Experimental linear-optical implementation of a multifunctional optimal qubit cloner

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We present the first experimental implementation of a multifunctional device for the optimal cloning of one to two qubits. Previous implementations have always been designed to optimize the cloning procedure with respect to one single type of a priori information about the cloned state. In contrast, our all-in-one implementation is optimal for several prominent regimes such as universal cloning, phase-covariant cloning, and also the first ever realized mirror phase-covariant cloning, when the square of the expected value of Paulis Z operator is known in advance. In all these regimes the experimental device yields clones with almost maximum achievable average fidelity (97.5% of theoretical limit). Our device has a wide range of possible applications in quantum information processing, especially in quantum communication. For instance, one can use it for incoherent and coherent attacks against a variety of cryptographic protocols, including the Bennett-Brassard 1984 protocol of quantum key distribution through the Pauli damping channels. It can be also applied as a state-dependent photon multiplier in practical quantum networks.

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Introduction. One of the most fundamental laws of nature, the so-called no-cloning theorem, states that an unknown quantum state cannot be perfectly copied. This fact has an imminent impact on quantum information processing. For instance, it allows designing inherently secure cryptographic protocols [1] or assures the impossibility of superluminal communication [2]. Although perfect quantum copying is impossible, one can still investigate how well such an operation can be approximated within the limits of physical laws. Despite some very intense research in this domain, many aspects of state-dependent quantum cloning have not yet been fully investigated.

Quantum cloning is one of the most intriguing topics in quantum physics. It is important not only because of its fundamental nature but also because of its immediate applications to quantum communications, including quantum cryptography. Similar to other important quantum information processing protocols, quantum cloning has undergone considerable development over the past two decades. The first design of an optimal cloning machine was suggested by Bužek and Hillery [3]. The cloner is called optimal when it gives the best results allowed by quantum mechanics. Moreover universal cloning (UC) should operate equally well for all possible qubit states [4–8]. In contrast, limiting cloning to a specific subset of qubit states, one can achieve a more precise cloning operation. A prominent example of this situation is phase-covariant cloning (PCC), where only qubit states with equal superposition of |0⟩ and |1⟩ are considered [9–12].

In this Rapid Communication we address a question that is interesting from both conceptual and practical points of view: how well a can quantum state be cloned if some a priori information about the state is known? Theoretical investigation of this issue led to quantifying

![FIG. 1: (Color online) Overview of various distributions on Bloch’s sphere describing a priori knowledge of qubits and the corresponding optimal cloning machines which are special cases of our multifunctional cloner: (a) uniform distribution can be cloned by the UC, (b) qubits on the equator of the Bloch sphere can be cloned by the standard PCC, (c) union of the set of qubits for the generalized PCC and its equator-plane reflection can be cloned by the MPCC, and (d) any set of qubits of unknown phase symmetric about any equator plane can be cloned by the generalized MPCC.](image-url)
the information known about the cloned state in terms of axially symmetric distributions on the Bloch sphere \cite{16}. This class of distributions contains an important subclass of distributions which are mirror symmetric with respect to the equatorial plane (see Fig. 1). It is therefore convenient to define mirror phase-covariant cloning (MPCC) as a strategy for cloning states with this kind of \textit{a priori} information \cite{17}.

We hereby present an implementation of the MPCC, and we also demonstrate that the same setup can be used for optimal cloning in other prominent regimes such as universal cloning and phase-covariant cloning. We show that the assumptions regarding the symmetry of the set of qubits cloned in an optimal way by the MPCC can be relaxed to include a wider class of qubit distributions that do not need to be axially symmetric. Finally, we demonstrate, for the example of the PCC for an arbitrary polar angle on the Bloch sphere \cite{15}, that our device can be also used as an optimal axially symmetric cloner for which the mirror-symmetry condition is not necessary. Mirror phase-covariant cloning. In our experiment we cloned the polarization state of a single photon given by

