Throughput Analysis for Relay-Assisted Millimeter-Wave Wireless Networks

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Abstract—In this work, we analyze the throughput of random access multi-user relay-assisted millimeter-wave wireless networks, in which both the destination and the relay have multi-packet reception capability. We consider a full-duplex network cooperative relay that stores the successfully received packets in a queue, for which we analyze the performance. Moreover, we study the effects on the network throughput of two different strategies, by which the source nodes transmit either a packet to both the destination and the relay in the same timeslot by using wider beams (broadcast approach) or to only one of these two by using narrower beams (fully directional approach). We consider the inter-beam interference at the receiver and show the optimal strategy with respect to several system parameters, e.g., positions and number of the nodes.

I. INTRODUCTION

Given the exponential growth of data rate and connections for the fifth generation (5G) of wireless networks, millimeter-wave (mm-wave) communications technology has attracted the interest of many researchers in the past few years. The abundance of spectrum resource in the mm-wave frequency range (30-300 GHz) could help to deal with the longstanding problem of spectrum scarcity. However, the signal propagation in the mm-wave frequency range is subject to more challenging conditions in comparison to lower frequency communications, especially in terms of path loss and penetration loss, which causes frequent communication interruptions.

Several solutions have been proposed in order to overcome the blockage issue, e.g., cell densification, multi-connectivity and relaying techniques. Though relay has been extensively analyzed for microwave frequencies [1]–[5], mm-wave communications present some peculiarities that make further analysis necessary, e.g., directional transmissions. These, in contrast to the broadcast transmissions (mainly used for lower frequency bands), use narrow beams with higher beamforming gain in order to overcome the path loss issue. By using these transmissions (fully directional approach, FD), a user (UE) sends a packet either to the relay or to the destination (mm-wave access point, mmAP). On the other hand, in the broadcast communication case (BR), a packet that is sent by a UE can be received by both the relay and the mmAP in the same timeslot.

In this work, we consider a mm-wave wireless network with one mmAP and one network cooperative relay, which operates in a decode and forward manner and is equipped with a queue. We analyze the network aggregate throughput for relaying techniques for both fully directional and broadcast transmissions by taking into account the different beamforming gains and interference caused by these two approaches. Moreover, when the UEs use a broadcast approach and the transmission to the destination fails, the relay stores the packets (that are correctly decoded) in its queue and is responsible to transmit it to the destination. This technique is also known as network level cooperation relaying [2]–[5].

A. Related Work

The benefits of relaying techniques for mm-wave wireless networks have been discussed in several works, e.g., [6]–[11]. The authors of [6] and [7] use stochastic geometry to analyze the system performance for a relay-assisted mm-wave cellular network. They show a significant improvement in terms of signal-to-interference-plus-noise ratio (SINR) distribution and coverage probability. Moreover, the second work further analyzes several relay selection techniques, which represent important aspects in a relay-assisted wireless network. These are further analyzed in [8], which proposes a two-hop relay selection algorithm for mm-wave communications that takes into account the dependency between the source-destination and relay-destination paths in terms of line-of-sight (LOS) probability. The work in [9] considers a joint relay selection and mmAP association problem. In particular, the authors propose a distributed solution that takes into account the load balancing and fairness aspects among multiple mmAPs. Other works, e.g., [10] and [11], focus on relaying techniques for device-to-device (D2D) scenarios and analyze, by using stochastic geometry, the coverage probability and the relay selection problem, respectively.

The authors of [12] analyze the tradeoff between mm-wave relay and microwave frequency transmissions for a two-hop half duplex relay scenario. They study the throughput and delay for a single source-destination pair and a single relay, which can transmit on mm-wave frequencies or by using microwave frequencies when the direct path is blocked. A model similar to ours, though for microwave frequencies, is considered in [4].
B. Contributions

As introduced above, we provide the analysis of the aggregate network and per-user throughput for random access multi-user cooperative relaying mm-wave wireless networks by considering two different approaches, i.e., fully directional and broadcast. To the best of our knowledge, analysis for this particular problem setup has not been yet investigated. The UEs, independently, choose to transmit by following one of the approaches and we identify the optimal strategy with respect to system parameters. Namely, we investigate when for the UEs is more beneficial to transmit simultaneously to both the mmAP and the relay by using wider beams, or when instead it is better to use narrow beams with higher beamforming gain and transmit either to the mmAP or the relay. Furthermore, by using queueing theory, we study the performance characteristic of the queue at the relay, for which we derive the stability condition, as well as the service and the arrival rate.

