Solitons in thermal media with periodic modulation of linear refractive index

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We address the existence and properties of solitons in thermal media with periodic modulation of linear refractive index. Many kinds of solitons in such optical lattices, including symmetric and antisymmetric lattices, are found under different conditions. We study the influence of the refractive index difference between two different layers on solitons. It is also found that there do not exist cutoff value of propagation constant and soliton power for shifted lattice solitons. In addition, the solitons launched away from their stationary position may propagate without oscillation when the confinement from lattices is strong.

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I. INTRODUCTION

Nonlocal solitons have been studied in many physical systems, such as photorefractive crystals [1], nematic liquid crystals [2–3], lead glasses [4, 5], Kerr-like media [6], materials with quadratic nonlinearities [7–10], atomic vapors [11], and Bose-Einstein condensates [12, 13], etc. They show some special and novel properties both in experiment and theory, for instance, the large phase shift [14], self-induced fractional Fourier transform [15], attraction between two dark solitons [16, 17], etc. Recently, nonlocal surface solitons propagating along the interface formed by a nonlocal nonlinear medium and a linear medium are found and observed [18, 19]. This kind of solitons presents strong asymmetry including their profiles and nonlinear refractive indices because of the different boundary conditions. They are stable only when the number of intensity peaks is less than three [20].

Solitons are also found in some media with periodic nonlinearity [21–23]. Recently, solitons in nonlocal nonlinear lattices are studied, such as odd, even, dipole, tripoles, and twisted solitons [24–27]. For lattice solitons formed in nonlocal media with an imprinted modulation of linear refractive index, there exists a cutoff point of propagation constant due to the existence of the bandgap structure of periodic lattice [28]. In thermal lattices with periodic nonlinearity, there can be found asymmetric solitons, which is shifted solitons. There exist a cutoff value of propagation constant and a minimal power for this kind of solitons in such lattices [29]. It is found that, in thermal nonlinear lattices, solitons can exist anywhere inside a band and two families of solutions bifurcate from each band edge because of the infinite range of nonlocality and the presence of boundaries [30]. At the same time, surface lattice solitons are found at the interface between a uniform medium and a nonlinear lattice or at the edge of layered nonlocal media [31, 32]. Such solitons present strongly asymmetric shapes and they are stable in large parts of their existence region. There is no restriction on the number of peaks in stable surface solitons localized at a thermally insulating interface between layered thermal media and a linear dielectric, which is quite different from that in uniform nonlocal media [24]. Interface solitons propagating at the interface of one-dimensional thermal nonlinear media with a step in linear refractive index have been studied in [33]. It is note that interface solitons in such media are asymmetric because of the difference of the linear refractive index. For tripoles and quadrupole interface solitons, there exist two different types of solutions under some special conditions [34].

In this paper, we study the propagation of solitons in the semi-infinite band gap of thermal media with periodic distribution of linear refractive index. In our model, the period modulation function of linear index is a step function. We find the odd, even, and shifted solitons in this kind of media. In addition, there do not exist cutoff value of propagation constant for shifted solitons, which is different from that in thermal media with periodic nonlinearity. We also study the influence of the index-difference of the lattices and the boundaries on solitons.

II. THEORETICAL MODEL

We consider a (1+1)-dimensional layered thermal sample occupying the region \(-L \leq x \leq L\) imprinted a linear refractive index lattice. A laser beam propagating along the \(z\) axis is governed by the nonlinear Schrödinger equation (NNLSE) for the dimensionless amplitude \(q\) of the light field, coupled to the equation for normalized nonlinear refractive index variation \(n\),

\[
\frac{i}{a} \frac{\partial q}{\partial z} + \frac{1}{2} \frac{\partial^2 q}{\partial x^2} + nq + \delta(x) n_a q = 0, \quad (1)
\]

\[
\frac{\partial^2 n}{\partial x^2} = -|q|^2, \quad (2)
\]

where \(x\) and \(z\) stand for the transverse and longitudinal coordinates scaled to the beam width and the diffraction...
position length, $n_d$ represents the difference between the linear refractive indices, $\delta(x)$ is a periodic function for the modulation of the linear refractive index. In our model, we study the cases of $\delta(x) = H[\cos(\pi x/d)]$ and $\delta(x) = H[\sin(\pi x/d)]$, which present the symmetric and antisymmetric lattices, respectively. Here $H[\xi]$ is the Heaviside step function defined as

$$H[\xi] = \begin{cases} \text{1, } & \xi \geq 0, \\ 0, & \xi < 0. \end{cases}$$

$d$ is the layer width and the lattice period is $2d$. For simplicity and without losing of generality, we use integer value of $d$ in this paper, which possibly leads to the incomplete layers at the two boundaries. Anyway, the forth kind of Eq. (1) represents the periodic modulation of two different linear refractive index in the sample. This kind of modulation is different from that in Ref. [23]. When $\delta(x) = 1$, it represents the higher linear refractive index, while $\delta(x) = 0$, it represents the lower linear refractive index. If $n_d = 0$, Eq. (11) represents the NNLSE in uniform thermal media.

