Covers of Query Results

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Abstract

We introduce succinct lossless representations of query results called covers that are subsets of these results, yet allow for constant-delay enumeration of all result tuples.

We first study covers whose structures are given by hypertree decompositions of join queries. For any join query, we give asymptotically tight size bounds for the covers of the query results and show that such covers can be computed in worst-case optimal time (up to a logarithmic factor in the database size). For acyclic join queries, we can compute covers compositionally from covers for subqueries using plans with a new operator called cover-join.

We then generalise covers from join queries to functional aggregate queries, which express a host of computational problems, e.g., evaluation of queries with joins and aggregates, in-database optimization, matrix chain multiplication, and inference in probabilistic graphical models.

1 Introduction

This paper introduces succinct lossless representations of query results called covers. Given a database and a join query or, more generally, a functional aggregate query (FAQ) [17], a cover is a subset of the query result that, together with a (hyper)tree decomposition of the query [13], recovers the query result. Covers enjoy desirable properties.

First, they can be more succinct than the explicit listing representation of the query result. For a join query $Q$, database $D$, and a tree decomposition $T$ of $Q$ with fractional hypertree width $w$, a cover over $T$ has size $O(|D|^w)$. In contrast, there are arbitrarily large databases for which the listing representation of the query result has size $\Omega(|D|^{\rho^*})$, where $\rho^*$ is the fractional edge cover number of $Q$ [4]. The gap between the fractional hypertree width and the fractional edge cover number can be as large as the number of relation symbols in $Q$. For an FAQ $\varphi$, any cover of its result can be computed in time $O(|D|^w \log |D|)$, where $w$ is the FAQ-width [17] of $\varphi$. FAQs can express aggregates over database joins [6], in-database optimization [24, 2], matrix chain multiplication, and inference in probabilistic graphical models.

Second, the tuples in the query result can be enumerated from one of its covers with linearithmic time pre-computation and constant delay. This is not the case for the simple representation defined by the pair of database and join query (unless $W[1]=\text{FPT}$) [25]. The benefits of covers over the latter representation seem less apparent for acyclic queries, for which both representations share the same linear-size bound and desirable enumeration complexity [5]. For acyclic joins, the question thus becomes why to succinctly represent a query result by one relation instead of the pair of a set of relations, i.e., the input database, and the query. We next highlight three practical benefits. Covers readily provide tuples in the query result without the need to compute the join. This improves cache locality for subsequent operations, e.g., aggregates, since we only need to read in tuple by tuple from the cover instead of reading tuples from different relations and then joining them. Similarly, covers provide access locality for disk operations since tuples from the cover are stored on the same disk page, whereas tuples from different relations are stored on different pages. Furthermore, covers are samples of the query result that disregard the uninformative yet exhaustive pairings...
brought by Cartesian products. In exploratory data analysis, such pairings may be overwhelming to the user as they lead to blowup in size and require the user to discover the Cartesian products in the listed result. Presenting the user with several relations and the query would rely on the user to figure out possible tuples in the query result, which may be undesirable. A cover, in contrast, is a compact relation that absorbs the user from ad-hoc join computation of several relations. Finally, subsequent processing may require a single relation as input, as it is the case for machine learning over joins [24]. Indeed, instead of learning regression models over the result of a join we can instead learn them over one of its covers with lower complexity.

Third, covers use the standard listing representation. Prior work introduced lossless representations of query results called factorized databases that achieve the same succinctness as covers, yet they are directed acyclic graphs that represent the query result as circuits whose nodes are data values or the relational operators Cartesian product and union [23]. The graph representation makes difficult their adoption as a data representation model by mainstream database systems that rely on relational storage (factorized computation is however used in relational systems [2]). A relational alternative to factorized databases, as metamorphosed in covers, can prove useful in a variety of settings. The intermediate results in query plans can be represented as covers. In distributed query plans, covers can encode succinctly the otherwise expensive intermediate query results that are communicated among servers in each round [26].

The contributions of this paper are as follows:

- Section 3 introduces covers of join query results over tree decompositions and their correspondence to minimal edge covers in the hypergraphs of the query results. We also give tight size bounds for covers and show that the tuples in the query result can be enumerated from any cover with linearithmic-time pre-computation and constant delay.

- Given a database and a join query, covers of its result can be computed in worst-case optimal time (modulo a log factor). Section 4 focuses on the compositionality of cover computation for acyclic join queries. We introduce cover-join plans to compute covers in time linearithmic in their sizes. A cover-join plan is a binary plan that follows the structure of a tree decomposition and computes covers over that decomposition. It uses a new cover-join operator that computes covers of the join of two input relations or covers for subqueries. We show that different plans may lead to different sets of covers and that there are covers that cannot be obtained using binary plans.

- Section 5 generalises our notion of covers from joins to functional aggregate queries by representing succinctly both the tuples and their associated aggregates in the query result.

Proofs and further details can be found in Appendix.

Related work. There are three strands of work directly related to our work: notions of cores in databases and graph theory; succinct representations of query results; and normal forms for relational data.

The notions of cores of graphs, queries, and universal solutions to data exchange problems revolve around smaller yet lossless representations that are homomorphically minimal subgraphs [16], subqueries [8], and universal solutions [11], respectively. A further application of graph cores is in the context of the Semantic Web, where cores of RDF graphs are used to obtain minimal representations and normal forms of such graphs [15]. Our notion of covers is different. Covers rely on tree decompositions of the query to achieve succinctness, and they only become lossless in conjunction with a tree decomposition. If we ignore the decomposition, the covers become lossy as they are subsets of the result. Whereas in data exchange all universal solutions have the same core (up to isomorphism), the result of a query may have exponentially many incomparable covers. While not a defining component of cores in data exchange, tree decompositions can help to derive improved algorithms for computing the core of a relational instance with labeled nulls under different classes of dependencies [12].

Covers are relational encodings of d-representations, a lossless graph-based factorization of the query result [23]. The structure of d-representations is given by variable orders called d-trees, which are an alternative syntax for tree decompositions. Whereas d-representations are lossless on their own, covers need the tree decomposition to fill in the missing tuples. Tree decompositions are the (data-independent) price to pay for achieving the (data-dependent) succinctness of factorized representations using the listing
representation. Both d-representations and covers achieve succinctness by avoiding the materialization of the Cartesian products of relations. Whereas the former encode the products symbolically and losslessly, the covers only keep a minimal subset of the product that is enough to reconstruct it entirely.

The goal of database design is to avoid redundancy in input data and existing normal forms achieve this by decomposing one relation into several relations guided by functional and join dependencies. Covers also exploit the join dependencies in the query result to avoid redundancy, but they do not decompose the result back into the (now globally consistent) input database. Like factorized representations, covers are a normal form for relations representing query results. From a cover of a join result over a tree decomposition, we can obtain a decomposition of the join result in project-join normal form (5NF) by taking one projection of the cover onto the attributes of each bag of the tree decomposition.

2 Preliminaries

Databases. We assume an ordered domain of data values. A relation schema is a finite set of attributes. For an attribute A, we denote by dom(A) its domain. A database schema is a finite set of relation symbols. A tuple t over a relation schema S is a mapping from the attributes in S to data values. A relation over a relation schema S is a finite set of tuples over S. A database D over a database schema S contains a relation R for each relation symbol R in S. For a database D, relation R, relation symbol R, and tuple t, we use S(D), S(R), S(R) and S(t), respectively, to refer to their schemas. The tuples t1, . . . , tn are joinable if πR(1) t1 = πR(2) t2 for all i, j ∈ [n] and S(1) t1 = S(2) t2. The size of a relation R (database D), denoted by |R| (|D|), is the number of its tuples (the sum of the sizes of its relations).

Join Queries. We consider (equi-)join queries, aka full conjunctive queries, in their general form σψ(R1 × . . . × Rn), where each Ri is a relation symbol and ψ is a conjunction of equalities of the form A1 = A2 with attributes A1 and A2. The transitive closure ψ+ of ψ under the equality on attributes defines the attribute equivalence classes: The equivalence class A of an attribute A is the set consisting of A and of all attributes equal to A in ψ+. We assume here that the equalities in ψ+ on attributes in the same relation are pre-computed (in one pass over each relation) and only focus on equalities on attributes from different relations. For a set S of attributes, S+ denotes the set of attributes transitively equivalent to those in S. The size |Q| of Q is the number of its relation symbols. The schema S(Q) = {Ri}i∈[n] of Q is the set of the relation symbols in Q. The set att(Q) = ∪i∈[n] S(Ri) of attributes of Q is the union of the schemas of its relation symbols. For a query Q, a database is globally consistent if there are no (dangling) tuples that do not contribute to Q’s result.

We require that all relation symbols in the query as well as all attributes occurring in the schemas of the relation symbols are distinct. To capture self-joins, we assume wlog that mappings of relation symbols to database relations are given together with the query and for each relation symbol there is a bijection between its attributes and the attributes of the corresponding database relation. We thus consider self-joins, but avoid the explicit use of aliases and renaming operators in the queries. Proposition 4 gives a transformation of equi-join queries into natural join queries having a one-to-one mapping between relation symbols in the query and relations in the database.

Hypergraphs. Let H be a multi-hypergraph (hypergraph for short) whose edge multiset E may contain multiple hyperedges (edges for short) with the same node set. A fractional edge cover for H is a function γ mapping each edge in H to a positive number such that Σe∈γ(e) ≥ 1 for each node v of H, i.e., the sum of the function values for all edges incident to v is at least 1. We define the weight of a fractional edge cover γ as weight(γ) = Σe∈Eγ(e). The fractional edge cover number ρ∗(H) of H is the minimum weight of fractional edge covers of H. It can be obtained from a fractional edge cover where the edge weights are rational numbers of bit-length polynomial in the size of H.

We use hypergraphs for queries and for relations representing their results. The hypergraph H of a query Q consists of one node A for each attribute A in Q and one edge S(R)+ for each relation symbol R ∈ S(Q). We define ρ∗(Q) = ρ∗(H).

Let R be a relation and P a set of (possibly overlapping) subsets of S(R) such that ∪S∈P S = S(R).
The hypergraph $H$ of $\mathbf{R}$ over $\mathcal{P}$ consists of one node for each distinct tuple in $\pi_5 \mathbf{R}$ for each attribute set $S \in \mathcal{P}$ and one edge for each tuple in $\mathbf{R}$. The edge for a tuple $t$ thus consists of all nodes for tuples $\pi_5(t)$ with $S \in \mathcal{P}$. We use $\text{tuple}(v)$ to denote the tuple represented by a node or edge $v$ in $H$. Given a subset $M$ of the edges in $H$, we define $\text{rel}(M) = \{\text{tuple}(e)\} \in M$ as the relation represented by $M$. The set $M$ is an edge cover of $H$ if each node in $H$ is contained in at least one edge in $M$. The set $M$ is a minimal edge cover if it is an edge cover and any of its strict subsets is not.

**Example 1.** Consider the path query $Q = \sigma_{\psi}(R_1 \times R_2 \times R_3)$, where the relation symbols $R_1$, $R_2$, and $R_3$ are over the schemas $\{A_1, A_2\}$, $\{A_3, A_4\}$, and $\{A_5, A_6\}$, and $\psi$ is the conjunction $A_2 = A_3 \land A_4 = A_5$. Figure 1 depicts in the top row a database of three relations that are instances of the three relation symbols, the query result and a subset of it. In the bottom row, the figure depicts the hypergraphs of $Q$ (and its tree decomposition defined below), of its result over the attribute sets $\{\{A_1, A_2, A_3\}, \{A_2, A_3, A_4, A_5\}, \{A_4, A_5, A_6\}\}$, and the hypergraph of a subset of the query result over the same attribute sets.

| $R_1$ | $R_2$ | $R_3$ | $Q(D)$ | $\text{rel}(M) \subset Q(D)$ |
|-------|-------|-------|--------|-----------------|
| $A_1$, $A_2$ | $A_3$, $A_4$ | $A_5$, $A_6$ | $A_1$, $A_2$, $A_3$, $A_4$, $A_5$, $A_6$ |
| $a_1$, $b_1$ | $b_1$, $c_1$ | $c_1$, $d_1$ | $a_1$, $b_1$, $b_1$, $c_1$, $c_1$, $d_1$ |
| $a_1$, $b_2$ | $b_2$, $c_2$ | $c_1$, $d_1$ | $a_1$, $b_1$, $b_1$, $c_1$, $c_1$, $d_2$ |
| $a_2$, $b_1$ | $b_3$, $c_3$ | $c_2$, $d_2$ | $a_2$, $b_1$, $b_1$, $c_1$, $c_1$, $d_2$ |
| $a_2$, $b_2$ | $b_4$, $c_4$ | $c_2$, $d_1$ | $a_2$, $b_2$, $b_2$, $c_2$, $c_2$, $d_1$ |

Figure 1: Top row: Database $\mathbf{D} = \{R_1, R_2, R_3\}$, the result $Q(D)$ of the path query $Q$ in Example 1 and a subset of $Q(D)$; bottom row: the hypergraph of $Q$, a tree decomposition $T$ of $Q$, the hypergraph of $Q(D)$ over attribute sets $S(T)$, and a minimal edge cover $M$ of this hypergraph.

**Query Decompositions.** A tree decomposition $T$ of the hypergraph $H$ of a query $Q$ is a pair $(T, \chi)$, where $T$ is a tree and $\chi$ a function mapping each node in $T$ to some $V^+$ where $V$ is a subset of the nodes of $H$. For a node $t \in T$, the set $\chi(t)$ is called a bag. It satisfies two properties. **Coverage:** For each edge $e$ in $H$, there must be a node $t \in T$ with $e \subseteq \chi(t)$. **Connectivity:** For each node $v$ in $H$, the set $\{t \mid t \in T, v \in \chi(t)\}$ must be non-empty and form a connected subtree in $T$. The schema of $T$ is the set of its bags: $S(T) = \{\chi(t) \mid t \in T\}$. The attributes of $T$ are defined by $\text{att}(T) = \bigcup_{B \in S(T)} B$. A tree decomposition is called reduced if it has not any bag which is the subset of some other bag.

A fractional tree decomposition [14] of a hypergraph $H$ is a triple $(T, \chi, \{\gamma_t\}_{t \in T})$ where $(T, \chi)$ is a tree decomposition of $H$ and for each node $t \in T$, $\gamma_t$ is a fractional edge cover for the subgraph of $H$ restricted
to $\chi(t)$. We define the (fractional hypertree) width of the fractional tree decomposition $(T, \chi, \{\gamma_t\}_{t \in T})$ as $\max_{t \in T}\{\text{weight}(\gamma_t)\}$. The width $\text{fhtw}(T)$ of a tree decomposition $T = (T, \chi)$ is defined as the minimal possible width for any extension of $T$ by weight functions $\{\gamma_t\}_{t \in T}$. Likewise, the width $\text{fhtw}(H)$ of a hypergraph $H$ (or query $Q$) is the minimal possible width of any tree decomposition of $H$ (or $Q$’s hypergraph). Any tree decomposition $T$ of $H$ can be turned into a reduced tree decomposition $T'$ of $H$ with $\text{fhtw}(T) = \text{fhtw}(T')$ ([18], Part C). If not stated otherwise, in the rest of this work we assume, without loss of generality, that all considered tree decompositions are reduced.

The hypergraph $H$ (or query $Q$) is $\alpha$-acyclic, or simply acyclic in this paper, if it has a tree decomposition in which each bag is contained in an edge of $H$ [7]. Such a tree decomposition is called a join tree. The width of any acyclic hypertree (query) is one.

**Example 2.** Figure 1 gives the hypergraph (left, bottom row) of the path query from Example 1 along with one of its tree decompositions. This tree decomposition has width one, since each bag is included in one edge of the hypertree. The tree decomposition, where the top two bags are merged into one, has width two.

For queries with cycles, e.g., Loomis-Whitney queries [21], the fractional width can be smaller than the integral width. For instance, the triangle query (Loomis-Whitney query over three relations) has a fractional width of 3/2 and an integral width of two [4].

**Computational model.** We use the uniform-cost RAM model [3] where data values as well as pointers to databases are of constant size. Our analysis is with respect to data complexity where the query is assumed fixed. We use $\mathcal{O}$ to hide a log $|D|$ factor.

**Result-preserving Transformations.** We use the following two transformations, where $(Q, T, D)$ denotes a triple of a join query $Q$, a tree decomposition $T$ of $Q$, and a database $D$.

**Proposition 3.** Given $(Q, T, D)$, we can compute $(Q', T, D')$ with size and in time $O(|D|)$ such that $Q'$ is a natural join query, $T$ is a tree decomposition of $Q'$, and $Q'(D') = Q(D)$.

