Leading and Non-leading Singularities
in Gauge Theory Hard Scattering

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Abstract

This talk reviewed some classic results and recent progress in the
resummation of leading and nonleading enhancements in QCD cross
sections and of poles in dimensionally-regularized hard-scattering am-
plitudes.

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1 Introduction

Interest in perturbative methods for QCD and related gauge theories arises from the phenomenology of high energy scattering, and also from the study of weak-strong duality, as inspired by string theory. In the following, I’ll review some methods and techniques that have a long history but remain of continuing interest, along with a few recent advances. The talk starts with a perspective on the place of perturbation theory in an asymptotically free theory, goes on to recall ideas of factorization and resummation in perturbative QCD, which leads to a review of one of its classic successes, so-called $Q_T$ resummation. It concludes with applications of these same ideas to dimensionally-regulated amplitudes for the scattering of massless partons, which have been the subject of much recent work.

2 How We Use Perturbative QCD

It’s worth recalling that despite the early successes of asymptotic freedom \[1, 2\] as a qualitative explanation of scaling, the applicability of perturbative methods beyond the parton model was met with a fair amount of skepticism. The underlying problems, of course, remain with us. First, confinement ensures that the quantities we would most naturally compute in perturbative QCD (pQCD), time-ordered products of fields,

$$
\int d^4x \ e^{-iq\cdot x} \langle 0 | T[\phi_a(x)\ldots] | 0 \rangle,
$$

have no $q^2 = m^2$ poles for any field (particle) $\phi_a$ that transforms nontrivially under color, while the “physical” poles at $q^2 = m_{\pi}^2$, for example, in

$$
\int d^4x \ e^{-iq\cdot x} \langle 0 | T[\pi(x)\ldots] | 0 \rangle,
$$

are not accessible to perturbation theory directly. And yet we use asymptotic freedom, up to power-suppressed corrections,

$$
Q^2 \hat{\sigma}_{SD}(Q^2, \mu^2, \alpha_s(\mu)) = \sum_n c_n(Q^2/\mu^2) \alpha_s^n(\mu) + O(1/Q^p)
$$

$$
= \sum_n c_n(1) \alpha_s^n(Q) + O(1/Q^p),
$$

for single-scale cross sections $\sigma(Q)_{SD}$, so long as they are finite in the zero-mass limit in perturbation theory, a property known as “infrared safety”.
Various total and jet cross sections as well as predictions based on evolution are of this type, and their phenomenological successes are well-known.

So, what are we really calculating? In many cases, we are computing matrix elements for color singlet currents, of the general form

\[ \int e^{-iq \cdot x} \langle 0 | T[J(x)J(0) \ldots] | 0 \rangle, \quad (4) \]

related to observables by the optical theorem. Of course, the optical theorem requires a complete sum over final states. But, in fact, there is another class of infrared (IR) safe color singlet matrix elements, related to jets and event shapes, that have received attention of late. These matrix elements accompany currents with the energy-momentum tensor, \( T_{\mu \nu} \), schematically,

\[ \lim_{R \to \infty} R^2 \int dx_0 \int d\hat{n} f(\hat{n}) e^{-iq \cdot y} \langle 0 | J(0)T[\hat{n}_i T_0(x_0, R \hat{n}) J(y)] | 0 \rangle, \quad (5) \]

with \( f(\hat{n}) \) a “weight” that controls the contributions of particles flowing to infinity in different directions, \( \hat{n} \). With the operator \( T_{0i} \) placed at infinity, these matrix elements rather directly represent the action of a calorimeter. If the weight is a smooth function of angles, then even though the matrix elements for individual final states have IR divergences in general, they cancel in sums over collinear splitting/merging and soft parton emission, precisely because these rearrangements respect energy flow. We regularize these divergences dimensionally (typically) and “pretend” to calculate the long-distance enhancements only to cancel them in infrared safe quantities.

