Galactic rotation curves vs. ultralight dark matter II

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We look for and place observational constraints on the imprint of ultralight dark matter (ULDM) soliton cores in rotation-dominated galaxies. Extending previous analyses, we find a conservative, model-independent constraint which disfavours the soliton-host halo relation found in numerical simulations over a broad range in the ULDM particle mass \( m \). Combining the observational constraints with theoretical arguments for the efficiency of soliton formation via gravitational dynamical relaxation, our results disfavour ULDM from comprising 100% of the total cosmological dark matter in the range \( 10^{-24} \text{ eV} \lesssim m \lesssim 10^{-20} \text{ eV} \). The constraints probe the ULDM fraction down to \( f \lesssim 0.3 \) of the total dark matter.

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I. INTRODUCTION

Ultralight bosonic fields offer a plausible candidate for dark matter (DM). The wave nature of such ultralight dark matter (ULDM) may manifest itself in a variety of astrophysical settings [1–37]. A wide range of the particle mass, \( 10^{-25} \lesssim m \lesssim 10^{-19} \text{ eV} \), can be probed via observations of galaxies [4–23, 36] (for reviews, see [3, 24]). At the lower end in \( m \), Ref. [23] argued that strong gravitational lensing by massive elliptical galaxies is sensitive to ULDM as light as \( m \sim 10^{-25} \text{ eV} \), making up a small fraction of order 10% of the total cosmological DM. At the higher end, Ref. [6] suggested that ULDM-induced dynamical heating in small satellite galaxies may probe \( m \sim 10^{-19} \text{ eV} \). Many studies highlighted \( m \sim 10^{-22} \text{ eV} \), for which ULDM was suggested as a solution to small-scale puzzles facing cold dark matter (CDM) [1, 4, 37]. However, with further scrutiny, this proposal became increasingly implausible. Notably, the possibility that ULDM at \( m \sim 10^{-22} \text{ eV} \) comprises the majority of the DM is in tension with Lyman-\( \alpha \) analyses [25–30], as well as with the analysis of stellar and gas kinematics in low-surface-brightness galaxies (LSBGs) [7, 8] and dwarf galaxies [36].

An important prediction of ULDM, discovered in numerical simulations [4, 5], is the formation of a “soliton” density core in the halo center. The soliton is a ground state configuration of the equations of motion. Ref. [5] found that the soliton mass in their simulations is related to the host halo via the so-called soliton-host halo relation. Refs. [7, 8] (see also [38]) showed that the soliton-host halo relation is equivalent to the equilibration of specific kinetic energy in the soliton and in the halo, \( K/M_{\text{soliton}} \approx K/M_{\text{halo}} \). The soliton-host halo relation is likely the result of gravitational dynamical relaxation [1, 39–43].

Ref. [7] used rotation curve data from the SPARC database [44] to look for the imprint of the solitons predicted by the soliton-host halo relation. The result was null, suggesting that – if the soliton-halo relation is correct

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1 Another cosmological bound comes from the cosmic microwave background [31] (and its combination with galaxy clustering [32]), and becomes more constraining than the Lyman-\( \alpha \) analyses for \( m \lesssim 10^{-24} \text{ eV} \).
2 See also [33, 34] for related discussion.
– then ULDM in the range $10^{-22} \lesssim m \lesssim 10^{-21}$ eV is excluded. In the current work we expand on the work done in [7] in a number of aspects.

First, whereas [7] reported only a crude estimate of the observationally disfavoured range in $m$, we perform a systematic scan of the data, resulting in broader and more comprehensive limits. A summary of our results is shown in Fig. 1. The blue region combines the constraints from all of the SPARC galaxies; each thin line corresponds to a single individual galaxy. On the y-axis, we use the soliton mass $M$, normalized to the mass specified by the soliton-halo relation, $M_{\text{SH}}$. We allow an uncertainty of a factor of 2, up or down [5, 7], in $M_{\text{SH}}$; this is represented by the red band. On the x-axis we show the ULDM mass $m$. Where the blue region dips below the red band, which happens for $3 \times 10^{-24} < m < 2 \times 10^{-20}$ eV, the soliton-halo relation is in conflict with the data.

FIG. 1: The combined 95\% C.L. constraints (solid blue) of SPARC galaxies on the mass of the soliton $M$, normalized by that predicted by soliton-host halo relation, $M_{\text{SH}}$. Each blue line corresponds to a galaxy. The blue dashed lines highlight analytical approximations, valid at small and large $m$. The red band comes from allowing $M_{\text{SH}}$ to vary by up to a factor of 2 up or down.

It is important to note that in many galaxies, the soliton-halo relation is not generally expected to hold for $m \gtrsim 10^{-21}$ eV. The relation was only tested by numerical simulations (of galaxies similar to typical SPARC systems) for $m \sim 10^{-22}$ eV; when $m$ is increased, the efficiency of dynamical relaxation diminishes, and eventually one expects the relation to break down because the soliton does not have enough time to form during the age of the galaxy. From this perspective, the $m \gtrsim 10^{-21}$ eV part of the excluded range in Fig. 1 may not be very informative. Nevertheless, as we explain later on, the full excluded range (including the higher $m$ range) is still of phenomenological interest. First, it is an observational constraint, and can be considered as a null search for the soliton imprint, putting aside theoretical bias. Second, we shall see that in a class of models (inspired by the “string axiverse” scenario [2]) in which more than one species of ULDM coexist, dynamical relaxation could become unexpectedly efficient, and populate the soliton state even for high-$m$ fields: in other words, there are in fact interesting and well-motivated theoretical models that could produce a soliton even for $m \gtrsim 10^{-21}$ eV.

