Analytical description of the interaction process of a rigid vibratory drum with a compacted surface using the general dynamics equation

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Abstract. This study has solved the task of creating the theory of the soil compaction by a rigid vibratory drum on the basis of the usage of the general dynamics equation. When rolling over the deformable soil the vibratory drum performs a compaction operation and represents a mechanical system that is influenced by stationary dynamic forces. A design diagram has been created for the consideration of dynamic interaction processes of the vibratory drum with the compacted soil, where the main parameters are represented by the angle $\alpha$ of the vibratory drum interaction with the soil, angle $\beta$ of the inclined plane, which simulates the vibratory drum resistance rolling during the soil compaction and vibratory drum geometric parameters. The general dynamics equation allows to obtain an analytical solution of the problem of the vibratory drum interaction with the soil in the form of the sum of elementary works of the tractive force $T$, load $Q$ on the wheel axles, soil reaction forces on the vibratory drum, and rolling resistance forces. The solution is presented in the equation form of the sum of the elementary works of all the indicated forces, which is equal to zero. Analytical dependences of the vibratory drum rolling moment, rolling resistance force and vibratory drum rolling resistance coefficient over the deformable soil in relation to construction soils of the 1st – 4th strength categories have been obtained. The obtained results can be used when designing vibration rollers for the compaction of deformable media.

Key-words: general dynamics equation, work of force, possible displacements, rolling resistance moment, rolling resistance coefficient.

1. Introduction
The process of the layer-by-layer compaction of soils by the road roller vibratory drum in the construction industry is not sufficiently studied, as there are no analytical solutions which use laws of mechanics. The soil compaction technological process requires further improvement on the basis of the use of modern research methods, laws of mechanics and general dynamics equation. The scientific study of the interaction of the vibratory drum with the soil during compaction is primarily connected with the creation of a design diagram of the studied mechanical process. Any design scheme represents a model which reflects certain mechanical movements and it is adequate to a real process relative to a definite number of indicators.

For the soil compaction process by the vibratory drum, scientific studies are known, where sufficiently complex design diagrams are used which take into consideration numerous features of the considered processes of the vibratory drum interaction with the soil. However, the use of such design diagrams leads to obtaining complex differential equations which don’t have analytical conclusions and solutions.
R. Bernstein [1], M.N. Letoshnev [2] and others have developed mathematical models of the rigid wheel interaction with the soil, which were further improved by the Russian and foreign scientists E.C. Woods. [3], M. G. Bekker [4], V. F. Babkov [5], N.A. Ulyanov [6], N.Ya. Kharkhuta [7], N.A. Tsytovich [8] and many others. The models of the wheel interaction with the soil of these authors possess a unified scientific basis.

Among the known formulas of the rigid wheel rolling resistance over the soil, empirical formula of M.N. Letoshnev [2] deserves consideration. If we apply this formula for the rolling of a rigid cylindrical vibratory drum, then it will take the following form:

$$P_f = \frac{Q^2}{\sigma_0 LD}, \quad (1)$$

where $P_f$ – vibratory drum rolling resistance; $Q$ – vertical load on the vibratory drum; $L$ – vibratory drum width; $D$ – vibratory drum diameter; $\sigma_0$ – specific pressure under the vibratory drum.

From formula (1), we can obtain the formula for the determination of the moment $M_k$ of the rigid vibratory drum rolling over the deformable soil

$$M_k = 0.5 \frac{Q^2}{\sigma_0 L}. \quad (2)$$

Studies [9-13] are devoted to the pneumatic wheel interaction with a support surface. At a high pressure inside a tire on loose soil, the pneumatic wheel movement is similar to the rigid vibratory drum movement over loose soil, i.e. the improvement of the rigid vibratory drum rolling theory is an urgent problem, both for a rigid vibratory drum and for a deformable pneumatic tire.

For the improvement of energy-intensive technological processes of the soil and material compaction a theory is required, allowing to establish connection between physical processes of the soil compaction with geometric dimensions and energy consumable for the compaction process.

2. Problem statement

When using the general dynamics equation to develop a methodology that analytically describes the soil compaction process, and which allows to determine the moment and force of the vibratory drum rolling resistance over the deformable soil, depending on the load on the vibratory drum axis $Q$, geometric parameters of the vibratory drum, initial stress $\sigma_0$, and soil deformation modulus $E_0$.

3. Theory

Figure 1 shows the process of the soil compaction by a vibratory drum, as a vertical indentation of the soil volume of the $ABD$ section, located under the arc $AB$, into the soil half-space.

