Thermodynamics of Spinor Quintom

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Abstract

We discuss in this paper the thermodynamics of dark energy with Quintom matter in spinor scenario. (1). We investigate the conditions of validity of the Generalized Second Law of thermodynamics in the four evolutionary phases of Spinor Quintom-B model. (2). We take thermodynamic stability of the combination between Spinor Quintom dark energy (DE) and the generalized Chaplygin Gas perfect fluid into account, and we find that in the case of $\beta > 0$ and $0 < T < T_0$, the system we consider is thermodynamically stable.

1 Introduction

There is mounting data from type Ia supernovae and cosmic microwave background (CMB) radiation and so on[1,2,3,4] have provided strong evidences for the present spatially flat and accelerated expanding universe, corresponding to $\ddot{a} > 0$, which is dominated by dark sectors. Combined analysis of the above cosmological observations support that the energy of our universe is occupied by dark energy(DE) about 73%, dark matter about 23% and only about 4% of usual baryon matter which can be described by the well known particle theory. In the context of Friedmann-Robertson-Walker(FRW) cosmology, the evolution of scale factor is governed by the temporal part of Einstein equation $3\ddot{a}/a = -4\pi G(\rho + 3p)$, this acceleration may be attributed to the exotic form of negative pressure satisfying $p < -3\rho$, the so-called DE. So
far, The nature of dark energy remains a mystery. To describe the property of this component, a significant parameter $w = \frac{p}{\rho}$, called equation of state, was introduced. And it is need to be $w < -\frac{1}{3}$ theoretically. Basis on different evolution of the EoS we can obtain different candidate for dark energy. Currently, it is widely taken as the candidate for a small cosmological constant $\Lambda$ (or vacuum energy) with equation of state $w = -1$ as well as a dynamical component such as the Quintessence with $-1 < w < 1$, Phantom with $w < -1$, K-essence with both $w \geq -1$ and $w < -1$ but never crossing $-1$. Although the recent fits to the data in combination of WMAP, the recently released 182 SNIa Gold sample and also other cosmological observational data show remarkably the consistence of the cosmological constant, it is worth of noting that a class of dynamical models with the equation-of-state (EoS) across $-1$ Quintom is mildly favored. In the literature there have been a lot of theoretical studies of Quintom-like models. Especially, a No-Go theorem has been proved to constrain the model building of Quintom, and according to this No-Go theorem there are models which involve higher derivative terms for a single scalar field, models with vector field, making use of an extended theory of gravity, non-local string field theory, and others (see e.g.). The similar work applied in scalar-tensor theory is also studied in Ref.

Despite many works have been done in pursuit of establishing concrete model to understanding the theoretical nature and origin of this special fluid, there are a number of recent papers commit themselves to investigating the thermal properties of DE fluid. The thermodynamics of de Sitter space-time was first investigated by Gibbons and Hawking and extended the study to quasi-de Sitter space-time. Based on an assumption that DE is a thermallized ensemble at certain temperature with an associated thermodynamical entropy, Ref. made various aspects of the thermodynamic discussions. The papers have investigated the generalized second law of thermodynamics of modified gravity. In the literature, the thermodynamics of Quantum Gravity has been investigated. Ref. consider the apparent horizon of the Friedmann-Robertson-Walker universe as a thermodynamical system and investigate the thermodynamics of LQC in the semiclassical region.
Previously, it have been considered that a Quintom dark energy with non-regular spinor matter[50]. In succession, to understand the possible combinations among different types of Quintom model in spinor field we study the implications of cosmic duality with this class of models and realize additional Quintom models by the aid of this dual properties. In the meantime, we also perform the statefinder diagnostic for this Spinor Quintom model[51]. In this paper, we will discuss the thermodynamics of the Spinor Quintom model. From the thermodynamical point of view, our universe can be considered as a thermodynamical system filled by DE perfect fluid, we will examine the Generalized Second Law of thermodynamics(GSL) and thermodynamic stability in this system. This letter is organized as follows. In section 2, we investigate the validity of GSL in spinor field with Quintom model of dark energy, we indicate that the conditions under which the GSL can be satisfied. In section 3, we explore the conditions for thermodynamic stability of the combination between Quintom model with spinor field and the Generalized Chaplygin Gas(GCG) perfect fluid. Section 4 contains discussions and conclusions.

2 Generalized Second Law of Thermodynamics in a System Filled with Spinor Quintom Matter

One of the distinguishing features of the driver of current accelerating expansion, the alleged DE, lies in violating the strong energy condition, $\rho + 3p > 0$[3, 52]. As a result of the dependence on theoretical models this strength of acceleration is a question in debating. While most model independent analysis suggest that it to be below the De Sitter value[53], it is certainly true that the body of observational data allows for a wide parameter space compatible with an acceleration larger than de Sitter’s[7, 54]. If eventually it is proven to be the case, this dark component would violate not only the strong energy condition $\rho + 3p > 0$ but also the dominated energy condition $\rho + p > 0$. In the literature, component with the above specialities was dubbed Phantom[7, 55], suffering from a long list of pathologies such as quantum instabilities[56, 57] which leads to superluminal sound speed and causes a super accelerating universe ending in a big rip or big crunch along the cosmic evolution. Attracting many attentions,
the interesting fluid has been widely discussed recent years\cite{58, 59}, and Ref.\cite{37, 60} investigated the thermodynamics on phantom dark energy dominated universe. The thermodynamics of DE with constant EoS in the range of $-1 < w < -\frac{1}{3}$ was considered in \cite{61}, and that of K-essence also was studied in Ref.\cite{45}.

