Personalized advice for enhancing well-being using automated impulse response analysis — AIRA

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Abstract

The attention for personalized mental health care is thriving. Research data specific to the individual, such as time series sensor data or data from intensive longitudinal studies, is relevant from a research perspective, as analyses on these data can reveal the heterogeneity among the participants and provide more precise and individualized results than with group-based methods. However, using this data for self-management and to help the individual to improve his or her mental health has proven to be challenging.

The present work describes a novel approach to automatically generate personalized advice for the improvement of the well-being of individuals by using time series data from intensive longitudinal studies: Automated Impulse Response Analysis (AIRA). AIRA analyzes vector autoregression models of well-being by generating impulse response functions. These impulse response functions are used in simulations to determine which variables in the model have the largest influence on the other variables and thus on the well-being of the participant. The effects found can be used to support self-management.

We demonstrate the practical usefulness of AIRA by performing analysis on longitudinal self-reported data about psychological variables. To evaluate its effectiveness and efficacy, we ran its algorithms on two data sets ($N = 4$ and $N = 5$), and discuss the results. Furthermore, we compare AIRA’s output to the results of a previously published study and show that the results are comparable. By automating Impulse Response Function Analysis, AIRA fulfills the need for accurate individualized models of health outcomes at a low resource cost with the potential for upscaling.

Index terms — impulse response; automated; vector autoregression; simulation; personalized advice; electronic diary study data

1 Introduction

Perspectives on eHealth (computer aided health care) and mHealth (mobile health) have changed greatly in the last decade \cite{1,2}. The modern individual possesses more mobile technology and ‘smart’ devices than ever before \cite{3}. Some of these devices, such as smartphones and tablets, enable people to have Internet access during a large part of their daily life, allowing the use of new and more accurate methods to perform assessments in health care and health research \cite{4}. Research for which a large number of assessments need to be conducted (possibly multiple times per day) can now be performed digitally using mobile technology. Technology enables researchers to carry out studies on a larger scale than would have been possible using traditional methods (i.e., collecting data using pencil and paper, using conventional postal methods for sending the data, and processing this data manually). Moreover, the use of technology can provide means for interactive and automated analysis methods. Using an automated method for analyzing the data might even be inevitable in large-scale studies. As
vast amounts of data are collected for ever larger groups of people, the data might grow too large for manual analysis. Additionally, manual analysis can result in inconsistent or opinionated outcomes, which can be reduced by automatizing the procedure.

1.1 Personalizing mental health research

In recent years, it has increasingly been acknowledged that mental health research requires a person-centered approach and should not focus only on group-based averages [5, 6, 7]. Since group-based analysis applies to the average person, several researchers have argued that conclusions from group-based analysis might not hold for any individual [8, 9]. Instead of using a few data points of many people as the starting point for research, the focus needs to shift towards using many data points of individuals. This allows for studying changes within the individual (within-person variability). Several studies have shown that mental health and ill-health are dynamic phenomena that vary highly between individuals and over time [10, 11, 12, 13]. A technique to assess this within-person variability of mental health is the Ecological Momentary Assessment method (EMA) [14], in which a longitudinal data set is created by asking the individual to repeatedly complete the same assessment, for a certain period of time (e.g., in the morning, afternoon, and evening, for thirty days in a row).

1.2 Analyzing diary study data

When questionnaires are filled out in sequence the data is a time series. A popular technique for analyzing multivariate, equally spaced time series data is vector autoregression (VAR) [15]. VAR can be used to fit a multivariate regression model; a model in which the outcome of one variable (e.g., ‘concentration’) is regressed on the outcomes of several other variables (e.g., ‘self-esteem’ and ‘agitation’). The VAR model itself is a set of such multivariate regression equations for a system of two or more variables, where each variable in the system is regressed on its own time-lagged values and the time-lagged values of all other variables in the system [10]. That is, a variable $x$ at time $t$ is predicted by the same variable $x$ at time $t-1, t-2, \ldots, t-p$ and by other variables at time $t-1, t-2, \ldots, t-p$. The number of measurements used to look back in time ($p$) are called lags in time series parlance.

VAR models allow for determining Granger causality [17]. A variable $x$ Granger causes another variable $y$, if and only if the variance of $y$ can be better explained by lagged values of $y$ and lagged values of another variable ($x$), than lagged values of $y$ alone [17]. Granger causal relations can be depicted by means of a weighted directed graph. Fig. 1 gives an example of such a graph, in which the nodes represent the variables (which can be self-reported psychological variables, such as measured in the present study; physiological variables as read from sensors, or other variables), the connections (or edges) represent the significant directed relations between the variables and the thickness of an edge represents the strength of the relation. In this image, the green nodes represent variables that participants usually experience as a positive phenomenon and the red nodes represent variables that participants usually experience as a negative phenomenon. In Fig. 1 for example, an increase in the variable ‘agitation’ at time $t-1$ Granger causes an increase in ‘rumination’ and a decrease in ‘self-esteem’, ‘concentration’, ‘cheerfulness’, and ‘eating candy’ at time $t$. The figure only shows relations present at lag 1.

Graphs like the one depicted in Fig. 1 encompass relevant information regarding the interactions in a VAR model that could be of interest to the participant. However, these graphs lack several important features to serve as a means to provide advice on how to improve the participant’s well-being. Firstly, participants may have a hard time understanding these graphs [13]. This can be attributed to the conceptual complexity of the different edge and node types in the graph. Secondly, these graphs give a general overview of the coefficients in a VAR model by providing an edge-focused representation. Although such a representation gives information about the individual relations between nodes, it remains complicated to interpret the model as a whole, especially with respect to the temporal interplay between the nodes in the model. The VAR coefficients are meaningless to interpret individually, as it is the VAR model as a whole that describes the complete dynamic behavior of the variables in the system [10].
1.3 Automated Impulse Response Analysis

In the present work, we describe Automated Impulse Response Analysis (AIRA), an approach to automatically generate advice for improving a participant’s well-being using VAR models derived from EMA data. AIRA creates advice by simulating the interactions between variables in a VAR model (i.e., showing what would happen to $y$ when variable $x$ increases). The technique AIRA uses is called Impulse Response Function (IRF) analysis. IRF analysis allows us to shock (that is, give an instantaneous exogenous impulse to) certain variables to see how this shock propagates through the various (time-lagged) relations in the VAR model. In other words, IRF shows how variables respond to an impulse applied to other variables [16]. AIRA generates the impulse response functions for each of the equations in a VAR model, and analyzes these IRFs to automatically generate personalized advice. AIRA uses and partly extends some of our previous work; the automatic creation of VAR models [18]. The fact that AIRA analyzes the VAR model as a whole enables AIRA to be a more appropriate and more precise technique for analyzing a VAR model than a mere manual inspection of said model. A second novelty of the present work is an implementation of VAR and IRF analysis in the JavaScript-language. As far as we know, this is the first openly available cross-platform web-based implementation of its kind. The JavaScript implementation can be useful for calculating VAR models or IRFs in the browser, or on a server running for example NodeJS, which can aid upscaling.

