On the new COMPASS measurement of the deuteron spin-dependent structure function $g_1^d$

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Abstract

Very recently, a new measurement of the deuteron spin-dependent structure function $g_1^d(x)$ was reported by the COMPASS group. A main change from the old SMC measurement is a considerable improvement of the statistical accuracy in the low $x$ region $0.004 < x < 0.03$. We point out that the new COMPASS data for $g_1^d(x)$ as well as their QCD fits for $\Delta \Sigma$ and $\Delta s + \Delta \bar{s}$ are all remarkably close to our theoretical predictions given several years ago based on the chiral quark soliton model.

If the intrinsic quark spin carries little of the total nucleon spin, what carries the rest of the nucleon spin? Quark orbital angular momentum (OAM) $L^Q$? Gluon OAM $L^g$? Or gluon polarization $\Delta g$? That is a still unsolved fundamental puzzle of QCD [1]. Toward the solution of the problem, remarkable progress has been made for the past few years. First, the new COMPASS measurement of the quasi-real photoproduction of high-$p_T$ hadron pairs indicates that $\Delta g$ cannot be very large at least below $Q^2 \leq 3$ GeV$^2$ [2]. (The small gluon polarization is also indicated by PHENIX measurement of neutral pion double longitudinal spin asymmetry in the proton-proton collisions [3] and also by the STAR measurement of the double longitudinal spin asymmetry in inclusive jet production in polarized proton-proton collisions [4],[5].) There also appeared an interesting paper by Brodsky and Gardner [6], in which, based on the conjecture on the relation between the Sivers mechanism [7] and the quark and gluon OAM [8], it was argued that small single-spin asymmetry observed by the COMPASS collaboration on the deuteron target is an indication of small gluon OAM. These observations together with the progress of the physics of generalized parton distribution functions [9] arose a growing interest on the role of quark OAM in the nucleon.

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The possible importance of quark OAM was pointed out many years ago based on the chiral soliton picture of the nucleon: first within the Skyrme model [10], second within the chiral quark soliton model (CQSM) [11]. According to the latter paper, the dominance of quark OAM is inseparably connected with collective motion of quarks in the rotating hedgehog mean field. The CQSM predicts at the model energy scale around 600 MeV that $\Delta \Sigma$ is around 0.35, while $2L_q$ is around 0.65. The CQSM also reproduces well the spin structure functions for the proton, the neutron and the deuteron [12], [13]. Very recently, a new measurement of the deuteron spin structure function $g_1^d(x, Q^2)$ was reported by the COMPASS group [14]. One should recognize that the precise measurement of $g_1^d(x, Q^2)$ is of crucial importance, because, aside from small effects of $s$-quark polarization as well as the nuclear binding effects etc., it is just proportional to the isosinglet quark helicity distribution, the integral of which gives the intrinsic quark-spin contribution to the total nucleon spin.

The purpose of the present paper is to point out that the new COMPASS data for $g_1^d(x)$ improved in the small-$x$ region turns out to be quite close to our theoretical predictions given several years ago based on the CQSM [12], [13]. We also compare our predictions for the polarized strange quark distribution $\Delta s(x)$ with the recent QCD fits at the next-to-leading order (NLO) performed by the COMPASS group, to find that they are order of magnitude consistent. We shall also see that the net longitudinal quark polarization $\Delta \Sigma$ as well as the strange quark polarization $\Delta s + \Delta \bar{s}$ in the nucleon extracted from the recent QCD fits by the CAMPASS [15] and HERMES [16] collaborations are not only mutually consistent but also surprisingly close to the predictions of the CQSM.

In QCD, the longitudinal spin structure functions for the proton and the neutron are given as

$$g_1^{p/n}(x) = \frac{1}{9} \left( C_{NS} \otimes \left[ \pm \Delta q_3 + \frac{1}{4} \Delta q_8 \right] + C_S \otimes \Delta \Sigma + 2N_f C_g \otimes \Delta g \right),$$

where $C_{NS}, C_S, C_g$ are the nonsinglet, singlet and gluon Wilson coefficients, while the symbol $\otimes$ represents the convolution with the quark and gluon distribution functions:

$$\Delta q_3(x) \equiv (\Delta u(x) + \Delta \bar{u}(x)) - (\Delta d(x) + \Delta \bar{d}(x)), \quad \Delta q_s(x) \equiv (\Delta u(x) + \Delta \bar{u}(x)) + (\Delta d(x) + \Delta \bar{d}(x)) - 2(\Delta s(x) + \Delta \bar{s}(x)), \quad \Delta \Sigma(x) \equiv (\Delta u(x) + \Delta \bar{u}(x)) + (\Delta d(x) + \Delta \bar{d}(x)) + (\Delta s(x) + \Delta \bar{s}(x)).$$