We assume the two boundaries are thermally conductive and the laser beam is far away from the boundaries, so the boundary conditions are $q(\pm L) = 0$ and $n(\pm L) = 0$. We search for soliton solutions of Eq. (11) numerically in the form $q(x, z) = w(x) \exp(izb)$, where $w(x)$ is a real function, $b$ is the propagation constant. The iterative method is used to get numerical solutions for different $n_d$, $d$ and $b$ as shown in Figs. 11, 12. To elucidate the linear stability, we search for the perturbed solution of Eq. (11) in the form $q = (w + u + iv) \exp(izb)$, the real part $u(x, z)$ and the imaginary part $v(x, z)$ of the perturbation can grow with a complex rate $\sigma$ upon propagation. Substituting the perturbed solution into Eq. (11), one can get the linear eigenvalue problem around stationary solution $w(x)$,

$$\begin{align*}
\sigma u &= -\frac{1}{2} \frac{d^2v}{dx^2} + bv - nv - \delta(x)n_d u, \\
\sigma v &= \frac{1}{2} \frac{d^2u}{dx^2} - bu + nu + \delta(x)n_d u + w \Delta n,
\end{align*}$$

where $\Delta n = -2 \int_{-L}^{L} G(x, x')w(x')u(x')dx'$ is the refractive index perturbation, the response function $G(x, x') = (x + L)(x' - L)/(2L)$ for $x \leq x'$ and $G(x, x') = (x' + L)(x - L)/(2L)$ for $x \geq x'$. $\sigma_r$ (real part of $\sigma$) represents the instability growth rate.

**III. LATTICE SOLITONS**

First, we study the solitons in the case of $\delta(x) = H[\cos(\pi x/d)]$ (symmetric lattices) and $d = 2$ as shown in Fig. 1. When $n_d > 0$ the center layer is with higher refractive index, and the number of higher-index layers is odd. In Figs. 1(a) and 1(c), there exist odd solitons in the center of the sample because intensity peaks always reside in the higher-index of the layers. Their profiles are similar to those in other types of optical lattices, while the distribution of nonlinear refractive indices are different [31, 32]. When $n_d < 0$ the center layer is with lower refractive index, and the number of higher-index layers is even. In Figs. 1(b) and 1(d), one can find even solitons, including symmetric ones [Fig. 1(b)] and antisymmetric ones [Fig. 1(d)], at the center of the sample. As we know that the beam width decreases as the propagation constant increases. In optical lattices, the smaller the $b$ is, the more layers the soliton profile occupies. That is the number of the intensity peaks decreases as the propagation constant increases [Figs. 1(b) and 1(d)].

The linear stability analysis shows that fundamental and tripole lattice solitons in Figs. 1(a) and 1(c) are stable in their existence domain for $n_d > 0$. Energy flows $U = \int_{-\infty}^{\infty} |q|^2 dx$ of fundamental and tripole solitons are monotonically growing functions of $b$ as shown in Fig. 1(c). However, when $n_d < 0$, the even and dipole solitons, as shown in Figs. 1(b) and 1(d), are unstable for $n_d = -0.5$ [Fig. 1(f)]. We find that the antisymmetric dipole solitons as shown in Fig. 1(d) can be stable for $n_d < -0.6$ due to the confinement of the deep lattice, whereas the even solitons as shown in Fig. 1(b) are unstable for any value of $n_d$.

Then, we discuss the solitons formed in the antisymmetric lattices. The modulation of linear refractive index is governed by $\delta(x) = H[\sin(\pi x/d)]$ and lattice structure is shown in Fig. 2 with $d = 2$. If there is only one period in the sample, i.e. $d = L$, the antisymmetric lattice
will be the same as the model in [36]. Here solitons are always asymmetric because of the antisymmetry of lattices. Figures 2(a) and 2(b) show the fundamental and even asymmetric solitons, respectively. One can see that the main intensity peaks reside in the higher-index layers, and the larger the propagation constant the higher the degree of asymmetry. Figures 2(c) and 2(d) show the high-order asymmetric solitons at the same propagation constant. One can see that the degree of the restriction of the lattices is proportional to $|n_d|$, and the asymmetry of the solitons is almost independent of $n_d$. The fundamental and the high-order asymmetric solitons shown in Figs. 2(a), 2(c) and 2(d) are stable in their whole existence domain, while the even solitons shown in Fig. 2(b) are unstable.

