On the necessity to reconsider the role of "action-at-a-distance" in the problem of the electro-magnetic field radiation produced by a charge moving with an acceleration along an axis

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**Summary.** - Some inadequacy in the traditional description of the phenomenon of electro-magnetic field radiation created by a point charge moving along a straight line with an acceleration is found and discussed in this paper in detail. The possibility of simultaneous coexistence of Newton instantaneous long-range interaction and Faraday-Maxwell short-rang interaction is pointed out.

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1. Introduction

The problem of interaction at a distance was raised for the first time more than 300 years ago by Newton in the first edition of his book "Matematical origins of natural philosophy" and has not lost relevance nowadays (see e.g.[1]). The question concerning the choice of one or another conception of interaction at a distance, namely - Newton instantaneous long-rang interaction (NILI) or Faraday-Maxwell short-rang interaction (FMSI), - seems to have been solved finally in favour of the latter one. But lately some authors (see e.g.[2-4,6]) have time and again resorted to NILI - a concept which was given up by contemporary physics long ago.

The necessity of introducing (or, at least, taking into account) NILI is based either on the possible incompleteness of Maxwell theory [3] or some possible inaccuracy of the main theses of the Special relativistic theory (SRT) [2,6].

Sometimes correspondent conclusions are connected with the experiments: for example, the results of Graneau experiments [5] are interpreted in [3] as an indication of the existence of a difference between electrical lepton-lepton and hadron-hadron interactions. The author [3] explains this difference by the existence of NILI, although in this case the energy transfer occurs with some delay, because it is carried out by the exchange with the zero energetic quantum-mechanical background. Thus, we see that the author, to avoid coming in conflict with SRT, has to resort to the help of quantum mechanics when discussing a completely nonquantum problem. At the same time, a number of authors indicate the incompleteness of Maxwell theory without referring to the NILI problem [7-9].

In this short note we shall use a simple mental experiment to show that in the case of rectilinear accelerated motion of a charge Maxwell theory cannot give a completely correct description of the process until NILI is not taken into account.

2. A mental experiment

Let a charge $q$ move in a reference laboratory system with a constant velocity $V$ along the positive direction of the $X$-axis. Then, let us consider the
electric field $\mathbf{E}(\mathbf{R})$ in a general point $\mathbf{R} = (x, y, z)$. It is straightforward to show applying the STR transformations, that the electric field $\mathbf{E}(\mathbf{R})$ is directed radially along the vector $\mathbf{R}$, since the delay effect is absent in our case of constant speed $\mathbf{V}$. (Note that in the case of an accelerated motion the field $\mathbf{E}$ is not directed radially everywhere, only in the direction of charge movement [10]). It can be easily shown [10] that the module $E(R) = |\mathbf{E}(\mathbf{R})|$ of the electric field in the point $\mathbf{R}$ of the reference system is given by:

$$E(R) = \frac{q(1 - \beta^2)}{R^2(1 - \beta^2 \sin^2 \alpha)^{3/2}}, \quad (1)$$

where $R(t) = |\mathbf{R}| = [(x - Vt)^2 + y^2 + z^2]^{1/2}$ is the distance between the charge and a point of observation $P$ lying on the $X$-axis, $\beta = V/c$, $c$ being the velocity of light, $\alpha$ is the angle between the vectors $\mathbf{V}$ and $\mathbf{R}$, $\mathbf{V} = |\mathbf{V}|$ and $t$ is the time in the reference system. In the case under consideration the coordinates $y$ and $z$ are equal to zero, and $|x - Vt|$ represent the distance between the charge and the point $P$ in the reference laboratory system.

Now let us apply the concepts of momentum and energy densities to our ”moving” field. The momentum density is given by

$$\mathbf{p} = \frac{1}{4\pi c} \mathbf{S}, \quad \mathbf{S} = \frac{c}{4\pi} [\mathbf{E}, \mathbf{H}], \quad (2)$$

where $\mathbf{S}$ is the Pointing-Umov (energy-flux) vector. The energy density is

$$W = \frac{E^2 + H^2}{8\pi}, \quad (3)$$

and the energy conservation condition for the electro-magnetic field, in the differential form, is:

$$\frac{\partial W}{\partial t} = -\nabla \cdot \mathbf{S}. \quad (4)$$

In the case of the charge movement considered here the change of $W$ with time on the left-hand side of (4) is:

$$\frac{\partial W}{\partial t} = \frac{q^2 V^2 (1 - \beta^2)}{2\pi(x - Vt)^5}. \quad (5)$$

Since $\mathbf{H} \equiv 0$ along the direction of the charge motion, which follows from the Maxwell equations, the vector $\mathbf{S}$, as well as the momentum density (2), turn out to be also zero along the same axis.
But what will happen if we suddenly accelerate the charge in the direction of the axis $X$? In this case expressions (2), (3) and (4) must be true as previously everywhere including the axis $X$. In the classic electrodynamics an electric field created by an arbitrarilly moving charge is given by the following expression:

$$E = q \frac{(R - R \frac{V}{c})(1 - \frac{V^2}{c^2})}{(R - R \frac{V}{c})^3} + q \frac{[R, [(R - R \frac{V}{c}), \frac{\dot{V}}{c}]]}{(R - R \frac{V}{c})^3}$$

(6)

