ANALYSIS OF THE EFFECT OF PROPELLANT TEMPERATURE ON INTERIOR BALLISTICS PROBLEM

C. Evci1,*, H. Işık2

ABSTRACT

This study investigates the effect of conditioning temperature of double base propellants on the interior ballistic parameters such as burning gas temperature, barrel wall temperature, pressure and stresses generated in the barrel. Interior ballistic problem was solved employing experimental, numerical and analytical methods with a thermo-mechanical approach. Double base propellants were conditioned at different temperatures (52, 35, 21, 0, -20, -35, -54°C). The maximum pressure in the barrel and projectile muzzle velocity were measured for all the propellants by conducting shooting tests with a special test barrel using 7.62x51 mm NATO ammunition. Vallier-Heydenreich method was employed to determine the transient pressure distribution along the barrel. The temperature of burnt gases was calculated by using Noble-Abel equation. The heat transfer analysis was done using the commercial software ANSYS to get the transient temperature and stress distributions. Temperature distribution through the barrel wall thickness was validated using a FLIR thermal imager. Radial, circumferential and axial stresses and corresponding equivalent Von Misses stresses were determined numerically and analytically. The results of the analytical solution for stress analysis validated the finite element solution of interior ballistic problem. Increasing the initial temperature of the propellant resulted in higher temperature and pressure inside the barrel which in turn increased the stresses in the barrel.

Keywords: Interior Ballistics, Heat Transfer, Stress Analysis, Experimental Verification

INTRODUCTION

A weapon system consists of many complex subsystems and designing them requires solving very complex processes. One of the most critical parts of such studies is the firing tests that have to be conducted. Also, the establishment of a test set up and conducting the firing tests and measurements are dangerous. Moreover, calibrating the set up accurately requires special work.

The barrel is one of the most critical parts while designing a weapon system. To examine the firing process that occur in a barrel is quite complex. Designing a barrel requires various internal ballistic parameters to be calculated. Determination of the inner gas pressure distribution along the barrel and the muzzle velocity are the most important steps before going on to the next stage. The gas pressure is influenced by many parameters such as propellant properties and climatic conditions. When the propellant is ignited in the combustion chamber the gas temperature increases rapidly and heat is transferred in milliseconds into the barrel which gives rise to high temperatures and stresses in the barrel. It is very difficult to estimate the temperature and stress distribution through the inner surface of the barrel. Especially, in multiple firings the barrel is exposed to heating and structural problems as a result of the high temperatures and pressures such as corrosion and wear. Therefore, it is very important to know both the mechanical and thermal behavior of a barrel.

Several studies have been carried out on examining the internal ballistics of a barrel. Jaramaz et. al. [1] studied the two-phase flow in a barrel theoretically and experimentally. Akçay and Yükselen [2] carried out an experimental and a numerical study to calculate the temperature distribution through a barrel. Sun and Zhang [3] have studied the maximum transient temperature changes in a barrel. Nelson and Ward [4] tried to determine the transient heat transfer into the barrel. Conroy [5] developed a ballistic code for modelling two-phase heating of a barrel. Mishra et. al. [6] developed a numerical code to calculate the heat transfer to the barrel. Cronemberger et. al. [7] studied theoretically and experimentally the interior ballistics of a 7.62 mm rifle. Hill and Conner [8] developed a model for transient heat transfer through the barrel wall. Değirmenci and Dirikolu [9] determined the convection heat transfer coefficient of the gases for calculating the temperature distribution in a barrel by using a thermochemical approach. Şentürk et. al. [10] studied interior ballistics of a 7.62 mm gun barrel using
experimental, numerical and analytical methods with a thermo-mechanical approach. Değirmenci et. al. [11] studied the combustion characteristics of double base propellants with various grain sizes and determined the temperatures and stresses in a barrel by using thermo-mechanical approach.

