Abstract

We present complete one-loop radiative corrections to the decay rate of a top quark into a charged Higgs boson and a bottom quark, and for the decay of a charged Higgs boson into leptons. The results are discussed in the framework of the two Higgs boson extension of the Standard Model suggested by supersymmetry. The effect of electroweak corrections after exclusion of universal corrections $\Delta r$ is found to decrease the partial width of the top quark typically by 5%.

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1 Introduction

Due to the expected large mass of the top quark and its possible large Yukawa coupling to Higgs bosons, decays of this particle (once it is observed, presumably at the Tevatron) can give us an insight into the Higgs sector and the mechanism of mass generation. A topic of particular importance is the number of Higgs doublets. The supersymmetric extensions of the Standard Model, for example, predicts existence of at least two Higgs doublets. In such scenarios, in addition to the charged Goldstone boson of the standard electroweak theory there would be a physical charged scalar particle $H^\pm$. Its presence could influence the rate of top quark decay and even open up a new decay channel.

If the charged Higgs boson is heavier than the top quark, its effect on the decay rate of the top will only be in the virtual corrections to the standard process $t \to W^+ b$. This has been examined in ref. [1, 2], and in some models the effect was found to be large, of the order of several percent. On the other hand, if the decay of the top into the charged Higgs and a bottom quark is kinematically allowed, it can become the dominant decay channel, especially if the ratio of vacuum expectation values of the two doublets is such that the Yukawa coupling to the top is not suppressed. It is this scenario that is the topic of the present work. We examine the effects of first order electroweak corrections on the width of the decay $t \to H^+ b$ in the two Higgs doublet extension of the Standard Model suggested by supersymmetry [3, 4]. In this model one of the Higgs fields, $H_1$, is responsible for giving masses to down-quarks, and the other one, $H_2$, to up-quarks. The ratio of the expectation values of these two fields is denoted by $\tan \beta = v_2/v_1$. In the present paper we consider the range of small values of $\tan \beta$, in which the mass of the bottom quark can be safely neglected, which considerably simplifies the calculations.

Radiative corrections to the decay $t \to H^+ b$ have been subject of several recent publications. The QCD corrections have been studied in ref. [5, 6]. An analysis of effects of the mass of the $b$ quark and a comparison of corrections to the main decay channels $t \to H^+ b$ and $t \to W^+ b$ has been done in ref. [7], where further references on this subject can be found.

In the electroweak sector the corrections have been studied only to the order $O(\alpha m_t^2/m^2_W)$. They have been calculated in ref. [8] and further analyzed in [9]. Such corrections would be dominant if the top quark was much heavier than the $W$ boson. However in view of the expected mass of the
top quark of the order of \((1.5 - 2)m_W\) it is important to compute also the remaining corrections not involving the top quark mass, as well as the effect of real photon radiation.

This paper is organized as follows: the next section explains the renormalization scheme and various kinds of corrections. Section 3 discusses cancellation of infrared and ultraviolet divergences, especially the quadratic ones. Calculation of virtual corrections to vertices and evaluation of the bremsstrahlung are explained in sections 4 and 5. Section 6 presents final results; previously unpublished formulas for renormalization constants are collected in the Appendix.

2 Renormalization scheme

At the tree level the decay rate for \(t \to H^+ b\) is obtained from the Feynman rule for the \(tbH^+\) vertex:

\[
\Gamma^{(0)}(t \to H^+ b) = \frac{\alpha m_t^3}{16 m_W^2 s_W} \cot^2 \beta \left(1 - \frac{m_{H^+}^2}{m_t^2}\right)^2 .
\]

