Quantum model for magnetic multivalued recording in coupled multilayers

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**Abstract**

In this paper, we discuss the possibilities of realizing the magnetic multi-valued (MMV) recording in a magnetic coupled multilayer. The hysteresis loop of a double-layer system is studied analytically, and the conditions for achieving the MMV recording are given. The conditions are studied from different respects, and the phase diagrams for the anisotropic parameters are given in the end.
Many efforts have been devoted to the studies of magnetic multilayers because there have been lots of fascinating behaviors displayed in such systems [1-4]. One of the important applications of magnetic multilayers in technology is that they can be used as recording media for memory devices. In such materials, the hysteresis loop of one domain should be rather rectangular in order that two messages can be recorded in the “spin-up” and “spin-down” states, respectively. Recently, much attention has been paid to increase the density of the recording media. One of the proposals is to diminish the size of the domain. However, the recording density will eventually come to a limit following this way, so that one must try to find new approaches. A simple idea is that: if more messages (than two) can be recorded in one domain, the recording density will be highly improved even though the domain’s size remains the same. This is just the idea of the magnetic multi-valued (MMV) recording which is believed to be the next strategy of high density recording and has attracted much attention from both experimental and theoretical sides. The MMV recording requires that more (than two) metastable phases which are stable enough to record messages must exist in the system; therefore, the hysteresis loop for such material should contain more (than one) sharp steps. Experimentally, the MMV recording was firstly confirmed by the field modulation method on disks of bilayers [3] or island on thin layers [4]. However, the theoretical origin is not yet clear.

More recently, a quantum theory of the coercive force [5] has been established for magnetic systems on the basis of some previous works [6-8]. The concept of metastable state was adopted and the magnon excitation gap was found to be an appropriate order parameter to monitor the stability of the metastable state and to determine the coercive force of the magnetic systems [5]. The quantum approach enables one to study the hysteresis behaviors of a magnetic system from a micromagnetic view, and some interesting effects in double-film structures had been discussed by this method [5].

The present letter is devoted to proposing a theoretical possibility of achieving the MMV recording in magnetic multilayers with the help of the quantum method. The main idea is to find more metastable states in such systems. It is found that the MMV recording can be realized if the interlayer coupling and the perpendicular anisotropic constants may satisfy some conditions.

In this paper, a double-layer system will be investigated analytically. The Hamiltonian can be given by:

\[ H = -\frac{1}{2} \sum_{m,m'} \sum_{\mathbf{R}, \mathbf{R}' \neq \mathbf{R}} I_{m,m'}(\mathbf{R}, \mathbf{R}') \mathbf{S}_m(\mathbf{R}) \cdot \mathbf{S}_{m'}(\mathbf{R}') - h \sum_{m, \mathbf{R}} S^z_m(\mathbf{R}) \]

\[ -\sum_{m} \sum_{\mathbf{R}} D_m(S^z_m(\mathbf{R}))^2, \]  

(1)

where the subscripts \( m, m' \) are the number of layers, and \( \mathbf{R}, \mathbf{R}' \) are the vectors of lattices on the \( x-y \) plane. \( I_{m,m'}(\mathbf{R}, \mathbf{R}') \) are the exchange parameters and only the nearest-neighbor interaction is considered. The single-ion anisotropy is the “easy-axis” case \((D_m > 0)\), and the “easy-axis” is perpendicular to the film. The spins and the anisotropies in different layers are different. It is supposed that \( S_1 > S_2 \) without losing any generality.

