Thermodynamic-temperature data from 30 K to 200 K

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Abstract

New measurements of thermodynamic temperature \( T \) with Dielectric-Constant Gas Thermometry (DCGT) were performed at PTB from 50 K to 200 K. Particular care was taken to check for possible systematic sources of errors by performing experiments applying three working gases, namely helium, neon, and argon, the polarizability of which differs by a factor of up to eight. Together with former DCGT values of thermodynamic temperature the new results yield a consistent dataset in the range from 30 K to 200 K. This dataset is in good agreement with the newest results of Acoustic Gas Thermometry (AGT) and Refractive-Index Gas Thermometry (RIGT), which have quite different sources of uncertainty compared with DCGT. The combination of these DCGT, AGT, and RIGT data with the ‘Estimates of the differences between thermodynamic temperature and the ITS-90’, being as an appendix of the ‘Mise en pratique for the definition of the kelvin in the SI’ the present-day recommendation of the Consultative Committee for Thermometry, yields a new function \( T - T_{90} \) versus ITS-90 temperature \( T_{90} \) for the range from 35 K to 195 K, the uncertainty of which is reduced by a factor up to about four.

Keywords: thermodynamic temperature, primary thermometry, dielectric-constant gas thermometry, International Temperature Scale, ITS-90

(Some figures may appear in colour only in the online journal)

1. Introduction

The appendix ‘Estimates of the differences between thermodynamic temperature and the ITS-90’ [1] of the Mise en Pratique (‘practical realisation’) of the definition of the kelvin [2] gives recommended estimates of the differences between thermodynamic temperature \( T \) and temperature \( T_{90} \) on the International Temperature Scale of 1990 (ITS-90) [3] at a predefined set of temperatures (‘base temperatures’), including the fixed points of the ITS-90 and additional points that are often secondary reference points. Additionally, for the temperature ranges from 8 K to 273 K and 273 K to 1358 K, respectively, functions \( T - T_{90} \) versus \( T_{90} \) are given. (The function for the low-temperature range is called in the present paper \( (T - T_{90})_{LT2011} \).) These estimates and functions allow corrections to be made to \( T_{90} \) values when accurate measurements of \( T \) are needed. The appendix is based on an evaluation of the existing thermodynamic data finished in 2010 by a working group of the Consultative Committee for Thermometry (CCT) [4]. In some temperature ranges below the triple point of water, the uncertainty of the estimates approaches 2 mK, while the absolute differences \( T - T_{90} \) are smaller than 10 mK. To overcome this unfavourable situation of high relative uncertainty, thermodynamic temperature measurements were performed mainly at three Metrological Institutes [5–7].

In this paper, the results of most recent measurements with the Dielectric-Constant Gas Thermometer (DCGT) performed at PTB are described in section 2. The new data \( T_{DCGT} - T_{90} \) (\( T_{DCGT} \) is the DCGT temperature value) are combined in section 3 with former thermodynamic results obtained with DCGT at PTB, which are summarised in [8], but not
considered in [4], to obtain a consistent function $T_{\text{DCGT}} - T_{\text{IS}}$ versus $T_{\text{IS}}$ for the temperature range from about 30 K to 200 K, called $(T - T_{\text{IS}})_{\text{ARD2020}}$. Finally, in section 4, this function is in turn combined with the ‘Estimates of the differences between thermodynamic temperature and the ITS-90’ [1] as well as with thermodynamic data obtained applying Acoustic Gas Thermometry (AGT) at the National Physical Laboratory (NPL), UK [5], and Refractive-Index Gas Thermometry (RIGT) at the National Research Council (NRC), Canada [6]. The final result is a function $T - T_{\text{IS}}$ versus $T_{\text{IS}}$ for the temperature range from 35 K to 195 K with reduced uncertainty that is called $(T - T_{\text{IS}})_{\text{ARD2020}}$, where ARD is an abbreviation for AGT-RIGT-DCGT.

2. Recent thermodynamic DCGT data

2.1. DCGT method

DCGT is based on replacing the density in the equation of state of a gas by the dielectric constant $\varepsilon$. The dielectric constant is determined via the change of the capacitance $C(p)$ of a suitable capacitor measured with $(C(p))$ and without the measuring gas $(C(0))$. For performing primary thermometry in the temperature range from 50 K to 200 K applying helium, neon, and argon as measuring gases at pressures up to 0.3 MPa, the following fourth-order working equation for the pressure $p$ considers sufficiently the interaction of the gas particles:

$$p = A_1 (\mu + A_2 \mu^2 + A_3 \mu^3 + A_4 \mu^4)$$  \hspace{1cm} (1)

with

$$\Delta C/C = (C(p) - C(0))/C(0) = \varepsilon_r - 1 + \varepsilon_r \kappa_{\text{eff}} \rho p,$$ \hspace{1cm} (2)

and

$$\mu = \Delta C/C / (\Delta C/C + 3),$$ \hspace{1cm} (3)

as well as

$$A_1 = (A_\varepsilon/RT + \kappa_{\text{eff}}/3)^{-1},$$ \hspace{1cm} (4)

where $A_\varepsilon$ is the molar polarizability, $R$ is the molar gas constant, and $\varepsilon_r = \varepsilon/\varepsilon_0$ ($\varepsilon_0$ is the electric constant). The linear term describes the ideal-gas behaviour. Its coefficient $A_1$ contains the target measuring quantity, the thermodynamic temperature $T$, $\kappa_{\text{eff}}$ denotes the effective compressibility of the capacitor, which describes the change of the capacitance only due to the mechanical deformation caused by the measuring gas. (The term ‘effective’ indicates the fact that each capacitor is a composite because small pieces of insulator materials are necessary to isolate the electrodes electrically.) As shown in [9], the coefficients $A_2$, $A_3$, and $A_4$ of the second-, third- and fourth-order terms, respectively, can be expressed as functions of $A_1$, $A_\varepsilon$, and $\kappa_{\text{eff}}$ as well as of both the density and dielectric virial coefficients, which describe the interaction of the gas particles.

