A Theory of Gamma-Ray Bursts

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ABSTRACT

Recent observations and theoretical considerations have linked gamma-ray bursts with ultra-bright type Ibc supernovae (‘hypernovae’). We here work out a specific scenario for this connection. Based on earlier work, we argue that especially the longest bursts must be powered by the Blandford-Znajek mechanism of electromagnetic extraction of spin energy from a black hole. Such a mechanism requires a high angular momentum in the progenitor object. The observed association of gamma-ray bursts with type Ibc supernovae leads us to consider massive helium stars that form black holes at the end of their lives as progenitors. In our analysis we combine the numerical work of MacFadyen & Woosley with analytic calculations in Kerr geometry, to show that about $10^{53}$ erg each are available to drive the fast GRB ejecta and the supernova. The GRB ejecta are driven by the power output through the open field lines threading the black hole, whereas the supernova can be powered both by the shocks driven into the envelope by the jet, and by the power delivered into the disk via field lines connecting the disk with the black hole. We also present a much simplified approximate derivation of these energetics.

Helium stars that leave massive black-hole remnants can only be made in fairly specific binary evolution scenarios, namely the kind that also leads to the formation of soft X-ray transients with black-hole primaries, or in very massive WNL stars. Since the binary progenitors will inevitably possess the high angular momentum we need, we propose a natural link between black-hole transients and gamma-ray bursts. Recent observations of one such transient,
GRO J1655–40/Nova Scorpii 1994, explicitly support this connection: its high space velocity indicates that substantial mass was ejected in the formation of the black hole, and the overabundance of $\alpha$-nuclei, especially sulphur, indicates that the explosion energy was extreme, as in SN 1998bw/GRB 980425. Furthermore, X-ray studies of this object indicate that the black hole may still be spinning quite rapidly, as expected in our model. We also show that the presence of a disk during the powering of the GRB and the explosion is required to deposit enough of the $\alpha$ nuclei on the companion.

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1. Introduction

The discovery of afterglows to gamma-ray bursts has greatly increased the possibility of studying their physics. Since these afterglows have thus far only been seen for long gamma-ray bursts (duration $\gtrsim 2$ s), we shall concentrate on the mechanism for this subclass. The shorter bursts (duration $\lesssim 2$ s) may have a different origin; specifically, it has been suggested that they are the result of compact-object mergers and therefore offer the intriguing possibility of associated outbursts of gravity waves. (Traditionally, binary neutron stars have been considered in this category (Eichler et al. 1989, Janka et al. 1999). More recently, Bethe & Brown (1998) have shown that low-mass black-hole, neutron-star binaries, which have a ten times greater formation rate and are stronger gravity-wave emitters, may be the more promising source of this kind.)

An important recent clue to the origin of long bursts is the probable association of some of them with ultra-bright type Ibc supernovae (Galama et al. 1998, Bloom et al. 1999, Galama et al. 2000). The very large explosion energy\footnote{Höflich et al. (1999) have proposed that the explosion energy was not much larger than usual, but that the explosion was very asymmetric; this model also provides a reasonable fit to the light curve of SN 1998bw.} implied by fitting the light curve of SN 1998bw, which was associated with GRB 980425, indicates that a black hole was formed in this event (Iwamoto et al. 1998). This provides two good pieces of astrophysical information: it implicates black holes in the origin of gamma-ray bursts, and it demonstrates that a massive star can explode as a supernova even if its core collapses into a black hole.

In this paper, we start from the viewpoint that the gamma-ray burst is powered by
electromagnetic energy extraction from a spinning black hole, the so-called Blandford-Znajek (1977) mechanism. This was worked out in detail by Lee, Wijers, & Brown (1999), and further details and comments were discussed by Lee, Brown, & Wijers (2000), who built on work by Thorne et al. (1986) and Li (2000). They have shown that with the circuitry in a 3+1 dimensional description using the Boyer-Lindquist metric, one can have a simple pictorial model for the BZ mechanism.

The simple circuitry which involves steady state current flow is, however, inadequate for describing dissipation of the black hole rotational energy into the accretion disk formed from the original helium envelope. In this case the more rapidly rotating black hole tries to spin up the inner accretion disk through the closed field lines coupling the black hole and disk. Electric and magnetic fields vary wildly with time. Using the work of Blandford & Spruit (2000) we show that this dissipation occurs in an oscillatory fashion, giving a fine structure to the GRB, and that the total dissipation should furnish an energy comparable to that of the GRB to the accretion disk. We use this energy to drive the hypernova explosion.

Not any black-hole system will be suitable for making GRB: the black hole must spin rapidly enough and be embedded in a strong magnetic field. Moreover, the formation rate must be high enough to get the right rate of GRB even after accounting for substantial collimation of GRB outflows. We explore a variety of models, and give arguments why some will have sufficient energy and extraction efficiency to power a GRB and a hypernova. We argue that the systems known as black-hole transients are the relics of GRBs, and discuss the recent evidence from high space velocities and chemical abundance anomalies that these objects are relics of hypernovae and GRBs; we especially highlight the case of Nova Scorpii 1994 (GRO J1655−40).

The plan of this paper is as follows. We first show that it is reasonable to expect similar energy depositions into the GRB outflow and the accretion disk (Sect. 2) and discuss the amount of available energy to be extracted (Sect. 3). Then we show the agreement of those results with the detailed numerical simulations by MacFadyen & Woosley, and use those simulations to firm up our numbers (Sect. 4). We continue by presenting a simple derivation of the energetics that approximates the full results well (Sect. 5). Finally, we discuss some previously suggested progenitors (Sect. 6) and present our preferred progenitors: soft X-ray transients (Sect. 7).
2. Simple Circuitry

Although our numbers are based on the detailed review of Lee, Wijers, & Brown (1999), which confirms the original Blandford-Znajek (1977) paper, we illustrate our arguments with the pictorial treatment of Thorne et al. (1986) in “The Membrane Paradigm”. Considering the time as universal in the Boyer-Lindquist metric, essential electromagnetic and statistical mechanics relations apply in their 3+1 dimensional manifold. We summarize their picture in our Fig. [Fig.]

The surface of the black hole can be considered as a conductor with surface resistance \( R_{BH} = 4\pi/c = 377 \text{ ohms} \). A circuit that rotates rigidly with the black hole can be drawn from the loading region, the low-field region up the axis of rotation of the black hole in which the power to run the GRB is delivered, down a magnetic field line, then from the North pole of the black hole along the (stretched) horizon to its equator. From the equator we continue the circuit through part of the disk and then connect it upwards with the loading region. We can also draw circuits starting from the loading region which pass along only the black hole or go through only the disk, but adding these would not change the results of our schematic model.

Using Faraday’s law, the voltage \( V \) can be found by integrating the vector product of charge velocity, \( \vec{v} \), and magnetic field, \( \vec{B} \), along the circuit:

\[
V = \int [\vec{v} \times \vec{B}] \cdot d\vec{l},
\]

(1)

(\( d\vec{l} \) is the line element along the circuit). Because this law involves \( \vec{v} \times \vec{B} \) the integrals along the field lines make no contribution. We do get a contribution \( V \) from the integral from North pole to equator along the black hole surface. Further contributions to \( V \) will come from cutting the field lines from the disk. We assume the field to be weak enough in the loading region to be neglected.

The GRB power, \( E_{GRB} \), will be

\[
\dot{E}_{GRB} = I_{BH+D}^2 R_L
\]

2

(2)

where \( R_L \) is the resistance of the loading region, and the current is given by

\[
I_{BH+D}^2 = \left( \frac{V_D + V_{BH}}{R_D + R_{BH} + R_L} \right)^2.
\]

(3)

(The index BH refers to the black hole, L to the load region, and D to the disk.)

The load resistance has been estimated in various ways and for various assumptions by Lovelace, MacAuslan, & Burns (1979) and by MacDonald & Thorne (1982), and by
Phinney (1983). All estimates agree that to within a factor of order unity \( R_L \) is equal to \( R_{BH} \).