It is customary to assume that the structure functions $g_1^p(x)$ and $g_1^n(x)$ on proton and neutron targets are related to that of the deuteron by the relation

$$g_1^d(x) = \frac{1}{2} \left( g_1^p(x) + g_1^n(x) \right) \left( 1 - \frac{3}{2} \omega_D \right),$$

with $\omega_D$ the $D$-state admixture to the deuteron wave function. Instead of $g_1^d(x)$, it is then more convenient to use $g_1^N(x) \equiv g_1^d(x)/(1 - \frac{3}{2} \omega_D)$, in which the correction for the $D$-state admixture in the deuteron state has been taken into account.
At the leading order (LO), $g_{1}^{p}$ and $g_{1}^{n}$ reduce to

$$g_{1}^{p}(x) = \frac{1}{9} \left[ 4 \Delta u(x) + \Delta d(x) - \Delta s(x) \right],$$

$$g_{1}^{n}(x) = \frac{1}{9} \left[ \Delta u(x) + 4 \Delta d(x) - \Delta s(x) \right].$$

(6)

(7)

Assuming that the polarizations of the $s$- and $\bar{s}$-quarks are small, we therefore have an approximate relation

$$g_{1}^{N}(x) \approx \frac{5}{36} \Delta \Sigma(x),$$

(8)

with

$$\Delta \Sigma(x) \approx (\Delta u(x) + \Delta \bar{u}(x)) + (\Delta d(x) + \Delta \bar{d}(x)),$$

(9)

which denotes that $g_{1}^{N}(x)$ is proportional to the isosinglet quark helicity distribution, the integral of which gives the intrinsic quark-spin contribution to the nucleon spin sum rule. This is of course exact only at the leading-order QCD and under the assumption of small strange quark polarization. Still, it clearly indicates the importance of precise measurements of spin-dependent structure function of the deuteron $g_{1}^{d}(x)$, which has recently been carried out by the COMPASS group [15] and also by the HERMES group [16].

Before comparing the predictions of the CQSM with the new COMPASS data for $g_{1}^{d}(x)$, several remarks are in order. Our predictions are based on the longitudinally polarized quark distributions evaluated in [12] within the framework of flavor $SU(2)$ CQSM and those evaluated in [13] within the framework of flavor $SU(3)$ CQSM. The $SU(2)$ CQSM is essentially parameter free, since its only one model parameter, i.e. the dynamical quark mass $M$ [18] was already fixed to be $M \approx 375$ MeV from the analyses of low energy nucleon observables. On the other hand, the $SU(3)$ CQSM contains one additional parameter, i.e. the mass difference $\Delta m_{s}$ between the strange and nonstrange quarks. In [13], the value of $\Delta m_{s}$ was fixed to be $\Delta m_{s} \approx 100$ MeV so as to reproduce the empirical unpolarized distribution of strange quarks. In the case of $SU(2)$ model, we regard the theoretical quark distributions $\Delta u(x), \Delta d(x), \Delta \bar{u}(x)$, and $\Delta \bar{d}(x)$ as initial parton distributions prepared at the low energy model scale around $Q_{\text{ini}}^{2} = 0.3$ GeV$^{2} \approx (600$ MeV$)^{2}$, following the spirit of the QCD analysis by Glück, Reya and Vogt [17]. The polarized strange quark distributions $\Delta s(x)$ and $\Delta \bar{s}(x)$ as well as the polarized gluon distribution $\Delta g(x)$ are all set zero at this low energy scale. We then solve the standard DGLAP equation at the NLO to obtain the parton distributions and the relevant structure functions at the high energy scale. In the case of $SU(3)$ model, only a difference is that it can provide us with the theoretical polarized strange quark distributions $\Delta s(x)$ and $\Delta \bar{s}(x)$ as well at the initial model energy scale.

Fig.1 shows the comparison between our predictions for $x g_{1}^{d}(x, Q^{2})$ given several years ago and the new COMPASS data (filled and open circles) together with the old SMC data (open squares). The solid and the dashed curves respectively stand for the predictions of the $SU(3)$
and $SU(2)$ CQSM evolved to the energy scale $Q^2 = 3 \text{GeV}^2$, which is the average energy scale of the new COMPASS measurement. The long-dashed curve shown for reference is the next-to-leading order QCD fit by the COMPASS group [15]. As one can see, the new COMPASS data show a considerable deviation from the central values of the old SMC data in the small $x$ region. One finds that our predictions are consistent with the new COMPASS data especially in the small $x$ region.

