The Single Machine Total Weighted Tardiness Problem—Is it (for Metaheuristics) a Solved Problem?

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1 Introduction

The single machine total weighted tardiness problem (SMTWTP) is a well-known planning problem from operations research, engineering and computer science. It is characterized by the assignment of starting times for a given set of jobs on a single processor, minimizing the (weighted) tardy completion of the jobs with respect to given due dates. In this sense, the tardiness is given an economical interpretation, referring to the consideration of costs which consequently have to be avoided as much as possible. Although most practical problems involve multiple resources (e.g. machines), many problems can be decomposed into a subsequently solved series of single-machine problems, as for example done in the famous shifting bottleneck heuristic [2]. The effective solution of each single-machine subproblem is therefore a relevant aspect for solving more complex models.

Besides the problem’s relevance with respect to the applicability in real-world situations, the SMTWTP is computationally challenging, as it has been proven to be strongly \( \mathcal{NP} \)-hard [13]. While some exact methods are available, many successful solution approaches are based on heuristics [1]. More recently, several different metaheuristics have been developed for the SMTWTP, successfully solving benchmark instances from the scientific literature. Important work includes simple local search [15], Evolutionary Algorithms [9], Ant Colony Optimization [5, 10, 17], Iterated Local Search [11], and Simulated Annealing [18]. A particularly successful neighborhood search technique for the SMTWTP is Iterated Dynasearch [8], which uses dynamic programming to determine an optimal series of moves to be executed simultaneously.

While it has been pointed out that available benchmark instances are comparably easily solvable by local search [11], several recent publications aim to push the scientific knowledge even further by proposing more refined metaheuristics [3, 6, 7, 14, 19, 20]. Common to these approaches is the fact that they lead to rather impressive results within the chosen experimental settings. From this perspective, and taking into account the remarks of [11], we need to raise the question whether recent results are so surprising after all. Or is it possible that, despite the problem’s complexity, the SMTWTP is a ‘solved’ problem for metaheuristics? Are there ‘difficult’ instances, and if so,
how can they be described in terms of their technical properties? Can we draw conclusions on how to organize future research?

The article is organized as follows. Section 2 presents a quantitative model for the SMTWTP and describes an appropriate representation (coding) for the alternatives. Besides, benchmark instances from the literature are briefly introduced. The following Sections 3 and 4 revisit local search for the SMTWTP, namely hillclimbing and Variable Neighborhood Search [12]. The identification of comparable ‘difficult’ instances is derived from the experiments, and a closer analysis of these data sets follows in Section 5. Conclusions are presented in Section 6.

2 Problem statement and benchmark data

2.1 Quantitative model

In the SMTWTP, a set of jobs \( J = \{J_1, \ldots, J_n\} \) needs to be processed on a single machine. Each job \( J_j \) consists of a single operation only, involving a processing time \( p_j > 0 \) \( \forall j = 1, \ldots, n \). The relative importance of the jobs is expressed via a nonnegative weight \( w_j > 0 \) \( \forall j = 1, \ldots, n \). Processing on the machine is only possible for a single job at a time, excluding parallel processing of jobs. Each job \( J_j \) is supposed to be finished before its due date \( D_j \). If this is not the case, a tardiness \( T_j \) occurs, measured as \( T_j = \max\{s_j + p_j - D_j; 0\} \), where \( s_j \) denotes the starting time of job \( j \). The overall objective of the problem is to find a feasible schedule \( x \) minimizing the total weighted tardiness \( TWT \), i.e. \( \min TWT = \sum_{j=1}^n w_j T_j \).