$$|\psi\rangle = \cos(\theta/2)|H\rangle + \sin(\theta/2)e^{i\phi}|V\rangle,$$

where $|H\rangle$ and $|V\rangle$ are the horizontal and vertical polarizations, respectively. In accord with the original definition \cite{15}, we assume $\langle \hat{\sigma}_z \rangle^2 = \cos^2 \theta_{\text{eff}}$ is the only \textit{a priori} information known about the cloned state, where $\hat{\sigma}_z$ denotes the third Pauli operator. A geometrical interpretation of the set of states of fixed $\cos^2 \theta_{\text{eff}}$ is shown in Fig. 1(c). It has been recently demonstrated \cite{16} that the MPCC can also be applied to a wider class of qubit distributions $g(\theta, \phi)$ shown in Fig. 1(d). Consequently, the optimal cloner for a set of qubits given by a distribution $g(\theta, \phi)$ is an MPCC set for an axial angle $\theta_{\text{eff}}$ defined as $\langle \cos^2 \theta \rangle = \cos^2 \theta_{\text{eff}}$, where the angle bracket stands for averaging over the distribution. Moreover, we note that the mirror-symmetry condition can be weakened and the MPCC transformation can be used as an optimal cloning transformation for other sets of qubits which are not axially symmetric and do not exhibit the mirror-symmetry, but rather fulfill the following conditions:

$$\int_0^{2\pi} [g(\theta, \phi) + g(\pi - \theta, \phi)] e^{i\phi} d\phi = 0,$$

$$\int_0^{2\pi} g(\theta, \phi) d\phi = \int_0^{2\pi} g(\pi - \theta, \phi) d\phi,$$

where $g$ is a distribution of qubits on the Bloch sphere and $n = 1, 2$. Therefore, any MPCC optimal for some $\theta_{\text{eff}}$ is also optimal for a wider class of distributions which do not need to be axially symmetric or mirror symmetric but fulfill Eqs. 2. The above-mentioned arguments considerably broaden the usefulness of the presented device.

Experimental setup. Our experimental setup is depicted in figure 2. First, the cloned and ancillary photon states are prepared by means of half and quarter wave plates. Then the cloning operation is performed by overlapping the two photons on a special unbalanced polarization-dependent beam splitter (PDBS). Subsequently, each of the two photons undergoes polarization sensitive filtering (transmittance $\tau$) using the beam divider assemblies (BDA1 and BDA2) placed in each of the output modes of the beam splitter. The PDBS employed in this scheme has different transmittances for horizontal ($\mu$) and vertical ($\nu$) polarizations. The transmittances should be given by

$$\mu = \frac{1}{2} \left( 1 + \frac{1}{\sqrt{3}} \right), \quad \nu = \frac{1}{2} \left( 1 - \frac{1}{\sqrt{3}} \right),$$

but due to manufacturing imperfections the observed transmittances of our PDBS were $\mu = 0.76$ and $\nu = 0.18$. Please note that this imperfection can be corrected without loss of fidelity through suitable filtering at the expense of a lower success rate (see Ref. 22).

The beam-divider assembly is depicted in more detail in Fig. 2(b). It is composed of two beam dividers (BDa and BDb) used to separate and subsequently combine horizontal and vertical polarizations. A neutral density filter (F) with tunable transmittance $\tau$ is positioned between the two beam dividers so that one of the paths (polarizations) is attenuated while the other remains intact. Also a half-wave plate (HWPb) is placed between the beam dividers swapping the polarizations and thus allowing them to be combined at the second beam divider (BDb). To control attenuation of each polarization by the neutral density filter, we envelope the beam-divider assembly by two half-wave plates (HWPa and HWPe). The beam-divider assembly is equivalent to a

FIG. 2: (Color online) (a) Scheme of the experimental setup as described in the text. (b) Detailed scheme of the beam-divider assembly. HWP, half-wave plate; QWP, quarter wave-plate; PDBS, polarization dependent beam splitter; BDA, beam divider assembly; F, neutral density filter; PBS, polarizing beam splitter; D, single-photon detector; BD, beam divider.
Mach-Zehnder interferometer, and by means of a piezo-driven tilt of one of the beam dividers, we can set an arbitrary phase shift between the two paths (polarizations).

In the ideal case, having \( \mu + \nu = 1 \), the setup operates as follows. A separable two-photon state \( |H_1 H_2\rangle \) (indices denote the mode number) is generated in the process of the type-I spontaneous parametric down conversion using a LiIO\(_3\) crystal pumped by cw Kr\(^+\) laser at 413 nm of 150-mW optical power. These photons are brought to the input of the setup via single-mode fibers. The parameters to be set for the MPCC and UC regimes are just specific cases of the MPCC setting as we discuss later.