In a similar fashion, some power will be deposited into the disk

\[
\dot{E}_{\text{disk}} = I_{BH+D}^2 R_D
\] (4)

but this equilibrium contribution will be small because of the low disk resistance \( R_D \).

Blandford & Spruit (2000) have shown that important dissipation into the disk comes through magnetic field lines coupling the disk to the black hole rotation. As shown in Fig. 2 these lines, anchored in the inner disk, thread the black hole.

The more rapidly rotating black hole will provide torques, along its rotation axis, which spin up the inner accretion disk, in which the closed magnetic field lines are anchored. With increasing centrifugal force the material in the inner disk will move outwards, cutting down the accretion. Angular momentum is then advected outwards, so that the matter can drift back inwards. It then delivers more matter to the black hole and is flung outwards again. The situation is like that of a ball in a roulette wheel (R.D. Blandford, private communication). First of all it is flung outwards and then drifts slowly inwards. When it hits the hub it is again thrown outwards. The viscous inflow time for the fluctuations is easily estimated to be

\[
\tau_d \sim \Omega_{\text{disk}}^{-1} \left( \frac{r}{H} \right)^2 \alpha_{\text{vis}}^{-1}
\] (5)

where \( H \) is the height of the disk at radius \( r \), \( \Omega_{\text{disk}} \) its angular velocity, and \( \alpha_{\text{vis}} \) is the usual \( \alpha \)-parameterization of the viscosity. We choose \( \alpha_{\text{vis}} \sim 0.1 \), \( r/H \sim 10 \) for a thin disk and then arrive at \( \tau_d \sim 0.1 \text{s} \). We therefore expect variability on all time scales between the Kepler time (sub-millisecond) and the viscous time, which may explain the very erratic light curves of many GRBs.

We suggest that the GRB can be powered by \( \dot{E}_{\text{GRB}} \) and a Type Ibc supernova explosion by \( \dot{E}_{\text{SN}} \) where \( \dot{E}_{\text{SN}} \) is the power delivered through dissipation into the disk. To the extent that the number of closed field lines coupling disk and black hole is equal to the number of open field lines threading the latter, the two energies will be equal. In the spectacular case of GRB 980326 (Bloom et al. 1999), the GRB lasts about 5 s, which we take to be the time that the central engine operates. We shall show that up to \( \sim 10^{53} \text{erg} \) is available to be delivered into the GRB and into the accretion disk, the latter helping to power the supernova (SN) explosion. This is more energy than needed and we suggest that injection of energy into the disk shuts off the central engine by blowing up the disk and thus removing the magnetic field needed for the energy extraction from the black hole. If the magnetic
field is high enough the energy will be delivered in a short time, and the quick removal of the disk will leave the black hole still spinning quite rapidly.

3. Energetics of GRBs

The maximum energy that can be extracted from the BZ mechanism (Lee, Wijers, & Brown 1999) is

\[(E_{\text{BZ}})_{\text{max}} \simeq 0.09 \, M_{\text{BH}} c^2. \quad (6)\]

This is 31% of the black hole rotational energy, the remainder going toward increasing the entropy of the black hole. This maximum energy is obtained if the extraction efficiency is

\[\epsilon_\Omega = \frac{\Omega_{\text{disk}}}{\Omega_H} = 0.5. \quad (7)\]

In Appendix A we give numerical estimates for this ratio for various \(\omega = \Omega_{\text{disk}}/\Omega_K\) and various radii in the region of parameter space we consider. As explained in Section 2 we expect the material in the inner disk to swing in and out around the marginally stable radius, \(r_{\text{ms}}\). It can be seen from the Table 2 and Appendix A that the relevant values of \(\epsilon_\Omega\) are close to that of eq. (7).

For a 7\(M_\odot\) black hole, such as that found in Nova Sco 1994 (GRO J1655−40),

\[E_{\text{max}} \simeq 1.1 \times 10^{54} \text{ erg}. \quad (8)\]

We estimate below that the energy available in a typical case will be an order of magnitude less than this. Without collimation, the estimated gamma-ray energy in GRB990123 is about 4.5 \(\times\) 10^{54} erg (Andersen et al. 1999). The BZ scenario entails substantial beaming, so this energy should be multiplied by \(d\Omega/4\pi\), which may be a small factor (perhaps 10^{-2}).

The BZ power can be delivered at a maximum rate of

\[P_{\text{BZ}} = 6.7 \times 10^{50} \left(\frac{B}{10^{15} \text{G}}\right)^2 \left(\frac{M_{\text{BH}}}{M_\odot}\right)^2 \text{ erg s}^{-1}, \quad (9)\]

(Lee et al. 1999) so that high magnetic fields are necessary for rapid delivery.

The above concerns the maximum energy output into the jet and the disk. The real energy available in black-hole spin in any given case, and the efficiency with which it can be extracted, depend on the rotation frequency of the newly formed black hole and the disk or torus around it. The state of the accretion disk around the newly formed black hole, and
the angular momentum of the black hole, are somewhat uncertain. However, the conditions should be bracketed between a purely Keplerian, thin disk (if neutrino cooling is efficient) and a thick, non-cooling hypercritical advection-dominated accretion disk (HADAF), of which we have a model (Brown, Lee & Bethe 2000). Let us examine the result for the Keplerian case. In terms of
\[
\tilde{a} \equiv \frac{Jc}{M^2G},
\] (10)
where \( J \) is the angular momentum of the black hole, we find the rotational energy of a black hole to be
\[
E_{\text{rot}} = f(\tilde{a})Mc^2,
\] (11)
where
\[
f(\tilde{a}) = 1 - \sqrt{\frac{1}{2}(1 + \sqrt{1 - \tilde{a}^2})}.
\] (12)
For a maximally rotating black hole one has \( \tilde{a} = \frac{1}{2} \).

We begin with a neutron star in the middle of a Keplerian accretion disk, and let it accrete enough matter to send it into a black hole. In matter free regions the last stable orbit of a particle around a black hole in Schwarzschild geometry is
\[
r_{\text{iso}} = 3R_{\text{Sch}} = \frac{6GM}{c^2}.
\] (13)
This is the marginally stable orbit \( r_{\text{ms}} \). However, under conditions of hypercritical accretion, the pressure and energy profiles are changed and it is better to use (Abramowicz et al. 1988)
\[
r_{\text{iso}} \gtrsim 2R_{\text{Sch}}.
\] (14)
With the equal sign we have the marginally bound orbit \( r_{\text{mb}} \). With high rates of accretion we expect this to be a good approximation to \( r_{\text{iso}} \). The accretion disk can be taken to extend down to the last stable orbit (refer to Appendix B for the details).

\(^2\) As an aside, we note a nice mnemonic: if we define a velocity \( v \) from the black-hole angular momentum by \( J = MR_{\text{Sch}}v \), so that \( v \) carries the quasi-interpretation of a rotation velocity at the horizon, then \( \tilde{a} = 2v/c \). A maximal Kerr hole, which has \( R_{\text{event}} = R_{\text{Sch}}/2 \), thus has \( v = c \). For \( \tilde{a} \lesssim 0.5 \), the rotation energy is well approximated by the easy-to-remember expression \( E_{\text{rot}} = \frac{1}{2}Mv^2 \).
We take the angular velocity to be Keplerian, so that the disk velocity $v$ at radius $2R_{\text{Sch}}$ is given by

$$v^2 = \frac{GM}{2R_{\text{Sch}}} = \frac{c^2}{4},$$  \hspace{1cm} (15)$$
or $v = c/2$. The specific angular momentum, $l$, is then

$$l \geq 2R_{\text{Sch}}v = 2\frac{GM}{c},$$  \hspace{1cm} (16)$$
which in Kerr geometry indicates $\tilde{a} \sim 1$. Had we taken one of the slowest-rotating disk flows that are possible, the advection-dominated or HADAF case (Narayan and Yi 1994, Brown, Lee & Bethe 2000), which has $\Omega^2 = 2\Omega_K^2/7$, we would have arrived at $\tilde{a} \sim 0.54$, so the Kerr parameter will always be high.

