Loop-Induced CP Violation in the Gaugino and Higgsino Sectors of Supersymmetric Theories

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ABSTRACT

We show that the gaugino and higgsino sectors of supersymmetric theories can naturally acquire observable CP violation through radiative effects which originate from large CP-violating trilinear couplings of the Higgs bosons to the third-generation scalar quarks. These CP-violating loop effects are not attainable by evolving the supersymmetric renormalization-group equations from a higher unification scale down to the electroweak one. We briefly discuss the phenomenological consequences of such a scenario, and as an example, calculate the two-loop contribution to the neutron electric dipole moment generated by the one-loop chromo-electric dipole moment of the gluino.

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Supersymmetric (SUSY) theories, including the minimal supersymmetric Standard Model (MSSM), predict several new unsuppressed CP-violating phases which generally lead to too large contributions to the electric dipole moments (EDM’s) of the neutron and electron [1, 2, 3, 4, 5, 6]. Several suggestions have been made in the literature to overcome such a CP crisis in SUSY theories. Apart from the cancellation mechanism proposed in [3], an interesting solution to the above CP-crisis problem is to assume that the first two generations are either very high above the TeV scale [4, 7] or they do not involve CP-violating phases in their trilinear couplings $A_f$ to the Higgs bosons [6]. In the latter SUSY framework, with all scalar quarks much below the TeV scale, we also have to require that the SU(3)$_c$, SU(2)$_L$ and U(1)$_Y$ gaugino masses: $m_{\tilde{g}}$, $m_{\tilde{W}}$ and $m_B$, as well as the higgsino-mass term $\mu$ be real parameters. Then, for both of the above scenarios, only the scalar-top ($\tilde{t}$) and bottom quarks ($\tilde{b}$) may be considered as potential mediators of CP non-conservation. Here, we shall adopt this minimal model of SUSY CP violation. We will not address the issue concerning the underlying mechanism of SUSY breaking that leads to the above low-energy scenario, as it is beyond the scope of the present study (see also [7]).

In the above discussion, the CP-violating phases were defined in the weak basis in which the soft-SUSY-breaking parameter $m_{12}^2$ of Higgs mixing is real. At the tree level, such a convention leads to positive vacuum expectation values $v_1$ and $v_2$ for the two Higgs doublets $\Phi_1$ and $\Phi_2$ of the MSSM, where the relative phase $\xi$ between $v_1$ and $v_2$ vanishes. If this phase convention is to be adopted order by order in perturbation theory [8], then the formally radiatively-induced phase $\xi$ can always be eliminated by an appropriate choice of the renormalization counter-term (CT) Im$m_{12}^2$. This latter CT is related to CP-odd tadpole graphs of the pseudoscalar Higgs bosons, and is important in rendering the CP-violating Higgs scalar-pseudoscalar transitions ultra-violet finite [8] in Higgs-boson mass spectrum analyses [9, 10]. However, it should be stressed that unlike Re$m_{12}^2$, Im$m_{12}^2$ does not directly enter the renormalization of the physical kinematic parameters of the MSSM, such as the charged Higgs-boson mass and $\tan \beta = v_2/v_1$. Consequently, no additional CP violation can be generated through the phase $\xi$ in the effective chargino and neutralino mass matrices.

In this paper, we shall study a novel mechanism of radiatively inducing CP violation in the gaugino and higgsino sectors of the MSSM. In these sectors, CP violation is communicated by trilinear interactions of scalar top and bottom quarks to the Higgs bosons. It is interesting to remark that this loop-induced CP violation cannot be achieved by running the renormalization-group (RG) equations from a higher unification scale down to the

*Nevertheless, it might still be necessary to assume that CP violation in the $K^0\bar{K}^0$ system is described by the usual Cabbibo-Kobayashi-Maskawa matrix.
electroweak energies [11]. Even though the discussion is confined to the MSSM, the results of the analysis can straightforwardly be generalized to other SUSY extensions of the SM.

