Lepton-pair Čerenkov radiation emitted by tachyonic neutrinos: Lorentz-covariant approach and IceCube data

Ulrich D. Jentschura\textsuperscript{1} and Robert Ehrlich\textsuperscript{2}

\textsuperscript{1}Department of Physics, Missouri University of Science and Technology, Rolla, Missouri 65409, USA
\textsuperscript{2}Department of Physics, George Mason University, Fairfax, Virginia 22030, USA

Abstract: Current experiments do not exclude the possibility that one or more neutrinos are very slightly superluminal or that they have a very small tachyonic mass. Important bounds on the size of a hypothetical tachyonic neutrino mass term are set by lepton pair Čerenkov radiation (LPCR), i.e., by the decay channel $\nu \rightarrow e^+ e^- \nu$ which proceeds via a virtual $Z^0$ boson. Here, we use a Lorentz-invariant dispersion relation which leads to very tight constraints on the tachyonic mass of neutrinos; we also calculate decay and energy loss rates. A possible cutoff seen in the IceCube neutrino spectrum for $E_\nu > 2$ PeV, due to the potential onset of LPCR, is discussed.

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I. INTRODUCTION

The early arrival of a neutrino burst from the 1987A supernova [1] still motivates speculations about a possible superluminal nature of neutrinos, even if it is generally assumed that the delay in the arrival of electromagnetic radiation (light) is caused by the time the shock wave from the core collapse needs in order to reach the surface of the exploding star. If neutrinos are ever so slightly superluminal, then they may emit Čerenkov radiation in the form of light lepton pairs. In this paper, we attempt to answer three questions: (i). How would the energy threshold for the decay channel $\nu \rightarrow e^+ e^- \nu$ (lepton pair Čerenkov radiation, LPCR) have to be calculated if we assume a strictly Lorentz-covariant, space-like dispersion relation for the relevant neutrino flavor eigenstate? (ii). How would the decay rate and the energy loss rate have to be calculated under this assumption? Can the tachyonic Dirac equation [2–5] and its bispinor solutions [6, 7] be used in that context? (iii). What implications could be derived for astrophysics, under the assumption that a possible cutoff seen by IceCube for neutrinos with energies $E_\nu > 2$ PeV is confirmed by future experiments?

Theoretical arguments can be useful in restricting the possible degree of superluminality of neutrinos and maximum attainable neutrino velocities [8–10]. In Refs. [8, 9], a Lorentz-noncovariant dispersion relation $E_\nu = |\vec{p}| v_\nu$ was used, where $v_\nu > c$ is a constant parameter. This assumption leads to an energy-dependent effective “mass” square $E_\nu^2 - \vec{p}^2 \approx E_\nu^2 (v_\nu^2 - 1) v_\nu^{-2} \equiv m_\nu^{\text{eff}}$. The effective mass $m_\nu^{\text{eff}} = E_\nu \sqrt{1 - v_\nu^{-2}}$ then grows linearly with the neutrino energy. (Natural units with $\hbar = c = \epsilon_0 = 1$ are used in this paper, yet we shall include explicit factors of $c$ when indicated by the context.) Indeed, at the time, a best fit to the available experimental neutrino mass data including the initial OPERA claim [11] suggested the conceivable existence of an “energy-dependent mass” of the neutrino, as evidenced in Fig. 1 of Ref. [12]. The choice of the relation $E_\nu = |\vec{p}| v_\nu$ made in Ref. [8] was consistent with the need to model the initial OPERA claim [11], and is perfectly compatible with the concept of perturbative Lorentz breaking terms in the neutrino sector [9]. A Dirac-type equation leading to the Lorentz-noncovariant dispersion relation used by Cohen and Glashow [8] can be obtained [9] from the current operator given in Eq. (2) of Ref. [13] upon a particular choice of the $c_\mu$ parameters in the generalized fermionic current operator (in the notation adopted in Ref. [13]). Then, assuming a constant neutrino speed $v_\nu > c$, one can effectively describe the apparent absence of an energy dependence of the deviation of the neutrino speed from the speed of light $v_\nu \approx \text{const} > c$ (in the range $5$ GeV $< E_\nu < 50$ GeV), according to the (falsified) initial claim made by OPERA [11], while remaining compatible with the framework of perturbative Lorentz breaking [13].

