Identifying Hub Wastewater Propagation Chains in China’s National Economic System: A ModelCoupled Input-Output Analysis with Graphical Theory

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Abstract: Wastewater propagation chains (WPCs) measure inter-sector average propagation lengths (APL) of wastewater discharge. To achieve sustainable wastewater management, one needs to understand the propagation mechanisms by identifying WPCs at a national level over time. However, the traditional model of identifying WPCs is prone to retaining APLs with lower values but larger wastewater discharge intensities, ignoring many linkages whereby intensities are less than a preset threshold. Nevertheless, these overlooked linkages are valuable in understanding wastewater propagation mechanisms. This study proposed a new model coupled input-output analysis with the graphical theory, called the average propagation lengths-hub covariance graph (APL-HCG). This model can investigate WPCs where the closeness of sector linkages exceeds the preset thresholds. Furthermore, it is capable of retaining linkages for identifying hub wastewater propagation chains (HWPCs). Based on APL-HCG, the resultant HWPCs are decomposed as separated sub-chains which are basically composed of linkages among certain significant sectors belonging to the secondary industry or the tertiary industry. Scenario analyses show that HWPCs are effective in reducing wastewater discharge in the national economic system. The total wastewater discharge would decrease by 1.36%, 2.53%, 2.46%, and 2.11% if we reduced 10% of the final demand of all sectors in HWPCs in 2002, 2007, 2012, and 2017. The APL-HCG model outperforms the traditional model on WPCs by 0.14%, 1.61%, 0.47%, and 0.10%, respectively. The APL-HCG model is 0.21%, 0.68%, 0.70%, and 0.35% better than the scenario of random sampling with the number of sectors equal to HWPCs, respectively. Certain policy implications were provided to reduce wastewater effectively at the national level.

Keywords: wastewater reduction; hub wastewater propagation chains; average propagation lengths-hub covariance graph (APL-HCG) model; input-occupancy-output analysis

1. Introduction

The environmental and social-economic sustainability of water use relies heavily on the appropriate treatment of wastewater. Despite the scale of urban wastewater treatment reaching 200 million cubic meters (m$^3$) per day by 2021 [1], the issue of wastewater reduction remains a big challenge to China. In 2017, China directly discharged 69.97 billion m$^3$ of wastewater in its intermediate production and final consumption [2]. Rapidly growing quantities of sewage sludge emerged as another challenge related to wastewater [3]. The increase in wastewater and sewage sludge discharge contaminates freshwater, increases the need for greywater to dilute polluted water, exacerbates water scarcity and ecosystem degradation [4] and, even more seriously, affects human health. In recent years, wastewater treatment has attracted much attention for sustainable water development. The Chinese
government has drawn up focused proposals to improve the quality and efficiency of wastewater treatment, such as the 'Three-year Action Plan' for specific secondary industries, tertiary industries, and water-related sectors [5] as well as the 'Guiding Opinions on Promoting the Utilization of Wastewater Resources' for sewage recycling [6].

It is necessary to analyze the mechanisms of wastewater propagation among sectors in China in order to explore the theoretical foundations for implementing the above policies and promoting sustainable wastewater management. Additionally, it will facilitate the understanding of the wastewater propagation mechanisms by identifying key wastewater propagation chains at a national scale. Since each economic sector generates direct wastewater discharge in the production process due to the final demand of the sector, the sector has indirect wastewater discharge from the upstream production process to the downstream production process up to consumers [7]. Thus, the wastewater propagation chain is comprised of linked relationships that causes fluctuation in wastewater discharges from directly related sectors when there is a change in the production processes or demand in a specific sector. The changes in wastewater discharges from these related sectors will further lead to changes in wastewater discharges from other sectors; it extends the production chains in input-output analysis, which characterizes the economic distances among sectors [8–11]. Moreover, the linkages in wastewater propagation chains measure the wastewater propagation distance from one sector to another through production connections.

The economic sectors, as the primary sources of wastewater discharge, bear the responsibility to reduce it. With the adjustment of the industrial structure, the connections between sectors are evolving [12]. Scientific measurement of wastewater discharge in each sector and rational analysis of sectoral wastewater discharge relationship can provide scientific theoretical basis and practical policy support for reducing wastewater discharge in China. In addition, such measurements have the potential to enable simulation of wastewater reduction scenarios at the national level. Furthermore, it is necessary to identify certain sectors in the chains, which are densely linked with other sectors in terms of wastewater discharge. Correctly identifying sectors around dense linkages would help retain more linkages and thus enhance the effectiveness of implementing wastewater reduction measures in recognized chains. Therefore, an in-depth study of wastewater propagation chains that retain more useful wastewater discharge linkages is necessary.

