X rays on quantum mechanics: Pauli Exclusion Principle and collapse models at test

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Abstract.
In the last decades huge theoretical effort was devoted to the development of consistent theoretical models, aiming to solve the so-called “measurement problem”. Among these, the Dynamical Reduction Models possess the unique characteristic to be experimentally testable, thus enabling to set experimental upper bounds on the reduction rate parameter $\lambda$ characterizing these models. By analysing the X-ray spectrum emitted by an isolated slab of Germanium, we set the most stringent limit on the $\lambda$ parameter up to date.

1. Introduction
In spite of its enormous success the Quantum Mechanics is still generating intensive debates related to its possible limits and/or interpretation. In this paper we present experimental tests on one of the most fascinating items of the quantum theory: the measurement problem.

Indeed, the results of the measurements, related to the collapse of the wave function, generated long ago the so-called “measurement problem”, which still continues to be one of the main items of studies in quantum theory.
Among the proposed possible solutions to the “measurement problem”, the collapse models occupy a special place, since they are proven to be mathematically consistent and so far in agreement with all known experimental predictions. In general, collapse models provide a theoretical framework for understanding how classical world emerges from quantum mechanics. Their dynamics practically preserves quantum linearity for the microscopic systems, but becomes strongly nonlinear when moving towards macroscopic scale. These models make precise predictions and the aim of this paper is to present a physical system that has the potential to experimentally test these predictions.

The conventional approach to test collapse models is to generate spatial superpositions of mesoscopic systems and examine the loss of interference, while environmental noises are under control. Naturally oscillating systems create quantum superpositions, and thus represent a natural case-study for testing quantum linearity. Neutrinos, neutral mesons and chiral molecules are such systems. However, the collapse models can not be tested with neutrinos and the effect, stronger for neutral mesons, is still beyond experimental reach, while chiral molecules can offer promising candidates for testing collapse models [2].

The best testing ground is offered by the spontaneous emission of radiation, predicted by the collapse models, which is used below to settle the most stringent limit on the collapse models’ characterizing parameter $\lambda$.

We will introduce the physics behind collapse models (Sect. 2) and present the spontaneous radiation emission of free electrons (Sect. 3). Then (Sect. 4) we show our limit on the reduction rate parameter $\lambda$, and conclude with Sect. 5.

2. The Dynamical Reduction Models and the $\lambda$ parameter
The recent development of experimentally testable, mathematically consistent Dynamical Reduction Models (DRM), as a possible solution of the measurement problem strongly renewed the interest of the scientific community for the foundations of Quantum Mechanics (QM).

The DRM were born to answer the long standing problem of measurement in QM and to settle in a more natural way its role as a theory of both micro and macroscopic phenomena. A scheme for an ideal measurement process was already used by von Neumann [3] to show that the linear nature of the Schrödinger equation enables for the superposition of macro-object states (a general demonstration was more recently obtained in [4]). Under the assumption that QM is a complete theory only two ways out exist:

- the existence of two dynamical regimes: a) the quantum states evolution governed by the Schrödinger equation, unitary and linear and, thus, deterministic b) the measurement process, governed by the wave packet reduction principle, which is non-linear and intrinsically stochastic,

or

- a non-linear stochastic modification of the Hamiltonian dynamics.

The first consistent and satisfying DRM, known as Quantum Mechanics with Spontaneous Localization (QMSL), was developed by Ghirardi, Rimini and Weber [5]. According to the QMSL model, particles undergo a spontaneous localizations around definite positions, following a Possion distribution characterized by a mean frequency $\lambda$ that was set to $10^{-16}$ s$^{-1}$. During 1989-90 the efforts of Ghirardi, Rimini, Weber and Pearle [6] led to the development of the CSL model (Continuous Spontaneous Localization). The CSL theory is based on the introduction of new, non linear and stochastic terms in the Schrödinger equation. These terms induce, for the state vector, a diffusion process, which is responsible for the wave packet reduction (for reviews and references see [7]). In the framework of CSL theory, Q. Fu [8] demonstrated that the particle
interaction with the stochastic field, besides collapsing the state vector on the position basis, causes an enhancement of the energy expectation value. This implies, for a charged particle, the emission of electromagnetic radiation (known as spontaneous radiation) not present in the standard QM. A measurement of the emitted radiation rate thus enables to set a limit on the $\lambda$ parameter characterizing these models.

By comparing the expected emission rate with the radiation emitted by an isolated slab of Germanium at 11 KeV [9], Fu obtained an upper limit on the reduction rate parameter. In the next sections the arguments of Fu will be described and a more refined analysis of the X-ray emission spectrum measured by the IGEX experiment [10] will be presented [11], which improves the precedent limit by a factor 20.

3. The spontaneous emission rate
The radiation spectrum spontaneously emitted by a free electron, as a consequence of the interaction with the stochastic field, was calculated by Fu [8] in the framework of the non-relativistic CSL model, and it is given by:

$$\frac{d\Gamma(E)}{dE} = \frac{e^2\lambda}{4\pi^2a^2m^2E}$$

where $m$ represents the electron mass, $E$ is the energy of the emitted photon, $\lambda$ and $a$ are respectively the reduction rate parameter and the correlation length of the reduction model (the latter is generally accepted to be of the order of $a = 10^{-7}m$).

