Spherically-symmetric gravitational fields in the metric-affine gauge theory of gravitation

A. V. Minkevich$^{1,2}$, Yu. G. Vasilevski$^1$

$^1$Department of Theoretical Physics, Belarussian State University, Minsk, Belarus.
$^2$Department of Physics and Computer Methods, University of Warmia and Mazury in Olsztyn, Poland.
E-mail: minkav@bsu.by; awm@matman.uwm.edu.pl

Abstract

Geometric structure of spherically-symmetric space-time in metric-affine gauge theory of gravity is studied. Restrictions on curvature tensor and Bianchi identities are obtained. By using certain simple gravitational Lagrangian the solution of gravitational equations for vacuum spherically-symmetric gravitational field is obtained.

As it is known, the application of gauge approach to gravitational interaction leads to generalization of Einsteinian theory of gravitation. At present there are different gauge theories of gravitation in dependence on using gauge group corresponding to gravitational interaction. The metric-affine gauge theory of gravitation (MAGT) is one of the most general gauge theories of gravity and it is based on the group of affine transformations $\text{A}(4,\mathbb{R})$ as gauge group $[1]$. In MAGT space-time continuum possesses curvature, torsion and nonmetricity, and as sources of gravitational field are energy-momentum tensor and so-called hypermomentum which is generalization of spin-momentum tensor of Poincare gauge theory of gravitation (PGT).

The geometric structure of space-time in MAGT is determined by three tensors: metrics $g_{\mu\nu}$, torsion $S^\lambda_{\mu\nu}$ and nonmetricity $Q^\lambda_{\mu\nu}$. By using the system of spherical coordinates $^{1}\mu, \nu, ...$ are holonomic indices; $i, k, ...$ are anholonomic (tetrad) indices. Numerical tetrad indices are denoted by means of a sign $^\wedge$ over them.
we find anholonomic connection:

$$\lambda_{\mu\nu}$$

where holonomic connection $\Gamma_{\mu\nu}^\lambda$ and $Q_{\mu\nu}$ in spherically-symmetric case was studied in Ref. [3]. The torsion is determined by 8 functions $S_i = S_i(r, t)$ $(i = 1, 2, \ldots, 8)$ and nonmetricity — by 12 functions $Q_k = Q_k(r, t)$ $(k = 0, 1, \ldots, 11)$. Namely nonvanishing components of tensors $S_{\mu\nu}$ and $Q_{\mu\nu}$ are:

$$S_{001} = S_1, \quad S_{212} = S_2, \quad S_{101} = S_3, \quad S_{202} = S_4, \quad S_{313} = S_2 \sin^2 \theta,$$

$$S_{303} = S_4 \sin^2 \theta, \quad S_{032} = S_5 \sin \theta, \quad S_{132} = S_6 \sin \theta,$$

$$S_{302} = -S_{203} = S_7 \sin \theta, \quad S_{312} = -S_{213} = S_8 \sin \theta,$$

$$Q_{000} = Q_0, \quad Q_{001} = Q_1, \quad Q_{010} = Q_2, \quad Q_{011} = Q_3, \quad Q_{110} = Q_4,$$

$$Q_{111} = Q_5, \quad Q_{022} = Q_6, \quad Q_{122} = Q_7, \quad Q_{220} = Q_8, \quad Q_{221} = Q_9,$$

$$Q_{023} = -Q_{032} = Q_{10} \sin \theta, \quad Q_{123} = -Q_{132} = Q_{11} \sin \theta,$$

$$Q_{033} = Q_6 \sin^2 \theta, \quad Q_{133} = Q_7 \sin^2 \theta, \quad Q_{330} = Q_8 \sin^2 \theta, \quad Q_{331} = Q_9 \sin^2 \theta.$$

Note that functions $S_i$ $(i = 5, 6, 7, 8)$ and $Q_k$ $(k = 10, 11)$ have pseudoscalar character. All other components of tensors $S_{\mu\nu}$ and $Q_{\mu\nu}$ vanish, with the exception of components connected with components (2) – (3) by symmetry properties of torsion $S_{\lambda\mu\nu} = -S_{\lambda\nu\mu}$ and nonmetricity $Q_{\lambda\mu\nu} = Q_{\mu\lambda\nu}$.

By choosing diagonal tetrad $h^i_\mu$ corresponding to metrics (1)

$$h^i_\mu = \text{diag}(e^\frac{r}{2}, e^\frac{\lambda}{2}, r, r \sin \theta),$$

we find anholonomic connection:

$$A^{ik}_\mu = h^{k\nu}(\partial_\mu h^i_\nu - h^i_\lambda \Gamma^\lambda_{\nu\mu}),$$

where holonomic connection $\Gamma^\lambda_{\mu\nu} = \{^\lambda_{\mu\nu}\} + S^\lambda_{\mu\nu} + S_{\mu\nu}^\lambda + S_{\nu\mu}^\lambda + \frac{1}{2}(Q_{\mu\nu}^\lambda - Q_{\mu\lambda}^\nu - Q_{\nu\lambda}^\mu)$ and $\{^\lambda_{\mu\nu}\}$ are Christoffel symbols. Nonvanishing components of connection $A^{ik}_\mu$ are:

$$A^{00}_0 = A_0, \quad A^{00}_1 = A_1, \quad A^{10}_0 = A_2, \quad A^{10}_1 = A_3, \quad A^{20}_2 = A_4,$$

$$A^{30}_3 = A_4 \sin \theta, \quad A^{0i}_0 = A_5, \quad A^{0i}_1 = A_6, \quad A^{1i}_0 = A_7,$$

$$A^{1i}_1 = A_8, \quad A^{2i}_2 = A_9, \quad A^{3i}_3 = A_0 \sin \theta, \quad A^{02}_2 = A_{10},$$

$$A^{03}_0 = A_10, \quad A^{13}_0 = A_11, \quad A^{23}_2 = A_{12}, \quad A^{33}_3 = A_{13}. $$
\( A^{\hat{0}3}_3 = A_{10} \sin \theta, \ A^{i} \hat{2} = A_{11}, \ A^{i} \hat{3} = A_{11} \sin \theta, \ A^{\hat{2}2}_0 = A^{\hat{3}3}_0 = A_{12}, \)
\( A^{\hat{2}2}_1 = A^{\hat{3}3}_1 = A_{13}, \ A^{\hat{0}3}_2 = A_{14}, \ A^{\hat{0}2}_3 = -A_{14} \sin \theta, \ A^{\hat{1}3}_2 = A_{15}, \)
\( A^{\hat{1}2}_3 = -A_{15} \sin \theta, \ A^{\hat{2}3}_0 = -A^{\hat{3}2}_0 = A_{16}, \ A^{\hat{2}3}_1 = -A^{\hat{3}2}_1 = A_{17}, \)
\( A^{\hat{3}0}_2 = A_{18}, \ A^{\hat{2}0}_3 = -A_{18} \sin \theta, \ A^{\hat{3}1}_2 = A_{19}, \ A^{\hat{2}1}_3 = -A_{19} \sin \theta, \)

where explicit form of functions \( A_i \ (i = 0, 2, \ldots 19) \) is:

\[
A_0 = \frac{1}{2} e^{-\nu} Q_0, \quad A_1 = \frac{1}{2} e^{-\nu} Q_1, \quad A_2 = \frac{1}{2} e^{-\nu} (Q_1 - 2Q_2 + 4S_1 - e^{\nu} \nu'), \\
A_3 = -\frac{1}{2} e^{-\nu} Q_4 + 4S_3 + e^{\nu} \lambda, \quad A_4 = -\frac{1}{2} e^{-\nu} (Q_3 - 4S_4), \\
A_5 = \frac{1}{2} e^{-\nu} (-Q_1 - 4S_1 + e^{\nu} \nu'), \quad A_6 = \frac{1}{2} e^{-\nu} (-2Q_3 + Q_4 - 4S_3 + e^{\nu} \lambda), \\
A_7 = \frac{1}{2} e^{-\nu} Q_4, \quad A_8 = \frac{1}{2} e^{-\nu} Q_5, \quad A_9 = \frac{1}{2} e^{-\nu} (2r + Q_9 - 4S_2), \\
A_{10} = \frac{1}{2} e^{-\nu} (Q_3 - 2Q_6 - 4S_4), \\
A_{11} = -\frac{1}{2} e^{-\nu} (2r - 2Q_7 + Q_9 - 4S_2), \quad A_{12} = \frac{1}{2u} Q_8, \\
A_{13} = \frac{1}{2r} Q_9, \quad A_{14} = \frac{1}{2} e^{-\nu} S_6, \quad A_{15} = -\frac{1}{2} e^{-\nu} S_6, \\
A_{16} = \frac{1}{2r} (Q_{10} - S_5 - 2S_7), \quad A_{17} = \frac{1}{2r} (Q_{11} - S_6 - 2S_8), \\
A_{18} = \frac{1}{2} e^{-\nu} (Q_{10} - S_5), \quad A_{19} = -\frac{1}{2} e^{-\nu} (Q_{11} - S_6). 
\]

