Higgs Self-Coupling in $\gamma\gamma$ Collisions

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“The changing of bodies into light, and light into bodies, is very conformable to the course of nature, which seems delighted with transmutations.”

Isaac Newton
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Abstract: To establish the Higgs mechanism experimentally, one has to determine the Higgs self-interaction potential responsible for the electroweak symmetry breaking. This requires a measurement of the trilinear and quadrilinear self-couplings of the Higgs particle, as predicted by the Standard Model (SM). We propose to measure the trilinear Higgs self-coupling in $\gamma\gamma$ collisions just above the kinematic threshold $E_{\text{thr}} = 2M_H$, where $M_H$ is the Higgs mass. Our calculation reveals that the sensitivity of the cross-section $\sigma_{\gamma\gamma \to HH}$ to the Higgs self-coupling is maximal near the $2M_H$ threshold for $M_H = 115 - 150$ GeV, and is larger than the sensitivities of $\sigma_{e^+e^- \to ZHH}$ and $\sigma_{e^+e^- \to \nu\bar{\nu}HH}$ to this coupling for $2E_e \leq 700$ GeV. We envisage to (a) study $\gamma + \gamma \to H$ by constructing an X-band $e^-e^-$ linac and a terawatt laser system in order to produce Compton-scattered $\gamma$-ray beams for a 160-GeV photon collider ($2E_e = 200$ GeV); (b) add a positron source and repeat all measurements done at LEP and SLC with much better precision; and (c) subsequently install 70-MeV/m rf cavities in order to study $e^+ + e^- \to H + Z$, $e^+ + e^- \to t\bar{t}$ and $\gamma + \gamma \to H + H$ at $2E_e \lesssim 350$ GeV. The total length of the linac would be about 7 km.

1 Introduction

Enormous progress has been made in the field of high-energy, or elementary particle, physics over the past four decades. The existence of a subnuclear world of quarks and leptons, whose dynamics can be described by quantum field theories possessing gauge symmetry (gauge theories) has been firmly established. The Standard Model (SM) of particle physics gives a coherent quantum-mechanical description of electromagnetic, weak and strong interactions based on fundamental constituents — quarks and leptons — interacting via force carriers — photons, $W$ and $Z$ bosons, and gluons.

In this model, the relativistic theory of photons and electrons (called quantum electrodynamics, or QED) is a consequence of a spontaneously broken symmetry in a theory in which the weak and electromagnetic interaction are initially unified and the corresponding force carriers (gauge bosons) are massless. To account for the observed mass spectrum of the field quanta, one postulates the existence of a Higgs field, which is a scalar under spatial rotations but is a weak isodoublet. Like the graviton and the gauge bosons of the Standard Model, the Higgs boson mediates a fundamental force of nature. The coupling of the Higgs field to the vector fields that mediate the electroweak interaction is arranged so as to give the $W$ and $Z$ masses in the $10^2$ GeV range, while maintaining the photon mass at zero. The Higgs field thus provides the mechanism for electroweak symmetry breaking.

All of the couplings of the Higgs boson to fermions and gauge bosons are completely determined in terms of coupling constants and fermion masses. The coupling of a fermion to the scalar Higgs field is proportional to the mass of the fermion. The Higgs boson mass represents a free parameter of the model.

The $W$- and $Z$-boson masses are related by the electroweak mixing angle $\theta_W$ (also called the Weinberg angle): $M_W = M_Z \cos \theta_W$. The fact that interactions of all gauge bosons are determined by the electric charge and one free parameter, $\theta_W$, means that the Standard Model is a (partially) unified theory of the weak and electromagnetic interactions.
The SU(2)$_L$ $\otimes$ U(1)$_Y$ gauge invariance of the SM requires masses of the gauge bosons to be zero, since the presence of a mass term $M^2 A_\mu A^\mu$ for these particles would render the model non-invariant under the gauge transformation $A_\mu \rightarrow A_\mu - \partial_\mu \chi(x)$. In order to provide a mechanism for the generation of particle masses without violating the gauge invariance of the model, a complex scalar SU(2) doublet with four real fields and hypercharge $Y = 1$ is introduced:

$$\Phi \equiv \left( \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right)$$

