Generation of Coherent Extreme-Ultraviolet Radiation Carrying Orbital Angular Momentum

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Modern generation free-electron lasers (FELs) delivering high-brightness optical beams in the extreme ultraviolet (XUV) [1] and x-ray regions [2, 3] have become indispensable tools for probing structural and chemical properties of matter at femtosecond temporal and nanometer spatial resolutions [4]. At present, transverse radiation profiles from FELs working at saturation are limited to a fundamental Gaussian-like mode with no azimuthal phase variation. This is true for FELs based on self-amplified spontaneous emission (SASE), where the amplification starts from electron shot-noise [5–14], as well as for seeded FELs, such as those based on high-gain harmonic-generation (HGHG), where the amplification process is triggered by a coherent input signal [1, 15, 16].

We propose an effective scheme for the generation of intense coherent extreme ultraviolet light beams carrying orbital angular momentum (OAM). The light is produced by a high-gain harmonic-generation free-electron laser (FEL), seeded using a laser pulse with a transverse staircase-like phase pattern. During amplification, diffraction and mode selection drive the radiation profile towards a dominant OAM mode at saturation. With a seed laser at 260 nm, gigawatt power levels are obtained at wavelengths approaching those of soft x-rays. Compared to other proposed schemes to generate OAM with FELs, our approach is robust, easier to implement, and can be integrated into already existing FEL facilities without extensive modifications of the machine layout.

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exploited in order to generate optical vortices at high harmonics of the seed laser, with wavelengths approaching those of soft x-rays. The method should, in principle, be straightforward to implement at existing seeded FEL user facilities, without the need for major machine upgrades.

The scheme is shown in Fig. 1. The main difference with respect to the standard HGHG setup [41] is the use of an optical phase mask in order to create a transverse phase modulation in the seed laser profile. Naively, the simplest way to produce an XUV/x-ray optical vortex with this setup would be to seed the FEL directly with an OAM beam, by using a spiral phase plate as the phase mask [42]. However, this approach fails at short wavelengths. To see why, let us look at the HGHG process in more detail. During the interaction of the seed laser with the e-beam in the first undulator, called the modulator, the seed properties, including the transverse helical phase dependence, are imprinted onto the e-beam as an energy modulation. Therefore, the longitudinally microbunched beam coming out of the dispersive section, which follows the modulator, also carries the OAM phase signature. The spatial microbunching in the longitudinal direction contains significant harmonic components. However, the topological charge $l_n$ of higher harmonics is multiplied with the harmonic number $n$ [39]: i.e., $l_n = ln$, where $l$ is the topological charge of the seed. This results in a high-order OAM mode at the entrance of the second undulator, called the radiator, which is tuned to $\lambda = \lambda_s/n$, where $\lambda_s$ is the seed laser wavelength. Due to a lower energy spread) with a transverse rms size of 150 $\mu$m, and peak current of 1.5 kA was injected into a 1-m-long modulator tuned to a high-order OAM mode at saturation.

The idea behind our approach is the following: instead of a helical transverse phase profile, a four quadrant staircase-like phase structure is imprinted onto an axially symmetric e-beam in the modulator. The resulting transverse distribution of electrons in phase can be represented by the following matrix:

$$B_m = \begin{bmatrix} \frac{\pi}{2} & 0 \\ \frac{\pi}{2} & \frac{\pi}{2} \end{bmatrix},$$

meaning simply that the electrons with the azimuthal coordinate between $-\pi/4$ and $\pi/4$ have a relative phase of 0, the electrons with $\pi/4 \leq \phi < 3\pi/4$ have a relative phase of $\pi/2$ and so on. Following frequency up-conversion at the radiator entrance, the phase distribution is multiplied by $n$, giving:

$$B_r = n \begin{bmatrix} \frac{\pi}{2} & 0 \\ \frac{\pi}{2} & \frac{\pi}{2} \end{bmatrix} \mod 2\pi,$$

which for odd $n = 2k+1$, where $k$ is an integer, becomes:

$$B_r = \begin{cases} \begin{bmatrix} 1 & 0 \\ \frac{\pi}{2} & 0 \end{bmatrix} = B_m, & \text{for even } k \\ \begin{bmatrix} \frac{3}{2} & 0 \\ \pi & 1 \frac{\pi}{2} \end{bmatrix}, & \text{for odd } k. \end{cases}$$

For the case of odd $k$, the elements in the main diagonal are interchanged, meaning that the transverse microbunching phase in the e-beam increases in the clockwise instead of anticlockwise direction.

