Computer Simulation of Non-Uniform Multiple Slip in Face Centered Cubic Bicrystals*

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Non-uniform multiple slip near a grain boundary plane in face centered cubic bicrystals was examined analytically. Mathematical models for movement and interaction of dislocations were introduced to have a quantitative constitutive relation describing plastic deformation of face centered cubic crystals. Then, a new computer code for finite element structure analyses based on the relation was developed. By applying the code to deformation analyses of isoaxial bicrystals of the nominal strain incompatible type, the formation process and the structure of shear strain in the multiple slip region were examined. Results are summarized as follows:

(1) Very early in the activation of secondary slip systems, shear strain on the systems is on the order of $10^{-7}$ and it is about 1/100 of that on the primary system.

(2) At the deformation stage when the nominal strain is about 12 times as large as that at the elastic limit, the multiple slip region consists of a rather uniform distribution of shear strain on the primary slip system and non-uniform shear strain on the secondary systems. The latter strain ranges from $10^{-7}$ to a value comparable to that on the primary system.

(3) When the interaction intensities between moving and forest dislocations are doubled, the propagation rate of the multiple slip region and shear strain on the secondary systems are reduced to about 1/1.6 and 1/2, respectively.

(4) The propagation rate of double slip is about 1/30 of the rate of single slip. This ratio does not depend significantly on the interaction intensity of dislocations.

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I. Introduction

Strain hardening is one of the most important phenomena when metal crystals are deformed. Experimental observations on deformed single- or polycrystals have shown that the deformation consists of non-uniform slip on many slip systems. Therefore, non-uniform multiple slip must be studied in detail to understand strain hardening. A number of important points remain for further investigation, such as how stresses are distributed in crystal grains or how much strain occurs on each slip system during the process of non-uniform multiple slip. These points are difficult to examine experimentally; they must be subjected to theoretical or numerical analysis.

A theoretical basis for slip deformation analysis of crystals was first described by Taylor and the framework for crystal plasticity analyses was provided by Hill. But the constitutive relation obtained by Hill does not refer to physical entities of the crystal plasticity such as strain hardening of each slip system or mutual interaction of slip systems. In the present study, we describe strain hardening and interaction of slip systems of face centered cubic crystals through mathematical models of dislocation behaviour in deforming crystals. By basing the models in the framework of crystal plasticity analyses, a technique is derived to analyse the non-uniform multiple slip deformation of crystals quantitatively. The obtained constitutive relation is verified by comparing the experimental and calculated stress-strain relationship of single crystals under a simple applied load. The results of the finite
element analyses of the non-uniform multiple slip of bicrystals are given and the shear strain on slip systems is discussed.

II. The Constitutive Relation and Models for Behaviour of Dislocations

First, we assume that the slip systems of face centered cubic (abbreviated to f.c.c. hereafter) crystals are {111}, <110> and their activation is given by Schmid’s law. Denoting components of the stress tensors and the critical resolved shear stress for the slip system \( (n) \) as \( \sigma_{ij} \) and \( \theta^{(n)} \) respectively, the Schmid conditions for active slip systems are given as follows:

\[
P(n)_{ij} \sigma_{ij} = \theta^{(n)} \tag{1}
\]

\[
P(n)_{ij} \sigma_{ij} = \theta^{(n)}. \tag{2}
\]

Subscripts such as \( i \) and \( j \) in eqs. (1) and (2) follow the summation convention hereafter and the superscript dot (‘.’) is used to indicate the increment of the quantity. Tensor \( P^{(n)}_{ij} \) is defined by

\[
P^{(n)}_{ij} = \frac{1}{2} \left( v_{i}^{(n)} b_{j}^{(n)} + v_{j}^{(n)} b_{i}^{(n)} \right). \tag{3}
\]

It is the outward normal of the yield hyperplane in the stress space for the slip system \( (n) \) whose slip plane normal and slip direction are \( v \) and \( b \), respectively. The strain increment \( \dot{\epsilon}_{ij} \) is a sum of the elastic component \( \dot{\epsilon}_{ij}^{e} \) and the plastic component \( \dot{\epsilon}_{ij}^{p} \).

\[
\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^{e} + \dot{\epsilon}_{ij}^{p}. \tag{4}
\]

Each component of the strain increment is related to stress increment and shear strain increment \( \dot{\gamma}^{(n)} \).

\[
\dot{\epsilon}_{ij}^{e} = S_{ijkl}^{e} \dot{\sigma}_{kl}
\]

\[
\dot{\epsilon}_{ij}^{p} = \sum_{n} \dot{\gamma}^{(n)} P_{ij}^{(n)}
\]

\( S_{ijkl}^{e} \) is the elastic compliance tensor. Equation (5) is the elastic constitutive relation in the incremental form.

