Circular dichroism in nonlinear electron-positron pair creation

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Abstract. The trident pair production by X-ray photons in the presence of a powerful laser field is considered in the framework of strong-field quantum electrodynamics. The dependence of differential probability rates of the process on the handedness of circular polarizations of X-ray and laser photons is illustrated. It is revealed that circular dichroism effects are most pronounced for short pulse durations of the driving fields.

1. Introduction
The possibility of converting energy into mass is one of the most fundamental issues of contemporary physics, as it provides a test of nonlinear quantum electrodynamics (QED). It led Sauter and Schwinger [1, 2] to their prediction of electron-positron \((e^-e^+)\) pair creation from vacuum in the presence of a static electric field. The following investigation of \(e^-e^+\) pair production by means of laser radiation started in the 1960s [3, 4, 5] (for recent reviews, see, Refs. [6, 7]). While the problem was of purely theoretical interest at that time, currently there is a clear perspective for corresponding experimental studies due to a remarkable and still ongoing progress in high-power laser technology. The reason is that field intensities well beyond \(10^{20}\ W/cm^2\) are available today in many laboratories (see, e.g. [8, 9]). Note that in the near future laser intensities exceeding \(10^{22}\ W/cm^2\) are expected to become also available (see, for instance, ELI and XCELS proposals [10, 11]).

Current technologies allow, in principle, to study experimentally \(e^-e^+\) pair creation in powerful laser fields. The reason is that relevant field intensities are Doppler upshifted when a highly relativistic particle beam counterpropagates an intense laser pulse (in the rest frame of target particles the laser field intensity is enhanced by \(4\gamma^2\) where \(\gamma \gg 1\) is the Lorentz factor). This has led to the first, and until recently, only experimental observation of laser-induced \(e^-e^+\) pair production [12]. In the experiment performed at the Stanford Linear Accelerator Center (SLAC) [12], the detected pairs were attributed to a nonlinear Breit-Wheeler process in which a high-energy photon generated by Compton backscattering collided with multiple laser photons. Another scenario of laser-induced pair creation exploiting the relativistic particle beams is the nonlinear Bethe-Heitler process, where the pairs are created directly in collisions of a laser and relativistic particle beams. Even though the latter process appears as a promising tool for efficient generation of electron-positron pairs, it is still awaiting experimental demonstration.

In our previous papers on nonlinear Bethe-Heitler process, we anticipated that in order to treat this process in strong laser fields it is important to account for a finite mass of the colliding
target [13, 14, 15, 16, 17]. This would be particularly important when we consider the pair creation by an impact of a strong laser field on a beam of electrons – even lighter particles than nuclei. Let us note that in the famous SLAC experiment [12], electrons carrying energy of almost 50 GeV were colliding with an intense laser pulse. For such energetic electrons, the laser field strength that they experience is enhanced by a factor of $10^5$, which would make it possible to perform the respective Bethe-Heitler–type experiment with presently available laser sources. We find this crucial to consider the situation similar to [13, 14, 15, 16, 17] but with an electron beam instead, i.e., the so-called trident pair production [18, 19, 20]. In contrast to aforementioned studies, we shall model the driving field as a two-color pulse train. Each color field will consist of a sequence of pulses (see, our previous papers on strong-field QED processes [21, 22, 23] as well): a high-frequency field will mimic X-rays while a low-frequency field will describe the laser beam, both being circularly polarized. This will allow us to study circular dichroism effects in the trident pair creation.

Regarding convention, in the theoretical formulas we keep $\hbar = 1$. However, we present our numerical results in relativistic units such that $\hbar = c = m_e = 1$, where $m_e$ is the electron mass.