The above equations show that the transverse microbunching structure is preserved for odd harmonics even after frequency up-conversion. The odd harmonics therefore carry the same staircase-like transverse phase pattern, which determines the spatial properties of the radiation at the radiator entrance. Because this initial bunching distribution contains a strong helical component, the radiation profile evolves into a dominant $l = 1$ OAM mode at saturation. With an initial Gaussian transverse seed profile with $\lambda_s = 260$ nm and e-beam parameters corresponding to modern seeded FELs, optical beams carrying orbital angular momentum at XUV wavelengths can be generated.

Fig. 2 a) provides details on the phase mask inserted into the beam path of the seed laser with an initial Gaussian transverse intensity distribution. The lens, placed just after the phase mask, performs a Fourier transform of the transversely modulated electric field distribution at the focal plane, located in the middle of the modulator. This gives a transverse phase profile of the seed laser similar to the one represented by the matrix $B_m$.

Due to diffraction, the staircase-like phase structure is maintained only near the focal plane of the seed. However, by choosing a relatively long focal distance (10 m), the integration of the seed laser electric field along the focal plane, located in the middle of the modulator, gives a phase profile which closely resembles that given by $B_m$. Despite the long focal distance, with seed laser powers around 1 GW or less, the bunching amplitude after the dispersive section is still sufficiently high to trigger the amplification of the $l = 1$ OAM mode in the radiator.

The construction of microbunching in the modulator and FEL amplification in the radiator was studied in details using GENESIS [44]. A 1.3 GeV e-beam (0.1% energy spread) with a transverse rms size of 150 $\mu$m, normalized emittance $\epsilon = 5.0 \mu$m, and peak current of 1.5 kA was injected into a 1-m-long modulator tuned to the fundamental of the seed laser with $\lambda_s = 260$ nm and power of 1 GW. Fig. 3 a) shows the transverse dependence of the amplitude and phase (argument) of the local
FIG. 2. a) The optical phase mask (right) used to modulate the transverse intensity profile of an initially Gaussian seed laser (left). b) Transverse intensity (left) and phase (right) distributions of the integrated seed laser field along the modulator.

The microbunching factor defined as $b(\vec{r}) = \langle \exp[i\theta_i(\vec{r})] \rangle$, where $\theta_i$ is the ponderomotive phase of the $i$-th particle and the brackets denote the ensemble average over all the particles at a certain transverse position $\vec{r}$ in the e-beam. The bunching distribution was evaluated after the beam passed through the dispersive section with $R_{06} = 50 \, \mu m$. The figure demonstrates that the transverse properties of the seed laser are efficiently transferred onto the e-beam. The microbunching distribution is maintained after frequency up-conversion when the e-beam enters the radiator, as shown in Fig. 3 b). Due to the fact that the transverse microbunching phase profile is not perfectly flat inside the four quadrants, the phase structure also contains high frequency components. However, since the microbunching amplitude is relatively low in these areas, the generation of the OAM mode is not affected, as demonstrated in the following.

Fig. 4 shows the evolution of the radiation profile in a 20 m long radiator, tuned to the 7th harmonic of the seed laser, i.e. $\lambda = 37.1 \, nm$. The initial field ($z = 1 \, m$) has a phase and intensity structure determined by the microbunching distribution after frequency up-conversion (cf. Fig. 3 b)), clearly showing the four-quadrant staircase-like phase dependence. Since the initial radiation profile is not a guided FEL radiation mode with a self-similar intensity distribution, it will evolve due to diffraction and amplification in the radiator. Initially, at the radiator entrance, the bunching construction can be considered as rigid [11], therefore the evolution of the radiation profile is mainly governed by diffraction and linear amplification of the field. This is exemplified in the middle panel of Fig. 4 where the radiation profile has evolved from a four-quadrant/four-sources-like distribution to a profile highly resembling a ring-like intensity pattern with a helical phase dependence. As the e-beam enters the second half of the radiator, the FEL starts operating in the high-gain regime. Here, the transverse mode selection process [5–14] favors radiation profiles of lower modes, such as the fundamental Gaussian-like and the $l = 1$ OAM modes. Generally, in the high-gain regime and for a sufficiently long radiator, only the mode with the highest growth rate (Gaussian-like non OAM mode) will remain at saturation. However, using the seeding scheme described above, diffraction in the first part of the radiator gives rise to a strong $l = 1$ mode at the beginning of the exponential growth regime. Therefore, the $l = 1$ OAM mode reaches saturation well before the fundamental mode. As other higher modes are filtered out due to lower growth rates, the amplification process drives the radiation profile towards a dominant guided OAM mode with unit topological charge at saturation, as shown in the right panel of Fig. 4.

