Asymptotic expansion in the fission process

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Asymptotic expansion of the Fokker-Planck equation in terms of the strength of the fluctuation has been carried out. The mean and the variance of the total kinetic energies of the fission fragments have been calculated and compared with the experimental values.

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I. INTRODUCTION

At present it is commonly agreed upon that the fission process is a dissipative phenomena, where initial energy of the collective variables get dissipated into the internal degrees of freedom of nuclear fluid giving rise to the increase in internal excitation energy causing evaporation of a large number of precision neutrons, which can not be explained by standard statistical model calculations. As dissipation is referred to the interaction of the system coordinate with the large number of degrees of freedom of the surrounding reservoir, this process is always associated with the fluctuation of relevant physical observable. This mesoscopic description is inevitable once the fluctuations of the observables are amenable to experimental observation.

Keeping this fact in mind, the fission process is pictured as follows; the collective variable such as elongation axis acts as a 'Brownian particle' interacting stochastically with large number of internal nucleonic degrees of freedom constituting the surrounding 'bath'. Several workers solve either the Langevin equation [1–3], or multidimensional Fokker-Planck equation [4,5], which is a differential version of Langevin equation, in order to study the time evolution of probability distribution function and have calculated the mean as well as the variance of the total kinetic energies of the fission fragments.

The experimental data show that the root mean square deviation of kinetic energy is always very small compared to the mean kinetic energy (∼ 0.1). The question naturally arises whether one can utilise this simple fact in the theoretical scheme instead of solving the Langevin equation or corresponding Fokker-Planck equation in detail. In this spirit, we assume that the full solution of the Fokker-Planck equation admits an asymptotic expansion in terms of strength of the fluctuation. Admittedly, when the fluctuation is ignored one obtains the deterministic picture and the machinery to handle such situation is to solve the Euler-Lagrange equation of motion. This was done in our previous works [6–8], where dissipation was generated through nonconservative Rayleigh function and the total kinetic energies, the fission yields and neutron multiplicities were calculated. However, in the above scheme, there had been no scope to study the effect of fluctuations originating from the stochastic dynamics of fission on fission observables. Here, we report a modified scheme where fluctuations have been included in the fission dynamics by making asymptotic expansion of the probability distribution function in terms of intensity or strength of the fluctuations as argued before. Thus, the picture we adopt here, is as follows: Due to smallness of the relative fluctuation, the process is grossly described in terms of macroscopic equation. The stochastic description is introduced by studying the evolution of the probability distribution of narrow width over its deterministic values.

In what follows, in Sec.II we describe briefly the procedure and derive the equations of the moments. The results of the calculations are discussed in Sec.III. Finally, concluding remarks are given in Sec.IV.

II. ASYMPTOTIC EXPANSION OF THE FOKKER-PLANCK EQUATION

A. The method

As argued before, the mesoscopic description of the fission process begins with a set of Langevin equations:

\[ \dot{X}_i = h_i\{X\}, \{Y\} + \eta_i(t) \]  (2.1a)

\[ \dot{Y}_i = H_i\{X\}, \{Y\} \quad : i = 1, 2, 3, ..., N \]  (2.1b)

where \( h_i \) and \( H_i \) are given functions of the stochastic collective variables \( X_1, X_2, ..., X_N \) and \( Y_1, Y_2, ..., Y_N \) in the fission process and \( \eta_i(t) \) refers to the driving noise term associated with the interaction of the \( i \)th collective variable.
with the reservoir constituting nucleonic degrees of freedom. For simplicity, we assume the noise to be a gaussian white with zero mean and decoupled for different degrees of freedom with auto-correlation functions given by

\[ < \eta_i(t) > = 0, \quad < \eta_i(t) \eta_j(t') > = D_i(y_i) \delta(t - t') \delta_{ij}, \]  

(2.2)

where \( D_i(y_i) \) is the diffusion coefficient associated with \( i \)th variable, depending only on the sample space \( y_i \) for the stochastic variable \( Y_i \).

