A Rotating Proposer Mechanism for Team Formation

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1 The Rotating Proposer Mechanism

A rotating proposer mechanism is a way to operationalize sequential proposer games in mechanism design for team formation. Formally define a team formation mechanism. A team formation mechanism $M$ maps every preference profile $\succ$ to a partition $\pi$ of the set of players, i.e. $\pi = M(\succ)$. Our goal is to exhibit such a mechanism, and analyze its properties. The mechanism, termed Rotating Proposer Mechanism (RPM), implements the subgame perfect Nash equilibrium of the sequential proposer game with each proposer able to make an offer to each possible team, in which all proposals are accepted (thus, only a single offer is actually made). In this equilibrium, whenever it’s a player $i$’s turn to propose, $i$ makes a proposal to her most preferred team among those that would be accepted.

For any profile, if all players report their preferences truthfully, equilibrium outcomes of the game have a number of good properties which are thereby inherited by RPM. Of particular note is that RPM is individually rational, Pareto optimal, and implements iterated matching of soulmates (IMS) (see \cite{6}). However, it is also immediate from known results that the RPM mechanism is not in general strategyproof (this would conflict with individual rationality and implementing IMS \cite{6}).

The loss of incentive compatibility seems problematic. However, one side-effect of RPM implementing IMS is that RPM is strongly incentive compatible\footnote{More precisely, truth telling is a strong ex post Nash equilibrium.} and yields a unique core team structure on a restricted class of preference domains for which IMS always matches all players \cite{6}. As an example, these domains include other well-known restrictions on preferences, such as top coalition \cite{3} and common ranking \cite{5} properties.

This, however, would seem to limit its practical consideration, as such restrictions can rarely be guaranteed or verified. Moreover, we wish to make stronger efficiency claims than Pareto optimality, and also view fairness as an important criterion. For the former, we are particularly interested in utilitarian social welfare, a much stronger criterion than Pareto efficiency. We will also consider several notions of fairness discussed below. While we cannot make strong theoretical guarantees about these for broad realistic preference domains, we consider such properties empirically.

While RPM is a rather intuitive mechanism, it is quite challenging to implement the associated subgame perfect Nash equilibrium. In particular, the size of the backward induction search tree is $O(2\sum_{i=1}^{n}|T_i|)$. Even in the roommate problem, in which the size of teams is at most two, computing SPNE is $O(2^n)$. We address this challenge in three ways: (1) preprocessing and pruning to reduce the search space, (2) approximation for the roommate problem, and (3) a general heuristic implementation.

1.1 Preprocessing and Pruning

One of the central properties of RPM is that it implements iterative matching of soulmates. In fact, it does so in every subgame in the backwards induction process. Now, observe that computing the subset of teams produced through IMS is $O(n^3)$ in general, and $O(n^2)$ for the roommate problem, and is typically much faster in practice. We therefore use it as a preprocessing step both initially (reducing the number of players we need to consider in backwards induction) and in each subgame of the backwards induction search tree (thereby pruning irrelevant subtrees).

Because IMS preprocessing is computationally efficient, it is always applied before any of the approximate/heuristic versions of RPM below, with the direct consequence that even these approximate versions implement IMS.
We show the computational value of IMS in preprocessing and pruning using synthetic preference profiles based on the generative scale-free model.

Figure 1 shows the ratio of time consumed by RPM with IMS to that without IMS. In all cases, we see a clear trend that using IMS in preprocessing and pruning has increasing importance with increased problem size.

1.2 Approximate RPM for the Roommate Problem

Using IMS for preprocessing and pruning does not sufficiently speed up RPM computation in large-scale problem instances. Thus, we next developed a parametric approximation of RPM that allows us to explicitly trade off computational time against approximation quality. We leverage the observation that the primary computational challenge of applying RPM to the roommate problem is determining whether a proposal is to be accepted or rejected. If we are to make this decision without exploring the full game subtree associated with it, considerable time can be saved. Our approach is to use a heuristic to evaluate the “likely” opportunity of getting a better teammate in later stages: if this heuristic value is very low, the offer is accepted; if it is very high, the offer is rejected; and we explore the full subgame in the balance of instances.

