Research Article
A Numerical Approach for an Unsteady Tangent Hyperbolic Nanofluid Flow past a Wedge in the Presence of Suction/Injection

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The analysis of unsteady tangent hyperbolic nanofluid flow past a wedge with injection-suction, because of its beneficial uses, has gained a lot of attention. The present study is mainly concerned with tangent hyperbolic nanofluid (non-Newtonian nanofluid). First, we have converted the system of partial differential equations (PDEs) to a system of ordinary differential equations (ODEs) with the help of appropriate similarity transformations. Boundary conditions are also transformed by utilizing suitable similarity transformation. Now, for the obtained ODEs, we have used the numerical technique bvp4c and investigated the velocity, temperature, and concentration profiles. The accuracy of the flow model is validated by applying MAPLE d-solve command having good agreement while comparing the numerical results obtained by bvp4c for both suction and injection cases. The effects of distinct dimensionless parameters on the various profiles are being analyzed. The novel features such as thermophoresis and Brownian motion are also discussed to investigate the characteristics of heat and mass transfer. Graphical representation of the impact of varying parameters and the solution method for the abovementioned model is thoroughly discussed. It was observed that suction or injection can play a key role in controlling boundary layer flow and brings stability in the flow. It was also noticed that by increasing the Darcy number, velocity profile increases in both injection-suction cases.

1. Introduction

The heat transfer plays an important role in manufacturing industries. It has effect on cost and quantity of production and is also a challenge for effective heating and cooling effects in high-tech manufacturing industries and many other big industries. A conventional heat transfer fluid has poor thermal conductivity as compared with the solids like water and engine oils [1]. In all of these, thermal conductivity of heat transfer fluids plays a vital role in heat development equipment’s. Choi [2] firstly gave the concept of nanofluids; nanofluids are the fluids whose particles size is less than 100 nm. In many previous years, researchers gave more attention to the nanofluids because of various reasons, for example, heat transfer performance and enhancement of thermal conductivity. Buogiorno [3] explained the extraordinary thermal conductivity of nanofluids with boundary layer (BL) mass and heat transfer phenomena. Sarviya and Fuskele [4] reviewed the brief description of thermal conductivity of nanofluids. Prabhakar et al. [5] studied the effects of inclined Lorentz forces for zero normal fluxes of nanoparticles at the stretching sheet. A brief study on BL tangent hyperbolic flow of nanofluid is explained in these articles [6–17]. Fluids in which viscosity remain constant, no matter the shear applied on the fluid with constant temperature, e.g, water, air, gasoline, and alcohol On the other hand, non-Newtonian fluids are the fluids in which when stresses applied, viscosity will be changed, for example, quicksand, cornflour-water, silly putty, ketchup, cream, paint, glue, detergents, and molten plastics. Non-newtonian fluids are more complex to deal than the Newtonian fluids [18]. They are so difficult to deal with resulting equations and classical Newtonian models. The tangent hyperbolic model was the famous non-Newtonian model that was first introduced by Pop and Ingham [19] with the extraordinary uses in modern industries. Reddy et al. [20], Khan et al. [21], Mondal et al. [22], and Jabeen et al. [23] presented their numerical studies on MHD flows of tangent
hyperbolic nanofluid over stretched sheet. Similarly, many applications of flow over a wedge-shaped surface are in scientific and manufacturing industries, for example, hydrodynamics, magnetohydrodynamics, and crude oil exploration and extraction.

