Mathematical model of the short circuit process in the electrical network with self-recovering current-limiting device with a liquid metal

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Abstract. The article shows that the high speed and current-limiting properties of a self-recovering current-limiting device with a liquid metal cause interest in their use in power supply systems. However, the level of research, both in our country and abroad, does not allow to proceed to the creation of industrial designs of such kind of current-limiting devices. More research is needed. To study the process of a short circuit in the AC (an alternating-current network) proposed a mathematical model that allows precise, detailed and deployed to investigate their current-limiting properties. The model takes into account the resistance of the electric arc during the switching process. Period of the burning time of the arc can be divided into infinitely small areas, arc resistance each of which is assumed to be linear.

1. Introduction

Self-recovering current-limiting device with a liquid metal consists of a self-resetting liquid metal fuse and a connected shunt resistor parallel to it. The level of research of a self-recovering current-limiting device today, both in the Russian Federation and abroad, does not allow the development of industrial designs for use in power supply systems. Such studies are conducted by individual initiative groups in different countries of the world. These are Japan, USA, Germany, China, RF, etc. [1-8]. To date, it has been established [1, 3, 7, 9, 10, 11, 12] that the self-recovering current-limiting devices with a liquid metal and devices based on them have higher current-limiting properties and speed than the electrical apparatus mastered by industry. This arouses interest and need for the development and continuation of research in the direction of creating prototypical and industrial samples of the self-recovering current-limiting devices and determining the area of their application.

A self-resetting liquid metal fuse is an electrical apparatus in which a liquid metal or alloy is used as a fusible element (indium, gallium, mercury, alkali metals, etc.) located in the channel of the dielectric bushing [1, 8, 13]. When flowing through the liquid metal fuse insert short-circuit current, it evaporates, causing an explosive increase in pressure. At high pressure, metal vapors have significant resistance. As a result, the current is sharply limited. Figure 1 shows a sample [14], developed at the Department of "Power Supply" of Ulyanovsk State Technical University.
2. Mathematical model of the short circuit process

The model includes a network replacement circuit with a current-limiting device based on the self-resetting liquid metal fuse in short-circuit mode is shown in Figure 1. It consists of an alternating voltage source $G$, a switching device $Q_1$, a network inductance $L$, a network active resistance $R$, a resistance of a fuse insert of the self-resetting liquid metal fuse $R_f$, an arc resistance $R_d$ that occurs when the self-resetting liquid metal fuse operates, a shunt resistance $R_{sh}$. The position of the key $Q_1$ “closed” corresponds to the occurrence of short-circuit current and its flow in the network. The position of the key $Q_1$ is “open” to its termination. The proposed model provides for the disabling of the short-circuit current (opening of the $Q_1$ key) at the moment of the second natural zero crossing. When using high-speed switching devices, it is not advisable to consider the short circuit process for a time longer than the frequency of the network. When using devices with a longer or shorter time of action, the model can be amended.

During the time from the onset of short-circuit to the natural passage of current through zero, there are three main stages of the process. Each stage in the equivalent circuit (see Figure 2) corresponds to the state of the key $Q_2$ at a specific position of 1, 2 or 3. At any of the stages the short circuit passes through the inductance of the network and the resistance $R_k$, whose value is determined by the state of the key $Q_2$.

![Figure 1. A self-resetting liquid metal fuse. Where: 1, 2 - contact conclusions; 3 - damping unit; 4 - housing in the form of a sleeve of a dielectric material; 5 - cylinder of dielectric material; 6 - capillary channels; 7 - liquid metal.](image)

At the first stage, the $Q_1$ key is in state 1. When the $Q_1$ key is closed, the current-limiting device turns on in a short-circuit. Since the shunt resistance is large compared to the resistance of the self-resetting liquid metal fuse, the short-circuit current $i_k$ passes through the fuse-link, the resistance of which is assumed to be zero. In this case $i_k = i_f$ (see Figure 2) the resistance of the short circuit $R_k$ is the resistance of the source $R_k = R$. The duration of the stage $\Delta t$ is determined by the period $0 \leq t \leq t_1$, where $t_1$ is the time, which includes the heating time of the fuse-link to the boiling point, the time of transition from the liquid to the vapor state. This time precedes the formation of high-resistance metal vapor resistance and the occurrence of an electric arc. Using the terminology for ordinary fusible inserts, we call this time pre-arc.