Based on the relation between the event of horizon and the thermodynamics of a black hole assumed by Bekenstein in 1973 \cite{62}, the event of horizon of a black hole is a measure of the entropy of the black hole. This idea has been generalized to horizons of cosmological models, so that each horizon corresponds to an entropy. Correspondingly, the second law of thermodynamics was modified in the way that in generalized form, the sum of all time derivative of entropies related to horizons plus time derivative of normal entropy must be positive, i.e., the sum of entropies must be increasing function of time. Ref. \cite{63} investigated the validity of Generalized Second Law (GSL) for the cosmological models which departs slightly from de Sitter space. Ref.\cite{38} explore the thermodynamics of dark energy taking into account the existence of the observer’s event horizon in accelerated universes. The conditions of validity of generalized second law in phantom dominated era was studied in \cite{39}. The validity of the Generalized Second Law of thermodynamics for the Quintom model of dark energy with two scalar fields without coupling potential term was considered by \cite{41}. In this section, we will discuss the validity of the Generalized Second Law of thermodynamics for a Quintom dominated universe in spinor field.

To begin with the discussion, we deal with the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) space-time, then the space-time metric as

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2.$$  

(1)

Assuming that the dynamics of gravity is governed by the Einstein-Hilbert action, for a spinor minimally coupled to general relativity\cite{64, 65, 66}, we have,

$$S = S_\psi + S_m - \frac{1}{6} \int d^4x \sqrt{-g}R.$$  

(2)

where $R$ is the scalar curvature, $S_\psi$ is given by the the Dirac action and $S_m$ describes additional matter fields, such as scalar fields and gauge fields\footnote{Here, we postulate symmetries, diffeomorphism and local Lorentz invariance.}.

We consider the spinor component as the thermodynamical system we may discuss, which is filled with Quintom dark energy fluid. In the aid of the dynamics of a spinor
field which is minimally coupled to Einstein’s gravity, we can write down the following Dirac action in a curved space-time background

\[
S_\psi = \int d^4x \, e \left[ \frac{i}{2} (\bar{\psi} \Gamma^\mu D_\mu \psi - D_\mu \bar{\psi} \Gamma^\mu \psi) - V \right] = \int d^4x \, e \, \mathcal{L}_\psi,
\]

(3)

Here, \(e\) is the determinant of the vierbein \(e^a_\mu\) and \(V\) stands for any scalar function of \(\psi, \bar{\psi}\) and possibly additional matter fields. We will assume that \(V\) only depends on the scalar bilinear \(\bar{\psi} \psi\). From the expression of the Dirac action, we have the energy density and the pressure of the spinor field:

\[
\rho_\psi = T^0_0 = V, \quad p_\psi = -T^i_i = V' \bar{\psi} \psi - V, \quad (4, 5)
\]

For a gauge-transformed homogeneous and a space-independent spinor field, the equation of motion of spinor reads

\[
\dot{\psi} + \frac{3}{2} H \psi + i \gamma^0 V' \psi = 0, \quad (6)
\]

\[
\dot{\bar{\psi}} + \frac{3}{2} H \bar{\psi} - i \gamma^0 V' \bar{\psi} = 0, \quad (7)
\]

where a dot denotes a time derivative while a prime denotes a derivative with respect to \(\bar{\psi} \psi\), and \(H\) is Hubble parameter.

In the framework of FRW cosmology, the Friedmann constraint equation will be

\[
H^2 = \frac{1}{3} \rho_\psi, \quad (8)
\]

From the equation of motion of spinor and the Friedmann constraint equation, we can obtain the the derivative of Hubble parameter with respect to time,

\[
\dot{H} = \frac{\dot{\rho}_\psi}{6H} = \frac{V' \bar{\psi} \psi}{2}. \quad (9)
\]

So we have

\[
\rho_\psi + p_\psi = -2\dot{H}. \quad (10)
\]

\[\text{Note that we use units } 8\pi G = \hbar = c = 1 \text{ and all parameters are normalized by } M_p = 1/\sqrt{8\pi G} \text{ in the letter.}\]
According to the Gibbons equation

\[ Tds = dE + p_\psi dV = (p_\psi + \rho_\psi) dV + V dp_\psi, \]  

