Comparative Study of Ranking Methods for Fuzzy Transportation

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Abstract
There are several methods that are used to solve the traditional transportation problems whose units of supply, demand quantities, and cost transportation are known exactly. These methods obtain basic solution, and develop it to the best solution through a series of consecutive calculations to obtain the optimal solution. The steps are more complex with fuzzy variables, so this paper presents the disadvantages of solutions of the traditional ways with existence of variables in the fuzzy form.

This paper also presents a comparison between the results that emerged after using different conversion ranking formulas to convert from fuzzy form to crisp form on the same numerical example with a full fuzzy form. The problem has been then converted into a linear programming model, and the BIG-M method to be later used to find the optimal solution that represents the number of units transferred from processing or supply centers to a number of demand centers based on the known cost of transportation.

Achieving the goal of the problem is by finding the lowest total transportation cost, while the comparison is based on that value. The results are presented in a comprehensive table that organizes data and results in a way that facilitates quick and accurate comparison. An amendment to one of the order formats was suggested, because it has different results compared to other formulas. One of the ranking equations is modified, because it has different results compared to other methods.

Keywords: Trapezoidal Intuitionistic Fuzzy Numbers, Fuzzy Transportation Problems, Ranking Function, Linear Programming Model.

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1. Introduction

Transportation problem is classified as an important linear programming model which is solving means finding the optimal solution that represents the final optimum value of the total cost of transportation problems. Researchers [1] showed that the first transportation model was presented by Hitchcock. In 1965, the theory of fuzzy set was presented by [2]; whereas, the concepts of uncertainty and fuzzy set were developed by many researchers [3].

In general, the transportation model "classic model" represents the known data in the problem which is the cost of transportation of one unit from supply center to demand center. This model is solved by many different methods to find an optimal solution, such as lower cost LCM, north-west corner NWM, Vogel approximated method VAM, and stepping stone method SSM [4]. All these famous methods looking for an optimal distribution way to transport units among cells of the model table with lowest total cost value.

Solving the model means finding the number of units that are transported from the number (i) of appropriate distribution supply centers to a number (j) of appropriate demand centers, so that the goal is to get the lowest cost of transferred units. These costs are organized in a table which is appropriate to the total number of distribution centers and the number of demand centers as described in Table-1 [5].

Table 1 - Transportation model

| T | D1 | D2 | D3 | ... | Dn |
|---|----|----|----|-----|----|
| S1 | C11 | C12 | C13 | C14 | C1n |
|    | x11 | x12 | x13 |     | x1n |
| S2 | C21 | C22 | C23 | C24 |     |
|    | x21 | x22 | x23 |     |     |
| Sm | Cm1 | Cm2 |     |     |     |
| Di | D1 | D2 | D3 | ... | Dn |
| Demand |     |     |     |     |     |

where: \( x_{ij} \) is a number of units which transported from \((i^{th})\) source to \((j^{th})\) demand.

\( C_{ij} \) is a transportation cost for one unit from \((i^{th})\) source to \((j^{th})\) demand.

\( S_i \) is a number of unit which are available at \((i^{th})\) source.

\( D_j \) is a number of unit which are demanded from \((j^{th})\) destination.

2. Basic concepts

In this section, some definitions represent basic information of the proposed comparison [6,7].

**Definition 1:** A function \( \Re \): \( H(\tilde{x}) \rightarrow R \) be a ranking function, where \( H(\tilde{x}) \) is known by a set of fuzzy numbers into real numbers, such that \( \Re \) is mapping each fuzzy number (triangular, trapezoidal or pentagon) into real numbers line.

**Definition 2:** Let \( \tilde{X} \) subset of universal set of real numbers \( R \) then it is said to be fuzzy set number if its membership function \( \mu_{\tilde{X}}(x) \) mapping domain element \( x \in X \) to closed interval [0, 1].

Membership function has the following properties.

1- It is represented by piecewise continues function or discrete points.

2- It holds a convex function property.

3- It is defined by many kinds of parameters as triangular, trapezoidal, pentagonal or octagonal [8].
4- If there exists \( m_0 \in X \) such that \( \mu_{\tilde{X}}(m_0) = 1 \) then \( \bar{X} \) is said to be normal.

