Peccei-Quinn invariant extension of the NMSSM

Kwang Sik Jeong, Yutaro Shoji, Masahiro Yamaguchi

Department of Physics, Tohoku University, Sendai 980-8578, Japan
E-mail: ksjeong@tuhep.phys.tohoku.ac.jp, yshoji@tuhep.phys.tohoku.ac.jp, yama@tuhep.phys.tohoku.ac.jp

Abstract: We study a Peccei-Quinn invariant extension of the next-to-minimal supersymmetric Standard Model (NMSSM), which turns out to be free from the tadpole and domain wall problems. Having a non-renormalizable coupling to the axion superfield, the SM singlet added to the Higgs sector can naturally generate an effective Higgs $\mu$ term around the weak scale. In the model, the lightest neutralino is dominated by the singlino, which gets a mass only through mixing with the neutral Higgsinos. We explore the phenomenological consequences resulting from the existence of such a relatively light neutralino. The coupling of the SM singlet to the Higgs doublets is constrained by the experimental bound on the invisible $Z$-boson decay width. Under this constraint, we examine the properties of the SM-like Higgs boson paying attention to its mass and decays. We also demonstrate a UV completion of the model in SU(5) grand unified theory with a missing-partner mechanism.

Keywords: Supersymmetry breaking, Supersymmetric Standard Model
1. Introduction

The next-to-minimal supersymmetric Standard Model (NMSSM) introduces a SM singlet \( S \) to explain the origin of a supersymmetric Higgs \( \mu \) term of the MSSM \([1]\). However, if \( S \) is a true singlet under all symmetries, it becomes difficult to embed the NMSSM into a more fundamental theory such as a grand unified theory (GUT). This is because the GUT partners of the MSSM Higgs doublets also couple to \( S \) and radiatively generate a large tadpole for \( S \), destabilizing the gauge hierarchy \([2]\). Non-renormalizable interactions are another source of large tadpoles \([3]\). A symmetry under which \( S \) transforms non-trivially can solve the tadpole problem, but generally introduces another problem. If one considers a discrete symmetry, dangerous domain walls would be formed in the early universe \([4, 5]\). On the other hand, a global symmetry spontaneously broken by \( S \) and the Higgs doublets would give rise to an unacceptable visible axion. Additional structure is thus needed to generate \( \mu \) dynamically from the coupling of \( S \) to the Higgs doublets while providing a viable framework for the grand unification.

In this paper, we point out that the difficulties arising due to the singlet \( S \) can be avoided in a Peccei-Quinn invariant extension of the NMSSM (PQ-NMSSM) where the PQ symmetry is spontaneously broken at a scale much higher than the weak scale by the axion
superfield. The PQ symmetry forbids the generation of large tadpoles for $S$ while solving the strong CP problem. Furthermore, the domain wall problem can be resolved in the presence of PQ messengers that couple to the axion superfield. It also turns out that a non-renormalizable coupling of $S$ to the axion superfield naturally leads $S$ to get a vacuum expectation value around the weak scale.

The Higgs and neutralino sectors are considerably modified by the addition of $S$. In the PQ-NMSSM, the lightest neutralino consists mostly of the singlino because it acquires a small mass through mixing with the neutral Higgsinos after the electroweak symmetry breaking. The presence of such a relatively light neutralino leads to phenomenological consequences different from other NMSSM models. In particular, the LEP bound on the invisible $Z$-boson decay width places a stringent constrain on the coupling of $S$ to the Higgs doublets if the decay mode is kinematically allowed. This constraint becomes important at large $\tan \beta$. We also note that loops involving the Yukawa coupling of the singlino give an additional positive contribution to the SM-like Higgs mass. This contribution is insensitive to $\tan \beta$, and arises when the Higgsinos are lighter than other MSSM particles. Another interesting feature is that the decay of the SM-like Higgs boson into a pair of the lightest neutralino can be dominant at low $\tan \beta$.

