Irreversibility in physics stemming from unpredictable symbol-handling agents

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ABSTRACT

The basic equations of physics involve a time variable $t$ and are invariant under the transformation $t \rightarrow -t$. This invariance at first sight appears to impose time reversibility as a principle of physics, in conflict with thermodynamics. But equations written on the blackboard are not the whole story in physics. In prior work we sharpened a distinction obscured in today’s theoretical physics, the distinction between obtaining evidence from experiments on the laboratory bench and explaining that evidence in mathematical symbols on the blackboard. The sharp distinction rests on a proof within the mathematics of quantum theory that no amount of evidence, represented in quantum theory in terms of probabilities, can uniquely determine its explanation in terms of wave functions and linear operators. Building on the proof we show here a role in physics for unpredictable symbol-handling agents acting both at the blackboard and at the workbench, communicating back and forth by means of transmitted symbols. Because of their unpredictability, symbol-handling agents introduce a heretofore overlooked source of irreversibility into physics, even when the equations they write on the blackboard are invariant under $t \rightarrow -t$. Widening the scope of descriptions admissible to physics to include the agents and the symbols that link theory to experiments opens up a new source of time-irreversibility in physics.

Keywords: symbol, logical synchronization, live clock, evidence vs. explanation, unpredictability, irreversibility.

1. INTRODUCTION

In 1950’s, Schödinger complained that the physics he helped make has no place in it to represent himself\textsuperscript{[1]} These days Bayesians view probabilities as numbers assigned by agents, and ‘Quantum Bayesians’ ascribe the choice of wave functions to agents.\textsuperscript{[2]} In quantum information science, Alice and Bob are spoken of as agents. These ways of speaking of agents are a start that we propose to advance by recognizing structure necessary to the functioning of agents in a communications network.

I want to think of myself as an agent to be represented in physics. Not all of me, but a slice of my activity. Which slice? The slice in which something I do might matter to physics. Is there any such...
slice? An answer to this question came in 2005 with the proof within quantum theory that whatever
evidence one may have leaves open an infinite set of inequivalent explanations, so that an investigator
faces a choice undetermined by logic: picking an explanation takes a guess. Furthermore the guess
enters the design and operation of physical experiments, suggesting that acts of guessing might be
worth representing as acts of an agent expressed within theoretical physics. With this in mind, the
idea came in 2002 of picturing myself as an investigator seated at the console of a classical process-
control computer (CPC) managing an experiment—programming and running algorithms that control
computer-mediated feedback loops, sending commands to actuators on the workbench, recording oc-
currences of outcomes, and modifying guessed hypotheses.

The design and operation of an experiment brings explanations written in symbols on the black-
board into contact with the devices on the workbench. In designing an experiment or in interpreting its
results, an investigator faces choices of explanations and of physical devices, and the investigator must
choose, and, on occasion, modify, which explanation to link to which arrangement of devices. The
activity of such an investigator is recordable in files of a CPC used both to calculate with the equations
and to manage the instruments. By noticing that equations and instruments make contact in a CPC, we
imagine the traffic in symbols transmitted by the CPC to and from the workbench and the blackboard
or our own imagination as a heretofore unrecognized part of physics.

On what basis can we argue for including traffic in transmitted symbols, including numerals, as part
of physics? In special relativity Einstein defined spacetime in terms of numerical clock readings at the
transmission of a signal from one clock and the reception of a signal at another clock. Hence implicit
in the concept of spacetime is the propagation of signals that carry symbols, namely the numerals by
which a clock reading can be expressed. On this basis we claim the right to bring the propagation
of signal-carrying symbols along with the symbols carried into theoretical physics, and to introduce
agents to transmit and to receive symbols. Here we introduce what we call a symbol-handling agent
as a role played sometimes by a person and other times by an automated device. In developing a
notion of symbol-handling agents as elements of discourse in physics, we want to keep these agents
“lean.” We begin with the idea that an agent makes unpredictable responses to unpredictable inputs.
We limit ourselves as much as we can to this focus on unpredictability, attributing to symbol-handling
agents neither values nor volition. By keeping agents “lean” in this way we soften a boundary between
animate and inanimate to show a wealth of unsuspected structure, relevant to both persons and to
computer-based machinery as agents. Next comes the question: what difference to physics does it
make to allow for symbol-handling agents in physical descriptions? An answer worked out below is
that the introduction of symbols such as numerals along with symbol-handling agents brings a new
source of irreversibility into physics.