Another aspect which we extend over Ref. [7], is to use the theoretical analysis of dynamical relaxation in order to derive constraints also in the case where ULDM comprises just a fraction $f < 1$ of the total cosmological DM.

An outline of the rest of this paper is as follows. In Sec. II, after briefly reviewing the soliton–host halo relation, we derive observational constraints on this relation using SPARC data. Most of our results are model-independent and conservative, in that we only consider a rotation curve to constrain a soliton feature if the soliton feature, by itself and without considering any additional mass component, overshoots the velocity data. In Sec. III we use theoretical considerations of soliton growth by dynamical relaxation in order to convert the observational constraints on soliton mass into constraints on the ULDM-to-total-DM fraction, $f$. In Sec. IV we comment on implications when ULDM is comprised of more than one species (“more than one $m^i$”), and on a possible caveat related to soliton random walk. We conclude in Sec. V. In App. A we consider more realistic fitting procedures, including estimates of additional DM and baryonic mass components, to complement the more conservative analysis of the main text. App. B outlines how the dynamical relaxation estimates can be generalised to a scenario with more than one species of ULDM.

II. ULDM VS. ROTATION CURVE DATA

A. Recap: soliton-halo relation

Numerical simulations of ULDM [4, 5] found an empirical relation, connecting the soliton to its host halo. While the soliton-halo relation was originally reported as a relation between the soliton mass and the host halo mass, Refs. [7, 8] showed that the reported relation is precisely equivalent to a more physically tractable equality between the specific kinetic energy stored in the soliton and that in the host halo:

$$K_M\Big|_{\text{sol}} \approx \frac{K_M}{M_{\text{halo}}}.$$  \hspace{1cm} (1)

Here, $K$ is the kinetic energy, and $M$ is the total mass in each component (the soliton core, and the host halo) of the density profile.

Phenomenologically, Eq. (1) implies that the peak rotation velocity of test particles, induced by the soliton gravitational potential, should be close to the peak rotation velocity induced by the host halo [7]. This shape information on the velocity curve makes the soliton-halo relation an easy observational target.
Theoretically, Eq. (1) is suggestive of quasi-equilibrium or approximate thermalisation between ULDM particles in the halo and in the soliton structure. Such a behaviour is consistent with the outcome of dynamical relaxation [39, 43], that is much more efficient in ULDM than in CDM models due to the formation of ULDM interference patterns or granules, acting as massive quasi-particles [1, 13].

In much of our analysis, we will use Eq. (1) as a benchmark for comparison of the soliton prediction of ULDM with observations. It is therefore important to emphasise that the soliton–halo relation as expressed by Eq. (1) is not without dispute. Ref. [45], in particular, reported a different relation; however, it was shown in Ref. [7] (see Sec III.B there) that the soliton–halo relation of [45] amounts to precisely equating the entire total energy of the halo with that of the soliton; suggesting that the initial conditions adopted in [45] were not realistic. More in general, additional numerical and analytical tests of Eq. (1) would be important: our results strongly highlight this fact.

B. Looking for solitons in SPARC

We use rotation curve data from the SPARC database [44] to look for the imprint of solitons. The database consists primarily of observationally inferred rotation curve data, along with model results aiming to separate the contribution of baryons (stellar disk and bulge, as well as gas), for 175 nearby galaxies.

In our main and most conservative pass on the data, we ignore the modelling attempts to identify the baryonic contribution to the rotation curve. Instead, to constrain the allowed $M_{\text{sol}}$ in a given galaxy, we perform a “one-sided” test, where a soliton contribution is excluded if it alone overshoots some portion of the rotation curve data to some specified significance. This approach is equivalent to modelling a soliton together with an arbitrary background profile, where the background profile can be adjusted to fit any velocity bin that the soliton-induced velocity undershoots. The only assumption we make for the (otherwise unspecified) background component is that it gives a positive contribution to the rotation velocity.

The radial mass profile due to the soliton, $M(r)$, is given by

$$M(r', M_{\text{sol}}, m) = \int_0^{r'} \rho_{\text{sol}}(r; M_{\text{sol}}, m) 4\pi r'^2 dr,$$  \hspace{1cm} (2)

where the soliton density profile is given approximately by [5]

$$\rho_{\text{sol}}(r) \approx \frac{\rho_{\text{sol}}(0)}{(1 + 0.091(r/r_c)^2)^8},$$  \hspace{1cm} (3)

$$\rho_{\text{sol}}(0) \approx 0.083 \left(\frac{M_{\text{sol}}}{M_\odot}\right) \left(\frac{r_c}{\text{kpc}}\right)^{-3} \frac{M_\odot}{\text{kpc}^3},$$

for a soliton total mass $M_{\text{sol}}$. The characteristic radius $r_c(M_{\text{sol}}, m)$ is given by

$$r_c \approx 2.28 \left(\frac{M_{\text{sol}}}{10^{11} M_\odot}\right)^{-1} \left(\frac{m}{10^{-22} \text{eV}}\right)^{-2/3} \text{pc}. \hspace{1cm} (4)$$

The soliton profile is then controlled by two parameters, $M_{\text{sol}}$ and $m$. In our analysis we scan a fixed grid in $m$, determining the limit on $M_{\text{sol}}$ for each value of $m$.