Following Refs. 5-6, we will introduce the local coordinates (LC) system \( \{\hat{x}_m, \hat{y}_m, \hat{z}_m\} \). The spin components in the LC system will have the following relations with those in the original one: \( S^x_m = \cos \theta_m S^x_m + \sin \theta_m S^y_m \), \( S^y_m = S^y_m(\mathbf{r}) \), \( S^z_m = \cos \theta_m S^z_m - \sin \theta_m S^x_m \). In order to study the ground state properties and the low-lying spin-wave excitations, one can apply the usual spin-Bose transformation such as Holstein-Primakoff (H-P) [9] or the complete Bose transformations (CBT) [10] to the spin operators in the LC system \( \{S^x_m, S^y_m, S^z_m\} \). In a harmonic approximation, the H-P transformation and the CBT yield the same results. Then, after the LC transformation and the Bose transformation, the Hamiltonian becomes:

\[ H = U_0 + H_1 + H_2 + \cdots, \]  

(2)
Equations above are just the same as the condition of $H_1 = 0$. The harmonic part of Hamiltonian can be exactly diagonalized by a generalized Bogolyubov transformation:

$$H_2 = \sum_{m,m'} \sum_k F_{m,m'}(k, \theta) a_{m'}^+(k) a_{m'}(k) + \sum_{m,m'} \sum_k G_{m,m'}(k, \theta) [a_{m'}^+(k) a_{m'}^-(\mathbf{k}) + a_m(k) a_m^-(\mathbf{k})],$$

(3)

in which the coefficients $F_{m,m'}$ and $G_{m,m'}$ are defined by:

$$F_{m,m}(k, \theta) = I_{m,m} Z S_m (1 - \gamma_k) - D_m (S_m - \frac{1}{2}) (\sin^2 \theta_m - 2 \cos^2 \theta_m) + \sum_{m'_{\theta}} S_{m,m'} \cos(\theta_m - \theta_m') + h \cos \theta_m,$$

(4)

$$F_{m,m'}(k, \theta) = -\frac{1}{2} I_{m,m'} \sqrt{S_{m} S_{m'}} [1 + \cos(\theta_m - \theta_m')]$$

(5)

$$G_{m,m}(k, \theta) = -\frac{1}{4} \sqrt{2 S_m (2 S_m - 1)} D_m \sin^2 \theta_m,$$

(6)

$$G_{m,m'}(k, \theta) = \frac{1}{4} I_{m,m'} \sqrt{S_{m} S_{m'}} [1 - \cos(\theta_m - \theta_m')]$$

(7)

Here, $\gamma_k = (1/Z) \sum_\delta \exp(i \mathbf{k} \cdot \delta)$ where the summation $\delta$ runs over the $Z$ nearest-neighbors of a given site in the $x$-$y$ plane.

In a first order approximation, spin configuration $\{\theta_m\}$ can be obtained by minimizing the ground state energy $U_0$: $\delta U_0 / \delta \theta_m = 0$, which yield the following equations:

$$\sum_{m'} I_{m,m'} S_{m'} \sin(\theta_m - \theta_m') + h \sin \theta_m + D_m (2 S_m - 1) \sin \theta_m \cos \theta_m = 0 \quad m = 1, 2, \ldots$$

(8)

Equations above are just the same as the condition of $H_1 = 0$. The harmonic part of Hamiltonian can be exactly diagonalized by a generalized Bogolyubov transformation:

$$a_{m'}^+(k) = \sum_n U_{m,n}(k) a_n^+(k) + \sum_n V_{m,n}(k) a_n(-k)$$

(9)

$$a_m(-k) = \sum_n U_{m,n}(k) a_n(-k) + \sum_n V_{m,n}(k) a_n^+(k),$$

(10)

so that we get finally:

$$H = U_0' + \sum_k \epsilon_m(k) a_{m'}^+(k) a_{m'}(k) + \cdots$$

(11)

where the magnon excitation energy $\epsilon_m(k)$ in Eq. (11) and the coefficients $(U_{m,n}, V_{m,n})$ in Eqs. (8)-(10) can be obtained from the eigenvalues and the eigenvectors of the following matrix

$$\hat{\mathcal{H}}(k) = \begin{pmatrix} \hat{\mathcal{F}}(k) & i 2 \hat{\mathcal{G}}(k) \\ i 2 \hat{\mathcal{G}}^*(k) & -\hat{\mathcal{F}}(k) \end{pmatrix}.$$ 

(12)

The elements of the sub-matrices $\mathcal{F}(k)$ and $\mathcal{G}(k)$ in matrix $\mathcal{H}(k)$ are $F_{m,m'}(\theta, k)$ and $G_{m,m'}(\theta, k)$ defined in Eqs. (9)-(11), respectively [5].

