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Boston University
Low-Temperature Spin Diffusion in a Spin-Polarized Fermi Gas

D. I. Golosov and A. E. Ruckenstein
Department of Physics, Rutgers University, Piscataway, NJ 08855-0849, U.S.A.

We present a finite temperature calculation of the transverse spin-diffusion coefficient, $D_\perp$, in a dilute degenerate Fermi gas in the presence of a small external magnetic field, $H$. While the longitudinal diffusion coefficient displays the conventional low-temperature Fermi-liquid behavior, $D_\parallel \propto T^{-2}$, the corresponding results for $D_\perp$ show three separate regimes: (a) $D_\perp \sim H^{-2}$ for $T \ll H$; (b) $D_\perp \sim T^{-2}$, $D_\perp/D_\parallel \neq 1$ for $T \gg H$ and large spin-rotation parameter $\xi \gg 1$, and (c) $D_\perp = D_\parallel \propto T^{-2}$ for $T \gg H$ and $\xi \ll 1$. Our results are qualitatively consistent with the available experimental data in weakly spin-polarized $^3$He and $^3$He–$^4$He mixtures.

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The unusual features of spin dynamics in spin-polarized quantum systems have been intensively studied since the pioneering paper of Leggett and Rice on spin diffusion in normal liquid $^3$He. The main effect arises from the observation that the presence of a molecular field (induced by the applied magnetic field) leads to an additional precession of the spin-current which in steady state acquires a component perpendicular to the magnetization gradient; through the continuity equation this results in an anomalous reactive component (damped spin-wave) to spin transport. This “spin-rotation” effect is also present in the case of spin-polarized Boltzmann gases.

From a microscopic point of view, it was expected that a number of qualitatively novel phenomena might arise in the case of sufficiently high polarizations. A natural suggestion, made by Meyerovich, was that low-temperature spin-diffusion becomes highly anisotropic for finite polarizations. More precisely, processes involving spin-flips make use of the phase-space volume between the two distinct Fermi surfaces (for “up” and “down” spins) leading to a finite scattering rate in the limit of $T \to 0$. This is in contrast with processes involving scattering in the vicinity of each of the Fermi surfaces which are subject to the phase-space restrictions of unpolarized Fermi liquids and are thus characterized by the conventional Fermi-liquid behavior of scattering rates, $\propto T^2$. Meyerovich’s suggestion was recently supported by measurements of the transverse spin-diffusion coefficient in weakly polarized liquid $^3$He. Theoretically, $D_\perp$ was calculated in the dilute gas limit at $T = 0$ and only a tentative estimate based on a variational solution of the Boltzmann equation is available for $T \neq 0$.

In this letter we present the analytical calculation of the finite temperature behavior of $D_\perp$ for a dilute, weakly polarized Fermi gas, in the s-wave approximation by solving the appropriate kinetic equations exactly. To the extent to which in the limit of small polarizations strong interactions only lead to constant renormalizations of the weakly interacting result we expect that our findings should also apply to the case of weakly polarized $^3$He. Indeed, as explained below, the detailed low $(H,T)$ behavior of our expression for $D_\perp$ is consistent with the small systematic deviations (lying within the error-bars) in the results of Ref.
from previous theoretical estimates. We note that the exact solution of the kinetic equations does not merely lead to quantitative renormalizations of the transport coefficients; rather, it brings out new qualitative effects. In particular, due to the existence of two dimensionless parameters, the low-temperature behavior of transverse diffusion displays two crossovers, the first at \( T \sim H \) and the other at \( T \propto \sqrt{H\epsilon_F} \) (\( \epsilon_F \) is the Fermi energy). Our finding, not appreciated in previous work on the subject, is that spin diffusion becomes isotropic (i.e., \( D_\perp = D_\parallel \)) only for \( T \) above the second crossover.