The curvature tensor can be calculated according to his definition:

\[
F^{ik}_{\mu \nu} = 2\partial_{\mu} A^{ik}_{\nu} + 2A^{i}_{\mu\nu} A^{ik}_{\mu}. 
\]

In considered case the curvature is determined by 27 functions \( F_i \ (i = 0, 1, \ldots 26) \) depending on functions \( \nu, \lambda, S_i, Q_k \):
Explicit form of functions $F_i$ is

$$F_0 = \frac{1}{2}e^{-\frac{\lambda}{2}(\lambda + \nu)} [-4Q_3S_1 - e^\lambda (\dot{Q}_1 - Q_0' + Q_0\nu' + Q_2\dot{\lambda} - Q_1\dot{\nu}) + Q_2(2Q_3 - Q_4 + 4S_3) + Q_3(e^\nu\nu' - Q_1)],$$

$$F_1 = \frac{1}{4}e^{-(\lambda + 2\nu)}[Q_0(Q_4 - 2Q_3 - 4S_3 + e^\lambda \dot{\lambda}) + Q_1(4Q_1 + 4S_1 + e^\lambda \lambda')] + \frac{1}{4}e^{-(2\lambda + \nu)}\{Q_5(Q_1 + 4S_1) + Q_4^2 + e^\lambda [4\dot{Q}_3 - 2\dot{Q}_4 + 8\dot{S}_3 - 2Q_1' - 8S_1' + 4S_1(\lambda' + \nu') - e^\nu(\lambda'\nu' + \nu'^2 + 2\nu'\nu) - (4S_3 + 2Q_3 - Q_4)(\dot{\lambda} + \nu')] - Q_4(2Q_3 + 4S_3) - e^\nu Q_5\nu'],$$

$$F_2 = -\frac{1}{4\pi}e^{-(\lambda + \nu)}\{r^2(2r - 2Q_7 + Q_9 - 4S_2)(4S_1 + Q_1 - e^\nu\nu') + e^\lambda [4S_5(Q_1 - S_5 - 2S_7) - Q_8^2 + 2r^2(\dot{Q}_8 - 2\dot{Q}_6 - 4S_4 + 2S_4\nu') + Q_8(4S_4 + 2Q_6 - r^2\nu) + 2r^2Q_6\nu)] - \frac{1}{4\pi}e^{-2\nu}Q_0(2Q_6 - Q_8 + 4S_4),$$

$$F_3 = \frac{1}{4\pi^2}e^{-\frac{\lambda}{2}(\lambda + \nu)}[\frac{4}{r^2}S_5(Q_{11} - S_6 + 2S_8) - 8S_4^2 + 8S_4\nu' + 2(Q_6' - 2Q_6 + 2\nu'Q_6 + \nu'Q_8)] + \frac{1}{4\pi^2}e^{-\frac{\lambda}{2}\lambda - \nu}(2r + Q_9 - 2Q_7 - 4S_4)(Q_4 - 2Q_3 - 4S_3 + e^\lambda \dot{\lambda}) - \frac{1}{4\pi^2}e^{-\frac{\lambda}{2}(\lambda + \nu)}(2Q_6 - Q_8 + 4S_4)[Q_1 + e^\nu(\frac{2}{r} + \frac{1}{r^2}Q_9 - \nu')],$$

$$F_4 = \frac{1}{4}e^{-(\lambda + 2\nu)}[Q_5^2 + Q_0(Q_4 - 4S_3 + e^\lambda \dot{\lambda}) + Q_1(4S_1 + 2Q_2 - e^\nu(\lambda' - 2e^\nu\nu'))] + \frac{1}{4}e^{-(2\lambda + \nu)}\{Q_5(Q_1 + 4S_1) + Q_4^2 + e^\lambda [2\dot{Q}_4 - 8\dot{S}_3 + 4S_3(\dot{\lambda} + \nu') + e^\lambda (\lambda^2 + 2\dot{\lambda} - \dot{\lambda}\nu - Q_4\nu + 2Q_1' - 4Q_9 + 8S_3' + (2Q_2 - 4S_1)(\lambda' + \nu') + e^\nu(\lambda'\nu' - \nu'^2 + 2\nu'\nu)] - 2Q_2Q_5 - 4Q_4S_3 - e^\nu Q_5\nu'],$$

(9)