Note that we use the self-gravitating soliton profile, without including the distortion of the profile due to the presence of the non-ULDM background density. The effect of the background density was studied in detail in Ref. [8]. The general results of that analysis indicate that the self-gravitating soliton profile is a good approximation to the actual profile, as long as the background mass component contained within the soliton core radius is smaller than the total soliton mass. While this assumption can be violated when the soliton mass is small, it is valid in the region of the $M_{\text{sol}}$, $m$ parameter space that saturates our bound.

The results of our analysis are shown in Fig. 1. The parameter region over which the data is most sensitive to the soliton-halo relation is $m \sim 10^{-21} - 10^{-22}$ eV. As discussed in [7, 8], we find no convincing hint for the soliton bump in any well-resolved, DM-dominated rotation curve. We therefore present exclusion limits, extending the discussion in [7].

For very large and very small values of $m$, we can understand the scaling of the exclusion curves in Fig. 1 analytically. This is highlighted in Fig. 1 as dashed lines for one sample galaxy. At low $m$ the constraints are dominated by the largest radius data bin, $r_1$, which falls inside the soliton core. The largest radius bin therefore constrains

$$M_{\text{sol}}(r_1) < r_1 V_{\text{obs}}^2(r_1)/G = \text{Const.},$$

where $V_{\text{obs}}$ is the observed rotation velocity. In this regime, the enclosed soliton mass is

$$M_{\text{sol}}(r_1) \approx (r_1/r_c)^3 M_{\text{sol}} \propto M_{\text{sol}}^4 m^6 \text{ (using Eq. (4)).}$$

In the plot, we show the scaled ratio, $M_{\text{sol}}/M_{\text{SH}}$, where $M_{\text{SH}} \propto m^{-1} M_{\text{halo}}^{1/3} [4, 5]$. Therefore, the constraint on the ratio $M_{\text{sol}}/M_{\text{SH}}$ in the low $m$ region of Fig. 1 follows $M_{\text{sol}}/M_{\text{SH}} \propto (m^{-6/4})/(m^{-1}) \propto m^{-1/2}$. At large $m$ the constraint is dominated by the innermost data bin $r_1$, and the soliton potential is approximately that of a point mass. The data then constrains the total soliton mass $M_{\text{sol}}$, so the constraint on $M_{\text{sol}}/M_{\text{SH}} \propto m$.

It is interesting to note that in the small-$m$ region $m \lesssim 10^{-24}$ eV, the data (including potential sensitivity from many galaxies) is compatible with a soliton saturating the soliton-halo relation. Observationally, this reflects the fact that many galaxies in the SPARC data base display rotation curves that scale linearly with radius, $V_{\text{obs}} \propto r$, consistent with the total density profiles of these galaxies.
forming large-radius cores. The shallow slope of these rotation curves suggest low-density, large-radius cores; to attribute such cores to ULDM, one would be forced to require $m \lesssim 10^{-24}$ eV. Such light ULDM is in strong contradiction with cosmological Ly-$\alpha$ data, unless the ULDM makes up just a small fraction of the total DM, $f \lesssim 0.2$. If that was the case, it is unclear how the main 80% of the DM disappears from these galaxies.

In the following subsections we discuss a number of additional points related to the constraints in Fig. 1.

1. **Realistic background density**

   ![Graph](image)

   **FIG. 2:** 95% CL limit on $M_{\text{sol}}/M_{\text{SH}}$ vs. $m$, obtained with different modelling of the background density profile for a sample galaxy (NGC100). Horizontal dashed line marks the soliton-halo relation. Green solid line shows the limit obtained with arbitrary background profile: this is the procedure we refer to in the main text. In addition, we also show results where the background DM is fitted with NFW (red) or Burkert (blue) profiles, matching the soliton and background halo components (solid) as well as simply adding the components on top of each other (dashed).

   The analysis leading to Fig. 1 was conservative, in that we only considered the impact of the soliton when it overshoots the rotation velocity data, allowing an unspecified background density profile to fit underpredicted velocity bins. In App. A we study how the limits change when considering more realistic background profiles. The exercise involves a statistical fit of the velocity profile, deriving the constraints on ULDM from a log-likelihood ratio. In addition to the soliton component, we include the following ingredients: (i) we add the baryonic contribution to the velocity curve, using the gas, disk, and bulge models from SPARC [44], and allowing the mass-to-light ratios of the disk and the bulge to vary freely in the fit; (ii) we consider two models for the DM contribution, in addition to the soliton: an NFW profile [47], and a cored Burkert profile [48]. These are matched to the soliton feature in different ways.

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2. **Impact of baryons**

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   **FIG. 3:** Same as Fig. 1, but colour-coding the baryonic fraction $M_{\text{bar}}/M_{\text{tot}} |_{r_{\text{peak}}}$ (see text around Eq. (5)) for each galaxy.

