Robust tracking control for two classes of variable stiffness actuators based on linear extended state observer with estimation error compensation

Jishu Guo*, Junmei Guo and Zhongjun Xiao

Abstract
In this article, a novel robust tracking control scheme based on linear extended state observer with estimation error compensation is proposed for the tracking control of the antagonistic variable stiffness actuator based on equivalent nonlinear torsion spring and the serial variable stiffness actuator based on lever mechanism. For the dynamic models of these two classes of variable stiffness actuators, considering the parametric uncertainties, the unknown friction torques acting on the driving units, the unknown external disturbances acting on the output links and the input saturation constraints, an integral chain pseudo-linear system with input saturation constraints and matched lumped disturbances is established by coordinate transformation. Subsequently, the matched lumped disturbances in the pseudo-linear system are extended to the new system states, and we obtain an extended integral chain pseudo-linear system. Then, we design the linear extended state observer to estimate the unknown states of the extended pseudo-linear system. Considering the input saturation constraints in the extended pseudo-linear system and the estimation errors of the linear extended state observer with fixed preset observation gains, the adaptive input saturation compensation laws and the novel estimation error compensators are designed. Finally, a robust tracking controller based on linear extended state observer, sliding mode control, adaptive input saturation compensation laws, and estimating error compensators is designed to achieve simultaneous position and stiffness tracking control of these two classes of variable stiffness actuators. Under the action of the designed controller, the semi-global uniformly ultimately bounded stability of the closed-loop system is proved by the stability analysis of the candidate Lyapunov function. The simulation results show the effectiveness, robustness, and adaptability of the designed controller in the tracking control of these two classes of variable stiffness actuators. Furthermore, the simulation comparisons show the effectiveness of the proposed estimation error compensation measures in reducing the tracking errors and improving the disturbance rejection performance of the controller.

Keywords
Variable stiffness actuator, linear extended state observer, disturbance observer, sliding mode control, input saturation compensation, estimation error compensation, robust tracking control

Introduction
The variable stiffness actuator (VSA) has such characteristic that the output link position and the joint output stiffness are independently controllable. The inherent
flexibility and joint stiffness adjustability make the VSA suitable for bionic robot arms, rehabilitation training equipments, lower-limb exoskeletons, and robotic-assisted walking devices. The tracking control of the output link position and the joint output stiffness are two basic control objectives of the VSA. Good output link position and joint output stiffness tracking control will help to improve the actuation performance of the VSA. Therefore, the tracking control of the position and stiffness of the VSA has important research significance.

In order to realize tracking control of the VSAs, many control strategies have been investigated over several years. The proportional derivative (PD) controllers are used for tracking control of the position and stiffness of the VSA-II, the actuator with adjustable stiffness (AwAS), the AwAS-II and the Compact-VSA. The simultaneous and independent control of the output link position and joint output stiffness of the variable stiffness actuator-2 designed by University of Twente (vsaUT-II) is realized by using two proportional integral differential (PID) controllers. The PD feedback plus feedforward controllers are used for both position and stiffness regulation of the serial variable stiffness actuator (SVSA) and the SVSA-II. A gain scheduling controller based on a set of linear quadratic regulators is designed to regulate both stiffness and position of the VSAs in series configurations using lever mechanisms. A command filtered backstepping approach is designed for the tracking control of the multi-degree-of-freedom (DOF) variable stiffness robots. The feedback linearization approach is designed to achieve tracking control of the desired position and stiffness of the antagonistic VSA and the SVSA. The feedback linearization controller with integral terms and the sliding mode controller with integral terms are designed for the tracking control of the position and stiffness of the VSAs, and the integral control is used to eliminate the tracking errors caused by model uncertainties and disturbances. A robust control method combining adaptive control and mapping filtered forwarding technique is developed for tracking control of the output link position of the antagonistic tendon-driven joint. An adaptive neural network control approach combining disturbance observer (DOB) is proposed for tracking control of the output link position of the robotic systems based on variable stiffness joints. A low-complexity state feedback controller with prescribed performance of the tracking errors and increased robustness to external perturbation is developed for the tracking control of the output link position of the multi-DOF variable stiffness actuated robots. The adaptive robust tracking controller based on feedback linearization and radial basis function neural network (RBFNN) is proposed for simultaneous position and stiffness tracking control of the VSAs based on lever mechanisms.

In the above studies, many control strategies do not take specific measures to ensure the robustness of the tracking control of the VSAs. The control methods using integral terms to eliminate tracking errors of the VSAs may have undesirable transient tracking performances. The control schemes designed by Zhang et al. and Psomopoulos et al. are mainly aimed at the tracking control of the output link position of the multi-DOF robot based on the variable stiffness joints, not at the simultaneous position and stiffness tracking control of the single VSA. The robust tracking controller designed by Lu et al. is used to realize tracking control of the output link position of the antagonistic tendon-driven joint, and this controller is not suitable for simultaneous position and stiffness tracking control of the VSA. The RBFNN is used to ensure the robustness of the controller but the control input saturation problem is not considered in the design of the controller. As mentioned above, the current studies on the robust tracking control of the position and stiffness of the VSAs are not sufficient. Therefore, in order to improve the actuation performance of the VSAs, it is necessary to further study the robust tracking control method for the VSAs.

In the current studies on robust tracking control, DOB, extended state observer (ESO) are two popular methods to effectively deal with the uncertainties and the disturbances in the systems. Generally, the ESOs can be classified into the linear extended state observers (LESOs) and the nonlinear extended state observers (NESOs). The gain parameters of the NESOs are usually difficult to set, especially for the high-order systems. Compared with the LESO, the LESO has the advantage of convenient gain parameter setting, and the control schemes based on the LESO have been applied to various systems. For the antagonistic VSAs based on equivalent nonlinear torsion springs (ENTSs) in the presence of parametric uncertainties, unknown friction torques, unknown external disturbance, and input saturation constraints, a combination of the LESO, the sliding mode control (SMC), and the adaptive input saturation compensation (ISC) is designed to achieve the simultaneous position and stiffness tracking control of the VSAs. However, in the above studies, the estimation errors were not considered explicitly in the DOB-based control or the LESO-based control.

Similar to the gain settings of the LESO in many other types of control systems, the LESO with fixed preset observation gains is used in the tracking control of the antagonistic VSA based on ENTS. However, the LESO with fixed preset observation gains always has estimation errors, especially when the reference trajectories change abruptly (i.e. the tracking errors are large), or the time-varying disturbances are imposed on the system, or the disturbances in the system are large, or the disturbances imposed on the system suddenly change. Increasing the gains of the LESO is beneficial to reducing the estimation errors, but the gains of the LESO should not be set too large. Excessive LESO gains may amplify the influence of measurement noises on tracking performance, or may lead to instability of the system. Considering that the estimation errors of the LESO with fixed preset observation gains will inevitably have a negative impact on the tracking
accuracy of the VSA, and in order to improve the robustness of the tracking control, it is necessary to design the estimation error compensation (EEC) measures to reduce the tracking errors of the system outputs (i.e. position and stiffness) and improve the disturbance rejection performance of the controller.

Among many kinds of VSAs, the antagonistic VSAs based on ENTS and the SVSAs based on lever mechanisms are two classes of widely studied VSAs. In this article, in order to demonstrate the adaptability of the designed controller, the system models of these two classes of VSAs are studied as examples. To the best of our knowledge, this is the first study to design a robust tracking controller based on LESO with EEC for the antagonistic VSA based ENTS and the SVSA based on lever mechanism. The main contributions of this article are stated as follows.

1. The novel estimation error compensators are developed to approximate the estimation errors of the traditional LESO with fixed preset observation gains. The proposed estimation error compensators are inspired by the stable single input single output (SISO) DOB. Each estimating error compensator has the characteristic of simple parameter adjustment, and the performance of each estimation error compensator can be tuned easily by only one parameter. The estimated values provided by the estimation error compensators are introduced into the controller to reduce the system output tracking errors. The design of the estimation error compensator reflects the flexibility and expansibility of the application of SISO DOB. As far as our knowledge goes, this is the first study to design robust tracking controller by skillfully combining the SISO DOB and the LESO. By combining the SISO DOB with the LESO, the tracking accuracy and disturbance rejection performance of the controller are improved.

2. A novel robust tracking control approach based on LESO with EEC is proposed for the tracking control of these two classes of VSAs (i.e. the antagonistic VSA based on ENTS and the SVSA based on lever mechanism) in the presence of parametric uncertainties, unknown friction torques, unknown external disturbances, and control input saturation. Furthermore, the semi-global ultimate uniformly bounded stability of the closed-loop systems is proved based on the stability analysis of the candidate Lyapunov function, and the tuning rules of the controller parameters are discussed. Finally, the simulation results show the effectiveness, robustness, and good adaptability of the designed controller. The proposed design idea of reducing the tracking error can be extended to the tracking control of other types of systems to improve the tracking accuracy and disturbance rejection performance of the controller.

This article is organized as follows. The system models of these two classes of VSAs (i.e. the antagonistic VSA based on ENTS and the SVSA based on lever mechanism) are described in the second section. The coordinate transformation and the LESO design for these two classes of VSAs are presented in the third section. The novel robust tracking controller based on the LESO with EEC is designed in the fourth section. The simulation studies are carried out in the fifth section. Finally, conclusions are drawn in the sixth section.

System models of these two classes of VSAs

In this section, the dynamic models of the antagonistic VSAs based on ENTSs and the SVSAs based on lever mechanisms are shown. Although the stiffness characteristics of the ENTSs of the antagonistic VSAs may be different, the antagonistic VSAs based on the ENTSs have the same structural form of the dynamic model. In this article, in order to select representative antagonistic VSA without losing generality, the dynamic model of antagonistic VSA based on equivalent exponential torsion spring (EETS) is selected as an example.

For the SVSAs based on lever mechanisms, although the existing VSAs of this class have different mechanical implementation schemes of variable stiffness mechanisms, all of these VSAs have the same transmission scheme design with series configuration, and these VSAs have the same structural form of the dynamic model. As a representative device in the class of the SVSAs based on lever mechanisms, the highly integrated Compact-VSA is chosen as the study object. It should be noted that although the dynamic models of the antagonistic VSA based on EETS (EETS-based VSA) and the Compact-VSA are taken as examples in the controller design, the proposed controller will be applicable to other antagonistic VSAs based on ENTS and the SVSAs based on lever mechanisms.

System dynamic model of the antagonistic VSAs based on ENTS

The nominal dynamic model of the antagonistic VSA based on ENTS can be written as

\[
\begin{align*}
M\ddot{\alpha} + D\dot{\alpha} + \varphi(\alpha, \beta, q) + E_g\sin(q) &= 0 \\
J_m\ddot{\beta} + D_m\dot{\beta} + \psi(\beta, q) &= \tau_eta
\end{align*}
\]

where the \(M\) is the inertia constant of the output link of the joint, \(J_m\) is the inertia of the elastic actuation unit, \(D\) is the damping coefficient of the output link, \(D_m\) is the damping coefficient of the elastic actuation unit, \(E_g = mgd\) is the gravity effect acting on the output link, \(m\) is the equivalent mass of the output link, \(d\) is the distance from the rotating...
axis of the output link to its center of mass, $q$ is the angular position of the output link, $\alpha$ and $\beta$ are the angular positions of two elastic actuating units, respectively. $\tau_\alpha$ and $\tau_\beta$ are the torques commanded to the elastic actuating units, respectively. $\phi(\alpha, q)$ and $\psi(\beta, q)$ are the elastic actuating torques acting on the output link by the two elastic actuating units. $\varphi(\alpha, \beta, q)$ is the combined effect of the elastic actuating torques acting on the output link of the VSA.