Researchers commonly apply input-output analysis to identify key sectors and paths in resource and environment studies. Wang et al. [13] developed a new hypothesis containing an extraction method to explore key sectors and paths in the Chinese energy sectors and then applied it to the same problem of SO2 emissions across economic sectors in Shandong Province [14]. Zhao et al. [15] used a modified hypothesis extraction method to analyze CO2 linkages in South Africa and China [16]. Guo et al. [17] applied an elasticity approach to define China’s key energy and CO2 emissions sectors. However, these key sector and path identification methods heavily rely on quantifying the impact of a specific sector on other sectors or the impact of other sectors on a particular sector. We refer to the effect of a sector affecting other sectors as external impacts and the impact of other sectors on a given sector as internal impact. They do not reveal the integrated effects of external and internal impacts in identifying key sectors and paths.

In economics, the average propagation lengths (APL), a method firstly proposed by Dietzenbacher et al. [10,11], is a popular method to quantify linkages among sectors [8,9,18–21]. An APL indicates how close two sectors are to each other. It is usually an integer. An APL value of 1 indicates that 2 sectors are closely linked. If there is a change in production or consumption in one sector, the other closely related sectors will be the first to see the most affects. In other words, APL is the average number of steps in which one sector directly or indirectly affects the production of another sector. For example, production in the agriculture sector may directly affect the output in the service sector. It could also affect the production of poultry and livestock sector first and then affect the service sector’s output through poultry and livestock production. Then APL is the average lengths of all
these possible affecting paths. It was later extended to environmental issues, especially the analysis of waste propagation chains, since waste, such as wastewater and sludge, is also generated in the production process, with one sector affecting the output of another sector. Tu [7] used the traditional APL method to analyze wastewater propagation chains in the Beijing-Tianjin-Hebei region. However, this classic model has a drawback in that it tends to select links with lower APL values but larger wastewater discharge intensities. Moreover, many links with intensities smaller than a preset threshold are thus overlooked.

The graphical theory is often used to model the dependence among random variables in statistics. For example, covariance models the correlations among random variables. The sectors in the input-output table can be seen as random variables in a covariance graph. The relationships of external and internal impacts correspond to out-edges and in-edges, respectively. Tan et al. [22] proposed the hub penalty function into the problem of covariance matrix estimation. Moreover, the estimated covariance matrix structure specifies the correlation patterns, as previously Chaudhuri et al. [23] and Xue et al. [24] developed new methods to estimate the covariance matrix with zeros and the sparse large covariance matrix, respectively.

This research aimed to find a method to identify key wastewater propagation chains at a national scale in order to provide a scientific theoretical basis and practical policy support for effectively reducing wastewater discharge. Inspired by graphical theory, this study utilized the idea of a covariance graph model to overcome the shortcomings of the traditional Tu’s model [7] and identify more effective intersectoral wastewater propagation chains. The APL-HCG model can address the shortcoming of the traditional Tu’s model [7] shortcoming that ignores many linkages when identifying wastewater propagation chains. It captures linkage correlation by considering the closeness of sectoral linkages, not only external impact but also internal impact relationships. Hub wastewater propagation chains are identified by retaining linkages whose correlation values in the estimated APL covariance matrix are greater than a preset threshold. The case studies show that the identified hub wastewater propagation chains have better wastewater reduction effects. The contributions of our works are as follows: (1) we proposed a new coupled model to identify wastewater propagation chains; (2) we identified hub wastewater propagation chains that are more effective in reducing wastewater in China in 2002, 2007, 2012, and 2017.

The following contents of this paper are presented as follows. The second section introduces the data and the average propagation lengths coupled hub covariance graph (APL-HCG) model used to identify hub wastewater discharge propagation chains. The third section gives the results of the hub wastewater propagation chains obtained by applying APL-HCG to the national input-occupancy-output (IOO) tables and compares them with the results of the traditional Tu’s model [7]. The third section also presents simulations of wastewater reduction scenario on the hub wastewater propagation chains. The fourth section discusses the limitations of the APL-HCG model, its extension to other pollutant indicators, and the support of current research for the APL-HCG results. The final section summarizes the advantages of the proposed model, the results compared with the traditional model and provides some policy implications.