(1)

in analogy with the $K^0$ system. The reason we need two complex fields rather than one is that three degrees of freedom are required to generate the masses of three gauge fields (the $W^\pm$ and the $Z^0$). The remaining degree of freedom will show up as a neutral massive Higgs field. The Higgs doublet conveniently serves two purposes, for it can also give mass to the fermions. The dynamics of the field $\Phi$ is described by the Lagrangian

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi)$$

(2)

where $(D_\mu \Phi)^\dagger (D^\mu \Phi)$ is the kinetic-energy term of $\mathcal{L}_\Phi$ and

$$V(\Phi^\dagger \Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

(3)

is the Higgs self-interaction potential with a minimum at

$$\langle \Phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/\lambda}$$

(4)

The quantity $v$ is the vacuum expectation value of $\Phi$. In the unitary gauge,

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

(5)

where $H$ is the physical Higgs field. The potential $V(\Phi)$ gives rise to terms involving only the scalar field $H$:

$$V_H = \frac{1}{2} (2\lambda v^2) H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$$

(6)

From this we infer that the Higgs mass

$$M_H = \sqrt{2\lambda v}$$

(7)

is related to the quadrilinear self-coupling strength $\lambda$. The trilinear self-coupling of the Higgs field is given by

$$\lambda_{HHH} = \lambda v = \frac{M_H^2}{2v}$$

(8)

and the self-coupling among four Higgs fields by

$$\lambda_{HHHH} = \frac{\lambda}{4} = \frac{M_H^2}{8v^2}$$

(9)

Evidently, the Higgs self-couplings are uniquely defined by the mass of the Higgs boson. The following definitions are often used: $\lambda_{HHHH} \rightarrow 3! \lambda_{HHH}$ and $\lambda_{HHHH} \rightarrow 4! \lambda_{HHHH}$.

The covariant derivative in (2) reads

$$D_\mu = i\partial_\mu + g T_a W^a_\mu - g' \frac{Y}{2} B_\mu$$

(10)
where $T_a$ ($a = 1, 2, 3$) denote the isospin generators of the SU(2)$_L$ gauge group, $Y$ represents the U(1)$_Y$ hypercharge generator, $g$ and $g'$ are the electroweak couplings, and $W^a_\mu$ and $B_\mu$ are the gauge fields associated with the two symmetry groups, respectively. Upon introducing the physical Higgs field (5) and transforming the electroweak eigenstates $W^a_\mu$ and $B_\mu$ to the mass eigenstates, the kinetic term in (2) can be expressed as

$$ (D_\mu \Phi)^\dagger (D^\mu \Phi) = \frac{1}{2} (\partial_\mu H)^2 + \frac{g^2}{4} (v + H)^2 \left( W^+_\mu W^-_\mu + \frac{Z_\mu Z^\mu}{2 \cos^2 \theta_W} \right) $$

where

$$ \cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad e = g \sin \theta_W = g' \cos \theta_W \quad \text{(12)} $$

($e$ is the electric charge). A comparison with the usual mass terms for the charged and neutral vector bosons reveals that

$$ M_W = \frac{g v}{2}, \quad M_Z = \frac{g v}{2 \cos \theta_W} = \frac{M_W}{\cos \theta_W} \quad \text{(13)} $$

From (11) we also infer that the Higgs-gauge boson interaction strengths are

$$ \lambda_{HWW} \equiv \frac{g^2 v}{2} = \frac{2 M_W^2}{v}, \quad \lambda_{HZZ} \equiv \frac{g^2 v}{8 \cos^2 \theta_W} = \frac{M_Z^2}{v} \quad \text{(14)} $$

and

$$ \lambda_{HWW} \equiv \frac{g^2}{4} = \frac{M_W^2}{v^2}, \quad \lambda_{HZZ} \equiv \frac{g^2}{8 \cos^2 \theta_W} = \frac{M_Z^2}{2v^2} \quad \text{(15)} $$

We can relate $v$ to the Fermi constant $G_F = 1.16639 \times 10^{-5}$ GeV as

$$ \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} = \frac{1}{2v^2} \quad \text{(16)} $$