However, while there are advantages to assuming a Lorentz-nonvariant dispersion relation for superluminal neutrinos (such as the preservation of the timelike positive quantity $E_\nu^2 - \vec{p}^2 > 0$), there are also a number of disadvantages. For example, if the dispersion relation $E_\nu = |\vec{p}| v_\nu$ holds in one particular Lorentz frame, then under a Lorentz boost, in general, one has $E_\nu' \neq |\vec{p}'| v_\nu$ in the moving frame [8, 9]. In order to illustrate the consequences of Lorentz noncovariance, let us consider a boost along the positive z axis into a frame which moves with velocity $u = c_2/v_\nu < c$. A particle moving along the positive z axis of the lab frame with four-momentum $p^\mu = (|\vec{p}| v_\nu, |\vec{p}| \hat{z})$ is mapped onto $p'^\mu = (|\vec{p}'| \sqrt{v_\nu^2 - 1}, 0)$ and thus is “at rest” in the moving frame. However, the general dispersion relation in the moving frame,

$$E_\nu' = \frac{p'_z}{2v_\nu} - \frac{(p_x^2 + p_y^2 + p_z^2)}{2p_z} v_\nu \quad (p_z' \neq 0), \quad (1)$$

is much more complicated. (Throughout this paper, we denote the spatial components of the four-vector $p^\mu = (E_\nu, \vec{p})$ by $\vec{p}$ and keep $|\vec{p}|$ explicitly, in order to avoid confusion between $p^2 = \vec{p}^2 p_\mu$ with $p^2 = |\vec{p}|^2$.)

An alternative, commonly accepted dispersion relation for so-called tachyons (these are space-like, faster-than-light particles described by a Lorentz-invariant wave
equation) reads as $E_{\nu}^2 = \vec{p}_{\nu}^2 - m_{\nu}^2$, i.e., it is the “normal” dispersion relation with the negative sign of the mass square term (see Refs. [2–7, 14–23]). Here, we calculate the threshold energy and the decay rate under the assumption of a Lorentz-invariant dispersion relation for the neutrino. We find that the alternate dispersion relation imposes a tight restriction on superluminality, and has important phenomenological implications for neutrino masses.

II. DISPERSION RELATIONS AND THRESHOLDS

For tachyonic particles, starting from the pioneering work of Sudarshan et al. [14–16], continuing with the works of Feinberg [17, 18], and including the tachyonic neutrino hypothesis [2–6, 19–23], the following dispersion relation has been assumed for the tachyonic (space-like) solutions,

$$E_{\nu} = \gamma_{\nu} m_{\nu}, \quad |\vec{p}_{\nu}| = \gamma_{\nu} m_{\nu} v_{\nu}, \quad \gamma_{\nu} = \frac{1}{\sqrt{1 - v_{\nu}^2}}. \quad (2a)$$

$$|\vec{p}_{\nu}| = E_{\nu} v_{\nu}, \quad p^\mu p_\mu = E_{\nu}^2 - \vec{p}_{\nu}^2 = -m_{\nu}^2, \quad (2b)$$

where we use the suggestive subscript $\nu$ for “neutrino”. These relations imply that $|\vec{p}| = E_{\nu} v_{\nu}$ instead of $E_{\nu} = |\vec{p}| v_{\nu}$. Here, the tachyonic Lorentz factor appears which is $\gamma_{\nu} = 1/\sqrt{1 - v_{\nu}^2}$. Tachyonic and tardyonic dispersion relations are unified upon assuming an imaginary value for $m$ in the tachyonic case (starting from the tardyonic case, one has $E = m/\sqrt{1 - v^2} \to im/\sqrt{1 - v^2} = m/\sqrt{v^2 - 1}$, where the latter equation holds for tachyons). With the standard definitions of $\vec{p}$ and $E_{\nu}$, one has $|\vec{p}_{\nu}| = \gamma_{\nu} m_{\nu} v_{\nu} = E_{\nu} v_{\nu}$ for both tardyons and tachyons.

