Abstract

We compare three different statistical models for the equation of state (EOS) of stellar matter at subnuclear densities and temperatures (0.5-10 MeV) expected to
occur during the collapse of massive stars and supernova explosions. The models introduce the distributions of various nuclear species in nuclear statistical equilibrium, but use somewhat different nuclear physics inputs. It is demonstrated that the basic thermodynamical quantities of stellar matter under these conditions are similar, except in the region of high densities and low temperatures. We demonstrate that mass and isotopic distributions have considerable differences related to the different assumptions of the models on properties of nuclei at these stellar conditions. Overall, the three models give similar trends, but the details reflect the uncertainties related to the modeling of medium effects, such as the temperature and density dependence of surface and bulk energies of heavy nuclei, and the nuclear shell structure effects. We discuss importance of new physics inputs for astrophysical calculations from experimental data obtained in intermediate energy heavy-ion collisions, in particular, the similarities of the conditions reached during supernova explosions and multifragmentation reactions.

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1 Introduction

It is known that the short-range strong interactions between nucleons can lead to a fast equilibration in violent nuclear reactions. Therefore, statistical models have proved to be very successful for interpretation of nuclear reactions at various energies. They are widely used for description of the fragment production when one or several equilibrated sources can be identified. Originally, this concept was proposed by Niels Bohr [1] for description of a compound nucleus decaying via evaporation of light particles or fission. Recently, it has been demonstrated that the concept of statistical equilibrium can be successfully used even for violent multifragmentation reactions leading to production of many intermediate mass fragments [2, 3, 4].

Moreover, the nuclear statistical equilibrium is established in many astrophysical processes, when the characteristic time for nuclear transformations is much shorter than those of these processes. For example, one of the most spectacular astrophysical events is a core-collapse supernova explosion, with a huge energy release of about 100 MeV per nucleon [5, 6]. When the core of a massive star collapses, it may reach baryon densities which are several times larger than the normal nuclear density $\rho_0 \approx 0.15$ fm$^{-3}$ (i.e., $m_N \rho_0 \approx 2.5 \cdot 10^{14}$ g/cm$^3$). The repulsive nucleon-nucleon interactions give rise to a bounce of the central core and the creation of a shock wave propagating through the in-falling stellar material. Many supernova simulations show that soon after the starting, the bounce shock turns into a stalled shock wave due to energy loss by dissociation of heavy nuclei in the in-falling material and by neutrino emission [7, 8]. If, at a later evolution, this shock wave will revive and produce a matter flow with positive velocities, it will lead to the ejection of the star’s envelope observed as a supernova explosion. During the collapse and subsequent explosions, the temperatures and the densities reach $T \approx (0.5–30)$ MeV.
and $\rho \approx (10^{-10} - 3)\rho_0$, respectively. It is expected that the nuclear statistical equilibrium is established under these conditions.

As demonstrated by several studies (see, e.g., refs. [9, 10, 11, 12, 13]), present hydrodynamical simulations of core-collapse supernovae in spherical symmetry are not able to produce successful explosions for most of the progenitors, except for the smallest progenitor masses in the range of 8 to 10 $M_\odot$. [14, 15]. Multidimensional effects like fluid instabilities, convection and rotation can successfully lead to explosions [16, 17, 18, 19, 20], but presently the results of different groups have not yet converged. On the other hand, it is known that the nuclear composition, i.e., the mass fractions of nuclei and nucleons, is also extremely important for understanding the physics of supernovae. In particular, the weak reaction rates and energy spectra of emitted neutrinos are very sensitive to the presence of heavy nuclei (see, e.g., [21, 22, 23, 24]). The nuclear distributions are especially relevant for weak reactions. For example, the new treatments of electron captures on heavy nuclei in Refs. [25, 26, 27] and inelastic neutrino–nucleon (nuclei) scattering in Ref. [28] are all based on the distributions of nuclei. The effect of nuclear composition can be even more important if the properties of nuclei (for example, their symmetry energy) embedded in supernova medium will be modified and this will lead to a dramatic change in the rates of neutrino reactions and the electron absorption [29]. In this case the energy deposition from neutrino processes will essentially change, as well as the electron fraction, which will influence the dynamics of the collapse and explosion. Nuclear reactions in the supernova environment are also important, because supernova explosions may be considered as breeders for creating chemical elements, heavier than Fe and Ni. Pronounced peaks in the element abundances can be explained by neutron capture reactions in s- and r-processes [30, 31]. It was always expected that suitable conditions for the r-process were provided by free neutrons abundantly produced in supernova environments together with appropriate seed nuclei. However, this is still a topic of current research, as typical core-collapse supernova simulations do not lead to a successful r-process. Alternatively, also neutron star mergers are considered as a source of r-process elements.

One of the initial EOSs for supernova matter, which is frequently used in supernova simulations, was proposed in Refs. [32, 33] many years ago. It includes nucleons, alpha particles, and heavy nuclei in statistical equilibrium. It is obtained under the assumption that the whole ensemble of hot heavy nuclei, i.e., with mass number $A$ larger than 4, can be replaced by a single “average” nucleus. The same assumption was also used in the EOS within a relativistic mean-field (RMF) approach given in Ref. [34]. Recent models [29, 35, 36, 37, 38, 39, 40], include ensembles with various nuclear species, which are most important for the accurate description of the neutrino transport during supernova explosion processes, though the distribution of heavy nuclei may have only a minor direct effect on thermodynamic quantities at some particular conditions [41].

Presently, it is under discussion if only the nuclei in long-lived states known from terrestrial experiments should be included in this ensemble (see refs. [36, 12, 13]), or particle-unstable states and new unknown exotic nuclei should be considered to have a contribution, as well [29, 35, 37, 38]. The last hypothesis is quite justified at temperatures higher than 1 MeV, when the nuclear shell structure is washed out and the liquid-drop
description becomes more adequate. Moreover, at high temperature and large baryon densities $\rho_B \approx 10^{-3} - 10^{-1} \rho_0$, the nuclei can still interact with surrounding species and experience thermal expansion. The properties of nuclei in such environments may differ significantly from the ones obtained in low-energy nuclear experiments. This is a challenging problem for both theoretical and experimental nuclear physics.

Properties of strongly-interacting nuclear matter have been studied experimentally and theoretically for a long time. On a qualitative level, the phase diagram of symmetric and neutron-rich nuclear matter is understood rather well (see, e.g., refs. [32, 49, 50]). It is commonly accepted that the nuclear phase diagram contains a liquid-gas phase coexistence line with a critical temperature $T_c$ about 10–20 MeV. In many models this phase transition is of first-order, if only nuclear matter is considered, and Coulomb interactions are ignored, see e.g., [51, 52]. Note that the formation of nuclei in astrophysical environment involves the Coulomb and other interactions existing in finite nuclei, where some features of the first order phase transition may be preserved. The critical temperature extracted from experiments on finite nuclei [53, 54] is in the range $T_c \approx 17 – 20$ MeV. In the present work we consider explicitly the ensemble of nuclei embedded in the background of nucleons and electrons. Still the comparison with the nuclear matter phase diagram gives important insight into the overall behavior of the supernova EOS.

In order to compare the thermodynamical conditions obtained in nuclear reactions and in supernovae, we show the phase diagram for symmetric and asymmetric nuclear matter in Fig. 1 for a range of densities and temperatures appropriate for core-collapse supernova explosions. The electron fraction $Y_e$, which is equal to the total proton fraction, in the supernova core varies from 0.1 to 0.5. In the two-phase coexistence region (below the solid and dot-dashed lines) at densities $\rho \approx 0.3 - 0.8 \rho_0$ the matter should be in a mixed phase, which is strongly inhomogeneous with intermittent dense and dilute regions. In the coexistence region at lower densities, $\rho < 0.3 \rho_0$ the nuclear matter breaks up into compact nuclear droplets surrounded by nucleons. These relatively low densities dominate during the main stages of stellar collapse and explosion. Under such conditions one can expect a mixture of nucleons and light and heavy nuclei.

In Fig. 1 we demonstrate also isentropic trajectories in nuclear matter (dashed curves) and the trajectories of density and temperature inside the supernova core, which are taken from the supernova simulation of the star with 15 solar masses [12] (red dotted curves). The snapshots in the supernova dynamics are selected for three stages before, at, and after the core bounce. In the gravitational collapse from the iron core of the star (BB: just before bounce), the density and temperature roughly follows the isentropic curves $S/B=1-2$. At the core bounce, when the central density increases just above the nuclear matter density $\rho_0$, the temperature of the inner core becomes higher than 10 MeV due to the passage of the shock wave (CB: core bounce). The temperature of the whole supernova core is still high at 150 ms after the core bounce (PB: post bounce). The shock wave is stalled around 130 km in this 1D calculation, which does not lead to an explosion.