![Figure 1: The predictions of the $SU(2)$ and $SU(3)$ CQSM in comparison with the new COMPASS data for $x g_1^d(x)$ (filled circles) and their NLO QCD fits (long-dashed curve). The two COMPASS points at low $x$ (low $Q^2$), which are not included in their QCD fits, are also shown by open circles. Here, the theoretical predictions correspond to the fixed energy scale $Q^2 = 3 \text{GeV}^2$, which corresponds to the average $Q^2$ of the COMPASS data, while the COMPASS points are given at the $\langle Q^2 \rangle$ where they were measured. The old SMC data [19] transformed to the corresponding COMPASS points are also shown by open squares, for reference.](image)

This tendency can more clearly be seen in the comparison of $g_1^N(x)$ illustrated in Fig. 2. The filled circles here represent the new COMPASS data for $g_1^N(x)$ evolved to the common energy scale $Q^2 = 3 \text{GeV}^2$, while the long-dashed curve is the result of the next-to-leading order QCD fit by the COMPASS group at the same energy scale. The corresponding predictions of the $SU(3)$ and $SU(2)$ CQSM are represented by the solid and dashed curves, respectively. For the quantity $g_1^N(x)$, the experimental uncertainties are still fairly large in the small $x$ region.
Still, one can say that the predictions of the CQSM is qualitatively consistent with the new COMPASS data as well as their QCD fit. The COMPASS group also extracted the matrix element of the flavor-singlet axial charge $a_0$ \[^{[15]}\] , which can be identified with the net longitudinal quark polarization $\Delta \Sigma$ in the MS factorization scheme. Taking the value of $a_8$ from the hyperon beta decay, under the assumption of $SU(3)$ flavor symmetry, they extracted from the QCD fit of the new COMPASS data for $g_1^N(x)$ the value of $\Delta \Sigma$ as

$$\Delta \Sigma(Q^2 = 3 \text{ GeV}^2)_{COMPASS(A)} = 0.35 \pm 0.03 \text{ (stat.)} \pm 0.05 \text{ (syst.)}. \quad (10)$$

On the other hand, the same quantity derived from the fits to all $g_1$ data is a little smaller

$$\Delta \Sigma(Q^2 = 3 \text{ GeV}^2)_{COMPASS(B)} = 0.30 \pm 0.01 \text{ (stat.)} \pm 0.02 \text{ (evol.)}. \quad (11)$$

A similar analysis was also reported by the HERMES group \[^{[16]}\]. Their result is

$$\Delta \Sigma(Q^2 = 5 \text{ GeV}^2)_{HERMESS} = 0.330 \pm 0.011 \text{ (theor.)} \pm 0.025 \text{ (exp.)} \pm 0.028 \text{ (evol.)}. \quad (12)$$

Main changes of these new QCD analyses from the old SMC analysis \[^{[19]}\] are considerable reduction of error bars and upward shift of the central values. Moreover, the results of the two groups for $\Delta \Sigma$ look mutually consistent within the reduced error bars. We now compare these
new results with the prediction of the $SU(3)$ CQSM given in our previous papers. Shown in Fig. 3 are the prediction of the CQSM for $\Delta \Sigma$ and $\Delta g$ as functions of the energy scale $Q^2$. They are obtained by solving the standard DGLAP equation at the NLO with the prediction of the model as the initial condition given at the scale $Q^2_{ini} = 0.30 \text{GeV}^2 \simeq (600 \text{MeV})^2$. Since the CQSM is an effective quark model, which contains no gluon degrees of freedom, $\Delta g$ is simply assumed to be zero at the initial scale. One sees that the new COMPASS and the HERMES results for $\Delta \Sigma$ are surprisingly close to the prediction of the CQSM. Also interesting is the longitudinal gluon polarization $\Delta g$. In spite that we have assumed that $\Delta g$ is zero at the starting energy, it grows rapidly with increasing $Q^2$. As pointed out in [20], the growth of the gluon polarization with $Q^2$ can be traced back to the positive sign of the anomalous dimension $\gamma_{gg}^{(0)}$. The positivity of this quantity dictates that the polarized quark is preferred to radiate a gluon with helicity parallel to the quark polarization. Since the net quark spin component in the proton is positive, it follows that $\Delta g > 0$ at least for the gluon perturbatively emitted from quarks. The growth rate of $\Delta g$ is so fast especially in the relatively small $Q^2$ region that its magnitude reaches around $(0.3 - 0.4)$ already at $Q^2 = 3 \text{GeV}^2$, which may be compared...
with the estimate given by the COMPASS group [15]:

\[ \Delta g(Q^2 = 3 \text{ GeV}^2)_{\text{COMPASS}} \simeq (0.2 - 0.3). \]  

