Band gap of two-dimensional layered cylindrical photonic crystal slab and slow light of W1 waveguide

Zhi-Wei Wang · Ya-Ting Xiang · Hai-Feng Zhang

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Abstract
In this paper, we apply the scatterers of cylindrical rings to a two-dimensional photonic crystals slab. The effects of the number of layers, the thickness, the index, and the height of the cylindrical layers on the photonic band gaps (PBGs) of such slab with different lattice arrangements are studied. It turns out that our new structure helps to obtain a large range of the PBGs. The maximum bandwidth is obtained with the value of 0.117 (2πc/a). The PBGs are moved to the lower frequencies with the augment of thickness, refractive index, and height. The choice of height, refractive index, and thickness is a trade-off, and adding the number of dielectric layers is not always positively correlated with the area of PBGs. In addition, in the W1 waveguide with a triangular lattice layout, we obtain a slow light of 0.026 × c. Compared with the square lattice, the triangular lattice is more suitable for slowing down the speed of light.

Keywords Two-dimensional photonic crystals slab · Photonic crystal waveguide · Photonic band gap · Slow light · Plane wave expansion method

1 Introduction
Photonic crystals (PCs) are a new concept proposed based on the study of the propagating characteristics of light in periodic dielectric materials (Yablonovitch 1987; John 1987). This special structure, which can be artificially changed and manufactured, has attracted much attention since its appearance in 1987. The periodic potential creates a band gap in the Bloch plane, and the PCs prevent the waves falling on this gap from propagating through its structure. This property, known as the photonic band gaps (PBGs), is the most fundamental property of PCs. Researchers have proposed many optical devices based on the PBGs peculiarity of PCs, just like filters (Takano et al. 2005), low-threshold lasers (Ellis et al. 2011), sensors (Chow et al. 2005), waveguides (Mcnab et al. 2003), and antennas (Temelkuran et al. 2000). We can set the dielectric periodically in different dimensions, and PCs are thus divided into three dimensions. One-dimensional PCs (1D-PCs) (Winn...
et al. 1998) have been fully discussed and studied in the past 30 years. The tremendous traits of three-dimensional PCs (3D-PCs) enable them to have wider application prospects (Lin et al. 1998). However, manufacturing 3D-PCs is difficult to realize. In this way, studying the two-dimensional PCs (2D-PCs) is of real significance. The problems of the traditional 2D-PCs require that the medium has an infinite height in the third dimension, which cannot be satisfied in reality. As a consequence, it is more practical to research the two-dimensional PCs slab (2D-PCS) (Chow et al. 2000). Although 2D-PCS are arranged periodically in two dimensions, this is actually a three-dimensional problem. The PBGs of 2D-PCS refer to the frequency range where no guided modes exist, which differ from the traditional 2D-PCs (Johnson et al. 1999). Hereinafter, the plane-wave expansion (PWE) method is used to analyze the PBGs characteristics of 2D-PCS. The light cone is the essential feature of the 2D-PCS, which is different from the ordinary 2D-PCs. It can be treated as the lowest energy band in the system, and only the band below the light cone can be considered to propagate in the slab (Johnson et al. 1999). In general, the inversion symmetry of the guided modes in the system is the problem we need to pay attention to. We can distinguish the parity of the guided modes according to whether the symmetry is odd or even. Odd symmetry corresponds to odd modes, and even symmetry corresponds to even, and the even and odd modes are the analog of the TM and TE modes, respectively. Johnson et al. (1999) (In this paper, the even modes are called TM-like modes, the odd modes are called TE-like modes, and the definition of TE mode and TM mode will be mentioned later.)

Previous applications of 2D-PCS are mostly focused on the defect states, and there are few reports on the tuning of the slab mode gap by structural parameters in recent years. In 2005, S. Y. Shi et al. presented a revised formulation of plane wave method for band structure calculation of PCs for dispersive material which provided a theoretical basis for subsequent studies (Shi et al. 2005). At the same time, I. S. Maksymov et al. studied the PBGs in nonlinear PCS, they focus on the red-shifted phenomenon in the Kerr-nonlinear PCS, but unfortunately, no large bandwidth was obtained (Maksymov et al. 2005). In 2013, T. F. Khalkhali et al. researched the PCS of circular air holes in an anisotropic Te background. However, in their work, the maximum bandwidth is 0.061 (2\pi c/a) (Khalkhali et al. 2013). In the same year, D. Liu et al. studied the PBGs in PCS of core–shell-type dielectric nanorod heterostructures, they only discussed the structure of one layer of rings and did not extend the structure to multiple layers of rings (Liu et al. 2013).

