Gauge Coupling Unification from Unified Theories in Higher Dimensions

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Abstract

Higher dimensional grand unified theories, with gauge symmetry breaking by orbifold compactification, possess $SU(5)$ breaking at fixed points, and do not automatically lead to tree-level gauge coupling unification. A new framework is introduced that guarantees precise unification — even the leading loop threshold corrections are predicted, although they are model dependent. Precise agreement with the experimental result, $\alpha_s^{\text{exp}} = 0.117 \pm 0.002$, occurs only for a unique theory, and gives $\alpha_s^{\text{KK}} = 0.118 \pm 0.004 \pm 0.003$. Remarkably, this unique theory is also the simplest, with $SU(5)$ gauge interactions and two Higgs hypermultiplets propagating in a single extra dimension. This result is more successful and precise than that obtained from conventional supersymmetric grand unification, $\alpha_s^{\text{SGUT}} = 0.130 \pm 0.004 \pm \Delta_{\text{SGUT}}$. There is a simultaneous solution to the three outstanding problems of 4D supersymmetric grand unified theories: a large mass splitting between Higgs doublets and their color triplet partners is forced, proton decay via dimension five operators is automatically forbidden, and the absence of fermion mass relations amongst light quarks and leptons is guaranteed, while preserving the successful $m_b/m_\tau$ relation. The theory necessarily has a strongly coupled top quark located on a fixed point and part of the lightest generation propagating in the bulk. The string and compactification scales are determined to be around $10^{17}$ GeV and $10^{15}$ GeV, respectively.
1 Introduction

Weak scale supersymmetry not only provides a framework for electroweak symmetry breaking, but also leads to a successful unification of gauge couplings at extremely high energies. If this picture of a supersymmetric desert is correct, then the scale of gauge coupling unification certainly heralds the threshold for some new physics. The desert will end with the appearance of some more unified theory. What is this new physics just above the supersymmetric desert? There are two conventional answers.

From the bottom up viewpoint, a supersymmetric grand unified theory is the simplest interpretation of gauge coupling unification \[1, 2\]. It gives an elegant explanation for charge quantization and the pattern of quark and lepton gauge quantum numbers. Furthermore, it can lead to a reduction of parameters in the flavor sector leading to quark and lepton mass relations, and is a near perfect home for the see-saw mechanism for generating small, non-zero Majorana neutrino masses. However, considerable obstacles are encountered in constructing simple and realistic 4D grand unified theories, even when low energy supersymmetry is included. Chief amongst these are the mass splitting of Higgs doublets from their color triplet partners, proton decay induced by dimension five operators and the observed breaking of $SU(5)$ symmetry in the light quark and lepton masses. Indeed, the simplest supersymmetric $SU(5)$ theory is excluded by the limit on the proton lifetime. Furthermore, as we discuss in detail shortly, the prediction from gauge coupling unification is not in precise agreement with data.

The second conventional answer is that string theory is just above the gauge coupling unification scale, without any energy interval with a 4D grand unified gauge symmetry. This top down approach is the only serious contender for a quantum theory of gravity. It has brought several new ideas relevant for the problems of gauge symmetry breaking, doublet-triplet splitting and fermion mass relations \[3, 4\]. However, at present there is a barrier preventing a connection to the low energy domain: the problem of finding a consistent solution of string theory having a fully realistic low energy spectrum.

In a previous paper \[5\], we have introduced a third possibility for the new unified physics which lies above the desert and leads to gauge coupling unification: a higher dimensional grand unified theory compactified on an orbifold, with boundary condition breaking of the gauge symmetry. This really is a new alternative — there is no limit of the theory in which it contains the usual 4D grand unified theories. It is not at all obvious that this new bottom up approach leads to gauge coupling unification, even at tree level, because there is local breaking of $SU(5)$ gauge invariance at the orbifold fixed points. To recover gauge coupling unification, we find that the string scale must be considerably above the compactification scale. The large energy interval where physics is described by a higher dimensional grand unified theory...
distinguishes this scheme from the conventional string theory picture. It is remarkable that the three problems of 4D grand unified theories are all elegantly solved in this scheme. The orbifold projection of the unwanted color triplet Higgs zero mode, previously used in the context of string theory, transfers elegantly to the case of higher dimensional field theory [8]. Lighter generations do not have unified fermion mass relations if they reside in the bulk, and dimension five proton decay from colored Higgsino exchange is automatically removed by the form of the Higgsino mass matrix determined by higher dimensional spacetime symmetry [4]. We call this third framework for physics beyond the desert Kaluza-Klein (KK) grand unification.

In this paper we attempt to identify the leading candidate theory within KK grand unification. We seek the simplest theory that gives a precise prediction for gauge coupling unification, without any significant dependence on unknown threshold corrections from high energies, and also solves the three problems of 4D grand unification. This is clearly very ambitious. In the case of 4D grand unification, unknown high energy threshold corrections to gauge coupling unification are present, and are assumed to eliminate the discrepancy between theory and experiment. These corrections can arise from any number of complications to the spectrum of the theory. In higher dimensions we find a contrasting result: a completely predictive and reliable framework for gauge coupling unification follows from imposing a crucial new assumption of strongly coupled gauge interactions at the string scale. Remarkably, precise agreement with data implies an essentially unique theory, which is also the simplest. This again contrasts with the 4D case, where the simplest theory is excluded from proton decay. The theory also provides a new arena for an understanding of quark and lepton masses: at least some of the hierarchy amongst fermion masses arises from the large volume of the bulk. Furthermore, the theory gives a clearly successful correlation: heavier fermions should display SU(5) mass relations while lighter ones should not.

The key ingredient to uncovering this higher dimensional unified theory is gauge coupling unification, so we start by reviewing the situation in 4D grand unified theories [7, 8]. The electroweak gauge couplings are now so well measured, that we choose to input them from data and give a prediction for the QCD coupling at the scale of the Z mass, which has a measured value $\alpha_s^{\exp} = 0.117 \pm 0.002$ [6]. Assuming the standard model holds to extremely high energies of order $10^{15}$ GeV, the grand unified prediction is

$$\alpha_s^{\text{GUT}} = 0.077 \pm \Delta_{\text{GUT}}.$$  \hspace{1cm} (1)

The physics at the weak scale and in the desert up to the unified mass scale is assumed known, so that this part of the calculation has essentially no uncertainty. The quantity $\Delta_{\text{GUT}}$ arises from the physics at the unified mass scale, which is not well known. This threshold correction must be very large, correcting the leading order prediction by 50%. One therefore concludes
that in this theory there is no precise prediction of the QCD coupling: the threshold correction depends on some continuous parameter of the unified theory which simply has to be chosen to give the experimental result.

The situation is greatly improved in supersymmetric grand unified theories \[1, 2\]. Superpartners at the TeV scale modify the radiative corrections to the gauge couplings, leading to the prediction \[10\]

\[
\alpha_s^{\text{SGUT}} = 0.130 \pm 0.004 \pm \Delta_{\text{SGUT}}. \tag{2}
\]

The first uncertainty arises from variations in the superpartner spectrum at the TeV scale, while the second uncertainty arises from the unknown spectrum of states at the unification scale. While a non-zero threshold correction is necessary for agreement with data, one can take the viewpoint that such corrections are typically expected. This prediction is a crucial part of the motivation for the 4D grand unification paradigm, but a purist might still object that in any particular model the correction \(\Delta_{\text{SGUT}}\) depends on unknown continuous parameters, so that formally there is no prediction. In the minimal \(SU(5)\) supersymmetric unified theory there is only one such parameter, which can be taken to be the mass of the heavy color triplet partners of the Higgs doublets. Unfortunately, the sign of the correction is such that the value of the Higgs triplet mass needed for the prediction for the QCD coupling to agree with data is less than the unified mass scale, and is excluded by the experimental limit on the proton lifetime. There are no known models where \(\Delta_{\text{SGUT}}\) can either be successfully predicted from theory or constrained from independent data. As the experimental data on gauge couplings has improved, the requirement for a large value of \(\Delta_{\text{SGUT}}\) has become more pronounced. For example, if superheavy \(5 + \bar{5}\) chiral multiplets of \(SU(5)\) are added, a unit logarithmic mass splitting between doublet and triplet components gives \(\Delta_{\text{SGUT}} \approx 0.003\). The data requires several such multiplets, a large Casimir, or a large mass splitting.

In higher dimensional gauge theories, the grand unified symmetry can be broken by orbifold boundary conditions. At first sight this is a disaster. The orbifold contains fixed sub-spaces on which the unified symmetry is explicitly broken. Local gauge kinetic terms on these sub-spaces lead to a tree-level violation of gauge coupling unification, so that the prediction for the QCD coupling is completely lost. In Ref. \[5\] we overcame this difficulty by requiring the bulk to have a large volume, and introduced a new framework for gauge coupling unification in higher dimensional unified theories, which we pursue further here. The improvements from supersymmetry are retained, with superpartner contributions to the evolution of gauge couplings from

\footnote{At first sight it appears that this prediction is accurate at the 10\% level. However, this understates the significance of the result. Viewing the prediction as a correlation in the plane of the QCD coupling and weak mixing angle, one finds the accuracy to be at about the 1\% level \[11\]. This is then the most significant prediction of any of the 18 free parameters of the standard model, explaining the wide attention it has received.}
the weak scale to some high energy scale $M_c$. At this scale the picture is greatly altered by the opening up of extra spatial dimensions: one expects threshold corrections at $M_c$ and power-law running of the three gauge couplings above $M_c$ up to the string scale $M_s$. The crucial new ingredient follows from the restricted set of unified gauge transformations in the bulk, which are determined by the orbifold boundary conditions [5]. This bulk symmetry ensures that the leading, power-law evolution is $SU(5)$ symmetric. However, fixed points in the bulk do not respect the full unified gauge symmetry and lead to an additional evolution: a non-universal logarithmic running of the standard model gauge couplings in the energy region above $M_c$.

