Arrival-Time Detection and Ultrasonic Flow-Meter Applications

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Abstract. The Danfoss problem on ultrasonic flow measurement has been separated into three parts each handled by a subgroup of the authors listed above. The first subgroup deals with a presentation of modelling equations describing the physics of ultrasonic flow meters employing reciprocal ultrasonic transducer systems. The mathematical model presented allows the electrical output signal to be determined corresponding to any time-dependent electrical input signal. The transducers modelled consist of a piezoceramic material layer and a passive acoustic matching layer. The second subgroup analyzes the possibility of coding the input signal so as to simplify arrival-time detection by refining the coded input sequence in the received signal. The narrow-band nature of the transducers makes this problem non-trivial but suggestions for improvement are proposed. The analysis given is based on traditional auto- and cross-correlation techniques. The third subgroup attempts to improve existing correlation methods in determining arrival-time detection of signals. A mathematical formulation of the problem is given and the application to a set of real signals provided by Danfoss A/S is performed with good results.

1. Introduction
An investigation of possible ways for the electronic excitation of ultrasonic transducer reciprocal systems is presented with emphasis to arrival-time detection accuracy for use in ultrasonic flow meters. This is done in three steps: (1) by analyzing the physics of ultrasonic reciprocal systems allowing for material-property differences between the various layers in the transducers \cite{1,2,3} and the possibility to excite the system with a general input electrical signal (this includes the development of a MatLab reciprocal-transducer program), (2) by examining the influence of changing the input electrical signal using different coded signal inputs so as to detect more accurately and unambiguously the arrival time of the received signal based on auto- and cross-correlation techniques, and (3) propose new signal-processing methods and check them for consistency versus typical experimentally acquired ultrasonic signals.

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2. **Theoretical Modelling of Reciprocal Transducer Systems**

Figure 1 illustrates the transmission of an imposed voltage signal \( u(t) \) across the transmitter electrodes to the received voltage signal \( y(t) \) generated across the receiver electrodes. Henceforth, the transfer function components \( H_1(t) \), \( H_2(t) \), and \( H_3(t) \) account for individual contributions to the total transfer function \( H(t) \) from the transmitting transducer, the sound transmission between the transmitter and receiver, and the receiving transducer, respectively, i.e.,

\[
H(t) = H_3(t)H_2(t)H_1(t),
\]

satisfying

\[
H(t) = \frac{y(t)}{u(t)}.
\]

![Figure 1. Schematic of the transfer function for the reciprocal transducer system setup.](image1)

Since the system modelled is a linear system, we can determine \( y(t) \) from \( u(t) \) as follows:

\[
u(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(t) \exp(i\omega t) dt, \]
\[
y(\omega) = H_3(\omega)H_2(\omega)H_1(\omega)u(\omega) = H(\omega)u(\omega), \]
\[
y(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y(\omega) \exp(-i\omega t) d\omega. \]

(3)
In the following, we shall disregard the influence of the sound transmission part: \( H_2(\omega) \) as it merely represents a time delay between the output from the transmitter (aperture pressure) and the input to the receiver (incoming pressure wave) in the case of a vanishing fluid flow. In other words, we assume \( H_2(\omega) = 1 \). Next, we concentrate on determining \( H_1(\omega) \) with the remark that \( H_3(\omega) \) can be found following a similar procedure.

2.1. Determination of \( H_1(\omega) \)

A schematic figure of the transmitter is given in Figure 2. The governing differential equations are Newton’s Second Law, the definition of strain, and Poisson’s Equation:

\[
\begin{align*}
\rho \frac{\partial u}{\partial t} &= \frac{\partial T}{\partial z} \\
\frac{\partial u}{\partial z} &= \frac{\partial S}{\partial t} \\
\frac{\partial D}{\partial z} &= \rho_{\text{free}} = 0,
\end{align*}
\]

where \( \rho, \rho_{\text{free}}, u, T, S, D, t, \) and \( z \) are the mass density, free electric carrier density, particle velocity, stress, strain, electric displacement, time, and position, respectively. Employing the constitutive relation of a 1D piezoelectric material

\[
T = -hD + c^D S,
\]

where \( h \) and \( c^D \) are the piezoelectric \( h \) constant and bulk modulus, respectively, and Ohm’s Law:

\[
V(t) = Z_1 I + \int_{e_1}^{e_2} E(z,t) dz,
\]

with \( V(t), Z_1, I, e_1, e_2, \) and \( E(z,t) \) the input voltage signal, the electric impedance connected in series with the transmitting transducer, the current, \( z \)-position of the left electrode, \( z \)-position of the right electrode, and the electric field, respectively, a complete set of equations needed so as to specify the dynamics of the transmitter circuit has been formulated.