The following Figure-1 presents function of trapezoidal fuzzy numbers.

\[ \mu_{\tilde{X}}(x) \]  

\[ 0 \quad x_1 \quad x_2 \quad m_0 \quad x_3 \quad x_4 \quad X \quad R \]

**Figure 1**-Function of trapezoidal fuzzy numbers

Definition 3: A fuzzy numbers set \( X \) is said to be triangular fuzzy numbers and expressed by \( (x_1, x_2, x_3) \) where \( x_1, x_2, x_3 \) are real numbers and its membership function \( \mu_{\tilde{X}}(x) \) is written as follows [3]:

\[
\mu_{\tilde{X}}(x) = \begin{cases} 
\frac{a-x_1}{x_2-x_1} & \text{if } x_1 \leq \alpha < x_2 \\
1 & \text{if } \alpha = x_2 \\
\frac{x_2-a}{x_3-x_2} & \text{if } x_2 \leq \alpha < x_3 \\
0 & \text{otherwise}
\end{cases}
\]  

(1)

Definition 4: A fuzzy numbers set \( X \) is said to be trapezoidal fuzzy number and expressed by \( (x_1, x_2, x_3, x_4) \) where \( x_1, x_2, x_3, x_4 \) are real numbers and its membership function \( \mu_{\tilde{X}}(x) \) is formed as follows:

\[
\mu_{\tilde{X}}(x) = \begin{cases} 
\frac{a-x_1}{x_2-x_1} & \text{if } x_1 \leq \alpha < x_2 \\
1 & \text{if } x_2 \leq \alpha < x_3 \\
\frac{x_4-a}{x_4-x_3} & \text{if } x_3 \leq \alpha < x_4 \\
0 & \text{otherwise}
\end{cases}
\]  

(2)

Other definitions such as Pentagonal, octagonal, etc. are defined similarly [9].

3. Mathematical Model and Environment of The fuzzy Transportation

Transportation problem and its available data include three main parts which follow the model of linear programing. The first part of transportation problems related to existence of the objective function that contains the total cost of transportation which depends on the number of units \( (x_{ij}) \) and costs \( C_{ij} \) that were assigned previously for each cell in the model of transportation problem. The objective function of linear programing is satisfied in terms of the first part that has the following form:

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} \cdot x_{ij}
\]  

(3)

The second part is satisfied within the form of constraints to the sum of the required units that have been transported. Note that the number of these units cannot be more than number of available supply units [2].

\[
\sum_{j=1}^{n} x_{j} \leq S_{i}
\]  

(4)
Also, the number of units equipped not less than the number of units required from demand centers.

\[ \sum_{i=1}^{m} x_{ij} \geq D_j \]  \hspace{1cm} (5)

In general, in transportation model, the number of available units in the supply sources is equal to the number of total demand \([6]\).

\[ \sum_{i=1}^{m} S_i = \sum_{i=1}^{n} D_j \]  \hspace{1cm} (6)

The last requirement of the whole linear programming based on the meaning of non-negativity which is satisfied due to the numbers that are used real and positive units.

\[ x_{ij} \geq 0; \hspace{1cm} \text{for all} \hspace{0.5cm} i, j \]  \hspace{1cm} (7)

The general mathematical formula for linear programming is represented by the following transportation model \([10]\).

Minimize \((Z)\) : \( Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} * x_{ij} \)

Subject to constraints:

\[
\begin{align*}
\sum_{j=1}^{n} x_{ij} & \leq S_i; \hspace{1cm} i = 1, 2, 3 \ldots m \\
\sum_{i=1}^{m} x_{ij} & \geq D_j; \hspace{1cm} j = 1, 2, 3 \ldots n \\
\sum_{i=1}^{m} S_i & = \sum_{j=1}^{n} D_j; \hspace{1cm} \forall i, j \\
x_{ij} & \geq 0; \hspace{1cm} \forall i, j
\end{align*}
\]  \hspace{1cm} (8)

In many transportation problems the decision maker has no proven and uncertain information about the number of units that are available for transportation from supply centers and the number of requirements for all the following expressions \((x_{ij}), (C_{ij}), (S_i), (D_j)\). The fact above is depending on the nature of the topic on which the problem was designed, and can represent these data with triple (triangular) points \((C_{ij}^1, C_{ij}^2, C_{ij}^3)\) trapezoidal points \((C_{ij}^1, C_{ij}^2, C_{ij}^3, C_{ij}^4)\) pentagonal point or more \([6,8]\).