For determining $A_1$, isotherms have to be measured. Then, a polynomial fit without a constant term according to the working equation (1) to the obtained $p$ versus $\mu$ dependence is performed. The calculation of $T$ from the fitting coefficient $A_1$ requires knowing $R$ and $A_\varepsilon$ with sufficiently small uncertainty. ($\kappa_{\text{eff}}$ must be determined individually for the measuring capacitor, see the following section.) The newest values of the fundamental constants including $R$ are published in [10]. For helium, a sufficiently accurate value for the static electric dipole polarizability of a gas particle has been obtained from ab initio calculations [11–15]. This value, having a relative uncertainty below one part per million (1 ppm), yields together with the newest values of the fundamental constants $A_{\text{He}}^{4\text{He}} = 5.1725409(5) \times 10^{-7}$ m$^3$ mol$^{-1}$ for the heavy helium isotope $^4\text{He}$ (the number in brackets is the standard uncertainty given as multiple of the last digit). For neon and argon, the uncertainty of theoretical calculations of the polarizability is yet too large for primary thermometry. Experimental values having the smallest uncertainty have been obtained by DCGT at the triple point of water [9]: $A_{\text{Ne}}^{9\text{Ne}} = 9.9471(24) \times 10^{-7}$ m$^3$ mol$^{-1}$ and $A_{\text{Ar}}^{4\text{Ar}} = 4.140686(10) \times 10^{-6}$ m$^3$ mol$^{-1}$.

2.2. Experimental setup

The DCGT setup of the second generation (DCGT2), leading to the present results, is already described extensively in [16] and in full detail in [17]. Therefore, in this section only a very short summary of the major changes and improvements achieved, in comparison with this original equipment, is given.

(1) For stabilising temperatures between 50 K and 200 K at a level of a few 10 $\mu$K, a tank cryostat with four cooling stages (nitrogen tank, helium tank and two pumped helium vessels) was used. The cryostat has five modes of operation: (i) liquid helium (lHe) and liquid nitrogen (lN$_2$) in the two tanks, respectively, and pumping of the two vessels filled with lHe from the tank via capillaries having well-dimensioned flow impedances; (ii) lHe and lN$_2$ in the two tanks and empty vessels; (iii) pumped solid nitrogen in the helium tank and lN$_2$ in the nitrogen tank; (iv) lN$_2$ in both tanks; (v) circulation of temperature-controlled alcohol through a cooling tube, which is normally used for fast cooling with liquid nitrogen. For covering the temperature range from 50 K to 200 K modes (ii) to (v) were necessary. The temperature control must be optimised individually for the different modes. This includes the temperature distribution along the pressure-sensing tubes, which go completely through vacuum. The temperature $T_{\text{IS}}$ according to the ITS-90 was measured with three capsule-type standard platinum resistance thermometers (CSPRTs) situated at opposite sides of the measuring copper block housing in between two measuring capacitors. The calibration of the CSPRTs is described in [18, 19]. The CSPRTs carrying an ITS-90 calibration were compared at all measuring temperatures, and the maximum difference between their readings was below 50 $\mu$K. At all isotherm temperatures, the CSPRTs showed an agreement between 30 $\mu$K and 50 $\mu$K. These comparison results were used as an estimate of thermal gradients in the system and included in the uncertainty component ‘Fit coefficient $A_1$’ of the budget in table 3.
The relative uncertainty of the pressure measurement was decreased to 3.6 parts per million (3.6 ppm) by calibrating the piston-cylinder unit (PCU) of the pressure balance used, which has a nominal effective area of 3 cm², against the special PCUs characterised for the measurement of the Boltzmann constant [20].

The equipment used for measuring capacitance changes was developed as described in [7, 21], which resulted in a relative uncertainty of one part per billion (1 ppb).

The two newest-generation cylindrical measuring capacitors called C1 and C2 have the same general design as those used by Luther et al [22]. The electrodes and the housing were machined from the copper–beryllium–cobalt alloy C17500 (nominal content of (0.4–0.7)% beryllium and (2.4–2.7)% cobalt). The inner electrode has a length of 45 mm and an outer diameter of 17 mm. The inner length of the outer electrode is 70 mm and its inner diameter is 22 mm. The electrodes are fixed on a grounded bed-plate using mica disks of thickness 0.1 mm for electrical isolation. The determination of \( \kappa_{\text{eff}} \) in dependence on temperature as described in [23] is based on the measurement of isotherms at the TPW as starting point, see [8]. Furthermore, it was necessary to measure the following properties of the electrode material as described in [23]: specific heat capacity (77 K to room temperature), thermal expansion (2 K to 300 K), and adiabatic bulk modulus (230 K to 330 K).

DCGT measurements cause extreme requirements regarding the purity of the measuring gas especially for helium. Impurities should not cause a relative change of the result by more than 1 ppm. To prevent contamination of the measuring gas during handling, gas purifiers (getter and adsorber) have been incorporated in the ultra-high-purity gas-handling system. (The specified upper limit for water and the other relevant impurities beside noble gases is 10 parts per billion (ppb) for the getter, and for the adsorber, the specified upper limit is 0.1 ppb.) After each measurement of an isotherm, which lasted usually one day, the measuring gas was analysed with the aid of a mass spectrometer to check for a possible contamination especially due to outgassing from the different pieces inside the experimental chamber (the detection limit for noble gas contaminations is 10 ppb). The most problematic impurity is water because it has a polarizability 160 times larger than that of helium. Thus, the detection limit of 20 ppb would cause an uncertainty component of order 1 ppm applying a rectangular distribution. But additional analysis of helium gas remaining in the system for weeks led to an even lower uncertainty component. Therefore, considering the specification of the adsorber, 1 ppm is a reliable upper estimate for an overall uncertainty component including all relevant impurities.

2.3. Isotherm data

New triplets of temperature \( T_{90} \), pressure \( p \) and dielectric constant (represented by the DCGT measuring quantity \( \mu \approx (\varepsilon_r - 1)/3 \), see equations 2 and 3) were measured with the working gases helium (purity 99.99999 %) at 50 K, 51 K, 60 K, 70 K, 79 K, 100 K, 120 K, and 200 K, and argon (purity 99.99999%) at 200 K. The complete dataset is given in table 1, in which the temperature is given on the ITS-90 \( (T_{90}) \) together with its standard uncertainty \( (\mu(T_{90})) \), and the \( p \) and \( \mu \) values are in each case the mean of the specified number \( n \) of single values. (The overall number of final triplets amounts to 276.) The uncertainty values \( \mu(T_{90}) \) have been estimated applying the procedures recommended in [24] for propagating the uncertainty and considering the non-uniqueness. Based on recent international intercomparisons, especially on a star intercomparison of sealed triple-point cells [25], the standard uncertainty of the fixed-point temperatures used for the calibration of CSPRTs amounts to 0.1 mK. The relative standard uncertainty of the pressure measurement is dominated by the uncertainty of the calibration of the pressure balance used (3.6 ppm, see section 2.2), whereby the resolution of the balance of 0.1 Pa is included in the Type A uncertainty component ‘Fit coefficient \( A_1 \)’ (see table 3). Furthermore, the uncertainty of the correction of the pressure difference caused by the gas column (head correction) has been included in table 3. The relative uncertainty of the measurement of capacitance changes of order 1 ppb yields a relative uncertainty of the \( \mu \) values of 10⁻⁹/\( \mu \). But in reality, the uncertainty of a single \( \mu \) value is only partly relevant for the temperature result (see the Type B component “Susceptibility measurement (capacitance change)” in table 3). This is because several pairs of \( p \) and \( \mu \) are fitted on an isotherm, which results in a Type A uncertainty component for the errors of the inductive voltage divider used for balancing the capacitance bridge. This component is included in the component ‘Fit coefficient \( A_1 \)’. The agreement of the zero-pressure capacitance ratio before and after the isotherm measurement was in all cases below the detection limit of the capacitance bridge.