**Proposition 4.** Given $(Q, T, D)$, we can compute $(Q', T, D')$ with size $O(|D|^{\text{fhtw}(T)})$ and in time $\tilde{O}(|D|^{\text{fhtw}(T)})$ such that $Q'$ is an acyclic natural join query, $T$ is a tree decomposition of $Q'$, $D'$ is globally consistent with one relation per bag in $T$, and $Q'(D') = Q(D)$.

**Example 5.** Consider the path query $Q$, tree decomposition $T$, and database $D$ in Example 2. Proposition 3 rewrites $Q$ into $Q' = \bigwedge_{i \in [3]} R_i'$, where $S(R_1') = \{A_1, A_2, A_3\}$, $S(R_2') = \{A_2, A_3, A_4, A_5\}$ and $S(R_3') = \{A_4, A_5, A_6\}$. The corresponding relations in the new database are extensions of the original relations from Figure 1 to match the new schemas, e.g., in $R_1'$ the column for $A_3$ is a copy of the column for $A_2$. The application of Proposition 3 to $(Q', T, D')$ leaves $Q'$ unchanged. The database in Figure 1 (and its above extension) is not globally consistent, since it contains tuples (under the thin lines) that do not contribute to the result. We remove these dangling tuples to make it consistent.

Consider now the bowtie query $Q_\bowtie = \bigwedge_{i \in [6]} R_i$ with schemas $S(R_1) = \{A_1, A_2\}$, $S(R_2) = \{A_2, A_3\}$, $S(R_3) = \{A_1, A_3\}$, $S(R_4) = \{A_1, A_4\}$, $S(R_5) = \{A_4, A_5\}$, and $S(R_6) = \{A_1, A_5\}$, already rewritten using Proposition 3. A tree decomposition with the lowest width of 3/2 has two bags $S_1 = \{A_1, A_2, A_3\}$ and $S_2 = \{A_1, A_4, A_5\}$, one for each clique (triangle) in the query. We materialize the bags in new relations $B_1$ and $B_2$ with schemas $S_1$ and respectively $S_2$. The new query is then $Q'_\bowtie = B_1 \bowtie B_2$, where the relation symbols $B_1$ and $B_2$ are mapped to the relations $B_1$ and respectively $B_2$. The relations $B_1$ and $B_2$ are consistent in the sense that each tuple in $B_1$ has at least one joinable tuple in $B_2$ and vice versa.

## 3 Covers for Join Queries

In this section we introduce the notion of covers of join query results along with a characterization of their size bounds, connection to minimal edge covers for hypergraphs of join query results, and complexity for enumeration of the result tuples from a cover.

We use $(Q, T, D)$ to denote a triple of join query $Q$, tree decomposition $T$ of $Q$, and database $D$. For an instance $(Q, T, D)$, covers of the query result $Q(D)$ are relations that are minimal while preserving the query result $Q(D)$ in the following sense.
Definition 6 (Result Preservation). A relation $C$ is result-preserving with respect to $(Q, \mathcal{T}, D)$ if its schema $\mathcal{S}(C)$ is $\text{att}(Q)$ and $\pi_B C = \pi_B Q(D)$ for each $B \in \mathcal{S}(T)$.

In other words, for the set $B$ of attributes at each bag in a tree decomposition $\mathcal{T}$ of $Q$, both the relation $C$ and the query result $Q(D)$ have the same projection onto $B$. This also means that the natural join of these projections of $C$ is precisely $Q(D)$.

Proposition 7. Given $(Q, \mathcal{T}, D)$, a relation $C$ with schema $\text{att}(Q)$ is result-preserving with respect to $(Q, \mathcal{T}, D)$ if and only if $\forall B \in \mathcal{S}(T) \quad \pi_B C = Q(D)$.

We further say that $C$ is a minimal result-preserving relation if it is result-preserving with respect to $(Q, \mathcal{T}, D)$ and this is not the case for any strict subset of it.

We can now define the notion of covers of query results.

Definition 8 (Covers). Given $(Q, \mathcal{T}, D)$, a relation $C$ is a cover of the query result $Q(D)$ over the tree decomposition $\mathcal{T}$ if it is a minimal result-preserving relation with respect to $(Q, \mathcal{T}, D)$.

Example 9. Figure 1 gives the tree decomposition $\mathcal{T}$ of a path query and one cover $\text{rel}(M)$ of the query result over $\mathcal{T}$. We give below four relations that are subsets of the query result. The relations $C_1$ and $C_2$ are covers, while the relations $N_1$ and $N_2$ are not:

| $C_1$ | $C_2$ | $N_1$ | $N_2$ |
|-------|-------|-------|-------|
| $A_1 A_2 A_3 A_4 A_5 A_6$ | $A_1 A_2 A_3 A_4 A_5 A_6$ | $A_1 A_2 A_3 A_4 A_5 A_6$ | $A_1 A_2 A_3 A_4 A_5 A_6$ |
| $a_1 b_1 b_1 c_1 c_1 d_2$ | $a_1 b_1 b_1 c_1 c_1 d_2$ | $a_1 b_1 b_1 c_1 c_1 d_1$ | $a_1 b_1 b_1 c_1 c_1 d_1$ |
| $a_2 b_1 b_1 c_1 c_1 d_1$ | $a_2 b_1 b_1 c_1 c_1 d_1$ | $a_2 b_1 b_1 c_1 c_1 d_2$ | $a_2 b_1 b_1 c_1 c_1 d_2$ |
| $a_1 b_2 b_2 c_2 c_2 d_1$ | $a_2 b_2 b_2 c_2 c_2 d_1$ | $a_1 b_2 b_2 c_2 c_2 d_1$ | $a_1 b_2 b_2 c_2 c_2 d_1$ |
| $a_2 b_2 b_2 c_2 c_2 d_1$ | $a_2 b_2 b_2 c_2 c_2 d_2$ | $a_2 b_2 b_2 c_2 c_2 d_2$ | $a_1 b_2 b_2 c_2 c_2 d_1$ |

To check the minimal result-preservation property, we take projections onto the bags $B_1 = \{A_1, A_2, A_3\}$, $B_2 = \{A_2, A_3, A_4, A_5\}$, and $B_3 = \{A_4, A_5, A_6\}$. The relation $N_1$ is not result-preserving, because $(a_2, b_2, b_2) \notin \pi_{B_1} N_1$. Even though $N_2$ has more tuples than the other relations above, it is not result preserving for the same reason as $N_1$.

Consider now the coarser tree decomposition $\mathcal{T}'$ with bags $B_{1,2}' = \{A_1, A_2, A_3, A_4, A_5\}$ and $B'_3 = \{A_4, A_5, A_6\}$. The three covers over $\mathcal{T}$ discussed above are also covers over $\mathcal{T}'$. The query result is the only cover over the coarsest tree decomposition $\mathcal{T}''$ with only one bag.

Example 10. A query result may admit exponentially many covers over the same tree decomposition. Consider for instance the simple product query $R_1 \times R_2$ with relations $R_1$ and $R_2$ of size two and respectively $n > 1$. The query result has size $2^n$. To compute a cover, we pair the first tuple in $R_1$ with any non-empty and strict subset of the $n$ tuples in $R_2$, while the second tuple in $R_1$ is paired with the remaining tuples in $R_2$. There are $2^n - 2$ possible covers. The empty and the full sets are missing from the choice of a subset of $R_2$ as they would mean that one of the two tuples in $R_1$ would have to be paired with tuples in $R_2$ that are already paired with the other tuple in $R_1$ and that would violate the minimality criterion of the covers. All covers have size $n$ and none is contained in the other.

We next give a characterization of covers via the hypergraph of the query result.

Proposition 11. Given $(Q, \mathcal{T}, D)$, a relation $C$ is a cover of $Q(D)$ over $\mathcal{T}$ if and only if the hypergraph of $Q(D)$ over the attribute sets $\mathcal{S}(T)$ has a minimal edge cover $M$ with $\text{rel}(M) = C$.

Example 12. Figure 1 gives a minimal edge cover $M$ of the hypergraph of the result of the path query and the corresponding cover $\text{rel}(M)$. By removing any edge from $M$, it is not anymore an edge cover. Likewise, by removing the tuple corresponding to that edge from $\text{rel}(M)$, it is not anymore a cover since it is not result preserving. By adding an edge to $M$ or the corresponding tuple to $\text{rel}(M)$, they are not anymore minimal.
We now turn our investigation to sizes and first note the following immediate property.

**Proposition 13.** Given \(Q, T, D\), each cover of \(Q(D)\) over \(T\) is a subset of \(Q(D)\).

An implication of Proposition 13 is that the covers cannot be larger than the query result. However, they can be much more succinct. The following theorem relies on the insight that for any cover \(C\) of some \(Q(D)\) over some \(T\), it holds \(\max_{B \in S(T)} |\pi_B Q(D)| \leq |C| \leq \Sigma_{B \in S(T)} |\pi_B Q(D)| - |S(T)| + 1\) (Observation 34 in Appendix A).

**Theorem 14.** Let \(Q\) be a join query and \(T\) a tree decomposition of \(Q\).

(i) For any database \(D\), each cover of \(Q(D)\) over \(T\) has size \(O(|D|^{fhtw(T)})\).

(ii) There are arbitrarily large databases \(D\) such that each cover of \(Q(D)\) over \(T\) has size \(\Omega(|D|^{fhtw(T)})\).

The size gaps between query results and their covers can be arbitrarily large. For any join query \(Q\) and database \(D\), it holds that \(|Q(D)| = O(|D|^\rho^*(Q))\) and there are arbitrarily large databases \(D\) for which \(|Q(D)| = \Omega(|D|^\rho^*(Q))\). For acyclic queries, the fractional edge cover number \(\rho^*\) can be as large as \(|Q|\), while the fractional hypertree width is one. Section 4 shows that the same gap also holds for time complexity.

**Example 15.** We continue Example 9. The tree decomposition \(T\) has width one, which is minimal. The covers over \(T\), such as \(C_1\) and \(C_2\), have sizes upper bounded by the input database size. The minimum size of a cover over \(T\) is the maximum size of a relation used in the query (assuming the relations are globally consistent). In contrast, there are arbitrarily large databases of size \(N\) for which the query result has size at least \(N^2\).

Proposition 11 and Theorem 14 give alternative characterizations of the size of a cover of a query result. The former gives it as the size of a minimal edge cover of the hypergraph of the query result over the attribute sets given by the bags of a tree decomposition \(T\), while the latter states it using the fractional hypertree width of \(T\) or alternatively the maximum fractional edge cover number over all the bags of \(T\).

This size gap between query results and their covers is precisely the same as for query results and their factorized representations called \(d\)-representations [23]. In this sense, covers can be seen as relational encodings of factorized representations of query results. We can easily translate covers into factorized representations, as shown next.

**Proposition 16.** Given \(Q, T, D\), any cover \(C\) of the query result \(Q(D)\) over \(T\) can be translated into a \(d\)-representation of \(Q(D)\) of size \(O(|C|)\) and in time \(O(|C|)\).

The above translation allows us to extend the applicability of covers to known workloads over factorized representations. Recent workloads include in-database optimization problems [2] and in particular learning regression models [22]. Nevertheless, it is practically desirable to process such workloads directly on covers, since this would avoid the indirect via factorized representations that comes with extra space cost and non-relational data representation. Aggregates, which are at the core of such workloads, can be computed directly on covers by joint scans of the projections of the cover onto the bags of the tree decomposition; alternatively, they can be computed by expressing any cover as the natural join of its bag projections and then pushing the aggregates past the join.

Despite their succinctness over the explicit listing of tuples in a query result, any cover of the query result can be used to enumerate the result tuples with constant delay and extra space (data complexity) following linear-time pre-computation. In particular, the delay and the space are linear in the number of attributes of the query result which is as good as enumerating directly from the result. This complexity follows from Proposition 11 and enumeration for factorized representations [23] with constant delay and extra space.

**Corollary 17** (Proposition 11, Theorem 4.11 [24]). Given \(Q, T, D\), the tuples in \(Q(D)\) can be enumerated from any cover \(C\) of \(Q(D)\) over \(T\) with \(O(|C|)\) pre-computation time and \(O(1)\) delay and space.
An alternative way to achieve constant-delay enumeration with $\tilde{O}(|C|)$ pre-computation is by noting that the acyclic join queries considered in this paper are free-connex and thus allow for enumeration with constant delay and $\tilde{O}(|D|)$ pre-computation \cite{25}. An acyclic conjunctive query is called free-connex if its extension by a new relation symbol covering all attributes of the result remains acyclic \cite{25}. Moreover, given a cover $C$ over a tree decomposition $T$, the natural join of the projections of $C$ onto the bags of $T$ is an acyclic query that computes the original query result (cf. Proposition \[7\]).

4 Computing Covers using Cover-Join Plans

In this section, we give a compositional approach for computing covers in case of acyclic join queries in time $\tilde{O}(|D|)$. We design so-called cover-join plans to compute covers for a triple $(Q, T, D)$. The cover-join plans follow the structure of $T$ and use a new binary operator called cover-join, which takes two relations and computes a cover of their join. This plan-based approach is in the spirit of standard query evaluation in relational databases. It is modular in the sense that to compute a cover of the query result, it suffices to compute a cover of the join of two relations, then a cover of the join of the previous cover and another relation, and so on. This is practical since such plans can be easily computed using existing relational query engines extended with the new cover-join operator. We also investigate the ability of such plans to define the entire space of possible covers and show that, due to their binary nature, they cannot recover all possible covers over $T$. Furthermore, different plans for the same tree decomposition may lead to different covers. Plans that are built with cover-join operators but do not follow the structure of a tree decomposition may be unsound as they do not necessarily construct covers.

Unless stated otherwise, we assume in this section that $Q$ is an acyclic natural join query, $D$ is a globally consistent database, and there are one-to-one mappings from the relations symbols in $Q$ to the relations in $D$ and also to the bags in $T$. Following Proposition \[4\] this is without loss of generality since the case of arbitrary join queries can be reduced to the case of acyclic join queries, albeit with a complexity overhead. Nevertheless, our approach can compute covers for arbitrary join queries worst-case optimally (modulo a logarithmic factor).

4.1 The Cover-Join Operator

The building block of our approach to computing covers is the binary cover-join operator.

Definition 18 (Cover-Join). Given $(Q, T, D)$ with $Q = R_1 \bowtie R_2$, $S(T) = \{S(R_1), S(R_2)\}$, and $D = \{R_1, R_2\}$, the cover-join of $R_1$ and $R_2$, denoted by $R_1 \bowtie R_2$, computes a cover of $Q(D)$ over $T$.

Following the alternative characterization of covers by minimal edge covers (Proposition \[14\]), the cover-join defines the relation $rel(M)$ of a minimal edge cover $M$ of the hypergraph $H$ of the result of the join $R_1 \bowtie R_2$ over the attribute sets $\{S(R_1), S(R_2)\}$. The hypergraph $H$ is bipartite and consists of disjoint complete bipartite subgraphs. Since a cover is a minimal edge cover, it corresponds to a bipartite subgraph with the same number of nodes but a subset of the edges, where all paths can only have one or two edges. A cover cannot have unconnected nodes, since it would not be an edge cover. A path of three (or more) edges violates the minimality of the edge cover: Such a path $a_1 - b_1 - a_2 - b_2$ in a bipartite graph covers the four nodes, yet a minimal cover would only have the two edges $a_1 - b_1$ and $a_2 - b_2$.

We can compute a cover of a join of two relations $R_1$ and $R_2$ in time $\tilde{O}(R_1 + R_2)$, since it amounts to computing a minimal edge cover in a collection of disjoint complete bipartite graphs that encode the join result. The smallest size of a cover is given by the edge covering number of the bipartite graph representing the join result, which is the maximum of the sizes of the two sets of nodes in the graph \[19\]. The largest size is achieved in case one of the two node sets has size one, in which case this is paired with all nodes in the second set. In case both sets have more than one node, the largest size is achieved when we pair one node from one of the two node sets with all but one node in the second set and then the remaining node in the second set with all but the already used node in the first set.
For the analysis in this paper, we assume that our cover-join algorithm may return any cover of the natural join of two relations. In practice, however, it makes sense to compute a cover of minimum size. We choose this cover as follows: For each complete bipartite hypergraph in the join result with node sets $V_1$ and $V_2$ such that $|V_1| \leq |V_2|$, we choose a minimum edge cover by pairing each node in $V_1$ with one distinct node in $V_2$ and all remaining nodes in $V_2$ with one node in $V_1$.

