Trajectory Planning of Robot-assisted Abrasive Cloth Wheel Polishing Blade based on Flexible Contact

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Trajectory planning of robot-assisted abrasive cloth wheel polishing blade based on flexible contact

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Abstract
Industrial robot-assisted abrasive cloth wheel (ACW) accurately polish blades is considered to be a challenging task, and it is necessary to realize the digitalization of the process. Due to the flexible contact characteristics of abrasive cloth wheel and the change of blade surface curvature, the amount of microscopic material removal and the topographical errors of the blade surface are not uniform. So the surface roughness value is larger. In this paper, considering the flexible deformation of the blade and the abrasive cloth wheel during polishing contact, the polishing contact model of the blade and the abrasive cloth wheel is established and simulated. Firstly, based on the Preston equation and Hertz contact theory, the material removal profile in the contact area of the blade and the abrasive cloth wheel is analyzed. Secondly, the step length is optimized by considering the deformation of the abrasive cloth wheel in the \(u\) direction, and the line spacing is optimized by considering the material removal uniformity in the \(v\) direction. The NURBS curve is used to extract the blade polishing area curve and generate the polishing trajectory data points. Then, two methods of rigid trajectory planning and flexible adaptive trajectory planning of abrasive cloth wheel are simulated and analyzed by offline programming software. Finally, experiments were carried out on a four-station wheel changing polishing platform. Simulation and experiments results show that the proposed flexible adaptive trajectory planning method can make the surface roughness of the convex and the concave \(R_s \leq 0.3\mu m\), the surface roughness of the leading and trailing edge \(R_s \leq 0.2\mu m\), the total polishing efficiency increased by about 9.4%.

Keywords
Robot-assisted polishing  Abrasive cloth wheel (ACW)  Blade  Flexible contact  NURBS curve  Trajectory planning

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1 Introduction

The blade is the core component of the engine, and its polishing quality directly affects the service life and work efficiency of the engine. Due to the complex contour shape of the blade and high processing requirements, the traditional manual polishing is faced with high pollution, high working hours, high technical experience dependence and low processing consistency. Although the multi-axis CNC machining technology improves the defects caused by manual polishing, its high cost, low flexibility, and limited working space restrict the diversified development of blade polishing. In recent years, industrial robots have been widely used in the processing field. For the polishing of complex free-form surface parts, the robot processing system has high efficiency, good flexibility, large operating space, low cost, and combined force/torque sensor can ensure the processing quality and consistency of the blade. Many scholars have studied the polishing blades of industrial robots with abrasive belts. Zou et al. [1] used abrasive belt floating compensation technology to polish aluminum alloy blades to improve the polishing accuracy and surface roughness $R_a \leq 0.4 \mu m$. Li et al. [2] used abrasive belts to polish nickel-based superalloy blades, adopted parameter control methods and established a multi-parameter experimental platform (polishing force, vibration and temperature), obtained optimized surface parameters, and improved the polished surface quality. Wang et al. [3] proposed an abrasive belt polishing path planning method for precise material removal based on Hertz contact theory. Lv et al. [4] carried out adaptive trajectory planning for the leading and trailing edges of the blades by using robot abrasive belt polishing.

The above literature use robot abrasive belts to polish the blades. Although the abrasive belts has a certain flexibility, the conformal characteristics and tool integration of its flexible contact are not as good as the abrasive cloth wheel (ACW). The ACW can reach any narrow working space with small size and simple structure. It is flexible and can realize the conformal contact between the blade and the surface of the ACW. The contact deformation generated in the local area can adapt to the geometric contour of the blade surface, effectively avoiding the phenomenon of “under-polishing” and “over-polishing” of the leading and trailing edges. Huai et al. [5] used the ACW to polish the leading and trailing edges of the integral blisk to predict and optimize the surface roughness, and concluded that the influencing factors were the granularity of the ACW, speed, contact compression, and robot feed speed, and surface roughness $R_a \leq 0.4 \mu m$ after polishing. Lin et al. [6] optimized the process parameters for polishing the integral blisk of the ACW, and obtained the optimized stable polishing interval by orthogonal experiment, the compression amount is 0.6-0.8mm, and the spindle speed is 5000-6000r/min, the feed speed is 200-400mm/min. The above integral blisk with the ACW polishing can better adapt to the geometric profile of leading and trailing edges, and obtain better polishing surface quality, but its application is a five-axis CNC machine tool, and the flexibility of the machine tool movement space is small, the size and variety of processing blades are small. Therefore, this article attempts to use the robot-assisted abrasive cloth wheel (ACW) polishing method to polish the blades, which better combines the large working space and low cost of industrial robots with the conformability and flexibility of the ACW polishing tool on the blades. Reasonable trajectory planning can realize the processing requirements of blades of different sizes, and has high polishing accuracy and stability.
Many scholars have studied the trajectory planning of blade free-form surface processing. Ji [7-9] et al. studied the curve interpolation of the blade surface, using adaptive NURBS curve, cubic B-spline, seven-degree B-spline and other trajectory interpolation algorithms to make the robot's trajectory planning speed, acceleration and jerk are bounded and continuous to ensure smooth and efficient tracking performance and motion characteristics during robot trajectory planning. Ma [10-11] et al. selected key contact points on the 3-D model and refined to produce sensitive target points, so that the polishing track points and curves evenly cover the complex free-form surface, and the experiment verified that the precision polishing effect was achieved. The trajectory planning of the robot polishing free-form surface mentioned above is analyzed from the perspective of graphics and geometry, and it lacks consideration of the impact of dynamic contact force and material removal in the polishing process. Yang et al. [12-16] analyzed the contact particle movement to determine the contact pressure distribution, considering the elastic modulus and Poisson's ratio of the contact wheel, and its root mean square error (RMSE) and mean absolute percentage error (MAPE) values significantly reduced. Wang et al. [17-18] used a flexible inclined polishing disc to study the pressure and velocity distribution of the material removal area under three conditions of flat, convex, and concave surfaces. Simulations and experiments verified the accuracy of the method.

