Pattern Division Multiple Access Featuring Amplify-and-Forward Relaying in an Uplink Network

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ABSTRACT NOMA is one of key enabling technologies for prosumer energy management. We studied co-operative techniques useful for establishing pattern division multiple access (PDMA) in an uplink network. In particular, we examined amplify-and-forward (AF) relaying with PDMA (AF-PDMA) over Rayleigh channels. To characterize the performance of AF-PDMA, we derived outage probability and system throughput data, and performed diversity analysis. To verify the performance advantages afforded by the AF-PDMA scheme, we allocated fixed power values to users when comparing the performance to that of a non-cooperative PDMA scheme, an AF relay with orthogonal multiple access (AF-OMA) scheme, and an OMA scheme. Monte Carlo simulation revealed that AF-PDMA outperformed the other three schemes in terms of outage probability and system throughput.

INDEX TERMS Internet of Things, pattern division multiple access, amplify-and-forward, uplink network.

I. INTRODUCTION With the rapid development of renewable energy, the carbon emission is reduced but the randomness is also brought [1], [2]. Distributed renewable energy resources have an adverse effect on energy management in microgrids [3]. With respect to microgrids, the optimal synergistic operation of distributed energy hubs is of vital importance [4]. The development of distributed energy hubs has made it possible for traditional passive consumers evolving into active prosumers. Internet of Things (IoTs) is of vital importance in prosumer monitoring, management and control. Communication security is very important for energy security [5]. Research has been conducted in this field of enquiry. With the development of large-scale multi input and multi output (MIMO) and IoTs, prosumer energy efficiency optimization will be a challenge for the future development of communication technology, in which multiple users can reuse non-orthogonal multiple access (NOMA) technology on the same subchannel shows great attraction.

With the advent of the information age, intelligent terminals, and mobile internet services, mobile communication systems face enormous challenges, given, the explosive growth of mobile data services in which large numbers of smart terminals are connected to a mobile internet. Existing 4G systems use orthogonal frequency division multiplexing multiple access (OFDMA) to avoid multi-access interference, such that reception is relatively simple. However, orthogonal systems lack the capacity boundary of a multi-user channel [6]. When many users seek access simultaneously, a multi-user communication system featuring orthogonal multiple access (OMA) does not allow for optimal spectral utilization. NOMA schemes have attracted significant attention, given their superior spectral efficiency and massive connectivity [7], [8]. Using NOMAs, signals from multiple users are superimposed and multiplexed using the same radio resources, but with different power factors based on the various channel conditions. Combining NOMAs with new technologies can further improve system performance [9]; NOMAs featuring relaying have attracted much attention. Ding et al. developed a cooperative NOMA (Co-NOMA) scheme [10], wherein users with better channel conditions serve as relaying nodes assisting those with poor...
channel conditions. Yue et al. used a NOMA with a fixed-gain amplify-and-forward (AF) system to relay over Nakagami-m fading channels; the user order was sorted [11]; the effect of relay selection (RS) on CO-NOMA performance was investigated [12].

Many researchers have contributed to the pattern division multiple access (PDMA) field [13]–[20]. Chen et al. [13] developed the basic PDMA concept, as well as PDMA framework, key technologies, PDMA system design, and performance evaluation method. Ren et al. [14] used a pattern matrix to design a PDMA for 5G uplink applications in massive machine type communication (mMTC) and enhanced mobile broadband (eMBB) scenarios. In [15], a novel method to assess the performance of PDMA link was developed; the performance of a belief propagation (BP) receiver was estimated using a genie-aided interference cancellation receiver as the upper bound and only a single fitting parameter was required. Dai et al. [16] used a complexity-constrained capacity-achieving method to design PDMA, and verified its performance via link- and system-level simulations. Zeng et al. [17] employed joint pattern assignment and power allocation to optimize the total throughput of all users in downlink PDMA system. Tang et al. [18] investigated applications of grant-free (GF) transmission to PDMA, and discussed the definition, latency, resource allocation, and transmission mechanism of GF-PDMA in great detail. Co-PDMAs featuring decode-and-forward (DF) relaying were proposed in [19] and [20]. However, use of successive interference cancellation (SIC) will have negative effects when DF relaying does not decode the signal, thus compromising system performance. AF relaying avoids this because no signal is decoded at the relay node. We were motivated by the above findings to contribute to this field of research. The main contribution of this paper is that we provide a novel model that combining PDMA technology with AF relaying in uplink network.

II. SYSTEM MODEL
A. SYSTEM DESCRIPTION
We consider a half-duplex, dual-hop communication scenario, a PDMA uplink network has multiple users, one base station (BS), and one AF relay.

