Finite Element Simulation and Shape Optimization of Acoustic Horn

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Abstract: Horn refers to the tube with continuous change of cross-sectional area, which can improve the matching of diaphragm and air load, so as to improve the electro-acoustic conversion efficiency. For long-distance strong sound equipment, the purpose of using horn is to pursue higher sound pressure level, and control directivity and smoothness of sound pressure level response. In order to obtain higher sound pressure level and sharper directivity, parametric curve expressions of conical, exponential, hyperbolic and parabolic horn were derived. By using COMSOL multiphysics software, two-dimensional axisymmetric finite element simulation models of conical, exponential, hyperbolic and parabolic horn were established. Combined with if + end if programming function of the software, the effects of different cross-section shapes of horn on far-field sound pressure level and directivity were analyzed. The simulation results show that the parabolic horn can achieve a higher sound pressure level, and the sound pressure level is more than 3dB higher than the other three in the frequency band above 2000Hz. In the aspect of directivity, the beam width of parabolic horn is the smallest and more stable, and the overall directivity is the strongest. By using the shape optimization function of the software and constructing 8-order Bernstein polynomials, based on the parabolic horn, the shape of the horn was optimized for the most sensitive frequency range of 2000-4000Hz, and the sound pressure level of the optimized horn was significantly improved in a large frequency range. This paper can provide technical reference for the design and development of horn loudspeaker.

1. Introduction

Most of the frequently used long range acoustic device can be classified as horn loudspeaker, whose main components are electroacoustic transducer and horn [1]. The horn refers to the pipe with continuous change of cross-section area, which can improve the matching between diaphragm and air load, thus improving the efficiency of electro-acoustic conversion. For long range acoustic device, the purpose of using horn is to pursue higher sound pressure level (SPL), and control directivity and smoothness of sound pressure level response [2].

The length, throat radius, mouth radius and cross-section shape of the horn all affect the propagation characteristics of sound wave in the horn [3]. For general horn loudspeakers, the length, throat radius and mouth radius of the horn are limited by the size of transducer, the volume and weight of the device and other factors, so it is of great significance to study the influence of cross-section shape on the sound transmission characteristics of the horn.
The finite element method is a low-cost and high-efficiency numerical analysis method for computer modeling and simulation, which is more and more applied to the analysis and solution of loudspeaker design analysis [4-5], horn shape optimization and other acoustic problems. Erik Büngtsson et al. used the finite element method to study the shape optimization of acoustic horn on how to improve the impedance matching of air load, and the obtained horn has good impedance characteristics [6]; David J Murphy and Rick Morgan used the finite element analysis in ANSYS and COMSOL to model and simulate the constant directional horn, and compared the simulation results with the acoustic measurement results [7]; With COMSOL software, Chris Smolen proposed a three-dimensional optimization method for loudspeaker horn to maintain a smooth response at a given angle. The optimized horn shape is close to a "oblate sphere" [8].

In this paper, the finite element method is used for modeling and simulation, which can visually show the sound field in the horn, and provide technical reference for the design of horn loudspeaker.

2. Finite element simulation model of horn

According to the different cross-section shape, the horn can be divided into cone, exponential, hyperbolic and parabola and so on. V. Salmon uses formula (1) to represent the horn system with the change of cross-sectional area, as shown in Figure 1.

\[ S = S_0 (c h \delta x + T s h \delta x) \]  

(1)

Where \( T \) and \( \delta \) are parameters indicating the shape of the horn, \( S_0 \) is the area of the horn throat.

![Figure 1. A group of horns with variable section area](image)

Reference[6][7][9]. COMSOL multiphysics software is used to establish the two-dimensional axisymmetric simulation model in \( rz \) plane coordinates as shown in Figure 2, in which the vertical red line is the axis of symmetry and the horizontal red line is infinite baffle. The wave source is excited by a plane wave of 1Pa at the waveguide, as shown by the red arrow in Figure 2, the horn radiates sound waves from the infinite baffle to the semi open space.

The lower side of the horizontal red line is waveguide and horn, and the upper side is air domain and perfect matching layer (PML). PML is used to simulate infinite open domain. The horn boundary and the outside of PML are set as the hard sound field boundary. In order to achieve higher calculation accuracy, the minimum wavelength should contain at least 5-6 grids. The maximum frequency of this study is 8000Hz, corresponding to the minimum wavelength of 4.3cm, so the maximum cell size of the grid is set to 7mm, and the shape of the grid is free triangle. The grid in Figure 2 is enlarged for illustration.
3. The influence of the cross-section shape of the horn on the sound pressure level and directivity

Parametric scanning method is used to study the influence of different cross-section shapes on sound pressure level and directivity, focusing on four types: conical, exponential, hyperbolic and parabolic. For the convenience of comparison, the horn length, throat radius and mouth radius are fixed in the simulation, and only the cross-section shape is changed. The initial values of each dimension are shown in Table 1.

| Table 1. The initial values |
|----------------------------|
| Length $L$/m | Throat radius $r_0$/m | Mouth radius $r_1$/m |
| 0.3          | 0.03                  | 0.18                 |

According to equation (1) and reference [10], the expressions of parametric curves for different horns are obtained.