The Fokker-Planck equation corresponding to the Langevin equation (2.1) is

\[
\frac{\partial f(x, y, t)}{\partial t} = - \sum_i \left[ \frac{\partial(h_i f)}{\partial x_i} + \frac{\partial(H_i f)}{\partial y_i} - (1/2)D_i(y_i) \frac{\partial^2 f}{\partial x_i^2} \right].
\]  

(2.3)

The quantity \( f(x, y, t) \) is the probability density function depending on the variables \( x_1, x_2, ..., x_N, y_1, y_2, ..., y_N \) and time \( t \) explicitly. If we are interested in finding the time evolution of the conditional probability distribution function then we have to solve Eq.(2.3) with initial values \( x_i(0) = x_i^0, y_i(0) = y_i^0, \forall i, \) at \( t = 0 \). That is, we have to solve Eq.(2.3) for those realisations which are known to start from these specific points in the whole sample space.

The asymptotic expansion was developed by van Kampen [9] for constant diffusion constant. The method consists of writing the stochastic variables as the sum of deterministic value and a fluctuating part at each time.

Next, we introduce the new distribution function \( Q(x, y, t) \) around their deterministic values \( \{ \bar{x}_i \}, \{ \bar{y}_i \} \). Next, we introduce the new distribution function \( Q \) depending only on the variables \( \{ \zeta_i \}, \{ \xi_i \} \) and \( t \). The normalisation condition suggests that the \( f \) and \( Q \) will be related by

\[ f(x, y, t) = \prod_{i=1}^N [D_i(\bar{y}_i)]^{-1} Q(\{ \zeta_i \}, \{ \xi_i \}, t) \]  

(2.5)

Substituting Eq.(2.4) in the Fokker-Planck equation(2.3), making Taylor expansion of \( h(\{ x \}, \{ y \}), H(\{ x \}, \{ y \}) \) around \( \{ \bar{x} \}, \{ \bar{y} \} \) and collecting coefficients of various order of \( D(\bar{y}_i) \), we could generate a hierarchy of equations. As expected, the first set would give rise to the equation of motion for \( \{ \bar{x} \} \) and \( \{ \bar{y} \} \).

\[
\dot{x}_i = h_i(\{ \bar{x} \}, \{ \bar{y} \}) \quad i \]  

(2.6a)

\[
\dot{y}_i = H_i(\{ x \}, \{ y \}) \quad : \forall i
\]  

(2.6b)

Eqs.(2.6) are the Euler-Lagrange equation for deterministic motion. These equations are to be solved with initial conditions \( \{ \bar{x}(0) \} = \{ x^0 \}, \{ \bar{y}(0) \} = \{ y^0 \} \). Next, we are going to calculate the conditional probability distribution \( f(\{ x \}, \{ y \}, t \mid \{ x^0 \}, \{ y^0 \}, 0) \) or \( Q(\{ \zeta \}, \{ \xi \}, t \mid 0, 0, 0) \).

Assuming the variation of diffusion coefficient over the narrow width of the distribution function at any instant of time to be \( \bigcirc(D) \), we could replace the second Fokker-Planck coefficient \( D(y) \) by \( D(\bar{y}) \) at each instant of time. This assumption makes the calculation extremely simple. Collecting coefficients \( \bigcirc(D^0) \), we get back quasilinear Fokker-Planck equation for \( Q \):

\[
\frac{\partial Q}{\partial t} + \sum_i \left[ \frac{\partial(\bar{y}_i) Q}{\partial \bar{y}_i} \right] Q = - \sum_i \left[ a_i \frac{\partial(\zeta_i Q)}{\partial \zeta_i} + b_i \frac{\partial(\xi_i Q)}{\partial \zeta_i} + c_i \frac{\partial(\zeta_i Q)}{\partial \xi_i} + d_i \frac{\partial(\zeta_i Q)}{\partial \xi_i} - (1/2) \frac{\partial^2 Q}{\partial \zeta_i^2} \right].
\]  

(2.7)

where \( a_i, b_i, c_i, d_i \) are given by

\[
a_i = \left( \frac{\partial h}{\partial \bar{x}_i} \right) \bigg| \frac{\bar{D}(\bar{y}_i)}{2D(\bar{y}_i)} \]  

(2.8a)

\[
b_i = \frac{\partial h}{\partial \bar{y}_i}
\]  

(2.8b)
\[ c_i = \frac{\partial H}{\partial \tilde{y}_i} - \left( \frac{\dot{D}(\tilde{y}_i)}{2D(\tilde{y}_i)} \right) \]  
\[ d_i = \frac{\partial H}{\partial \tilde{x}_i} \]  

