PLASTIC RESPONSE OF A 2D AMORPHOUS SOLID TO QUASI-STATIC SHEAR:
II - DYNAMICAL NOISE AND AVALANCHES IN A MEAN FIELD MODEL

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Abstract

We build a minimal, mean-field, model of plasticity of amorphous solids, based upon a phenomenology of dissipative events derived, in a preceding paper [A. Lemaître, C. Caroli, arXiv:0705.0823] from extensive molecular simulations. It reduces to the dynamics of an ensemble of identical shear transformation zones interacting via the dynamic noise due to the long ranged elastic fields induced by zone flips themselves. We find that these ingredients are sufficient to generate flip avalanches with a power-law scaling with system size, analogous to that observed in molecular simulations. We further show that the scaling properties of avalanches sensitively depend on the detailed shape of the noise spectrum. This points out the importance of developing a realistic coarse-grained description of elasticity in these systems.
I. INTRODUCTION

Much effort has recently been devoted to the development of theoretical descriptions of plasticity of amorphous media. They aim at proposing macroscopic constitutive laws consistent with the microscopic information emerging from a wealth of numerical results accumulated over the past twenty years. In particular, it is now well established that plastic deformation in these systems proceeds via irreversible sudden rearrangements of small clusters of atoms. Even at zero temperature, these flips occur intermittently. These empirical facts are the basis of two recently proposed phenomenologies: the STZ (shear transformation zone) theory \[1\] and SGR (soft glass rheology) model \[2\]. Both represent the sheared disordered solid as a set of spatially random and independent ”zones” or ”traps” of small size embedded in a homogeneous background. These structures are metastable, so that, when loaded elastically by the external driving strain, in the absence of noise elastic loading would stop at an instability threshold where they flip into an unstressed state. Moreover, in both models, structural disorder gives rise to a noise acting in parallel with advective loading, thus resulting in intermittent flips occurring before absolute instability is reached. An important point is that they introduce noise via Arrhenius factors associated with a constant, strain-rate independent effective temperature.

In an attempt at testing the validity of the assumption concerning flip independence, Maloney and Lemaître \[3\] later carried out extensive numerical simulations on 2D glasses of various sizes $L \times L$ in the athermal quasi-static regime ($T = 0$; vanishing strain rate $\dot{\gamma} \to 0$), hereafter abbreviated as AQS. In this regime, pure elastic, reversible, loading is interrupted by randomly spaced discontinuous stress drops associated with the (quasi-instantaneous) plastic events. They found that these events can be interpreted as avalanches involving a varying number $n$ of elementary zone flips. In the stationary state, the avalanche size $n$, hence the stress drop amplitude $\Delta \sigma$ are broadly distributed, and their averages are system-size dependent. The average avalanche size scales roughly as $\langle n \rangle \sim L^4$. Distributed avalanches have also been found by Bailey et al in 3D simulations \[5\], with an approximate scaling $\langle n \rangle \sim L^{3/2}$.

In the 2D simulations, flips show up clearly as elastic quadrupolar structures in the energy and the non-affine displacement field, consistent with their representation as shear transformations of Eshelby-like inclusions \[6\]. Avalanches are known to result from long range
interactions. In the present case interzone elastic couplings mediated by the background medium are thus likely to be responsible for the avalanche behavior.

Motivated by this analysis, we have performed in a previous, companion, paper an extensive numerical study of the evolution with strain of the non-affine field in a 2D LJ glass. It substantiates the importance of interzone elastic couplings and leads to the following picture, consistent with that proposed already long ago by Argon et al: via its associated quadrupolar field, a zone flip induces, at any other zone site in the system, a shift of the strain level whose amplitude and sign depend on the relative position of target and source. These shifts may bring some zones past their instability threshold, hence triggering an avalanche. For the zones which do not take part in an avalanche, the flip-induced elastic signals act as an intrinsic dynamical noise the frequency of which scales as the strain rate $\dot{\gamma}$. The statistical study of particle motion shows that this noise dominates largely over fluctuations associated with non affine deformations during the elastic episodes separating the plastic events.