AIRA generates several types of advice answering the following questions: (i) Which of the variables has the largest effect on my well-being?, (ii) How long is $Y$ affected by an increase in $X$?, and (iii) What can I do to change a certain $Y$ variable? Firstly, AIRA shows how well each of the modeled variables can be used to improve all other variables in the network, by summing over the effects variables have on the other variables. For this type of advice, we consider an improvement of the complete network an improvement of the participant’s well-being in general. Secondly, AIRA can provide insight into the duration of an effect, giving insight in the persistence of a perceived relation. Thirdly, AIRA allows the participant to select a variable he or she would like to improve and by how much, after which AIRA will try to find a suitable solution to achieve this improvement. AIRA will iterate over all other variables and estimate for each of these variables how large a change is needed to achieve the desired effect.

\[^1\text{Website: http://nodejs.org}\]
This paper is organized as follows. Section 2 gives an overview of related work. Section 3 illustrates the concept of AIRA presenting its mathematical foundation. Section 4 describes AIRA by presenting pseudocode of the algorithms. Section 5 describes the experimental results acquired when evaluating AIRA. We also evaluate the implementation of AIRA by comparing its analysis with a manually performed analysis. Section 6 discusses the results. Section 7 concludes the work and provides direction for future work.

2 Related work

IRF is a technique used in several fields, including digital and analog signal processing [19], control theory [20], psychiatry [21], econometrics [22], and even for the quality assurance of fruit [23]. Each field applies IRF in a differently, but they all revolve around a comparable principle. IRFs are used to determine how a model or system, or part of that model or system, responds to a sudden large change or shock on one or more of the variables.

2.1 Personalized feedback on EMA data

Several applications exist that can provide users with feedback based on diary studies or other longitudinal health data collected by them. The feedback and advice provided in EMA studies can roughly be split into two categories: real-time advice and post-hoc advice. The type of advice chosen for a study depends on the goal of the study.

Real-time advice can be used to intervene directly in the EMA study. For example, Hareva et al. present advice to a participant by means of applying a severity threshold to the EMA results [24]. They describe a use case of their system for smoking cessation in which an email is automatically sent whenever a participant has smoked fewer cigarettes than a set threshold in order to encourage the behavior of smoking less. Real-time advice triggered by diary data has also been applied in treating childhood overweight [25]. In a study by Bauer et al., juvenile participants sent weekly text messages (SMS) describing various parameters (such as emotion and eating behavior), after which an algorithm would automatically compare these results to the preceding week and create advice [25]. Furthermore, Ebner-Priemer and Trull reviewed the use of diary studies in psychopathology research [26]. They discuss EMA studies about mood-disorder and mood dysregulation (mainly bipolar disorder, depression, and borderline personality), and also methods for providing feedback on these data.

Post-hoc advice is advice offered after completing the study. One of the advantages of this method is that elaborate statistical analysis can be performed, since all collected data can be used for analysis (instead of a small window of data). Furthermore, post-hoc feedback can be used in cases where the goal is to map the baseline behavior of a participant, not affected by interventions such as real-time feedback. To the best of our knowledge, only a few studies exist to date that provide personalized, post-hoc advice based on the dynamic relations between the variables in an EMA study. In an electronic diary study performed by Booij et al., participants received a post-hoc personal report on their daily activity and mood patterns [27]. Van Roekel et al. provided participants of their electronic diary study with a written report and gave them lifestyle advice based on VAR models and the variables most promising for inducing a change in pleasure [28]. Two of our own studies that provide such post-hoc advice are HowNutsAreTheDutch [12, 29] and Leeplezie (English: ‘Joys of living’). HowNutsAreTheDutch is a Dutch mental health study that, inter alia, offers an EMA study. In HowNutsAreTheDutch, a participant receives post-hoc feedback mainly consisting of a graph as shown in Fig. 1 combined with a static description text. The second example, Leeplezie, is a study focusing on the elderly population of the Netherlands. During the study, participants receive minimal feedback consisting of basic charts intended to motivate (but not influence) the users to complete the next assessments. At the end of a study, Leeplezie participants who finished with enough completed measurements, receive feedback comparable to that received by the HowNutsAreTheDutch participants. Both studies provide feedback based on VAR models, but merely present Granger causality networks as-is (as shown in Fig. 1), without performing analysis on the VAR models. Furthermore,
the feedback provided in these studies is mainly descriptive and does not provide concrete examples on how participants can enhance their well-being, unlike AIRA.

2.2 Vector autoregression in psychopathology research

Several studies exist in mental health research in which data is longitudinally collected from individuals. Often, the analysis in these studies involves techniques designed for group studies, instead of purely individual-based analysis methods [7]. These methods are fine to use for drawing conclusions at the general level, but they might not hold at the individual level [8, 9].

One analysis method applicable for EMA studies focusing purely on the individual is VAR. The use of VAR in mental health research is relatively new. Several researchers have incorporated this type of analysis (e.g., Dugas et al. [31], Snippe et al. [32] and Van Gils et al. [33]), including some of the authors of the present work [10, 21]).

The manual creation of VAR models can be challenging; for example, researchers have to select the appropriate variables and decide upon lag length. In [18], we describe a method to automate VAR modeling named Autovar, which can solve these issues by selecting some of the optimal parameters for the model configuration. Autovar can automatically create and evaluate VAR models based on EMA data. The models offered by Autovar adhere to assumptions of a valid VAR model, such as the stability assumption, the white noise assumption, the homoskedasticity assumption, and the normality assumption [18]. It reveals the Granger causality found in the data and generates a network graph. However, Autovar does not generate specific advice nor does it apply IRF to the created models. The creation of VAR models themselves, is outside the scope of the present work, see for instance [16] and [18].

3 From Variable Selection to Advice Generation

AIRA uses IRF analysis to determine the effect each variable has on the other variables in the model for generating advice. The outcome of this analysis is then converted by AIRA to several types of advice for the participant. The advice describes in several ways which of the variables can best be adjusted in order to achieve the desired effect. The advice generation process of AIRA can be subdivided into four phases: (a) initialization, (b) simulation, (c) variable selection, and (d) advice generation. The AIRA process is illustrated in Fig. 2.

3.1 Initialization

In the first phase, AIRA converts the VAR model into a vector moving average (VMA) representation. This VMA representation shows how the model responds to changes in the residuals or to exogenous shocks on the model (i.e., shocks from outside of the model) [16 p. 37]. As an example, Eq. 1A shows a basic var($p$) model (a VAR model with $p$ lags), Eq. 1B shows the vma($\infty$) representation of the same model.

\[
Y_t = c + B^1 Y_{t-1} + \cdots + B^p Y_{t-p} + \xi X + \tilde{e}_t
\]

\[
Y_t - d = \tilde{e}_t \cdot (I_m + \left( \sum_{i=1}^{k} C_{1,i}L \right) + \left( \sum_{i=1}^{k} C_{2,i}L^2 \right) + \cdots )
\]

In Eq. 1A, $Y_t$ is a vector containing $m$ variables ($\vec{v}$) at time $t$, $p$ is the number of lags in the model (i.e., the number of previous measurements included in the model for the prediction of the current variable), $c$ is a vector containing $m$ constant terms, $X$ are the exogenous variables (i.e., variables that can influence the model, but cannot be influenced by the model, such as the day of the week or the weather), $\xi$ is the coefficient matrix for the exogenous variables, $\tilde{e}_t$ is a vector containing the error in the model (i.e., all variance left in the data not explained by the model), and each $B^1, B^2, \cdots, B^p$ is
A coefficient matrix for a specific lag in time. Each entry ($\beta$) of one of the matrices $B^p$ is a coefficient for one variable predicting another value at a specific lag. Each matrix in $B^p$ has the order variable $\times$ coefficients. That is, each row of a $B^p$ matrix represents the coefficients of lag $p$ for the variable in that row (e.g., $\beta_{i,j}^p$ is the scalar coefficient for predicting variable $i$ using variable $j$, at lag $p$).

