NO MOTIONS OF BODIES PRODUCE GW’S

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Abstract. A close comparison between Maxwell field and Einstein field makes conceptually and immediately evident that in general relativity (GR) no motions of bodies can generate gravitational waves (GW’s).

1. – Let us consider a continuous “cloud of dust” characterized by a material energy tensor \( \rho u^j u^k \), \((c = 1), (j, k = 0, 1, 2, 3)\); \( \rho \) is the invariant mass density and \( u^j \) is the four-velocity. We assume first that this “dust” is electrically charged – with an invariant charge density \( \sigma \) –, and that the gravitational interaction between its particles is negligible. Thus we have a total mass tensor \( T^{jk} \) given by

\[
T^{jk} = \rho u^j u^k + S^{jk},
\]

where \( S^{jk} \) is the energy tensor of Maxwell field. Suppose that the spatio-temporal substrate is a Minkowskian manifold, which is referred to a system of general co-ordinates \( x^0, x^1, x^2, x^3 \). If a colon denotes a covariant differentiation, we have, as it can be formally proved (see sect. A.2 of Appendix A):

\[
T^{jk}_{:k} = 0,
\]

i.e. the differential conservation law of tensor \( T^{jk} \).

From

\[
4\pi S^{jk} := -F^j_r F^{kr} + \frac{1}{4} f^{jk} F_{rs} F^{rs},
\]

where \( F^{jk} \) is the e.m. field and \( f^{jk} \) the metric tensor –, taking into account Maxwell equations

\[
F_{jkr} + F_{krj} + F_{rjk} = F_{jkr} + F_{krj} + F_{rjk} = 0,
\]

(the comma denotes ordinary differentiation), and

\[
F^{jk}_{:k} = 4\pi \sigma u^i,
\]

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we obtain

\[ S^{jk} = 4\pi F^{jk} \sigma u_k. \]

Thus eqs. (2) give

\[ \rho u^j u^k_{;j} + F^{kj} \sigma u_j = 0, \]

which represent the equations of motion of the charged particles; they are an analytical consequence of differential conservation eqs. (2) and of Maxwell equations (4) and (5). \[ \text{(1)} \]

The theoretical existence of the electromagnetic waves is an analytical consequence of the above equations, as it is known.

2. – We consider now a “dust”, the particles of which interact only gravitationally; a physical example: the solar system. According to GR we have \((G = 1)\):

\[ R^{jk} - \frac{1}{2} g^{jk} R = -8\pi \rho u^j u^k; \]

by virtue of Bianchi relations, the (left-hand side),\( k \) vanishes identically; consequently:

\[ (\rho u^j u^k)_{;k} = 0; \]

from which:

\[ (\rho u^j)_{;j} = 0; \]

i.e. the mass conservation, and

\[ u^j u^k_{;j} = 0; \]

i.e. the geodesic equations of motion. Clearly, the geodesic motions **cannot** generate GW’s.

Remark the fundamental difference with Maxwell case of sect.1.: in lieu of eqs. (3), we have here:

\[ (R^{jk} - \frac{1}{2} g^{jk} R)_{;k} = 0; \]

i.e. relations which do not involve explicitly the four-velocity \( u^j \). Accordingly, in lieu of eqs. (7), we have the simple geodesic eqs. (11).

3. – For an electrically charged “dust”, the particles of which interact gravitationally (and electromagnetically), we obtain the following equations of motion:

\[ \rho u^j u^k_{;j} + F^{kj} \sigma u_j = 0; \]
i.e. four equations which are formally analogous to eqs. (7). Here the motions are not geodesic. However, no GW can be emitted, for the following reason.

Assume, for simplicity’s sake only, that \( \rho \) vanishes everywhere in space-time, except for a thin tube of world lines. Suppose further that at a given time \( t = t' \) the particle \( P \) extending over the tube begins to emit GW’s; let \( K'(t') \) be the set of kinematical elements (velocity, acceleration, time derivative of the acceleration, etc.) of \( P \) at \( t = t' \). Consider now another “dust”, identical to the previous one, but such that its charge density \( \sigma \) is equal to zero, and immerse it in a suitable “external”, “fixed” gravitational field. It is obvious that at some time \( t'' \) particle \( P \) will have a set of kinematical elements, say \( K''(t'') \), which is equal to the above \( K'(t') \). But now the particle \( P \) describes a tube of geodesic lines, and therefore no GW can be emitted.

We see in particular that the current conviction according to which an accelerated mass sends forth GW’s is false. This belief was originated in the old times by a partial analogy between Maxwell theory and the linearized version of GR. [1].

