Merger rate density of binary black holes formed in open clusters

Jun Kumamoto,1 * Michiko S. Fujii,1 and Ataru Tanikawa,2,3

1 Department of Astronomy, Graduate School of Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
2 Department of Earth Science and Astronomy, College of Arts and Sciences, The University of Tokyo, 3-8-1 Komaba, Meguro-ku, Tokyo 153-8902 Japan
3 RIKEN Center for Computational Science, 7-1-26 Minatojima-minami-machi, Chuo-ku, Kobe, Hyogo 650-0047, Japan

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

Several binary black holes (BBHs) have been observed using gravitational wave detectors. For the formation mechanism of BBHs, two main mechanisms, isolated binary evolution and dynamical formation in dense star clusters, have been suggested. Future observations are expected to provide more information about BBH distributions, and it will help us to distinguish the two formation mechanisms. For the star cluster channel, globular clusters have mainly been investigated. However, recent simulations have suggested that BBH formation in open clusters is not negligible. We estimate a local merger rate density of BBHs originated from open clusters using the results of our N-body simulations of open clusters with four different metallicities. We find that the merger rate per cluster is the highest for our 0.1 solar metallicity model. Assuming a cosmic star formation history and a metallicity evolution with dispersion, we estimate the local merger rate density of BBHs originated from open clusters to be $\sim 70 \text{ yr}^{-1} \text{Gpc}^{-3}$. This value is comparable to the merger rate density expected from the first and second observation runs of LIGO and Virgo. In addition, we find that BBH mergers obtained from our simulations can reproduce the distribution of primary mass and mass ratio of merging BBHs estimated from the LIGO and Virgo observations.

Key words: gravitational waves – methods: numerical – stars: black holes

1 INTRODUCTION

The first two runs of gravitational wave detectors, LIGO and Virgo, have detected ten binary black hole (BBH) mergers (Abbott et al. 2019a). Most of the detected BBHs have masses of a few times $10 M_\odot$ (Abbott et al. 2016a,b, 2017a,b,c). Before the detection of gravitational waves, black holes with such a mass was not known. The formation mechanism of these BBHs is still unclear.

There are two major scenarios for the origin of such BBHs. One is a common envelope and mass transfer evolution of isolated field binaries (e.g. Tutukov et al. 1997; Bethe & Brown 1998; Dominik et al. 2012; Kimugawa et al. 2014; Belczynski et al. 2016; Giacobbo et al. 2018; Bavera et al. 2019). In this scenario, some heavy stars are born in binary. As the massive stars evolve, the orbital separation shrinks due to the common envelope. Therefore, BBHs formed via this process have a semi-major axis smaller than the initial binary separations.

The other scenario is the dynamical formation due to three-body encounters in the core of star clusters (Portegies Zwart & McMillan 2000) or galactic nuclei (O’Leary et al. 2009; Antonini & Rasio 2016). The core of globular clusters ($10^5-10^6 M_\odot$) has long been investigated as a formation site of BBHs in many previous works (e.g. Portegies Zwart & McMillan 2000; O’Leary et al. 2006; Sadowski et al. 2008; Downing et al. 2010, 2011; Banerjee et al. 2010; Tanikawa 2013; Bae et al. 2014; Rodriguez et al. 2015, 2016; Fujii et al. 2017; Park et al. 2017; Askar et al. 2017; Hong et al. 2018; Zevin et al. 2019).

On the other hand, open cluster (10$^3-10^4 M_\odot$) has not been expected as a main formation site of BBHs merging in the Hubble time because of the fewer number of massive stars and their shallower gravitational potential. However, the number of open clusters would have been an order of magnitude larger than that of globular clusters when they formed (Portegies Zwart et al. 2010). Although there may be few BBHs formed from one open cluster, more BBHs may be formed from open clusters throughout the Universe. Actually, the formation of BBHs in open clusters has been investigated by some previous works (e.g. Ziosi et al. 2014; Goswami et al. 2014; Mapelli 2016; Banerjee 2017, 2018a,b; Rastello et al. 2019; Di Carlo et al. 2019; Kumamoto et al. 2019; Bouffanais et al. 2019).

© 2019 The Authors

E-mail:kumamoto@astron.s.u-tokyo.ac.jp

* Corresponding author
In Kumamoto et al. (2019, hereafter Paper I), we found a new channel for the formation of merging BBHs in open clusters. In the case of the open cluster with the half-mass density of $10^4 M_\odot \text{pc}^{-3}$, the core-collapse time is shorter than the lifetime of massive main-sequence. Then, massive main-sequence stars can form binaries in the dense core of the cluster before they evolve to black holes. Some of these binaries experience common envelope evolution and evolve to tight BBHs.

The merger rate density in the local Universe is calculated by integrating the merger rate density of BBHs ejected from clusters formed in each redshift (Fujii et al. 2017). While most of the globular clusters are formed more than 12 Gyr ago, open clusters are expected to form in any redshift. Therefore, the contribution of BBHs originated from the open cluster to the local merger rate density can be more significant. However, black hole mass strongly depends on metallicity, and we know that there is a cosmic metallicity evolution (Madau & Fragos 2017). Therefore, black holes formed in a lower redshift tend to be less massive due to stronger stellar wind of metal-rich stars.