**Proposition 19.** Given $(Q, T, D)$ where $Q = R_1 \bowtie R_2$, $S(T) = \{S(R_1), S(R_2)\}$, and $D = \{R_1, R_2\}$, there is a cover $C$ of $Q(D)$ over $T$ that can be computed in time $\mathcal{O}(|R_1| + |R_2|)$ and has size $\max\{|R_1|, |R_2|\} \leq |C| \leq |R_1| + |R_2| - 1$.

**Example 20.** Consider again the product $R_1 \times R_2$ in Example 19 where $R_1 = [2]$, $R_2 = [n]$ with $n > 1$, $S(R_1) = \{A\}$ and $S(R_2) = \{B\}$. Examples of covers of size $n$ over the tree decomposition $T$ with bags $\{A\}$ and $\{B\}$ are: $\{(1, i) | i \in \{n\} - \{k\}\} \cup \{(2, k)\}$ for any $k \in [n]$; $\{(1, i) | i \in [k]\} \cup \{(2, j + k) | j \in [n-k]\}$ for any $k \in [n-1]$. If $R_1 = [m]$ with $m > n$, then examples of covers over $T$ of minimum size $m$ are: $\{(i, i) | i \in \{k-1\}\} \cup \{(k-1+i, k+i) | i \in \{n-k\}\} \cup \{(n-1+i, k) | i \in \{m-n+1\}\}$ for any $k \in [n]$. A cover over $T$ of maximal size $n + m - 2$ is: $\{(1, i) | i \in \{n-1\}\} \cup \{(j+1, n) | j \in \{m-1\}\}$. Below are depictions of the complete bipartite graph corresponding to the query result for $n = 4$ and $m = 5$, where the edges in a minimal edge cover are solid lines and all other edges are dotted. The left minimal edge cover corresponds to a cover over $T$ of minimum size $m = 5$, while the right minimal edge cover corresponds to a cover over $T$ of maximum size $n + m - 2 = 7$.

![Diagram](image)

**4.2 Cover-join Plans**

We now compose cover-join operators into so-called cover-join plans to compute covers for acyclic natural join queries. Before we define them, we need to introduce a bit of notation.

For a tree decomposition $T$, we write $T = T_1 \circ T_2$ if the tree $T$ of $T$ can be split into two non-empty subtrees of decompositions $T_1$ and $T_2$ that are connected by a single edge in $T$.

**Definition 21 (Cover-Join Plan).** Given $(Q, T, D)$, a cover-join plan for $Q$ over $T$, denoted by $\phi_T$, is defined recursively as follows:

- If $T$ has one node and $Q = R$, then $\phi_T = R$. The plan $\phi_T$ returns $R$.
- If $T = T_1 \circ T_2$, $Q = Q_1 \bowtie Q_2$, $T_i$ is a tree decomposition of $Q_i$, and $\phi_{T_i}$ is a cover-join plan for $Q_i$ over $T_i$, then $\phi_T = \phi_{T_1} \bowtie \phi_{T_2}$. The plan $\phi_T$ returns the result of $R_1 \bowtie R_2$, where $R_i$ is the relation returned by the sub-plan $\phi_{T_i}$.

A cover-join plan is thus a function that maps triples $(Q, T, D)$ to relations.

**Lemma 22** states that a cover-join plan computes a cover of $Q(D)$ over $T$. In case the query $Q$ is the identity on $R$, the plan returns the relation $R$ since a cover of a relation is the relation itself. In case $Q$ is a join, the plan computes a cover of $Q(D)$ over $T$ given that covers of $Q_i(D)$ over $T_i$ are computed by $\phi_{T_i}$.

**Lemma 22.** Given $(Q, T, D)$ with $D = \{R_i\}_{i \in [n]}$, any cover-join plan for $Q$ over $T$ computes a cover $C$ of $Q(D)$ over $T$ in time $\mathcal{O}(|C|)$ and with size $\max_{i \in [n]} |R_i| \leq |C| \leq \sum_{i \in [n]} |R_i| - |S(T)| + 1$.

**Lemma 22** states several remarkable properties of cover-join plans.

First, they compute covers compositionally: To obtain a cover of the entire query result it is sufficient to compute covers of subquery results. More precisely, for a cover-join plan $\phi_{T_1} \bowtie \phi_{T_2}$, the sub-plans $\phi_{T_i}$
and $\varphi_{T_2}$ compute covers for the subqueries defining the join of the relations with schemas from $S(T_1)$ and respectively $S(T_2)$ over the tree decompositions $T_1$ and respectively $T_2$. Then, the plan $\varphi_{T_1} \otimes \varphi_{T_2}$ computes a cover for the join of the relations with schemas from $S(T) = S(T_1) \cup S(T_2)$ over the tree decomposition $T_1 \circ T_2$.

Second, it does not matter which cover we pick at each cover-join operator, the final result is still a cover. These two properties rely on the global consistency of the database and the plans using the tree decomposition. Covers are lossy representations and chaining cover-joins may otherwise lose result tuples by pairing critical tuples with dangling tuples in case of arbitrary databases. Furthermore, query plans that do not follow the structure of a tree decomposition may be unsound, cf. Example 25. Note that although each cover-join operator computes a cover of minimum size for the join of two input relations, the overall cover computed by a cover-join plan over some tree decomposition $\mathcal{T}$ may not be a cover of minimum size of the query result over $\mathcal{T}$, cf. Example 23 in Appendix D.3.

Third, it does not matter which cover-join plan we choose for a given tree decomposition, the resulting covers are computed with the same time guarantee that depends on the tree decomposition (here, of width one since the query $Q$ is acyclic). Nevertheless, different plans over the same tree decomposition may lead to different covers, cf. Example 24.

**Example 23.** A tree decomposition $\mathcal{T}$ that admits several splits can define many possible plans. For instance, the tree decomposition $\mathcal{T}$ in Figure 1 for the natural join path query $R_1' \bowtie R_2' \bowtie R_3'$ admits two possible splits that lead to the plans $\varphi_1 = (R_1' \bowtie R_2') \bowtie R_3'$ and $\varphi_2 = R_1' \bowtie (R_2' \bowtie R_3')$. The relations $R_i'$ are those in Figure 1 now calibrated (by removing dangling tuples ) and extended (by accommodating equivalent attributes), as per Proposition 4.

The covers computed by the sub-plans $R_1' \bowtie R_2'$ and $R_2' \bowtie R_3'$ correspond to full join results, since all join values in $R_2'$ only occur once. By taking any possible cover at each cover-join operator in the plans, both plans yield the same four possible covers of the query result: One of them is $rel(M)$ in Figure 1 and two of them are $C_1$ and $C_2$ in Example 9. The last cover is not depicted, it is the same as $C_1$ with the change that the values $d_1$ and $d_2$ are swapped between the first two rows.

An immediate corollary of Proposition 4 and Lemma 22 is that we can compute a cover of the result of an arbitrary join query over any of its tree decompositions in time proportional to the size of the cover, up to a logarithmic factor. Given $(Q, T, D)$, a cover of $Q(D)$ over $\mathcal{T}$ can be computed in three main steps: (i) construct $(Q', T, D')$ such that $Q'$ is an acyclic natural join query and there are one-to-one mappings from the relation symbols in $Q'$ to the relations in $D'$ and to the bags in $T$; (ii) turn $D'$ into a globally consistent database $D''$ with respect to $Q'$; (iii) execute on $D''$ a cover-join plan for $Q'$ over $\mathcal{T}$.

**Theorem 24 (Proposition 4 Lemma 22).** Given $(Q, T, D)$ where $Q$ is any join query and $D$ is an arbitrary database, any cover-join plan for $Q$ over $\mathcal{T}$ computes a cover $C$ of $Q(D)$ over $\mathcal{T}$ of size $O(|D|^{\operatorname{htw}(\mathcal{T})})$ and in time $O(|D|^{\operatorname{htw}(\mathcal{T})})$.

Since there are arbitrarily large databases for which the size bounds on covers are tight (Theorem 14), the cover-join plans, together with a worst-case optimal algorithm for materializing bags [21], represent a worst-case optimal algorithm for computing covers.

We conclude this section with three insights into the ability of cover-join plans to compute covers. We give an example of a cover-join plan that does not follow the structure of a tree decomposition and that yields an empty result, which makes it unsound. We then note the incompleteness of our cover-join plans due to the binary nature of the cover-join operator. We give an example of a cover that cannot be computed with our cover-join plans, but can be computed using a multi-way cover-join operator. Finally, we give an example showing that distinct cover-join plans over the same (or also distinct) tree decompositions can yield incomparable sets of covers.

**Example 25 (Unsound plan).** Consider the following globally consistent database with relations $R_1$, $R_2$, and $R_3$ and four covers computed by cover-joining two of the three relations:
We would like to compute covers of the path query expressed by the natural join $R_1 \Join R_2 \Join R_3$. Following Definition 21, the plan $(R_1 \Join R_2) \Join R_3$ would require a split $T_{1,3} \circ T_2$ with $S(T_{1,3}) = \{S(R_1), S(R_3)\}$ and $S(T_2) = \{S(R_2)\}$ of a tree decomposition. However, there is no tree decomposition that allows such a split.

The cover-join $R_1 \Join R_3$ computes one of the two covers $C_{1,3}$ and $C'_{1,3}$. The result of the join of $C'_{1,3}$ and $R_2$ is empty and so is the cover-join. This means that this plan does not always compute a cover, which makes it unsound.

This problem cannot occur with cover-join plans over tree decompositions of our path query. The only cover-join plans for our path query (up to commutativity) are $(R_1 \Join R_2) \Join R_3$ and $R_1 \Join (R_2 \Join R_3)$. The only cover of $R_1 \Join R_2$ is $C_{1,2}$ above, which can be cover-joined with $R_3$. The only cover of $R_2 \Join R_3$ is $C_{2,3}$ above, which can be cover-joined with $R_1$.

**Example 26 (Cover-Join Incompleteness).** Consider the following globally consistent database with relations $R_1$, $R_2$, and $R_3$ and one cover $K$ of the product $R_1 \times R_2 \times R_3$:

| $R_1$ | $R_2$ | $R_3$ | $K$ |
|------|------|------|-----|
| $A$  | $B$  | $C$  | $A$ $B$ $C$ |
| $a_1$ | $b_1$ | $c_1$ | $a_1$ $b_1$ $c_1$ |
| $a_2$ | $b_2$ | $c_2$ | $a_1$ $b_2$ $c_2$ |

A tree decomposition of the product query can have up to three bags.

In case of three bags, each bag consists of exactly one attribute. There are three possible cover-join plans (up to commutativity): $\varphi_1 = R_1 \Join (R_2 \Join R_3)$, $\varphi_2 = R_2 \Join (R_1 \Join R_3)$ and $\varphi_3 = R_3 \Join (R_1 \Join R_2)$. None of these plans can yield the cover $K$ above. As discussed after Definition 18, a minimal edge cover corresponding to a cover computed by a binary cover-join operator can only have paths of one or two edges. For instance, $\pi_{A,B}K$, which should correspond to a cover of $R_1 \Join R_2$, has the path of three edges $b_2 - a_1 - b_1 - a_2$. The cover-join $R_1 \Join R_2$ would not create this path since it corresponds to a non-minimal edge cover. Similarly, $\pi_{A,C}K$ and $\pi_{B,C}K$ have paths of three edges.

For tree decompositions with two bags, two of the three attributes are in the same bag. Without loss of generality, assume $A$ and $B$ are in the same bag. Following Proposition 4, this bag is covered by a new relation $R_{1,2}$ that is the product of $R_1$ and $R_2$. This means that $K$ has to be the cover of $R_{1,2} \Join R_3$, yet $\pi_{A,B}K$ is not $R_{1,2}$!

The tree decomposition with one bag consisting of all three attributes has this bag covered by a new relation that is the product of the three relations. This relation is the full Cartesian product of the three relations. However, $\pi_{A,B,C}K$ does not equal this new relation.

We conclude that the cover $K$ cannot be computed using cover-join plans over tree decompositions of the product query.

**Example 27 (Incomparable Sets of Covers).** Consider the following globally consistent database with relations $R_1$, $R_2$, and $R_3$:

| $R_1$ | $R_2$ | $R_3$ | $C$ | $C_{1,2}$ | $C'_{1,2}$ |
|------|------|------|-----|---------|----------|
| $A$  | $B$  | $C$  | $A$ $B$ $C$ | $A$ $B$ | $A$ $B$ |
| $a_1$ | $b_1$ | $c_1$ | $a_1$ $b_1$ $c_1$ | $a_1$ $b_1$ | $a_1$ $b_2$ |
| $a_2$ | $b_2$ | $c_2$ | $a_2$ $b_2$ $c_2$ | $a_2$ $b_2$ | $a_2$ $b_1$ |
We consider the query \( Q = R_1 \times R_2 \times R_3 \) and its tree decomposition \( T \) which consists of the path \( \{ A \} - \{ B \} - \{ C \} \). There are (up to commutativity) two possible cover-join plans for \( Q \) over \( T \): \( \varphi_1 = R_1 \bowtie (R_2 \bowtie R_3) \) and \( \varphi_2 = (R_1 \bowtie R_2) \bowtie R_3 \).

The relation \( C \) above is a cover of the result of \( Q \) computed by \( \varphi_1 \), which cover-joins \( R_1 \) and a cover of the join of \( R_2 \) and \( R_3 \). This cover cannot be computed by \( \varphi_2 \). Indeed, \( \varphi_2 \) first cover-joins \( R_1 \) and \( R_2 \), yielding \( C_{1,2} \) or \( C'_{1,2} \) as the only possible covers. Then, cover-joining any of them with \( R_3 \) does not yield the cover \( C \) since \( \pi_{A,B}C \) is different from both \( C_{1,2} \) and \( C'_{1,2} \). Similarly, \( \varphi_2 \) computes covers that cannot be computed by \( \varphi_1 \).

## 5 Covers for Functional Aggregate Queries

In the following, we write \( a_S \) to indicate that \( a \) is a tuple over the attribute set \( S \). A functional aggregate query (FAQ) has the form [17] (slightly adapted to our notation):

\[
\varphi(a_{A_1},\ldots,a_{A_f}) = \bigoplus_{f+1 \in \text{dom}(A_{f+1})} (f+1) \cdots \bigoplus_{n \in \text{dom}(A_n)} (n) \bigotimes_{S \in \mathcal{E}} \psi_S(a_S), \tag{1}
\]

- \( H = (\mathcal{V}, \mathcal{E}) \) is the multi-hypergraph of the query with \( \mathcal{V} = \{A_i\}_{i \in \llbracket n \rrbracket} \).
- \( \text{Dom} \) is a fixed (output) domain, such as \{true,false\}, \{0,1\}, or \( \mathbb{R}^+ \).
- \( \{A_1,\ldots,A_f\} = \mathcal{V}_{\text{tree}} \) is the set of result or free attributes; all other attributes are bound.
- For each attribute \( A_i \) with \( i > f \), \( \oplus(i) \) is a binary (aggregate) operator on the domain \( \text{Dom} \). Different bound attributes may have different aggregate operators.
- For each attribute \( A_i \) with \( i > f \), either \( \oplus(i) = \otimes \) or \((\text{Dom}, \oplus(i), \otimes)\) forms a commutative semiring with the same additive identity \( 0 \) and multiplicative identity \( 1 \) for all semirings.
- For every hyperedge \( S \in \mathcal{E} \), \( \psi_S : \prod_{A \in S} \text{dom}(A) \to \text{Dom} \) is an (input) function.

FAQs are a semiring generalization of aggregates over join queries, where the aggregates are the operators \( \oplus(i) \) and the natural join is expressed by \( \boxtimes_{S \in \mathcal{E}} \psi_S(a_S) \). The relational encoding \( R_{\psi} \) of a function \( \psi_S \) is a relation over the schema \( S \cup \{\psi_S(S)\} \) which consists of all input-output pairs for \( \psi_S \) where the output is non-zero, i.e., \( R_{\psi_S} \) contains a tuple \( a_S | S_{\psi_S(S)} \) if and only if \( \psi_S(a_S) = a_{\psi_S(S)} \neq 0 \). The database underlying an FAQ \( \varphi \) is defined as \( D_{\varphi} = \{R_{\psi_E} \mid E \in \mathcal{E}\} \). We say that \( T \) is a tree decomposition of \( \varphi \) if \( T \) is a tree decomposition of the hypergraph \( H \) of \( \varphi \). Given an FAQ \( \varphi \) along with the relational encodings of its input functions, the FAQ-problem is to compute the result of \( \varphi \), i.e., the relational encoding of \( \varphi \).

Each FAQ \( \varphi \) has an FAQ-width \( \text{faqw}(\varphi) \) which is defined similarly to the fractional hypertree width of the hypergraph of \( \varphi \). For instance, in case where all attributes of \( \varphi \) are free, \( \text{faqw}(\varphi) \) is equal to the fractional hypertree width of the hypergraph of \( \varphi \).

