Jet quenching and broadening: the transport coefficient $\hat{q}$ in an anisotropic plasma

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Abstract

The jet quenching parameter $\hat{q}$ is analyzed for a quark jet propagating in an anisotropic plasma. The momentum anisotropy is calculated at high temperature of the underlying quark-gluon plasma. $\hat{q}$ is explicitly estimated in leading-logarithmic approximation by the broadening of the massless quark interacting via gluon exchange. A plasma instability is present. Strong indications are found that $\hat{q}$ is increasing with increasing anisotropy. Possible implications for the saturation scale $Q_s$ in $A−A$ collisions are pointed out.

I. INTRODUCTION

Jet quenching in $Au − Au$ collisions at RHIC [1, 2], i.e. the suppression of $\pi^0$ and $\eta$ production at large transverse momenta in central collisions when compared with scaled measurements in $p − p$ collisions [3] is considered as one of the important observations in favour of the production of a plasma state [4, 5].

The experimental data from RHIC are used to determine the actual value of the transport coefficient $\hat{q}$, which controls the radiative energy loss, responsible for jet quenching in the induced gluon bremsstrahlung picture [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

Within the kinetic theory framework the coefficient (for an isotropic medium) is estimated by

$$\hat{q} = \rho \int d^2 q_\perp q_\perp^2 \frac{d\sigma}{d^2 q_\perp}, \quad (1)$$

where $\rho$ is the number density of the constituents of the medium, and $\frac{d\sigma}{d^2 q_\perp}$ is the differential scattering cross section of the parton (massless quark or gluon) on the medium.

It is important to keep in mind that $\hat{q}$ not only determines the energy loss, but it is also related to $p_\perp$-broadening (per unit length) of energetic partons propagating in the medium/plasma [6].

Because of many theoretical uncertainties in the determination of $\hat{q}$ its value maybe quoted to be in a broad range of $0.5−20 \text{ GeV}^2/\text{fm}$, see e.g. [13, 19, 20].

It has been found (see e.g. [21, 22, 23, 24, 25, 26, 27, 28, 29, 30]) that the physics of anisotropic plasmas differs from that of isotropic ones, because of the presence of plasma instabilities in the former. Due to these observations it requires to reanalyze $\hat{q}$ in the context of anisotropic plasmas. Momentum broadening in a homogeneous but locally anisotropic high-temperature system for a heavy quark induced by collisions has been discussed recently [31] (see also [27, 32, 33]). Here the case for massless partons is of relevance.

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In the following the anisotropic medium is modeled as a high temperature gas. As pointed out in [31] the temperature is kept as a placeholder of the correct hard scale, like the saturation scale $Q_s$, present in the anisotropic system. These assumptions allow an analytic and tractable treatment, at least for small anisotropies, considered in this paper. In detail the main effect is due to the propagation of the dominantly soft gluon in this medium.

II. SETUP

In the following momentum broadening (per unit pathlength) for a massless quark in the high energy limit (eikonal limit) in a homogeneous but anisotropic high temperature plasma is studied. The following kinematical setup is used: the initial energetic (hard) quark propagates with momentum $p^\mu$ along the z-axis, whereas the beam nuclei collide along the y-axis, which is the direction of anisotropy, denoted by the three-dimensional unit vector $n = (0, 1, 0)$.

In the limit under consideration momentum broadening $q_\perp$ takes place only in the $x-y$ plane, $q_\perp^2 = q_x^2 + q_y^2$.

Instead of kinetic theory [32, 34] $\hat{q}$ is calculated in the thermal field theory approach [35] using the notation and convention of [36], generalizing the expression given by Eq. (1) to include the finite temperature dependence. The coefficient $\hat{q}$ is expressed by the thermal quark self-energy, however weighted by $q_\perp^2$,

$$\hat{q} = \frac{1}{4p^0} \text{Tr} \left[ \not{\hat{p}} \Sigma_w(p) \right],$$

where explicitly

$$\Sigma_w(p) = g^2 C_F \int \frac{d^4q}{(2\pi)^3} q_\perp^2 \left[ \gamma^\mu \not{S}^\rightarrow_F(p') \gamma^\nu \Delta^\rightarrow_{\mu\nu} \right].$$

