deriving from the preliminary status of quantum-gravity theories have been circumvented by developing direct phenomenological descriptions of the relevant phenomena. I propose to apply the same strategy to the description of the noise induced in interferometers by quantum gravity.

As mentioned in Section 1, my task is partly facilitated by the fact that in order to guide interferometric studies of foam it is only necessary to estimate a relatively simple (single-variable) function: the power spectrum \(\rho_\nu(f)\) of the strain noise \[16, 17\]. In fact, the strain noise power spectrum, through its dependence on the frequency \(f\) at which observations are performed, contains the most significant information on the distance fluctuations, such as the mean square deviation (which is given by the integral of the power spectrum over the bandwidth of operation of the detector), and is the quantity against which the observations are compared.

The quantum-gravity-induced strain noise should depend only on the Planck length, the speed-of-light constant \(c (c \simeq 3 \cdot 10^8 m/s)\), and, perhaps, a length scale characterizing the properties of the apparatus with respect to quantum gravity. This renders dimensional-analysis arguments rather powerful: if the analysis of a given quantum-gravity approach allowed at least the identification of the qualitative nature of the distance fluctuations (e.g., random walk) then quite a lot of guidance could be provided to experimentalists using simple dimensional arguments. This is the central point being made in this Article and it will be illustrated in a few examples relevant for quantum gravity, also showing that in these examples the sensitivity of planned interferometers can be significant.

The first qualitative picture that I want to analyze dimensionally is one in which the foam-induced noise is white (noise with constant, \(f\)-independent, power spectrum). White noise is to be expected whenever \[16, 17\] the relevant stochastic phenomena are such that there is no correlation between one fluctuation and the next, an hypothesis which appears to be rather plausible for the case of foam-induced distance fluctuations. The hypothesis that foam-induced noise be white is also consistent with the intuition emerging from analogies \[48\] between thermal environments and the environment provided by foam as a (dynamical) arena for physical processes. According to these studies one can see foam-induced noise as essentially analogous to thermal noise in various physical contexts (such as electric circuits), which is indeed white whenever the bandwidth of interest is below some characteristic (resonant) frequency. In the case of foam-induced noise the characteristic frequency (which should be somewhere in the neighborhood of the quantum-gravity frequency scale \(c/L_p\)) would be

---

\[\text{Here the analogy with the strategy adopted in proton-decay experiments is very direct.}\]

\[\text{Strain here has the standard engineering definition } h \equiv \Delta L/L \text{ in terms of the displacement } \Delta L \text{ in a given distance } L.\]
much higher than the frequencies of operation of our interferometers, and foam noise would be white at those frequencies.

Within a white-noise model, by observing that the strain noise power spectrum carries dimensions of $Hz^{-1}$, one is naturally led to the estimate

$$\rho_h(f) = \text{constant} \sim \frac{L_p}{c} \sim 5 \cdot 10^{-44} Hz^{-1}.$$  \hspace{1cm} (1)

I also observe that, as mentioned, the frequencies we can access experimentally are much smaller than $c/L_p$, white noise is actually the only admissible structure for foam-induced strain noise within the hypothesis that this noise be independent of the characteristics of the apparatus which is used as a space-time probe. In fact this hypothesis implies that $\rho_h$ can only depend on its argument $f$, on the Planck length and on the speed-of-light constant, and therefore the most general low-frequency expansion is of the type

$$\rho_h(f) = a_0 \frac{L_p}{c} + a_1 \left( \frac{L_p}{c} \right)^2 f + a_2 \left( \frac{L_p}{c} \right)^3 f^2 + ...$$  \hspace{1cm} (2)

where the $a_i$ are numerical coefficients and all monomials of the type $f^{-|n|}$ were not included in the expansion because they would require coefficients of the type $L_p^{-|n|+1}$ (which are inconsistent with the fact that quantum-gravity effects must disappear in the limit $L_p \to 0$). For $f \ll c/L_p$ the expansion (2) is well approximated by its first term, which corresponds to the dimensional estimate (1). From the point of view of experimental tests it is also important to consider the value of the coefficient $a_0$, i.e. to take into account the inherent uncertainty associated with the dimensional estimate (1). In this type of studies based on dimensional analysis, the natural guess, which often turns out to be correct, is that coefficients such as $a_0$ are of order 1, but it is not uncommon to find a disagreement between the dimensional estimate and the experimental result of a few orders of magnitude. In testing (1) we shall therefore be looking for sensitivities extending a few orders of magnitude below the $L_p/c$ level.

