Light element variations in globular clusters via nucleosynthesis in black hole accretion discs

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ABSTRACT

Ancient globular clusters contain multiple stellar populations identified by variations in light element (e.g., C, N, O, Na). Though many scenarios have been suggested to explain this phenomenon, all are faced with challenges when compared with all the observational evidence. In this Letter, we propose a new scenario in which light element variations are determined by nucleosynthesis in accretion discs around black holes. Since the black holes form after a few Myrs, the cluster is expected to still be embedded in a gas rich environment. By using a simplified accretion model which assumes virial temperatures, we show that the correct light element anti-correlations could be produced in accretion flows around stellar-mass black holes. Assuming a Kroupa IMF, each black hole would only have to process $\approx 300 M_{\odot}$ of material in order to produce multiple populations; over a period of 1 Myr this corresponds $\sim 10^{-4} M_{\odot} \text{yr}^{-1}$, which is within the range of values typically assumed for the formation of massive stars.

Key words: (Galaxy:) globular clusters: general, stars:chemically peculiar, accretion, accretion discs

1 INTRODUCTION

Ancient globular clusters have been found to exhibit light element variations in the form of anticorrelation in $\text{Na} - \text{O}$ and $\text{N} - \text{C}$, where some stars show enrichment in $\text{Na} (\text{N})$ and depletion in $\text{O} (\text{C})$ relative to field stars with the same metallicity (see Gratton, Carretta & Bragaglia 2012; Bastian & Lardo 2017, for a review). Evidence has also been found of multiple populations in young massive clusters, with ages in the range 2-8 Gyr, see Krause et al (2016) for a review. So far no evidence of multiple populations have been found in massive clusters (with mass $\sim 10^5$) younger than $\sim 2$ Gyr (e.g., Mucciarelli et al 2008, 2014; Martocchia et al 2017).

A number of scenarios have been put forward to explain this phenomenon, usually involving pollution by a first generation of stars. The most studied scenario involves pollution by slow moving winds from AGB stars (see e.g. Cottrell & Da Costa 1981; D’Ercole et al 2008; Bekki 2017), which cools and collects in the center of the cluster to form the polluted stars. However, this scenario requires clusters to have a significantly larger initial mass, several times the present value (D’Ercole et al 2008; Conroy 2012).

Other possible sources of pollution include fast Rotating Massive Stars (Decressin et al 2007), Interacting Binaries (de Mink et al 2009), stripped envelopes of high mass stars (Prantzos & Charbonnel 2006; Elmegreen 2017) and a single Very Massive Star (Denissenkov & Hartwick 2014). It has also been suggested by Marcolini et al (2009) that the order in which the populations form may be reversed, this scenario involves a pre-enrichment phase with Type II supernovae as well as local enrichment by a Type Ia SN and AGB stars. Another possibility is that the enriched stars are not the result of multiple epochs of star-formation but the result of the pollution of protoplanetary discs of low mass stars (Bastian et al 2013).

All current scenarios face challenges when compared to all the observational constraints (e.g., see Fig. 6 in Bastian & Lardo 2017). One challenge for all models is the ability to create discrete sub-populations (e.g., Milone et al 2015); there is also a need for stochasticity in the pollution process in order to explain the high degree of cluster-to-cluster variations (Bastian, Cabrera-Ziri & Salaris 2015; Milone et al 2015). Another constraint is that properties of the pollution mechanism seem to be correlated to the cluster mass; for example both the fraction of polluted stars and the size of the abundance spreads is correlated with the mass of the cluster (Milone et al 2017).

In this Letter we propose that light element variations could be generated through nucleosynthesis in accretion
2 BLACK HOLE ACCRETION SCENARIO

Assuming a Kroupa (2001) IMF, the total number of black holes \(N_{bh}\) expected to form is \(N_{bh} \approx 2.2 \times 10^{-3} N_{cl}\), where \(N_{cl}\) is the total number of stars in the cluster (assuming a mass range of \(0.1 - 100.0 M_\odot\) and that all stars over \(25M_\odot\) become black holes). Since the black holes are formed after a few \(M yr\), we assume that the conditions are similar to when star formation began, i.e. the environment is gas rich. The black holes will then begin to accrete the ambient pristine gas, which could experience high temperatures near the black hole and significant nucleosynthesis may occur, see Section 2.1. If the accretion rates are high enough most of the accreted gas will escape in outflows, where it will mix with pristine gas.

