Sudden expansion of Mott insulators in one dimension

L. Vidmar,1, 2 S. Langer,3 I. P. McCulloch,4 U. Schneider,5 U. Schollwöck,1 and F. Heidrich-Meisner1

1Department of Physics and Arnold Sommerfeld Center for Theoretical Physics, Ludwig-Maximilians-Universität München, D-80333 München, Germany
2J. Stefan Institute, SI-1000 Ljubljana, Slovenia
3Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA
4School of Physical Sciences, The University of Queensland, Brisbane, QLD 4072, Australia
5Department of Physics, Ludwig-Maximilians-Universität München, D-80799 München, Germany

We investigate the sudden expansion dynamics of bosons and fermions in a homogeneous lattice using exact numerical methods. As a main result, we show that in one dimension, both bosonic and fermionic Mott insulators expand with the same velocity, irrespective of the interaction strength. They therefore expand like noninteracting spinless fermions or, equivalently, hard-core bosons (HCB), both of which are integrable systems. Moreover, we investigate the effect of breaking the integrability of HCB: While such a system exhibits ballistic dynamics in one dimension, we obtain strong deviations from this behavior on a two-leg ladder. This is consistent with a recent experiment that studied the dimensional crossover from one to two dimensions.

Introduction—The possibility of realizing various many-body Hamiltonians in the laboratory with ultracold atomic gases [1] allows one to address outstanding questions from condensed matter theory in well-controlled experiments. In the context of low-dimensional systems with strong correlations, our main interest is in two topics, namely non-equilibrium dynamics [2–4] and transport properties [5–9]. In the former case, theorists seek to understand the relaxation processes and the conditions for thermalization [10, 11] whereas in the latter case, qualitative questions such as whether transport is ballistic or rather diffusive in microscopic models remain actively debated (see [12–15] and references therein).

In both contexts, one-dimensional (1D) models such as the Heisenberg model, the Fermi-Hubbard model (FHM) or hard-core bosons (HCB) [16] are believed to exhibit anomalous behavior, including, for example, non-ergodic dynamics [17], non-thermal stationary states (see, e.g., [3, 18]), or ballistic transport with divergent dc-conductivities despite the presence of interactions [12–15]. This is often traced back to the integrability of these models [17]. A number of examples, such as dissipationless energy transport in Heisenberg chains [12–15], convincingly demonstrate that integrable systems are ideal candidates in which to search for deviations from generic behavior (i.e., finite dc-conductivities and diffusive transport), precisely because of the existence of additional (usually local) conservation laws [12–14]. An unambiguous test for, e.g., ballistic dynamics in condensed matter systems is difficult since one needs to account for impurities or phonons [19] which can in principle break most non-trivial conservation laws. Nonetheless, there are many intriguing experimental results, in particular for low-dimensional quantum magnets [20], that are speculated to be related to the existence of such conservation laws for the underlying spin Hamiltonians. Many non-integrable 1D models seem to exhibit dynamics that are compatible with diffusion [21], although several exceptions have also been identified [22].

In this work we consider a non-equilibrium problem, namely the sudden expansion of interacting particles into a homogeneous lattice, as sketched in Fig. [1]. The expansion is induced by suddenly quenching the trapping potential, which is present in all cold atomic gas experiments, to zero. Our study is motivated by two recent experiments that have realized sudden expansion in optical lattices [7, 8]. For fermions in 2D, the data and theoretical modeling suggest that the dynamics in the high-density regime is diffusive [7]. In the case of bosons [8], dramatic differences were observed between strongly interacting particles in 1D versus 2D: In 1D, the expansion of HCB is ballistic (and fast for initial states with one boson per site), whereas in 2D, the atomic cloud barely expands at all, similar to the diffusive dynamics observed for fermions. The reason for the ballistic expansion in 1D is indeed the integrability of HCB in 1D [23]. Due to an interaction quench performed simultaneously with the removal of the trap, this latter experiment [8] did not probe the low-energy dynamics at small and intermediate interaction strengths. In another recent experiment, the expansion of bosons in a 3D array of tunnel-coupled 1D tubes was studied [9], focusing on the transverse dy-
In this Letter, we address two objectives. First, we disentangle the effects of the trap opening from the interaction quench by studying the expansion dynamics of bosons starting from the ground state of the trapped gas. Surprisingly, we find the same ballistic expansion at sufficiently long times for all initial Mott insulators (MI): The asymptotic expansion velocity is independent of the interaction strength, provided it is larger than the critical interaction strength that separates the bosonic MI from the superfluid (SF) in 1D. This closely resembles the expansion from a fermionic MI in 1D [24], suggesting a deeper connection between these two cases in their asymptotic dynamics. Second, we investigate how breaking the integrability of HCB in 1D affects the ballistic expansion dynamics. While the asymptotic dynamics remain unchanged when going to finite interactions, coupling chains to ladders results in a qualitatively different behavior.

