Nonlinear control of active power filter based on LQR control

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Abstract: In order to improve the ability of real-time tracking and compensating harmonic of three-phase active power filter (APF), a nonlinear sliding mode control strategy of APF is proposed based on the state feedback linearization theory. In order to improve the dynamic performance and tracking ability of APF system, the affine nonlinear model of APF system is established, and the nonlinear control strategy of current inner loop is designed. The control parameters are adjusted by LQR method. In order to adjust and stabilize the DC voltage of the system, the slip mode control (SMC) is used to design the voltage outer loop. The simulation and experimental platform are built. The experimental results show that the method has good dynamic response, steady-state characteristics and robustness.

1. Introduction

With the development of economy, power electronic equipment is widely used in the production of various industries. However, most of the power electronic equipment belongs to nonlinear load, which will cause grid current distortion and threaten the safe operation of other equipment. Compared with various harmonic compensation methods, active power filter (APF) has strong ability to track and compensate harmonics in real time, which is a commonly used harmonic compensation method in low-voltage distribution network.

At present, APF control strategy is mainly based on inner loop current control and outer loop voltage control double loop PI control [1-3], which has high reliability. In three-phase APF control system, in order to achieve zero static error control, the control model is usually converted to dq coordinate system for PI control, but it brings about cross coupling between d-axis and q-axis. In dq coordinate system, feedforward decoupling control algorithm based on PI regulator can be used to counteract the influence of cross coupling term on control performance, but the dynamic response is not obvious and the control accuracy needs to be further improved. In recent years, more and more attention has been paid to the nonlinear design method based on state feedback accuracy. The global or partial linearization of the nonlinear system can be realized through coordinate transformation, and then the control system design can significantly improve the dynamic performance and control accuracy of the system on the basis of ensuring the steady-state performance of the system.

In this paper, a nonlinear sliding mode control strategy for active power filter (APF) in distribution network is proposed. The outer voltage loop adopts sliding mode control, which can stabilize and adjust the DC bus voltage; The inner current loop is mainly used to track and compensate harmonics, so nonlinear design is carried out based on the precise linearization theory of state feedback, and the parameters of the controller are adjusted according to the linear quadratic theory, and the simulation model is built. The results show that the total harmonic distortion rate (THD) of the grid current after
APF compensation is less than 5% with rectifier load. The proposed method has better dynamic response, steady-state characteristics, and robustness.

2. APF structure and mathematical model

The main circuit structure of three-phase three-wire APF is shown in Fig. 1. \(e_a, e_b, e_c\) are grid voltage, \(i_{ca}, i_{cb}, i_{cc}\) are output compensation current of APF, \(i_{La}, i_{Lb}, i_{Lc}\) are load current, \(L\) and \(R\) are filter inductors and equivalent resistance, \(C\) is DC side capacitors, and their capacitance values are equal.

![Figure.1 Schematic diagram of APF](image)

According to the above figure, the mathematical model of the \(dq\) axis of APF is the equation (1), the \(\omega\) is fundamental angular frequency, \(s_d, s_q\) are the \(d\) and \(q\) axis components of the switching function, \(i_{cd}, i_{cq}\) are the compensation current of APF in \(dq\) coordinate system, and \(e_d, e_q\) are the grid voltage in \(dq\) coordinate system.

\[
\frac{di_{cd}}{dt} = \frac{R}{L} i_{cd} + \omega i_{cq} + \frac{u_{dc}}{L} s_d - \frac{1}{L} e_d \\
\frac{di_{cq}}{dt} = \frac{R}{L} i_{cq} - \omega i_{cd} + \frac{u_{dc}}{L} s_q - \frac{1}{L} e_q \\
\frac{du_{dc}}{dt} = -i_{cd} s_d - i_{cq} s_q \\
\frac{dt}{C_{dc}} = -i_{cd} s_d - i_{cq} s_q
\]

(1)