Eq. (1b) shows the same model as Eq. (1a). However, in this case the model is recentered around its equilibrium values, and it is converted to a function of the error term in the model [16]. The VMA representation allows one to investigate the standardized impact of a shock on the model. For more information on the VAR to VMA model conversion, see [Brandt and Williams] [16]. In Eq. (1b), $L^k$ is the lag operator which shifts the variable that it is multiplied by with $k$ steps, i.e., $L^k x_t = x_{t-k}$. $C_1$, $C_2$, etc., are the VMA coefficient matrices of the model. Each $C_i$ represents the $i^{th}$ row of $C$, each $C_{i,j}$ a specific element from that row. $C$ itself is a block lower triangular matrix, as shown in Eq. (2). $C$ is contained in itself, as each row of $C_i$ contains all preceding entries in $C$ (i.e., $C_{i-1}$, $C_{i-2}$ until $C_0$). The reason for this recursion is that an effect at time $t$ is also affected by all of the preceding effects ($t-1$, $t-2$, etc.). Theoretically, the $C$ matrix can contain an infinite number of rows. However, when using a stable VAR model the responses will eventually converge to zero [16]. This number of rows can be limited to a certain number of steps ($k$), also known as horizon. The horizon is the total
number of steps the IRF model will predict. The \( \vec{e}_t \) vector \((m \times 1)\) originates from an \( m \times k \) matrix \( E \) that contains the error terms for each variable corresponding to a variable at the same location in \( \vec{v} \). When performing IRF analysis, the \( \vec{e}_t \) term is replaced by a vector of shocks \( \vec{s} \), which is a not-lagged vector of structural shocks to the model. In this work, we assume shocks of unit size, i.e., each entry in this vector is either 1 (if a variable receives a shock) or 0 (if a variable does not receive a shock). Since the effects are standardized, this corresponds to a \( \sigma \) (standard deviation) increase.

Eq. [11] does not take into account the contemporaneous effects, and captures these effects in the error term. It is often problematic to determine the direction of the effect for the contemporaneous effects, as they merely show the correlation, and therefore the direction remains unclear. If one has a hypothesis regarding the directionality of these contemporaneous effects they could be implemented by using a technique named orthogonalized IRF (OIRF) \[16\]. In that case, \( I_m \) (the \( m \times m \) identity matrix) should be changed to a matrix representing the contemporaneous effects, e.g., by computing the first matrix from the Cholesky decomposition of the error covariance matrix and possibly testing all directions for the effects, or by using theoretical domain knowledge for the directions \[16\] \[22\]. \( d \) is the VAR constant term divided by the vector autoregressive lag polynomial.

### 3.2 Simulation

In the second phase, AIRA runs IRF calculations for each of the variables in the model. AIRA simulates an impulse on one variable \((x)\) in the model and registers how all other variables respond. The response of each variable is used to determine the effect of a variable on the other variables in the model. Besides using all responses caused by an impulse, AIRA can also be configured to consider only the effects which have a certainty of at least 95%, by bootstrapping the model \[34\]. The methods in this phase allow for determining the variables most suitable for influencing other variables and serves as a preprocessing step for generating advice.

An example of an IRF of the network model used in Fig. [1] is shown in Fig. [3]. The figure shows how a shock on ‘agitation’ (the green line) affects the other variables in the model. The shock is a \( \sigma \) increase of the variable ‘agitation’ (at time \( t = 0 \)). The other variables respond from \( t = 1 \) onwards (as contemporaneous relations are neglected in this example). The shock on ‘agitation’ has an effect on most of the variables in the model: a negative effect on ‘eating candy’, ‘cheerfulness’, ‘self-esteem’, and ‘concentration’, and a positive effect on ‘rumination’. The effect vanishes after \( \approx 7 \) time steps.

The response of one variable to changes in another variable can be calculated using Eq. [3] in which \( x \) is the index of the variable receiving the shock and \( y \) the index of the variable for which the response is analyzed. The outcome of this equation is a vector with the response of \( y \), where each entry is a response on the horizon \((k)\). The remainder of the equation is similar to Eq. [11], however, in this equation, \( \vec{\alpha} \) is a vector of zeros, with a 1 on the index of the variable to shock \((x)\).

\[
irf(x, y, k) = [(\psi_0 \cdot \vec{\alpha}(x))_y, \ldots, (\psi_k \cdot \vec{\alpha}(x))_y]^T
\]

\[
\psi_i = \begin{cases} 
I_m & \text{if } i = 0 \\
 \sum_{j=1}^i C_{i,j} & \text{if } i \geq 1 
\end{cases}
\]  

(3)

In addition, AIRA applies a form of cumulative IRF analysis to determine the total response a variable has on another variable. In cumulative IRF analysis, the response of a variable is summed to a total value, as shown in Eq. [4]. The cumulative IRF is equivalent to the net area under the curve (AUC), where areas corresponding to a response less than zero are subtracted from areas corresponding to responses higher than zero. Using the example IRF in Fig. [3] the cumulative response is the green areas minus the red areas. The definition of the IRF as shown in Eq. [3] takes all responses into account, which might be too optimistic and cause inaccuracies. These inaccuracies may cause small insignificant effects to add up to a large, seemingly significant effect. To circumvent this, AIRA can be configured so that it only considers significant effects by bootstrapping the results \[16\] \[34\] \[35\] and only use effects significantly different from 0. In Fig. [3] the darker areas depict the significant areas. The dashed lines indicate the 95% confidence interval.
Figure 3: An example of the responses of six variables from a \textit{var}(1) model after a shock on the variable ‘agitation’, corresponding to the network shown in Fig. 1.

Figure 4: Artificial example of the \textit{AUC} to demonstrate the response to an impulse.
Eq. 4 shows the calculation of the cumulative IRF, where \( x \) is the variable that is shocked and \( y \) is the variable the response is analyzed on. It sums all responses on the horizon \( k \). The advantage of using this cumulative approach is that we obtain a single value representing the total effect of a single variable on another variable in the model, whilst taking into account all other interrelated variables.

\[
irf_{cum}(x, y, k) = \sum_{j=0}^{k} irf(x, y, k)_j
\]  

(4)

3.3 Variable selection

In the third phase, AIRA selects the variable that is most suitable for adjusting the other variables in the model. AIRA determines the net effect one variable has on all other variables. By using the total AUC, including the negative effects, AIRA gives an estimate of the net effect a variable has. The function for calculating the net effect of a single variable \((x)\) is shown in Eq. 5. In Eq. 5 \( k \) is the horizon over which the effect is calculated, and \( m \) is the number of variables in the model. The result of this equation is the net effect variable \( x \) has on the other variables. This result is contained in a vector of size \( m - 1 \).

\[
irf_{total}(x, k) = \sum_{i=1}^{m} irf_{cum}(x, i, k)
\]  

(5)

