Two-Loop $Z_4$ Dirac Neutrino Masses and Mixing, with Self-Interacting Dark Matter

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Abstract

Choosing how gauge $U(1)_\chi$ breaks in the context of $SO(10) \rightarrow SU(5) \times U(1)_\chi$, $Z_4$ lepton number may be obtained which maintains neutrinos as Dirac fermions. Choosing $\Delta(27)$ as the family symmetry of leptons, tree-level Dirac neutrino masses may be forbidden. Choosing a specific set of self-interacting dark-matter particles, Dirac neutrino masses and mixing may then be generated in two loops. This framework allows the realization of cobimaximal neutrino mixing, i.e. $\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, $\delta_{CP} = \pm \pi/2$, as well as the desirable feature that the light scalar mediator of dark-matter interactions decays only to neutrinos, thereby not disrupting the cosmic microwave background (CMB).
1 Introduction

The fundamental issue of whether neutrinos are Majorana remains open, without any incontrovertible experimental evidence that they are so, i.e. no definitive measurement of a nonzero neutrinoless double beta decay. If they are Dirac, for each left-handed $\nu_L$ observed in weak interactions, there must be a corresponding right-handed $\nu_R$, which has no interactions within the standard model (SM) of quarks and leptons. To justify its existence, the canonical choice is to extend the SM gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ to the left-right symmetry $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)/2}$. In that case, the $SU(2)_R$ doublet $(\nu, e)_R$ is required, and the charged $W^\pm_R$ gauge boson is predicted along with a neutral $Z'$ gauge boson.

A more recent choice is to consider $U(1)_\chi$ which comes from $SO(10) \rightarrow SU(5) \times U(1)_\chi$, with $SU(5)$ breaking to the SM at the same grand unified scale. Assuming that $U(1)_\chi$ survives to an intermediate scale, the corresponding $Z_\chi$ gauge boson has prescribed couplings to the SM quarks and leptons, which allow current experimental data to put a lower bound of about 4.1 TeV [1, 2] on its mass. In this scenario, $\nu_R$ is a singlet and it exists for the cancellation of gauge anomalies involving $U(1)_\chi$. Using this new framework, new insights into dark matter [3, 4] and Dirac neutrino masses [5, 6] have emerged.

To break $U(1)_\chi$, a singlet scalar is the simplest choice, but it must not couple to $\nu_R^\dagger \nu_R$, or else a Majorana mass for $\nu_R$ would be generated. This simple idea was first discussed [7] in 2013 in the general case of singlet fermions charged under a gauge $U(1)_X$. If a scalar with three units of $X$ charge is used to break it, these fermions with one unit of $X$ charge would not be able ever to acquire Majorana masses. Hence a residual global $U(1)$ symmetry remains. This idea is easily applicable to lepton number [8] as well.

In the SM, the Yukawa couplings linking $\nu_L$ to $\nu_R$ through the SM Higgs boson must be
very small if neutrinos are Dirac. To avoid these tiny tree-level couplings, some additional symmetry is often assumed which forbids them. However, since neutrinos are known to have mass, this symmetry cannot be exact. Indeed, Dirac neutrino masses may be generated radiatively as this symmetry is broken softly by dimension-three terms. For a generic discussion, see Ref. [9], which is fashioned after that for Majorana neutrinos [10]. In some applications [11, 12, 13], the particles in the loop belong to the dark sector. This is called the scotogenic mechanism, from the Greek 'scotos' meaning darkness, the original one-loop example [14] of which was applied to Majorana neutrinos.

Instead of the \textit{ad hoc} extra symmetry which forbids the tree-level couplings, unconventional assignments of the gauge charges of $\nu_R$ may be used [8, 15, 16, 17, 18] instead. However, a much more attractive idea is to use a non-Abelian discrete family symmetry, which is softly broken in the dark sector. In particular, $\Delta(27)$ [19, 20, 21, 22, 23] has been shown to be useful in achieving the goal of having scotogenic Dirac neutrino masses with a mixing pattern [24, 25, 26] called cobimaximal [27, 28, 29, 30, 31, 32], i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm \pi/2$, which is consistent with present neutrino oscillation data [33] for $\delta_{CP} = -\pi/2$.