Following Ref. 5, the minimum value of the magnon excitation energy $\epsilon_m(k)$ is defined as the gap:

$$\Delta(h) = \text{Min}[\epsilon_m(k)].$$

Eqs. (11) may have many solutions corresponding to various possible spin configurations. For every solution of Eqs. (8), one can calculate the magnon excitation gap $\Delta(h)$ following the method described above. According to Ref. 5, if the gap $\Delta(h)$ is positive, the state described by such a solution is a metastable one since a variation from this state must cost energy. However, when the gap comes to
zero even negative at a field $h_c$, such a state will no longer be metastable and a transition from this state to another metastable one will take place. Thus, in the case that there are many metastable states existing in the system, the MMV recording is possible to take place.

Eq. (8) has two kinds of solutions: the trivial solutions (i.e. $\theta_m = 0$ or $\pi$, $m = 1, 2$) which correspond to the aligned spin states; the nontrivial solutions (i.e. $\theta_m \neq 0$ or $\pi$, $m = 1, 2$) to the canted spin states. Subsequently, we will discuss both the aligned spin states and the canted spin states, and discuss which state the system will transit to if the current state is unstable.

The following notations will be used. $I_{m,m'}(R, R') = J$, $I_{m,m'}(R, R) = I$ and $D_m(2S_m - 1) = \tilde{D}_m$.

The exchange interaction within a layer should be the ferromagnetic type ($J > 0$). However, both the ferromagnetic and the antiferromagnetic types of interlayer exchange coupling will be discussed (i.e. $I > 0$ or $I < 0$).

The aligned spin states:

In such a system, four aligned states are possible. They are illustrated in figure 1.

For A configuration, we have $\theta_1 = 0$, $\theta_2 = 0$. From Eqs. (4)-(7), we obtain

$$\hat{F}(k) = \left( \begin{array}{cc}
IS_2 + \tilde{D}_1 + h + JZS_2(1 - \gamma_k) & -I\sqrt{S_1S_2} \\
-I\sqrt{S_1S_2} & IS_1 + \tilde{D}_2 + h + JZS_1(1 - \gamma_k)
\end{array} \right)$$ (13)

and

$$\hat{G}(k) = 0$$ (14)

According to Eqs. (11)-(12), we find that the excitation energy $\epsilon_m(k)$ are just the eigenvalues of the matrix $\hat{F}(k)$. Thus we obtain:

$$\Delta_A(h) = h + \frac{1}{2} [\tilde{D}_1 + \tilde{D}_2 + IS_1 + IS_2 - \sqrt{(IS_2 - IS_1 + \tilde{D}_1 - \tilde{D}_2)^2 + 4I^2S_1S_2}]$$ (15)

From the discussions above, one may find that the system in A configuration will be stable only in the case that

$$h \geq h^0_c, \quad (16)$$

where

$$h^0_c = \frac{1}{2} [-IS_1 - IS_2 - \tilde{D}_1 - \tilde{D}_2 + \sqrt{(IS_2 - IS_1 + \tilde{D}_1 - \tilde{D}_2)^2 + 4I^2S_1S_2}].$$ (17)

It is similar for B configuration. One may find that the stable region for B configuration is

$$h^2_c \leq h \leq h^1_c, \quad (18)$$

where

$$h^1_c = \frac{1}{2} [IS_2 - IS_1 + \tilde{D}_2 - \tilde{D}_1 + \sqrt{(\tilde{D}_1 + \tilde{D}_2 - IS_1 - IS_2)^2 - 4I^2S_1S_2}],$$

$$h^2_c = \frac{1}{2} [IS_2 - IS_1 + \tilde{D}_2 - \tilde{D}_1 - \sqrt{(\tilde{D}_1 + \tilde{D}_2 - IS_1 - IS_2)^2 - 4I^2S_1S_2}].$$ (19)

Considering the symmetry between A, B states and C, D states, it is very easy to understand that the stable region of C and D states are $[-h^1_c, -h^2_c]$ and $(-\infty, -h^0_c]$, respectively.