We start our discussion by considering the gluino sector. In the convention in which the gluino mass is real, i.e. \( \arg(m_{\tilde{g}}) = 0 \), CP violation is induced by the interaction Lagrangian:

\[
\mathcal{L}_{\tilde{g}q} = -\sqrt{2}g_s \sum_{q=t,b} \left( \tilde{q}_L \tilde{g} \gamma^a \frac{\lambda^a}{2} P_L q - \tilde{q}_R \tilde{g} \gamma^a \frac{\lambda^a}{2} P_R q \right) + \text{h.c.,}
\]

where \( P_{L(R)} = (1 - (+(-)\gamma_5))/2 \) is the left- (right-) handed chirality projection operator and \( \lambda^a \), with \( a = 1, \ldots, 8 \), are the usual SU(3) Gell-Mann matrices. In fact, CP violation enters through the mixing of the weak \((\tilde{q}_L, \tilde{q}_R)\) with mass eigenstates \((\tilde{q}_1, \tilde{q}_2)\). Such a mixing of states is described by the unitary transformation:

\[
\begin{pmatrix}
\tilde{q}_L \\
\tilde{q}_R
\end{pmatrix} = U^q \begin{pmatrix}
\tilde{q}_1 \\
\tilde{q}_2
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
0 & e^{i\delta_q}
\end{pmatrix} \begin{pmatrix}
\cos \theta_q & \sin \theta_q \\
-\sin \theta_q & \cos \theta_q
\end{pmatrix} \begin{pmatrix}
\tilde{q}_1 \\
\tilde{q}_2
\end{pmatrix},
\]

where \( \delta_q = \arg(A_q - \mu^* R_q) \), with \( R_t = \cot \beta \) and \( R_b = \tan \beta \), is a non-trivial CP-violating phase. The gluino mass \( m_{\tilde{g}} \) receives a finite radiative shift from diagrams depicted in Fig. 3.
which can easily be calculated to be

$$\delta m_\tilde{g} = \frac{\alpha_s}{2\pi} m_t \sin \theta_t \cos \theta_t e^{-i\delta t} \int_0^1 dx \ln \left[ \frac{m^2 x + M^2_{\tilde{g}}(1-x)}{m^2 x + M^2_{\tilde{g}}(1-x)} \right].$$  (3)

It is obvious that only scalar-top quarks are of relevance, as the effect of the scalar bottom quarks is suppressed by a factor $m_b^2/m_t^2$. For relatively large values of the scalar-top mixing, i.e. $M_{\tilde{t}_2}/M_{\tilde{t}_1} \sim 5$, $\theta_t \approx 45^\circ$, we find a radiatively-induced phase for $m_\tilde{g}$: $\text{arg}(m_\tilde{g}) \approx 10^{-2}m_t/m_\tilde{g}$. Even though $\text{arg}(m_\tilde{g})$ may formally be re-absorbed into the renormalization of $m_\tilde{g}$, its presence will, however, manifest itself in related CP-violating higher-point correlation functions\textsuperscript{4}, such as the chromo-electric dipole moment (CEDM) form factor of the gluino described by the effective Lagrangian $\mathcal{L}^\text{CEDM}_{\tilde{g}} = \frac{1}{4} d_\tilde{g}^{abc} f^{abc} g^{i\sigma\mu\nu} \gamma_5 \tilde{g}^c F^a_{\mu\nu}$, where $F^a_{\mu\nu}$ is the gluon field strength tensor and

$$\frac{d^2(p^2)}{g_s} = \frac{\alpha_s}{4\pi} m_t \sum_{i=1,2} \text{Im} \left(U^i_{1i} U^i_{2i}\right) \int_0^1 dx \frac{x}{m_i^2 x + M^2_{\tilde{t}_i}(1-x) - p^2 x(1-x)}. \quad \text{(4)}$$

Notice that the gluino being a Majorana particle with internal degrees of freedom, i.e. colour, can only possess a non-vanishing CEDM; it cannot have an EDM. Furthermore, at two loops, the CEDM form factor of Eq. (4) can in turn give rise to a CEDM of a light quark $d_q^C$, as shown in Fig. 2. The contribution to CEDM of a light quark $d_q^C$ has been calculated to give

$$\frac{d_q^C}{g_s} = \frac{3\alpha^2}{16\pi^2 m_\tilde{g}^2} \sum_{i,j=1,2} \text{Im} \left(U^i_{1i} U^j_{2i}\right) \text{Re} \left(U^i_{1j} U^j_{2j}\right) H \left(\frac{M^2_{\tilde{g}}}{m^2_{\tilde{g}}}, \frac{m^2_{\tilde{g}}}{m^2_{\tilde{g}}}, \frac{M^2_{\tilde{g}}}{m^2_{\tilde{g}}} \right),$$  (5)

where the two-loop function $H(a, b, c)$ is defined as

$$H(a, b, c) = \int_0^1 dx \frac{x}{(1-a)^2} \left[ \frac{1-a}{x(1-x) - bx - c(1-x)} + \frac{a \ln a}{ax(1-x) - bx - c(1-x)} \right] - \frac{(1-a)^2 \ln \left(\frac{bx + c(1-x)}{x(1-x)}\right)}{[ax(1-x) - bx - c(1-x)][x(1-x) - bx - c(1-x)]^2}. \quad \text{(6)}$$

