Surface diffuseness anomaly in heavy-ion potentials for large-angle quasielastic scattering

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Recent high precision experimental data for heavy-ion fusion reactions at subbarrier energies systematically show that a surprisingly large surface diffuseness parameter for a Woods-Saxon potential is required in order to fit the data. We point out that experimental data for quasi-elastic scattering at backward angles also favor a similar large value of surface diffuseness parameter. Consequently, a double folding approach fails to reproduce the experimental excitation function of quasielastic scattering for the $^{16}$O + $^{154}$Sm system at energies around the Coulomb barrier. We also show that the deviation of the ratio of the quasielastic to the Rutherford cross sections from unity at deep subbarrier energies offers an unambiguous way to determine the value of the surface diffuseness parameter in the nucleus-nucleus potential.

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The nucleus-nucleus potential is the primary ingredient in nuclear reaction calculations. Its nuclear part has often been parametrized as a Woods-Saxon form. Elastic and inelastic scattering are sensitive mainly to the surface region of the nuclear potential, where the Woods-Saxon parametrization has a simple exponential form. This fact has been exploited to study the surface property of nuclear potential. Usually, the best fit to experimental data for scattering is obtained with a diffuseness of around 0.63 fm. This value is consistent with a double folding potential, and seems to be well accepted.

In marked contrast, recent high precision experimental data for heavy-ion fusion reactions at energies around the Coulomb barrier suggest that a much larger value of diffuseness, ranging from 0.75 to 1.5 fm, is required to fit the data. (See Ref. for a detailed systematic study). The Woods-Saxon potential which fits elastic scattering overestimates fusion cross sections at energies both above and below the Coulomb barrier, having an inconsistent energy dependence to the experimental fusion excitation function. When the height of the Coulomb barrier is fixed, the larger diffuseness parameter leads to the smaller barrier position and the smaller barrier curvature (thus the larger tunneling region). The main effect on the fusion cross sections comes from the barrier position and the tunneling width of the barrier at energies above and below the Coulomb barrier, respectively. A large diffuseness parameter appears to be desirable in both these aspects. The reason for the large discrepancies in diffuseness parameters extracted from scattering and from fusion analyses, however, has not yet been understood.

The purpose of this paper is to discuss the dependence of quasielastic excitation function at a large scattering angle on the surface diffuseness parameter in a nucleus-nucleus potential. The quasielastic cross section is defined as the sum of the cross sections of elastic, inelastic, and transfer reactions. Its excitation function at backward angles provides complementary information to the fusion process. It therefore offers an ideal test ground for a large diffuseness parameter suggested by the recent fusion data. This is particularly of interest in connection to the steep falloff phenomena of fusion cross sections at deep subbarrier energies observed recently in several systems. This is so because the measurement of quasielastic scattering is experimentally much easier than that of fusion reaction, especially at deep subbarrier energies. Contrary to what one might expect, we demonstrate below that the surface diffuseness parameter which fits the experimental data of quasielastic scattering is consistent with the one for fusion, rather than the commonly accepted value for scattering.

As a concrete example, let us consider the $^{16}$O+$^{154}$Sm reaction. Neglecting the finite excitation energy of the ground state rotational band in the target nucleus, the cross sections for fusion and quasielastic scattering are given by

\[
\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta_T) \sigma_{\text{fus}}(E; \theta_T),
\]

and

\[
\sigma_{\text{qel}}(E, \theta) = \int_0^1 d(\cos \theta_T) \sigma_{\text{qel}}(E, \theta; \theta_T),
\]

respectively, in the isocentrifugal approximation, where one neglects the angular momentum transfer in the centrifugal potential $\theta$ and $\theta_T$ are the scattering angle and the orientation angle of the deformed target with respect to the projectile direction, respectively. $\sigma_{\text{fus}}(E; \theta_T)$ and $\sigma_{\text{qel}}(E, \theta; \theta_T)$ are the fusion and the elastic cross sections for the angle dependent potential $V(r, \theta_T)$ given by

