Abstract. In general there are a large number of light scalar fields in the theories going beyond the standard model, such as string theory, and some of them can be considered as curvatons candidates. For simplicity, we assume that all curvatons have the same decay rate and suddenly decay into radiation at the same time. In order to distinguish this scenario from the more general case, we call it the ‘$N$-vaton’ scenario. We use $\delta N$ formalism to calculate the primordial power spectrum and bispectrum in the $N$-vaton model and investigate various bounds on the non-Gaussianity parameter $f_{NL}$. A red-tilted primordial power spectrum and a large value of $f_{NL}$ can be obtained naturally if the curvature perturbation generated by inflaton also makes a significant contribution to the primordial power spectrum. For a realistic $N$-vaton model, we suppose that the axions in the Kachru–Kallosh–Linde–Trivedi compactifications of type IIB string theory are taken as the curvatons and a rich phenomenology is obtained.

Keywords: CMBR theory, cosmological perturbation theory, string theory and cosmology, physics of the early universe

ArXiv ePrint: 0807.1567
1. Introduction

Most inflation models predict a nearly Gaussian distribution of the primordial curvature perturbation. Deviations from an exactly Gaussian distribution are characterized by a dimensionless parameter $f_{\text{NL}}$ [1]. In the case of the single-field inflation model, $f_{\text{NL}} \sim \mathcal{O}(n_s - 1)$ [2], which is constrained by WMAP ($n_s = 0.960^{+0.014}_{-0.013}$) [3] to be much less than unity. A Gaussian distribution of the primordial curvature perturbation is still consistent with WMAP five-year data [3]:

$$-9 < f_{\text{NL}}^{\text{local}} < 111 \quad \text{and} \quad -151 < f_{\text{NL}}^{\text{equil}} < 253 \quad (95\% \text{ CL}),$$

where ‘local’ and ‘equil’ indicate the shapes of the non-Gaussianity. In [4] the authors reported that a positive large non-Gaussianity

$$27 < f_{\text{NL}}^{\text{local}} < 147$$

is detected at 95% C.L. Planck is expected to bring the uncertainty of $f_{\text{NL}}^{\text{local}}$ to less than 5 [5]. If a large value of $f_{\text{NL}}$ is confirmed by the forthcoming cosmological observations, the simplest model of inflation will be ruled out and some very important new physics of the early universe will show up.

In general a large number of light scalar fields are expected in the theories beyond the standard model, such as string theory. The consistent perturbative superstring theory can only live in ten-dimensional spacetime. To connect string theory with experiments, string theory must be compactified on some six-dimensional manifold and many dynamical moduli fields emerge in four dimensions. The typical number of moduli...
fields is \( N \sim O(10^2-10^3) \). One can expect the expectation values of some of these scalar fields to be displaced from the minimum of their potential due to the quantum fluctuations during inflation. Usually these scalar fields are subdominant during inflation and their fluctuations are initially of isocurvature type. After the end of inflation they are supposed to completely decay into thermalized radiation before primordial nucleosynthesis and thus the isocurvature perturbations that they generate are converted to final adiabatic perturbations. These scalar fields are called curvatons.

A curvaton mechanism for generating an initially adiabatic perturbation deep in the radiation era is proposed in [6]–[9]. The primordial curvature perturbation in a curvaton model with a single curvaton has been discussed in [6]–[13]. If many curvatons make contributions to the primordial density perturbation, the calculation becomes much more complicated [14]–[16]. Since a curvaton model can give a large positive local-type non-Gaussianity, recently some topics related to curvaton models have been discussed [17]–[24].

In the single-curvaton model \( f_{\text{NL}} \) is inversely proportional to the fraction of curvaton energy density in the energy budget at the epoch of curvaton decay. The smaller the energy density of the curvaton, the larger the non-Gaussianity. Since the curvaton mass is smaller than the Hubble parameter \( H_* \) during inflation, or equivalently its Compton wavelength is large compared to the curvature radius of the de Sitter space \( H_*^{-1} \), the gravitational effects play a crucial role in the behavior of the curvaton field in such a scenario. The typical energy density of the curvaton field in such a background is roughly \( H_*^4 \), which leads to an upper bound on \( f_{\text{NL}} \) [17]:

\[
\frac{f_{\text{NL}}}{\text{order} 1 < 522 \cdot r^{1/4}}
\]

Since a multiplicity of scalar fields is generally expected, we focus on the multi-curvaton scenario in this paper. Here we consider a special case in which all of the curvatons have the same decay rate and their masses are larger than the decay rate. In order to simplify the calculation of the primordial curvature perturbation, we assume that all of curvatons suddenly decay into radiation at the same time. We give a name, the ‘\( N \)-vaton’ scenario, to this. As a realistic \( N \)-vaton model, the axions in the KKLT compactification of type IIB string theory are suggested to be curvatons. On the basis of the random matrix theory, the mass spectrum of the axion obeys the Marcenko–Pastur law and a rich phenomenology of this model is shown.

Our paper is organized as follows. In section 2, we use the \( \delta \mathcal{N} \) formalism [25]–[27] to calculate the primordial curvature perturbation on large scales in the \( N \)-vaton model. The various bounds on \( f_{\text{NL}} \) are discussed in section 3. In section 4 we consider a more general case where the curvature perturbation generated by inflaton cannot be ignored and we find that the spectral index of the primordial power spectrum can be red-tilted naturally. In section 5, we propose a realistic \( N \)-vaton model in which the axions in the KKLT compactification of type IIB string theory are taken as curvatons. At the end we give some discussion of the \( N \)-vaton model in section 6.

## 2. Primordial curvature perturbation

In this paper we consider that the inflaton \( \phi \) and curvatons \( \sigma_i \) are decoupled from each other. The action takes the form

\[
S = \frac{M^2}{2} \int d^4 x \sqrt{-g} R + \int d^4 x \sqrt{-g} \left[ \frac{1}{2} \dot{\phi}^2 + \sum_{i=1}^{N} \frac{1}{2} \dot{\sigma}_i^2 - V(\phi, \sigma_i) \right],
\]

(2.1)
where \( M_p = 2.438 \times 10^{18} \) GeV is the reduced Planck scale and the potential \( V(\phi, \sigma_i) \) is given by
\begin{equation}
V(\phi, \sigma_i) = V(\phi) + \frac{1}{2} \sum_{i=1}^{N} m_i^2 \sigma_i^2.
\end{equation}

During inflation the total energy density is dominated by the inflaton potential \( V(\phi) \) and the dynamics of the system is described by the equation of motion of the inflaton and the Friedmann equation:
\begin{equation}
\ddot{\phi} + 3H \dot{\phi} + \frac{dV(\phi)}{d\phi} = 0,
\end{equation}
\begin{equation}
H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{3M_p^2} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right).
\end{equation}

We also define some slow-roll parameters, such as
\begin{equation}
\epsilon = \frac{M_p^2}{2} \left( \frac{V'(\phi)}{V(\phi)} \right)^2, \quad \eta = M_p^2 \frac{V''(\phi)}{V(\phi)}.
\end{equation}

If \( \epsilon \ll 1 \) and \( |\eta| \ll 3 \), inflaton slowly rolls down its potential.

In this paper we expand any field or perturbation at each order \( (n) \) as follows:
\begin{equation}
\zeta(t, x) = \zeta^{(1)}(t, x) + \sum_{n=2}^{\infty} \frac{1}{n!} \zeta^{(n)}(t, x).
\end{equation}

We assume that the first-order term \( \zeta^{(1)} \) is Gaussian and higher order terms describe the non-Gaussianity of the full non-linear \( \zeta \). Working in the framework of Fourier transformation of \( \zeta \), the primordial power spectrum \( P_\zeta \) is defined by
\begin{equation}
\langle \zeta(k_1)\zeta(k_2) \rangle = (2\pi)^3 P_\zeta(k_1) \delta^3(k_1 + k_2),
\end{equation}
and the primordial bispectrum takes the form
\begin{equation}
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^3 B_\zeta(k_1, k_2) \delta^3(k_1 + k_2 + k_3).
\end{equation}

The amplitude of the bispectrum relative to the power spectrum is parameterized by the non-Gaussianity parameter \( f_{NL} \), i.e.
\begin{equation}
B_\zeta(k_1, k_2) = \frac{6}{5} f_{NL} [P_\zeta(k_1)P_\zeta(k_2) + 2 \text{ permutations}].
\end{equation}

The primordial density perturbation can be described in terms of the non-linear curvature perturbation on uniform density hypersurfaces [28]
\begin{equation}
\zeta(t, x) = \delta N(t, x) + \frac{1}{3} \int_{\rho(t)}^{\rho(t, x)} \frac{d\bar{\rho}}{\bar{\rho} + \bar{\rho}},
\end{equation}
where \( N = \int H \, dt \) is the integrated local expansion, \( \bar{\rho} \) is the homogeneous density in the background model, \( \bar{\rho} \) is the local density and \( \bar{\rho} \) is the local pressure.