Since interactions of $S$ are controlled by the PQ symmetry, it is possible to embed the PQ-NMSSM into GUT models without the tadpole problem. The associated UV completion is then related to the doublet-triplet splitting problem. We find that incorporating the PQ symmetry in a missing-partner model for supersymmetric SU(5) GUT can lead to the PQ-NMSSM at low energy scales. In addition, it naturally achieves a phenomenologically acceptable value of the axion decay constant as $F_a \sim \sqrt{M_{\text{SUSY}} M_{\text{Pl}}}$ with $M_{\text{SUSY}}$ being the SUSY breaking scale. The PQ symmetry is also important for suppressing harmful dimension 5 operators for proton decays.

This paper is organized as follows. In the next section, we present the model and discuss its general properties and various constraints on the singlet couplings. Then, in section 3, we construct a low energy effective theory below the SUSY breaking scale to examine the phenomenological aspects resulting from the existence of a singlino-like light neutralino. Section 4 is for the discussion on how to UV complete the model. The PQ-NMSSM can arise as a low energy theory of a missing-partner GUT model. Section 5 is the conclusion.

2. PQ-invariant extension of the NMSSM

In this section, we extend the NMSSM to incorporate the PQ symmetry and study the properties of the model. The PQ-invariant extension turns out not only to provide a solution to the strong CP problem but also to solve both the tadpole and domain wall problems. We also examine constraints on the coupling of $S$ to the Higgs doublets.

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$^1$Extensions of the MSSM with a SM singlet have been received revived attention after the first results on the Higgs search at the LHC had been announced.
2.1 Model

In the PQ-NMSSM, an effective Higgs $\mu$ term is generated by the vacuum expectation value of $S$ which couples to the Higgs doublets and to the axion superfield $X$ through the PQ-invariant interactions

$$\mathcal{L} = \int d^2 \theta \lambda S H_u H_d + \int d^4 \theta \frac{X^2}{M_{Pl}} S + \text{h.c.}, \quad (2.1)$$

for $(X, S, H_u, H_d)$ carrying the PQ charges as $(1, 2, -1, -1)$. All other terms involving $S$ are forbidden by the PQ symmetry in the renormalizable superpotential. A superpotential term $X^2 H_u H_d / M_{Pl}$, which is allowed by $U(1)_{PQ}$, can be removed by a holomorphic redefinition of $S$ without loss of generality. Here we have taken such a field basis.

In the following, we assume that a mechanism to stabilize $X$ is operative with its vacuum expectation value fixed at $10^{10-12}$ GeV as required by cosmological and astrophysical observations. As suppressed by $M_{Pl}$, the interactions between $X$ and other scalar fields rarely affect the saxion potential at $|S| \ll |X| \ll M_{Pl}$. The Higgs potential can thus be examined by replacing $X$ with its vacuum expectation value. Then, the involved mass parameters are determined by the SUSY breaking scale $M_{SUSY}$ and the axion decay constant $F_a \sim |X|$. We note that, when treating $X$ as a spurion field, the present model can be regarded as the nMSSM,\(^2\) where the superpotential contains an effective tadpole for $S$:

$$W_{\text{eff}} = \lambda S H_u H_d + m_0^2 (1 + \theta^2 B_\kappa) S, \quad (2.2)$$

with $B_\kappa \sim M_{SUSY}$ and\(^3\)

$$m_0^2 \sim \kappa M_{SUSY} \frac{F_a^2}{M_{Pl}}. \quad (2.3)$$

The appearance of $m_0^2$ and $B_\kappa$ terms can be understood by promoting $\kappa$ to a function depending on SUSY breaking fields in a hidden sector. It is obvious that the model does not suffer from the tadpole problem because a tadpole for $S$ requires a higher dimensional coupling of $S$ to $X$ as dictated by $U(1)_{PQ}$. For $F_a = 10^{10-12}$ GeV, $M_{SUSY} \sim 1$ TeV and $\kappa$ less than order unity, the value of $m_0$ can naturally be around the weak scale. Hence, it is natural to expect that electroweak symmetry breaking would occur at the correct scale.

In fact, the same spirit is shared with the Kim-Nilles mechanism\(^2\) that explains the smallness of $\mu$ in extensions of the MSSM with $U(1)_{PQ}$. Since $S$ and $H_{u,d}$ carry $U(1)_{PQ}$ charges and develop vacuum expectation values, the Higgs and neutralino sectors have small mixing with $X$ suppressed by $F_a$.