Given an FAQ \( \varphi \) and the relational encodings of the input functions in \( \varphi \), the InsideOut algorithm [17] solves the FAQ-problem as follows. First, it eliminates all bound attributes along with their corresponding aggregate operators by performing equivalence-preserving transformations on \( \varphi \). Then, it computes the relational encoding of the remaining query. The algorithm runs in time \( \tilde{O}(|D_{\varphi}|^{\text{faqw}(\varphi)} + Z) \) where \( Z \) is the size of the output.

We can compute a cover of the result of a given FAQ \( \varphi \) in time \( \tilde{O}(|D_{\varphi}|^{\text{faqw}(\varphi)}) \), hence, our construction time does not depend on the the size of the relational encoding of \( \varphi \). In a nutshell, our strategy is as follows. First, we eliminate all bound attributes in \( \varphi \) by using InsideOut resulting in an FAQ \( \varphi' \). Then, we take a tree decomposition \( T \) of \( \varphi' \). Just like constructing bag relations in case of join queries (as in the proof of Proposition [2]), we compute bag functions \( \beta_B, B \in \mathcal{S}(T) \), with \( \varphi'(a_{\text{tree}}) = \bigotimes_{B \in \mathcal{S}(T)} \beta_B(a_B) \). Finally, we compute a cover of the join result of the relational encodings of the bag functions over the extension of \( T \) that contains, for each bag \( B \), the attribute \( \beta_B(B) \) for the values of the function \( \beta_B \). Keeping the
Example 28. We consider the following FAQ \( \varphi \) over the sum-product semiring \((\mathbb{N}, +, \cdot)\) (for simplicity we skip the explicit iteration over the domains of the attributes in \( \varphi \)):

\[
\varphi(a, b, d) = \sum_{c, e, f, g, h} \psi_1(a, b, c) \cdot \psi_2(b, d, e) \cdot \psi_3(d, e, f) \cdot \psi_4(f, h) \cdot \psi_5(e, g),
\]

where \( \varphi, \psi_1, \psi_2, \psi_3, \psi_4 \) and \( \psi_5 \) are over \( \{A, B, D\}, \{A, B, C\}, \{B, D, E\}, \{D, E, F\}, \{F, H\} \) and \( \{E, G\} \), respectively. We first run \textsc{InsideOut} on \( \varphi \) to eliminate the bound attributes and obtain the following FAQ:

\[
\varphi'(a, b, d) = \left( \sum_c \psi_1(a, b, c) \right) \cdot \left( \sum_e \psi_2(b, d, e) \right) \cdot \left( \sum_f \psi_3(d, e, f) \right) \cdot \left( \sum_h \psi_4(f, h) \right) \cdot \left( \sum_g \psi_5(e, g) \right).
\]

We consider the tree decomposition \( T \) of \( \varphi' \) with two bags \( B_1 = \{A, B\} \) and \( B_2 = \{B, D\} \) and bag functions \( \psi_6 \) and respectively \( \psi_{10} \). Then, we execute the cover-join plan \( R_{\psi_6} \bowtie R_{\psi_{10}} \) over the extended tree decomposition \( T' \) with bags \( \{A, B, \psi_6(A, B)\} \) and \( \{B, D, \psi_{10}(B, D)\} \). While the computation of the result of \( \varphi' \) can take quadratic time, the above cover-join plan takes linear time. We exemplify the computation of the cover-join plan. Assume the following tuples in \( \psi_6 \) and \( \psi_{10} \), where \( \gamma_1, \ldots, \gamma_4, \delta_1, \ldots, \delta_4 \in \mathbb{N} \):

| \( A \) | \( B \) | \( \psi_6(A, B) \) | \( B \) | \( D \) | \( \psi_{10}(B, D) \) | \( A \) | \( B \) | \( D \) | \( \psi_6(A, B) \) | \( \psi_{10}(B, D) \) |
|---|---|---|---|---|---|---|---|---|---|
| \( a_1 \) | \( b_1 \) | \( \gamma_1 \) | \( b_1 \) | \( d_1 \) | \( \delta_1 \) | \( a_1 \) | \( b_1 \) | \( d_1 \) | \( \gamma_1 \) | \( \delta_1 \) |
| \( a_2 \) | \( b_1 \) | \( \gamma_2 \) | \( b_1 \) | \( d_2 \) | \( \delta_2 \) | \( a_2 \) | \( b_1 \) | \( d_2 \) | \( \gamma_2 \) | \( \delta_2 \) |
| \( a_3 \) | \( b_2 \) | \( \gamma_3 \) | \( b_2 \) | \( d_3 \) | \( \delta_3 \) | \( a_3 \) | \( b_2 \) | \( d_3 \) | \( \gamma_3 \) | \( \delta_3 \) |
| \( a_4 \) | \( b_2 \) | \( \gamma_4 \) | \( a_4 \) | \( b_2 \) | \( d_3 \) | \( \gamma_4 \) | \( \delta_3 \) |

The relation \( C \) is one of the two possible covers obtained from the cover-join plan. The cover carries over the aggregates in columns \( \psi_6(A, B) \) and \( \psi_{10}(B, D) \), one per bag of \( T' \). The aggregate of the first tuple in \( C \) is \( \gamma_1 \cdot \delta_1 \) (where \( \gamma_1 \otimes \delta_1 \) under a semiring with multiplication \( \otimes \)).

The following theorem relies on Lemma [23] and Theorem [24] that give an upper bound on the time complexity for constructing covers of join results.

Theorem 29. For any FAQ \( \varphi \), a cover of the result of \( \varphi \) can be computed in time \( \widetilde{O}(|D\varphi|^{\text{faqw}(\varphi)}) \).

Any enumeration algorithm for covers of join results can be used to enumerate the tuples of an FAQ result from one of its covers. We thus have the following corollary:

Corollary 30 (Corollary [17]). Given a cover \( C \) of the result of an FAQ \( \varphi \), the tuples in the result of \( \varphi \) can be enumerated with \( \widetilde{O}(|C|) \) pre-computation time and \( \mathcal{O}(1) \) delay and space.

6 Conclusion

Results of join and functional aggregate queries entail redundancy in both their computation and representation. In this paper we propose the notion of covers of query results that reduce such redundancy. While covers can be more succint than the query results, they nevertheless enjoy desirable properties such as
listing representation and constant-delay enumeration of result tuples. For a given database and a join or functional aggregate query, the query result can be normalised as a globally consistent database over an acyclic schema. Covers represent one-relational, lossless, linear-size encodings of such normalised databases.

**Definition 31.** borged /ˈbɔrd/ : Buy One Relation, Get Entire Database!

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A Further Preliminaries

We introduce necessary notation for the proofs in the following sections and state an observation on size bounds of covers.

Restrictions of queries and databases Given a query $Q$, a database $D$ and a subset $X$ of the attributes in $Q$, the $X$-restrictions of $Q$ and $D$ are obtained by restricting them to the attributes in $X^+$, i.e., to the attributes transitively equivalent to those in $X$. When defining $X$-restrictions, we have to be precise with respect to the mappings from the relation symbols and attributes in $Q$ to the relations and attributes in $D$. In the following, we use the notation $(Q, D, \lambda, \{\mu_R\}_{R \in S(Q)})$ to indicate that $\lambda$ is a mapping from $S(Q)$ to $S(D)$ and $\mu_R$ is a bijection from $S(R)$ to $S(\lambda(R))$ for each $R \in S(Q)$.

Definition 32 (Restrictions). Let $\xi = (Q, D, \lambda, \{\mu_R\}_{R \in S(Q)})$ be a quadruple with $Q = \sigma_\psi(R_1 \times \ldots \times R_n)$. The $X$-restriction $(Q_X, D_X, \lambda_X, \{\mu^X_R\}_{R \in S(Q_X)})$ of $\xi$ is defined as follows:

- $Q_X = \sigma_{\psi_X}(R^X_1 \times \ldots \times R^X_i)$ where $\psi_X$ and each $R^X_i$ are the restrictions of $\psi$ and $R_i$ to the attributes in $X^+$.
- $D_X = \{R^X_i\}_{i \in [n]}$ where each $R^X_i$ is the restriction of $\lambda(R_i)$ to the attributes in $\{\mu_{R_i}(A) \mid A \in S(R^X_i)\}$.
- $\lambda_X = \{R^X_i \mapsto R^X_i\}_{i \in [n]}$, and
- $\mu^X_R$ is the restriction of $\mu_R$ to $S(R^X_i)$ for each $i \in [n]$.

The query $Q_X$ and the database $D_X$ are called the $X$-restrictions of $Q$ and $D$, respectively.

Observe that in case $Q = R_1 \bowtie \ldots \bowtie R_n$ is a natural join query, the $X$-restriction of $Q$ is just $Q_X = R^X_1 \bowtie \ldots \bowtie R^X_n$ where each $R^X_i$ is the restriction of $R_i$ to the attributes in $X$.

We demonstrate restrictions by an example.

Example 33. Let $\xi = (Q, D, \lambda, \{\mu_R\}_{R \in S(Q)})$ be a quadruple. Assume that $Q = \sigma_\psi(R_1 \times R_2 \times T)$ where $S(R_1) = \{A_1, B_1, C_1\}$, $S(R_2) = \{A_2, B_2, C_2\}$, $S(T) = \{A, B, C, D\}$ and $\psi$ is the conjunction $A = A_1 \land B_1 = B_2 \land C = C_2 \land C_1 = D$. Let $D = \{R, T\}$ with $S(R) = \{A, B, C\}$ and $S(T) = \{A, B, C, D\}$. Let $X = \{A, B, C\}$. The $X$-restriction $(Q_X, D_X, \lambda_X, \{\mu^X_R\}_{R \in S(Q_X)})$ of $\xi$ is defined as follows:

- $Q_X = \sigma_{\psi_X}(R_1^X \times R_2^X \times T^X)$ where $S(R^X_1) = \{A_1, B_1\}$, $S(R^X_2) = \{B_2, C_2\}$, $S(T^X) = \{A, C\}$ and $\psi_X$ is the conjunction $A = A_1 \land B_1 = B_2 \land C = C_2$;
- $D_X = \{R^X_1, R^X_2, T^X\}$ where $R^X_1 = \pi_{\{A, B\}} R$, $R^X_2 = \pi_{\{B, C\}} R$ and $T^X = \pi_{\{A, C\}} T$;
- $\lambda_X = \{R^X_1 \mapsto R^X_1\}_{i \in [2]} \cup \{T^X \mapsto T^X\}$;
- $\mu^X_R = \{A \mapsto A, B \mapsto B, C \mapsto C\}$, $\mu^X_{R_1} = \{B_2 \mapsto B, C_2 \mapsto C\}$ and $\mu^X_{T^X} = \{A \mapsto A, C \mapsto C\}$.

Size bounds for covers

Observation 34. For any $(Q, T, D)$ and any cover $C$ of $Q(D)$ over $T$, it holds $\max_{B \in S(T)} |\pi_B Q(D)| \leq |C| \leq \sum_{B \in S(T)} |\pi_B Q(D)| - |S(T)| + 1$.

The first inequality holds due to $C$ being result-preserving with respect to $(Q, T, D)$. The second inequality is implied by Proposition 11 since the hypergraph $H = (V, E)$ of $Q(D)$ over $S(T)$ must have a minimal edge cover $M$ with $rel(M) = C$. If $V$ is non-empty, then, $M$ must contain at least one hyperedge $e$ which contains for each $B \in S(T)$, a node corresponding to a tuple in $\pi_B Q(D)$. Each other hyperedge $e'$ in $M$ must cover at least one node in $V \setminus \{e\}$ which is not covered by any other hyperedge in $M$. Otherwise, $M \setminus \{e\}$ would be an edge cover, which is a contradiction to the minimality of $M$. Since $V \setminus \{e\}$ contains $|V| - |S(T)|$ nodes, the total number of edges in $M$ is upper-bounded by $|V| - |S(T)| + 1$. As $|V| = \sum_{B \in S(T)} |\pi_B Q(D)|$ and $|M| = |C|$, we derive that the number of tuples in $C$ is upper-bounded by $\sum_{B \in S(T)} |\pi_B Q(D)| - |S(T)| + 1$. 

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B  Missing Details and Proofs of Section 2

B.1  Proof of Proposition 3

Proposition 3   Given $(Q, T, D)$, we can compute $(Q', T, D')$ with size and in time $O(|D|)$ such that $Q'$ is a natural join query, $T$ is a tree decomposition of $Q'$, and $Q'(D') = Q(D)$.

The query $Q$ has the form $\psi(R_1 \times \cdots \times R_n)$, where $\psi$ is a conjunction of equality conditions. The relation symbols as well as all attributes occurring in the schemas of the relation symbols are pairwise distinct. Given an equivalence class $A$ of attributes in $Q$, we let $\phi_A = \bigwedge_{A_i, A_j \in A} A_i = A_j$. Then, given the set $\{A_i\}_{i \in [n]}$ of all equivalence classes in $Q$, the conjunction $\bigwedge_{i \in [n]} \phi_{A_i}$ is the transitive closure $\psi^+$ of $\psi$ in $Q$.

Construction. The query $Q'$ has one relation symbol $R'_i$ for each relation symbol $R_i$ in $Q$ such that $S(R'_i) = S(R_i)^+$. We thus have $Q' = R'_1 \times \cdots \times R'_n$, where the equality conditions in the transitive closure of $\psi$ are now expressed by natural joins in $Q'$.

For the sake of simplicity, we describe the construction of $D$ in two steps. Let $\lambda^{(Q,D)}$ be the function mapping the relation symbols in $Q$ to the relations in $D$. Moreover, for each $R_i \in S(Q)$, let $\mu^{(Q,D)}_{R_i}$ be the bijection function mapping the attributes in $R_i$ to the attributes in $\lambda^{(Q,D)}(R_i)$. First, we construct a database $D_1$ which for each $R_i \in S(Q)$, has a relation $R_i$, which results from $\lambda^{(Q,D)}(R_i)$ by replacing each attribute $A$ by the attribute $B$ with $\mu^{(Q,D)}_{R_i}(B) = A$. We define a mapping $\lambda^{(Q,D)}(Q, D_1)$ from relation symbols in $Q$ to relations in $D_1$ by $\lambda^{(Q,D)}(Q, D_1) = \{R_i \mapsto R_i' \mid R_i \in S(Q)\}$. For each $R_i \in S(Q)$, we also define a bijective attribute mapping $\mu^{(Q,D)}_{R_i}(D_1) = \{A \mapsto A \mid A \in S(R_i)\}$.

We obtain the database $D'$ from $D_1$ by replacing each relation $R_i$ by a relation $R'_i$ as defined above. The relation $R'_i$ is a copy of $R_i$ extended with one new column for each attribute $A$ in $S(R'_i) \setminus S(R_i)$ transitively equal to $A$. The mapping from relation symbols in $Q'$ to relations in $D'$ is defined by $\lambda^{(Q,D')}(Q, D') = \{R'_i \mapsto R'_i' \mid R'_i \in S(Q')\}$. The bijective mapping from the attributes of a relation symbol $R'_i$ to the attributes of the relation $R'_i$ is defined by $\mu^{(Q,D')}(R'_i') = \mu^{(Q,D)}(Q, D_1) \cup \{A \mapsto A \mid A \in S(R'_i)\}$.

$T$ is a tree decomposition of $Q'$. By construction, $Q$ and $Q'$ have the same set of attributes and thus the same equivalence classes of attributes. Moreover, the transitive closures of the schemas of relation symbols are identical: For any pair of relation symbols $R_i \in S(Q)$ and $R'_i \in S(Q')$, it holds that $S(R'_i)^+ = S(R'_i)$. The hypergraphs of $Q'$ and $Q$ are thus the same as they have the same nodes, which are the attributes in $Q$ and $Q'$ respectively, and the same hyperedges, which are the transitive closures $S(R'_i)^+$ and $S(R'_i)^+$ respectively. This means that the tree decomposition $T$ of $Q$ is also a tree decomposition of $Q'$.

$Q'(D') = Q(D)$. Database $D_1$ results from $D$ by, basically, making for each relation $R$ as many copies as the number of relation symbols in $Q$ mapped to $R$. Hence, it easily follows $Q(D_1) = Q(D)$. Thus, it remains to show $Q'(D') = Q(D_1)$.

We first treat the special case when $Q$ is a Cartesian product, i.e., it does not contain any equality conditions. Then, $Q' = Q$ and each relation in $D'$ is an exact copy of a relation in $D_1$. Hence, $Q'(D') = Q(D_1)$ holds trivially. We next consider the case when $Q$ has equality conditions.