2. Data and Methodology

2.1. Data

Zheng [25] compiled and released input-occupancy-output (IOO) tables for 49 sectors in China in 2002, 2007, 2012, and 2017. Zheng [25] combined some sectors of China’s input-output tables published by the National Bureau of Statistics of China for 123 sectors in 2002, 135 sectors in 2007, 139 sectors in 2012, and 149 sectors in 2017, while other sectors were split. The electricity and heat production and supply sector and water production and supply sector were split into four sectors: hydropower, water transportation, water supply, and sewage treatment. See Appendix A for the structure description of the tables and Appendix B for sector name and sector codes of the 49 sectors. The National Bureau of Statistics in China issued the total wastewater discharge volume and the wastewater
discharge volume of some secondary industries. They were split into wastewater occupancy of each sector by Zheng [25]. Except for the catering sector, wastewater occupancy of each sector under the input matrix excludes municipal wastewater (domestic sewage). The adjustment of livestock water from domestic to agricultural water in 2012 does not affect the following calculation because the wastewater occupancy of the five agricultural sectors in the sequential IOO tables is set to zero due to the lack of wastewater discharge data from the agricultural sector.

2.2. Methodology

Given the insights from traditional model for identifying wastewater propagation chains, we intend to introduce an APL-based covariance graph model with hubs (HCG) [22] to retain more key linkages. In the covariance graph model setting, the covariance between sector i and sector j is essentially the covariance between the APL vector from sector i to all sectors and the APL vector from sector j to all sectors. If sector i and sector j are dependent in the sense of APL, the covariance between them is not zero. We refer to this as APL-HCG model. It attempts to couple HCG with the average propagation lengths (APL). Since we can use the input-output table from a column perspective or a row perspective, we can calculate the wastewater discharge APL by following the backward APL formula (4.5) and forward APL formula (4.10) in the paper of Tu [7]. They are called the Leontief model and the Ghosh model, respectively. The APL of the Leontief model is the backward APL or Leontief APL, which is given by

\[ H = \hat{a}_w (I - A)^{-1} [(I - A)^{-1} - I] \]  

\[ Q = \hat{a}_w [(I - A)^{-1} - I] \]  

where I denotes the identity matrix, A = Z\hat{x}^{-1} is the Leontief technical coefficient matrix, Z is the intermediate input matrix (see Appendix A), \( \hat{x} \) is the diagonal matrix of total input vector X (see Appendix A). \( \hat{a}_w = W D_1 \hat{x}^{-1} \) is the diagonal matrix of direct wastewater discharge coefficients, and WD1 is the wastewater discharge occupancy vector (see Appendix A). \( V = (v_{ij})_{i,j=1}^n \) denotes the Leontief type of APL matrix or backward APL matrix, then

\[ v_{ij} = \begin{cases} \frac{h_{ij}}{\hat{q}_{ij}}, & \text{if } \hat{q}_{ij} > 0 \\ 0, & \text{if } \hat{q}_{ij} = 0 \end{cases} \]  

Let B = \( \hat{x}^{-1}Z \) be the Ghosh technical coefficient matrix, and

\[ \hat{H} = (I - B)^{-1} [(I - B)^{-1} - I] \hat{a}_w \]  

\[ \hat{Q} = [(I - B)^{-1} - I] \hat{a}_w \]  

Correspondingly, \( \hat{V} = (\hat{v}_{ij})_{i,j=1}^n \) denote the Ghosh type of APL matrix or forward APL matrix, then

\[ \hat{v}_{ij} = \begin{cases} \frac{\hat{h}_{ij}}{\hat{q}_{ij}}, & \text{if } \hat{q}_{ij} > 0 \\ 0, & \text{if } \hat{q}_{ij} = 0 \end{cases} \]  

Each element \( \hat{v}_{ij} \) of the forward APL matrix \( \hat{V} \) gives the average number of steps in which sector i’s production affects sector j’s wastewater discharge. The \( v_{ij} \) of backward APL matrix V provides the average number of steps in which sector i is involved sector j’s wastewater discharge due to sector j’s demand for sector i’s products. Theoretically, \( \hat{V} = V \).

