A $U$-Spin Anomaly in Charm CP Violation

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Abstract

Recent LHCb data shows that the direct CP asymmetries of the decay modes $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^+K^-$ have the same sign, violating an improved $U$-spin limit sum rule in an unexpected way at 2.1σ. From the new data, we determine for the first time the imaginary part of the CKM-subleading, $U$-spin breaking $\Delta U = 1$ correction to the $U$-spin limit $\Delta U = 0$ amplitude. The imaginary part of the $\Delta U = 0$ amplitude is determined by $\Delta a_{\text{dir}}^{\text{CP}}$. The corresponding strong phases are yet unknown and could be extracted in the future from time-dependent measurements. Assuming $\mathcal{O}(1)$ strong phases due to non-perturbative rescattering, we find the ratio of $U$-spin breaking to $U$-spin limit contributions to the CKM-subleading amplitudes to be $(173^{+85}_{-74})\%$. This highly exceeds the Standard Model (SM) expectation of $\sim 30\% U$-spin breaking, with a significance of 1.95σ. If this puzzle is confirmed with more data in the future, in the SM it would imply the breakdown of the $U$-spin expansion in CKM-subleading amplitudes of charm decays. The other solution are new physics models that generate an additional $\Delta U = 1$ operator, leaving the $U$-spin power expansion intact. Examples for the latter option are an extended scalar sector or flavorful $Z'$ models.

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I. INTRODUCTION

After the discovery of CP violation in charm decays \cite{1}, recently, there have again been several important advances in the measurement of mixing and CP violation in charm decays \cite{2-9}, see Ref. \cite{10} for most recent world averages and global fits. Also theoretically, charm CP violation obtains a lot of attention right now \cite{11-25}, see earlier Refs. \cite{26-55}. The most recent news is the first evidence of a non-vanishing CP asymmetry

\[ a_{\text{dir}}^{CP} (f) \equiv \frac{|A(D^0 \to f)|^2 - |A(D^0 \to \bar{f})|^2}{|A(D^0 \to f)|^2 + |A(D^0 \to \bar{f})|^2} \]  

in a single decay \cite{2}, namely \( D^0 \to \pi^+\pi^- \). The knowledge of both CP asymmetries \cite{2}

\[ a_{\text{dir}}^{CP}(D^0 \to K^+K^-) = (7.7 \pm 5.7) \cdot 10^{-4} , \]  

\[ a_{\text{dir}}^{CP}(D^0 \to \pi^+\pi^-) = (23.2 \pm 6.1) \cdot 10^{-4} \]  

gives important advantages compared to the combination

\[ \Delta a_{\text{dir}}^{CP} \equiv a_{\text{dir}}^{CP}(D^0 \to K^+K^-) - a_{\text{dir}}^{CP}(D^0 \to \pi^+\pi^-) \]  

only. The reason is that a separate measurement of both CP asymmetries allows to test the \( U \)-spin expansion in the amplitude contributions which are relatively suppressed by Cabibbo-Kobayashi-Maskawa (CKM) matrix elements compared to the leading singly-Cabibbo-suppressed (SCS) amplitude. In fact, we can now probe the \( U \)-spin limit sum rule for the sum of CP asymmetries \cite{35, 42, 50, 53}

\[ \Sigma a_{\text{dir}}^{CP} \equiv a_{\text{dir}}^{CP}(D^0 \to K^+K^-) + a_{\text{dir}}^{CP}(D^0 \to \pi^+\pi^-) = 0 , \]  

which is violated at \( 2.7 \sigma \) \cite{2}. Remarkably, Eq. (1.5) predicts that \( a_{\text{dir}}^{CP}(D^0 \to K^+K^-) \) and \( a_{\text{dir}}^{CP}(D^0 \to \pi^+\pi^-) \) have opposite signs, but in fact the measurement shows that they have the same sign.

An improved version of the sum rule Eq. (1.5) is given as \cite{35, 42, 50, 53}

\[ \frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)}_{U\text{-spin limit}} \equiv - \frac{a_{\text{dir}}^{CP}(D^0 \to \pi^+\pi^-)}{a_{\text{dir}}^{CP}(D^0 \to K^+K^-)} . \]  

The sum rules Eqs. (1.5, 1.6) belong to a category of \( U \)-spin sum rules which are based on the complete interchange of \( s \) and \( d \) quarks \cite{56, 58}. Inserting the experimental measurements listed in Table I below, we obtain

\[ \frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} = 2.81 \pm 0.06 \]  

(1.7)
and

$$- \frac{a_{CP}^{dir}(D^0 \to \pi^+\pi^-)}{a_{CP}^{dir}(D^0 \to K^+K^-)} = -3.01_{-0.95}^{+0.05},$$

(1.8)

i.e. altogether

$$- \frac{\Gamma(D^0 \to K^+K^-) a_{CP}^{dir}(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-) a_{CP}^{dir}(D^0 \to \pi^+\pi^-)} = -0.93_{-0.41}^{+0.62} \neq 1.$$  

(1.9)

The improved $U$-spin limit sum rule Eq. (1.6) is broken at 2.1σ, because Eq. (1.9) has the “wrong” sign. While $U$-spin breaking is expected, because $U$-spin is only an approximate symmetry of QCD, the amount of breaking goes beyond the Standard Model expectations of $\varepsilon \sim m_s/\Lambda_{QCD} \sim 30\%$ at 1.9σ.

In this article, we analyze the implications of the new charm CP measurements in more detail, extracting the CKM-subleading $\Delta U = 1$ contributions to the amplitudes of $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays. In the SM these are generated from the tensor product of the $U$-spin limit $\Delta U = 0$ operator with the $U$-spin breaking triplet operator [25, 59].