In this article we propose a minimal model which incorporates as simply as possible the essential features of this phenomenology in terms of a set of identical spatially random zones embedded in a 2D elastic continuum, driven by external loading toward their instability threshold and coupled via flip-induced quadrupolar elastic fields.

In Section II we define our model in detail. We discuss the various relevant time scales in steady plastic flow, which allows us to identify, for finite systems, a size-dependent quasi-static regime where avalanches can be considered instantaneous. We then study numerically, in a mean-field approximation for the elastic noise, the steady state dynamics for different system sizes. We find that dissipation occurs via broadly distributed flip avalanches, the average size of which exhibits a power law scaling with system size: $<n> \sim L^\beta$. However, the exponent, $\beta \approx 0.3$, is definitely smaller than the value, of order 1, measured in reference.

So, in Section III we discuss the various simplifications involved in our modelling of zones and of their elastic field. This leads us to test a second version of the model in which we assume empirically a gaussian spectrum for the elastic noise. While preserving the existence of the avalanche dynamics, this modified model turns out to predict the correct power law scaling for $<n>$. We further show that, in this case, an analytical estimate based on a Fokker-Planck-like approximation yields the same prediction for the scaling exponent.

Although still preliminary, these results lead us to conclude that phenomenologies of
plasticity of amorphous media should definitely incorporate into their basic ingredients the strain-rate dependent elastic noise generated by the zone flips themselves.

II. ELASTICALLY COUPLED ZONES AND AVALANCHES:

A. The model

We consider an ensemble of $N$ identical zones of size $a$, shear elastic modulus $\mu$, randomly distributed with a fixed density $\rho$ in a 2D elastic medium of lateral size $L$. The average distance between nearest zones $d$ is fixed and given by: $d^2 = L^2/N = a^2/\rho$. The elastic state of each zone is characterized by an internal strain $\epsilon_i$, which measures the departure from its zero stress state. The $\epsilon_i$'s lie below a common instability threshold $\epsilon_c$. The athermal system is driven by external shear at rate $\dot{\epsilon}$, which advects all $\epsilon_i$'s. When a zone reaches $\epsilon_c$, it disappears while releasing an amount of internal strain $\Delta \epsilon_0$. At the same time another one is created, at an uncorrelated position, with zero initial stress (hence zero internal strain).

During a zone flip the cluster of atoms forming the zone jump into a configuration compatible with the externally imposed strain. This process relaxes the intra-zone stress and, at the same time, deforms the surrounding elastic medium. Following Picard et al [9], its field can be represented as due to two force dipoles (Figure 1). We take for this strain field its expression in an infinite medium. At relative position $\mathbf{r} = (r, \theta)$ from the flipping center:

$$\Delta \epsilon(\mathbf{r}) = \frac{2 a^2 \Delta \epsilon_0}{\pi r^2} \cos 4\theta$$

(1)

where $\theta$ is measured from the shearing direction.

It has quadrupolar symmetry, hence zero average. $\Delta \epsilon_0$ and $\epsilon_c$ can be related using the following argument: the amount of stress $\mu \Delta \epsilon_0$ released inside the zone by a flip gives rise to a spatially averaged, macroscopic stress drop $\Delta \sigma_0 = \kappa \frac{a^2}{L^2} \mu \Delta \epsilon_0$, where the number $\kappa = \mathcal{O}(1)$ depends on the shape of the system. From now on, we assume that $\kappa = 1$. In stationary
state, over a large strain interval, the average number of zone flips is $N \Delta \epsilon / \epsilon_c$. The associated macroscopic plastic stress release $N \Delta \sigma_0$ must balance the increase of elastic stress, $\mu \Delta \epsilon$, due to loading. Hence

$$\frac{\Delta \epsilon_0}{\epsilon_c} = \frac{L^2}{Na^2} = \frac{d^2}{a^2}$$

(2)

The duration $\tau_0$ of a flip is controlled by the time necessary to radiate elastic energy out of the zone region, i.e. by radiative acoustic damping, so $\tau_0 \sim a/c_s$, with $c_s$ a sound speed. This acoustic signal, emitted from site $r_i$, propagates throughout and modifies the strains of all other zones $j$ which adjust, over a time $\sim \tau_0$, to this space-dependent shift $\Delta \epsilon(r_{ij})$. Flip signals thus constitute a noise acting on the $\epsilon_i$’s.