Among them, the state quantity \(X = [x_1, x_2]^T = [i_{cd}, i_{cq}]^T\), the switch function as the input value \(U = [u_1, u_2]^T = [s_d, s_q]^T\), and the output value is \(Y = [x_1, x_2]^T\). The nonlinear equation is obtained as follows.

\[
\dot{X} = \begin{bmatrix} \frac{-R}{L} x_1 + \omega x_2 - \frac{1}{L} e_d \\ -\omega x_1 - \frac{R}{L} x_2 - \frac{1}{L} e_q \end{bmatrix} + \begin{bmatrix} \frac{u_{dc}}{L} \\ 0 \end{bmatrix} U
\]

\[
Y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X
\]

(2)

3. The controller design of APF

3.1. design of current inner loop LQR of APF

Linearize the original nonlinear system.

\[
Z = AZ + BV
\]

(3)

Let the correlation matrix of equation (3) be

\[
A = \begin{bmatrix} -k & 0 \\ 0 & -k \end{bmatrix}, B = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}
\]

(4)
The negative feedback system of equation (5) is designed to minimize the objective function value of equation (6)

\[ V = -KZ \quad (5) \]

\[ J = \int_0^\infty (Z^T QZ + V^T RV) \, dt \quad (6) \]

The weight matrix \( Q \) and \( R \) are semi positive definite matrices, and the state feedback gain matrix is as follows:

\[ K = R^{-1} B^T P \quad (7) \]

Where \( P \) can be obtained by Riccati equation:

\[ A^T P + PA - PBR^{-1} B^T P + Q = 0 \quad (8) \]

The matrix \( P \) can be obtained through the weight matrix \( Q \), and then \( P \) is brought into equation (8) so that \( R \) is equal to the identity matrix. Thus, the state feedback control law of linear system is

\[ V = -R^{-1} B^T PZ = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}Z \quad (9) \]

### 3.2. voltage outer loop design of APF

Analysis Equation (1), the control quantity \( s_d \) stabilizes the DC voltage \( u_{dc} \), the control quantity \( s_q \) compensates the reactive current. When \( u_{dc} \) and \( i_{cq} \) are selected as output, equation (1) can be rewritten as follows:

\[
\frac{d}{dt} \begin{bmatrix} i_{cq} \\ u_{dc} \end{bmatrix} = \begin{bmatrix} \frac{-\omega i_{cd}}{L} + \frac{R_{eq}}{L} + \frac{u_{dc}}{L} - \frac{e_q}{L} \\ -\frac{s_q i_{cd}}{C_{dc}} - \frac{u_{dc}}{C_{dc}} - \frac{s_q i_{cd}}{L} + \frac{s_q i_{cd}}{C_{dc}} - \frac{u_{dc}}{C_{dc}} \end{bmatrix} - \frac{R}{C_{dc}} (s_d i_{cd} + s_q i_{cd}) + \omega L (-s_d i_{cd} + s_q i_{cd}) + \frac{u_{dc}}{C_{dc}} (s_d^2 + s_q^2) \]

\[ \frac{L}{C_{dc}} \quad (10) \]

Define error variables:

\[ e = \begin{bmatrix} e_1^T \\ e_2 \\ e_3 \\ \dot{u}_{dc} - \dot{u}_{dc} \end{bmatrix} = \begin{bmatrix} i_{cq} - \dot{i}_{cq} \\ u_{dc} - \dot{u}_{dc} \\ e_3 \\ \dot{u}_{dc} - \dot{u}_{dc} \end{bmatrix} \quad (11) \]

Therefore, the relative orders of \( e_1 \) and \( e_2 \) are 1 and 2 respectively, so the sliding surfaces \( S_1 (e_1, t) \) and \( S_2 (e_2, e_3, t) \) are selected as

\[ S_1 = k_1 e_1 = 0 \]

\[ S_2 = k_2 e_2 + k_3 e_3 = e_2 + \beta e_3 = 0 \quad (12) \]