Considering the uncertainties of the system dynamic model parameters, the unknown friction torques (i.e. $\tau_f(\alpha)$ and $\tau_f(\beta)$), the unknown external disturbance (i.e. $\tau_{\text{ext}}$) acting on the output link and the control input saturation, the actual dynamic model of the antagonistic VSA based on the ENTSs can be rewritten in the following form

\[
\begin{align*}
M_\alpha \ddot{q} + D_q\dot{q} + \varphi(\alpha, \beta, q) + E_{gr}\sin(q) &= \tau_{\text{ext}} \\
J_m \ddot{\alpha} + D_m \dot{\alpha} + \phi(\alpha, q) + \tau_{\text{fa}} &= \text{sat}(\tau_\alpha) \\
J_m \ddot{\beta} + D_m \dot{\beta} + \psi(\beta, q) + \tau_{\text{fb}} &= \text{sat}(\tau_\beta)
\end{align*}
\]

where the actual model parameters (i.e. $M_\alpha$, $D_q$, $E_{gr}$, $J_m$, $D_m$) and the unknown bounded parameter perturbation (i.e. $\Delta$) can be expressed in the following form

\[
\begin{align*}
\Delta M &= M_1 - M; \quad \Delta J_m = J_{mt} - J_m; \quad \Delta D_q = D_{qt} - D_q \\
\Delta D_m &= D_{mt} - D_m; \quad \Delta E_{gr} = E_{gr} - E_g
\end{align*}
\]

The control input saturation models are described in equation (4). The actual control input of the dynamic model of the antagonistic VSA based on ENTS is $\text{sat}(\tau_i)$ ($i = \alpha$, $\beta$). The $\tau_{\text{max}}$ and $\tau_{\text{min}}$ are the known bounds of the $\tau_i$ ($i = \alpha$, $\beta$)

\[
\text{sat}(\tau_i) = \begin{cases} 
\tau_{\alpha_{\text{max}}}; & \text{if } \tau_\alpha \geq \tau_{\alpha_{\text{max}}} \\
\tau_\alpha; & \text{if } \tau_{\alpha_{\text{min}}} < \tau_\alpha < \tau_{\alpha_{\text{max}}} \\
\tau_{\alpha_{\text{min}}}; & \text{if } \tau_\alpha \leq \tau_{\alpha_{\text{min}}} 
\end{cases}
\]

\[
\text{sat}(\tau_\beta) = \begin{cases} 
\tau_{\beta_{\text{max}}}; & \text{if } \tau_\beta \geq \tau_{\beta_{\text{max}}} \\
\tau_\beta; & \text{if } \tau_{\beta_{\text{min}}} < \tau_\beta < \tau_{\beta_{\text{max}}} \\
\tau_{\beta_{\text{min}}}; & \text{if } \tau_\beta \leq \tau_{\beta_{\text{min}}} 
\end{cases}
\]

The antagonistic EETS-based VSA is a representative device in the class of antagonistic VSA. In this article, the tracking control of the EETS-based VSA is used to show the tracking performance of the designed controller. As presented in Guo and Tian,\textsuperscript{40} the elastic actuating torques (i.e. $\phi(\alpha, q)$ and $\psi(\beta, q)$) and the joint output stiffness $k_{\eta_0}$ of the antagonistic EETS-based VSA are given by

\[
\phi(\alpha, q) = ae^{b(\alpha + qt)} - a; \quad \psi(\beta, q) = ae^{b(\beta + qt)} - a \\
\varphi(\alpha, \beta, q) = \phi(\alpha, q) - \psi(\beta, q) \\
k_{\eta_0} = \frac{\partial \varphi(\alpha, \beta, q)}{\partial q} = abe^{b(\alpha + qt)} + abe^{b(\beta + qt)} \geq k_{\eta_0}
\]

where $k_{\eta_0}$ is the initial joint stiffness corresponding to $\alpha = \beta = q = 0$, $a = 0.1753$ and $b = 1$ are the predefined parameters of the EETS. Assuming that the output shaft of the EETS-based VSA is always in the antagonistic state, and the initial amounts of pretension of the two EETSs are defined as $\alpha_0 = 2.3$ rad and $\beta_0 = 2.3$ rad, respectively. The assumed nominal parameter values of the dynamic model of the EETS-based VSA are presented as follows: $M = 0.075$ kg · m$^2$, $J_m = 0.015$ kg · m$^2$, $D_q = 1.5$ N · m · s/rad, $D_m = 0.075$ N · m · s/rad, $m = 2$ kg, $d = 0.15$ m, $g = 9.8$ N/kg and $E_g = mgd = 2.94$ N · m.

### System dynamic model of the SVSAs based on lever mechanisms

The VSA with series configuration based on lever mechanism consists of a primary motor, used to adjust the angular position of the output link of the joint through the equivalent elastic transmission unit, and of a secondary motor, used to regulate the equivalent stiffness of elastic transmission unit by the equivalent lever mechanism. The nominal dynamic model of the SVSA based on lever mechanism\textsuperscript{7–9,11–13,16,23} is given by

\[
\begin{align*}
M_\alpha \ddot{q} + D_q\dot{q} + \tau_c + E_{gr}\sin(q) &= \tau_{\text{ext}} \\
J_p \ddot{\theta}_p + D_p \dot{\theta}_p - \tau_c &= \tau_p \\
J_s \ddot{\theta}_s + D_s \dot{\theta}_s + \tau_r &= \tau_s
\end{align*}
\]

where $M$, $J_p$, and $J_s$ are the equivalent inertia moments of the system; $D_q$, $D_p$, and $D_s$ are the viscous friction coefficients; $\tau_c$ is the elastic actuating torque, $\tau_r$ is the coupled elastic torque produced by the angular deflection of the elastic transmission that acts against the joint stiffness control component, $\tau_p$ and $\tau_s$ are the control torques associated with the joint position control component and the joint stiffness control component, respectively; $q$ is the angular position of the output link of the joint; $\theta_p$ and $\theta_s$ are the angular positions associated with the joint position control component and the joint stiffness control component, respectively.

Considering the parametric uncertainties, the unknown external disturbance (i.e. $\tau_{\text{ext}}$), the unknown friction torques (i.e. $\tau_f(p)$ and $\tau_f(s)$), and the control input saturation constraints (i.e. $\text{sat}(\tau_p)$ and $\text{sat}(\tau_s)$), the actual dynamic model of the SVSA based on lever mechanism can be rewritten as

\[
\begin{align*}
M_\alpha \ddot{q} + D_q\dot{q} + \tau_c + E_{gr}\sin(q) &= \tau_{\text{ext}} \\
J_p \ddot{\theta}_p + D_p \dot{\theta}_p - \tau_c &= \text{sat}(\tau_p) \\
J_s \ddot{\theta}_s + D_s \dot{\theta}_s + \tau_r &= \text{sat}(\tau_s)
\end{align*}
\]
\[
\Delta M = M_1 - M; \quad \Delta J_p = J_{pt} - J_p; \quad \Delta J_s = J_{st} - J_s \\
\Delta D_q = D_{q2} - D_q; \quad \Delta D_p = D_{p2} - D_p; \quad \Delta D_s = D_{st} - D_s \\
\Delta E_g = E_{gt} - E_g
\]

(8)

The Compact-VSA\(^9\) is a representative device in the class of the SVSAs based on lever mechanisms, and the tracking control of the Compact-VSA is used to show the tracking performance of the designed controller. For the Compact-VSA system, the elastic torque \(\tau_e\), the coupled elastic torque \(\tau_r\), and the joint output stiffness \(k_{qj}\) are given by

\[
\tau_e = \frac{2K_s\delta^2\phi}{(\Delta - \delta)^2}; \quad \tau_r = \frac{2K_s n^2 \theta_s \phi^2 \Delta^3}{(\Delta - n\theta_s)^3}; \quad k_{qj} = \frac{2K_s \delta^2 \Delta^2}{(\Delta - \delta)^2}
\]

(9)

where \(\phi = q - \theta_s\) is the angular difference caused by the elastic transmission, \(\Delta(0.015 \text{ m})\) is the length of the lever arm, \(n(0.006 \text{ m/rad})\) is the transmission ratio between the rack and pinion mechanism, \(\delta_1 = n\theta_s\) is the distance from the pivot point to the center of rotation of the joint, \(K_s(10,000 \text{ N/m})\) is the equivalent spring rate, \(m(2 \text{ kg})\) is the assumed mass of the output link, \(a(0.2 \text{ m})\) is the assumed distance from the rotating axis of the output link to its center of mass, \(E_g = mgd\) is the gravity effect acting on the output link, and \(k_{qj}\) is the joint output stiffness. Other model parameters of the Compact-VSA are presented as follows: \(D_p(10.2763 \text{ N·m·s/rad}), D_q(0.1316 \text{ N·m·s/rad}), D_g(0.2 \text{ N·m·s/rad}(\text{assumed value})), M(0.072 \text{ kg·m}^2), J_p(0.1055 \text{ kg·m}^2), J_s(0.000795 \text{ kg·m}^2)\).

State space equation of the antagonistic VSA based on ENTS

For the dynamic model of the antagonistic VSA based on ENTS shown in equation (2), let the system be \(q = x_1, \dot{q} = x_2, \alpha = x_3, \dot{\alpha} = x_4, \beta = x_5, \dot{\beta} = x_6\), that is, \(x = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [q, \dot{q}, \alpha, \dot{\alpha}, \beta, \dot{\beta}]^T\), then the state space equation is given by

\[
\dot{x} = f(x) + g(x)\text{sat}(u) + d_w
\]

\[
y = h(x) = \left[ \begin{array}{c}
h_1(x) \\
h_2(x) \\
k_{qj}\end{array} \right] = \left[ \begin{array}{c}
q \\
\dot{q} \\
k_{qj}\end{array} \right];
\]

(10)

where \(x \in \mathbb{R}^6\) is the system state vector, \(\text{sat}(u) \in \mathbb{R}^2\) is the control input vector, \(d_w \in \mathbb{R}^6\) is the composite disturbance vector, \(\Delta u_i\) is the difference between the actual control input \(u_i = \alpha, \beta\). The \(f(x) \in \mathbb{R}^6\) and \(g(x) \in \mathbb{R}^{6×2}\) shown in equation (10) are given by equation (11)

\[
f(x) = \left[ \begin{array}{c}
-D_q x_2 - \frac{\phi(x_1, x_3, x_5)}{M} - \frac{E_g \sin(q)}{M} \\
x_4 \\
-D_m x_2 - \frac{\phi(x_1, x_3)}{J_m} \\
x_6 \\
\end{array} \right]
\]

(11)

The composite disturbance vector \(d_w \in \mathbb{R}^6\) shown in equation (10) can be expressed as

\[
\left[ \begin{array}{c}
d_{w1} \\
d_{w2} \\
d_{w3} \\
d_{w4} \\
d_{w5} \\
d_{w6} \\
\end{array} \right] = \left[ \begin{array}{c}
-\frac{\Delta M x_2}{M} - \frac{\Delta D_q x_2}{M} - \frac{\Delta E_g}{M} \sin(q) + \frac{\tau_{\text{ext}}}{M} \\
0 \\
-\frac{\Delta J_m}{J_m} x_4 - \frac{\Delta D_m}{J_m} x_4 - \frac{\tau_{f\alpha}}{J_m} \\
0 \\
-\frac{\Delta J_m}{J_m} x_6 - \frac{\Delta D_m}{J_m} x_6 - \frac{\tau_{f\beta}}{J_m} \\
\end{array} \right]
\]

(12)

State space equation of the SVSA based on lever mechanism

Invoking the actual dynamic model of the SVSA based on lever mechanism shown in equation (7) and let the system be \(x = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [q, \dot{q}, \theta_p, \theta_p, \theta_s, \dot{\theta}_s]^T\), then the state space equation of the SVSA is given by

\[
\dot{x} = f(x) + g(x)\text{sat}(u) + d_w
\]

\[
y = h(x) = \left[ \begin{array}{c}
h_1(x) \\
h_2(x) \\
k_{qj}\end{array} \right] = \left[ \begin{array}{c}
q \\
\dot{q} \\
k_{qj}\end{array} \right];
\]

sat(u) = \left[ \begin{array}{c}
\text{sat}(\tau_{\alpha}) \\
\text{sat}(\tau_{\beta}) \\
\text{sat}(u_{\alpha}) \\
\text{sat}(u_{\beta}) \\
\end{array} \right] = \left[ \begin{array}{c}
u_{\alpha} + \Delta u_{\alpha} \\
u_{\beta} + \Delta u_{\beta} \\
\end{array} \right]
\]

(13)

where \(x \in \mathbb{R}^6\) is the system state vector, \(\text{sat}(u) \in \mathbb{R}^2\) is the control input vector, \(\Delta u_i\) is the difference between the
actual control input $sat(u_i)$ and the designed control input $u_i(i = p, s)$. The disturbance vector $d_w \in \mathbb{R}^6$ is given by

\[
\begin{bmatrix}
    d_{w1} \\
    d_{w2} \\
    d_{w3} \\
    d_{w4} \\
    d_{w5} \\
    d_{w6}
\end{bmatrix} =
\begin{bmatrix}
    -\frac{\Delta M}{M} x_2 - \frac{\Delta D_q}{M} x_2 - \frac{\Delta E_g}{M} \sin(x_1) + \frac{\tau_{ext}}{M} \\
    0 \\
    -\frac{\Delta J_p}{J_p} x_4 - \frac{\Delta D_p}{J_p} x_4 - \frac{\tau_{fp}}{J_p} \\
    0 \\
    -\frac{\Delta J_s}{J_s} x_6 - \frac{\Delta D_s}{J_s} x_6 - \frac{\tau_{fs}}{J_s} \\
    0
\end{bmatrix}
\]  

(14)

### Coordinate transformation and LESO design for these two classes of VSAs

In this section, by using the coordinate transformation, the state space models with composite disturbances and input saturation constraints shown in equations (10) and (13) can be transformed into an integral chain pseudo-linear system with matched lumped disturbances and

input saturation constraints. Subsequently, an extended integral chain pseudo-linear system is obtained by treating the matched lumped disturbances as the new system states, and the LESO is designed to estimate the unknown system states of the extended integral chain pseudo-linear system.

