Coalescence Phenomenon in the Three Photon Quantum Interferences

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Abstract. We present the theoretical study of three-photon interference in the six ports Mach-Zehnder Interferometer (6p-MZI). The 6p-MZI consist of two lossless symmetrically balanced tritters and three independently controllable phase modulators inserted in each photon line path between the tritters. Three photons (Fock state) are injected onto the first tritter input ports and measured in second tritter output ports. By independently varying the phases states of the three photons, one can control the detection probability of photon numbers at final tritter output ports. The calculation results demonstrated that, at the specific combination of phase states, the three photons interference exhibited a coalescence phenomenon, which is the three photons detected at the same output port. Interestingly, the probability of detecting photons in all combination modes of |0, 0, 3⟩, |0, 3, 0⟩, and |3, 0, 0⟩ are monotonously in the equal values.

1. Introduction

Multi-photon quantum interference has been one of the most attractive research fields to be explored both in fundamental quantum optics and in practical implementations of quantum technologies. In 1987 [1], Chung Ki Hong, et al. was reported experimental results on two indistinguishable photons interference on a beam splitter. This effect is known as Hong-Ou-Mandel (HOM) interference.

Zukowski, et al [2] have conducted research related to multiport beam splitters for particles with more than two dimensions. This article also discusses the tritter which is formed from the beam splitter arrangement. Furthermore, the research have been done by Reck [3] used 2 tritter and 3 phase shifts with one input field. Spagnolo et al [4] conducted research using a tritter made with a directional coupler.

In this paper, we theoretically studied three-photon interference in the six ports Mach-Zehnder Interferometry (6p-MZI). In the calculation, the 6p-MZI was implemented by modeling that a couple of three ports lossless beam splitters (tritters) and three phase-modulators were expressed in the form of unitary operators. To independently control the phase differences among the arms, phase modulators were positioned in each photon line-path inside the 6d-MZI. Our theoretical results were explicitly shown that the probability of detecting coalescence three-photon at any combination modes of |0, 0, 3⟩, |0, 3, 0⟩, and |3, 0, 0⟩ modes were monotonously in equal values at all phase states.
2. Theory and Calculation

2.1. Fock state

Let us introduce Fock state, $|n\rangle$, as the eigenstates of the photon-number operator $\hat{n}$,

$$\hat{n}|n\rangle = n|n\rangle,$$

where $\hat{n}$ is photon number operator. Equation (1) clearly implies that Fock states have a perfectly fixed photon number. Since $\hat{a}|n\rangle$ is an eigenstate of $\hat{n}$ with eigenvalue of $n - 1$, and $\hat{a}^\dagger|n\rangle$ is an eigenstate of $\hat{n}$ with eigenvalue of $n + 1$, hence the fundamental relations of lowering and rising photon states can be obtained as,

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle,$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle,$$

where $\hat{a}$ and $\hat{a}^\dagger$ are annihilation operator and creation operator, respectively, to lowers and riser the photon number in the integer steps. Operators $\hat{a}$ and $\hat{a}^\dagger$ do not commute. From Equation (3) with an initial vacuum state, the exited state derived as,

$$|n\rangle = (\hat{a}^\dagger)^n \sqrt{n!/}|0\rangle.$$  

Finally, for the whole Hilbert space, Fock state must be completed by

$$\sum_{n=0}^{\infty} |n\rangle\langle n| = 1$$

$$\langle n|n'\rangle = \delta_{nn'},$$

where $\delta_{nn'}$ is Kronecker delta function, defined as $\delta_{nn'} = 1$, if $n = n'$, ($= 0$, otherwise).

2.2. Symmetric Balanced Tritter

Tritter is a beamsplitter that consists of three input ports and three output ports. Symmetry balanced tritter is an ideal tritter characterized by its ability to split photon entering in anyone of input port into anyone of output port in the equal probability.