\(^2\)The nMSSM\(^1\)\(^6\)\(^7\)\(^8\)\(^9\) assumes specific discrete $R$ symmetries to ensure the absence of large tadpoles for $S$. A general discussion on the phenomenological aspects of the nMSSM for small $\tan \beta$ can be found in \(^11\)\(^12\)\(^13\)\(^14\). Some cosmological issues have also been discussed in \(^15\)\(^16\). Neglecting small mixing with the axion superfield, the Higgs and neutralino sectors of the PQ-NMSSM have the same phenomenological properties as the nMSSM. However, the cosmological properties can be different depending on the cosmological evolution of the saxion. A continuous symmetry to restrict couplings of $S$ in the NMSSM has been introduced in \(^20\), where it is explicitly broken only by a linear superpotential of $S$.\(^3\)
Let us now examine the vacuum structure of the low energy effective theory given by (2.2). Including soft SUSY breaking terms, the scalar potential of the extended Higgs sector reads

\[
V = \frac{1}{8}(g^2 + g'^2)(|H_u|^2 - |H_d|^2)^2 + \frac{1}{2}g^2|H_u \dagger H_d|^2 \\
+ |\lambda H_u H_d + m_0^2|^2 + |\lambda|^2|S|^2(|H_u|^2 + |H_d|^2) \\
+ m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + m_S^2|S|^2 + (A_\lambda \lambda S H_u H_d - B_\kappa m_0^2 S + \text{h.c.}),
\]

(2.4)

where \(m_0^2\) is a soft scalar mass squared, and \(A_\lambda\) is a soft \(A\)-parameter. The potential contains four complex parameters, \(\lambda, A_\lambda, m_0^2\) and \(B_\kappa\). Among them, \(\lambda\) and \(m_0^2\) can be made real and positive by a field redefinition of \(H_{u,d}\) and \(X\). Furthermore, if \(\arg(A_\lambda) = \arg(B_\kappa)\), one can rotate away the phases of \(A_\lambda\) and \(B_\kappa\) by redefining \(S\). We will assume this is the case, for which CP invariance is preserved in the Higgs sector and there is no mixing between scalar and pseudo-scalar fields. From the above scalar potential, it is straightforward to get the conditions for electroweak symmetry breaking. Similarly as in the MSSM, two of them can be written

\[
\frac{1}{2} M_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2, \\
\sin 2\beta = \frac{2b_{\text{eff}}}{m_{H_d}^2 + m_{H_u}^2 + 2\mu_{\text{eff}}^2 + \lambda^2 v^2},
\]

(2.5)

for \(\mu_{\text{eff}}\) and \(b_{\text{eff}}\) defined by

\[
\mu_{\text{eff}} = \lambda v_S, \quad b_{\text{eff}} = \lambda(A_\lambda v_S + m_0^2),
\]

(2.6)

where \(\langle |H_u^0| \rangle = v \sin \beta\) and \(\langle |H_d^0| \rangle = v \cos \beta\) with \(v = 174\) GeV. The value of \(|S|\) at the vacuum is fixed as

\[
v_S = \frac{A_\lambda \lambda v^2 \sin 2\beta + 2B_\kappa m_0^2}{2(m_S^2 + \lambda^2 v^2)}.
\]

(2.7)

The tree-level mass matrices for the scalar fields are presented in the appendix A.

To explore the global structure of the potential, one can substitute \(S\) by the solution of \(\partial_S V = 0\). Then, the Higgs potential (2.4) is written

\[
V = V|_{S=0} - \frac{|A_\lambda \lambda H_u H_d - B_\kappa m_0^2|^2}{m_S^2 + |\lambda|^2(|H_u|^2 + |H_d|^2)},
\]

(2.8)

which increases monotonically along the \(D\)-flat direction \(|H_u^0| = |H_d^0|\) when

\[
R_1 \geq 1 \text{ and } 3R_1 \geq 2 + R_2, \quad \text{or} \quad 1 \geq R_1^3 \geq R_2,
\]