As early as 1992, Erdogan et al. studied the propagation features of TE mode in one-dimensional cylindrical PCs (Erdogan and Hall 1992). In 1994, Ping et al. extended their work to the sphere and studied the propagation and application of electromagnetic waves in the system of planar, cylindrical, and spherical dielectric layers (Ping 1994). Their work laid the foundation for subsequent research, which was carried out one after another (Chen et al. 2009a,b; Li et al. 2011). So far, this kind of 1D-PCs has not been investigated in the two-dimensional case, so it is necessary to study the PBGs features of the 2D-PCS with the scatterers of cylindrical rings.

As shown in Fig. 1a, the cylinder is proposed against an air background. In former studies, researchers were keen on introducing defects into the slab (Säynätjoki et al. 2007; Mulot et al. 2007; Kiyota et al. 2006). In this paper, we not only study the tuning effect of different structural parameters on different mode gaps but also introduce line defects into the 2D-PCS to observe the propagation of light in line defects. The W1 waveguide is constructed by introducing circuit defects in the perfect PCs. This waveguide provides two forms for realizing slow light (Krauss 2007). The first form is called backscattering. When the amplitude and phases of backward and forward propagating light meet the coherent
Fig. 1  a The schematic diagram of photonic crystal slab in an air background with finite height, b the structure of scatterers, the schematic diagram of c the square lattice arrangement, d the triangular lattice arrangement with five layers ($N=5$) of dielectric and its first Brillouin zone, e the photonic crystal slab filled with air rings (blue areas) on GaAs (red areas) dielectric background, and the dispersion relations of the photonic crystal slab filled with air rings which arranged in the square lattice f even modes and g odd modes and $N=5$, $h=0.2 \times a$, $d=0.05 \times a$
conditions, standing waves will be formed. Light in the waveguide behaves as if it was taking three steps forward and two steps back. The second form of slow light is omnidirectional reflection. The light is reflected back and forth perpendicular to the direction of the wave’s advance. The result of this phenomenon is that the forward component of the light is very small. As is known to all, the 2D-PCS restricts the light obviously, and the total reflection is used to limit the behavior of light in the third dimension. Theoretically, the W1-type waveguide composed of the 2D-PCS is suitable for realizing slow light.

Thus, it is still of practical consequence to study the PBGs peculiarities of the cylindrical stratified structure PCs and the group velocity in the W1 waveguide. In this work, the composition of the scatterer is GaAs. The PWE method is used to study the effects of different structural parameters on the different modes of the 2D-PCS, and compare the traits of the PBGs in the triangular and square lattices. After introducing the line defect, the group velocity of $0.026 \times c$ in the triangular lattice is obtained.

2 Structure design and simulation

2.1 Band structure theory

As mentioned above, the scatterer is placed against an air background and the height of the cylinder along the $y$-axis is $h$ in Fig. 1a. The complementarity of the similar structures in Fig. 1a is displayed in Fig. (e), which are formed by adding several layers of air rings to the dielectric.

When the scatterer has five layers of air rings ($N=5$), the thickness of each layer of air is $0.05 \times a$ and $d=0.2 \times a$. The band structures of the odd and even modes under the square lattice arrangement are shown in Fig. 1f and Fig. 1g. For the even modes, a PBG extends from $0.5750 \times (2\pi c/a)$ to $0.6634 \times (2\pi c/a)$, with a width of $0.0884 \times (2\pi c/a)$. When it comes to the odd modes, the number of PBGs is increased to two, with widths of $0.0116 \times (2\pi c/a)$, and $0.1083 \times (2\pi c/a)$, respectively. The band with the lowest frequency is denoted by band 0, the first (1st) PBG is distributed at $0.3624–0.3740 \times (2\pi c/a)$, and the second (2nd) PBG is located in $0.5760–0.6843 \times (2\pi c/a)$. The PBGs characteristics of 2D-PCS with air rings in the dielectric background are also significant to explore. This kind of 2D-PCS type with the dielectric background will be discussed in detail in another work with the application of Maxwell’s Fisheye lens which is based on graded PCs.

In this paper, we discuss the structure in the air background. The structure of each scatterer is shown in Fig. 1b, and the dielectric constant of the structure presents a periodic distribution along any axis of the $x$–$z$ plane. Meanwhile, the single scatterer is placed in the form of 2D-PCS as shown in Fig. 1a. The lattice constant is $a$, $RN$ is the outer radius of the $N$th shell, and the inner radius is $rN$. The red area is GaAs, the blue is air, and the thickness of the GaAs column in each layer is $RN-rN=d=0.05 \times a$. Schematic diagrams of scatterers with five dielectric layers arranged into the square lattice and triangular lattice are shown in Figs.1c and d, respectively. It can be seen that when the number of layers is $N=5$, each scatterer is tangent to the other. In this paper, the maximum value of $N$ is 5. The curve of the dielectric constant of GaAs changing with wavelength is shown in Fig. 1e, when the wavelength of incident light is light $\lambda=1$ μm, the dielectric constant of GaAs is 12.3131, the refractive index $n$ of GaAs is 3.502.

