Effects of updating rules on the coevolving prisoner’s dilemma

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Abstract

We studied the effect of three strategy updating rules in coevolving prisoner’s dilemma games where agents (nodes) can switch both the strategy and social partners. Under two node-based strategy updating rules, strategy updating occurs between a randomly chosen focal node and its randomly selected neighbour. The focal agent becomes the strategy recipient and may imitate the strategy of the neighbour according to the payoff difference, i.e. voter-model-like dynamics (VMLD), or becomes a strategy donor and thus may be imitated by the neighbour, i.e. invasion-process-like dynamics (IPLD). For edge-based updating rules, one edge is selected, and the roles of the two connected nodes (donor or recipient) are randomly decided, i.e. edge-based dynamics (EBD). A computer simulation shows that partner switching supports the evolution of cooperation under VMLD, which has been utilised in many studies on spatial evolutionary games, whereas cooperators often vanish under IPLD. The EBD results lie between these two processes. This difference was prominent among nodes with large degrees, and the resulting degree heterogeneity differed based on the update rules. In addition, partner switching induced a non-monotonic relationship between the fraction of cooperators and intensity of selection under VMLD and EBD and a weak or strong selection supported cooperation. In contrast, only a strong selection supported cooperators under IPLD. Our results imply that the direction of imitation is quite important for understanding the evolutionary process of cooperation.

Introduction

The evolution of cooperation is an actively studied subject in physical and biological science [1–3]. In social interactions, cooperators must pay a cost for the benefit of others. Despite the benefit of mutual cooperation, natural selection appears to hinder the evolution of cooperation because non-cooperative individuals can receive the benefit of cooperation without bearing the cost of a cooperative act. The prisoner’s dilemma is a widely adopted framework that represents this social dilemma. In the prisoner’s dilemma game, two players (agents) simultaneously choose whether to cooperate (C) or defect (D). They will receive $R$ if both choose cooperation and $P$ if both choose defection. If one player cooperates and the other one defects, the cooperator receives $S$ and the defector receives $T$. Because the order of the payoff is $T > R > P > S$, players should choose $D$ regardless of the partner’s choice if they wish to maximise their own payoff. This temptation leads to mutual defection, although the realised payoff ($P$) is smaller than the result of mutual cooperation ($R$). However, this prediction contradicts the widely observed cooperation in actual human society.

Many models have been proposed to study the evolutionary origin of cooperation, including the effect of the network (spatial) structure. In their pioneering work, Nowak and Sigmund [4] showed that cooperation proliferates if the players are located on a two-dimensional lattice. Subsequent studies introduced complex networks that reflect the characters of real-world networks. Notably, a scale-free network gives a unified explanation on the emergence of cooperation in the prisoner’s dilemma as well as other games [5–7]. These studies highlight the importance of the heterogeneity in degree (i.e. the number of neighbours of each node). Following works examined the robustness of this phenomenon under wider conditions [8–11]. In addition, the effect of the network structure on the evolution of cooperation was investigated in other networks, including random regular graphs [12], small world networks [13][14] and actual social networks...
Furthermore, the role of networks in resolving social dilemma was investigated in combination with other mechanisms, including voluntary participation, heterogeneous teaching activity, time scale for strategy updating, payoff aspiration, conformity and punishment. In many of these models, the criterion of the continuation of the relationship depends on the agents’ strategy or payoff from the game. These studies showed that the possibility of partner switching (link adaptation) greatly enhances the evolution of cooperation compared to static graphs. The effect of coevolution was also studied with other games, including the snowdrift game, stag hunt game and ultimatum game.

In many of these studies, it is assumed during strategy evolution one randomly chosen focal agent (node) decides whether to imitate the strategy of a randomly chosen neighbour by comparing their payoff from games. This means that the role of a focal agent is fixed to a strategy recipient, whereas that of a neighbour is fixed to a strategy donor. Previous literature has also considered the opposite situation. For example, previous study found that cooperation is enhanced on various lattices if the focal agent is a recipient as opposed to a donor. The effect of updating rules was also examined in models other than the prisoner’s dilemma, and the ability of various updating rules to support the fixation of an advantageous mutant was investigated. In addition, a recent study showed that strategy updating rules can change the consequences of the evolutionary process in a well-mixed population with mutations.