3 Factorization and Resummation

Beyond the relatively limited class of cross sections that are directly IR safe, the predictive power of pQCD depends on factorization [4, 5]. From factorization we can derive the evolution familiar from deep-inelastic scattering and other single-scale problems, and generalizing this viewpoint, we can motivate resummations of enhancements in multiscale problems. A factorized cross section takes the general form

\[ Q^2 \sigma_{\text{phys}}(Q, m) = \omega_{\text{SD}}(Q/\mu, \alpha_s(\mu)) \otimes f_{\text{LD}}(\mu, m) + \mathcal{O}(1/Q^p) \], \quad (6)
where $\mu$ is a factorization scale, $m$ represents IR scales, perturbative or non-perturbative, and where $\otimes$ represents a convolution, typically in parton fraction or transverse momentum, often accurate to power corrections as shown. Speculations on new physics are contained $\omega_{SD}$, as perturbative (as in SUSY) or nonperturbative (as in technicolor) extensions of the Standard Model; $f_{LD}$ represents parton distributions of various sorts, universal among cross sections sharing the same factorization.

The familiar “DGLAP” evolution equations [6] can be derived from factorization, just by observing that physical cross sections cannot depend on the choice of factorization scale

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\text{phys}}(Q, m),$$

which, combined with (6) leads to a separation of variables,

$$\mu \frac{d}{d\mu} \ln f = -P(\alpha_s(\mu)) = -\mu \frac{d}{d\mu} \ln \omega,$$

where the “separation constant” $P$ can depend only on the variables held in common between the short- and long-distance functions in the factorized expression, $\alpha_s$ and the convolution variable(s).

The solutions to evolution equations like Eq. (8) are examples of resummation, in this case summarizing leading (and nonleading) logarithms of $Q$,

$$\ln \sigma_{\text{phys}}(Q, m) \sim \exp \left\{ \int_{Q} d\mu' \frac{P(\alpha_s(\mu'))}{\mu'} \right\}.$$  

This result is most familiar in the form of DGLAP evolution; as we shall see, however, its applications are even more wide-ranging.

This sequence of methods and results: factorization $\rightarrow$ evolution $\rightarrow$ resummation, varies between observables, and must be verified for each case. Such verifications, or “factorization proofs” [4, 5, 7], rely in general on four features of gauge theory: (1) The operator product expansion, according to which short-distance dynamics in $\omega_{SD}$ is incoherent with long-distance dynamics; (2) Jet-jet factorization, or the mutual incoherence of the dynamics of particles with $v_{\text{rel}} = c$; (3) Jet-soft factorization, by which wide angle soft radiation depends only on the overall color flow in jets [8, 9]; (4) Dimensionless couplings and renormalizability, which ensure that infrared singularities are no worse than logarithmic [10].
4 The Classic Case: $Q_T$ Resummation

What makes factorization necessary, and evolution and resummation so rewarding, is that every final state from a hard scattering carries the imprint of QCD dynamics from all distance scales. We will illustrate how these ideas play out in the classic application of resummed pQCD, the transverse momentum distribution for Dell-Yan pairs [9, 11].

We start with the transverse momentum distribution at order $\alpha_s$ for the purely partonic process

$$q(p_1) + \bar{q}(p_2) \rightarrow \gamma^*(Q) + g(k).$$

At lowest order (LO), $k = -Q_T$, and the partonic cross section is free of infrared divergences. The corresponding factorized expression for the LO hadronic cross section is

$$\frac{d\sigma_{NN \rightarrow \mu^+ \mu^- + X}(Q, p_1, p_2)}{dQ^2 d^2Q_T} = \int_{\xi_1, \xi_2} \sum_{a=q\bar{q}} \frac{d\hat{\sigma}_{a\bar{a} \rightarrow \mu^+ \mu^-}(Q) + X(Q, \mu, \xi_1 p_1, Q_T)}{dQ^2 d^2Q_T} \times f_{a/N}(\xi_1, \mu) f_{\bar{a}/N}(\xi_2, \mu).$$

The LO diagrams for the measured-$Q_T$ cross section are shown in Fig. 1, where the short-distance factor (the analog of $\omega_{SD}$ above) is

![Figure 1: LO gluon emission diagrams for $\hat{\sigma}$, Eq. (12).](image)

$$\frac{d\hat{\sigma}_{q\bar{q} \rightarrow \gamma^* g}^{(1)}}{dQ^2 d^2Q_T} = \sigma_0 \frac{\alpha_s C_F}{\pi^2} \left(1 - \frac{4Q_T^2}{(1-z)^2 \xi_1 \xi_2 S}\right)^{-1/2} \times \left[\frac{1}{Q_T^2} \frac{1+z^2}{1-z} - \frac{2z}{(1-z)Q^2}\right].$$
with $\sigma_0$ the LO total cross section, this expression, and the corresponding factorized cross section (11), is well-defined as long as $Q_T \neq 0$ and $z = Q^2/\xi_1\xi_2 S \neq 1$.

