A Bayesian Residual Transform for Signal Processing
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Multi-scale decomposition has been an invaluable tool for the processing of physiological signals. Much focus in multi-scale decomposition for processing such signals have been based on scale-space theory and wavelet transforms. By contrast, Bayesian-based multi-scale decomposition for processing physiological signals is less explored and ripe for investigation. In this study, we investigate the feasibility of utilizing a Bayesian-based method for multi-scale signal decomposition called Bayesian Residual Transform (BRT) for the purpose of physiological signal processing. In BRT, a signal is modeled as the summation of stochastic residual processes, each characterizing information from the signal at different scales. A deep cascading framework is introduced as a realization of the BRT. Signal-to-noise ratio (SNR) analysis using electrocardiography (ECG) signals was used to illustrate the feasibility of using the BRT for suppressing noise in physiological signals. Results in this study show that it is feasible to utilize the BRT for processing physiological signals for tasks such as noise suppression.

Introduction

Physiological signals are signals that are measured from sensors that are either placed on or implanted into the body. Such physiological signals include those obtained using electromyography (EMG), electrocardiography (ECG), electroencephalography (EEG), photoplethysmography (PPG), and ballistocardiography (BCG). The processing and interpretation of such signals is challenging due to a number of different factors. For example, it is often difficult to obtain high-fidelity physiological signals due to noise, resulting in low signal-to-noise ratio (SNR). Traditionally, signal averaging and linear filters such as band-reject and band-pass filters have been used to process such physiological signals to suppress noise; however, such approaches have also been shown to result in signal degradation \cite{1,2}. As such, more advanced methods for handling such physiological signals are desired.

Multi-scale decomposition has become an invaluable tool for the processing of physiological signals. In multi-scale decomposition, a signal is decomposed into a set of signals, each characterizing information about the original signal at a different scale. A common signal processing task that multi-scale decomposition has shown to provide significant benefits is noise suppression, based on the notion that the information pertaining to the noise component would be largely characterized by certain scales that are separate from the scales characterizing the desired signal. Much of literature in multi-scale decomposition for physiological signal processing has focused on scale-space theory \cite{3-8} and wavelet transforms \cite{9-17}. Let $t$ denote time and $j$ denote scale. In scale-space theory \cite{5}, a signal $f(t)$ is decomposed into a single-parameter family of $n$ signals, denoted by $L$, with a progressive decrease in fine scale signal information between successive scales:

$$L = \{l_j(t)|0 \leq j \leq n - 1\},$$ \hspace{1cm} (1)

where $l_j(t)$ is the signal at the $j^{th}$ scale, and $l_0(t) = f(t)$. In wavelet decomposition \cite{18,19}, a signal $f(t)$ is decomposed into a set of wavelet coefficients $c_{j,k}(t)$ obtained using a wavelet transform $W$: 

\begin{align*}
L = \{l_j(t)|0 \leq j \leq n - 1\},
\end{align*}
\[ c_{j,k}(t) = W_{\psi,f}(a,b)(t) \]  \hspace{1cm} (2)

where \( \psi \) is the wavelet, \( a = 2^{-j} \) is the dyadic dilation, and \( b = k2^{-j} \) is the dyadic position. By contrast, Bayesian-based multi-scale decomposition for processing physiological signals is less explored and hence is ripe for further investigation. In this study, we investigate the feasibility of utilizing a new Bayesian-based method for multi-scale signal decomposition called Bayesian Residual Transform (BRT) for the purpose of physiological signal processing.