![Figure 1. Design diagram of the road roller vibratory drum during compaction process.](image-url)
The vibratory drum moves under the action of the horizontal force $T$, at the same time the vertical force $Q$ on the vibratory drum axle performs the compaction process. The vibratory drum turning at the angle $\alpha$ creates the horizontal displacement by the value $x=r\sin\alpha$. When the vibratory drum turns at the angle $\alpha$, the soil crushing occurs under the $AB$ arc, which for small angles $\alpha$ can be replaced with the $AB$ chord with sufficient accuracy for technical calculations.

Figure 1 shows the parameters: $z_s$ – vibratory drum settlement; $\alpha$ – interaction angle of the vibratory drum with the soil; $\beta$ – angle of the $AB$ chord, forming a reduced plane, which moves translationally while the vibratory drum rolling, and it is a support for the moving vibratory drum; $h_0$ – thickness of the compacted soil layer.

The vibratory drum rolling resistance when moving along an inclined plane with the angle $\beta$, created by the vertical load $Q$ on the vibratory drum axle represents an analogue of the numerical value of the rolling resistance force $P_f$.

The following regularity of the relationship between small values of the angles $\alpha$ and $\beta$ has been established, at the same time the angle $\alpha$ is twice as much as the value of the angle $\beta$.

The value of the angle $\beta$ is determined by the geometric formulas

$$\beta = \arctg\frac{1-\cos \alpha}{\sin \alpha} \quad \text{or} \quad \beta = \arcsin\frac{1-\cos \alpha}{2\sin \frac{\alpha}{2}}.$$  \hspace{1cm} (3)

The contact area of the vibratory drum with the soil is equal to the projection of the contact surface on the horizontal plane.

Let’s consider in more details the physical and mechanical nature of phenomena, arising during the soil compaction by the vibratory drum (Figures 1, 2).

Figure 1 shows that the resistance force $P_f$ is also equal to zero in the absence of the shearing force $T$. The vibratory drum contact point $A$ is shifted to the left to the position $B'$, which means that for a stationary vibratory drum the contact length has the value $b'+b$. However, for a moving wheel, when the force $T$ and corresponding force $P_f$ appear, the contact form is changed. The section $b'$ of the contact length is decreased and the section $b$ is increased. Thus, the developed design diagram (see Figure 1) of the vibratory drum interaction with the soil is based on the following initial positions.

During the soil compaction by the vibratory drum, the vibratory drum vertical settlement $z_s$ occurs, the soil strength is increased, at the same time the soil acquires elastic properties, which are equal to 2–6% of the total soil strength. Therefore, the obtained design diagram in Figure 1 turns out to be the most adequate for technical calculations in comparison with design diagrams which are currently published in the technical literature.

The considered regularities in Figure 1 are turned out to be valid for the final stage of the compaction process as well (Figure 2), when the roller settlement is equal to 0.

Figure 2. Design diagram of the road roller vibratory drum at the final compaction stage
In Figure 2 at the final stage of the soil compaction by the vibratory drum in the contact area at the $AB$ arc section the soil elastic vertical compression process $z_y$ occurs, at the same time elastic deformations of the initial flat soil surface restoration under the vibratory drum take place at the $AB'$ arc section.

For the vibratory drum moving under the action of the force $T$, the vibratory drum contact with the soil has the length $b' + b$. However, at the $AB'$ arc section of the vibratory drum contact with the soil, the soil reaction forces on the vibratory drum are insignificant, as the separation of the vibratory drum from the soil occurs.

As per the analytical mechanics classification, the vibratory drum movement during the soil compaction represents a stationary dynamic process for a mechanical system with two degrees of mobility: $x$ – translational movement coordinate; $\phi$ – vibratory drum rotation coordinate.

The general dynamics equation allows to solve dynamics problems of mechanical systems on the basis of the principle of virtual displacements. For Figure 1, general dynamics equation has the form

$$
\sum \delta A_k^F + \sum \delta A_k^R + \sum \delta A_k^\Phi = 0.
$$

where \( \sum \delta A_k^F \) – sum of elementary works of active forces; \( \sum \delta A_k^R \) – sum of elementary works of constrain reaction forces; \( \sum \delta A_k^\Phi \) – sum of elementary works of inertial forces.

In equation (4) the constraints are considered as ideal ones, at the same time friction and resistance forces have been transferred to the category of specified forces and are taken into consideration as active forces.

During the movement of a mechanical system with ideal constraints, at any time the sum of elementary works of all applied active forces and all inertial forces at any possible movement of the system is equal to zero.

The soil compaction process by the vibratory drum is a stationary dynamic process, where acting forces are stationary ones, that’s why there are no inertia forces in equation (4), and work of inertia forces is equal to zero \( \sum \delta A_k^\Phi = 0 \).

