Tetrahedral and spherical representations of the periodic system

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Abstract The s, p, d and f blocks of the elements, as delimited by Charles Janet in 1928, can be represented as parallel slices of a regular tetrahedron. They also fit neatly on to the surface of a sphere. The reasons for this are discussed and the possible objections examined. An attempt is made to see whether there are philosophical implications of this unexpected geometrical regularity. A new tetrahedral design in transparent plastic is presented.

Keywords Periodic system · Electronic structure · Status of helium

Plato, in his dialogue *Timaeus*, supposed that the atoms of the four elements were in the form of four of the regular polyhedra, while the fifth—the dodecahedron—‘was used in the delineation of the inhabited world’. After the initial shock of learning that there are at least 120 existing or possible elements, he might have been pleased to know that they can be represented as four parallel and equally spaced slices of a regular tetrahedron. This has been demonstrated by Valery Tsimmerman, an American engineer, Fig. 1. The tetrahedral symmetry had also been noticed by Jess Tauber in 1979 (personal communication) and 2000.

Tsimmerman’s starting point was the blocks of chemical elements as delimited by Janet (1928), a French engineer and amateur biologist and geologist, who turned his mind to the Periodic System at the age of 78. Janet arrived at his ‘left-step table’ by moving what is now called the s block from the left-hand side of the long form of the table to the right-hand side, placing H and He at its head and making Li and Be into the second period. In consequence his periods are different from those in other tables; B to Mg are pushed down into the third period, and there are in all eight periods. There is no reason to object to this; the delimitation of periods is arbitrary; the Greek *periodos* means simply ‘coming around’.

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Found Chem (2018) 20:111–120
https://doi.org/10.1007/s10698-017-9299-y
The sequence of elements is a continuum and there are different ways—at least six published—of cutting it up into repeating sections. As Mendeleev (in Jensen 2002, p. 56) wrote: ‘the series of elements is uninterrupted and corresponds, to a certain degree, to a spiral function.’ In the heaviest elements periodic behaviour appears to break down (Scerri 2013, pp. 24–26).

At the start of Janet’s eighth period stood the actinides (widely thought at the time to be part of the d block), in which he included Ac to U and added places for elements 93–102, anticipating Seaborg by 15 years. For completeness he extended the eighth period to
element 120, which, for arithmetic reasons, he considered the heaviest possible; if ever it is created it should be called Janetium. His rearrangement (Fig. 2) was undertaken in order to remove the gaps within periods, but he noted that it achieved greater geometrical regularity, giving four pairs of rows with 2, 8, 18 and 32 elements, these numbers being twice the squares of the first four integers (because the widths of the blocks are the successive odd numbers, 1, 3, 5, and 7, whose cumulative sums are the squares in question).

Janet’s death in 1932 prevented him from hearing of the discovery of deuterium, whose existence he had predicted, and of the positron—key to the anti-matter periodic table that he had envisaged. Because his work was self-published and poorly distributed, his remarkable legacy remained largely unknown even to French-speaking chemists, to whose professional bodies he did not belong. As for English-speaking chemists, they knew of him only through a poor and badly edited translation (Janet 1929). The left-step table was proposed again by a physicist, Simmons (1947, 1948) with inadequate acknowledgement of Janet.

Janet’s table was highly regarded by J. W. van Spronsen and Edward Mazurs, who both adopted Janet’s blocks in their own designs (Mazurs 1974, p. 121 and end fold-out; van Spronsen 1969, endpapers) though Mazurs indicated the irregularities of electronic structure within the blocks, and van Spronsen placed the s block on the left (and later reverted to the traditional treatment of He). In the twenty first century the Janet table was revived by Katz (2001) and received an exhaustive treatment by Henry Bent (2006). It was considered favourably by Scerri (2007). I have discussed Janet’s overall contribution (Stewart 2010).

Janet’s table has not displaced the long form introduced by Werner (1905), now usually printed with the f block footnoted. Mazurs (pp. 63, 175–180) traced the germ of this to Mendeleev himself, who did not like it because of the gaps within periods, which meant that ‘the centre of the table would be nearly empty’ (Jensen 2002, pp. 34–35). In the currently most widespread form of the medium-long form, published by Web Elements, the blocks are the same as Janet’s, apart from He, but in the order s–f–d–p. Logically, the blocks should be either in the sequence s–p–d–f, the order in which they first appear, or in Janet’s f–d–p–s, their order in the longest periods.

