Constraining heavy neutral gauge boson $Z'$ in the 3 - 3 - 1 models by atomic parity violation in Cesium and proton

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The atomic parity violation (APV) is analyzed in the framework of the models based on SU(3)$_C$ × SU(3)$_L$ × U(1)$_X$ (3-3-1) gauge group, including the 3-3-1 model with CKS mechanism (331CKS) and the general 3-3-1 models with arbitrary $\beta$ (3-3-1$\beta$) with three Higgs triplets. We will show that at the TeV scale, the mixing among neutral gauge bosons plays valuable effect. Within the present APV data of Cesium and proton we get the lowest mass value of the extra heavy neutral gauge boson is 1.27 TeV. The results derived from the APV data, perturbative limit of Yukawa coupling of the top quark, and the relevant Landau poles favor the models with $\beta = \pm \frac{1}{\sqrt{3}}$ and $\beta = 0$ while rule out the ones with $\beta = \pm \sqrt{3}$. In addition, there are some hints showing that in the 3-3-1 models, the third quark family should be treated differently from the first two.

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I. INTRODUCTION

Nowadays, the experimental data on neutrino masses and mixing as well as on Dark Matter (DM) lead to fact that the Standard Model (SM) must be extended. Among the beyond SM extensions, the models based on the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge group [1–7] (3 - 3 - 1 models) are attractive in the following senses. First of all, these models are concerned with the search of an explanation for the number of fermion generations to be three, when the QCD asymptotic freedom is combined. Some other advantages of the 3-3-1 models are: i) the electric charge quantization is solved [8, 9], ii) there are several sources of CP violation [10, 11], and iii) the strong-CP problem is solved due to the natural Peccei-Quinn symmetry [12–15].

There are two main versions of the 3 - 3 - 1 models which are depended on the parameter $\beta$ in the electric charge operator

$$Q = T_3 + \beta T_8 + X.$$  \hspace{1cm} (1)

If $\beta = \sqrt{3}$, this is the minimal version [2–4], and $\beta = -\frac{1}{\sqrt{3}}$ corresponds to the 3-3-1 model with right-handed neutrinos [1, 5–7].

At present, we still face an old problem of explanation of hierarchies and structure of the fermion sector. However, in the above models, most researches on the 3-3-1 models are not concerned with vast different masses among the generations (see references in Ref.[16]). It is well known that the Yukawa interactions are not enough for producing fermion masses and mixings. According our best of knowledge, the first work for solving the mentioned puzzles in quark sector is in Ref. [17] named Froggatt-Nielsen mechanism. Recently, the new mechanism based on sequential loop suppression mechanism, is more natural since its suppression factor is arisen from loop factor $l \approx (1/4\pi)^2$. The above mentioned mechanism

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is called by CKS - the names of its authors [18]. The Froggatt-Nielsen mechanism was implemented to the 3-3-1 model in Ref. [19]. In recent work Ref. [16] the CKS mechanism has been implemented to the 3-3-1 model with $\beta = -\frac{1}{\sqrt{3}}$, and it is interesting to note that the derived model is renormalizable. We name it the 3-3-1 CKS model for short. In the Ref. [20], the Higgs and gauge sectors of the model are explored. From the experimental data on the $\rho$ parameter, the bound on the scale of the first step of the spontaneous symmetry breaking (SSB) in the 3-3-1 CKS is in the range of 6 TeV [20]. There also exist helpful relations among masses of gauge bosons, this is essential point for the model phenomenology.

At present, the new neutral gauge boson $Z'$ is a very attractive subject in Particle Physics due to potential discovery of right-handed neutrino through its mediation [21]. Within its mass around 2.5 TeV, the simulation shows that it may be discovered at the LHC. Hence it is necessary to study more deeply different aspects to fix the mass as well as properties of the mentioned object. To fix the model parameters, ones often look at the famous data such as the $\rho$ parameter, mass differences of neutral mesons, and deviation of nuclear weak charge, etc. So, in this paper we focus on the latter subject.

The parity violation in weak interactions was known for long time ago. In the SM, it can be seen from the APV caused by the neutral gauge boson $Z$. In the beyond Standard Model (BSM), the APV gets additional contribution from new heavy neutral gauge bosons $Z'$. Therefore, the data on APV, especially of the Cesium ($^{133}\text{Cs}$) being stable atom, is an effective channel for probing the new neutral gauge boson $Z'$. This is our aim in this work.

The experimental data on the APV in Cesium atom [22] has caused extensive interest and reviews [23–28]. Parity violation in the SM results from exchanges of weak gauge bosons, namely, in electron-hadron neutral-current processes. The parity violation is due to the vector axial-vector interaction in the effective Lagrangian. The measurement is stated in terms of the nuclear weak charge $Q_W$, which parameterizes the parity violating Lagrangian. Due to the extra neutral gauge bosons, in the BSM, the nuclear weak charge of an isotope (X) gets additional value which is called by deviation defined as follows

$$
\Delta Q_W(^A_ZX) \equiv Q_W^{\text{BSM}}(^A_ZX) - Q_W^{\text{SM}}(^A_ZX).
$$

(2)

For the concrete stable isotope Cesium (Cs), it is reported recently from experiment as [29, 30]

$$
Q_W^{\text{exp}}(^{133}\text{Cs}) = -72.62 \pm 0.43.
$$

(3)
Comparing to the SM prediction $Q_{W}^{SM}(^{133}_{55}Cs) = -73.23 \pm 0.01$ \cite{30,31} yields the deviation $\Delta Q_{W}$ as follows \cite{29}

$$
\Delta Q_{W}(^{133}_{55}Cs) \equiv Q_{W}^{exp}(^{133}_{55}Cs) - Q_{W}^{SM}(^{133}_{55}Cs) = 0.61 \pm 0.43,
$$

which is $1.4\,\sigma$ away from the SM prediction. This value has been widely used for analysis of possible new physics, where it is assumed that the BSM can be explained the experimental APV data $Q_{W}^{BSM}(^{1}_{2}X) \equiv Q_{W}^{exp}(^{1}_{2}X)$.

On the other hand, the nuclear weak charge of an atomic is formulated as a function of the two independent contributions of light quarks $u$ and $d$, the experimental APV data of the two distinguishable isotopes will result in different allowed regions of the parameter space defined by a BSM. Hence, combining result of allowed regions from experimental data of APV of Cesium and proton will be more strict than the private one. The latest experimental result of APV measurements of proton $^{1}_{1}p$ is $Q_{W}^{exp}(^{1}_{1}p) = 0.0719 \pm 0.0045$ \cite{32}, which was shown to be in great agreement with the SM prediction, $Q_{W}^{SM}(^{1}_{1}p) = 0.0708 \pm 0.0003$. The deviation from the SM is

$$
\Delta Q_{W}(^{1}_{1}p) = 0.0011 \pm 0.0045. 
$$

Considering a BSM containing an additional heavy neutral gauge boson $Z'$ apart from the SM one $Z$, a theoretical deviation of $Q_{W}$ from the SM prediction for an isotope $^{4}_{2}X$ is given by

\begin{equation}
\Delta Q_{W}^{BSM}(^{4}_{2}X) \simeq \left[ 2Z - A + 4Z \left( \frac{s_{\mu}^{1}}{1 - 2s_{W}^{2}} \right) \right] \Delta \rho \\
+ 4s_{\phi} \left\{ (A + Z) [g_{A}(e)g_{V}^{'}(u) + g'_{A}(e)g_{V}(u)] \\
+ (2A - Z) [g_{A}(e)g_{V}^{'}(d) + g'_{A}(e)g_{V}(d)] \right\} \\
- 4 \left( \frac{M_{Z_{1}}^{2}}{M_{Z_{2}}^{2}} \right) [(A + Z)g_{A}(e)g_{V}^{'}(u) + (2A - Z)g'_{A}(e)g_{V}(d)]
\end{equation}

where $s_{\phi} \equiv \sin \phi$ corresponds to the $Z - Z'$ mixing of the SM and new heavy neutral gauge bosons $Z$ and $Z'$ that create the two physical states $Z_{1,2}$ with masses $M_{Z_{1,2}}$.

Notations in Eq. (6) are based on the vector-axial (V-A) currents of neutral gauge bosons defined by the well-known Lagrangian

\begin{equation}
L_{Vff} = \frac{g}{2c_{W}} \sum_{f} \overline{f} \gamma^{\mu}(g_{V}(f) - \gamma_{5}g_{A}(f))fZ_{\mu} \\
+ \frac{g}{2c_{W}} \sum_{f} \overline{f} \gamma^{\mu}(g_{V}^{'}(f) - \gamma_{5}g'_{A}(f))fZ_{\mu}'.
\end{equation}
where the summation is taken over the fermions of the BSM, $g = e/s_W$ is the $SU(2)_L$ gauge coupling of the SM.