2 Outline of Model

Following Refs. [6, 23], the interplay between $U(1)_\chi$ and $\Delta(27)$ is used for restricting the interaction terms among the various fermions and scalars. The irreducible representations of $\Delta(27)$ and their character table are given in Ref. [19]. Note that if a set of 3 complex fields transforms as the 3 representation of $\Delta(27)$, then its conjugate transforms as $3^*$, which is distinct from 3. The basic multiplication rules are

$$3 \times 3 = 3^* + 3^* + 3^*, \quad 3 \times 3^* = \sum_{i=1}^{9} 1_i.$$

(1)

The particles of this model are shown in Table 1.
In the notation above, all fermion fields are left-handed. The usual right-handed fields are denoted by their charge conjugates. The SM particles transform under $U(1)_\chi$ according to their $SO(10)$ origin, as well as the particles of the dark sector ($\sigma, N, N^c, S, E_{1,2}$). The input family symmetry is $\Delta(27)$. The gauge $U(1)_\chi$ is broken by $\zeta_4$. The allowed terms $\zeta_2^2\zeta_4^*$ and $\sigma^2\zeta_4^*$ imply that a residual $Z_4^L$ symmetry \cite{34,35,36,37,38} remains for lepton number as shown in Table 1. The dark symmetry is simply $R_\chi = (-1)^{Q_x+2j}$ as pointed out recently \cite{3}. Note that the dark scalar $\sigma$ is also a lepton \cite{39} because it has the same $Z_4^L$ charge as $\nu^c$. The complete Lagrangian is invariant under gauge $U(1)_\chi$ in all its terms, as well as $\Delta(27)$ in all the dimension-four terms. Whereas the breaking of gauge $U(1)_\chi$ must only be spontaneous, through the vacuum expectation values of $\zeta_4$ and $\Phi_{1,2}$, the breaking of $\Delta(27)$ is both spontaneous, through the vacuum expectation values of $\Phi_{1,2}$, and explicit, through the soft dimension-three terms $\sigma_j\sigma_k\zeta_4^*$, as shown below.

From Table 1, the Yukawa term $ee^c\phi_1^0$ is allowed, but $\Delta(27)$ forbids $\nu\nu^c\phi_2^0$, hence neu-
trinos do not have tree-level Dirac masses. Moreover, the usual dimension-five operator for Majorana neutrino mass, i.e. $\nu\nu\phi_2^0\phi_2^0$, is forbidden as well as the usual singlet Majorana mass term $\nu^c\nu^c$. Without $U(1)_\chi$, $\nu^c\nu^c$ would be a soft term breaking $\Delta(27)$ and would then have been allowed by itself. To obtain Dirac neutrino masses, the fermion doublets $E_{1,2}$ and singlets $S, N, N^c$ with even $Q_\chi$ as well as the scalar singlet $\sigma$ with odd $Q_\chi$ are added. They belong to the dark sector because SM fermions have odd $Q_\chi$ and the SM Higgs doublet has even $Q_\chi$, as explained in Ref. [3]. With the above particle content, Dirac neutrino masses cannot be generated in one loop, but are possible in two loops with the soft breaking trilinear couplings of $\sigma_j\sigma_k\zeta_2^*$, as shown in Fig. 1.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Two-loop diagram for scotogenic $U(1)_\chi$ Dirac neutrino masses and mixing.}
\end{figure}

In the above, only $\phi_2^0$ is shown, but it can be replaced by $\phi_1^0$. Since $\phi_2^0$ is a 3 under $\Delta(27)$, its components are denoted as $\phi_{2,i}^0$ with $i = 1, 2, 3$. The dark scalars and fermions have allowed interactions with $\nu, \nu^c$ under $U(1)_\chi$. The dimension-four terms, i.e. $\nu\sigma E_2^0$, $\nu^c\sigma N$, $E_2^0\bar{\phi}_2^0 S$, $SN\zeta_2$, respect both $U(1)_\chi$ and $\Delta(27)$. The dimension-three scalar trilinear couplings $\sigma_j\sigma_k\zeta_2^*$ respect $U(1)_\chi$ but not $\Delta(27)$.

Consider now the spontaneous breaking of $U(1)_\chi$. First, because $\nu^c \sim 3^*$ under $\Delta(27)$ and has $Q_\chi = -5$, it is protected from acquiring a tree-level Majorana mass. Choosing $\zeta_4$ (instead of $\zeta_2$) to have a nonzero vacuum expectation value then makes the residual lepton symmetry $Z_4^L$, through the allowed couplings $\zeta_2^2\zeta_4^*$ and $\sigma^2\zeta_2^*$, with their connections to leptons from $\nu^c\sigma N$ and $\nu\sigma E_2^0$. 

