GLUONIC EFFECTS IN η- and η’-NUCLEON AND NUCLEUS INTERACTIONS

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Gluonic degrees of freedom play an important role in the masses of the η and η’ mesons. We discuss η- and η’-nucleon and nucleus interactions where this glue may be manifest. Interesting processes being studied in experiments are η’ production in proton-nucleon collisions close to threshold and possible η–nucleus bound-states.

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1 The axial U(1) problem

Gluonic degrees of freedom play an important role in the physics of the flavour-singlet \( J^P = 1^+ \) channel [1] through the QCD axial anomaly [2]. The most famous example is the axial U(1) problem: the masses of the η and η’ mesons are much greater than the values they would have if these mesons were pure Goldstone bosons associated with spontaneously broken chiral symmetry [3]. This extra mass is induced by non-perturbative gluon configurations and the QCD axial anomaly [2].

Spontaneous chiral symmetry breaking is associated with a non-vanishing chiral condensate

\[
\langle \text{vac} | \bar{q}q | \text{vac} \rangle < 0. \tag{1}
\]

The non-vanishing chiral condensate also spontaneously breaks the axial U(1) symmetry so, naively, we expect a nonet of would-be pseudoscalar Goldstone bosons: the octet associated with chiral \( SU(3)_L \otimes SU(3)_R \) plus a singlet boson associated with axial U(1) — each with mass squared \( m^2_{\text{Goldstone}} \sim m_q \) where \( m_q \) denotes the light and strange quark masses. The pions and kaons are described well by this theory. The masses of the η and η’ mesons are about 300-400 MeV too big to fit in this picture without additional physics. One needs extra mass in the singlet channel associated with non-perturbative gluon configurations and the QCD axial anomaly [2]. The strange quark mass induces considerable η-η’ mixing. For free mesons the η–η’ mass matrix (at leading order in the chiral expansion) is

\[
M^2 = \begin{pmatrix}
\frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2 & -\frac{2}{3}\sqrt{2}(m_K^2 - m_\pi^2) \\
-\frac{2}{3}\sqrt{2}(m_K^2 - m_\pi^2) & \left[\frac{2}{3}m_K^2 + \frac{1}{3}m_\pi^2 + \tilde{m}_{\eta_0}^2\right]
\end{pmatrix}. \tag{2}
\]
Here $\tilde{m}_{\eta_0}^2$ denotes the gluonic mass contribution in the singlet channel. It has a rigorous interpretation through the Witten-Veneziano mass formula [6, 7] and is associated with non-perturbative gluon topology, related perhaps to confinement [8] or instantons [9]. When we diagonalize this matrix

$$
|\eta\rangle = \cos \theta \ |\eta_8\rangle - \sin \theta \ |\eta_0\rangle \\
|\eta'\rangle = \sin \theta \ |\eta_8\rangle + \cos \theta \ |\eta_0\rangle
$$

with

$$
\eta_0 = \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} + s\bar{s}), \quad \eta_8 = \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s})
$$

we obtain values for the $\eta$ and $\eta'$ masses

$$
m_{\eta',\eta}^2 = \left( m_K^2 + \tilde{m}_{\eta_0}^2 / 2 \right) \pm \frac{1}{2} \sqrt{(2m_K^2 - 2m_\pi^2 - \frac{1}{3} \tilde{m}_{\eta_0}^2)^2 + \frac{8}{9} \tilde{m}_{\eta_0}^4},
$$

The physical mass of the $\eta$ is close to the octet mass $m_{\eta_8} = \sqrt{\frac{4}{3} m_K^2 - \frac{1}{3} m_\pi^2}$, within a few percent. However, to build a theory of the $\eta$ treating it as a pure octet state risks losing essential physics associated with the singlet component and axial U(1) dynamics. In the absence of the gluonic term ($\tilde{m}_{\eta_0}^2$ set equal to zero), one finds $m_{\eta'} \sim \sqrt{2m_K^2 - m_\pi^2}$ and $m_{\eta} \sim m_\pi$. That is, without extra input from glue, in the OZI limit, the $\eta$ would be approximately an isosinglet light-quark state ($\frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}|$) degenerate with the pion and the $\eta'$ would be a strange-quark state $|s\bar{s}|$ — mirroring the isoscalar vector $\omega$ and $\phi$ mesons. The gluonic mass term is vital to understanding the physical $\eta$ and $\eta'$ mesons.  

