An explanatory study of $^{11}$Li $\beta$ decay into $^9$Li and deuteron

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Abstract

The $\beta$-decay of $^{11}$Li into $^9$Li and $d$ is studied using simple halo wave functions of $^{11}$Li. The sensitivity of the transition probability is elucidated on a description of the halo part and on a choice of the potential between $^9$Li and $d$.

Experiment with radioactive nuclear beams led to the discovery of light nuclei with unusually large matter radii near the neutron drip-line. The nucleus $^{11}$Li has received much attention as one of those typical systems which have neutron halo-like structure. The direct information on the extended structure was derived from the enhancement of the interaction cross sections. Momentum distributions of fragments have offered another useful way of probing the correlated structure of the halo part and testing model descriptions. Beta-delayed particle emission from the nuclei near the drip-line also provides us with a unique probe for the halo structure. The $^6$He $\beta$ decay into $\alpha$ and $d$ with the branching ratio, $P_d=(7.6\pm0.6)\cdot10^{-6}$, for $E_d>350$keV [2] is a good example and has recently been studied by several authors [3–6]. It was stressed in ref. [4], among others, that this decay is quenched by a cancellation between the internal and halo contributions to the Gamow-Teller(GT) matrix element and shows an extreme sensitivity to the halo description up to large distances. The
The purpose of this letter is to do an explanatory analysis on the $^{11}\text{Li}$ $\beta$ decay into $^9\text{Li}$ and $d$. This decay attracts much attention and is expected to give important information on the halo structure of $^{11}\text{Li}$.

Most of the $\beta$ decay($\sim 97\%$) of $^{11}\text{Li}$ takes place to low-lying states in $^{11}\text{Be}$. A small branch of the $\beta$ decay involves delayed $n$, $t$ and $\alpha$ emission. Because of the experimental difficulty there is no direct evidence for the $\beta$-delayed $d$ emission from $^{11}\text{Li}$. There is, however, only one experiment [7] that has measured the $t+(d)$ spectrum down to 350 keV although no separation was made between $t$ and $d$. The experiment indicates that two separate components appear in the spectrum. The intensity at lower energy may be attributable to the branch of the deuterons because the $Q$ values for $^8\text{Li}+t$ and $^9\text{Li}+d$ decay channels are 4.96 MeV and 2.76 MeV, respectively. The lower limit on the branching ratio determined from the experiment is $P_{t+(d)}=(1.8\pm 0.3)\cdot 10^{-4}$ and a tentative value for the deuteron branch is obtained as $P_d=(1.0\pm 0.3)\cdot 10^{-4}$.

Despite numerous investigations the understanding of the structure of $^{11}\text{Li}$ is still imperfect. This is in sharp contrast to the case of $^6\text{He}$ in which a $^4\text{He}+n+n$ three-body model works nicely. The three-body configuration of $^9\text{Li}+n+n$ is certainly an important ingredient for the $^{11}\text{Li}$ structure but there are at least two basic points to be settled before a more satisfactory description of $^{11}\text{Li}$ is made, i.e., the role of the polarizability of $^9\text{Li}$ and the feature of the $^9\text{Li}-n$ interaction. At present we have to use those very simple wave functions for $^{11}\text{Li}$ which assume a $^9\text{Li}$ core, and our study is therefore of qualitative nature.

We assume that the $\beta$-delayed $d$ emission proceeds through the so-called direct decay, i.e., a transition from $^{11}\text{Li}$ to $^9\text{Li}+d$ continuum. In this $\beta$ decay the isospin changes and the total angular momentum, $J_f$, in the final channel may be $J_f=1/2$, $3/2$, or $5/2$. Since the decay $Q$-value is small, we assume the relative motion between $^9\text{Li}$ and $d$ is an $s$-wave and independent of $J_f$. Under these assumptions the decay transition probability per time and energy units, $dW/dE$, is calculated by

\[
\frac{dW}{dE} = \frac{mc^2}{\pi^4 \hbar^2} G^2_{\beta f}(Q-E)B_{GT}(E),
\] (1)
where \( m \) is the electron mass, \( v \) the relative velocity between \(^9\text{Li}\) and \( d \), \( G_\beta = 2.996 \times 10^{-12} \) the dimensionless \( \beta \)-decay constant, and \( f \) is the Fermi integral. The GT reduced matrix element is expressed as