For this reason we now concentrate on the MPCC setting. The polarization of the first (cloned) photon is set in such a way that it belongs to one of the parallels of latitude on the Bloch sphere with a given polar angle \( \theta \) [see Eq. (1)]. The second (ancillary) photon remains either horizontally polarized or is randomly swapped to vertical polarization. After this preparation stage the two photons are coherently overlapped at the PDBS. Depending on the polarization of the ancillary photon, we perform subsequent transformation. If the ancillary photon remains horizontally polarized we set the half-wave plates (HWPa1 and HWPa2) in front of the beam dividers to 45\(^\circ\) so that the vertical polarization is attenuated in both beam-divider assemblies. The level of transmittance \( \tau \) of the filters F is set according to the relation

\[
\tau = \frac{(1 - \Lambda^2)(1 - 2\mu)^2}{2\mu\Lambda^2}, \quad \Lambda = \sqrt{\frac{1 + \cos^2\theta}{2\sqrt{P}},}
\]

where \( P = 2 - 4\cos^2\theta + 3\cos^4\theta \). Additionally, we also set a phase shift \( \pi \) between horizontal and vertical polarization in both output modes. In the case of the ancillary photon being vertically polarized we set the half-wave plates HWPa1 and HWPa2 to 0\(^\circ\) and this time subject the horizontal polarization to the same filtering as given by Eq. (4). Also we set the phase shift between the polarizations to zero and rotate the half-wave plates HWPa1 and HWPa2 to 45\(^\circ\), thus canceling the polarization swap exercised by the half-wave plates HWPa1 and HWPa2 (inside the beam-divider assemblies).

Finally, the two-photon state polarization analysis is carried out by measuring the rate of two-photon coincidences for all combinations of single-photon projections to horizontal, vertical, diagonal, anti-diagonal linear, and right and left circular polarizations [19]. We can then estimate the two-photon density matrix using a standard maximum likelihood method [20].

In order to use the setup for the PCC, one just needs to set all the parameters as if performing the MPCC set for the latitude angle \( \theta = \pi/2 \). In case of the PCC there is no need to randomly swap the horizontal and vertical ancillae. In this case we know the hemisphere to which the cloned states belong so we can simply use the closer ancilla (horizontal for northern hemisphere and vertical for southern hemisphere).

A similar analysis can be carried out to determine that the setup actually performs the UC if set to the same parameters as for the MPCC with the polar angle \( \theta = \arccos(\sqrt{3}/3) \). In this regime a random swap between horizontal and vertical ancillae is also required.

**Experimental results.** In order to verify the versatile nature of the cloner, we performed a series of measurements in three regimes: PCC, MPCC, and UC. These regimes differ just in the amount of a priori knowledge about the cloned state. For the PCC and MPCC we verified the theoretical prediction of maximally achievable average fidelity as a function of polar angle \( \theta \). For all polar angles (except the poles) we estimated the fidelities of both clones for four different equally distributed input states.

![FIG. 3: (Color online) Experimental fidelity \( F_{ex} \) for the PCC regime depicted against theoretical prediction \( F_{th} \) (solid curve). The short-dashed curve indicates the fidelity drops to 97.5% with respect to the corresponding theoretical value. The theoretical fidelity \( F_{UC} \) for the UC is also depicted (long-dashed curve). For the generalized PCC the hemisphere of the cloned qubit is known (the reverse is true for the MPCC); we choose the ancilla deterministically [22] (horizontal for northern hemisphere and vertical for southern hemisphere), and we set transmittances of filters as for the MPCC tuned for the angle \( \theta = \pi/2 \).](image-url)

![FIG. 4: (Color online) Same as in Fig. 3 but for the MPCC.](image-url)
The observed values are depicted in Fig. 5 for the PCC and similarly in Fig. 4 for the MPCC regime. For UC (when we set $\tau = |\arccos(\sqrt{3}/3)|$) we cloned six input states: horizontally, vertically, right and left circularly, diagonally and anti diagonally polarized states. The average fidelity obtained in the UC mode is 81.5\%.

The vast majority of the experimentally obtained fidelities in all regimes reached or surpassed 97.5\% of their theoretical prediction leading only to a very small experimental error.

Additional measurement of the success probability was performed for the case of MPCC. The success probability as a function of polar angle $\theta$ is depicted in Fig. 5. Note that success probability strongly depends on the splitting ratio of the beam splitter. Its theoretical prediction is given by

$$P_{\text{th}} = \frac{(1 - 2\mu)^2}{2 + \mu \nu \kappa}, \quad (5)$$

where $\kappa = (2\mu - 1)/(1 - 2\nu)$. The presented theoretical value is therefore calculated for the above-mentioned transmittances of the beam splitter used. In order to determine the success probability of the scheme we measured the coincidence rate of the setup set to perform MPCC and also the calibration coincidence rate (all the filters and beam splitter were removed). The ratio of these two rates determines the success probability calibrated for “technological losses” (inherent losses due to back reflection or systematic error of all the components) [18]. For more details see Ref. [22].