5
3 Neutrino Mixing

Using the decomposition $3 \times 3^*$ and $\langle \phi^0_{2,i} \rangle = v_i$, with $l_1, l_7, l_4$ as defined in Ref. [19], instead of the usual $l_1, l_2, l_3$ of the original $A_4$ model [40] of neutrino mixing, the charged-lepton mass matrix is given by

\[
M_l = \begin{pmatrix}
  f_e v_1^* & f_\mu v_3^* & f_\tau v_2^* \\
  f_e v_2^* & f_\mu v_1^* & f_\tau v_3^* \\
  f_e v_3^* & f_\mu v_2^* & f_\tau v_1^*
\end{pmatrix} = \begin{pmatrix}
  m_e & 0 & 0 \\
  0 & m_\mu & 0 \\
  0 & 0 & m_\tau
\end{pmatrix},
\]

(2)

where $v_2 = v_3 = 0$ has been assumed for the spontaneous breaking of $\phi^0_2$ (or $\bar{\phi}^0_1$). This $M_l$ is diagonal and different from that of Ref. [40]. It allows also three independent masses for the charged leptons, and the emergence of lepton flavor triality [41, 42] in the Yukawa interactions of the three charged leptons with the three Higgs doublets.

The $\nu \sigma E^0_2$ couplings obey $\Delta(27)$ according to

\[
(3 \times 3) \times 3 = (3^* + 3^* + 3^*) \times 3 = 1 + 1 + 1.
\]

The three $\Delta(27)$ invariants are

\[
111 + 222 + 333, \quad 123 + 231 + 312, \quad 132 + 321 + 213.
\]

(4)

However, since only $\phi^0_{2,1}$ has a nonzero vacuum expectation value, only $E^0_{2,1}$ matters in the above. Hence only the 111, 231, and 321 couplings contribute to the radiative neutrino mass matrix of Fig. 1. Because the charged-lepton mass matrix is diagonal, all three couplings may be chosen real by absorbing their phases. One magnitude may also be arbitrarily chosen. The coupling matrix linking $\nu_i$ to $\sigma_j$ is then

\[
\begin{pmatrix}
  a & 0 & 0 \\
  0 & 0 & c \\
  0 & s & 0
\end{pmatrix},
\]

(5)

where $c^2 + s^2 = 1$. The soft breaking of $\Delta(27)$ occurs at the $\sigma_j \sigma_k \zeta^*_2$ trilinear vertex. Choosing the residual $1 - 1$ and $2 - 3$ exchange symmetry with complex conjugation [25], this $3 \times 3$
coupling matrix is of the form
\[
\begin{pmatrix}
  b & e & e^* \\
  e & d & f \\
  e^* & f & d^*
\end{pmatrix},
\]
where \(b, f\) are real. The resulting Dirac neutrino mass matrix is proportional to their product
\[
\mathcal{M} = \begin{pmatrix}
  a & 0 & 0 \\
  0 & 0 & c \\
  0 & s & 0
\end{pmatrix} \begin{pmatrix}
  b & e & e^* \\
  e & d & f \\
  e^* & f & d^*
\end{pmatrix} = \begin{pmatrix}
  ab & ae & ae^* \\
  ce & cf & cd^* \\
  sc & se & sf
\end{pmatrix}.
\]
This is diagonalized on the left by the unitary neutrino mixing matrix \(U_{\nu}\), which may be obtained by considering the Hermitian matrix
\[
\mathcal{M} \mathcal{M}^\dagger = \begin{pmatrix}
  a^2(b^2 + 2|e|^2) & ac(be + fe + de^*) & as(be^* + fe^* + d^*) \\
  ac(be^* + fe^* + d^*) & c^2(|e|^2 + f^2 + |d|^2) & sc(e^2 + 2fd^*) \\
  as(be + fe + de^*) & sc(e^2 + 2fd) & s^2(|e|^2 + |d|^2 + f^2)
\end{pmatrix}.
\]
Rewriting
\[
\mathcal{M} \mathcal{M}^\dagger = \begin{pmatrix}
  A & \sqrt{2c}|D|e^{i\theta_D} & \sqrt{2s}|D|e^{-i\theta_D} \\
  \sqrt{2c}|D|e^{-i\theta_D} & 2c^2B & -2sc|E|e^{2i\theta_E} \\
  \sqrt{2s}|D|e^{i\theta_D} & -2sc|E|e^{-2i\theta_E} & 2s^2B
\end{pmatrix}
\]
\[
= \begin{pmatrix}
  1 & 0 & 0 \\
  0 & e^{i\theta_E} & 0 \\
  0 & 0 & e^{-i\theta_E}
\end{pmatrix} \begin{pmatrix}
  A & \sqrt{2c}|D|e^{i\theta} & \sqrt{2s}|D|e^{-i\theta} \\
  \sqrt{2c}|D|e^{-i\theta} & 2c^2B & -2sc|E| \\
  \sqrt{2s}|D|e^{i\theta} & -2sc|E| & 2s^2B
\end{pmatrix} \begin{pmatrix}
  1 & 0 & 0 \\
  0 & e^{-i\theta_E} & 0 \\
  0 & 0 & e^{i\theta_E}
\end{pmatrix},
\]
and removing the diagonal phases on both sides, where \(\theta = \theta_D + \theta_E\), the mass-squared matrix becomes
\[
\mathcal{M} \mathcal{M}^\dagger = \begin{pmatrix}
  A & \sqrt{2c}D & \sqrt{2s}D^* \\
  \sqrt{2c}D^* & 2c^2B & -2scE \\
  \sqrt{2s}D & -2scE & 2s^2B
\end{pmatrix},
\]
where
\[
A = a^2(b^2 + 2|e|^2), \quad B = (1/2)(|d|^2 + |e|^2 + f^2),
\]
\[
D = (a/\sqrt{2})|be + fe + de^*|e^{i\theta}, \quad E = (1/2)|e^2 + 2fd|.
\]
If \(c = s = 1/\sqrt{2}\), then \(\mathcal{M} \mathcal{M}^\dagger\) is diagonalized by a cobimaximal \(U_{\nu}\), as shown below.
Multiplying Eq. (10) with \( c = s = 1/\sqrt{2} \) on the left by