(11)

combined with the above relations and the expression of volume \( V = \frac{4}{3} \pi R_H^3 \) \((R_H\) is the event of the horizon), we may rewriting the first law of thermodynamics as,

\[ Tds = -2 \dot{H} d(\frac{4}{3} \pi R_H^3) + \frac{4}{3} \pi R_H^3 d\rho_\psi = -8\pi R_H^2 \dot{H} dR_H + 8\pi HR_H^3 dH, \]  

(12)

where \( T \) is the temperature of Spinor Quintom fluid. Therefore, the derivative of normal entropy is given as follows:

\[ \dot{s} = \frac{ds}{dt} = \frac{1}{T} 8\pi \dot{H} R_H^2 (HR_H - \dot{R}_H). \]  

(13)

Now we turn to consider the entropy corresponding to the event horizon. The definition of event of horizon in a de Sitter spacetime is

\[ R_H = a(t) \int_t^\infty \frac{dt'}{a(t')} . \]  

(14)

So the time derivative of event of horizon in a spinor field approaching to de Sitter space satisfies the following equation:

\[ \dot{R}_H = \dot{a}(t) \int_t^\infty \frac{dt'}{a(t')} + a(t) \int_t^\infty \frac{dt'}{a(t')} = HR_H - 1. \]  

(15)

When \( HR_H \geq 1 \) the EoS of spinor larger than \(-1\), corresponding a Quintessence dominant universe\[63\], while \( HR_H \leq 1 \) for a Phantom phase\[39\]. Then we can write the final form of the time derivative of normal entropy of the Spinor Quintom matter,

\[ \dot{s} = \frac{8\pi R_H^2 \dot{H}}{T} . \]  

(16)

As we well know, the entropy is proportional to the area of its event of horizon. If the horizon entropy corresponding with \( R_H \) is defined as \( s_H = \pi R_H^2 \), the GSL can be stated as:

\[ \dot{s} + s_H = \frac{8\pi R_H^2 \dot{H}}{T} + 2\pi R_H \dot{R}_H \geq 0. \]  

(17)

In the following, we will take the Quintom-B model realized in\[50\] to discuss the validity of GSL in spinor field. The temperature of Spinor Quintom-B is assumed to be positive.
(1). Phantom dominated evolution:
In this phase $\dot{R}_H \leq 0$, so $s_H \leq 0$. From $V' < 0$ one can get $\dot{H} > 0$. So the condition for validity of GSL can be expressed as:

$$\dot{H} \geq \left| \frac{\dot{R}_H T}{4R_H^4} \right|. \quad (18)$$

(2). Quintessence dominated evolution:
In this period of evolution $\dot{R}_H \geq 0$, then we have a negative time derivative of Hubble parameter but that of horizon entropy is not a negative value. Thus the condition for validity of GSL is:

$$|\dot{H}| \leq \frac{\dot{R}_H T}{4R_H^4}. \quad (19)$$

(3). Phase transition from Phantom to Quintessence:
At the transition point, we have $w = -1$ and $V' = 0$, that is to say $\dot{H} = 0$, so $\dot{s} = 0$. Assuming that the event horizon $R_H$ varies continually, one may expect that $\dot{R}_H = 0$ in transition time, so the horizon entropy is continuous and differentiable. Therefore, to realize the transition, it need to be continuous and differentiable in transition time for the total entropy of the universe.

(4). The final phase--an approximate de Sitter universe:
In such a state, the temperature is

$$T = \frac{bH}{2\pi}, \quad (20)$$

where $b$ is a parameter. During this period, the universe lies in the Quintessence phase, so

$$b \geq \frac{8\pi |\dot{H}| R_H}{HR_H^4}, \quad (21)$$

in de Sitter space-time case $R_H = \frac{1}{H}$, one can get $b \geq 8\pi$, which should be satisfied if GSL is valid.

In conclusion, one can find that the conditions for the validity GSL of Spinor Quintom model are similar to that of the Quintom DE model constructed by two scalar fields without coupling potential term which was considered in [41].
3 Thermodynamic Stability of The Combination between Spinor Quintom and GCG Perfect Fluid

Since the Chaplygin gas was generalized people have made many correlative studies\cite{70,71} to reconcile the standard cosmological model with observations. Ref.\cite{43} discusses the behavior of temperature and the thermodynamic stability of a generalized Chaplygin gas considering only general thermodynamics, the corresponding thermal equation of state for the GCG and analyzed its temperature behavior as well as its thermodynamic stability considering both adiabatic and thermal equations of state. While in the literature\cite{44}, Chaplygin gas was modified again, and a scenario was set up to determine the corresponding thermal equation of state of the modified Chaplygin gas (MCG) and it reveals that the MCG only presents thermodynamic stability during any expansion process if its thermal equation of state depends on temperature only, $P = P(T)$. Moreover, the modified Chaplygin gas may cool down through any thermodynamic process without facing any critical point or phase transition. We have established a combination between Chaplygin gas and Spinor Quintom in\cite{50}, in this section we will investigate the thermodynamic stability in a universe filled with the fluid combined between Quintom and GCG in spinor field.