Since it does not involve any explicit dependence on the structure of the apparatus being used to probe space-time, the estimate (1) can be tested using any detector with sensitivity to distance strain, such as interferometers and resonant-bar detectors. Remarkably, in spite of the smallness of the effects predicted, these types of experiments are reaching such a high level of sensitivity that (1) is going to be completely tested within a few years.

Denoting with $\rho_h^{TOT}$ the total strain noise power spectrum observed by the experiments, the present level of interferometric data is best characterized by the results obtained by the 40-meter interferometer [49] at Caltech and the TAMA interferometer [50] at the Mitaka campus of the Japanese National Astronomical Observatory, both reaching $\rho_h^{TOT}$ of order $10^{-40} Hz^{-1}$ (the lowest level has been achieved by TAMA around 1kHz: $\rho_h^{TOT} \sim 3 \cdot 10^{-41} Hz^{-1}$). Even more remarkable is the present sensitivity $\rho_h^{TOT} \simeq 5 \cdot 10^{-43} Hz^{-1}$ of resonant-bar detectors such as NAUTILUS [51] (which achieved it near 924Hz). This is already quite close to the estimate $L_p/c$ of (1). We are already probing a potentially interesting region and in order to complete a satisfactory test of the estimate (1) we only need to improve the sensitivity by a few orders of magnitude (in order to exclude also the possibility that the coefficient $a_0$ be somewhat smaller than 1).

This will be accomplished in the near future. Planned upgrades of the NAUTILUS resonant-bar detector are expected [52] to reach sensitivity at the level $7 \cdot 10^{-45} Hz^{-1}$. The LIGO/VIRGO generation of interferometers [53, 54] should achieve sensitivity of the order of $10^{-44} Hz^{-1}$ within a year or two, during its first phase of operation. A few years later, with the space interferometer LISA [55] and especially with the “advanced phase” [52, 53]
of the LIGO/VIRGO interferometers, another significant sensitivity improvement should be achieved: according to recent estimates it should be possible to reach sensitivity levels in the neighborhood of $10^{-48}\text{Hz}^{-1}$, more than four orders of magnitude below the $L_p/c$ estimate!

This expected experimental progress is described in the figure together with the $L_p/c$ white-noise level and the analogous noise-level estimates that can be obtained by assuming instead that the foam-induced noise be of “random-walk” type (i.e. with $f^{-2}$ frequency dependence of the power spectrum). Through the example of random-walk noise the figure shows that the sensitivity of modern interferometers is significant also with respect to non-white models of foam-induced noise. In the case in which the foam-induced distance fluctuations are of random-walk type, the corresponding strain noise should necessarily depend on some experiment-characteristic length scale $\Lambda$ (differently from the case of $L_p$-linear white noise). In fact, if random-walk noise depended only on $f$, $c$, and $L_p$, the strain-noise spectrum would have to be of the type $\rho_h \sim cL_p^{-1}f^{-2}$, in contradiction with the fact that any Planck-scale effect should vanish in the limit $L_p \to 0$. A model with random-walk strain noise linearly suppressed by the Planck length would have to predict a power spectrum of the form $\rho_h \sim cL_p f^{-2}\Lambda^{-2}$. Our capability to test such a model is to be described with the range of values of $\Lambda$ which we can exclude. As shown in the figure, for the $L_p$-linear random-walk-noise model the excluded range of values of $\Lambda$ extends all the way up to values of $\Lambda$ of the order of the length of the arms of the interferometer. In the random-walk case we will soon even reach some sensitivity to models with effects quadratically suppressed by the Planck length; in fact, as shown in the figure, the LISA interferometer will be able to test the possibility of noise levels of the type $\rho_h \sim cL_p^2 f^{-1}\Lambda^{-2}$, at least in the case in which the scale $\Lambda$ is identified with the wavelength of the beam used by LISA (which is, however, one of the smallest length scale that characterize the experimental setup of LISA). Since other quantum-gravity-motivated experimental programmes can only achieve sensitivity to effects linear in the Planck length, LISA’s capability to reach some level of “$L_p^2$ sensitivity” will mark the beginning of another significant phase in the search of quantum properties of space-time.