4. Shortcoming of the Existing Methods

There are several methods of solution apply algorithms similar to those used in traditional problems, and develop it to include fuzzy data after definition of some operations and properties. Meanwhile, some of shortcoming points arise while applying the algorithms.

1- The algorithms of the famous methods to obtain the basic solution for traditional transportation problem are incompetent when it used to solve a model that contains fuzzy triangular, trapezoidal or pentagonal data \([8]\). Additionally, some of these problems its data consist of two sets of membership and non-membership, and this resulted in increases the complexity of arithmetical operations \([5]\).

2- The algorithms of the developed methods for solving the fuzzy data need to have many additional calculations in order to obtain the basic solution, and then develop it to reach the optimal solution \([11]\).

3- Some researchers used the general model of linear programming to solve the fuzzy model by dividing it into problems equal to the number of variables in a single cell. This procedure doubles the number of iterations that used in the algorithm of solution \([12]\).

4- While applying some original algorithms to solve a fuzzy transportation problem because of using subtraction operations, some negative numbers appear in the occupied cells that represent the number of transferred units according to transportation problem model. The negative signal is not realistic and not correspond to the nature of used data\([8]\).

5. Ranking Functions (R):

In order to avoid the shortcoming that were presented by solving the transportation model which includes data in the form of fuzzy numbers, the ranking function is used for the purpose of converting the data of the problem from fuzzy number to crisp number \((R)\). Thus, ranking function shortens the procedures to reach to the optimal solution. The problem is first converted into a linear programming problem, and then is solved by using a software program \((TORA)\) that characterized by precision and the lowest number of procedures.

To study the results and compare the elements of optimal solution in every format of ranking formulas, the following numerical example in the Table-2 shows a full fuzzy formula data of transportation problem with parameters designed as trapezoidal form.
Table 2-Data transportation problem represents full fuzzy

| T₂ | D₁ | D₂ | D₃          | Availability |
|----|----|----|-------------|--------------|
| S₁ | 5  | 6  | 8          | 10          | 16           | 18          | 20          | 22          | 17           | 18          | 20          | 25          |
| S₂ | 37 | 38 | 40         | 42          | 28           | 29           | 30          | 32          | 52           | 53          | 55          | 57          |
| S₃ | 18 | 19 | 20         | 22          | 22           | 23           | 25          | 27          | 32           | 33          | 35          | 37          |
| Demand | 8 | 9  | 10         | 12          | 5            | 6            | 7           | 8           | 12           | 13          | 14          | 16          |

The value of the objective function Z that obtained from using ranking formula, should be between the objective function of first parameters x₁ as the lower limit Zₐ in Table-3 and objective function of the fourth parameters x₄ as the upper limit Zᵣ.

Table 3-The first parameter x₁ of numerical example

| T₁ | D₁ | D₂ | D₃ | Availability |
|----|----|----|----|--------------|
| S₁ | 5  | 7  | 16 | 17           |
| S₂ | 37 | 28 | 52 | 45           |
| S₃ | 18 | 22 | 32 | 46           |
| Demand | 8 | 5  | 12 |              |

The optimal solution of lower value Zₐ is:

x₁₃ = 12, x₁₁ = 8, x₁₂ = 5, x₁₄ = 45, Sx₁₄ = 38.

Zₐ = 16*12+18 *8 +7 *5+ 22*0 =371

The value of objective function (in case upper value x₄ ) is in the Table-4

Table 4-The fourth parameter x₄ of numerical example

| T₄ | D₁ | D₂ | D₃ | Availability |
|----|----|----|----|--------------|
| S₁ | 10 | 12 | 22 | 25           |
| S₂ | 42 | 32 | 57 | 55           |
| S₃ | 22 | 27 | 37 | 55           |
| Demand | 12 | 8  | 16 |              |

Then the optimal solution of upper value Zᵣ is:

x₁₁ = 1, x₁₂ = 8, x₁₃ = 16, x₁₄ = 55, x₁₅ = 44

Zᵣ = 16*12+18 *8 +7 *5+ 22*11 = 700

Therefore, the value of the objective function Z with any ranking formula must be

371 ≤ Z ≤ 700.