In order to create the enriched population of stars each black hole would have to process

\[
f_s f_d \frac{M_{cl}}{\epsilon_{sf} N_{bh}} = 300 M_\odot,
\]

where \(f_s\) is the fraction of enriched stars, \(f_d\) is the dilution factor and \(\epsilon_{sf}\) is the star formation efficiency of the enriched gas i.e. the fraction of enriched gas converted into stars. For the above estimate we have used \(f_s = 0.5\), which is within the range of present day values (see e.g. Milone et al 2017). We have also set \(f_d = 0.5\) and \(\epsilon_{sf} = \frac{1}{4}\), however these values uncertain as both are likely to depend on the properties of the outflow from the accretion disc. Note that \(M_{cl}/N_{bh} \approx 5 \times 10^{5}\) m and is independent of the cluster’s mass.

The average accretion rate required over \(1 M yr\) to process the material is \(\sim 10^{-4} M_\odot yr^{-1}\), a value which is within the range used in theories for massive star formation (e.g., see Tan et al 2014). Note that in this scenario there is a natural dilution mechanism, as the polluted material escapes from the disc in outflows and mixes with pristine gas. If we assume that accretion rates are higher in more massive clusters, then the amount of dilution decreases with increasing cluster masses as required to match observations.

Above we have assumed that all the black holes are retained within the system. It is possible that a large fraction of black holes are retained if they form via “failed supernova” or if the natal kicks are substantially reduced by a large amount of fall-back (e.g. see Fryer 1999; Fryer & Kalogera 2001; Heger et al 2003; Fryer et al 2012). However the model does not require that all the black holes are retained as the enrichment process could last over a number of \(M yr\), which only requires a fraction of black holes to active in the enrichment process.

Since we are assuming that the most massive stars \((\gtrsim 40 M_\odot)\) form via direct collapse, there is no supernova and the formation of the black hole will have little effect on the surrounding gas. For stars in the range \(25 - 40 M_\odot\) we assume that there is a large amount of fall-back, which substantially weakens the supernova. Therefore we expect little contamination from p-process elements and for the supernova to have little effect on gas until around \(\sim 8M yr\) when supernova form with masses \(\gtrsim 25M_\odot\) occur. However, the cluster may experience gas expulsion as a result of energy injection by the accreting stellar-mass black holes (Krause et al 2012).

In many cases it has been found that the enriched stars are more centrally concentrated than stars with primordial composition (see e.g. Lardo et al 2011). In this scenario we are not required to assume any degree of primordial mass segregation, as dynamical friction will naturally segregate the massive progenitor stars to the center of the cluster on a time scale of a few \(M yr\).

2.1 Nucleosynthesis in Accretion Discs

The topic of accretion discs around black holes is a rich and interesting subject, see Abramowic & Fragile (2013) for a review on the subject. One of the most studied accretion disc models is \(\alpha\)-discs of Shakura & Sunyaev (1973). The model is geometrically thin, optically thick and uses a dimensionless constant, \(\alpha\), to parameterize uncertainty over the viscosity mechanism. The model also admits analytical solutions under assumption on the pressure and opacity (for the general relativistic versions see Novikov & Thorne 1973). However the model is only valid at low accretion rates \(L \lesssim 0.3L_{Edd}\), as at higher rates the assumption that the accretion is radiatively efficient breaks down as cooling by advection becomes more important. In other models, advection-dominated accretion flows (ADAFs) advection is the dominant cooling mechanism (e.g. see Narayan & Yi 1994; Narayan & McClintock 2008). These models have nearly virial temperatures, low density and low accretion rates \(L \sim 0.01L_{Edd}\).