Although the antagonistic VSAs based on ENTSs and the SVSAs based on lever mechanisms have different variable stiffness principles and variable stiffness mechanisms, these two classes of VSAs have the same structural forms of the state space models. Therefore, the designed LESO will be applicable to these two classes of VSAs.

### Coordinate transformation and system model transformation

By using the coordinate transformation shown in equation (15), equations (10) and (13) can be transformed into a pseudo-linear system, as shown in equation (16). The matched lumped disturbance vector $v_d \in \mathbb{R}^2$ is given by equation (17). The description about the matrices $A_o$, $B_o$, and $C_o$ can be referred to Guo and Tian.

For the antagonistic VSA based on ENTS, the new saturation control input $sat(v_c) \in \mathbb{R}^2$ is given by
where \( \Delta v_c \) is the saturation constraint. The LESO with input saturation constraints is proposed such that the position \( \hat{\xi}_e \) and the observer gain matrix.

The characteristic equations of the LESO are given by

\[
\begin{align*}
\dot{\xi}_e &= A_e \xi_e + B_c \text{sat}(v_c) + B_{de} h_c \\
y_e &= C_e \xi_e
\end{align*}
\]

where \( \xi_e \in \mathbb{R}^8 \) is the state vector, the \( h_{eq} \) and the \( h_{kej} \) denote the derivatives of the \( \xi_{eq} \) and the \( \xi_{kej} \), respectively. The \( \xi_e \in \mathbb{R}^8 \), \( h_e \in \mathbb{R}^2 \), \( A_e \in \mathbb{R}^{8 \times 8} \), \( B_e \in \mathbb{R}^{8 \times 2} \), \( C_e \in \mathbb{R}^{2 \times 8} \), and \( B_{de} \in \mathbb{R}^{8 \times 2} \) can be referred to Guo and Tian.\(^{40}\)

**LESO design**

For the extended integral chain pseudo-linear system expressed in equation (22), the LESO is designed as
used to estimate unknown system states in the extended pseudo-linear system and the adaptive ISC laws based on SMC are used to deal with the control input saturation constraints. Furthermore, a novel compensation method for the estimation errors of the LESO with fixed preset observation gain is proposed to reduce the tracking errors. The semi-global eventually uniformly bounded stability of the system under the designed controller is proved by the stability analysis based on the candidate Lyapunov function.

For the purpose of the stability analysis, the assumptions and the lemma are presented as follows.

**Assumption 1.** The reference trajectories and the derivatives of the reference trajectories are all bounded.

**Assumption 2.** For the input–output differences shown in equation (16) (i.e. \( \Delta v_{eq} \) and \( \Delta v_{kej} \)), there exist unknown bounded positive constants \( \bar{b}_k \) and \( \bar{b}_k \) such that \( \| \Delta v_{eq} \| \leq \bar{b}_q \), and \( \| \Delta v_{kej} \| \leq \bar{b}_k \).

**Lemma 1.** The inequality \( 0 \leq |\eta| - \eta \tanh(\frac{\eta}{2}) \leq 0.2785\varepsilon \) holds for any \( \varepsilon > 0 \) and for any \( \eta \in \mathbb{R}^{40} \).

**Algorithm.** This section describes in detail the newly proposed tracking control scheme based on LESO with EEC for achieving simultaneous position and stiffness tracking control of the VSAs.

**Step 1:** Considering the state vector \( \xi_e \in \mathbb{R}^8 \) and the estimated state vector \( \hat{\xi}_e \in \mathbb{R}^8 \) shown in equation (23), the state estimation errors \( \xi_{eqm} (m = 1, 2, 3, 4, 5) \) and \( \xi_{kej} (n = 1, 2, 3) \) are represented as

\[
\begin{align*}
\hat{\xi}_{q1} &= \xi_{q1} - \hat{\xi}_{q1}; \hat{\xi}_{q2} = \xi_{q2} - \hat{\xi}_{q2}; \hat{\xi}_{q3} = \xi_{q3} - \hat{\xi}_{q3}; \hat{\xi}_{q4} = \xi_{q4} - \hat{\xi}_{q4} \\
\hat{\xi}_{q5} &= \xi_{q5} - \hat{\xi}_{q5}; \hat{\xi}_{kej1} = \xi_{kej1} - \hat{\xi}_{kej1}; \hat{\xi}_{kej2} = \xi_{kej2} - \hat{\xi}_{kej2}; \hat{\xi}_{kej3} = \xi_{kej3} - \hat{\xi}_{kej3}
\end{align*}
\]

(25)

The tracking errors of the output link position and the joint output stiffness of the VSA are represented as

\[
\begin{align*}
e_{q1} &= \dot{\hat{\xi}}_{q1} - q_d; e_{q2} = \dot{\hat{\xi}}_{q2} - q^{(1)}_d; e_{q3} = \dot{\hat{\xi}}_{q3} - q^{(2)}_d; e_{q4} = \dot{\hat{\xi}}_{q4} - q^{(3)}_d \\
e_{k1} &= \dot{\hat{\xi}}_{kej1} - k_{ejd}; e_{k2} = \dot{\hat{\xi}}_{kej2} - k^{(1)}_{ejd}
\end{align*}
\]

(26)

where \( q_d \) is the desired differentiable position trajectory, \( q^{(i)}_d (i = 1, 2, 3, 4) \) is the \( i \)th order time derivative of the desired position trajectory, \( k_{ejd} \) is the desired differentiable joint stiffness trajectory, and \( k^{(i)}_{ejd} (i = 1, 2) \) is the \( j \)th order time derivative of the desired joint stiffness trajectory.

Invoking the extended integral chain pseudo-linear system (22), the LESO (23), the state estimation errors (25), and the tracking errors (26), the first-order time derivative of the estimated system states can be expressed as

\[
\begin{align*}
\dot{\hat{\xi}}_{q2} &= \hat{\xi}_{q2} + \beta_{01} \hat{\xi}_{q1}; \dot{\hat{\xi}}_{q3} = \hat{\xi}_{q3} + \beta_{02} \hat{\xi}_{q1}; \dot{\hat{\xi}}_{q4} = \hat{\xi}_{q4} + \beta_{03} \hat{\xi}_{q1}; \dot{\hat{\xi}}_{q5} = \text{sat}(v_{cq}) + \hat{\xi}_{q5} + \beta_{04} \hat{\xi}_{q1} \\
\dot{\hat{\xi}}_{kej1} &= \hat{\xi}_{kej1} + \beta_{06} \hat{\xi}_{kej1}; \dot{\hat{\xi}}_{kej2} = \text{sat}(v_{cke}) + \hat{\xi}_{kej2} + \beta_{07} \hat{\xi}_{kej1}
\end{align*}
\]

(27)

The first-order time derivative of \( e_{q1} \) is given by equation (28) where \( \delta_{q1} \) represents the estimation error

\[
\begin{align*}
\dot{\hat{\xi}}_{q1} &= \hat{\xi}_{q1} - q^{(1)}_d; \dot{\hat{\xi}}_{q2} + \beta_{01} \hat{\xi}_{q1} - q^{(1)}_d = e_{q2} + \hat{\delta}_{q1} \\
\dot{\delta}_{q1} &= \beta_{01} \hat{\delta}_{q1}
\end{align*}
\]

(28)

The state estimation error compensator shown in equation (29) is used to estimate the state estimation error \( \hat{\delta}_{q1} \) and obtain the estimated value \( \hat{\delta}_{q1} \). The new estimation error is represented as \( \hat{\delta}_{q1} = \delta_{q1} - \hat{\delta}_{q1} \), and \( \delta_{q1} \) is the output of the state estimation error compensator shown in equation (29)

\[
\begin{align*}
\dot{\hat{\delta}}_{q1} &= -\gamma_{q1}\delta_{q1} - \gamma_{q1} \left( e_{q2} + \gamma_{q1} e_{q1} \right) \\
\hat{\delta}_{q1} &= s_{q1} + \gamma_{q1} e_{q1}; \delta_{q1}(0) = 0; \hat{\delta}_{q1}(0) = 0; \gamma_{q1} > 0
\end{align*}
\]

(29)

**Step 2:** The first-order time derivative of \( e_{q2} \) is given by equation (30) where \( \delta_{q2} \) represents the estimation error

\[
\begin{align*}
\dot{\hat{\delta}}_{q2} &= \hat{\xi}_{q2} - q^{(2)}_d; \dot{\hat{\xi}}_{q3} - q^{(2)}_d + \beta_{02} \hat{\xi}_{q1} = e_{q3} + \delta_{q2} \\
\delta_{q2} &= \beta_{02} \hat{\delta}_{q1}
\end{align*}
\]

(30)

The state estimation error compensator shown in equation (31) is used to estimate the unknown state estimation error \( \delta_{q2} \) and the estimated value \( \hat{\delta}_{q2} \) can be obtained. The
new estimation error is represented as \( \tilde{\delta}_q = \delta_q - q_d \), and \( s_q \) is the output of the state estimation error compensator shown in equation (31)

\[
\begin{align*}
\dot{s}_q &= -\gamma s_q \gamma q_d + \gamma q_3 e_q \\
\delta_q &= s_q + \gamma q_2 e_q; s_q(0) = 0; \delta_q(0) = 0; \gamma q_2 > 0
\end{align*}
\] (31)

**Step 3:** The first-order time derivative of \( e_q \) is given by equation (32) where \( \delta_q \) represents the estimation error

\[
\begin{align*}
\dot{e}_q &= \tilde{\xi}_3 - q_d \\
\delta_q &= \beta_0 q_3 
tilde{\xi}_q^4)
\end{align*}
\] (32)

The state estimation error compensator shown in equation (33) is used to estimate the unknown state estimation error \( \delta_q \) and the estimated value \( \tilde{\delta}_q \) can be obtained. The new estimation error is represented as \( \delta_q = \delta_{q3} - \delta_{q4} \), and \( e_q \) is the output of the state estimation error compensator shown in equation (33)

\[
\begin{align*}
\dot{s}_3 &= -\gamma q_3 s_q - \gamma q_3 [w_q + \gamma q_3 e_q] \\
\delta_q &= s_3 + \gamma q_3 e_q; s_3(0) = 0; \delta_q(0) = 0; \gamma q_3 > 0
\end{align*}
\] (33)

\[
\begin{align*}
\dot{s}_q &= c_{q1} e_q + c_{q2} e_q + c_{q3} e_q + e_q \\
\dot{q}_q &= c_{q1} (e_q + e_q) + c_{q2} (e_q + e_q) + c_{q3} (e_q + e_q) + \text{sat}(v_{cq}) + \tilde{\xi}_q q_d + \delta_q q_d + v_q \\
\delta_q &= c_{q1} \delta_q + c_{q2} \delta_q + c_{q3} \delta_q + \delta_q; \text{sat}(v_{cq}) = v_{cq} + \Delta v_{cq}
\end{align*}
\] (36)

where \( \delta_q \) represents the estimation error, and the parameters \( c_{q1} (i = 1, 2, 3) \) have to be designed such that the characteristic equation (37) is Hurwitz

\[
p(s) = s^3 + c_{q3} s^2 + c_{q2} s + c_{q1} = 0
\] (37)

The state estimation error compensator shown in equation (38) is used to estimate the state estimation error \( \delta_v \) and obtain the estimated value \( \tilde{v} \). The new estimation error is represented as \( \delta_v = \delta_v - \delta_v \), and \( s_v \) is the output of the state estimation error compensator shown in equation (38)

\[
\begin{align*}
\dot{s}_v &= -\gamma v_s s_v - \gamma v_s \left( c_{q1} (e_q + \delta q) + c_{q2} (e_q + \delta q) + c_{q3} (e_q + \delta q) + \text{sat}(v_{cq}) + \tilde{\xi}_q q_d \right) \\
\delta_v &= s_v + \gamma v_s q_v; s_v(0) = 0; \delta_v(0) = 0; \gamma v_s > 0
\end{align*}
\] (38)

**Step 4:** The first-order time derivative of \( e_{q4} \) is given by equation (34) where \( \delta_{q4} \) represents the estimation error

\[
\begin{align*}
\dot{e}_{q4} &= \tilde{\xi}_{q4} - q_d + \text{sat}(v_{cq}) + \tilde{\xi}_{q5} - q_d + \delta_{q4} \\
\delta_{q4} &= \beta_{q4} \tilde{\xi}_{q4}
\end{align*}
\] (34)

The state estimation error compensator shown in equation (35) is used to estimate the state estimation error \( \delta_{q4} \) and obtain the estimated value \( \tilde{\delta}_{q4} \). The new estimation error is represented as \( \delta_{q4} = \delta_{q4} - \delta_{q4} \), and \( s_{q4} \) is the output of the state estimation error compensator shown in equation (35)

\[
\begin{align*}
\dot{s}_{q4} &= -\gamma q_4 s_{q4} - \gamma q_4 \left[ \text{sat}(v_{cq}) + \tilde{\xi}_{q5} - q_d + \gamma q_4 e_{q4} \right] \\
\delta_{q4} &= s_{q4} + \gamma q_4 e_{q4}; s_{q4}(0) = 0; \delta_{q4}(0) = 0; \gamma q_4 > 0
\end{align*}
\] (35)

**Step 5:** In order to deal with the control input saturation constraints in the extended pseudo-linear system and prove the semi-global ultimate uniformly bounded stability of the closed-loop system based on the candidate Lyapunov function, the adaptive ISC law based on SMC is adopted in the design of the controller. The sliding mode surface (i.e. \( s_s \)) for the angular position tracking of the output link of the VSA is represented as

\[
\text{sat}(v_{cq}) = v_{cq} + \Delta v_{cq}
\]
\[ v_{c_q} = -c_{q_1} (e_{q_2} + \hat{\delta}_{q_1}) - c_{q_2} (e_{q_3} + \hat{\delta}_{q_2}) - c_{q_3} (e_{q_4} + \hat{\delta}_{q_3}) - \hat{\xi}_{q_5} + q^{(4)}_{d} - \hat{\delta}_{q_4} - \mu_{q} \hat{s}_{q} - \hat{b}_{q} \tanh \left( \frac{s_{q}}{\rho_{q}} \right) \]  
\[ \dot{\hat{b}}_{q} = \gamma_{q} \left[ s_{q} \tanh \left( \frac{s_{q}}{\rho_{q}} \right) - k_{q} \hat{b}_{q} \right] ; \hat{b}_{q}(0) = 0; \gamma_{q} > 0; k_{q} > 0; \rho_{q} > 0 \]  

where \( \hat{b}_{q} \) is the estimate of the \( b_{q} \); \( \mu_{q} > 0, \gamma_{q} > 0, k_{q} > 0 \) are the positive constants to be designed; and the estimation error for the \( \hat{b}_{q} \) is represented as \( \hat{b}_{q} = \hat{b}_{q} - b_{q} \).

