Effect of disorder on the vortex-lattice melting transition

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We use a three dimensional stacked triangular network of Josephson junctions as a model for the study of vortex structure in the mixed state of high $T_c$ superconductors. We show that the addition of disorder destroys the first order melting transition occurring for clean samples. The melting transition splits in two different (continuous) transitions, occurring at temperatures $T_i$ and $T_p$ ($> T_i$). At $T_i$, the perpendicular-to-field superconductivity is lost, and at $T_p$ the parallel-to-field superconductivity is lost. These results agree well with recent experiments in $Y$ $BaCuO$.

Although the existence of a melting transition of the vortex structure of high $T_c$ superconductors is now accepted, the nature of the transition and its dependence on different parameters is not completely clear in spite of the big deal of theoretical and experimental work performed. Experimentally, it is known that some external parameters can have strong effect on the behavior of the system. Some of these parameters are the magnetic field, the anisotropy, and the quenched disorder. It was found that in some cases (low fields and weak disorder) the transition is first order whereas in other cases (high fields or strong disorder) it is continuous.

Theoretical studies on the nature of the transition focus on the problem of whether it is a single transition at which superconducting coherence is lost along all directions of the sample, or two successive transitions where perpendicular and parallel to field coherence is lost in two sequential steps. In addition, different possibilities for the transition such as first order, second order, or crossovers have been proposed. Theoretical treatments have been based on models that can usually account only for part of the experimental data. The results depend on the parameters used and sometimes they look contradictory.

In this paper we fill part of the gaps between theory and experiment by studying the three dimensional (3D) Josephson junction array (JJA) model in the presence of an external magnetic field and with different kinds of disorder. We show that when temperature increases this model gives different results depending on the disorder: for low disorder we obtain a single first order transition where superconducting coherence is lost in all directions, whereas for high disorder superconducting coherence is lost in two successive continuous steps, first perpendicularly and then parallel to the applied magnetic field. This behavior is closely related to that found in experiments.

The equations for the 3D JJA model are:

\[ j^{ii'} = I_c^{ii'} \sin \left( \varphi^i - \varphi^{i'} - A^{ii'} \right) + \frac{\phi_0}{2\pi R_0} \frac{\partial (\varphi^i - \varphi^{i'})}{\partial t} + \vartheta^{ii'}(t) \]

(1)

\[ \sum_{i'} j^{ii'} = j_{ext}^{i} \]

(2)

where $\varphi^i(t)$ is the phase of the superconducting order parameter, $I_c$ and $R_0$ are the critical current and normal resistance of each junction, and $\phi_0$ is the flux quanta. Eq. (1) gives the current $j^{ii'}$ between nearest neighbors nodes $i$ and $i'$. Here $A^{ii'}$ is the vector potential of the external magnetic field, and $\vartheta^{ii'}(t)$ is an uncorrelated gaussian noise which incorporates the effect of temperature. Eq. (2) ensures the current conservation on each node, and $j_{ext}^{i}$ is the external current applied at node $i$. We study this model on a stack of triangular two-dimensional planes coupled by vertical links.

In all the simulations shown below the external current $j_{ext}$ (which is used to calculate resistivities) is about $\sim 10^{-2}$ of the Josephson junction critical current in the corresponding direction. If we want to minimize the pinning effect of the subjacent JJA lattice we have to use a magnetic field $H$ as low as possible. However, a low value of $H$ increases the sample size needed to get good statistics. We have used a magnetic field $H$ equal to $1/6$ flux quanta per plaquette. This value of field generates an Abrikosov lattice which is commensurate with the subjacent triangular JJA lattice and is a compromise between low fields and not too long computation time. Self-inductance effects are not considered -this is equivalent to take the magnetic penetration depth $\lambda \rightarrow \infty$. When we simulate anisotropic systems we diminish the c axis mean critical current of the junctions and at the same time increase the c axis elemental resistance by the same factor $\eta^2$. $\eta^2 = \left< I_c^\parallel \right> / \left< I_c^\perp \right>$ where $\parallel$ and $\perp$ indicate parallel an perpendicular to the $ab$ plane, and $\langle \ldots \rangle$ indicates the mean value on the sample. Boundary conditions (BC).
are used as follows: when external current is applied in the $x$ direction we take open BC for the $x$ direction and periodic BC for $y$ and $z$ directions; when external current is applied in the $z$ direction we take pseudo periodic BC for the $z$ direction [10] and periodic BC for $x$ and $y$ direction. In all cases the temperature is measured in units of the mean Josephson energy of the in-plane junctions.