Electroweak corrections modify the values of parameters in the vertex: the coupling constant \(e\), masses \(m_W\), \(m_Z\) and \(m_t\), and the angle \(\beta\). It is also necessary to calculate effects of the real photon radiation, virtual corrections to the vertex (triangle diagrams) and the renormalization of wave functions of the charged Higgs and of the quarks \(t\) and \(b\). On the one loop level there are also contributions from the mixing of the charged Higgs with the \(W\) boson. Finally, since we are going to work in the 't Hooft-Feynman gauge, we have to include the mixing between \(H^+\) and the charged Goldstone boson \(G^+\). The one loop correction to the decay rate can be written in the following
form:

\[
\Gamma^{(1)}(t \to H^+ b) = 2\Gamma^{(0)}(t \to H^+ b) \left( \frac{\delta e}{e} - \frac{\delta s_W}{s_W} \frac{\delta m_t}{m_t} - \frac{\delta m_W}{m_W} + \frac{\delta \cot \beta}{\cot \beta} \right. \\
+ \left. \frac{1}{2} \delta_{\text{REAL}}^t + \frac{1}{2} \delta_{\Delta}^t + \frac{1}{2} \delta Z_L^\nu + \frac{1}{2} \delta Z_R^\tau + \frac{1}{2} \delta Z_H + \delta_{\text{MIX}}^t \right). \tag{3}
\]

\(\delta_{\text{REAL}}^t, \delta_{\Delta}^t\) and \(\delta_{\text{MIX}}^t\) denote contributions of the real photon radiation, triangle diagrams and mixing of \(H^+\) with \(W^+\) and with \(G^+\) respectively. For the renormalization of the angle \(\beta\) we employ the prescription introduced by Méndez and Pomarol \cite{10, 11}, with a small modification. It is assumed that the value of \(\beta\) will be extracted from the leptonic decay channel of the charged Higgs boson. Since the coupling is proportional to the mass, the dominant decay will be into a \(\tau\) lepton and its neutrino. The renormalization of the angle \(\beta\) is fixed by the condition that radiative corrections to the vertex \(\tau\nu \tau H\) vanish. However, the renormalization constant for \(\beta\) defined in this way is infrared divergent; this problem was not addressed in the original papers \cite{10, 11}, because only the fermionic loop corrections were discussed there. The infrared divergence could also be removed in the suitable process of extracting the value of the \(\beta\) angle from the experimental measurement of the decay width of the charged Higgs boson. For the purpose of the current calculation it is convenient to include the effect of the real photon radiation in definition of \(\delta \beta\). The one loop correction to the decay rate of the charged Higgs into tau and the neutrino can be written in analogy to the top decay:

\[
\Gamma^{(1)}(H^- \to \tau \bar{\nu}_\tau) = 2\Gamma^{(0)}(H^- \to \tau \bar{\nu}_\tau) \left( \frac{\delta e}{e} - \frac{\delta s_W}{s_W} \frac{\delta m_\tau}{m_\tau} - \frac{\delta m_W}{m_W} \right. \\
- \left. \frac{\delta \cot \beta}{\cot \beta} + \frac{1}{2} \delta_{\text{REAL}}^\tau + \frac{1}{2} \delta_{\Delta}^\tau + \frac{1}{2} \delta Z_L^\nu + \frac{1}{2} \delta Z_R^\tau + \frac{1}{2} \delta Z_H + \delta_{\text{MIX}}^\tau \right). \tag{4}
\]

The notation here is analogous to the formula \(\text{(3)}\). Since the coupling of the charged Higgs to leptons is proportional to \(\tan \beta\), the effect of renormalization of \(\beta\) has an opposite sign in the two decays under consideration. The reason for this is that in both cases we have only one fermion with non-negligible mass, but they have opposite values of the weak isospin.

The condition of vanishing of radiative corrections to the tau channel of the decay of the charged Higgs allows us to express the renormalization
constant of the $\beta$ angle in terms of corrections to the $H\tau\nu_\tau$ vertex. This leads to the following formula for the relative correction to the rate $t \rightarrow H^+ b$:

$$\Delta \Gamma \equiv \frac{\Gamma^{(1)} (t \rightarrow H^+ b)}{\Gamma^{(0)} (t \rightarrow H^+ b)} = 2 \left( \frac{\delta e}{e} - \frac{2 \delta s_W}{s_W} \right) \frac{\delta m_\tau}{m_\tau} + \frac{\delta m_t}{m_t} - \frac{\delta m_\tau^2}{m_\tau^2} \frac{\delta m_t^2}{m_t^2}$$

$$+ \frac{1}{2} \delta^t_{\text{REAL}} + \frac{1}{2} \delta^\tau_{\text{REAL}} + \delta^\tau_\Delta + \delta^\tau_{\Delta} + \frac{1}{2} \delta Z^L + \frac{1}{2} \delta Z^R$$

$$+ \frac{1}{2} \delta Z^L + \frac{1}{2} \delta Z^R + \delta Z_H + \delta Z^H + \delta^t_{\text{MIX}} + \delta^\tau_{\text{MIX}} \right). \quad (5)$$

