Theoretical study of non-Newtonian micropolar nanofluid flow over an exponentially stretching surface with free stream velocity

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Abstract
The computational analysis of the second-order micropolar stagnation point flow of nanofluid over an exponentially permeable stretching sheet is considered. The freestream velocity with the thermal slip effects is taken into account in this analysis. This model is developed on the basis of flow assumptions and reduced into partial differential equations before applying the boundary layer approximations. The governing equations as a mathematical model are simplified with the help of suitable transformations. The differential system is further solved by using the bvp4c. Both graphs and tables are used to report observations. The skin friction and Nusselt number are reported for both weak and strong concentrations. The magnitude of skin friction is noticed greater for strong concentration in comparison with weak concentrations. Subject to couple stress, the values of skin friction are relatively high for the case of weak concentration in comparison with strong concentration. Micropolar profile admit the direct relation toward micropolar parameter and micro-gyration parameter. Both Sherwood number and Nusselt number admits higher values for strong concentration as compared to weak concentration.

Keywords
Non-Newtonian fluid, micropolar fluid, exponential stretching, numerical technique, slip effects

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Introduction
The study of non-Newtonian fluids acknowledged notable attention by the researchers because of a wide range of applications. Owning to such importance as yet numerous non-Newtonian fluid models are introduced to encounter the flow field properties of fluids like the flow of an incompressible second-order fluid past a stretched sheet was considered by Rajagopal et al.¹

The issue involved continuous extrusion of a polymer sheet from a die, which is a polymer processing application. Bujurke et al.² investigated second order fluid flow along with heat transfer aspects by use of
Noll and Coleman constitutive equations. This flow's boundary layer properties were determined. Both thermal and velocity boundary layer thickness were closely examined. Pontrelli3 examined the steady fluid flow of a second-grade homogeneous incompressible fluid across a stretching sheet. The flow partial differential equations are solved by using the collocation method. The solution dependency on the non-Newtonian parameter was explored, and the shear stress was provided along with several velocity distribution. The two dimensional flow of a viscoelastic fluid past a moving sheet was studied by Sadeghy and Sharifi.4 They utilized the boundary layer approach for their investigation. HAM solution for second grade fluid over a porous plate was solved by Hayat and Khan.5 Fetecau6 discussed the solutions of second grade fluid for unsteady and unidirectional flows. Khan et al.7 investigated the physical aspects of third grade nanoparticles with optimization of entropy. Flow of a viscoelastic fluid toward a fixed plate with non-axisymmetric Homann stagnation point was studied by Mahapatra and Sidui.8 Shamshuddin et al.9 examined the impacts of power law fluid model under the viscous dissipation with Hall current at exponential stretching sheet. Shamshuddin et al.10 considered the impacts of Joule heating and viscous dissipation on magnetohydrodynamics convective flow of a nanofluid past a stagnation point micropolar fluid flow with non Darcy porous medium having homogeneous-heterogeneous reactions. Dawar et al.11 analyzed the effects of MHD micropolar nanofluid chemically over stretching surface. They also highlighted the effects of variable heat sink/source and velocity slips effects for micropolar nanofluid. The recent developments in this regard can be assessed in References.13–15

The fluid with nanosized particles is termed as nanofluid. Owning to importance of nanofluids various authors reported their finds like Rahman et al.16 discussed the suspended nanoparticles in the fluid flow past a porous exponentially stretching/shrinking by using the Buongiorno’s model. Thermal radiation, magnetic field effects with slip flow over a stretching/shrinking sheet with nanofluid was concluded by Abdul Hakeem et al.17 Viscous dissipation and second order slip of melting heat transfer on magnetohydrodynamics convective flow of a nanofluid past a stretching sheet was studied by Mabood and Das.18 Kadir et al.19 investigated the effects of von Karman swirling bioconvection over a rotating disk with the nanofluid flow. Beg et al.20 offered the experimental results about the lubricity and rheology of drilling liquids enriched having nanoparticles. Shamshuddin et al.21 examined the time dependent micropolar fluid flow over rotating disk under the multi-physicochemical magnetic field. Numerical outcomes of nanofluid effects discussed by Kumar and Sokhal.22 Khan et al.23 studied the features of activation energy and Wu’s slip of bioconvection with magnetized couple stress nanofluid. Nadeem et al.24 examined the impacts of CNTs with base fluid at rotating disk. Abbas et al.25 reported the impact of induced MHD micropolar fluid flow with viscous dissipation over an exponentially stretching sheet. The thermal slip effects and suction/injection are also considered. The mathematical model is developed using the boundary layer approximation under the flow assumptions. The developed differential model is solved through the numerical technique namely bvp4c. Before our study, no one highlighted the effects on the boundary layer flow discussed. The ultimate results are shared by using tables and graphs.

