Regulatory performance of two different tuning methods for milk cooling control system

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Abstract. Temperature is one of the essential factors for bacterial growth in milk. The higher the temperature, the faster the growth of bacteria. It is empirically proven that the bacteria will stop growing at the temperature of about 4°C. In this research, continuous milk cooling process was simulated and then controlled by using Proportional - Integral (PI) feedback control system. The regulatory performance of the two different tuning methods were then analyzed (i.e Tyreus - Luyben and Hagglund – Astorm). Their SSE (sum squared of errors) were compared. It was found that Tyreus – Luyben method gave better regulatory performance than Hagglund – Astorm.

1. Introduction
Milk is collected, transported and delivered to milk cooling centres in a number of ways, as shown in Figure 1.

Warm fresh milk should be cooled immediately after milking to preserve the quality and prevent spoilage. Cooling to 4°C within 3 to 4 hours is essential, but more rapid cooling is much preferred [1].
Normally, milk arrives at the milk cooling centres in the morning and late in the afternoon / early evening. Therefore, batch milk cooling is performed as such in cooperatives (i.e Koperasi Unit Desa SAE in Pujon, KUD Batu and elsewhere). However, in this study continuous milk cooling process was simulated as it was more applicable for larger capacity and it gave more rapid cooling.

Proportional Integral (PI) controllers were the most commonly used form of Proportional Integral Derivative (PID) controllers, accounting for over 90% of industrial PID applications. Tuning a PI controller involved setting the controller gain \( k_c \) and the reset time \( \tau_i \) [7]. There was a strong evidence that PI and PID controllers remained poorly understood and, in particular, poorly tuned in many applications [9].

The aim of this research was to compare regulatory performance of two different tuning methods (i.e Tyreus - Luyben and Hagglund – Astorn) for controller parameter determination while servo performance for this application had been published elsewhere [2].

2. Simulation

The system being studied was the milk cooling system at Koperasi Unit Desa SAE Pujon [2] as shown Figure 2 and 3.

\[ T_0(s) = \frac{Q(s)}{wTC_p} \cdot \frac{T_{0,s} - T_{i,s}}{w_s} \]

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Where:

- \( W_s = \) milk flowrate (kg/min)
- \( T_{0,s} = \) steady state temperature of inlet milk (°C)
- \( T_{i,s} = \) steady state temperature of outlet milk (°C)
- \( m = \) mass of inlet milk (kg)
Figure 4. Continuous milk cooling system

The data for milk is as follows:

\[ C_p = \text{heat capacity of milk} = 3.93 \frac{kJ}{kg.K} \]  \hspace{1cm} (2)

\[ \rho = \text{density of milk} = 1027 \frac{kg}{m^3} \]  \hspace{1cm} (3)

Table 1 shows data at steady state.

| Parameter | Value | Unit |
|-----------|-------|------|
| \( V_{milk} \) | 2500 | L |
| \( T_i \) | 36 | °C |
| \( T_0 \) | 4 | °C |
| \( T_1 \) | -13 | °C |
| \( T_2 \) | 0 | °C |

**Tyreus – Luyben Tuning Method**

PI or PID parameters can be determined by using equation as shown in Table 2 [3-4], where:

- \( k_u \) = ultimate gain
- \( P_u \) = ultimate period

\( P_u \) and \( k_u \) are similar to those in Ziegler – Nichol method and detail can be read elsewhere [5-7].

| Controller | \( k_c \) | \( \tau_I \) | \( \tau_D \) |
|------------|---------|-------------|-------------|
| PI         | \( \frac{k_u}{3.2} \) | 2.2\( P_u \) | -           |
| PID        | \( \frac{k_u}{2.2} \) | 2.2\( P_u \) | \( \frac{P_u}{6.3} \) |
Hagglund – Astorm Tuning Method
Setting PI controller for Hagglund – Astorm is using equation below [8]:

\[
k_c = \frac{1}{K} \left( 0.14 + 0.28 \frac{\tau}{\theta} \right)
\]

\[
\tau_I = \theta \left( 0.33 + \frac{6.8\tau}{10\theta + \tau} \right)
\]

Where:
- \(K\) = process gain
- \(\theta\) = process time delay
- \(\tau\) = process time constant

3. Results and Discussion
Tyreus-Luyben is quite similar to Ziegler Nichols method. There are two methods in obtaining \(k_{cu}\) (ultimate gain) and \(P_u\) (ultimate period). For simplicity, we chose plotting Bode diagram of open loop system (which depends on parameters of the process) [5]. Therefore, both Tyreus-Luyben and Hagglund-Astorn were basically model based control methods. The values for time delay were assumed ranging from 1 to 5 min so that both tuning methods could be applied. Table 3 shows the values for controller parameters obtained.