As will be seen later, the mixing can be described by one constant $\delta_{\text{MIX}}$ defined so that

$$\delta^\tau_{\text{MIX}} + \delta^t_{\text{MIX}} = \frac{\cot \beta - \tan \beta}{m_{H^+}^2 - m_W^2} \delta_{\text{MIX}}. \quad (6)$$

The renormalization of the electroweak parameters is done in the on-shell scheme of ref. [12, 2, 13]. In particular, for the weak coupling constant $e/s_W$ we have:

$$\frac{\delta e}{e} - \frac{\delta s_W}{s_W} \equiv \delta Z_e + \frac{\delta m_Z^2}{2m_Z^2} - \frac{\delta m_Z^2 - \delta m_W^2}{2(m_Z^2 - m_W^2)}$$

$$= \frac{1}{2} \frac{\partial \Sigma^{A_A}(s)}{\partial s} \bigg|_{s=0} - \frac{s_W}{c_W} \frac{\Sigma^{A_Z}(0)}{m_Z^2} + \frac{\delta m_Z^2}{2m_Z^2} - \frac{\delta m_Z^2 - \delta m_W^2}{2(m_Z^2 - m_W^2)}. \quad (7)$$

This leads to the final formula from which we are going to calculate the one loop corrections:

$$\Delta \Gamma = 2 \left( 2 \delta Z_e + \frac{\delta m_\tau}{m_\tau} + \frac{\delta m_t}{m_t} - \frac{\delta m_\tau^2}{m_\tau^2} - \frac{\delta m_t^2}{m_t^2} \right) \frac{\delta m_Z^2 - \delta m_W^2}{(m_Z^2 - m_W^2)}$$

$$+ \frac{1}{2} \delta^t_{\text{REAL}} + \frac{1}{2} \delta^\tau_{\text{REAL}} + \delta^\tau_\Delta + \delta^\tau_{\Delta} + \frac{1}{2} \delta Z^L + \frac{1}{2} \delta Z^R$$

$$+ \frac{1}{2} \delta Z^L + \frac{1}{2} \delta Z^R + \delta Z_H + \delta Z_H + \frac{\cot \beta - \tan \beta}{m_{H^+}^2 - m_W^2} \delta_{\text{MIX}} \right). \quad (8)$$

Many details and explicit formulas for some of the renormalization constants can be found in ref. [14]. There are no external Higgs particles in processes described in that reference, so the wave function renormalization of the charged Higgs boson and mixing with $W^+$ and Goldstone boson was not included. The relevant formulas can be found in the appendix of the present work.
3 Remarks on cancellation of divergences

In the calculation of electroweak corrections to decays $t \rightarrow H^+b$ and $H^- \rightarrow \tau\bar{\nu}_\tau$ one encounters three kinds of infinite quantities: infrared divergences, and logarithmic and quadratic ultraviolet divergences. The infrared divergent integrals result from the radiation of soft and collinear photons from external charged particles. They are cancelled in the calculation of the total decay rate by wave function renormalization constants of the Higgs boson and of fermions, as well as by corrections to the Higgs-fermion vertex. For the purpose of the present calculation the infrared divergence was regularized by introducing a small mass $\lambda$ of the photon. All phase space integrals relevant to this problem have been listed in ref. [13].

The ultraviolet divergent integrals are regularized dimensionally. In this scheme, the quadratic divergences show up as poles at number of dimensions $n = 2$. They originate from tadpole diagrams and from the fermionic loop contribution to charged Higgs - Goldstone boson mixing. Some individual non-tadpole diagrams in boson self energies also contain quadratic divergences, but the relevant sums of diagrams are free from them (in the \'t Hooft-Feynman gauge), just like in the Standard Model [15]. Goldstone bosons are absent in the unitary gauge and there all the tadpole contributions cancel out. The problem is more delicate in the \'t Hooft-Feynman gauge, in which the present calculation is done\(^2\).