The following various ranking formulas are applied on the same numerical example to convert the data from fuzzy to crisp form.

5.1. The first formula of ranking function:

Let $(\tilde{x})$ be a fuzzy number then $R(\tilde{x})$ represents the Robust ranking technique for trapezoidal numbers [3,13].

\[ R(\tilde{x}) = \int_0^1 0.5 \left( s^l_{\tilde{x}} , s^u_{\tilde{x}} \right) d\alpha = \langle x_1, x_2, x_3, x_4 \rangle \]

where \((s^l_{\tilde{x}}, s^u_{\tilde{x}}) = [(x_2 - x_1) \alpha + x_1, x_4 - (x_4 - x_3) \alpha] \]

Then \(R(x_1, x_2, x_3, x_4) = \int_0^1 0.5 \left( (x_2 - x_1) \alpha + x_1, x_4 - (x_4 - x_3) \alpha \right) d\alpha \)

For example $R(5, 6, 8, 10) = \int_0^1 0.5 \left( 6 - 5 \right) \alpha + 5,10 - (10 - 8) \alpha \right) d\alpha$

\[ = \int_0^1 0.5 \left( \alpha + 5 + 10 - 2\alpha \right) d\alpha \]

\[ = 0.5 \left[ 15 - 1/2 \right] = 29/4 = 7.25 \]

The ranking formula is applied on all data of the problem. Then, the results appeared in crisp form, and placed on a similar Table-5.
The problem is converted into a linear programming problem with constraints equal to the number of sources, other constraints equals to the number of demand centers and non-negative constrains as shown in the following formula [1].

Minimize \( Z \): 
\[ Z = \sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij} x_{ij} \]

Subject to constraints:
\[ \sum_{j=1}^{3} x_{ij} \leq S_i \; ; \text{such that } i = 1,2,3 \]
\[ \sum_{i=1}^{3} x_{ij} \geq D_j \; ; \text{such that } j = 1,2,3 \]
\[ x_{ij} \geq 0 \; ; \forall i,j \]

Minimize(\( Z \)): 
\[ Z = 7.25 x_{11} + 9.25 x_{12} + 19 x_{13} + 39.25 x_{21} + 29.75 x_{22} + 54.25 x_{23} + 19.75 x_{31} + 24.25 x_{32} + 34.25 x_{33} \]

Subject to constraints:
\[ x_{11} + x_{12} + x_{13} \leq 20 \]
\[ x_{21} + x_{22} + x_{23} \leq 49.25 \]
\[ x_{31} + x_{32} + x_{33} \leq 49.25 \]
where: \( x_{ij} \geq 0 \; ; \forall i,j \; ; i,j = 1,2,3 \)

The problem is solved after that by software (TORA program).

The optimal solution \( x_{12} = 6.25 \), \( x_{13} = 13.75 \), \( x_{31} = 9.75 \), \( x_{32} = 0.25 \)
The value of objective function \( Z = 9.25 \times 6.25 + 19 \times 13.75 + 19.75 \times 9.75 + 24.25 \times 0.25 = 517.69 \).

When the problem solved by Least Cost Method, the results as follow:
The basic solution is \( x_{11} = 9.75 \), \( x_{12} = 6.5 \), \( x_{13} = 3.75 \), \( x_{33} = 10 \).
The value of objective function \( Z = 9.25 \times 7.25 + 9.25 \times 6.5 + 19 \times 3.75 + 24.25 \times 0.25 = 536.93 \).

Then the solution improved by a Stepping Stone Method, and the obtained solution shown in the Table-6

### Table 5- Application of Rupust ranking formula to convert to crisp form

| T | \( D_1 \) | \( D_2 \) | \( D_3 \) | Availability |
|---|---|---|---|---|
| \( S_1 \) | 7.25 | 9.25 | 19 | 20 |
| \( S_2 \) | 39.25 | 29.25 | 54.25 | 49.25 |
| \( S_3 \) | 19.75 | 24.25 | 34.25 | 49.25 |
| Demand | 9.75 | 6.5 | 13.75 | |

The optimal solution is \( x_{11} = 0.25 \), \( x_{12} = 6.0 \), \( x_{13} = 13.75 \), \( x_{22} = 0.5 \), \( x_{31} = 9.5 \)
The value of objective function is:
\[ Z = \sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij} x_{ij} = 7.25 \times 0.25 + 9.25 \times 6.0 + 19 \times 13.75 + 29.25 \times 0.5 + 19.75 \times 9.5 = 520.81 \]

It is obvious from the results of the total cost \( Z \) by using the TORA Program is lower than the cost produced by using the Least Cost Method and then Stepping Stone Method.