In Ref.\cite{50}, we take the form of potential as $V = \frac{1}{\sqrt{1+\beta V_0}}(\bar{\psi}\psi)^{1+\beta} + c$, and obtain the EoS of GCG model

$$p = -\frac{c}{\rho^\beta}$$  \hspace{1cm} (22)

where parameter $\beta$ is a constant and positive $\beta > 0$ and $c$ is also positive and a universal constant\cite{43}. Here we consider a closed thermodynamic system full of dark energy fluid, in which the combination of Spinor Quintom with GCG play the role. Assuming the internal energy $U$ and pressure $p$ as only the functions of their natural viables entropy $s$ and volume $V$: $U = U(s, V), p = p(s, V)$, and the energy density of DE fluid is

$$\rho = \frac{U}{V}.$$  \hspace{1cm} (23)

From general thermodynamics\cite{72,73}, we know that

$$\left(\frac{\partial U}{\partial V}\right)_s = -p.$$  \hspace{1cm} (24)
Combined the above there equations, we can get the following form,

\[
\frac{\partial U}{\partial V}_s = c \frac{V^\beta}{U^\beta},
\] (25)

and the expression of the internal energy of this system is also given by its solution,

\[
U = 1 + \beta \sqrt{cV^{1+\beta} + b},
\] (26)

where \( b = b(s) \) is an integration parameter. It can be proven that even \( c = c(s) \) is not a universal constant, the above expression remains valid. The Eq. (25) also can be written as [13]:

\[
U = V^{1+\beta} \sqrt{c[1 + (\frac{\sigma}{V})^{1+\beta}]},
\] (27)

where parameter \( \sigma^{1+\beta} = \frac{b}{c} \). Then we may deduce the expressions of energy density and pressure with respect to this parameter,

\[
\rho = \frac{1+\beta}{\sqrt{c[1 + (\frac{\sigma}{V})^{1+\beta}]}},
\] (28)

\[
p = -\frac{1+\beta}{\sqrt{1 + (\frac{\sigma}{V})^{1+\beta}}}.\] (29)

By these two equations, we could understand the behavior of both past and future of our universe. In the early time with small scale factor and volume, the energy density and pressure behave as the below form:

\[
\rho \approx c^{1+\beta} \frac{\sigma}{V},
\] (30)

\[
p \approx c^{1+\beta} (\frac{V}{\sigma})^\beta \sim 0,
\] (31)

corresponding to a high energy density and approximative pressureless matter dominated phase. During this period the energy density reduces as its entropy and volume adiabatically. Along with the cosmological expansion through to some late times, these two parameter are approximate respectively to

\[
\rho \approx c^{1+\beta} + \frac{c^{1+\beta}}{1 + \beta (\frac{\sigma}{V})^{1+\beta}},
\] (32)

\[
p \approx c^{1+\beta}.
\] (33)

During this period of the evolution, the total system can be seen as constituted by two components: one with constant energy density and the other with an alterable energy
density with respect to volume. While for a large value of scale factor, the energy density may rather lower and EoS is \( p = -\rho = c^{1+\beta} \) which is a de Sitter Space-time. Consequently, we realize a transformation from dust-like matter dominated universe to a de Sitter phase in the point of view of thermodynamics.

In what follows, we will extensively examine the conditions for the thermodynamic stability of this combined system.

(1). We determine how the pressure change with volume through the adiabatic expansion. Using Eq. (28), one can get

\[
(\frac{\partial p}{\partial V})_s = \beta \frac{p}{V} [1 - \frac{1}{1 + (\frac{\beta}{V})^{1+\beta}}],
\]

\( (34) \)

it is obvious that we exclude the case of \( \beta = 0 \) due to a constant pressure and the disappearing derivative. While in the case of \( \beta > 0 \) the above derivative is always negative value.

(2). To make a system stable, it is necessary for the thermal capacity at constant volume to be positive \( c_V > 0 \), the pressure reduces as volume at constant temperature, as well.

For this purpose, we calculate the formula of temperature \( T \) and entropy \( s \) to determine how the temperature depends on its entropy and volume. In the thermodynamics and statistical physics, the temperature of a system is defined as:

\[
T = (\frac{\partial U}{\partial s})_V, \tag{35}
\]

combined with the expression of internal energy, the formula of temperature can be written as follows\[^{[43]}\]:

\[
T = \frac{1}{1+\beta} (cV^{1+\beta} + \varepsilon)^{-\frac{\beta}{1+\beta}} (V^{1+\beta} \frac{dc}{ds} + \frac{d\varepsilon}{ds}). \tag{36}
\]

Clearly, if we take parameter as both \( c \) and \( \varepsilon \) are universal constant, the temperature equals to 0 for any value of pressure and volume. As a result, the isotherm \( T = 0 \) is simultaneously an isentropic curve at \( s = const \), which violates the third law of thermodynamics\[^{[43]}\]. Taking this factor into account, we choose \( c \) as a universal constant and \( \frac{d\varepsilon}{ds} > 0 \). From dimensional analysis it can be understood that \( \varepsilon \) has a dimension of energy, \( [\varepsilon]^{1+\beta} = [U] \). In this case, we take it as\[^{[43]}\]

\[
b = (T_0s)^{1+\beta}, \tag{37}
\]
so,
\[
\frac{d\varepsilon}{ds} = (1 + \beta)(T_0 s)^\beta T_0. \tag{38}
\]