In order to investigate the local merger rate density of BBHs originated from open clusters in each redshift, we perform $N$-body simulations of open clusters with four different metallicity models. From the results of our simulations, we estimate local merger rate density of BBHs originated from open clusters formed at each cosmic time.

The structure of this paper is as follows. We describe our simulation methods and models in section 2. In Section 3, we investigate the properties of binary black holes formed in our simulations. In Section 4, we calculate the merger rate from each model and estimate the local merger rate density from our simulation results. Conclusions are in sections 5.

## 2 METHODS AND MODELS

We simulated open cluster models changing metallicity in addition to Model A in Paper I and investigated the formation of BBHs. A summary of our simulations is following.

### 2.1 Initial conditions

We set up four cluster models of which metallicities are $Z = 0.002, 0.005, 0.01$ and 0.02. Table 1 summarise our models. We set the initial cluster mass ($M_{\text{cl,ini}}$) to be $2500 M_\odot$, which is the same as Model A in Paper I and named as Model Z0002 in the present paper. The number of runs ($N_{\text{run}}$) per model depends on the metallicity. For more metal-rich models, we adopt a larger $N_{\text{run}}$ because heavier BHs are expected to form less in more metal-rich clusters due to stronger stellar wind (see also subsection 2.3).

As the initial density profile of cluster, we adopt Plummer profile (Plummer 1911);

$$
\rho(r) = \frac{3M_{\text{cl,ini}}}{4\pi r_p^3} \left(1 + \frac{r^2}{r_p^2}\right)^{-5/2},
$$

where $r_p$ is a half-mass radius. We set $r_{\text{hm}}$ to be 0.31 pc so that the initial half-mass density ($\rho_{\text{hm}} = 3M_{\text{cl,ini}}/8\pi r_{\text{hm}}^3$) is $10^4 M_\odot \text{pc}^{-3}$. This half-mass density is similar to those of observed densest young massive clusters and higher than those of currently observed open clusters (Portegies Zwart et al. 2010). However, even if we set such an initial density higher than current values, open clusters experience core-collapse in a time shorter than the first supernova explosion (3–4 Myr), and their densities immediately drop to the current density of observed typical open clusters (Fujii & Portegies Zwart 2016).

The initial mass of each stellar particle is given randomly from the Kroupa initial mass function (Kroupa 2001). The lower and upper limit of the stellar mass are set to be $m_{\text{min}} = 0.08 M_\odot$ and $m_{\text{max}} = 150 M_\odot$, respectively. In this case, the expected average stellar mass is $\langle m \rangle = 0.586 M_\odot$. Thus, the initial number of particles is given as

$$
N_{\text{ini}} = \frac{M_{\text{cl,ini}}}{\langle m \rangle} = 4266.
$$

The half-mass relaxation time is calculated from the half-mass density as:

$$
\tau_{\text{rh}} = 0.711 \left(\frac{N}{\log(0.4N)}\right) \left(\frac{\rho_{\text{hm}}}{M_\odot \text{pc}^{-3}}\right)^{-0.5} \text{Myr.}
$$

The core-collapse time is correlated with the relaxation time, and the correlation factor depends on the ratio between the maximum and average masses of stars in the system. In our models, $m_{\text{max}}/\langle m \rangle > 50$, and we obtain

$$
\tau_{cc} \sim 0.07 \tau_{\text{rh,ini}}.
$$

from Gürkan et al. (2004); Fujii & Portegies Zwart (2016). Therefore, the core-collapse time of our models is estimated to be $\sim 0.7$ Myr.

We did not assume any primordial binaries. In Paper I, we argued that primordial binaries would not affect much on the formation rate of merging BBHs in open clusters for the following reason. The number of massive BHs ($\sim 20–30 M_\odot$) in open clusters is limited because of their total mass, and therefore, only a few massive BHs can merge within the Hubble time. Even without primordial binaries, massive stars tend to form a massive binary in the core of star clusters. Thus, we did not include primordial binaries in our simulation.

### 2.2 N-body simulations

We perform simulations of star clusters using a direct $N$-body simulation code, NBODY6++GPU (Wang et al. 2015). This code is an MPI-parallelised and GPU enabled version of NBODY6 (Aarseth 1999). We perform our simulations using GPU cluster SGI Rackable C1102-GP2 (Reedbus-L) in the Information Technology Center, The University of Tokyo.

| Table 1. Models. |
|------------------|
| $M_{\text{cl,ini}}[M_\odot]$ | $Z$ | $N_{\text{run}}$ |
| Model Z0002 | $2.5 \times 10^3$ | 0.002 | 360 |
| Model Z0005 | $2.5 \times 10^3$ | 0.005 | 500 |
| Model Z001 | $2.5 \times 10^3$ | 0.01 | 1000 |
| Model Z002 | $2.5 \times 10^3$ | 0.02 | 1000 |
The motions of individual stars are integrated by a fourth-order Hermite scheme (Makino & Aarseth 1992). In our simulations, binaries dynamically formed via three-body encounters. Some of these binaries have time steps much smaller than the time-scale of cluster evolution. Such hard binaries are integrated using KS regularization (Kustaanheimo & Stiefel 1965; Mikkola & Aarseth 1993).

Since the core-collapse time of our models is ~0.7 Myr, most BBHs are formed within a few hundred Myr. In such a short time, tidal disruption of star clusters due to galactic tidal fields would not much affect the internal structure of star clusters. We, therefore, do not assume external tidal force.

