A No-Scale Supergravity Realization of the Starobinsky Model

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We present a model for cosmological inflation based on a no-scale supergravity sector with an SU(2,1)/U(1) Kähler potential, a single modulus $T$ and an inflaton superfield $\Phi$ described by a Wess-Zumino model with superpotential parameters $(\mu, \lambda)$. This model yields a scalar spectral index $n_s$ and a tensor-to-scalar ratio $r$ that are compatible with the Planck measurements for values of $\lambda \simeq \mu/3M_P$. For the specific choice $\lambda = \mu/3M_P$, the model is a no-scale supergravity realization of the $R + R^2$ Starobinsky model.

The initial release of cosmic microwave background (CMB) data from the Planck satellite\textsuperscript{11} confront theorists of cosmological inflation\textsuperscript{2,3} with a challenge. On the one hand, the data have many important features that are predicted qualitatively by the inflationary paradigm. For example, there are no significant signs of non-Gaussian fluctuations or hints of non-trivial topological features such as cosmic strings, and the spectrum of scalar density perturbations exhibits a significant tilt: $n_s \simeq 0.960 \pm 0.007$, as would be expected if the effective scalar energy density decreased gradually during inflation. On the other hand, many previously popular field-theoretical models of inflation are ruled out by a combination of the constraint on $n_s$ and the tensor-to-scalar ratio $r < 0.08$ as now imposed by Planck et al.: see, e.g., Fig. 1 of\textsuperscript{11}. The only model with truly successful predictions displayed in Fig. 1 of\textsuperscript{11} is the $R^2$ inflation model of Starobinsky\textsuperscript{4}.

Since the field energy during the inflationary epoch is typically $\ll M_P^4$, it is natural to study renormalizable models, i.e., some combination of $\phi^2$, $\phi^3$ and $\phi^4$ in the single-field case. In this spirit, it was shown in\textsuperscript{5} that a single-field model with a potential of the form

$$V = A\phi^2(v - \phi)^2$$

(1)

could easily produce Planck-compatible values of $(n_s, r)$ for a suitable number of e-folds before the end of inflation $N \sim 50$ to 60. This simple symmetry breaking potential has a long pedigree, having been proposed initially in\textsuperscript{6} (for a review, see\textsuperscript{2}) where it was argued that sufficient inflation required $v > M_P$.

We think that a natural framework for constructing inflationary models is supersymmetry. As we pointed out in\textsuperscript{7}, in addition to all the well-known reasons for postulating low-scale supersymmetry, the small values of the quartic and quadratic couplings that would be required in a successful inflationary model become technically natural in the presence of low-scale supersymmetry. Small values of $\delta \rho/\rho$ become technically natural if low-scale supersymmetry is invoked\textsuperscript{7}, and if the GUT Higgs is distinguished from the singlet field that produces inflation, that later became known as the inflaton\textsuperscript{8}.

The simplest globally-supersymmetric model is the Wess-Zumino model with a single chiral superfield $\Phi$, which is characterized by a mass term $\tilde{\mu}$ and a trilinear coupling $\lambda$, with the superpotential

$$W = \frac{\tilde{\mu}}{2} \Phi^2 - \frac{\lambda}{3} \Phi^3.$$  (2)

As was discussed in\textsuperscript{5}, the effective potential of the Wess-Zumino model reduces to\textsuperscript{11} when the imaginary part of the scalar component of $\Phi$ vanishes, in which case this model yields Planck-compatible inflation.