A schedule for a particular problem can be interpreted as a vector of starting times of the jobs, \( x = (s_1, \ldots, s_n) \). We assume that processing starts at time 0, thus \( s_j \geq 0 \) \( \forall j = 1, \ldots, n \). A possible overlapping of jobs on the machine is avoided by the formulation of disjunctive side constraints:

\[
s_j \geq s_k + p_k \lor s_j + p_j \leq s_k \forall j, k = 1, \ldots, n, j \neq k.
\]

As the objective function of the single machine total weighted tardiness problem is a \textit{regular} function [4], it is known that at least one active schedule exists which is also optimal. A schedule is called \textit{active} if, for a given sequence of jobs, all operations are started as early as possible, thus avoiding all unnecessary in-between waiting times (delays). The problem of finding an optimal schedule may therefore be reduced to the problem of finding an optimal sequence of jobs. A given sequence is represented by a permutation \( \pi = \{\pi_1, \ldots, \pi_n\} \) of the job indices. Each element \( \pi_i \) in \( \pi \) stores the index of the job which is to be processed as the \( i \)th job in the processing sequence. The permutation of indices is then ‘decoded’ into a schedule by assigning \( s_{\pi_1} = 0 \) and computing the values of \( s_{\pi_i} = s_{\pi_{i-1}} + p_{\pi_{i-1}} \forall i = 2, \ldots, n \).

Obviously, this leads to an active schedule without any waiting times between jobs.

2.2 Benchmark instances from the literature

Optimization approaches for the SMTWTP are commonly verified using the benchmark instances of [9]. The authors presented 375 data sets with varying characteristics. For values of \( n = 40 \), \( n = 50 \), and \( n = 100 \), 125 instances are each proposed. The computation of the processing times \( p_j \) is randomly drawn from a uniform distribution \([1, 100]\), while the weights are taken from \([1, 10]\). Depending on the relative range of due dates \( RDD \) and the average tardiness factor \( TF \), the due
dates are randomly computed as integer values within \( P \left(1 - TF - \frac{RDD}{2}\right), P \left(1 - TF + \frac{RDD}{2}\right) \), where \( P = \sum_{j=1}^{n} p_j \). Five instances have been computed for each combination of \( RDD \) and \( TF \): 

\( RDD \in \{0.2, 0.4, 0.6, 0.8, 1.0\}, TF \in \{0.2, 0.4, 0.6, 0.8, 1.0\} \).

All data sets are available in the OR-Library under \[\text{http://people.brunel.ac.uk/~mastjjb/jeb/info.html}\].

For the ones with \( n = 40 \) and \( n = 50 \), optimal solutions are reported in the literature, lacking their final proof of optimality. It should be noticed however, that the results for the larger data sets are commonly assumed to be optimal, as, despite rather active research in this area, there has not been any improvement of the best-known upper bounds within past ten years.

### 3 Hillclimbing revisited

#### 3.1 Configuration

Despite the existence of advanced metaheuristics for the SMTWTP, we first investigate the effectiveness of simple local search in a hillclimbing framework. In these experiments, we are interested in finding out how good results may get when only applying rather basic search strategies. In conclusion, such experiments present a lowest possible benchmark for any alternative technique, and allow an insight into the difficulty of different benchmark data sets.

The implemented hillclimbing algorithm is based on a best-improvement move strategy, investigating all neighboring alternatives and carrying out a move that realizes the largest possible improvement. Applied neighborhood operators are exchange (EX), forward shift (FSH) and backward shift (BSH). EX exchanges two jobs at the positions \( i \) and \( j \) in \( \pi \), FSH moves a job from position \( i \) to \( j \) with \( i < j \), and BSH moves a job from \( j \) to \( i \), \( i < j \). 100 test runs have been carried out for each data set/configuration of the algorithm, each starting from a (different) random job permutation. Naturally, the test runs terminate with the identification of a local optimum.

#### 3.2 Results

Table 1 shows the number of instances which have been solved to optimality in at least one of the 100 test runs, depending on the chosen neighborhood operator and the size of the instances \( n \).

| Operator | \( n = 40 \) | \( n = 50 \) | \( n = 100 \) |
|----------|------------|------------|-------------|
| EX       | 108        | 85         | 48          |
| FSH      | 86         | 65         | 31          |
| BSH      | 48         | 35         | 13          |

It is interesting to see that the results differ rather drastically with respect to the chosen neighborhood structure. EX clearly leads to the best results, followed by FSH and BSH. As it has to be expected, the number of data sets that have been solved to optimality decreases with increasing...
Nevertheless, a large portion of the 125 instances is solved for each value of $n$, even in case of the largest value $n = 100$. It becomes apparent that the benchmark instances given in [9] contain a number of data sets that are less appropriate for state-of-the-art metaheuristics, as they are comparatively easily solvable using EX-hillclimbing.

