Abstract

We argue that an exact gauge invariance may disable some generic features of the Standard Model which could otherwise manifest themselves at high energies. One of them might be related to the spontaneous Lorentz invariance violation (SLIV) which could provide an alternative dynamical approach to QED and Yang-Mills theories with photon and non-Abelian gauge fields appearing as massless Nambu-Goldstone bosons. To see some key features of the new physics expected we propose partial rather than exact gauge invariance in an extended SM framework. This principle applied, in some minimal form, to the weak hypercharge gauge field $B_\mu$ and its interactions leads to SLIV with $B$ field components appearing as the massless Nambu-Goldstone modes, and provides a number of distinctive Lorentz beaking effects. Being naturally suppressed at low energies they may become detectable in high energy physics and astrophysics. Some of the most interesting SLIV processes are considered in significant detail.
1 Introduction

It is now generally accepted that internal gauge symmetries form the basis of modern particle physics being most successfully realized within the celebrated Standard Model of quarks and leptons and their fundamental strong, weak and electromagnetic interactions.

On the other hand, postulated local gauge symmetries, unlike global symmetries, represent redundancies of the description of a theory rather than being “true” symmetries. Indeed, as has been discussed time and again (for some instructive example, see [1]), the very existence of an exact gauge invariance means that there are more field variables in the theory than are physically necessary. Usually, these superfluous degrees of freedom are eliminated by some gauge-fixing conditions which have no physical meaning in themselves and actually are put by hand. Instead, one could think that these extra variables would vary arbitrarily with the time so that they could be made to serve in description of some new physics.

One of possible ways for such new physics to appear may be linked to the idea that local symmetries and the associated masslessness of gauge bosons have in essence a dynamical origin rather than being due to a fundamental principle, as was widely contemplated over the last several decades [2, 3, 4]. By analogy with a dynamical origin of massless scalar particle excitations, which is very well understood in terms of spontaneously broken global internal symmetries, the origin of massless gauge fields as vector Nambu-Goldstone (NG) bosons could be related to the spontaneous violation of Lorentz invariance which is in fact the minimal spacetime global symmetry underlying the elementary particle physics. This approach providing a valuable alternative framework to quantum electrodynamics and Yang-Mills theories has gained new impetus [4] in recent years.

However, in contrast to the spontaneous internal symmetry violation which is readily formulated in gauge invariant theories, the spontaneous Lorentz invariance violation (SLIV) implies in general an explicit breakdown of gauge invariance in order to physically manifest itself. Indeed, the simplest model for SLIV is given by a conventional QED type Lagrangian extended by an arbitrary vector field potential energy terms

$$U(A) = \frac{\lambda}{4} \left( A_\mu A^\mu - n^2 M^2 \right)^2$$

which are obviously forbidden by a strict $U(1)$ gauge invariance of the starting Lagrangian. Here $n_\mu$ ($\mu = 0, 1, 2, 3$) is a properly-oriented unit Lorentz vector, $n^2 = n_\mu n^\mu = \pm 1$, while $\lambda$ and $M^2$ are, respectively, dimensionless and mass-squared dimensional positive parameters. This potential means that the vector field $A_\mu$ develops a constant background value $\langle A_\mu \rangle = n_\mu M$ and Lorentz symmetry $SO(1, 3)$ breaks at the scale $M$ down to $SO(3)$ or $SO(1, 2)$ depending on whether $n_\mu$ is time-like ($n_\mu^2 > 0$) or space-like ($n_\mu^2 < 0$). Expanding the vector field around this vacuum configuration,

$$A_\mu(x) = n_\mu (M + \phi) + a_\mu(x) , \quad n_\mu a^\mu = 0$$

1 Independently of the problem of the origin of local symmetries, Lorentz violation in itself has attracted considerable attention as an interesting phenomenological possibility that may be probed in direct Lorentz non-invariant, while gauge invariant, extensions of QED and Standard Model (SM) [5, 6, 7].

1
one finds that the $a_\mu$ field components, which are orthogonal to the Lorentz violating direction $n_\mu$, describe a massless vector Nambu-Goldstone boson, while the $\phi(x)$ field corresponds to a Higgs mode. This minimal polynomial extension of QED, being sometimes referred to as the “bumblebee” model, is in fact the prototype SLIV model intensively discussed in the literature (see [3] and references therein).

So, if one allows the vector field potential energy like terms (1) to be included into the properly modified QED Lagrangian, the time-like or space-like SLIV could unavoidably hold thus leading to photon as the massless NG boson in the symmetry broken SLIV phase. If this SLIV pattern is taken as some generic feature of QED the gauge principle should be properly weakened, otherwise this feature might be disabled. It is clear that this type of reasoning can be equally applied to any model possessing, among others, some local $U(1)$ symmetry which is properly broken by the corresponding gauge field terms in the Lagrangian. Remarkably, just the $U(1)$ local symmetry case with its gauge field (2) possessing one Higgs and three Goldstone components (being equal to number of broken Lorentz generators) appears to be optimally fitted for the physically valuable SLIV mechanism\(^2\). Actually, if things were arranged in this way, one could have indeed an extremely attractive dynamical alternative to conventional QED and/or Standard Model that could be considered in itself as some serious motivation for a status of an overall gauge symmetry in them to be properly revised.

In this connection, we propose partial rather than exact gauge invariance in the Standard Model according to which, while the electroweak theory is basically $SU(2) \times U(1)_Y$ gauge invariant being constructed from ordinary covariant derivatives of all fields involved, the $U(1)_Y$ hypercharge gauge field $B_\mu$ field is allowed to form all possible polynomial couplings on its own and with other fields invariants. So, the new terms in the SM Lagrangian, conditioned by the partial gauge invariance, may generally have a form

$$- U(B) + B_\mu \Upsilon^\mu(f, h, g) + B_\mu B_\nu \Theta^{\mu\nu}(f, h, g) + \cdots$$

where $U(B)$ contains all possible $B$ field potential energy terms, the second term in (3) consists of all vector type couplings with the SM fields involved (including left-handed and right-handed fermions $f$, Higgs field $h$ and gauge fields $g$), the third term concerns possible tensor like couplings, and so on. These new terms (with all kinds of the $SU(3)_c \times SU(2) \times U(1)_Y$ gauge invariant tensors $\Upsilon^\mu$, $\Theta^{\mu\nu}$ etc.) "feel” only $B$ field gauge transformations while remaining invariant under gauge transformations of all other fields. Ultimately, just their sensitivity to the $B$ field gauge transformations leads to physical Lorentz violation in SM. Indeed, the constant part of the vector field SLIV pattern (2) can be treated in itself as some gauge transformation with gauge function linear in coordinates, $\omega(x) = (n_\mu x^\mu) M$, and therefore, this violation may physically emerge only through the terms like those in (3) which only possess partial gauge invariance (PGI).

The paper is organized as follows. In section 2 we make some natural simplification of a general PGI conjecture given above in (3) and find an appropriate minimal form for PGI. We exclude the accompanying SLIV Higgs component in the theory going to the nonlinear

\(^2\)Note in this connection that SLIV through the condensation of non-Abelian vector fields would lead to a spontaneous breakdown of internal symmetry as well (see some discussion in [9]) that could make our consideration much more complicated.
realization of Lorentz symmetry and propose that all possible vector and tensor couplings in the PGI expansion (3) are solely determined by the SM Noether currents. When only vector couplings are taken in (3) this leads to the simple nonlinear Standard Model (NSM) which is considered in detail in the next section 3, and its physical Lorentz invariance and observational equivalence to the conventional SM is explicitly demonstrated. In section 4 we will mainly be focused on the extended NSM (ENSM) with the higher dimensional tensor coupling terms included. By contrast, they lead to the physical SLIV with a variety distinctive Lorentz breaking effects in a laboratory some of which are considered in detail. And, finally, in section 5 we conclude.

2 Partial Gauge Invariance Simplified

Generally, the PGI conjecture, as formulated in (3), may admit too many extra terms in the SM Lagrangian. However, one can have somewhat more practical choice for PGI. This is related to the way SLIV is realized in SM and a special role which two SM Noether currents, namely, the total hypercharge current and the total energy-momentum tensor of all fields involved may play in formulation of the PGI conjecture (3). We give below a brief discussion of each of terms in (3) and try to make some possible simplifications.

2.1 Nonlinear Lorentz realization

The first thing of interest in (3) is the potential energy terms $U(B)$ for the SM hypercharge gauge field $B_\mu$, which are like those we had in (1) for QED. Though generally just these terms cause a spontaneous Lorentz violation, their physical effects are turned out to be practically insignificant unless one considers some special SLIV interplay with gravity [8, 10] at the super-small distances, or a possible generation of the SLIV topological defects in the very early universe [11]. Actually, as in the pure SLIV QED case [12], one has an ordinary Lorentz invariant low energy physics in an effective SLIV SM theory framework. The only Lorentz breaking effects may arise from radiative corrections due to the essentially decoupled superheavy (with the SLIV scale order mass) Higgs component contributions, which are generally expected to be negligibly small at lower energies.

For more clearness and simplicity, we completely exclude this vector field Higgs component in the theory going to the nonlinear realization of Lorentz symmetry through the nonlinear $\sigma$-model for the hypercharge gauge field $B_\mu$, just as it takes place in the original nonlinear $\sigma$-model [15] for pions. Actually, for the pure QED case this has been done by Nambu long ago [13] (see also [14] for some recent discussion). Doing so in the SM framework, particularly in its hypercharge sector, one immediately comes to the $B$ field

\footnote{Notice that these currents are proposed to be used in the form they have in an ordinary rather than extended Standard Model (for further discussion, see subsection 2.3).}

\footnote{This correspondence with the nonlinear $\sigma$ model for pions may be somewhat suggestive, in view of the fact that pions are the only presently known NG bosons and their theory, chiral dynamics [15], is given by the nonlinearly realized chiral $SU(2) \times SU(2)$ symmetry rather than by an ordinary linear $\sigma$ model.}
constraint  
\[ B_\mu^2 = n^2 M^2. \]  
This constraint provides in fact the genuine Goldstonic nature of the hypercharge gauge field appearing at the SLIV scale \( M \), as could easily be seen from an appropriate \( B \) field parametrization,  
\[ B_\mu = b_\mu + \frac{n_\mu}{n^2}(M^2 - n^2 b_\mu^2)^{1/2}, \quad n_\mu b_\mu = 0 \]  
with the pure NG modes \( b_\mu \) and an effective Higgs mode (or the \( B \) field component in the vacuum direction) being given by the square root in (4). Indeed, both of these SLIV patterns in the SM framework, linear and nonlinear, are equivalent in the infrared energy domain, where the Higgs mode is considered to be infinitely massive. We consider for what follows just the nonlinear SM (or NSM, as we call it hereafter) where SLIV is related to an explicit nonlinear constraint put on the hypercharge gauge field (4) rather than to a presence of its potential energy terms in the SM Lagrangian. We show later in section 3 that this theory with the corresponding Lagrangian \( \mathcal{L}_{NSM} \) written in the pure NG modes \( b_\mu \) (5) is physically equivalent to an ordinary SM theory.

### 2.2 The minimal PGI

Further, we propose for the second term in the PGI extension of SM (3) that it is solely given by \( B \) field dimensionless couplings with the total hypercharge current \( J^\mu(f, h, g) \) of all matter fields involved. Namely, \( T^\mu \) in (3) is replaced by \( g' J^\mu \), where \( g' \) is some coupling constant. However, the inclusion of these couplings into the NSM Lagrangian \( \mathcal{L}_{NSM} \) would only redefine the original hypercharge gauge coupling constant \( g' \) which is in essence a free parameter in SM. This means that for the basic theory with dimensionless coupling constants the partial gauge invariance is really indistinguishable from an ordinary gauge invariance due to which SLIV can be gauged away in the basic NSM, as we shall see in the next section. Otherwise, the large Lorentz breaking effects would make the whole model absolutely irrelevant. From the above reasoning the second term in (3) will be simply omitted in the subsequent discussion.