Basic principle of PDMA can be concluded as follows. PDMA is based on the joint optimization design of transmitter and receiver, with unequal diversity at the transmitter side and post-detection (quasi-) equal diversity at the receiver side. PDMA pattern defines the sparse mapping from data to a group of resources on multiple signal domains (including time, frequency, power, and spatial domain) to distinguish different users [14].

As shown in Figure 1, users are divided into several groups according to PDMA pattern matrix. A PDMA pattern is associated with each user. The users in the different groups are allocated different time and frequency resources, while users in the same group share the same resources. The channels from the users to the relay, and from the relay to the BS, are frequency-nonselective and undergo independent Rayleigh fading. The channels between the users and the BS, the AF relaying node and the BS, and the users and the AF relaying node are denoted as $h_{mn}$ and $\mu_{mn}$, and $(mn \in \{ub, rb, ur\})$, respectively, where $i$ and $j$ denote the $i$th resource element (RE) and the $j$th user, respectively. Without loss of generality, we assume that all nodes have a single antenna.

The signal at the receiver (from the transmitter) can be expressed as (1):

$$
y = H_{PDMA}x + n = \sum_{k=1}^{K} \text{diag}(h_k)g_k x_k + n$$

$$= \tilde{h}_k x_k + \sum_{i \neq k} \tilde{h}_i x_i, \quad i, k = 1, 2, \ldots, K \quad (1)$$

where

$$H_{PDMA} = H_{CH} \odot G_{PDMA}^{[N,K]} = [h_1 \ h_2 \ \cdots \ h_K] \odot [g_1 \ g_2 \ \cdots \ g_K]$$

$$= [\tilde{h}_1 \ \tilde{h}_2 \ \cdots \ \tilde{h}_K] = \begin{bmatrix}
h_{11}g_{11} & h_{12}g_{12} & \cdots & h_{1K}g_{1K} \\
h_{21}g_{21} & h_{22}g_{22} & \cdots & h_{2K}g_{2K} \\
\vdots & \vdots & & \vdots \\
h_{N1}g_{N1} & h_{N2}g_{N2} & \cdots & h_{NK}g_{NK}
\end{bmatrix} \quad (2)$$

is the PDMA-equivalent channel response matrix from users to the receiver; this is an $N \times K$ vector. $x = [x_1 \ x_2 \ \cdots \ x_K]$ is a $K \times 1$ modulation symbol vector transmitted by $K$ users, and $x_k$ is the modulation symbol of the $k$th user; $y$ is an $N \times 1$ received signal vector using $N$ resources. $n$ denotes additive white Gaussian noise (AWGN) and is modeled as $n \sim CN(0, N_0 I_{N \times N})$. $H_{CH} = [h_1 \ h_2 \ \cdots \ h_K]$ is the channel response matrix from users to the BS. We assume that all users are located in a homogeneous environment, and that the signal from each user to the BS has independent identically distributed (i.i.d), frequency non-selective Rayleigh fading and AWGN. The $N \times 1$ channel vector between the $k$th user.
and the $n^{th}$ RE at the BS is denoted by $h_k = [h_{1k}, h_{2k}, \ldots, h_{NK}]$. where $h_{ik}$ is modeled by $h_{ik} \sim \text{CN}(0, \mu)$.

$$G_{\text{PDMA}}^{[N,K]} = [g_1, g_2, \ldots, g_k] = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1K} \\ g_{21} & g_{22} & \cdots & g_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N1} & g_{N2} & \cdots & g_{NK} \end{bmatrix}$$

(3)

denotes a PDMA pattern matrix with dimensions of $N \times K$, where $K/N$ is the system overload rate (SOR); $g_k$ is the PDMA pattern used by the $k^{th}$ user.

### B. SIGNAL MODEL

The entire communication process is completed over two time slots. Note that the superimposed signals use the same channel for the $k^{th}$ user.

During the first time slot, users directly transmit superimposed signals to the relay node and the BS. The received signals at the BS and node can be written as (4) and (5), respectively:

$$y^{ub}_{k} = \tilde{h}^{ub}_{k} x_k + \sum_{i \neq k} \tilde{h}^{ub}_{i} x_i + n^{ub}_{k}, \quad (i, k = 1, 2, 3)$$  

(4)

$$y^{ur}_{k} = \tilde{h}^{ur}_{k} x_k + \sum_{i \neq k} \tilde{h}^{ur}_{i} x_i + n^{ur}_{k}, \quad (i, k = 1, 2, 3)$$  