In steady state, the average flip rate in the whole system is

$$R_{\text{flip}} = \delta t_{\text{flip}}^{-1} = \frac{N \dot{\epsilon}}{\epsilon_c} = \frac{L^2 \dot{\epsilon}}{d^2 \epsilon_c}$$

(3)

If flips occur independently, i.e. in the absence of avalanches, the noise correlation time is $\tau_0$, and the QS regime, where flips can be assumed instantaneous, corresponds to $\delta \tau_{\text{flip}} \gg \tau_0$, that is to:

$$\dot{\epsilon} << \dot{\epsilon}_{\text{flip}} = \frac{c_s}{a} \epsilon_c \frac{d^2}{L^2}$$

(4)

For a glass-like system, with $\epsilon_c \sim 1\%$, $a \sim 1\text{nm}$, zone density $a^2/d^2 \sim 10^{-1}$, lateral size $L \sim 1\text{mm}$, this yields the loose criterion $\dot{\epsilon} << 1$.

Now, a first elastic noise signal may drive some $\epsilon_j$’s beyond $\epsilon_c$, hence trigger secondary flips, thus initiating an avalanche whose duration $\tau_{\text{av}}$ is set by sound propagation. For a very conservative estimate, we take the average distance between successive flips to be $L$. This leads to a duration $\tau_{\text{av}} = < n > L/c_s$, with $< n >$ the average avalanche size. It must be compared with the average time interval $10$ between avalanches $\delta t_{\text{av}}$, given by:

$$\delta t_{\text{av}}^{-1} = \frac{N \dot{\epsilon}}{< n > \epsilon_c}$$

(5)

The QS condition then becomes

$$\dot{\epsilon} << \frac{\epsilon_c c_s}{N L} = \frac{a}{L} \dot{\epsilon}_{\text{flip}}$$

(6)

Let us emphasize here an important point, usually ignored in related earthquake models centered on the issue of criticality: the quasi-static range is limited by the acoustic delays controlling avalanche spreading, hence shrinks with system size.
We will assume from now on that the QS condition is fulfilled. We thus model the noise by instantaneous shifts $\delta \epsilon_i$ of the zone strains. Moreover, we treat our model, now coined “E model”, in the mean-field approximation, i.e. assume that the $\delta \epsilon_i$’s are independent random variables, which amounts to neglecting space correlations between flip centers. That is, we take the spectrum $\Pi_E(\delta \epsilon)$ of these noise signals to be that due to a spatially uniform distributions of sources truncated at the average distance between nearest zones $d$.

$$\Pi_E(\delta \epsilon) = \frac{1}{\pi (L^2 - d^2)} \int_d^L r dr \int_{-\pi}^{\pi} d\theta \delta (\Delta \epsilon(r) - \delta \epsilon)$$

where $\Delta \epsilon(r)$ given by equation (1).

