Helicity amplitudes in the $\bar{B} \to D^*\bar{\nu}_\tau\tau$ decay with V-A breaking in the quark sector

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Abstract In view of the recent measurement of the $F^{D^*}_L$ magnitude in the $\bar{B} \to D^*\bar{\nu}_\tau\tau$ reaction we evaluate this magnitude within the standard model and for a family of models with the $\gamma^{\mu} - \alpha\gamma^{\mu}\gamma_5$ current structure for the quarks for different values of $\alpha$. At the same time we evaluate also the transverse contributions, $M = -1$, $M = +1$, and find that the difference between the $M = -1$ and $M = +1$ contributions is far more sensitive to changes in $\alpha$ than the longitudinal component. These findings should be looked as an incentive to measure the transverse helicities which are bound to be a far more sensitive magnitude to possible new physics than $F^{D^*}_L$.

1 Introduction

The measurement of vector polarization in $B$ decays with vectors in the decay products has captured the attention of the physics community as promising tools of information on possible physics beyond the standard model (BSM) [1–21]. In particular the information on the helicity amplitudes in the $B \to D^*\bar{\nu}_l\tau$ has been advocated in [22,23] as useful tools to explore physics BSM.

In a recent paper [24] the Belle Collaboration has reported the first measurement of the $D^*$ polarization in the $\bar{B} \to D^*\bar{\nu}_\tau\tau$ decay, providing a value for $F^{D^*}_L$, the fraction of the longitudinal polarization contribution to the total, of

$$F^{D^*}_L = 0.60 \pm 0.08 \pm 0.04 .$$

In the standard model (SM), $F^{D^*}_L$ is about 0.45 [23,25–28], with the most recent predictions giving 0.441 ± 0.006 [27] and 0.457 ± 0.010 [28]. Models BSM with scalar and tensor contributions can produce sizeable changes in $F^{D^*}_L$, as shown in [23,26–29].

In the present work we want to show that for some family of models BSM the fraction $F^{D^*}_L$ is rather insensitive to changes from the SM, while the transverse polarizations are more sensitive and in particular the difference between the $M' = -1$ and $M' = +1$ components is very sensitive to these changes.

In a former paper [30] we studied the helicity amplitudes of the $B \to D^*\bar{\nu}_l$ transition for a model in which the quark current is given by

$$Q^\mu = \langle \bar{u}_c | [1 - \alpha\gamma_5] | u_b \rangle ,$$

where $\alpha = 1$ for the SM. The calculations are done using a quark model for the operators [31], with a mapping of the quark momenta to those of the mesons consistent with heavy quark symmetry [32–34]. The longitudinal, $M' = 0$, and two transverse $M' = -1$, $M' = +1$ polarization contributions, taking the $z$ axis along the $D^*$ direction in the $\bar{\nu}_l$ rest frame, were evaluated. It was found that for different values of $\alpha$ the magnitude most sensitive to the change was the difference between the $M' = -1$ and $M' = +1$ contributions.

The model of [30] neglects the contribution of intrinsic quark form factors, which are claimed to approximately cancel in the ratios evaluated there.

In view of the measurement of [24], we find most opportune to test this model with this measurement, extending our model of the $B \to D^*\bar{\nu}_\tau\tau$ reaction to the $B \to D^*\bar{\nu}_\tau\tau$ one.
2 Formalism

In the present work we will study the $B \rightarrow D^* \bar{v} l$ decay, which is depicted in Fig. 1 for $B^- \rightarrow D^{*0} \bar{v} l^-$. We use the same nomenclature as in [30] where a study of the meson decays $J \rightarrow \bar{v} l J'M'$ was done, where $J(M(J'M'))$ are the modulus and third component of the initial (final) meson spin, and the rates for the different third components in the $J = 0, J' = 1$ case were evaluated.

The differential width for $B \rightarrow D^* \bar{v} l$ is given by

$$\frac{d\Gamma}{dM_{\text{inv}}^{(vl)}} = \frac{2m_\nu m_l}{(2\pi)^3} \frac{1}{4M_B^2} p'_{D^*} \tilde{p}_\nu \sum |t|^2,$$

(3)

where $p'_{D^*}$ is the $D^*$ momentum in the $B$ rest frame and $\tilde{p}_\nu$ the $\bar{v}$ momentum in the $\bar{v} l$ rest frame,

$$p'_{D^*} = \frac{\lambda^{1/2}(m_B^2, M_{\text{inv}}^{(vl)}, m_{D^*}^2)}{2m_B},$$

$$\tilde{p}_\nu = \frac{\lambda^{1/2}(M_{\text{inv}}^{(vl)}, m_\nu^2, m_l^2)}{2M_{\text{inv}}^{(vl)}}.$$

(4)

After considering right-handed quark currents in terms of $\alpha$, we find for the different helicity contributions ($M' = 0, \pm 1$):

