A Microscopic Three-Cluster Model with Nuclear Polarization applied to the Resonances of $^7$Be and the Reaction $^6$Li$(p,^3$He)$^4$He.

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Abstract

A microscopic model for three-cluster configurations in light nuclei is presented. It uses an expansion in terms of Faddeev components for which the dynamical equations are derived. The model is designed to investigate binary channel processes in a compound system. Gaussian and oscillator bases are used to expand the wave function and to represent appropriate boundary conditions. We study the effect of cluster polarization on ground and resonance states of $^7$Be, and on the astrophysical $S$-factor of the reaction $^6$Li$(p,^3$He)$^4$He.

Key words: cluster model, polarization, resonance, astrophysical $S$-factor, Faddeev amplitude
PACS: 21.60.Gx, 24.10.-i, 26., 25.40.Ep

1. Introduction

Many of the light nuclei are weakly bound. Such nuclei can change their size and shape considerably when interacting with other nuclei. We refer to this phenomenon as cluster polarization. One expects cluster polarization to play a role in reactions which involve light nuclei with small separation energy such as the deuteron, $^6$Li, $^7$Li, and so on. One also expects the effects to be more pronounced at small energies of the colliding nuclei, due to the longer interaction time intervals.

Two different methods have been used to date to take into account polarization of interacting clusters. The first, introduced by Tang et al. [1,2,3,4,5,6], considers internal monopole excitations to describe the polarization. The second, introduced by the
Kiev-Antwerp collaboration [7,8,9,10,11,12,13,14,15], is based on collective monopole and quadrupole polarizations of the compound nucleus.

In this paper we introduce a new approach in which we expand the three-cluster many-particle wave function into Faddeev components. This approach allows us to describe the proper boundary conditions for both binary and three-cluster channels. We also introduce two different expansion schemes: a Gaussian basis to describe bound two-cluster subsystems, and an oscillator basis to describe the relative motion of the third cluster with respect to the two-cluster subsystem. The Gaussian basis reproduces the intricate and complicated two-cluster bound-state behavior with a limited number of terms, and is thus suited to describe cluster polarization. The oscillator basis on the other hand allows for the proper representation of the scattering boundary condition in the matrix form of the Schrödinger equation. We derive a set of equations for the Faddeev components within the Coupled Channels Formalism.

We apply this approach to cluster polarization in \(^7\)Be. This system exhibits a well determined set of bound and resonance states, and has been thoroughly studied by many microscopical methods [16,17,18,19,20,21,22,23,24,25]. Moreover, two reactions which are connected to this nucleus, \(^3\)He\((\alpha,\gamma)^7\)Be and \(^6\)Li\((p,^4\)He\)\(^4\)He, are important in astrophysical models [20,27]. The former reaction has been extensively investigated by different microscopic and semi-microscopic methods and is involved in the solar neutrino problem [23,24,21,28,29,30]. The latter reaction is connected to the big-bang nucleosynthesis and determines the abundance of light elements in the universe. It has received much less attention in the literature. It was investigated within a three-cluster microscopic model in an astrophysically relevant energy range [16], and also in a multi-configuration resonating group model [18] for a wide energy range.

We model the \(^7\)Be nucleus using a many-channel cluster wave function containing both two-cluster and three-cluster components. As we wish to consider both the two-cluster components \(^4\)He\(^3\)He and \(^6\)Li\(^p\), we use the \(^4\)He\(^d\(^p\) three-cluster configuration.

In the reaction \(^6\)Li\((p,^3\)He\)\(^4\)He one structureless subsystem, the proton, and three cluster subsystems \(^6\)Li, \(^4\)He and \(^3\)He are involved. They are connected with the lowest binary channels \(^4\)He\(^3\)He and \(^6\)Li\(^p\) which define the main properties of the bound and some resonance states of \(^7\)Be.

Only 1.5 MeV is necessary to split the \(^6\)Li nucleus in \(^4\)He and \(^d\). To disintegrate the \(^3\)He nucleus in a deuteron and a proton the total energy of \(^3\)He has to exceed 5.5 MeV. To split the \(^4\)He nucleus into a proton and \(^3\)H already more than 20 MeV is needed. This leads one to expect that cluster polarization of the \(^6\)Li and \(^3\)He nuclei will be important and should be taken into account for the low-energy states of \(^7\)Be, and that the polarization of \(^4\)He can be neglected.

The new method presented in this paper achieves two goals: (1) it allows to study the polarizability of weakly bound two-cluster systems induced by an incident cluster, and (2) it provides a description of the resonance structure of the resulting three-cluster system.
2. The three-cluster model

2.1. Model space and Hamiltonian

We introduce our approach for the general case of \( s \)-shell clusters, but it can in principle be extended to cover clusters with an arbitrary number of nucleons. It is analogous to the one formulated in [31] and [32]. However, in those contributions a bi-oscillator basis was used to study the three-cluster interaction, while we will include both Gaussian and oscillator basis states.

A Gaussian basis is a multi-parameter variational basis that can reproduce complicated and intricate inter-cluster wave functions with few terms, thus achieving high numerical precision with low computational complexity. It has been considered on different occasions in microscopic calculations, and found to be very efficient for bound states, even for loosely bound nuclei with proton and neutron excess [33, 34, 35, 36, 37, 38]. Its drawback is the non-orthogonality of the basis functions that can lead to numerical instabilities. We will adopt this basis to represent the (weakly) bound two-cluster subsystems in the three-cluster description.

The oscillator basis is suitable for the description of bound as well as scattering bound-ary conditions [39, 40, 41, 42, 43], and the corresponding matrix form of the Schrödinger equation [44, 45] is similar to the \( R \)-matrix theory for nuclear reactions. Due to the orthogonality of the basis functions it does not suffer from numerical instabilities but it converges more slowly than the Gaussian basis. It was shown that an acceptable precision for light \( p \)-shell nuclei can be achieved with 30 to 50 oscillator functions [10, 13, 46]. In some model situations this number can be further reduced, even down to 3 or 5 functions, as was shown in [47]. The calculation of matrix elements of different operators between oscillator functions can be done with the technique of the Generalized Coherent States [48, 10, 49], which leads to recurrence relations for the matrix elements. We will consider this basis to describe the scattering component in the three-cluster system.