We first show $Q'(D') \subseteq Q(D_1)$. Assume there is a tuple $t$ that is contained in $Q'(D')$. Then, $t = \times_{i \in [n]} t_i$ is the natural join of tuples $t_i \in R'_i$. Let $A$ be any equivalence class of attributes in $Q'$. By construction, whenever one of these attributes occur in the schema of a relation $R'_i$, so are the others. Furthermore, their values are the same in any tuple of $R'_i$. Since $t$ is a join of tuples $t_i$, it follows that all attributes in $A$ have the same value in $t$ and therefore $\sigma_A(t) = t$. This holds for all equivalence classes of attributes, so $\sigma_{\psi^+}(t) = t$ and thus $\psi(t) = t$. This means that $t \in Q(D_1)$.

We now show $Q(D_1) \subseteq Q'(D')$. Assume there is a tuple $t$ that is in $Q(D)$. This means that $t = \times_{i \in [n]} t_i$ is a product of tuples $t_i \in R_i$, $\sigma_{\psi^+}(t) = t$ and in particular $\sigma_{\psi^+}(t) = t$ for each equivalence class $A$ in $Q$. We extend each tuple $t_i$ with values for all attributes in the class $A$ whenever $S(t_i) \cap A \neq \emptyset$: Let $t'_i$ be the extension of $t_i$. Then, $t = \times_{i \in [n]} t'_i$. All attributes in $A$ thus have the same value in $t'_i$. Since by
construction the relation \( R'_i \) is an extension of \( R_i \) with same-valued columns for all attributes in \( A \) whenever \( S(R_i) \cap A \neq \emptyset \), it follows that \( t'_i \in R'_i \). Thus, \( t \in Q'(D') \).

**Size of the construction.** The query \( Q' \) has the same number of relation symbols as \( Q \).

The number of relations in \( D_1 \) is the same as the number of relation symbols in \( Q \). Moreover, each relation in \( D_i \) is a copy of a relation in \( D \). Hence, \( |D_1| = O(|D|) \). The relations \( R'_i \) in \( D' \) are copies of relations \( R_i \) in \( D_1 \) with additional columns. Each \( R'_i \) in \( D' \) has a new column for each attribute which is not in \( S(R_i) \) but equivalent to an attribute in \( S(R_i) \). This transformation does not increase the size of the relations. Altogether, we have \( |D'| = O(|D|) \).

**Construction time.** The database \( D_1 \) evolves from \( D \) by duplicating each relation in \( D \) at most \( |Q| \) times. Hence, \( D_1 \) can be constructed in linear time. Each relation \( R'_i \) in \( D' \) can be constructed from \( R_i \) in \( D_1 \) by a single pass through the tuples in \( R_i \). For each tuple, we choose for each new attribute \( A \) in \( R'_i \) but not in \( R_i \), an equivalent attribute in \( R_i \) and copy its value to the \( A \)-column. Thus, the transformation from \( D_1 \) to \( D' \) can also be done in linear time.

### B.2 Proof of Proposition 4

**Proposition 4.** Given \((Q, T, D)\), we can compute \((Q', T, D')\) with size \( O(|D|^{\text{frtw}(T)}) \) and in time \( O(|D|^{\text{frtw}(T)}) \) such that \( Q' \) is an acyclic natural join query, \( T \) is a tree decomposition of \( Q' \), \( D' \) is globally consistent with one relation per bag in \( T \), and \( Q'(D') = Q(D) \).

The construction is standard in the literature \[1, 24\]. For convenience, we describe the main ideas.

**Construction.** The construction comprises three transformation steps. We first construct a triple \((Q'', T, D'')\) by using Proposition 3. In the second step, we compute \( R_B = Q''_B(D''_B) \) (recall that \( Q''_B \) and \( D''_B \) are \( B \)-restrictions of \( Q'' \) and \( D'' \), respectively) for each \( B \in S(T) \). Given such a relation \( R_B \), let \( R_B \) be a relation symbol with \( S(R_B) = S(R_B) \). We define \( Q'_i = \{ Q''_B \}_{B \in S(T)} \) and \( D''' = \{ R_B \}_{B \in S(T)} \) and map each relation symbol \( R_B \) to the relation \( R_B \). In the third transformation step, we execute a semi-join programme on \( D''' \) to turn it into a database \( D' \) which is pairwise consistent, i.e., \( D' \) does not contain any pair of relations such that one of them contains any tuple which cannot be joined with any tuple from the other relation. To achieve pairwise consistency, it is not necessary to consider all pairs of relations in \( D''' \). It suffices to execute a bottom-up and a subsequent top-down traversal in \( T \). During each traversal, we delete for each father-child pair \( B_1, B_2 \) of bags, all tuples in each of the two relations \( R_{B_1}, R_{B_2} \) which do not have any join partner in the other relation.

**Proof.**

**Step 1:** We first show \( Q'(D') = Q(D) \). By Proposition 3, \( Q(D) = Q''(D'') \). Since the last transformation step only deletes tuples in \( D''' \) which do not contribute to the result \( Q'(D'') \), we have \( Q''(D'') = Q'(D') \). It remains to show \( Q''(D'') = Q'(D) \). Let \( t'' = \pi_i \in [i] \in R_i \). Due to the construction in the proof of Proposition 3, there is a one-to-one mapping from relation symbols \( R_i \) in \( Q'' \) to relations \( R_i \) in \( D'' \). Likewise, due to our construction, there is a one-to-one mapping from relation symbols \( R_B \) in \( Q' \) to relations \( R_B \) in \( D'' \). For the sake of simplicity, we furthermore assume for both queries that each attribute of a relation symbol is mapped to the same attribute of the corresponding relation in the database.

We first show \( Q'_B(D''_B) \subseteq Q''(D'') \). Let \( t \in Q''(D'') \). Since \( \pi_B Q'(D'') \subseteq Q''_B(D''_B) \), it follows that \( \pi_B t \in Q''_B(D''_B) \) for each \( B \in S(T) \). Hence, \( \pi_B t \in R_B \) for each \( B \in S(T) \). This means that \( t \in Q'(D') \).
We now show $Q'(D^{''}) \subseteq Q''(D'')$. Let $t \in Q'(D^{''})$. By definition, $\pi_Bt \in R_B$ for each $B \in S(T)$. By the fact that the attributes of each relation symbol in $Q''$ are covered by at least one bag of $T$ and by the construction of the relations $R_B$, it holds that $\pi_{S(R_i)}t \in R_i$ for each $i \in [n]$. This implies $t \in Q''(D'')$.

Construction size. By Proposition 3, the size of $D''$ is linearly bounded in $|D|$. Each relation $R_B$ in $D''$ has size $O(|D_B|^{\rho^*(Q''_B)})$. Since $\text{htw}(T) = \max_{B \in S(T)}\{\rho^*(Q''_B)\}$, it follows that the size of $D''$ is $O(|D|^{\text{htw}(T)})$. The semi-join program on $D''$ does not increase the size of the database. The size of $Q'$ is $O(|Q|)$. Altogether, the size of $(Q', T, D')$ is $O(|D|^{\text{htw}(T)})$.

Construction time. By Proposition 3, $D''$ can be computed in time $O(|D|)$. Each relation $R_B$ in $D''$ is computable in time $O(|D_B|^{\rho^*(Q''_B)})$. By $\text{htw}(T) = \max_{B \in S(T)}\{\rho^*(Q''_B)\}$, we derive that the computation time for $D''$ is $O(|D|^{\text{htw}(T)})$. During the semi-join program on $D''$, we can achieve consistency between each pair $R_{B_1}, R_{B_2}$ of father-child relations as follows. We first sort both relations on the join attributes. In a subsequent scan we delete in each of the relations each tuple with no join partner in the other relation. Hence, the semi-join program can be realised in time $O(|D|^{\text{htw}(T)})$. It follows that the overall running time is $O(|D|^{\text{htw}(T)})$.

C Missing Details and Proofs of Section 3

C.1 Proof of Proposition 7

Proposition 7. Given $(Q, T, D)$, a relation $C$ with schema $\text{att}(Q)$ is result-preserving with respect to $(Q, T, D)$ if and only if $\forall_{B \in S(T)} \pi_B C = Q(D)$.

Following Proposition 8 we assume without loss of generality that $Q$ is a natural join query.

Proof of the "⇒"-direction. Assume that $C$ is result-preserving with respect to $(Q, T, D)$. We show in two steps that $\forall_{B \in S(T)} \pi_B C = Q(D)$.

- $\forall_{B \in S(T)} \pi_B C \subseteq Q(D)$: Let $t$ be an arbitrary tuple from $\forall_{B \in S(T)} \pi_B C$. This means that $\pi_Bt \in \pi_B C$ for every $B \in S(T)$. Since $C$ is result-preserving with respect to $(Q, T, D)$, we derive that $\pi_Bt \in \pi_B Q(D)$ for every $B \in S(T)$. By the definition of tree decompositions, for every relation symbol $R$ in $Q$, there is at least one bag of $T$ containing all attributes of $R$. Hence, $\pi_{S(R)}t \in \pi_{S(R)}Q(D)$ for every $R \in S(Q)$. It follows that $t$ is included in $Q(D)$. Thus, $\forall_{B \in S(T)} \pi_B C \subseteq Q(D)$.

- $Q(D) \subseteq \forall_{B \in S(T)} \pi_B C$: Let $t \in Q(D)$. It follows that $\pi_Bt \in \pi_BQ(D)$ for every $B \subseteq S(Q(D))$, hence, for every $B \in S(T)$. Due to result-preservation of $C$ with respect to $(Q, T, D)$, this implies that $\pi_Bt \in \pi_B C$ for every $B \in S(T)$ which means that $t \in \forall_{B \in S(T)} \pi_B C$. Hence, $Q(D) \subseteq \forall_{B \in S(T)} \pi_B C$.

Proof of the "⇐"-direction. Assume that $\forall_{B \in S(T)} \pi_B C = Q(D)$. Given any $B \in S(T)$, we show in two steps that $\pi_B C = \pi_B Q(D)$.

- $\pi_B C \subseteq \pi_B Q(D)$: Let $t$ be an arbitrary tuple from $\pi_B C$. This means that there is a tuple $t' \in C$ with $\pi_B t' = t$. Since $\pi_B t' \in \pi_B C$ for each $B' \in S(T)$, we derive that $t' \in \forall_{B' \in S(T)} \pi_B C$. Using our assumption $\forall_{B' \in S(T)} \pi_B C = Q(D)$, we get $t' \in Q(D)$. From the latter and the fact that $t = \pi_B t'$, it follows $t \in \pi_B Q(D)$. Altogether, we conclude $\pi_B C \subseteq \pi_B Q(D)$.

- $\pi_B Q(D) \subseteq \pi_B C$: Let $t$ be an arbitrary tuple from $\pi_B Q(D)$. This means that there is a tuple $t' \in Q(D)$ with $\pi_B t' = t$. By assumption, $t' \in \forall_{B' \in S(T)} \pi_B C$. Since $B$ is an element of $S(T)$, the latter implies $\pi_B t' = \pi_B C$. Altogether, we get $\pi_B Q(D) \subseteq \pi_B C$.

C.2 Proof of Proposition 11

Proposition 11. Given $(Q, T, D)$, a relation $C$ is a cover of $Q(D)$ over $T$ if and only if the hypergraph of $Q(D)$ over the attribute sets $S(T)$ has a minimal edge cover $M$ with $\text{rel}(M) = C$.
Let $H = (V, E)$ be the hypergraph of $Q(D)$ over $S(T)$. Let $\text{tuple}_V$ and $\text{tuple}_E$ be the restrictions of the mapping $\text{tuple}$ (as defined in Section 2) to $V$ and $E$, respectively. The mapping $\text{tuple}_V$ is a bijective mapping from $V$ to $\bigcup_{B \in S(T)} \pi_B Q(D)$. Likewise, $\text{tuple}_E$ is a bijection from $E$ to $\bigcup_{B \in S(T)} \pi_B Q(D)$ with $\text{tuple}_E(e) = \bigcup_{v \in e} \text{tuple}_V(v)$ for any $e \in E$. Furthermore, $rel$ is a bijection from subsets of $E$ to subsets of $\bigcup_{B \in S(T)} \pi_B Q(D)$ with $rel(M) = \{ \text{tuple}_E(e) \}_{e \in M}$ for any $M \subseteq E$. It suffices to show that a relation $C$ is result-preserving with respect to $(Q, T, D)$ if and only if $rel^{-1}(C)$ is an edge cover of $H$. The minimality conditions in the proposition follow from the fact that $rel^{-1}(C') \subseteq rel^{-1}(C''')$ for any $C' \subseteq C''$.

Proof of the $\Rightarrow$ direction. Let $C$ be a result-preserving relation with respect to $(Q, T, D)$. For the sake of contradiction, assume that $rel^{-1}(C) = M$ is not an edge cover of $H$. This means that $H$ contains a node $v$, which is not covered by any edge in $M$. Let $t_v = \text{tuple}_V(v)$. By construction of $H$, $t_v \in \pi_B Q(D)$ for some $B \in S(T)$. By the definition of $rel$, it follows that $C$ does not contain any tuple $t$ with $\pi_B t = t_v$, which is a contradiction to the assumption that $C$ is result-preserving with respect to $(Q, T, D)$.

Proof of the $\Leftarrow$ direction. Let $M$ be an edge cover of $H$ and $rel(M) = C$. We observe that $\text{att}(C) = \text{att}(T) = \text{att}(Q)$. For the sake of contradiction, assume that $C$ is not result-preserving with respect to $(Q, T, D)$. This means that either (i) there is a tuple $t_B \in \pi_B Q(D)$ for some $B \in S(T)$ such that $C$ does not contain any tuple $t$ with $\pi_B t = t_B$, or (ii) there is a tuple $t \in C$ with $\pi_B t \notin \pi_B Q(D)$ for some $B \in S(T)$. In case of (i), it follows from the definition of $rel$ that there is a node $\text{tuple}^{-1}(t_B) \in V$ which is not covered by any edge in $M$. This, however, is a contradiction to the assumption that $M$ is an edge cover. In case of (ii), it follows that $M$ contains an edge $\text{tuple}^{-1}(t)$ containing a node $v_B$ such that $\text{tuple}_V(v_B) \notin \pi_B Q(D)$. The latter is a contradiction to the construction of $H$.

C.3 Proof of Proposition 13

Proposition 13. Given $(Q, T, D)$, each cover of $Q(D)$ over $T$ is a subset of $Q(D)$.

Let $C$ be a cover of $Q(D)$ over $T$ and let $t \in C$ be an arbitrary tuple from $C$. We show that $t$ must be included in $Q(D)$. For each $B \in S(T)$, let $t_B = \pi_B t$. It holds that $t = \bigcup_{B \in S(T)} t_B$. As $C$ is result-preserving with respect to $(Q, T, D)$, $t_B$ must be included in $\pi_B Q(D)$ for each $B \in S(T)$. Since by Proposition 7 $Q(D) = \bigcup_{B \in S(T)} \pi_B Q(D)$, it follows that $t$ is included in $Q(D)$.

C.4 Proof of Theorem 14

Theorem 14. Let $Q$ be a join query and $T$ a tree decomposition of $Q$.

(i) For any database $D$, each cover of $Q(D)$ over $T$ has size $O(|D|^{\text{fhtw}(T)})$.

(ii) There are arbitrarily large databases $D$ such that each cover of $Q(D)$ over $T$ has size $\Omega(|D|^{\text{fhtw}(T)})$.

Our proof relies on the results that for any join query $Q$ and database $D$, it holds $|Q(D)| = O(|D|^{\rho^*(Q)})$ and there are arbitrarily large databases $D$ with $|Q(D)| = \Omega(|D|^{\rho^*(Q)})$.

Let $T = (T, \chi)$ (recall that $\chi$ is a function mapping each node in the tree $T$ to its bag $B = V^+_{\chi}$ where $V$ is a subset of the nodes of $H$). Let $(T, \chi, \{\gamma_t\}_{t \in T})$ be a fractional tree decomposition with minimal width obtained from $T$. Given a node $t'$ in $T$ with $\chi(t') = B$ for some $B$, we recall that $\text{weight}(\gamma) \ge \rho^*(Q_B)$. Moreover, if $\text{weight}(\gamma)$ is maximal over all weight functions $\{\gamma_t\}_{t \in T}$, then $\text{weight}(\gamma) = \rho^*(Q_B) = \text{fhtw}(T)$.