**Step 7:** The first-order time derivative of \( \delta_{k_1} \) is given by equation (40) where \( \delta_{k_1} \) represents the estimation error

\[ \dot{\delta}_{k_1} = \frac{\gamma_{k_1}}{\beta_{00}} \left( -s_{k_1} - \hat{\delta}_{k_1} \right) \]  

The state estimation error compensator (41) is used to estimate the unknown state estimation error \( \delta_{k_1} \) and the estimated value \( \hat{\delta}_{k_1} \) can be obtained. The new estimation error is represented as \( \hat{\delta}_{k_1} = \delta_{k_1} - \hat{\delta}_{k_1} \), and \( s_{k_1} \) is the output of the state estimation error compensator shown in equation (41)

\[ s_{k_1} = -\gamma_{k_1} \delta_{k_1} - \gamma_{k_1} \left( e_{k_2} + \gamma_{k_1} e_{k_1} \right) ; \delta_{k_1}(0) = 0; \hat{\delta}_{k_1}(0) = 0; \gamma_{k_1} > 0 \]  

**Step 8:** The first-order time derivative of \( \delta_{k_2} \) is given by equation (42) where \( \delta_{k_2} \) represents the estimation error

\[ \dot{\delta}_{k_2} = \frac{\gamma_{k_2}}{\beta_{00}} \left( -s_{k_2} - \hat{\delta}_{k_2} \right) \]  

\[ s_{k_2} = c_{k_2} e_{k_1} + e_{k_2} ; \delta_{k_1}(0) = 0 ; \gamma_{k_1} > 0 \]

The state estimation error compensator (43) is used to estimate the unknown state estimation error \( \delta_{k_2} \) and obtain the estimated value \( \hat{\delta}_{k_2} \). The new estimation error is represented as \( \hat{\delta}_{k_2} = \delta_{k_2} - \hat{\delta}_{k_2} \), and \( s_{k_2} \) is the output of the state estimation error compensator shown in equation (43)

\[ s_{k_2} = -\gamma_{k_2} s_{k_2} - \gamma_{k_2} \left( c_{k_2} + \hat{\delta}_{k_2} \right) \]  

\[ \hat{\delta}_{k_2}(0) = 0; \delta_{k_2}(0) = 0; \gamma_{k_2} > 0 \]  

**Step 9:** The sliding mode surface \( s_{k} \) used for joint output stiffness tracking is defined as

\[ s_{k} = c_{k_1} e_{k_1} + e_{k_2} ; \delta_{k_1}(0) = 0 \]

\[ s_{k_2} = -\gamma_{k_2} s_{k_2} - \gamma_{k_2} \left( c_{k_2} + \hat{\delta}_{k_2} \right) \]  

\[ \delta_{k_2}(0) = 0; \hat{\delta}_{k_2}(0) = 0; \gamma_{k_2} > 0 \]  

**Step 10:** The control law \( v_{c_{k_2}} \) and the adaptive ISC law \( \hat{b}_{k} \) are given by

\[ v_{c_{k_2}} = -c_{k_2} (e_{k_2} + \hat{\delta}_{k_1}) + k^{(2)}_{c_{k_2}} (e_{k_2} + \hat{\delta}_{k_2}) - \hat{\delta}_{k_2} - \delta_{k_2} - \mu_{k} s_{k} - \hat{b}_{k} \tanh \left( \frac{s_{k}}{\rho_{k}} \right) \]  

\[ \dot{\hat{b}}_{k} = \gamma_{k} \left[ s_{k} \tanh \left( \frac{s_{k}}{\rho_{k}} \right) - k_{k} \hat{b}_{k} \right] ; \hat{b}_{k}(0) = 0; \gamma_{k} > 0; k_{k} > 0; \rho_{k} > 0 \]

where \( \hat{b}_{k} \) is the estimate of \( b_{k} \); \( \mu_{k} > 0 \) is a positive constant, and the estimation error for \( \hat{b}_{k} \) is \( \hat{b}_{k} = \hat{b}_{k} - b_{k} \).
In this section, the stability of the closed-loop system under the proposed controller is analyzed. Invoking equations (28) to (46) and Lemma 1, the related calculations shown in equations (47) to (56) can be obtained

\[
\begin{align*}
\dot{\delta}_{q_1} \dot{\delta}_{q_1} &= \ddot{\delta}_{q_1} \left( \delta_{q_1} - \gamma_{q_1} \dot{\delta}_{q_1} \right) \leq -\gamma_{q_1} |\dot{\delta}_{q_1}|^2 + |\dot{\delta}_{q_1}| |\ddot{\delta}_{q_1}| \leq -\frac{\gamma_{q_1}}{2} |\dot{\delta}_{q_1}|^2 + \frac{1}{2\gamma_{q_1}} |\ddot{\delta}_{q_1}|^2 \tag{47} \\
\dot{\delta}_{q_2} \dot{\delta}_{q_2} &= \ddot{\delta}_{q_2} \left( \delta_{q_2} - \gamma_{q_2} \dot{\delta}_{q_2} \right) \leq -\gamma_{q_2} |\dot{\delta}_{q_2}|^2 + |\dot{\delta}_{q_2}| |\ddot{\delta}_{q_2}| \leq -\frac{\gamma_{q_2}}{2} |\dot{\delta}_{q_2}|^2 + \frac{1}{2\gamma_{q_2}} |\ddot{\delta}_{q_2}|^2 \tag{48} \\
\dot{\delta}_{q_3} \dot{\delta}_{q_3} &= \ddot{\delta}_{q_3} \left( \delta_{q_3} - \gamma_{q_3} \dot{\delta}_{q_3} \right) \leq -\gamma_{q_3} |\dot{\delta}_{q_3}|^2 + |\dot{\delta}_{q_3}| |\ddot{\delta}_{q_3}| \leq -\frac{\gamma_{q_3}}{2} |\dot{\delta}_{q_3}|^2 + \frac{1}{2\gamma_{q_3}} |\ddot{\delta}_{q_3}|^2 \tag{49} \\
\dot{\delta}_{q_4} \dot{\delta}_{q_4} &= \ddot{\delta}_{q_4} \left( \delta_{q_4} - \gamma_{q_4} \dot{\delta}_{q_4} \right) \leq -\gamma_{q_4} |\dot{\delta}_{q_4}|^2 + |\dot{\delta}_{q_4}| |\ddot{\delta}_{q_4}| \leq -\frac{\gamma_{q_4}}{2} |\dot{\delta}_{q_4}|^2 + \frac{1}{2\gamma_{q_4}} |\ddot{\delta}_{q_4}|^2 \tag{50} \\
\dot{\delta}_{w_1} \dot{\delta}_{w_1} &= \ddot{\delta}_{w_1} \left( \delta_{w_1} - \gamma_{w_1} \dot{\delta}_{w_1} \right) \leq -\gamma_{w_1} |\dot{\delta}_{w_1}|^2 + |\dot{\delta}_{w_1}| |\ddot{\delta}_{w_1}| \leq -\frac{\gamma_{w_1}}{2} |\dot{\delta}_{w_1}|^2 + \frac{1}{2\gamma_{w_1}} |\ddot{\delta}_{w_1}|^2 \tag{51}
\end{align*}
\]


\[ s_k \delta_k + \frac{1}{\gamma_k} \hat{b}_k \hat{b}_k = s_k \left[ \Delta v_{ck} + \tilde{\delta}_{vk} - \mu_k s_k - \hat{b}_k \tanh \left( \frac{s_k}{\rho_k} \right) \right] + \frac{1}{\gamma_k} \hat{b}_k \left[ -\gamma_k \left( s_k \tanh \left( \frac{s_k}{\rho_k} \right) - k_s \tilde{b}_k \right) \right] \]

\[ \leq |s_k| \hat{b}_k + |s_k| |\tilde{\delta}_{vk}| - \mu_k |s_k|^2 - s_k \hat{b}_k \tanh \left( \frac{s_k}{\rho_k} \right) \left( \frac{s_k}{\rho_k} \right) - k_s \tilde{b}_k^2 + k_s \tilde{b}_k \hat{b}_k \]

\[ = -\frac{\mu_k}{2} |s_k|^2 + \left( \frac{\mu_k}{2} |s_k|^2 + |s_k| |\tilde{\delta}_{vk}| \right) + \left( |s_k| \hat{b}_k - s_k \hat{b}_k \tanh \left( \frac{s_k}{\rho_k} \right) \right) + (-k_s \tilde{b}_k^2 + k_s \tilde{b}_k \hat{b}_k) \]

\[ \leq -\frac{\mu_k}{2} |s_k|^2 + \frac{1}{2\mu_k} |\tilde{\delta}_{vk}|^2 + 0.2785 \tilde{b}_k \rho_k + \left( -k_s \tilde{b}_k^2 + k_s \tilde{b}_k \right) \]

Let the candidate Lyapunov function \( V = V_q + V_k \) be

\[ V_q = \frac{1}{2} \tilde{\delta}_{q1}^2 + \frac{1}{2} \tilde{\delta}_{q2}^2 + \frac{1}{2} \tilde{\delta}_{q3}^2 + \frac{1}{2} \tilde{\delta}_{q4}^2 + \frac{1}{2} \tilde{\delta}_{q5}^2 + \frac{1}{2} \tilde{\delta}_{q6}^2 + \frac{1}{2} |s_k|^2 + \frac{1}{2\gamma_q} \tilde{b}_q^2 \]

\[ V_k = \frac{1}{2} \tilde{\delta}_{k1}^2 + \frac{1}{2} \tilde{\delta}_{k2}^2 + \frac{1}{2} \tilde{\delta}_{k3}^2 + \frac{1}{2} \tilde{\delta}_{k4}^2 + \frac{1}{2} \tilde{\delta}_{k5}^2 + \frac{1}{2} \tilde{\delta}_{k6}^2 + \frac{1}{2 \gamma_k} \tilde{b}_k^2 ; \ V = V_q + V_k \]

