Production of optical phase space vortices with non-locally distributed mode converters

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Abstract

Optical vortices have been observed in a wide variety of optical systems. They can be observed directly in the wavefront of optical beams, or in the correlations between pairs of entangled photons. We present a novel optical vortex which appears in a non-local plane of the two-photon phase space, composed of a single degree of freedom of each photon of an entangled pair. The preparation of this vortex can be viewed as a ‘non-local’ or distributed mode converter. We show how these novel optical vortices of arbitrary order can be prepared in the spatial degrees of freedom of entangled photons.

Keywords: optical vortices, orbital angular momentum, spontaneous parametric down-conversion

1. Introduction

Optical vortices arise from phase singularities, typically associated to the orbital angular momentum (OAM) of light. The OAM of light beams is a classical feature that has only been studied in detail recently [1]. Among other applications [2], a key motivation for this study is related to applications in quantum information [3]. These potential applications have motivated the study of the OAM in the context of the spatial correlations between photon pairs produced in spontaneous parametric down-conversion (SPDC) [4–6]. Experimentally, entanglement in the OAM of photon pairs has been observed [7–13] and quantified in recent experiments [14, 15]. These correlations are a consequence of the conservation of the OAM in the SPDC process, which arises from the transfer of the angular spectrum of the pump beam to the wavefunction of the down-converted photons [16, 17].

Investigations of OAM correlations between down-converted photons generally rely on one of two types of detection systems. In the first type, holographic masks and single mode optical fibres are used to project the down-converted fields onto modes with well-defined OAM [7], such as the Laguerre–Gaussian (LG) modes. In other experiments, point-like detectors are scanned in the transverse detection plane, and the characteristic doughnut modes are observed in the coincidence count distributions [9, 11]. In [9, 11], one detector was kept fixed while the other was scanned in the two-dimensional detection plane. The intensity distribution of a doughnut mode does not appear in the single-photon counts but rather only in the joint distribution of coincidence counts. The correlations observed with these two types of detection systems have been shown to be characteristic of entanglement.

A different kind of non-local optical vortex has been recently observed [18]. Here the term ‘non-local’ is related to the two-dimensional space in which the optical vortex appears. A doughnut-shaped coincidence distribution and phase dislocation was observed, but the pump beam was not prepared in an LG mode and neither of the twin photons was projected onto states with well-defined OAM. In all previous experiments, OAM correlations were observed only if either the pump, or at least one of the down-converted photons was prepared or projected onto a state with OAM different from zero. This is not the case in [18], where the optical vortex appears in the coordinate plane composed of the position variable of photon 1 and the wavevector variable of photon 2.
In this paper, we generalize the scheme employed in [18] and show that the presence of the optical vortex is due to the implementation of a non-local mode converter. We demonstrate that it is possible to create non-local optical vortices of arbitrary order through manipulation of the pump laser beam. In addition, we show that this can be done separately and simultaneously in two spatial dimensions.

2. Non-local optical vortex

Figure 1 shows a sketch of the experimental setup used in [18] to produce a non-local optical vortex. The pump laser is prepared in a first order Hermite–Gaussian mode (HG) HG_{10}, which is achieved by passing half of the Gaussian laser beam through a thin glass slide so that a phase difference of \( \pi \) is produced between the two halves of the pump beam. Further propagation in free space provides the spatial filtering necessary to clean up the beam, producing a good quality HG_{10} mode. This beam is then focused onto a nonlinear crystal with a cylindrical lens and produces entangled photons through spontaneous parametric down-conversion (SPDC). Twin photons with the same wavelength are collected and detected through narrow band interference filters. For photon 1, an imaging system is used to map the source distribution \((x, y)\) onto the detection plane. For photon 2, a Fourier lens maps the wavevector \((q_x, q_y)\) distribution of the source onto the detection plane. Both detectors are scanned in a line along the vertical (either \(x\) or \(q_x\)) detection axis, giving a two-dimensional non-local distribution of coincidence counts. A typical distribution is sketched in figure 1, showing the doughnut shape that is an indication of the presence of an optical vortex. For comparison, figure 1 also sketches the coincidence distribution when both photons are measured in the position basis \(x\). We see that in this case the coincidence distribution mimics the pump beam inside the crystal [16]. It is important to notice that the pump beam is not prepared in an LG mode and the down-converted photons are not projected onto LG modes.

The simple observation of a doughnut shape is not enough to prove the existence of the optical vortex. It is also necessary to obtain information about the azimuthal phase distribution. In [18], this was done through a double slit interference experiment. For a classical light beam, the phase singularity associated to an LG beam can be evidenced by observing a shift in the interference fringes [19], as illustrated in the inset of figure 1. This method was used to evidence the phase dependence of the non-local vortex in [18]. As shown in figure 1, a double slit aperture is inserted into the path of photon 1 and the detector is scanned to register the interference fringes in the coincidence distribution for two different positions of detector 2. The measurements are performed for two configurations of the pump beam, first pumping with a zero order Gaussian beam and second with the HG_{10} mode. In the first case, the sets of observed fringes...
are in phase, while in the second the observed fringes are out of phase (see figure 1), as in the case of a classical LG₀₀ beam.