In 1937, Hartree [24] studied solutions on wedge. A brief study was devoted to analysis the differential parameters on the wedge flow. For example, the authors in [25–28] explained the heat transfer phenomena in wedge surface. Only few studies on unsteady MHD flow of tangent nanofluid along a wedge are in the literature, i.e., Kebede et al. [29] used a method to give the approximation of two-dimensional tangent hyperbolic Nanofluids over a stretched wedge in a porosity medium. We use the numerical method to approximate the solution of unsteady tangent hyperbolic flow over a wedge with suction-injection, and then we compared the results of numerical methods that we used to calculate the solution of nonlinear equations [30–32]. In this study, we have discussed numerical approximation of heat and mass transfer of a tangent hyperbolic nanofluid flow over a wedge shaped surface along with injection-suction. Taking into account rising demand of modern technology, every field of engineering and science uses nanoparticles to enhance their performance and minimize the cost of production. Fluids that have been traditionally used for heat transfer application in manufacturing industries have rather a low thermal conductivity. We used nanofluids which enhance the thermal conductivity behavior and rate of heat transfer as compared with the conventional fluids. Fluids, in general, may reveal both Newtonian and non-Newtonian behavior. Newtonian fluids behavior is observed when there is linear relationship in between the shear stress and the rate of deformation; otherwise, for any deviation for this behavior, fluids are treated as non-Newtonian fluids. It is so difficult to comprehend all the particular behaviors of non-Newtonian fluids as compared with the Newtonian fluids. Many of the fluids used in industries for the manufacturing of paint, honey, toothpaste, butter, shampoo, custard, soap solution, and so on do not obey Newton’s viscosity law. It is very difficult to analyze all the characteristics of such fluids. So, in many non-Newtonian formulations in the prior survey, tangent hyperbolic nanofluids are the famous non-Newtonian model that we considered here. A system of partially differential equations is converted to a system of ODEs under appropriate similarity variables. Numerical-based approaches are applied for numerical computations for the solution of nonlinear equations. It is expected that the convergence of these methods will give better results than the other methods.

The numerical method always works in iterations and is easy to compute. When the analytical solution does not exist to solve a system, then numerical solution is preferred. These methods take short time to execute and cast less computational cost. MAPLE d solves basically a method to obtain the numerical solution for both initial and boundary value problems. This method is also an effective solver, with an approach towards the increasing Convergence criteria. Bvp4c is a fourth-order numerical method to solve boundary value ordinary differential equation in MATLAB software. The computed numerical results are demonstrated for a variety of nondimensional constraints in the equations. It is used for different parametric values, involving slip parameter, Prandtl number, Brownian movement variable \(N_b\) and \(N_t\), the thermophoresis variable, the Schmidt number \(S_c\), and the ratio stretching parameter \(S_c\). Also, \(M\) is the material fluids parameter, and \(n\) is the microrotation parametric influence. The MATLAB BVP solver bvp4c basically a typical finite difference code that deals with the three-stage LobattoIIIa formulation approach, which is used to obtain numerical results using bvp4c methodology.

2. Constitutive Relation for Tangent Hyperbolic Nanofluid

The non-Newtonian fluids have number of formulations in the literature that exhibit different rheological fluid behaviors. Amongst them, the tangent hyperbolic fluid is one of an interesting non-Newtonian four constant fluid formulation which basically narrates a behavior of shear thinning. It means that the viscosity declines as the rate of shear is enhanced while the apparent viscosity varies cautiously in between zero and infinite rate of shear.

So, for tangent hyperbolic fluid, the constitutive relation is given as follows:

\[
\tau = [\mu_\infty + (\mu_o + \mu_\infty) \tanh (\Gamma \bar{\omega})^n] \bar{\omega}. \tag{1}
\]

In the above expression, ‘\(n\)’ indicates the power law constant, \(\gamma\) represents the time constant, \(\tau\) denotes extra stress tensor whereas zero and infinite shear rate of viscosity is referred here as \(\mu_o\) and \(\mu_\infty\), respectively. Also, \(\bar{\omega}\) is defined as follows:

\[
\bar{\omega} = \sqrt{\frac{1}{2} \sum_i \sum_j \bar{\omega}_{ij} \bar{\omega}_{ji}} \tag{2}
\]