After inflation, the inflaton decays into radiations which dominate the total energy density of our universe. In the radiation dominated era the Hubble parameter \( H \) goes like \( \sim a^{-2} \). Once the Hubble parameter drops below the mass of the curvaton field the field
The \( N \)-vaton model starts to oscillate. Non-linear evolution of the values of the curvatons on a large scale is possible if the potential of the curvatons deviates from a purely quadratic potential away from their minima \([29,30]\). Thus, in general, the initial amplitude of curvaton oscillations \( \sigma_{i,o} \) is some function of the field value \( \sigma_{i,*} \) at the Hubble exit\(^1\):

\[
\sigma_{i,o} = g_i(\sigma_{i,*}). \tag{2.11}
\]

When the curvaton starts to oscillate about the minimum of its potential, but before it decays, it behaves like pressureless dust \((\rho_{\sigma_{i,o}} \sim a^{-3})\) and the non-linear curvature perturbation on uniform curvaton density hypersurfaces is given by

\[
\zeta_{\sigma_{i,o}}(t, x) = \delta N(t, x) + \int_{\bar{\rho}_{\sigma_{i,o}}(t)}^{\rho_{\sigma_{i,o}}(t, x)} \frac{d\rho_{\sigma_{i,o}}}{3\bar{\rho}_{\sigma_{i,o}}}. \tag{2.12}
\]

The curvaton density on spatially flat hypersurfaces is

\[
\rho_{\sigma_{i,o}}|_{\delta N=0} = e^{3\zeta_{\sigma_{i,o}}} \bar{\rho}_{\sigma_{i,o}}. \tag{2.13}
\]

The quantum fluctuations in a weakly coupled field, such as a curvaton, at Hubble exit during inflation are expected to be well described by a Gaussian random field \([31]\). So we have

\[
\sigma_{i,*} = \bar{\sigma}_{i,*} + \delta \sigma_{i,*}, \tag{2.14}
\]

without higher order non-linear terms. During the curvaton oscillation we expand the energy density \( \rho_{\sigma_{i,o}} = \frac{1}{2}m_i^2\bar{\sigma}_{i,o}^2 \) and \( \zeta_{\sigma_{i,o}} \) to second order:

\[
\rho_{\sigma_{i,o}} = \bar{\rho}_{\sigma_{i,o}} \left[ 1 + 2X_i + (1 + h_i)X_i^2 \right], \tag{2.15}
\]

\[
\zeta_{\sigma_{i,o}} = \zeta_{\sigma_{i,o}}^{(1)} + \frac{1}{2}\zeta_{\sigma_{i,o}}^{(2)}, \tag{2.16}
\]

where \( \bar{\rho}_{\sigma_{i,o}} = \frac{1}{2}m_i^2\bar{\sigma}_{i,o}^2 \), \( \bar{\sigma}_{i,o} \equiv g_i(\sigma_{i,*}) \), and

\[
X_i = \frac{\delta \sigma_{i,o}^{(1)}}{\bar{\sigma}_{i,o}}, \tag{2.17}
\]

\[
h_i = \frac{g_i g_i''}{g_i^2}. \tag{2.18}
\]

Here the prime denotes the derivative with respect to \( \sigma_{i,*} \). Order by order, from equation (2.13) we have

\[
\zeta_{\sigma_{i,o}}^{(1)} = \frac{2}{3}X_i, \tag{2.19}
\]

\[
\zeta_{\sigma_{i,o}}^{(2)} = -\frac{3}{2} (1 - h_i) \left( \zeta_{\sigma_{i,o}}^{(1)} \right)^2. \tag{2.20}
\]

In the \( N \)-vaton model, we assume that the curvatons have the same decay rate \( \Gamma_{\sigma} \). When the Hubble parameter drops below \( \Gamma_{\sigma} \), all of the curvatons decay into radiations. In order to get analytic expressions, we work in the sudden-decay approximation which means that all of the curvatons suddenly decay into radiations at the time \( t_D \) when \( H = \Gamma_{\sigma} \).

\(^1\) In this paper the subscript \(*\) denotes a quantity evaluated at the Hubble exit.
For simplicity, we assume $m_i > \Gamma_\sigma$ for $i = 1, 2, \ldots, N$ and then all of the curvatons begin oscillating before they decay.

The curvaton decay hypersurface is a uniform density hypersurface and thus from equation (2.10) the perturbed expansion on this hypersurface is $\delta N = \zeta$, where $\zeta$ is the total curvature perturbation at the curvaton decay hypersurface. Before the curvatons decay, there have been radiations produced by decay of the inflaton. Since the equation of state of the radiation is $p_r = \rho_r / 3$, the curvature perturbation related to radiations is

$$\zeta_r = \zeta + \frac{1}{4} \ln \frac{\rho_r}{\bar{\rho}_r}.$$  (2.21)

The curvatons behave like pressureless dust ($p_\sigma = 0$) and thus

$$\zeta_{\sigma,i,o} = \zeta + \frac{1}{3} \ln \frac{\rho_{\sigma,i,o}}{\bar{\rho}_{\sigma,i,o}}.$$  (2.22)

In the absence of interactions, the curvature perturbations $\zeta_r$ and $\zeta_{\sigma,i,o}$ are each conserved and the above two equations can be written as

$$\rho_r = \bar{\rho}_r e^{4(\zeta_r - \zeta)},$$  (2.23)

$$\rho_{\sigma,i,o} = \bar{\rho}_{\sigma,i,o} e^{3(\zeta_{\sigma,i,o} - \zeta)}.$$  (2.24)

At the time of curvaton decay, the total energy density $\rho_{\text{tot}}$ is conserved, i.e.

$$\rho_r(t_D, x) + \sum_{i=1}^N \rho_{\sigma,i,o}(t_D, x) = \bar{\rho}_{\text{tot}}(t_D).$$  (2.25)

Requiring that the total energy density is uniform on the decay surface, we have

$$(1 - \Omega_{\sigma,D}) e^{4(\zeta_r - \zeta)} + \sum_{i=1}^N \Omega_{\sigma,i,D} e^{3(\zeta_{\sigma,i,o} - \zeta)} = 1,$$  (2.26)

where $\Omega_{\sigma,D} = \bar{\rho}_{\sigma,D}/\bar{\rho}_{\text{tot}}$ is the fraction of the curvaton energy density in the energy budget at the time of curvaton decay, and

$$\Omega_{\sigma,D} \equiv \sum_{i=1}^N \Omega_{\sigma,i,D}.$$  (2.27)

Actually $\zeta_r$ is generated by the fluctuation of inflaton $\phi$ during inflation, namely $\zeta_r = \zeta_\phi$. In the $N$-vaton scenario, usually we assume that the curvature perturbation caused by inflaton is relatively small and can be neglected. The more general case with $\zeta_r = \zeta_\phi \neq 0$ is discussed in appendix A. Here we consider $\zeta_r = 0$ and equation (2.26) gives

$$e^{4\zeta} - \left( \sum_{i=1}^N \Omega_{\sigma,i,D} e^{3\zeta_{\sigma,i,o}} \right) e^{\zeta} + \Omega_{\sigma,D} - 1 = 0.$$  (2.28)

Order by order, from equation (2.28) we have

$$\zeta^{(1)} = A \sum_{i=1}^N \Omega_{\sigma,i,D} \zeta_{\sigma,i,o}^{(1)}.$$  (2.29)
The $N$-vaton

\[ \zeta^{(2)} = \frac{1}{4 - \Omega_{\sigma,D}} \left[ \frac{9}{2} \sum_{i=1}^{N} \Omega_{\sigma_i,D} (1 + h_i) \left( \zeta_{\sigma_i,o}^{(1)} \right)^2 - \left( 8 + \Omega_{\sigma,D} \right) \left( \zeta^{(1)} \right)^2 \right], \quad (2.30) \]

where

\[ A = \frac{3}{4 - \Omega_{\sigma,D}}. \quad (2.31) \]

The total curvature perturbation up to second order is

\[ \zeta = \zeta^{(1)} + \frac{1}{2} \zeta^{(2)} = A \sum_{i=1}^{N} \Omega_{\sigma_i,D} \zeta_{\sigma_i,o}^{(1)} + \frac{3A}{4} \sum_{i=1}^{N} \Omega_{\sigma_i,D} (1 + h_i) \left( \zeta_{\sigma_i,o}^{(1)} \right)^2 \]

\[ - \left( 1 + \frac{A}{2} \Omega_{\sigma,D} \right) A^2 \left( \sum_{i=1}^{N} \Omega_{\sigma_i,D} \zeta_{\sigma_i,o}^{(1)} \right)^2. \quad (2.32) \]