(2.9)

where \(R_{1,2}\) are defined by \(R_1 m_S^2 = |(2\mu_{\text{eff}} - A_\lambda) m_S^2 + (2\mu_{\text{eff}} - A_\lambda \sin 2\beta) \lambda^2 v^2|^{2/3}\) and \(R_2 m_S^2 = (2\mu_{\text{eff}} - A_\lambda)^2 - 2(m_{H_u}^2 + m_{H_d}^2 + 2\mu_{\text{eff}}^2 - 2b_{\text{eff}}^2)\). If the above condition is not satisfied, the
potential may develop another minimum away from the weak scale.\footnote{Actually a minimum of the potential does not lie in the D-flat direction unless $m_{\tilde{\chi}_1}^2 \approx m_{\tilde{\chi}_1}^2$. However, as will be shown in the appendix, a minimum other than the electroweak vacuum, if exists, is located near the D-flat direction for much of the parameter space. This justifies our approach of examining the D-flat direction to see when there can be another minimum. See also\cite{24}, where the stability of the electroweak vacuum has been examined within the framework of the effective Lagrangian beyond the MSSM.} The involved soft parameters are then constrained by the requirement that the electroweak vacuum should be a global minimum. For $m_{\tilde{\chi}_1}^2 \sim M_{\text{SUSY}}^2 \gg \mu_{\text{eff}}^2$, which is the case we shall focus on, the stability condition \( (2.9) \) requires $A_\lambda^2 \lesssim m_\lambda^2$ or $m_\lambda^2 \lesssim A_\lambda^2 \lesssim m_A^2$ with $m_A$ being the mass of the CP-odd neutral Higgs boson. Keeping this in mind, we will consider also the case with $A_\lambda \sim M_{\text{SUSY}}$, which is favored to avoid large mixing of the SM-like Higgs scalar with the singlet scalar when $\mu_{\text{eff}}$ and $\tan \beta$ are large.

An important consequence of $U(1)_{\text{PQ}}$ is the appearance of a relatively light neutralino with a large singlino component. This is because the PQ symmetry prevents the singlino $\tilde{S}$ from having a supersymmetric mass. The lightest neutralino is mostly singlino if the masses of the bino $\tilde{B}$ and wino $\tilde{W}$ are larger than $\lambda v$, as is the case for $\lambda \lesssim 1$. The neutralino mass matrix for $(\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S})$ is given by

\[
\begin{pmatrix}
M_{\tilde{B}} & 0 & -M_Z \sin \theta_W \cos \beta & M_Z \sin \theta_W \sin \beta & 0 \\
0 & M_{\tilde{W}} & M_Z \cos \theta_W \cos \beta & -M_Z \cos \theta_W \sin \beta & 0 \\
-M_Z \sin \theta_W \cos \beta & M_Z \cos \theta_W \cos \beta & 0 & -\mu_{\text{eff}} & -\lambda v \sin \beta \\
M_Z \sin \theta_W \sin \beta & -M_Z \cos \theta_W \sin \beta & -\mu_{\text{eff}} & 0 & -\lambda v \cos \beta \\
0 & 0 & -\lambda v \sin \beta & -\lambda v \cos \beta & 0
\end{pmatrix},
\]

(2.10)

where $\theta_W$ is the weak mixing angle. If we write the lightest neutralino as a linear combination of $(\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S})$:

\[
\tilde{\chi}_1^0 = N_{\tilde{B}}^1 \tilde{B} + N_{\tilde{W}}^1 \tilde{W}^0 + N_{\tilde{H}_d}^0 \tilde{H}_d^0 + N_{\tilde{H}_u}^0 \tilde{H}_u^0 + N_{\tilde{S}}^0 \tilde{S},
\]