In this expression, \(\Pi\) is defined as follows:

\[
\Pi = \frac{1}{2} \text{tr}((\nabla V) + (\nabla V)^T)^2. \tag{3}
\]

Since we are considering the behavior of shear thinning in case of tangent hyperbolic fluid, therefore, for the case of \(\Gamma \bar{\omega} < 1\), then the extra stress tensor which is denoted by \(\tau\) is typically reduced to the following:

\[
\tau = \mu_o [1 + n (\Gamma \bar{\omega} - 1) \bar{\omega}] \tag{4}
\]

3. Mathematical Formulation

Consider a tangent hyperbolic fluid flow occurring in a two-dimensional laminar way of mode across a wedge with injection-suction. The origin of Cartesian coordinate systems is established at vertex of the wedge, by using the
Cartesian coordinate system which is \((x, y)\). The \(x\)-axis is lying along the surface of the wedge whereas the \(y\)-axis is lying perpendicular to the surface of wedge. Assume that the external flow moves with a free stream velocity of \(U_e(x, t) = (bx^m/1 - ct)\) and the fluid enters the wedge boundary layer with a wall velocity of \(U_w(x, t) = (ax^m/(1 - ct))\) subject to the magnetic domain of \(B = (0, B_o)\) acting perpendicularly to the surface of wedge, where \(t\) representing the time variable while \(a, b, m,\) and \(c\) depict constants assuming \(a > 0\) and \(a < 0\) that demonstrates the rates of stretching and shirking of wedge, respectively. Here, \(m\) is Falkner–Skan constants defined as \(m = (\beta/(2 - \beta))\) and also described as a total angle of wedge \(\Omega\) by \(\beta = (\Omega/n)\). Suppose the temperature of the surface \(T_w\) and concentration \(C_w\) of the wedge vary in the form of power-law as \(T_w = T_{\infty} + (bx^m/(1 - ct))\) and \(C_w = C_{\infty} + (bx^m/(1 - ct))\) as shown in Figure 1.

The equations, provided as follows, express the conservation rules regulating flow phenomena when all of the following assumptions are taken into account [29]:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= \frac{\partial u_e}{\partial t} + U_e \frac{\partial u_e}{\partial x} + \gamma \left( 1 - n \right) + \sqrt{2} n f \frac{\partial T}{\partial y} + \frac{\alpha B^2}{\rho} + \mu (u - U_e) \\
&+ g \left[ \beta_1 (T - T_{\infty}) - \beta_c (C - C_{\infty}) \right] \sin \left( \frac{\Omega}{2} \right), \\
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2} + r \left[ \frac{\partial C}{\partial y} \frac{\partial T}{\partial v} + \frac{\partial T}{\partial v} \frac{\partial C}{\partial y} \right] T_{\infty} - \frac{16 \alpha^* T_{\infty}^3}{3 k_c \rho C} \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B^2}{\rho C_p} (u - U_e)^2 \\
&+ \frac{v}{\gamma} \left[ \frac{1 - \gamma}{\gamma} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\Gamma^2 (\partial u)}{\sqrt{2}} \right]^{2} + \frac{Q_o}{(\rho C_p)} (T - T_{\infty}), \\
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} &= D_c \frac{\partial^2 C}{\partial y^2} + D_{\infty} \frac{\partial^2 T}{\partial y^2}.
\end{align*}
\]

The components of velocity are \((u, v)\) along \(x\)- and \(y\)-axis. \(C\) and \(T\) are the nanofluid concentration and temperature in the BL region. \(\mu\) and \(\rho\) are the dynamic viscosity and density of the nanofluids. \(\nu = \mu/\rho\) is the kinematic viscosity, \(n\) is the power law index. The numbers \(\Gamma, g, K_c, \beta_1,\) and \(\beta_c\) signify material constant with time dependency, the gravitational acceleration magnitude, the porous medium permeability, volumetric thermal, and the expansion coefficient of concentration, respectively. Mean absorption and Stefan–Boltzmann constants are \(k^*\) and \(\alpha^*,\) respectively; and the coefficient \(Q_o\) represents the generation of heat when \(Q_o > 0\) and the absorption of heat when \(Q_o < 0.\)