\[
V(r, \theta_T) = V_N(r, \theta_T) + V_C(r, \theta_T),
\]
at potential with the surface diffuseness parameter \( \beta \) with

\[
\sigma_{\text{qel}}(\theta) = \left( \frac{\beta}{\sqrt{1 + \beta^2}} \right) \sigma_R(\theta),
\]

obtained with a Woods-Saxon potential with \( \beta = 0.306 \) and \( \beta_4 = 0.05 \) by using the Woods-Saxon potential with the surface diffuseness parameter \( a \) of 1.05 fm, while the dashed line with \( a = 0.65 \) fm. The result of the double folding potential with the density-dependent M3Y interaction is denoted by the thin solid line. The experimental data are taken from Refs. [9, 15], where the quasi-elastic excitation function around \( E = 65 \) MeV is due to the transfer process [15], which is not included in the present calculations.

For a single channel problem, the ratio of the elastic to the Rutherford cross sections at backward angles is given by [14, 15]

\[
\frac{d\sigma}{d\theta_R}(E, \theta) \sim 1 + \frac{V_N(r_c)}{k_a} \sqrt{2\alpha \pi k \eta} E, \tag{6}
\]

at energies well below the Coulomb barrier, where the tunneling probability is exponentially small (see Ref. [16] for a more general formula which is valid also at higher energies). This formula is obtained with the semiclassical perturbation theory by assuming that the nuclear potential \( V_N(r) \) is proportional to \( \exp(-r/a) \) around the distance of closest approach, that is, \( r_c = (\eta + \sqrt{\eta^2 + \lambda^2}) / k \), where \( \eta \) is the Sommerfeld parameter and \( \lambda_c = \eta \cot(\theta/2) \). The deviation of the ratio of the cross sections at subbarrier energies from unity is therefore sensitive only to the surface property of nuclear potential, and provides a relatively model independent way to study the effect of surface diffuseness parameter. In order to demonstrate that the surface diffuseness is indeed more

FIG. 1: The ratio of quasielastic to the Rutherford cross sections at \( \theta_{\text{lab}} = 170 \) deg (the upper panel) and the fusion cross section (the lower panel) for the \(^{16}O + \text{Sm}\) reaction. The solid line is obtained with the orientation-integrated formula with \( \beta_2 = 0.306 \) and \( \beta_4 = 0.05 \) by using the Woods-Saxon potential with the surface diffuseness parameter \( a \) of 1.05 fm, and the dashed line with \( a = 0.65 \) fm. The result of the double folding potential with the density-dependent M3Y interaction is denoted by the thin solid line. The experimental data are taken from Refs. [9, 15].

\[
V_N(r, \theta_T) = \frac{-V_0}{1 + \exp((r - R - R_T) \sum \beta \lambda Y_{\lambda}(\theta_T))/a}, \tag{4}
\]

\[
V_C(r, \theta_T) = \frac{Z_P Z_T e^2}{r} \sum \left( \beta_0 + \frac{2}{\sqrt{5}} \beta_2 \delta \lambda,2 \right) \times \frac{3Z_P Z_T e^2}{2\lambda + 1} \frac{R_T^2}{r^{\lambda+1}} Y_{\lambda}(\theta_T). \tag{5}
\]

\( V_C(r, \theta_T) \) is the Coulomb correction to \( V_N(r, \theta_T) \).