The wave function for \( s \)-shell clusters can be written as

\[
\Psi = \mathcal{A} \left\{ (\Phi_1(A_1) \Phi_2(A_2) \Phi_3(A_3))^S \left[ f_1^L(x_1, y_1) + f_2^L(x_2, y_2) + f_3^L(x_3, y_3) \right] \right\}^J \tag{1}
\]

where \( \Phi_\alpha (A_\alpha) \) is a shell-model wave function for the internal motion of cluster \( \alpha \) (\( \alpha = 1, 2, 3 \)) and \( f_\alpha^L(x_\alpha, y_\alpha) \) is a Faddeev component. The first factor describes the internal cluster motion and has total orbital angular momentum \( L = 0 \) because of the \( s \)-shell clusters, and only its total spin quantum number is indicated. The second factor represents the relative inter-cluster motion, and is responsible for the total orbital angular momentum \( L \). We consider an \( LS \) coupling scheme so that \( L \) and \( S \) couple to the total angular momentum \( J \).

It is well known that Faddeev components are very suitable for implementing the necessary boundary conditions for binary as well as for three-cluster channels [50].

In the Faddeev component \( f_\alpha^L(x_\alpha, y_\alpha) \), \( x_\alpha \) is the Jacobi vector proportional to the distance between the \( \beta \) and \( \gamma \) clusters (\( \alpha, \beta \) and \( \gamma \) form a cyclic permutation of 1, 2 and 3), while \( y_\alpha \) is the Jacobi vector connecting the \( \alpha \) cluster to the center of mass of the \( \beta \) and \( \gamma \) clusters.
\[ x_\alpha = \sqrt{\frac{A_\beta A_\gamma}{A_\beta + A_\gamma}} \left( \frac{1}{A_\beta} \sum_{j \in A_\beta} r_j - \frac{1}{A_\gamma} \sum_{k \in A_\gamma} r_k \right) \quad (2) \]

\[ y_\alpha = \frac{A_\alpha (A_\beta + A_\gamma)}{A_\alpha + A_\beta + A_\gamma} \left( \frac{1}{A_\alpha} \sum_{i \in A_\alpha} r_i - \frac{1}{A_\beta + A_\gamma} \left[ \sum_{j \in A_\beta} r_j + \sum_{k \in A_\gamma} r_k \right] \right) \quad (3) \]

For each Faddeev component we use bi-spherical harmonics

\[ f^L_\alpha (x_\alpha, y_\alpha) = \sum_{\lambda_\alpha, l_\alpha} f^{(\lambda_\alpha, l_\alpha; L)}_\alpha (x_\alpha, y_\alpha) \{ Y_{\lambda_\alpha} (\vec{x}_\alpha) Y_{l_\alpha} (\vec{y}_\alpha) \}_{LM} \quad (4) \]

which lead to the four quantum numbers \( \lambda_\alpha, l_\alpha, LM \). The parity of the three-cluster states is then determined by the partial angular momenta: \( \pi = (-)^{\lambda_\alpha + l_\alpha} \).

The radial part of the Faddeev components \( f^{(\lambda_\alpha, l_\alpha; L)}_\alpha \) is obtained by using products of Gaussian basis functions \( \{ G_{\lambda_\alpha} (x_\alpha, b_{\nu_\alpha}) \} \) and oscillator basis functions \( \{ \Phi_{n_\alpha l_\alpha} (y_\alpha, b) \} \), where

\[ \Phi_{n_\ell} (y, b) = (-1)^n \frac{1}{b^{3/2}} N_{n_\ell} \rho^L L_n^{l+1/2} \left( \rho^2 \right) \exp \left( -\rho^2 / 2 \right) \ Y_{l_\ell} (\hat{y}) \]

\[ = \Phi_{n_\ell} (y, b) Y_{l_\ell} (\hat{y}) \quad \left( \rho = \frac{y}{b}, \ N_{n_\ell} = \sqrt{\frac{2 \Gamma (n + 1)}{\Gamma (n + l + 3/2)}} \right) \]

represents an oscillator function and

\[ G_{\lambda} (x, b_{\nu}) = \frac{1}{b^{3/2}} \sqrt{\frac{2}{\Gamma (\lambda + 3/2)}} \rho^\lambda \exp \left\{ -\frac{1}{2} \rho^2 \right\} \ Y_{\lambda} (\hat{x}) \]

\[ = G_{\lambda} (x, b_{\nu}) Y_{\lambda} (\hat{x}) \quad \left( \rho = \frac{x}{b_{\nu}} \right) \]

stands for a Gaussian function. One then immediately obtains the radial part

\[ f^{(\lambda_\alpha, l_\alpha; L)}_\alpha (x_\alpha, y_\alpha) = \sum_{n_\alpha, l_\alpha} C^{(\alpha)}_{n_\alpha l_\alpha} G_{\lambda_\alpha} (x_\alpha, b_{\nu_\alpha}) \Phi_{n_\alpha l_\alpha} (y_\alpha, b) \quad (7) \]