Having already discussed the possibility of white noise and random-walk noise, let us consider just one more significant candidate: “$f^{-1}$ noise”. It is in fact quite common in other noisy physical contexts to find a noise contribution with $f^{-1}$ spectrum, and it appears reasonable at this early stage of analysis to consider the possibility that also quantum-gravity-induced strain noise might have this characteristic behaviour. Actually, some studies reported in Refs. appear to provide preliminary evidence of the possibility that a minimum ingredient of quantum gravity is sufficient for generating $f^{-1}$ noise. In fact, I interpret the results obtained in Refs. as an indication that the minimum of distance fluctuations one would expect in a quantum-gravity theory, the fluctuations induced by the presence of gravitons, is characterized by $f^{-1}$ noise. Graviton effects would lead to $f^{-1}$ noise which is quadratically suppressed by the Planck length ($\rho_h \sim L_p^2 f^{-1}\Lambda^{-2}$), and this is beyond the reach of forthcoming experiments, unless the characteristic scale $\Lambda$ is to be identified with a rather short length (e.g., the wavelength of the beam used in the interferometer).

Examples of $f^{-1}$ noise are not reported in the figure (in order to maintain the number of lines in the figure to a level that allows easy consultation); however, the careful reader should realize that the structure of $f^{-1}$ noise is somewhat intermediate between the case of white ($f^0$) noise and random-walk ($f^{-2}$) noise. It is therefore relatively straightforward to deduce from the information provided in the figure, and from the type of dimensional-analysis considerations I reported above for white and random-walk noise, that forthcoming experiments also have good sensitivity to some plausible candidates of foam-induced $f^{-1}$ noise.
Figure 1: A qualitative (at best semi-quantitative) comparison between the sensitivity of certain interferometers and the types of strain noise power spectra ($\rho_h$) I am considering. We expect significant progress from the level of sensitivity ("PRESENT") of interferometers already in operation, to the first phase of LIGO/VIRGO interferometers ("LIGOI"), then to the second phase of LIGO/VIRGO ("LIGOII"), and finally to LISA ("LISA"). The horizontal line marks the noise level corresponding to $L_p/c$. The line "RW1" is representative of the random-walk scenario which is linearly suppressed by the Planck length and is proportional to the square of the inverse of the length of the arms of the interferometer. In spite of the Planck-length suppression the line RW1 is above (and therefore inconsistent with) the noise levels achieved by "PRESENT" interferometers (and by resonant-bar detectors already in operation such as NAUTILUS [51], which achieved sensitivity $5 \cdot 10^{-43} \text{Hz}^{-1}$, near 924 Hz). The figure also shows that with LISA we will achieve the capability to start the exploration of some scenarios with quadratic suppression by the Planck length: the line "RW2" corresponds to random-walk noise quadratically suppressed by the smallness of the Planck length and proportional to the cubic power of the inverse of the wavelength of the beam.
Some relevant issues for theoretical physics

The main point of this Article, discussed in the preceding Section (extending the analysis already reported in Ref. [15]), is that the phenomenology of quantum-gravity-induced distance fluctuations can be quite effective starting from very simple qualitative information on the nature of the fluctuations. The two key ingredients are: (a) the general picture of the fluctuation mechanism, and (b) the value of an experiment-characteristic length scale $\Lambda$. Since at present even this qualitative information is not available in the various approaches to quantum gravity, in the preceding Section I considered 3 plausible scenarios for ingredient (a) (white noise, random-walk noise and $f^{-1}$ noise) and 2 plausible values of the scale $\Lambda$ (the length of the arms of the interferometer and the wavelength of the beam). The number of scenarios to be considered will of course be sharply reduced as soon as some clear theory indications on the ingredients (a) and (b) are obtained.