In the next section we introduce a model of signal-handling agents, focused on displaying an
unsuspected structure of logical synchronization necessary for symbols transmitted by one agent to be
recognizable to another agent. In Sec. we show how unpredictable, symbol-handling agents imply
irreversibility. In Sec. we review the notion of “agent” that has been rejected in physics for three
centuries but is now, in modified form, attracting advocates.
2. MODEL OF A SYMBOL-HANDLING AGENT

By *symbols*, we mean recognizable elements of communication among agents, human or not, that can lead to action involving energy supplied not by the symbols themselves, nor by the agent that sends them, but by the receiving agent. (I can speak in words as symbols to ask you to move; if I just push you, that is not communication by use of symbols.) We conceive of a symbol-handling agent, human or not, that computes with numerals, communicates sequences of symbols, especially numerals, with other agents, and takes unpredictable symbolic input and also interacts, energetically rather than symbolically, with an environment.

We posit that a symbol-handling agent is equipped with a memory that at any moment may be found in one of some number of states. How a receiving agent responds to a symbol is contingent on the contents of the memory of the agent. Examples of symbols are (1) instances of letters, words, and numerals, expressed as written characters, (2) electronic impulses carrying bits 0 and 1 in a computer, and (3) molecules and their constituent parts (e.g. nucleobases of DNA) involved in biological signaling. Symbols used by people convey elements of thought, often prompting actions and emotions. And beyond ‘elements of thought’, symbols convey the calculational traffic to be found both in man-made computers and in the biological processes of living creatures. We speak of a symbol (as something that can be recognized) carried by a signal (as something physical that can be measured). It is important to distinguish between ‘recognizing a symbol’ from ‘measuring a signal that carries the symbol’. For example, the symbols 0 and 1 are carried over transmission lines of a computer chip by electromagnetic signals. A programmer is indifferent to how such signals vary in their electrical characteristics and their timing within allowed tolerances, while an electrical circuit designer has to attend to the details of the physical signals that carry what the programmer views as symbols. In particular, the circuit designer must provide for maintaining the signal within allowable tolerances, which requires that the computer hardware measure and respond to small timing variations to which the programmer is oblivious, but on which the operation of the computer depends. It is noteworthy that symbols carried by signals with large tolerances can take part in mechanisms that act with tight tolerances. For example the electronic signals in a computer can vary over a fairly wide range in amplitude and timing without spoiling the exactness of arithmetical operations of the computer.

Picture agents at the blackboard reading and writing explanations in mathematical symbols, and picture agents at the workbench manipulating arrangements of physical devices. Communication of symbols between agents at the workbench and agents at the blackboard links evidence to explanations. As will be described later, agents maintain a structure of logical synchronization with one another, needed for symbols to be recognized. For this the agents respond to unpredictable, measured phases of reception of signals, which requires their measuring the signals, rather than just recognizing the symbols.

