Causality in 1+1 Dimensional Yukawa Model-II

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Abstract

We discuss the limits $g \to \text{large, } M \to \text{large}$ with $\frac{g^2}{M} = \text{const.}$ of the 1 + 1 dimensional Yukawa model. We take into account conclusion of the results on bound states of the Yukawa Model in this limit (obtained in [7]). We find that model reduces to an effective nonlocal $\phi^3$ theory in this limit. We observe causality violation in this limit. We discuss the result.

1 Introduction

The (local) Standard model (SM) has, in particular, been looked upon [1] as an effective field theory, an approximation to the actual field theory. The underlying theory could, for example, be a composite model and the observed SM particles then may be composites of the underlying constituents [2]. Thus, then SM particles will generally have a finite size and are hence likely to exhibit non-local interactions. The local SM then is an approximation; as the effects of non-locality are such as to be normally ignorable at the
present energies. Causality violations are likely to be associated with non-local interactions \[3\]. At least, there are likely to be quantum violations of causality \[5, 3\]. Now, the effects of non-locality can possibly become visible at LHC. Simple model calculations show that a causality violation effects could be observed around energy scale \( \lesssim \Lambda \), the mass scale in theory such as the scale of compositeness \[3, 4, 6\].

In this work, we would like to construct a simple model that embodies this hypothesis and study the phenomenon. To this end, we study a simple 1+1 dimensional Yukawa theory; in a certain limit of its parameters. We show that there is a limit of parameters of the theory so that there is a simple non-local effective field theory. Causality violation is observed in the model.

To understand how this is possible, we employ the results we have obtained earlier \[7\]. We have studied elsewhere the bound-state formation in the model in this limit and have shown that the effective model can be interpreted as a field theory of a bound state. We study causality in such a model. It is suggested that a non-trivial mechanism that leads to bound states can lead to causality violation.

Even though the discussion is confined to Yukawa model, similar effects should be observed in realistic composite models \[2\] involving gauge fields.

In section 2 the Yukawa model in 1+1 dimensions is discussed. Section 2.1 deals with the motivation as to causality violation in the bound states. In section 3 condition of causality due to Bogoliubov and Shirkov is briefly reviewed. In section 4.1 we show, by using the power counting, that only one loop 2-point is divergent, 3-point function remains of fixed magnitude and n-point function \((n \geq 4)\) will tend to vanish in the large \(M\) limit. Moreover the contribution to a given n-point function from higher and higher loops fall off faster. Section 4.3 describes 3-point function which turns out to be an effective nonlocal field theory in this limit. In Section 4.4 we find that the proper 4-point function vanishes in the limit \(M \to \infty\) and 3-point function in one-loop approximation alone survives in this limit. In Section 5 we
calculate the commutator and show that causality is violated in this model.

2 The Model

We shall consider the Yukawa model in 1+1 dimensions:

\[
\mathcal{L} = \bar{\psi} \left[ i \partial - M + g \phi \right] \psi + \frac{1}{2} \partial \mu \phi \partial^{\mu} \phi - \frac{1}{2} m_0^2 \phi^2
\]  

(1)

We note that \( \psi \) is of dimension \( \frac{1}{2} \), \( \phi \) is dimensionless and \( g \) and \( M \) have dimension 1 each. We shall assume that the interaction is normal-ordered, (so that the tadpole diagrams are eliminated). Among the remaining diagrams, the one-loop two-point function (i.e. self-energy) alone is divergent: See section 4.1. We shall, to begin with, regularize it by a cut-off \( \Lambda \). We shall be interested in a particular limit of parameters of the theory [7]. We shall consider the possibility that the fermion-antifermion mass \( M \) is very large and at the same time the coupling constant \( g \) is large, leading to a large attractive Yukawa potential. We then found that [7] the ground state is a non-relativistic heavy-quark-like bound state, and it alone is stable; from among (a large number of) bound states. We shall find that despite the large coupling, for a certain relation between \( g \) and \( M \), the higher loop diagrams are all smaller and smaller and the perturbation series is in fact convergent.