In contrast to the preceding literature that examined the evolutionary process on static networks, our current work investigated the effect of strategy updating rules in coevolutionary games. Specifically, we studied the effect of the combination of link adaptation and strategy updating rules. Three updating rules used in this study arose from research on network interactions. Node-based strategy updating occurred under the first two rules: voter-model-like dynamics (VMLD) and invasion-process-like dynamics (IPLD). Specifically, under these two dynamics, one node (agent) is selected randomly, and then one neighbour of that node (agent) is selected randomly. Strategy updating occurs by comparing the payoff of these two agents. Under VMLD, agent copies the strategy of neighbour with higher probability if agent earns a larger payoff when compared with agent . In contrast, under IPLD, neighbour may imitate the strategy of agent . Therefore, a randomly chosen neighbour is served as a strategy donor under VMLD and a strategy recipient under IPLD. The last rule is edge-based dynamics (EBD). Under this rule, one link is selected randomly, and the role of the two connected agents (donor or recipient) is randomly assigned. The payoff of these two agents is compared, and a recipient copies the donor’s strategy with higher probability if a donor earns a larger payoff.

Here, we first detail our coevolutionary model and the three strategy update rules. We next report the results of a computer simulation. Lastly, we discuss the implication of our results for the modelling of the evolutionary process of human cooperation.

**Model**

Let us assume that agents are located on a (social) network defined by the neighbours of each node. Links between nodes represent the social relationship. Initially, each agent has the same number of edges that are randomly linked to other nodes. Half of the agents, who are chosen randomly, are cooperators, and the rest are defectors. We denote agents’ strategy by the two-dimensional vector . Agent is a cooperator if and a defector if . In this numerical simulation, the payoff matrix is given by:

\[ C \quad D \\
C \quad 1 \quad 0 \\
D \quad b \quad 0 \]

where is the temptation to defect . In each time step, a strategy updating event or a partner switching (link adaptation) event occurs.

Strategy updating events occur with probability . We used three types of strategy updating mechanisms. The first one is VMLD. Under this rule, one node is chosen randomly, and one of the neighbour’s agents is also selected randomly. Then, each agent plays the prisoner’s dilemma game with their neighbours and collects a payoff:

\[ \Pi_k = \sum_{l \in N_k} s_k^T A s_l, \]
where \( N_k \) is the set of agent \( k \)’s neighbours. Agent \( i \) decides whether to copy \( j \)’s strategy based on their accumulated payoff. Specifically, agent \( i \) copies the strategy of agent \( j \) with a probability based on the following equation:

\[
P(s_i \leftarrow s_j) = \left[ 1 + \exp(-\beta(\Pi_j - \Pi_i)) \right]^{-1}.
\]

The value of \( \beta \) represents the intensity of selection (\( \beta \to 0 \) implies random drift, whereas \( \beta \to \infty \) implies imitation dynamics).

The second rule is IPLD. Under this rule, two agents (\( i \) and \( j \)) are selected and accumulate a payoff (\( \Pi_i \) and \( \Pi_j \)) in the same manner as VMLD. The difference is that the neighbour \( j \) copies the strategy of agent \( i \) with the following probability:

\[
P(s_j \leftarrow s_i) = \left[ 1 + \exp(-\beta(\Pi_i - \Pi_j)) \right]^{-1}.
\]

Therefore, the chosen neighbour (\( j \)) serves as a potential strategy donor under VMLD and a potential recipient under IPLD.

The third rule is EBD. Under this rule, one edge (\( E_{ij} \)) is chosen randomly and each agent’s role in strategy updating (donor or recipient) is randomly assigned. Each agent plays the prisoner’s dilemma with their neighbours and collects a payoff. Next, a recipient (\( i \)) copies the strategy of the donor (\( j \)) with probability \( P(s_i \leftarrow s_j) \). Unlike the previous literature, which assumes that agent \( j \) copies \( i \)’s strategy unless imitation by agent \( i \) occurs [42], we limited the number of potential recipients to one to enable comparisons with the other two updating rules. Note that both agent \( i \) and \( j \) have the possibility to copy the strategy of the other agent \textit{ex ante} under EBD, whereas the role in the strategy updating is fixed under VMLD and IPLD.

