Quantization of strongly interacting phonons.

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The assumption is considered that the strong interaction between phonons makes a certain contribution to the formation of Cooper pairs. Heisenberg's old idea about the quantization of strong nonlinear fields using the Tamm-Dankoff method is discussed. The approximate solution method of infinite Tamm-Dankoff equations system is suggested. This allows us to obtain an equation for the fixed deformation of the lattice between two Cooper electrons. Such deformations can introduce a significant contribution to the energy of Cooper pairs. The possible approximate model of the appearance of the flux tube (nonlocal object) in the SU(2) Yang-Mills theory is considered. The similar mechanism can play important role in High-$T_c$ superconductivity if there is the strong nonlinear potential.

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I. INTRODUCTION

In Ref. [1] a string model of High-$T_c$ superconductivity was suggested. This model is based on the proposal that phonons have a strong interaction among themselves. In this case a tube filled with phonons appears between Cooper electrons. This is similar to the flux tube filled with gluons between quarks in quantum chromodynamic. Thus, this model is based on the existence of strong interaction between the quantum - carrier of the interaction. The possibility of such an interaction between phonons in superconductors is discussed in Ref. [2]. Such a strong interaction can obstruct the application of Feynman diagram techniques in this case. Some time ago W. Heisenberg had conceived of the difficulties in applying an expansion in small parameters to quantum field theories having strong interactions. He had investigated the Dirac equation with nonlinear terms (Heisenberg equation) (see, for example, Ref's [3] - [4]). In these papers he repeatedly underscored that a nonlinear theory with a large parameter requires the introduction of another quantization rule. He worked out a quantization method for strong nonlinear field unusing the expansion in a small parameter (Tamm-Dankoff method). It is possible that in High-$T_c$ superconductivity the interaction between phonons is strong making it necessary take into account the interaction between phonons to correctly calculate the energy of the Cooper pairs.

II. HEISENBERG QUANTIZATION OF FIELD WITH STRONG INTERACTION

Heisenberg’s basic idea proceeds from the fact that the n-point Green functions must be found from some infinity differential equations system derived from the field equation for the field operator. For example, we present Heisenberg quantization for nonlinear spinor field.

The basic equation (Heisenberg equation) has the following form:

$$\gamma^\mu \partial_\mu \psi(x) - i^2 \Im \left[ \psi(\bar{\psi}\psi) \right] = 0,$$

(1)

where $\gamma^\mu$ are Dirac matrices; $\psi(x)$ is the field operator; $\bar{\psi}$ is the Dirac adjoint spinor; $\Im[\psi(\bar{\psi}\psi)] = \psi(\bar{\psi}\psi)$ or $\psi\gamma^5(\bar{\psi}\gamma^5\psi)$ or $\psi\gamma^\mu(\bar{\psi}\gamma_\mu\psi)$ or $\psi\gamma^\mu\gamma^5(\bar{\psi}\gamma_\mu\gamma^5\psi)$. Heisenberg emphasizes that the 2-point Green function $G_2(x_2, x_1)$ in this theory differs strongly from the propagator in linear theory. This difference lies in its behaviour on the light cone. $G_2(x_2, x_1)$ oscillates strongly on the light cone in contrast to the propagator of the linear theory which has a $\delta$-like singularity. Heisenberg then introduces the $\tau$ functions:

$$\tau(x_1,x_2 \ldots | y_1 y_2 \ldots) = < 0| T\psi(x_1)\psi(x_2) \ldots \psi^*(y_1)\psi^*(y_2) \ldots | \Phi >,$$

(2)

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where $T$ is the time ordering operator, $|\Phi>$ is a system state characterized by the fundamental Eq. (1). Relationship (2) allows us to establish a one-to-one correspondence between the system state $|\Phi>$ and the function set $\tau$. This state can be defined using the infinite function set of (3). Applying Heisenberg’s equation (1) to (2) we can obtain the following infinite equations system:

$$l^{-2} \gamma_{(r)}^\mu \frac{\partial}{\partial x^\mu} \tau(x_1 \ldots x_n | y_1 \ldots y_n) = \Im \left[ \tau(x_1 \ldots x_n x_r | y_1 \ldots y_n y_r) \right] + \delta(x_r - y_1) \tau(x_1 \ldots x_{r-1} x_{r+1} \ldots x_n | y_2 \ldots y_{r-1} y_{r+1} \ldots y_n) + \delta(x_r - y_2) \tau(x_1 \ldots x_{r-1} x_{r+1} \ldots x_n | y_1 y_2 \ldots y_{r-1} y_{r+1} \ldots y_n) + \ldots \quad (3)$$

Heisenberg then employs the Tamm-Dankoff method for getting approximate solutions to the infinite equations system of (3). The key to this method lies in the fact that the system of equation has an approximate solution derived after cutting off the infinite equation system (3) to a finite equation system.