The different types of tadpole diagrams in the two Higgs doublet model are shown in figure (8). The external particle can be one of the CP even neutral Higgs bosons, $H^0$ or $h^0$. These diagrams contribute to mass renormalization of external fermions, to $\delta m_W$ and $\delta m_Z$, and to the mixing between the Higgs boson and Goldstone and $W$ bosons. The quadratic divergence from the fermionic loop in figure (8a) cancels the one from the fermionic contribution to the Higgs-Goldstone mixing shown in figure (7a). The sum of contributions of the remaining, bosonic tadpole diagrams, is free from quadratic divergences. The logarithmic divergences of tadpole diagrams are cancelled by loop diagrams of Higgs-Goldstone boson mixing depicted in figures (5a) and (7b,c). The sum of bosonic loops of Higgs-$W$ boson mixing is finite.

\(^2\)A discussion of tadpole diagrams with a fermion loop can be found in ref. [16] which also contains further references.
4 Vertex corrections

Electroweak corrections to vertices are of two kinds: there are modifications of the values of parameters determining the strength of the coupling and relations among them, and triangle diagrams. It is this second type which will be considered in this section. The basic types of triangle diagrams contributing to decays of the $t$ quark and the charged Higgs boson are depicted in figures (3) and (4). Since the number of diagrams of is fairly large it is most convenient to employ the method of standard matrix elements (see ref. [13] for a review and further references). The principle of this method is to calculate coefficients in a representation of an invariant matrix element in form of a sum over certain standard tensors, which depend only on the Lorentz structure of the process. In particular, in the case of scalar-fermion interaction, there are only two standard matrix elements:

$$M^L = \bar{u}(p)Lu(q),$$
$$M^R = \bar{u}(p)Ru(q),$$

where $L = (1 - \gamma_5)/2$. Born amplitude of the decay of the Higgs boson into leptons is proportional to $M^L$, and since on the level of one-loop corrections we need to compute only the interference of triangle and tree diagrams, it is sufficient to evaluate only the $M^L$ component of the triangle diagrams. Analogously, in the case of the top quark decay, we need the $M^R$ part only.

The resulting formulas are quite space consuming and will not be shown here, because in contrast to the two point functions their applicability in other contexts is rather limited. However, in Table 1 we list concrete particle assignments to the general diagrams of figures (3) and (4) together with explicit expressions of their ultraviolet divergent parts.

Complete analytic formulas are obtained using *FeynArts* (also used to illustrate the present paper) and *FeynCalc* [17, 18]. Fortran output of these programs is evaluated using the library *FF* [19].
| Diagram (Figure No.) | Particle assignments | X   | Y   | Z   | Residuum                                                                 |
|----------------------|----------------------|-----|-----|-----|--------------------------------------------------------------------------|
| 3(a)                 | t                    | H^+ | H^0 | 0   |                                                                           |
| 3(a)                 | t                    | H^+ | H^0 | 0   |                                                                           |
| 3(a)                 | t                    | G^+ | A^0 | 0   |                                                                           |
| 3(b)                 | t                    | H^+ | γ   | 2/3 |                                                                           |
| 3(b)                 | t                    | H^+ | Z   |       | $-1/3 + s_W^2/(3c_W^2)$                                                  |
| 3(c)                 | t                    | W^+ | H^0 |      | $-\sin \alpha \sin(\beta - \alpha)/(4s_W^2 \cos \beta)$               |
| 3(c)                 | t                    | W^+ | h^0 |      | $\cos \alpha \cos(\beta - \alpha)/(4s_W^2 \cos \beta)$               |
| 3(c)                 | t                    | W^+ | A^0 | 1/3 |                                                                           |
| 3(c)                 | b                    | γ   | H^+ |      |                                                                           |
| 3(c)                 | b                    | Z   | H^+ |      | $(2s_W^2 - 3)(s_W^2 - c_W^2)/(12s_W^2 c_W^2)$                           |
| 3(d)                 | γ                    | b   | t   |      | $-8/9$                                                                  |
| 3(d)                 | Z                    | b   | t   |      | $(12 - 8s_W^2)/(9c_W^2)$                                                 |
| 4(a)                 | γ                    | τ   | H^- | 1   |                                                                           |
| 4(a)                 | Z                    | τ   | H^- |      | $(s_W^2 - c_W^2)/(2c_W^2)$                                               |
| 4(b)                 | W                    | τ   | H^0 |      | $\cos \alpha \sin(\beta - \alpha)/(4s_W^2 \sin \beta)$               |
| 4(b)                 | W                    | τ   | h^0 |      | $\sin \alpha \cos(\beta - \alpha)/(4s_W^2 \sin \beta)$               |
| 4(b)                 | W                    | τ   | A^0 | 1/3 |                                                                           |
| 4(b)                 | Z                    | ν_τ | H^- |      | $(c_W^2 - s_W^2)/(2s_W c_W)^2$                                           |
| 4(c)                 | Z                    | ν_τ | τ   | 2/c_W^2 |                                                                                  |