![Fig. 2 Janet’s table with a half-square per element](image-url)
The standard form has varied over the years, with some uncertainty over the limits of the f and d blocks, La sometimes replacing Lu in the d-block, La to Lu sometimes forming a 15-element-wide f block, and Sc and Y sometimes floating over the f block. People have routinely placed He over the noble gases in the p block on the basis of its inertness, though its ground-state electrons are undeniably in the 1s orbital. By a different reasoning, some chemists and some authors put H above the halogens, again in the p block, on the analogy between hydrides and halides, and for the neatness of keeping it beside He. Others have left it floating above the p block. In fact it behaves more like C, with predominantly covalent bonding, which is to be explained by the fact that both elements are half way between an empty and a full shell. Indeed, M. W. Cronyn (2003) proposed placing it at the head of the C group in the table.

The traditional placement of He above Ne is a relic of earlier classifications, which derived places in the system from chemical behaviour, much of which is now explained by electronic structure. Elements behave ‘nobly’ if they have complete ns$^2$ and np$^6$ electrons, which together form an impregnable ‘eightsome’—well, impregnable in Ne, in which alone the sixth p electron completes a shell. Completion of subsequent subshells also seems to have some influence. The third shell is completed not in Ar but by the 3d$^{10}$ electron in Zn, the fourth shell not in Kr but by the 4f$^{14}$ electron in ytterbium, and the fifth shell is never completed at all; there is no 5g$^{18}$. Even Ar has been shown to form a couple of compounds at very low temperatures, and Kr, Xe and Rn are progressively less ‘noble’. Hg, with a complete d subshell, has a ‘pseudo-noble’ monatomic gas. The noble behaviour of He is explained by the fact that the first shell is complete with only two electrons; there is no 1p subshell to make an ‘eightsome’.

Part of the objection to the placing of He—and H—above Li and Be is that otherwise the s-block consists entirely of strongly electropositive metals. However, the presence of two non-metals there should be no more disquieting than the fact that half the p block, including the whole of the first row, consists of non-metals. In any case, metallic behaviour is partly a matter of temperature and pressure. If there is ‘metallic hydrogen’ in the hearts of giant planets, it will be a plasma of naked protons and electrons, not a molten metal.

Tsimmerman saw that if Janet’s table is redrawn with one square per orbital, as in Fig. 2 above, the height of each block is proportionate to the number of values of the principal quantum number, $n$,—2, 4, 6, or 8 for f, d, p or s respectively, while the width of the block is equal to the number of values of the magnetic number, $m_l$,—7, 5, 3 or 1, in the same order. This means that the perimeters of all four Janet blocks are the same: 2 × 9 units, the unit being the side of an ‘orbital square’. This means that they can be represented as parallel slices of a regular tetrahedron (of edge 9), of which it is a peculiarity that all cross-sections parallel with two opposite edges have the same perimeter.

Strictly speaking, the squares do not usually correspond to orbitals, since the differentiating electrons of consecutive elements go into empty orbitals as long as any are available. For example, B and C are inscribed in two halves of the same square although their differentiating electrons are in different orbitals. Of course, the very idea of a ‘differentiating electron’ is a concession to the notion that electrons have identities. Use of a half-square per element also has the desirable consequence of reducing the width to height ratio of the table to 2:1, obviating the need to footnote the f block.

Once Janet had realized that his blocks corresponded with quantum theory (Janet 1930), he saw that $n + 1$ is the same for successive subshells in a period (using Bohr’s values for the second quantum number; $k = 1 + 1$), so that the values of $n$ increase as those of $l$ decrease. He considered this an invariable rule and expressed it in a diagram of subshell filling (Fig. 3). He ‘corrected’ the data of Bohr and Stoner to make them fit his rule,
considering them to be errors of measurement. This was the ‘Madelung Rule’, 6 years before Madelung.

