Covariant Isobar Model for $K^+\Lambda$ Electroproduction

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Abstract. Kaon electroproduction process $e + p \rightarrow e' + K^+ + \Lambda$ has been investigated by using a covariant isobar model constructed from the appropriate Feynman diagrams. The corresponding propagators and vertex factors are obtained from our previous analysis. For the electromagnetic form factor we adopt the standard dipole one with the cutoff considered as a free parameter. We include nucleon resonances with spins up to 9/2. All nucleon resonances listed in the particle listings of the Particle Data Group are considered in this investigation. The unknown parameters, including the electromagnetic and hadronic coupling constants, are extracted by fitting the predicted observables to the currently available experimental data.

1. Introduction
In the previous study we have successfully produced an isobar model for kaon photoproduction $\gamma + p \rightarrow K^+ + \Lambda$. Two models with different propagator and vertex formalisms for nucleon resonances with spins up to 5/2 were proposed [1]. The models were tested by fitting the calculated observables to the experimental data and adjusting the unknown parameters. The best model obtained in this study was used in the subsequent investigation, where the nucleon resonances with spins up to 9/2 were included [2]. The formulation is presented in terms of form functions $A_i$ for each of the nucleon resonances. In the recent study, hyperon resonances with spins up to 3/2 were also added in the model [3].

In this paper we present the result of extending the model to include the electroproduction process. The formulation can be made simple by considering the process of kaon virtual photoproduction. In this case both photo- and electroproductions can be analyzed simultaneously.

One of the interesting properties in kaon electroproduction is that there exist electromagnetic form factors which cannot contribute in kaon photoproduction. These form factors can be used for studying the internal structures of kaon and hyperon, which is not possible in the pion electroproduction. A similar investigation has been performed, but very close to threshold [4]. As a consequence, the studied electromagnetic form factors are also very limited. Extending the electroproduction model to the higher energy region is therefore very demanded.

Technically, extending the formulation of photoproduction to electroproduction is the same as adding the longitudinal terms. In terms of form functions $A_i$, this means that we add the longitudinal form functions $A_5$ and $A_6$. On the other hand, the transverse terms coming from the photoproduction can be fixed by our previous photoproduction model. The longitudinal coupling constants and the form factor cutoffs can be set as free parameters during the fitting process. We note that there exist 1274 experimental data points in the $K^+\Lambda$ electroproduction.
In the present work we use the dipole form factor for the hyperon and nucleon resonances. For the $K_1$ and $K^*$ intermediate states the monopole form factor is considered. Experimental data are taken from the CLAS measurements [5, 6, 7].

2. Kinematics
The process of kaon electroproduction on the nucleon can be written as

$$e(k_i) + p(p) \rightarrow e'(k_f) + K^+(q) + \Lambda(p_\Lambda),$$

(1)

where the momenta of involved particles are explicitly indicated. From Eq. (1) it is obvious that $k_i - k_f$ denotes the momentum of virtual photon. Note that the square of this momentum does not equal to zero as in the case of photoproduction.

The transition amplitude obtained from the appropriate Feynman diagrams can be decomposed into the gauge and Lorentz invariant matrices $M_i$ through

$$\mathcal{M} = \bar{u}_\Lambda(p_\Lambda) \sum_{i=1}^{6} A_i(s, t, k^2) M_i u_p(p_p),$$

(2)

where $s$, $t$ and $u$ are the Mandelstam variables, defined by

$$s = (k + p_p)^2, \quad t = (k - q)^2, \quad u = (k - p_\Lambda)^2,$$

(3)

while the gauge and Lorentz invariant matrices $M_i$ are given by [4]

$$M_1 = \frac{1}{2} \gamma_5 (\not{k} - \not{k'}),$$
$$M_2 = \gamma_5 [(2q - k) \cdot \epsilon P \cdot k - (2q - k) \cdot k P \cdot \epsilon],$$
$$M_3 = \gamma_5 (k \cdot \epsilon k - \not{k} \cdot \not{\epsilon} k),$$
$$M_4 = i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu} q^{\nu} \epsilon^{\rho} k^{\sigma},$$
$$M_5 = \gamma_5 [q \cdot \epsilon k^2 - q \cdot k - k_\epsilon \cdot \epsilon],$$
$$M_6 = \gamma_5 (\not{k} \cdot \epsilon \not{k} - \not{k^2} \epsilon),$$

