Abstract

We use the ‘branes within branes’ approach to study the appearance of stable \((p - 2)\)-branes and unstable \((p - 1)\)-branes in type II string theory from \(p\)-brane–\(p\)-antibrane pairs. Our goal is to describe the emergence of these lower dimensional branes from brane-antibrane pairs in string theory using a tractable gauge theory language. This is achieved by suspending the original \(p\)-brane–\(p\)-antibrane pair between two \((p + 2)\)-branes, and describing its dynamics in terms of the worldvolume gauge theory on the spectator \((p + 2)\)-branes. Instantons, monopoles, sphalerons and their higher-dimensional generalizations in this worldvolume gauge theory correspond to stable (BPS) and unstable (non-BPS) branes in string theory. Collisions of stable branes with corresponding antibranes and production of lower-dimensional branes in string theory are described in a straightforward way in gauge theory. Tachyonic modes on the \(p\)-brane–\(p\)-antibrane worldvolume do not appear in our analysis since we work on the worldvolume of the spectator \((p + 2)\)-branes. Our results on brane descent relations are in agreement with Sen’s tachyon condensation approach.
1 Introduction

Much of our current quantitative understanding of non-perturbative string theory centers on stable BPS-saturated D-branes and appeals to powerful constraints imposed by supersymmetry. One way to go beyond this and to introduce inter-brane interactions and time-dependent processes involving branes is to consider non-supersymmetric brane-antibrane systems. D-branes and D-antibranes carry opposite RR charges and are not protected by supersymmetry. These branes and antibranes are expected to scatter and to annihilate each other in string theory as particles or extended objects do in quantum field theory.

Most of the recent progress in this subject follows the approach initiated by Sen \cite{Sen1, Sen2, Sen3, Sen4} and based on tachyon condensation in brane-antibrane systems. In this approach the perturbative instability of the brane-antibrane pair at short distances \cite{Sen5} manifests itself as the appearance of a tachyonic mode of the fundamental F1 string which is stretched between the brane and the antibrane. This gives rise to a tachyon field, which is a complex scalar living on the worldvolume of the coincident brane-antibrane configuration, and transforming in the bi-fundamental $U(1) \times U(1)$ representation of the gauge theories on the brane and the antibrane. The idea is that the tachyon condenses and causes the annihilation of the brane-antibrane configuration to the vacuum. More precisely, Sen conjectures that the tachyon potential is of the 'Mexican Hat' type such that the unstable configuration at the top of the potential corresponds to the coincident brane-antibrane configuration, and the ground state is the closed string vacuum with no branes or open strings left.

Importantly, stable and unstable lower-dimensional D-branes can now appear as solitons in the tachyon field \cite{Sen2}. Stable branes appear as co-dimension-2 topological solitons in the brane-antibrane worldvolume, and the unstable branes are co-dimension-1 unstable classical solutions. This implies that all branes in e.g. type II string theory can be obtained from annihilations of the highest-dimensional D9 and anti-D9-branes. An elegant brane classification follows from this \cite{Sen6} and is based on K-theory.

In Sen’s scenario, the complex tachyon serves as the Higgs field which spontaneously breaks the $U(1) \times U(1)$ gauge theory on the brane-antibrane worldvolume to a diagonal $U(1)$. The fate of this remaining $U(1)$ is a little less clear, as it is supposed to have completely disappeared in the string vacuum. It was argued in \cite{Sen7} that the diagonal $U(1)$ is not seen in the vacuum because it is confined due to the condensation of different tachyons living this time on the D-branes stretching between the brane-antibrane pair.

The main motivation of this paper is to gain further insights into brane-antibrane systems and to find independent confirmations of the results of Sen’s approach using a different language — gauge theory.
Very recently the authors of Ref. [8] have made interesting progress in this direction by seeing signs of the tachyon condensation from the gauge theory perspective. More precisely Ref. [8] studied the gauge theory living on the worldvolume of two intersecting branes. When the intersection angle $\theta$ is close to $\pi$, the configuration becomes the brane-antibrane pair. The difficulty in using the results of this approach is that for $\theta \sim \pi$ the worldvolume theory is not described by a gauge theory. Hence, the authors of Ref. [8] had to work with small values of $\theta$ (which is more like a brane-brane configuration rather than a brane-antibrane) where the gauge theory description is valid, and to extrapolate their findings to a regime of interest, $\theta \sim \pi$. We will avoid this difficulty altogether by introducing additional – spectator – branes and working with the gauge theory on their worldvolume.

We will use a version of a ‘branes within branes’ approach where the $p$-brane–$p$-antibrane pair is suspended between two $(p+2)$-branes. The dynamics of the $p$-brane–$p$-antibrane annihilation can then be described in terms of the worldvolume gauge theory on the $(p+2)$-branes. Descent relations between stable branes which follow from Sen’s approach [6, 3] imply that lower-dimensional branes can be produced in brane-antibrane annihilations. We want to understand this in a gauge theory language, where one might naively expect that, for example, instanton-antiinstanton configurations and monopole-antimonopole configurations annihilate each other completely into a perturbative vacuum.

In fact, Taubes [9, 10] showed long time ago that monopole-antimonopole classical configurations can be used to construct non-contractible loops which (as will be explained below) give rise to instanton and sphaleron solutions. This observation of Taubes will be at the heart of the SYM analysis in this paper.

In Section 2 we will outline the Taubes construction applied to the $\mathcal{N} = 4$ SYM and explain how it links together all three types of the brane solutions in SYM in four dimensions. In the second half of Section 2 we switch to type IIB string theory and embed gauge instantons, monopoles and sphalerons into string theory as D-branes within branes. This allows us to embed the non-contractible monopole-antimonopole loop in string theory.

In the second half of the paper (Section 3) we will explain how to generalize and incorporate this construction to higher-dimensional D-branes in type II string theory. We will describe how the lower-dimensional stable branes are produced in brane-antibrane annihilation processes and derive the descent relations between branes. We will also explain how these relations incorporate stable non-Dirichlet branes, such as F1, NS5 and the S-dual of the D-instanton. Section 4 presents our conclusions and some open questions.
2 Instantons and sphalerons from monopoles in SYM and in string theory

The field theory considered in this section is the $\mathcal{N} = 4$ supersymmetric $SU(2)$ gauge theory in Minkowski and also in Euclidean 4-dimensional spacetimes. Greek indices, $\mu, \nu$, will refer to spacetime components, $\mu, \nu = 0, 1, 2, 3$ in Minkowski and $\mu, \nu = 1, 2, 3, 4$ in Euclid. Latin indices, $m, n$, label spatial directions, $m, n = 1, 2, 3$. This gauge theory in the Coulomb phase is embedded in type IIB string theory at low energies ($\alpha' \to 0$) as the worldvolume theory on two parallel D3-branes with the relative separation $2\pi\alpha' v$ along the perpendicular to the branes direction, e.g. $x_9$.

2.1 The monopole

First we recall some basic facts about ‘t Hooft–Polyakov monopoles [11, 12, 13]. The standard BPS monopole solution in a static Hedgehog gauge is [13]

$$\Phi^{\text{mono}}(x_n) = \frac{1}{g} \left( g v |x| \coth(g v |x|) - 1 \right) \frac{x_a}{|x|^2} \frac{\tau^a}{2},$$

$$A^{\text{mono}}_m(x_n) = \frac{1}{g} \left( 1 - \frac{g v |x|}{\sinh(g v |x|)} \right) \epsilon_{mna} \frac{x_n}{|x|^2} \frac{\tau^a}{2}. \quad (2.1)$$

Here $v$ is the vacuum expectation value (vev) of the adjoint scalar field $\Phi$, which follows from the large-$|x|$ asymptotics of the solution, $\Phi^{\text{mono}}(a) \to v x^a / |x|$, the distance in 3D space is denoted as $|x| = \sqrt{x_m x_m}$, and $\tau^a$ are the three Pauli matrices.