4. – The line of reasoning of previous sect. 2. can be extended to the case of a “dust” characterized by a mass tensor \( T^{jk} = (\mu + p)v^j u^k + pg^{jk} \), where \( \mu \) and \( p \) are scalars connected by an equation of state \( \mu = \varphi(p) \) [2]. The differential conservation relations of GR

\[
T^{jk}_{:k} = 0
\]

(14)

give the Eulerian equations of motion of perfect fluid hydrodynamics. Obviously, the motions of the particles are not geodesic; however, with an argument similar to that of sect. 2. we may conclude that no emission of GW’s is possible.

5. – In Maxwell theory the divergence of the e.m. energy tensor is zero only in the absence of charges and currents; in general, it is equal to \( 4\pi F^{jk} J_k \), cf. eqs. (6). On the contrary, in GR we have the fundamental circumstance that the divergence of the left-hand side of (8) is always (identically) equal to zero:

\[
(R^{jk} - \frac{1}{2} g^{jk} R)_{;k} = 0 ;
\]

(15)

in the last analysis, the absence of a “mechanism” for the emission of GW’s can be ascribed to eqs. (15). –

Consider, quite generally, a generic continuous medium; the differential conservation equations

\[
T^{jk}_{:k} = 0
\]

(16)

yield – completely or partially – its equations of motion, which in general are not geodesic. However, the kinematical elements of these motions are not different from the kinematical elements of purely geodesic motions,
and therefore no GW is sent forth – not even by catastrophic astrophysical perturbations.

The mass tensor $T^{jk}$ is the sum of a material (*stricto sensu*) energy tensor and of the energy tensors of all the fields, *except* the metric tensor $g_{jk}$, which is the substance of spacetime. This very special character of $g_{jk}$ is responsible for the fact that the undulatory solutions of Einstein equations are destitute of a physical reality.

5. – We have seen that, contrary to what occurs in Maxwell theory for the e.m. waves, in GR no motions of bodies can give origin to GW’s. In my opinion, the previous considerations are rather stringent. Moreover, their correctness is indirectly confirmed by various other proofs of the same result [3]. There are many roads to Rome.

Unfortunately, the astrophysical community is still very far from Caput mundi, and has entered into a blind alley [4]. Getting rid of current mythological ideas on GW’s – and BH’s – will be a painful operation.

**APPENDIX A**

A.1. – Let us consider the two following integrals, $I$ and $I'$, over a generic spatio-temporal region $D$:

$$I = \int_D R\sqrt{-g}\,dx$$

with $R = R^{ik}g_{jk}$, and $dx \equiv dx^0dx^1dx^2dx^3$, and

$$I' = \int_D L\,dx$$

where $L$ is a scalar density, which does not contain derivatives of the $g_{jk}$ – for simplicity’s sake. $L$ is a function of the physical quantities of the system – as four-velocities, e.m. fields, hydrodynamical observables, *etc.* – and of their derivatives. $L$ must be such that

$$\frac{\partial L}{\partial g_{jk}} = T^{jk}\sqrt{-g} \ .$$

Consider now the variations of $I$ and $I'$, say $\delta*I$ and $\delta*I'$, generated by a variation $\delta*g_{jk}$ of the metric tensor, which is induced by an *infinitesimal* co-ordinate transformation

$$x'^j = x^j + \epsilon^j(x) \ ,$$

such that the functions $\epsilon^j(x)$ vanish at the bounding surface of $D$.

By well known computations [5], the invariance conditions $\delta*I = \delta*I' = 0$ yield respectively:

$$(R^{jk} - \frac{1}{2}g^{jk}R)_{;k} = 0 \ ,$$
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\[ T^{jk}_{\cdot k} = 0 \]

i.e. the differential conservation equations of the tensor \([R^{jk} - (1/2)g^{jk}R]\) and of the mass tensor \(T^{jk}\).

Remarkable facts: i) the physical results (21) and (22) are a mere consequence of the above formal invariance; ii) they have been deduced in a way that is fully independent of Einstein equations

\[ R^{jk} - \frac{1}{2}g^{jk}R = -8\pi T^{jk} \]

iii) eqs.(21) have been deduced without using Bianchi relations.

A.2. – Another remarkable fact is the following. Consider the case for which the gravitational interactions are negligible, and consider a formulation of SR in arbitrary co-ordinates \(x^0, x^1, x^2, x^3\).

The differential conservation equations for the mass tensor \(T^{jk}\) can be derived in this way: choose, as in A.1., a scalar density \(L\) such that

\[ \frac{\partial L}{\partial g_{jk}} = T^{jk} \sqrt{-f} \]

where \(f := \det \|f_{jk}\|\), and \(f_{jk}(x)\) is the metric tensor; the condition \(\delta_{\ast} I' = 0\), induced by \(\delta_{\ast} f_{jk}\), gives – exactly as in A.1. –:

\[ T^{jk}_{\cdot k} = 0 \]

This means that, contrary to a diffuse opinion, also in SR eqs.(25) are a mere consequence of a simple invariance property of \(I'\). The merit of this outcome pertains only to the formulation in general co-ordinates \(x^0, x^1, x^2, x^3\).