The prediction for the QCD coupling therefore involves three terms: supersymmetric evolution from $M_Z$ to $M_c$ involving a very large logarithm $\ln(M_c/M_Z)$, threshold contributions at $M_c$ and $M_s$, and a moderately large logarithmic term proportional to $\ln(M_s/M_c)$ originating from the KK towers. It is this structure that allows a completely predictive framework for gauge coupling unification. Requiring the gauge coupling to be strongly coupled at the string scale suppresses unknown contribution from ultraviolet physics to a negligible level [12]. The discrepancy between the usual supersymmetric prediction, Eq. (2), and data is then provided by the second KK logarithm, which is smaller than the first supersymmetric logarithm, but larger than the non-logarithmic threshold corrections. It is remarkable that even though there are many theories of this type, depending on the number of compact dimensions, the nature of the orbifold and the gauge group, we find that only one gives precise agreement with data. This theory is also the minimal possibility, with gauge group $SU(5)$ in 5D broken to the standard model on the orbifold $S^1/Z_2$, giving

$$\alpha_{KK}^s = 0.118 \pm 0.004 \pm 0.003,$$

where the first error bar is the uncertainty from the superpartner spectrum, which can be eliminated by future measurements, and the second error is from residual uncertainties from physics at $M_s$. It is important that the theory does not contain any free parameter that can be used to adjust the prediction for the QCD coupling. The masses and couplings of the unified scale particles are all fixed by the orbifold boundary conditions, and the ratio $M_s/M_c$ is determined by the strong coupling requirement. The essential features of this theory are illustrated in Fig. 2.

In section 2 we define a framework which leads to a precise prediction for the QCD coupling, and find that the compactification which breaks the unified gauge symmetry involves at most two extra dimensions. In section 3 we show that this framework precisely accounts for the data only if this compactification is on $S^1/Z_2$ and the unified gauge group is $SU(5)$. The size of the theoretical uncertainties are discussed. In section 4 a fully realistic $SU(5)$ model is explored, concentrating on the solution to the three outstanding problems of 4D supersymmetric grand
unified theories: the splitting between doublet and triplet Higgs masses, dimension five proton decay and the absence of $SU(5)$ mass relations for the first two generations. We investigate to what extent fermion masses and mixings can be understood from locality in the bulk, and briefly mention possible signatures from dimension six proton decay, which occurs only via flavor mixing. In section 3 we comment on the possible origin of our theory from string theory. Conclusions are drawn in section 6.

2 The Framework

KK grand unification provides a third possibility for physics above the supersymmetric desert. Here we push this idea further and propose a completely predictive framework for gauge coupling unification, which follows from the additional assumption that the theory is strongly coupled at the string scale. This assumption, which we find quite plausible, ensures that threshold corrections from the string scale are highly suppressed, and determines the size of the leading loop corrections below the string scale. Our framework is defined by the following five elements:

- We introduce two mass scales $M_c$ and $M_s$, rather than a single unification mass scale.
- The scale, $M_c = 1/R$, characterizes the size of $d$ extra spatial dimensions. The structure of this $d$ dimensional compact space is chosen so that the framework leads to a precise prediction for gauge coupling unification without significant sensitivity to unknown ultra-violet physics. (We will find that $M_c$ must be taken very large, of order $10^{15}$ GeV, but it differs from the usual unification mass scale.)
- The effective theory above $M_c$ is a higher dimensional grand unified theory with gauge group $G$. This gauge group is broken at $M_c$, by boundary conditions of the extra spatial dimensions, to the standard model gauge group (together with possible extra factors).
- The effective theory below $M_c$ is the minimal supersymmetric standard model (MSSM).
- $M_s$ is the scale at which the effective higher dimensional theory is embedded into a more fundamental theory such as string theory. We identify $M_s$ as the scale where gauge interactions of $G$ become strongly coupled: $C(g^2/16\pi^2)(M_s/M_c)^d \simeq 1$, where $C$ is a group theoretical factor appearing in the loop expansion. (For example, in the case of $G = SU(N)$, $C \simeq N$.) Since $g$ evolves slowly up to energies very close to $M_s$, we may estimate $M_s$ by taking $g$ to be the 4D gauge coupling at the scale $M_c$, $g \simeq 0.7$, giving $(M_s/M_c)^d \simeq 300/C$. (Note that $d$ cannot be taken too large, otherwise the two masses $M_c$ and $M_s$ do not represent different scales.)

This framework appears to be very broad, encompassing many possibilities for $G$, $d$ and the compact space. From the viewpoint of gauge coupling unification it is convenient to divide
the set of higher dimensional unified theories into four types, I — IV. In Fig. 1 the evolution of the three standard model gauge couplings \( g_i \) are illustrated for each type of theory, showing the behavior in the energy intervals \( M_Z \) to \( M_c \) and \( M_c \) to \( M_s \). In general, the prediction for gauge coupling unification depends not only on the zero modes, but on the entire towers of KK modes. We therefore discuss the gauge symmetry structure of the entire higher dimensional theory.

Compactification is obtained by imposing a set of identifications on the space of the extra dimensions: \( y \rightarrow k(y) \). To break the unified gauge symmetry, while leaving an unbroken subgroup, the gauge fields are chosen to be even or odd under each such identification: \( A_{\mu}^{a \pm}(x, y) \rightarrow \pm A_{\mu}^{a \pm}(x, k(y)) \). This set of gauge fields results only if the underlying gauge symmetry of the theory has the form: \( \xi^{a \pm}(x, y) \rightarrow \pm \xi^{a \pm}(x, k(y)) \), which we refer to as restricted gauge symmetry [5]. At a typical location in the bulk, all gauge parameters \( \xi^a \) are non-zero, so that the bulk is \( G \) invariant.

In type I theories the \( d \) dimensional space is a manifold. This means that all identifications are freely acting: there is no point in the bulk for which \( k(y) = y \). The full gauge invariance of \( G \) acts locally at every point in the manifold, so that the gauge couplings unify at \( M_c \), as shown in Fig. 1a. There are no \( G \)-violating effects at distances below \( R \). The prediction for the QCD gauge coupling is the usual one for supersymmetric unification, Eq. (2). However, the threshold corrections \( \Delta_{SGUT} \) from \( M_c \) can now be computed. They arise from the KK excitations of the Higgs and gauge multiplets and are much too small to explain the difference between the prediction, \( 0.130 \pm 0.004 \), and the data, \( 0.117 \pm 0.002 \). If extra bulk multiplets are added, the boundary conditions on the manifold will lead to additional zero modes which are not in complete \( SU(5) \) multiplets, with disastrous results for gauge coupling unification. Theories compactified on a manifold could become interesting if experiments find the spectrum of superpartners to be far from that of theoretical expectations, so that weak scale threshold corrections bring the prediction for standard supersymmetric unification into agreement with data. Even in this case, however, obtaining a realistic low energy theory with chiral fermions may be difficult on such smooth spaces.

The remaining theories are all compactified on orbifolds, and easily lead to low energy chiral theories. Orbifolds result when there is at least one identification having \( k(\bar{y}) = \bar{y} \), and we call \( \bar{y} \) a fixed sub-space or a brane. The gauge parameters and associated gauge fields which are odd under this identification vanish on the fixed sub-space, \( \xi^{a-}(\bar{y}) = 0 \). On \( \bar{y} \), the gauge symmetry is broken from \( G \) to a subgroup \( H \) generated by \( \xi^{a+} \). Hence, the restricted gauge symmetry, resulting from the orbifold boundary conditions, allows local \( G \) violation [5]. Since we are using an effective field theory viewpoint, the most general set of \( H \) invariant operators occurs on the orbifold fixed sub-spaces, leading to explicit, local breaking of \( G \). In particular, there
a) Type I theories

b) Type II theories

c) Type III theories

d) Type IV theories

Figure 1: The running of the difference of the three gauge couplings, $\eta_i \equiv \alpha_i^{-1} - \alpha_1^{-1}$, below and above $M_c$. In type I and III theories, the gauge couplings unify at $M_c$ and $M_s$, respectively. In type II theories, the successful prediction is typically destroyed by the non-universal tree-level and power-law running contributions to $\eta_i$. (In the case that the tree-level contributions are small, the situation is similar to that in type I theories.) In type IV theories, a naive extrapolation of the low energy gauge couplings leads to an approximate unification at a scale between $M_c$ and $M_s$, giving a small deviation from the case of single-scale unification.
are fixed sub-spaces giving kinetic energy operators for the standard model gauge fields with non-unified coefficients \( \frac{1}{\tilde{g}_i^2} \). At first sight, such “non-universal fixed sub-spaces” ruin gauge coupling unification. However, we find that this need not be the case, although these branes do play an important role for gauge coupling unification, at both tree and quantum levels.

To see the effect of local \( G \) violation, we first consider the effective action at the scale \( M_s \). No matter what physics occurs above \( M_s \), the restricted gauge symmetry ensures that the \((4 + d)\)-dimensional bulk is \( G \) symmetric and all \( G \)-violating effects appear only on \( G \)-violating branes. Therefore, the most general form for the gauge kinetic energy is given by

\[
S = \int d^4x d^d y \left[ \frac{1}{g_{4+d}^2} F^2 + \delta^{(d-\delta)}(y - \bar{y}) \frac{1}{\tilde{g}_i^2} F_i^2 \right],
\]

where the first term is a \( G \)-invariant bulk gauge kinetic energy, while the second term represents non-unified kinetic operators located on a non-universal brane of dimensions \( 4 + \delta \). (In general, there are contributions from several non-universal branes and also from \( G \)-symmetric branes.)