Many ways can be used to calculate the structures of energy bands, such as the transfer matrix method, finite-difference time-domain method, finite-difference frequency-domain
method, and PWE (Kuzmiak and Maradudin 1997). Hereinafter, we will use the PWE method to solve problems.

In PCs, Maxwell’s equations can be used in describing the propagation of electromagnetic waves

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]  (1-a)

\[ \nabla \cdot \vec{B} = 0 \]  (1-b)

\[ \nabla \times \vec{H} = \frac{\partial \vec{B}}{\partial t} + j \]  (1-c)

\[ \nabla \cdot \vec{D} = \rho \]  (1-d)

In our discussions, the scatterer is composed of GaAs, which is linear and isotropic, and we can get:

\[ \vec{D} = \varepsilon_0 \varepsilon(\vec{x}) \vec{E}(\vec{x}) \exp(-i\omega t) \]  (2-a)

\[ \vec{B} = \mu_0 \vec{H}(\vec{x}) \exp(-i\omega t) \]  (2-b)

Combining Eq. (2-a) and Eq. (2-b) with Maxwell’s equations, the vectorial Helmholtz equation can be written as

\[ \nabla \times \left[ \frac{1}{\varepsilon(\vec{x})} \cdot \nabla \times \vec{H} \right] = \frac{\omega^2}{c^2} \vec{H}(\vec{x}) \]  (3-a)

\[ \nabla \times \left[ \frac{1}{\varepsilon(\vec{x})} \cdot \nabla \times \vec{E} \right] = \frac{\omega^2}{c^2} \vec{E}(\vec{x}) \]  (3-b)

The solutions to Eq. (3-a) and Eq. (3-b) are the dispersion relationship between vector \( \vec{k} \) and frequency \( \omega \). The medium dielectric constant can be expressed by \( \varepsilon_m \), and the dielectric constant of air is 1, so we get

\[ \vec{\varepsilon}(\vec{r}) - 1 = (\varepsilon_m - 1) \sum \vec{u}(\vec{r} - \vec{R}) \]  (4)

When \( \vec{r} - \vec{R} \) outside the cycle column, \( u(\vec{r} - \vec{R}) = 0 \), otherwise, \( u(\vec{r} - \vec{R}) = 1 \).

\[ s \cdot \vec{\varepsilon}(\vec{G}) = \int_{s} \vec{\varepsilon}(\vec{r}) \cdot \exp(i\vec{G} \cdot \vec{r}) \, d\vec{r} \]  (5)

We assume that the dielectric constant is independent of the \( y \) axis, the TE modes use nonzero \((H_y, H_z, E_x)\) form, while TM modes with nonzero \((H_y, E_x, E_z)\). Hence, we get the solution to the scalar wave equation by TE modes:
\[(\nabla_i^2 + \frac{\omega_i^2}{c^2} \varepsilon)E = 0 \] (6-a)

while, TM-like modes satisfy the relationship:

\[\nabla_t \cdot \left( \frac{1}{\varepsilon_i} \cdot \nabla_t \cdot H_y \right) + \frac{\omega_i^2}{c^2} H_y = 0 \] (6-b)

Due to \[\varepsilon_i(\vec{r}) = \varepsilon_i(\vec{r} + \vec{R})\], we take the Fourier expansion of this relation, and get \(H_y\) equation:

\[\frac{\partial}{\partial t} \left[ \frac{1}{\varepsilon(\vec{r})} \cdot \frac{\partial H_y(\vec{r})}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{\varepsilon(\vec{r})} \cdot \frac{\partial H_y(\vec{r})}{\partial y} \right] = \frac{\omega^2}{c^2} H_y(\vec{r}) \] (7)

From Bloch’s theorem:

\[H_y(\vec{r}) = \sum_{\vec{k}} H(\vec{k} + \vec{G}) \cdot \exp \left[ i \cdot \left( \vec{k} + \vec{G} \right) \cdot \vec{r} \right] \] (8)

Substitute Eq. (6-a) and Eq. (8) into Eq. (7), and the eigenequation of the TM will be expressed as:

\[\sum_{\vec{G}} \left( \vec{k} + \vec{G} \right) \cdot \left( \vec{k} + \vec{G}' \right) \varepsilon^{-1} \left( \vec{G} - \vec{G}' \right) \cdot H \left( \vec{k} + \vec{G}' \right) = \frac{\omega^2}{c^2} H \left( \vec{k} + \vec{G} \right) \] (9-a)

The intrinsic equation of the TE mode is

\[\sum_{\vec{G}} \left| \vec{k} + \vec{G} \right| \cdot \varepsilon^{-1} \cdot \left( \vec{G} + \vec{G}' \right) \cdot E \left( \vec{k} + \vec{G}' \right) = \frac{\omega^2}{c^2} E \left( \vec{k} + \vec{G} \right) \] (9-b)

### 2.2 Slow light theory

As shown in Fig. 2, we replaced the middle row of columns with a solid cuboid as the defect column with the same height of \(h\). Its side lengths in the \(x-z\) cross-section are \(Dx\) and \(Dz\), corresponding to the \(x\)-axis and \(z\)-axis respectively.

