Short-range YN interactions in the Quark Cluster Model

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A phenomenological model for the hyperon-nucleon interactions is constructed by using the quark cluster model approach to the short-distance baryon-baryon interactions. The model contains the SU(3) symmetric meson exchange interaction at large distances and the quark-exchange short-distance interaction. The main feature of the model is that strong channel dependences of the short range repulsions due to the quark model symmetry. It is pointed out that two channels, \((I, S) = (1/2, 0)\) and \((3/2, 1)\), of the S-wave sigma-nucleon interactions have extremely strong repulsions at short-distances.

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1. Introduction

Various phenomenological models of the hyperon (Y) – nucleon (N) interactions are available for studying the structure of hypernuclei\(^1,2\). They are mostly based on the meson exchange potentials with the SU(3) flavor symmetry. At short distances the meson-exchange potentials are supplemented by strong repulsions, which are similar to the well-known NN repulsive core. The models contain many arbitrary parameters concerning the short-range repulsion, which are determined by the very-limited experimental data of the YN scatterings.

On the other hand, the quark model description of the short-range repulsion between two nucleons is quite successful\(^3,4\). It has been demonstrated in the quark cluster model (QCM) calculations that the color-magnetic gluon exchange and the quark antisymmetrization in the valence quark dynamics provides a non-local soft repulsive core, which can reproduce the N-N scattering S matrices for energies up to 300-400 MeV. When QCM is applied to other two-baryon systems, the same mechanism yields strong short-range repulsions in most of the two ground-state baryon systems, including Λ-N and Σ-N\(^5\). We find that the quark exchange effects (due to the antisymmetrization) show distinctive spin-isospin dependences especially for the Σ-N interactions. Such strong channel dependences have not been considered or taken into account in the conventional YN potential models. Thus it is quite interesting to see whether the YN interaction model that incorporates the quark-exchange mechanism can achieve similar successes in explaining the YN two-body data as the conventional models.

The aim of this report is to show how one can construct a phenomenological potential model for the hyperon-nucleon interaction incorporating both the meson exchanges and the quark-gluon effects. Such a model enables us to analyze experimental YN scattering data and to determine whether the quark-exchange mechanism is indeed at work for the hyperon-nucleon systems. This is important further for the study of double strange systems, such as Λ-Λ, N-Ξ and the H dibaryon, because the interactions of \(S = -2\) two-baryon systems are not yet directly accessible in experiment and thus require theoretical predictions.
2. Mechanisms of short-range repulsion in the quark model

The short-distance repulsions between baryons seem universal for most two-baryon interactions. It is, for instance, known from the study of hypernuclei that the hyperon-nucleon interactions contain a short-range repulsion similar to the nuclear force. This universality can be accounted for by the simple quark model, which provides us with two distinct mechanisms for the short-distance repulsion[3,4].

The first (I) is due to the Pauli exclusion principle among the valence quarks. It can produce a strong repulsion between two baryons where quark distributions overlap with each other. The strength of the repulsion can roughly be estimated from the eigenvalues of the normalization integral kernel for the two-baryon system,

\[ \langle B_1 B_2 \delta(R - S) | A | B_1 B_2 \chi(R) \rangle = \int \langle B_1 B_2 \delta(R - S) | A | B_1 B_2 \delta(R - S') \rangle \chi(S') dS' = e \chi(S) \]  

\[ \text{(1)} \]

where \( A \) is the quark antisymmetrization operator for all the six quarks and \( \chi(R) \) denotes the relative \( B_1 - B_2 \) wave function. \( N(S, S') \) is called the normalization integral kernel of the resonating group method. The Pauli forbidden state yields \( e = 0 \), because the antisymmetrized state vanishes. One obtains \( e = 1 \), if no antisymmetrization is considered. In general, the eigenvalue \( e \) gives a good indication of the “forbiddenness” of the two-baryon system. Namely, if \( e < 1 \), the channel has a “partially forbidden” state and the baryonic potential has a repulsion at short distances (or actually \( R = 0 \)).

Table 1 shows the smallest eigenvalue \( e \) for various \( S \)-wave \( YN \) systems. They are evaluated for the simple harmonic oscillator quark model. One finds that two \( N \Sigma \) \( (L = 0) \) channels, \( N \Sigma \) \( (S = 0, I = \frac{1}{2}) \) and \( N \Sigma \) \( (S = 1, I = \frac{3}{2}) \), have small eigenvalues, \( e = 1/9 \) and \( 2/9 \) respectively. The corresponding eigen-function \( \chi \) is the harmonic oscillator \( 0s \) function, which results in \( |B_1 B_2 \chi(R)\rangle = |(0s)^6\rangle \). Thus each of these channels has an almost forbidden state, which will cause a strong short-range repulsions[3].