After briefly reviewing the application of SU(3)$_F$ methods in charm decays in Sec. II, we summarize our notation in Sec. III. In Sec. IV we recapitulate how to completely solve the system of two-body $D^0$ decays to kaons and pions. We also show explicitly how to extract in principle the strong phases of the CKM-subleading $\Delta U = 1$ and $\Delta U = 0$ hadronic matrix elements from time-dependent CP violation. In Sec. V we present our numerical results. Finally, in Sec. VI we give predictions and options for interpretations in terms of new physics models that can be tested with future and more precise data. We conclude in Sec. VII.

II. REVIEW OF SU(3)$_F$-BREAKING IN CHARM DECAYS

The application of SU(3)$_F$ methods in particle physics have their roots in spectroscopy, namely the “eightfold way” for the description of the spectrum of the meson and baryon octets [60, 61]. In spectroscopy, SU(3)$_F$ has proven to be an extremely useful ordering principle. For example, SU(3)$_F$-limit predictions agree with the baryon octet mass splitting with an accuracy of 10% [62]. Furthermore, the Gell-Mann–Okubo mass formula [60, 63] demonstrated that by including SU(3)$_F$-breaking effects in a systematic way the precision of predictions can be significantly improved. We know therefore that SU(3)$_F$ is a very trustable
technique for the particle spectrum. The question is if the same applies also to decay rates, in particular for charm decays.

The nominal size of $SU(3)_F$-breaking for decay amplitudes can be estimated from the ratio of the decay constants \[ \frac{f_K}{f_\pi} = 1 \sim 0.2. \] \hfill (2.1)

Now, two important examples where it looks naively as if $U$-spin is broken by $O(1)$ are given in terms of the ratios

\[ \frac{\mathcal{B}(D^0 \to K^+K^-)}{\mathcal{B}(D^0 \to \pi^+\pi^-)} \sim 3, \] \hfill (2.2)

\[ \frac{\mathcal{B}(D^0 \to K_SK_S)}{\mathcal{B}(D^0 \to K^+K^-)} \sim 0.03. \] \hfill (2.3)

In the strict $SU(3)_F$ limit, neglecting also differences from phase space effects, we have:

\[ \frac{\mathcal{B}(D^0 \to K^+K^-)}{\mathcal{B}(D^0 \to \pi^+\pi^-)} = 1, \] \hfill (2.4)

\[ \frac{\mathcal{B}(D^0 \to K_SK_S)}{\mathcal{B}(D^0 \to K^+K^-)} = 0, \] \hfill (2.5)

in clear contradiction with the experimental measurements Eqs. (2.2) and (2.3). However, already in Ref. [65] it was realized that Eq. (2.2) can actually be explained by $\varepsilon \sim 30\%$ $SU(3)_F$-breaking on the amplitude. This can be understood as follows: Already for $\varepsilon \sim 30\%$, very roughly the ratio of branching ratios can be estimated as

\[ \frac{(1 + \varepsilon)^2}{(1 - \varepsilon)^2} \sim 3. \] \hfill (2.6)

That means, Eq. (2.2) can be consistently explained with $SU(3)_F$ breaking of $\sim 30\%$ on the amplitude. Note that the rough illustration Eq. (2.6) also demonstrates that higher order contributions may be important, as at linear order the left-hand side of Eq. (2.6) results in $\sim 2$, and only at $O(\varepsilon^2)$ it reaches $\sim 3$. Below, we extract first and second order $U$-spin breaking from branching ratio data, see Eqs. (3.4)–(3.7) and Table III. For the CKM-leading amplitudes, which dominate the branching ratios, our results support that $\varepsilon \sim 0.3$ and $\varepsilon^2 \sim 0.1$, consistent with the $U$-spin power counting.

Coming now to the second example, as $\mathcal{B}(D \to K_SK_S)$ vanishes in the $SU(3)_F$ limit, we can estimate the corresponding amplitude-level $SU(3)_F$ breaking roughly as

\[ \varepsilon' \sim \sqrt{\frac{\mathcal{B}(D^0 \to K^0K^0)}{\mathcal{B}(D^0 \to K^+K^-)}} \sqrt{\frac{2\mathcal{B}(D^0 \to K_SK_S)}{\mathcal{B}(D^0 \to K^+K^-)}} \sim 0.26, \] \hfill (2.7)
again consistent with the nominal size of SU(3)$_F$ breaking. Here we use the experimental values\cite{66}

\begin{align}
B(D^0 \to K_SK_S) &= (1.41 \pm 0.05) \cdot 10^{-4}, \quad (2.8) \\
B(D^0 \to K^+K^-) &= (4.08 \pm 0.06) \cdot 10^{-3}, \quad (2.9)
\end{align}

and, due to Bose symmetry, see e.g. Ref.\cite{37}

\begin{align}
A(D^0 \to K_SK_S) &= -\frac{1}{\sqrt{2}}A(D^0 \to \overline{K}^0K^0). \quad (2.10)
\end{align}

It follows that Eqs. (2.2, 2.3) can not be used as an argument that SU(3)$_F$ is broken at $\mathcal{O}(1)$ for charm decays.

We note that if one would adopt additional theory assumptions in terms of a $1/N_c$ power counting\cite{28,67} on top of the SU(3)$_F$ expansion, in the $1/N_c$ limit one can factorize the tree amplitude of non-leptonic charm decays, see e.g. Ref.\cite{38}. However, in this case the factorizable $U$-spin breaking of the tree amplitudes alone does not suffice in order to explain the SU(3)$_F$ breaking in Eq. (2.2)\cite{68}. In the topological diagram approach, besides the tree amplitude $T$, the branching ratios of $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ depend also on exchange diagrams $E$ and SU(3)$_F$-breaking combinations of penguin contractions of the tree operator $P_{\text{break}}$; see the parametrizations in Refs.\cite{32,38}. Therefore, under the assumption of a $1/N_c$ power counting, in order to explain Eq. (2.2), additional contributions to the SU(3)$_F$-breaking have to come from these contributions. At first glance this seems counterintuitive, as $E$ and $P_{\text{break}}$ are formally $1/N_c$ suppressed relative to $T$, which would also affect the possible amount of SU(3)$_F$ breaking. However, there are two contributions to these respective topological diagrams, which are both suppressed by $1/N_c$, and which stem from the Hamiltonian\cite{38}

\begin{align}
H_W \propto C_1Q_1 + C_2Q_2, \quad (2.11)
\end{align}

where

\begin{align}
C_1 &\sim \mathcal{O}(1/N_c), & C_2 &\sim \mathcal{O}(1), \quad (2.12) \\
\langle PP'|Q_1|D \rangle &\sim \mathcal{O}(1), & \langle PP'|Q_2|D \rangle &\sim \mathcal{O}(1/N_c). \quad (2.13)
\end{align}