(13)

Also interesting to investigate is the COMPASS fits for the polarized strange quark distributions, extracted from the difference between \( \Delta \Sigma(x) \) and \( \Delta q_8(x) \). They performed two next-to-leading order fits corresponding to positive and negative gluon polarizations. The long-dashed curve in Fig 4 shows the polarized strange quark distribution \( x \Delta s(x) \) at \( Q^2 = 3 \) GeV\(^2\) corresponding to the fits with \( \Delta g > 0 \), while the solid curve represents the corresponding predictions of the \( SU(3) \) CQSM. For comparison, we also show the corresponding distributions from the DNS2005 [21] and LSS2005 [22] QCD fits. Note that, the flavor symmetry of the polarized strange sea, i.e. \( \Delta s(x) = \Delta \bar{s}(x) \) is assumed in all the above three QCD fits. On the other hand, within the CQSM, as was pointed out in [13], the longitudinal strange quark polarization is almost solely born by the \( s \)-quark and the polarization of \( \bar{s} \)-quark is very small, Bearing this fact in mind, one sees that the result of the new COMPASS fits for \( x \Delta s(x) \) is definitely negative and its magnitude is qualitatively consistent with the prediction of the CQSM as well as with the DNS2005 and LSS2005 QCD fits.

Figure 4: The prediction of the \( SU(3) \) CQSM for the polarized strange quark distribution \( x \Delta s(x) \) is compared with the recent QCD fits by the COMPASS group (long-dashed curve). The corresponding distributions from the DNS2005 and the LSS2005 fits are also shown for comparison by the dash-dotted and dashed curves, respectively.
The net strange quark polarization $\Delta s + \Delta \bar{s}$, or the first moment of $\Delta s(x) + \Delta \bar{s}(x)$ extracted by the COMPASS and the HERMES group may also be interesting to see. The COMPASS group obtained
\[
(\Delta s + \Delta \bar{s})(Q^2 \to \infty)_{\text{COMPASS}} = -0.08 \pm 0.01 \text{ (stat.)} \pm 0.02 \text{ (stat.)},
\]
while the result of the HERMES analysis is
\[
(\Delta s + \Delta \bar{s})(Q^2 = 5 \text{ GeV}^2)_{\text{HERMES}} = -0.085 \pm 0.013 \text{ (theor.)} \pm 0.008 \text{ (exp.)} \pm 0.028 \text{ (evol.)}.
\]
One finds that the results of the two semi-empirical fits are not only mutually consistent but also they are surprisingly close to the the corresponding prediction of the $SU(3)$ CQSM given by
\[
(\Delta s + \Delta \bar{s})(Q^2 = 5 \text{ GeV}^2)_{\text{CQSM}} = -0.082.
\]
A sizable polarization of the strange sea appears to contradict the indication of the semi-inclusive DIS analysis [23]. However, we believe that our understanding of the semi-inclusive processes has not reached the precision of inclusive DIS physics yet. We also emphasize that the large and negative polarization of the strange quarks is not a crucial factor for our resolution scenario of the nucleon spin puzzle. This is clear from the fact that the flavor $SU(2)$ CQSM, which naturally predicts zero strange polarization at the model energy scale, already explains small $\Delta \Sigma$. In fact, aside from very small $SU(3)$ breaking effect, which turns out to be the order of 0.01, both of the $SU(2)$ CQSM and the $SU(3)$ CQSM gives exactly the same answer for $\Delta \Sigma$ as
\[
\Delta \Sigma[SU(2)] \equiv \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} = 0.35, \quad (17)
\]
\[
\Delta \Sigma[SU(3)] \equiv \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} = 0.35. \quad (18)
\]
Since $\Delta s + \Delta \bar{s} < 0$ in the $SU(3)$ CQSM, this means that $\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d}$ in the $SU(2)$ model is smaller than that in the $SU(3)$ CQSM, while keeping the equality
\[
\Delta \Sigma[SU(2)] = \Delta \Sigma[SU(3)]. \quad (19)
\]
In any case, the above explanation clearly shows that the unique feature of the CQSM, which can reproduce very small $\Delta \Sigma$, is not crucially dependent on the negative polarization of strange quarks, but it rather comes from the basic dynamical assumption of the model, i.e. the physical picture of the nucleon as a rotating hedgehog, which naturally generates large quark orbital angular momentum. As a consequence, the HERMES result, even though it is assumed to be correct, would not change the main conclusions of the present paper, i.e. the resolution scenario of the nucleon spin puzzle based on the importance of the quark orbital angular
momentum. For other resolution scenarios of the nucleon spin puzzle, we refer to the recent workshop summary [24].