Although the above literature analyze the pressure and velocity distribution in the contact area, they lack the guidance of CAE analysis results to guide the generation of robot trajectory planning. Therefore, this paper takes into account the influence of elastic deformation, conducts CAE modeling of the contact process of blades and the ACW, analyzes the contact profile and pressure distribution of the contact area, and proposes a flexible adaptive trajectory planning based on contact theory and material removal profile. The planning algorithm realizes the CAE simulation analysis to guide the CAD trajectory planning to improve the accuracy of the industrial robot-assisted the ACW polishing blade. The main research content process is shown in Figure 1. Firstly, CAE simulation is used to analyze the shape of the contact area between the blade and the ACW, and the contact area model is established. Then, the material removal amount in the contact area is analyzed, and the material removal amount model is established. When planning the trajectory, considering the influence of contact theory and the amount of material removal, the polishing trajectory is generated and imported into the robot control cabinet for experimental verification, and finally the surface roughness topography is analyzed.

**Fig.1** Flow chart of robotic abrasive cloth wheel polishing trajectory planning

### 2 Contact analysis of blade and abrasive cloth wheel (ACW)

#### 2.1 The structure of blade and abrasive cloth wheel (ACW)

**2.1.1 Blade structure and related data**

Taking the blade of a certain type of steam turbine as the research object, it is formed by precision casting before polishing and the material is high-temperature alloy. Its structure is shown in Figure 2(a). The blade length is 205mm, width is 77mm, leading edge thickness is 3mm, trailing edge is 1.3mm, leading and trailing edge thickness is large, the overall rigidity is high, and when the polishing force is large, it is not easy to produce vibration and processing deformation. The overall structure of the blade is composed of leading edge, trailing edge, convex, concave and tenon. The profile 1 is fitted by the spline curve of leading edge, convex, concave, and the trailing edge is formed by a circular arc with radius of 1.3mm. Each curve satisfies G2 continuity at the intersection point, and its curvature comb is shown in Figure 2(b). Contour 1-Contour 4 are the intersection lines
with the solid surface of the blade when four equally spaced cross-sections of the blade are taken, and the blade body is fitted by a certain number of contour lines. The surface roughness value $R_d$ of the blade surface measured by PerimeterM2 is between 1.0-1.5μm, and the surface roughness $S_d$ measured by Leica super optical profiler 5X is about 3.353μm.

2.1.2 The structure and characteristics of the ACW

The ACW is an abrasive tool formed by sticking sand cloth closely arranged and at a certain angle with the central axis on the mandrel. There are various sizes and can be customized. The polishing material removal is completed by the polishing action of the abrasive grains on the sheet, as shown in Figure 3. The industrial robot-assisted the ACW polishing process has the following advantages:

(1) Flexible polishing. As shown in Figure 3, the emery cloth sheet of the ACW is in a bent state when it is stationary. When rotating at high speed, the emery cloth sheet stretches and the radius increases. Under the action of the polishing force, the sheet is bent and deformed. The base material of the sheet is flexible. High-strength cloth base material, the adhesive has elasticity, and the polishing flexibility is greater.

(2) Good spatial accessibility. The Kawasaki industrial robot RS020N has a maximum arm span of 1725mm, six degrees of freedom of rotation, and can reach any direction and position of the machining coordinate system. It adopts trajectory planning to effectively avoid interference, and is especially suitable for polishing of blades with complex structures.