\[
B_{GT}(E) = 6\lambda^2 < g_E | \psi_{eff} >^2, \tag{2}
\]

where \( \lambda = -1.25 \) is the ratio of the axial-vector to vector coupling constants. The radial part of the \(^9\text{Li}-d\) relative motion function \( g_E \) is normalized as

\[
g_E(R) \rightarrow R^{-1} (F_0(kR) \cos \delta + G_0(kR) \sin \delta), \tag{3}
\]

where \( k \) is the wave number of the relative motion with energy \( E \) and \( \delta \) is the \( s \)-wave phase shift at energy \( E \). The effective wave function \( \psi_{eff} \) is defined as

\[
\psi_{eff}(R) = \int_0^\infty F_d(r) \chi_0(r, R) r^2 dr, \tag{4}
\]

where \( F_d(r) \) is the radial part of the deuteron wave function. The function \( \chi_0(r, R) \) is the \( s \)-wave part in both of the coordinates, \( r \) and \( R \), of a two-neutron reduced amplitude \( \chi(r, R) \) defined by

\[
\chi(r, R) = (55)^{1/2} < \phi_{J=\frac{3}{2}}^{M}(\text{Li}), S M_S = 00, T M_T = 11 | \Psi_{\frac{3}{2}}^{M}(\text{Li}) >, \tag{5}
\]

where \( r \) is the relative coordinate between the two neutrons and \( R \) is the relative coordinate between \(^9\text{Li}\) and the center-of-mass of the two neutrons.

We used the \(^9\text{Li}-d\) relative motion function \( g_E \) generated from the folding potential of Watanabe type

\[
V(R) = \frac{1}{4\pi} \int F_d^2(r) \left[ U_p(|R + \frac{r}{2}|) + U_n(|R - \frac{r}{2}|) \right] dr, \tag{6}
\]

where \( U_p(U_n) \) is a \( p(n)\)-\(^9\text{Li}\) optical potential of Woods-Saxon form. We assumed \( U_p = U_n \) and took into account only the real part of the central potential \( V_0 = 53 \text{ MeV} \). Dependence of the transition probability on the optical potential parameters, the diffuseness \( a \) and the radius parameter \( r_0 \), will be discussed later. The Coulomb potential of uniform charge distribution
was used. As stated in the beginning our purpose is not to make a quantitative prediction but to learn the sensitivity of the spectrum and the branching ratio on the halo structure and the continuum wave function. We consider following simple two-neutron reduced amplitudes which were employed in the analysis \[9\] of the momentum distribution of a \(^9\)Li fragment arising from the \(^{11}\)Li reaction: (i) a \((p_{1/2})^2_{J=0}\) harmonic-oscillator shell model(SM) wave function, (ii) a di-neutron cluster model(CM) wave function, and (iii) a hybrid model(HM) wave function. For each case the function \(\chi_0\) is expressed in terms of a nodeless harmonic oscillator function, \(F(b,r) = \frac{2\pi}{-\frac{1}{4}}^{1/2} b^{-3/2} \exp(-r^2/2b^2)\) \[4\]:

\[
\chi_{0}^{SM}(r, R) = \frac{1}{6} b_0^{-2}(r^2 - 4R^2) F(2^{1/2} b_0, r) F(2^{-1/2} b_0, R),
\]

\[
\chi_{0}^{CM}(r, R) = F(b_1, r) F(b_2, R).
\]

The parameters, \(b_0\) and \((b_1,b_2)\), are chosen to reproduce the measured interaction cross section of \(^{11}\)Li on \(^{12}\)C at an incident energy of 800 MeV/nucleon. See Table 1 of ref. \[9\] for the values. These two-neutron halo wave functions, however, cannot explain the observed momentum distribution of \(^9\)Li in the \((^{11}\)Li, \(^9\)Li) reaction. In case (iii) a simple wave function that reproduces the momentum distribution was constructed \[9\] by taking a linear combination of the shell model and cluster model wave functions: \(\chi_0^{HM}(r, R) = \varepsilon_1 \chi_0^{SM}(r, R) + \varepsilon_2 \chi_0^{CM}(r, R)\) with \(\varepsilon_1 = 0.79\) and \(\varepsilon_2 = -0.41\). The hybrid model is considered "best" among three cases.