The formula (6) has been checked in details by us (see appendix A) based on original calculation in Ref. [33] that concerned for $U(1)$ gauge extensions of the SM. However, it is also valid for other non-Abelian gauge extensions including 3-3-1 models [34-41]. Specially, the formulas for arbitrary $\beta$ given in Ref. [35] was corrected in Ref. [40] following a recent correction of $Z - Z'$ mixing angle [42], but new constraints on APV of Cesium Eq. (3) was not used. Additionally, using the same notation our formula (6) contains two factors 4 instead of 16 in the expression of the nuclear weak charge used in Ref. [40].

Taking into account the SM gauge couplings

\[ g_A(e) = -\frac{1}{2}, \quad g_V(u) = \frac{1}{2} - \frac{4s_W^2}{3}, \quad g_V(d) = -\frac{1}{2} + \frac{2s_W^2}{3}; \]  

the experimental value of the Weinberg angle at the $M_Z$ scale [30] $s_W^2 = 0.23122$, \( \left( \frac{s_W^2}{1 - 2s_W^2} \right) = 0.0994544 \); and the scale dependence of the gauge couplings $g$ in Eq. (7), the expression (6) is written in the more general form

\[ \Delta Q_{W}^{BSM}(A_{X}) \simeq - (A - 2.39782 \times Z) \Delta \rho \]

\[ - 2s_A \{ A [2g_V'(d) + g_V'(u) + g_A'(e)] \}
\]

\[ - Z [g_A'(e) \times 1.07512 + g_V'(d) - g_V'(u)] \} \times \frac{g(M_{Z1})}{g(M_{Z2})}
\]

\[ - 4g_A'(e) \left( \frac{M_{Z1}^2}{M_{Z2}^2} \right) \{ A [2g_V'(d) + g_V'(u)] + Z [g_V'(u) - g_V'(d)] \} \times \frac{g^2(M_{Z1})}{g^2(M_{Z2})}, \]

where $g(M_{Z1,2})$ are respective gauge couplings of the $Z_{1,2}$ at their mass scales. We emphasize that Eq. (9) contains major improvements from the original version [33], see detailed discussion in appendix A. The above formula is also applicable for the models based on $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge group, where effect of scale dependence was mentioned but the $Z - Z'$ mixing was ignored [39, 41]. The subject was also considered earlier in Refs. [34, 37], but for only the minimal and economical 3-3-1 versions, respectively. The formula (9) is different from those used to investigate APV in 3-3-1 models in Refs. [40], where the scale dependence of neutral gauge couplings are also taken into account. Furthermore, we want to re-investigate the APV in the light of new experimental result of both nuclear weak charge and rho parameter [30]. Instead of Ref. [40], where only model C introduced in Ref. [43] was paid attention using the APV of $Q_W(Cs)$, we will discuss on allowed regions.
of the three parameter spaces corresponding three models A, B, and C, based on the latest experimental data of both \(Q_W(Cs)\) and \(Q_W(p)\). The effects of the perturbative limit of top quark Yukawa coupling on the parameter space will also be mentioned.

The further plan of this paper is as follows. Sect. [II] is devoted to the 3-3-1 CKS model where the particle content is introduced. In this section, the gauge boson masses and mixing are also discussed, and the couplings between neutral gauge bosons \(Z\) and \(Z'\) and fermions are presented. In Sect. [II C], we consider the deviation of nuclear weak charge for Cesium in the 3-3-1 CKS, from which the lower bound on the \(M_{Z_2}\) is derived. Sect. [III] is devoted for the model 3-3-1\(\beta\) \[35, 44\]. In this section, we will focus on different kinds of quark assignments listed in Ref. [35], where the heavy flavor quarks \(t\) and \(b\) behave differently from other ones (representation A) or the light quarks \(u\) and \(d\) do the same (representation C). The analytic expressions of the deviations \(\Delta Q_{W}^{BSM}(AZX)\) predicted by the models will be combined with the latest APV data to investigate allowed regions of the parameter spaces, which will can result in the possibility of surviving or ruling out the model under consideration. We make a conclusion in the last section - section [IV]. Two appendices show in detailed steps to derive the analytic expressions of the nuclear weak charges in the general case and the particular case of the 3-3-1\(\beta\) model.

### II. ATOMIC PARITY VIOLATION IN THE 3 - 3 - 1 CKS MODEL

In this section the needed ingredients for investigating the APV predicted by the 3-3-1 CKS model are discussed.

#### A. Particle content

As in the ordinary 3-3-1 model without exotic electric charges, the quark sector contains two quark generations transforming as antitriplet and one remaining generation transforming as triplet under \(SU(3)_L\) subgroup. The other extra quarks transform as singlet under above mentioned subgroup. The quantum numbers of the quark sector are summarized in Table [I].

As seen from Table [I] in the model under consideration, all extra quarks have electric charges of quarks in the SM. As shown in Ref. [16], the spontaneous symmetry breaking (SSB) provides masses for only extra quarks as well as top quark. The remaining quarks get
TABLE I: Quark assignments under $SU(3)_L, U(1)_X, U(1)_{L_g}, Z_4, Z_2$ and the values of generalized lepton number $L_g$ (all quarks are in triplets under $SU(3)_C$)

|         | $Q_{1L}$ | $Q_{2L}$ | $Q_{3L}$ | $U_{1R}$ | $U_{2R}$ | $U_{3R}$ | $T_R$ | $D_{1R}$ | $D_{2R}$ | $D_{3R}$ | $J_{1R}$ | $J_{2R}$ | $\bar{T}_{1L}$ | $\bar{T}_{1R}$ | $\bar{T}_{2L}$ | $\bar{T}_{2R}$ | $B_L$ | $B_R$ |
|---------|----------|----------|----------|----------|----------|----------|-------|----------|----------|----------|----------|----------|-----------------|-----------------|-----------------|-----------------|-------|-------|
| $SU(3)_L$ | $3^*$    | $3^*$    | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $rac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| $X$     | 0        | 0        | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $rac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| $L_g$   | $\frac{2}{3}$ | $\frac{2}{3}$ | $-\frac{2}{3}$ | 0        | 0        | $-2$      | 0        | 0        | 2        | 2        | 0        | 0        | 0               | 0               | 0               | 0               | 0     | 0     |
| $Z_4$   | $-1$     | $-1$     | $1$       | $1$      | $1$      | $-i$     | $1$      | $1$      | $1$      | $-1$     | $-1$     | $i$      | $1$              | $i$              | $1$              | $1$              | $-1$  | $-1$  |
| $Z_2$   | 1        | 1        | 1         | 1        | $-1$     | $-1$     | 1        | 1        | $-1$     | $-1$     | 1        | 1        | 1               | 1               | 1               | 1               | 1     | 1     |

masses by radiative corrections. To explain why top quark gets mass at the tree level but bottom quark does not get, the reason lies in the behaviour of their right-handed components under the symmetry $Z_2$: $U_{3R}$ is odd, while $D_{3R}$ is even. It is crucial for forbiddance from unaimed terms.

The content of the leptonic sector is summarized in Table II. As in the previous sector, the extra leptons: $E_i, i = 1, 2, 3$, $N_i, i = 1, 2, 3$ and $\Psi_R$ get masses at the tree level. Table II also shows that under the $Z_2$, right-handed components of the charged leptons in the second (muon) and the third (tauon) generations are even, while for the first generation, it is odd. That is why tauon and muon get masses at the one-loop level, but the electron gets mass at two-loop correction [16]. Table II also shows that the extra neutral leptons $N_i, i = 1, 2, 3$ have lepton number opposite to those of ordinary leptons.

The Higgs sector contains three scalar triplets: $\chi$, $\eta$ and $\rho$ and seven singlets $\varphi^0_1, \varphi^0_2, \xi^0, \phi^+_1, \phi^+_2, \phi^+_3$ and $\phi^+_4$. The content of the Higgs sector is presented in Table III.

It is to be noted that, in contradiction with ordinary 3-3-1 model, the neutral component of the $\rho$ triplet does not have a vacuum expectation value (VEV). That is why, the charged leptons do not get masses at the tree level. From Table III it follows that $\chi$ triplet has generalized lepton number $L_g$ [16] [45] different from those of $\eta$ and $\rho$ triplets. This leads to the fact that the bottom elements of the $\eta$ and $\rho$ triplets as well as two first rows of the $\chi$ have lepton number equal to 2, the same as $\phi^+_i, i = 2, 3, 4$ and $\xi$ do.