These conditions inside the supernova core pass through interesting regions of the phase diagram. The BB and CB trajectories go through the multifragmentation region, which motivates us to assess the supernova conditions by the multifragmentation reactions.
taking place in heavy-ion collisions of intermediate energy. The trajectories after the bounce (CB and PB) traverse the phase boundary between the mixture of nuclei and the nucleon gas. It is also interesting that this region is dominated by light nuclei ($^4$He and lighter ones) \[38, 55, 56, 57, 58, 59\]. Since the dynamics of the shock wave is affected by the interaction of neutrinos with nucleons and nuclei, it is important to determine the composition of hot dense matter in this region. In fact, the heating through neutrino absorption contributes to the revival of shock wave together with the hydrodynamical instabilities in 2D and 3D supernova simulations. Large abundance of light nuclei ($A = 2, 3$) may contribute to the modifications of heating rate and/or energy spectrum through neutrino reactions with these nuclei \[60, 61\].

Our goal is to compare three different models for the EOS of stellar matter at sub-saturation densities. The calculations are done for wide intervals of $T$, $\rho$ and $Y_e$. All the three EOS models investigated in the present article, contain detailed information about the nuclear composition as they are built from a statistical distribution of different nuclear species. In the present paper we do not try to give preference to any of them, by keeping in mind that properties of nuclei in stellar medium and interactions between nuclear species in diluted matter are not known sufficiently well. We rather want to identify how the different model assumptions affect thermodynamic quantities and the nuclear composition. In the future we plan to construct a unified EOS which includes the components of these models which are best verified by theoretical and experimental studies. We believe that this kind of comparison of the EOSs will be very useful for people working on nuclear astrophysics. We clearly demonstrate the similarities and differences of the model results, and discuss their physical reasons. It is necessary to mention that all models treat electrons and photons in the same way, and differences come entirely due to a different description of nuclei and the underlying nuclear interactions. Therefore, if the photon and lepton contributions dominate, all models give similar results.

This article is organized as follows. In section 2, we explain the formalism of the three EOSs respectively. The comparison of results is presented in section 3, where we show predicted mass and isotope distributions, their moments and fractions, as well as thermodynamic quantities. The paper is wrapped up with a summary and some discussions in section 4.

## 2 Statistical models for supernova matter

First, we should remark about experimental possibilities for studying fragmented nuclear matter at subsaturation density. Traditionally, only theoretical approaches which use nuclear forces extracted from experimental study of nuclear structure were applied for this purpose. However, as became clear after last 20 years of intensive experimental studies, many nuclear reactions lead to the formation of thermalized nuclear systems characterized by subnuclear densities and temperatures of 3-8 MeV. De-excitation of such systems goes through nuclear multifragmentation, i.e. break-up into many excited fragments and
nucleons. We emphasize that thermal and chemical equilibrium can be established in many multifragmentation reactions, and it is generally accepted by the scientists involved in the field. Transport theoretical calculations of central heavy-ion collisions around the Fermi-energy (i.e., with energies of 20–50 MeV per nucleon) predicts momentum distributions of nucleons which are similar to equilibrium ones after first $\approx 100$ fm/c (see, e.g., [62] and other references in the topical issue [63]). In peripheral heavy-ion collisions there is a midrapidity region which can be associated with a dynamical fragment formation, however, the projectile and target excited residues represent thermal sources, which later on expand and break-up into many fragments. The final proof of equilibration was provided by numerous comparisons of statistical models with experimental data: These models perfectly describe many characteristics of nuclear fragments observed in multifragmentation experiments: multiplicities of intermediate mass fragments, charge and isotope distributions, event by event correlations of fragments (including fragments of different sizes), their angular and velocity correlations, and other observables [3, 4, 47, 48, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77]. The parameters of nuclear matter at the moment of formation of hot fragments, such as temperature and density, can also be established reliably in experiment by using observed relative velocities of fragments, and relative isotope yields [63]. Typical conditions associated with multifragmentation reactions are indicated by the shaded area in Fig. 1. These reactions give us a chance to access hot nuclei in the environment of other nuclear species in thermodynamical equilibrium as we expect in supernova matter. The properties of these nuclei can be directly extracted from experimental data and then this information can be used for more realistic calculations of nuclear composition in stellar matter. As one can estimate from Fig. 1 in the course of massive star collapse the stellar nuclear matter passes exactly through the multifragmentation region with typical entropy per baryon $S/B = 1 - 4$.

Below we consider three typical models for supernova matter, which are based on the assumption of statistical equilibrium: The Statistical Model for Supernova Matter (SMSM) [29, 37], the statistical model of Hempel and Schaffner-Bielich (HS) [38] and the statistical model of Furusawa, Yamada, Sumiyoshi and Suzuki (FYSS) [35]. In this section we briefly describe the models emphasizing their similarities and differences.

### 2.1 EOS – SMSM

This model is a generalization of well-known statistical multifragmentation model (SMM) [1] for astrophysical conditions. The SMM is one of the most successful models used for the theoretical description of multifragmentation reactions as demonstrated in the above cited publications, and it can be easily extended to describe clusterized stellar matter, what is not possible with other statistical models, e.g., MMMC [3], designed for finite nuclear systems only.

We give a brief description of the SMSM’s nuclear part, which is important for comparison with other models. The full description of the model including electron, photon, and neutrino contributions (in thermal equilibrium) one can find in Ref. [29]. Generally,
the system is characterized by the temperature $T$ and baryon density $\rho_B$ and electron fraction $Y_e = \rho_e/\rho_B$, where $\rho_e$ is the net electron density. The nuclear component of supernova matter is represented as a mixture of gases of different species $(A, Z)$ including nuclei and nucleons. It is convenient to introduce the numbers of particles of different kind $N_{AZ}$ in a normalization volume $V$. Then the total free energy density of this mixture can be written as

$$f = \frac{1}{V} \sum_{AZ} N_{AZ} \left\{ -T \cdot \left[ \ln \left( \frac{g^0_{AZ} V_f A^{3/2}}{N_{AZ} \lambda^2_T} \right) + 1 \right] + F_{AZ} \right\}$$ (1)

The first term comes from the translational motion of particles, and the second term is associated with the binding energy and excitation of nuclear fragments ($A > 4$). $g^0_{AZ}$ is the ground-state degeneracy factor of species $(A, Z)$, $\lambda_T = \left( \frac{2\pi \hbar^2}{m_N T} \right)^{1/2}$ is the nucleon thermal wavelength, $m_N \approx 939$ MeV is the average nucleon mass. $V_f$ is the so-called free volume, which accounts for the finite size of nuclear species. The SMSM does not include excited states explicitly, but incorporates a temperature dependence of the nuclear free energy $F_{AZ}$ of nuclei as will be presented below. We assume that all nuclei have normal nuclear density $\rho_0$, so that the proper volume of a nucleus with mass $A$ is $A/\rho_0$. At low densities the finite-size corrections can be included via the excluded volume approximation, $V_f/V \approx (1 - \rho_B/\rho_0)$. This approximation is commonly accepted in statistical models, and it is considered as a reasonable one at densities $\rho_B < 0.1 \rho_0$. Some information about the free volume at higher densities can be extracted from analysis of experimental data obtained in multifragmentation reactions [66]. Eq. (1) can also be written in the following form:

$$f = \frac{1}{V} \sum_{AZ} N_{AZ} \left\{ F_{AZ}^0 - T \ln(V_f/V) + F_{AZ} \right\}$$ (2)

where

$$F_{AZ}^0 = -T \cdot \left[ \ln \left( \frac{g^0_{AZ} V A^{3/2}}{N_{AZ} \lambda^2_T} \right) + 1 \right]$$ (3)

corresponds to the translational energy of an ideal gas without the excluded volume correction.

The numbers of nuclear species are constrained by the conditions for baryon number conservation and electro-neutrality

$$\rho_B = \frac{1}{V} \sum_{AZ} A N_{AZ} , \quad \rho_Q = \frac{1}{V} \sum_{AZ} Z N_{AZ} - \rho_e = 0$$ (4)

We have performed calculations within the Grand Canonical approximation. In this case the conservation laws (4) are fulfilled only for the mean values $< N_{AZ} >$. These values are obtained by minimizing the free energy (1) under constraints (4). The resulting expression is

$$< N_{AZ} > = g^0_{AZ} V_f A^{3/2} \lambda^3_T \exp \left[ -\frac{1}{T} (F_{AZ} - \mu_{AZ}) \right]$$ (5)
where
\[ \mu_{AZ} = A\mu_B + Z\mu_Q \]  
(6)
is the chemical potential of species \((A, Z)\), \(\mu_B\) and \(\mu_Q\) are chemical potentials responsible for the baryon number and charge conservation. For protons \(\mu_p = \mu_B + \mu_Q\), for neutrons \(\mu_n = \mu_B\). Below we use average densities \(\rho_{AZ} = \langle N_{AZ} \rangle / V\).

