Enhanced method for multiscale wind simulations over complex terrain for wind resource assessment

A Flores-Maradiaga 1,2,3, R Benoit 2, C Masson 2

1 Department of Mechanical Engineering, Technical University Federico Santa Maria, Av. España 1680, casilla 110-V, Valparaíso, Chile
2 École de Technologie Supérieure, Université du Québec, 1100 Notre-Dame Ouest, H3C1K3, Montréal, Québec, Canada

E-mail: alex.floresm@usm.cl

Abstract. Due to the natural variability of the wind, it is necessary to conduct thorough wind resource assessments to determine how much energy can be extracted at a given site. Lately, important advancements have been achieved in numerical methods of multiscale models used for high resolution wind simulations over steep topography. As a contribution to this effort, an enhanced numerical method was devised in the mesoscale compressible community (MC2) model of the Meteorological Service of Canada, adapting a new semi-implicit scheme with its imbedded large-eddy simulation (LES) capability for mountainous terrain. This implementation has been verified by simulating the neutrally stratified atmospheric boundary layer (ABL) over flat terrain and a Gaussian ridge. These preliminary results indicate that the enhanced MC2-LES model reproduces efficiently the results reported by other researchers who use similar models with more sophisticated sub-grid scale turbulence schemes. The proposed multiscale method also provides a new wind initialization scheme and additional utilities to improve numerical accuracy and stability. The resulting model can be used to assess the wind resource at meso- and micro-scales, reducing significantly the wind speed overestimation in mountainous areas.

1. Introduction

Multiscale simulations of the atmospheric boundary layer (ABL) over complex terrain are aimed to yield high fidelity wind mapping and more accurate wind resource assessment when compared with commercial modeling systems. This type of models has become a dominant trend in wind engineering, introducing interesting new features such as the aero-structural dynamics of wind turbines [1-2]. However, the biggest challenge for successful multiscale simulations is the capability to reproduce the ABL separation as well as turbulence generated near steep slopes. Numerical errors in the surface layer and the computational instability associated to conforming grid deformations are issues that require robust numerical algorithms capable of solving transient flow phenomena [3-4].

Therefore, the objective of this study is to validate some general improvements that can be implemented in many atmospheric models for ABL simulations over mountainous areas. These enhancements are particularly useful for those solvers that apply a semi-implicit (SI) time discretization to solve the compressible non-hydrostatic Navier-Stokes equations [5]. The new SI scheme and the large-eddy simulation (LES) method have been adapted for complex terrain in the mesoscale compressible community (MC2), including the classic Smagorinsky and Deardorff constant coefficient

3 Author to whom any correspondence should be addressed. E-mail: alex.floresm@usm.cl
sub-grid scale (SGS) schemes for turbulence parameterization [6]. Since both SGS schemes yield very close results, only the Smagorinsky scheme will be presented and discussed.

For the verification process, first, the results of a neutral ABL over flat terrain are discussed to calibrate the modeling parameters and define the mesh resolution sensitivity. Then, the results of a canonical case simulating the neutral ABL over a Gaussian ridge are presented to analyze the model’s response in presence of terrain-induced forcing, as presented in [7]. Based on this analysis, some recommendations are drawn for future work on multiscale models used in wind resource assessment.

2. Model equations and numerical enhancements

In general terms, the MC2 kernel solves the governing equations by separating the material derivatives applied on the prognostic variables (dΨ/dt) and linear terms (L, treated implicitly) from the non-linear terms (R, treated explicitly), external forcing and source terms (F), expressed in matrix form as

\[
\frac{d\Psi}{dt} + L = R + F.
\]

The semi-implicit semi-Lagrangian (SISL) discretization is applied on the first three terms of matrix system (1) to calculate the fluid particle’s trajectory over three time-levels and, then, the external forcing and source terms are added in a fractional-step procedure [5-6]. Thus, the fundamental improvement introduced in system (1) is the restructuration of the linear L and non-linear R terms, in order to remove the computational mode and terrain-induced noise in the new formulation. As explained in [6], the momentum and heat turbulent fluxes are included in the F terms after the governing equations are filtered. These turbulent fluxes need to be modeled using a particular SGS scheme in order to close the equation system.