The nucleosynthesis usually is not included in models of accretion discs, however there has been some work on the topic. It has already been shown by Mukhopadhyay & Chakrabarti (2000) that significant nucleosynthesis could take place around a \(10M_\odot\) and \(10^6M_\odot\) black holes if the viscosity is low enough. Also Arai & Hashimoto (1992) and Hu & Peng (2008) considered a thick accretion disc around a \(10M_\odot\) black hole with very low viscosity and showed that elements up to Fe could be produced.

In order to estimate the yields from an accreting black hole we will use a highly simplified model. We will start by assuming that the temperature is approximately virial, i.e.

\[
t_{vir} = \frac{GM_{bh} m_p}{6 k r} \approx 2 \times 10^{18} t^{-1},
\]

using \(M_{bh} = 10 M_\odot\). The correct anti-correlations are produced at roughly \(5 \times 10^7 K\) corresponding to a radius of \(\approx 5.4 \times 10^{11}\) cm. We will consider a shell from \(T = 2.5 \times 10^7 - 7.5 \times 10^7 K\) and assume that yields will be similar to that of the composition being evolved at fixed \(p\) and \(T = 5 \times 10^7 K\). Radial velocities \(v_r\) are calculated by \(\frac{dr}{dt}\) where \(dr\) is the width of a shell. In order to calculate yields using the publicly available nuclear reaction network Torch by Timmes (1999)\(^1\), the initial composition used is the same as that given in (Decressin et al 2007, see their Table 3) corresponding to a metallicity of \([Fe/H] = -1.5\). The initial and final mass fractions of C, N, O and Na are given in Table 1.

The results are shown in Fig. 1, as long as the flow is slow enough (for a given mean density) both Na and N are significantly enriched by a factor \(10^{1.8}\) and \(10^{1.48}\).

\(^1\) http://cococubed.asu.edu/code_pages/burn.shtml
The processed material will need to be removed from the accretion flow in order to produce the enriched stars. A simple model for the outflow assumes $M \propto r^{s}$, where $s$ is constant in the range $0 < s < 1$. This is the same treatment as in the advection-dominated inflow-outflow solution (ADIOS) of Blandford & Begelman (1999). Using $s = 1$ (Begelman 2012), then the outflow from the region of the accretion disc considered above is approximately 70% of the inflow. However this still allows for a small amount of enrichment in heavier elements from the inner part of the disc. This means the scenario could potentially explain small Fe spreads measured in some globular clusters (e.g. M22 Da Costa et al 2009; Marino et al 2009, 2011) without the need for retained supernova ejecta.

### 2.2 Dynamical considerations

In order to achieve the necessary accretion rates we are assuming that the environment remains highly inhomogeneous, i.e. the black holes are embedded in massive clumps $\sim 1000 M_\odot$. If there is global virial equilibrium between the gas and stars, we would expect the black holes to accrete at the Bondi-Hoyle accretion rate (Bondi & Hoyle 1944)

$$M = \frac{4\pi G^2 M_{bh}^2}{(c_s^2 + v^2)^2} \approx 2 \times 10^{-8} \left( \frac{M_{bh}}{10^6 M_\odot} \right)^2 \left( \frac{M}{10^6 M_\odot} \right)^{-1/4} \left( \frac{r_h}{1 \text{pc}} \right)^{-1/4} \text{M}_{\odot} \text{yr}^{-1}$$

where $c_s$ is the sound speed, $v$ the velocity of the black hole, $\rho$ the gas density, $M_{bh}$ the mass of the black hole, $M$ is the total cluster mass including gas and stars. We have also used $\dot{v}^2 = \dot{v}^2 = 0.45 GM/\rho r$ and $c_s^2 \approx \dot{v}^2 / 3$. The resulting expected accretion rates are 4 orders of magnitude lower than those considered in Section 2. It is possible that accretion rates maybe enhanced if the black holes are accreting through a disc (e.g. see Bonnell et al 1997, 2001) or if the gas is compressed into thin filaments or sheets by stellar winds (Krause et al 2013).

