Unintegrated gluon distribution of a nucleus and photon-jet correlations

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An impact parameter dependent unintegrated gluon distribution is constructed as a solution of a nonlinear evolution equation with realistic Glauber–Gribov input. Photon–jet correlations in pA collisions in the proton fragmentation region are proposed as a direct probe of the nuclear unintegrated glue.

1. Dipole scattering amplitude vs. unintegrated glue

At high energies, where the relevant transverse momenta of partons fulfill $\Lambda_{QCD} \ll p_\perp \ll \sqrt{s}$, we should take them explicitly into account. The most appropriate description is one in terms of unintegrated parton (dominantly gluon) distributions. The latter are related to observables by the so-called $k_\perp$-factorization. If we are in a regime where multiple scattering/saturation effects are large, such as in the case of a nuclear target, the linear $k_\perp$ factorization breaks down and observables are in general nonlinear functionals the unintegrated glue [12]. The nuclear unintegrated glue $\phi(b, x, p)$ is defined in terms of the forward scattering amplitude of a $q\bar{q}$ dipole $r$ at impact parameter $b$, $\Gamma(b, x, r)$:

$$
\int \frac{d^2 r}{(2\pi)^2} \Gamma(b, x, r)e^{-ipr} = (1 - w_0(b, x))\delta(2)(p) - \phi(b, x, p),
$$

where $w_0(b, x)$ has the meaning of Bjorken’s gap survival probability. To construct the nuclear unintegrated glue, one starts at moderately small $x \sim x_A \sim 0.01$, where only the $q\bar{q}$ state is coherent over the whole nuclear size. Then the dipole amplitude for the nuclear target takes the simple Glauber–Gribov form:

$$
\Gamma(b, x_A, r) = 1 - \exp[-\sigma(x_A, r)T_A(b)/2]
$$

where $T_A(b)$ is the nuclear matter density. The nuclear unintegrated glue can then be expressed as an expansion over multiple convolutions of the free–nucleon unintegrated glue $f(x, p)$ [3]:

$$
\phi(b, x_A, p) = \sum w_j(b, x_A)f^{(j)}(x_A, p)
$$

Here

$$
f^{(j)}(x_A, p) = \int [\prod_i d^2\kappa_i f(x_A, \kappa_i)]\delta^{(2)}(p - \sum \kappa_i)
$$

is the collective glue of $j$ overlapping nucleons, and

$$
w_j(b, x_A) = \frac{\nu_A^j(b, x_A)}{j!}\exp[-\nu_A(b, x_A)],
$$

is the probability for $j$ nucleons to contribute. It depends on the effective opacity

$$
\nu_A(b, x_A) = \frac{1}{2}\alpha_S(g^2)\sigma_0(x_A)T_A(b),
$$

where $\sigma_0(x) = \int d^2pf(x, p)$ is a nonperturbative parameter, the cross section of a large color dipole. Interestingly, the expansion [3] also has a simple unitarity cut interpretation: namely $\phi(b, x, p)$ is proportional to the quasielastic quark–nucleus cross section, and the $j$-th term in eq.(3) is the contribution from $j$ cut Pomerons [4].

2. Nonlinear evolution of the unintegrated glue

At smaller values of $x$ the multigluon-Fock states $q\bar{q}, q\bar{q}g_1g_2 \ldots q\bar{q}g_1 \ldots g_n$ with strongly ordered light-cone momentum fractions $z_n \ll z_{n-1} \ldots \ll z_1 \ll 1$ must be accounted for. While for the free nucleon target the effect of all