Hence,

$$ v = (\sqrt{2} G_F)^{-1/2} \approx 246 \text{ GeV} \quad \text{(17)} $$

The coupling between the Higgs boson and any fermion $f$ is given by the interaction Lagrangian

$$ \mathcal{L}_{Hff} = -\frac{m_f}{v} H \bar{\psi}_f \psi_f \quad \text{(18)} $$

The corresponding coupling strength is

$$ \lambda_{Hff} = \frac{m_f}{v} \quad \text{(19)} $$

The Higgs boson can also couple to two photons. The decay $H \to \gamma \gamma$ does not occur at lowest level in the Standard Model because photons couple to charge and the Higgs boson is neutral. The decay proceeds through spin-1/2, spin-1 and spin-0 loops. The width is determined by

$$ \Gamma(H \to \gamma \gamma) = \frac{\alpha^2 G_F}{128 \pi^3 \sqrt{2}} M_W^3 \left| \sum_{i=1}^{n} N_{ci} Q_i^2 F_i \right|^2 $$

where $N_{ci}$ is the color multiplicity of particle $i$ (3 for quarks and 1 otherwise), $Q_i$ is the electric charge in units of $e$, and $F_i$ are some functions of $4m_i^2/M_W^2$. The $H \to \gamma \gamma$ decay mode evidently probes the existence of heavy charged particles. When the particle in the loop is much heavier than the Higgs boson, $F_0 \to -1/3$, $F_{1/2} \to -4/3$ and $F_1 \to 7$. Note the opposite sign between fermion and $W$ loops.
To summarize, Higgs production and decay processes can be computed in the SM unambiguously in terms of the Higgs mass alone. The Higgs-boson coupling to fermions and gauge bosons is proportional to the particle masses. We thus infer that the Higgs boson will be produced in association with heavy particles, and will decay into the heaviest particles that are kinematically accessible.

The discovery of a Higgs boson with a mass below about 135 GeV might indicate that the Standard Model is embedded in a supersymmetric theory. The minimal supersymmetric extension of the Standard Model (MSSM) introduces two SU(2) doublets of complex Higgs fields, whose neutral components have vacuum expectation values \( v_1 \) and \( v_2 \). In this model, spontaneous electroweak symmetry breaking results in five physical Higgs-boson states: two neutral scalar fields \( h^0 \) and \( H^0 \), a pseudoscalar \( A^0 \) and two charged bosons \( H^\pm \). This extended Higgs system can be described at ‘tree level’ by two parameters: the ratio \( \tan \beta \equiv v_2/v_1 \), and a mass parameter, which is generally identified with the mass of the pseudoscalar boson \( A^0 \), \( M_{A^0} \). While there is a bound of about 135 GeV on the mass of the lightest CP-even neutral Higgs boson \( h^0 \) \cite{1,2}, the masses of the \( H^0, A^0 \) and \( H^\pm \) bosons may be as large as 1 TeV.

The trilinear self-coupling of the lightest MSSM Higgs boson at ‘tree level’ is given by

\[
\lambda_{hhh} = \frac{M_A^2}{2v} \cos 2\alpha \sin(\beta + \alpha),
\]

where

\[
\tan 2\alpha = \tan 2\beta \frac{M_A^2 + M_{H^\pm}^2}{M_A^2 - M_{H^\pm}^2}.
\]

We see that for arbitrary values of the MSSM input parameters \( \tan \beta \) and \( M_A \), the value of the \( h^0 \) self-coupling differs from that of the SM Higgs boson. However, in the so-called ‘decoupling limit’ \( M_A^2 \sim M_{H^0}^2 \sim M_{H^\pm}^2 \gg v^2/2 \), the trilinear and quadrilinear self-couplings of the lightest \( CP \)-even neutral Higgs boson \( h^0 \) approach the SM value. The inclusion of one-loop MSSM Higgs-sector corrections and \( \mathcal{O}(m_t^4) \) Yukawa corrections does not lead to any significant deviations from the SM prediction \cite{3}. As a result, the \( h^0 \)-boson self-interactions in the ‘decoupling limit’ do not differ appreciably from those of the SM Higgs particle.

In the non-supersymmetric two-Higgs-doublet model (the simplest extension of the SM), large one-loop effects can occur. For charged Higgs bosons with masses of about 400 GeV, the decay widths of \( h^0 \to \gamma \gamma, h^0 \to \gamma Z \) and \( h^0 \to b\bar{b} \) may differ from the SM values by as much as \( 10\% \sim 25\% \) \cite{4}. In this model, the non-decoupling effects of the additional heavier Higgs bosons in loops can produce \( \mathcal{O}(100\%) \) deviations of the effective \( h^0 h^0 h^0 \) self-coupling from the SM prediction, even if the Higgs couplings to gauge bosons and fermions are almost SM-like \cite{5}.