**Methods**

**Bayesian Residual Transform.**

A full derivation of the proposed Bayesian Residual Transform (BRT) can be described as follows. In the BRT, a signal \( f(t) \) is modeled as the summation of \( n \) stochastic processes, each characterizing signal information from the signal at increasingly coarse scales:

\[ f(t) = \sum_{i=1}^{n} r_i(t), \]  \hspace{1cm} (3)

where \( r_i = \{ r_i(t) | t \in T \} \) is a stochastic process characterizing the signal information at the \( i \)th scale following a distribution \( p(r_i(t)) \). As such, the goal of the BRT (denoted by the function \( B \)) is to decompose a signal \( f(t) \) into the set of \( n \) stochastic processes \( r_1(t), r_2(t), \ldots, r_n(t) \) given assumed distributions \( p(r_1(t)), p(r_2(t)), \ldots, p(r_n(t)) \):

\[ \{r_1(t), r_2(t), \ldots, r_n(t)\} = B(f(t)|p(r_1(t)), p(r_2(t)), \ldots, p(r_n(t))). \]  \hspace{1cm} (4)

Determining the set of stochastic processes characterizing the signal information at the different scales and whose sum is equal to \( f(t) \) (i.e., Eq. 3) is a highly challenging problem, and as such with the BRT we wish to introduce a deep cascading framework to solve this problem in a more tractable manner, where a stochastic process at a particular scale is computed based on computations performed at a previous scale. Let us first rewrite Eq. 3 as follows:

\[ f(t) = \sum_{i=2}^{n} r_i(t) + r_1(t), \]  \hspace{1cm} (5)

\[ r_1(t) = f(t) - \sum_{i=2}^{n} r_i(t), \]  \hspace{1cm} (6)

since \( f(t) = \sum_{i=1}^{n} r_i(t) \) (based on the condition established in Eq. 3),

\[ r_1(t) = \sum_{i=1}^{n} r_i(t) - \sum_{i=2}^{n} r_i(t), \]  \hspace{1cm} (7)

It can be observed from Eq. 7 that the stochastic process \( r_1(t) \) can be treated as the residual between the summation of all stochastic processes at scales \([1, n]\) and the summation of all stochastic processes at scales \([2, n]\). Taking this concept further, the stochastic process at scale \( j \) can be expressed as

\[ r_j(t) = \sum_{i=j}^{n} r_i(t) - \sum_{i=j+1}^{n} r_i(t), \]  \hspace{1cm} (8)
Hence, a stochastic process \( r_j(t) \) can be expressed in a recursive form based on knowledge of: i) the summation of all stochastic processes at scales \([j, n]\), and ii) the summation of all stochastic processes at scales \([j + 1, n]\). Given this formulation, we can then establish a deep cascading framework as follows. Let \( f_{\Sigma,j}(t) \) denote a signal representing the summation of all stochastic processes at scales \([j, n]\):

\[
f_{\Sigma,j}(t) = \sum_{i=j}^{n} r_i(t)
\]  

(9)

Given Eq. 9 and the fact that \( f(t) = f_{\Sigma,1}(t) \) (based on the condition established in Eq. 3), we can rewrite Eq. 5 as

\[
f_{\Sigma,1}(t) = f_{\Sigma,2}(t) + r_1(t).
\]  

(10)

Hence, one can treat this as an inverse problem of estimating \( f_{\Sigma,2}(t) \) given \( f_{\Sigma,1}(t) \), with the analytical solution given by the conditional expectation \( E(f_{\Sigma,2}(t)|f_{\Sigma,1}(t)) \) [20]:

\[
\hat{f}_{\Sigma,2}(t) = E(f_{\Sigma,2}(t)|f_{\Sigma,1}(t)) = \int_{\hat{f}_{\Sigma,2}(t)} f_{\Sigma,2}(t)p(f_{\Sigma,2}(t)|f_{\Sigma,1}(t))df_{\Sigma,2}(t)
\]  

(11)

where \( p(f_{\Sigma,2}(t)|f_{\Sigma,1}(t)) \) is the conditional probability of \( f_{\Sigma,2}(t) \) given \( f_{\Sigma,1}(t) \). Therefore, given \( \hat{f}_{\Sigma,2}(t) = E(f_{\Sigma,2}(t)|f_{\Sigma,1}(t)) \), one can substitute \( \hat{f}_{\Sigma,2}(t) \) for \( f_{\Sigma,2}(t) \) and plug in \( f_{\Sigma,1}(t) = \sum_{i=1}^{n} r_i(t) \) in Eq. 7 to obtain \( r_1(t) \) as:

\[
r_1(t) = f_{\Sigma,1}(t) - E(f_{\Sigma,2}(t)|f_{\Sigma,1}(t)).
\]  