2.1 Violation of Causality in Bound States

Suppose a scalar bound state of fermions is formed at the CM at \( x \) at time \( t \). Consider the commutator,

\[ \left[ \phi(x), \phi(y) \right] \]
Where $\phi(x)$ may be expressed as:

$$
\phi(x) = \int \bar{\psi}(\xi)\psi(\eta)f(\xi, \eta)d\xi; \quad \frac{\xi + \eta}{2} = x
$$

$$
= \int \bar{\psi}(x-w)\psi(x+w)f(x-w, x+w)dw; \quad \text{Putting } \xi = x-w
$$

The function $f$ is related to the wavefunction of the bound state. Now,

$$
[\phi(x), \phi(y)] = \int dwdz [\bar{\psi}(x-w)\psi(x+w), \bar{\psi}(y-z)\psi(y+z)]
$$

$$
\times f(x-w, x+w)f(y-z, y+z)
$$

$$
= \int dwdz \{\bar{\psi}(x-w)\{\psi(x+w), \bar{\psi}(y-z)\}\psi(y+z)
$$

$$
- \bar{\psi}(y-z)\{\psi(y+z), \bar{\psi}(x-w)\}\psi(x+w)
$$

$$
\times f(x-w, x+w)f(y-z, y+z)
$$

In the above commutator $[\phi(x), \phi(y)]$ even if $(x-y)^2$ is space-like, the fermions in the two bound states can still be at time-like distances. So, they will contribute to the commutator. If $(x-y)^2 < 0$ is varied, the region in spacetime over which the anticommutator is nonzero will be varied along with. It is unlikely for all values of $(x-y)^2 < 0$, the commutator will not be equal to zero. [A similar argument has been briefly discussed by K. Akama et al [8].]

3 Condition of Causality section

Bogoliubov and Shirkov [9] have formulated conditions for causality. For this formulation, they introduce an $x$-dependent coupling $g(x)$ in an intermediate stage and find:

$$
\frac{\delta}{\delta g(x)} \left( \frac{\delta S(g)}{\delta g(y)} S^\dagger(g) \right) = 0 \quad x \sim y; \ x_0 < y_0
$$

(2)
where $x \sim y$ means that $x$ is spacelike with respect to $y$. They expand the $S$-operator as,

$$S[g(x)] = 1 + \sum_{n=1}^{\infty} \int \prod_{i=1}^{n} dx_i g(x_1) g(x_2) \cdots g(x_n) \frac{S_n(x_1, x_2, \ldots, x_n)}{n!}$$

and expand (2) in powers of $g$. Among these conditions, the first one, in terms of the above $S'_i$s, is:

$$S_2(x, y) = S_1(x) S_1(y) \quad x_0 > y_0$$

$$= S_1(y) S_1(x) \quad y_0 > x_0$$

If for a given $x$ and $y$ with $(x - y)^2 < 0$, there are Lorentz frames, in which either of the above conditions are fulfilled. Assuming the covariance of the $S$–matrix, this, in particular, implies,

$$0 = [S_1(x), S_1(y)] \quad (x - y)^2 < 0 \quad (3)$$

and

$$0 = H_1(x, y) \equiv S_2(x, y) - T[S_1(x) S_1(y)] \quad x_0 \neq y_0 \quad (4)$$

As $S_2(x, y)$ is given by a covariant expression (say that is obtained from a path-integral expression) which we denote as below,

$$S_2(x, y) = T^*[S_1(x) S_1(y)]$$

so that

$$0 = H_1(x, y) = T^*[S_1(x) S_1(y)] - T[S_1(x) S_1(y)] \quad (5)$$

The above is a necessary condition for causality to hold. In other words, if any matrix element of $H_1$ is non-zero, then that is sufficient for CV.
4 Calculations

4.1 The Power Counting

Consider a diagram with \( n \) external scalars, \( L \) fermion loops and \( V \) vertices and \( I_F \) internal fermion lines and \( I_B \) internal boson lines (\( I_F + I_B \equiv I \)). (There are no scalar loops as there is no scalar interaction term.) For such a diagram,

\[
3V = 2I + n
\]
\[
L = I - V + 1
\]

i.e. \( 2L - I = 2 - \frac{1}{2}(n + V) \) and
\[
D = 2L - I_F - 2I_B = 2L - I - I_B
\]
\[
= 2 - \frac{1}{2}(n + V) - I_B
\] (6)

It also follows that,
\[
V = 2L + n - 2
\]
\[
n + V = 2(L + n - 1)
\] (7)

Ignoring the tadpole, the largest value that \( D \) takes is zero and is for the one loop two-point function diagram. This diagram alone is divergent.