The precision electroweak data obtained over the past sixteen years consists of over a thousand individual measurements. Many of these measurements may be combined to provide a global test of consistency with the Standard Model. The best constraint on \( M_H \) is obtained by making a global fit to the data, which yields \( M_H = 91^{+58}_{-37} \) GeV \cite{6}. The precision electroweak data, therefore, strongly suggest that the most likely mass for the SM Higgs boson is just above the limit of 114.4 GeV set by direct searches at the LEP \( e^+e^- \) collider \cite{7}.

The next crucial step in our investigation of the Standard Model would be to discover the Higgs boson and determine its properties. Ideally, one would like to determine, in a model-independent way, the mass, total width, spin, parity and \( CP \) properties of the Higgs boson, as well as its tree-level and one-loop induced couplings. In contrast to any anomalous couplings of the gauge bosons, an anomalous self-coupling of the Higgs particle would contribute to electroweak observables only at two-loop and higher orders, and is therefore virtually unconstrained by the current precision measurements \cite{8}.
2 Raisons d’être for a photon collider

Once the Higgs boson has been discovered, a thorough exploration of the Higgs sector of the Standard Model will be undertaken with hadron, $e^+e^-$ and photon colliders. The rich set of final states in $\gamma\gamma$, $pp$ and $e^+e^-$ collisions will play an essential role in measuring the mass, two-photon width, spin and parity of the Higgs boson, which are difficult to determine with only one initial state. By combining data from $e^+e^-$ and $\gamma\gamma$ collisions, the total decay width of the Higgs boson can be determined in a model-independent way with a precision of about 10% (see [9] and references therein).

Since photons couple directly to all fundamental fields carrying the electromagnetic current (leptons, quarks, $W$ bosons, supersymmetric particles), $\gamma\gamma$ collisions provide a comprehensive means of exploring virtually every aspect of the SM and its extensions. The production mechanisms in $e^+e^-$ collisions are often more complex and model-dependent. In $\gamma\gamma$ collisions, the Higgs boson will be produced as a single resonance in a state of definite CP, which is perhaps the most important advantage over $e^+e^-$ annihilations, where this $s$-channel process is highly suppressed. For the Higgs-boson mass in the range $115−200$ GeV, the effective cross-section for $\gamma\gamma\rightarrow H$ is about an order of magnitude larger than that for Higgs production in $e^+e^-$ annihilations. In this mass range, the process $e^+e^\rightarrow ZH$ requires considerably higher centre-of-mass (CM) energies than $\gamma\gamma\rightarrow H$. Since $\gamma\gamma\rightarrow H$ proceeds through a ‘loop diagram’ and receives contributions from all particles with mass and charge, this mode is a powerful probe of new physics beyond the SM. Moreover, we find that the sensitivity of the cross-section $\sigma_{\gamma\gamma\rightarrow HH}$ to the trilinear Higgs self-coupling is maximal near the $2M_H$ threshold for $M_H = 115−150$ GeV, and is larger than the sensitivities of $\sigma_{e^+e^-\rightarrow ZHH}$ and $\sigma_{e^+e^-\rightarrow \nu\bar{\nu}HH}$ to this coupling for $2E_e \lesssim 700$ GeV.

3 Higgs-pair production in $\gamma\gamma$ and $e^+e^-$ collisions

The production of a pair of SM Higgs bosons in photon-photon collisions

$$\gamma\gamma \rightarrow HH \quad (23)$$

which is related to the Higgs-boson decay into two photons, is due to $W$-boson and top-quark box and triangle loop diagrams. The total cross-section for $\gamma\gamma\rightarrow HH$ in polarized photon-photon collisions, calculated at the leading one-loop order [10] as a function of the $\gamma\gamma$ centre-of-mass energy and for $M_H = 115−150$ GeV, is shown in Fig. 1a. The cross-sections calculated for equal $(J_z = 0)$ photon helicities, $\sigma_{\gamma\gamma\rightarrow HH}(J_z = 0)$, and for different values of $M_H$ rise sharply above the HH-threshold, and each has a peak value of about 0.4 fb at a $\gamma\gamma$ centre-of-mass energy of 400 GeV. In contrast, the cross-sections $\sigma_{\gamma\gamma\rightarrow HH}(J_z = 2)$ rise much slower with energy, because a pair of Higgs bosons is produced in a state with orbital angular momentum of at least 2; each of these cross-sections reaches a value of about 0.4 fb at a $\gamma\gamma$ centre-of-mass energy of 800 GeV.