FIG. 1. Resistivity along the $ab$ plane for a decoupled system of $18 \times 18 \times 18$ planes (open symbols), and for a three dimensional system with different anisotropies (full symbols, anisotropies $\eta^2 = 5$, 10, 20, and 50 from right to left). The first order melting temperatures $T_m$ for the different anisotropies are indicated by arrows. Temperature is measured in units of the Josephson energy of the in-plane junctions.

We consider first the behavior of a system of uncoupled (triangular) planes when the $c$ coupling is turned on. It has been found that a single plane has a weakly first order transition [11]. Our results for the resistivity of this system are shown in Fig. 1 (open symbols). Within the numerical precision we cannot distinguish from the behavior of the resistivity between a continuous transition and a weak first order transition occurring at $T \sim 0.23$ [12]. We will refer to this type of behavior as a continuous transition, indicating that no discontinuities are observed in the $\rho(T)$ curve. Both continuous (second order) phase transitions and crossovers are in this category. When the coupling is increased we clearly see a well defined transition temperature where the resistivity has a jump. This is a first order transition as found in [11] and [13]. In addition, the $c$ axis resistivity (not shown) has also a jump at the transition temperature, indicating that the superconductor coherence is lost discontinuously and at the same temperature $T_m$ in all directions. Hysteresis loops, which for the sake of clarity are not shown in the figure, are observed for all the cases studied but for the uncoupled planes. We conclude that the $c$ axis coupling transforms a continuous transition into a first order transition [14]. Note that for high anisotropies, the resistivity for $T > T_m$ is very close to the value corresponding to decoupled planes, whereas for $T < T_m$ it is clearly different from that value. This indicates that an effective decoupling of the planes is occurring at $T_m$. As we argue below, the nature of the first order transition in thick samples is different from that of an isolated plane.

It has been found experimentally that disorder alters the previous picture. In fact, for samples with twin boundaries the transition in $\rho_{ab}$ is continuous [7]. To study the effect of impurities in our model we allow the critical currents $I_c$ of each junction to vary randomly between two fixed values $I_c^{\text{min}}$ and $I_c^{\text{max}}$. This generate pinning centers close to the junctions with low critical currents. All the results shown in the simulations with disorder correspond to $D \equiv (I_c^{\text{max}} - I_c^{\text{min}}) / (I_c^{\text{max}} + I_c^{\text{min}}) = 0.5$.

FIG. 2. Resistivities along the $ab$ plane (full symbols) and along the $c$ direction for an isotropic $18 \times 18 \times 18$ sample and different configurations of the disorder. For the clean sample hysteresis loops around $T_m$ upon heating and cooling are shown. For the disordered case the approximate values of $T_i$ and $T_p$ are indicated.

The effect of disorder depends on its spatial correlation. We consider three different possibilities: point (uncorrelated), columnar (correlated along $c$ axis), and pla-
narrow (correlated in ‘twin boundaries’ perpendicular to the applied current) defects. For the uncorrelated disorder no correlation on the values of \( L \) for different junctions are set in. For the columnar pinning all junction having the same \( ab \) plane coordinates have the same critical currents, whereas there is no correlation for junctions with different \( ab \) plane coordinates. For the twin boundaries case all junctions with the same \( x \) coordinate \( (j_{\text{ext}} \parallel x) \) have the same critical currents, whereas there is no correlation for junctions with different \( x \) coordinates.

The results for \( \rho_{ab} \) obtained for the different types of disorder are illustrated in Fig. 2 (full symbols) for an isotropic \( \langle I_{c}^{\parallel} \rangle = \langle I_{c}^{-} \rangle \) sample. For the clean sample the hysteretic loop is shown, indicating again a first order transition at \( T = T_{m} \). Although hysteresis in the resistivity is not an unequivocal indication of a first order transition, we checked that the transition is in fact first order by using the histogram technique \([15,9]\). In addition other indication of loose of coherence (in particular the helicity modulus) give results which are in agreement with the conclusions obtained from resistivity measurements. In all disordered cases the transition in \( \rho_{ab} \) is continuous, and we denote the transition temperature as \( T_{i} \). Point defects and twin boundaries decrease the transition temperature, whereas columnar defects increase it. This is consistent with the fact that columnar pinning generates the strongest pinning.