Mathematical formulation

Non-Newtonian stagnation point micropolar fluid flow over an exponential stretching surface is considered. Both suction and injection are considered and specifically, \( V_w < 0 \) signifies the injection flow and \( V_w > 0 \) symbolizes the suction flow. The nanoparticles are considered suspended in the flow regime. Buongiorno fluid model is used in this analysis. The geometry of problem is given in Figure 1. The flow field is mathematically modeled and the ultimate flow differential system for micropolar fluid (Rehman et al.,30 Dero et al.)31 is concluded as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_e \frac{\partial U_e}{\partial x} + \left( \frac{\mu + k}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \alpha_1 \left( \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right) + \frac{k}{\rho} \frac{\partial N}{\partial y}, \quad (2)
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\rho C_p}{\rho C_f} \left[ D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_e} \left( \frac{\partial T}{\partial y} \right)^2 \right] + \left( \frac{\mu + k}{\rho C_f} \right) \left( \frac{\partial T}{\partial y} \right)^2, \quad (3)
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left( \frac{\partial^2 C}{\partial y^2} \right) + \frac{D_T}{T_e} \frac{\partial^2 T}{\partial y^2}, \quad (4)
\]
(1)–(5) and we have, endpoint conditions of the fluid flow are defined as

\[ \frac{\partial N}{\partial x} + u \frac{\partial N}{\partial y} = \gamma^* \frac{\partial N}{\partial y} - \frac{k \partial u}{\partial y} \frac{2k}{\partial N}, \tag{5} \]

here, equation (1) represents the continuity equation, equation (2) represents the momentum equation, equation (3) represents the energy equation, equation (4) represents the concentration equation, and equation (5) represents the micropolar equation respectively. The suitable transformations are as follows:

\[ v = V_w, \quad u = U_w, \quad T = T_w + L \frac{\partial T}{\partial y}, \quad D_n \frac{\partial C}{\partial y} = 0, \quad N = -n \frac{\partial u}{\partial y} \text{ at } y \to 0, \]

\[ u = u_c, \quad T = T_c, \quad C = C_c, \quad N = 0 \text{ as } y \to \infty. \tag{6} \]

The suitable transformations are applied on equations (1)–(5) and we have,

\[ (1 + K)F^{'''} + \beta_1 \left( 2\eta F^{''} + 5F'F^{''} + 3F''F' - FF'' \right) + KH' - 2F'F' + FF'' = 0, \tag{8} \]

\[ \frac{1}{P_r} \frac{\partial \theta}{\partial y} + \frac{N_b \theta^2}{N_f} \frac{\partial \theta}{\partial y} + \theta' F' + Ec(1 + K)F''F'' = 0, \tag{9} \]

\[ \phi'' + \frac{N_i}{N_b} \phi' - Sc \left( 2F' \phi - \phi' F \right) = 0, \tag{10} \]

\[ \left( 1 + \frac{K}{2} \right) H'' + FH' - 3F' H - (2H + F'') = 0. \tag{11} \]

The reduced conditions are

\[ F(0) = S, \quad \theta(0) = 1 + \lambda \theta(0), \]

\[ F'(0) = \gamma', \quad N_b \phi'(0) + N_f \theta(0) = 0, \]

\[ H(0) = -nF'(0), \]

\[ F' (\infty) = 1, \quad \theta (\infty) = 0, \quad \phi'(\infty) = 0, \quad H(\infty) = 0, \tag{12} \]

where, \( F(\eta) \) and \( \theta(\eta) \) are the velocity and temperature of the fluid. \( \phi(\eta) \) and \( H(\eta) \) are micropolar and concentration profiles. \( S > 0 \) presented the suction and \( S < 0 \) represents the injection. The shear stress and wall heat stress on the permeable sheet are defined as follows:

\[ C_{xw} = \frac{u_w}{ho_w U_w^2}, \quad C_m = \frac{\tau_m}{\rho_w U_w^2}, \quad Nu_x = \frac{xyw}{k_f (T_w - T_c)}, \]

\[ \tau_m = -\rho_f \left( 1 + \frac{K}{2} \right) \left( \frac{\partial N}{\partial y} \right)_{y \to 0}, \]

\[ q_w = -k_f \left( \frac{\partial T}{\partial y} \right)_{y \to 0}, \quad q_m = -D_n \left( \frac{\partial C}{\partial y} \right)_{y \to 0}, \]

\[ \tau_w = \left( \left[ \mu + k \right] \frac{\partial u}{\partial y} + \left( 2 \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \left( \frac{\partial F}{\partial x} \right) \right)_{y \to 0}. \tag{13} \]