As we know in [38], a soliton launched away from the sample center will oscillate periodically because of the restoring boundary force. In our model, we simulate the beam propagations with finite difference method in the symmetric lattices. Solitons launched away from the sample center undergo oscillation, as shown in Figs. 3(a) and 3(b), when the boundary force is stronger than the restoring force exerted by two boundaries due to the shift of the soliton center. Figure 3(c) shows that late irregularly and destroy themselves for small $n_d$ and $d$. However, stationary high-order solitons can form for large $n_d$ and $d$ when they are launched away from the sample center.

Based on the above analysis, when the constraint force of the lattices is large, solitons can form asymmetrically away from the sample center, that is shifted solitons. Figures 4(a) and 4(b) show the profiles of fundamental, dipole, tripole, and quadrupole shifted solitons at the same conditions. Their mass center, defined as $x_m = \int_{-\infty}^{\infty} x |q|^2 dx / \int_{-\infty}^{\infty} |q|^2 dx$, are 3.1, 6.8, 3.9, and 7, respectively. This kind of solitons exists when the constraint force of the higher-index waveguide is stronger than the restoring force exerted by two boundaries due to the shift of the soliton center. Figure 4(c) shows that
their energy flows are monotonically growing function of $b$ and there do not exist a cutoff value of $b$ for any shifted solitons. It is found that there exist a minimal power and a cutoff value of $b$ for shifted solitons in the model in [32], because the modulation in their model is induced by the nonlinearity, which will become weak for small propagation constant and low soliton power. However, in our model, the modulation of linear refractive index does not depend on the constant propagation, so there does not exist a cutoff value of $b$ for shifted solitons. For fundamental, dipole, and tripole shifted solitons shown in Figs. 4(a) and 4(b), they are all stable in their whole existence domain, while for the quadrupole shifted solitons, there exists an instability region shown in Fig. 4(d). We can see that the instability region is dependent on $n_d$, and the instability region decreases as increasing of $|n_d|$. As $|n_d| > 8$, almost all solitons are stable.

Another type of shifted solitons formed away from the sample center is shown in Fig. 5. They are separated by the central layers and located in any higher-index layer, including symmetrical [the red lines in Figs. 5(a) and 5(b)], antisymmetrical [the black lines in Figs. 5(a) and 5(b)], and asymmetrical [Fig. 5(c)] ones, because of the restraint force of lattices and the existence of boundaries. Almost all intensity peaks reside in the higher-index layers for this type of solitons, and they do not affect other peaks during the propagation due to the restriction of the lattices. For this reason, the solitons shown in Figs. 5(a)-5(c) are stable including the quadrupole shifted solitons, which is different form that in Fig. 4(b). Their propagations are shown in Figs. 5(d)-5(f). One can see from Fig. 5(f) that the two intensity peaks are stable and have no impact on each other, even if they are far away from the sample center.

Finally, we discuss the influence of the index-difference on solitons with $d$ and $b$ fixed in the antisymmetrical lattices. When $n_d$ is small, as shown in Fig. 6(a), there are three main intensity peaks with a significant part of the field residing in the lower-index layers and exist several sub-peaks. That is because the higher-index layers present a little restriction at small $n_d$. As $n_d$ increases [Fig. 6(b)], the part of the field residing in the lower-index layers decreases, and the middle intensity peak decreases gradually. When $n_d = 10$, almost all soliton power resides in the higher-index layer and the middle peak disappears as shown in Fig. 6(c). It shows that the restriction of the higher-index layers increases as $n_d$ increases. Figure 6(d) shows the relationship between $n_d$ and beam width and the energy flow. One can see that the energy flow is proportional to $n_d$, while the beam width changes non-monotonically with increasing $n_d$. When $n_d \leq 1.5$, the beam width decreases monotonically as $n_d$ increases as shown in Fig. 6(d). The main reason is that the widths of two lateral peaks decrease as $n_d$ increases, while the width of the middle peak is unchanged [Fig. 6(a)]. When $n_d \geq 1.5$, the beam width increases monotonically as $n_d$ increases [Fig. 6(d)].
cause the widths of two lateral peaks are unchanged, while the width of the middle peak decreases gradually, as asymmetric [Figs. 2 and 6]. For comparison, soliton profiles in symmetric lattices are symmetric when the constraint force is weaker than the restoring force [Figs. 11], whereas they can be either symmetric [Fig. 5] or asymmetric [Figs. 4 and 6] when the constraint force is stronger than the restoring boundary force for large $n_d$ [Fig. 3]. For solitons in antisymmetric lattices, however, they are always asymmetric [Figs. 2 and 6].

IV. CONCLUSION

To conclude, we have studied the propagation of solitons in thermal media with periodic distribution of linear refractive index including symmetric and antisymmetric lattices. In this kind of optical lattice, we found fundamental, odd, even, and shifted solitons. The shifted solitons are found do not exist cutoff value of propagation constant and power, which is different from that in thermal lattices with periodic nonlinearity. We also studied the solitons launched away from their stationary position. Their propagation is related to the difference of the refractive indices.

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