Why is TeV scale a geometric mean of neutrino mass and GUT scale?

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Abstract – Among three typical energy scales, a neutrino mass scale ($m_\nu \sim 0.1$ eV), a GUT scale ($M_{GUT} \sim 10^{16}$ GeV), and a TeV scale ($M_{NP} \sim 1$ TeV), there is a fascinating relation, i.e. $M_{NP} \simeq \sqrt{m_\nu \cdot M_{GUT}}$. The TeV scale, $M_{NP}$, is a new physics scale beyond the standard model which is regarded as supersymmetry in this letter. We suggest a simple supersymmetric neutrinophilic Higgs doublet model, which realizes the above relation dynamically as well as the suitable $m_\nu$ through a tiny vacuum expectation value of the neutrinophilic Higgs doublet without additional scales other than $M_{NP}$ and $M_{GUT}$. A gauge coupling unification, which is an excellent feature in the supersymmetric standard model, is preserved automatically in this setup.

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Introduction. – There are three typical energy scales, a neutrino mass scale ($m_\nu \sim 0.1$ eV), a GUT scale ($M_{GUT} \sim 10^{16}$ GeV), and a TeV scale ($M_{NP} \sim 1$ TeV) which is a new physics scale beyond the standard model (SM) and regarded as supersymmetry (SUSY) in this letter. Among these three scales, we notice a fascinating relation,

$$M_{NP}^2 \simeq m_\nu \cdot M_{GUT}. \tag{1}$$

Is this relation an accident, or is it providing a clue to the underlying new physics? We take a positive stance toward the latter possibility.

As for a neutrino mass $m_\nu$, its smallness is still a mystery, and it is one of the most important clues to find new physics. Among a lot of possibilities, a neutrino-Higgs doublet model suggests an interesting explanation of the smallness by a tiny vacuum expectation value (VEV) $[1,2,5,9–15]$. This VEV from a neutrino-Higgs doublet is of $O(0.1)$ eV which is the same as the neutrino mass, so that it implies the Dirac neutrino $[3,4,6,8]$.

Thus, the neutrino mass is much smaller than other fermions, since its origin is the tiny VEV from the different (neutrinophilic) Higgs doublet. The introduction of $Z_2$-symmetry distinguishes the neutrino-Higgs doublet from the SM-like Higgs doublet, where $m_\nu$ is surely generated only through the VEV of the neutrinophilic Higgs doublet. The SUSY extension of the neutrinophilic doublet model is considered in refs. $[7,11,12,15]$. Since the neutrino Yukawa couplings are not necessarily tiny anymore, some related researches have been done, such as, collider phenomenology $[8,10]$, low-energy thermal leptogenesis $[11,12]$, cosmological constraints $[13]^2$, and so on.

On the other hand, the SUSY is the most promising candidate as a new physics beyond the SM because of the excellent success of gauge coupling unification. Thus, the SUSY SM well fits the GUT scenario as well as the existence of a dark-matter candidate.

There are some attempts that try to realize the relation in eq. (1). One example is to derive $m_\nu$ from a higher-dimensional operator in the SUSY framework $[16]$. Another example is to take a setup of matter localization $[17]$ in a warped extra dimension $[18]$. Both scenarios are interesting, but the model in this letter is much simpler and contains no additional scales other than $M_{NP}$, $m_\nu$, and $M_{GUT}$.

In this letter, we suggest a simple SUSY neutrinophilic Higgs doublet model, which dynamically realizes the relation of eq. (1). Usually, SUSY neutrino-Higgs doublet models have tiny mass scale of soft $Z_2$-symmetry breaking

1 In refs. $[1,2,5,9–12]$, the Majorana neutrino scenario is considered through the TeV scale seesaw with a neutrino-Higgs doublet VEV of $O(1)$ MeV.