After dimensionlized the above equations, we have

\[ Re_{f}^{1/2} C_f = (1 + (1 - n)K)F''(0) + \beta_1(7F''(0)F''''(0) - F''(0)F''''(0)) \]

\[ Re_{c} C_m = (1 + K/2) H'(0) \]

\[ Re_{c}^{1/2} Nu_x = -\theta'(0), \]

\[ Re_{c}^{1/2} Sh_c = -\phi(0). \tag{14} \]

**Numerical procedure**

Equations (8)–(12) are nonlinear and hence cannot be solved exactly. Therefore we seek a numerical solution by using bvp4c through Matlab. The initial value problem subject to the above dimensionless system is as follows:

\[ y(1) = F(\eta); \quad y(2) = F'(\eta); \quad y(3) = F''(\eta); \]

\[ y(4) = F'''(\eta); \quad y(5) = yy' \quad y(6) = F''(\eta); \]

\[ \theta(\eta) = y(5); \quad \theta'(\eta) = y(6); \quad \theta''(\eta) = yy'2; \]

\[ \phi(\eta) = y(7); \quad \phi'(\eta) = y(8); \quad \phi''(\eta) = yy'3; \]

\[ H(\eta) = y(9); \quad H'(\eta) = y(10); \quad H''(\eta) = yy'4; \]
\[
\begin{align*}
\nu_1 &= \left(\frac{1}{\beta y(1)}\right)((1 + K)y(4) + \beta_1(2y(3)y(4)) \\
&\quad + 3y(2)y(4) + 3y(3)y(3)) \\
&\quad + Ky(10) - 2y(2)y(2) + y(1)y(3)), \\
\nu_2 &= -Pr(N_b y(6)y(6) + N_d y(6)y(8) - 2y(5)y(2) \\
&\quad + y(1)y(6) + Ec(1 +Ky(3)y(3)), \\
\nu_3 &= -\left(\frac{N}{N_b}\right)y_2 - Sc(2y(2)y(7) - y(1)y(8)), \\
\nu_4 &= \left(1 + \frac{K}{2}\right)^{-1}(y(1)y(10) - 3y(2)y(9)) \\
&\quad - K(2y(9) + y(3))).
\end{align*}
\]

Dimensionless boundary conditions takes the form
\[
\begin{align*}
y(1) - S; & \quad y(5) - 1 - \lambda y(6); \\
y(2) - \gamma; & \quad N_b y(0)(8) + N_d y(0)(6); \\
y(9) + ny(3); & \quad yinf(2) - 1; \quad yinf(5); \quad yinf(7); \quad yinf(9).
\end{align*}
\]

It is important to note that the following values are used of default parameters
\[
\beta_1 = 0.1, \quad K = 0.1, \quad Pr = 1.5, \quad N_b = 0.1, \quad N_d = 0.1, \quad S = 0.1, \\
Ec = 0.1, \quad Sc = 0.2, \quad n = 0.5, \quad \lambda = 0.1, \quad and \gamma = 0.1.
\]

### Results and discussion

The dimensionless system of nonlinear differential equations are solved through numerical technique. The physical effects of involving parameters under the flow assumptions are discussed in the form of tables and figures. The impact of flow variables on \(F'(0), H'(0), \phi'(0),\) and \(\theta'(0)\) are highlighted in Tables 1 and 2 for both cases of strong and weak concentration. The variations of the couple stress and skin friction for different values of \(S\) for both cases of weak and strong concentrations are presented in Table 1. It is seen that higher values of \(S\) reduced the values of skin friction and the couple stress in both cases of strong and weak concentration. The value of skin friction is significant for the case of strong concentration as compared to weak concentration. The impact of stretching parameter on couple stress and skin friction is offered in Table 1 for both cases of strong and weak concentration. The increasing stretching parameter decline the skin friction and enhances the couple stress. The impacts of the material parameter on couple stress and skin friction is also reveals in Table 1. The couple stress and skin friction enhances due to increase in material parameter. The impacts of the micropolar parameter are reveals in Table 1. Both skin friction and couple stress admits significant variations toward micropolar parameter. The impact of variations of the suction parameter on Sherwood and Nusselt numbers are reported in Table 2. The increasing suction parameter improved the Nusselt number and Sherwood number in both cases for strong and weak concentration. The impact of increasing values of Brownian motion parameter is also offered in Table 2. The increasing values of Brownian motion parameter cause decline in the Nusselt number and Sherwood number in both cases for weak and strong concentrations. The influence of thermophoresis parameter on the Sherwood and Nusselt numbers is presented in Table 2. The increasing values of thermophoresis parameter improved the Sherwood and Nusselt numbers in both cases for strong and weak concentration. The influence of stretching parameter on both numbers is also offered in Table 2. Significant impact of the stretching parameter on the Nusselt number and Sherwood number is noticed for both cases namely strong and weak concentration. The impact of variation of material parameter