In order to produce a more formal expression of a symbol-handling agent, we turn to Turing’s 1936 paper in which he introduced Turing machines, thereby establishing computability as a discipline within mathematics. In a side remark, he briefly introduced an alternative machine called a choice machine, contrasted with the usual Turing machine that Turing called an a-machine:

If at each stage the motion of a machine . . . is completely determined by the configuration, we shall call the machine an “automatic machine” (or a-machine). For some purposes we
might use machines (choice machines or c-machines) whose motion is only partially de-
termined by the configuration . . . . When such a machine reaches one of these ambiguous
configurations, it cannot go on until some arbitrary choice has been made by an exter-
nal operator. This would be the case if we were using machines to deal with axiomatic
systems.\textsuperscript{9}

Our model of a symbol-handling agent amounts to a choice machine modified in two ways. The
first modification provides for a choice machine not only to receive symbols but to transmit then,
so that modified choice machines can communicate symbols with one another and with an external
environment. The second modification provides the choice machine with a clock that steps it, with
the tick rate of the clock adjusted by commands issuing from the choice machine itself. This is our model
of symbol-handling agent. In this model, a symbol-handling agent ticks like a clock, computes like a
digital computer, and transmits and receives sequences of symbols, in some cases unpredictably. To
assure the unpredictability of a symbol-handling agent, we posit that an “external operator” chooses
a symbol and writes it onto the scanned square of the agent’s choice machine \textit{privately}, in the sense
that the symbol remains unknown to other agents unless and until the symbol-handling agent that
receives the chosen symbol reports it to others. In past work we described how such symbol-handling
agents, which in that work we called \textit{live clocks}\textsuperscript{7} or \textit{open machines}\textsuperscript{10} generate evidence on which
implementations of spacetime coordinates depend.

2.1 Logical synchronization

One can describe the operation of a Turing machine as one might describe the play of a chess game by
stating the conditions of the chess pieces on the board over a sequence of moments interspersed by the
moves of the players; indeed, Turing speaks moments interspersed by moves. For the choice machines
adapted to communicate as symbol-handling agents, one must attend in more detail to the cycle of
operation that encompasses a single moment along with a single move. We speak of phases of the
cycle, thought of as indicated by an imagined clock hand that rotates during a cycle once around a dial.
For a symbol to be received by a symbol-handling agent, the signal carrying the symbol must arrive
within a suitable phase of the cyclical operation of the agent. Otherwise the agent “drops the ball”. An
agent $B$ receiving symbols from an agent $A$ under this condition is said to be \textit{logically synchronized}
to agent $A$. As discussed in \cite{7,10}, maintenance of logical synchronization requires feedback loops that
respond to the measured deviation of the arrival of a signal from a desired aiming phase. This measured
deviation in the phase of arrival of a symbol-carrying signal is invisible to the Turing-machine logic
of an agent, because any physical implementation of that logic must tolerate variations within certain
limits in the phase of arrival of a symbol-carrying signal. Thus logical synchronization that depends
on responding to phases requires extra physical circuitry not encompassed in Turing’s mathematics.

Because of the use of feedback to maintain logical synchronization among the symbol-handling
agents of an experiments, the lag in responding to measured deviations limits physically possible
behavior. For this reason, it can be important to respond rapidly to the physical phase at which a
signal carrying a symbol arrives within a computational cycle of an agent, which precludes waiting for
an analog to digital conversion; the agent has to respond to the phase of an arriving signal energy in
relation to a clock pulse, and the phase to which the agent responds in immediate, physical, and \textit{not} a
number.
Another way to see the non-numerical aspect of the phase of arrival of a signal is to examine the fan-out of a symbol-carrying signal from one agent to two other agents, and to contrast the behavior of phases with that of symbols in this fan-out. Apart from occasional device failures, the receiving agents fed by a fan-out agree about what symbol they receive. In contrast, once the phases of arrival of the symbol-carrying signal has been clocked by an agent and expressed in numbers (e.g., analog to digital conversion), the clocks of the two receiving agents will generally disagree about those phases. We call such expectation of unpredictable disagreement idiosyncrasy. Idiosyncrasy due to jitter in phase arrivals is intrinsic to logical synchronization.

The need for agents to react to idiosyncratic physical phases prior to their expressions in numerals has an intriguing implication: the behavior of symbols that makes makes mathematical calculations dependable depends on the handling of unpredictable phases by symbol-handling agents. This interdependence between numerical symbols and non-numerical phases seems to be a largely or completely overlooked feature of the relation between mathematics and physics.