(5)

where $z^{ub}_{k}$ and $z^{ur}_{k}$ represent the noise-plus-interference encountered by the $k^{th}$ user at the BS and the relay. The covariance of $z_k$ is explicitly calculated as:

$$KZ_k = N_0 I_N + \sum_{i \neq k} P_i \tilde{h}_i (\tilde{h}_i)^H. \quad (6)$$

According to [17], the SINR of the $k^{th}$ user is given by:

$$\gamma^{ub}_{k} = P_k (\tilde{h}^{ub}_{k})^H (K_{\text{eq}}^{ub})^{-1} \tilde{h}^{ub}_{k}, \quad (7)$$

$$\gamma^{ur}_{k} = P_k (\tilde{h}^{ur}_{k})^H (K_{\text{eq}}^{ur})^{-1} \tilde{h}^{ur}_{k}. \quad (8)$$

During the second time slot, the relay re-modulates the recovered data $y^{rb}_{k}$ and transmits these to the BS after inserting a CP with an amplifying gain factor $G = \sqrt{P/(P|h^{rb}| + N_0)}$.

AT the BS, the signal received from the relay is given by:

$$y^{rb}_{k} = G \tilde{h}^{rb}_{k} \otimes \left( \tilde{h}^{ur}_{k} x_k + \sum_{i \neq k} \tilde{h}^{ur}_{i} x_i + n^{ur}_{k} \right) + n^{rb}_{k}, \quad (i, k = 1, 2, 3).$$  

(9)

The covariance of $z^{ur}_{k}$ is explicitly calculated as (10):

$$KZ_k^{ur} = y^2 \sum_{i=k+1}^{K} a_i (h^{ur}_{i} \circ h^{rb}) (h^{ur}_{i} \circ h^{rb})^H + \gamma (h^{ur}_{i} h^{ur}_{i})^H + h^{rb} (h^{rb})^H + I_N. \quad (10)$$

The SINR of the $k^{th}$ user is given by:

$$\gamma^{rb}_{k} = P_k (\tilde{h}^{rb}_{k})^H (K_{\text{eq}}^{rb})^{-1} \tilde{h}^{rb}_{k}. \quad (11)$$

where $y^{rb}_{k}$ and $n$ are $2 \times 1$ vectors of the signals and noise, respectively, transmitted from the relay node to the BS on two REs, $\tilde{h}^{rb}_{k}$ is a $2 \times 1$ vector denoting the channel response for the $k^{th}$ user from the relay node to the BS.

### III. MATH PERFORMANCE ANALYSIS

#### A. OUTAGE PERFORMANCE

Here, we derive close-form expressions for user outage probability (OP). We combine BS data with user observations and information on the relay node, and use different combinations thereof during the final slot. Outage occurs if neither the direct nor the relay transmission succeeds, expressed as follows:

$$O_k = O^{ub}_k \times O^{rb}_k,$$  

(12)

where $O^{ub}_k$ and $O^{rb}_k$ denote the OP for a BS that cannot directly detect the $k^{th}$ user signal, or for a BS that cannot detect the signal from the relay node. We define $E^{ub}_k = \{\gamma^{ub}_{k} < \varphi_k\}$ and $E^{rb}_k = \{\gamma^{rb}_{k} < \varphi_k\}$ as outage events occurring when the BS receives the $k^{th}$ user signal directly and from the relay node, respectively. $\varphi_k$ denotes the targeted SINR for the $k^{th}$ user; $\varphi_k = 2^{2R_k} - 1$, and $R_k$ is the target data rate (TDR) for the $k^{th}$ user. $(E^{ub}_k)^c$ and $(E^{rb}_k)^c$ are the complementary sets of $E^{ub}_k$ and $E^{rb}_k$ respectively; we thus obtain:

$$P_r \left( (E^{rb}_k)^c \right) = 1 - P_r \left( E^{rb}_k \right) = P_r \left( \gamma^{rb}_{k} > \varphi_k \right),$$  

(13)

$$P_r \left( (E^{ub}_k)^c \right) = 1 - P_r \left( E^{ub}_k \right) = P_r \left( \gamma^{ub}_{k} > \varphi_k \right).$$  

(14)

In the first time slot, the BS uses a successive interference canceling (SIC) algorithm to detect direct user signals and relay node will amplify the received signal. At BS, the outage of the $k^{th}$ user will be absent when the message of that user and the prior $k - 1$ user have both been detected successfully, and interference from the prior user has been removed completely; the OP that the BS will detect based on the direct message from the $k^{th}$ user can be expressed as:

$$O^{ub}_k = \left[ 1 - P_r \left( (E^{ub}_1)^c \cap \cdots \cap (E^{ub}_{k-1})^c \cap (E^{ub}_k)^c \right) \right].$$  

(15)