Cone horn:

$$ r = 0.5z + 0.03 $$

Exponential horn:

$$ r = \left[ r_0^2 \exp\left(\frac{z \ln(S_i / S_o)}{L}\right) \right]^{1/2} $$

Hyperbolic horn:

$$ r = a \times \left( 3e^{z/aL} + e^{-z/aL} \right) / 4 $$

Here  

$$ a = \frac{4r_i / r_0 + \left((4r_i / r_0)^2 - 12\right)^{1/2}}{6} $$

Parabola horn:

$$ r = (kz + r_0^2)^{1/2} $$

The corresponding geometric model is shown in Figure 3.
Let us define a parameter $n$. When $n$ is 1, 2, 3 and 4, the geometric models of conical, exponential, hyperbolic and parabolic horn are established respectively. “If + end if” programming function of the software can be used to achieve this purpose. The flow chart of parametric scanning is shown in Figure 4.

**Figure 3.** Simplified representations of two dimensional axisymmetric geometric model of different horns

![Diagram of geometric models](image)

**Figure 4.** The flow chart of parametric scanning

The frequency band of this study is set at 100hz-8khz, and 100Hz is taken as a step.

3.1 *Comparison - On-axis SPL*

Figure 5 shows the SPL frequency response curves of different horns at 1meter in front of the horn axis.
Figure 5. Comparison of SPL at 1 meter of different horns

At low frequency, the sound resistance of the horn is very small, and the acoustic reactance is very dominant. At this time, the radiated sound is very small. With the increase of the frequency of the external signal, the sound resistance increases gradually, the sound reactance decreases slowly, and the sound from the horn also increases from small to large. The SPL frequency response characteristics of exponential and hyperbolic horn are almost the same in size and change synchronously. The high frequency response of cone horn is poor. In the frequency band above 2000Hz, the response of parabolic horn is obviously better than the other three, and it is often higher than 3dB.

3.2 Comparison – Beam width

As shown in Figure 6, the beam width of each horn at different frequencies is calculated when the sound pressure level drops by 3dB at a distance of 1 meter.

Figure 6. Comparison of beam width of different horns

It can be seen from the figure that in the case of low frequency, each horn has no obvious directivity due to the larger wavelength of sound wave, while the directivity of medium and high frequency is enhanced. Similar to the sound pressure level, the directivity of exponential and hyperbolic horn also shows the characteristics of little difference and synchronous change. The beam width of cone horn fluctuates greatly and its directivity is poor. The beam width of parabolic horn is the smallest and more stable, and the overall directivity is the strongest. The directivity of each horn can also be seen directly from the radiation pattern. Figure 7 shows the radiation pattern at 4000Hz.
4. Shape optimization of horn

Through the above analysis, we can see that parabolic horn can produce higher sound pressure level and sharper directivity, which is suitable for use in strong sound equipment. Next, based on the parabolic horn, the shape of the horn is optimized for the most sensitive frequency range of 2000–4000 Hz.

According to Weierstrass approximation theorem, continuous functions on closed intervals can be uniformly approximated by polynomial series. A set of Bernstein Polynomials of order \( n \) is constructed, whose values represent the deviation degree of the optimized shape from the original parabola in the \( z \) direction. Different shapes can be produced by changing the parameters of polynomials. Calculate the value of the objective function corresponding to different shapes, and find the best result through the optimization algorithm. When using higher order, the freedom of the objective function is larger and the final value may be better, but it will also make the optimization process more susceptible to influence and produce the shape unsuitable for production. The 8th Order Bernstein polynomials are used in this optimization.

The shape optimization module of COMSOL multiphysics software is used to complete the relevant settings, and the MMA algorithm is used to solve the problem. Figure 8 shows the choice of free form domain and polynomial boundary.
Figure 8. The choice of free form domain and polynomial boundary

Optimization objective: calculate the sound pressure level at 1 meter in front of the horn axis at the frequency of 2000Hz, 2500 Hz, 3000 Hz, 3500 Hz and 4000 Hz, and maximize its average value. The optimized shape boundary is shown in Figure 9.

Figure 9. The optimized shape boundary

Figure 10 shows the SPL frequency response curve of the optimized horn. It can be seen that the SPL in the frequency range of 2400Hz-6600Hz has been optimized, which is about 1dB higher than that of parabolic horn. Figure 11 shows the three-dimensional distribution of optimized SPL at 4000Hz.

Figure 10. The SPL frequency response curve of the optimized horn
5. Conclusion
In this paper, the two-dimensional axisymmetric finite element simulation model of the horn was established by using COMSOL multiphysics software. The influence of the horn cross-section shape on the SPL and directivity was analyzed. The simulation results show that parabolic horn can get higher SPL and sharper directivity than conical, exponential and hyperbolic horn. Based on the parabolic horn, the shape of the horn was optimized by using the shape optimization function of the software. The SPL of the optimized horn in a large frequency range was significantly improved.

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