   In Fig. 3 we repeat Fig. 1, colour-coding the limit from each galaxy according to the importance of the baryonic contribution in the rotation curve. The baryonic contribution is estimated via

   $$M_{\text{bar}}/M_{\text{tot}} |_{r_{\text{peak}}} = \frac{V_{\text{bar}}^2}{V_{\text{obs}}^2} |_{r_{\text{peak}}}, \quad (5)$$

   where $r_{\text{peak}}$ is the bin with maximal rotation velocity. For the purpose of this estimate we fix the mass-to-light ratios as $\Upsilon_{\text{disk}} = 0.5$ and $\Upsilon_{\text{bulge}} = 0.5$ (see App. A for details). We see that the constraints derive mostly from DM-dominated galaxies, with $M_{\text{bar}}/M_{\text{tot}} |_{r_{\text{peak}}} \ll 1$.

3. **Statistical significance**

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   **FIG. 4:** Same as Fig. 1, but combining the data from all of the SPARC galaxies, allowing an unspecified background density profile to fit underpredicted velocity bins. In App. A we study how the limits change when considering more realistic background profiles. The exercise involves a statistical fit of the velocity profile, deriving the constraints on ULDM from a log-likelihood ratio. In addition to the soliton component, we include the following ingredients: (i) we add the baryonic contribution to the velocity curve, using the gas, disk, and bulge models from SPARC [44], and allowing the mass-to-light ratios of the disk and the bulge to vary freely in the fit; (ii) we consider two models for the DM contribution, in addition to the soliton: an NFW profile [47], and a cored Burkert profile [48]. These are matched to the soliton feature in different ways.

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\[ \frac{\tau}{\text{yr}} \approx 10 \left( \frac{m}{10^{-22} \text{ eV}} \right)^3 \left( \frac{\sigma}{0.1 \text{ km} \text{s}^{-1}} \right)^6 \left( \frac{M_{\odot}}{\rho} \right)^2 \left( \frac{3}{\ln \Lambda} \right). \]

The numerical factor \( b \approx 0.7 \) is calibrated by numerical simulations \([39]\) (see also \([41–43]\)). We estimate the Coulomb log as \( \ln \Lambda = \ln (m\sigma R) \), where \( R \) is the characteristic radius of the system. Note that Eq. (6) is expected to become inaccurate for \( \ln \Lambda \lesssim 1 \).

Eq. (6) shows that over wide regions in the density profile of typical galaxies (specifically, typical SPARC galaxies referred to later on in this work), \( \tau \) can become much shorter than the age of the galaxy.

The relaxation time becomes longer if ULDM comprises only a fraction \( f < 1 \) of the total density \( \rho \); in that case, we should replace \( \rho \rightarrow f \rho \) in Eq. (6) \([23]\) (see also App. B).

If the relaxation time is much shorter than the age of a galaxy, then we expect that a soliton should form. Once the soliton specific kinetic energy saturates the value corresponding to the soliton-halo relation, Eq. (1) (that is, once \( M_{\text{sol}} \) grows to saturate \( M_{\text{SH}} \)), the soliton growth by dynamical relaxation becomes quenched and slows down considerably. This scenario, which is understood theoretically, is consistent with the results of numerical simulations \([4, 5, 39, 41, 42]\).

On the other hand, a relaxation time longer than the age of a galaxy may mean that a soliton could not have formed in the galaxy. In such a system, we do not translate the observational constraints on \( M_{\text{sol}} \) to a constraint on total ULDM fraction \( f \).

Following Ref. \([23]\) we suggest a concrete, approximate criterion, to see if a given galaxy should be expected to have formed a soliton of mass \( M_{\text{sol}} \) by dynamical relaxation. To this end, we define two characteristic radii:

- \( r_{\text{supply}} \): If solitons grow by accreting mass from an initially stochastic halo, then to assemble a soliton of mass \( M_{\text{sol}} \), field needs to be accreted from a radius that is at least as large as \( r_{\text{supply}} \), defined by \( \int r_{\text{supply}} 4\pi r^2 \rho_X(r) dr = M_{\text{sol}} \), where \( \rho_X \) is the initial ULDM halo density profile. If ULDM makes up only a fraction of the total DM density, then only the ULDM part should be included in \( \rho_X \). In particular, if the ULDM fraction \( f \) is decreased, then \( r_{\text{supply}} \) must increase, to compensate for the over-all smaller ULDM density by drawing mass from larger distances.

To make an analytic estimate, if \( \rho_X \) follows an NFW profile, then for \( r \ll r_s \) we have \( \rho_X \propto f/r \) and \( M_{\text{sol}} \propto f r_{\text{supply}}^2 \). Since \( M_{\text{SH}} \propto 1/m \), we find \( r_{\text{supply}} \propto (mf)^{-1/2} \). If, for a very massive soliton, the process extends out to the region \( r \sim r_s \), where \( \rho_X \propto f/r^3 \), a similar consideration gives \( r_{\text{supply}} \propto (mf)^{-1} \).

- \( r_{\text{relax}} \): The process of soliton growth should only be efficient within a region of the halo for which the dynamical relaxation time is shorter than the age of the system. Defining the boundary of that region by \( r_{\text{relax}} \), we have: \( \tau_{\text{relax}}(r_{\text{relax}}) = t_{\text{gal}} \), with \( t_{\text{gal}} \) the age of the galaxy. As discussed above, if ULDM makes up only a fraction \( f < 1 \) of the total DM density, then \( \tau \) in Eq. (6) is increased as \( \tau \rightarrow \tau/f^2 \). Thus, making \( f \) smaller has the effect of pushing \( r_{\text{relax}} \) further in to a smaller radius in the halo, to compensate for the smaller \( f \) by a larger density (for simplicity, in

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5 Dynamical relaxation starting from a stochastic initial state is not necessarily the only channel to form solitons. In principle, a coherent soliton core could exist in halo centres from the early structure formation stage.
this argument we assume a roughly constant velocity dispersion $\sigma$; in practice, we use a prescription to estimate $\sigma(r)$ from the observed velocity data, to be explained shortly below.

