Phase-space representation of non-classical behaviour of scalar wave-fields

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Abstract. The modelling of optical fields by using radiant and virtual point sources for the spatial coherence wavelets in the phase-space representation evidences some effects, conventionally attributed to non-classical correlations of light, although such type of correlations are not explicitly included in the model. Specifically, a light state is produced that has similar morphology to the Wigner Distribution Function of the well-known quantum Schrödinger cat and squeezed states.

1. Introduction

The manipulation of the quantum states of light is a subject of growing interest, with topics as the production of only-one photon sources [1, 2, 3], the analysis of the quantum interference mechanisms like the Schrödinger cat states of light [4] and those derived from the matter-radiation interaction [5] for instance. They are promising topics for technological applications in quantum computation and information processing [6], quantum teleportation [7] and quantum opto-electronic systems [8]-[11].

In spite of the accepted quantum nature of topics as the Schrödinger cat states, a classical approach to them seems to be possible by adding novel considerations to the phase-space representation of the optical wave-field, like the spatial coherence wavelets emitted by both radiant and virtual point sources [12]. It could reevaluate the actual limits of the classical theories and the real grounds of the physical behaviour of light. For instance, the analysis presented in this paper suggests that the Schrödinger cat states are actually originated by the spatial coherence state of light.

In the phase-space representation [13] of optical fields in arbitrary states of spatial coherence recently proposed, the spatial coherence wavelets and the marginal power spectrum [14] are responsible for the propagation of power and correlation features of the field. These descriptors are emitted by source pairs, gathered in classes determined by the pair separations [15]. Such description is compatible to the second order spatial coherence theory of the optical field [16], and provides new insight about the interference and the diffraction phenomena, as discussed.
in the present work. For instance, the marginal power spectrum provides a ray-map with two kinds of rays, i.e. carrier and 0-\(\pi\) modulating rays, defined in the phase-space [17, 18]. Such rays are quite different from their geometrical counterparts, derived from the eikonal theory or Hamilton’s characteristic functions [19], i.e. they are actually radiometric entities capable of describing diffraction and interference [17, 20].

Each kind of rays is emitted by a specific set of point sources [12], i.e. radiant point sources for carrier rays and virtual point sources for 0-\(\pi\) modulating rays. These sets of point sources are optically disjoint, in such a way that they can be regarded as allocated over two distinct layers of the space, which eventually can be brought together onto the same plane in specific arrangements. In such cases, dual point sources are induced by the coincidence of radiant and virtual sources at the same point on the plane.

The very pertinence of this model has been assured by its successfully description of most the properties and behaviour of both scalar and random electromagnetic fields [12, 21]-[28]. However, it reveals some unexpected features of the field, morphologically similar to the quantum squeezed states and the Schrödinger cat states, without appealing to explicit quantum constrains, as discussed in the following.

2. The Young’s experiment

Such unexpected behaviour can be analysed by modelling the one-dimensional Young’s experiment, performed by arranging a double slit mask at the AP, with parameters depicted in Figure 1. Centre and difference coordinates [14] \((\xi_A, \xi_D)\) and \((x_A, x_D)\) are used for univocally specified the positions of pairs of points at the mask (aperture plane AP) and the detector (observation plane OP) respectively.

\[ S_{\text{diff}}(\xi_A, x_A) = S_0 \left[ 1 + 2 \int_{|\xi_D|\neq 0}^{2|\xi_D|} \mu(\xi_A + \xi_D/2, \xi_A - \xi_D/2) \right] \]

Figure 1. One-dimensional double slit mask for Young’s experiment with diffraction effects. The shadowed regions describe the structured spatial coherence supports for a) the diffraction of light through the slits and b) for the interference between the contributions of the slits.

