B-Reconstruction Methods via Geometro-Kinematic Constraints (I)

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Decay channels with attractive branching ratios, or interesting physics, are recovered by substituting “missing” particles ($\gamma, \nu, \pi^0$, etc) with combined geometric and kinematic constraints. The “Sliding Vertex” method is shown in this part-I, for reconstructing strongly boosted $B_s^0$ decays - at the LHC.

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Precision B-Physics in today’s experiments [1] relies heavily on exclusively reconstructed decay chains. There are however decay modes that are not readily exclusive-reconstructable, but very attractive from the view point of the physics, or that of a large branching ratio - im-paired in reconstruction usually by missing particles. Methods for constraining and recovering information in missing particle events have been explored before with kinematic fitting [2,3] and in a variety of other contexts, a good collection thereof being [4, 5].

Consider for instance the $B_s^0 \to D_s^- K^+$ decay with the subsequent $D_s^- \to K^+ K^- \pi^- (\pi^0)$ decay. This mode is simple and attractive for measuring $\gamma_{\text{CKM}}$, however its branching ratio is rather fair. If we consider the related mode, with $\pi^0$ in the final state, the branching ratio is ca. 3.5 times larger, a real feat. Evidently, the $\pi^0$ can be reconstructed in the E-calorimeter, albeit with less resolution than for tracking. In principle the reconstruction with the $\pi^0$ is very attractive, but seemingly somewhat impractical.

There is however enough information in the detector to reconstruct the decay without the $\pi^0$: first there are the 4-momentum conservation laws in the $B_{VTX}$ and $D_{VTX}$, and second, the $B_{VTX}$ must lie on the $\pi^+$ track. Figure 1 shows the topology of the event (in the absence of magnetic field - such as for LHCb, although, given the small track bending over the vertexing region, a solution in the form of an $O_{III}$ helical correction exists also for ATLAS). The evident question arises, whether such methods reject not only background, but also “sister physics” (in the case of the above mode the significantly more abundant $B_s^0 \to D_s^- \pi^+$ decay, with the $\pi^+$ mistaken by the PID system as a $K^+$), which play a more dominant role. Such is the case of decays coming from $B_d^0$ which to a large degree shadow (ca. 3:1) the $B_s^0$ decays through similar mass ratios between mother and daughters. From the start it should be stated that this is the most difficult part and that possible solutions lie in the avenues of tracking/vertexing resolutions, of (semi)-leading (bi-)particle effects in the aforementioned decays and of different branching ratios (sometimes smaller) for $B_d^0$ for the exact event topology as $B_s^0$. In fact $B_d^0$ contaminates $B_s^0$ events more heavily with different topologies (where 1 track is lost, or which have 1-2 extra $\pi^0$’s).

Kinematic Constraints - are the 4-momentum conservation laws in the $B_{VTX}$ and $D_{VTX}$. For the $B_{VTX}$ the useful part is:

$$(E_\pi + E_D)^2 = M_B^2 + (p_D^2 + p_\pi^2 + 2 p_D p_\pi \cos \theta_{D\pi})$$

where $\cos \theta_{D\pi} = \frac{1}{p_D} (E_D / \beta_\pi - Q_{D\pi})$

with $Q_{D\pi} = \frac{1}{p_\pi} \Delta_{D\pi}^2$ and $\Delta_{D\pi}^2 = \frac{1}{2} (M_B^2 - M_D^2 - m_\pi^2)$ (1)

and for the $D_{VTX}$:

$$(E_\pi - E_v)^2 = m_0^2 + (p_D^2 + p_v^2 - 2 p_D p_v \cos \theta_{Dv})$$

where $\cos \theta_{Dv} = \frac{1}{p_D} (E_D / \beta_v - Q_{Dv})$

with $Q_{Dv} = \frac{1}{p_v} \Delta_{Dv}^2$ and $\Delta_{Dv}^2 = \frac{1}{2} (M_B^2 + m_v^2 - m_0^2)$ (2)

where “0” is the missing neutral, “v” the sum of visible

![FIG. 1: Topology of $B_s^0 \to D_s^- K^+$ decay with the subsequent $D_s^- \to K^+ K^- \pi^- (\pi^0)$](image)

The tracks are shown in the absence of magnetic field - such as for LHCb, although, given the small track bending over the vertexing region, a solution in the form of an $O_{III}$ helical correction exists also for ATLAS.
particles in the \( \text{D}_{\text{VTX}} \) and the rest of the notations are self-evident.