In this study, three-dimensional transient heat transfer and stress analysis of a 7.62 mm rifle were analyzed with a thermo-mechanical approach to see the whole thermo-mechanical effect on the barrel. The firing tests were conducted with double based 7.62x51 mm standard NATO ammunition. After obtaining experimentally the maximum pressure and muzzle velocity in the test barrel, the pressure distribution and projectile velocity along the barrel were determined for various climatic conditions. Then, the temperature of burnt gases along the barrel was determined using Noble-Abel equation. The convection heat transfer coefficient was determined by using Vielle’s burning law and put into ANSYS finite element solver to obtain the temperature distribution. The temperature distribution through the barrel wall was validated by the temperature readings taken at the outer barrel surface using a thermal imager. At the end, pressure and temperature data along the barrel were employed in 3-D stress analysis to get the stresses. The results of the finite element (ANSYS) and analytical solutions were closely conforming to each other. Consequently, a thermo-mechanical approach covering heat transfer and stress analysis was carried out in this study using experimental, numerical and analytical methods.

THEORETICAL FORMULATION

Interior ballistics theory is concerned with all of the phenomena that occur in a barrel during a firing process. When the propellant is ignited, the burnt gases fill whole of the combustion chamber in milliseconds and create very high pressure and temperature. The projectile is moved forward under the backside pressure till the muzzle of the barrel with high acceleration. Meanwhile, heat transfer takes place into the barrel. To calculate the heat transfer, convection heat transfer coefficient needs to be found. After getting the maximum pressure and muzzle velocity experimentally for various climatic conditions the Vallier-Heydenreich method [7,10] was used to calculate the pressure distribution and the projectile velocity along the barrel.

Then, the Noble-Abel equation was used to calculate the gas temperature in the barrels [10],

\[ P(V_t - \eta) = m_g RT_g \]  
(1)

where \( P \) is the gas pressure in the barrel, \( V_t \) is the total volume, \( m_g \) is the mass of gases, \( T_g \) is the gas temperature, \( R \) is the gas constant and \( \eta \) is the covolume which can be obtained by using closed bomb vessel [12,13] or generally taken as the specific volume [14].

The total volume in Equation (1) is determined as [10],

\[ P \left[ V_c + V(x) - m_p \left( \varphi \eta + \frac{1 - \varphi}{\rho} \right) \right] = m_p \varphi RT_g \]  
(2)

where \( V_c \) is the combustion chamber volume, \( V(x) \) is the expanded volume in the barrel as the projectile moves, \( m_p \) is the mass of propellant, \( \varphi \) is the percentage of the burned propellant, \( \rho \) is the gas density and \( x \) is distance moved by the projectile.

According to the Vieille’s burning law [15] the burning rate is,

\[ \frac{dS}{dt} = \beta P^n \]  
(3)

where \( \beta \) and \( n \) are the constants depending on the propellant respectively [9, 10].

The convection heat transfer coefficient of the gases \((h_g)\) is calculated from [16],

\[ Nu = \frac{h_g L}{k} = 0.023 Re^{0.8} Pr^{0.3} \]  
(4)
where $Re$ is the Reynolds number and $Pr$ the Prandtl number, $D$ is the barrel inside diameter and $k$ is the thermal conductivity. All the thermodynamic properties of burnt gases used in above equations are calculated according to the gas mixture law.

The 3-D transient heat transfer equation in cylindrical coordinates and the initial/boundary conditions are written in the form of,

$$\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left( k \frac{\partial T}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \bar{q} = \rho c \frac{\partial T}{\partial t}$$

(5)

for $t = 0$ and $r_i \leq r \leq r_o$, $T(r, t) = T_\infty$

(6)

for $t > 0$, $h_g (T_g - T_{wi}) = -k \frac{\partial T}{\partial r} \bigg|_{r=r_i}$

$$h_a (T_{wo} - T_\infty) = -k \frac{\partial T}{\partial r} \bigg|_{r=r_o}$$

(7)

where $k$ is the thermal conductivity ($W/mK$), $\bar{q}$ is the heat generation per unit volume ($W/m^3$), $\rho$ is the density, $t$ the time, $T$ is the temperature distribution, $c$ is the specific heat (J/kgK), $r$, $z$, $\theta$ are the radial, axial and circumferential coordinates, $r_i/r_o$ is the inner/outer radii of the barrel, $h_g/h_a$ is the gas/air convection heat transfer coefficient, $T_{wi}/T_{wo}$ is the inner/outer barrel surface temperatures, $T_g$ is the inner gas temperature and $T_\infty$ is the temperature of the air nearby the outer barrel surface. The radiation heat transfer is neglected.

