Electro-magnetic Aharonov-Bohm effect in a 2-D electron gas ring

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We define a mesoscopic ring in a 2-dimensional electron gas (2DEG) interrupted by two tunnel barriers, enabling us to apply a well-defined potential difference between the two halves of the ring. The electron interference in the ring is modified using a perpendicular magnetic field and a bias voltage. We observe clear Aharonov-Bohm oscillations up to the quantum Hall regime as a function of both parameters. The electron transport time between the barriers is found to increase with the applied magnetic field. Introducing a scattering model, we develop a new method to measure the non-equilibrium electron dephasing time, which becomes very short at high voltages and magnetic fields. The relevance of electron-electron interactions is discussed.

Aharonov and Bohm predicted that the phase of an electron wave is affected by both magnetic and electric potentials, being observable in an interference experiment [1]. A magnetic flux, Φ, threading a loop leads to an oscillating contribution to the conductance with period Φ0 = h/e: the magnetic Aharonov-Bohm (AB) effect. In a solid state device, AB oscillations are observable at mesoscopic scale, where quantum coherence is preserved. The effect was first observed in a metal ring [2] and later in 2DEG rings [3].

In the case of the electrostatic AB effect, as originally suggested, the electron phase is affected by an electrostatic potential, although the electrons do not experience an electric field [6]. The required geometry is difficult to realize and so far only geometries have been investigated where an electric field changes the interference pattern [4,5]. In Ref. [7] a ring geometry interrupted by tunnel barriers was suggested, where a bias voltage, V, leads to an electrostatically controlled AB effect with predictions very similar to the original proposal [1]. This effect was observed in a disordered metal ring with two tunnel junctions [6].

In this Letter, we report the observation of an electrostatic AB effect in a quasi-ballistic ring-shaped 2DEG system interrupted by tunnel barriers. There are two important distinctions from metal systems that play a key role in the present study. First, only a few electron modes are involved in transport, in contrast to hundreds or thousands in a typical metal wire. Second, in a 2DEG we can enter the quantum Hall regime using a perpendicular magnetic field, B.

We observe both a magnetic and electrostatic AB effect up to the edge channel regime (filling factor ν = 3). The electron transport time between the barriers increases with B. We ascribe this to the bending effect of the Lorentz force on the electron trajectories. Importantly, the electrostatic oscillations form a new tool to determine the non-equilibrium electron dephasing time, τφ. The possibility to estimate the electron dephasing time at a controlled, finite energy and magnetic field distinguishes this method from weak localization experiments used to estimate the (equilibrium) electron dephasing time [4].

![Image](https://example.com/image.png)

**FIG. 1.** (a) Atomic force microscope image of the device. The AB ring is defined in a 2DEG by dry etching (dark regions, depth ∼75 nm). The 2DEG with electron density 3.4 × 10^15 m^−2 is situated 100 nm below the surface of an AlGaAs/GaAs heterostructure. In both arms of the ring (lithographic width 0.5 µm; perimeter 6.6 µm) a barrier can be defined by applying negative voltages to the gate electrodes Vgl and Vgr. The other gates are not used. (b) Schematic energy picture at one of the barriers (see text). (c) Differential AB conductance, G_AB, as function of B at 15 mK for different source drain voltages, V, separated by 5.3 µV. The traces have a vertical offset. Three values for V are indicated on the right. The dashed line highlights a phase change by a change in the amplitude and sign of the AB oscillations, indicating an electrostatic AB effect.
We find that $\tau_0$ decreases as a function of $B$ and $V$ and reaches $\tau_0|\varepsilon| \approx \hbar$ at the highest $B$, with $\varepsilon$ the electron energy measured from the Fermi level. This demonstrates the increasing importance of many-body effects at quantizing magnetic fields where the Fermi-liquid picture and the quasi-particle concept are at the edge of applicability.

Our device (Fig. 1a) consists of an AB ring defined in a 2DEG [10]. The gate electrodes $V_{gu}$ and $V_{gl}$ define a barrier in each arm of the ring. The effective width of the arms supports $N_e \approx 10$ transport channels (i.e. the conductance of the ring without barriers is approximately $10e^2/h$ at $B = 0$ T). In addition to the dc bias voltage, $V$, we apply a relatively small ac voltage (3 to 10 $\mu$V) between source and drain contacts. We measure the differential conductance, $G = dI/dV$, using a lock-in technique in a dilution refrigerator with a base temperature of 15 mK.