To close this section, we remind that after SSB, the charged and non-Hermitian gauge
TABLE II: Lepton assignments under $SU(3)_L, U(1)_X, U(1)_L_g, Z_4, Z_2$ and the values of generalized lepton number $L_g$ (all leptons are singlet under $SU(3)_C$)

|        | $L_{1L}$ | $L_{2L}$ | $L_{3L}$ | $e_{1R}$ | $e_{2R}$ | $e_{3R}$ | $E_{1L}$ | $E_{2L}$ | $E_{3L}$ | $E_{1R}$ | $E_{2R}$ | $E_{3R}$ | $N_{1R}$ | $N_{2R}$ | $N_{3R}$ | $\Psi_R$ |
|--------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $SU(3)_L$ | 3       | 3       | 3       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       |
| $X$     | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | -1       | -1       | -1       | -1       | -1       | -1       | -1       | 0       | 0       | 0       | 0       | 0       | 0       |
| $L_g$   | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | 1       | 1       | 1       | 1       | 1       | 1       | 1       | -1      | -1      | -1      | 1       | 1       | 1       |
| $Z_4$   | $i$     | $i$     | $i$     | $-i$    | $-i$    | $-i$    | $i$     | $i$     | $-i$    | $-i$    | $i$     | $i$     | $i$     | $i$     | $i$     | $i$     |
| $Z_2$   | -1      | 1       | 1       | -1      | 1       | 1       | -1      | 1       | 1       | -1      | -1      | -1      | -1      | 1       | 1       | -1      |

TABLE III: Scalar assignments under $SU(3)_L, U(1)_X, U(1)_L_g, Z_4, Z_2$ and the values of generalized lepton number $L_g$ (all leptons are singlet under $SU(3)_C$)

|        | $\chi$ | $\eta$ | $\rho$ | $\phi_1^0$ | $\phi_2^0$ | $\phi_1^+$ | $\phi_2^+$ | $\phi_3^+$ | $\phi_4^+$ | $\xi^0$ |
|--------|--------|--------|--------|------------|------------|------------|------------|------------|------------|--------|
| $SU(3)_L$ | 3       | 3       | 3       | 1          | 1          | 1          | 1          | 1          | 1          | 1       |
| $X$     | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $\frac{2}{3}$ | 0         | 0         | 1          | 1          | 1          | 1          | 0       |
| $L_g$   | $\frac{1}{3}$ | $-\frac{2}{3}$ | $-\frac{2}{3}$ | 0        | 0         | 0         | -2         | -2         | -2         | -2      |
| $Z_4$   | 1       | 1       | -1      | -1        | i         | i         | -1         | -1         | 1          | 1       |
| $Z_2$   | -1      | -1      | 1       | 1         | 1         | 1         | -1         | -1         | -1         | -1      |

Bosons get masses as below [20]

$$m_W^2 = \frac{g^2}{4} v_\eta^2, \quad M_{X^0}^2 = \frac{g^2}{4} (v_\chi^2 + v_\eta^2), \quad M_Y^2 = \frac{g^2}{4} v_\chi^2,$$

(10)

where we have used the following notations

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (A_{\mu1} \mp iA_{\mu2}), \quad Y_\mu^\pm = \frac{1}{\sqrt{2}} (A_{\mu6} \pm iA_{\mu7}), \quad X_\mu^0 = \frac{1}{\sqrt{2}} (A_{\mu4} - iA_{\mu5}).$$

(11)

From (11), the following consequences are in order

$$v_\eta = v = 246 \text{ GeV},$$

(12)

$$M_{X^0}^2 - M_Y^2 = m_W^2.$$  

(13)

Note that the value $\Delta Q_W$ depends on couplings of neutral gauge bosons $Z$ and $Z'$ with light quark $u$ and $d$. Hence, we turn into neutral current sector of the model.
B. Neutral currents

Looking at Eq. (A15), one recognizes that some couplings between fermions and neutral gauge bosons $Z, Z'$ enter to the discrepancies. The needed interactions between fermions and gauge bosons are followed from a piece

$$L_{\text{fermion \& gauge boson}} \supset \sum_i \bar{f} T^\mu \gamma^\mu D \mu f.$$  \hspace{1cm} (14)

Here, the covariant derivative is defined by

$$D \mu = \partial \mu - ig A_{\mu a} T^a - ig_X T^9 B_{\mu},$$  \hspace{1cm} (15)

where $g$ and $g_X$ are the gauge coupling constants of the $SU(3)_L$ and $U(1)_X$ groups, respectively. Here, $T^a \ (a = 1, 2, ..., 9)$ are the generators of the $SU(3)$ group with gauge bosons $A_{\mu a}$. Corresponding to the $SU(3)_L$ representations, namely triplet, antitriplet, or singlet of the fermion, $T^a = \frac{1}{2} \lambda^a, -\frac{1}{2} \lambda^T_a$, or 0. Furthermore, we choose the $U(1)_X$ generator as $T^9 = 1/\sqrt{6}$ diag(1,1,1) for both triplet and antitriplet, while $T^9 = 1/\sqrt{6}$ for singlets. For the convenience, one rewrite (15) as follows

$$D \mu = \partial \mu - ig P^{CC}_\mu - ig P^{NC}_\mu,$$  \hspace{1cm} (16)

where

$$P^{CC}_\mu = \sum_{a=1,2,4,5,6,7} T^a A_{\mu a},$$  \hspace{1cm} (17)

and $P^{NC}_\mu$ is determined from diagonal generators, namely

$$P^{NC}_\mu = \sum_{a=3,8} T^a A_{\mu a} + t X T^9 B_{\mu}, \quad t = \frac{g_X}{g} = \frac{3\sqrt{2} \sin \theta_W (M_{Z'})}{\sqrt{3 - 4 \sin^2 \theta_W (M_{Z'})}}.$$  \hspace{1cm} (18)

Since atom cesium is only composed of light quarks, namely $u$ and $d$ quarks and electron, therefore, we just need to deal with the above mentioned fermions. The coupling constants relevant for calculations of APV in the cesium atom for the SM and the 3 - 3 - 1 CKS model are presented in Table IV.

In the limit $v_\chi \gg v_\eta$, the $Z - Z'$ mixing angle is

$$\tan \phi \simeq \frac{(1 - 2s^2_W) \sqrt{3 - 4s^2_W}}{4c^4_W} \left( \frac{v_\eta^2}{v_\chi^2} \right).$$  \hspace{1cm} (19)
TABLE IV: Vector and axial-vector coupling constants relevant for APV in the models SM and 3-3-1 CKS

|                     | Standard Model | 3-3-1 CKS model |
|---------------------|----------------|-----------------|
| $g_A(e) = -\frac{1}{2}$ | $g'_A(e) = +\frac{1}{2\sqrt{3-4s^2_W}}$ |
| $g_V(u) = \frac{1}{2} - \frac{4s^2_W}{3}$ | $g'_V(u) = \frac{-3+8s^2_W}{6\sqrt{3-4s^2_W}}$ |
| $g_V(d) = -\frac{1}{2} + \frac{2s^2_W}{3}$ | $g'_V(d) = \frac{-3+2s^2_W}{6\sqrt{3-4s^2_W}}$ |

C. Deviation of nuclear weak charge in the 3-3-1 CKS model

Let us note that one of the most important observables is the $\rho$ parameter defined as

$$\rho = \frac{m_W^2}{c_W^2 M_{Z_1}^2},$$

where $\rho = 1$ for the SM. Let us analyze the expression in [9] with $\Delta \rho \equiv \rho - 1$ for a BSM. The $\Delta \rho$ is determined by

$$\Delta \rho \simeq \alpha T,$$

where $\alpha$ is the fine structure constant and $T$ is one of the Peskin-Takeuchi parameters [46]. The latter is given by

$$T = T_{ZZ'} + T_{oblique},$$

where the contribution from $Z - Z'$ mixing $T_{ZZ'}$ is as follows

$$T_{ZZ'} \simeq \frac{\tan^2 \phi}{\alpha} \left( \frac{M_{Z_2}^2}{M_{Z_1}^2} - 1 \right).$$

The $T_{oblique}$ being an oblique correction, is the model dependent.