Now the leading behavior for $Q_T \ll Q$ can be found by considering the $z$ integral. When $Q^2/S$ is not too close to unity, the phase space factor in (12) and the parton distribution functions (PDFs) can be treated as nearly constant over the physical range of $z$, which then gives a logarithmic integral,

$$\frac{1}{Q_T^2} \int_{Q^2/S}^{1-|Q_T/Q|} \frac{dz}{1-z} \sim \frac{1}{Q_T^2} \ln \left[ \frac{Q}{|Q_T|} \right].$$

This approximation gives a neat prediction for $Q_T$ dependence at fixed $Q$,

$$\frac{d\sigma_{NN \rightarrow \mu^+ \mu^- +X(Q,Q_T)}}{dQ^2d^2Q_T} \sim \frac{\alpha_s C_F}{\pi} \frac{1}{Q_T^2} \ln \left[ \frac{Q}{|Q_T|} \right] \times \sum_{a=q\bar{q}} \int_\xi_1 \xi_2 \frac{d\hat{\sigma}_{a\bar{a} \rightarrow \mu^+ \mu^-(Q)+X(Q,\mu)}}{dQ^2} f_{a/N}(\xi_1,\mu) f_{\bar{a}/N}(\xi_2,\mu),$$

which we can compare, for example, to the transverse momentum of the $Z$ boson at the Tevatron. As can be seen from Fig. 2 taken from Ref. [12], a simple $\ln Q/Q_T$-dependence works pretty well for “large” $Q_T$, less than but...
of the order of $Q = m_Z$, but at smaller $Q_T$ the distribution reaches a maximum, then decreases near the “exclusive” limit, at $Q_T = 0$, corresponding to parton model kinematics. Indeed, most events are at “low” $Q_T \ll m_Z$, where the LO cross section diverges. To understand the distribution in this range, we turn to transverse momentum resummation, which, as we shall see, controls logarithms of $Q_T$ to all orders in $\alpha_s$. As suggested above, we can resum logarithms of $Q_T$ by developing variant factorizations and separations of variables.

In brief, the factorization we will exhibit reflects a relatively simple physical picture. The active quark and antiquark arrive at the point of annihilation with nonzero transverse momenta, due to gluons radiated in the transition from the initial state. Now before the collision, the quark and antiquark radiate independently, reflecting a lack of overlap between their Coulomb fields. Similarly, after the collision, final-state radiation occurs too late to affect the cross section, that is, the net probability of annihilation into an electroweak vector boson with a given $Q_T$. These considerations are summarized by $Q_T$-factorization, in the form \[11\]

$$\frac{d\sigma_{NN\to QX}}{dQ^2d^2Q_T} = \int d\xi_1 d\xi_2 d^2k_{1T} d^2k_{2T} d^2k_s T \delta^2(Q_T - k_{1T} - k_{2T} - k_s T)$$

$$\times \sum_{a=qq} H_{a\bar{a}}(\xi_1 p_1, \xi_2 p_2, Q, n)_{a\bar{a}} Q + X$$

$$\times \mathcal{P}_{a/N}(\xi_1, p_1 \cdot n, k_{1T}) \mathcal{P}_{\bar{a}/N}(\xi_2, p_2 \cdot n, k_{2T}) U_{a\bar{a}}(k_{sT}, n).$$

(15)

Here the $\mathcal{P}$'s are new transverse momentum-dependent PDFs, and in the general case we also need a new function labelled, $U$, a soft function that describes wide-angle radiation. Symbolically, in the spirit of the general factorization, Eq. (6), we can write

$$\frac{d\sigma_{NN\to QX}}{dQ^2d^2Q_T} = \sum_{a=qq} H_{a\bar{a}} \times \mathcal{P}_{a/N}(\xi_1, p_1 \cdot n, k_{1T}) \mathcal{P}_{\bar{a}/N}(\xi_2, p_2 \cdot n, k_{2T})$$

$$\otimes_{\xi, k_{T}} U_{a\bar{a}}(k_{sT}, n).$$

(16)

What we are going to do is derive the $k_T$ dependence of the $\mathcal{P}$'s from this relation. For the purposes of this talk, we proceed intuitively and with broad strokes; much more careful analyses can be found in \[9, 11, 13\].

