The Effects of the Little Higgs Model on the Decay Width $Z \rightarrow e^+e^-$

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Abstract.
In the framework of the Little Higgs Model (LHM), we calculate the decay width $\Gamma(Z_1 \rightarrow e^+e^-)$ with corrections of QED and QCD. We analyze this with recent data from LEP and compute the contribution of the model. We find that the deviations of the decay width of reaction $Z_1 \rightarrow e^+e^-$ from its SM value are relatively large in the parameter space preferred by the electroweak precision data. Furthermore, with reasonable free parameter values, the absolute value of the relative correction parameter $\delta \Gamma/\Gamma_{SM}$ is of $15\% - 50\%$. The experimental measurement values might generate constraints on the free parameters of the LHM.

1. Introduction
There are a number of scenarios for new physics beyond the Standard Model (SM) [1]. The most famous is the supersymmetric scenario. One of the major motivations for physics beyond the Standard Model is to resolve the hierarchy and fine-tuning problems between the electroweak scale and the Planck scale. Supersymmetric theories introduce an extended space-time symmetry and quadratically divergent quantum corrections are canceled due to the symmetry between the bosonic and fermionic partners. Technicolor theories introduce new strong dynamics at scales not much higher than the electroweak scale, thus defer the hierarchy problem. Theories with TeV scale quantum gravity reinterpret the problem completely by lowering the fundamental Planck scale. Recently, an alternative called the Little Higgs Model (LHM) [2, 3, 4, 5] has been proposed. The little Higgs model offers an alternative route to the solution of the hierarchy problem of the SM, reviving the idea that the Higgs doublet is a pseudo Goldstone boson of some global symmetry which is spontaneously broken at a TeV scale. The key feature of this kind of model is that the Higgs boson is a pseudo-Goldstone boson of an approximate global symmetry which is spontaneously broken by a vev at a scale of a few TeV and is thus naturally light. In the LHM, a set of new heavy gauge bosons ($A_2, Z_2, W_2$) and a new heavy vector-like quark (T) are introduced which cancel the quadratic divergence induced by SM gauge boson loops and the top quark loop, respectively. The distinguishing features of this model are the existence of these new particles and their couplings to the light Higgs. The measurement of these couplings and new particle effects might prove the existence of the little Higgs mechanism [6]. The global symmetry breaking scale is expected to be $\lesssim 10 \ TeV$ so the little Higgs model will be relevant for the hierarchy.

In this work, we calculate the decay width of the process $Z_1 \rightarrow e^+e^-$ in the LHM. When
compared to the process \( Z_1 \rightarrow e^+e^- \) in the SM, the process in the LHM receives the additional contribution arising from the vector and axial-vector couplings as well as the parameters of the LHM [7]. We find that, with reasonable values of the free parameters, the deviation of the decay width \( \delta \Gamma / \Gamma_{SM} \) from its SM is of 15% - 50%.

We also study the effects of the little Higgs model in the reaction \( Z_1 \rightarrow e^+e^- \). The leptonic \( Z_1 \) decays are free from the long distance effects and are thus clean.

Processes measured near the resonance have served to set bounds on the parameters of the model. This partial decay occurs in the resonance zone. As a consequence, the process is independent of the mass of the additional \( Z \) heavy gauge boson which appears in these kind of models. They also carry considerable information about the free parameters of the model used. Therefore, it is worthwhile to analyze these decay processes in the context of the new physics models.

This paper is organized as follows: In Sec. 2 we present the expressions for the decay width \( Z_1 \rightarrow e^+e^- \) including QED and QCD corrections is given by:

\[
\Gamma(Z_1 \rightarrow e^+e^-) = \frac{G_F M_Z^2}{6\pi\sqrt{2}} \left[ (g_V^l)^2 + (g_A^l)^2 - 2(g_V^l)(g_A^l) \frac{v^2}{f^2} \left( -c_W x_Z W' c/2s + \frac{s_W x_Z W'}{s'c'}(2y_e - \frac{9}{5} + \frac{3}{2}e^2) \right) \right] - 2(g_A^l)^2 \frac{v^2}{f^2} \left( c_W x_Z W' c/2s + \frac{s_W x_Z W'}{s'c'}(-\frac{1}{5} + \frac{1}{2}e^2) \right) \left( 1 + \delta \rho + \delta \rho_e + \delta_{QED} \right). (1)
\]

The vector and axial-vector \( Z_1l\bar{l} \) couplings \( g_V^l \) and \( g_A^l \) compare one-loop and higher electroweak and internal QCD corrections through the form factors \( \delta \rho_l \) and \( k_l \), which can be written as:

\[
g_V^l = \sqrt{\alpha}(\frac{1}{2} - 2\sin^2\theta_{eff})^l, \quad g_A^l = \sqrt{\alpha}(\frac{1}{2}), \quad (2)
\]

with \( \sin^2\theta_{eff} = k_l\sin^2\theta_W \). The term \( \delta \rho \) is the deviation from the SM prediction for the \( \rho \) parameter \( \rho = M_Z \cos \theta/M_W = 1 + \delta \rho \), taking into account contributions of the gauge group structure of LHM only. The contribution to \( \delta \rho \) is given by

\[
\delta \rho \approx -\frac{v^2}{8f^2} \left[ 1 + 5(e^2 - s^2)^2 \right]. \quad (3)
\]