(10)
\[ F_5 = \frac{1}{2}e^{-\frac{1}{2}(\lambda+\nu)}[Q_3(e^\nu \nu' - Q_1 - 4S_1) + e^\nu(Q_5\dot{\lambda} - \dot{Q}_5 + Q_4' - Q_4\lambda')] + Q_2(2Q_3 - Q_4 + 4S_3 - e^\lambda \dot{\lambda})], \]
\[ F_6 = -\frac{1}{4r^4}e^{-\frac{1}{2}(\lambda+\nu)}[-r^2e^{-\lambda}Q_4 + Q_8 + r^2\dot{\lambda}](2r - 2Q_7 + Q_9 - 4S_2) + S_6(-4Q_{10} + 4S_5 + 8S_7) + r^2(4\dot{Q}_7 - 2\dot{Q}_9 + 8\dot{S}_2) + r^2(2Q_6 - Q_8 + 4S_4)(-\nu' + e^{-\nu}Q_1 + 4e^{-\nu}S_1 - 2e^{-\nu}Q_2)], \]
\[ F_7 = -\frac{1}{4r^4}[e^{-\lambda r^2}Q_5(2r - 2Q_7 + Q_9 - 4S_2) + e^{-(\lambda+\nu)}r^2(2Q_6 - Q_8 + 4S_4)(4S_3 - Q_4 - e^\lambda \dot{\lambda})] - \frac{1}{4r^4}e^{-\lambda}[Q_9(4r + Q_9 - 4S_2) - 8rS_2 + 4S_6(S_6 - Q_{11} + 2S_8) + 2r^2(2Q_7' - Q_9' + 4S_2' + \lambda' r + \frac{1}{2}\lambda' Q_9 - 2\lambda' S_2) - 2Q_7(Q_9 + 2r + r^2\lambda)], \]
\[ F_8 = \frac{1}{4r^4}e^{-\nu}[(Q_8 - 4S_4)(Q_8 + e^{-\nu}r^2Q_0 - r^2\dot{\nu}) - 4(Q_{10} - S_5)(Q_{10} - S_5 - 2S_7) + 2r^2(Q_8 - 4S_4) + e^{-\lambda r^2}(2r + Q_9 - 4S_2)(Q_1 - 2Q_2 + 4S_1 - e^\nu \nu')], \]
\[ F_9 = \frac{1}{4r^4}e^{-\frac{1}{2}(\lambda+\nu)}[(Q_8 - 4S_4)(Q_9 - 2r + e^{-\nu}r^2Q_1 - r^2\nu') - 4(Q_{10} - S_5)(Q_{11} - S_6 - 2S_8) - e^{-\lambda r^2}(2r + Q_9 - 4S_2)(Q_4 - 4S_3 + e^\lambda \dot{\lambda})] + 2r^2(Q_9 - 4S_4)(Q_1 - 2Q_2 + 4S_1 - e^\nu \nu'), \]
\[ F_{10} = -\frac{1}{4r^4}e^{-\frac{1}{2}(\lambda+\nu)}[(2r + Q_9 - 4S_4)](Q_8 - e^{-\lambda r^2}(Q_4 + e^\lambda \dot{\lambda})] - 4(Q_{11} - S_6)(Q_{10} - S_5 - 2S_7) + 2r^2(\dot{Q}_9 - 4\dot{S}_2) + e^{-\nu r^2}(Q_8 - 4S_4)(Q_1 + 4S_1 - e^\nu \nu')], \]
\[ F_{11} = -\frac{1}{4r^4}e^{-\lambda}[4r^2 - 4(Q_{11} - S_6)(Q_{11} - S_6 - 2S_8) + e^{-\nu r^2}(Q_8 - 4S_4)(2Q_3 - Q_4 + 4S_3 - e^\lambda \dot{\lambda}) + 2r^2(Q_9 - 4S_4) + (2r + Q_9 - 4S_2)(Q_9 - 2r - r^2e^{-\lambda}Q_5 - r^2\lambda')], \]
\[ F_{12} = -\frac{1}{2r^3}e^{-\frac{1}{2}(\lambda+\nu)}[2Q_8 + r(\dot{Q}_9 - Q_8)], \]
\[ F_{13} = -\frac{1}{r^4} + \frac{1}{4r^4}e^{-\lambda}[(2r + Q_9 - 4S_2)(2r - 2Q_7 + Q_9 - 4S_2) + 4S_6(S_6 - Q_{11})] - \frac{1}{4r^4}e^{-\nu}[Q_8 - 4S_4)(Q_8 - 2Q_6 - 4S_4) + 4S_5(S_5 - Q_{10})], \]
\[ F_{14} = \frac{1}{r^4}e^{-\nu}[Q_{10}(Q_8 - 2Q_6 - 4S_4) + 2Q_6S_5], \]
\[ F_{15} = -\frac{1}{r^4}e^{-\frac{1}{2}(\lambda+\nu)}[Q_{11} - S_6)(Q_8 - 2Q_6 - 4S_4) + S_5(2r + Q_9 - 4S_2)], \]
\[ F_{16} = -\frac{1}{r^4}e^{-\frac{1}{2}(\lambda+\nu)}[(Q_{10} - S_5)(2r - 2Q_7 + Q_9 - 4S_2) + S_6(Q_8 - 4S_4)], \]
\[ F_{17} = \frac{1}{r^4}e^{-\lambda}[Q_{11}(2r - 2Q_7 + Q_9 - 4S_2) + 2Q_7S_6]. \]
\begin{align*}
F_{18} &= \frac{1}{2r}e^{-\nu}\{(Q_8 - 4S_4)(Q_{10} - S_5 - 2S_7) + 2r^2(\dot{Q}_{10} - \dot{S}_5) + (Q_{10} - S_5)[Q_8 + e^{-\nu}r^2(Q_0 - e^\nu\dot{\nu})] + e^{-\nu}r^2(Q_{11} - S_6)(Q_1 - 2Q_2 + 4S_1 - e^\nu\dot{\nu})\}, \\
F_{19} &= \frac{1}{2r}e^{-\frac{1}{2}(\lambda + \nu)}\{(Q_{10} - S_5)[Q_9 - 2r + e^{-\nu}r^2(Q_1 - e^\nu\dot{\nu})] + (Q_8 - 4S_4)
(Q_{11} - S_6 - 2S_8) - e^{-\nu}r^2(Q_{11} - S_6)(Q_4 - 4S_3 + e^\nu\dot{\nu}) + 2r^2(Q_{10} - S_5')\}, \\
F_{20} &= -\frac{1}{2r}\{e^{-2\nu}r^2Q_0S_5 - e^{-(\lambda + \nu)}r^2S_6(Q_1 + 4S_1) + e^{-\nu}[-Q_{10}(2Q_6 - Q_8 + 4S_4) - 2Q_8S_7 + (S_5 + 2S_7)(4S_4 + 2Q_6) - 2r^2\dot{S}_5 + r^2S_5'] + e^{-\lambda}r^2S_6'\}, \\
F_{21} &= \frac{1}{2r}e^{-\frac{1}{2}\lambda - \frac{1}{2}\nu}S_6[r^2(2Q_3 - Q_4 + 4S_3) + e^{\lambda}(Q_8 - 2Q_6 - 4S_4 - r^2\dot{\lambda})] + \\
&\quad \frac{1}{r^2}e^{-\frac{1}{2}(\lambda + \nu)}\bigl[\frac{1}{2}Q_{11}(2Q_6 - Q_8 + 4S_4) + S_8(Q_8 - 2Q_6 - 4S_4) + r^2S_5'\bigl] - \\
&\quad \frac{1}{2r}e^{-\frac{1}{2}\lambda - \frac{1}{2}\nu}S_5[r^2Q_1 + e^\nu(Q_9 + 2r + r^2\dot{\nu})], \\
F_{22} &= -\frac{1}{2r}e^{-\frac{1}{2}(\lambda + \nu)}\{(Q_{11} - S_6)(e^{-\nu}r^2Q_4 + r^2\dot{\lambda} - Q_8) - (2r + Q_9 - 4S_2)(Q_{10} - S_5 - 2S_7) - 2r^2(\dot{Q}_{11} - \dot{S}_6) + e^{-\nu}r^2(Q_{10} - S_5)(-Q_1 - 4S_1 + e^\nu\dot{\nu})\}, \\
F_{23} &= -\frac{1}{2r}e^{-\lambda}[(Q_{11} - S_6)(2r - Q_9 + e^{-\nu}r^2Q_5 + r^2\lambda') - (2r + Q_9 - 4S_2)(Q_{11} - S_6 - 2S_8) + e^{-\nu}r^2(Q_{10} - S_5)(2Q_9 - Q_4 + 4S_3 - e^\lambda\dot{\lambda}) - 2r^2(Q_{11} - S_5)] \\
F_{24} &= \frac{1}{2r}e^{-2\lambda - \nu}\{e^{\lambda}r^2S_5(Q_4 - 4S_3) - r^2e^\nu Q_5S_6 + e^{\lambda + \nu}[2S_6(Q_7 + 2S_2) + (Q_{11} - 2S_8)(2r - 2Q_7 + Q_9 - 4S_2) + r^2S_6\lambda' - 2r^2S_6'] + e^{2\lambda}r^2S_5\dot{\lambda}\}, \\
F_{25} &= \frac{1}{2r}e^{-\frac{1}{2}(\lambda + \nu)}[Q_8S_6 + (2r - 2Q_7 + Q_9 - 4S_2)(Q_{10} - 2S_7 - e^\nu\dot{S}_5) + \\
&\quad r^2(Q_6\dot{\lambda} - 2\dot{S}_6) + r^2S_5(2Q_2 - Q_1 - 4S_1 + e^\nu\dot{\nu})]\} - \frac{1}{2r}e^{-\frac{1}{2}(3\lambda + \nu)}Q_4S_6, \\
F_{26} &= \frac{1}{r^2}e^{-\frac{1}{2}(\lambda + \nu)}[2Q_{10} - 2S_5 - 4S_7 + r(\dot{S}_6 - 2\dot{S}_8 - Q_{10}' + S_5' + 2S_7')].
\end{align*}

Let us consider Bianchi identities, which can be written in the following form:

\[ \varepsilon^{\sigma\lambda\mu\nu} \nabla_\lambda F^{i}_{k\mu\nu} = 0. \] (11)

where \( \nabla \) is differential operator defined analogously to covariant derivative determinant by means of Cristoffel symbols or connection \((-A^i_{\lambda\mu})\) in case of holonomic and anholonomic indices respectively:

\[ \nabla_\lambda F^{i}_{k\mu\nu} = \partial_\lambda F^{i}_{k\mu\nu} + A^i_{\lambda\lambda}F^i_{l\mu\nu} - A^i_{\lambda\lambda}F^l_{k\mu\nu} - \left\{ \varepsilon^{\sigma}_{\mu\lambda} F^i_{k\sigma\nu} - \varepsilon^{\sigma\lambda\nu}_{\nu\lambda} F^i_{k\mu\sigma} \right\}, \] and \( \varepsilon^{\sigma\lambda\mu\nu} \) is Levi-Chivita symbol.