Assume that the two different curvatons are uncorrelated with each other; then

\[ \langle \zeta_{\sigma_i,o}^{(1)}(k_1) \zeta_{\sigma_j,o}^{(1)}(k_2) \rangle = (2\pi)^3 P_{\sigma_{i,o}}(k_1) \delta_{ij} \delta^3(k_1 + k_2). \quad (2.33) \]

Using equation (2.29), we can easily calculate the primordial power spectrum:

\[ P_\zeta = A^2 \sum_{i=1}^{N} \Omega_{\sigma_i,D}^2 P_{\sigma_{i,o}}. \quad (2.34) \]

For convenience, we introduce a new parameter $\alpha_i$ as follows:

\[ P_{\sigma_{i,o}} = A^{-2} \alpha_i P_\zeta. \quad (2.35) \]

The constraint on the coefficients $\alpha_i$ is

\[ \sum_{i=1}^{N} \Omega_{\sigma_i,D} \alpha_i = 1. \quad (2.36) \]

Similarly we can also calculate the primordial bispectrum:

\[ \langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = \frac{3A^3}{4} \sum_{i,j,k=1}^{N} \Omega_{\sigma_i,D} \Omega_{\sigma_j,D} \Omega_{\sigma_k,D} \langle \zeta_{\sigma_i,o}^{(1)}(k_1) \zeta_{\sigma_j,o}^{(1)}(k_2) \zeta_{\sigma_k,o}^{(1)}(k_3) \rangle \]

\[ - \left( 1 + \frac{A}{2} \Omega_{\sigma,D} \right) A^4 \sum_{i,j,k,l=1}^{N} \Omega_{\sigma_i,D} \Omega_{\sigma_j,D} \Omega_{\sigma_k,D} \Omega_{\sigma_l,D} \]

\[ \times \langle \zeta_{\sigma_{i,o}}^{(1)}(k_1) \zeta_{\sigma_{j,o}}^{(1)}(k_2) \zeta_{\sigma_{k,o}}^{(1)}(k_3) \rangle \zeta_{\sigma_{l,o}}^{(1)}(k_4) \zeta_{\sigma_{l,o}}^{(1)}(k_5) \zeta_{\sigma_{l,o}}^{(1)}(k_6) \]

\[ + 2 \text{ permutations of } \{k_1, k_2, k_3, k_4, k_5, k_6\}, \quad (2.37) \]

where $*$ denotes a convolution as follows:

\[ \langle \zeta_{\sigma_{i,o}}^{(1)}(k) \zeta_{\sigma_{j,o}}^{(1)}(k) \rangle \zeta_{\sigma_{l,o}}^{(1)}(k - q) = \frac{1}{(2\pi)^3} \int d^3q \zeta_{\sigma_{i,o}}^{(1)}(q) \zeta_{\sigma_{j,o}}^{(1)}(q). \quad (2.38) \]
After straightforward calculations, we get
\[
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = \left[ \frac{3}{2A} \sum_{i=1}^{N} \Omega_{\sigma_i,D}^3 q_i^2 (1 + h_i) - (2 + A\Omega_{\sigma,D}) \right] \\
\times (2\pi)^3[P_\zeta(k_1)P_\zeta(k_2) + 2 \text{ permutations}] \delta^3(k_1 + k_2 + k_3).
\] (2.39)

Using the definition of $f_{\text{NL}}$ in equation (2.9), we have
\[
f_{\text{NL}} = \frac{5}{4A} \sum_{i=1}^{N} \Omega_{\sigma_i,D}^3 q_i^2 (1 + h_i) - \left( \frac{5}{3} + \frac{5A}{6} \Omega_{\sigma,D} \right).
\] (2.40)

For the single curvaton, the solution of equation (2.36) is $\alpha = 1/\Omega_{\sigma,D}^2$ and then $f_{\text{NL}}^{\text{single}} = (5/(4f_D))(1 + h) - \frac{5}{3} - (5f_D/6)$, where $f_D = A\Omega_{\sigma,D}$. This is the same as the result in the literature.

To compare with the cosmological observations, we introduce a ‘dimensionless’ power spectrum $P_\zeta$ which is defined by
\[
\langle \zeta(k_1)\zeta(k_2) \rangle = \frac{2\pi^2}{k_1^3} P_\zeta \delta^3(k_1 + k_2).
\] (2.41)

The power spectrum of $\delta_{\sigma_i,*}$ is given by
\[
P_{\delta_{\sigma_i,*}} = \left( \frac{H_i}{2\pi} \right)^2.
\] (2.42)

According to equations (2.17) and (2.19), we have
\[
P_{\zeta_{\sigma_i,o}} = \frac{4}{9\pi^2} P_{\delta_{\sigma_i,*}} = \frac{1}{9\pi^2} q_i^2 H_i^2,
\] (2.43)

where
\[
q_i = g_i'/g_i.
\] (2.44)

The value of $\alpha_i$ takes the form
\[
\alpha_i = A^2 P_{\zeta_{\sigma_i,o}} / P_\zeta = \frac{A^2 q_i^2 H_i^2}{9\pi^2 P_\zeta}.
\] (2.45)

On the basis of equation (2.34), the amplitude of the primordial power spectrum $P_\zeta$ becomes
\[
P_\zeta = \frac{A^2}{9\pi^2} \sum_{i=1}^{N} \Omega_{\sigma_i,D}^2 q_i^2 H_i^2.
\] (2.46)

In [3] the WMAP normalization of the primordial power spectrum is
\[
P_{\zeta,\text{WMAP}} = 2.457^{+0.092}_{-0.093} \times 10^{-9}.
\] (2.47)

On the other hand, the amplitude of primordial power spectrum generated by the inflaton is
\[
P_\phi = \frac{H_i^2/M_p^2}{8\pi^2\epsilon}.
\] (2.48)
which should be much smaller than $P_{\zeta, \text{WMAP}}$, namely

$$H_* \ll 4.4 \times 10^{-4} \sqrt{\epsilon} M_p.$$  

(2.49)

Gravitational wave perturbation (tensor perturbation) is also generated during inflation in the $N$-vaton model. The tensor perturbation depends only on the inflation scale and its amplitude is given by

$$P_T = \frac{H_*^2/M_p^2}{\pi^2/2}.$$  

(2.50)

Usually we define a new parameter named the tensor–scalar ratio $r$ to measure the amplitude of the tensor perturbations:

$$r = \frac{P_T}{P_\zeta}.$$  

(2.51)

So the Hubble parameter during inflation is related to the tensor–scalar ratio by

$$H_* = \frac{\pi}{\sqrt{2}} P_1^{1/2} r_{1/2} M_p.$$  

(2.52)

Using WMAP normalization (2.47), we get $H_* = 10^{-4} r^{1/2} M_p$. If the density perturbation is dominated by inflaton fluctuation, we have $r = 16\epsilon$. In the curvaton/$N$-vaton scenario, the density perturbation caused by the inflaton is subdominant, and thus the inflation scale should be relatively low, i.e. $r < 16\epsilon$.

3. The bound on the non-Gaussianity parameter $f_{\text{NL}}$

In this section we consider the case in which the Hubble parameter is roughly a constant during inflation. If $\epsilon = -\dot{H}/H^2$ is large, the variation of the inflaton is larger than the Planck scale [32]. Usually this kind of inflation model cannot be embedded into string theory [33]–[38]. For simplicity we also assume that the curvaton values do not evolve between Hubble exit during inflation and the beginning of their oscillations. So we have $\sigma_{i,o} = g_i(\sigma_{i,*}) = \sigma_{i,*}$, and thus $q_i = 1/\sigma_{i,*}$ and $h_i = 0$. Here we are interested in the case of large non-Gaussianity. From equation (2.40), a large non-Gaussianity can be obtained if $\alpha_i \gg 1$, but $\Omega_{\sigma,D}$ is not necessarily required to be much smaller than 1. For example, if one or more coefficients $\alpha_i$ are large enough and $\Omega_{\sigma_i,D} \alpha_i$ takes a finite value for $\Omega_{\sigma_i,D} \ll 1$, $f_{\text{NL}}$ can be large even when $\Omega_{\sigma,D} = 1$ because $f_{\text{NL}} \sim (\Omega_{\sigma_i,D} \alpha_i)^2/\Omega_{\sigma,D}$. This case can be achieved only for multiple curvaton. In the single-curvaton model, $\Omega_{\sigma,D}^2 \alpha = 1$ and $f_{\text{NL}} \sim 1/\Omega_{\sigma,D}$. However whether the above conditions can be naturally realized in a concrete $N$-vaton model is still an open question and we will return to this problem in some future work. Here we only give a brief discussion for the case with two curvaton, in appendix B.