### 2.3 Stellar evolution

**NBODY6++GPU** contains a stellar evolution model, **SSE** (Hurley et al. 2000). This model provides the time evolution of stellar radius, mass, and luminosity of each star depending on metallicity. We transported an updated mass loss model (Belczynski et al. 2010), which is contained in the latest version of NBODY6, to the stellar evolution model in NBODY6++GPU. For binaries evolution, **NBODY6++GPU** contains a binary evolution model (Tout et al. 1997), which is an algorithm for rapid evolution binary star following the common envelope and mass transfer.

The most important effect of metallicity is mass loss due to stellar winds. Therefore, the black hole mass also strongly depends on metallicity. Figure 1 shows relations between zero-age main-sequence stellar and black hole masses for each metallicity in our model. More metal-rich stars evolve into less massive black holes because of the stronger stellar wind.

In our simulation, we do not assume natal kicks caused by asymmetric supernovae explosion for simplification. The natal kicks may affect the ejection rate of BBHs (Tanikawa 2013), but relatively massive black holes (10–20 $M_\odot$) tend to retain in star clusters (Morscher et al. 2013). Even if some black holes are ejected from the star clusters due to the natal kicks, the hardening of BBHs in open clusters mainly proceed via interactions with the other stars rather than black holes. More discussion on this point is described in section 4.2 of Paper I.

### Table 2. Number of BBHs

| Model | $N_{BBH}$ | $N_{BBH}/N_{run}$ | $N_{merg}$ | $N_{merg}/N_{run}$ |
|-------|-----------|--------------------|-----------|--------------------|
| Z0002 | 338       | 0.939              | 37        | 0.103              |
| Z0005 | 487       | 0.974              | 17        | 0.034              |
| Z001  | 988       | 0.988              | 32        | 0.032              |
| Z002  | 877       | 0.877              | 7         | 0.007              |

| a number of BBHs | b number of merging BBHs |

We obtained in total ~300–1000 BBHs per model\(^1\). In Table 2, we show the number of BBHs ($N_{BBH}$) formed in each model. About one BBH per one cluster formed in our simulations. We investigate the properties of these BBHs.

Figure 2 shows the cumulative distributions of BBHs formed (and ejected) in our simulations as a function of primary mass ($M_1$), mass ratio ($q$), orbital eccentricity ($e$), and semi-major axis ($a$). While dashed curves are for BBHs which experienced common envelope, solid ones are for BBHs which did not.

The cumulative distribution of the primary mass of ejected BBHs in panel (a) of Figure 2. Primary mass of BBHs formed in metal-poorer model tends to be greater than that in the metal-richer model. This trend results from the black hole mass formed from massive main-sequence (See Figure 1). The cumulative distributions of $q$ and $e$ do not show remarkable differences between metallicity models. Most of BBHs which experience common envelope has zero-eccentricity.

Semi-major axis of BBHs which experience the common envelope tends to be shorter than those of BBHs without common envelope evolution. Interestingly, the semi-major axis of BBHs without common envelope formed in the metal-richer cluster model are shorter than those in metal-poorer cluster model. These binaries (binaries without common envelope evolution) formed via purely dynamical interactions. If the cluster mass is the same, the binding energy of ejected binaries should be similar among these models. Therefore, BBHs with a smaller $M_1$ tend to have a smaller value of $a$. Since the BH masses tend to be lower in more metal-rich clusters, binaries dynamically formed in clusters with a higher-metallicity tend to have a shorter semi-major axis.

We calculate merger time from the parameters of ejected binaries using the following equation

\[^1\] We obtained not only BBHs but also black hole–main-sequence binaries, which are one of the targets of the Gaia mission (Gaia Collaboration et al. 2016). Shikuchi et al. (in prep) investigated the detectability of these binaries by Gaia from our simulation results.
Figure 2. Cumulative distribution for $M_1$ (panel (a)), $q$ (panel (b)), $e$ (panel (c)) and $a$ (panel (d)) of BBHs formed in each metallicity model. Solid and dashed lines show the distribution of BBHs evolved with and without a common envelope.

(Peters & Mathews 1963):

$$t_{GW} = \frac{5}{256} \frac{c^5}{G^3 M_1^3 q (1 + q)} g(e)$$

$$\sim 1.2 \left( \frac{M_1}{30 M_\odot} \right)^{\frac{3}{2}} \left( \frac{a}{0.1 \text{ AU}} \right)^4 g(e) \frac{q (1 + q)}{q (1 + q)} \text{ Gyr},$$

where

$$g(e) = \frac{(1 - e^2)^{3.5}}{1 + (73/24)e^2 + (37/96)e^4}.$$

Here, $c$ and $G$ are light speed and gravitational constant, respectively.

Figure 3 shows the cumulative distribution of the merger time for all models. Solid curves show the results from each model in our simulation. Dashed curves are fitted results using the following function:

$$N(t_{GW} < t) = N_0 \ln \left( \tau^{-1} t + 1 \right),$$

where $N_0$ and $\tau$ are fitting parameters. The fitted equations are also shown in each panel of Figure 3.

The number of BBHs merged within 14 Gyr (hereafter, merging BBHs), $N_{\text{BBH,merge}}$, in each model is summarised in Table 2. The number of merging BBHs per cluster is more significant in the metal-poorer model.