The canted spin states

For every trivial solution, non-trivial solutions can be bifurcated from them at some fields. Around the bifurcation points, the variations of the angles from the trivial solution should be very small. Thus,
it is reasonable to linearize the non-linear equations \(\delta F\) to study the behaviors of the non-trivial solutions around the bifurcation point. Taking a configuration as an example, we have: \(\sin \theta_m \sim \theta_m\). Then around the bifurcation point, equations \(\delta F\) will be linearized as:

\[
\begin{align*}
\delta F_{1,1} &= -\frac{1}{2} IS_2 (\theta_1 - \theta_2)^2 - \frac{1}{2} h_0^2 \frac{\theta_1^2}{2} \\
\delta F_{2,2} &= -\frac{1}{2} IS_1 (\theta_1 - \theta_2)^2 - \frac{1}{2} h_0^2 \frac{\theta_2^2}{2} \\
\delta F_{1,2} &= \delta F_{2,1} = -\frac{1}{4} \sqrt{\delta S_1 \delta S_2} (\theta_1 - \theta_2)^2
\end{align*}
\]

(28) (29) (30)

Since \(\theta_1, \theta_2\) can be substituted by the solution of the linearized equations \(\delta F\) in the critical point \(h_0^c\) in a first order approximation, they must have the following relation:

\[
\frac{\theta_1}{\theta_2} = \frac{IS_2}{IS_2 + D_1 + h_0^c} = \frac{IS_2}{F_{1,1}^0}
\]

(31)

Thus all the terms in Eqs. (28)- (30) can be obtained after extracting a common parameter \(\theta_2^2\) through use of Eq. (31). For example,

\[
\delta F_{1,1} = \left[-\frac{1}{2} IS_2 \left(1 - \frac{IS_2}{F_{1,1}^0}\right)^2 - \frac{h_0^c}{2} \frac{IS_2}{F_{1,1}^0} \right] \theta_2^2
\]

(32)
So, based on the perturbation theory, the magnon excitation gap $\Delta(h_c^0)$ of the canted spin state at the critical point $h_c^0$ in the first order approximation can be presented as:

$$\Delta(h_c^0) \simeq \delta F_{1,1} + \delta F_{2,2} - \frac{1}{F_{1,1}^0 + F_{2,2}^0}[(\delta F_{1,1} - \delta F_{2,2})(F_{1,1}^0 - F_{2,2}^0) + 4\sqrt{T^2S_1S_2}\delta F_{1,2}].$$

(33)

$\Delta(h_c^0)/\theta_2^2$ must now be a definite value determined by the parameters. The non-trivial spin state which is bifurcated from the B configuration can be studied similarly, and the magnon excitation gap $\Delta(h_c^2)$ for such a canted spin states can also be derived following the same procedure.

The conditions for MMV recording

We have studied both the aligned spin state and the canted one. In order to realize the MMV recording, the four aligned spin states must be overlapping with each other. Thus, it is required that:

$$h_c^2 < h_c^0 < h_c^1.$$  

(34)

On the other hand, to be used for recording, the hysteresis loop should be as sharp as possible. Otherwise, it may cause difficulty to distinguish two messages. Thus, the canted spin states should not exist:

$$\Delta(h_c^0) < 0, \quad \Delta(h_c^2) < 0.$$  

(35)

(34)-(35) are just the conditions for realizing the MMV recording in a double-layer structure. The conditions are the complicated relations between the single-ion anisotropy parameters ($D_1$, $D_2$), the exchange interaction parameter ($I$) and the spins ($S_1$, $S_2$). We will study them from different respects.