If we assume that the clumps are disrupted and global virial equilibrium is achieved on the half mass relaxation time scale ($t_{rh}$), we can impose a constraint on the cluster’s mass. A commonly used definition for $t_{rh}$ is

$$t_{rh} = \frac{0.138 \Lambda^{-1/4} r_h}{(G \bar{n})^{1/4}} \ln \Lambda$$

where $\ln \Lambda$ is the Coulomb logarithm, $\bar{n}$ is the average stellar mass and $r_h$ is the half mass radius (Heggie & Hut 2003). However the theory behind this definition is based on the assumption that all particles have the same mass. It was shown by Breen & Heggie (2012a,b) and independently by Fujii & Portegies Zwart (2014) that for a multimass system it is more appropriate to calculate $t_{rh}$ using an effective number of particles (i.e. $N_{ef} = M_{ef}/m_{max}$), which is if we ignore the variation of the Coulomb logarithm is the same as the dynamical friction timescale, i.e.

$$t_{ef, rh} \approx \frac{\bar{m}}{m_{max}} t_{rh}$$

Using this definition, if we require that $t_{ef,rh} \geq 10 \text{Myr}$ to allow sufficient time for the enriched stars to form than $M_{cl} \geq 10^3$, assuming that $r_h = 1 \text{pc}$, $m_{max} = 10 M_\odot$ and $\bar{m} = 0.86$. For systems with $M_{cl} \lesssim 10^5$ relaxation is expected

### Table 1. $\alpha$ element symbol, $M_{fr,i}$ initial mass fraction, $M_{fr,f}$ final mass fraction and $\log_{10}(M_{fr,f}/M_{fr,i})$ enrichment factor.

| $\alpha$ | $M_{fr,i}$ | $M_{fr,f}$ | $\log_{10}(M_{fr,f}/M_{fr,i})$ |
|----------|------------|------------|-------------------------------|
| C        | $3.50 \times 10^{-5}$ | $3.39 \times 10^{-6}$ | -1.01                        |
| N        | $1.03 \times 10^{-5}$ | $3.13 \times 10^{-4}$ | 1.48                         |
| O        | $3.00 \times 10^{-4}$ | $3.02 \times 10^{-4}$ | -2.00                        |
| Na       | $3.30 \times 10^{-7}$ | $1.94 \times 10^{-3}$ | 1.76                         |

a dilution factor of $f_d = 0.5$, this will produce spreads of 1.5 and 1.1 dexs. Similarly the depletion in O and C are approximately $10^{-2}$ and $10^{-1}$, times the initial values. In a dilution model so long as the final abundances of O and C are small, the variation is independent of the final values and is approximately $\log_{10}(1 - f_d)$. Note that the amount of enrichment and depletion will depend on the initial composition and will decrease with increasing initial abundances.
to impede the star formation process. This also suggests a possible dynamical explanation for the correlation found between $f_\ast$ and cluster mass.

### 2.3 More massive black holes as the polluters

The size of the region within the accretion discs we are considering scales linearly with mass, for an IMBH this would be a by factor $10 \sim 100$ larger. Therefore, this will allow significant nucleosynthesis to take place for higher radial velocities. However, temperatures within the original region would be a factor $10 \sim 100$ hotter and one might expect a more significant production of heavier elements. The IMBH would also need accretion rates $\sim 10^3 M_\odot$ in order to enrich enough material, which would require very high average accretion rates $\sim 10^{-1} M_\odot \text{yr}^{-1}$ if the enrichment process takes $1 \text{Myr}$. However, if the black hole is fed with massive clumps of $\sim 10^3 M_\odot$ then it is only required to accrete $10^5$ clumps, which would lead to discrete episodes of star formation. If the light element variations are in fact generated in accretion discs, this may allow for the possibility of identifying IMBH through light element variations (assuming the IMBH is present during star formation). Furthermore, if one considers super-massive black holes also as polluters responsible for the large population of N-rich stars in bulge/inner halo Schiavon et al (2017), then there is a common pollution mechanism at work in these systems.

