Mott Insulator to high $T_c$ Superconductor via Pressure
Resonating Valence Bond theory and prediction of new Systems

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Mott insulator superconductor transition, via pressure and no external doping, is studied in orbital non degenerate spin-$\frac{1}{2}$ systems. It is presented as another RVB route to high $T_c$ superconductivity. We propose a 'strong coupling' hypothesis which helps to view first order Mott transition as a self doping process that also preserves superexchange on metal side. We present a generalized t-J model where a conserved $N_0$ doubly occupied ($e^-$) sites and $N_0$ empty sites ($e^+$) hop in the background of $N-2N_0$ singly singly occupied (neutral) sites in a lattice of N sites. An equivalence to the regular t-J model is made and some old and new systems are predicted to be candidates for pressure induced high $T_c$ superconductivity.

I. INTRODUCTION

Bednorz-Muller’s discovery [1] of high temperature superconductivity in doped $La_2CuO_4$ and Anderson’s resonating valence bond (RVB) theory [2] initiated a new interest in Mott insulators as a novel quantum state. In RVB theory the pre-existing singlet correlations among electron spins in a spin-$\frac{1}{2}$ Mott insulator readily become the superconducting correlations on doping. The RVB mean field theory [2], gauge theory [3] and later developments [4] have given results that are in qualitative and sometimes quantitative agreement with many experimental results.

Motivated by high $T_c$ superconductivity in cuprates, RVB theory has so far focussed on the metallization of Mott insulating state by external doping. However, we know that there are three families of ‘commensurate’ tight binding systems that undergo Mott insulator (spin-Peierls or antiferromagnetic order) to superconductor transition under pressure or chemical pressure and no external doping: i) quasi 1 dimensional (TMSTF)$_2$, Bechgaard salt family [5] ii) quasi 2 dimensional $\kappa$-(BEDT-TTF)$_2$, ET-salt family [6] and iii) 3 dimensional fullerites [7,8]. For ET and Bechgaard salts a single band repulsive Hubbard model at half filling is known to be a right model [10,11].

As antiferromagnetism (more correctly, enhanced singlet correlations [13]) are present in the insulating side we study Mott transition in spin-$\frac{1}{2}$ orbitally non-degenerate systems from RVB theory point of view. By looking at a body of experimental results and theories on Mott transition [9] in real systems and using the first order character of the Mott transition we propose a ‘strong coupling’ hypothesis; it states that a generic Mott transition in real systems is to a (strong coupling) metallic state with superexchange. This hypothesis allows us to view the conducting state as a self doped Mott insulator that has very nearly the same superexchange $J$ as the insulator and a fixed (conserved) number $N_0$ of delocalized doubly occupied sites and $N_0$ empty sites. This enables us to propose a generalized t-J model, where a fixed number $N_0$ of doubly occupied sites ($e^-$) and $N_0$ empty sites ($e^+$) hop in the background of $N-2N_0$ singly singly occupied (neutral) sites that have superexchange interaction among themselves. Here N is the number of lattice sites. In determining the total number of mobile charges $2N_0$, that is the amount of self doping, large range coulomb interaction plays an important role.

The issue of RVB superconductivity is solved by transforming our generalized t-J model containing $N_0$ holes and $N_0$ doubly occupied sites in a Mott insulator into a t-J model that contains either $2N_0$ holes or $2N_0$ doubly occupied sites. So our model also exhibits superconductivity to the extent the corresponding ordinary t-J model exhibits superconductivity. Encouraged by our theory we make certain predictions about possibility of pressure induced superconductivity in a family of compounds: i) old ones such as three dimensional $CuO$, layered $La_2CuO_4$, infinite layer $CaCuO_4$, insulating Ti and Hg cuprates and YBCO and ii) new ones such as $La_2CuS_2O_7$, $La_2CuS_4$, $CaCuS_2$ with $CuS_2$ planes or their selenium analogues, to mimic chemical pressure along the ab-plane.

It should be pointed out that, 1d Mott transition and various Hubbard model based theories exist in the literature [10–12] for the Bechgaard, ET salts and fullerites. Our view point emerging from ‘strong coupling’ hypothesis and the resulting generalized t-J model emphasizes that the physics of the conducting state is also determined by a strong coupling physics with superexchange and the consequent RVB physics.