(3) Cold polishing. When the ACW rotates at a high speed, as shown in Figure 3, the gap between the emery cloth sheets is formed, similar to the surface of the grinding wheel, which has many cooling grooves, which can function as a fan, and it is not easy to cause the surface of the blade to burn.

(4) High efficiency. The flexibility of the ACW increases the contact area between it and the blade, and the contour of the abrasive grains on the emery cloth sheet makes the grinding ability stronger than that of the grinding wheel. Although the linear velocity of the ACW is lower than the abrasive belt during polishing, its flexibility is greater than that of the abrasive belt, which can increase the contact area with the blade, and its polishing efficiency is about 2/3 of the abrasive belt.

2.2 Establishment and simulation of contact model between the blade and the ACW

The CAE simulation software ABAQUS is used to analyze the polishing contact process of the blade and the ACW. In the simulation environment, firstly, the digital model of the ACW and the blade is established, and the material properties of the blade and the ACW are input, including the elastic modulus $E/\rho$ and Poisson's ratio $\nu_1/\nu_2$. Assemble and establish an analysis step to make the blade and the ACW contact and apply load. Both the blade and the ACW are divided into hexahedral meshes, and the finite element calculation is performed to obtain the stress distribution in the contact area as shown in Figure 4.

Analyzing Figure 4, it can be seen that the contact area is elliptical, with its semi-major axis is a, and semi-minor axis is b. Through multiple simulation analyses, the length of the ellipse is related...
to the normal force applied during the polishing contact, the elastic modulus of the blade and the ACW material, the radius of curvature, and the width of the ACW. The center of the ellipse is the center of the polishing contact area. The polishing pressure at the center of the circle is the largest, that is, the material removal per unit time is the largest, and it gradually decreases toward the ellipse boundary, that is, the material removal rate decreases sequentially, and the polishing pressure at the edge is zero. That is, the amount of material removal in the contact area is zero.

2.3 Contact area analysis

According to the simulation analysis in Fig. 4 and combined with the Preston equation, the material removal rate is proportional to the polishing pressure and the relative speed, and its expression is formula (1).

\[ \frac{dh}{dt} = k_p \cdot p \cdot v_r \]  

(1)

where \( k_p \) is the Preston coefficient, \( p \) is the pressure distribution at the contact area between ACW and blade, \( v_r \) is the relative speed between ACW and blade, and is given by \( v_r = v_s \pm v_f \), and \( v_s \) represents the linear velocity of ACW, unit is m/s; \( v_f \) represents the robot feed speed, unit is mm/s, “±” represents the same/reverse direction between ACW steering and the robot feed speed.

Assuming within \( dt \) time, the actual arc length of the industrial robot end polished at the feed speed \( v_f \) is \( dl \), then \( dl = v_f \cdot dt \), formula (1) becomes formula (2).

\[ \frac{dh}{dl} = \frac{k_f \cdot p \cdot v_r}{v_f} \]  

(2)

Analysis formula (2) shows that the material removal rate per unit length is proportional to the polishing pressure, the relative speed between ACW and blade, and inversely proportional to the feed speed of the robot. According to Figure 4, it can be seen that the polishing pressure distribution in the contact area is approximately elliptical, and the contact process satisfies the conditions of Hertz's law. The formula for the boundary conditions is:

\[ \left( \frac{x}{a} \right)^2 + \left( \frac{y}{b} \right)^2 = 1 \]  

(3)

\[ a = \left[ \frac{3k^2 \varepsilon(k)F_n R}{\pi E_y} \right]^{1/2} \quad b = \left[ \frac{3k^2 \varepsilon(k)F_n R}{\pi E_y} \right]^{1/2} \]  

(4)

\[ R = \frac{1}{k_1 + k_2} \quad \frac{1}{E_y} = \frac{1 - v_1}{E_1} + \frac{1 - v_2}{E_2} \]  

(5)

\[ k \approx 1.0339 \left( \frac{k_2}{k_1} \right)^{0.66} \]  

(6)

\[ \varepsilon(k) \approx 1.00003 + 0.5968 \left( \frac{k_2}{k_1} \right) \]  

(7)

\[ \left\{ \begin{array}{l} k_1 + k_2 = \frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_5} + 2 \left( \frac{1}{R_1} - \frac{1}{R_5} \right) \cos \gamma \cos \gamma \cos \gamma \cos \gamma \cos \gamma \cos \gamma \cos \gamma \cos \gamma \end{array} \right\}^{1/2} \]  

(8)