Then, based on equations (47) to (57), the first-order time derivative of the candidate Lyapunov function \( V = V_q + V_k \) is given by equations (58) and (59)

\[ \dot{V} = \dot{V}_q + \dot{V}_k \]

\[ \leq -\frac{\gamma_q}{2} |\tilde{\delta}_{q1}|^2 - \frac{\gamma_q}{2} |\tilde{\delta}_{q2}|^2 - \gamma_q \tilde{\delta}_{q3}^2 - \gamma_q \tilde{\delta}_{q4}^2 - \gamma_q \tilde{\delta}_{q5}^2 - \gamma_q \tilde{\delta}_{q6}^2 - \frac{\mu_q}{2} |s_k|^2 - \frac{k_q}{2} \tilde{b}_q^2 + Q_q + \cdots \]

\[ -\frac{\gamma_k}{2} |\tilde{\delta}_{k1}|^2 - \frac{\gamma_k}{2} |\tilde{\delta}_{k2}|^2 - \gamma_k \tilde{\delta}_{k3}^2 - \gamma_k \tilde{\delta}_{k4}^2 - \frac{\mu_k}{2} |s_k|^2 - \frac{k_s}{2} \tilde{b}_k^2 + Q_k \]

\[ \leq -r_F V + Q_q ; Q = Q_q + Q_k \]

\[ Q_q = \frac{1}{2 \gamma_q} |\tilde{\delta}_{q1}|^2 + \frac{1}{2 \gamma_q} |\tilde{\delta}_{q2}|^2 + \frac{1}{2 \gamma_q} |\tilde{\delta}_{q3}|^2 + \frac{1}{2 \gamma_q} |\tilde{\delta}_{q4}|^2 + \frac{1}{2 \gamma_q} |\tilde{\delta}_{q5}|^2 + \frac{1}{2 \gamma_q} |\tilde{\delta}_{q6}|^2 + \frac{0.2785 \tilde{b}_q \rho_q + k_q \tilde{b}_q^2}{2} \]

\[ Q_k = \frac{1}{2 \gamma_k} |\tilde{\delta}_{k1}|^2 + \frac{1}{2 \gamma_k} |\tilde{\delta}_{k2}|^2 + \frac{1}{2 \gamma_k} |\tilde{\delta}_{k3}|^2 + \frac{1}{2 \gamma_k} |\tilde{\delta}_{k4}|^2 + \frac{1}{2 \gamma_k} |\tilde{\delta}_{k5}|^2 + \frac{1}{2 \gamma_k} |\tilde{\delta}_{k6}|^2 + \frac{0.2785 \tilde{b}_k \rho_k + k_s \tilde{b}_k^2}{2} \]

The controller parameters are set as shown in equation (60)

\[ r_F = 2 \min \left\{ \frac{\gamma_q}{2}, \frac{\gamma_q}{2}, \frac{\gamma_q}{2}, \frac{\gamma_q}{2}, \frac{\gamma_q}{2}, \frac{\gamma_q}{2}, \frac{\gamma_k}{2}, \frac{\gamma_k}{2}, \frac{\gamma_k}{2}, \frac{\gamma_k}{2}, \frac{\gamma_k}{2}, \frac{\gamma_k}{2}, \frac{\gamma_k}{2}, \frac{\gamma_k}{2}, \frac{\gamma_k}{2}, \frac{\gamma_k}{2}, \frac{\gamma_k}{2}, \frac{\gamma_k}{2}, \frac{\gamma_k}{2} \right\} \]

Assuming that \( V(0) \leq p \), by choosing the parameter \( r_F \geq Q/q \), the inequality shown in equation (61) can be established according to the boundedness theorem

\[ V(t) \leq V(0) e^{-r_F t} + \frac{Q}{r_F} (1 - e^{-r_F t}) \Rightarrow \lim_{t \to \infty} V(t) \leq \frac{Q}{r_F} ; r_F > 0 \]

For the antagonistic VSA based on ENTS, the control input is represented as
For the SVSA based on lever mechanism, the control input is represented as

$$
\begin{bmatrix}
    u_s \\
    u_f
\end{bmatrix}
= \begin{bmatrix}
    L_{g_a} h_1(x) & L_{g_a}^2 h_1(x) \\
    L_{g_b} h_2(x) & L_{g_b}^2 h_2(x)
\end{bmatrix}^{-1}
\begin{bmatrix}
    v_{cq} - L_f^2 h_1(x) \\
    v_{cke} - L_f^2 h_2(x)
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
    \text{sat}(u_s) \\
    \text{sat}(u_f)
\end{bmatrix}
$$

For the SVSA based on lever mechanism, the control input is represented as

$$
\begin{bmatrix}
    u_p \\
    u_s
\end{bmatrix}
= \begin{bmatrix}
    L_{g_a} h_1(x) & L_{g_a}^2 h_1(x) \\
    L_{g_b} h_2(x) & L_{g_b}^2 h_2(x)
\end{bmatrix}^{-1}
\begin{bmatrix}
    v_{cq} - L_f^2 h_1(x) \\
    v_{cke} - L_f^2 h_2(x)
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
    \text{sat}(u_p) \\
    \text{sat}(u_s)
\end{bmatrix}
$$

The schematic diagram of the proposed LESO + SMC + ISC + EEC controller is shown in Figure 1.

**Remark 1.** The proposed estimating error compensators are shown in equations (29), (31), (33), (35), (38), (41), (43), and (45), respectively. It is necessary to point out that the estimation error compensator designed in this article has the same structural form as the stable SISO DOB,²⁴⁻²⁸ but its control input setting is different from that of the traditional SISO DOB. The purpose of the definition and setting of the control input of the estimation error compensator is to obtain the estimation value of the estimation error of the LESO. Based on the stability analysis of the candidate Lyapunov function, the semi-global ultimate uniformly bounded stability of the closed-loop control system with EEC is proved. To the best of our knowledge, this is the first study to compensate the estimation errors of the traditional LESO with fixed observation gains by using the SISO DOB.

**Remark 2.** The estimated errors $\delta_{qi}(i = 1, 2, 3, 4)$, $\delta_{kj}(j = 1, 2)$, $\delta_{vq}$, and $\delta_{vk}$ are shown in equation (64). Considering that the gains $\beta_{01}$, $\beta_{02}$, $\beta_{03}$, and $\beta_{04}$ in the state estimation errors increase in turn (i.e. $\beta_{01} < \beta_{02} < \beta_{03} < \beta_{04}$), the gains $\gamma_{q1}$, $\gamma_{q2}$, $\gamma_{q3}$, and $\gamma_{q4}$ of the estimation error compensators should be set to increase in turn (i.e. $\gamma_{q1} < \gamma_{q2} < \gamma_{q3} < \gamma_{q4}$). Since $\delta_{vq}$ is a function of...
the new estimation errors $\tilde{\xi}_{qi} (i = 1, 2, 3, 4)$ and the control gains $e_{qi} (j = 1, 2, 3)$, the gain parameter $\gamma_{vq}$ of the estimation error compensator used to estimate $\delta_{vq}$ can be set to relatively large values. Because the LESO gain $\beta_{0q}$, $\beta_{qy}$ and control gain $c_{k1}$ are relatively small, the gain parameters $\gamma_{k1}$, $\gamma_{k2}$, and $\gamma_{vq}$ of the estimation error compensators can be set to relatively small values. In addition, the gains of the estimation error compensators should not be too large to avoid the peak phenomenon of the control input

$$
\begin{align*}
\delta_{x1} &= \beta_{01}\tilde{\xi}_{x1}; \\
\delta_{x2} &= \beta_{q2}\tilde{\xi}_{x1}; \\
\delta_{x3} &= \beta_{03}\tilde{\xi}_{x1}; \\
\delta_{x4} &= \beta_{04}\tilde{\xi}_{x1};
\end{align*}
$$

Remark 3. As shown in equation (61), the convergence rate of the tracking errors is mainly determined by the parameter $r_F$. Since $r_F$ is related to parameters $\gamma_{qi} (i = 1, 2, 3, 4)$, $\gamma_{vq}$, $\mu_q$, $k_q\gamma_{qf}$, $\gamma_{kq}$, $\gamma_{k2}$, $\gamma_{k1}$, $\mu_k$, and $k_s\gamma_{sk}$ then larger $\gamma_{qi}$, $\gamma_{vq}$, $\mu_q$, $k_q\gamma_{qf}$, $\gamma_{kq}$, $\gamma_{k2}$, $\gamma_{k1}$, $\mu_k$, and $k_s\gamma_{sk}$ will lead to larger $r_F$ and faster convergence rate of the tracking errors from the definition of parameter $r_F$. Considering the input saturation constraints in the system model, the ISC measures are introduced in the controller design. From the controllability of the VSA system, the assumption that $[\Delta v_{cq}]$ and $[\Delta v_{kek}\xi]$ are always bounded is reasonable. For the designed adaptive ISC laws with sliding mode surfaces as input, the parameters in this article are directly set as $2r_F = k_q\gamma_{qf}$ and $2r_F = k_s\gamma_{sk}$. According to the expressions of $Q_q$ and $Q_k$ shown in equation (59), it can be concluded that $k_q$, $k_s$, $\mu_q$, and $\mu_k$ should be set to relatively small values, $\gamma_{q} = 2r_F/k_q$ and $\gamma_{k} = 2r_F/k_s$ should take relatively large values. In addition, the tank $(\frac{B}{P})$ and the sign($s_i$) are very close to each other if $\rho_i \ll |s_i|$ ($i = q, k$). The parameters $\rho_q$ and $\rho_k$ should take small values with respect to the variation ranges of $s_q$ and $s_k$, respectively.

Remark 4. As shown in equations (62) and (63), in order to obtain the control input $sat(u)$, the decoupling matrix $G(x)$ must be nonsingular. The determinant of the decoupling matrix of the EETS-based VSA is shown in equation (65), and it is always nonsingular due to the exponential properties of the EETS.\textsuperscript{15} As presented in Guo and Tian,\textsuperscript{46} the maximum angular displacement of the EETS is defined as $\theta_{max} = 4.5$ rad. Therefore, due to the mechanical limitation, the ranges of variation of the composite states are limited to $0 < (\beta_0 + x_5 - x_1) \leq 4.5$ rad and $0 < (\beta_0 + x_5 - x_1) \leq 4.5$ rad, respectively. In order to ensure that the two EETSs of the antagonistic VSA are always in the antagonistic actuation state, the initial pre- tension of the EETS is defined as $\alpha_0 = \beta_0 = 2.3$ rad $> \theta_{max}/2$. In addition, the joint stiffness reference trajectory of the EETS-based VSA is always defined as $k_{qj0} \leq k_{qj} \leq k_{qj, max}$ (i.e. 3.497 $\leq k_{qj} \leq 28.7$ Nm/rad). The determinant of the decoupling matrix $G(x)$ of the Compact-VSA\textsuperscript{5} is given by equation (66). It can be observed that if the adjustment range of the joint output stiffness of the Compact-VSA is defined as $0 < k_{qj} < \infty$, the determinant of the decoupling matrix of the Compact-VSA is different from zero, and the decoupling matrix $G(x)$ of the Compact-VSA is always nonsingular

$$
det(G(x)) = - \frac{2g^2}{Jn^2M}b(h(\beta_0 + x_5 - x_1) + \beta(n_0 + x_1 + x_5))
$$

$$
det(G(x)) = \frac{8\Delta^3K_x^2n^4x_3^5}{JpJ_nM(\Delta - x_3)^5} = \frac{8\Delta^3K_x^2n}{JpJ_nM(\Delta - 0_1)^5}
$$

Remark 5. The LESO + SMC + ISC + EEC controller designed in this article is shown in equations (39) and (46). In order to demonstrate the effectiveness of the proposed EEC measures in reducing the system output tracking errors, the LESO + SMC + ISC controller shown in equation (67) is used to compare with the LESO + SMC + ISC + EEC controller. In order to further demonstrate the compensation effect of the proposed EEC measures and reduce the complexity of the controller, the LESO + SMC controller shown in equation (68) and the LESO + SMC + EEC controller shown in equation (69) are used for comparison in simulation. Note that the calculations of the actual control inputs corresponding to these three controllers can refer to equations (62) and (63)

$$
v_{eq} = -c_{q1}e_{q2} - c_{q2}e_{q3} - c_{q3}e_{q4} - \tilde{\xi}_{q5} - q_{d} - \mu_q s_q - \hat{b}_q \tanh \left( \frac{s_q}{\rho_q} \right)
$$

$$
v_{ekj} = -c_{k1}e_{k2} + \frac{k_{eq}}{c_{k2}} - \tilde{\xi}_{ekj} - \mu_k s_k - \hat{b}_k \tanh \left( \frac{s_k}{\rho_k} \right)
$$

(67)
Table 1. Definitions of the composite disturbances imposed on the EETS-based VSA system.

|          | 0–5 s | 5–10 s | 10–17 s | 17–20 s |
|----------|-------|--------|---------|---------|
| $\Delta D_q$ | $0.65 \times D_q$ | $0.75 \times D_q$ | $0.65 \times D_q$ | $0.70 \times D_q$ |
| $\Delta M$ | $0.55 \times M$ | $0.60 \times M$ | $0.65 \times M$ | $0.55 \times M$ |
| $\Delta D_m$ | $0.60 \times D_m$ | $0.65 \times D_m$ | $0.75 \times D_m$ | $0.65 \times D_m$ |
| $\Delta J_m$ | $0.55 \times J_m$ | $0.65 \times J_m$ | $0.60 \times J_m$ | $0.65 \times J_m$ |
| $\Delta E_q$ | $0.55 \times E_q$ | $0.65 \times E_q$ | $0.70 \times E_q$ | $0.75 \times E_q$ |
| $\tau_{\text{ext}}$ | $2.75$ N·m | $5.75$ N·m | $4.5$ N·m | $3.75$ N·m |
| $\tau_{f_{\text{so}}}$ | $0.65$ N·m | $0.75$ N·m | $0.85$ N·m | $0.95$ N·m |
| $\tau_{f_{\text{si}}}$ | $0.55$ N·m | $0.95$ N·m | $0.65$ N·m | $0.75$ N·m |

EETS: equivalent exponential torsion spring; VSA: variable stiffness actuator.