To enhance the model’s numerical stability, an eigenmode analysis was applied on the former equation system of MC2 v4.9.8 [5], discretized with the original semi-implicit scheme (O-SI), which revealed that the temperature divergence contributes significantly to the origin of numerical noise [3]. The system includes the momentum, energy and continuity equations, respectively,

\[
\begin{align*}
\frac{d\mathbf{v}}{dt} & = \nabla \gamma \mathbf{k} \cdot \mathbf{P} + \mathbf{k} \cdot (b \mathbf{g} - \gamma \mathbf{S}) = -f \mathbf{k} \times \mathbf{v} - \frac{b}{g} (\nabla \gamma \mathbf{k}) \cdot \mathbf{P} + \mathbf{f}, \\
\frac{d}{dt} (b \mathbf{g} - \gamma \mathbf{S}) + \nabla \cdot \mathbf{w} = -b \left[ \beta \mathbf{w} + \frac{R}{c_r} \nabla \cdot \mathbf{v} \right] + \gamma \mathbf{Q}, \\
\frac{d}{dt} \left( \frac{P}{c_r^2} \right) + \nabla \cdot \mathbf{w} = \frac{Q}{c_r^2 T},
\end{align*}
\]

where \(d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla\) represents the material derivative, \(\mathbf{v} = (u, v, w)\) the velocity, \(\mathbf{k}\) is the vertical direction unit vector, \(f\) the Coriolis parameter, \(P = RT \ln(p')\) the generalized pressure, \(p' = p - p_0\) the pressure perturbation, \(b = g (T'/T)\) the buoyancy force, \(T' = T - T_*\) the temperature perturbation, \(\mathbf{f} = (f_x, f_y, f_z)\) the non-conservative forces and \(\mathbf{Q}\) heat sources. The main variables are complemented with the speed of sound \(c_r^2 = (c_p/c_v)^2 (RT)\) and the natural oscillation frequency \(N_2^2 = g \gamma_* = g (\beta_A + \gamma_A) = g^2 / c_p T_*\), two fundamental parameters among with the constants \(\beta_A = \partial \ln(T_*/c_\gamma)\) and \(\gamma_A = g / c_p T_*\).

By applying the new semi-implicit discretization (N-SI) scheme proposed by Flores-Maradiaga et al. [3], system (2) is reformulated with a numerically stable structure of the non-linear terms related to the buoyancy, as follows

\[
\begin{align*}
\frac{d\mathbf{v}}{dt} & = \nabla \gamma \mathbf{k} \cdot \mathbf{P} + \mathbf{k} \cdot (b \mathbf{g} - \gamma \mathbf{S}) = -f \mathbf{k} \times \mathbf{v} - \frac{b}{g} (\nabla \gamma \mathbf{k}) \cdot \mathbf{P} + \mathbf{f}, \\
\frac{d}{dt} (b \mathbf{g} - \gamma \mathbf{S}) + \nabla \cdot \mathbf{w} = -b \left[ \beta \mathbf{w} + \frac{R}{c_r} \nabla \cdot \mathbf{v} \right] + \gamma \mathbf{Q}, \\
\frac{d}{dt} \left( \frac{P}{c_r^2} \right) + \nabla \cdot \mathbf{w} = \frac{Q}{c_r^2 T},
\end{align*}
\]
\[
\frac{dv}{dt} + (\alpha + 1) \left( \nabla P + k \hat{b} \right) - (\gamma_s + \gamma_s) k P = -f k \times v + \frac{\beta_s}{g} k P + \Gamma,
\]
\[
\frac{d}{dt} \left[ (\alpha + 1) \hat{b} - \gamma_s P \right] + \frac{N_s^2 w}{\alpha + 1} = -\left( \frac{\alpha}{\alpha + 1} \right) N_s^2 w + \gamma_s Q,
\]
\[
\frac{d}{dt} \left( \frac{P}{c_s^2} \right) + \nabla \cdot v - \frac{g}{(\alpha + 1)c_s^2} w = \left( \frac{\alpha}{\alpha + 1} \right) \frac{g}{c_s^2} w + \frac{Q}{c_s^2} T,
\]
where the overbar \([\quad]\) denotes the implicit time averaging operator for terms solved over three time levels, \(\alpha = T'/T\) is the temperature perturbation ratio and \(\hat{b} = g(T'/T)\) is the new buoyancy definition. Equation system (3) constitutes the enhanced kernel of MC2 v4.9.9, which contains explicitly treated terms appropriately modified to recover the linearity in the hydrostatic relationship ensuring the model’s numerical stability in the presence of steep topography.

