EXCLUSIVE PRODUCTION OF PION PAIRS WITH LARGE INVARIANT MASS IN NUCLEUS-NUCLEUS COLLISIONS

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The cross section for exclusive $\pi^+\pi^-$ and $\pi^0\pi^0$ meson pairs production in peripheral ultrarelativistic heavy-ion collisions is calculated at the energy available at the CERN Large Hadron Collider, i.e., $\sqrt{s_{NN}} = 3.5$ TeV. The cross section for elementary $\gamma\gamma \to \pi\pi$ process is calculated with the help of the pQCD Brodsky-Lepage approach with the distribution amplitude used recently to describe the pion transition form factor measured by the BaBar Collaboration.

1 Introduction

Figure 1: The Feynman diagram for the formation of the pion pair. The $A_1$ and $A_2$ letters denote the $^{208}Pb$ nuclei.

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It is known that ultrarelativistic colliding heavy ions are a source of high-energy $\gamma\gamma$ collisions. We present realistic cross section for exclusive electromagnetic production of two neutral and two charged pions in coherent photon-photon processes in ultrarelativistic heavy-ion collisions. We consider $PbPb \rightarrow PbPb\pi^+\pi^-$ and $PbPb \rightarrow PbPb\pi^0\pi^0$ reactions. In Fig. 1 we show the basic mechanism of the exclusive production of $\pi^+\pi^-$ and $\pi^0\pi^0$ meson pairs. To calculate the correct cross section, we have to take into account several important factors. First we include realistic charge densities in nuclei. The validity of this ingredient has been presented in our previous publications where we have studied the production of $\rho^0\rho^0$ [1], $\mu^+\mu^-$ [2], heavy-quark heavy-antiquark [3] as well as $D\bar{D}$ [4] pairs. The next step is a correct description of the elementary cross section. This was done using the approach proposed by Brodsky and Lepage. They made a first prediction of the LO pQCD approach [5]. The pQCD amplitude for the $\gamma\gamma \rightarrow \pi\pi$ reaction depends on the pion distribution amplitude. It was believed for long time that the pion distribution amplitude is close to the asymptotic form. This turned out to be inconsistent with recent results of the BaBar Collaboration [6] for the pion transition form factor for large photon virtualities.

2 The elementary cross section for $\gamma\gamma \rightarrow \pi\pi$

\begin{equation}
M(\lambda_1, \lambda_2) = \int_0^1 dx \int_0^1 dy \, \phi_\pi(x, \mu_x^2) \, T^{\lambda_1 \lambda_2}_H(x, y, \mu^2) \, \phi_\pi(y, \mu_y^2) \times F_{eq}^{pQCD}(t, u), \tag{1}
\end{equation}

where $\mu_{x/y} = \min(x/y, 1 - x/y) \sqrt{s(1 - z^2)}$; $z = \cos \theta$ [5]. We take the helicity dependent hard scattering amplitudes from Ref. [7]. These scattering amplitudes are different.
for $\pi^+\pi^-$ and $\pi^0\pi^0$. The extra form factor in Eq. 1 was proposed in Ref. [8]:

$$F^{pQCD}_{\text{reg}}(t, u) = \left[ 1 - \exp \left( \frac{t - t_m}{\Lambda_{\text{reg}}^2} \right) \right] \left[ 1 - \exp \left( \frac{u - u_m}{\Lambda_{\text{reg}}^2} \right) \right], \quad (2)$$

where $t_m = u_m$ are the maximal kinematically allowed values of $t$ and $u$. This form factor excludes the region of small Mandelstam $t$ and $u$ variables which is clearly of nonperturbative nature.

![Figure 3: The quark distribution amplitude of the pion.](image)