2 The setup in refs. $[13]$ is different from the usual neutrinophilic Higgs doublet models, since it includes a light Higgs particle.
(ρ, ρ′ = O(10) eV in refs. [11,12,15]). This additional tiny mass scale plays a crucial role in generating the tiny neutrino mass; however, its origin is completely unknown (and assumption). In other words, the smallness of mν is just replaced by that of Z2-symmetry breaking mass parameters, and this is not an essential explanation of tiny mν. This is a common serious problem which exists in neutrinoophilic Higgs doublet models in general. The present model solves this problem, in which two scales, M_{GUT} and M_{NP}, induce the suitable magnitude of mν dynamically, and it does not require any additional scales, such as O(10) eV. This is one of the excellent points in our model.

The potential of the Higgs doublets is given by

\[ V = \left( |\mu|^2 + |\rho|^2 \right) H_u^\dagger H_u + \left( |\mu|^2 + |\rho|^2 \right) H_d^\dagger H_d + \frac{g_2^2}{2} \left( H_u^\dagger \frac{1}{2} H_u - \frac{1}{2} H_d + H_u^\dagger \frac{1}{2} H_d - \frac{1}{2} H_u^\dagger \frac{1}{2} H_d \right)^2 + \sum_a \frac{g_a^2}{2} \left( H_u^\dagger \rho^a H_u + H_d^\dagger \tau^a H_d + \frac{1}{2} H_u^\dagger H_d \right) + H_u^\dagger \rho^a H_u - \frac{1}{2} m_{\tilde{H}}^2 H_u^\dagger H_u + m_{\tilde{H}}^2 H_d^\dagger H_d + m_{\tilde{H}}^2 H_u^\dagger H_d + B \tilde{H}^\dagger H_u - B \tilde{H}^\dagger H_d - \mu^* \tilde{H}^\dagger H_u - \mu \tilde{H}^\dagger H_d - M^* \rho^a \tilde{H}^\dagger H_u + \text{h.c.}, \right) \]

where \( \tau^a \) and \( \rho^a \) mean a generator and the cross product of SU(2), respectively. \( m_{\tilde{H}}^2, m_{\tilde{H}}^\dagger, m_{\tilde{H}}^\dagger, \tilde{B}, \tilde{B}' \), \( B \), and \( B' \) are soft SUSY breaking parameters of order \( O(1) \) TeV. We assume \( -m_{\tilde{H}}^2 < 0 \) for the suitable electroweak symmetry breaking, and real VEVs as \( \langle H_u \rangle = v_u, \langle H_d \rangle = v_d \), \( \langle \bar{H}_u \rangle = v_{\bar{u}}, \langle \bar{H}_d \rangle = v_{\bar{d}} \) in neutral components. Now, let us examine whether we can really obtain the suitable magnitudes of VEVs as \( v_{u,d} = O(10^2) \) GeV and \( v_{u,d'} = O(0.1) \) eV or not. By taking \( \mu, \rho, B \)-parameters to be real, and denoting \( M_2^2 = \mu^2 + \rho^2 - m_{\tilde{H}}^2(\mu < 0), M_2^2 \equiv \mu^2 + \rho^2 - m_{\tilde{H}}^2(\mu > 0), M_2^2 \equiv M_2^2 + \rho^2 + m_{\tilde{H}}^2(\mu \gtrless M_2^2 > 0), M_2^2 \equiv M_2^2 + \rho^2 + m_{\tilde{H}}^2(\mu \lesssim M_2^2 > 0), \) and \( M_2^2 \equiv M_2^2 + \rho^2 + m_{\tilde{H}}^2(\mu \lesssim M_2^2 > 0) \), the stationary conditions of \( \frac{\partial V}{\partial v_u} = 0, \frac{\partial V}{\partial v_d} = 0, \frac{\partial V}{\partial v_{\bar{u}}} = 0, \frac{\partial V}{\partial v_{\bar{d}}} = 0 \) are given by

\[ 0 = M_2^2 v_u + \frac{1}{4} (g_1^2 + g_2^2) v_u (v_u^2 - v_d^2 + v_{\bar{u}}^2 - v_{\bar{d}}^2) + B \mu v_u - B \rho v_{\bar{u}} - (\mu \rho + M \rho) v_u, \]