As most atmospheric models, MC2 employs a curvilinear terrain-following coordinate system defined in terms of a height-based monotonic transformation, such as the Gal-Chen height [8]

\[
Z(X, Y, z) = \left[ \frac{z - h(X, Y)}{H - h(X, Y)} \right] H,
\]

where \(z\) is the local Cartesian height, \(h(X, Y)\) is the topographic height and \(H\) is the height of model rigid lid. Hence, the model’s kernel (3) is transformed with the metric tensor transformation based on equation 4. A comprehensive explanation of this procedure is provided in [5].

Based on the Boussinesq hypothesis and using Einstein’s notation, the turbulent stresses and heat fluxes can be expressed in terms of eddy- and heat-mixing coefficients \((\mu = \rho K_M; \Pr = \frac{K_m}{K_T})\), the resolved strain rate \(S_{ij} = \frac{1}{2} \left[ \hat{\varepsilon}_{ij} / \hat{\varepsilon}_{ij} + \hat{\varepsilon}_{ij} / \hat{\varepsilon}_{ij} \right]\), the sub-filter turbulent kinetic energy \(\tilde{\rho}k = \frac{1}{2} \rho \tilde{u}_i \tilde{u}_i\) and the potential temperature gradient, such that

\[
-\rho \tilde{x}_i \tilde{x}_j = \tau_{ij} = 2\mu S_{ij} - 2/3(\mu S_{ij} - \tilde{\rho}k) \quad \text{and} \quad -\rho \tilde{u}_i \theta' = \mu / \Pr \hat{\theta} / \hat{\varepsilon}_{ij}\].

Here the tilde represents the application of the implicit filter when the solution is projected onto the numerical grid. Even though MC2 solves the compressible equations, for low Mach-number atmospheric flows the incompressible assumption can be enforced. Thus, the volumetric part of the Reynolds stress tensor \(\tau_{ij}\) (i.e., \(2/3(\mu S_{ij} - \tilde{\rho}k)\)) is added to the pressure and solved directly.

The computation on terrain-conforming grids of the deviatoric part of the Reynolds tensor (i.e., \(2\mu S_{ij}\) included among the turbulent diffusion terms \(F_{turb}\)) requires its transformation based on equation 4. The resulting horizontal tendencies \(F_{turb}^H\), affected by complex terrain, become

\[
F_{turb}^H = \begin{bmatrix}
F_{turb-x}^H \\
F_{turb-y}^H \\
F_{turb-z}^H \\
0
\end{bmatrix} = \begin{bmatrix}
\rho \left[ \frac{\partial A_{\tilde{u}_i}}{\partial X} + \frac{\partial A_{\tilde{u}_i}}{\partial Y} + \frac{1}{\partial Z} \right] \\
\rho \left[ \frac{\partial A_{\tilde{u}_i}}{\partial X} + \frac{\partial A_{\tilde{u}_i}}{\partial Y} + \frac{1}{\partial Z} \right] \\
\rho \left[ \frac{\partial A_{\tilde{u}_i}}{\partial X} + \frac{\partial A_{\tilde{u}_i}}{\partial Y} + \frac{1}{\partial Z} \right] \\
\rho \left[ \frac{\partial A_{\tilde{u}_i}}{\partial X} + \frac{\partial A_{\tilde{u}_i}}{\partial Y} + \frac{1}{\partial Z} \right]
\end{bmatrix},
\]