For mathematical simplicity and without lack of generality, let us assume the diffraction of a uniform scalar wave field of constant power \(S_0\) over each slit. Its marginal power spectrum (also called ray-map [14]) can be expressed as \(S(\xi_A, x_A) = S_{\text{diff}}(\xi_A - (b + a)/2, x_A) + S_{\text{diff}}(\xi_A + (b + a)/2, x_A) + S_{\text{int}}(\xi_A, x_A)\), with

\[ S_{\text{diff}}(\xi_A, x_A) = S_0 \left[ 1 + 2 \int_{|\xi_D|\neq 0}^{2|\xi_D|} \mu(\xi_A + \xi_D/2, \xi_A - \xi_D/2) \right] \]
\[
\times \cos \left( \frac{k}{\lambda} \xi_{Dx_A} - \alpha(\xi_A + \xi_D/2, \xi_A + \xi_D/2) \right) d^2\xi_D [\text{rect}(\xi_A/a)]
\]

where \( \text{rect}(\xi_A/a) = 1 \) for \( |\xi_A| \leq a/2 \) and equal to null otherwise, describing the diffraction of the scalar wave field through any of the slits; and

\[
S_{int}(\xi_A, x_A) = 2S_0 \text{rect}(\xi/A) \int_{b+2|\xi_A|}^{2a+b-2|\xi_A|} |\mu(\xi_A + \xi_D/2, \xi_A - \xi_D/2)| d\xi_D
\]

\[
\times \cos \left( \frac{k}{\lambda} \xi_{Dx_A} - \alpha(\xi_A + \xi_D/2, \xi_A - \xi_D/2) \right) d\xi_D
\]

describing the interference between the contributions provided by the both slits. The quantity \( \mu(\xi_A + \xi_D/2, \xi_A - \xi_D/2) = |\mu(\xi_A + \xi_D/2, \xi_A - \xi_D/2)| \exp(i\alpha(\xi_A + \xi_D/2, \xi_A - \xi_D/2)) \) in equations 1 is the complex degree of spatial coherence [16] of the optical wave field at the AP. It is worth noting that there are both radiant and virtual emitters associated to \( S_{dif}(\xi_A, x_A) \) while there are only virtual emitters associated to \( S_{int}(\xi_A, x_A) \) for \( b > 0 \).

Taking into account that \( S(x_A) = \left( \frac{1}{\lambda z} \right)^2 \int_{\text{AP}} S(\xi_A, x_A) d\xi_A \) determines the power spectrum of the field recorded by the detector at the OP [14], it follows \( S(x_A) = S_{rad}(x_A) + S_{virt}(x_A) = S_{dif}^{(rad)}(x_A) + S_{dif}^{(virt)}(x_A) + S_{int}^{(virt)}(x_A) \), where

\[
S_{rad}(x_A) = S_{dif}^{(rad)} = \left( \frac{1}{\lambda z} \right)^2 2aCS_0 \geq 0
\]

is the uniform contribution of the radiant emitters within both diffracting slits onto the power spectrum at any point \( x_A \) on the OP, and \( S_{virt}(x_A) = S_{dif}^{(virt)}(x_A) + S_{int}^{(virt)}(x_A) \) the contribution of the virtual emitters related to both the diffraction of the field through any slit and the interference between the contributions of both slits, onto the power spectrum at any point \( x_A \) on the OP, i.e.

\[
S_{dif}^{(virt)}(x_A) = \left( \frac{1}{\lambda z} \right)^2 \int_{-(b+a)/2}^{-(a+b)/2} S_{dif}^{(virt)}(\xi_A + (b+a)/2, x_A) d\xi_A
\]

\[
\quad + \left( \frac{1}{\lambda z} \right)^2 \int_{b/2}^{a+b/2} S_{dif}^{(virt)}(\xi_A - (b+a)/2, x_A) d\xi_A
\]

and

\[
S_{int}^{(virt)}(x_A) = \left( \frac{1}{\lambda z} \right)^2 \int_{-a/2}^{a/2} S_{int}^{(virt)}(\xi_A, x_A) d\xi_A
\]

More insight about the diffraction and interference involved in the Young’s experiment with scalar wave fields in any state of spatial coherence is reached by considering separately the terms 3 to 5 of the power spectrum at the OP. Indeed, according to the conservation law of the total energy of the field [17] the total radiant energy is only determined by equation 3 independently of the spatial coherence state of the wave field, i.e. \( \int_{\text{OP}} S(x_A) dx_A = \int_{\text{OP}} S_{rad}(x_A) dx_A = S_{dif}^{(rad)} \int_{\text{OP}} dx_A = 2aS_0 \). Furthermore, the terms 4 and 5 can take on positive and negative values, depending on the spatial coherence state of the field. The positive values of \( S_{dif}^{(virt)}(x_A) \)
determines the main maximum of the diffraction envelope of the power spectrum at the OP, while the negative values extend over the region outside of the main maximum, determining the points of null power of the diffraction envelope, its side lobes and its convergence to null in that region. In contrast, positive and negative values of \( S_{int}(x_A) \) strongly oscillates within the diffraction envelope, without exceeding in magnitude the value of the diffraction envelope.