Setup—We investigate the setup shown in Fig. 1 described by the Bose-Hubbard model (BHM), unless stated otherwise:

\[ H = -J \sum_{\langle i,j \rangle} (b_i^\dagger b_j + h.c.) - J_\perp \sum_{\langle i,j \rangle \perp} (b_i^\dagger b_j + h.c.) + \frac{U}{2} \sum_i n_i(n_i - 1) + V_{\text{trap}} \theta(-t) . \]

\( n_i = b_i^\dagger b_i \) represents the density (\( b_i \) is the boson annihilation operator at site \( i \)), \( U \) is the on-site interaction strength, and \( \langle i, j \rangle \parallel (\perp) \) indicates a summation over nearest neighbors along (perpendicular to) the chain. We set the lattice spacing to unity. To obtain a chain [Fig. 1(a)], we set \( J_{\perp} = 0 \), while \( J_{\parallel} > 0 \) corresponds to a two-leg ladder [Fig. 1(b)]. The trapping potential \( V_{\text{trap}} \) has the form of an infinitely deep box of length \( L_{\text{box}} \) and is switched off at \( t = 0 \). Unless stated otherwise, we prepare the system at \( t < 0 \) in the ground state of the Hamiltonian Eq. 1 at density \( n = N/L_{\text{box}} = 1 \). We calculate the ground state as well as the time evolution via the adaptive time-dependent DMRG method [25, 26]. The time \( t \) is measured in units of \( 1/J \) (\( h = 1 \)).

We investigate the expansion dynamics in both real and momentum space, yielding complementary information. The time-dependent radius of the density distribution is defined as \( R^2(t) = \frac{1}{N} \sum_i \langle n_i(t) \rangle (i - i_0)^2 \), where \( i_0 \) represents the center of mass. The corresponding radial velocity \( v_r(t) \) is defined through the reduced radius \( \dot{R}(t) = \frac{\sqrt{R^2(t) - R^2(0)}}{\Delta t} \) as \( v_r(t) = \partial R(t)/\partial t \). Another measure of the expansion velocity is related to the momentum distribution function (MDF) \( n_k = \frac{1}{N} \sum_{l,m} e^{-ik(l-m)} \langle b_l^\dagger b_m \rangle \): We define the average velocity as \( v_{av,k}(t) = \frac{1}{N} \sum_k n_k(t) v_{av,k} \), where \( v_{av,k} = 2J \sin k \).

Expansion of HCB on a chain—In order to illustrate the rich phenomenology of this non-equilibrium problem, we first discuss the expansion of HCB \( U/J = \infty \), \( (b_i^\dagger)^2 = 0 \) in Eq. 1, which is the only integrable point of the 1D BHM besides \( U/J = 0 \). We start from a product of local Fock states, \( \phi_{\text{local}} = \prod_i b_i^\dagger(0) \).

A 1D system of HCB can be mapped onto noninteracting spinless fermions [23] such that \( H_{\text{HCBO}} = \sum_k \varepsilon_k n_k^f \), where \( \varepsilon_k = -2J \cos k \). The occupations of fermionic momenta are conserved quantities and, as a consequence, the particle current \( j_k = \sum_k \varepsilon_k n_k^f \) is conserved as well. In linear response, the conductivity thus reads \( \text{Re} \sigma(\omega) = D \delta(\omega) \) with a nonzero Drude weight \( D \) at finite temperatures. The diffusion constant, which is related to the dc-conductivity via the Einstein relation, therefore also diverges, indicating ballistic transport [12, 15].

For noninteracting fermions, \( \bar{R}(t) = v_{av}^f t \) with \( v_{av}^f = v_{av} \), hence both expansion velocities are time-independent quantities fully determined by the initial state. For \( n = 1, n_k^f \) is flat and \( v_{av} = v_{av}^f = \sqrt{2J} \) [Fig. 2]. Since for HCB, \( n_k^f \), these strongly interacting bosons expand ballistically with \( \bar{R} = v_{av}^f t \) as well [8]. In contrast to spinless fermions, the MDF of HCB is, however, not conserved. This is illustrated in Fig. 2 where the non-monotonic behavior of \( v_{av}(t) \) [main panel] is a direct consequence of the changes in \( n_k(t) \) [insets]. The initial increase of \( v_{av}(t) \) reflects the dynamical quasi-condensation at finite momenta \( k = \pm \pi/2 \) visible in the middle inset [25, 29]. At larger times, however, \( v_{av}(t) \) decreases, since in the \( t \to \infty \) limit the MDF of HCB tends to \( n_k^f \), the MDF of spinless fermions. This process is called dynamical fermionization [30, 33] and it results in \( v_{av}(t \to \infty) = \sqrt{2J} \) for our initial conditions. To summarize, the measurement of \( v_{av}(t) \) for HCB is sensitive to the emergence of both dynamical quasi-condensation and fermionization.

Expansion velocity for \( U/J < \infty \) on a chain—For density \( n = 1 \), there is a quantum phase transition at \( U_c/J \approx 3.4 \) from the SF to the MI [34]. At \( U/J = 0 \), the bosons quasi-condense at \( k = 0 \), leading to \( \bar{R} = v_r t \)
than the time scales that control $\delta k < \pi / 2$. Few bosons indicate that the final MDF is particle-hole symmetric, i.e., $n_{\pi/2 < \delta k} - n_{\pi/2} = n_{\pi/2} - n_{\pi/2 < \delta k}$ ($0 < \delta k < \pi / 2$), resulting in the observed $v_{av}(t \to \infty)$.

The calculation of the radial velocity $v_r$ relies on the analysis of $\dot{R}(t)$, which after a transient, much shorter than the time scales that control $v_{av}(t)$, becomes linear in $t$ [see Fig. 2 for details]. In Fig. 2 (circles), we show $v_r$ as a function of $U/J$ for the expansion from the ground state. Surprisingly, we observe that $v_r = \sqrt{2}/J$ is constant in the entire MI phase. This behavior is in clear contrast to that in the SF phase, where $v_r$ monotonically decreases to zero.