Our starting point is the kinetic equation for the Wigner transform of the transverse component of the density matrix (in the frame rotating at the bare Larmor frequency), \( n_{\uparrow\downarrow}(t,\vec{r},\vec{p}) \), where \( t,\vec{r} \) are the center of mass time and space coordinates and \( \vec{p} \) is the relative momentum vector:

\[
\left( \frac{\partial}{\partial t} + \frac{\vec{p}}{m} \frac{\partial}{\partial \vec{r}} \right) n_{\uparrow\downarrow}(t,\vec{r},\vec{p}) = n_{\uparrow\downarrow}(t,\vec{r},\vec{p}) \int \left\{ i \left[ \frac{4\pi a}{m} + B(\vec{p},\vec{p}') \right] (n_{\uparrow'} - n_{\downarrow'}) - A(\vec{p},\vec{p}') \right\} \frac{d^3 p'}{(2\pi)^3} - \\
- \int \left\{ i \left[ \frac{4\pi a}{m} + B(\vec{p}',\vec{p}) \right] (n_{\uparrow} - n_{\downarrow}) - A(\vec{p}',\vec{p}) \right\} n_{\uparrow\downarrow}(t,\vec{r},\vec{p}') \frac{d^3 p'}{(2\pi)^3} .
\] (1)

Here \( n_{\uparrow\downarrow}^\pm = [\exp((\epsilon_p \mp H/2 - \epsilon_F)/T) + 1]^{-1} \) are the equilibrium distribution functions for up- and down-spin particles, respectively, \( a \) is the s-wave scattering length, and \( \epsilon_p = p^2/2m \). Throughout this paper we use energy units for the field \( H \) and we set \( \hbar = 1 \). The two terms in the r.h.s. of equation (1) correspond to direct and exchange two-particle scattering processes, described by the finite-temperature many-body \( T \)-matrix,

\[
T(\vec{p}_1,\vec{p}_2; (g_0,\vec{g})) = -\frac{4\pi a}{m} \left( \frac{4\pi a}{m} \right)^2 \int \left[ \frac{\pi i}{2} (1 - \frac{n_{\uparrow}^\pm - n_{\downarrow}^\pm}{g_{\uparrow}^\pm - g_{\downarrow}^\pm})^2 \delta \left( g_0 + \epsilon_F - \frac{g^2 + k^2}{2m} \right) - \\
- \mathcal{P} \frac{1 - n_{\uparrow}^\pm - n_{\downarrow}^\pm}{g_0 - g^2/m + 2\epsilon_F - k^2/m} + \mathcal{P} \frac{m}{p^2 - k^2} \right] \frac{d^3 k}{(2\pi)^3} ,
\] (2)

where \( (g_0,\vec{g}) \) is the 4-momentum of the center of mass, the incoming particles have momenta \( \vec{g} \pm \vec{p}_1 \), and the outgoing \( -\vec{g} \pm \vec{p}_2 \).

In the kinetic equation (1), the functions \( A(\vec{p},\vec{p}') \) and \( B(\vec{p},\vec{p}') \) contain all effects of second order in \( apF, pF = \sqrt{2m\epsilon_F} \). In the case of low temperature and small polarization, \( T, H \ll \epsilon_F \), these take the form,
\[ A(\vec{p}, \vec{p}') \approx \frac{\pi a^2}{p} \left( \frac{(p^2 + p'^2)}{m} - 4\epsilon_F \right) \cdot \left( n^\uparrow_{\vec{p}'} + n^\downarrow_{\vec{p}'} - \frac{1}{1 - \exp \left[ \frac{p^2 + p'^2 - 2\epsilon_F}{2mT} \right]} \right), \]  

(3)

\[ B(\vec{p}, \vec{p}') \approx \frac{2a^2}{m} \left( -4p_F + |\vec{p} - \vec{p}'| \log \left| \frac{|\vec{p} - \vec{p}'|}{|\vec{p} - \vec{p}'| - |\vec{p} + \vec{p}'|} \right) \right). \]  

(4)

A more detailed derivation of the kinetic equations will be given in a longer publication.

To compute the transverse spin-diffusion coefficient we will solve the kinetic equations (1) and extract the steady state transverse spin-current driven by a constant magnetization gradient. For simplicity we will consider the case in which \( \vec{M}(x) \equiv \vec{M}(x) \) with \( \partial \vec{M}(x)/\partial x = \text{const} \). We will also restrict ourselves to the low temperature, weakly polarized limit, \( T, H \ll \epsilon_F \), in which case a complete analytical solution is possible in the s-wave approximation.

