Pattern Synthesis of MIMO Radar Based on Immune Differential Evolution Algorithm

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Abstract. In order to optimize the pattern synthesis of multiple input and multiple output (MIMO) radar, immune mechanism is adopted to overcome the premature risk of differential evolution (DE) algorithm, namely immune differential evolution (IDE). Firstly, the modeling of MIMO radar is introduced by encoding the position of array in the binary. Secondly, the immune mechanism is employed to improve the DE. In IDE, two parameters are self-adapted for the mutation operator by immune mechanism to enhance the convergence ability of DE, including scaling factor and crossover rate. Several experiments are conducted to analysis the performance of IDE. The simulation results show that the IDE with variable parameters can get optimal results in MIMO radar with lower the peak side-lobe level (PSLL) and maintain the diversity with stronger convergence ability and shorter calculation speed.

Keywords: Multiple Input and Multiple Output (MIMO) radar; Pattern synthesis; Immune mechanism; Differential evolution.

1. Introduction

With the recognition of various advantages, such as strong parameter estimation and high space resolution, and so on, there is more and more research aiming at multiple input and multiple output (MIMO) radar[1][2]. It launches synchronously diversity waveform and receives the echo signal by multiple antennas. In order to get high performance of MIMO radar, the location of each antenna and the waveform of MIMO radar should be properly dealt with, such as adjust the arrangement of antenna and suitable coding format to lower the peak side-lobe level (PSLL).

In earlier studies, in order to keep diversity and get the global optimization results to suitable for MIMO radar optimization problem, several hybrid and improved intelligent algorithms are proposed and simulated, such as GA-PSO[3], differential particle swarm optimization (DPSO)[4], and differential evolution algorithm (DEA). By introducing some other mechanisms to basic DEA can get better performance[5], and the simulation result shown lower PSLL can be achieved, but this research didn’t considered the resolution. By combing the DEA and differential set (DS), DSDE[6] located the elements at the fixed grid and adjusted the excitation amplitude. CWO was proposed in 2018, which combined chaotic theory and whale algorithm[7]. Both DSDE and CWO can only get better performance at the cost of running time. Although the improved GA, PSO, CWO and DE can get better result and avoid the risk of premature in some distance at the cost of computation or running time, and they cannot easily and quickly get the global best due to the cross and mutation strategy.
Hence, it is necessary to study a more efficient algorithm for MIMO radar to optimize the pattern synthesis. In recent years, some researchers simulated the immune strategy of bio-system, and proposed immune algorithm, which can get high performance in dealing with the optimal solutions[8], and the immune algorithm was adopted to improve performance in the field of objective optimization[9], community detection in complex network[10], and so on. ADE-MOIA[11] was proposed to optimize population diversity and the convergence speed by combing adaptive DE and multi-objective immune. Based on the above researches, the basic DEA is improved by the immune mechanism, namely immune differential evolution (IDE). The novel algorithm is aimed at optimizing the MIMO radar arrays for pattern synthesis with the advantages of better convergence ability and calculation speed.

The mainly work of IDE is organized as follows. The modeling of MIMO radar is discussed in section 2, by encoding in the binary. The process of IDE is introduced in section 3. On the one hand, DE has contribution to keep the diversity. On the other hand, several parameters are self-adapted for the mutation operator by immune mechanism to enhance the convergence ability of DE. Several experiments are simulated to show the better performances of IDE in section 4, including convergence ability and calculation speed, lower PSLL and narrower 3dB bandwidth and null depth. Section 5 contains the conclusion.

2. MIMO Model

Figure 1 is a model of MIMO radar model, and there are $N_T \bullet N_R$ different transmission paths, where $N_T, N_R$ tand for the paths between the transmitting arrays nd the target, the target and the receiving arrays, generally. Assume that detection parameters, namely $d, \lambda, R$ and $D$, stand for the array spacing, wavelength, target distance and the target length, thus the detection parameters meet the limit $d > \lambda R / D$. Generally, the orthogonal code is adopted and the minimum distance between arrays is set as $\lambda / 2$ to avoid superposition and interference.