From now on, we focus solely on the case with $\Omega_{\sigma,D} \ll 1$ for simplicity. In this case a large non-Gaussianity is also expected. Now that $A = 3/4$, the amplitude of the primordial power spectrum and the non-Gaussianity parameter in equations (2.46) and (2.40) are respectively simplified to

$$P_\zeta = \frac{1}{16\pi^2} \sum_{i=1}^{N} \Omega_{\sigma_i,D}^2 \frac{H_*^2}{\sigma_{i,*}^2},$$  

(3.1)
\[ f_{NL} = \frac{5}{3} \sum_{i=1}^{N} \Omega_{\sigma_i, D}^3 \alpha_i^2. \]  \hfill (3.2)

Since \( \alpha_i \) is constrained only by equation (2.36), usually we need more information if we want to constrain the non-Gaussianity parameter \( f_{NL} \).

### 3.1. The lower bound on \( f_{NL} \)

Let us introduce a very useful inequality:

\[ \sum_{i=1}^{N} u_i^2 \cdot \sum_{j=1}^{N} v_j^2 \geq \left( \sum_{i=1}^{N} u_i v_i \right)^2, \]  \hfill (3.3)

where \( u_i \geq 0 \) and \( v_i \geq 0 \) for \( i = 1, 2, \ldots, N \). The equality in equation (3.3) is satisfied only when \( u_i/u_j = v_i/v_j \) for \( i, j = 1, 2, \ldots, N \). Using this inequality, we immediately find

\[ \sum_{i=1}^{N} \Omega_{\sigma_i, D} \sum_{j=1}^{N} \Omega_{\sigma_j, D}^3 \alpha_j^2 \geq \left( \sum_{i=1}^{N} \Omega_{\sigma_i, D}^2 \alpha_i \right)^2. \]  \hfill (3.4)

Taking equation (2.36) into account, we find that the non-Gaussianity parameter \( f_{NL} \) in equation (3.2) is bounded from below, namely,

\[ f_{NL} \geq \frac{5}{3 \Omega_{\sigma, D}}. \]  \hfill (3.5)

The equality is satisfied when \( \alpha_i \Omega_{\sigma_i, D} = \theta \) which is a constant. We can easily check this. In this special case the solution of equation (2.36) is given by

\[ \theta = 1/\Omega_{\sigma, D}, \]  \hfill (3.6)

and then

\[ \alpha_i = \frac{1}{\Omega_{\sigma_i, D} \Omega_{\sigma, D}}. \]  \hfill (3.7)

Substituting this solution into equation (3.2), we get

\[ f_{NL} = \frac{5}{3 \Omega_{\sigma, D}}. \]  \hfill (3.8)

Keeping \( \Omega_{\sigma, D} \) fixed, the non-Gaussianity parameter \( f_{NL} \) in the \( N \)-vaton model is not less than that in the single-curvaton model.
3.2. The upper bound on $f_{NL}$

In this subsection we take more information into account. Because we focus solely on the limit of $\Omega_{\sigma,D} \ll 1$, the radiation produced by the inflaton is always dominant before the curvaton decay. After that, curvatons oscillate around their minima $\sigma_i = 0$ and their energy density decreases as $a^{-3}$. Once the Hubble parameter drops below $\Gamma_\sigma$, the curvaton energy is converted into radiation. Like in the arguments in \[8,10\], the energy density parameter $\Omega_{\sigma,D}$ at the time of curvaton decay is given by

$$\Omega_{\sigma,D} = \frac{\sigma_{i,*}^2}{6M_p^2} \left( \frac{m_i}{\Gamma_\sigma} \right)^{1/2}.$$  (3.9)

Substituting the above equation into equation (3.1), the amplitude of the primordial power spectrum becomes

$$P_\zeta = \frac{H_*^2}{(24\pi)^2 M_p^4 \Gamma_\sigma} \sum_{i=1}^{N} m_i \sigma_{i,*}^2,$$  (3.10)

or, equivalently,

$$\sum_{i=1}^{N} m_i \sigma_{i,*}^2 = (24\pi)^2 P_\zeta M_p^4 \Gamma_\sigma / H_*^2.$$  (3.11)

The WMAP normalization gives a constraint on $\sum_{i=1}^{N} m_i \sigma_{i,*}^2$.

In this section $A = 3/4$, $g_i(\sigma_{i,*}) = \sigma_{i,*}$ and $q_i = 1/\sigma_{i,*}$. Equation (2.45) is simplified to

$$\alpha_i = \frac{1}{16\pi^2} \frac{H_*^2 / \sigma_{i,*}^2}{P_\zeta},$$  (3.12)

and then

$$\alpha_i \Omega_{\sigma,D} = \frac{r}{192} \sqrt{\frac{m_i}{\Gamma_\sigma}}.$$  (3.13)

If $m_i = m$ for $i = 1, 2, \ldots, N$, $\alpha_i \Omega_{\sigma,D}$ is a constant and the inequality in equation (3.5) is saturated. Now we have

$$\theta = \frac{r}{192} \sqrt{\frac{m}{\Gamma_\sigma}},$$  (3.14)

and

$$f_{NL} = \frac{5}{576} r \sqrt{\frac{m}{\Gamma_\sigma}}.$$  (3.15)

In general, different curvatons $\sigma_i$ have different masses $m_i$. Using equations (3.2), (3.9) and (3.12), we find that the non-Gaussianity parameter $f_{NL}$ takes the form

$$f_{NL} = 3 \times 10^{-7} P_{\zeta}^{-2} \frac{H_*^4}{M_p^6 \Gamma_\sigma^{3/2}} \sum_{i=1}^{N} m_i^{3/2} \sigma_{i,*}^2.$$  (3.16)

When $m_i = m$ for $i = 1, 2, \ldots, N$, we can easily check that this result is the same as (3.15).
How one determines the value of \( \sigma_{i,*} \) is a crucial problem in the curvaton/ \( N \)-vaton model. In the literature, the \( \sigma_{i,*} \) are taken as free parameters. At a classical level, this is correct. However, for a scalar field \( \chi \) in de Sitter space, if its mass is much smaller than \( H_* \), its Compton wavelength is large compared to the curvature radius of the background \( H_*^{-1} \) and the gravitational effects may play a crucial role in its behavior. In \cite{39}–\cite{41} the authors explicitly showed that the quantum fluctuation of a light scalar field \( \chi \) with mass \( m_\chi \) in de Sitter space gives it a non-zero expectation value of \( \chi^2 \):

\[
\langle \chi^2 \rangle = \frac{3}{8\pi^2} m_\chi^2.
\] (3.17)

This result is reliable for a light scalar field with \( m_\chi \ll \sqrt{2} H_* \) in a long-lived, quasi-de Sitter inflation. Here we also ignore the possible corrections from the cubic or higher power terms in the curvaton potential. So the typical or average energy density of the scalar field \( \chi \) is \( \frac{3}{8\pi^2} m_\chi^2 \). Since the masses of curvatons are assumed to be much smaller than \( H_* \), the total energy density of the curvatons can be estimated as \( \frac{3}{8\pi^2} NH_*^4 \), which implies

\[
\sum_{i=1}^{N} m_i^2 \sigma_{i,*}^2 = \frac{3NH_*^4}{8\pi^2}.
\] (3.18)

Using the inequality (3.3), and equations (3.11) and (3.18), we have

\[
\sum_{i=1}^{N} m_i^{3/2} \sigma_{i,*}^2 \leq 6\sqrt{6}(NP_\xi \Gamma_\sigma)^{1/2} H_* M_p^2.
\] (3.19)

We see that the non-Gaussianity parameter \( f_{NL} \) in equation (3.16) is bounded from above:

\[
f_{NL} \leq 4.41 \times 10^{-6} P_\xi^{-3/2} N^{1/2} \frac{H_*^5}{M_p^4 \Gamma_\sigma}.
\] (3.20)

The inequality (3.20) is saturated when these curvaton fields have the same mass: \( m_1 = m_2 = \cdots = m_N = m \). Now \( \alpha_i \Omega_{\sigma,D} \) is a constant and the lower bound (3.5) is also saturated. One point that we want to stress is that \( \Omega_{\sigma,D} \) is not kept fixed. Keeping the inflation scale \( H_* \) (or tensor–scalar ratio \( r \)), the number of curvatons \( N \) and the curvaton decay rate \( \Gamma_\sigma \) fixed, the non-Gaussianity \( f_{NL} \) is maximized in the case where all of the curvatons have the same mass. We discuss this special case in section 3.4 in detail.

