The error modeling and measurement of 3-axis machine tools

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Abstract. Machine tools’ accuracy is key characteristic, which may affect processing quality. Error compensation is an efficient and economical way to upgrade machine tools’ precision. The paper focuses on the basic research of error compensation, which includes source of errors analysis, error modelling and errors acquirement. Thus, 21 geometric errors are sought out which have strong influence on the precision; relationship between errors and compensation is built; the measurements for all 21 errors (including the vertical roll error) are presented, by the experiments; the feasibility is validated.

Key words: Error compensation; geometric errors; measurement; machine tool.

1. Introduction
With the development of industrialization, products become more complicated, more delicate and more diverse, which calls for high quality machine tools, especially those of high accuracy. machine tools. So how to improve machine tools’ precision is a hot topic among researches.

Generally speaking, there are 2 methods to enhance the accuracy. One is error avoiding, which needs to design elaborately, produce exactly and install carefully. As such, the cost is huge, while it will encounter a bottleneck when the machine’s accuracy has reached at a certain level. Thus, it is not an optimal choice. The alternative method is error compensation. It gives up the concept of reducing errors, but it makes use of new adding errors to offset original errors.

2. The steps to improve the accuracy of machine tools by error compensation.
Three steps are indispensable for machine tools’ error compensation. The first step is the analysis of the source of errors, which is aimed to seek out the important errors. These errors may affect the machine tools’ accuracy badly. The second step is errors modelling, which is used to establish the relationship between involved errors. The last but not least step is errors acquirement, which represents measurement. All the procedure is given in Fig.1.
3. Analysis of Source of Errors

Many errors may affect machine tools’ precision, such as geometric errors, thermal errors, etc. More details are shown in Fig.2. According to previous research[1-2], geometric errors play an important role in machine tools’ precision, so the paper focuses on them.

For 3-axis machine tools, there are 3 translational axes in each of them, which are X-axis, Y-axis and Z-axis. Ideally, each axis has only one degree of freedom, so it can move or rotate in one direction. However, the inescapable errors make it can slightly move or rotate in every direction. According to theory of degree of freedom, each object without constrains has 6 degrees of freedom, therefore, each axis may have 6 types of errors. Take X-axis for example, the positioning error \( \delta_x(y) \), the horizontal straightness error \( \delta_h(y) \), the vertical straightness error \( \delta_v(y) \), the roll error \( \epsilon_r(y) \), the yaw error \( \epsilon_y(y) \) and the pitch error \( \epsilon_z(y) \). The positioning error and straightness errors are translational errors and the rest three are angular errors[3].

The errors of rest translational axes—Y-axis and Z-axis are similar to the ones of X-axis, so they won’t be repeated here.
Besides the each axis errors, the errors between axes should be introduced, which are squareness errors—\( \varepsilon_{xy} \), \( \varepsilon_{yz} \), and \( \varepsilon_{xz} \). Therefore, for 3-axis machine tools, there are 21 geometric errors in total.

4. Error Modelling

3-axis machine tools have several different structures, which make the models various. Even so, the principles of modelling are similar, hence the structure of TYXZ machine tool is taken for example to elaborate the modelling process. Shown in Fig.3, 0 is the fixed base, 1 is machining workpiece, 2 is the gantry which can move along Y-axis, 3 is the sliding block, which run along X-axis, 4 represents the column which is parallel to Z-axis, and 5 is the cutting tool.