Applying Eq. (9) for Cesium yields

$$\Delta Q_W^{(133\text{Cs})} = -1.12004 \times \Delta \rho$$

$$-s_{\phi} \left[ 422 g''_{V}(d) + 376 g''_{V}(u) + 147.737 g'_{A}(e) \right] \times \frac{g(M_{Z_2})}{g(M_{Z_1})}$$

$$-g'_{A}(e) \left[ 84.4 g''_{V}(d) + 752 g''_{V}(u) \right] \left( \frac{M_{Z_2}^2}{M_{Z_1}^2} \right) \times \frac{g^2(M_{Z_2})}{g^2(M_{Z_1})}.$$  

Taking values $g'_{A}(e), g'_{A}(d),$ and $g'_{A}(u)$ from Table IV we get an expression for $\Delta Q_W^{(133\text{Cs})}$ predicted by the 3-3-1 CKS model

$$\Delta Q_W^{\text{CKS}}^{(133\text{Cs})} = -1.12004 \times \alpha (T_{ZZ'}^{\text{CKS}} + T_{oblique}^{\text{CKS}})$$
\[ + \left[ s_\phi \times 122.655 \times \frac{g(M_{Z_2})}{g(M_{Z_1})} + 120.743 \left( \frac{M_{Z_2}^2}{M_{Z_1}^2} \right) \times \frac{g^2(M_{Z_2})}{g^2(M_{Z_1})} \right]. \quad (25) \]

Looking at Eq. (25), we see that when \( M_{Z_2}^2 \to \infty \), the value \( \Delta Q^{CKS}_{W}(\frac{133}{55}Cs) \) can be negative. However, it is very tiny. According to Ref. [35], in the minimal model, the first term \( \propto -0.01 \), while in Ref. [42], the \( T_{\text{oblique}} \) is neglected. Following recent experimental data of \( \Delta \rho \), which is in order of \( \mathcal{O}(10^{-4}) \), we accept the assumption in Ref. [42].

The APV of proton is determined as

\[ \Delta Q^{CKS}_{W}(p) = 1.140\Delta \rho + \left[ 0.437 \times \frac{g(M_{Z_2})}{g(M_{Z_1})} + 0.777 \times \frac{g^2(M_{Z_2})}{g^2(M_{Z_1})} \right] \left( \frac{M_{Z_1}^2}{M_{Z_2}^2} \right). \quad (26) \]

For the model under consideration, the oblique correction has the same form given in Ref. [20, 47]. Combining with Eq. (13), ones get [20]

\[
\Delta \rho_{CKS} \simeq \tan^2 \phi \left( \frac{M_{Z_2}^2}{M_{Z_2}^2} - 1 \right) + \frac{3\sqrt{2}G_F}{16\pi^2} \left[ 2M_{Y_+}^2 + m_W^2 - \frac{2M_{Y_+}^2(M_{Y_+}^2 + m_W^2)}{m_W^2} \ln \left( \frac{M_{Y_+}^2 + m_W^2}{M_{Y_+}^2} \right) \right] \\
- \frac{\alpha(m_Z)}{4\pi s_W^2} \left[ t_W^2 \ln \left( \frac{M_{Y_+}^2 + m_W^2}{M_{Y_+}^2} \right) + \frac{m_W^4}{2(M_{Y_+}^2 + m_W^2)^2} \right],
\]

where \( \alpha(m_Z) \approx \frac{1}{128} \) [30].

In Fig. [1] we have plotted \( \Delta Q^{CKS}_{W}(Cs) \) and \( \Delta Q^{CKS}_{W}(p) \) as functions of the extra neutral gauge boson \( Z_2 \) mass. It follows that the allowed values of the \( Z_2 \) mass is \( 1.27 \text{ TeV} \leq M_{Z_2} \leq 2.66 \text{ TeV} \). This range is less restrict than that from the \( \rho \) data [20] but it does not contradict it.
III. ATOMIC PARITY VIOLATION IN THE 3 - 3 - 1 MODELS FOR ARBITRARY BETA

Let us briefly resume particle content of the model 3-3-1 [35]. Here the $\beta$ is defined in Eq. (1). The leptons lie in the $SU(3)_L$ triplet as follows

$$l_aL = (\nu_a, \ e_a, \ E^Q_a)^T \sim \left(1, 3, -\frac{1}{2} - \frac{\beta}{2\sqrt{3}}\right),$$

where $a = 1, 2, 3$ is generation index. This choice of lepton representation was called the model $F_2$ [42]. On the other hand, there exit models (model $F_1$) that $l_aL$ are antitriplets, but it can be shown that they are always equivalent to some models with left-handed lepton triplets, in the sense that both have the same physics [40, 48]. Therefore, it is enough to focus on only the model $F_2$.

The chiral anomaly free requires the number of fermion triplets to be equals to that of fermion antitriplets. Therefore, in the model under consideration, one generation of quarks transforms as $SU(3)_L$ triplets and two others transform as $SU(3)_L$ antitriplets. However, it is free to assign to quarks, provided the anomaly free.

Here we adapt to notations in tables 1, 2, and 3 of Ref. [35]. In particular, we consider the models containing just three Higgs triplets defined in Refs. [35, 49], for example those given in Table 3 of Ref. [35]. There are three different left-handed quark assignments, where the third, second or first left-handed quark family is assigned as triplet, three respective models reps. A, B, and C were introduced in Table 2 of Ref. [35]. Remind that the right-handed fermions are $SU(3)_L$ singlets.

Note that the VEV of $\chi$ triplet provides masses of new particles, namely the exotic quarks and lepton as well as new gauge bosons: $Z'$ and bilepton gauge bosons $X$ and $Y$. Remember that the bottom element of $\chi$ does not carry lepton number, while the similar elements of $\eta$ and $\rho$ triplets have lepton number equal to two. This means that only scalar components without lepton number can have VEV. In practice, to make the charged Higgs bosons having the integer value, the parameter $\beta$ can take some special values only.

The masses and mixing of the neutral gauge bosons are presented in appendix B. The needed gauge couplings used to determine $Q_W$ are given in Table V, where only two models A and C with different assignments of the first quark family are considered. The two models A and B have the same assignments of the first quark family, leading to the same APV result.
The 3 - 3 - 1 model (rep. A)
The 3 - 3 - 1 model (rep. C)

In the numerical calculation, we will use $t$ consequence, $\Gamma$ satisfy the perturbative limit: $\beta$ to investigate the APV using the formula expressing the mixing $Z - Z'$ in terms of the model parameter $\beta$ [42]. The detailed steps to derive $\Delta Q_{W}^{331}(Cs)$ are shown in appendix B. Contribution from $\Delta \rho$ will be neglected. The relevant $Z'$ couplings are given in Table V.

With $M_{Z'}^2 \ll M_W^2$, the $Z - Z'$ mixing angle $\phi$ can be formulated as follows [42]

$$s_\phi \simeq \tan \phi \simeq \frac{c_\phi}{3} \sqrt{f(\beta)} \left[ \frac{M_{Z}^2}{M_{Z'}^2} \right],$$

(29)

where

$$f(\beta) = \frac{1}{1 - (1 + \beta^2)^2}, \quad c_\phi \equiv \cos(2\beta) = \frac{1 - \rho^2}{1 + \rho^2}, \quad t_\phi \equiv \tan \beta \phi \equiv \frac{\rho}{\eta}.$$  

(30)

In the numerical calculation, we will use $\frac{M_{Z}^2}{M_{Z'}^2} \approx \frac{M_{Z1}^2}{M_{Z2}^2}$.

The parameter $t_\phi$ in Eq. (30) is constrained from the Yukawa couplings of the top quark in the third family, as in the well-known two Higgs doublet models (2HDM), for example see a review in Ref. [50]. Depending on the model A (B, C), where left-handed top quarks are in triplets (anti-triplets), they get tree level mass mainly from the coupling to $\eta$ ($\rho$) [35]. Especially, the top quark mass is $m_t \simeq \Gamma t \times \frac{v_{(\rho)}}{\sqrt{2}}$, where the Yukawa coupling should satisfy the perturbative limit: $|\Gamma t| < \sqrt{4\pi}$, resulting in a lower bound $v_{\rho(\eta)} > \frac{m_t}{\sqrt{2}4\pi}$. As a consequence, $t_\phi$ is constrained as

$$s_\phi = \frac{v_\rho}{\sqrt{v_\rho^2 + v_\eta^2}} = \frac{gv_\rho}{2M_W} > \frac{g}{2M_W} \times \frac{m_t}{\sqrt{2}\pi} \simeq 0.28 \Rightarrow t_\phi > t_0 = \sqrt{\frac{1}{1 - 0.28^2} - 1} \simeq 0.29$$

(31)

