Student visualization in solving geometry problem: the case of reflection

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Abstract. Visualization is a mental act representing a visual information. In solving geometry problem, we need visualization to identify and illustrate the information. This study aimed to analyse student visualization in solving the reflection problems. We studied a female high school athlete at grade eleven. There were three steps in this study. First, we designed two reflection problems. Next, we assessed and interviewed her by semi structure interviewed. Finally, we analysed student spatial visual comparing to types of visualization by Presmeg. Lot of athletes do not have time enough in learning about some topic in their class, so we have to find an appropriate approach to led her in learning activity for creating an efective learning. We found that she has two types of visualization that are pattern imagery and kinaesthetic imagery. In the future, we can design instructional tasks that are convenient for this visualizer.

1. Introduction
In mathematics, there are two representations that are internal and external [1]. Internal representations consist of the illustration in the mind related with personalized, and construct images and external representations consist of symbols in mathematics or graphics that was seen on a paper or a computer screen [1][2]. Internal representation consist of the process of illustrate information (encoding) and assume or conclude the information constructed by the encoding (decoding) [1]. Visualization is a mental act representing a visual information [3][4]. Visualization leads someone in drawing an illustration on paper or computer screen. Visual image are internal representations that illustrate an image in the mind appearing to be similar to real objects [5]. There are two types of visual representation: (1) pictorial representation is a mental act encoding a problem in a picture or illustration; (2) schematic representation is a mental act relating some visual representation in solving the problems [6].

Visualization is shown from student communication the information [2][7]. Visualization can be identified in three condition: (1) internalizing and eksternalizing an object acquired from through experinces and knowledges; (2) connecting objects and symbols based on last experinces and knowledges; and (3) finding a new symbol [2]. Visualization involves four processes: (1) showing an representative image; (2) investigating an representative image to solve problems; (3) holding a representative image from other mental operations (avoiding the bias); (4) transforming an representation image [5].
In solving mathematical problem, Presmeg postulated five types visualization: (1) concrete imagery, (2) pattern imagery, (3) memory image of formulae, (4) kinaesthetic imagery, and (5) dynamic (moving) imagery. First, concrete imagery is a mental act in illustrating objects in the mind that can help someone to understand some concept. Second, pattern imagery is a mental act that interpret the concept in pattern. Third, memory image of formulae is a type of visualization that someone imagines the formulae of some concept in the mind. Fourth, kinaesthetic imagery is a type of visualization that someone use their body to help them interpret some concept. The last, dynamic imagery is a type of visualization that someone represent the concepts by changing them to another concept [8].

Visualization is an important skill in geometry topics [9]. Visualization influences student performance in solving mathematics problems, one of them is geometry problem [10]. The shapes of geometry is defined by: (1) a mental image confined by a definition; (2) creating the shapes is only a concrete form, incarnated of it; and (3) the mental image of them is an representation model of them [11]. In the other hand, the shapes of geometry is an abstract concept in the mind confined by a definition. Visualization is a process of mental generating and manipulating an image in two and three dimensional objects that shifting (translation), coordinating, and merging them [5]. Visualization elevate students’ performance figuring shape in an illustration, depicting visual information, relating some of concepts, and reflecting [12]. Reflection is one concept of geometry and fragment of test assessment measuring visual ability [13]. It means we can measure student visualization by using assessment test consisting of reflection problems. We interviewed some teachers in senior high school and they said that the students had a trouble in understanding geometric transformation. The teachers just gift the formula, not leading students in building their knowledge.

Sport training may develop athletes spatial ability in relation [14]. This ability may be used in the visual spatial tasks. Visual spatial ability of female athletes is worse than male athletes [15] and high school athletes performed worse than collegiate athletes [16]. Hence, the purpose of this research is to analyse student visualization of a female high school athlete in geometric transformation that is reflelction. From the assessment we want to know: (1) how she understand and visualize the question; and (2) how she conclude her visualization.

2. Methods

This research aimed to analyse student visual thinking in solving geometry problems. We studied one of high school students at grade eleven who is a female athlete. There were three steps in this research. First, we designed two problems (Table 1) about reflection. In problem 1, we led her to find the reflection of a dot. In problem 2, we led her to find the reflection of a line. We assumed that she had studied geometric transformation. Both of problems we led her to find the formula of a reflection either a dot or a line. Next, we assessed her and she tried to solving the assessments. And then, we led her in answering the problems and we interviewed her to know her reasoning in solving the problems. And finally, we analysed her answers and compared with visualization analysis that was adapted from Presmeg [8]. Five types of visualization are concrete imagery, pattern imagery, memory image of formulae, kinaesthetic imagery, and dynamic (moving) imagery. Then, we analysed how her make the conclusion about the problem from her visualization. Then we compared her visualization to types of representation by Hegarty and Kozhevnikov [6].