$g$ is the QCD coupling, $C_F = (N_c^2 - 1)/2N_c$ is the colour factor of the quark coupled to the gluon. The hard quark with momentum $p' = p + q$ in the medium remains on-shell at temperature $T = 0$,

$$S^\rightarrow_F(p') = 2\pi p' \delta_+(p'^2) = 2\pi q_\perp^2 \delta(p'^2).$$

The gluon propagates with momentum $q^\mu$ through the thermal medium,

$$\Delta^\rightarrow_{\mu\nu} = -2(1 + f(q^0)) \text{Im} \Delta_{\mu\nu},$$

with the Bose-Einstein distribution $f(q^0) = (\exp(q^0/T) - 1)^{-1}$. $\Delta_{\mu\nu}$ denotes the gluon propagator, which is treated in detail in Appendix A.

In the relativistic case and for small momentum transfer, $-q^2/2 = p \cdot p' \simeq q_\perp^2/2$, and with $p \cdot q = p^0(q^0 - \vec{v} \cdot \vec{q}) \simeq 0$, $\vec{v} = \vec{p}/p^0 = (0, 0, 1)$, one may approximate

$$\hat{q} = -\frac{g^2 C_F}{2p^0} \text{Im} \int \frac{d^4q}{(2\pi)^4} q_\perp^2 2\pi \delta_+((p + q)^2)(1 + f(q^0)) \text{Tr} \left[ \not{p} \gamma^\mu (\not{q} + \not{\vec{q}}) \gamma^\nu \right] \Delta_{\mu\nu},$$

by
\[ \hat{q} = -g^2 C_F \text{Im} \int \frac{d^4q}{(2\pi)^4} q_+^2 \frac{2T}{q^0} \frac{2\pi \delta(q^0 - \hat{q})}{(p^0)^2} \frac{p \cdot \Delta \cdot p}{q^2} , \]  
when \( q^0 \ll T \) and \( q^0/p^0 \to 0 \). Concerning kinematics: \( \hat{v} \cdot \hat{q} = |\hat{q}| \cos \theta_{pq} = q^0, q_\perp = |\hat{q}|^2(1 - x^2) \) with \( x = q^0/|\hat{q}| \).

As already pointed out in [31] this equation (7) tells us that all the information about the medium, either isotropic or anisotropic, is contained in the imaginary part of the gluon propagator (Appendix A).

### III. HARD-THERMAL-LOOP SELF-ENERGY IN AN ANISOTROPIC PLASMA

In the hard-thermal-loop approximation [37] the retarded gauge-field self-energy is given by [38]
\[ \Pi^{\mu\nu}(q) = g^2 \int \frac{d^3 p}{(2\pi)^3} v^{\mu} \frac{\partial n(\vec{p})}{\partial p^3} \left( g^{\nu\beta} - \frac{v^{\nu} p^{\beta}}{q \cdot v + i\epsilon} \right) , \]  
where \( v^{\mu} \equiv (1, \vec{p}/|\vec{p}|) \) is a light-like vector describing the propagation of a plasma particle in space-time. The self-energy is symmetric, \( \Pi^{\mu\nu}(q) = \Pi^{\nu\mu}(q) \), and transverse, \( q^\mu \Pi^{\mu\nu}(q) = 0 \).

In order to determine \( \Pi^{\mu\nu} \) the phase space distribution function has to be specified. The following form is used,
\[ n(\vec{p}) = n_{iso} \left( \sqrt{\vec{p}^2 + \xi(\vec{p} \cdot \vec{n})^2} \right) . \]  
Thus, \( n(\vec{p}) \) is obtained from an isotropic distribution \( n_{iso}(|\vec{p}|) \) by removing particles with a large momentum component along \( \vec{n} \) [23, 24, 26]. In a more general approach the distribution \( n_{iso} \) need not necessarily be thermal, as assumed here.

The functions \( \alpha, \beta, \gamma \) and \( \delta \), introduced in the Appendix A in Eq. (38) are obtained from Eq. (8). Explicit expressions maybe found in [23, 24, 31].