Although the 1D BHM is non-integrable for any value of $U/J$ other than zero or infinity, the expansion velocity of HCB is indistinguishable from those extracted from the long-time behavior of $\dot{R}(t)$ for $U/J > U_c / J$. In addition, the density profiles $\langle n_k(t) \rangle$ for any $U/J > U_c / J$ differ from those of HCB by less than $10^{-4}$ per site at any time during the expansion [36]. Therefore, $v_r = \sqrt{2}/J$ is a hallmark feature of the whole MI phase in 1D.

It is very instructive to compare these findings to the FHM. The inset of Fig. 3 (triangles) shows $v_r$ for the expansion from the ground state of the 1D FHM, which is a MI for any $U/J > 0$ (data from Ref. [24]). Such expansions always exhibit $\dot{R}(t) = v_t t$ and $v_r / J = \sqrt{2}$, irrespective of $U/J$ [24]. Remarkably, both fermionic and bosonic MI expand with the same expansion velocity. Even more so, the density profiles for the expansion for any MI are identical for bosons and fermions (see Fig. 3 in [30]) and therefore cannot be distinguished from non-interacting fermions. As an explanation, we conjecture that the asymptotic velocity distribution $n_{\phi_{\text{Fock}}}$ is independent of $v_k$ and identical for all MI in 1D, consistent with the asymptotic behavior of $v_{av}(t)$.

FIG. 3. (Color online) Single chain: Radial velocity of bosons and fermions. Main panel: Radial velocity $v_r / J$ versus $U/J$ for the BHM, where the initial state is the ground state (g.s.; circles) or $|\phi_{\text{Fock}}\rangle$ (squares, from Ref. [8]). Inset: $v_r / J$ versus $U/J$ for the FHM, where the initial state is the g.s. (triangles) or the Néel state $|\phi_{\text{Néel}}\rangle$ (diamonds). All data is extrapolated to $N \to \infty$ [35].

We note that recent exact results for the Lieb-Liniger model suggest that a dynamical renormalization occurs during the sudden expansion [33], which states that the asymptotic dynamics of a repulsively interacting Bose gas is, for any interaction strength, governed by the behavior of noninteracting fermions. Considering that the density decreases as the gas expands, it is conceivable that the dynamics of the BHM can be described by the Lieb-Liniger model at long times. A formal proof of this interpretation is left for future research.

Our results for bosons complement a recent experimental and numerical study [8] where the initial state was $|\phi_{\text{Fock}}\rangle$ and an additional quench from infinite to finite $U/J$ was performed simultaneously with the removal of the external potential. The numerical results for the radial velocities $v_r$ (squares) from Ref. [8] for such an initial state are included in Fig. 3. In this case, $v_r / J = \sqrt{2}$ at large $U/J$ and at $U/J = 0$, with a minimum at $U/J \sim 4$. The expansion velocities for the two different initial states, i.e., the ground state and $|\phi_{\text{Fock}}\rangle$, exhibit strong differences both in the vicinity of $U \sim U_c$ and at $U = 0$, where the difference is due to the different initial $n_k$. Since $|\phi_{\text{Fock}}\rangle$ is not the ground state of the trapped gas, the excess energy compared to the ground state strongly influences the dynamics (see Ref. [8] for a discussion). We define the excess energy as $\delta E = E_{\text{exp}} - E_0$, where $E_{\text{exp}}$ is the total energy during the expansion and $E_0$ is the energy of the ground state of the initial trap. For the expansion from $|\phi_{\text{Fock}}\rangle$, $\delta E = |E_0|$, hence $\delta E / N \to 2J$ at $U/J = 0$. For $U/J \to \infty$, in contrast, the ground state approaches $|\phi_{\text{Fock}}\rangle$ and so $\delta E \to 0$ [35]. This is why the interaction quench has virtually no effect at large $U \gg U_c$. The behavior at $U \sim U_c$ is, however, counter-intuitive: the gas with the higher energy per particle [i.e., the expansion from $|\phi_{\text{Fock}}\rangle$] expands slower. One has to bear in mind, though, that changing temperature (or in our case the excess energy) can in principle quantitatively alter transport coefficients such as the diffusion constant. In particular, this may affect the time and length scales on which diffusion sets in. The intermediate regime $0 < \delta E < |E_0|$ can be studied experimentally by preparing the gas close to the ground state and subjecting it to heating.

It is plausible to speculate whether the dynamics of the
FHM at intermediate \(U\) differs from those of the BHM for the combination of interaction quench and trap removal with \(\delta E > 0\), since the former model is integrable and the latter is not. However, this expectation is not supported by our analysis of expansion dynamics when we calculate the expansion dynamics of an initial Néel state \(|\phi_{N\uparrow}\rangle = \prod_{i=0}^{n-1} c_i^\dagger c_{i+1}^\dagger |\emptyset\rangle\) in the 1D FHM [37]. This state corresponds to \(|\phi_{\text{rock}}\rangle\) used in the case of the BHM, and is a ground state only for \(U/J = \infty\) (see [38] for details). Interestingly, the dependence of \(v_c/J\) on the interaction strength (diamonds in the inset of Fig. 3) shares striking similarities with that of the 1D BHM. Since the 1D FHM is integrable for any \(U/J\), these results indicate that integrability per se does not imply ballistic and fast expansions.

**Expansion of HCB on a ladder**—For 1D systems the conservation of \(n_L^c\) is the core reason for the ballistic dynamics of strongly interacting particles. We next demonstrate that coupling chains to a ladder system, Fig. 1(b), has a dramatic effect for HCB, expanding from \(|\phi_{\text{rock}}\rangle\) [38]. In contrast to 1D systems, HCB on a ladder can be mapped to interacting spinless fermions [39], and the model is nonintegrable as soon as \(J_{\perp}/J > 0\).