In spherically-symmetric case Bianchi identities are reduced to 20 relations:
\[
\begin{align*}
  &e^{\frac{1}{2}(\lambda + \nu)}(4F_{14} + 2r^{2}F'_{14}) + e^{\lambda}[2F_{19}(2Q_{6} - Q_{8} + 4S_{4}) - \\
  &2F_{21}(Q_{8} - 4S_{4}) + 4F_{3}(Q_{10} - S_{5}) - 4F_{9}S_{5} + r^{2}\dot{\lambda}(F_{15} + F_{16})] + \\
  &r^{2}[(F_{15} + F_{16})(Q_{4} - 4S_{3}) - 2F_{16}Q_{3}] = 0, \\
  &-rF_{15}(e^{\lambda}rQ_{1} + e^{\nu}rQ_{5}) + e^{\lambda+\nu}[2F_{21}(2r + Q_{9} - 4S_{2}) + 4rF_{15} - 4F_{3}(Q_{11} - \\
  &S_{6}) + 2r^{2}F'_{15}] + e^{\frac{1}{2}(\lambda + \nu)}[-e^{\lambda}(2F_{23}(2Q_{6} - Q_{8} + 4S_{4}) + \\
  &4F_{11}S_{5}) + r^{2}(F_{17} + F_{14})(Q_{4} - 2Q_{3} - 4S_{4} + e^{\lambda}\dot{\lambda})] = 0, \\
  &rF_{16}(e^{\lambda}rQ_{1} + e^{\nu}rQ_{5}) + e^{\lambda+\nu}[2rF_{16} + F_{19}(2r - 2Q_{7} + Q_{9} - 4S_{2}) + \\
  &2F_{9}S_{6} + r^{2}F'_{16}] + e^{\frac{1}{2}(\lambda + \nu)}[r^{2}(F_{17} + F_{14})(Q_{4} - 4S_{3} + e^{\lambda}\dot{\lambda}) + \\
  &e^{\lambda}F_{24}(8S_{4} - 2Q_{8} + 4Q_{10} - 4S_{5})] = 0, \\
  &4rF_{17} + 2(F_{24} - F_{23})(2r - 2Q_{7} + Q_{9} - 4S_{2}) + 4F_{24}Q_{7} - 4F_{7}(Q_{11} - S_{6}) + \\
  &4F_{11}S_{6} + e^{-\frac{1}{2}(\lambda + \nu)}r^{2}[(F_{15} + F_{16})(Q_{4} - 4S_{3} + e^{\lambda}\dot{\lambda}) - 2F_{16}Q_{3}] + 2r^{2}F'_{17} = 0, \\
  &4r(F_{14} - F_{17}) - 4F_{23}Q_{7} + 2(F_{23} - F_{24})(2r + Q_{9} - 4S_{2}) + \\
  &e^{\frac{1}{2}(\lambda - \nu)}[4F_{19}Q_{6} + 2(F_{19} + F_{21})(-Q_{8} + 4S_{4}) + \\
  &4F_{3}(Q_{10} - S_{5}) - 4F_{9}S_{5}] + 4F_{7}(Q_{11} - S_{6}) - 4F_{11} + 2r^{2}(F'_{14} - F'_{17}) = 0, \\
  &4rF_{13} + (F_{11} - F_{7})(2r + Q_{9} - 4S_{2}) - 2F_{11}Q_{7} + \\
  &e^{\frac{1}{2}(\lambda - \nu)}[(Q_{8} - 4S_{4})(F_{9} - F_{3}) - 2F_{9}Q_{6} - 2F_{21}(Q_{10} - S_{5}) - 2F_{19}S_{5}] - \\
  &2F_{24}(Q_{11} - S_{6}) + 2F_{23}S_{6} + 2r^{2}F'_{13} = 0, \\
  &2e^{\frac{1}{2}(\lambda + \nu)}[(4S_{4} - Q_{8})(F_{18} + F_{20}) + 2Q_{6}F_{18} + 2F_{2}(Q_{10} - S_{5}) - \\
  &2F_{8}S_{5} + r^{2}F'_{14}] - r^{2}(F_{15} + F_{16})(Q_{1} + 4S_{1} - e^{\nu}r') + 2r^{2}F_{15}Q_{2} = 0, \\
  &-r^{2}F_{15}(e^{\lambda}Q_{0} + e^{\nu}Q_{4}) + 2e^{\frac{1}{2}(\lambda + 3\nu)}[F_{20}(2r + Q_{9} - 4S_{2}) - 2F_{2}(Q_{11} - S_{6})] - \\
  &2e^{\lambda+\nu}[F_{22}(2Q_{6} - Q_{8} + 4S_{4}) + 2F_{10}S_{5} - r^{2}F'_{15}] - \\
  &e^{\frac{1}{2}(\lambda + \nu)}r^{2}(F_{14} + F_{17})(Q_{1} + 4S_{1} - e^{\nu}r') = 0,
\end{align*}
\]
\[ r^2 F_{16}(e^\lambda Q_0 + e^\nu Q_4) + 2e^{\frac{1}{2}(\lambda + 3\nu)}[F_{18}(2r - 2Q_7 + Q_9 - 4S_2) + F_8S_6] + \\
2e^{\lambda + \nu}[-F_{25}(Q_8 - 4S_4) + 2F_6(Q_{10} - S_5) + r^2F_{16}] - \\
e^{\nu}[4F_6(S_6 - Q_{11}) + 2(F_{25} - F_{22})(2r + Q_9 - 4S_2) + 4F_{22}Q_7 + \\
4F_{10}S_6 + r^2\nu'(F_{15} + F_{16})] + r^2[2F_{15}Q_2 - \\
(Q_1 + 4S_1)(F_{15} + F_{16})] + 2e^{\frac{1}{2}(\lambda + \nu)}r^2F_{17} = 0, \\
e^{\frac{1}{2}(\nu - \lambda)}[(F_6 - F_{10})(2r + Q_9 - 4S_2) + 2F_{10}Q_7 + 2F_{25}(Q_{11} - S_6) - 2F_{22}S_6] + \\
(F_2 - F_8)(Q_8 - 4S_4) + 2F_6Q_6 + 2F_{20}(Q_{10} - S_5) + 2F_{18}S_5 - 2r^2F_{13} = 0, \\
e^{\nu}(F_{21}(Q_{10} - S_5 - 2S_\gamma) + r^2(-2\dot{F}_3 + F_6\dot{\lambda})] = 2e^{\nu}r^2F_6(2Q_3 - Q_4 + 4S_3) = 0, \\
\frac{1}{r}e^{\frac{1}{2}\nu}[F_{20}(-2r + Q_9) + F_1S_6 - 2F_2(Q_{11} - S_6 + 2S_8) - r^2(2F_{20} + \\
F_{20}\nu') - e^{\frac{1}{2}\lambda - \nu}rF_{21}Q_0 + e^{-\frac{3}{2}\nu}r[F_{20}Q_1 + F_{24}(-Q_1 + 4S_1 + e^{\nu}\nu')] + \\
\frac{1}{r}e^{\frac{1}{2}(\nu - \lambda)}[-2F_{21}Q_8 + F_{26}(-2Q_6 + Q_8 - 4S_4) - 2S_5(F_0 + F_{12}) + 2F_3(Q_{10} - \\
S_5 - 2S_\gamma) + r^2(2\dot{F}_{21} + F_6\dot{\lambda})] - e^{-\frac{1}{2}\lambda}rF_{25}(-2Q_3 + Q_4 - 4S_3 + e^{\lambda}\dot{\lambda}) = 0, \\
e^{\lambda + \nu}[(F_{12} - F_5)(2r - 2Q_7 + Q_9 - 4S_2) + 2F_{26}S_6 - 2F_{25}(Q_{11} - S_6 - 2S_8) + \\
2r^2F_6 + F_6(-Q_9 + 2r + r^2\nu')] + e^{\frac{1}{2}(\lambda + \nu)}r^2[Q_4(F_2 - F_7) - 4F_2S_3] + \\
e^{\frac{1}{2}(\lambda + \nu)}[2F_4(Q_6 + 2S_4) + Q_8(F_7 - F_{4}) + 2F_{24}(Q_{10} - S_5 - 2S_7) + r^2(-2\dot{F}_7 + \\
\dot{\lambda}F_2 - \dot{\lambda}F_7)] + e^{\lambda}r^2F_3(Q_1 - 2Q_2 + 4S_1 + e^{\nu}\nu') + e^{\nu}r^2F_6Q_5 = 0,
In the case of vanishing pseudoscalar functions $S_i (i = 5, \ldots, 8)$ and $Q_i (i = 10, 11)$ the curvature tensor is determined by 14 functions $\tilde{F}_i (i = 0, 1, \ldots, 13)$ (otherwise functions $F_i (i = 0, 1, \ldots, 13)$).
where explicit form of functions $\tilde{F}_i$ is:

$$
\tilde{F}_0 = \tfrac{1}{2} \{e^{-\frac{3}{2}(\lambda+2\nu)} \{p + Q_2(2Q_3 - Q_4 + 4S_3)\} + \\
e^{-\frac{1}{2}(\lambda + 3\nu)}(\dot{Q}_1 - Q_2^2 + Q_1\dot{\lambda} + Q_1^2\dot{\nu} + Q_0 - Q_0\nu')\} + e^{-\frac{1}{2}(3\lambda + \nu)} \{Q_3\nu'\},
$$

$$
\tilde{F}_1 = \tfrac{1}{2} \{e^{-\lambda - 2\nu} \{Q_1^2 + Q_0(-2Q_3 + Q_4 - 4S_3 + e^\lambda) + 4Q_1S_1\} + \\
e^{-2\lambda - \nu} \{Q_5(Q_1 + 4S_1 - \nu'\nu') + Q_4(Q_4 - 2Q_3 - 4S_3)\} + e^{-(\lambda + \nu)} \{Q_3 - \\
2Q_4 + 8S_3 + Q_1\nu' + (\dot{\lambda} + \dot{\nu})(4S_3 - 2Q_3) + Q_4\dot{\lambda} - 2Q_1 - \\
8S_1 + 4S_1(\nu' + \nu') + e^\lambda(-\dot{\lambda}^2 - \dot{\lambda}\dot{\nu} - 2\dot{\lambda})\} - e^{-\lambda}(\lambda\nu' + \nu'^2 + 2\nu'')\},
$$