More precisely, consider an arbitrary offer from player \(i\) to another player \(j\). Given the subgame of the corresponding RPM, let \(U_j(i)\) denote the set of feasible teammates that \(j\) prefers to \(i\), and let \(U_j(j)\) be the set of feasible teammates who \(j\) prefers to be alone. We can use these to heuristically compute the likelihood \(R_j(i)\) that \(j\) can find a better teammate than the proposer \(i\):

\[
R_j(i) = \frac{|U_j(i)|}{|U_j(j)|} \cdot \frac{1}{|U_j(i)|} \sum_{k \in U_j(i)} \left(1 - \frac{|U_k(j)|}{|U_k(k)|}\right) = \frac{1}{|U_j(j)|} \sum_{k \in U_j(i)} \left(1 - \frac{|U_k(j)|}{|U_k(k)|}\right)
\]

Intuitively, we first compute the proportion of feasible teammates that \(j\) prefers to \(i\). Then, for each such teammate \(k\), we extract the proportion of feasible teammates who are not more preferred by \(k\) than the receiver \(j\). Our heuristic then uses an exogenously specified threshold, \(\alpha\), \((0 \leq \alpha \leq 0.5)\) as follows. If \(R_j(i) \leq \alpha\), player \(j\) accepts the proposal, while if \(R_j(i) \geq 1 - \alpha\), the proposal is rejected. In the remaining cases, our heuristic proceeds with evaluating the subgame at the associated decision node. Consequently, when \(\alpha = 0\), it is equivalent to the full backwards induction procedure, and computes the exact RPM. Note that for any \(\alpha\), this approximate RPM preserves IR, and we also maintain IMS by running it as a preprocessing step.

The parameter \(\alpha\) of our approximation method for RPM in the roommate problem allows us to directly evaluate the trade-off between running time and quality of approximation; small \(\alpha\) will lead to less aggressive use of the acceptance/rejection heuristic, with most evaluations involving actual subgame search, while large \(\alpha\) yields an increasingly heuristic approach for computing RPM, with few subgames fully explored.

\[2\]The simulations described in this section were run on a 2.6 GHz Intel Core i5 Mac machine with 8 GB RAM.
Figure 2a depicts the fraction of time consumed by RPM with different values of $\alpha$ compared to exact RPM (when $\alpha = 0$) on scale-free networks ($m = 3$). Based on this figure, even a comparatively small value of $\alpha$ dramatically decreases computation time.

Figure 2b compares similarity of the final team partition when using the heuristic compared to the exact RPM. Notice that even for high values of $\alpha$, there is a significant overlap between the outcomes selected by RPM with and without the heuristic. We note that $\alpha = 0.1$ appears to trade off approximation quality and running time particularly well: for comparatively sparse networks (i.e., $m = 2$) it yields over 99% overlap with exact RPM (this proportion is only slightly worse for denser networks), at a small fraction of the running time. Henceforth, we use $\alpha = 0.1$ when referring to the approximate RPM in the remainder of this section.

1.3 Heuristic Rotating Proposer Mechanism (HRPM)

Unlike the roommate problem, general team formation problems have another source of computational complexity: the need to iterate through the combinatorial set of potential teams to propose to. Moreover, evaluating acceptance and rejection becomes considerably more challenging. We therefore develop a more general heuristic which scales far better than the approaches above, but no longer has the exact RPM as a special case. We term the resulting approximate mechanism **Heuristic Rotating Proposer Mechanism (HRPM)**, and it assumes that the sole constraint on teams is their cardinality and that preferences can be represented by an additively separable utility function \[3\]. With the latter assumptions, we allow preferences over teams to be represented simply as preference orders over potential teammates, avoiding the combinatorial explosion in the size of the preference representation.

In HRPM, each proposer $i$ attempts to add a single member to their team at a time in the order of preferences over players. If the potential teammate $j$ accepts $i$’s proposal, $j$ is added to $i$’s team, and $i$ proposes to the next prospective teammate until either the team size constraint is reached, or no one else who $i$ prefers to being alone is willing to join the team. Player $j$’s decision to accept or reject $i$’s proposal is based on calculating $R_j(l)$ for each member $l$ of $i$’s current team $T$ using Equation 1 and then computing the average for the entire team, $R_j(T) = \frac{1}{|T|} \sum_{l \in T} R_j(l)$ (see Algorithm 1 for the precise description of HRPM). We then use an exogenously specified threshold $\beta \in [0, 1]$, where $j$ accepts if $R_j(T) \leq \beta$ and rejects otherwise. The advantage of HRPM is that the team partition can be found in $O(\omega n^2)$, where $\omega$ is the maximum team size. The disadvantage, of course, is that it only heuristically implements RPM. Crucially, it does preserve IR, and IMS is implemented as a preprocessing step.

