States localized on a boundary of the time-dependent parity-breaking medium

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Abstract

We consider the massive vector field propagating in the inhomogeneous parity-breaking medium, such as the dense hot hadronic matter with chiral imbalance. The transition between the regions with approximately constant values of the parity-breaking parameter allows for the states localized on such boundary to occur. The adiabatic change of the background introduces either decay or the amplification of the localized states. We also discuss the non-adiabatic destruction of these bound states.

1 Introduction

It is a well-known feature of the Standard model all the parities $\mathcal{P}$, $\mathcal{C}$, $\mathcal{T}$ and $\mathcal{CP}$ are non-conserved [1]. Many models of the new physics predict new sources of the $\mathcal{CP}$ violation. This may affect the physics at the energies much lower than the masses of the new particles through the radiative loop corrections. Thus, the searches of the $\mathcal{CP}$-violating effects in the collider, atomic and molecular experiments [2–8] constitute an important strategy for the exploration of the physics beyond the Standard model. One of the ways the $\mathcal{CP}$-symmetry may be violated is through the $\theta$-term of the gluon field,

$$S = -\frac{\theta}{16\pi^2} \int d^4x \text{Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}],$$

$$\tilde{G}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}$$

(1)
When $\theta$ is constant, this term happens to be a topological charge. Thus, it does not appear in the perturbative processes, however it changes the nonperturbative dynamics. One of the effects is the emergence of the electric dipole moment (EDM) for the neutron [9]. Its measurement puts very strong constraints on the value of $\theta$ which constitutes a so-called strong $\mathcal{CP}$-problem [10,11].

However, even if fundamentally $\theta$ is zero, the strong interaction dynamics may admit the local $\mathcal{P}$ and $\mathcal{CP}$-violation [12–16]. Such an effect may occur when the large fluctuations of the gluon field produce metastable configurations with nonzero value of the axial topological charge that is tied through the anomaly to the quark axial charge. The emergence of the parity-violating regions may occur within the fireballs of very hot and very dense hadronic medium approaching a phase transition curve of the QCD phase diagram. Such conditions may be achieved in the heavy-ion collisions at NICA, STAR, PHOENIX, SPS and CBM experiments. The manifestations of the QCD local parity violation include the chiral magnetic effect in the peripheral collisions [17–24] and the excessive production of the dileptons in the central collisions [25–29]. The chiral imbalance environment may also produce exotic decays of mesons [30,31].

The presence of the chiral imbalance background affects the propagation and interaction of the effective photon and meson particles. The finite temperature and axial chemical potential as well as the local nature of the fireball result in the Lorentz symmetry breaking. This is associated with the local violation of the $\mathcal{CPT}$ invariance.

The fireball of chiral matter exists only for a short time. After the cooldown of the medium and disappearance of the local chiral imbalance, the effective vacuum state transforms into a squeezed state of ordinary photons and mesons [32].

Similar situation may occur on different scales if the strong $\mathcal{CP}$-problem is solved with help of the axion particle. In that scenario, the constant $\theta$ is promoted to the condensate of the pseudoscalar axion field. The axion field must also have similar $\mathcal{CP}$-violating interaction with the electromagnetic field. Local variations of the condensate of the axion particles may play the role of the Dark matter. In the vicinity of the very dense stars the condensate may be especially high [33]. The passage of the photons through the spatial boundary of such an axion lump was investigated in [32]. Those results also give a rough idea on the similar passage of the vector mesons through the spatial boundary of the static hadronic medium.