In formulas (3)-(8), \( a \) and \( b \) are the length of long and short semi-axes of the ellipse, \( k \) is the ratio
of long and short semi-axes, $\varepsilon(k)$ is the second type of elliptic integral, and $F_n$ is the normal contact force between ACW and blade, $R$ is the relative radius of curvature, $k_1$ is the relative principal curvature of the ACW, unit (mm), $k_2$ is the relative principal curvature of the blade, unit (mm), $E_c$ is the relative contact elastic modulus, unit (MPa); where $E_1$ and $\nu_1$, $E_2$ and $\nu_2$ represent the elastic modulus and Poisson's ratio of the ACW and blade respectively, $R_1$ and $R_1'$ are the main radius of curvature of the ACW at the contact point, $R_2$ and $R_2'$ is the main curvature radius of the blade at the contact point, $\gamma$ is the angle between the direction where the main curvature of the ACW and the blade is in contact. $x_i$ and $y_i$ are the coordinate values of any point in the contact area. According to Hertz contact theory, the pressure distribution of any point in the contact area is expressed as formula (9).

$$p(x, y) = \frac{3F_n}{2\pi a \cdot b} \left[1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2\right]^{\frac{3}{2}}$$

(9)

**2.4 Profile model analysis of material removal**

The polishing removal depth of the blade surface directly affects the material removal rate of the blade surface, and the shape of the contact area is approximately elliptical. A micro element in the elliptical contact area is selected for analysis. As shown in Figure 5, the material removal amount at the polishing contact point A is:

$$h(x) = \frac{2 \cdot k \cdot \nu}{v_f} \int_{y=0}^{h} p(x, y) dy$$

(10)

Integrate formula (10) and substitute formula (4) and (9) into formula (10) to get the polishing depth of the contact point.

$$h(x) = \frac{2 \cdot k \cdot \nu}{v_f} \cdot \frac{3F_n}{2\pi a \cdot b} \int_{y=0}^{h} \left[1 - \left(\frac{x}{a}\right)^2 \cdot b^2 - y^2\right]^{\frac{3}{2}} dy$$

$$= \frac{3F_n \cdot k \cdot \nu}{4v_f} \left[\frac{\pi E_c (k_1 + k_2)}{3k^2 c(k) F_n} \right]^{\frac{3}{2}} \left[1 - \left(\frac{x}{a}\right)^2 \right]$$

(11)

The depth of material removal at the center contact point of the polishing area is the largest, and the maximum is

$$h_{\text{max}} = \frac{F_n^{\frac{3}{2}} \cdot k \cdot \nu}{v_f} \left[\frac{9\pi E_c (k_1 + k_2)}{64k^2 c(k)}\right]^{\frac{3}{2}}$$

(12)

**Fig.5** Distribution of material removal elements in the contact area

Analysis formula (12) shows that the maximum removal depth of the contact point $O$ is related to the polishing contact normal force, relative speed, robot feed speed, relative elastic modulus, and relative principal curvature, which are basically consistent with the simulation results in Fig. 4. For the selected polishing ACW and blade, the relative elastic modulus $E_c$ and the main curvature of the ACW are fixed. For a small range, the radius of curvature of the blade is considered to be approximately constant. Comprehensive analysis shows that the factors affecting the maximum material removal depth are normal contact force, relative speed and robot feed speed.

**3 Trajectory planning algorithm**

When planning the trajectory of the blade, it is necessary to consider the material removal rate in the contact area of the blade. A reasonable step length and line spacing can obtain a uniform material
removal rate on the blade surface.

3.1 Calculate the step length and line spacing

According to the contact model established during the simulation in Fig. 4, the step length and the line spacing are calculated by lateral polishing. Use equal chord height error to calculate the step length in the $u$ direction, and use equal residual height to calculate the line spacing in the $v$ direction. Because the ACW is a flexible emery cloth sheet, it is different from the contact process of rigid polishing wheel. During the contact process, the polishing contact point will produce visible elastic deformation. The magnitude of the elastic deformation is related to the normal contact force, which determines the material removal profile, therefore, it is necessary to calculate the polishing step length and the line spacing based on the contact deformation of the ACW. Figure 6 is a schematic diagram of the step length calculation.

**Fig.6** Step-length calculation considering flexible deformation of the wheel

It can be seen from Figure 6 that the material removal at the polishing contact point A of the blade is the largest, $\varepsilon'$ is the maximum removal depth of the blade, its size is $h_{max}$, and:

$$\varepsilon = \varepsilon' + \delta$$

In formula (13), $\delta$ is the material removal depth when the blade is in initial contact, $\varepsilon$ is the material removal depth when the ACW is in contact with the blade at the intermediate contact time, and $\varepsilon'$ is the material removal depth when the ACW deformation is considered.

$$\delta = \frac{3\xi(k)F_s}{2\cdot \pi \cdot E_s \cdot a}$$

(14)

$$\xi(k) = 1.5277 + 0.6023\ln\left(\frac{k_2}{k_1}\right)$$

(15)

Substitute formula (12) (14) into formula (13) to obtain formula (16)

$$\varepsilon = \varepsilon - \delta = \frac{F_s^{\frac{1}{2}} \cdot k_p \cdot v_f}{v_f} \left[\frac{9\pi E_s (k_1 + k_2)}{64k^2 \varepsilon(k)}\right]^{\frac{1}{2}} - \frac{3\xi(k)F_s}{2\cdot \pi \cdot E_s \cdot a}$$

(16)

The calculation of the polishing step length when the ACW and the blade are in rigid contact is shown in Figure 7, and the formula is (17).