### 3.3. The adiabatic condition

In \cite{21} the author pointed out that the curvaton model is free from the constraint of isocurvature perturbation from WMAP \cite{3} if the cold dark matter (CDM) is not the direct decay product of the curvatons and the CDM is generated after the curvatons decay completely. This is also the case for the \( N \)-vaton. Denote as \( H_{\text{cdm}} \) the Hubble parameter when the CDM is generated; thus \( H_{\text{cdm}} < \Gamma_\sigma \). The Hubble parameter \( H_{\text{cdm}} \) is related to the temperature \( T_{\text{cdm}} \) at the epoch of CDM creation by \( H_{\text{cdm}} = T_{\text{cdm}}^2 / M_p^2 \).

\[ Journal of Cosmology and Astroparticle Physics 09 (2008) 017 (stacks.iop.org/JCAP/2008/i=09/a=017) \] 12
The curvaton

\[ \Gamma_{\sigma} > \frac{T_{\text{cdm}}^2}{M_{p}}. \]  

(3.21)

Combining this with equation (3.20), we find

\[ T_{\text{cdm}} < 1.87 \times 10^{12} N^{1/4} r^{5/4} f_{\text{NL}}^{-1/2} \text{ GeV}, \]

(3.22)

where we use equation (2.52) and the WMAP normalization \( P_{\zeta} = P_{\zeta,\text{WMAP}} \). In [42], the relationship between \( T_{\text{cdm}} \) and the mass of the CDM \( M_{\text{cdm}} \) is roughly given by

\[ M_{\text{cdm}} \approx 20 T_{\text{cdm}}. \]

(3.23)

So the mass of the CDM is bounded from above:

\[ M_{\text{cdm}} < 3.7 \times 10^{13} N^{1/4} r^{5/4} f_{\text{NL}}^{-1/2} \text{ GeV}. \]

(3.24)

For example, for \( N \sim 10^{3}, r \sim 10^{-4} \) and \( f_{\text{NL}} \sim 50 \), the mass of the CDM is less than \( 3 \times 10^{8} \text{ GeV} \). On the other hand, \( f_{\text{NL}} \) is bounded by \( 1/M_{\text{cdm}} \) from above.

3.4. The case with \( m_i = m \) for \( i = 1, 2, \ldots, N \)

In this case the constraint coming from the amplitude of the power spectrum (3.11) and the estimate of the total energy density of the curvaton during inflation (3.18) are respectively simplified to

\[ \sigma_T^2 \equiv \sum_{i=1}^{N} \sigma_{i,*}^2 = (24\pi)^2 P_{\zeta} M_{p}^4 \Gamma_{\sigma} H^2 / m, \]

(3.25)

and

\[ \sigma_T^2 = \frac{3 N H^4}{8 \pi^2 m^2}. \]

(3.26)

According to the above two equations, we find that the curvaton mass \( m \) is related to the curvaton decay rate \( \Gamma_{\sigma} \) by

\[ m = 6.68 \times 10^{-6} P_{\zeta}^{-1} N^{1/4} \frac{H^6}{M_{p}^4 \Gamma_{\sigma}}. \]

(3.27)

Keeping \( \Gamma_{\sigma} \) fixed, the mass of the curvatons \( m \) in the \( N \)-vaton is \( N \) times that in the single-curvaton model.

In section 3.2, we argue that the upper bound on the non-Gaussianity parameter \( f_{\text{NL}} \) in equation (3.20) is saturated when the curvatons have the same mass and now we have

\[ f_{\text{NL}} = 4.41 \times 10^{-6} P_{\zeta}^{-3/2} N^{1/2} \frac{H^5}{M_{p}^4 \Gamma_{\sigma}}. \]

(3.28)

Keeping \( \Gamma_{\sigma} \) and \( H_{*} \) fixed, the larger the number of curvatons, the larger the non-Gaussianity parameter \( f_{\text{NL}} \). On the other hand, if \( N \) is fixed, the smaller \( \Gamma_{\sigma} \), the larger \( f_{\text{NL}} \). However, like for the argument in [29], the curvaton decay rate is larger than the
gravitational strength decay rate, i.e.

$$\Gamma_\sigma > \frac{1}{c^4 M_p^2} \frac{m^3}{N^{3/4} H_*^{3/2}}$$

(3.29)

where $c$ is supposed to be an order 1 coefficient which we do not know exactly. The curvaton decay rate cannot be arbitrary small. Substituting equation (3.27) into (3.29), we find that $\Gamma_\sigma$ is bounded from below by the number of curvatons:

$$\Gamma_\sigma > 1.3 \times 10^{-4} c^{-1} P_\zeta^{-3/4} N^{3/4} \frac{H_*^{3/2}}{M_p^{7/2}}.$$  

(3.30)

The lower bound on the curvaton decay rate rises as the number of curvatons increases. Combining this with equation (3.28), we find that $f_{NL}$ is bounded by the tensor–scalar ratio from above:

$$f_{NL} < 0.034 c P_\zeta^{-3/4} N^{-1/4} \frac{H_*^{1/2}}{M_p^{1/2}} = 10^3 c \left( \frac{r}{N} \right)^{1/4}.$$  

(3.31)

For $N = 1$, our result is the same as that in [17] where we went beyond the sudden-decay approximation and ignored the coefficient $3/8 \pi^2$ when we estimated the expectation value of the square of the curvaton field. Here we introduce an order 1 coefficient $c$ to encode the uncertain coefficient in the calculations. On the other hand, using equations (3.27), (3.28) and (3.31), we obtain

$$f_{NL} < \frac{c^{2/3}}{P_{\zeta}^{2/3}} \left( \frac{m/N}{1.3 \times 10^3 M_p} \right)^{1/3} = c^{2/3} \left( \frac{m/N}{2 \times 10^4 \text{ GeV}} \right)^{1/3}.$$  

(3.32)

Requiring the mass of each curvaton to be smaller than the Hubble parameter $H_*$ leads to another bound on $f_{NL}$, i.e.

$$f_{NL} < 2.3 \times 10^{3/2} \frac{r^{1/6}}{N^{1/3}}.$$  

(3.33)

If $r > 2 \times 10^4/(c^4 N)$, the constraint in equation (3.33) is more restrictive than that in (3.31). To summarize, the bound on the non-Gaussianity parameter $f_{NL}$ is given by

$$f_{NL} < \min \left[ 10^3 c \left( \frac{r}{N} \right)^{1/4}, 2.3 \times 10^{3/2} \frac{r^{1/6}}{N^{1/3}} \right] .$$  

(3.34)

We see that the constraint on $f_{NL}$ in the N-vaton model is more stringent than that in the single-curvaton model. The reason is that the energy density of each curvaton during inflation is roughly $H_*^4$ and thus the total energy density of the curvatons in the N-vaton model is much larger than the single-curvaton energy density. The larger the number of curvatons, the larger $\Omega_{\sigma,D}$. Since $f_{NL} \sim 1/\Omega_{\sigma,D}$, the non-Gaussianity is suppressed in the N-vaton model by the number of curvatons $N$. A reasonable estimate of the number of curvatons in string theory might be $10^3$ and $r \leq 10^{-3}$ if we require the variation of the inflaton be smaller than the Planck scale [34]. If so, $f_{NL} \leq 32c$. Usually $f_{NL}$ in the N-vaton model should be less than $10^2$. For $f_{NL} > 10$, $r > 10^{-8} N$ and $m > 10^7 N$ GeV. Typically we have $N \sim 10^3$ and then $m > 10^{10}$ GeV, $r > 10^{-5}$, which implies $H_* > 10^{12}$ GeV.
3.5. The $N$-vaton versus the single-curvaton model

According to previous discussion, the non-Gaussianity parameter $f_{NL}$ in the $N$-vaton model is larger than that in the single-curvaton model if the curvaton decay rate is kept fixed, and the maximum value of $f_{NL}$ in the $N$-vaton case is obtained when all of the curvatons have the same mass. The maximum value of $f_{NL}$ is $\sqrt{N}$ times that in the single-curvaton model; however, now the curvaton mass in the $N$-vaton case is $N$ times that in the single-curvaton model. The requirement that the curvaton decay rate be larger than the gravitational strength decay rate in the $N$-vaton model becomes much more stringent than that in the single-curvaton model. That is why the upper bound on $f_{NL}$ in equation (3.31) is suppressed by a factor $1/N^{1/4}$.