Table 2: Number of instances solved to optimality by hillclimbing depending on $RDD$ and $TF$

|       | EX |       |       |       |       | FSH |       |       |       |       | BSH |       |       |
|-------|----|-------|-------|-------|-------|-----|-------|-------|-------|-------|-----|-------|-------|
| $RDD$ | $TF$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
| 0.2   |     | 11 | 13 | 7 | 10 | 15 | 5 | 6 | 3 | 7 | 13 | 0 | 0 | 0 | 2 | 12 |
| 0.4   |     | 11 | 9 | 6 | 11 | 15 | 12 | 8 | 8 | 6 | 9 | 3 | 0 | 0 | 4 | 9 |
| 0.6   |     | 15 | 6 | 3 | 9 | 11 | 15 | 4 | 4 | 3 | 9 | 12 | 0 | 0 | 5 | 6 |
| 0.8   |     | 15 | 4 | 4 | 7 | 12 | 15 | 6 | 5 | 2 | 6 | 13 | 0 | 0 | 1 | 7 |
| 1.0   |     | 15 | 8 | 5 | 8 | 11 | 15 | 11 | 3 | 1 | 6 | 13 | 4 | 0 | 1 | 4 |

A closer analysis indicates possible characteristics of relatively ‘hard’ versus ‘easy’ instances. Table 2 shows the number of instances solved to optimality depending on the values of $RDD$ and $TF$. As we here aggregate over all settings of $n$, the maximum possible value in each cell is 15.

It is possible to see that instances generated by assuming either low or high values of $TF$, i.e. $TF \in \{0.2, 1.0\}$, turn out to be comparatively easily solvable using simple local search. With respect to the range of due dates $RDD$, the tendency towards difficult versus easy data sets is less obvious. In combination with values of $TF = 0.2$, high values of $RDD$ such as $RDD \in \{0.6, 0.8, 1.0\}$ lead to easy instances. For a value of $TF = 1.0$, the choice of a low $RDD$, i.e. $RDD \in \{0.2, 0.4\}$, results in easier data sets.

On the other hand, values such as $TF = 0.6$ show the tendency to produce relatively difficult instances, at least for simple local search based on hillclimbing.

4 Variable neighborhood search

4.1 Configuration

On the basis of past studies of local search for the SMTWTP [11], and following the outcomes of the previous section, we investigate two different configurations of Variable Neighborhood Search (VNS). The first, VNS-1, applies the basic neighborhood operators in order of EX$\rightarrow$FSH$\rightarrow$BSH, while the second, VNS-2, implements the reverse order BSH$\rightarrow$FSH$\rightarrow$EX. Knowing that there appears to be a relative order of neighborhood operators, it will be interesting to further investigate the effects of the different configurations of the VNS algorithms.

All neighborhoods are searched in a best-move-fashion, thus searching the entire neighborhood. As usual in VNS, the neighborhood is switched to the succeeding operator if the active one fails to improve the current solution. Search terminates with the identification of a local optimum, which is, in case of VNS, an alternative being locally optimal with respect to all considered neighborhoods. Again, 100 test runs have been carried out for each benchmark instance/VNS-configuration, each starting from a (different) random job permutation.

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4.2 Results

Table 3 shows the number of instances which have successfully been solved by the VNS algorithms in at least a single out of the 100 test runs.

Table 3: Number of instances for which optimal/best known solutions have been found by VNS

| Algorithm         | n = 40  | n = 50  | n = 100 |
|-------------------|---------|---------|---------|
| VNS-1 (EX→FSH→BSH)| 121     | 114     | 108     |
| VNS-2 (BSH→FSH→EX)| 125     | 124     | 117     |

We can see that VNS-2 is able to solve more instances than VNS-1. It can be suspected that this behavior originates from the relative order of neighborhood operators. VNS-2 starts with the weak operator BSH, and then continues to relatively better neighborhoods. Therefore, an improvement of solutions being locally optimal to the initially searched neighborhoods appears to be more likely as in the case of VNS-1, where search is organized starting with relatively good operators.