\[
U_{TBM}^\dagger = \begin{pmatrix}
\sqrt{2}/3 & -1/\sqrt{6} & -1/\sqrt{6} \\
1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\
0 & i/\sqrt{2} & -i/\sqrt{2}
\end{pmatrix}
\]

and on the right by \( U_{TBM} \), a real matrix is obtained, with

\[
M_{11}^2 = \frac{1}{3}(2A - 4D_R + B - E), \quad M_{22}^2 = \frac{1}{3}(A + 4D_R + 2B - 2E),
\]

\[
M_{12}^2 = M_{21}^2 = \frac{\sqrt{2}}{3}(A + D_R - B + E),
\]

\[
M_{33}^2 = B + E, \quad M_{13}^2 = M_{31}^2 = \frac{2D_I}{\sqrt{3}}, \quad M_{23}^2 = M_{32}^2 = \sqrt{2}D_I/\sqrt{3}.
\]

Since a real matrix is diagonalized by an orthogonal matrix \( O \), the product

\[
U_{\nu} = U_{TBM}O
\]

is easily shown to have the property of \(|U_{\mu i}| = |U_{\tau i}|\) for \( i = 1, 2, 3 \), which is the necessary and sufficient condition for cobimaximal mixing, i.e. \( \theta_{13} \neq 0, \theta_{23} = \pi/4, \) and \( \delta_{CP} = \pm \pi/2 \).

Using the fact that \( U_{TBM} \) is a good approximation of the experimental data, the orthogonal matrix may be written as

\[
O = \begin{pmatrix}
1 & s_1 & s_2 \\
-s_1 & 1 & s_3 \\
-s_2 & -s_3 & 1
\end{pmatrix}
\]

where

\[
s_1 = \frac{M_{12}^2}{M_{22}^2 - M_{11}^2} = \frac{-\sqrt{2}F}{3D_R - F},
\]

\[
s_2 = \frac{M_{13}^2}{M_{33}^2 - M_{11}^2} = \frac{2D_I}{\sqrt{3}(B + E - A + D_R + F)},
\]

\[
s_3 = \frac{M_{23}^2}{M_{33}^2 - M_{22}^2} = \frac{\sqrt{2}D_I}{\sqrt{3}(B + E - A - 2D_R + 2F)} \approx \frac{s_2}{\sqrt{2}},
\]

with \( F = (A + D_R - B + E)/3 \). Since \( M_{22}^2 - M_{11}^2 \approx 7.37 \times 10^{-5} \text{ eV}^2 \) and \( M_{33}^2 - M_{11}^2 \approx 2.56 \times 10^{-3} \).
eV², the above implies |F| << D_R << |B + E - A|. Now
\[
U_{\nu} = \begin{pmatrix}
\frac{\sqrt{2/3}(1 - s_1/\sqrt{2})}{\sqrt{1/3}(1 + \sqrt{2}s_1)} & \frac{\sqrt{1/3}(1 + \sqrt{2}s_1)}{\sqrt{1/3}(1 - s_1/\sqrt{2}) + is_2/\sqrt{2}} & \frac{\sqrt{1/3}(\sqrt{2}s_2 + s_3)}{is_3/\sqrt{2}} - i/\sqrt{2}