With these assumptions the solution of the kinetic equation (1) can be taken to be of the form, \( n^\uparrow\downarrow(t, \vec{r}, \vec{p}) = g(t, x, p) + f(t, x, p) \cos \psi \), where \( p = |\vec{p}| \) and \( \cos \psi = \vec{p} \cdot \hat{x}/p \). Performing explicit integrations in equation (1) then leads to the two coupled kinetic equations,

\[
\frac{\partial g}{\partial t} + \frac{p}{2m} \frac{\partial f}{\partial x} + \frac{2a}{\pi m} i \int_0^\infty g(t, x, q)q^2 dq(n^\uparrow_p - n^\downarrow_p) - \frac{4\pi a}{m} i(N^\uparrow - N^\downarrow)g = -I_{rel}[g] - iI_{sr}[g]
\]  

(5)

and

\[
\frac{\partial f}{\partial t} + \frac{p}{m} \frac{\partial g}{\partial x} - \frac{4\pi a}{m} i(N^\uparrow - N^\downarrow)f = -\frac{I_{rel}[f \cos \psi]}{\cos \psi} + iI_{sr}[f \cos \psi] \equiv -I_{rel1}[f] - iI_{sr1}[f].
\]  

(6)

In (5-6) \( I_{rel} \) and \( I_{sr} \) represent the relaxational and spin-rotation parts of the “collision integral”,

\[
I_{rel}[W(\vec{p})] = W(\vec{p}) \int A(\vec{p}, \vec{p}') \frac{d^3p'}{(2\pi)^3} - \int A(\vec{p}', \vec{p})W(\vec{p}') \frac{d^3p'}{(2\pi)^3},
\]  

(7)

\[
I_{sr}[W(\vec{p})] = -W(\vec{p}) \int B(\vec{p}, \vec{p}')(n^\uparrow_{\vec{p}'} - n^\downarrow_{\vec{p}'}) \frac{d^3p'}{(2\pi)^3} + (n^\uparrow_{\vec{p}} - n^\downarrow_{\vec{p}}) \int B(\vec{p}, \vec{p}')W(\vec{p}') \frac{d^3p'}{(2\pi)^3}. \]  

(8)
As can be shown by analysing the eigenvalue spectrum of the relaxational part of the collision integral $I_{rel1}[f]$ the spin current decays to its steady state value in a microscopic time scale, $\tau_\perp$, beyond which the time derivative terms in equations (5–6) can be omitted. In addition, since $\partial \tilde{M}_\perp(x)/\partial x = \text{const}$ or equivalently $\partial g/\partial x$ is independent of $x$, equation (5) implies $\partial f/\partial x = 0$. The solution of (5) can then be shown to be

$$g_0(x,p) = G(x)(n^\uparrow_\vec{p} - n^\downarrow_\vec{p}).$$  \hfill (9)

in which case the equation for $f(p)$ can be written as

$$v_F \frac{\partial G}{\partial x}(n^\uparrow_\vec{p} - n^\downarrow_\vec{p}) - i\Omega^{(1)} f(p) = -I_{rel1}[f(p)] - iI_{sr1}[f(p)].$$  \hfill (10)

Here, $v_F$ is the Fermi velocity and $\Omega^{(1)} = (8\pi a/m) M_\parallel \approx 2a p_F H/\pi$ represents the leading correction to the precession frequency of the spin current, $J^-_\perp \equiv J^-_\uparrow - iJ^-_\downarrow = \int_0^\infty f(p)p^3dp/6\pi^2m$ ($M_\parallel$ is the longitudinal magnetization induced by the field $H$). Note that no such correction appears in the equation for the transverse magnetization itself, $M^-_\perp \equiv M^x_\perp - iM^y_\perp = \int_0^\infty g(p)p^2dp/4\pi^2$, i.e., there is no renormalization of Larmor precession, as expected from rotational invariance of the inter-particle interactions.