We apply some boundary conditions on the above four PDEs. The boundary conditions near and far from the wedge’s surface are used as follows [29]:

\[
\begin{align*}
u = U_w(x, t), \quad v = V_w(t), \quad -k \frac{\partial T}{\partial y} &= h_f (T_w - T) , \\
-D_c \frac{\partial C}{\partial y} &= h_c (C_w - C) \quad \text{at} \ y = 0, \\
u \rightarrow U_e(x, t) = \frac{bx^m}{1 - ct} \rightarrow T_{\infty}, C \rightarrow C_{\infty} \quad \text{at} \ y \rightarrow \infty,
\end{align*}
\]

where \(V_w = V_o/\sqrt{1 - ct}\) represents the transpiration velocity also representing the transmission of mass at the stretching wedge surface with \(V_o\) as the constant velocity value, and convective heat and mass transfers coefficients are represented by \(h_f\) and \(h_c\), respectively.

The transformation functions are as follows [29]:
Here, $\eta$ denote the nondimensional quantity for transformation purpose and $\psi$ is the stream function with following factors $u = (\partial \psi / \partial y)$ and $v = - (\partial \psi / \partial x)$. The dimensionless velocity and thermal and concentration profiles are represented by $f(\eta)$ and $\theta(\eta)$ and $\phi(\eta)$, respectively.

By the similarity transformation into the governing equations where the continuity equation as (5) and the equations as (6) to (8) came up in the nondimensional system of ordinary differential equations as follows:

\[
\left\{ \begin{array}{l}
\eta = y \sqrt{\frac{(1 + m)U_e}{2\nu(1 - ct)}}, \\
\psi(x, y, t) = \sqrt{\frac{2\nu U_e}{1 + m}} f(\eta), \\
\theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \\
\phi(\eta) = \frac{C - C_{\infty}}{C_w - C_{\infty}}.
\end{array} \right.
\tag{10}
\]

where prime' indicates the differentiation with respect to $\eta$. And we have some dimensionless parameters which are as follows:

\[
\begin{align*}
\left(1 - n\right) + nWe \sqrt{1 + m f''} &+ f f'' + \frac{2m}{m + 1} \left(1 - f''^2\right) + \frac{A}{m + 1} \left(2 - 2f'' - nf''^2\right) + \frac{2m}{m + 1} (M + Da) (1 - f') + (Gr\theta + Gc\phi) \sin \left(\frac{\omega}{2}\right) = 0, \\
\frac{1}{Pr} \left(1 + \frac{4}{3} \text{Ray} \right) \theta'' + N_t \theta' \phi' + N_t \theta'' + f \theta'' - \frac{2m}{m + 1} f' \theta' - \frac{A}{m + 1} (\theta' + 2\theta) + \frac{2m}{m + 1} M Ec \left(f'^2 - 2f' + 1\right) + Ec \left(1 - n\right) f''^2 + nWe f''^3 &+ \frac{2m}{m + 1} Q = 0, \\
\phi'' + PrLe \left(f \theta' - \frac{2m}{m + 1} f' \phi - \frac{A}{m + 1} (\eta \theta' + 2\theta)\right) + \left(\frac{N_t}{Nb}\right) \theta'' = 0,
\end{align*}
\tag{11}
\]
Biot numbers are as follows:

\[
Bi_1 = \frac{h_f}{k_f} \sqrt{\frac{2\nu}{U_e(m+1)}},
\]

\[
Bi_2 = \frac{h_t}{D_a} \sqrt{\frac{2\nu}{U_e(m+1)}}.
\]