\frac{-\sqrt{1/6}(1 + \sqrt{2}s_1) + is_2/\sqrt{2}}{\sqrt{1/3}(1 - s_1/\sqrt{2}) + is_2/\sqrt{2}} & \frac{\sqrt{1/3}(1 - s_1/\sqrt{2}) - is_3/\sqrt{2}}{\sqrt{1/3}(1 - s_1/\sqrt{2}) + is_2/\sqrt{2}} & i/\sqrt{2}

\frac{-\sqrt{1/6}(1 + \sqrt{2}s_1) - is_2/\sqrt{2}}{\sqrt{1/3}(1 - s_1/\sqrt{2}) - is_3/\sqrt{2}} & \frac{\sqrt{1/3}(1 - s_1/\sqrt{2}) - is_3/\sqrt{2}}{-\sqrt{1/6}(1 + \sqrt{2}s_1) + is_2/\sqrt{2}} & -i/\sqrt{2}
\end{pmatrix}
\]  
(22)

Note first that \(U_{\tau_i} = U_{\mu i}^*\). Multiplying the third row by \(-1\) and the third column by \(i\), the PDG convention of \(U_{\nu}\) is obtained, with \(\delta_{CP} = -\pi/2\) for
\[
s_{13} = \frac{\sqrt{2}s_2}{\sqrt{3}} + \frac{s_3}{\sqrt{3}} = \frac{\sqrt{2}D_I}{\sqrt{3}(B + E - A)} > 0. \quad (23)
\]

At the same time, \(\theta_{23} = \pi/4\), and
\[
\tan\theta_{12} = \frac{1 + \sqrt{2}s_1}{\sqrt{2}(1 - s_1/\sqrt{2})}. \quad (24)
\]

Using \(\sin^2\theta_{12} = 0.297\), the above implies \(s_1 = -0.039\). Note that the deviations from \(\tan^2\theta_{12} = 1/2\) due to \(s_2\) and \(s_3\) are quadratic. For \(s_{13}^2 = 0.0215\), their contributions shift \(s_1\) to \(-0.041\).

To see how deviations from cobimaximal mixing occur, let \(\sqrt{2}s - 1 = \epsilon\) and \(\sqrt{2}c - 1 = -\epsilon\), then
\[
\Delta(MM^\dagger) = \begin{pmatrix}
0 & -\epsilon D & \epsilon D^* \\
-\epsilon D^* & -2\epsilon B & 0 \\
\epsilon D & 0 & 2\epsilon B
\end{pmatrix}. \quad (25)
\]

Multiplying on the left by \(U_{TB M}^\dagger\) and on the right by \(U_{TB M}\), this becomes
\[
i\epsilon \begin{pmatrix}
0 & -\sqrt{2}D_I & -(2/\sqrt{3})(B - D_R) \\
\sqrt{2}D_I & 0 & \sqrt{2/3}(2B + D_R) \\
(2/\sqrt{3})(B - D_R) & -\sqrt{2/3}(2B + D_R) & 0
\end{pmatrix}. \quad (26)
\]

The additional mixing contributions analogous to \(s_{1,2,3}\) are thus
\[
i\epsilon_1 = \frac{-i\epsilon\sqrt{2D_I}}{3D_R - F}, \quad (27)
i\epsilon_2 = \frac{-2i\epsilon(B - D_R)}{3(B + E - A + D_R + F)}, \quad (28)
i\epsilon_3 = \frac{\sqrt{2i\epsilon(2B + D_R)}}{3(B + E - A - 2D_R + 2F)} \simeq -\sqrt{2i\epsilon_2}. \quad (29)
\]
Numerically, $\epsilon_1$ is enhanced by $\sqrt{3}s_{13}\Delta m_{31}^2/\Delta m_{21}^2 \simeq 8.82$, but not $\epsilon_2$. Hence the rotation matrix of Eq. (18) due to $s_{1,2,3}$ is replaced with