Introducing the Gaussian-Oscillator bi-spherical expansion in the total wave function \( \Psi^J \) in (1) leads to the form

\[ \Psi^J = \sum_{\alpha} \sum_{\lambda_\alpha, l_\alpha} \sum_{n_\alpha, l_\alpha} C^{(\alpha)}_{n_\alpha l_\alpha} \left\{ [\Phi_1 (A_1) \Phi_2 (A_2) \Phi_3 (A_3)]_S \left[ G_{\lambda_\alpha} (x_\alpha, b_{\nu_\alpha}) \Phi_{n_\alpha l_\alpha} (y_\alpha, b) \right]_L \right\}^J \]

\[ = \sum_{\alpha} \sum_{\lambda_\alpha, l_\alpha} \sum_{n_\alpha, l_\alpha} C^{(\alpha)}_{n_\alpha l_\alpha} \left\{ [\Phi_1 (A_1) \Phi_2 (A_2) \Phi_3 (A_3)]_S \left[ G_{\lambda_\alpha} (x_\alpha, b_{\nu_\alpha}) \Phi_{n_\alpha l_\alpha} (y_\alpha, b) \right]_L \right\} \]

\[ \times \left\{ Y_{\lambda_\alpha} (\vec{x}_\alpha) Y_{l_\alpha} (\vec{y}_\alpha) \right\}_L \quad (8) \]

and will subsequently be referred to using the acronym GOB.
The microscopic hamiltonian for a three-cluster configuration can be written as

\[ \hat{H} = \hat{T} + \hat{V} = \sum_{\alpha=1}^{3} \hat{H}^{(1)}_{\alpha} + \hat{T}_r + \sum_{\alpha} \hat{V}_\alpha \] (9)

i.e. a sum of three single-cluster hamiltonians \( \hat{H}^{(1)}_{\alpha} \) describing the internal structure of each cluster, and a term responsible for the inter-cluster dynamics. The latter consists of the kinetic energy operator for relative motion of clusters \( \hat{T}_r \) and the potential energy of the interaction between clusters. This hamiltonian can be also expressed as a sum of one two-cluster hamiltonian, and terms representing the interaction of the third cluster with the two-cluster subsystem:

\[ \hat{H} = \hat{H}^{(2)}_{\alpha} + \hat{H}^{(1)}_{\alpha} + \hat{T}_r + \sum_{\beta \neq \alpha} \hat{V}_\beta. \] (10)

The terms appearing in (9) and (10) are easily expanded in terms of the particle operators:

\[ \hat{T}_r = \frac{\hbar^2}{2m} \Delta_x + \frac{\hbar^2}{2m} \Delta_y, \] (11)

\[ \hat{T}_\alpha = \frac{\hbar^2}{2m} \Delta_y, \] (12)

\[ \hat{H}^{(1)}_{\alpha} = \sum_{i \in A_\alpha} \hat{T}(i) + \sum_{i < j \in A_\alpha} \hat{V}(ij), \] (13)

\[ \hat{H}^{(2)}_{\alpha} = \sum_{i \in A_\alpha} \hat{T}(i) + \sum_{i < j \in A_\alpha \pm A_\gamma} \hat{V}(ij), \] (14)

\[ \hat{V}_\alpha = \sum_{i \in A_\alpha} \sum_{j \in A_\gamma} \hat{V}(ij). \] (15)

The wave function of a two-cluster subsystem

\[ \psi^{J_{\alpha} \lambda}_{J} = \mathcal{A}\left\{ [\Phi_{\beta} (A_\beta) \Phi_{\gamma} (A_\gamma)]^{S_\alpha} \phi_{\lambda_\alpha} (x_{\alpha}) Y_{\lambda_\alpha} (\tilde{x}_{\alpha}) \right\}^{J_\alpha}. \] (16)

is expanded in Gaussian cluster functions

\[ \psi^{J_{\alpha} \lambda}_{J} = \sum_{\nu} D^{(\alpha)}_{\lambda_\alpha, \nu} \chi_{\nu, \alpha}^{J_{\alpha} \lambda_{\alpha}}, \] (17)

where

\[ \chi_{\nu, \alpha}^{J_{\alpha} \lambda_{\alpha}} = \mathcal{A}\left\{ [\Phi_{\beta} (A_\beta) \Phi_{\gamma} (A_\gamma)]^{S_\alpha} G_{\lambda_\alpha} (x_{\alpha}, b_\nu) Y_{\lambda_\alpha} (\tilde{x}_{\alpha}) \right\}^{J_\alpha}. \] (18)

The bound states \( E^{(\alpha)}_\sigma \) of this subsystem (\( \sigma = 0 \) is the ground state, \( \sigma > 0 \) are excited or pseudo-bound states), and their corresponding eigenstates \( \phi^{(\alpha, \sigma)}_{\lambda_\alpha} \) are defined by \( \{D^{(\alpha, \sigma)}_{\lambda_\alpha, \nu}\} \) and can be obtained by solving the corresponding generalized eigenvalue problem

\[ \sum_{\nu=1}^{N_\alpha} \langle \nu, \alpha | \hat{H}^{(2)}_{\alpha} - E^{(\alpha)}_\sigma | \nu, \alpha \rangle D^{(\alpha, \sigma)}_{\lambda_\alpha, \nu} = 0 \] (19)
The number of terms in (18), and correspondingly the number of eigenstates, is chosen for sufficient convergence of the ground state, and depends on the two-cluster subsystem labeled by $\alpha$. The corresponding wave functions for the two-cluster relative motion are

$$\sum_{\nu_{\alpha}=1}^{N_{G}^{(G)}} D_{\lambda_{\alpha}, \nu_{\alpha}}^{(\sigma, \alpha)} G_{\lambda_{\alpha}}(x_{\alpha}, b_{\nu_{\alpha}}) = \phi_{\lambda_{\alpha}}^{(\alpha, \sigma)}(x_{\alpha})$$

(20)

Because of the Pauli principle between nucleons, it is hard to unambiguously derive a set of Faddeev type equations for the Faddeev three-cluster amplitudes $f_{\alpha}(x_{\alpha}, y_{\alpha})$ in a fully microscopic three-cluster description. An attempt to achieve this has recently been proposed in [51]. In the current paper we solve the Schrödinger equation through the traditional coupled channels formalism [52, 53] to obtain the $f_{\alpha}(x_{\alpha}, y_{\alpha})$ amplitudes.

2.2. Boundary conditions

In this paper we focus on the energy range between the ground state of $^7\text{Be}$ and the three-cluster threshold for $^4\text{He} + d + p$ disintegration. We therefore only have to consider binary scattering and reaction channels, and can neglect three-cluster decay. Thus only two-cluster asymptotics need to be included in the boundary conditions. In this case $x_{\alpha} \ll y_{\alpha}$, i.e. one cluster is at a large distance of the other two clusters, and the latter will constitute a bound two-cluster subsystem.