In Eq. (15) we encounter new invariants, $p_i \cdot n$, formed from a fixed vector $n^\mu$. We can think of $n^\mu$ as being used to apportion real and virtual gluons of
momentum \( k \) into the various factors in (15), according to the scheme:

\[
p_a \cdot k < n \cdot k \quad \Rightarrow \quad k \in \mathcal{P}_a
\]

\[
p_a \cdot k, \ p_a \cdot k > n \cdot k \quad \Rightarrow \quad k \in U.
\] (17)

It is the variables \( p_a \cdot n \) that will play the role of factorization scales. Before reviewing this analysis, we go to impact parameter space, replacing the convolution in \( k_{i,T} \) by a product after the Fourier transform with \( e^{i\vec{Q}\cdot\vec{b}} \), giving, in place of (15),

\[
\frac{d\sigma}{dQ^2} \rightarrow QX(Q, b) \frac{dQ}{2} = \int d\xi_1 d\xi_2 \ H(\xi_1 p_1, \xi_2 p_2, Q, n) a_{\bar{a}} \rightarrow Q + X \times \mathcal{P}_{a/N}(\xi_1, p_1 \cdot n, b) \mathcal{P}_{\bar{a}/N}(\xi_2, p_2 \cdot n, b) U_{a\bar{a}}(b, n). \] (18)

We are now ready once again to resum by separating variables.

The physical impact parameter cross section of Eq. (18) is independent of both \( \mu_{\text{ren}} \) and of the vector \( n^\mu \). As a result, we have two equations that express this independence,

\[
\mu_{\text{ren}} \frac{d\sigma}{d\mu_{\text{ren}}} = 0, \quad n^\alpha \frac{d\sigma}{dn^\alpha} = 0. \] (19)

These equations represent the scale variation and the boost invariance of the theory. The solutions to pairs of equations of this kind were developed in this context by Collins and Soper [9] and by Sen [14].

Now variations from the jets must cancel variations from the short-distance function \( H \) and from the soft function \( U \), which depend on different variables. This analysis gives

\[
p \cdot n \left. \frac{\partial}{\partial p \cdot n} \ln \mathcal{P}(p \cdot n/\mu, b\mu) \right| \left. = \frac{1}{2} G(p \cdot n/\mu, \alpha_s(\mu)) + \frac{1}{2} K(b\mu, \alpha_s(\mu)), \right) \] \] (20)

where \( G \) matches \( H \), and \( K \) matches \( U \). On the other hand, renormalization is independent of \( n^\mu \), which implies

\[
\mu \left. \frac{\partial}{\partial \mu} [G(p \cdot n/\mu, \alpha_s(\mu)) + K(b\mu, \alpha_s(\mu))] \right| = 0, \] (21)

from which we find

\[
\mu \left. \frac{\partial}{\partial \mu} G(p \cdot n/\mu, \alpha_s(\mu)) \right| = \gamma_K(\alpha_s(\mu)) = -\mu \left. \frac{\partial}{\partial \mu} K(b\mu, \alpha_s(\mu)) \right|.
\] (22)
It is the combination of Eqs. (20) and (22) that gives the basic results. We solve Eq. (22) first,\[ G(p \cdot n/\mu, \alpha_s(\mu)) + K(b\mu, \alpha_s(\mu)) = G(1, \alpha_s(p \cdot n)) + K(1, \alpha_s(1/b)) - \int_{1/b}^{p \cdot n} \frac{d\mu'}{\mu'} \gamma_K(\alpha_s(\mu')). \] (23)

Inserting this result in the consistency equation (20) for the jet enables us to integrate $p \cdot n$ and get double logs in $b$, which, when inverted back to $Q_T$ space, produce the leading behavior $\alpha_n \ln \frac{1}{b}$ at each order in $\alpha_s$, along with nonleading contributions (which require an analysis of the soft function $U$). When carried out in detail (with attention paid to nonperturbative corrections from large $b$), this approach can describe the data of Fig. 2 all the way to $Q_T = 0$. [12, 15] The resulting expression can be summarized as\[ \frac{d\sigma_{NN_{\text{res}}}}{dQ^2 d^2 Q_T} = \sum_a \int \frac{d^2 b}{(2\pi)^2} e^{iQ_T \cdot \vec{b}} \exp \left[ E_{a\bar{a}}^{\text{PT}}(b, Q, \mu) \right] \times \sum_{a=bq} \int \xi_1 \xi_2 H(\xi_1 p_1, \xi_2 p_2, Q, n)_{a\bar{a} \rightarrow Q+X} f_{a/N}(\xi_1, 1/b) f_{\bar{a}/N}(\xi_2, 1/b), \] (24)