Again, the number of solved instances is decreasing with increasing \( n \). However, the results remain on a rather high level despite the growing difficulty of the benchmark instances. Only few instances remain unsolved.

Table 4: Number of instances solved to optimality by VNS depending on RDD and TF

| RDD: | VNS-1 | VNS-2 |
|------|-------|-------|
| TF:  |       |       |
| 0.2  | 15    | 15    |
| 0.4  | 13    | 15    |
| 0.6  | 15    | 15    |
| 0.8  | 15    | 15    |
| 1.0  | 15    | 15    |

More detailed results are given in Table 4 which shows the number of solved instances with respect to the choices of RDD and TF. In comparison to the results obtained by hillclimbing, a similar pattern arises in the investigation of VNS. Again, both small and high values of TF lead to easy instances. In the light of the overall impressive results of VNS however, this pattern is less obvious. Knowing that VNS-2 successfully solved 97.6% of the benchmark instances, only few instances are left that can be considered to be ‘difficult’.

Besides the best-case behavior of the algorithms, the average quality of the local optima over all 100 test runs is interesting. Table 5 gives the results with respect to this.

Table 5: Average deviation from the optimal/best known solution

| Algorithm         | n = 40 | n = 50 | n = 100 |
|-------------------|--------|--------|---------|
| VNS-1 (EX→FSH→BSH)| 2.50% | 2.36% | 2.10% |
| VNS-2 (BSH→FSH→EX)| 0.99% | 1.45% | 0.98% |

Also on an average level, VNS-2 leads to better results than VNS-1. Moreover, the average.
deviation from the optimal/best known solutions is rather small, which indicates that VNS is indeed an effective approach for most instances from the literature.

Unfortunately however, a considerable dispersion can be identified within these results. As shown in Table 6, the average results deviate for some few instances significantly more than in the majority of data sets. Even for some smaller instances, i.e. \( n = 40 \) and \( n = 50 \), considerable deviations can be found in some cases.

Table 6: Average deviation of VNS-2 from the optimal/best known solutions given in percent

|       | \( n = 40 \)       |       | \( n = 50 \)       |       | \( n = 100 \)       |
|-------|---------------------|-------|---------------------|-------|---------------------|
|       | TF: 0.2 0.4 0.6 0.8 1.0 |       | TF: 0.2 0.4 0.6 0.8 1.0 |       | TF: 0.2 0.4 0.6 0.8 1.0 |
| RDD:  | 0.2 6.7 0.4 0.2 0.2 0.0 |       | 0.4 0.4 0.1 0.0 0.0 0.0 |       | 0.4 0.2 0.2 0.2 0.0 0.0 |
|       | 0.4 0.0 0.3 0.6 0.1 0.0 |       | 13.9 1.2 0.4 0.1 0.0 0.0 |       | 1.1 1.1 0.4 0.1 0.0 0.0 |
|       | 0.6 0.0 4.6 0.9 0.0 0.0 |       | 0.0 3.1 1.8 0.2 0.0 0.0 |       | 0.0 2.5 0.6 0.1 0.0 0.0 |
|       | 0.8 0.0 9.7 0.3 0.0 0.0 |       | 0.0 10.3 0.8 0.2 0.0 0.0 |       | 0.0 15.9 1.1 0.1 0.0 0.0 |
|       | 1.0 0.0 0.0 0.6 0.1 0.0 |       | 0.0 0.0 1.9 0.1 0.1 0.1 |       | 0.0 0.2 0.3 0.1 0.0 0.0 |

Combining the results of Table 5 and 6 an interesting observation emerges. While VNS-2 successfully solves the instances with \( TF = 0.2/RDD = 0.2 \), \( TF = 0.4/RDD = 0.6 \), \( TF = 0.4/RDD = 0.8 \) in the best case, the results show a relative high deviation from the optimal/best known results in the average case. On the other hand, instances that are not solved in the best case, i.e. the ones with \( TF = 0.6/RDD = 0.8 \), are approximated very nicely in average.