However, the aforementioned papers consider only the very edge of the transition region where $\theta$-parameter behaves linearly. This may be valid only for a very high energy particles that have short wavelengths, and, thus, insensitive to the large scale behavior. In contrast, particles with lower energies should be sensitive to the entire transition area. As will be demonstrated in this paper, the transition between two plateau regions may contain the localized states of the massive vector field. While the model we considered is simplified, this implies that the fireball of the chiral hadronic matter possesses the boundary currents. We also consider the influence of the temporal evolution of the medium which results in the amplification or the decay of the boundary states.
2 Vector field in the parity breaking medium

As in [32] we start with the massive electrodynamics model with local parity-violating term,

\[ S = \int d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \theta(x) F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{2} m^2 A^\mu A_\mu + A^\mu \partial_\mu B + \frac{1}{2} B^2 \right) \]  

(2)

This model is related to the vector dominance model in the presence of the chiral imbalance as described in [34]. The auxiliary field \( B \) is introduced to employ the St"uckelberg mechanism for self-consistency of the massive vector field theory [35,36]. The equations of motion for this field yield,

\[ (\Box + m^2) B = 0, \quad \partial_\mu A^\mu = B \]  

(3)

where \( \Box = \partial_\mu \partial^\mu \). From that we can derive the condition,

\[ (\Box + m^2)(\partial_\mu A^\mu) = 0, \]  

(4)

and omit \( B \) from our consideration. The equations of motion for \( A^\mu \) take the form,

\[ (\Box + m^2) A^\mu = \epsilon^{\mu\alpha\beta}(\partial_\alpha \theta)(\partial_\beta A^\mu) \]  

(5)

We will assume that the boundary thickness of the chiral breaking matter region is much smaller than its size. Then we can approximate \( \theta \) by a function only of \( t \) and \( z \). In the components the equations take the form,

\[ (\Box + m^2) A^t = - (\partial_z \theta) \left[ \partial_x A^y - \partial_y A^x \right], \]  

(6)

\[ (\Box + m^2) A^z = (\partial_t \theta) \left[ \partial_x A^y - \partial_y A^x \right], \]  

(7)

\[ (\Box + m^2) A^x = (\partial_t \theta) \left[ \partial_y A^z - \partial_z A^y \right] + (\partial_z \theta) \left[ \partial_t A^y + \partial_y A^x \right], \]  

(8)

\[ (\Box + m^2) A^y = - (\partial_t \theta) \left[ \partial_x A^z - \partial_z A^x \right] - (\partial_z \theta) \left[ \partial_t A^x + \partial_x A^t \right]. \]  

(9)

As the translational invariance in \( x \) and \( y \) directions is not broken it is useful to rewrite these equations in the momentum representation,

\[ A^\mu(t,x,y,z) = \int d^2k A^\mu(t,z) \exp(ik_xx + ik_yy), \]  

(10)

It is also convenient to use the circular components,

\[ A_\pm = A^x \pm iA^y, \quad k_\pm = k_x \pm ik_y \]  

(11)
Then the equations take the form,

\[(\Box + m^2)A^t = -\frac{1}{2}(\partial_z\theta)\left[k_-A_+ - k_+A_\right],\]  
(12)

\[(\Box + m^2)A^z = \frac{1}{2}(\partial_t\theta)\left[k_-A_+ - k_+A_\right],\]  
(13)

\[(\Box + m^2)A_\pm = \pm k_\pm \left[(\partial_t\theta)A^z + (\partial_z\theta)A^t\right]
\pm i\left[(\partial_\theta)\partial_z A_\pm - (\partial_z\theta)\partial_\theta A_\pm\right],\]  
(14)

For the simplicity we will consider the case of the momentum directed perpendicularly to the boundary i.e. $k_\pm = 0$, assuming that the results may serve as an approximation for sufficiently low momenta in the $x$ and $y$ directions. Then the $A_\pm$ decouple both from the equations on $A^t$ and $A^z$, and from the condition (4).

The $(t, z)$ sector does not depend on $\theta$ and corresponds to the longitudinal polarization of the massive photon in the empty space. In contrast the equation for the transverse polarizations looks like,

\[(\partial^2_t - \partial^2_z + m^2)A_\pm = \pm i\left[(\partial_\theta)\partial_z A_\pm - (\partial_z\theta)\partial_\theta A_\pm\right],\]  
(15)

As $A_-$ satisfies the same equation as $A_+$ but for the reversed time, we will consider only $A_+$ from now on.