where
where \( G_0 \equiv \partial z / \partial Z \), \( G_1 \equiv \partial z / \partial X \) and \( G_2 \equiv \partial z / \partial Y \) are the metric coefficients and \( \pi = T/\theta \) is the dry-air Exner function. The turbulent fluxes \((A_g, B_g)\) are then related with the eddy-mixing coefficients and the map scale factor \((S = m^2)\) in a gradient form such as, for example,

\[
A_{uv} = \rho K_M S \left[ \frac{\partial \tilde{v}}{\partial X} + \frac{G_1}{G_0} \frac{\partial \tilde{v}}{\partial Z} + \frac{G_2}{G_0} \frac{\partial \tilde{v}}{\partial Y} \right],
\]

\[
B_u = \rho K_M \left[ \frac{\partial \tilde{v}}{\partial X} \right].
\]

As mentioned in the introduction, the classical Smagorinsky (SMAG) [9] and the Deardorff (TKE) [10] SGS schemes have been implemented in MC2 as constant coefficient parametrizations with stability functions based on the Richardson’s number \((Ri = N^2/S^2)\) with \(N^2 = (g/\theta)(\partial \theta / \partial z)\) as the natural frequency of the air parcel’s oscillation. However, only the SMAG will be presented since both models yield very close results. The SMAG closure is formulated with [6]

\[
K_M = \lambda f_m S, \quad K_r = \lambda f_h S,
\]

\[
\lambda = \left[ \frac{1}{C_s \Delta + \kappa |z + z_0|^2} \right]^{1/2},
\]

\[
f_m = \begin{cases} 
(1-16Ri)^{1/2}, & 0 < Ri \\
(1-Ri/Ri_c)^4, & 0 \leq Ri \leq Ri_c
\end{cases}
\]

\[
f_h = \begin{cases} 
(1-40Ri)^{1/2}, & 0 < Ri \\
Pr_l \left(1-Ri/Ri_c\right)^4, & 0 \leq Ri \leq Ri_c
\end{cases}
\]

where \( \lambda \) is the characteristic length scale, \( f_m \) and \( f_h \) are the stability functions for momentum and heat transport, respectively, \( S = 2(S_y - \delta_y/2S_y) \) is the corresponding strain rate tensor modulus for compressible flow simulations [11-12], \( C_s \) is the Smagorinsky coefficient, \( \Delta = \left(\Delta_x \Delta_y \Delta_z\right)^{1/3} \) is the filter length scale, \( \kappa = 0.4 \) is the von Kármán constant, \( z_0 \) is the aerodynamic roughness length, \( Ri_c \) is the critical Richardson number. Although the shortcomings of the constant coefficient Smagorinsky-based scheme are well known [13-16], it still constitutes an important and necessary step towards better wind modeling. A comprehensive explanation on the SGS scheme details and implementation for MC2 is provided by [6].

3. Validation of the enhanced multiscale method

To verify the quality of the results and parametric adjustments for multiscale modeling, firstly, a neutral ABL is simulated over flat terrain to reproduce the canonical Ekman layer test, discussed by several researchers with diverse LES approaches [14-17]. Once a smooth transition between the previous and upgraded versions of MC2 is achieved we proceed to test the model with a neutral ABL over a Gaussian transverse ridge and compare with the results in [7]. For both tests, the flow is driven by a large scale
pressure gradient with a Coriolis parameter of \( f = 10^{-4} \text{s}^{-1} \) to maintain a balance with a geostrophic wind of \( (U_g, V_g) = (10.0, 0.0) \text{m s}^{-1} \).