(12)

Given \( \hat{f}_{\Sigma,2}(t) \) computed to obtain \( r_1(t) \), we can express \( r_2(t) \) based on Eq. 8 as:

\[
\hat{f}_{\Sigma,2}(t) = f_{\Sigma,3}(t) + r_2(t).
\]  

(13)

Rearranging the terms of Eq. 13 gives us:

\[
\hat{f}_{\Sigma,2}(t) = f_{\Sigma,3}(t) + r_2(t),
\]  

(14)

which can similarly be treated as an inverse problem of estimating \( f_{\Sigma,3}(t) \) given \( \hat{f}_{\Sigma,2}(t) \), with the analytical solution given by the conditional expectation \( E(f_{\Sigma,3}(t)|\hat{f}_{\Sigma,2}(t)) \), resulting in:

\[
r_2(t) = \hat{f}_{\Sigma,2}(t) - E(f_{\Sigma,3}(t)|\hat{f}_{\Sigma,2}(t)).
\]  

(15)

Generalizing this, \( r_j(t) \) at scale \( j \) can be obtained by

\[
r_j(t) = \hat{f}_{\Sigma,j}(t) - E(f_{\Sigma,j+1}(t)|\hat{f}_{\Sigma,j}(t)),
\]  

(16)

where

\[
E(f_{\Sigma,j+1}(t)|\hat{f}_{\Sigma,j}(t)) = \int_{f_{\Sigma,j+1}(t)} f_{\Sigma,j+1}(t)p(f_{\Sigma,j+1}(t)|\hat{f}_{\Sigma,j}(t))df_{\Sigma,j+1}(t)
\]  

(17)

with Eq. 16 conforming to the form expressed in Eq. 8. Hence, given Eq. 16, we have a deep cascading framework for the BRT where we can obtain the residual process at scale \( j \) (i.e., \( r_j(t) \)) given estimates made to obtain the residual process at scale \( j - 1 \) (i.e., \( \hat{f}_{\Sigma,j}(t) \)). Furthermore, since the residual process at scale \( j - 1 \) (i.e., \( r_{j-1}(t) \)) is not involved in the computation of the residual process at scale \( j \) (i.e., \( r_j(t) \)) (only \( \hat{f}_{\Sigma,j}(t) \) obtained from previous cascading step is), the information
In this study, to illustrate the feasibility of utilizing the BRT for processing physiological signals, SNR Analysis using ECG signals is introduced. Based on Eq. [16], the deep cascading framework for the forward Bayesian Residual Transform (BRT) is illustrated in Fig. 1a.

Due to the condition of the summation of stochastic processes at all scales being equal to signal $f(t)$ (Eq. [3]), the inverse BRT is simply the summation of all residual processes $r_1(t), r_2(t), \ldots, r_n(t)$:

$$f(t) = B^{-1}(r_1(t), r_2(t), \ldots, r_n(t)) = \sum_{i=1}^{n} r_i(t). \quad (18)$$

The inverse Bayesian Residual Transform (inverse BRT) procedure is illustrated in Fig. 1b.

**Realization of Bayesian Residual Transform.**

In this study, we implement a realization of the BRT by assuming that the residual processes at scales $[1, n-1]$ are Gaussian processes with zero mean ($r_j(t) \sim N(0, \lambda_j)$), and the $n^{th}$ residual process is a Gaussian process with non-zero mean ($r_n(t) \sim N(\mu, \lambda_n)$). Under such a scenario, we compute $E(f_{\sum,j+1}(t)|\hat{f}_{\sum,j}(t))$ based on nonparametric kernel regression [21,22] as:

$$E(f_{\sum,j+1}(t)|\hat{f}_{\sum,j}(t)) = \frac{\sum_{i \in W_t} K(\hat{f}_{\sum,j}(t) - \hat{f}_{\sum,j}(t_i)) f_{\sum,j+1}(t)}{\sum_{i \in W_t} K(\hat{f}_{\sum,j}(t) - \hat{f}_{\sum,j}(t_i))}, \quad (19)$$

where $W_t$ denotes a time window centered at time $t$ and $K$ is a kernel function. Here, we employ the following Gaussian kernel function:

$$K(\hat{f}_{\sum,j}(t) - \hat{f}_{\sum,j}(t_i)) = e^{-\frac{1}{2\lambda_j} (f_{\sum,j}(t) - f_{\sum,j}(t_i))^2} \quad (20)$$