Figure 4 shows the primary mass ($M_1$), mass ratio ($q$), semi-major axis ($a$), and eccentricity ($e$) of merging BBHs ejected from open clusters. BBHs which experienced common envelope have a distribution different from those which did not. In general, merging BBHs experienced common envelope phase have a short semi-major axis and nearly zero eccentricity. However, some BBHs which experienced common envelope phase dynamically interact with other single stars, and as a result, the eccentricity is pumped up. These eccentric BBHs can merge within 14 Gyr because of their high eccentricity (see eq. (7)). Furthermore, some of them result in an exchange of a member with an encountering single black hole during three-body encounters.

Merging BBHs which experienced dynamical interactions after their common envelope phase had a relatively large semi-major axis compared with those which can merge just after the common envelope evolution. Because of their relatively large cross-section, they could interact with other stars and change their orbital parameters. The semi-major axis of BBHs which experienced common envelope phase distribute $-1.5 \lesssim \log a \lesssim 2$ (panel (d) of Figure 2). Among them, only BBHs which could experience dynamical interactions and get an eccentricity high enough to merge within 14 Gyr appear as a merging BBH.

We also find a few merging BBHs which did not ex-
experience common envelope evolution (dynamically formed BBHs). All of these merging BBHs have a semi-major axis larger than those of merging BBHs which experienced common envelope phase. The eccentricities of the dynamically formed BBHs are almost one (actually, \( e > 0.995 \) in our results). Because of such high eccentricities, they can merge within 14 Gyr in spite of their relatively large semi-major axis.

In Figure 4, we also show the number distribution of merging BBHs for each parameter:

\[
\nu = \frac{N_{\text{bin}}}{N_{\text{BBH}}},
\]

(10)

where \( N_{\text{bin}} \) is the number of merging BBH included each parameter bin. The bin sizes are equal to \( 5M_\odot \), 0.2, 1 and 0.2 for \( M_1 \), \( q \), \( \log(a) \) and \( e \).

4 LOCAL MERGER RATE DENSITY

4.1 Integration

A local merger rate density of BBHs originated from star clusters is written as,

\[
R = \int dM_\text{cl} \int dZ \int dt_L D(Z, M_\text{cl}, t_{GW} = t_L) \frac{dN_{\text{cl}}}{dZ dM_\text{cl}}(t_L, Z, M_\text{cl}).
\]

(11)

where \( t_L \) is lookback time, and \( D(Z, M_\text{cl}, t_{GW}) \) is the merger rate of BBHs originated from one cluster, which is having metallicity, \( Z \), and mass, \( M_\text{cl} \), and merging after \( t_{GW} \) from the cluster formation. In order to obtain a current merger rate, we need to count merger rates of BBHs with \( t_{GW} = t_L \). In equation (11), \( dN_{\text{cl}}(t_L, Z, M_\text{cl})/dZ dM_\text{cl} \) is the formation rate density of cluster with a metallicity of \( Z \) and a mass of \( M_\text{cl} \).

We first derive \( D(Z, M_\text{cl}, t_{GW}) \) from the delayed-time distribution obtained from our simulations. From the fitted function to the cumulative distribution of merger time shown in Figure 3, we can obtain the delayed-time distribution for each metallicity model as

\[
D(Z, M_\text{cl} = 2500M_\odot, t_{GW}) = \left( \frac{dN(t_{GW} < t)}{dt} \times \frac{1}{N_{\text{run}}} \right)
\]

(12)

In Figure 5, we show the equation for each metallicity and those obtained from the distribution of BBHs formed in our simulations. We confirmed that the fitted functions are consistent with the simulation results.

Next, we estimate the formation rate density of clusters. The following equation shows the formation rate density of the cluster with a mass of \( M_\text{cl} \):

\[
\frac{dN_{\text{cl}}}{dZ dM_\text{cl}}(t_L, Z, M_\text{cl}) = \frac{f_3(M_\text{cl}) d\Psi(t_L, Z)}{M_\text{cl}}.
\]

(13)

Here, \( \Psi(t_L, Z) \) is a comoving formation rate density of star which is having metallicity, \( Z \), and \( f_3(M_\text{cl}) \) is the fraction of
stellar mass formed as star cluster having the mass, $M_{\text{cl}}$. If we assume that all stars are formed as members of clusters with a mass range of $10^2$ to $10^6$, then we can write as

$$\int_{10^2 M_\odot}^{10^6 M_\odot} f_{\text{cl}}(M_{\text{cl}})dM_{\text{cl}} = 1. \quad (14)$$

In addition, we assume that the cluster mass function follow to $M_{\text{cl}}^{-2} \left( f_{\text{cl}}(M_{\text{cl}})/M_{\text{cl}} \propto M_{\text{cl}}^{-2} \right)$, therefore,

$$f_{\text{cl}}(M_{\text{cl}}) = \frac{1}{4 \ln 10} M_{\text{cl}}^{-1}. \quad (15)$$

With this assumption, we estimate $R_{\text{OC}}$, which is the local merger rate density of BBHs formed in open clusters with a mass of $10^5 M_\odot$ to $10^4 M_\odot$, as

$$R_{\text{OC}} = \frac{1}{4 \ln 10} \int_{10^5 M_\odot}^{10^6 M_\odot} dM_{\text{cl}} \int dZ \int d\ln \frac{D(Z, M_{\text{cl}}; t_L)}{M_{\text{cl}}^2} \frac{d\Psi(t_L, Z)}{dZ}. \quad (16)$$