To identify the diffusion coefficient we want to match the conventional form \cite{1,10} of the macroscopic constitutive relation,

$$J^-_\perp = -\frac{D_\perp}{1 - i\xi} \frac{\partial}{\partial x} M^-_\perp$$  \hfill (11)

to the one implied by the microscopic equation (10). Here, $D_\perp$ is the transverse spin diffusion coefficient while $\xi$ is referred to as the “spin-rotation” parameter. The non-zero imaginary part of the denominator in (11) reflects the fact that, due to the spin-rotation effect, the spin current is not parallel (in spin-space) to the driving magnetization gradient. We recall that in the relaxation-time approximation $\xi \approx \Omega^{(1)} \tau_{relax}$, where $\tau_{relax}$ is the corresponding relaxation time. We note from the outset that, except in the high field regime, $H \gg T$, the relaxation-time approximation breaks down and the above parametrization of $\xi$ is inapplicable. We are now in position to compute $D_\perp$ and $\xi$ as functions of $T$ and $H$. As we show below, the calculation is tractable analytically in both “high”- and “low”-field limits:
High-Field Behavior, $H/T \gg (ap_F)(T/\epsilon_F)$: In this case the relaxational term in (10) is small, $I_{rel}[f] \ll \Omega^{(1)}f$, and the current $\vec{J}_\perp$ is almost perpendicular to the magnetization gradient. In this limit the solution of equation (10), $f_D(p)$, can be obtained iteratively:

$$f_D(p) = -i v_F \frac{\partial G}{\partial x}(n^\uparrow_p - n^\downarrow_p) - i v_F \frac{\partial G}{\partial x} I_{rel1}[n^\uparrow_p - n^\downarrow_p] - i v_F \frac{\partial G}{\partial x} I_{sr1}[n^\uparrow_p - n^\downarrow_p].$$

(12)

Upon integration this leads to

$$J_\perp[f_D(p)] = -i v_F \frac{\partial G}{\partial x} J_\perp[n^\uparrow_p - n^\downarrow_p] \left\{ 1 + \frac{1}{\Omega^{(1)}} \frac{J_\perp[I_{sr1}[n^\uparrow_p - n^\downarrow_p]]}{J_\perp[n^\uparrow_p - n^\downarrow_p]} - i \frac{J_\perp[I_{rel1}[n^\uparrow_p - n^\downarrow_p]]}{\Omega^{(1)}} \frac{J_\perp[n^\uparrow_p - n^\downarrow_p]}{J_\perp[n^\uparrow_p - n^\downarrow_p]} \right\},$$

(13)

where $J_\perp[W(p)] = \int W(p) p^3 dp / 12\pi^2 m$. In (12) and (13) we identify $J_\perp[I_{sr1}[n^\uparrow_p - n^\downarrow_p]]/J_\perp[n^\uparrow_p - n^\downarrow_p] = -\Omega^{(2)}$ with the second order correction to the spin current precession frequency, and $J_\perp[I_{rel1}[n^\uparrow_p - n^\downarrow_p]]/J_\perp[n^\uparrow_p - n^\downarrow_p] = \tau_D(H,T)$ with the “diffusion time”. We note that the “diffusion time”, $\tau_D$, is not the relaxation time of the spin current usually involved in the phenomenological discussions: the latter describes the relaxation of the distribution function towards the steady state solution, $f_D(p)$ and would arise as the eigenvalue of the relaxational part of the collision operator (see below). With these definitions, (11) and (13) immediately yield,

$$D_\perp = \frac{v_F^2 \tau_D}{3} = \frac{3\pi v_F^2}{8ma^2(H^2 + 4\pi^2 T^2)},$$

$$\xi = (\Omega^{(1)} + \Omega^{(2)})\tau_D = \frac{9H v_F}{4a(H^2 + 4\pi^2 T^2)}[1 + \frac{2}{5\pi} ap_F(1 - \frac{4}{3} \log 2)].$$

(14)  

(15)

The first thing to stress is that the behaviour of $D_\perp$ in (14) differs significantly from that of the longitudinal spin diffusion coefficient [11], $D_\parallel \approx (v_F^2 / 8\pi ma^2 T^2)C(-1/3)$, where $C(-1/3) \approx 0.843$ is the Brooker-Sykes coefficient [12]. The origin of this effect is the different phase space restrictions associated with the scattering processes leading to transverse! and longitudinal spin diffusion in the high-field limit. While collisions responsible for longitudinal spin diffusion are restricted to energies within $k_B T$ of each of the (“up” or “down”) Fermi surfaces, those leading to transverse spin diffusion involve spin-flips which can also take advantage of the full region enclosed between the two surfaces. Thus, the expression for $D_\perp$
in (14) involves the phase-space for scattering generated both by the magnetic field as well as by the thermal smearing of the individual Fermi surfaces. As a result, the transverse diffusion coefficient in (14) remains finite at $T = 0$ as was predicted by Meyerovich [4] and recently confirmed experimentally [6].