Meanwhile, a clear signal of the physical Lorentz violation inevitably occurs when one goes beyond the minimal theory to also activate the higher dimensional tensor couplings in (3) that leads to the extended nonlinear SM (or ENSM). For further simplicity, these couplings are proposed to be determined solely by the total energy-momentum tensor \( T^{\mu\nu} \) of all fields involved, namely, \( \Theta^{\mu\nu} = (\alpha/M_P^2) T^{\mu\nu} \). So, the lowest order ENSM which conforms with the chiral nature of SM and all accompanying global and discrete symmetries, is turned out to include the dimension-6 couplings of the type

\[ \mathcal{L}_{ENS} = \mathcal{L}_{NSM} + \frac{\alpha}{M_P^2} B_\mu B_\nu T^{\mu\nu}(f, g, h) \]  

describing at the Planck scale \( M_P \) the extra interactions of the hypercharge gauge fields with the energy-momentum tensor bilinears of matter fermions, and gauge and Higgs bosons.\(^5\)

---

\(^5\)Actually, as in the pion model, one can go from the linear model for SLIV to the nonlinear one taking the corresponding potential, similar to the potential (4), to the limit \( \lambda \to \infty \).
(with the dimensionless coupling constant $\alpha$ indicated), respectively. The $T^{\mu\nu}$ tensor in (6) is proposed to be symmetrical (in spacetime indices) and $SU(3)_c \times SU(2) \times U(1)_Y$ gauge invariant according to our basic conjecture (3). So, the physical Lorentz violation, in a form that follows from this minimal PGI determined by the SM Noether currents, appears to be naturally suppressed thus being in a reasonable compliance with current experimental bounds. Nonetheless, as we show in section 4, the extra couplings in (6) may lead, basically through the deformed dispersion relations of all matter and gauge fields involved, to a new class of processes which could still be of a distinctive observational interest in high energy physics and astrophysics. As to the higher dimensional couplings in (3), we assume that they are properly suppressed or even forbidden if one takes a minimal choice for PGI (6) to which we follow here.

### 2.3 How the minimal PGI works

Now, one can readily see that the SM hypercharge current $J^\mu$ and energy-momentum tensor $T^{\mu\nu}$ used above as the only building blocks for the simplified version of ENSM (6) may really determine some minimal gauge symmetry breaking mechanism in the theory. Indeed, they are changed when SM is modified by the tensor type couplings so that one has new conserved Noether currents $J^\mu'$ and $T^{\mu\nu'}$ in ENSM

\[
J^\mu' = J^\mu + \kappa B^\mu B^\rho J^\rho , \quad (7)
\]
\[
T^{\mu\nu'} = T^{\mu\nu} + \kappa \left( B^\mu B^\rho T^{\rho\nu} + B^\nu B^\rho T^{\rho\mu} \right) , \quad (8)
\]

(where $\kappa$ stands for $\alpha / M^2$), while the old currents $J^\mu$ and $T^{\mu\nu}$ participating in the couplings (1) are only approximately conserved, $\partial_\mu J^\mu = O(\kappa)$ and $\partial_\mu T^{\mu\nu} = O(\kappa)$. Nonetheless, this appears enough to have, in turn, the "almost" gauge invariant field equations in ENSM. Indeed, the Lagrangian density (6) varies to the taken accuracy into some total derivative

\[
\delta \mathcal{L}_{ENSM} = \kappa \partial_\mu (R_\nu T^{\mu\nu}) + O(\kappa^2) \quad (9)
\]

where $R_\nu$ stands for the properly defined integral function

\[
R_\nu(x) = \int^x dx^\rho (B^\mu \partial_\rho \omega + B_\rho \partial_\rho \omega + \partial_\rho \omega \partial_\rho \omega) \quad (10)
\]

conditioned by the corresponding $B$ field gauge transformations

\[
B_\mu \rightarrow B_\mu + \partial_\mu \omega(x) . \quad (11)
\]

The gauge function $\omega(x)$ is an arbitrary function, only being restricted by the requirement to conform with the $B$ field constraint (4)

\[
(B_\mu + \partial_\mu \omega)(B^\mu + \partial^\mu \omega) = n^2 M^2 , \quad (12)
\]

due to which the introduced integral function $R_\nu$ (10) has to be divergenceless

\[
\partial^\nu R_\nu = B_\mu^2 - n^2 M^2 = 0 . \quad (13)
\]
Should a solution to the constraint equation (11) exist for some class of finite gauge functions \( \omega(x) \) the reverse would also be true: requiring the above approximate gauge invariance (8) of the Lagrangian under transformations (10) one comes to the minimal ENSM (6). The actual physical equivalence of NSM determined by the SLIV constraint (4) to an ordinary SM theory, that is explicitly demonstrated in section 3, shows that such gauge function may really exist. So, the partial gauge invariance in a minimal form taken above tends to appear reasonably well defined at least at the classical level.

2.4 The metric expansion viewpoint

It is conceivable, on the other hand, that the extra interaction terms in \( \mathcal{L}_{ENSM} \) might arise as remnants of some operator expansion of the metric tensor \( g_{\mu\nu}(x) \) into all possible tensor-valued covariants which could generally appear in quantum gravity. For metric correlated with the total energy-momentum tensor \( T^{\mu\nu}(f, g, h) \) of SM (or NSM in the nonlinear Lorentz realization case) this expansion

\[
g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_P + \alpha B_\mu B_\nu/M_P^2 + \beta W_\mu W_\nu/M_P^2 + \cdots
\]

may include, along with the Minkowski metric tensor \( \eta_{\mu\nu} \) and graviton field \( h_{\mu\nu} \), as is usually taken in a weak gravity approximation, the SM gauge and matter field covariants as well (\( \alpha, \beta, ... \) are coupling constants). As a result, once SLIV occurs with \( B \) field developing a constant background value (5) the conventional SM interactions appear to be significantly modified at small distances presumably controlled by quantum gravity.

2.5 Running to low energies

Now let us concretize the form of the minimal ENSM theory given above (6). First of all, note that the Lagrangian containing part \( -\eta^{\mu\nu}\mathcal{L}_{NSM} \) in the total energy-momentum tensor \( T^{\mu\nu}(f, g, h) \) appears unessential since it only leads to a proper redefinition of all fields involved regardless their properties under SM. Actually, the contraction of this part with the shifted hypercharge gauge field \( B_\mu \) in (5) gives in the lowest order the universal factor

\[
1 - \alpha M^2 n^2 M_P^2
\]

to the whole nonlinear SM Lagrangian \( \mathcal{L}_{NSM} \) considered. So, we will consider only "the Lagrangian subtracted" energy-momentum tensor \( T^{\mu\nu} \) in what follows (leaving the former notation for it).

The more significant point concerns the running of the coupling constant \( \alpha \) for the basic extra interaction of ENSM (6). It is clear that even if one starts with one universal constant at the Planck scale \( M_P \), it will appear rather different for matter fermions (\( \alpha_f \)), gauge fields (\( \alpha_g \)) and Higgs boson (\( \alpha_h \)) being appropriately renormalized when running down to lower energies. Moreover, each of these constants is further split for different fermion

---

6This confirms that the constraint (4) in itself may well only be some particular gauge choice in SM to which just NSM corresponds (for more discussion, see section 3).
and gauge multiplets in SM that is determined, in turn, by the corresponding radiative corrections. For example, one could admit that quarks and leptons have equal $\alpha$-coupling ($\alpha_f$) in the Planck scale limit. However, due to radiative corrections this coupling constant may split into two ones - one for quarks ($\alpha_q$) and another for leptons ($\alpha_l$), respectively, that could be in principle calculated. Apart from that, there appear two more coupling constants, namely, those for left-handed quarks and leptons ($\alpha_{ql}$, $\alpha_{ll}$) and right-handed ones ($\alpha_{qr}$, $\alpha_{lr}$). We will take into account some difference between $\alpha$-couplings of quarks and leptons but will ignore such a difference for left-handed and right-handed fermions of the same species. Indeed, the associated radiative corrections, which basically appear due to the chirality-dependent weak interactions in SM, are expected to be relatively small. So, practically there are only four effective coupling constants at normal laboratory energies, $\alpha_f$ ($f = q, l$), $\alpha_g$ and $\alpha_h$, in the theory with one quark-lepton family. In other words, the total energy-momentum tensor $T^{\mu\nu}(f, g, h)$ in the basic ENSM coupling (6) breaks into the sum

$$T^{\mu\nu}(f, g, h) = \frac{\alpha_f}{\alpha} T^{\mu\nu}_f + \frac{\alpha_g}{\alpha} T^{\mu\nu}_g + \frac{\alpha_h}{\alpha} T^{\mu\nu}_h$$  

(15)

of the energy-momentum tensors of matter fermions, and gauge and Higgs bosons, respectively, when going from the Planck scale down to low energies. However, the different fermion quark-lepton families may still have rather different $\alpha$-couplings that could eventually lead to the flavor-changing processes in our model (some interesting examples are discussed in section 4).

3 Nonlinear Standard Model

In contrast to the spontaneous violation of internal symmetries, SLIV seems not to necessarily imply a physical breakdown of Lorentz invariance. Rather, when appearing in a minimal gauge theory framework, this may eventually result in a noncovariant gauge choice in an otherwise gauge invariant and Lorentz invariant theory. This is what just happens in a simple class of QED type models [13, 14] having from the outset a gauge invariant form, in which SLIV is realized through the "length-fixing" field constraint of the type (4) rather than due to some vector field potential energy terms. Remarkably, this type of model makes the vector Goldstone boson a true gauge boson (photon), whereas the physical Lorentz invariance is left intact. Indeed, despite an evident similarity with the nonlinear $\sigma$-model for pions, the nonlinear QED theory ensures that all the physical Lorentz violating effects prove to be non-observable. Particularly, it was shown, first only in the tree approximation [13], that the nonlinear constraint (4) implemented as a supplementary condition into the standard QED Lagrangian appears in fact as a possible gauge choice for the vector field $A_\mu$. At the same time the $S$-matrix remains unaltered under such a gauge convention. Really, this nonlinear QED contains a plethora of Lorentz and $CPT$ violating couplings when it is expressed in terms of the pure NG photon modes according to the constraint.

---

7Note that, apart from the proposed generic Planck scale unification of the PGI couplings in ENSM (6) there could be some intermediate grand unification [16] and/or family unification (for some example, see [17]) in the theory. These extra symmetries will also influence their running down to low energies.
condition being similar to (4). However, the contributions of these Lorentz violating couplings to physical processes completely cancel out among themselves. So, SLIV was shown to be superficial as it affects only the gauge of the vector potential $A_{\mu}$, at least in the tree approximation [13].

Some time ago, this result was extended to the one-loop approximation [14]. It was shown that the constraint like (4), having been treated as a nonlinear gauge choice for the $A_{\mu}$ field at the tree (classical) level, remains as a gauge condition when quantum effects in terms of the loop diagrams are taken into account as well. So, one can conclude that physical Lorentz invariance is left intact in the one-loop approximation in the nonlinear QED taken in the flat Minkowski spacetime.

