Duality Symmetries and Topology Change in String Theory

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ABSTRACT

Duality symmetries for strings moving in non-trivial spacetime backgrounds are analysed. It is shown that for backgrounds generated from WZW and coset CFT models such duality symmetries are exact to all orders in string perturbation theory. Their implications for string dynamics in non-trivial/singular spacetimes are discussed.

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Introduction

Strings, being extended objects, sense the target space, into which they are embedded, in a different way than point particles. In a compact space this difference appears because, strings, except from their local excitations that mimic point particle behavior ("momentum" modes), have "winding" excitations where the string wraps around non-contractible cycles of the manifold. The masses of momentum modes are inversely proportional to the volume of the manifold, whereas those of the winding modes are proportional to the volume, since it costs energy in order to stretch the string. Moreover, the string contains oscillating modes that respond to background fields differently than the center of mass of the string. In certain cases, the physics of string propagation remains invariant under a reorganization of the one string Hilbert space and a specific change in the background. This symmetry is known as duality. In the simplest possible example, that of a string moving on a circle, it was observed that the spectrum of the theory with radius $R$ and that with radius $1/R$ are identical, once we interchange winding and momentum modes, [1].

It turns out that such duality symmetries exist (semiclassically) for all backgrounds with isometries, [2]. In CFT, some of these symmetries were identified as different abelian gaugings of a WZW theory, [3], and this was generalized to abelian gaugings, [4], and organized into (semi-classical) O(d,d,Z) type symmetries, [5], mimicking the situation for flat backgrounds. Moreover, for coset models, such duality symmetries exist also for backgrounds without any isometries, [3,6]. A careful analysis of the underlying CFT structure, revealed that most of these semiclassical symmetries (pertaining to compact cosets) are indeed exact in string theory, and they are intimately related to the affine Weyl symmetries of the "parent" theory, the WZW model.

In the non-compact case, the affine Weyl group is not a manifest symmetry but it can be shown that a particular kind of duality, axial-vector duality, [3] is still a symmetry, [7].

At the semiclassical level, provided there is an abelian isometry, the duality transformation can be effected by gauging this isometry and adding also a langrange multiplier coupled to the field strength of the gauge field, [2]. Integrating out the langrange multiplier, forces the gauge field to be pure gauge which can, subsequently, be gauged away, giving back the original model. On the other hand, one can gauge fix to a unitary gauge and then integrate out the gauge field (which appears quadratically in the action). In this way, a different (dual) sigma model action is obtained (the measure can be also taken care off, effectively changing the dilaton). Modulo global properties, the original and the dual action describe the same theory.

Duality in WZW and Coset Models

Applying this procedure to the WZW theory, using the isometries giving rise to the Cartan subalgebra of the current algebra, we can verify that the dual action is similar to the original one up to periodicities of the Cartan angles, [3]. The dual metric has Taub-
Nut type of singularities. At the level of the Hilbert space, these duality transformations correspond to Weyl transformations of the (left $\times$ right) current algebra. Thus, the duality group is $W_L \times W_R/W_D$, where $W_{L,R}$ are the left(right) Weyl groups of the (finite) Lie algebra of the WZW model and $W_D$ is the diagonal Weyl group (whose action corresponds to reparametrizations of the action) [6]. Duality here is an exact symmetry, since $W_L, W_R$ transformations are symmetries for any highest weight representation of any compact or non-compact semi-simple affine Lie algebra, and thus, for the associated WZW theory, whose spectrum is built from such representations.

From the WZW theory we can built other CFTs by projections. The simplest such projection corresponds to constraining the affine currents of a subalgebra, known as coset construction, [8]. The $\sigma$-model action of coset models is obtained by gauging the appropriate subgroup of the WZW model. Gauging different dual versions of the WZW model, dual versions of the coset model are obtained. In most cases, this duality exists for coset actions without isometries, [6].