The internal excitations of nuclei play an important role in regulating fragment’s abundance, since they increase significantly their entropy. We calculate the internal excitation energy of nuclei by assuming that they have the same internal temperature as the surrounding medium. In this case not only particle-stable states but also particle-unstable states will contribute to the excitation energy and entropy. This assumption can be justified by the dynamical equilibrium of nuclei in the hot environment [29], and is supported by both many comparisons of SMM with experimental data on multifragmentation and direct experimental measurements [78]. Moreover, in the supernova environment both the excited states and the binding energies of nuclei will be strongly affected by the surrounding matter. By this reason, we find it more appropriate to use an approach which can easily be generalized to include in-medium modifications of nuclear properties. Namely, the internal free energy of species \((A, Z)\) with \(A > 4\) is parameterized in the spirit of the liquid-drop model, which has been proved to be very successful in nuclear physics [1, 4]:

\[ F_{AZ}(T, \rho) = F^B_{AZ} + F^S_{AZ} + F^{\text{sym}}_{AZ} + F^C_{AZ}. \]  
(7)

Here the right-hand side contains, respectively, the bulk, the surface, the symmetry and the Coulomb terms. The first two terms are temperature-dependent and are motivated by properties of nuclear matter corresponding to the liquid-gas phase transition [3], the third one is temperature independent:

\[ F^B_{AZ}(T) = \left( -w_0 - \frac{T^2}{\varepsilon_0} \right) A, \]  
(8)
\[ F^S_{AZ}(T) = \beta_0 \left( \frac{T^2_c - T^2}{T^2_c + T^2} \right)^{5/4} A^{2/3}, \]  
(9)
\[ F^{\text{sym}}_{AZ} = \gamma \frac{(A - 2Z)^2}{A}. \]  
(10)

Here \(w_0 = 16\) MeV, \(\varepsilon_0 = 16\) MeV, \(\beta_0 = 18\) MeV, \(\gamma = 25\) MeV, and \(T_c = 18\) MeV are the model parameters which are extracted from nuclear phenomenology and provide a good description of multifragmentation data [4, 65, 66, 67, 68, 69, 70]. However, these parameters can be easily adopted to the new conditions expected in stellar environment. Nucleons and light fragments with \(A \leq 4\) are considered as structure-less particles characterized only by exact masses and proper volumes [4]. For them we adopt \(F_{AZ} = -B_{AZ} + F^C_{AZ}\), where \(B_{AZ}\) is the measured binding energy. In the electrically-neutral environment the fragment Coulomb energy \(F^C_{AZ}\) is modified by the screening effect of electrons. In the SMSM it is calculated by using the Wigner-Seitz approximation [29, 32] for all fragments and charged
particles (including protons).

\[
F_{AZ}^C(\rho) = \frac{3}{5} c(\rho) \frac{(eZ)^2}{r_0 A^{1/3}}, \tag{11}
\]

\[
c(\rho) = \left[ 1 - \frac{3}{2} \left( \frac{\rho_e}{\rho_0p} \right)^{1/3} + \frac{1}{2} \left( \frac{\rho_e}{\rho_0p} \right) \right], \tag{12}
\]

where \( r_0 = 1.17 \text{ fm} \) and \( \rho_0p = (Z/A)\rho_0 \) is the proton density inside the nuclei. The screening function \( c(\rho) \) is 1 at \( \rho_e = 0 \) and 0 at \( \rho_e = \rho_0p \). To simplify calculations one can use an approximation \( \rho_e/\rho_0p = \rho_B/\rho_0 \), as in ref. \[32\], which works well when neutrons are mostly bound in nuclei, and leads to very similar results in many cases. The average densities of all nuclear species are calculated self-consistently by taking into account the relations between their chemical potentials. We perform calculations for all fragments with \( 1 \leq A \leq 1000 \) and \( 0 \leq Z \leq A \). This restriction on the size of nuclear fragments is fully justified in our case, since fragments with larger masses \( (A > 1000) \) can be produced only at very high densities \( \rho \gtrsim 0.3\rho_0 \) \[6, 32\], which are appropriate for regions deep inside of protoneutron stars, and which are not considered in this model. The SMSM EOS tables (like Shen \[34\] or HS \[38\]) are currently under construction for publication, and will be available in the Internet soon.\[2\]

2.2 EOS – HS

In the HS model for the EOS, matter is described as an ensemble of nucleons and nuclei in nuclear statistical equilibrium (NSE), whereas interactions of the nucleons and excluded volume corrections are implemented. We remark that tabulated versions of the EOS-HS have recently been applied in core-collapse supernova simulations \[55\]. These tables are available online for five different parameterizations of relativistic mean-field (RMF) interactions.\[3\] In Ref. \[57\] it was shown that the HS model gives a similar description of the medium effects on light clusters like two quantum many-body models. This model is in good agreement with experimental data for the equilibrium constant \( K_c \) at densities between 0.02 and 0.03 \( \text{fm}^{-3} \) and temperatures around 10 MeV \[79\]. In the following, we give a brief summary of the HS model, all details can be found in Ref. \[38\].

In the HS model nuclei are treated as non-relativistic classical particles with Maxwell-Boltzmann statistics. For the description of nuclei the experimentally measured masses from the atomic mass table 2003 from Audi, Wapstra, and Thibault \[80\] are used. Shell effects are thus naturally included. In addition, for the masses of experimentally unknown nuclei we use results of theoretical nuclear structure calculations via the nuclear mass table of Geng et al. \[81\]. We have chosen this mass table because it was calculated with the TMA interactions, which are very similar to the TM1 interaction used in the paper for the unbound nucleons. Other mass tables can also be used, however, we have found that

\[2\]See http://fias.uni-frankfurt.de/physics/mishus/research/smsm/

\[3\]See http://phys-merger.physik.unibas.ch/~hempel/eos.html.
the selection of the mass table which provides very similar ground state masses (with the differences around \( \approx 1 \text{ MeV} \)) does not change the overall behavior of the EOS model and its main characteristic features. This mass table lists 6969 even-even, even-odd and odd-odd nuclei, extending from \(^{16}\text{O}\) to \(^{331}\text{Fm}\) from slightly above the proton to slightly below the neutron drip line. However, nuclei beyond the neutron drip line have been excluded in the EOS calculation of HS. This is done because nuclear structure calculations are not very reliable beyond the neutron drip-line, and to have a clear physical criteria which nuclei to be included. The total binding energy of nucleus \((A, Z)\) will be denoted \(B_{AZ}\) hereafter. The total mass of a nucleus is set by its rest-mass and its binding energy, \(M_{AZ} = M_0^{AZ} - B_{AZ}\).

Differently than in SMSM, temperature effects in HS are implemented via a temperature-dependent internal partition function \(g_{AZ}(T)\). It represents the sum over all excited states of a hot nucleus. The semi-empirical expression from Ref. [82] is used:

\[
g_{AZ}(T) = g_{AZ}^0 + \int_0^{E_{max}} dE^* e^{-E^*/T} \exp\left(\sqrt{2a(A)E^*}\right),
\]

with \(g_{AZ}^0\) denoting the spin-degeneracy of the ground-state, as before in SMSM. In the original reference of the HS model [38] \(E_{max}^{AZ} = \infty\) was chosen. For the supernova simulations in Ref. [55] HS changed the value of \(E_{max}^{AZ}\) to the binding energy of the nucleus, \(E_{max}^{AZ} = B_{AZ}\), which is also used here. It means that only excited states are considered which are still bound, to avoid an overestimation of internal excitation energies at large temperatures. It is well known that in the low temperature limit Eq.(13) leads to the \(T^2\) term in the free energy, as also assumed in the SMSM, Eq.(8). However, in the SMSM the level density parameter \(a\) is taken about twice smaller than the empirical value (\(\approx A/8 \text{ MeV}^{-1}\)) because of the additional contribution from the surface term, Eq.(9).

In the HS model, nuclear matter is described as a chemical mixture of different nuclear species and nucleons. To assure the disappearance of nuclei above saturation density \(\rho_0\), an excluded volume approach is used. Like in SMSM, this is done via the free volume fraction \(V_f/V\). However, in HS the treatment of unbound nucleons is different from Boltzmann particles as taken in SMSM. In HS and also in FYSS, interactions of the unbound nucleons are taken into account with an RMF model. The RMF parameter set TM1 was chosen [83] in order to use the same approach for the nucleons as in the EOS of Shen et al. [34]. Nucleons are assumed to be situated outside of nuclei, and therefore the filling factor of the nucleons \(\eta = 1 - \sum_{AZ} A\rho_{AZ}/\rho_0\) is introduced. It relates the total number densities of neutrons \(\rho_n\) and protons \(\rho_p\), respectively, with the local number densities outside of nuclei \(\rho'_n\) and \(\rho'_p\), by \(\rho_{n/p} = \rho'_{n/p}\eta\).