In this paper, the wastewater propagation chains are constructed by the forward APL (V).
Here we use the estimated covariance of APL to indicate the sector association. The covariance of APL indicates whether the change in direction of the closeness degree of the sector association of wastewater discharge between the two sectors is consistent and also indicates the degree of correlation between the sector linkages of wastewater discharge; the higher the degree of correlation, the more important. APL-HCG retains the linkages whereby the closeness of the sector association of wastewater discharge is more significant than the preset threshold. Formulaically, we estimated a sparse covariance matrix of APLs through a specific hub penalty function as in Tan et al. [22], which is given by

\[
P(C) = \min_{W,Z : C = Z + W + W^T} \left\{ \lambda_1 \|Z - \text{diag}(Z)\|_1 + \lambda_2 \|W - \text{diag}(W)\|_1 + \lambda_3 \sum_{j=1}^{p} \|(W - \text{diag}(W))_j\|_2 \right\}
\]  

(7)

where \(C\) is the covariance matrix of the APL to be estimated, and the symmetric matrix \(Z\) is mainly used to introduce the sparsity into the covariance matrix and is to be estimated. Each non-zero element in \(Z\) implies that the closeness of the links between two sectors is significant. The non-zero columns of \(W\) represent sectors that densely surround many linkages and have a considerable degree of linkage closeness to their linked sectors. Hence, Formula (7) has the advantage of encouraging the estimation of sparse links, which contains some sectors that densely surround many linkages.

Specifically, let \(C = (c_{ij})_{ij=1}^{n}\), \(c_{ij}\) is the estimated covariance of the \(i\)'th column of forward APL (\(\tilde{V}\)) and the \(j\)'th column of forward APL (\(\tilde{V}\)). Moreover, \(S = (s_{ij})_{ij=1}^{n}\), where \(s_{ij} = \mathbb{E}\left[ (\tilde{v}_i - \bar{v}_i)(\tilde{v}_j - \bar{v}_j)^T \right]\), which gives the empirical covariance of the \(i\)'th column of forward APL (\(\tilde{V}\)) and the \(j\)'th column of forward APL (\(\tilde{V}\)). We calculate the empirical covariance matrix based on input-output tables. \(\mathbb{E}\) is the mathematical expectation operation, and \(\bar{v}_i\) and \(\bar{v}_j\) are the mean of the average number of steps in which sector \(i\) affects all sectors and sector \(j\) affects all sectors, respectively. The larger the value of \(s_{ij}\), the more significant the sector association of wastewater discharge between sector \(i\) and sector \(j\). Suppose \(s_{ij}\) is negative, the sector association of wastewater discharge between the two sectors changes in the opposite direction. Otherwise, the direction of change is the same. We wish the estimated covariance matrix \(C\) to be as close as possible to the empirical covariance matrix \(S\). The re-estimated covariances retained after the sparsity of \(C\) indicates which sector links of wastewater discharge are significant. Therefore, we choose the square loss function \(\ell(C, S) = \|C - S\|_F^2 / 2\). Finally, the general convex optimization problem of the APL hub covariance graphical (APL-HCG) model is given as follows,

\[
\min_{C, W, Z} \left\{ \frac{1}{2} \|C - S\|_F^2 + \lambda_1 \|Z - \text{diag}(Z)\|_1 + \lambda_2 \|W - \text{diag}(W)\|_1 + \lambda_3 \sum_{j=1}^{p} \|(W - \text{diag}(W))_j\|_2 \right\}
\]

subject to \(C = Z + W + W^T\)  

(8)

The objective function in Formula (8) is a joint optimization framework of loss of function plus penalty terms, which is widely used in the machine learning community. Formula (8) shows that a certain proportion of elements in \(C\) should be zero, provided \(C\) and \(S\) are as close as possible. That is, we keep the linkages with the highest degree of sector correlations. The indexes of all of the elements in the estimated covariance matrix \(C\) greater than a preset threshold indicate the APLs that should be kept. We construct the adjusted APL matrix \(\hat{M}\) as follows,

\[
\hat{m}_{ij} = \begin{cases} 
\text{int}(\tilde{v}_{ij}), & \text{if } c_{ij} \geq a \\
0, & \text{if } c_{ij} < a
\end{cases}
\]

(9)

The adjusted APLs in the matrix \(\hat{M}\) construct the hub wastewater propagation chains (HWPCs). Here, we refer to the propagation chains calculated by Tu [20], using the traditional model as wastewater propagation chains (WPCs). The estimated covariance...
matrix C can be considered a wastewater discharge intensity matrix that integrates the possible correlations of all of the sector links. It is used to select the APLs that should eventually be retained. The convex optimization problem (2) can be solved by the ADMM algorithm described by Tan et al. [22].