Further accretion will add angular momentum to the black hole at a rate determined by the angular velocity of the inner disk. The material accreting into the black hole is released by the disk at $r_{\text{iso}}$, where the angular momentum delivered to the black hole is determined. This angular momentum is, however, delivered into the black hole at the event horizon $R_{\text{Sch}}$, with velocity at least double that at which it is released by the disk, since the lever arm at the event horizon is only half of that at $R_{\text{Sch}}$, and angular momentum is conserved. With more rapid rotation involving movement towards a Kerr geometry where the event horizon and last stable orbit coincide at

$$r_{\text{iso}} = R_{\text{event}} = \frac{GM}{c^2}.$$  \hspace{1cm} (17)$$
Although we must switch over to a Kerr geometry for quantitative results, we see that $\tilde{a}$ will not be far from its maximum value of unity. Again, for the lower angular-momentum case of a HADAF, the expected black-hole spin is not much less.

4. Comparison with Numerical Calculation

Our schematic model has the advantage over numerical calculations that one can see analytically how the scenario changes with change in parameters or assumptions. However, our model is useful only if it reproduces faithfully the results of more complete calculations which involve other effects and much more detail than we include. We here make comparison with Fig.19 of MacFadyen & Woosley (1999). Accretion rates, etc., can be read off from their figure which we reproduce as our Fig.4. MacFadyen & Woosley prefer $\tilde{a}_{\text{initial}} = 0.5$ (We have removed their curve for $\tilde{a}_{\text{initial}} = 0$). This is a reasonable value if
the black hole forms from a contracting proto-neutron star near breakup. MacFadyen & Woosley find that $\tilde{a}_{\text{initial}} = 0.5$ is more consistent with the angular momentum assumed for the mantle than $\tilde{a}_{\text{initial}} = 0$. (They take the initial black hole to have mass $2M_\odot$; we choose the Brown & Bethe (1994) mass of $1.5M_\odot$.) We confirm this in the next section.

After 5 seconds (the duration of GRB 980326) the MacFadyen & Woosley black hole mass is $\sim 3.2M_\odot$ and their Kerr parameter $\tilde{a} \sim 0.8$, which gives $f(\tilde{a})$ of our eq.(12) of 0.11. With these parameters we find $E = 2 \times 10^{53}$ erg, available for the GRB and SN explosion.

One can imagine that continuation of the MacFadyen & Woosley curve for $M_{BH}(M_\odot)$ would ultimately give something like our $\sim 7M_\odot$, but the final black hole mass may not be relevant for our considerations. This is because more than enough energy is available to power the supernova in the first 5 seconds; as the disk is disrupted, the magnetic fields supported by it will also disappear, which turns off the Blandford-Znajek mechanism.

Power is delivered at the rate given by eq. (11). Taking a black hole mass relevant here, $\sim 3.2M_\odot$, we require a field strength of $\sim 5.8 \times 10^{15}$ G in order for our estimated energy ($4 \times 10^{52}$ erg) to be delivered in 5 s (the duration of GRB 980326). For such a relatively short burst, we see that the required field is quite large, but it is still not excessive if we bear in mind that magnetic fields of $\sim 10^{15}$ G have already been observed in magnetars (Kouveliotou 1998, 1999). Since in our scenario we have many more progenitors than there are GRBs, we suggest that the necessary fields are obtained only in a fraction of all potential progenitors.

Thus we have an extremely simple scenario for powering a GRB and the concomitant SN explosion in the black hole transients, which we will discuss in Section 7.2. After the first second the newly evolved black hole has $\sim 10^{53}$ erg of rotational energy available to power these. The time scale for delivery of this energy depends (inversely quadratically) on the magnitude of the magnetic field in the neighborhood of the black hole, essentially that on the inner accretion disk. The developing supernova explosion disrupts the accretion disk; this removes the magnetic fields anchored in the disk, and self-limits the energy the B-Z mechanism can deliver.

5. An Even More Schematic Model

Here we calculate the energy available in a rotating black hole just after its birth (before accretion adds more). Our model is to take a $1.5M_\odot$ neutron star which co-rotates with the inner edge of the accretion disk in which it is embedded. The neutron star then collapses to a black hole, conserving its angular momentum. Since the accretion disk is
neutrino cooled, but perhaps not fully thin, its angular velocity will be somewhere between the HADAF value and the Keplerian value. We parameterize it as $\Omega = \omega \Omega_K$, where $\omega = 1$ for Keplerian and $\omega = \sqrt{2/7} \sim 0.53$ for the HADAF.

The moment of inertia, $I$, of a neutron star is well fitted for many different equations of state with the simple expression

$$I = \frac{0.21 MR^2}{1 - 2GM/Rc^2}$$

(Lattimer & Prakash 2000). With $J = \omega I \Omega_K$ and a neutron star of $1.5M_\odot$, with a radius of 10 km, we find

$$\tilde{a}^2 = \left(\frac{Jc}{GM^2}\right)^2 = 0.64\omega^2.$$  

We choose $\omega \simeq 1.0$ to roughly reproduce the MacFadyen & Woosley value of $\tilde{a}$, see our Fig. [3]. We do not really believe the disk to be so efficiently neutrino cooled that its angular velocity is Keplerian; i.e. $\omega = 1$, but it may be not far from it. Our $\omega$ should be more properly viewed as a fudge factor which allows us to match the more complete MacFadyen & Woosley calculation. MacFadyen & Woosley find that, while the accretion disk onto the black hole is forming, an additional solar mass of material is added to it “as the dense stellar core collapses through the inner boundary at all polar angles”. We shall add this to our $1.5M_\odot$ and take the black hole mass to be $2.5M_\odot$. We neglect the increase in spin of the black hole by the newly accreted matter; this is already included in the MacFadyen & Woosley results. For $\tilde{a}^2 = 0.64$ we find $f(\tilde{a}^2) = 0.11$, so that the black hole rotation energy becomes

$$E_{BZ} = 1.5 \times 10^{53} \text{ erg}$$

in rough agreement with the estimates of MacFadyen & Woosley in the last section.

6. Previous Models

6.1. Collapsar

We have not discussed the Collapsar model of Woosley (1993), and MacFadyen & Woosley (1999). In this model the center of a rotating Wolf-Rayet star evolves into a black hole, the outer part being held out by centrifugal force. The latter evolves into an accretion disk and then by hypercritical accretion spins the black hole up. MacFadyen & Woosley point out that “If the helium core is braked by a magnetic field prior to the supernova
explosion to the extent described by Spruit & Phinney (1998) then our model will not work for single stars.” Spruit & Phinney argue that magnetic fields maintained by differential rotation between the core and envelope of the star will keep the whole star in a state of approximately uniform rotation until 10 years before its collapse. As noted in the last section, with the extremely high magnetic fields we need the viscosity would be expected to be exceptionally high, making the Spruit & Phinney scenario probable. Livio & Pringle (1998) have commented that one finds evidence in novae that the coupling between layers of the star by magnetic fields may be greatly suppressed relative to what Spruit & Phinney assumed. However, we note that even with this suppressed coupling, they find pulsar periods from core collapse supernovae no shorter than 0.1 s. Independent evidence for the fact that stellar cores mostly rotate no faster than this comes from the study of supernova remnants: Bhattacharya (1990, 1991) concludes that the absence of bright, pulsar-powered plerions in most SNRs indicates that typically pulsar spin periods at birth are no shorter than 0.03–0.05 s. Translated to our black holes, such spin periods would imply $\dot{a} \lesssim 0.01$, quite insufficient to power a GRB. As a cautionary note, we might add that without magnetic coupling the cores of evolved stars can spin quite rapidly (Heger et al. 2000). This rapid initial spin may be reconciled with Bhattacharya’s limit if r-mode instabilities cause very rapid spindown in the first few years of the life of a neutron star (e.g., Heger, Langer, & Woosley 2000, Lindblom & Owen 1999).