Then the formulae of temperature and entropy of this system can be written as:
\[
T = T_0^{1+\beta} s^\beta [e V^{1+\beta} + (T_0 s)^{1+\beta}]^{-\frac{\beta}{1+\beta}}, \tag{39}
\]
\[
s = \frac{1}{c_{1+\beta}} \frac{T_0^{\frac{1}{1+\beta}}}{(T_0^\beta - T^{1+\beta})^{\frac{1}{1+\beta}}} V. \tag{40}
\]

A stable thermodynamic system requires a positive and finite entropy, which requests that the temperature satisfy
\[
0 < T < T_0. \tag{41}
\]

By the definition of \(c_V\) and the formulae of temperature and entropy, one can rewrite \(c_V\) as,
\[
c_V = \frac{1}{\beta T_0} \frac{c^{\frac{1}{\beta}} V}{[1 - (\frac{T}{T_0})^{\frac{1}{1+\beta}}]^\frac{1+\beta}{1+\beta}} (\frac{T}{T_0})^{\frac{1}{\beta}}, \tag{42}
\]

Thus, When \(\beta > 0\) and \(0 < T < T_0\), one can get a positive \(c_V\).

Correspondingly, we can obtain the expression of pressure,
\[
p = -c^{\frac{1}{1+\beta}} [1 - (\frac{T}{T_0})^{\frac{1+\beta}{1+\beta}}]^{\frac{\beta}{1+\beta}}, \tag{43}
\]

It can be seen that the pressure is only the function of temperature, so \((\frac{\partial p}{\partial V})_T > 0\) is satisfied.

In a word, in the case of \(\beta > 0\) and \(0 < T < T_0\), the system we consider is thermodynamically stable.

4 Conclusion and Discussions

To summarize, we have investigated the thermodynamics of Quintom DE dominant thermodynamical system in spinor field. Firstly, we show the conditions in which the total entropy may not decrease with time not only in Phantom and Quintessence phase but also at the transition time and the final approximative de Sitter phase. We set up the similar conditions to a Quintom universe with two scalar fields without coupling potential term. In the second place, we, using general thermodynamics, explore the thermodynamic stability of a system full of the DE fluid combined Spinor
Quintom with GCG, and we conclude that in a certain range of temperature, i.e., $0 < T < T_0$, this system remains thermodynamically stable without any limitation on pressure.

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References

[1] S. Perlmutter et al., Astrophys. J. 483, 565 (1997); Adam G. Riess et al., Astrophys. J. 116, 1009 (1998).

[2] D. N. Spergel et al., Astrophys. J. Suppl. 148, 175 (2003).

[3] A. G. Riess et al., Astrophys. J. 607, 665 (2004).

[4] U. Seljak et al., Phys. Rev. D 71, 103515 (2005) [arXiv:astro-ph/0407372].

[5] C. Wetterich, Nucl. Phys. B 302, 668 (1988).

[6] B. Ratra and P. J. E. Peebles, Phys. Rev. D 37, 3406 (1988).

[7] R. R. Caldwell, Phys. Lett. B 545, 23 (2002) [arXiv:astro-ph/9908168].

[8] C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. D 63, 103510 (2001) [arXiv:astro-ph/0006373].

[9] T. Chiba, T. Okabe and M. Yamaguchi, Phys. Rev. D 62, 023511 (2000) [arXiv:astro-ph/9912463].

[10] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 170, 377 (2007) [arXiv:astro-ph/0603449].

[11] E. Komatsu et al. [WMAP Collaboration], arXiv:0803.0547 [astro-ph].

[12] A. G. Riess et al., arXiv:astro-ph/0611572.

[13] B. Feng, X. L. Wang and X. M. Zhang, Phys. Lett. B 607, 35 (2005) [arXiv:astro-ph/0404224].

[14] G. B. Zhao, J. Q. Xia, H. Li, C. Tao, J. M. Virey, Z. H. Zhu and X. Zhang, Phys. Lett. B 648, 8 (2007) [arXiv:astro-ph/0612728].

[15] G. B. Zhao, J. Q. Xia, B. Feng and X. Zhang, Int. J. Mod. Phys. D 16, 1229 (2007) [arXiv:astro-ph/0603621].

[16] Y. Wang and P. Mukherjee, Astrophys. J. 650, 1 (2006) [arXiv:astro-ph/0604051].
[17] J. Q. Xia, Y. F. Cai, T. T. Qiu, G. B. Zhao and X. Zhang, Int. J. Mod. Phys. D 17, 1229 (2008) arXiv:astro-ph/0703202.