2.4. Data evaluation and uncertainty estimation

All isotherms were fitted to polynomials of second, third, and fourth order according to the working equation characterised briefly in section 2.1. (In view of the limited number of triplets and the spacing of the measuring temperatures, only single-isotherm fits were performed.) The \( A_1 \) values resulting from fits of second order cannot be used directly for determining the thermodynamic temperature value \( T_{\text{DCGT}} \) because they are distorted by neglecting the third-order term, which is mainly connected with the third density virial coefficient \( C \). But these values can yield a useful additional information if they are corrected for the influence of the third coefficient in equation (1) \( (A_1 A_3) \). A derivation of the complete formula is given in [9]. Neglecting higher order terms in \( \kappa_{\text{eff}} \), the third dielectric virial coefficient as well as the combination of the second density and dielectric virial coefficients, this term has the form \( A_1^3 (RT)^{-2} C \). Theoretical values of the third virial coefficient as constraints were taken from [26] for helium, from [27] for neon, and from [28] for argon (for more details see also [9]). For the evaluation with the constraints, a relative uncertainty component of 5% was added for all three gases. This is far above the theoretical estimates and, therefore, sufficiently conservative. The resulting correction of the \( A_1 \) values from
The new DCGT dataset between 50 K and 200 K is listed. For each isotherm, the specific capacitor C1 and C2, respectively, as well as the associated compressibility and the uncertainty is listed. Furthermore, the triplets $T_{90}$, $p$ and $\mu$ are given, were $\mu$ is the measuring quantity used in the DCGT measurements (see section 2.1, equation (3)). The $p$ and $\mu$ values are in each case the mean of $n$ single values.

The nominal purity stated by the manufacturer, before further purifying procedures (see text), is listed, whereby 7N stands for 99.99999% of the measuring gas and 5.3N stands for 99.9993%. The reference temperature $T_{90}$ is the PTB realisation of the ITS-90 applying capsule-type standard platinum resistance thermometers. The temperature $T_{DCGT}$ is the weighted mean of the results of second, third and fourth order fits (for more details see text).

$u(T_{90})$ and $u(T_{DCGT})$ as well as $u(T - T_{90})$ are the individual standard uncertainties of the temperatures $T_{90}$, $T_{DCGT}$ and the difference $T - T_{90}$, respectively (for more details see table 3).

| Gas | Capacitor | $T_{90}$ (K) | $\kappa_{eff} \times 10^{12}$ | $T_{DCGT}$ (K) | $u(T_{90})$ (mK) | $u(T_{DCGT})$ (mK) | $u(T - T_{90})$ (mK) | $n$ | $p$ (Pa) | $\mu \times 10^4$ |
|-----|-----------|--------------|-----------------|----------------|------------------|-----------------|------------------|-----|-----------|------------------|
| He C1 | 49.83501 | 0.13 | 0.30 | 7 | 45 205.44 | 0.5634483 |
| He C1 | 50.78686 | 0.12 | 0.32 | 4 | 45 205.19 | 0.5528771 |
| He C1 | 50.78509 | 0.12 | 0.44 | 5 | 45 225.42 | 1.0690589 |
| Ne C1 | 50.78686 | 0.12 | 0.44 | 5 | 191 289.79 | 1.7643593 |
| Ne C2 | 50.78686 | 0.12 | 0.33 | 4 | 45 204.74 | 0.9096661 |
| Ne C2 | 50.78686 | 0.12 | 0.74 | 6 | 45 224.60 | 1.7643593 |
| 7N | −2.353(4) | 49.83315 | 0.26 | 2 | 191 292.23 | 2.3768872 |
| 7N | −2.353(4) | 50.78509 | 0.30 | 2 | 191 289.79 | 2.3321979 |
| 5.3N | −2.353(4) | 50.78504 | 0.42 | 2 | 191 289.79 | 2.3321979 |
| 7N | −2.323(5) | 50.78480 | 0.30 | 2 | 191 286.91 | 2.6869113 |
| 5.3N | −2.323(5) | 50.78398 | 0.73 | 2 | 191 373.31 | 6.0100773 |