2. Theory

We describe the external electromagnetic field by the following four-vector potential,

$$A^\mu(k \cdot x) = A^\mu_L(k \cdot x) + A^\mu_X(k \cdot x),$$

which represents the laser field, $A^\mu_L(k \cdot x)$, and the X-ray field, $A^\mu_X(k \cdot x)$, both corresponding to pulse trains. More precisely,

$$A^\mu_i(k \cdot x) = A_{0i} f_i(k \cdot x) \left[ \varepsilon^\mu_1 \sin(N_{i,\text{osc}} k \cdot x) \cos \delta_i - \varepsilon^\mu_2 \cos(N_{i,\text{osc}} k \cdot x) \sin \delta_i \right]$$

for $i \in \{L, X\}$. $A_{0i}$ defines the field strength, $N_{i,\text{osc}}$ is the number of cycles within one pulse, while $f_i(k \cdot x)$ defines the wave envelope. In the following, we shall assume that

$$f_L(k \cdot x) = \sin^2 \left( \frac{k \cdot x}{2} \right),$$
$$f_X(k \cdot x) = \sin^2 \left( \frac{N_{X,\text{osc}} k \cdot x}{2N_{X,\text{osc}}} \right).$$

$\varepsilon_{1,2} = (0, \varepsilon_{1,2})$, present in Eq. (2), are linearly polarized four-vectors satisfying the conditions: $\varepsilon^\mu_1 = -\varepsilon^\mu_2 = -\varepsilon_2 = -1$ and $\varepsilon_1 \cdot \varepsilon_2 = -\varepsilon_1 \cdot \varepsilon_2 = 0$. In addition, $\delta_i$ describes the ellipticity of the $i$-th field. For our further purpose, we note that for $\delta_i = -\pi/4$ the electromagnetic field (2) becomes left-handed circularly polarized while for $\delta_i = \pi/4$ it becomes right-handed circularly polarized. If $\omega$ is the fundamental field frequency and $\mathbf{n}$ is the field propagation direction, the wave four-vector becomes $k = (\omega/c)(1, \mathbf{n})$.

Note that for the vector potential defined by Eqs. (1) and (2) it holds that $k \cdot A_i = 0$ and $k \cdot \mathbf{n} = 0$. Based on these properties, one can derive exact solutions of the Dirac equation coupled to the electromagnetic field; the so-called Volkov states [24]. These laser-dressed states labeled by four-momentum $p$ and spin projection on a quantization axis $\lambda$ read

$$\psi^{(\beta)}_{p\lambda}(x) = \sqrt{\frac{m_e c^2}{VE_p}} \left( 1 - \frac{\beta e}{2k \cdot p} \mathcal{A}^\mu \right) u^{(\beta)}_{p\lambda} e^{-i\beta \mathcal{S}^{(\beta)}_{p\lambda}(k \cdot x)},$$

after being normalized in volume $V$. Here, the electron charge is $e < 0$ whereas $\beta = +1$ stands for an electron and $\beta = -1$ for a positron. In Eq. (5), $u^{(\beta)}_{p\lambda}$ is a four-spinor satisfying the field-free
Figure 1. (Color online) The Feynman diagrams for the trident pair production. The fermion lines (double solid lines) represent the Volkov states for electrons and positrons in the background laser field.

equations, \((p - \beta mc)v^{(\beta)}_{pA} = 0\), while the phase

\[
S^{(\beta)}_p = p \cdot x + \int_{k-x} \phi \left[ \beta e \frac{A(\phi) \cdot p}{k \cdot p} - \frac{(eA)^2(\phi)}{2k \cdot p} \right]
\]  

has a meaning of the classical action of a particle (antiparticle) in the electromagnetic field.

The probability amplitude of \(e^-e^+\) pair production by the impact of a powerful laser field on a relativistic electron, to the leading order in fine structure constant \(\alpha\), has the form

\[
S_R = \mathcal{A}(q_i, q_f, p_{e+}, p_{e-}) - \mathcal{A}(q_i, p_{e-}, p_{e+}, q_f),
\]  

where the first term relates to the direct diagram whereas the second term relates to the exchange diagram, as shown in Fig. 1. Both terms can be written in a compact form as

\[
\mathcal{A}(q_{in}, q_{out}, p_{in}, p_{out}) = -4\pi i\alpha \int d^4x \int d^4y j_{q_{out}, q_{in}}^{(++)\mu}(x)D_{\mu\nu}(x-y)j_{p_{out}, p_{in}}^{(+-)\nu}(y),
\]