Conclusions. Our implementation presents a concept of a multifunctional cloner optimized for quantum communication purposes with respect to a priori information about transmitted states and communication channels. We have experimentally verified the versatile nature of the proposed cloner. It performs at about 97.5\% of the theoretical limit for all three regimes tested (UC, PCC, and MPCC). Thus, in contrast to previous implementations, it can be used in attacks against a variety of quantum cryptographic protocols at once [21]. Some of its capabilities cannot be provided by previous cloners, especially for communication through the Pauli damping channels [22]. Potential applications of our approach can also include practical quantum networks based on state-dependent photonic multipliers or amplifiers. We therefore conclude that this device can be an efficient tool for a large set of quantum communication and quantum engineering applications requiring cloning.

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Supplementary material:

OPTIMALITY PROOF OF THE CLONERS

Average fidelity of cloning a set of qubits

While considering optimal symmetric $1 \rightarrow 2$ cloning the optimized figure of merit is the average single-copy fidelity. We express the average fidelity as $F = \text{Tr}(R\chi)$, where the operator $\chi$ is isomorphic to a completely-positive trace-preserving map $[1]$ (CPTP) which performs the cloning operation and $R = (\rho^T \otimes (\rho \otimes 1 + 1 \otimes \rho))/2$, where $\rho = |\psi\rangle \langle \psi|$ is the density matrix of a qubit from the cloned set. The matrix $R$ is given explicitly for any phase-covariant cloner as:

$$R = \frac{1}{8} \begin{pmatrix}
8c_2 & 0 & 0 & 0 & s_1^2 & s_1^2 & 0 \\
0 & 4c_2^2 & 0 & 0 & 0 & 0 & s_1^2 \\
0 & 0 & 4c_2^2 & 0 & 0 & 0 & s_1^2 \\
0 & 0 & 0 & 2s_1^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2s_1^2 & 0 & 0 \\
s_1^2 & 0 & 0 & 0 & 0 & 4s_2^2 & 0 \\
s_1^2 & 0 & 0 & 0 & 0 & 0 & 4s_2^2 \\
0 & s_1^2 & s_1^2 & 0 & 0 & 0 & 8s_2^2
\end{pmatrix},$$

where $s_i^j = \langle \sin^i(\theta)/i \rangle$ and $c_i^j = \langle \cos^j(\theta)/j \rangle$ for $i, j = 1, 2, 3, \ldots$, and the angle bracket stands for averaging over the input qubit distribution $g(\theta, \phi)$. We find the optimal cloning map $\chi$ by maximising the functional $F$.

The necessary conditions for optimality of the MPCC

First, let us note that the optimal cloning map $\chi$ must satisfy the following symmetry conditions imposed by the symmetry of a mirror phase-covariant set of qubits, i.e., $\forall \phi \in [0, 2\pi] |R_x(\phi)^* \otimes R_z(\phi)_{\otimes 2}, \chi \rangle = 0$ and $|\sigma_{x^3}, \chi \rangle = 0$. Second, we assume symmetric cloning, and therefore we require that both clones have the same fidelity. Thus, we demand $|\text{in} \otimes \text{SWAP, \chi} \rangle = 0$. Moreover, we can show that elements of $\chi$ must be real since maximized fidelity depends linearly only on the real part of the off-diagonal elements. We must also remember that $\chi$ must preserve trace, i.e., $\text{Trout}[\chi] = \mathbb{1}$. All the above conditions imply the following form of the map $\chi$ being a mixture of two CPTP maps:

$$\chi = (1 - p)$$

$$+ p(|011\rangle \langle 011| + |100\rangle \langle 100|).$$

where $\Lambda^2 + \Lambda^2 = 1$ and $1 \geq \Lambda$, $p \geq 0$ are free parameters. Since we can write the cloning fidelity as $F = (1 - p)F_\Lambda + ps^2/2$ it is apparent that in order to achieve maximal fidelity assuming $F > 1/2$ we must set $p = 0$. Now, we can derive the expression for $\Lambda$ demanding that $dF/d\Lambda = 0$. As one of the solutions we obtain

$$\Lambda(c_i^2) = \sqrt{\frac{1}{2} + \frac{c_i^2}{2\sqrt{p}}},$$

where $P = 2 - 4c_i^2 + 3c_i^4$. However, at this point we cannot conclude that it is optimal.