$$
\tilde{F}_2 = -\frac{1}{4\nu} \{e^{-(\lambda + \nu)}r^2(2r - 2Q_7 + Q_9 - 4S_2)(Q_1 + 4S_1 - \nu'\nu') + \\
e^{-\nu}[(-Q_8 + 4S_4 + 2Q_6)(Q_8 + r^2\dot{\nu'}) + r^2(-4\dot{Q}_6 + 2\dot{Q}_8 - \\
8\dot{S}_4)] + e^{-2\nu}Q_0(2Q_6 - 4S_4)\},
$$

$$
\tilde{F}_3 = \frac{1}{4\nu} \{-e^{-\frac{3}{2}(\lambda + \nu)}\{2r + Q_5\}(2Q_6 - 8S_4 - 4Q_6' - 2Q_8') - 4\nu'Q_6' + 2\nu'Q_8') + e^{-\frac{1}{2}(3\lambda + \nu)}(2r - 2Q_7 + Q_9 - 4S_2)(-2Q_3 + \\
Q_4 - 4S_1 + e^\lambda\dot{\lambda}) + e^{-\frac{1}{2}(\lambda + 3\nu)} \{2Q_6 - 4S_4\}(Q_1 + 4S_1 - \nu'\nu')\},
$$

$$
\tilde{F}_4 = \frac{1}{2} \{e^{-\lambda - 2\nu} \{Q_0(Q_4 - 4S_3 + e^\lambda) + Q_1(Q_1 - 2Q_2 + 4S_1 - \nu'\nu' - \\
2e^\nu\nu')\} + e^{-2\lambda - \nu} \{Q_4(Q_4 - 4S_3) + Q_5(Q_1 + 4S_1 - 2Q_2)\} + e^{-\lambda - \nu} [2\dot{Q}_4 - \\
8\dot{S}_3 + 4S_3(\dot{\lambda} + \dot{\nu}) - Q_4\dot{\nu} + 2Q_1 - 4Q_2 + 8S_1' + (\nu' + \nu')(2Q_2 - 4S_1)] + \\
e^{-\nu}(\dot{\lambda}^2 - \dot{\lambda}\dot{\nu} + 2\dot{\lambda}) - e^{-2\lambda}Q_5\nu' + e^{-\lambda}(\lambda\nu' - \nu'^2 - 2\nu'')\},
$$

(14)
\[
\mathcal{F}_5 = \frac{1}{2} e^{-\frac{i}{2}(\lambda + \nu)} [-Q_3 (Q_1 + 4S_1) + e^\nu (-\dot{Q}_5 + Q_5 \dot{\lambda} + Q_4 \lambda' + Q_3 \nu')] + Q_2 (2Q_3 - Q_4 + 4S_3 - e^\lambda \dot{\lambda})],
\]
\[
\mathcal{F}_6 = -\frac{1}{4r} \{ r^2 e^{-\frac{i}{2}(\lambda + 3\nu)} (2Q_6 - Q_8 + 4S_4) (Q_1 + 4S_1 - e^\nu \nu' - 2Q_2) + e^{-\frac{i}{2}(\lambda + \nu)} [(2r - 2Q_7 + Q_9 - 4S_2) (Q_8 + r^2 \lambda - e^{-\lambda - r^2} Q_4) + r^2 (4\dot{Q}_7 - 2\dot{Q}_9 + 8\dot{S}_2)] \},
\]
\[
\mathcal{F}_7 = -\frac{1}{4r} \{ -e^{-2\lambda} r^2 (Q_5 - e^\lambda \lambda') (2r - 2Q_7 + Q_9 - 4S_2) + e^{-\lambda - r^2} (2Q_6 - Q_8 + 4S_4) (Q_1 + 4S_1 - e^\lambda \lambda') + 4S_2 - 8rS_2 + e^\lambda r^2 (4\dot{Q}_7 - 2\dot{Q}_9 + 8\dot{S}_2) - Q_7 (2r + Q_9) \},
\]
\[
\mathcal{F}_8 = \frac{1}{4r} \{ e^{-\nu} (Q_8 + e^{-\nu} r^2 Q_0 - r^2 \nu') (Q_8 - 4S_4) + e^{-\nu - r^2} (Q_8 - 4S_4) + e^{-\nu} r^2 (Q_9 + 4S_2) (Q_8 - 4S_4) \}.
\]
\[
\mathcal{F}_9 = \frac{1}{4r} e^{-\frac{i}{2}(\lambda + \nu)} [ (Q_8 - 4S_4) (Q_9 - 2r + e^{-\nu} r^2 Q_1 - r^2 \nu') - e^{-\lambda - r^2} (2r + Q_9 - 4S_2) (Q_4 - 4S_3 + Q_2 + 4S_1 - e^\lambda \dot{\lambda}) + 2e^\lambda r^2 (Q_8 - 4S_4) ]
\]
\[
\mathcal{F}_{10} = -\frac{1}{4r} e^{-\frac{i}{2}(\lambda + \nu)} [(2r + Q_9 - 4S_2) (Q_8 - e^{-\lambda - r^2} Q_4 - r^2 \dot{\lambda}) + 2r^2 (Q_9 - 4S_2) + e^{-\nu} r^2 (Q_8 - 4S_4) (Q_1 + 4S_1 - e^\nu \nu')]
\]
\[
\mathcal{F}_{11} = -\frac{1}{4r} e^{-\lambda} r^2 [4 + e^{-\nu} (Q_8 - 4S_4) (2Q_3 - Q_4 + 4S_3 - e^\lambda \dot{\lambda}) + 2Q_9 - 8S_2] + (2r + Q_9 - 4S_2) (Q_9 - 2r - e^{-\lambda - r^2} Q_5 + r^2 \lambda')
\]
\[
\mathcal{F}_{12} = \frac{1}{2r} e^{-\frac{i}{2}(\lambda + \nu)} [2Q_8 + r (\dot{Q}_9 - Q_9')],
\]
\[
\mathcal{F}_{13} = -\frac{1}{4r} [4r^2 - e^{-\lambda} (2r + Q_9 - 4S_2) (2r - 2Q_7 + Q_9 - 4S_2) + e^{-\nu} (Q_8 - 4S_4) (-2Q_6 + Q_8 - 4S_4)].
\]

In this case Bianchi identities are reduced to 6 following relations:

\[
-4r \mathcal{F}_{13} + (2r + Q_9 - 4S_2) (\mathcal{F}_7 - \mathcal{F}_{11}) + 2 \mathcal{F}_{11} Q_7 + e^{\frac{i}{2}(\lambda - \nu)} [(Q_8 - 4S_4) (\mathcal{F}_3 - \mathcal{F}_9) + 2 \mathcal{F}_9 Q_6] - 2r^2 \mathcal{F}_{13}' = 0,
\]
variation we get 16 h-equations:

\[ e^{i\nu} [\tilde{F}_2 (\frac{1}{r} Q_9 - 2 - rv') + \frac{1}{r} \tilde{F}_1 (2r - 2Q_7 + Q_9 - 4S_2) - 2r \tilde{F}_2] - e^{\frac{i}{2} \nu} r \tilde{F}_3 Q_0 + r e^{-\frac{i}{2} \nu} [\tilde{F}_2 Q_1 + \tilde{F}_7 (Q_1 - 4S_1 + e^\nu v') + e^{\frac{i}{2} \nu} \{ \frac{1}{r} [ - \tilde{F}_3 Q_8 + (\tilde{F}_12 - \tilde{F}_0)(2Q_6 - Q_8 + 4S_4)] + r(2\tilde{F}_3 + \tilde{F}_3 \lambda) \} - e^{-\frac{i}{2} \nu} r \tilde{F}_3 (Q_2 + 4S_2) + \frac{1}{r} [\tilde{F}_6 Q_9 + (\tilde{F}_12 + \tilde{F}_5)(2r - 2Q_7 + Q_9 - 4S_2)] + e^{-\frac{i}{2} \nu} r \tilde{F}_3 (Q_2 - 4S_2 + e^\nu \lambda)] - e^{-\nu} \{ [\tilde{F}_6 Q_5 + e^{\frac{i}{2} \nu} \{ \frac{1}{r} [ - \tilde{F}_7 Q_8 - \tilde{F}_1 (2Q_6 - Q_8 + 4S_1)] + 2r \tilde{F}_7 + r \tilde{F}_7 \lambda] \} = 0, \tag{15} \]

As result of \( f_0 = (16\pi G)^{-1}, \) G is Newton’s gravitational constant; \( f, a, k, m \) are indefinite parameters, \( F = F^\mu \nu \). The gravitational equations can be obtained by variation of total action integral

\[ I = \int \delta^4 x h(L_G + L_m) \tag{17} \]

\( (L_m \) is Lagrangian of matter and \( h = det(h^i_\mu)) \) with respect \( t h^i_\mu \) and \( A^i_\mu \). As result of variation we get 16 h-equations:

\[ H^i_\mu - \nabla^i \sigma^i_\mu = t^i_\mu, \tag{18} \]

and 64 A-equations:

\[ 2\nabla^i \tilde{\phi}^{i\mu} + \sigma^{i\mu} = -J^i_\mu, \tag{19} \]
were \( H_{\mu}^i = h^{-1}(\delta L_\text{G} / \delta h_i^\mu) \), \( \sigma_{\mu}^{i\nu} = (\partial L_\text{G} / \partial S_{\mu}^{i\nu}) \), \( \varphi_{ik}^{\mu\nu} = (\partial L_\text{G} / \partial F_{ik}^{\mu\nu}) \), \( t_i^\mu = -h^{-1}(\delta L_m / \delta h_i^\mu) \), \( J_{ik}^{\mu} = -h^{-1}(\delta L_m / \delta A_{ik}^{\mu}) \).

