Role of recursive cubic spline interpolation method and convolution filter in image processing

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Abstract. During this article, we suggested a RCSIM through Convolution filter. This filter explores the images to enhance and improve the efficiency of both gray scale and color images. Initially we identified the noises in the images and simultaneously, the corresponding pixels are noticed to improve that particular part of that image using edge detection algorithm. The pixel values of the image is converted as the window size of the matrix. This matrix should be a \((n \times n)\) square matrix and then we applied the convolution filtering techniques to the \((n \times n)\) matrix. The development of the (input) image through the convolution filter yields the feature map. The feature map accentuates the uniqueness of the original image. At the same time, the RCSIM is used to interrupt the mathematical data to the feature map. The proposed approach reduces the sound and protects the efficiency of the unique image.

1. Introduction

In the numerical field of Numerical investigation, spline introduction is a type of addition [1-3]. Spline instrument utilizes a strategy that assessments esteem utilizing a mathematical function that limits overall surface that goes precisely through the info focuses Mathematically portray such a bend with piecewise cubic polynomial capacity whose first and second subsidiaries are continuous, over the different bend areas.

A cubic spline is a spline built of piecewise third request polynomials which go through a bunch of control focuses. The second subordinate of every polynomial is usually set to zero toward the end focuses, since this gives a limit condition that finishes the arrangement of conditions [4-7].

Rest of the article is structured as develops, we implemented RCSIM in section 2. The section 3, we broadened technique throughout various streaming methods like edge detection, convolution leak and mathematical model have been demonstrated. Lastly, accomplishes the article.

2. Cubic spline interpolation method

Characterizing a capacity that takes on determining qualities at indicated focuses [8-12]. It tends to be utilized to create rough a confounded capacity of the stretch. Accept information focuses introduced, exact in subsequent estimate. Quantity of focuses is associated with smooth bends in cubic spline addition [13-15]. This spline comprises loads joined to a level surface at the focuses to be associated. The focuses are mathematical information.

\[
p_{i}(x) = \begin{cases} 
\eta_{1}(x) & \text{if } x_{1} \leq x \leq x_{2} \\
\eta(x) & \text{if } x_{2} \leq x \leq x_{3} \\
\vdots & \\
\eta_{n-1}(x) & \text{if } x_{n-1} \leq x \leq x_{n} 
\end{cases}
\]  

(1)
\[ \eta_i(x) = a_i x^2 + b_i x + c_i + d_i \]  

(2)

Where \( a_i, b_i, c_i \) and \( d_i \) are obscure constants. The 'n' number of realizing information focuses on having 'n-1' stretches, which is appeared in Figure 1. Cubic splines utilizing third request polynomials are extremely well known by and by on the grounds that they guarantee that the first and second subordinates are persistent.

\[ \varphi_i(x) = \frac{\pi''(x_{i-1})}{6(x_i-x_{i-1})} (x_i-x)^2 + \frac{\pi''(x_i)}{6(x_i-x_{i+1})} (x-x_i)^2 + \left( \frac{\pi''(x_{i-1}) - \pi''(x_i)}{6(x_{i-1}-x_i)} \right) (x_i-x) + \left( \frac{\pi(x_i) - \pi(x_{i-1})}{6(x_i-x_{i-1})} \right) (x-x_{i-1}) \]  

(3)

Condition (3) contains just two questions \( \eta''(x_i) \) and \( \eta''(x_{i-1}) \), the second subordinates toward the end purposes of the span \( (x_i - x_{i-1}) \) which are resolved structure condition (2)

\[ (x_i - x_{i-1}) \eta''(x_i) + 2(x_{i+1} - x_i) \eta''(x_{i+1}) = \frac{6}{(x_{i+1} - x_i)} [\eta(x_{i+1}) - \eta(x_i)] + \frac{6}{(x_i - x_{i-1})} [\eta(x_i) - \eta(x_{i-1})] \]  

(4)

The condition (4) composed for \( n-1 \) inside focuses brings about \( n-1 \) synchronous conditions for second subordinates. Settling this concurrent condition encourages one to discover the questions \( \eta''(x_i) \) and \( \eta''(x_{i-1}) \) the primary focal points of spline interjection are its strength and count effortlessness.

2.1 Recursive cubic spline interpolation method (RCSIM)

Spline insertion is a helpful procedure to introduce between realized information focuses because of its steady and smooth attributes. These two qualities are utilized to eliminate the clamor. The salt and pepper commotion can take the dim levels the arbitrary esteemed drive, clamor can take any an incentive somewhere in the range of 0 and 255. This paper examines the expulsion of salt and pepper commotion in pictures utilizing a recursive spline interjection channel. The recursive spline addition channel eliminates the salt and pepper commotion and holds the perfection of the first image.