We remind that here the value of $E$ is taken in a moment of time $t$ and the values of $R, V$ and $\dot{V}$ are taken in a former moment of time $t_0 = t - \tau$, where $\tau$ is ”retarded time”. In our approach since all the vectors are collinear, the second term in (6) is canceled, and we obtain

$$E(t) = q \frac{(1 - \frac{V^2(t_0)}{c^2})}{x^2(t_0)(1 - \frac{V(t_0)}{c})^2} \hat{i},$$

(7)

where $\hat{i}$ is an unit vector along the $X$-axis. In the case $V = const$ it is easy to prove that (7) can be reduced to (1). But the vectors $S$, and consequently $p$, are identically zero along the whole axis $X$. On the other hand, from (3) and (4) we see that $W$ and $\frac{\partial W}{\partial t}$ must differ from zero everywhere along $X$ and there is a linear connection between $W$ and $E^2$. I.e. conflict takes place: if, for example, the charge is vibrating in some mechan way along the axis $X$, then the value of $W$ (which is a point function like $E$) on the same axis will be also oscillating. Then the question arises: how does the point of observation, lying at some fixed distance from the charge on the continuation of axis $X$, ”know” about the charge vibration? The fact of ”knowing” is obvious.

The presence of ”retarded time” $\tau$ in (7) indicates that along the $X$-axis a longitudinal perturbation should be spreaded with energy transfer (contrary to Eq.(2)). And since the energy-flux vector $S$ is the product of the energy density and its spreading velocity

$$S = W \mathbf{v}$$

(8)

(here $\mathbf{v}$ is the velocity of the perturbation spread), then we can assume, for instance, that this velocity equals zero everywhere along $X$ except the region where the charge is localised, i.e. the energy transfer or radiation transfer is not carried out along $X$! It is known that Maxwell equations forbid the
spreading of any longitudinal electro-magnetic perturbation in vacuum. But P.A.M. Dirac writes ([11], p.32): "As long as we are dealing only with transverse waves, we cannot bring in the Coulomb interactions between particles. To bring them in, we have to introduce longitudinal electromagnetic waves... The longitudinal waves can be eliminated by means of mathematical transformation. ...Now, when we do make this transformation which results in eliminating the longitudinal electromagnetic waves, we get a new term appearing in the Hamiltonian. This new term is just the Coulomb energy of interaction between all the charged particles:

\[ \sum_{(1,2)} \frac{e_1 e_2}{r_{12}} \]

...This term appears automatically when we make the transformation of the elimination of the longitudinal waves."

But in this term "the delay effect" is not taken into account! So if we place a test charge \( q_0 \) on the axis \( X \) at some fixed distance from the vibrating charge \( q \), then the test charge will "feel" the influence of the charge \( q \) in an unknown way! Dirac writes [11]: "...but it also means a rather big departure from relativistic ideas". Now if \( W \) in (8) is supposed to be zero, then the question on the meaning of \( v \) loses sense. And we have to assume that energy is not stored in the field along \( X \). Moreover, calculations made in the book [12] (see also [10] ch.IV, § 33) can give us some indirect proof that the "own" field of charge particles does not directly contain energy. Indeed, it possible to show that the total 4-momentum of the system of charge particles interacting with the electro-magnetic field

\[
P_i = \sum_{\alpha} p_i^\alpha + \int_V \Theta_{i4} dV, \tag{9}\]

where \( \Theta_{i4} \) is the symmetric 4-momentum tensor of electro-magnetic field [10,12], is represented by the sum of the 4-momentums of free particles and free field. We must note that such a field is always transversal in vacuum. The analogous statement is true for the 4-angular-momentum, i.e. it is just the sum of the 4-angular-momentums of free particles and free field [12].
3. Discussion

In one of the latest works [13], the authors also discuss the paradox which is considered in our paper. They note quite truely that if one decomposes the total eletric field in terms of its transverse and longitudinal components, one must deal with the fact that the longitudinal component is propagated *instantaneously*. Then, *imposing* the same condition on the longitudinal component as on the transverse one about the limit of the spread velocity of the interaction, they could demonstrate that a space-time transverse electric field appears, which contains a term that exactly cancels the instantaneous longitudinal electric field. However, in their speculations the authors made the obvious logical error: the absence of the instantaneous ”action-at-a-distance” was derived from the *hypothesis* of its no-existence (see Eq.(8) in[13]).

It follows from the mental experiment considered above that an *instantaneous long-range interaction* must exist as a *direct consequence* of the Maxwell theory. Indeed, we found that the energy (or radiation) transfer is not carried out along the X-axis. Nevertheless, placing a test charge on the axis at some fixed point away from the vibrating charge $q$, we must observe an *influence* of the latter which *cannot be explained satisfactorily staying* on the position of the FMSI.

Of course, it is quite desirable to save *both* the STR and the Maxwell theory. On the other hand, an instantaneous long-range interaction must also exist. That is why it seems reasonable to introduce a certain *principle of electrodynamical supplementarity*. According to this, both pictures, the NILI and the FMSI, have to be considered as two *supplementary* descriptions of one and the same reality. Each of the descriptions is only *partly* true. In other words, both Faraday and Newton in their external argument about the nature of interaction at a distance turned out right: instantaneous long-range interaction takes place not *instead of*, but *along with* the short-range interaction in the classic field theory.

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