With probability \( w \), partner switching (link adaptation) events occur. The basic idea of the partner switching mechanism used in our current work stems from Zimmermann et al. [56]. In their pioneering work on the coevolutionary prisoner’s dilemma game, these authors reported that cooperation can evolve if a defective agent deters the relationship with another defective agent. Here, we also assumed that edges between two defectors are unstable and that one of the defectors tries to deter the relationship. Specifically, in link adaptation events, one edge \( E_{ij} \) is chosen randomly. If the chosen edge represents a \( D \rightarrow D \) interaction, one randomly chosen agent \( i(j) \) stops the interaction with the current partner \( j(i) \) and constructs a new link with a randomly chosen agent. If the selected edge represents other situations (\( C \rightarrow C \) or \( C \rightarrow D \)), nothing occurs during that period.

Notably, we do not directly replicate the model of Zimmermann et al. For example, we use the Fermi function in strategy updating events even though it was not used in the original work because the effect of the intensity of selection is the quantity of interest in our research. In addition, we used asynchronous updating, whereas the original paper used synchronous updating, and agents who changed their strategy from \( C \) to \( D \) deterred the relationship. Because of these differences, our results should not be regarded as a validation or a criticism of the work by Zimmerman et al. Here, our goal was to examine the effects of strategy updating mechanisms.

### Results

To investigate the effects of updating rules in coevolving prisoner’s dilemma games, we conducted a numerical simulation. Each simulation run continued for \( 2 \times 10^7 \) periods, and the values of the following \( 10^6 \) periods were recorded to compute the average frequency of cooperators, unless one strategy dominated the population. We conducted 1000 independent simulations for each combination of parameters, and calculated the mean values of the simulation results. These values were always utilised unless otherwise stated.

The emerging pattern as a function of the temptation to defect (\( b \)) is shown in fig. 1. As expected, a larger temptation to defect induces less cooperation regardless of the strategy updating rule. Here, our interest was to compare the different strategy updating rules. VMLD helps the evolution of cooperation under a wider range of the temptation to defect, whereas cooperation deteriorates rapidly under IPLD. The result for the EBD lies between these two results.

We next examined the effect of the frequency of partner switching. Figure 2 shows the resulting proportion of cooperators as a function of the frequency of link adaptation (\( w \)) for different strategy updating rules. Panel (a) shows that even with a small probability of link adaptation, cooperation is greatly enhanced under VMLD and EBD, as evidenced by a small temptation to defect (\( b = 1.05 \)). In contrast, under IPLD, we observed a non-monotonic relationship between \( w \) and the proportion of cooperators. Notably, the evolution of cooperation is impeded with a higher frequency of partner switching.
Panel (b) shows the results of a larger temptation to defect \((b = 1.3)\). The effect of the frequency of link adaptation is consistent with the aforementioned results. Cooperation can evolve easily even with small \(w\) under VMLD, whereas partner switching does not help the evolution of cooperation under IPLD. The results for EBD lie between these two results. Under EBD, an extremely larger \(w\) deteriorates cooperation, suggesting that defectors can find cooperative partners using larger opportunities for partner switching.

Figures 1 and 2 show that cooperators flourish in the order of VMLD, EBD and IPLD. We can explain this pattern by considering how often the cooperators are chosen as the potential strategy donor or recipient. In the coevolving prisoner’s dilemma, it is often assumed that defectors are more likely to lose edges. Cooperators tend to have larger number of neighbours, and therefore cooperators are more likely to be chosen as the neighbour of the focal agent under VMLD and IPLD. Under VMLD, because the randomly chosen neighbour serves as the potential donor of the strategy, cooperators have a greater chance to be imitated. This advantage for cooperators was previously observed, i.e., the result that cooperators flourish despite the smaller average payoff when compared with defectors [45]. In contrast, because the neighbour serves as the potential strategy recipient under IPLD, cooperators have more opportunity to imitate others’ strategy. This difference in the frequency of cooperators becoming a donor (recipient) of the strategy supports (hinders) the evolution of cooperation under VMLD (IPLD). Because cooperators have a greater chance to imitate and to be imitated under EBD, the result lies between the other two processes.