3. Results
3.1. Hub Wastewater Propagation Chains by APL-HCG

Figures 1–4 show HWPCs constructed by the matrix M in 2002, 2007, 2012, and 2017. The linkages among sectors are non-zero APLs identified by APL-HCG. Each pair of the indexes in the matrix M where the adjusted APL is not zero implies a directed edge. Therefore, every edge in Figures 1–4 has a retained non-zero APL. Different edge colors correspond to different APL values. A directed edge on Figures 1–4 reflects that a wastewater linkage exists between the two sectors represented by endpoints of the edge. For example, in Figure 1, the APL from sector 45 to sector 28 is 1, which indicates that sector 45 directly affects the wastewater discharge of sector 28. The APLs between sector 36 and sector 44 are 3, which means that sector 36 indirectly affects the wastewater discharge of sector 44 through the other two intermediate sectors and vice versa. In Appendix B, we visualized the WPCs of the traditional model. The information of its edges is the same as HWPCs. To facilitate a comparison between HWPCs and WPCs, we set thresholds to make the number of sectors contained in HWPCs and WPCs in the same year equal. There are 14, 14, 15 and 15 retained sectors in 2002, 2007, 2012, and 2017. The selected linkages of HWPCs are highly different from that of WPCs.

Figure 1. The hub wastewater propagation chains that APL-HCG selected in 2002. The orange, dark blue and green edges mean that their APL value is 1, 2 and 3, respectively. Each vertex in figures represents a sector, e.g., S47 in Figure 1 corresponds to Sewage Treatment explained in Appendix B. Dark red nodes indicate sectors that densely surround many linkages with a considerable degree of closeness to their linked sectors. The blue nodes imply that there are no surrounding dense linkages in the results.
Figure 2. The hub wastewater propagation chains that APL-HCG selected in 2007. The orange and dark blue edges mean that their APL value is 1 and 2, respectively. Each vertex in figures represents a sector explained in Appendix B. Dark red nodes indicate sectors that densely surround many linkages with a considerable degree of closeness to their linked sectors. The blue nodes imply that there are no surrounding dense linkages in the results.

Figure 3. The hub wastewater propagation chains that APL-HCG selected in 2012. The orange, dark blue and green edges mean that their APL value is 1, 2 and 3, respectively. Each vertex in figures represents a sector explained in Appendix B. Dark red nodes indicate sectors that densely surround many linkages with a considerable degree of closeness to their linked sectors. The blue nodes imply that there are no surrounding dense linkages in the results.

Since the re-estimated covariances retained after sparsity in C indicate which sectoral linkages of wastewater discharge are significant, we can find sector groups with close sectoral linkages within each group. We can also compare the estimated covariance matrices based on input-output tables from different years to reflect the dynamic process of evolving trends in sectoral linkages. Appendix C shows that the two common big sector groups with large values in C are separated, representing the significant closeness of sectoral linkages in 2002, 2007, 2012, and 2017. The first group consists of sectors 19–25, all of which are secondary industries. The second group contains 19 sectors, with sectors 31–49 belonging to the tertiary industry. However, from 2002 to 2017, the estimated closeness of sectoral wastewater discharge linkages in the first group decreased, while that of the second increased. By 2017, sectors 20-24 formed the first group. We can use this information as a specific theoretical reference for the implementation of wastewater reduction policies.
in the secondary, tertiary, and water-related sectors, such as those mentioned in the 'Three-year Action Plan for Quality and Efficiency Improvement of Urban Sewage Treatment (2019–2021)' [5].

Figure 4. The hub wastewater propagation chains that APL-HCG selected in 2017. The orange, dark blue and green edges mean that their APL value is 1, 2 and 3, respectively. Each vertex on figures stands for a sector explained in Appendix B. Dark red nodes indicate sectors that densely surround many linkages with a considerable degree of closeness to their linked sectors. The blue nodes imply that there are no surrounding dense linkages in the results.

3.2. Simulating the Wastewater Reduction Effects of HWPCs

To evaluate the effects of wastewater reduction in the HWPCs, we assume that all sectors in the HWPCs simultaneously reduce their final demand by 10%. Then, we calculate how much wastewater is reduced in the national economic system (Decrease Volume) and its percentage to the total wastewater discharge (Decreased Rate). Table 1 shows the wastewater reduction quantities and percentage of wastewater reduction for a 10% final demand reduction strength. The same scenario applies to the WPCs that calculated by the traditional model (Table 2). The comparison of Tables 1 and 2 shows that the wastewater reduction performance of reducing 10% final demand in all sectors of HWPCs is superior to that of the same scenario implementing to all sectors of WPCs by 0.14%, 1.61%, 0.47%, and 0.10% in 2002, 2007, 2012, and 2017.