We will show in section 4 that this experiment can be viewed as the implementation of mode conversion between a non-local HG and an LG mode. To do so, let us briefly review mode conversion of HG and LG beams.

3. Mode conversion between Hermite–Gaussian and Laguerre–Gaussian modes

In classical optics, the Hermite–Gaussian (HG) and Laguerre–Gaussian (LG) beams are solutions to the paraxial Helmholtz equation [20] in Cartesian and cylindrical coordinates, respectively. We will write the HG modes as HG_nm = HG_n(x)HG_m(y), where n (m) is the number of zeros of the Hermite polynomials Hₙ(x) (Hₘ(y)). The order of the HG modes is n + m. The LG modes can be written as LG_p(x, y), where the order is |ℓ| + 2p. It is well known that the Laguerre–Gaussian beams carry orbital angular momentum of ℓħ per photon [1].

Beijersbergen et al showed how one can exploit the Gouy phase of a paraxial beam to convert HG modes into LG modes or vice versa [21]. The LG modes, for example, can be described as a superposition of HG modes of the same order. A simple example is the mode LG₀₁, which is given by the superposition of HG₁₀ and HG₀₁ with a relative phase of π/2: LG₀₁ = 1/√2(HG₁₀ + iHG₀₁). At the same time, the diagonal HG mode DHG is defined as

\[
\text{DHG}_{nm}(x, y) = \text{HG}_n(x + y)\text{HG}_m(x - y)/(\sqrt{2}) \quad (1)
\]

and DHG₀₁ is given by DHG₀₁ = 1/√2(HG₁₀ + iHG₀₁). One can see that the DHG₀₁ mode can be converted into the LG₀₁ mode by introducing a relative phase between the HG components. This relative phase can be introduced using a mode converter, which is a set of cylindrical lenses which produces an astigmatic region [21], giving rise to the relative phase shift.

In general, the DHG modes are given by [21]

\[
\text{DHG}_{nm}(x, y) = \sum_{j=0}^{N_{nm}+m} b(n, m, j)\text{HG}_{N-j, j}(x, y), \quad (2)
\]

while the higher-order modes LG_p are given by

\[
\text{LG}_p(x, y) = \sum_{j=0}^{N_{nm}+m} i^j b(n, m, j)\text{HG}_{N-j, j}(x, y), \quad (3)
\]

with ℓ = n − m, p = min(n, m) and b(n, m, j) = \frac{([-N_{nm}-j])^{j/2}}{\sqrt{j!}(1-\sqrt{j})(1+j)^{m-j}}|_{j=0}. For the higher-order beams, each component DHG_{N-j, j} in expansion (2) picks up a phase i^j upon propagation through the astigmatic region of the mode converter [21]. In this way, one can transform a general DHG_nm mode into an LG_p mode of the same order.

4. Non-local conversion of higher-order modes in two dimensions

In this section, we will show a general method of mode conversion using entangled photons, such as those produced in SPDC. This method is a ‘non-local’ or distributed mode converter, since the LG mode is generated in a non-local phase space of the spatial degrees of freedom of the entangled photons. This generalizes our previous result reported in [18] to the case of higher-order modes and two spatial dimensions.

A fundamental characteristic of the SPDC process is that the transverse profile of the pump field is transmitted to the two-photon state [16]. In the monochromatic and paraxial approximations, the two-photon state produced by SPDC can be written as

\[
\Psi(q_1, q_2) = v(q_1 + q_2)\gamma(q_1 - q_2), \quad (5)
\]

where v is the angular spectrum of the pump beam and γ is the phase matching function [16]. Let us assume that the down-converted photons are degenerate and quasi-collinear, so that the phase matching function simplifies to [17]: \( γ(q) \propto \sin[\lambda L (q_x^2 + q_y^2)/8\pi], \)

where \( \lambda \) is the wavelength of the pump beam and \( L \) is the length of the crystal. Using a Gaussian spatial filter on the down-converter photons described in [18] we can tailor the phase matching function \( γ(q) \) so that it can be well approximated by a Gaussian: \( γ(q) \approx HG_{000}(q, \lambda, w), \)

where the waist is given by \( w \approx 8\pi/\lambda L. \) Let us further assume that the pump beam is prepared in an HG mode \( v(q) = HG_{nm}(q, \lambda, w_0), \) which is characterized by the wavelength \( \lambda \) and waist \( w_0 \) of the pump beam. In wavevector representation, the HG beams are