Writing \( V(t) \) in terms of its Fourier components:

\[
\begin{align*}
V(t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} V(\omega) \exp(-i\omega t) d\omega \\
V(\omega) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} V(t) \exp(i\omega t) dt,
\end{align*}
\]

and differentiating Equation (8) with respect to time allows a linear set of algebraic equations to be obtained for each frequency component \( V(\omega) \):

\[
\begin{align*}
T_L &= c^D_1 (A_2 + A_1) - h_1 D \\
\frac{T_L}{r_L} &= c^D_1 \left( \frac{A_2}{r_1} - \frac{A_1}{r_1} \right) \\
c_2^D (A_4 + A_3) &= c_1^D (A_2 \exp(-ik_1 l_1) + A_1 \exp(ik_1 l_1)) \\
&\quad - h_1 D \\
c_2^D \left( \frac{A_4}{r_2} - \frac{A_3}{r_2} \right) &= c_1^D \left( \frac{A_2 \exp(-ik_1 l_1)}{r_1} - \frac{A_1 \exp(ik_1 l_1)}{r_1} \right)
\end{align*}
\]
Figure 3. Output electrical signal for the case of a PZT5 piezoceramic transducer with steel matching layers (1 MHz frequency input, burst of eight sinus periods).

\[ TR = c_2^D \left( A_4 \exp(-ik_2l_2) + A_3 \exp(ik_2l_2) \right) \]
\[ -T_R = c_2^D \left( \frac{A_4 \exp(-ik_2l_2)}{r_2} - \frac{A_3 \exp(ik_2l_2)}{r_2} \right) \]
\[ -i\omega V(\omega) = -AZ_1 \omega^2 D - i\omega \frac{l_1}{\epsilon_1^S} D - h_1 \left[ c_2^D \left( \frac{A_4}{r_2} - \frac{A_3}{r_2} \right) - c_1^D \left( \frac{A_2}{r_1} - \frac{A_1}{r_1} \right) \right], \quad (11) \]

where \( A_i (i = 1, 2, 3, 4), r_i (i = 1, 2), T_L(T_R), r_L(r_R), A, l_i (i = 1, 2), \) and \( \epsilon_1^S \) are the wave amplitudes for the four forward and backward propagating strain waves shown in Figure 2, the acoustic impedance of layer 1 and layer 2 materials, the stress wave amplitude at the left (right) aperture, the acoustic impedance of the left (right) approximately semi-infinite materials surrounding the transmitter, transducer cross-sectional area, the layer lengths of material 1 and 2, and the dielectric constant of the piezoelectric material at constant strain, respectively. In obtaining the set of Equations (11), use has been made of the relations:

\[ I = A \frac{\partial D}{\partial t}, \]
\[ D = \epsilon_S^S E + \frac{\hbar}{\epsilon_S^S} S, \quad (12) \]

where the expression for the current assumes that displacement currents are of main importance in an ultrasonic transducer operating at ultrasonic frequencies (close to 1 MHz).

The solution to Equation (11) determines \( T_R \) which (apart from a sign change) is the input pressure wave amplitude on the receiving transducer, i.e., \( H_1(\omega) \) is found. A similar analysis of the receiver circuit allows the voltage component \( V_{rec}(\omega)d\omega \) generated across the receiver electrodes to be calculated, i.e., \( H_3(\omega) \) and \( H(\omega) \) are found.

2.2. Case studies

In Figures 3 and 4, calculated output signals \( y_1(t) \) and \( y_2(t) \) are shown for the input voltage signal cases:

\[ u_1(t) = \sin(\omega_0t) \times H(4\mu sec > t > -4\mu sec), \quad (13) \]
Figure 4. Output electrical signal for the case of a PZT5 piezoceramic transducer with steel matching layers (overlap of 50 kHz frequency input to the main 1 Mhz frequency input, burst of eight sinus periods).

and

\[ u_2(t) = \sin(\omega_0 t) \times \sin(\pi/2 + \omega_0 t/20) \times H(4\mu \text{sec} > t > -4\mu \text{sec}), \]  

(14)

respectively. In Equations (13) and (14), \( \frac{\omega_0}{2\pi} = 1 \text{ MHz} \) and

\[ H(a < t < b) = 1, \text{ if } a < t < b, \]
\[ H(a < t < b) = 0, \text{ elsewhere.} \]  

(15)

We finally note that the program delivered to Danfoss Flow Division can tackle any time-dependent voltage input \( u(t) \).