According to [21], we can obtain the CDF expression of $|h^{ub}_{k}|$ as follows:

$$F_{|h^{ub}_{k}|} (x) = 1 - e^{-x^2}.$$  

(16)
According to (7), the SNR expressions for users which BS detect the direct message from the $k^{th}$ user are derived as follows:

$$
\gamma_{1}^{ub} = \alpha_{1}\gamma \left[ h_{1}^{ub} \right]^{H} \left[ \alpha_{2}\gamma h_{1}^{ub} + 1 \right]^{-1} h_{1}^{ub}
$$

$$
\gamma_{2}^{ub} = \alpha_{2}\gamma \left[ h_{2}^{ub} \right]^{H} \left[ 1 \right]^{-1} h_{2}^{ub},
$$

(17)

$$
\gamma_{3}^{ub} = \alpha_{3}\gamma \left[ h_{3}^{ub} \right]^{H} \left[ 1 \right]^{-1} h_{3}^{ub},
$$

(19)

and

Then, according to (10), we have

$$
\text{Pr}\left(\left(E_{1}^{ub}\right)^{c} \cap \left(E_{2}^{ub}\right)^{c} \cap \left(E_{3}^{ub}\right)^{c}\right) = \text{Pr}\left(\begin{cases}
\frac{\alpha_{1}\gamma \left| h_{1}^{ub} \right|^{2}}{\alpha_{2}\gamma \left| h_{1}^{ub} \right|^{2} + 1} + \frac{\alpha_{1}\gamma \left| h_{2}^{ub} \right|^{2}}{\alpha_{3}\gamma \left| h_{2}^{ub} \right|^{2} + 1} > \phi_{1}, \\
\frac{\alpha_{2}\gamma \left| h_{1}^{ub} \right|^{2}}{\alpha_{3}\gamma \left| h_{1}^{ub} \right|^{2} + 1} + \frac{\alpha_{1}\gamma \left| h_{2}^{ub} \right|^{2}}{\alpha_{2}\gamma \left| h_{2}^{ub} \right|^{2} + 1} > \phi_{2}, \\
\frac{\alpha_{3}\gamma \left| h_{1}^{ub} \right|^{2}}{\alpha_{2}\gamma \left| h_{1}^{ub} \right|^{2} + 1} + \frac{\alpha_{1}\gamma \left| h_{2}^{ub} \right|^{2}}{\alpha_{3}\gamma \left| h_{2}^{ub} \right|^{2} + 1} > \phi_{3}, \\
\end{cases}\right)
$$

(20)

where $a = \left[ \alpha_{1}(\alpha_{2} + \alpha_{3}) - \alpha_{2}\alpha_{3}\phi_{1}\right] \gamma^{2},$

$b = \left[ 2\alpha_{1} - (\alpha_{2} + \alpha_{3})\phi_{1} \right] \gamma,$

c = -\phi_{1},$

$d = b^{2} - 4ac > 0,$

$x_{1} = \frac{-b + \sqrt{d}}{2a},$

$x_{2} = \frac{-b - \sqrt{d}}{2a},$

$\tau_{1} = \max\{0, x_{1}\}, \tau_{2} = \max\{0, x_{2}\}.$

$$
\text{Pr}\left(\left(E_{1}^{ub}\right)^{c} \cap \left(E_{2}^{ub}\right)^{c} \cap \left(E_{3}^{ub}\right)^{c}\right) = \text{Pr}\left(\begin{cases}
\frac{\alpha_{1}\gamma \left| h_{2}^{ub} \right|^{2}}{\alpha_{2}\gamma \left| h_{2}^{ub} \right|^{2} + 1} + \frac{\alpha_{1}\gamma \left| h_{3}^{ub} \right|^{2}}{\alpha_{3}\gamma \left| h_{3}^{ub} \right|^{2} + 1} > \phi_{1}, \\
\frac{\alpha_{2}\gamma \left| h_{2}^{ub} \right|^{2}}{\alpha_{3}\gamma \left| h_{2}^{ub} \right|^{2} + 1} + \frac{\alpha_{1}\gamma \left| h_{3}^{ub} \right|^{2}}{\alpha_{2}\gamma \left| h_{3}^{ub} \right|^{2} + 1} > \phi_{2}, \\
\frac{\alpha_{3}\gamma \left| h_{2}^{ub} \right|^{2}}{\alpha_{2}\gamma \left| h_{2}^{ub} \right|^{2} + 1} + \frac{\alpha_{1}\gamma \left| h_{3}^{ub} \right|^{2}}{\alpha_{3}\gamma \left| h_{3}^{ub} \right|^{2} + 1} > \phi_{3}, \\
\end{cases}\right)
$$