Table 1 illustrates the results above. Figures in the first column on the left are the ray maps produced by double slit masks, whose transmission function \( t(\xi) \) is depicted on the top of each figure (i.e. slit width \( a \) and distance \( b \) between the consecutive slit edges). Diffraction and interference contributions, \( S_{dif}(x_A, x_A) \) and \( S_{int}(x_A, x_A) \) respectively, are delimited by white bars in each ray map. The coordinate origin for both \( x_A \) and \( x_A \) is at the midpoint of each map. The profiles of the power spectra that arrive onto the OP are shown in the last column on the right respectively. Their energies can be recorded by square modulus detectors placed at the corresponding planes. The profiles of the diffraction and interference contributions to the power spectra at the OP are also sketched in the second and third columns from the left respectively, and the diffraction contribution includes the energies provided by both the radiant and the virtual emitters placed within the slits.

Rows 1 to 4 show the changes in the diffraction of a completely spatially coherent wave field (i.e. \(|\mu(\xi_A + \xi_A/2, \xi_A - \xi_D/2)| = 1\); it is also assumed that \( \alpha(\xi_A + \xi_D/2, \xi_A - \xi_D/2) = 0 \) for simplicity and without lack of generality) due to the increasing of the slit width \( a \) by maintaining the distance between the slit centres unchanged. According to equations 1 and 2, the diffraction contribution of each slit to the ray map takes the form

\[
S_{dif}(\xi_A, x_A) = S_0a \frac{\sin \left[ \frac{kb}{2}(a - 2|x_A|)x_A \right]}{kbx_A} \text{rect}(\xi_A/a)
\]  

while the interference contribution due to both slits will be given by

\[
S_{int}(\xi_A, x_A) = 2S_0a \text{rect}(\xi_A/a) \left[ \frac{\sin \left[ \frac{kb}{2}(2a + b - 2|x_A|)x_A \right]}{kbx_A} - \frac{\sin \left[ \frac{kb}{2}(b + 2|x_A|)x_A \right]}{kbx_A} \right]
\]  

The integration of equations (6) and (7) following equations (3) to (5) gives the power spectrum produced by the Young’s experiment at the OP. Row 1 is corresponding to the mask with the narrowest slits that leads to \( \text{rect}(\xi_A/a) \approx \delta(\xi_A) \), \( S_{dif}(\xi_A, x_A) \approx S_0a \left[ 1 - \frac{(\frac{ka}{z})^2x_A^2}{6} \right] \delta(\xi_A) \), \( S_{int}(\xi_A, x_A) \approx 4aS_0 \left[ 1 - \frac{(\frac{ka}{z})^2x_A^2}{6} \right] \cos \left( \frac{kb}{z}x_A \right) \delta(\xi_A) \), with \( \delta(\xi_A) \) a Dirac’s delta. Thus, the ray map on row 1 is described by

\[
S(\xi_A, x_A) \approx aS_0 \left\{ \delta \left( \xi_A - \frac{b + a}{2} \right) + \delta \left( \xi_A + \frac{b + a}{2} \right) + 4\cos \left( \frac{kb}{z}x_A \right) \delta(\xi_A) \right\}
\]

\[
-\frac{a}{6} \left( \frac{ka}{z} \right)^2 S_0x_A^2 \left\{ \delta \left( \xi_A - \frac{b + a}{2} \right) + \delta \left( \xi_A + \frac{b + a}{2} \right) + 16\cos \left( \frac{kb}{z}x_A \right) \delta(\xi_A) \right\}
\]