First, consider the set of \( n \)-point one-loop diagrams with \( n \geq 3 \). For such a diagram, \( V = n \) and \( D = 2 - n \). The diagrams are then finite and proportional to \( \frac{g^n}{M^{n-2}} \). Suppose, we consider the limit of large \( g \) and \( M \), such that \( \frac{g^3}{M} = constant = C \). Then, the 3-point function will remain of fixed magnitude and the \( n \)-point function (\( n \geq 4 \)) will tend to vanish as \( M^{2 - \frac{2}{3}n} \).

Moreover, for the diagrams with higher number of loops, they will go as:

\[
\frac{g^V}{M^{D}} \sim M^{\frac{1}{3}V + D} \sim M^{-(\frac{L-1}{3})-2(\frac{M}{L})-I_B}
\]

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Thus, the contributions to a given n-point function from higher and higher loops fall off faster.

4.2 Two-point Function

We have noted that the one-loop two point function is the only primitively divergent diagram. On calculation, we find:

\[ i \Sigma(p) = -g^2 \int \frac{d^2 k}{(2\pi)^2} \frac{1}{(k^2 - M^2)((k + p)^2 - M^2)} Tr \left[ (\not{k} + M)(\not{k} + \not{p} + M) \right] \]

\[ = \frac{ig^2}{4\pi} \int_0^1 d\alpha \left[ \ln \left( \frac{\Lambda^2 + M^2 - \alpha(1 - \alpha)p^2}{M^2 - \alpha(1 - \alpha)p^2} \right) - \frac{2\Lambda^2}{\Lambda^2 + M^2 - \alpha(1 - \alpha)p^2} \right] \]

For \( p^2 = 0 \), we have,

\[ \Sigma(0) = \frac{g^2}{4\pi} \left[ \ln \left( \frac{\Lambda^2 + M^2}{M^2} \right) - \frac{2\Lambda^2}{\Lambda^2 + M^2} \right] \quad (8) \]

We next look at the correction when \( p^2 \neq 0 \). Assuming \( p^2 \ll \Lambda^2 \ll M^2 \), we find,

\[ -\Sigma(0) = -\frac{g^2}{4\pi} \left[ \ln \left( \frac{\Lambda^2 + M^2}{M^2} \right) - \frac{2\Lambda^2}{\Lambda^2 + M^2} \right] \]

\[ \simeq \frac{g^2}{4\pi} \left[ \frac{\Lambda^2}{M^2} \right] \]

This is the first order quantum correction to \( m^2 \); and using \( g = (CM)^{1/3} \), and \( \Lambda \sim M^{5/9} \) this falls of as \( M^{-1/9} \). Also,

\[ \Sigma(p) - \Sigma(0) = \frac{g^2}{4\pi} \int_0^1 d\alpha \alpha(1 - \alpha) \frac{p^2}{M^2} \left[ 1 - \frac{M^2}{\Lambda^2 + M^2} + \frac{2\Lambda^2 M^2}{(\Lambda^2 + M^2)^2} \right] \quad (9) \]

\footnote{This condition requires explanation: it is given later.}
Using $g = (CM)^{1/3}$, we find that this leading contribution falls of faster than $\frac{p^2}{M^2} M^{-2/3}$. Thus, the quantum corrections, both to $m^2$ and the propagator vanish in this limit. In this we have crucially made use of $\Lambda \sim M^{5/9}$ which has arisen from our study of the bound states in this model ([7]): the basic assumptions are (i) in this limit, all states except the ground bound state are absolutely unstable, (ii) the propagator is saturated by this state, (iii) the wavefunction is dominated by momenta $\leq M^{5/9}$.