(21)

where $\zeta = \frac{\alpha_{2}}{\alpha_{3} \gamma}, \zeta_{1} = \max\{\frac{\alpha_{2}}{\alpha_{3} \gamma}, x_{1}\}, \zeta_{2} = \max\{\frac{\alpha_{2}}{\alpha_{3} \gamma}, x_{2}\}.$

$$
\text{Pr}\left(\left(E_{1}^{ub}\right)^{c} \cap \left(E_{2}^{ub}\right)^{c} \cap \left(E_{3}^{ub}\right)^{c}\right) = \text{Pr}\left(\begin{cases}
\frac{\alpha_{1}\gamma \left| h_{2}^{ub} \right|^{2}}{\alpha_{2}\gamma \left| h_{2}^{ub} \right|^{2} + 1} + \frac{\alpha_{1}\gamma \left| h_{3}^{ub} \right|^{2}}{\alpha_{3}\gamma \left| h_{3}^{ub} \right|^{2} + 1} > \phi_{1}, \\
\frac{\alpha_{2}\gamma \left| h_{2}^{ub} \right|^{2}}{\alpha_{3}\gamma \left| h_{2}^{ub} \right|^{2} + 1} + \frac{\alpha_{1}\gamma \left| h_{3}^{ub} \right|^{2}}{\alpha_{2}\gamma \left| h_{3}^{ub} \right|^{2} + 1} > \phi_{2}, \\
\frac{\alpha_{3}\gamma \left| h_{2}^{ub} \right|^{2}}{\alpha_{2}\gamma \left| h_{2}^{ub} \right|^{2} + 1} + \frac{\alpha_{1}\gamma \left| h_{3}^{ub} \right|^{2}}{\alpha_{3}\gamma \left| h_{3}^{ub} \right|^{2} + 1} > \phi_{3}, \\
\end{cases}\right)
$$

(22)
The SNR expressions for user 2 and user 3 that signal from the AF relaying node are derived as expressions (29) and (30), respectively.

\[ \gamma^{ur}_2 = \alpha_2 \frac{\gamma^2 |h^{ur}_2|^2 |h^{br}_2|^2}{\gamma |h^{ur}_2|^2 + \gamma |h^{br}_2|^2 + 1} = \alpha_2 Z_2 \]  
(29)

Similar to \( \gamma^{ur}_2 \), we have

\[ \gamma^{ur}_3 = \alpha_3 \frac{\gamma^2 |h^{ur}_3|^2 |h^{br}_3|^2}{\gamma |h^{ur}_3|^2 + \gamma |h^{br}_3|^2 + 1} = \alpha_3 Z_3. \]  
(30)

To make the SNR expression more concise, we define function \( Z_k \) as follows:

\[ Z_k = \gamma \frac{\gamma^2 |h^{ur}_k|^2 |h^{br}_k|^2}{\gamma |h^{ur}_k|^2 + \gamma |h^{br}_k|^2 + 1}. \]  
(31)

The CDF expression of \( Z_k \) is defined as expression (53).

\[ F_{Z_k}(x) = Pr \left( \frac{\gamma^2 |h^{ur}_k|^2 |h^{br}_k|^2}{\gamma |h^{ur}_k|^2 + \gamma |h^{br}_k|^2 + 1} < x \right) \]
\[ = 1 - e^{-\tau \left( x + \frac{1}{\mu^{ur}_k \gamma} \right)} \sqrt{4\tau (\gamma \tau + 1) \frac{4\tau (\gamma \tau + 1)}{\mu^{br}_k \mu^{ur}_k \gamma} K_1 \left( \frac{4\tau (\gamma \tau + 1)}{\mu^{br}_k \mu^{ur}_k \gamma} \right)}. \]  
(32)

where \( \tau = x / \gamma \), \( K_1(\cdot) \) is the first-order modified Bessel function of the second kind. To render the OP expression more concise, we define the function \( G_k(x) = F_{Z_k}(x \gamma) \) when \( x > 0 \) and \( G_4(0) = 0 \). Therefore, we have

\[ G_k(x) = 1 - e^{-x \left( \frac{1}{\mu^{ur}_k \gamma} + \frac{1}{\mu^{br}_k \gamma} \right)} \sqrt{4\tau (\gamma \tau + 1) \frac{4\tau (\gamma \tau + 1)}{\mu^{br}_k \mu^{ur}_k \gamma} K_1 \left( \frac{4\tau (\gamma \tau + 1)}{\mu^{br}_k \mu^{ur}_k \gamma} \right)}. \]  
(33)

Then, according to (18), we have

\[ Pr \left( \left( E^{rb}_1 \right)^c \right) = Pr \left\{ \frac{\alpha_1 Z_1}{\alpha_2 Z_1 + 1} + \frac{\alpha_1 Z_1}{\alpha_3 Z_1 + 1} > \phi_1 \right\} \]
\[ = Pr \left\{ \left[ \frac{\alpha_1 (\alpha_2 + \alpha_3) - \alpha_2 \alpha_3 \phi_1}{\alpha_2 + \alpha_3} \right] Z_1 > \phi_1 - 1 \right\} \]
\[ = \begin{cases} 
G_1(\tau_2) - G_1(\tau_1), & \text{if } a < 0 \\
1 + G_1(\tau_1) - G_1(\tau_2), & \text{if } a > 0, \end{cases} \]  
(34)

where \( \tau_1 = \max\{x, 1\}, \tau_2 = \max\{0, x\} \).