The rest of the paper is organized as follows: in Section II we describe the system model and the assumptions. In Section III we present the queue analysis at the relay with two UEs and in Section IV we generalize these results and evaluate the aggregate network throughput for N UEs. In Section V we illustrate the results and performance evaluation and Section VI concludes the paper.

II. SYSTEM MODEL AND ASSUMPTIONS

A. Network Model

We consider a set of symmetric UEs $N$, with cardinality $N$. We assume multiple packet reception capability both at the mmAP and the relay ($R$), which are equipped with hybrid beamformers and they can form multiple beams at the same time [13]. The UEs are equipped with analog beamformers, which can form one beam at a time. We assume slotted time and each packet transmission takes one timeslot. The relay has no packets of its own, but it stores the successfully received packets from the UEs in a queue, which has infinite size [1]. We assume that acknowledgements (ACKs) are instantaneous and error free and packets received successfully are deleted from the queues of the transmitting nodes, i.e., UEs and $R$.

Users and relay transmit a packet with probabilities $q_u$ and $q_r$, respectively. As mentioned previously, a UE can transmit by using either the fully directional (FD) or the broadcast (BR) approach with probabilities $q_u$ and $q_u (q_u = 1 - q_{ub})$, which are conditioned to the event that a packet is transmitted. In turn, when a UE uses the FD approach, it transmits either to the mmAP or to $R$ with probabilities $q_{ud}$ and $q_{ur} (q_{ud} = 1 - q_{ur})$, respectively, which are conditioned to the event that a packet is transmitted by using the FD approach. In the BR case, $R$ stores the successfully received packets only when these are not received by the mmAP and the relay always uses directional communications to forward them to the mmAP.

In Fig. 1 we illustrate an example of the FD and BR approaches, where $d_{ur}$ and $d_{ud}$ represent the distances between the UE and $R$ and between the UE and the mmAP, respectively. The parameter $\theta_{ij}$ is the angle formed by $R$ and the mmAP with a UE as vertex and $\theta_{BR}$ is the beamwidth. Hereafter, we indicate the probability of the complementary event by a bar over the term (e.g., $q_u = 1 - q_{ub}$). Moreover, we use superscripts $f$ and $b$ to indicate the FD and BR transmissions, respectively.

B. SINR Expression and Success Probability

A packet is successfully received if the SINR is above a certain threshold $\gamma$. Ideally, multiple transmissions at the receiver side of a node do not interfere when they are received on different beams. However, in real scenarios, interference cancelation techniques are not perfect and we introduce a coefficient $0 \leq \alpha \leq 1$ that models the interference between received beams. The cases $\alpha = 0$ and $\alpha = 1$ represent perfect interference cancelation and no interference cancelation, respectively. Moreover, we assume that an FD transmission to the mmAP does not interfere with the packet transmitted to $R$ and vice-versa. On the other hand, when a UE uses a BR approach, its transmission interferes with the transmissions of the other UEs for both the mmAP and $R$.