### 4.3 Computational effort

The following Table 7 gives an overview about the required computational effort for finding a local optimum, depending on the neighborhood operator/ metaheuristic and the problem size \( n \).

Table 7: Average number of evaluations for finding a local optimum

|       | \( n = 40 \) | \( n = 50 \) | \( n = 100 \) |
|-------|--------------|--------------|--------------|
| EX    | 23,147       | 46,355       | 416,920      |
| FSH   | 24,866       | 51,471       | 513,764      |
| BSH   | 29,563       | 62,637       | 693,206      |
| VNS-1 | 26,389       | 52,692       | 471,406      |
| VNS-2 | 39,847       | 85,420       | 950,904      |

Clearly, the length of the hillclimbing runs significantly increases with \( n \), and it becomes clear that for large values of \( n \), the identification of even a local optimum becomes intractable. On the other hand, the basic operators EX, FSH and BSH do not show bigger differences. On the contrary, the relatively best-performing neighborhood EX requires less computations for finding (better) local optima. Consequently, the effort for the execution of VNS-1 is much smaller in comparison to VNS-2, as the latter implementation first invests in the computationally demanding identification of qualitatively inferior local optima for BSH.

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5 A first analysis of ‘difficult’ instances

While the results of the experiments above clearly show that already simple local search strategies are rather effective for the SMTWTP, some few instances remain which can be considered to be relatively ‘difficult’. In fact, one of the data sets with \( n = 50 \) and three with \( n = 100 \) have not been solved in at least a single run of any local search strategy. For some other instances with \( n = 100 \), an optimal solution has been found, however only in a single test run. The following Table gives an overview about these instances and some of their properties.

| Instance | \( n \) | RDD | \( TF \) | Distinct optimal alt. | Entropy | Entropy of 100 alt. |
|----------|-------|-----|-------|----------------------|--------|---------------------|
| 50_109   | 50    | 1.0 | 0.4   | \( > 1,000,000 \)   | 9.42E−11| 0.0036              |
| 100_20   | 100   | 0.2 | 0.8   | \( \geq 2,304 \)   | 4.52E−06| 0.0018              |
| 100_42   | 100   | 0.4 | 0.8   | \( \geq 23,552 \)  | 9.60E−08| 0.0026              |
| 100_44   | 100   | 0.4 | 0.8   | \( \geq 17,280 \)  | 1.24E−07| 0.0021              |
| 100_45   | 100   | 0.4 | 0.8   | \( \geq 52,992 \)  | 1.92E−08| 0.0028              |
| 100_86   | 100   | 0.8 | 0.6   | \( \geq 100,000 \) | 6.10E−09| 0.0027              |
| 100_88   | 100   | 0.8 | 0.6   | \( \geq 100,000 \) | 6.31E−09| 0.0028              |
| 100_111  | 100   | 1.0 | 0.6   | \( \geq 3,480 \)   | 1.81E−06| 0.0018              |
| 100_113  | 100   | 1.0 | 0.6   | \( \geq 100,000 \) | 9.17E−09| 0.0037              |
| 100_118  | 100   | 1.0 | 0.8   | \( \geq 256 \)     | 5.51E−04| 0.0033              |

Obviously, difficult instances are generated by assuming medium values of \( TF \), i.e. mainly \( TF = 0.6 \) and \( TF = 0.8 \). With respect to the choice of the \( RDD \), the tendency towards difficult data sets is less obvious.

After having been able to identify optimal alternatives even for the instances that remained unsolved by VNS and hillclimbing, we tried to estimate the number of distinct global optima for each of the ‘difficult’ data sets. A large number of distinct optimal alternatives has been found. In some cases even, the computation of distinct global optima has been externally terminated after identifying 1,000,000 (the case of \( n = 50 \)) or 100,000 (the cases of \( n = 100 \)) alternatives. Despite the fact that the search space comprises \( n! \) alternatives, these numbers are surprisingly high. Although identifying an optimal alternative still is a complex issue in general, we are able to show that there is a rather large number of distinct global optima.