It should be noted that, even for $T \gg H$ (but still in the high field limit, $H \gg (a_F)T^2/\epsilon_F$) the transverse spin diffusion coefficient $D_\perp \approx (3v_F^2/32\pi ma^2T^2) \approx 0.890D_\parallel$ still differs from the longitudinal one. This is contrary to the commonly implied belief [7,15] that the crossover to the isotropic regime of spin diffusion occurs for $T \sim H$. As we will see below $D_\perp$ and $D_\parallel$ become equal only at much lower fields, $H \sim (a_F)T^2/\epsilon_F$.

Crossover and Low-Field Behavior, $H/T \sim (a_F)T/\epsilon_F$: In this region, $I_{rel1}[f(p)] \sim \Omega^{(1)} f(p) \gg I_{sr1}[f(p)]$ and, therefore, the spin-rotation terms in the collision integral may be omitted. Furthermore, one may also set $H = 0$ in evaluating the functional $I_{rel1}$. In terms of reduced variables, $\eta(p) = (\epsilon_p - \epsilon_F)/T$ (and $f(\eta(p)) \equiv f(p)$), $h = H/2T \ll 1$, and $\Gamma_0 = 2a^2m/3\pi$, the steady state equation (10) then becomes,

$$-\frac{v_F}{T^2} \frac{\partial \cal{G}}{\partial x} \left( \frac{1}{e^{\eta-h}+1} - \frac{1}{e^{\eta+h}+1} \right) = 3\Gamma_0 f(\eta) \left( \eta^2 + \pi^2 - \frac{i\Omega^{(1)}}{3\Gamma_0 T^2} \right) - 2\Gamma_0 \int_{-\infty}^{\infty} (\eta + \sigma) f(\sigma) \left( \frac{1}{e^{\sigma}+1} + \frac{1}{e^{\eta+\sigma}-1} \right) d\sigma. \quad (16)$$

Our analysis makes use of the methods of Reference [12] and begins by transforming (16) into the differential equation,

$$F''(k) - \pi^2 \gamma^2 F(k) - \frac{2}{3} \mathrm{sech}^2 \pi k F(k) = \frac{\pi hv_F}{3\Gamma_0 T^2} \frac{\partial \cal{G}}{\partial x} \frac{\cos kh}{\cosh \pi k} \quad (17)$$

for the function $F(k) = \int_{-\infty}^{\infty} e^{ik\eta} f(\eta) \cosh(\eta/2)d\eta$. Here $\gamma^2 = 1 - i\Omega^{(1)}/3\pi^2\Gamma_0 T^2 \approx 1 - 2iH\epsilon_F/\pi^2 T^2 a_F$.

The solutions to (17) may be expressed in terms of Gegenbauer polynomials [13] with a complex index,

$$F(k) = \sum_{n=0}^{\infty} F_n \phi_n(k), \quad \phi_n(k) = (\cosh \pi k)^{-\gamma} C_n^{\gamma + \frac{1}{2}}(\tanh \pi k). \quad (18)$$

where
\[ F_n = -\frac{v_F h}{\pi^2 \Gamma_0 T^2} \frac{\partial G}{\partial x} \frac{g_n}{\left(\frac{2}{3} + (\gamma + n)(\gamma + n + 1)\right)} \].

(19)

and

\[ g_n = \frac{\pi^{5/2}(n + \gamma + \frac{1}{2})\Gamma(\gamma + \frac{1}{2})}{3 \cos \left(\frac{\pi}{2}\right) \Gamma\left(\frac{1}{2} - \frac{\gamma}{2}\right) \Gamma\left(1 + \gamma + \frac{\gamma}{2}\right) \Gamma\left(1 + \frac{\gamma}{2} + \frac{\gamma}{2}\right) \Gamma\left(\frac{1}{2} - \frac{\gamma}{2} - \frac{n}{2}\right)}.