Here, we assume that the dependence of $D(Z, M_{\text{cl}}; t_L)$ on $M_{\text{cl}}$ is negligible in this cluster mass range, we can rewrite equation (16) as

$$R_{\text{OC}} = \frac{9 \times 10^{-4}}{4 M_\odot \ln 10} \int dZ \int d\ln D(Z, t_{\text{GW}} = t_L) \frac{d\Psi(t_L, Z)}{dZ}. \quad (17)$$

Hereafter, we omit the argument $M_{\text{cl}}$ in function.
$D(Z, M_{cl}, t_{GW})$ for simplification. In order to calculate the integral part of this equation, we apply two methods. The first one is to assume that all star clusters which were born in the same era have the same averaged metallicity at that time (single metallicity evolution model). The other is to consider metallicity dispersion in each formation era as well as the averaged metallicity evolution (metallicity dispersion model). The merger rate density calculated using each model will be described in subsection 4.2 and 4.3, respectively. In addition, we calculate the differential merger rate density of $M_1$ and $q$ for each model.

### 4.2 Single metallicity evolution model

First, we assume that all stars born at the same era have the same metallicity and that the metallicity evolve with time. In order to connect the metallicity of star clusters to their birth time, we use a relation between metallicity ($Z$) and redshift ($z$) (Madau & Fragos 2017),

$$\log \left( \frac{Z}{Z_\odot} \right) = 0.153 - 0.074z^{1.34}. \quad (18)$$

Thus, stars formed at any lookback time ($t_L$) have the metallicity ($Z(t_L)$), so that,

$$\frac{d\Psi(t_L, Z)}{dZ} = \Psi(t_L)\delta(Z(t_L)), \quad (19)$$

where $\delta$ is the Dirac delta function. Substituting this for equation (17), we obtain

$$R_{OC} = \frac{9 \times 10^{-4}}{4M_\odot \ln 10} \int dZ' \int dt_L D(Z', t_L)\Psi(t_L)\delta(Z(t_L))$$

$$= \frac{9 \times 10^{-4}}{4M_\odot \ln 10} \int dt_L D(Z(t_L), t_L)\Psi(t_L). \quad (20)$$

The star formation rate (SFR) density, $\Psi(t_L)$, is given as a function of redshift ($z$) (Madau & Fragos 2017), such as:

$$\Psi(z) = 0.01 \frac{(1 + z)^{2.6}}{1 + ((1 + z)/3.2)^{0.2}} M_\odot \text{yr}^{-1} \text{Mpc}^{-3}. \quad (21)$$

#### 4.2.1 Merger rate density for each metallicity

We calculate $D(Z, t_{GW})\Psi(t_L)$ from our simulations and equation (21). Although $D$ is a function of metallicity ($Z$), it is known that the typical metallicity in the Universe evolves. We can calculate the local merger rate of BBHs, originated from those clusters, per cluster, $D(Z, t_{GW} = t_L)$, from the functions shown in Figure 5. Table 3 shows the results of these calculations. We obtain $3.3 \times 10^{-12}$ yr$^{-1}$ for Model Z0002, and this value is the largest among our models. The value of $D(Z, t_L)$ tends to decrease as metallicity increase. For Model Z002, $D(0.02, t_L)$ is only 10% of that for Model Z0002.
The interpolated and extrapolated

Figure

about 20 times greater than that for Model Z0002 (z = 13.1 Gyr). Therefore, the contribution to the local merger rate of the metal-richer cluster may be greater because of the more active star formation.

We calculate D(Z, tL)Ψ(z) for each metallicity in our simulations, and the results are shown in Table 3. These values present the contribution to the local merger rate from clusters which have metallicity Z and are born in unit time and unit volume. If we compare the total merger rate including cosmic SFR density, the contribution from metal-rich clusters (Models Z001 and Z002) is larger than that from metal-poor clusters (Models Z0001 and Z0002) due to the cosmic SFR density.

Table 3. Our results (Mobj = 2500M⊙).

| Z     | zα | tL(Gyr) | D(Z, Mobj = 2500M⊙, tGW = tL) [10^-11 M⊙ yr^-1] | Ψ(z) [10^-5 M⊙ yr^-1 Gpc^-3] | D(Z, 2500M⊙, tL)Ψ(z) [10^-5 M⊙ yr^-1 Gpc^-3] |
|-------|----|---------|---------------------------------------------|-----------------------------|---------------------------------------------|
| Model Z0002 | 0.002 | 7.76 | 13.1 | 0.33 | 0.00547 | 1.8 |
| Model Z0005 | 0.005 | 5.66 | 12.8 | 0.081 | 0.0145 | 1.2 |
| Model Z001 | 0.01 | 3.87 | 12.2 | 0.13 | 0.0422 | 5.4 |
| Model Z002 | 0.02 | 1.72 | 9.97 | 0.037 | 0.0988 | 3.7 |

α redshift calculated from Z and equation (18)  
β lookback time at redshift z in the case of (h, ΩM, ΩΛ) = (0.7, 0.3, 0.7)  
γ local merger rate calculated from functions shown in figure 5 for each metallicity  
δ SFR density calculated from equation (21)

On the other hand, SFR density (Ψ(z)) has a peak at z = 2.0 (tL = 10.5 Gyr, see Figure 6). The value of Ψ(z) when Model Z001 like cluster was born (z = 3.87, tL = 12.2 Gyr), is about 20 times greater than that for Model Z0002 (z = 7.76, tL = 13.1 Gyr). Therefore, the contribution to the local merger rate of the metal-richer cluster may be greater because of the more active star formation.