#### A. isotropic case

For reference the case for \( \xi = 0 \) is first considered, for which the propagator is given by Eq. (52), containing the unscreened transverse mode and the Debye screened longitudinal one. Therefore dynamical screening due to Landau damping [36], as e.g. done in [34], has to be included.

To leading logarithm order (LL) in \( \ln \frac{T}{m_D} \simeq \ln \frac{1}{g} \) - in the following the only limit of interest - one may keep only terms linear in \( x \) in the denominator of the propagator in Eq. (52), and approximate
\[ \text{Im} \left[ \frac{\hat{p} \cdot A \cdot \hat{p}}{q^2 - \Pi_T} + \frac{\omega^2 \hat{p} \cdot B \cdot \hat{p}/q^2}{q^2 - \Pi_L} \right] , \]  
with \( \omega = q^0, \hat{p}^\mu = p^\mu/p^0 \), by
\[ \text{Im} \left[ \frac{(1 - x^2)}{q^2 - i\text{Im}\Pi_T} + \frac{1}{q^2 + m_D^2 - i\text{Im}\Pi_L} \right] \sim \]  
(11)
Using Eq. (54).

Performing the integrations to LL order with \( q = |\vec{q}| \leq T \) gives finally for a hard quark jet

\[
\hat{q}_{iso} = \frac{g^2 C_F m_D^2 T}{2\pi} \ln \frac{T}{m_D},
\]

which coincides with the kinetic result Eq. (61) given in Appendix B. Dynamical screening indeed guarantees this finite answer.

**B. anisotropic case**

In the hot anisotropic system (\( \xi \neq 0 \)) the gluon propagator of the form of Eq. (49) is the relevant one, such that

\[
\hat{p} \cdot \Delta \cdot \hat{p} = \Delta_A \left[ 1 - \frac{x^2}{1 - \hat{q}_y^2} \right] + \Delta_G \left[ x^2 (q^2 - \alpha - \gamma) + \frac{x^2 \hat{q}_y^2}{1 - \hat{q}_y^2} (\omega^2 - \beta) - 2x^2 \hat{q}_y \delta \right],
\]

with the transverse momentum component \( \hat{q}_y = \frac{\vec{q} \cdot \vec{n}}{|\vec{q}|} = q_y / |\vec{q}| \) into the direction of anisotropy.

First, the contribution of \( \Delta_A \) to \( \hat{q} \) of Eq. (7) is considered, which even to LL accuracy shows the possible presence of the plasma instability. The contribution reads

\[
\hat{q}_A = -\frac{g^2 C_F T}{4\pi^3} \int d\Omega q \frac{1 - x^2}{x} \left[ 1 - \frac{x^2 \hat{q}_y^2}{1 - \hat{q}_y^2} \right] I(x, \alpha),
\]

noting that \( x = \frac{\vec{v} \cdot \vec{q}}{|\vec{q}|} \) does depend only on angles, as does \( \hat{q}_y \), which are integrated by the angular integration in (15).

In performing the \( |\vec{q}| = q \)-integration

\[
I(x, \alpha) = \text{Im} \int_0^T q^3 dq \Delta_A(x, q, \alpha) = -\text{Im} \int_0^T dq \frac{q^3}{q^2(1 - x^2) + \text{Re} \alpha + i\text{Im} \alpha},
\]

it has to be kept in mind that \( \text{Im} \alpha \) is non-vanishing because of Landau damping, however, \( \text{Im} \alpha \propto x \) for \( x \to 0 \), and that \( \text{Re} \alpha \) is negative for \( \xi > 0 \) in a range of \( x \) values, \( |x| < 1 \), as e.g. shown in Fig. 1a of [24]. Therefore, in this range, including the point \( x = 0 \), there is a pole present in the integrand of Eq. (16), characteristic of the anisotropy.