Based on these assumptions, HS derive the following free energy density \(f\):

\[
f = \eta_f^{RMF}(T, \rho'_n, \rho'_p) + \sum_{A>1,Z} \rho_{AZ} \left(F_{AZ}^{00} - T \ln(V_f/V) + F_{AZ}\right).
\]

The part with the sum over experimentally known and theoretically calculated nuclei \((A, Z)\) with \(A > 1\), has formally the same structure as in the free energy density of the
SMSM (Eq. (2)). Still there are some important differences, which we want to discuss now. $F_{AZ}^{t0}$ is the translational free energy of a heavy nucleus:

$$F_{AZ}^{t0} = -T \cdot \left[ \ln \left( \frac{g_{AZ}(T)}{\rho_{AZ} \lambda_{AZ}^3} \right) + 1 \right], \quad \lambda_{AZ} = \left( \frac{2\pi\hbar^2}{M_{AZ} T} \right)^{1/2},$$

(15)

which is similar as in the SMSM, with the negligible difference that the masses $M_{AZ}$ appear instead of $m_N$. However, the use of $g_{AZ}(T)$ in HS instead of $g_{AZ}^0$ in SMSM is an important difference, because $g_{AZ}(T)$ carries the main temperature dependence of heavy nuclei in HS. The excluded volume term $-T \ln(V_f/V)$ in Eq. (14) is the same as in the SMSM. The internal free energy of the nucleus $(A, Z)$,

$$F_{AZ} = -B_{AZ} + F_{AZ}^{C},$$

(16)

is significantly different compared to the liquid-drop formulation of the SMSM (see Eq. (7)), mainly because of the binding energies $B_{AZ}$ which are based on experimental data and nuclear structure calculations. In SMSM the free energy includes a temperature dependence in agreement with properties of matter in the region of nuclear liquid-gas phase transition. The Coulomb energy $F_{AZ}^{C}$ is described as in the SMSM, except that the constant part of the heavy nucleus is already included in $B_{AZ}$. The contribution of unbound nucleons in HS (the first term in Eq. (14)) is described separately by the RMF model with excluded volume corrections. Here the filling factor $\eta$ appears in front of the pure RMF contribution of the nucleons $f_{RMF}$ which is set by their local number densities. The filling factor $\eta$ and the other excluded volume term $-T \ln(V_f/V)$ play an important role in the HS model, because they assure the disappearance of nuclei and a continuous transition to uniform nucleon matter at high densities. The abundances of all nuclei and unbound nucleons are determined by the chemical equilibrium condition (6) and the conservation laws (4). All other thermodynamic variables are then derived from Eq. (14) in a thermodynamic consistent way.

### 2.3 EOS – FYSS

The formulation of the FYSS model is based on the NSE description using the mass formula for nuclei up to the atomic number of 1000 under the influence of surrounding nucleons and electrons. The mass formula is based on experimental data on nuclear binding energies that allow us to take into account nuclear shell effects. An extended liquid-drop model is used to describe the medium effects, and in particular, formation of the pasta phases. Because of this combination, the free energy of a multi-component system can reproduce the ordinary NSE results at low densities and make a continuous transition to the EOS for supra-nuclear densities. The details are given in Ref. [35]. FYSS model is being improved in the important points which appear in this comparison work.

Below, we give a short description of FYSS EOS. Assuming NSE, the abundances of nuclei as a function of $\rho_B, T$ and $Y_p$ are calculated by minimizing the model free energy
with respect to the number densities of nuclei and nucleons under the constraints (4), and (6). The free energy density of FYSS consists of contributions from unbound nucleons outside nuclei and the summation of translational \( F_{AZ}^t \), bulk \( F_{AZ}^B \), Coulomb \( F_{AZ}^C \) and surface energies \( F_{AZ}^S \) of all nuclei \((Z \leq 1000, N \leq 1000)\).

The free energy density of the unbound nucleons, \( f_{p,n} \), is calculated by the RMF with TM1 parameter set, which is the same as Ref. [34]. FYSS takes into account the excluded-volume effect of free nucleons through \( f_{p,n} = \eta f_{RMF}(T, \rho'_n, \rho'_p) \). This formula of the free energy density of the unbound nucleons is the same as in the HS model. On the other hand, the excluded volume effect for nuclei is different from that of the HS and SMSM models. FYSS assumes the nuclear translational motion contribution is calculated from Maxwell-Boltzmann statistics, however, the translational free energy of nuclei are suppressed by an additional volume factor \( F_{AZ}^t = F_{AZ}^t_0 V_f/V \). Furthermore, \( F_{AZ}^t_0 \) is slightly different from the expression in HS, because the mass term appearing in the thermal wavelength \( \lambda_{AZ} \) contains liquid-drop modifications [35]. The contribution from the excited states of nuclei to the free energy is included in the \( F_{AZ}^t_0 \) through \( g_{AZ}(T) \). This is the same formula as the HS model but the upper limit of the integral is set to infinity, \( E_{AZ}^{\text{max}} = \infty \), being different from the HS model.

To obtain the bulk energy of the nuclei, the experimental mass data [80] are used at low densities whenever available. These experimental mass data are the same as in HS, but the theoretical data [81] included in HS are not used in FYSS. At high densities, where the nuclear structure is affected by the presence of other nuclei, nucleons and electrons, the bulk energy of the nuclei is approximated by interpolation of the value obtained experimentally and the value derived theoretically from the RMF between \( 10^{12} \text{g/cm}^3 \) and the nuclear saturation density, \( \sim 10^{14.2} \text{g/cm}^3 \). The experimental and theoretical bulk energies are combined by the relation \( F_{AZ}^B = M_{AZ} - [F_{AZ}^C]_{\text{vacuum}} - [F_{AZ}^t]_{\text{vacuum}} \) and \( F_{AZ}^B = AF_{RMF}(\rho_{0AZ}, T, Z/A) \) where \( \rho_{0AZ} \) is the saturation density of the nuclei. \( F_{RMF}(\rho_B, T, Y_p) \) is the free energy per baryon predicted by the RMF. \( \rho_{0AZ}(T) \) is set to the saturation density to have the lowest free energy of the RMF theory \( F_{RMF}(\rho_B, T, Z/A) \) for given \( T \) and \( Z/A \). More neutron-rich nuclei at higher temperature have lower saturation density. Note that the proton fraction in this expression is not the one for the whole system but the one for each nucleus and that this bulk energy includes the symmetry energy. For very heavy and/or very neutron-rich nuclei with no experimental mass data available, the RMF is used for the evaluation of the bulk energy at any density.

As in SMSM and HS models, the Coulomb energy of nuclei is calculated using the Wigner-Seitz approximation. The outside unbound protons are included in the charge neutrality condition in addition to bound protons inside nuclei and uniformly distributed electrons.

The surface energy of nuclei is given by the product of the nuclear surface area and the surface tension.

\[
F_{AZ}^S = 4\pi R_{AZ}^2 \sigma_{AZ} \left(1 - \frac{\rho'_p + \rho'_n}{\rho_0}\right)^2
\]  

(17)
\[ \sigma_{AZ} = \sigma_0 - \frac{A^{2/3}}{4\pi R_{AZ}^2} [S_s(1 - 2Z/A)^2], \]  

(18)

where \( R_{AZ} = (3/4\pi V_{AZ})^{1/3} \) is the radius of nucleus \((A, Z)\) at baryon density \( \rho \) and \( \sigma_0 \) denotes the surface tension for symmetric nuclei. The surface tension \( \sigma_{AZ} \) includes the surface symmetry energy, where neutron-rich nuclei have lower surface tensions than the symmetric nuclei. The values of the constants, \( \sigma_0 = 1.15\text{MeV/fm}^3 \) and \( S_s = 45.8\text{MeV} \), are adopted from Ref. [33]. The last factor, \( \left(1 - (\rho_p' + \rho_n')/\rho_0\right)^2 \), is introduced to take into account the effect that the surface energy should be reduced as the density contrast decreases between the nucleus and the nucleon vapor. This surface energy has a dependence on the neutron-richness of nuclei and density of outside unbound nucleons. Note that contrary to the SMSM this model has no temperature dependence of the surface free energy.

The free energy density of the FYSS model is

\[ f = \eta f^{RMF}(T, \rho'_n, \rho'_p) + \sum_{AZ} \rho_{AZ} \left\{ F_{AZ}^{0} V_f / V + F_{AZ} \right\}. \]  

(19)

The last term in the summation is the internal free energy of the nucleus \((A, Z)\);

\[ F_{AZ} = F_{AZ}^B + F_{AZ}^C + F_{AZ}^S. \]  

(20)

The bulk energies include the symmetry energies as previously stated and this internal free energies \( F_{AZ} \) are equal to the experimental mass of the nucleus in the vacuum limit. Other thermodynamical quantities can be calculated from partial derivative of the minimized free energy explained above. In brief, the special feature of this model is to include in the calculations nuclear shell effects and very heavy nuclei \((Z > 100)\) by using the experimental mass data and the theoretical mass formula. Furthermore, FYSS assumes that each nucleus enters the nuclear pasta phase individually when the volume fraction in the Wigner-Seitz cell \( u_{AZ} \approx (A/\rho_{0AZ}(T))/(Z/\rho_e) \), i.e., the ratio of the nuclear volume by the cell volume, reaches 0.3 and that the bubble phase is realized when it exceeds 0.7. The intermediate states \((0.3 < u_{AZ} < 0.7)\) are simply interpolated as other pasta phases. Under this assumption more neutron-rich nuclei go into the pasta phases at lower densities than symmetric nuclei, since the volume fractions of neutron-rich nuclei \( u_{AZ} \) are larger than that of symmetric nuclei. At high temperatures, the saturation densities \( \rho_{0AZ}(T) \) are low and the volume fractions \( u_{AZ} \) are large compared with \( u_{AZ} \) at low temperature and the same density.

