Analysis of Covid-19 in India using mathematical modeling

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Abstract. An infectious disease COVID-19 is caused by virus called coronavirus. The corona virus transmitted from individual to individual through droplets / contact. The first conformed case of India was on 30 January 2020 occurred in Kerala. As still no vaccine is available for controlling it so from initial stage government of India started different initiatives like lockdown, work from home, suspension of train services and international flights, implemented rules and regulation such as limiting the number of passengers in the vehicle, banning on social gathering to control on this virus. Ministry of Health and Family Welfare (MoHFW) was launched a nationwide training for front line health worker on COVID19. Hence the implication of the study becomes very crucial. In this paper the special case of India is considered from the initial period to October 2020. Based on available data, the analysis of this viruses is studied also the different cases of stability is checked by considering logistic growth in prey population. The parameter which used to study this case are predators, prey, time, prey’s growth rate, attack rate, predator or consumer mortality rate, conversion efficiency and maximum curing capacitive rate.

Key words: COVID-19, growth rate, attack rate, mortality rate

1. Introduction:

A newly detected corona virus is first found in china in December 2019. The disease caused by this virus is COVID-19. This disease spread from person to person and no vaccine was available so research across globe was started about this deadly acting virus. Prime Minister of India announced first lockdown of 21 days under Disaster management act, 2005. Following are month wise figures of corona cases and death due to COVID 19 in India.[5]

Till 28 March 2020, total cases have been reported were 909 and 19 were reported deaths. As of 26 April 2020, registered cases were 26,917 out of which 826 are number of deaths. As on 24 May 2020, Active cases were 73,560, and no. of death are 3867. 
As on 28June 2020, Active cases were 203,051 and 16,095 Deaths.
As on 26July 2020, there were 467,882 active cases, and 32,063 Deaths.
As on 24 August 2020, 710,771 were active cases and 57,542 Deaths
As of 27September 2020, recorded cases were 5,992,532 and 94,503 deaths.
As of 25 October 2020, cases were 7,864,811 and 118,534 deaths

2. Predator-prey dynamics

Here Prey is considered as population of India and Predator is assumed as COVID 19 patient. Here the study of predator – prey dynamics in India is considered. This is characterized by
population density of Predator and Prey, by considering case when the COVID start in India here the peak Predator oscillation lagging behind the peak prey which is based on the lockdown.[6]

Following assumptions are made for this model:
1) The prey populations will increase in exponential manner in absence of predators.
2) The population of predator will suffer in cutting of population of prey.
3) Corona can hit an infinite quantity of populations.
4) No environmental complexity.

The characteristics that improve predator COVID19 power to affect the prey populace will be selected for in the predator, while the characteristics that improve the human ability to avoid being suffer from this virus will be selected for in the prey.

To formulate this model, here two species X and Y are considered.
Where, X (acts as a prey) = Population in India
Y (acts as a predator) = Number of COVID 19 patients.
Here, model is made from a pair of differential equations that describe dynamics of Virus & people affected by it starting from first phase to third phase in India.
The following parameters are considered to form the model.
- X = population of India.
- Y = Number of COVID 19 patient
- t = time
- R = 1% = growth rate of population of India (Prey)
- A = 1.1% = attack rate
- M = 6% = decline rate of COVID19 (Predator)
- C = 3.2% = conversion efficiency (predator efficiency at turning food into offspring)

2.1 When there is no interaction between non affected people and virus affected patients

Here we expect that without predation means COVID 19, the population will increase exponentially. The rate of increase of the prey population with respect to time is given by:
\[
\frac{dX}{dt} = RX
\]  
In the absence of prey, the differential equation of the predator population is
\[
\frac{dY}{dt} = -MY
\]  
Which shows that the number of covid cases will be expected to decline exponentially.

2.2 When there is interaction between prey and predator

When prey and predator interact with each other then more will get infected by corona and that affect, growth rate of people because of COVID 19 patient increases.

But in presence of predators (COVID-19 patients), the prey population is fend off from increase in exponential order. In this case prey population dynamics can be modified in differential equation as:
\[
\frac{dX}{dt} = RX - AXY
\]  
The term AXY shows that dropping in the population of prey because of predation are proportional to the product of prey and predator plentiful.
The predator population in the presence of prey is

\[ \frac{dY}{dt} = CAXY - MY \]  

(4)

Where, \( C = \) conversion efficiency, \( A = \) attack rate

It means when the people are in contacts with each other than this decrease is opposed by birth rate \( CAXY \) of predator. As \( X, Y \) increase they come across incessant, but the actual consumption rate is based on the attack rate. Equations (3) and (4) represent Prey and predator population model in presence of each other.

The cyclical relation between these two dependent variables is forecasted by model. As the number of COVID 19 patient’s increases then affecting rate \( (AXY) \) become strengthen, this rises in \( (AXY) \) result in shrink in the number of prey \( (X) \). Also prey take proper care to avoid COVID 19, which result in \( (AXY) \) to decrease. This drop in \( AXY \) is able to regain the prey population \( X \) and hence prey population \( X \) increases. This increase in \( X \) effect in increase \( Y \) and repetition of cycle begins again.