If we assume

\[
\dot{\theta}^{(m)} = \sum_{m} \dot{h}^{(nm)} \dot{\gamma}^{(m)}
\]

as a strain hardening rule, then the following elasto-plastic constitutive relation is obtained:

\[
\dot{\epsilon}_{ij} = S_{ijkl}^{e} \dot{\sigma}_{kl}
\]

\[
S_{ijkl}^{e} = S_{ijkl}^{e} + \sum_{n} \left( \frac{1}{h^{(nm)}} \right) \dot{P}_{ij}^{(n)} P_{kl}^{(m)}
\]

where \( n \) and \( m \) are active systems.

The critical resolved shear stress for the \( (n) \) slip system of a crystal of a certain deformation history is assumed to be a function of shear strain on twelve slip systems as follows:

\[
\theta^{(n)} = \theta_{0} + \sum_{m=1}^{12} \Omega^{(nm)} F(\dot{\gamma}^{(m)}).
\]

Here, \( \theta_{0} \) is a constant which represents the Peierls force or dragging force on dislocations due to solute atoms and so on. \( \Omega^{(nm)} F(\dot{\gamma}^{(m)}) \) represents the contribution of the slip system \( (m) \) to \( \theta^{(n)} \), where \( F(\dot{\gamma}^{(m)}) \) is a function of the shear strain on the slip system \( (m) \) and \( \Omega^{(nm)} \) is a constant which does not depend on the deformation history, but is defined by a geometrical relationship between the slip systems \( (m) \) and \( (n) \). From the viewpoint of a dislocation, the function \( F(\dot{\gamma}^{(m)}) \) is related to the density of the accumulated dislocations on the slip system \( (m) \), and \( \Omega^{(nm)} \) describes the intensity of the dislocation interaction between moving ones on the \( (n) \) slip system and accumulated ones on the \( (m) \) slip system.

Intensity of dislocation interactions must be evaluated according to types of the interaction. Interactions of dislocations in the f.c.c. crystals are classified into six types. Figure 1(a) shows the classifications on the Thompson tetrahedra. That is, we take the interaction intensity between dislocations on the same slip system as a unit of the interaction intensity and according to the other types of dislocation interactions, the interaction intensities are denoted as \( R_{1} \), \( R_{2} \), \( R_{3} \), \( R'_{3} \) and \( R_{4} \) (Fig. 1(a)). For example, the interaction intensity between dislocations on the primary and a co-planar system is \( R_{1} \) times larger than the interaction between dislocations on the same slip system. When the interaction of dislocations produces the Lomer-Cottrel (L-C) sessile dislocations, the interaction intensity is \( R_{1} \). Interaction intensity \( R_{1} \), \( R_{2} \), \( R_{3} \), \( R'_{3} \) or \( R_{4} \) is assigned to component \( \Omega^{(nm)} \) according to the combination of slip systems \( (n) \) and \( (m) \). Figure 1(b) shows the matrix \( \Omega^{(nm)} \) in which slip systems are denoted by the Schmid-Boas notation in place of \( (n) \) or
The relationship between the intensities is connected to the latent hardening characteristics of the slip systems. If we assume the isotropic hardening postulate whereby interaction intensities of dislocations are the same for all combinations of slip systems, the relationship between the intensities is:

\[ R_1 = R_2 = R_3 = R_4 = 1. \]  
(10a)

A more realistic relationship can be assumed if we refer to experimental results on the latent hardening of f.c.c. crystals. Results by Jackson and Basinski\(^\text{(7)}\) suggest that

\[ 1 \leq R_1 \leq R_2 \leq R_3, \quad R_3 = R_4 = R_1. \]  
(10b)

Absolute values for \( R_1 \) to \( R_4 \) are not clearly known yet, but in this paper using reported values of the latent hardening ratio\(^\text{6,7}\) we assume values between 1.0 to 2.0.