It is necessary to note that a method of solution to Eq. (3) is not important for us. For example, we can try to determine the Green functions using the numerical lattice calculations. Here the important point is the following: The technique of expansion in small parameters (Feynman diagrams) can not be employed for strong nonlinear fields.

For the definition of $<\phi>$ here $\phi$ is the quantum chromodynamic it is the nonabelian $SU(3)$ gauge field – the gluons. This nonlinear wave is not a wave between Cooper electrons but wave moving together with Cooper pairs.

Thus, in this model it is assumed that operators of strong nonlinear sound waves must satisfy the following equation (which is implied from the Lagrangian (4)):

$$\Box \hat{\varphi} = \hat{\varphi} (\hat{\varphi}^2 - \phi_0^2), \quad (5)$$

The multitime formalism of Heisenberg’s method (when in $\tau(t_1, t_2, \cdots), t_1 \neq t_2 \neq \cdots$) allows us investigate the scattering processes in quantum theory. The simultaneous formalism (when $\tau(t_1, t_2, \cdots), t_1 = t_2 = \cdots = t$) allows us to calculate the mean value of the field, the energy, or any combination of field powers. It is easy to see that the mean value $<\varphi>=<0|\hat{\varphi}(x)|0>$ satisfies the following equation:

$$\Box <\varphi(x)> = <\varphi^3(x)> - \varphi_0^2 <\varphi(x)> \quad (6)$$

For the definition of $<\varphi^3(x)>$ we turn to Eq.(4) and obtain ($<\varphi^3(x)> = \tau(xxx)$ in Heisenberg’s notation):

$$\Box <\varphi^3(x)> = 3 ( <\varphi^5(x)> - \varphi_0^2 <\varphi^3(x)> ), \quad (7)$$

Here $<\varphi^5(x)> = \tau(xxxxx)$. Analogously it can be used to derive the infinite equation system for calculating $<\varphi^n(x)>$. To first approximation we can solve this equation system using the following assumption:
then we can derive the equation investigated in Ref. [1]. But now we can take another interpretation of the function, \( \psi(x) = < \varphi(x) > \), as a fixed deformation of lattice.

It should be pointed out that the investigation of \( \tau(x_1 x_2 \cdots) = < \varphi^n(x) > \) gives us information about the mean value of the field \( \varphi(x) \). For an investigation of questions on the scattering or interaction of phonons it is necessary to explore the functions, \( \tau(x_1 x_2 \cdots) = < 0|\varphi(x_1)\cdots\varphi(x_n)|0 > \). In following section we discuss the possible mechanism of the nonlocal object appearance in the strong nonlinear theory on example of SU(2) nonabelian Yang-Mills theory. If such a mechanism exist then it can act in the High-\( T_c \) superconductivity if there we have the strong nonlinear potential interaction between phonons.

### IV. POSSIBLE MECHANISM OF THE TUBE RISE IN SU(2) YANG - MILLS THEORY

In this section we propose a model of the tube origin in such nonlinear theory as nonabelian Yang - Mills theory. At first we consider classical cylindrically symmetric solution in this theory [3]. The Yang - Mills equations have the following view:

\[
\nabla_\mu F^\mu \nu = 0
\]

(9)

here \( \nabla_\mu = \partial_\mu + ig[A_\mu, F_{\mu \nu}] \) is covariant derivative; \( F^\mu \nu = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \) is the tensor of Yang - Mills field; \( A_\mu \) is SU(2) gauge potential.

Ansatz for our goals we have as:

\[
A^1_\rho = f(\rho),
\]

(10)

\[
A^2_\rho = v(\rho),
\]

(11)

\[
A^3_\rho = \rho w(\rho),
\]

(12)

here \( \rho, z, \varphi \) are cylindrical coordinate system. After substitution into Eq. (9) we have the following equation:

\[
f'' + \frac{f'}{\rho} = fv^2,
\]

(13)

\[
v'' + \frac{v'}{\rho} = -vf^2,
\]

(14)

here for simplicity we put \( w = 0 \). The solutions of these classical equations have been investigated in Ref. [3]. The asymptotical behaviour of these solutions is following:

\[
f \approx 2 \left[ x + \frac{\cos(2x^2 + 2\phi_1)}{16x^3} \right],
\]

(15)

\[
v \approx \sqrt{2} \frac{\sin(x^2 + \phi_1)}{x},
\]