2 Glue and $\eta$ and $\eta'$ nucleon interactions

Given that glue plays an important role in the masses of the $\eta$ and $\eta'$ mesons, it is worthwhile and interesting to look for possible manifestations of gluonic effects in dynamical processes involving these mesons. In the rest of this paper we consider $\eta$ and $\eta'$ production in proton-nucleon collisions close to threshold, and possible $\eta$–nucleus bound-states. These systems are being studied at COSY and GSI. We note that the $\eta'$–nucleon coupling constant is related, in part, to the flavour-singlet axial-charge extracted from polarized deep inelastic scattering experiments [14] – for a recent review see [15].

2.1 $\eta$ and $\eta'$ production in proton-nucleon collisions close to threshold

Since the singlet components of the $\eta$ and $\eta'$ couple to glue, it is natural to consider the process where glue is excited in the “short distance” ($\sim 0.2$fm) interaction region of a proton-nucleon collision and then evolves to become an $\eta'$ in the final state [16]. This gluonic induced production

$\text{Taking the value } \tilde{m}_{\eta_0}^2 = 0.73\text{GeV}^2 [7] \text{ in the leading-order mass formula, Eq.}(5) \text{ gives agreement with the physical masses at the 10% level. The corresponding } \eta - \eta' \text{ mixing angle } \theta \approx -18^\circ \text{ is within the range } -17^\circ \text{ to } -20^\circ \text{ obtained from a study of various decay processes in } [10, 11]. \text{ Closer agreement with the physical masses can be obtained by introducing the singlet decay constant } F_0 \neq F_\pi \text{ and including higher-order mass terms in the chiral expansion } [12, 13].$
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3 mechanism is extra to the contributions associated with meson exchange models [17–19]. Given the large gluonic effect in the mass, there is no reason, a priori, to expect it to be small. The contribution to the matrix elements for $\eta'$ and $\eta$ production is weighted by the singlet-component projection-factors $\cos \theta$ for the $\eta'$ and $\sin \theta$ for the $\eta$ where $\theta$ is the $\eta - \eta'$ mixing angle. The angle $\theta \sim -20$ degrees means that gluonic induced production should be considerably enhanced in $\eta'$ production compared to $\eta$ production.

What is the phenomenology of this gluonic interaction?

Since glue is flavour-blind the gluonic production process has the same size in both the $pp \rightarrow pp\eta'$ and $pn \rightarrow pn\eta'$ reactions. CELSIUS [20] have measured the ratio $R_\eta = \sigma(pp \rightarrow pp\eta)/\sigma(pp \rightarrow pp\eta')$ for quasifree $\eta$ production from a deuteron target up to 100 MeV above threshold. They observed that $R_\eta$ is approximately energy-independent $\simeq 6.5$ over the whole energy range. The value of this ratio signifies a strong isovector exchange contribution to the $\eta$ production mechanism [20]. This experiment is being repeated for $\eta'$ production. The cross-section for $pp \rightarrow pp\eta'$ close to threshold has been measured by the COSY-11 Collaboration [21] who are now measuring the $pn \rightarrow pn\eta'$ process [22]. In the extreme scenario that the glue-induced production saturated the $\eta'$ production cross-section, the ratio $R_{\eta'} = \sigma(pn \rightarrow pn\eta')/\sigma(pp \rightarrow pp\eta')$ would go to one after we correct for the final state interaction [19, 23] between the two outgoing nucleons. In practice, we should expect contributions from both gluonic and meson-exchange type mechanisms. It will be interesting to observe the ratio $R_{\eta'}$ and how it compares with $R_\eta$.