3. Coded signals

Next, we present an investigation of the possibilities of coding the input signal to the transmitting transducer for improving identification of the transmitted signals and thus the time of flight.

The basis for this investigation is a simple electrical model of a pair of ultrasonic transducers modelled in MatLab [4, 5].

The following systems are modelled (refer to Figures 5-11):

1. Set of transducers with a centre frequency at 1 MHz
   - a. Input signal is a single-frequency signal
   - b. Input signal is a chirp signal
   - c. Input signal is a two-frequency signal

All signals in the above cases 1a)-1c) are folded with an envelope (either a square- or a sine wave).

2. Set of transducers with different centre frequencies
   - a. Transducer offset +/- 3 % (from 1 MHz)
b. Transducer offset +/- 5 % (from 1 MHz)
c. Transducer offset +/- 10 % (from 1 MHz)

All the above cases 2a)-2c) are investigated with dual frequency signals where the two frequencies are at the centres of the transducer as well as offset further to the 3dB crossing. All cases are for both types of envelopes. Furthermore, in case 2b), a chirp signal has been used and a few tests were made with noise added.

In Figure 5, a block diagram of the setup is shown.

**Figure 5.** Block diagram in MatLab/Simulink of the setup.

The transfer function for the set of transducers having a centre frequency at 1 MHz is shown in Figure 6.

**Figure 6.** Bode diagram of the system transfer function. The upper curve is a plot of the magnitude as calculated in dB from −200 to 0 (y axis). The lower curve is a plot of the phase from −360 to +180 degrees (y axis). Both curves are plotted vs. frequency in the range 100 kHz to 10 MHz.

Employing an enveloped input signal as shown in Figure 7(left) we obtain the output signal shown in Figure 7(right). The output signal is approximately of half the amplitude as compared to the output signal corresponding to a rectangular envelope.
Figure 7. Left plot: Input electrical signal (in the range −2 V to +2 V) plotted vs. time in the range 0 to 50 microsecs. Right plot: Output electrical signal (in the range: −0.04 V to +0.04 V) plotted vs. time in the range 0 to 50 microsecs.

Figure 8. Bode diagram of the system transfer function. The upper curve is a plot of the magnitude as calculated in dB from −180 to −20 (y axis). The lower curve is a plot of the phase from −360 to +180 degrees (y axis). Both curves are plotted vs. frequency in the range 100 kHz to 10 MHz.

Using a set of transducers with different resonance frequencies, say approximately +/- 5%, we get a transfer function as shown in Figure 8.

This combination is, of course, making the set of transducers slightly more broad-banded thus decreasing the signals moderately but - as a gain - increasing the frequency-band, allowing for a signal in the range +/- app. 8% relative to the original centre frequency to pass through the system.

Using an input signal consisting of two single frequencies, 0.96 and 1.04 MHz, respectively, convoluted with a sine envelope as shown in Figure 9, we obtain a fairly well-behaved output signal as shown in the upper right part of the same figure. The autocorrelation and cross-correlation shown in the lower part of Figure 9 is quite a bit more pronounced or sharper compared to the auto- and cross-correlation obtained with the 1 MHz single frequency system.
Figure 9. Upperleft plot: Input electrical signal (in the range $-4 \text{ V}$ to $+4 \text{ V}$) plotted vs. time in the range 0 to 50 microsecs. Upperright plot: Output electrical signal (in the range: $-0.04 \text{ V}$ to $+0.04 \text{ V}$) plotted vs. time in the range 0 to 50 microsecs. Lowerleft plot: Auto-correlation vs. sampling number (20 samples per microsecond; the $y$ axis is from $-0.08$ to $+0.08$) and the $x$ axis is from 0 to 2500 samples. Lowerright plot: Cross-correlation vs. sampling number (20 samples per microsecond). The $y$ axis ranges from -8 to 8 and the $x$ axis ranges from 0 to 2500 samples.

which, of course, is an improvement.