Fig.1 illustrates how the functions \(g_E\) and \(\psi_{eff}\) contribute to the GT matrix element for \(E=0.5\) MeV ( \(a=0.6\) fm, \(r_0=1.2\) fm). The integral, \(I(R) = \int_0^R g_E(R') \psi_{eff}(R') R'^2 dR'\), for three effective wave functions is displayed in the lower part of the figure. The limit of \(I(R)\) for \(R \rightarrow \infty\) appears in eq.(2). The GT value gains most of the contribution in the region of \(R=4\sim10\) fm. It has negligible contribution from the interior part of the wave function, which is understood by the fact that the \(^9\)Li-\(d\) relative motion function exhibits more oscillatory behavior in the interaction region than the \(\alpha-d\) case. We can conclude that the \(\beta\) decay of \(^{11}\)Li into \(^9\)Li and \(d\) probes mainly the halo part of the \(^{11}\)Li wave function. This is in
sharp contrast to the case of $^6$He, where the cancellation in the GT matrix element occurred between the internal and external parts of the $^6$He wave function.

We have compared the energy dependence of the transition probability for three model wave functions. All three cases yielded similar spectrum although the magnitudes of the transition probability differed within a factor of ten at the peak values.

Table I lists the branching ratio calculated with the HM wave function for various values of the potential parameters, $a$ and $r_0$. Since no $^9$Li+$d$ scattering data are available, we cannot select a good set of the parameters. The potential with $r_0=1.3$ fm is ruled out, however, because it leads to a smaller branching ratio than expected from experiment. Increasing $r_0$ or $a$ leads to a stronger potential. The branching ratio of order $10^{-6}$ is obtained when the third node of $|g_E|$ is pulled inside at shorter distance and the overlap integral with $\psi_{eff}$ becomes smaller as the potential becomes more attractive. Fig.2 shows the dependence of the transition probabilities on the potential parameters. The energy at which the peak of the spectrum occurs is dependent on the choice of the potential. Measurements with good statistics down to the lower energy will be very informative in this respect.

In conclusion we have examined the $\beta$ decay of $^{11}$Li into $^9$Li and $d$ in order to shed light on the role of the halo structure of $^{11}$Li. We find that the $\beta$-decay matrix elements are determined practically by the halo part of distances, $R \geq 4$ fm. The branching ratio and the $^9$Li-$d$ relative motion energy at which the transition probability reaches a maximum are sensitive to the choice of the optical potential. A standard set of the potential yields the branching ratio of order $10^{-4}$.

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TABLE I. Dependence of the branching ratio of the $^{11}\text{Li}$ $\beta$ decay into $^9\text{Li}$ and $d$ on a set of the parameters, $a$ and $r_0$ (in fm), of the optical potential. The hybrid model wave function is used.

| $r_0$ | $a=0.5$ | $a=0.6$ | $a=0.7$ |
|-------|---------|---------|---------|
| 1.1   | $1.4\times10^{-4}$ | $2.2\times10^{-4}$ | $4.7\times10^{-4}$ |
| 1.2   | $5.2\times10^{-4}$ | $0.5\times10^{-4}$ | $1.0\times10^{-6}$ |
| 1.3   | $1.1\times10^{-6}$ | $5.2\times10^{-6}$ | $9.5\times10^{-6}$ |
FIGURES

FIG. 1. The contribution of the initial and final wave functions to the GT matrix element. Absolute values of $R_{gE}(R)$ for $E = 0.5$ MeV and of $R_{\psi_{eff}}(R)$ and the integral $I(R)$ are drawn. See text for the definition of $I(R)$.

FIG. 2. The transition probability per time and energy units as a function of the relative motion energy $E$ for different sets of the optical potential parameters. The values of $a$ and $r_0$ are given in units of fm. The hybrid model wave function is used.