In spherically-symmetric case with vanishing pseudoscalar torsion and nonmetricity functions the system of gravitational equations (18) – (19) is reduced to 19 differential equations:

\[
2f_0(\tilde{F}_9 - \tilde{F}_{20} - \tilde{F}_{17}) + f[4\tilde{F}_9 - \tilde{F}_{14} - (2\tilde{F}_1 - \tilde{F}_4)^2 + 4(\tilde{F}_20 + \tilde{F}_{17})^2 - 8\tilde{F}_9(\tilde{F}_{20} + \tilde{F}_{17}) + 4\tilde{F}_8(2\tilde{F}_1 - \tilde{F}_4) - 4\tilde{F}_8^2 + 2\tilde{F}_{14}(\tilde{F}_4 - 2\tilde{F}_1 + 2\tilde{F}_8)] + k\{e^{-\lambda-2\nu}Q_2^2 - e^{-3\nu}Q_0^2 + e^{-2\lambda-\nu}Q_3^2 - e^{-3\lambda}Q_4^2 + \frac{\nu}{r}[2e^{-\nu}Q_0^2 - 2e^{-\lambda}Q_7(Q_7 + 2Q_9)]\} + \frac{m}{r}\{e^{-\lambda-2\nu}(Q_1Q_2 + 2Q_2S_1 - Q_0Q_4) + \frac{1}{2r}(e-\lambda - \nu Q_2 - e^{-2\lambda}Q_5) + e^{-2\lambda-\nu}(Q_3Q_4 - Q_1Q_5 - 4Q_3S_3) + \frac{\nu}{r}[e^{-\lambda-\nu}(Q_4Q_6 - Q_6S_3 - Q_1Q_7 + Q_3Q_8 - 4Q_3S_4)] - e^{-2\nu}Q_0Q_8 + 4e^{-2\lambda}Q_5S_2 + e^{-\nu}Q_6\dot{\lambda} - 2e^{-\lambda}Q_7 + e^{-\lambda}Q_7\lambda] + \frac{4}{r^2}[e^{-\nu}Q_0(Q_8 - 4S_4) + 4e^{-\lambda}Q_7S_2] + e^{-2\nu}Q_0\dot{\lambda} + e^{-2\lambda}(3Q_5\lambda' - 2Q_5') + e^{-\lambda-\nu}(Q_3\dot{\lambda} + 2Q_2' - Q_2\lambda' - 2Q_2\nu')] + \frac{\nu}{r}\{e^{-\lambda-\nu}(\frac{4}{r}S_1 - S_3\lambda + 2S_1' - S_1\lambda' - 2S_1\nu') + e^{-\lambda-2\nu}(Q_1S_1 + 2S_1^2) - \frac{1}{r^2}[4e^{-\lambda}S_2^2 + 2e^{-\nu}(Q_8S_4 - 2S_1^2)] + e^{-2\lambda-\nu}(2S_3^2 - Q_4S_3)\} = t_0^6,
\]

\((\tilde{F}_5 - \tilde{F}_{13})[2f_0 + 4f(2\tilde{F}_19 - \tilde{F}_{14} - 2\tilde{F}_20 - 2\tilde{F}_1 + \tilde{F}_4 - 2\tilde{F}_{17} + 2\tilde{F}_8)] + 2k[e^{-\frac{1}{2}(\lambda+5\nu)}Q_0Q_2 - e^{-\frac{3}{2}(\lambda+\nu)}Q_2(Q_3 + Q_4) + e^{-\frac{1}{2}(5\lambda+\nu)}Q_4Q_5 + \frac{2}{r^2}e^{-\frac{1}{2}(\lambda+\nu)}Q_7Q_8 + \frac{m}{r}\{e^{-\frac{1}{2}(\lambda+5\nu)}Q_0(Q_2 - Q_1 - 4S_1) + e^{-\frac{3}{2}(\lambda+\nu)}(Q_3Q_1 - 2Q_3Q_2 + Q_0Q_5 + 4S_1Q_3 - 4S_1Q_4 + 4Q_2S_3) + \frac{1}{r^2}e^{-\frac{1}{2}(\lambda+3\nu)}[2Q_6(Q_1 - 2Q_2 + 4S_1) + 2Q_6Q_7 + 2Q_2Q_8 + 8Q_2S_1 + r^2(Q_2\dot{\lambda} - 2\dot{Q}_2 + 2Q_2\nu + Q_0\nu')] + e^{-\frac{1}{2}(3\lambda+\nu)}[2\dot{Q}_5 - 3Q_5\dot{\lambda} - \frac{1}{r^2}(2Q_5Q_8 + Q_4S_2) - Q_3\nu')] + \frac{1}{r^2}e^{-\frac{1}{2}(\lambda+\nu)}(4\dot{Q}_7 - \frac{4}{r}Q_7Q_8 - 2Q_7\dot{\lambda} - Q_6\nu')] + \frac{2}{r^2}[e^{-\frac{1}{2}(\lambda+\nu)}Q_8S_2 - e^{-\frac{1}{2}(\lambda+5\nu)}Q_0S_1 + e^{-\frac{3}{2}(\lambda+\nu)}S_3(2Q_2 - Q_1 - 4S_1) + e^{-\frac{1}{2}(\lambda+3\nu)}(S_1\dot{\lambda} - 2\dot{S}_1 + 2S_1\nu') + e^{-\frac{1}{2}(3\lambda+\nu)}S_3\nu'] = t_1^6\),