### 3 Comparison of results

In this section we compare predictions of the three models for the nuclear composition and general thermodynamical properties of stellar matter. We present the results of SMSM, HS and FYSS for the EOS at baryon densities \( \rho/\rho_0 = 10^{-3}, 10^{-2} \) and \( 10^{-1} \), and
temperatures $T = 0.5–10$ MeV. Since the full $\beta$-equilibrium is unlikely to be established in a supernova, we have adopted the fixed electron fractions $Y_e = 0.2$ and 0.4, which are typical for this scenario. The calculations with $\beta$-equilibrium, which may be appropriate for a neutron star crust, one can find, e.g., in Refs. 29 37.

### 3.1 Mass distributions

Let us start with the analysis of mass distribution of nuclear species produced in stellar matter. Detailed comparison of mass distributions predicted by our models is presented in Figs. 2–7. It includes yields at various densities $\rho$, electron fraction $Y_e$ and temperature $T$. These distributions contain important information about fragmentation of matter in the nuclear liquid-gas phase transition (coexistence region). The concept of statistical equilibrium assumes a continuous interaction between fragments via specific microscopic processes, like absorption and emission of neutrons, which provide equilibration (see, e.g., discussion in Ref. 29). In this situation the nuclei can remain hot and have modified properties and masses, which are different from the cold isolated nuclei.

Figs. 2 and 3 present results obtained at the lowest density under investigation $\rho/\rho_0 = 10^{-3}$. In this case the residual interaction between nuclear species should be minimal, though in-medium mass modifications are still possible. At very low temperatures the SMSM predicts a Gaussian-like distribution for heavy nuclei. In this case an approximation of a single heavy nucleus adopted in the Lattimer and Swesty 33 and Shen et al. 34 EOSs may work reasonably well for calculations of thermodynamical characteristics of matter. However, already at $T \gtrsim 1$ MeV the gap between the Gaussian peak and light clusters and nucleons is essentially filled by nuclei of intermediate masses leading to characteristic U-shaped distributions. These distributions are also typical for the onset of the liquid-gas phase transition in finite nuclei 4. On the other side of the Gaussian peak for big fragments, there is an continuous exponential fall of fragment yields with $A$. The difference from a single nucleus case is even more evident if we include nuclear shells, as done in HS and FYSS models.

We remind that shell effects in masses of cold isolated nuclei may survive in nuclei at low temperatures ($T < 1–2$ MeV) and low densities of matter. This can be seen in the distributions of HS and FYSS in Fig. 2. The peaks of the distributions occur around the well-known neutron magic numbers due to the increased binding energies. For most conditions the results of HS and FYSS look similar to the Gaussian distributions of SMSM with additional peaks on top. Only for $T = 0.5$ and 1 MeV with $Y_e = 0.2$ some interesting features occur. For $T = 1$ MeV and $Y_e = 0.2$, the yields of FYSS are similar to those of HS up to $A \sim 90$. In the intermediate mass range $90 < A < 130$ the yields of HS are several orders of magnitude larger. These are the neutron-rich nuclei contained in the theoretical nuclear structure calculations of Geng et al. 81, but which are not in the experimental compilation of Audi et al. 80. The jump around $A \sim 130$ in FYSS is caused by the transition to the liquid-drop formulation for exotic nuclei. The differences visible for $T = 0.5$ MeV and $Y_e = 0.2$ can be explained in the same way. For these conditions,
the HS model mainly contains nuclei with binding energies from the theoretical nuclear structure calculation, which are absent in FYSS. Because the nuclei found in FYSS are not yet described by the liquid-drop model, this leads to a sharp peak for a certain nucleus with experimentally measured binding energy. The drop in HS around $A \sim 120$ occurs because extremely neutron-rich nuclei with large mass numbers are not included. This drop does not occur for $T = 1$ MeV, because the temperature increases the unbound neutron mass fraction and thereby decreases the asymmetry of heavy nuclei. For the larger values of $Y_e = 0.4$ in Fig. 2 no unexpected features occur, because nuclei have a smaller asymmetry and they are well inside the region of nuclei with measured binding energies.

At temperature $T = 2$ MeV we obtain plateau-like mass distributions in all three models. They are well known from nuclear multifragmentation studies [4] and can be connected to the liquid-gas phase transition. Furthermore, it is impossible to describe it with a single nucleus approximation. As shown in Ref. [29] this phase transition can be driven by both temperature and density. Besides changing mass distributions qualitatively, at this point one can observe other critical phenomena as maximum heat capacity (i.e., a plateau-like behavior of the caloric curve), a minimum of exponent $\tau$ in the power-law mass distribution $A^{-\tau}$ of intermediate mass fragments, large fluctuations in fragment sizes, etc., which exist in both finite and infinite systems (see, e.g., discussions in Refs. [70, 84, 85]).

At high temperatures ($T \geq 3$ MeV) all models predict disintegration of nuclear matter into small fragments and their yields decrease exponentially with mass number $A$. These results demonstrate that the transition from heavy nuclei (droplets of nuclear liquid) to lightest fragments and nucleons (nuclear gas) always proceeds through the same sequence of mass distributions: U-shape, power-law, and exponential fall-off, both with increasing temperature and decreasing density. Presence of mass shell effects in nuclei and other differences in their description do not influence this general evolution. For example, larger yields of heavy fragments in SMSM shown in Fig. 3 can be explained by a difference from other models in the calculation of binding energies and of the internal excitation energy of intermediate-mass fragments leading to their higher entropy. For $A \leq 4$, where all the models use almost the same binding energies, the yields are rather similar.

As was mentioned, at larger densities the mean-field effects start to play an important role in the HS and FYSS cases, and this influences the yields of nuclei. However, as we see from Figs. 4–7 the general trends for the mass distributions do not change. It is natural that at high density heavy nuclei can be produced, up to $A \sim 500$. The temperature associated with the plateau-like behavior of mass distributions increases to $\approx 3$ MeV at $\rho/\rho_0 = 10^{-2}$, and to 4–6 MeV at $\rho/\rho_0 = 10^{-1}$ (as expected, at large $\rho$ it becomes more sensitive to the model). The temperature observed experimentally in multifragmentation reactions is around $T \approx 5$ MeV [85].

Apart from the overall similar behavior some interesting new features can be noticed in Figs. 4–7. In the upper and middle right panels of Fig. 4 HS and FYSS give distributions with two separate peaks, corresponding to nuclei with magic neutron numbers 50 and 82. Obviously such a bimodal distribution cannot be captured by the average value or by a single representative heavy nucleus. The upper left panel of Fig. 4 shows the mass yields
for very asymmetric, cold nuclear matter. Compared to the lower density shown in the upper left panel of Fig. 2, the neutron yield has further increased and neutrons start to become degenerate. This would occur e.g. in the crust of a neutron star. In HS, shell effects are still dominant, leading to even sharper peaks than in Fig. 2 because of the decreased role of temperature at higher density. Furthermore, for such high neutron abundances there are only few nuclei in the mass tables of HS with a suitable asymmetry. Note that the most abundant nucleus is at a similar $A$ for the maximum yield of the SMSM. Contrary to HS, the nuclei of FYSS are described by the liquid-drop formula, leading to Gaussian distributions like in the SMSM, but with smaller masses due to reduced surface energy.

It is interesting to investigate differences between calculations with electron fractions $Y_e=0.2$ and 0.4. The calculations with large electron fraction (respectively total proton fraction) are more reliable, since nuclei with the corresponding $Z/A$ ratios have been studied in experiments. However, because of the electron capture at subnuclear densities, nuclei with low proton fractions may occur in supernova matter too. Regarding core-collapse supernovae, heavy nuclei are most important for the collapse phase before bounce. Here one has typically moderate asymmetries of $0.2 \lesssim Y_e \lesssim 0.4$, see e.g. Refs. [9, 12, 55]. After the shock has formed, the temperature increases significantly and heavy nuclei are dissociated, as we also find in our comparison. In the late post bounce phase there appear regions with low $Y_e \sim 0.1$ in the shock-heated matter, but then the mass fraction of heavy nuclei is rather small, around $10^{-3}$ to $10^{-2}$. Therefore neutrino reactions with unbound neutrons and protons are more important. In the later stages of the protoneutron stars’ cooling large asymmetries and moderate temperatures can be realized together, when the system approaches the equilibrium configuration of the neutron stars’ inner crust. At a small electron fraction the SMSM can give very large and very neutron-rich nuclei which are not present in HS and FYSS tables. Due to the limitation of mass tables, one can see clear cuts of mass distributions in the HS calculations. The SMSM considers the whole ensemble of nuclei produced in stellar matter, without any additional constraint on their masses and charges. The universal liquid-drop approximation is used to describe their properties, and this is the reason why all SMSM distributions are smooth. The FYSS model also considers all possible exotic nuclei and they are calculated by a liquid-drop description when the mass or charge number of a nucleus is not contained in the table. As a result of the transition to the liquid-drop description some FYSS distributions becomes also smooth at low $Y_e$. It is also interesting that the mass numbers of the most abundant nuclei are different between the SMSM and FYSS model for $Y_e = 0.2$ as shown in Fig. 6 and Fig. 7. This results from the difference of the two liquid-drop descriptions. The surface tensions of nuclei of SMSM depend on temperature but do not depend on neutron-richness of nuclei. On the other hands FYSS assumed the surface tension depending on neutron-richness but independent of temperature. For example, the lower surface tensions of neutron-rich nuclei of FYSS increase abundances of lighter nuclei in neutron-rich environment with $Y_e = 0.2$, as shown in the left panel of Fig. 6.