Where \( k_1 \) and \( \beta = k_3/k_2 \) are feedback coefficients. By substituting the equation (11) into (12), it is concluded that:

\[ S_2 = (u_{dc} - \dot{u}_{dc}) - \beta \left( \frac{s_d i_{cd}}{C_{dc}} + \frac{s_q i_{cq}}{C_{dc}} \right) - \beta \frac{u_{dc}}{C_{dc}} = 0 \quad (13) \]

Transformation:

\[ S_2 = (u_{dc} - \dot{u}_{dc}) - \beta \left( \frac{s_d i_{cd}}{C_{dc}} + \frac{s_q i_{cq}}{C_{dc}} \right) - \beta \frac{u_{dc}}{C_{dc}} \quad (14) \]

definition:

\[ \dot{i}_{cd} = (u_{dc} - \dot{u}_{dc}) - \frac{\beta}{C_{dc}} s_d i_{cd} - \beta \frac{u_{dc}}{C_{dc}} \quad (15) \]

Therefore, the synovial surface of the system can be redefined as:
Assuming that the three-phase voltage is symmetrical, there are:

\[
\begin{align*}
S_1 &= (i_{cq} - i_{\text{ref}}) = 0 \\
S_2 &= (i_{cd} - i_{\text{ref}}) = 0
\end{align*}
\]  

(16)

Therefore (15) can be defined as:

\[
\begin{align*}
\frac{s_q}{s_d} &= \sqrt{3} u_{sc} / u_{sb} \\
\frac{s_q}{s_d} &= 0
\end{align*}
\]  

(17)

Therefore, (15) can be defined as:

\[
i_{\text{ref}} = \left( u_{sb} - u_{sc} \right) \frac{C_{dc} u_{sb}}{\sqrt{3} \beta u_s}
\]  

(18)

Therefore, the control schematic diagram of Fig. 2 can be obtained.

![Control schematic diagram](image)

**Figure 2** Control schematic diagram

### 4. Simulation Analysis

In order to verify the effectiveness of LQR and synovial control strategy, a simulation model is built in MATLAB / Simulink, and the parameters are shown in Table 1. The harmonic source is an uncontrollable diode rectifier bridge. When it is set at 0.3s, the load fluctuates. The reference current is the harmonics detected by \( i_{p-q} \) method. Take \( \lambda_1 = \lambda_2 = 0.45, \lambda_3 = 0.1 \).

| Parameters       | Value   |
|------------------|---------|
| Grid Voltage     | 380V    |
| DC Voltage       | 800V    |
| Capacitor \( C \)| 3000\( \mu \)F |
| Inductor         | 4mH     |
| Resistor         | 0.01\( \Omega \) |
| Load Resistor    | 10\( \Omega \) |

By setting the parameters above the simulation model and running the simulation in Simulink, we can get the simulation diagram of Fig. 3, which can reflect the dynamic performance and stability performance of the algorithm.
Figure 3 (a) is the three-phase load current; (b) is the three-phase current after compensation; (c) is the DC side voltage control; (d) the total harmonic distortion rate of the current.

It can be seen from (a) that the harmonics of the power grid are 5, 7, 11, 13. (b) for the result of using LQR algorithm to control APF for compensation, the current waveform is close to sine wave, and has fast dynamic performance. When the load changes, it can track and compensate the harmonic well. The THD of (f) is 2.07%, which meets the harmonic requirements of power grid. The control method has good tracking performance and can keep a small tracking error.

5. Conclusion
In this paper, a nonlinear model of active power filter (APF) in distribution network is established based on state feedback precise linearization theory, and a nonlinear sliding mode control method is proposed. This method weakens the influence of circuit parameters and system disturbance on steady-state and dynamic performance. Simulation and experimental results show that the proposed control strategy can accurately and timely track and compensate the non-linear performance Harmonics generated by linear loads.

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