3 Adiabatically changing parity-violating background

First, we will consider the equation (15) in the adiabatic regime, when $\theta$ is changing in time slowly compared to the frequency of the wave whereas its spatial variation is sufficiently high compared to the wavelength. Then we will assume that the solution behaves as,

\[A_+ = a(t, z) \exp\left(-i \int dt \omega(t)\right),\]  
(16)

where the profile function $a$ and the frequency $\omega(t)$ are slowly varying at the same rate as $\theta$.

Then in the leading approximation the $i(\partial_\theta)$ term may be neglected. We will denote the solutions in this approximation with subscript 0,

\[a(t, z) \simeq a_0(t, z), \quad \omega(t) \simeq \omega_0(t)\]  
(17)

and the equation becomes,

\[-\partial^2_z a_0 + \left[m^2 - \omega_0^2 + \omega_0(\partial_z \theta)\right]a_0 = 0,\]  
(18)

which looks like a Schrödinger equation though with the potential proportional to the spectral parameter $\omega_0$.

Consider the boundary between regions of approximately homogeneous $\theta$. Let us consider the following shape of $\theta$,

\[\theta(t, z) = \bar{\theta}(t) + \Delta\theta(t) \cdot \tanh\left[\mu(t)\left(z - z_0(t)\right)\right],\]  
\[\theta \xrightarrow{z \to -\infty} \bar{\theta} - \Delta\theta, \quad \theta \xrightarrow{z \to +\infty} \bar{\theta} + \Delta\theta,\]  
(19)
where all the functions only slowly depend on time. For this shape of \( \theta \) the potential becomes the Pöschl-Teller one \[37\],

\[
- \partial_z^2 + m^2 - \omega_0^2 + \frac{\omega_0 \mu \cdot \Delta \theta}{\cosh^2 \mu (z - z_0)} \]  

\[a_0 = 0, \quad (20)\]

As at large \( z \) the potential becomes constant \(-\varepsilon \equiv -(\omega_0^2 - m^2)\) the potential has the continuous spectrum of the wavelike solutions for \( \varepsilon > 0 \) i.e. \( |\omega_0| > m \).

If \( \varepsilon < 0 \) i.e. in the band \( |\omega_0| < m \) the bound states may exist. This is indeed true for the case of \( \omega \mu \cdot \Delta \theta < 0 \) when the last term gives a potential well. Then, as well-known for the Pöschl-Teller potential the bound states correspond to,

\[
\varepsilon_n = -\mu^2 (\lambda - n - 1)^2, \quad \lambda (\lambda - 1) = \left| \frac{\omega_0 \Delta \theta}{\mu} \right| \quad (21)
\]

where \( n \) is integer from 0 to the largest integer \( n_{\text{max}} \leq \lambda - 1 \). As \( \varepsilon = \omega_0^2 - m^2 \) this turns into equation on \( \lambda \),

\[
\lambda^4 - 2\lambda^3 \left( 1 + \Delta \theta^2 \right) + 2(n + 1)\Delta \theta^2 \lambda^2 + \Delta \theta^2 \left( (n + 1)^2 - \frac{m^2}{\mu^2} \right) = 0 \quad (22)
\]

When this equation has a real solution \( \lambda > 1 \) the bound state exists. From the discussion above it is evident that \( n_{\text{max}} (n_{\text{max}} + 1) \leq |m \Delta \theta / \mu| \).

The normalized \( n = 0 \) bound state takes the form,

\[
a_{0,n=0} = \frac{C}{\cosh^{\lambda_0 - 1} \mu (z - z_0)}, \quad C^2 = \frac{\mu}{\sqrt{\pi}} \frac{\Gamma \left( \frac{1}{2} \right)}{\Gamma (\lambda_0 - 1)}. \quad (23)
\]

The examples of the spectrum for \( (19) \) are presented on Fig. 1 and Fig. 2. Only solutions with \( \omega_0 < 0 \) are included, as otherwise the potential introduces a barrier instead of the well and no bound state exist. One can see that the levels appear with growth of \( \Delta \theta \). The phenomena of emergence or disappearance of the levels must lead to the breakdown of the adiabatic approximation and will be explored below.