Table 1. Wastewater reduction effects to the entire economic system by reducing 10% final demand on sectors in HWPCs.

| Year | Decrease Volume (Unit: $10^8$ m$^3$) | Decrease Rate (%) |
|------|-------------------------------------|-------------------|
| 2002 | 5.95                                | 1.36              |
| 2007 | 14.07                               | 2.53              |
| 2012 | 16.81                               | 2.46              |
| 2017 | 14.73                               | 2.11              |
Table 2. Wastewater reduction effects to the entire economic system by reducing 10% final demand on sectors in WPCs.

| Year | Decrease Volume (Unit: 10^8 m$^3$) | Decrease Rate (%) |
|------|----------------------------------|------------------|
| 2002 | 5.35                             | 1.22             |
| 2007 | 5.09                             | 0.92             |
| 2012 | 13.64                            | 1.99             |
| 2017 | 14.04                            | 2.01             |

Then, we randomly sample the same number of sectors as HWPCs to simulate 10% final demand reduction effects. The results are listed in Table 3. Compared with results in Table 1, in 2002, 2007, 2012, and 2017, it outperforms the randomly sampled sectors by 0.21%, 0.68%, 0.70%, and 0.35%, respectively. It indicates that HWPCs outperforms the randomly sampled sectors’ effects on reducing wastewater discharge. Therefore, by promoting wastewater discharge reduction around HWPCs, the wastewater discharge of sectors on HWPCs can be improved, and the direct and embodied wastewater discharge of related sectors can be reduced by the extensive direct linkages of sectors on HWPCs to other sectors.

Table 3. Wastewater reduction effects to the entire economic system by reducing 10% final demand on sampled sectors.

| Year | Decrease Volume (Unit: 10^8 m$^3$) | Decrease Rate (%) |
|------|----------------------------------|------------------|
| 2002 | 5.06                             | 1.15             |
| 2007 | 10.30                            | 1.85             |
| 2012 | 12.06                            | 1.76             |
| 2017 | 12.06                            | 1.76             |

4. Discussion

Currently, there are few studies on wastewater reduction paths. This paper proposed APL-HCG, which is markedly different from the traditional Tu’s model [7], to explore hub wastewater propagation chains. The APL-HCG relies on a preset threshold to identify HWPCs. However, the results of a smaller threshold cover the results of a larger threshold. In other words, the HWPCs with a larger threshold are a subset of the HWPCs with a smaller threshold. Without a threshold, a 10% reduction in final demand of the HWPCs sector has a wastewater reduction effect of 26.78, 33.16, 35.33, and 33.54 in unit 10^8 m$^3$ for the whole economic system, representing 6.09%, 5.96%, 5.16%, and 4.79% of the total wastewater discharge in 2002, 2007, 2012, and 2017. Zheng et al. [26] noted that primary manufacturing of foods, clothing, wood, and paper dominated the wastewater discharge from 2002–2015 in Guangdong Province. At the national level, sector 10 and sector 11 are densely surrounded by many linkages and have a considerable degree of closeness to their linking sectors. Xiao et al. [3] calculated the wastewater and sludge footprints of the Chinese city of Xiamen, which discharged 2.11 × 10^8 m$^3$ of wastewater in 2012. They found that ‘Education services’, ‘Grain mill products’ and food-related sectors all had significant direct wastewater discharge and their wastewater footprints were also very considerable. Tu [7] analyzed inter-industry linkages of wastewater discharge in the Beijing-Tianjin-Hebei region in China, based on the traditional model. ‘Petroleum, coking products and nuclear fuel processed products’, ‘Hotel and catering’, ‘Electricity and heat production and supply’, ‘Textile’, ‘Papermaking, printing and cultural, educational and sporting goods’ and ‘Food manufacturing and tobacco processing’ were anchored as key sectors for wastewater discharge. Zheng et al. [27] indicated that China’s Food & Tobacco sector still needs to significantly improve resource utilization efficiency through technological innovations and a green food industry chain. Furthermore, the results of this study showed that sector 10, named Food Manufacturing and Tobacco Processing, was a
recognized significant sector in the HWPCs in 2007, 2012 and 2017. The results of these existing studies did not provide a linkage relationship among the sectors. However, the key sectors identified by the results of current studies are in reasonably comparable sector categories with the sectors on the HWPCs of this study, and therefore they can support the conclusions that it has reached.