The standard model gauge couplings in the equivalent 4D KK theory are obtained by integrating over the \( d \) extra dimensions:

\[
\frac{1}{g_i^2} = \frac{V}{g_{4+d}^2} + \frac{V'}{\tilde{g}_i^2},
\]

where \( V \) is the volume of the bulk, and \( V' \) the volume of the non-universal brane (\( V' = 1 \) if \( \delta = 0 \)). Now, since the theory is assumed to be strongly coupled at \( M_s \), both bulk and brane gauge couplings are reliably estimated as \( g_{4+d} \approx \bar{g}_i \approx 4\pi \) in units of \( M_s \).\footnote{They are estimated, for example, by considering loop diagrams in the equivalent 4D KK theory. In the 4D picture, the bulk term gives gauge kinetic terms with KK momentum conservation, while the brane ones give terms with KK momentum violation. After diagonalizing these kinetic terms, the gauge couplings among KK towers are obtained. Requiring that contributions from all loop diagrams become comparable at the scale \( M_s \) (i.e. the theory is strongly coupled at \( M_s \)), we obtain the result \( g_{4+d} \approx \bar{g}_i \approx 4\pi \) in units of \( M_s \), neglecting group theoretical factors of order unity.} Thus the tree-level values of the 4D gauge couplings at \( M_s \) are given by

\[
\frac{1}{g_i^2} \approx \frac{V}{16\pi^2} + c_i \frac{V'}{16\pi^2},
\]

where \( c_i \approx 1 \) represent non-universal coefficients. Here and below, \( V \) and \( V' \) are given in units of \( M_s \). The requirement of our framework that gauge coupling unification is insensitive to unknown ultraviolet physics translates to the simple requirement that \( V'/V \) is sufficiently small for all non-universal branes.\footnote{In general, if the bulk and brane gauge kinetic terms are comparable at the string scale (i.e. \( g_{4+d} \approx \bar{g}_i \) in units of \( M_s \)), the non-universal contribution to the zero mode gauge couplings from the brane terms is suppressed compared with the universal bulk contribution by a factor of \( V'/V \). A formal understanding of this fact is given as follows. Since the non-universal brane kinetic operators, \( \delta^{(d-\delta)}(y - \bar{y}) F_i^2 \), have higher mass...} This becomes easier to satisfy as the dimension of the branes is...
reduced. Furthermore, since the value of the unified 4D gauge coupling is of order unity, the volume of the bulk is large in fundamental units: $V \approx 100$.

Having obtained gauge coupling unification at tree level at $M_s$, we turn to the quantum effects below $M_s$ that result from non-universal branes. Consider first the case of such a brane with dimension $\delta > 0$. At one loop, the scaling from $M_s$ to $M_c$ can give a non-universal correction to $1/g_i^2$ by an amount $(1/16\pi^2)(M_s/M_c)^{\delta} \approx V'/16\pi^2$. Thus the requirement that the tree-level unification of gauge couplings is insensitive to unknown physics at $M_s$ also guarantees insensitivity at the loop level to non-universal branes of $\delta > 0$. The relative power-law running of $g_i$ induced by such branes is found, perhaps surprisingly, to be unimportant. Type II theories are defined as those having non-universal branes with $\delta > 0$, but none with $\delta = 0$. These theories typically do not satisfy the condition of ultraviolet insensitivity, and the successful prediction is destroyed by the non-universal tree-level and power-law running contributions. Even in the case that the condition is satisfied, they predict $\alpha_s = 0.130 \pm 0.004$ like type I theories on manifolds, and hence are excluded for conventional superpartner spectra.

The remaining theories are those which possess $\delta = 0$ non-universal fixed points. The crucial aspect of these fixed points is that they induce a non-universal logarithmic running of the gauge couplings; from $M_s$ to $M_c$, $1/g_i^2$ is corrected by an amount $(1/16\pi^2)\ln(M_s/M_c)$. Since the non-universal tree correction factor is $1/16\pi^2$, the loop contribution dominates by $\ln(M_s/M_c)$. We find a remarkable result: only in the case of $\delta = 0$ are the relative corrections to the gauge couplings dominated by loop rather than tree effects. Furthermore, since the loop effects are logarithmic, they can be reliably computed in the effective theory. The unknown contributions from $M_s$ are suppressed relative to this calculable term by $1/\ln(M_s/M_c)$.

We thus concentrate on theories having non-universal branes with $\delta = 0$, in the hope that the logarithmic running above $M_c$ will lead to a precise agreement of gauge coupling unification with data. For the case $d > 2$, this does not happen. To obtain a supersymmetric theory below $M_c$, the unified theory in $4 + d$ dimensions must be supersymmetric. Supersymmetry in high dimensions is very constraining, corresponding to several supersymmetries in the 4D picture. If $d > 2$, each excited KK level of the equivalent 4D theory has 4D supersymmetry with $N \geq 4$, and hence does not contribute to the running of $g_i$. The evolution of $g_i$ above $M_c$ is only due to the zero modes, which by construction are those of the MSSM with $N = 1$, and hence is the same as the evolution below $M_c$. For $d > 2$ gauge coupling unification mimics the conventional supersymmetric case, with $M_s$ as the unification scale. This also occurs for $d = 1, 2$ if the dimensions (i.e. they are more irrelevant in the Wilsonian sense) than the $G$-preserving bulk kinetic term, $F^2$, by an amount $d - \delta$ corresponding to the dimension of the delta function, the effect of the former is suppressed relative to the latter at lower energies $\mu$. The suppression factor is given by $(\mu/M_s)^{d-\delta}$, which exactly gives the transverse volume factor $V'/V$ at the compactification scale, $\mu \approx M_s V^{-1/d} \approx M_s V'^{-1/\delta}$, that is, the relevant scale for the zero mode gauge fields.
higher dimensional theory has more than one supersymmetry. These theories we call type III, and the coupling evolution is shown in Fig. 1c.

Therefore, we find that a precise and successful prediction for gauge coupling unification is possible only for a 5D or 6D unified theory, with minimal amount of supersymmetry, compactified on an orbifold having non-universal fixed points, but not non-universal fixed lines. (Non-universal fixed lines in 6D give too large unknown corrections to $1/g_i^2$, of size $1/4\pi$.) The picture of gauge unification for these type IV theories is shown in Fig. 1d. This result is consistent with the requirement that $M_s$ and $M_c$ are well separated: in the minimal case of $G = SU(5)$, for instance, there are about 60 (8) KK excitations of each zero mode for $d = 1$ (2), so that there is a substantial energy interval having physics described by a higher dimensional field theory. This is quite unlike the case of string theory compactified on a 6D compact space with comparable sizes for the six dimensions. For strongly coupled string theory there are 2 KK excitations for each zero mode [13], and fewer for the case of weak coupling [3, 4].

The leading correction to gauge coupling unification arises from the logarithmic contribution to the running above $M_c$, as shown in Fig. 1l, and is proportional to $\ln(M_s/M_c)$. There are further corrections which are not logarithmically enhanced: threshold corrections from $M_s$ and $M_c$. Those from $M_s$, representing unknown physics from higher energies, correct $1/g_i^2$ by $1/16\pi^2$, leading to an uncertainty in $\alpha_s$ of $\pm 0.002$. As for threshold corrections from $M_c$, they arise from any multiplets that are $SU(5)$ split in any particular model. In the higher dimensional picture, they are represented by non-local operators spread out in the extra dimensions. While there are no contributions from matter, even if they arise from several multiplets of $G$, there are contributions from the multiplets of $G$ which contain the zero mode Higgs doublets and standard model gauge fields. These contributions are computed and included in our result, even though they correct $1/g_i^2$ only by $1/16\pi^2$ and thus are small. There can be no other split multiplets with zero modes, otherwise physics below $M_c$ would not be described by the MSSM, and these additional zero modes would ruin the success of supersymmetric unification, Eq. (2). However, there could be extra bulk multiplets with brane mass terms giving the zero modes a mass of order $M_c$. We assume that the dominance of the $\ln(M_s/M_c)$ correction is not spoiled by a large number of such multiplets.

3 Determining the Orbifold and Gauge Group

In this section, we calculate the correction to $\alpha_s$ proportional to $\ln(M_s/M_c)$ for type IV theories and determine the structure of the theory above $M_c$ using the experimental value of $\alpha_s$. We first derive a general formula for the KK tower contribution to $\alpha_s$. In type IV theories, the
low-energy values for the three gauge couplings are given by

\[
\frac{1}{g_i^2}(\mu) = \frac{1}{g_s^2} + \sum_{k=1}^{d} \frac{c_kb^{(k)}}{8\pi^2 k} \left[ \left( \frac{M_s}{M_c} \right)^k - 1 \right] + \frac{\tilde{b}_i}{8\pi^2} \ln \frac{M_s}{M'_c} + \frac{b'_i}{8\pi^2} \ln \frac{M'_c}{\mu} + \frac{\Delta_i}{8\pi^2},
\]

where \(b'_i\) are the \(\beta\)-function coefficients for the MSSM, \((b'_1, b'_2, b'_3) = (33/5, 1, -3)\), and \(\tilde{b}_i\) those for the theory above \(M_c\); \(g_s\) is the unified gauge coupling at \(M_s\), \(c_k = \pi^{k/2}/\Gamma(1+k/2)\), and \(\Delta_i\) represent the effects of threshold corrections from \(M_s\) and \(M_c\). The power-law terms proportional to \((M_s/M_c)^k - 1\) come from the running of the bulk gauge coupling (or gauge kinetic terms on \(G\)-symmetric fixed lines) and thus must be universal, while the term proportional to \(\ln(M_s/M'_c)\) comes from the running of gauge kinetic terms localized on the (non-universal) fixed points and can depend on \(i\) \([5]\). Here, we have matched the logarithmic contribution in higher dimensions to that in 4D at the scale \(M'_c = M_c/\pi\), which represents the length scale of extra dimensions. This is the natural scale for the matching, as indicated by summing up leading-log contributions from KK towers \([14]\). Using the above equations, we obtain one relation among \(g_i\)’s

\[
\frac{1}{g_3^2} = \frac{12}{7} \frac{1}{g_2^2} - \frac{5}{7} \frac{1}{g_1^2} + \frac{\tilde{b}}{8\pi^2} \ln \frac{M_s}{M'_c} + \frac{\Delta}{8\pi^2},
\]

at any scale \(\mu (\ll M'_c)\). Here, \(\tilde{b}\) and \(\Delta\) are defined by

\[
\tilde{b} = \tilde{b}_3 - \frac{12}{7} \tilde{b}_2 + \frac{5}{7} \tilde{b}_1,
\]

and \(\tilde{b}, \tilde{b}_i \to \Delta, \Delta_i\). Note that the dependence on \(b^{(k)}\) drops out since the power-law pieces are universal.