$$U_\epsilon = \begin{pmatrix}
1 - \epsilon_1^2/2 & s_1 + i\epsilon_1 & s_2 + i\epsilon_2 \\
-s_1 + i\epsilon_1 & 1 - \epsilon_1^2/2 & s_3 + i\epsilon_3 \\
-s_2 + i\epsilon_2 & -s_3 + i\epsilon_3 & 1
\end{pmatrix}, \quad (30)$$

and $U_{\nu} = U_{TBM} U_\epsilon$ instead of Eq. (17). The various entries of $U_{\nu}$ are thus

$$U_{e3} = s_{13}, \quad U_{\mu 3} = \frac{-i}{\sqrt{2}} \left( 1 - \sqrt{3} \epsilon_3 \right), \quad (31)$$

$$U_{e2} = \frac{1}{\sqrt{3}} \left( 1 - \frac{\epsilon_1^2}{2} + \sqrt{2}s_1 \right) + i\frac{\sqrt{2}}{3}\epsilon_1, \quad (32)$$

$$U_{\mu 2} = \frac{1}{\sqrt{3}} \left( 1 - \frac{\epsilon_1^2}{2} - \frac{s_1}{\sqrt{2}} + \sqrt{\frac{3}{2}}\epsilon_3 \right) - \frac{i\epsilon_1}{\sqrt{6}}. \quad (33)$$

To obtain $\delta_{CP}$, the identity

$$J_{CP} = Im(U_{\mu 3} U_{e3}^* U_{e2} U_{\mu 2}^*) = c_{13}s_{12}c_{12}s_{23}c_{23}s_{13}c_{13} \sin \delta_{CP} \quad (34)$$

is used, where

$$s_{23} = \frac{1}{\sqrt{2}} \left( 1 - \sqrt{3} \frac{2}{\epsilon_3} \right), \quad c_{23} = \frac{1}{\sqrt{2}} \left( 1 + \sqrt{\frac{3}{2}} \epsilon_3 \right), \quad (35)$$

$$s_{12} = \frac{1}{\sqrt{3}} \left( 1 + \sqrt{2}s_1 + \frac{\epsilon_1^2}{2} \right), \quad c_{12} = \sqrt{\frac{2}{3}} \left( 1 + \frac{s_1}{\sqrt{2}} - \frac{\epsilon_1^2}{4} \right). \quad (36)$$

This implies

$$\sin \delta_{CP} = -\left( 1 - \frac{7\epsilon_1^2}{4} \right). \quad (37)$$

Using $B = A + E + D_R - 3F$, the deviations of $s_{23}^2$ and $\sin \delta_{CP}$ from $1/2$ and $-1$ are given by

$$1 - 2s_{23}^2 = \frac{4}{\sqrt{3}} \left( \frac{A + E}{2E} \right) \epsilon, \quad 1 + \sin \delta_{CP} = \frac{7}{4}(8.82)^2\epsilon^2. \quad (38)$$

As an example, for $\epsilon = 0.02$ and $A = E$, $s_{23}^2 = 0.48$ and $\sin \delta_{CP} = -0.95$. 

10
4 Dark Sector

In this two-loop model, the scalar $\sigma$ is a pure singlet. This means that it interacts with quarks not through the $Z$ boson, but rather the $Z_\chi$ gauge boson. The lightest of $\sigma_{1,2,3}$ is dark matter. Its annihilation to the light scalar mediator $\zeta_2$ is a well-known mechanism for generating the correct dark-matter relic abundance of the Universe.

At the mass of 150 GeV, the constraint on the elastic scattering cross section of $\sigma_1$ off nuclei per nucleon is about $1.5 \times 10^{-46}$ cm$^2$ from the latest XENON result \[43\]. This puts a lower limit on the mass of $Z_\chi$, i.e.

$$\sigma_0 = \frac{\mu_\sigma^2}{64\pi} \left[ Z f_P + (A - Z) f_N \right]^2 < 1.5 \times 10^{-10} \text{ pb},$$

(39)

where $\mu_\sigma$ is the reduced mass of $\sigma_1$, and

$$f_P = g_{Z_\chi}^2 f_\sigma (2 u_V + d_V)/M_{Z_\chi}^2, \quad f_N = g_{Z_\chi}^2 f_\sigma (u_V + 2 d_V)/M_{Z_\chi}^2,$$

(40)

and $Z = 54$, $A = 131$ for xenon. In $U(1)_\chi$, the vector couplings are

$$f_\sigma = -\sqrt{5}/8, \quad u_V = 0, \quad d_V = -1/\sqrt{10}.$$  

(41)

Using $\alpha_\chi = g_{Z_\chi}^2/4\pi = 0.0154$ from Ref. \[3\], the bound $M_{Z_\chi} > 24$ TeV is obtained.