\( R_T \) is the radius parameter of the potential as well as the deformation parameters. The discrepancy between the experimental data and the theoretical curve for the quasielastic scattering function around \( E = 65 \) MeV is due to the fusion process [15], which is not included in the present calculations.
influential than the channel coupling effect to quasila-
estic scattering at low energies, Figure 2 shows the effect
of deformation of the target nucleus on the quasielastic
cross sections. We find that the effect is negligible at
deep subbarrier energies, and the role played by the sur-
face diffuseness parameter is indeed identified unambigu-
ously. The strongest energy dependence of the cross sec-
tion ratio comes from the exponential factor, \( \exp(-r_c/a) \),
in the nuclear potential \( V_N(r_c) \). The larger value of dif-
huseness parameter results in the stronger energy depen-
dence, and thus the larger deviation of the ratio from
unity. The measured quasielastic cross sections at ener-
gies between 35 and 55 MeV are clearly inconsistent with
\( a=0.65 \text{ fm} \). As in subbarrier fusion reactions, a larger dif-
huseness parameter seems to be required in order to fit
the experimental data.

For completeness of our study, we next examine the
performance of a double folding potential \[24, 25, 26\] for
the subbarrier reactions. In order to construct a nucleus-
nucleus potential with the double folding procedure, we
assume a deformed Fermi function for the (intrinsic) tar-
get density,

\[
\rho_T(r) = \frac{\rho_0}{1 + \exp[(r - R - R \sum \beta \lambda Y_0(r))/a]}.
\]  

We use the same parameters as in Ref. \[24\], including
the \( \beta_2 \) and \( \beta_4 \) deformations. We numerically expand
Eq. \(7\) into multipoles up to \( L=6\), and construct the
double folding potential for each multipole components,
leading to an orientation dependent potential which cor-
responds to Eq. \[3\]. We use the same (spherical) density
for \( ^{16}\text{O} \) as in Ref. \[28\]. For an effective nucleon-nucleon in-
teraction, we use the density-dependent Michigan three-
range Yukawa (DDM3Y) interaction \[29\], together with
the zero-range approximation for the exchange contribu-
tion (See Ref. \[27\] for the parameters). We introduce an
overall scaling factor to the nuclear potential so that the
barrier height is the same as that of the Woods-Saxon
potentials. The cross sections computed with the dou-
ble folding potential thus obtained are denoted by the
thin solid line in Fig. 1. Those are similar to the re-
sults of the Woods-Saxon potential with the diffuseness
parameter of \( a=0.65 \text{ fm} \). In particular, compared with
the experimental data, the double folding potential leads
to a much weaker fall off of quasielastic cross sections at
energies well below the Coulomb barrier. Evidently, the
double folding model does not provide a good represen-
tation both for the quasielastic scattering and the fusion
reaction at subbarrier energies.

In summary, we have studied the sensitivity of large
angle quasielastic scattering to the surface diffuseness
parameter in the nucleus-nucleus potential. We have ar-
gued that the deviation of the ratio of quasielastic to the
Rutherford cross sections from unity at deep subbarrier
energies is sensitive mainly to the surface property of
nuclear potential, and thus provides a useful way to de-
termine the value of surface diffuseness parameter. Using
this fact, we have shown that the experimental excitation
function for quasielastic scattering at energies around the
Coulomb barrier can be reproduced only when a much
larger diffuseness parameter is used in a Woods-Saxon
potential than the commonly accepted value, that is,
around 0.63 fm. This finding is consistent with a recent
observation in heavy-ion subbarrier fusion reactions. It
would be helpful to perform other quasi-elastic measure-
ments at deep subbarrier energies, so that a systematic
study for the diffuseness parameter for scattering process
is possible.

We have also discussed the applicability of a double
folding potential in quasielastic scattering. We have
shown that the cross sections obtained with the double
folding potential is similar to the one obtained with a
Woods-Saxon potential whose surface diffuseness param-
erter is around 0.65 fm. Consequently, the double fold-
ing potential does not reproduce the experimental ex-
citation function for large angle quasielastic scattering
around the Coulomb barrier. This may appear rather
surprising, given that a double folding approach has en-
joyed success in reproducing an angular distribution for
elastic and inelastic scattering in many systems. In order
to reconcile this apparent contradiction, a more careful
investigation, e.g., for the energy dependence of a double
folding potential due to the exchange contribution would
be necessary. We will report this in a separate paper.

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