\[ 0 = M_2^2 v_d - \frac{1}{4} (g_1^2 + g_2^2) v_u (v_u^2 - v_d^2 + v_{\bar{u}}^2 - v_{\bar{d}}^2) + B \mu v_u - B \rho v_{\bar{u}} - (\mu \rho + M \rho) v_u, \]

\[ 0 = M_2^2 v_{\bar{u}} + \frac{1}{4} (g_1^2 + g_2^2) v_u (v_u^2 - v_d^2 + v_{\bar{u}}^2 - v_{\bar{d}}^2) + B' M v_u - B' \rho v_{\bar{u}} - (\mu \rho + M \rho) v_u, \]

\[ 0 = M_2^2 v_{\bar{d}} - \frac{1}{4} (g_1^2 + g_2^2) v_u (v_u^2 - v_d^2 + v_{\bar{u}}^2 - v_{\bar{d}}^2) + B' M v_u - B' \rho v_{\bar{u}} - (\mu \rho + M \rho) v_u, \]
respectively. Since $M$ is a GUT scale and $v_\nu, v_{\nu'} \ll v_u, v_d$, eqs. (6) and (7) become

$$0 = M v_\nu - \rho v_u, \quad 0 = M v_{\nu'} - \rho' v_d$$

in the leading order, respectively. These are just the relation in eq.(1) (Here we neglect one order magnitude between $M_{NP}$ and the weak scale.) This is what we want to derive, and the VEVs of neutrinoophilic Higgs fields become

$$v_\nu = \frac{\rho v_u}{M}, \quad v_{\nu'} = \frac{\rho' v_d}{M}. \quad (9)$$

It is worth noting that they are induced dynamically through the stationary conditions in eqs. (6) and (7), and their magnitudes are surely of $O(\rho M)$ eV. As for $v_{u,d}$, by use of eq. (9), eqs. (4) and (5) become

$$0 = (M_u^2 - \rho^2) v_u - \frac{1}{4} (g_1^2 + g_2^2) v_u v_{\nu}^2 + B \mu v_d,$$

$$0 = (M_d^2 - \rho'^2) v_d - \frac{1}{4} (g_1^2 + g_2^2) v_d v_{\nu'}^2 + B \mu v_u, \quad (10)$$

in the leading order, respectively. Then, the MSSM Higgs fields take VEVs as

$$v^2 \approx \frac{2}{g_1^2 + g_2^2} \left( \frac{M_u^2 - M_d^2}{\cos 2\beta} - (M_u^2 + M_d^2) \right),$$

$$\sin 2\beta \approx \frac{2B\mu}{M_u^2 + M_d^2}, \quad (12)$$

where $v^2 = v_u^2 + v_d^2$, $\tan \beta = v_u/v_d$, $M_u^2 \equiv M_\nu^2 - \rho^2$ and $M_d^2 \equiv M_{\nu'}^2 - \rho'^2$. They mean slight modifications of VEVs for $H_u$ and $H_d$.

Since the masses of neutrinoophilic Higgs doublets $H_\nu$ and $H_{\nu'}$ are super-heavy as the GUT scale, there are no other vacua (such as, $v_{u,d} \sim v_{\nu,\nu'}$) except for $v_{u,d} \gg v_{\nu,\nu'}$ [15]. Also, their heaviness guarantees the stability of the VEV hierarchy, $v_{u,d} \gg v_{\nu,\nu'}$, against radiative corrections [14,15]. It is because, in the effective potential, $H_u$ and $H_{\nu'}$ inside loop diagrams are suppressed by their GUT scale masses. Anyhow, we stress again that the relation of eq. (1) is dynamically obtained in eq. (8).

As for the gauge coupling unification, it is preserved automatically in our setup, since fields, except for the MSSM, have masses of order the GUT scale.