The distribution amplitudes are subjected to the ERBL pQCD evolution [9, 10]. The scale dependent quark distribution amplitude of the pion [11, 12] can be expanded in term of the Gegenbauer polynomials:

$$\phi_\pi(x, \mu^2) = \frac{f_\pi}{2\sqrt{3}} C_{3/2}^{\alpha} (2x - 1) a_n(\mu^2), \quad (3)$$

where the expansion coefficients $a_n(\mu^2)$ depend among others on the form of the distribution amplitude $\phi_\pi(x, \mu_0^2)$. Different distribution amplitudes have been used in the past [5, 13, 12]. Wu and Huang [14] proposed recently a new solution:

$$\phi_\pi(x, \mu_0^2) = \frac{\sqrt{3} A m_q \beta}{2\sqrt{2}\pi^{3/2} f_\pi} \sqrt{x(1-x)} \left( 1 + B \times C_2^{3/2} (2x - 1) \right)$$

$$\times \left[ \text{Erf} \left( \sqrt{\frac{m_q^2 + \mu_0^2}{8\beta^2 x(1-x)}} \right) - \text{Erf} \left( \frac{m_q^2}{8\beta^2 x(1-x)} \right) \right]. \quad (4)$$
This pion distribution amplitude at the initial scale \( \mu_0^2 = 1 \text{ GeV}^2 \) is controlled by the parameter B. It has been found that the BABAR data at low and high momentum regions can be well described by setting B to be around 0.6. As seen from Fig. 3, this pion distribution amplitude is rather close to the well know Chernyak-Zhitnitsky \(^{15}\) distribution amplitude.

Finally, the total (angle integrated) cross section can be calculated as:

\[
\sigma (\gamma \gamma \to \pi \pi) = \int \frac{2\pi}{64\pi^2W_0^2} \frac{p}{q^2} \sum_{\lambda_1, \lambda_2} |M (\lambda_1, \lambda_2)|^2 \, dz .
\] (5)

3 The nuclear cross section for the \( \text{PbPb} \to \text{PbPb} \pi \pi \) process

Figure 4: The nuclear (upper lines) and elementary (lower lines) cross section as a function of photon–photon subsystem energy \( W_{\gamma \gamma} \) in the b-space EPA.

The nuclear cross section has been calculated with the help of b-space equivalent photon approximation (EPA). This approach allows to separate peripheral collisions of nuclei \( b > R_1 + R_2 \approx 14 \text{ fm} \). A compact formula for calculating the total cross section takes the form:

\[
\sigma (\text{PbPb} \to \text{PbPb} \pi \pi; W_{\gamma \gamma}) = \int \hat{\sigma} (\gamma \gamma \to \pi \pi; W_{\gamma \gamma}) \theta (|b_1 - b_2| - 2R_A) \\
\times N (\omega_1, b_1) N (\omega_2, b_2) 2\pi b \, d\vec{b} \, dB_x \, dB_y \frac{W_{\gamma \gamma}^2}{2} \, dW_{\gamma \gamma} \, dY .
\] (6)
The details of its derivation can be found in our last papers [1, 2, 3].

In Fig. 4 we show distribution in the two-pion invariant mass. Here we have taken experimental limitations usually used for the $\pi\pi$ production in $e^+e^-$ collisions. In the same figure we show our results for the $\gamma\gamma$ processes extracted from the $e^+e^-$ collisions together with the corresponding nuclear cross sections for $\pi^+\pi^-$ (left panel) and $\pi^0\pi^0$ (right panel) production. We show the results for the case when we include the extra form factor in Eq. 2 (solid lines) and for the case when $F_{reg}^{pQCD}(t, u) = 1$ (dashed lines). One can see that a difference occurs only at small energies which is not the subject of the present analysis. Above $\sqrt{s_{\gamma\gamma}} > 3$ GeV the two approaches coincide. By comparison of the elementary and nuclear cross sections we see a large enhancement of the order of $10^4$ which is somewhat less than $Z_1^2Z_2^2$ one could expect from a naive counting.

4 Outlook

In Fig. 5 we show the ratio of the cross section for the $\gamma\gamma \rightarrow \pi^0\pi^0$ process to that for the $\gamma\gamma \rightarrow \pi^+\pi^-$ process. The dashed line represents the hand-bag model [16] result and the solid lines is for the Brodsky-Lepage pQCD approach. For larger $z = \cos \theta$ the ratio is smaller which means that the ratio is $z$ dependent. The ratio is practically independent of the collision energy. In the present calculations the $z$-averaged ratio for
$|\cos \theta| < 0.6$ is about 0.12. The experimental data points are in between the predictions of the BL pQCD approach and the hand-bag model which further clouds the situation. More results one can find in our last paper [17].

From Fig. 5 we can conclude that other mechanisms are necessary to correctly describe the elementary cross section. We are in the process of including pion exchange, resonances and high-energy $\pi\pi$ rescatterings.

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