The configuration (2.1) can be embedded into the $\mathcal{N} = 4$ SYM theory as

$$\phi_1 = \Phi^{\text{mono}}(x_n), \quad \phi_2 = \phi_3 = \ldots = \phi_6 = 0, \quad A_0 = 0, \quad A_m = A^{\text{mono}}_m(x_n). \quad (2.2)$$

Here the $\mathcal{N} = 4$ SYM is on the Coulomb branch with one of the six real scalar fields, i.e. $\phi_1$, having a nonzero vev$^1 v$. This breaks gauge $SU(2)$ spontaneously to $U(1)$. Expressions (2.2) give a time-independent classical solution, with finite energy, and one unit of magnetic charge of the unbroken $U(1)$. The monopole solution (2.1) or (2.2) is topological in nature, its magnetic charge is the winding number of $S^2 \to S^2$. Here the first $S^2$ is the 2-sphere at the boundary of $R^3$ as $|x| \to \infty$, and the second $S^2 = SU(2)/U(1)$ is the manifold of the asymptotic values of the scalar field $\phi_1^a$ as $\sum_{a=1}^3 \langle (\phi_1^a)^2 \rangle = v^2$.

As already mentioned, the monopole solution (2.1) is written in the static Hedgehog gauge,

$$\langle A_0 \rangle = 0, \quad \langle \phi_1^a \rangle = v x^a / |x|. \quad (2.3)$$

$^1$R symmetry $SO(6)$-rotations can always be used to single out any one of the six scalars. For definiteness we will always choose to give the vev to $\phi_1$. 

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It is more convenient for our purposes to gauge-transform (2.1) to the unitary gauge where
\[ \langle \phi_1^a \rangle = v \delta^{a3} . \] (2.4)
This can be achieved by using a gauge transformation which transforms the unit vector \( x^a / |x| \) into the unit vector along the third direction, \( \delta^{a3} \). This gauge-transformation is singular along the ray emitted from the monopole centre and introduces the Dirac string into the regular monopole configuration (2.1). This string, however, is clearly a gauge artifact and will be largely ignored in what follows.

The scalar field of the monopole in the singular unitary gauge is purely Abelian, i.e. aligned with the vev in (2.4), and is of the form
\[ \phi_1 = \Phi_{\text{mono}}(x) = \frac{1}{g} \left( g v |x| \coth(g v |x|) - 1 \right) \frac{1}{|x|} \frac{\tau^3}{2} , \] (2.5)
while the gauge field \( A_m \) contains Abelian \( (\propto \tau^3) \) as well as non-Abelian \( (\propto \tau^{1,2}) \) components. The unitary gauge form of the monopole (2.5) will be required below for a realization of the monopole in type IIB string theory as a D1-brane suspended between the two parallel D3-branes.

For future reference we note here that the SYM-monopole is a BPS-saturated configurations in the sense that the monopole is annihilated by half of the sixteen supercharges of the \( \mathcal{N} = 4 \) theory. An easy way to see it is to note that if one renames the scalar monopole component in (2.1) as \( A_4 \), the corresponding field-strength \( F_{\mu\nu} \) (made out of \( A_\alpha, A_4 \)) is self-dual, \( F_{\mu\nu} = * F_{\mu\nu} \) for the monopole solution. This implies that eight of the supercharges of the \( \mathcal{N} = 4 \) theory will annihilate the bosonic monopole, and further eight supercharges will give eight adjoint fermion zero modes of the monopole. There are precisely two fermion zero modes for each of the four flavours \( I = 1, \ldots, 4 \):
\[ \lambda^I_\alpha = \frac{i}{2} \xi_\beta^I (\sigma^\mu \bar{\sigma}^\nu)_{\alpha}^\beta F_{\mu\nu} , \] (2.6)
where \( \xi_\beta^I \) are the Grassmann collective coordinates for the four spinor supercharges \( Q^I_\beta \) and \( \sigma^\mu \) and \( \bar{\sigma}^\nu \) are the four Pauli matrices.

So far we have been discussing the monopole with magnetic charge +1. Being BPS-saturated, single monopoles do not interact with each other and multi-monopole configurations can be constructed. General multi-monopole solutions follow from the Nahm construction [14]. There are also antimonopole solutions with negative magnetic charges, they are obtained from monopole solutions by switching the sign of the scalar field. We also note that there is a Coulomb attraction between monopoles and antimonopoles at large distances, and no static monopole-antimonopole configuration exists as a classical solution. There are, of course, time-dependent monopole-antimonopole solutions which describe a classical scattering process. Such solutions, in principle, can be constructed numerically.
The single-instanton configuration \([15, 16]\) in Euclidean \(N = 4\) theory is

\[
A_{\mu}^{\text{inst}}(x) = \frac{2}{g} \frac{\rho^2}{x^2(x^2 + \rho^2)} \bar{\eta}_{\mu\nu}^a x^\nu \tau^a, \tag{2.7}
\]

\[
\phi_1^{\text{inst}}(x) = v \frac{x^2}{x^2 + \rho^2} \tau^3.
\]

where \(\bar{\eta}_{\mu\nu}^a\) is the 't Hooft \(\bar{\eta}\)-symbol \([16]\), and \(\rho\) is the instanton scale size.\(^2\) In (2.7) we showed only the non-vanishing bosonic fields and set fermionic collective coordinates to zero. For future reference, we note that the instanton scalar field in (2.7) is purely Abelian and is already in the unitary gauge (2.4).

The instanton (2.7) is a time-dependent field configuration with finite Euclidean action,

\[
S_E = \frac{8\pi^2}{g^2} + 4\pi^2 v^2 \rho^2. \tag{2.8}
\]

For a non-zero vev, \(S_E\) explicitly depends on \(\rho\) and thus, \(\rho\) is not an exact zero mode of the instanton, and (2.7) is not an exact solution of equations of motion for \(\rho > 0\) (at \(\rho = 0\) the instanton is singular). We recall that for nonzero vev a nontrivial regular solution cannot exist, due to Derrick’s theorem: for any putative solution one can lower the action further simply by shrinking the configuration. One way to fix this problem was found by Affleck \([17]\). For a brief practical review with an application see sections 3 and 4 of \([18]\). The idea is as follows: a new operator, or Affleck constraint, is introduced into the action by means of a Faddeev-Popov insertion of unity. If this operator is of suitably high dimension, Derrick’s theorem is avoided, and the instanton stabilizes at a fixed scale size \(\rho\). The integration over the Faddeev-Popov Lagrange multiplier in the path integral can then be traded off for the integration over \(\rho\). The now-stable solutions are known as constrained instantons.

The detailed shape of the constrained instanton depends on a choice of constraint. But it turns out that only the short-distance regime, \(x \ll 1/M_W\), and the long-distance regime, \(x \gg 1/M_W\), of the instanton are important. (Here \(M_W = g v\) is the \(W\)-boson mass.) In particular, the instanton measure and action depend only on the short-distance instanton (2.7), while the low-energy fields in the instanton background require the long-distance instanton – see \([19]\) and references therein.