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| Gas   | Capacitor | $T_{90}$ (K) | $u(T_{90})$ (mK) | $u(T_{DCGT})$ (mK) | $u(T - T_{90})$ (mK) | $n$ | $p$ (Pa) | $\mu \times 10^4$ |
|-------|-----------|--------------|------------------|-------------------|---------------------|-----|---------|----------------|
| He C1 | 59.78451  | 0.10         | 0.39             | 4                 | 45.20716            | 0.4696628 |
|       |           |              |                  |                   | 74426.64            | 0.7727734 |
|       |           |              |                  |                   | 103644.95           | 1.0755239 |
|       |           |              |                  |                   | 132865.61           | 1.3779511 |
|       |           |              |                  |                   | 162082.96           | 1.6799814 |
| 7 N   | −2.356(4) | 59.78264     | 0.38             |                   | 191301.84           | 1.9816823 |
|       |           |              |                  |                   | 229520.59           | 2.2830260 |
|       |           |              |                  |                   | 249739.91           | 2.5840319 |
|       |           |              |                  |                   | 278960.15           | 2.8846933 |
|       |           |              |                  |                   | 308179.00           | 3.1849806 |
| Ne C1 | 59.78451  | 0.10         | 0.64             | 4                 | 45.21837            | 0.9066230 |
|       |           |              |                  |                   | 74444.36            | 1.4947857 |
|       |           |              |                  |                   | 103668.49           | 2.0846445 |
|       |           |              |                  |                   | 132896.76           | 2.6763490 |
|       |           |              |                  |                   | 162122.14           | 3.2697392 |
| 5.3N  | −2.356(4) | 59.78150     | 0.63             |                   | 191347.43           | 3.8648760 |
|       |           |              |                  |                   | 220572.25           | 4.4617822 |
|       |           |              |                  |                   | 249798.49           | 5.0605337 |
|       |           |              |                  |                   | 279025.27           | 5.6610526 |
|       |           |              |                  |                   | 308260.32           | 6.2635284 |
| He C2 | 59.78451  | 0.10         | 0.39             | 5                 | 45.20735            | 0.4696696 |
|       |           |              |                  |                   | 74444.95            | 1.4947857 |
|       |           |              |                  |                   | 103644.72           | 1.0755331 |
|       |           |              |                  |                   | 132864.95           | 1.3779639 |
|       |           |              |                  |                   | 162083.89           | 1.6800167 |
| 7N    | −2.326(5) | 59.78250     | 0.38             |                   | 191303.33           | 1.9817276 |
|       |           |              |                  |                   | 220522.22           | 2.2830742 |
|       |           |              |                  |                   | 249740.56           | 2.5840692 |
|       |           |              |                  |                   | 278960.30           | 2.8847249 |
|       |           |              |                  |                   | 308179.90           | 3.1850183 |
| Ne C2 | 59.78451  | 0.10         | 0.71             | 9                 | 45.21857            | 0.9066217 |
|       |           |              |                  |                   | 74444.86            | 1.4948312 |
|       |           |              |                  |                   | 103669.61           | 2.0847239 |
|       |           |              |                  |                   | 132897.63           | 2.6764440 |
|       |           |              |                  |                   | 162123.82           | 3.2698723 |
| 5.3N  | −2.326(5) | 59.78170     | 0.71             |                   | 191349.70           | 3.8650520 |
|       |           |              |                  |                   | 220575.79           | 4.4620283 |
|       |           |              |                  |                   | 249802.97           | 5.0607806 |
|       |           |              |                  |                   | 279030.00           | 5.6613501 |
|       |           |              |                  |                   | 308257.62           | 6.2636526 |
| He C1 | 69.73874  | 0.10         | 0.55             | 9                 | 45.20444            | 0.4025816 |
|       |           |              |                  |                   | 74422.83            | 0.6624407 |
|       |           |              |                  |                   | 103638.69           | 0.9219933 |
|       |           |              |                  |                   | 132856.97           | 1.1812959 |
|       |           |              |                  |                   | 162073.58           | 1.4402964 |
| 7N    | −2.360(4) | 69.73531     | 0.54             |                   | 191290.24           | 1.6990188 |
|       |           |              |                  |                   | 220507.96           | 1.9574824 |
|       |           |              |                  |                   | 249724.10           | 2.2156557 |
|       |           |              |                  |                   | 278940.55           | 2.4735506 |
|       |           |              |                  |                   | 308157.44           | 2.7311461 |
Table 1. Continue.

| Gas  | Capacitor | $T_{90}$ (K) | $u(T_{90})$ (mK) | $u(T_{DCGT})$ (mK) | $n$ | $p$ (Pa) | $\mu \times 10^4$ |
|------|-----------|-------------|-----------------|-------------------|----|---------|-----------------|
| Ne C1 | 69.73874 | 0.10 | 0.57 | 5 | 45 217.56 | 0.7764175 |
|      |           |             |                 |                   |    |         | 74 443.70     | 1.2793542 |
|      |           |             |                 |                   |    |         | 103 667.87    | 1.7831210 |
|      |           |             |                 |                   |    |         | 132 894.99    | 2.2878135 |
|      |           |             |                 |                   |    |         | 162 120.37    | 2.7933543 |
| 5.3N | −2.360(4) | 69.73599    | 0.57            |                   |    |         | 191 345.25    | 3.2997599 |
|      |           |             |                 |                   |    |         | 220 570.11    | 3.8070459 |
|      |           |             |                 |                   |    |         | 249 796.28    | 4.3152127 |
|      |           |             |                 |                   |    |         | 279 018.97    | 4.8242100 |
|      |           |             |                 |                   |    |         | 308 247.68    | 5.3341683 |
| He C2 | 69.73874 | 0.10 | 0.53 | 12 | 45 204.56 | 0.4025880 |
|      |           |             |                 |                   |    |         | 74 421.70     | 0.6624311 |
|      |           |             |                 |                   |    |         | 103 636.77    | 0.9219879 |
|      |           |             |                 |                   |    |         | 132 855.35    | 1.1812941 |
|      |           |             |                 |                   |    |         | 162 072.06    | 1.4403034 |
| 7N  | −2.330(5) | 69.73585    | 0.52            |                   |    |         | 191 288.14    | 1.6990289 |
|      |           |             |                 |                   |    |         | 220 504.16    | 1.9574862 |
|      |           |             |                 |                   |    |         | 249 720.68    | 2.2156702 |
|      |           |             |                 |                   |    |         | 278 938.58    | 2.4735767 |
|      |           |             |                 |                   |    |         | 308 156.12    | 2.7311985 |
| Ne C2 | 69.73874 | 0.10 | 0.84 | 6  | 45 218.74 | 0.7764528 |
|      |           |             |                 |                   |    |         | 74 444.61     | 1.2793951 |
|      |           |             |                 |                   |    |         | 103 668.95    | 1.7831864 |
|      |           |             |                 |                   |    |         | 132 896.69    | 2.2879108 |
|      |           |             |                 |                   |    |         | 162 121.97    | 2.7934543 |
| 5.3N | −2.330(5) | 69.73497    | 0.84            |                   |    |         | 191 347.75    | 3.2998897 |
|      |           |             |                 |                   |    |         | 220 573.36    | 3.8072202 |
|      |           |             |                 |                   |    |         | 249 799.79    | 4.3154414 |
|      |           |             |                 |                   |    |         | 279 027.69    | 4.8245507 |
|      |           |             |                 |                   |    |         | 308 251.69    | 5.3344382 |
| He C1 | 78.55801 | 0.10 | 0.54 | 6  | 45 204.02 | 0.3573624 |
|      |           |             |                 |                   |    |         | 74 422.40     | 0.5880590 |
|      |           |             |                 |                   |    |         | 103 637.15    | 0.8184986 |
|      |           |             |                 |                   |    |         | 132 855.47    | 1.0487376 |
|      |           |             |                 |                   |    |         | 162 072.35    | 1.2787367 |
| 7N  | −2.364(4) | 78.55456    | 0.53            |                   |    |         | 191 288.27    | 1.5084967 |
|      |           |             |                 |                   |    |         | 220 504.55    | 1.7380358 |
|      |           |             |                 |                   |    |         | 249 720.44    | 1.9673488 |
|      |           |             |                 |                   |    |         | 278 938.12    | 2.1964390 |
|      |           |             |                 |                   |    |         | 308 154.12    | 2.4252800 |
| Ne C1 | 78.55801 | 0.10 | 0.76 | 8  | 45 217.07 | 0.6888615 |
|      |           |             |                 |                   |    |         | 74 442.12     | 1.1347113 |
|      |           |             |                 |                   |    |         | 103 665.54    | 1.5810168 |
|      |           |             |                 |                   |    |         | 132 892.43    | 2.0278524 |
|      |           |             |                 |                   |    |         | 162 117.00    | 2.4751515 |
| 5.3N | −2.364(4) | 78.55501    | 0.75            |                   |    |         | 191 341.44    | 2.9229240 |
|      |           |             |                 |                   |    |         | 220 566.41    | 3.3711908 |
|      |           |             |                 |                   |    |         | 249 792.10    | 3.8199373 |
|      |           |             |                 |                   |    |         | 279 017.42    | 4.2691708 |
|      |           |             |                 |                   |    |         | 308 242.63    | 4.7188454 |
Table 1. Continue.