2.2 Problem formulation

In the event that the preparing pixel is other than '0' or '255', at that point the pixel is without clamor. In any case the handling pixel is uproarious, which is prepared. The means of the RCSIM are clarified as follows. First we take the window size as n xn matrix and the processing pixel is considered as \( Q_{ij} \). If \( Q_{ij} \) is 0 or 255 then \( Q_{ij} \) is noisy pixel otherwise it is noise free pixel and its value is left unvaried. If the
selected pixel does not having all the elements as o’s and 255’s , then remove all 0’s and 255’s and apply the convolution technique then we arrive a feature map.

\[ y = \begin{cases} \phi(x) & \text{if } 0 < x < 255 \\ \phi(x) & \text{otherwise} \end{cases} \]  

(5)

3. Filtering techniques
This is a local task wherein each out pixel is a weighted amount of neighboring information pixels. The framework of weight is known as the convolution part, otherwise called the channel. Convolution is a significant activity in picture and sign handling. Convolution works on two signs (in 1D) or two pictures (2D). We can think of one as the information picture (or signal) and the other (called the bit) as a “channel” on the information picture. Creating a yield picture (so convolution accepts two pictures as info and produces as yield) Convolution is an the incredibly significant ideas in numerous territories of science and designing.

3.1 Numerical illustration: (1)

![Figure 2. Image](image)

|   | 2 | 8 | 5 | 3 |
|---|---|---|---|---|
| 4 | 7 | 2 | 1 |
| 0 | 4 | 2 | 1 |
| 1 | 0 | 2 | 7 |

In the event that the preparing pixel in the chose window size is (4x4) square grid. At that point, we will produce an element map through the convolution channel activity of this picture.

![Figure 3. Four by four pixel image](image)
We utilize the difficulty leaks demonstrated at this point.

\[ \begin{array}{cccc}
2 & 8 & 5 & 3 \\
4 & 7 & 2 & 1 \\
0 & 4 & 2 & 1 \\
0 & 2 & 7 & \\
\end{array} \]

**Figure 4. The convolution operation starts at the upper –left corner.**

\[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

Activity amount results to components that are situated in similar situations as the two matrices. The aftereffect of 9 in figure 4 is determined as \((2 \times 1) + (8 \times 0) + (4 \times 0) + (7 \times 1) = 9\)

\[ \begin{array}{cccc}
2 & 8 & 5 & 3 \\
4 & 7 & 2 & 1 \\
0 & 4 & 2 & 1 \\
1 & 0 & 2 & 7 \\
\end{array} \]

**Figure 5. The second convolution operation**

\[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]

\[ \begin{array}{cccc}
2 & 8 & 5 & 3 \\
4 & 7 & 2 & 1 \\
0 & 4 & 2 & 1 \\
1 & 0 & 2 & 7 \\
\hline
9 & 10 & \\
\end{array} \]

**Figure 6. The third convolution operation**
Figure 7. The convolution operation starts over from the left
It rehashes a similar cycle until the element guide of the given channel is produce, as appeared in figure 8

In a similar way, preparing second convolution channel delivers the element map share in figure 9.

To find \( \eta''(5) \eta''(6) \), we get,
From equation (4) we get,
\( \eta''(5) + 4 \eta''(6) = -6 \)
\( \eta''(4) + 4 \eta''(5) + \eta''(6) = 66 \)
\( \eta''(3) + 4 \eta''(4) + \eta''(5) = -50 \)

\[ \varphi_1(x) = \frac{\eta''(x_i-1)}{6(x_i-x_{i-1})} (x_i-x)^2 + \frac{\eta''(x_i)}{6(x_i-x_{i-1})} (x-x_i)^2 \]
\[ + \left[ \frac{\eta''(x_i-1)}{(x_i-x_{i-1})} - \frac{\eta''(x_i)}{6} \right] (x_i-x) \]
\[ + \left[ \frac{\eta'(x_i)}{(x_i-x_{i-1})} - \frac{\eta''(x_i)}{6} \right] (x-x_i) \]

Interpolate at 5.5, here \( x_i = 6 \) \( x_{i+1} = 7 \) \( x_{i-1} = 5 \)

\( \varphi(5.5) = 4 + \eta''(5)(-0.0625) + \eta''(6)(-0.0625) \) (6)