As in the monopole case earlier, the instanton solution is topologically stable, but

\(^2\)Other bosonic collective coordinates of the \(SU(2)\) instanton are the global \(SU(2)\) rotations of the gauge field only, and the four-translations of the gauge and the Higgs field together. Together with \(\rho\) this makes \(1 + 3 + 4 = 8\) bosonic zero modes.
instantons are governed by $S^3$ spheres. Instanton topological charge,

$$Q = \frac{1}{16\pi^2} \int F_{\mu\nu}^* F_{\mu\nu} \, d^4x,$$

(2.9)
is the winding of $S^3 \rightarrow S^3$ also known as Pontryagin number. The first $S^3$ is the large-$|x|$ sphere of Euclidean 4D spacetime, and the second $S^3$ is $SU(2)$ (it arises from the requirement that $A_\mu$ goes to a pure gauge at large values of $x$ as a necessary condition for the finiteness of the action).

Since the instanton field-strength is self-dual, instantons are BPS-saturated. As in the monopole case, there are precisely eight exact (adjoint) fermion zero modes in the instanton background.$^3$ Multi-instanton solutions follow from the ADHM formalism$^{20,21,22}$ – see$^{19}$ for a review and applications. Finally, instanton-antiinstanton configurations are not classical solutions at finite separations; instantons and antiinstantons interact and annihilate into a perturbative vacuum. This is different from the time-dependent monopole-antimonopole case earlier.$^4$

2.3 Barrier penetration and sphalerons

In gauge theory, the instanton solution in the $A_4 = 0$ gauge mediates the transition between two topologically distinct vacua – e.g. from a trivial vacuum to a vacuum with a winding number one. Instanton contributions to Euclidean path integrals correspond to tunneling transitions between these two vacua$^{25}$.

When the vev is non-zero, there is a barrier between the vacua which corresponds to an unstable classical solution, the sphaleron$^{26}$. The sphaleron solution can be determined as the maximal energy configuration along the non-contractible loop of field configurations starting and ending in the vacuum. More concretely, consider a continuous path made of finite energy field configurations, which starts at the trivial vacuum and terminates in the vacuum with winding number one. Find the point with maximal energy on each of these paths and find such a path where this energy is minimal. This ‘minimax’ procedure determines the saddle-point solution on top of the minimal energy path. This solution is a sphaleron and it has precisely one negative mode. When the two vacua at the beginning and at the end of the path are identified, the path becomes a non-contractible loop as depicted in Figure 1.

This discussion can be complemented by an argument due to Taubes$^{10}$ who has shown rigorously that the sphaleron solution exists in the gauge theory with an adjoint

$^3$For the single instanton there are also eight quasi-zero fermion modes, which are lifted by the vev $v$.

$^4$Instanton-antiinstanton configurations at finite separations can be seen and rigorously defined as solutions to the valley equation of Yung – see$^{23,24}$ for the formalism and applications to gauge theories.
Figure 1: A path made of finite-energy classical field configurations interpolating between two topologically distinct vacua. When the vacua are identified in the picture on the right, the path becomes a non-contractible loop. The loop with the minimal value of the maximal energy will pass through the sphaleron solution.

Higgs. This establishes the existence of the sphaleron solution in the $\mathcal{N} = 4$ SYM theory. An explicit form of the sphaleron in $\mathcal{N} = 4$ can in principle be found numerically by choosing a suitable family of non-contractible loops.

It is clear from the above discussion that the sphaleron is not a topological solution, it is unstable as it decays along the non-contractible loop to the vacuum. At the same time, the non-contractible loop itself has a topology of the instanton. Hence, instantons and sphalerons are intimately related to each other via the notion of non-contractible loops. The instanton corresponds to the loop with the minimal Euclidean action, and the sphaleron to the loop with the minimal maximal energy. What is most remarkable, however, is the observation of Taubes [9, 10] that the non-contractible loop itself is constructed from the monopole-antimonopole pair.

In the following subsection we will outline the Taubes construction and explain how it links together all three types of the brane solutions in the $\mathcal{N} = 4$ SYM.

2.4 Non-contractible monopole loop in gauge theory

We will now describe the key idea which is at the heart of the Taubes construction [9, 10] of a non-contractible monopole loop in a gauge theory with an adjoint scalar field. Applied directly to the $\mathcal{N} = 4$ SYM in the Coulomb phase, this construction derives the very existence of sphaleron and instanton solutions in this theory starting from the monopole solution (or more precisely, a combination of 't Hooft-Polyakov monopoles with the net magnetic charge equal to zero). In Section 3 this line of reasoning will become our starting point in explaining how lower-dimensional branes are produced in annihilations...
of brane-antibrane pairs of higher dimension.

The idea of Taubes was to construct a representative of a homotopy class of non-contractible loops in the $\mathcal{N} = 4$ classical field configuration space from the ’t Hooft-Polyakov monopole solution (2.1). The element of this homotopy class which has the lowest Euclidean action along the loop is the instanton solution, and the element with the lowest maximal energy of a point on the loop is the sphaleron saddle-point solution.

First we consider the Hedgehog gauge where the topology is more obvious, and then will recast the same argument in the unitary gauge which is more suited for branes within branes applications in string theory.

We recall from (2.1) that at large distances, $x \gg 1/M_W$, from the monopole centre, the monopole scalar field is an $S^2$ hedgehog while the antimonopole has the reversed picture of field-lines, see Figure 2. Now consider a composite configuration made out of the monopole

![Figure 2: Monopole and antimonopole scalar field components in the hedgehog gauge.](image)

and the antimonopole at a large separation as shown on Figure 3. This configuration is a fixed-time snap-shot of the corresponding time-dependent classical solution with the net monopole charge zero. If brought together, the monopole field would cancel the antimonopole precisely, leaving perturbative vacuum (plus radiation if the collision occurs in real time). Now we want to continuously deform the configuration in a non-trivial way. We start rotating the monopole along the axis of the configuration throat where the field-lines match, while keeping the antimonopole fixed. At any time during this rotation, the long-range (Abelian) fields still match, but not the short-range (non-Abelian) fields. Thus, the monopoles would not annihilate if brought together since their non-Abelian fields would not cancel. This remains so until the rotation completes a full circle, and we arrive at the original configuration where all fields match and which can be shrunk to a perturbative vacuum. By starting in the vacuum, then continuously deforming it to produce a classical monopole-antimonopole pair, then separating the pair, then rotating one of the monopoles by $2\pi$ and, finally, bringing them together to annihilate, we create a non-contractible monopole-antimonopole loop in the classical configuration space. Since the antimonople can be described as the monopole moving backwards in time, this loop can also be viewed as a single monopole making a full circle in space, rotating at the same
Figure 3: Non-contractible monopole-antimonopole loop in the hedgehog gauge. The monopole is rotated by $2\pi$ while the antimonopole is fixed.

time by a full rotation.\footnote{Remarkably, it turns out that this non-contractible loop has the topology of the instanton. The $S^3$-sphere associated with the instanton is a twisted product of $S^1$ (the monopole field rotation) and the $S^2$-sphere formed by the monopole scalar hedgehog field. $S^3$ exhibited as a twisted $S^1$ bundle over $S^2$ is the Hopf fibration. The topological charge associated with the non-contractible monopole loop is the Pontryagin index \cite{9}.}

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The same picture can also be described in physical terms in the unitary gauge. In the isospin space the monopole in the unitary gauge has Abelian (isospin-3) and non-Abelian (isospin-1,2) field components as indicated on Figure 4. The monopole and the antimonopole have long-range Abelian and short-range non-Abelian interactions. The isospin-3 interaction, $V_{\text{long}}(r) = -2/r$, is long-range and is always attractive.\footnote{The factor of two comes from adding the Higgs-mediated to the gauge-mediated Coulomb interaction.} The isospin-1,2 interaction is short-range, but also depends on the relative orientation $\theta$ of the isospin-1 and isospin-2 components. Attraction changes to repulsion as $\theta$ varies from $-\pi$ to 0. The