Table 1: Particle contents of triangle diagrams. The last column shows the coefficient of $\frac{a}{4\pi} \frac{\tau}{2-n}$ in $\delta^i_\Delta$ and $\delta^r_\Delta$. 
5 Real photon radiation

Triangle diagrams discussed in the previous section are infrared divergent due to exchange of soft photons. These divergences are cancelled by bremsstrahlung processes depicted in figures (1) and (2). These diagrams can be easily evaluated in terms of phase space integrals listed in ref. [13]. We give here as an example an explicit formula for the width of the process $H^- \rightarrow \tau \bar{\nu} \tau \gamma$:

$$\Gamma (H^- \rightarrow \tau \bar{\nu} \tau \gamma) = \frac{e^4 m^2 \tan^2 \beta}{2^7 \pi^3 s_W^5 m_H - m_W^5} \left( |A|^2 + |B|^2 + A^*B + B^*A \right),$$

(10)

where $A$ and $B$ denote the amplitudes corresponding to diagrams in figure (2), for which we have:

$$|A|^2 = 4m^2_H -(m^2_\tau - m^2_{H^-}) I_{00} + 2(m^2_\tau - 3m^2_{H^-}) I_0 - 2I,$$

$$|B|^2 = 4m^2_H (m^2_\tau - m^2_{H^-}) I_{11} + 4m^2_1 I_1 - 2I^0_1,$$

$$A^*B + B^*A = 4(m^4_\tau - m^4_{H^-}) I_{01} + 2(m^2_\tau + m^2_{H^-}) I_0 - 4m^2_{H^-} I_1 + 2I.$$ (11)

Integrals $I$ are taken from ref. [13], where explicit expressions can be found. Here we only quote the definition:

$$I_{i_1,..,i_m}^{j_1,..,j_m} = \frac{1}{\pi^2} \int \frac{d^3p_1}{2p_{10}} \frac{d^3p_2}{2p_{20}} \frac{d^3q}{2q_0} \delta^4 (p_0 - p_1 - p_2 - q) \frac{(\pm 2qp_{j_1})...(\pm 2qp_{j_m})}{(\pm 2qp_{i_1})...(\pm 2qp_{i_n})},$$

(12)

where we $q$, $p_0$, $p_1$ and $p_2$ denote momenta of the photon, Higgs boson, tau and neutrino respectively, and the signs should be chosen in the following way: minus sign if $i_k$ or $j_k$ is zero, plus in all other cases. Functions $I_{00}$, $I_{01}$ and $I_{11}$ contain infrared divergences, regularized by introducing a small mass of the photon $\lambda$. If the mass of neutral particle in the final state is small, the representation of these functions given in [13] becomes numerically unstable and it is more convenient to use the corresponding formulas from ref. [20].