First, we study what is the requirement for the interlayer exchange parameter $I$ if the two magnetic layers are determined. The following model will be investigated:

Model 1: $S_1 = 3$, $S_2 = 1$, $D_1/D_2 = 2.0$

The critical fields $h_c^0$, $h_c^1$, $h_c^2$ have been shown together as functions of $I/D_2$ in figure 2. One may find that the exchange parameter should satisfy $I_c^1 < I < I_c^2$ for condition (34). If the exchange coupling is the ferromagnetic case and is very strong ($I > I_c^1$), the B and C states can not be stable at all since the two magnetic layers are unwilling to antiparallel with each other (fig. 2). In figure 3, $\Delta(h_c^0)$ is shown with respect to $I/D_2$ in order to study the stabilities of the canted spin state bifurcated from a configuration. One may find that: only when the interlayer coupling is the antiferromagnetic case and is stronger than a critical value ($|I| < |I_c^0|$), could this canted spin states appear. If the coupling is the ferromagnetic case, this canted spin state is not able to be metastable. $\Delta(h_c^2)$ has also been studied for model 1, and it is always negative. In all, the exchange coupling should be very strong compared to the anisotropy in order to realize the MMV recording. An example has been shown in figure 4 where $I/D_2 = 0.5$. The multi-step shape of the hysteresis loop can clearly observed.

However, there remains a question. At the field $h_i^0$ where A spin configuration is no longer stable, B and D spin states are both stable. Why the system will transit to B spin configuration instead of D configuration (fig. 4)? This question can be answered by studying the magnon excitation spectrum. According to Eqs. (3)-(12), one can get the concrete forms of the Bose operators $a_m(k)$. We only study the lowest mode of spin wave, so that $k = 0$. Suppose $m = 1$ without losing generality, thus

$$\epsilon_1(h_c^0) = \Delta(h_c^0) = 0,$$

$$\alpha_1 = U_{1,1}a_1 + U_{1,2}a_2$$

(36)

(37)

where

$$U_{1,1} = \frac{I\sqrt{S_1S_2}}{\sqrt{(IS_2 + h_c^0 + D_1)^2 + I^2S_1S_2}}$$

(38)

$$U_{1,2} = \frac{IS_2 + h_c^0 + D_1}{\sqrt{(IS_2 + h_c^0 + D_1)^2 + I^2S_1S_2}}$$

(39)
Since the excitation energy of this mode is zero, if there are any kind of fluctuations, the Bosons at this mode must be greatly excited without costing energy. The current spin configuration will be completely destroyed because of the excitations. Noting \( \alpha_1 \) is a linear combination of \( a_1, a_2 \), the quantities \( |U_{1,m}|^2 \) may be understood as the possibilities of the Bosons in the \( m \)th layer to be excited, thus they must can be considered as the possibilities of the spins in the \( m \)th layer to turn flipping. In figure 5, the two quantities \( |U_{1,m}|^2 \) are shown together as functions of \( I/D_2 \) for Model 1. One may find that in the region where the MMV recording is permitted, \( |U_{1,1}|^2 \sim 0 \) while \( |U_{1,1}|^2 \sim 1 \). Thus, at the field \( h_0^0 \) where the A configuration is not able to be stable, the spins in 2nd layer are mostly likely to turn flipping while those in the 1st layer are not likely to do so. So, in this case, the system will transit to B configuration instead of D configuration. One may also find that if the interlayer exchange is the ferromagnetic case and is very strong (\( I \gg D_2 \)), the two quantities will be close. Thus, the two magnetic layers are willing to turn flipping together because of the strong interlayer coupling. The MMV recording is not able to be realized then. By the way, if there is no coupling between the two magnetic layers (\( I = 0 \)), we find that \( |U_{1,1}|^2 = 0 \) and \( |U_{1,2}|^2 = 1 \). This is easy to be understood. Because the two layers are not coupled, they can be treated independently. In this case, one may find that \( h_0^0 \) and \( h_2^2 \) are just the coercive forces of the two layers. So, when the external field reaches \( h_0^0 \), the 2nd layer will turn over while the first layer will not.