The cross-sections for equal photon helicities are of special interest, since only the $J_z = 0$ amplitudes contain contributions with trilinear Higgs self-coupling. By adding to the SM Higgs potential [8] a gauge-invariant dimension-6 operator $(\Phi^4\Phi)^3$, one can introduce a gauge-invariant anomalous trilinear Higgs coupling $\delta\kappa$ [10]. For the reaction (23), the only effect of such a coupling in the unitary gauge would be to replace the trilinear $HHH$ coupling of the SM, Eq. (8), by an anomalous Higgs self-coupling

$$\tilde{\lambda}_{HHH} = (1 + \delta\kappa)\lambda_{HHH} \quad (24)$$
Figure 1: (a) The total $\gamma \gamma \to HH$ cross-section as a function of the $\gamma \gamma$ centre-of-mass energy for $M_H=115, 120, 130, 140$ and 150 GeV. Contributions for equal ($J_z = 0$) and opposite ($J_z = 2$) photon helicities are shown separately.

(b) The cross-sections for HH production in $\gamma \gamma$ collisions for anomalous trilinear Higgs self-couplings $\delta \kappa = 0, \pm 1, \pm 0.3$.

The dimensionless anomalous coupling $\delta \kappa$ is normalized in such a way that $\delta \kappa = -1$ exactly cancels the SM HHH coupling. The cross-sections $\sigma_{\gamma \gamma \to HH}$ for various values of $\delta \kappa$ are shown in Fig. 1b.

In an experiment to measure the trilinear Higgs self-coupling, the contribution from $\gamma \gamma \to HH$ for opposite photon helicities represents an irreducible background. Clearly, the optimal energy for such measurements would be somewhere between the production threshold and 400 GeV. In order to ascertain the potential of $\gamma \gamma$ colliders for measuring an anomalous Higgs self-coupling, one must take into account the fact that photon spectra will not be monochromatic [11]. The cross-section for Higgs-pair production in polarized $\gamma \gamma$ collisions is given by

$$\sigma_{\gamma \gamma \to HH} = \int \frac{y^2}{4M_H^2/s} \frac{dL_{\gamma \gamma}}{d\tau} \left[ \frac{1}{2} \left( 1 + \langle \xi_2^{(1)} \xi_2^{(2)} \rangle \right) \sigma_{++}(\hat{s}) + \frac{1}{2} \left( 1 - \langle \xi_2^{(1)} \xi_2^{(2)} \rangle \right) \sigma_{+-}(\hat{s}) \right]$$

(25)

where

$$\frac{dL_{\gamma \gamma}}{d\tau} = \int_{\tau/y_m}^{y_m} \frac{dy}{y} f_{\gamma}(x,y) f_{\gamma}(x,\tau/y),$$

$$\tau = \frac{\hat{s}}{s}, \quad 0 \leq y = \frac{E_\gamma}{E_e} \leq y_m = \frac{x}{x+1}, \quad x \equiv \frac{4E_e\omega_0}{m_e^2}.$$  

(26)

Here $E_e$ is the energy of the electron beam, $\omega_0$ is the laser photon energy, $f_\gamma(x,y)$ is the photon momentum distribution function and $\langle \xi^{(1,2)} \rangle$ are mean photon helicities [11]; $\sigma_{++}$ and $\sigma_{+-}$ are the cross-sections for Higgs-pair production, calculated assuming monochromatic photons with total helicities $J_z = 0$ and $J_z = 2$, respectively. As usual, the dimensionless parameter $x$ has been
set to 4.8 ($y_m \approx 0.8$) to avoid undesirable backgrounds. In what follows we shall assume 90% polarization for electron beams and 100% for laser beams, in a configuration that maximizes the $\gamma\gamma$ luminosity for $J_z = 0$ in the high-energy part of the photon spectrum [12]:

$$L_{\gamma\gamma} = \int_{\tau=(0.8 y_m)^2}^{y_m^2} d\tau \frac{dL_{\gamma\gamma}}{d\tau} \approx \frac{1}{3} L_{e^+e^-}. \quad (27)$$

Figure 2: For the process $\gamma\gamma \rightarrow \text{HH}$, the number of standard deviations from the SM prediction for event rates, defined by Eq. (28), is plotted as a function of the $e^-e^-$ centre-of-mass energy assuming a $\gamma\gamma$ luminosity $L_{\gamma\gamma} = 300$ fb$^{-1}$.