According to the theory of array antenna, direction functions of radar sub-array can be set as the following equation, where (1) and (2) stand for the transmitter and the receiver individually.

$$f_T(u) = \left| \sum_{i=0}^{N_T-1} A_{Ti} \exp\left( j \frac{2\pi}{\lambda} \alpha_{Ti} u \right) \right|$$

$$f_R(u) = \left| \sum_{k=0}^{N_R-1} A_{Rk} \exp\left( j \frac{2\pi}{\lambda} \alpha_{Rk} u \right) \right|$$

(1) (2)
In the equation, variable \( u = \sin \phi - \sin \psi \) and meets \( u \in [-2, 2] \) for any \( \phi, \psi \in [-\pi, \pi] \), where \( \varphi \) denotes the angle between plane wave and line formation normal, and \( \varphi_0 \) stands for antenna beam pointing. Moreover, the direction functions of radar sub-array satisfy symmetry about \( u = 1 \), which can be expressed as \( f(1 + \Delta u) = f(1 - \Delta u) \). Hence, the scope of research can be narrowed to \( 0 \leq u \leq 1 \). At the transmitting terminal, the position of \( i-th \) array is denoted as \( \alpha_{Ti} \), and the array excitation amplitude of \( \alpha_{Ti} \) is \( A_{Ti} \). At the receiving terminal, the position of \( k-th \) array is denoted as \( \alpha_{Rk} \), and the array excitation amplitude of \( \alpha_{Rk} \) is \( A_{Rk} \). According to the basic theory of MIMO radar, the direction function of MIMO radar can be described as the following equation.

\[
f_{\text{MIMO}}(u) = \left| \sum_{j=0}^{N_{T} - 1} A_{Tj} \exp(j \frac{2\pi}{\lambda} \alpha_{Tj} u) \cdot \sum_{k=0}^{N_{R} - 1} A_{Rk} \exp(j \frac{2\pi}{\lambda} \alpha_{Rk} u) \right|
\]

Due to the orthogonality of antenna signals in MIMO radar, the gain of the wide beam formed in the detection space is low, instead of narrow beam with high gain. The direction function of MIMO radar can be equivalent to be the Kronecker product\(^{[12]}\) of the sub-array direction function, as shown in (4), which simplifies the calculation of (3) in the application of pattern synthesis.

\[
f_{\text{MIMO}}(u) = \left| \prod_{j=0}^{N_{T} - 1} A_{Tj} \exp(j \frac{2\pi}{\lambda} \alpha_{Tj} u) \right| \left| \prod_{k=0}^{N_{R} - 1} A_{Rk} \exp(j \frac{2\pi}{\lambda} \alpha_{Rk} u) \right|
\]

(4)

3. Pattern Synthesis Based on IDE

3.1 MIMO Radar Binary Modeling

Owing to the discreteness of the optimization problem of MIMO radar sparse array, the arrays’ positions of sending terminal and receiving terminal are necessary to be encoded in binary, and the excitation amplitude of array is initialized randomly. Firstly, the array apertures of transmitting terminal and receiving terminal are set as \( \eta_r \) and \( \eta_T \) respectively. Secondly, the regulation is appointed that "1" means having array in the grid position, and "0" means none. Thirdly, the expression \( \Xi^M_{N}(a, b) \) indicates that there are \( N \) elements in total, and the value of "\( a \)" in the vector has \( M \) elements, and the other \( N - M \) elements are valued of "\( b \)". Hence, the binary code vector \( S_T \) of transmitting array can be expressed as (5), and the binary code vector \( S_R \) of receiving array can be expressed as (6). Moreover, a novel vector of MIMO array is expressed as (7) by merging (5) and (6) together.

\[
S_T = \{1 \ \Xi^{N_T-2}_{N_T/\eta_T-1}(1,0) \ 1\}
\]

(5)

\[
S_R = \{1 \ \Xi^{N_R-2}_{N_R/\eta_R-1}(1,0) \ 1\}
\]

(6)

\[
S_M = \{1 \ \Xi^{N_T-2}_{N_T/\eta_T-1}(1,0) \ 1 \ 1 \ \Xi^{N_R-2}_{N_R/\eta_R-1}(1,0) \ 1\}
\]

(7)
Generally, the fitness function is related to the side-lobe level of MIMO radar. $f_{CL}$ and $f_{SL}$ are used to stand for the main-lobe level and the side-lobe level respectively. So the normalized peak side-lobe level can be expressed as

$$PSLL = 20 \cdot \lg \frac{\max(f_{SL})}{\max(f_{CL})}$$

(8)

The fitness function can be shown as (9) if null depth, 3dB bandwidth, and resolution are taken in consideration.

$$f = \omega_1 |PSLL - ESLL| + \omega_2 |MNUL - ENUL| + \omega_3 |NU_SD| + \omega_4 |BW|$$

(9)

where target peak side-lobe level is indicated as $ESLL$. Average null depth and target null depth are expressed as $MNUL$ and $ENUL$. Besides, $NU_SD$ indicates null variance namely the resolution, and $BW$ is the bandwidth, and $\omega_1, \omega_2, \omega_3, \omega_4$ respectively stand for the weighting factor of side-lobe level, null level, $NU_SD$ and $BW$. Moreover the summation of them is 1, the bigger of a weighting factor means the more of a parameter was considered to get better convergence results.