6.2. Coalescing Low-Mass Black Holes and Helium Stars

Fryer & Woosley (1998) suggested the scenario of a black hole spiraling into a helium star. This is an efficient way to spin up the black hole.

Bethe & Brown (1998) evolved low-mass black holes with helium star companion, as well as binaries of compact objects. In a total available range of binary separation $0.04 < a_{13} < 4$, low-mass black-hole, neutron-star binaries were formed when $0.5 < a_{13} < 1.4$ where $a_{13}$ is the initial binary separation in units of $10^{13}$ cm. The low-mass black hole coalesces with the helium star in the range $0.04 < a_{13} < 0.5$. Binaries were distributed logarithmically in $a$. Thus, coalescences are more common than low-mass black-hole, neutron-star binaries by a factor of $\ln(0.5/0.04)/\ln(1.9/0.5) = 1.9$

In Bethe & Brown (1998), the He-star, compact-object binary was disrupted $\sim 50\%$ of the time by the He-star explosion. This does not apply to the coalescence. Thus, the rate of low-mass black-hole, He-star mergers is 3.8 times the formation rate of low-mass black-hole, neutron-star binaries, or

$$R = 3.8 \times 10^{-4} \text{ yr}^{-1}$$ (21)
in the Galaxy. The estimated empirical rate of GRBs, with a factor of 100 for beaming, is $10^{-5}$ yr$^{-1}$ in the Galaxy (Appendix C of Brown et al. 1999). Thus, the number of progenitors is more than adequate.

In Bethe & Brown (1998) the typical black hole mass was $\sim 2.4M_\odot$, somewhat more massive than their maximum assumed neutron star mass of $1.5M_\odot$. As it enters the helium star companion an accretion disk is soon set up and the accretion scenario will follow that described above, with rotating black holes of various masses formed. Brown, Lee, & Bethe (2000) find that the black hole will be spun up quickly. We have not pursued this scenario beyond the point that it was developed by Fryer & Woosley (1998).

### 7. Soft X-ray Transients as Relics of Hypernovae and GRB

#### 7.1. Our Model: Angular Momentum

We favor a model of hypernovae similar to MacFadyen & Woosley (1999) in that it involves a failed supernova as a centerpiece. But, in distinction to MacFadyen & Woosley, our initial system is a binary, consisting of a massive star A (which will later become the failed SN) and a lighter companion B, which serves to provide ample angular momentum.

Failed supernovae require a ZAMS mass of $20 - 35M_\odot$, according to the calculations of Woosley & Weaver (1995) as interpreted by Brown, Lee, & Bethe (1999). The limits 20 and $35M_\odot$ are not accurately known, but it is a fairly narrow range, so we shall in many of our calculations assume a “typical” ZAMS mass of $25M_\odot$. The heavy star A must not be in a close binary because then its hydrogen envelope would be removed early in its evolution and therefore the star would lose mass by wind at a very early stage and become a low-mass compact object (Brown, Weingartner, & Wijers 1996). Instead, we assume a wide binary, with a separation, $a$ in the range

$$a = 500 - 1000R_\odot,$$

so star A evolves essentially as a single star through its first few burning stages. It is essential that most of the He core burning is completed before its hydrogen envelope is removed (Wellstein & Langer 1999; Heger & Wellstein 2000). We assume the initial distance $a$ between the two stars to be in this range. When star A fills its Roche lobe, the companion, star B, will spiral inwards.

The initiation and early development of the common envelope has been best treated by Rasio & Livio (1996). This is the only phase that can at present be modeled in a realistic way. They find a short viscous time in the envelope, but emphasize that numerical
viscosity may play an important role in their results. However, we believe the viscosity to be large. Torkelsson et al. (1996) showed the Shakura-Sunyaev (1973) viscosity parameter, $\alpha_{SS}$, to range from 0.001 to 0.7, with the higher values following from the presence of vertical magnetic fields. Since in our Blandford-Znajek model extremely high magnetic fields $\sim 10^{15}$ G are needed in the He envelope to deliver the energy rapidly, we believe $\alpha_{SS}$ to be not much less than unity. Given such high viscosities, it seems reasonable to follow the Rasio-Livio extrapolation, based on a short viscous transport time, to later times.

The most significant new result of Rasio & Livio “is that, during the dynamical phase of common envelope evolution, a corotating region of gas is established near the central binary. The corotating region has the shape of an oblate spheroid encasing the binary (i.e., the corotating gas is concentrated in the orbital plane).”

A helium core, which we deal with, is not included in their calculations, because they do not resolve the inner part of the star numerically. However, since the physics of the spiral-in does not really change as it proceeds past the end of their calculations, it seems most likely that during further spiral-in, the spin-up of material inside the orbit of the companion will continue to be significant.

Star B will stop spiraling in when it has ejected the H envelope of A. Since we assume that all stars A have about the same mass, and that $a_i$ is very large, we expect

$$\frac{M_B}{a_f} \simeq \text{const.} \tag{23}$$

From section 7.2 we conclude that $a_f$ is a few $R_\odot$ for $M_B = (0.4 - 1) M_\odot$. Now the He cores of stars of ZAMS mass $M = 20 - 35 M_\odot$ have a radius about equal to $R_\odot$. Therefore small $M_B$ stars will spiral into the He core of A. There they cannot be stopped but will coalesce with star A. However, they will have transmitted their angular momentum to star A.

Star B of larger mass will stop at larger $a_f \gg R_\odot$. It is then not clear whether they will transfer all of their angular momentum to star A. In any case, they must generally wait until they evolve off the main sequence into the subgiant or possibly even the giant stage before they can fill their Roche Lobes and later accrete onto the black hole resulting from star A.

The Kepler velocity of star B at $a_f$ is

$$V_K^2 = G \frac{M_B}{a_f}\tag{24}$$

We estimate the final mass of A, after removal of its hydrogen envelope, to be about $10 M_\odot$; then

$$V_K \simeq 1.2 \times 10^8 a_{f,11}^{-1/2} \text{ cm s}^{-1}, \tag{25}$$
where $a_{f,11}$ is $a_f$ in units of $10^{11}$ cm. The specific angular momentum of B is then

$$j(B) = a_f V_K = 1.2 \times 10^{19} a_{f,11}^{1/2} \text{ cm}^2 \text{ s}^{-1}. \quad (26)$$

If B and A share their angular momentum, the specific angular momentum is reduced by a factor $M_B/(M_{A,f} + M_B)$ which we estimate to be $\sim 0.1$. Since $a_f$ should be $\gtrsim 3 R_\odot$ (See Table I), the specific angular momentum of A should be

$$j(A) \gtrsim 10^{18} \text{ cm}^2 \text{ s}^{-1}. \quad (27)$$

Star B has now done its job and can be disregarded.

7.2. Supernova and collapse

Star A now goes through its normal evolution, ending up as a supernova. But since we have chosen its mass to be between 20 and 35$M_\odot$, the SN shock cannot penetrate the heavy envelope but is stopped at some radius

$$R_{SN} \approx 10^{10} \text{ cm}, \quad (28)$$

well inside the outer edge of the He envelope. We estimate $R_{SN}$ by scaling from SN 1987A: in that supernova, with progenitor mass $\sim 18 M_\odot$, most of the He envelope was returned to the galaxy. The separation between compact object and ejecta was estimated to occur at $R \sim 5 \times 10^8$ cm (Woosley 1988, Bethe 1990) at mass point $1.5 M_\odot$ (gravitational). Woosley and Weaver (1995) find remnant masses of $\sim 2 M_\odot$, although with large fluctuations, for ZAMS masses in the range 20–35$M_\odot$, which go into high-mass black holes. From table 3 of Brown, Weingartner, and Wijers (1996) we see that fallback between $R = 3.5$ and $4.5 \times 10^8$ cm is $0.03 M_\odot$. Using this we can extrapolate to $R = 10^{10}$ cm as the distance within which matter has to begin falling in immediately in our heavier stars, to make up a compact object of $2 M_\odot$. Unlike in 1987A the shock energy in the more massive star does not suffice to eject the envelope beyond this point, and the remaining outer envelope will also eventually fall back.