Also, \( \delta_{QED} \) accounts for the final state photon radiation,

\[
\delta_{QED} = \frac{3\alpha(s)}{4\pi} Q^2, \quad (4)
\]

where \( \alpha \) is the QED coupling computed at the energy scale \( s \), while \( Q \) is the lepton charge.

In order to obtain a prediction for the standard model partial \( Z_1 \) decay width into \( e^+e^- \), we take the input parameters [8], \( M_Z = 91.187 \text{ GeV}, G = 1.16637 \times 10^{-5} \text{ GeV}^{-2}, \alpha(M_Z) = 1/128.95 \) and \( \sin^2\theta_W = 0.22335 \). These parameters can be used to obtain the form factors for the decay \( Z_1 \rightarrow e^+e^- \) using the Zfitter package [9], yielding

\[
\delta \rho_e = 0.00531, \quad (5)
\]

and
Figure 1. The relative correction $\delta \Gamma / \Gamma_{SM}$ as a function of the scale of energy $f$ for $c = 0.5$ and different values of the mixing parameter $s$. Starting from the top, the curves are for $s = 0, 0.4, 0.6, 0.8, 1$.

$$\sin^2 \theta_{W(\text{eff})} = 0.2315,$$

which translates into $\kappa_e = 1.0367$. Plugging these parameters into Eq. (1) together with the limit $f \to \infty$ and $\delta \rho \to 0$, we obtain the standard model prediction $\Gamma(Z_1 \to e^+ e^-) = 83.99$ MeV.

3. Results and Conclusions

Our numerical results are summarized in Figs. 1-3. To see the dependence of relative correction on the parameter $f$, in Fig. 1 we plot $\delta \Gamma / \Gamma_{SM}$ as a function of the scale of energy $f$ for $c = 0.5$ and different values of the mixing parameter $s = 0.2, 0.4, 0.6, 0.8, 1$. We can see that the absolute value of the relative correction decreases as $f$ increases. The curves also demonstrate that the effect of the LHM is not sensitive to $f$ in the range of $f \leq 6.5$ TeV. This is because, in general, the extra contribution of the LHM to the decay width $\Gamma(Z_1 \to e^+ e^-)$ is proportional to a factor of $1/f^2$. In this case, the absolute value of $\delta \Gamma / \Gamma_{SM}$ is in the range of $40\%$ in most of the parameter space.

The relative correction $\delta \Gamma / \Gamma_{SM}$ as a function of the scale of energy $f$ for $s = 0.5$ and different values of $c = 0.2, 0.4, 0.6, 0.8, 1$ is presented in Fig. 2. In this figure we can see that the absolute value of the relative correction $\delta \Gamma / \Gamma_{SM}$ decreases when the scale energy $f$ increases and is sensitive to the mixing parameter $c$. For $c = 0.2$, the absolute value of $\delta \Gamma / \Gamma_{SM}$ is in the range of $2\% - 50\%$.

Finally, the graphic in Fig. 3 shows the allowed values for $f$, $s$ and $c$ that can be developed allowed by the decay width $\Gamma^{exp} = (83.984 \pm 0.086)$ MeV with $95\%$ C.L.. We see that the values for $f$, $s$ and $c$ which can develop are in complete agreement with those reported in the literature.

The SM gauge boson $Z_1$ is now abundantly produced at the LHC and will be as well at the future high energy linear $e^+ e^-$ collider experiments. It is possible to examine its properties with unprecedented precision. We calculate the decay width correction of the little Higgs model of the process $\Gamma(Z_1 \to e^+ e^-)$. We find that the correction is significant even when we consider the constraint of electroweak precision data on the parameters. In the favorable parameter space, the absolute value of the relative correction parameter $\delta \Gamma / \Gamma_{SM}$ is of $15\% - 50\%$. We conclude that the future experiments at the ILC could determine the effects on the $\Gamma(Z_1 \to e^+ e^-)$ decay width contributed by the LHM in some parameter space or put more stringent constraints on the LHM parameters.
Figure 2. The relative correction $\delta\Gamma/\Gamma_{SM}$ as a function of the scale of energy $f$ for $s = 0.5$ and different values of the mixing parameter $c$. Starting from the top, the curves are for $c = 0.2, 0.4, 0.6, 0.8, 1$.

Figure 3. Possible values for $s$, $c$ and $f$ that can be developed allowed by the decay width $\Gamma_{exp}(Z_1 \to e^+e^-)$ with 95% C.L..

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