A priori, it is unclear how the two terms of order $\mathcal{O}(1/N_c)$ from Eq. (2.11) interfere. This depends on the assumptions one makes about the respective matrix elements and can at
this time not be determined from first principles. The fit in Ref. [38] shows the existence of a solution that is compatible with \(1/N_c\) counting, namely when both contributions interfere constructively, see Fig. 3(c) therein. This leads then to a large \(E/T\) ratio, see also Refs. [52, 68, 69].

The fit result can be understood already when considering the single decay mode \(B(D \to K_SK_S)\), which only depends on \(SU(3)_F\)-breaking exchange diagrams. Eq. (2.7) determines their relative size as \(\sim 0.26\). In case of a constructive interference of the matrix elements of Eq. (2.11), together with the estimates Eq. (2.13), we obtain the rough estimate

\[
\varepsilon(|C_1| + |C_2/N_c|) \sim 0.24, \tag{2.14}
\]

where we use \(C_2 = 1.2\) and \(C_1 = -0.4\) [38]. The estimate Eq. (2.14) reproduces the measurement Eq. (2.7) up to 8\%. At the same time, the fit in Ref. [38] finds that the large \(SU(3)_F\)-breaking exchange diagrams together with the broken penguin can also explain Eq. (2.2).

Global fits in the pure group-theoretical approach [35] agree with the approach employing topological diagrams [38] in that the maximal needed linear \(SU(3)_F\) breaking in the CKM-leading amplitudes is given as \(\varepsilon \sim 30\%\).

We can test the \(SU(3)_F\) expansion also beyond linear order breaking effects. For the ratio

\[
R_{DPP} = \frac{|A(D^0 \to K^+K^-)/(V_{cs}V_{us})| + |A(D^0 \to \pi^+\pi^-)/(V_{cd}V_{ud})|}{|A(D^0 \to K^+\pi^-)/(V_{cd}V_{us})| + |A(D^0 \to K^-\pi^+)/(V_{cs}V_{ud})|} - 1 \tag{2.15}
\]

the \(SU(3)_F\) expansion predicts that it is proportional to second order \(SU(3)_F\)-breaking effects [31, 32, 50]

\[
R_{DPP}^{th} = \mathcal{O}(\varepsilon^2). \tag{2.16}
\]

The experimental branching ratio measurements give

\[
R_{DPP}^{exp} = 0.046 \pm 0.008, \tag{2.17}
\]

confirming the theory prediction Eq. (2.16). If \(U\)-spin breaking were \(\mathcal{O}(1)\), the second order \(U\)-spin breaking contributions that are isolated in Eq. (2.17) would still be \(\mathcal{O}(1)\). Instead, it is, as expected, consistent with \(\mathcal{O}(\varepsilon^2)\).

Although we see many examples where the \(SU(3)_F\) expansion, including \(U\)-spin, is applied with great success, it is still an open question how trustable it is in general. Therefore, we seek to test the validity of \(U\)-spin at every possible opportunity.
Below, from recent data, we identify a new puzzle that appears in the CKM-subleading amplitude contributions to charm decays, as opposed to the CKM-leading contributions discussed above.

III. NOTATION

We use the notation of Ref. [11] which we shortly summarize in this section. In the SM, the Hamiltonian of SCS charm decays has the $U$-spin structure

$$\mathcal{H}_{\text{eff}} \sim \Sigma(1, 0) - \frac{\lambda_b}{2}(0, 0), \quad (3.1)$$

with $(i, j) = O_{U=i,j}^U$ and the CKM matrix element combinations

$$\Sigma \equiv \frac{V_{cs}^* V_{us} - V_{cd} V_{ud}}{2}, \quad -\frac{\lambda_b}{2} \equiv -\frac{V_{cb}^* V_{ub}}{2} = \frac{V_{cs}^* V_{us} + V_{cd}^* V_{ud}}{2}. \quad (3.2)$$

Amplitudes of SCS charm decays can then be written as

$$\mathcal{A} = \Sigma A_\Sigma - \frac{\lambda_b}{2} A_b. \quad (3.3)$$

We use the following parametrization of $U$-spin related two-body $D^0$ decays to kaons and pions [11, 32]

$$\mathcal{A}(K\pi) = \mathcal{A}(D^0 \to K^+ \pi^-) = V_{cs} V_{us}^* \left( t_0 - \frac{1}{2} t_1 \right), \quad (\text{CF}) \quad (3.4)$$

$$\mathcal{A}(\pi\pi) = \mathcal{A}(D^0 \to \pi^+ \pi^-) = -\Sigma^* \left( t_0 + s_1 + \frac{1}{2} t_2 \right) - \lambda_b^* \left( p_0 - \frac{1}{2} p_1 \right), \quad (\text{SCS}) \quad (3.5)$$

$$\mathcal{A}(KK) = \mathcal{A}(D^0 \to K^+ K^-) = \Sigma^* \left( t_0 - s_1 + \frac{1}{2} t_2 \right) - \lambda_b^* \left( p_0 + \frac{1}{2} p_1 \right), \quad (\text{SCS}) \quad (3.6)$$