We assume that the links between all pairs of nodes are independent and can be in two different states, LOS and non-line-of-sight (NLOS). Specifically, LOS$_{ij}$ and NLOS$_{ij}$ are the events that node $i$ is in LOS and NLOS with node $j$ with probabilities $P($LOS$_{ij})$ and $P($NLOS$_{ij})$, respectively. Usually $R$ is represented by a node that is placed in a position that guarantees the LOS with the mmAP, namely, $P($LOS$_{rd}) = 1$. In order to compute the SINR for link $ij$, we first identify the sets of interferers that use FD and BR transmissions, which are $I_f$ and $I_b$, respectively. Then, we partition each of them into the sets of nodes that are in LOS and NLOS with node $j$. These sets are $I_{f,los}$ and $I_{b,los}$ for the UEs that use the FD approach and $I_{f,nlos}$ and $I_{b,nlos}$ for the UEs that use the BR transmissions. Thus, when node $i$ is in LOS with node $j$, we can write the SINR, conditioned to $I_{f,los}, I_{f,nlos}$, $I_{b,los}$, and $I_{b,nlos}$, as in (1).
The beamforming gain of the transmitter and the receiver are $g_i$ and $g_j$, respectively. These are computed in accordance to the ideal sectored antenna model [14], which is given by: $g_i = g_j = \frac{2\pi}{\theta_{BW}}$ in the main lobe, and 0 otherwise. The term $h_l(i,j)$ is the path loss on link $ij$ when this is in LOS. The transmit and the noise power are $p_t$ and $p_N$, respectively. The terms $p_{r/t}(i,j)$ and $p_{r/t'}(i,j)$ represent the received power by node $j$ from node $i$, when the first is in LOS and NLOS, respectively. Note that similar expressions of the SINR can be derived also in case of BR and NLOS.

Finally, the success probabilities for a packet sent on link $ij$ by using FD and BR transmissions are represented by the terms $P^{s}_{ij}/P_{ij}$ and $P^{b}_{ij}/P_{ij}$, respectively. Here, we consider only the conditioning on the sets $I_f$ and $I_b$ because we average on all the possible scenarios for the LOS and NLOS link conditions. The expression for the FD approach and $N$ UEs is given in Appendix A.

III. PERFORMANCE ANALYSIS FOR THE RELAY QUEUE

In order to compute the aggregate network and per-user throughput, in this section, we evaluate the service rate, $\mu$, for the queue at $R$, for which we further analyze the arrival rate, $\lambda$, and the stability condition. Namely, we present the results for two UEs to give insights to understand the throughput analysis, which is generalized for $N$ UEs in Section [IV]. First, we compute $\lambda$ as follows:

$$\lambda = P(Q = 0)\lambda_0 + P(Q \neq 0)\lambda_1$$

$$= P(Q = 0) \sum_{k=1}^{2} kr_k^0 + P(Q \neq 0) \sum_{k=1}^{2} kr_k^1,$$

where $r_k^0$ and $r_k^1$ are the probabilities to receive $k$ packets at $R$ in a timeslot when the queue is empty or not, respectively. These two events occur with probabilities $P(Q = 0)$ and $P(Q \neq 0)$ and present different arrival rates, $\lambda_0$ and $\lambda_1$, respectively. Indeed, when the queue is not empty, $R$ may transmit and interfere with the other transmissions to the mmAP. Now, considering all the possible combinations for the two UEs scenario, where $R$ can receive at maximum two packets per timeslot, we can compute $\lambda_0$ and $\lambda_1$. Moreover, for the success probability expressions, we explicitly identify the nodes that belong to the sets $I_f$ and $I_b$. However, since the UEs are symmetric, it is sufficient to indicate the number of UEs that are interfering and whether $R$ is transmitting; i.e., we indicate with $\{I_f, r\}^f$ and $\{I_f\}^r$ the sets of interferers that use FD transmissions when $R$ is transmitting or not, respectively, and with $\{r\}^f$ the set of interferers when only the relay is transmitting. Therefore, we obtain:

$$\lambda_0 = 2q_ufqfP_{uf} + 2q_ufqafP_{uf}P_{ud}$$

$$+ q_ufqafqafq_{af}^2 + 2P_{uf}P_{uf}P_{uf}P_{ud}$$

$$+ 2P_{uf}P_{uf}P_{uf}P_{ud} + P_{uf}P_{uf}P_{uf}P_{ud}$$

$$+ P_{uf}P_{uf}P_{uf}P_{ud} + 2P_{uf}P_{uf}P_{uf}P_{ud} + 2P_{uf}P_{uf}P_{uf}P_{ud}$$