### 4 Correction due to the \( \partial_t \theta \) term

At small times we may approximate the influence of the \( \partial_t \theta(t, z) \equiv v(t, z) \) term with the adiabatically changing perturbation that changes the frequencies and profile functions,

\[
\omega = \omega_0 + \omega_1, \quad a = a_0 + a_1, \quad \omega_1/\omega_0, \quad a_1 \sim |v|/\omega_0 \sim \mu_5/m \quad (24)
\]
The next order equation takes the form,

$$\left[\omega_0^2 - \partial_z^2 + m^2 + \omega (\partial_z \theta)\right]a_1 = iv(\partial_z a_0)$$

$$-2\omega_0 \omega_1 a_0 - \omega_1 (\partial_z \theta) a_0$$  \hspace{1cm} (25)

Let $a$ be one of the localized bound states we found in the preceding section. For simplicity we will choose it to be real (which can always be done in this dimensionality by the appropriate phase transformation). If we multiply this equation on $a_0^*$ and integrate over $z$, l.h.s. vanishes thanks to the Hermiticity of the leading order operator and the equation (18). From that we derive,

$$\omega_1 = i \frac{\int_{-\infty}^{+\infty} dz \nu a_0 \partial_z a_0}{\int_{-\infty}^{+\infty} dz (2\omega_0 + \partial_z \theta) a_0^2}$$

$$= -i \frac{\int_{-\infty}^{+\infty} dz (\partial_z v) a_0^2}{2 \int_{-\infty}^{+\infty} dz (2\omega_0 + \partial_z \theta) a_0^2}$$  \hspace{1cm} (26)

From this it is evident that $\omega_1$ is purely imaginary. This should not be a surprise as the perturbation is non-Hermitian,

$$(iv\partial_z)^\dagger = i\partial_z v = i(\partial_z v) + iv\partial_z$$  \hspace{1cm} (27)

Therefore the $\partial_t \theta$ term leads to the decay or the amplification of the bound states that were stable in the leading approximation.

Suppose for simplicity that in (19) only $\bar{\theta}$ and $\Delta \theta$ depend on time. Then for the level $n = 0$ we get from (23),

$$\omega_{1,n=0} = -i \lambda^{-1} \frac{\mu(\partial_t \Delta \theta)}{2\omega_0 + \lambda^{-1} \mu \Delta \theta}$$  \hspace{1cm} (28)
5 Schrödinger form of the evolution equation

For various purposes we would like to rewrite our problem as an ordinary Schrödinger equation. To do so we introduce,

\[
\Psi = \begin{pmatrix} A_+ \\ i\dot{A}_+ \end{pmatrix}
\]  

then the equation (15) takes the form,

\[
i\partial_t \Psi = \mathcal{H}\Psi,
\]

(30)

The time-dependent and non-Hermitian Hamiltonian is written as,

\[
\mathcal{H} = \begin{pmatrix} 0 & 1 \\ -\partial_z^2 + m^2 - i(\partial_t\theta)\partial_z & (\partial_z\theta) \end{pmatrix},
\]

(31)

and satisfies the time-dependent pseudo-Hermiticity relation [38],

\[
\rho \mathcal{H} = \mathcal{H}^\dagger \rho - i(\partial_t\rho),
\]

(32)

where the matrix \( \rho \) is given by,

\[
\rho = \begin{pmatrix} -(\partial_z\theta) & 1 \\ 1 & 0 \end{pmatrix}
\]

(33)

Therefore we may construct the inner product,

\[
\left( \Psi_1, \Psi_2 \right) = \int_{-\infty}^{+\infty} dz \, \Psi_1^\dagger \rho \Psi_2
\]

\[
= \int_{-\infty}^{+\infty} dz \left( iA_{1,+}^*A_{2,+} - iA_{1,+}^*A_{2,+} - (\partial_z\theta)A_{1,+}^*A_{2,+} \right),
\]