Suppose we compute \(\alpha_s\) from the observed electroweak gauge couplings \(g_1\) and \(g_2\), using Eq. (8). Then, the obtained value \(\alpha^\text{KK}_s\) is in general different from the value \(\alpha^\text{SGUT,0}_s\) obtained in the case where the couplings unify at a single scale \(M_u\) without any threshold correction (which corresponds to setting \(M_s = M_c = M'_c = M_u\) and \(\Delta_i = 0\) in Eq. (7)). The difference \(\delta\alpha_s \equiv \alpha^\text{KK}_s - \alpha^\text{SGUT,0}_s\) is given by

\[
\delta\alpha_s = -\frac{1}{2\pi} \alpha_s^2 \left( \tilde{b} \ln \frac{M_s}{M'_c} + \Delta \right),
\]

at leading order in \(\delta\alpha_s\). An important point is that \(\delta\alpha_s\) is dominated by the first logarithmic term. As we discussed before, threshold corrections from both \(M_s\) and \(M_c\) are under control and actually represented by \(\Delta = O(1)\). Since \(\ln(M_s/M'_c) \simeq 5/d\) for \(C = 5\), we find that \(\delta\alpha_s\) is reliably estimated by knowing the \(\beta\)-function coefficients \(\tilde{b}_i\), especially when \(d = 1\).

Let us now calculate \(\tilde{b}_i\) in various type IV models. This can be done easily by using a diagrammatic technique. A remarkable thing is that \(\tilde{b}_i\) do not depend on the detailed structure
of the models. They depend on only two things: the discrete symmetry used to define an orbifold and the higher dimensional multiplet containing the low-energy Higgs doublets. The basic idea is the following. Consider an orbifold $M/K$, where $M$ is a manifold and $K$ is a discrete group with $n_K$ elements. Then, we find a close relationship between the set of KK modes of $M/K$ and those of $M$: for each non-zero mode of $M/K$, there are $n_K$ corresponding modes of $M$ that are taken into each other by the elements of $K$. (For the example $M = S^1$ and $K = Z_2$: while $S^1$ has two states $e^{\pm iny/R}$ at each non-zero energy, only one linear combination, $\cos(ny/R)$ or $\sin(ny/R)$, is available for a field on $S^1/Z_2$, since it must be either $+$ or $-$ under the $Z_2$ orbifold symmetry $y \rightarrow -y$.) Hence, apart from the zero modes, the contribution to the gauge coupling running from some KK tower on $M/K$ is a fraction $1/n_K$ of the contribution from the corresponding KK tower on $M$. However, we know that KK towers on $M$ produce only universal running, since $M$ has no fixed points at which local $G$ violation may occur (type I theories). This means that the non-universal running on $M/K$ is simply caused by a “mismatch” of the zero modes between the two towers: the zero mode contribution on $M/K$ is not necessarily $1/n_K$ of that on $M$. This observation allows a very simple result for the values of $\tilde{b}_i$ on $M/K$: if some gauge component $U$ of a $(4 + d)$-dimensional supermultiplet has the zero mode on $M/K$, it contributes to $\tilde{b}_i$ by

$$\tilde{b}_i = b_i^{\hat{U}0} - \frac{1}{n_K} b_i^U,$$  \hspace{1cm} (11)

where $b_i^{\hat{U}0}$ are the 4D $\beta$-function coefficients from the zero mode, while $b_i^U$ are those from the excited KK level of $U$.

To see how this works explicitly, let us consider 5D models on $S^1/Z_2$ in which gauge group $G$ is broken by orbifold boundary conditions. We first consider a gauge multiplet $V^A = \{V^A, \Sigma^A\}$, where $A$ is the gauge index and fields in the curly bracket represent 4D $N = 1$ superfields ($V$ and $\Sigma$ are vector and chiral superfields in the adjoint representation, respectively). According to the transformation property under translation $y \rightarrow y + 2\pi R$, the gauge multiplet $V^A$ is divided into two classes: $V^a(y + 2\pi R) = V^a(y)$ with KK masses $m_n = n/R$ and $V^\hat{a}(y + 2\pi R) = -V^\hat{a}(y)$ with KK masses $m_n = (n + 1/2)/R$, where $n$ takes integer values with $n \geq 0$. Now, consider relating this KK tower to the corresponding KK tower on $S^1$, as discussed in the previous general analysis. Suppose each KK level of $V^\hat{a}$ contributes to the running of $g_i$ with 4D $\beta$-function coefficients $b_i = b_i^X$. Then, as far as the runnings of $g_i$ are concerned, this KK tower, $(m_n, b_i) = ((n + 1/2)/R, b_i^X) \ (n \geq 0)$, is equivalent to the tower $T^X : (m_n, b_i) = ((n + 1/2)/R, b_i^X/2) \ (-\infty < n < \infty)$ that would be obtained on $S^1$. The same rearrangement

4 In type II theories, the “mismatch” consists of a sum of KK towers characteristic of manifolds with dimensions less than $d$. In general, we can easily read off full gauge coupling running equations, including the coefficients of non-universal power-law pieces, from this decomposition.
is possible for $\mathcal{V}^a$, but in this case there is an extra subtlety coming from the presence of zero modes: the 4D $\beta$-function coefficients of the zero modes, $b_i^{A_0}$, are in general different from half of those of the excited modes, $b_i^A$. Thus the equivalent pattern for $\mathcal{V}^a$ consists of the tower $\mathcal{T}^A : (m_n, b_i) = (n/R, b_i^A/2)$ ($-\infty < n < \infty$) and an additional “effective zero mode” with $b_i = b_i^{A_0} - b_i^A/2$. This explicitly shows that the KK tower on $S^1/Z_2$ is equivalent to the tower $\{\mathcal{T}^X, \mathcal{T}^A\}$ on $S^1$, apart from a slight mismatch represented by the “effective zero mode”. Since the pattern of the tower, $\{\mathcal{T}^X, \mathcal{T}^A\}$, is completely the same as that obtained on $S^1$ (type I theories), they contribute only to $G$-symmetric power-law piece with $b^{(1)} = (b_i^X + b_i^A)/2$, which means that the $G$-violating piece entirely comes from the “effective zero mode” and is given by $\tilde{b}_i = b_i^{A_0} - b_i^A/2$. This provides an explicit example of the general result, Eq. (11), with $n_K = 2$.

The result can be simplified further by observing that there are simple relations between $b_i^{A_0}$ and $b_i^A$, which depend only on zero mode representations; if the zero modes come from $V$ and $\Sigma$, they are given by $b_i^A = (2/3)b_i^{A_0}$ and $b_i^A = -2b_i^{A_0}$, respectively. Therefore, we finally obtain the simple result for the contribution to $\tilde{b}_i$ from the gauge multiplet: $\tilde{b}_i = (2/3)b_i^{A_0}$ and $2b_i^{A_0}$ in the case of $V$ and $\Sigma$ zero modes, respectively. The contribution from a hypermultiplet, $\mathcal{H} = \{\Phi, \Phi^c\}$, can be worked out similarly ($\Phi$ and $\Phi^c$ represent 4D $N = 1$ chiral superfields). The result is given by $\tilde{b}_i = b_i^{H_0} - b_i^H/2$, where $b_i^{H_0}$ and $b_i^H$ are the 4D $\beta$-function coefficients of the zero modes and the excited modes, respectively. However, since $b_i^H = 2b_i^{H_0}$ in the hypermultiplet case, we find that it does not contribute to $\tilde{b}_i$, $\tilde{b}_i = 0$, if the extra dimension is $S^1/Z_2$.

The extension to the case with $d > 1$ is straightforward. In the case of $d = 2$, the KK pattern is plotted in the two-dimensional momentum space $(p_5, p_6)$. The only extra complication, compared with the case of $d = 1$, is that the original orbifold KK towers do not necessarily fill half of this momentum plane; if the orbifold is $T^2/Z_m$, they fill only $1/m$ of the plane. By repeating a similar analysis to the $d = 1$ case, we obtain $\tilde{b}_i = (1 - 2/3m)b_i^{A_0}, (1 + 2/m)b_i^{A_0}$ and $(1 - 2/m)b_i^{H_0}$ in the case of $V, \Sigma$ and $\Phi$ zero modes, respectively. Since the difference comes only from the fraction of momentum plane filled by KK towers, the $d = 1$ case is reproduced by setting $m = 2$ in these expressions.

With the above knowledge of $\tilde{b}_i$’s, we can calculate $\tilde{b}$ from Eq. (12) in any type IV theories. In our framework the massless sector consists of the MSSM states, so that we only have to consider the KK excitations for these states. First, the matter fields do not contribute to $\tilde{b}$ even if they live in the bulk, since they fill complete $SU(5)$ multiplets and so are the excited states $\tilde{b}_1 = \tilde{b}_2 = \tilde{b}_3$. The gauge fields come from $V \subset \mathcal{V}$, so their contribution is given by $\tilde{b}_1 = \tilde{b}_2 = \tilde{b}_3$.

