An existence theorem for steady Navier-Stokes equations in the axially symmetric case

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Abstract. We study the nonhomogeneous boundary value problem for the Navier-Stokes equations of steady motion of a viscous incompressible fluid in a bounded three-dimensional domain with multiply connected boundary. We prove that this problem has a solution in some axially symmetric cases, in particular, when all components of the boundary intersect the axis of symmetry.

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1. Introduction

Let \( \Omega \) be a bounded domain in \( \mathbb{R}^3 \) with Lipschitz boundary \( \partial \Omega = \Gamma_0 \cup \ldots \cup \Gamma_N \), consisting of \( N + 1 \) disjoint connected components \( \Gamma_j \). Consider the stationary Navier–Stokes system with nonhomogeneous boundary conditions

\[
\begin{aligned}
- \nu \Delta \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p &= 0 \quad \text{in} \ \Omega, \\
\text{div} \ \mathbf{u} &= 0 \quad \text{in} \ \Omega, \\
\mathbf{u} &= \mathbf{a} \quad \text{on} \ \partial \Omega.
\end{aligned}
\]  

(1.1)

The continuity equation (1.12) implies the compatibility condition

\[
\int_{\partial \Omega} \mathbf{a} \cdot \mathbf{n} \, dS = \sum_{j=0}^{N} \int_{\Gamma_j} \mathbf{a} \cdot \mathbf{n} \, dS = \sum_{j=0}^{N} F_j = 0
\]  

(1.2)

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