Finally, the residual at scale $n$ (i.e., $r_n(t)$) can be set as $E(f_{\sum,n}(t)|\hat{f}_{\sum,n-1}(t))$, which is computed at the step where $r_{n-1}(t)$ is computed. By setting $r_n(t) = E(f_{\sum,n}(t)|\hat{f}_{\sum,n-1}(t))$, the condition of the summation of signal decompositions at all scales being equal to signal $f(t)$ (i.e., Eq. [3]) is satisfied.

**Noise suppression.**

In this study, we wish to illustrate the feasibility of utilizing the BRT for processing physiological signals through the task of noise suppression. As such, we establish a simple approach to noise suppression of signals using the BRT for illustrative purposes as follows. We first perform the forward BRT on the signal $f(t)$ to obtain $n$ residual signals characterizing signal information at different scales ($r_1(t), r_2(t), \ldots, r_n(t)$). At scale $j$, we estimate the noise threshold $\theta_j$ as the estimated standard deviation, as expressed by:

$$\theta_j = MAD(r_j)/0.6745, \quad (21)$$

where $MAD$ is the median absolute deviation. Based on $\theta_j$, noise thresholding is achieved to obtain noise-suppressed residual signal $r_j'(t)$ by:

$$r_j'(t) = \begin{cases} 
0 & \text{if } -\theta_j < r_j(t) < \theta_j \\
 r_j(t) & \text{otherwise}
\end{cases} \quad (22)$$

Finally, the inverse BRT (Eq. [18]) is performed on the set of $n$ noise-suppressed residual signals at the different scales ($r_1'(t), r_2'(t), \ldots, r_n'(t)$) to produce the noise-suppressed signal $f'(t)$.

**SNR Analysis using ECG signals.**

In this study, to illustrate the feasibility of utilizing the BRT for processing physiological signals,
we performed a SNR analysis using electrocardiography (ECG) signals to study the performance of the BRT for the task of noise suppression. ECG signals from the MIT-BIH Normal Sinus Rhythm Database \cite{23} were used in this study to perform the SNR analysis. This database consists of 18 ECG recordings (recorded at a sampling rate of 128 Hz) of subjects conducted at the Arrhythmia Laboratory in the Beth Israel Deaconess Medical Center. The subjects were found to have no significant arrhythmias. A total of 18 low-noise segments of 10 seconds was extracted, one from each recording, based on visual inspection to act as the baseline signals for evaluation. To study noise suppression performance at different SNR levels, each of the 18 baseline signals were contaminated by white Gaussian noise to produce noisy signals with SNR ranging from 12 dB to 2.5 dB (with 20 different noisy signals at each SNR), resulting in 3960 different signal permutations used in the analysis. For comparison purposes, wavelet denoising methods with the following shrinkage rules were also used \cite{25}: i) Stein’s Unbiased Risk (SURE), ii) Heuristic SURE (HSURE), iii) Universal (UNI), and iv) Minimax (MINIMAX). To quantitative evaluate noise suppression performance, we compute the SNR improvement as follows \cite{26}:

$$SNR_I = 10 \log \left( \frac{\sum_t (f(t) - f_b(t))^2}{\sum_t (f'(t) - f_b(t))^2} \right)$$

(23)

where $f(t)$, $f_b(t)$, and $f'(t)$ are the noisy, baseline, and noise-suppressed signals obtained using a noise suppression method, respectively.