We have already mentioned that the problem of the adequate description of nuclei under these extreme conditions, leading to modified binding energies and other properties, should be addressed in future studies. In this work we have taken into account only some
of them which were generally discussed previously: the screening of Coulomb energy is calculated in the Wigner-Seitz approximation is considered in all three models, the temperature dependence of bulk and surface energies, as well as disappearance of shell effects at high excitations are taken into account in SMSM, the density dependence of the bulk and surface energies is included in the FYSS. Other modifications are now under intensive theoretical and experimental investigation in multifragmentation reactions, for example, a decrease of the symmetry energy of fragments \[16\,17\,18\], and reduction of the binding energies of light clusters \[56\,87\,88\] in-medium. A calculation within a self-consistent microscopic approach has been performed recently in \[89\]. Presently, we can only declare significant differences between mass distributions of SMSM, HS and FYSS models, which use different assumptions on properties of fragments, especially at low electron fractions, low temperatures and high densities. We note that these differences are also manifested in the behavior of nuclear pressure, see Fig. 22 (a)-(b).

3.2 Mass fractions

The mass fractions of light and heavy nuclei can be easily calculated from mass distributions presented above. The light nuclear species \((A \leq 4)\) present the gas phase of nuclear matter, and they are mainly responsible for the nuclear pressure at high temperatures, together with unbound nucleons. In addition, many important reactions involve nucleons and alpha-particles, so it is crucial to know their abundances in stellar matter. We show the mass fractions of neutrons \(X_n\), protons \(X_p\), and alpha particles \(X_\alpha\) in Figs. 8, 9, and 10, respectively. All three models give similar results, except for \(\rho/\rho_0 = 10^{-1}\) and \(Y_e=0.2\) and at higher temperatures \(T > 5\) MeV and \(Y_e = 0.4\). In Fig. 8 it is clear that the number of free neutrons decreases with increasing density, reflecting the formation of very heavy nuclei and the transition to the liquid phase at \(\rho \rightarrow \rho_0\). The matter is mainly composed of heavy nuclei at low temperatures \((T < 1\) MeV), however, the free neutrons are also present for small \(Y_e = 0.2\) (top panel of Fig. 8), since the nuclear symmetry energy suppresses accumulation of neutrons in heavy nuclei. This can also be linked to the difference between the proton and neutron chemical potentials, which is much larger for \(Y_e = 0.2\) than for \(Y_e = 0.4\), see Figs. 23 and 24.

With increasing temperature heavy nuclei gradually disintegrate into \(\alpha\)'s, neutrons and protons. For this reason in Fig. 10 one can see a so-called "rise-and-fall" behavior of \(X_\alpha\), which occurs actually for both increasing temperature and decreasing density. By disintegration of nuclei one can also explain an increase of mass fractions of protons \(X_p\), which should reach \(Y_e\) values at very high temperatures (Fig. 8). We remark that other light nuclei, like deuterons or tritons, can appear with large abundance. For \(T \geq 5\) MeV and at sufficiently low densities these light nuclei can be even more abundant than alpha particles \[29\,87\,55\,56\,57\,58\,59\]. In the present study, to be consistent with other works, we only show the mass fraction of alpha particles. The information from mass fractions is complementary to the one from mass distributions. It shows that at low densities like \(\rho/\rho_0 = 10^{-3}\) the disintegration of matter into light particles happens already

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at temperatures \( T \approx 1 - 2 \) MeV, while at subnuclear densities \( (\rho \approx 10^{-1}\rho_0) \) some heavy nuclei survive even at high temperatures, though they become very excited. We show the mass fraction of heavy nuclei \( X_{\text{heavy}} (A > 4) \) in Fig. 11. At higher temperatures all EOS models give essentially different results. It is important to note that these “heavy” nuclei at high temperatures are actually close to the lower bound \( A = 5 \), i.e., they are light and intermediate mass nuclei. At high temperature the fractions of heavy nuclei in SMSM are higher than in HS and FYSS, since SMSM takes into account the temperature dependences of bulk and surface energies of such nuclei. The FYSS and HS also include the contribution to the free energy due to the internal excitations, but only in the bulk term. In SMSM the surface contribution is important and it increases for light nuclei.

3.3 Moments of mass and charge distributions

One can define a mean mass number of the heavy fragments \( \langle A_h \rangle \) (taking into account only nuclei with \( A > 4 \)). Actually, in the case of a sharp Gaussian mass distribution, it can be used for characterization of the nuclear liquid phase. It is also important for comparison of our approaches with models assuming a single nucleus approximation. Souza et al. have estimated [90] that the mass number of the single nucleus approximation is systematically over-predicted compared to the average of a nuclear distribution calculated with otherwise the same nuclear physics inputs. We have analyzed the \( \langle A_h \rangle \) evolution with respect to different temperatures for various densities and electron fractions in Fig. 12. As expected, these values decrease with \( T \), and then, beyond some value, decrease rather slowly, and go to a nearly constant value. This suggests that the vaporization process becomes dominant around this point. It can be expected that all models should approach the limiting value of \( \langle A_h \rangle = 5 \) with increasing temperature. For the temperatures shown in Fig. 12 this happens in all models, but only for the two lower densities. Note that the mass fractions of these nuclei are significantly decreased at the same time, see Fig. 11. For \( \rho/\rho_0 = 0.1 \), the average mass numbers remain larger than five for all models even at \( T = 10 \) MeV. Here it is found that FYSS gives the largest \( \langle A_h \rangle \) around 10. These differences are a result of the different temperature and mass dependence of the free energies of nuclei in the models, as was discussed previously: In SMSM this is done via the temperature dependence of the liquid-drop formula, and in HS via the temperature-dependent internal partition function. In FYSS also the temperature-dependent internal partition function is used, but without an integration cut for high excitation energies. Furthermore, at large densities when nuclei are affected from the liquid-drop modifications of FYSS, there is an additional temperature dependence of the bulk energy of nuclei, calculated from the RMF model. In Fig. 13 we show the average charge of heavy nuclei \( \langle Z_h \rangle \), which shows a similar trend as Fig. 12.

The standard deviation of the average mass of heavy nuclei, \( \sigma_{A_h} = \sqrt{\langle A_h^2 \rangle - \langle A_h \rangle^2} \) is shown in Fig. 14. This figure gives us valuable information about the character of the nuclear distributions. It is seen for higher densities that all three models have a maximum around \( T = 5 \) MeV which exactly corresponds to the temperatures obtained
in investigations of nuclear multifragmentation of finite nuclei. The values of \( \sigma_{A_h} \) in this region vary between 50 and 100 for different models. Interestingly, the mass fractions of heavy nuclei are still dominant for these conditions. This large spread in the mass distribution could be important for neutrino reactions in core-collapse supernova. For lower densities the maximum \( \sigma_{A_h} \) values shift to lower temperatures, which correspond to plateau-like mass distributions discussed above. Again, all EOSs demonstrate a similar behavior except for higher density \( \rho/\rho_0 = 10^{-1} \).

3.4 Isotopic distributions

To get further insight into characteristics of nuclear species we have investigated the isotopic yields for selected elements with charges \( Z = 8, 26, \) and 50 described by the three models, see Figs. 15–20. We have selected only the cases, where the yields of isotopes are larger than \( \sim 10^{-12} \). The study of isotopic yields helps to understand the differences observed for summed quantities like mass yields or mass fractions discussed before. Indeed we observe similar features, but now in more detail. Generally, the SMSM gives Gaussian type distributions, which are the consequence of the liquid-drop description of fragments. At some conditions HS and FYSS results are similar because they use the same tabulated binding energies. This can be seen e.g. for the oxygen isotopes shown in Fig. 15. We remark that no theoretically calculated binding energies are included in HS for oxygen. Therefore the used tabulated binding energies in FYSS and HS are identical, and one finds almost identical isotope distributions. For the larger densities shown in Fig. 16 differences between HS and FYSS emerge even for nuclei which are contained in the mass table. This happens because in FYSS the bulk energies of these nuclei experience the liquid-drop medium modifications. In the HS case there are cuts, since the tables contain only a limited number of nuclei. This can also be seen in Fig. 16 e.g. in the lower left panel, where the maximum \( A \) of oxygen isotopes considered in HS is 28. In the FYSS model, nuclei which are not in the mass table are described exclusively with the RMF liquid-drop model. This concerns all oxygen isotopes with \( A > 28 \). Jumps occur at the boundaries between the tabulated nuclei and the nuclei which are described by the liquid-drop model. Comparing HS and FYSS, it is very interesting to realize that the yields of HS seem to connect relatively smoothly with the liquid-drop yields of FYSS for \( A > 28 \). This could be a result of rearrangement effects to satisfy the baryon and charge conservations (4) in the nuclear distributions. Apart from the upper right panels in Figs. 15 and 16 the purely liquid-drop description of the SMSM gives significantly larger oxygen yields, with up to four orders of magnitude difference. In addition to the different description of medium effects, this also represents the differences in the nuclear binding energies. The nuclear structure effects included in HS and FYSS can be identified, e.g., by the even-odd staggering and the favored appearance of \(^{16}\text{O}\) clearly visible at lower temperature.