We consider here in this section the nonlinear Standard Model (or NSM) treated merely as SM with the nonlinear constraint (4) put on the hypercharge gauge field $B_{\mu}$. We show that NSM despite many generic complications involved (like as the spontaneous breaking of the internal $SU(2) \times U(1)_Y$ symmetry, the diverse particle spectrum, mixings in gauge and matter sectors, extension by the PGI vector couplings etc.) appears observationally equivalent to the ordinary SM, just like what happens in the above mentioned nonlinear QED case. Actually, due to the SLIV constraint (4), physical $B$ field components convert into the massless NG modes which, after an ordinary electroweak symmetry breaking, mix with a neutral $W^3$ boson of $SU(2)$ leading, as usual, to the massless photon and massive $Z$ boson. When expressed in terms of the pure NG modes NSM, like the nonlinear QED, contains a variety of Lorentz and CPT violating couplings. Nonetheless, all SLIV effects turn out to be strictly cancelled in all lowest order processes some of which are considered in detail below.

### 3.1 Hypercharge vector Goldstone boson

We start with the Standard Model where, for simplicity, we restrict ourselves to the electron family only

$$L = \left( \begin{array}{c} \nu_e \\ e \end{array} \right)_{L} , \ e_{R}$$

that can be then straightforwardly extended to all matter fermions observed. For the starting hypercharge gauge field $B_{\mu}$ expressed in terms of its Goldstone counterpart $b_{\mu}$ (5) one has in the leading order in the inverse SLIV scale $1/M$

$$B_{\mu} = b_{\mu} + \frac{n_{\mu}}{n^2} M - \frac{b_{\nu} b_{\mu}}{2M} n_{\nu}$$

so that in the same order the hypercharge field stress-tensor $B_{\mu\nu}$ amounts to

$$B_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu} = b_{\mu\nu} - \frac{1}{2M} (n_{\nu} \partial_{\mu} - n_{\mu} \partial_{\nu}) (b_{\rho})^2 .$$

---

8Note that the extension of NSM by the extra vector couplings in (4) which according to the minimal PGI (section 2) are solely determined by the total hypercharge current are simply absorbed in NSM only leading to the redifinition of the hypercharge coupling constant.
When one also introduce the NG modes $b_\mu$ in the hypercharge covariant derivatives for all matter fields involved one eventually comes to the essentially nonlinear $b$ field theory sector in the Standard Model due to which it is now called the nonlinear SM or NSM.

This model might seem unacceptable since it contains, among other terms, the inappropriately large (the SLIV scale $M$ order) Lorentz violating fermion and Higgs fields bilinears which appear when the starting $B$ field expansion [17] is applied to the corresponding couplings in SM. However, due to partial gauge invariance, according to which all matter fields remain to possess the covariant derivatives, these bilinears can be gauged away by making an appropriate field redefinition according to

\[(L, e_R, H) \longrightarrow (L, e_R, H) \exp(i \frac{Y_{L,R,H}}{2} g' n^\mu M(n_\mu \epsilon^\mu))\]  

(19)

So, one eventually comes to the nonlinear SM Lagrangian

\[\mathcal{L}_{NSM} = \mathcal{L}_{SM}(B_\mu \rightarrow b_\mu) + \mathcal{L}_{nSM}\]  

(20)

where the conventional SM part being expressed in terms of the the hypercharge NG vector boson $b_\mu$ is presented in $\mathcal{L}_{SM}(B_\mu \rightarrow b_\mu)$, while its essentially nonlinear couplings are collected in $\mathcal{L}_{nSM}$ written in the taken order $O(1/M)$ as

\[2M\mathcal{L}_{nSM} = -(n_\mu \partial_\mu (b_\mu^2) + \frac{1}{2} g' b_\nu b_\nu \gamma^\mu n_\mu L + g' b_\nu \gamma^\mu n_\mu e_R - i \frac{g'}{2} [H^+(n_\mu \partial_\mu H) - (n_\mu \partial_\mu H^+)H]\]  

(21)

Note that the SLIV conditioned ”gauge” $n_\mu b^\mu = 0$ [5] for the $b$-field is imposed everywhere in the Lagrangian $\mathcal{L}_{NSM}$. Moreover, we take the similar axial gauge for $W^i$ bosons of $SU(2)$ so as to have together

\[n_\mu W^{i\mu} = 0 , \quad n_\mu b^\mu = 0 .\]  

(22)

in what follows. As a result, all terms containing contraction of the unit vector $n_\mu$ with electroweak boson fields will vanish in the $\mathcal{L}_{NSM}$.

We see later that NSM, despite the presence of particular Lorentz and CPT violating couplings in its essentially nonlinear part [21], does not lead in itself to the physical Lorentz violation until the extra PGI couplings appearing in the ENSM Lagrangian [6] start working.

3.2 Electroweak symmetry breaking in NSM

At much lower energies than the SLIV scale $M$ a conventional spontaneous breaking of the internal symmetry $SU(2) \times U(1)_Y$ naturally holds in NSM. This appears when the Higgs field $H$ acquires the constant background value through its potential energy terms

\[U(H) = \mu_H^2 H^+ H + (\lambda/2)(H^+ H)^2 , \quad \mu_H^2 < 0\]  

(23)
in the electroweak Lagrangian. Due to the overall axial gauge adopted \(^{(22)}\) there is no more a gauge freedom \(^9\) in NSM to exclude extra components in the \(H\) doublet. So, one can parametrize it in the following general form

\[
H = \frac{1}{\sqrt{2}} \left( \frac{\phi}{(h + V)} e^{i\xi/V} \right), \quad V = (-\mu_H^2/\lambda)^{1/2} \tag{24}
\]

The would-be scalar Goldstone bosons, given by the real \(\xi\) and complex \(\phi(\phi^*)\) fields, mix generally with \(Z\) boson and \(W(W^*)\) boson components, respectively. To see these mixings one has to write all bilinear terms stemming from the starting Higgs doublet Lagrangian which consists of its covariantized kinetic term \(|D_{\mu}H|^2\) and the potential energy part \(^{(23)}\). Putting them all together one comes to

\[
(\partial^{\mu}h)^2/2 + \mu_h^2h^2/2 + |M_WW_{\mu} - i\partial_{\mu}\phi|^2 + (M_ZZ_{\mu} + \partial_{\mu}\xi)^2/2 \tag{25}
\]

where we have used the usual expression for Higgs boson mass \(\mu_h^2 = \lambda |\mu_H^2|\), and also the conventional expressions for \(W\) and \(Z\) bosons

\[
(W_{\mu}, W^*_{\mu}) = (W^1_{\mu} \pm iW^2_{\mu})/\sqrt{2}, \quad Z_{\mu} = \cos \theta W^3_{\mu} - \sin \theta b_{\mu}, \quad \tan \theta = g'/g \tag{26}
\]

(\(\theta\) stands for electroweak mixing angle). They acquire the masses, \(M_W = gV/2\) and \(M_Z = gV/2 \cos \theta\), while an orthogonal superposition of \(W^3_{\mu}\) and \(b_{\mu}\) fields, corresponding to the electromagnetic field

\[
A_{\mu} = \cos \theta b_{\mu} + \sin \theta W^3_{\mu} \tag{27}
\]

remains massless, as usual. Then to separate the states in \(^{(26)}\) one needs to properly shift the \(\xi\) and \(\phi\) modes. Actually, rewriting the mixing terms in \(^{(25)}\) in the momentum space and diagonalizing them by the substitutions \(^{[18]}\)

\[
\phi(k) \rightarrow \phi(k) + M_W \frac{k_{\nu}W^\nu(k)}{k^2}, \quad \xi(k) \rightarrow \xi(k) - iM_Z \frac{k_{\nu}Z^\nu(k)}{k^2} \tag{28}
\]

one has some transversal bilinear forms for \(W\) and \(Z\) bosons and the new \(\phi(k)\) and \(\xi(k)\) states

\[
\left| -k_{\mu}\phi(k) + M_W \left( g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2} \right) W^\nu(k) \right|^2 + \frac{1}{2} \left[ -ik_{\mu}\xi(k) + M_Z \left( g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2} \right) Z^\nu(k) \right]^2 \tag{29}
\]

to be separated. As a result, the NSM Lagrangian with the gauge fixing conditions \(^{(22)}\) included determines eventually the propagators for massless photon and massive \(W\) and \(Z\)

\(^{9}\)This kind of SM with all gauge bosons taken in the axial gauge was earlier studied \(^{[18]}\) in an ordinary Lorentz invariant framework. Also, the SLIV conditioned axially gauged vector fields in the spontaneously broken massive QED was considered in \(^{[14]}\).
bosons in the form

\[ D_{\mu\nu}^{(\gamma)}(k) = \frac{-i}{k^2 + i\epsilon} \left( g_{\mu\nu} - \frac{n_{\mu}k_{\nu} + k_{\mu}n_{\nu}}{(nk)} + \frac{n^2 k_{\mu}k_{\nu}}{(nk)^2} \right), \]

\[ D_{\mu\nu}^{(W,Z)}(k) = \frac{-i}{k^2 - M^2_{W,Z} + i\epsilon} \left( g_{\mu\nu} - \frac{n_{\mu}k_{\nu} + k_{\mu}n_{\nu}}{(nk)} + \frac{n^2 k_{\mu}k_{\nu}}{(nk)^2} \right). \]

(where \((nk)\) stands, as usual, for a contraction \(n_{\mu}k^{\mu}\)). Meanwhile, propagators for massless scalar fields \(\phi\) and \(\xi\) amount to

\[ D^{(\phi)}(k) = \frac{i}{k^2}, \quad D^{(\xi)}(k) = \frac{i}{k^2}. \]

These fields correspond to unphysical particles in a sense that they could not appear as incoming or outgoing lines in Feynman graphs. On the other hand, they have some virtual interactions with Higgs boson \(h\), and \(W\) and \(Z\) bosons that will be taken into account when considering the corresponding processes (see below).

Apart from the bilinear terms (25), some new field bilinears appear from the \(n\)-oriented Higgs field covariant derivative term \(|n^\lambda D_\lambda H|^2\) (see below Eq. (33)) when the this NSM is further extended to ENSM (6). They amount to

\[ \delta_h [(n_{\mu}\partial^\mu h)^2 + |n_{\mu}\partial^\mu\phi|^2 + (n_{\mu}\partial^\mu\xi)^2] \]

where \(\delta_h = \alpha_h (M^2 / M_P^2)\). Inclusion of the last two terms in the procedure of the \(\phi - W\) and \(\xi - Z\) separation discussed above will change a little the form of their propagators (30, 31). We do not consider this insignificant change here.