Assume that the array amplitude vector of a full vector is shown as (10), in which the parameters’ values go from 0 to 1. By initializing the excitation of array with the hadamard product of $M_S$ and $M_E$, the amplitude of MIMO array can be sown as (11).

$$E_M = rand ((1, N_R + N_T))$$

(10)

$$A_M = E_M \circ S_M$$

(11)

where ($\circ$) is hadamard product operator. To get better optimization result, the vector $A_g = E_g \circ S_g$ and $A_f = E_f \circ S_f$. Finally, the objective function is evaluated for MIMO radar pattern synthesis with the following equation

$$fit = f / \max(f)$$

(12)

3.2. Immune Differential Evaluation Algorithm

So far, there is no clearly definition of immune algorithm, but most of the research about immune algorithm was conducted on the GA. However, GA usually uses binary coding, and DE uses real number coding method. In this research, a new method which introduces immune mechanism to DE rather than GA was proposed due to the amplitude vector, which can be used to solving the concrete multiple extreme optimization problem.

The realization of IDE can be expressed as:

(1) Initial the population

Assume that the $i$-th genes of the $j$-th antibody in the $k$-th generation of the population $x(k)$ meets the limit: $\{x(k) | x_{i,j}(k) \in [lb, ub]\}$, where $i = 1, 2, \cdots, M$, $j = 1, 2, \cdots, N$, $lb$ and $ub$ are the lower band and the upper band of each genes, and the amount of antibodies in the population and genes in each antibody are expressed as $M$ and $N$. So the initial gene of each antibody in the population is that:

$$x_{i,j}(0) = lb + rand(0,1) \cdot (up - lb)$$

(13)

(2) Evaluate the population
In order to optimize the MIMO radar, IDE adopts Equation (12) to measure the population in the optimization of MIMO radar, which indicates the affinity of the antibody and the antigen in a body.

(3) Generate new population
Both immune algorithm and differential evolutionary algorithm are based on the organic evolution phenomenon, in which the fittest body can survive, while the ones which cannot fit the environment would be eliminated. So the new population is generated by this law.

① Mutation
To make the population fit the environment, a body can be muted to survive or eliminate, and the whole bodies in the population can also be muted into new form. If the survival ones are more than the eliminated ones in the new form, the population will survive, otherwise the population will finally be eliminated. So the mutation of the population in IDE is divided into two parts, and one is the mutation of the body, and the other is the mutation of the population, which are expressed in (14) and (15):

\[ y_{i,k}(g) = y_{r_i,k}(g) + F_A \cdot [y_{r_i,k}(g) - y_{r_i,k}(g)] \] (14)

\[ v_h(g + 1) = y_h(g) + F \cdot [y_{h,g}(g) - y_{h,g}(g)] \] (15)

These two equation indicates the gene mutation of the \( k \)-th body and the mutation of the \( h \)-th generation population respectively. In order to avoid the premature of DE, the mutation parameters \( F_A \) and \( F \) are renewed with the generation under the global best and the local best antibody in the population. However, (14) would change the sparsity of the MIMO radar which is designed by (7) and (11), so it is forced to thin randomly to fit (7).

② Crossover
The rules of crossover are described as the following equation (16), including \( g \)-th generation population and the temporary population, where the integer \( j_{\text{rand}} \) is limited as \( 1 \leq j_{\text{rand}} \leq D \).

\[ u_{j,g}(g + 1) = \begin{cases} v_{j,g}(g + 1) & \text{[} \text{rand}(0,1) \leq CR] \text{or}\left[j = j_{\text{rand}}\right] \\ x_{j,g}(g) & \text{others} \end{cases} \] (16)

(4) Selection
A selection operator is necessary to IDE, and greedy selection method is applied. The adopted selection method is expressed as equation (17), and the fitness of \( u \) is evaluated through \( \text{fit}(u) \).

\[ x_{j,g}(g + 1) = \begin{cases} u_{j,g}(g + 1) & \text{fit}[u_{j,g}(g + 1)] \leq \text{fit}[x_{j,g}(g)] \\ x_{j,g}(g) & \text{others} \end{cases} \] (17)

4. Experiments and Numerical Results

4.1. Experiments and Numerical Results
In order to know the convergence ability of IDE, two experiments were conducted to show the difference of constant mutation parameters and variable mutation parameters. Firstly, the values of common parameters in this two experiments were shown as Table 1. Secondly, we assume that the number of constant parameters to be selected are \( F_A = 0.5 \) and \( F = 0.3 \). Thirdly, \( F_A = 1 - 1/\exp(gb) \) and \( F = 1 - 1/\exp(lb) \) are set as the variable mutation parameters, where \( gb \) and \( lb \) are respectively global best and local best. Figure 2 is the computation result.