4.3 Three-point Function

It turns out that the three-point function is the only $O(1)$ vertex in this limit. It is a non-local field theory with,

$$S^{(3)} \equiv \int d^2 x_2 \mathcal{L}^{(3)} (x_2) = \int d^2 x_1 d^2 x_2 d^2 x_3 \phi (x_1) \phi (x_2) \phi (x_3) F (x_1 - x_2; x_3 - x_2)$$

with

$$FT \{ F (x_1 - x_2; x_3 - x_2) \} = \tilde{F} [p_1, p_3] = -g^3 \frac{1}{2\pi M} \int d\alpha d\beta \frac{M^4}{[M^2 - \alpha (1 - \alpha) p_3^2 - \beta (1 - \beta) p_1^2 - 2\alpha\beta p_1 p_3]^2}$$

Causality is preserved only if,

$$[\mathcal{L}^{(3)} (x_1), \mathcal{L}^{(3)} (x_2)] = 0 \quad \text{whenever} \quad (x_1 - x_2)^2 < 0. \quad (10)$$

We note that in the strict limit $M \to \infty$, $\tilde{F} \to const.$ and $S^{(3)}$ becomes a local 3-point interaction, and in this limit, (10) is automatically fulfilled.
4.4 Four-point Function

The proper four-point function behaves like \( \frac{g^4}{M^2} \sim M^{-2/3} \). In the limit \( M \to \infty \), the function vanishes. The leading term in a Taylor expansion of \( \frac{g^4}{M^2} F \left( \frac{p^2}{M^2} \right) \) is of the order \( M^{-8/3} \).

Three-point function in one-loop approximation is what survives in the limit we are interested in. We find:

\[
i \Gamma^{(3)} = -ig^3 \int \frac{d^2k}{(2\pi)^2} Tr \left[ (k + M)(k + p_1 + M)(k + p_2 + M) \right]
= -2ig^3 \int \frac{d^2k}{(2\pi)^2} \frac{M^3 + 3Mk^2 + 2M(p_1 + p_2)k + Mp_1^2 + M_{p_1.p_2}}{((k + p_1)^2 - M^2)((k + p_1 + p_2)^2 - M^2)}
\approx \left( \frac{-g^3}{2\pi M} \right) \int_0^1 du \int_0^{1-u} dv \frac{M^4}{[M^2 + u(1-u)p_1^2 + v(1-v)p_2^2 + 2uvp_1.p_2]^2}
\]

5 Evaluation of the Commutator: \[ \mathcal{L}^{(3)}(x), \mathcal{L}^{(3)}(y) \]

We have,

\[
C(x, y) = \mathcal{L}^{(3)}(x), \mathcal{L}^{(3)}(y)
= \int d\xi_1 d\xi_2 \varphi(x + \xi_1) \varphi(x + \xi_2) F(\xi_1, \xi_2), \int d\eta_1 d\eta_2 \varphi(y + \eta_1) \varphi(y + \eta_2) F(\eta_1, \eta_2)
= \int d\xi_1 d\xi_2 d\eta_1 d\eta_2 F(\xi_1, \xi_2) F(\eta_1, \eta_2) [\varphi(x) \varphi(x + \xi_1) \varphi(x + \xi_2), \varphi(y) \varphi(y + \eta_1) \varphi(y + \eta_2)]
\]

After expanding the commutator:

\[
C(x, y) = \int \{ \varphi(x) \varphi(x_1) \varphi(y) \varphi(y_1) [\varphi(x_2), \varphi(y_2)] + \varphi(x) \varphi(x_1) \varphi(y) [\varphi(x_2), \varphi(y_1)] \varphi(y_2) \\
+ \varphi(x) \varphi(x_1) [\varphi(x_2), \varphi(y)] \varphi(y_1) \varphi(y_2) + \varphi(x) \varphi(y) \varphi(y_1) [\varphi(x_1), \varphi(y_2)] \varphi(x_2) \\
+ \varphi(x) \varphi(y) [\varphi(x_1), \varphi(y_1)] \varphi(y_2) \varphi(x_2) + \varphi(x) [\varphi(x_1), \varphi(y)] \varphi(y_1) \varphi(y_2) \varphi(x_2)
\]

\[
9
\]
\begin{align*}
&+ \varphi(y)\varphi(y_1) [\varphi(x), \varphi(y_2)] \varphi(x_1)\varphi(x_2) + \varphi(y) [\varphi(x), \varphi(y_1)] \varphi(y_2)\varphi(x_1)\varphi(x_2) \\
&+ [\varphi(x), \varphi(y)] \varphi(y_1)\varphi(y_2)\varphi(x_1)\varphi(x_2)] F(\xi_1, \xi_2)F(\eta_1, \eta_2)d\xi_1d\xi_2d\eta_1d\eta_2
\end{align*}