| \(S\) | \(y\) | \(\beta_1\) | \(K\) | \(n = 0.0\) | \(n = 0.5\) |
|-----|-----|-----|-----|---------|---------|
|     |     |     |     | \(F'(0)\) | \(H'(0)\) | \(F'(0)\) | \(H'(0)\) |
| 0.1 | 0.2 | 0.1 | 0.2 | 1.8241 | -0.0571 | 1.8194 | 1.3828 |
| 0.2 | -   | -   | -   | 1.8204 | -0.0590 | 1.8157 | 1.4230 |
| 0.3 | -   | -   | -   | 1.8166 | -0.0608 | 1.8119 | 1.4639 |
| 0.5 | 0.1 | -   | -   | 2.0192 | -0.0474 | 2.0140 | 1.5175 |
| -   | 0.2 | -   | -   | 1.8241 | -0.0571 | 1.8194 | 1.3828 |
| -   | 0.3 | -   | -   | 1.6287 | -0.0668 | 1.8194 | 1.3828 |
| -   | 0.4 | -   | -   | 1.8241 | -0.0571 | 1.8194 | 1.3828 |
| -   | 0.5 | -   | -   | 1.8297 | -0.0581 | 1.8211 | 1.3841 |
| -   | 0.6 | -   | -   | 1.8345 | -0.0592 | 1.8225 | 1.3852 |
| -   | 0.7 | -   | -   | 1.8429 | -0.0875 | 1.8403 | 1.3006 |
| -   | 0.8 | -   | -   | 1.8241 | -0.0571 | 1.8194 | 1.3828 |
| -   | 0.9 | -   | -   | 1.8079 | -0.0305 | 1.8015 | 1.4550 |
The influence of Schmidt number on both numbers is offered in Table 2. Increasing values of Schmidt number cause decline in Nusselt number and Sherwood number for both cases namely strong and weak concentration. The influence of micropolar parameter on the Nusselt number and Sherwood number is offered in Table 2. Increasing values of micropolar parameter enhances which enhance the Sherwood and Nusselt numbers for both cases namely strong and weak concentration.

The impacts of the involving parameter on velocity, temperature, concentration, and micropolar profile are highlighted in the Figures 2 to 12. Figure 2 shows the impacts of $K$ on the velocity. It is noted that velocity decreases with increase in micropolar parameter. Physically, vortex viscosity enhances which declines the velocity profile. The influence of material parameter on the velocity profile is given in Figure 3. Figure 3 indicates that velocity of the fluid declines by increasing the values of the material parameter $\beta_1$ of the fluid. The second grade fluid parameter enhances which enhance the viscosity of fluid and as a result the fluid velocity decline. Figure 4 is presented the effects of stretching parameter on the velocity function. The velocity function declines due to enhancing the values of stretching parameter. The variation of the suction parameter with velocity function which reveals in Figure 5. The velocity function found to be decline behavior when the values of suction parameter enhances. Figure 6 indicates the relation between the temperature profile and Prandtl number. The temperature function reduced when values of Prandtl number increase. Physically the thermal diffusivity decreases as the Prandtl number increases. 

### Table 2. Numerical results of Sherwood Number and Nusselt Number for different values of involving physical parameters in both cases of strong ($n = 0.0$) and weak ($n = 0.5$) concentrations.