### 3 DISCUSSION

We have argued that the light element variations observed in globular clusters could be generated in accretion discs around stellar-mass black holes. In this scenario the black holes form from the earliest massive stars after a few $\sim M_\odot$yr and are born into a gas rich environment consisting of a hierarchical structure with many dense clumps (McKee & Ostriker 2007; Tan et al 2014). The black holes are expected to experience similar accretion rates as are expected during massive star formation; the accreted gas is subject to high temperatures in the inner part of the disc after which it escapes in outflows. If temperatures in the accretion disc are high enough significant nucleosynthesis will occur and the required light element variations could be generated. The polluted material escapes from the disc in outflows which mix with pristine gas, from which the enriched stars are formed.

Using a highly simplified model we have shown that a population of stellar-mass black holes could produce sufficient material to produce a polluted population of stars with $Na \sim O$ and $N \sim C$ anticorrelation, which is the main signature of multiple populations. These results need to be compared with more detailed numerical simulations of accretion flow and further work is needed to develop aspects of the model. The aim of this Letter is to show that such a model is plausible and warrants further study.

Although more detailed investigation is required to gain a better understanding of light element yields, and indeed if sufficient nucleosynthesis does actually occur in accretion discs, the model presented in this Letter has many promising features. First, since pollution takes place over a short time scale $\lesssim 10\text{Myr}$ the populations are co-evolved. There is not a mass budget problem since the black holes process the pristine gas and there is a natural dilution mechanism as outflows from the accretion disc mix the enriched gas with pristine gas. Since accretion rates and times depend on the properties of the host system, the fraction of enriched stars and abundances are expected to correlate with the cluster mass.

It has been recently suggested by Carretta, Bragaglia & Lucatello (2018) that simple dilution models can not explain the light elements variations observed in NGC 2808 and that polluters of different masses are required. This might not be an issue for the model proposed here for two reasons. First, for the simple scenario of steady steady accretion the velocity of winds from the disc increases towards the center. Such a condition may allow to keep the material polluted by the inner and outer parts of the disc as separated. Therefore a single black hole could produce stars with different composition. Second, in reality the composition of the polluted material is likely to depend on accretion rates and may be subject to instabilities, which would also produce stars with different composition that would be allowed for a simple dilution models. Instabilities in the accretion flows, may also lead to discreteness and stochasticity in the enrichment process.

Although star formation in this model is continuous this does necessarily imply that the light element enrichment is continuous. If the outflow from black holes are compressing the surrounding gas inducing star formation, then there may be a discrete jump in enrichment. Further research on star formation in the vicinity of an accreting black hole would be an interesting avenue to further develop the model. Interestingly, if the enriched stars are formed in the vicinity of accreting black holes then their stellar masses may be limited by competitive accretion (Bonnel et al 2001). A lower maximum stellar-mass for enriched stars may explain why multiple populations have not been observed in young massive clusters younger than $\sim 2\text{Gyr}$.

If we consider a cluster of $N = 10^6$ stars and if all the black holes are accreting at the Eddington limit, then the luminosity would be $\sim 10^{45}$ erg/s, i.e. at the top end of the range for an ultra-luminosity X-ray source ULXs or the lower end of an active galactic nuclei (AGN). However if the gas column density is high enough, then these sources would be unobservable except possibly during the brief period when most of the cluster gas has been expelled but the accretion discs have not been exhausted. However, if black holes are dynamically ejected with their accretion discs intact, then they will become visible once free of the cluster. A single black hole would itself appear as a ULX if it is accreting at super Eddington rates or through anisotropic emission of radiation (King et al 2001). Indeed a physical association between the ULXs and super star clusters in large starburst galaxies has been suggested (e.g. see Kaaret, Ward & Zezsa 2004; Poutanen et al 2013). However, note that the nature of ULXs is still an open question and most probably consists of a class of objects (for a recent review see Kaaret, Feng & Roberts 2017).
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