At $R_{SN}$, the specific angular momentum of Kepler motion around a central star of mass $10 M_\odot$ is, cf. eq.(27)

$$j_K(10 M_\odot) = 1.2 \times 10^{19} R_{f,11}^{1/2} \text{ cm}^2 \text{ s}^{-1} = 4 \times 10^{18} \text{ cm}^2 \text{ s}^{-1}. \quad (29)$$

In reality, at this time the central object has a mass $M \sim 1.5 M_\odot$ (being a neutron star) and since $j_K \sim V_K \sim M^{1/2}$

$$j_K(1.5 M_\odot) = 1.5 \times 10^{18} \text{ cm}^2 \text{ s}^{-1}. \quad (30)$$
The angular momentum inherent in star A, eq. (27), is therefore greater than the Kepler angular momentum. This would not be the case had our initial object been a single star, a collapsar. (The collapsar may work none the less, but our binary model is more certain to work.)

The supernova material is supported by pressure inside the cavity, probably mostly due to electromagnetic radiation. The cavity inside $R_{SN}$ is rather free of matter. After a while, the pressure in the cavity will reduce. This may happen by opening toward the poles, in which case the outflowing pressure will drive out the matter near the poles and create the vacuum required for the gamma ray burst. Reduction of pressure will also happen by neutrino emission. As the pressure gets reduced, the SN material will fall in toward the neutron star in the center. But because the angular momentum of the SN material is large (eq. 27) the material must move more or less in Kepler orbits; i.e., it must spiral in. This is an essential point in the theory.

If $j(A)$ is less than $j_K$ at $R_{SN}$, the initial motion will have a substantial radial component in addition to the tangential one. But as the Kepler one decreases, cf. eq. 29, there will come a point of $r$ at which $j_K = j(A)$. At this point an accretion disk will form, consisting of SN material spiraling in toward the neutron star. The primary motion is circular, but viscosity will provide a radial component inward

$$v_r \sim \alpha v_K$$

(31)

where $\alpha$ is the viscosity parameter. It has been argued by Brandenburg et al. (1996) that $\alpha \sim 0.1$ in the presence of equipartition magnetic fields perpendicular to the disk, and it may be even larger with the high magnetic fields required for GRBs. Narayan & Yi (1994) have given analytical solutions for such accretion disks. The material will arrive at the neutron star essentially tangentially, and therefore its high angular momentum will spin up the neutron star substantially. Accretion will soon make the neutron star collapse into a black hole. The angular momentum will be conserved, so the angular velocity is increased since the black hole has smaller radius than the neutron star. Thus the black hole is born with considerable spin.

A large fraction of the material of the failed supernova will accrete onto the black hole, giving it a mass of order $7M_\odot$. All this material adds to the angular momentum of the black hole since all of it has the Kepler velocity at the black hole radius. Our estimates show that the black hole would be close to an extreme Kerr hole (Section 6), were it to accrete all of this material. It may, however, be so energetic that it drives off part of the envelope in the explosion before it can all accrete (see Section 6).
7.3. Soft X-ray Transients with Black-Hole Primaries

Nine binaries have been observed which are black-hole X-ray transients. All contain a high-mass black hole, of mass $\sim 7M_\odot$. In seven cases the lower-mass companion (star B) has a mass $\lesssim M_\odot$. The two stars are close together, their distance being of order $5R_\odot$. Star B fills its Roche Lobe, so it spills over some material onto the black hole. The accretion disk near the black hole emits soft X rays. Two of the companions are subgiants, filling their Roche lobes at a few times larger separations from the black hole.

In fact, however, the accretion onto the central object is not constant, so there is usually no X-ray emission. Instead, the material forms an accretion disk around the black hole, and only when enough material has been assembled, it falls onto the black hole to give observable X rays. Hence, the X-ray source is transient. Recent observation of a large space velocity of Cygnus X-1 (Nelemans et al. 1999) suggests that it has evolved similarly to the transient sources, with the difference that the companion to the black hole is an $\sim 18M_\odot$ O star. The latter pours enough matter onto the accretion disk so that Cyg X-1 shines continuously. We plan to describe the evolution of Cyg X-1 in a future paper (Brown et al. 2000).

Table 1 is an abbreviated list of data on transient sources. A more complete table is given in Brown et al. (1999b). Two of the steady X-ray sources, in the LMC, have been omitted, because we believe the LMC to be somewhat special because of its low metallicity; also masses, etc., of these two are not as well measured. Of the others, 6 are main-sequence K stars, one is main-sequence M, and the other two have masses greater than the Sun. The masses given are geometric means of the maximum and minimum masses given by the observers. The distance $a$ between the black hole and the optical (visible) star is greater for the heavier stars than for the K- and M stars (except the more evolved one of them) as was expected in Section 7.1 for the spiraling in of star B. The table also gives the radius of the Roche Lobe and the specific orbital angular momentum of star B.

Five K stars have almost identical distance $a \sim 5R_\odot$, and also Roche Lobe sizes, $\sim 1.0R_\odot$. These Roche Lobes can be filled by K stars on the main sequence. The same is true for the M star. Together, K and M stars cover the mass range from 0.3 to 1$M_\odot$. The two heavier stars have Roche Lobes of 3 and 5$R_\odot$ which cannot possibly be filled by main-sequence stars of mass $\sim 2M_\odot$. We must therefore assume that these stars are subgiants, in the Herzsprung gap. These stars spend only about 1% of their life as subgiants, so we must expect that there are many “silent” binaries in which the 2$M_\odot$ companion has not yet evolved off the main sequence and sits well within its Roche lobe, roughly 100 times more. The time as subgiants is even shorter for more massive stars; this explains their absence among the transient sources.
Therefore we expect a large number of “silent partners”: stars of more than $1 M_\odot$, still on their main sequence, which are far from filling their Roche Lobe and therefore do not transfer mass to their black hole partners. In fact, we do not see any reason why the companion of the black hole could not have any mass, up to the ZAMS mass of the progenitor of the black hole; it must only evolve following the formation of the black hole. It then crosses the Herzsprung gap in such a short time, less than the thermal time scale, that star A cannot accept the mass from the companion, so that common envelope evolution must ensue. If we include these ‘silent partners’ in the birth rate, assuming a flat mass ratio distribution, we enhance the total birth rate of black-hole binaries by a factor 25 over the calculations by Brown, Lee, & Bethe (1999).

On the lower mass end of the companions, there is only one $M$ star. This is explained in terms of the model of Section 7.1 by the fact that stars of low mass will generally spiral into the He core of star A, and will coalesce with A, see below eq. (23), so no relic is left. (Since the core is left spinning rapidly, these complete merger cases could also be suitable GRB progenitors.) As the outcome of the spiral-in depends also on other factors, such as the initial orbital separation and the primary mass, one may still have an occasional survival of an $M$ star binary (note that the one $M$ star companion is $M_0$, very nearly in the K star range).

The appearance of the black hole transient X-ray binaries is much like our expectation of the relic of the binary which has made a hypernova: a black hole of substantial mass, and an ordinary star, possibly somewhat evolved, of smaller mass. We expect that star B would stop at a distance $a_f$ from star A which is greater if the mass of B is greater (see Section 7.1). This is just what we see in the black-hole binaries: the more massive companion stars ($\sim 2 M_\odot$) are further from the black hole than the K stars. We also note that the estimated birth rate of these binaries is high enough for them to be the progenitors of GRB, even if only in a modest fraction of them the conditions for GRB powering are achieved.