The Higgs doublets $H_u$ and $H_d$, there are three possibilities: they can originate from $\Phi \subset \mathcal{H}$ or $\Sigma \subset \mathcal{V}[15]$, or can be brane fields $\mathcal{B}$ localized on a non-universal fixed point [16]. In each case, their contribution is given by $\tilde{b}_1 = \tilde{b}_2 = \tilde{b}_3 = (3/5 - 6/5m, 1 - 2/m, 0)$, $\tilde{b}_1 = \tilde{b}_2 = \tilde{b}_3 = (3/5 + 6/5m, 1 + 2/m, 0)$, and
Table 1: The values of $\tilde{b}$ in general type IV theories, where $H_u$ and $H_d$ represent two Higgs doublets in the MSSM. The values for the $S^1/Z_2$ case correspond to setting $m = 2$ in the $T^2/Z_m$ case.

\[
\begin{array}{|c|c|c|c|}
\hline
 & H_u, H_d \subset \mathcal{H} & H_u, H_d \subset \mathcal{V} & H_u, H_d = \mathcal{B} \\
S^1/Z_2 & 6/7 & -12/7 & -3/7 \\
T^2/Z_m & 12/7m & -24/7m & -6/7m \\
\hline
\end{array}
\]

$(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (3/5, 1, 0)$, respectively. Adding all together, we finally obtain the values of $\tilde{b}$ for general type IV theories. The result is summarized in Table 1. Since the derivation is general, $T^2$ could be replaced by any two-dimensional compact manifold, and $Z_m$ by any discrete group with $m \geq 2$ elements.

We now have a list for $\delta \alpha_s$ in all type IV theories, which must be compared with the value, $\alpha_s^{\text{exp}} - \alpha_s^{\text{SGUT},0} \simeq -0.013 \pm 0.004$. Here we used $\alpha_s^{\text{SGUT},0}$ to represent the value obtained including full two-loop effects, $\alpha_s^{\text{SGUT},0} \simeq 0.130 \pm 0.004$. Using $M_s/M_c' \simeq \pi(300/C)^{1/d}$, we calculate $\delta \alpha_s$ for $C = 5$ from the first term of Eq. (11). For $d = 1$, $\delta \alpha_s \simeq (-0.010, +0.020, +0.005)$ and for $d = 2$ $\delta \alpha_s \simeq (-0.012/m, +0.024/m, +0.006/m)$ for the cases of $(H_u,d \subset \mathcal{H}, \subset \mathcal{V}, = \mathcal{B})$. Does the logarithmic running between $M_c$ and $M_s$ resolve the discrepancy between $\alpha_s^{\text{SGUT},0}$ and $\alpha_s^{\text{exp}}$? In the cases that the Higgs doublets originate from a $(4+d)$-dimensional gauge multiplet or are brane fields the answer is clearly no, since the negative sign of $\tilde{b}$ leads to an even larger discrepancy. The Higgs must be bulk hypermultiplet fields, and furthermore, a precise agreement with data is only possible for the single case of $d = 1$.

The case of $d = 2$ and low $m$ cannot be excluded, although it is certainly not preferred. For example, an $SO(10)$ theory on $T^2/Z_2$ ($d = 2, m = 2, C = 8$) leads to a central prediction of $\alpha_s \simeq 0.124$ at leading logarithm. Measurements of the superpartner masses will determine whether such theories are excluded or not. They require very characteristic supersymmetry breaking parameters such as squarks and gluinos above 1 TeV and/or highly non-universal gaugino masses. It is significant that the theories which are excluded or disfavored by gauge coupling unification generically pose problems for model building: models with the Higgs originating from gauge multiplets have difficulties in obtaining sizable Yukawa couplings (or must have 6D $N = 2$ supersymmetry, but then they are Type III theories); if the Higgs

\footnote{In principle, we could add more bulk hypermultiplets if they have brane mass terms giving the zero modes a mass of order $M_c$. For $m = 2$, which includes all theories with $d = 1$, they do not contribute to $\tilde{b}_i$ and thus do not affect the values of $\delta \alpha_s$ or $M_c$ obtained here. For $d = 2$, $m > 2$ (and for some of type III theories), these additional multiplets could contribute to $\tilde{b}_i$. While we cannot exclude these more complicated theories, they are ad hoc, and reminiscent of fixing up non-supersymmetric theories by populating the desert with additional split multiplets. The same is true for adding extra multiplets on branes with local $G$ violation.}
doublets are on the brane then they are typically not subject to charge quantization; and if \(d = 2\) it is generically difficult to satisfy stringent constraints from anomaly cancellation in the 6D bulk.

We conclude that our framework strongly favors \(d = 1\), and therefore the gauge group \(G = SU(5)\), since it is the largest group that can be broken to the standard model gauge group by compactifying on \(S^1/Z_2\). (Note that larger gauge groups are also disfavored from the fact that they have larger values of \(C\) and thus lead to smaller values of \(\ln(M_u/M'_u)\) and \(|\delta\alpha_s|\).) The bulk matter content is also fixed to be two Higgs hypermultiplets, up to a possibility of putting matter in the bulk, since otherwise there remain unwanted massless fields at low energies. The compactification scale \(M_c\) is calculated using Eq. (7) as \(M_c = \pi M_u(M'_c/M_s)^{5/7} \approx 10^{15}\) GeV. Therefore, we finally arrive at the following picture: there is a large energy interval ranging from \(10^{15}\) to \(10^{17}\) GeV in which the physics is described by a higher dimensional grand unified field theory, and it must actually be a 5D \(SU(5)\) theory with the two Higgs hypermultiplets propagating in the bulk. The overall physical picture is summarized in Fig. 2.
Here we make one brief comment. In section 2 we defined our framework to have $d$ extra dimensions of comparable radii. It is important to note that there is no need for the radii to be comparable, rather this was a simplifying assumption. For example, to obtain a tree-level gauge coupling unification at $M_s$, we require only that the volume of the bulk is large, and much larger than the volume of non-universal branes. The addition of extra radii will result in extra parameters entering the prediction for the QCD gauge coupling, so that it is possible to get a continuum of results, interpolating between the discrete possibilities that follow for a single radius. For example, 6D $SO(10)$ theories compactified on an asymmetric space, $R_5 \neq R_6$, can interpolate $\alpha_s$ predictions from that of $d = 2$ to that of $d = 1$, and therefore can agree well with data. Nevertheless, it is still true that the best fit value of $\alpha_s$ is obtained for $R_5 \gg R_6$, in which case the effective theory in the energy region between $R_5^{-1}$ and $R_6^{-1}$ is 5D $SU(5)$ ($\times U(1)$), and the physics picture is almost that of Fig. 2 with $M_c \to R_5^{-1}$ and $M_s \approx R_6^{-1}$. Obviously, in this case, what we previously said as a fixed point can actually be a fixed line in 6D with a sufficiently small volume $V' \approx 1$.

Having obtained a detailed picture of the structure of the theory around the unified scale, we here discuss uncertainties for the present analysis. First, there is an uncertainty for the ratio $M_s/M_c$ due to a universal power-law running of $g_i$ above $M_c$. However, its effect on $\delta\alpha_s$ is small; for instance, even a factor 2 uncertainty in $M_s/M_c$ gives only about 15% uncertainty in $\delta\alpha_s$. Second, there are higher loop field theory contributions just below $M_s$, which are no longer loop suppressed since the theory is becoming strongly coupled. However, this strongly coupled physics is occurring only over a very small energy interval, and the theory is weakly coupled in most of the energy region between $M'_c$ and $M_s$ where the logarithmic contribution comes from. This is because in the 5D picture the couplings in the theory have negative mass dimensions, and in the 4D picture the number of KK states circulating in the loop decreases with decreasing energies so that loop expansion parameters in the theory become small by powers of $(\mu/M_s)$. Therefore, we expect that these contributions are no larger than those from the physics above $M_s$ encoded in the operators localized on the non-universal fixed points [12]. We estimate the resulting uncertainty to be about $\ln(3)/\ln(60\pi) \approx 20\%$, assuming that the strong coupling physics makes the one-loop value of $\tilde{b}$ unreliable above $M_s/3$. There are also uncertainties in $\Delta$ which represents the threshold corrections from $M_s$ and $M_c$.

Thus, here we take $\Delta = 0.84 \pm 1$, where the error represents the threshold correction from $M_s$ and a renormalization scheme dependence. Adding up all together, we finally obtain the value $\delta\alpha_s = -0.012 \pm 0.002$, which is translated into the prediction of our framework given in Eq. (3). It is important to notice that the difference, $\delta\alpha_s$, from the conventional one-scale
unification comes dominantly from the logarithmic evolution between $M'_c$ and $M_s$. Unlike the threshold correction $\Delta_{SGUT}$ in the usual framework, there is no free parameter which can be chosen to adjust $\delta \alpha_s$; the masses and couplings for the particles around the unification scale are all determined by the orbifold compactification. We have achieved a significant improvement over the conventional supersymmetric grand unification framework.

4 The $SU(5)$ Model in Five Dimensions

We have shown that our framework, a higher dimensional unified theory breaking on an orbifold, correctly predicts the QCD coupling only in the unique situation that the orbifold is $S^1/Z_2$ and the unified gauge symmetry is $SU(5)$. In this section we discuss features of this 5D $SU(5)$ theory, showing that other aspects are also determined. We explicitly present a completely realistic theory, which illustrates how various problems in 4D supersymmetric grand unified theories are elegantly solved in the KK grand unification framework.