(20)

Note, that since the functions \( f(\eta) \) and \( F(k) \) are even and \( \phi_n(-k) = (-1)^n \phi_n(k) \), only even \( n \) terms contribute to the sum in \( \text{[18]} \).

The transverse spin current can then be evaluated as

\[ J_\perp = \left(T p_F^2 / 12\pi^2\right) \int_{-\infty}^{\infty} dk F(k) / \cosh \pi k = \left(T p_F^2 / 12\pi^2\right) \sum_{n=0}^{\infty} F_n a_n, \]

(21)

with

\[ a_n = \frac{\pi \Gamma(\gamma + 1) \Gamma(2\gamma + n + 1)}{n! \cos \left(\frac{\pi}{2}\right) \Gamma(2\gamma + 1) \Gamma\left(\frac{1}{2} - \frac{n}{2}\right) \Gamma(\gamma + \frac{\gamma}{2} + 1) \Gamma\left(\frac{\gamma}{2} + \frac{n}{2} + 1\right) \Gamma\left(\frac{1}{2} - \frac{n}{2} - \frac{\gamma}{2}\right)}.

(22)

This allows us to write the final expression,

\[ \frac{D_\perp}{1 - i\xi} = \frac{1}{4\pi ma^2 T^2} \sum_{n=0}^{\infty} \frac{g_n a_n}{\frac{2}{3} + (\gamma + n)(\gamma + n + 1)}, \]

(23)

from which we can extract \( D_\perp \) and the spin-rotation parameter \( \xi \). The low field limit, \( H << (a p_F) T^2 / \epsilon_F \) corresponds to \( \gamma \to 1 \) in which case \( \text{[23]} \) yields, \( D_\perp = D_\parallel = (v_F^2 / 8\pi ma^2 T^2) C(-1/3) \). In this limit the spin-rotation parameter is given by \( \xi \approx 0.139 H \epsilon_F / T^2 (a p_F) << 1 \), reflecting the fact that the spin current becomes parallel (in spin space) to the driving magnetization gradient. The detailed behavior of \( D_\perp \) as a function of \( T/H \), which clearly displays the two crossovers discussed above, is shown in Figure 1.

Although the calculations presented above are restricted to low densities, we expect that our qualitative arguments, based on the existence of two independent parameters, \( \xi \) and \( H/T \), should also hold in the Fermi liquid regime, \( T, H << \epsilon_F \). The \( s \)-wave approximation, which made our analytical calculations possible, raises more serious issues, especially concerning the detailed behavior of \( D_\perp \) in the crossover regions. In particular, we would expect that the region in which \( D_\perp \propto T^{-2} \) with \( D_\perp \neq D_\parallel \) should shrink.
Some comments are in order concerning the possible relevance of our findings to experiment. First we note that the available experimental data in weakly polarized $^3$He deviate systematically from the naive theoretical fit which uses a single adjustable parameter ($T_a$ in [3]) to cover the entire temperature range including both, $\xi > 1$ and $\xi < 1$. Much better agreement is obtained by restricting the fit to the $\xi \leq 1$ region with an overall prefactor smaller than the one implied by fitting to the value of $D_\parallel$ in the low field, high temperature regime (we use for the inverse overall prefactor the value $A \approx 7.7 \times 10^5$ sec/cm$^2$K$^2$ instead of $5.8 \times 10^5$ sec/cm$^2$K$^2$ as in Reference [3] and for the “anisotropy temperature” $T_a \approx 12.5$ mK instead of 16.4 mK). This is consistent with our picture, with $D_\perp < D_\parallel$ for $\xi \geq 1$ and $T >> H$. In addition, although it appears that the region between the two crossovers cannot be clearly identified – most likely due to large Fermi liquid renormalization effects – the isotropic limit is indeed reached in the regime $\xi < 1$ [6]. In principle, our calculations should be more relevant to the measurements in dilute $^3$He – $^4$He mixtures. Although in the available data (for 18% $^3$He) the crossover to the isotropic limit occurs for $\xi \sim 1$ with $H << T$, the temperature is not sufficiently far below $\epsilon_F$ and, moreover, the polarization is somewhat high, $\sim 25\%$. Nevertheless, for $\xi > 1$ $D_\perp \propto D_\parallel$ with the ratio $D_\perp / D_\parallel$ slightly less than unity [14,15]. Also, the measured $T$ dependence of the “spin-rotation” parameter, $\xi$, near the crossover to the isotropic (“low-field”) limit is qualitatively consistent with our results in both the data of reference [14] and those obtained in the degenerate regime of more concentrated solutions (2.6% $^3$He) [17] with lower polarization ($\sim 2\%$). In both situations, the crossover to the isotropic regime can be clearly distinguished. However, in the former case the polarization was rather high ($\sim 25\%$) and the “low-field” crossover itself occurs beyond the degenerate limit. Also, there is a large discrepancy in the magnitude of the shift of $\xi T^2$ in the latter case (see Figure 2) which can be attributed to Fermi liquid renormalizations anticipated in high concentration solutions. To sharpen the identification of two crossovers the data of reference [17] should be extended to lower temperatures (to study the $H/T \sim 1$ behavior). Quantitative comparisons could be made only in the more dilute case of Reference [14] where lower field and lower temperature experiments should be
performed.