In order to obtain the threshold energy for the LPCR decay $\nu \to e^+ e^- \nu$, we use the following conventions (see Fig. 1), inspired by Chap. 10 of Ref. [24], and define $E_1 = \sqrt{\vec{p}_{\nu}^2 - m_{\nu}^2}$ and $E_3 = \sqrt{\vec{p}_{3\nu}^2 - m_{\nu}^2}$ as the oncoming and outgoing neutrino energies, with $q = (E_1, \vec{p}_1) - (E_3, \vec{p}_3)$ being the four-momentum of the $Z^0$. Pair production threshold is reached for $q^2 = 4m_{\nu}^2$ and $\cos \theta = \vec{p}_1 \cdot \vec{p}_3/|(|\vec{p}_1||\vec{p}_3|)| = 1$. For collinear geometry, with all momenta pointing along the z axis, we have

$$q^2 = \left( \sqrt{p_{1z}^2 - m_{\nu}^2} - \sqrt{p_{3z}^2 - m_{\nu}^2} \right)^2 = \left( p_{1z} - p_{3z} \right)^2 = 4m_{\nu}^2. \quad (3)$$

Furthermore, threshold obviously requires $E_3 = 0$. (This is possible for tachyonic particles, when $|\vec{p}_{3\nu}| = p_{3z} = m_{\nu}$.) In this limit, the tachyonic particles becomes infinitely fast, and loses all of its energy, which implies that it is impossible to detect it [25]. The counterintuitive loss of energy for tachyons under acceleration is a consequence of standard tachyonic kinematics [2, 6, 7, 14–18, 26–28].

When the relations $E_3 = 0$ and $|\vec{p}_{3\nu}| = p_{3z} = m_{\nu}$ are substituted into Eq. (3), this yields

$$p_{1z}^2 - m_{\nu}^2 - (p_{1z} - m_{\nu})^2 = 4m_{\nu}^2. \quad (4)$$

Identifying $p_{1z} = |\vec{p}_{\text{th}}|$, with the threshold momentum, one easily finds

$$|\vec{p}_{\text{th}}| = 2m_{\nu}^2 + m_{\nu}. \quad (5)$$

The threshold energy is then easily found as

$$E_{\text{th}} = \sqrt{p_{\text{th}}^2 - m_{\nu}^2} = \frac{2m_{\nu}}{m_{\nu}} \sqrt{m_{\nu}^2 + m_{\nu}^2} \approx \frac{2m_{\nu}^2}{m_{\nu}}. \quad (6)$$

Because we are using a tachyonic dispersion relation, the threshold energy can be expressed a function of only the mass parameters. Larger tachyonic masses $m_{\nu}$ lead to lower threshold energies. In view of the tachyonic dispersion relation $m_{\nu} = E_{\text{th}} / \sqrt{v_{\text{th}}^2 - 1}$, where $v_{\text{th}}$ is the neutrino velocity at threshold, we may convert the threshold energy into a function of the electron mass and the neutrino threshold velocity. For given $E_{\nu}$, the limit $m_{\nu} \ll m_e$ is equivalent to the limit $v_{\text{th}}^2 - 1 = \delta_{\text{th}} \to 0$ because $m_{\nu} = E_{\nu} / \sqrt{v_{\text{th}}^2 - 1}$. In this limit, we have

$$E_{\text{th}} \approx \frac{2m_{\nu}^2}{m_{\nu}} = \frac{2m_e}{E_{\text{th}} / \sqrt{v_{\text{th}}^2 - 1}}$$

$$\Rightarrow E_{\text{th}} \approx \frac{\sqrt{2} m_e}{(v_{\text{th}}^2 - 1)^{1/4}}. \quad (7)$$

Substituting the exact dispersion relation into the threshold condition $E_{\text{th}} = 2m_e / \sqrt{m_e^2 + m_{\nu}^2}$, and solving for $E_{\text{th}}$, one obtains

$$E_{\text{th}} = \sqrt{2} m_e \left( 1 + \frac{v_{\text{th}}}{\sqrt{v_{\text{th}}^2 - 1}} \right)^{1/2}$$