A.3. – The equations of motion of a considered material continuum can be derived (completely or partially) from eqs.(22) in GR and from eqs.(25) in SR.

The nonlinearity of Einstein eqs.(23) has nothing to do with this fundamental result – contrary to a widespread belief.

A.4. – A final remark. I have emphasized that eqs.(21) have been here derived without using Bianchi identities. However, the present deduction depends on the relativistic dichotomy between the gravitational potential \(g_{jk}\) and the other fields. Otherwise, in lieu of the two conditions \(\delta_{\ast} I = 0\) and \(\delta_{\ast} I' = 0\), we ought to write the unique (and weaker) condition \(\delta_{\ast} (I + I') = 0\).
Let us reconsider the case (see sect. 1.) of a continuous charged “dust” – whose particles (of finite size) interact only electromagnetically –, when the Minkowskian spacetime is described by the customary metric tensor $\eta_{jk}$ such that: $\eta_{rs} = 0$ for $r \neq s$, $\eta_{00} = 1$, $\eta_{11} = \eta_{22} = \eta_{33} = -1$.

A pedestrian repetition of the reasoning of sect. 1. – with the only substitution of the ordinary differentiation for the covariant one – leads us to conclude that the equations of motion of the charged particles

\[ \rho u^j u^k_j + F^{kj} \sigma u_j = 0 \]

are a mere consequence of the definition of the mass tensor $T^{jk}$ [eqs. (1)] and of Maxwell field equations. This result is generally ignored in the treatises dealing the electromagnetic theory. On the contrary, in the standard formulations of Maxwell electrodynamics it is affirmed that the law of motion of the charges is independent of the field equations.

It is interesting that both in GR and in SR the equations of motion of any material continuum are only analytical consequences of the definition of the mass tensor $T^{jk}$ and of Einstein and/or Maxwell field equations.

Parergon

The searchers of GW’s and of BH’s have entered into a cul-de-sac, owing to their reluctance to abandon erroneous loci communes concerning the real physical meaning of GR. A conceptually inadequate “Vulgate”, based on second-hand works, has got the upper hand. The papers quoted in [4] are an example of this situation. (Stat pro ratione libido).

From the “Conclusions” of the first paper in [4]: “Two different astrophysical searches were performed: an all-sky search aimed at signals from isolated neutron stars and an orbital parameter search aimed at signals from the neutron star in the binary system ScoX-1. Both searches also cover a wide range of possible emission frequencies: a 568.8 Hz band for the isolated pulsar search and two 20-Hz bands for the ScoX-1 search. – The sensitivity of these analyses makes the detection of a signal extremely unlikely. As a consequence the main goal of the paper is to demonstrate an analysis method using real data […]” (An implicit admission of another fiasco).

From p.54 of the second paper in [4]: “The coalescence of two relativistic stars (double neutron star or black hole/neutron star binary mergers) is the end result of 0.1-1 Gyr of orbital decay caused by the emission of gravitational waves. This paroxysmal event should also give rise to a black hole surrounded by a torus of matter at nuclear densities, possibly producing relativistic jets that are less energetic and shorter lived than those of collapsars and originating short GRBs.” (A report from the dream-land: a bundle of unfounded conjectures).
References

[1] See also: H. Weyl, *Amer. J. Math.*, **66** (1944) 591; A. Loinger, *Spacetime & Substance*, Vol.5, No.2 (22), 2004, p.53.

[2] See e.g. V. Fock, *The Theory of Space, Time and Gravitation* (Pergamon Press, Oxford, etc.) 1964, sect.32 and Appendix C.

[3] Cf. A. Loinger, *arXiv:physics/0603214 v1* (March 25th, 2006) – in course of publication on *Spacetime & Substance – and references therein*.

[4] See e.g.: The LIGO Scientific Collaboration, *arXiv:gr-qc/0605028 v1* (May 4th, 2006); G. Chincarini et al., *The Messenger*, **123** (March 2006) 54.

[5] Cf. e.g.: E. Schrödinger, *Space-Time Structure* (Cambridge University Press, Cambridge) 1960, p.93 seqq.; P.A.M. Dirac, *General Theory of Relativity* (J. Wiley and Sons, New York, etc.) 1975, pp.59 and 60. See also: A. Loinger, *Nuovo Cimento*, A110 (1997) 341; Idem, *ibidem*, A112 (1999) 407.

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