**Implementation details.**

The BRT is implemented in MATLAB (The MathWorks, Inc.), with the nonparametric conditional expectation estimates implemented in C++ and compiled as a dynamically linked MATLAB Executable (MEX) to improve computational speed. The only free parameters of the implemented realization of the BRT are the standard deviations used to model the residual processes (e.g., $\lambda$), the number of scales $n$, and time window size, which can be adjusted by the user to find a tradeoff between noise suppression quality and computational costs. For the SNR analysis of ECG signals, $\lambda$ is set equally for all scales to the standard deviation of $f(t)$ for simplicity, $n$ is set at 6 scales, and the time window size is set to 0.1s. For this configuration, the current implemented realization of the BRT can process a 1028-sample signal in <1 second on an Intel(R) Core(TM) i5-3317U CPU at 1.70GHz CPU. For the wavelet-based methods tested (SURE, HSURE, UNI, and MINIMAX), as implemented in MATLAB (The MathWorks, Inc.), soft thresholding with the Coiflet3 mother wavelet at 6 scales and single level rescaling was used as it was found to provide superior results for ECG noise suppression \cite{27}.

**Results**

To illustrate the feasibility of utilizing the BRT for processing physiological signals, such as for the task of noise suppression, we first performed the BRT on two test signals: i) a noisy periodic test signal, and ii) a noisy piece-wise regular test signal. The multi-scale signal decomposition using the BRT on a noisy periodic test signal is shown in Fig. 2. Here, a baseline test signal (Fig. 2a) is contaminated by a zero-mean Gaussian noise process to produce a noisy signal (Fig. 2b) and then decomposed using the BRT at different scales (Figs. 2c-h). It can be observed that the noise process contaminating the signal is well characterized in the decompositions at the lower (finer) scales (scales 1 to 3), while the structural characteristics of the test signal is well characterized in the decompositions at the higher (coarser) scales (scales 4 to 6).
The multi-scale signal decomposition using the BRT on a noisy piece-wise regular test signal (generated using [24]) is shown in Fig. 3. As with the previous example, a baseline test signal (Fig. 3a) is contaminated by a zero-mean Gaussian noise process to produce a noisy signal (Fig. 3b) and then decomposed using the BRT at different scales (Figs. 3c-h). It can be observed that, as with the periodic signal example, the noise process contaminating the signal is well characterized in the decompositions at the lower (finer) scales (scales 1 to 2), while the structural characteristics of the test signal is well characterized in the decompositions at the higher (coarser) scales (scales 3 to 6). Furthermore, more noticeable here than in the periodic signal example, it can be seen that that the decomposition at each scale exhibits good signal structural localization. Therefore, given the ability of the BRT to decouple the noise process from the true signal into different scales, as illustrated in both the periodic and piece-wise regular test signals, the BRT has the potential to be useful for performing noise suppression on signals while preserving inherent signal characteristics.

In this study, to illustrate the feasibility of utilizing the BRT for processing physiological signals, we introduced a simple thresholding approach to noise suppression using the BRT for illustrative purposes (see Methods). We then performed a quantitative SNR analysis using electrocardiography (ECG) signals from the MIT-BIH Normal Sinus Rhythm Database [23] to study the performance of the BRT for the task of noise suppression, where the SNR improvement (see Methods for formulation) is assessed for a range of noisy signals generated from 18 ECG recordings, with input SNR ranging from 12 dB to 2.5 dB (with 20 different noisy signals at each SNR), resulting in 3960 different signal permutations used in the analysis. For comparison purposes, wavelet denoising methods with the following shrinkage rules were also used [25]: i) Stein’s Unbiased Risk (SURE), ii) Heuristic SURE (HSURE), iii) Universal (UNI), and iv) Minimax (MINIMAX).

A plot of the mean SNR improvement of the tested methods vs. the different input SNRs ranging from 12 dB to 2.5 dB is shown in Fig. 4a. It can be observed that the noise-suppression method using the BRT provided strong SNR improvements across all SNRs, comparable to SURE and higher than the other 3 tested methods. Typical results of noise-suppressed signals produced by the method using the BRT are shown in Fig. 4c and Fig. 4e (corresponding to two different 12 dB noisy input signals shown in Fig. 4b and Fig. 4d, respectively). Visually, it can be seen that the BRT was effectively used to produce signals with significantly reduced noise artifacts while preserving signal characteristics. Results in this study show that it is feasible to utilize the BRT for processing physiological signals for tasks such as noise suppression.