Proof of statement (i). Let $C$ be a cover of $Q(D)$ over $T$ and let $t$ be an arbitrary node from $T$ with $\chi(t) = B$ for some $B$. It holds $|Q_B(D_B)| = O(|D_B|^{\rho^*(Q_B)})$ [4], thus, $|Q_B(D_B)| = O(|D|^{\text{fhtw}(T)}) = O(|D|^{\text{fhtw}(T)})$. Since $|\pi_B Q(D)| = |Q_B(D_B)|$ (Proposition 3.2 of [23]), it follows that $|\pi_B Q(D)| = O(|D|^{\text{fhtw}(T)})$. Using Observation 33 we conclude $|C| \le \sum_{B \in S(T)} |\pi_B Q(D)| = O(|D|^{\text{fhtw}(T)})$.

Proof of statement (ii). Let $t$ be a node in $T$ such that $\gamma_t$ has maximal weight and let $\chi(t) = B$. There are arbitrarily large databases $D'$ such that $|Q_B(D')| = \Omega(|D'|^{\rho^*(Q_B)}) = \Omega(|D'|^{\text{fhtw}(T)})$ [4]. For each such database $D'$, there exists a database $D$ with $|D| = O(|D'|)$ and $|\pi_B Q(D)| = \Omega(|Q_B(D)|) = \Omega(|D'|^{\text{fhtw}(T)})$.

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(Lemma 7.18 of [23]). This means that there are arbitrarily large databases $D$ such that $|\pi_B Q(D)| = \Omega(|D|^{\text{htw}(T)})$. Due to Observation [33], each cover $C$ of $Q(D)$ over $T$ must be at least of size $|\pi_B Q(D)|$, hence, $|C| = \Omega(D^{\text{htw}(T)})$.

C.5 Proof of Proposition [16]

We first give a brief introduction to d-representations; for a more detailed description, we refer the reader to the original publication [23]. We then discuss a translation from covers to d-representations. Our treatment focuses on natural join queries (following Proposition 3).

$$
\begin{array}{|c|c|c|}
\hline
R_1 & R_2 & R_3 \\
A & B & B & C \\
\hline
a_1 b_1 & b_1 c_1 & c_1 d_1 \\
a_2 b_1 & b_2 c_1 & c_1 d_2 \\
a_3 b_2 & & \\
a_4 b_2 & & \\
\hline
\end{array}
\begin{array}{|c|c|c|c|}
\hline
Q(D) & C \subseteq Q(D) \\
A & B & C & D \\
\hline
a_1 b_1 c_1 d_1 & a_1 b_2 c_1 d_1 \\
a_1 b_1 c_1 d_2 & a_2 b_1 c_1 d_2 \\
a_1 b_1 c_1 d_1 & a_2 b_1 c_1 d_1 \\
a_2 b_2 c_1 d_1 & a_2 b_2 c_1 d_1 \\
a_3 b_2 c_1 d_1 & a_3 b_2 c_1 d_1 \\
a_4 b_2 c_1 d_2 & a_4 b_2 c_1 d_2 \\
\hline
\end{array}
$$

Figure 2: Database $D = \{R_1, R_2, R_3\}$, the result $Q(D)$ of the path query $Q = R_1 \bowtie R_2 \bowtie R_3$, a cover $C \subseteq Q(D)$, a tree decomposition $T$ of $Q$ and an equivalent d-tree $T'$.

D-Representations

D-representations form a factorized representation system for relational data. A d-representation is a set of named relational algebra expressions $\{N_1 := E_1, \ldots, N_n := E_n\}$, where each $N_i$ is a unique name (or a pointer) and each $E_i$ is a relational algebra expression with unions, Cartesian products, singleton relations, i.e., unary relations with one tuple, and name references in place of singleton relations. The size $|E|$ of a d-representation $E$ is the total number of its singletons, union symbols, product symbols and occurrences of expression names.

**Example 35.** Consider the two relations $\{(i, 1) \mid i \in [m]\}$ and $\{(1, j) \mid j \in [n]\}$ over the schemas $\{A, B\}$ and $\{B, C\}$, respectively, for $m, n \geq 1$. Their natural join $\{(i, 1, j) \mid i \in [m], j \in [n]\}$ has schema $\{A, B, C\}$ and is of size $m \cdot n$. A possible d-representation for this join result is the expression $\langle B : 1 \rangle \times ((A : 1 \cup \ldots \cup A : m) \times (C : 1 \cup \ldots \cup C : n))$ which is of size $O(m + n)$. Sub-expressions of the form $\langle A : i \rangle$ are singleton relations.

The nesting structure of the d-representations for query results is given by so-called d-trees. A d-tree of a query $Q$ is a special tree decomposition of $Q$ where each bag is partitioned into one attribute $A$, called
Given any $A$ and the column for the attribute maps as relations whose columns are distinctly separated into those for the key attributes (the map keys).

The parse graph follows the structure of the d-tree. At the top level we have a union of $B$-values. Then, given any $B$-value, the $A$-values are independent of the values for $C$ and $D$. Therefore, under each $B$-value, the $A$-values are represented in a different branch than the values for $C$ and $D$. Within the branches for $C$ and $D$, the values are first grouped by $C$ and then by $D$. Informations about keys allow the usage of pointers across branches which leads to further succinctness. Observe that the key of attribute $D$ consists of $C$. Therefore, all $C$-nodes with equal values point to the same union of $D$-values. In our example, both $c_1$-nodes point to the same set $\{d_1, d_2\}$ of $D$-values.

The cover $C$ from Figure 2 can be mapped immediately to the parse graph: Under each product node, we need to take a minimum number of combinations of its children that ensures that every value under the product node occurs in one of these combinations. To enumerate the tuples in the query result, it suffices to choose in turn one branch of each union node and all branches of each product node. For instance, the left product node represents the combinations of $\{a_1, a_2\}$ with $\{d_1, d_2\}$, together with the values $b_1$ and $c_1$. There are four combinations, so four tuples in the result. The first two tuples in the cover represent two of them, yet they are sufficient to cover all these tuples.

The encoding of a d-representation as a set of maps consists of one (multi)map $m_A$ for each bag attribute $A$: $m_A$ maps tuples over the attributes in $\text{key}(A)$ to (possibly several) values of $A$. Figure 3 shows these maps as relations whose columns are distinctly separated into those for the key attributes (the map keys) and the column for the attribute $A$ itself (the map payload). We have, for instance, $m_A(b_1) = a_1$ and $m_A(b_1) = a_2$, whereas $m_C(b_1) = c_1$. Since $\text{key}(A) = \{B\}$ and there are two $B$-values in the d-representation leading to the sets $\{a_1, a_2\}$ and $\{a_3, a_4\}$, respectively, $m_A$ maps the $B$-value $b_1$ to both $A$-values $a_1$ and $a_2$ and the $B$-value $b_2$ to both $A$-values $a_3$ and $a_4$. 

Figure 3: A d-representation encoded as a parse graph and as a set of multimaps.
Translating covers into d-representations

Figure 4 gives an algorithm that constructs from a cover over a tree decomposition an equivalent d-representation. Both the cover \( C \) and the output d-representation are for a query result \( Q(D) \) of a query \( Q \). The tree decomposition \( T \) is for the query \( Q \).

The algorithm creates a multimap for each attribute \( A \) and populates it with mappings from tuples over the keys of \( A \) to the values of \( A \) as encountered in the tuples of the cover.

**Example 37.** We consider the cover \( C \) over the tree decomposition \( T \) and the d-tree \( T' \), which is equivalent to \( T \), in Figure 2. According to the algorithm in Figure 4 the cover \( C \) is translated into a d-representation over \( T' \) as follows. After reading the first tuple \((a_1, b_1, c_1, d_1)\), we add

- \( () \mapsto b_1 \) to \( m_B \),
- \( b_1 \mapsto a_1 \) to \( m_A \),
- \( b_1 \mapsto c_1 \) to \( m_C \), and
- \( c_1 \mapsto d_1 \) to \( m_D \)

where \( () \) means the empty tuple. After processing the second tuple \((a_2, b_1, c_1, d_1)\), we only change \( m_A \) by adding

- \( b_1 \mapsto a_2 \) to \( m_A \).

After the third tuple \((a_3, b_2, c_1, d_2)\), we add the following new mappings:

- \( () \mapsto b_2 \) to \( m_B \),
- \( b_2 \mapsto a_3 \) to \( m_A \),
- \( b_2 \mapsto c_1 \) to \( m_C \), and
- \( c_1 \mapsto d_2 \) to \( m_D \).

After reading the last tuple \((a_4, b_2, c_1, d_2)\), we add the following new mapping:

- \( b_2 \mapsto a_4 \) to \( m_A \).
Proof of Proposition 16

Proposition 16. Given \((Q, T, D)\), any cover \(C\) of the query result \(Q(D)\) over \(T\) can be translated into a \(d\)-representation of \(Q(D)\) of size \(O(|C|)\) and in time \(O(|C|)\).

Using the algorithm in Figure 4, we construct from \(C\) and \(T\) a \(d\)-representation of \(Q(D)\) encoded as a set \(M\) of mapping. Recall that the constructed \(d\)-representation is over a \(d\)-tree \(T'\) equivalent to \(T\).

Correctness of the construction. For each \(m_A \in M\), we denote by \(R_A\) the relational encoding of \(m_A\) as presented in Figure 3. For each bag attribute \(A\) in \(T'\), the set \(\{A\} \cup \text{key}(A)\) constitutes a bag in the signature \(S(T')\) of \(T'\). We write \(B_A\) to express that the bag attribute of \(B_A\) is \(A\). By the definition of \(d\)-representations, the query result represented by the map set \(M\) is \(R = \bigcup_{B_A \in S(T')} R_A\) [23]. It remains to show that \(R = Q(D)\). By construction of the maps in \(M\), we have \(R_A = \pi_{B_A} C\) for each \(B_A\). For each \(B_A \in S(T')\), there is a \(B \in S(T)\) with \(B_A \subseteq B\) (proof of Proposition 9.3 of [23]). Hence, by the definition of covers, we have \(R_A = \pi_{B_A}(C) = \pi_{B_A} Q(D)\) for each \(B_A\). As \(T'\) is a valid tree decomposition of \(Q\), it follows from Proposition 16 that \(\bigcup_{B_A \in S(T')} \pi_{B_A} C = Q(D)\). Since for each \(B_A\), we have \(\pi_{B_A} C = R_A\) and \(R = \bigcup_{B_A \in S(T')} R_A\), it follows \(R = Q(D)\).

Construction size and translation time. The number of the maps in \(M\) is bounded by the number of attributes in \(C\). We consider the cover \(C\) sorted using a topological order of the tree decomposition \(T'\), so that inserts into the multimaps become appends (alternatively, inserts in sorted order would take logarithmic time in the number of entries). For each tuple in \(C\) we insert at most one tuple in the multimap of each attribute. Thus, the overall size of the set of multimaps, and thus of the \(d\)-representation, is \(O(|C|)\) with respect to data complexity (the linear factor in the number of attributes is ignored). The data complexity of the overall translation time is thus \(O(|C|)\).

D Missing Details and Proofs of Section 4

Following the prerequisites in Section 4, the proofs of Proposition 19 and Lemma 22 assume that in any given triple \((Q, T, D)\), the query \(Q\) is an acyclic natural join query, \(D\) is a globally consistent database with one relation per bag in \(T\) and there is a one-to-one mapping from relation symbols \(R\) in \(Q\) to relations \(R\) in \(D\). For the sake of simplicity, we further assume without loss of generality that each relation symbol \(R\) in \(Q\) has the same set of attributes as \(R\) and each attribute in \(R\) is mapped to the same attribute in \(R\).

In the following, we say that two relations \(R_1\) and \(R_2\) are consistent if the database \(\{R_1, R_2\}\) is globally consistent with respect to the query joining \(R_1\) and \(R_2\).

D.1 Proof of Proposition 19

Proposition 19. Given \((Q, T, D)\) where \(Q = R_1 \bowtie R_2\), \(S(T) = \{S(R_1), S(R_2)\}\), and \(D = \{R_1, R_2\}\), there is a cover \(C\) of \(Q(D)\) over \(T\) that can be computed in time \(\tilde{O}(|R_1| + |R_2|)\) and has size \(\max\{|R_1|, |R_2|\} \leq |C| \leq |R_1| + |R_2| - 1\).

By Proposition 11, a relation \(C\) is a cover of \(Q(D)\) over \(T\) if and only if the hypergraph \(H\) of \(Q(D)\) over the attribute sets \(\{S(R_1), S(R_2)\}\) has a minimal edge cover \(M\) with \(\text{rel}(M) = C\). The hypergraph \(H\) is a collection of disjoint complete bipartite subgraphs. The set of nodes of each such subgraph corresponds to a maximal subset of tuples of the input relations agreeing on the join attributes. A minimal edge cover of \(H\) is a collection of minimal edge covers for these subgraphs. We construct a cover \(C\) of minimum size such that each maximal subset of tuples in \(C\) agreeing on the join attributes corresponds to a minimal edge cover of one of the complete bipartite subgraphs of \(H\).

Construction. Let \(A\) be the set of common attributes of \(R_1\) and \(R_2\). For \(i \in \{1, 2\}\) and \(t \in \pi_A R_i\), we call \(\sigma_{A=t} R_i\) the \(t\)-block in \(R_i\) and denote its size by \(n'_t\). Since \(R_1\) and \(R_2\) are consistent, for each \(t\)-block in \(R_1\), there must be a corresponding \(t\)-block in \(R_2\), and vice-versa. First, the algorithm sorts \(R_1\) and \(R_2\) with respect to the values of the attributes in \(A\). After sorting, the \(t\)-blocks occur in the same order in both
relations. The cover \( C \) is constructed by performing the following procedure for each pair of corresponding t-blocks in \( R_1 \) and \( R_2 \). Without loss of generality, assume \( n_1^j \geq n_2^j \). For each \( j, j \)-th tuple \( t' \) in the t-block of \( R_1 \) is combined with the \( j \)-th tuple \( t'' \) in the t-block of \( R_2 \) resulting in a new tuple \( t' \times t'' \). Then, all remaining tuples in the t-block of \( R_1 \) are combined with the \( n_2^j \)-th tuple in the t-block of \( R_2 \). All new tuples are added to \( C \).

**Construction time.** The sorting phase can be realised in time \( \tilde{O}(|R_1| + |R_2|) \). The phase for constructing the new tuples can be done in one pass over the sorted relations. Hence, the overall running time of the described algorithm is \( O(|R_1| + |R_2|) \).

**Size of the Cover.** The size bounds \( \max\{|R_1|, |R_2|\} \leq |C| \leq |R_1| + |R_2| - 1 \) follow from Observation 33 and the assumption that \( R_1 \) and \( R_2 \) are consistent, so we have \( \pi_{S(Q)}(D) = R_i \) for each \( i \in \{1, 2\} \).

Our algorithm above constructs a specific cover. Other covers can be constructed within the same time bounds. We exemplify the construction of some further covers following different patterns. In our construction above, after combining the first \( n_2^j - 1 \) tuples in the t-block of \( R_1 \) with the first \( n_2^j - 1 \) tuples in the t-block of \( R_2 \), we combined the last tuple in the t-block of \( R_2 \) with all remaining tuples in the t-block of \( R_1 \). Alternatively, we can fix any tuple \( t' \) in the t-block of \( R_2 \), combine the first \( n_2^j - 1 \) tuples in the t-block of \( R_1 \) with all tuples besides \( t' \) in the t-block of \( R_2 \) and then combine the remaining tuples in the t-block of \( R_1 \) with \( t' \).

### D.2 Proof of Lemma 22

**Lemma 22.** Given \((Q, T, D)\) with \( D = \{R_i\}_{i \in [n]} \), any cover-join plan for \( Q \) over \( T \) computes a cover \( C \) of \( Q(D) \) over \( T \) in time \( \tilde{O}(|C|) \) and with size \( \max_{i \in [n]} |\pi_{R_i} Q| \leq |C| \leq \sum_{i \in [n]} |R_i| - |S(T)| + 1 \).

Any cover-join plan for \( Q \) over \( T \) computes a cover \( C \) of \( Q(D) \) over \( T \). Let \( \varphi \) be a cover-join plan for \( Q \) over \( T \). We show by induction on the structure of \( \varphi \) that for any sub-plan \( \varphi' \) of \( \varphi \), the following holds: If there is a sub-tree \( T' \) of \( T \) and a sub-query \( Q' \) of \( Q \) such that \( \varphi' \) is a cover-join plan for \( Q' \) over \( T' \), then, \( \varphi' \) returns a cover of \( Q'(\{R\}_{R \in S(Q')}) \) over \( T' \).

For the base case, assume that \( \varphi' \) consists of a single relation symbol \( R \). Then, \( \varphi' \) returns the relation \( R \). By Definition 8, \( R \) is the unique cover of the result of the identity query \( R \) over the tree decomposition consisting of a single bag with the attributes of \( R \).