| No | Problem | Answer |
|----|---------|--------|
| 1  | Find the reflection of $A (2,1)$ when the line of reflection is $x$-axis. Can you find the formula? | $(2, -1)$ |
| 2  | Find the reflection of line $y = 2x$ when the line of reflection is $y$-axis. Can you find the formula? | $y = -2x$ |
3. Result and Discussion
Based on neuropsychological research, female high school athletes had worse performance than the others in visual spatial [15][16]. So, we (I) assessed one of high school students at grade eleven who is a female athlete named Ratisa (R). She was 17 years old. We assumed that she had learned and understood geometric transformation already. Before we gave her assessment, we ensured that she understood this topic. She said that lot of athletes do not have time enough in learning about some topic in their class. It was happened because some of them had to concern for their olympiade. After that, we gave her the test assessments and we interviewed her to know her knowledge about reflection and how she solved the problems.

I: Have you learned reflection? Do you still remember about reflection?
R: I am not sure. I think, I forget it.
I: So, if I ask you about reflection, what do you think about it?
R: I think, reflection is like a mirror. When we look into a mirror, we can see that our shadow is looked like us but it is in opposite side. And I know that our shadow is the same distance from the mirror as we are.
I: Yes, right. So, if we give you some problems about reflection, can you find the reflection of a dot or a line?
R: I am not sure. I forget the formula.
I: Do you still remember about Cartesian graph?
R: Yes.
I: Do you think that you can find the reflection of a dot or a line by using a Cartesian graph?
R: Oo. Let me try.

So, we gave her the test assessments. She solved the problems on a Cartesian graph and found the reflection of dot (Figure 1) and line (Figure 2) that we gave.

![Figure 1](image1.png)

Translate:
The reflection is \((2, -1)\)

Figure 1. Student Visualization in Solving Problem 1

After she finished the assessment test, we corrected her answers and interviewed her again to know her reasoning in solving the problem and to know her visualization. First, we analysed her answer in problem 1 (Figure 1).

I: In problem 1, how do you find the reflection?
R: I just need to find the shadow of A when x-axis is the line of reflection. (She rotated the paper)
I: Why did you rotate the paper?
R: Because I have to make sure that the line of reflection is a vertical line, so I can reflect the object correctly.
We were curious about her visualization, then we gave her the other question. We asked her how to find the reflection of $A$ when the line of reflection is $y$-axis and she tried to figure it and answered the reflection is $(-2,1)$.

$I$ : Oke. If the line of reflection is $y$-axis, can you find the reflection of $A$?
$R$ : Ya. $(-2,1)$
$I$ : You do not require to rotating the paper because the line of reflection is a vertical line, right?
$R$ : Yes.

![Image of the visualization process](image)

**Figure 2.** Student Visualization in Solving Problem 2

Next, we analysed her answer in problem 2 (Figure 2). When we interviewed her, she solved problem 2 using the same method when she solved problem 1. Then, we asked her the reflection of the line when the line of reflection is $x$-axis.

$I$ : So, if the line of reflection is $x$-axis and you want to find the reflection in problem 2, what should you do? (R showed the solution by rotating the paper).
$I$ : Oke. So, from the problems, can you find the formula of reflection?
$R$ : Yes. If the line of reflection is $x$-axis, then $y$ is changed. So, the formula is $(x, -y)$. And if the line of reflection is $y$-axis, then $x$ is changed. So, the formula is $(-x, y)$. It is easier than I knew before.

Actually, she was not sure that she had learned this topic already. Then, we helped her solving the problems. We used an approach leading her to build her understanding from geometric form to find the formula in algebra and we led her from a concrete form to more abstract. Geometry is a basic concept of algebra [17] and a good learning is based on students experiences [18]. By this approach, we found her visualization. Ozel, Larue, and Molinaro found that athletes have a visualization better than non athletes [14]. Then, we find that she had two types of visualization that are kinaesthetic imagery and pattern imagery. Kinaesthetic imagery was marked by rotating the paper to find the reflection and pattern imagery was marked by finding the formula from her activity. Presmeg defined that kinaesthetic imagery is a visualization marked by moving involving muscular activity and pattern imagery is a visualization by patterning the relationship of objects [8]. It was happened because she could not remember the words [3]. Pattern imagery guided her to find the relationship of the object with the formulae [3]. Kinaesthetic and pattern imagery were collaborated and transformed be a vivid imagery, one of efficacy of visual processing [3]. Finally, she found the formula in algebra because her representation of the reflection in geometry. It was line with research by Ozel, Larue, and Molinaro [14] argued that athletes sport training may develop their spatial ability and it was line with research by Ginn and Pickens argued that women with spatial experience in athletics have a good performance in visualization [19].

4. Conclusion
As an athlete, she had a good visualization and we found two types of visualization that she had. We found kinaesthetic imagery and pattern imagery. She constructed her knowledge from her visualization and her experience in depicting the reflection on Cartesian graph. For the further, we can design and develop instructional task and we can choose instructional media in learning activity based on her visualization that it can develop her geometry and algebra ability.

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