**SU(5) GUT embedded model.** – The model we suggested has the GUT scale mass of the neutrinoophilic Higgs doublets in eq. (2), so that it is natural to embed the model in a GUT framework. Let us consider SU(5) GUT. A superpotential of a Higgs sector at the GUT scale is given by

$$W_{SU(5)}^{GUT} = M_0 \text{tr} \Sigma^2 + \lambda \text{tr} \Sigma^3 + H \Sigma \bar{H} + \Phi_1 \Sigma \Phi_1 - M_1 H \bar{H} - M_2 \Phi_1 \Phi_1,$$  

$$+ \Phi_1 \Phi_1 - M_1 H \bar{H} - M_2 \Phi_1 \Phi_1,$$  

$$M_0 \text{tr} \Sigma^2 + \lambda \text{tr} \Sigma^3 + H \Sigma \bar{H} + \Phi_1 \Sigma \Phi_1 - M_1 H \bar{H} - M_2 \Phi_1 \Phi_1,$$  

$$\Phi_1 \Phi_1 - M_1 H \bar{H} - M_2 \Phi_1 \Phi_1.$$  

where $\Sigma$ is an adjoint Higgs field whose VEV reduces the GUT gauge symmetry to the SM. $\Phi_1$ ($\Phi_1$) is a neutrinoophilic Higgs field of (anti-)fundamental representation, which contains $H_{\nu}$ ($H_{\nu'}$) in the doublet component (while the triplet component is denoted as $T_\nu$ ($T_{\nu'}$)). $\Phi_1$ and $\Phi_1$ are odd under the $Z_2$-parity. $H (H)$ is a Higgs field of (anti-)fundamental representation, which contains $H_u$ ($H_d$) in the doublet component (while the triplet component is denoted as $T (T)$). The VEVs of $\Sigma$ and $M_{0,1,2}$ are all of $O(10^{16})$ GeV, thus we encounter the so-called triplet-doublet (TD) splitting problem. Some mechanisms have been suggested for a solution of TD splitting, but here we show a case that the TD splitting is realized just by a fine-tuning between $(\Sigma)$ and $M_1$. That is, $(\Sigma) - M_1$ induces GUT scale masses of $T, \bar{T}$, while weak scale masses of $H_u, H_d$. This is a serious fine-tuning, so that we cannot expect that a simultaneous fine-tuned cancellation also happens between $(\Sigma)$ and $M_2$. Thus, we consider the case in which the TD splitting only works in $H$ and $\bar{H}$, while it does not work in $\Phi_1$ and $\Phi_1$. This situation makes eq. (13) become

$$W_{SU(5)}^{eff} = \mu H_u H_d + MH_u H_{\nu'} + M'T\bar{T} + M''T_\nu \bar{T}_{\nu'}, \quad (13)$$

This is the effective superpotential of the Higgs sector below the GUT scale, and $M, M', M''$ are of $O(10^{16})$ GeV, while $\mu = O(1) \text{TeV}$.

Now let us consider the origin of soft $Z_2$-parity breaking terms, $\rho H_u H_{\nu'}$ and $\rho' H_d H_{\nu'}$, in eq. (2). They play a crucial role in generating the marvelous relation in eq. (1) as well as a tiny Dirac neutrino mass. Since the values of $\rho, \rho'$ are of order $O(1) \text{TeV}$, they might be induced from the SUSY breaking effects. We can consider some possibilities for this mechanism. One example is to introduce a singlet $S$ with odd $Z_2$-parity. The superpotential including $S$ below the GUT scale is given by

$$W_{SU(5)}^{eff} = \mu H_u H_d + MH_u H_{\nu'} + M'T\bar{T} + M''T_\nu \bar{T}_{\nu'},$$

$$\mu S^2 + \frac{1}{2} S^2 = SH_u H_{\nu'} - SH_d H_{\nu'} - ST\bar{T} - ST_\nu \bar{T}_{\nu'}, \quad (14)$$