(2.11)

then we find

\[
N_{\tilde{S}}^0 = 1 - \frac{1}{2}(1 + \cdots) \epsilon_{\tilde{H}}^2,
\]

\[
N_{\tilde{W}}^0 = -\epsilon_{\tilde{H}}^2 \sin \beta + \cdots,
\]

\[
N_{\tilde{H}_d}^0 = -\epsilon_{\tilde{H}}^2 \cos \beta + \cdots,
\]

\[
N_{\tilde{B}}^1 = -\epsilon_{\tilde{H}}^2 \epsilon_{\tilde{B}}^2 (1 + \cdots),
\]

\[
N_{\tilde{H}_u}^0 = \epsilon_{\tilde{H}}^2 \epsilon_{\tilde{W}}^2 (1 + \cdots),
\]

(2.12)

for $\epsilon_{\tilde{H}}^2 \ll 1$ and $|\epsilon_{\tilde{B}}, \tilde{W}| \ll 1$. Here the epsilon parameters are defined by

\[
\epsilon_{\tilde{H}} \equiv \frac{\lambda v}{\mu_{\text{eff}}}, \quad \epsilon_{\tilde{B}} \equiv \frac{g v \cos 2 \beta}{\sqrt{2} M_{\tilde{B}}}, \quad \epsilon_{\tilde{W}} \equiv \frac{g v \cos 2 \beta}{\sqrt{2} M_{\tilde{W}}},
\]

(2.13)

and the ellipsis indicates terms of higher orders in $\epsilon_{\tilde{H}}^2$ or $\epsilon_{\tilde{B}}, \tilde{W}$. One can see that $\epsilon_{\tilde{H}}^2 = 0$ is needed to make $\chi_1^0$ massive through mixing. In the following discussion, we will neglect small gaugino components of $\chi_1^0$ since it does not change our results substantially. Then, one can find

\[
m_{\chi_1^0} \simeq 2 \left( \mu_{\text{eff}} N_{\tilde{H}_d}^0 N_{\tilde{H}_u}^0 + \lambda v N_{\tilde{H}_d}^0 N_{\tilde{H}_u}^0 \cos \beta + \lambda v N_{\tilde{H}_d}^0 N_{\tilde{H}_u}^0 \sin \beta \right)
\]

\[
= \frac{\lambda^2 v^2}{\mu_{\text{eff}}} \sin 2 \beta \left( 1 - \frac{\lambda^2 v^2}{\mu_{\text{eff}}} + O \left( \frac{\lambda^4 v^4}{\mu_{\text{eff}}^4} \right) \right),
\]

(2.14)
As we will discuss later, the singlino-like neutralino with a small mass can considerably change the phenomenological properties of the model.

Meanwhile, there can exist PQ messengers $\Psi + \bar{\Psi}$ which are vector-like under the SM gauge group and obtain heavy masses from the coupling $X\Psi\bar{\Psi}$ in the superpotential. Such interaction can play an important role in the saxion stabilization because it induces a radiative potential for the saxion after SUSY breaking. The presence of PQ messengers also helps to avoid the domain wall problem. Let us consider $N_\Psi$ pairs of $\Psi + \bar{\Psi}$ forming $5 + \bar{5}$ representation under SU(5), for which the gauge coupling unification is preserved. Then, the domain wall number is given by

$$N_{\text{DW}} = |N_\Psi - 6|. \quad (2.15)$$

This implies that the domain wall problem can be resolved for $N_\Psi = 5, 7$. For other cases with $N_{\text{DW}} \neq 1$, the formation of dangerous domain walls can still be avoided if the saxion is displaced far from the origin after the inflation ends so that the PQ symmetry is not restored at high temperatures [23].

### 2.2 Constraints on the model parameters

Since $S$ modifies the Higgs and neutralino sectors, it is of importance to explore constraints on the singlet couplings $\lambda, A_\lambda, B_\kappa$ and $m_S^2$. Here we focus on the case with $0.1 \lesssim \lambda \lesssim 1$ at the weak scale as would be natural because an effective $\mu$ term is generated as $\mu_{\text{eff}} = \lambda S$ with $S$ fixed around $M_{\text{SUSY}}$. Let us first examine the mixing of the singlet scalar with the Higgs doublets. After taking the rotation of $(H_u^0, H_d^0)$ by an angle $\beta$, the mass matrix for the CP-even scalar fields has