| Gas | Capacitor | $T_{90}$ (K) | $\kappa_{\text{eff}} \cdot 10^{12}$ | $T_{\text{DCGT}}$ (K) | $u(T_{\text{DCGT}})$ (mK) | $u(T - T_{90})$ (mK) | $p$ (Pa) | $\mu \times 10^4$ |
|-----|-----------|--------------|---------------------------------|----------------|----------------|----------------|--------|----------------|
| He  | C2        | 78.55801     | 0.10                            | 0.64            | 9              | 45 203.61       | 0.3573666 |
|     |           |              |                                 |                 |                | 74 442.48       | 0.5880574 |
|     |           |              |                                 |                 |                | 103 635.26      | 0.8184914 |
|     |           |              |                                 |                 |                | 132 853.02      | 1.0487399 |
|     |           |              |                                 |                 |                | 162 069.68      | 1.2787340 |
| 7N  | $-2.334(5)$ | 78.55409     | 0.64                            |                 |               | 191 284.27      | 1.5084941 |
|     |           |              |                                 |                 |                | 220 500.61      | 1.7380355 |
|     |           |              |                                 |                 |                | 249 718.78      | 1.9673577 |
|     |           |              |                                 |                 |                | 278 933.91      | 2.1964356 |
|     |           |              |                                 |                 |                | 308 149.81      | 2.4252716 |
| Ne  | C2        | 78.55801     | 0.10                            | 0.61            | 8              | 45 216.43       | 0.6888707 |
|     |           |              |                                 |                 |                | 74 442.48       | 1.1347557 |
|     |           |              |                                 |                 |                | 103 663.99      | 1.5810490 |
|     |           |              |                                 |                 |                | 132 891.36      | 2.0279234 |
|     |           |              |                                 |                 |                | 162 116.19      | 2.4752410 |
| 5.3N| $-2.334(5)$ | 78.55335     | 0.60                            |                 |               | 191 339.37      | 2.9230122 |
|     |           |              |                                 |                 |                | 220 564.01      | 3.3712994 |
|     |           |              |                                 |                 |                | 249 787.77      | 3.8200444 |
|     |           |              |                                 |                 |                | 279 012.46      | 4.2692890 |
|     |           |              |                                 |                 |                | 308 236.72      | 4.7190588 |
| He  | C1        | 100.49553    | 0.16                            | 0.54            | 5              | 45 202.94       | 0.2793061 |
|     |           |              |                                 |                 |                | 74 418.80       | 0.4596403 |
|     |           |              |                                 |                 |                | 103 633.61      | 0.6398190 |
|     |           |              |                                 |                 |                | 132 851.11      | 0.8198664 |
|     |           |              |                                 |                 |                | 162 064.39      | 0.9997353 |
| 7N  | $-2.375(4)$ | 100.49025    | 0.51                            |                 |               | 191 282.50      | 1.1794926 |
|     |           |              |                                 |                 |                | 220 496.45      | 1.3590777 |
|     |           |              |                                 |                 |                | 249 713.10      | 1.5385278 |
|     |           |              |                                 |                 |                | 278 928.31      | 1.7178210 |
|     |           |              |                                 |                 |                | 308 147.49      | 1.8969080 |
| Ne  | C1        | 100.49553    | 0.16                            | 0.69            | 12             | 45 215.38       | 0.5380612 |
|     |           |              |                                 |                 |                | 74 440.13       | 0.8859592 |
|     |           |              |                                 |                 |                | 103 663.35      | 1.2339358 |
|     |           |              |                                 |                 |                | 132 889.36      | 1.5820350 |
|     |           |              |                                 |                 |                | 162 113.03      | 1.9302186 |
| 5.3N| $-2.375(4)$ | 100.49076    | 0.68                            |                 |               | 191 337.44      | 2.2784994 |
|     |           |              |                                 |                 |                | 220 561.74      | 2.6268792 |
|     |           |              |                                 |                 |                | 249 786.06      | 2.9753578 |
|     |           |              |                                 |                 |                | 279 010.54      | 3.3239169 |
|     |           |              |                                 |                 |                | 308 235.24      | 3.6725821 |
| He  | C2        | 100.49553    | 0.16                            | 0.58            | 10             | 45 203.35       | 0.2793141 |
|     |           |              |                                 |                 |                | 74 419.78       | 0.4596527 |
|     |           |              |                                 |                 |                | 103 634.46      | 0.6398368 |
|     |           |              |                                 |                 |                | 132 852.28      | 0.8198836 |
|     |           |              |                                 |                 |                | 162 068.09      | 0.9997832 |
| 7N  | $-2.345(5)$ | 100.49014    | 0.55                            |                 |               | 191 283.76      | 1.1795165 |
|     |           |              |                                 |                 |                | 220 499.61      | 1.3591335 |
|     |           |              |                                 |                 |                | 249 715.83      | 1.5385672 |
|     |           |              |                                 |                 |                | 278 931.87      | 1.7178675 |
|     |           |              |                                 |                 |                | 308 148.43      | 1.8970288 |
| Purity | Gas | Capacitor | $T_{90}$ (K) | $u(T_{90})$ (mK) | $u(T_{DCGT})$ (mK) | $u(T - T_{90})$ (mK) | $n$ | $p$ (Pa) | $\mu \times 10^4$ |
|--------|-----|-----------|-------------|------------------|-------------------|---------------------|-----|----------|--------------|
| Ne C2  | 100.49553 | 0.16 | 0.72 | 15 | 45 215.32 | 0.5380638 |
|        |         |         |          |                  |                   |                     |     |          |              |
|        |         |         |          |                  |                   |                     |     |          |              |
|        |         |         |          |                  |                   |                     |     |          |              |
| 5.3N   | 100.49095 | 0.70 | 191 336.16 | 2.2785445 | 220 560.06 | 2.6269271 | 249 784.14 | 2.9754233 | 279 008.54 | 3.3239978 | 308 233.13 | 3.6726806 |
|        |         |         |          |                  |                   |                     |     |          |              |
| Ne C2  | 120.72688 | 0.21 | 0.93 | 7 | 45 209.71 | 0.4476758 |
|        |         |         |          |                  |                   |                     |     |          |              |
|        |         |         |          |                  |                   |                     |     |          |              |
| 5.3N   | 120.72078 | 0.91 | 162 092.96 | 1.6650224 | 191 313.39 | 1.8943267 | 220 532.65 | 2.1836295 | 249 757.94 | 2.4729759 |
|        |         |         |          |                  |                   |                     |     |          |              |
| He C1  | 200.08784 | 0.21 | 1.42 | 5 | 45 201.01 | 0.1401276 |
|        |         |         |          |                  |                   |                     |     |          |              |
|        |         |         |          |                  |                   |                     |     |          |              |
| 7N     | 200.08352 | 1.40 | 191 275.45 | 0.5923383 | 220 489.61 | 0.6826588 | 249 704.56 | 0.7729472 | 278 919.24 | 0.8631889 | 308 133.75 | 0.9533962 |
|        |         |         |          |                  |                   |                     |     |          |              |
| Ne C1  | 200.08784 | 0.21 | 1.46 | 4 | 45 205.70 | 0.2698690 |
|        |         |         |          |                  |                   |                     |     |          |              |
|        |         |         |          |                  |                   |                     |     |          |              |
| 5.3N   | 200.08414 | 1.45 | 191 296.84 | 1.1411692 | 220 512.44 | 1.3152687 | 249 731.97 | 1.4893265 | 278 950.84 | 1.6633257 | 308 167.01 | 1.8372804 |
|        |         |         |          |                  |                   |                     |     |          |              |
| Ar C1  | 200.08784 | 0.21 | 1.61 | 8 | 45 208.07 | 1.1263472 |
|        |         |         |          |                  |                   |                     |     |          |              |
|        |         |         |          |                  |                   |                     |     |          |              |
| 7N     | 200.08283 | 1.60 | 191 306.16 | 4.7867784 | 220 523.98 | 5.5226013 | 249 742.89 | 6.2597047 | 278 961.88 | 6.9980711 | 308 181.28 | 7.7377618 |
Table 1. Continue.