$$= \begin{cases} \frac{\sqrt{2} m_e}{\delta_{\text{th}}^{1/4}} & \delta_{\text{th}} \ll 1 \\ 2m_e + \frac{m_e}{4\delta_{\text{th}}} & \delta_{\text{th}} \gg 1 \end{cases}. \quad (8)$$
For collinear incoming and outgoing neutrinos, threshold for pair production is reached at \( q^2 = (E_1 - E_2)^2 - (p_{1z} - p_{3z})^2 = (p_{1z} - p_{3z})^2 (v_e^2 - 1) = 4m_e^2 \), from which one derives (setting \( p_3 = 0 \)) the following threshold values (in agreement with Ref. [8]),

\[
|\vec{p}_1|_{\text{th}} = \frac{2m_e}{\sqrt{v_e^2 - 1}}, \quad (E_1)_{\text{th}} = \frac{2m_e v_e}{\sqrt{v_e^2 - 1}}.
\]  

Here, \( G_F \) is Fermi's coupling constant and the \( u \) and \( v \) are the standard fundamental positive-energy and negative-energy bispinor solutions of the Dirac equation [29]. The invariant matrix element is

\[
\mathcal{M} = \frac{G_F}{\sqrt{2}} \left[ \bar{\eta}(p_3) \gamma_\lambda (1 - \gamma^5) u(p_1) \right] \times \left[ \bar{\eta}(p_4) (c_V \gamma_\lambda - c_A \gamma_\lambda \gamma^5) v(p_2) \right].
\]

(10)

Here, \( c_V \approx 0 \), and \( c_A \approx -1/2 \) [see Eq. (5.57) on p. 153 of Ref. [30]]. Following [9], we now make the additional assumption that the functional form of the projector sum over spin orientations remains the same as for the ordinary Dirac equation even if the underlying dispersion relation is Lorentz-nonvariant (for a general discussion on such models see Ref. [31, 32]). In this case, the sum over final state and the averaging over the initial spins leads to

\[
\frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 = 64 G_F^2 (p_1 \cdot p_2) (p_3 \cdot p_4). 
\]

This enters the lab-frame expression for the decay rate [33],

\[
\Gamma = \frac{G_F^2}{12 \pi^2 (2E_1)} \int \frac{d^3 p_3}{2E_3} \int \frac{d^3 p_2}{2E_2} \int \frac{d^3 p_4}{2E_4} \left( p_1 \cdot p_3 q^2 + 2 (p_1 \cdot q) (p_3 \cdot q) \right),
\]

(11)

where \( q = p_1 - p_3 \). The azimuthal symmetry suggests the use of cylindrical coordinates. The domain of integration contains, for given \( p_1 = (p_{1z}, v_1, 0, 0) \), all permissible \( p_3 = p_{3\rho} \hat{\rho} + p_{3z} \hat{z} \), where \( p_{3\rho} = |\vec{p}_3| (v_\rho, \phi, v_\rho) \). With \( v_{\text{th}} = \frac{|\vec{p}|}{v_{\text{th}}} \), the momentum transfer is

\[
q^2 = -(p_{1z} - p_{3z})^2 - p_{3\rho}^2 + (p_{1z} - \sqrt{p_{3\rho}^2 + p_{3z}^2})^2 v_\rho^2,
\]

(12)

where we require \( q^2 > 4m_e^2 \approx 0 \). Solving Eq. (12) for \( p_{3\rho} \), one obtains the boundary of the region of permissible \( \vec{p}_3 \) vectors. An example is given in Fig. 2(a), in the form of a “sharpened ellipsoid” with a “sharp” top near \( p_{3z} \to p_{1z} \), and a “rounded” bottom with \( p_{3\rho} \to 0 \), and \( p_{3z} \to -(\nu - 1)/(\nu + 1) p_{1z} \). After a somewhat tedious integration over the allowed \( \vec{p}_3 \) vectors, one obtains

\[
\Gamma = \frac{G_F^2}{2688 \pi^3 v_\rho} \approx \frac{1}{14} \frac{G_F^2}{192 \pi^3}.
\]

\[
\frac{dE_{\nu}}{dx} \approx \frac{G_F^2}{96 \pi^4 (2E_{\nu})} \int_{q^2 > 0} d^3 p_3 \frac{2E_3}{2E_4} (E_{\nu} - E_3)
\]

\[
\times \left[ (p_1 \cdot p_3) q^2 + 2 (p_1 \cdot q) (p_3 \cdot q) \right],
\]

\[
= - \frac{G_F^2}{86016 \pi^3} \frac{\rho_0^2 \delta_3^3}{v_\rho} \approx - \frac{25}{448} \frac{G_F^2}{192 \pi^3}.
\]