In Figs. 3(a)–(c) we present density profiles \(\langle n_i(t)\rangle\) for \(J_{\perp}/J = 0, 0.5\) and 1, respectively. At \(J_{\perp}/J = 0\), the expanding cloud develops two well-defined wings, as expected for ballistic dynamics in 1D [39, 40]. At \(J_{\perp}/J = 1\), on the other hand, there remains a stable core of particles in the center of the lattice which barely delocalizes. This suggests that the expansion is qualitatively different from the one observed for strongly interacting particles in 1D, and it may indicate diffusive dynamics [2, 39].

Next we study how the expansion velocity changes as \(J_{\perp}/J\) is varied continuously from 0 to 1. In Fig. 4 (squares) we show the core expansion velocity \(v_c = \partial r_c(t)/\partial t\). \(r_c(t)\) is the half-width-at-half-maximum of the density distribution, which has been measured in recent experiments [7, 8]. Remarkably, \(v_c\) exhibits a sharp drop at intermediate \(J_{\perp}/J \approx 0.5\) from \(v_c/J \approx 2\) to a vanishing \(v_c/J \approx 0\) at large \(J_{\perp}/J\). For \(J_{\perp}/J \lesssim 0.4\), \(v_c\) detects the fast wings observed in Fig. 3(a), which expand with \(v/J \approx 2\). For \(J_{\perp}/J \gtrsim 0.6\), however, the formation of a stable core dominates \(v_c\) and renders it small. Comparing different particle numbers suggests that a sharp drop persists around \(J_{\perp}/J \approx 0.5\) as \(N\) increases [39]. In the inset of Fig. 5 we show the radial velocity \(v_r\), which in contrast exhibits a smooth decrease. Since \(v_r\) is derived from a sum over the whole density profile, it measures how the relative contribution of the fast ballistic wings decreases when \(J_{\perp}/J\) increases. In notable contrast to 1D HCB, on a ladder, \(R(t)\) is not linear in time even for \(J_{\perp} \sim 0.2\), but undergoes transient dynamics [36].

The experimental results [8] for the 1D-2D crossover are included in Fig. 5 (diamonds) for comparison. It is very intriguing that in both cases, a ladder and a 2D system, a sufficiently large \(J_{\perp}/J\) results in a very slow expansion and a stable high-density core. Despite the quantitative differences between the dimensional crossover and a ladder concerning the \(v_c = v_c(J_{\perp}/J)\) curves, the conservation of the fermionic momenta \(n_i^f\) of strictly 1D HCB is violated as soon as \(J_{\perp}/J > 0\), which we argue is the common reason for the strong changes of the expansion dynamics compared to 1D HCB. An example for \(n_i^f(t)\) for \(J = J_{\perp}\) is shown in [36].

Our results identify HCB on a ladder as an ideal testbed of integrability breaking for experimental studies. The required homogeneous ladder potentials can be readily realized by combining the superlattice technique of [41] with the control of the external confinement demonstrated in [7], provided that the transverse potential created by the superlattice is overall anticonfining [42].

**Conclusions**—We studied the expansion dynamics of bosons and fermions in 1D and of HCB on ladders. Remarkably, we observe that both bosonic and fermionic MI expand expand with the same fast expansion velocity in 1D and show identical density profiles, independent of the interaction strength \(U/J > U_c/J\). The fast and ballistic dynamics of HCB, which are integrable in 1D, therefore persist to finite values of \(U/J < \infty\), whereas coupling chains to a ladder changes the qualitative behavior dramatically.

![FIG. 4. (Color online) Two-leg ladder: Density of HCB for the expansion from \(|\phi_{\text{rock}}\rangle\). (a) \(J_{\perp}/J = 0\), (b) \(J_{\perp}/J = 0.5\), (c) \(J_{\perp}/J = 1\) \((N = 8)\).](image-url)

![FIG. 5. (Color online) Two-leg ladder: Expansion velocities of HCB. Main panel: Core velocities \(v_c/J\) vs \(J_{\perp}/J\). Inset: Radial velocity \(v_r/J\) vs \(J_{\perp}/J\). Diamonds: Experimental (E) results [8] for the 1D-2D crossover at \(U/J = 20\). All other data represent theoretical (T) results for HCB \((N = 12)\).](image-url)
Acknowledgments.—We thank N. Andrei, C. Bolech, and M. Rigol for fruitful discussions, and we thank S. Thwaites and J. P. Ronzheimer for a critical reading of the manuscript. L.V. is supported by Alexander von Humboldt Foundation. S.L. was financially supported by FOR 801.

For the Deutsche Forschungsgemeinschaft (DFG) and the Discovery Projects funding scheme (Project No. Centre of Excellence for Engineered Quantum Systems Humboldt Foundation. S.L. was financially supported through FOR 801.

and M. Rigol for fruitful discussions, and we thank S. Thwaites and J. P. Ronzheimer for a critical reading of the manuscript. L.V. is supported by Alexander von Humboldt Foundation. S.L. was financially supported by FOR 801.

and M. Rigol for fruitful discussions, and we thank S. Thwaites and J. P. Ronzheimer for a critical reading of the manuscript. L.V. is supported by Alexander von Humboldt Foundation. S.L. was financially supported through FOR 801.
B 61, 12474 (2000).

[35] K. Rodriguez, S. Manmana, M. Rigol, R. Noack, and A. Muramatsu, New J. Phys. 8, 169 (2006).

[36] Supplementary material: Finite-size analysis, extraction of velocities, expansion for fermions from product states, discussion of a harmonic trap, and additional figures.