1. $M' = 0$

$$\sum |t|^2 = \frac{m_l^2}{m_\nu m_l} M_{\text{inv}}^{(vl)} - m_l^2 \left[ AA' (B + B')^2 \right]^2 \alpha^2$$

$$+ \frac{2}{m_\nu m_l} \left( \tilde{E}_\nu \tilde{E}_l + \frac{1}{3} \tilde{p}_\nu^2 \right) \left[ AA' (1 + B B' p^2)^2 \right]^2 \alpha^2.$$

(5a)

2. $M' = 1$

$$\sum |t|^2 = \frac{2}{m_\nu m_l} \left( \tilde{E}_\nu \tilde{E}_l + \frac{1}{3} \tilde{p}_\nu^2 \right)$$

$$\times \left[ AA' ((1 - B B' p^2) \alpha + (B p - B' p)) \right]^2.$$

(5b)

3. $M' = -1$

$$\sum |t|^2 = \frac{2}{m_\nu m_l} \left( \tilde{E}_\nu \tilde{E}_l + \frac{1}{3} \tilde{p}_\nu^2 \right)$$

$$\times \left[ AA' ((1 - B B' p^2) \alpha - (B p - B' p)) \right]^2.$$

(5c)

Following the approach of [30], the above matrix elements are evaluated in the frame where the $\bar{v} l$ system is at rest, where $p_B = p_{D^*} = p$, with $p$ given by

$$p = \frac{\lambda^{1/2}(m_B^2, M_{\text{inv}}^{(vl)}, m_{D^*}^2)}{2M_{\text{inv}}^{(vl)}} \times \left[ AA' ((1 - B B' p^2) \alpha - (B p - B' p)) \right]^2.$$

(6)

where $M_{\text{inv}}^{(vl)}$ is the invariant mass of the $v l$ pair. For $B$ and $D^*$ mesons we have

$$A = \sqrt{E_B + m_B}, \quad B = \frac{1}{m_B + E_B};$$

$$A' = \frac{\sqrt{E_{D^*} + m_{D^*}}}{2m_{D^*}}, \quad B' = \frac{1}{m_{D^*} + E_{D^*}}.$$

(7)

and $\tilde{E}_\nu, \tilde{E}_l$ are the energies of $\bar{v}$ and lepton, respectively,

$$\tilde{E}_\nu = \frac{M_{\text{inv}}^{(vl)} + m_\nu^2 - m_l^2}{2M_{\text{inv}}^{(vl)}}, \quad \tilde{E}_l = \frac{M_{\text{inv}}^{(vl)} + m_l^2 - m_\nu^2}{2M_{\text{inv}}^{(vl)}}.$$

(8)

3 Results

We present results of $d\Gamma/dM_{\text{inv}}^{(vl)} (v l \equiv \bar{v} l \tau)$ for different $M' = \pm 1, 0$ and the sum

$$R = \frac{d\Gamma}{dM_{\text{inv}}^{(vl)}} \bigg|_{M' = 0} + \frac{d\Gamma}{dM_{\text{inv}}^{(vl)}} \bigg|_{M' = -1} + \frac{d\Gamma}{dM_{\text{inv}}^{(vl)}} \bigg|_{M' = +1}.$$

(9)

In Fig. 2 we show the results for the different contributions of $\alpha = 1$ (SM) and in Fig. 3 we show the same results but normalized to the total.

In Fig. 4 we show the results of Fig. 3 for different values of $\alpha$. We can see that all magnitudes change with $\alpha$, but the
most spectacular is the difference between the $M' = -1$ and $M' = +1$ contributions, which change sign when $\alpha$ changes sign.

The magnitude $F_{L}^{D^*}$ is obtained integrating $d\Gamma/dM_{inv}^{(\nu l)}$ over the $M_{inv}^{(\nu l)}$ variable and dividing by the total $\Gamma$. We show the results for $F_{L}^{D^*}$ in Table 1 for different values of $\alpha$.

We can see that for $\alpha = 1$ (SM) we obtain

$$F_{L}^{D^*} = 0.456,$$

which is in remarkable agreement with the result of [28]. We should note that in [30] no free parameters nor fit to data are used, but ratios are expected to be relatively accurate. The results in Table 1 are interesting because we observe that $F_{L}^{D^*}$ is very insensitive to the value of $\alpha$. We can claim that for this family of models, $F_{L}^{D^*}$ is not a good magnitude to test contributions beyond SM.

