APPROXIMATE ESTIMATES OF THE TEMPERATURE STATE OF CERAMIC NUCLEAR FUEL IN CYLINDRICAL FUEL ELEMENTS AND THE INFLUENCE OF PROCESSES AND PARAMETERS OF A NUCLEAR REACTOR CORE

The approximate mathematical model of the temperature state of ceramic nuclear fuels in cylindrical fuel elements was proposed in the form of linear ordinary differential equation and the boundary conditions. The theory of heat conduction and assumptions about the axial symmetry and absence of heat flows along axis of fuel element, which allow to simplify the common equations in cylindrical coordinates, are the basis of the proposed simplified mathematical model for approximate estimating the temperature state of the nuclear fuel. The intensity of volume heat sources in fuel element was taken into account by using the average values corresponding with the heat power and the structural characteristics of a nuclear reactor core. The conception about the heat transfer coefficient was used for modeling interaction between the fuel and the heat carrier. This heat transfer coefficient depends on characteristic sizes and heat conductions of constituted materials of the fuel element and allows to estimate influence of these on the temperature state of the nuclear fuel. The analytical solution for the temperature of a ceramic fuel in cylindrical fuel elements was obtained and was used for researching. It was shown that the heat conductivity of the fuel has significantly influences both the average temperature and the difference between the inner and outer temperatures in the fuel pellet. At the same time, other parameters have significant influence only on the average temperature of the fuel pellet. Due to these, it is necessary to consider the temperature dependence of the thermal conductivities of the materials constituted the fuel elements for more precisely estimation the temperature state of the fuel pellets, which will lead to nonlinear equations will required the numerical methods for their solving.

**Key words**: ceramic nuclear fuel, fuel elements, temperature state, volume heat sources, stationary thermal conductivity, thermal conductivity, heat transfer coefficient, heat transfer condition, boundary conditions, integration constants.

**Introduction**

The operability of fuel elements of industrial nuclear reactors is primarily limited by the temperature state of their structural elements due to understandable natural causes, occurring from purposes of the fuel element and its operating conditions in a core of nuclear reactors.

Nuclear power reactors, using cylindrical fuel rods with ceramic nuclear fuel, are the most common in the world nuclear power industry at present, which is well-known [1, 2]. The temperature state of structural elements significantly limits the operability of cylindrical fuel rods [1–3]. Therefore the study of the fundamental regularities of the temperature state in fuel rods is of considerable interest for nuclear power industry and engineering. Taking into account the noted circumstances, it seems to be of current interests the theme of this article, in which approximate estimates were obtained and some fundamental regularities of the temperature state of ceramic nuclear fuel in cylindrical fuel elements were established. Obtained results can be used also for benchmarking the more complicated approaches [4].

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Purposes of the article

In cylindrical fuel rods of a widely-used design, the temperature state of a ceramic nuclear fuel is determined by several heat exchange processes, including the heat transfer between the fuel and the gap gas, the heat conductivity in the gap gas, the heat transfer from the gap gas to the shell, the thermal conductivity in the shell, and the heat transfer from the shell to the moving coolant. A reliable theoretical assessment of the temperature state of a ceramic nuclear fuel requires consideration of all heat exchange processes in a fuel element, which leads to the complicated mathematical models in general. At the same time, it is necessary to have the approximate quantitative assessments, which will allow to understand the general fundamental qualitative regularities of the temperature state of a ceramic nuclear fuel in fuel rods. Such approximate assessments must be based on the simplified mathematical models, providing opportunities for obtaining the exact analytical solutions, which will allow to use these approximate estimations for constructing and benchmarking the more complicated mathematical models will require the numerical methods. Thus, the purpose of this article is to build the simplified quantitative assessments of the temperature state of the ceramic nuclear fuel in cylindrical fuel rods for benchmarking further the results, based on the more complicated mathematical models.

Mathematical model of heat conductivity for simplified assessments of temperature state

The ceramic nuclear fuel, made as the pellets, is contained inside the cladding of the fuel element as shown on the fig. 1. The length \(L\) of the fuel element is significantly greater than the external radius \(R_e\) of their cladding (fig. 1); the gap-2 between the pellet-1 and the internal surface of the cladding-3 is filled by the gas with good heat conductivity, usually by the helium; it is possible that the central hole in the fuel pellet is absent, i.e. \(R_g = 0\). The heat, producing inside the volume of the fuel pellets due to the nuclear fission reaction, is transferred to the moving heat carrier thru the gaseous gap and the cladding of the fuel element; the heat carrier moves up along the axis of the fuel element from down-edge cap toward the upper-edge cap of the cladding. The difference between the average temperature inside the volume of the fuel pellets and the average temperature of the heat carrier is about 1000 K, but the difference between the average temperatures of the heat carrier on the level of the upper- and down-edge caps is about 50 K. As result of these circumstances and well-known the Fourier’s Law of heat conduction, the heat flows along the longitudinal axis are significantly less than the heat flows along the radius of the fuel element with direction denoted as \(r\) on the fig. 1; the circumferential heat flows are significantly smaller the radial heat flows too due to the longitudinal flows of the heat carrier are the almost symmetrical. Thus, all these circumstances allow to assume that the temperature of the fuel pellets is depended on the radial coordinate only at least in the central part of the fuel element far enough from the down and upper edges of the cladding. Further, the stationary temperature states only will be considered, and the well-known equation of heat conduction, which represent the temperature state of the ceramic nuclear fuel pellet in the fuel rod, can be wrote considering with the simplifying assumptions in cylindrical coordinates as follows [5]:

\[
d\frac{dT}{dr} + \frac{1}{r} \frac{dT}{dr} = -\frac{Q}{\lambda_f}, \quad R_h < r < R_f, \tag{1}
\]

where \(T\) is the temperature in a point of the fuel pellet; \(Q\) is the intensity of volume heat sources due to the nuclear fission reactions; \(\lambda_f\) is the average heat conductivity of the ceramic nuclear fuel; \(R_h\) and \(R_f\) are the internal and external radii of the pellet (fig. 1).

Considering the thermal equilibrium on the inner surface \(r = R_h\) and heat transfer on the outer surface \(r = R_f\), the boundary conditions, for equation (1) defining the temperature state of the fuel pellet can be represented in the form [5]:

\[
d\left(\frac{dT}{dr}\right)_{R_h} = 0, \quad -\lambda_f \frac{dT}{dr} = k h_{RC} \left(T - T_{HC}\right), \tag{2}
\]

where \(k\) is the heat transfer coefficient from the fuel pellet to the heat carrier; \(T_{HC}\) is the temperature of the heat carrier.

It is possible to obtain the analytical solution of the heat conduction linear equation (1) considering with the boundary conditions (2):

\[
T = T_{HC} - \frac{Q}{4\lambda_f} r^2 + \frac{QR_h^2}{2\lambda_f} \ln \frac{r}{R_f} + Q \cdot C, \tag{3}
\]

Fig. 1 – The typical scheme of the fuel element, containing the ceramic nuclear fuel:

1 – pellet of ceramic nuclear fuel; 2 – gaseous gap; 3 – cladding; 4 – moving heat carrier; 5 – upper edge cap; 6 – down edge cap
where \( C = \frac{R_f^2}{4k_f} + \frac{R_f}{2k} - \frac{R_g^2}{2kR_f} \).

The average magnitude of the intensity of volume heat sources due to the nuclear fission reactions can be estimated as:

\[
Q = \frac{W}{\pi(kR_f - R_g^2)l},
\]

where \( W \) is the heat power of the reactor; \( n \) is the number of the fuel elements in the core of the nuclear reactor.

The heat transfer coefficient from the fuel pellet to the heat carrier is defined by the widths and the heat conductivities of the gaseous gap and the wall of the cladding, as well as the heat transfer from the cladding to the heat carrier [6]:

\[
k = \left( \frac{R_f}{\lambda_g} \ln \frac{R_g}{R_f} + \frac{R_f}{\lambda_c} \ln \frac{R_g}{R_f} + \frac{R_f}{\alpha R_c} \right)^{-1},
\]

where \( R_g \) is the radius of the gaseous gap and \( R_f \) is the radius of the cladding (see fig. 1); \( \lambda_g \) is the heat conductivity in the gaseous gap and \( \lambda_c \) is the heat conductivity of the cladding; \( \alpha \) is the heat transfer coefficient between the cladding and the heat carrier.

Relations (3)–(5) represent the approximate quantitative estimation of the temperature state of the ceramic nuclear fuel in the fuel rod of nuclear reactor. These relations (3)–(5) give possibilities for researching influences on the temperature state of the fuel pellets such factors as the heat conductivity of the fuel, the heat conductivity of the gaseous gap, the heat conductivity of the cladding as well as the heat transfer from the cladding to the heat carrier.

Results of researches the temperature state of ceramic nuclear fuel in VVER-1000

Industrial power nuclear reactor VVER-1000 is the most widely used in the Eastern-European countries and is the basis for designing the next generation of power nuclear reactor for industrial purposes [7]. The parameters in relations (3)–(5), which are corresponded to the VVER-1000 nuclear reactor, are follows [7, 8]:

\[ Q = 3000 \text{ MW}, \quad n = 50856, \quad L = 3530 \text{ mm}, \quad T_{HC} = 583 \text{ K} \]

\[ R_f = 3765 \text{ mm}, \quad R_g = 386 \text{ mm}, \quad R_c = 455 \text{ mm} \]

\[ \lambda_f = 5 \frac{\text{W}}{\text{m} \cdot \text{K}}, \quad \lambda_g = 0.3 \frac{\text{W}}{\text{m} \cdot \text{K}}, \quad \lambda_c = 20.5 \frac{\text{W}}{\text{m} \cdot \text{K}}. \]  

The As is well-known [9], the heat transfer coefficient between the cladding and the heat carrier is about

\[ 33 \leq \alpha \leq 35 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}. \]

Next, it will be considered separately the fuel pellets with \( R_b = 0 \) and \( R_b = 1.15 \text{ mm} \) [7].