Standard thought experiment of Mott transition is an adiabatic expansion of a cubic lattice of hydrogen atoms forming a metal. Electron density decreases on expansion and Thomas-Fermi screening length increases; when it becomes large enough to form the first electron-hole bound state, there is a first order transition to a Mott insulating state, at a critical value of the lattice parameter $a \approx 4a_B$, where $a_B$ is the Bohr radius. The charge gap jumps up from zero to a finite Mott-Hubbard gap across the transition (figure 1a), by a feedback process that crit-
ically depends on the long range part of the coulomb interaction, as emphasized by Mott [9].

Experimentally known Mott transitions are first order and the insulating side close to the transition point usually have a substantial Mott-Hubbard gap; in oxides this gap is often of the order of an eV. In organics, where the band width are narrow ≈ 0.25eV the Mott Hubbard gap also has similar value. In view of the finite Mott Hubbard gap, the magnetism on the Mott insulating side is well described by an effective Heisenberg model with short range superexchange interactions. There are no low energy charge carrying excitations. That is, we have a strong coupling situation.

\[ N_{d}(e^{-}) + N_{d}(e^{+}) \]

\[ \sum_{i} N_{d}(e^{-}) = N_{d}(e^{+}) \]

Here \( N_{d}(e^{-}) = N_{d}(e^{+}) \) are the number of doubly occupied \((e^{-})\) and number of empty sites \((e^{+})\); total number of lattice sites \( N = \) total number of electrons. Optimal carrier density \( x_{0} \equiv 2k_{F} \alpha_{s} \) is determined by long range part of coulomb interaction and superexchange energy. b) and c) Schematic picture of the real part of the frequency dependent conductivity on the insulating and metallic side close to the Mott transition point in a real system. W is the band width.

What is interesting is that this strong coupling situation continues on the metallic side as shown by optical conductivity studies for example in Bechgaard [14] and ET salts: one sees a very clear broad peak (a high energy feature) corresponding to the upper Hubbard band both in the insulating and conducting states. The only difference in the conducting state is the appearance of Drude peak, whose strength and shape gives an idea of number of free carriers that have been liberated (figure 1b and 1c). As the location and width of the Hubbard band has only a small change across the transition, one may conclude that the local quantum chemical parameters such as the hopping matrix elements t’s and Hubbard U (corresponding superexchange J) remain roughly the same. This is the basis of our ‘strong coupling’ hypothesis: a generic Mott insulator metal transition in real system is to a (strong coupling) metallic state that contains superexchange.

As superexchange survives in the conducting state, two neighboring singly occupied sites of net charge \((0,0)\) can not decay into freely moving doubly occupied and empty sites \((e^{-}, e^{+})\). Conversely a pair of freely moving doubly occupied and empty sites cannot annihilate each other and produce a bond singlet (figure 2). (Recall that in a free fermi gas, where there is no superexchange, the above processes freely occur). Superexchange and long range part of the coulomb interactions determine the number of self doped carriers \(2N_{d}\) and their conservation.

\[
\begin{array}{lllll}
0 & 0 & e^{-} & e^{+} \\
1 & 1 & \equiv & \equiv & 2 \\
2 & 2 & \equiv & \equiv & 0
\end{array}
\]

FIG. 2. If superexchange survives on the metallic side, a pair of neighboring singly occupied sites can not decay into freely moving doubly occupied and empty sites. The converse is also true.

The above arguments naturally leads to a generalized t-J model for the conducting side in the vicinity of the Mott transition point

\[
H_{tJ} = -\sum_{ij} t_{ij} P_{d} c_{i\sigma} \bar{c}_{j\sigma} P_{d} - \sum_{ij} t_{ij} P_{e} e_{i\sigma} \bar{e}_{j\sigma} P_{e} + h.c. \\
- \sum_{ij} J_{ij} (S_{i} \cdot S_{j} - \frac{1}{4} n_{i} n_{j}),
\]

operating in a subspace that contains a fixed number \(N_{0}\) of doubly occupied and \(N_{0}\) empty sites. The projection operators \(P_{d}\) and \(P_{e}\) allows for the hopping of a doubly occupied and empty sites respectively in the background \(N - 2N_{0}\) of singly occupied sites. Here \(N\) is the total number of electrons, which is the same as the number of lattice sites. As the Mott-Hubbard gap is the smallest at the Mott transition point, higher order superexchange processes may also become important and contribute to substantial non neighbor \(J_{ij}\)’s.