To make an analytic estimate, assuming $\sigma \approx \text{Const.}$ one finds $r_{\text{relax}} \propto \int \frac{\rho(r')M(r')}{r'^2} dr'$ for $r \ll r_s$, and $r_{\text{relax}} \propto f^{1/2}/(m^3\alpha^6)^{1/4}$ at $r \sim r_s$. In the numerical computation, we take the initial ULDM density to follow an NFW profile, and estimate $\sigma$ using Jeans modelling, discussed below.

A rough criterion for the formation of a soliton with mass $M_{\text{sol}}$ is:

$$r_{\text{relax}}(M_{\text{sol}}, m, f) > r_{\text{supply}}(M_{\text{sol}}, m, f).$$

We take this as a condition for the applicability of the soliton-host halo relation.

To estimate the local velocity dispersion in the relaxation time in Eq. (6), we solve the Jeans equation for self-gravitating NFW halo, assuming isotropic velocity dispersion [49]

$$\sigma^2(r) = \frac{G}{\rho(r)} \int \frac{\rho(r')M(r')}{r'^2} dr',$$

where $M(r) = \int r^6 dr' \rho(r')$ is the enclosed mass. For NFW, this integral has an analytic solution. To speed-up the numerical analysis, we use an approximate form for $\sigma$, $\sigma(r)/V_{\text{circ}}(r) \approx 0.55 + 0.2 \exp(-r/2r_s) + 0.2 \exp(-2r/r_s) + 0.6 \exp(-8r/r_s)$, where $r_s$ is the transition scale in NFW as defined in Eq. (A2), $V_{\text{circ}}(r)$ is the circular velocity. The approximation differs from the exact solution by less than 2% in the range of $0.05 r_s < r < 10 r_s$.

In Fig. 5 we show the different scales as functions of the ULDM fraction $f$, for one sample galaxy, setting $m = 10^{-22}$ eV. We also show the core radius $r_c$ of a soliton that satisfies $M_{\text{sol}} = M_{\text{SH}}$ for this system. In this galaxy, according to the criterion Eq. (7), ULDM with the prescribed value of $m$ can be expected to form a soliton saturating the soliton-halo relation only for $f \gtrsim 0.3$. For smaller values of $f$, the soliton-host halo relation may break down as dynamical relaxation becomes inefficient.

Using the criterion Eq. (7), we can translate the observational constraints of Sec. II B into constraints on the ULDM fraction $f$. For each value of $m$, we scrutinise the SPARC database and find the smallest value of $f$ for which: (i) solitons with a mass $M_{\text{sol}} = M_{\text{SH}}$ are in tension with the data to some specified statistical significance, as in Fig. 4; (ii) the condition Eq. (7) is satisfied, for all of the galaxies that yield this tension. We show the result of this exercise in Fig. 6, where we also explore the sensitivity of our results to the details of the relaxation condition. The left panels show the disfavoured range of $f$ vs. $m$, using Eq. (7). The right panels use a stricter condition, $r_{\text{relax}} > 2r_{\text{supply}}$. In the upper panels we take the soliton-halo relation as in tension with data when $M_{\text{sol}} < M_{\text{SH}}$ at 95% confidence level, while in the lower panels we use a stricter criterion $M_{\text{sol}} < 0.5 M_{\text{SH}}$. For comparison, we also display the cosmological Lyman-$\alpha$ constraints.

IV. ADDITIONAL COMMENTS

A. More than one $m$

If one species of ULDM exists, there may just as well be more than one [50]; indeed, this could be the expectation in scenarios such as that advocated in [2]. It is therefore quite relevant to check if the constraints we derived so far could become weakened by the presence of additional species of ULDM. We try to address this question in this section. Our analysis suggests that the constraints derived under the assumption of only one species of ULDM are, in fact, more likely to become even tighter, if additional species exist. Moreover, additional, even sub-dominant species of ULDM could open up new regions of the parameter space for which observational imprints in galaxy kinematics could be sought after. The reason this happens is dynamical relaxation, which could become more efficient with additional ULDM components.

If more than one species of ULDM exists, then quasi-particles of one species should also induce dynamical relaxation on the other species. We can define the relaxation time, $\tau_{ij}$, of species $i$ due to the gravitational interaction with $j$. Estimating the relaxation process as coming from 2-body encounters between ULDM quasi-particles [1, 13] shows that $\tau_{ij} \approx \tau_{jj}$ (see App. B for a derivation). Note that $\tau_{ii}$ is given by Eq. (6), with $\rho \rightarrow f_i\rho$ and $m \rightarrow m_i$. The effective relaxation time of a species should thus be given by $\tau_1 = (\sum_j \tau_{ij}^{-1})^{-1}$. For example, if two dominant species of ULDM exist in the system, we can estimate the effective relaxation time of species 1 as (taking $b\sqrt{2} \approx 1$)

$$\tau_1 \approx \frac{1}{12\pi^3} \frac{m_i^3\sigma^6}{G^2\rho^2} \frac{1}{f_i^2 \ln \Lambda_1} \left[1 + \left(\frac{f_2/f_1}{(m_2/m_1)^3} X^2\right)^2\right]^{-1},$$