While waiting for some quantum-gravity approaches to reach this type of capability, we can at least attempt to establish whether a given quantum-gravity approach would support some of the hypothesis for the experiment-characteristic length scale $\Lambda$. Two clear candidates are the length $L$ of the arms of the interferometer and the wavelength $\lambda_0$ of the beam, but there are several other length scales that characterize and interferometer, and, even focusing only on $L$ and $\lambda_0$, it cannot be a priori excluded that the value of $\Lambda$ (rather than being one of the simple options $\Lambda \sim L$ or $\Lambda \sim \lambda_0$) be given by some combination of $L$ and $\lambda_0$, e.g. $\Lambda \sim L^2/\lambda_0$.

It is also important to extend the analysis of the better-developed quantum-gravity approaches, with the objective of establishing whether they predict a genuinely quantum space-time, i.e. establishing whether the emerging picture of space-time requires something beyond the propagation of gravitational particles in an otherwise classical space-time. The latest results on “critical superstrings” and “loop quantum gravity” provide encouragement for the idea of a genuinely quantum space-time. These results suggest that at the quantum level space-time has some elements of discreteness and/or noncommutativity [21, 24, 25], and it appears unlikely that these features could be faithfully described by the propagation of some new particles in an otherwise classical space-time (continuous and commutative, and such that there is no preferred frame for its description).

Another issue which could be discussed already at the qualitative level concerns the key point of the discussion in the preceding Section: could space-time fluctuations be of a type that averages out only in the sense of the mean and not in the sense of the standard deviation? The mentioned quantum-gravity scenarios with some random-walk elements [11, 41, 42] provide encouragement for this possibility, but more work is needed, especially in order to develop suitable effective theories.

One more point that deserves mention is the one concerning whether or not there are some “energy constraints” that could be imposed on the structure of the quantum-gravity-induced interferometric noise. Energy considerations are clearly non-trivial in quantum gravity (and general relativity) because one should not rely on a given background space-time. Effects describable in terms of particle propagation (e.g., graviton propagation) in a given background space-time can be easily analyzed from the point of view of energy considerations, but, as mentioned, it appears likely [18] that quantum gravity would predict also effects that cannot be described in terms of particle propagation in a given background space-time. For this second class of effects energy considerations are highly nontrivial, for example it is easy to conceive [18] space-time fluctuations that effectively “carry” negative energy. For pictures

---

5In the formalisms we ordinarily consider, such as the one of field theory, effects on very short distances do not leave any trace on larger scales. This is a key aspect of the renormalization procedure and of certain types of coarse graining. For the fluctuations here considered this should correspond to the requirement that the mean vanishes and the standard deviation does not grow with time.
of space-time fluctuations that “carry” positive energy a strong constraint comes from the
requirement not to overclose the Universe, a requirement which can be formalized through
the formula [52]
\[
\frac{\rho_{gw}}{\rho_c} \approx \frac{10^{36}}{h_0^2} \int_{f_{\text{min}}}^{f_{\text{max}}} df \left( \frac{f}{1\text{Hz}} \right)^2 \rho_h(f),
\]
(3)
where \(\rho_{gw}\) is the energy density of the stochastic background of gravity waves, \(\rho_c\) is the
value of the critical energy density for closing the Universe, and \(h_0\) is a parameter which at
present we can only constrain to the interval \(0.50 < h_0 < 0.85\) (reflecting the experimental
uncertainty in the Hubble constant).

Among the phenomenological models considered in the preceding Section, this require-
ment can be applied straightforwardly only to the one in which \(f^{-1}\) noise originates from
the properties of gravitons (which indeed are effects associated with the propagation of new
particles in a classical background space-time). It is easy to see that the fact that \(f^{-1}\) noise
tends quickly to 0 at high frequencies leads to automatic compliance with the requirement.
For the other scenarios considered in the preceding Section, which should not be associated
with particle propagation in a background space-time, it is not clear whether one should be
allowed to enforce analogous energy requirements. In addition to the mentioned fact that en-
ergy considerations become highly nontrivial when there is no background space-time, these
requirements must also be treated prudently because the proper way to understand quantum
noise in interferometers is that this noise is a property of the apparatus\(^6\) (not a property
of “empty” space), and therefore it is plausible that bounds based on energy considerations
should involve some estimate of the energy that the fluctuations carry locally at the lab site
(rather than considering some integration over the whole Universe).