The consequence of this for the periodic system is made clearer if the rows are made to represent electron shells (Arabic numerals) instead of periods (Roman numerals)—a proposal made by Mazurs (1974, p. 134). Tsimmerman produced a version of this with orbital squares, inverting the table so that the rows read from the bottom upwards. In a further modification, he rotated his first version clockwise through 90°, so that the rows became columns and vice versa, with the f block towering like a skyscraper over the symmetrical design. There is logic in this, the s block being, so to speak, at the base of the build-up of elements, but it is disconcerting for anybody familiar with conventional tables—in other words 99% of chemists. In Fig. 4, ‘Janet Rajeuni’ (Janet Rejuvenated), I present this without the inversion and rotation, to restore the original rows and columns.

This makes clear why the Janet rule produces blocks with identical perimeters. As each new sub-shell is added on the left, it entails the addition of one subshell to each of the columns to its right. Thus 2p brings in its wake 3s; 3p brings 4s; 3d brings 4p and 5s: 4d
brings 5p and 6s and so on, until 5f brings 6d, 7p and 8s. The number of values of \( n \) in each block increases by one each time, and the sum of that and the number of values of \( m_l \) remains the same. If the fifth shell were expanded to subshell 5g, it would bring in its wake 6f, 7d, 8p and 9s (which, of course, is science fiction), the sum of the number of values of \( n \) and of \( m_l \) would become respectively 1 + 9, 3 + 7, 5 + 5, 7 + 3 and 9 + 1, the perimeters of all the blocks would be 20, and they could be represented as five slices of a tetrahedron of edge 10. Tetrahedral symmetry is in the nature of the system.

This representation fits neatly on to a sphere, with the top and bottom of the s block near the poles and shells 4 and 5 wrapped round the equator. It is 8 orbital units high and 16 wide, so a circumference of 16 units would in theory be exactly right, but in practice 20 allows for the joins within periods to be shown. I have called this ‘the Chemosphere’ (Plate 1). Like a spiral, it shows the continuous nature of the series of elements, with the help of a few arrows. As far as I know, the periodic system has never had a satisfactory spherical representation. An attempt by Friend (1925) completely confused the elements of the different blocks. An evocative impression was given in an advertisement by Union Carbide (1960). A spherical representation is not in the nature of the system; if the series of elements ended with Ba, the ‘equatorial’ shell would be only 9 units wide and the ‘pole to pole’ s block 6 high; if a g block were added, the ‘equatorial’ shell would be 25 units wide and the s block only 9 units high.

It may seem like ‘cheating’ to add elements 119 and 120 to the s block, but only 20 years ago the bottom row of the p block was empty. There is no reason why these elements should not be created. The main problem is to add enough neutrons. The ratio of neutrons to protons in stable nuclei steadily increases with atomic number. For \(^{208}\text{Pb}\) \( N/Z = 1537 \); for \(^{232}\text{Th}\) \( N/Z = 1578 \). From this dizzy height we descend to \(^{285}\text{Cn}\), with a half-life of 29s and \( N/Z = 1.545 \). For \(^{290}\text{Fl}\) with a half-life of 19s, \( N/Z \) is still lower: 1.544. \(^{294}\text{Og}\), with a half-life in milliseconds, has 176 neutrons, which give it a miserable \( N/Z \) ratio of 1.492. There is a long way to go; perhaps synthesis needs to take place in a dense ‘gas’ of cold neutrons. But of course there is no reason why the series of elements that can be created on earth should end neatly with the end of a shell or period.

The main objection to a tetrahedral or spherical representation is that the blocks are idealized. The Janet/Madelung rule is only a rule of thumb, and there are increasing numbers of exceptions as one moves down the table and to the left (Scerri 2013 bis). In the f and d blocks, the energetic order of f, d and s orbitals depends on several factors including...
screening by core shells and the oxidation state of the atom. The blocks are based on the behaviour of single, neutral atoms in the ground state, and in reality atoms are rarely in this condition (Schwarz and Rich 2010). La has a differentiating electron in a d orbital—as does Gd—but it would be confusing to count both La and Lu as belonging to the d block. There are good reasons for giving the place to Lu: it follows the lanthanoid contraction, so its relation to Y is as close as that of Hf to Zr; indeed it was discovered in yttria, while La was first found in ceria. An argument for making La the first element in the f block is that then, except in Gd, the pth element in the series has p f electrons.