**Fig.7** Step-length calculation of rigid polishing wheel

In $\triangle ABO$, according to the Pythagorean Theorem,

$$S = 2 \left[ R^2 - (R - a)^2 \right]^{\frac{1}{2}}$$

(17)

Simplify formula (17) to get

$$S = (8R_a)\varepsilon^{\frac{1}{2}}$$

(18)

Substituting formula (16) into formula (18) to obtain the calculation formula (19) of polishing line spacing considering the elastic deformation of the ACW.

$$S = 8R_a \left[ \frac{F_s^{\frac{1}{2}} \cdot k_p \cdot v_f}{v_f} \left[\frac{9\pi E_s (k_1 + k_2)}{64k^2 \varepsilon(k)}\right]^{\frac{1}{2}} - \frac{3\xi(k)F_s}{2\cdot \pi \cdot E_s \cdot a} \right]^{\frac{1}{2}}$$

(19)

According to formula (19), the following can be obtained: assuming that the blade and ACW
material, the spindle speed, the robot feed speed, the relative principal curvature of the contact point and other process parameters are known, the step length is related to the normal contact force of the polishing. Without affecting the polishing accuracy, the greater the normal contact force, the greater the elastic deformation of the ACW, the longer the polishing contact step, and the higher the corresponding polishing efficiency. The most notable feature of industrial robot-assisted ACW polishing is flexible polishing, and the influence of elastic deformation needs to be considered when calculating the polishing line spacing. Figure 8 is the calculation of the polishing line spacing, \( w \) is the width of the ACW, \( L \) is the polishing line spacing, \( R_{i-1}/R_i/R_{i+1} \) is the radius of curvature of the adjacent points of the blade, and \( 2\theta \) is the angle of the blade corresponding to the width of ACW, \( \varepsilon \) is the compression of the ACW perpendicular to the direction of the polishing vector.

In \( \triangle DBO \) \( w = 2 \left[ R_i^2 - (R_i - \varepsilon)^2 \right]^{\frac{1}{2}} \)

(20)

The relationship between the width of the ACW and the polishing line spacing is formula (21)

\[ L = w \cdot \cos \theta \]

(21)

Substituting formulas (14) and (20) into formula (21) can obtain formula (22).

\[ L = \left[ 8R \left( \frac{3kF_n}{2 \cdot \pi \cdot E_a} \right) \right]^{\frac{1}{2}} \cdot \cos \theta \]

(22)

Analyzing formula (22), given the relative elastic modulus of the polishing contact and the curvature of the polishing contact point, the polishing line spacing is related to the normal contact force \( F_n \), the major semi-axis of the ellipse in the contact area, and the central angle of the adjacent polishing contact points. In the \( v \) direction, the blade curvature change is small and almost zero, so \( \cos \theta \approx 1 \). Therefore, the polishing line spacing is related to the normal contact force. After theoretical calculation and simulation analysis, as shown in formula (9) and Figure 4, the normal polishing force at the center of the ellipse contact area is the largest, the material removal depth is the deepest, and the normal polishing force at the edge of the ellipse contact area is zero, the amount of material removal is zero. In order to improve the consistency of the polished surface quality, the difference in the amount of material removal at different points within the polishing width of the ACW should be reduced. Since the average polishing force \( F_{nm} \) in the contact area is \( \pi4 \approx 0.785 \) times of the maximum polishing force, that is, \( F_{nm} = 0.785F_{nm\text{max}} \), the polishing line spacing of the ACW is determined to be 0.785 times the original polishing bandwidth, and the formula (23) as the polishing line spacing in simulation and experiment.

\[ L = 0.785W \]

(23)

**Fig. 8** Line spacing calculation diagram of flexible wheel

Formulas (19) and (22) are the calculation formulas for the polishing step length and line spacing considering the elastic contact deformation of the ACW. Under the premise that the polishing contact force and the long and short axis of the polishing contact ellipse are known, the step length and line spacing can be obtained. Table 1 shows the process parameters during polishing. The main curvature of the blade concave is selected as \( k_1 = \frac{1}{230} \text{mm} \), \( k_2 = \frac{1}{320} \text{mm} \), the main curvature of the ACW is \( k_1 = \frac{1}{110} \text{mm} \), \( k_2 = 0 \) as the research object. Table 2 is the calculated comparison value of the step length and line spacing before and after the improvement of this point.
Table 1 Parameters of polishing process

Table 2 Step-length/ line-spacing at that point before/ after improvement

Analyzing Table 2 shows that, under the premise of not affecting the polishing efficiency, compared with the rigid tool to calculate the step length and line spacing, the flexible adaptive trajectory planning algorithm is adopted to select the known polishing point on the blade, The polishing step length is increased from 9.25mm to 13.09mm, and the line spacing is reduced from 20mm to 15mm.