| Gas | Capacitor | \( T_{90} \) (K) | \( u(T_{90}) \) (mK) | \( u(T_{DCGT}) \) (mK) | \( u(T - T_{90}) \) (mK) | \( n \) | \( p \) (Pa) | \( \mu \times 10^4 \) |
|-----|-----------|-----------------|-----------------|-----------------|-----------------|---------|-----------|---------------|
| Purity | \( \kappa_{\text{eff}} \cdot 10^{12} \) | \( T_{DCGT} \) (K) | \( T_{DCGT} \) (mK) | \( T_{DCGT} \) (mK) | \( T_{DCGT} \) (mK) |
| He | C2 | 200.08784 | 0.21 | 2.10 | 8 | 45.201.40 | 0.1401330 |
| | | | | | | | 74.416.94 | 0.2306585 |
| | | | | | | | 103.630.30 | 0.3211356 |
| | | | | | | | 132.846.82 | 0.4115757 |
| | | | | | | | 162.061.65 | 0.5019877 |
| 7N | \(-2.407(3)\) | 200.08439 | 2.09 | 191.276.32 | 0.5923526 |
| | | | | | | | 220.490.22 | 0.6826650 |
| | | | | | | | 249.704.92 | 0.7729558 |
| | | | | | | | 278.919.73 | 0.8632096 |
| | | | | | | | 308.134.54 | 0.9534087 |
| Ne | C2 | 200.08784 | 0.21 | 1.64 | 5 | 45.207.72 | 0.2698888 |
| | | | | | | | 74.431.69 | 0.4442941 |
| | | | | | | | 103.647.27 | 0.6185914 |
| | | | | | | | 132.867.40 | 0.7928694 |
| | | | | | | | 162.074.17 | 0.9670116 |
| 5.3N | \(-2.407(3)\) | 200.08246 | 1.63 | 191.290.48 | 1.1411635 |
| | | | | | | | 220.509.01 | 1.3152743 |
| | | | | | | | 249.724.57 | 1.4893396 |
| | | | | | | | 278.939.72 | 1.6633215 |
| | | | | | | | 308.153.24 | 1.8372484 |
| Ar | C2 | 200.08784 | 0.21 | 1.59 | 5 | 45.208.23 | 1.1263579 |
| | | | | | | | 74.428.21 | 1.8559566 |
| | | | | | | | 103.646.18 | 2.5867560 |
| | | | | | | | 132.867.46 | 3.3188772 |
| | | | | | | | 162.086.31 | 4.0522039 |
| 7N | \(-2.407(3)\) | 200.08313 | 1.57 | 191.304.91 | 4.7867740 |
| | | | | | | | 220.524.05 | 5.5226275 |
| | | | | | | | 249.743.07 | 6.2597425 |
| | | | | | | | 278.963.85 | 6.9981987 |
| | | | | | | | 308.182.83 | 7.7378457 |

second-order fits in temperature equivalent is largest for Neon at 51 K with 6 mK. It is continuously decreasing to a level of 0.7 mK with raising temperature. For argon, the correction at 200 K amounts to 5 mK, and for helium it ranges from 2.5 mK to 0.4 mK. The smallness of the correction together with the very conservative uncertainty attributed to it leads to the fact that for this approach, no high-level \textit{ab initio} calculations are needed. Thus, this approach would also work with semi-classical calculations available since many years. Therefore, it is intentionally different from other approximations using high-level \textit{ab initio} calculations as constraints for both the second and the third virial coefficient, where the corrections due to real-gas properties are on a level of 1 K or more [6]. Nevertheless, the use of highly-accurate \textit{ab initio} values not only for the density but also for the dielectric virial coefficients of neon and argon is continuously improving [29, 30], and it will allow similar evaluation techniques, which are at the moment mostly restricted to helium.

For each isotherm, the \( A_1 \) value obtained with the constraint was used together with the \( A_1 \) values of the third- and fourth-order fits to calculate a weighted mean, with the weights being the inverse uncertainty estimates squared. Since the three single values are largely correlated, the uncertainty of the result was not estimated by the weighted-mean uncertainty, but by the smallest uncertainty of the three \( A_1 \) values. This approach corresponds to the case of full correlation.