3. REVERSIBILITY AND IRREVERSIBILITY IN PHYSICS: THE IMPACT OF ACCEPTING AGENTS

The basic equations of physics involve a global time coordinate \( t \) and are invariant under the transformation \( t \to -t \). This invariance appears at first sight to impose time reversibility as a principle of physics, in conflict with the phenomenological equations postulated in thermodynamics and statistical mechanics that assert, as amply confirmed by experiment, that heat flows from hot to cold and not the other way around. We suggest that the inference that the invariance in the equations implies a corresponding invariance in the physical world comes from a traditional way of viewing the relation between equations on the blackboard and physical behavior on the workbench. One thinks of this relation in terms of error. The idea is that numbers from theory differ from numbers expressing evidence by some “error,” hopefully small. If one assumes that the relation between explanatory equations and the evidence explained is encompassed by the notion of error, the invariance of equations involving a time variable \( t \) under the transformation \( t \to -t \) indeed implies time reversibility as a principle of physics.

But if, as discussed above, one accepts unpredictability in symbol-handling agents necessary to the linking of the blackboard of theory with the workbench of experiment, there is long-overlooked room for irreversibility in physics. To see this source of irreversibility, consider the symbol-handling agent functioning as a process-control computer. Whatever can be computed from fixed inputs can be computed logically reversibly, as follows from Bennett’s introduction of a logically reversible Turing machine. Logical reversibility, however, cannot work for a symbol-handling agent because, being a choice machine, its inputs are neither fixed nor predictable: one cannot “back compute” how some unpredictable “external operator” makes its choice of symbol to write on the scanned square of the symbol-handling agent. The agent responds to symbols put into it unpredictably, so that an attempt to run a program backwards encounters needs to determine what generated an unpredictable symbol. The need cannot be met, from which we conclude that the operation of an agent is intrinsically irreversible, thereby introducing an overlooked source of irreversibility into physics.

Given that \( t \) as a variable global time coordinate is much used in expressing motion, there is still the question: is this necessary; can one formulate dynamics without a global \( t \)-variable? Indeed one can,
albeit at a certain cost. As described in [7, 10], one can locate happenings near the symbol-handling agents of a network by introducing a “local” variable $t_A$ for the clock reading of (adjustable) clock of the agent $A$ of the network. As described in the cited references, one needs to attribute to each agent’s clock an unadjusted (possibly idiosyncratic) reference. That is, one views the adjustable clock as moving a clock hand linked through an adjustable “gear box” to an unadjusted reference clock hand. One expresses the unadjusted hand by a variable $t_{A,0}$, related to the adjusted hand by a function $t_{A,0}(t_A)$. One also needs functions that express signal propagation. To this end one introduces the reading of the adjustable clock of an agent $B$ at the receipt of a signal transmitted from agent $A$ at a reading $t_A$ of $A$’s adjustable clock. This is expressed in [7, 10] by a relation denoted $\vec{AB}$. These functions define graphs having vertices labeled by readings of agents’ clocks. These graphs display communications among the symbol-handling agents of a network, and the graphs offer a reference system for locating events close to ticks of the clocks of agents, a basis that assumes no spacetime manifold, let alone a metric tensor field. In particular such a reference system serves to express evidence of the readings of physical clocks that can be used to support or to refute one or another assumption of a spacetime manifold with an assigned metric tensor field [10].

If one invokes the optional hypothesis of spacetime manifold with a metric by which to determine signal propagation, one can express the functions listed above as functions of spacetime coordinates $x$, e.g. $t_A(x)$, $t_{A,0}(x)$, etc., thereby achieving a much more economical representation of location, but then one loses the structure of clock readings related by signal propagations that allows for testing the hypothesis. And in some cases, spacetime coordinates are unavailable or fail to supply a basis for location that is available from a network of symbol-handling agents [7, 10].