Evaluation gives

\[
I(x, \alpha) = \frac{\text{Im} \alpha}{2(1 - x^2)^2} \left\{ \frac{1}{2} \ln \frac{T^2(1 - x^2) + \text{Re} \alpha^2 + |\alpha|^2}{|\alpha|^2} \left[ \text{Im} \alpha \right] \right\} + \frac{\text{Re} \alpha}{\text{Im} \alpha} \left[ \arctan \frac{\text{Im} \alpha}{T^2(1 - x^2) + \text{Re} \alpha} - \arctan \frac{\text{Im} \alpha}{\text{Re} \alpha} + \pi \Theta(-\text{Re} \alpha) \right].
\]
In the LL approximation this gives

\[
I(x, \alpha) \simeq \frac{\text{Im} \alpha}{(1 - x^2)^2} \left[ \frac{1}{2} \ln \frac{T}{m_D} + \frac{\pi \text{Re} \alpha}{2 \text{Im} \alpha} \Theta(-\text{Re} \alpha) \right],
\]

where the second term has to be kept, because it reflects a singularity for \(\text{Im} \alpha \propto x \to 0\) due to the anisotropy \(\xi > 0\).

To be explicit, the small \(\xi\) behaviour is considered, where

\[
\alpha = \Pi_T(x) + \xi \left\{ \frac{x^2}{6} (5\hat{q}_y^2 - 1)m_D^2 - \frac{\hat{q}_y^2}{3} m_D^2 \\
+ \frac{1}{2} \Pi_T(x)[3\hat{q}_y^2 - 1 - x^2(5\hat{q}_y^2 - 1)] \right\},
\]

which shows that for \(x \to 0\)

\[
\text{Re} \alpha \simeq -\frac{1}{3} \xi \hat{q}_y^2 m_D^2,
\]

indeed negative for \(\xi > 0\), and

\[
\text{Im} \alpha \simeq -\frac{\pi}{4} x(1 - x^2)m_D^2 \left\{ 1 + \frac{\xi}{2}[3\hat{q}_y^2 - 1 - x^2(5\hat{q}_y^2 - 1)] \right\}.
\]

Following [31] the contribution from the first term in Eq. (18) to \(\hat{q}_A\) is denoted as regular. After performing the angular integrations in Eq. (15) it leads at LL order with \(T >> m_D\) to

\[
\hat{q}_A^{\text{reg}} = \frac{g^2 C_F m_D^2 T}{8\pi} \ln \frac{T}{m_D} (1 + O(\xi^2)) ,
\]

with no contribution at first order in \(\xi\).

Turning to the regular contribution due to \(\Delta G\) one first obtains

\[
\hat{q}_G = \frac{g^2 C_F T}{8\pi^3} \int d\Omega_q \, x(1 - x^2) \\
\times \text{Im} \int dq^2 q^2 \frac{(1 - x^2)q^2 + \alpha + \gamma - \frac{x^2\hat{q}_y^2}{1 - \hat{q}_y^2}(q^2 - \tilde{\beta}) + 2\hat{q}_y \tilde{\delta}}{x^2(1 - x^2)(\beta - q^2)q^2 + x^2(\alpha + \gamma)(\beta - q^2) - \tilde{\delta}^2(1 - \hat{q}_y^2)}.
\]

The regular LL part reads after the \(q\)-integration

\[
\hat{q}_G^{\text{reg}} = -\frac{g^2 C_F T}{4\pi^3} \ln \frac{T}{m_D} \int d\Omega_q \, \frac{1}{x(1 - x^2)} [(1 - x^2)\text{Im} \tilde{\beta} + 2(1 - x^2)\hat{q}_y \text{Im} \tilde{\delta} + \frac{x^2\hat{q}_y^2}{1 - \hat{q}_y^2} \text{Im}(\alpha + \gamma)] ,
\]

with (21) and

\[
\text{Im} \tilde{\beta} = -\frac{\pi}{2} x m_D^2 \left\{ 1 + \xi [2\hat{q}_y^2 - 1 - x^2(3\hat{q}_y^2 - 1)] \right\},
\]

\[
\text{Im}(\alpha + \gamma) = -\frac{\pi}{4} x(1 - x^2)m_D^2 \left\{ 1 + \frac{\xi}{2}[5\hat{q}_y^2 - 3 - x^2(7\hat{q}_y^2 - 3)] \right\},
\]

\[
\text{Im} \tilde{\delta} = -\frac{\pi}{4} x(1 - x^2)m_D^2 \xi (1 - 4x^2)\hat{q}_y ,
\]

(24)
taken from [24, 31].