Because of the $\zeta_2^* \sigma_1 \sigma_1$ interaction, $\sigma_1$ is a self-interacting dark-matter (SIDM) candidate \[44\] which has been postulated to explain the flatness of the core density profile of dwarf galaxies \[45\] and other related astrophysical phenomena. The light scalar mediator $\zeta_2$ transforms as $-1$ under $Z_L^4$ lepton symmetry and decays only to $\nu^c \nu^c$ in one loop as shown in Fig. 2, where the allowed Majorana mass term $NN\langle \zeta_4 \rangle$ has been used. It does not disrupt \[46\] the cosmic microwave background (CMB) \[47\], thus eluding the stringent constraint \[48\] due to the enhanced Sommerfeld production of $\zeta_2$ at late times if it decays to electrons and photons, as in most proposed models. This problem may also be solved if
the light mediator is stable \cite{49,50,51} or if it decays into $\nu \nu$ through a pseudo-Majoron in the singlet-triplet model of neutrino mass \cite{52}. A much more natural solution is for it to decay into $\nu^c \nu^c$ as first pointed out in the prototype model of Ref. \cite{53} and elaborated in Refs. \cite{3,5,6}. Here it is shown how it may arise in the scotogenic Dirac neutrino context using $U(1)_{\chi}$ as well as $\Delta(27)$. The generic connection of lepton parity to simple models of dark matter was first pointed out in Ref. \cite{39}. Typical mass ranges for $\sigma_1$ and $\zeta_2$ are

\begin{equation}
100 < m_\sigma < 200 \text{ GeV}, \quad 10 < m_\zeta < 100 \text{ MeV},
\end{equation}

as shown in Ref. \cite{53}, where details of relic abundance and the required elastic cross section for SIDM are explicitly given.

\section{Concluding Remarks}

A recent insight concerning lepton number symmetry is that it could be $Z_N$ with $N \neq 2$. This paper shows explicitly a model with $Z_4^L$ lepton symmetry in the context of $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{\chi}$, where $U(1)_{\chi}$ comes from $SO(10) \rightarrow SU(5) \times U(1)_{\chi}$, and the non-Abelian discrete symmetry $\Delta(27)$ as its family symmetry. With the particle content of Table 1, where $U(1)_{\chi}$ is spontaneously broken by $\zeta_4$ and $\Delta(27)$ explicitly broken by the soft trilinear $\sigma_j \sigma_k \zeta_2^*$ scalar vertex, Dirac neutrino masses are radiatively generated in two loops through the dark sector, which consists of particles odd under $R_{\chi} = (-1)^{Q_x+2j}$. A pattern of
neutrino mixing is obtained which fits the cobimaximal hypothesis, i.e. \( \theta_{13} \neq 0, \theta_{23} = \pi/4, \delta_{\text{CP}} = \pm \pi/2 \), with possible deviations shown in Eq. (38). The lightest of the scalar singlets \( \sigma_{1,2,3} \) is self-interacting dark matter, with \( \zeta_2 \) as its light scalar mediator which decays only to two neutrinos.

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References

[1] ATLAS Collaboration, M. Aaboud et al., JHEP 1710, 182 (2017).
[2] CMS Collaboration, A. M. Sirunyan, A. Tumasyan et al., JHEP 1806, 120 (2018).
[3] E. Ma, Phys. Rev. D98, 091701(R) (2018).
[4] E. Ma, LHEP 2.1, 103 (2019); arXiv:1810.06506 [hep-ph].
[5] E. Ma, LHEP 2.1, 109 (2019); arXiv:1811.09645 [hep-ph].
[6] E. Ma, Phys. Lett. B793, 411 (2019).
[7] E. Ma, I. Picek, and B. Radovcic, Phys. Lett. B726, 744 (2013).
[8] E. Ma and R. Srivastava, Phys. Lett. B741, 217 (2015).
[9] E. Ma and O. Popov, Phys. Lett. B764, 142 (2017).
[10] E. Ma, Phys. Rev. Lett. 81, 1171 (1998).
[11] P.-H. Gu and U. Sarkar, Phys. Rev. D77, 105031 (2008).
[12] Y. Farzan and E. Ma, Phys. Rev. D86, 033007 (2012).
[13] C. Bonilla, E. Ma, E. Peinado, and J. W. F. Valle, Phys. Lett. B762, 214 (2016).
[14] E. Ma, Phys. Rev. **D73**, 077301 (2006).