[18] M. Z. Li, B. Feng and X. M. Zhang, JCAP 0512, 002 (2005) arXiv:hep-ph/0503268.

[19] C. Armendariz-Picon, JCAP 0407, 007 (2004) arXiv:astro-ph/0405267; H. Wei and R. G. Cai, Phys. Rev. D 73, 083002 (2006) arXiv:astro-ph/0603052.

[20] R. G. Cai, H. S. Zhang and A. Wang, Commun. Theor. Phys. 44, 948 (2005) arXiv:hep-th/0505186; P. S. Apostolopoulos and N. Tetradosis, Phys. Rev. D 74, 064021 (2006) arXiv:hep-th/0604014; K. Bamba, C. Q. Geng, S. Nojiri and S. D. Odintsov, arXiv:0810.4296 [hep-th].

[21] I. Y. Aref’eva, A. S. Koshelev and S. Y. Vernov, Phys. Rev. D 72, 064017 (2005) arXiv:astro-ph/0507067; S. Y. Vernov, arXiv:astro-ph/0612487; A. S. Koshelev, JHEP 0704, 029 (2007) arXiv:hep-th/0701103.

[22] Z. K. Guo, Y. S. Piao, X. M. Zhang and Y. Z. Zhang, Phys. Lett. B608, 177 (2005) arXiv:astro-ph/0410654.

[23] X. F. Zhang, H. Li, Y. S. Piao and X. M. Zhang, Mod. Phys. Lett. A 21, 231 (2006) arXiv:astro-ph/0501652; Z. K. Guo, Y. S. Piao, X. Zhang and Y. Z. Zhang, Phys. Rev. D 74, 127304 (2006) arXiv:astro-ph/0608165.

[24] Y. F. Cai, H. Li, Y. S. Piao and X. M. Zhang, Phys. Lett. B 646, 141 (2007) arXiv:gr-qc/0609039.

[25] B. Feng, M. Li, Y. S. Piao and X. Zhang, Phys. Lett. B 634, 101 (2006) arXiv:astro-ph/0407432; H. Wei and R. G. Cai, Phys. Rev. D 72, 123507 (2005) arXiv:astro-ph/0509328; X. Zhang, Phys. Rev. D 74, 103505 (2006) arXiv:astro-ph/0609699.

[26] Y. F. Cai, T. Qiu, Y. S. Piao, M. Li and X. Zhang, JHEP 0710, 071 (2007) arXiv:0704.1090 [gr-qc].

[27] Y. F. Cai, M. Z. Li, J. X. Lu, Y. S. Piao, T. T. Qiu and X. M. Zhang, Phys. Lett. B 651, 1 (2007) arXiv:hep-th/0701016.
quintom is found to be able to give a bouncing solution and its perturbations combine aspects of singular and nonsingular bounce models, see for example: Y. F. Cai, T. Qiu, R. Brandenberger, Y. S. Piao and X. Zhang, JCAP 0803, 013 (2008) [arXiv:0711.2187 [hep-th]]; Y. F. Cai, T. T. Qiu, J. Q. Xia and X. Zhang, arXiv:0808.0819 [astro-ph]; Y. F. Cai and X. Zhang, arXiv:0808.2551 [astro-ph].

W. Zhao, and Y. Zhang, Phys. Rev. D 73, 123509 (2006) [arXiv:astro-ph/0604460]; H. Mohseni Sadjadi and M. Alimohammadi, Phys. Rev. D 74, 043506 (2006) [arXiv:gr-qc/0605143]; E. O. Kahya and V. K. Onemli, Phys. Rev. D 76, 043512 (2007) [arXiv:gr-qc/0612026]; Y. F. Cai and Y. S. Piao, Phys. Lett. B 657, 1 (2007) [arXiv:gr-qc/0701114]. R. Lazkoz, G. Leon and I. Quiros, Phys. Lett. B 649, 103 (2007) [arXiv:astro-ph/0701353]; H. Zhang and Z. H. Zhu, arXiv:0704.3121 [astro-ph]; T. Qiu, Y. F. Cai and X. M. Zhang, arXiv:0710.0115 [gr-qc]; M. R. Setare, J. Sadeghi and A. Banijamali, Phys. Lett. B 669, 9 (2008) [arXiv:0807.0077 [hep-th]].

H. H. Xiong, T. Qiu, Y. F. Cai and X. Zhang, arXiv:0711.4469 [hep-th]; H. H. Xiong, Y. F. Cai, T. Qiu, Y. S. Piao and X. Zhang, Phys. Lett. B 666, 212 (2008) [arXiv:0805.0413 [astro-ph]]; S. Li, Y. F. Cai and Y. S. Piao, arXiv:0806.2363 [hep-ph]; S. Zhang and B. Chen, Phys. Lett. B 669, 4 (2008) [arXiv:0806.4433 [hep-ph]].