Figure 1c shows the differential AB conductance, $G_{AB}$, versus $B$ and $V$, around 1.9 T. $G_{AB}$ is extracted from the measured $G$, by fitting a polynomial to the smoothly varying background in $G$. Subtraction of this polynomial yields the oscillating part, $G_{AB}$. The barriers are tuned such that the conductance through one arm is $0.2 e^2/h$. $G_{AB}$ as function of magnetic field exhibits AB oscillations with a period $B_0 = 1.0$ mT, in good agreement with $h/(eS)$, where $S$ is the area enclosed by the ring. The effect of the dc bias voltage, $V$, is schematically shown in Fig. 1b. The transmitted part of the electron wave has energy $\varepsilon_R$, whereas the reflected part has $\varepsilon_L$ with respect to the Fermi level in each half of the ring (note $eV = \varepsilon_R - \varepsilon_L$ and $|\varepsilon_L|, |\varepsilon_R|, |eV| \ll E_F$). The energy difference leads to a phase difference $\Delta \Phi_V = 2\pi V t_0/h$, where $t_0$ is the ($B$-dependent) time the reflected and transmitted electron waves spend in their respective parts of the ring before they interfere [7]. We thus expect electrostatic AB oscillations by changing $V$.

The dashed line in Fig. 1c highlights the effect of the applied dc voltage, $V$. The red curve at $V = 48 \mu$V has a minimum when it crosses the dashed line. By decreasing $V$ the amplitude of the AB oscillations decreases. The two blue curves near $V = 24 \mu$V show a case where the amplitudes are small and the sign of the amplitude changes. At $V = 0 \mu$V, the amplitude is maximal. Comparing the 48 and 0 $\mu$V traces, the AB oscillation has acquired a change in phase by $\pi$. This continues by another phase shift of $\pi$ when $V$ is decreased to -42 $\mu$V. Thus, the electrostatic period, $V_0$, is 90 $\mu$V at this magnetic field. This corresponds to an electron travel time $t_0 = 45$ ps. We note that the Onsager-Büttiker symmetry relation $G(B) = G(−B)$ for a 2-terminal measurement [1] only holds at small bias voltages. Therefore, the phase of the oscillations at the field of the dashed line is not restricted to 0 or $\pi$.

Figure 2 shows a color scale plot of $G_{AB}$ versus $B$ and $V$ for five different $B$ ranges. The main features of the results are as follows. The amplitude of the AB oscillations is relatively small ranging from $\sim 0.01G$ to $\sim 0.1G$. The period of the oscillations in $G_{AB}(V)$ decreases with $B$, as is clearly seen by comparing the different panels in Fig. 2. For $B = 0.4$ T, 1.9 T, 3.6 T, and, less clearly, for $B = 4.7$ T, the phase of the magnetic oscillations exhibits sharp $\pi$-shifts at lower voltages. At higher voltages, the phase changes more smoothly. At $B = 2.5$ T, the phase changes smoothly over the entire voltage range. We return to the phase evolution at different $B$ and $V$ later, when we discuss our scattering model.

The AB amplitude decreases with $V$ and, the higher $B$, the faster the decrease. The decreasing AB amplitude is clearly seen in Fig. 3 where we plot the normalized AB conductance, $G_{AB}(V)/G_{AB}(V = 0)$, versus $V$ at $B = 1.9$ T. We stress that Fig. 3 is obtained by taking a single horizontal cut, i.e. for a single $B$ with no averaging [2], from the $B = 1.9$ T panel of Fig. 2 (but over a larger voltage range). Note that $G_{AB}(V)/G_{AB}(V = 0)$ can be negative since it only represents the oscillating part of the total conductance. The curve is approximately symmetric in $V$ ($G_{AB}(V) \approx G_{AB}(−V)$), indicating that within the range of applied voltages the electron travel time, $t_0$, hardly changes with $V$. Thus, the electrostatic period does not vary within the applied bias window and we cannot attribute the decrease with $V$ in Fig. 3 to self-averaging of trajectories with different periods. We
therefore believe that the only possible mechanism for the decrease is electron dephasing due to inelastic processes.