To investigate the noise immunity of the 'broadband system' 100% noise is added to the input signal. Superimposing the noise to a two-frequency input signal of 0.94 MHz and 1.06 MHz we get an output signal and a cross-correlation as shown in Figure 10. Even in this case the cross-correlation seems fairly pronounced while having a maximum peak easy to identify. Reducing the noise to a more moderate level of 10%, commonly seen in applications, and using a chirped signal varying from 0.94 MHz to 1.06 MHz we get an output signal and a cross-correlation as seen in Figure 11. Again a fairly sharp cross-correlation is obtained indicating that a coded transmitted signal could lead to improvements in the identification of received signal arrival times, although the amplitude is somewhat reduced.

4. Signal-processing methods for use in determining arrival times of ultrasonic signals

The third subgroup of the Danfoss Flow Division ultrasonic arrival-time group has examined possibilities for improving on the signal processing methods for detecting ultrasonic signals. It is necessary to introduce appropriate notation for the analysis described in this Section.

- All the calculations will be done exclusively in $\mathbb{C}^L$ using the dot-product and the Euclidean
Figure 10. Left plot: Output electrical signal (in the range: $-0.1$ V to $+0.1$ V) plotted vs. time in the range 0 to 50 microsecs. Right plot: Cross-correlation vs. sampling number (20 samples per microsecond). The $y$ axis ranges from $-8$ to $+8$ and the $x$ axis ranges from 0 to 2500 samples.

Figure 11. Left plot: Output electrical signal (in the range $-0.02$ V to $+0.02$ V) plotted vs. time in the range 0 to 50 microsecs. Right plot: Cross-correlation vs. sampling number (20 samples per microsecond). The $y$ axis ranges from $-2.5$ to $+2.5$ and the $x$ axis ranges from 0 to 2500 samples.

norm:

$$\|f\|_2 = \sqrt{\sum_{j=0}^{L-1}|f(j)|^2},$$

where $f$ is a signal of length $L$. 

• For all \( t = 0, \ldots, L - 1 \), the translation operator \( \tau_t : \mathbb{R}^L \to \mathbb{R}^L \) (or \( \mathbb{C}^L \to \mathbb{C}^L \)) is defined by
\[
(\tau_t f)(j) = f(j - t), \quad j = 0, \ldots, L - 1.
\]
(17)

Note that since the signals are of finite length, the translation is really a circular shift of the signal.

• For all \( k = 0, \ldots, L - 1 \) the modulation operator \( m_k : \mathbb{C}^L \to \mathbb{C}^L \) is defined by
\[
(m_k f)(j) = e^{2\pi i kj/L} f(j), \quad j = 0, \ldots, L - 1.
\]
(18)

The Discrete Fourier transform \( \mathcal{F} : \mathbb{C}^L \to \mathbb{C}^L \) is defined by
\[
(\mathcal{F} f)(k) = \frac{1}{\sqrt{L}} \sum_{j=0}^{L-1} f(j)e^{-2\pi i kj/L}.
\]
(19)

Because of this normalization: \( \|\mathcal{F} f\|_2 = \|f\|_2 \).

• The following relations hold for interchanging the operators:
\[
\mathcal{F}\tau_t f = m_{-t}\mathcal{F} f
\]
(20)

and
\[
m_k\tau_t f = e^{2\pi i kt/L}\tau_t m_k f
\]
(21)

4.1. The problem
The approach is to consider existing real-world data and to try to improve existing correlation methods. We present essentially a mathematical formulation of the problem and the application to some provided real signals. First, two real signals of length \( L \) are given. The upstream signal is denoted by \( y_1 \in \mathbb{R}^L \) and the downstream signal by \( y_2 \in \mathbb{R}^L \). Next, we propose to model the main difference between \( y_1 \) and \( y_2 \) as a time shift \( \Delta t \), or equivalently, \( y_1 \approx \tau_{\Delta t} y_2 \).

The problem with this model is that the signals may be noisy, and the tails of the signal do not match very well. For illustration, some real world signals can be seen in Figure 12.

For each example, two signals are displayed. The first signal is the one which traveled against the flow direction (labelled “upstream”) and the second signal is the signal which traveled along the flow direction (labelled “downstream”). The signals correspond to a location after passage through the tube.

The signals are not very noisy and they consist mainly of one frequency close to 1 MHz. The differences between the up- and downstream signals are clearly visible.

The solution we propose is to try to match the signals in an appropriate norm that focuses on the part of the signals which are clearly identifiable.