However, global symmetry is not enough. In the context of early-Universe cosmology one should certainly include gravity and hence construct a locally-supersymmetric model, i.e., upgrade to supergravity\textsuperscript{10}. The first attempt at constructing an inflationary model in $N = 1$ supergravity proposed a generic form for the superpotential for a single inflaton\textsuperscript{11}, the simplest form being $W = m^2(1 - a\Phi)^2$\textsuperscript{12}. As discussed in\textsuperscript{13}, while this relatively simple model is capable of sufficient inflation, it is an example of accidental inflation in the sense that the coefficient of the linear term in the superpotential, $a$, must be extremely close to unity. This model has also become one of Planck’s casualties. The scalar-to-tensor ratio in this model is very small, but the value of...
In a supergravity model with a generic Kähler potential for the chiral supermultiplets there are quadratic $|\phi|^2$ terms in the effective potential that destroy its suitability for inflation, an obstacle known as the $\eta$-problem \cite{15}. As was pointed out in \cite{15}, a natural solution to this problem is offered by no-scale supergravity \cite{16}, in which there is a non-compact SU(N,1)/U(1) symmetry, such quadratic terms are suppressed, and the effective scalar potential resembles that in a globally-supersymmetric model. Other no-scale supergravity approaches have also been proposed \cite{17}, as well as models based on a non-compact Heisenberg symmetry \cite{18}, a shift symmetry \cite{19,21}, or string theory \cite{22}. The SU(N,1) model \cite{15} was based on the superpotential \cite{15} which we identify as the inflaton field, with the Kähler function $K$ where

\begin{equation}
K = \ln(\phi - \phi^\dagger/4) + \text{similar predictions for the inflationary parameters as the minimal } N = 1 \text{ model discussed above. This too is an example of accidental inflation } \cite{15} \text{ and a small change in the coefficient of the quartic term would lead to parameters consistent with Planck data } \cite{1}.
\end{equation}

In this paper we show how one can elevate the simplest globally-supersymmetric Wess-Zumino inflationary model of \cite{5} to a no-scale supergravity version (NSWZ). Concretely, we study a model in which the inflaton superfield is embedded in an SU(2,1)/U(1) no-scale supergravity sector together with a modulus field $T$ and find a range of the parameters where it is compatible with the Planck data \cite{1}. Quite remarkably, as we show, the NSWZ model is the conformal equivalent of an $R + R^2$ model of gravity for one specific value of $\mu/\lambda$, so that in this case our realization of inflation in the NSWZ model is equivalent to the Starobinsky model of inflation \cite{1}.

We first recall the basic relevant formulae governing the kinetic term and the effective potential of scalar fields $\phi$ in $N = 1$ supergravity, specializing to the no-scale case with non-compact SU(N,1)/U(1) symmetry. The scalar sector may be characterized in general by a hermitian Kähler function $K$ and a holomorphic superpotential $W$ via the combination $G \equiv K + \ln W + \ln W^\dagger$. The kinetic term is then given by

\begin{equation}
\mathcal{L}_{KE} = \left( \partial_\mu \phi^\ast, \partial_\mu T^\ast \right) \left( \begin{array}{c} 3 \\ (T + T^\ast - |\phi|^2/3)^2 \end{array} \right) \left( \begin{array}{c} (T + T^\ast)/3 - \phi \\ -\phi^\ast/1 \end{array} \right) \frac{\partial^\mu \phi}{\partial^\mu T},
\end{equation}

and the effective potential becomes

\begin{equation}
V = \frac{\hat{V}}{(T + T^\ast - |\phi|^2/3)^2} : \hat{V} \equiv \left| \frac{\partial W}{\partial \phi} \right|^2.
\end{equation}

In early no-scale models \cite{15,18} it was assumed that $K$ was fixed so that the potential up to a re-scaling was simply $\hat{V}$. Here we assume that the $T$ field has a vacuum expectation value (vev) $2(\Re T) = c$ and $(\Im T) = 0$ that is determined by non-perturbative high-scale dynamics, as in the KKLT \cite{23} or KL models \cite{24,25}. In this case, we may neglect the kinetic mixing between the $T$ and $\phi$ fields in \cite{1}, and are left with the following effective Lagrangian for the inflaton field $\phi$:

\begin{equation}
\mathcal{L}_{eff} = \frac{c}{(c - |\phi|^2/3)^2} |\partial_\mu \phi|^2 - \frac{\hat{V}}{(c - |\phi|^2/3)^2}.
\end{equation}

We assume as in \cite{5} the minimal Wess-Zumino superpotential \cite{2} for the inflaton field.