where

\[
D_{\mu\nu}(x-y) = \int \frac{d^4K}{(2\pi)^4} \overline{D}_{\mu\nu}(K) e^{-iK \cdot (x-y)},
\]

\[
\overline{D}_{\mu\nu}(K) = -\frac{1}{K^2} \left( g_{\mu\nu} - \frac{K_{\mu}K_{\nu}}{K^2} \right),
\]

is the photon propagator, with the gauge fixing constant \(\xi_G\). Eq. (8) contains also four-currents \(j_{q_{out}, q_{in}}^{(++)\mu}(x)\) and \(j_{p_{out}, p_{in}}^{(+-)\nu}(y)\) which are expressed in terms of Volkov solutions (5),

\[
j_{q_{out}, q_{in}}^{(++)\mu}(x) = \bar{\psi}_{q_{out}, q_{in}}^{(+)}(x) \gamma^\mu \psi_{q_{out}, q_{in}}^{(+)}(x),
\]  

\[
j_{p_{out}, p_{in}}^{(+-)\nu}(y) = \bar{\psi}_{p_{out}, p_{in}}^{(-)}(y) \gamma^\nu \psi_{p_{out}, p_{in}}^{(-)}(y),
\]
Next, we introduce the Fourier expansion of both four-currents,

\[
j^{(\pm)}_{\mu}(x) = \frac{m_e c^2}{\sqrt{E_{q_{\text{out}}} E_{q_{\text{in}}}}} \sum_{M=-\infty}^{\infty} C^\mu_M(q_{\text{out}}, q_{\text{in}}) e^{-i(Mk+\vec{q}_{\text{in}}-\vec{q}_{\text{out}})x},
\]

\[
j^{(\pm)}_{\mu}(y) = \frac{m_e c^2}{\sqrt{E_{p_{\text{out}}} E_{p_{\text{in}}}}} \sum_{L=-\infty}^{\infty} F^\mu_L(p_{\text{out}}, p_{\text{in}}) e^{-i(Lk-\vec{p}_{\text{in}}-\vec{p}_{\text{out}})y},
\]

where \(C^\mu_M(q_{\text{out}}, q_{\text{in}})\) and \(F^\mu_L(p_{\text{out}}, p_{\text{in}})\) are the corresponding Fourier coefficients and where the four-momenta of particles dressed by the laser field are defined, in general, as

\[
\vec{p} = p - \frac{e^2((A_L + A_X)^2)}{2k \cdot p} \vec{k}.
\]

Here, \((...)\) means the average over \(k \cdot x\) from 0 to 2\(\pi\). Plugging these expansions and the definition of the photon propagator into Eq. (7), we obtain

\[
\mathcal{A}(q_{\text{in}}, q_{\text{out}}, p_{\text{in}}, p_{\text{out}}) = \frac{-4\pi i}{\sqrt{E_{q_{\text{out}}} E_{q_{\text{in}}} E_{p_{\text{out}}} E_{p_{\text{in}}}}} \left(\frac{m_e c^2}{2\pi}\right)^2 \int d^4K \delta(Mk + K + \vec{q}_{\text{in}} - \vec{q}_{\text{out}}) \delta(Lk - K - \vec{p}_{\text{in}} - \vec{p}_{\text{out}}) C^\mu_M(q_{\text{out}}, q_{\text{in}}) \tilde{D}_\mu(K) F^\mu_L(p_{\text{out}}, p_{\text{in}}),
\]

where the remaining integral can be performed exactly. This leads to the following expansion,

\[
\mathcal{A}(q_{\text{in}}, q_{\text{out}}, p_{\text{in}}, p_{\text{out}}) = \sum_{N=-\infty}^{\infty} \delta(\vec{q}_{\text{in}} - \vec{q}_{\text{out}} - \vec{p}_{\text{in}} - \vec{p}_{\text{out}} + Nk) A_N(q_{\text{in}}, q_{\text{out}}, p_{\text{in}}, p_{\text{out}}),
\]