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multigluon Fock states is summed up by the linear BFKL–equation, no such simple procedure exists for the nuclear target. Simply iterating the first step of small–$x$ evolution, one would arrive at the Balitsky–Kovchegov (BK) equation. In terms of $\phi$ it can be put in the form

$$\frac{\partial \phi(\nu_A, x, p)}{\partial \log(1/x)} = K_{BFKL} \otimes \phi(\nu_A, x, p) + Q[\phi](\nu_A, x, p),$$

with a linear, BFKL, piece, and a quadratic “gluon fusion” term $Q[\phi]$. The latter has a “real” as well as a “virtual” piece $\propto \phi(\nu_A, x, p)$. The numerical treatment is much simplified by the observation, that the real piece can in fact be written as a square:

$$\int d^2q d^2\kappa \phi(\nu_A, x, q) \left[ K(p + \kappa, p + q) - K(p, p + q) \right] \phi(\nu_A, x, \kappa)$$

$$\propto \left( \int d^2\kappa \phi(\nu_A, x, \kappa) \left[ \frac{p}{p^2 + \mu_G^2} - \frac{p + \kappa}{(p + \kappa)^2 + \mu_G^2} \right] + Q[\phi](\nu_A, x, p^2) \right),$$

and can be viewed as a diffractive cut of a triple Pomeron contribution. At large $p^2$, above the saturation scale, we obtain for the fusion term

$$Q[\phi](\nu_A, x, p) \propto -\frac{1}{p^2} \left( \int d^2\kappa \phi(\nu_A, x, \kappa) \right)^2$$

$$- \phi(\nu_A, x, p^2) \int \frac{d^2\kappa}{\kappa^2} \int \frac{d^2q}{q^2} \phi(\nu_A, x, q^2).$$

Notice that it is pure higher twist, and involves only the “anticollinear” integration domain $\kappa^2 > p^2$. In particular it cannot be written in terms of the square of the integrated gluon distribution.

In the practical solution of the evolution equation eq. (5) one integrates over all transverse momenta, from soft to hard, and some regularization in the infrared domain is inevitable. For example, we freeze the running coupling $\alpha_S$ at small momenta. It is also necessary to introduce a finite gluon correlation radius $\mu_G^{-1}$ to remove unphysical long-range gluon exchange contributions. Notice that this can be done without upsetting the gauge cancellations in the kernel, as it would be the case e.g. for a sharp momentum cutoff.

Results from a numerical solution are shown in fig. 1. The boundary condition at $x_A = 0.01$ was constructed in terms of a free–nucleon glue fitted to HERA data. We used $\mu_G^2 = 0.5 \text{ GeV}^2$. We plot $p^2 \phi(\nu_A, x, p) \propto \partial G/\partial p^2$, which is proportional to the phase space density of gluons. Instead of impact parameter $b$ we changed variables to the effective opacity $\nu_A$. Larger $\nu_A$ corresponds to more central, and smaller $\nu_A$ to more peripheral collisions.

The position of the maximum in fig. 1 is a good definition of the saturation scale. We observe,
that it is a function of the opacity as well as of \( x \). It increases for more central collisions (where absorption is stronger) as well as for smaller \( x \). Notice that the evolution of the saturation scale with \( \nu A \) is a result of the dynamics, and does not involve an assumption its impact-parameter dependence. For the boundary condition, the saturation scale is in the soft (for small \( \nu A \)) to semi-hard (for large \( \nu A \)) regime. We need to go to very small \( x \sim 10^{-6} \) to finally leave the soft region behind. Notice however that for most observables an average over \( \nu A \) is implied, and the contribution from the impulse approximation (or lowest order in \( j \)) is never really small (see e.g. [1,6]).

3. Photon-Jet correlations

It would now be helpful to find an observable which allows to map the \( p \)–dependence of the unintegrated glue, and to determine the saturation scale as directly as possible. We have no space here to explain here in detail the formalism of nonlinear \( k_{\perp} \)–factorization, which principal feature is that cross sections of hard processes in a nuclear environment are highly nonlinear functionals (quadratures) of the collective nuclear unintegrated glue [17,6,8]. The emerging nonlinear \( k_{\perp} \)–factorization formulas for the hard dijet spectra we shown to fall into several universality classes depending on color properties of the underlying pQCD subprocess [2,6,8]. These classes differ in the character of incoherent initial state and final state interactions and coherent distortions of two-parton Fock states of the incident parton. Of special interest are reactions in which both the direct photon and the accompanying (balancing, recoiling) quark jet are observed. We shall refer to such final states as dijets. The production of such \( q\gamma \) dijets can be viewed, in the nucleus rest frame, as an an excitation of the \( q\gamma \) Fock state of the incident quark, \( q^* \rightarrow q\gamma \). As with all dijet observables [12,6,8], the dijet spectrum is obtained in terms of multiparton S–matrices of four (\( q\gamma q\gamma \)–), three(\( q\gamma q \)–) and two(\( qq \)–) parton states. In distinction to purely QCD–processes, however there is a great simplification due to the fact that the \( \gamma \) does not interact through multigluon exchanges. As a consequence, the problem abelianizes, and all multiparton S–matrices can be reduced to the ones of \( q\bar{q} \)–states. The emerging dijet spectrum is then a linear functional of the unintegrated glue.