The sufficient conditions for optimality of the MPCC

As noted by Audenaert and De Moor 2 the problem of designing an optimal cloning machine can be solved by means of semidefinite programming. Moreover, it was noted that as long as $\chi$ is a CPTP map in a convex set, there are only global extrema. It can be shown that $\chi$ maximizes fidelity $F = \text{Tr}[R\chi]$ if the following conditions are satisfied:

$$(A - R)\chi = 0,$$

$$(9) \quad A - R \geq 0,$$

where $A = \lambda \otimes \mathbb{1} \geq 0$ is a positive semidefinite matrix of Lagrange multipliers ensuring that $\chi$ is CPTP map, i.e., $\text{Trout}(\chi) = \mathbb{1}_{\text{in}}$. The operator $\lambda = \text{Trout}(R\chi)$ is derived by demanding that the variance of fidelity $F$ over $\chi$ should be equal to zero. If the condition (9) is satisfied, then for any CPTP map $\chi$ we obtain $\text{Tr}[(A - R)\chi] \geq 0$. It also follows from the trace preservation condition that $\text{Tr}(A\chi) = \text{Tr}\lambda$. Hence, the fidelity $F$ is bounded by the trace preservation condition and $F \leq \text{Tr}[\lambda]$. If inequality is saturated by $\chi$, then $\chi$ represents the optimal cloning transformation.

For MPCC we have

$$\lambda = \frac{1}{4}[(1 + c_i^2)\Lambda^2 + \bar{\Lambda}^2 + \sqrt{2}(1 - c_i^2)\bar{\Lambda}] \mathbb{1}_{\text{in}},$$

where $\bar{\Lambda} = \text{Tr}[\chi]$. Hence, the fidelity $F$ is bounded by the trace preservation condition and $F \leq \text{Tr}[\lambda]$. If inequality is saturated by $\chi$, then $\chi$ represents the optimal cloning transformation.
Henceforth, it can be easily shown that $\text{Tr} \lambda - F = 0$. The eigenvalues of operator $\Delta = A - R$ can be expressed in terms of $R$ matrix elements in the following way:

$$\begin{align*}
\delta_1 &= \frac{1}{2} \left( F - \frac{1}{2} \right), \\
\delta_2 &= \frac{1}{2} \left( F - \frac{1 - c_1^2}{2} \right), \\
\delta_{3,4} &= \frac{1}{2} \left( F - R_{1,1} - R_{2,2} \pm \bar{R} \right),
\end{align*}$$

(12)

where $\bar{R}^2 = (R_{1,1} - R_{2,2})^2 + 8R_{1,6}^2$. All the eigenvalues are double degenerated. Moreover, we have

$$F = R_{1,1} + R_{2,2} + \bar{R}. \quad (13)$$

Thus, $\delta_3 = F - (2 + c_1^2)/4$ and $\delta_4 = 0$. Since $F > 3/4$, $\forall \delta_i \geq 0$, we conclude that $\Delta$ is a positive semidefinite matrix. Thus, we have shown that the conditions (9) and (10) are satisfied, which completes the proof.

### COMPENSATING FOR IMPERFECT TRANSMITTANCES

In our case the equation relating beam-splitter transmittances ($\mu + \nu = 1$) does not hold and we have $\mu + \nu \neq 1$. Hence additional filtering operations are required in order to maintain the maximum achievable fidelity of the setup. This additional filtering manifests itself in two ways. First, one needs to unbalance the ancilla-dependent filtering performed by filters $F$ in both BDAs. We require $\tau_1 = \tau$ and $\tau_2 = \omega \tau$ for the BDA1 and BDA2, respectively, where

$$\omega = \frac{\tau_2}{\tau_1} = \frac{\mu \nu}{(1 - \mu)(1 - \nu)}. \quad (14)$$