The term on the first line of this expression is corresponding to the ray map for a Young experiment with delta-like slits, which enclose only radiant emitters, and then turn on only virtual emitters for interference. The term on the second line describes the change of the ray map due to a small growth in slit width, which increases the number of radiant emitters within the slits.
Now, they are capable of turning on some virtual emitters for diffraction. This fact introduces a parabolic disturbance on the term on the first line, along the \( x_A \)-axis of the OP. The virtual emitters within the slits increase \( S_{\text{dif}}(x_A) \) in the middle region at the AP and decrease it at the pattern edges. The correlations between the radiant emitters in both slits turn on the virtual emitters of \( S_{\text{int}}(x_A) \), whose positive and negative modulating energies are cosine-like distributed on the OP, but with parabolic increments at the middle region and decrements at the pattern edges. Therefore, \( S(x_A) \) will be a high contrasted cosine-like pattern of fringes with a parabolic disturbance, which can be minimized if the observation region on the OP is small enough. As the slit width increases, more radiant emitters are included within, so that more virtual sources are turned on for both \( S_{\text{dif}}(x_A) \) and \( S_{\text{int}}(x_A) \). Accordingly, both the increments and decrements of \( S_{\text{dif}}(x_A) \) become more significant, in such a way that \( S_{\text{dif}}(x_A) \) acquires a main maximum in a delimited region around the midpoint of the OP and decreasing side lobes. The bigger the slits the narrower the main maximum. It is worth noting that \( S_{\text{dif}}(x_A) \geq 0 \) and that \( S_{\text{dif}}(x_A) = S(x_A) \) for completely spatially incoherent wave fields. In addition, the magnitudes of the positive and negative modulating energies of \( S_{\text{int}}(x_A) \) significantly growth within the region of the main maximum of \( S_{\text{dif}}(x_A) \) and appreciably diminish outside this region, in such a way that \( S(x_A) \) will be a high contrasted cosine-like pattern of fringes modulated by the profile of \( S_{\text{dif}}(x_A) \).

It is worth noting that the distribution of the positive and negative modulating energies of \( S_{\text{int}}(x_A) \) resembles the Schrödinger cat state between mixed states of a quantum system, i.e. because of its spatial coherence state, the scalar wave field at both slits of the Young’s mask mixes through its propagation to the OP in a similar way as the quantum cat state. This mixture is modelled in terms of the virtual point sources turned on at the middle region of the mask, due to the correlations between pairs of radiant emitters placed at different mask slits. Consequently, the profiles on the third column from the left in Table 1 are corresponding to the positive and negative modulating energies emitted onto the OP by the whole set of virtual point sources associated to the mixture of the fields that emerge from the both slits.

The spatial distribution of these energies must fulfil the condition [17]

\[
\int_{\text{OP}} \cos \left( \frac{k}{\lambda} \xi_D \cdot r_A - \frac{k}{\lambda} \xi_A \cdot r_A - \alpha (\xi_A + \xi_D/2, \xi_A - \xi_D/2) - \Delta \phi \right) d^2r_A = 0 \quad (9)
\]

Therefore, the phase of the complex degree of spatial coherence \( \alpha(\xi_A + \xi_D/2, \xi_A - \xi_D/2) \) and/or the transmission phase difference at pairs of points on the different apertures of the mask \( \Delta \phi \) in the cosine function of this equation can be used in order to shift the distribution of such modulating energies. For instance, a shift by \( \pi \) means that positive energies are turned in negative ones and vice versa. This behaviour is compatible with the description of the mentioned Schrödinger cat state, although it was obtained without appealing to quantum premises, but only by describing the spatial coherence state of the wave-field in terms of a layer of virtual point sources and the modulating 0-\( \pi \) rays they emit, novel concepts that formalize a classical field behaviour. Specifically, the set of virtual point sources associated to the correlations of radiant point sources placed in different mask slits, and their modulating 0-\( \pi \) rays epitomize the Schrödinger cat state of the state mixture of the fields at the slits. It seems to reveal that the Schrödinger cat state actually results from the spatial coherence state of the wave-field, as expressed in equation 2 for the regarded classical Young’s experiment. It is also remarkable that this result is not a secondary derivation or peculiarity of the model but a substantially part of it, because it is very necessary to appropriately describing the interference in the Young’s experiment.