6 Results and discussion

Following ref. [2, 14], the electroweak correction can be expressed by comparing the one-loop decay width to the Born rate parameterized by Fermi
coupling constant $G_F$ instead of the fine structure constant $\alpha$:

$$\Gamma^{(0)}(G_F) = \frac{\Gamma^{(0)}(\alpha)}{1 - \Delta r}, \quad (13)$$

where $\Delta r$ denotes radiative corrections to the muon decay, from which Fermi constant is determined. Such representation has the advantage of including large corrections due to fermion loops in the Born rate. In the present renormalization scheme, based on the condition of vanishing of radiative corrections to the $H^+\tau\nu_\tau$ vertex, the effect of coupling constant renormalization is doubled (see equation 8), and one ought to subtract $2\Delta r$ in order to cancel the fermion loop contribution from universal corrections. This is due to the fact that in order to avoid the artificially large corrections one has to parameterize both decay rates $\Gamma^0(H^\pm \to \tau\nu_\tau)$ and $\Gamma^0(t \to H^+b)$ by $G_F$. At this point our analysis differs from ref. [8]. For moderate values of $\tan \beta > 1$ the corrections consist typically of -4% bosonic contributions and +7% from fermion loops. This last part is cancelled by subtraction of $\Delta r$, so that the fermionic contribution to the corrections becomes slightly negative. This can be seen in figure 11.

Numerical evaluation of corrections to the decay width $\Gamma^{(0)}(G_F)$ proceeds in the following way. The set of input parameters consists of $m_Z$, $G_F$, $\alpha$, masses of fermions and CKM matrix elements; values of them are taken from a recent review [13]. All the numerical results are presented for mass of the top quark equal 140 GeV. In addition we need two parameters of the Higgs sector: we choose angle $\beta$ and mass of the charged Higgs boson. Masses of the remaining Higgs particles and angle $\alpha$ are found using the formulas of ref. [4]. Mass of the $W$ boson is found by solving a nonlinear equation [2]:

$$m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right) = \frac{\pi \alpha}{\sqrt{2}G_F} \frac{1}{1 - \Delta r}. \quad (14)$$

Finally, using this value of $m_W$, we find $\Delta r$ and $\Delta \Gamma$. The resulting corrections $\Delta \Gamma = \Delta \Gamma - 2\Delta r$ are plotted as a function of mass of the charged Higgs boson in figure 11 and as function of $\tan \beta$ in [8].

Similarly to the case of the decay $t \to W^+b$ [2], the corrections become large when mass of the lighter CP even neutral Higgs boson $h^0$ is small. In particular, they diverge at the point $\tan \beta = 1$ where $m_{h^0} = 0$. This divergence should be cancelled by adding width of the decay $t \to H^+bh^0$,
just like the infrared divergence due to virtual photon exchange is cancelled by the real photon radiation. As $\tan \beta$ becomes larger (or smaller) than 1, mass of $h^0$ increases, and at the point where it reaches $m_{H^+} - m_{W^+}$, amplitudes of both decays $H^\pm \to \tau \nu_\tau$ and $t \to H^+ b$ have singularities which show up as discontinuities of the derivative of the one-loop decay rate and can be noticed on the diagrams; the value of $\tan \beta$ where this happens is close to 1 for light $H^+$, and gets further away as $H^+$ becomes heavier. The corresponding cusps on the diagrams are easier to recognize for $\tan \beta > 1$, but they are present also in the region of $\tan \beta < 1$.

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**A Renormalization constants**

In this Appendix we list those of renormalization constants of the 2 Higgs doublet model which have not been published so far. We first give expressions for the wave function renormalization of the charged Higgs boson and then analyze various contributions to the mixing of Higgs boson with $W$ and Goldstone boson $\delta_{MIX}$. The results are given in terms of standard Passarino-Veltman integrals \([21]\), using the conventions of ref. \([13, 14]\), where many useful properties of these functions have been collected.

The wave function renormalization constant of the charged Higgs boson gets contributions from diagrams with fermion, scalar and vector-scalar loops.
To make the formula more compact it is convenient to introduce the notation:

\[ \lambda (m_i, m_j, m_k) \equiv m_i^4 + m_j^4 + m_k^4 - 2m_i^2m_j^2 - 2m_i^2m_k^2 - 2m_j^2m_k^2. \]  