2 Properties of Exact and Approximate RPM

Over truthful preference reports, RPM inherits the properties of the game, including IR, IMS, and Pareto efficiency. In general, however, these properties conflict with incentive compatibility. Moreover, when it comes to efficiency, Pareto optimality is a weak criterion and we would wish to know how well a mechanism fairs in terms of stronger efficiency criteria, such as utilitarian social welfare (with cardinal preferences). Fairness, too, is an important consideration in matching, particularly when it comes to forming teams. Next, we explore these issues using empirical tools.
Algorithm 1 Heuristic Rotating Proposer Mechanism (HRPM)

**input:** \((N, \succeq, O), \omega, \beta\)
**return:** Team formation outcome \(\pi\)

1: \(\pi = \emptyset\)
2: while \(O\) is non-empty do
3: \(i \leftarrow \text{the first player in } O\)
4: \(\pi_i \leftarrow \{i\}\)
5: while \(|\pi_i| < \omega\) do
6: if \(\succ_i\) is empty or the first player in \(\succ_i\) is \(i\) then
7: \(O \leftarrow O \setminus \{i\}\)
8: break
9: player \(i\) proposes to the first player \(j\) in \(\succ_i\)
10: \(R_j(\pi_i) = \frac{1}{|\pi_i|} \sum_{l \in \pi_i} \frac{|U_j(l)|}{|U_k(l)|} \sum_{k \in U_j(l)} \left(1 - \frac{|U_k(j)|}{|U_k(k)|}\right)\)
11: if \(R_j(\pi_i) \leq \beta\) then \(\triangleright\) player \(j\) accepts the proposal
12: \(\pi_i \leftarrow \pi_i \cup \{j\}\)
13: remove \(j\) from \(O\) and \(N\)
14: remove \(j\) from \(\succ_k\) for each player \(k \in N\)
15: remove \(i\) from \(O, N\) and \(\succ_k\) for each player \(k \in N\)
16: while \(N\) is non-empty do \(\triangleright\) add singletons into the outcome.
17: pick an arbitrary instance \(i\) from \(N\)
18: remove \(i\) from \(O\) and \(N\)
19: return \(\pi\)

2.1 Empirical Methodology

In our empirical assessments, we use both synthetic and real hedonic preference data. In both cases, preferences were generated based on a social network structure in which a player \(i\) is represented as a node and the total order over neighbors is then generated randomly. Non-neighbors represent undesirable teammates (\(i\) would prefer being alone to being teamed up with them).

The networks used for our experiments were generated using the following models:

- **Scale-free network:** We adapt the Barabási-Albert model [1] to generate scale-free networks. For each \((n, m)\), where \(n\) is the number of players, \(m\) denotes the density of the network, we generate 1,000 instances of networks and profiles.

- **Karate-Club Network [9]:** This network represents an actual social network of friendships between 34 members of a karate club at a US university, where links correspond to neighbors. We generate 100 preference profiles based on the network.

Finally, we used a Newfrat dataset [8] that contains 15 matrices recording weekly sociometric preference rankings from 17 men attending the University of Michigan. In order to quantitatively evaluate both the exact and approximate variants of RPM, the ordinal preferences \(\succ\) have to be converted to cardinal ones \(u_i(\cdot)\), upon which both mechanisms operate. For this purpose, we introduce a scoring function suggested by [4] to measure a player’s utility. To compute a player \(i\)’s utility of player \(j\) we adopt normalized Borda scoring function, defined as \(u_i(j) = g(r) = 2(k - r + 1)/k - 1\), where \(k\) is the number of \(i\)’s neighbors, and \(r \in \{1, \ldots, k\}\) is the rank of \(j\) in \(i\)’s preference list. Without loss of generality, for every player \(i\) we set the utility of being a singleton \(u_i(i) = 0\). We assume that the preferences of players are additively separable [3], which means that a player \(i\)’s utility of a team \(T\) is \(u_i(T) = \sum_{j \in T} u_i(j)\).
2.2 Incentive Compatibility

In spite of the known impossibility results, the fact that RPM is not incentive compatible may be intuitively surprising, given that it implements an equilibrium of the complete information game. To gain further intuition into this, consider the following example.