where we wrote the Coulomb logarithm $X = (\ln \Lambda_1/\ln \Lambda_2)$.
FIG. 6: Cosmological ULDM fraction $f$, disfavoured by combining SPARC data together with the condition of sufficiently fast dynamical relaxation, for a range of ULDM particle mass $m$. For the relaxation criterion, in the left panel we impose $r_{\text{relax}} > r_{\text{supply}}$ (c.f. Eq. (7)). The right panel tests a stricter version of the criterion, with $r_{\text{relax}} > 2r_{\text{supply}}$. We put bounds on $f$ in the mass range where the rotation curve data is in tension with the soliton-halo relation at 95% confidence level. In the top panel we impose $M_{\text{sol}} < M_{\text{SH}}$ (mass range below the dashed line in Fig. 1); in the bottom panel we impose $M_{\text{sol}} < 0.5 M_{\text{SH}}$ (mass range below the whole red band in Fig. 1).

and $\ln \Lambda_i = \ln \Lambda_1 + \ln(m_i/m_1)$.

Up to the logarithmic correction, the relaxation time due to species $i$ scales as $m_i^3/f_i^2$. This means that the presence of even a small amount ($f_2 \ll 1$) of “spectator” ULDM species, if $(m_1/m_2)^3 > (f_2/f_1)^2$. What cuts-off this potential enhancement of relaxation is the Coulomb log: Eq. (6) should break down for $\ln \Lambda \lesssim 1$. Thus the effect can only take place as long as $m_2 \gg 1/(\sigma R) \approx 4 \times 10^{-23} \left(\frac{\text{1 kpc}}{R}\right) \left(\frac{50 \text{ km/s}}{\sigma}\right) \text{ eV}$.

As an aside, note that in a multi-species scenario of axion-like particles, the cosmological relic abundance of each species is expected in the minimal vacuum misalignment mechanism to satisfy $\Omega_i = f_i \Omega_{\text{DM}} \propto \theta_i^2 F_i^2 m_i^{1/2}$, where $F_i$ is the axion decay constant and $\theta_i$ is a vacuum misalignment angle (expected to be of order unity for initial conditions set before inflation). Assuming that $F_i \sim F$ is roughly universal among the different species, and neglecting $O(1)$ differences in initial misalignment angles, we find the parametric dependence of the factor in the parentheses of Eq. (9)

$$\left[ \sum_j \frac{(f_j/f_1)^2}{(m_j/m_1)^3} \right]^{-1} \sim \left[ \sum_j \frac{m_i^2}{m_j^2} \right]^{-1} \sim \frac{\min(m)^2}{m_1^2}.$$  \hspace{1cm} (10)

The species participating in the sum are those for which $m_i \gg 1/(\sigma R)$. Even with this condition, it is possible in principle for this factor to enhance the efficiency of dynamical relaxation, compared to naive expectations with a single species of ULDM.

Suppose there is one species of ULDM with $m_1$, $f_1$, and a second species with $m_2 < m_1, f_2$. This set-up could lead to stronger constraints on $f_1$, compared to the single species scenario. Fig. 7 demonstrates this point. The region inside the gray-colored contour corresponds to single-species relaxation discussed previously. The total blue-shaded region is the constraint on $f_1$ vs $m_1$, that would be obtained...
FIG. 7: Demonstration of the impact of a light "spectator" ULDM species with mass $m_2 = 10^{-23}$ eV and DM fraction $f_2 = 0.1$ on the dynamical relaxation of a second species with $m_1$. The region inside the gray contour shows the constraint derived in the $m_1, f_1$ plane, neglecting the impact of the $m_2$ species. All blue shaded region shows the constraint when the $m_2$ species is accounted for in the relaxation time estimate. The sharp cut-off at the right edge of the blue region for each galaxy is due to the soliton-halo relation becoming compatible with the data at high $m_1$.

if, in the relaxation time computation, we include an additional species of ULDM at $m_2 = 10^{-23}$ eV and $f_2 = 0.1$, consistent with the Lyman-α limit.

B. Soliton random walk

Throughout our analysis we considered the soliton to be at rest at the bottom of the host halo gravitational potential well. Simulations in Ref. [51] (see also discussion in [15, 16, 52]) suggest that, instead, the soliton may be constantly moving in a random walk at the centre of the halo. For the halo studied in [51], which was intended to mimic the dwarf galaxy Eridanus II with a virial mass of order $\sim 10^{10} M_\odot$, the range of the soliton motion was found to be of the same order as the soliton core radius, with a time scale of order the gravitational dynamical time scale.

Soliton random walk [51] could affect our constraints because it would induce a time-varying potential. Dedicated simulations would be needed to conclusively check the effect, and we think that our results strongly motivate such dedicated simulations. This said, we suspect that it is unlikely to ameliorate our bounds significantly. The first point to make is that the benchmark soliton–halo relation is in a rather significant tension w.r.t. many rotation curves. Judging from Figs. 1 and 4, even at 10σ CL, with a conservative treatment of the background mass profile of galaxies, the soliton–halo relation overpredicts the rotation velocity of many galaxies by factors of a few\(^6\). The soliton-induced “bump” in the rotation velocity of a star peaks at a radius $x_{\text{peak}} \approx 2x_c$ (see, e.g. Ref. [7]), where $x_c$ is the core radius as defined in [4, 5]. Displacing the soliton by $2x_c$ would decrease the soliton-induced rotation velocity at its former peak position by about 40%, compared to the factor of few mismatch noted above. The second point is that soliton random walk in 3D in the central region of a cold stellar disk, like those of some low-surface-brightness, low-dispersion galaxies in SPARC [44] (see discussion in [8]), is likely to heat-up and disperse such cold disks, analogously to the effect found in [51] when considering the nuclear cluster of Eridanus II. Investigating this effect further is beyond our present scope, but we suspect that it may amplify, rather than ameliorate, the tension for ULDM in disk galaxies.