4. DISCUSSION

4.1 Objections to the notion of an “agent”

Why would theoretical physicists so often think with the relation between the blackboard and the workbench as if error were all that mattered? For one thing, one often needs to attend to errors as gaps between numbers predicted in theory and numbers reported from experiments. We suggest, however, that this way of thinking comes from a tradition established with Descartes and reinforced by Hume, a tradition of rejecting any concept of agent or agency in theoretical physics. For that reason introducing a concept of agents, in particular the symbol-handling agents introduced above, has a residual antagonism to overcome.

In the Merriam-Webster dictionary we find the following relevant definition of ‘agent’.

agent: something that produces or is capable of producing a certain effect; an active or efficient cause.

Because of the association of “agent” with “cause,” a distaste for “cause” rubs off on “agent”. Traditionally in physics one avoids using or even discussing notions of “cause” and “agency.” Although not much discussed in physics, these notions are discussed by philosophers and historians, to which we now refer. The notion of an agent as an “active cause” runs into a logical swamp announced in Hume’s demonstration that causality is unprovable, to which Einstein agrees [13]. Russell [14], as did Locke before him, says that the conception of cause likely comes from volition, i.e. from ones experience of
doing something that one wants to do, and is applied to other agents by analogy; for Hume ‘cause’
is a feeling or intuition stemming from habitual association. As Russell puts it in discussing Hume’s
objections to the conception of cause,

I think perhaps the strongest argument on Hume’s side is to be derived from the character
of causal laws in physics. It appears that simple rules of the form “A causes B” are never
to be admitted in science, except as crude suggestions in early stages. The causal laws
by which such simple rules are replaced in well-developed sciences are so complex that
no one can suppose them given in perception; they are all, obviously, elaborate inferences
from the observed course of nature. I am leaving out of account modern quantum theory,
which reinforces the above conclusion. So far as the physical sciences are concerned,
Hume is wholly in the right: such propositions as “A causes B” are never to be accepted,
and our inclination to accept them is to be explained by the laws of habit and association.

In her recent book, “The Restless Clock,” J. Riskin starts a discussion of agency with: “By ‘agency’
I mean . . . a capacity to . . . do things in a way that is neither predetermined nor random. Its opposite is
passivity.” In the next paragraph she writes:

. . ., the scientific principle banning ascriptions of agency to natural things supposes a
material world that is essentially passive. This principle came into dominion around the
middle of the seventeenth century, during the period that historians generally identify as
the origin moment of modern science, or the New Science as its inventors called it. It is
the informing axiom of a mechanistic approach to science. Mechanism, the core paradigm
of modern science from the mid-seventeenth century onward, describes the world as a
machine—a great clock, in seventeenth- and eighteenth-century imagery—whose parts
are made of inert matter, moving only when set in motion by some external force, such as
a clockmaker winding the spring. According to this originally seventeenth-century model,
a mechanism is something lacking agency, produced and moved by outside forces; and
nature, as a great mechanism, is similarly passive. Assuming that living beings are part of
nature, according to this model, they too must be rationally explicable without appeal to
intentions or desires, agency or will [15], p.3.

4.2 Rehabilitating a “lean” agency in physics, based on recognizing unpredictability

Opening a breach against this resistance, authors of papers on quantum information science habitually
speak of communications between agents Alice and Bob, sometimes subject to eavesdropping by Eve
and Charlie, etc.; however, these authors speak in an offhand way, as a convenience, without claiming
that agents are necessary to the physics described, nor do they touch on logical synchronization.
Similarly, synthetic biology papers deal with living cells as computational agents responding to unpredictable inputs, but specify the agents only vaguely, without facing the question whether speaking of agency is a minor convenience or the introduction of a significant novelty. We attribute the announcement of occurrences of quantum outcomes to symbol-handling agents, leading of course to one aspect of the introduction of irreversibility. This aspect of irreversibility was introduced over twenty years
ago as ‘quantum jumps’ that, that admit the unpredictability of measurements into physics, but without explicit mention of any notion of agents, and again without pointing to the logical synchronization needed for the functioning of symbol-handling agents.\textsuperscript{15}