Table 2. Definitions of the composite disturbances imposed on the Compact-VSA system.

|          | 0–5 s | 5–10 s | 10–17 s | 17–20 s |
|----------|-------|--------|---------|---------|
| $\Delta D_q$ | $0.70 \times D_q$ | $0.65 \times D_q$ | $0.75 \times D_q$ | $0.65 \times D_q$ |
| $\Delta D_p$ | $0.75 \times D_p$ | $0.70 \times D_p$ | $0.65 \times D_p$ | $0.60 \times D_p$ |
| $\Delta D_s$ | $0.75 \times D_s$ | $0.70 \times D_s$ | $0.75 \times D_s$ | $0.65 \times D_s$ |
| $\Delta M$ | $0.50 \times M$ | $0.65 \times M$ | $0.60 \times M$ | $0.55 \times M$ |
| $\Delta J_p$ | $0.55 \times J_p$ | $0.65 \times J_p$ | $0.60 \times J_p$ | $0.55 \times J_p$ |
| $\Delta J_s$ | $0.55 \times J_s$ | $0.70 \times J_s$ | $0.65 \times J_s$ | $0.55 \times J_s$ |
| $\Delta E_q$ | $0.55 \times E_q$ | $0.60 \times E_q$ | $0.75 \times E_q$ | $0.55 \times E_q$ |
| $\tau_{\text{ext}}$ | $3.5$ N·m | $5.5$ N·m | $8.5$ N·m | $3.5$ N·m |
| $\tau_{f_{\text{sp}}}$ | $0.45$ N·m | $0.75$ N·m | $0.95$ N·m | $0.55$ N·m |
| $\tau_{f_{\text{si}}}$ | $0.65$ N·m | $0.35$ N·m | $0.55$ N·m | $0.45$ N·m |

VSA: variable stiffness actuator.

Simulation and analysis of the effect of EEC on tracking control

In this section, the performance of the proposed controller based on LESO with EEC is demonstrated by MATLAB simulation. The controlled system dynamic models are provided by equations (10) and (13), respectively. The purpose of simulation studies is to show that the designed controller can achieve simultaneous position and stiffness robust tracking control of these two classes of VSAs. Furthermore, the purpose of simulation comparison is to verify and demonstrate that the proposed novel EEC measures can effectively improve the tracking accuracy and disturbance rejection performance of the controller. In addition, sufficient simulation comparisons show that the proposed robust tracking controller based on LESO with EEC measures has good adaptability in the simultaneous position and stiffness tracking control applications of these two different classes of VSAs.

For the antagonistic EETS-based VSA and the serial Compact-VSA based on lever mechanism, accurate system dynamic models are difficult to obtain, and the uncertainties of model parameters are unavoidable. Considering the inaccuracy of dynamic model parameters (i.e. the unknown parameter perturbations), the unknown friction torques, and the unknown external disturbance, the predefined system disturbances shown in Tables 1 and 2 are used to demonstrate the effectiveness and robustness of the designed controller. It should be noted that in order to demonstrate the robustness of the controller, the predefined disturbances shown in Tables 1 and 2 are set to be different in four different time periods.

Simulation and analysis of the effect of EEC on tracking control of the antagonistic VSA based on EETS corresponding to comparison settings 1

In this simulation, the purpose of simulation comparison is to show the effect of the proposed novel EEC measures on tracking performance of the EETS-based VSA. In order to reduce the complexity of the controller, two kinds of controllers without ISC, namely, LESO + SMC controller shown in equation (68) and LESO + SMC + EEC controller shown in equation (69), are used for simulation comparison. Compared with the LESO + SMC + EEC controller, the LESO + SMC controller only lacks the EEC term.

In the simulation, the initial system states of the EETS-based VSA and the initial states of the LESO are defined as $x(0) = [0, 0, 0, 0, 0, 0]^T$ and $\hat{\xi}_c(0) = [\hat{\xi}_q(0), 0, 0, 0, \hat{\xi}_q(0), 0, 0, 0]^T$, respectively. The reference trajectories are given by $q_d = 1.75 + 1.5 \times (t > 75) - 1 \times (t > 155)$ and $\hat{v}_d = \hat{v}_d + 10 + 2.5 \times (t > 12.5) - 5 \times (t > 185)$, respectively. Note that $1.5 \times (t > 75) = 1.5$ when time $t > 75$. The initial joint stiffness is
The control input constraints of the EETS-based VSA are defined as \( \text{sat}(u_a) \in [-13 \text{ N} \cdot \text{m}, 13 \text{ N} \cdot \text{m}] \) and \( \text{sat}(u_i) \in [-13 \text{ N} \cdot \text{m}, 13 \text{ N} \cdot \text{m}] \). The parameters of LESO + SMC controller and LESO + SMC + EEC controller are selected as \( \beta_{01} = 300, \beta_{02} = 36,000, \beta_{03} = 2.16 \times 10^8, \beta_{04} = 6.48 \times 10^7, \beta_{05} = 7.776 \times 10^6, \beta_{06} = 150, \beta_{07} = 7500, \beta_{08} = 1.25 \times 10^5, c_{q1} = 1000, c_{q2} = 300, \gamma_{q1} = 30, c_{k1} = 50, r_{F} = 5, \gamma_{q2} = 10, \gamma_{q3} = 20, \gamma_{q4} = 30, \gamma_{q5} = 50, \gamma_{k1} = 5, \gamma_{k2} = 15, \) and \( \gamma_{k3} = 30 \).

The main simulation results under the designed LESO + SMC controller and the LESO + SMC + EEC controller are depicted in Figure 2. It can be observed that the angular position tracking error of the output link under the action of the LESO + SMC controller is relatively large due to the large predefined system disturbances, while with the designed LESO + SMC + EEC controller, the angular position of the output link can track the reference position trajectory with relatively small tracking error. For the stiffness tracking response curve of the EETS-based VSA under the LESO + SMC + EEC controller, the stiffness tracking error can be observed during \( t = 5-5.5 \) s and \( t = 10-10.5 \) s due to the change of predefined system disturbances. Moreover, small stiffness tracking error can be observed during the period of \( t = 7-8 \) s, which is caused by the change of angular position tracking trajectory, and the small angular position tracking error can be observed during \( t = 12.7-13.4 \) s due to the change of joint stiffness tracking trajectory. Therefore, the position tracking response curve and the stiffness tracking response curve interact with each other. As shown in Figure 2(b), the
control inputs of the EETS-based VSA under the LESO + SMC + ISC controller are fluctuated on small ranges during the period of $t = 5–5.3$ s and $t = 10–10.3$ s, which is caused by the sudden change of the predefined disturbances, and the control inputs are compensated by the estimation error compensators. As shown in Figure 2(c), the response curves of the composite states are all within the mechanical allowable ranges (i.e. $0 < \alpha_0 + x_1 + x_3 \leq 4.5$ rad, $0 < (\beta_0 + x_5 - x_1) \leq 4.5$ rad).

From the simulations shown in Figure 2, it can be concluded that the proposed EEC measures are helpful to reduce the system output tracking errors and improve the disturbance rejection performance of the controller. Moreover, the influence of the EEC measures on the control input is acceptable.

**Simulation and analysis of the effect of EEC on tracking control of the antagonistic VSA based on EETS corresponding to comparison settings 2**

In order to further demonstrate the effect of estimated error compensation measures on tracking performance, the simulations under the LESO + SMC + ISC controller\(^4^0\) and the LESO + SMC + ISC + EEC controller are shown in Figure 3. The parameters of the adaptive ISC laws for these two controllers are all set as $\gamma_q = 150$, $\gamma_k = 150$, $k_q = 2R_1/\gamma_q$, $k_s = 2R_1/\gamma_k$, $\rho_q = 60$, and $\rho_k = 30$. The other parameters of these two controllers, the reference trajectories, the ranges of the control inputs, and the predefined disturbances are all set to be the same as those set in the

**Guo et al.**

---

**Figure 3.** Comparison of response curves of the EETS-based VSA corresponding to the LESO + SMC + ISC controller\(^4^0\) and the LESO + SMC + ISC + EEC controller: (a) comparison of output response curves of the EETS-based VSA, (b) comparison of response curves of the control inputs for the EETS-based VSA, and (c) comparison of response curves of the composite states for the EETS-based VSA. Note that the predefined system disturbances are shown in Table 1. LESO: linear extended state observer; SMC: sliding mode control; ISC: input saturation compensation; EEC: estimation error compensation; EETS: equivalent exponential torsion spring; VSA: variable stiffness actuator.
control of the antagonistic VSA based on EETS corresponding to comparison settings 1” subsection.

As shown in Figure 3, the slight angular position tracking error and the stiffness tracking error can be observed during $t = 5–6\ s$ and $t = 10–11\ s$, which are caused by predefined system disturbance. Moreover, the angular position tracking errors during $t = 12.5–13.5\ s$ and $t = 18–19\ s$ are caused by the change of the stiffness reference trajectory, and the stiffness tracking errors during $t = 7–8\ s$ and $t = 15–16\ s$ are caused by the change of the position reference trajectory. In order to show more clearly the effect of the EEC measures on tracking performance, the tracking errors and the controlling energy consumptions are shown in Table 3. The abbreviations in Table 3 are defined as

$$
\begin{align*}
\text{IAEQ} &= \int_{0}^{T} |q - q_{d}|dt, \quad \text{IAEK} = \int_{0}^{T} |k_{ej} - k_{ejd}|dt, \\
\text{IAU}_\alpha &= \int_{0}^{T} |\text{sat}(u_{\alpha})|dt, \quad \text{IAU}_\beta = \int_{0}^{T} |\text{sat}(u_{\beta})|dt, \\
\text{SIAE} &= \text{IAEQ} + \text{IAEK}, \\
\text{SIAU} &= \text{IAU}_\alpha + \text{IAU}_\beta,
\end{align*}
$$

Note that the total simulation time is selected as $T = 20\ s$.

According to the simulation comparisons shown in Figure 3 and Table 3, it can be concluded that the proposed EEC measures are beneficial to reduce the system output tracking errors, and the introduction of the EEC measures does not make the response of the control input worse.

### Table 3. Comparison of the effects of EEC on tracking performance of the EETS-based VSA.

|               | LESO + SMC + ISC | LESO + SMC + ISC + EEC |
|---------------|------------------|-------------------------|
| IAEQ          | 3.3343           | 1.9604                  |
| IAEK          | 3.5803           | 1.9452                  |
| SIAE          | 6.9146           | 3.9056                  |
| IAU$_\alpha$  | 167.4728         | 167.4721                |
| IAU$_\beta$   | 133.4454         | 133.7947                |
| SIAU          | 300.9183         | 301.2668                |

**Note:**
- LESO: linear extended state observer; SMC: sliding mode control; ISC: input saturation compensation; EEC: estimation error compensation; EETS: equivalent exponential torsion spring; VSA: variable stiffness actuator.

### Simulation and analysis of the effect of EEC on tracking control of the antagonistic VSA based on EETS corresponding to comparison settings 3

In order to further demonstrate the robustness of the designed controller and the effectiveness of the proposed novel EEC measures in reducing tracking errors, the simulation results with time-varying external disturbances based on the LESO + SMC + ISC (WODEG) controller and the LESO + SMC + ISC + EEC (WODEG) controller are depicted in Figure 4. Note that WODEG (without considering the damping coefficients and the gravity effect parameter) indicates that the friction damping coefficients and the gravity effect parameter are set to zero directly in the calculation process of the controller, namely, $D_q = \dot{q} = 0$. Although the actual friction damping coefficients (i.e. $D_q$ and $\dot{q}m$) and the gravity effect parameter (i.e. $E_g$) in the system dynamic model are not zero and subject to uncertainties in the simulation, this setting ($\dot{q} = \dot{q}m = E_g = 0$) can reduce the computational burden of the controller. Moreover, the reference trajectories are reselected as $q_d = 1.5 + 2\sin(t)(t > 3\ s) - 2\sin(t)(t > 13\ s)$ and $k_{ejd} = k_{ej0} + 10 + 4\sin(t)(t > 6.5\ s) - 4\sin(t)(t > 16\ s) + 4\times(t > 17.5\ s)$ to show that the designed controller can track the position and stiffness simultaneously.

In this simulation, the characteristic equations of the LESO are set to $(s_q + 90)^5 = 0$ and $(s_q + 70)^3 = 0$, and the gain parameters of the LESO can be obtained according to equation (24). The other parameters of these two controllers (i.e. LESO + SMC + ISC (WODEG) controller and LESO + SMC + ISC + EEC (WODEG) controller) and the ranges of the control inputs are all set to be the same as those set in the “Simulation and analysis of the effect of EEC on tracking control of the antagonistic VSA based on EETS corresponding to comparison settings 1” subsection. It should be noted that in this simulation, the unknown external disturbance $r_{ext}$ is defined as the time-varying disturbance in the period of $0–10\ s$ to further verify the robustness of the designed controller and the effectiveness of the estimated error compensation measures, that is, $r_{ext} = 2.5 + 1.5\sin(2t)$ for $t = 0–5\ s$, $r_{ext} = 4.5 + 2\sin(2t)$ for $t = 5–10\ s$. The other system disturbances still refer to Table 1.