(34)
that is conserved by the Hamiltonian $\mathcal{H}$ despite its non-Hermiticity. Obviously this is a generalization of the ordinary Klein-Gordon inner product and is not positive definite. However, we may restrict our consideration to the positive-norm sector of solutions. On that subspace the equation (15) corresponds to the unitary evolution. The Hamiltonian can be related to the Hermitian Hamiltonian,

$$h = \eta \mathcal{H} \eta^{-1} + i(\partial_t \eta) \eta^{-1},$$

where we introduced (restricted to the positive norm subspace),

$$\rho = \eta \dagger \eta,$$

The fact that $\mathcal{H}$ is not similar to $h$ in case of time-dependent $\rho$ means that the spectra of these two operators are not equivalent. Hence, the eigenvalues of $\mathcal{H}$ (that are frequencies $\omega$ considered above) are not guaranteed to be real, and its eigenvectors,

$$\mathcal{H} \Psi_n = \omega_n \Psi_n,$$

with different $\omega$ are not orthogonal even with respect to the inner product (34),

$$\left(\Psi_m, \Psi_n\right) = \frac{i}{\omega_m - \omega_n} \int_{-\infty}^{+\infty} dz \Psi_m^\dagger (\partial_t \rho) \Psi_n,$$

which is obtained from the relation (32).

6 Adiabatic regime breakdown

Using the results from the previous section we may determine the moment of the adiabatic regime breakdown following similar steps to the ordinary Schrödinger case. At every moment we decompose the solution into a superposition of the eigenvectors of $\mathcal{H}$ which will for now consider to be a discrete set,

$$\Psi = \sum_n c_n(t) \Psi_n e^{-i \int dt \omega_n}$$

Where we assume that each $\Psi_n$ has unit norm with respect to (34). Then the equation (30) results in,

$$\sum_n \left( \dot{c}_n(t) \Psi_n + c_n(t) \dot{\Psi}_n \right) \exp \left( -i \int dt \omega_n \right) = 0$$

where $\dot{\Psi}_n$ is not an evolution described by (30) but a change of $\mathcal{H}$ eigenbasis with time. We are interested when the terms mixing different $c_n$ become significant. For that we multiply this equation on $\Psi_m$ using the inner product (34),

$$\dot{c}_n = - \sum_{m \neq n} N_{mn} \exp \left( i \int dt (\omega_m^* - \omega_n) \right)$$

We are interested in the mixing terms,

$$N_{nm} = \dot{c}_n \left( \Psi_m, \Psi_n \right) + c_n \left( \Psi_m, \dot{\Psi}_n \right)$$
In the adiabatic regime \( \dot{c}_n \simeq 0 \) so we will neglect the first term. Taking a time derivative of (37) and using (38) we get,

\[
N_{nm} \simeq c_n \left[ \frac{i\omega}{(\omega_m^* - \omega_n)^2} \int_{-\infty}^{+\infty} dz \Psi_m^\dagger(\partial_t \rho) \Psi_n \\
- \frac{1}{\omega_m^* - \omega_n} \int_{-\infty}^{+\infty} dz \Psi_m^\dagger(\partial_t \mathcal{H}) \Psi_n \right]
\]  

(43)

As usual, the breakdown of the adiabatic approximation should happen when levels become too close to each other. In other case this happens when \( \Delta \theta \) drops sufficiently for the level to approach the continuum spectrum near the point \( |\omega| = m \) when the level ceases to exist as can be seen on the Figs. 1 and 2. As can be seen from (21) for the \( n \)-th level that corresponds to,

\[
\Delta \theta \simeq \Delta \theta_n = n(n+1) \frac{\mu}{m}
\]

(44)

Let us assume that,

\[
\Delta \theta = \Delta \theta_n + \vartheta, \quad \vartheta = -2 \mu_5 t
\]

(45)

where \( \mu_5 \) is the axial chemical potential inside the fireball. Let us define \( |\omega| = m - \delta \) and assume that both \( \delta \) and \( \vartheta \) are small. Then from (21) we get,

\[
\delta \simeq \frac{1}{2} \frac{m}{(2n+1)^2} \vartheta^2
\]

(46)

Which results in the following estimates,

\[
\left| \frac{i\omega}{(\omega_m^* - \omega_n)^2} \int_{-\infty}^{+\infty} dz \Psi_m^\dagger(\partial_t \rho) \Psi_n \right| \sim 16 \frac{\mu^2_5 (2n+1)^2}{m \vartheta^2},
\]