Example 1. Consider a roommate problem with 3 players having the following preferences:

1: {1, 2} ≻1 {1, 3} ≻1 {1}
2: {2, 3} ≻2 {2, 1} ≻2 {1}
3: {3, 1} ≻3 {3, 2} ≻3 {1}

Suppose that the order in RPM is \( O = (1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3) \). In the subgame perfect Nash equilibrium of the corresponding RPG, 1 will propose to \{1, 2\}, 2 will accept, and the resulting teams are \{1\}, \{3\}. This is because 2 is 1’s most preferred roommate, and if 2 rejects, then 1 would offer to 3 who would accept (since they like 1 more than 2), and 2 would be left alone.

Now, if player 3 misreports preferences to claim that she prefers 2 to 1, then 2 and 3 are soulmates and would be matched, with the resulting outcome \{(1), {2, 3}\}. The latter outcome is clearly preferred by 3, and consequently 3 has the incentive to lie.

Despite the general failure of incentive compatibility in RPM, we now explore empirically how frequently this failure actually occurs. We use the roommate problem, as in this case the special structure of RPM allows us to use Algorithm 2 to compute an upper bound on the number of players who could possibly benefit by misreporting preferences. In applying the algorithm, we use \( T_i \) to denote the set of feasible teammates (since teams are of size at most 2).

At the high level, this algorithm considers all the players who have accepted or rejected a proposal and checks whether reversing this decision improves their outcomes. The following theorem shows that this method indeed finds the upper bound on the number of untruthful players.

**Theorem 1.** Algorithm 2 returns an upper bound on the number of players who can gain by misreporting their preferences.

**Proof.** We divide the players into proposers and receivers. Proposers are those who propose in RPM and were thus teamed up (including singleton teams). Receivers accept or reject someone’s offer.

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### Algorithm 2 Computing Upper Bound of Untruthful Players

**input:** \((N, \succ, T, O)\), teammate vector \(\text{teammate}[]\) which results from RPM  
**return:** number of potential untruthful players \(\text{Sum}\)

1: \(\text{sum} \leftarrow 0\)
2: \(\textbf{while } |O| \geq 2 \textbf{ do}\)
3: \(\text{proposer} \leftarrow \text{the first player in } O\)
4: \(\text{receiver} \leftarrow \text{teammate}[\text{proposer}]\)
5: \(\textbf{for } \text{player } i \in T_{\text{proposer}} \textbf{ do}\)
6: \(\text{if } i \succ_{\text{proposer}} \text{receiver} \text{ and } \text{proposer} \succ_i \text{teammate}[i] \text{ then}\)
7: \(\text{sum} \leftarrow \text{sum} + 1\)  \(\triangleright i \text{ is potentially untruthful}\)
8: \(\textbf{for } \text{player } j \in T_{\text{receiver}} \textbf{ do}\)
9: \(\text{if } j \succ_{\text{receiver}} \text{proposer} \text{ and } \text{receiver} \succ_j \text{teammate}[j] \text{ then}\)
10: \(\text{sum} \leftarrow \text{sum} + 1\)  \(\triangleright \text{receiver} \text{ is potentially untruthful}\)
11: \(\text{remove } \text{proposer} \text{ and } \text{receiver} \text{ from } N, O \text{ and } T\)
12: \(\text{return } \text{sum}\)

There are 4 possible cases:
1. A proposer $i$ untruthfully reveals her preference and remains a proposer. As RPM implements subgame perfect Nash equilibrium in the corresponding subgame, the proposer $i$ can match with the best roommate among those accept her proposals by acting truthfully. Consequently, $i$ cannot improve by lying.

2. A receiver $j$ untruthfully reveals her preference and is still a receiver. In this case, if $j$ has an incentive to lie, there has to be a proposer $i'$ who prefers $j$ to her teammate under RPM, while $j$ must prefer $i'$ to her teammate. Steps 4 – 7 in Algorithm 2 count all such instances.

3. A proposer $i$ untruthfully reveals her preference and becomes a receiver. In this case, if $i$ has an incentive to untruthfully reveal her preference, there has to be a proposer $i'$ who prefers $i$ to their teammate under RPM, and who also prefers to her teammate. Steps 4 – 7 in Algorithm 2 count all such instances.

4. A receiver $j$ untruthfully reveals her preference and becomes a proposer. In this case, if $j$ has an incentive to misreport her preference, there must be a receiver $j'$ who prefers $j$ to her teammate, while $j$ must prefer $j'$ to her teammate. Steps 8 – 10 in Algorithm 2 count all such instances.