For large values of the Jacobi vector $y_{\alpha}$, the function $f_{\alpha}^{(\lambda_{\alpha}, l_{\alpha}; L)}(x_{\alpha}, y_{\alpha})$ asymptotically factorizes as

$$f_{\alpha}^{(\lambda_{\alpha}, l_{\alpha}; L)}(x_{\alpha}, y_{\alpha}) \approx \phi_{\lambda_{\alpha}}^{(\alpha, \sigma)}(x_{\alpha}) \left[ S_{c_{0}, c_{\alpha}} \psi_{l_{\alpha}}^{(-)}(p_{\alpha} y_{\alpha}) - S_{c_{0}, c_{\alpha}} \psi_{l_{\alpha}}^{(+)}(p_{\alpha} y_{\alpha}) \right]$$

(21)

for continuum states and

$$f_{\alpha}^{(\lambda_{\alpha}, l_{\alpha}; L)}(x_{\alpha}, y_{\alpha}) \approx -\phi_{\lambda_{\alpha}}^{(\alpha, \sigma)}(x_{\alpha}) \left[ S_{c_{0}, c_{\alpha}} \psi_{l_{\alpha}}^{(+)}(-i |p_{\alpha}| y_{\alpha}) \right]$$

(22)

for bound states. The entrance channel is denoted by $c_0$, and $c_{\alpha}$ refers to the current channel where $\alpha$ stands short for all necessary $(\lambda_{\alpha}, l_{\alpha}, \ldots)$ quantum numbers. The momentum $p_{\alpha}$ is defined by

$$p_{\alpha} = \sqrt{\frac{2m}{\hbar^2} \left( E - E_{\sigma}^{(\alpha)} \right)}$$

(23)

and the bound state energy $E_{\sigma}^{(\alpha)}$ of the two-cluster subsystem determines the threshold energy of the $c_{\alpha}$ channel.

This factorization of the wave function (4) also occurs in the expansion coefficients $\{C_{\nu_{\alpha}, \lambda_{\alpha}, n_{\alpha}, l_{\alpha}}^{(\alpha)}\}$. The asymptotic region in this representation is connected to large values of $n_{\alpha}$ (for more details see [39, 40, 47, 10]), and there the coefficients factorize as:

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The dynamic equations decrease to zero as \( n \) tends to infinity.

The many-channel equations of the GOB model can be solved in three stages. In the first step the Schrödinger equation for all two-cluster subsystems is solved. This is done by diagonalizing the \( N_\nu \times N_\nu \) matrix of the two-cluster Hamiltonian

\[
\left\langle \nu_\alpha, \lambda_\alpha | \tilde{\mathcal{H}}^{(2)}(2) | \tilde{\nu}_\alpha, \lambda_\alpha \right\rangle
\]

between the cluster Gaussian functions of \( \nu_\alpha \). The discrete set of eigenvalues \( E^{(\nu)}_\sigma \) correspond to bound states, or to pseudo-bound states above the threshold that are artifacts of the diagonalization in a finite basis. The eigenstate wave function is \( \{ C^{(\nu)}_{\nu_\alpha, \lambda_\alpha; n_\alpha, l_\alpha} \} \) for scattering and bound states in the \( c_\alpha \) binary channel.

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In the second step the block matrix of the total three-cluster Hamiltonian

\[
\left\langle \nu_\alpha, \lambda_\alpha; n_\alpha, l_\alpha | \tilde{\mathcal{H}} | \nu_\beta, \lambda_\beta; n_\beta, l_\beta \right\rangle
\]

is transformed to the representation of two interacting clusters using the aforementioned eigenfunctions. One obtains

\[
\left\langle \sigma_\alpha, \lambda_\alpha; n_\alpha, l_\alpha; \alpha | \tilde{\mathcal{H}} | \sigma_\beta, \lambda_\beta; n_\beta, l_\beta; \beta \right\rangle
\]

where

\[
\left\langle \sigma_\alpha, \lambda_\alpha; n_\alpha, l_\alpha; \alpha | \tilde{\mathcal{H}} | \sigma_\beta, \lambda_\beta; n_\beta, l_\beta; \beta \right\rangle
\]

\[
= \sum_{\nu_\alpha=1}^{N_\nu} \sum_{\nu_\beta=1}^{N_\nu} D^{(\sigma_\alpha, \nu_\alpha)}_{\nu_\alpha, \nu_\beta} \left\langle \nu_\alpha, \lambda_\alpha; n_\alpha, l_\alpha | \tilde{\mathcal{H}} | \nu_\beta, \lambda_\beta; n_\beta, l_\beta \right\rangle D^{(\sigma_\beta, \nu_\beta)}_{\nu_\beta, \nu_\alpha}
\]

This new representation exhibits the correct asymptotic behavior for large values of \( n_\alpha \) and \( n_\beta \), in the sense that off-diagonal matrix elements coupling different channels, decrease to zero as \( n_\alpha \) and \( n_\beta \) tend to infinity.
Asymptotically the matrix has a tri-diagonal form from the kinetic energy of the relative motion of the clusters. The diagonal matrix elements represent the interaction within a given channel.

The third step in our approach consists of solving the set of equations

$$\sum_{c_\alpha} \sum_{n_\alpha=0}^\infty \left\langle \sigma_\alpha, \lambda_\alpha; n_\alpha, l_\alpha; \alpha \left| \hat{H} - E \right| \sigma_\alpha, \lambda_\alpha; n_\alpha, l_\alpha; \alpha \right\rangle C^{(c_\alpha)}_{n_\alpha l_\alpha} = 0 \quad (31)$$

taking into account the appropriate boundary conditions to obtain either scattering or bound state solutions. This means that the solutions have to match the conditions (24) or (25) respectively, beyond some matching point \( N_i \) that separates the internal and asymptotic parts of the wave function. Thus e.g. for scattering we look for solutions for the form

$$\left\{ C^{(c_{\alpha})}_{n_\alpha l_\alpha} \right\} = \left\{ C^{(c_0)}_{n_\alpha}, C^{(c_1)}_{n_\alpha}, \ldots, C^{(c_{N_i})}_{n_\alpha} \right\},$$