with a “Sudakov” exponent that, as anticipated, links large and low virtuality,\[ E_{q\bar{q}}^{\text{PT}} = -\int_{1/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[ 2A_q(\alpha_s(\mu)) \ln \left( \frac{Q^2}{\mu^2} \right) + 2B_q(\alpha_s(\mu)) \right], \] (25)

where [11] $B_q$ is related to $(K + G)_{\mu=p-n}$ and at lowest order $A_q = \gamma_K/2$, and where the lower limit $1/b$ of the integral in the exponent generates the leading logarithmic $Q_T$ dependence.

5 Poles in Color Exchange Amplitudes

Color exchange is a feature central to the analysis of hard scattering and jet and heavy particle production at hadron colliders. As intermediate results in calculations of short-distance functions, and as a subject of interest in their own right, multiloop scattering amplitudes in dimensional regularization have received considerable attention [16, 17, 18, 19]. We conclude with a sketch of
how the general methods described above lead to important results for these amplitudes.

We consider a partonic process, denoted: \( f_A(p_A, r_A) + f_B(p_B, r_B) \rightarrow f_1(p_1, r_1) + f_2(p_2, r_2) + \ldots \), where we restrict ourselves to wide-angle scattering. The amplitude for any such process can be expanded in a basis of color tensors \( c_L \) linking the external partons,

\[
\mathcal{M}^{[f]}_{\{r_i\}} \left( p_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) = \mathcal{M}^L_{\{r_i\}} \left( p_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) (c_L)_{\{r_i\}}, \quad (26)
\]

with infrared singularities regularized by going to \( 4 - 2\epsilon \) dimensions with \( \epsilon < 0 \), after renormalization has been performed. Examples of the \( c_L \)'s are singlet and octet exchange in the \( s \)-channel of quark-antiquark scattering. We need to control poles in \( \epsilon \) for factorized calculations at fixed order, and, for resummation, to all orders.

Double logs and poles in dimensional regularization are associated with leading regions \([10, 20]\) in the loop momentum space for arbitrary graphical contributions to the amplitude. These take the general form shown in Fig. 3. Leading regions are characterized by jet subdiagrams, consisting of lines parallel to the external momenta \( p_i \), a short-distance subdiagram \((H)\), with only lines off-shell by order of the momentum transfer(s), and a soft subdiagram \((S)\) with lines whose momenta vanish. Historically, it was the
Figure 4: Soft-jet factorization for wide-angle scattering.

In summary, we can write a factorized expression for $M$,

$$M_L^{[f]} \left( p_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) = \prod_{f=A,B,1,2} J_f^{[\text{virt}]} \left( \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) 
\times S_{LI}^{[f]} \left( p_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) h_I^{[f]} \left( \phi_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right),$$

(27)

where the jet functions $J_f$ for parton $f$ can be identified with the square roots of the corresponding singlet form factors, $\sqrt{\Gamma_{\text{singlet}}(Q^2)}$ [18], the soft functions are matrices labelled by color exchange (singlet, octet . . . ), and all factors require dimensional regularization. We return to the soft function $S^{[f]}$ below.
The same analysis as for Drell-Yan $Q_T$ described above, starting with factorization and arriving at resummation, gives the following explicit expression \[16\] modeled on the work of Collins and Soper \[9\] and of Sen \[14\]:

$$
\Gamma \left( \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon \right) = \exp \left\{ \frac{1}{2} \int_0^{-Q^2} \frac{d\xi^2}{\xi^2} \left[ K \left( \epsilon, \alpha_s(\mu^2) \right) \right] \right. \\
+ \left. \mathcal{G} \left( -1, \alpha_s \left( \xi^2, \epsilon \right), \epsilon \right) + \frac{1}{2} \int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \gamma_K \left( \alpha_s \left( \lambda^2, \epsilon \right) \right) \right\} ,
$$