The wall friction \( C_f \), the local Nusselt number \( Nu_x \), and Sherwood number \( Sh_x \) are given as follows:

\[
C_f = \frac{\tau_w}{\rho_f U_e^2},
\]

\[
Nu_x = \frac{xq_w}{a(T_w - T_{co})},
\]

\[
Sh_x = \frac{xq_m}{D_b(C_w - C_{co})},
\]

where

\[
\tau_w = \mu \left[ (1 - n) \frac{\partial u}{\partial y} + \frac{m}{\sqrt{2}} \left( \frac{\partial u}{\partial y} \right)^2 \right]_{y=0},
\]

\[
q_w = - \left[ (\alpha + \frac{16\sigma^T_{co}}{3C_p} \sqrt{\rho}) \frac{\partial T}{\partial y} \right]_{y=0},
\]

\[
q_m = -D_b(\frac{\partial C}{\partial y}) \bigg|_{y=0}.
\]

We get the following relationships by substituting and applying the similarity transformation:

\[
\sqrt{Re C_f} = \sqrt{1 + m(1-n) + \frac{1}{2} n We f''(0) f''(0)},
\]

\[
\frac{Nu_x}{\sqrt{Re}} = - \sqrt{1 + m} \left( 1 + \frac{4}{3} \frac{Rd}{\nu} \right) \theta'(0),
\]

\[
\frac{Sh_x}{\sqrt{Re}} = - \sqrt{1 + m} q'_m(0),
\]

where \( Re = (U_e x / \nu) \) is the local Reynold’s number.

### 4. Results and Discussion

Under the boundary layer approximations, the dimensional form of a system of PDEs is yielded against the numerous distinct profiles which are transformed into ODEs by utilizing suitable similarity transformation. To explore the numerical results of velocity, temperature, and concentration profile, a numerical solution with the help of the bvp4c algorithm using MATLAB and MAPLE d-solve command has been constructed. Table 1 shows that present results are in good agreement with previous studies reported in [11, 25, 29]. In Tables 2–4, the effects of dimensionless...
parameters upon local Sherwood $# Sh_x$, Nusselt $# N_{ux}$, and skin friction $C_{fx}$ are summarized.

A graphical demonstration for the behavior of emerging pertinent parameters associated different profiles is disclosed. Figure 2 demonstrates the effects of $M$ on velocity distribution. It can be seen that velocity profile increases by enhancing the values of $M$ in the presence of suction which is physically fit with the argument that the decelerating force

| $m$ | $f'' (0)$ | $f'' (0)$ | $f'' (0)$ | $f'' (0)$ | $f'' (0)$ | $f'' (0)$ | $f'' (0)$ | $f'' (0)$ | $f'' (0)$ |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0   | 0.4696    | 0.4696    | 0.4688    | 0.4228    | 0.2326    | 0.4697    | 0.4205    | 0.2323    |           |
| 1/23 | -0.5690  | 0.5693    | 0.4598    | 0.2348    | 0.5690    | 0.4542    | 0.2391    |           |
| 1/11 | 0.6550    | 0.6554    | 0.4818    | 0.2453    | 0.6550    | 0.4847    | 0.2457    |           |
| 1/7  | 0.7320    | 0.7322    | 0.5090    | 0.2516    | 0.7319    | 0.5134    | 0.2523    |           |
| 1/5  | 0.8021    | 0.8021    | 0.5402    | 0.2669    | 0.8021    | 0.5403    | 0.2588    |           |
| 1/3  | 0.9277    | 0.9277    | 0.5904    | 0.2699    | 0.9277    | 0.5904    | 0.2716    |           |
| 1/2  | 1.0389    | 1.0389    | 0.6369    | 0.2822    | 1.0389    | 0.6366    | 0.2842    |           |
| 1.00 | 1.2326    | 1.2326    | 0.7202    | 0.3063    | 1.2326    | 0.7204    | 0.3086    |           |
| 5.00 | 1.5504    | 1.5503    | 0.8648    | 0.5317    | 1.5503    | 0.8648    | 0.5345    |           |
| 100.0 | 1.6794   | 1.6794    | 0.9252    | 0.3726    | 1.6794    | 0.9252    | 0.3748    |           |