$$\mathcal{A}(\pi K) = \mathcal{A}(D^0 \to \pi^+ K^-) = V_{cd} V_{us}^* \left( t_0 + \frac{1}{2} t_1 \right), \quad (\text{DCS}) \quad (3.7)$$

where the decays are classified according to their suppression with Wolfenstein-$\lambda$ as Cabibbo-favored (CF), SCS and doubly-Cabibbo suppressed (DCS). The subscripts of the parameters in Eqs. (3.4)–(3.7) indicate the corresponding order in the $U$-spin expansion. We employ the normalized parameters

$$\tilde{t}_1 \equiv \frac{t_1}{t_0}, \quad \tilde{t}_2 \equiv \frac{t_2}{t_0}, \quad \tilde{s}_1 \equiv \frac{s_1}{t_0}, \quad \tilde{p}_0 \equiv \frac{p_0}{t_0}, \quad \tilde{p}_1 \equiv \frac{p_1}{t_0}. \quad (3.8)$$
The amplitudes are normalized such that

\[ \mathcal{B}(D \rightarrow PP') = |A|^2 \cdot \mathcal{P}(D, P, P'), \]  
\[ \mathcal{P}(D, P, P') = \frac{\tau_D}{16\pi m_D^3} \sqrt{(m_D^2 - (m_P - m_{P'})^2)(m_D^2 - (m_P + m_{P'})^2)}, \]

\[ |A|^2 \cdot \mathcal{P}(D, P, P'), \]  
\[ \mathcal{P}(D, P, P') = \tau_D \frac{16}{16\pi m_D^3} \sqrt{(m_D^2 - (m_P - m_{P'})^2)(m_D^2 - (m_P + m_{P'})^2)}, \]

and \[29, 36, 53\]

\[ a_{\text{dir}}^{\text{CP}} = \text{Im} \left( \frac{\lambda_b}{\Sigma} \right) \text{Im} \left( \frac{A_b}{A_{\Sigma}} \right). \]

Furthermore, we write the amplitudes without CKM factors as \( A(f) \) for CF and DCS decays and \( A(f) \equiv A_{\Sigma}(f), A_b(f) \) for SCS decays.

Following Ref. [11] we also use the observable combinations

\[ R_{K\pi} \equiv \frac{|A(K\pi)|^2 - |A(\pi K)|^2}{|A(K\pi)|^2 + |A(\pi K)|^2}, \]  
\[ R_{KK,\pi\pi} \equiv \frac{|A(KK)|^2 - |A(\pi\pi)|^2}{|A(KK)|^2 + |A(\pi\pi)|^2}, \]

\[ R_{KK,\pi\pi,K\pi} \equiv \frac{|A(KK)|^2 + |A(\pi\pi)|^2 - |A(K\pi)|^2 - |A(\pi K)|^2}{|A(KK)|^2 + |A(\pi\pi)|^2 + |A(K\pi)|^2 + |A(\pi K)|^2}. \]

The strong phase between CF and DCS \( D^0 \) decays is defined as

\[ \delta_{K\pi} \equiv \text{arg} \left( \frac{A(D^0 \rightarrow K^+\pi^-)}{A(D^0 \rightarrow K^-\pi^+)} \right). \]

For convenience, we define the strong phases \( \delta_{KK} \) and \( \delta_{\pi\pi} \) slightly different from Ref. [11] as

\[ \delta_{KK} \equiv \text{arg} \left( \frac{A_b(KK)}{A_{\Sigma}(KK)} \right), \]  
\[ \delta_{\pi\pi} \equiv \text{arg} \left( \frac{A_b(\pi\pi)}{A_{\Sigma}(\pi\pi)} \right). \]

**IV. SOLVING FOR UNDERLYING THEORY PARAMETERS**

In the convention of Ref. [11], the parametrization Eqs. (3.4)–(3.7) has the following eight real parameters, not counting the normalization \( t_0 \):

\[ \text{Re}(\mathbf{t}_1), \text{Im}(\mathbf{t}_1), \tilde{t}_2, \bar{s}_1, \text{Re}(\mathbf{p}_0), \text{Im}(\mathbf{p}_0), \text{Re}(\mathbf{p}_1), \text{Im}(\mathbf{p}_1). \]
We can solve the complete system to order $\mathcal{O}(\varepsilon^2)$ as follows \[11\]

\[
\text{Re}(\tilde{t}_1) = -R_{K\pi},
\]
\[
\text{Im}(\tilde{t}_1) = -\tan(\delta_{K\pi}),
\]
\[
\tilde{t}_2 = 2R_{KK,\pi\pi,K\pi} - \frac{1}{4}R_{KK,\pi\pi}^2 + \frac{1}{4}R_{K\pi}^2 + \frac{1}{4}\tan^2(\delta_{K\pi}),
\]
\[
\tilde{s}_1 = -\frac{1}{2}R_{KK,\pi\pi},
\]
\[
\text{Im}(\tilde{p}_0) = \frac{1}{4\text{Im}(\lambda_b/\Sigma)}\Delta a_{CP}^{\text{dir}},
\]
\[
\text{Im}(\tilde{p}_1) = \frac{1}{2\text{Im}(\lambda_b/\Sigma)}\left(\Sigma a_{CP}^{\text{dir}} + \frac{1}{2}R_{KK,\pi\pi}\Delta a_{CP}^{\text{dir}}\right),
\]
\[
\text{Re}(\tilde{p}_0) = \frac{1}{4}\left(\text{Re}\left(\frac{A_b(D^0 \to K^+K^-)}{A\Sigma(D^0 \to K^+K^-)}\right) - \text{Re}\left(\frac{A_b(D^0 \to \pi^+\pi^-)}{A\Sigma(D^0 \to \pi^+\pi^-)}\right)\right),
\]
\[
\text{Re}(\tilde{p}_1) = \frac{1}{2}\left(\text{Re}\left(\frac{A_b(D^0 \to K^+K^-)}{A\Sigma(D^0 \to K^+K^-)}\right) + \text{Re}\left(\frac{A_b(D^0 \to \pi^+\pi^-)}{A\Sigma(D^0 \to \pi^+\pi^-)}\right)\right)
+ \frac{1}{4}R_{KK,\pi\pi}\left(\text{Re}\left(\frac{A_b(D^0 \to K^+K^-)}{A\Sigma(D^0 \to K^+K^-)}\right) - \text{Re}\left(\frac{A_b(D^0 \to \pi^+\pi^-)}{A\Sigma(D^0 \to \pi^+\pi^-)}\right)\right).
\]