For calculating \( T_{DCGT} \) from the weighted-mean \( A_1 \) value, the effective compressibility \( \kappa_{\text{eff}} \) of the measuring capacitor is needed. Its determination in dependence on temperature is briefly described in section 2.2. The resulting consistent dataset \( T_{DCGT} - T_{90} \) in dependence on \( T_{90} \) obtained in the range from 50 K to 200 K with DCGT is given in table 2 together with the standard uncertainty \( u(T_{DCGT} - T_{90}) \) as well as the measuring gas, and the capacitor used. New data is listed for 50 K, 51 K, 60 K, 70 K, 79 K, 100 K, 120 K and 200 K. If different results exist, the weighted mean has been estimated in steps considering correlations. The isotherms obtained with neon at the triple point of argon at 84 K are already given in [8] but the evaluation is shifted due to the new determination of the polarizability of neon with reduced uncertainty [31]. Therefore, the weighted-mean value listed in table 2 for 84 K and, due to an additional neon isotherm (the helium isotherms at 120 K are already listed in [8]), the value at 120 K are revised compared with [8].
Uncertainty budgets to estimate $u(T_{\text{DCGT}} - T_{90})$ have been established individually. Examples are given in table 3 for the three temperatures 51 K, 100 K, and 200 K. In accordance with a recommendation of the Consultative Committee for Thermometry, the non-uniqueness is included in the estimation of the uncertainty of the realisation of the ITS-90 via the calibration of standard platinum resistance thermometers [24].

3. Consistent dataset and function $T_{\text{DCGT}} - T_{90}$ versus $T_{90}$ from 30 K to 200 K

The data presented in table 2 of reference [8] for $T_{\text{DCGT}} - T_{90}$ and $u(T_{\text{DCGT}} - T_{90})$ in the temperature range from 28.5 K to 140 K were not considered in [4]. The combination of this data with the results listed in table 2 of the present paper yields, therefore, a consistent dataset for the temperature range from about 30 K to 200 K that is independent from the evaluation in [4].

For further comparison with literature data, the new consistent dataset was fitted with a fourth-order polynomial. Both unweighted and weighted fits have been compared against a spline interpolation. The squared invers uncertainty estimates were weighted and unweighted fits have been compared against a spline interpolation. Figure 3 shows the final interpolation function $T_{\text{DCGT}} - T_{90}$ versus $T_{90}$ being the fourth-order polynomial obtained from the unweighted fit:

$$ (T - T_{90})_{\text{PTB2020}}/\text{mK} = \sum_{i=0}^{4} a_i (T_{90}/\text{K})^i $$

with $a_0 = 0.3260$, $a_1 = 0.013628$, $a_2 = -0.001506$, $a_3 = 1.0079 \times 10^{-5}$, $a_4 = -1.7443 \times 10^{-8}$, which is valid for the temperature range from 30 K to 200 K.

Figure 1 shows also data points $(T_{\text{DCGT}} - T_{90}; T_{90})$ together with bars representing their standard uncertainty estimates $u(T_{\text{DCGT}} - T_{90})$. The data points taken from literature are discussed in the following section.

4. Updated function $T - T_{90}$ versus $T_{90}$ from 35 K to 195 K

For updating the information on $T - T_{90}$ versus $T_{90}$, it seems to be the easiest and most clearly arranged way to combine the recommendations of the Consultative Committee for Thermometry with the new data at the ‘base temperatures’ selected in [4]. Rourke [6] published three $T - T_{90}$ values measured with RIGT directly at such temperatures, see table 4. To deduce corresponding values from the results obtained with AGT [5], a fourth-order polynomial was fitted to the data points, as it was done to obtain the function $(T_{\text{DCGT}} - T_{90})_{2000}$ (equation 5). This fitting polynomial and the function $(T_{\text{DCGT}} - T_{90})_{\text{PTB2020}}$ were used to calculate the values given in table 4 for AGT and DCGT, respectively. In both cases uncertainty estimates were deduced applying spline interpolations.

Furthermore, in table 4, weighted-mean values for $T - T_{90}$ are given for the following two combinations: (1): AGT, DCGT, and RIGT; (2): AGT, DCGT, RIGT, and CCT (values from [4]). The weighted-mean values obtained for combination (2) were approximated (with the constraint $(T - T_{90}) = 0$ mK at 273.16 K) by the polynomial

$$ (T - T_{90})_{\text{ARD2020}}/\text{mK} = \sum_{i=0}^{4} b_i (T_{90}/\text{K})^i $$

with $b_0 = 0.861199$, $b_1 = -0.023377$, $b_2 = -0.000588$, $b_3 = 1.293 \times 10^{-5}$, $b_4 = 8.277 \times 10^{-9}$, which is valid for the temperature range from 35 K to 195 K. Polynomial (6) is shown in figure 2 together with the function $(T - T_{90})_{\text{PTB2011}}$ from [4] and a spline interpolation between the weighted-mean values for combination (1) given in table 4. The corresponding shaded areas display the confidence interval corresponding to the standard uncertainty.

Figure 2 gives the impression that the estimates of the thermodynamic temperature, being based on the critical review of previous $T - T_{90}$ determinations performed by a working group of the Consultative Committee for Thermometry [4], are slightly too low, but the order of magnitude of the uncertainty is realistic. The new data, combined in the weighted-mean values for combination (1), have an uncertainty, which is
Table 3. Single-isotherm uncertainty budgets for the difference values $T_{\text{DCGT}} - T_{90}$ determined in this paper. The estimates are given in mK at selected temperatures as representative examples.

| Component                        | 51 K | 51 K | 100 K | 100 K | 200 K | 200 K | 200 K |
|----------------------------------|------|------|-------|-------|-------|-------|-------|
| Measuring gas                    | He   | Ne   | He    | Ne    | He    | Ne    | Ar    |
| Measuring capacitor              | C1   | C2   | C1    | C2    | C1    | C2    | C1    |

**Type A component**
- Fit coefficient $A_1$:
  - 0.22

**Type B estimates**
- Susceptibility measurement (capacitance change): 0.02
- Determination of the effective compressibility: 0.06
- $T_{90}$ (realisation of the ITS-90): 0.12
- Pressure (effective area): 0.18
- Head correction $^b$: 0.04
- Impurities in the measuring gas: 0.05
- Surface layers (impurities): 0.03
- Polarizability (He: theory, Ar and Ne: experiment): 0.01
- Combined standard uncertainty of $T_{\text{DCGT}} - T_{90}$ $^c$: 0.32

$^a$These uncertainty estimates are based on recommendations of the Consultative Committee for Thermometry, calibrations and comparisons of pressure balances for determining the effective area, on literature data for the typical thickness of impurity layers on the capacitor plates, and analysis data (mass spectroscopy) for impurity concentrations in the measuring gas.