$$\left\{ \sqrt{2r_{n_\alpha}} S_{c_\alpha, c_{\alpha}} \psi_{l_\alpha}^{(-)} (p_\alpha r_{n_\alpha}), - S_{c_\alpha, c_{\alpha}} \psi_{l_\alpha}^{(+)} (p_\alpha r_{n_\alpha}) \right\}; n_\alpha > N_i \right\} \quad (32)$$

where only the internal coefficients need to be determined. For simplicity we assume that \( N_i \) is identical for all channels. Inserting (32) into (31) then leads to

$$\sum_{c_\alpha} \sum_{n_\alpha \leq N_i} \left\langle \sigma_\alpha, \lambda_\alpha; n_\alpha, l_\alpha \left| \hat{H} - E \right| \sigma_\alpha, \lambda_\alpha; n_\alpha, l_\alpha \right\rangle C^{(c_\alpha)}_{n_\alpha l_\alpha} - \sum_{c_\alpha} S_{c_\alpha, c_{\alpha}} V^{(+)}_{c_\alpha, n_\alpha, c_{\alpha}} = - \sum_{c_\alpha} \delta_{c_\alpha, c_{\alpha}} V^{(-)}_{c_\alpha, n_\alpha, c_{\alpha}} \quad (33)$$

where e.g.

$$V^{(+)}_{c_\alpha, n_\alpha, c_{\alpha}} = V^{(-)}_{c_\alpha, n_\alpha, c_{\alpha}}$$

$$= \sum_{n_\alpha > N_i} \left\langle \sigma_\alpha, \lambda_\alpha; n_\alpha, l_\alpha \left| \hat{H} - E \right| \sigma_\alpha, \lambda_\alpha; n_\alpha, l_\alpha \right\rangle \sqrt{2r_{n_\alpha}} \psi_{l_\alpha}^{(+)} (p_\alpha r_{n_\alpha}) \quad (34)$$

The solution of (33) then provides the explicit many-channel scattering wave function. If the total number of binary channels is \( N_c \), there are \( N_c \cdot N_i + N_c \cdot N_e \) equations for \( N_c \cdot N_i \) expansion coefficients of the internal part of the wave function, and \( N_c \cdot N_e \) equations for the determination of the \( S \)-matrix.

3. Results and discussion

3.1. Parameters of the calculation

In the current calculations we consider a Minnesota nucleon-nucleon potential (MP) for which we take the central part from [55], and the spin-orbital part from [56] (data set IV). The exchange parameter \( u \) is fixed at \( u = 0.956 \) to reproduce the relative positions of the \( ^6Li + p \) and \( ^4He + ^3He \) thresholds.

To fix the oscillator bases we use a the same oscillator radius for both the \(^4He\) and deuteron clusters. We determine it by minimizing the energy of the three-cluster threshold
$^4He + d + p$, and obtain a value of $b = 1.311$ fm. Table 1 shows a good agreement of the computed threshold energies of $^6Li + p$ and $^4He + d + p$ compared to experiment.

Table 1
$^6Li + p$ and $^4He + d + p$ threshold, w.r.t. the $^4He + ^3He$ threshold (MeV).

| Threshold  | MP         | Experiment [57] |
|------------|------------|-----------------|
| $^6Li + p$ | 4.015      | 4.020           |
| $^4He + d + p$ | 5.852      | 5.493           |

To fix the set of Gaussian wave functions we follow the procedure of [34,58], and parametrize a set of widths $b_\nu$ with two variational and parameters $a_0$ and $q$ as

$$b_\nu = a_0 q^{\nu - 1}, \quad \nu = 1, 2, \ldots$$

This has been used in [34] and [58] to obtain the ground state energy of $^6He$.

3.2. Two-cluster subsystem properties

We first elaborate on the merits of the Gaussian basis for the two-cluster subsystems. To confirm its rapid convergence rate, we compare in Fig. 1 the ground state energy of $^6Li$ for both the Gaussian basis ($a_0 = 1.0$ fm and $q = 1.8$) and the oscillator basis ($b = 1.311$ fm). We have taken the latter value considered in this paper, although it is not necessarily the optimal choice for this particular system. One notices that convergence is reached with only 4 Gaussian functions, compared to more than 20 oscillator functions.

![Fig. 1. $^6Li$ ground state energy with MP as a function of the number of Gaussian (GB) and oscillator (OB) cluster states.](image)

For the ground state of $^3He$ in the two-cluster $d + p$ model, a similar situation occurs, and 4 Gaussian functions are sufficient with the $a_0 = 0.9$ fm, $q = 1.8$ parametrization.
In Table 2 we reproduce the energies of bound and pseudo-bound states of $^6\text{Li}$ and $^3\text{He}$ obtained with four Gaussian functions.

Table 2
Spectrum of bound and pseudo-bound state of $^6\text{Li}$ and $^3\text{He}$ clusters.

| $\sigma$ | $^3\text{He}^6\text{Li}; L = 0$ | $^6\text{Li}; L = 2$ |
|------|----------------|----------------|
| 0    | 5.852          | 0.000          |
| 1    | 1.421          | 0.000          |
| 2    | 8.765          | 0.000          |
| 3    | 48.774         | 0.000          |

A standard Resonating Group Method (RGM) description takes one cluster function, i.e. a single (oscillator) shell-model many-particle wave function, to describe the internal structure of the interacting clusters. In this approximation the bound state energy of $^6\text{Li}$ is 8.800 MeV, and -3.018 MeV for $^3\text{He}$, which is way above the values obtained in Table 2 as was to be expected from Fig. 1.

To obtain stable results in the three-cluster model of $^7\text{Be}$ for its weakly bound state, as well as for the elastic and inelastic scattering parameters, about 100 oscillator states must be considered in the calculation. This guarantees the unitarity of the calculated S-matrix with high precision better than 0.1%.