### 3.3 SLIV interactions in NSM

#### 3.3.1 The gauge interactions

The Goldstone \(b\)-field interactions are given by the Lagrangian \(L_{NSM}\) (20) and particularly by its pure nonlinear part \(L_{nSM}\) (21) which includes in the leading order in \(1/M\) the trilinear self-interaction term of the new hypercharge vector field \(b_\mu = \cos \theta A_\mu - \sin \theta Z_\mu\) and, besides, the quadrilinear couplings of this field with left-handed and right-handed fermions, and Higgs boson. All of them have Lorentz noncovariant (preferably oriented) form and, furthermore, they violate \(CPT\) invariance as well. For the Higgs boson part in \(L_{nSM}\) one has in the leading order in \(1/M\) using the parametrization (24)

\[ L_{nSM}(H) = \frac{1}{2M} g' (b_\mu)^2 \left[ (h + V) (n_{\mu}\partial^\mu)\xi - \frac{i}{2} [\phi^*(n_{\mu}\partial^\mu)\phi - \phi(n_{\mu}\partial^\mu)\phi^*] \right]. \]

so that the quadrilinear interactions of \(b_\mu\) field with the would-be Goldstone bosons \(\xi\) and \(\phi(\phi^*)\) inevitably emerge. For the properly separated \(\phi - W\) and \(\xi - Z\) states, which is reached by the replacements (28), there appear trilinear and quadrilinear couplings between all particles involved in the Higgs sector (photon, \(W\), \(Z\), Higgs bosons, and \(\phi\) and \(\xi\) fields) as directly follows from the Lagrangian (33) taken in the momentum space after corresponding substitutions of (28) and \(b_\mu = \cos \theta A_\mu - \sin \theta Z_\mu\), respectively.
3.3.2 Yukawa sector

Now let us turn to the Yukawa sector whose Lagrangian is

\[
\mathcal{L}_{\text{Yuk}} = -G \left[ \bar{L} H e_R + \bar{e}_R H^+ L \right] = -G \sqrt{2} \left[ (h + V) \bar{e}e + i \xi \bar{e} \gamma^5 e + \bar{e}_R \Phi^* \nu_1 + \bar{\nu}_1 \Phi e_R \right]
\]  (34)

Due to the \( \xi \) field redefinition (28) there appears one extra (Yukawa type) \( Z \) boson coupling, which in the momentum space has the form

\[
\mathcal{L}_{\text{Yuk}}(Zee) = -\frac{G}{\sqrt{2}} M_Z \frac{k_\nu Z^\nu (k)}{k^2} \bar{e} \gamma^5 e = -\frac{g}{2 \cos \theta} m_e \frac{k_\nu Z^\nu (k)}{k^2} \bar{e} \gamma^5 e
\]  (35)

The similar extra coupling appears for the charged \( W \) boson as well when it is separated from the \( \phi \) field due to the replacement (28).

3.4 Lorentz preserving SLIV processes

We show now by a direct calculation of some tree level amplitudes that the physical Lorentz invariance being intact in the massless nonlinear QED [13, 14] is still survived in the nonlinear SM. Specifically, we will calculate matrix elements of two SLIV processes naturally emerging in NSM. One of them is the elastic photon-electron scattering and another is the elastic \( Z \) boson scattering on an electron.

3.4.1 Photon-electron scattering

This process in lowest order is concerned with four diagrams one of which is given by the direct contact photon-photon-fermion-fermion vertex generated by the \( b^2 \)-fermion-fermion coupling in (21), while three others are pole diagrams where the scattered photon and fermion exchange a virtual photon, \( Z \) boson and \( \xi \) field, respectively. Their vertices are given, apart from the standard gauge boson-fermion couplings in \( \mathcal{L}_{\text{SM}}(B_\mu \to b_\mu) \) (20), by the SLIV \( b^3 \) and \( b^2 \)-fermion couplings in (21) and by the \( b^2 \)-\( \xi \) coupling in (33), and also by Yukawa couplings (34, 35).

So, one has first directly from the \( b^2 \)-fermion coupling the matrix element corresponding to the contact diagram

\[
\mathcal{M}_c = i \frac{3g}{4M} \sin \theta \cos \theta (\epsilon_1 \epsilon_2) \bar{\nu}_2 \gamma^\rho n_\rho (1 + \gamma^5) u_1
\]  (36)

when expressing it through the weak isotopic constant \( g \) and Weinberg angle \( \theta \) (where \( \epsilon_1 \epsilon_2 \) stands for a scalar product of photon polarization vectors \( \epsilon_{1\mu} \) and \( \epsilon_{2\mu} \)).

Using then the vertex for the ordinary SM photon-electron coupling,

\[
- g \sin \theta \gamma^\mu
\]  (37)

together with vertex corresponding to the SLIV three-photon coupling,
\[- \frac{i}{M} \cos^3 \theta [(nq)q_\nu g_{\lambda\rho} + (nk_1)k_{1\lambda}g_{\nu\rho} + (nk_2)k_{2\rho}g_{\nu\lambda}] \] (38)

(where \(k_{1,2}\) are ingoing and outgoing photon 4-momenta and \(q = k_2 - k_1\), while \((nk_{1,2})\) and \((nq)\) are their contractions with the unit vector \(n\)) and photon propagator (30), one comes to the matrix element for the first pole diagram with the photon exchange

\[ M_{p1} = -i \frac{g}{M} \cos^3 \theta \sin \theta (\epsilon_1 \epsilon_2) \bar{u}_2 \gamma^\mu n_\mu u_1 \] (39)

Analogously, combining the joint vertex for the Lorentz invariant \(Z\) boson-fermion couplings which include both an ordinary SM coupling and extra Yukawa coupling (35) appearing due to a general parametrization (24),

\[ i \frac{g}{2 \cos \theta} \left[ \frac{1}{2} \gamma^\mu (3 \sin^2 \theta - \cos^2 \theta + \gamma^5) - m_e \gamma^5 \frac{q_\mu}{q^2} \right] \] (40)

with the vertex for the SLIV photon-photon-\(Z\) boson coupling,

\[ i \frac{\cos^2 \theta \sin \theta}{M} \left[ (1 - \frac{M^2}{q^2})(nq)q_\nu g_{\lambda\rho} + (nk_1)k_{1\lambda}g_{\nu\rho} + (nk_2)k_{2\rho}g_{\nu\lambda} \right], \] (41)

one finds the matrix element corresponding to the second pole diagram with the \(Z\)-boson exchange

\[ M_{p2} = -i \frac{g}{2M} \sin \theta \cos(\epsilon_1 \epsilon_2) \bar{u}_2 [\gamma^\mu n_\mu (1 - 2 \cos 2\theta + \gamma^5) / 2 + \gamma^5 (nq) m_e / q^2] u_1 \] (42)

where was also properly used Dirac equation for on-shell fermions and \(Z\)-boson propagator (30).

And lastly, the third pole diagram with the \(\xi\) field exchange include two vertices, the first corresponds to Yukawa \(\xi ee\) coupling (34),

\[ \frac{g}{2 \cos \theta} \frac{m_e}{M} \gamma^5 \] (43)

while the second to the SLIV \(\xi\)-photon-photon one (33)

\[ M_Z \cos^2 \theta \sin \theta (\epsilon_1 \epsilon_2) (nq) \] (44)

that leads, using the \(\xi\) field propagator (31), to the matrix element

\[ M_{p3} = i \frac{g}{2M} \frac{m_e}{q^2} \sin \theta \cos(\epsilon_1 \epsilon_2) (nq) \bar{u}_2 \gamma^5 u_1 \] (45)

Putting together all these contributions one can readily see that the total SLIV induced matrix element for the Compton scattering taken in the lowest order precisely vanishes,

\[ M_{SLIV}(\gamma + e \rightarrow \gamma + e) = M_c + M_{p1} + M_{p2} + M_{p3} = 0. \] (46)
3.4.2 Z boson scattering on electron

For this process there are similar four diagrams - one is the $Z$-$Z$-fermion-fermion contact diagram and three others are pole diagrams where the scattered $Z$ boson and fermion exchange a virtual photon, $Z$ boson and $\xi$ field, respectively. Their vertices are also given by the corresponding couplings in the nonlinear SM Lagrangian terms \([20, 21, 33, 34, 35]\). One can readily find that the matrix elements for the contact and pole diagrams differ from the similar diagrams in the photon scattering case only by the Weinberg angle factor

\[
M'_c = \tan^2 \theta M_c, \quad M'_{pi} = \tan^2 \theta M_{pi} \quad (i = 1, 2, 3)
\]

so that we have the vanished total matrix element in this case as well

\[
M_{SLIV}(Z + e \rightarrow Z + e) = M'_c + M'_{p1} + M'_{p2} + M'_{p3} = 0.
\]

3.4.3 Other processes

In the next order $O(1/M^2)$ some new SLIV processes, such as photon-photon, $Z$-$Z$, photon-$Z$ boson scatterings, also appear in the tree approximation. Their amplitudes are related, as in the above, to photon, $Z$ boson and $\xi$ field exchange diagrams and the contact $b^4$ interaction diagrams following from the higher terms in $M^2$ in the Lagrangian \([21]\). Again, all these four diagrams are exactly cancelled giving no the physical Lorentz violating contributions. Actually, this argumentation can be readily extended to the SLIV processes taken in any tree-level order in $1/M$.

Most likely, a similar conclusion can be derived for SLIV loop contributions as well. Actually, as in the massless QED case considered earlier \([13]\), the corresponding one-loop matrix elements in NSM may either vanish by themselves or amount to the differences between pairs of the similar integrals whose integration variables are shifted relative to each other by some constants (being in general arbitrary functions of external 4-momenta of the particles involved) that in the framework of dimensional regularization leads to their total cancellation. So, NSM not only classically but also at quantum level appears to be physically indistinguishable from a conventional SM. This, in turn, means that the SLIV condition \([4]\) taken in the Standard Model is merely reduced to a possible gauge choice for the hypercharge gauge field $B_\mu$, while the $S$-matrix remains unaltered under such a gauge convention.

4 Extended Nonlinear Standard Model

We now turn to the extended NSM (or ENSM) with the higher-dimensional tensor coupling terms included, namely in some minimal form they have in equations \([6]\) and \([13]\). In contrast to the PGI vector couplings which are simply absorbed in NSM (only leading to an insignificant redefinition of hypercharge coupling constant), these terms lead, as we show here, to the physical SLIV with a number of specific Lorentz breaking effects appearing through the slightly deformed dispersion relations for all SM fields involved. Being naturally suppressed at low energies these effects may become detectable in high energy physics and
astrophysics. They include a considerable change in the Greisen-Zatsepin-Kouzmin (GZK) cutoff for ultra-high energy (UHE) cosmic-ray nucleons, possible stability of high-energy pions and weak bosons and, on the contrary, instability of photons, very significant increase of the radiative muon and kaon decays, and some others. In this connection, the space-like Lorentz breaking effects, due to a possible spatial anisotropy of which the current observational limitations appear to be much weaker, may be of special interest. Relative to the previous pure phenomenological studies [6, 7], our semi-theoretical approach allows us to be more certain in predictions or check up some ad hoc assumptions made till now.

4.1 The basic bilinear and trilinear terms

So, we proceed to a systematic study of the total ENSM Lagrangian (6). First, we express the new PGI terms in (6) through the hypercharge NG modes \( b_\mu \). Using again the equations (17) and (18) in which, however, due to the high dimensionality of the tensor PGI couplings considered, the terms of the order \( O(1/M) \) are omitted, we have

\[
\mathcal{L}_{ENSM} = \mathcal{L}_{NSM} + \frac{\alpha}{M_P} [b_\mu b_\nu + n^2 (n_\mu b_\nu + n_\nu b_\mu) M + n_\mu n_\nu M^2] T^{\mu \nu}(f, g, h). \tag{49}
\]

Here the total energy-momentum tensor \( T^{\mu \nu}(f, g, h) \) is taken as a sum given in (15) with the corresponding (“the Lagrangian subtracted”) energy-momentum tensors of fermions \( T^{\mu \nu}_f \), gauge fields \( T^{\mu \nu}_g \) and Higgs boson \( T^{\mu \nu}_h \), respectively

\[
T^{\mu \nu}_f = \frac{i}{2} \left[ \bar{L} \gamma^{(\mu} D^{\nu)} L + \bar{\epsilon}_R \gamma^{(\mu} D^{\nu)} \epsilon_R \right],
\]

\[
T^{\mu \nu}_g = -B^{\mu \rho} B^{\nu}_\rho - W^{(i) \mu \rho} W^{(i) \nu}_\rho,
\]

\[
T^{\mu \nu}_h = (D^{\mu} H)^+ D^{\nu} H + (D^{\nu} H)^+ D^{\mu} H
\]

which all are symmetrical (in spacetime indices) and gauge invariant. One can then use that the \( W^i_\mu \) bosons \((i = 1, 2, 3)\) likewise the NG field \( b_\mu \) are taken in the axial gauge (22), due to which one has one noticeable simplification - their preferably oriented covariant derivatives amount to ordinary derivatives

\[
n_\mu D^\mu (b, W^i) = n_\mu \partial^\mu . \tag{51}
\]

Eventually, the total Lagrangian (6) with all leading couplings involved comes to the sum

\[
\mathcal{L}_{ENSM} = \mathcal{L}_{NSM} + \mathcal{L}_{ENSM2} + \mathcal{L}_{ENSM3} \tag{52}
\]

where the nonlinear SM Lagrangian \( \mathcal{L}_{NSM} \) up to the \( 1/M \) order terms was discussed above (20, 21), while for the new terms in the extended Lagrangian \( \mathcal{L}_{ENSM} \) we have only included the bilinear and trilinear terms in fields involved, \( \mathcal{L}_{ENSM2} \) and \( \mathcal{L}_{ENSM3} \), respectively. Just these terms could determine the largest deviations from a conventional SM.