From eqs. (7) and (9) the hardening coefficient \( h^{(nm)} \) is given as follows:

\[ h^{(nm)} = \Omega^{(nm)} \frac{\partial F(\gamma^{(m)})}{\partial \gamma^{(m)}}. \]  
(11)

And if the shear strain is a function of the density of accumulated dislocations \( \rho^{(m)} \),

\[ \gamma^{(m)} = f(\rho^{(m)}) \]  
(12)

we can write the function \( F \) of eq. (9) from the well known relationship of dislocation theory, as

\[ F(\gamma^{(m)}) = F(f(\rho^{(m)})) = \mu b \sqrt{\rho^{(m)}}. \]  
(13)

Here, \( \mu \) is the shear modulus, \( b \) is the magnitude of the Burgers vector and \( a \) is a dimensionless number which is about 0.1\(^\text{8}\).

Using eq. (12), eq. (11) is written as

\[ h^{(nm)} = \Omega^{(nm)} \frac{\partial F(\rho^{(m)})}{\partial \rho^{(m)}} \frac{\partial \rho^{(m)}}{\partial \gamma^{(m)}}. \]  
(14)

The function \( f \) is determined by models for movement of dislocations. If we suppose that the plastic slip occurs by a free expansion of dislocation loops and the accumulated dislocation loops are rectangular shaped with an aspect ratio \( 1: \alpha \) as illustrated in Fig. 2 then

\[ \gamma^{(m)} = \frac{2\alpha}{(1+\alpha)^2} bL \rho^{(m)}. \]  
(15)

Dislocation loops are assumed here to pile up on the slip system at an average distance \( L \) from the dislocation source.

The mean free flight length \( L \) of dislocation loops also depends on the deformation history. Seeger et al.\(^\text{9}\) proposed a model for the mean free flight distance of dislocations in single crystals under tensile deformation. In their model, the mean free flight distance is kept constant during deformation stage I and with the onset of deformation stage II it begins to decrease in a manner inversely proportional to the shear strain on the slip system. We can
not use this model for the multiple slip analyses in its original form because it assumes a single slip. Some modifications are needed (Fig. 3). At first, the mean free flight distance is assumed to start decreasing with the onset of multiple slip. If the shear strain amounts to a prescribed value (known as the limit of easy slip and denoted by \( \gamma^* \)) before multiple slip takes place, \( L \) also begins to decrease, just as in the original model. Then a second modification is necessary; when \( L \) decreases, its function is inversely proportional to \( \sum \gamma^{(m)} \). That is, for the case of single slip,

\[
L = \begin{cases} 
L_0 & ; \gamma^{(m)} < \gamma^* \\
\Lambda & \gamma^{(m)} > \gamma^* 
\end{cases}
\]

assuming that \((m)\) is the active system. When more than one slip system operates, the mean free flight distance is given by

\[
L = \frac{\Lambda}{\sum_m \gamma^{(m)} - (\gamma^D - \Lambda / L_0)}.
\]

Here, \( \gamma^D \) is the shear strain when the multiple slip begins and \( \Lambda \) is a constant.

If eq. (15) is integrated with an initial dislocation density (which is assumed to be known) for an integral constant, the current dislocation density \( \rho \) on each slip system is obtained. With \( \rho \) and \( L \) determined by eqs. (15), (16) and (17), eqs. (13) and (15) are substituted into eq. (14) to evaluate the hardening coefficient. Constitutive relation (8) is quantitatively determined by the hardening coefficient. It is applied to structure analysis of non-uniform multiple slip of crystals.

III. Results and Discussion

1. Verification of the constitutive relation

Some parameters are used to describe the models for dislocation behaviour. First we examine the effect of these parameters on results of deformation analyses while calculating the stress-strain curves for tensile deformation of single crystals\(^\dagger\) at single and double slip orientations. Then comparisons are made with experimental results.

Figure 4 shows calculated stress-strain curves with four values for the initial mean free flight distance \( L_0 \). The elastic compliance data for copper\(^{(10)}\) are used in the calculation. Parameters are given in the figure caption. The graph shows that the larger \( L_0 \) is, the smaller the strain hardening ratio in deformation stage I is. The experimental points are tensile deformation results of pure copper at 4 K\(^{(11)}\). There is good agreement between calculated and experimental results when \( L_0 \) is 2000 \( \mu \)m. On the other hand, the initial mean free flight distance has been measured experimentally: reported values for \( L_0 \) in pure copper crystals are 1000 \( \mu \)m\(^{(12)}\), 1400 \( \mu \)m\(^{(13)}\), 2000 \( \mu \)m\(^{(14)}\) or 5000 \( \mu \)m\(^{(15)}\). These values are just or very close to the value 2000 \( \mu \)m with which the calculated stress-strain relationship agrees well with experimental one.