TABLE V: Vector and axial-vector coupling constants relevant for APV of the 3 - 3 - 1 $\beta$ model

| Standard Model | The 3 - 3 - 1 model (rep. A) | The 3 - 3 - 1 model (rep. C) |
|----------------|-----------------------------|-----------------------------|
| $g_A(e) = -\frac{1}{2}$ | $g'_A(e) = \frac{1-(1+\sqrt{3}\beta)\sqrt{2}}{2\sqrt{3}v(1+(1+\beta^2)^2)}$ | $g'_A(e) = \frac{1-(1+\sqrt{3}\beta)\sqrt{2}}{2\sqrt{3}v(1+(1+\beta^2)^2)}$ |
| $g_V(u) = \frac{1}{2} - \frac{4s_{3/2}^2}{3}$ | $g'_V(u) = \frac{-3+3\sqrt{3}\beta}{6\sqrt{3}v(1+(1+\beta^2)\sqrt{s_{3/2}^2})}$ | $g'_V(u) = \frac{3-3\sqrt{3}\beta}{6\sqrt{3}v(1+(1+\beta^2)\sqrt{s_{3/2}^2})}$ |
| $g_V(d) = -\frac{1}{2} + \frac{2s_{3/2}^2}{3}$ | $g'_V(d) = \frac{-3+3\sqrt{3}\beta}{6\sqrt{3}v(1+(1+\beta^2)\sqrt{s_{3/2}^2})}$ | $g'_V(d) = \frac{3-3\sqrt{3}\beta}{6\sqrt{3}v(1+(1+\beta^2)\sqrt{s_{3/2}^2})}$ |
for top quark in anti-triplet (models B and C) and
\[ c_v = \frac{v_\eta}{\sqrt{v_\rho^2 + v_\eta^2}} = \frac{g v_\eta}{2M_W} > 0.28 \Rightarrow t_v < \sqrt{\frac{1}{0.28^2}} - 1 \approx 3.43 = t_0^{-1} \] (32)
for top quark in triplet (model A). The constraint of \( t_v \) in 3-3-1 models is similar to the 2HDMs [50]. We will use \( t_v \leq 3.4 \) for model A and \( t_v \geq 0.3 \) for models B, C.

In the numerical investigation, we will look for allowed regions satisfying three constraints of APV data of Cs and proton; and the perturbative limit of Yukawa coupling of the top quark. We will concentrate on the two models A and C. The allowed regions predicted by model B will be addressed based on the APV results of the model A and the condition (31). Numerical results are presented as follows

A. APV in the 3-3-1 model with \( \beta = \pm \sqrt{3} \)

1. The model with exotic leptons

The model we mention here is not the minimal 3-3-1 because the third components of lepton triplets are the exotic ones. The numerical results are illustrated in Fig. 2. We used the numerical values of the \( SU(2)_L \) gauge boson couplings and the Weinberg angle relating with \( Z' \) given in Ref. [41], where the renormalization group evolutions are taken into account. It also gives a consequence that the limit for perturbative calculations requires \( M_{Z_2} \leq 4 \text{ TeV} \). In the models under consideration, the relation between \( g_X \) and \( g \) is determined by Eq. (B1) from which the Landau pole arises at \( s_W^2 = 1/(1 + \beta^2) \). For the \( \beta = \pm \sqrt{3} \), the models lose their perturbative character at the scale around 4 TeV [41, 43, 51–53]. We accept that the models will be rule out if there are not any regions satisfying \( M_{Z_2} \leq 4 \text{ TeV} \).

From Fig. 2 we get the lowest value of \( M_{Z_2} \) given in Table VI. The following remarks

| \( t_v \) | 0 | 0.3 | 1 | 3.4 | 50 |
|----------|---|-----|---|-----|----|
| A        | 5.37 | 5.35 | 5.24 | 5.12 | 5.10 |
| C        | Excl. | Excl. | Excl. | 4.24 | 5.43 |

| \( t_v \) | 0 | 0.3 | 1 | 3.4 | 50 |
|----------|---|-----|---|-----|----|
| A        | 10.84 | 10.38 | 7.66 | 3.05 | 0.14 |
| C        | Excl. | Excl. | Excl. | Excl. | Excl. |

are in order:
FIG. 2: $\Delta Q_{W}^{331}(Cs)$ as a function of the $Z_{2}$ mass with $\beta = \pm \sqrt{3}$, predicted by rep. A (C) in the left (right) panel. The two red dotted lines present two lower and upper experimental bounds of $\Delta Q_{W}(Cs)$. We use $s_{W}^{2}(M_{Z_{2}}) = 0.246$ and $g = 0.636$ [41].

1. For $\beta = -\sqrt{3}$, the model rep. A always predicts the lowest allowed value of $M_{Z_{2}}$ around 5 TeV, where the perturbative property of the model is lost. The same conclusion for the model rep. C for $t_{v} = 50$ or $t_{v} \leq 1$.

2. For $\beta = +\sqrt{3}$, the model rep. C is excluded for all values of $t_{v}$.

3. The value $t_{v} = 3.4$ is survived for two models: rep. C with $\beta = -\sqrt{3}$ ($M_{Z_{2}} \geq 4.24$) and rep. A with $\beta = \sqrt{3}$ ($M_{Z_{2}} \geq 3.05$). Combing with the condition of Yukawa coupling of top quark and APV data of proton, the allowed regions are more strict, see Fig. 3. The $M_{Z_{2}}$ values must satisfy $M_{Z_{2}} \geq 4$ TeV for model rep. A and $M_{Z_{2}} \geq 4.5$ TeV for model rep. C. Hence the lower bounds from combined data are more constrained than those obtained from the single data of APV of Cs.
FIG. 3: Allowed regions in the plane $M_{Z_2} - t_v$ predicted by rep. A (C) with $\beta = \sqrt{3}$ ($\beta = -\sqrt{3}$), where the orange region is excluded by $t_v \leq 3.4$ ($t_v \geq 0.3$). The green and yellow regions are excluded by the APV data of Cs and proton, respectively.

2. The minimal 3-3-1 model

Apart from the case of $\beta = -\sqrt{3}$ mentioned above, another model with $\beta = -\sqrt{3}$ but no new charged lepton, i.e. the third components of the lepton triplets are conjugations of right-handed SM charged leptons, is well known as the minimal 3-3-1 model (M331). The gauge couplings relevant to the APV are given in table VII.

| Standard Model | rep. A | rep. C |
|----------------|--------|--------|
| $g_A(e) = -\frac{1}{2}$ | $g'_A(e) = -\frac{\sqrt{1 - 4s^2_W}}{2\sqrt{3}}$ | $g'_A(e) = -\frac{\sqrt{1 - 4s^2_W}}{2\sqrt{3}}$ |
| $g_V(u) = \frac{1}{2} - \frac{4s^2_W}{3}$ | $g'_V(u) = \frac{-1 + 6s^2_W}{2\sqrt{3}\sqrt{1 - 4s^2_W}}$ | $g'_V(u) = \frac{1 + 4s^2_W}{2\sqrt{3}\sqrt{1 - 4s^2_W}}$ |
| $g_V(d) = -\frac{1}{2} + \frac{2s^2_W}{3}$ | $g'_V(d) = -\frac{1}{2\sqrt{3}\sqrt{1 - 4s^2_W}}$ | $g'_V(d) = \frac{1 - 2s^2_W}{2\sqrt{3}\sqrt{1 - 4s^2_W}}$ |

The numerical results are illustrated in Fig. 4. It can be seen that all curves are out of the allowed range given by experiment in the framework of rep. A. In contrast, there still exist
FIG. 4: $\Delta Q_{M331}^{M_{Z2}}(Cs)$ as a function of the $Z_2$ mass, predicted by the M331 model with the case of rep. A (C) in the left (right) panel. We have used $s_{W}^{2}(M_{Z2}) = 0.246$ and $g = 0.636$ [41] allowed $M_{Z2}$ values in the rep. C. Furthermore, small allowed $M_{Z2}$ corresponds to small $t_v$. Some specific limits are resumed in Table VIII.

| $t_v$ | 0 | 0.3 | 1 | 3.4 | 50 |
|-------|---|-----|---|-----|----|
| A     | excl. | excl. | excl. | excl. | excl. |
| C     | [3.11, 7.47] | [7.66, 18.41] | [10.40, 24.99] | [10.83, 26.04] |

We see that the data on APV of Cesium excludes the M331 model with rep. A, but still allows rep. C with some small $t_v$, for example $m_{Z2} \geq 3.11$ TeV with $t_v = 0.3$. Combining with the conditions of $t_v \geq 0.3$ and APV of proton will gives a more strict lower bound $m_{Z2} \geq 4$ TeV, see Fig. 5. The lower bound of $M_{Z2}$ obtained from the APV of proton is more strict than that of Cs.