### 5.2. The second formula of ranking function:

This formula is applied on the original problem [12]

\( \mathfrak{R}(\tilde{x}) \) where \( \tilde{x} = (x_1, x_2, x_3, x_4) \)

where:
\[ \mathfrak{R}(\tilde{x}) = (x_1, x_2, x_3, x_4) = \frac{1}{2}(x_1 + x_2) + \frac{1}{4}(x_4 - x_3) = \frac{(2x_1+2x_2+x_4-x_3)}{4} \]

In another form of the same formula:
\[ (\mathfrak{R}\tilde{x}) = \mathfrak{R}(m,n,\alpha,\beta) = (4x_1 + 3x_2 + 2x_3 + x_4)/4 \]
where \( x_1 = m - \infty, \ x_2 = \infty, \ x_3 = n - m, \ x_4 = \beta \)
\[ R(\bar{x}) = R(x_1, x_2, x_3, x_4) = \frac{4(x_1-x_3)+3(x_3+2(x_2-x_4)+x_4)}{4} \] (10)

By applying the same trapezoidal fuzzy example:
\[ R(5,6,8,10) = \frac{4(5-8)+3(8)+2(6-5)+10}{4} = \frac{-12+24+12}{4} = \frac{24}{4} = 6 \]

Likewise, all data in table 5 is converted by using the current ranking formula. The results are then converted into a linear programming model and by using TORA Program to obtain the optimal solution as shown in the following Table-7.

**Table 7** - Data and solution by using the second ranking formula

| T7 | D1 | D2 | D3 | dummy | Availability |
|----|----|----|----|-------|--------------|
| S1 | 6  | 8  | 17.5 | 0     | 18.75        |
| S2 | 38 | 29 | 26   | 9     | 47.25        |
| S3 | 19 | 23 | 33   | 0     | 47.25        |
| Demand | 9 | 5.75 | 13 | 85.5 | 113.25 |

\( x_{11} = 9, \ x_{12} = 5.75, \ x_{13} = 4, \ x_{23} = 9 \)

The value of objective function:
\[ Z = \sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij} x_{ij} = 6 * 9 + 8 * 5.75 + 17.5 * 4 + 26 * 9 = 444.5 \]

**5.3. The third formula of ranking function:**
\[ R(\bar{x}) \text{ where } \bar{x} = (x_1, x_2, x_3, x_4) [9,14] \]
\[ R(\bar{x}) = R(x_1, x_2, x_3, x_4) = (x_1 + x_2 + x_3 + x_4)/4 \]
(11)

For example \( R(\bar{x}) = (5, 6, 8, 10) = (5 + 6 + 8 + 10)/4 = 7.25 \)

By applying the same steps as in the second model of the ranking function, the following results are obtained in T8

**Table 8** - Data and solution by using the third ranking formula

| T8 | D1 | D2 | D3 | dummy | Availability |
|----|----|----|----|-------|--------------|
| S1 | 7.25 | 9.25 | 19 | 13.75 | 0 | 20 |
| S2 | 39.25 | 29.75 | 54.25 | 0 | 49.25 | 49.25 |
| S3 | 19.75 | 24.25 | 34.25 | 0 | 39.25 | 49.25 |
| Demand | 9.75 | 6.5 | 13.75 | 88.5 | 118.5 |

\( x_{13} = 13.75, \ x_{31} = 9.75, \ x_{12} = 6.25, \ x_{32} = 0.25 \)

The value of objective function is
\[ Z = \sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij} x_{ij} = 6.25 * 9.25 + 19 * 13.75 + 19.75 * 9.75 + 0.25 * 24.25 = 517.69 \]

**5.4. The fourth formula of ranking function:**
\[ R(\bar{x}) \text{ where } \bar{x} = (x_1, x_2, x_3, x_4) [9] \]
\[ R(\bar{x}) = R(x_1, x_2, x_3, x_4) = (x_1 + 2x_2 + 2x_3 + x_4)/6 \]
(12)