In a subsequent study, APL-HCG can identify hub propagation chains for pollutant indicators such as COD, sewage sludge, and wastewater treatment sub-processes. The data for these indicators are first collected and compiled into occupancy by sector under the input matrix of the input-output table. Then, the direct consumption factors of each sector are calculated based on the occupancy of each sector for pollutant indicators. Finally, we substitute the direct consumption factors into the backward or forward APL formula introduced in the Methodology section. Equations 1–9 are the main steps in calculating the HWPCs for these indicators. There are three tuning parameters in Formula (7), and the results will vary as these three parameters vary. In this study, some sectors that densely surround many linkages and have a considerable degree of closeness with the sectors they link to always stay the same, except for appropriate changes in the parameters. Therefore, we choose the tuning parameters to make the number of these sectors both reasonable and interpretable. In addition, we set the linkages selection thresholds so that the number of sectors included in the HWPCs reached around 15, which facilitates the interpretation and comparison of the results. These empirical heuristics avoid the flexibility associated with tuning parameters. In addition to the flexibility of tuning parameters, the APL-HCG model has a wide range of applications and also has the value of extension.

5. Conclusions

Aiming to find the most interpretable and effective paths for wastewater reduction, we take advantage of the input-output analysis and covariance graphical model to explore and pinpoint the important sectors and propagation chains of wastewater discharge. We proposed a new general model APL-HCG (average propagation lengths coupled hub covariance graph [22]), where the APL covariance matrix carries the closeness of the sectoral linkages. We also simulated the wastewater reduction effect of identified wastewater propagation chains. This approach differs from previous studies.

The proposed APL-HCG model estimates the closeness of sectoral linkages, represented by the estimated APL covariance among sectors. The APL-HCG identifies effective hub wastewater propagation chains at the national level in 2002, 2007, 2012, and 2017. The scenario analysis shows that the HWPCs outperform the WPCs of the traditional Tu’s model [7] by 0.14%, 1.61%, 0.47%, and 0.10% for wastewater reduction in 2002, 2007, 2012, and 2017. In addition, the estimated covariances show information on the grouping of sectors, which the closeness of sector linkages within each group is substantial. Additionally, the changes in sectoral groupings reflect the dynamic process of evolving trends in sectoral linkages. The APL-HCG model is also applicable to other pollutant indicators.

Policymakers may conduct wastewater reduction policies based on HWPCs to reduce direct and implied wastewater discharge throughout the economic system. The focus of reducing wastewater discharge could capture the sectors in the hub wastewater propagation chains screened in this study for which the collaborative treatment is effective. In the collaborative treatment of the sectors in the wastewater propagation chains, we can deploy a combination of centralized and decentralized wastewater treatment and appropriately increase the fines imposed for unit wastewater discharged. Liu et al. [28] indicated that it is optimal to use different wastewater treatment modes in each sector. Therefore, a combined wastewater management mode can be implemented for the sectors on the identified HWPCs. The sectors of secondary industries such as sector 20, sector 21, sector 46, and sector 47 can adopt a centralized wastewater treatment mode. Conversely, the tertiary industry sectors such as sector 36 on the identified HWPCs are best implemented with a decentralized wastewater treatment mode. Furthermore, the results presented in this paper imply specific sectors where fines per unit of wastewater discharge should be increased; Liu et al. [28]
and Yu et al. [29] suggest that this is a possible intervention to limit wastewater discharge in China. In addition, encouraging sectors on HWPCs to improve water-use efficiency and enhance the application of advanced treatment of wastewater technologies (e.g., advanced treatment [30], coupled microalgal-bacterial biofilms [31], chemically enhanced primary treatment combined with side stream partial nitritation/anammox (PN/A) [32], nanomaterial enhanced treatment techniques [33]) throughout the production and wastewater treatment process may reduce future wastewater discharge. As Lin [30] summarized, it is inevitable to replace simple treatment with advanced treatment to improve the water quality of wastewater treatment when considering the trade-off between wastewater treatment and environmental impacts. Akao et al. [31] stated that it efficiently reduces wastewater pollutants and saves energy costs by incorporating a coupled microalgal-bacterial biofilm treatment in wastewater ponds. Paulu et al. [32] noted that the implementation of mainstream PN/A could reduce the overall environmental impact and thus replace the activated sludge process completely.