This upper bound obtains for both the exact and approximate versions of RPM, including HRPM. Next we evaluate the incentives to misreport preferences using our RPM approximations in the context of the roommate problem.

Table 1: Average upper bound of untruthful players for (Approximate) RPM

| $n$ | $m = 2, \alpha = 0$ | $m = 2, \alpha = 0.1$ | $m = 3, \alpha = 0$ | $m = 3, \alpha = 0.1$ |
|-----|------------------|----------------------|-------------------|---------------------|
| 20  | 0.015% 0.013% 0.015% 0.115% | 0.013% 0.10% 0.107% 0.103% | 0.002% 0.072% 0.085% 0.103% | 0.002% 0.038% 0.085% 0.103% |
| 30  | 0.008% 0.011% 0.015% 0.085% | 0.004% 0.022% 0.038% 0.085% | 0.01% 0.037% 0.076% 0.085% | 0.008% 0.024% 0.076% 0.085% |
| 40  | 0.011% 0.010% 0.015% 0.085% | 0.029% 0.029% 0.038% 0.085% | 0.01% 0.037% 0.076% 0.085% | 0.008% 0.024% 0.076% 0.085% |
| 50  | 0.010% 0.013% 0.015% 0.085% | 0.036% 0.036% 0.038% 0.085% | 0.01% 0.037% 0.076% 0.085% | 0.008% 0.024% 0.076% 0.085% |
| 60  | 0.011% 0.013% 0.015% 0.085% | 0.036% 0.036% 0.038% 0.085% | 0.01% 0.037% 0.076% 0.085% | 0.008% 0.024% 0.076% 0.085% |
| 70  | 0.011% 0.013% 0.015% 0.085% | 0.036% 0.036% 0.038% 0.085% | 0.01% 0.037% 0.076% 0.085% | 0.008% 0.024% 0.076% 0.085% |
| 80  | 0.011% 0.013% 0.015% 0.085% | 0.036% 0.036% 0.038% 0.085% | 0.01% 0.037% 0.076% 0.085% | 0.008% 0.024% 0.076% 0.085% |

Table 2: Lower bound of profiles where every player is truthful for (Approximate) RPM

| $n$ | $m = 2, \alpha = 0$ | $m = 2, \alpha = 0.1$ | $m = 3, \alpha = 0$ | $m = 3, \alpha = 0.1$ |
|-----|------------------|----------------------|-------------------|---------------------|
| 20  | 99.7% 99.6% 99.5% 99.9% | 99.7% 99.7% 99.4% 99.8% | 99.6% 99.2% 99.2% 99.2% | 99.7% 99.7% 99.4% 99.8% |
| 30  | 99.7% 99.7% 99.4% 99.8% | 98.8% 98.8% 98.1% 97.2% | 99.6% 99.2% 99.2% 99.2% | 99.7% 99.7% 99.4% 99.8% |
| 40  | 97.9% 98.8% 97.1% 98.1% | 97.8% 98.4% 98.3% 98.3% | 97.9% 98.4% 98.3% 98.3% | 97.9% 98.4% 98.3% 98.3% |
| 50  | 97.8% 98.6% 96.8% 96.2% | 96.3% 95.1% 92.9% 92.9% | 97.9% 98.4% 98.3% 98.3% | 97.9% 98.4% 98.3% 98.3% |

Table 1 presents the upper bound on the number of players with an incentive to lie, as a proportion of all players, on scale-free networks. We observe that the upper bound is always below 0.2%, and is even lower when the networks are sparse ($m = 2$). On the Karate club data, we did not find any player with an incentive to lie in test cases when we apply (Approximate) RPM. On the Newfrat data, the upper bounds are less than 7% and 0.4% when we apply RPM with and without heuristics, respectively. In addition, we also computed the lower bound on the fraction of preference profiles where truth telling is a Nash equilibrium (Table 2). We find that without the heuristic, when $m = 2$ (sparse networks), RPM is incentive compatible in more than 99% of the profiles; and when $m = 3$ (the networks are comparatively dense), RPM is truthful at least 96% of the time.