Note that $\omega = 1$ in the ideal case for $\mu + \nu = 1$ and $\omega = 0.726$ for the applied PDBS. Second, the realization of the MPCC with the PDBS where $\mu + \nu \neq 1$ requires applying an additional unconditional filtering. This filtering is polarization-dependent and is performed regardless of the state of the ancillary photon. The polarization dependent transmittances $\tau_H$ and $\tau_V$ for the $H$ and $V$-polarized photons, respectively, need to satisfy the following relation:

$$\kappa = \frac{\tau_V}{\tau_H} = \frac{2\mu - 1}{1 - 2\nu}, \quad (15)$$

where $\kappa (\kappa = 1$ for $\mu + \nu = 1$) is a constant value fixed by the parameters of the PDBS (in our case $\kappa = 0.838$), and both $\tau_H$ and $\tau_V$ should have the largest possible values in order to maximize the efficiency of the setup. Therefore, we apply an additional unconditional filtering only for the $V$-polarized photons since the optimal transmittances are $\tau_V = \kappa$ and $\tau_H = 1$. Please note that for our PDBS $\kappa < 1$ and in the opposite case the best choice of the parameters is $\tau_V = 1$ and $\tau_H = 1/\kappa$. Moreover, if there are any other systematic uniform polarization-dependent losses $\tau'_H$ and $\tau'_V$, we can compensate for them by setting $\kappa = \tau'_H/\tau'_V \times (2\mu - 1)/(1 - 2\nu)$.

To summarize the above-mentioned corrections, the overall filtering operations in the first mode are described by

$$\tau_{1,H} = \tau_0^{\delta V,s} \text{ and } \tau_{1,V} = \kappa \tau_0^{\delta H,s}, \quad (16)$$

and in the second mode by

$$\tau_{2,H} = (\omega \tau)^{\delta V,s} \text{ and } \tau_{2,V} = \kappa (\omega \tau)^{\delta H,s}, \quad (17)$$

where $\delta_{V,s}$ ($\delta_{H,s}$) is Kronecker’s delta and is equal to 1 iff the polarization $s$ of the ancillary photon is $V$ ($H$).

To implement the required filtering, additional polarization-independent filters $F_A1$ and $F_A2$ are placed at the output modes. These two filters together with the filters in both BDAs are sufficient to perform filtering operation described by Eqs. (16) and (17).

The usage of additional filtering saves the maximum achievable fidelity at the expense of lowering the success probability of the scheme. Using PDBA transmittances and the parameter $\Lambda$ of the cloned state one can express the expected success probability of the scheme in the form of

$$P_{th} = (1 - 2\mu)^2/2 + \mu \nu \tau \kappa, \quad (18)$$

where $\kappa = (2\mu - 1)/(1 - 2\nu)$.

### IMPLEMENTING THE GENERALIZED PCC AND AXISYMMETRIC CLONING

By using the same setup we implemented the generalized PCC (see Tab. I) which is a special case of the axisymmetric cloner described in Ref. [3]. In order to perform arbitrary axisymmetric cloning we set parameters of filters according to the following relations:

$$\begin{cases}
\tau_{1,H} = \frac{(\cos \alpha_+ \sin \alpha_-)}{(\sin \alpha_+ \cos \alpha_-)} \frac{2(1-\mu)(1-\nu)}{(1-2\mu)^2} & \text{for } s = H, \\
\tau_{1,V} = \frac{(\cos \alpha_- \sin \alpha_+)}{(\sin \alpha_- \cos \alpha_+)} \frac{2\mu \nu}{(1-2\mu)^2}
\end{cases}$$

(19)

and

$$\begin{cases}
\tau_{2,H} = \frac{(\sin \alpha_+ \cos \alpha_-)}{(\cos \alpha_+ \sin \alpha_-)} \frac{2(2\nu-1)^2}{(2\nu-1)^2} & \text{for } s = H, \\
\tau_{2,V} = \frac{(\sin \alpha_- \cos \alpha_+)}{(\cos \alpha_- \sin \alpha_+)} \frac{2(2\nu-1)^2}{(2\nu-1)^2}
\end{cases}$$

(20)

where $\alpha_\pm$ is given by Eq. (14) from [3] and $s = H, V$ stands for polarization of the ancillary state. For the generalized PCC we picked $s$ deterministically. We set $\alpha_+ = \pi/2$ and $\alpha_- = 0$ for $s = H$ when we cloned a
qubit from the northern hemisphere, alternatively we set \( \alpha_+ = 0 \) and \( \alpha_- = \pi/2 \) for \( s = V \) each time when the cloned qubit was from the southern hemisphere. Please note that we could have also picked ancillary state at random (as for the MPCC), but then we would have had to block all the output modes for half of the cases.

