Rapid broadband characterization of scattering medium using hyperspectral imaging: supplementary material

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This document provides supplementary information to “Rapid broadband characteriza-tion of scattering medium using hyperspectral imaging,” https://doi.org/10.1364/optica.6.000274. We provide detailed description on the hyperspectral imaging system calibration and data processing.

1. HYPERSONTICAL IMAGING CALIBRATION

Hyperspectral images captured by the camera CAM2 (see Figure 2 of the main manuscript) contain both spatial and spectral data within one captured frame. A calibration matrix $M$ establishes a linear correspondence between the raw CAM2 image $I$ and the spatio-spectral data $S$:

$$S(N_{ss}, 1) = M(N_{ss}, N_{pixels}) I(N_{pixels}, 1).$$  (S1)

Here $I$ is the vectorized CAM2 image of size $(N_{pixels}, 1)$, which is multiplied by the calibration matrix $M$ of size $(N_{ss}, N_{pixels})$ to obtain the vectorized spatio-spectral data $S$ of size $(N_{ss}, 1)$. $S$ is then reshaped into dimensions $(N_{spectral}, N_{spatial})$, with $N_{spectral} \times N_{spatial} = N_{ss}$.

The calibration procedure to determine the matrix $M$ is elucidated in the Figure S1. First, the laser in a CW-mode is scanned across several wavelengths to obtain a series of monochromatic calibration images (see Fig. S1A). The positions of the peaks are used to locate the rectangular zone delimiting each linear spectrum (see Fig. S1B). The rectangles are then divided into $N_{spectral}$ equally spaced spectral bins (see Fig. S1C), and the pixels within a given bin are assigned into the corresponding element of $S$.

Mathematically, a row of matrix $M_{ij}$ used to calculate a given element $S_j$ from $I_i$ consists mostly of zeros, apart from the few nonzero elements corresponding to this spatio-spectral bin of $I_i$. To minimize quantization effects, each bin was normalized by the number of contained pixels that can fluctuate. The matrix $M$ is very sparse ($\sim 1/N_{ss} \sim 6 \cdot 10^{-5}$ nonzero elements) and is represented as such for computational purposes (Matlab has a dedicated data type for sparse matrices).

In the Figure S2 we show the raw and treated images for different experimental phases for the scattering medium with $N_{\lambda} = 80$. First row (see Fig. S2A1-D1) corresponds to the broadband random speckle originating from the scattering medium. Individual line spectra on the raw image from CAM2 lack both spatial and spectral correlations. Figure 2B1 shows all the spectra aligned with respect to the wavelength. It can be seen from the image that the speckle is indeed broadband (13.8 nm measured FWHM). Red line indicates the 800 nm spectral bin whose intensity is used to create a spatial image Fig. S2C1. The Fig. S2D1 shows the reference image from CAM1 that is not spectrally resolved.

Second row (see Figs. S2A2-D2) shows hyperspectral images after spatio-spectral focusing (Section 2A of the main manuscript). Raw (Fig. S2A2) and spectrally-aligned (Fig. S2B2) hyperspectral images show very narrow bandwidth of the spatial focus. Total spectral intensity (sum over all spectral bins) is used to reconstruct the spatial intensity map Fig. S2C2. Reference camera image Fig. S2D2 confirms spatial focusing.

Finally, the third row (see Figs. S2A3-D3) shows hyperspectral images after temporal pulse recompression (Section 2B of the main manuscript). Raw (Fig. S2A2) and spectrally-aligned


**Fig. S1.** Hyperspectral calibration. A) Image of a monochromatic pattern at 800 nm. B) 790 nm (blue), 800 nm (green) and 810 (red) patterns within delimiting rectangles corresponding to spatial spectra. C) Individual spatio-spectral bins visualized via random RGB color-coding. Size and shape non-uniformity of individual bins can be seen. All the images are cropped $\sim 3 \times$ from the original for better visibility.

**Fig. S2.** Hyperspectral imaging of $N_\lambda = 80$ medium. (row 1) random speckle, (row 2) single spatio-spectral focus, (row 3) temporally recompressed broadband pulse. A) Raw image from CAM2 showing individual linear spectra with hexagonal arrangement. B) Spectra for all 198 spatial pixels extracted from A) using the calibration matrix $M$. C) Spatial intensity map for 800 nm spectral bin (1), and total spectrum (2), (3). D) Corresponding image on the CAM1. White box delimits the region captured by the HSI. All values are in arbitrary units, scaled from 0 to 1.
(Fig. S2B2) hyperspectral images show the same spatial localization as in Figs. S2A2-D2, but a broader spectrum. In contrast to the random speckle whose spectral intensity is non-uniform (Fig. S2B1), the spectra present relatively uniform intensity due to the locked global phase between spectral components. Total spectral intensity (sum over all spectral bins) is used to reconstruct spatial intensity map Fig. S2C3. Reference camera image Fig. S2D3 confirms spatial focusing.

Note the presence of a weak spurious signal of diagonal arrangement on Fig. S2CB2 and Fig. S2CB3. This manifests a weak coupling between neighboring line spectra in the raw images.

2. TEMPORAL FOCUSING

An ultrashort pulse of light whose spectral bandwidth $\Delta \lambda_{\text{laser}}$ is larger than the correlation bandwidth of the medium $\delta \lambda_m$, is temporally distorted. $\delta \lambda_m$ can be retrieved from the output speckle intensity correlation function $C(\lambda)$ as defined in [1].

Similarly in the temporal domain, the same pulse, characterized by a Fourier-limited duration $\delta t$, is stretched after its propagation through the medium [3]. Resulting temporal profiles vary across different spatial locations, and averaging this temporal speckle over spatial positions yields the time-of-flight (TOF) distribution.

The relationship between the medium correlation wavelength $\delta \lambda_m$ and the duration of the output pulse is derived in [2]. More precisely, the TOF distribution has a complex shape dictated by the diffusion theory. The long-time behavior of this curve is dominated by an exponential decay with the decay time $\tau$, which is characteristic of the time stretch of the pulse, yet easily extractable from the curve. It has been demonstrated [2] that this decay time is, within 10%, equal to the inverse spectral bandwidth of the medium: $\tau \simeq 1/\omega_m$. Therefore, smaller correlation bandwidth $\delta \lambda_m$ corresponds to the proportionally longer duration of the stretched pulse $\tau$.

As $\omega = 2\pi c/\lambda$, therefore $\delta \omega_m = 2\pi \delta \lambda_m c/\lambda^2$, together with the expression for $\tau$ yields:

$$\tau = \lambda^2 / (2\pi \delta \lambda_m) \quad (S2)$$

The experimentally measured TOF intensity curves for the media with $N_{\lambda} = 25$ and $N_{\lambda} = 80$ are shown in Fig. S3. From the linear fit in log-linear scale we extract $\tau = 0.66$ ps and $\tau = 2.01$ ps, respectively. The decay times predicted by the Eq. S2 from the experimentally measured $\delta \lambda_m$ are 0.64 ps and 2 ps, respectively, which is fully consistent with the experimental data.

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