(16)

here \( \phi_{1,2} \) are some constants, \( x = \rho/\rho_0 \) is dimensionless radius. We see that \( f(\rho) \) is increases as \( r \) and \( v(\rho) \) is very strong oscillating function. According to Heisenberg ideas of quantization of strong nonlinear theory (Heisenberg equation and maybe Yang - Mills theory) we have to write Yang-Mills equations as equation for \( \hat{A}_\mu \) operator. In order that make the quantization of this cylindrically symmetric object we try to shorten the number of degrees of freedom which is necessary for approximate description of such object to a minimum in the following manner:

1. The degrees of freedom (10)-(11) contribute significantly to the tube formation.

2. According to (13)-(16) we suppose that asymptotically \( f(\rho) \) function is almost classical degrees of freedom but \( v(\rho) \) function is quantum degrees of freedom.

According to Heisenberg ideas the Yang-Mills equations for \( \hat{f}(\rho) \) and \( \hat{v}(\rho) \) operators have the following view:

\[
\hat{f}'' + \frac{\hat{f}'}{x} = \hat{f} \hat{v}^2,
\]

(17)

\[
\hat{v}'' + \frac{\hat{v}'}{x} = -\hat{v} \hat{f}^2,
\]

(18)
here (′) is derivative with respect to x and. Taking into account assumption (2) we have
\[ f'' + \frac{f'}{x} = f < v^2 > , \]  
(19)
\[ \hat{v}'' + \frac{\hat{v}'}{x} = -\hat{v} f^2 , \]  
(20)
For receiving the equation for \(< v^2 >\) we act on \(\hat{v}^2(r)\) by operator: \((\frac{d^2}{dx^2} + \frac{1}{x} \frac{d}{dx})\):
\[ (\hat{v}'') + \frac{1}{x} (\hat{v}') = -2\hat{v} f^2 + 2\hat{v}'^2 \]  
(21)
Averaging (21) we have equation for \(< v^2 >\):
\[ < v^2 >'' + \frac{1}{x} < v^2 >' = -2 < v^2 > f^2 + 2 < v^2 > \]  
(22)
This equation is nonclosed: we have to have once more equation for defining \(< v^2 >\). We suppose that \(< v^2 > \approx \alpha < v^2 >\) where \(\alpha\) is some constant. In this case we have the following closed equation set:
\[ < v^2 >'' + \frac{1}{x} < v^2 >' = 2 < v^2 > (1 - f^2) , \]  
(23)
\[ f'' + \frac{1}{x} f' = f < v^2 > \]  
(24)
here following renaming is made: \(\alpha^2 x^2 \rightarrow x^2\), \(< v^2 > / \alpha \rightarrow < v^2 >\), \(f/\alpha \rightarrow f\). Only asymptotical solution of these equations is interesting for us. It is easy to see that by \(x \rightarrow \infty\) we have:
\[ < v^2 > \approx v_0^2 \exp(-\gamma x) \sqrt{x} , \]  
(25)
\[ f \approx f_\infty + f_0 \exp(-\gamma x) \sqrt{x} , \]  
(26)
\[ f_0 = \frac{f_\infty v_0^2}{2(1 - f_\infty^2)} , \quad \gamma = \sqrt{2(1 - f_\infty^2)} . \]
Thus we see that the strong nonlinearity can a cause of the appearance of nonlocal object as tube and so on. On the basis of this approximate model we can suppose that the strong interaction between phonons in High \(T_c\) superconductivity can lead to the nonlocal object appearance, like to the flux tube in quantum chromodynamic, too.

[1] Dzhunushaliev V.D., Phys.Rev., B54, 10121(1996)
[2] H.R.Krishnamurty, D.M.Newns, P.C.Pattnail, C.C.Tsuei, C.C. Chi, Phys. Rev., B49, 3520(1994).
[3] W.Heisenberg, Nachr. Akad. Wiss. Göttingen, N8, 111(1953); W.Heisenberg, Zs. Naturforsch., 9a, 292(1954); W. Heisenberg, F. Kortel und H. Mütter, Zs. Naturforsch., 10a, 425(1955); W.Heisenberg, Zs. für Phys., 144, 1(1956); P.Askali and W.Heisenberg, Zs. Naturforsch., 12a, 177(1957); W.Heisenberg, Nucl. Phys., 4, 532(1957); W. Heisenberg, Rev. Mod. Phys., 29, 269(1957)
[4] W.Heisenberg, Introduction to the unified field theory of elementary particles., Max - Planck - Institut für Physik und Astrophysik, Interscience Publishers London, New York, Sydney, 1966.
[5] V.D. Dzhunushaliev, “Confining properties of the classical SU(3) Yang-Mills theory”, hep-th/9611096.