COMPARISON OF IBM-2 CALCULATIONS WITH X(5) CRITICAL POINT
SYMmetry FOR LOW-LYING STATES IN $^{144-154}$Nd

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Abstract- The X(5) would take place when moving continously from the pure U(5)
symmetry to the SU(3) symmetry and it implies a definite relations among the level
energies and among the E2 transition strengths. It was recently shown that a signature
of phase transition is observed in the chain of Sm, Mo and Nd isotopes, where
$^{152}$Sm, $^{164}$Mo and $^{150}$Nd display the predicted features of the X(5) symmetry and mark
therefore the critical point. However, more detailed studies and experiments are needed
to get ideas about this signature. Without entering into detail we have firstly compared
the results obtained in our previous study [15] of $^{144-154}$Nd with that of the limits in X(5)
symmetry and then given a clear description about the validity of the Hamiltonian
parameters used in the study. At the end, we have concluded that some of Nd isotopes
display X(5) symmetry features.

Key Words- critical symmetry, interacting boson model, even Nd.

1. INTRODUCTION

Dynamic symmetries have provided a useful tool to describe properties of
several physical systems [1]. The most notable examples are the dynamic symmetries of
the interacting boson model [1,2] in nuclear physics and those of the vibron model [1,3]
in molecular physics. There three possible dynamic symmetries generally labeled by the
first subalgebra U(5):harmonic vibrator [4], SU(3);symmetrically deformed rotor [5]
and SO(6);triaxially soft rotor [6]. Nuclei may display behavior near these idealized
limits and it is a recent approach to apply the ideas of a phase transition of the nuclear
shape [1,7]. Definition of critical points of the shape change is stated as new
benchmarks and the transition from a spherical harmonic vibrator to an axially
deformed rotor has been described analytically [7] by introducing a dynamical
symmetry, denoted as X(5). This dynamical symmetry arises when the potential in the
Bohr Hamiltonian [5] is decoupled into two components-an infinite square well
potential for the quadrupole deformation parameter $\beta$ and a harmonic potential well for the triaxiality deformation parameter $\gamma$ [8]. The signature of a phase transition between collective vibrator and axially deformed rotor has received considerable attention, in the frame of critical point properties in transitional nuclei [9]. The X(5) would take place when moving continously from the pure U(5) symmetry to the SU(3) symmetry and it implies a definite relations among the level energies and among the E2 transition strengths.

It was recently shown that a signature of phase transition is observed in the chain of Sm [7,10], Mo [11] and Nd [12-14] isotopes, where $^{152}$Sm, $^{104}$Mo and $^{150}$Nd display the predicted features of the X(5) symmetry and mark therefore the critical point. However, more detailed studies and experiments are needed to get ideas about this signature. Without entering into detail, which can be found in the Ref.[7], we can give purpose of the present study as follows; (i) To compare the results obtained in Ref.[15] with the limits of X(5) symmetry, (ii) To give a clear descripion about the validity of the Hamiltonian parameters used in this study, (iii)To get a brief conclusion about the relation between $^{144-154}$Nd and X(5) symmetry.

The outline of the remaining part of this paper is as follows; An approximate IBM-2 formulation is given without entering into detail and theoretical background is reviewed in section 2. The calculated $R_{4/2}$, $R_{0/2}$ and B(E2) values are compared with the results of some neighboring nuclei and with that of X(5) limits in section 3. The last section contains some concluding remarks.

2. THEORETICAL BACKGROUND

IBM Hamiltonian takes different forms [16] depending on the regions (SU(5), SU(3), SO(6)) of the traditional IBA triangle. The Hamiltonian that we consider is in the form of [17],

$$H = H_{sd} + \Sigma \theta_l [d^+ d^+ d^+]^{(L)} [\tilde{d} \tilde{d} \tilde{d}]^{(L)}$$  

where $H_{sd}$ is the standard Hamiltonian of the IBM [18,19],

$$H_{sd} = \varepsilon_d \eta_d + \kappa Q \cdot Q + \kappa' L \cdot L + \kappa'' P \cdot P + q_3 T_3 \cdot T_3 + q_4 T_4 \cdot T_4$$  