For the mechanical system, we can compose equations relative to the sum of elementary works in the $x$ and $\phi$ – coordinates. The vibratory drum horizontal movement along the $x$ axis when turning at the angle $\alpha$ performs the technological operation of the soil compaction. When the vibratory drum moves by the value $\delta x = r \sin \alpha$, the soil under the arc of the $AB$ contact is crushed by vertical compression, at the same time the crushed soil columns are subject to the vertical compression.

Figure 1 and Figure 2 show the angles $\alpha$ and $\beta$ as significantly increased, in fact, when compacted, they represent small values, at the same time the $AB$ cord and arc differ insignificantly. The vertical reaction force $R_{zs}$ is formed under the vibratory drum on the soil side, which according to the Newton’s third law is equal to the vertical force $Q$ on the vibratory drum axle, i.e. $R_{zs} = Q$.

In contrast to working procedure [14], in this study the points $A$ and $B$, connecting the end and beginning of the contact area, allow to simulate the calculation process of the vibratory drum displacement force along this inclined plane.

The process of the vibratory drum upward movement along the $AB$ inclined plane and process of crushing the soil under the vibratory drum are appeared to be oppositely directed. As a result of this, vibratory drum $l$ performs a translational horizontal movement together with the force $Q$, at the same time the reaction force $R_{zs}$ according to the Newton’s third law is equal to the force $Q$, i.e. $Q = R_{zs}$, which performs the soil compaction work.

The horizontal force $T$ on the wheel axle creates the horizontal force $P_f$ of rolling resistance in the vibratory drum contact with the soil, which are also equal to each other $P_f = T$.

The considered condition allows to compose a general dynamics equation with the following assumption: we express the infinitesimal displacements $\delta x$, $\delta \phi$ through the finite turning angle $\alpha$.

The sum of works of active forces and reaction forces when turning the vibratory drum at the angle $\alpha$ is equal to zero.
\[ P_f r \sin \alpha - Q(r - r \cos \alpha) - Q \sin \beta \cdot r \sin \alpha = 0. \] (5)

The first summand in equation (5) represents the work of the rolling resistance force \( P_f \) on the displacement \( \delta x = r \sin \alpha \). The second summand represents the work of the soil crushing force on the vertical displacement \( z_s \). The third summand represents the work of the component of the resistance force \( Q \sin \beta \) to the vibratory drum movement along the inclined \( AB \) plane.

From equation (5) the vibratory drum rolling resistance moment along the deformable soil has the form:

\[ M_k = \frac{Q(1 - \cos \alpha)}{\sin \alpha} + Qr \sin \beta. \] (6)

From equation (6) we’ll obtain

\[ M_k = Q \cdot r \left( \frac{(1 - \cos \alpha) + \sin \beta \sin \alpha}{\sin \alpha} \right). \] (7)

From formulas (6), (7) it is possible to determine the vibratory drum rolling resistance force

\[ P_f = \frac{M_k}{r} = Q \cdot r \left( \frac{(1 - \cos \alpha) + \sin \beta \sin \alpha}{\sin \alpha} \right). \] (8)

And obtain the rolling resistance coefficient

\[ f = \frac{P_f}{Q} = \frac{(1 - \cos \alpha) + \sin \beta \sin \alpha}{\sin \alpha}. \] (9)

### 4. Results discussion

The study of the process of the soil compaction by a static vibratory drum starts with the determination of the initial contact length of the vibratory drum with the soil for the initial compaction conditions by the formula

\[ b' = \frac{Q}{L \sigma_0}. \] (10)

The contact width \( b' \) allows to determine the initial angle of the vibratory drum interaction with the soil

\[ \alpha' = \arcsin \frac{b'}{r}. \] (11)

We determine the initial soil settlement

\[ z_s' = r(1 - \cos \alpha'). \] (12)

In the compaction process an additional component of the load \( \Delta Q \) appears, caused by the soil strength increase

\[ \Delta Q = L r z_s E_0 \cos \alpha', \] (13)

where \( E_0 \) – soil deformation modulus.

Let’s determine the final width of the contact \( b \) after the vibratory drum turning at the angle \( \alpha \)

\[ b = \frac{Q - \Delta Q}{L \sigma_0}. \] (14)

The contact width \( b \) allows to determine the interaction angle of the vibratory drum with the soil
\[ \alpha = \arcsin \frac{b}{r}. \]  

(15)

We determine the vibratory drum settlement \( z_s \) by the formula

\[ z_s = r (1 - \cos \alpha). \]  

(16)

Using formulas (7) and (8) we determine the moment and force of the vibratory drum rolling resistance over the deformable soil, as per formula (9) – the trolling resistance coefficient \( f \).