13
\begin{align*}
& 2(\tilde{F}_{15} - \tilde{F}_9)[f_0 + 2f(2\tilde{F}_{19} - \tilde{F}_{14} - 2\tilde{F}_{29} - 2\tilde{F}_1 + 2\tilde{F}_4 - 2\tilde{F}_{17} + \\
& 2\tilde{F}_8)] + 2k [e^{-\frac{i}{2}(\lambda + \nu)Q_3(O_1 + Q_2)} - e^{-\frac{i}{2}(\lambda + 5\nu)Q_0Q_1} - \\
& e^{-\frac{i}{2}(5\lambda + \nu)Q_3Q_5 - \frac{2}{7}e^{-\frac{i}{2}(\lambda + \nu)Q_6Q_9} + \frac{2}{7}e^{-\frac{i}{2}(\lambda + \nu)}(2Q_2Q_3 - \\
& Q_2Q_4 - Q_6Q_5 + 4Q_3S_1 - 4Q_1S_3 + 4Q_2S_3) + e^{-\frac{i}{2}(5\lambda + \nu)Q_5(Q_4 - Q_3 - 4S_3) + \\
& e^{-\frac{i}{2}(\lambda + \nu)}\left[\frac{8}{7}Q_6 + \frac{4}{7}Q_6Q_9 + \frac{3}{7}(Q_7\lambda - 2Q_6 + Q_6\nu')\right] + e^{-\frac{i}{2}(3\lambda + \nu)}\left[\frac{2}{7}(Q_5Q_6 - \\
& 2Q_3Q_7 + Q_4Q_7 + Q_3Q_9 + 4Q_3S_2 - 4Q_7S_3) + Q_5\lambda - 2Q_3^3 + 2Q_3\lambda' + Q_3\nu'] + \\
& e^{-\frac{i}{2}(\lambda + 3\nu)}[2Q_0' - 3Q_0\nu' - \frac{2}{7}(Q_0Q_9 + 4Q_1S_4) - Q_2\lambda)] + \frac{6}{7}[e^{-\frac{i}{2}(\lambda + \nu)}S_1(2Q_3 - \\
& Q_4 + 4S_3) + e^{-\frac{i}{2}(3\lambda + \nu)}(\frac{4}{7}S_3 + 2S_3' + 2S_3\lambda' - S_3\nu') + e^{-\frac{i}{2}(\lambda + \nu)Q_5S_3 - \\
& e^{-\frac{i}{2}(\lambda + \nu)}(\frac{4}{7}S_4 + \frac{2}{7}Q_9S_4) - e^{-\frac{i}{2}(\lambda + 3\nu)S_1}\lambda] = t_0^1, \\
& 2f_0(\tilde{F}_8 - \tilde{F}_1 - \tilde{F}_{20}) + f[8\tilde{F}_{20}\tilde{F}_1 - \tilde{F}_{14} - 4\tilde{F}_{19} + 4\tilde{F}_1^2 - 4\tilde{F}_{19}(\tilde{F}_4 - 2\tilde{F}_{17}) + \\
& 2\tilde{F}_4(2\tilde{F}_{19} + \tilde{F}_4 - 2\tilde{F}_7) - (\tilde{F}_4 - 2\tilde{F}_{17})^2 - 8\tilde{F}_1\tilde{F}_8 + 4(\tilde{F}_{20} - \tilde{F}_8)^2] + \\
& k\{e^{-3\nu}Q_0^2 - e^{-\lambda - 2\nu}Q_2^2 - e^{-2\lambda - \nu}Q_3^2 + e^{-3\lambda}Q_5^2 + \frac{1}{7}e^{-\nu}(2Q_0^2 + 4Q_6Q_8) - \\
& \frac{3}{7}e^{-\nu}Q_9^2] + m\{e^{-\lambda - \nu}[(\frac{1}{1})Q_2 - \frac{1}{27}Q_4Q_6 - 4Q_7Q_9 - 4Q_7S_1 + \\
& 4Q_2S_2] + \frac{1}{2}Q_3 - \frac{1}{2}Q_3\lambda - \frac{1}{2}Q_3\nu + \frac{1}{2}Q_2\nu'] + \\
& \frac{1}{4}e^{-\lambda - 2\nu}(Q_0Q_4 - Q_1Q_2 - 4Q_2S_1) + \frac{1}{4}e^{-2\lambda - \nu}(Q_1Q_5 - Q_3Q_4 + \\
& 4Q_3S_3) - e^{-2\lambda}Q_5(\frac{1}{1} + \frac{2}{7}Q_9 + \frac{2}{7}Q_0\nu') + e^{-\lambda}Q_7(\frac{4}{7}S_2 - \frac{2}{7} - \frac{1}{7}Q_9 - \frac{2}{27}Q_9) + \\
& e^{-2\nu}(\frac{2}{7}Q_6S_4 - \frac{1}{2}Q_0 + \frac{2}{7}Q_0\nu) + \frac{1}{7}e^{-\nu}(Q_6 - \frac{1}{7}Q_0S_4 - \frac{1}{7}Q_6\nu)\} + a[\frac{1}{7}e^{-\lambda}S_2(2 + \\
& \frac{1}{7}Q_9 - \frac{2}{7}S_2) - e^{-\lambda - 2\nu}(\frac{1}{7}Q_1S_1 + S_1^2) + e^{-2\lambda - \nu}(\frac{1}{7}Q_4S_3 - S_3^2) + \frac{2}{7}e^{-\nu}S_1^2 + \\
& e^{-\lambda - \nu}(S_3\lambda - S_3 + \frac{1}{2}S_3\nu + \frac{1}{2}S_1\nu')] = t_1^1,
\end{align*}
\begin{align*}
f_0(\hat{F}_4 - \hat{F}_1 + \hat{F}_{19} - \hat{F}_{17} + \hat{F}_8 - \hat{F}_{14}) + f[\hat{F}_{14}^2 - 4\hat{F}^2_{20} - \\
4\hat{F}_{20}\hat{F}_1 - 2\hat{F}_1\hat{F}_4 + \hat{F}_4^2 + (4\hat{F}_{20} + 2\hat{F}_4)(\hat{F}_{19} - \hat{F}_{17} + \hat{F}_8) - \\
2\hat{F}_{14}(\hat{F}_{19} - \hat{F}_1 + \hat{F}_4 - \hat{F}_{17} + \hat{F}_8)] + k[e^{-3\nu}Q_0^2-
\begin{align*}
e^{-\lambda-2\nu}(2Q_1Q_2 + Q_2^2) + e^{-2\lambda-\nu}(Q_3^2 + 2Q_3Q_4) - e^{-3\lambda}Q_5^2 + \frac{2}{7\nu}e^{-\nu}Q_6Q_8-
\frac{2}{7\nu}e^{-\lambda}Q_7Q_9 + m\{e^{-\lambda-\nu}[(\frac{2}{7\nu})Q_2 + \frac{1}{\nu}(Q_7S_1 - \frac{1}{4}Q_3Q_8 - \frac{1}{2}Q_2Q_9 - \\
Q_7S_4 - Q_7S_3 - Q_3S_4) + \frac{1}{2}Q_3 - \frac{1}{4}Q_3\hat{\lambda} - \frac{1}{4}Q_3\nu + \frac{1}{2}Q_2 - \\
\frac{1}{2}Q_2\lambda' - \frac{1}{4}Q_2\nu'] + e^{-2\lambda}[Q_5(\frac{1}{4\nu}Q_9 + \frac{1}{\nu}S_2 - \frac{1}{2\nu} + \\
\frac{1}{2\nu}\lambda' - \frac{1}{2\nu}\nu') - \frac{1}{2\nu}Q_9]^2 + e^{-2\nu}[Q_0(\frac{1}{4\nu}Q_8 + \frac{1}{\nu}S_4 - \frac{1}{4}\lambda + \frac{3}{4}\nu)] - \\
\frac{1}{2}\hat{Q}_0 + \frac{1}{2\nu}e^{-\nu}[2\hat{Q}_6 + Q_6(\hat{\lambda} - \hat{\nu} - \frac{1}{4\nu}Q_8)] + e^{-\lambda-2\nu}(Q_0S_3 - Q_2S_1) + \\
e^{-2\lambda-\nu}(Q_9S_1 - Q_3S_3)] + a[\frac{1}{2\nu}e^{-\lambda}(2S_2' - S_2\lambda + S_2\nu' - \frac{2}{\nu}S_2 - \frac{1}{\nu}Q_9S_2) - \\
e^{-3\lambda}S_2^2 + e^{-2\lambda-\nu}S_3^2 + \frac{1}{2\nu}e^{-\nu}(\frac{1}{\nu}Q_8S_4 - 2\hat{S}_4 - S_4\lambda + S_4\nu)] = t_2^2, \\
4e^{-\frac{3}{2}\nu}kQ_0 + (e^{-\lambda-\nu}Q_3 + \frac{2}{\nu}e^{-\frac{3}{2}\nu}Q_6)[f_0 + 2f(2F_{19} - F_{14} - 2F_0 - \\
2F_1 + F_4 - 2F_{17} + 2F_8)] = J_{00}^0, \\
-\frac{1}{2}e^{-\frac{3}{2}\lambda-\nu}Q_2[2f_0 + 4f(2F_{19} - F_{14} - 2F_0 - 2F_1 + F_4 - 2F_{17} + 2F_8) - \\
1 + 8k] - \frac{1}{2}\frac{e^{-\frac{3}{2}\lambda}Q_5 - \frac{1}{4}e^{-\frac{3}{2}\lambda-\nu}(e^\nu Q_7 - a\nu^2 S_1)}{J_{00}^1, \\
\frac{1}{2}e^{-\frac{3}{2}\lambda-\nu}[4k(Q_1 + Q_2) + 2aS_1] + [\frac{1}{2}e^{-\frac{3}{2}\lambda-\nu}Q_1 + \frac{1}{2}e^{-\frac{3}{2}\lambda}Q_5 + \frac{1}{4}e^{-\frac{1}{2}\lambda}(2Q_7 - Q_9 + \\
4S_2)] [f_0 + 2f(2F_{19} - F_{14} - 2F_0 - 2F_1 + F_4 - 2F_{17} + 2F_8)] + 2fe^{-\frac{3}{2}\lambda}(2F_{19}^2 - \\