[15] C.-Y. Yao and G.-J. Ding, Phys. Rev. **D97**, 095042 (2018).

[16] C. Bonilla, S. Centelles-Chulia, R. Cepedello, E. Peinado, and R. Srivastava, [arXiv:1812.01599](https://arxiv.org/abs/1812.01599) [hep-ph].

[17] J. Calle, D. Restrepo, C. E. Yaguna, and O. Zapata, Phys. Rev. **D99**, 075008 (2019).

[18] A. Dasgupta, S. K. Kang, and O. Popov, [arXiv:1903.12558](https://arxiv.org/abs/1903.12558) [hep-ph].

[19] E. Ma, Mod. Phys. Lett. **A21**, 1917 (2006).

[20] I. de Medeiros Varzielas, S. F. King, and G. G. Ross, Phys. Lett. **B648**, 201 (2007).

[21] E. Ma, Phys. Lett. **B723**, 161 (2013).

[22] A. Aranda, C. Bonilla, S. Morisi, E. Peinado, and J. W. F. Valle, Phys. Rev. **D89**, 033001 (2014).

[23] E. Ma, [arXiv:1905.01535](https://arxiv.org/abs/1905.01535) [hep-ph].

[24] K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. **B552**, 207 (2003).

[25] W. Grimus and L. Lavoura, Phys. Lett. **B579**, 113 (2004).

[26] R. N. Mohapatra and C. C. Nishi, Phys. Rev. **D86**, 073007 (2012).

[27] E. Ma, Phys. Rev. **D92**, 051301(R) (2015).

[28] X.-G. He, Chin. J. Phys. **53**, 100101 (2015).

[29] E. Ma, Phys. Lett. **B752**, 198 (2016).

[30] E. Ma, Phys. Lett. **B755**, 348 (2016).

[31] P. M. Ferreira, W. Grimus, D. Jurciukonis, and L. Lavoura, JHEP **1607**, 010 (2016).

[32] W. Grimus and L. Lavoura, Phys. Lett. **B774**, 325 (2017).

[33] T2K Collaboration, K. Abe et al., Phys. Rev. Lett. **121**, 171802 (2018).

[34] J. Heeck and W. Rodejohann, Europhys. Lett. **103**, 32001 (2013).
[35] J. Heeck, Phys. Rev. D88, 076004 (2013).

[36] S. Centelles-Chulia, E. Ma, R. Srivastava, and J. W. F. Valle, Phys. Lett. B767, 209 (2017).

[37] S. Centelles-Chulia, R. Srivastava, and J. W. F. Valle, Phys. Lett. B761, 431 (2016).

[38] M. Hirsch, R. Srivastava, and J. W. F. Valle, Phys. Lett. B781, 302 (2018).

[39] E. Ma, Phys. Rev. Lett. 115, 011801 (2015).

[40] E. Ma and G. Rajasekaran, Phys. Rev. D64, 113012 (2001).

[41] E. Ma, Phys. Rev. D82, 037301 (2010).

[42] Q.-H. Cao, A. Damanik, E. Ma, and D. Wegman, Phys. Rev. D83, 093012 (2011).

[43] E. Aprile et al. (XENON Collaboration) Phys. Rev. Lett. 121, 111302 (2018).

[44] A. Kamada, M. Kaplinghat, A. B. Pace, and H.-B. Yu, Phys. Rev. Lett. 119, 111102 (2017).

[45] F. Donato et al., Mon. Not. Roy. Astron. Soc. 397, 1169 (2009).

[46] S. Galli, F. Iocco, G. Bertone, and A. melchiorri, Phys. Rev. D80, 023505 (2009).

[47] P. A. R. Ade et al., PLANCK Collaboration, Astron. Astrophys. 594, A13 (2016).

[48] T. Bringmann, F. Kahlhoefer, K. Schmidt-Hoberg, and P. Walia, Phys. Rev. Lett. 118, 141802 (2017).

[49] E. Ma, Phys. Lett. B772, 442 (2017).

[50] E. Ma, LHEP 1.2, 1 (2018).

[51] M. Duerr, K. Schmidt-Hoberg, and S. Wild, JCAP 1809, 033 (2018).

[52] E. Ma and M. Maniatis, JHEP 1707, 140 (2017).

[53] E. Ma, Mod. Phys. Lett. A33, 1850226 (2018).