Optimization of phase masks using simulated annealing algorithm for mode conversion

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Abstract. The simulated annealing algorithm was applied for optimizing binary phase masks used in the conversion of optical modes through spatial light modulation in free space. The method changes the phase distribution to be displayed on a spatial light modulator, in such a way maximizes the correlation between de converted mode and the theoretical mode. The method allowed the optimal conversion of the linearly polarized modes through a diffractive process. The analysis of the correlations between obtained and theoretical modes showed the effectiveness of the method and its capability to generate optical modes similar to those in an optical fiber. The optimized phase masks could be applied in a dynamic and arbitrary mode converter.

1. Introduction

Optical communications are the basis of current communication due their capacity to transmit data at high rates for long distances with low losses \cite{1,2}. Nevertheless, the increasing demand of transmission and availability of information implies the continuous improvement of transmitters, data channels, detectors and so on. Advances in optical fiber technology, erbium-doped fiber amplifiers (EDFA) \cite{3}, and the exploitation of all the physical dimensions of light for new multiplexing and modulation techniques, as wavelength-division multiplexing (WDM) \cite{3}, polarization division multiplexing (PDM) \cite{4}, among others, had satisfied the data transmission until now. However, these techniques and technologies are reaching their fundamental limits \cite{5-7}.

As a feasible solution to this problem, the use of the spatial domain of the light has been proposed as a multiplexing method, called spatial division multiplexing (SDM) \cite{6,8}. SDM establish spatially distinguishable data paths through a physical channel \cite{5,6}.

SDM has been implemented using multicore optical fiber (MOF), multimode fiber (MMF) \cite{9,10}, single mode fiber (SMF), photonic bandgap fibers, few-mode fiber (FMF) \cite{5,6} and free space \cite{7}. When the spatial modes of propagation of light are used as multiplexing method, SDM is called mode division multiplexing (MDM). Those modes could be the linearly polarized modes (LP modes), Laguerre-Gauss modes (LG modes), or any other optical mode capable of propagating through the optical link. By taking advantage of common optical fibers and a simple framework, LP modes in FMF are one of the most promising method for implementing SDM by means of MDM.

Mode conversion is a key process in MDM \cite{11}. It converts an initial mode in a desired mode through changes in phase and amplitude. There are several modal conversion techniques and they could be classified as static, and dynamic techniques. The first, include diffractive optical elements (DOE) \cite{7},...
[12], phase mask [13-15] and fiber Bragg gratings (FGB) [16,17]. The seconds, include mechanical and thermal actuators [17-20], and liquid crystal spatial light modulation (LC-SLM) [11,21].

Dynamic modal converter usually uses phase mask displayed on LC-SLM. Nevertheless, because this kind of device works in free space, morphologic of converted modes differ from the desired mode due to the boundary conditions are different from those inside the fiber core.

In this work, a simulated process of mode conversion using binary phase masks on a spatial light modulation (SLM) device was made. The phase masks were optimized using simulated annealing (SA) algorithm and allowed the optimal conversion of LP modes from an optical fiber through a diffractive process in free space. The analysis of the correlations between obtained and theoretical modes showed the effectiveness of the method and its capability to generate optical modes similar to those in a waveguide. The optimization is only needed to be performed once. Then, phase masks could be applied in a dynamic mode converter, as part of an MDM system. Furthermore, using binary phases reduces the computational cost in the optimization, without significantly compromising the conversion performance.

2. Theory

Binary phase masks consist of a spatial distribution of binary phase values \{0, \pi\}, addressed on a SLM. This phase transforms the incident light to a desired phase and amplitude outgoing distribution. Every mode conversion needs a specific phase mask.