| $S$ | $N_b$ | $N_t$ | $\gamma$ | $\beta_1$ | $Sc$ | $\lambda$ | $Ec$ | $K$ | $Pr$ | $\theta'(0)$ | $\phi'(0)$ | $\theta'(0)$ | $\phi'(0)$ |
|-----|-------|-------|--------|--------|-----|--------|-----|-----|-----|--------|--------|--------|--------|
| 0.1 | 0.3   | 0.2   | 0.3    | 0.1    | 0.2 | 0.1    | 0.2 | 6.0 | -1.3491 | -0.4497 | -1.2924 | -0.4308 |
| 0.2 | -     | -     | -      | -      | -   | -      | -   | -   | -1.4591 | -0.4864 | -1.4052 | -0.4684 |
| 0.3 | -     | -     | -      | -      | -   | -      | -   | -   | -1.5651 | -0.5217 | -1.5144 | -0.5048 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.8339 | -0.6120 | -1.7615 | -0.5872 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.5567 | -0.5189 | -1.4922 | -0.4974 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3491 | -0.4497 | -1.2924 | -0.4309 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3055 | -0.4352 | -1.2468 | -0.4156 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3491 | -0.4497 | -1.2924 | -0.4308 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3912 | -0.4637 | -1.3383 | -0.4461 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.4025 | -1.4025 | -1.2937 | -1.2937 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.4013 | -0.7007 | -1.2930 | -0.6645 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.4002 | -0.4667 | -1.2930 | -0.6645 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3491 | -0.4497 | -1.2924 | -0.4308 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3492 | -0.4498 | -1.2923 | -0.4307 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3493 | -0.4499 | -1.2922 | -0.4306 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3509 | -0.4503 | -1.2924 | -0.4308 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3491 | -0.4497 | -1.2930 | -0.4301 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3472 | -0.4491 | -1.2881 | -0.4294 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3509 | -0.4503 | -1.3489 | -0.4496 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3216 | -0.4455 | -1.3196 | -0.4399 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.2923 | -0.4308 | -1.2903 | -0.4301 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3509 | -0.4503 | -1.3489 | -0.4496 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3216 | -0.4405 | -1.3196 | -0.4399 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.2923 | -0.4308 | -1.2903 | -0.4301 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.2924 | -0.4308 | -1.3497 | -0.4499 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.2923 | -0.4308 | -1.3489 | -0.4496 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.2922 | -0.4308 | -1.3482 | -0.4494 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.2923 | -0.4308 | -1.3489 | -0.4496 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3158 | -0.4386 | -1.3753 | -0.4584 |
|     | -     | -     | -      | -      | -   | -      | -   | -   | -1.3377 | -0.4459 | -1.4000 | -0.4667 |
increases. Figure 7 presented the impact of variation of Eckert number on temperature function. The increases in Eckert number cause decrease in the temperature of the fluid. Eckert number indicates the kinetic energy of the flow relative to the boundary layer enthalpy differences. Eckert number shows an important role in high speed flows. Figure 8 shows influence of thermophoresis on temperature function. The temperature function curves increasing due to higher values of thermophoresis parameter. Figure 9 illustrates the effects of thermal slip parameter on the temperature function. The increasing thermal slip parameter cause decline in the temperature function. Figure 10 shows that influence of Brownian motion parameter on temperature function. The temperature function enhances due to higher values of Brownian motion parameter because the fluid temperature enhances due to the increase in the kinetic energy. Figure 11 reveals micropolar profile enhancement due to higher values of micro-gyration. Figure 12 shows the variation of the micropolar profile toward micropolar parameter. We found that micropolar profile is increasing function of micropolar parameter.

**Conclusions**

Non-Newtonian micropolar fluid flow along with suspended nanoparticles subject to an exponentially stretching surface under the stagnation point flow is
considered in the current analysis. The key outcomes are as follows:

- The values of skin friction are higher in magnitude for the case of weak concentration ($n = 0$) in comparison with strong concentration ($n = 0.5$).
- The magnitude of couple stress is observed higher for the case of strong concentration ($n = 0.5$) as compared to weak concentration ($n = 0$).
- Significant variations are noticed for Nusselt and Sherwood numbers toward involved flow parameters subject to strong and weak cases.
Fluid velocity is found as increasing function of $g$ while opposite is the case for $S$, $K$ and $\beta_1$.

Fluid temperature admits direct relation toward Ec, $N_t$, and $Nb$ but inverse is the case for Pr and $\lambda$.

Micropolar profile shows direct relation toward $n$ and $K$.

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Appendix

Notations

\( \beta_1 \) material parameter

\( Pr \) Prandtl number

\( N_t \) thermophoresis parameter

\( Ec \) Eckert number

\( n \) micro-gyration parameter

\( \gamma \) stretching parameter

\( \mu \) viscosity of fluid

\( D_T \) coefficient of thermophoresis diffusion

\( \alpha_1 \) material constant

\( \alpha \) thermal diffusivity of fluid

\( n \) micro-gyration

\( K \) micro-polar parameter

\( N_b \) Brownian motion parameter

\( S \) suction/injection

\( Sc \) Schmidt number

\( \lambda \) thermal slip

\( k \) vertex viscosity of fluid

\( \rho \) density of fluid

\( T_c \) ambient temperature

\( D_B \) coefficient of Brownian diffusion

\( \gamma^* \) spin gradient viscosity