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{Monopole and antimonopole field components in the unitary gauge}
\end{figure}
total potential energy at intermediate distances can be represented as follows\[9\]

\[
V(r) = -\frac{2}{r} \left( 1 - e^{-Mwr} \left( \frac{1}{2} + \cos \theta \right) \right).
\]

(2.10)

The non-contractible loop,

\[
l(\tau) = (A^r_\mu(x_n), \phi^\tau(x_n)),
\]

(2.11)
is a continuous family of static finite-energy configurations parameterized by \(\tau\), see Figure 5. At the initial value of \(\tau = \tau_0\) one starts from the vacuum, \((0, v^{\frac{3}{2}})\), and as \(\tau\) increases, creates from the vacuum the monopole-antimonopole configuration. The monopole-antimonopole parametrization \((r, \theta)\) is initially \((0, -\pi)\) and changes to \((R, -\pi)\) as \(\tau\) grows from \(\tau_0\) to \(\tau_1\). As \(\tau\) continues to increase we keep \(r = R\) fixed and gauge-rotate the monopole by increasing \(\theta\) continuously from \(-\pi\) to 0 at \(\tau = \tau_2\), and then further to \(+\pi\) at \(\tau = \tau_3\). Finally, after completing the full rotation of the monopole, we bring the configuration \((R, \pi)\) to the vacuum \((0, \pi)\) as \(\tau\) reaches its final value \(\tau_4\). This loop is non-contractible since for any fixed value of \(\tau\) in the vicinity of \(\tau_2\), the monopoles cannot be brought together, as their non-Abelian interaction is repulsive. Of course this loop is completely identical to the loop in the Hedgehog gauge discussed earlier. When the loop

\[
\tau_0 \rightarrow \tau_1
\]

\[
\tau_1 \rightarrow \tau_3
\]

\[
\tau_3 \rightarrow \tau_4
\]

Figure 5: Monopole-antimonopole non-contractible loop in the unitary gauge.

parameter \(\tau\) is identified with the Euclidean time \(x_4\), the non-contractible loop is in the same homotopy class as the 1-instanton solution; its topology is characterized by Pontryagin number equal to one. When \(\tau\) is interpreted as \(x_4\) one can calculate the Euclidean
action, $S_E$, along the loop as $\int d\tau E$. Since the energy $E$ of the configuration for each $\tau$ is finite, and since $\tau$ varies in a finite interval, the action $S_E$ is finite for our loop. Now, by continuously deforming the trial loop described above, one can find the loop which minimizes $S_E$. It is the instanton.

Similarly, in the same homotopy class we can look for the loop which now has the lowest maximal energy, i.e. for every loop find the value of $\tau$, $\tau = \tau_*$, for which the energy $E$ is maximal (for our trial loop it is at $\tau_* = \tau_2$) and then choose the loop which minimizes the value of $E(\tau_*)$. The corresponding configuration is a static saddle-point solution of equations of motion. It is the sphaleron, and its energy represents the top of the barrier under which the instanton tunnels.

In conclusion we comment that in this construction, the loop parameter $\tau$ is not to be interpreted as the real time $x_0$. The real-time classical process of monopole-antimonopole creation and subsequent annihilation cannot start from the vacuum as the energy is not conserved. In fact, the energy varies continuously along the loop, from zero to the sphaleron mass and then back to zero. This also fits with the fact that the instanton is the imaginary-time solution of classical equations. We will add the real time dimension to this discussion in Section 3.

### 2.5 Monopoles and instantons as branes within branes

Some of the most remarkable developments in instanton calculus in gauge theory came with the realization that the ADHM formalism \[20, 21, 22\] arises naturally in the context of string theory \[27, 28, 29, 30, 31\]. Instantons in the $\mathcal{N} = 4$ gauge theory in four dimensions correspond precisely to the boundstates of D($-1$)-branes on the worldvolume of coincident or parallel D3-branes. Instanton solutions in gauge theory are localized objects in space and time, and so is the point-like D($-1$)-brane within the 4-dimensional worldvolume of D3-branes. More generally, the standard 4D instanton solutions embedded in a higher-dimensional gauge theory, are realized in string theory as D$p$-branes within D($p + 4$)-branes. The low energy collective dynamics of $N$ coincident D($p + 4$)-branes in Type II string theory is described by a $U(N)$ SUSY gauge theory in $p + 5$-dimensions with 16 supercharges. An instanton in the worldvolume theory of the D($p + 4$)-branes is a soliton which has 4 transverse directions in the higher dimensional brane, i.e. it is a $p$-brane. Remarkably, it is precisely a D$p$-brane bound to the D($p + 4$)-branes. In general $k$ D$p$-branes bound to the $N$ higher dimensional D($p + 4$)-branes correspond to a charge $k$ instanton in a $U(N)$ SUSY gauge theory. The gauge theory and the D-brane realizations of instantons are both BPS-saturated configurations.

Not only the ADHM multi-instanton gauge field can be re-derived in string theory using a brane-probe approach \[28, 30\], but also the $k$-instanton integration measure and action in the $U(N)$ $\mathcal{N} = 4$ gauge theory is identical to the partition function of $k$ D($-1$)-
branes within the $N$ D3-branes in type IIB string theory \cite{31} (for a review see Section 10 of \cite{19}).

For now let us set $p = -1$ so that the worldvolume theory on $D(p + 4)$-branes is a (3+1)-dimensional gauge theory. For two coincident D3-branes we have the $U(2)$ (or decoupling the overall $U(1)$ factor, the $SU(2)$) $\mathcal{N} = 4$ gauge theory in the conformal phase. The instanton is the D($-1$)-brane lying within the worldvolume of the coincident D3-branes. An interesting question to ask is what happens to this geometrical picture when the two D3-branes are separated, i.e. when we the $\mathcal{N} = 4$ SYM develops a nonzero vev $v$. In other words, for a general $p$, how is the $(p + 1)$-dimensional worldvolume of the $D_p$-brane situated in relation to the separated $(p + 5)$-dimensional worldvolumes of two $D(p + 4)$-branes?

Before addressing this it will be useful to recall the realization of the monopole in type IIB string theory. The monopole is a D1-brane suspended between the two parallel D3-branes. This is a BPS configuration in string theory as it preserves eight supersymmetries, just as the $\mathcal{N} = 4$ monopole. The worldvolume theory on D3-branes is the $\mathcal{N} = 4$ gauge theory, and the two diagonal elements of $2\pi \alpha' \langle \phi_1 \rangle = 2\pi \alpha' v \tau^3 / 2$ are identified with the positions of the D3-branes along the external direction. On the D3-brane worldvolume the ends of the D1-brane span the world-line of a particle – the SYM monopole.

Similarly to the instanton case, this realization of the monopole is straightforwardly generalized by T-duality to a $D_p$-brane stretched between two $D(p + 2)$-branes.

The precise correspondence between the boundstate of $k$ D1-branes suspended between the D3-branes in IIB string theory and the $k$-monopole solution in gauge theory was established in \cite{32} by identifying the moduli space of the brane boundstates with the classical moduli space of the Nahm multi-monopole \cite{14} in SYM. In addition, similarly to the instanton case before, the monopole gauge field itself can be read off from the brane configuration using the brane-probe analysis.

This ‘brane within branes’ realization of the monopole in string theory is also confirmed/illustrated via a simple geometrical picture \cite{33}. Since in the vacuum the scalar field $\phi_1$ represents the separation between the D3-branes, the scalar monopole field (2.5) should correspond to the deformation of the D3-branes pulled by the monopole D1-string. Following \cite{33} we show the D-monopole profile by plotting (2.5) in Figure 6. The presence of the cusp where the two D3-branes meet in this picture is interpreted as the D1-string suspended between the two D3-branes. The D3-branes are pulled together by the tension of the D1-string stretched between them, which shrinks to a point.