Because $D_q = \dot{q}m = E_g = 0$ is set directly in the calculation of the controller, and the time-varying external disturbance is imposed on the system model, the tracking errors under the LESO + SMC + ISC (WODEG) controller are relatively large as shown in Figure 4(a). In addition, the observation of Figure 4(c) shows that the response curves of the composite states of the EETS-based VSA under the LESO + SMC + ISC (WODEG) controller have exceeded the mechanical allowable ranges, which will lead to worse tracking effect in practical control. Compared with the tracking response curves under the LESO + SMC + ISC (WODEG) controller, the tracking errors under the LESO + SMC + ISC + EEC (WODEG) controller are relatively small, and the control inputs change more smoothly, and the composite states of the EETS-based VSA change within the mechanical allowable ranges (i.e. $0 < (\alpha_0 + x_1 + x_3) \leq 4.5\ rad$, $0 < (\beta_0 + x_5 - x_1) \leq 4.5\ rad$).

From the simulation results shown in Figure 4, it can be observed that the LESO + SMC + ISC + EEC (WODEG) controller can simultaneously realize the tracking control of position and stiffness of the EETS-based VSA although the nominal values of the viscous friction coefficients and the gravitational effect parameter are directly defined as zero in the process of controller calculation. The comprehensive comparison of Figure 4 and Table 4 shows the robustness of the LESO + SMC + ISC + EEC (WODEG) controller.
controller and the effectiveness of the proposed EEC measures in reducing the system output tracking error and improving the disturbance suppression performance of the controller. According to the simulation settings and simulation results in this section, it is worth mentioning that the proposed novel EEC measures will play a more significant role in reducing the system output tracking errors when the system is subject to time-varying external disturbance.

**Figure 4.** Comparison of response curves of the EETS-based VSA corresponding to the LESO + SMC + ISC (WODEG) controller and the LESO + SMC + ISC + EEC (WODEG) controller: (a) comparison of output response curves of the EETS-based VSA, (b) comparison of response curves of the control inputs for the EETS-based VSA, and (c) comparison of response curves of the composite states for the EETS-based VSA. LESO: linear extended state observer; SMC: sliding mode control; ISC: input saturation compensation; EEC: estimation error compensation; EETS: equivalent exponential torsion spring; VSA: variable stiffness actuator; WODEG: without considering the damping coefficients and the gravity effect parameter.

**Table 4.** Comparison of the effects of EEC on tracking performance of the EETS-based VSA.

|                      | LESO + SMC + ISC (WODEG) | LESO + SMC + ISC + EEC (WODEG) |
|----------------------|--------------------------|---------------------------------|
| IAEQ                 | 5.8012                   | 1.6220                          |
| IAЕK                 | 9.7642                   | 3.0044                          |
| SIAE                 | 15.5653                  | 4.6264                          |
| IAU\(\alpha\)       | 164.4722                 | 162.2904                        |
| IAU\(\beta\)        | 145.0213                 | 147.5101                        |
| SIAU                 | 309.4935                 | 309.8005                        |

LESO: linear extended state observer; SMC: sliding mode control; ISC: input saturation compensation; EEC: estimation error compensation; EETS: equivalent exponential torsion spring; VSA: variable stiffness actuator; WODEG: without considering the damping coefficients and the gravity effect parameter.

In order to validate the effectiveness and adaptability of the proposed EEC measures in tracking control of the Compact-VSA corresponding to comparison settings I, the simulation and analysis of the effect of EEC on tracking control of the Compact-VSA corresponding to comparison settings I was conducted. According to the simulation settings and simulation results in this section, it is worth mentioning that the proposed novel EEC measures will play a more significant role in reducing the system output tracking errors when the system is subject to time-varying external disturbance.

Simulation and analysis of the effect of EEC on tracking control of the Compact-VSA corresponding to comparison settings I

In order to validate the effectiveness and adaptability of the proposed EEC measures in tracking control of the
Compact-VSA and reduce the complexity of the controller, the simulation comparison based on the LESO + SMC controller and the LESO + SMC + EEC controller is depicted in Figure 5. In order to demonstrate that the designed controller can achieve simultaneous position and stiffness tracking control of the Compact-VSA, the combined reference trajectories (i.e. the combination of step trajectories and sinusoidal trajectories) are selected in this simulation.

In this simulation, the initial system states are set to $x(0) = [0, 0, 0, 0, 0]^T$ and the initial joint stiffness of the Compact-VSA is $k_{ej0} = 87 \text{ N} \cdot \text{m/rad}$. The control input constraints are defined as $\text{sat}(u_p) \in [-35 \text{ N} \cdot \text{m}, 35 \text{ N} \cdot \text{m}]$ and $\text{sat}(u_q) \in [-10 \text{ N} \cdot \text{m}, 10 \text{ N} \cdot \text{m}]$. The reference trajectories are $q_d = 1.5 + \sin(t) \cdot (t > 3.5s) + 1 \cdot (t > 7s) - \sin(t) \cdot (t > 16s)$ and $k_{ejd} = k_{ej0} + 15 + 8\sin(t) \cdot (t > 6.5s) - 8\sin(t) \cdot (t > 18s)$, respectively. The parameters of the LESO + SMC controller and the LESO + SMC + EEC controller are selected as $\beta_01 = 1000, \beta_02 = 4 \times 10^5, \beta_03 = 8 \times 10^7, \beta_04 = 8 \times 10^9, \beta_05 = 3.2 \times 10^{11}, \beta_06 = 600, \beta_07 = 1.2 \times 10^5, \beta_08 = 8 \times 10^6, c_{q1} = 2.7 \times 10^4, c_{q2} = 2.7 \times 10^3, c_{q3} = 90, c_{k1} = 100, r_F = 5, \mu_q = r_F, \mu_k = r_F, \gamma_{q1} = 10, \gamma_{q2} = 15, \gamma_{q3} = 20, \gamma_{q4} = 30, \gamma_{q5} = 50, \gamma_{k1} = 5, \gamma_{k2} = 15, \text{ and } \gamma_{k3} = 30$. The predefined disturbance settings can be referred to Table 2. In this simulation, large predefined system composite disturbances are imposed on the Compact-VSA, and the predefined composite disturbances are set to sudden change at $t = 5s, t = 10s$, and $t = 17s$. Therefore, in order to obtain relatively good tracking performance, the large observation gains need to be set for the LESO.

As shown in Figure 5(a), compared with the output response curves based on LESO + SMC controller, the output response curves based on LESO + SMC + EEC controller can achieve better tracking performance.
controller can track the reference trajectories more quickly. Due to large tracking errors, large system disturbances and EEC provided by estimation error compensators, the response curves of the control inputs of the Compact-VSA under the LESO + SMC + EEC controller are fluctuated during the initial stages of tracking. During $t = 5–6$ s, $t = 10–10.5$ s, and $t = 17–17.5$ s, the stiffness tracking errors under the LESO + SMC + EEC controller can be observed, which are caused by the sudden change of the system disturbances. During $t = 3.5–4$ s and $t = 7–7.5$ s, small stiffness tracking errors can also be observed due to the change of position tracking trajectory. As shown in Figure 5(c), the variation ranges of response curves of the lever arm and the angular deflection of the Compact-VSA are all within the mechanical allowable range (i.e. $0 < \delta_1 < 0.015$ m and $-0.35 \text{ rad} \leq \phi \leq 0.35 \text{ rad}$).

The abbreviations in Table 5 are defined as

$$\text{IAU} = \int_0^T \left| \text{sat}(u_p) \right| dt, \quad \text{IAUS} = \int_0^T \left| \text{sat}(u_s) \right| dt, \quad \text{and} \quad \text{SIAU} = \text{IAUP} + \text{IAUS},$$

respectively. The comparison results of Figure 5 and Table 5 show the effectiveness of the proposed EEC measures in reducing the system output fluctuations.

**Table 5.** Comparison of the effects of EEC on tracking performance of the Compact-VSA.

|               | LESO + SMC | LESO + SMC + EEC |
|---------------|------------|------------------|
| $\text{IAEQ}$ | 2.2162     | 1.7098           |
| $\text{IAEK}$ | 61.5462    | 9.5738           |
| $\text{SIAE}$ | 63.7624    | 11.2836          |
| $\text{IAUP}$ | 227.3696   | 211.4690         |
| $\text{IAUS}$ | 22.0121    | 21.4076          |
| $\text{SIAU}$ | 249.3816   | 232.8766         |

LESO: linear extended state observer; SMC: sliding mode control; EEC: estimation error compensation; VSA: variable stiffness actuator.

**Figure 6.** Comparison of response curves of the Compact-VSA corresponding to LESO + SMC + ISC (WODEG) controller and LESO + SMC + ISC + EEC (WODEG) controller: (a) comparison of output response curves of the Compact-VSA, (b) comparison of response curves of the control inputs for the Compact-VSA, and (c) comparison of response curves of the lever arm and the angular deflection of the Compact-VSA. Note that the $\tau_{\text{ext}}$ is reset to time-varying external disturbance. LESO: linear extended state observer; SMC: sliding mode control; ISC: input saturation compensation; EEC: estimation error compensation; VSA: variable stiffness actuator; WODEG: without considering the damping coefficients and the gravity effect parameter.
tracking error and improving the disturbance rejection performance of the controller. In addition, although the EETS-based VSA and the Compact-VSA have completely different variable stiffness principles and dynamic model structures, the designed LESO + SMC + EEC controller can still achieve simultaneous tracking control of the position and stiffness of the Compact-VSA system. The simulation results show good adaptability of the designed controller with EEC measures.

**Simulation and analysis of the effect of EEC on tracking control of the Compact-VSA corresponding to comparison settings 2**

Similar to the simulation comparison shown in the “Simulation and analysis of the effect of EEC on tracking control of the antagonistic VSA based on EETS corresponding to comparison settings 3” subsection, in this simulation, $D_q = D_y = D_z = E_y = 0$ is set directly in the calculation of the controller to reduce the computational burden of the controller. In order to further demonstrate the robustness of the designed controller and the effectiveness of the proposed EEC measures in reducing system output tracking errors, the simulation comparison under LESO + SMC + ISC (WODEG) controller and LESO + SMC + ISC + EEC (WODEG) controller is carried out. In this simulation, the step reference trajectories $q_{ref} = 1.5 + 1.5 \times (t > 9s) - 1 \times (t > 13s) - 1 \times (t > 15.5s)$ and $k_{sid} = k_{sid} \cdot 10 + 10 \times (t > 7s) + 10 \times (t > 12s) - 20 \times (t > 18.5s)$ are set up. Note that the predefined external disturbance imposed on the Compact-VSA is reset to time-varying disturbance to demonstrate the robustness of the controller. Also, the state observer with fixed preset observation gains, the adaptive ISC laws for these two controllers are designed. Finally, a robust tracking controller based on LESO with EEC has good adaptability and the ability to deal with large model uncertainties.

### Table 6. Comparison of the effects of EEC on tracking performance of the Compact-VSA.

| Parameter | LESO + SMC + ISC (WODEG) | LESO + SMC + ISC + EEC (WODEG) |
|-----------|---------------------------|---------------------------------|
| IA EQ     | 2.4026                    | 1.8104                          |
| IA EK     | 22.7513                   | 10.7121                         |
| SIAE      | 25.1539                   | 12.5225                         |
| IAJP      | 153.8342                  | 140.6004                        |
| IAJUS     | 17.9396                   | 17.7705                         |
| IA JU     | 171.7738                  | 158.3709                        |

LESO: linear extended state observer; SMC: sliding mode control; ISC: input saturation compensation; EEC: estimation error compensation; VSA: variable stiffness actuator; WODEG: without considering the damping coefficients and the gravity effect parameters.

### Conclusion

In this article, a robust tracking controller based on the LESO with EEC is designed to achieve simultaneous position and stiffness tracking control of the EETS-based VSA\textsuperscript{15,20,46} (i.e. a representative VSA in the class of antagonistic VSAs based on ENTS) and the Compact-VSA\textsuperscript{9} (i.e. a representative VSA in the class of SVSAs based on lever mechanisms). Firstly, considering the parametric uncertainties, the unknown friction torques acting on the driving units, the unknown external disturbance acting on the output link, and the input saturation constraints in the system dynamic models of these two classes of VSAs (i.e. the antagonistic VSAs based on ENTSs and the SVSAs based on lever mechanisms), the state space models with composite disturbances and input saturation constraints are proposed. Then, by using the nonlinear coordinate transformation, the state space models of these two classes of VSAs are transformed into integral chain pseudo-linear systems with the same structural form. Subsequently, by extending the matched lumped disturbances in the integral-chain pseudo-linear system to two new system states, an extended integral chain pseudo-linear system is obtained, and a LESO with input saturation constraints is designed to estimate the unknown states of the extended pseudo-linear system. Considering the input saturation constraints in the extended pseudo-linear systems and the estimation errors of the LESO with fixed preset observation gains, the adaptive ISC laws and the novel estimation error compensators are designed. Finally, a robust tracking controller based on LESO, SMC, ISC, and EEC is obtained. To the best of our
knowledge, this is the first study to design the estimation error compensators for the estimation errors of the LESO with fixed observation gains, and the design inspiration of the estimation error compensators come from the stable SISO DOB. The tracking errors of the system can be effectively reduced by skillfully combining the LESO and the SISO DOB. The proposed estimation error compensator innovatively extends the DOB to the error estimation application of the LESO, and this error compensation design idea can be applied to more general system tracking control based on ESO\textsuperscript{29–40} (i.e., LESO or NESO) or DOB.\textsuperscript{26–28} Based on the stability analysis of the candidate Lyapunov function, the semi-global ultimate uniformly bounded stability of the closed-loop system under the designed LESO + SMC + ISC + EEC controller is proved, and the guidelines for controller parameter settings are discussed.