The orbifold boundary conditions are unique. The orbifold reflection of $Z : y \rightarrow -y$ preserves $SU(5)$, having parities for gauge and hypermultiplets of $V = \{V(+) , \Sigma(-)\}$ and $\mathcal{H} = \{\Phi(+) , \Phi^c(-)\}$. Under translation $T : y \rightarrow y + 2\pi R$, the $SU(5)$ is broken by the action of $P = \text{diag}(+,+,+,−,−)$ on a 5-plet. In addition, bulk hypermultiplets can have extra factors $\eta_\Phi = \pm 1$ under the translation. These are the most general boundary conditions preserving 4D $N = 1$ supersymmetry. Specifically, the boundary conditions for the gauge and hypermultiplets are written as

\[
\begin{align*}
(V^{(\pm)} , \Sigma^{(\pm)}) (x^\mu , y) & = \pm (V^{(\pm)} , \Sigma^{(\pm)}) (x^\mu , y + 2\pi R), \\
(\Phi^{(\pm)} , \Phi^{(\pm)}c) (x^\mu , y) & = \pm (\Phi^{(\pm)} , \Phi^{(\pm)}c) (x^\mu , y + 2\pi R),
\end{align*}
\]

where we have labelled the standard model gauge multiplets and the broken $SU(5)$ gauge multiplets as $V(+) , \Sigma(+)$ and $V(-) , \Sigma(-)$, respectively; $\Phi(+)$ (\Phi(−)) represents the components of $\Phi$ that are even (odd) under the action of $P$.

We have shown that the two Higgs doublets of the MSSM must arise from bulk hypermultiplets, rather than as fields localized at the $y = \pi R$ brane, which respects only the standard model gauge symmetry. Thus we introduce two Higgs hypermultiplets $\{H, H^c\}$ and $\{\bar{H}, \bar{H}^c\}$ in the bulk, which transform as $5$ and $\bar{5}$ under $SU(5)$. These Higgs multiplets must have $\eta_H = \eta_{\bar{H}} = -1$ to have massless Higgs doublets. (In the present notation, $H(+) \text{ and } H^{(−)} \text{ (} \bar{H}(+) \text{ and } \bar{H}^{(−)} \text{) represent triplet and doublet components, } H_T \text{ and } H_D \text{ (} \bar{H}_T \text{ and } \bar{H}_D \text{), of } H \text{ (} \bar{H} \text{).}

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6 This is equivalent to the boundary conditions of Ref. [6] described in terms of $Z$ and $Z' = ZT$ as $S^1/(Z_2 \times Z_2')$. 

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Table 2: The transformation properties for the bulk fields under the orbifold reflection and translation. Here, we have used the 4D $N = 1$ superfield language. The fields written in the $(\mathcal{Z}, \mathcal{T})$ column, $\varphi$, obey the boundary condition $\varphi(y) = \mathcal{Z}\varphi(-y) = \mathcal{T}\varphi(y + 2\pi R)$. The masses for the corresponding KK towers are also given $(n = 0, 1, \cdots)$.

respectively.) Then, from Eqs. (12, 13), we obtain the following KK towers for the gauge and Higgs fields: the standard model gauge vectors $V_{321}$ with masses $n/R$, joined at $n \neq 0$ levels by the $N = 2$ partners $\Sigma_{321}$; the broken $SU(5)$ vectors $V_X$, joined by their $N = 2$ partners $\Sigma_X$, of mass $(n + 1/2)/R$; two Higgs doublets $H_D$ and $\bar{H}_D$ with masses $n/R$, joined at $n \neq 0$ levels by the $N = 2$ partners $H_D^c$ and $\bar{H}_D^c$; and two Higgs triplets $H_T$ and $\bar{H}_T$ with masses $(n + 1)/R$ joined by their $N = 2$ partners $H_T^c$ and $\bar{H}_T^c$. These KK towers are summarized in Table 2. The Higgs KK towers do not have zero modes for the color triplet states $[5]$, and the KK excitations do not lead to proton decay from dimension five operators because their mass term takes the form $HH^c + \bar{H}\bar{H}^c$ rather than $H\bar{H}$ $[5]$. It is precisely these vector and Higgs towers which lead to the successful prediction for the QCD coupling.

To preserve the $SU(5)$ understanding of matter quantum numbers the quarks and leptons should either be in the bulk or reside at the $SU(5)$ preserving brane at $y = 0$ $[3]$. Yukawa interactions are forbidden by 5D supersymmetry from appearing in the bulk Lagrangian, and hence must be brane localized. If quarks and leptons are on the brane, they fill out 4D chiral multiplets which are 10 or 5 representations of $SU(5)$: $T$ and $F$. The Yukawa interactions are located on the $y = 0$ brane as

$$S = \int d^4x \, dy \, \delta(y) \left[ \int d^2\theta \left( y_T TTH + y_F TF\bar{H} \right) + h.c. \right]. \quad (14)$$

Since the full $SU(5)$ symmetry is operative at $y = 0$, we have $SU(5)$ mass relations for the quarks and leptons localized on the brane. The resulting 4D Yukawa couplings are suppressed by a factor of $1/(M_s R)^{1/2}$ due to the Higgs wavefunctions being spread out over the bulk. On the other hand, if quarks and leptons are in the bulk, they arise from hypermultiplets: $\{T, T^c\} + \{T', T'^c\}$ and $\{F, F^c\} + \{F', F'^c\}$ with $\eta_T = \eta_F = 1$ and $\eta_{T'} = \eta_{F'} = -1$. We find from Eq. (13) that a generation $q, u, d, l, e$ arise from the zero modes of bulk fields as $T(u, e), T'(q), F(d)$ and $F'(l)$. (Note that $T^{(+)} = T_{U,E}$, $T^{(-)} = T_Q$, $F^{(+)} = F_D$, $F^{(-)} = F_L$, $F'^{(+)} = F_L'$, $F'^{(-)} = F_D'$.)
and similarly for $T'$ and $F'$, where $T_{Q,U,E}$ ($F_{D,L}$) are the components of $T$ ($F$) decomposed into irreducible representations of the standard model gauge group. The tower structure for these fields is given in Table 2.) Since $q$ and $u, e$ ($d$ and $l$) come from different hypermultiplets, the broken gauge boson exchange does not lead to proton decay. The Yukawa couplings are introduced on the $y = 0$ brane as

$$
S = \int d^4x\, dy\, \delta(y) \left[ \int d^2\theta \left( y_T^1 T T H + y_T^2 T T' H + y_T^3 T T' T' H + y_F^1 T F \bar{H} + y_F^2 T F' \bar{H} + y_F^3 T' F \bar{H} + y_F^4 T' F' \bar{H} \right) + \text{h.c.} \right].
$$

(15)

Although these are $SU(5)$ symmetric interactions, the quarks and leptons do not respect $SU(5)$ mass relations because the down-type quark and charged lepton masses come from $y_F^2$ and $y_F^3$ couplings, respectively, and are not related by the $SU(5)$ symmetry. Moreover, since the matter wavefunctions are also spread out in the extra dimension, the resulting 4D Yukawa couplings receive a stronger suppression, a factor of $1/ (M_s R)^{3/2}$, than in the case of brane matter. Thus we find a clearly successful correlation between the mass of the fermion and whether it has $SU(5)$ mass relations — heavier fermions display $SU(5)$ mass relations while lighter ones do not. Of course, if we have both bulk and brane matter, we can also write down the Yukawa couplings that mix them, on the $y = 0$ brane.

We now discuss an important issue of what brane localized operators can be introduced in our theory. The 5D restricted gauge symmetry alone allows many unwanted operators on the branes. For instance, the operators $[H \bar{H}]_{\theta^2}$ and $[F H]_{\theta^2}$ give a large mass, of order the unified scale, for the Higgs doublets destroying the solution to the doublet-triplet splitting problem, $[T F F]_{\theta^2}$ causes disastrous dimension four proton decay, and $[Q Q Q L]_{\theta^2}$ induces too rapid dimension five proton decay. In addition, if matter is located in the bulk, we could also have $SU(5)$ non-invariant operators on the $y = \pi R$ brane, such as $[T_Q T_Q \bar{H}_T]_{\theta^2}$ and $[T_Q F_L H^c_T]_{\theta^2}$, which reintroduce the problem of dimension five proton decay caused by colored Higgsino exchange. Remarkably, however, the structure of the theory allows a mechanism that simultaneously suppresses all these unwanted operators.

Since the bulk Lagrangian has higher dimensional supersymmetry, it possesses an $SU(2)_R$ symmetry. It also has an $SU(2)_H$ flavor symmetry rotating the two Higgs hypermultiplets in the bulk. After the orbifolding, these two $SU(2)$ symmetries are broken to two $U(1)$ symmetries, one from $SU(2)_R$ and one from $SU(2)_H$. A particularly interesting symmetry is the diagonal subgroup of these $U(1)$ symmetries, which we call $U(1)_R$ symmetry since it is an $R$ symmetry rotating the Grassmann coordinate of the low energy 4D $N = 1$ supersymmetry. We can extend this bulk $U(1)_R$ symmetry to the full theory by assigning appropriate charges to the brane localized quark and lepton superfields, and use it

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7 We could also introduce Yukawa couplings that do not respect the $SU(5)$ symmetry, on the $y = \pi R$ brane.
Table 3: $U(1)_R$ charges for 4D vector and chiral superfields.

|   | $V$ | $\Sigma$ | $H$ | $H^c$ | $\bar{H}$ | $H^c$ | $T$ | $T^c$ | $F$ | $F^c$ | $N$ | $N^c$ |
|---|-----|---------|-----|-------|--------|------|----|------|----|------|----|------|
| $U(1)_R$ | 0   | 0       | 2   | 0     | 2      | 1    | 1  | 1    | 1  | 1    | 1  | 1    |

to constrain possible forms of brane localized operators. The resulting $U(1)_R$ charges are given in Table 3, where $T$ and $F$ (and $N$) represent both brane and bulk matter. Imposing this $U(1)_R$ symmetry on the theory, we can forbid unwanted operators while keeping the Yukawa couplings. The dimension four and five proton decays are prohibited, and the $R$-parity violating operators are absent since $U(1)_R$ contains the usual $R$ parity as a discrete subgroup. After supersymmetry breaking, this $U(1)_R$ symmetry is broken (presumably to its $R$-parity subgroup), generating gaugino masses and the supersymmetric mass term for the two Higgs doublets ($\mu$ term) of the order of the weak scale. Since the breaking scale is small, however, it will not reintroduce the problem of proton decay. It is interesting to note that the spacetime symmetries of the theory allows a bulk mass term of the form $[H \bar{H} - H^c \bar{H}^c]_{\theta^2}$, coupling the two Higgs hypermultiplets. This would remove the Higgs doublets from the low energy theory and reintroduce dimension five proton decay from colored Higgsino exchange. The $U(1)_R$ symmetry also forbids this bulk mass term, providing a complete solution to the doublet-triplet and proton decay problems.