### 7.4. Nova Scorpii 1994 (GRO J1655-40)

Nova Sco 1994 is a black hole transient X-ray source. It consists of a black hole of $\sim 7 M_\odot$ and a subgiant of about $2 M_\odot$. Their separation is $17 R_\odot$. Israeli et al. (1999) have analyzed the spectrum of the subgiant and have found that the $\alpha$-particle nuclei O, Mg, Si, and S have abundances 6 to 10 times the solar value. This indicates that the subgiant has been enriched by the ejecta from a supernova explosion; specifically, that some of the ejecta of the supernova which preceded the present Nova Sco (a long time ago) were intercepted by
star B, the present subgiant. Israeli et al. (1999) estimate an age since accretion started from the assumption that enrichment has only affected the outer layers of the star. We here reconsider this: the time that passed since the explosion of the progenitor of the black hole is roughly the main-sequence lifetime of the present subgiant companion, which given its mass of $\sim 2M_\odot$ will be about 1 Gyr. This is so much longer than any plausible mixing time in the companion that the captured supernova ejecta must by now be uniformly mixed into the bulk of the companion. This rather increases the amount of ejecta that we require the companion to have captured. (Note that the accretion rate in this binary is rather less than expected from a subgiant donor, though the orbital period leaves no doubt that the donor is more extended than a main-sequence star (Regős, Tout, and Wickramasinghe 1998). It is conceivable that the high metal abundance has resulted in a highly non-standard evolution of this star, in which case one might have to reconsider its age.)

The presence of large amounts of S is particularly significant. Nomoto et al. (2000) have calculated the composition of a hypernova from an $11M_\odot$ CO core, see Fig. 4. This shows substantial abundance of S in the ejecta. Ordinary supernovae produce little of this element, as shown by the results of Nomoto et al. (2000) in Fig. 4. The large amount of S, as well as O, Mg and Si we consider the strongest argument for considering Nova Sco 1994 as a relic of a hypernova, and for our model, generally.

Fig. 4 also shows that $^{56}$Ni and $^{52}$Fe are confined to the inner part of the hypernova, and if the cut between black hole and ejecta is at about $5M_\odot$, there will be no Fe-type elements in the ejecta, as observed in Nova Scorpii 1994. By contrast hypernova 1998bw shows a large amount of Ni, indicating that in this case the cut was at a lower included mass.

The massive star A in Nova Sco will have gone through a hypernova explosion when the F-star B was still on the main sequence, its radius about $1.5R_\odot$. Since the explosion caused an expansion of the orbit, the orbital separation $a$ was smaller at the time of the supernova than it is now, roughly by a factor

$$a_{\text{then}} = a_{\text{now}}/(1 + \Delta M/M_{\text{now}}).$$

(32)

($\Delta M$ is the mass lost in the explosion; see, e.g., Verbunt, Wijers, and Burm 1990). With $\Delta M \sim 0.8M_{\text{now}}$, as required by the high space velocity, this means $a_{\text{then}} = 10R_\odot$. Therefore the fraction of solid angle subtended by the companion at the time of explosion was

$$\frac{\Omega}{4\pi} = \frac{\pi(1.5R_\odot)^2}{4\pi(10R_\odot)^2} \approx 6 \times 10^{-3}.$$  

(33)

Assuming the ejecta of the hypernova to have been at least $5M_\odot$ (Nelemans et al. 1999),
the amount deposited on star B was

\[ M_D \gtrsim 0.03M_\odot. \]  

(34)

The solar abundance of oxygen is about 0.01 by mass, so with the abundance in the F star being 10 times solar, and oxygen uniformly mixed, we expect \( 0.1 \times 2.5 = 0.25M_\odot \) of oxygen to have been deposited on the companion, much more than the total mass it could have captured from a spherically symmetry supernova. \([\text{Si/O}]\) is 0.09 by mass in the Sun, and \([\text{S/O}]\) is 0.05, so since the over-abundances of all three elements are similar we expect those ratios to hold here, giving about \( 0.02M_\odot \) of captured Si and \( 0.01M_\odot \) of captured S. We therefore need a layer of stellar ejecta to have been captured which has twice as much Si as S, at the same time as having about 10 times more O. From fig. 4, we see that this occurs nowhere in a normal supernova, but does happen in the hypernova model of Nomoto et al. (2000) at mass cuts of \( 6M_\odot \) or more. This agrees very nicely with the notion that a hypernova took place in this system, and that the inner \( 7M_\odot \) or so went into a black hole.

What remains is to explain how the companion acquired ten times more mass than the spherical supernova model allows, and once again we believe that the answer is given in recent hypernova calculations (MacFadyen and Woosley 1999, Wheeler et al. 2000): hypernovae are powered by jet flows, which means they are very asymmetric, with mass outflow along the poles being much faster and more energetic than along the equator. The disk provides a source for easily captured material in two ways: First, it concentrates mass in the equatorial plane, which will later be ejected mostly in that plane. Second, the velocity acquired by the ejecta is of the order of the propagation speed of the shock through it. This propagation speed is proportional to \( \sqrt{P_2/\rho_1} \), where \( P_2 \) is the pressure behind the shock and \( \rho_1 \) the density ahead of it. The driving pressure will be similar in all directions (or larger, due to the jet injection, in the polar regions), whereas the disk density is much higher than the polar density. Hence, the equatorial ejecta will be considerably slower than even normal supernova ejecta, greatly increasing the possibility of their capture by the companion. Other significant effects of the disk/jet geometry are (1) that the companion is shielded from ablation of its outer layers by fast ejecta, which is thought to occur in spherical supernovae with companion stars (Marietta, Burrows & Fryxell 2000) and (2) that there is no iron enrichment of the companion, because the iron —originating closest to the center— is either all captured by the black hole or ejected mainly in the jet, thus not getting near the companion (Wheeler et al. 2000; note that indeed no overabundance of Fe is seen in the companion of GRO J1655−40).

For the companion to capture the required \( 0.2–0.3M_\odot \) of ejecta it is sufficient that the ejecta be slow enough to become gravitationally bound to it. However, the material may not stay on: when the companion has so much mass added on a dynamical time scale it
will be pushed out of thermal equilibrium, and respond by expanding, as do main-sequence stars that accrete mass more gradually on a time scale faster than their thermal time scale (e.g., Kippenhahn & Meyer-Hofmeister 1977). During this expansion, which happens on a time scale much longer than the explosion, the star may expand beyond its Roche lobe and transfer some of its mass to the newly formed black hole. However, because the dense ejecta mix into the envelope on a time scale between dynamical and thermal, i.e., faster than the expansion time, this back transfer will not result in the bulk of the ejecta being fed back, though probably the material lost is still richer in heavy elements than the companion is now. Since the outer layers of the star are not very dense, and the mass transfer is not unstable because the black hole is much more massive than the companion, the total amount of mass transferred back is probably not dramatic. However, the expansion does imply that the pre-explosion mass of the companion was somewhat higher than its present mass, and that the amount of ejecta that needs to be captured in order to explain the abundances observed today is also somewhat higher than the present mass of heavy elements in the companion.

A further piece of evidence that may link Nova Sco 1994 to our GRB/hypernova scenario are the indications that the black hole in this binary is spinning rapidly. Zhang, Cui, & Chen (1997) argue from the strength of the ultra-soft X-ray component that the black hole is spinning near the maximum rate for a Kerr black hole. However, studies by Sobczak et al. (1999) show that it must be spinning with less than 70% maximum. Gruzinov (1999) finds the inferred black hole spin to be about 60% of maximal from the 300 Hz QPO. Our estimates of the last section indicate that enough rotational energy will be left in the black hole so that it will still be rapidly spinning.

We have already mentioned the unusually high space velocity of $-150 \pm 19$ km s$^{-1}$. Its origin was first discussed by Brandt et al. (1995), who concluded that significant mass must have been lost in the formation of the black hole in order to explain this high space velocity: it is not likely to acquire a substantial velocity in its own original frame of reference, partly because of the large mass of the black hole. But the mass lost in the supernova explosion is ejected from a moving object and thus carries net momentum. Therefore, momentum conservation demands that the center of mass of the binary acquire a velocity; this is the Blaauw–Boersma kick (Blaauw 1961, Boersma 1961). Note that the F-star companion mass is the largest among the black-hole transient sources, so the center of mass is furthest from the black hole and one would expect the greatest kick. Nelemans et al. (1999) estimate the mass loss in this kick to be $5 - 10 M_\odot$.