In Eq. 5 the \( irf_{cum} \) function uses a VAR model that handles ‘positive’ and ‘negative’ variables differently. Whether a variable is ‘positive’ or ‘negative’ is defined by its interpretation, that is, variables are considered positive or negative with respect to the well-being of a participant. For instance, a model might include two variables: ‘agitation’ and ‘cheerfulness’, in which ‘Agitation’ is considered a negative variable and ‘cheerfulness’ is interpreted as a positive one. A variable deemed positive (‘cheerfulness’) is presumably preferred to be increased, while a negative variable (‘agitation’) is preferred to be decreased. To deal with this dichotomy, a transformation is performed on the negative variables using the \( \Gamma \) and \( E \) matrices; two matrices that convert the variables of a VAR model so that they are always positive. That is, negative variables change sign so their interpretation switches from an increase to a decrease of said variable. Each entry of the \( \Gamma \) matrix is in \( \{1, -1\} \) and is created using \( \vec{v}_{labels} \), a vector representing the interpretation of a variable. That is, \( \vec{v}_{labels}_i = 1 \) if the variable on position \( i \) in \( \vec{v} \) (a vector containing the variables in the model) is positive, and \( \vec{v}_{labels}_i = -1 \) if \( \vec{v}_i \) is negative. The \( E \) matrix is a matrix of which the rows of a negative endogenous variable are negative. Eq. 6 shows the creation of these matrices. This example shows \( \vec{v}_{labels} \) built using a \( \vec{v} \) with 6 elements: \( \vec{v} = [\text{eating candy (negative)}, \text{cheerfulness (positive)}, \text{agitation (negative)}, \text{concentration (positive)}, \text{rumination (negative)}, \text{self-esteem (positive)}]^T \). The \( l \) variable denotes the number of exogenous variables in the model. After the transformation the \( \vec{v}_{labels} \) can be considered a vector of all ones.

\[
\vec{v}_{labels} = [-1, 1, -1, 1, -1, 1]^T
\]

\[
\Gamma = \vec{v}_{labels} \cdot \vec{v}_{labels}^T
\]

\[
E = \vec{v}_{labels} \cdot \vec{1}_{l}^T
\]  

(6)

These \( \Gamma \) and \( E \) matrices are used to calculate the Hadamard product of \( \Gamma \) and the \( B \) matrices \((\Gamma \circ B^p)\) and \( E \) and the \( \xi \) matrices \((E \circ \xi)\) of Eq. 1a. This equation is then used as input for the \( irf_{cum} \) function. AIRA can now determine the total effect of each variable on other variables, and therewith calculate the effect of a variable on the network as a whole.
3.4 Advice generation

In the last phase, AIRA generates the actual advice for the participant. The procedures of the previous phases are combined and personalized advice is constructed. AIRA currently generates three types of advice: (i) most influential node in network, (ii) percentage effect, and (iii) length of an effect.

3.4.1 Most influential node in network

AIRA identifies the variable with the largest net positive effect that can best be used for changing other variables. The advice describes how a participant can have the largest positive influence on his or her well-being. For example, if we determine that overall an increase in activity has a positive effect on the network, the advice would be: ‘If you were to increasing your amount of activity, this seems to positively affect your well-being’. The well-being of a participant is expressed by all variables in the network model. An increase in the network as a whole is considered an increase in the well-being of the participant. The generated advice consists of a sorted list of all variables and the extent to which they positively (i.e., increase one’s well-being), negatively (i.e., decrease one’s well-being), or neutrally (i.e., do not affect one’s well-being) impact the network. The net effect of a variable is calculated by summing the cumulative IRFs, as described in Eq. 5. By using the net effect approach, we offer a method to deal with the issue where different lags of a variable have conflicting signs for effects. For example, a variable can have a positive coefficient for a variable on the first lag, but a negative effect in the second one. AIRA balances these effects by using the net effect.

3.4.2 Length of effect

AIRA shows the participant how long an impulse is estimated to have an effect on the other variables in the model. For calculating the length of an effect AIRA uses the (bootstrapped) IRF as input, and determines how long, in minutes, a response of a variable remains (significantly) different from zero. The length is calculated by multiplying the EMA measurement interval with the number of steps on the horizon for which the effect is (significantly) greater or less than zero. For example, if an impulse on ‘activity’ has an effect smaller than zero on ‘depression’ for two time-steps and the measurement interval is 6 hours, AIRA would state that a one standard deviation increase on ‘activity’ has a negative effect on ‘depression’ for approximately 720 minutes (or 12 hours). Furthermore, it determines how long the effective horizon is with respect to the given impulse. That is, after how many steps all the (significant) effects have converged to zero.

3.4.3 Percentage effect

AIRA generates specific advice showing what a participant could do in order to improve a single specific variable in the network by a specified percentage. For example, imagine a participant that would like to increase his or her ‘cheerfulness’ variable by 10%. AIRA can determine how to achieve this increase by advising the participant to either increase or decrease certain other variables in the model. Advice could then be generated as follows: ‘In order to increase your cheerfulness by 10%, you can either increase your concentration by 20% or decrease your agitation by 33%.’ In order to calculate such advice, AIRA uses Eq. 7 provided that $irf_{cum}(y, x) \neq 0$, $\hat{y} \neq 0$, and $\sigma_x \neq 0$.

$$
\Delta_y = \frac{(\hat{k} \cdot \hat{x}) \cdot \Delta \cdot \sigma_y}{\frac{\Delta y}{y \cdot irf_{cum}(y, x, \hat{k}) \cdot \sigma_x}} (\forall y \mid \vec{v}_y \in \vec{v}, y \neq x)
$$

In Eq. 7 $\Delta \cdot 100$ is the desired percentage for increasing ($\Delta > 0$) or decreasing ($\Delta < 0$) variable $x$, $\Delta_y \cdot 100$ is the calculated percentage effect $y$ has on $x$. $\hat{x}$ and $\hat{y}$ represent the average scores for the variables $x$ and $y$ of a participant respectively, $\sigma_x$ and $\sigma_y$ are the standard deviations of respectively variables $x$ and $y$, and $\hat{k}$ is the effective horizon over which the effect is calculated. The effective horizon is the number of steps on the horizon when the response of a shock has not yet converged to zero. The calculation is performed for all variables in the model ($\vec{v}_y \in \vec{v}$) not equal to the variable to
improve \((y \neq x)\), and the advice is established from the outcomes for each variable. The total effect \(y\) has on \(x\) is used in the calculation.

As an example, assume a person having a variable \(x\) (the variable a person would like to change) with a mean value of 50, and \(\sigma_x = 15\). Assuming normality, an increase of one standard deviation would mean an increase of 15, i.e., an increase of \(\frac{15}{x} \times 100 = 30\%\). Assume this person would like to increase the value of \(x\) by \(10\%\), we would require an increase of \(\frac{1.1x}{\sigma_x} = \frac{1}{5}\), i.e., a \(\frac{1}{5}\) standard deviation increase causes an increase of \(10\%\) with respect to the mean of \(x\). Secondly, assume this person also has the variables \(\{u, y, z\}\) that affect \(x\) (so \(\vec{v} = \{u, x, y, z\}\)). For now, we only consider one of these variables, namely \(y\). Assume that we have calculated that a unit (one standard deviation) impulse in \(y\) has a unit increase in \(x\) over an arbitrary horizon of 10 steps, i.e., \(irf_{cum}(y, x, 10) = 1\). Since the IRF are standardized, this corresponds to a one standard deviation response. For each step, the effect of \(y\) on \(x\) is therefore on average \(\frac{\text{effect}}{\text{horizon}}\). Recalling that the person would like an increase in \(x\) of \(10\\%\), equal to a \(\frac{1}{3}\) standard deviation increase on a single step. As \(y\) has an average effect of \(\frac{1}{10}\) standard deviation on \(x\) per step, we require a difference of \(\frac{3^{-1}}{10} = \frac{3}{10}\) standard deviations in \(y\). Because the impulse is standardized, we can determine the exact percentage of change needed in \(y\) with respect to the average of \(y, \bar{y}\). Thus, the required change was \(\frac{3}{10} \times \sigma_y\), which is \(\frac{3}{10} \times \sigma_y \times \frac{100 \% \text{ required change}}{y}\).