Giving agency a fundamental place in physics would be a big change. We propose that the proof of a gap between evidence and its explanations that has to be bridged by a guess is a sufficient reason to at least re-open the discussion of agency. The model outlined above of symbol-handling agents and their dependence on logical synchronization adds force to this proposal. It is worth noting that the introduction of unpredictability is free of any need for ‘volition’, let along ‘values’, on the part of a symbol-handling agent. For example, a symbol-handling agent fed an input from an unpredictable photo-detector evokes no need to speak of volition, nor do we need to speak of such an agent as “wanting to do something”; recognizing its unpredictability suffices to assure its “agency” in the leaned-down sense that we have explicated in the sections above.

The recognition of the physical reference systems as constructions involving symbol-handling agents with their adjustable clocks brings into a modern technical context the contention of Saint Augustine of time as a construction. As Russell quotes a translation of the Confessions of Saint Augustine, “Time was created when the world was created.” \textsuperscript{14} (p. 353). As we argued in \textsuperscript{7,10}, the “time” relevant to physics is created not just when the world was created, but is constructed to serve particular purposes. For many purposes, the time disseminated by the National Institutes of Science and Technology (NIST) or by other standards organizations is convenient and adequate; however, some investigations, including some yet to be designed, require, in place of “NIST time,” a special-purpose network of symbol-handling agents logically synchronized to one another.

Acceptance of the construction of networks of logically synchronized symbol-handling agents to serve as reference frames for particular investigations would shift the ground on which physics stands, by doing away with “time” as an indispensable externality. A partial recognition of the construction needed to produce “time” is found among those who develop and implement international time standards. For example, David Allan wrote:

\begin{quote}
The fact is that time as we now generate it is dependent upon defined origins, a defined resonance in the cesium atom, interrogating electronics, induced biases, timescale algorithms, and random perturbations from the ideal. Hence, at a significant level, time—as man generates it by the best means available to him—is an artifact. Corollaries to this are that every clock disagrees with every other clock essentially always, and no clock keeps ideal or “true” time in an abstract sense except as we may choose to define it.\textsuperscript{17}
\end{quote}

In spite of this known role of people in generating the “NIST time,” it seems to us that physics is haunted by the ghost of old sense of “time” (or in a relativistic setting, spacetime) as an invisible “ether” in which we all live but can’t see. This ghost is inherited from Descartes and from Newtonian physics—the idea that our bodies and all that we investigate physically are made of bits and pieces that passively respond “in time” to forces, with no room in physics for our own agency. We contend that although the concept of “ether” was discredited by special and general relativity, it still exerts its mythical power, distracting investigators from opportunities to take responsibility for generating the reference systems for their particular investigations, e.g. by designing and implementing networks of
symbol-handling agents. Accepting the primacy of locally constructed, special-purpose networks of logically synchronized symbol-handling agents to serve as reference frames for particular investigations would shift the ground on which physics stands, by doing away with “time” as an indispensable externality.

Reference frames provided by networks of symbol-handling agents, with their feedback loops that respond to unpredictable events in order to try to maintain logical synchronization, constitute tools permitting a range of experimental investigations. Such tools have been used for a long time, and, for example, are used in the Laser Interferometer Gravitational-Wave Observatory (LIGO).

In conclusion, we recognize that:

1. something announces unpredictable outcomes of measurements; and
2. something puts unpredictable equations on the blackboard.

These two propositions raise the question for theoretical physics: Do we as physicists want to ignore this something? Can we overcome our prejudice against agency in physical explanations to extend the reach of basic physics beyond time-reversible equations to encompass new sources of irreversibility? We close with a last question: How may recognizing the dependence of symbolic communication on physical, non-numeric phase management impact collaboration between mathematicians and experimental physicists?

Acknowledgment

Our thanks to Jessica Riskin for an exchange of emails concerning the historical pulling and tugging over the role in physics, or lack of it, for an agent.

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