Based on its structure, there are two types of tritter, a planar tritter [2] and a triangle tritter [5, 6]. Planar tritter is an array of conventional beam splitters and phase shifters in such that they function as a three-mode splitter (see Ref.[2]). On the other hand, the triangle tritter consists of three parallel directional couplers. To enable in distributing each input photon to the three outputs with equal probability, the mutually interfere in the three-arm directional coupler by evanescent field interaction must happen in particular propagation length. In this calculation, we employed an ideal symmetric balanced triangle tritter with the unitary matrix, $\hat{U}_T$, given by [4],

$$\hat{U}_T = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & e^{\frac{2\pi}{3}} & e^{\frac{4\pi}{3}} \\ e^{\frac{2\pi}{3}} & 1 & e^{\frac{2\pi}{3}} \\ e^{\frac{4\pi}{3}} & e^{\frac{2\pi}{3}} & 1 \end{pmatrix}.$$  

2.3. Phase modulator

The phase modulator used to control the relative phase difference among photons entering ports of the tritter. For this purpose, independently controllable phase modulator can be written in
the form of unitary operator $\hat{U}_P$ as,

$$
\hat{U}_P = \begin{pmatrix}
    e^{-i\phi_1} & 0 & 0 \\
    0 & e^{-i\phi_2} & 0 \\
    0 & 0 & e^{-i\phi_3}
\end{pmatrix},
\tag{8}
$$

where $\phi_1, \phi_2,$ and $\phi_3$ are the phase shifts introduced in path-1, path-2, and path-3 of tritter, respectively.

2.4. Six-ports Mach-Zehnder interferometer

Figure 1 illustrate the model of the six-port Mach-Zehnder interferometer (6p-MZI) used in the calculation. Since first and second tritters are identical, the 6p-MZI unitary operator, $\hat{U}$, immediately can be written as

$$
\hat{U} = \hat{U}_T \hat{U}_P \hat{U}_T,
\tag{9}
$$

where $\hat{U}_T$ and $\hat{U}_P$ are unitary operators of lossless symmetric balanced tritter (Equation (7)) and phase modulator (Equation (8)).

Figure 1. Six ports Mach-Zehnder interferometer model used in the calculation. Phase modulator $U_P$ inserted in between two tritters $U_{T,1}$ and $U_{T,2}$, where $U_{T,1} = U_{T,2} = U_T$. Fock state $|\psi_{\text{in}}\rangle = |1, 1, 1\rangle$ were pumped into the input ports of tritter $U_{T,1}$. The output photon $|\psi_{\text{out}}\rangle$ was measured at output port of tritter $U_{T,2}$. Probability photon-number at the output tritter $U_{T,2}$ was strongly depend on the relative phase-difference in the $U_P$.

In order to determine the the probability of detecting photon after interference process in the 6p-MZI, first, the $|\psi_{\text{out}}\rangle$ must be obtained. Using equation (9) with input state $|\psi_{\text{in}}\rangle$, the final output state $|\psi_{\text{out}}\rangle$ can be calculated from

$$
|\psi_{\text{out}}\rangle = \hat{U} |\psi_{\text{in}}\rangle,
\tag{10}
$$

where the input photon state $|\psi_{\text{in}}\rangle = |1\rangle_{k1}|1\rangle_{k2}|1\rangle_{k3}$ is Fock states, with $k_i$ represent photon mode at input port $i$. In the Heisenberg picture, the output operators $b_i^\dagger$ and the input operators $a_i^\dagger$ are connected by a 6p-MZI unitary transformation (see Ref.[7]), Equation (9), in the form

$$
b_i^\dagger = \sum_{j} \hat{U}_{ij} a_j^\dagger, \quad \text{with } i = 1, 2, 3.
\tag{11}
$$

By introducing Equation (11), the output state $|\psi_{\text{out}}\rangle$ of an input of Fock state $|\psi_{\text{in}}\rangle$ in the 6p-MZI model (Fig. 1) can be written as,

$$
|\psi_{\text{out}}\rangle = \hat{U} |1\rangle_{k1}|1\rangle_{k2}|1\rangle_{k3}
= \hat{U} a_1^\dagger a_2^\dagger a_3^\dagger |0\rangle_{k1}|0\rangle_{k2}|0\rangle_{k3},
\tag{12}
$$

where $\hat{U}$ is unitary operator of the six-ports Mach-Zehnder interferometer, Equation (9).
The probability to detect any modes of photon in the output 6p-MZI can be found by solving

\[ P_{(r,s,t)}(\phi_1, \phi_2, \phi_3) = |\langle r,s,t | \psi^{\text{out}} \rangle|^2, \tag{13} \]

where \( \langle r,s,t \rangle \) is photon modes, with \( r, s, \) and \( t \) are photon number at port 1, 2, and 3, respectively. Hence, \( r, s, \) and \( t \) can be 0, 1, 2, or 3. In the present work, we focused on the analyzing probability to detect photon in the coalescence modes, therefore in the following sections, we will only presented the calculation results for the \( P_{(0,0,3)}, P_{(0,3,0)}, \) and \( P_{(3,3,0)} \).