(47)

\[
\left| \frac{1}{\omega_m^* - \omega_n} \int_{-\infty}^{+\infty} dz \Psi_m^\dagger \rho(\partial_t \mathcal{H}) \Psi_n \right| \sim \frac{4 \mu_5 (2n+1)^2}{\vartheta^2}
\]

(48)

To break the adiabatic regime one of these contributions must be comparable to the energy i.e. \( \sim m \). This gives,

\[
\vartheta_{n.a.} \sim \max \left[ \left( \frac{4 \mu_5}{m} (2n+1) \right)^{2/3}, (2n+1) \sqrt{\frac{4 \mu_5}{m}} \right]
\]

(49)

The numerical simulations (using second order finite difference on a finite interval of \( z \) with Dirichlet boundary conditions) for \( n = 1 \) state with \( \Delta \theta = -(2 \mu_5 t) h(-t) \), where \( h(t) \) is a Heaviside function show what happens after the adiabatic regime breakdown (Fig. 3). During the adiabatic regime as \( \Delta \theta \) approaches \( \Delta \theta_n \) the bound state profile widens. However, near \( \Delta \theta \simeq \Delta \theta_n + \vartheta_{n.a.} \) this widening stops and the bound state transforms into two unbounded wavepackets moving to opposite directions. Their combined momentum representation is close to the momentum representation of the bound state at the adiabatic regime breaking point. Indeed, if we estimate the \( \vartheta \) for which the location of the peak of the spectrum for \( a_{0,n=1} \)
Figure 3: After the breakdown of the adiabatic regime the localized state (solid) transitions into two unbound wavepackets (dashed) travelling towards opposite directions. This particular plot corresponds to $m = 3, \mu = 1, \mu_5 = 0.05$.

coincides with the numerically obtained one, we get the values close to the estimate (49) (see Figs. 4 and 5).

Figure 5: Comparison between adiabatic regime breaking point estimated from the peak of the numerically obtained spectrum and estimate (49) for $m = 3\mu$.

7 Conclusions

We have demonstrated that the massive vector field on a parity-violating background may possess localized states on the boundary of the parity-violating medium. While greatly simplified,
this model may imply that a similar statement is valid for the vector meson states in a chiral-breaking medium [34]. The time evolution of the background means that the localized states are not exactly stationary. This makes possible for the boundary currents to appear in the transition regions between the domains with different values of the parity-breaking parameter.

Presently, only vector meson masses \( m \approx 770 \) MeV are available from the experimental data whereas only theoretical estimates exist for the value of the axial chemical potential \( \mu_5 \) or the associated parity-breaking topological charge fluctuations based on different approaches [39–44]. If the axial chemical potential at hadronization is as low as \( \mu_5 \approx 0.2T \) [45] with \( T \sim 150 \) MeV then it is indeed much smaller than \( m \) so \( \theta \) may be assumed to change slowly. We assume that the spatial parameter \( \mu \) also should lie in the range 10 \( \sim \) 1000 MeV.

Our treatment would be valid only as long as the adiabatic approximation holds. However, as we noted, it may be expected to break during the evolution of the parity-breaking medium leading to the annihilation of the boundary currents into the ingoing and outgoing particles with potentially rich phenomenology. The spectrum of the resulting unbound particles is determined by the bound state profile at the point of the adiabatic regime breakdown, which is determined by the instant \( \mu_5 \) and \( \mu \) values rather than by a history of the background evolution. This suggests that such explosion processes may appear as exotic spikes in the vector meson production.

At this point, however, it is extremely hard to make specific conclusions about experimental implications of the existence and explosions of such boundary currents. For that, one also has to study the interaction between the bound and unbound states, and investigate the possible pumping of the boundary currents by the background. It is also important to study the backreaction of such processes on the parity-breaking background evolution. We employ the generalized chiral perturbation theory and vector meson dominance model [34,46–48] to explore these questions in [49].

Figure 4: Comparison between adiabatic regime breaking point estimated from the peak of the numerically obtained spectrum and estimate (49) for \( m = \mu \)
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