In terms of standard deviations, the discrepancy between the SM prediction for event rates and that for zero HHH coupling is defined by

$$\#\text{STD} = \frac{|\sigma(\delta_\kappa = 0) - \sigma(\delta_\kappa = -1)|}{\sqrt{\sigma(\delta_\kappa = 0)^2}} \sqrt{L_{\gamma\gamma}}. \quad (28)$$

(see Fig. 2). Here the cross-sections are given by Eq. (25), an efficiency of 100% is assumed and the $\gamma\gamma$ luminosity is taken to be 300 fb$^{-1}$ (see (27)). For $M_H = 120$ GeV, a maximum sensitivity of almost $9\sigma$ is achieved at a $e^-e^-$ centre-of-mass energy of 370 GeV. An effect of more than $5\sigma$ is seen already at energies above 310 GeV. Note that the abscissa in Fig. 2 shows the $e^-e^-$ CM energies. For instance, $E_{e^-e^-} = 310$ GeV corresponds to a maximum $\gamma\gamma$ CM energy of $y_m E_{e^-e^-} \approx 254$ GeV, which is just 14 GeV above the HH-threshold. Of course, the numbers shown in Fig. 2 represent the maximum achievable sensitivity assuming 100% detection and reconstruction efficiencies and no backgrounds. In reality, the sensitivity to an anomalous Higgs self-coupling will be considerably worse. Nevertheless, Fig. 2 shows that in photon-photon collisions the optimum $e^-e^-$ CM energy
for measuring the trilinear Higgs self-coupling is rather low (between 300 and 400 GeV) for a Higgs-boson mass $M_H \leq 130$ GeV.

It is well known that hadron colliders are not well suited for measuring the self-coupling of the Higgs boson if $M_H \leq 140$ GeV \[13\]. The potential of a future $e^+e^-$ collider for determining the $HHH$ coupling has therefore been closely examined \[14, 15, 16, 17, 18\]. The trilinear Higgs-boson self-coupling can be measured either in the double Higgs-strahlung process

$$e^+e^- \rightarrow ZHH$$  \hspace{1cm} (29)

or in the $W$-boson fusion reaction

$$e^+e^- \rightarrow \nu_e\bar{\nu}_eHH.$$  \hspace{1cm} (30)

The total cross-section for the Higgs-pair production in $e^+e^-$ collisions, calculated for unpolarized beams, is presented in Fig 3a. If the electron beams are 100% polarized, the cross-section for the reaction (29) will approximately stay the same, but the cross-section for the $W$-fusion process (30) will be twice as large. The cross-sections shown in Fig 3a were calculated at ‘tree level’ using the program CompHEP \[19\]. The effect of full $\mathcal{O}(\alpha)$ electroweak radiative corrections to the process (29) has been shown to be small around the peak of the corresponding cross-section \[18\]. From Fig 3a, we infer that the SM cross-section for the process (29) exceeds 0.1 fb at 400 GeV for $M_H=120$ GeV, and reaches a broad maximum of about 0.2 fb at a $e^+e^-$ centre-of-mass energy of 550 GeV. The SM cross-section for the $W$-boson fusion process (30) stays below 0.1 fb all the way up to $E_{e^+e^-} \approx 1$ TeV.

Figure 3: (a) The total cross-sections for $e^+e^- \rightarrow ZHH$ and $e^+e^- \rightarrow \nu_e\bar{\nu}_eHH$ as functions of the $e^+e^-$ centre-of-mass energy for $M_H=120$ GeV and anomalous trilinear Higgs self-couplings $\delta\kappa = 0, -1$. (b) For HHH production in $e^+e^-$ collisions, the number of standard deviations from the SM prediction for event rates, defined analogously to Eq. (28), is plotted as a function of the $e^+e^-$ centre-of-mass energy assuming an $e^+e^-$ luminosity $L_{e^+e^-} = 1000$ fb$^{-1}$. 