$\Pi_E(\delta \epsilon)$, plotted on Figure 2, is size-dependent, and has variance

$$M_2(N) = \frac{2\epsilon_c^2}{\pi^2 N}$$

It exhibits a narrowly peaked structure, associated with distant zones, and vanishes beyond the cut-off $(2a^2\Delta \epsilon_0/\pi d^2)$, so that all its moments are finite. Yet, it presents broad, power law, tails. For example, its first half moment

$$M_{1+}^E = \int_0^\infty d(\delta \epsilon) \delta \epsilon \Pi_E(\delta \epsilon) = \frac{8\epsilon_c}{\pi} \frac{\ln N}{N}$$

At this stage, the model can be summarized into the following two-step algorithm. Since, in the QS regime, the dynamics between instantaneous avalanches reduces to steady drift of all $\epsilon_i$’s at constant loading rate:
(i) Starting from an initial configuration where all $\epsilon_i < \epsilon_c$, we first identify $\epsilon_M = Max(\epsilon_i)$. We shift all $\epsilon_i$ by $\Delta \epsilon_{drift} = \epsilon_c - \epsilon_M$, and the first avalanche is triggered. Then:

(ii) Zone $M$ flips, i.e. is removed, while a new one is introduced at zero strain. This first flip emits a noise signal which randomly shifts all the other zones: $\epsilon_i \rightarrow \epsilon_{i1} = \epsilon_i + \delta \epsilon_i$, where the $\delta \epsilon_i$’s are independent and distributed according to $\Pi_E(\delta \epsilon)$. If all $\epsilon_{i1} < \epsilon_c$, we are back to step (i). Otherwise, an avalanche starts: we count the number $q_1$ of zones which flip at this stage and are replaced by new, unstrained, ones.

(iii) Each zone must now receive $zq_1$ signals, which we treat successively. The first of these yields a new shifted configuration $\{\epsilon_{i2}\}$ which produces $q_2$ new flips. $z$ is then updated to $z \rightarrow z - 1 + q_1$, etc. . . The avalanche stops when $z$ vanishes. Its size is $n = 1 + \sum q_i$.

B. Numerical results

In order to study the statistical properties of our model in steady state, we eliminate the initial transients ($\epsilon/\epsilon_c < 2$) and perform ergodic averaging over long strain intervals involving $\gtrsim 10^5$ avalanches.

As shown on Figure 3, their size distribution $\varpi_E(n)$ depends only weakly on the size $N$
for small $n (\lesssim 5)$. For $n \gtrsim 5$, it presents a quasi-exponential tail, which broadens noticeably with increasing $N$. This size dependence reflects into the growth with $N$ of the average avalanche size (see Figure 4) which we find to fit closely, over the whole $N$-range (two decades) a power law behavior:

$$\langle n \rangle \sim N^{\alpha_E} \quad \alpha_E = 0.147 \quad (10)$$

When plotting $\langle n \rangle \varpi_E(n)$ versus $n/\langle n \rangle$ (see insert of Figure 3) it turns out, however, that $\varpi_E(n)$ does not obey a simple scaling. Even though power law like decay may be identified on a limited, small-$n$, range, this by no means allows us to conclude to self-criticality – at variance with a previous claim by Chen et al on a related earthquake model [11].

We must now confront the above results with those of the molecular simulations. Clearly, our highly simplified model shows that long range interzone elastic couplings indeed produce broadly distributed avalanches with an average size growing as a power law of system size. However, at this stage, agreement is merely qualitative, since (i) The scaling exponent $\alpha_E$ differs from the simulation value $\alpha_{\text{sim}} \simeq 1/2$. (ii) In the molecular simulations [3], rescaling avalanche sizes by their $N$-dependent average results in a rather good collapse of data, which does not hold in the model.

FIG. 4: Average avalanche size $\langle n \rangle$ vs $N$ for the E model.
III. DISCUSSION

A. Approximations of the E model

The above comparison leads us to discuss in more detail the assumptions involved in our
minimal model.

First of all, let us consider more closely our representation of elastic couplings in our
finite system. When approximating the static elastic propagator by its value for an infinite
medium, we neglect the finite global elastic recoil which necessarily accompanies, when
driving at imposed strain, the macroscopic stress release after each flip. This recoil, of order
$1/L^2$, should be modelled as a common backward shift, $(-\xi/N)$ which should be added to
the noise $\delta\varepsilon_i$. We have rerun the E model for $\xi = 1$ and 2. We find that a finite recoil leaves
the power law scaling of $\langle n \rangle$ (equation (10)) unchanged, the intuitively expected avalanche
size reduction showing up only in a slow decrease with $\xi$ of the prefactor.