Gluonic induced production appears as a contact term in the axial U(1) extended chiral Lagrangian for low-energy QCD [16].

2.2 $\eta$-nucleus bound-states

New experiments at the GSI will employ the recoilless $(d, ^3He)$ reaction to study the possible formation of $\eta$ meson bound states inside the nucleus [24, 25], following on from the successful studies of pionic atoms in these reactions [26]. The idea is to measure the excitation-energy spectrum and then, if a clear bound state is observed, to extract the in-medium effective mass, $m^*_\eta$, of the $\eta$ in nuclei through performing a fit to this spectrum with the $\eta$-nucleus optical potential.

Meson masses in nuclei are determined from the scalar induced contribution to the meson propagator evaluated at zero three-momentum, $\vec{k} = 0$, in the nuclear medium. Let $k = (E, \vec{k})$ and $m$ denote the four-momentum and mass of the meson in free space. Then, one solves the equation

$$k^2 - m^2 = \text{Re} \, \Pi(E, \vec{k}, \rho)$$

for $\vec{k} = 0$ where $\Pi$ is the in-medium $s$-wave meson self-energy and $\rho$ is the nuclear density. Contributions to the in medium mass come from coupling to the scalar $\sigma$ field in the nucleus in mean-field approximation, nucleon-hole and resonance-hole excitations in the medium. The $s$-wave self-energy can be written as [27]

$$\Pi(E, \vec{k}, \rho) \bigg|_{\vec{k} = 0} = -4\pi\rho \left( \frac{b}{1 + b(\frac{1}{m})} \right).$$

(7)

Here $b = a(1 + \frac{m}{M})$ where $a$ is the meson-nucleon scattering length, $M$ is the nucleon mass and the mean inter-nucleon separation is $\langle \frac{1}{r} \rangle$. Attraction corresponds to positive values of $a$. The denominator in Eq.(7) is the Ericson-Ericson double scattering correction.
The in-medium mass $m^*_\eta$ is sensitive to the flavour-singlet component in the $\eta$, and hence to the non-perturbative glue associated with axial U(1) dynamics. An important source of the in-medium mass modification comes from light-quarks coupling to the scalar $\sigma$ mean-field in the nucleus. Increasing the flavour-singlet component in the $\eta$ at the expense of the octet component gives more attraction, more binding and a larger value of the $\eta$-nucleon scattering length, $a_{\eta N}$. Since the mass shift is approximately proportional to the $\eta$–nucleon scattering length, it follows that the physical value of $a_{\eta N}$ should be larger than if the $\eta$ were a pure octet state.

This physics has been investigated by Bass and Thomas [28]. QCD arguments suggest that the gluonic mass term is suppressed at finite density due to coupling to the $\sigma$ mean-field in the nucleus. Phenomenology is used to estimate the size of the effect in the $\eta$ using the Quark Meson Coupling model (QMC) of hadron properties in the nuclear medium [30]. Here one uses the large $\eta$ mass (which in QCD is induced by mixing and the gluonic mass term) to motivate taking an MIT Bag description for the $\eta$ wavefunction, and then coupling the light (up and down) quark and antiquark fields in the $\eta$ to the scalar $\sigma$ field in the nucleus working in mean-field approximation [30]. The strange-quark component of the wavefunction does not couple to the $\sigma$ field. $\eta - \eta'$ mixing is readily built into the model.