A special form of duality is obtained for coset models where the gauged subgroup contains a U(1) factor. In such a case, one has the option of gauging either the axial or the vector subgroup of the original $U(1)_L \times U(1)_R$ subalgebra. The $\sigma$-model actions of these two gauged models are generically different but it can be shown that the models are dual to each other, [3]. This type of duality is known as axial-vector duality and at the semiclassical level is powerful in generating different types of backgrounds, [4]. It would seem that Weyl symmetry of the affine algebra is enough to guarantee that axial-vector duality is an exact symmetry. The story is more complicated though. It turns out, [6] that the underlying symmetry of current algebra responsible for axial-vector duality is affine-Weyl symmetry, $\hat{W}_L \times \hat{W}_R$. For integrable (unitary) representations of (compact) affine algebras the affine Weyl group is a symmetry and so is axial-vector duality. I conjecture that the most general (dimension preserving) duality group $D^g$ of a compact, unitary $g$-WZW model is isomorphic to $\hat{W}_L^g \times \hat{W}_R^g \times \hat{A}^g/W^g$ where $\hat{A}^g$ is the group of external automorphisms of the current algebra $\hat{g}$. The action of $A^g$ in $\sigma$-model language is not yet known.

For compact cosets, the full duality group is obtained by reduction of the WZW one. In particular, there are subgroups $D^h$ of $D^g$ for every reductive subalgebra $h \subset g$. The quotient $D^g/D^h$ is the maximal duality group of the coset CFT $G/H$.

The full set of duality symmetries of WZW and coset models generates dualities for other theories, which are continuously connected to them by marginal perturbations. Consider such a theory, which is connected to a WZW (or coset) model via a marginal perturbation $g \int O_{1,1}$ where $O_{1,1}$ is a (1,1) operator of the WZW (or coset) theory. Self-duality at the WZW (coset) point implies that the line of theories parametrized by $g$

\[\text{There are more general projections though, preserving conformal invariance, [9].}\]

\[\text{By the term “dimension preserving” I exclude the “collapse” phenomenon that happens for example at WZW of level 1 or rank level duality, where in both cases the dual theories have a target space with different dimension.}\]
is equivalent to the one generated by $\int O_{1,1}$. The background interpretation of the two perturbations is generically different. Thus, in the full moduli space, duality symmetries can be generated by the self-duality symmetries at special points (WZW and cosets). It is an open question in the non-flat case, if this construction exhausts all duality symmetries of the moduli space.

The situation for $G$ non-compact (or non-integer level of compact $G$) is more complicated and axial-vector equivalence not obvious (although it still works semiclassically). In such a case, the affine Weyl group maps most representations to different ones. Thus, a priori, without knowledge of the spectrum of representations duality is at stake. The only way for it to survive is, if the spectrum is organized into complete orbits of the affine Weyl group.

It can be shown, [7], that even for non-compact abelian cosets, axial-vector duality remains an exact symmetry. I will focus on the simplest non-trivial case, that of $SL(2,R)/U(1)$, since it contains all the important ingredients of the general case.

The key step is to construct the $\sigma$-model description of the line of CFTs (parametrized by a positive real number $R$) generated by the $\int J^3\bar{J}^3$ perturbation of the WZW model. This $\sigma$-model can be found by imposing the following three requirements:

1) It has a $U(1)_L \times U(1)_R$ chiral symmetry along the hole line.
2) At any $R$ the variation of the action is of the form $\int J^3(R)\bar{J}^3(R)$.
3) At $R=1$, it reduces to the known $\sigma$-model action for $SL(2,R)$.

These three requirements fix the $\sigma$-model action (and measure) uniquely and this action is correct to all order in $\alpha'$ (in a certain scheme). This action can be found in [7]. Around $R=1$, the theories at $1+\delta R$ and $1-\delta R$ are equivalent since they are related by a left Weyl transformation in the WZW theory, $J^3 \rightarrow -J^3$, $\bar{J}^3 \rightarrow \bar{J}^3$. This infinitesimal duality transformation propagates along the line and its finite form is $R \rightarrow 1/R$.