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The cross-sections for the processes (29) and (30), computed for $\delta \kappa = 0, -1$ and $M_H = 120$ GeV, are shown in Fig. 3a. The number of standard deviations from the SM prediction for event rates, defined analogously to Eq. (28), is shown in Fig. 3 for $M_H = 120$ and 150 GeV. We again assume an efficiency of 100%, but the $e^+e^-$ luminosity is taken to be 1000 fb$^{-1}$. The polarization of the electron beams was chosen to be 90%. For $M_H = 120$ GeV, a maximum sensitivity of about $6\sigma$ is achieved at a centre-of-mass energy of 500 GeV in the double Higgs-strahlung process (29). An effect of more than $5\sigma$ is seen at energies above 400 GeV. A comparison of Fig. 2 and Fig. 3b reveals that the optimum $e^+e^-$ centre-of-mass energy for measuring the Higgs self-coupling in the reaction (29) is about 500 GeV, significantly higher than the corresponding energy in $\gamma\gamma$ collisions.

4 Backgrounds

We shall present an order-of-magnitude estimate of the most important backgrounds to the process $\gamma\gamma \rightarrow HH$. The dominant background is the $W$-boson pair production $\gamma\gamma \rightarrow W^+W^-$, with the total cross-section of 70 pb at 300 GeV. However, by imposing the invariant mass cut

$$|M(q\bar{q}) - M_H| < 5 \text{ GeV} \quad (31)$$

the $W^+W^-$ background can be reduced by about four orders of magnitude. In order to suppress this background even further, one could rely on the fact that the predominant decay mode of the SM Higgs boson with $M_H = 115$ to 130 GeV is into a pair of $b$ quarks, with a SM branching ratio that decreases from 73% to 53% as the Higgs-boson mass increases. This is the dominant decay mode also of the MSSM $h^0$ boson for various values the MSSM parameters, in particular for $\tan\beta > 1$. In order to select the $HH \rightarrow b\bar{b}b\bar{b}$ events, we require that at least three jets be identified as originating from $b$-quarks. If we assume that the standard method used for tagging $b$-hadrons at the LEP $e^+e^-$ collider [20] can also be used at a photon collider, then the sample tagged as $b$-quark would have the following flavour composition: 4.3% light quarks, 10.4% $c$-quarks and 85.4% $b$-quarks [20]. The requirement that at least three jets originating from $W^\pm$ decays be identified as $b$-jets would suppress $\gamma\gamma \rightarrow W^+W^-$ by another three orders of magnitude, to a level well below the HH signal.

Table 1: The cross-sections for the production of four heavy quarks in unpolarized $\gamma\gamma$ collisions for $E_{\gamma\gamma} = 250$ and 300 GeV.

| E_{\gamma\gamma} = 250 (300) GeV | $\sigma_{tot}$ (fb) | $|\cos \theta_{q\bar{q}}| < 0.9$ | $|\cos \theta_{q\bar{q}}| < 0.9$ |
|---|---|---|---|
| M_H = 120 GeV | | | |
| $\gamma\gamma \rightarrow b\bar{b}b\bar{b}$ | 360 (380) | 5.0 (3.9) | 0.015 (0.015) |
| $\gamma\gamma \rightarrow b\bar{b}c\bar{c}$ | 9400 (9800) | 66 (52) | 0.13 (0.16) |
| $\gamma\gamma \rightarrow c\bar{c}c\bar{c}$ | 81000 (83000) | 150 (120) | 0.24 (0.26) |

The next most significant background is the direct production of four heavy quarks in photon-photon collisions. The total cross-sections for $b\bar{b}b\bar{b}$, $b\bar{c}c\bar{c}$ and $c\bar{c}c\bar{c}$ production are shown in Table 1. These cross-sections do not decrease with energy and are quite large. For instance, the cross-section for the production of four $c$-quarks is even larger than that for $W^+W^-$ production. Since the cross-sections calculated for two-photon helicities $J_z = 0$ and $J_z = 2$ have similar magnitudes,
the polarization of photon beams does not lead to a reduction in the four-quark background. The $b$ and $c$ quarks are produced mostly in the forward or backward direction. As shown in Table 1, a simple angular cut

$$|\cos \theta_{q, \bar{q}}| < 0.9 \quad (32)$$

suppresses these backgrounds by at least two orders of magnitude. Near the HH production threshold, the angular distribution of $b$-jets originating from Higgs-boson decays is isotropic, and the efficiency of the angular cut (32) is about 80%. Since the cross-sections for quark production are still much larger than the cross-section for double Higgs-boson production after the cut (32), the invariant-mass cut (31) should also be imposed. As shown in Table 1, after these two cuts the cross-section for $b\bar{b}b\bar{b}$ production is already an order of magnitude smaller than $\sigma_{\gamma\gamma \rightarrow HH}$, and the cross-sections for $bbc\bar{c}$ and $cc\bar{c}\bar{c}$ production are of the same order as $\sigma_{\gamma\gamma \rightarrow HH}$. The additional requirement that at least three jets be identified as $b$-jets would suppress these cross-sections well below that of the signal. After the invariant-mass cut (31), the angular cut (32) and the $b$-tagging requirement, the reconstruction efficiency for the HH final state is about 50%. A more thorough study will definitely improve this number.