4 Algorithms

AIRA is freely available (open source) from \texttt{http://frbl.eu/aira}. We implemented two versions of the proof-of-concept algorithm, one version in JavaScript (\texttt{https://github.com/frbl/aira}), a language designed to run client-side in a web browser, and a version in the R-language (\texttt{https://github.com/frbl/airaR}), a software environment for statistical computing. JavaScript has the advantage that the calculations can be performed on the client and do not require any computation on the server. JavaScript also provides interactivity in the client’s browser; it allows for live updating the browser’s DOM (Document Object Model) of the web-page, enabling animations and interactivity. Furthermore, the JavaScript implementation could aid the use of AIRA on a large scale, as the implementation can be used on a back-end server (for example using NodeJS) or in the browser. A live demo can be found at \texttt{http://frbl.eu/aira}.

Each of the algorithms used for generating advice in AIRA is elaborated in the following sections. The algorithms used for performing the IRF calculations and for converting the VAR model to a VMA representation are provided in Appendix \[\text{A}\] In all examples, \(m\) denotes the number of variables in the model and \(p\) the number of lags.

4.1 Selecting Variables and Determining Advice

The advice generation of AIRA is split up into three parts: (i) ‘most influential node in network’ (determining the net effect of a variable on well-being), (ii) ‘length of effect’ (presenting the duration of a significant effect), and (iii) ‘percentage effect’ (giving advice on how each of these variables can actually be changed). The pseudocode of the algorithm used for the first type of advice is provided in Algorithm \[\text{1}\] The result of this function is an associative array \((r)\) of cumulative effects (positive or negative) for each variable, sorted by descending absolute value. The algorithm iterates over all variables in the model (line 3). From line 6–14 it determines the net cumulative IRF effect a variable has on each other variable, and stores summed effect per variable in \(r\).

Besides showing the net effect a variable has on well-being, AIRA shows how long an impulse has a (significant) response. AIRA makes it insightful what the effect duration would be when an impulse is given to another variable. The algorithm iterates over all steps on the horizon (line 7–22). For each step it then checks whether the effect differs (significantly) from zero (line 8). It does so until an effect has been found. When this first effect has been found, a flag \((\text{effect\_started})\) is toggled (line 12), and the start and direction of the effect are estimated. The direction is either positive or negative, and is determined in line 9. If the effect does not start in the first step, the effect is linearly interpolated.
Algorithm 1 Determines the most influential variable.

1: function DetermineMostInfluentialNode(C, k)

arguments C are the VMA coefficients as returned by Algorithm 4, k is the horizon to forecast.

2: r ← associative array

3: for x ← 1, x ≤ m do

4: r_x ← 0

5: eff ← CalculateIRF(α(x), C, k)

6: for y ← 1, y ≤ m do

7: if x ≠ y then

8: for l ← 1, l ≤ k do

9: r_x ← r_x + (eff_y)_l

10: l ← l + 1

11: end for

12: end if

13: y ← y + 1

14: end for

15: x ← x + 1

16: end for

17: r ← Sort r based on the absolute values

18: return r

19: end function

to the expected moment it passed a threshold (line 11). This continues until the effect converges or exceeds the threshold. If this is the case, the algorithm linearly interpolates the point where it exceeded the threshold (line 16). This result is stored, in terms of the total duration of the effect (line 18) and the total time the effect took to converge to zero (line 19). In case the chosen horizon was too small, and the effect did not have enough time to converge, the total effect and effective horizon are determined in lines 23–26. The algorithm returns several values on line 27. Firstly it returns the total time of the effect, that is, the length of the effect multiplied by the interval between measurements, yielding the total length of the effect in minutes. Secondly it returns the total length of the effect, that is, the number of steps where the (significant) effect is non-zero. Lastly it returns the total effective horizon. That is, the total horizon over which the effect is non-zero (i.e., the last step where there is a (significant) effect. It defaults to the horizon if the effect does not converge within the given horizon).

Lastly, AIRA shows what a participant can do to adjust said variables. Algorithm 3 shows the pseudocode designed for generating this advice. The AVG and SD functions used in the algorithm give respectively the average and the standard deviation of the values of the answers, as measured during the diary study. The algorithm iterates over all variables in the model (line 3), and for each variable the algorithm determines the length of the effect, and the net effect the current variable (y) has on the variable to be changed (x) (lines 5–6). In lines 7 to 12, this net effect is converted to a percentage of the average value of the variable and stored as a result, according to Eq. 7. The algorithm allows to filter out effects lower than a threshold θ (expressed in terms of standard deviations, line 7), as the percentage needed for a change with a low effect might become unrealistically large.

4.2 Time complexity

We determined the time complexity of each of the algorithms presented in Section 4 using the Big-O notation technique (denoted using O). The time complexity of the algorithms for generating the IRF functions are provided Appendix B. These time complexities describe the upper bound of the processing time of the algorithm when the input size grows infinitely large.

For determining the most influential node in the model (Algorithm 1), the algorithm calculates the total effect each variable has on all other variables and therefore loops over all m variables in the model m times. For each of these variables, it determines the IRF once. Finally, it iterates over each
Algorithm 2 Determines the actual effect length.

1: function DetermineLengthOfEffect(x, y, inter, k)  
   arguments x is the variable that receives the shock, y is the variable we measure the response on, inter is the interval with which the data was sampled, and k the horizon to forecast.
2:   \( \vec{\gamma} \leftarrow \text{IRF}(x, y, k) \)
3:   start, end \( \leftarrow 0 \)
4:   effect_started \( \leftarrow \text{FALSE} \)
5:   total, effective_horizon, d \( \leftarrow 0 \)
6:   threshold \( \leftarrow 1e^{-4} \)
7:   for \( i \leftarrow 1, i \leq k \) do
8:     if \( |\vec{\gamma}_i| > \text{threshold} \) then
9:       d \( \leftarrow ((\vec{\gamma}_i > \text{threshold}) \ ? -1 : 1) \)
10:      if \( i > 1 \land \neg \text{effect_started} \) then
11:         start = \( i - \frac{(\vec{\gamma}_i + d \cdot \text{threshold})}{(\vec{\gamma}_i - \vec{\gamma}_{i-1})} \)
12:         effect_started \( \leftarrow \text{TRUE} \)
13:     end if
14:   else
15:     if \( \text{effect_started} \) then
16:       end = \( i - 1 + (1 - \frac{(\vec{\gamma}_i + d \cdot \text{threshold})}{(\vec{\gamma}_i - \vec{\gamma}_{i-1})}) \)
17:       effect_started \( \leftarrow \text{FALSE} \)
18:       total \( \leftarrow \text{total} + (end - start) \)
19:       effective_horizon \( \leftarrow \text{end} \)
20:     end if
21:   end if
22: end for
23: if \( \text{effect_started} \) then
24:   total \( \leftarrow \text{total} + (k - start) \)
25:   effective_horizon \( \leftarrow k \)
26: end if
27: return \[\text{total} \cdot \text{inter}, \text{total}, \text{effective_horizon}\]
28: end function

calculated response on the horizon of length \( k \). The total time complexity of determining the advice is therefore \( O(m \cdot (O(\text{CalculateIRF}) + mk)) \) or \( O(m \cdot (k^2m^2 + mk)) \), which reduces to \( O(k^2m^3) \), where \( m \) is the number of variables in the model and \( k \) is the horizon of the IRF.