V. CONCLUSIONS

We used galaxy rotation curves to look for and constrain ultralight dark matter (ULDM), following and extending earlier work by [7, 8]. The analysis is independent from and complementary to cosmological bounds in the literature. As already shown in [7, 8], the soliton-halo relation found in simulations is strongly disfavoured by the data in the range of ULDM particle mass around $m \sim 10^{-22}$ eV, where it was directly tested in numerical experiments. Here we have shown that the data disfavours the soliton-halo relation over a broad range, $10^{-24} \text{eV} < m < 2 \times 10^{-20} \text{eV}$. In much of this range, the relation was not directly tested numerically; however, theoretical analysis of soliton formation via gravitational dynamical relaxation suggests that in many galaxies, a soliton adhering to the soliton-halo relation should indeed form. While turning this argument into a robust constraint would require dedicated simulations, we believe that it (i) provides adequate motivation for the search in the data, and (ii) having done the search, the lack of significant soliton features disfavours (if indeed not robustly excludes) ULDM in a broad range of $m$.

As an aside, we argued that the presence of multiple species of ULDM, as might be expected in the string axiverse scenario, could lead to dynamical relaxation becoming more efficient than would be naively estimated in case the ULDM makes up just a fraction $f < 1$ of the total cosmological DM. This suggests that having “more than one $m$” could open up unexpected regions in parameter space where the signature of an ULDM soliton might be meaningfully sought after.

\(^6\) A factor of $\sim 3$ overdensity of the velocity comes from a factor of $\sim 10$ overprediction of the central mass of the halo, which is what the y-axis of Fig. 4 shows.
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Appendix A: Modelling a soliton with realistic background profiles

The constraints we considered in Sec. II B and most of the main text were based on a conservative analysis, in which no attempt was made to fit the actual rotation curve data, and an ULDM soliton was disfavoured only if the soliton-induced rotation velocity by itself overshoots the data. In reality, of course, we expect additional contributions to the rotation curve, coming from baryonic matter as well as from ULDM outside of the soliton core, or perhaps non-ULDM components of DM in scenarios in which \( f < 1 \). The goal of this appendix is to estimate the impact of such additional mass components on the analysis.

Regarding the baryonic mass, the SPARC database \[44\] includes model estimates of the baryon-induced rotation velocity components, with radial profiles anchored to stellar \((3.6\mu)\) surface brightness and HI column density data:

\[
V_{\text{bar}}^2(r) = |V_{\text{gas}}(r)|V_{\text{gas}}(r) + Υ_{\text{disk}}|V_{\text{disk}}(r)| + Υ_{\text{bulge}}|V_{\text{bulge}}(r)|, \tag{A1}
\]

We allow the mass-to-light ratios \( Υ_{\text{disk,bulge}} \) to vary in the fit. The gas component is held fixed as given in SPARC\[7\].

We will consider two models for the DM or ULDM outside of the soliton region: an NFW profile \[47\], and a Burkert profile \[48\].

\textbf{a. Soliton + NFW}

In addition to the baryonic contributions and the soliton core, this model includes an NFW density profile:

\[
ρ_{\text{NFW}}(r) = \frac{ρ_s r_s}{r (1 + r/r_s)^2}. \tag{A2}
\]

\( ρ_{\text{NFW}} \) has two parameters, which we take to be the NFW radius \( r_s \), and the concentration parameter \( c \), related to the density parameter \( ρ_s \) via \( ρ_s = ρ_c(200c^3)/\left(3\ln(1 + c) - c/(1 + c)\right) \), with \( ρ_c \) the critical density.

We consider two versions of the model. In the first, we simply add the NFW component in addition to the soliton profile. This way, even in the region where the soliton profile dominates the density, the two DM components overlap. This scenario may be quite relevant, if ULDM makes up just a fraction of the total DM.

In the second version of the model, we match the density of the soliton and NFW profiles at a transition radius \( r_t \), where \( ρ_{\text{sol}}(r_t) = ρ_{\text{NFW}}(r_t) \), and consider the NFW (soliton) component only outside (inside) of \( r_t \) (in case \( ρ_{\text{sol}} \) is subdominant everywhere, \( r_t = 0 \), we use only the NFW profile):

\[
ρ_{\text{DM}}(r) = \begin{cases} 
ρ_{\text{NFW}}(r), & r > r_t \\
ρ_{\text{sol}}(r), & r < r_t
\end{cases} \tag{A3}
\]

In both versions, the total DM mass profile has four free parameters, \( M_{\text{sol}}, m, c, r_s \). We define the total model-predicted velocity as

\[
V_{\text{th}}^2(r; θ_{\text{th}}; θ_ν) = \frac{GM_{\text{DM}}(r; θ_{\text{th}}; θ_ν)}{r} + V_{\text{bar}}^2(r; θ_ν), \tag{A4}
\]

where \( θ_{\text{th}} = \{M_{\text{sol}}, m\} \) and \( θ_ν = \{c, r_s, Υ_{\text{disk}}, Υ_{\text{bulge}}\} \). This is compared with the observed velocity data using

\[
χ^2(θ_{\text{th}}; θ_ν) = \sum_i \left( \frac{V_{\text{th}}(r; θ_{\text{th}}; θ_ν) - V_{\text{obs}}(r_i)}{\Sigma_i} \right)^2, \tag{A5}
\]

where \( V_{\text{obs}}(r_i) \) and \( \Sigma_i \) are the measured rotation velocity and standard deviation in the \( i \)-th radius bin. The summation is over the radial data bins.