Table 2: Calculation of $(Re_x)^{1/2}C_{fx}$ for different auxiliary parameters.

| $S$ | MATLAB | MAPLE |
|-----|--------|-------|
| $M$ |        |       |
| 0   | 1.213  | 0.918 |
| 1.5 | 1.3681 | 1.1726|
| 3.5 | 1.6366 | 1.4458|
| 5.5 | 1.8641 | 1.6758|
| $W_e$ |       |       |
| 0.2 | 1.1476 | 0.9372|
| 0.6 | 1.1487 | 0.9453|
| 1.2 | 1.1613 | 0.9572|
| 1.8 | 1.1746 | 0.9604|
| $A$ |        |       |
| 0   | 1.1054 | 0.9006|
| 0.2 | 1.1729 | 0.9729|
| 0.4 | 1.2393 | 1.0438|
| 0.6 | 1.3042 | 1.1128|
| $D_a$ |      |       |
| 0   | 1.1213 | 0.9180|
| 0.5 | 1.2095 | 1.0096|
| 1.0 | 1.2914 | 1.0940|
| 1.5 | 1.3680 | 1.1725|
| $\delta$ | |       |
| 0.1 | 1.2461 | 1.0174|
| 0.2 | 1.1412 | 0.9381|
| 0.3 | 1.0286 | 0.8513|
| 0.4 | 0.9087 | 0.7571|
| $G_r$ |        |       |
| 0.1 | 1.1013 | 0.8955|
| 0.4 | 1.1298 | 0.9266|
| 0.8 | 1.1673 | 0.9672|
| 1.2 | 1.2043 | 1.0071|
| $G_c$ |        |       |
| 0.1 | 1.1279 | 0.9220|
| 0.4 | 1.1396 | 0.9371|
| 0.8 | 1.1551 | 0.9570|
| 1.2 | 1.1705 | 0.9768|
shows lesser domination in the entire flow region while an opposite trend followed up by temperature and concentration profile can be seen in Figures 3 and 4. We is the division of relaxation time and viscous forces of fluid. The effects of various profiles, i.e., $f'$, $\theta$, and $\phi$ are exhibited in Figures 5–7. It is observed that the hiking (We) increases and the velocity of fluid starts to decline. On contrary, the temperature and concentration got increased which is due to the fact that the augmented Weissenberg number causes an increase in the relaxation time of the fluid flow which results in thicker nanofluid and enhanced resistance in the flow.

Figures 8–10 demonstrate that an increment in the $(A)$ outcome is the hike in the velocity of fluid; however, it ultimately decreases the temperature and concentration profiles.

The permeability of the medium is analyzed in the form of $(D_{\text{eff}})$ whose effects are crucially important on the fluid’s velocity distribution as depicted in Figure 11. It can be measured that the increasing values of the Darcy number cause a hike in the velocity of nanoparticles. It is evident in the practical phenomenon as the increased values in this parameter yield hikc in porous medium that turns out as the minimization in the hurdles based along the entire flow path. This permits the free flow of the fluid with increased fluid velocity in the BL region.

The effects of the non-Newtonian fluid are significantly important in the region of BL. These impacts are intimates in the form of $(n)$, and results on the velocity profiles are exhibited in Figure 12. The velocity profile depicts an
increasing trend with the gradual increment in distinct values of parameter. The reason behind this, as the parametric value start increasing, the fluid’s nature sharply changes, i.e., transmission between shear thinning and shear thickening state. The results given in Figures 13 and 14 show the influences of thermal and concentration distributions with the amendments in $N_b$ parameter. The collisions between the nanoparticles increased due to which they start to move in an irregular manner that trivially rises the temperature and concentration profile.