The above conclusions are supported by Fig. 5 where we plot the power, modulus of the longitudinal bunching factor for the fundamental mode $b_0 = \langle \exp(i\theta_i) \rangle$, and
FIG. 4. Evolution of the transverse radiation profile in the radiator.

FIG. 5. Evolution of the FEL power (top) and the bunching factors (bottom) along the radiator.

the helical bunching factors for the two higher (OAM) modes with \( l = 1, 2 \), defined as \( b_l = \langle \exp(\text{i}l\theta - \text{i}l\phi) \rangle \). Here the brackets denote the ensemble average over all the particles in the e-beam. The figures show the evolution of the radiation from the initial quadratic regime, where the bunching factors remain almost constant, into the high-gain regime where the modes are amplified. Throughout the amplification process, the bunching at the fundamental and \( l = 2 \) modes remains below \( \approx 5\% \) of the bunching factor \( b_1 \) corresponding to the \( l = 1 \) mode. This shows that with our seeding scheme a strong \( l = 1 \) component is excited at the radiator entrance. The intensity as well as \( b_1 \) attain maximum values at the radiator exit as the \( l = 1 \) OAM mode reaches saturation at a gigawatt power level.

Figs. 4 and 5 demonstrate the power and robustness of our approach. By simply using an optical phase mask in order to inject an e-beam with a strong helical bunching component into the radiator, an OAM mode with gigawatt power levels is obtained at saturation. Even though the seed laser does not imprint a pure \( l = 1 \) OAM profile onto the e-beam, this mode emerges as the dominant one automatically at saturation due to diffraction and mode selection in the radiator.

Our method can be readily extended to higher harmonics. For shorter wavelengths, the radiator length at which a pure \( l = 1 \) OAM mode is obtained scales with the Rayleigh range, which is proportional to \( 1/\lambda \), assuming a constant beam size of the seed laser. Nevertheless, by adjusting the modulator length, seed waist and power, optical vortices at wavelengths approaching those of soft x-rays can be generated efficiently with e-beam parameters of modern seeded FELs.

The above results were obtained for the case of a planar radiator. We have also performed simulations using a helical radiator where, due to a shorter gain length, we could relax the condition on the electron peak current required to reach saturation (1 kA instead of 1.5 kA). At this point we would like to stress that using a movable phase mask and a variable polarizing undulator [45] our method allows to independently control OAM and spin, thereby enabling complete control over the total angular momentum carried by an FEL-light beam.

Compared to other approaches where, e.g. a helical modulator is used to generate OAM beams [37], our technique is much more general. By modifying the phase mask, the seed laser can be manipulated in order to change the transverse properties of the FEL light at saturation. With a two-quadrant phase mask and a phase jump of \( \pi \), a FEL profile close to a TEM\(_{01}\) is obtained at high harmonics of the seed laser (not shown). Our technique based on an optical phase mask therefore opens up new opportunities that will allow to manipulate the spatio-temporal properties of modern and future light sources delivering high photon fluxes at XUV and soft x-rays wavelengths.

Finally, we propose to use our scheme in high-order harmonic generation (HHG) in gases. With our setup the generation of well-defined vortex beams with unit topological charge could be extended into the so-called linear propagation regime [46], where the propagation instabilities which lead to formation of low-order vortices [47] are not important. Even though the transverse mode selection process is absent in HHG, the propagation effects alone could result in a well-defined dominant \( l = 1 \) OAM mode in far field. Extensive simulations are currently underway to corroborate our hypothesis.

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