To better study the potential for the inflaton, we first transform $\phi$ to the field $\chi$:

\begin{equation}
\phi = \sqrt{3} c \tanh \left( \frac{\chi}{\sqrt{3}} \right). \tag{7}
\end{equation}

With this field redefinition, the Lagrangian becomes

\begin{equation}
\mathcal{L}_{eff} = \sec^2(\sqrt{3}(\chi - \chi^\ast)/\sqrt{3}) \left| \partial^a \chi \right|^2 - \left( \frac{3}{c} - \frac{\partial^a \chi^\ast}{\partial^a \chi} \right) \cosh(\chi/\sqrt{3}) \left( \sinh(\chi/\sqrt{3}) - \sqrt{3} c \lambda \sinh(\chi/\sqrt{3}) \right)^2.
\end{equation}

Clearly the vev of the $T$ field can be absorbed into the definition of the mass and, writing $\hat{m} = \mu \sqrt{c}/3$, the potential becomes

\begin{equation}
V = \mu^2 \left| \sinh(\chi/\sqrt{3}) \left( \cosh(\chi/\sqrt{3}) - \frac{3\lambda}{\mu} \sinh(\chi/\sqrt{3}) \right) \right|^2. \tag{9}
\end{equation}

Writing $\chi$ in terms of its real and imaginary parts: $\chi = (x + iy)/\sqrt{2}$ and, for reasons which will become clear, considering the specific case where the quartic coupling $\lambda = \mu/3$ (in Planck units), so that we have

\begin{equation}
\mathcal{L}_{eff} = \frac{1}{2} \sec^2(\sqrt{2}/3y) \left( \partial_x x \right)^2 + \left( \partial_y y \right)^2 - \frac{\mu^2 e^{-\sqrt{2}/3x}}{2} \sec^2(\sqrt{2}/3y) \left( \cosh(\sqrt{2}/3x) - \cos(\sqrt{2}/3y) \right)^2.
\end{equation}

The imaginary part of the inflaton is fixed to $y = 0$ by the potential, having a mass $m_y = \mu/\sqrt{3}$ during inflation when $x$ is large and $m_y = \mu/\sqrt{6}$ at the end of inflation when $x = 0$. Thus we expand the Lagrangian about $y = 0$, in which case we have minimal kinetic terms for $x$ and $y$, accompanied by derivative interaction terms. The
potential for the real part of the inflaton now takes the form
\[ V = \mu^2 e^{-\sqrt{2/3}x} \sinh^2 (x/\sqrt{6}). \] (11)

This potential is depicted in Fig. 1 where we also display the potential for values of \( \lambda \) slightly perturbed from the nominal value of \( \mu/3 \).

![FIG. 1. The potential \( V \) in the NSWZ model for choices of \( \lambda \sim \mu/3 \) in Planck units, as indicated.](image)

We use the standard slow-roll expressions for the tensor-to-scalar ratio \( r \) and the spectral index \( n_s \) for the scalar perturbations in terms of the slow-roll parameters \( \epsilon, \eta \) [9], which we evaluate in terms of the canonically-normalized field \( x \). In the NSWZ model described above the vev of \( T \) is absorbed in the definition of the mass parameter \( \mu \), which is determined by the normalization of the quadrupole. For the special case \( \lambda = \mu/3 \), we have
\[ A_s = \frac{V}{24 \pi^2 \epsilon} = \frac{\mu^2}{8 \pi^2} \sinh^4 (x/\sqrt{6}), \] (12)
implying a value \( \mu = 2.2 \times 10^{-5} \) in Planck units for \( N = 55 \). \( \mu \) varies between 1.8 - 3.4 \times 10^{-5} over the range of NSWZ models considered here. Setting the remaining NSWZ parameter \( \lambda = \mu/3 \), we have
\[ \epsilon = \frac{1}{3} \text{csch}^2 (x/\sqrt{6}) e^{-\sqrt{2/3}x}, \] (13)
\[ \eta = \frac{1}{3} \text{csch}^2 (x/\sqrt{6}) \left( 2e^{-\sqrt{2/3}x} - 1 \right), \] (14)
which allows us to determine the quantities \((n_s, r)\), once the value of the field \( x \) is fixed by requiring \( N = 50 - 60 \) e-folds. The nominal choice of \( N = 55 \) yields \( x = 5.35 \), \( n_s = 0.965 \), and \( r = 0.0035 \).