For determining the length of the effect a variable \( x \) has on a variable \( y \) (Algorithm 2), the algorithm first calculates the IRF of this effect \( (O(\text{IRF})) \). The algorithm itself then loops over each of the \( k \) steps on the horizon \( (O(k)) \). As calculating the IRF has a higher complexity than \( O(k) \), the upper bound for the complexity of this algorithm is \( O(\text{IRF}) = O(k^2m^2) \).

Determining the percentage advice as listed in Algorithm 3 considers the effect all variables have on a single variable. Hence, for this algorithm we do not need to loop over all variables more than once, making the time complexity \( O(m \cdot (O(DetermineLengthOfEffect) + O(\text{IRF}_{cum}))) = O(m \cdot (k^2m^2 + k^3m^2)) \). When one would apply dynamic programming to cache the IRF calculation, the time complexity could be reduced to \( O(2mk + k^2m^2) = O(k^2m^2) \), as the IRF then only has to be calculated once for all variables.

We can now define the total time complexity of \( \text{aira} \) as the maximum complexity of its algorithms. The algorithm with the highest complexity is the algorithm for calculating Algorithm 3 (DeterminePercentageEffect). This results in the time complexity of \( \text{aira} \) being \( O(m^3k^3) \).

The algorithms presented in this section do allow for parallelization on several levels. For example, the most complex algorithm (Algorithm 3) can be optimized by parallelizing its outer loop, in which it iterates over the \( m \) variables in the model. This reduces the complexity of \( \text{aira} \) by a factor \( m \),
Algorithm 3 Determines the percentage of change needed per variable in order to induce a change of a certain percentage in another variable.

1: function DeterminePercentageEffect(perc, x, θ, k)
   arguments perc is the percentage with which the variable to change (x) needs to be changed. θ is a threshold of minimal effect needed in the variables, and k the horizon to forecast.
2:   results ← associative array
3:   for y ← 1, y ≤ m do
4:     if y ≠ x then
5:        k ← DetermineLengthOfEffect(y, x, 0, k)
6:        n.e ← IRFcum(y, x, ̂k)
7:        if n.e > θ then
8:           Δ ← perc · 100⁻¹
9:           ζ ← AVG(x) · ̂k · Δ · SD(y)
10:          ̂ζ ← ζ · (n.e⁻¹ · AVG(y⁻¹ · SD(x⁻¹))
11:          results_y ← ζ · 100
12:     end if
13:   end if
14:   y ← y + 1
15: end for
16: return results
17: end function

resulting in a time complexity of $O(m^2k^3)$ for AIRA on systems with at least m threads available for parallel execution.

The practical performance of AIRA is acceptable and usable for general use. For instance, calculating advice for a model of six variables and a horizon of twenty steps took less than a second, as measured on both a modern laptop and a tablet.

5 Experimental results

We performed several experiments to evaluate the effectiveness and accuracy of AIRA. These experiments show possible use cases of AIRA and give an impression of the three advice types AIRA can generate. We ran each of the aforementioned algorithms on data sets from two studies. Furthermore, we compared AIRA’s results of the LENGTHOFEFFECT-algorithm to the results of an earlier study.

The first dataset we used originated from the HowNutsAreTheDutch study, from which we randomly selected five participants having more than 85% or 77 completed measurements (N = 164, henceforth referred to as the HND data set) [12]. For these five participants, we fitted VAR models using the AutovarCore procedure [36]. AutovarCore is a faster and more efficient version of the original Autovar procedure [18, 36]. We used three variables recorded in this study: (i) feeling gloomy, (ii) relaxation, and (iii) feeling inadequate. These variables were selected so our experiment contains both positive and negative variables.

The second data set was retrieved from the study performed by Rosmalen et al. (henceforth referred to as the Rosmalen data set) [10]. Rosmalen et al. investigate the relation between depression and activity using VAR and IRF analysis. In their work, they describe for one subject how long significant effects on activity and depression remain by inspecting the IRF.

5.1 Most influential node

We ran Algorithm 1 on the HND data set to demonstrate a particular use case of AIRA. We loaded each model in AIRA and applied the procedure as described in Algorithm 1, DETERMINEMOSTINFLUENTIALNODE. We marked ‘feeling gloomy’ and ‘feeling inadequate’ as negative variables and only
used effects having a confidence of at least 95% by bootstrapping the results 200 times. The results are shown in Table 1. Note that the ‘feeling gloomy’ and ‘feeling inadequate’ variables have been converted to positive variables (resp. ‘feeling less gloomy’ and ‘feeling less inadequate’).

|                | Feeling less gloomy | Relaxation | Feeling less inadequate |
|----------------|---------------------|------------|------------------------|
| Person 1       | 0.000               | -0.061     | 0.045                  |
| Person 2       | 0.230               | 0.000      | 0.000                  |
| Person 3       | 0.000               | 0.000      | 0.057                  |
| Person 4       | 0.000               | 0.000      | 0.000                  |
| Person 5       | 0.015               | 0.410      | 0.000                  |

Table 1: Effects of feeling less gloomy, relaxation and feeling less inadequate on well-being, in terms of standard deviations.

These results show several interesting findings. First of all, they emphasize the difference between the participants. Some participants show similar responses, but some participants are rather deviant or even have opposite responses. For person 2 and person 5 ‘feeling less gloomy’ seems to have a positive effect on ‘well-being’, whereas the effect is neutral for the other participants. For person 1 ‘relaxation’ seems to have a negative effect on ‘well-being’ whereas this effect is positive for person 5. ‘Feeling less inadequate’ seems to have a small positive effect for person 1 and person 3. The variables do not seem to have an effect on the well-being of person 4.

5.2 Length of the effect

In the second evaluation, we compared AIRA’s method for determining the length of an effect to the results of the manual analysis by Rosmalen et al. [10] (the Rosmalen data set). In this experiment, AIRA was configured to use orthogonalized IRF similar to Rosmalen et al. We compared the contemporary effects where activity is assumed to precede depression with those of Rosmalen et al. (see [10] for more information).

During the experiment, we noticed differences between our outcomes and those of Rosmalen et al. These differences mainly originate from differences in the statistical packages used. Firstly, there is a small difference between the VAR coefficients generated for AIRA (in R using the `vars` package) and the models used by Rosmalen et al. (as generated using Stata 11.0). Secondly, there seem to be differences in the confidence intervals for IRF as calculated in Stata compared to those calculated using the R `vars` package. These differences seem to be caused by the different methods used by Stata and R to calculate the VAR models and error bands for IRF (see [37] and the Stata Time series Manual for details). Lastly, the method for calculating the confidence intervals uses bootstrapping, which depends on a random component that also contributes to the differences.