Following the common practice for mesoscale models, these simulations are initialized with a wind sounding based on the wind speed, wind direction, velocity gradient and thermal stratification provided by a global atmospheric model. Also, the analytical Ekman layer velocity profile is initially perturbed with random fluctuations ranging between \( \pm 0.1 \text{m s}^{-1} \) to generate sufficient instabilities for a fully developed turbulent flow field. The simulations are carried out during \( tf = 30 \) time cycles (equivalent to \( 300,000 \) s with \( \Delta t = 4 \text{s} \)) on a \( 4032 \text{m} \times 2016 \text{m} \times 1008 \text{m} \) C-type grid (depicted in Figure 1) with spatial resolution of \( \Delta x = \Delta y = 32 \text{m} \) and \( \Delta z = 4 \text{m} \).

The Arakawa’s C-type staggered grid is usually employed in atmospheric models because it eases the calculations of the fluid particle’s trajectory and velocity through the grid cells [18], which is particularly useful for semi-Lagrangian solvers, as in MC2 and other models. Depending on the transport phenomenon calculation, the horizontal velocity components and/or the pressure are taken at the cell faces and center points (on the “momentum levels”) to calculate their respective gradients. The vertical velocity, temperature and buoyancy, as well as the TKE, are located at the center of the lower and upper cell faces (also called “thermodynamic levels”). The mixing coefficients, map scale factors and Coriolis parameter are stored at the corners.

These tests are performed with periodic boundary conditions on the lateral sections, a wave-damping condition for the upper layer (c.f., the Shuman’s boundary condition [19], equation 5) and, to account for rough terrain, the surface momentum fluxes are given by a logarithmic drag law at each grid point as

\[
\overline{uw} = C_d \frac{1}{k} \ln \left( \frac{z + z_0}{z_0} \right)^2, \tag{9}
\]

Here \( \langle \rangle \) represents the averaging operation on each model level, and the constant drag coefficients are computed at the height of the first momentum level (i.e., \( z = \Delta z/2 \)) with a fixed roughness length of \( z_0 = 0.1 \text{m} \) over the whole domain surface.
3.1 Ekman layer flow over flat terrain

In this section we will discuss the first and second order statistics obtained for the classical Ekman layer, and compare with those reported in the literature. Based on these preliminary results we will proceed to calibrate the model to ensure quality simulations over topography. One of the fundamental parameters for turbulence modeling is the Smagorinsky coefficient \( c_s \), which will be tested in a range of 0.125 to 0.225 to define a balance between energy dissipation and numerical noise control.

![Figure 2. Mean streamwise and spanwise velocity components for the Ekman layer test.](image)

![Figure 3. Resolved and sub-grid streamwise momentum fluxes for the Ekman layer test.](image)

As it can be observed in Figure 2, over flat terrain the mean values for the streamwise \( U \) and spanwise velocity \( V \) components have the expected behavior and compare well with the literature [14, 16]. Since this ABL flows predominantly in the streamwise direction, mainly the \( U \) component balances the large scale pressure gradient in the entrainment zone. On the other hand, the transverse velocity displays the rotational effect of the Coriolis force on the air parcels. However, still no clear distinction can be drawn for the best suited \( c_s \) value.