In the IBA-2 model the neutrons' and protons' degrees of freedom are taken into account explicitly. Thus the Hamiltonian [2] can be written as,

$$H = \varepsilon_{\pi} n_{\pi \pi} + \varepsilon_{\nu} n_{\nu \nu} + \kappa Q_{\pi} Q_{\nu} + V_{\pi \nu} + V_{\nu \nu} + M_{\pi \nu}$$  

where $n_{dp}$ is the neutron (proton) d-boson number operator.

$$n_{dp} = d^+ \tilde{d} , \rho = \pi , \nu$$  

$$\tilde{d}_{\rho \rho} = (-1)^m d_{\rho \rho}$$  

$$\tilde{d}_{\rho m} = (-1)^m d_{\rho m}$$
where $s^+_{\rho}, d^+_{\rho m}$ and $s^-, d^-_{\rho m}$ represent the $s$ and $d$-boson creation and annihilation operators. The rest of the operators in the equation (3) are defined as

$$Q_\rho = (s^+_{\rho} d^+_{\rho} + d^+_{\rho} s^+_{\rho})^{(2)} + \chi_\rho (d^+_{\rho} d^+_{\rho})^{(2)}$$

$$V_{\rho \rho} = \sum_{L=0,2,4} C_{L,\rho} \left( (d^+_{\rho} d^+_{\rho})^{(L)} (d^+_{\rho} d^+_{\rho})^{(L)} \right)^{(0)} ; \rho = \pi, \nu$$

(5)

and

$$M_{\pi \nu} = \sum_{L=1,3} \xi_L (d^+_{\pi} d^+_{\pi})^{(L)} (d_{\nu} d_{\nu})^{(L)} + \xi_2 (s_{\pi} d_{\pi} - s_{\pi} d_{\pi})^{(2)} (s_{\nu} d_{\nu} - s_{\nu} d_{\nu})^{(2)}$$

(6)

In this case $M_{\pi \nu}$ affects only the position of the non-fully symmetric states relative to the symmetric ones. For this reason $M_{\pi \nu}$ is often referred to as the Majorana force.

The electric quadrupole (E2) transitions are one of the important factors within the collective nuclear structure. In IBM-2 model, the general linear E2 operator is expressed as [2],

$$T(E2) = e_\nu T_\nu (E2) + e_\pi T_\pi (E2)$$

$$= e_\nu Q_\nu + e_\pi Q_\pi$$

(7)

In these expression $\chi_\rho$ is an adimensional coefficient and $e_\rho$ is the effective quadrupole charges. Below we show how $B(E2; J \rightarrow J')$ prescription is implemented in formulation.

$$B(E2; J \rightarrow J') = \frac{1}{2J+1} |(J') \langle T(E2) || J \rangle|^2$$

(8)

### 3. RESULTS AND DISCUSSION

The energy ratios $R_{4/2} = E(4^+_{1})/E(2^+_{1})$ and $R_{0/2} = E(0^+_{1})/E(2^+_{1})$ are characteristics of different collective motions of the nucleus [8,18]. So, we firstly examined the energies of the yrast sequences in some even-even Nd nuclei. Table 1 shows the most appropriate Hamiltonian parameters of calculations for examining $^{144-154}$Nd nuclei.
The experimental signatures for X(5) behavior are the following [8]. (a) The energies of the yrast states, E(J^+_r), should show characteristic ratios lying between those of a vibrator and a rotor; (b) The strength of transitions between yrast states as reflected in the B(E2;J→J-2) values should increase with angular momentum J at a rate intermediate between the values for a vibrator and rotor; (c) The position of the first excited collective 0^+_2 state is 5.67 times the energy of 2^+_1 level; (d) the nonyrast states based on the 0^+_2 level have larger energy spacings than the yrast sequence; (e) The B(E2;J→J-2) values for intrasequence transitions should be lower for the nonyrast sequence relative to those of the yrast sequence; (f) intersequence B(E2) values should show a characteristic pattern. We will use all of the above points in our search for Nd nuclei of A≤140 displaying behavior similar to the X(5) predictions.