F_{19}' - 2F_0' - 2F_1' + F_4' - 2F_{17}' + 2F_8') = J_{01}^0, \\
\frac{1}{2}e^{-\frac{1}{2}\nu}(Q_8 - 2S_4) - \frac{1}{2}e^{-\nu}Q_0 - \frac{1}{2}e^{-\lambda-\nu}Q_4][f_0 + 2f(2F_{19} - F_{14} - 2F_0 - \\
2F_1 + F_4 - 2F_{17} + 2F_8)] - 2e^{-\lambda-\nu}k(Q_3 + Q_4) + \frac{1}{2}e^{-\lambda-\nu}S_3 + \frac{1}{4}e^{-\frac{1}{2}\nu}S_4 - \\
f_0 e^{-\frac{3}{2}\nu}(2\hat{F}_{19} - \hat{F}_{14} - 2\hat{F}_0 - 2\hat{F}_1 + \hat{F}_4 - 2\hat{F}_{17} + 2\hat{F}_8) = J_{01}^1, \\
(\frac{1}{2}e^{-\lambda-\nu}Q_4 - \frac{1}{2}e^{-\frac{3}{2}\nu}Q_0 - 2e^{-\lambda-\nu}S_3 - \frac{2}{\nu}e^{-\frac{3}{2}\nu}S_4)[f_0 + 2f(2F_{19} - F_{14} - 2F_0 - \\
2F_1 + F_4 - 2F_{17} + 2F_8)] - \frac{1}{2}\frac{e^{-\frac{3}{2}\nu}k(Q_0 + Q_8) + \frac{1}{2}e^{-\frac{1}{2}\nu}S_4 + 2e^{-\lambda-\nu}S_3}{J_{02}^2, \\
2fe^{-\frac{3}{2}\nu}(2\hat{F}_{19} - \hat{F}_{14} - 2\hat{F}_0 - 2\hat{F}_1 + \hat{F}_4 - 2\hat{F}_{17} + 2\hat{F}_8) = J_{02}^2, \\
\end{align*}
\[
\left(\frac{1}{2}e^{-\frac{1}{2}\lambda-\nu}Q_1 + \frac{1}{2}e^{-\frac{1}{2}\lambda}Q_5 + \frac{1}{2}e^{-\frac{1}{2}\lambda}Q_9 - \frac{1}{2}e^{-\frac{1}{2}\lambda}S_2\right)[f_0 + 2f(2F_{19} - F_{14} - 2F_{20} - 2F_1 + F_4 - 2F_{17} + 2F_8)] + 2e^{-\frac{1}{2}\lambda-\nu}k(Q_1 + Q_2) - 2e^{-\frac{1}{2}\lambda}(2F_{19} - F_{14} - 2F'_{20} - 2F'_{1} + F_4 - 2F_{17} + 2F_8)] = J_{10}^0,
\]
\[
\left[\frac{1}{2}e^{-\frac{1}{2}\nu}(2Q_6 - Q_8 + 4S_4) - \frac{1}{2}e^{-\frac{1}{2}\lambda}Q_0 - \frac{1}{2}e^{-\lambda-\frac{1}{2}\nu}Q_1\right][f_0 + 2f(2F_{19} - F_{14} - 2F_{20} - 2F_1 + F_4 - 2F_{17} + 2F_8)] + 2f(2F_{19} - F_{14} - 2F_{20} - 2F_1 + F_4 - 2F_{17} + 2F_8)] + \]
\[
\frac{1}{2}e^\lambda - \frac{\nu}{2}Q_3(1 + 4k) + 4kQ_4 - 3S_3(1 + a) + \frac{1}{2}e^{-\frac{1}{2}\nu}(2S_4 - Q_6) = J_{10}^1,
\]
\[
e^{-\lambda - \frac{1}{2}\nu}Q_3[f_0 + 2f(2F_{19} - F_{14} - 2F_{20} - 2F_1 + F_4 - 2F_{17} + 2F_8)] + \]
\[
e^{-\lambda - \frac{1}{2}\nu}(4kQ_3 + aS_3) = J_{11}^0,
\]
\[
\left(\frac{2}{2}e^{-\frac{1}{2}\lambda}Q_7 - e^{-\frac{3}{2}\lambda-\nu}Q_2\right)[f_0 + 2f(2F_{19} - F_{14} - 2F_{20} - 2F_1 + F_4 - 2F_{17} + 2F_8)] - 4e^{-\frac{3}{2}\lambda}kQ_5 - 2e^{-\frac{3}{2}\lambda-\nu}S_1 - \frac{1}{2}e^{-\frac{1}{2}\lambda}S_2 = J_{11}^1,
\]
\[
\left[\frac{1}{2}e^{-\frac{1}{2}\lambda}Q_5 - \frac{1}{2}e^{-\frac{3}{2}\lambda-\nu}(Q_1 + 4S_1) - \frac{7}{2}e^{-\frac{1}{2}\lambda}S_2\right][f_0 + 2f(2F_{19} - F_{14} - 2F_{20} - 2F_1 + F_4 - 2F_{17} + 2F_8)] - 2f e^{-\frac{3}{2}\lambda}(2F'_{19} - F'_{14} - 2F'_{20} - 2F'_1 + F'_4 - 2F'_{17} + 2F'_8) + \]
\[
\frac{1}{2}e^{-\frac{3}{2}\lambda}[2S_2 - k(Q_7 + Q_9)] + 2e^{-\frac{3}{2}\lambda-\nu}S_1 = J_{12}^2,
\]
\[
\left[\frac{1}{2}e^{-\lambda - \frac{1}{2}\nu}(2Q_3 - Q_4 + 4S_3) - \frac{1}{2}e^{-\frac{3}{2}\nu}Q_0 + \frac{1}{2}e^{-\frac{1}{2}\nu}(Q_6 - Q_8 + 2S_4)\right][f_0 + 2f(2F_{19} - F_{14} - 2F_{20} - 2F_1 + F_4 - 2F_{17} + 2F_8)] + 2f e^{-\frac{1}{2}\nu}(2F_{19} - \]
\[
\hat{F}_{14} - 2F_{20} - 2\hat{F}_1 + \hat{F}_4 - 2\hat{F}_{17} + 2\hat{F}_8) + e^{-\frac{1}{2}\nu}Q_0 + e^{-\lambda - \frac{1}{2}\nu}(2S_3 - Q_3) + \]
\[
\frac{1}{2}e^{-\frac{1}{2}\nu}[2Q_8 - 2Q_6(1 + k) + S_4(1 + a)] = J_{20}^2,
\]
\[
[f_0 + 2f(2F_{19} - F_{14} - 2F_{20} - 2F_1 + F_4 - 2F_{17} + 2F_8)]
\]
\[
[e^{-\frac{1}{2}\lambda-\nu}(\frac{1}{2}Q_1 - Q_2 + 2S_1) + \frac{1}{2}e^{-\frac{3}{2}\nu}Q_5 + \frac{1}{2}e^{-\frac{1}{2}\nu}(Q_7 - Q_9 + 2S_2)] + 2f e^{-\frac{3}{2}\lambda}(2F_{19} - F_{14} - 2F_{20} - 2F_1 + F'_4 - 2F'_{17} + 2F'_8) + \]
\[
2F'_{17} + 2F'_8) + e^{-\frac{1}{2}\lambda-\nu}(Q_2 + 2S_1) - e^{-\frac{3}{2}\nu}Q_5 - \]
\[
\frac{1}{2}e^{-\frac{1}{2}\nu}[2Q_7(1 + k) + 2kQ_9 - S_2(4 + a)] = J_{21}^2,
\]
\[
\frac{1}{2}e^{-\frac{1}{2}\nu}(4kQ_6 + aS_4) + \frac{1}{2}e^{-\frac{1}{2}\nu}Q_6[f_0 + 2f(2F_{19} - F_{14} - 2F_{20} - 2F_1 + F_4 - 2F_{17} + 2F_8)] = J_{22}^0,
\]
\[
\frac{1}{2}e^{-\frac{i}{2}\lambda}Q_5 - \frac{1}{2}e^{-\frac{i}{2}\lambda - \nu}Q_2 - \frac{1}{2}e^{-\frac{i}{2}\lambda}Q_7(4k - 1) + aS_2) - \\
\frac{1}{r^2}e^{-\frac{i}{2}\lambda}Q_7[f_0 + 2f(2F_{19} - F_{14} - 2F_{20} - 2F_1 + F_4 - 2F_17 + 2F_8)] = J_{22}^1.
\]

In the case \(t_i^\mu = 0\), \(J_{ik}^\mu = 0\) this system of equations is satisfied by vanishing torsion and nonmetricity and vacuum Schwarzchild metrics:

\[
g_{\mu\nu} = \text{diag}((1 - r_g/r), -(1 - r_g/r)^{-1}, -r^2, -r^2 \sin^2 \theta), (r_g = \text{const}).
\]  

(21)

References

[1] Hehl F. W., McCrea G. D., Mielke E. W. and Ne’eman Y. Metric-Affine Gauge Theory of Gravity: Fields Equations, Noether Identities, World Spinors, and Breaking of Dilaton Invariance. 1995, \textit{Phys. Rep.} \textbf{258}, 1.

[2] Minkevich A. V., Garkun A. S. Homogeneous isotropic models in the metric-affine gauge theory of gravity. 2000, \textit{Class. Quantum Grav.} \textbf{17}, p.3045-3054.

[3] Minkevich A. V. Spherically-symmetric spaces in the metric-affine gauge theory of gravitation, 1995, \textit{Vestnik BGU, ser. 1}, No 3, pp. 30-33.