\[ Pr \left( \left( E^{rb}_1 \right)^c \cap \left( E^{rb}_2 \right)^c \cap \left( E^{rb}_3 \right)^c \right) \]
\[ = Pr \left\{ \frac{\alpha_1 Z_2}{\alpha_2 Z_2 + 1} + \frac{\alpha_1 Z_2}{\alpha_3 Z_2 + 1} > \phi_1, \alpha_2 Z_2 > \phi_2 \right\} \]
\[ = \begin{cases} 
G_2(\xi_2) - G_2(\xi_1), & \text{if } a < 0 \\
1 + G_2(\xi_1) - G_2(\xi_2), & \text{if } a > 0, \end{cases} \]  
(35)

where \( \xi = \max\{\phi_2, \tau_1\}, \xi_1 = \max\{\phi_2, \tau_2\}, \xi_2 = \max\{\phi_2, \tau_3\} \).

For an AF-OMA, the OP close-form expressions in the final slot are:

\[ \mathcal{O}_o = \mathcal{O}^{rb}_o \times \mathcal{O}^{ub}_o \]
\[ = \left( 1 - e^{-\tau \left( \frac{1}{\mu^{rb}_o} + \frac{1}{\mu^{ub}_o} \right)} \sqrt{4\tau (\gamma \tau + 1) \frac{4\tau (\gamma \tau + 1)}{\mu^{br}_o \mu^{ur}_o \gamma} K_1 \left( \frac{4\tau (\gamma \tau + 1)}{\mu^{br}_o \mu^{ur}_o \gamma} \right)} \right) \times \left( 1 - e^{-\frac{\varphi_o}{\mu^{br}_o \gamma}} \right). \]  
(43)

where \( \tau = \frac{\varphi_o}{\mu^{br}_o \gamma} \).
B. THROUGHPUT PERFORMANCE

System throughput is one of the most important indicators of the performance of a wireless network. From the above expressions of OP [(40)-(42)], we derive an AF-PDMA system throughput expression as follows:

\[ C_{\text{sum}} = R_1 (1 - O_1) + R_2 (1 - O_2) + R_3 (1 - O_3) \]  

(44)

For the PDMA pattern matrix \( G_{\text{PDMA}} \), the data from three users are multiplexed on two REs. To ensure a fair comparison, the throughput of an AF-OMA featuring signal multiplexing on two REs is given by (45):

\[ C_o = 2R_o (1 - O_o) \]  

(45)

C. DIVERSITY PERFORMANCE

Here, we derive the diversity order achieved by users based on the above analytical results. The diversity order is:

\[ \text{div} = - \lim_{\gamma \to \infty} \frac{\log (O (\gamma))}{\log (\gamma)} \]  

(46)

First, the relationship between the roots of the quadratic equation and the signal-to-noise ratio (SNR) is investigated. When the SNR tends to infinity, the roots are proportional to the reciprocal of SNR:

\[ x_{1,2} \gamma \to \infty = -b \pm \sqrt{\Delta} \propto \frac{1}{\gamma} \]  

(47)

Second, we investigate the relationship between the function \( G_k (\varphi) \) and the SNR. As \( e^x \simeq 1 + x \) and \( K_1 (x) \simeq 1/x \) when \( x \to 0 \), the function \( G_k (\varphi) \) is proportional to the reciprocal of SNR, and can be expressed as:

\[ G_k (\varphi) \gamma \to \infty \simeq 1 - e^{-\varphi \gamma (1/\mu_k^b + 1/\mu_k^u)} \propto \varphi / \gamma (1/\mu_k^b + 1/\mu_k^u) \]  

\[ \propto 1/\gamma^2 \]  

(48)

Finally, substituting (47) and (48) into (40)-(42), we obtain the approximate OPs for the three users as follows:

\[ O_1 \gamma \to \infty \approx \left(1 - \frac{\xi_1}{\gamma} \left(\frac{1}{\mu_1^b} + \frac{1}{\mu_1^u}\right) + \frac{\xi_2}{\gamma} \left(\frac{1}{\mu_2^b} + \frac{1}{\mu_2^u}\right)\right) \propto \frac{1}{\gamma^3}, \text{ if } a < 0 \]  

(49)

\[ O_2 \gamma \to \infty \approx \left(1 - \frac{\xi_1}{\gamma} \left(\frac{1}{\mu_1^b} + \frac{1}{\mu_1^u}\right) + \frac{\xi_2}{\gamma} \left(\frac{1}{\mu_2^b} + \frac{1}{\mu_2^u}\right)\right) \propto \frac{1}{\gamma^3}, \text{ if } a > 0 \]  

(50)

IV. NUMERICAL RESULTS

This section may be divided by subheadings. It should provide a concise and precise description of the experimental results, their interpretation as well as the experimental conclusions that can be drawn.