It is acknowledged that the group velocity is used to describe the slow light (Gersen et al. 2005; Li et al. 2008). The group velocity can be obtained by derivative of the dispersion relation, the propagation velocity, and direction of the amplitude envelope and can be expressed by Eq. (10).

\[v_g = \frac{d\omega}{dk} = \frac{c}{n_g} \] (10)

where, \(c\) represents the speed of light in vacuum, \(n_g\) is used to express the group index, \(\omega\) is the angular frequency, and \(k\) on behalf of the wave vector. The group index can be portrayed by Eq. (11).
Analysis and discussion

3.1 The discussions of the characteristics of PBGs

3.1.1 The influences of $N$ on the band structures of the square and triangular cases

Figure 3 displays the dispersion relation of even modes in a square lattice arrangement with the wavelength of incident light $\lambda = a$. Meanwhile, the value of $h$ is chosen as $a$.

$$n_g = c \cdot \frac{dk}{d\omega}$$  \hspace{1cm} (11)
the thickness of the medium ring is \(0.05 \times a\). We can see that when there is only a hollow cylinder, only three bands exist, and they are close to the light cone. As shown in Fig. 3a, only one PBG with a width of 0.0171 \((2\pi c/a)\) appears, which is distributed among 0.6817–0.6988 \((2\pi c/a)\). From Fig. 3b, when \(N\) adds to 2, the number of PBGs also increases to two, with widths of 0.0587 \((2\pi c/a)\) and 0.0127 \((2\pi c/a)\), respectively. The band with the lowest frequency is denoted by band 0, the first (1st) PBG (bands 1–2) is

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**Fig. 3** The dispersion relations of the even mode when the inserted scatterers are arranged in the square lattice, and \(h = a, \ d = 0.05 \times a\). a \(N = 1\), b \(N = 2\), c \(N = 3\), d \(N = 4\) and e \(N = 5\), f the map of gap with the increase of \(N\) and the solid icon represents the lower bound, while the hollow icon represents the upper bound.
distributed at 0.6318–0.6905 (2π/a) and the second (2nd) PBG (bands 2–3) is located in 0.6934–0.7061 (2π/a). Figures 3c–e demonstrate the PBGs characteristics in the case of \( N=3,4,5 \). If \( N=4 \), the third (3rd) PBG is the maximum gap, which frequency distributed 0.6131 (2π/a) to 0.6790 (2π/a), and the width is 0.0659 (2π/a); while at \( N=5 \), the corresponding frequency scope covers 0.6026–0.6615 (2π/a), and the width is dipped to 0.0588 (2π/a). However, despite this, the width in the low-frequency area reaches the maximum value at the case of \( N=5 \), with a width of 0.0481 (2π/a) and extends 0.3762 (2π/a) from 04.243 (2π/a). The map of PBGs with the increase of \( N \) with the square lattice is shown in Fig. 3f. The results exhibit that the PBGs occur a redshift with the augment of the number of dielectric layers. However, the gap width does not always accession with the increase in \( N \).

Figure 4 manifests the dispersion relation of the odd modes of the square lattice in \( N=1,2,3,4 \). Similar to Figs. 3a, 4a presents that when the filling rate is small, there are fewer modes that can propagate in the slab, and a gap is located in 0.6992–0.7070 (2π/a) is produced. The filling rate is small, the Bragg scattering effect of the structure is very weak, the ability of the system to control light is limited, the band is close to that of the light cone, few guide modes can exist in the system, and the remaining modes that cannot exist in the system (the region outside the light cone) are considered to leak out of the system. In Figs. 4b and c, the width of the PBG, whose location is covered in 0.6012–0.6995 (2π/a), increases to 0.0983 (2π/a) regarding \( N=2 \); as \( N \) grows to 3, the number of PBGs fortifies with 3, which distributed among 0.4976–0.5489 (2π/a), 0.5566–0.6057 (2π/a) and 0.6109–0.6331 (2π/a), and the width of each PBG is smaller than that when \( N=2 \). Figure 4d describes the dispersion relationship if \( N=4 \), a gap of 0.076 (2π/a) width comes out in the high-frequency range, the others are situated in 0.4663–0.4924 (2π/a) and 0.4450–0.4624 (2π/a). Unfortunately, no PBG is reported in the case of \( N=5 \). Figure 4f depicts the map of PBGs of odd modes with the square layout in \( N=1,2,3,4 \). We conclude that the PBGs move to the low-frequency region, the width of 1st PBG increases first and then decreases, the width of 2nd PBG diminishes, while the width of 3rd PBG tends to be wider.