The second mechanism (II) for the short range repulsion is driven by the hyperfine
Table 1: The smallest eigenvalues of the normalization kernel and the effective core radius for various S-wave $YN$ systems. The “type” indicates the origin of the short-range repulsion, either from the first (I) or the second (II) mechanisms. See the text for the effective core radius.

| $BB'$     | $(J,I)$   | $e$ | type | effective core radius |
|-----------|-----------|-----|------|-----------------------|
| $N\Lambda$ | $(0,\frac{1}{2})$ | 1   | II   | 0.40 fm               |
| $N\Sigma$  | $(0,\frac{1}{2})$ | $\frac{1}{9}$ | I    | 0.68 fm               |
| $N\Lambda$ | $(1,\frac{1}{2})$ | 1   | II   | 0.34 fm               |
| $N\Sigma$  | $(1,\frac{1}{2})$ | 1   | II   | 0.30 fm               |
| $N\Sigma$  | $(0,\frac{3}{2})$ | 1   | II   | 0.48 fm               |
| $N\Sigma$  | $(1,\frac{3}{2})$ | $\frac{2}{9}$ | I    | 0.67 fm               |

Interaction between quarks:

$$V_{\text{CMI}} = -\frac{\alpha_s}{4} \sum_{i<j} \frac{2\pi}{3m_i m_j} (\lambda_i \cdot \lambda_j) (\vec{\sigma}_i \cdot \vec{\sigma}_j) \delta(\vec{r}_{ij})$$  \hspace{1cm} (2)

which is considered to come from the color magnetic part of a gluon exchange between quarks. The importance of this interaction in the baryon spectrum is manifested, for instance, in $N - \Delta$, and $\Lambda - \Sigma$ mass differences, and the negative neutron mean charge square radius.

The importance of the hyperfine interaction in the short-range $NN$ interaction has been pointed out in the quark cluster model calculation[3,5]. One finds that the spin-spin interaction (2) produces a short-range repulsion not only for $NN$ but also for other baryon-baryon interactions, such as $N\Lambda$ and $N\Sigma$. Such calculations also indicate that the Pauli exclusion principle (mechanism I) gives in general a stronger short-range repulsion than the hyperfine interaction (II).
3. Quark cluster model with the Nijmegen meson exchange potential

The quark cluster model (QCM) is most suitable in exploring the above mechanisms of the short-range repulsion in two-baryon systems[3]. The model incorporates the full antisymmetrization among valence quarks and the quark exchange interactions induced by the one-gluon exchange. Our aim is to construct a realistic $YN$ interaction based on QCM, which incorporates the quark exchange interaction at short distances and the meson exchange potential at larger distances[6].

First, we consider a valence quark model with a Hamiltonian,

$$ H = K + V_{\text{CONF}} + V_{\text{OGE}} $$

(3)

where $K$ is the nonrelativistic quark kinetic energy term, $V_{\text{CONF}}$ stands for a quark confinement potential and $V_{\text{OGE}}$ is the Fermi-Breit potential for the one gluon exchange. We employ the resonating group method (RGM) wave function for the six-quark system, given by

$$ \Phi_{BB'}(1 \sim 6) = A[\phi_B(1 \sim 3) \phi_{B'}(4 \sim 6) \chi(R)] $$

(4)

and solve the RGM integral equation, with the kernels $H$ (Hamiltonian) and $N$ (Normalization):

$$ \int [H(R, R') - E N(R, R')] \chi(R') dR' = 0 $$

(5)

Nonlocality of the RGM equation comes from the antisymmetrization of the quarks.

In order to describe the long-range part of the baryon-baryon interaction, we additionally need the meson exchange potentials. We keep the SU(3) symmetry for the meson-baryon couplings. Indeed, the $YN$ potential models, such as the Nijmegen models[1] and Jülich models[2], are based on the SU(3) symmetry. In this study, we employ the meson-exchange part of the Nijmegen potential model D and instead of using the hard cores in the original model, superpose it with the quark exchange interaction at the short distance. We introduce to the QCM equation (3) the meson exchange potential, which
is borrowed from the Nijmegen model D in this study. This can be done by adding an integral kernel for the meson exchange potential, given by

\[ V(R, R') \equiv \int dR'' N^{1/2}(R, R'') V_f(R'') N^{1/2}(R'', R') \]  

where \( V_f \) is the Nijmegen meson exchange potential with the appropriate form factor. The form factor is chosen so as to be consistent with the quark wave function of the baryon,

\[ V_f(R) \equiv \int \rho(x; R/2) V_N(x - y) \rho(y; -R/2) \, dx \, dy \]

where \( V_N \) is the original Nijmegen D potential without the repulsive core and \( \rho(x; R/2) \) stands for the quark density of the baryon centered at \( R/2 \). In the QCM calculation, we employ the Gaussian for the internal quark wave functions of the baryon for simplicity, and thus the corresponding form factor is given also by a Gaussian.