E. Elizalde, S. Nojiri and S. D. Odintsov, Phys. Rev. D 70, 043539 (2004) arXiv:hep-th/0405034.

G. W. Gibbons and S. W. Hawking, Phys. Rev. D 15, 2738 (1977).

M. D. Pollock and T. P. Singh, Class. Quant. Grav. 6 (1989) 901.

A. V. Frolov and L. Kofman, JCAP 0305, 009 (2003) arXiv:hep-th/0212327.

I. Brevik, S. Nojiri, S. D. Odintsov and L. Vanzo, Phys. Rev. D 70, 043520 (2004) arXiv:hep-th/0401073. S. Nojiri and S. D. Odintsov, Phys. Rev. D 70, 103522 (2004) arXiv:hep-th/0408170.

J. A. S. Lima and J. S. Alcaniz, Phys. Lett. B 600, 191 (2004) arXiv:astro-ph/0402265.
[37] P. F. Gonzalez-Diaz and C. L. Siguenza, Nucl. Phys. B 697, 363 (2004) [arXiv:astro-ph/0407421]. S. H. Pereira and J. A. S. Lima, arXiv:0806.0682 [astro-ph].

[38] G. Izquierdo and D. Pavon, Phys. Lett. B 633, 420 (2006) [arXiv:astro-ph/0505601].

[39] H. Mohseni Sadjadi, Phys. Rev. D 73, 063525 (2006) [arXiv:gr-qc/0512140].

[40] M. R. Setare and S. Shafei, JCAP 0609, 011 (2006) [arXiv:gr-qc/0606103].

[41] M. R. Setare, Phys. Lett. B 641, 130 (2006) [arXiv:hep-th/0611165].

[42] B. Wang, Y. Gong and E. Abdalla, Phys. Rev. D 74, 083520 (2006) [arXiv:gr-qc/0511051]. Y. Gong, B. Wang and A. Wang, Phys. Rev. D 75, 123516 (2007) [arXiv:gr-qc/0611155]. B. Wang, C. Y. Lin, D. Pavon and E. Abdalla, Phys. Lett. B 662, 1 (2008) [arXiv:0711.2214 [hep-th]].

[43] F. C. Santos, M. L. Bedran and V. Soares, Phys. Lett. B 636 (2006) 86.

[44] F. C. Santos, V. Soares and M. L. Bedran, Phys. Lett. B 646, 215 (2007).

[45] N. Bilic, arXiv:0806.0642 [gr-qc].

[46] A. Sheykhi and B. Wang, arXiv:0811.4477 [hep-th].

[47] A. Sheykhi and B. Wang, arXiv:0811.4478 [hep-th].

[48] V. Husain and R. B. Mann, arXiv:0812.0399 [gr-qc].

[49] L. F. Li and J. Y. Zhu, arXiv:0812.3544 [gr-qc].

[50] Y. F. Cai and J. Wang, Class. Quant. Grav. 25, 165014 (2008) [arXiv:0806.3890 [hep-th]].

[51] Jing Wang and Shi-ping Yang, (submit to Phys. Lett. B)

[52] S. Perlmutter, et al., Nature, 391, 51 (1998)
[53] R. A. Daly and S. G. Djorgovski, Astrophys. J. 597, 9 (2003)  
[arXiv:astro-ph/0305197]. M. V. John, Astrophys. J. 614, 1 (2004)  
[arXiv:astro-ph/0406444]. S. Nesseris and L. Perivolaropoulos, Phys. Rev. D 70, 043531 (2004)  
[arXiv:astro-ph/0401556]. Y. Wang and M. Tegmark, Phys. Rev. D 71, 103513 (2005)  
[arXiv:astro-ph/0501351].

[54] S. Hannestad and E. Mortsell, JCAP 0409, 001 (2004)  
[arXiv:astro-ph/0407259]. J. Q. Xia, G. B. Zhao, B. Feng, H. Li and X. Zhang, Phys. Rev. D 73, 063521 (2006)  
[arXiv:astro-ph/0511625].

[55] R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003)  
[arXiv:astro-ph/0302506].

[56] S. M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D 68, 023509 (2003)  
[arXiv:astro-ph/0301273]. J. M. Cline, S. Jeon and G. D. Moore, Phys. Rev. D 70, 043543 (2004)  
[arXiv:hep-ph/0311312].

[57] P. H. Frampton, Mod. Phys. Lett. A 19, 801 (2004)  
[arXiv:hep-th/0302007].

[58] M. P. Dabrowski, T. Stachowiak and M. Szydlowski, Phys. Rev. D 68, 103519 (2003)  
[arXiv:hep-th/0307128]. G. W. Gibbons,  
[arXiv:hep-th/0302199]. A. E. Schulz and M. J. White, Phys. Rev. D 64, 043514 (2001)  
[arXiv:astro-ph/0104112]. P. Singh, M. Sami and N. Dadhich, Phys. Rev. D 68, 023522 (2003)  
[arXiv:hep-th/0305110]. S. Nojiri and S. D. Odintsov, Phys. Lett. B 562, 147 (2003)  
[arXiv:hep-th/0303117]. S. Nojiri and S. D. Odintsov, Phys. Lett. B 565, 1 (2003)  
[arXiv:hep-th/0304131]. Z. K. Guo, Y. S. Piao and Y. Z. Zhang, Phys. Lett. B 594, 247 (2004)  
[arXiv:astro-ph/0404225]. S. Nojiri, S. D. Odintsov and O. G. Gorbunova, J. Phys. A 39, 6627 (2006)  
[arXiv:hep-th/0510183].