Let us now try to list the various simplifications underlying our representation of the
coupled zones. They can be separately into approximations concerning respectively (a) the
zones themselves and (b) the elasticity of the embedding medium.

(a) Zones:

We have taken them to be identical, i.e. to have the same threshold strain $\varepsilon_c$, and the
same shear modulus $\mu$, which we have assumed to be constant up to $\varepsilon_i = \varepsilon_c$. However,
clearly, in atomically disordered systems, both $\mu$ and $\varepsilon_c$ depend on details of the internal
structure of the zone and of its immediate vicinity. So, these two parameters are certainly
distributed about characteristic averages. For example, a signature of the spread of $\mu$ is
the observation in the LJ simulations [7] of instances in which a zone ”overtakes” another
one during an elastic loading episode. Moreover, we know [12] and have checked in ref. [7]
that significant elastic softening occurs near the threshold, where a metastability barrier
vanishes.

On the other hand, it was observed in [7] that, frequently, a first flip does not result in
the disappearance of the zone. Rather, this persists after a finite strain release, several flips
being needed for it to finally ”die out”. Describing this behavior would demand a multistate
zone model.

(b) Elastic couplings:
We have represented the embedding elastic medium as a homogeneous continuum, and have taken for the elastic field (equation (1)) its expression for an infinite system. We believe that this last approximation does not affect avalanche size scaling. Indeed, an elementary dimensional analysis for the stain field \( \Delta \varepsilon \) in a finite \( L \times L \) box shows that all its moments retain the same \( L \) dependence as those of the \( \Pi_E \) spectrum (equation (7)) used in Section II.

Note however that the homogeneous continuum approximation itself is probably overschematic, in particular at short distances, where numerical studies of the response to a localized force have shown that it is dominated by disorder-induced fluctuations [13]. This certainly contributes to a decrease of the tail of noise spectrum.

Pending more quantitative information about these finer disorder effects, we now propose to test the robustness of our minimal model by investigating the avalanche statistics under the empirical assumption of a gaussian elastic noise spectrum.

**B. Avalanches in a gaussian noise model**

In this "G model", we choose the gaussian noise spectrum \( \Pi_G(\delta \varepsilon) \) to have the same variance \( M_2(N) \) as that (equation (8)) for the E model. The algorithm is then implemented as already described.

Here again, we find broadly distributed avalanches, whose average size increases with \( N \) The data, shown on Figure 5 are consistent with the asymptotic (see insert) power-law

![Graph showing average avalanche size \( \langle n \rangle \) vs \( N \) for model G. The dashed line has slope 1/2.](image-url)
FIG. 6: Distribution $\varpi(n)$ of avalanche sizes, for model G and all system sizes. Inserts: $\langle n \rangle \varpi(n)$ vs $n/\langle n \rangle$.

behavior:

$$\langle n \rangle \sim N^{\alpha_G} \quad \alpha_G = 1/2$$

Moreover, Figure 6 shows that the rescaling of the size distribution $\varpi_G(n)$ as $\langle n \rangle^{-1} f(n/\langle n \rangle)$ leads to good collapse. So, agreement with the results of the molecular simulations is much more satisfactory than was the case for the E model.

We now attempt to clarify how the noise spectrum affects so significantly the avalanche behavior. We plot on Figure 7 the strain (ergodic) averages of the distributions $p_{E,G}(\epsilon)$ of zone strains in steady state for the two models and various system sizes $N$ ranging from 250 to 32000. For each model, $p$ converges rapidly almost everywhere toward a limit curve. This reflects into a very weak size dependence of the macroscopic stress $\bar{\sigma} = 2\mu\langle \epsilon_i \rangle$, which varies by less than 0.4% (G model) and 0.02% (E model) when $N$ increases from 1000 to 32000.