Assume now that \( \varphi' \) is of the form \( \varphi_1 \Pi \varphi_2 \). By definition of cover-join plans, there are subtrees \( T_1 \) and \( T_2 \) of \( T \) and sub-queries \( Q_1 \) and \( Q_2 \) of \( Q \) such that \( T = T_1 \circ T_2 \), \( Q = Q_1 \bowtie Q_2 \), each \( T_i \) is a tree decomposition of \( Q_i \), and each \( \varphi_i \) is a cover-join plan for \( Q_i \) over \( T_i \). By induction hypothesis, each \( \varphi_i \) returns a cover \( C_i \) of \( Q_i(D_i) \) over \( T_i \) where \( D_i = \{R\}_{R \in S(Q_i)} \). Due to our prerequisites in Section 3, the cover-join operator, as defined in Definition 15, is only applicable on consistent relations. In case that \( C_1 \) and \( C_2 \) are consistent, \( \varphi' \) returns a cover \( C \) of the join result \( C_1 \bowtie C_2 \) over the tree decomposition with bags \( S(C_1) \) and \( S(C_2) \). We proceed as follows. First, we show that \( C_1 \) and \( C_2 \) are consistent. Then, we prove that the cover-join result \( C \) of \( C_1 \) and \( C_2 \) must be a cover of \( Q_1 \bowtie Q_2(D_1 \cup D_2) \) over \( T \circ T_2 \), that is, \( C \) is result-preserving with respect to \( (Q_1 \bowtie Q_2, T_1 \bowtie T_2, D_1 \cup D_2) \) and it is minimal in this respect.

- \( C_1 \) and \( C_2 \) are consistent: Let \( B_1 \) and \( B_2 \) be the two bags incident to the single edge connecting \( T_1 \) and \( T_2 \) and let \( B \) be the set of common attributes of these bags. Let \( A \) be the set of common attributes of \( C_1 \) and \( C_2 \). We first show that \( A \subseteq B \). Let \( A \notin A \). Since each \( C_i \) is a cover over \( T_i \), there must be at least one bag in \( T_1 \) and at least one bag in \( T_2 \) containing \( A \). Due to the connectivity property of tree decompositions, \( A \) must occur in all bags along the single path between these bags. Since \( B_1 \) and \( B_2 \) are on this path, both must include \( A \). Hence, \( A \subseteq B \).

Due to our assumption that \( D \) is globally consistent and contains one relation per bag of \( T \), there must be two consistent relations \( R_1 \in D_1 \) and \( R_2 \in D_2 \) such that \( R_1 \) covers the attributes in \( B_1 \) and \( R_2 \) covers the attributes in \( B_2 \). Since, furthermore, each \( C_i \) is result-preserving with respect to \( (Q_i, T_i, D_i) \), it holds \( \pi_B C_1 = \pi_B R_1 = \pi_B R_2 = \pi_B C_2 \). As \( A \subseteq B \), the relations \( C_1 \) and \( C_2 \) must be consistent.
• C is result-preserving with respect to \((Q_1 \bowtie Q_2, T_1 \bowtie T_2, D_1 \cup D_2)\): Let \(R\) be an arbitrary relation in \(D_1 \cup D_2\). Without loss of generality, assume that \(R \in D_1\) (the other case is handled along the same lines). Since, by induction hypothesis, \(C_1\) is result-preserving with respect to \((Q_1, T_1, D_1)\) and the database is globally consistent, we have \(R = \pi_{S(R)}C_1\). Since the cover-join result \(C\) of \(C_1\) and \(C_2\) preserves \(C_1\) and \(C_2\), we have \(R = \pi_{S(R)}C\).

• \(C\) is a minimal result-preserving relation with respect to \((Q_1 \bowtie Q_2, T_1 \bowtie T_2, D_1 \cup D_2)\): For the sake of contradiction, assume that \(C\) is not minimal in this respect. This means that there is a tuple \(t^-\in C\) such that \(C' = C \setminus \{t^-\}\) is still result-preserving with respect to \((Q_1 \bowtie Q_2, T_1 \bowtie T_2, D_1 \cup D_2)\). It follows that \(\pi_{S(C')}C'\) is result-preserving with respect to \((Q_1, T_1, D_1)\) for each \(i \in [2]\). Observe that the minimal edge cover \(M\) with \(rel(M) = C\) in the hypergraph of \(C_1 \bowtie C_2\) over the attribute sets \(\{S(C_1), S(C_2)\}\) must contain an edge \(e^-\) connecting \(\pi_{S(C_1)}t^-\) and \(\pi_{S(C_2)}t^-\). This implies that \(M\) cannot have two further edges \(e_1\) and \(e_2\) such that \(e_1\) covers \(\pi_{S(C_1)}t^-\) and \(e_2\) covers \(\pi_{S(C_2)}t^-\). Indeed, in this case, \(M' = \{e^-\}\) would be an edge cover, contradicting the minimality of \(M\). Hence, there is no tuple \(t \neq t^-\) in \(C\) with \(\pi_{S(C)}t = \pi_{S(C_2)}t\) or there is no tuple \(t \neq t^-\) in \(C\) with \(\pi_{S(C_2)}t = \pi_{S(C_1)}t\). It follows that \(\pi_{S(C_1)}(C \setminus \{t^-\}) \subset \pi_{S(C_1)}C\) or \(\pi_{S(C_2)}(C \setminus \{t^-\}) \subset \pi_{S(C_2)}C\). Using the consistency of \(C_1\) and \(C_2\), we obtain \(\pi_{S(C_1)}(C \setminus \{t^-\}) \subset \pi_{S(C_1)}C = C_1\) or \(\pi_{S(C_2)}(C \setminus \{t^-\}) \subset \pi_{S(C_2)}C = C_2\). However, as we noticed that \(\pi_{S(C)}(C \setminus \{t^-\})\) is result-preserving with respect to \((Q_1, T_1, D_1)\) for each \(i \in [2]\), the statement of the last sentence contradicts the induction hypothesis that each \(C_i\) is a minimal result-preserving relation with respect to \((Q_i, T_i, D_i)\).

**Size of \(C\).** From the global consistency of \(D\) and Observation 33, it follows for any cover \(C\) of \(Q(D)\) over \(T\) that \(\max_{i \in [n]} |R_i| \leq |C| \leq \sum_{i \in [n]} |R_i| - |S(T)| + 1\).

**Computation time for \(C\).** By Proposition 19 we can design an algorithm for the cover-join operator which for every two input covers \(C_1\) and \(C_2\), computes a cover-join result of size \(O(|C_1| + |C_2|)\) and in time \(O(|C_1| + |C_2|)\). Hence, given a triple \((Q, T, D)\) and a cover-join plan \(\varphi\) for \(Q\) over \(T\), starting from the innermost expressions of \(\varphi\), we can compute a cover \(C\) of \(Q(D)\) over \(T\) in time \(O(|C|)\).

### D.3 Cover-Join Plans Delivering Covers of Non-Minimum Size

**Example 38** (Plans delivering non-minimum covers). We consider the natural join query \(Q = R_1 \bowtie R_2 \bowtie R_3\), the globally consistent database \(D = \{R_1, R_2, R_3\}\) where each \(R_i\), \(i \in \{1, 2, 3\}\), is the instantiation of \(R_i\), and the tree decomposition \(T\) with three bags \(\{A, B\}, \{B, C\}\) and \(\{C, D\}\).

| \(R_1\) | \(R_2\) | \(R_3\) | \(C\) | \(C_{12}\) | \(C'\) |
|---|---|---|---|---|---|
| \(A\) | \(B\) | \(C\) | \(D\) | \(A\) | \(B\) | \(C\) | \(D\) | \(A\) | \(B\) | \(C\) | \(D\) |
| \(a_1\) \(b_1\) | \(b_1\) \(c_1\) | \(c_1\) \(d_1\) | \(a_1\) \(b_1\) \(c_1\) \(d_1\) | \(a_1\) \(b_1\) \(c_1\) | \(a_1\) \(b_1\) \(c_1\) \(d_1\) |
| \(a_2\) \(b_1\) | \(b_1\) \(c_2\) | \(c_2\) \(d_2\) | \(a_2\) \(b_1\) \(c_1\) \(d_1\) | \(a_2\) \(b_1\) \(c_1\) | \(a_2\) \(b_1\) \(c_1\) \(d_1\) |
| \(a_3\) \(b_1\) | \(b_1\) \(c_2\) | \(c_2\) \(d_2\) | \(a_3\) \(b_1\) \(c_2\) \(d_2\) | \(a_3\) \(b_1\) \(c_2\) | \(a_3\) \(b_1\) \(c_2\) \(d_2\) |

The relation \(C\) is a cover of \(Q(D)\) over \(T\). It follows from Observation 33 that every cover of \(Q(D)\) over \(T\) must have size at least three. Hence, \(C\) is a minimum-sized cover of \(Q(D)\) over \(T\).

We take the cover-join plan \((R_1 \bowtie R_2) \bowtie R_3\) for \(Q\) over \(T\) and assume that the cover-join operator computes for each two input relations \(R\) and \(R'\), a minimum-sized cover of \(R \bowtie R'\) over the tree decomposition with bags \(S(R)\) and \(S(R')\). Then, a possible output of the sub-plan \((R_1 \bowtie R_2)\) is the relation \(C_{12}\). A possible result of the cover-join of the latter relation with \(R_3\) is the relation \(C'\). While \(C'\) is a valid cover of \(Q(D)\) over \(T\), it is not a minimum-sized cover of \(Q(D)\) over \(T\).
D.4 Missing Details and Proofs in Section 5

For the rest of this section we fix an FAQ $\varphi$ as written in [1]. Given the hypergraph $H$ of the query and an attribute set $U$, we denote by $H_U$ the hypergraph obtained from $H$ by restricting each hyperedge in $H$ to the attributes in $U$.

D.4.1 Recap on FAQs

Indicator projections are used in the InsideOut algorithm [17] which solves the FAQ-problem. They will also occur in our construction of FAQ-covers.

**Definition 39 (Indicator projections).** Given two attribute sets $S$ and $T$ with $S \cap T \neq \emptyset$ and a function $\psi_S$, the function $\psi_{S/T} : \prod_{A \in (S \cap T)} \text{dom}(A) \rightarrow \text{Dom}$ defined by

$$\psi_{S/T}(a_{S\cap T}) = \begin{cases} 1 & \exists b_S \text{ s.t. } \psi_S(b_S) \neq 0 \text{ and } a_{S\cap T} = b_{S\cap T}, \\ 0 & \text{otherwise} \end{cases}$$

is called the indicator projection of $\psi_S$ onto $T$.

In particular, if $S \subseteq T$, then $\psi_{S/T}(a_S) = 1$ if and only if $\psi_S(a_S) \neq 0$.

**Equivalent attribute orderings.** A $\varphi$-equivalent attribute ordering $\tau = \tau(1), \ldots, \tau(n)$ is a permutation of the indices of the attributes in $\mathcal{V}$ satisfying the following conditions:

(a) $\{A_{\tau(1)}, \ldots, A_{\tau(f)}\} = \{A_1, \ldots, A_f\}$ and

(b) $\varphi'(a_{\{A_{\tau(1)}, \ldots, A_{\tau(f)}\}}) = \bigoplus_{a_{\tau(f+1)} \in \text{dom}(A_{\tau(f+1)})} \ldots \bigoplus_{a_{\tau(n)} \in \text{dom}(A_{\tau(n)})} \otimes_{S \in \mathcal{E}} \psi_S(a_S)$

is equivalent to $\varphi$ irrespective of the definition of the input functions $\psi_S$.

We denote by $\text{EVO}(\varphi)$ the set of all $\varphi$-equivalent attribute orderings.

**The InsideOut algorithm** Given an FAQ $\varphi$ and a $\varphi$-equivalent attribute ordering, the InsideOut algorithm computes the relational encoding of $\varphi$. The algorithm first rewrites the query according to the given attribute ordering and then processes the resulting query in two phases: *bound attribute elimination* and *output computation*. We sketch the main steps of the algorithm on input $\varphi$ and the attribute ordering which corresponds to the identity permutation. Thus, the initial rewriting step does not change the structure of $\varphi$.

In the bound attribute elimination phase, the algorithm eliminates attributes $A_{f+1}, \ldots, A_n$ along with their corresponding aggregate operators in reverse order. When eliminating an attribute $A_j$ it distinguishes between the cases whether $\bigoplus^{(j)}$ is different from $\otimes$ or not. We demonstrate the two cases in the elimination step for $A_n$. In case that $\bigoplus^{(n)}$ is different from $\otimes$, the algorithm first rewrites the query as follows

$$\bigoplus_{a_{f+1} \in \text{dom}(A_{f+1})} \ldots \bigoplus_{a_n \in \text{dom}(A_n)} \otimes_{S \in \mathcal{E}} \psi_S(a_S)$$

$$= \bigoplus_{a_{f+1} \in \text{dom}(A_{f+1})} \ldots \bigoplus_{a_{n-1} \in \text{dom}(A_{n-1})} \otimes_{S \in \mathcal{E} \setminus \partial(n)} \psi_S(a_S) \otimes \left( \bigoplus_{a_n \in \text{dom}(A_n)} \otimes_{S \in \partial(n)} \psi_S(a_S) \right),$$

where $\partial(n) = \{S \in \mathcal{E} \mid A_n \notin S\}$ and $U_n = \bigcup_{S \in \partial(n)} S$. The correctness of the rewriting follows from the distributivity of $\otimes$ over $\bigoplus^{(n)}$. Then, the algorithm computes the relational encoding of a function $\psi_{U_n \setminus \{A_n\}}'$ such that replacing $\delta$ by $\psi_{U_n \setminus \{A_n\}}'$ does not change the semantics of $\varphi$. Observe that the cartesian product of the domains of the attributes in $U_n \setminus \{A_n\}$ can contain tuples $a_{U_n \setminus \{A_n\}}$ such that (i) there is a $\psi_S$ with
\( S \in \mathcal{E}\setminus \partial(n) \), \( S \cap (U_n \setminus \{A_n\}) \neq \emptyset \) and (ii) there is no \( b_S \) which agrees with \( a_{U_n \setminus \{A_n\}} \) on the common attributes and \( \psi_S(b_S) \neq 0 \). Such tuples will not occur in the final result. To rule them out in advance, indicator projections are used inside \( \psi'_{U_n \setminus \{A_n\}} \). The function \( \psi'_{U_n \setminus \{A_n\}} \) is precisely defined as

\[
\psi'_{U_n \setminus \{A_n\}}(a_{U_n \setminus \{A_n\}}) = \bigoplus_{a_n \in \text{dom}(A_n)} (n) \left[ \bigotimes_{S \in \partial(n)} \psi_S(a_S) \right] \otimes \left[ \bigotimes_{S \notin \partial(n)} \psi_S/_{U_n} (a_S \cap U_n) \right].
\]

The computation of the relational encoding of this function requires the computation of the join of the relational encodings of the functions \( \psi_S \) with \( S \in \partial(n) \) and the indicator projections. The computation time for this elimination step is \( \widetilde{O}(|D_e|^{\varphi(H_{U_n})}). \)

In case that \( \bigoplus (n) \) is equal to \( \bigotimes \), the formula is rewritten as follows

\[
\bigoplus_{a_{f+1} \in \text{dom}(A_{f+1})} \cdots \bigoplus_{a_n \in \text{dom}(A_n)} \bigotimes_{S \in \mathcal{E}} \psi_S(a_S)
\]

\[
= \bigoplus_{a_{f+1} \in \text{dom}(A_{f+1})} \bigotimes_{a_{n-1} \in \text{dom}(A_{n-1})} \bigotimes_{a_n \in \text{dom}(A_n)} \bigotimes_{S \in \mathcal{E}} \psi_S(a_S)
\]

\[
= \bigoplus_{a_{f+1} \in \text{dom}(A_{f+1})} \bigotimes_{a_{n-1} \in \text{dom}(A_{n-1})} \bigotimes_{a_n \in \text{dom}(A_n)} \bigotimes_{S \in \partial(n)} \psi_S(a_S), \quad S \notin \partial(n)
\]

where \( \partial(n) \) is defined as above. Then, the algorithm computes for each \( S \notin \partial(n) \), a function \( \psi'_S \) equivalent to \( \psi^{|\text{dom}(A_n)|}_S \) and for each \( S \in \partial(n) \), a function \( \psi^{|\text{dom}(A_n)|}_{S \setminus \partial(n)} \) equivalent to \( \delta^S \). This elimination step can be realised in time \( \widetilde{O}(|D_e|) \).