In order to check the adequacy of obtained formulas (7) and (8), comparisons are made with formulas (1) and (2), which are adapted for the vibratory drum moving over the soil surface.

5. Consideration of the results

The Table shows the study results of the compaction process of construction soils of the 1st – 4th strength categories [15] by a vibratory drum, which width and diameter are \( L \times D = 2.14 \times 1.55 \) m.

**Table 1. Results of the analytical study of the compaction process of soils of the 1st – 4th categories by a rigid vibratory drum**

| No | Soil category | \( Q \), kN | \( \sigma_0 \), MPa | \( E_0 \), MPa | \( b' \), m | \( \alpha' \), grad |
|----|---------------|-------------|-----------------|-------------|------------|---------------|
| 1  | I             | 240         | 0.524           | 2.096       | 0.2130     | 15.955        |
| 2  | I             | 240         | 2.091           | 33.560      | 0.0534     | 3.950         |
| 3  | II            | 500         | 2.620           | 52.400      | 0.0888     | 6.577         |
| 4  | II            | 500         | 4.190           | 134.000     | 0.0555     | 4.107         |
| 5  | III           | 600         | 4.718           | 170.252     | 0.0592     | 4.377         |
| 6  | III           | 600         | 8.381           | 537.000     | 0.0333     | 2.462         |
| 7  | IV            | 800         | 8.908           | 605.990     | 0.0418     | 3.090         |
| 8  | IV            | 800         | 18.340          | 2567.000    | 0.0203     | 1.500         |

Continuation of the Table

| No. | \( z'_s \), m | \( \Delta Q \), N | \( b \), m | \( \alpha \), grad | \( z_s \), m | \( \beta \), grad |
|-----|---------------|----------------|---------|----------------|------------|---------------|
| 1.  | 0.0299        | 28659.51       | 0.1876  | 14.008        | 0.0230     | 7.004         |
| 2.  | 0.0018        | 7090.90        | 0.0518  | 3.833         | 0.0017     | 1.9165        |
| 3.  | 0.0051        | 50998.82       | 0.0797  | 5.903         | 0.0041     | 2.9515        |
| 4.  | 0.0020        | 31821.60       | 0.0520  | 3.845         | 0.0017     | 1.9225        |
| 5.  | 0.0023        | 48943.77       | 0.0543  | 4.020         | 0.0019     | 2.01          |
| 6.  | 0.0007        | 27512.65       | 0.0318  | 2.349         | 0.0007     | 1.1745        |
| 7.  | 0.0011        | 61305.85       | 0.0386  | 2.853         | 0.0010     | 1.4265        |
| 8.  | 0.0003        | 29741.55       | 0.0195  | 1.444         | 0.0002     | 0.722         |

Continuation of the Table

| No. | \( M_s \), N·m by formula (7) | \( M_s \), N·m by formula (2) | \( P_f \), N by formula (8) | \( P_f \), N by formula (1) | Discrepancy of results, % | Coefficient \( f \) |
|-----|-------------------------------|-------------------------------|-----------------|-----------------|--------------------------|----------------|
| 13  | 28317.70                      | 25563.64                      | 36538.97        | 32985.35        | 9.73                     | 0.1522        |
| 14  | 6638.77                       | 6406.19                       | 8566.15         | 8266.06         | 3.50                     | 0.0357        |
| 15  | 22026.18                      | 22190.66                      | 28420.88        | 28633.11        | 0.74                     | 0.0568        |
| 16  | 13877.12                      | 13875.78                      | 17905.96        | 17904.24        | 0.01                     | 0.0358        |
| 17  | 17457.99                      | 17745.00                      | 22526.44        | 22896.78        | 1.62                     | 0.0375        |
The Table shows the results of checking the adequacy of the performed studies of $M_k$ (column 13), which were compared with the values of $M_k$ (column 14). The average discrepancy of the results for construction soils (column 17) doesn’t exceed 2.5663%.

Figure 3 shows the nonlinear dependence of the rolling resistance coefficient $f$ on the angle $\alpha$ of the vibratory drum contact with the support surface.

![Figure 3. Dependence of the rolling resistance coefficient $f$ on the angle $\alpha$ for different soils:](image)

1. For the first time the analytical dependence of the rolling resistance coefficient $f$ has been obtained as the function of the angle $\alpha$ of the vibratory drum contact with the soil and angle $\beta$ of the inclined plane. For the first time the value of the rolling resistance coefficient $f$ has been tied to the construction soils of the $1^{\text{st}}$ – $4^{\text{th}}$ strength categories.

2. In the zone of elastic deformations the rolling resistance coefficient $f$ represents a nonlinear function.

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