Method and Algorithm for Estimating the Maximum Total Error of an Automotive LiDAR

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Abstract. The article proposes a method and an appropriate algorithm for estimating the error of the measurements of the LiDAR as a technical measurement system to “include” this evaluation in the given accuracy. A generic model for the LiDAR’s measurements is described, a mathematical model of the measurements with the total values of random and systematic errors is given. Based on this method the algorithm for estimating the maximum total error (not exceeding the accuracy of the measurement) is formed. The algorithm complies both with the mathematical description presented in the article, and the methodology described in Russian standard GOST R 8.736-2011. On the basis of the calculated estimate of the weighted average error, it is possible to construct a technical device, that provides a three-fold reserve in terms of measurement accuracy.

1. Introduction

Recently, the sphere of research and development of the elements of “smart city” has received a big boost. This concept is the basis of modernization of the current urban infrastructure; the result of its implementation will be a city of the future with the maximum use of digital technologies, broadband access to the Internet and local area networks, as well as the “Internet of things” implementation [1-3]. “Smart cities” imply the use of the fleets of unmanned autonomous vehicles (AVs) capable of functioning both independently and in swarms for cargo and passenger transportation [4].

One of the main systems of the AVs is the navigation system that provides the central computer (processing unit) with data on the current position of the vehicle itself (local and global positioning), as well as information about the surrounding environment (location of other traffic participants, pedestrians, transport infrastructure, objects of the “smart city” etc.) [5-7]. In turn, one of the key elements of the AV navigation system are automotive LiDARs, calculating the distance to the objects by using time delays or phase shifts of the received (reflected from the objects) signal, which forms the corresponding cloud of points [8-10].

The LiDARs systems are constantly improving, and one can now speak of the centimeter accuracy and maximum distance ranges of up to 200 meters. The accuracy of AV positioning is also subject to relevant standards. For example, the preliminary Russian standard PNST 440-2020 “Information technology. Smart city. ICT indicators” [11], based on the international standard ISO/IEC 30146:2019 “Information technology – Smart city ICT indicators” [12], defines, that AVs’ positioning accuracy requirement should be centimeters rather than meters.
In general, modern LiDARs can be considered to satisfy these accuracy values. In particular, Yeong et al. in article [13] presented the main specifications of the considered LiDARs, including the exact characteristics of mechanical scanning and solid state ones. For example, LeiShen’s C32-151A and C16-700B (±0.02 m) show the best precise characteristics among those considered, while the worst values demonstrate IBEO LUX 4L Standard, LUX HD, and LUX 8L LiDARs (±0.1 m). Concerning the minimum and maximum range thresholds RoboSense RS-Lidar32 is the winner (0.4 – 200 m), while Hokuyo YVT-35LXF0 solid state LiDAR shows the worst range values (among the considered LiDARs) – (0.3 – 35 m).

Improving accuracy is a key issue for developers of such systems. One of the main problems is the very physical component of the LiDAR as a measuring device itself.

It is known that measurements with the required accuracy are quite complex and not always feasible. For the measurement procedure it is considered that the true value of the measured parameter is in the vicinity of the resulting estimate and differs with the estimate of the measurement error. However, this is possible with minor discrepancies between the model parameters and the real properties of the object; only when the value of the inconsistency does not exceed the tolerance of the measurement error, then the required accuracy can be measured. As a result, it is possible to formulate a metrological principle: the measurement with a given accuracy is feasible when the measurable property of an object can be set as a constant parameter of its model.

The uncertainty of the measurement result is most often characterized by the margin of the error of the measurement result corresponding to the given probability. It is clear that the less the uncertainty quantitative value of the measurement, the better the quality of these measurements, i.e. it is possible to introduce accuracy as a measurement characteristic. In fact, accuracy is the parameter that reflects the proximity of the measurement result to the true measured value [14], and it is not quantifiable. Only if the true value is replaced by the real value, the accuracy can be expressed as inverse to the module of the relative error:

$$T = \frac{\Delta \text{value}}{\text{Qvalue}}^{-1}$$  \hspace{1cm} (1)

But there is also such a characteristic of the measurement as the reproducibility of the measurements, otherwise they lose their meaning. Measurements may not be produced if the accuracy of the measurements is too high. Therefore, when specifying the accuracy of the measurements, the developers should be guided by the following aspects:

- the purpose of the conducted measurements;
- the reproducibility of the measurements;
- the difference between the true and the real value.

The aim of this work is to create an algorithm for estimating the error of the measurements of the LiDAR as a technical measurement system, as well as calculate the maximum estimate of the total error that will not exceed the accuracy of the measurement. Even at the stage of the development this will allow to “encapsulate” the error itself in the accuracy of the measurements without creating additional errors.

2. Problem statement

To formulate a problem, a generalized measurement model must be presented. Having the model of the process, it is easier to construct a mathematical model of measurements. When obtaining measurement information about the object, a huge amount of data with both useful signal and fluctuating noise is received on inputs and is given as outputs (Figure 1).
In accordance with the proposed generalized model of the measurement process (Figure 1), it is possible to present its mathematical model as follows:

\[ Z(t) = A Y(t) + B X(T) + W \Psi(t) \]  \hspace{1cm} (2)

where \( A, B, W \) are matrices of factors of vectors \( Y(t), X(t), \) and \( \Psi(t) \); \( \Psi(t) \) – the total sum of systematic and random errors.

The total error can be expressed as follows:

\[ \Psi(t) = \Theta_\Sigma * S_v \]  \hspace{1cm} (3)

where \( \Theta_\Sigma \) is the total sum of systematic error; \( S_v \) – the total estimate of standard deviation of random errors.