**MEASURING THE SUCCESS PROBABILITY**

In order to measure success probability we need to estimate the inherent technological losses of the scheme and the initial photonic rate. The technological losses occur as a result of detector efficiencies, fiber coupling losses or back reflections. The coincidence rate \( C_{\text{clon}} \) measured at the end of the working cloner can be expressed as

\[
C_{\text{clon}} = P_{\text{ex}} \tau_{\text{tech}} C_{\text{init}},
\]

where \( P_{\text{ex}} \) denotes the success probability of the cloning scheme, \( \tau_{\text{tech}} \) denotes the transmittance of the setup due to technological losses and \( C_{\text{init}} \) is the initial rate of photon pairs from the source. To compensate for the technological losses and the initial photon rate we use the following calibration procedure: PDBS is placed on a translation state allowing us to shift it slightly so that the reflected beam is no longer coupled. We use \( |\psi\rangle \) for the input state knowing that the beam splitter would decrease the coincidence rate by the factor of \( 1/\mu^2 \). In this configuration we remove all the neutral density filters and measure the calibration coincidence rate \( C_{\text{calib}} \) at the end of the scheme. One can clearly see that

\[
C_{\text{calib}} = \mu^2 \tau_{\text{tech}} C_{\text{init}}
\]

so the success probability of the cloning operation can be expressed by combining Eqs. (21) and (22).

\[
P_{\text{ex}} = \mu^2 \frac{C_{\text{clon}}}{C_{\text{calib}}}
\]

This equation allows us to obtain the success probability of the cloning operation from the measurement of two coincidence rates: the first is the coincidence rate of the working cloner and the second is the calibration coincidence rate. Note that Eqs. (18) and (23) describe the same quantity.

**MEASURED VALUES**

Our detailed summary of measured and predicted results is presented in Tables I, II, and III. In Fig. 6 we show how the cloning fidelity of the MPCC varies with phase \( \varphi \).

| Angle \( \theta \) | \( F_{\text{ex}} \) [%] | \( F_{\text{th}} \) [%] | \( P_{\text{ex}} \) [%] | \( P_{\text{th}} \) [%] |
|------------------|----------------|----------------|----------------|----------------|
| 0                | 99.6 ± 0.4    | 100.0          | 10.5 ± 2.8     | 13.3           |
| \( \pi/12 \)     | 95.6 ± 1.7    | 97.0           | 10.6 ± 1.9     | 13.3           |
| \( 0.43 \)       | 89.6 ± 0.4    | 90.4           | 10.4 ± 1.5     | 13.5           |
| \( \pi/5 \)      | 86.1 ± 1.6    | 87.4           | 9.6 ± 0.9      | 14.3           |
| \( \pi/4 \)      | 81.9 ± 2.0    | 84.1           | 14.0 ± 2.9     | 16.2           |
| \( 0.95 \)       | 80.2 ± 1.5    | 83.3           | 19.5 ± 3.5     | 18.6           |
| \( 3\pi/8 \)     | 82.3 ± 1.3    | 84.0           | 23.7 ± 1.5     | 21.7           |
| \( \pi/2 \)      | 84.1 ± 0.5    | 85.4           | 24.8 ± 0.1     | 24.0           |
| \( 5\pi/8 \)     | 82.3 ± 1.3    | 84.0           | 23.7 ± 1.5     | 21.7           |
| \( 2\pi/3 \)     | 80.2 ± 1.5    | 83.3           | 19.5 ± 3.5     | 18.6           |
| \( 2.19 \)       | 81.9 ± 2.0    | 84.1           | 14.0 ± 2.9     | 16.2           |
| \( 4\pi/5 \)     | 86.1 ± 1.6    | 87.4           | 9.6 ± 0.9      | 14.3           |
| \( 2.71 \)       | 86.9 ± 0.4    | 90.4           | 10.4 ± 1.5     | 13.5           |
| \( 11\pi/12 \)   | 95.6 ± 1.7    | 97.0           | 10.6 ± 1.9     | 13.3           |
| \( \pi \)        | 99.6 ± 0.4    | 100.0          | 10.5 ± 2.8     | 13.3           |

**Table I:** Summarized data for the PCC regime. \( F_{\text{ex}} \) denotes experimentally estimated average fidelity for a given polar angle \( \theta \) on the Bloch sphere and \( F_{\text{th}} \) is the theoretical prediction. Note that the error estimated as RMS is just indicative of the actual error, because it does not take into account the physical properties of fidelity.