The calculated and experimental energy values are given in table 2 with R_{4/2} and R_{0/2} ratios. Fig.1 shows the R_{4/2} and R_{0/2} ratios as a function of neutron number changing from 84 to 94. An harmonic vibrator should have R_{4/2} = 2.00, an axially symmetric rotor has R_{4/2}=3.33, while X(5) behavior should have R_{4/2}=2.91. The searched nuclei in the present study have 2.00 ≤ R_{4/2} ≤ 2.96. As it is seen from the table 2 the calculated and experimental energy values are very close to X(5) predictions for ^{150}Nd, especially. Around N=90, the positions of the excited 0^+_2 states are also close to the X(5) prediction and we note that the spacings in the excited sequence follow the expected behavior.

In table 3, we present the calculated data with available experimental ones for ^{144-154}Nd. In addition, fig.2 shows the energies of the yrast sequences (normalized to the energy of

| \( \frac{A}{Z} \) X | N_t | N_v | N | Fit Number | \( \epsilon \) | \( \kappa \) | \( J_r \) | \( J_v \) | C_{L_r} | C_{L_v} |
| --- | --- | --- | --- | -- | --- | --- | --- | --- | --- | --- |
| 144 \(_{60}\) Nd | 84 | 5 | 1 | 6 | Fit 1 | 0.87 | -0.095 | -1.2 | -1.2 | 0.90 | 0.00 |
| 146 \(_{60}\) Nd | 86 | 5 | 2 | 7 | Fit 1 | 0.81 | -0.098 | -1.1 | -1.2 | 0.90 | -0.60 |
| 148 \(_{60}\) Nd | 88 | 5 | 3 | 8 | Fit 2 | 0.73 | -0.085 | 1.2 | 1.1 | 0.00 | 0.00 |
| 150 \(_{60}\) Nd | 90 | 5 | 4 | 9 | Fit 1 | 0.66 | -0.070 | -1.1 | -1.2 | 0.00 | 0.00 |
| 152 \(_{60}\) Nd | 92 | 5 | 5 | 10 | Fit 2 | 0.70 | -0.090 | 1.0 | 0.8 | 0.00 | 0.00 |
| 154 \(_{60}\) Nd | 94 | 5 | 6 | 11 | Fit 1 | 0.55 | -0.091 | -1.2 | -1.2 | 0.90 | -0.80 |
| 156 \(_{60}\) Nd | 96 | 5 | 7 | 12 | Fit 2 | 0.45 | -0.080 | 1.2 | 0.4 | 0.00 | 0.00 |

The calculated and experimental energy values are given in table 2 with R_{4/2} and R_{0/2} ratios. Fig.1 shows the R_{4/2} and R_{0/2} ratios as a function of neutron number changing from 84 to 94. An harmonic vibrator should have R_{4/2} = 2.00, an axially symmetric rotor has R_{4/2}=3.33, while X(5) behavior should have R_{4/2}=2.91. The searched nuclei in the present study have 2.00 ≤ R_{4/2} ≤ 2.96. As it is seen from the table 2 the calculated and experimental energy values are very close to X(5) predictions for ^{150}Nd, especially. Around N=90, the positions of the excited 0^+_2 states are also close to the X(5) prediction and we note that the spacings in the excited sequence follow the expected behavior.

In table 3, we present the calculated data with available experimental ones for ^{144-154}Nd. In addition, fig.2 shows the energies of the yrast sequences (normalized to the energy of
their respectively $2^+_1$ levels) in those nuclei and compare them with the expected behavior for an harmonic vibrator, an axially deformed rotor, and the X(5) prediction.

### Table 2. The calculated and experimental energy values of $^{144-154}\text{Nd}$ nuclei with $R_{4/2}$ and $R_{0/2}$ ratios. $R_{4/2}$ values and ground-state band energies in $^{144-154}\text{Nd}$ are compared to relevant analytical models, X(5), and IBM where these apply. X(5) values are taken from Refs. [21] and IBM values are taken from our previous study refered as [15]. Experiments are taken from Refs.[23-36].