Without changing the parameters of the structure, the wavelength of the incident light is still \( a \), and the scatterers are arranged into triangular lattice form. Figure 5 illustrates the situation of PBGs in even modes under a triangular lattice setting. Figure 5a tells us that in terms of \( N=1 \), a PBG extends from 0.6463 (2π/a) to 0.6610 (2π/a), with a width of 0.0147 (2π/a). However, no PBGs are found in the case of \( N=2 \) in Fig. 5b. It can be seen from Fig. 5c that four PBGs come into being, and their frequency scope is 0.5222–0.5327 (2π/a), 0.5509–0.5852 (2π/a), 0.6460–0.6647 (2π/a), respectively. And the widest one is covered 0.6055–0.6458 (2π/a) with a width of 0.0404 (2π/a). To keep boosting the value of \( N \), and the PBGs will degrade and move to the low-frequency sphere. Figure 5e illustrates the case of \( N=5 \), the number of the PBGs decreases to 3, the locations of the PBGs are 0.4065–0.4196 (2π/a), 0.5573–0.5712 (2π/a) and 0.6064–0.6194 (2π/a). Figure 5f reveals the map of PBGs of even modes with an augment of \( N \), the changing trend of PBGs is almost the same as that of the square lattice, except that the gap is missing at \( N=2 \).

The odd modes of triangular lattice arrangement are slightly different from that of the square lattice. First of all, it can be seen from Fig. 6a that the PBG width somewhat increases, but it is no longer near the top of the light cone, and distributed at 0.5372–0.5732 (2π/a) with a bandwidth of 0.0359 (2π/a). Figures 6b–c demonstrate the width of the PBGs and the number of PBGs are increased, and the PBGs are caused a redshift. If \( N=2 \), a PBG in the range of 0.4525–0.5180 (2π/a) will be reported. With regard to \( N=3 \), the
number of the PBGs grows to 3, which is located in $0.4097–0.4188 \, (2\pi c/a)$, $0.4991–0.5521 \, (2\pi c/a)$ and $0.5561–0.6085 \, (2\pi c/a)$, respectively. As $N$ gets larger, the PBGs blue shift, and the width and number tail off simultaneously. In the case of $N=4$, as displayed in Fig. 6d, three PBGs are observed with the scope of $0.4933–0.5095 \, (2\pi c/a)$, $0.5649–0.5745 \, (2\pi c/a)$ and $0.6354–0.6422 \, (2\pi c/a)$. As $N$ adds to 5 in Fig. 6e, the number of PBGs reduces to 2, whose locations are $0.5347–0.5476 \, (2\pi c/a)$. According to the distribution of PBGs shown

**Fig. 4** The band structures of the odd Mode in different medium layers $N, d=0.05 \times a, h=a$, a $N=1$, b $N=2$, c $N=3$, and d $N=4$, e the map of gap with the increase of $N$ and the solid icon represents the lower bound, while the hollow icon represents the upper bound.
Fig. 5 The dispersion relationships of the even mode when the medium column is arranged in the triangular lattice, the thickness of the medium layer of \( d = 0.05 \times a \). a \( N = 1 \), b \( N = 2 \), c \( N = 3 \), d \( N = 4 \), and e \( N = 5 \). f the map of gap with the increase of \( N \) and the solid icon represents the lower bound, while the hollow icon represents the upper bound

in Fig. 6f, in the arrangement of triangular lattice, the PBGs of the odd modes have the best characteristics at \( N = 3 \).

In Fig. 1c and d, we can see that the first Brillouin region of the two lattices is not the same, which results in the distinctions of the reciprocal lattice loss space of the two lattices. Therefore, the Bragg scattering of the two lattices is not the same, and PBGs
show diverse characteristics. This can also be explained by eigenequations Eq. (9-a) and Eq. (9-b). Since the first Brillouin region of the two lattices is not identical, the wave vectors along the first Brillouin region are not the same when solving the eigenmodes, and the values of the eigenmodes of the two lattices are not the same, and PBGs will be different.