Using LP modes imply the conversion between LP modes. In this way, differences between the converted modes and theoretical modes must be reduced by optimization, taking as figure of merit the theoretical expressions of the LP modes, as follow in Equation (1).

\[
E_{ml} = \begin{cases} 
J_m \left( \frac{r}{a} U_{ml} \right) \cos(\theta) & r \leq a \\
\frac{J_m(U_{ml})}{K_m(W_{ml})} K_l \left( \frac{r}{a} W_{ml} \right) \cos(\theta) & r > a
\end{cases}
\]  

(1)

where \(E_{ml}\) is the electric field amplitude of a linearly polarized beam propagating on the z positive direction, \(J_m\) is the Bessel function of first kind, \(K_m\) is the modified Bessel function of second kind; \(m\) and \(l\) designate azimuthal and radial mode number, respectively; \(a\) is the radius of the fiber core, \(U_{ml}\) is the transverse wavevector, and \(W_{ml}\) is the cladding transverse evanescent propagation constant [22].

2.1. Optimization of phase masks using simulated annealing

The conversion of optical modes using binary phases displayed on a SLM implies to take the light beam out of the fiber and its propagation in free space. In this way, due to the differences in the propagation conditions of the light between the waveguide and free space, converted modes will be different from the theoretical LP modes, which only exist in a confined medium. On the other hand, a mode converter is expected to generate optical modes as close as possible to the guided ones, in such a way they can propagate in the optical fiber. Thus, the optimization process is intended to reduce the difference between converted and theoretical modes by changing the phase mask used in the spatial modulation of the incident mode.

The mode conversion is performed by adding some spatial phase distribution to an incident mode to obtain a desired mode. This phase mask is the phase difference between the incident and desired mode. In the particular case of LP modes, the phase differences are binary phase distributions, and are the starting point of the optimization. We propose an algorithm based in SA for optimization of the binary phase mask.

The SA is a metaheuristic algorithm based on solids cooling process. The process starts taking a system, assumed with a high temperature \(T\) and energy \(E\) (initial or worse case), and stops when \(T\) is equal to zero. The objective function is maximized or minimized. In our case, the correlation between
obtained and corresponding theoretical mode is maximized making changes over the initial phase. While the temperature reduces, the correlation increases. If at a point, the correlation is lower than the correlation of the previous step, the algorithm uses Boltzmann distribution to decide if a bad solution is accepted or rejected. If rejected the change in the step is ignored and try to find another [11,23,24].

Boltzmann distribution is defined in Equation (2).

$$P = e^{-\frac{\Delta E}{T}},$$  \hspace{1cm} (2)

where $\Delta E$ is the energy change. Each iteration influences the velocity of temperature decreasing and modifies $\Delta E$, causing a diminution of the probability of accepting bad solutions.

The similarity measurement in the optimization process is the Pearson correlation coefficient, defined as Equation (3) [23-25].

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\left(\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^{\frac{1}{2}}\left(\sum_{i=1}^{n}(y_i - \bar{y})^2\right)^{\frac{1}{2}}},$$  \hspace{1cm} (3)

where $x_i, y_i$ is the intensity of the i-th pixel of the theoretical and current mode, respectively. $\bar{x}$ and $\bar{y}$ are the mean intensity.

2.2. Mode conversion

The modal conversion setup using SLM consists of a 4f system. Figure 1 shows a schematic 4f system, in which both lenses act as an optical Fourier transform ($\mathcal{F}$). The incident mode gets on the SLM, where the phase necessary to convert the mode is added through the phase mask ($H$) displayed on the SLM. In this way, the outgoing beam will have a morphological distribution, described by Equation (4).

$$MD = \mathcal{F}(\mathcal{F}(IM) * H),$$  \hspace{1cm} (4)

where MD is the converted mode, and IM, the incident mode.

![Figure 1. Schematic setup of a mode converter using a SLM in free space. In a 4f system, the incident mode is converted in a desired mode by changing its phase in the central Fourier plane, in which a SLM displays a phase mask.](image)

3. Optimization

The morphological difference between the obtained modes without optimization and the desired modes, lies in the change in the boundary conditions between free space and a waveguide. In order to obtain similar modes, it is necessary to optimize the phase masks on the SLM. In Figure 2, the process is explained in a schematic process. The algorithm begins finding the initial phase mask (M1), calculated by subtracting the phases of the theoretical incident and desired modes. Then, a modal conversion takes place using M1 in Equation (4). Afterwards, a single random pixel of the mask is changed to the opposite binary value, generating a modified mask (M2). A modal conversion is calculated using M2 to correlate
the resulting mode with the theoretical desired mode. The correlation coefficient between the mode obtained with M1 and the theoretical mode, will be identified as the previous correlation (C_p); meanwhile the correlation coefficient obtained with M2 will be identified as current correlation (C_c). Both correlations are calculated using Equation (3).