Table 3 presents the upper bound on the number of untruthful players for HRPM (still for the roommate problem). Even with this heuristic, we can see that fewer than 5% of the players have any incentive to misreport preferences in all cases.
Table 3: Average upper bound of untruthful players for HRPM

| n   | 20  | 30  | 40  | 50  | 60  | 70  | 80  |
|-----|-----|-----|-----|-----|-----|-----|-----|
| m = 2, β = 0.5 | 1.44% | 1.77% | 1.71% | 2.00% | 2.09% | 2.16% | 2.06% |
| m = 2, β = 0.6 | 1.62% | 1.83% | 1.96% | 2.09% | 2.25% | 2.11% | 2.11% |
| m = 3, β = 0.5 | 2.99% | 3.36% | 3.76% | 3.90% | 4.18% | 4.02% | 4.33% |
| m = 3, β = 0.6 | 3.44% | 3.69% | 3.97% | 3.98% | 4.40% | 4.24% | 4.52% |

2.3 Efficiency

In terms of social welfare, ex post Pareto optimality, satisfied by both random serial dictatorship (RSD) [2] and RPM, is a very weak criterion. Moreover, it is not necessarily satisfied by our approximations of RPM. Conversion of ordinal to cardinal preferences allows us to empirically consider utilitarian social welfare, a much stronger criterion commonly used in mechanism design with cardinal preferences.

We define utilitarian social welfare as \( \frac{1}{|N|} \sum_{i \in N} u_i(\pi_i) \), where \( \pi_i \) is the team that \( i \) was assigned to by the mechanism.

![Figure 3: Utilitarian social welfare for roommate problem](image)

Figures 3a and 3b depict the average utilitarian social welfare for RSD and RPM in the roommate problem on scale-free networks, Karate club networks, and the Newfrat data. In all cases, RPM yields significantly higher social welfare than RSD, with 15% – 20% improvement in most cases. These results are statistically significant \((p < 0.01)\). Furthermore, there is virtually no difference between exact and approximate RPM.

![Figure 4: Utilitarian social welfare for trio-roommate problem](image)

For the trio-roommate problem (in which the maximum size of team is 3), we compare HRPM \((\beta = 0.6)\) with RSD on the same data sets. Figures 4a and 4b show that HRPM yields significantly higher social welfare than RSD in all instances, and HPRM performs even better when the network is comparatively dense \((m = 3)\) in the scale-free
network). All results are statistically significant \( p < 0.01 \).

### 2.4 Fairness

A number of measures of fairness exist in prior literature. One common measure, envy-freeness, is too weak to use, especially for the roommates problem: every player who is not matched with her most preferred other will envy someone else. Indeed, because RPM matches soulmates—in contrast to RSD, which does not—it already guarantees the fewest number of envious players in the roommates problem. We consider two alternative measures that aim to capture different and complementary aspects of fairness: the Gini coefficient, representing the inequality among values of player utilities, and the correlation between utility and rank in the random proposer order (i.e., Pearson correlation).

![Figure 5: Gini coefficient for the roommate problem](image)

The Gini coefficient measures the inequality of player’s utilities. It is extracted based on the Lorenz curve \[7\]. A Gini coefficient of zero expresses perfect equality, where all the players have the same utility, while a Gini coefficient of one expresses maximal inequality among values (e.g., for a large number of players, where one team is composed of soulmates and all the players are matched to their least preferred team).

Correlation between utility and rank considers each random ranking of players in \( O \) used for both RSD and RPM, along with corresponding utilities \( u_i(\pi) \) of players for the partition \( \pi \) generated by the mechanism, and computes the correlation between these. It thereby captures the relative advantage that someone has by being earlier (or later) in the order to propose than others, and is a key cause of ex post inequity in RSD. We view the correlation measure as perhaps the most meaningful criterion of fairness for mechanisms based on random player rankings: for example, someone who is extremely unpopular is likely to have lower utility than others, but that’s likely to remain the case for any team formation mechanism with good efficiency properties. On the other hand, this may be relatively invariant of the ex post position that the player has in the order of proposers.

![Figure 6: Pearson Correlation for the roommate problem](image)

\[3\] The proportion of the total utility of the players that is cumulatively earned by the bottom \( x \% \) of the population.
Our experiments on the roommate problem show that RPM is significantly more equitable than RSD on scale-free networks (Figures 5a and 6a), as well as on the Karate club network and Newfrat dataset (Figures 5b and 6b). The differences between exact and approximate RPM are negligible in most instances.

In the trio-roommate problem, HRPM ($\beta = 0.6$) is much more equitable than RSD as shown in Figures 7 and 8. These results are statistically significant ($p < 0.01$).

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