[37] F. Heidrich-Meisner, S. R. Manmana, M. Rigol, A. Muramatsu, A. E. Feiguin, and E. Dagotto, Phys. Rev. A 80, 041603 (2009); J. Kajala, F. Massel, and P. Törmä, Phys. Rev. Lett. 106, 206401 (2011); D. Karlsson, C. Verdozzi, M. Odashima, and K. Capelle, EPL 93, 23003 (2011); S. Kellner, I. P. McCulloch, and F. Marquardt, preprint arXiv:1301.6951.

[38] The expansion of bosons in the 1D-2D crossover was also studied in I. Hen and M. Rigol, Phys. Rev. Lett. 105, 180401 (2010).

[39] M. Polini and G. Vignale, Phys. Rev. Lett. 98, 266403 (2007).

[40] S. Langer, F. Heidrich-Meisner, J. Gemmer, I. McCulloch, and U. Schollwöck, Phys. Rev. B 79, 214409 (2009); S. Langer, M. Heyl, I. P. McCulloch, and F. Heidrich-Meisner, 84, 205115 (2011).

[41] Y.-A. Chen, S. D. Huber, S. Trotzky, I. Bloch, and E. Altman, Nature Phys. 7, 61 (2011).

[42] This can easily be fulfilled by using a blue-detuned short period lattice and suitable beam waists.
**Supplementary material**

For the Bose-Hubbard model on a single chain, we describe how we calculate expansion velocities and clarify the finite-size effects. Furthermore, we briefly discuss the case where the initial confining potential is given by a harmonic trap instead of a box trap. For the Fermi-Hubbard model on a single chain, we discuss the influence of the initial non-eigenstates on the results presented in the inset of Fig. 3. Finally, we comment on the calculation of expansion velocities and finite-size effects for hard-core bosons on a two-leg ladder.

**BOSE-HUBBARD MODEL ON A SINGLE CHAIN**

**Radial velocities**

In this Section we clarify how the radial velocities \( v_r \) of the Bose-Hubbard model on a single chain, presented in Fig. 2 are extracted from real-time calculations using the t-DMRG method with a discarded weight \( \eta \leq 10^{-4} \). In all our data, we used a time step \( \Delta tJ = 1/16 \) and checked different values of \( N_{\text{max}} \) to verify that the results are independent of \( N_{\text{max}} \) (\( N_{\text{max}} \) denotes the maximal number of bosons per site used in t-DMRG calculations). We limit the analysis to the expansion from the ground state, while the analysis of the expansion from a product of local Fock states was performed in Ref. 8.

Unless stated otherwise, the confining potential at \( t < 0 \) is represented by a box trap, see also Fig. 1. A box trap for a single chain is defined as

\[
V_{\text{trap}}^{(b)} = V_b \left( \sum_{i=1}^{n_a} n_i + \sum_{i=n_a}^{L} n_i \right),
\]

where \( V_b = 10^3 J \) and \( n_a < n_b \). The initial density is \( n = N/L_{\text{box}} = 1 \), where \( N \) represents the total number of bosons and the box size is given by \( L_{\text{box}} = i_b - i_a - 1 \).

![Figure S2](image)

**FIG. S2.** (Color online) Single chain: Expansion dynamics of interacting bosons from the ground state. (a) Time-dependence of the radial velocity \( v_r(t) = \partial \tilde{R}(t)/\partial t \). At \( U/J = 1 \), we show results for \( N = 6, 8, 10, 12, 14 \) particles with \( N_{\text{max}} = 5 \). At \( U/J = 4 \), the four nearly overlapping curves show results for \( N = 6, 8, 10, 12 \) particles with \( N_{\text{max}} = 5, 5, 4, 3 \), respectively. For the data shown in this figure, a discarded weight \( \eta = 10^{-5} \) was used. Horizontal solid lines on the right-hand-side of the figure indicate the extrapolated values of \( v_r(t) \) at \( t J \gg 1 \). These values (with the corresponding error bars) are used in (b) where the finite-size scaling is performed. (b) \( v_r(N) \) for different number of particles \( N \). Large symbols at \( 1/N \sim 0 \) denote the extrapolated values when \( N \rightarrow \infty \). These values (with the corresponding error bars) are used in Fig. 3 (circles) of the main text where \( v_r \) vs \( U/J \) is plotted.

The radius \( \tilde{R}(t) \) of the expanding cloud of bosons for different values of \( U/J \) is shown in Fig. S1. We see that the radius can be approximated by \( \tilde{R}(t) \propto t \) in a wide time-window, including short times just after the sudden release of the trap. Such a time dependence of \( \tilde{R}(t) \) is similar to the case of 1D two-component Fermi gases studied in Ref. 23.

![Figure S1](image)

**FIG. S1.** (Color online) Single chain: Expansion dynamics of interacting bosons from the ground state. Time dependence of the radius \( \tilde{R}(t) \). We used \((N,N_{\text{max}}) = (14,5) \) for \( U/J = 1 \) and \((N,N_{\text{max}}) = (10,4) \) for \( U/J = 4 \). Circles: Expansion of hard-core bosons (HCB) for which \( \tilde{R}(t) = \sqrt{2} J t \).
$v_r(t)$ becomes stationary increases for smaller $U/J$. We define the radial velocity, presented in the main panel of Fig. S2 as the asymptotic value $v_r = v_r(t \rightarrow \infty)$ when $N \rightarrow \infty$.