$$t_{pd} = t_1$$  \hspace{1cm} (1)

It is defined as the time for which the current value of the thermal integral

$$W = \int i_f^2 dt$$  \hspace{1cm} (2)

from the current flowing through the self-resetting liquid metal fuse will reach the value equal to the value of the pre-arc integral:

$$W = W_{pd} = A \cdot S^2$$  \hspace{1cm} (3)
In expression (3), \( S \) is the minimum section of the fuse-link; \( A \) is constant for the material of the fusible link value [1]. Equation (2), (3) is solved relatively \( t \) by numerical methods. When the current value changes \( i_k = i_f \) at each step of the iterative process, the current value of \( W \) is calculated and compared with \( W_{prd} \). When condition (3) is met, the current value \( t \) is equated to \( t_1 \). The solution is found if the difference between \( W \) and \( W_{prd} \) does not exceed the specified error. The value of the current \( i_k = i_f \) in this case is a limited self-resetting liquid metal fuse short-circuit current (transmitted current).

The second stage of the process takes place on a time interval \( t_1 < t \leq t_2 \). At this stage, the formation of an electric arc begins in the liquid metal vapor formed as a result of the heating of the fuse link. In some literature sources, this stage is called the transition phase of preliminary arc formation [15]. The process of pre-arc formation is poorly understood, even in conventional fuses. To a greater extent, the same can be said about the self-resetting liquid metal fuse. Many models do not take into account this stage, which can result in certain errors. The authors propose to present the process at this stage as simplified as the occurrence of a vapor tube with high resistance, which sharply limits the short circuit current. At the end of the stage, the applied voltage causes the breakdown of the vapor tube formed and the formation of an electric arc.

In accordance with the assumption at this stage, the electrical branch with the self-resetting liquid metal fuse is broken, which is equivalent to switching the key \( Q_2 \) from position 1 to position 2 (see Figure 2). The resistance of the circuit short circuit \( R_k = R + R_{Sh} \). Short circuit current is limited by shunt resistance. The duration of the second stage \( \Delta t_2 \) is determined by the duration of the existence of the vapor lock.

The third stage occurs on the time interval from \( t_2 \) to \( t_3 \). At this stage, at the initial moment of time, the vapor lock is broken, and an electric arc occurs, which is equivalent to switching the key \( Q_2 \) from position 2 to position 3. At the end of the stage at the moment of time \( t_3 \), the electric arc corresponding to the natural current zero crossing is extinguished. The end time of the third stage is determined by monitoring the calculated current values in the iterative process and comparing them with a zero value. The zero current value corresponds to the end time of the third stage. The duration of the stage is equal time \( \Delta t_3 \). Part of the short-circuit current at this stage flows through the arc resistance in the self-resetting liquid metal fuse. The other part is through shunt resistance. Short-circuit current is limited by two parallel-connected resistances \( R_d \) and \( R_{Sh} \).

![Figure 2. The equivalent circuit of the short circuit process in the network with a current-limiting device based on the self-resetting liquid metal fuse. Where: \( i_k \) - short circuit current, \( i_f \) - current flowing through the self-resetting liquid metal fuse.](image-url)
The resistance of the electric arc $R_d$ is non-linear. It depends on the current and voltage applied to the self-resetting liquid metal fuse $U_{SH}$. Assuming that this resistance does not change when the voltage $U_{SH}$ changes during an infinitely small time interval $\Delta t$ corresponding to the step of dividing the entire time of the stage into $n$ parts, the short circuit process at this stage can be represented in accordance with the replacement circuit shown in figure 3. In this scheme, the nonlinear resistance $R_d$ is represented by a set of linear resistances $R_{dj}$ that are alternately connected to the circuit using the $Q_j$ key with a time interval $\Delta t$. At the third stage, at each $j$-volume its interval $\Delta t$, the resistance of the short circuit $R_k$ consists of two series-connected sections. One of them source resistance $R_s$, the second is a parallel connection of resistances $R_{SH}, R_{d,j}$. In this case $R_k = R_{k,j} = R + \frac{R_{SH} \cdot R_{d,j}}{R_{SH} + R_{d,j}}$.

The third stage will end when the arc is extinguished during the natural passage of current through zero. What follows is the disconnection of the short circuit by key $Q_j$.

As a result, it turned out that the short circuit process is divided into a series of successive periods of time: $\Delta t_1, \Delta t_2$ appropriate to the duration of the first and second stages of the process; segments $\Delta t_{3,j}$ ($j=1,2...n$), each of which is part of stage 3 of the arc stage of the process in the first half-time.