3.2 Constant feed rate interpolation calculation

Let $p(u)$ to be the spline curve, and the time function $u$ is the curve parameter, let $u(t_i) = u_i$,

$$u(t_{i+1}) = u_{i+1}$$

$$P(u) = (x(u), y(u), z(u))$$  \hspace{1cm} (24)

Perform a second-order Taylor expansion of the parameter $u_i$ to time $t$ to get the next interpolation point $u_{i+1}$.

$$u_{i+1} = u_i + \frac{du}{dt} \left|_{u_i} \right| \left( t_{i+1} - t_i \right) + \frac{1}{2} \frac{d^2 u}{dt^2} \left|_{u_i} \right| \left( t_{i+1} - t_i \right)^2 + H.O.T$$  \hspace{1cm} (25)

In the formula, $u_i$ — the curve parameter $i$ corresponding to the first interpolation point;

$u_{i+1}$ — The curve parameter $i+1$ corresponding to the first interpolation point;

$t_i$ — The interpolation time $i$ corresponding to the first interpolation point;

$t_{i+1}$ — The interpolation time $i+1$ corresponding to the first interpolation point;

$H.O.T$ — High-order infinitesimals.

The interpolation speed of the servo motors of each joint of the robot is:

$$V(u_i) = \frac{dP(u)}{dt} \bigg|_{u_i}$$

According to formula (25),

$$\frac{du}{dt} \bigg|_{u_i} = \frac{V(u_i)}{\sqrt{\left(x'(u)\right)^2 + \left(y'(u)\right)^2 + \left(z'(u)\right)^2}}$$  \hspace{1cm} (27)

Derivation of formula (26) can be obtained,

$$\frac{d^2 u}{dt^2} \bigg|_{u_i} = \frac{V^2(u_i)}{\left(\frac{dP(u)}{du}\right)^2} = -\frac{V^2(u_i)}{\left(\frac{dP(u)}{du}\right)^2} \left(\frac{d^2 P(u)}{du^2}\right)$$  \hspace{1cm} (28)

NURBS curve interpolation period $t_s$ is the difference between adjacent interpolation points,
\( t_s = t_{i+1} - t_i \), substituting formula (27)(28) into formula (25), ignoring high-order infinitesimals, and obtaining NURBS curve second-order approximate expression of interpolation (29), the second-order accuracy can satisfies the interpolation accuracy of industrial robots.

\[
U_{i+1} = U_i + \frac{V^2(u_i) \cdot t_i}{2 \left( \frac{dP(u)}{dt} \right)_{u=u_i}} \cdot t_i^2
\]

4 Simulation and experimental verification

4.1 Algorithm simulation

Use offline programming software SprutCAM to verify the algorithm. First, import the Kawasaki industrial robot RS020N, the four-station polishing platform, the blade, and the ACW into the simulation environment. Carry out the calibration of the ACW tool TCP point and the three characteristic points of the blade, and use the quaternion method to calculate the characteristic points of the blade, so that the blade imported into the simulation environment is consistent with the actual installation environment. Then extract the NURBS curve of the blade surface, set the processing environment and process parameters, simulate and verify the collision, unreachable, and over-limit, and select the normal direction of the contact point between the blade and the ACW as the polishing posture. Fig. 9(a) is a simulation diagram of NURBS curve trajectory planning, and Fig. 9(b) is a simulation diagram of flexible adaptive NURBS curve trajectory planning.

Fig. 9 Simulation comparison before and after improvement of NURBS curve algorithm

Analyzing the simulation results of the trajectory planning in Fig.10, after the algorithm is improved, the polishing time of the blade convex is shortened, and the polishing trajectory points are reduced.

Fig. 10 Comparison polishing time and track point before/after improvement

The polishing time of the blade convex before the algorithm improvement is 339s, and the polishing time is 215s after the improvement. The polishing efficiency is increased by 36.58\%. The number of polishing nodes is 614 before the improvement and 364 after the improvement, and the number of nodes is reduced by 40.72\%. The polishing time of the blade concave is 263s before the algorithm is improved and 160s after the improvement. The polishing efficiency is increased by 39.13\%. The number of polishing nodes is 478 before the improvement and 368 after the improvement, and the number of nodes is reduced by 23.01\%. The polishing time of the leading edge is 291s before the algorithm is improved, and it is 347s after the improvement. The polishing time is increased. This is because the curvature of the adjacent line spacing of the leading and trailing edges changes greatly, that is, the central angle of adjacent curvatures is larger and the line spacing is small. Therefore, the number of polishing nodes is 528 before the improvement, 665 after the improvement, the trailing edge polishing time before the improvement is 371s, after the improvement is 423s, the number of polishing nodes is 678 before the improvement, 768 after the improvement, the total polishing time before the improvement 1264s, the total polishing time after the improvement is 1145s, the total number of polishing nodes before the improvement is 2298, the total number of polishing nodes after the improvement is 2165, the number of nodes is reduced by 5.79\%, and the total polishing time after the algorithm is improved is reduced 119s.
4.2 Experimental verification