Note that $\tan\delta_{K\pi} \approx \delta_{K\pi}$. Furthermore, we have to $\mathcal{O}(\varepsilon^2)$:

\[
\frac{1/2\text{Im}(\tilde{p}_1)}{\text{Im}(\tilde{p}_0)} = \frac{\Sigma a_{CP}^{\text{dir}}}{\Delta a_{CP}^{\text{dir}}} + \frac{1}{2}R_{KK,\pi\pi}.
\]

The parameters $\text{Re}(\tilde{p}_0)$ and $\text{Re}(\tilde{p}_1)$ can be determined from time-dependent measurements. In the following, we write the equations for $\text{Re}(\tilde{p}_0)$ and $\text{Re}(\tilde{p}_1)$ in a more convenient form in terms of the phases $\cot\delta_{KK}$ and $\cot\delta_{\pi\pi}$. These are related to the parametrization Eqs. (3.4)–(3.7) as

\[
\cot\delta_{KK} = \frac{\text{Re}(A_b(KK)/A\Sigma(KK))}{\text{Im}(A_b(KK)/A\Sigma(KK))},
\]
\[
\cot\delta_{\pi\pi} = \frac{\text{Re}(A_b(\pi\pi)/A\Sigma(\pi\pi))}{\text{Im}(A_b(\pi\pi)/A\Sigma(\pi\pi))},
\]

and can be obtained from the subleading, non-universal contributions to the time-dependent CP violation observable $\Delta Y_f$, where \[6\]

\[
A_{CP}(f,t) \approx a_{CP}^{\text{dir}} + \Delta Y_f \frac{t}{\tau_{D^0}},
\]

and to very good precision \[6\]

\[
\Delta Y_f = x\sin \phi - y \left( \left| \frac{q}{p} \right| - 1 \right) + ya_{CP}^{\text{dir}}(f) \left( 1 + \frac{x}{y} \cot \delta_f \right).
\]
Here, \( x, y, |q/p| \) and \( \phi \) are the parameters of \( D^0 - \bar{D}^0 \) mixing, see Refs. \[6, 21, 70, 71\] for details. Rearranging Eq. (4.13), we extract \( \cot \delta_f \) from \( \Delta Y_f \) as

\[
\cot \delta_f = \frac{y}{x} \left( \frac{\Delta Y_f - x \sin \phi + y (|q/p| - 1)}{ya_{CP}^{\text{dir}}(f)} - 1 \right).
\]  

(4.14)

In terms of \( \cot \delta_{KK}, \cot \delta_{\pi\pi}, \Delta a_{CP}^{\text{dir}} \) and \( \Sigma a_{CP}^{\text{dir}} \) we obtain the following expressions for \( \text{Re}(\tilde{p}_0) \) and \( \text{Re}(\tilde{p}_1) \) to order \( \mathcal{O}(\varepsilon^2) \):

\[
\text{Re}(\tilde{p}_0) = \frac{1}{8 \text{Im}(\lambda_b/\Sigma)} \Delta a_{CP}^{\text{dir}} (\cot \delta_{KK} + \cot \delta_{\pi\pi}) ,
\]

(4.15)

\[
\text{Re}(\tilde{p}_1) = \frac{1}{8 \text{Im}(\lambda_b/\Sigma)} \Delta a_{CP}^{\text{dir}} (\cot \delta_{\pi\pi}(R_{KK,\pi\pi} - 2) + \cot \delta_{KK}(R_{KK,\pi\pi} + 2)) + \frac{1}{4 \text{Im}(\lambda_b/\Sigma)} \Sigma a_{CP}^{\text{dir}} (\cot \delta_{KK} + \cot \delta_{\pi\pi}) .
\]

(4.16)

V. NUMERICAL RESULTS

For the numerical determination of the hadronic matrix element parameters of the parametrization Eqs. (3.4)–(3.7) we employ the experimental input data in Tables \[III\] and apply Eqs. (4.2)–(4.7) and (4.14)–(4.16). From the branching ratio measurements we obtain the combinations

\[
R_{K\pi} = -0.08 \pm 0.01 ,
\]

(5.1)

\[
R_{KK,\pi\pi} = 0.532 \pm 0.008 ,
\]

(5.2)

\[
R_{KK,\pi\pi,K\pi} = 0.083 \pm 0.008 .
\]

(5.3)

From time-dependent CP violation we obtain for the strong phases

\[
\cot \delta_{KK} = -28^{+61}_{-126} , \quad \cot \delta_{\pi\pi} = -28^{+30}_{-36} ,
\]

(5.4)

i.e., basically no constraint. This is understandable from the fact that the phases only contribute to the subleading, final-state dependent contributions of \( \Delta Y_f \), and at the current precision \( \Delta Y_{K+K-} \) and \( \Delta Y_{\pi+\pi-} \) do not yet show a significant final-state dependence.