We can now compare this picture to the D-instanton within the separated D3-branes. Since the gauge theory is now in the Coulomb phase, the vev $v$ is non-zero and we should use the constrained instanton solution (2.7). Plotting the scalar field in (2.7) in Figure 7 we find a qualitatively different picture from Figure 6. There is no cusp at the point
Figure 6: The monopole as the D1-string suspended between two D3-branes. Its tension makes the D1-string shrink to a point and at the same time pulls the D3-branes together as two cusps which are attached to the D-string.

where the D3-branes meet. This is consistent with the fact that the D(−1)-brane is a point-like defect and not a string as in Figure 6. For a general p we see in Figure 8 that the instanton is a Dp-brane parallel to the D(p + 4)-branes which is situated exactly half way between them. The D(p + 4)-branes are attracted by the instanton, but the instanton worldvolume lies entirely along the larger branes. Hence, the instanton is a brane within branes and the monopole is a brane stretched between branes. (For simplicity we will continue referring to both realizations as branes within branes.)

The worldvolumes of D3-branes are smoothly deformed by the instanton between them with the curvature of the deformation determined by the SYM instanton size ρ. It is also known that when ρ → 0 the D(−1)-brane in string theory can escape from the D3-branes [28]. This corresponds to a phase transition from the Higgs to the Coulomb phase of the combined Dp/D(p + 4) gauge theory.

The reader might ask whether the constrained nature of the SYM instanton solution can affect the details of the instanton-brane picture on Figures 7 and 8. The answer is simple: since the short-distance scalar field in (2.7) is purely Abelian, and since the long-distance field is always Abelian, the scalar field in (2.7) can be used at all distances, due to Affleck’s patching conditions [17] between the short- and the long-distance regimes. It should also be added that in the string theory realization of the instanton, in general there is no arbitrariness associated with a choice of Affleck constraints, only the short-distance instanton is relevant for the instanton partition function [19].

We note that the topology of all possible static stable solutions on stacks of parallel D-branes was studied exhaustively by Semenoff and Zarembo in [34]. In the present paper we will need only the well-known examples of such solutions – the D-instantons
Figure 7: D-instanton as the $D(-1)$-brane placed between two $D3$-branes. The $D3$-branes are smoothly deformed and meet at the location of the $D(-1)$.

and monopoles and their T-dual generalizations. As mentioned earlier, our main goal is in studying time-dependent processes involving solutions with opposite charges – brane-antibranes – and the appearance of lower-dimensional branes from brane-antibrane pairs.

Finally, we want to comment on the relation between classical branes and their quantum excitations. Classically, the monopole solution in gauge theory represents a non-trivial vacuum. Particle excitations appear from the lowest lying normal modes when we first-quantize around this vacuum. The eight fermion zero modes \(^{(2.6)}\) are essential in this set-up\(^7\) since their creation operators acting on the monopole vacuum fill in precisely the right number of states to give a vector $\mathcal{N} = 4$ supermultiplet, for a review see e.g. \([35]\). In particular, there are precisely as many particle states in the monopole supermultiplet as in the supermultiplet of the $W$-boson $W^+$. This is one of the key elements \([36]\) in support of the electric-magnetic self-duality conjecture \([37]\) of the $\mathcal{N} = 4$ SYM. When the $\mathcal{N} = 4$ SYM is realized as the worldvolume theory on D3-branes, this electric-magnetic duality becomes a part of the S-duality of type IIB string theory. In particular, monopoles (D1-strings suspended between D3-branes) can be interchanged with electrically charged $W$-bosons (fundamental F1-strings stretched between D3-branes). The D1-strings and the F1-strings are dual to each other as classical extended objects; the duality between the monopole supermultiplet of states and the supermultiplet of $W^+$ then arises from quantizing the D1 and the F1 strings (with the usual difficulty that when the electric

\(^7\)Bosonic zero modes of the monopole also play a role. They are the 3-translations in space and the $U(1)$ global rotations in the unbroken gauge group. When quantized, the latter give the tower of electrically charged dyons which are important for the full $SL(2,\mathbb{Z})$ duality of the theory.
modes are weakly coupled, the magnetic ones are strongly coupled and vice-versa). The analysis in this paper involves classical branes; their particle excitations would arise from the first quantization and will not be relevant for the branes from branes programme we want to pursue.

2.6 Imaginary time and real time processes through the sphaleron barrier

Let us first consider the evolution in imaginary time of parallel (or coincident) D3-branes passing through the D-instanton. The D-instanton is a D(−1)-brane which is located at a point in space and time within the worldvolume swept by the evolution of the 3-space-dimensional D3-branes, as shown in Figure 1. Before the encounter with the instanton, the worldvolume fields on the D3-brane are in a vacuum state. Passing through the instanton, the fields on the D3-worldvolume change, and at a late (Euclidean) time settle to another vacuum state, as illustrated in Figure 9. The two classical vacua are topologically distinct, their \( S^3 \rightarrow S^3 \) winding numbers differ by one. This picture is a brane realization of the gauge theory instanton which describes the tunneling process between topologically distinct vacua. When vev is non-zero, there is a barrier between the vacua (as in Figure 1) which corresponds to the unstable classical solution in gauge theory, the sphaleron [26].

If one attempts to analytically continue the instanton (2.7) to Minkowski space one encounters two immediate problems: (1) the instanton becomes complex and (2) it is not
PSfrag replacements

SYM already by Harvey, Horava and Kraus in [39]. These authors emphasized that this IIA/B theory. Unstable D-branes in string theory have been identified with sphalerons in $p$ in the type IIB theory. By T-dualizing we can add common spatial dimensions to the

decay and produce an exploding shell in the infinite future. This process is illustrated in Figure 10 which is interpreted as the production of an unstable D0 sphaleron brane in the type IIB theory. By T-dualizing we can add common spatial dimensions to the D3 and the D0 world-volumes, producing unstable D$p$-branes with $p$ odd/even in type IIA/B theory. Unstable D-branes in string theory have been identified with sphalerons in SYM already by Harvey, Horava and Kraus in [39]. These authors emphasized that this

point-like anymore, but lives on the lightcone.

A better way to describe real time instanton-like processes is to look for genuine Minkowski-space classical solutions which change the vacuum. A class of such solutions with finite energy was found in [38]. These solutions describe spherical shells of radiation first imploding, collapsing and then expanding in time. The region inside and outside the shell is a vacuum. As the solution evolves, the vacuum inside the shell can change to a topologically different vacuum (i.e. vacuum changes before and after the collapse of the shell). This process inside the D3-branes is illustrated in the second picture on Figure 9. The time-dependent solution lives on the light-cone (in the absence of vevs), in this way it resembles the Wick-rotated instanton, but there are no complexities in genuine real-time solutions.

When the $\mathcal{N} = 4$ theory is in the Coulomb phase one can fine-tune the incoming classical radiation to pass precisely through (or close to) the top of the spaleron barrier separating different vacua. In the infinite past one would start with an imploding classical solution which at time zero will collapse to the sphaleron. The spahleron will eventually decay and produce an exploding shell in the infinite future. This process is illustrated in Figure 10 which is interpreted as the production of an unstable D0 sphaleron brane in the type IIB theory. By T-dualizing we can add common spatial dimensions to the D3 and the D0 world-volumes, producing unstable D$p$-branes with $p$ odd/even in type IIA/B theory. Unstable D-branes in string theory have been identified with sphalerons in SYM already by Harvey, Horava and Kraus in [39]. These authors emphasized that this

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identification leads to a highly nontrivial vacuum structure of string theory.

2.7 Interpretation of the non-contractible monopole loop in imaginary time

Returning now to the monopole-antimonopole loop of section 2.4, we recall that the loop parameter $\tau$ should not be thought of as the real time $x_0$. The real time dimension will be added to the loop in Section 3.