In order to demonstrate the effectiveness, robustness, and adaptability of the designed LESO + SMC + ISC + EEC controller and the effect of the estimated error compensation measures on tracking performance, the simulation studies are carried out. The main purposes of these simulations are presented in the following three aspects:

1. The first purpose of these simulations is to show the effect of the proposed EEC measures on tracking performance. The simulation comparisons include: the tracking response curves under the LESO + SMC controller are compared with those under the LESO + SMC + EEC controller, and the tracking response curves under the LESO + SMC + ISC controller\textsuperscript{40} are compared with those under the LESO + SMC + ISC + EEC controller; and the tracking response curves under the LESO + SMC + ISC (WODEG) controller are compared with those under the LESO + SMC + ISC + EEC (WODEG) controller. These simulation comparison studies demonstrate the effectiveness of the proposed EEC measures in reducing the system output tracking errors.

2. The second purpose of these simulations is to demonstrate the robustness and effectiveness of the designed LESO + SMC + ISC + EEC controller in tracking control. In particular, the simulations under the LESO + SMC + ISC + EEC (WODEG) controller demonstrate the ability of the designed controller to deal with large model uncertainties. The simulation results show that the designed controller can achieve simultaneous position and stiffness tracking control of these two classes of VSAs (i.e., the antagonistic VSA based on ENTS and the SVSA based on lever mechanism).

3. The third purpose of these simulations is to demonstrate the extensive adaptability of the designed LESO + SMC + ISC + EEC controller in the tracking control of these two classes of VSAs. As the representatives of the antagonistic VSA based on ENTS and the SVSA based on lever mechanisms, although the EETS-based VSA and the Compact-VSA have different dynamic models and variable stiffness principles, the designed controller is suitable for the tracking control of these two VSAs. This is because although the system dynamic models of the EETS-based VSA and the Compact-VSA are different, they can be transformed into extended integral chain pseudo-linear systems with the same structural type. Because the decoupling matrices of these two VSAs are both nonsingular, the designed controller based on LESO with EEC can achieve simultaneous position and stiffness tracking control.

It should be noted that when the control scheme proposed in this article is implemented, the high-performance CPU, the high-speed data acquisition, and communication hardware equipment should be used in the hardware design to ensure the real-time control. In addition, in the calculation of the LESO + SMC + ISC + EEC (WODEG) controller, the friction damping coefficients and the gravity effect parameters are set to zero directly. This setting will help to reduce the computational burden of the controller, and the simulation results show that the controller still has good robust tracking performance under this setting (i.e., $D_q = D_m = D_p = D_s = E_g = 0$ is set directly in the calculation process of the controller). Because there is no experimental platform, only simulation studies are carried out in this article. It is the future work to verify the tracking control performance of the designed controller through real experiments.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD
Jishu Guo https://orcid.org/0000-0002-6653-3227

References
1. Wolf S, Grioli G, Eiberger O, et al. Variable stiffness actuators: review on design and components. IEEE/ASME Trans Mechatronics 2016; 21(5): 2418–2430.
2. Liu L, Leonhardt S, and Misgeld BJE. Experimental validation of a torque-controlled variable stiffness actuator tuned by gain scheduling. IEEE/ASME Trans Mechatronics 2018; 23(5): 2109–2120.
3. Li Z and Bai S. A novel revolute joint of variable stiffness with reconfigurability. Mech Mach Theory 2019; 133: 720–736.
4. Cestari M, Merodio DS, Arevalo JC, et al. An adjustable compliant joint for lower-limb exoskeletons. IEEE/ASME Trans Mechatronics 2015; 20(2): 889–898.

5. Molto M, Cavallo G, Bace T, et al. Variable stiffness ankle actuator for use in robotic-assisted walking: control strategy and experimental characterization. Mech Mach Theory 2019; 134: 604–624.

6. Schiavi R, Grioli G, Sen S, et al. VSA-II: a novel prototype of variable stiffness actuator for safe and performing robots interacting with humans. In: Proceedings of the 2008 IEEE international conference on robotics and automation, Pasadena, CA, USA, 19–23 May 2008, pp. 2171–2176. Washington DC, USA: IEEE Computer Society.

7. Jafari A, Tsagarakis NG, and Caldwell DG. A novel intrinsically energy efficient actuator with adjustable stiffness (AwAS). IEEE/ASME Trans Mechatronics 2013; 18(1): 355–365.

8. Jafari A, Tsagarakis NG, Sardellitti I, et al. A new actuator with adjustable stiffness based on a variable ratio lever mechanism. IEEE/ASME Trans Mechatronics 2014; 19(1): 55–63.

9. Tsagarakis NG, Sardellitti I, and Caldwell DG. A new variable stiffness actuator (CompAct-VSA): design and modeling. In: Proceedings of the 2011 IEEE/RSJ international conference on intelligent robots and systems, San Francisco, CA, USA, 25–30 September 2011, pp. 378–383. Washington DC, USA: IEEE Computer Society.

10. Groothuis SS, Rusticelli G, Zucchelli A, et al. The vsaUT-II: a novel rotational variable stiffness actuator. In: Proceedings of the 2012 IEEE international conference on robotics and automation, Saint Paul, MN, USA, 14–18 May 2012, pp. 3355–3360. Washington DC, USA: IEEE Computer Society.

11. Sun J, Guo Z, Zhang Y, et al. A novel design of serial variable stiffness actuator (SVSA) based on an Archimedean spiral relocation mechanism. IEEE/ASME Trans Mechatronics 2018; 23(5): 2121–2131.

12. Sun J, Guo Z, Sun D, et al. Design, modeling and control of a novel compact, energy-efficient, and rotational serial variable stiffness actuator (SVSA-II). Mech Mach Theory 2018; 130: 123–136.

13. Sardellitti I, Medrano-Cerda GA, Tsagarakis NG, et al. Gain scheduling control for a class of variable stiffness actuators based on lever mechanisms. IEEE Trans Robot 2013; 29(3): 791–798.

14. Petir F, Daasch A, and Albu-Schäffer A. Backstepping control of variable stiffness robots. IEEE Trans Control Syst Technol 2015; 23(6): 2195–2202.

15. Palli G. Model and control of tendon actuated robots. PhD Thesis, Università degli Studi di Bologna, Italia, 2007.

16. Flacco F. Modeling and control of robots with compliant actuation. PhD Thesis, SAPIENZA Università di Roma, Roma, 2012.

17. Buondonno G and Luca AD. Efficient computation of inverse dynamics and feedback linearization for VSA-based robots. IEEE Robot Autom Lett 2016; 1(2): 908–915.

18. Choi J, Park S, Lee W, et al. Design of a robot joint with variable stiffness. In: Proceedings of the 2008 IEEE international conference on robotics and automation, Pasadena, CA, USA, 19–23 May 2008, pp. 1760–1765. Washington DC, USA: IEEE Computer Society.

19. Palli G and Melchiorri C. Robust control of robots with variable joint stiffness. In: Proceedings of the 2009 international conference on advanced robotics, Munich, Germany, 22–26 June 2009, pp. 1–6. Washington DC, USA: IEEE Computer Society.

20. Lu H, Zhang X, and Huang X. Robust adaptive control of antagonistic tendon-driven joint in the presence of parameter uncertainties and external disturbances. J Dyn Syst Meas Control 2017; 139: 101003-1–101003-10.

21. Zhang L, Li Z, and Yang C. Adaptive neural network based variable stiffness control of uncertain robotic systems using disturbance observer. IEEE Trans Ind Electron 2017; 64(3): 2236–2245.

22. Psomopoulos E, Theodorakopoulos A, Doulgeri Z, et al. Prescribed performance tracking of a variable stiffness actuated robot. IEEE Trans Contr Syst Technol 2015; 23(5): 1914–1926.

23. Guo Z, Pan Y, Sun T, et al. Adaptive neural network control of serial variable stiffness actuators. Complexity 2017; 2017: 1–9.

24. Chen WH, Yang J, Guo L, et al. Disturbance-observer-based control and related methods—an overview. IEEE Trans Ind Electron 2016; 63(2): 1083–1095.

25. Chen WH. Disturbance observer based control for nonlinear systems. IEEE/ASME Trans Mechatronics 2004; 9(4): 706–710.

26. Li Y, Zeng M, An H, et al. Disturbance observer-based control for nonlinear systems subject to mismatched disturbances with application to hypersonic flight vehicles. Int J Adv Robot Syst 2017; 14(2): 1–10.

27. Yang J, Zolotas A, Chen WH, et al. Robust control of non-linear MAGLEV suspension system with mismatched uncertainties via DOBC approach. ISA Trans 2011; 50: 389–396.

28. Ren J and Yang D. Disturbance observer–based control of flexible hypersonic flight vehicle. Int J Adv Robot Syst 2017; 14(2): 1–9.

29. Liu C, Liu G, and Fang J. Feedback linearization and extended state observer-based control for rotor-AMs system with mismatched uncertainties. IEEE Trans Ind Electron 2017; 64(2): 1313–1322.

30. Li S, Yang J, Chen WH, et al. Generalized extended state observer based control for systems with mismatched uncertainties. IEEE Trans Ind Electron 2012; 59(12): 4792–4802.

31. Talole SE, Kolhe JP, and Phadke SB. Extended-state-observer-based control of flexible-joint system with experimental validation. IEEE Trans Ind Electron 2010; 57(4): 1411–1419.

32. Du B, Wu S, Han S, et al. Application of linear active disturbance rejection controller for sensorless control of internal permanent-magnet synchronous motor. IEEE Trans Ind Electron 2016; 63(5): 3019–3027.
33. Ren C, Li X, Yang X, et al. Extended state observer-based sliding mode control of an omnidirectional mobile robot with friction compensation. *IEEE Trans Ind Electron* 2019; 66(12): 9480–9489.
34. Cui M, Liu W, Liu H, et al. Extended state observer-based adaptive sliding mode control of differential-driving mobile robot with uncertainties. *Nonlinear Dyn* 2016; 83: 667–683.
35. Xing HL, Li DH, Li J, et al. Linear extended state observer based sliding mode disturbance decoupling control for nonlinear multivariable systems with uncertainty. *Int J Control Autom Syst* 2016; 14(4): 967–976.
36. Sayem AHM, Cao ZW, and Man ZH. Model free ESO-based repetitive control for rejecting periodic and aperiodic disturbances. *IEEE Trans Ind Electron* 2017; 64(4): 3433–3441.
37. Alonge F, Cirrincione M, D’Ippolito F, et al. Robust active disturbance rejection control of induction motor systems based on additional sliding-mode component. *IEEE Trans Ind Electron* 2017; 64(7): 5608–5624.
38. Cui RX, Chen LP, Yang CG, et al. Extended state observer-based integral sliding mode control for an underwater robot with unknown disturbances and uncertain nonlinearities. *IEEE Trans Ind Electron* 2017; 64(8): 6785–6795.
39. Wang JX, Li SH, Yang J, et al. Extended state observer-based sliding mode control for PWM-based DC–DC buck power converter systems with mismatched disturbances. *IET Control Theory Appl* 2015; 9(4): 579–586.
40. Guo J and Tian G. Mechanical design and robust tracking control of a class of antagonistic variable stiffness actuators based on the equivalent nonlinear torsion springs. *Proc Inst Mech Eng I: J Syst Control Eng* 2018; 232(10): 1337–1355.
41. Boukadida W, Benamor A, Messaoud H, et al. Multi-objective design of optimal higher order sliding mode control for robust tracking of 2-DoF helicopter system based on metaheuristics. *Aerosp Sci Technol* 2019; 91: 442–455.
42. Esmaeili N, Alfi A, and Khosravi H. Balancing and trajectory tracking of two-wheeled mobile robot using backstepping sliding mode control: design and experiments. *J Intell Robot Syst* 2017; 87(3–4): 601–613.
43. Rafiq M, Rehman S, Rehman F, et al. A second order sliding mode control design of a switched reluctance motor using super twisting algorithm. *Simul Model Pract Theory* 2012; 25: 106–117.