On substituting the values of all parameters in equations (3) and (4) we get,

\[ \frac{dX}{dt} = X - 1.1XY \]

\[ \frac{dY}{dt} = (3.2)(1.1)XY - (0.06)Y \]

For the steady state of this model,

\( X - 1.1XY = 0 \Rightarrow X = 0 \) or \( Y = 0.9091 \)

\( 3.2(1.1)XY - 0.06Y = 0 \Rightarrow Y = 0 \) or \( X = 0.017 \)

Hence the steady state points are \((0, 0)\) & \((0.17, 0.9091)\).

The Jacobian of the system is given as:

\[
J = \begin{bmatrix}
1 - 1.1Y & -1.1X \\
3.52Y & 3.52Y - 0.06
\end{bmatrix}
\]  

(5)

2.2.1 Analysis of Steady state condition for \((0, 0)\)

Equation (5) is

\[ J = \text{diag}[1, -0.06] \]

Since \( J \) is diagonal matrix, the eigen values are 1, -0.06, so the steady state \((0, 0)\) is unstable because one eigen value is positive.

This is not favorable condition as far as COVID 19 is considered.

2.2.2 Analysis of Steady state condition for \((0.17, 0.9091)\)

Equation (5) is

\[ J = \begin{bmatrix}
0 & -0.187 \\
3.200032 & 0
\end{bmatrix} \]

Characteristic equation is \( \lambda^2 + 0.5984 = 0 \) \( \Rightarrow \lambda = \pm i\sqrt{0.5984} \)

As the eigen values are imaginary so this steady state is stable but not asymptotically stable.

3. Logistic growth in prey population
\[
\frac{dX}{dt} = RX \left[ 1 - \frac{X}{K} \right] - AXY 
\]

Where \( K \) is maximum carrying capacity

\[
\frac{dY}{dt} = CA\bar{Y} - MY 
\]

For analysis of this, consider the steady state of this model

\[
(6) \Rightarrow \begin{bmatrix} RX - \frac{RX^2}{K} \end{bmatrix} - AXY = 0 \Rightarrow X \left\{ R - \frac{RX}{K} - AY \right\} = 0 \Rightarrow X = 0 \text{ or } R - \frac{RX}{K} - AY = 0 
\]

\[
(7) \Rightarrow \text{CA\bar{Y}-MY=0} \Rightarrow Y (\text{CA-X-M}) = 0 \Rightarrow Y = 0 \text{ or } \frac{M}{CA} 
\]

After solving this simultaneous equations, we get the values of \((X,Y)\) as \((0,0)\), \((K,0)\), \((0,\frac{M}{CA})\), \((\frac{M}{CA},0)\).

Here \( K > \frac{M}{CA} > 0 \)

The Jacobian matrix is given by

\[
\begin{bmatrix} \frac{R}{A} \left[ 1 - \frac{M}{CAK} \right] \end{bmatrix} 
\]

3.1 Analysis of Steady state condition for \((0,0)\)

The steady state condition \((0,0)\) is not possible as far as COVID case is consider.

3.2 Analysis of Steady state condition for \((K,0)\)

The Jacobian for the condition \((K,0)\) is

\[
\begin{bmatrix} -R & -AK \end{bmatrix} 
\]

As \( K > \frac{M}{CA} \), so \( \text{CA-K-M} \) is positive so the system is unstable.

3.3 Analysis of Steady state condition for \(\left(\frac{M}{CA}, \frac{R}{A} \left[ 1 - \frac{M}{CAK} \right] \right)\)

For steady state, \(\begin{bmatrix} \frac{M}{CA}, \frac{R}{A} \left[ 1 - \frac{M}{CAK} \right] \end{bmatrix}\), The Jacobian is

\[
\begin{bmatrix} \frac{-MR}{CAK} & \frac{-M}{C} \\ \frac{CR(1 - \frac{M}{CA})}{M} & 0 \end{bmatrix} 
\]

Here trace of matrix is negative and the determinant is positive, so both the eigen values are negative and hence the system is stable.

So in absence of COVID 19 or when the vaccine is available on COVID 19 then we will get stable condition of human being.
4. Result
In article 2, model for corona with exponential growth is considered. Equations (1) and (2) represent model when there is no interaction between prey and predator while Equations (3) & (4) gives model in presence of each other. Solving this equations stability of model is checked under different conditions also analysis of model is done.
In article 3. Logistic growth of prey is considered and discuss the case under different conditions.

5. Conclusion
In this paper we have taken the cases for the period from March 2020 to October 2020 with the conclusions that if predators are less in number then there is an increase in prey population. As prey population increase, it increases connection between the people so result in number of cases of COVID increase that means predators size increase. Which shows that these are cyclic in nature biologically.
The actual system in such a way that it should converge to steady state.
In case of logistic growth, we get asymptotically stable case.

6. References
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