(13)

III. DECAY RATE AND TIME-LIKE NONCOVARIANT DISPERSION RELATION

Given the complexities of calculating the decay rate due to LPCR using a tachyonic dispersion relation, it is extremely useful to first discuss the case of a Lorentz noncovariant form \( E_{\nu} = \frac{|\vec{p}|}{v_{\text{th}}} v_\nu \), using lab frame variables. For collinear incoming and outgoing neutrinos, threshold for pair production is reached at \( q^2 = (E_1 - E_2)^2 - (p_{1z} - p_{3z})^2 = (p_{1z} - p_{3z})^2 v_e^2 - 1 = 4m_e^2 \), from which one derives (setting \( p_3 = 0 \)) the following threshold values.

\[
|\vec{p}_1|_{\text{th}} = \frac{2m_e}{\sqrt{v_e^2 - 1}}, \quad (E_1)_{\text{th}} = \frac{2m_e v_e}{\sqrt{v_e^2 - 1}}.
\]
for the energy loss per unit length, confirming the results given in Eq. (2) and (3) of Ref. [8], and in Ref. [9]. This confirmation of the results given in Ref. [8] (under the assumptions made in the cited paper, namely, the dispersion relation $E_V = |\vec{p}| v_0$), but using a different method, namely, phase-space integration directly in the laboratory frame, encourages us to apply the same method to the calculation of the tachyonic neutrino decay rate, where the use of the laboratory frame is indispensable. The confirmation also underlines the consistency of the theoretical formalism under a change of the assumptions made in the calculation.

IV. DECAY RATE AND SPACE–LIKE COVARIANT DISPERSION RELATION

For an incoming tachyon, the particle state (space-like neutrino) may transform into an antiparticle state upon Lorentz transformation, and its trajectory may reverse the time ordering (see Fig. 3). Thus, the interpretation of a tachyonic neutrino state as a particle or antiparticle may depend on the frame of reference, and we should calculate the process directly in the lab frame. The necessity to transform certain tachyonic particle field operators into antiparticle operators under Lorentz boosts has been stressed in Refs. [6,17,18]. Incoming and outgoing states are required to be above-threshold positive-energy states in the lab frame (causality and tachyonic trajectories are discussed in Refs. [2,14–18] and Appendix A.2 of Ref. [34]).

We consider the matrix element

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left[ \pi^T(p_3) \gamma_\lambda (1 - \gamma^5) u^T(p_1) \right] \times \left[ \bar{\pi}(p_4) \left( c_V \gamma^\lambda - c_A \gamma^\lambda \gamma^5 \right) v(p_2) \right].$$

Here, the $u^T(p_1)$, and $u^T(p_3)$ are Dirac spinor solutions of the tachyonic Dirac equation [6, 7]. The bar denotes the Dirac adjoint. In the helicity basis (see Chap. 23 of Ref. [35] and Refs. [6, 7]), these are given by

$$u^T_\sigma(p) = \left( \frac{\sqrt{|\vec{p}|^2 + m a_\pm(\vec{p})}}{\pm \sqrt{|\vec{p}|^2 + m a_\mp(\vec{p})}} \right),$$

where the $a_\pm(\vec{p})$ are the fundamental helicity spinors (see p. 87 of Ref. [29]). Following [6, 7, 19], we use the tachyonic sum rule of the fundamental tachyonic bispinor solutions [see Eq. (34a) of Ref. [6]]

$$\sum_\sigma (-\sigma) u^T_\sigma(p) \otimes \bar{u}^T_\sigma(p) \gamma^5 = 0 ,$$