$$+ \sum_{i,j}(P_{uf}P_{uf}P_{uf}P_{ud} + 2P_{uf}P_{uf}P_{uf}P_{ud})$$

$$= \lambda.$$
Then, we can study the evolution of the queue at the relay, modelled as a discrete time Markov Chain (DTMC), as represented in Fig. 2. The terms $p_k^1$ and $p_k^2$ are the probabilities that the queue size increases by $k$ packets in a timeslot when the queue is empty or not, respectively; these probabilities are derived in Appendix B. Moreover, by omitting the details for sake of space, we compute $P(Q = 0)$ by considering the Z-transformation of the steady-state distribution vector $[16]$: 

$$P(Q = 0) = \frac{p_{i-1} - p_i - 2p_1^2}{p_{i-1} - p_i - 2p_1 + \lambda_0}.$$  

(7)

IV. THROUGHPUT ANALYSIS

In this section, we derive the network aggregate throughput, $T$, for $N$ UEs by generalizing the results obtained in Section III. When the queue at $R$ is stable, $T$ can be expressed as $T = NT_u$, where $T_u$ represents the per-user throughput. This is composed by two terms, $T_{ud}$ and $T_{ur}$, which represent the contributions to $T_u$ given by the packets transmitted directly to the mmAP or by $R$, respectively. Thus, $T$ is given by: 

$$T = NT_u = N(T_{ud} + T_{ur}).$$  

(8)

On the other hand, when the queue at $R$ is unstable, the aggregate throughput becomes: 

$$T = NT_{ud} + \mu_r.$$  

(9)

Now, we analyze $T_{ud}$ and $T_{ur}$. We indicate with $m$ the number of UEs that interfere and with $i$ the number of those that use FD transmissions ($m-i$ UEs use the BR approach). A certain number, $j$, of FD interferers transmit to $R$ and $i-j$ to the mmAP. Therefore, $T_{ud}$ and $T_{ur}$ are given by:

$$T_{ud} = (1 - q_r P(Q \neq 0)) T_{u0}^0 + q_r P(Q \neq 0) T_{u1}^1,$$

(10)

$$T_{ur} = q_u q_{uf} q_{ur} \sum_{m=0}^{N-1} \binom{N-1}{m} q_m^u q_u^{N-1-m}$$

$$\times \sum_{i=0}^{m} \binom{m}{i} q_i^u q_{ur}^{m-i} \sum_{j=0}^{i} \binom{i}{j} q_j^u q_{ud}^{i-j}$$

(11)

$$\times P_{ur/(j)}^{i-j,(m-i)}, \quad + \quad (1 - q_r P(Q \neq 0)) T_{ur}^0$$

$$+ \quad q_r P(Q \neq 0) T_{ur}^1,$$

where $P(Q = 0)$ is derived by following the same methods used in Section III but for $N$ UEs. This is given by:

$$P(Q = 0) = \frac{p_{i-1}^1 - \sum_{k=1}^{N} k \cdot p_k^1}{p_{i-1}^1 - \sum_{k=1}^{N} k \cdot p_k^1 + \lambda_0},$$

(12)

Finally, we derive the terms $T_{ud}^0$ and $T_{ud}^1$:

$$T_{ud}^0 = q_u q_{uf} q_{ud} \sum_{m=0}^{N-1} \binom{N-1}{m} q_m^u q_u^{N-1-m}$$

$$\times \sum_{i=0}^{m} \binom{m}{i} q_i^u q_{ud}^{m-i} \sum_{j=0}^{i} \binom{i}{j} q_j^u q_{ud}^{i-j}$$

(15)

$$\times P_{ud/(i-j)}^{i-j,(m-i)}, \quad + \quad (1 - q_r P(Q \neq 0)) T_{ur}^0$$

$$+ \quad q_r P(Q \neq 0) T_{ur}^1,$$

V. NUMERICAL RESULTS

In this section, we provide numerical results to validate the performance analysis derived in the previous sections. In order to compute the LOS and NLOS probabilities and the path loss, we use the 3GPP model for urban micro cells (UMi) in outdoor street canyon environment [17]. More precisely, the path loss depends on the height of the mmAP, 10 m, the
height of the UE, 1.5 m, the carrier frequency, $f_c = 30$ GHz and the distance between the transmitter and the receiver. The transmit and the noise power are set to $P_t = 24$ dBm and $P_N = -80$ dBm, respectively. Then, the SINR in (17) and the success probability in (17) are numerically computed.