The angular integration in Eq. (24) then gives

\[ \hat{q}_{reg}^{\text{GL}} = \frac{3g^2 C_F m_D^2 T}{8\pi} \ln \frac{T}{m_D} (1 + O(\xi^2)). \]  
(26)

Summing the two terms Eqs. (22) and (26) the LL transport coefficient in the limit of small \( \xi \) up to \( O(\xi) \) becomes

\[ \hat{q}_{\text{reg}}^{\text{aniso}} = \hat{q}_{A}^{\text{reg}} + \hat{q}_{G}^{\text{reg}} = \frac{g^2 C_F m_D^2 T}{2\pi} \ln \frac{T}{m_D} (1 + O(\xi^2)). \]  
(27)

For \( \xi = 0 \) the value for \( \hat{q}_{\text{iso}} \), Eq. (13), is recovered. This result for \( \hat{q}_{\text{reg}}^{\text{aniso}} \) in Eq. (27) does agree with the result given for \( \kappa_{\perp}^{\text{reg}} + \kappa_{z}^{\text{reg}} \) in Eq. (14) of [31] when the velocity \( v \) is taken to be \( v = 1 \) for a massless quark jet. Because of this agreement, which is not unexpected, one may try to extrapolate \( \hat{q}_{\text{reg}}^{\text{aniso}} \) for \( v \rightarrow 1 \) from the numerical values of \( \kappa_{\perp,z}^{\text{reg}} \) summarized in Table 1 of [31] even for large values of \( \xi \) by

\[ \frac{\hat{q}_{\text{reg}}^{\text{aniso}}}{\hat{q}_{\text{iso}}} \simeq 1, \]  
(28)

almost independent of \( \xi \leq 100 \).

Next the anomalous contribution [31] due to the second term of Eq. (18) is evaluated. In LL order only the behaviour for \( x \rightarrow 0 \) is relevant. With Eq. (20) it gives

\[ \hat{q}_{\text{anom}}^{\alpha} \simeq \frac{g^2 C_F m_D^2 T}{24\pi^2} \xi \int d\Omega_q \frac{\hat{q}_y^2}{q_x}, \]  
(29)

inducing a logarithmic singularity with \( x = \cos \theta_{pq} \). The contribution to \( \hat{q}_{G}^{\text{anom}} \) starts at \( O(\xi^2) \).

A short way to see the anomalous contribution is to sum the singular parts in the gluon propagator for the static case in terms of the masses, here explicitly quoted from [24] in the limit of small anisotropy \( \xi \rightarrow 0 \) scaled with respect to the Debye mass Eq. (55), i.e. \( \hat{m} = m/m_D \),

\[ \hat{m}_\alpha^2 = -\frac{\xi}{6} (1 + \cos 2\theta_n), \]

\[ \hat{m}_\gamma^2 = \frac{\xi}{3} \sin^2 \theta_n, \]  
(30)

and

\[ \hat{m}_+^2 = 1 + \frac{\xi}{6} (3 \cos 2\theta_n - 1), \]

\[ \hat{m}_-^2 = -\frac{\xi}{3} \cos 2\theta_n, \]  
(31)

where the angle is given by \( \cos \theta_n = \hat{q}_y \).

Since \( m_\alpha^2 \) and \( m_\gamma^2 \) are negative the pole contributions (with the Feynman prescription) read

\[ \hat{q}_{\text{anom}} = \frac{g^2 C_F T}{8\pi^3} \Im \int d\Omega_q \int \frac{q^2 dq^2}{x} \left[ \frac{1}{q^2 + m_\alpha^2 - i\epsilon} - \frac{1}{m_+^2 - m_-^2} \frac{m_+^2 - m_-^2}{q^2 + m_+^2 - i\epsilon} \right]. \]  
(32)
The term of \( O(\xi) \) comes from the first pole term and leads to Eq. (29), whereas the second term is obviously of \( O(\xi^2) \).