**Author Contributions:** X.L. (Xuefeng Li) proposed the main ideas, designed the APL-HCG model, compiled program to estimate parameters, got results, analyzed the results and drafted the manuscript. X.L. (Xiuli Liu) suggested to simulating the effects of hub wastewater propagation chains. She provided modification ideas and methods, guided to revise the manuscript. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by grant from National Natural Science Foundation of China under Grant No. 71874184.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** The data presented in this study are available on request from the corresponding author.

**Conflicts of Interest:** The authors declare no conflict of interest.

**Appendix A**

Table A1. China’s input-occupancy-output tables format for 2002, 2007, 2012, and 2017 in Zheng [25].

| Input | Intermediate Input Demand | Final Demand | Total Output |
|-------|--------------------------|--------------|--------------|
| Value-added | Z | f | X |
| Total Input | V | |
| Occupancy | Wastewater Discharge | WD<sub>T</sub> | WD<sub>T</sub> | WD |

**Appendix B**

Table A2. Sectors in Zheng [25].

| Sector Code | Sector Name | Sector Code | Sector Name |
|-------------|-------------|-------------|-------------|
| S1          | Agricultural Products | S26 | Waste Resources and Waste Material Recycling Processed Products |
| S2          | Forest Products | S27 | Metal Products, Machinery and Equipment Repair Services |
| Sector Code | Sector Name                                         | Sector Code | Sector Name                                         |
|-------------|-----------------------------------------------------|-------------|-----------------------------------------------------|
| S3          | Animal Husbandry Products                           | S28         | Electricity, Steam and Hot Water Production and Supply (Excluding Water and Electricity) |
| S4          | Fishery Products                                    | S29         | Gas Production and Supply                           |
| S5          | Agriculture, Forestry, Animal Husbandry and Fishery Service Products | S30         | Construction                                         |
| S6          | Coal Mining and Washing Products                    | S31         | Merchandise Transportation and Storage (Excluding Water Transportation) |
| S7          | Oil and Gas Extraction Products                     | S32         | Postal Service                                       |
| S8          | Metal Mining Products                               | S33         | Business                                             |
| S9          | Non-metallic Mining Products                        | S34         | Catering                                             |
| S10         | Food Manufacturing and Tobacco Processing           | S35         | Information Transmission, Computer Services and Software |
| S11         | Textile                                             | S36         | Finance and Insurance                                |
| S12         | Textile Clothing, Shoes, Hats, Leather, Down and Their Products | S37         | Real Estate                                          |
| S13         | Wood Processing and Furniture                       | S38         | Scientific Research and Technical Services           |
| S14         | Papermaking, Printing and Cultural, Educational and Sporting Goods | S39         | Environment and Public Facilities Management         |
| S15         | Petroleum, Coking Products and Nuclear Fuel Processed Products | S40         | Resident Services, Repairs and Other Services        |
| S16         | Chemical Products                                   | S41         | Education                                            |
| S17         | Non-metallic Mineral Products                       | S42         | Health and Social Work                               |
| S18         | Metal Smelting and Calendered Products              | S43         | Culture, Sports and Entertainment                    |
| S19         | Metal Products                                      | S44         | Public Administration, Social Security and Social Organization |
| S20         | Mechanical Equipment                                | S45         | Hydropower                                           |
| S21         | Transportation Equipment                            | S46         | Water Supply                                         |
| S22         | Electrical Machinery and Equipment                  | S47         | Sewage Treatment                                     |
| S23         | Communication Equipment, Computers and Other Electronic Equipment | S48         | Water Transportation                                 |
| S24         | Instrumentation                                     | S49         | Water Management                                     |
| S25         | Other Manufactured Products                         |             |                                                     |
Appendix C

Figure A1. Wastewater propagation chains that obtained by the traditional model [7]. Each vertex in sub-figures represents a sector, e.g., S43 in sub-figure (a) corresponds to Culture, Sports and Entertainment in Appendix B. The orange, blue and green edges mean that their APL value is 1, 2 and 3, respectively.
Appendix D

Figure A2. The estimated covariance matrix that obtained by APL-HCG in 2002, 2007, 2012, and 2017. Each solid dot in sub-figures represents the estimated covariance of linkages between the sectors corresponding to the row and column position is not zero. Each red rectangle marks a sector group, and the estimated covariance of the linkages among most sectors within the group is not zero. We use solid dots of different colors to indicate different sector groups.

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