The 5f series has an even more ragged start. Both Ac and Th are differentiated by 6d electrons, and Pa, U and Np continue to place electrons in 6d as well as 5f orbitals. Cm, like Gd, is differentiated by a d-electron. If Ac is made the first of the series, it is true for only 8 actinides—just over half—that the pth element has p f electrons. Again the boundary between the f and d blocks is obscured. It is evident that irregularities increase as

Plate 1 Figure 4 projected on to the surface of a sphere
one moves down and across in the Janet table. If ever a g block were reached, most of its elements might have mixed d, f and g electrons.

However, idealization is inherent in the making of models. As Alfred Korzybski used to say ‘the map is not the territory’. The division into blocks is justified by their distinctive nature: s is characterized, except in H and He, by highly electropositive metals; p by a range of very distinctive metals and non-metals, many of them essential to life; d by metals with multiple oxidation states; f by metals so similar that their separation is problematic. Useful statements about the elements can be made on the basis of the block they belong to and their position in it, for example highest oxidation state, density, melting point... Electronegativity is rather systematically distributed across and between blocks. Overall, the Janet rule applies to most of the commonest elements and about nine tenths of them all, which is rather good as models go.

Plato got it the wrong way round. The physical universe ‘explores’ possible forms and ‘selects’ those that can survive. The human mind abstracts from them ideal forms and describes them mathematically. For example the sphere has emerged over and over again as the form that minimizes potential energy at its surface—in star and planet, in raindrop and volvox, in soap-bubble and water-melon—but it is because concrete spheres have come into existence that the qualities of the sphere can be abstracted. No physical sphere corresponds exactly to the mathematical abstraction, but that does not invalidate the equations. The sphere is exceptional in its exactness; most forms are less determinate and correspond less perfectly with any mathematical definition. Einstein’s caution is appropriate: ‘As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.’

No data have had to be ‘tortured’ to produce Tsimmerman’s tetrahedron, but is it a mere accident or does it have a deeper significance? A partial explanation is that the second and third quantum numbers both derive from the first. The azimuthal number—l—can take only the values 0 to n – 1, but in practice l = 3 is the maximum. The magnetic number—m_l—takes the values −l through 0 to l, i.e. −n + 1 to n − 1. It is not surprising that the interplay of a limited number of closely related small integers produces a systematic pattern. Pythagoras would have been pleased.

Tsimmerman has speculated on the meaning of his model (personal communication): ‘The dimensions of the blocks are dictated by the number of values of m_l and number of values of n. “We know that n governs quantization of energy. Quantization of the possible orientations of L [orbital angular momentum] with respect to an external magnetic field is often referred to as space quantization.”’ (Serway and Jewett 2004: p. 1369). That is, m_l stands for space quantization. Therefore, the Perimeter Rule reflects a direct relationship between energy and space. I think that this could have some significance.’

Tsimmerman arrived at his tetrahedral arrangement in 2007 and published it on the internet (see references). He finally commissioned a three-dimensional version in 2014—the Adomah Periodic Cube. The tetrahedron is contained in a circumscribed cube of crystal glass, 78 mm to the edge; it has to be small because of its density (2.53). The blocks of elements are etched internally by laser. This object is remarkably beautiful—more like a gem than a scientific model. From very close, the names and symbols of the elements are perfectly legible. However, Tsimmerman uses his inverted and rotated version of Janet’s table, which turns the rows into columns and vice versa.

For legibility, a larger model is needed. I have produced an acrylic version 180 mm to the edge, which makes it more than twelve times the volume of the glass cube. It can be taken apart and reassembled, allowing the blocks to be viewed separately for educational purposes. It is not simply an enlargement. I have restored the rows and columns familiar to
chemists. Topologically this makes no difference, but I hope it will be easier to read. I was unsure what to call it. Tsimmerman himself hesitated between calling his cube ‘periodic’ or ‘quantum’. In the end I have preferred to evoke the latter concept. I also wanted to give equal weight to the tetrahedron and the cube or hexahedron, two of Plato’s ‘elemental’ solids; so I have called it ‘the QuantaHedron’ (Plate 2).

**Acknowledgements** I am grateful to Valery Tsimmerman for permission to reproduce Fig. 1.

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