It is instructive to study the behavior of \( \rho_{c} \) with disorder. In Fig. 2 (open symbols) we show the values of \( \rho_{c} \) for the same disorder configurations as before. The main result is that \( \rho_{c} \) also becomes continuous in the presence of disorder, and the transition temperature (that we denote \( T_{p} \) \([8,10]\) and experimentalists \( T_{th} \) \([13]\)) is higher than the corresponding \( T_{i} \). The temperature \( T_{p} \) is higher than \( T_{m} \) for samples with columnar defects and twin boundaries, whereas is lower than \( T_{m} \) for point defects. This indicates that point defects diminish the coherence along the \( c \) direction, whereas the other two types of defects enhance it. Note also that the absolute difference \( T_{p} - T_{i} \) is the highest for the twinned sample. This is due to the fact that planar defects parallel to the vortex movement have opposite effects on \( T_{i} \) and \( T_{p} \): \( T_{i} \) diminishes because there are paths of easy movement for the vortices, and the rigidity of the ideal Abrikosov lattice has been destroyed, and \( T_{p} \) increases because the defects are \( c \) axis correlated, and thus they enhance the coherence along the \( c \) direction.

We found that if the value of the disorder \( D \) is smaller than a critical value \( D_{c} (D_{c} \sim 0.3 \) for isotropic samples) the first order transition persist and the coherence in all directions is lost at the melting temperature. Strong disorder \( (D > D_{c}) \) is necessary to produce the behavior of Fig. 2.

Our results are in complete agreement with recent results on \( YBaCuO \) samples, where a first order transition is observed in clean samples and two transitions are observed with \( T_{i} < T_{m} < T_{p} \) in twinned samples \([8,10]\). It has also been shown that point defects destroy the first order transition \([8,10]\), but unfortunately pseudo DC transformer or direct \( c \) axis resistivity measurements were not performed on these samples.

We checked that the transitions at \( T_{i} \) and \( T_{p} \) occurring in our triangular samples with disorder have the same features described in references \([8,10]\) for the case of a square lattice of junctions: the transition at \( T_{i} \) is a thermal depinning of vortex lines, which is probably related to the large amount of disorder \([13]\): the transition at \( T_{p} \) is a percolation phase transition of the vortex lattice perpendicularly to the applied field. Also we have checked that around \( T_{p} \), \( \rho_{c}(T) \) satisfies the percolation scaling laws of \([17]\). In addition, the structure of the lattice for \( T_{i} < T < T_{p} \) is that of a disentangled vortex liquid. The range \( T_{p} - T_{i} \) in which this disentangled liquid exists shrinks to zero when increasing the thickness of the sample \([10,17]\).

When two transitions at different temperatures \( T_{i} \) and \( T_{p} \) occur, it is natural that both of them are continuous: at \( T_{i} \) the system changes from a solid to a liquid of \( c \) axis correlated (and thus effectively two dimensional) vortices. This transition is qualitatively similar to the fusion of a two dimensional system, and it is expected to be continuous. At \( T_{p} \) the transition is driven by a percolation of vortex loops, which is a second order (continuous) phase transition.

Experimental and theoretical evidence suggest that the transition is first order only when the superconducting coherence is lost in all directions at the same temperature. This suggests that the first order melting in clean samples is a consequence of the interplay between two different transitions: the depinning of single vortex lines -which drives the crossover occurring at \( T_{i} \), and the percolation of vortex loops between planes -which is responsible for the appearance of dissipation along the \( c \) direction at \( T_{p} \). These two transitions may cooperate and merge onto a single one for clean samples: when percolation occurs the planes decouple, and this reduces the effective value of \( T_{i} \). In turn, when vortices depin, vortex loops between planes are more easily generated due to the screening effect of mobile vortices, thus reducing \( T_{p} \). The combination of these two effects can generate an instability that drives the transition first order \([19]\).