After the elimination of all bound attributes we are left with a formula \( \varphi'_{A_1, \ldots, A_f} \) without any bound attributes. In the output computation phase the algorithm first computes (a factorized representation of) the set of tuples \( a_{A_1, \ldots, A_f} \) for which \( \varphi'_{A_1, \ldots, A_f}(a_{A_1, \ldots, A_f}) \neq 0 \) and then reports the output.

Before giving the overall running time of \textsc{InsideOut}, we introduce elimination hypergraph sequences corresponding to attribute orderings.

**Elimination hypergraph sequence** Given a \( \varphi \)-equivalent attribute ordering \( \tau = \tau(1), \ldots, \tau(n) \), we recursively define the elimination hypergraph sequence \( H^1, \ldots, H^T \) associated with \( \tau \). For each \( j \) with \( n \geq j \geq 1 \), we additionally define two sets \( U_j \) and \( \partial^T(j) \). In the following, for the sake of readability, we skip the superscript \( \tau \) in our notation.

We set \( H_n = (V_n, \mathcal{E}_n) = H \) and define \( \partial(n) = \{ S \in \mathcal{E}_n \mid A_{\tau(n)} \in S \} \) and \( U_n = \bigcup_{S \in \partial(n)} S \).

For each \( j \) with \( n - 1 \geq j \geq 1 \), we define:

- If \( \bigoplus (\tau(j+1)) = \bigotimes \), then, \( V_j = \{A_{\tau(1)}, \ldots, A_{\tau(j)}\} \) and \( \mathcal{E}_j \) is obtained from \( \mathcal{E}_{j+1} \) by removing \( A_{\tau(j+1)} \) from all edges in \( \mathcal{E}_{j+1} \).

- Otherwise, \( V_j = \{A_{\tau(1)}, \ldots, A_{\tau(j)}\} \) and \( \mathcal{E}_j = (\mathcal{E}_{j+1} \setminus \partial(j+1)) \cup (U_{j+1} \setminus \{A_{\tau(j+1)}\}) \).

We further set \( \partial(j) = \{ S \in \mathcal{E}_j \mid A_{\tau(j)} \in S \} \) and \( U_j = \bigcup_{S \in \partial(j)} S \).

**Running time of \textsc{InsideOut}** For a \( \varphi \)-equivalent attribute ordering \( \tau \), let \( K = [f] \cup \{ j \mid j > f, (\tau(j)) \neq \bigotimes \} \). The FAQ-width of \( \tau \) is defined as \( \text{faqw}(\tau) = \max_{j \in K} \{ \rho^*(H_{U_j}) \} \). For a given \( \tau \), \textsc{InsideOut} runs in time \( \widetilde{O}(|D_e|^{\text{faqw}(\tau)} + Z) \) where \( Z \) is the size of the output. The FAQ-width of \( \varphi \) is defined as \( \text{faqw}(\varphi) = \min_{\tau \in \text{EVO}(\varphi)} \{ \text{faqw}(\tau) \} \). Hence, given the best attribute ordering (i.e., with smallest FAQ-width), the running time of \textsc{InsideOut} is \( \widetilde{O}(|D_e|^{\text{faqw}(\varphi)} + Z) \).
From attribute orderings to tree decompositions  We say that \( T \) is a tree decomposition of \( \varphi \) if \( T \) is a tree decomposition of the hypergraph \( H \) of \( \varphi \).

Proposition 40 \cite{IS}, Proposition C.2. For any FAQ \( \varphi \) without bound attributes and any \( \varphi \)-equivalent attribute ordering \( \tau \), one can construct a tree decomposition \( T \) of \( \varphi \) with \( \text{fhtw}(T) \leq \text{faqw}(\tau) \).

**D.4.2 Covers for FAQs**

Given two input functions \( \psi_S \) and \( \psi_T \) with \( T \subseteq S \), we can always compute the function \( \psi_S' = \psi_S \otimes \psi_T \) in time \( \mathcal{O}(|R_{\psi_S}| + |R_{\psi_T}|) \) and replace \( \psi_S \otimes \psi_T \) by \( \psi_S' \) without changing the semantics of the FAQ. To do this, we first sort the relational encodings \( R_{\psi_S} \) and \( R_{\psi_T} \) of \( \psi_S \) and \( \psi_T \) on the attributes in \( T \). During a subsequent scan through both relations we add for each pair \( a_{S \cup \{\psi_S(S)\}} \in R_{\psi_S} \) and \( b_{T \cup \{\psi_T(T)\}} \in R_{\psi_T} \) with \( a_T = b_T \), the tuple \( c_{S \cup \{\psi_S(S)\}} \) with \( c_S = a_S \) and \( c_{\psi_S(S)} = \psi_S(a_S) \otimes \psi_T(b_T) \) to the relational encoding of \( \psi_S' \). Hence, in the following we assume, without loss of generality, that \( \varphi \) does not contain any function whose attributes are included in the attribute set of another function.

**Bag functions** Given an FAQ \( \varphi \) without bound attributes and a tree decomposition of \( \varphi \), we define bag functions which are the counterparts of bag relations in case of join queries. Our goal is to define for each bag \( B \) of \( T \), a function \( \beta_B \) such that \( \varphi(a_V) = \bigotimes_{B \in S(T)} \beta_B(a_B) \). While in case of join queries it is harmless to include all relations sharing attributes with \( B \) into the join computing the bag relation of \( B \), in case of FAQs we have to be a bit careful. Including the same input function into the computation of bag functions of several bags can violate the above equality. Therefore, in the definition below we use a mapping from input functions to bags. To keep the sizes of the bag functions small we also use indicator projections which achieve pairwise consistency between relational encodings of bag functions sharing attributes.

**Definition 41 (Bag functions).** Given an FAQ \( \varphi \) without bound attributes and a tree decomposition \( T \) of \( \varphi \), a set \( \{\beta_B\}_{B \in S(T)} \) is called a set of bag functions for \( \varphi \) and \( T \) if there is a mapping \( m : E \rightarrow S(T) \) such that \( S \subseteq m(S) \) for each \( S \in E \) and \( \beta_B \) is defined by

\[
\beta_B(a_B) = \bigotimes_{S \in E : S \cap B \neq \emptyset} \psi_{S/B}(a_B \setminus S) \otimes \bigotimes_{S \in m(S) = B} \psi_S(a_S)
\]

for each \( B \in S(T) \).

We define \( B(\varphi, T) = \{\beta_B\}_{B \in S(T)} \mid \{\beta_B\}_{B \in S(T)} \text{ is a set of bag functions for } \varphi \text{ and } T \} \).

Note that since each hyperedge in the hypergraph of \( \varphi \) must be included in at least one bag of the tree decomposition, one can always find a mapping \( m \) meeting the condition given in the above definition. Observe also that for bags \( B \) to which no input function is mapped, the function \( \beta_B \) is just the product of indicator projections of all \( \psi_S \) sharing attributes with \( B \) onto \( B \).

**Observation 42.** Given an FAQ \( \varphi \) without bound attributes, a tree decomposition \( T \) of \( \varphi \) and a set \( \{\beta_B\}_{B \in S(T)} \in B(\varphi, T) \), it holds

\[
\varphi(a_V) = \bigotimes_{B \in S(T)} \beta_B(a_B).
\]

Given \( \{\beta_B\}_{B \in S(T)} \in B(\varphi, T) \), we denote by \( \text{ext}(T, \{\beta_B\}_{B \in S(T)}) \) the tree decomposition obtained from \( T \) by adding into each bag \( B \) the attribute \( \beta_B(B) \). Observe that if \( T \) is a tree decomposition of \( \varphi(a_V) = \bigotimes_{B \in S(T)} \beta_B(a_B) \), then \( \text{ext}(T, \{\beta_B\}_{B \in S(T)}) \) is a tree decomposition of the query joining the relational encodings of the functions \( \beta_B \). Moreover, \( T \) and \( \text{ext}(T, \{\beta_B\}_{B \in S(T)}) \) have the same fractional hypertree width.

**Covers of FAQ results** We again turn towards the general case where FAQs can contain bound attributes also. Let \( \tau = \tau_1 \tau_2 \) be a \( \varphi \)-equivalent attribute ordering where \( \tau_1 \) consists of the free and \( \tau_2 \) consists of the bound attributes in \( \varphi \). By \( \tau_{\text{tree}} \) we denote the FAQ constructed by the InsideOut algorithm after eliminating
all bound attributes in \( \varphi \) according to the ordering \( \tau_2 \). We write \((\varphi, \tau, T)\) to express that \( \varphi \) is an FAQ, \( \tau \) is a \( \varphi \)-equivalent attribute ordering and \( T \) is a tree decomposition of \( \varphi_{\text{free}}^{\neg} \) with \( \text{fhtw}(T) \leq \text{faqw}(\tau_1) \). Note that due to Proposition 40 such a tree decomposition is always constructible.

**Definition 43** (Covers of FAQ results). Given \((\varphi, \tau, T)\), a relation \( C \) is a cover of the results of \( \varphi \) over \( T \) induced by \( \tau \) if there is a set \( \{ \beta_B \}_{B \in S(T)} \in B(\varphi_{\text{free}}^{\neg}, T) \) such that \( C \) is a cover of the join of the relations \( \{ R_{\beta_B} \}_{B \in S(T)} \) over \( \text{ext}(T, \{ \beta_B \}_{B \in S(T)}) \).

We call \( \{ \beta_B \}_{B \in S(T)} \) the set of bag functions underlying \( C \).

Observe that if \( C \) is a cover over \( T \) with underlying bag functions \( \{ \beta_B \}_{B \in S(T)} \), then, \( \pi_{\text{free}} C \) must be a cover of \( \pi_{\text{free}} R_{\beta_B} \) over \( T \).

The following Proposition relies on Lemma 22 and Theorem 24 which give an upper bound on the time complexity for constructing covers of join results.

**Proposition 44.** Given \((\varphi, \tau, T)\), a cover of the result of \( \varphi \) over \( T \) induced by \( \tau \) can be computed in time \( \tilde{O}(|D_{\varphi}|^{\text{faqw}(\tau_1)}) \).

**Proof. Construction.** Let \( \tau = \tau_1 \tau_2 \) where \( \tau_1 \) consists of the free and \( \tau_2 \) consists of the bound attributes in \( \varphi \). We first run InsideOut on \( \varphi \) according to the attribute ordering \( \tau_2 \) until all bound attributes are eliminated and we get \( \varphi_{\text{free}}^{\neg} \). Then, we construct a set \( \{ \beta_B \}_{B \in S(T)} \in B(\varphi_{\text{free}}^{\neg}, T) \) of bag functions. Finally, using a cover-join plan as introduced in Definition 21 we construct a cover \( C \) of the join of the relations \( \{ R_{\beta_B} \}_{B \in S(T)} \) over \( \text{ext}(T, \{ \beta_B \}_{B \in S(T)}) \).

**Construction time.** The FAQ \( \varphi_{\text{free}}^{\neg} \) can be computed in time \( \tilde{O}(|D_{\varphi}|^{\text{faqw}(\tau_2)}) \) and relational encodings of the input functions in \( \varphi_{\text{free}}^{\neg} \) have size \( \tilde{O}(|D_{\varphi}|^{\text{faqw}(\tau_1)}) \) \[17\]. The construction of the bag functions \( \{ \beta_B \}_{B \in S(T)} \) can be realised via the computation of the bag relations of \( T \). By Proposition 1 the size of the relational encodings of these bag functions is \( \tilde{O}(|D_{\varphi}|^{\text{fhtw}(T)}) \) and their computation time is \( \tilde{O}(|D_{\varphi}|^{\text{fhtw}(T)}) \). By Theorem 22 \( C \) can be computed in time \( \tilde{O}(|\cup_{B \in S(T)} R_{\beta_B}|) \). Hence, the time for computing \( C \) from \( \varphi_{\text{free}}^{\neg} \) is \( \tilde{O}(|D_{\varphi}|^{\text{fhtw}(T)}) \).

Since \( \text{faqw}(\tau) = \max_{1 \leq i \leq 2} \{ \text{faqw}(\tau_i) \} \) and \( \text{fhtw}(T) \leq \text{faqw}(\tau_1) \) (by construction), the overall computation time is \( \tilde{O}(|D_{\varphi}|^{\text{faqw}(\varphi)}) \).

**Theorem 29** is an immediate corollary:

**Theorem 29.** For any FAQ \( \varphi \), a cover of the result of \( \varphi \) can be computed in time \( \tilde{O}(|D_{\varphi}|^{\text{faqw}(\varphi)}) \).

### 4.3.3 Enumeration of tuples in FAQ results using covers

Any enumeration algorithm on covers of join results can easily be turned into an enumeration algorithm on covers of FAQ-results. Assume that \( C \) is a cover of the result of the FAQ \( \varphi \) over some tree decomposition \( T \) (induced by some attribute ordering). Let \( \{ \beta_B \}_{B \in S(T)} \) be the underlying set of bag functions. Recall that the set of attributes of \( C \) is \( V_{\text{free}} \cup \{ \beta_B(B) \}_{B \in S(T)} \) and the set of attributes of the relational encoding of \( \varphi \) must be \( V_{\text{free}} \cup \{ \varphi(V_{\text{free}}) \} \). To enumerate the relational encoding of \( \varphi \), we can run any enumeration algorithm on \( C \) with respect to \( \text{ext}(T, \{ \beta_B \}_{B \in S(T)}) \) and adapt its output as follows. For each output tuple \( \mathbf{v}_{\text{free}} \cup \{ \beta_B(B) \}_{B \in S(T)} \), we output the tuple \( \mathbf{b}_{\text{free}} \cup \{ \varphi(V_{\text{free}}) \} \) which agrees with \( \mathbf{a}_{\text{free}} \cup \{ \beta_B(B) \}_{B \in S(T)} \) on \( V_{\text{free}} \) and where the \( \varphi(V_{\text{free}}) \)-value is given by \( \bigotimes_{B \in S(T)} \beta_B(B) \).

The following proposition shows that by this strategy we indeed enumerate the relational encoding of \( \varphi \).

**Proposition 45.** Given \((\varphi, \tau, T)\), let \( C \) be a cover of the result of \( \varphi \) over \( T \) induced by \( \tau \) and let \( \{ \beta_B \}_{B \in \tau} \) be the set of bag functions underlying \( C \). It holds

\[
\varphi(a_{\text{free}}) = v \neq 0 \text{ for some } v \in \text{Dom}
\]

if and only if

\[
\exists \mathbf{b}_{\text{free}} \cup \{ \beta_B(B) \}_{B \in \tau} \in \pi_B.R_{\{ \beta_B(B) \}} C, \quad a_{\text{free}} = b_{\text{free}} \text{ and } \bigotimes_{B \in \tau} \beta_B(B) = v.
\]
Proof. Let \( \varphi_{\text{free}} = \bigotimes_{S' \in E'} \psi_{S'} \). Then,

\[
\varphi(a_{\text{free}}) = v \neq 0
\]

\[
\iff \bigotimes_{S' \in E'} \psi_{S'}(a_{S'}) = v \neq 0
\]

\[
\iff \bigotimes_{B \in S(T)} \beta_B(a_B) = v \neq 0
\]

\[
\iff \exists b_{\text{free}} \cup \{\beta_B(B)\} \cup \{\beta_B(B)\} \in \bigotimes_{B \in S(T)} R_{\beta_B}, \quad a_{\text{free}} = b_{\text{free}} \quad \text{and} \quad \bigotimes_{B \in S(T)} b_{\beta_B(B)} = v
\]

\[
\iff \exists b_{\text{free}} \cup \{\beta_B(B)\} \cup \{\beta_B(B)\} \in \bigotimes_{B \in S(T)} \pi_{B \cup \beta_B(B)} C, \quad a_{\text{free}} = b_{\text{free}} \quad \text{and} \quad \bigotimes_{B \in S(T)} b_{\beta_B(B)} = v.
\]

Equivalence (1) holds by the correctness of the InsideOut algorithm. The second equivalence holds by Observation 42. Equivalence (3) follows from the simple observation that the product of functions corresponds to the join of their relational encodings. The last equivalence follows from Proposition 7 which guarantees that \( \bigotimes_{B \in S(T)} R_{\beta_B} \) is equal to \( \bigotimes_{B \in S(T)} \pi_{B \cup \beta_B(B)} C \).

Thus, our enumeration result for covers of join results carries over to covers of FAQ-results.

**Corollary 30.** (Corollary 17, Proposition 45). Given a cover \( C \) of the result of an FAQ \( \varphi \), the tuples in the result of \( \varphi \) can be enumerated with \( \tilde{O}(|C|) \) pre-computation time and \( O(1) \) delay and space.