Fig. 6 The dispersion relationships of the odd mode when the medium column is arranged in the triangular lattice, the thickness of the medium layer of $d=0.05 \times a$. a $N=1$, b $N=2$, c $N=3$, d $N=4$, and e $N=5$, f the map of gap with the increase of $N$ and the solid icon represents the lower bound, while the hollow icon represents the upper bound.
3.1.2 The influences of the height of dielectric layer $h$ on the PBGs of square and triangular cases

Figure 8 exhibits the tuning effect of the cylinder height on the 1st PBG. In this discussion, $n=3.502$, $d=0.05 \times a$, $N=2$, $h$ is changed as a variable. The results tell the truths that the bandwidths will increase with the augment of $h$, but when $h$ adds to a certain value, the bandwidths will gradually decrease. This is consistent with Ref. (Johnson et al. 1999) If the rods are too high, the energy required to produce the higher-order mode is small. Little height increments add horizontal nodal planes in the vertical direction, and higher-order modes can be easily created. These higher-order modes have slightly higher energy than the lowest-order modes, preventing band gaps from forming. The lowest higher-order odd state is of the first type, and is depicted in Fig. 7a–b.

If the height of the rod is too low, the equivalent refractive index of the system decreases to the degree where it is only slightly greater than the background dielectric constant, shown in Fig. 7.(c)-(d), this greatly weakened the control of the light. Although the guided modes will still exist in the system, they are close to the light cone edge and guided very weakly, the existence of any band gap is very small.

Figure 8a shows how the bandwidth of the 1st PBG changes with $h$ under the square lattice arrangement. For the even modes, the 1st PBG is bands 2–3, the maximum bandwidth is 0.058 (2$\pi c/a$), corresponding to a height of 1.02 $\times a$. The 1st PBG of the mixed modes is bands 2–3 [even] and bands 2–3 [odd]. For the case of the odd modes, the maximum bandwidth is located in (2.34, 0.07). Since the width of even modes is narrower, the width of mixed modes is completely determined by even modes.

The results of the triangular lattice arrangement are described in Fig. 8b. The 1st PBG of the mixed modes with the triangular lattice is between bands 3–4 (even) and 2–3 (odd). However, the 1st PBG of the odd modes is bands 0–1, while the bands 2–3 for the even modes. The maximum bandwidth of the even modes is obtained at 1.28 $\times a$. When $h=2.225 \times a$, the maximum bandwidth is obtained with the value of 0.117 (2$\pi c/a$).

![Contour map of $E_y$ at $x=0.5$](image)

![Contour map of $E_y$ at $z=0$](image)

![Contour map of $H_y$ at $x=0.5$](image)

![Contour map of $H_y$ at $z=0$](image)

Fig. 7 The vertical cross section of $E_y$ for the rod structure in a $y–z$ plane b $x–y$ plane, and the vertical cross section of $H_y$ for the rod structure in a $y–z$ plane b $x–y$ plane
3.1.3 The influences of the thickness of dielectric layer $d$ on the PBGs of square and triangular cases

In this part, the value of $N$ is still 2, $h$ is set to the value of the lattice constant $a$, $n = 3.502$, and the value of $d$ is changed. It is found that the value of $d$ is a trade-off consideration, and increasing the value of $d$ is not conducive to the fortify of the bandwidth.

Figure 9a–c demonstrates the effect of changing $d$ on bandwidth under the layout of the square lattice. Herein, we focus on the 1st PBG (bands 1–2) of all modes to discuss

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Fig. 8 The features of PBGs varying with different $h$, under square lattice arrangement a the odd modes, b the even modes, c the mixed modes while the triangular lattice layout are depicted in d the odd modes, e the even modes, f the mixed modes. And the units of $d$ is $a$, and $h = a$, $N = 2$
the effect of the thickness of the circular cylinder on the width of the PBGs. It should be noted that for odd modes, there was a gap (bands 0–1) at $d = 0.1 \times a$ with a lower frequency than we previously thought for the “1st PBG”, so the "1st PBG" we studied here should actually be called 2nd PBG. However, the real "1st PBG (bands 0–1)" only appears when $d$ is near several discrete values of $0.1 \times a$, so it is meaningless to take this gap (bands 0–1) as the object of discussion. Therefore, the first appearing "PBG (bands
1–2)" is considered to be the real “1st PBG” in this paper, because, in a considerable abscissa span, it has been acting as the “1st PBG”.

For the 1st PBG of the odd modes, in Fig. 9a, the vertex of the curve is located in (0.05, 0.0983). Regarding the case of the 1st PBG of the even modes, the corresponding vertex coordinate is (0.0308, 0.0226) in Fig. 9b. In terms of the mixed modes, which means there are both odd modes and even modes, from Fig. 9c, the maximum bandwidth of the 1st PBG is 0.0415 \((2\pi c/a)\) and the corresponding \(d\) is 0.0444 \(x a\).