The understanding of these delicate issues is however very important for the development
of the phenomenological approach here advocated. It would not affect models with noise that
decreases rapidly at high frequencies (just like in the case of \(f^{-1}\) noise, also random-walk
noise complies automatically with requirements of the type (3)), but significant constraints
could emerge (if at all applicable) for models in which there is no sharp decrease at high
frequencies, such as the white-noise model.

6 Key challenges for the experimental programme

As for the other research lines of quantum-gravity phenomenology, these preliminary analy-
eses of quantum-gravity-induced distance fluctuations (and the associated strain noise) should
be interpreted as an invitation to experimentalists to keep a vigilant eye on possible anom-
aliies that could be interpreted as manifestations of the Planck-scale-structure of space-time.
Quantum-gravity models are not yet ready for making definite predictions, but one can ex-
amine some qualitative aspects of a candidate new phenomenon and then make dimensional-
analysis considerations in order to establish whether relevant experiments have a chance
to uncover effects originating at Planckian distances. As mentioned, at least in one case,
the mentioned studies of observed threshold anomalies, the fact that experimentalists were
alerted by this type of considerations has led to exciting developments [7, 8, 34, 35], and it
\(^6\)Interferometric noise is always correctly understood as a property of the interferometer with respect
to the relevant physical processes. This is true [16] of the well-understood noise sources originating from
classical-physics phenomena (e.g., thermal effects) and from phenomena of ordinary (non-gravita-
tional) quantum mechanics (the mentioned combination of photon shot noise and radiation pressure noise), and it
is here naturally assumed to be true of the possible contribution to noise resulting from quantum properties
of space-time.
appears now reasonable to hope that other quantum-gravity-phenomenology research lines might stumble upon analogous experimental results.

In the case of interferometers (and resonant detectors) the task of keeping a vigilant eye on possible quantum-gravity anomalies is particularly challenging. In fact, quantum gravity motivates the search for excess noise, but excess noise of non-quantum-gravity origin is very common in these experiments. Experimentalists make a large effort of predicting all noise sources, but when the machines finally are in operation it is quite natural to find that one of the noise sources was underestimated. Upon encountering excess noise the first natural guess is that the excess be due to ordinary/conventional physics rather than new physics. In this respect one first point to be remarked is that even just the determination of upper bounds on quantum-gravity-induced noise can be valuable at this stage of development of quantum gravity. This research field has been for a long time a theoretical exploration of a territory that was completely uncharted experimentally, and it is therefore quite important that a few experiments are now providing at least some guidance in the form of upper limits naturally expressable in terms of the Planck length \[2, 5, 6\]. Conservative upper limits on quantum-gravity-induced excess noise can be set straightforwardly \[6, 9, 11\] by making conservative (lower bounds) estimates of conventional noise and observing that any quantum-gravity-induced noise could not exceed the difference between the noise observed and the corresponding conservative estimate of conventional noise.

While upper limits are important, the possibility of discovering quantum-gravity-induced noise would of course be more exciting. As mentioned, the difficulties involved in accurate predictions of noise levels of conventional origin combine with the scarce information on the structure to be expected for quantum-gravity noise in a way that renders such discoveries very problematic; however, it is worth noticing that there are certain characteristics of quantum-gravity-induced noise which could plausibly be used in order to identify it. For example, from the description given in the preceding Sections it is clear that in an interferometer quantum-gravity-induced noise might roughly look like a stochastic background of gravity waves with the important characteristic of the absence of long-distance correlations. Excess noise in the form of a stochastic background of gravity waves is predicted also by other new-physics proposals\[7\] but typically these other new-physics proposals predict long-distance correlation \[3, 52\]. This important difference could be used to distinguish experimentally between noise induced by genuinely quantum properties of space-time and other excess-noise new-physics proposals.