\[
\int_{\text{OP}} \cos \left( \frac{k}{\lambda} \xi_D \cdot r_A - \frac{k}{\lambda} \xi_A \cdot r_A - \alpha (\xi_A + \xi_D/2, \xi_A - \xi_D/2) - \Delta \phi \right) d^2r_A = 0 \quad (9)
\]
Table 1. Young’s experiment with diffraction effects. Ray-maps in the first column of the left; diffraction envelope of the power spectrum at the OP in the second column from the left; modulating power of the interference pattern in the third column and power spectrum of the interference pattern recorded by a detector at the OP in the column on the right. All quantities in arbitrary units.

| $S(\xi_A, x_A)$ (arbitrary units) | $S_{dif}(x_A) = S_{dif}^{(rad)} + S_{dif}^{(vert)}$ (arbitrary units) | $S_{int}(x_A) = S_{int}^{(vert)}(x_A)$ (arbitrary units) | $S(x_A) = S_{dif}(x_A) + S_{int}(x_A)$ (arbitrary units) |
|----------------------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| ![Ray-map](image1)               | ![Diffraction envelope](image2)                | ![Modulating power](image3)                  | ![Power spectrum](image4)                      |
| ![Ray-map](image5)               | ![Diffraction envelope](image6)                | ![Modulating power](image7)                  | ![Power spectrum](image8)                      |
| ![Ray-map](image9)               | ![Diffraction envelope](image10)               | ![Modulating power](image11)                 | ![Power spectrum](image12)                     |

(1)  

(2)  

(3)
\[
S(\kappa, \lambda) = S_{\text{dif}}(\kappa, \lambda) + S_{\text{int}}(\kappa, \lambda)
\]

\[
S_{\text{dif}}(\kappa, \lambda) = S_{\text{virt}}(\kappa, \lambda) + S_{\text{int}}(\kappa, \lambda)
\]

\[
S_{\text{int}}(\kappa, \lambda) = S_{\text{virt}}(\kappa, \lambda)
\]
\[ S(e, \xi, \lambda) = S_{\text{diff}}(e, \lambda) + S_{\text{int}}(e, \lambda) \]
On the other hand, the region of the main maximum of $S_{\text{dif}}(x_A)$ is corresponding to the region enclosed by the dotted rectangle in each ray map (the whole ray map in row 1 is enclosed by the rectangle); it also concentrates the significant positive and negative modulating energies of $S_{\text{int}}(x_A)$. Therefore, the bigger the slits the narrower such region of the ray-map. This feature resembles the quantum squeezing as discussed in the following.

Profiles in columns 2 to 4 from the left of Table 1 point out that the region that concentrates the significant values of both the radiant and the modulating energies of the power spectrum at the OP is corresponding to the support of the main maximum of the diffraction pattern. It is delimited by the first zeroes of equation 6, i.e. $x_A^{(0)}(\xi_A) = \pm 0.5 \frac{\lambda z}{a - 2|\xi_A|}$, so that $\triangle x_A(\xi_A) = \frac{\lambda z}{a - 2|\xi_A|}$ (Figure 2), whose minimum value is $\triangle x_A^{(\text{min})} = \triangle x_A(0) = \frac{\lambda z}{a}$, i.e. $\triangle x_A(\xi_A) \geq \frac{\lambda z}{a}$. Thus, the smaller the slit width the bigger the support of the main maximum of diffraction; in addition, the smaller the slit width the faster the growth of the support size in the surrounding of $\xi_A = 0$.

![Figure 2. Support of the main maximum of the diffraction pattern by a slit as function of the slit width and its growth with the coordinate $\xi_A$.](image)

Now, let us consider the ray-tracing from the AP to the OP as resulting from an ensemble of statistical realizations, each one consisting in tracing all the rays (both carrier and modulating ones) that travel for the AP to the OP at a given instant. The addition of the energies of all the rays arriving to the same point $x_A$ on the OP gives the power spectrum at that point. Two radiant sources emitting at the same time, can turn on a virtual source at this time depending on the correlation between their emissions. So, the probability of such activation will be close to one for fully spatially coherent wave fields and close to zero for fully spatially incoherent fields. Therefore, the ray emissions can randomly fluctuate from a realization to the next. In this sense, diffraction virtual sources cannot be turned on within ideal delta-like slits, but interference virtual sources do, at the midpoint between the slits. Accordingly:

- There are up to three well defined starting points at the AP for all rays of any realization produced by a mask with ideal delta-like slits, i.e. one at each slit for the corresponding radiant point source, and one in the midpoint between the slits for the unique virtual point source. The radiant sources at the slits emit only carrier rays that belong to $S_{\text{dif}}(\xi_A)$, and the virtual source emits only 0-$\pi$ modulating rays that belong to $S_{\text{int}}(\xi_A, x_A)$. 