The non-contractible monopole loop is interpreted as a classical ‘process’ of instanton creation in imaginary time $\tau = x_4$. More precisely this loop is a blow-up of the instanton which shows the instanton constituents. Since the instanton is a (Euclidean) time-dependent configuration, it should be thought of as a ‘process’ in $x_4$ rather than a ‘particle’. The monopole loop is the instanton and it shows that the instanton ‘process’ is made out of constituent monopole particles which are first created from the vacuum and then annihilate each other in imaginary time. It was suspected for a long time [40] that in gauge theories instantons should be thought of as composite states of more basic configurations referred to as ‘instanton quarks’. We conclude that the monopoles are the instanton quarks.

In a context, when one of the dimensions is finite, this conclusion was tested via explicit calculations of the gluino condensate in our earlier work [41, 42]. Note that there is an alternative way to construct a non-contractible monopole loop when there is a compact dimension. Instead of introducing the Taubes winding by $U(1)$-gauge rotating the
monopole, one can wind the monopole worldline along a compact direction \[44, 43, 45, 41\]. Such a non-contractible monopole loop also gives rise to an instanton solution in agreement with Taubes arguments. More concretely, in \[43\] it was verified that the instanton on partially compactified D-branes is a composite configuration made of monopoles with the net magnetic charge zero and with one unit of winding along the compactified worldvolume direction. In \[45\] the periodic instanton in high-temperature QCD was identified as a composite monopole-antimonopole configuration. In Ref. \[41\] the instanton solution in $\mathcal{N} = 1$ pure SYM on $\mathbb{R}^3 \times S^1$ was decomposed into its constituents: magnetic monopoles (with unit net winding around $S^1$ and vanishing net magnetic charge). On $\mathbb{R}^3 \times S^1$ these monopoles have finite Euclidean action, since the dimension compactified on $S^1$ is finite. These monopoles are the elementary semiclassical configurations contributing to the path integral. The gluino condensate can be calculated exactly on these monopole configurations, and the results \[41, 42\] are in complete agreement with the known values\(^8\) of the gluino condensate for all classical gauge groups.

Our SYM construction of the non-contractible monopole-antimonopole loop is embedded into type IIB string theory in an obvious way following the discussion in the previous subsection. It is realized as the D1-brane–D1-antibrane non-contractible loop suspended between two D3-branes. The worldvolumes of the D3-branes are Euclidean i.e. the D3-branes are S-branes \[10\] spanning the $x_1, \ldots, x_4$ purely spatial dimensions. The loop parameter is identified with the spatial dimension $x_4$, hence, D1-branes are also S-branes. This D1-brane–D1-antibrane non-contractible loop within two D3-branes is sketched in Figure 11. By minimizing the action of the brane-configurations in Figure 12 we obtain the D(−1)-brane within the D3-branes – the string instanton.

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\[\begin{array}{c}
\begin{array}{c}
\text{D1-brane} \\
x_3
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\text{D1-antibrane} \\
x_4
\end{array}
\end{array}\]

Figure 11: The D1-brane–D1-antibrane non-contractible loop as seen from the worldvolume of one of the D3-branes. Red lines trace the ends of the D1 and anti-D1 strings as they evolve in $x_4$. The circular arrow denotes the $U(1)$ twist of the D1-brane in the D3-brane worldvolume.

\(^8\)Meaning the correct values obtained in the weakly coupled theory – the WCI results – see \[41\] for more detail.
3 D(p − 2)-branes from Dp-brane–Dp-antibrane annihilation

3.1 D0-brane from D2-brane–D2-antibrane annihilation

Now we are ready to add a real time dimension to the purely Euclidean considerations of the previous section. Let us consider the deformation of Figure 11 depicted on Figure 12. This Figure can be seen as a sequence of snap-shots in time starting at $t = 0$ with Figure 11, and evolving backwards in time to the far separated D2-brane and D2-antibrane at $t = -T_0$. The Euclidean time evolution of the monopole-antimonopole configuration of the previous section is now replaced by the extend of D2-branes in the spatial $x_4$ direction.

The time-dependent process we want to consider is the annihilation of the D2- and the anti-D2-branes suspended between two fixed D4-branes. At an early time, $t = -T_0$, the 2-branes describe two parallel two-dimensional surfaces in $(x_4, x_9)$ at a large separation from each other along, say, $x_3$. The two D4-branes span three spatial dimensions $(x_1, x_2, x_3)$ and are located at a fixed distance $2\pi\alpha'v$ away from each other along $x_9$. We further require that this initial configuration is prepared in such a way that one of the D2-branes is gauge-rotated in the D4-worldvolume by a U(1) gauge transformation $U(x_4)$ with the winding number one.\(^9\) This gauge twisting cannot be removed with a global $U(1)$ gauge transformation in the D4 worldvolume as it corresponds to a relative gauge orientation factor of the D2-anti-D2 pair.

Evolving this initial configuration forward in time, the D2-brane annihilates the D2-antibrane at $t = 0$, and leaves behind (amid perturbative radiation) a D0-brane whose worldline is along the $t \geq 0$ ray in the COM frame of the collision as in Figure 8. The D0-worldline is parallel to the D4-worldvolumes, it is the instanton of subsection 2.5. The D0-brane appears as a topological soliton in the worldvolume theory of the D4-branes from the process of the D2-anti-D2 annihilation. The RR-charge of this D0-brane is the winding number of the gauge twist of the D2-anti-D2 pair.

In the approach of Sen \(^2\) the D0-brane is the topological soliton or kink of the complex tachyon field which lives on the worldvolume of the D2-brane-antibrane pair. The appearance and stability of the kink depend crucially on the hypothesis of the tachyon condensation and on the form of the conjectured tachyon potential. In our approach we choose to work on the worldvolume of the external D4-branes, and the tachyon, which lives on the worldvolume of the D2-anti-D2 pair, does not make an appearance. As already stated earlier, the D0-brane charge in our approach originates from the D2 gauge twisting on the D4-worldvolume.

\(^9\)By winding number $n$ we mean $U(x_4) = \exp[i\Lambda(x_4)\tau^3/2]$, with $\Lambda(\infty) - \Lambda(0) = 2\pi n$. 

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Figure 12: Annihilation of the D2-anti-D2 brane configuration suspended between D4-branes. Due to the $U(1)$ twist, denoted by the circular arrow, this annihilation process leads to a formation of the D0-brane at $t > 0$.

In general, it is clear that if one requires the infinite ends of the D2 and the anti-D2 branes to completely annihilate each other at time $t > 0$, the $U(1)$ gauge twisting must have an integer (or vanishing) winding number. Another way to think about it is to imagine the $x_4$ dimension being compactified. If one does not wish to impose such a restriction, it is natural to expect that as $t \to 0$ and the D2-branes approach each other, a generic gauge twisting will cluster into a product of gauge transformations, each with a support in a local region and an integer winding number in this region. D0 and anti-D0 branes will be produced locally in these local regions with the charges prescribed by the local winding numbers. The ends of the D2-branes at infinity (in $x_4$) would not match and will live some radiation debris.
3.2 Generalization to $D_p$-brane–$D_p$-antibrane pairs suspended between $D(p + 2)$-branes

The treatment of higher dimensional cases is a straightforward generalization of the picture developed in the previous subsection. It proceeds by adding $p - 2$ common spatial dimensions to all of the worldvolumes of the branes and antibranes involved.