The convection coefficient $h_a$ for low speed (~1.0-2.0 m/s) air flow around the barrel was calculated from Eq. (8) for cross flow over a cylinder [16].

$$N u_b = \frac{h_a D}{k} = 0.683 Re^{0.466} Pr^{1/3}$$

(8)

where $Nu_b$ is the Nusselt number for the barrel, $D$ is the barrel outer diameter, $Re$ is the Reynolds number and $Pr$ is the Prandtl number. The convection coefficient for the air flow around the barrel was obtained nearly as 25 W/m²K throughout the numerical analysis.

Using the generalized Hooke’s law, stress-strain relations in the elastic region can be expressed by the following set of equations

$$\varepsilon_r = \frac{1}{E} \left( \sigma_r - v \sigma_\theta - v \sigma_z \right) + \alpha T$$

(9)

$$\varepsilon_\theta = \frac{1}{E} \left( \sigma_\theta - v \sigma_r - v \sigma_z \right) + \alpha T$$

(10)

$$\varepsilon_z = \frac{1}{E} \left( \sigma_z - v \sigma_r - v \sigma_\theta \right) + \alpha T$$

(11)

where $\varepsilon_r$, $\varepsilon_\theta$, $\varepsilon_z$ are the strain components and $\sigma_r$, $\sigma_\theta$, $\sigma_z$ are the stress components in the radial, circumferential and axial directions, respectively, $E$ is the modulus of elasticity, $v$ is the Poisson’s ratio and $\alpha$ is the thermal expansion coefficient.

Mechanical and thermal loading of the barrel case may be assumed to be axisymmetric. The barrel with an open end is axially free to expand and it is assumed that axial strain ($\varepsilon_z$) is constant.
\[ \varepsilon_z = \frac{1}{E} (\sigma_z - v\sigma_r - v\sigma_\theta) + aT = d \]  

By means of Eq.12, the stress component in the axial direction is obtained as

\[ \sigma_z = v(\sigma_r + \sigma_\theta) + (d - \alpha T)E \]  

Inserting the expression given by Equation 13 for the axial stress into Equation 9 and Equation 10 yields

\[ \varepsilon_r = \frac{1}{E} [(1 - v^2)\sigma_r - v(1 + v)\sigma_\theta - v dE + (1 + v) E \alpha T] \]  

\[ \varepsilon_\theta = \frac{1}{E} [(1 - v^2)\sigma_\theta - v(1 + v)\sigma_r + (1 + v) E \alpha T - v dE] \]  

Equation 14 and Equation 15 are used together to get the radial and circumferential stresses as following

\[ \sigma_r = \frac{(1 - v)E}{(1 + v)(1 - 2v)} \varepsilon_r + \frac{vE}{(1 + v)(1 - 2v)} \varepsilon_\theta - \frac{E \alpha T}{(1 - 2v)} + \frac{vdE}{(1 + v)(1 - 2v)} \]  

\[ \sigma_\theta = \frac{(1 - v)E}{(1 + v)(1 - 2v)} \varepsilon_\theta + \frac{vE}{(1 + v)(1 - 2v)} \varepsilon_r - \frac{E \alpha T}{(1 - 2v)} + \frac{vdE}{(1 + v)(1 - 2v)} \]  

where the radial and circumferential strains are expressed by

\[ \varepsilon_r = \frac{du}{dr} \]  

\[ \varepsilon_\theta = \frac{u}{r} \]  

where \( u \) is the radial displacement and \( r \) is the radial coordinate. The equilibrium equation in the radial direction is given by

\[ \frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \]  

Substituting Equation 16 and Equation 17 in Equation 20 the following form of differential equation is obtained

\[ \frac{d^2u}{dr^2} + \frac{1}{r} \frac{du}{dr} = \frac{E (1+v) \alpha(T_i-T_o)}{\ln \left( \frac{a}{b} \right)} \frac{1}{r} \]  

where \( T_i \) and \( T_o \) are the inner and outer surface temperatures of the barrel, \( a \) and \( b \) are the inner and outer radii.