7 Closing remarks

Encouraged by the exciting developments that have recently emerged in another quantum-gravity-phenomenology research line \[8, 34, 35\], I have here outlined a phenomenological approach to the description of quantum-gravity-induced noise. Based on the information contained in the figure it appears that forthcoming experiments will start constraining quite severely model-building for quantum-gravity-induced noise.

These opportunities are directly associated with the fact that interferometers are preparing to reach sensitivity at or below \(10^{-44} \text{Hz}^{-1}\) over a relatively wide (combining LIGO/VIRGO and LISA) range of frequencies. It is quite amusing to notice that experimentalists have been preparing for these sensitivity levels in response to classical-physics theoretical studies showing that the strain noise power spectrum should be reduced at or below the level \(10^{-44} \text{Hz}^{-1}\)

\[\text{The early-Universe quantum-gravity effects considered in Ref. 3 would have eventually given rise to a stochastic background of gravity waves, which presently could play the role of noise in the observation of the gravity waves produced by other astrophysical phenomena. The distance fluctuations induced by the stochastic background of gravity waves considered in Ref. 3 would have long distance correlations 32.}\]
in certain frequency windows in order to allow the discovery of classical gravity waves. It is a remarkable numerical accident that the result of these classical-physics studies, involving several length scales such as the distance between the Earth and potential sources of gravity waves, has pointed us toward a sensitivity level which I here observed to be also naturally described in terms of the intrinsically quantum scale $L_p/c$. 
References