In addition, for a correct operation of the flow meter, the time difference \( \Delta t \) must be determined with very high precision. In other words, the error on \( \Delta t \) must be less than the sampling distance. In this study, we have not tried to obtain such a high precision. However, we have managed to match the signals according to a precision within one wavelength (which for these examples is close to 20 samples). When \( \Delta t \) is determined with a better precision than one wavelength, the precision can be improved by studying the zero-crossings of the signal.

4.2. Time-frequency analysis
The Discrete Short Time Fourier Transform (DSTFT) with window \( g \in \mathbb{C}^L \) is given by
\[
: \mathbb{C}^L \to \mathbb{C}^{L \times L}, \quad (V_g f)(m, n) = f \cdot m_n \tau_n g.
\]
(22)
Since the DSTFT maps $\mathbb{C}^L$ into $\mathbb{C}^{L \times L}$, it is a highly redundant transform. To avoid some of this redundancy, we are restricted to only the coefficients situated on a regularly spaced grid, i.e., to consider $c \in \mathbb{C}^{M \times N}$ given by

$$c(m, n) = f \cdot m_{mb}^{\tau} g,$$

for some integers $a, b \in \mathbb{N}$ for which $L = Ma = Nb$ where $M, N \in \mathbb{N}$.

Depending on the parameters $a, b$ and the window $g$ it might be possible to reconstruct the original window from the coefficients $c$. This is studied in the theory of Gabor frames, see e.g., [8, 7].

### 4.3. Upstream-downstream flows time shift versus $l^2$-norms

In this subsection, we are interested in evaluating the time shift $\Delta t$ between the upstream and the downstream flows as given above with respect to different norms. $\Delta t$ is then calculated by minimizing, given the norm chosen, the difference between two signal data obtained for the two flows. The following subsections list the minimized quantities for five norms.

#### 4.4. The standard $l^2$-norm

The first choice is to use the standard Euclidean norm. The optimization problem to solve becomes:

$$\Delta t = \arg \min_t \|y_1 - \tau_t y_2\|_2$$
arg \min_t \sum_{j=0}^{L-1} |y_1(j) - y_2(j - t)|^2, \quad (24)

or, equivalently, using the DFT

\[
\Delta t = \arg \min_t \|\mathcal{F}y_1 - m \cdot \mathcal{F}y_2\|_2
\]

\[
= \arg \min_t \sum_{j=0}^{L-1} \left| (\mathcal{F}y_1)(j) - e^{-2\pi i j t/L} (\mathcal{F}y_2)(j) \right|^2. \quad (25)
\]

4.5. Weighted $l^2$-norm
Since the tails of the signals often differ, we propose a weighted $l^2$-norm, where $w \in \mathbb{R}^L$ is a weight-function that picks out the front of the signal by some heuristic approach. The optimization problem becomes:

\[
\Delta t = \arg \min_t \sum_{j=0}^{L-1} w(j) |y_1(j) - y_2(j - t)|^2. \quad (26)
\]

In order to roughly determine where the signal starts, we used the following primitive heuristic: Find the point in time where the signal first time goes above 50% of the maximum amplitude, and subtract four full wavelengths from this. This approach worked well for the few examples we considered. The important point to note is that while it is very difficult to determine the exact starting point of the signal, it is far easier to determine it plus/minus a few wavelengths.

4.6. The $l^2$-norm of a noise-filtered signal
The signals are very well localized in frequency around 1 MHz. In order to remove the noise, the standard technique of filtering the DFT can be used. The filtered signals $\tilde{y}_1$ and $\tilde{y}_2$ are determined by

\[
(\mathcal{F} \tilde{y}_1)(\omega) = \alpha(\omega) (\mathcal{F}y_1)(\omega), \quad (27)
\]

and similarly for $y_2$. Here $\alpha$ is a weight function that picks out the sought frequencies. The optimization problem becomes

\[
\Delta t = \arg \min_t \sum_{j=0}^{L-1} |\tilde{y}_1(j) - \tilde{y}_2(j - t)|^2, \quad (28)
\]

or equivalently,

\[
\Delta t = \arg \min_t \sum_{k=0}^{L-1} \left| (\alpha \mathcal{F}y_1)(k) - e^{-2\pi i k t/L} (\alpha \mathcal{F}y_2)(k) \right|^2. \quad (29)
\]