Figure 5 gives calculated stress-strain curves

\(^\dagger\) If we assume a uniform slip in the single crystal specimen, the stress distribution is uniform too. In that case the structure analysis technique does not have to be utilized to describe the deformation. We can repeat the procedure that a stress increment (increment of applied load divided by the cross-sectional area of the specimen) is given to the right hand side of eq. (8a) and the strain increment is calculated accordingly.
2. Analyses of non-uniform multiple slip in bicrystals

Geometry of the employed bicrystal specimen and coordinate system are shown in Fig. 6(a). Aspect ratio of the specimen is 1:3:1. The grain boundary plane lies on \( y = 0 \) between grain 1 in \( y < 0 \) and grain 2 in \( y > 0 \). Crystal orientation of grains 1 and 2 is shown in Fig. 6(b). That is the bicrystal is iso-axial and the tensile axis \((y\)-axis\) of both grains have the same orientation. Relative rotation about the tensile axis of the component grains is \( 90^\circ \).

Arrangement of primary slip systems in both grains is illustrated in Fig. 6(a) too. As shown in this figure, the slip plane normal and slip direction do not have any component for the \( x \) or \( z \) direction in grains 1 or 2, respectively. Therefore, strain components induced by the activation of primary slip systems are \( \varepsilon_{yy}, \varepsilon_{zz} \) and \( \varepsilon_{yz} \) for grain 1 and \( \varepsilon_{xx}, \varepsilon_{yy} \) and \( \varepsilon_{xy} \) for grain 2. On the other hand, according to the compatibility condition\(^{(16)}\) \( \varepsilon_{xx}, \varepsilon_{zz} \) and \( \varepsilon_{zx} \) must be continuous on the grain boundary plane. But as \( \varepsilon_{zx} \) does not occur due to an operation of the primary system, as mentioned above, the employed bicrystal is a pure normal strain incompatible type for the slip deformation on the primary slip system.

The elastic anisotropy effect on the compatibility on grain boundary planes has already been widely studied\(^{(16)-(19)}\). So the effect of elastic anisotropy is not considered in the present paper. Elastic compliance data used in this study are \( S_{11} = 10 \), \( S_{12} = -3 \) and \( S_{44} = 26 \times 10^{-12} \).
m²/N). Here \( S_{11}, \) etc. are the elastic compliance values corresponding to the crystal cubic axis. In this combination of elastic compliance, the elastic anisotropy ratio \( (2(S_{11} - S_{12}) / S_{44}) \) is unity and the elastic anisotropy effect does not occur.

With regard to parameters for the movement of dislocations, we use the same data as used in the preceding section, except for the dislocation interaction intensities \( R_1-R_4. \) These are given by two different sets of data. The first set of data is given by eq. (10a) in which the crystal grains have isotropic hardening characteristics. We call these bicrystals of the isotropic hardening type or simply, isotropic hardening bicrystals. The second set of data is \( R_1=1.01, R_2=2.022, R_3=2.024 \) and \( R'_3=R_4=R_3; \) intensity of the interaction with forest dislocations is about twice as large as the interaction with dislocations on the same slip plane. This set of data satisfies eq. (10b) and gives the crystal grains latent hardening characteristics. So we call these bicrystals of the latent hardening type or simply, latent hardening bicrystals.

Deformation of the specimen is analysed by the finite element method which is constructed on the base of eq. (8). The specimen is divided into 450 (5\( \times \)18\( \times \)5) elements. We use combined elements of eight nodes. On the grain boundary plane, displacement is continuous. While strictly speaking traction is not, in the sense of numerical analyses it is taken as continuous too. A compressive load is applied in such a manner that uniform relative displacement in the \( y \) direction is given between two surfaces normal to the \( y \) axis. To avoid the constraint effect of grip rotation, no constraint, except the relative compressive displacement, is applied on the loading ends. The deformation process during the increase of the applied load is analysed by the incremental technique.