Let us consider first these bilinear terms. One can readily see that they appear from a contraction of the last term \( n_\mu n_\nu M^2 \) in the square bracket in (49) with energy momentum
tensors $T_{f,g,h}$. As a result, one finally comes to the bilinear terms collected in

$$
\mathcal{L}_{ENSM2} = i \delta_f \left[ \mathcal{L} \left( \gamma^\mu n_\mu n_\nu \partial^\nu \right) L + \bar{e}_R \left( \gamma^\mu n_\mu n_\nu \partial^\nu \right) e_R \right] - \delta_g n_\mu n_\nu \left( B_{\mu \rho} B_{\nu}^\rho + W_{(i)}^{(\mu \rho)} W_{(i)}^{(\nu)} \right) + 2 \delta_h \left| n_\nu \partial^\nu H \right|^2
$$

containing the presumably small parameters $\delta_{f,g,h} = \alpha_{f,g,h} M^2 / M_P^2$ since the SLIV scale $M$ is generally proposed to be essentially lower than Planck mass $M_P$. These bilinear terms modify dispersion relations for all fields involved, and lead, in contrast to the nonlinear SM given by Lagrangian $\mathcal{L}_{NSM}$ \cite{20, 21}, to the physical Lorentz violation (see below).

Let us turn now to the trilinear Lorentz breaking terms in $\mathcal{L}_{ENSM}$. They emerge from the contraction of the term $n^2 (n_\mu b_\nu + n_\nu b_\mu)M$ in the square bracket in (49) with the energy-momentum tensors $T_{f,g,h}$. One can see that only contractions with derivative terms in them give the nonzero results so that we have for the corresponding couplings for fermions

$$
\mathcal{L}_{ENSM3} = n^2 \delta_f b_\mu \left[ \mathcal{L} \left( \gamma^\mu n_\mu \partial^\nu + \gamma^\nu n_\nu \partial^\mu \right) L + i \bar{e}_R \left( \gamma^\mu n_\mu \partial^\nu + \gamma^\nu n_\nu \partial^\mu \right) e_R \right].
$$

They present in fact the new type of interaction of the hypercharge Goldstone vector field $b_\mu$ with the fermion matter which does not depend on the gauge constant value $g'$ at all. Remarkably, the inclusion of other quark-lepton families into the consideration will necessarily lead to the flavour-changing processes once the related mass matrices of leptons and quarks are diagonalized. The point is, however, that all these coupling in (53) are further suppressed by the SLIV scale $M$ and, therefore, may only become significant at superhigh energies being comparable with this scale. In this connection, the flavour-changing processes stemming from the less suppressed bilinear couplings (53) appear much more important. We will consider these processes later.

### 4.2 Modified dispersion relations

The bilinear terms collected in the Lagrangian $\mathcal{L}_{ENSM2}$ lead, as was mentioned above, to modified dispersion relation for all fields involved.

#### 4.2.1 Fermions

Due to the chiral fermion content in the Standard Model we use for what follows the chiral basis for $\gamma$ matrices

$$
\gamma_\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^\mu \equiv (1, \sigma^i), \quad \overline{\sigma}^\mu \equiv (1, -\sigma^i)
$$

and take the conventional notations for scalar products of 4-momenta $p_\mu$, unit Lorentz vector $n_\mu$ and four-component sigma matrices $\sigma^\mu(\overline{\sigma}^\mu)$, respectively, i.e.

$$
p^2 \equiv p_\mu p^\mu, \quad (np) \equiv n_\mu p^\mu, \quad \sigma \cdot p \equiv \sigma^\mu p_\mu \quad \text{and} \quad \sigma \cdot n \equiv \sigma^\mu n_\mu.
$$

We will discuss below Lorentz violation (in a form conditioned by the partial gauge invariance) in the chiral basis for fermions in some detail.\footnote{Remind that the fermion parameter $\delta_f = \alpha_f M^2 / M_P^2$ is generally different for quarks and leptons, and also depends on the quark-lepton family considered, whereas the parameter $\delta_g = \alpha_g M^2 / M_P^2$ is taken to be the same for all SM gauge bosons.}
Neutrino. The Lorentz noncovariant terms for neutrino and electron in $\mathcal{L}_{ENSM2}$ has a form

$$i\tilde{f}[\bar{\nu}^{\nu}(\gamma^\rho n_\rho)n^\lambda \partial_\lambda \nu + \bar{e}_L^{\nu}(\gamma^\rho n_\rho)n^\lambda \partial_\lambda e_L + \bar{e}_R^{\nu}(\gamma^\rho n_\rho)n^\lambda \partial_\lambda e_R]$$  (56)

So, the modified Weyl equation for the neutrino spinor $u_\nu(p)$ in the momentum space, when one assumes the standard plane-wave relation

$$\nu(x) = u_\nu(p) \exp(-ip_\mu x^\mu) \quad (p_0 > 0) \ ,$$  (57)

simply comes in the chiral basis for $\gamma$ matrices (55) to

$$[(\sigma \cdot p) + \delta f(\sigma \cdot n)(np)]u_\nu(p) = 0$$  (58)

In terms of the new 4-momentum

$$p'_\mu = p_\mu + \delta f(np)n_\mu$$  (59)

it acquires a conventional form

$$(\sigma \cdot p')u_\nu(p) = 0$$  (60)

So, in terms of the "shifted" 4-momentum $p'_\mu$ the neutrino dispersion relation satisfies a standard equation $p'^2 = 0$ that gives

$$p'^2 = p^2 + 2\delta f(np)^2 + \delta^2 n^2(np)^2 = 0$$  (61)

while the solution for $u_\nu(p')$, as directly follows from (60), is

$$u_\nu(p) = \sqrt{\sigma \cdot p'}\xi$$  (62)

where $\xi$ is some arbitrary two-component spinor.

Electron. For electron, the picture is a little more complicated. In the same chiral basis one has from the conventional and SLIV induced terms (56) the modified Dirac equations for the two-component left-handed and right-handed spinors describing electron. Indeed, assuming again the standard plane-wave relation

$$e(x) = \begin{pmatrix} u_L(p) \\ u_R(p) \end{pmatrix} \exp(-ip_\mu x^\mu) \ , \ p_0 > 0$$  (63)

one comes to the equations

$$ (\sigma \cdot p')u_L = mu_R$$  (64)

$$ (\sigma \cdot p')u_R = mu_L$$

where we have written them in terms of 4-momenta $p'$

$$p'_\mu = p_\mu + \delta f(np)n_\mu$$  (65)
being properly shifted in the preferred spacetime direction. Proceeding with a standard squaring procedure one come to another pair of equations

\begin{align}
(\sigma \cdot p')(\sigma \cdot p')u_L &= m^2 u_L \\
(\bar{\sigma} \cdot p')(\sigma \cdot p')u_R &= m^2 u_R
\end{align}

being separated for left-handed and right-handed spinors. So, in terms of the "shifted" 4-momentum $p'_\mu$ the electron dispersion relation satisfies a standard equation $p'^2 = m^2$ that gives

\begin{equation}
p'^2 = p^2 + 2\delta f(np)^2 + \delta^2 n^2 (np)^2 = m^2
\end{equation}

while the solutions for $u_L(p)$ and $u_R(p)$ spinors in the chiral basis taken are

\begin{equation}
u_L(p) = \sqrt{\sigma \cdot p'} \xi, \quad u_R(p') = \sqrt{\bar{\sigma} \cdot p'} \xi
\end{equation}

where $\xi$ is some arbitrary two-component spinor.

Further, one has to derive the orthonormalization condition for Dirac four-spinors $u(p) = (u_L(p), u_R(p))$ in the presence of SLIV and also the spin summation condition over all spin states of a physical fermion. Let us propose first the orthonormalization condition for the helicity eigenspinors $\xi^s$

\begin{equation}
\xi^s \xi^{s'} = \delta^{ss'}
\end{equation}

where index $s$ stands to distinguish the "up" and "down" states. In consequence, one has for the Hermitian conjugated and Dirac conjugated spinors, respectively,

\begin{equation}
u^s(p) u^{s'}(p) = 2[p_0 + \delta_f(np)n_0] \delta^{ss'}, \quad \bar{\sigma} u^{s'} = 2m \delta^{ss'}
\end{equation}

Note that, whereas the former is shifted in energy $p_0$ for a time-like Lorentz violation, the latter appears exactly the same as in the Lorentz invariant theory for both the time-like and space-like SLIV.

Analogously, one has the density matrices for Dirac spinors allowing to sum over the polarization states of a fermion. The simple calculation using the unit "density" matrix for the generic $\xi^s$ spinors

\begin{equation}
\xi^s \xi^{s\dagger} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\end{equation}

(summation in the index $s$ is supposed) finally gives

\begin{equation}
u^s(p) \bar{\sigma}^s(p) = \begin{pmatrix} m & \sigma \cdot p' \\ \sigma \cdot p' & m \end{pmatrix} = \gamma^\mu [p_\mu + \delta_f(np)n_\mu] + m
\end{equation}

when writing it in terms of the conventional Dirac $\gamma$ matrices \cite{[55]}.
Positron. In conclusion, consider antifermions in the SLIV extended theories. As usual, one identifies them with the negative energy solutions. Their equations in the momentum space appear from the plane-wave expression with the opposite sign in the exponent

\[ e(x) = \left( \frac{v_L(p)}{v_R(p)} \right) \exp(ip_\mu x^\mu), \quad p_0 > 0 \]  (73)

and actually follow from the equations \(^{54}\), if one replaces \( m \rightarrow -m \). As a result, their solution have a form

\[ v_L(p) = \sqrt{\sigma \cdot p'} \chi, \quad v_R(p) = -\sqrt{\sigma \cdot p'} \chi \]  (74)

where \( \chi \) stands for some other spinors which are related to the spinors \( \xi^s \). This relation is given, as usual, by charge conjugation \( C \)

\[ \chi^s = i\sigma_2(\xi^s)^* \]  (75)

where the star means the complex conjugation\(^{11}\). This form of \( \chi^s \) says that this operation actually interchanges the "up" and "down" spin states given by \( \xi^s \). All other equations for positron states described by the corresponding four-spinors \( v^s(p) \), namely those for the normalization

\[ v^s_1(p)v^s_1(p) = 2[p_0 + \delta_f(np)n_0]\delta^{ss'}, \quad \bar{v}^sv^s = -2m\delta^{ss'} \]  (76)

and density matrices

\[ u^s(p) \bar{u}^s(p) = \begin{pmatrix} -m & \sigma \cdot p' \\ \bar{\sigma} \cdot p' & -m \end{pmatrix} = \gamma^\mu[p_\mu + \delta_f(np)n_\mu] - m \]  (77)

also straightforwardly emerge. One can notice, that, apart from the standard sign changing before the mass term all these formulas are quite similar to the corresponding expressions in the positive energy solution case.