with $P = \frac{1}{2}(p_p + p_\Lambda)$, and $\epsilon_{\mu\nu\rho\sigma}$ is the four dimensional Levi-Civita tensor. The form functions $A_i$ are given in our previous works [1, 2, 3].

3. Cross section and Form Factor for kaon electroproduction
For kaon electroproduction the cross section can be separated into the transversely unpolarized (T), longitudinally polarized (L), transversely polarized (TT) and the interference between transversely and longitudinally polarized (LT) cross sections. In this work we define

$$\sigma_T = \frac{d\sigma_T}{d\Omega}, \quad \sigma_{TT} = \frac{d\sigma_{TT}}{d\Omega}, \quad \sigma_{LT} = \frac{d\sigma_{LT}}{d\Omega}, \quad \sigma_L = \frac{d\sigma_L}{d\Omega}, \quad \sigma_{LT} = \frac{d\sigma_{LT}}{d\Omega}, \quad \sigma_U = \sigma_T + \epsilon \sigma_L,$$

(4)

where $\sigma_U$ is the unpolarized differential cross section. These cross sections can be calculated analytically from $A_i$. They are fitted to experimental data by using the CERN Minuit code by adjusting the longitudinal coupling constants.

The electromagnetic form factors are taken in the form of monopole and dipole. The standard monopole form factor is used for kaon resonances $K^*$ and $K_1$, where

$$F^{K^*}(Q^2) = \left(1 + \frac{Q^2}{M_{K^*}^2}\right)^{-1}, \quad F^{K_1}(Q^2) = \left(1 + \frac{Q^2}{M_{K_1}^2}\right)^{-1},$$

(5)
Table 1. Coupling constants and hadronic form factor cutoff fixed by the photoproduction process.

| Parameter          | Value      | Parameter          | Value  |
|--------------------|------------|--------------------|--------|
| $g_{KNN}/\sqrt{4\pi}$ | $-3.43$    | $G_{K1}^V/4\pi$   | $-0.07$ |
| $g_{K\Sigma N}/\sqrt{4\pi}$ | $1.30$     | $G_{K1}^V/4\pi$   | $4.40$  |
| $G_{K^*}^V/4\pi$    | $0.22$     | $\Lambda_B$       | $0.70$  |
| $G_{K^*}^T/4\pi$    | $0.36$     | $\Lambda_R$       | $1.07$  |

with $Q^2 = -k^2$. For baryon resonances we use dipole form factors

$$F^{N^*}(Q^2) = \left(1 + \frac{Q^2}{\Lambda_N^2}\right)^{-2}, \quad F^{Y^*}(Q^2) = \left(1 + \frac{Q^2}{\Lambda_Y^2}\right)^{-2}$$

(6)

where the cutoffs $\Lambda_i$ are considered as free parameters determined from fitting the experimental data.

Note that the hadronic coupling constants have been fixed by the photoproduction process. Few of them are listed in Table 1. On the other hand, the longitudinal coupling constants, i.e., the coupling constants that exist in the longitudinal terms, are considered as free parameters. They are fitted to around 1274 kaon electroproduction data with $Q^2 = 0.65 - 2.55$ GeV$^2$. In this model we include nucleon resonances with spins up to $9/2$ and hyperon resonances with spins up to $3/2$. These resonances have the status of two stars or more in Particle Data Book (PDG) [9].