The abundances of the nuclei with \( Z = 8 \) of the FYSS model are increasing with mass number in left panel of Fig 16. It is a very interesting effect related to RMF description of nuclear clusters and properties of nucleons in dilute matter. One can see that at these
conditions neutrons are accumulated in clusters considerably more than in the SMSM case. The isotope distribution of oxygen with a peak at \( A \approx 40 \) can be realized as effective reduction of the symmetry energy for these nuclei (see, Eq. (10)). Such a trend have been already reported in analysis of some experimental data on multifragmentation [46, 47, 48, 72, 73, 91, 92].

The isotopic yields of iron nuclei are shown in Figs. 17 and 18. In HS, the theoretical nuclear structure calculations extend up to \( ^{92}\text{Fe} \). In FYSS, tabulated binding energies from Ref. [80] are only available up to \( ^{72}\text{Fe} \), and isotopes with mass numbers larger than 72 are calculated by the liquid-drop formulas. This leads to jumps in isotope distributions. Such jumps are avoided in the HS model by using binding energies from theoretical nuclear structure calculations for exotic nuclei. However, it is found that the end of the mass table is reached, e.g., in the upper left panel of Fig. 18. Compared with the SMSM, the shapes of the iron isotope distributions predicted in HS and FYSS are much more similar than for oxygen. The mean mass numbers of isotopes in the three models are rather close to each other. However, in SMSM the isotope yields are several orders of magnitude larger than in FYSS or HS, that is related to the mass distribution discussed above.

In Fig. 19 we show the isotope distributions for tin nuclei, which are qualitatively rather similar to the iron case. Similar results as in the previous figures are obtained. In Fig. 20 we show yields of iron and tin isotopes, for the most extreme conditions, namely \( \rho/\rho_0 = 0.1 \) and \( T = 5 \) MeV. There are rather large differences for all three models. For low \( Y_e \), the limited range of mass numbers in HS is apparent. SMSM obtains overall larger yields, as before. Also the mean values predicted by the three models are slightly shifted. Large discontinuities are again found in the FYSS calculations because of utilizing different descriptions of fragments for different mass regions, as previously noted. The abundance curves in SMSM at low \( Y_e \) are steeper than in FYSS since FYSS includes RMF calculations of nuclear binding energies leading to the difference in their symmetry energies.

### 3.5 Thermodynamical properties

Finally, we present thermodynamical characteristics of stellar matter predicted by the three models. In Fig. 21 we show the nuclear entropy per baryon as a function of temperature. As seen, the nuclear entropy per baryon is increasing with temperature, and all models give more or less similar results. This conclusion remains true if we extend comparison to other similar models as in Refs. [36, 13]. The largest differences occur at the highest density considered, \( \rho/\rho_0 = 0.1 \). SMSM tends to give the largest entropies for \( T \geq 5 \) MeV because of entropy accumulation in internal excitation of fragments. In Fig. 11 one sees that the composition is dominated by heavy nuclei for these densities. Here we have to be more specific: Fig. 12 shows that \( \langle A \rangle_{\text{heavy}} \) drops below 100 for \( T > 3 \) MeV and even below 20 for \( T > 5 \) MeV. Thus the differences observed for the entropy for \( T \geq 5 \) MeV can be traced back to the different description of the free energies of low-mass nuclei in the three models. One also sees in Fig. 11 that in the SMSM the
mass fraction of "heavy" nuclei remains close to one, even for temperatures of 10 MeV. This agrees with the finding that the entropy in SMSM is slightly increased, indicating a temperature dependence which favors intermediate mass nuclei in SMSM. In the other two models, which both use the internal partition function, a decrease of $X_{\text{heavy}}$ is observed for temperatures larger than 5 MeV and the results are more similar.

In Fig. 22(a)-(b), we present the contributions to pressure caused by single nucleons and nuclei produced after clusterization of nucleons. In the following we call it pure nuclear pressure. The differences between the models are seen especially at the largest density. They are mainly caused by differences in fractions of nucleons and nuclei discussed previously. The mean-field effects taken into account in FYSS and HS also influence the results. In addition, in the supernova environment free electrons existing together with nuclear clusters modify the Coulomb energy and lead to a negative Coulomb pressure. The physical reason is that electrons compensate the positive charge of nuclei, and the Coulomb energy of Wigner-Seitz cells decrease when the density increases. This Coulomb pressure can be calculated as:

$$P_C = \rho_B \sum_{AZ} \rho_{AZ} \frac{\partial F_{AZ}^C}{\partial \rho_B},$$

(21)

where the expression (12) for $F_{AZ}^C$ is used. The total nuclear pressure, which is the sum of the pure nuclear pressure and $P_C$, is demonstrated in Fig. 22(c)-(d) as a function of temperature. It is important that at $T > \sim 5\text{MeV}$, when matter nearly completely dissociates into nucleons and lightest clusters, $P_C$ is close to zero. In this region the total nuclear pressure coincides with the pure nuclear pressure. The Coulomb pressure becomes very important when heavy clusters dominate in the system. One can see in Fig. 22(c)-(d) that at low temperatures and high density the total nuclear pressure may be negative. In this case the nuclear clusterization is favorable for the collapse, and all three models show a similar behavior. However, the positive pressure of the relativistic degenerate electron Fermi-gas is considerably (more than an order of magnitude) larger. Therefore the total pressure $P_{\text{tot}}$ given by the sum of the total nuclear, electron and photon pressures in such environment will always be positive and the condition of thermodynamical stability ($\partial P_{\text{tot}}/\partial \rho \geq 0$) will be fulfilled.

The chemical potentials of both protons and neutrons are very important for all electron- and neutrino-induced reactions, which play an important role in supernova dynamics. They are shown in Figs. 23 and 24. One can see that results of SMSM, HS and FYSS EOS agree well and show the same behavior with density and temperature. However, there are deviations at the highest density $\rho/\rho_0 = 10^{-1}$ and high temperatures. They may be transformed into essential differences in mass and isotope distributions of produced fragments. It is interesting that $\mu_p$ obeys a rather similar trend for HS and FYSS, but in SMSM there is a strong decrease observed with increasing temperature. This should also be a result of the different description of heavy fragments, as discussed.

\[\text{Note that in Fig. 4 of Ref. [29] an inconsistent comparison was made between the SMSM pure nuclear pressure and the total nuclear pressure of the Shen model [34].}\]
before. As one can see in Fig. 9, HS and FYSS predict roughly 10% of unbound protons for $T = 10$ MeV, which is about one magnitude larger than in SMSM. This means that more protons are bound in SMSM nuclei. Also we can see $\mu_n$ of FYSS are lower than in SMSM and HS at $\rho/\rho_0 = 10^{-1}$, high temperatures and $Y_e = 0.2$. This is because more neutrons are bound in FYSS nuclei, as shown in Fig 8. These differences may lead to significant effects in the weak reactions.

4 Conclusions

We believe that the present comparative study of the three models, SMSM, HS and FYSS for the EOS will be very instructive for understanding the differences and similarities in their predictions. These results can be used in hydrodynamical simulations of the collapse of massive stars and their subsequent explosions. The intervals of temperatures $T=0.5–10$ MeV and densities $\rho/\rho_0 = 10^{-1} – 10^{-3}$ at electron fractions $Y_e=0.2, 0.4$ were under investigation. These conditions can occur during the main stages of the supernova process. All the three models give the detailed information about the nuclear mass and isotope distributions. In supernovae, the presence of heavy nuclei and their distributions are most important for electron captures and neutrino trapping during the collapse phase. We found that the width of the mass distributions can become even larger than 100 units, indicating substantial differences to the commonly used single nucleus approximation. It would be interesting to investigate the impact of the different EOSs and nuclear distributions on numerical simulations of the core-collapse supernovae in the future.