Besides the sheer amount of distinct global optima, we have to question whether these alternatives share similar characteristics, or whether they are rather different. One possibility for such an analysis is based on measuring the entropy of the codings as described in [16]. In this approach, we measure the distribution of precedence relations between jobs in a finite set of solutions. Possible values range from 1 for an entirely random population to 0 for identical codings. As stated in [16], let \( \omega_{ijk} \) denote the observed number of solutions with \( \text{prec}_i(j, k) = \text{true} \) in a pool of size \( \mu \). The entropy \( E_{ijk} \) of this precedence relation of operations is given as \( E_{ijk} = -\frac{1}{\log \sqrt{2}} \frac{\omega_{ijk}}{\mu} \log \frac{\omega_{ijk}}{\mu} \), and the overall entropy is computed as the average over all \( E_{ijk}, (j \neq k) \).

As shown in Table the entropy of the set of optimal alternatives turns out to be extremely small. The obtained values arise from two observations. First, the codings of the alternatives are indeed rather similar. Second, the huge number of distinct global optima leads to the computation...
of, in average, such small entropies. As the latter circumstance biases the analysis of the entropy, we also computed the entropy of a subset of 100 randomly chosen distinct global optima, given in the rightmost column of Table 8. Again, the observed values are very small, demonstrating that in fact the optimal alternatives, while generally being distinct, really are ‘micro-mutations’ of each other.

6 Summary and conclusions

The current article presented an investigation of some local search heuristics for the single machine total weighted tardiness problem. In particular, we considered several issues raised in previous work, indicating that existing benchmark instances are not sufficiently challenging for modern metaheuristics. Therefore, our experiments first employed a rather simple hillclimbing algorithm. In these first experiments, we have been able to identify an order of neighborhood operators with respect to their relative performance. For the SMTWTP, the exchange neighborhood operator leads to significantly better results than shift operators.

Despite the problem’s complexity, many data sets have successfully been solved by simple hillclimbing. We also have been able to show that the control parameters used for the generation of the data sets have a significant influence on the difficulty of the benchmark instances.

Extended experiments have been carried out using Variable Neighborhood Search. On the basis of the experiments with the hillclimbing algorithms, an order of neighborhood operators within the VNS metaheuristics has been tested. The obtained results are indeed impressive. Even for the larger-sized instances, only few cases remain unsolved, and the average deviations to the optimal/best known solutions are surprisingly small.

A first investigation of data sets that, during the experiments, turned out to be ‘difficult’ followed. We have been able to show that a large number of distinct global optima exists for these instances. To some extent we may reason that the sheer number of global optima makes the instances attractive for local search.

Several conclusions arise from the experiments. While we cannot generally state that the SMTWTP is a solved problem as such, existing benchmark instances indeed appear to be unsuitable for the verification of novel metaheuristics. In this light, recent results are not too surprising, as even rather simple search strategies will lead to very close approximations of the optimal solutions of the established data sets. Before presenting novel metaheuristics for the SMTWTP and testing them on existing data sets, future research should therefore address the following open issues first:

1. **Proposition of challenging benchmark instances.**
   Clearly, bigger data sets are needed, significantly increasing the set of $n$ jobs from $n = 100$ to a larger number. This would prevent the application of simple hillclimbing/VNS algorithms as the computation of a locally optimal alternative becomes (prohibitively) costly. When designing such instances, values of $TF \in \{0.6, 0.8\}$ should be favored, while $TF \in \{0.2, 1.0\}$ should be avoided.

2. **Theoretical foundation/explanation of the obtained results.**
   While the observation made with respect to the different behavior of the neighborhood operators is interesting, there appears to be a lack of theoretical foundation of these results.

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believe it would be beneficial to explain the obtained results from a theoretical point of view, as opposed to a purely experimentally-driven research.

A first attempt in this direction could investigate the relative impact of the neighborhoods on the derived neighboring solutions. Based on such an investigation, it might be possible to conclude in general on the modifying impact of operators on alternatives.

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