3. Results and Discussion

The probability to observe photon in the modes \( |0,0,3\rangle, |0,3,0\rangle, \) and \( |3,0,0\rangle \) were calculated using equations (13) and (12). Interestingly, we realized that the calculation results for all three coalescence photons modes \( P_{(0,0,3)}, P_{(0,3,0)}, \) and \( P_{(3,3,0)} \) show an identical probability result,

\[ P_{(3,0,0)}(\phi_1, \phi_2, \phi_3) = |\langle \{3,0,0\} | \psi^{\text{out}} \rangle|^2 \]

\[ = \frac{72}{729} + \frac{12}{729} \left( \cos(3(\phi_1 - \phi_2)) + \cos(3(\phi_1 - \phi_3)) + \cos(3(\phi_2 - \phi_3)) \right) \]

\[ - \frac{4}{81} \left( \cos\left(\frac{4\pi}{3} + \phi_2 + \phi_3 - 2\phi_1\right) + \cos\left(\frac{4\pi}{3} + \phi_1 + \phi_3 - 2\phi_2\right) \right) \]

\[ + \cos\left(\frac{4\pi}{3} + \phi_2 + \phi_1 - 2\phi_3\right) \]. \tag{14} \]

Here, \( P_{(3,0,0)}(\phi_1, \phi_2, \phi_3) \) is to represent all \( P_{(0,0,3)}, P_{(0,3,0)}, \) and \( P_{(3,3,0)} \), whereas \( |\{3,0,0\}\rangle \) is representing all mode \( |0,0,3\rangle, |0,3,0\rangle, \) and \( |3,0,0\rangle \).

Figure 2. Typical probabilities of detecting photon from six-ports Mach-Zehnner interferometer as function of phase of path 1 \( \phi_1 \), for three differences phase combination \( \phi_2 \) an \( \phi_3 \). (a) for \( \phi_2 = \phi_3 = 0 \), (b) for \( \phi_2 = \frac{2\pi}{3} \) and \( \phi_3 = 0 \), and (c) for \( \phi_2 = \frac{\pi}{3} \) and \( \phi_3 = 0 \).

Figure 2 shows the typical theoretical results based on Equation (14), where the probability for detecting photon at output 6p-MZI, \( P_{(3,0,0)}(\phi_1, \phi_2, \phi_3) \) were plot as a function of phase \( \phi_1 \). For all figures, phases \( \phi_3 \) set to be 0, while \( \phi_2 = 0, \frac{2\pi}{3}, \) and \( \frac{\pi}{3} \) for Figure 2(a), (b) and (c), respectively.

From Figure 2, we observed that the maximum probability of mode \( |\{3,0,0\}\rangle \) was only \( \frac{2}{3} \). It indicates that, in the three-photon interference, the coalescence mode was less significant compared to other possible modes. Interestingly, however, the possibility to detect three-photon coalescence from each three output ports were always in the same probability. For instant, let us look at Figure 2(a), at \( \phi_1 = \frac{2\pi}{3} \) the probability is equal 0. It means that at phase configuration
\( \phi_1 = \frac{2}{3}\pi, \phi_2 = 0, \) and \( \phi_3 = 0, \) all outputs have not detected any coalescence three photons. This situation is also valid for any other phase configurations.

It is clearly stated in the Equation (14) and shown in Figure 2, the probability of detecting coalescence photons for all modes are identical in all time. These results reveal the significant impact of the quality of the devices on the interference process in the 6p-MZI. In this calculation, the phase modulator and tritter were assumed to be ideally lossless and symmetrical, which enable us to distribute each input photon to the three outputs interferometer in the equal probability. Therefore, as expected, the monotonously coalescence photons for modes \( |0, 0, 3\rangle, |0, 3, 0\rangle, \) and \( |3, 0, 0\rangle \) are detected at all phase configurations.

4. Conclusion
We have theoretically studied the three-photon interferences in the six ports Mach-Zehnder interferometer using three photons Fock state. Our calculation results confirmed that the possibility of detecting three-photons coalescence in all modes was always in equal probability, with the maximum value of \( \frac{2}{9} \). Our results also suggested that the monotonous coalescence three photons in the six ports MZI is only possible to be realized by using perfect devices, such as lossless and ideally symmetrical tritter.

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