In view of the above, we consider it well established that Nova Sco 1994 is the relic of a hypernova. We believe it highly likely that the other black-hole transient X-ray
sources are also hypernova remnants. We believe it likely that the hypernova explosion was accompanied by a GRB if, as in GRB980326, the energy was delivered in a few seconds. It is not clear what will happen if the magnetic fields are so low that the power is delivered only over a much longer time. There could then still be intense power input for a few seconds due to neutrino annihilation deposition near the black hole (Janka et al. 1999), but that may not be enough for the jet to pierce through the He star and cause a proper GRB (MacFadyen and Woosley 1999). At this point, we recall that the GRB associated with SN1998bw was very sub-luminous, $10^5$ times lower than most other GRB. While it has been suggested that this is due to us seeing the jet sideways, it is in our view more likely that the event was more or less spherical (Kulkarni et al. 1998) and we see a truly lower-power event. A good candidate would be the original suggestion by Colgate (1968, 1974) of supernova shock break-out producing some gamma rays. Indications are that the expansion in SN1998bw was mildly relativistic (Kulkarni et al. 1998) or just sub-relativistic (Waxman and Loeb 1999). In either case, what we may have witnessed is a natural intermediate event in our scenario: we posit that there is a continuum of events varying from normal supernovae, delivering 1 foe more or less spherically in ten seconds, to extreme hypernovae/GRB that deliver 100 foes in a highly directed beam. In the middle, there will be cases where the beam cannot pierce through the star, but the total energy delivered is well above a supernova, with as net result a hypernova accompanied by a very weak GRB.

7.5. Numbers

Nearly all observed black hole transient X-ray sources are within 5 kpc of the Sun. Extrapolating to the entire Galaxy, a total of 8,800 black-hole transients with main-sequence K companions has been suggested (Brown, Lee, & Bethe 1999).

The lifetime of a K star in a black hole transient X-ray source is estimated to be $\sim 10^{10}$ yr (Van Paradijs 1996) but we shall employ $10^9$ yr for the average of the K-stars and the more massive stars, chiefly those in the “silent partners”. In this case the birth rate of the observed transient sources would be

$$\lambda_K = \frac{10^4}{10^9} = 10^{-5} \text{per galaxy yr}^{-1}. \quad (35)$$

We see no reason why low-mass companions should be preferred, so we assume that the formation rate of binaries should be independent of the ratio

$$q = \frac{M_{B,i}}{M_{A,i}}. \quad (36)$$

In other discussions of binaries, e.g., in Portegies Zwart & Yungelson (1998), it has often been assumed that the distribution is uniform in $q$. This is plausible but there is no proof.
Since all primary masses $M_A$ are in a narrow interval, 20 to $35M_\odot$, this means that $M_B$ is uniformly distributed between zero and some average $M_A$, let us say $25M_\odot$. Then the total rate of creation of binaries of our type is

$$\lambda = \frac{25}{0.7}\lambda_K = 3 \times 10^{-4}\text{ galax}^{-1}\text{ yr}^{-1}. \quad (37)$$

This is close to the rate of mergers of low mass black holes with neutron stars which Bethe & Brown (1998) have estimated to be

$$\lambda_m \simeq 2 \times 10^{-4}\text{ galax}^{-1}\text{ yr}^{-1}. \quad (38)$$

These mergers have been associated speculatively with short GRBs, while formation of our binaries is supposed to lead to "long" GRBs (Fryer, Woosley, & Hartmann 1999). We conclude that the two types of GRB should be equally frequent, which is not inconsistent with observations. In absolute number both of our estimates eqs. (37) and (38) are substantially larger than the observed rate of $10^{-7}$ galax$^{-1}$ yr$^{-1}$ (Wijers et al. 1998); this is natural, since substantial beaming is expected in GRBs produced by the Blandford-Znajek mechanism. Although we feel our mechanism to be fairly general, it may be that the magnetic field required to deliver the BZ energy within a suitable time occurs in only a fraction of the He cores.

8. Discussion and Conclusion

Our work here has been based on the Blandford-Znajek mechanism of extracting rotational energies of black holes spun up by accreting matter from a helium star. We present it using the simple circuitry of “The Membrane Paradigm” (Thorne et al. 1986). Energy delivered into the loading region up the rotational axis of the black hole is used to power a GRB. The energy delivered into the accretion disk powers a SN Ib explosion.

We also discussed black-hole transient sources, high-mass black holes with low-mass companions, as possible relics for both GRBs and Type Ib supernova explosions, since there are indications that they underwent mass loss in a supernova explosion. In Nova Sco 1994 there is evidence from the atmosphere of the companion star that a very powerful supernova explosion (‘hypernova’) occurred.

We estimate the progenitors of transient sources to be formed at a rate of 300 GEM (Galactic Events per Megayear). Since this is much greater than the observed rate of GRBs, there must be strong collimation and possible selection of high magnetic fields in order to explain the discrepancy.
We believe that there are strong reasons that a GRB must be associated with a black hole, at least those of duration several seconds or more discussed here. Firstly, neutrinos can deliver energy from a stellar collapse for at most a few seconds, and sufficient power for at most a second or two. Our quantitative estimates show that the rotating black hole can easily supply the energy as it is braked, provided the ambient magnetic field is sufficiently strong. The black hole also solves the baryon pollution problem: we need the ejecta that give rise to the GRB to be accelerated to a Lorentz factor of 100 or more, whereas the natural scale for any particle near a black hole is less than its mass. Consequently, we have a distillation problem of taking all the energy released and putting it into a small fraction of the total mass. The use of a Poynting flux from a black hole in a magnetic field (Blandford & Znajek 1977) does not require the presence of much mass, and uses the rotation energy of the black hole, so it provides naturally clean power.

Of course, nature is extremely inventive, and we do not claim that all GRBs will fit into the framework outlined here. We would not expect to see all of the highly beamed jets following from the BZ mechanism head on, the jets may encounter some remaining hydrogen envelope in some cases, jets from lower magnetic fields than we have considered here may be much weaker and delivered over longer times, etc., so we speculate that a continuum of phenomena may exist between normal supernovae and extreme hypernovae/GRBs. This is why we call our effort “A Theory of Gamma Ray Bursts” and hope that it will be a preliminary attempt towards systematizing the main features of the energetic bursts.

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**A. Estimates of** $\epsilon_\Omega = \Omega_{\text{disk}}/\Omega_H$

We collect here useful formulas needed to calculate $\epsilon_\Omega = \Omega_{\text{disk}}/\Omega_H$. First of all

\[
\Omega_H = \frac{\bar{a}}{1 + \sqrt{1 - \bar{a}^2}} \left( \frac{c^3}{2MG} \right) = \frac{\sqrt{2} \bar{a}}{1 + \sqrt{1 - \bar{a}^2}} \left( \frac{R}{R_{\text{Sch}}} \right)^{3/2} \Omega_K
\]

(A1)
\[ \Omega_{\text{disk}} = \omega \Omega_K \left[ 1 + \tilde{\alpha} \frac{GM}{c^2 \sqrt{c^2 R^3}} \right]^{-1} \]

\[ = \omega \Omega_K \left[ 1 + \tilde{\alpha} \left( \frac{R_{\text{Sch}}}{2R} \right)^{3/2} \right]^{-1} \]  

(A2)

where \( \Omega_K \equiv \sqrt{GM/R^3} \) and \( \omega \) is dimensionless parameter \((0 < \omega < 1)\). Thus

\[ \frac{\Omega_{\text{disk}}}{\Omega_H} = \omega \frac{1 + \sqrt{1 - \tilde{\alpha}^2}}{\sqrt{2} \tilde{\alpha}} \left( \frac{R_{\text{Sch}}}{R} \right)^{3/2} \left[ 1 + \tilde{\alpha} \left( \frac{R_{\text{Sch}}}{2R} \right)^{3/2} \right]^{-1}. \]  

(A3)

The numerical estimates are summarized in Table 2 for various \( \omega \) and radii.