**B. APV in the 3-3-1 model with $\beta = \pm \frac{1}{\sqrt{3}}$**

Regarding the couplings of $Z'$ at $M_{Z'} = \mathcal{O}(1)$ TeV, we will use $g(M_{Z2}) = 0.633$, $s_{W}^{2}(M_{Z2}) = 0.249$ for $\beta = 0, \pm \frac{1}{\sqrt{3}}, \pm \frac{2}{\sqrt{3}}$ [41, 52, 53, 54]. The numerical results obtained from APV of Cesium are shown in Fig. 6. Some limits for $\beta = \pm \frac{1}{\sqrt{3}}$ are explicitly presented in Table IX. One gets the following results

1. For both $\beta = \pm \frac{1}{\sqrt{3}}$, the model rep. A survives with all $t_v$. The allowed values of $M_{Z2}$ decrease with increasing $t_v$. 
FIG. 5: Allowed regions in the plane $M_{Z^2} - t_v$ predicted by rep. C of the M331. The orange region is excluded by $t_v \geq 0.3$. The green and yellow regions are excluded by the APV data of Cs and proton, respectively.

| $\beta = -\frac{1}{\sqrt{3}}$ | $t_v$ | 0     | 0.3   | 1     | 3.4   | 50     |
|-------------------------------|------|------|------|------|------|------|
| A                             | [1.11, 2.66] | [1.06, 2.57] | [0.86, 2.07] | [0.58, 1.39] | [0.51, 1.23] |
| C                             | Excl. | Excl. | Excl. | Excl. | [0.35, 0.85] | [0.57, 1.37] |
| $\beta = +\frac{1}{\sqrt{3}}$ | A    | [1.39, 3.34] | [1.33, 3.20] | [1.00, 2.39] | [0.45, 1.08] | [0.23, 0.55] |
| C                             | Excl. | Excl. | Excl. | Excl. | [0.29, 0.70] |

2. The model C survives with only large $t_v$ and small $M_{Z^2} \leq 1.5$ TeV.

**C. APV in the 3-3-1 model with $\beta = 0$**

The 3-3-1 model with $\beta = 0$ has been recently constructed in Ref. [55]. The numerical results are shown in the Fig. [7]. Result is resumed in Table [X]. We see the similarity with the cases $\beta = \pm \frac{1}{\sqrt{3}}$. The difference is that the allowed ranges of $M_{Z^2}$ drift increasingly for $\beta$ changing from $-\frac{1}{\sqrt{3}}$ to $\frac{1}{\sqrt{3}}$.

There are some common properties for model rep. A, that we can see from all the above
FIG. 6: $\Delta Q_{W}^{331}(Cs)$ as a function of the $Z_2$ mass with $\beta = \pm \frac{1}{\sqrt{3}}$, predicted by rep. A (C) in the left (right) panel.

FIG. 7: $\Delta Q_{W}^{331}(Cs)$ as a function of the $Z_2$ mass with $\beta = 0$, predicted by rep. A (C) in the left (right) panel.

plots. Namely, the lower bounds of $M_{Z_2}$ involved with the APV of Cs are always increased corresponding to the decreasing $t_v$. As a result, an illustration of the allowed regions is shown in Fig. 8 for $\beta = 0$. Hence, perturbative condition $t_v \leq 3.4$ excludes regions of small $M_{Z_2}$. In the region with small $t_v \to 0$, the APV data of proton gives more strict lower bounds than the APV data of Cesium, see again Fig. 8. The largest allowed values of $M_{Z_2}$
TABLE X: Allowed range of $M_{Z_2}$ for $\beta = 0$

| $t_v$ | 0   | 0.3 | 1   | 3.4 | 50  |
|-------|-----|-----|-----|-----|-----|
| A     | [1.18, 2.83] | [1.13, 2.72] | [0.87, 2.09] | [0.47, 1.13] | [0.35, 0.84] |
| C     | excluded | excluded | excluded | [0.14, 0.33] | [0.41, 0.98] |

FIG. 8: Allowed regions in the plane $M_{Z_2} - t_v$, predicted by rep. A with $\beta = 0$. The orange, green and yellow regions are excluded by the condition $t_v \leq 3.4$, the APV data of Cs and proton, respectively. The blue region is excluded by the condition $t_v \geq 0.3$.

is around 2.8 TeV. It increases to 4.65 TeV for $\beta = \frac{2}{\sqrt{3}}$.

Regarding to model rep. B, which has the same results of APV, but the allowed regions satisfy $t_v > 0.3$, which can be seen in Fig. 8. The model B excludes regions containing large $M_{Z_2}$.

Illustrations for allowed regions predicted by models rep. C with $\beta = 0, -\frac{1}{\sqrt{3}}$ are shown in Figs. 9. The APV data of proton excludes small $m_{Z_2}$ and $t_v$, which is more strict than the pertubative limit of top quark coupling. The allowed regions predict only small values $M_{Z_2} < 1.5$ TeV. For other $\beta$ satisfying $|\beta| < \sqrt{3}$, the situations are similar to the mentioned illustrations, but the upper bounds of $M_{Z_2}$ may reach larger value of 2.5 TeV.
FIG. 9: Allowed regions in the plane $M_{Z_2} - t_v$, predicted by rep.C. The green and yellow regions are excluded by the APV data of Cs and proton, respectively.

IV. CONCLUSIONS

The effects of the APV in Cesium and proton on the parameter spaces of 3-3-1 models are discussed under the current experimental APV data and the perturbative limit of the Yukawa coupling of the top quark. Within a recently proposed 3-3-1 CKS, we get the lowest value of $M_{Z_2}$ is 1.27 TeV. This limit is slightly lower than that concerned from the LHC searches, $B$ decays or rho parameter data.

We have also performed studies on other versions of the 3-3-1 models with three Higgs triplets. Here are main conclusions:

• $\beta = \pm \sqrt{3}$, the regions with $M_{Z_2} < 4$ TeV are excluded in the frameworks of all models reps. A, C and M331. They are ruled out when the perturbative calculation limit are required, where the Landau pole of the models happens at the scale around 4 TeV. The singe APV data of Cesium rules out only three cases of the model C with $\beta = -\sqrt{3}$, the model rep. A with $\beta = \sqrt{3}$, and the M331 rep. A. Other cases are ruled out based on the APV data of proton and top quark couplings limit.

• For $|\beta| < \sqrt{3}$, for example $\beta = 0, \pm 1/\sqrt{3}$, the allowed regions are affected significantly by the APV data of proton, namely it result in the lower bounds of $M_{Z_2}$ more strict than those obtain from the APV data of Cesium. This point was not mentioned previously.
• For \( \beta = 0, \pm \frac{1}{\sqrt{3}} \), the model rep. C favors the regions with only small \( M_{Z_2} < 1.5 \) TeV.

• For \( \beta = \pm \frac{1}{\sqrt{3}} \), the model reps. A gives larger allowed values of \( M_{Z_2} \). This model will not be ruled out by other constraints from LHC, where \( M_{Z_2} \geq 2.5 \) TeV with assumption that \( Z_2 \) does not decay to heavy fermions \([56, 57]\), or all heavy fermion masses are 1 TeV \([58]\). A reasonable lower bound were acceptable in literature \( M_{Z_2} \geq 1 \) TeV \([41, 59]\).

• The model rep. B also survives, although the perturbative limit of the Yukawa couplings of the top quark gives constraints on the allowed regions with large \( M_{Z_2} \).

From our discussion, we emphasize that the information of APV data of proton and the perturbative limit of top quark Yukawa couplings are as important as that obtained from the APV of Cesium, therefore all of them should be discussed simultaneously to constrain the parameter space of the 3-3-1 models. The numerical calculations have also shown that the allowed regions predicted by the two models reps. B and C disfavor the large \( M_{Z_2} \) hence they may be rule out by future constraints from colliders such as LHC, especially the model rep. C. While the model rep. A may still be survived, resulting in that the heaviest quark family must treat differently from the remaining.

The APV data is consistent with data on masse difference of neutral meson \([60]\) in the sense that the third family should be treated different from the first two. This also gives a reason why the top quark is so heavy.

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Appendix A: Derivation of nuclear weak charge in the models with extra neutral gauge boson

Nowadays, a lot of beyond Standard Models contain extra neutral gauge bosons associated with new diagonal generators such as \( T_8, T_{15} \) or extra generators of the new \( U(1)_N \)
groups. The above mentioned neutral gauge bosons will give contribution to the atomic parity violation. So we will provide a detailed analysis of the APV in the light of extra gauge bosons.

Some authors use the notations with different coefficients and signs associated with axial part ($\gamma_5$). Here we point out the relation among the notations.