The situations of a triangular lattice are shown in Fig. 9d–f. And just like the square lattice, there’s an optimal value for \(d\) for the triangular lattice arrangement. In this part, the width of the 1st PBG is discussed. The peak value of the odd modes, the even modes, and the mixed modes is located in (0.041, 0.0715), (0.1062, 0.0574), and (0.0534, 0.0423), which exhibited in Fig. 9d, e and f, respectively.

### 3.1.4 The influences of the index of dielectric layer \(n\) on the PBGs of square and triangular cases

The influence of refractive index change on PBG is shown in Fig. 10. In this discussion, \(d = 0.05 \times a\), \(h = a\), \(N = 2\). The 1st PBG (bands 2–3) of the odd modes, even modes, and the mixed modes are exhibited in Fig. 10a–c, respectively. The PBGs of the mixed modes are almost determined by the even modes (the bandwidth of the even modes is smaller than the odd modes), which have the same maximum with the location of (3.6, 0.0485). The maximum width of the odd modes with a value of 0.0995 \((2\pi c/a)\) is also reported at \(n = 3.6\).

For the case of the triangular lattice layout, the 1st PBG (bands 0–1) of the even and odd modes will be discussed. The first PBG of the mixed modes is bands 2–3 (odd) and bands 0–1 (even). Similarly, the bandwidth of the odd modes obtained under the triangular lattice arrangement is still large, while the variation trend of the mixed modes is determined by the even modes. As illustrated in Fig. 10d, the maximum bandwidth of the odd modes is 0.0412 \((2\pi c/a)\) and the maximum bandwidth of the mixed modes is 0.0332 \((2\pi c/a)\).

### 3.2 Group velocity discussion of introducing line defects

The semiconductor processing technology is becoming more and more mature. In this paper, GaAs is selected as the material to make waveguides. The lattice constants \(a = 1 \mu m\), \(N = 3\), \(d = 0.05 \times a = 0.05 \mu m\), \(\lambda = 1 \times a = 1 \mu m\). At this wavelength, the refractive index of GaAs is 3.509. The height \(h = 2 \times a = 2 \mu m\) in the square lattice arrangement and \(h = 1.5 \times a = 1.5 \mu m\) in the triangular lattice form. As the above parameters are set, we find a PBG among 0.3462–0.3657 \((2\pi c/a)\) in the case of square lattice layout, and a PBG over 0.3629–0.3946 \((2\pi c/a)\) in the case of triangular lattice settlement. Line defects as shown in Fig. 2a are introduced into the slab, and the values of \(D_x\) and \(D_z\) are changed to calculate the group velocity of the guided modes.

In the case of the square lattice layout, the slow light with the minimum peak value of 0.072 \(c\) in the waveguide is achieved, and the corresponding normalized frequency is 0.352 \((2\pi c/a)\), where \(D_x = 0.5 \times a = 0.5 \mu m\), \(D_z = 0.7 \times a = 0.7 \mu m\).

If \(D_x = D_z = 0.6 \times a = 0.6 \mu m\), the peak value of the \(v_g\) is 0.078 \(c\) with the normalized frequency is 0.349 \((2\pi c/a)\). At the normalized frequency 0.359 \((2\pi c/a)\), the peak value of 0.119 \(c\) is observed with \(D_x = D_z = 0.6 \times a = 0.9 \mu m\). At the case of \(D_x = D_z = 0.6 \times a = 0.8 \mu m\), the peak value is located in (0.348, 0.118), and when
$D_x = 0.5 \times a = 0.5 \, \mu m$, $D_z = 0.7 \times a = 0.8 \, \mu m$, the peak value of $0.19 \times c$ w come into being with the location of $0.352 \left(2\pi c / a\right)$.

Because of the existence of the light cone, the starting point of the guided modes of the planar waveguide is not at the boundary of the Brillouin zone, so the group velocities of the guided modes do not always arise from 0. Since the position of the guided mode appears close to $M = (\pi / a, 0)$ and is approaching the Brillouin zone boundary, the group velocities of the most guided modes are maximized at the beginning and then gradually die down.

The triangular lattice is more beneficial for slowing the speed of the light than the case of the square lattice. The minimum group velocity peak value of $0.026 \times c$ is acquired at

Fig. 10 The features of PBGs varying with different $n$, under square lattice arrangement a the odd modes, b the even modes, c the mixed modes while the triangular lattice layout are depicted in d the odd modes, e the even modes, f the mixed modes. And the units of $d$ is $a$, and $h = a$, $N = 2$. 

$D_x = 0.5 \times a = 0.5 \, \mu m$, $D_z = 0.7 \times a = 0.8 \, \mu m$, the peak value of $0.19 \times c$ w come into being with the location of $0.352 \left(2\pi c / a\right)$.