On the other hand, we consider the masses of different curvatons to be quite different from each other. For simplicity, we use the estimate $\sigma_{i,*} \sim H^2/m_i$. According to equations (3.10) and (3.16), the contributions to the amplitude of the primordial power spectrum and the non-Gaussianity parameter from curvaton $\sigma_i$ are respectively $P_{\zeta,i} \sim H^6/(\mathcal{M}_p^4\Gamma_\sigma m_i)$ and $f_{NL,i} \sim H^8/(P_{\zeta}^2\mathcal{M}_p^6\Gamma_\sigma^3/2 m_i^{1/2})$. If the lightest curvaton $\sigma_L$ is much lighter than other curvatons, the total primordial power spectrum and non-Gaussianity are roughly contributed by $\sigma_L$. Now the $N$-vaton model is reduced to the single-curvaton model and $f_{NL} \sim m_L/(P_{\zeta}^{1/2}H_*).$ Similarly, requiring $\Gamma_\sigma > m_L^3/(c^4\mathcal{M}_p^2)$ yields $f_{NL} < 10^{10}c^{1/4}.$

4. The spectral index and non-Gaussianity

The spectral index of the primordial power spectrum generated by curvatons is defined as

$$n_s^{nc} \equiv 1 + \frac{d\ln P_{\zeta}^{nc}}{d\ln k} = 1 - 2\epsilon + 2\eta_{\sigma\sigma},$$

where

$$\eta_{\sigma\sigma} = \sum_{i=1}^{N} \Omega^2_{\sigma_i,D} \frac{1}{3H^2_*} \frac{d^2V(\sigma_i)}{d\sigma_i^2} = \sum_{i=1}^{N} \Omega^2_{\sigma_i,D} \frac{m_i^2}{3H^2_*}.$$

If the primordial power spectrum is dominated by the curvature perturbation generated by curvatons, $n_s = n_s^{nc}$. The masses of curvatons are assumed to be much smaller than $H_*$ and then $\eta_{\sigma\sigma} \simeq 1 - 2\epsilon$. For $n_s = 0.96$, $\epsilon = 0.02$, which might be realized in landscape inflation [43]–[45] or monodromies [46]. However, in this case the Hubble parameter $H$ during inflation cannot be taken as a constant any longer. In [24], we showed that the curvaton values depend on the initial condition of inflation which should be fine-tuned to achieve a suitable amplitude of the primordial power spectrum and non-Gaussianity parameter $f_{NL}$. This is quite unnatural. Usually a close to scale-invariant curvature perturbation generated by the curvaton is expected in the curvaton/$N$-vaton scenario.

On the other hand, the spectral index of the primordial power spectrum generated by the inflaton is

$$n_s^{inf} \equiv 1 + \frac{d\ln P_{\zeta}^{inf}}{d\ln k} = 1 - 6\epsilon + 2\eta.$$

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In some inflation models, $\epsilon \simeq 0$, but the order of magnitude of $\eta$ can be $-\mathcal{O}(10^{-1})$ to $-\mathcal{O}(10^{-2})$. If the inflaton fluctuation makes a significant contribution to the total primordial power spectrum, a red-tilted primordial power spectrum may be obtained. We calculate the curvature perturbation for this scenario in appendix A. Introduce a parameter $\beta$ to measure the relative amplitude of the power spectrum generated by the curvatons:

$$\beta = \frac{P_{\zeta}^{\text{nc}}}{P_{\zeta}^{\text{tot}}},$$

and then

$$P_{\zeta}^{\text{inf}} = (1 - \beta) P_{\zeta}^{\text{tot}}.$$  

(4.5)

Now the spectral index becomes

$$n_s \equiv 1 + \frac{\frac{d}{d \ln k} P_{\zeta}^{\text{tot}}}{\frac{d}{d \ln k} k} = \beta n_s^{\text{nc}} + (1 - \beta)n_s^{\text{inf}}$$

$$= 1 - (6 - 4\beta)\epsilon + 2\beta\eta_{\sigma\sigma} + 2(1 - \beta)\eta,$$

(4.6)

where $\alpha_i$ in equation (4.2) should be replaced by $\gamma_i$. We consider $\epsilon \ll 1$ and $\eta_{\sigma\sigma} \ll 1$ and thus

$$n_s \simeq 1 + 2(1 - \beta)\eta.$$  

(4.7)

For $\beta = 0.8$ and $\eta = -0.1$, $n_s \simeq 0.96$.

Since the total primordial power spectrum is not generated only by curvatons in this scenario, some formulations in section 3 should be modified by some powers of $\beta$: $f_{NL}$ is replaced by $f_{NL}/\beta^2$ and the WMAP normalization becomes $P_{\zeta} = \beta P_{\zeta,\text{WMAP}}$. For example, equation (3.34) is changed to

$$f_{NL} < \min \left[ 10^3 \beta^{5/4} \frac{r}{N} \right]^{1/4}, 2.3 \times 10^3 \beta^{4/3} \epsilon^{2/3} \frac{r^{1/6}}{N^{1/3}}].$$

(4.8)

For $\beta = 0.8$, the bound on the non-Gaussianity does not change so much and a large value of $f_{NL}$ is still achieved naturally. But we need to stress that a large value of $f_{NL}$ is obtained only when $r$ is not too small.

The size of non-Gaussianity generated by the inflaton is controlled by a factor $(1 - \beta)^2$ in equation (A.18) and rich phenomena are expected in this mixed scenario. A large value of $f_{NL}^{\text{inf}}$ may be detectable if $(1 - \beta)$ is not so small. Another bonus of this mixed scenario is that if the adiabatic fluctuation generated by the inflaton is big compared to that from the curvaton ($\beta \sim 0$), for our model the constraint on the isocurvature perturbation from [3] is relaxed, even in the case where dark matter was generated before the decay of the curvaton.

5. The random matrix and the typical mass spectrum in string theory

Axions are typically present in large numbers in string compactifications, and even when all other moduli are stabilized, the axion potentials remain rather flat as a consequence of well-known non-renormalization theorems [47]. Following [48] the potential for $N$ axions,
\[ V(\varphi) = \sum_{i=1}^{N} \Lambda_i^4 \left[ 1 - \cos \left( \frac{\varphi_i}{f_i} \right) \right], \quad (5.1) \]

where \( f_i \) is the axion decay constant and \( \Lambda_i \) is the dynamically generated scale of the axion potential that typically arises from an instanton expansion. Redefine the axion field as \( \sigma_i \equiv \varphi / f_i \); then the Lagrangian for small axion displacements \( \sigma_i \ll M_p \) in [49] is given by

\[ \mathcal{L} = \sum_{i=1}^{N} \left[ \frac{1}{2} \left( \partial \sigma_i \right)^2 - \frac{1}{2} m_i^2 \sigma_i^2 \right]. \quad (5.2) \]

In [48, 49] the axion fields are taken as inflatons and the value of \( \sigma_i \) is larger than \( M_p / \sqrt{N} \), but smaller than \( M_p \). In [35] we argued that the vacuum expectation value of \( \sigma_i \) is bounded by \( M_p / \sqrt{N} \) from above and this inflation model might be inconsistent with a full quantum theory of gravity. In this section we suggest that these axion fields play the role of the curvatons, not inflatons.

It is still difficult to explicitly calculate the mass of the axion in the context of KKLT moduli stabilization [50]. However, in [49] the authors found an essentially universal probability distribution for the mass square of axions, as the Marcenko–Pastur law:

\[ p(m^2) = \frac{1}{2\pi v} \sqrt{\frac{b - m^2/\bar{m}^2}{m^2}} \left( \frac{m^2/\bar{m}^2 - a}{m^2} \right), \quad (5.3) \]

for \( a \leq m^2/\bar{m}^2 \leq b \), where

\[ a = (1 - \sqrt{v})^2, \quad (5.4) \]
\[ b = (1 + \sqrt{v})^2. \quad (5.5) \]

The shape of the distribution depends only on a single parameter, \( v \), which is determined by the dimensions of the Kähler and complex structure moduli spaces. This distribution is universal because it does not depend on specific details of the compactification, such as the intersection numbers, the choice of fluxes, and the location in moduli space. It is also insensitive to superpotential corrections. But we cannot determine the overall mass scale from string theory. In a KKLT compactification of type IIB string theory, there are \( h_{1,1} \) axions, and \( h_{1,1} + h_{2,1} + 1 \) is the total dimension of the moduli space (Kähler, complex structure, and dilaton), so

\[ v = \frac{h_{1,1}}{h_{1,1} + h_{2,1} + 1}. \quad (5.6) \]

In [49] the authors argued that the models with \( v = \frac{1}{2} \) are strongly favored.