We pursue the following two-step process to calculate $v_r$: (i) For a fixed number of particles $N$, we obtain $v_r(N) = v_r(N; t \rightarrow \infty)$ by taking the value at the largest time available from our simulations, provided that the change of $v_r(t)$ in the last few time units (typically 5-10 time units) is below 1%. This is shown in Fig. S2(a). In addition, Fig. S2(a) reveals that $v_r(N)$ is essentially $N$-independent for $U/J = 4$, while this is no longer the case for $U/J = 1$: (ii) We perform a fit $v_r(N) = \alpha \frac{1}{\sqrt{N}} + \beta$, yielding the desired value $v_r = \beta$. The fits are shown in Fig. S2(b). Both steps produce an uncertainty which is then assigned to $v_r$ (in case no error bar is indicated in the figures, it implies that it is smaller than the size of the symbol).

**Density profiles**

In Fig. S3(a) we show density profiles $\langle n_i(t) \rangle$ for different $U/J$. We plot $\langle n_i(t) \rangle$ at $tJ = 25$ in Fig. S3(a), showing that curves at $U > U_c$ are virtually identical. The same effect can be observed for any time reached in our simulations.

We further calculate the deviation of the density profiles at a finite $U/J < \infty$ from those of hard-core bosons by defining

$$\xi(U/J) = \frac{\sum_i |\langle n_i(t) \rangle_{U/J} - \langle n_i(t) \rangle_{HCB}|}{NL}.$$  

We obtain $\xi(U > U_c) \lesssim 10^{-4}$ for all $U > U_c$ up to the longest times reached in the simulations. This result supports our conjecture that the time-evolution of densities for the expansion from a bosonic MI is governed by the dynamics of spinless fermions, i.e., it cannot be distinguished from that case by measuring only densities or the radius.

**Excess energy for the expansion from $|\phi_{\text{Fock}}\rangle$**

Here we comment on the influence of the interaction quench performed simultaneously with the removal of the trap as realized in [8]. In this case, the energy of the system exceeds the one of the ground state. We define the excess energy as $\delta E = E_{\text{exp}} - E_0$, where $E_{\text{exp}}$ is the total energy during the expansion and $E_0$ is the ground-state energy. In the main part of the Letter, we discussed the expansion from a product of local Fock states $|\phi_{\text{Fock}}\rangle = \prod_i b_i^\dagger |\emptyset\rangle$, which is characterized by $E_{\text{exp}} = 0$, hence $\delta E = |E_0|$. In Fig. S4 we plot $\delta E/J$, which is a monotonic function of $U/J$. In the limit $U/J = 0$, $\delta E/N \rightarrow 2J$ as $N \rightarrow \infty$, while in the opposite limit $U/J \rightarrow \infty$, $\delta E/J \rightarrow 0$. Therefore, in the latter limit the ground state has a large overlap with $|\phi_{\text{Fock}}\rangle$, hence the interaction quench has little effect on the expansion dynamics.

**Bose-Hubbard model on a single chain: the case of a harmonic trap**

We briefly discuss the influence of the choice of the trapping potential for $t < 0$. We focus on a harmonic trap, since it models the situation realized in experiments.
on optical lattices [7,8]. A harmonic trap for a single chain is defined as

$$V_{\text{trap}}^{(h)} = V_h \sum_{i=1}^{L} n_i \left( i - \frac{L + 1}{2} \right)^2.$$  \hfill (S3)

The effective density of particles is defined as $\rho = N/\sqrt{V_h}$. In analogy to the studies for the box trap, we are interested in values of $\rho$ such that the initial ground state is either a superfluid state with $\langle n_i \rangle < 1$ for any site $i$, or a Mott insulating state with $\langle n_i \rangle = 1$ in the center of the trap and $\langle n_i \rangle < 1$ on the edges.

Our study reveals that the core observations that we made for the box trap (compare Fig. 3(a) of the main text) carry over to the expansion from the harmonic trap. In particular, the Mott insulating phase is characterized by $v_f/J = \sqrt{2}$. In fact, the Mott plateau with $v_f/J = \sqrt{2}$ can be observed in the case when $U/J$ is fixed and $U/J$ varied, as well as in the opposite case when $U/J$ is fixed and $\rho$ varied.

In Fig. S5 we present the time-dependence of the radius $R(t)$ for $U/J = 8$ and $\rho = 3.16$. For these parameter values, the ground state is a Mott insulator with $\langle n_i \rangle = 1$ in the center of the trap (the initial density profile is shown in the inset of Fig. S5). As a main result, we show that the characteristic expansion velocity $v_f/J = \sqrt{2}$ of the Mott insulator emerges both in the box trap as well as in the harmonic trap. In the main panel of Fig. S5, squares represent the numerical data while the solid line represents $R(t) = \sqrt{2}J t$.

FERMI-HUBBARD MODEL ON A SINGLE CHAIN

As an important part of our discussion in the main part of the Letter, we compared our results for the expansion of bosons with the expansion of a two-component Fermi gas. The Fermi-Hubbard model on a single chain is defined as

$$H_{FH} = -J \sum_{i=1}^{L-1} \sum_{\sigma \in \{\uparrow, \downarrow\}} (c_{i,\sigma}^\dagger c_{i+1,\sigma} + \text{h.c.}) + U \sum_{i=1}^{L} n_{i,\uparrow} n_{i,\downarrow} + V_{\text{trap}} \theta(-t),$$ \hfill (S4)

where $n_{i,\sigma} = c_{i,\sigma}^\dagger c_{i,\sigma}$ represents the on-site density of a single fermionic component. We restrict our discussion to the box trap only, i.e., $V_{\text{trap}} = V_{\text{trap}}^{(b)}$ as given by Eq. (S2).