Where \( x_i = x + \xi_i \) and \( y_i = y + \eta_i \). The last term reads:

\[ C_9(x, y) \equiv \int [\varphi(x), \varphi(y)] \varphi(y_1)\varphi(y_2)\varphi(x_1)\varphi(x_2)] F(\xi_1, \xi_2)F(\eta_1, \eta_2)d\xi_1d\xi_2d\eta_1d\eta_2 \]

\( C_9(x, y) \) is zero for \([\varphi(x), \varphi(y)] \) vanishes for \( x \sim y \). We are now left with the eight terms:

\[ C(x, y) = \int d\xi_1d\xi_2d\eta_1d\eta_2F(\xi_1, \xi_2)F(\eta_1, \eta_2) \]

\( \times \) \( \{ \varphi(x)\varphi(x_1)\varphi(y)\varphi(y_1) \} \) \( \varphi(x_2), \varphi(y_2)] + \varphi(x)\varphi(x_1)\varphi(y)\varphi(y_2) [\varphi(x_2), \varphi(y_1)] \\
+ \varphi(x)\varphi(x_1)\varphi(y)\varphi(y_2) [\varphi(x_2), \varphi(y)] + \varphi(x)\varphi(y)\varphi(y_1)\varphi(x_2) [\varphi(x_1), \varphi(y_2)] \\
+ \varphi(x)\varphi(y)\varphi(y_2)\varphi(x_2) [\varphi(x_1), \varphi(y_1)] + \varphi(x) [\varphi(x_1), \varphi(y)] \varphi(y_1)\varphi(y_2)\varphi(x_2) \\
+ \varphi(y)\varphi(y_1)\varphi(x_1)\varphi(x_2) [\varphi(x), \varphi(y_2)] + \varphi(y)\varphi(y_2)\varphi(x_1)\varphi(x_2) [\varphi(x), \varphi(y_1)] \}

Let us consider the first three terms of \( C(x, y) \) together:

\[ C_{123}(x, y) \equiv \int d\xi_1d\xi_2d\eta_1d\eta_2F(\xi_1, \xi_2)F(\eta_1, \eta_2) \]

\( \times \) \( \{ \varphi(x)\varphi(x_1)\varphi(y)\varphi(y_1) \} \) \( \varphi(x_2), \varphi(y_2)] + \varphi(x)\varphi(x_1)\varphi(y)\varphi(y_2) [\varphi(x_2), \varphi(y_1)] \\
+ \varphi(x)\varphi(x_1)\varphi(y)\varphi(y_2) [\varphi(x_2), \varphi(y)] \}

\[(11)\]

The scalar field operators can be expanded in terms of creation and annihilation operators. Each term in the r.h.s. of \((11)\) will have sixteen independent terms comprising creation and annihilation operators. \( C_{123}(x, y) \) can be therefore expressed as a collection of the sixteen independent terms. Non-vanishing of any term for \( x \sim y \) will signal causality violation. Let us consider the specific term belonging to \( C_{123}(x, y) \) as follows:
\[ C'_{123}(x, y) \]
\[ = \int \frac{dp_1 dp_2 dq_1 dq_2 dq_3}{(2\pi)^5 \sqrt{2\omega_{p1} 2\omega_{p2}}} \left[ \frac{a_{p1}^\dagger a_{p2}^\dagger a_{q1}^\dagger a_{q2}^\dagger}{\sqrt{2\omega_{q1} 2\omega_{q2} 2\omega_{q3}}} e^{ip_1 x + ip_2 x_1 + i q_1 y + i q_2 y_1} \sin(q_3(x - y + \xi_2 - \eta_2)) \right] \]
\[ + \frac{1}{\sqrt{2\omega_{q1} 2\omega_{q2} 2\omega_{q3}}} a_{p1}^\dagger a_{p2}^\dagger a_{q1}^\dagger a_{q3}^\dagger e^{ip_1 x + ip_2 x_1 + i q_1 y + i q_2 y_2} \sin(q_2(x - y + \xi_2 - \eta_1)) \]
\[ + \frac{1}{\sqrt{2\omega_{q2} 2\omega_{q3} 2\omega_{q1}}} a_{p1}^\dagger a_{p2}^\dagger a_{q2}^\dagger a_{q3}^\dagger e^{ip_1 x + ip_2 x_1 + i q_2 y_1 + i q_3 y_2} \sin(q_1(x - y + \xi_2)) \]
\[ \times (-2i) F(\xi_1, \xi_2) F(\eta_1, \eta_2) d\xi_1 d\xi_2 d\eta_1 d\eta_2 \]