Note that in principle there is a further opportunity for constraining the strong phases of \( \tilde{p}_{0,1} \) by extracting \( \cot \delta_{KK} \) and \( \cot \delta_{\pi\pi} \) from future precision determinations of the isolated mixing parameters \( y_{CP}^{KK} \) and \( y_{CP}^{\pi\pi} \), where these phases also appear in subleading, final-state dependent contributions, see Refs. \[21, 70, 72\] for details. Recently, LHCb measured the
combinations \( y_{CP}^{KK} - y_{CP}^{K\pi} \) and \( y_{CP}^{\pi\pi} - y_{CP}^{K\pi} \) \([9]\). However, like \( \Delta Y_{K+K^-} \) and \( \Delta Y_{\pi+\pi^-} \), they do not yet show a significant final-state dependence.

Our results for all parameters in Eq. (3.8) are given in Table III. As a result of Eq. (5.4), we have basically no information on the real parts \( \text{Re}(\tilde{p}_0) \) and \( \text{Re}(\tilde{p}_1) \). We will therefore not include them in the discussion any further.

We make now the following assumption:

- Due to non-perturbative rescattering \([11]\), the phases of \( \tilde{p}_0 \) and \( \tilde{p}_1 \) are \( \mathcal{O}(1) \), resulting in \( |\text{Im}(\tilde{p}_1)/\text{Im}(\tilde{p}_0)| \approx |\tilde{p}_1|/|\tilde{p}_0| \).

With future data on time-dependent CP violation this assumption can be tested and improved. From Eq. (4.10), it follows then for the ratio of the magnitude of the \( U \)-spin breaking contribution to the CKM-subleading amplitude \( A_\theta(\pi\pi) \) \( (A_\theta(KK)) \) to the corresponding \( U \)-spin limit contribution:

\[
\frac{1/2|\tilde{p}_1|}{|\tilde{p}_0|} \approx \left| \frac{1/2\text{Im}(\tilde{p}_1)}{\text{Im}(\tilde{p}_0)} \right| = 1.73^{+0.85}_{-0.74}, \tag{5.5}
\]

which deviates at 1.95\( \sigma \) from the SM expectation of \( \mathcal{O}(30\%) \). Eq. (5.5) is our main result. We illustrate Eq. (5.5) and the dependence of Eq. (4.10) on \( \Sigma a_{CP}^{\text{dir}} \) in Fig. 1.

The found \( U \)-spin breaking of \((173^{+85}_{-74})\%) \) may lead to the question if the \( U \)-spin power counting used for its extraction in Eqs. (4.2–4.7, 4.15, 4.16) is actually still valid. Note that Eqs. (4.2, 4.7, 4.15, 4.16) are all formally valid at \( \mathcal{O}(\varepsilon^2) \). Now, if \( \text{Im}(\tilde{p}_1) \) breaks the power counting by being \( \mathcal{O}(1) \) instead of \( \mathcal{O}(\varepsilon) \), these equations have the following power counting:

- Eqs. (4.2, 4.5, 4.7) for the extraction of \( \text{Re}(\tilde{t}_1), \text{Im}(\tilde{t}_1), \tilde{t}_2, \tilde{s}_1 \), and \( \text{Im}(\tilde{p}_1) \) are in this case still valid at \( \mathcal{O}(\varepsilon^2) \).

- Eqs. (4.6, 4.16) for the extraction of \( \text{Im}(\tilde{p}_0) \) and \( \text{Re}(\tilde{p}_1) \) are in this case valid at \( \mathcal{O}(\varepsilon) \).

- Eq. (4.15) obtains \( \mathcal{O}(1) \) corrections, \( i.e. \) is broken in this case and can no longer be used for the extraction of \( \text{Re}(\tilde{p}_0) \).

Note that also Eq. (4.10) is formally valid at \( \mathcal{O}(\varepsilon^2) \) and is still valid at \( \mathcal{O}(\varepsilon) \) when \( \text{Im}(\tilde{p}_1) \sim \mathcal{O}(1) \). The above implies that for \( \text{Im}(\tilde{p}_1) \sim \mathcal{O}(1) \) the methodology of Sec. IV still enables a consistent parameter extraction with the exception of the parameter \( \text{Re}(\tilde{p}_0) \). However, with current data we have in any case no sensitivity to this parameter. As can be seen from Table III, the other \( U \)-spin breaking parameters are consistent with the \( U \)-spin power counting.
FIG. 1. Illustration of the dependence of $1/2 \text{Im}(\tilde{p}_1)/\text{Im}(\tilde{p}_0)$ on $\Sigma a_{CP}^{\text{dir}}$ according to Eq. (4.10). For this illustration we fix $\Delta a_{CP}^{\text{dir}}$ and $R_{KK,\pi\pi}$ to their central values and vary $\Sigma a_{CP}^{\text{dir}}$ away from its measured value (blue). In red we show the current experimental data for $\Sigma a_{CP}^{\text{dir}}$ and the resulting value for $1/2 \text{Im}(\tilde{p}_1)/\text{Im}(\tilde{p}_0)$ including $1\sigma$ errors. For the estimate of the region of $30\%$ $U$-spin breaking (yellow) we assume that the strong phases are $\mathcal{O}(1)$, such that $|1/2 \text{Im}(\tilde{p}_1)/\text{Im}(\tilde{p}_0)| \approx 1/2 |\tilde{p}_1|/|\tilde{p}_0| \leq 30\%$. 