| Angle \( \theta \) | \( F_{\text{ex}} \) [%] | \( F_{\text{th}} \) [%] | \( P_{\text{ex}} \) [%] | \( P_{\text{th}} \) [%] |
|------------------|----------------|----------------|----------------|----------------|
| 0                | 99.8 ± 0.4    | 100.0          | 10.5 ± 2.8     | 13.3           |
| \( \pi/12 \)     | 99.3 ± 0.4    | 99.8           | 10.8           | 13.4           |
| \( \pi/5 \)      | 98.0 ± 0.8    | 98.8           | 10.8           | 13.4           |
| \( \pi/3 \)      | 95.7 ± 0.8    | 95.3           | 10.8           | 13.4           |
| \( 3\pi/8 \)     | 92.4 ± 1.5    | 93.4           | 10.8           | 13.4           |
| \( \pi/2 \)      | 88.7 ± 1.1    | 89.4           | 10.8           | 13.4           |
| \( \pi/2 \)      | 84.1 ± 0.5    | 85.4           | 10.8           | 13.4           |
| \( 5\pi/8 \)     | 87.9 ± 0.7    | 89.4           | 10.8           | 13.4           |
| \( \pi/3 \)      | 95.0 ± 0.8    | 95.3           | 10.8           | 13.4           |
| \( 4\pi/5 \)     | 97.9 ± 0.7    | 93.4           | 10.8           | 13.4           |
| \( 11\pi/12 \)   | 98.4 ± 1.0    | 99.8           | 10.8           | 13.4           |
| \( \pi \)        | 99.8 ± 0.4    | 100.0          | 10.8           | 13.4           |

**Table II:** Same as in Table I but for the MPCC regime. Moreover \( P_{\text{ex}} \) and \( P_{\text{th}} \) denote experimental and theoretical success probabilities.

**THE MPCC AND PAULI DAMPING CHANNEL**

Here, we show that our implementation of the MPCC can be interpreted as a quantum simulation of the Pauli dampening channel, where an error (bit-flip error, phase-flip error or both) occurs with some probability. This correspondence can lead to immediate applications of the proposed device for quantum eavesdropping. The density
| Polarization state | $F_{ex}$ [%] | $F_{th}$ [%] |
|--------------------|--------------|-------------|
| horizontal         | 80.2 ± 3.1   | 83.3        |
| diagonal           | 81.5 ± 1.5   | 83.3        |
| anti-diagonal      | 81.3 ± 0.2   | 83.3        |
| right-circular     | 82.5 ± 1.4   | 83.3        |
| left-circular      | 80.1 ± 0.9   | 83.3        |
| vertical           | 83.2 ± 0.3   | 83.3        |

TABLE III: Same as in Tables I and II but for the UC regime and various polarization states.

matrices of both clones are the same as the density matrix of the copied state transmitted via the noisy channel,

$$\hat{\rho}_{\text{out}} = \alpha_+ \hat{\rho}_{\text{in}} + \frac{\Lambda^2}{4} (\hat{\sigma}_x \hat{\rho}_{\text{in}} \hat{\sigma}_x + \hat{\sigma}_y \hat{\rho}_{\text{in}} \hat{\sigma}_y) + \alpha_- \hat{\sigma}_z \hat{\rho}_{\text{in}} \hat{\sigma}_z,$$

where $\alpha_+ = \frac{1 + \Lambda^2 + 2\sqrt{2}\Lambda^2}{4}$ and $\Lambda^2 + \overline{\Lambda}^2 = 1$. The parameter $\Lambda$ depends on the distribution $g$ of the cloned qubits and $\hat{\rho}_{\text{in}} = |\psi\rangle\langle\psi|$. In the special case for $\Lambda^2 = 2/3$, the channel becomes so-called depolarizing channel, where the probability of all errors is the same and equal to $1/12$. In such case the corresponding cloning machine is the UC. Moreover, for $\Lambda^2 = 1$ the channel becomes a dephasing channel (only the phase-flip error can occur) and the corresponding cloning is optimized for covariant cloning of the eigenstates of the phase-flip operator $|0\rangle$ and $|1\rangle$ and, thus, those two states can be perfectly copied or transmitted through the lossy channel.

FIG. 6: Phase (angle $\varphi$) dependence of fidelity of the MPCC for the selected values (see Table II) of angle $\theta$.

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