In Figure 3, both the resolved and sub-grid scale momentum fluxes reveal that smaller values of \( c_s \) tend to solve more flow structures and rely less on the turbulence modeling. One can also infer that higher values of \( c_s \) enhance the numerical dissipation near the surface, which is one of the main reasons for the overshoot of the non-dimensional gradient of the mean streamwise velocity \( \Phi_M = (\kappa z / u_*) \partial U / \partial z \). The \( \Phi_M \) overshoot is presented on Figure 4, where one can relate its magnitude reduction with higher values of the sub-grid parametric constant \( c_s \). However, since the influence of the \( \Phi_M \) overshoot must be kept the closest possible to the surface (i.e., within the surface layer \( z/H < 0.1 \)) and its magnitude should approximate a unit [14-17], it was determined that an appropriate value for subsequent simulations is \( c_s = 0.175 \).
In Figure 5 the influence of the grid aspect ratio \( AR = \Delta x / \Delta z \) is examined for coarse \( (\Delta x = 36 \text{ m}, 72 \text{ m}, 144 \text{ m} \text{ and } \Delta z = 9 \text{ m}) \) and fine \( (\Delta x = 16 \text{ m}, 32 \text{ m}, 64 \text{ m} \text{ and } \Delta z = 4 \text{ m}) \) grid resolutions. With a ratio of \( AR = 16 \) or greater the \( \Phi_M \) overshoot increases near the wall, which indicates there is a spurious energy backscatter [16-17]. Unfortunately, when the horizontal mesh size is refined yielding \( AR = 4 \) or less the \( \Phi_M \) overshoot expands upwards exceeding the surface layer height, a condition which has been proven to generate spurious energy dissipation [17]. Therefore, a balanced option with \( AR = 8 \) seems appropriate for the following simulations.

Also in Figure 5 one can compare the effect of the grid resolution in the model’s response, taking \( \Phi_M \) as an analysis parameter. Clearly the magnitude of the velocity gradient’s overshoot is very close between both coarse and fine resolutions. Consequently, the velocity profiles can be expected to be similar and proportional by a scale factor. However, as Brasseur and Wei [17] recommend, it is convenient to keep the \( \Phi_M \) overshoot as close to the ground as possible to reduce the numerically induced friction. Thus, a fine grid of \( \Delta x = \Delta y = 32 \text{ m} \text{ and } \Delta z = 4 \text{ m} \) seems to be the best choice in this modeling conditions.

The Ekman layer case is repeated using the two semi-implicit time discretization methods, presented in section 2 (i.e., O-SI and N-SI), to assess their influence on the ABL turbulence modeling. Energy spectra in Figure 6 show very similar behavior between both, only with slightly less dissipation for N-SI on the higher wavenumbers that correspond to microscale energy-dissipating structures near the surface. On a similar manner, Figure 7 illustrates how close the velocity variances are with different model combinations, a behavior also seen for the turbulent kinetic energy and momentum fluxes. Evidently, the turbulent flow properties are conserved with the new MC2-LES implementation, which indicates that an accurate solution of the canonical Ekman layer is achieved as compared to the literature [14-17].
3.2 Wind simulations over complex terrain

A transverse Gaussian ridge is chosen as the topographic obstacle, inspired in the tests presented in [7]. A simple expression of this topographic profile, taking $h_m$ as the maximum terrain height and $a/2$ as the mountain half-length (c.f. Figure 8), is given by

$$h(x) = h_m \exp \left[ -\left( \frac{x}{a/2} \right)^2 \right].$$

Thus, a neutral ABL simulation over a transverse Gaussian ridge, with $h_m = 50$ m, $a = 512$ m and maximum slope of 0.2 ($\theta \approx 11.3^\circ$), is performed with the same lateral, top and bottom boundary conditions. Also, the same large-scale geostrophic forcing is imposed to drive the flow. Near the ground the wall stress corresponds to the prescribed drag law of equation 9, fixing $z_0 = 0.1$ m for the whole domain surface. The results are going to be compared at the same positions with Kirkil et al. [7] at the hill top (apex), lee-side and downstream valley, as shown in Figure 9.

As Allen and Brown [20] point out, for laminar boundary layers the flow displacement near the hill causes a pressure field alteration that in turn drives the flow in a recirculating loop. On the contrary, for turbulent boundary layers the separation point and recirculation bubble is a complex singularity which cannot be solely explained in terms of the terrain-induced displacement. The ABL separation not necessarily occurs at the point of zero wall stress and/or may not be located at the surface. Turbulent boundary layer experiments over rough hills have proven that in the recirculation zone the velocity profiles do not follow the log law and, thus, the SGS modeling is a real challenge [20-21].
On Figure 10 it is clearly seen that the interaction of neutrally stratified wind with a moderately steep mountain ridge causes, first, a flow acceleration at the apex, then, a deceleration and/or flow inversion in the lee-side due to the adverse pressure gradient and, finally, a mild acceleration with the progressive reestablishment of the velocity profile. Sometimes, a drastic change from a favorable to an unfavorable pressure gradient causes flow reversal, separation and recirculation. Usually, in the leeward valley the separation bubble reattaches allowing the wind to recover its momentum.