| Time Delay | Controller Parameters | Tyreus - Luyben | Hagglund - Astorn |
|------------|-----------------------|-----------------|------------------|
| 1          | \(k_c\) \[min\]       | -10209          | -2834            |
|            | \(\tau_I\) \[min\]    | 4.2             | 5.4              |
| 2          | \(k_c\) \[min\]       | -6200           | -1441            |
|            | \(\tau_I\) \[min\]    | 6.9             | 8.7              |
| 3          | \(k_c\) \[min\]       | -4894           | -977             |
|            | \(\tau_I\) \[min\]    | 8.8             | 10.9             |
| 4          | \(k_c\) \[min\]       | -3674           | -745             |
|            | \(\tau_I\) \[min\]    | 11.7            | 12.6             |
| 5          | \(k_c\) \[min\]       | -2954           | -606             |
|            | \(\tau_I\) \[min\]    | 5.5             | 14.0             |

Most commercial PID controllers use a controller gain, \(k_c\) (or proportional band, PB) that is expressed as a standard dimensionless %. But \(k_c\) actually has units of (% of CO signal)/(% of PV signal). In a precise mathematical world, these units do not cancel \(^{10}\). However, as most process control paper do \([5,6,11]\), we also did not show the unit.

In this research, direct acting controller was applied, but as we can see from its transfer function: \(\frac{T_o}{Q}\) was negative, therefore the controller gain should be negative. Tuning parameter was based on servo problem but tested for regulatory problem. Hence the process model: \(\frac{T_o}{Q}\) was still used.

At certain times, the temperature of inlet milk (\(T_i\)) was disturbed in the form of step function of -2°C (form its steady state value) at \(t = 35\) min and at 3°C at \(t = 60\) minutes. When interference was
given, the output would move away from its setpoint but would be back immediately. These disturbances may affect the output milk temperature and the amount of heat released ($Q$).

Figure 5 shows profile of output milk temperature ($T_0$) and its setpoint when such disturbances of inlet milk temperature ($T_i$) occurred. Profile of heat released ($Q$) was also shown. This figure was for one min time delay assumption and using Tyreus-Luyben. Profile for $T_0$ and $Q$ under similar disturbances are shown in Figure 6 for Hagglund–Astorm tuning method. Again, one minute time delay assumption was applied. Simulations were then continued for other time delay values (i.e. time delay = 2 to 5 minutes). The figures similar to figure 5 and 6 could also be generated. The sum squared of errors (SSE) values for both method and all time delay assumptions are listed in Table 4. Both methods were successful in term of disturbance rejection, but Tyreus – Luyben gave smaller values of SSE.

![Figure 5](image1.png)

**Figure 5.** Profile of outlet milk temperature ($T_0$) and heat released ($Q$) for 1 min time delay assumption using Tyreus-Luyben tuning method

![Figure 6](image2.png)

**Figure 6.** Profile of outlet milk temperature ($T_0$) and heat released ($Q$) for 1 min time delay assumption using Hagglund–Astorm tuning method
Table 4 also shows that small value of time delay assumption were preferred since the higher the value, the higher their SSE values (other error definitions for comparing the performance of different tuning methods such as IAE, ITAE, ISE could also be applied [6]). Looking at the controller parameters between the two methods in Table 3, it seems Tyreus – Luyben has higher values of $k_c$ and lower values of $\tau_i$ compared with one of Hagglund - Astorn. Therefore PI controller based on Tyreus - Luyben should be better than that of Hagglund - Astorn.

| Time Delay (min) | SSE Tyreus - Luyben | SSE Hagglund - Astorn |
|------------------|---------------------|-----------------------|
| 1                | 0.0953              | 1.4872                |
| 2                | 0.4117              | 8.1972                |
| 3                | 0.8057              | 17.5491               |
| 4                | 1.7406              | 27.2457               |
| 5                | 3.0064              | 36.5148               |

4. Conclusions
Feedback control system using PI controller has been applied for continuous milk cooling process. Two methods of tuning have been investigated (i.e. Tyreus – Luyben and Hagglund – Astorn) and both gave satisfactory results for regulatory performance. However, it seems that Tyreus- Luyben was slightly better because it gave smaller SSE values than Hagglund Astorn. It is also recommended that small time delay assumption is used instead of higher values.

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