[1] R. Colella, A.W. Overhauser and S.A. Werner, Phys. Rev. Lett. 34 (1975) 1472.
[2] J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, Nucl. Phys. B241 (1984) 381.
[3] R. Brustein, M. Gasperini, M. Giovannini and G. Veneziano, Phys. Lett. B361 (1995) 45.
[4] D.V. Ahluwalia, Mod. Phys. Lett. A13 (1998) 1393.
[5] G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, D.V. Nanopoulos and S. Sarkar, astro-ph/9712103, Nature 393 (1998) 763.
[6] G. Amelino-Camelia, gr-qc/9808029, Nature 398 (1999) 216.
[7] T. Kifune, Astrophys. J. Lett. 518 (1999) L21.
[8] G. Amelino-Camelia and T. Piran, astro-ph/0008107.
[9] G. Amelino-Camelia, Are we at the dawn of quantum-gravity phenomenology?, Lect. Notes Phys. 541 (2000) 1-49.
[10] G. Amelino-Camelia, “Planck-length phenomenology”, gr-qc/0008010.
[11] G. Amelino-Camelia, Phys. Rev. D62 (2000) 024015.
[12] Y.J. Ng and H. van Dam, Found. Phys. 30 (2000) 795.
[13] A. Campbell-Smith, J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B466 (1999) 11.
[14] Hong-wei Yu and L.H. Ford, gr-qc/9907037.
[15] G. Amelino-Camelia, A phenomenological description of space-time noise in quantum gravity, Nature (2001) in press.
[16] Saulson P.R., Fundamentals of interferometric gravitational wave detectors (World Scientific, Singapore, 1994).
[17] Radenka V., Low-Noise Techniques in Detectors, Ann. Rev. Nucl. Part. Sci. 38 (1988) 217.
[18] G. ’t Hooft, Class. Quantum Grav. 16 (1999) 3263.
[19] D.V. Ahluwalia, Phys. Lett. B339 (1994) 301.
[20] G. Amelino-Camelia, Mod. Phys. Lett. A13 (1998) 1319.
[21] C. Rovelli and L. Smolin, Phys. Rev. D52 (1995) 5743.
[22] J. Lukierski, A. Nowicki and H. Ruegg, Ann. Phys. 243 (1995) 90.
[23] G. Amelino-Camelia, J. Lukierski and A. Nowicki, Int. J. Mod. Phys. A14 (1999) 4575; G. Amelino-Camelia and S. Majid, Int. J. Mod. Phys. A15 (2000) 4301.
[24] A. Bilal, Fortsch. Phys. 47 (1999) 5.
[25] N. Seiberg and E. Witten,  hep-th/9908142, JHEP 9909 (1999) 032.
[26] P. Huet and M.E. Peskin, Nucl. Phys. B434 (1995) 3.
[27] J. Ellis, J. Lopez, N. Mavromatos, D. Nanopoulos and CPLEAR Collaboration, Phys. Lett. B364 (1995) 239.
[28] V.A. Kostelecky and R. Potting, Phys. Lett. B381 (1996) 89.
[29] F. Benatti and R. Floreanini, Nucl. Phys. B 488, 335 (1997).
[30] G. Amelino-Camelia and F. Buccella, Mod. Phys. Lett. A15 (2000) 2119.
[31] G. ’t Hooft, Class. Quant. Grav. 13 (1996) 1023.
[32] R. Gambini and J. Pullin, Phys. Rev. D59 (1999) 124021.
[33] B.E. Schaefer, Phys. Rev Lett. 82 (1999) 4964; S.D. Biller et al, Phys. Rev. Lett. 83 (1999) 2108.
[34] R. Aloisio, P. Blasi, P.L. Ghia and A.F. Grillo, Phys. Rev. D62 (2000) 053010.
[35] R.J. Protheroe and H. Meyer, Phys. Lett. B493 (2000) 1.
[36] S. Coleman, S.L. Glashow, Phys. Rev. D59 (1999) 116008.
[37] G. Veneziano, Phys. Lett. B265 (1991) 287; M. Gasperini and G. Veneziano, Astropart. Phys. 1 (1993) 317.
[38] J.A. Wheeler, in Relativity, groups and topology, (eds. B.S. De Witt & C.M. De Witt) (Gordon and Breach, New York, 1963).
[39] S.W. Hawking, Spacetime foam, Nucl. Phys. B144 (1978) 349.
[40] A. Ashtekar, C. Rovelli and L. Smolin, Phys. Rev. Lett. 69 (1992) 237.
[41] F. Markopoulou and L. Smolin, Phys. Rev. D58 (1998) 084033.
[42] R. Loll, J. Ambjorn and K.N. Anagnostopoulos, Nucl. Phys. Proc. Suppl. 88 (2000) 241.
[43] M.B. Green, J.H. Schwarz, & E. Witten, Superstring theory (Cambridge Univ. Press, Cambridge, 1987).
[44] J. Polchinski, String theory (Cambridge Univ. Press, Cambridge, 1998).
[45] A. Ashtekar, Quantum mechanics of geometry, gr-qc/9901023.
[46] M. Gaul and C. Rovelli, Loop Quantum Gravity and the Meaning of Diffeomorphism Invariance, Lect. Notes Phys. 541 (2000) 277-324.
[47] L. Smolin, The new universe around the next corner, Physics World 12 (1999) 79-84.
[48] L.J. Garay, Space-time foam as a quantum thermal bath, Phys. Rev. Lett. 80 (1998) 2508.
[49] A. Abramovici et al, Phys. Lett. A218 (1996) 157.
Updated information on the progress of observations performed by the TAMA interferometer can be found at the WWW site [http://tamago.mtk.nao.ac.jp/]

Astone P. et al, Phys. Lett. B385 (1996) 421.

M. Maggiore, Physics Reports 331 (2000) 283.

A. Abramovici et al, LIGO: The Laser Interferometer Gravitational-Wave Observatory, Science 256 (1992) 325-333. (Updated information on expected sensitivity of an advanced phase of the LIGO interferometer can be found at WWW site [http://www.ligo.caltech.edu/~ligo2/].)

B. Caron et al, The Virgo interferometer, Class. Quantum Grav. 14 (1997) 1461-1469. (Details on the sensitivity objectives of VIRGO can be found at the WWW site [http://www.virgo.infn.it/].)

K. Danzmann, LISA: Laser interferometer space antenna for gravitational wave measurements, Class. Quantum Grav. 13 (1996) A247.

M.-T. Jaekel and S. Reynaud, Europhys. Lett. 13 (1990) 301.

M.-T. Jaekel and S. Reynaud, Phys. Lett. B185 (1994) 143.