where the running coupling is treated as $\epsilon$-dependent. All levels of exponentiating poles are generated by the anomalous dimensions $\mathcal{G}$, $K$ and $\gamma_K = -\mu dK/d\mu$. (The functions are $\mathcal{G}$ and $K$ are related to, but not identical with the analogous functions above.) The relations of such QCD results to supersymmetric Yang-Mills theories were explored in several talks at this workshop (see also the recent review by Alday and Roiban \[23\]). Double poles are generated from $\gamma_K$, which is familiar as the so-called "cusp" anomalous dimension \[24\]. A complete portrait of single poles at each order requires the $\epsilon$-dependent function $\mathcal{G}$ \[25\], which also generates finite coefficient functions in $\Gamma_{\text{singlet}}$ \[26\]. To find a field-theoretic interpretation for $\mathcal{G}$, we once again turn to the factorization approach, this time for the singlet form factor itself, as illustrated in Fig. 5. In the figure, soft radiation is organized in a singlet product of light-like Wilson lines,

$$
\mathcal{S} \left( \alpha_s(\mu^2), \epsilon \right) = \langle 0 | \Phi_{\beta_2}(\infty, 0) \Phi_{\beta_1}(0, -\infty) | 0 \rangle ,
$$

where $\Phi_\beta(\infty, 0) \equiv P \exp[-ig \int_0^\infty d\lambda \beta \cdot A(\lambda \beta)]$, and where we may take $\beta_1 \cdot \beta_2 = 1$. Such an expectation value obeys \[27\]

$$
\mu \frac{d}{d\mu} \log \mathcal{S} \left( \alpha_s(\mu^2), \epsilon \right) = G_{eik} \left( \alpha_s(\mu^2) \right) - \frac{1}{2} \int_0^{\mu^2} \frac{d\xi^2}{\xi^2} \gamma_K \left( \alpha_s \left( \xi^2, \epsilon \right) \right) ,
$$

in terms of the same $\gamma_K$ and a new anomalous dimension $G_{eik}$ that organizes non-collinear poles.

Following this analysis, the full $\mathcal{G}$ for the form factor in Eq. (28) can be written as \[25\]

$$
\mathcal{G} = 2B + G_{eik} + \beta(g) \frac{\partial}{\partial g} C(\alpha_s(Q)) ,
$$

with $B$ the $N$-independent coefficient in spin-$N$ leading-twist operators for parton $i$, and with $C$ the short-distance function shown in Fig. 5. Similar
combinations have been encountered in analyses of deep-inelastic scattering and Drell Yan in Refs. [28, 29, 30].

The remainder of the dimensional dependence in the general amplitude, Eq. (27) is generated by a matrix of anomalous dimensions for the soft functions [31, 18]

\[
S^f(\frac{Q^2}{\mu^2} , \alpha_s(\mu^2), \epsilon) = P \exp \left[ -\frac{1}{2} \int_0^{Q^2} \frac{d\tilde{\mu}^2}{\tilde{\mu}^2} \Gamma^f_S(\alpha_s(\tilde{\mu}^2, \epsilon)) \right].
\]

(32)

The one-loop expressions for arbitrary \( \Gamma^f_S \) were computed in [31], and the two-loop expressions in [19]. Remarkably, the one- and two-loop contributions are proportional [19],

\[
\Gamma_S = \frac{\alpha_s}{\pi} \left( 1 + \frac{\alpha_s}{2\pi} K \right) \Gamma_S^{(1)} + \cdots,
\]

(33)

with with the same constant, \( K = C_A(67/18 - \zeta_2) - (5/9)n_f \), that appears
\[ \gamma_K = 2C_i \frac{\alpha_s}{\pi} \left(1 + \frac{\alpha_s}{2\pi} K\right) + \ldots \] (34)

This suggests an exact one-loop anomalous dimension, supplemented by a "CMW" scheme for \( \alpha_s \) \[32\]. If this conjecture turns out to hold, there is a deep simplicity inherent in infrared vector exchange, even in QCD.

6 Summary

I have shown how the factorization properties of gauge theories serve as keys to resummation. For double-logarithmic, or "Sudakov" corrections, resummation follows from two equations, one associated with boost invariance, and another with scale variations (scale invariance for conformal theories). The basic factorization structure and its consequences are not limited to weak coupling. Whether at weak or strong coupling, many of the the long-distance properties of gauge theories can be organized quite explicitly in both cross sections and the perturbative S-matrix.

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