By looking at the form of the action at the end points ($R=0, \infty$) we observe that at $R=0$ the theory is a direct product of a free non-compact boson and the (uncorrelated) vectorial $SL(2,R)/U(1)$ coset model. At $R=\infty$ the theory also factorizes directly into a free non-compact boson and the axial $SL(2,R)/U(1)$ coset model. In view of the $R \rightarrow 1/R$ duality this shows that the axial and vector $SL(2,R)/U(1)$ models are equivalent. This can be generalized to all non-compact abelian cosets, [7].

**Topology change**

A slightly modified version of the model discussed in the previous section can provide a simple example of smooth topology change in string theory. The modification amounts to a blowing up (via a $GL(2,R)$ transformation) of the shrinking $S^1$. The line element of the metric is

$$ds^2(\alpha) = \frac{k}{\Delta(\alpha)}[(1-\Sigma)d\theta_1^2 + (1+\Sigma)d\theta_2^2] + kdx^2$$  \hspace{1cm} (0.1)\footnote{For flat toroidal backgrounds, the full duality symmetry is generated this way, [10].}
where
\[ \Sigma = \cos 2x, \quad \Delta = \cos^2 \alpha (1 + \Sigma) + (\cos \alpha + k \sin \alpha)^2 (1 - \Sigma) \] (0.2)
and the SL(2,R) case is obtained for \( x \to ix \). At \( \alpha = 0 \) the metric describes an \( S^3 \) (or pseudo-sphere in the SL(2,R) case). This deforms continuously until \( \alpha = \pi/2 \) where the metric becomes
\[ ds^2(\alpha = \pi/2) = \frac{1}{k} [d\theta_1^2 + \frac{1+\Sigma}{1-\Sigma} d\theta_2^2] + k dx^2. \] (0.3)
with the topology of \( D_2 \times S^1 \), where \( D_2 \) is a 2-disk. This gives an example of a smooth topology change. What is also interesting here is that the \( \alpha = 0 \) neighborhood is mapped via an \( O(2,2) \) duality transformation to that around \( \alpha = \pi/2 \). Thus, this maps a region where topology change occurs to one where topology is not changed at all. A similar phenomenon of topology change happens in more complicated examples of CY compactifications, [11].

**Comments on the Physics of Duality**

The main lesson from duality and related symmetries is that the background fields do not determine uniquely the spectrum and physics of string theory. Duality can be viewed as a tiny (unbroken) part of the huge string gauge symmetry, whose glory remains obscure to our days. Another way to state this, is, that different modes of the string feel different geometry. Thus, the background interpretation of string vacua should be used with care in order to ascertain the physics. The only cases were the description is reliable is the large volume limit of compact manifolds, and at the asymptotically flat region of non-compact manifolds. Once at a region of finite curvature, the geometrical description of string theory breaks down. As we have seen, even topology is not preserved under duality and there are continuous families of ground states in string theory where topology changes without the occurrence of anything catastrophic.

An example of how this type of symmetry can affect string propagation, can be given (heuristically) as follows\[\footnote{This argument is advanced in collaboration with C. Kounnas.}]: Consider a string background which is highly curved or even singular (semiclassically) in a certain region, (the 2-d black hole, [12] is such an example but one needs to add a few extra dimensions in order to have non-trivial massive states). In the asymptotic region, (which is obtained by some spacetime-depended radius becoming very large), one has quantum numbers for asymptotic states that correspond roughly to windings and momenta. Momentum states are the only low energy states in this region. Consider a momentum mode travelling towards the high curvature region. Its effective mass starts growing as it approaches large curvatures. At some point it becomes energetically possible for it to decay to winding states which, in this region, start having effective masses that are lower than momentum modes. In such backgrounds (unlike flat ones) winding and momentum are not separately conserved so that such a transition is possible. The reason for this is that there is a non-trivial dilaton field and thus, winding and momentum conservation is broken by the screening operators which transfer it to discrete states localized at the high curvature region. An alternative interpretation of
this, is that particles interact with such localized states losing momentum (in discrete steps) and gaining winding number.

Once such a momentum to winding mode transition happens in the strongly curved region, the winding state sees a different geometry, namely the dual one and thus continues to propagate further into the strong curvature region since it feels only the (weak) dual curvature.

This type of picture implies that the physics of string black holes will be qualitatively different that their classical general relativity counterparts.
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