However, the flow reversal and separation phenomenon is not evident for this turbulent ABL presumably because the ridge is not steep enough. In spite of this, the results compare well with those reported by Kirkil et al. [7] (represented with K12 in Figure 10), who used both the standard SMAG SGS model and the dynamic reconstruction model (DRM). As declared and proven in [7], the DRM is more sophisticated but computationally demanding, and mainly devised to predict the TKE production and inertial range scaling of the power spectra. Hence, with MC2-LES similar results can be achieved with lower computational effort, ensuring numerical stability and accuracy with the N-SI scheme.

In Figure 11 the resolved and SGS momentum fluxes are compared for both SI schemes on the same positions as before (c.f. Figure 9). It is noted that at the summit (Figure 11-a), where the flow is naturally accelerated, the combination of the N-SI and SMAG scheme resolves better the turbulent fluxes since the resolved part is closer to the surface. Yet the cross points between the resolved and sub-filter fluxes occur almost at the same heights, with the N-SI scheme the model is able to perform stably without smoothing filters and it resolves small-scale structures with less dependence on the SGS parametrization.

Lastly, in figure 12 we present the instantaneous streamwise velocity component obtained with both model versions at 10 m above the ground. Although similar, certain qualitative differences can be recognized between these surface layers as a result of the MC2-LES enhancements. Both model versions yield the expected wind streaks generated by the rotational Coriolis effect, as seen in Kirkil et al. [7], but only with the N-SI scheme the flow pattern is properly confined to the area where the wind interacts with the topographic slope. Namely, with O-SI scheme the wind expands outside the expected influence zone, showing some spurious acceleration in the upward slope.

It was also noticed that the N-SI scheme maintains the model numerically stable for long term simulations and yields similar lee-side flow patterns seen in several experimental campaigns [1]. With the O-SI scheme a fractional step smoothing is always necessary to filter the computational mode in order to maintain the model numerically stable, and it usually overestimates the hill-top winds due to spurious acceleration in the upwind slope.
Additional to the improved time discretization scheme and the LES adaptation to terrain-following grids, the proposed method provides:

- A wind initialization routine that accounts for atmospheric thermal stratification,
- A novel energy-conserving Robert-Asselin-Williams [22] frequency filter used for cases with very steep mountain slopes,
- A routine for uncoupling the temperature and pressure in order to solve for the thermodynamic variables without affecting the momentum calculations in neutral cases,
- An alternative surface stress calculation for complex terrain that accounts for the vertical velocity component on terrain-conforming grids, and
- Short analytical test routines for validation of mountain wave and turbulence diffusion calculations.

4. Conclusions
The statistical analysis of the ABL simulations in terms of the vertical profiles of the mean velocity components, non-dimensional gradient of the mean streamwise velocity, energy spectra, momentum fluxes and velocity variances indicates that over flat terrain there is good agreement with the similarity solution, but no significant improvements are achieved with the proposed multi-scale method. However, in presence of mountainous terrain it effectively reduced the numerical errors and instability compared to the former model version.

With the model enhancements the neutral ABL past a Gaussian ridge generates the expected acceleration at the hill top, reproduces well the flow patterns on the downwind lee-side and valley, solves better the small structures close to the surface and keeps the numerical stability for long-term integrations. The proposed enhancements reduce the need of excessive mesh refinement and, as compared to more sophisticated but computationally demanding methods, the proposed approach performs efficiently.

Although modest these improvements are a milestone for the wind engineering community and could be considered an important step towards more accurate wind resource assessment over complex topography. In future publications this method will be compared to dynamic models, and real experimental data will be used for further validation.

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