Figure 7 exhibits the calculated shear strain

\[ \text{Shear strain} \]

\[ \times 10^{-5} \]

11.97
5.75
2.58
1.05
0.36
0.08
0

Fig. 7 Slip on the primary slip system and localized slip on secondary (critical) system in the vicinity of the grain boundary plane. Distribution on the \( y-z \) (\( x=0 \)) plane.

\[ \text{Nominal strain} \]

1.10
1.73
2.17
2.84
4.44
7.80
\( \times 10^{-5} \)

\[^\dagger\] Generally, this equation is non-symmetric \( (S_{ijkl} \neq S_{klji}) \) since eq. (14) shows \( h^{(mm)} \neq h^{(mm)}. \) The usual finite element codes for structure analyses assume symmetricity and they cannot be used for the crystal plasticity analyses.
distribution on the primary (B4) and on the critical (A3) slip systems. The isotropic hardening postulate (eq. 10(a)) is used in this analysis. The figure shows the two-dimensional aspect of the strain distribution on the $y-z$ ($x=0$) cross-section. The nominal strain values which are given at the bottom of the figure are the relative (compressive) displacement divided by the original distance between the loaded ends.

As the deformation proceeds, slip on the primary system spreads from the top and the bottom ends to the grain boundary plane, while the slip on the critical system starts at an edge of the grain boundary plane and spreads out into the grains. When examined in detail, analyses results show that the first activation of the critical system takes place near points A, B, C and D (shown in Fig. 6(a)). Then, they spread along lines AB, BC, etc. and after that they proceed into the grains. The stress distribution just before activation of the critical system which occurs when the nominal strain is equal to $1.1 \times 10^{-5}$, is described as follows. That is, near the grain boundary plane normal stresses are generated. In grains 1 and 2, the stresses are $\sigma_{xx}$ and $\sigma_{zz}$, respectively. These stresses have positive values and range from 6–10% of the nominal stress (which is the applied load to generate the relative displacement divided by the original cross-sectional area). Along lines AB and CD, shear stress $\sigma_{yz}$ occurs locally in grain 1 and at the same time along BC and AD, $\sigma_{xy}$ occurs in grain 2. The shear stress distribution has peaks at A, B, C and D. From these we can tell that the activation of the secondary slip systems is connected to these shear stresses which occur along the intersection of the grain boundary plane and free surfaces.

In the grain, stresses other than $\sigma_{yy}$ rapidly vanish. This feature of the stress distribution is very similar to that of elastic stress distribution in bicrystals of the elastically nominal strain incompatible type(17). This fact suggests that distribution characteristics of stresses due to non-uniform deformation do not depend on the cause of the non-uniform deformation, i.e. changes in the elastic property or discontinuous slip.

Figure 8(a) and (b) illustrate spacial frequency of shear strain on the active systems. The ordinates of the histograms are the shear strain on slip systems and the abscissas give volume fraction of regions where the magnitude of shear strain lies within a certain range. For example, the volume of the region where shear strain $\gamma^{(B4)}$ on the B4 (primary)

\[\text{Fig. 8 Histograms of shear strain on slip systems when the dislocation interaction is isotropic. (a) At an early stage of activation of the secondary system (nominal strain}=1.73\times10^{-5}). \text{ (b) When the nominal strain is about } 7.80\times10^{-5}.\]

\[\text{The volume fraction is evaluated for each slip system as the volume of a region where the slip system is active divided by the volume of the specimen. Therefore the sum of the volume fraction can be greater than 100% in the case of multiple slip. In the figure, the volume fraction is rounded off to a whole percent.}\]
slip system satisfies $-4.75 < \log (g^{(B4)}) < -4.5$ is about 62% of the specimen volume when the nominal strain of the specimen is $1.73 \times 10^{-5}$ (Fig. 8(a)).

As shown in Fig. 8(a), the shear strains on the secondary slip system (which is the critical slip system A3 in this case) are mostly on the order of $10^{-7}$ and this is about one hundred times smaller than the shear strain on the primary system. Further deformation makes the magnitude of the shear strain on secondary slip systems vary widely according to the site of activation and the type of secondary system. Figure 8(b) gives an example of the frequency of shear strain at a deformation stage when the nominal strain of the specimen is $7.8 \times 10^{-5}$. On secondary slip systems of A2, A3 (critical systems) or C1 (conjugate system) the magnitude of the strain lies between $10^{-7}$ and $10^{-4.25}$ without any steep peak. This appearance of the shear strain frequency histogram suggests that the spatial distribution of the shear strain is very non-uniform. Development of this non-uniform strain distribution can be attributed to a mechanism in which a site where the activation of the secondary slip system has taken place early, tends to have a continuously active slip system and a large shear strain will be generated. On the other hand, on a newly activated slip system or near a propagating front of the multiple slip region the shear strain is at an order of $10^{-7}$ which is almost the same as that generated by the first activation of the secondary slip system.