1. Notations

For convenience to apply the results into our calculations, we review here more detailed steps to derive analytic formulas of $\Delta Q_W(\frac{1}{2}X)$. However, we firstly consider the case with just one extra neutral gauge boson $Z'$. After some steps of diagonalization in neutral gauge boson sector, we come to two states $Z$ and $Z'$ with Lagrangian \cite{7}. It is emphasized that the $Z$ and $Z'$ are mixed and the physical states are a result of the last step of diagonalization which is discussed latter. In conventional way, the $Z$ and $Z'$ are mixing with an angle $\phi$; and the consequence is a pair of the physical bosons $Z_1$ and $Z_2$. Relations between the notations in Eq. \cite{7} and those mentioned in Ref. \cite{33} are

\begin{align}
\frac{g_V(f)}{g_a(f)} &= \frac{2v_f}{2a_f}, \quad \frac{g'_V(f)}{g'_a(f)} = \frac{-2v'_f}{2a'_f}. \quad (A1)
\end{align}

We will base on the approach to derive the deviation comparing with the results given by G. Altarelli et al. \cite{33}. The equivalence of the neutral gauge boson states between our notation and those in Ref. \cite{33} are \cite{A1} and

\begin{align}
(Z, Z') &\equiv (Z_0, Z'_0), \quad (Z_1, Z_2) \equiv (Z, Z'), \quad \xi_0 \equiv \phi, \quad \tilde{g} = g' = g_{tW}.
\end{align}

The mixing matrix $O$ relating two base of neutral gauge bosons are:

\begin{align}
O &= \begin{pmatrix}
 c_{\xi_0} & -s_{\xi_0} \\
 s_{\xi_0} & c_{\xi_0}
\end{pmatrix} \equiv \begin{pmatrix}
 c_{\phi} & -s_{\phi} \\
 s_{\phi} & c_{\phi}
\end{pmatrix}, \quad (A2)
\end{align}

which give $(Z_1, Z_2)^T = O(Z, Z')^T$. We will use our notations in the following calculations.

Lagrangian containing gauge couplings of neutral gauge bosons in the basis $(Z, Z')$ is

\begin{align}
\mathcal{L}_{Vff}^{BSM} &= J_\mu Z^\mu + J'_\mu Z'_\mu \\
&\equiv \frac{g}{2c_W} \sum_f \overline{f} \gamma^\mu [g_V(f) - \gamma_5 g_A(f)] f Z_\mu + \frac{g}{2c_W} \sum_f \overline{f} \gamma^\mu [g'_V(f) - \gamma_5 g'_A(f)] f Z'_\mu. \quad (A3)
\end{align}
On the other hand, in terms of physical neutral gauge boson mediations $Z_1$ and $Z_2$, this Lagrangian can be written as follows

$$
\mathcal{L}_{Vff}^{\text{BSM}} = \frac{g}{2c_W} \sum_f \bar{f} \gamma^\mu [g_V^{(1)}(f) - \gamma^5 g_A^{(1)}(f)] f Z_{1\mu} + \frac{g}{2c_W} \sum_f \bar{f} \gamma^\mu [g_V^{(2)}(f) - \gamma^5 g_A^{(2)}(f)] f Z_{2\mu},
$$

(A4)

where the couplings $g_V^{(1)}(f)$, $g_V^{(2)}(f)$, $g_A^{(1)}(f)$ and $g_A^{(2)}(f)$ are gauge couplings of the physical states of neutral gauge boson, which will be determined as functions of $g_{V,A}(f)$ and $g'_{V,A}(f)$.

Eq. (A4) gives the following effective Lagrangian for a quark $f = u, d$:

$$
\mathcal{L}^f_{\text{eff}} = \frac{g^2}{4c_W M_Z^2} (\bar{e} \gamma^\mu \gamma^5 e) \left( \bar{f} \gamma^\mu f \right) \left( g_A^{(1)}(e) g_V^{(1)}(f) + g_A^{(2)}(e) g_V^{(2)}(f) \frac{M_Z^2}{M_{\phi}^2} \right)
$$

$$
= + \frac{G_F}{\sqrt{2}} (\bar{e} \gamma^\mu \gamma^5 e) \left( \bar{f} \gamma^\mu f \right) \times 2 \rho \left( g_A^{(1)}(e) g_V^{(1)}(f) + g_A^{(2)}(e) g_V^{(2)}(f) \frac{M_Z^2}{M_{\phi}^2} \right)
$$

$$
\equiv - \frac{G_F}{2\sqrt{2}} (\bar{e} \gamma^\mu \gamma^5 e) \left( \bar{f} \gamma^\mu f \right) \times C_1^{\text{BSM}}(f),
$$

(A5)

where we have denoted

$$
C_1^{\text{BSM}}(f) \equiv -4 \rho \left( g_A^{(1)}(e) g_V^{(1)}(f) + g_A^{(2)}(e) g_V^{(2)}(f) \frac{M_Z^2}{M_{\phi}^2} \right).
$$

(A6)

The parameter $\rho$ is defined in Eq. (20). Then a nuclear atom $\frac{A}{2}X$ with $Z$ protons and $N = A - Z$ neutrons consisting of $(2Z + N)$ quarks $u$ and $Z + 2N$ quark $d$ in the first family has a nuclear weak charge determined as follows [61]

$$
Q_{W}^{\text{BSM}}(\frac{A}{2}X) = [(2Z + N)C_1^{\text{BSM}}(u) + (Z + 2N)C_1^{\text{BSM}}(d)],
$$

(A7)

In the SM, it has only neutral boson $Z \equiv Z_1$ with mass $M_Z \equiv M_{Z_1}$, while $\frac{M_{Z_1}}{M_{Z_2}} = 0$, $g^{(1)}_{V,A}(f) = g_{V,A}(f)$ with $f = e, u, d$. It can be derive that $\rho = 1$ and $C_1^{\text{SM}}(f) \equiv -4 g_A(e) g_V(f)$, resulting to the popular value APV of $^{133}_{55}C$'s used to compare with experiments, namely

$$
Q_{W}^{\text{SM}}(\frac{133}{55}C) = -73.8684.
$$

(A8)

The latest value of $Q_{W}^{\text{SM}}(\frac{133}{55}C)$ including other loop contributions is given in Ref. [30].

From the $Z - Z'$ mixing matrix $O$ given in (A2), the states $Z$ and $Z'$ are written as functions of $Z_{1,2}$. Inserting them into (A3) then identifying the two Lagrangians (A3) and (A4), we obtain:

$$
g_A^{(1)}(f) = c_\phi g_A(f) - s_\phi g_A'(f), \quad g_V^{(1)}(f) = c_\phi g_V(f) - s_\phi g_V'(f),
$$
\[ g^{(2)}_A(f) = s_\phi g_A(f) + c_\phi g'_A(f), \quad g^{(2)}_V(f) = s_\phi g_V(f) + c_\phi g'_V(f). \]  \tag{A9}

Now, \( C^{\text{BSM}}_1(f) \) is determined as follows:

\[
C^{\text{BSM}}_1(f) = -4\rho \left[ (c^2 + s^2 \frac{M_{Z_1}^2}{M_{Z_2}^2}) g_A(e)g_V(f) - [g_A(e)g'_V(f) + g'_A(e)g_V(f)] \left( 1 - \frac{M_{Z_1}^2}{M_{Z_2}^2} \right) s_\phi c_\phi + \left( s^2 + c^2 \frac{M_{Z_1}^2}{M_{Z_2}^2} \right) g'_A(e)g'_V(f) \right]. \tag{A10}
\]

To keep the approximation up to order of \( O\left(\frac{M_{Z_1}^2}{M_{Z_2}^2}\right) \), we take \( c_\phi \approx 1 \), \( s_\phi^2 \approx 0 \) in the first term of expression in (A10) because \( s_\phi \sim O\left(\frac{M_{Z_1}^2}{M_{Z_2}^2}\right) \). Hence, \( g^{(1)}_A(e)g^{(1)}_V(f) \approx g_A(e)g_V(f) - [g_A(e)g'_V(f) + g'_A(e)g_V(f)]s_\phi \). In contrast, the second term of (A6) is simple, \( g^{(2)}_A(e)g^{(2)}_V(f) \approx g'_A(e)g'_V(f) \).

Thus

\[
C^{\text{BSM}}_1(f) = -4\rho \left[ g_A(e)g_V(f) - [g_A(e)g'_V(f) + g'_A(e)g_V(f)] s_\phi + (\frac{M_{Z_1}^2}{M_{Z_2}^2}) g'_A(e)g'_V(f) \right] + O\left(\frac{M_{Z_1}^4}{M_{Z_2}^4}\right). \tag{A11}
\]

Let us now deal with a derivation of nuclear weak charge

\[
\Delta Q^{\text{BSM}}_W(ZX) = Q^{\text{BSM}}_W(XZ) - Q^{\text{SM}}_W(XZ) = -4 \left\{ \left( \frac{N-Z}{4} + Zs_W^2 \right) \rho - \frac{N-Z}{4} + Zs_W^2 \right. \\
- s_\phi ((2Z+N) [g_A(e)g'_V(u) + g'_A(e)g_V(u)] \\
+ (Z + 2N) [g_A(e)g'_V(d) + g'_A(e)g_V(d)]) \\
+ \left( \frac{M_{Z_1}^2}{M_{Z_2}^2} \right) [(2Z+N)g'_A(e)g'_V(u) + (Z + 2N)g'_A(e)g'_V(d)] \\
+ O\left(\frac{M_{Z_1}^4}{M_{Z_2}^4}\right), \tag{A12}
\]

where we have used the SM couplings of the electron, quarks \( u \) and \( d \) given in Table IV and \( \rho s_\phi \approx s_\phi, \rho \approx \frac{M_{Z_1}^2}{M_{Z_2}^2} \approx \left(\frac{M_{Z_1}^2}{M_{Z_2}^2}\right) \).