where \(M + L = N\) can be interpreted as a net number of photons exchanged with the laser field by target and product particles. The delta function in Eq. (17) expresses the four-momentum conservation condition, while \(A_N(q_{\text{in}}, q_{\text{out}}, p_{\text{in}}, p_{\text{out}})\) is the probability amplitude of the trident process by \(N\)-photon absorption:

\[
A_N(q_{\text{in}}, q_{\text{out}}, p_{\text{in}}, p_{\text{out}}) = \frac{-4\pi i}{\sqrt{E_{q_{\text{out}}} E_{q_{\text{in}}} E_{p_{\text{out}}} E_{p_{\text{in}}}}} \left(\frac{m_e c^2}{2\pi}\right)^2 \int d (2\pi)^4 t_N(q_{\text{in}}, q_{\text{out}}, p_{\text{in}}, p_{\text{out}}),
\]

where \(t_N(q_{\text{in}}, q_{\text{out}}, p_{\text{in}}, p_{\text{out}}) = \sum_{L} C^\mu_{N-L}(q_{\text{out}}, q_{\text{in}}) \tilde{D}_\mu(Lk - \vec{p}_{\text{in}} - \vec{p}_{\text{out}}) F^\mu_L(p_{\text{out}}, p_{\text{in}}).

Let us define the total probability rate of trident pair production by a two-color pulse train:

\[
W_N = \frac{1}{2} \sum_{\lambda_i, \lambda_e, \lambda_{-\sigma}} \int V \frac{d^3q_{\text{in}}}{(2\pi)^3} \frac{d^3p_{\text{in}+}}{(2\pi)^3} \frac{d^3p_{\text{in}-}}{(2\pi)^3} \frac{d^3q_{\text{out}}}{(2\pi)^3} \frac{d^3p_{\text{out}+}}{(2\pi)^3} \frac{d^3p_{\text{out}-}}{(2\pi)^3} \delta(\vec{q}_{\text{in}} - \vec{q}_{\text{out}} - \vec{p}_{\text{in}} - \vec{p}_{\text{out}} + Nk)
\times (2\pi)^4 V |A_N(q_{i}, q_{f}, p_{e+}, p_{e-}) - A_N(q_{i}, p_{e-}, p_{e+}, q_{f})|^2,
\]

where the coefficient in front of the integral stands for an average over the initial and summation over the final spin degrees of freedom. We also define the probability rates related to the direct and exchange diagrams:

\[
W_N^{\text{(direct)}} = \frac{1}{2} \sum_{\lambda_i, \lambda_e, \lambda_{-\sigma}} \int V \frac{d^3q_{\text{in}}}{(2\pi)^3} \frac{d^3p_{\text{in}+}}{(2\pi)^3} \frac{d^3p_{\text{in}-}}{(2\pi)^3} \frac{d^3q_{\text{out}}}{(2\pi)^3} \frac{d^3p_{\text{out}+}}{(2\pi)^3} \delta(\vec{q}_{\text{in}} - \vec{q}_{\text{out}} - \vec{p}_{\text{in}} - \vec{p}_{\text{out}} + Nk)
\times (2\pi)^4 V |A_N(q_{i}, q_{f}, p_{e+}, p_{e-})|^2
\]
Following Ref. [13], one can partially perform integrals in Eqs. (19), (20), and (21) by using the corresponding four-momentum conservation condition. As a result, one can represent $W_N$ in Eq. (19) as

$$W_N = \sum_\ell \int dE_{q_i} d^2\Omega_{q_i} d^2\Omega_{p_{e^+}} d^2\Omega_{p_{e^-}} \frac{d^5 W_N^{(\ell)}}{dE_{q_i} d^2\Omega_{q_i} d^2\Omega_{p_{e^+}} d^2\Omega_{p_{e^-}}},$$

(22)

which implicitly defines the $N$-photon differential probability rate of the trident process. Here, $\ell$ labels solutions of the four-momentum conservation condition: $\vec{q}_i - \vec{q}_f - \vec{p}_{e^+} - \vec{p}_{e^-} + N\vec{k} = 0$ where, in the most general case, $\ell = 1, 2, 3, 4$. Similarly one can define the $N$-photon differential probability rates for the direct and exchange contributions to the process, which is based on
Figure 3. (Color online) The same as in Fig. 2 but both driving fields consist of much shorter pulses such that $N_{L,\text{osc}} = 2$ and $N_{X,\text{osc}} = 20$.