Hereafter, we show the aggregate network and per-user throughput ($T$ and $T_u$) while varying several parameters. Unless otherwise specified, we set $d_{ur} = 30$ m, $d_{rd} = 50$ m, $\gamma = 10$ dB and $\alpha = 0.1$. Moreover, we set either $\theta_{BW} = 5^\circ$ or $\theta_{BW} = \theta_{rd}$ for the FD and BR approaches, respectively. In Fig. 3 we show $T$ (solid lines) and $T_u$ (dotted lines) while varying the number of UEs for several UE transmit probability values, i.e., $q_u$. For $q_u = 0.1$ the queue at $R$ is always stable, in contrast, for $q_u = 0.3$ and $q_u = 0.9$ the queue becomes unstable at $N = 12$ and $N = 3$, respectively. This can be observed by the slope of the curves. Indeed, above a certain number of UEs, $T$ reaches almost the maximum value and for $q_u = 0.3$ it is almost constant for the whole range of $N$. On the other hand, for $q_u = 0.9$ we have that above a certain number of UEs, $T$ decreases. Namely, at this point the queue becomes again stable, because high values of $N$ and $q_u$ lead to high interference that decreases the number of packets successfully received by $R$ and the mmAP. Moreover, the increase of the interference with $N$ causes the monotonic decreasing behavior of $T_u$.

In Fig. 4 we show the aggregate throughput, $T$, while varying the probability of using the FD approach, $q_{uf}$, and $\theta_{rd}$. Hereafter, we set $q_u = 0.1$ and $N = 10$. We can observe that the optimal choice of $q_{uf}$ depends on $\theta_{rd}$. Namely, for small values of $\theta_{rd}$, a broadcast approach is more preferable, which corresponds to small values of $q_{uf}$. Indeed, in this case, we can use a narrow beam with high beamforming gain to transmit simultaneously to $R$ and the mmAP. In contrast, for higher values of $\theta_{rd}$, the optimal value of $q_{uf}$ becomes 1, which corresponds to always using the FD approach. Furthermore, it is possible to observe that for $q_{uf} = 1$, $T$ increases with $\theta_{rd}$. This is caused by the interference of $R$ on the communications between the UEs and the mmAP.

This phenomenon can be better observed in Fig. 5. This shows both the aggregate throughput directly transmitted to the mmAP and by using $R$, i.e., $T_d$ and $T_r$, respectively, while varying $\theta_{rd}$ for several values of $q_{uf}$. Larger values of $\theta_{rd}$ correspond to longer distances between $R$ and the mmAP, i.e., $d_{rd}$. Since this link is always in LOS, the success probability for a packet transmitted from $R$ to the mmAP, and so $T_r$ (dotted lines), are barely affected by increasing the link length. In contrast $T_d$ (solid lines) increases for wider $\theta_{rd}$ because the interference caused by $R$ decreases. For $0 < q_{uf} < 1$, $T_d$ and $T_r$ have a non-monotonic behavior. Namely, initially, as $\theta_{rd}$ increases, $T_d$ decreases because of two reasons. First, the beamforming gain of the broadcast transmissions decreases, and so the success probability for a packet sent by using the BR approach. At the same time, the packets that are not successfully received by the mmAP may be stored in the queue at $R$. This explains the increase of the interference at the receiver side of the mmAP caused by the relay, which represents the second reason for the decrease of $T_d$. Indeed, at the beginning, $T_r$ increases. However, above a certain value of $\theta_{rd}$, $T_r$ starts decreasing because wider beams with lower beamforming gains are not enough to overcome the path loss.