4.2.2 Gauge bosons

To establish the form of modified dispersion relations for gauge fields one should take into account, apart from their standard kinetic terms in the NSM Lagrangian \(^{20}\), the quadratic terms appearing from SLIV \(^{53}\). The modification of photon kinetic term appears from the modifications of kinetic terms for \( B \) and \( W^3 \) gauge fields both taken in the axial gauge. These terms, due to the invariant quadratic form of the "bilinear" SLIV Lagrangian \( \mathcal{L}_{ENSM2} \) \(^{53}\), lead to similar modifications for photon \(^{27}\) and \( Z \) boson \(^{20}\). So, constructing kinetic terms for the photon in the momentum space one readily finds its modified dispersion relation

\[ k^2 + 2\delta_g(nk)^2 = 0, \quad \delta_g = \alpha_g(M^2/M_P^2) \]  (78)

\(^{11}\)This conforms with a general definition of the \( C \) conjugation for the Dirac spinors as an operation \( u(p)^T = C\bar{u}(p)^T = i\gamma_2u^*(p_0,p_1) \), where one identifies \( u(p)^T = v(p) \), while \( C \) matrix is chosen as \( i\gamma_0\gamma_2 \).
while its SLIV modified propagator has a form

$$D_{\mu\nu} = \frac{-i}{k^2 + 2\delta_g(nk)^2 + \epsilon^2} \left[ g_{\mu\nu} - \frac{1}{1 + 2\delta_g n^2} \left( n_{\mu}k_{\nu} + k_{\mu}n_{\nu} - n_{\mu}k_{\nu}(nk)^2 + 2\delta_g n_{\mu}n_{\nu} \right) \right] \quad (79)$$

This satisfies the conditions

$$n^\mu D_{\mu\nu} = 0, \quad k^\mu D_{\mu\nu} = 0 \quad (80)$$

where the transversality condition in (80) is imposed on the photon "mass shell" which is now determined by the modified dispersion relation (78). Clearly, in the Lorentz invariance limit ($\delta_g \to 0$) the propagator (79) goes into the standard propagator taken in an axial gauge [30].

**W and Z bosons.** Analogously, constructing the kinetic operators for the massive vector bosons one has the following modified dispersion relations for them

$$k^2 + 2\delta_g(nk)^2 = M^2_{Z,W} \quad (81)$$

To make the simultaneous modification of their propagators, one also should take into account the terms emerged from the Higgs sector. These terms appear when, through the proper diagonalization, the Higgs bilinears decouple from those of the massive W and Z bosons. Due to their excessive length we do not present their modified propagators here.

**4.2.3 Higgs boson**

For Higgs boson (with 4-momentum $k_\mu$ and mass $\mu_h$), we have from the properly modified Klein-Gordon equation, appearing from its basic Lagrangian [25] taken together with the last term in $\mathcal{L}_{ENSM2}$ (53), the dispersion relation

$$k^2 + 2\delta_h(nk)^2 = \mu_h^2, \quad \delta_h = \alpha_h(M^2/M_P^2) \quad (82)$$

**4.3 Lorentz breaking SLIV processes**

We are ready now to consider the SLIV contributions into some physical processes. They include as ordinary processes where the Lorentz violation gives only some corrections, being quite small at low energies but considerably increasing with energy, so the new processes being entirely determined by SLIV in itself. Note that the most of these processes were considered hitherto [6, 7] largely on the pure phenomenological ground. We discuss them here in the ENSM framework which contains only four effective SLIV parameters $^{10}$ $\delta_f$ ($f = q, l$), $\delta_g$ and $\delta_h$ rather than a variety of phenomenological parameters introduced for each particular process individually [7]. Indeed, one (or, at most, two) more fermion parameters should be added in our case too when different quark-lepton families and related flavor-changing processes are also considered.

Another important side of our consideration is that for every physical process we take into account, together with the direct contributions of the SLIV couplings in the Lagrangian, the Lorentz violating contributions appearing during the integration over the
phase space. The latter for the most considered processes is still actually absent in the literature. Specifically, for decay processes, we show that when there are identical particles (or particles belonging to the same quark-lepton family) in final states one can directly work with their SLIV shifted 4-momenta (see, for example (67)) for which the standard dispersion relations hold and, therefore, the standard integration over the phase space can be carried out. At the same time, for the decaying particles by themselves the special SLIV influenced quantity called the ”effective mass” may be introduced. Remarkably, all such decay rates in the leading order in the SLIV δ-parameters are then turned out to be readily expressed in terms of the standard decay rates, apart from that the masses of decaying particles are now replaced by their ”effective masses”.

Our calculations confirm that there are lots of the potentially sensitive tests of the Lorentz invariance, especially at superhigh energies $E > 10^{18}eV$ that is an active research area for the current cosmic-ray experiments [21]. Some of them will be considered in detail below.

4.3.1 Higgs boson decay into fermions

We start by calculating the Higgs boson decay rate into an electron-positron pair. The vertex for such process is given by the Yukawa coupling

$$\frac{G}{\sqrt{2}} h \bar{e} e$$

with the coupling constant $G$. Properly squaring the corresponding matrix element with the electron and positron solutions given above (63, 73) one has

$$|M_{he\bar{e}}|^2 = \frac{G^2}{2} \left( \text{Tr}[(p'_\mu \gamma^\mu)(q'_\nu \gamma^\nu)] - 4m^2 \right)$$

$$= 2G^2(p'_\mu q'^\mu - m^2)$$

where $p'_\mu$ and $q'_\mu$ are the SLIV shifted 4-momenta of electron and positron, respectively, defined as

$$p'_\mu = p_\mu + \delta_f (np)n_\mu$$

$$q'_\mu = q_\mu + \delta_f (nq)n_\mu$$

We use then the conservation law for the original 4-momenta of Higgs boson and fermions

$$k_\mu = p_\mu + q_\mu$$

since just these 4-momenta rather than their SLIV shifted 4-momenta (for which the above conservation law only approximately works) still determine the spacetime evolution of all freely propagating particles involved. Rewriting this relation as

$$k_\mu + \delta_f (nk)n_\mu = p'_\mu + q'_\mu$$
and squaring it one has, using the relations (82) and (85),

\[ p'_\mu q'^\mu = \mu_h^2 - (\delta_h - \delta_f)(nk) - m^2 \]

that finally gives for the matrix element (84)

\[ |M_{he}|^2 = G^2(\mu_h^2 - 4m^2) \]  

(88)

where we have denoted by \( \mu_h^2 \) the combination

\[ \mu_h^2 = \mu_h^2 - 2(\delta_h - \delta_f)(nk)^2. \]  

(89)

This can be considered as an ”effective” mass square of Higgs boson which goes to the standard value \( \mu_h^2 \) in the Lorentz invariance limit. One can also introduce the corresponding 4-momentum

\[ k'_\mu = k_\mu + \delta_f(nk)_\mu, \quad k'^2_\mu = \mu_h^2 \]  

(90)

which differs from the 4-momentum determined due the Higgs boson dispersion relation (82).

So, Lorentz violation due to the matrix element is essentially presented in the ”effective” mass of the decaying Higgs particle. Let us turn now to the SLIV part stemming from an integration over the phase space of the fermions produced. It is convenient to come from the deformed original 4-momenta \( (k_\mu, p_\mu, q_\mu) \) to the shifted ones \( (k'_\mu, p'_\mu, q'_\mu) \) for which fermions have normal dispersion relations given in (85). Actually, possible corrections to such momentum replacement are quite negligible\(^\text{12}\) as compared to the Lorentz violations stemming from the ”effective” mass (89) where they are essentially enhanced by the factor \((nk)^2\). Actually, writing the Higgs boson decay rate in the shifted 4-momenta we really come to a standard case, apart from that the Higgs boson mass is now replaced by its ”effective” mass (see below). So, for this rate we still have

\[ \Gamma_{he} = \frac{G^2(\mu_h^2 - 4m^2)}{32\pi k'_0} \int \frac{d^3p'd^3q'}{p'_0q'_0} \delta^4(k' - p' - q') \]  

(91)

Normally, in a standard Lorentz-invariant case this phase space integral comes to \( 2\pi \). Now, for the negligible fermion (electron) mass, \( \mu_e^2 >> m^2 \), one has using the corresponding energy-momentum relations of particles involved,

\[ \int \frac{d^3p'd^3q'}{p'_0q'_0} \delta^4(k' - p' - q') \simeq 2\pi \frac{k'_0}{\sqrt{\mu_e^2}} \]  

(92)

that for Higgs boson rate eventually gives

\(^{12}\)Actually, there is the following correspondence between shifted and original momenta when integrating over the phase space: for the delta functions this is \( \delta^4(k' - p' - q') = (1 + \delta_f)^{-1} \delta^4(k - p - q) \) (for both time-like and space-like SLIV), while for the momentum differentials there are \( \frac{d^3p'd^3q'}{p'_0q'_0} = (1 + \delta_f)^{-3} \frac{d^3p' d^3q'}{p_0q_0} \) (time-like SLIV) and \( \frac{d^3p'd^3q'}{p'_0q'_0} = (1 + \delta_f)^{2} \frac{d^3p' d^3q'}{p_0q_0} \) (space-like SLIV). So, one can use in a good approximation the shifted momentum variables instead of the original ones.
\[ \Gamma_{\text{he}} \simeq \frac{G^2}{16\pi} \sqrt{\frac{\mu_h^2}{\mu^2}} \simeq \Gamma^0_{\text{he}} \left[ 1 - (\delta_h - \delta_f) \frac{(nk)^2}{\mu_h^2} \right] \] (93)

The superscript "0" in the decay rate \( \Gamma \) here and below belong to its value in the Lorentz invariance limit. Obviously, the SLIV deviation from this value at high energies depends on a difference of delta parameters. In the time-like SLIV case for energies \( k_0 > \mu_h/|\delta_h - \delta_f|^{1/2} \) this decay channel breaks down, though other channels like as \( h \rightarrow 2\gamma \) (or \( h \rightarrow 2 \) gluons) may still work if the corresponding kinematical bound \( \mu_h/|\delta_h - \delta_f|^{1/2} \) for them is higher. For the space-like SLIV the effective delta parameter becomes dependent on the orientation of momentum of initial particle as well, and if, for example, \( \vartheta \) is the angle between \( -\vec{k} \) and \( -\vec{n} \), the threshold energy is given by \( k_0 > \mu_h/|\delta_h - \delta_f| \cos^2 \vartheta |^{1/2} \). So, the decay rate may acquire a strong spatial anisotropy at ultra-high energies corresponding to standard short-lived Higgs bosons in some directions and, at the same time, to unusually long-lived bosons in other ones.