Discussion

In this study, the feasibility of employing a Bayesian-based approach to multi-scale signal decomposition introduced here as the Bayesian Residual Transform for use in the processing of physiological signals. The Bayesian Residual Transform decomposes a signal into a set of stochastic residual processes, each characterizing information from the signal at different scales. This allows information at different scales to be decoupled for the purpose of signal analysis and, for the purpose of noise suppression, allows for information pertaining to the noise process contaminating the signal to be separated from the rest of the signal characteristics. This trait is important for performing noise suppression on signals while preserving inherent signal characteristics. SNR analysis using a set of ECG signals from the MIT-BIH Normal Sinus Rhythm Database at different noise levels demonstrated that it is feasible to utilize the BRT for processing physiological signals for tasks such as noise suppression.

Given the promising results, we aim in the future to investigate alternative adaptive thresholding schemes for the task of noise suppression in physiological signals characterized by nonstationary noise processes, so that one can better adapt to the nonstationary noise statistics embedded at different
scales. Moving beyond low-level signal processing tasks such as noise suppression, we aim with our future work to investigate and devise methods for multi-scale analysis of a signal using the Bayesian Residual Transform, which could in turn lead to improved features for signal classification. Finally, we aim to investigate the extension and generalization of the Bayesian Residual Transform for dealing with high-dimensional physiological signals such as vectorcardiographs (VCG) \[28\], as well as dealing with high-dimensional medical imaging signals such as diffusion weighted magnetic resonance imaging (DWI), dynamic contrast enhanced MRI (DCE-MRI) \[30\], and correlated diffusion imaging \[31\].

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Author contributions

A.W. conceived and designed the BRT method. A.W. and X.W. worked on formulation and derivation of solution for the BRT method. A.W. performed the data processing. All authors contributed to writing the paper and to the editing of the paper.

Competing Financial Interests

The authors declare no competing financial interests.
Figure 1: Bayesian Residual Transform framework. (a) forward BRT. (b) inverse BRT.
Figure 2: Example of multi-scale signal decomposition using the BRT. (a) Baseline periodic test signal. (b) Noisy input signal with zero-mean Gaussian noise. (c)-(h) Signal decompositions using the BRT at different scales. It can be observed that the noise process contaminating the test signal is well characterized in the decompositions at the lower (finer) scales (scales 1 to 3), while the structural characteristics of the test signal is well characterized in the decompositions at the higher (coarser) scales (scales 4 to 6).
Figure 3: Example of multi-scale signal decomposition using the BRT. (a) Baseline piece-wise regular test signal. (b) Noisy input signal with zero-mean Gaussian noise. (c)-(h) Signal decompositions using the BRT at different scales. It can be observed that, as with the periodic signal example, the noise process contaminating the signal is well characterized in the decompositions at the lower (finer) scales (scales 1 to 2), while the structural characteristics of the test signal is well characterized in the decompositions at the higher (coarser) scales (scales 3 to 6). Furthermore, more noticeable here than in the periodic example, it can be seen that that the decomposition at each scale exhibits good signal structural localization.
Figure 4: Application of the BRT on ECG signals. (a) A plot of the mean SNR improvement vs. the different input SNRs ranging from 12 dB to 2.5 dB for the MIT-BIH Normal Sinus Rhythm Database for the tested methods. Noise-suppression method using the BRT provided strong SNR improvements across all SNRs, with performance comparable to SURE and higher than the other 3 tested methods. (b) Noisy input signal with SNR=12 dB, and (c) noise-suppressed results using BRT for b. (d) Another noisy input signal with SNR=12 dB, and (e) noise-suppressed results using BRT for d. The results produced using the BRT has significantly reduced noise artifacts while the signal characteristics are preserved.