In the general case, at any time interval of the short circuit process, the sequence number of which is denoted as $l$ transient equation is as follows:

$$U_m \cdot \sin(\omega \cdot t + \psi) = L \frac{di}{dt} + R_{k,l} \cdot i_k$$

where $U_m$ - the amplitude value of the voltage source; $\psi$ - the initial phase of the source, at the moment of closing the key $Q_j$; $L$ - network inductance; $R_{k,l}$ - active resistance of the short circuit, conforming to the considered time interval under the number $l$.

The solution of equations at each time interval of the short circuit process is:

$$i_k = I_{i,m} \cdot \sin(\omega \cdot t + \psi - \phi_l) - [I_{i,m} - I_{i,m} \cdot \sin(\omega \cdot t_{i,m} + \psi - \phi_l)] \cdot e^{-\frac{t_{i,m}}{\tau_l}}$$

In expression (5):

- $I_{i,m}$ - the time start of the time interval under the number $l$;
- $\tau_l$ - the time constant of the short circuit, corresponding to each time interval under the number $l$;
- $\phi_l$ - phase angle between current and voltage in the short circuit with parameters, appropriate to each time interval under the number $l$ in the steady state;
- $I_{i,m}$ - the estimated amplitude of the steady-state current in the short circuit with parameters appropriate to each time interval under the number $l$. 

Figure 3. Equivalent circuit of the arc stage.
The current value $I_{l,n}$ at the beginning of each time interval under the number $l$ is equal to the current value at the end of the previous time interval under the number $(l-1)$.

The expression for the voltage on $R_{sh}$ at any time interval $l$ has the following form:

$$u_{sh} = i_k \cdot (R_{k,l} - R)$$

The equations described above represent a mathematical model of a short circuit process in a network with a current-limiting device based on the self-resetting liquid metal fuse. The results of the calculation using the mathematical model are presented in Figure 4. The results are presented in the form of dependences of the current in the network $i_k = f(t)$, the current through the self-resetting liquid metal fuse $i_f = f(t)$, the current through the shunting resistance $i_{sh} = f(t)$, as well as the voltage drop on the shunting resistance $U_{sh} = f(t)$ against time, taking into account the considered stages of the process. The network parameters in the equivalent circuit in the test example consistent with the voltage of the source 240 V (effective value), $\cos\varphi = 0.19$ the value of the maximum shock current of short circuit 52.6 kA at the initial phase of the voltage source $\psi = 90^\circ$.

![Figure 4. Calculated dependencies in equivalent circuit’s arms.](image)

In the calculations in the equivalent circuit, parameters of the self-resetting liquid metal fuse with a nominal current of 60 A, a fusible mercury insert, and a narrow neck of 0.7 mm in diameter were used.
The value of the shunt resistance is taken equal to 0.2 ohms. The time intervals of each of the stages of the short-circuit process are selected so that they can be clearly distinguished in the figure. The resistances of the self-resetting liquid metal fuse arc at each of the intervals of the arc burning stage are chosen commensurate with the value of the shunt resistance. The arc resistance at intervals \( \Delta t_{3,1}, \Delta t_{3,2}, \Delta t_{3,3}, \Delta t_{3,4} \) is assumed to be equal to \( 7 \cdot R_{SH}, 6 \cdot R_{SH}, 4 \cdot R_{SH}, 6 \cdot R_{SH} \), respectively.

3. Conclusions

(1) High performance and current-limiting properties of current limiters based on the self-resetting liquid metal fuse generate interest in the development and use to limit short-circuit currents in power supply systems. The level of research, both in our country and abroad, does not allow us to proceed to the creation of industrial designs of current limiters based on the self-resetting liquid metal fuse.

(2) The most complete scientific justification in the technical literature received the option of using the self-resetting liquid metal fuse in combination with a shunt resistance and an additional switching device. The scientific groundwork in this respect is closest to the practical output. However, the mathematical model by which the available results are obtained is imperfect. It allows you to get reliable and fairly accurate results only in a limited scope.

(3) To study the short circuit process in an alternating current network with a self-resetting liquid metal fuse shunted by a resistor, a new mathematical model has been proposed, which, unlike the known one, allows a more qualitative study of the current-limiting properties of the device based on the self-resetting liquid metal fuse.

4. References

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