The behavior observed in Fig. 4 and 5 are further corroborated by Fig. 6. Here, we show $T$ while varying the probability to transmit at the relay $q_{ur}$ and $\theta_{rd}$, when an FD approach is used, i.e., $q_{uf} = 1$. As explained above, for an FD approach and short distances ($d_{ur} = 30$ m and $d_{rd} = 50$ m), we can note that we have higher values of $T$ as $\theta_{rd}$ increases. Furthermore, we can observe that, as the interference of $R$ on the mmAP decreases ($\theta_{rd}$ increases), the optimal value of $q_{ur}$ for maximizing $T$ changes. More precisely, for small values

\begin{align*}
T & = \text{[graph]} \\
T_u & = \text{[graph]} \\
T_d & = \text{[graph]} \\
T_r & = \text{[graph]}
\end{align*}
of $\theta_{rd}$ and $d_{rd}$ it is better to transmit with higher probability to $R$. In contrast, when $\theta_{rd}$ increases, and the interference of $R$ decreases, lower values of $q_{ur}$ provide a better throughput.

This is more evident for longer distances as shown in Fig. 7. However, in this case, $T$ decreases as $\theta_{rd}$ increases. Indeed, the transmissions between the UEs and the mmAP are barely affected by the interference of $R$ and the link path loss between $R$ and the mmAP is dominant. This decreases the success probability for a packet sent from $R$ to the mmAP and makes the queue at $R$ not stable when $q_{ur}$ is above certain values. This value is $q_{ur} = 0.3$ for $\theta_{rd} = 30^\circ$ and it decreases as $\theta_{rd}$ increases.

Finally, in Fig. 8 we show $T$ while varying $q_{uf}$ for several values of $d_{ur}$ and $d_{ud}$, when $\theta_{rd} = 30^\circ$. It is possible to observe that for short distances (blue curve), the optimal value of $q_{uf}$ is smaller than 0.5. This means it is better to use a BR approach because of the small path loss of the links UE-mmAP and UE-$R$. In contrast, when the distances increase, the transmissions need higher beamforming gain and the FD approach is preferable.

VI. Conclusion

We presented a throughput analysis for relay assisted mm-wave wireless networks where the UEs can transmit either by using a fully directional or broadcast approach by setting the beamwidth accordingly. We have shown that the optimal strategy (values of $q_{uf}$ and $q_{ur}$) highly depends on the network topology, e.g., $d_{ud}$, $d_{ar}$ and $\theta_{rd}$. We also evaluated the performance of the relay queue by deriving the stability conditions and the arrival and service rates. Moreover, we have shown how the interference of the relay and the link path loss represent the main issues to the success probability for short and long distances, respectively.

Future work will investigate also the behavior of the throughput with respect to other parameters, e.g., SINR threshold $\gamma$, the number of UEs $N$ and the inter-beam interference cancellation technique parameter $\alpha$. Moreover, further extensions include analyzing the optimal strategy in terms of $q_{a}$, $q_{uf}$ and $q_{ur}$ in order to minimize the packet delay by taking into account the beam alignment phase duration.

APPENDIX A

Hereafter, we derive the expression for the success probability for the link $ij$ with $N$ symmetric UEs, conditioned to the sets $|I_f|$ and $|I_h|$. We average on all the possible scenarios for the LOS and NLOS links. We consider that $k$ and $h$ UEs over $|I_f|$ and $|I_h|$ interferers, respectively, are in LOS. Thus, the success probability can be derived as follows:

$$P_{ij/I_f,I_h} = P(L_{ij})P(SINR_{ij/I_f,I_h} \geq \gamma |L_{ij})$$

$$+ P(NLOS_{ij})P(SINR_{ij/I_f,I_h} \geq \gamma |NLOS_{ij})$$

$$= P(LOS_{ij}) \left[ \sum_{k=0}^{[I_f]} \binom{[I_f]}{k} P(LOS_{ij})^k P(NLOS_{ij})^{[I_f]-k} \right]$$

$$\times \sum_{h=0}^{[I_h]} \binom{[I_h]}{h} P(LOS_{ij})^h P(NLOS_{ij})^{[I_h]-h} \times P(SINR_{ij/I_f,I_h,I_{fa},I_{bt},I_{bn}} \geq \gamma |L_{ij})$$