As we said above, the merging of \( T_{i} \) and \( T_{p} \) onto a single first order melting temperature \( T_{m} \) is observed only for clean samples. However, if the thickness of the system is lower than a minimum value two different transitions at \( T_{i} \) and \( T_{p} \) are observed. In particular, in our simulations the thickness of the system has to be greater than ten planes on isotropic samples in order to obtain a first order transition. For thinner samples we observe two separate and continuous transitions. One can make a rough estimate of the dependence of the critical thick-
ness on field in the following way. Previous results [10,17] indicate that $T_p$ depends on thickness as

$$k_BT_p \sim \Delta/\ln(L_c/d),$$  \hspace{1cm} (3)

with $d$ a distance of the order of the interlayer spacing, $L_c$ the thickness of the sample, and $\Delta$ an energy which is the order of the energy necessary to make two nearest vortices touch each other. Being $\Delta$ proportional to the distance between vortices we can write $\Delta = e_0\phi_0\sqrt{H}$, with $e_0$ an energy scale factor related to the linear energy density of a vortex line. To be able to merge onto a single transition, $T_i$ and $T_p$ should be of the same order. Having into account that $T_i$ is rather thickness independent [14,17] we obtain the minimum value $L_{\text{min}}$ necessary to have a first order transition as

$$L_{\text{min}} = d \exp \left( e_0\phi_0/\sqrt{H}k_BT_i \right).$$  \hspace{1cm} (4)

This expression shows that $L_{\text{min}} \rightarrow \infty$ for $H \rightarrow 0$, indicating that very thick samples are necessary to observe the first order transition at very low fields. This is probably not an experimental limitation due to the small value of $d$, but it should be taken into account in numerical simulations at low fields.

We have concentrated on the effect of disorder on the melting transition at a fixed value of the external field. Finding the behavior of the system when changing the magnetic field $H$ is difficult because of commensurability effects between the vortex lattice and the mesh. However, we can choose to change at the same time the magnetic field and the discretization parameter, in such a way that for any real field we have $1/6$ flux quanta per plaquette. The change in the discretization produces a change in the anisotropy $\eta$ and the disorder of the system. For a real change in $H$ of the form $H \rightarrow H\delta$ with $\delta \simeq 1$, the anisotropy changes as $\eta \rightarrow \eta\delta^{1/2}$ [20]. The change in the disorder may depend on the particular realization. In the simplest case of uncorrelated disorder it can be shown that $D$ changes as $D \rightarrow D\delta^{1/2}$. We conclude that we can study the $H-T$ phase diagram of high-$T_c$'s by analyzing the $\eta - D$ phase diagram along lines of fixed value of $\eta/D$. Preliminary results indicate that the overall properties of the $H-T$ phase diagrams of $\text{YBa}_2\text{Cu}_3\text{O}_7$ and $\text{BiSr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$ are well reproduced [18]. In particular, from these arguments and Fig. 2 we can conclude that the jump in the resistivity at the first order melting should decrease when increasing $H$.

In summary, we have presented results obtained using the three dimensional JJA model that reproduce very well the behavior of the vortex lattice in high-$T_c$ superconductors. For clean samples the system has a first order melting transition at temperature $T_m$, at which the superconducting coherence is lost discontinuously in all directions. In the presence of disorder the transition separates in two continuous ones occurring at temperatures $T_i$ and $T_p$ ($> T_i$). At $T_i$ perpendicular-to-field correlation is lost due to depinning of vortices from the pinning centers. At $T_p$ parallel-to-field correlation is lost due to a percolation transition of the vortex lattice. In addition, relative values of $T_i$, $T_p$, and $T_m$ depend on the kind of disorder considered.

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Fig. 1

$\rho_{ab}$ (arb. units)

$T$

Fig. 1
Fig 2

- Clean
- Point defects
- Columnar defects
- Twin boundaries defects

\( \rho_{ab}, \rho_c \) (arb. units)

Temperature (T) range: 0.8 to 1.4

- \( T_i \) for different types of defects
- \( T_p \) for different types of defects

Note: The diagram shows the behavior of \( \rho_{ab}, \rho_c \) with temperature for clean, point defects, columnar defects, and twin boundaries defects.