We calculate D(Z, tL)Ψ(z) for each metallicity in our simulations, and the results are shown in Table 3. These values present the contribution to the local merger rate from clusters which have metallicity Z and are born in unit time and unit volume. If we compare the total merger rate including cosmic SFR density, the contribution from metal-rich clusters (Models Z001 and Z002) is larger than that from metal-poor clusters (Models Z0001 and Z0002) due to the cosmic SFR density.

4.2.2 Local merger rate density

In order to calculate the equation (20), we have to integrate D(Z(tL), tL)Ψ(tL). However, the delayed distribution function D(Z(tL), tL) is dispersively given from our simulations. We, therefore, interpolate and extrapolate the data points obtained from four our simulation models as follows:

\[
D(Z, t_{GW}) = \begin{cases} 
D_1 & \text{for } Z < 0.002, \\
0.005 - Z & 0.003 D_1 + Z - 0.002 & 0.003 & D_2 & \text{for } 0.002 \leq Z < 0.005, \\
0.01 - Z & 0.005 D_2 + Z - 0.005 & 0.005 & D_3 & \text{for } 0.005 \leq Z < 0.01, \\
0.02 - Z & 0.01 D_3 + Z - 0.01 & 0.01 & D_4 & \text{for } 0.01 \leq Z < 0.02, \\
D_4 & \text{for } 0.02 \leq Z.
\end{cases}
\]  

(22)

Here,

\[
D_1 = D(Z = 0.002, t_{GW}),
\]  

(23)

\[
D_2 = D(Z = 0.005, t_{GW}),
\]  

(24)

\[
D_3 = D(Z = 0.01, t_{GW}),
\]  

(25)

\[
D_4 = D(Z = 0.02, t_{GW}).
\]  

(26)

The interpolated and extrapolated D(Z, tGW = tL) as a function of tL is shown by the solid curve in the top panel of Figure 6. In the same panel, the dashed line shows the SFR density, Ψ(tL).

Figure 6. Top panel shows D(Z, tGW = tL) (solid line) and Ψ(tL) (dashed line) calculated from equation (22) and (13) as a function of tL. Bottom panel shows the product of D(Z, tL) and Ψ(tL).

The bottom panel of Figure 6 shows D(Z, tGW = tL)Ψ(z(tL)) as a function of tL. This distribution has a maximum value between tL = 11 and 12 Gyr and decreases toward the present time because the SFR density decreases. There is a small peak at tL ~ 13 Gyr. This peak results from the distribution of D(Z, tL), in which D(Z = 0.01, tL) is larger D(Z = 0.005, tL) contrary to the number of merging BBH per cluster decreases as metallicity increases. This may be due to the relatively small number of runs. We obtained up to tens of merging BBHs for each model, but these may not be enough to fit a function to obtain delayed-time distribution (see Fig. 5). However, the contribution to the estimation of the merger rate density would not be large even if the values of D(Z, tL) slightly changed.

We finally integrate D(Z, tL)Ψ(z(tL)) and estimate the
local merger rate density of BBHs ejected from open clusters using equation (20) as
\[ R_{\text{OC}} \sim 35 \text{ yr}^{-1} \text{Gpc}^{-3}. \] (27)

We apply the delayed distribution function at \( Z = 0.02 \) to calculate merger rate density for \( t_L < 9.97 \text{ Gyr} \), although the averaged metallicity of this time is larger than \( Z = 0.02 \). Because of this, we might overestimate \( R_{\text{OC}} \). On the other hand, Abbott et al. (2019b) have estimated the BBH merger rate density from the first and second observing runs of LIGO and Virgo as
\[ R_{\text{obs}} = 64.0^{+17.5}_{-33.0} \text{ yr}^{-1} \text{Gpc}^{-3}, \] (28)
in their Model A. Our estimation corresponds to \( \sim 55\% \) of this value.

### 4.2.3 Differential merger rate density

We also give the distribution of \( M_1 \) and \( q \) of BBHs expected to merge in the local Universe. From equations (17) and (10), the differential merger rate density is obtained as
\[ \frac{dR_{\text{OC}}}{dM_1} = \frac{1}{W_{M_1}} \frac{9 \times 10^{-4}}{4\pi G^2 \ln 10} \int dM_1 \varphi(M_1) D(Z(t_L), t_L) \Psi(t_L). \] (29)
and
\[ \frac{dR_{\text{OC}}}{dq} = \frac{1}{W_q} \frac{9 \times 10^{-4}}{4\pi G^2 \ln 10} \int dM_1 \varphi(q) D(Z(t_L), t_L) \Psi(t_L). \] (30)
for \( M_1 \) and \( q \), respectively. Here, \( W_{M_1} \) and \( W_q \) are the bin sizes for \( \varphi(M_1) \) and \( \varphi(q) \), respectively. The results obtained from our simulations are shown as histograms in Figure 7. We also plot the differential merger rate density expected from 10 BBH mergers detected in O1 and O2 of LIGO and Virgo (Model A of Abbott et al. 2019b).