4.3.2 Weak boson decays

Analogously, one can readily write the \( Z \) and \( W \) boson decay rates into fermions replacing in standard formulas the \( Z \) and \( W \) boson masses by their "effective masses" which similar to (89) are given by

\[ M^2_{Z,W} \simeq M^2_{Z,W} - 2(\delta_g - \delta_f)(nk)^2 \] (94)

Therefore, for the \( Z \) boson decay into a neutrino-antineutrino pair one has again the factorized expression in terms of the Lorentz invariant and SLIV contributions

\[ \Gamma_{Z\nu\bar{\nu}} \simeq \frac{g^2}{96\pi \cos^2 \vartheta} \sqrt{M^2_Z} \simeq \Gamma^0_{Z\nu\bar{\nu}} \left[ 1 - (\delta_g - \delta_f) \frac{(nk)^2}{M^2_Z} \right] \] (95)

For the \( Z \) decay into massive fermions (with masses \( m << \sqrt{M^2_Z} \) one has a standard expression though with the "effective" \( Z \) boson mass square \( M^2_Z \) inside rather than an ordinary mass square \( M^2_Z \)

\[ \Gamma_{Zee} = \frac{g^2(1 + r)}{96\pi \cos^2 \vartheta} \sqrt{M^2_Z} = \Gamma^0_{Zee} \left[ 1 - (\delta_g - \delta_f) \frac{(nk)^2}{M^2_Z} \right] \] (96)

where, for certainty, we have focused on the decay into an electron-positron pair and introduced, as usual, the electroweak mixing angle factor with \( r = -4 \sin^2 \vartheta \cos 2\vartheta \). One can see that in the leading order in \( \delta \)-parameters the relation between the total decay rates \( \Gamma_{Z\nu\bar{\nu}} \) and \( \Gamma_{Zee} \) remains the same as in the Lorentz invariant case.

As to the conventional \( W \) boson decay into an electron-neutrino pair, one can write in a similar way

\[ \Gamma_{W\nu\bar{\nu}} \simeq \frac{g^2}{48\pi} \sqrt{M^2_W} \simeq \Gamma^0_{W\nu\bar{\nu}} \left[ 1 - (\delta_g - \delta_f) \frac{(nk)^2}{M^2_W} \right] \]

So, again as was in the Higgs boson case, the \( Z \) and \( W \) bosons at energies \( k_0 > M^2_{Z,W}/|\delta_g - \delta_f| \) tend to be stable for the time-like SLIV or decay anisotropically for the space-like one.
4.3.3 Photon decay into electron-positron pair

Whereas the above mentioned decays could contain some relatively small SLIV corrections, the possible photon decay, which we now turn to, is entirely determined by the Lorentz violation. Indeed, while physical photon remain massless, its "effective" mass, caused by SLIV, may appear well above of the double electron mass that kinematically allows this process to go.

The basic electromagnetic vertex for fermions in SM is given, as usual

\[-(ie)\bar{e}e\gamma^{\mu}e\]  \hspace{1cm} (97)

where we denoted electric charge by the same letter \(e\) as the electron field variable \(e(x)\) and introduced the photon polarization vector \(\epsilon^\mu(s)\). The fermion dispersion relations in terms of the SLIV shifted 4-momenta and the photon "effective" mass have the form (similar to those in the above cases)

\[p'^2_\mu = q'^2_\mu = m^2, \quad M^2_\gamma \simeq 2(\delta_f - \delta_g) (n^n k_\mu)^2 \equiv k'^2_\mu\]  \hspace{1cm} (98)

Consequently, for the square of the matrix element one has

\[|M_{\gamma e e}|^2 = 4e^2 \left[2(p'\epsilon)(q'\epsilon) - \epsilon^2_\mu(m^2 + (p'q'))\right]\]  \hspace{1cm} (99)

\[\epsilon_\mu(s)\epsilon_\nu(s) = -g^{\mu\nu} + \frac{1}{1 + 2\delta g n^2} \left(\frac{n_\mu k_\nu + k_\mu n_\nu}{(nk)} - n^2 k_\mu k_\nu \right)\]  \hspace{1cm} (101)

one finally comes to the properly averaged square of the matrix element

\[\overline{|M_{\gamma e e}|}^2 = \frac{4e^2}{3}(M^2_\gamma + 2m^2)\]  \hspace{1cm} (102)

Therefore, for a calculation of the photon decay rate there is only left an integration over phase space

\[\Gamma_{\gamma e e} = \frac{e^2}{24\pi^2 k_0'} (M^2_\gamma + 2m^2) \int \frac{d^3p'd^3q'}{p_0'q_0} \delta^4(k' - p' - q')\]  \hspace{1cm} (103)

that in a complete analogy with the above Higgs boson decay case (91) leads in the limit \(M^2_\gamma >> m^2\) to the simple answer

\[\Gamma_{\gamma e e} \simeq \frac{e^2}{12\pi} \sqrt{M^2_\gamma} \simeq \frac{e^2}{12\pi} \sqrt{2|\delta|k_0}\]  \hspace{1cm} (104)
where $\delta = \delta_f - \delta_g$ for the time-like violation and $\delta = (\delta_f - \delta_g) \cos^2 \vartheta$ for the space-like one with an angle $\vartheta$ between the preferred SLIV direction and the starting photon 3-momentum. Note that, though, as was indicated in [7], the detection of the primary cosmic-ray photons with energies up to $20 \, \text{TeV}$ sets the stringent limit on the Lorentz violation, this limit belongs in fact to the time-like SLIV case giving $|\delta_f - \delta_g| < 10^{-15}$ rather than to the space-like one which in some directions may appear much more significant.

4.3.4 Radiative muon decay

In contrast, the muon decay process $\mu \to e + \gamma$, though being kinematically allowed, is strictly forbidden in the ordinary SM and is left rather small even under some of its known extensions. However, the Lorentz violating interactions in our model may lead to the significant flavor-changing processes both in lepton and quark sector. Particularly, they may raise the radiative muon decay rate up to its experimental upper limit $\Gamma_{\mu e\gamma} < 10^{-11} \Gamma_{\mu e\nu\nu}$. The point is that the "effective" mass eigenstates of high-energy fermions do not in general coincide with their ordinary mass eigenstates. So, if we admit that, while inside of the each family all fermions are proposed to have equal SLIV $\delta$-parameters, the different families could have in general the different ones, say, $\delta_e, \delta_\mu$ and $\delta_\tau$ for the first, second and third family, respectively. As a result, diagonalization of the fermion mass matrices will then cause small non-diagonalities in the energy-dependent part of the fermion bilinears presented in the $\mathcal{L}_{\text{ENSM2}}$ [53], even if initially they are taken diagonal.

Let us consider, as some illustration, the electron-muon system ignoring for the moment possible mixings of electrons and muons with tau leptons. To this end, the Lagrangian $\mathcal{L}_{\text{ENSM2}}$ is supposed to be extended so as to include the muon bilinears as well. Obviously, the leading diagrams contributing into the $\mu \to e + \gamma$ are in fact two simple tree diagrams where muon emits first photon and then goes to electron due to the "Cabibbo rotated" bilinear couplings [53] or, on the contrary, muon goes first to electron and then emits photon. Let us ignore this time the pure kinematical part of the SLIV contribution following from the deformed dispersion relations of all particles involved thus keeping in mind only its "Cabibbo rotated" part in the properly extended bilinear couplings [53]. In this approximation the radiative muon decay rate is given by

$$
\Gamma_{\mu e\gamma} = \frac{e^2}{32\pi} \frac{(p)^3}{m_\mu} (\delta_\mu - \delta_e)^2 \sin^2 2\varphi
$$

where $p$ is the muon 4-momentum and $\varphi$ is the corresponding mixing angle of electron and muon. Taking for their starting mass matrix $m_{ab}$ the Hermitian matrix with a typical $m_{11} = 0$ texture form [22]

$$
m_{ab} = \begin{pmatrix}
0 & b \\
b & c
\end{pmatrix},
$$

one has

$$\sin^2 2\varphi = \frac{4m_e}{m_\mu}.
$$

So, though the decay rate [115] is in fact negligibly small when muon is at rest, this rate increases with the cube of the muon energy and becomes the dominant decay mode at
sufficiently high energies. If we admit that there are still detected the UHE primary cosmic ray muons possessing energies around $10^{19}\text{eV}$ \cite{21} the following upper limit for the SLIV parameters stems

$$|\delta_\mu - \delta_e| < 10^{-24}$$  \hspace{1cm} (107)

provided that the branching ratio $\Gamma_{\mu e\gamma}/\Gamma_{\mu e\nu}$ at these energies is taken to be of the order one or so. This suggests, as one can see, a rather sensitive way of observation of a possible Lorentz violation through the search for a lifetime anomaly of muons at ultra-high energies.

4.3.5 The GZK cutoff revised

One of the most interesting examples where a departure from Lorentz invariance can essentially affect a physical process is the transition $p + \gamma \rightarrow \Delta$ which underlies the Greisen-Zatsepin-Kouzmin cutoff for UHE cosmic rays \cite{19}. According to this idea primary high-energy nucleons ($p$) should suffer an inelastic impact with cosmic background photons ($\gamma$) due to the resonant formation of the first pion-nucleon resonance $\Delta(1232)$, so that nucleons with energies above $\sim 5 \cdot 10^{19}\text{eV}$ could not reach us from further away than $\sim 50\text{ Mpc}$. During the last decade there were some serious indications \cite{20} that the primary cosmic-ray spectrum extends well beyond the GZK cutoff, though presently the situation is somewhat unclear due to a certain criticism of these results and new data that recently appeared \cite{21}. However, no matter how things will develop, we could say that according to the modified dispersion relations of all particles involved the GZK cutoff will necessarily be changed at superhigh energies, if Lorentz violation occurs.

Actually, one may expect that the modified dispersion relations for quarks will, in turn, change dispersion relations for composite hadrons (protons, neutrons, pions, $\Delta$ resonances etc.) depending on a particular low-energy QCD dynamics appearing in each of these states. In general, one could accept that their dispersion relations have the same form (67) as they have for elementary fermions, only their SLIV $\delta$ parameters values may differ. So, for the proton and $\Delta$ there appear equations,

$$P_{p,\Delta}^2 = m_{p,\Delta}^2 - 2\delta_{p,\Delta} (nP_{p,\Delta})^2 = m_{p,\Delta}^2$$  \hspace{1cm} (108)

respectively, which determine their deformed dispersion relations and corresponding ”effective” masses (where $P_{\mu} = (E, \vec{P})$ stands for the associated 4-momenta). To proceed, one must replace the fermion masses in a conventional proton threshold energy for the above mentioned process

$$E_{\mu} \geq \frac{m_{\Delta}^2 - m_{p}^2}{4\omega}$$  \hspace{1cm} (109)

by their ”effective” masses $m_{p,\Delta}^2$. The target photon energies $\omega$ in (109) are vanishingly small ($\omega \sim 10^{-4}\text{eV}$) and, therefore, its SLIV induced ”effective mass” can be ignored that gives an approximate equality of the fermion energies, $E_{\Delta} = E_{p} + \omega \equiv E_{p}$. As a result, the modified threshold energy for the UHE proton scattering on the background photon via the intermediate $\Delta$ particle production is happened to be
\[ E_p \geq \frac{m_\Delta^2 - m_p^2}{2\omega + \sqrt{4\omega^2 + 2(\delta_\Delta - \delta_p)(m_\Delta^2 - m_p^2)}} \] (110)

Obviously, if there is time-like Lorentz violation and, besides, \( \delta_p - \delta_\Delta > 2\omega^2/(m_\Delta^2 - m_p^2) \) this process, as follows from (110), becomes kinematically forbidden at all energies. For other values of \( \delta \) parameters one could significantly relax the GZK cutoff. The more interesting picture seems to appear for the space-like SLIV with \( \delta_p - \delta_\Delta > 2\omega^2/(m_\Delta^2 - m_p^2) \cos^2 \vartheta \), where \( \vartheta \) is the angle between the initial proton 3-momentum and preferred SLIV direction fixed by the unit vector \( \vec{n} \). Actually, one could generally observe different cutoffs for different directions, or not to have them at all for some other directions thus permitting the UHE cosmic-ray nucleons to travel over cosmological distances.