Based on the naive quark model and after including QCD renormalization effects, the contribution to the neutron EDM may be estimated to be

$$\frac{d_n^C}{e} \approx \left(\frac{g_s(M_Z)}{g_s(\Lambda)}\right)^{120/23} \left[ \frac{4}{9} \left(\frac{d_u^C}{g_s}\right)_\Lambda + \frac{2}{9} \left(\frac{d_d^C}{g_s}\right)_\Lambda \right],$$  (7)

where $g_s$ and the $u$- and $d$-quark masses in $d_u^C$ and $d_d^C$ are calculated at the scale $\Lambda$: $m_u(\Lambda) = 7$ MeV, $m_d(\Lambda) = 10$ MeV, and $g_s(\Lambda)/(4\pi) = 1/\sqrt{6}$ \textsuperscript{5}.\footnote{As was discussed in \textsuperscript{2}, $\text{arg}(m_\tilde{g})$ may also contribute to the CP-violating phase of QCD $\tilde{\theta}$. Here we shall not deal with this strong CP problem, but merely assume that $\tilde{\theta}$ is eliminated by a chiral rotation of the quark fields.}
Table 1: Numerical estimates of the gluino-induced CEDM contribution to the neutron EDM, assuming moduli-universal trilinear couplings: $|A_f| = 1$ TeV, with arg$(A_{t,b}) = 90^\circ$ and arg$(A_{u,d}) = 0$, and non-universal soft-scalar-quark masses: $\tilde{M}_{tL} = \tilde{M}_{tR} = 500$ GeV and $\tilde{M}_{u,d} = \tilde{M}_{uL,dL} = \tilde{M}_{uR,dR}$ as given in the table.

In Table 1, we exhibit numerical estimates of $d_n^C/e$, for a non-universal scenario for the scalar-quark masses with moduli-universal trilinear couplings. We assume that the first two generations possess no CP violation, and the only non-vanishing CP phase is arg$(A_{t,b}) = 90^\circ$. We find that the EDM contributions increase almost linearly with $\tan \beta$ and, as expected, decouple when the first-generation scalar-quark masses $\tilde{M}_{u,d}$ become large. The additional EDM contributions to $d_n$ are comparable to its present experimental upper limit: $|d_n| < 6.3 \times 10^{-26}$ \cite{12}, for $\tilde{M}_{u,d} \lesssim 300$ GeV, and relatively large scalar-top mixing. In particular, it would be interesting to remark that for the same values of the $\mu$-term and the soft-SUSY-breaking parameters, the two-loop EDM effects induced by the gluino CEDM are comparable in size to the two-loop EDM contributions due to Weinberg’s three-gluon operator, for $m_{\tilde{g}} \gtrsim 400$ GeV \cite{3, 5}, and to the two-loop Barr-Zee-type EDM graphs \cite{6}, for ‘CP-odd’ scalar masses larger than 700 GeV.

In the following, we will focus our attention on the chargino sector. Significant CP violation in this sector can be induced by the interactions of the charged gaugino, $\tilde{W}^T = (\tilde{w}^+, \tilde{w}^-)$, and higgsino, $\tilde{H}^T = (\tilde{h}_1^+, \tilde{h}_1^-)$ with $\tilde{t}$, $\tilde{b}$ and $t$, $b$. Specifically, the interaction Lagrangian of interest to us reads:

$$\mathcal{L}^{(\tilde{W}, \tilde{H})_{\text{int}}} = -g_w (\tilde{b}_L^* \overline{W} P_L t + \tilde{t}_L^* \overline{W}^C P_L b) + h_b (\tilde{b}_R^* \overline{H} P_L t + \tilde{t}_L^* \overline{H}^C P_R b)$$

$$+ h_t (\tilde{b}_L^* \overline{H} P_R t + \tilde{t}_R^* \overline{H}^C P_L b) + \text{h.c.}$$

(8)