After solving for the radial displacement and then the radial and circumferential strains, stress components in the radial, circumferential and axial directions are obtained as follows

\[ \sigma_r = \frac{P_i}{n^2-1} + \frac{Ea}{2(1-v)} \frac{n^2T_0-T_i}{n^2-1} + \frac{Ea}{4(1-v)} \frac{\Delta T}{\ln n} + \frac{b^2}{(n^2-1)r^2} \left[ -P_i + \frac{Ea\Delta T}{2(1-v)} \right] - \frac{Ea}{2(1-v)} \left[ T_i - \frac{\Delta T}{\ln n} \left( \ln \frac{r}{a} \frac{1}{2} \right) \right] \]  

2130
\[
\sigma_\theta = \frac{P_i}{n^2-1} + \frac{E\alpha}{2(1-\nu)} \frac{n^2T_0-T_i}{n^2-1} + \frac{E\alpha}{4(1-\nu)} \frac{\Delta T}{\ln n} - \frac{b^2}{(n^2-1)r^2} \left[ -P_i + \frac{E\alpha\Delta T}{2(1-\nu)} + \frac{1}{2(1-\nu)\ln n} \right] + \frac{1}{2(1-\nu)\ln n} \left( 1 + \ln \frac{r}{a} \right) \]
\[
\sigma_z = \frac{a\Delta T a^2}{(1-\nu)(b^2-a^2)} - \frac{2vP_i}{E(n^2-1)} + \frac{\nu\alpha}{1-\nu} \frac{n^2T_0-T_i}{n^2-1} + \frac{\nu\alpha}{2(1-\nu)\ln n} \frac{\Delta T}{(1-\nu)} - \alpha \left( T_i - \frac{(T_i-T_0)}{\ln n} \frac{r}{a} \right) \]
\]
where \(P_i\) is the internal pressure in the barrel, \(n=a/b\) and \(\Delta T = T_i - T_o\).

The equivalent stress of the principal stresses which are radial, circumferential and axial stresses (given by Equation 22, Equation 23 and Equation 24, respectively) in the barrel can be obtained by using the Von Mises theory as

\[
2\sigma_E^2 = (\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_r - \sigma_z)^2
\]

**EXPERIMENTAL STUDY**

The firing tests were conducted with double based 7.62x51 mm standard NATO ammunition conditioned at -54°C, -35°C, -20°C, 0°C, +21°C, +35°C and +54°C. Ballistic tests were conducted according to the NATO standards [17]. The maximum pressure in the barrel was measured by using piezoelectric transducer and the projectile velocity was measured by the method of parallel velocity plates located at a distance of 23.77 m as seen in Figure 1.

Although, they are manufactured in the same conditions specified with a particular lot number, propellants show slightly different characteristics. Therefore, 10 rounds have been fired in every separate firing test and the mean values of pressure and velocity data were calculated to use as input values in the empirical, numerical and analytical calculations.

The firing tests were conducted as single and multiple (20 rounds) firings. The results for the temperature distribution at the outer barrel surface measured for both tests with a FLIR thermal imager came out to be very accurate.

\[\text{(a)}\quad \text{(b)}\]

**Figure 1.** Firing test setup, (a) Test barrel, (b) G3 Rifle

**NUMERICAL ANALYSIS**

The three-dimensional model of the test barrel was drawn by using the ANSYS program. To model the projectile location transiently, the barrel was cut into 19 parts beginning from the rifled section of barrel and the mesh was formed around the barrel as seen in Figure 2. As seen in the figure the parts around the maximum pressure were put closer to each other.
For the thermal analysis, all the inner temperatures and convection coefficient values of the separate 19 sections of three-dimensional barrel geometry were put into ANSYS solver and the convection coefficient for the ambient air was taken constant as 25 W/m²K throughout the calculations.