Eq.(2.7) suggests that 

\[ Q(\{\zeta\}, \{\xi\}, t) = \prod_j Q_j(\zeta_j, \xi_j, t) \]  

where the distribution function \( Q_j \) for each \( j \) satisfies the similar equation written below without the subscript:

\[ \frac{\partial Q}{\partial t} + \left( \frac{\dot{D}(\tilde{y})}{D(\tilde{y})} \right) Q = -[a \frac{\partial (\xi Q)}{\partial \xi} + b \frac{\partial (\zeta Q)}{\partial \zeta} + c \frac{\partial (\xi Q)}{\partial \xi} + d \frac{\partial (\zeta Q)}{\partial \zeta} - (1/2) \frac{\partial^2 Q}{\partial \zeta^2}] \]  

subject to the initial condition

\[ Q(\zeta, \xi, t = 0) = \delta(\zeta)\delta(\xi) \]  

The solution of Eq.(2.10) is given by

\[ Q(\zeta, \xi, t) = \int \int \exp \left\{ ik\zeta + il\xi + \left[ \frac{g(t)k^2 + G(t)l^2 + 2c(t)kl}{2D(t)} \right] \right\} dkdld \]  

where \( g(t), G(t), c(t) \) satisfy the set of coupled first order differential equations:

\[ \frac{\dot{g}}{2} = \left( \frac{\partial h}{\partial x} \right) g + \left( \frac{\partial h}{\partial y} \right) c + \frac{D}{2} \]  
\[ \frac{\dot{G}}{2} = \left( \frac{\partial H}{\partial y} \right) G + \left( \frac{\partial H}{\partial x} \right) c \]  
\[ \dot{c} = \left( \frac{\partial h}{\partial x} \right) c + \left( \frac{\partial h}{\partial y} \right) G + \left( \frac{\partial H}{\partial y} \right) c + \left( \frac{\partial H}{\partial x} \right) g \]  

with the initial conditions

\[ g(0) = G(0) = c(0) = 0 \]  

Once \( Q(\zeta, \xi, t) \) is known, from Eq.(2.9) and Eq.(2.5) the full conditional probability distribution function \( f(\{x\}, \{y\}, t | \{x^0\}, \{y^0\}, 0) \) is known. Integrating this function over all variables except one, say \( x_i \), one identifies \( g_i(t) \) as the variance of the stochastic variable \( X_i \).

\[ < (X_i - \langle X_i \rangle)^2 > = g_i(t) \]  

We note that the homogeneity of Eq.(2.10) suggests that \( < \zeta(t) > = < \xi(t) > = 0 \), or the average of the variables \( X \) and \( Y \) at any time will be determined by their deterministic values obtained by solving Euler-Lagrange equation (2.6). Similarly one observes from Eq.(2.12),

\[ < (X_i - \langle X_i \rangle) > < (Y_i - \langle Y_i \rangle) > = c_i(t) \]  
\[ < (Y_i - \langle Y_i \rangle)^2 > = G_i(t) \]

**B. Application to the fission process**

In the fission process, we choose the elongation axis and velocity associated with it as the stochastic variables interacting with a large number of internal nucleonic degrees of freedom constituting a heat bath at temperature \( T \) determined by the excitation energy available to it. We further assume that the ‘collisional’ time scale of the nucleonic degrees of freedom is much shorter than the time scale of the macroscopic evolution of the collective variable so that at each instant of time the heat bath is assumed to be in quasi-stationary equilibrium.
As stated before the Euler-Lagrange equation(2.6) was solved in our earlier works [8]. To avoid repetition we deliberately omit the procedure and scheme to solve that equation. For the sake of completeness we merely write that equation and refer to our previous paper to clarify the details.

Giving correspondence to the terminology used in this paper, we associate

\[ Y = r, X = \dot{r} \]

Thus we have

\[ H(x, y) = x = \dot{r} \] (2.18a)

\[ h(r, \dot{r}) = \frac{L^2}{\mu r^3} - \gamma \dot{r} - \frac{\partial (V_C + V_N)}{\partial r} \bigg|/\mu \] (2.18b)

The quantities \( V_C, V_N \) represent the Coulomb and nuclear interaction potentials, respectively. The quantity \( \gamma, \mu \) and \( L \) refer to friction coefficient, reduced mass associated with the fissioning liquid drop and the relative angular momentum, respectively [8].