**B. Spin-up of Black Holes by Accretion**

The specific angular momentum and energy of test particles in Keplerian circular motion, with rest mass \( \delta m \), are

\[ \tilde{E} \equiv \frac{E}{\delta m} = c^2 \left[ \frac{r^2 - R_{\text{Sch}}r + a \sqrt{R_{\text{Sch}}r/2}}{r(r^2 - \frac{3}{2} R_{\text{Sch}}r + a \sqrt{2 R_{\text{Sch}}r})^{1/2}} \right], \]

\[ \tilde{l} \equiv \frac{l}{\delta m} = c \sqrt{\frac{R_{\text{Sch}}r}{2}} \left[ \frac{(r^2 - a \sqrt{2 R_{\text{Sch}}r} + a^2)}{r(r^2 - \frac{3}{2} R_{\text{Sch}}r + a \sqrt{2 R_{\text{Sch}}r})^{1/2}} \right]. \]  

(B1)

where \( R_{\text{Sch}} = 2GM/c^2 \) and BH spin \( a = J/Mc = \tilde{\alpha}(GM/c^2) \). The accretion of \( \delta m \) changes the BH’s total mass and angular momentum by \( \Delta M = \tilde{E} \delta m \) and \( \Delta J = \tilde{l} \delta m \). The radii of marginally bound \( (r_{mb}) \) and stable \( (r_{ms}) \) orbits are given as

\[ r_{mb} = R_{\text{Sch}} - a + \sqrt{R_{\text{Sch}}(R_{\text{Sch}} - 2a)} \]

\[ r_{ms} = \frac{R_{\text{Sch}}}{2} \left( 3 + Z_2 - [(3 - Z_1)(3 + Z_1 + 2 Z_2)]^{1/2} \right) \]

\[ Z_1 = 1 + \left( 1 - \frac{4a^2}{R_{\text{Sch}}^2} \right)^{1/3} \left[ \left( 1 + \frac{2a}{R_{\text{Sch}}} \right)^{1/3} + \left( 1 - \frac{2a}{R_{\text{Sch}}} \right)^{1/3} \right] \]

\[ Z_2 = \left( \frac{4a^2}{R_{\text{Sch}}^2} + Z_1^2 \right)^{1/2}. \]  

(B2)

The numerical values of the specific angular momentum and energy of test particles are summarized in Table 3 and Fig.5. In Fig.6, we test how much mass we need in order to spin up the non-rotating black hole up to given \( \tilde{\alpha} \). Note that the last stable orbit is
almost Keplerian even with the accretion disk, and we assume 100% efficiency of angular momentum transfer from the last stable Keplerian orbit to BH. In order to spin-up the BH up to $\tilde{a} = 0.9$, we need $\sim 68\%$ ($52\%$) of original non-rotating BH mass in case of $r_{ls} = r_{ms} (r_{mb})$. For a very rapidly rotating BH with $\tilde{a} = 0.99$, we need 122% and 82%, respectively. For $r_{ls} = r_{ms}$, there is an upper limit, $\tilde{a} = 0.998$, which can be obtained by accretion (Thorne 1974). In the limit where $r_{ls} = r_{mb}$, however, spin-up beyond this limit is possible because the photons can be captured inside thick accretion disk, finally into BH (Abramowicz et al. 1988).

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| Spectral Type       | $M_B [M_{\odot}]$ | $a [R_{\odot}]$ | $R_L [R_{\odot}]$ | $j_{B,\text{orb}} [10^{19} \text{ cm}^2 \text{ s}^{-1}]$ |
|---------------------|-------------------|-----------------|-------------------|-------------------------------------|
| 5 K-type            | 0.4 − 0.9         | 4.5 ± 0.8       | 0.9 ± 0.2         | 1.5 ± 0.6                           |
| 1 K-type            | 0.8               | 34              | 6.1               | 5.7                                 |
| 1 M-type            | 0.5               | 3.1             | 0.6               | 1.5                                 |
| F (Nova Scorpii)    | 2.2               | 16              | 4.7               | 1.9                                 |
| A2 (1543-47)        | 2.0               | 9.0             | 2.6               | 1.4                                 |

Table 1: Properties of transient X-ray sources

| $\omega$ | $\tilde{a}$ | $r = r_{mb} (\tilde{a})$ | $r = r_{ms} (\tilde{a})$ | $r = 2 R_{Sch}$ | $r = 3 R_{Sch}$ | $\Omega_{disk}/\Omega_H$ |
|-----------|-------------|--------------------------|--------------------------|-----------------|-----------------|--------------------------|
| 1.0       | 0.80        | 1.00                     | 0.69                     | 0.45            | 0.26            |                          |
| 0.9       | 0.72        | 0.99                     | 0.63                     | 0.49            | 0.27            |                          |
| 0.8       | 0.64        | 0.93                     | 0.58                     | 0.51            | 0.29            |                          |
| 0.7       | 0.56        | 0.89                     | 0.54                     | 0.53            | 0.30            |                          |
| 0.6       | 0.48        | 0.84                     | 0.50                     | 0.55            | 0.31            |                          |
| 0.5       | 0.40        | 0.80                     | 0.46                     | 0.57            | 0.32            |                          |

Table 2: Estimates of $\epsilon_\Omega = \Omega_{disk}/\Omega_H$ as a function of spin parameter and radius, where $r_{mb}$ is the marginally bound radius and $r = r_{ms}$ the marginally stable radius.
Table 3: Properties of Schwarzschild & Kerr BH. a) $r_{ls} = r_{ms}$ case: 6% (42%) of energy can be released during the spiral-in for Schwarzschild (maximally-rotating Kerr) BHs. b) $r_{ls} = r_{mb}$ case: The released energy during the spiral-in is almost zero.

|        | $a[GM/c^2]$ | $l[GM/c]$ | $r[R_{\text{Sch}}]$ | $\dot{e}[c^2]$ |
|--------|-------------|------------|----------------------|-----------------|
| $r_{ms}$ | 0           | $2\sqrt{3} \approx 3.46$ | 3                  | $\sqrt{8/9} \approx 0.943$ |
|         | 1           | $2/\sqrt{3} \approx 1.15$ | 1                  | $\sqrt{1/3} \approx 0.577$ |
| $r_{mb}$ | 0           | 4          | 2                    | 1               |
|         | 1           | 2          | 1                    | 1               |

Rotating Black Hole

Fig. 1.— The black hole in rotation about the accretion disk. A circuit, in rigid rotation with the black hole is shown. This circuit cuts the field lines from the disk as the black hole rotates, and by Faraday’s law, produces an electromotive force. This force drives a current. More detailed discussion is given in the text.

Fig. 2.— Magnetic field lines, anchored in the disk, which thread the black hole, coupling the disk rotation to that of the black hole.
Fig. 3.— Time evolution of BH mass and angular momentum taken from Fig. 19 of MacFadyen & Woosley 1999. The upper panel shows the increase in the Kerr parameter for various models for the disk interior to the inner boundary at 50 km. “Thin” (dash-dot), neutrino-dominated (thick solid) and advection dominated (short dash) models are shown for initial Kerr parameter $\tilde{a}_{\text{init}} = 0.5$. The lower panel shows the growth of the gravitational mass of the black hole. The short-dashed line shows the growth in baryonic mass of the black hole since for a pure advective model no energy escapes the inner disk.
Fig. 4.— The isotopic composition of ejecta of the hypernova ($E_K = 3 \times 10^{52}$ erg; left) and the normal supernova ($E_K = 1 \times 10^{51}$ erg; right) for a $16M_\odot$ He star, from Nomoto et al. (1999). Note the much higher sulphur abundance in the hypernova.
Fig. 5.— Specific angular momentum and energy of test particle in units of $[GM/c]$ and $[c^2]$. BH spin $a$ is given in unit of $[GM/c^2]$. For the limiting values at $a = 0$ and $GM/c^2$, refer Table 3.
Fig. 6.— Spinning up of Black Holes. The BH spin $a$ is given in units of $[GM/c^2]$ and $\delta m$ is the total rest mass of the accreted material.