Finally, we study what kinds of materials can be used for MMV recording. Suppose \( S_1 = S_2 = S, D'_m = (2S-1)D_m/|I|S \), the phase diagrams for \( D'_1 - D'_2 \) plane are given in ferromagnetic case (figure 6) and in antiferromagnetic case (figure 7), respectively. The two cases are quite different. To realize the MMV recording, the anisotropies for the two materials can not be very close (\( D'_1 \sim D'_2 \)) if the exchange is the ferromagnetic case (fig. 6), while there is no such restriction for the antiferromagnetic case (fig. 7). However, a common requirement in the two cases is still a weak interlayer exchange interaction.

If the structure is more complicated, one may easily understand that there may be more sharp steps in the hysteresis loop. Thus even more messages can be recorded in one domain. The extension from the present double-layer system to a complicated one is rather straightforward.

Conclusion

In conclusion, we have analytically studied the hysteresis loop of a double-layer magnetic system. We find that when the interlayer coupling and the anisotropies of the two materials satisfy some complicated conditions, more metastable states will be possible to appear, and the magnetic multi-valued recording may be realized. The conditions are discussed from different respects, and the permitted values of the anisotropies for realizing the MMV recording are presented.

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References

[1] R E Camley and R L Stamps, J. Phys.: Condens. Matter 5 3727 (1993).
[2] S. Ohnuki, K. Shimazaki, N. Ohta and H. Fujiwara, J. Magn. Soc. Jpn, 15 Supplement No. S1, 399 (1991).
[3] K. Shimazaki, M. Yoshihiro, O. Ishizaki, S. Ohnuki and N. Ohta, J. Magn. Soc. No. S1, 429 (1995).
[4] S. Gadetsky, T. Suzuki, J. K. Erwin and M. Mansuripur, J. Magn. Soc. No. S1, 91 (1995).
[5] Lei Zhou and Ruibao Tao, “Quantum theory of the coercive force and the capping effect for magnetic multilayer” Phys. Rev. B (1996), to appear in 01Oct issue.
[6] Ruibao. Tao, Xiao Hu and Yoshiyuki Kawazoe, Phys. Rev. B 52 6178 (1995).
[7] X. Hu and Kawazoe, J. Appl. Phys., 75 6486 (1994).
[8] X. Hu and Y. Kawazoe, Phys. Rev. B 49 3294 (1994).
[9] T. Holstein and H. Primakoff, Phys. Rev 59 1098 (1940).
[10] Ruibao Tao, J. Phys. A: Math. and Gen. 27 6374 (1994).

Captions:

Figure 1: Aligned spin configurations in a double-layer magnetic system.

Figure 2: Critical fields $h_c^0$, $h_c^1$ and $h_c^2$ as functions of the interlayer coupling constant for the double-layer system.

Figure 3: $\Delta(h_c^0)/\theta_2^2$ as the function of the interlayer coupling constant for the double-layer system.

Figure 4: Hysteresis loop of a double-layer magnetic system with a ferromagnetically interlayer coupling constant $I/D_2 = 0.5$.

Figure 5: The possibilities for the two magnetic layers to turn flipping ($|U_{1,1}|^2$, $|U_{1,2}|^2$) as functions of the interlayer exchange parameter.

Figure 6: Permitted values of the two anisotropies for realizing the MMV recording in the ferromagnetic coupling case. One can achieve the MMV recording in region I, and can not do so in region II.

Figure 7: Permitted values of the two anisotropies for realizing the MMV recording in the antiferromagnetic coupling case. One can achieve the MMV recording in region I, and can not do so in region II.