Eqs. (20) and (21). These quantities are plotted below to illustrate a possibility of observing circular dichroism in the trident process.

3. Numerical illustration

We present the results calculated in the reference frame in which the initial electron is at rest, meaning that $\mathbf{q}_i = \mathbf{0}$. In the considered setup, we also assume that the electromagnetic field propagates against the $z$-axis, such that $\mathbf{k} = -\frac{2}{c} e_z$, while the final electron is detected with the momentum $\mathbf{q}_f = -m_e c e_z$. The parameters describing the laser field are $\omega_L = N_{L,\text{osc}} \omega = m_e c^2$ and $\mu_L = \frac{|eA_{0L}|}{m_e c} = 0.2$, whereas for the X-ray field: $\omega_X = N_{X,\text{osc}} \omega = 10 \omega_L$ and $\mu_X = \frac{|eA_{0X}|}{m_e c} = 0.01$. The results presented below correspond to the case when the energy of $30 \omega_L$ is absorbed from the electromagnetic field. Since the $e^- e^+$ pairs are mainly generated in the direction of the external field propagation (cf., in Refs. [13, 14]), we assume that the polar angle of the produced electron is $\theta_{e^-} = 0.975 \pi$. In Fig. 2, we plot the dependence of differential probability rates on the azimuthal angle of the produced electron $\varphi_{e^-}$ in the case when a single modulation of the laser field contains 16 cycles ($N_{L,\text{osc}} = 16$) while a single modulation of the X-ray field contains 160 cycles ($N_{X,\text{osc}} = 160$). Each panel in Fig. 2 is for a different polarization setup. The upper left panel, labeled as “LL”, is for the case when both fields are left-handed circularly polarized. The upper right panel, labeled as “LR”, is for the case when the laser field is left-handed- while the X-ray field is right-handed circularly polarized. The same labeling scheme relates to the lower panels. Moreover, in each panel we present the results for the direct process (dashed red line), for the exchange process (dotted-dashed green line), and for the complete process with both contributions included (solid blue line). One can conclude from Fig. 2 that for the given parameters the exchange diagram contributes very little to the trident pair creation.
Nevertheless, the probability amplitude of the exchange process interfere constructively with the probability amplitude of the direct process, which holds for any polarization of the driving fields. It follows from Fig. 2 that the distributions for “LL” and “RR” polarizations, as well as for “LR” and “RL” polarizations, are the same. In other words, the probability rates do not change under a simultaneous change of handedness of both polarized fields. Let us compare the distributions corresponding to only one different handedness of either of the polarized fields, for instance, for “LL” and “LR” polarizations. One can see a qualitative difference between these distributions, despite a similar magnitude. It happens that more pronounced circular dichroism effects are observed if both external fields are composed of much shorter pulses. This is illustrated in Fig. 3 for $N_{\text{L,osc}} = 2$ and $N_{\text{X,osc}} = 20$ with the remaining parameters the same as in Fig. 2. In this case, the rates with different handedness of the polarized X-ray field are similar in shape but they differ in magnitude substantially. What is more striking, however, is that the rates seem to depend significantly on the polarization of the laser field (compare, for instance, the results for “LL” and “RL” polarizations). This is the main reason for differences between the rates for “LL” and “RR”, as well as “LR” and “RL”, polarizations.

4. Conclusions
We have presented preliminary results on the trident pair production in the presence of an X-ray field and a laser field, both modeled as circularly polarized pulse trains. We have demonstrated a sensitivity of the differential probability rates of pair creation to the handedness of the polarized driving fields. As we have shown, such circular dichroism is stronger for shorter pulses consisting the trains. A more complete analysis of this effect will be presented elsewhere.

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