**Geometric Constraints** - are supplying the missing, 3rd equation to the above set. Due to momentum conservation in the \( \text{B}_{\text{VTX}} \) “D”, “π” and “q” (see figure 1) lie in the same plane: \( \vec{n}_D = \lambda \vec{n}_\pi + \mu \vec{n}_q \), where \( \vec{n} \) are unit vectors and \( \lambda, \mu \) constants. The cosines from the kinematic relations are: \( \cos\theta_{\text{D}} = \vec{n}_D \cdot \vec{n}_\pi \) and \( \cos\theta_{\text{D}_\nu} = \vec{n}_D \cdot \vec{n}_\nu \), respectively:

\[
\lambda = \frac{1}{\Delta} [(v\mu)\cos\theta_{\text{D}} - (q\pi)\cos\theta_{\text{D}_\nu}]
\]

\[
\mu = \frac{1}{\Delta} [-(v\mu)\cos\theta_{\text{D}} + \cos\theta_{\text{D}_\nu}]
\]

(3)

where \( (ab) = \vec{n}_a \cdot \vec{n}_b \) and \( \Delta = (v\mu) - (v\pi)(q\pi) \). The “closure” equation is \( |\vec{n}_D|^2 = 1 = \lambda^2 + \mu^2 + 2(q\pi)\lambda\mu \).

In terms of the geometric constraints the kinematic section condenses to \( E^2_D - M^2_D = (\lambda p_D)^2 + (\mu p_D)^2 + 2(q\pi)(\lambda p_D)(\mu p_D) \) which can be solved in favor of \( E_D \) as a second order equation. The two fold ambiguity resulting therefrom is lifted through a (2D) pointback to IP criterion for the \( \text{B}^\mu_i \) (after vertex determination).

At this point all kinematic quantities are known [8].

**Vertex Determination** - once \( \text{D}_{\text{VTX}} \) is known the 3D vectors of all particles are known. The \( \text{B}_{\text{VTX}} \) \( \vec{b} \) is the locus that takes the best-shot at the:

1. IP in the \( \vec{n}_B \) direction: \( \vec{b} - \lambda_B \vec{n}_B = \text{IP} \pm \sigma_{\text{IP}} \)

2. \( \text{D}_{\text{VTX}} \) in the \( \vec{n}_D \) direction: \( \vec{b} - \lambda_D \vec{n}_D = \text{D}_{\text{VTX}} \pm \sigma_D \)

3. \( \pi^+ \) track \( \sigma_\pi \) tube: \( \vec{b} - \lambda_\pi \vec{n}_\pi = \vec{r}_\text{IP} \pm \sigma_D \)

Mathematically this means - using the \( \sigma_i \) error matrices:

\[
\begin{align*}
\langle \vec{b} - \vec{v}_B - \lambda_B \vec{n}_B | \sigma^2_{\text{IP}} | \vec{b} - \vec{v}_B - \lambda_B \vec{n}_B \rangle + \\
\langle \vec{b} - \vec{v}_D - \lambda_D \vec{n}_D | \sigma^2_D | \vec{b} - \vec{v}_D - \lambda_D \vec{n}_D \rangle + \\
\langle \vec{b} - \vec{r}_\text{IP} - \lambda_\pi \vec{n}_\pi | \sigma^2_\pi | \vec{b} - \vec{r}_\text{IP} - \lambda_\pi \vec{n}_\pi \rangle = \text{min} \quad (4)
\end{align*}
\]

By differentiating to find the minimum, the “sliding” along each direction is:

\[
\lambda_i = \langle \vec{n}_i | \sigma_i^{-2} | \vec{b} - \vec{v}_i \rangle / \langle \vec{n}_i | \sigma_i^{-2} \rangle 
\]

(5)

The vertex solution is then:

\[
| \vec{b} \rangle = \left[ \sum_i \sigma_i^{-2} \left( 1 - \left| \frac{\vec{n}_i}{\sigma_i^{-2}} \right| \langle \vec{n}_i | \sigma_i^{-2} \rangle \right) \right]^{-1} \vec{w} \]  \quad (6)

where:

\[
| \vec{w} \rangle = \sum_i \sigma_i^{-2} \left( 1 - \left| \frac{\vec{n}_i}{\sigma_i^{-2}} \right| \langle \vec{n}_i | \sigma_i^{-2} \rangle \right) \vec{v}_i \]  \quad (7)

The math in itself is straightforward, however computer implementation proved to be somewhat of a hassle. Code that implements vectors and matrices is rather slow due to multiple inheritances, unnecessary functions, etc.

We coded our own MXV4 namespace that addressed these issues and performed in speed.