We here comment on neutrino masses. Small neutrino masses are generated through the see-saw mechanism [19], if we introduce right-handed neutrino superfields. They could be either brane fields, $N$, or bulk fields, $\{N, N^c\}$ with $\eta_N = 1$. The Yukawa couplings, $[F NH]_{\theta^2}$, and Majorana masses, $[NN]_{\theta^2}$, are written on the brane. The $U(1)_R$ charges for these fields are given in Table 3.

Now we ask how we can determine the location of matter fields. Since our framework gives $M_c \approx 10^{15} \text{ GeV}$, the $X$ gauge bosons are considerably lighter, of mass about $10^{15} \text{ GeV}$, than in the case of 4D supersymmetric grand unification. This makes dimension six proton decay a non-trivial issue in our theory. We find that the quarks and leptons of the first generation coming from a 10 representation must be bulk fields, since otherwise the $X$ gauge boson exchange would induce proton decay at too rapid a rate.\footnote{The authors of Ref. [18] did not consider the possibility of bulk matter, and hence concluded that unification in 5D did not improve the 4D unification prediction for the QCD coupling.} We will say that $T_1$ is in the bulk, although we really mean the combination $\{T_1, T_1^c\} + \{T_1', T_1'^c\}$. On the other hand, the top quark must arise from a brane field $T_3$. If the top quark were a bulk mode, it would have a mass suppressed by a factor of $1/(M_s R)^{3/2}$, which gives too light a top quark even in the case that the Yukawa coupling is strong. With $T_3$ on the brane, strong coupling leads to a top Yukawa coupling of the
low energy theory of $4\pi/(M_s R)^{1/2} \approx 1$, giving a top quark mass of the observed size. Thus we are able to derive the location of both the first and third generation $10$’s, and we find that at least some aspects of flavor physics are associated with the geometry of the orbifold, and with strong coupling. Arguments can be made for the location of the rest of the quarks and leptons, although these are not strict requirements. For example, the rest of the third generation, $F_3$, is best placed on the brane, giving the successful $SU(5)$ mass prediction for $m_b/m_\tau$. On the other hand, some (or all) of lighter generations are located in the bulk so that it does not exhibit unwanted $SU(5)$ mass relations.

For the lighter two generations we mention two interesting possibilities. The large $\nu_\mu\nu_\tau$ mixing, observed in atmospheric neutrino fluxes, suggests that $F_2 \supset \nu_\mu$ is also on the brane so that it has a large mixing with $F_3 \supset \nu_\tau$. With these assignments the location of the rest of the second generation is fixed: $T_2$ must be in the bulk, otherwise all the second and third generation fermions would be on the brane, leading to an incorrect $SU(5)$ mass prediction between the strange quark and the muon. A bulk location for $T_2$ is in any case desired, since it leads to small CKM mixing between second and third generations, $V_{cb} \approx \epsilon$, and to a mass hierarchy $m_\mu/m_\tau, m_s/m_b \approx \epsilon$ and $m_c/m_t \approx \epsilon^2$. It is interesting to note that, in the case that all the Yukawa couplings of the heaviest two generations are strongly coupled, $\epsilon \approx (M_c/M_s)^{1/2} \approx 0.1$, so that all the above relations are good at the factor of 3 level. This requires the large $m_t/m_b$ ratio to result from a large ratio of electroweak vacuum expectation values, $\tan \beta$. The only remaining question is the location of $F_1$, which is not constrained by proton decay. One possibility is that all three $F_i$’s are on the brane. One might expect this to give large angle solar neutrino oscillations. However, such a location implies that the first two generations are not distinguished by their spatial location. Thus the hierarchies of the masses and mixings of the first two generations must come from elsewhere. Another possibility is for $F_1$ to be in the bulk. In this case the hierarchies $m_d/m_s, m_e/m_\mu \approx \epsilon$, but the smallness of the Cabibbo angle, and, more importantly, of $m_u/m_c$ are not explained. It appears that many, but not all, aspects of flavor can be understood from this single extra dimension [2, 20].

Another possibility is to have two generations on the brane and one in the bulk. The bulk generation must be interpreted as $T_1, F_2$, rather than $T_1, F_1$, which would lead to an incorrect relation for $m_s/m_\mu$. The extra dimension cannot explain all aspects of flavor — some additional ingredient is needed. In the present case a very simple approximate flavor symmetry is sufficient. The brane fields, with flavor charges in parentheses, are $T_3(0), F_3(1), T_2(1), F_1(1)$, while the bulk fields are $T_1(1), F_2(0)$. The size of entries in the Yukawa matrices are determined by a combination of factors of the volume of the bulk, $\epsilon$, and the size of the flavor symmetry
breaking parameter, $\delta$. In the case that $\epsilon \approx \delta$, it gives the following Yukawa matrices:

$$
\mathcal{L}_4 \approx \begin{pmatrix} T_1 & T_2 & T_3 \end{pmatrix} \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} H + \epsilon \begin{pmatrix} T_1 & T_2 & T_3 \end{pmatrix} \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} \bar{H},
$$

where only underlined entries must respect $SU(5)$ relations. This well reproduces the qualitative pattern of the observed quark and lepton masses and mixings: $m_t : m_c : m_u \approx 1 : \epsilon^2 : \epsilon^4$, $m_b : m_s : m_d \approx m_\tau : m_\mu : m_e \approx 1 : \epsilon : \epsilon^2$, $(V_{us}, V_{cb}, V_{ub}) \approx (\epsilon, \epsilon, \epsilon^2)$, $m_b/m_\tau \simeq 1$, and $m_s/m_\mu \neq 1$. The neutrino mixing angles are expected to be bi-maximal, giving large angle solutions to the solar neutrino problem. Unlike our earlier example, the present matter configuration has a local cancellation of gauge anomalies and does not need a Chern-Simons term. The two examples also have very different power-law running of the 4D gauge coupling above $M_c$. With $T_2$ in the bulk this couplings blows up at about $40 M_c$; this reduces $\ln(M_s/M'_c)$ below our central value, but gives a prediction for the QCD coupling within our quoted uncertainty. With $T_2$ on the brane, the one-loop coefficient of the power-law running vanishes. Of course, the 5D coupling is still strong at $M_s$.

The location of $T_2$ is very important for gauge boson mediated proton decay, which only occurs via brane localized $T_1$. In our first flavor model, $T_2$ is in the bulk, so that proton stability is expected ($\tau_p \approx 10^{39} - 10^{41}$ years). In the second model, $T_2$ is on the brane, leading to proton decay in the Cabibbo suppressed channels $\mu^+ K^0$ and $K^+ \bar{\nu}_\tau$, at an interesting rate for future experiments ($\tau_p \approx 10^{33} - 10^{35}$ years).

5 Relation to String Theory

Superstring theories are formulated in 10D and therefore require compactification on a 6D space. Most work on compactification has concentrated on a 6D space which is close to symmetrical, with the six radii all comparable in size. Also much attention has been paid to perturbative heterotic string theory. The message of this paper is that string theory should be strongly coupled, and compactified on a highly asymmetric space, with one radius, $R$, much larger than the others, $\bar{R}$. The mass scale of the strongly coupled string theory is near $10^{17}$ GeV.\footnote{Another possibility is that the string theory is just perturbative, but close to being strongly coupled. In the case of the heterotic $E_8 \times E_8'$ theory this is not possible, as the string scale is $(\sqrt{\alpha}/2) M_{Pl} \approx 10^{18}$ GeV, an order of magnitude larger than $M_s$. Our theory may be realized in brane world scenarios.}

The value of the unified gauge coupling requires the volume of the 6D compact space to be $\approx 60$, using fundamental units of $M_s$. Much of this volume will arise from the large dimension, so that $\ln(M_s/M_c)$ is large enough to correct the usual supersymmetric prediction of $\alpha_s$. However, the other radii need not be exactly $1/M_s$; the present uncertainty from the
superpartner thresholds allows for $\tilde{R}$ to be somewhat larger than $1/M_s$. Measurements of superpartner masses would place tighter restrictions on this.

At distances larger than $\tilde{R}$, the 5D effective theory has $SU(5)$ gauge interactions propagating in spacetime $M^4 \times S^1/Z_2$. Bulk modes include hypermultiplets for two $5$ of Higgs fields, and for $(10, 10')$ which has the lightest generation as zero modes. The remaining matter fields may be bulk or brane modes, except for the top quark, which must be contained in a brane $10$. Unlike attempts to get 4D grand unified theories from string theory, there should not be adjoint or other fields for breaking the unified gauge symmetry. Rather there must be an $SU(5)$ breaking twist in the translation boundary condition for $S^1$. It will be interesting to pursue 10D string models which reduce to the above 5D $SU(5)$ theory below $\tilde{R}^{-1}$.