The small relative amplitude of the AB oscillations indicates strong elastic scattering in the arms, whereas the quickly decreasing AB amplitude with V indicates strong inelastic scattering. Since these scattering mechanisms were not included in the model of Ref. 14, we introduce a simple interference model including dephasing, proceeding along the lines of the general scattering formalism 14.

We assume single-channel tunnel junctions with transmission amplitudes \( t_1 \) and \( t_2 \), respectively. A small AB amplitude implies that the amplitudes of propagating waves between the junctions, \( r_{12}^L \) or \( r_{12}^R \) (see Fig. 4), are small so that we only consider scattering processes of first order in \( r \). The oscillating part of the transmission probability, \( T_{AB} \), is built up from the pairwise interference of the electron trajectories shown in Fig. 4. Collecting the contributions at a given energy, we obtain

\[
T_{AB} = 2\text{Re}(t_1^* t_2 e^{i\Phi_{AB}} (r_{12}^L + r_{12}^R))
\]  

We assume right-left symmetry 14 and specify the energy dependence of the propagation amplitudes as follows

\[
r_{12}^L(21) = r_{12}(+e^{i\pi t_0/h}); \quad r_{12}^R(21) = r_{12}(-e^{i\pi t_0/h})
\]  

where \( r_{12}(+) \) correspond to (counter)clockwise propagation of electrons along the ring (remind that \( eV = \varepsilon_R - \varepsilon_L \)). For the differential conductance this yields (assuming \( t_1 = t_2 \))

\[
G_{AB}(V)/G = |r_-|^2 \cos(eVt_0/h) \cos(2\pi\Phi/\Phi_0 + \Phi_a) + |r_+|^2 \cos(eVt_0/h) \cos(2\pi\Phi/\Phi_0 + \Phi_b) + 2|r_+|r_- \cos(eVt_0/h + \Phi_c) + \sin(eVt_0/h + \Phi_c)eVt_0/h \cos(2\pi\Phi/\Phi_0 + \Phi_d)
\]  

where \( \Phi_{a,b,c,d} \) are contributions to the phase that vary slowly with \( B \) in comparison to \( \Phi/\Phi_0 \) (e.g. due to bending of the electron trajectories). At small \( B \) time reversibility holds, implying \( r_+ = r_- \) and \( \Phi_{a,b,d} = 0 \). Under these conditions, the phase of the magnetic oscillations only changes by \( \pi \) as a function of \( V \). Interestingly, the same happens at high \( B \). In this case Lorentz bending suppresses counterclockwise propagation, so that \( r_+ \gg r_- \). The result is then dominated by the second term in Eq. 3. For a B-interval in which \( \Phi_b \) can be considered constant, we find again that the phase of the magnetic oscillations only changes by \( \pi \) as a function of \( V \). In the intermediate regime \( r_+ \approx r_- \), and \( \Phi_{a,b,c,d} \) vary slowly with \( B \). The phase of the magnetic oscillations in this case continuously shifts with voltage, saturating at \( V \gg h/\tau_\phi e \).

To model electron dephasing due to inelastic processes, we assume that the electron amplitudes at a given energy are suppressed by a factor \( \exp(-t/2\tau_\phi) \), \( \tau_\phi(e) \) being the energy-dependent dephasing time. In analogy with results found for a disordered electron gas in the quantum Hall regime 15, disordered metal-like systems 16, composite fermions 17 and a Luttinger liquid 18, we propose a dephasing time proportional to energy, \( h/\tau_\phi(e) = 2\alpha e \). Here \( \alpha \) is a dimensionless factor of the order of the dimensionless 2DEG conductance, \( G/(e^2/h) \). A small value for \( \alpha (\alpha \ll 1) \) and large conductances \((G \gg e^2/h) \) correspond to vanishing electron-electron interactions 14. On the contrary, \( \alpha \approx 1 \) and \( G \approx e^2/h \) signal the importance of many-body effects. In the limit of \( r_+ \gg r_- \) our model for the dephasing gives

\[
G_{AB}(V)/G = |r_+|^2 \cos(eVt_0/h) - \alpha \sin(eVt_0/h)\cos(2\pi\Phi/\Phi_0 + \Phi_b)
\]  