In a previous $^3\text{He} + ^3\text{He}$ two-cluster model for $^7\text{Be}$ within a standard RGM approach [10], [14] stable bound and continuous results were obtained with 30 to 50 oscillator states.

By taking into account cluster polarization, the GOB model allows for more spatially dispersed clusters. This is confirmed by calculating the root-mean-square-radius $R_m$ of the two interacting clusters. For $^3\text{He}$, a one cluster function approach yields $R_m = 1.311$ fm, while $R_m = 1.696$ fm with four Gaussian functions. For $^6\text{Li}$, these values are respectively $R_m = 1.650$ fm and $R_m = 2.288$. It is therefore natural that more oscillator states are necessary to properly reach the asymptotic region, because of the relatively large distances between the clusters compared to their sizes.

3.3. The Spectrum of $^7\text{Be}$

In Table 3 we display the energy of the $3/2^-$ (bound) ground state, and the energies and widths of the resonance states of $^7\text{Be}$ obtained with the MP interaction. All the energies are relative to the $^4\text{He} + ^3\text{He}$ threshold. There is good agreement between theory and experiment for the $3/2^-$ ground state. It is however slightly overbound by

| State | Theory | Experiment [57] |
|------|--------|----------------|
| $L = 1, J_z = 3/2^-$ | -1.702 | -1.587 |
| $L = 3, J_z = 7/2^-$ | 2.820 + 0.130 (2.983 ± 0.05) + i (0.175 ± 0.007) |  |
| $L = 3, J_z = 5/2^-$ | 5.040 + 1.343 (5.143 ± 0.10) + i | 1.20 |
0.115 MeV, and the spin-orbital splitting energy is 0.16 MeV less than the experimental value. The energies and widths of the lowest two resonances (7/2− and 5/2−) are very close to the experimental value. The positions of the ground state and resonances of 7Be are displayed in Fig. 2.

![Fig. 2. Spectrum (relative to the 4He + 3He threshold) of the ground and resonance states of 7Be in the GOB model with MP interaction. Theory (right) and experiment (left).](image)

We now turn to the effect of the polarization of the two-cluster subsystems 6Li and 3He on the ground state energy of 7Be. We do so comparing results with (marked ”Y”) and without (marked ”N”) polarization of the subsystem in Table 4. We suppress the polarization by using only a single function instead of all eigenfunctions of the corresponding two-cluster hamiltonian in the calculations. This corresponds to a rigid cluster throughout the calculation, whereas using the full set of eigenfunctions allows for adapting the size and shape of the subsystem to the presence of the third cluster. One notices that the polarization of 6Li has a stronger impact than that of 3He.

| Polarization (Y: included, N: suppressed) effect on the 7Be ground state energy. |
|--------------------------------|
| 4He 6Li E(MeV)                |
| N N  -0.971                   |
| Y N  -1.413                   |
| N Y  -1.666                   |
| Y Y  -1.702                   |

Using the same (Y,N) approach we look at the effect of cluster polarization on the phase shift of 4He + 3He elastic scattering with total momentum $J^\pi = 7/2^− (L = 3)$ in Fig. 3. Similarly, in Table 4 we look at the effect on the parameters for the 7/2− and 5/2− resonance states.
Fig. 3. Polarization effects on the phase shift of $^4\text{He} + ^3\text{He}$ elastic scattering.

The cluster polarization substantially decreases the resonance energy and width, which points to an increase in the effective interaction between the clusters. The same observation has been made in \[7,8,9,10,11,13,14,48\] for collective monopole and quadrupole polarizations. One again notices that the polarization of the $^6\text{Li}$ cluster is more important than that of the $^3\text{He}$ cluster. The resonance properties, obtained with $^6\text{Li}$ polarization, are only marginally different from those calculated with both $^6\text{Li}$ and $^3\text{He}$ polarizations.

Table 5

|         | $J^\pi = 7/2^-$ | $J^\pi = 5/2^-$ |
|---------|-----------------|-----------------|
| $^3\text{He}$, $^6\text{Li}$, $^N$ | $E + i\Gamma$(MeV) | $E + i\Gamma$(MeV) |
| N, N   | 4.01 + i0.51    | 5.86 + i2.42    |
| Y, N   | 3.69 + i0.37    | 5.74 + i2.24    |
| N, Y   | 2.87 + i0.14    | 5.08 + i1.38    |
| Y, Y   | 2.82 + i0.13    | 5.04 + i1.34    |

3.4. $^7\text{Be}$ ground state properties

The electromagnetic observables can be calculated using the explicit ground state wave function. In Table 6 we list root-mean-square radii (proton ($R_p$), neutron ($R_n$) and mass ($R_m$)) and the quadrupole moments (proton ($Q_p$), neutron ($Q_n$) and mass ($Q_m$)) for the $3/2^-$ ground state. As one expects, the proton radius is larger than the neutron one. The quadrupole moment is an indicator of the deformation of the nucleus, and a negative
value corresponds to a prolate deformation. The differences in these moments reflect the pronounced cluster structure of the $^7\text{Be}$ ground state.

Table 6

| $J^\pi$ | GOB | MCRGM \cite{16} | SCRGM \cite{25} | SVM \cite{36} | SM \cite{59} | Experiment |
|--------|------|----------------|----------------|-------------|-------------|------------|
| $R_p$ (fm) | 2.457 | 2.74 | 2.41 | 2.342 | 2.53±0.03 \cite{60} |
| $R_n$ (fm) | 2.263 | 2.50 | 2.31 |          |             |
| $R_m$ (fm) | 2.375 | 2.36 |      |             |             |
| $Q_p$ (e·fm$^2$) | -6.245 | -6.4 | -6.125 | -6.11 | -5.153 |
| $Q_n$ (e·fm$^2$) | -3.739 |      |      |      |             |
| $Q_m$ (e·fm$^2$) | -9.984 |      |      |      |             |

In Table 6 we compare our GOB results with those of the multi-channel RGM of Arai et al (MCRGM) \cite{16}, of the single-channel RGM of Kajino et al. (SCRGM) \cite{25}, of the Stochastic Variational Method (SVM) \cite{36} and of the Shell Model (SM) calculations \cite{59}.