While solving for variances we appeal to Eq.(2.13). In that set the diffusion coefficient \( D \) is evaluated employing Einstein’s fluctuation dissipation theorem. The set thus reads

\[
\begin{align*}
\dot{g}(t) &= 2 \frac{\partial h}{\partial r} g(t) + 2 \frac{\partial h}{\partial r} c(t) + 2 \gamma(r) T(r)/\mu^2 \\
\dot{G}(t) &= 2 c(t) \\
\dot{c}(t) &= \frac{\partial h}{\partial r} c(t) + 2 \frac{\partial h}{\partial r} G(t) + g(t)
\end{align*}
\] (2.19a)

(2.19b)

(2.19c)

with the initial conditions (2.14). The initial conditions of \( r \) and \( \dot{r} \) for solving Eq.(2.18) are [8]

\[ r(t=0) = r_{\text{min}}, \dot{r}(t=0) = \left( \frac{E^* R_N}{2\mu} \right)^{1/2} \] (2.20)

where the fissioning nucleus starts from the minimum of the potential energy surface, the quantity \( R_N \) is a random number between 0 and 1 from uniform probability distribution and \( E^* \) is the available energy.

Solving Eq.(2.18) and Eqs.(2.19) simultaneously with initial conditions (2.14) and (2.20) we generate the conditional probability distribution function \( f(r, \dot{r}, t | r(t=0), \dot{r}(t=0), 0) \). The probability distribution function \( f(r, \dot{r}, t) \) could be obtained as

\[ f(r, \dot{r}, t) = \int f(r, \dot{r}, t | r(t=0), \dot{r}(t=0), 0) f(r(t=0), \dot{r}(t=0), 0) dr(t=0) d\dot{r}(t=0) \] (2.21)

where \( f(r(t=0), \dot{r}(t=0), 0) \) is the probability distribution of position and velocity of the stochastic variables at the initial time. As described by the initial condition (2.20), this can be represented as

\[ f(r(t=0), \dot{r}(t=0), 0) = \delta(r(t=0) - r_{\text{min}}) \times f(\dot{r}(t=0)) \] (2.22)

Here, we assumed that each fissioning nucleus in the ensemble starts from a fixed initial position but with different partioning of initial excitation energy [8]. Finally, substitution of Eq.(2.22) in Eq.(2.21) would give

\[ f(r, \dot{r}, t) = \sum_{R_N} f(r, \dot{r}, t | r_{\text{min}}, \left( \frac{E^* R_N}{2\mu} \right)^{1/2}, 0) \] (2.23)

### III. RESULTS AND DISCUSSIONS

The temporal evolutions of the variables \( g(t) \), \( c(t) \), \( G(t) \) along the fission trajectory have been computed by solving numerically the set of Eqs. (2.19c). The results are plotted in Fig. 1 for a representative system \( ^{16}\text{O} + ^{124}\text{Sn} \). It is seen from the figure that, initially, all of them increase steeply and then their magnitudes become nearly constant throughout the rest of the trajectory. Furthermore, the calculation shows that
As envisaged earlier, the result clearly shows that is also seen to increase steeply at the beginning and then it becomes nearly constant throughout the rest of the time.

\[
\frac{c^2(t)}{g(t)G(t)} \ll 1
\]  

(3.1)

This implies that the correlation of position and velocity of the elongation variable (\(r\)) is much smaller compared to their respective variances. This fact simplifies our calculation of variance of energy of the fission fragments. The variance of energy and average of kinetic energy at scission point are given by

\[
\sigma_E^2 = \langle \mu \dot{r} \rangle^2 g(t) + [\frac{\partial(V_C + V_N)}{\partial r}]^2 G(t)
\]  

(3.2a)

\[
\langle E(t) \rangle = \mu g(t_{sc})/2 + E_{det}
\]

(3.2b)

where \(t_{sc}\) is the time at scission point and \(E_{det}\) is the deterministic value of total fragment kinetic energy (TKE) after scission and \(\sim 100 - 200\) MeV. In deriving the above approximate results Eqs. (3.2), we utilise our observation (3.1). It is further assumed that the variation of the potential over the narrow width of the probability distribution is small so that the average of the potential is approximated as the value of the potential at the mean position. The variation of the kinetic energy variance \(\sigma_E^2\) as a function of time has also been displayed in Fig. 1. The value of \(\sigma_E^2\) is also seen to increase steeply at the beginning and then it becomes nearly constant throughout the rest of the time.