where $p = (E, \vec{p})$ is the four-momentum, and $\sigma$ is a helicity quantum number. We refer to Refs. [6, 7] for a thorough discussion; roughly speaking, the factor $(-\sigma)$ in Eq. (16) restores the correct sign in the calculation of the time-ordered product of tachyonic field operators (the propagator), for the contribution of all virtual degrees of freedom of the tachyonic field [see Eqs. (46)–(57) and Eq. (73)–(75) of Ref. [7]]. The $\gamma^5$ matrix in Eq. (16) is a part of the natural Dirac “adjoint” for the tachyonic spinor. Namely, the adjoint equation to the tachyonic Dirac equation, $(i \gamma^\mu \partial_\mu - \gamma^5 m) \psi(x) = 0$, reads as $[\bar{\psi}(x) \gamma^5] (i \gamma^\mu \partial_\mu - \gamma^5 m) \psi(x) = 0$. As explained in Eqs. (73)–(75) of Ref. [7], right-handed particle and left-handed antiparticle states (those with the “wrong” helicity) are excluded from the physical spectrum of the tachyonic field by a Gupta–Bleuler condition; these cannot contribute to the oncoming and outgoing neutrino states in Fig. 1 [while they do contribute to the virtual states, i.e., the propagator, see Eqs. (46)–(57) of Ref. [7]]. Both the incoming as well as the outgoing neutrinos in Fig. 1 are real rather then virtual neutrinos. Hence, in order to calculate the LPCR decay rate, we use the modified sum over tachyonic spinors

$$\sum_\sigma u^T_\sigma(p) \otimes \bar{u}^T_\sigma(p) = (1 + \gamma^5 \not\! \! \! \not p) (\not\! \! \! \! \not p - \gamma^5 m) \gamma^5 ,$$

where $\sigma = (1,0,0,0)$ is a time-like unit vector, $\not\! \! \! \not p/|\vec{p}|$ is the unit vector in the $\vec{p}$ direction, and upon promotion to a four-vector, we have $\not\! \! \! \! \not p = (0, \vec{p})$, so that $1 + \gamma^5 \not\! \! \! \! \not p = 1 - \Sigma \not\! \! \! \! \not p/|\vec{p}|$ becomes a left-handed helicity projector.

We thus calculate with an incoming, positive-energy, left-helicity tachyonic neutrino. One obtains the modified sum over spins $
abla_{\text{spins}}$ in the matrix element,

$$\nabla_{\text{spins}} \left| \mathcal{M} \right|^2 = \frac{G_F^2}{2} \text{Tr} \left[ \frac{1}{2} (1 + \gamma^5 \not\! \! \! \! \not p_3) (\not\! \! \! \! \not p_3 - \gamma^5 m) \gamma^5 \right] K^{\lambda \rho} ,$$

where $K^{\lambda \rho} = \text{Tr}\left[(\not\! \! \! \! \not p_4 + m_\nu) (c_V \gamma^\lambda - c_A \gamma^\lambda \gamma^5) (\not\! \! \! \! \not p_2 + m_\nu) (c_V \gamma^\rho - c_A \gamma^\rho \gamma^5)\right]$ is the familiar trace from the outgoing fermion pair. The decay rate is given by Eq. (11),
under the replacement \( \frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 \to \sum_{\text{spins}} |\mathcal{M}|^2 \).

The integrals over the momenta of the outgoing fermion pair (d3p2 and d3p4) are done using (p22 = p4 = m2)

\[
J_{\lambda\rho} (q) = \int \frac{d^3 p_2}{2E_2} \int \frac{d^3 p_4}{2E_4} \delta (q - p_2 - p_4) (p_{2\lambda} p_{4\rho})
= \frac{\pi}{24} \sqrt{1 - \frac{4m_2^2}{q^2}} \left[ g_{\lambda\rho} (q^2 - 4m_2^2) + 2q_{\lambda\rho} \left( 1 + \frac{2m_2^2}{q^2} \right) \right].
\]

(19)

It remains to analyze the domain of allowed \( \vec{p}_3 \) vectors [see the “cupola structure” in Fig. 2(b)], which is defined by the requirement \( q^2 > 4m_2^2 \), for \( p_i^2 = (\sqrt{p_{1z}^2 - m_2^2}, 0, 0, p_{1z}) \). The dispersion relation \( E_\nu = \sqrt{p_3^2 - m_2^2} \) implies that

\[
q^2 = 2 \left( \sqrt{E_1^2 + m_2^2} \sqrt{E_3^2 + m_2^2} \cos \theta - E_1 E_3 - m_2^2 \right).
\]