4.2.1 Selection of the ACW

The polishing adopts the ACW, which is installed at the end of the four-station wheel-changing polishing platform. The diameter, width, and abrasive of the ACW are selected according to the curvature of the blade and the polishing process. The radius of curvature of the blade convex is between 14mm-1988mm, the radius of curvature of the blade concave is between 15mm-1316mm, the radius of curvature of the leading edge is about 2.1-3.5mm, and the trailing edge is about 1.3mm. The curvature radius of the blade convex and concave varies greatly. The red dividing line divides the curvature radius of the blade convex and concave into two parts, as shown in Figure 11. The curvature radius of near the red dividing line of the blade convex is about 80mm. Considering the influence of polishing surface quality and polishing efficiency, when polishing the blade convex, the diameter of the ACW is ∅80mm, the contact wheel width is 20mm, and the coarse polishing abrasive number is 320#. The number of finely polishing abrasive is 600#. The leading and trailing edges adopt ACW with the diameter of ∅80mm, contact width of 20mm, coarse polishing is 320#, and fine polishing is 600#.

Fig. 11 Curvature diagram of blade

In consideration the principle of avoiding interference in blade concave polishing, the diameter of the ACW is selected according to the minimum curvature is ∅30mm, the contact width is 20mm, the coarse polishing abrasive is 320#, and the fine polishing abrasive is 600#. The blade polishing process is shown in Figure 12.

Fig. 12 Flow chart of polishing process

4.2.2 Experimental device

Figure 13 is an experimental device for the robot to polish complex free-form blades, including the Kawasaki RS020N six-degree-of-freedom industrial robot, with terminal load of 20kg, working space range is 1725mm, and repeat positioning accuracy is 0.05mm. The six-dimensional force/torque sensor (ATI delta SI330-30) is installed at the end of the sixth joint flange of the robot to measure the contact force during the polishing process. The data processing module uses TCP communication to transmit the polishing force to the robot control cabinet. The machine uses the force control algorithm to adjust the robot's end pose in real time to maintain constant polishing normal contact force. The blade clamp is installed at the end of the force sensor, the blade is installed in the clamp, and the ACW is used as the polishing tool. The four-station wheel changing platform adopts PLC control to replace the polishing wheel in real time according to the different parts of the polishing blade. The leading and trailing edge polishing force is about 10N, the blade concave and concave polishing force is about 30N.

(1. Kawasaki industrial robot RS020N 2. Robot E control cabinet 3. Teach pendant 4. Gas storage 5. ATI six dimensional force/moment sensor 6. Fixture 7. Abrasive cloth wheel 8. Blade 9.PLC control cabinet 10. PC 11. Offline simulation software 12.Four-station polishing platform 13. Air dryer 14. Pan-rui air compressor)

Fig. 13 Industrial robot polishing blade device

4.3 Result Analysis
Figure 14/Figure 15 compare the surface quality of various parts of the blade before and after polishing when the industrial robot assists the ACW to polish the blade. The blade concave and concave, leading edge and trailing edge are smoother than before polishing, achieving the mirror polishing effect. Figure 16 is the surface topography analysis before and after polishing with Leica surface profiler. Figure 16(a) (c) is the surface topography before polishing. The maximum unit of the three-dimensional topography is 90μm, Figure 16(b) (d) is the surface morphology after polishing, the maximum unit of the three-dimensional morphology is 60μm, and the three-dimensional topography of the blade surface after polishing is basically stable at 5-10μm.