To remove the bias of differing statistical packages, we refitted each of the models from Rosmalen et al. using the R `vars` package and compared AIRA with these models. The IRF time plots of these models are shown in Appendix C and are nearly identical to the results of Rosmalen et al. as shown in Figure 2 on page 7 of [10].

Table 2 shows the outcome of AIRA. The first two columns (annotated with (1)) show the predictions of AIRA, the third and fourth column (annotated with (2)) show the predictions based on the VAR models in the work of Rosmalen et al., and the last two columns (annotated with (3)) show the results of a manual inspection of the VAR models from Rosmalen et al. refitted using the `vars` package. The results between AIRA and the refitted models are very similar, and it can be argued that the results of AIRA are in fact more precise as AIRA applies linear interpolation to estimate the exact length of the effect.
Table 2: Comparison between the outcomes of AIRA (1) and results from the study by Rosmalen et al. (2,3) [10]. The table shows both the results from the paper (2) and from the Rosmalen VAR models fitted using the VARS package (3).

|     | A → D (1) | D → A (1) | A → D (2) | D → A (2) | A → D (3) | D → A (3) |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| pp1 | 1746.2 min. (1.2 days) | 0 min. (0 days) | 5 days | 0 days | 1 days | 0 days |
| pp2 | 639.4 min. (0.4 days) | 0 min. (0 days) | 3 days | 0 days | 1 days | 0 days |
| pp4 | 0 min. (0 days) | 11081.7 min. (7.7 days) | 2 days | 6 days | 0 days | 7 days |
| pp5 | 0 min. (0 days) | 0 min. (0 days) | 0 days | 1 days | 0 days | 0 days |

5.3 Percentage effect

In the third evaluation, we evaluate Algorithm 3 using the HND data set. For each of the modeled variables, we determined how well each of the other variables in the model could be used to increase the positive variables or decrease the negative variables by 10%. That is to say, we determined how well ‘activity’ and ‘relaxation’ can be used to reduce ‘feeling nervous’ by 10%. If the found effect was negligible, or if there was no effect at all, the variables were assumed to require an infinite increase in order to change the variable. For each of the results we only used the effects having a certainty of at least 95% (by bootstrapping 200 times). The results of the algorithm are (only showing percentages ≥−1000% and ≤1000%): Person 2 can decrease feeling inadequate by changing feeling gloomy by -89.89%, Person 5 can decrease feeling inadequate by changing relaxation by 206.38%. In this example, AIRA was able to generate advice for person 2 and person 5. These results show that for person 2, in order to decrease ‘feeling inadequate’ by 10%, he or she would need to feel approximately 90% less gloomy than now. Person 5 could decrease his or her feeling of ‘inadequacy’ by 10%, by ‘relaxing’ approximately twice as much. Although these outcomes might be hard to interpret in the present form, they can give an idea which of the variables would be the best fit for intervening on another variable.

6 Discussion

AIRA provides tailored advice about mental health and well-being, using data collected in a diary study. AIRA allows participants to inspect how relations between their psychological, and how they interact over time by means of irf. We performed several experiments to demonstrate the performance of AIRA. In our experiment, presented in Section 5.2, we showed that the results from the automated analysis of AIRA are comparable to earlier work and arguably more precise.

In mental health research, few case studies (generally consisting of one to five participants) have previously applied IRF analysis to diary data (e.g., Rosmalen et al. [10] and Hoenders et al. [21]). AIRA includes several methods to perform an IRF analysis similar to the one used previously, but instead of using manual analysis, AIRA applies an automated technique. To the best of our knowledge, AIRA is the first approach for automatically generating IRF-based advice.

The analysis of IRF and VAR models leaves room for ample discussion. Firstly, a point of discussion is whether or not to include contemporaneous effects in IRF analysis [15, 16]. Because the directionality of contemporaneous effects has no foundation in reality and therefore lacks conceptual justification, the use of contemporaneous effects is controversial. Although some of these directions can be determined using theoretical domain knowledge, this might not hold for the individual. Therefore, AIRA currently does not base its advice on contemporaneous effects.

Secondly, the use of IRF and VAR models in the context of psychopathology research can be a point of discussion. Both VAR models and IRF analysis are not originally developed to be used with EMA data. In this study, we assume that a VAR model fit to EMA data is representative for a period that exceeds the EMA study itself, and is representative for future events. Treating the model as a linear and time invariant system enables us to perform such analysis. We based this assumption on several previous studies also applying VAR analysis to psychological research data (e.g., [10, 21, 33]).
We should note that, as for many statistical techniques, a key requirement for AIRA to work is that the model comprises relevant variables. When the model merely consists of variables not having meaningful connections, not having meaningful variance, or variables that cannot be influenced AIRA will not function properly.

Thirdly, when a VAR model has more than one lag, it might happen that the sign of a VAR coefficient (i.e., the direction of the effect) is not equal for all lags. For example, a variable may have a negative effect on another variable in one lag but a positive effect in a different lag. In this scenario, it is difficult to determine whether the effect of the variable should be considered positive or negative when basing this decision merely on the VAR model. In AIRA, we circumvent this issue by using IRF, and moreover, by summing the cumulative IRF values to obtain a net effect value. This net effect value considers the entire horizon for determining an effect to be either a net gain or a net loss. That is, if the positive effects outweigh the negative effects, the relation is considered positive (and vice versa for negative effects). This approach is novel and specific to AIRA.

Fourthly, the creation of the VAR model. Although the creation of VAR models is deliberately left out of the present work, there are a few important points one needs to take into account. The estimation of a VAR model entails various caveats, such as the selection of the correct lag length and the equidistant measuring of the EMA data set. When fitting a VAR model to data, these are properties to take into account. For the present work, we relied on previous work allowing the creation of VAR models to be performed automatically [18].

The performance of AIRA with respect to the calculation time is sufficient for practical use. Basic experiments show that the calculation of an advice using a VAR model consisting of six variables and a horizon of twenty steps happens in less than a second on a modern laptop. Furthermore, our time complexity analysis implies that this performance scales well within acceptable boundaries for practical use. AIRA is implemented in the R-language and in JavaScript, and can therefore run in any browser on any modern operating system (mobile or desktop). It should be noted that the running time for computing advice using bootstrapped IRF analysis increases linearly with the (constant) number of bootstrap iterations.

Objectively establishing the correctness of any algorithm that provides suggestions or advice is a complex issue. AIRA forms no exception. While the implemented formulas may be mathematically correct, we have not yet evaluated the practical utility of the advice generated by AIRA in terms of clinical accuracy, understandability, and helpfulness for the individual. Hence, whether the advice actually contributes to the well-being of an individual remains to be shown in future research.

In a broader perspective, AIRA could be a useful asset in the current field of mental health research and clinical practice. AIRA could be used on large scale platforms as a decision support tool, and as a means to give automatic personalized advice on mental health and well-being, perhaps on a national scale such as in the HowNutsAreTheDutch project [12].