Here, \( \theta \) is the polar angle in spherical coordinates,

\[
p_i^\nu = (E_3, |\vec{p}_3| \sin \phi \cos \varphi, |\vec{p}_3| \sin \phi \sin \varphi, |\vec{p}_3| \cos \theta).
\]

Pair production threshold is reached, for given \( E_1 \) and \( E_3 \), by solving Eq. (20) for \( u = \cos \theta \), setting \( q^2 = 4m_2^2 \). After a somewhat tedious integration over the allowed \( \vec{p}_3 \) vectors (no masses can be neglected), one obtains

\[
\Gamma = \begin{cases} 
\frac{G_F^2 m_\nu^3}{128 \pi^3 m_2^2} (E_\nu - E_{\text{th}})^2 & E_\nu \gtrsim E_{\text{th}} \\
\frac{G_F^2 m_\nu^4}{288 \pi^3} E_\nu & E_\nu \ll E_{\text{th}} 
\end{cases}
\]

(22a)

for the decay rate, and

\[
\frac{dE_\nu}{dx} = \begin{cases} 
\frac{G_F^2 m_\nu^5}{64 \pi^3} (E_\nu - E_{\text{th}})^2 & E_\nu \gtrsim E_{\text{th}} \\
\frac{G_F^2 m_\nu^4}{144 \pi^3} E_\nu^2 & E_\nu \gg E_{\text{th}} 
\end{cases}
\]

(22b)

for the energy loss rate. In the high-energy limit, one may (somewhat trivially) rewrite the expressions as follows (\( m_\nu = E_1 \sqrt{\delta} \)),

\[
\Gamma = \frac{G_F^2 E_\nu^5 \delta_\nu^2}{288 \pi^3}, \quad \frac{dE_\nu}{dx} = \frac{G_F^2 E_\nu^5 \delta_\nu^2}{144 \pi^3}, \quad E_\nu \gg E_{\text{th}}. \quad (23)
\]

These results confirm that it is possible to use the tachyonic bispinor formalism [2–7] for the calculation of decay rates of tachyonic particles.

V. CONSTRAINTS ON THE MASS OF A TACHYONIC NEUTRINO

Our threshold relation Eq. (8) is based on a Lorentz-covariant dispersion relation. Only neutrinos with \( E_\nu < E_{\text{th}} = \sqrt{2m_\nu/\delta_{\text{th}}} \) survive the possibility of generalized leptonic Čerenkov radiation over a sufficiently long path length. The hypothetical observation of an absence of neutrinos above some energy \( E_{\text{th}} \) could thus be interpreted as a constraint on the neutrino mass. Let us assume a neutrino mass of \( m_\nu = X eV \), where \( X \) is generally assumed to be of order unity or less. Then, threshold is reached for \( m_\nu = X eV \), \( \delta_{\text{th}} = 3.67 \times 10^{-24}X^3 \), and \( E_{\text{th}} = \frac{222}{\sqrt{X}} \) GeV.

The IceCube experiment [36, 37] has observed 37 neutrinos having energies \( E_\nu > 10 \text{ TeV} \) during 3 years of data taking. Three of these events had energies \( E_\nu > 1 \text{ PeV} \), and one (often referred to as “Big Bird”) had \( E_\nu = (2.004 \pm 0.236) \text{ PeV} \). According to the IceCube collaboration [37], the spectrum of the 37 neutrinos is well fitted by a slope \( \sim E_\nu^{-2} \), which includes astrophysical as well as background atmospheric neutrinos, the latter being exclusively below 0.4 PeV. However, their best fit to the spectrum predicts 3.1 additional events for \( E_\nu > 2 \text{ PeV} \), and yet none were seen. Preliminary data for the fourth year includes 17 additional events, with none seen for \( E_\nu > 1 \text{ PeV} \) [38]. These facts suggest to the IceCube authors [36, 37] the possibility that there may be a cutoff for the spectrum for neutrinos above \( E \approx 2 \text{ PeV} \). The hypothesis is given further support by models which show that the Glashow resonance [39] (resonant \( \pi_\nu e^- \to W^- \to \text{anything} \) should add between zero and three times the number of events that appear in the interval 1 PeV < \( E_\nu < 2 \text{ PeV} \) as part of a broad peak centered around 6.3 PeV [40]. While evidence for the cutoff is disputed and alternative explanations have been proposed [41], the significance of such a cutoff has been analyzed in the light of superluminal neutrinos [42, 43].