To conclude, the new measurements of the deuteron spin-structure function $g_1^d(x)$ carried out by the COMPASS group as well as by the HERMES group achieved a remarkable improvement in the accuracy of the experimental data, especially in the low $x$ region, as compared with the existing old data. As an important outcome, our knowledge on the net quark helicity contribution $\Delta \Sigma$ to the total nucleon spin has been improved to a large degree. As we have pointed out, the value of $\Delta \Sigma$ extracted from the new QCD fits by the COMPASS and the HERMES groups is around $0.3 \sim 0.35$, which is surprisingly close to the prediction of the CQSM. Now that the role of quark helicity contribution to the nucleon spin sum rule has been understood fairly well, we come back to the question: what carry the rest of the nucleon spin? The CQSM claims that the role of quark orbital momentum is important at least at the low energy scale of nonperturbative QCD around $Q^2 \simeq (600 \text{MeV})^2$. (Although this is a highly model-dependent statement, we can give a kind of model-independent analysis, based only upon some reasonable theoretical postulates, which supports the importance of quark OAM at the low energy scale [25],[26].) We hope that this unique prediction of the CQSM will be verified by the near-future measurement of the generalized parton distribution functions of the nucleon with enough precision.

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References

[1] EMC Collaboration, J. Ashman et al., Phys. Lett. B206 (1988) 364 ; Nucl. Phys. B328 (1989) 1.
[2] COMPASS Collaboration, E.S. Ageev et al., Phys. Lett. B633 (2006) 25.
[3] PHENIX Collaboration, K. Boyle et al., nucl-ex/0606008.
[4] STAR Collaboration, J. Kiryluk et al., hep-ex/0512040.
[5] STAR Collaboration, R. Fatemi et al., nucl-ex/0606007.
[6] S.J. Brodsky and S. Gardner, hep-ph/0608219.
[7] D.W. Sivers, Phys. Rev. D41 (1990) 83 ; Phys. Rev. D43 (1991) 261.
[8] M. Burkardt, Phys. Rev. D66 (2002) 114005.

[9] X. Ji, J. Phys. G24 (1998) 1181.

[10] S.J. Brodsky, J. Ellis, and M. Karliner, Phys. Lett. B206 (1988) 309.

[11] M. Wakamatsu and H. Yoshiki, Nucl. Phys. A524 (1991) 561.

[12] M. Wakamatsu and T. Kubota, Phys. Rev. D60 (1999) 034020.

[13] M. Wakamatsu, Phys. Rev. D67 (2003) 034005 ; Phys. Rev. D67 (2003) 034006.

[14] COMPASS Collaboration, E.S. Ageev et al., Phys. Lett. B612 (2005) 154.

[15] COMPASS Collaboration, V.Yu. Alexakhin et al., hep-ex/0609038.

[16] HERMES Collaboration, A. Airapetian et al., hep-ex/0609039.

[17] M. Glück, E. Reya, and A. Vogt, Z. Phys. C67 (1995) 433.

[18] D.I. Diakonov, V.Yu. Petrov, and P.V. Pobylitsa, Nucl. Phys. B306 (1988) 809.

[19] SMC Collaboration, B. Adeva et al., Phys. Rev. D58 (1998) 112001.

[20] H.-Y. Cheng, Int. J. Mod. Phys. A11 (1996) 5109.

[21] D.de Florian, G.A. Navarro, and R. Sassot, Phys. Rev. D71 (2005) 094018.

[22] E. Leader, A.V. Sidorov, and D.B. Stamenov, Phys. Rev. D73 (2006) 034023.

[23] HERMES Collaboration, A. Ackerstaff et al., Phys. Lett. B464 (1995) 123.

[24] S.D. Bass and C.A. Aidala, hep-ph/0606269.

[25] M. Wakamatsu, Phys. Rev. D72 (2005) 074006.

[26] M. Wakamatsu and Y. Nakakoji, Phys. Rev. D74 (2006) 054006.