Figures 15 and 16 show the directly proportional behavior of temperature and concentration profile in response to $N_t$ (thermophoresis parameter). It can be observed that with the increasing behavior of $N_t$, the coming nanoparticles transfer from the hot surface towards cold ambient fluid state due to which the temperature rises in the boundary

Table 4: Calculation of $(Re_x)^{-1/2}Sh_x$ for different auxiliary parameters.

| $M$  | $S = 0.2$ | $S = -0.2$ | $S = 0.2$ | $S = -0.2$ |
|------|-----------|------------|-----------|------------|
| 0.0  | 0.4052    | 0.3653     | 0.4052    | 0.3653     |
| 1.5  | 0.4079    | 0.3701     | 0.4079    | 0.3701     |
| 3.5  | 0.4104    | 0.3745     | 0.4104    | 0.3745     |
| 5.5  | 0.4124    | 0.3777     | 0.4124    | 0.3777     |
| $\lambda$ |          |            |           |             |
| 0.0  | 0.4020    | 0.3586     | 0.4020    | 0.3586     |
| 0.2  | 0.4088    | 0.3725     | 0.4088    | 0.3725     |
| 0.4  | 0.4157    | 0.3852     | 0.4157    | 0.3852     |
| 0.6  | 0.4223    | 0.3965     | 0.4223    | 0.3965     |
| $N_b$ |          |            |           |             |
| 0.1  | 0.4054    | 0.3657     | 0.4054    | 0.3657     |
| 0.2  | 0.4143    | 0.3687     | 0.4143    | 0.3687     |
| 0.3  | 0.4172    | 0.3697     | 0.4172    | 0.3697     |
| 0.4  | 0.4187    | 0.3702     | 0.4187    | 0.3702     |
| $N_t$ |          |            |           |             |
| 0.1  | 0.4054    | 0.3657     | 0.4054    | 0.3657     |
| 0.2  | 0.3893    | 0.3616     | 0.3893    | 0.3616     |
| 0.3  | 0.3742    | 0.3586     | 0.3742    | 0.3586     |
| 0.4  | 0.3601    | 0.3567     | 0.3601    | 0.3567     |
| $Sc$ |          |            |           |             |
| 0.5  | 0.2916    | 0.2828     | 0.2916    | 0.2828     |
| 1.0  | 0.3514    | 0.3278     | 0.3514    | 0.3278     |
| 1.5  | 0.3839    | 0.3510     | 0.3839    | 0.3510     |
| 2.0  | 0.4054    | 0.3657     | 0.4054    | 0.3657     |

Figure 2: Velocity profile versus $M$.

Figure 3: Temperature profile versus $M$. 

\[ f' (\eta) \]

\[ \theta (\eta) \]
Figure 4: Concentration profile versus $M$.

Figure 5: Velocity profile versus $W_e$.

Figure 6: Temperature profile versus $W_e$.

Figure 7: Concentration profile versus $W_e$.

Figure 8: Velocity profile versus $A$.

Figure 9: Temperature profile versus $A$. 
Figure 10: Concentration profile versus $A$.

Figure 11: Velocity profile versus $Da$.

Figure 12: Velocity profile versus $n$.

Figure 13: Temperature profile versus $Nb$.

Figure 14: Concentration profile versus $Nb$.

Figure 15: Temperature profile versus $Nt$. 
layer of fluid. Hence, the thickness of thermal boundary layer is improved. Also, the nanoparticles concentration is increased in the fluid, so the concentration boundary layer is enhanced significantly.

Figure 17 highlights the behavior of the Prandtl number (Pr) on thermal profile. It can be seen that with higher values of Pr, the temperature profile ultimately declines. Thus, it has an inverse relation with thermal diffusivity. So, increase in Pr delineates the thermal graph.

Effects of behavior of different profiles against radiation parameter are demonstrated in Figure 18. It is indicated that the increase in $R$ inspires significant hike in the thermal BL region. Physical agreement can be seen in the existence of thermal radiation relates to high kinetic energy of the molecules in the entire flow field.