4. Numerical Result

The extracted longitudinal coupling constants along with the form factor cutoffs are listed in Table 2. Note that $G_N^{(2)}$ is the longitudinal coupling constant for the nucleon resonance with spin 1/2. This coupling constant does not contribute to the photoproduction. In contrast to this, the electromagnetic coupling $G_N^{(1)}$ is adopted from the photoproduction result. A complete report regarding the hyperon and nucleon resonances will be published later because the number of resonances included is very large. From Table 2 we can determine whether or not the resonances have large contribution to this process. For instance, the $K^*$ cutoff and the first cutoffs of $N(1700)$ and $N(1720)$ indicate small contributions.

There are many possibilities to compare the results of the present work with experimental data. In this work, we only present one of them, i.e., the unpolarized differential cross sections in Figure 1. They are compared with experimental data from the CLAS collaboration.

In Fig. 1 we present $\sigma_U$ in three different angles and three different momentum transfers. The upper panels show $\sigma_U$ at $Q^2 = 0.65$ GeV$^2$. The middle panels are $\sigma_U$ at $Q^2 = 1.55$ GeV$^2$ and the bottom panels are $\sigma_U$ at $Q^2 = 2.55$ GeV$^2$. From these figures we found that the model have a same pattern with experimental data. The model can perfectly reproduce the experimental data at $\theta_K = 60^\circ$. However, in the backward and forward angles there are some data points which are underestimated by the model, especially in the forward angle. Furthermore, we see that the cross section increases with increasing energy in forward angles. However, a fair comparison cannot be easily performed, because of the lack of experimental data in the high energy region.

5. Summary and Conclusions

The isobar model for kaon photoproduction has been expanded to the electroproduction. Compared to experimental data the model calculation is in good agreement. The result also
Figure 1. Unpolarized differential cross section obtained in the present work (solid lines) compared with the CLAS experimental data (solid circles) as a function of the total c.m. energy $W$ for different kaon angles. Panels (a), (b), and (c) exhibit that the data are taken with the momentum transfer $Q^2 = 0.65 \text{ GeV}^2$, $1.55 \text{ GeV}^2$, and $2.55 \text{ GeV}^2$, respectively.

indicates that each of the resonances contributes to this process. In the future, we will explore different types of form factors in order to achieve a better agreement with experimental data.

6. ACKNOWLEDGMENTS
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| Parameter | Value | Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|-----------|-------|
| $\Lambda_{K^*}^{(1)}$ | 0.002 | $\Lambda_{N(1700)}^{(1)}$ | 0.002 | $\Lambda_{N(1860)}^{(1)}$ | 0.002 |
| $\Lambda_{K1}$ | 0.482 | $\Lambda_{N(1700)}^{(2)}$ | 0.792 | $\Lambda_{N(1860)}^{(2)}$ | 0.564 |
| $G_{N(1440)}^{(2)}/4\pi$ | 0.453 | $\Lambda_{N(1700)}^{(3)}$ | 0.834 | $\Lambda_{N(1860)}^{(3)}$ | 0.478 |
| $\Lambda_{N(1440)}^{(1)}$ | 0.660 | $\Lambda_{N(1700)}^{(4)}$ | 0.878 | $\Lambda_{N(1860)}^{(4)}$ | 0.450 |
| $\Lambda_{N(1440)}^{(2)}$ | 2.000 | $\Lambda_{N(1720)}^{(1)}$ | 0.002 | $\Lambda_{N(1860)}^{(1)}$ | 0.353 |
| $G_{N(1650)}^{(2)}/4\pi$ | 0.085 | $\Lambda_{N(1720)}^{(2)}$ | 0.002 | $\Lambda_{N(1990)}^{(2)}$ | 0.639 |
| $\Lambda_{N(1650)}^{(1)}$ | 0.813 | $\Lambda_{N(1720)}^{(3)}$ | 1.718 | $\Lambda_{N(1990)}^{(3)}$ | 0.240 |
| $\Lambda_{N(1650)}^{(2)}$ | 0.650 | $\Lambda_{N(1720)}^{(4)}$ | 0.002 | $\Lambda_{N(1990)}^{(4)}$ | 0.932 |

Table 2. The extracted longitudinal coupling constants and resonance cutoffs. These parameters are extracted from fitting to experimental data with $\chi^2/N = 1.73$. 


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