Figure 2. The mesh formed around the barrel

**RESULTS AND DISCUSSION**

**Thermal Analysis**

Before the thermal analysis, the gas pressures for various climatic conditions of propellant ranging from -54 °C to +52 °C were calculated as shown in Figure 3. It was observed that the maximum pressure for the initial propellant temperature range of -54 °C to +52 °C increased from 306 MPa to 367 MPa. Meanwhile, the gas temperature inside the barrel increased from 2478 K to 2974 K as seen in Figure 4. The inner and outer temperature values for the whole barrel were analyzed three-dimensionally with all the necessary data put into the ANSYS solver. During testing, the ambient temperature was measured as 16 °C whereas the surface temperature of the barrel was 18 °C. The firings with 20 shots were completed in 2 seconds. The 3-D temperature distribution for initial propellant temperature of 21°C obtained from ANSYS solver for multiple 20 shots are shown in Figure5. The temperature rise in the combustion chamber and the gun barrel is one of the most important design criteria that must be accounted for. Lastly, the inner barrel surface temperature distribution along the barrel for multiple 20 shots was determined as seen in Figure 6. The maximum temperature point for different climatic conditions was observed to be obtained nearly at the maximum pressure section.

Figure 3. Pressure distribution along the barrel of multiple 20 shots
Figure 4. Gas temperature distribution along the barrel of multiple 20 shots

Figure 5. Half section view of the temperature distribution along the barrel corresponding to multiple 20 shots

Figure 6. The inner surface temperature distribution along the barrel for multiple 20 shots

After getting the inner barrel surface temperature distribution, heat transfer analysis was performed to find the temperature distribution on the barrel outer surface. The results of temperature distribution along the outer...
barrel surface as obtained from ANSYS solver come out to be very close to that measured using a FLIR thermal imager (See Figure 7).

![Figure 7. Outer surface temperatures obtained from ANSYS solver and thermal imager for multiple 20 shots](image)

**Stress Analysis**

Equivalent maximum stresses in 19 sections of the barrel were calculated using the Von Mises theory. Figure 8 presents the stress distribution along the barrel for multiple 20 shots. Examining this figure, it is evident that equivalent stresses decreased from the chamber to the muzzle as a consequence of higher pressure and temperature close to the chamber. Equivalent stresses along the barrel axis increased with increasing initial propellant temperature. Although this result may be an expected one, it was perfectly proved experimentally.

![Figure 8. Stress distribution along the barrel for multiple 20 shots](image)

**CONCLUSION**

The effect of the initial propellant temperature on the interior ballistics parameters were comprehensively investigated using experimental, numerical and analytical methods with a thermo-mechanical approach. Temperature distribution of burnt gasses and the pressure inside the barrel were determined along the barrel axis. Obtaining the internal gas temperature information, thermal analysis was performed using the commercial software ANSYS solver to get the inner and outer surface temperatures of the barrel which were then used together with the pressure data to calculate the radial, circumferential and axial stress components in the barrel. The equivalent maximum Von Mises stresses were calculated analytically along the barrel axis. Temperature and
pressure along the barrel as well as the equivalent stresses decreased from the chamber to the muzzle. Increasing the initial propellant temperature resulted in greater stress components in the barrel which in turn caused increased equivalent stresses along the barrel.

**ACKNOWLEDGEMENT**

The authors would like to express their thanks to MKE Gazi Fişek Fabrikasi for its valuable support.