| J   | X(5) | $^{144}\text{Nd}$ | $^{146}\text{Nd}$ | $^{148}\text{Nd}$ | $^{150}\text{Nd}$ | $^{152}\text{Nd}$ | $^{154}\text{Nd}$ |
|-----|------|----------------|----------------|----------------|----------------|----------------|----------------|
|     | EXP  | IBM | EXP  | IBM | EXP | IBM | EXP | IBM | EXP | IBM | EXP | IBM | EXP  |
| $2^+_1$ | 0.122 | 0.697 | 0.694 | 0.453 | 0.451 | 0.302 | 0.327 | 0.130 | 0.127 | 0.072 | 0.085 | 0.071 | 0.078 |
| $4^+_1$ | 0.354 | 1.315 | 1.414 | 1.043 | 1.026 | 0.752 | 0.748 | 0.381 | 0.376 | 0.237 | 0.246 | 0.233 | 0.231 |
| $6^+_1$ | 0.661 | 1.791 | 2.223 | 1.780 | 1.728 | 1.280 | 1.265 | 0.720 | 0.735 | 0.484 | 0.480 | 0.477 | 0.459 |
| $8^+_1$ | 1.033 | - | 3.105 | 2.555 | 2.545 | 1.857 | 1.876 | 1.130 | 1.202 | 0.810 | 0.802 | 0.805 | 0.779 |
| $10^+_1$ | 1.465 | - | 4.060 | 3.248 | 3.467 | 2.472 | 2.577 | 1.599 | 1.767 | - | 1.194 | - | 1.650 |
| $12^+_1$ | 1.954 | - | 5.014 | - | 4.481 | - | 3.381 | - | 2.473 | - | 1.832 | - | 1.761 |
| $14^+_1$ | 2.499 | - | - | 5.575 | - | 4.311 | - | 3.273 | - | 2.555 | - | 2.282 |
| $R_{4/2}$ | 2.910 | 1.890 | 2.000 | 2.300 | 2.280 | 2.490 | 2.290 | 2.930 | 2.960 | 3.290 | 2.890 | 3.280 | 2.960 |
| $R_{0/2}$ | 5.800 | 2.900 | 2.100 | 2.100 | 2.200 | 3.000 | 2.700 | 5.200 | 5.800 | - | 5.900 | - | 6.300 |

In fig.3 we present the $B(E2;J \rightarrow J-2)$ reduced transition strength which is normalized to their respective $B(E2; 2^+_1 \rightarrow 0^-_1)$ values and again compare them with the expected behavior for an harmonic vibrator, an axially deformed rotor, and the X(5) prediction.

It is clear from table 2 and fig.2 that the Nd nuclei with yrast energies that closely follow the X(5) prediction. However, as can be seen from fig.3, in most of the cases X(5) behavior can be excluded on the basis of the deduced yrast $B(E2;J \rightarrow J-2)$ values.

In fig.4 we compare the relative $\gamma$-band energies in $^{144-154}\text{Nd}$ with X(5), with an axial rotor and with neighboring $^{146-156}\text{Sm}$ nuclei. The results are quite interesting. They provide an extensive test of X(5) for the $\gamma$ degree of freedom and the calculated values exhibit a good agreement with experimental ones. Moreover, as it is seen from the fig, X(5) agrees well with the data for the $\gamma$-band energies. However, $^{144}\text{Nd}$, $^{146}\text{Nd}$ and $^{148}\text{Nd}$ deviate from X(5) slightly in the direction of the rotor. This striking disagreements need to be better understood.
Fig. 1. The $R_{4/2}$ and $R_{0/2}$ ratios as a function of neutron number changing from 84 to 94.
Table 3. The calculated and experimental intrasequence B(E2;J→J-2) values for $^{144-154}$Nd. E2 transition rates are compared with the X(5) limit [1]. The B(E2) values are in Weisskopf units ($1 e^2 b^2 = 2.62 \times 10^{-3} W.u.$). Experiments are taken from ref.[35-39].