$$\begin{align*}
(M_H^2)_{11} &= M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta, \\
(M_H^2)_{13} &= \lambda v (2\mu_{\text{eff}} - A_\lambda \sin 2\beta), \\
(M_H^2)_{33} &= m_S^2 + \lambda^2 v^2,
\end{align*} \quad (2.16)$$

where $(M_H^2)_{11}$ constitutes an upper bound on the mass of the lightest CP-even Higgs boson at the tree-level. In the following, we would like to consider the situation that the SM-like Higgs boson has negligible contamination from the singlet scalar. For $0.1 \lesssim \lambda \lesssim 1$, this is achieved when

$$\frac{\mu_{\text{eff}}^2}{m_S^2} \left| 1 - \frac{A_\lambda \sin 2\beta}{2\mu_{\text{eff}}} \right| \ll 1, \quad (2.17)$$

with $m_S^2$ being of the order of $M_{\text{SUSY}}^2$. It is thus found that, if $A_\lambda$ is larger than $2\mu_{\text{eff}}$, the mixing can get a sizable suppression at some region of $\tan \beta$. One would otherwise need $\mu_{\text{eff}}^2 \ll m_S^2$ to suppress the mixing.

In the PQ-NMSSM, a stringent constraint on $\lambda$ comes from the experimental bound on the $Z$-boson invisible decay rate because the PQ symmetry makes $\tilde{\chi}_1^0$ light. The singlino mixes with neutral Higgsinos to induce the interaction $\tilde{\chi}_1^0 \sigma^\mu \tilde{\chi}_1^0 Z_\mu$ [24], through which $Z$...
can invisibly decay into pairs of the lightest neutralino. The coupling for this interaction is given by

\[ g_{N_1}^2 = \frac{g}{2 \cos \theta_W} \left( |H_u^0|^2 - |H_d^0|^2 \right) \approx \frac{g}{2 \cos \theta_W} \frac{\lambda^2 v^2 \cos 2\beta}{\mu_{\text{eff}}^2}, \]  

(2.18)

where the last approximation is valid for small \( \lambda v / \mu_{\text{eff}} \). Hence, at large \( \tan \beta \), the interaction gets strong while the mass of \( \tilde{\chi}_1^0 \) becomes small. The above coupling mediates the \( Z \) decay into \( \tilde{\chi}_1^0 \) with

\[ \Gamma_{Z \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0} = \frac{g_{Z_1}^2}{24 \pi} M_Z^3 \approx 25 \beta_{\text{eff}}^2 \left( \frac{\lambda}{0.8} \right)^4 \left( \frac{\cos 2\beta}{0.8} \right)^2 \left( \frac{200 \text{ GeV}}{\mu_{\text{eff}}} \right)^4 \text{MeV}, \]  

(2.19)

if \( M_Z > 2m_{\tilde{\chi}_1^0} \). Here \( \beta_Z = (1 - 4m_{\tilde{\chi}_1^0}^2 / M_Z^2)^{1/2} \) is the velocity of \( \tilde{\chi}_1^0 \) in the rest frame of \( Z \). The process \( Z \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \) contributes to the invisible \( Z \) decay and is tightly constrained by the LEP data to occur with a small rate, \( \Gamma_{Z \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0} \lesssim 2 \text{ MeV} \). This translates into

\[ \lambda \lesssim 0.4 \left( \frac{\mu_{\text{eff}}}{200 \text{ GeV}} \right) \left( \frac{0.8}{\cos 2\beta} \right)^{1/2}. \]  

(2.20)

The above constraint on \( \lambda \) around the weak scale becomes important for large values of \( \tan \beta \). To kinematically forbid the mode \( Z \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \), we need

\[ \lambda \gtrsim 0.7 \left( \frac{\mu_{\text{eff}}}{200 \text{ GeV}} \right)^{1/2} \left( \frac{0.6}{\sin 2\beta} \right)^{1/2}, \]  

(2.21)

which is possible for \( \lambda \lesssim 1 \) at low \( \tan \beta \). For instance, at small \( \tan \beta \) around 2, the LEP limits on the \( Z \) invisible width exclude values of \( \mu_{\text{eff}} \) in the range between 196 GeV and 260 GeV for \( \lambda \approx 0.6 \) \cite{26}.