The question about the treatment of the logarithmic singularity in Eq. (29) arises. At least three possibilities to cut-off the singularity may be discussed:

(i) One may follow the detailed and plausible arguments given in [31] that this soft singularity is screened by \( O(g^3) \) terms in the gluon propagator, i.e. beyond the HTL approximation under discussion. It leads to the replacement of \( \text{Im} \alpha \) in the second term in the denominator of Eq. (18) by \( \text{Im} \alpha \sim x \to x + cg \), i.e. it is suggestive to cut the singularity in (29) by

\[
\xi \int \frac{dx}{x} \to 2\xi \int \frac{dx}{x + cg} \sim 2\xi \ln \frac{1}{g} \sim 2\xi \ln \frac{T}{m_D}.
\]

This way a finite result is obtained,

\[
\hat{q}_A^{\text{anom}} \sim \frac{g^2 C_F m_D^2 T}{2\pi} \ln \frac{T}{m_D} \frac{\xi}{6},
\]

which shows a positive, but weak dependence on \( \xi \) as a sign of the anisotropy for \( \xi > 0 \).

(ii) The origin of the \( 1/x \) singularity is traced back to the Bose-Einstein distribution \( f(q^0) \sim T/q^0 \) in Eq. (5). Pragmatically, in the anisotropic case, this behaviour could be modified by \( q^0 \to \sqrt{(q^0)^2 + \xi (\vec{n} \cdot \vec{q})^2} = q \sqrt{x^2 + \xi \hat{q}^2}, \quad q^0 > 0 \). On mass-shell this replacement gives the distribution in Eq. (9), and leads to

\[
\xi \int \frac{dx}{x} \to 2\xi \ln \frac{1}{g \xi}.
\]

(iii) To form the anisotropic configuration in momentum space a characteristic time scale is present of the order \( \tau_c \sim O(1/g\xi T) \), for not too large \( \xi \). It is then natural to cut the energies of the constituents in the heat bath by \( |q^0| \geq 1/\tau_c > g \xi T \), and

\[
\xi \int \frac{dx}{x} \to 2\xi \ln \frac{1}{g \xi}.
\]

In summary all three options Eqs. (33 - 36) lead to a positive contribution of \( O(\xi) \) at LL order to \( \hat{q}_{\text{aniso}} \).

IV. CONCLUSION

Because of the approximations, keeping only leading logarithmic order terms, a detailed quantitative study is not aimed. Qualitatively, however, the main result obtained strongly indicates that the instability due to the anisotropy leads to \( \hat{q}_{\text{aniso}} > \hat{q}_{\text{iso}} \). For small anisotropies it turns out to be an effect of \( O(\xi) \), which should be taken into account in the future phenomenological comparison with experimental jet quenching data, although the actual numerical value depends on the full treatment including the non-leading terms.

Independent information on \( \hat{q}_{\text{aniso}} \) maybe found in the following references:

i) A result \( \hat{q}_{\text{aniso}} \geq \hat{q}_{\text{iso}} \) is found in a numerical simulation in [33] treating jet broadening in an unstable non-Abelian (\( SU(2) \)) plasma. At early evolution times \( \hat{q}_{\text{aniso}} \simeq \hat{q}_{\text{iso}} \simeq 2.2 \text{GeV}^2/fm \), whereas at times when the instability is acting there are numerical indications that \( \hat{q}_{\text{aniso}} \) increases when compared to \( \hat{q}_{\text{iso}} \).
ii) The transport coefficient \( \hat{q} \) of a fast parton (gluon) propagating in an expanding turbulent quark-gluon plasma is estimated in \([5, 39, 40]\). The coefficient called anomalous in the quoted references (but not to be confused with the one in Eq. (15)) is parametrically given by

\[
\hat{q}_A \simeq g \xi \frac{2}{3} \frac{m_D^2}{T},
\]

therefore \( \hat{q}_A / \hat{q}_{iso} \simeq \frac{\xi \frac{2}{3}}{g} > 1 \), even significantly larger than one for small coupling \( g \) (assuming \( \xi \) not very small).

These results together indeed show that \( \hat{q}_{\text{aniso}} > \hat{q}_{\text{iso}} \).