4.7. Weighted $l^2$-norm of noise-filtered signal
By combining the two previous methods, we obtain a joint time and frequency-filtering method. The corresponding optimization problem becomes

\[
\Delta t = \arg \min_t \sum_{j=0}^{L-1} w(j) |\tilde{y}_1(j) - \tilde{y}_2(j - t)|^2, \quad (30)
\]

where $\alpha$, $w$, $\tilde{y}_1$, and $\tilde{y}_2$ are defined as previously.
4.8. The $l^2$-norm of a selected part of the DSTFT

Alternatively, one can choose to consider only a cut-out of a sub-sampling of the DSTFT. The optimization problem to be solved is

$$\Delta t = \arg \min_t \sum_{m=m_1}^{m_2} \sum_{n=n_1}^{n_2} |y_1 \cdot m_{mb}\tau_{na}g - \tau_2y_2 \cdot m_{mb}\tau_{na}g|^2,$$

where the integers $m_1, m_2, n_1,$ and $n_2$ are chosen by some heuristic approach such that they pick out the most informative part of the time frequency plane.

The method simply cuts out a rectangular part of the time-frequency plane. The place to cut is determined by the same heuristic approach as in the previous methods.

Using Equation (21),

$$\tau_{t+pa}y_2 \cdot m_{mb}\tau_{na}g = e^{-pamb/L}\tau_{t}y_2 \cdot m_{mb}\tau_{a(n-p)}g.$$

This shows that it is not necessary to compute $c_t(m, n) = \tau_ty_2 \cdot m_{mb}\tau_{na}g$ for all $t$, but only so for $t = 0, ..., a - 1$, and then use that $c_1(m, n) = e^{-pamb/L}c_t(m, n - p)$ so as to compute the remaining coefficients. In Ref. [9] it is shown how to compute $y_1 \cdot m_{mb}\tau_{na}g$ for all $m = 0, ..., M - 1$ and $n = 0, ..., N - 1$ using $O(LNd)$ flops, where $d$ is the greatest common denominator of $b$ and $N$.

4.9. An example

As an application of the presented method, the example used is “Signal 3” from Figure 12 with added noise. The noise-filled signals can be seen in Figure 13.

Figure 14 shows how the time filtering and frequency filtering works.

The noise reduction removes the noise efficiently, but it also changes the envelope of the signal slightly. The time-filtering picks out the front of the signal by some degree of success.

The results for the different norms are shown on Figure 15. The signal appears almost identical to itself when shifted by a single wavelength and the fine details are smeared out due to noise. When no filtering is applied, the consequence is that the correct solution is hardly visible. Time filtering improves the case, and time-frequency filtering even more so. However, the method based on the DSTFT does not show an improvement over the simple time-frequency method.
Figure 14. The figure shows how the various filtering methods work. The first plot shows the original signal, the second plot shows the signal after noise reduction overlaid with the window to use for the time filtering. The third and fourth plot show how the signals look after the two different ways of time/frequency filtering have been applied. Signals are plotted in arbitrary units vs. sampling number (20 samples per microsec).

Figure 15. Each plot shows one of the four norms around the best ($\Delta t = 25$) and second-best ($\Delta t = 5$) local minimum. The $x$ axis ranges from 1 to 7 and the $y$ axis ranges from 1 to 100 (upper left) and 0.1 to 10 (other three plots) in semi-logarithmic plots.
5. Conclusion
The first subgroup on the Danfoss Flow Division problem dealt with obtaining a general model based on the physics of piezoceramic structures and matching layers. The model allows any time-dependent input to be analyzed using Fourier-transform techniques. A MatLab model is constructed for this purpose and some examples are considered. The second subgroup analyzed the influence of coding the input signal so as to obtain an unambiguous and accurate detection of arrival times based on traditional auto- and cross-correlation techniques. The main conclusion of this subgroup is that in the case with a set of narrowband transducers it is almost impossible to get improvements with a coded input signal since the response is almost exclusively at the centre frequency. Best compromise seems to be a dual frequency input 1 MHz +/- 5% with pair of transducers with centre frequencies at 1 MHz +/- 5% as well. Alternatively a swept frequency signal from 950 kHz to 1050 kHz may be used with the same set of transducers. The third subgroup has demonstrated on a real-world example that the new suggested methods work. However, since the examples given to us are few and the heuristics involved are many, we cannot say if these ideas can be used for production. Much further testing is needed: To determine the best choice of parameters (windows sizes and shapes, heuristics to find the beginning of the signal etc.), and also to determine which of the methods to use. The three subgroups have developed computer codes in MatLab used in the calculation of results presented (computer codes have been transferred to Danfoss A/S).

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