Expansion from the ground state at $n = 1$

The expansion of a two-component gas expanding from the Mott insulator was studied in [24]. Figure S4(b) shows the density profiles for various values of $U$ at time $tJ = 10$. These are all virtually identical. Since the density profiles of spinless fermions ($U = \infty$) are identical to the ones of hard-core bosons, this implies that the density profiles for the expansion from any 1D bosonic or fermionic Mott insulator are practically indistinguishable.
Expansion from product states

The expansion dynamics as a function of \( n \) and \( U/J \) from the ground state of the Fermi-Hubbard model, Eq. (S4), was investigated in detail in Ref. [23, 29, 37]. In this work we are interested in the expansion from a product of local Fock states, in analogy to the boson experiment [8]. For the case of two-component fermions we have an arbitrary product of local Fock states \(|\phi_w^{(j)}\rangle\) has \( W \) domain walls if

\[
\sum_{i=1}^{L_{\text{box}}-1} \sum_{\sigma \in \{\uparrow, \downarrow\} } \langle \phi_w^{(j)} | c_{i,-\sigma} c_{i+1,\sigma} c_{i+1,-\sigma} | \phi_w^{(j)} \rangle = W. \tag{S5}
\]

To decrease boundary effects we construct the initial states as a superposition of all states having \( W \) domain walls

\[
|\phi_W\rangle = \frac{1}{\sqrt{N_W}} \sum_{j=1}^{N_W} |\phi_w^{(j)}\rangle, \tag{S6}
\]

where \( N_W \) denotes the total number of states having \( W \) domain walls for a fixed density and value of \( L_{\text{box}} \). For \( W_{\text{max}} = L_{\text{box}} - 1 \), the initial state is the Néel state \(|\phi_N\rangle\). We investigate systems with a fixed particle number \( N = 10 \) and zero magnetization (\( N_\uparrow = N_\downarrow = 5 \)).

Radial velocities \( v^c_r \) for the different initial states described above are shown in Fig. S5(a). There are two common limits for all initial states: (i) For noninteracting fermions (\( U/J = 0 \)) the radial and average expansion velocities are equal and time-independent, \( v^c_r = v^c_w \).

Furthermore, since the initial MDF of any initial state is flat, it follows that \( v^c_r = \sqrt{2J} \). (ii) In the opposite limit \( U/J \to \infty \), the charge dynamics of the system become identical for any initial state \(|\phi_w\rangle\) irrespective of the magnetization including, in particular, the state with maximal magnetization, i.e., single-component (spinless) fermions. Therefore, the expansion is again ballistic with \( v^c_r = \sqrt{2J} \). Note that these two limits behave analogously to the 1D Bose-Hubbard model as discussed in Ref. [8].

The regime of intermediate \( U/J \) exhibits the strongest \( W \)-dependence. If the initial wavefunction contains states with large domains, i.e., \( W \ll L_{\text{box}} \) their local configuration resembles that of spinless fermions (characterized by \( v^c_r/J = \sqrt{2} \)) for any \( U/J \). Indeed, our results for small \( W \) approach this limit. On the other hand, the minimum of \( v^c_r = v^c_r(U/J) \) is the lowest for the initial Néel state (\( W_{\text{max}} = L_{\text{box}} - 1 \)). Note that the energies of all the initial states are degenerate, \( E_{\text{init}} = 0 \). For the initial states studied here, the excess energy \( \delta E \) monotonically increases with decreasing \( U/J \). In the extreme limits, \( \delta E/JN = 4/\pi \) at \( U/J = 0 \) (as \( N \to \infty \)), while \( \delta E = 0 \) at \( U/J = \infty \). This effect influences the expansion velocities at large \( U/J \), where the deviation of \( v^c_r \) from \( \sqrt{2J} \) follows the trend of the excess energy given to the system. In the opposite limit of \( U/J \to 0 \), the noninteracting approach is followed, which again yields \( v^c_r = \sqrt{2J} \). As a consequence, \( v^c_r \) has a minimum at intermediate \( U/J \) and the dip becomes more pronounced for larger \( W \). Note, though, that the \( W \)-dependence that is evident in our data shown in Fig. S6 implies that dynamics measure through \( v^c_r \) does not only depend on the excess energy \( \delta E \) since all initial states described by Eq. (S6) have the same \( \delta E \).

Further insight into the expansion dynamics is provided by the calculation of double occupancies during the expansion,

\[
n_{\downarrow\downarrow}(t) = \frac{1}{N} \sum_{i} \langle c_{i,\downarrow}^\dagger c_{i,\downarrow} c_{i+1,\downarrow}^\dagger c_{i+1,\downarrow} \rangle, \tag{S7}
\]

which obey \( n_{\downarrow\downarrow}(t = 0) = 0 \) for any initial state \(|\phi_w\rangle\) used in our study. In Fig. S6(b) we plot \( n_{\downarrow\downarrow}(t) \) at time \( tJ = 1 \) for different \( U/J \). Since the total energy is conserved during the expansion, the formation of double occupancies is possible only at the expense of reduced kinetic energy. At a fixed time after opening the trap, \( n_{\downarrow\downarrow} \) decreases monotonously as a function of \( U/J \). Furthermore, the formation of double occupancies can, on short timescales, only occur at sites with antiparallel neighboring spins, which in turn decreases the expansion velocity at finite \( U/J \). Hence, \( n_{\downarrow\downarrow}(t) \) increases as a function of \( W \), as observed in Fig. S6(b).

**HARD-CORE BOSONS ON A TWO-LEG LADDER**

Radial and core velocities

For a two-leg ladder, we study two different expansion velocities originating from complementary measures of the expanding cloud of particles. Besides the radius \( R(t) \), we also calculate \( r_c(t) \), which is defined as the half-width-at-half-maximum of the density distribution \( \langle n_i(t) \rangle \). In the case when the half of the maximal local density is measured at more than one site, \( r_c \) corresponds to the outermost site. This definition follows Ref. [8].