Other potential background sources are $\gamma\gamma \rightarrow b\bar{b}Z$, $\gamma\gamma \rightarrow c\bar{c}Z$, $\gamma\gamma \rightarrow q\bar{q}'W$, $\gamma\gamma \rightarrow W^+W^-Z$ and $\gamma\gamma \rightarrow ZZ$ processes. We believe that appropriate invariant-mass and angular cuts, as well as the $b$-jet tagging requirement, would suppress these backgrounds to a manageable level.

## 5 The proposed facility

We propose the construction of an X-band $e^-e^-$ linac (based on the JLC design) and a terawatt laser system (based on the Mercury architecture) in order to produce Compton-scattered $\gamma$-ray beams for a photon collider [9]. The key advantage of using $e^-e^-$ beams is that they can be polarized to a high degree (about 90%).$^1$ In $\gamma\gamma$ collisions, a light Higgs boson can be detected either as a peak in the invariant mass distribution or by conducting an energy scan exploiting the sharp high-energy edge of the $\gamma\gamma$ luminosity distribution [9]. The proposed facility would use 40-MeV/m rf cavities in a 7-km tunnel to reach a centre-of-mass energy $2E_e = 200$ GeV ($E_{\gamma\gamma} \approx 160$ GeV). It would be capable of producing around $10^4$ light Higgs bosons per year.

We envisage to add a positron source to the linac, turning it into a high-luminosity $e^+e^-$ collider [22]. Such a machine would operate in a wide energy range, from the $Z^0$ peak to well above the $WW$ threshold. High-precision studies of electroweak physics provide a natural complement to the direct searches for the Higgs boson. In principle, all measurements done at LEP and SLC could be repeated at the proposed $e^+e^-$ collider with much better accuracy. Assuming a geometric luminosity $\mathcal{L}_{e^+e^-} \approx 5 \times 10^{33}$ cm$^{-2}$s$^{-1}$ at the $Z^0$ resonance, and the cross-section $\sigma_z \approx 30$ nb, about $2 \times 10^9$ $Z^0$ events would be produced in an operational year of $10^7$ s, which is approximately 200 times the entire LEP statistics. Moreover, about $10^6$ $W$ bosons could be detected near the $W$-pair threshold at the optimal energy point for measuring the $W$-boson mass. This would open new opportunities for high-precision electroweak studies [23].

$^1$ Both the energy spectrum and polarization of the backscattered photons depend strongly on the polarizations of the incident electrons and laser photons. By polarizing the incident beams one can tailor the photon energy distribution to one’s needs [21]. In a collision of two photons, the possible helicities are 0 or 2. For example, the Higgs boson is produced in the $J_z = 0$ state, whereas the background processes $\gamma\gamma \rightarrow b\bar{b}, c\bar{c}$ are suppressed for this helicity configuration. The circular polarization of the photon beams is therefore an important asset, for it can be used both to enhance the signal and suppress the background.
In order to study $e^+ e^- \rightarrow H + Z$, $e^+ e^- \rightarrow t\bar{t}$ and $\gamma + \gamma \rightarrow H + H$, we propose to install 70-MeV/m rf cavities in the same tunnel once the technology for their production becomes available. The maximum centre-of-mass energy would then be $2E_e \approx 350$ GeV ($E_{\gamma\gamma} \approx 280$ GeV), sufficiently high to produce $t\bar{t}$ pairs. From a scan of the $t\bar{t}$ production cross-section in the $t$-pair threshold region, the top-quark mass could be measured with $10^2$ MeV accuracy.

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Erratum: In Ref. [9] on page 8, the sentence beginning with “The rich set of final states in $\gamma\gamma$, $e\gamma$ and $e^-e^-$ collisions...” should read “The rich set of final states in $\gamma\gamma$, $pp$ and $e^+e^-$ collisions...”.