\[ \text{Dependence on } \Sigma a_{CP}^{\text{dir}} \]

\[ \text{30\% U-spin breaking} \]

\[ \text{Data} \]

\[ \begin{array}{cccccc}
-0.006 & -0.004 & -0.002 & 0.000 & 0.002 & 0.004 & 0.006 \\
-3 & -2 & -1 & 0 & 1 & 2 & 3 \\
\end{array} \]
| Direct CP Asymmetries |
|------------------------|
| $a_{CP}^{dir}(D^0 \rightarrow K^+K^-)$ | $(7.7 \pm 5.7) \times 10^{-4}$ [2] |
| $a_{CP}^{dir}(D^0 \rightarrow \pi^+\pi^-)$ | $(23.2 \pm 6.1) \times 10^{-4}$ [2] |
| $\rho$ | 0.88 [2] |

| Time-dependent CP Violation |
|----------------------------|
| $\Delta Y_{K^+K^-}$ | $(-2.3 \pm 1.5 \pm 0.3) \times 10^{-4}$ [3] |
| $\Delta Y_{\pi^+\pi^-}$ | $(-4.0 \pm 2.8 \pm 0.4) \times 10^{-4}$ [3] |

| $D^0 - \bar{D}^0$ Mixing Parameters |
|-------------------------------------|
| $x$ | $(0.409^{+0.048}_{-0.049}) \times 10^{-2}$ [10] |
| $y$ | $(0.615^{+0.056}_{-0.055}) \times 10^{-2}$ [10] |
| $\delta_{K\pi}$ | $(7.2^{+7.9}_{-9.2})^\circ$ [10] |
| $|q/p|$ | $0.995 \pm 0.016$ [10] |
| $\phi$ | $(-2.5 \pm 1.2)^\circ$ [10] |

| Branching Ratios |
|------------------|
| $B(D^0 \rightarrow K^+K^-)$ | $(4.08 \pm 0.06) \times 10^{-3}$ [66] |
| $B(D^0 \rightarrow \pi^+\pi^-)$ | $(1.454 \pm 0.024) \times 10^{-3}$ [66] |
| $B(D^0 \rightarrow K^+\pi^-)$ | $(1.363 \pm 0.025) \times 10^{-4}$ [66] |
| $B(D^0 \rightarrow K^-\pi^+)$ | $(3.947 \pm 0.030) \times 10^{-2}$ [66] |

| Further Numerical Inputs |
|--------------------------|
| Im ($\lambda_b/\Sigma$) | $(-6.0 \pm 0.3) \times 10^{-4}$ [66] |

**TABLE I.** Experimental input data. We include the correlation between $a_{CP}^{dir}(D^0 \rightarrow K^+K^-)$ and $a_{CP}^{dir}(D^0 \rightarrow \pi^+\pi^-)$ which is given by $\rho$. We also include the correlations between the mixing parameters $x$, $y$, $\delta_{K\pi}$, $|q/p|$, and $\phi$, which are given in Table II. We symmetrize all errors of the input data if applicable. Note that for $a_{CP}^{dir}(D^0 \rightarrow K^+K^-)$ and $a_{CP}^{dir}(D^0 \rightarrow \pi^+\pi^-)$ we use directly the most recent preliminary measurements by LHCb which contain all Run-1 and Run-2 measurements. Note further that the fit results by the Heavy Flavor Averaging Group (HFLAV) for the $D^0 - \bar{D}^0$ mixing parameters [10] do not yet include these latest results for the direct CP asymmetries. Both of these points can be improved in the future with updates of the world averages and global fits [10, 73-78].
TABLE II. Correlation matrix for the needed $D^0 - \bar{D}^0$ mixing parameters from Ref. [10].

|     | $x$    | $y$          | $\delta_{K\pi}$ | $|q/p|$ | $\phi$ |
|-----|--------|--------------|------------------|--------|--------|
| $x$ | 1.0    | $-0.075$     | $-0.029$         | $-0.122$ | 0.087  |
| $y$ | $-0.075$ | 1.0          | $0.970$          | $-0.035$ | 0.071  |
| $\delta_{K\pi}$ | $-0.029$ | $0.970$      | 1.0              | $-0.043$ | 0.079  |
| $|q/p|$ | $-0.122$ | $-0.035$     | $-0.043$         | 1.0    | 0.558  |
| $\phi$ | 0.087   | 0.071        | 0.079            | 0.558  | 1.0    |

TABLE III. Results for hadronic matrix elements of the $U$-spin expansion Eqs. (3.4)–(3.7), as extracted from the experimental data in Tables I and II.

|     |       |       |       |       |       |
|-----|-------|-------|-------|-------|-------|
| Re($\tilde{t}_1$) | $0.083 \pm 0.010$ |   |   |   |   |
| Im($\tilde{t}_1$) | $-0.11^{+0.15}_{-0.16}$ |   |   |   |   |
| $\tilde{t}_2$ | $0.101^{+0.019}_{-0.016}$ |   |   |   |   |
| $\tilde{s}_1$ | $-0.2658^{+0.0040}_{-0.0039}$ |   |   |   |   |
| Im($\tilde{p}_0$) | $0.66 \pm 0.13$ |   |   |   |   |
| Im($\tilde{p}_1$) | $-2.27^{+0.96}_{-0.98}$ |   |   |   |   |
| Re($\tilde{p}_0$) | $-18^{+23}_{-47}$ |   |   |   |   |
| Re($\tilde{p}_1$) | $64^{+56}_{-55}$ |   |   |   |   |
VI. PREDICTIONS AND NEW PHYSICS INTERPRETATIONS

The large $U$-spin breaking of $(173^{+85}_{-71})\%$ that we find in Eq. (5.5) indicates large contributions from $\Delta U = 1$ operators in the CKM-subleading amplitude of SCS charm decays. This leads to an $O(1)$ breaking of the $U$-spin limit sum rule \[33, 42, 50, 53\]

$$\frac{\Gamma(D^0 \rightarrow K^+ K^-)}{\Gamma(D^0 \rightarrow \pi^+ \pi^-)} = -\frac{a_{CP}^{dir}(D^0 \rightarrow \pi^+ \pi^-)}{a_{CP}^{dir}(D^0 \rightarrow K^+ K^-)} ,$$

(6.1)

see the discussion in Sec. [0]. As the decays $D^0 \rightarrow K^+ K^-$ and $D^0 \rightarrow \pi^+ \pi^-$ are also connected to a wider class of decays via SU(3)$_F$ symmetry, we expect that the $U$-spin limit sum rule \[33, 42, 50, 53\]

$$\frac{\Gamma(D^+ \rightarrow \bar{K}^0 K^+)}{\Gamma(D^+_s \rightarrow K^0 \pi^+)} = -\frac{a_{CP}^{dir}(D^+_s \rightarrow K^0 \pi^+)}{a_{CP}^{dir}(D^+ \rightarrow \bar{K}^0 K^+)} ,$$