In Figure 19, effects of the Ec can be observed on temperature profile. It is depicted that the increment in dissipation parameter produces frictional heating force which is to be stored in the fluid layer. Such frictional heating enhanced the thickness of temperature distribution at the surface of the wedge.

Figure 20 is plotted to emanate the effects on concentration profile for distinct parametric values of the Schmidt number (Sc). The different values for Sc are chosen to be 0.5, 1.0, 1.5, and 2.0 to indicate the diffusing chemical species in gases like $H_2$, $H_2O$, and $NH_3$. It is to be noted that the graph of incremented (Sc) tends to decrease fluid concentration.

Figure 21 illustrates that temperature profile is increasing functions of the heat source parameter.

The behavior of parameter $\delta$ on velocity profile is demonstrated in Figure 22. It is of highly consideration that velocity profile got enhanced by increasing the parameter $\delta$.

The results portrayed in Figures 23 and 24 represent the effects of thermal and concentration buoyancy parameters on velocity profile. It can be observed that the buoyancy parameters make substantial variation in velocity profile.
Figure 20: Concentration profile versus $\text{Sc}$.

Figure 21: Temperature profile versus $Q$.

Figure 22: Velocity profile versus $\delta$.

Figure 23: Velocity profile versus $\text{Gr}$.

Figure 24: Velocity profile versus $\text{Gc}$.

Figure 25: Velocity profile versus $m$. 
The influences on the numerous distinct profiles are determined for of wedge angle in Figures 25–27. Figures 25–27 depict that the velocity of fluid is increased while opposite trend is carried out by the other two profiles. Physically this corresponds to the fact that the increment in wedge angle parameter causes increase in the pressure of fluid. Moreover, changes in the hiking changing of wedge angle parameter influence the temperature near the surface of the wedge.

5. Conclusion

Unsteady, two-dimensional tangent hyperbolic fluid of nanofluid with injection-suction is studied. Following results were investigated:

(i) With the increase in $m$, the velocity profile shows behavior of increasing velocity with injection suction. And temperature and concentration profiles show the behavior of decreasing temperature and concentration with injection-suction.

(ii) By increasing the Weissenberg number parameter ($We$), the velocity profile decreases and temperature and concentration profile will increase with injection-suction. The Weissenberg number is the significance in the study of flow of non-Newtonian fluids, depending upon the rheology and ratio of elastic to viscous forces.

(iii) With the increase in the unsteadiness parameter ($A$), the velocity profile shows that velocity increases and temperature concentration profiles show that both temperature and concentration decrease with injection-suction.

(iv) With the increase in the Darcy number ($Da$), velocity profile shows that velocity increases with injection-suction.

(v) With the increase in the power law index parameter ($n$), velocity profile increases with injection-suction.

(vi) By increasing the Brownian motion parameter ($Nb$), temperature profile shows that temperature increases and concentration profile shows that concentration decreases with injection-suction.

(vii) By increasing the thermophoresis parameter ($Nt$), temperature and concentration profiles show that both are increased with injection-suction.

(viii) By increasing radiation parameter ($R$), temperature profile shows that temperature decreases with injection-suction.

(ix) By increasing the Eckert number parameter ($Ec$), the temperature profile shows that temperature will increase with injection-suction.

(x) By increasing the Schmidt number parameter, the concentration profile shows that concentration decreases with injection-suction.

(xi) By increasing the heat source ($Q$), the temperature profile shows that temperature increases with injection-suction.

(xii) By increasing the velocity ratio parameter ($d$), velocity profile will show that velocity increases with injection-suction.

(xiii) With the increment in $m$, the velocity profile shows that velocity increases with injection-suction while temperature and concentration profile show that both temperature and concentration decrease with injection-suction.

Data Availability

The data set used to support the results and conclusion is included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.
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