Our scattering model provides an explanation for the observed experimental results. The model accounts for the abrupt \( \pi \)-phase changes observed in Fig. 2 at the lowest \( (B = 0.4 \text{ T and } 1.9 \text{ T}) \) and highest \((B = 3.6 \text{ T and } 4.7 \text{ T}) \) magnetic fields (provided \( V \) is small). In the intermediate regime \((B = 2.5 \text{ T}) \) the model explains also why the phase varies more continuously. The well-defined period \( t_0 \) probably indicates the formation of an edge channel connecting the junctions. However, the magnitude of the AB oscillations is small. This signals a strong scattering to and from the edge channel involving almost localized states at higher magnetic field. One can estimate \( |r_+|^2 \) as a classical probability, assuming uniform distribution

\[
\text{FIG. 3. Normalized AB conductance, } G_{AB}(V)/G_{AB}(V = 0), \text{ versus } V \text{ at } B = 1.9 \text{ T. The amplitude decreases rapidly with } V \text{ and, contrary to the magnetic AB effect, only a few oscillations can be observed. The red curve is a fit with Eq. 4, } \alpha = 0.26, \text{ to } t_0 = 45 \text{ ps. The left inset shows the values of the travel time } t_0 \text{ versus } B \text{ as extracted from fits with Eq. 4, varying from } 18 \text{ ps at } B = 0.4 \text{ T to } 60 \text{ ps at } B = 4.7 \text{ T. (L}/e\text{F} = 13 \text{ ps, with } L \text{ the distance between the barriers and } e\text{F} \text{ the Fermi velocity). The values of } t_0 \text{ in the edge channel regime are fitted to a line with a slope of } 6 \text{ ps/T. The right inset shows the extracted values for } \alpha = h/2e\tau_\phi(e). \text{ The values of } \alpha \text{ increase with } B, \text{ indicating an increasing importance of many-body effects at quantizing magnetic fields.}
\]
over the transport channels in the ring and 1/3 suppression due to scattering near the openings to the source and drain leads. This gives $|r_{e}|^2 \approx 1/3N_{tr} = 0.03$, which is in agreement with the experimental value of $G_{AB}(V = 0)/G \approx 0.02$ at low $B$.

We use Eq. 4 to fit the experimental $V$-dependence of the AB amplitude, as shown in Fig. 3 for the particular case $B = 1.9$ T. The reasonable quality of the fit supports our model for the dephasing time. From such fits we extract values for $t_0$ and $\alpha$ at various magnetic fields. The left inset to Fig. 3 shows our results for the $B$-dependence of $t_0$. The values in the edge channel regime are fitted to a straight line with a slope of $6/\hbar$ over the transport channels in the ring and 1/3 suppression due to scattering near the openings to the source and drain leads. This gives $|r_{e}|^2 \approx 1/3N_{tr} = 0.03$, which is in agreement with the experimental value of $G_{AB}(V = 0)/G \approx 0.02$ at low $B$.

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In the right inset to Fig. 3 we show our results for $\alpha = \hbar/2\pi \tau_0(\varepsilon)$. The $\alpha$ values are rather high and increase with $B$. At $\varepsilon = 10 \mu$eV, we find $\tau_0 = 180$ ps at $B = 0.4$ T and $\tau_0 = 65$ ps at $B = 4.7$ T. The usual Fermi-liquid theory assumes well-defined quasi-particles, corresponding to $\alpha \ll 1$. The fact that we observe $\alpha \approx 1$ signals the importance of many-body effects (electron-electron interactions) in our sample. The Fermi-liquid theory here is on the edge of applicability. We attribute this to significant scattering in the arms of the ring. For a clean 2DEG one expects no significant many-body effects until $\nu < 1$ ($B_{\nu=1} = 14$ T in our 2DEG). One can consider $\alpha$ as an effective dissipative conductance in units of $\hbar^2/\varepsilon$. In our case, the relevant conductance is that of the arms of the AB ring, which is of the order of $\hbar^2/\varepsilon$ at the highest magnetic fields. This is consistent with the observed $\alpha$ values.

In conclusion, using an electro-magnetic AB effect, we find a new method to determine the non-equilibrium electron dephasing time in a 2DEG, which becomes very short at high voltages and magnetic field.

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![FIG. 4. Schematic diagrams of interfering electron trajectories contributing (in first order) to the oscillating part of the transmission probability. At $B = 0$, the pairs of trajectories shown in (a)-(d) contribute equally, whereas at high magnetic field only the pair shown in (a) is relevant (B points downwards).](image)

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