In general, the number of axions is roughly \( \mathcal{O}(10^2–10^3) \) in string theory. So

\[ \frac{1}{N} \sum_{i=1}^{N} m_i^{2k} \equiv \langle m^{2k} \rangle \simeq \int_{a\bar{m}^2}^{b\bar{m}^2} m^{2k} p(m^2) \, dm^2 = \bar{m}^{2k} s(k, v) \quad (5.7) \]
up to the order of $1/N$ which can be safely neglected in our analysis, where $k$ is just a number and
\[
s(k, v) = \frac{1}{2\pi v} \int_a^b x^{k-1} \sqrt{(b-x)(x-a)} \, dx.
\] (5.8)

The function $s(k, v)$ has some interesting properties:
\[
s(0, v) = s(1, v) = s(k, 0) = 1.
\] (5.9)

Since $s(1, v) = 1$, \(\langle m^2 \rangle = \bar{m}^2\) which denotes the overall mass scale. We also define
\[
\frac{1}{N} \sum_{i=1}^N m_i^2 \sigma_i^2 \equiv \langle m^2 \sigma^2 \rangle = \langle m^2 \langle \sigma^2 \rangle \rangle.
\] (5.10)

Here we have $\langle \sigma^2 \rangle = 3H_\ast^4/(8\pi^2 m^2)$ and then
\[
\sum_{i=1}^N m_i^2 \sigma_i^2 = \frac{3N}{8\pi^2} H_\ast^4 \bar{m}^{2(k-1)} s(k-1, v).
\] (5.11)

Because $s(0, v) = 1$, $\sum_{i=1}^N m_i^2 \sigma_i^2 = 3N H_\ast^4/(8\pi^2)$ and equation (3.18) is automatically satisfied. Here we assume that $\sigma_i \ll f_i$. Otherwise, the quartic or higher power correction terms from the expansion of the axion potential will be important. In string theory, the axion decay constant $f_i$ is generically large [51] and our assumption of $\sigma_i \ll f_i$ is reasonable.

According to equations (3.10) and (3.16), we can easily calculate the amplitude of the primordial power spectrum and the non-Gaussianity parameter generated by curvatons:
\[
P_{n_c}^\zeta = 6.68 \times 10^{-6} s(-1/2, v) \frac{NH_\ast^6}{M_p \Gamma_\sigma \bar{m}},
\] (5.12)
\[
f_{n_c}^{\text{NL}} = 1.14 \times 10^{-8} s(-1/4, v) (P_{n_c}^\zeta)^{-2} \frac{NH_\ast^8}{M_p^6 \Gamma_\sigma^{3/2} \bar{m}^{1/2}}.
\] (5.13)

Here there are three unknown scales, $H_\ast$, $\Gamma_\sigma$ and $\bar{m}$, which still cannot be determined by microscopic physics. Canceling $\Gamma_\sigma$, we have
\[
f_{n_c}^{\text{NL}} = 0.66 f_1(v) (P_{n_c}^\zeta)^{-1/2} N^{-1/2} \frac{\bar{m}}{H_\ast},
\] (5.14)

where
\[
f_1(v) = s(-1/4, v)/s^{3/2}(-1/2, v).
\] (5.15)

On the other hand, canceling $\bar{m}$ yields
\[
f_{n_c}^{\text{NL}} = 4.41 \times 10^{-6} f_2(v) (P_{n_c}^\zeta)^{-3/2} N^{1/2} \frac{H_\ast^5}{M_p^4 \Gamma_\sigma},
\] (5.16)

with
\[
f_2(v) = s(-1/4, v)/s^{1/2}(-1/2, v).
\] (5.17)
We have \( f_1(1/2) = 0.76 \) and \( f_2(1/2) = 0.98 \). The behaviors of \( f_1(v) \) and \( f_2(v) \) are shown in figure 1. When \( v \to 0 \), the mass gap of the curvatons disappears and we can expect our model to reduce to the case in section 3.4 where all of the curvatons have the same mass. Since \( s(k, 0) = 1 \) and then \( f_1(0) = f_2(0) = 1 \), we see that both the amplitude of the primordial power spectrum and the non-Gaussianity parameter are really the same as those in section 3.4. On the other hand, in section 3.2, we find that the non-Gaussianity parameter \( f_{NL} \) is maximized when \( m_1 = m_2 = \cdots = m_N = m \) for \( H_\ast, \Gamma_\sigma \) and \( N \) fixed. This model is really consistent with our analysis: \( f_2(v) \) approaches its maximum value when \( v \to 0 \). Here we also know how the mass scale \( \mbar \) varies with \( v \). For a given \( f_{NL}^c \), \( \mbar \sim 1/f_1(v) \) rises as \( v \) increases.

In this case, we can also calculate \( \eta_{\sigma \sigma} \), i.e.

\[
\eta_{\sigma \sigma} = \frac{1}{3} f_1(v) \frac{\mbar^2}{H^2}, \tag{5.18}
\]

where

\[
\tau(v) = s(1/2, v)/s(-1/2, v). \tag{5.19}
\]

The function \( \tau(v) \) is illustrated in figure 2 and \( \tau(1/2) = 0.73 \).

In general, curvaton fluctuations only contribute to a part of the total primordial power spectrum, \( P_{\zeta}^c = \beta^2 P_{\zeta}^{tot} \), and then \( f_{NL} \simeq \beta^2 f_{NL}^c \). Similarly, requiring \( \Gamma_\sigma > \mbar^3/(c^4 M_p^2) \) yields

\[
f_{NL} < \min \left[ 10^3 \beta^{3/4} c d_1(v) \left( \frac{r}{N} \right)^{1/4}, 2.3 \times 10^3 \beta^{1/3} c^{2/3} d_2(v) \frac{r^{1/6}}{N^{1/3}} \right], \tag{5.20}
\]
The functions $d_1(v)$ and $d_2(v)$ are shown in figure 3, and $d_1(1/2) = 0.81$ and $d_2(v) = 0.79$. Again we see that our results are exactly reduced to the case in section 3.4 when the parameter $v$ approaches zero.

$$d_1(v) = s(-1/4, v)/s^{5/4}(-1/2, v),$$

$$d_2(v) = s(-1/4, v)/s^{4/3}(-1/2, v).$$
5.1. Comparison to experiments

In this subsection we focus on how to compare our model to experiments. We consider the case with $P^\text{tot}_\zeta = \beta P^\text{tot}_\zeta$. There are eight parameters:

- inflation: $H_*, \epsilon, \eta$
- $N$-vaton: $N, \Gamma_\sigma, \bar{m}, v$
- ratio parameter: $\beta$.

Using equation (5.18), we have $\bar{m}/H_* = \sqrt{3\eta_{\sigma\sigma}/\tau(v)}$ and then $f_{\text{NL}} = 1.14\beta^{3/2}f_1(v)(P^\text{tot}_\zeta)^{-1/2}N^{-1/2}\sqrt{\eta_{\sigma\sigma}/\tau(v)}$. (5.23)

Since $P^\text{inf}_\zeta = (H_*^2/M_p^2)/(8\pi\epsilon) = (1 - \beta)P^\text{tot}_\zeta$, the relationship between the tensor–scalar ratio $r$ and $\epsilon$ is given by $r = 16(1 - \beta)\epsilon$. (5.24)

The spectral index is given in equation (4.6) as $n_s = 1 - (6 - 4\beta)\epsilon + 2\beta\eta_{\sigma\sigma} + 2(1 - \beta)\eta$. (5.25)

To summarize, there are four quantities which can be measured by experiments: $P^\text{tot}_\zeta$, $f_{\text{NL}}(\beta, v, N, \eta_{\sigma\sigma})$, $r(\beta, \epsilon)$ and $n_s(\beta, \epsilon, \eta, \eta_{\sigma\sigma})$. For a given inflation model, which means that $\epsilon$ and $\eta$ are given, the parameters $\eta_{\sigma\sigma}, N$ and $\beta$ can be determined by experiments for the preferred model with $v = 1/2$. Furthermore, if the number of curvatons is given by string theory, we can check whether our model is consistent with experiments. For example, let us consider an inflation model with $\epsilon = 10^{-4}$ and $\eta = -0.1$, and we also assume $\bar{m}/H_* \ll 1$ (or $\eta_{\sigma\sigma} \ll 1$). The tensor perturbation is too small to be detected. For $n_s = 0.96$, $P^\text{tot}_\zeta = 2.457 \times 10^{-9}$, $f_{\text{NL}} = 30$, $v = 1/2$ and $N = 10^3$, we find $\beta = 0.8$ and $\eta_{\sigma\sigma} = 0.0042$. Now $r = 3.2 \times 10^{-4}$, $H_* = 4.36 \times 10^{12}$ GeV and then $\bar{m} = 5.7 \times 10^{11}$ GeV.

6. Discussion

In this paper we explicitly calculate the primordial curvature perturbation in the $N$-vaton model. Multiplicity of light scalar fields is generic in the theories going beyond the standard model. Even though the total energy density of these light scalar fields is subdominant during inflation, the perturbation that they produce can dominate the density perturbation on large scales. We also suggest a realistic $N$-vaton model in which the axions in the KKLT compactification of type IIB string theory are taken as curvatons, and a rich phenomenology is shown. If a large local-type non-Gaussianity is confirmed by the forthcoming experiments, it could shed light on these light scalar fields.