The resulting process in the type II string theory is a mutual annihilation of a $D_p$-brane with a $D_p$-antibrane suspended between two $D(p + 2)$-branes and with a non-trivial $U(1)$ net winding along the $x_4$ direction. After the annihilation one is left with $D(p - 2)$-branes within the spectator $D(p + 2)$-branes. The $D(p - 2)$-charges are determined by the $U(1)$ winding in the $D(p + 2)$-worldvolume.

The highest dimension we can address in this way is dictated by the dimensionality of the $D(p+2)$-branes. In type IIA theory we can have a D6-anti-D6 pair suspended between two D8-branes and leading to stable D4-branes within the same spectator branes. We denote this process as,

$$\text{IIA} : D_8 - [D_6 - \overline{D}_6] \rightarrow D_8 - [D_4] - D_8. \quad (3.1)$$

By successive applications of T-duality in type II string theory (or by dimensional reduction in the 9-dimensional gauge theory) we can access all the lower cases up to,

$$\text{IIB} : D_3 - [D_1 - \overline{D}_1] \rightarrow D_3 - [D(-1)] - D_3. \quad (3.2)$$

Before we list classify the brane descent relations obtained in this way we want to pause to examine the process which is S-dual to the last equation,

$$\text{IIB} : D_3 - [F_1 - \overline{F}_1] \rightarrow D_3 - [\tilde{D}(-1)] - D_3. \quad (3.3)$$

3.3 S-duality in IIB and the S-dual of the D-instanton

The type IIB superstring theory is believed to be invariant under $SL(2, Z)$ duality transformations \[47\] \[48\] \[49\]. We will need to consider here only the S-duality generator of this group which interchanges the fundamental string, $F_1$, with the D1-string. The fate of other branes in type IIB is as follows: the D3-brane is mapped to itself and the D5-brane is interchanged with the solitonic NS-5 brane,

$$D_1 \rightarrow \tilde{D}_1 = F_1, \quad F_1 \rightarrow \tilde{F}_1 = D_1, \quad (3.4)$$
$$D_3 \rightarrow \tilde{D}_3 = F_3, \quad (3.5)$$
$$D_5 \rightarrow \tilde{D}_5 = NS_5, \quad NS_5 \rightarrow \tilde{NS}_5 = D_5. \quad (3.6)$$
One way to derive these relations is by calculating tensions of these branes,

\[ \tau_{Dp} = \frac{1}{(2\pi)^p \alpha'^{p+1} g_{st}} , \quad \tau_{F1} = \frac{1}{2\pi \alpha'} , \quad \tau_{NS5} = \frac{1}{(2\pi)^5 \alpha'^3 g_{st}^2} , \] (3.7)

and using the S-duality dictionary

\[ g_{st} \to \tilde{g}_{st} = \frac{1}{g_{st}} , \quad \alpha' \to \tilde{\alpha}' = \alpha' g_{st} , \] (3.8)

to equate them, see e.g. [50, 51].

An interesting question to ask is what is the S-dual of the D(−1)-brane (and similarly of its magnetic dual D7 brane). We will argue now that the S-dual of the D-instanton is a new (−1)-brane in the type IIB theory, which we denote as D(−1) (and similarly there is a second 7-brane D7). First, it is obvious from the first equation in (3.7) and the dictionary (3.8) that the tensions of the D(−1)-brane, and of the D7 brane are different from the D(−1) and the D7 tensions,

\[ \tau_{D(1)} = \frac{2\pi}{g_{st}} = 2\pi g_{st} \] , \[ \tau_{D(1)} = \frac{2\pi}{g_{st}} , \] (3.9)

\[ \tau_{D7} = \frac{1}{(2\pi)^7 \alpha'^4 g_{st}} \] , \[ \tau_{D7} = \frac{1}{(2\pi)^7 \alpha'^4 g_{st}} . \] (3.10)

It also appears that the S-dual of the instanton, the D(−1)-brane, has a ‘perturbative’ tension \( \propto g_{st} \) in terms of the parameters of the original theory. Are there really two types of (−1)-branes in type IIB?

If we look at the D-instanton as a classical supergravity solution [52], we discover that it is mapped to itself under the S-duality transformation

\[ \tau(x) \longrightarrow -\frac{1}{\tau(x)} \] , \text{ where } \tau(x) = C^{(0)}(x) + i e^{-i\phi(x)} . \] (3.11)

In other words, it appears that there is only one type of D-instanton solution in type IIB supergravity. Technically, the instanton components of the dilaton, \( \phi(x) \), and of the RR-scalar field, \( C^{(0)}(x) \), change non-trivially under (3.11), but when they are combined into \( \tau(x) \), simplifications occur. The instanton solution in [52] is constructed in such a way that the complexified scalar field \( \tau(x) \) is actually a constant, \( \tau_{D-{\text{inst}}}(x) = \langle \tau \rangle \), and the spacetime-dependent contributions in \( \phi(x) \) and \( C^{(0)}(x) \) cancel each other. Hence, there is only one instanton solution in supergravity, but under the S-duality transformation, the asymptotic value of \( \tau(x) \) changes,

\[ \langle \tau \rangle \longrightarrow \langle \tilde{\tau} \rangle = -\frac{1}{\langle \tilde{\tau} \rangle} . \] (3.12)

S-duality is not a symmetry of the theory. Under the transformation (3.11) one supergravity formulation goes to a different one. Both formulations have the same Lagrangian,
but different values of $\langle \tau \rangle$ which are related via (3.12). These two theories describe the same physics in terms of different degrees of freedom, $\tau(x)$ in the first version, and $\tilde{\tau}(x)$ in the S-dual version. There is an instanton solution in each of these theories, which has the same algebraic form when expressed in terms of the fundamental fields of each theory. But the D-instanton and its S-dual are different objects with different tensions, in agreement with (3.9).

Exactly the same conclusion is reached in the $\mathcal{N} = 4$ SYM case (which of course is another simple limit of the type IIB string theory). There is an instanton solution (2.7) in the original formulation of the theory in terms of $A_\mu$ and $\phi$. The S-dual formulation of this theory is in terms of $\tilde{A}_\mu$ and $\tilde{\phi}$, which describe monopole degrees of freedom. This theory too has the instanton solution (2.7) but in terms of $\tilde{A}_\mu$ and $\tilde{\phi}$. The actions of these two solutions are related again via (3.9).

We get an insight into the nature of the second instanton by S-dualizing the original instanton when it is viewed as a composite configuration of monopoles or, equivalently D1-anti-D1 branes. The S-dual of the instanton is then the F1-anti-F1 configuration suspended between two self-dual D3-branes (with the $U(1)$-twist),

$$IIB : \text{D3} \rightarrow [\text{F1} - \text{F1}] - \text{D3} \rightarrow \text{D3} \rightarrow [\tilde{D}(-1)] - \text{D3}. \quad (3.13)$$

Since our approach is classical in nature, we think of fundamental strings F1 here as classical solutions to the Born-Infled action in the $\mathcal{N} = 4$ gauge theory, the BIons [53, 54, 55]. The classical BIons are S-dual to classical monopoles as F1-strings are S-dual to D1-strings. We conclude that there is an S-dual instanton, $\tilde{D}(-1)$, made of dual instanton quarks, which are $W^+$ and $W^-$ bosons.

### 3.4 Stable brane descent relations in type II

By successive applications of T-duality of type II theory to (3.1) and by using S-duality in type IIB theory as in the previous section, we can have a variety of processes describing the production of stable branes. These processes are summarized in Table 1.

The brane descent relations in Table 1 are in agreement with those obtained from Sen’s tachyon condensation approach [2, 6, 3], except that we cannot describe annihilations of D9, D8 and D7 branes. The highest dimension we can address is restricted by the dimensionality of spectator branes.