Here, we present numerical results verifying the validity of our theoretical AF-PDMA construct for an uplink wireless network. Without loss of generality, an AF-OMA, a non-cooperative PDMA, and an OMA serve as benchmarks for comparison. The simulation parameters are shown in Table 1. These cases are simulated using MATLAB R2013b. The simulation was carried out at a Dell workstation, whose processor is an Intel(R) Core (TM) i7-7820HQ CPU @ 2.90 GHz, with 8 GB of available memory.

Figure 2 plots the precise OP curves of three users of an AF-PDMA and an AF-OMA, derived via numerical simulation. Monte Carlo simulation revealed that the curves

| Parameter | Value |
|-----------|-------|
| Path loss exponent | \( \sigma = 2 \) |
| Distances from users to BS | \( d_{r1}^u = 1, d_{r2}^u = 0.5, d_{r3}^u = 0.3 \) |
| Distances from users to relay node | \( d_{r1}^s = 0.8, d_{r2}^s = 0.3, d_{r3}^s = 0.1 \) |
| Distances from relay nodes to the BS | \( d_{o1} = 0.2 \) |
| Target data rate | \( R_{r1} = 1, R_{r2} = 1.4, R_{o2} = 1.6, R_{o3} = 2 \) |
| Power allocation for users | \( \alpha_1 = 0.5, \alpha_2 = 0.3, \alpha_3 = 0.2, \alpha_4 = 1 \) |
perfectly matched the theoretical results. In the simulation, strong users (i.e., those near the BS; users 2 and 3) exhibit lower OPs than users of the AF-OMA; the OP of user 1 (on the edge of the AF-PDMA cell) is the poorest. Also, when several user quality of service (QoS) values are obtained simultaneously, an AF-PDMA affords better fairness than an AF-OMA.

Figure 3 shows the precise OPs of three users of an AF-PDMA and a PDMA employing Rayleigh fading channels. The blue curves represent the PDMA OPs and the red curves are the AFPDMA OPs. The AF-PDMA OPs fall rapidly in the high-SNR region. Also, the gaps between the AF-PDMA and PDMA OPs become larger as the SNR increases. The AF-PDMA outperforms the PDMA in terms of OP; an AF relay improves the reliability of a wireless network.

Figure 4 compares system throughput and SNRs among the four schemes. The precise curves of system throughput are plotted. The curves perfectly match the Monto Carlo simulation curves. All four curves exhibit throughput ceilings in the high-SNR regions, because OP trends to zero in these regions; throughput is determined only by target rate (the target rate sums to 4 in these simulations). In the low-SNR region, the AF-PDMA throughput is much larger versus the other three schemes.

Figure 5 plots a three-dimensional graphical of system throughput for AF-PDMA. The X-axis, Y-axis, and Z-axis represent the power allocation of user 1, SNR, and the system throughput for AF-PDMA, respectively. From the simulation, we can find that in the low SNR region, power allocation for user will affect the system throughput. When the power is allocated reasonably, the throughput can reach the maximum at the same value of SNR. However, when in the high SNR region, the system throughput ceiling exits which not be affected by power allocation. This confirms the conclusion in fig. 4 that throughput is only determined by the target rate in the high SNR region.

V. CONCLUSION

The proposed method is an IoTs based solution for prosumer monitoring, management and control. We explored the performance of a PDMA featuring AF relaying over Rayleigh fading channels. First, the outage behaviors of three
paired users using a PDMA pattern matrix and an AF relay protocol were researched in detail, and new closed-form OPs were derived. Second, based on these OPs, AF-PDMA system throughput was explored. When several user QoS values were satisfied simultaneously, AF-PDMA scheme was fairer than AF-OMA scheme. We allocated fixed powers to users when comparing performance among the four schemes (i.e., AF-PDMA, PDMA, AF-OMA, and OMA).

REFERENCES

[1] H. Wang, Z. Lei, X. Zhang, B. Zhou, and J. Peng, “A review of deep learning for renewable energy forecasting,” Energy Convers. Manag., vol. 198, Oct. 2019, Art. no. 111799.

[2] H. Z. Wang, G. B. Wang, G. Q. Li, J. C. Peng, and Y. T. Liu, “Deep belief network based deterministic and probabilistic wind speed forecasting approach,” Appl. Energy, vol. 182, pp. 80–93, Nov. 2016.