For \( M_1 \), our result shows a good agreement with the distribution estimated from the observed BBH mergers at the low mass end. However, the merger rate of massive (\( M_1 \gtrsim 20M_\odot \)) BBHs predicted from our simulation is much smaller than those expected from the observations. For \( q \), our differential merger rate densities are less than those of observation in most of \( q \), but the shapes are similar.

Here, the metallicity of star clusters was adopted by the mass-weighted average metallicity at each \( t_L \). However, the metallicity dispersion at each \( t_L \) should also be considered. In lower-metallicity clusters, the merger rate of BBHs from a cluster is larger than that of higher-metallicity clusters. In addition, the distribution of \( M_1 \) is flatter, if the metallicity is lower. Therefore, the models with metallicity dispersion may increase the total merger rate density and the number of merging BBHs with a larger \( M_1 \).

### 4.3 Metallicity dispersion model

#### 4.3.1 Local merger rate density with metallicity dispersion

Next, we estimate a local merger rate density with a cosmic metallicity evolution and the dispersion at each time. We add the metallicity dispersion by considering cosmic star formation density for stars with each metallicity and then integrating the results with respect to metallicity. The result should give us the total merger rate density with metallicity dispersion at each cluster formation time. With this assumption, \( R_{\text{OC}} \) in equation (17) is approximated as;
\[ R_{\text{OC}} \sim \frac{9 \times 10^{-4}}{4\pi G^2 \ln 10} \sum_{i=1}^{4} dM_1 D_i(t_{\text{GW}} = t_L) \Psi_i(t_L). \] (31)

where
\[ \Psi_1(t_L) = \int_{0.001}^{0.002} \frac{dZ}{dz} \frac{d\Psi}{dZ}(Z, t_L), \] (32)
\[ \Psi_2(t_L) = \int_{0.002}^{0.005} \frac{dZ}{dz} \frac{d\Psi}{dZ}(Z, t_L), \] (33)
\[ \Psi_3(t_L) = \int_{0.005}^{0.01} \frac{dZ}{dz} \frac{d\Psi}{dZ}(Z, t_L), \] (34)
\[ \Psi_4(t_L) = \int_{0.01}^{0.02} \frac{dZ}{dz} \frac{d\Psi}{dZ}(Z, t_L). \] (35)

Here, we divided the metallicity to four regions, for which we performed \( N \)-body simulations. By integrating these functions with respect to \( Z \), we can obtain the SFR den-
This value is twice as large as that obtained in the case without metallicity dispersion and in good agreement with that estimated from the observations (equation (28)).

4.3.2 Differential merger rate density with metallicity dispersion

We calculate the differential merger rate density for $M_1$ and $q$ in the case with metallicity dispersion similar to those in subsection 4.2. The results are shown in Figure 9.

The top panel shows $d\dot{N}_{\text{BBH}}/d\ln M_1$. Compared to the case without metallicity dispersion (see Figure 7), we find a larger number of merging BBHs for larger $M_1$. This is caused by the BBHs originated from lower-metallicity clusters formed at low-$z$, which were not considered the model without metallicity dispersion. Between 20$M_\odot$ and 30$M_\odot$, our results are consistent with the observation. Above 30$M_\odot$, however, the differential merger rate that we obtained is still several times smaller than that estimated from observed BBH mergers. The lack of massive BBHs in our models can be filled by BBHs originated from globular clusters (Rodriguez et al. 2016; Fujii et al. 2017), and/or isolated binaries (Dominik et al. 2015), formed in high redshift, at which the expected BH mass is higher.

Considering the metallicity dispersion, the differential merger rate density of $q$ increased overall and is consistent with the estimation from the observations, but still less at $q \sim 1$. BBHs with $q \sim 1$ tend to be formed in globular clusters and the field (isolated binary evolution), which we did not take into account in this study. Even if the number of equal-mass merging BBHs increases a few times when we add BBH populations formed in globular clusters and the field, the merger rate density would be consistent within the 90% confidence interval of the observation.

5 CONCLUSIONS

We performed a series of $N$-body simulation of open clusters with four different metallicity models, $Z = 0.002, 0.005, 0.01$, and $0.02$. Roughly one BBH per one cluster was formed in all cluster models.

We investigated the properties of BBHs formed in our simulations. In our simulations, the primary mass of BBHs formed in metal-richer open clusters tends to be smaller than that formed in metal-poorer clusters because of stronger stellar-wind mass-loss of metal-richer stars. The number of BBHs merging within cosmic time is the highest for the $Z = 0.002$ model. We also investigated the delayed-time distribution function of the merger time of BBHs formed in open cluster with each metallicity.

We also estimated a local merger rate density, considering the metallicity evolution in the Universe. We tried two assumptions. In the first one, we assumed the stellar metallicity is equal to the mass-weighted average metallicity in each era (single metallicity evolution model) (Madau & Fragos 2017). In the other one, we considered the metallicity dispersion in each era as well as metallicity evolution (Chruslinska & Nelemans 2019). With metallicity dispersion in each formation era, the merger rate density calculated from our results was $70\text{ yr}^{-1}\text{Gpc}^{-3}$, which is twice as large as the case without metallicity dispersion. The merger
rate density that we obtained is comparable to that estimated from the first and second runs of LIGO and Virgo (Abbott et al. 2019b). We found that the effect of lower-metallicity clusters formed at low-$z$ was especially significant.