On the free nucleon target, we obtain for the parton–level cross section:

\[
\frac{2(2\pi)^2 d\sigma_{N}(q \rightarrow q\gamma)}{dzd^2p^2 d\Delta} = f(x, \Delta) P_{\gamma q}(z) \times K(p, p - z\Delta),
\]

Figure 2. Top: \( R_{pA} \) at \( x = 0.01 \), Lower: \( R_{cp} \) for \( \nu_\geq = 8, \nu_\leq = 1 \) for different \( x \).
The decorrelation momentum \( \Delta \) out that the spectrum (6) is exact over the full back–to–back situation. It is worth to point measures the deviation of the dijet system from the photon’s transverse momentum \( \epsilon \), the photon emission of the scattered quark. It corresponds to the “monojet” configuration, where the \( q\gamma \) system. In particular notice the collinear pole at \( p = z\Delta \) from the final state photon emission of the scattered quark. It corresponds to the “monojet” configuration, where the \( q\gamma \) system recoils against a jet at a large distance in rapidity.

For the nuclear target we obtain:

\[
\frac{(2\pi)^2 d\sigma_A(q \to q\gamma)}{dx dp d^2 \Delta d^2 b} = \frac{\phi(\nu_A, x, \Delta) + w_0(\nu_A) \delta^{(2)}(\Delta)}{x P(q\gamma)(z) K(p, p - z\Delta)}.
\]

A main result of this work is that the decorrelation momentum distribution maps out the nuclear unintegrated glue. Notice that a potential diffractive contribution \( \propto w_0(\nu_A) \), which would violate the linear \( k_\perp \)-factorization, vanishes on a heavy nucleus, where the momentum transfer distribution is \( \delta \)-function–like. For a recent discussion of \( \gamma \)-particle correlations in the framework of the Color–Glass–Condensate model, see [9].

Nuclear effects are conveniently characterized by the ratio (here we stay at the parton–level throughout)

\[
R_{pA}(\nu_A, p, \Delta) = \frac{\frac{d\sigma_A}{T_A(b) dN}}{\nu_A f(x, \Delta)}.
\]

A similar ratio is the central–to–peripheral ratio, which involves only nuclear quantities:

\[
R_{cp}(\nu_>, \nu_<, p, \Delta) = \frac{\nu_< \phi(\nu_>, x, \Delta)}{\nu_> \phi(\nu_<, x, \Delta)}.
\]

Interestingly, both these ratios do not depend on the photon’s transverse momentum \( p \). We show the ratio \( R_{pA} \) at \( x = x_A \) for different opacities in the top panel of fig.2. We observe a shadowing at small values of \( \Delta \) and a Cronin–type peak which position reflects the \( \nu_A \)-dependent saturation scale. In the lower panel we show \( R_{cp} \) and its evolution with \( x \). While it displays the same Cronin–peak as \( R_{pA} \) for \( x = x_A \), the latter is entirely quenched at small \( x \).

4. Conclusions

We presented a nuclear unintegrated glue from the solution of a nonlinear small–\( x \) evolution equation. Photon–jet correlations in the proton fragmentation region of \( pA \) collisions have been suggested as a direct probe of the nuclear unintegrated glue. They may present an opportunity to measure the saturation scale at the LHC.

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