In \( A - A \) collisions the parton saturation scale \( Q_s \) \([41]\) is for a large nucleus \( A \) related to the jet quenching parameter (of gluons) \( \hat{q} \) by \( Q_s^2 = 2\hat{q}\sqrt{R^2 - b^2} \) \([42, 43, 44, 45]\), where \( R \) is the radius of the nucleus and \( b \) is the impact parameter of the collision. Because \( \hat{q} \) is affected by anisotropies in \( A - A \) collisions, it is not obvious that \( Q_s \) determined from small-\( x \) processes in proton/deuteron - nucleus collisions is indeed the same as in \( A - A \) collisions.

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V. APPENDIX

A. Propagator in covariant gauge in an anisotropic plasma

Here we summarize the results for the gluon self-energy \( \Pi^{\mu\nu} \) and the gluon propagator \( i\Delta^{\mu\nu}_{ab} \) (diagonal in colour) in covariant gauge in an anisotropic medium.

The self-energy is decomposed as

\[
\Pi^{\mu\nu} = \alpha A^{\mu\nu} + \beta B^{\mu\nu} + \gamma C^{\mu\nu} + \delta D^{\mu\nu}.
\]

The tensor basis of \( \Pi^{\mu\nu} \) is determined explicitly for anisotropic systems in \([24]\). We use here a basis appropriate for the covariant gauge, and define

\[
A^{\mu\nu} = -P^{\mu\nu} + \frac{\bar{K}^\mu \bar{K}^\nu}{K^2} = -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} + \frac{\bar{K}^\mu \bar{K}^\nu}{K^2},
\]

\[
B^{\mu\nu} = -\frac{q^2}{(q \cdot u)^2} \frac{\bar{K}^\mu \bar{K}^\nu}{K^2}.
\]

\[
C^{\mu\nu} = -\frac{\bar{n}^\mu \bar{n}^\nu}{\bar{n}^2},
\]

\[
D^{\mu\nu} = \bar{K}^\mu \bar{n}^\nu + \bar{K}^\nu \bar{n}^\mu,
\]

where \( u^\mu \) is the heat-bath vector, which in the local rest frame is given by \( u^\mu = (1, 0, 0, 0) \), and

\[
\bar{K}^\mu = q^\mu - \frac{q^2}{(q \cdot u)} u^\mu.
\]
The direction of anisotropy in momentum space is given by the vector
\[ n^\mu = (0, \vec{n}) . \] (44)

Defining the vector
\[ \tilde{q}^\mu = q^\mu - (q \cdot u) u^\mu , \] (45)
and the projector
\[ \tilde{P}^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu - \frac{\vec{q}^\mu \vec{q}^\nu}{q^2} , \] (46)
introduces the vector
\[ \tilde{n}^\mu = \tilde{P}^{\mu\nu} n_\nu . \] (47)

Both vectors \( \tilde{n}^\mu \) and \( \tilde{K}^\mu \) are orthogonal to \( q^\mu \), and \( \tilde{K} \cdot \tilde{n} = 0 \).

The inverse propagator in covariant gauge is then
\[ \left( \Delta^{-1} \right)^{\mu\nu} (q, \xi) = -q^2 g^{\mu\nu} + q^\mu q^\nu - \Pi^{\mu\nu}(q, \xi) - \frac{1}{\lambda} q^\mu q^\nu , \] (48)
where \( \lambda \) is the gauge fixing parameter. Upon inversion \[46\], the propagator is finally written as, with \( \omega = (q \cdot u) \),
\[ \Delta^{\mu\nu} = \Delta_A A^{\mu\nu} - C^{\mu\nu} + \Delta_G \left[ (q^2 - \alpha - \gamma) \frac{\omega^4}{q^4} B^{\mu\nu} + (\omega^2 - \beta) C^{\mu\nu} + \delta \frac{\omega^2}{q^2} D^{\mu\nu} \right] - \frac{\lambda}{q^4} q^\mu q^\nu , \] (49)
where
\[ \Delta_A^{-1} = (q^2 - \alpha) , \] (50)
and
\[ \Delta_G^{-1} = (q^2 - \alpha - \gamma)(\omega^2 - \beta) + \delta^2 \vec{q}^2 \vec{n}^2 . \] (51)