The GOB results are close to those of SVM \cite{36} and of the multi-channel RGM \cite{16}. These results have also obtained with the MP interaction.

The single-channel SCRGM results of \cite{25} have been obtained with the modified Hasegawa-Nagata potential. The ground state quadrupole moment is comparable, whereas the charge radius is much larger in the SCRG.

The Shell Model calculation of \cite{59} has been performed with the Bonn nucleon-nucleon potential. Its results for the charge radius and quadrupole moments are smaller than in the cluster models (MCRGM, SCRGM, SVM,GOB). This can be attributed to the fact that the Shell Model, involving a 10$h\Omega$ or 12$h\Omega$ state space, confines the inter-cluster distances in the $^7\text{Be}$ nucleus. In comparison, the GOB model uses a model space of 200$h\Omega$ for the inter-cluster behavior of the dominant $^4\text{He} + ^3\text{He}$ cluster configuration.

3.5. Comparing polarization methods

As discussed in the introduction, one can distinguish collective polarization (quadrupole and monopole) for the compound nucleus as a whole (QM), RGM models with monopole excitations of the individual clusters (MRGM), and cluster polarization as described in the GOB model. It is interesting to compare these different polarization methods to gauge their impact on the ground state energy.

A detailed study in MRGM is only available for $^7\text{Li}$ \cite{61}, so we investigate the three methods applied to this nucleus, as it should have deformation properties comparable to $^7\text{Be}$.

To make the comparison consistent, we use the MP interaction parameters of \cite{61}, i.e. $u = 1.0174$ and a spin-orbital strength of 0.821 (it is 1.0 for the other calculations in this paper, as suggested in \cite{59}). With this interaction, we have performed the QM calculations along the lines reported in \cite{15} and \cite{14}, a GOB calculations for a single $\alpha+t$ channel (SGOB) and a full GOB calculation as outlined in this paper. A standard RGM calculation without cluster polarization (SRGM) has been included as a benchmark.

In Table 7 the results for the different polarization models are displayed. One notices that the collective QM and SGOB models, which are comparable in terms of model space,
Table 7
Ground state energy of $^7$Li without polarization (Standard RGM) and with different types of polarization (see text)

| SRGM | MRGM | SGOB | QM | GOB |
|------|------|------|----|-----|
| -31.465 | -32.027 | -33.227 | -33.777 | -34.150 |

improve with respect to the MRGM that includes only monopole cluster polarization. Clearly the full GOB calculation (including cluster polarization of $^6$Li in the $^6$Li + n channel) provides the best results. This was to be expected from the above $^7$Be results.

We notice that in the MRGM calculation, the monopole polarization of $^3$He is obtained by using four Gaussian functions resulting in -5.906 MeV of binding energy. In the SGOB model four Gaussian cluster functions are used to describe cluster polarization of $^3$He, and yields a binding energy of -5.953 MeV. This confirms the above results.

We can thus state that the cluster polarization of the channel subsystems described in the GOB model indeed plays an important role in seven-nucleon systems, and currently represents most prominent polarization type.

3.6. Three-cluster geometry

The expansion coefficients $\{C^{(a)}_{\alpha,\gamma,\mu,\nu,\lambda}\}$ determine the three-cluster wave function $\Psi^J$ (3) and also the Faddeev components $f_{\alpha}(x_{\alpha},y_{\alpha})$ ($\alpha = 1, 2, 3$) (4). We use the latter representation to analyze the wave function.

The correlation function for the ground state is

$$P_{\alpha}(x_{\alpha},y_{\alpha}) = x_{\alpha}^2 y_{\alpha}^2 \int |f_{\alpha}(x_{\alpha},y_{\alpha})|^2 d\bar{x}_{\alpha} d\bar{y}_{\alpha}$$ (36)

where the integration runs over the unit vectors $\bar{x}_{\alpha}$ and $\bar{y}_{\alpha}$.

To interpret the the polarizability in terms of the three-cluster structure, we introduce the root-mean-square radii $R_{\alpha}$:

$$R_{\alpha}(y_{\alpha}) = \sqrt{\int d\bar{y}_{\alpha} \int x_{\alpha}^2 |f_{\alpha}(x_{\alpha},y_{\alpha})|^2 d\bar{x}_{\alpha}/N_{\alpha}(y_{\alpha})}$$ (37)

and

$$N_{\alpha}(y_{\alpha}) = \int d\bar{y}_{\alpha} \int |f_{\alpha}(x_{\alpha},y_{\alpha})|^2 d\bar{x}_{\alpha}$$ (38)

They represent the root-mean-square radius of a two-cluster subsystem as a function of its distance to the third cluster (i.e. the distance of the centre-of-mass of the two-cluster system to the centre-of-mass of the third cluster).

We introduce new coordinates $r_{\alpha}$ and $S_{\alpha}$ whose norms ($r_{\alpha}$ and $S_{\alpha}$ correspond to the distances between the centers of mass of the clusters, that allow for a proper interpretation of the quantities (36) and (37). They relate to the original Jacobi coordinates as:

$$x_{\alpha} = \sqrt{\frac{A_{\beta} A_{\gamma}}{A_{\beta} + A_{\gamma}}} r_{\alpha}, \quad y_{\alpha} = \sqrt{\frac{A_{\alpha} (A_{\beta} + A_{\gamma})}{A_{\alpha} + A_{\beta} + A_{\gamma}}} S_{\alpha}$$ (39)
Fig. 4. Correlation function for the $^4\text{He} + ^3\text{He}$ binary channel.