On the other hand the investigated conditions are interesting for understanding the nuclear liquid-gas phase transition which can be investigated in laboratories, e.g., in multifragmentation reactions induced by heavy-ion collisions. Generally, we have concluded that at low density and high temperatures ($T > 2$ MeV), the three models give similar results for basic thermodynamical quantities like pressure, entropy, mass fractions of neutrons, protons, alphas and heavy nuclei, and chemical potentials of protons and neutrons. It is interesting that the mass distributions and other characteristics of ensemble nuclei differ from each other especially at high subnuclear densities. The differences result mainly from the dependence of bulk and surface energies on temperature and neutron-richness, and presence or absence of shell effects. We point especially at differences in isotope distributions of produced nuclei. Many isotopes are important for calculating the rate of weak reactions. Since there is not enough data on the production of heavy nuclei in this region of the phase diagram, this striking difference between three models calls for new experimental investigations. Reactions of multifragmentation of excited neutron-rich nuclear systems into many fragments would be very suitable, since they provide a natural method for simulating conditions with low electron fraction. In supernova matter at subnuclear density we may expect nuclei with extremely high mass numbers and large isospin asymmetries. Therefore, these conditions should be initiated by collisions of large exotic and neutron-rich nuclei.

Regarding thermodynamic quantities like pressure or entropy, the largest differences
occur at high densities $\rho/\rho_0 \sim 0.1$ and high temperatures $T > 5$ MeV. For such conditions intermediate mass and light nuclei are most important. The medium modification of such nuclei turns out to be the most important element of uncertainty in calculations of the supernova EOS, in addition to the bare nucleon-nucleon interactions. Part of the medium modification is due to Pauli-blocking, which can be calculated with quantum statistical approaches [87]. One could try to incorporate these binding energy-shifts phenomenologically in statistical models as mass corrections. One can use also knowledge obtained from experimental studies of nuclear multifragmentation reactions. For example, recent analyses of multifragmentation data give evidences for reduction of the symmetry energy coefficient in mass formula of the nuclei in such hot environment [16, 17, 18, 22, 73, 91, 93]. This may serve as a guideline for future microscopic theories. A theoretical approach in this direction is the so-called “generalized RMF” model of Ref. [56], which includes mean-field interactions of light nuclei.

At low temperatures ($T < 2$ MeV) but high densities another important uncertainty comes from medium modifications of the nuclear binding energies of heavy nuclei. Part of it refers to the evolution of the nuclear shell effects. It is for many years under theoretical [44] and experimental [45] investigations in nuclear reactions. This problem is also related to the general question on properties of nuclei for from stability, including the effect of temperature and surrounding medium of unbound nucleons and light clusters. Theoretical studies in this direction have been done recently within Skyrme-Hartree-Fock calculations [94], RMF models in the Hartree approximation [95], and within a simple model with two-body momentum-dependent interaction [89]. However, these calculations are performed within the single nucleus approximation. Until now, no realistic quantum many-body model has been calculated for all supernova conditions which takes into account distributions of different nuclei (light and heavy). At densities even higher than the densities we considered in our work, i.e., above 0.1$\rho_0$, additional effects like the formation of large-scale non-spherical structures (usually denoted as the pasta phases) may set in [32, 94, 96]. This is obviously beyond the scope of the present article, but part of these effects are already included in a simplified manner in some EOS models, e.g., in the FYSS model.

The comparative analysis of several models made in this paper show the necessity of theoretical and experimental studies of nuclei in the wide region of nuclear chart and their consistent treatment. For the basic input it is necessary to have experimental data on nuclei toward the neutron-rich side as much as possible. It is also important to study the nuclear compositions and modification of properties of nuclei at sub-saturation densities in laboratory experiments, e.g., on heavy-ion collisions leading to multifragmentation reactions. We want to stress that excited nuclear matter created in these reactions in many respects is similar to supernova matter. As for theoretical side, it is necessary to construct a unified statistical approach taking into account ingredients which pass verification with experimental data at the corresponding thermodynamical conditions of nuclear matter. The binding energies and level densities of heavy nuclei at high temperature and density should utilize the medium modifications as predicted by microscopic theories. In addition, a consistent treatment of transformation of nuclei into ‘pasta’ and uniform matter is
crucial in application of EoS for supernova simulations. Such efforts are indispensable for understanding both for astrophysics and nuclear physics.

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Figure 1: Nuclear phase diagram in the 'temperature – baryon density' plane. Solid black and dashed-dotted purple lines indicate boundaries of the liquid-gas coexistence region for symmetric ($Y_e = 0.5$) and asymmetric matter ($Y_e = 0.2$) calculated with TM1 interactions [83]. The shaded area corresponds to typical conditions for nuclear multifragmentation reactions [29]. The dashed black lines are isentropic trajectories characterized by constant entropy per baryon, $S/B = 1, 2, 4$ and 6 calculated with SMSM [29]. The dotted red lines show model results of Ref. [12] for BB (just before the bounce), CB (at the core bounce) and PB (the post bounce) in a core-collapse supernova. (Color version online.)
Figure 2: Mass distributions of fragments produced in matter with temperatures $T = 0.5$, 1 and 2 MeV, electron fractions $Y_e = 0.2$ and 0.4, and density $\rho/\rho_0 = 10^{-3}$. (Color version online.)
Figure 3: Mass distributions of fragments produced in matter with temperatures $T = 3, 5$ and 10 MeV, electron fractions $Y_e = 0.2$ and 0.4, and density $\rho/\rho_0 = 10^{-3}$. (Color version online.)
Figure 4: Mass distributions of fragments produced in matter with temperatures $T = 0.5, 1$ and $2$ MeV, electron fractions $Y_e = 0.2$ and $0.4$, and density $\rho/\rho_0 = 10^{-2}$. (Color version online.)
Figure 5: Mass distributions of fragments produced in matter with temperatures $T = 3, 5$ and $10$ MeV, electron fractions $Y_e = 0.2$ and $0.4$, and density $\rho/\rho_0 = 10^{-2}$. (Color version online.)
Figure 6: Mass distributions of fragments produced in matter with temperatures $T = 0.5, 1$ and $2$ MeV, electron fractions $Y_e = 0.2$ and $0.4$, and density $\rho/\rho_0 = 10^{-1}$. (Color version online.)
Figure 7: Mass distributions of fragments produced in matter with temperatures $T = 3, 5$ and $10$ MeV, electron fractions $Y_e = 0.2$ and $0.4$, and density $\rho/\rho_0 = 10^{-1}$. (Color version online.)
Figure 8: Comparison of SMSM, HS, and FYSS model results for the average fraction of free neutrons as a function of temperature. (Color version online.)
Figure 9: Average fraction of protons as a function of temperature. (Color version online.)
Figure 10: Average fraction of alpha particles as a function of temperature. (Color version online.)
Figure 11: Average fraction of heavy particles \((A > 4)\) as a function of temperature. (Color version online.)
Figure 12: Average mass number of heavy nuclei ($A > 4$) as a function of temperature. (Color version online.)
Figure 13: Average proton number of heavy nuclei ($A > 4$) as a function of temperature. (Color version online.)
Figure 14: Dispersion of the mass number of heavy nuclei \((A > 4)\) as a function of temperature. (Color version online.)
Figure 15: Isotopic distributions of $Z = 8$ fragments produced in matter with temperatures $T = 1, 2$ and 3 MeV, electron fractions $Y_e = 0.2$ and 0.4, and density $\rho/\rho_0 = 10^{-3}$. (Color version online.)
Figure 16: Isotopic distributions of $Z = 8$ fragments produced in matter with temperatures $T = 2, 3$ and 5 MeV, electron fractions $Y_e = 0.2$ and 0.4, and density $\rho/\rho_0 = 10^{-1}$. (Color version online.)
Figure 17: Isotopic distributions of $Z = 26$ fragments produced in matter with temperatures $T = 0.5, 1$ and 2 MeV, electron fractions $Y_e = 0.2$ and 0.4, and density $\rho/\rho_0 = 10^{-3}$. (Color version online.)
Figure 18: Isotopic distributions of $Z = 26$ fragments produced in matter with temperatures $T = 1, 2$ and 3 MeV, electron fractions $Y_e = 0.2$ and 0.4, and density $\rho/\rho_0 = 10^{-2}$. (Color version online.)
Figure 19: Isotopic distributions of $Z = 50$ fragments produced in matter with temperatures $T = 1, 2$ and 3 MeV, electron fractions $Y_e = 0.2$ and 0.4, and density $\rho/\rho_0 = 10^{-2}$. (Color version online.)
Figure 20: Isotopic distributions of $Z = 26$ and $Z = 50$ fragments produced in matter with temperatures $T = 5$ MeV, electron fractions $Y_e = 0.2$ and 0.4, and density $\rho/\rho_0 = 10^{-1}$. (Color version online.)
Figure 21: Comparison of SMSM, HS, and FYSS model results for the total nuclear entropy per baryon as a function of temperature. (Color version online.)
Figure 22: Comparison of SMSM, HS, and FYSS model results for the pure ((a)-(b)) and total ((c)-(d)) nuclear pressure as a function of temperature. (Color version online.)
Figure 23: The chemical potential of protons as a function of temperature. (Color version online.)
Figure 24: The chemical potential of neutrons as a function of temperature. (Color version online.)