It is understandable, that the decreasing the heat conductivities It is understandable, that the decreasing the heat conductivities on the nuclear fuel, the gaseous gap and the cladding will be lead to increasing the temperature of the fuel pellets. The quantitative estimations for influences of these heat conductivities on the temperature state of fuel pellet without the central hole are obtained for data (6), (7) using the relations (3)–(5) and are given on the fig. 2. It is understood, that the heat transfer between the cladding and the heat carrier is have some influence on the temperature state of the fuel pellets too. The quantitative estimations of this influence, obtained using the simplified assessments (3)–(5) for data (6) for fuel pellets without the central hole, are shown on the fig. 3.

It is evidently, the central hole must have some influence on the temperatures of the fuel pellets. Quantitative assessments for influencing the central hole on the temperature state can be estimated using the results, presenting on the fig. 4, which are obtained considering with the simplified assessments (3)–(5).
Discussing the results for temperature state of pellets of ceramic nuclear fuel

The presented results (see fig. 2–4) about influencing of different factors on temperature state of pellets of nuclear fuel in fuel rods of nuclear reactors are corresponded for all well-known qualitative regularities and imaginations about the quantitative magnitudes, although all these results was obtained using the approximate assessments (3)–(5), based on the simplified mathematical model (1), (2). At the same time, these obtained results cannot be considered as the absolutely reliable assessments of the temperature state of the nuclear fuel pellets if fuel rods. The more reliable assessments of the temperature state of nuclear fuel pellets must be based on the most complicated mathematical models of the heat conductivity and the heat transfer in the area of the fuel rods to considering the assumptions more closely corresponding to the actual conditions during operating in the nuclear reactor core. Building such complicated mathematical models requires the sophisticated knowledge and approaches, which can be substantiated by benchmarking only. The approximate assessments (3)–(5) presented in this article can be used as the benchmarking for substantiated the most complicated mathematical models and approaches for estimating the temperature state of ceramic nuclear fuel in fuel rods of nuclear reactors. It is necessary to note, that the quantitative estimations of the temperature state of the fuel pellet inside the fuel rod are overvalued due to neglecting the axial heat flows, which must be directed outside the fuel elements pellets in the centre of the fuel rod, because the temperatures above and down theirs must be lower corresponding the axial distributions of the volume heat sources due to the axial distribution of the neutron field in the core.

Results about influencing the heat conductivities of the fuel element's materials on the temperature state of fuel pellets are shown that the heat conductivities of the gaseous gap and the cladding having influence on the average temperature only, but heat conductivity of the nuclear fuel having influence both on the average temperature and the difference between internal and external temperatures of the fuel pellet. The temperature state is especially sensitive to the heat conductivities of the nuclear fuel and the gaseous gap.

The heat transfer from the cladding to the heat carrier having influence on the average temperature of the nuclear fuel pellets inside the cladding of the fuel element. The central hole, made in the nuclear fuel pellets, leads to decreasing the temperature inside the pellet.

Conclusions

The approximate estimations for the temperature state of the ceramic nuclear fuel in cylindrical fuel rods of nuclear reactor is proposed in this article on the base of the simplified mathematical formulation of the heat conduction problem.

The approximate estimations for the temperature state of the ceramic nuclear fuel in cylindrical fuel rods of nuclear reactor is proposed in this article on the base of the simplified mathematical formulation of the heat conduction problem. It is shown that the proposed approximate estimations are corresponded to well-known qualitative regularities of influencing the parameters of fuel rods on the temperature state of the fuel pellets. At the same time, the quantitative estimations of the temperature state of the fuel pellet inside the fuel rod are overvalued due to neglecting the axial heat flows, which must be directed outside the fuel elements pellets in the centre of the fuel rod, because the temperatures above and down theirs must be lower corresponding the axial distributions of the volume heat sources due to the axial distribution of the neutron field in the core.

The main value of proposed approximate estimations is the opportunities of their using for benchmarking the most complicated mathematical models and approaches for assessment the temperature state of the ceramic nuclear fuel in fuel rods of nuclear reactors. Nevertheless on the base of the used simplified mathematical model it can be proposed to develop the improved approximate estimations considering with the axial heat flows corresponding with the axial distribu-
tions of the volume heat sources due to the axial distribution of the neutron field in the core. It can be supposed, that the quantitative magnitudes will be estimated more accurate due to these improved assessments.

It is necessary to consider the temperature dependences of the heat conductivities of the nuclear fuel and the gas in the gap to obtain the more precisely dependences of the heat conductivities of the nuclear reactors, because these heat conductivities having the most influencing on the temperature state of fuel pellets. This will be leaded to more complicate nonlinear differential equation of heat conductivity [4, 10].

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