As envisaged earlier, the result clearly shows that \(\frac{\sigma_E^2}{\mu g(t_{sc})} \ll 1\), which demonstrates the validity of asymptotic expansion in deriving the result instead of solving the Fokker-Planck equation in detail.

The theoretical predictions of \(\sigma_E(th)\) for the fission of several compound systems produced in the 158.8 MeV \(^{18}\)O and 288 MeV \(^{16}\)O reactions on various targets have been displayed in Fig. 1 along with the respective experimental data \([10]\) for comparison. The experimental data \(\sigma_E(exp)\) is represented by the filled circles and the solid curves are the results of the present calculations. It is seen from Fig. 1 that when the projectile energy (and vis-a-vis the excitation energy of the fused composite) is relatively lower (lower half), the calculated values are in fair agreement with the data. However, the calculation underpredicts the experimental value of \(\sigma_E\) for the heaviest target considered (\(^{238}\)U in the present case). With the increase in the projectile energy (and the excitation energy of the composite) (upper half of Fig. 1), the theoretical predictions are found to underestimate the corresponding experimental values and the discrepancy between the two increases with the increase in mass number. We have also studied the fragment mass asymmetry dependence of energy variance, \(\sigma_E^2(A_1, A_2)\) for some representative systems and the results are displayed in Fig. 3. It is seen from Fig. 3 that the theoretical values of variances have only a weak dependence on the fragment mass asymmetry.

In order to investigate into the discrepancy between the predicted and the experimental values of the TKE variance, it is observed that the experimental values of \(\sigma_E(exp)\) are usually obtained by averaging over the full mass yield spectrum. Therefore, \(\sigma_E(exp)\) consists of two terms, viz., (i) contributions arising due to stochastic fluctuations in the dynamics of fission process and (ii) contributions from the variation of the mean kinetic energy with the fragment mass asymmetry. So, \(\sigma_E(exp)\) may be written as [11].

\[
\sigma_E^2(exp) = \sigma_E^2 + \sigma_E^2(kin)
\]

(3.3a)

\[
\sigma_E^2 = \sum_{A_1} \sigma_E^2(A_1, A_2) \cdot Y(A_1)
\]

(3.3b)

\[
\sigma_E^2(kin) = \sum_{A_1} \left( \bar{E} - \langle E(A_1, A_2) \rangle \right)^2 \cdot Y(A_1)
\]

(3.3c)

where \(\sigma_E^2(A_1, A_2)\) and \(< E(A_1, A_2) >\) are the variances and mean values of the total kinetic energy of two fission fragments with mass numbers \(A_1\) and \(A_2\) (compound nucleus mass \(A_{CN} = A_1 + A_2\), \(\bar{E}\) being the average of \(< E(A_1, A_2) >\) over the normalised fragment mass yield, \(Y(A_1)\) with \(\sum_{A_1} Y(A_1) = 1\). It is, therefore, clear that for a proper comparison of the theoretical predictions of kinetic energy variance with the relevant experimental data, the theoretical numbers should be averaged over the respective fragment mass distribution. Moreover, the calculated value of \(\sigma_E^2(th)\) should be compared with the extracted experimental value of \(\sigma_E^2\) obtained by subtracting \(\sigma_E^2(kin)\) from \(\sigma_E^2(exp)\).