Fig. 5. Correlation function for the $^6\text{Li} + p$ binary channel
Fig. 4 displays the correlation function for the $^4\text{He} + ^3\text{He}$ channel and Fig. 5 for the $^6\text{Li} + p$ channel. One notices from Fig. 4 that the distance between $^4\text{He}$ and $^3\text{He}$ is approximately 8 fm, and much larger than the separation of $d$ and $p$. From Fig. 5 one observes that the binary cluster configuration $^6\text{Li} + p$ is surprisingly compact. Both the distance between $^4\text{He}$ and $d$ and between $^6\text{Li}$ and $p$ are around 1 fm. These different geometric configurations can be related to the ground state energy relative to the thresholds of the corresponding binary channel. Indeed, the ground state is positioned at -1.702 MeV from the $^4\text{He} + ^3\text{He}$ threshold and at -5.722 MeV with from the $^6\text{Li} + p$ threshold. This agrees with a very dispersed $^4\text{He} + ^3\text{He}$ and a compact $^6\text{Li} + p$ configuration.

![Graph](image_url)

**Fig. 6. Dependence of the root-mean square radius $R_\alpha$ of the two-cluster subsystems on the distance $S_\alpha$ from the third cluster.**

In Fig. 6 we show the dependence of the root-mean-square radius of the two-cluster subsystems on the distance from the third cluster. One notices how $^6\text{Li}$ strongly adapts its size when the proton is at large distances (more than 15 fm) from $^6\text{Li}$. When the $^6\text{Li}$ and the proton are near, the $^6\text{Li} 0^+$ ground state gets compressed approximately 1.5 times, and the $2^+$ excited state approximately two times. The $^3\text{He}$ nucleus, in a two-cluster configuration $d+p$, is strongly affected when the $\alpha$-particle is closer than 6 to 7 fm. Note that without polarization all three curves in Fig. 6 would be horizontal lines, as the two-cluster subsystems then have a constant size. The figure illustrates the impact of cluster polarization of $^6\text{Li}$ and $^3\text{He}$ in the description of $^7\text{Be}$.

3.7. The Reaction $^6\text{Li} (p, ^3\text{He}) ^4\text{He}$

The astrophysical $S$-factor of the reaction $^6\text{Li} (p, ^3\text{He}) ^4\text{He}$ is obtained from the total cross section, which has been computed with four total angular momentum $L = 0, 1, 2$
Fig. 7. Astrophysical $S$-factor of the reaction $^6$Li($p$,$^3$He)$^4$He for the dominant $L = 0$ component with and without $^3$He and $^6$Li polarization.

and 3. We have observed that zero angular momentum contribution dominates the low energy range $0 \leq E \leq 1$ MeV of the cross section.

Fig. 7 displays the $S$-factor of the $^6$Li($p$,$^3$He)$^4$He reaction calculated for $L = 0$. In contrast to the analysis of the ground state and resonance energies, the effects of cluster polarization are less evident here. In Fig. 7 it is seen that $^3$He polarization increases the $S$-factor, thus increasing the coupling between the channels. The $^6$Li polarization only has a small effect on the $S$-factor.

In Fig. 8 we compare the results for the $S$-factor with both $^6$Li and $^3$He cluster polarization with the available experimental data [62,63,64,65,66,67,68,69,70]. Notations and data are taken from [71] and the web site http://pntpm.ulb.ac.be. One observes that our model somewhat overestimates the $S$-factor in the low energy range [68]. A similar result was obtained in [16]. A possible reason is the lack of tensor components in the MP interaction. These would couple channels with different total spin and total angular momentum, and thus reduce the coupling between entrance and exit channels.

4. Conclusions

We have introduced a fully microscopic three-cluster model in which an expansion in terms of Faddeev components is used. The set of equations for the Faddeev amplitudes is derived and solved within the Coupled Reaction Channel Formalism. The dynamics of the three-cluster system is described by an effective two-body nucleon-nucleon potential, and takes into account the Pauli exchange principle correctly.

We have used two different expansion bases: (1) a Gaussian basis suitable for the description for the different two-cluster subsystems, and (2) an oscillator basis to in-
Fig. 8. S-factor of the reaction \(^{6}\text{Li} (p,^3\text{He})^4\text{He}\). Experimental data are taken from [62]-MA56, [63]-FA64, [64]-GE66, [67]-GO74, [68]-LI77, [69]-EL79, [70]-EN92.

corporate the appropriate boundary conditions for bound and continuous states in a many-channel compound system. Only few Gaussian states are required to describe the ground state of the (bound) two-cluster subsystems, such as \(^{6}\text{Li}\) which has a high degree of \(\alpha + d\) clusterization. This limits the computational effort involved in the calculations. As for the three-cluster results, many more oscillator states (typically 70 to 100) are needed to guarantee both the convergence of the ground state, and the unitarity of the many channel S-matrix to sufficient precision.

The method has been applied to investigate cluster polarizability in the ground and resonance states of \(^{7}\text{Be}\) for which the three-cluster configuration \(^4\text{He} + d + p\) was used. This provides for (i) the two binary channels \(^3\text{He} + ^4\text{He}\) and \(^6\text{Li} + p\), which are prominent in the low energy region of \(^7\text{Be}\), and (ii) two bound two-cluster subsystems (\(^6\text{Li}\) as \(^4\text{He} + d\), \(^3\text{He}\) as \(d + p\)). The latter are modeled with a Gaussian basis expansion, and allow to study the relative behavior of the containing clusters when \(^6\text{Li}\) (respectively \(^3\text{He}\)) collides with a proton (respectively an \(\alpha\)-particle). We refer to this behavior as “cluster polarization”. The inclusion of cluster polarization in \(^7\text{Be}\) leads to a strong decrease of the energy of the ground and resonance states, and reduces the resonance widths two to four times. The \(^6\text{Li}\) polarization is more important in this respect than the \(^3\text{He}\) one.

The effect of cluster polarizability was also studied in the reaction \(^6\text{Li} (p,^3\text{He})^4\text{He}\) by considering the S-factor, for which the \(^3\text{He}\) polarization was seen to be more important.
5. Acknowledgments

Support from the Fonds voor Wetenschappelijk Onderzoek Vlaanderen (FWO), G0120-08N is gratefully acknowledged. V. S. Vasilevsky is grateful to the Department of Mathematics and Computer Science of the University of Antwerp (UA) for hospitality. This work was supported in part by the Program of Fundamental Research of the Physics and Astronomy Department of the National Academy of Sciences of Ukraine.

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