Table 1 The fraction of $D^*$ longitudinal polarization $f_{L}^{D^*}$ with different values of $\alpha$.

| $\alpha$  | 0.5  | 0.8  | 1.0  | 1.2  | 1.5  | $-0.5$ |
|-----------|------|------|------|------|------|-------|
| $f_{L}^{D^*}$ | 0.415 | 0.448 | 0.456 | 0.461 | 0.465 | 0.415 |

On the other hand we see that the measurement of the two transverse components carries more information and we provide this information in Fig. 5, in which we show the results obtained for the different ratios, and the difference of $M' = -1$ and $M' = +1$ for different values of $\alpha$. One can see that the longitudinal component $M' = 0$ is less sensitive than any of the other two, and in particular the difference between the $M' = -1$ and $M' = +1$ components is the most sensitive magnitude. If we take the range of value $\alpha \in [0.8 - 1.2]$ the band of values for the $M' = 0$ contribution is quite narrow, while the band for the difference between the $M' = -1$ and $M' = +1$ components is considerably larger.

The form factors needed in [30] and here are evaluated using free quark spinors in the $\bar{u}l$ rest frame, where the $D^*$ and $B$ are moving with momentum $p$ given by Eq. (6). They do not incorporate the effects from the binding of the quark in the $B$ and $D^*$ mesons. In Ref. [30] they are compared with the empirical ones of [22,35] and differences are seen. Yet, it is also shown there that when evaluating ratios, as in Figs. 3, 4 and 5, the effect of the form factors largely cancel, and, in particular, close to the maximum invariant mass, the results with the form factors of [22,35] and those of [30] are practically indistinguishable. Thus, our predictions for the hypothetical case that $\alpha \neq 1$ are realistic. With this perspective, the conclusion that we draw from the present work is that the two transverse contributions are more sensitive to potential changes of the standard model than the longitudinal one recently measured. This should be seen as an incentive to measure these magnitudes experimentally, which is the message that we want to convey in the present paper.

4 Connection with the conventional formalism

We discussed that our formalism does not contain the form factors of the decay amplitudes and this is why we calculate ratios where their effects cancel to a good extend. This was sufficient to get the message of the work, which is that the transverse polarizations, and in particular the difference between the two transverse polarizations, are more sensitive to changes of the V-A structure than the longitudinal one. Yet, it is also interesting to show what are the predictions for these magnitudes if we use the conventional form factors used in the study of these reactions. For this purpose we use the formalism of [22,32,33,35,36] for the weak amplitudes, using the $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$ form factors. We take
Fig. 4 The same as Fig. 3 but for different $\alpha$.

\[ \frac{1}{R dM^{(\nu)}_{\text{inv}}} \]

- $\alpha = 0.5$
- $\alpha = 0.8$
- $\alpha = 1.0$
- $\alpha = 1.2$
- $\alpha = 1.5$
- $\alpha = -0.5$

\[ M_{\text{inv}}^{(\nu)} \text{ [GeV]} \]

\[ M_{\text{inv}}^{(\nu)} \text{ [GeV]} \]
The longitudinal $M' = 0$, two transverse $M' = \pm 1$ components and the difference for the range of $\alpha \in [0.8 - 1.2]$

the parametrization used in [22,35], which is also detailed in Ref. [30].

The way to connect our formalism with the one of [22,35] is given in Ref. [30] by means of the equations

$$AA' (1 + BB'p^2) \rightarrow \frac{E_{D'}}{m_{D'}} (m_B + m_{D'}) A_1 - \frac{2p^2(E_B - E_{D'})}{m_{D'}(m_B + m_{D'})} A_2, \quad (11)$$

$$AA' (B - B'p) \rightarrow \frac{2(E_{D'} - E_B)}{(m_B + m_{D'})} V_{p}. \quad (11)$$

which we use in Eqs. (5a, 5b, 5c).
Fig. 6 The same as Fig. 4 but with conventional formalism connected by Eq. (11)
Then we plot the results equivalent to Fig. 4 with these form factors and show the results in Fig. 6.

We can see that the results are very similar in the whole range of invariant masses, in particular in the region close to the maximum neutrino lepton invariant mass. This exercise serves to quantify our former claims that the formalism used produces realistic results in the ratios.

Once we have done this exercise, we can also plot the equivalent to Fig. 5, which we show in Fig. 7. We can see that the results are very similar, and once again, in the region close to the maximum neutrino lepton invariant mass they are practically indistinguishable.
5 Summary

As a summary, we have shown that the model used, which evaluates explicitly the operators and contains no free parameters, is in remarkable agreement for the value of $F_{L}^{D^{∗}}$ with the most sophisticated evaluations of the SM. In view of this, we extended the model to calculate the contributions of the longitudinal and transverse helicities of the $\bar{B} \to D^{∗} \bar{\nu}_{\tau} \tau$ reaction for a family of models BSM with right handed quark currents. We concluded that the measurement of the transverse helicity components in the $\bar{B} \to D^{∗} \bar{\nu}_{\tau} \tau$ reaction is a more promising tool than the longitudinal helicity in the search for potential extrapolations of the SM and strongly suggest to study these components experimentally.

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Data Availability Statement

This manuscript has no associated data or the data will not be deposited. [Authors comment: All data generated during this study are contained in this published article.]

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