The total sum of random errors is described as:

\[ S_v = S_{\xi_{out}} + S_{\xi_{in}} + S_{\xi_m} + S_{\xi_{add}} \]  \hspace{1cm} (4)

Systematic errors, in turn, break down to a number of components [15]:

\[ \Theta_\Sigma = \Delta_{const} + \Delta_r + \Delta_d + \sum_{i=1}^{l} \Delta_{i_{add}} \]  \hspace{1cm} (5)

where \( \Delta_{const} \) is the constant component; \( \Delta_r \) – the progressive component; \( \Delta_d \) – the dynamic component (due to the influence of speed of change of parameters; \( \sum_{i=1}^{l} \Delta_{i_{add}} \) – the combination of additional errors due to the influence of measured parameters.

The condition for satisfying the given accuracy is that the total measurement error should be as follows:

\[ T^{-1} \geq k(S_{\Psi(t)}) \]  \hspace{1cm} (6)

The coefficient \( k=3 \) in (6) is determined in accordance with the probability theory and mathematical statistics.

The reason for such limitation is the influence of a large number of error components on the measurement result, thus providing a margin of metrological reliability.

Before using the expression (3), the weights (significance) of systematic and random errors should be calculated as (7)-(8) respectively:

\[ g_{j\Theta} = \frac{s_{\Theta_j}^2}{\sum_{j=1}^{N} s_{\Theta_j}^2} \]  \hspace{1cm} (7)
These formulas will lead to a more accurate measurement result on one condition: the estimation of boundaries of systematical error and the estimation of standard deviations of random error have a uniform distribution. Given that most errors in digital measurements are uniformly distributed, these expressions will be valid for the most measurements.

Estimate the errors of the cumulative (combined) average. The weights of the combined measurement results will be considered as accurately known numerical factors (9)-(10).

\[
\Theta = \sum_{j=1}^{N} \frac{g_j}{\sum_{j=1}^{N} s_j^2} \Theta_j^2
\]

(9)

\[
S_v = \frac{\sum_{j=1}^{N} g_j^2 \Theta_j^2}{\sum_{j=1}^{N} g_j^2}
\]

(10)

Now calculate the total measurement error taking into account the formula (3) and the method described in Russian national standard GOST R 8.736-2011 [16]. The resulting estimate of the total error must be compared with the given accuracy.

3. The algorithm for calculating the maximum estimate of the total error

Before describing the algorithm for calculating the maximum estimate of the total error that will not exceed the accuracy of the measurement, an algorithm for calculating the limit values of the error components of the selected measurement system should be developed. This algorithm (Figure 2) will establish the systematic and random error of the selected measuring device for certain nominal values within the measurement range.

![Diagram](https://via.placeholder.com/150)

**Figure 2.** The algorithm for calculating the limit values of error components of the selected measuring device.
Block A.1 of the algorithm describes all input data: \( d_j \) – the selected measuring device; \( P_{\text{conf}} \) – the selected confidence level; \( t_{\text{conf}} \) – the statistical criterion depending on the number of measurements, the law of distribution and the chosen confidence level; \( y_k \) – the selected nominal values of the measured value, within the range of measurement; \( \sigma \) – standard deviation of the measured values of the technical device at the same nominal value; \( \Delta_{\text{const}}, \Delta_r, \Delta_d, \Delta_{\text{add}} \) – components of the systematic error of the selected technical device; \( M \) – the number of selected nominal values of a certain technical device (should be at least three: the beginning, the middle and the end of the measurement range); \( N \) – the Number of the selected technical devices to be tested; \( L \) – the number of the components of the systematic error.

Blocks A.2, A.3 and A.4 set the values of the corresponding “counters”. The limit value of the assessment of the standard deviation of the systematic error (in the sequence from block A.5 to block A.7) is then calculated. Block A.8 sets the limit value of the random component of the error, depending on the law of distribution and the confidence level. Block A.9 displays the data array (dataset) consisting of estimates of marginal standard deviations of random and systematic errors separately for each selected technical device.

The algorithm for calculating the maximum estimate of the total error (shown in Figure 3) can then be described, taking into account the data from the previous algorithm.

![Figure 3. The algorithm for calculating the maximum estimate of the total error.](image-url)
Block B.1 of the algorithm shown in Figure 3 also sets all input data needed: \( P_{\text{conf}} \) – the selected confidence level; \( N \) – the number of selected technical devices to be tested; \( k \) – known factor, which depends on the number of components (in our case – the number of marginal valuations of the standard deviations) and the given confidence level; \( S_{\theta_{\text{max}}} (d_i) \) – the marginal valuation of the standard deviation of the systematic error of the selected technical device; \( S_{\varepsilon_{\text{max}}} (d_i) \) – the marginal valuation of the standard deviation of the random error of the selected technical device.

Block B.2 sets the value of the corresponding counter. Block B.3 defines the reference coefficient based on the number of marginal estimates of the standard deviations of the systematic errors of a given technical device and the established confidence level. Blocks B.4 and B.5 calculate the weights of the marginal estimates of standard deviations of random and systematic errors, respectively. Next, blocks B.6 and B.7 establish the weighted average estimates of systematic and random components of the error. Block B.8 represents the overall weighted average estimate of the total sum of the error.

Based on the calculated value of the estimate of the weighted average error, it is possible to create a technical device which provide a three-fold reserve in terms of measurement accuracy, when used in measurement process.

4. Conclusion

The proposed method and the corresponding algorithm allow to estimate the measurement error of the measuring systems at the design stage so that the estimation of the error of measurement does not exceed the given accuracy. On the basis of the above method, the corresponding algorithm for calculating the maximum estimate of the total error is formed.

Further work implies the simulation of the used method, as well as the functioning of the proposed algorithm for a certain LiDAR system.

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