Let us add a few clarifying remarks here. First, we note that the plots in the paper [37] refer to the neutrino flux as a function of neutrino energy; the events were apparently sufficiently well reconstructed so that no excess neutrino energy in addition to the energy deposited inside the detector is expected. Our Fig. 4 is based on Fig. 4 of Ref. [37]. Meanwhile, members of the IceCube collaboration have presented preliminary evidence for a through-going muon of energy \( \geq (2.6 \pm 0.3) \text{ PeV} \) which could be interpreted as a decay product of a neutrino of even higher energy [44, 45]. If the through-going muon could indeed be assigned to an ultra-high-energy neutrino of non-atmospheric origin, then it would push the conceivable cutoff seen by IceCube to even higher energies, further constraining the tachyonic mass term of the relevant neutrino flavor. So far, the authors of Ref. [37] (see the right column on page 4 of Ref. [37]) observe that “this [the lack of high-energy events] may indicate, along with the slight excess in lower energy bins, either a softer spectrum or a cutoff at high energies.”

Assuming \( E_{\text{th}} \approx 2 \text{ PeV} \) we would find using Eq. (8) that \( \delta_{\text{th}} = (\sqrt{2m_\nu/E_{\text{th}}})^{-1} \approx 1.7 \times 10^{-38} \), and furthermore, that \( m_\nu = \sqrt{\delta_{\text{th}}E_{\text{th}} \approx 0.00026 \text{eV} \left( \text{i.e., } -m_2^2 \approx -6.8 \times 10^{-8} \text{eV}^2 \right) } \) for one or more of the three neutrino flavors (conceivably, the one with the smallest absolute
were generated using an assumed pure value of \( m^2 \). A shifted cutoff [44, 45] of \( E_{\text{th}} \approx 3 \text{PeV} \), would be consistent with a tachyonic neutrino mass of \( m_\nu = 0.00017 \text{eV} \). One might object that it is not possible to have one (or more) tachyonic flavor masses \( m^2 < 0 \) and satisfy both neutrino oscillation data and the recent findings from cosmology for the sum of the flavor masses, i.e., \( \Sigma m \approx 0.32 \text{eV} \) [46, 47]. However, such consistency can be achieved using 3 active-sterile \( \pm m^2 \) (tardyon-tachyon) neutrino pairs [48]. The curves in Fig. 4 were generated using an assumed pure \( E_\nu^{-2} \) power law for the flux \( N \) beyond the assumed threshold, \( E_{\text{th}} \). We then use our \( dE_\nu/dx \) formula [Eq. (22)] for \( E_\nu > E_{\text{th}} \) to find the modified \( N E_\nu^2 \) spectrum. Good agreement is found with the IceCube data at a threshold \( E_{\text{th}} = 2.5 \text{PeV} \), although much more statistics will be needed to determine if the cutoff is real.

VI. CONCLUSIONS

Three main conclusions of the current investigation can be drawn. (i) As described in Sec. II, the assumption of a Lorentz covariant, tachyonic dispersion relation leads to tight bounds on conceivable tachyonic neutrino mass terms. The tachyonic decay rate due to LPCR is most conveniently calculated in the laboratory frame, because the space-like kinematics involved in the process, which leads to a non-unique time ordering of the trajectories, as discussed in Sec. IV. (ii) We may apply the formalism of the tachyonic bispinor solutions of the tachyonic Dirac equation [2–5] recently developed in Ref. [6, 7, 19] to the calculation of the tachyonic neutrino decay, as outlined in Sec. IV. (iii). A comparison of recent IceCube data with the results for the calculated tachyonic decay rates reveals that a tachyonic neutrino could possibly explain a possible sharp cutoff in IceCube data, but only if the neutrino flavor involved has a very specific tachyonic mass. In a more general context, the calculation of tachyonic thresholds and decay rates based on Lorentz-covariant dispersion relations could be of phenomenological significance for string theories, some of which predict the existence of tachyons [49, 50]. The same is true for the precise calculation of the tail of the beta decay spectrum, which is influenced by a conceivably tachyonic neutrino mass term [51].

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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