We close with a brief comment about the time scale for the relaxation to the steady state solution. This relaxation proceeds exponentially with a transverse relaxation time $\tau_\perp$ defined (in the appropriate reference frame) by

$$ -\frac{1}{\tau_\perp} f(p) + i\tilde{\Omega}^{(2)} f(p) = -I_{rel1}[f] - iI_{sr1}[f], \quad \tilde{\Omega}^{(2)} \sim \Omega^{(2)}. \quad (24) $$

We have only solved this equation in the limit $H \gg T^2/\epsilon_F$ in which case the spin-rotation term $I_{sr1}$ results in the rapid oscillations of the distribution function $f(p)$ with a characteristic frequency $\sim \Omega^{(2)} >> 1/\tau_\perp$. Upon averaging these oscillations over time scales short compared to $\tau_\perp$, (24) reduces to an eigenvalue problem for the collision operator, $I_{rel1}$, $f(p)/\tau_\perp = I_{rel1}[f(p)]$. The result of this calculation, plotted in Figure 3, illustrates two important points: first of all, $\tau_\perp$ is different from the “diffusion time”, $\tau_D$. Moreover, its $T, H$ dependence cannot be reduced to the “conventional” form, $\tau_\perp^{-1} \propto AT^2 + BH^2$.

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FIGURES

FIG. 1. Spin diffusion coefficient in the high-field region $H \gtrsim T$. The transverse spin diffusion coefficient $D_\perp$ (equation (14)), the longitudinal coefficient $D_\parallel$, and the naive fit $\left(1/D_\perp(0) + 1/D_\parallel\right)^{-1}$ for $D_\perp(0) = 3\pi v_F^2/8ma^2H^2$ being the limiting value of $D_\perp$ at $T \to 0$ are represented by the solid, dashed and dotted lines, respectively. The inset shows the ratio $D_\perp/D_\parallel$ near the second (low-field) crossover, $T \sim T_c = \sqrt{H\epsilon_F/\alpha p_F}$.

FIG. 2. The low-field behaviour of the spin-rotation parameter $\xi$ in the concentrated $^3$He-$^4$He mixture. The points represent the experimental data of Reference [17], the dashed line – the $T^{-2}$ fit for the behaviour of $\xi$ below the crossover, and the solid line is our theoretical result for the $s$-wave approximation (with the values of $T$ and $\xi$ scaled by appropriate factors). Note, that the latter is not a straight line. On the inset, the theoretical curve for $\xi$ at $T \sim T_c = \sqrt{H\epsilon_F/\alpha p_F}$ is plotted in a different scale.

FIG. 3. The relaxation time of the transverse spin current plotted as a function of $T/H$. The solid line corresponds to the exact eigenvalue of collision integral and the dotted line – to the fit $\tau_\perp^{-1} = AT^2 + BH^2$, which differs from the exact value in the crossover region $H \sim T$. The dashed line represents the diffusion relaxation time $\tau_D$ (see equation (14)).