The power spectrum at the OP will be an interference pattern of cosine-like fringes without diffraction modulation, which extends over a broad region on this plane.

As the slit width increases, more radiant sources are included within the slits, and therefore:

- More starting points are enclosed in each slit, corresponding to the positions of both the radiant and the active virtual sources, associated to the diffraction. The same average number of carrier and $0-\pi$ modulating rays are emitted from the corresponding starting point within the slits if the wave field on the mask is uniform and its spatial coherence state over each slit is the same.
- Because of the emission of diffraction $0-\pi$ modulating rays, each slit provides a diffraction modulation on the power spectrum at the OP in each realization. Such modulations will be statistically identical if the slits have the same geometry. The modulation is smooth if few point sources are enclosed within the slits, i.e. if the slits are very narrow (figures on the row 1 of the Table 1), and becomes stronger as the slit width increases, concentrating the energy of the power spectrum in the main maximum of the diffraction pattern.
- By illuminating great enough slits by high spatially coherent scalar wave fields, a significant amount of virtual sources will be turned on within the slits in each realization, whose modulating $0-\pi$ rays will produce a regular sequence of points of zero energy in the power spectrum, that delimits the diffraction maximum and determine the decreasing side lobes of the same size. The positions of the energy zeroes depend on the slit width (i.e. the distance between them is inversely proportional to the slit width, as shown by figures in rows 2 to 4).
- Scalar wave fields of low spatial coherence are able to turn on only few virtual sources within the slits in each realization, so that the energy of the power spectrum is also concentrated in the main maximum of the diffraction pattern, but they cannot completely annihilate the energy at any point of the OP, as depicted in figures in the rows 7 to 9.
- The additional radiant emitters enclosed by the slits can turn on interference virtual emitters at new positions between the slits in each realization. So, starting points for interference $0-\pi$ modulating rays should be regarded within a region of the same size as the slits, centred at the midpoint between the slits.
- It is worth noting that the strength of the positive and negative modulating energies emitted by the virtual emitters is not uniform by high spatially coherent scalar wave fields. Indeed, it is stronger for the virtual emitters around the midpoints of the Young mask or within the slits than for the emitters at the edges of such regions. Consequently, the interference modulating energies are modulated in a similar fashion as the radiant energy does by the diffraction modulating energies, as depicted in figures on the rows 2 to 4 of Table 1.

According to the reasoning above, the expression $a \triangle x_A(0) = \lambda z$, obtained for high spatially coherent wave fields, points out that the area $a \triangle x_A(0)$ of the rectangles centred on the $\xi_A$-axis in both the diffraction and the interference regions of the ray map remains unchanged no matter the slit size. Thus, the greater the slit width the stronger the diffraction modulation and therefore the narrower the main diffraction maximum, where the radiant energy concentrates and the modulating energies have the higher strengths. This behaviour resembles the squeezing effect in parameters of quantum systems involved in uncertainty principles (as position and momentum, for instance), in which the broadening of the uncertainty of any of the parameters causes the squeezing on the uncertainty of the other one. This feature is also very necessary to properly describe the diffraction effects of the slit size onto the interference pattern in a Young’s experiment.
3. Conclusion

It was shown that the classical phase-space representation of optical fields, based on the Wigner Distribution Function named the marginal power spectrum, produces similar features to the Wigner Distribution Function of a cat state of light with “quantum interferences” and of a squeezed state that are not explicitly included in the model. Both features are close related to sets of virtual point sources and the modulating $0-\pi$ rays they emit, that represent the spatial coherence state of the field. It is remarkable that from only classical premises, this model is able to predict quantum-like behaviours of light after adopting the hypothesis of the existence of the virtual point sources, associated to the (classical) correlation properties of the field. Such unexpected result, supported by the analysis of the Young’s interference experiment in the framework of the spatial coherence wavelets, suggests that this “non-classical” behaviour is actually related to the spatial coherence state of the light, even in the classical context.

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