For example \( R(\bar{x}) = (5, 6, 8, 10) = (5 + 2 * 6 + 8 * 2 + 10)/6 = 7.20 \)

By applying the same steps as in the previous models of the ranking function. The results of optimal solutions are bolded in Table-9.
Table 9-Data and solution by using the fourth ranking formula

| T   | D1 | D2 | D3 | dummy | Availability |
|-----|----|----|----|-------|--------------|
| S1  | 7.67 | 9.67 | 19  | 13.7  | 0            |
|     |     | 6   |     |       | 19.7         |
| S2  | 39.17 | 29.67 | 54.17 | 0     | 49           |
| S3  | 19.67 | 24.17 | 34.17 | 0     | 38.6         |
| Demand | 9.67 | 6.5   | 13.67 | 87.6  | 117.5        |

\[ x_{12} = 6, \ x_{13} = 13.75, \ x_{31} = 9.7, \ x_{32} = 0.5 \]

The value of objective function:

\[ Z = \sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij}x_{ij} = 9.2 * 6 + 19 * 13.7 + 19.7 * 9.7 + 0.5 * 24.2 = 518.69 \]

**5.5. The fifth formula of ranking function [15]:**

\[
\Re(\bar{x}) = \sqrt{\varphi_1(\bar{x}) + \varphi_2(\bar{x})} \tag{13}
\]

where: \((\bar{x}) = (x_1, x_2, x_3, x_4)\) and \(\varphi_1(\bar{x}) = 1/3 \sqrt{\left(\sum_{i=1}^{4} x_i - x_4\right) / \left(\sum_{i=1}^{4} x_i \cdot x_3 - (x_1 + x_2)\right)}\)

\[
\varphi_2(\bar{x}) = 1/3 \sqrt{1 + \frac{x_3 - x_2}{(x_4 + x_3) - (x_1 + x_2)}}
\]

Similarly, the data is converted by using the fifth ranking formula in the Table-10

Table 10-data and solution by using the fifth ranking formula

| T10 | D1 | D2 | D3 | dummy | Availability |
|-----|----|----|----|-------|--------------|
| S1  | 2.77 | 3.117 | 4.406 | 0    | 4.54         |
|     | 1.91 | 2.63 |     |       |              |
| S2  | 6.3 | 5.49 | 7.39 | 0    | 7.06         |
|     | 7.06 |     |     |       |              |
| S3  | 4.49 | 4.97 | 5.89 | 0    | 7.06         |
|     | 3.77 | 2.09 |     |       |              |
| Demand | 3.11 | 2.63 | 3.77 | 9.15  | 18.66        |

When the current ranking formula is applied ranking function 13, the results are quite different from the results obtained from using other formulas in this paper. The reason for that is the incompatibility with the transport model data.

**5.6. The sixth formula of ranking function:**

\[
\Re(\bar{x}) = \frac{2x_1 + 7x_2 + 7x_3 + 2x_4}{18} \tag{14}
\]

where, \((\bar{x}) = (x_1, x_2, x_3, x_4)\), let \(w = 1\), (normalize fuzzy).

For example: \(\Re\left(\frac{32,33,35,37}{18}\right) = \frac{2*32+7*33+7*35+2*37}{18} = 13.27\)

By applying the same steps as in the previous models of the ranking function, the crisp results are placed in Table-11 The optimal solutions is bolded in same table.

Table 11-Data and solution by using the sixth ranking formula

| T11 | D1 | D2 | D3 | dummy | Availability |
|-----|----|----|----|-------|--------------|
| S1  | 2.76 | 3.54 | 7.38 | 0    | 7.56         |
|     | 5.29 |     |     |       |              |
| S2  | 5.21 | 11.51 | 21.04 | 18.99 | 18.99        |
|     | 18.99 |     |     |       |              |
| S3  | 7.63 | 9.38 | 13.27 | 0    | 18.88        |
|     | 14.88 |     |     |       |              |
| Demand | 3.74 | 2.53 | 5.29 | 33.87 | 45.43        |

\[ x_{12} = 2.27, \ x_{13} = 5.29, \ x_{31} = 3.74, \ x_{32} = 0.26 \]

The value of the objective function is:
Note that the current result of the total transportation cost $Z=78.05$ is quite different from the other results of the previous formulas, and it is out of the limits. The reason for that difference is finding the center of the trapezoidal shape that has been segmented in to triangles and then finding the center of the resulting triangles as shown in the Figure-2.