Our t-J model adapted to the self doped Mott insulator has a more transparent form in the slave boson representation \(c_{i\sigma} \equiv s_{i\sigma} d_{i} + \sigma s_{i\sigma} \bar{e}_{i}\). Here the chargeons \(d_{i}\)’s and \(e_{i}\)’s are hard core bosons that create doubly occupied sites \((e^{-})\) and empty sites \((e^{+})\) respectively. The fermionic spinon operators \(s_{i\sigma}\)’s create singly occupied sites with a spin projection \(\sigma\). The local constraint, \(d_{i}^\dagger d_{i} + e_{i}^\dagger e_{i} + \sum_{\sigma} s_{i\sigma} s_{i\sigma} = 1\), keeps us in the right Hilbert space.

In the slave boson representation our t-J model takes a suggestive form:

\[
H_{tJ} = -\sum_{ij} t_{ij}(d_{i}^\dagger d_{j}^\dagger \sum_{\sigma} s_{i\sigma} s_{j\sigma}^\dagger + e_{i}^\dagger e_{j}^\dagger \sum_{\sigma} s_{i\sigma} s_{j\sigma}) + h.c. \\
- \sum_{ij} J_{ij} b_{ij}^\dagger b_{ij}
\]

where \(b_{ij}^\dagger = \frac{1}{\sqrt{2}}(s_{i\uparrow}^\dagger s_{j\downarrow}^\dagger - s_{i\downarrow}^\dagger s_{j\uparrow}^\dagger)\) is a spin singlet spinon pair creation operator at the bond \(ij\). It is easily seen that the total number operator for doubly occupied sites \(\hat{N}_{d} \equiv \sum d_{i}^\dagger d_{i}\) and empty sites \(\hat{N}_{e} \equiv \sum e_{i}^\dagger e_{i}\) commute with the t-J Hamiltonian (equation 2):
\[
[H_{ij}, \hat{N}_d] = [H_{ij}, \hat{N}_e] = 0 \quad (3)
\]

That is, \( \hat{N}_d \) and \( \hat{N}_e \) are individually conserved. In our half filled band case \( N_d = N_e = N_0 \). (This special conservation law is true only for our effective t-J Hamiltonian and not for the original Hubbard model).

This conservation law allows us to make the following statement, which is exact for a particle-hole symmetric Hamiltonian and approximate for the asymmetric case: our generalized t-J model with a fixed number \( N_0 \) of doubly occupied sites and equal number \( N_0 \) of empty sites has the same many body spectrum as the regular t-J model that contains either \( 2N_0 \) holes or \( 2N_0 \) electrons. Symbolically it means that \( H_{ij}(N_0, 0) \equiv H_{ij}(2N_0, 0) \equiv H_{ij}(0, 2N_0) \). This means we can borrow all the known results of t-J model, viz. mean field theory, variational approach, numerical approach etc. and apply to understand the thermodynamic and superconductivity properties of our self doped Mott insulator. Response to electric and magnetic field perturbation has to be done separately as the \( d \) and \( e \) bosons carry different charges, \( e^- \) and \( e^+ \) respectively.

Another consequence of the above equivalence is shown in figure 3, where we have managed to draw the path of pressure-induced Mott transition in a Hubbard model phase diagram, even though Hubbard model does not contain the crucial long range interaction physics. The jump from B to C is the first order phase transition, remembering that in the presence of our new conservation law what decides the spectrum of our generalized t-J model is the total number of \( e^+ \) and \( e^- \) charge carriers in an equivalent regular t-J model. The horizontal jump is also consistent with our strong coupling hypothesis.

An important parameter in our modeling is the equilibrium total \( e^+ \) and \( e^- \) carrier concentration, \( x_0 \equiv \frac{2N_0}{N} \) in our self doped Mott insulator. This also controls the value of superconducting \( T_c \) we will get across the Mott transition point. Estimate of \( x_0 \) depends on the long range part of the coulomb interaction energy and also the short range superexchange energy; we will defer this discussion to a later publication. \( x_0 \) may also be determined from experiments such as frequency dependent conductivity by a Drude peak analysis.

Since we have reduced our self doped Mott insulator problem into a t-J model, superconducting \( T_c \) is determined by \( t, J \) and \( x_0 \), as in the t-J model. If exchange interaction contribution is comparable to the long range coulomb contribution, \( x_0 \) will be closer to value that maximizes superconducting \( T_c \). Another important point is the possibility of non nearest neighbor superexchange \( J_{ij} \), processes, which i) frustrate long range antiferromagnetic order to encourage spin liquid phase and ii) increase the superexchange energy contribution to the total energy; this could give a larger superconducting \( T_c \) across the Mott transition than expected from a t-J model with nearest neighbor superexchange. Perhaps an optimal self doping and sufficiently frustrated superexchange interactions is realized in \((NH_3)K_3C_60\) family [8], since Neel temperature \( T_N \approx 40 K \) and superconducting \( T_c \approx 30 K \) are comparable.