7 Conclusion and Future Work

The present work describes the Automatic Impulse Response Analysis (AIRA), an algorithm and related implementations on two platforms that can automatically provide feedback and advice on time series data such as diary data collected using the ecological momentary assessment method. Our paper described the basis and theoretical foundation of AIRA. We created a method to give specific advice on which variables require change, and with what percentage, in order to have the desired adjustment in other variables. Furthermore, we provided a proof-of-concept implementation of this algorithm for use in e-mental health platforms. AIRA provides an automated method to deal with the previously unsolved problem where the lags of a variable have conflicting effects (e.g., a positive effect on one lag and a negative effect on another lag). These mixed effects make it difficult to determine whether the overall effect of a variable is a net gain or a net loss. Using AIRA, these effects can be summarized, revealing the net effect.

Future versions of AIRA could include algorithms to determine which sequence of impulses over time, rather than which single impulse, has the desired effect. Moreover, the percentages currently
provided could be converted to an easier to understand format, such as time investment required to effectuate a change. In its current form, AIRA can be used by participants to find out how best to self-manage their well-being. This can be improved by allowing the individual to personally assign importance to the variables under study. We currently apply a general, binary approach to determining whether a variable is considered positive or negative. This self-management can be improved further by allowing users to provide a relative importance for each variable.

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A Impulse response calculation

AirA first converts VAR models into the VMA representation of the model. Algorithm 4 shows the pseudocode for determining the VMA coefficients. In this (and the other) examples, \( m \) denotes the number of variables in the model and \( p \) the number of lags. The R version of AirA uses the vars package to calculate these models [37]. The JavaScript version of AirA uses an implementation specially crafted for the present work. This JavaScript implementation for calculating the VMA models and running the IRF analysis is described in this appendix.

Firstly, the VAR coefficient matrix (of size \( m \times mp \)) in the VAR model is partitioned in separate matrices based on the lag, generating \( p \) matrices of size \( m \times m \), as shown in lines 4–8 of Algorithm 4. This allows for reusing them when determining the VMA coefficients. Secondly, AirA performs the transformation from VAR coefficients (the \( B \) matrices in the VAR equation) to the VMA coefficients (the \( C \) coefficient matrix in VMA representation), as shown in lines 9–17. Note that only the first \( k \) rows of \( C \) are converted, i.e., the specified horizon.

The \( \delta \) function used in this algorithm in line 9, 12, and 15 checks whether a coefficient matrix is available for the provided lag (i.e., a model with two lags has two coefficient matrices, one for each lag). This function is described in Eq. 8. The algorithm for the VMA coefficients is based on the work of [Brandt and Williams] [16] and on the work of Lütkepohl [34].
Algorithm 4 Finds the VMA coefficients of the VAR model.
1: function CALCULATEVMA(var._coef, p, k)
   arguments var._coef is the coefficient matrix from the VAR model (of size \(m \times mp\)), p is the number of lags, and k is the number of steps to forecast (the horizon).
2: \( B \leftarrow \) empty list of size \( p \)
3: \( C \leftarrow \) matrix of all zeroes of size \((k \times k)\) \(\triangleright\) the VMA coefficient matrix
4: for \( l \leftarrow 1, l \leq p \) do
5: \( x \leftarrow m \cdot (l - 1) + 1 \)
6: \( B_l \leftarrow \text{var._coef}(1...m),(x...x+m−1)\) \(\triangleright\) \( B \) is a list of matrices, each matrix being the coefficients for a different lag
7: \( l \leftarrow l + 1 \)
8: end for
9: \( C_{1,1} \leftarrow \delta(B,1) \)
10: for \( i \leftarrow 2, i \leq k \) do
11:   for \( j \leftarrow 1, j < i \) do
12:   \( C_{i,j} \leftarrow \delta(B_j) \cdot (\sum_{x=1}^{i-j} C_{i-j,x}) \)
13: \( j \leftarrow j + 1 \)
14: end for
15: \( C_{i,i} \leftarrow \delta(B, i) \)
16: \( i \leftarrow i + 1 \)
17: end for
18: return \( C \)
19: end function

\[
\delta(B, j) = \begin{cases} 
  m \times m \text{ zero matrix} & \text{if } j \geq B\text{.length} \\
  B_j & \text{otherwise} 
\end{cases}
\tag{8}
\]

After AIRA has calculated the vector moving average model, it performs the IRF analysis. Algorithm 5 lists the pseudocode used for running this analysis. The algorithm starts by defining an output matrix in line 2. In the case of orthogonalized IRF, the identity matrix used in line 3 \((I_m)\) should be replaced by the contemporaneous coefficient matrix. Orthogonalized IRF is currently only supported in the R version of AIRA. The actual impulse responses are defined in line 4–11. The shocks are multiplied with each of the VMA coefficients to determine the response and are calculated for each moment on the horizon.

Algorithm 5 Algorithm for calculating the IRF from shocks in \(\vec{s}\), for \(k\) steps in the future.
1: function CALCULATEIRF(\(\vec{s}\), C, k)
   arguments \(\vec{s}\) vector of length \(m\) containing the shocks, \(C\) are the VMA coefficients as returned by Algorithm 4 and \(k\) is the number of steps (horizon) to forecast.
2: \( Y \leftarrow \) empty matrix of size \((k \times m)\)
3: \( Y_1 = I_m \cdot \vec{s} \)
4: for \( t \leftarrow 2, t \leq k \) do
5: \( Y_t \leftarrow [0_1, \ldots, 0_m]^T \)
6: for \( i \leftarrow 1, i < t \) do
7: \( Y_t \leftarrow Y_t + (C_{t-1,i} \cdot \vec{s}) \)
8: \( i \leftarrow i + 1 \)
9: end for
10: \( t \leftarrow t + 1 \)
11: end for
12: return \(Y^T\)
13: end function
Figure 5: The IRF output of the models from [10], recalculated using the R vars package [37]. The red/dashed lines indicate the 95% confidence interval, the black line the IRF curve. Each of the examples has been bootstrapped 200 times. The area under the 95% confidence curves correspond to values presented in the main article.

B Time complexity

Algorithm 4 describes the conversion of VAR coefficients to VMA coefficients. In order to determine these VMA coefficients, the algorithm iterates over $k$ (the used horizon). In each iteration, the algorithm retrieves all previously created VMA coefficients $k$ times. This is done for all entries and therefore bounded by $k$. Each of the entries retrieved requires a matrix summation for each $m \times m$ matrix. Finally, this summed matrix is multiplied by another $m \times m$ matrix. The total time-complexity therefore has an upper bound of $O(k^2 m^3 + k^3 m^2)$.

The time complexity of the IRF calculation as shown in Algorithm 5 depends on the horizon and the number of variables in the model. The algorithm iterates over the horizon ($k$ steps). For each step on the horizon, it determines an effect at most $k$ times, where each effect calculation is a matrix-vector multiplication of at most $m \times m$ steps (the size of matrix $C_{t-1,i}$). Finally, each element of the resulting $m \times 1$ vector is added to the $Y_t$ vector. The total upper bound of the calculation of the IRF is therefore $O(k^2 m^2)$.

C Time plots

The plots of the impulse response functions used as input for the comparison of AIRA with the work of Rosmalen et al. are shown in Fig. 3. In Fig. 3, each PP represents a participant, the horizontal axis depicts the horizon of the IRF, and the vertical axis shows the variable from which the response is recorded. Fig. 5 shows the IRF graphs of four participants from Rosmalen et al. These IRF plots are similar to the time plots provided in the work of Rosmalen et al. (see Figure 2 on page 7 of [10]).