Figure S7(a) shows \( r_c(t) \) for different values of \( J_\perp/J \). After some short transient dynamics, considerable differences occur in the time dependence of \( r_c \). This can be understood from the structure of the density profile \( \langle n_i(t) \rangle \) shown in Fig. 4. We use the fitting function \( r_c(t) = v_c t + \gamma \) in the time interval \( 2 \leq tJ < 5 \) to avoid transient dynamics and the kinks in \( r_c(t) \).
result, the core velocity $v_c$, plotted in the main panel of Fig. 5, exhibits a strong dependence on $J_{\perp}/J$, ranging from $v_c/J \approx 2$ at $J_{\perp}/J = 0$ to $v_c/J \approx 0$ at $J_{\perp}/J = 1$.

For comparison we show $\tilde{R}(t)$ in Fig. S7(b), which essentially exhibits the same properties, however with a less dramatic drop of the expansion velocity as a function of $J_{\perp}/J$. As in the case for $r_c(t)$, $\tilde{R}(t)$ is not linear in time but undergoes transient dynamics. We obtain the radial velocity $v_r$ from fitting $\tilde{R}(t) = v_r t + \delta$ to the numerical data in the time interval $2 \leq t J < 5$. Results for $v_r$ are shown in the inset of Fig. S9. They suggest that due to the formation of the stable high-density core for $J_{\perp} \gtrsim J/2$ [see Fig. 4(b)], $v_r$ is decreased by a factor of $\sim 2.5$ with respect to the expansion on uncoupled chains, $J_{\perp} = 0$.

In Fig. S8 we show the dependence of the core velocity $v_c$ on the particle number $N$. Finite-size effects disappear as we approach the integrable limit $J_{\perp} = 0$ and become very small for $J_{\perp} \rightarrow J$. For the available values of $N$, the drop of $v_c$ from $2J$ to zero occurs at $J_{\perp}/J \approx 0.5$.

Mapping to spinless fermions

In the main part of the Letter we argued that hard-core bosons in 1D can be mapped to noninteracting spinless fermions such that $H_{\text{HC}} = \sum_k \varepsilon_k n_k^\dagger n_k$, where $\varepsilon_k = -2J \cos k$. Since the fermionic momenta $n_k^\dagger$ are conserved quantities, the expansion dynamics is ballistic, characterized in terms of expansion velocities as $\tilde{R}(t) = \sqrt{2J} t$.

On a two-leg ladder, it is still possible to map hard-core bosons to spinless fermions. However, it this case only a mapping to interacting spinless fermions is possible, since the sign of the hopping matrix element depends on the occupation number of other sites. In our calculation, we follow the numbering indicated in Fig. S9. The corresponding Hamiltonian reads

$$H = -J \sum_{i=1}^{2L-1} (c_i^\dagger c_{i+1} + \text{h.c.})$$

$$- J_\perp \sum_{i=1}^{L} (c_i^\dagger \prod_{j=i+1}^{2L-i} (1 - 2n_j) c_{2L+1-i} + \text{h.c.}) \tag{S8}$$

We define the momentum distribution function $n_k^f$ on a two-leg ladder with $L$ rungs as

$$n_k^f = \frac{1}{2} (\langle n_{k_x,k_y=0} + n_{k_x,k_y=\pi} \rangle) \tag{S9}$$

$$n_k^f = \frac{1}{2L} \sum_{\mathbf{r},\mathbf{r}^\prime} e^{i(\mathbf{r}-\mathbf{r}^\prime) \cdot \mathbf{k}} \langle c_{\mathbf{r}^\prime}^\dagger c_{\mathbf{r}} \rangle \tag{S10}$$

where $\mathbf{r}$ and $\mathbf{r}^\prime$ represent vectors associated to sites on the ladder. As the main difference in relation to the 1D system, $n_k^f$ is not conserved during the expansion. As an example, we show $n_k^f$ for $J_{\perp} = J$ at different times in Fig. S9.

FIG. S7. (Color online) Two-leg ladder: Expansion dynamics of hard-core bosons from a local product of Fock states, $|\phi_{\text{naa}}\rangle$. (a) Time dependence of the half-width-at-half-maximum $r_c(t)$ for different $J_{\perp}/J$. (b) Time dependence of the radius $\tilde{R}(t)$ for different $J_{\perp}/J$. We used $N = 12$ particles. The corresponding velocities $v_c$ and $v_r$ vs $J_{\perp}/J$ are shown in Fig. 5. Both velocities are extracted from a linear fit of $r_c(t)$ and $\tilde{R}(t)$ in the time interval $2 \leq t J < 5$. We show $v_c$ vs $J_{\perp}/J$ for different particle numbers $N$ in Fig. S8.

FIG. S8. (Color online) Two-leg ladder: Expansion dynamics of hard-core bosons from a local product of Fock states, $|\phi_{\text{naa}}\rangle$. Core expansion velocity $v_c$ as a function of $J_{\perp}/J$ for different particle numbers $N$. 

FIG. S9.
FIG. S9. (Color online) Two-leg ladder: Momentum distribution function $n^f_k(t)$, Eq. (S9), for interacting spinless fermions (which can be mapped to hard-core bosons), Eq (S8), at different times. The inset shows the fermionic counting applied in calculations on a two-leg ladder with $L$ rungs and $J_\perp = J$. We used $N = 4$ and $L = 16$. 