**Fig.14** Comparison blade convex / concave surface morphology before/after polishing

**Fig. 15** Surface morphology of the blade edge after polishing

**Fig.16** Morphology of blade surface

Figure 17 compares the surface roughness values $R_a$ of various parts of the blade before and after polishing, the measured surface roughness values $R_a \geq 1.0\mu m$ of the blade convex / concave before polishing, the surface roughness value before the algorithm is improved is about $R_a \leq 0.4\mu m$, and the surface roughness value after the algorithm is improved approximately $R_a \leq 0.3\mu m$, the polishing efficiency is increased by approximately 22%. The surface roughness value before the leading edge polishing is about $R_a \geq 0.5\mu m$, the surface roughness before the algorithm is improved is about $R_a \leq 0.3\mu m$, and the surface roughness after the algorithm is improved is about $R_a \leq 0.2\mu m$, and the polishing efficiency is reduced by 20%. The surface roughness of the trailing edge before polishing is about $R_a \geq 0.6\mu m$, the surface roughness before the algorithm is improved is about $R_a \leq 0.3\mu m$, and the surface roughness after the algorithm is improved is about $R_a \leq 0.2\mu m$, and the polishing efficiency is reduced by 12%. The test results fully proved that when considering the material removal in the flexible contact of contact area about the ACW, the surface roughness of the blade is reduced when the total polishing efficiency is increased by 9.4%.

**Fig. 17** Comparison the measured values of blade surface roughness

5 Conclusion

Based on the ABAQUS simulation results, combined with the Preston equation and Hertz contact theory, this paper analyzes the contact area between the ACW and the blade, uses the URBSS curve to extract the polishing data points on the blade surface, obtains the polishing step length and line spacing of the blade, and adopts the constant feed rate completes the interpolation calculation. The correctness of the flexible adaptive trajectory planning method is verified through offline simulation and experiment, and the following conclusions are drawn:

(1) Considering the flexible deformation of the ACW and the material removal rate for trajectory planning, the improved flexible adaptive trajectory planning algorithm reduces the polishing time of the blade convex and the concave from 602s to 375s, and increases the polishing time of the leading and trailing edges from 662s to 770s. The total polishing efficiency is increased by about 9.4%.

(2) Using flexible adaptive trajectory planning algorithm, the surface roughness $R_a \leq 0.3\mu m$ of the blade concave and convex, and the three-dimensional contour $\delta_s \leq 0.2\mu m$.

(3) The flexible ACW achieves the conformal contact polishing of the leading edge and the trailing...
edge, without "over-polishing" and "under-polishing" phenomenon, surface roughness $R_a \leq 0.2\mu m$, and three-dimensional contour $\delta_r \leq 0.2\mu m$.

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Fig. 1 Flow chart of robotic abrasive cloth wheel polishing trajectory planning
Fig. 2 3D model of blade
(a) 3D structure
(b) Blade curvature of profile 1
Fig. 3 Abrasive cloth wheel structure
Fig. 4 Stress simulation effect diagram in the contact area
Fig. 5 Distribution of material removal elements in the contact area
Fig. 6 Step-length calculation considering flexible deformation of the wheel
Fig. 7 Step-length calculation of rigid polishing wheel
Fig. 8 Line spacing calculation diagram of flexible wheel
Fig. 9 Simulation comparison before and after improvement of NURBS curve algorithm
Fig. 10 Comparison polishing time and track point before/after improvement
Fig. 11 Curvature diagram of blade
Fig. 12 Flow chart of polishing process
(1. Kawasaki industrial robot RS020N 2. Robot E control cabinet 3. Teach pendant 4. Gas storage 5. ATI six dimensional force/moment sensor 6. Fixture 7. Abrasive cloth wheel 8. Blade 9. PLC control cabinet 10. PC 11. Offline simulation software 12. Four-station polishing platform 13. Air dryer 14. Pan-rui air compressor)

**Fig. 13** Industrial robot polishing blade device
Fig.14 Comparison blade convex / concave surface morphology before/after polishing
Fig. 15 Surface morphology of the blade edge after polishing
Fig. 16 Morphology of blade surface
Fig. 17 Comparison the measured values of blade surface roughness
| Parameters                                      | Values                  |
|------------------------------------------------|-------------------------|
| Elastic modulus $E_1$, Poisson's ratio $\nu_1$ of ACW | 30e6 Pa, 0.05           |
| Elastic modulus $E_2$, Poisson's ratio $\nu_2$ of blades | 102040e6 Pa, 0.3        |
| Blades feed speed $v_f$                          | 0.2 mm/min              |
| Fast moving feed $v_k$                           | 1 mm/min                |
| ACW’s spindle speed $v_{s1}$                     | 12.57 m/s (convex/LE/TE) |
| ACW’s spindle speed $v_{s2}$                     | 4.7 m/s (concave)       |
| Normal contact force $F_{n1}$                    | 30 N (convex/concave)   |
| Normal contact force $F_{n2}$                    | 10 N (LE/TE)            |
| Chord height error $\epsilon'$                   | 0.08 mm                 |
| Scallop height                                   | 0.01 mm                 |
| Preston coefficient $k_p$                        | 1                       |
Table 2 Step-length/ line-spacing at that point before/ after improvement

| Parameters   | Before improvement | After improvement |
|--------------|--------------------|-------------------|
| Step-length  | 9.25mm             | 13.09mm           |
| Line-spacing | 20mm               | 15mm              |