For \( \xi = 0 \), we recover the isotropic propagator in covariant gauge \[37\]
\[ \Delta_{iso}^{\mu\nu} = \frac{1}{q^2 - \Pi_T A^{\mu\nu}} + \frac{1}{(q^2 - \Pi_L) q^2} B^{\mu\nu} - \frac{\lambda}{q^4} q^\mu q^\nu , \] (52)
where \( \Pi_L = \frac{q^2}{\tilde{q}^2} \Pi_L \) and \[36\]
\[ \Pi_T(\omega, \vec{q}) = \frac{m_D^2}{2} \frac{\omega^2}{q^2} \left[ 1 - \frac{\omega^2 - \vec{q}^2}{2 \omega |\vec{q}|} \log \frac{\omega + |\vec{q}|}{\omega - |\vec{q}|} \right] , \]
\[ \Pi_L(\omega, \vec{q}) = m_D^2 \left[ \frac{\omega}{2 |\vec{q}|} \log \frac{\omega + |\vec{q}|}{\omega - |\vec{q}|} - 1 \right] . \] (53)

In the soft \( \omega \) limit we remind that
\[ \Pi_T \rightarrow -i \pi \frac{\omega}{4 |\vec{q}|} \left( 1 - \frac{\omega^2}{|\vec{q}|^2} \right) m_D^2 , \quad \Pi_L \rightarrow -m_D^2 - i \pi \frac{\omega}{2 |\vec{q}|} m_D^2 . \] (54)

\( m_D \) is the isotropic Debye mass, which for a thermal system at temperature \( T \) with \( N_c \) colours and \( N_f \) light quark flavours is given by
\[ m_D^2 = \frac{g^2 T^2}{3} \left( N_c + \frac{N_f}{2} \right) . \] (55)
Eq. (49) agrees with the expression given in [47], when the following relations are used,

\[ \tilde{K}^\mu = -\frac{q^2}{(q \cdot u)}\tilde{m}^\mu, \tag{56} \]

and

\[ \tilde{n}^\mu = \tilde{n}_{DGS}^\mu - \frac{(q \cdot n)}{K^2}\tilde{K}^\mu, \tag{57} \]

where the tilde vector in [47] is given by

\[ \tilde{n}_{DGS}^\mu = n^\mu - \frac{q \cdot n}{q^2}q^\mu. \tag{58} \]

**B. Kinetic theory approach**

In the case of an isotropic medium the transport coefficient \( \hat{q} \) may be calculated using Eq. (1), where the differential cross section is related to the spin and colour summed (averaged) squared matrix element by

\[ \frac{d\sigma}{d^2q_\perp} = \frac{1}{16\pi^2s^2}|\mathcal{M}|^2, \tag{59} \]

with \( s \) the invariant energy squared.

When properly taking quantum statistics (Pauli-blocking and Bose-Einstein enhancement) into account, it is appropriate to modify (1) as

\[ \rho|\mathcal{M}|^2 \rightarrow \int \frac{d^3k}{(2\pi)^3} \left[ 4N_cN_f n_{FD}(1 - n_{FD})|\mathcal{M}|^2_{qq} + 2(N_c^2 - 1)n_{BE}(1 + n_{BE})|\mathcal{M}|^2_{qG} \right], \tag{60} \]

together with the scattering of the quark jet with the thermalised quarks and gluons in the hot medium, distributed according to the functions \( n_{FD} \) and \( n_{BE} \), respectively.

At leading order, after screening the effective forward scattering amplitudes [48] by the Debye mass \( 1/q_\perp^2 \rightarrow 1/(q_\perp^2 + m_D^2) \), the result is

\[ \hat{q}|_{\text{kinetic}} = \frac{g^2C_F m_D^2 T}{2\pi} \ln \frac{T}{m_D}, \tag{61} \]

in agreement with Eq. (13). This result agrees with the LL one for \( d < p_\perp^2 > /dt = 2 \kappa_T \) calculated in [32] (Eq. B32) taking the quark velocity to be \( v = 1 \).

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