(6.2)

is also broken at $O(1)$. In Ref. \[39\] improved versions of the sum rules Eqs. (6.1, 6.2) are formulated that account for the first order SU(3)$_F$ breaking effects from all topological diagrams except for the penguin contraction of tree operators ($P$ and $PA$ in the notation therein). Therefore, we predict that also the sum rules \[39\]

$$\frac{S(D^0 \rightarrow K^+ K^-) - S(D^0 \rightarrow \pi^+ \pi^-)}{e^{2i\delta(D^0 \rightarrow K^+ K^-)} - e^{2i\delta(D^0 \rightarrow \pi^+ \pi^-)}} - \frac{S(D^0 \rightarrow K^+ K^-) + \sqrt{2}S(D^0 \rightarrow \pi^0 \pi^0)}{e^{2i\delta(D^0 \rightarrow K^+ K^-)} - e^{2i\delta(D^0 \rightarrow \pi^0 \pi^0)}} = 0 ,$$

(6.3)

$$\frac{S(D^+ \rightarrow \bar{K}^0 K^+)}{e^{2i\delta(D^+ \rightarrow \bar{K}^0 K^+)} - e^{2i\delta(D^+_s \rightarrow K^0 \pi^+)} - \frac{S(D^+ \rightarrow \bar{K}^0 K^+)}{e^{2i\delta(D^+ \rightarrow \bar{K}^0 K^+)} - e^{2i\delta(D^+_s \rightarrow K^0 \pi^+)} = 0} (6.4)$$

are broken at $O(1)$. Here, $\delta(d) \equiv \arg(A_{CP}(d))$ and the function $S(d)$ can be found in Ref. \[39\]. Further SU(3)$_F$ sum rules are given in Refs. \[50, 79\]. For a general treatment of $U$-spin sum rules at any order see Ref. \[25\]. In light of the puzzle posed by the $U$-spin expansion of charm decays, also a further test of the respective isospin structure is very important \[55\].

As laid out in Ref. \[35\], new physics models with additional $\Delta U = 1$ operators, so-called \textquotedblleft $\Delta U = 1$ models" \[35\] can explain the breaking of Eqs. (6.1, 6.2) beyond the $U$-spin power counting. The same applies to Eqs. (6.3, 6.4). Such models generate additional effective operators with the flavor content $\bar{s}c\bar{u}s$ and/or $\bar{d}c\bar{u}d$ with non-universal coefficients. They can \textit{e.g.} arise from two-Higgs-doublet models (2HDMs) \[54\] or flavorful $Z'$ models \[12, 20, 54\]. Recently, in Ref. \[20\] it has been shown that $Z'$ models can induce large $U$-spin breaking between $a_{CP}^{dir}(D^0 \rightarrow K^+ K^-)$ and $a_{CP}^{dir}(D^0 \rightarrow \pi^+ \pi^-)$, depending on the charge assignments of the quarks under an additional $U(1)'$ group. With the new data, charm CP asymmetries
can be used effectively to probe and explore the parameter space of such models further. For example, the specific charge assignments of $Z'$ models considered in Ref. [20] lead to opposite signs for $a^{\text{dir}}_{CP}(D^0 \to K^+K^-)$ and $a^{\text{dir}}_{CP}(D^0 \to \pi^+\pi^-)$, see Fig. 3 therein, whereas the most recent data indicates $a^{\text{dir}}_{CP}(D^0 \to K^+K^-) > 0$ and $a^{\text{dir}}_{CP}(D^0 \to \pi^+\pi^-) > 0$.

The exploration of the $U$-spin puzzle with future and more precise measurements including sum rule tests is important for a complete understanding of the CKM-subleading amplitudes of SCS charm decays and in order to further probe the parameter space of $\Delta U = 1$ models.

VII. CONCLUSIONS

Assuming the Standard Model, from recent measurements of charm CP violation in the single decay channels $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ we extract for the first time the imaginary part $\text{Im}(\tilde{p}_1)$ of the $U$-spin breaking $\Delta U = 1$ contribution to the CKM-subleading amplitudes. We obtain

$$\frac{1/2 \text{Im}(\tilde{p}_1)}{\text{Im}(\tilde{p}_0)} = (-173^{+74}_{-85})\%,$$

(7.1)

where $\text{Im}(\tilde{p}_0)$ is the $U$-spin limit $\Delta U = 0$ contribution to the CKM-subleading amplitudes which is determined by $\Delta a^{\text{dir}}_{CP}$. The strong phases of $\tilde{p}_{0,1}$ are yet unknown. Assuming $\mathcal{O}(1)$ strong phases due to non-perturbative rescattering, the result implies very large $U$-spin breaking, which exceeds the SM expectation of $\sim 30\%$ by almost a factor six, at $1.95\sigma$.

It is crucial to probe this anomaly further with more data and test the $U$-spin expansion also in additional decays, using the sum rules listed in Sec. VI. Most importantly, we need improved time-dependent measurements, such that we can extract the strong phases of $\tilde{p}_{0,1}$ from data. In order to test the pattern of the SU(3)$_F$ expansion, measurements of CP asymmetries of basically all singly-Cabibbo suppressed decays are necessary.

We encourage experimental collaborations to extract the underlying theory parameters using the methodology described in Sec. IV directly from the data, enabling the most comprehensive treatment of all correlations.

If the $U$-spin anomaly is confirmed with more data in the future, this would imply either a breakdown of the $U$-spin expansion in the Standard Model, or a sign for new physics with an additional $\Delta U = 1$ operator, for example from additional scalar particles or a flavorful $Z'$. 
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