[59] X. H. Meng and P. Wang,  
[arXiv:hep-ph/0311070]. V. B. Johri, Phys. Rev. D 70, 041303 (2004)  
[arXiv:astro-ph/0311293]. L. P. Chimento and R. Lazkoz, Phys. Rev. Lett. 91, 211301 (2003)  
[arXiv:gr-qc/0307111]. L. P. Chimento and R. Lazkoz, Phys. Rev. Lett. 91, 211301 (2003)  
[arXiv:gr-qc/0307111]. L. P. Chimento and D. Pavon, Phys. Rev. D 73,
063511 (2006) [arXiv:gr-qc/0505096]. M. Sami and A. Toporensky, Mod. Phys. Lett. A 19, 1509 (2004) [arXiv:gr-qc/0312009]. M. Szydlowski, W. Czaja and A. Krawiec, Phys. Rev. E 72, 036221 (2005) [arXiv:astro-ph/0401293]. M. Bouhmadi-Lopez and J. A. Jimenez Madrid, JCAP 0505, 005 (2005) [arXiv:astro-ph/0404540]. Y. H. Wei and Y. Tian, Class. Quant. Grav. 21, 5347 (2004) [arXiv:gr-qc/0405038]. V. K. Onemli and R. P. Woodard, Phys. Rev. D 70, 107301 (2004) [arXiv:gr-qc/0406098]. P. F. Gonzalez-Diaz, TSPU Vestnik 44N7, 36 (2004) [arXiv:hep-th/0408225]. Y. H. Wei, Mod. Phys. Lett. A 20, 1147 (2005) [arXiv:gr-qc/0410050]. S. Nojiri and S. D. Odintsov, Phys. Rev. D 72, 023003 (2005) [arXiv:hep-th/0505215]. S. Capozziello, S. Nojiri and S. D. Odintsov, Phys. Lett. B 632, 597 (2006) [arXiv:hep-th/0507182]. S. Nojiri and S. D. Odintsov, Gen. Rel. Grav. 38, 1285 (2006) [arXiv:hep-th/0506212].

[60] Y. S. Myung, arXiv:0810.4385 [gr-qc].

[61] U. H. Danielsson, Phys. Rev. D 71, 023516 (2005) [arXiv:hep-th/0411172].

[62] J. D. Bekenstein, Phys. Rev. D 7 (1973) 2333.

[63] P. C. W. Davies, Class. Quant. Grav. 4, L225 (1987).

[64] C. Armendariz-Picon and P. B. Greene, Gen. Rel. Grav. 35, 1637 (2003) [arXiv:hep-th/0301129].

[65] B. Vakili, S. Jalalzadeh and H. R. Sepangi, JCAP 0505, 006 (2005) [arXiv:gr-qc/0502076].

[66] M. O. Ribas, F. P. Devecchi and G. M. Kremer, Phys. Rev. D 72, 123502 (2005) [arXiv:gr-qc/0511099].

[67] S. Weinberg, *Gravitation and Cosmology*, Cambridge University Press (1972).

[68] N. Birrell and P. Davies, *Quantum Fields in Curved Space*, Cambridge University Press (1982).

[69] M. Green, J. Schwarz, E. Witten, *Superstring Theory Vol. 2*, Chapter 12, Cambridge University Press (1987).
[70] A. Dev, D. Jain and J. S. Alcaniz, Phys. Rev. D 67, 023515 (2003) [arXiv:astro-ph/0209379]. M. C. Bento, O. Bertolami and A. A. Sen, Gen. Rel. Grav. 35, 2063 (2003) [arXiv:gr-qc/0305086]. L. Amendola, F. Finelli, C. Burigana and D. Carturan, JCAP 0307, 005 (2003) [arXiv:astro-ph/0304325]. R. Bean and O. Dore, Phys. Rev. D 68, 023515 (2003) [arXiv:astro-ph/0301308]. M. Szydlowski and W. Czaja, Phys. Rev. D 69, 023506 (2004) [arXiv:astro-ph/0306579]. T. Multamaki, M. Manera and E. Gaztanaga, Phys. Rev. D 69, 023004 (2004) [arXiv:astro-ph/0307533].

[71] L. P. Chimento, Phys. Rev. D 69, 123517 (2004) [arXiv:astro-ph/0311613].

[72] R. Kubo, Thermodynamics, North-Holland, Amsterdam, 1968.

[73] L.D. Landau, E.M. Lifschitz, Statistical Physics, third, ed., Course of Theoretical Physics, vol.5, Butterworth-Heinemann, London, 1984.