[3] V. V. S. N. Murty and A. Kumar, “Multi-objective energy management in microgrids with hybrid energy sources and battery energy storage systems,” Protection Control Modern Power Syst., vol. 5, no. 1, pp. 1–20, Dec. 2020.

[4] D. Xu, Q. Wu, B. Zhou, C. Li, L. Bai, and S. Huang, “Distributed multi-energy operation of coupled electricity, heating and natural gas networks,” IEEE Trans. Sustain. Energy, early access, Dec. 23, 2019, doi: 10.1109/TSTE.2019.2961432.

[5] H. Wang, J. Ruan, B. Zhou, C. Li, Q. Wu, M. Q. Raza, and G.-Z. Cao, “Dynamic data injection attack detection of cyber physical power systems with uncertainties,” IEEE Trans. Ind. Informat., vol. 15, no. 10, pp. 5505–5518, Oct. 2019.

[6] D. Tse and P. Viswanath, Fundamentals of Wireless Communication, Cambridge, U.K.: Cambridge Univ. Press, 2005.

[7] Z. Ding, Z. Yang, P. Fan, and H. V. Poor, “On the performance of non-orthogonal multiple access in 5G systems with randomly deployed users,” IEEE Signal Process. Lett., vol. 21, no. 12, pp. 1501–1505, Dec. 2014.

[8] Z. Ding, Y. Liu, J. Choi, Q. Sun, M. Elkashlan, I. Chih-Lin, and H. V. Poor, “Application of non-orthogonal multiple access in LTE and 5G networks,” IEEE Commun. Mag., vol. 55, no. 2, pp. 185–191, Feb. 2017.

[9] Y. Xu, G. Wang, L. Zheng, and S. Jia, “Performance of NOMA-based coordinated direct and relay transmission using dynamic scheme,” IET Commun., vol. 12, no. 18, pp. 2231–2242, Nov. 2018.

[10] Z. Ding, M. Peng, and H. V. Poor, “Cooperative non-orthogonal multiple access in 5G systems,” IEEE Commun. Lett., vol. 19, no. 8, pp. 1462–1465, Aug. 2015.

[11] X. Yue, Y. Liu, S. Kang, A. Nallanathan, and Z. Ding, “Exploiting full/half-duplex user relaying in NOMA systems,” IEEE Trans. Commun., vol. 66, no. 2, pp. 560–575, Feb. 2018.

[12] X. Yue, Y. Liu, S. Kang, A. Nallanathan, and Z. Ding, “Spatially random relay selection for Full/Half-duplex cooperative NOMA networks,” IEEE Trans. Commun., vol. 66, no. 8, pp. 3294–3308, Aug. 2018.

[13] S. Chen, B. Ren, Q. Gao, S. Kang, S. Sun, and K. Niu, “Pattern division multiple access—A novel nonorthogonal multiple access for fifth-generation radio networks,” IEEE Trans. Veh. Technol., vol. 66, no. 4, pp. 3185–3196, Apr. 2017.

[14] B. Ren, Y. Wang, X. Dai, K. Niu, and W. Tang, “Pattern matrix design of PDMA for 5G UL applications,” China Commun., vol. 13, no. 2, pp. 159–173, 2016.

[15] B. Ren, Y. Wang, S. Kang, X. Dai, X. Yue, W. Tang, and K. Niu, “Link performance estimation technique for PDMA uplink system,” IEEE Access, vol. 5, pp. 15571–15581, 2017.

[16] X. Dai, Z. Zhang, B. Bai, S. Chen, and S. Sun, “Pattern division multiple access: A new multiple access technology for 5G,” IEEE Wireless Commun., vol. 25, no. 2, pp. 54–60, Apr. 2018.

[17] J. Zeng, B. Liu, and X. Su, “Joint pattern assignment and power allocation in PDMA,” in Proc. IEEE 86th Veh. Technol. Conf. (VTC-Fall), Toronto, ON, Canada, Sep. 2017, pp. 1–5.

[18] W. Tang, S. Kang, B. Ren, and X. Yue, “Uplink grant-free pattern division multiple access (GF-PDMA) for 5G radio access,” China Commun., vol. 15, no. 4, pp. 153–163, Apr. 2018.

[19] W. Tang, S. Kang, and B. Ren, “Performance analysis of cooperative pattern division multiple access (Co-PDMA) in uplink network,” IEEE Access, vol. 5, pp. 3860–3868, 2017.

[20] W. Tang, S. Kang, X. Fu, X. Yue, and X. Zhang, “On the performance of PDMA with Decode-and-Forward relaying in downlink network,” IEEE Access, vol. 6, pp. 20113–20124, 2018.

[21] A. Goldsmith, Wireless Communications. Cambridge, U.K.: Cambridge Univ. Press, 2005.