If the self doping is small there will be competition from antiferromagnetic metallic phase, stripes and phase separation. For a range of doping one may also get superconductivity from inter plane/chain charge disproportionation. If self doping is very large then the effect of superexchange physics and the consequent local singlet correlations are diluted and the superconducting \( T_c \) will become low. This is the reason for the fast decrease of superconducting \( T_c \) with pressure in the organics.

In what follows we discuss some families of compounds, some old ones and some new ones and predict them to be potential high \( T_c \) superconductors, unless some crystallographic transitions or band crossing intervenes and change the valence electron physics drastically. \( CuO \), is the mother compound [15] of the cuprate high \( T_c \) family. It is monoclinic and \( CuO_2 \) ribbons form a 3 dimensional network, each oxygen being shared by two ribbons mutually perpendicular to each other. The square planar character from four oxygens surrounding a Cu in a ribbon isolates one non-degenerate valence \( d \)-orbital with a lone electron. This makes \( CuO \) an orbitally non-degenerate spin-\( \frac{1}{2} \) Mott insulator and makes it a potential candidate for our pressure route to high \( T_c \) superconductivity. The frustrated superexchange leads to a complex three dimensional magnetic order with a Neel temperature \( \sim 230 K \). These frustrations should help in stabilizing short range singlet correlations, which will help in singlet cooper pair delocalization on metatllization.

As far as electronic structure is concerned, the \( CuO_2 \) ribbons give \( CuO \) a character of coupled 1d chains. This
makes it some what similar to quasi one dimensional Bechgaard salts, which has a Mott insulator to superconductor transition, via an intermediate metallic antiferromagnetic state as a function of physical or chemical pressure. The intermediate metallic antiferromagnetic state represents a successful competition from nesting instabilities of flat fermi surfaces arising from the quasi one dimensional character. Once the quasi one dimensional character is reduced by pressure, nesting of fermi surface is also reduced and the RVB superconductivity takes over.

If manganite [16], a perovskite and fullerites [8] are any guidance, metallization should take place under a pressure of ~ tens of GPa’s. CuO should undergo a Mott insulator superconductor transition, perhaps with an intermediate antiferromagnetic metallic state. The superconducting temperature will be a finite fraction of the Neel temperature, as is the case with Bechgaard salts or K$_3$(NH$_3$)$_2$C$_60$. Thus an optimistic estimate of $T_c$ will be 50 to 100 K.

Similar statements can be made of the more familiar La$_2$CuO$_4$, insulating YBCO and the CuCaCuO$_2$, the infinite layer compound or the family of Mott insulating cuprates such as Hg and Tl based insulating cuprates. Infinite layer compound has the advantage of absence of apical oxygen and should be less prone to serious structural modifications in the pressure range of interest to us. The quasi 2d Hubbard model describing the CuO$_2$ planes does have an appreciable $t'$, making nesting magnetic instabilities weaker. Thus we expect that on metallization a superconducting state to be stabilized with a small or no antiferromagnetic metallic intermediate state.

The quasi 2d cuprates have a special advantage in the sense we may selectively apply ab-plane pressure in thin films by epitaxial mismatch and ab-plane compression. Apart from regular pressure methods, this method [17] should be also tried.

One way of applying chemical pressure in cuprates is to increase the effective electron band width by increasing the band parameters such as $t$ and $t'$ in the Hubbard model. This can be achieved by replacing oxygens in the CuO$_2$ planes (or in 3 dimensional CuO) by either sulfur or selenium, which because of the larger size of the bridging 3p or 4p orbitals increase the band width and at the same time should reduce the charge transfer or Mott-Hubbard gap. On partial replacement of oxygen, as CuO$_{2-x}$X$_x$ in the planes or CuO$_{1-x}$X$_x$ ($X = S, Se$) one might achieve metalization without doping.

Some possible new stoichiometric compounds are La$_2$Cu$_2$S$_2$O$_2$, La$_2$Cu$_2$S$_4$ and Cu$_2$Cu$_2$S$_2$ or their Se versions. Synthesizing these compounds may not be simple, as the filled and deep bonding state of oxygen 2p orbitals in CuO$_2$ play a vital role in stabilizing square planar coordination. With S or Se versions these bands will float up and come closer to the fermi level thereby making square structure less stable. Under pressure or some other non equilibrium conditions some metastable versions of these compounds may be produced. One could also optimize superconducting transition $T_c$ by a judicious combination of pressure induced self doping and external doping.

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