This tendency influences how the cooperators enjoy the benefit of a larger degree. In Fig. 3 we classified agents by their degree \((k_i)\) at \(t = 10^4\), and calculated the proportion of cooperators at \(t = 2 \times 10^4\). In the typical coevolving prisoner’s dilemma game, cooperators can achieve a larger degree size, which helps cooperators resist the invasion by defectors. This relationship is confirmed in panel (a), which shows the result when \(b = 1.05\) and \(w = 0.3\). In this setting, almost full cooperation was observed (panel (a) of Fig. 2) shows the corresponding final outcomes). Indeed, agents with a larger degree size were more likely to be a cooperator in the following periods regardless of the strategy updating rule.

In contrast, when \(b = 1.3\) and \(w = 0.5\), the relationship differed depending on the strategy updating rules (panel (b) of Fig. 2 shows the corresponding final outcomes).
Fig. 3 Frequency of cooperators at $t = 2 \times 10^4$ as a function of the each node’s degree at $t = 10^4$. Results are based on the accumulation of $10^4$ simulation runs. We report the results if the number of observations (nodes) exceeds 100 because of the fluctuations due to the small sample size. (a): Larger degree helps cooperators regardless of the strategy updating rule when $b = 1.05$ and $w = 0.3$. (b) Cooperators can exploit the benefit of larger degree in the order of VMLD, EBD and IPLD when $b = 1.3$ and $w = 0.5$. Fixed parameters: $N = 1000$, $\langle k \rangle = 8$, $\beta = 1$.

Under VMLD, where full cooperation was achieved, the same pattern was observed, i.e., a larger degree helped the cooperators. In contrast, under IPLD, where the dominance of defectors was observed, the relationship does not hold and cooperators do not enjoy the benefit of a larger degree. Under EBD, where a moderate level of cooperation was observed, the results lie between those of the other two rules, i.e., the relationship between degree and cooperation is confirmed but weaker than VMLD.

Whether cooperators can enjoy the benefit of a larger degree also affects the resulting network. Figure 4 shows the resulting normalised variance of degree ($\langle (k_i^2) - \langle k_i \rangle^2 \rangle / \langle k_i \rangle$). The combination of parameters in this figure corresponds to that of fig. 2. Panel (a) shows that degree heterogeneity becomes larger as the opportunities for link adaptation increases under VMLD and EBD. In contrast, a non-monotonic relationship, which is similar to that of fig. 2, was observed under IPLD. Under IPLD, cooperators with a larger degree had more opportunities to imitate defectors and lose edges, which prevents the appearance of nodes with a larger degree. Panel (b) shows that the degree heterogeneity becomes larger with smaller values of $w$ under VMLD compared to EBD. This fact corresponds to the result shown in panel (b) of fig. 2, i.e., that higher level of cooperation was achieved with a smaller $w$ under VMLD. Degree heterogeneity was almost suppressed under IPLD.

In addition, comparing figs. 2 and 4, the required degree heterogeneity for cooperators to proliferate was found to be different for the three strategy updating rules. For example, panel (a) shows that IPLD required a larger degree heterogeneity for the evolution of cooperation (e.g. $0.02 \leq w \leq 0.3$). Panel (b) shows that a smaller degree heterogeneity is sufficient for cooperators to proliferate under VMLD compared with EBD ($w \geq 0.54$). These observations corroborate the inference on the efficiency of each updating rule to help evolution of cooperation. Because cooperators have a relatively larger chance to influence (defective) neighbours under VMLD, a smaller neighbourhood size and accumulated payoff were sufficient for cooperators to proliferate. In contrast, cooperators were easily enforced the strategy by defectors and they needed a larger degree heterogeneity to survive under IPLD.