The spatial distribution of shear strain on the secondary slip system has peaks in the vicinity of grain boundary plane edges which we can see in Fig. 7. In the grain, the shear strain rapidly vanishes. These are common features for almost all secondary systems. By contrast, on the primary slip system, the shear strain near the grain boundary is small compared to that in the grain, however, a ratio of the maximum to the minimum of the strain is about ten. So the shear strain on the primary system can be said to be rather uniform. At a place very near to the grain boundary plane, the magnitude of the shear strain on the primary system is shown in Figs. 7 and 8(b) to be almost the same as that on secondary systems at a deformation stage when multiple slip has been developed to some degree. Therefore, the initial structure of the multiple slip region in bicrystals of this type is roughly described as a non-uniform distribution of shear strain on secondary slip systems being superimposed on a rather uniform distribution of shear strain on the primary system. The shear strain on secondary systems ranges from $10^{-7}$ to a value like that on the primary system. The extent of the multiple slip region and the magnitude of the shear strain there depend on the interaction intensity of dislocations too. Figure 9 shows the shear strain frequency for a bicrystal specimen of the latent hardening type which is deformed until the nominal strain is about $7.8 \times 10^{-5}$. Comparing this with Fig. 8(b), we recognize that the qualitative aspects do not change remarkably, but shear strain on the secondary slip systems is reduced to almost a half of the one in the bicrystals of the isotropic hardening type. The extent of the active region of secondary systems is smaller too. Figure 10 shows the volume fractions of single and double slip regions for isotropic and latent hardening bicrystals. For example, at a deformation stage when the nominal strain is $1 \times 10^{-4}$, 60% of the isotropic hardening
specimen is covered by single slip and 25% of the same specimen is covered by double slip. In the remaining 15%, three or more slip systems are active (this is omitted from the figure). Broken lines exhibit development of single and double slip regions in the bicrystal of the latent hardening type. This figure demonstrates that at a deformation stage of the same nominal strain, the volume fraction of the double slip region in the latent hardening type is about a half of the volume fraction in the isotropic hardening bicrystal.

From the slopes of these curves, we can compare the propagation rate of the single and double slip regions. That is, the propagation rates of both single and double slips in the isotropic hardening bicrystal are about 1.6 times larger than those in the latent hardening bicrystal and this indicates that the rate depends on the interaction intensity of dislocations, especially the interaction intensity with forest dislocations. The propagation rate of the double slip region is about 1/30 of the propagation rate of single slip region, in both the latent hardening and the isotropic hardening bicrystals.

IV. Summary

Mathematical models for dislocation behaviour were proposed for a constitutive relation describing plastic deformation of face centered cubic crystals. A new computer code was developed to analyse the non-uniform multiple slip of crystals quantitatively. Using the computer code, deformation of single crystals by uniform single and double slip was analysed. The calculated curves of the stress-strain relationship were ascertained as agreeing well with experimental results. Applying the computer code to isoaxial bicrystals of the nominal strain incompatible type, the formation process and the structure of shear strain in the multiple slip region were discussed. The effects of dislocation interaction intensity on the structure of shear strain were also examined. The main points are as follows.

1. In bicrystals of the isotropic hardening type, where mutual interactions between dislocations did not depend on the combination of dislocations, magnitude of shear strain on secondary slip systems was on the order of $10^{-7}$ at the onset of their activation and this was about 1/100 of the shear strain on the primary slip system.

2. When the nominal strain of the specimen was $7.8 \times 10^{-5}$, the structure of shear strain in the multiple slip region was roughly described as a non-uniform distribution of shear strain on the secondary slip system which was superimposed on a rather uniform distribution of shear strain on the primary system. Magnitude of shear strain on secondary systems ranged from $10^{-7}$ to a value comparable to that on the primary system.

3. Interaction intensity of moving dislocations with forest ones had a remarkable influence on the extent of the multiple slip region and on the magnitude of shear strain in that region. That is, when the interaction intensity was doubled, shear strain on the secondary slip system and the rate of slip propagation were reduced to a half and 1/1.6, respectively.

4. The propagation rate of double slip was about 1/30 of the rate of single slip. This ratio did not depend remarkably on the interaction intensity of dislocations.

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