**NOMENCLATURE**

- \( P \) Pressure in the barrel
- \( V_t \) Total volume
- \( \eta \) covolume
- \( m_g \) mass of gas
- \( R \) gas constant
- \( V_c \) combustion chamber volume
- \( V(x) \) Expanded volume in the barrel
- \( m_p \) propellant mass
- \( \varphi \) percentage of the burned propellant
- \( Nu \) Nusselt number
- \( Re \) Reynolds number
- \( Pr \) Prandtl number
- \( h_g/h_a \) convection heat transfer coefficients for gas and air
- \( D \) inner diameter of the barrel
- \( k \) thermal conductivity
- \( r_i/r_o \) inner/outer barrel radii
- \( T_{ao} \) air temperature nearby the barrel outer surface
- \( T_{wi}/T_{wo} \) wall temperatures for the inner/outers surfaces
- \( T_g \) gas temperature
- \( r, z, \theta \) radial, axial and circumferential coordinates
- \( \bar{q} \) heat generation per unit volume
- \( \rho \) density
- \( t \) time
- \( c \) specific heat
- \( s \) amount of reduction in ballistic size of propellant grains
- \( u \) radial displacement
- \( E \) modulus of elasticity
- \( \nu \) Poisson’s ratio
- \( \sigma \) stress
- \( \sigma_E \) equivalent Von Mises stress
- \( \varepsilon \) strain
- \( \alpha \) heat expansion coefficient

**REFERENCES**

[1] Jaramaz, S., Micković, D., & Elek, P. (2011). Two-phase flows in gun barrel: Theoretical and experimental studies. International Journal of Multiphase Flow, 37(5), 475-487.

[2] Akcay, M., & YÜKSELEN, M. A. (2014). Unsteady thermal studies of gun barrels during the interior ballistic cycle with non-homogenous gun barrel material thermal characteristics. J. Therm. Sci. Technol., 34(2), 75-81.

[3] Sun, Y., & Zhang, X. (2015). Transient heat transfer of a hollow cylinder subjected to periodic boundary conditions. Journal of Pressure Vessel Technology, 137(5), 051303.

[4] Nelson, C. W., & Ward, J. R. (1981). Calculation of heat transfer to the gun barrel wall (No. ARBRL-MR-03094). Army Ballistic Research Lab Aberdeen Proving Ground Md.
[5] Conroy, P. J. (1991). Gun tube heating (No. BRL-TR-3300). Army Ballistic Research Lab Aberdeen Proving Ground Md.

[6] Mishra, A., Hameed, A., & Lawton, B. (2010). Transient thermal analyses of midwall cooling and external cooling methods for a gun barrel. Journal of Heat Transfer, 132(9), 091901.

[7] Cronemberger, P. O., Lima Junior, E. P., Gois, J. A. M., & Caldeira, A. B. (2014). Theoretical and experimental study of the interior ballistics of a rifle 7.62. Engenharia Térmica (Thermal Engineering), 13(2), 20-27.

[8] Hill, R. D., & Conner, J. M. (2012). Transient heat transfer model of machine gun barrels. Materials and Manufacturing Processes, 27(8), 840-845.

[9] Değirmenci, E., & Dirikolu, M. H. (2012). A thermochemical approach for the determination of convection heat transfer coefficients in a gun barrel. Applied Thermal Engineering, 37, 275-279.

[10] Şentürk, A., İşık, H., & Evci, C. (2016). Thermo-mechanically coupled thermal and stress analysis of interior ballistics problem. International Journal of Thermal Sciences, 104, 39-53.

[11] Değirmenci, E., Evci, C., İşık, H., Macar, M., Yılmaz, N., Dirikolu, M. H., & Çelik, V. (2016). Thermo-mechanical analysis of double base propellant combustion in a barrel. Applied Thermal Engineering, 102, 1287-1299.

[12] Farrar, C. I., & Leeming, D. W. (1983). Military Ballistics a Basic Manual, Battlefield Weapons Systems and Technology, vol. 10. Royal Military College of Science, Shrivenham, UK.

[13] AMCP 706-150 (1965). “Interior ballistics of guns”, Ballistics series of the U.S. Army Materiel Command Engineering Design Handbook.

[14] John Corner. (1950). Theory of the interior ballistics of guns. Wiley.

[15] AMCP 706-247, “Design For Projection”, Engineering Design Handbook, Ammunition Series of the U.S. Army Materiel Command, 1973.

[16] Cengel, Y. A. (2003). Heat Transfer: A Practical Approach McGrawHill. New York, 492.

[17] AEP-97 “Multi calibre manual of proof and inspection (M-C MOPI) for 5.56 mm, 7.62 mm, 9 mm and 12.7 mm ammunition”, NATO Standard, NATO Army Armaments Group (NAAG), NATO Standardization Agency (NSA).