We have calculated \(\sigma_E^2(kin)\) for a few systems for which the experimental fragment mass yield data are available [11], taking \(< E(A_1, A_2) >\) from Viola systematics [12]. The results are given in Table 1 along with the values of \(\sigma_E^2\) extracted from the experimental data. It is evident from the Table 1 that the contribution to the variance from the difference of mean TKE with fragment mass asymmetry, \(\sigma_E^2(kin)\), increases with the increase in excitation energy and mass of the composite. For the lighter system e.g. \(O + Ag\), it is observed that this contribution is quite large due to the asymmetric nature of fission. Further, it is seen from the Table 1 that there is a very good agreement between the \(\sigma_E(th)\) and the \(\sigma_E\) extracted from experiment for the systems for which the mass fragment yield data are available. Moreover, the values \(\sigma_E\), i.e., (\(\sigma_E^2(exp) - \sigma_E^2(kin)\)), are also shown in Fig. 2 as open triangles and they agree very well with the predicted values of TKE variance.
IV. SUMMARY AND CONCLUSIONS

We have developed asymptotic expansion of the Fokker-Planck equation for the systems in which the relative fluctuation of collective variable is small and have derived the equation of variances. The formalism is applied to the case of fission where the fluctuation in total kinetic energy is small as compared to its mean value. The calculated variances, \( \sigma_E(\text{th}) \), underpredict the experimental data, \( \sigma_E(\text{exp}) \). The discrepancy between the two increases with the increase in the mass of the composite and the excitation energy of the composite. However, we emphasize that this discrepancy is not very surprising because the experimental values contain additional contributions from variation of the mean kinetic energy over full fragment mass yield distributions. Once this contribution is properly taken care of, the predicted TKE variances are found to agree quite well with the \( \sigma_E \) extracted from the experimental data for the systems where the fragment mass yield data are available. Therefore, for a more direct test of theoretical models it is necessary that experimental estimation of variances should not have admixture of other contributions arising due to the variation of mean kinetic energy over different mass yields. This may be achieved if measurements are done in smaller mass bins.

Thus, it may be concluded that the present approach is quite successful in reproducing the extracted TKE variances from the experimental data without going in to solving the Fokker-Planck equation in detail. In the present studies, the correlation of the position and velocity of the elongation axis has been found to be small. However, in the cases where such condition is not valid the energy variance still can be calculated by adding a term \( 2\mu^2 H(x,y) \) to \( \sigma^2 \), where \( H(x,y) \) is the function of the variables.

The procedure developed here could systematically generate higher order hierarchies for relatively larger fluctuations than the ones encountered in the present studies. In those cases one may have to solve the higher order equation which involve higher order derivatives of the functions \( h(x,y) \) and \( H(x,y) \), in general.

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FIG. 1. Variation of \( g(t) \), \( c(t) \), \( G(t) \) and \( \sigma_E^2 \) as a function of time \( t \) for the system \(^{16}\text{O} + ^{124}\text{Sn}\).

FIG. 2. Variation of \( \sigma_E \) as a function of target mass number, for 288 MeV \(^{16}\text{O} \) (upper half), and 158.8 MeV \(^{18}\text{O} \) (lower half) induced fission reactions. Filled circles correspond to the experimental data [10] and solid lines correspond to the present theoretical results. Open triangles are the modified results using Eq. (3.3) (see text).

FIG. 3. Variation of predicted values of \( \sigma_E \) as a function of fragment mass asymmetry, \( a_{\text{sym}} = |A_1 - A_2|/A_{\text{CN}} \).

| System | \( E_{\text{lab}} \) (MeV) | \( \sigma_E^2(\text{exp}) \) (MeV\(^2\)) | \( \Sigma_{A_1}(<E(A_1, A_2) > -E)^2Y(A_1) \) a | \( \sigma_E^2 \) (MeV\(^2\)) | \( \sigma_E \) (MeV) | \( \sigma_E(\text{th}) \) (MeV) |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|
| \(^{16}\text{O} + ^{124}\text{Sn}\) | 158.8 | 112.3 | 10.1 | 102.3 | 10.1 | 9.7 |
| \(^{18}\text{O} + ^{197}\text{Au}\) | 158.8 | 190.4 | 15.2 | 175.2 | 13.2 | 11.9 |
| \(^{16}\text{O} + ^{197}\text{Au}\) | 288.0 | 331.2 | 49.7 | 281.5 | 16.7 | 13.3 |

TABLE I. Calculation of TKE variance.
\begin{tabular}{ccccccc}
\hline
$^{16}$O + $^{109}$Ag & 288.0 & 225.0 & 142.8 & 82.2 & 9.0 & 8.3 \\
\hline
\end{tabular}

"See Eq.(3.3)"