$^b$This component is based on Monte-Carlo simulations performed with one hundred data sets randomized with the standard deviation of the input quantities.

$^c$This estimate considers correlations between the different experimental data.

Figure 1. Recent determinations of the difference $T - T_{90}$ between thermodynamic temperature, $T$, and temperature on the ITS-90, $T_{90}$, obtained by different primary-thermometry methods. The bars represent the confidence interval corresponding to the standard uncertainty. The weighted mean DCGT values $T_{\text{DCGT}} - T_{90}$ are marked as follows: Filled red dots: New data listed in table 2; Filled grey dots: Data published in [8]; Half grey and half red dots: Values from [8] readjusted at 84 K due to a new polarizability value of neon and at 120 K due to an additional neon isotherm, respectively. The red line represents a fourth-order polynomial obtained from an unweighted fit to the DCGT data from about 30 K to 200 K (Polynomial 5). The black line displays the best fit of a critical review of previous $T - T_{90}$ determinations performed by a working group of the Consultative Committee for Thermometry [4] (This function is called in the present paper $(T - T_{90})_{LT2011}$). In addition, literature data are included for comparison: Blue stars: AGT by Underwood et al 2016 [5]; Black filled squares: RIGT by Rourke 2020 [6].
### Table 4. Overview of results for the differences between thermodynamic temperature \(T\) and temperature \(T_{\text{90}}\) on the International Temperature Scale of 1990 (ITS-90) at ‘base temperatures’ selected in [4]. The second to seventh column contain recent data obtained by AGT [5], DCGT (this paper), and RIGT [6], respectively, and the accompanying standard uncertainty estimates \(u(T - T_{\text{90}})\). The AGT and DCGT data have been deduced applying fourth-order polynomials fitted to the experimental pairs \((T_{\text{90}}; (T - T_{\text{90}}))\), see text. Columns eight to eleven show weighted-mean values together with their uncertainty estimates. Combination (1) considers the information given in the second to seventh column. Combination (2) includes also the values from [4]. The \((T - T_{\text{90}})\) values for combination (2) have been approximated by function \((T - T_{\text{90}})_{\text{ARD2020}}\) (Polynomial 6). All differences and uncertainty estimates are given in mK.

| \(T_{\text{90}}/\text{K}\) | AGT \((T - T_{\text{90}})\) | DCGT \(u(T - T_{\text{90}})\) | RIGT \((T - T_{\text{90}})\) | Combination (1) \((T - T_{\text{90}})\) | Combination (2) \((T - T_{\text{90}})\) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 35              | -0.64           | 0.29            | -0.61           | -0.64           | -0.64           |
| 45              | -1.26           | 0.47            | -1.20           | -1.26           | -1.26           |
| 54              | -1.92           | 0.29            | -1.90           | -1.93           | -1.93           |
| 70              | -3.06           | 0.39            | -3.04           | -3.06           | -3.06           |
| 78              | -3.61           | 0.42            | -3.59           | -3.61           | -3.61           |
| 84              | -4.04           | 0.44            | -4.05           | -4.04           | -4.04           |
| 90              | -4.44           | 0.46            | -4.51           | -4.44           | -4.44           |
| 100             | -5.03           | 0.49            | -5.17           | -5.03           | -5.03           |
| 130             | -6.95           | 0.44            | -6.98           | -6.89           | -6.89           |
| 161             | -7.16           | 0.42            | -7.14           | -7.07           | -7.07           |
| 195             | -4.39           | 0.35            | -4.51           | -4.43           | -4.43           |

Figure 2. Estimates for the difference \(T - T_{\text{90}}\) between thermodynamic temperature, \(T\), and temperature on the ITS-90, \(T_{\text{90}}\). The two functions \((T - T_{\text{90}})_{\text{LT2011}}\) (from [4]) and \((T - T_{\text{90}})_{\text{ARD2020}}\) (Polynomial 6) are shown as black and red line, respectively. The corresponding shaded areas display the confidence intervals corresponding to the standard uncertainty obtained by spline interpolation. The symbols represent weighted-mean values of recent results obtained with AGT, DCGT, and RIGT, cf Table 4. They are differently marked depending on whether it is a pure DCGT input or a weighted mean between DCGT & RIGT, DCGT & AGT or DCGT & AGT & RIGT. The blue line is a spline interpolation between all symbols (combination (1)).

much smaller (up to a factor of about four) than the uncertainty estimates presented in [4]. This leads in turn to a significant reduction of the uncertainty of the updated function \(T - T_{\text{90}}\) versus \(T_{\text{90}}\) from 35 K to 195 K, called \((T - T_{\text{90}})_{\text{ARD2020}}\) (Polynomial 6), which approximates the overall weighted-mean values (combination (2)). It should be emphasised that considering the combined uncertainties, the DCGT results are consistent with the AGT ones. This is of crucial importance because the sources of error of these two primary-thermometry methods are quite different [32].
5. Summary and conclusions

New data is presented for the difference between the thermodynamic temperature measured with DCGT, $T_{DCGT}$, and the temperature on the ITS-90, $T_{90}$, at 50 K, 51 K, 60 K, 70 K, 79 K, 84 K, 100 K, 120 K, and 200 K. Particular care was taken to check for possible systematic sources of uncertainty by performing experiments applying three working gases, namely helium, neon, and argon, the polarizability of which differs by a factor of up to eight. The new data $T_{DCGT} - T_{90}$ are combined with former thermodynamic results obtained with DCGT at PTB to obtain a consistent function $T_{DCGT} - T_{90}$ versus $T_{90}$ for the temperature range from about 30 K to 200 K, called $(T - T_{90})_{PTB2020}$. In turn, this function is combined with the ‘Estimates of the differences between thermodynamic temperature and the ITS-90’ [1, 4] as well as with thermodynamic data obtained applying AGT [5] and RIGT [6]. The final result is an updated function $T - T_{90}$ versus $T_{90}$ for the temperature range from 35 K to 195 K with significantly reduced uncertainty (up to a factor of about four), called $(T - T_{90})_{ARD2020}$. Since the sources of error of the two primary-thermometry methods DCGT and AGT are quite different, the observed consistency is a good basis for a high quality of updating the $T - T_{90}$ estimates. The presented data as well as unpublished and therefore not included results from other institutes were driven by different joint European projects during the last years (for more details see [33–35]). The function $(T - T_{90})_{ARD2020}$ will be submitted to the Working Group for Contact Thermometry of the Consultative Committee for Thermometry (CCT-WG-CTh) for preparing new recommendations on $T - T_{90}$ versus $T_{90}$ in the low-temperature part of the ITS-90 (0.65 K to 273 K).

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