Denoting $\langle T \rangle = t, \langle \bar{T} \rangle = \bar{t}, \langle T_\nu \rangle = t_{\nu}, \langle \bar{T}_{\nu'} \rangle = \bar{t}_{\nu'},$ and $(S) = s$, the effective potential of the Higgs sector is given by

$$V^{eff} = |M_{\nu_{\nu'}} - sv_{\nu}|^2 + |M_{\nu_{\nu'}} - sv_{\nu}|^2$$

$$+ |\mu_{\nu_{\nu'}} - st_{\nu}|^2 + |\mu_{\nu_{\nu'}} - st_{\nu}|^2$$

$$+ |\mu_{\nu_{\nu'}} - st_{\nu}|^2 + |\mu_{\nu_{\nu'}} - st_{\nu}|^2$$

$$+ \mu_{\nu_{\nu'}} + 4s^2/\Lambda - v_{\nu_{\nu'}} - v_{\nu_{\nu'}} - t_{\nu} - t_{\nu}$$

$$- m_{\nu_{\nu'}}^2 v_{\nu_{\nu'}}^2 + m_{\nu_{\nu'}}^2 v_{\nu_{\nu'}}^2 + m_{\nu_{\nu'}}^2 v_{\nu_{\nu'}}^2 + m_{\nu_{\nu'}}^2 v_{\nu_{\nu'}}^2 + \cdots, \quad (16)$$

where we omit $D$- and $B$-terms for simplicity. The last two lines in eq. (16) are soft SUSY breaking mass squared
terms. Taking the parameter region of $-m'^2 \equiv -m_S^2 + 4\mu_S^2 < 0$, we obtain $s \sim \sqrt{\Lambda/32\mu_S} m'_S$. Then, when $m'_S \sim \mu_S \sim 1$ TeV and $\Lambda \sim 30$ TeV, the suitable $\rho$- and $\rho'$-terms in eq. (2) are effectively induced through $s \sim 1$ TeV. This vacuum also suggests the suitable magnitude of $\nu_{\mu}$-vevs $v_{\nu,\nu'} \sim 0.1$ eV as well as $t = t = t_{\nu} = t_{\nu'} = 0$. Unfortunately, the scale of $\Lambda \sim 30$ TeV is a little artificial in this example. But, this is around $M_{NP}$, and much better than inputting $\mathcal{O}(10)$ eV as the Z$_2$-parity breaking soft mass parameters. Another example is to take a non-canonical K"ahler potential of $[S(H_u H_v + H_u H_d) + h.c.]_D$, where the $F$-term of $S$ could induce the $\rho$- and $\rho'$-terms effectively through the SUSY breaking scale as in the Giudice-Masiero mechanism [22]. There might be other models which induce the $\rho$- and $\rho'$-terms in eq. (2) except for introducing a singlet $S$.

Summary. – Among three typical energy scales, a neutrino mass scale, a GUT scale, and a TeV (SUSY) scale, there is the marvelous relation in eq. (1). In this paper, we have suggested a simple supersymmetric neutrinoless Higgs doublet model, which realizes the relation of eq. (1) dynamically as well as the suitable $m_{\nu}$, through a tiny VEV of the neutrinoless Higgs doublet without additional scales other than $M_{NP}$ and $M_{GUT}$. Usually, SUSY neutrinoless doublet models have tiny mass scale of soft $Z_2$-symmetry breaking as $\rho, \rho' = \mathcal{O}(10)$ eV. This additional tiny mass scale plays a crucial role in generating the tiny neutrino mass; however, its origin is just an assumption. In other words, the smallness of $m_{\nu}$ is just replaced by that of $Z_2$-symmetry breaking mass parameters, and this is not an essential explanation of tiny $m_{\nu}$. This is a common serious problem which exists in neutrinoless Higgs doublet models in general. Our model has solved this problem, where two scales, $M_{GUT}$ and $M_{NP}$, naturally induce the suitable magnitude of $m_{\nu}$ through the relation of eq. (1), and it does not require any additional scales. A gauge coupling unification is also preserved automatically in our setup. We have also considered the embedding in $SU(5)$ GUT and some candidates for inducing the $Z_2$-symmetry breaking terms from the SUSY breaking effects.

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