Again, the graphs shown in Fig. 3 can give rise to CP-violating self-energy transitions among...
the chargino fields. In particular, Fig. 3(c), which represents an off-diagonal wave-function contribution to the $\tilde{W}\tilde{H}$ transition, becomes dominant; the other graphs are suppressed by powers of $m_b/m_t$. To have an estimate of CP violation, we consider the effective kinetic Lagrangian

$$\mathcal{L}_{\text{eff}}^{\text{kin}} = (\tilde{W}, \tilde{H}) \left( \begin{pmatrix} \tilde{\phi} - m_{\tilde{W}} \\ \Sigma_L \tilde{p} \rho - \frac{1}{\sqrt{2}} g_w v_1 P_R - \frac{1}{\sqrt{2}} g_w v_2 P_L \end{pmatrix} \right) \left( \begin{pmatrix} \tilde{W} \\ \tilde{H} \end{pmatrix} \right),$$

with

$$\Sigma_L(p^2 = 0) = \frac{3 g_w h_t}{32 \pi^2} \sin \theta_t \cos \theta_t e^{-i \delta_t} \ln \left( \frac{M_{\tilde{t}_1}^2}{M_{\tilde{t}_2}^2} \right).$$

Notice that $\Sigma_L$ should not be renormalized away, since the respective $\tilde{W}\tilde{H}$-wave-function CT would have hardly violated the original R-symmetry of the MSSM by a dimension-four operator; the R-symmetry is only softly broken in the MSSM by the gaugino masses, the trilinear couplings $A_f$ and $\mu$. Under the R-symmetry, matter superfields and gauginos carry R-charges, while Higgs superfields are neutral. After canonically normalizing the derivative part of $\mathcal{L}_{\text{eff}}^{\text{kin}}$, we obtain the effective CP-violating chargino mass matrix:

$$\mathcal{L}_{\text{eff}}^{\text{mass}} \approx (\tilde{W}_R, \tilde{H}_R) \left( \begin{pmatrix} m_{\tilde{W}} - \frac{1}{2} g_w v_2 \Sigma_L - \frac{1}{2} g_w v_1 \Sigma_L \\ \frac{1}{\sqrt{2}} g_w v_2 \Sigma_L \end{pmatrix} \begin{pmatrix} \tilde{W}_L \\ \tilde{H}_L \end{pmatrix} + \text{h.c.} \right).$$

From Eq. (11), it is not difficult to see that CP violation becomes of order unity in the chargino sector, when $v_1 \sim \mu \text{Im } \Sigma_L$. This last condition can be easily satisfied for relatively large scalar-top mixing. For example, for $M_{\tilde{t}_2}/M_{\tilde{t}_1} \sim 3$ and $\mu$, $A_t \sim 1 \text{ TeV (} \theta_t \approx 45^\circ)$, we have $\text{Im } \Sigma_L \sim 10^{-2}$ and $\mu \text{Im } \Sigma_L \sim 10 \text{ GeV}$, which is indeed comparable to $v_1$ for $\tan \beta > 2$.

It might now seem that the CP-violating phases in the effective chargino-mass matrix (11) could almost be absorbed by a field redefinition of the charginos. However, the same CP-violating phases will reappear in the interaction Lagrangian (8). In fact, CP-violating physical observables, such as the electron EDM, are invariant under such phase rotations. Therefore, chargino- and neutralino-mediated diagrams such as those presented in Fig. 4 are expected to give large EDM effects, comparable to the two-loop Higgs-boson contributions discussed in [6]. An extensive study of these novel EDM graphs will be given elsewhere [13].

In summary, we have explicitly demonstrated that sizeable CP-violating trilinear couplings of the Higgs bosons to third-generation scalar quarks can introduce observable CP violation into the gaugino and higgsino sectors of the MSSM, which is not attainable by an evolution of the SUSY RG equations. Based on this minimal CP-violating SUSY framework, we have shown that a new Higgs-independent class of two-loop graphs exists which may lead to potentially large contributions to the electron and neutron EDM’s. Finally,
it is important to stress that even though the two-loop gaugino- and higgsino-mediated contributions to EDMs only involve a single SUSY CP-violating phase, i.e. \( \arg A_t \), they can result in different signs related to the signs of the gaugino masses and the \( \mu \) parameter, and so allow for potential cancellations with other existing third-generation scalar-quark contributions studied in the literature \[3, 6\]. The latter may represent a technically natural as well as phenomenologically appealing solution to the CP-crisis problem in SUSY theories. In this context, it would be interesting to analyze in detail the phenomenological impact of loop-induced gaugino/higgsino-sector CP violation on \( B \)-meson decays, dark-matter searches and electroweak baryogenesis \[14\].
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