### 4.3.6 Other hadron processes

Some other hadron processes, like as the pion or nucleon decays, studied phenomenologically earlier [7] are also interesting to be reconsidered in our semi-theoretical framework. Departures from Lorentz invariance can significantly modify the rates of allowed hadron processes, such as \( \pi \to \mu + \nu \) and \( \pi \to 2\gamma \), at superhigh energies. In our model these rates can be readily written replacing the mass of the decaying pion by its “effective masses” being determined independently for each of these cases. So, one has them again in the above mentioned factorized forms (in the leading order in \( \delta \) parameters) as

\[ \Gamma_{\pi\mu\nu} \simeq \Gamma_{\pi\mu\nu}^0 \left[ 1 - (\delta_\pi - \delta_f)(nk)^2/m_\pi^2 \right] \] (111)

\[ \Gamma_{\pi\gamma\gamma} \simeq \Gamma_{\pi\gamma\gamma}^0 \left[ 1 - 3(\delta_\pi - \delta_g)(nk)^2/m_\pi^2 \right] \]

where we have used that their standard decay rates are proportional to the first and third power of the pion mass, respectively. Therefore, the charged pions at energies \( k_0 > m_\pi/\sqrt{\delta_\pi - \delta_f} \) and neutral pions at energies \( k_0 > m_\pi/\sqrt{3(\delta_\pi - \delta_g)} \) may become stable for the time-like SLIV or decay anisotropically for the space-like one. As was indicated in [7], even for extremely small \( \delta \)-parameters of the order \( 10^{-24} \div 10^{-22} \) this phenomenon could appear for the presently studied UHE primary cosmic ray pions possessing energies around \( 10^{19} \text{eV} \) and higher.

As in the lepton sector, there also could be the SLIV induced non-diagonal transitions in the quark sector leading to the flavor-changing processes for hadrons. The SLIV induced radiative quark decay \( s \to d + \gamma \) is of special interest. This could make the radiative hadron decays \( K \to \pi + \gamma \) and \( \Sigma(\Lambda) \to N + \gamma \) to become dominant at ultra-high energies just as it is for the radiative muon decay mentioned above. Again, an absence of kaons and hyperons at these energies or a marked decrease of their lifetime could point to the fact that Lorentz invariance is essentially violated.
5 Conclusion

We found it conceivable that an exact gauge invariance may disable some generic features of the Standard Model which could otherwise manifest themselves at high energies. In this connection, we have proposed the partial gauge invariance (or PGI) in SM [3] and found an appropriate minimal form for PGI [6]. This form depends on the way SLIV is realized in SM and a special role which may play the basic Noether currents, namely, the total hypercharge current and the total energy-momentum tensor in the partially gauge invariant SM. These currents and nonlinear Lorentz realization taken together are precisely the ingredients which appeared essential for our consideration. Just they provide the minimal PGI principle to be reasonably well defined at least at the classical level, as was argued in section 2.

In regard to the theory obtained, we showed first that in the simplest nonlinear SM extension (NSM) with the vector field "length-fixing" constraint \( B_\mu^2 = n^2 M^2 \) the spontaneous Lorentz violation actually holds (as it normally takes place for internal symmetries in any nonlinear \( \sigma \)-model type theory) due to which the hypercharge gauge field is converted into a vector Goldstone boson which having been then mixed with a neutral \( W^3 \) boson of \( SU(2) \) leads, as usual, to the massless photon and massive \( Z \) boson. However, in sharp contrast to an internal symmetry case, all observational SLIV effects in NSM are turned out to be exactly cancelled due to some remnant gauge invariance that is still left in the theory[13]. The point is that the SLIV pattern according to which just the vector field (rather than some scalar field derivative [24] or vector field stress-tensor [25]) develops the vacuum expectation value, taken as \( B_\mu(x) = b_\mu(x) + n_\mu M \), may be treated in itself as a pure gauge transformation with gauge function linear in coordinates, \( \omega(x) = (n_\mu x^\mu) M \). In this sense, the starting gauge invariance in SM, even being partially broken by the nonlinear field constraint, leads to the conversion of SLIV into gauge degrees of freedom of the massless NG boson \( b_\mu(x) \). This is what one could refer to as the generic non-observability of SLIV in a conventional SM. Furthermore, as was shown some time ago [26], gauge theories, both Abelian and non-Abelian, can be obtained by themselves from the requirement of the physical non-observability of SLIV, caused by the Goldstonic nature of vector fields, rather than from the standard gauge principle.

However, a clear signal of the physical Lorentz violation inevitably appears when one goes beyond NSM to include as well the higher dimensional tensor couplings in [3] that leads to the extended nonlinear SM (ENSM). These couplings according to the minimal PGI are proposed to be determined solely by the total energy-momentum tensor of all SM fields involved. So, the lowest order ENSM which conforms with the chiral nature of SM and all accompanying global and discrete symmetries, is turned out to include the dimension-6 couplings[14] given in (6). We showed then that this type of couplings lead, basically through

\[ \text{Example Equation} \]

13Remarkably, a similar nonlinear \( \sigma \)-model type modification of conventional Yang-Mills theories and gravity with the "length-fixing" constraint put on gauge fields appears again insufficient to lead to an actual physical Lorentz violation [23].

14Note that in the pure QED with vectorlike (rather than chiral) fermions the dimension-5 coupling of the type \( (1/M_P^2) A_\mu \bar{\psi} \partial_\mu \psi \) satisfying our partial gauge invariance conjecture could also appear [27]. However, for the conventional SM the minimal Lorentz breaking couplings are proved to be just the terms presented in the ENSM Lagrangian [6].
the deformed dispersion relations of the SM fields, to a new class of processes being of a distinctive observational interest in high energy physics and astrophysics some of which have been considered in significant detail. Such processes leading in themselves to sensitive tests of special relativity, may also shed some light on a dynamical origin of symmetries that may only appear, as we argued, if partial rather than exact gauge invariance holds in the Standard Model.

Though we were mainly focused here on the minimal PGI extension of SM, our conclusion is likely to largely remain in force for any other extension provided that they all are determined by the partial gauge invariance conjecture taken in its general form \([3]\). It is worth noting, however, that this conjecture has been solely formulated here for the hypercharge Abelian symmetry in SM. In this connection, further study of PGI in a wider context, particularly in conventional Yang-Mills theories and gravity seems to be extremely interesting.

Acknowledgments

One of us (J.L.C.) cordially thanks James Bjorken, Masud Chaichian, Ian Darius, Colin Froggatt, Roman Jackiw, Oleg Kancheli, Archil Kobakhidze, Rabi Mohapatra and Holger Nielsen for interesting correspondence, useful discussions and comments. Financial support from Georgian National Science Foundation (grants N 07_462_4-270 and Presidential grant for young scientists N 09_169_4-270) is gratefully acknowledged by authors.

References

[1] P.A.M. Dirac, Proc. Roy. Soc. 209A (1951) 292.
[2] J.D. Bjorken, Ann. Phys. (N.Y.) 24 (1963) 174.
[3] For some later developments, see
   T. Eguchi, Phys.Rev. D 14 (1976) 2755;
   H. Terazava, Y. Chikashige and K. Akama, Phys. Rev. D 15 (1977) 480;
   M. Suzuki, Phys. Rev. D 37 (1988) 210;
   V.A. Kostelecky and S. Samuel, Phys. Rev. D 39 (1989) 683.
[4] J.L. Chkareuli, C.D. Froggatt and H.B. Nielsen, Phys. Rev. Lett. 87 (2001) 091601;
   J.D. Bjorken, hep-th/0111196;
   A. Jenkins, Phys. Rev. D 69 (2004) 105007;
   V.A. Kostelecky, Phys. Rev. D 69 (2004) 105009;
   and some of references below [8, 9, 12, 13, 14, 26, 27].
[5] S. Chadha, H.B. Nielsen, Nucl. Phys. B 217 (1983) 125;
   S.M. Carroll, G.B. Field and R. Jackiw, Phys. Rev. D 41 (1990) 1231.
[6] D. Colladay and V.A. Kostelecky, Phys. Rev. D 55 (1997) 6760; D 58 (1998) 116002; V.A. Kostelecky and R. Lehnert, Phys. Rev. D 63 (2001) 065008.

[7] S. Coleman and S.L. Glashow, Phys. Lett. B 405 (1997) 249; Phys. Rev. D 59 (1999) 116008.

[8] S R. Bluhm and V. A. Kostelecky, Phys. Rev. D 71 (2005) 065008.

[9] J.L. Chkareuli and J.G. Jejelava, Phys. Lett. B 659 (2008) 754.

[10] B.M. Gripaios, JHEP 0410 (2004) 069.

[11] J.L. Chkareuli, A. Kobakhidze and R.R. Volkas, Phys. Rev. D 80 (2009) 065008; M.D. Seifert, Phys. Rev. D 82 (2010) 125015.

[12] Per Kraus and E.T. Tomboulis, Phys. Rev. D 66 (2002) 045015.

[13] Y. Nambu, Progr. Theor. Phys. Suppl. Extra 190 (1968).

[14] J.L. Chkareuli, C.D. Froggatt, R.N. Mohapatra and H.B. Nielsen, A.T. Azatov and J.L. Chkareuli, Phys. Rev. D 73 (2006) 065026; J.L. Chkareuli and Z.R. Kepuladze, Phys. Lett. B 644 (2007) 212.

[15] S. Weinberg, The Quantum Theory of Fields, v.2, Cambridge University Press, Cambridge, 2000.

[16] R.N. Mohapatra, Unification and Supersymmetry, Springer-Verlag, 1986.

[17] P. Ramond, hep-ph/9809459; J.L. Chkareuli, JETP Lett. 32 (1980) 671, Pisma Zh. Eksp. Teor. Fiz. 32 (1980) 684; J.L. Chkareuli, C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B 626 (2002) 307.

[18] C. Dams and R. Kleiss, Eur. Phys. Journ. C 34 (2004) 419.

[19] K. Greizen, Phys. Rev. Lett. 16 (1966) 748; G.T. Zatsepin and V.A. Kuz’min, JETP Lett. 41 (1966) 78.

[20] Fly’s eye Collab. (D.J. Bird et al.), Astrophys. Journ. 424 (1995) 144; AGASA Collab. (M. Takeda et al.), Phys. Rev. Lett. 81 (1998) 1163; Astropart. Phys. 19 (2003) 447.

[21] Pierre Auger Collab. (J. Abraham et al.), Astropart. Phys. 29 (2008) 243; Phys. Lett. B 685 (2010) 239.

[22] H. Fritzsch and Z.-Z. Xing, Prog. in Part. and Nucl. Phys. 45 (2000) 1; J.L. Chkareuli and C.D. Froggatt, Phys.Lett. B 450 (1999) 158.
[23] J.L. Chkareuli, J.G. Jejelava and G. Tashihvili, Phys. Lett. B 696 (2011) 124;
    J.L. Chkareuli, C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B 848 (2011) 498.

[24] N. Arkani-Hamed, H.-C. Cheng, M. Luty and J. Thaler, JHEP 0507 (2005) 029.

[25] J. Alfaro and L.F. Urrutia, Phys. Rev. D 81 (2010) 025007.

[26] J.L. Chkareuli, C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B 609 (2001) 46.

[27] J.L. Chkareuli, Z. Kepuladze and G. Tashihvili, Eur. Phys. Journ. C 55 (2008) 309.