\[
\text{HG}_{nm}(q_x, q_y) = D_{nm} H_n(\frac{w_0 q_x}{\sqrt{2}}) H_m(\frac{w_0 q_y}{\sqrt{2}}) \times e^{-\frac{(q_x^2 + q_y^2)}{w_0^2}} e^{-i\hat{a}(m+n+1)q(z)}, \quad (6)
\]

where \( w_0 \) is the beam waist, \( H_n \) is a Hermite polynomial and \( D_{nm} \) is a constant [22]. Mathematically, the HG beam with wavelength \( \lambda \) and waist \( w_0 \) can be rewritten as \( \text{HG}_{nm}(q, \lambda, w_0) = \text{HG}_{nm}(q/\sqrt{2}, 2\lambda, \sqrt{2}w_0). \) In this case the wavefunction is given by

\[
\Psi(q_1, q_2) = \text{HG}_{nm}(q_1 + q_2, q_1 - q_2) \times \text{HG}_{00}(\frac{q_1 + q_2}{\sqrt{2}}, \frac{q_1 - q_2}{\sqrt{2}}). \quad (7)
\]

From here on, all HG and LG functions are characterized by the wavelength of the down-converted photons \( \lambda_c = 2\lambda \) and waist \( w_c = \sqrt{2}w_0. \) Using \( \text{HG}_{nm}(x, y) = \text{HG}_n(x)\text{HG}_m(y) \) we
can rewrite
\[
\Psi(q_1, q_2) = \text{HG}_n(\frac{q_{x1} + q_{x2}}{\sqrt{2}}, \frac{q_{x1} - q_{x2}}{\sqrt{2}}) \\
\times \text{HG}_m(\frac{q_{y1} + q_{y2}}{\sqrt{2}}, \frac{q_{y1} - q_{y2}}{\sqrt{2}}). \tag{8}
\]
By the definition of the DHG modes (1), we recognize that wavefunction (8) is equivalent to
\[
\Psi(q_1, q_2) = \text{DHG}_0(n, q_{x1}, q_{x2}) \text{DHG}_0(m, q_{y1}, q_{y2}) \\
= \sum_{j=0}^{n} b(n, 0, j) \text{HG}_{n-j, j}(q_{x1}, q_{x2}) \\
\times \sum_{t=0}^{m} b(m, 0, t) \text{HG}_{m-t, t}(q_{y1}, q_{y2}) \\
= \sum_{j=0}^{n} b(n, 0, j) \text{HG}_{n-j}(q_{x1}) \text{HG}_{j}(q_{x2}) \\
\times \sum_{t=0}^{m} b(m, 0, t) \text{HG}_{m-t}(q_{y1}) \text{HG}_{t}(q_{y2}). \tag{9}
\]
Wavefunction (9) is a product of DHG modes which appear in non-local coordinate planes composed of one spatial dimension of each down-converted photon. Using expansions (2) and (3), we see that introducing relative phases of \(i^n \) and \(i^m \) in the \(x\) and \(y\) spatial dimensions of photon 2, these DHG modes can be converted into LG modes. These relative phases can be introduced by a Fourier transform operation on photon 2 as was done in [18]. Indeed, as the Hermite–Gaussian functions are eigenfunctions of the Fourier transform \(\mathcal{F}\), i.e. \(\mathcal{F}[\text{HG}_n] = i^n \text{HG}_n\), the two-photon wavefunction is transformed to
\[
\Phi(q_1, q_2) = \sum_{j=0}^{n} i^j b(n, 0, j) \text{HG}_{n-j, j}(q_{x1}, x_2) \\
\times \sum_{t=0}^{m} i^t b(m, 0, t) \text{HG}_{m-t, t}(q_{y1}, y_2) \\
= \text{LG}_m(q_{x1}, x_2) \text{LG}_n(q_{y1}, y_2). \tag{10}
\]
Thus, performing a bidimensional Fourier transform on photon 1 implements the required relative phases. This operation can be viewed as a non-local mode converter, where the astigmatism is introduced between the spatial degrees of freedom of photons 1 and 2, rather than the \(x\) and \(y\) coordinates of a single classical beam.

5. Conclusion
We have presented a method to produce non-local optical vortices using entangled photon pairs produced in parametric down-conversion. The vortex appears in a coordinate plane composed of one spatial degree of freedom of one photon and the corresponding transverse wavevector degree of freedom of the other photon. We have shown that this kind of vortex is obtained from a non-local or ‘distributed’ mode converter in which a single lens is placed in the path of each of the down-converted photons. Pumping a nonlinear crystal with an appropriate HG\(_nm\) beam produces a pair of non-local optical vortices of order \(n\) and \(m\) in different spatial directions.

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