| $J_i \rightarrow J_f$ | X(5) | $^{144}$Nd | $^{146}$Nd | $^{148}$Nd | $^{150}$Nd | $^{152}$Nd | $^{154}$Nd |
|----------------------|------|-----------|-----------|-----------|-----------|-----------|-----------|
|                      | EXP  | IBM       | EXP       | IBM       | EXP       | IBM       | EXP       | IBM       |
| $2^+_1 \rightarrow 0^+_1$ | 144  | 42        | 31        | 51        | 47        | 103       | 103       | 206       | 206       | -         | -         | 342       |
| $4^+_1 \rightarrow 2^+_1$ | 230  | 32        | 50        | 76        | 76        | 164       | 164       | 313       | 305       | -         | 381       | -         | 503       |
| $6^+_1 \rightarrow 4^+_1$ | 285  | -         | 57        | -         | 90        | -         | 176       | -         | 335       | -         | 494       | -         | 677       |
| $8^+_1 \rightarrow 6^+_1$ | 328  | -         | 55        | -         | 89        | -         | 180       | -         | 332       | -         | 493       | -         | 684       |
| $10^+_1 \rightarrow 8^+_1$ | 361  | -         | 44        | -         | 79        | -         | 168       | -         | 307       | -         | 462       | -         | 651       |
| $2^+_2 \rightarrow 0^+_2$ | 115  | -         | 3         | -         | 11        | -         | 13        | -         | 46        | -         | 77        | -         | 114       |
| $4^+_2 \rightarrow 2^+_2$ | 173  | -         | 29        | -         | 46        | -         | 88        | -         | 141       | -         | 190       | -         | 276       |
| $0^+_2 \rightarrow 2^+_1$ | 90   | -         | 41        | -         | 57        | -         | 88        | -         | 53        | -         | 31        | -         | 36        |
| $2^+_2 \rightarrow 0^+_1$ | 3    | 1.15      | 0.76      | 28        | 35        | 7.6       | 3.4       | 4.6       | 5.3       | -         | 26        | -         | 9.2       |
| $2^+_2 \rightarrow 2^+_1$ | 12   | 36        | 42        | 34        | 44        | 46        | 60        | 14        | 42        | -         | 191       | -         | 267       |
| $2^+_2 \rightarrow 4^+_1$ | 53   | -         | 1         | -         | 30        | -         | 2.6       | -         | 0.23      | -         | 1.5       | -         | 1         |
| $4^+_2 \rightarrow 2^+_1$ | 1.4  | -         | 0.4       | -         | 0.7       | -         | 0.0       | -         | 0.53      | -         | 1         | -         | 0.2       |
| $4^+_2 \rightarrow 4^+_1$ | 9    | -         | 22        | -         | 23        | -         | 34        | -         | 34        | -         | 122       | -         | 197       |
| $4^+_2 \rightarrow 6^+_1$ | 40   | -         | 0.3       | -         | 1.2       | -         | 2         | -         | 0.0       | -         | 0.04      | -         | 6         |
Fig. 2. The energies of the yrast sequences (normalized to the energy of their respectively $2^+_1$ levels) in $^{144-154}$Nd nuclei.
Fig. 3. The $B(E2; J \rightarrow J-2)$ reduced transition strength which is normalized to their respective $B(E2; 2_1^+ \rightarrow 0_1^+)$ values.
Fig. 4. Comparison of relative $\gamma$-band energies in $^{144-154}$Nd with X(5), with an axial rotor and with neighboring $^{146-156}$Sm nuclei.

3. CONCLUSION

We have searched the validity of our new parameters in IBM-2 formulation and theoretical background is reviewed for $^{144-154}$Nd. The calculated $R_{4/2}$, $R_{0/2}$ and B(E2) values are compared with the results of some neighboring nuclei and with that of X(5) limits. At the end, it was seen that some Nd nuclei, especially nuclei around N=90, with yrast energies follow the X(5) prediction closely. But X(5) behavior can be excluded on the basis of the deduced yrast B(E2;J→J-2) values in most of the cases. On the basis of the yrast state energies and yrast intraband transition strengths, the best candidates were performed to be $^{148}$Nd, $^{150}$Nd and $^{152}$Nd nuclei and the X(5) Picture reproduces the position of the first excited $0^+_2$ in the nuclei with N=90.
We suggest that future experiments should focus on more detailed measurements of the excited states in $^{148}$Nd and $^{152}$Nd. Moreover, the detailed information on states above the collective $0^+_2$ levels is needed. The present study will be important for understanding the collective excitations in transitional nuclei regarding the applicability of the IBM and the X(5) description.

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