We scan a grid of values of the ULDM particle mass \( m \in (10^{-24}, 10^{-19}) \) eV. For each value of \( m \), we allow the remaining model parameters to vary in the following range:

\[
M_{\text{sol}}/M_\odot \in (10^{4.5}, 10^{12}), \quad Υ_{\text{disk}} \in (0, 5), \quad r_s/kpc \in (5, 30), \quad Υ_{\text{bulge}} \in (0, 5), \quad c \in (5, 30)
\]

To constrain \( M_{\text{sol}} \), we perform a log-likelihood ratio test for \( M_{\text{sol}} \cup θ_ν \) separately for each value of \( m \), minimizing the \( χ^2 \) with respect to \( θ_ν \).

\textbf{b. Soliton + Burkert}

This model is identical to that in the previous section (including the two versions of adding the soliton term), apart from replacing the NFW density profile with the Burkert profile,

\[
ρ_{\text{Bkt}}(r) = \frac{ρ_0}{\left(1 + \frac{r}{r_0}\right)^3 \left(1 + \left(\frac{r}{r_0}\right)^2\right)}, \tag{A6}
\]

\[7\] Ref. \[8\] did an independent gas model reconstruction for a few sample galaxies, arriving at similar results to those reported in \[44\].
We express $\rho_0 = \rho_c \delta_0$, with $\rho_c$ the critical density of the universe. The total DM mass profile has four free parameters, $M_{\text{sol}}, m, \delta_0, r_0$. For each value of $m$ on a fixed grid $m \in (10^{-24}, 10^{-19})$ eV, we allow the remaining parameters to vary in the following range:

$$M_{\text{sol}}/M_\odot \in (10^{4.5}, 10^{12})$$
$$\Upsilon_{\text{disk}} \in (0, 5)$$
$$\Upsilon_{\text{bulge}} \in (0, 5)$$
$$\log_{10}(\delta_0) \in (-1, 6)$$
$$r_0/\text{kpc} \in (1, 60)$$

Appendix B: Relaxation of multiple axions

Ref. [1] pointed out that gravitational dynamical relaxation (see, e.g. [49] for textbook review) in an ULDM field can be understood effectively as being mediated by two-body scattering events of massive quasi-particles (QPs). The QPs arise from interference patterns in the field, with a characteristic coherence length of $\lambda_{\text{DB}} \sim 2\pi/(m\sigma)$. If the ULDM ambient density is $\rho$, the mass of each QP is of order $M_{\text{QP}} \sim (4\pi/3)\rho \lambda_{\text{DB}}^3 \sim 6 \times 10^{10} (\rho/0.1 M_\odot\text{pc}^{-3}) (50 \text{ km s}^{-1}/\sigma)^3 (10^{-22} \text{ eV/m})^3 M_\odot$. The effective QP description was made rigorous in analytical studies [13], and further elucidated and calibrated in numerical simulations [39].

Consider the case of just one species of ULDM, with particle mass $m$, ambient density $\rho$, and QP mass $M_{\text{QP}}$, and consider the motion of a single test particle (not necessarily ULDM), with mass $m_{\text{test}} \ll M_{\text{QP}}$, traversing this medium. The mean time between significant collisions of the test particle against QPs in the background is (ignoring order unity factors)

$$\tau \sim \frac{1}{n_{QP} \sigma b^2}, \quad \text{(B1)}$$

where $n_{QP} = \rho/M_{\text{QP}}$ is the QP number density, $\sigma$ is the velocity dispersion in the system (pertaining to the QPs and to the test particle alike), and $b$ is the impact parameter for a significant collision. We define significant collisions as collisions that change the velocity of the test particle by an order unity factor; thus,

$$b \sim \frac{GM_{\text{QP}}}{\sigma^2}. \quad \text{(B2)}$$

Inserting this into Eq. (B1), and using the definition of $M_{\text{QP}}$, we have

$$\tau \sim \frac{m^3 \sigma^6}{G^2 \rho^2}. \quad \text{(B3)}$$

Up to the Coulomb log, Eq. (B3) has the same parametric scaling as Eq. (6). Of course, the equations describe the same process; we could just as well have set the test particle mass to $m_{\text{test}} = m$, making it part of the ULDM. The numerical factors required to make Eq. (B3) precise were calibrated in [13, 39].

Using this understanding, the dynamical relaxation induced by one “spectator” species of ULDM, with particle mass $m_2$ and DM fraction $f_2$, onto another ULDM species with particle mass $m_1$, is simply obtained from Eq. (6), substituting $\rho \rightarrow f_2 \rho$ and $m \rightarrow m_2$. In the main text we referred to this “off diagonal” relaxation time as $\tau_{12}$. 

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