$$+ P(NLOS_{ij}) \left[ \sum_{k=0}^{[I_f]} \binom{[I_f]}{k} P(LOS_{ij})^k P(NLOS_{ij})^{[I_f]-k} \right]$$

$$\times \sum_{h=0}^{[I_h]} \binom{[I_h]}{h} P(LOS_{ij})^h P(NLOS_{ij})^{[I_h]-h} \times P(SINR_{ij/I_f,I_h,I_{fa},I_{bt},I_{bn}} \geq \gamma |NLOS_{ij}) \right].$$
The expressions $P(\text{SINR}^f_{ij}|x_f, z_i) \geq \gamma | \text{LOS}_{ij}$ and $P(\text{SINR}^f_{ij}|x_f, z_i) \geq \gamma | \text{NLOS}_{ij}$ are the probabilities, conditioned to the specific scenarios of interferers, $x_f$ and $z_i$, that the received SINR is above $\gamma$, when link $ij$ is in LOS and NLOS, respectively.

**APPENDIX B**

Hereafter, we derive the transition probabilities $p^0_1$ and $p^1_1$ for the two UEs case.

\[
p_1^1 = q_r \left[ p^f_{rd} \left( q_{ur}^{2} + 2q_{ur}q_{uf}q_{ur}P_{ur}^f + (q_{uf}q_{ur}q_{ur}P_{ur}^f)_{rd}^{f} \right) \right. \\
+ \left. (q_{uf}q_{ur}q_{ur}P_{ur}^f)_{rd}^{f} + 2q_{uf}q_{ur}P_{ur}^f \right] (18)
\]

\[
p_1^0 = 2q_{ur}q_{uf}q_{ur}P_{ur}^f + 2q_{ur}q_{uf}q_{ur}P_{ur}^f + 2q_{uf}q_{ur}q_{ur}P_{ur}^f \\
+ 2q_{uf}q_{ur}q_{ur}P_{ur}^f \left( 1 - P_{ur}^{b}P_{ad}(f)P_{ur}^f \right) \\
+ \left. P_{ur}^f \right] (19)
\]

\[
p_1^2 = q_r p_0^1 + q_r \left[ q_{ur}^{2} + q_{ur}^{2} + 2q_{ur}q_{uf}q_{ur}P_{ur}^f + (q_{uf}q_{ur}q_{ur}P_{ur}^f)_{rd}^{f} \right. \\
+ \left. (q_{uf}q_{ur}q_{ur}P_{ur}^f)_{rd}^{f} + 2q_{uf}q_{ur}P_{ur}^f \right] (20)
\]

\[
p_2^1 = \left( q_{uf}q_{ur}q_{ur}P_{ur}^f + (q_{uf}q_{ur}q_{ur}P_{ur}^f)_{rd}^{f} \right) \left( q_{uf}q_{ur}q_{ur}P_{ur}^f \right)
\]

\[
p_2^2 = \left( q_{uf}q_{ur}q_{ur}P_{ur}^f \right)^2 + \left( q_{uf}q_{ur}q_{ur}P_{ur}^f \right)^2
\]

\[
+ 2q_{uf}q_{ur}q_{ur}P_{ur}^f \left( 1 - P_{ur}^{b}P_{ad}(f)P_{ur}^f \right) \left( 1 - P_{ur}^{b}P_{ad}(f)P_{ur}^f \right)
\]

\[
(21)
\]

\[
\begin{aligned}
p_2^3 &= \left( q_{uf}q_{ur}q_{ur}P_{ur}^f \right)^2 + \left( q_{uf}q_{ur}q_{ur}P_{ur}^f \right)^2 \\
&+ 2q_{uf}q_{ur}q_{ur}P_{ur}^f \left( 1 - P_{ur}^{b}P_{ad}(f)P_{ur}^f \right) \left( 1 - P_{ur}^{b}P_{ad}(f)P_{ur}^f \right)
\end{aligned}
\]

\[
(22)
\]

**REFERENCES**

[1] G. Kramer, I. Marić, and R. D. Yates, “Cooperative communications,” *Found. Trends. Netw.*, vol. 1, no. 3, pp. 271–425, Aug. 2006.