If the stochastic field is assumed to be coupled to the particle mass density (mass proportional CSL model) (see for example [4]) then the previous expression for the emission rate (eqn. (1)) for electrons should be multiplied by the factor $\left(\frac{m_e}{m_N}\right)^2$, with $m_N$ being the nucleon mass. Using the measured radiation appearing in an isolated slab of Germanium [9] corresponding to an energy of 11 KeV, and employing eqn. (1), Fu obtained the following upper limit for $\lambda$:

$$\lambda < 0.55 \cdot 10^{-16}s^{-1}.$$  

(2)

Only the four valence electrons were considered to contribute to the measured X-ray emission; since their binding energy $\sim 10$ eV is orders of magnitude less than the emitted photons’ energy, they can be considered as quasi-free.

In Ref. [12] the author argues that, in evaluating his numerical result, Fu uses for the electron charge the value $e^2 = 17137.04$, whereas the standard adopted Feynman rules require the identification $e^2/(4\pi) = 17137.04$. We took into account this correction when evaluating the new limit on the collapse rate parameter.

4. A new limit on the $\lambda$ parameter
In order to reduce the possible biases introduced on the $\lambda$ value by the evaluation of the rate at one single energy bin, we decided to adopt a different strategy with respect to the analysis performed in [8]. The X-ray emission spectrum measured by the IGEX experiment [10] was fitted in the range $\Delta E = 4.5 \div 48.5$ KeV $\ll m$, compatible with the non-relativistic assumption (for electrons) used in the calculation of the predicted rate (eqn. (1)). IGEX is a low-background experiment based on low-activity Germanium detectors dedicated to the neutrinoless double beta decay ($\beta\beta^0\nu$) decay research. The data [13], used to extract a new upper limit on the collapse rate parameter, refers to 80 kg day exposure.

A Bayesian model is adopted to calculate the $\chi^2$ variable minimized to fit the X-ray spectrum, assuming the predicted (eqn. (1)) energy dependence:

$$\frac{d\Gamma(E)}{dE} = \frac{\alpha(\lambda)}{E}.$$  

(3)
The result of the performed fit is shown in Figure 1 (for more details see [11])

![Figure 1](image)

**Figure 1.** *Fit of the X ray emission spectrum measured by the IGEX experiment [10, 13], using the theoretical fit function eqn. (3).*

Considering as quasi-free the four valence electrons, consistently with Fu’s hypothesis, using the correct prescription $e^2/(4\pi) = 17137.04$ (see [12]), the following upper limits for the reduction rate parameter is obtained:

$$\lambda \leq 1.4 \cdot 10^{-17} \text{s}^{-1} \quad \text{non mass prop.}$$

$$\lambda \leq 4.7 \cdot 10^{-11} \text{s}^{-1} \quad \text{mass prop.} \quad (4)$$

if the stochastic field is not assumed to be coupled to the particle mass density (left) and for a mass proportional CSL model (right). If we consider in the calculation the 22 external electrons of the $^{32}\text{Ge}$ atom, down to the $3s$ orbit, based on the fact that the corresponding binding energy ($BE_{3s} = 180.1$ eV) is about 22 times smaller than the less energetic measured photons (the 22 outermost electrons can be then considered to be quasi-free) the limits on the reduction rate parameter become:

$$\lambda \leq 2.5 \cdot 10^{-18} \text{s}^{-1} \quad \text{non mass prop.} \quad (5)$$

$$\lambda \leq 8.5 \cdot 10^{-12} \text{s}^{-1} \quad \text{mass prop.} \quad (6)$$

The limits in eqns. (5 and 6) improves the precedent Fu’s limit, eqn. (2), by a factor 20. The estimated upper limits eqns. (5 and 6) for the rate parameter are to be compared with the values conventionally assumed in the specific collapse models:

$$\lambda_{QMSL} = 10^{-16} \text{s}^{-1}, \quad \lambda_{CSL} = 2.2 \cdot 10^{-17} \text{s}^{-1} \quad (7)$$
and with the values proposed, more recently, by S. Adler [12]. From eqn. (5) one concludes that if a mass proportional coupling is not taken into account, and by considering a “white” noise inducing the collapse, the upper limit on $\lambda$ strongly constrains the collapse models, being exceeded by both $\lambda_{QMSL}$ and $\lambda_{CSL}$.

5. Conclusions and perspectives
The collapse of the wave function and, more generally the “measurement problem” is one of the hottest topics in QM and triggers many new experimental activity. A possible mechanism inducing the collapse, the so called Continuous Spontaneous Localization (CSL), has an unique experimental signature: a spontaneous radiation emitted by (free) charged particles. A new limit on the free collapse frequency parameter $\lambda$, characterizing the CSL model, was obtained by performing an analysis of the IGEX experimental data. The $\lambda$ value is obtained to be $\lambda \leq 2.5 \cdot 10^{-18} \text{s}^{-1}$ if no mass dependence is considered, and $\lambda \leq 8.5 \cdot 10^{-12} \text{s}^{-1}$ if such a dependence is taken into consideration, to be compared with proposed CSL value ($\lambda_{CSL} = 2.2 \cdot 10^{-17} \text{s}^{-1}$).

We are presently exploring the possibility to perform a new dedicated measurement that will allow for more then 2-3 orders of magnitude improvement on the collapse rate parameter $\lambda$ which will impose a very stringent limit on the collapse models.

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