The bosonic contributions to the renormalization constant \( \delta Z_H \) is:

\[
\delta Z_H^{\text{bos}} = \frac{\alpha}{4\pi} \sum_{H=H^0, h^0, A^0} \left\{ \frac{1}{4m_W^2 s_W^2} \left( \delta_{HH^0} \sin^2(\beta - \alpha) + \delta_{Hh^0} \cos^2(\beta - \alpha) + \delta_{HA^0} \right) \right. \\
\left. \left[ 2m_W^2 B_0 \left( m_{H^+}^2, m_H, m_W \right) - \lambda (m_{H^+}, m_H, m_W) B_0' \left( m_{H^+}^2, m_H, m_W \right) \right] \right\} \\
+ \frac{(s_W^2 - c_W^2)^2}{4s_W^2 c_W^2} \left[ 2B_0 \left( m_{H^+}^2, m_{H^+}, m_Z \right) \\
+ \left( 4m_{H^+}^2 - m_Z^2 \right) B_0' \left( m_{H^+}^2, m_{H^+}, m_Z \right) \right] \\
+ 2B_0 \left( m_{H^+}^2, m_{H^+}, \lambda \right) + 4m_{H^+}^2 B_0' \left( m_{H^+}^2, m_{H^+}, \lambda \right) \\
- \frac{m_W^2}{s_W^2} \left\{ \left[ \cos(\beta - \alpha) - \frac{\cos 2\beta \cos(\beta + \alpha)}{2c_W^2} \right]^2 B_0' \left( m_{H^+}^2, m_{H^+}, m_{H^0} \right) \\
+ \left[ \sin(\beta - \alpha) + \frac{\cos 2\beta \sin(\beta + \alpha)}{2c_W^2} \right]^2 B_0' \left( m_{H^+}^2, m_{H^+}, m_{h^0} \right) \right\} .
\]  

The contribution of one generation of quarks is:

\[
\delta Z_H^{q} = \frac{\alpha}{4\pi} \frac{N_C}{2s_W^2 m_W^2} \left\{ - \left( m_d^2 \tan^2 \beta + m_u^2 \cot^2 \beta \right) B_0 \left( m_{H^+}^2, m_d, m_u \right) \\
+ \left[ \left( m_d^2 \tan^2 \beta + m_u^2 \cot^2 \beta \right) \left( m_d^2 + m_u^2 - m_{H^+}^2 \right) + 4m_d^2 m_u^2 \right] B_0' \left( m_{H^+}^2, m_d, m_u \right) \right\} .
\]  

Finally, the contribution of a lepton-neutrino pair is obtained from the formula (17) by taking \( N_C = 1, m_u = 0 \) and using:

\[
\frac{d}{ds} B_0 (s, 0, m) = \frac{1}{m^2 - s} \left\{ - \frac{m^2}{s} [B_0 (s, 0, m) - B_0 (0, 0, m)] + 1 \right\} .
\]
The result is:

\[
\delta Z_l^l = \frac{\alpha}{4\pi} \frac{m^2 \tan^2 \beta}{2 s_W^2 m_W^2} \left\{ - \frac{m^2}{m_{H^+}^2} \left[ B_0 \left( m_{H^+}^2, 0, m \right) - B_0 \left( 0, 0, m \right) \right] \right. \\
+ 1 - B_0 \left( m_{H^+}^2, 0, m \right) \right\}. \tag{19}
\]

The contribution of bosons to mixing can be represented by the following formula:

\[
\delta_{\text{MIX}}^{\text{bos}} = \frac{\alpha}{4\pi s_W^2} \sum_{H=H^0,H^0} \left[ \frac{\sin(\beta - \alpha) \cos(\beta - \alpha)}{4} \right] \left( \delta_{H,H^0} - \delta_{H,h^0} \right) \\
\cdot \left\{ \frac{(m_H^2 - m_W^2)^2}{m_{H^+}^2} \left( B_0 \left( m_{H^+}^2, m_H, m_W \right) - B_0 \left( 0, m_H, m_W \right) \right) \right. \\
+ \left( 2 m_{H^+}^2 + m_H^2 - 3 m_W^2 \right) B_0 \left( m_{H^+}^2, m_H, m_W \right) \right\} \\
+ \frac{m_H^2}{2 m_{H^+}^2} \left\{ \delta_{H,H^0} \sin(\beta - \alpha) \left( \cos(\beta - \alpha) - \frac{\cos 2\beta \cos(\beta + \alpha)}{2c_W^2} \right) - \delta_{H,h^0} \cos(\beta - \alpha) \left( \sin(\beta - \alpha) + \frac{\cos 2\beta \sin(\beta + \alpha)}{2c_W^2} \right) \right\} \\
\cdot \left( m_H^2 - m_{H^+}^2 \right) \left[ \left( 1 - \frac{m_{H^+}^2}{m_W^2} \right) B_0 \left( m_{H^+}^2, m_{H^+}, m_H \right) \right. \\
+ \frac{\cos 2\beta}{4c_W^2} \left( \sin(\beta - \alpha) \cos(\beta + \alpha) \delta_{H,H^0} + \sin(\beta + \alpha) \cos(\beta - \alpha) \delta_{H,h^0} \right) \\
\left. \cdot \left( m_{H^+}^2 - m_H^2 \right) B_0 \left( m_{H^+}^2, m_H, m_W \right) \right\} \\
\left. - \frac{\alpha}{4\pi \frac{1}{8s_W^2 c_W^2}} \left\{ \sin 2\beta \cos 2\beta \left[ 4A(m_W) - 4A(m_{H^+}) + A(m_Z) - A(m_{h^0}) \right] \right. \right. \\
+ \left( c_W^2 \sin 2\alpha \cos 2\beta + s_W^2 \cos 2\alpha \sin 2\beta \right) \left[ A(m_{H^0}) - A(m_{h^0}) \right] \right\} \\
\left. + \frac{g}{2m_W} \left[ \left( m_W^2 - m_{H^+}^2 + m_{h^0}^2 \right) \sin(\beta - \alpha) t_1 \right. \right. \\
\left. - \left( m_W^2 - m_{H^+}^2 + m_{h^0}^2 \right) \cos(\beta - \alpha) t_2 \right]. \tag{20}
\]

The last two lines in the above formula represent contributions of tadpole diagrams. Formulas for fermion loops are given below for $H - G$ and $H - W$.
mixing separately:

\[
\delta_{MIX}^{HG} = -\frac{\alpha}{4\pi} \frac{N_C s_W}{2 s_W m_W^2} \{ (-m_d^2 \tan \beta + m_u^2 \cot \beta) \\
\cdot [ (m_d^2 + m_u^2 - m_{H^+}^2) B_0 (m_{H^+}^2, m_u, m_d) + A(m_u) + A(m_d)] \\
+ 2m_u^2 m_d^2 (\tan \beta - \cot \beta) B_0 (m_{H^+}^2, m_u, m_d) \},
\]

\[
\delta_{MIX}^{HW} = -\frac{\alpha}{4\pi} \frac{N_C}{s_W^2} \left[ (m_d^2 \tan \beta + m_u^2 \cot \beta) B_1 (m_{H^+}^2, m_u, m_d) \\
+ m_u^2 \cot \beta B_0 (m_{H^+}^2, m_u, m_d) \right].
\]

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**Figure captions**

1. Real photon corrections to the decay of $t$ quark

2. Real photon corrections to the charged Higgs boson decay

3. Vertex corrections to the decay of $t$ quark

4. Vertex corrections to the charged Higgs boson decay

5. Momentum independent contributions to mixing

6. Mixing between the charged Higgs and the $W$ boson

7. Mixing between the charged Higgs and the Goldstone boson

8. Types of tadpole diagrams in 2HDM

9. Corrections $\Delta \Gamma$ plotted as a function of $\tan \beta$ for two different values of $m_{H^+}$: $m_{H^+} = 90$ GeV (solid line) and $m_{H^+} = 120$ GeV (dashed)

10. Corrections $\Delta \Gamma$ plotted as a function of $m_{H^+}$ for various values of $\tan \beta$: $\tan \beta = 0.5$ (solid line), $\tan \beta = 1.5$ (long dash) and $\tan \beta = 5$ (short dash)

11. Bosonic contributions to corrections $\Delta \Gamma$ (solid line) and the fermionic contributions from which twice the value of universal corrections $\Delta r$ was subtracted (dashed). Plotted as a function of $m_{H^+}$ for $\tan \beta = 1.5$