There may be several ways to realize our framework in string theory. One possibility is the strongly coupled $E_6 \times E_6'$ heterotic string theory, which can be viewed as a 11D supergravity theory having a large “gravity-only” dimension $[13]$. The resulting 11D theory can be written, using standard notation, as

$$S = \int d^4 x d^6 y dz \left\{ \frac{1}{2\kappa_5^2} \sqrt{g} R - \frac{1}{8\pi (4\pi \kappa_5^2)^{2/3}} \sqrt{g} \left( \delta(z) \text{tr} F^2 + \delta(z - \pi \rho) \text{tr} F'^2 \right) \right\}.$$  

(17)

The eleventh dimension $z$ has a radius $\rho$ of size

$$\frac{1}{\rho} = (8\pi^2 \alpha^{-1})^{3/2} \left( \frac{1}{V^{1/2} M_{Pl}^3} \right).$$  

(18)

where $\alpha \simeq 1/24$ is the unified gauge coupling. For a symmetrical 6D space, with six comparable radii $[13]$, $V \approx M_u^{-6}$ and $1/\rho \simeq 4 \times 10^{15}$ GeV. In our framework, the large asymmetry in the 6D compact space implies $V \approx M_s^{-5} M_c^{-1} \approx M_u^{-6} (M'_c/M_s)^{5/7}$, so that $1/\rho$ is increased to about $9 \times 10^{15}$ GeV. In summary: if our framework is described by strongly coupled heterotic string theory, the fundamental string scale is close to $10^{17}$ GeV, five dimensions of the compact space are close to this scale, but two have much larger radii, characterized by the mass scales $10^{16}$ GeV and $10^{15}$ GeV, respectively. They are both described by $S^1/Z_2$, but the former is a “gravity-only” dimension, while the latter allows propagation of $SU(5)$ gauge interactions.

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10 An interesting intermediate step for this construction might be a 6D $N = 2$ $SU(6)$ model along the line of Ref. [15], compactified on a $T^2/(Z_2 \times Z'_2)$ orbifold with $R_5 \gg R_6 \approx M_s^{-1}$. Imposing the boundary condition, $Z_5 = \text{diag}(+, +, +, +, +, +), Z_6 = \text{diag}(+, +, +, +, +, -), T_5 = \text{diag}(+, +, +, +, +, -), T_6 = \text{diag}(+, +, +, +, +, +)$ acting on 6 of $SU(6)$, the structure of the theory precisely reduces to that of the $SU(5)$ theory discussed here (with an additional $U(1)$) below $R_6^{-1}$. The $SU(6)$ gauge multiplet in 6D reproduces the 5D $SU(5)$ gauge multiplet (plus $U(1)$) with two Higgs hypermultiplets in a 5 representation. The matter fields are located on the $(x_5, x_6) = (0, 0)$ fixed point or the $x_6 = 0$ fixed line. The Yukawa couplings are introduced on the $(x_5, x_6) = (0, 0)$ brane.
6 Discussion and Conclusions

The QCD, weak and electromagnetic forces play very different roles in nature, and, at first sight, it seems very unlikely that they are all manifestations of a single unified interaction. Nevertheless, the structure of the standard model does allow an elegant picture of unification, although the resulting prediction for the QCD coupling is 50% from the observed value, as shown in Fig. 3. This picture of unification introduces several problems into the structure of the theory. First, the unification occurs at the enormous energy scale of \(10^{15}\) GeV, introducing a large hierarchy with the weak scale. Actually, there is already a hierarchy between the weak scale and the Planck mass, the scale at which gravity becomes strong, and it is, perhaps, disappointing that the unification scale is fully four orders of magnitude lower than the Planck mass. Other problems include excessive proton decay induced by the unified gauge bosons, breaking the unified gauge symmetry, and understanding why the Higgs doublets and their color triplet partners have a hierarchical mass splitting.

Weak scale supersymmetry has successfully addressed some of these problems, so that the MSSM up to a very high energy scale has become the standard paradigm for new physics. The superpartners of the gauge and Higgs bosons lead to a marked improvement in the prediction for the QCD coupling, as shown in Fig. 3. At the same time the unification scale is raised to \(2 \times 10^{16}\) GeV, removing the problem of gauge boson mediated proton decay, and diminishing the distance to the Planck scale. The superpartners also lead to a radiative stability of the hierarchy of mass scales. Despite these successes, an energy desert of 13 orders of magnitude is a startling conclusion, and should not be drawn lightly. There are scenarios without a desert: for example, large extra dimensions can lead to gravity getting strong at the TeV scale \[21\], and string theory can occur at the TeV scale \[22\]. There are many weak arguments against a low fundamental mass scale; for example, from proton decay, neutrino masses and inflation. The only argument which has real strength, since it is based on the numerical prediction of a measured quantity, is that of gauge coupling unification. On this score the low scale theories do poorly: typically they cannot yield a simple picture of coupling unification. Such a picture does exist in the case of power-law unification in higher dimensions \[23\], but the accuracy of the prediction is greatly weakened through high sensitivity to unknown ultraviolet physics. In the case of accelerated unification \[24\], an accurate prediction persists, but at the cost of multiple replications of the standard model gauge group. Nevertheless, it must be admitted that high scale gauge coupling unification is not perfect. The central value for the prediction of the QCD coupling is about 10% off, requiring large threshold corrections from the unified scale. For example, if superheavy \(5 + \bar{5}\) chiral multiplets of \(SU(5)\) are added, with a unit logarithmic mass splitting between doublet and triplet components, the threshold correction...
Figure 3: The predictions for $\alpha_s$ in the three frameworks: non-supersymmetric grand unification $\alpha_s^{GUT}$, supersymmetric grand unification $\alpha_s^{SGUT}$, and Kaluza-Klein grand unification $\alpha_s^{KK}$. Solid error bars represent the threshold corrections from the superpartner spectrum. Dotted error bars for $\alpha_s^{GUT}$ and $\alpha_s^{SGUT}$ represent threshold corrections from the unified scale corresponding to a heavy $5 + \bar{5}$ representation with unit logarithmic mass splitting between doublets and triplets. The dashed error bars represent possible dependence on models from physics at the unification scale: particle content, higher dimensional operators, coupling constants, etc, and have been arbitrarily normalized to bring $\alpha_s^{SGUT}$ in agreement with experiment. The dotted error bar for $\alpha_s^{KK}$ is the theoretical uncertainty (other than from superpartner masses) for our theory, as estimated in the text.
is small, $\Delta_{\text{SGUT}} \simeq 0.003$, as shown by the dotted uncertainty drawn in Fig. 3 for $\alpha_{\text{SGUT}}$. The correction required by data, shown by the dashed error bar in Fig. 3, is much larger. Furthermore, in supersymmetric theories the questions of breaking the unified gauge group and the mass splitting between Higgs doublets and triplets still remain, and the further problem of proton decay from dimension five operators is introduced. However, these are objections against 4D grand unified theories rather than high scale gauge coupling unification.

The discrepancy between the experimental values of the gauge couplings and the prediction from supersymmetric unification is usually ascribed to threshold corrections from the unified scale which depend on unknown parameters or moduli of the unified theory. In this paper we have taken an alternative viewpoint. We have discovered a new framework that offers the possibility of a reliably calculated, precision agreement with experimental data. The discrepancy of the standard supersymmetric prediction is accounted for by a moderately large logarithmic effect in a higher dimensional unified theory, with orbifold breaking of the gauge symmetry. The size of this logarithm is determined by the strong coupling requirement. Remarkably, this framework does correctly predict the central experimental value for the QCD coupling, as shown in Fig. 3. Furthermore, threshold corrections from the scale of the unified gauge boson masses are unambiguous and have been included. The remaining uncertainty from unknown physics at even higher energies can be reliably estimated to be small, as shown by the dotted error bar of Fig. 3 for $\alpha_{\text{KK}}$.

Since the framework allows for many possible models, each with differing coefficients of the logarithm, it is fair to question whether we have really predicted the data, or whether we have used the data to select a model. We view the situation as somewhat analogous to the case of supersymmetric unification. Within that framework there are many possible models, for example ones with $2n$ Higgs doublets, each giving a different prediction for the QCD coupling. Nevertheless, the addition of weak scale supersymmetry is viewed as highly significant because the simplest possible model is precisely the one that works best. All the more complicated theories are very much further from the data. We have found a similar situation to hold in the case of adding extra dimensions at the unified scale. The various models lead to a discrete set of predictions, yet it is only the simplest model, with one extra dimension and $SU(5)$ gauge group, that is able to precisely account for the data; the majority of models find a correction which is either too small or of the wrong sign. As precision electroweak measurements strengthened the case for the MSSM, future measurements of superpartner masses will further test minimal KK unification. In supersymmetric unification the crucial new running is induced by one set of superpartners for the minimal set of gauge and Higgs bosons. Adding extra dimensions, we find that precision unification follows from the running induced by one set of KK modes for the minimal gauge and Higgs bosons. In our view, this observation strengthens the case for a
high fundamental scale.

In this paper we have studied an alternative to 4D grand unification or string theory for physics just beyond the supersymmetric desert which leads to gauge coupling unification: Kaluza-Klein grand unification [5]. We have introduced a new framework which predicts the leading radiative corrections to gauge coupling unification from the high scale. There is an essentially unique theory which provides a precise and successful prediction for the QCD coupling. Not only does this improve on the prediction from conventional supersymmetric unification, but it also solves the three outstanding problems of 4D grand unified theories. The color triplet partners of the doublet Higgs bosons are projected out of the zero mode sector by the orbifold boundary condition, the underlying $R$ symmetry of the theory automatically removes all proton decay from dimension four and five operators, and light fermions are guaranteed not to have $SU(5)$ mass relations.

Furthermore, the addition of extra dimensions leads to new avenues of exploration for flavor. We find that the top quark is necessarily a brane mode, while part of the first generation is necessarily in the bulk. Some aspects of flavor must be associated with the geometry of the extra dimension. It is interesting that at the fundamental scale the top Yukawa interaction, and perhaps other flavor couplings, are strongly coupled. Finally, it is intriguing to note that the size of the neutrino mass suggested by atmospheric neutrino oscillations is related to the compactification scale, $M_c$, by $v^2/M_c$, where $v$ is the electroweak vacuum expectation value.

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