![Figure 2-Centroid Ranking Method](image)

Therefore, the ranking formula can be adjusted by removing the weight ratio ($7/18$) of the trapezoidal variables.

The adjusted form of the formula is: $\mathbb{R}(\bar{x}) = \frac{2x_1+7x_2+7x_3+2x_4}{18}$

The obtained results of the adjusted formula is $Z=516.31$ by applying the data of Table-12.

Table 12-Data and solution by using the adjusted formula of ranking

| T12 | D1 | D2 | D3 | Dummy | Availability |
|-----|----|----|----|-------|--------------|
| S1  | 7.11 | 9.11 | 19 | 0.0  | 19.44        |
|     |      | 5.83 |    |       |              |
| S2  | 39.11 | 29.61 | 54.11 | 0.0  | 48.83        |
|     |       | 13.61 |    |       |              |
| S3  | 19.61 | 24.11 | 34.11 | 0.0  | 48.56        |
|     | 9.61  | 0.67  |    |       |              |
| Demand | 9.61 | 6.5  | 13.61 | 87.11 | 116.83       |

Consequently, the obtained results are similar to the results obtained by applying other ranking equations, as shown in the column $7-[16^*]$ of Table-13.

6- Results

The aim of this study is to compare between various ranking formulas to obtain the optimal solution in order find the minimum value of total cost of transportation. The data and results that placed in the table for comparison and analysis, columns A-D are trapezoidal fuzzy numbers for numerical example, columns E-K represent the results of applying ranking formulas, column L represents result of LCM and column M represents result of SSM as shown in Table-13.

Table 13-results of various ranking formulas

| T11 | A | B | C | D | E | F | G | H | I | J | K | L | M |
|-----|---|---|---|---|---|---|---|---|---|---|---|---|---|
| X1  | 5 | 6 | 8 | 10 | 7.25 | 6 | 7.25 | 7.1667 | 2.777 | 7.11111 | 7.25 | 7.25 |
| C11 | 7 | 8 | 10 | 12 | 9.25 | 8 | 9.25 | 9.1667 | 3.117 | 9.11111 | 9.25 | 9.25 |
| C12 | 16 | 18 | 20 | 22 | 19 | 17.5 | 19 | 19 | 4.406 | 7.38889 | 19 | 19 | 19 |
| C13 | 37 | 38 | 40 | 42 | 39.25 | 38 | 39.25 | 39.167 | 6.302 | 15.2099 | 39.1111 | 39.25 | 39.25 |
| C21 | 28 | 29 | 30 | 32 | 29.75 | 29 | 29.75 | 29.667 | 5.495 | 11.5154 | 29.6111 | 29.75 | 29.75 |
| C22 | 52 | 53 | 55 | 57 | 54.25 | 53 | 54.25 | 54.167 | 7.397 | 21.0432 | 54.1111 | 54.25 | 54.25 |
7. Discussion and Conclusions

After studying the results and comparing them, the following are obtained:

7.1. Using varies ranking formulas shortens the steps and requirements of the solutions, as the model can be solved by using one parameter instead of using three or four parameters for fuzzy data

7.2. Solving the original problem of fuzzy numbers, the minimum numbers of all data. \( x_{ij}, C_{ij}, S_{ij}, D_{ij} \) are taking to form a problem its result represent the lower limit \( Z_{k} = 371 \) units of cost, while the maximum numbers of the data are also taking to form a problem its solving represent the upper limit \( Z_{d} = 700 \) units Table – 4, Table – 5.

7.3. The lowest value has been achieved when applying the ranking function (10) of column F in the table of results Table-13 The reason for that is the ranking function gives greater weight to the first and second elements of fuzzy number \((x_1, x_2)\) and less weight for other parameters \((x_3, x_4)\).

7.4. Data and results that are obtained by applying the fifth formula are not accepted depending on the nature of the model of transportation as they are out of the limits.

7.5. The value of the ranking function is dependent on the weight given to every element of the fuzzy numbers. In order to achieve the realism in transportation problem, the formula (14) is adjusted as formula(15).

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