In order to fit the spectral index from WMAP data, the inflaton fluctuation is still required to play a significant role in the total primordial power spectrum. Generally the tensor–scalar ratio is required to be not smaller than $10^{-5}$ if $f_{\text{NL}}^\text{local} > 10$. Many inflation models constructed in string theory, such as brane inflation [52, 53], happen on a quite low energy scale with $r \sim 10^{-10}$, which is too small to generate a large non-Gaussianity in the curvaton/ $N$-vaton scenario. How one constructs an inflation model with $r \sim \mathcal{O}(10^{-5} - 10^{-3})$ and $\eta \sim -0.1$ is still an open question.
In general, the curvaton decay rate mediated by particles of mass \( M_X \) is expected to be of order \( m^3/M_X^2 \). So it is natural to assume that the different curvatons have different decay rates and decay at different times, in particular for the case where they have different masses. The authors in [15] gave a concrete example to show that a large non-Gaussianity can be obtained even when the curvatons dominate the total energy density at the time of decays in the case of two curvatons. However if there are hundreds or thousands of curvatons, whether this enhancement of the non-Gaussianity is generic or not is still an open question. It would be worth studying this problem in the future.

Acknowledgments

We would like to thank Daniel Chung, Hironobu Kihara, Han-Tao Lu, K P Yogendran and Yu-Feng Zhou for useful discussions.

Appendix A. \( \zeta_r = \zeta_\phi \neq 0 \)

This is the most general case. We also expand the curvature perturbation \( \zeta_\phi \) to second order as follows:

\[
\zeta_\phi = \zeta_{\phi}^{(1)} + \frac{1}{2} \zeta_{\phi}^{(2)}.
\]

(A.1)

Now equation (2.26) becomes

\[
e^{4\zeta} = \left( \sum_{i=1}^{N} \Omega_{\sigma_i,D} e^{\kappa_{\sigma_i,o}} \right) e^{\kappa} + (1 - \Omega_{\sigma,D}) e^{4\zeta_\phi}.
\]

(A.2)

Order by order, from the above equation we have

\[
\zeta^{(1)} = A \sum_{i=1}^{N} \Omega_{\sigma_i,D} \zeta_{\sigma_i,o}^{(1)} + B \zeta_{\phi}^{(1)},
\]

(A.3)

and

\[
\zeta^{(2)} = \frac{3A}{2} \sum_{i=1}^{N} \Omega_{\sigma_i,D} (1 + h_i) \left( \zeta_{\sigma_i,o}^{(1)} \right)^2 - (2 + A \Omega_{\sigma,D}) A^2 \left( \zeta_{\sigma_i,o}^{(1)} \right)^2
\]

\[
- 8A^2 B \sum_{i=1}^{N} \Omega_{\sigma_i,D} \zeta_{\sigma_i,o}^{(1)} \zeta_{\phi}^{(1)} + B^2 C \left( \zeta_{\phi}^{(1)} \right)^2 + B \zeta_{\phi}^{(2)},
\]

(A.4)

where

\[
A = \frac{3}{4 - \Omega_{\sigma,D}}, \quad B = \frac{1 - \Omega_{\sigma,D}}{1 - \Omega_{\sigma,D}/4}, \quad \text{and} \quad C = \frac{3 \Omega_{\sigma,D}}{1 - \Omega_{\sigma,D}} A.
\]

(A.5)

The total curvature perturbation is given by

\[
\zeta = \zeta^{(1)} + \frac{1}{2} \zeta^{(2)}.
\]

(A.6)

We assume that all of these fields including curvatons and inflaton are independent. Thus the two different curvatons are not correlated with each other:

\[
\langle \zeta_{\sigma_i,o}^{(1)}(k_1) \zeta_{\sigma_j,o}^{(1)}(k_2) \rangle = (2\pi)^3 \delta^3(k_1) \delta_{ij} \delta^3(k_1 + k_2),
\]

(A.7)
and the curvatons are also decoupled from the inflaton $\phi$:
\[
\langle \zeta^{(1)}_{\sigma_{i,o}}(k_1) \zeta^{(1)}_{\phi}(k_2) \rangle = 0. \tag{A.8}
\]

The primordial power spectrum $\mathcal{P}_{\zeta_{\phi}}$ generated by the inflaton is defined as
\[
\langle \zeta^{(1)}_{\phi}(k_1) \zeta^{(1)}_{\phi}(k_2) \rangle = (2\pi)^3 \mathcal{P}_{\zeta_{\phi}} \delta^3(k_1 + k_2). \tag{A.9}
\]

The fluctuations of the curvatons and the inflaton contribute to the total primordial power spectrum which is given by
\[
\mathcal{P}_{\zeta}^{\text{tot}} = \mathcal{P}_{\zeta}^{\text{nc}} + \mathcal{P}_{\zeta}^{\text{inf}}, \tag{A.10}
\]
where
\[
\mathcal{P}_{\zeta}^{\text{nc}} = A^2 \sum_{i=1}^{N} \Omega_{\sigma_{i,D}}^2 \mathcal{P}_{\zeta_{i,o}}, \tag{A.11}
\]
is the total curvature perturbation generated by curvatons and
\[
\mathcal{P}_{\zeta}^{\text{inf}} = B^2 \mathcal{P}_{\zeta_{\phi}} \tag{A.12}
\]
is the curvature perturbation generated by the inflaton. For convenience, we introduce parameters $\beta$ and $\gamma_i$ as follows:
\[
\beta = \frac{\mathcal{P}_{\zeta}^{\text{nc}}}{\mathcal{P}_{\zeta}^{\text{tot}}}, \tag{A.13}
\]
\[
\mathcal{P}_{\zeta_{i,o}} = A^{-2} \gamma_i \mathcal{P}_{\zeta}^{\text{nc}}. \tag{A.14}
\]
Thus we have
\[
\sum_{i=1}^{N} \Omega_{\sigma_{i,D}}^2 \gamma_i = 1. \tag{A.15}
\]

When $\beta = 1$, all of the primordial power spectrum is generated by curvatons and $\gamma_i = \alpha_i$. Now we have
\[
\mathcal{P}_{\zeta_{i,o}} = A^{-2} \beta \gamma_i \mathcal{P}_{\zeta}^{\text{tot}}, \tag{A.16}
\]
and
\[
\mathcal{P}_{\zeta_{\phi}} = B^{-2}(1 - \beta) \mathcal{P}_{\zeta}^{\text{tot}}. \tag{A.17}
\]

Similarly, we can also calculate the total non-Gaussianity parameter $f_{\text{NL}}^{\text{tot}}$. Here we only give the result:
\[
f_{\text{NL}}^{\text{tot}} = \beta^2 f_{\text{NL}}^{\text{nc}} + \beta(1 - \beta) f_{\text{NL}}^{\text{cross}} + (1 - \beta)^2 f_{\text{NL}}^{\text{inf}}, \tag{A.18}
\]
where
\[
f_{\text{NL}}^{\text{nc}} = \frac{5}{4A} \sum_{i=1}^{N} \Omega_{\sigma_{i,D}}^3 \gamma_i^2 (1 + h_i) - \left( \frac{5}{3} + \frac{5A}{6} \Omega_{\sigma,D} \right), \tag{A.19}
\]
\[
f_{\text{NL}}^{\text{cross}} = -\frac{20}{3} A, \tag{A.20}
\]
\[
f_{\text{NL}}^{\text{inf}} = \frac{5}{6} C + \frac{1}{B} f_{\text{NL}}^{\text{inf}}, \tag{A.21}
\]
and $f_{\text{NL}}^{\text{inf}}$ is determined by concrete inflation models [54]–[63].
Appendix B. Another way to get a large non-Gaussianity in the $N$-vaton model

In this section, we consider that there are two curvatons whose masses are $m_1$ and $m_2$. Without loss of the generality, we assume $m_1 \geq m_2$. Once the Hubble parameter drops below $m_1$, the curvaton $\sigma_1$ starts to oscillate and its energy density goes like $\sim a^{-3}$. Similarly, when $H \sim m_2$, the curvaton $\sigma_2$ begins to oscillate. For simplicity, we assume that the universe is dominated by radiation before $\sigma_2$ starts to oscillate. The scale factor at the time when $\sigma_1$ starts to oscillate is denoted as $a = 1$, and then $a = \sqrt{m_1/m_2}$ when $H \sim m_2$. Since the energy density of an oscillating curvaton goes like $\sim a^{-3}$, the ratio of the energy densities of these two curvatons at the time of their decay is

$$x \equiv \frac{\rho_{\sigma_1,D}}{\rho_{\sigma_2,D}} \sim \sqrt{\frac{m_1}{m_2}} \frac{\sigma_{1,*}^2}{\sigma_{2,*}^2},$$

(B.1)

and then

$$\Omega_{\sigma_1,D} = \frac{x}{1 + x} \Omega_{\sigma,D}, \quad \Omega_{\sigma_2,D} = \frac{1}{1 + x} \Omega_{\sigma,D}. \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \qu
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