We have at \( x^0 = 0 \) and \( y^\mu = 0 \):

\[ C'_{123}(x^0 = 0, x, y^\mu = 0) \]
\[ = \int \frac{dp_1 dp_2 dq_1 dq_2 dq_3}{(2\pi)^5 \sqrt{2\omega_{p1} 2\omega_{p2}}} \left[ \frac{a_{p1}^\dagger a_{p2}^\dagger a_{q1}^\dagger a_{q2}^\dagger}{\sqrt{2\omega_{q1} 2\omega_{q2} 2\omega_{q3}}} \left( e^{i(p_1 + p_2 + q_3)x} \tilde{F}(-p_2, -q_3) \tilde{F}(-q_2, q_3) \right) \right] \]
\[ - e^{i(p_1 + p_2 - q_3)x} \tilde{F}(-p_2, q_3) \tilde{F}(-q_2, -q_3) + \frac{1}{\sqrt{2\omega_{q1} 2\omega_{q2} 2\omega_{q3}}} a_{p1}^\dagger a_{p2}^\dagger a_{q1}^\dagger a_{q3}^\dagger \left( e^{i(p_1 + p_2 + q_3)x} \tilde{F}(-p_2, -q_3) - e^{i(p_1 + p_2 - q_3)x} \tilde{F}(-p_2, q_3) \right) \]
\[ \times \left( e^{i(p_1 + p_2 + q_1)x} \tilde{F}(-p_2, -q_2) \tilde{F}(q_2, q_3) - e^{i(p_1 + p_2 - q_2)x} \tilde{F}(-p_2, q_2) \tilde{F}(q_2, -q_3) \right) \]
\[ + \frac{a_{p1}^\dagger a_{p2}^\dagger a_{q2}^\dagger a_{q3}^\dagger}{\sqrt{2\omega_{q2} 2\omega_{q3} 2\omega_{q1}}} \left( e^{i(p_1 + p_2 + q_3)x} \tilde{F}(-p_2, -q_1) - e^{i(p_1 + p_2 - q_3)x} \tilde{F}(-p_2, q_1) \right) \tilde{F}(q_2, -q_3) \]
\[ = \int \frac{dp_1 dp_2 dq_1 dq_2 dq_3}{(2\pi)^5 \sqrt{2\omega_{p1} 2\omega_{p2} 2\omega_{q1} 2\omega_{q2}}} \left[ \frac{a_{p1}^\dagger a_{p2}^\dagger a_{q1}^\dagger a_{q2}^\dagger}{2\omega_{q3}} \left( e^{i(p_1 + p_2 - q_3)x} - e^{i(p_1 + p_2 + q_3)x} \right) \right] \]
\[ \times \tilde{F}(-p_2, -q_3) \tilde{F}(-q_2, q_3) \]
\[ + \int \frac{dp_1 dp_2 dq_1 dq_2 dq_3}{(2\pi)^5 \sqrt{2\omega_{p1} 2\omega_{p2} 2\omega_{q1} 2\omega_{q3}}} \left[ \frac{a_{p1}^\dagger a_{p2}^\dagger a_{q2}^\dagger a_{q3}^\dagger}{2\omega_{q2}} \left( e^{i(p_1 + p_2 - q_3)x} - e^{i(p_1 + p_2 + q_3)x} \right) \right] \]
\[ \times \tilde{F}(-p_2, -q_2) \tilde{F}(q_2, -q_3) \]
\[ + \int \frac{dp_1 dp_2 dq_1 dq_2 dq_3}{(2\pi)^5 \sqrt{2\omega_{p1} 2\omega_{p2} 2\omega_{q1} 2\omega_{q3}}} \left[ \frac{a_{p1}^\dagger a_{p2}^\dagger a_{q2}^\dagger a_{q3}^\dagger}{2\omega_{q1}} \left( e^{i(p_1 + p_2 - q_1)x} - e^{i(p_1 + p_2 + q_1)x} \right) \right] \]
\[ \times \tilde{F}(-p_2, -q_1) \tilde{F}(q_2, -q_3) \]
Now, we can subject Taylor expansion to $C'_{123}(x^0 = 0, x, y^\mu = 0)$ around $x = 0$:

$$C'_{123}(x^0 = 0, x, y^\mu = 0) = C'_{123}(x^0 = 0, x = 0, y^\mu = 0) + x \frac{\partial}{\partial x} C'_{123}(x^0 = 0, x, y^\mu = 0) |_{x=0} + \ldots$$

We can have now,

$$\frac{\partial}{\partial x} C'_{123}(x^0 = 0, x, y^\mu = 0) |_{x=0} = \int \frac{dp_1 dp_2 dq_1 dq_2 dq_3}{(2\pi)^5 \sqrt{2\omega_{p_1} 2\omega_{p_2} 2\omega_{q_1} 2\omega_{q_2}}} \frac{a_{p_1} a_{p_2} a_{q_1} a_{q_2}}{2\omega_{q_3}} (-2q_3) \tilde{F}( -p_2, -q_3) \tilde{F}( -q_2, q_3)$$

$$+ \int \frac{dp_1 dp_2 dq_1 dq_2 dq_3}{(2\pi)^5 \sqrt{2\omega_{p_1} 2\omega_{p_2} 2\omega_{q_1} 2\omega_{q_2}}} \frac{a_{p_1} a_{p_2} a_{q_1} a_{q_3}}{2\omega_{q_2}} (-2q_2) \tilde{F}( -p_2, -q_2) \tilde{F}( q_2, -q_3)$$

$$+ \int \frac{dp_1 dp_2 dq_1 dq_2 dq_3}{(2\pi)^5 \sqrt{2\omega_{p_1} 2\omega_{p_2} 2\omega_{q_1} 2\omega_{q_2}}} \frac{a_{p_1} a_{p_2} a_{q_2} a_{q_3}}{2\omega_{q_1}} (-2q_1) \tilde{F}( -p_2, -q_1) \tilde{F}( q_2, -q_3)$$

Making the following interchanges in the above expression: ($q_2 \leftrightarrow q_3$) in the second term and ($q_1 \leftrightarrow q_3$) in the third term, $\frac{\partial}{\partial x} C'_{123}(x^0 = 0, x, y^\mu = 0) |_{x=0}$ takes the form:

$$\frac{\partial}{\partial x} C'_{123}(x^0 = 0, x, y^\mu = 0) |_{x=0} = \int \frac{dp_1 dp_2 dq_1 dq_2 dq_3}{(2\pi)^5 \sqrt{2\omega_{p_1} 2\omega_{p_2} 2\omega_{q_1} 2\omega_{q_2}}} \frac{a_{p_1} a_{p_2} a_{q_1} a_{q_2}}{2\omega_{q_3}} (-2q_3) (\tilde{F}( -p_2, -q_3) \tilde{F}( -q_2, q_3))$$

$$+ \tilde{F}( -p_2, -q_3) \tilde{F}( q_3, -q_2) + \tilde{F}( -p_2, -q_3) \tilde{F}( -q_2, -q_1))$$

Which is nonzero. Thus the commutator $[\mathcal{L}^{(3)}(x), \mathcal{L}^{(3)}(y)]$ does not vanish even for space-like separation and leads to the violation of causality in this model.
6 Discussion and Conclusions

We have discussed the particular limit of the Yukawa field theory in $1 + 1$ dimensions. We find that all n-point function except 2-point and 3-point functions tend to vanish into this limit. Higher order corrections to the 2-point function corrected for using bound state model results also tends to vanish in this limit. This theory reduces to effective nonlocal scalar field theory. Causality violation is observed in this model.

The original theory is causality preserving. So we may suspect the causality should be preserved in the effective theory. However this problem has been investigated recently [4], and it has been found that composite state model can give to the causality violation even if underlying field theory does not show any causality violation. The source of causality violation in this model exhibits bound states which we have taken into account while calculating at least propagator. This gives us only with effective $\phi^3$ interaction which is nonlocal and causality violating.

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