Different patterns also appeared for the effect of the intensity of selection ($\beta$). When $\beta = 0$, the payoff from games has a totally neutral effect on evolution. As is displayed in fig. 5, full cooperation is achieved under VMLD with a small opportunity for partner switching, whereas cooperation is suppressed under IPLD. The results under EBD lie between these results, i.e., neither cooperation nor defection is favoured. Without the influence of the payoff from the games, how often the cooperators are chosen as the strategy donor or recipient is critical in the evolution of cooperation.

Thus, VMLD, which has been commonly used in previous studies [60], can help the evolution of cooperation independently of the payoff because cooperators are more likely to be chosen as the strategy donor. Some previous
Fig. 5 Fractions of cooperators when the payoff has neutral effect ($\beta = 0$). Partner switching helps (hinders) cooperation under VMLD (IPLD). Neither cooperation nor defection is favored under EBD. Parameters: $N = 1000, \langle k \rangle = 8, \beta = 0$.

studies showed that weak selection favours cooperation in coevolving games \cite{38,39,44}. We infer that VMLD also played an important role in the evolutionary process, in addition to a coevolutionary mechanism.

We also show the effect of the intensity of selection ($\beta$) in fig. 6. Under VMLD, full cooperation is achieved more easily when selection is weak or strong, and a higher frequency of link adaptation is required with moderate values of $\beta$. Qualitatively the same pattern is observed under EBD; cooperation deteriorated when the values of $\beta$ are moderate, and a higher $w$ enhanced the evolution of cooperation even with a weaker intensity of selection. This pattern is contrary to the results under the static network, which showed that there exists an optimal intensity of selection in supporting evolution of cooperation \cite{65}. Under IPLD, stronger selection is required for cooperators to survive, especially when the high frequency of link adaptation hinders the evolution of cooperation.

The overall resulting pattern can be explained by the combination of the effects of partner switching and strategy updating rules. Partner switching is beneficial for cooperators due to the emerging degree heterogeneity \cite{66} and cooperators can gain a larger accumulated payoff by link adaptation unless $w$ is too large (see fig. 2). As a result, a stronger intensity of selection helps cooperators with larger a degree to maintain their strategy. However, because cooperators are sustained by VMLD (and to a lesser extent by EBD) when the intensity of selection is extremely weak, a stronger intensity of selection can reduce the advantage of the strategy updating rules. Indeed, a non-monotonic effect of $\beta$ was observed under VMLD and EBD. In contrast, IPLD hinders the evolution of cooperation, therefore only the combination of a stronger intensity of selection and appropriate frequency of partner switching helps cooperators.

**Discussion**

In this paper, we compared the three strategy updating rules, VMLD, IPLD and EBD, on the coevolutionary prisoner’s dilemma game. Our results show that VMLD, which were adopted in many previous studies, favour cooperation under a wide range of parameters. This is because cooperators who have larger number of neighbours are more likely to become a potential strategy donor. In contrast, cooperators are more likely to become a strategy recipient under IPLD, which prevents cooperators from enjoying the benefit of degree heterogeneity. Consequently, a higher frequency of partner switching sometimes deters the evolution of cooperation under IPLD. The results of EBD lie between these two outcomes. These differences also affect the resulting degree heterogeneity of the network. Furthermore, combined with the effect of the strategy updating rules, a non-monotonic relationship between the intensity of selection and proportion of the cooperators was observed under VMLD and EBD, whereas stronger selection favours cooperation under IPLD.

In previous studies, VMLD was often utilised as the strategy updating rule. In the context of modelling human behaviour, strategy updating rules can be regarded as the assumption for the social learning (imitation) process. Specifically, updating rules determine the direction of influence. In our simulation, strategy updating rules influenced the possibility that agents with a larger degree become a strategy donor (or recipient) and thus effect the resulting cooperation level. Therefore, understanding who will be more likely to imitate others and who will be imitated by others may be very important for studying the evolution of human cooperation.

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Fig. 6 Fractions of cooperators as a function of the intensity of selection ($\beta$). Non-monotonic relationship is observed and moderate intensity of selection hinders the evolution of cooperation for VMLD and EBD. Only strong selection helps cooperators under IPLD. Parameters: $N = 1000$, $\langle k \rangle = 8$, $b = 1.3$. 

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