When machining, the cutting tool should coincide with the workpiece at the certain point in ideal condition, however, in reality there always exists gap. In order to describe the difference, Denavit-Hartenberg matrixes and homogeneous transformations are utilized. The former is used to explain the position and gesture of workpiece and cutting tool in their own coordinate systems, while the latter can transform the position and the gesture between different coordinate systems.

\[
\begin{align*}
T_0 &= T_{02} T_{23} T_{34} T_{45} T_t \\
T_w &= T_{01} T_w \\
[\hat{E}] &= 0T_t - 0T_w
\end{align*}
\]

5. Measurement

All of 21 geometric errors can be gathered by laser interferometer (it is named laser interferometer method), except for vertical roll errors \( \varepsilon_z(z) \). The principle of laser interferometer is given in Fig.5. There are two reflectors, where one is fixed with splitter during measurement, and keeps static, while
the other is mobile reflector which moves with the motion axis. The beam comes from laser head to the fixed reflector where it meets with the splitter. The splitter separates the beams into two, with one beam going up to the fixed reflector and returning (shown in line 1), and the other beam going to the mobile reflector and turning back (shown in line 2). At the splitter, two beams will reunite into one beam which goes back to the detector (in laser head). The detector can monitor the interference phenomenon and it will record the times of interference when the mobile reflector moves from A to B. According the times, the distance L can be calculated [4].

![Fig.5 The principle of laser interferometer](image)

The vertical roll errors $\varepsilon_z(z)$ can be acquired by double dial indicators method. In order to explain the principle of the method, plate A is supposed to fix with Z-axis and be parallel to the ZX plate, given in Fig. 6a. When Z-axis moves, the plate A moves with it together. Two points, named m and n, exist in the plate A. They have distance L from Z-axis at the same level. Supposed that plate B is parallel to plate A and they have the distance t. When Z-axis moves, two geometric errors may influence the distance between the point n and the plate B (the distance is named $t_1$), as well as the distance between the point m and the plate B (the distance is named $t_2$). The two errors are Z-axis’ roll error $\varepsilon_z(z)$ and straightness error $\delta_z(z)$.

![Fig.6 The principle of double dial indicators method](image)

In order to gain the value of roll error $\varepsilon_z(z)$, the two errors’ characteristic should be analyzed. When Z-axis moves, $\delta_z(z)$ may increase or reduce the value of $t_1$ and $t_2$ together, while $\varepsilon_z(z)$ may change $t_1$ and $t_2$ in opposite directions. Therefore, according to the variation of $t_1$ (named $\Delta_m$) and $t_2$ (named $\Delta_n$), as well as the center to center distance (named L), the roll error can be calculated, given in Eq.1. The variation can be measured by dial indicator, while plate B is a high precision plate, shown in Fig. 6b.

$$\varepsilon_z(z) = \frac{\Delta_m - \Delta_n}{L}$$ (4)
6. Experiment
To verify the mentioned methods, experiments are done. Due to the limitation of experimental conditions, tests are carried out in 5-axis machine tools (the rotary axes are locked during measurements), shown in Fig.7. The range of Y-axis is from 0 to -1000mm, X-axis is from 0 to -1200mm, and Z-axis is from 0 to -500mm.

![The machine tool](image1)

**Fig.7** The machine tool

![The measurement of Y-axis’ positioning error](image2)

**Fig.8** The measurement of Y-axis’ positioning error

To avoid redundancy, each method gives one example to elaborate. For laser interferometer method, the positioning error of Y-axis is selected. The used laser interferometer is Renishaw’s XL-80, shown in Fig.8, and the results are given in Fig.9. The measurement is carried out from -1000mm to 0 mm, the max value is 0.027009mm and the backlash is 0.001500mm.

![The results of Y-axis’ positioning error](image3)

**Fig.9** The results of Y-axis’ positioning error

The double dial indicators method needs a high precision plate and two dial indicators, shown in Fig.10. Due to the limitation of installment, the measurement range is from -210mm to -550mm, the step is 20mm. The results are given in Fig.11.

![The measurement of Z-axis’ roll error](image4)

**Fig.10** The measurement of Z-axis’ roll error
7. Conclusion
In the paper, the source of 3-axis machine tools’ errors is analyzed and 21 geometric errors are found out which have strong effect on machine tools’ accuracy; the error model is established, which builds a bridge between errors and compensation; the errors acquirement methods (measurement methods) are proposed, by the experiments, the feasibility is validated. The results given by the methods can provide date basis for later compensation.

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