While $p_E$ and $p_G$ are similar in most of the $\epsilon$ range, they present significant differences in the two regions $\epsilon \sim 0$ and $\epsilon \sim \epsilon_c$. The peak in $p_E$ results from refeeding zones at $\epsilon = 0$ after flips. The larger an avalanche is, the more the corresponding peak is broadened by ulterior flips within the avalanche itself. We therefore attribute the washing out of the peak for the G model to the fact that it exhibits much larger avalanches. More significant for our purpose is the detailed behavior of $p$ near threshold, which reflects avalanche statistics.
FIG. 7: Steady state distribution $p$ of zone strains vs $\epsilon/\epsilon_c$ for: (a) E model; (b) G model. Inserts: blow-ups of the near-threshold region. The arrows indicate increasing values of $N = 250 \times 2^q$ ($q = 0, \ldots, 7$).

In particular, $p_c = p(\epsilon_c)$ is directly related to $\langle n \rangle$. Indeed, the average flip and avalanche rates verify $R_{flip} = \langle n \rangle R_{av}$. On the other hand, in steady state, the flux of zones which cross threshold under the effect of advective elastic loading, i.e. which initiate avalanches, is $f = R_{av} = \dot{\epsilon} p_c N$. So:

$$p_c = \frac{1}{\langle n \rangle \epsilon_c}$$

Relation (12) provides a consistency test of our calculations. We determine $p_c$ with the help of a second order polynomial extrapolation near $\epsilon_c$, with sampling intervals $1.25 \times 10^{-6} \epsilon_c$. We find that relation (12) holds within 1% for the G model and 3% for the E one.

In order to try and obtain analytical estimates for avalanche size scalings, we describe the evolution of $p(\epsilon)$ by the approximate master equation:

$$\frac{\partial p}{\partial t} = -\dot{\epsilon} \frac{\partial p}{\partial \epsilon} + \int_{-\infty}^{\epsilon_c} d\epsilon' p(\epsilon') w(\epsilon - \epsilon') - \Gamma p(\epsilon) + \frac{f}{N} \delta(\epsilon)$$

where $w$ is the single flip transition probability

$$w(\delta \epsilon) = N \frac{\dot{\epsilon}}{\epsilon_c} \Pi(\delta \epsilon)$$
with $i$ the noise distribution, and $\Gamma = \int_{-\infty}^{\infty} d(\delta \epsilon) w(\delta \epsilon)$. The delta term, proportional to the normalized zone flux $f/N = \dot{\epsilon}/\epsilon_c$, accounts for post-flip reinjection and ensures the conservation of zone number.

In this approximation, advection operates between all single flips, which amounts to neglecting intra-avalanche time correlations. This leads to an average advective $\epsilon$-shift during an avalanche $\Delta \epsilon_{\text{adv}} \sim R_{\text{av}} \sim \epsilon_c \langle n \rangle /N$, to be compared with the average diffusive broadening $\Delta \epsilon_{\text{diff}} \sim \sqrt{\langle n \rangle} M_2(N)$, with $M_2(N) = \epsilon_c^2/N$ the noise variance. Hence, $\Delta \epsilon_{\text{adv}} / \Delta \epsilon_{\text{diff}} \sim \sqrt{\langle n \rangle} \sim N^{-(1-\alpha)/2}$, which suggests that our approximation should improve in the large $N$ limit.

Integration of equation (13) in steady state yields:

$$\frac{\dot{f}}{N} = \frac{\dot{\epsilon}}{\epsilon_c} = \dot{\epsilon} p_c + \int_{-\infty}^{\epsilon_c} d\epsilon' \epsilon \int_{\epsilon_c}^{\infty} d\epsilon w(\epsilon - \epsilon')$$