Fig. 2 displays the predictions for \((n_s, r)\) of the NSWZ model for five choices of the coupling \( \lambda \) that yield \( n_s \in [0.93, 1.00] \) and \( N \in [50, 60] \). The last 50-60 e-folds of inflation arise as \( x \) rolls to zero from \( \sim 5.1 - 5.8 \), the exact value depending on \( \lambda \) and \( N \). As one can see, the values of \( \lambda \) are constrained to be close to the critical value \( \mu/3 \), for which we find extremely good agreement with the Planck determination of \( n_s \). The values of \( r \) are rather small for \( \lambda = \mu/3 \), varying over the range 0.0012 - 0.0084, in the models considered.

At first sight, this success might appear to be another example of accidental inflation [13] but, as we now show, this choice of \( \lambda \) has a more profound geometric interpretation. The alert reader may have noticed resemblances of both the potential shown in Fig. 1 and the values of \((n_s, r)\) found for the \( \lambda = \mu/3 \) model with results for inflation in the \( R + R^2 \) model proposed by Starobinsky [4]. To probe further this resemblance, we examine the generalization of the Einstein-Hilbert action to contain an \( R^2 \) contribution, where \( R \) is the scalar curvature,
\[ S = \frac{1}{2} \int d^4 x \sqrt{-g} (R + R^2/6M^2), \] (15)
where \( M \ll M_P \) is some mass scale. This theory is conformally equivalent to canonical gravity plus a scalar field \( \varphi \) [26]. Making the transformation \( \tilde{g}_{\mu\nu} = (1+\varphi/3M^2)g_{\mu\nu} \) and the field redefinition \( \varphi' = \sqrt{2/3} \ln (1 + \varphi/3M^2) \), we obtain the action
\[ S = \frac{1}{2} \int d^4 x \sqrt{-\tilde{g}} \left[ \tilde{R} + \left( \partial_\mu \varphi' \right)^2 - \frac{3}{2} M^2 (1 - e^{-\sqrt{2/3} \varphi'})^2 \right], \] (16)
corresponding to a potential
\[ V = \frac{3}{4} M^2 (1 - e^{-\sqrt{2/3} \varphi'})^2. \] (17)
The potential (17) is identical with the potential (11) along the real direction of the NSWZ model! Moreover, we have the identification \( M^2 = \mu^2/3 \), which = \( \hat{\mu}^2 \) for \( c = (T + T^*) = 1 \). Thus the Starobinsky mass \( M \) is directly related to the NSWZ mass \( \hat{\mu} \) in the superpotential [2].

We have shown in this paper that the simplest SU(2,1)/U(1) no-scale supergravity model with a single
modulus field $T$ and a single matter field $\phi$ with the simplest renormalizable Wess-Zumino superpotential, identified with the inflaton, is capable of yielding cosmological inflation with values of the scalar spectral tilt $n_s$ and the tensor-to-scalar ratio $r$ within the region favoured by Planck and other data at the 68% CL. Successful inflation is obtained for $\lambda \approx \mu/3$ in Planck units. This NSWZ model is a proof of the existence of acceptable models of inflation based on no-scale supergravity, and normally we would not advocate that its details should necessarily be taken literally. For example, a realistic no-scale model derived from a generic compactification of string theory would have more moduli fields, with many matter fields that could be the inflaton, with a superpotential more complicated than assumed here.

However, it is truly striking that the NSWZ model is conformally equivalent to the Starobinsky $R^2$ model [4] for the specific choice $\lambda = \mu/3$ in Planck units. This correspondence suggests that there is a profound geometric interpretation of this model that remains to be understood.

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