In Figure 2 shows the convergence of IDE algorithm with variable parameters and constant parameters through fitness curve. Firstly, the flat region in the fitness curve can be considered as the search temporarily trapped in local minimum. Both variable parameters and constant parameters used in IDE have the ability to jump out of local minimum, and converge as 14.73 and 15.25. As what’s expected, the fit values of variable parameters are lower than the fit values of constant parameters in the same degree, and it shows that the convergence ability of variable parameters is stronger.
More specifically, computational indexes and performance are listed in Table 2 of 50 times’ experiments, including PSLL, time cost and convergence rate. The data from Table 2 and Figure 2 confirms that IDE with variable parameters pattern has the lower peak side lobe level \( \text{PSLL}_{\text{Best}} = -39.69 \text{dB} \) than IDE with constant parameters pattern, which is \(-26.82 \text{dB}\), what indicate that the convergence performance of IDE with variable parameters pattern is better than that with constant parameters pattern, although the average running time and integration number are larger.

| Population | Max_gen | CR | \( \omega_1 \) | \( \omega_2 \) | \( \omega_3 \) | \( \omega_4 \) |
|------------|---------|----|-------------|-------------|-------------|-------------|
| 64         | 1000    | 0.5| 0.4         | 0.2         | 0.2         | 0.2         |

![Figure 2. The convergence result.](image)

4.2. Experiment of null depth optimization at specific angles

In this experiment, all the simulation parameters are also set as Table 1, and two simulation are conducted, one expect the null depth \( \pm 16^\circ, \pm 56^\circ \) and \( \pm 73^\circ \) which is shown in Figure 3(a), and the other expect the null depth is lower than -40dB at 0° and main lobe is at 90° which shown in Figure 3(b). It shows that IDE is suitable and effective for MIMO radar to optimize pattern synthesis.

![Figure 3. Null depth optimization at specific angle achieved by ID.](image)
4.3. Experiment Joint PSLL Optimization of Equation 9
As it indicated that the IDE of variable parameters pattern can get optimal results, the experiment of PSLL optimization is conducted by this pattern. Figure 4 and Table 3 are the simulation results of 50 times running of IDE for pattern optimization. The simulation results achieved by IDE of MIMO radar pattern optimization in Table 3 show that IDE with variable parameter joint optimization can perform well, where the lower peak side lobe level PSLL_{best} = -30.02 dB.

![Figure 4. Pattern optimization of MIMO radar achieved by IDE of variable parameter.](image)

| Parameters’ type | PSLL/dB | Average running time / s | Integration number |
|------------------|---------|--------------------------|--------------------|
| Best             | Average | Best | Average |
| variable         | -30.02  | -29.78 | 4.97 | 887 | 849 |

4.4. Performance of IDE
Table 4 shows comparison of DPSO, CDE, DSDE and IDE proposed in this article. Firstly, comparing the performances of the three indicators in Table 4, IDE is better than DPSO with lower PSLL value and running time. Secondly, the average PSLL value of IDE is larger than CDE, but IDE can take less running time to get optimal PSLL result. What means that IDE has the advantages of maintaining the diversity and avoiding risk of prematuration. Finally, IDE can perform better than DSDE in average PSLL value and running time, which could make up for the lack of the best PSLL. Comparative analysis means that the variable parameter FA of IDE algorithm has better optimization ability than DPSO, CDE and DSDE.

![Table 4. Performance of IDE compared to DPSO, CDE and DSDE.](image)

| Algorithms | DPSO[14] | CDE[16] | DSDE[18] | IDE    |
|------------|----------|---------|----------|--------|
| Average PSLL/d | -28.32 | -33.0045 | -30.04 | -30.82 |
| Best PSLL/d | -32.20 | -33.02 | -35.19 | -33.69 |
| Average Running Time/s | 21.45 | 11.23 | 10.03 | 4.02 |

5. Conclusions
In this paper, the mechanism of immune algorithm is introduced to DE, which renewed the convergence mechanism by changing the mutation method. According to experiment results showed that the novel IDE can lower running time comparing to DPSO, CDE and DSDE, on the basis of ensuring acceptable diversity and PSLL. Besides, the numerical results of the joint optimization of the position and the amplitude of MIMO radar’s arrays showed that the novel proposed algorithm can get better optimal results, which indicates that IDE algorithm can get better global results and avoid the premature of DE. Although, both the position and the amplitude of MIMO radar’s arrays have been taken into consideration, there is much to do, such as the joint optimization of MIMO by introducing...
random algorithm, such as immune algorithm and DE, into deterministic algorithm to get better results.

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