It might be tempting to dispose of the spectator branes altogether, but then we would not be able to use the SYM language of their worldvolume theory to describe annihilations of lower-dimensional branes. The spectator branes are certainly not necessary for brane-antibrane annihilations to occur and for smaller branes to be produced in string theory.
One would just need to use a different approach to describe this – the original tachyon condensation conjecture of Sen [2]. Our approach provides a complimentary picture to [2]. Using our method we are also able to gain insight into non-Dirichlet stable branes and the S-dual of the D-instanton.

4 Comments and open questions

We explained how stable \( q \)- and unstable \((q + 1)\)-branes in string theory appear from \((q + 2)\)-brane-antibrane pairs in the background of two stable \((q + 4)\)-branes (here \( q \) is even/odd for type IIA/B theory). These lower-dimensional branes appear as classical configurations in the SYM worldvolume theory on the \( D(q + 4)\)-branes.

More specifically, we argued that stable \( q \)-branes are produced in annihilation processes of \((q + 2)\)-branes with \((q + 2)\)-antibranes, and that their RR charge is carried by the \( U(1) \) winding of the \((q + 2)\)-brane in the \((q + 4)\)-brane worldvolume. The \( q \)-branes are stable BPS configurations and their RR charges are under control.\(^{10}\) The string theoretical interpretation of the \( U(1) \) winding is as follows: the \( U(1) \) gauge transformation of the \((q + 2)\)-branes corresponds to a rotation of the strings stretching between the \((q + 2)\)-

\(^{10}\)It is known [30] that the RR-charge of the \( Dq \)-brane within \( D(q + 4)\)-branes coincides with the instanton charge [24], and that the latter does appear from the non-contractible monopole-antimonopole loop [9] as discussed in subsection 2.4.

Table 1: Brane-antibrane annihilations to lower-dimensional branes.

| spectator branes | brane-antibrane | lower brane |
|------------------|-----------------|------------|
| \( 2 \times D8 \) | \( D6 - \overline{D6} \) | \( D4 \) |
| \( 2 \times D7 \) | \( D5 - \overline{D5} \) | \( D3 \) |
| \( 2 \times \tilde{D}7 \) | \( NS5 - \overline{NS5} \) | \( D3 \) |
| \( 2 \times D7 \) | \( NS5 - \overline{NS5} \) | \( D3 \) |
| \( 2 \times \tilde{D}7 \) | \( D5 - \overline{D5} \) | \( D3 \) |
| \( 2 \times D6 \) | \( D4 - \overline{D4} \) | \( D2 \) |
| \( 2 \times D5 \) | \( D3 - \overline{D3} \) | \( D1 \) |
| \( 2 \times NS5 \) | \( D3 - \overline{D3} \) | \( F1 \) |
| \( 2 \times D4 \) | \( D2 - \overline{D2} \) | \( D0 \) |
| \( 2 \times D3 \) | \( D1 - \overline{D1} \) | \( D(-1) \) |
| \( 2 \times D3 \) | \( F1 - \overline{F1} \) | \( \overline{D}(-1) \) |
brane and the two \((q+4)\)-branes, such that the strings ending on the upper and the lower \((q+4)\)-brane are rotated in opposite directions. What needs to be understood better is the relation of this \(U(1)\)-winding to the winding of the complex tachyon field in the ‘Mexican Hat’ tachyon potential proposed by Sen \[2\].

Tachyonic modes on the \((q+2)\)-brane–\((q+2)\)-antibrane worldvolume do not appear in our analysis since we work on the worldvolume of the spectator \((q+4)\)-branes. The forces between \((q+2)\)-branes and \((q+2)\)-antibranes at large separations are the standard brane-antibrane forces, as in \[5\], and in the presence of the \(D(q+4)\) spectator branes these forces are fully reproduced in gauge theory on the \(D(q+4)\)-worldvolume.

At small separations of the \((q+2)\)-brane–\((q+2)\)-antibrane the same physics can be described in two very different languages.

(1) In the language of tachyon condensation \[2\], a new tachyon channel opens up between the brane and the antibrane and the tachyon condensation describes the annihilation of the brane-antibrane pair into the true vacuum. The unstable configuration at the top of the tachyon potential corresponds to the coincident brane-antibrane configuration, and the ground state is the closed string vacuum. Lower-dimensional branes appear as solitons in the tachyon field.

(2) The second language is the gauge theory on the spectator \(D(q+4)\)-branes used in this paper. All other lower-dimensional branes are realized as classical solutions in this theory. At small separations, the fields representing the \((q+2)\)-brane and the \((q+2)\)-antibrane start eating each other leading to a destruction of the \((q+2)\)-brane-antibrane configuration. If the \(U(1)\)-winding is trivial, the destruction is complete and one ends up in the vacuum. For a non-vanishing \(U(1)\)-winding, one is left with topological solitons describing stable \(D(q)\)-branes within the spectator \(D(q+4)\)-branes.

In the tachyon condensation approach the brane and the antibrane are not modified even at zero separations. Instead, the tachyon channel opens up and the configuration becomes unstable. In the branes within branes approach, there are no tachyons, but the brane and the antibrane partially or completely annihilate each other as classical objects. It would be very interesting to understand better the precise relation between these two approaches.

Finally, we would like to comment on unstable branes. Unstable branes arise in our approach as unstable classical solutions in the worldvolume gauge theory. It should be clear from sections 2.3 and 2.4 that these sphaleron \(D(q+1)\)-branes are directly related to both: the monopole \(D(q+2)\)-branes, and the instanton \(Dq\)-branes. The existence of unstable \(D(q+1)\)-branes follows from the minimax procedure applied to the non-contractible \(D(q+2)\)-loop, and they represent the saddle-point configurations on top of the barrier under which the Euclidean \(Dq\)-branes tunnel. Whenever \(D\)-instanton \(S\)-branes are present, there exists a finite-energy real-time process which produces a sphaleron
solution of one dimension higher as in Figure 10. Stable D-branes can be produced in real-time collisions of higher branes as explained in section 3 and in Figure 12. However, unstable sphaleron-branes do not appear directly in these processes. Unstable branes in string theory have been identified with the SYM sphaleron solutions already in [39, 56], but as these branes carry no RR charges and are not BPS-protected, there is not much we can infer from the SYM side about them, apart from their existence.

The derivation of descent relations between the stable branes summarized in section 3.4 is one of the main results of this paper. These relations are in agreement with general K-theory considerations [6]. Two features of these relations are particularly interesting. First, is that the same \((q + 2)\)-brane-\((q + 2)\)-antibrane pair can produce different \(q\)-branes depending on the type of spectator branes used as in

\[
\text{D5} - [\text{D3} - \overline{\text{D3}}] - \text{D5} \rightarrow \text{D5} - [\text{D1}] - \text{D5}
\]

\[
\text{NS5} - [\text{D3} - \overline{\text{D3}}] - \text{NS5} \rightarrow \text{NS5} - [\text{F1}] - \text{NS5}
\]

It is not clear what would distinguish between these two processes in the absence of the spectator branes, and how the second process would arise in Sen’s approach.

The second interesting feature is the appearance of the S-dual of the instanton (and also of its magnetic dual) as in

\[
\text{D3} - [\text{D1} - \overline{\text{D1}}] - \text{D3} \rightarrow \text{D3} - [\text{D}(1)] - \text{D3}
\]

\[
\text{D3} - [\text{F1} - \overline{\text{F1}}] - \text{D3} \rightarrow \text{D3} - [\tilde{\text{D}}(1)] - \text{D3}
\]

and its interpretation in 3.3 as a non-contractible BIon-anti-Bion loop. The S-dual instanton \(\tilde{\text{D}}(1)\) is a point-like object with a perturbative action. Though a classical solution in the dual theory, it would be interesting to find its fully quantum interpretation in the original theory.

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