In addition, we investigated differential merger rate density for $M_1$ and $q$. We found that differential merger rate density calculated from our simulations generally agrees with that estimated from observed BBH mergers. However, BBH mergers with the primary mass larger than $30M_\odot$ in our simulations are several times less than the probability distribution estimated from the observed BBH mergers. Such BBHs may be formed in globular clusters and/or isolated field binaries formed at high-$z$.

ACKNOWLEDGEMENTS

This work is supported by JSPS KAKENHI Grant Number 17H06360, 19H01933, and 19K03907 and The University of Tokyo Excellent Young Researchers Program. Numerical calculations reported in this paper were supported by Initiative on Promotion of Supercomputing for Young or Women Researchers, Supercomputing Division, Information Technology Center, The University of Tokyo.

REFERENCES

Aarseth S. J., 1999, PASP, 111, 1333
Abbott B. P., et al., 2016a, Physical Review Letters, 116, 061102
Abbott B. P., et al., 2016b, Physical Review Letters, 116, 241103
Abbott B. P., et al., 2017a, Physical Review Letters, 118, 221101
Abbott B. P., et al., 2017b, Physical Review Letters, 119, 141101
Abbott B. P., et al., 2017c, ApJ, 851, L35
Abbott B. P., et al., 2019a, Physical Review X, 9, 031040
Abbott B. P., et al., 2019b, ApJ, 882, L24
Antonini F., Rasio F. A., 2016, ApJ, 831, 187
Asker A., Szudzik M., Gondek-Rosińska D., Giersz M., Bulik T., 2017, MNRAS, 464, L36
Bae Y.-B., Kim C., Lee H. M., 2014, MNRAS, 440, 2714
Banerjee S., 2017, MNRAS, 467, 524
Banerjee S., 2018a, MNRAS, 473, 909
Banerjee S., 2018b, MNRAS, 481, 5123
Banerjee S., Baumgardt H., Kroupa P., 2010, MNRAS, 402, 371
Barber S. S., et al., 2019, arXiv e-prints, p. arXiv:1906.12257
Belczynski K., Bulik T., Fryer C. L.,Ruiter A., Valsecchi F., Vink J. S., Hurley J. R., 2010, ApJ, 714, 1217
Belczynski K., Holz D. E., Bulik T., O’Shaughnessy R., 2016, Nature, 534, 512
Bethe H. A., Brown G. E., 1998, ApJ, 506, 780
Bouffanais Y., Mapelli M., Gerosa D., Di Carlo U. N., Giacobbo N., Berti E., Baibhav V., 2019, ApJ, 886, 25
Chruslinska M., Nelemans G., 2019, MNRAS, 488, 5300
Di Carlo U. N., Giacobbo N., Mapelli M., Pasquato M., Spera M., Wang L., Haardt F., 2019, MNRAS, 487, 2947
Dominik M., Belczynski K., Fryer C., Holz D. E., Berti E., Bulik T., Mendel I., O’Shaughnessy R., 2012, ApJ, 759, 52
Dominik M., et al., 2015, ApJ, 806, 263
Downing J. M. B., Benacquista M. J., Giersz M., Spurzem R., 2018, MNRAS, 407, 1946
Downing J. M. B., Benacquista M. J., Giersz M., Spurzem R., 2011, MNRAS, 416, 133
Fujii M. S., Portegies Zwart S., 2016, ApJ, 817, 4
Fujii M. S., Tanikawa A., Makino J., 2017, PASJ, 69, 94
Gaia Collaboration et al., 2016, A&A, 595, A1
Giacobbo N., Mapelli M., Spera M., 2018, MNRAS, 474, 2959
Goswami S., Kiel P., Rasio F. A., 2014, ApJ, 781, 81
Girkan M. A., Freitag M., Rasio F. A., 2004, ApJ, 604, 632
Hong J., Vesperini E., Asker A., Giersz M., Szudzik M., Bulik T., 2018, MNRAS, 480, 5645
Hurley J. R., Pols O. R., Tout C. A., 2000, MNRAS, 315, 543
Kinugawa T., Inayoshi K., Hotokezaka K., Nakauchi D., Nakamura T., 2014, MNRAS, 442, 2963
Kroupa P., 2001, MNRAS, 322, 231
Kumamoto J., Fujii M. S., Tanikawa A., 2019, MNRAS, 486, 3942
Kustaanheimo P., Stiefel E., 1965, Journal für die reine und angewandte Mathematik, 218, 204
Madau P., Fragos T., 2017, ArXiv e-prints, p. arXiv:1706.02616
Morscher M., Umbreit S., Farr W. M., Rasio F. A., 2013, ApJ, 763, L15
O’Leary R. M., Rasio F. A., Fregeau J. M., Ivanova N., O’Shaughnessy R., 2006, ApJ, 637, 937
O’Leary R. M., Kocsis B., Loeb A., 2009, MNRAS, 395, 2127
Park D., Kim C., Lee H. M., Bae Y.-B., Belczynski K., 2017, MNRAS, 469, 4665

Figure 9. Same as Figure 7, but for the model considering metallicity dispersion.