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The triangular lattice is more beneficial for slowing the speed of the light than the case of the square lattice. The minimum group velocity peak value of $0.026 \times c$ is acquired at
the normalized frequency of 0.375 \( (2\pi c/a) \), corresponding to \( D_x = D_z = 0.6 \times a = 0.6 \) \( \mu m \). When \( D_x = D_z = 0.6 \times a = 0.7 \) \( \mu m \) and \( D_x = D_z = 0.6 \times a = 0.8 \) \( \mu m \), the peak value of the \( v_g \) is suited in \((0.367, 0.065)\) and \((0.385, 0.084)\). In the context of \( D_x = 0.5 \times a = 0.6 \) \( \mu m \), \( D_z = 0.7 \times a = 0.8 \) \( \mu m \), the coordinates of peak value is \((0.378, 0.088)\), and if \( D_x = 0.5 \times a = 0.6 \) \( \mu m \) and \( D_z = 0.7 \times a = 0.9 \) \( \mu m \), the peak value of the \( v_g \) is located in \((0.372, 0.085)\).

Then, the FDTD method is used to analyze the distribution of the electric field in the waveguide. In Fig. 11d, The minimum group velocity peak value of 0.026 \( c \) is acquired at \( D_x = D_z = 0.6 \times a = 0.6 \) \( \mu m \). We use the FDTD method to observe the slow light phenomenon of the waveguide structure.

The waveguide is arranged in a triangular lattice and the side length of the line defect column \( D_x = D_z = 0.6 \times a = 0.6 \) \( \mu m \). The perfectly matched layers are used to be the boundary conditions. This high-loss material will absorb all electromagnetic waves without reflection (Chen et al. 1998; Berenger 1997; Shi et al. 2004). In this model, the light incident at a frequency of 0.373–0.380 \( (2\pi c/a) \).

Figure 12 illustrates the spatial distribution of the electric field component \( E_y \) at a frequency of 0.373 \( (2\pi c/a) \). The width of the field source is the same as that of the waveguides in the \( z \)-axis. The cases of the frequencies of 0.374–0.38 \( (2\pi c/a) \) are illustrated in Fig. 12. We can see that the electromagnetic wave is well restricted around the defect column. And only the electromagnetic waves that propagate along the line defects in the

Fig. 11 a The guided modes of waveguide in a square lattice arrangement, b the group velocity for square lattice arrangement, c the guided modes of waveguide in a triangular lattice arrangement, d the group velocity for triangular lattice arrangement
Fig. 12 The electric field distribution of the waveguide and the transmission curve at the frequency of a–b 0.373 ($2\pi c/a$), c–d 0.374 ($2\pi c/a$), e–f 0.375 ($2\pi c/a$), g–h 0.376 ($2\pi c/a$), and i–j 0.38 ($2\pi c/a$)
waveguide can travel through the system, while light from other locations is reflected. This proves the practical value of the proposed waveguide. Figure 12f shows the curve of waveguide transmittance changing with time at the frequency of 0.375 (2πc/a). Due to the slow light effect of the waveguide structure, the light propagating through the line defect has the delay effect, and the transmittance gradually increases to 100% after a period of time. When the frequency is changed to 0.38 (2πc/a), the light cannot propagate in the waveguide in Fig. 12i–g. Although the frequency falls within the PBG range of 0.3629–0.3946 (2πc/a), it is not the frequency of the guiding mode, shown in Fig. 11d. By comparing the transmission curves of the four frequencies, we find that 0.375 (2πc/a) has the best transmission effect, which is consistent with the results of the group velocity curve in Fig. 10d. This also confirms the practicality of the waveguide proposed by us from one aspect.

4 Conclusion

In this paper, the one-dimensional cylindrical PCs are extended to the two-dimensional level, GaAs material is chosen to fabricate the structure, and the effects of different structural parameters and the variation of incident wavelength on the PBGs characteristics of 2D-PCS are discussed in detail. The height, index, and thickness of the cylinder layer are compromised consideration, and the PBGs width will first augment and then reduce with the increase in the height index and thickness, and the PBGs will move to the low frequencies. Since the first Brillouin region of the two lattices is not identical, the wave vectors along the first Brillouin region are not the same when solving the eigenmodes, and the values of the eigenmodes of the two lattices are not the same, and PBGs will be different. In our model, the maximum bandwidth is obtained with a value of 0.117 (2πc/a). Finally, the line defect is introduced into the slab to construct the W1 waveguide. In the case of square lattice arrangement, the minimum group velocity peak value is 0.072 × c, and in the case of triangular lattice layout, it is 0.026 × c. As it turns out, the triangular lattice arrangement is more suitable for slowing down the speed of light. The results of FDTD show that the defect mode can propagate well along the defect line.

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