Exclusion Statistics of Composite Fermions

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Abstract

The exclusion statistics parameter of composite fermions is determined as an odd number ($\alpha = 3, 5, ...$). The statistics of composite fermion excitations at $\nu = \frac{n}{2pn+1}$ is rederived as $\alpha_{qe}^{CF} = 1 + \frac{2p}{2pn+1}$, $\alpha_{qh}^{CF} = 1 - \frac{2p}{2pn+1}$. The duality $\frac{1}{\alpha_{qe}(n,2p)} = \alpha_{qh}(n+1, 2p)$ is found. The distribution function for $\alpha = 3$ is obtained.

Introduction

The exclusion statistics was proposed by Haldane [1] as generalization of Pauli exclusion principle. The idea of Haldane is to define the change in the available particle space when particles are added to the system (or removed):

$$\Delta d_i = - \sum_j \alpha_{ij} \Delta N_j .$$

(1)

The generalization of the Haldane exclusion statistics was proposed by Wu [2] and others [3, 4]. The idea of generalization is to divide the available particle space for smaller cells (let us say with $k$ states). The number of many-particle states is given by:

$$\prod_i \left( d_{N_i} + N_i - 1 \right)$$

$$\prod_i \left( d_{N_i} + N_i - 1 \right)$$

(2)
where \( d_{N_i} = k - \alpha(N_i - 1), \alpha = \alpha_{ii}. \)

Johnson and Canright used the spherical geometry to find the statistics of Laughlin quasiparticles [6]. Analyzing numerical results they found the number of many-particle states of quasiholes (of the Laughlin \( 1/m \) state, \( m – \text{odd number} \)):

\[
\binom{N_e + N_{qh}}{N_{qh}} \tag{3}
\]

where \( N_{qh} \) is the number of quasiholes, \( 2S + 1 = m(N_e - 1) + 1 - N_{qh}, 2S \) is the number of flux quanta piercing the sphere. According to the definition (2) \( \alpha_{qh} = \frac{1}{m} \) [6]. The corresponding statistics parameter of Laughlin quasielectrons is \( \alpha_{qe} = 2 - \frac{1}{m} \). Here, we perform similar analysis in order to determine the exclusion statistics of composite fermions.

**Composite fermions**

The exact diagonalization results for the sphere can be interpreted in terms of composite fermions [7] if the effective field \( 2S^* = 2S - 2p(N_e - 1) \) is introduced. The composite fermion approach predicts the same angular momentum shell for quasiparticles [6], however, the main role play composite fermions (the number of composite fermions equals the number of electrons \( N_{CF} = N_e \)). The Eq. (3) is:

\[
\binom{2S^* + 1}{N_{qh}} \tag{4}
\]

and equals

\[
\binom{N_{CF} + 2S - m(N_{CF} - 1)}{N_{CF}} = \binom{2S + 1 + (1 - m)(N_{CF} - 1)}{N_{CF}}. \tag{5}
\]

According to the definition (2) the statistics parameter of composite fermions \( \alpha_{CF} = m \) (odd number \( \alpha_{CF} = 3, 5, ... \), for \( \alpha = 1 \) one has fermions).

Wu et al. [2] found the distribution function of \( \alpha \)-particles. The duality between \( \alpha \)-particles of \( \alpha = 1/m \) (holes) and \( \alpha = m \) was noticed by Nayak and Wilczek [3]. Our analysis reflects the duality between Laughlin quasiholes and composite fermions.

The distribution function is given by the set of equations [4]:

\[
n_i = \frac{1}{w + \alpha} \tag{6}
\]

\[
w^{\alpha}(1 + w)^{1-\alpha} = \xi \tag{7}
\]
\[ \xi = e^{\frac{\alpha}{1+2p}}. \]  

For example, for \( \alpha = 3 \) the solution is

\[ w = \frac{1}{s_+ + s_- - \frac{2}{3}} \]  

where

\[ s_\pm = \left[ \frac{1}{2\xi} + \frac{1}{27} \pm \sqrt{\frac{1}{\xi} \left( \frac{1}{\xi} + \frac{1}{27} \right)} \right]^\frac{1}{3}. \]

It is interesting to consider also the composite fermion excitations. Let us consider the filling \( \nu = \frac{n}{2p+1} \), then one gets \( n \) filled shells (in the field \( 2S^* = 2S - 2p(N_e - 1) \) the degeneration of the \( n \)-th effective shell is \( 2S^* + 2n - 1 \)). When one creates \( N_{qe} \) quasielectrons (of the state \( \nu = \frac{n}{2p+1} \)) in the \((n+1)\)-th level, the number of many-particle states is

\[ \binom{2S_{qe} + 2n + 1}{N_{qe}} \]  

and

\[ 2S_{qe} = -\frac{2p}{1+2pn} N_{qe} + \frac{2S - 2pn^2 + 2p}{1+2pn}. \]

If quasiholes are present in the \( n \)-th level then

\[ \binom{2S_{qh}^* + 2n - 1}{N_{qh}} \]  

and

\[ 2S_{qh}^* = \frac{2p}{1+2pn} N_{qh} + \frac{2S - 2pn^2 + 2p}{1+2pn}. \]

Hence, according to the definition (2)

\[ \alpha_{qe} = 1 + \frac{2p}{1+2pn}, \]  

\[ \alpha_{qh} = 1 - \frac{2p}{1+2pn} \]

as was first obtained in Ref. [8]. For \( n = 1 \) one gets the Laughlin states \( \frac{1}{m} = \frac{1}{2p+1} \). One can notice the duality between quasielectrons of the state \( \frac{n}{2p+1} \) and quasiholes of the state \( \frac{n+1}{2p(n+1)+1} \):

\[ \frac{1}{\alpha_{qe}(n, 2p)} = \alpha_{qh}(n + 1, 2p). \]

For example, consider quasielectrons at 1/3 and quasiholes at 2/5.
Conclusions

It is found that composite fermions can be described within the generalized exclusion statistics as particles with statistics parameter $\alpha$ being an odd number. Also, the exclusion statistics of composite fermion excitations is rederived as $\alpha_{qe} = 1 + \frac{2p}{1+2p}$, $\alpha_{qh} = 1 - \frac{2p}{1+2p}$. The duality $\frac{1}{\alpha_{qe}(n,2p)} = \alpha_{qh}(n + 1, 2p)$ is found.

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