4. Results

We have five parameters in the present model: the light quark mass \( m_q \), the ratio of the light and strange quark masses \( m_q/m_s \), the strength of linear confinement potential, \( a \), the strength of the one-gluon exchange potential, \( \alpha_s \), and the size parameter \( b \) for the Gaussian wave function of quarks in the baryon. In order to make the calculation consistent in kinematics, we choose \( m_q \) to be one-third of the average octet baryon mass, i.e., 383.7 MeV. The ratio of the light/strange quark masses is fixed to 0.6, which gives the correct \( \Lambda - \Sigma \) mass difference. The gluon coupling constant is chosen so as to reproduce the \( N - \Delta \) mass difference, and we also choose the confinement \( a \) so that the baryon state is stable against the breathing mode excitation, i.e., \( \partial E_B/\partial b = 0 \). The remaining parameter \( b \) is sensitive to the \( NN \) interaction, because it determines the size of the form factor and also the range of the quark exchange interaction. Therefore we leave this as a free parameter and use the \( NN \) scattering data to choose the best value for \( b \). The QCM calculation with the Nijmegen D meson exchange potential can fit the \( NN \) \(^1S_0\) scattering phase shift well for \( b = 0.56 \) fm. Then the other parameters are determined: \( a = 20.8 \) MeV/fm, \( \alpha_s = 1.85 \). Fig. 1 shows the fit of the \( NN \) \(^1S_0\) scattering phase shift
Figure 1: $^1S_0$ NN scattering phase shifts.

Figure 2: $^1S_0$ NA scattering phase shifts. The dotted and the dash-dotted curves are for the calculations without couplings to $N\Sigma$.

calculated with this choice of parameters. The fit is not very good for low energy. In order to improve the fit, we might adjust some of the meson exchange parameters, which at the present are taken from the Nijmegen D model.

We then calculate the scattering S matrices for various $YN$ systems in this model and find that the qualitative predictions given above are confirmed in the present model. Fig. 2 shows the $\Lambda N ~ ^1S_0$ scattering phase shifts. The result is compared with that for the original Nijmegen model D with the hard core. One sees that the $\Lambda N$ interaction in QCM is more attractive than that in the original Nijmegen model. This channel couples with the $\Sigma N ~ ^1S_0$ state. The effect of the coupling is significant especially for QCM.

Figs. 3 and 4 show the $\Sigma N$ scattering phase shifts for $I = \frac{1}{2}, ~ ^1S_0$ and $I = \frac{3}{2}, ~ ^3S_1$ channels. These are the channels where the Pauli principle expects the type I strong repulsion. We indeed obtain strongly repulsive phase shifts, far more repulsive than the original Nijmegen model, while QCM yields milder repulsions in the other $\Lambda N$ and $\Sigma N$ channels. Therefore these two Pauli-forbidden $\Sigma - N$ interactions are exceptional. This indicates that the $\Sigma - N$ interactions have strong spin-isospin dependences.

In Table 1, the properties of the short-distance $YN$ interactions are summarized in terms of the effective size of the repulsive core, defined by $d\delta/dk$ at $E_{lab} = 350$ MeV. One sees that the Pauli exclusion principle gives a stronger repulsion for the $N\Sigma$ ($S = 0$, ...)
Figure 3: $I = \frac{1}{2} S_0 N\Sigma$ scattering phase shifts with couplings to $N\Lambda$.

Figure 4: $I = \frac{3}{2} S_1 N\Sigma$ scattering phase shifts.

$I = \frac{1}{2}$ and $N\Sigma (S = 1, I = \frac{3}{2})$ channels, while the other channels show a mild repulsion which is generally softer than the original Nijmegen model D. The repulsion in the $N\Sigma (I = \frac{3}{2}, S_1)$ channel is as strong as that in the Nijmegen model F, which is known to provide not enough binding for $\Sigma$ to make a bound $\Sigma$ hypernuclei. Details of the model and the results will be published elsewhere[6].

5. Conclusion and Discussion

We present a quark model analysis of hyperon-nucleon interactions. The realistic $YN$ interactions, which are nonlocal due to the quark antisymmetrization effects, are proposed using the quark cluster model approach with the Nijmegen model D meson exchange potential. The main difference between the original Nijmegen model and our interaction arises in the spin-isospin dependence of the $YN$ short range interactions. Especially, $\Sigma N$ with $S = 0, I = 1/2$ and $S = 1$ and $I = 3/2$ have strong repulsion at the short distance in the quark model and may make the bound $\Sigma$ hypernuclei implausible.

The model is not complete yet. The final goal is a no-parameter model for the $YN$ (and $YY$) interactions based on the SU(3) symmetry for the meson-baryon coupling constants. The quark model parameters and the meson-baryon coupling for each SU(3) multiplet can be determined in the non-strange sector, that is the $NN$ scatterings. The present model borrows the meson-baryon couplings from the Nijmegen potential, whose short-
range behaviors are different from the quark cluster model. Thus the fit to the NN data is not as complete as the original potential. One has to adjust the coupling constants so as to reproduce the NN data. Work along this line in under way.

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