To continue, we check the shift of \( \delta(s_{W'}^2) \) introduced in Ref. [33]. Using the formula

\[
s_{W'}^2c_{W'}^2 = \frac{\mu^2}{\rho M_Z^2}, \quad \mu \equiv \frac{\pi \alpha}{\sqrt{2} G_F}, \tag{A13}
\]

where \( \mu \) and \( M_Z \) are fixed as experimental inputs. Defining \( x = s_W^2 \), with \( c_W^2 = 1 - x \), as a variable in the following intermediate steps, ones have

\[
(x - x^2)\rho = \text{const} \to 0 = \frac{\delta}{\delta x} [(x - x^2)\rho] = (1 - 2x)\rho + (x - x^2)\frac{\delta \rho}{\delta x}
\]
\[
\delta(s_W^2) = \delta x = -\frac{x - x^2}{(1 - 2x)\rho} \delta \rho \simeq -\frac{s_W^2 c_W^2}{c_{2W}} \Delta \rho.
\] (A14)

Here we have used that fact that \(\rho = 1 + \Delta \rho\) with \(\Delta \rho = \mathcal{O}\left(\frac{M_{Z1}^4}{M_{Z2}^4}\right)\). The result in Eq. (A14) is consistent with Eq. (2.13) of Ref. [33], but slight different from the expression used in Refs. [42, 62, 63].

To compare with the SM, we have to derive the deviation of \(s_W^2\) and \(\rho\) from the ones of the SM, namely \(\rho \to 1 + \Delta \rho\) and \(s_W^2 \to s_W^2 + \delta(s_W^2)\), where \(\delta(s_W^2)\) is given in (A14).

Applying the above procedure, we have

\[
\Delta Q_{BSM}^{\text{W}}(A_Z X) = (Z - N)(1 + \Delta \rho) - 4Z[s_W^2(1 + \Delta \rho) - \frac{s_W^2 c_W^2}{c_{2W}} \Delta \rho] - Z - N - 4Z s_W^2
\]

\[+ 4s_\phi \{(2Z + N) [g_A(e)g'_V(u) + g'_A(e)g_V(u)]
\]

\[+ (Z + 2N) [g_A(e)g'_V(d) + g'_A(e)g_V(d)]\}

\[-4 \left(\frac{M_{Z1}^2}{M_{Z2}^2}\right) [(2Z + N)g'_A(e)g_V(u) + (Z + 2N)g'_A(e)g'_V(d)] + \mathcal{O}\left(\frac{M_{Z1}^4}{M_{Z2}^4}\right).\] (A15)

Substituting \(N = A - Z\) into (A15), we obtain the expression (6) for \(\Delta Q_{BSM}^{\text{W}}(A_Z X)\). If the scale dependence of gauge couplings are taken into account, replacements need to be done in Eq. (7), namely \(g \to g(M_{Z1,2})\) for couplings of \(Z_{1,2}\), respectively. In addition, the factor in front of Eq. (A5) is always \(g^2(M_{Z1})\), corresponding to the \(M_{Z1}\) scale. Hence, the \(Z'\) couplings in (A15) should be replaced with \(g'_{A,V}(f) \to g'_{A,V}(f) \times \frac{g(M_{Z1})}{g(M_{Z1})}\), resulting in Eq (9).

To conclude this section, we note that the above procedure can be easily extended for the models with two or more extra gauge bosons, for instance the models based on the gauge group \(\text{SU}(3)_C \times \text{SU}(4)_L \times \text{U}(1)_X\) [64, 65]. In the framework of the 3-4-1 model, the APV has been considered in Ref. [66].

Appendix B: General discussions on recent 3-3-1 models

The APV can be considered in a more general class of 331 models with arbitrary parameter \(\beta\) defined the electric charge of the model in Eq. (1). We consider here the popular class of 3-3-1 models with three Higgs triplets, namely the 3-3-1 \(\beta\), where general analytic ingredient for determining APV such as the \(Z - Z'\) mixing \(s_\phi\) and heavy neutral gauge boson are well-known [35, 42]. Furthermore, the formula of APV for these models was mentioned [35, 40], but it needs to be improved, at least because of the mixing angle and the scale
dependence of the gauge couplings concerned in Ref. [42]. In addition, many new models with \( \beta \neq \pm \frac{1}{\sqrt{3}}, \pm \sqrt{3} \) such as \( \beta = 0, \pm \frac{2}{\sqrt{3}} \) discussed recently should be paid attention to (Ref. [42, 49, 55]). The APV relating with these models will be discussed in the following.

Three Higgs triplets are defined the same as those given in Table 3 of Ref. [35], except that the VEVs of neutral components are denoted as those in Ref. [42] for consistence with the definition of \( t_v \) appearing in Eq. (30). The standard definitions of covariant derivatives were given in Ref. [49], which are consistent with Eq. (18) and

\[
t \equiv \frac{g_X}{g} = \frac{\sqrt{6} s_W}{\sqrt{1 - (1 + \beta^2) s^2_W}}.
\]

The masses of the SM gauge bosons including \( W^\pm_\mu = \frac{w^\pm_\mu w^\mp_\mu}{\sqrt{2}} \) and \( Z_\mu \) are

\[
M^2_W = \frac{g^2 (v_\rho^2 + v_\eta^2)}{4}, \quad M^2_Z = \frac{M^2_{W}}{c^2_W}.
\]

After the breaking \( SU(3)_L \otimes U(1)_X \to U(1)_Q \), the model consists of three neutral gauge bosons including one massless photon, a SM boson \( Z_\mu \) and a new heavy \( Z'_\mu \) (Ref. [35])

\[
\begin{align*}
A_\mu &= s_W W^3_\mu + c_W \left( \beta t_W W^8_\mu + \sqrt{1 - \beta^2 t^2_W} B_\mu \right), \\
Z_\mu &= c_W W^3_\mu - s_W \left( \beta t_W W^8_\mu + \sqrt{1 - \beta^2 t^2_W} B_\mu \right), \\
Z'_\mu &= \sqrt{1 - \beta^2 t^2_W} W^8_\mu - \beta t_W B_\mu,
\end{align*}
\]

where the state \( Z'_\mu \) has an opposite sign with the choice in Ref. [35, 40, 42] in order to be consistent with the particular case of the 3-3-1 CKS model we mentioned above. In the limit \( v_\chi \ll v_\rho, v_\eta \), the \( Z - Z' \) mixing angle in Eq. (19) can be found as given in Eq. (29). We emphasize that this formula was introduced firstly in Ref. [42], which corrects the one in Ref. [35].

We note that our choice of the mixing matrix is

\[
C_{ZZ'} \equiv \begin{pmatrix} c_\phi & -s_\phi \\ s_\phi & c_\phi \end{pmatrix},
\]

which define the relation between two base of neutral gauge boson states: \((Z_1, Z_2)^T = C_{ZZ'} (Z, Z')^T\). The mixing angle \( \phi \) in this definition is different from that in Refs. [35, 40, 42] by a minus sign. Combining with the state \( Z' \) defined in this work, the formula (29)
determining \( \phi \) was found to be consistent with Ref. [42]. Based on this, the needed couplings can be calculated, as given in Table V, where our notations coincide with those Ref. [35]. We can see that the mixing angle \( \phi \) and couplings are consistent with the particular case of \( \beta = 0 \) and \( v_\rho = 0 \) we discussed above.

Now comparing with the result in table 4 of Ref. [35], we found an global opposite sign of \( Z' \) couplings, which can be removed by choosing the \( Z' \) state to have the same sign defined in Ref. [35]. But a minus sign will also appear in the right-handed side of Eq. (29). In conclusion, both signs of \( s_\phi \) and \( Z' \) couplings will be changed if the phase of the state \( Z' \) is changed, leading to the fact that the Eq. (6) is independent with the phase of \( Z' \).

Now we will pay attention to the \( \frac{133}{55} Cs \), where \( (A - 2.39782 \times Z) \Delta \rho \simeq 1.12 \Delta \rho = O(10^{-4}) \ll |\Delta Q(Cs)| \) following recent experimental results. Hence, in the framework of the 3-3-1 \( \beta \) model, the expression for APV of \( Cs \) is written as Eq. (25), based on Eq. (6), where the term depending on the \( \rho \) parameter can be ignored. For \( s_\phi \) given in Eq. (29), the respective \( Z' \) couplings are listed in Table V.

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