Since $w$ is peaked around zero, in the spirit of the Fokker-Planck approximation, we expand $p(\epsilon)$ close to $\epsilon_c$ to first order: $p(\epsilon) \simeq p_c + (\epsilon - \epsilon_c) p'_c$. Using $p_c = (\langle n \rangle \epsilon_c)^{-1}$, we obtain for the avalanche average size:

$$\langle n \rangle = \left[ 1 + \frac{NM_{1+}}{\epsilon_c} \right] \left[ 1 + \frac{NP_c' M_{2+}}{2} \right]^{-1}$$

where the (semi)-moments

$$M_{r+} = \int_{0}^{\infty} d(\delta \epsilon) w(\delta \epsilon) \delta \epsilon^r$$

For both models $M_{2+} = \epsilon_c^2/\pi^2 N$, while $M_{1+}^{(G)} = \epsilon_c/\pi \sqrt{\pi N}$ and $M_{1+}^{(E)}$ is given by equation (9).

If $p'_c$ converges towards a finite value $p'_c(\infty)$, equation (16) predicts that, for large systems,

- for the G model: $\langle n \rangle \sim N^{1/2}$
- for the E model: $\langle n \rangle \sim \log N$

While this prediction accounts satisfactorily for the numerical results for the gaussian model, we have checked (see also Figure 4) that the log scaling is ruled our by our data. The reason for this failure is illustrated by the insert of Figure 7 (top). For the E model, we find that, for increasing $N$, $p$ becomes increasingly steep in the near vicinity of $\epsilon_c$. In the $N$-range investigated, we see a marked, non-saturating, increase of $|p'_c|$ suggesting a possible divergence, higher derivatives increasing even faster. This highly singular behavior, reminiscent of that analyzed by Chabanol and Hakim [14] for a related model, clearly invalidates the above Fokker-Planck expansion for model E. Conversely, for the G model,
we find numerically that \( p' \epsilon^2 / \pi^2 \) does exhibit convergence, towards \( \approx -1.3 \). This regular behavior suggests that a Fokker-Planck expansion of the master equation (13) should be valid for model \( G \) in the large \( N \) limit. We have indeed checked that, for increasing \( N \), the steady state distribution \( p_G \) converges rapidly towards the solution of this FP equation.

So, while even a schematic representation of long range elastic couplings suffices to account for avalanches with power law size scaling, this discussion underscores that the detailed shape of the noise spectrum is of crucial importance. Indeed, not only does it affect the scaling exponent, but, as well, the scaling properties of the distribution of avalanche sizes.

We consider that the results presented here, though still preliminary, clearly show that the dynamical noise due to long range elastic couplings is a key ingredient that must be included in phenomenologies of plasticity of amorphous solids. As \( \dot{\gamma} \) increases beyond the limit of the QS regime, since avalanches are no longer separable, the spectrum of the flip-generated dynamical noise will of course change. The question of its evolution with \( \dot{\gamma} \), as well as that of its interplay with thermal noise at finite temperature remain for the moment completely open ones. Besides, the above discussion indicates two main routes for further investigation.

On the one hand, a more realistic modelization of elastic couplings in the presence of structural disorder is needed. Indeed, Leonforte et al [13, 15] have shown that the elastic response of amorphous solids self-averages into the continuum elastic response only beyond a length scale \( \xi \) of order \( \sim 20 \) atomic diameters \( a_0 \). For \( r < \xi \), the elastic response is dominated by non-affine effects. So, for the existing molecular simulations focussing on avalanche dynamics, where \( L \) is limited to \( \lesssim 50a_0 \), noise tails are very likely to be controlled by elastic non-affinity. This issue, as well as that of the statistical distribution of zone parameters (such as shear modulus and threshold strain), will demand the development of a coarse-grained description of the elasticity of glassy systems.

On the other hand, our mean-field approximation wipes out from the start the correlation anisotropy arising from the quadrupolar symmetry of elementary events, responsible for the preferential avalanche orientations observed in the glass simulations of Maloney et al [3] and Tanguy et al [16]. In order to evaluate the robustness of the mean-field scalings and also start addressing the issue of localization, full simulations of model \( E \) in the presence of rigid boundaries will be necessary.
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