\(|2| 0 | 2 | 7 \)
\(|9| 10 | 6 |
\(|8|

\( \otimes \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \)

\( \otimes \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \)

\[ = \]

\(|2| 8 | 5 | 3 \)
\(|4| 7 | 2 | 1 |
\(|0| 4 | 2 | 1 |
\(|1| 0 | 2 | 7 |

\(|2| 8 | 5 | 3 \)
\(|4| 7 | 2 | 1 |
\(|0| 4 | 2 | 1 |
\(|1| 0 | 2 | 7 |

\(|2| 8 | 5 | 3 \)
\(|4| 7 | 2 | 1 |
\(|0| 4 | 2 | 1 |
\(|1| 0 | 2 | 7 |

\(|10| 12 | 5 \)
\(|7| 6 | 3 |
\(|5| 2 | 3 |

\(|10| 12 | 5 \)
\(|7| 6 | 3 |
\(|5| 2 | 3 |
By elimination method, we get

\[ \eta^{(1)} = -4019.16, \eta^{(2)} = 1085.698, \eta^{(3)} = -287.32, \eta^{(4)} = 58.33, \eta^{(5)} = 2.312, \eta^{(6)} = -2.078 \]

And from (3.1) we have,

\[ \phi(5.5) = 4 \times 0.0625 (2.312) - 0.0625 (-2.078) \]

ie, \( \phi(5.5) = 3.9854 \)

3.2 Numerical illustration: (2)

On the off chance that the preparing pixel in the chose window size is (4x4) square network. Then, we will create an element map through the convolution channel activity of this picture.

|   |   |   |   |
|---|---|---|---|
| 7 | 4 | 3 | 2 |
| 5 | 9 | 3 | 11 |
| 45| 3 | 0 | 2 |
| 4 | 0 | 1 | 7 |

Figure 10. Image

Figure 11. Four by four-pixel image
Figure 12. The convolution operation starts at the upper–left corner.

![Image of convolution operation starting at the upper-left corner]

Figure 13. The second convolution operation

In a similar way, the third convolution activity is led as appeared in figure 3.4

![Image of convolution operation]

Figure 14. The third convolution operation

When the top line is done, the following lines begins once again from the left (see figure 15)

![Image of convolution operation]

Figure 15. The convolution operation starts over from the left

It rehashes a similar cycle until the element guide of the given channel is produce, as appeared in the figure 16.

![Image of convolution operation]

Figure 16. The feature map of the given filter has been completed

In a similar way, preparing second convolution channel creates the element map share in figure 17.

![Image of convolution operation]
Figure 17. The values depend on whether image matrix matches the convolution filter

\[ \varphi(3.5) = 30 + \eta(3) (-0.0625) + \eta(4) (-0.0625) \]  

To find \( \eta(3), \eta(4), \) we get, from equation (4)

\[ \eta(3) + 4 \eta(4) + \eta(5) = -6 \]  

ie \( \eta(5) = 0 \)

hence, \( \eta(3) + 4 \eta(4) = 318 \)  

\[ \eta(2) + 4 \eta(3) + \eta(4) = -582 \]  

\[ \eta(1) + 4 \eta(2) + \eta(3) = 336 \]  

\[ \eta(0) + 4 \eta(1) + \eta(2) = -60 \]  

\[ 4 \eta(1) + \eta(2) = -60 \]  

Resolving the instantaneous equations (14) to (17) and we obtain the \( \eta(3) \) and \( \eta(4) \) as

\[
\begin{pmatrix}
0 & 0 & \eta'(1) \\
1 & 1 & \eta'(2) \\
0 & 1 & \eta'(3) \\
1 & 4 & \eta'(4)
\end{pmatrix}
\begin{pmatrix}
318 \\
-582 \\
336 \\
-60
\end{pmatrix}
\]

By elimination method, we get \( \eta(1) = -52.854, \eta(2) = 133.7, \ eta(3) = -216.77, \eta(4) = 151.407 \)

And from (3.8) we have, 

\[ \varphi(3.5) = 4 - 0.0625 (-216.77) - 0.0625 (151.407) \]

ie, \( \varphi(3.5) = 34.085 \)

4. Conclusion
To improve the efficiency of the image, Recursive cubic spline interpolation method through Convolution filter is applied in both gray scale and color images. After identified the noise in the image, the consequent pixels are converted in matrix form. These matrices are imported by an edge detection algorithm along with convolution filter to overcome the noises of the images.

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