One special mention is with respect to the \( \pi \) track \( \sigma_{\pi}^{-2} \) matrix. This is not immediate two-fold: writing the \( \sigma_{\pi} \) and secondly, the tube is not cylindrical, rather elliptical in cross-section. This is due to to the fact that the track is determined by (approximately) circular errors in the VELO planes of LHCb (prototype address) which stand vertically. (This of course remains to be solved exactly for each detector where it is applied.) The \( \pi \) track tube is thus an infinitely long ellipsoid with unequal cross-section major axes:

\[
\sigma_{\pi}^{-2} = \frac{1}{\sigma_{\text{long}}^{-2}} | \vec{n}_\pi \rangle \langle \vec{n}_\pi | + \\
\sigma_{\text{minor}}^{-2} [ \vec{e}_z \times \vec{n}_\pi ] \langle \vec{e}_z \times \vec{n}_\pi | + \\
\sigma_{\text{major}}^{-2} | \vec{n}_\pi \times \vec{e}_z \times \vec{n}_\pi \rangle \langle \vec{n}_\pi \times \vec{e}_z \times \vec{n}_\pi | 
\]

(8)

where \( \sigma_{\text{minor}}^{-2} = \sigma_{\pi}^{-2} \) and \( \sigma_{\text{major}}^{-2} = (\vec{e}_z | \vec{n}_\pi \rangle)^{-2} \).

With this, the \( \pi \) track contribution in the \( \text{B}_{\text{VTX}} \) calculation reduces significantly (and this needs to be so implemented in the code in order to avoid singularities).

![FIG. 2: The xy- (top) and z-residuals (bottom) for the reconstructed \( \text{B}_{\text{VTX}} \) with respect to the Monte Carlo position. All dimensions are in \( \mu \text{m} \).](image-url)}

We tested the code on PYTHIA simulated data (smeared to give invariant mass resolutions as those expected in the LHCb detector).

The xy- and z-residuals for the reconstructed \( \text{B}_{\text{VTX}} \) with respect to the Monte Carlo position are shown in
The leading-particle effect is visible for the kaon-pair in the dynamics. Since the CM decay energy is 0.7-0.8 GeV some that can still play some role would be one due to dy-

FIG. 3: Momentum of $\pi^-$ coming from $D_s^-$ (blue) vs. that from $D^-$. The spectrum is harder for the latter, as the other two components do not necessarily each contain a valence quark of the decaying particle, leading to a more equitable energy distribution among daughter particles. This is not the case for the two kaons coming from a $D_s^-$ decay, which together take a greater share of the decay energy (leading-particle effect).

Leading particle effects - figure 2 shows the $\pi^-$ momentum (yellow) from $D^- \rightarrow \bar{\pi}^-\pi^-K^+$ of the decay $B^0_d \rightarrow D^-\pi^+$, where $\bar{\pi}^-$ is mis-ID‘ed as a kaon. Charge does not help in distinguishing, as both $B$ and $\bar{B}$ can be present, mass differences are small, hence the only factor that can still play some role would be one due to dynamics. Since the CM decay energy is 0.7-0.8 GeV some leading-particle effect is visible for the kaon-pair in a $D_s^-$ decay (each holding a valence quark of the $D_s^-$). In blue is the $\pi^-$ momentum coming from $D_s^-$, evidently softer (less available energy, as most is concentrated by the kaon pair).

Applications - the method aims evidently at physics analyses, however other useful events can also be recon-

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[1] Prominent dedicated B-Physics experiments are: BaBar (SLAC: 1999-running), Belle (KEK: 1999-running), HERA-B (DESY: 2000-2003), LHCb (CERN: 2008-running). A number of other experiments have significant B-Physics programmes: CDF (FNAL: E-741/1987-1992, E-775/1992-1996, E-980/2001-running), D0 (FNAL: E-740/1992-1996, E-823/2001-running), etc

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matic fitting algorithms and lessons learned from KWFIT Padua 2000, Computing in High Energy and Nuclear Physics, p. 135

[3] BaBar KinFitter, BAD #1061

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views Supplement Series, Physics, 2)

[5] N. I. Starkov and V. A. Ryabov, Bull. Lebedev Phys. Inst. 2000N12, 1 (2000) [Kratk. Soobshch. Fiz. 2000N12, 3 (2000)].

[6] MXV4 C++ namespace freely available from: http://cern.ch/modima/MXV4.tgz - implements the following classes: 3D vec, 4D vek, 3D mtx, and 4D lmx. The acronyms are self-evident (lmx = Lorentz mtx).

[7] SLD Collaboration. Phys. Rev. Lett. 78, 3442, (1997), M. Dima, SLAC-R-0505, (1997).

[8] Historically the solution was reached “sliding” the position of the $B_{VTX}$ track along the $p_T$ track until the $B^0_d$ mass and IP-

pointback where simultaneously met - which also lifted the two-fold ambiguity. In the newer version of “Sliding VTX” which we are here presenting, the $B_{VTX}$ is determined separately, after the kinematic quantities are known. Ancient history has it that “Sliding VTX” started in the equations of the “$p_T$ corrected B-mass” of SLD, SLAC-PUB-7170, hence its initial “SLD-ing” approach.