The mass for the $\eta$ in nuclear matter is self-consistently calculated by solving for the MIT Bag in the nuclear medium [30]:

$$m^*_\eta(\vec{r}) = \frac{2[a_P^2 \Omega^*_q(\vec{r}) + b_P^2 \Omega^*_s(\vec{r})] - z_\eta}{R^*_\eta} + \frac{4}{3} \pi R^*_\eta^3 B,$$

(8)

$$\frac{\partial m^*_j(\vec{r})}{\partial R_j} \bigg|_{R_j=R^*_j} = 0, \quad (j = \eta, \eta').$$

(9)

Here $\Omega^*_q$ and $\Omega^*_s$ are light-quark and strange-quark Bag energy eigenvalues, $R^*_\eta$ is the Bag radius in the medium and $B$ is the Bag constant. The $\eta - \eta'$ mixing angle $\theta$ is included in the terms $a_P = \frac{1}{\sqrt{3}} \cos \theta - \frac{1}{\sqrt{2}} \sin \theta$ and $b_P = \frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{3}} \sin \theta$ and can be varied in the model. One first solves the Bag for the free $\eta$ with a given mixing angle, and then turns on QMC to obtain the mass-shift. In Eq. (8), $z_\eta$ parameterizes the sum of the center-of-mass and gluon fluctuation effects, and is assumed to be independent of density [31].

The coupling constants in the model for the coupling of light-quarks to the $\sigma$ mean-field in the nucleus are adjusted to fit the saturation energy and density of symmetric nuclear matter and the bulk symmetry energy. The Bag parameters used in these calculations are $\Omega^*_q = 2.05$ (for the light quarks) and $\Omega^*_s = 2.5$ (for the strange quark) with $B = (170 \text{MeV})^4$. For nuclear matter density we find $\Omega^*_\eta = 1.81$ for the $1s$ state. This value depends on the coupling of light-quarks to the $\sigma$ mean-field and is independent of the mixing angle $\theta$.

Increasing the mixing angle increases the amount of singlet relative to octet components in the $\eta$. This produces greater attraction through increasing the amount of light-quark compared to strange-quark components in the $\eta$ and a reduced effective mass. Through Eq.(7) increasing
the mixing angle also increases the $\eta$-nucleon scattering length $a_{\eta N}$. We quantify this in Table 1 which presents results for the pure octet ($\eta = \eta_8$, $\theta = 0$) and the values $\theta = -10^\circ$ and $-20^\circ$ (the physical mixing angle). The values of $\text{Re}a_{\eta}$ quoted in Table 1 are obtained from substituting the in-medium and free masses into Eq. (7) with the Ericson-Ericson denominator turned-off, and using the free mass in the expression for $b$. The effect of exchanging $m$ for $m^*$ in $b$ is a 5% increase in the quoted scattering length. The QMC model makes no claim about the imaginary part of the scattering length. The key observation is that $\eta - \eta'$ mixing leads to a factor of two increase in the mass-shift of the $\eta$ meson and in the scattering length obtained in the model.\footnote{Because the QMC model has been explored mainly at the mean-field level, it is not clear that one should include the Ericson-Ericson term in extracting the corresponding $\eta$ nucleon scattering length. Substituting the scattering lengths given in Table 1 into Eq. (7) (and neglecting the imaginary part) yields resummed values $a_{\eta f} = a/(1 + b(1/\rho))$ equal to 0.44 fm for the $\eta$ with the physical mixing angle $\theta = -20^\circ$, with corresponding reduction in the binding energy.}

The density dependence of the mass-shifts in the QMC model is discussed in Ref. [30]. Neglecting the Ericson-Ericson term, the mass-shift is approximately linear. For densities $\rho$ between 0.5 and 1 times $\rho_0$ (nuclear matter density) we find

$$m^*_\eta/m_\eta \simeq 1 - 0.17 \rho/\rho_0$$

for the physical mixing angle $-20^\circ$. The scattering lengths extracted from this analysis are density independent to within a few percent over the same range of densities.

### 3 Conclusions and Outlook

Glue plays an important role in the masses of the $\eta$ and $\eta'$ mesons. New experiments are measuring the interactions of these mesons with nucleons and nuclei. The glue which generates a large part of the $\eta$ and $\eta'$ masses can contribute to the cross-section for $\eta'$ production in proton-nucleon collisions and to the possible binding energies of $\eta$ and $\eta'$ mesons in nuclei. It will be interesting to see the forthcoming data from COSY and GSI on these processes.

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