If \( C_C > C_p \), the result is accepted and M1 equals M2. In the case where \( C_c < C_p \), the algorithm uses equation 2 to calculate the probability P and a random probability R. If \( P > R \) the result is accepted and then M1 equals M2. If \( P < R \) the result is discarded and M2 equals M1. Finally, the temperature is decreased, and checked if it is greater or equal to zero. In the case the temperature is greater than zero, the process goes to the step in which a random single pixel is modified. If zero, the process stops and returns an optimized phase mask.

![Figure 2. Flowchart of the simulated annealing algorithm used to optimize phase mask for mode conversion. C_C is the current correlation, C_p is the previous correlation, P and R denote a Boltzmann distributed and a random probability, respectively.](image)

4. Results and discussion
The process was made using the fundamental LP_{01} as the incident mode, and the conversion gives higher-order modes. In Figure 3(a) shows theoretical modes calculated using Equation (1), which are the desired modes, specifically, LP_{11a}, LP_{11b}, and LP_{21}. Additionally, in Figure 3(b) the first column shows the initial phase masks without optimization, resulting of subtracting the theoretical phases of the incident and desired modes. The second column contains the modes obtained by modal conversion with the non-optimized masks. It is clear the morphological difference between these modes and the corresponding modes shown in Figure 3(a).

The two columns in Figure 3(c), show the optimized phase masks and their corresponding resulting modes. These modes are closer to the theoretical ones, compared with the modes shown in Figure 3(b). The quantitative analysis was performed through the correlation coefficients between the obtained modes, non-optimized and optimized, and their corresponding theoretical modes. The correlations were
calculated using the Pearson formula in Equation (3), and the results were presented in Figure 4 and Figure 5.

![Figure 3](image1.png)

**Figure 3.** Morphological analysis between desired modes and obtained modes through modal conversion using an optimized and non-optimized phase masks. (a) Desired mode, (b) obtained mode without optimized mask, and (c) obtained mode with optimized mask.

Figure 4 shows the bar diagram of the correlation coefficients, comparing the obtained modes before (blue) and after optimization (red). The non-optimized modes have a maximum of 72% correlation for low-order modes (LP<sub>21</sub>), and a minimum of 33% in the high-order LP<sub>51</sub> mode. Higher correlation values, between 88% and 98%, were obtained due to the optimization. Despite all modes were significantly improved, high-order modes had a more noticeable change. In Figure 5 is shown the correlation matrix between each desired theoretical mode with the obtained modes with optimized phase masks. A close to one diagonal shows the high similarity.

![Figure 4](image2.png)

**Figure 4.** Correlation between calculated and theoretical LP modes. Blue bar for modes without optimization, and red bar for modes with optimized masks.

![Figure 5](image3.png)

**Figure 5.** Correlation matrix between optimized and theoretical LP modes.
5. Conclusions
The simulated annealing algorithm was applied to optimize the phase masks needed for a mode converter based on a spatial light modulator in free space. The method allowed the generation of modes with high similarity to the modes guided in a few-mode optical fiber. Starting with correlations as low as 33%, and reaching correlations until 98%, the process is effective to correct the differences between guided modes and free-space modes.

The results show that a mode converter based on a SLM in free space could be implemented in a MDM system, since the optimized modes will have high coupling efficiency to an optical fiber, can be dynamically controlled in a SLM like a digital micromirror device, and the process could be applied to generate any kind of optical modes of propagation.

Despite the diffraction limitations of a binary phase mask, the high correlations achieved showed the approximation is suitable and has a lower computational cost in the optimization. For each mode, the optimization is only needed to be performed once. The optimized masks can then be programed to be displayed on a SLM.

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