[2] A. K. Sadek, K. J. R. Liu, and A. Ephremides, “Cognitive multiple access via cooperation: Protocol design and performance analysis,” *IEEE Transactions on Information Theory*, vol. 53, no. 10, pp. 3677–3696, Oct. 2007.

[3] N. Pappas, A. Ephremides, and A. Tragantidis, “Relay-assisted multiple access with multi-packet reception capability and simultaneous transmission and reception,” in *IEEE Information Theory Workshop*, Oct. 2011, pp. 578–582.

[4] N. Pappas, M. Kountouris, A. Ephremides, and A. Tragantidis, “Relay-assisted multiple access with full-duplex multi-packet reception,” *IEEE Transactions on Wireless Communications*, vol. 14, no. 7, pp. 3544–3558, July 2015.

[5] G. Papadimitriou, N. Pappas, A. Tragantidis, and V. Angelakis, “Network-level performance evaluation of a two-relay cooperative random access wireless system,” *Computer Networks*, vol. 88, pp. 187–201, Sept. 2015.

[6] B. Xie, Z. Zhang, and R. Q. Hu, “Performance study on relay-assisted millimeter wave cellular networks,” in *IEEE 83rd Vehicular Technology Conference (VTC Spring)*, May 2016, pp. 1–5.

[7] S. Biswas, S. Vuppala, J. Xue, and T. Ratnarajah, “On the performance of relay aided millimeter wave networks,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 10, no. 3, pp. 576–588, Apr. 2016.

[8] J. W. Sungok Kwon, “Relay selection for mmwave communications,” in *the 28th Annual IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (IEEE PIMRC)*, Oct. 2017, pp. 1–5.

[9] Y. Xu, H. Shokri-Ghadikolaei, and C. Fischione, “Distributed association and relaying with fairness in millimeter wave networks,” *IEEE Transactions on Wireless Communications*, vol. 15, no. 12, pp. 7955–7970, Dec. 2016.

[10] N. Wei, X. Lin, and Z. Zhang, “Optimal relay probing in millimeter-wave cellular systems with device-to-device relaying,” *IEEE Transactions on Vehicular Technology*, vol. 65, no. 12, pp. 10218–10222, Dec. 2016.

[11] S. Wu, R. Atar, N. Mastronarde, and L. Liu, “Coverage analysis of 6d relay-assisted millimeter-wave cellular networks,” in *IEEE Wireless Communications and Networking Conference (WCNC)*, Mar. 2017, pp. 1–6.

[12] R. Congiu, H. Shokri-Ghadikolaei, C. Fischione, and F. Santucci, “On the relay-fallback tradeoff in millimeter wave wireless systems,” in *IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS)*, Apr. 2016, pp. 622–627.

[13] S. Sun, T. S. Rappaport, R. W. Heath, A. Nix, and S. Rangan, “Mimo for millimeter-wave wireless communications: beamforming, spatial multiplexing, or both?” *IEEE Communications Magazine*, vol. 52, no. 12, pp. 110–121, Dec. 2014.

[14] T. Bai and R. W. Heath, “Coverage and rate analysis for millimeter-wave cellular networks,” *IEEE Transactions on Wireless Communications*, vol. 14, no. 2, pp. 1100–1114, Feb. 2015.

[15] R. M. Loynes, “The stability of a queue with non-independent inter-arrival and service times,” *Mathematical Proceedings of the Cambridge Philosophical Society*, vol. 58, no. 3, pp. 497–520, 1962.

[16] F. Gebali, *Analysis of Computer and Communication Networks*. New York, NY, USA: Springer-Verlag, 2010.

[17] 3GPP, “Study on channel model for frequencies from 0.5 to 100 GHz (release 14), 3gpp tr 38.901 v14.2.0,” Tech. Rep., Sept. 2017.