On the other hand, for the theory to remain perturbative up to \( M_{\text{GUT}} \), \( \lambda \) should be small enough at the weak scale. NMSSM models with \( \lambda S H_u H_d \) usually require \( \lambda \) less than 0.7 – 0.8 for \( \tan \beta \gtrsim 2 \). In models with a superpotential term \( S^3 \), the upper bound on \( \lambda \) decreases as the coupling for \( S^3 \) increases. However, the situation in the PQ-NMSSM is different because \( S^3 \) is absent and the PQ messengers with mass \( M_{\Phi} \propto |X| \) affect the running of gauge couplings. Above the scale \( M_{\text{GUT}} \), gauge couplings have larger values than in the MSSM and slow down the running of Yukawa couplings. This results in the increase of the perturbativity bound on \( \lambda \) by \( \delta \lambda \lesssim 0.1 \) \cite{20}. A large number of light PQ messengers are favored by raising the bound, but disfavored by the requirement of the perturbativity of gauge couplings up to \( M_{\text{GUT}} \). If there exists an extra gauge interaction, the perturbation theory would be valid below \( M_{\text{GUT}} \) for a larger value of \( \lambda \) at the weak scale.

Though we do not discuss it here in detail, there is also a constraint placed by cosmology. If \( \tilde{\chi}_1^0 \) is the lightest sparticle, its relic abundance should not exceed the measured amount of the dark matter. In the present model, the production of dark matter relies on the cosmological evolution of the saxion, which has a very flat potential generated after SUSY breaking and thus can play some non-trivial role in cosmology. On the other hand, the gravitino or axino can be lighter than \( \tilde{\chi}_1^0 \) depending on the mediation mechanism of SUSY breaking and on how the saxion is stabilized.
3. Low energy Higgs sector

In this section, we study the low energy Higgs sector. To see the impact of the PQ-NMSSM specific Higgs properties, we consider the decoupling limit of the MSSM where all heavy Higgs states decouple below $M_{\text{SUSY}}$ and thus one combination of $H_{u,d}$ behaves exactly like the SM Higgs scalar $H$. In such a situation, we include the singlet $S$ and construct a low energy effective theory below $M_{\text{SUSY}}$ to examine how much the model departs from the MSSM. The modification is mainly due to (i) the extra contribution to the Higgs quartic coupling, which is a general property of NMSSM models, and (ii) the presence of a light neutralino that is singlino-like, which is a consequence of the PQ symmetry.

3.1 Effective theory below the SUSY breaking scale

For $0.1 \lesssim \lambda \lesssim 1$ at the weak scale, $\mu_{\text{eff}}^2 \ll m_S^2$ is favored to suppress the mixing between $H$ and the singlet scalar. Here we consider such a case and assume that the MSSM sparticles other than Higgsinos obtain masses of the order of $M_{\text{SUSY}}$. The singlet scalar is also assumed to have $m_S^2 \sim M_{\text{SUSY}}^2$. For $\mu_{\text{eff}}$ less than $M_{\text{SUSY}}$, the low energy effective theory below $M_{\text{SUSY}}$ contains $\tilde{H}_{u,d}$ and $\tilde{S}$ in addition to the ordinary SM particles. The Lagrangian relevant to our analysis is given by

$$-L_{\text{eff}}^{\text{scalar}} = \frac{\lambda_H}{2} (|H|^2 - v^2)^2 + (y_t Q_L H^c + \mu_{\text{eff}} \tilde{H}_u \tilde{H}_d + y'_u H \tilde{H}_u \tilde{S} + y'_d H^c \tilde{H}_d \tilde{S} + \text{h.c.}),$$

where $H^c = -i \sigma_2 H^*$, and $y_t$ is the top-Yukawa coupling. The singlino Yukawa couplings at the SUSY breaking scale are

$$y'_u(M_{\text{SUSY}}) = \lambda \cos \beta, \quad y'_d(M_{\text{SUSY}}) = \lambda \sin \beta,$$

while the Higgs quartic coupling is given by

$$\lambda_H(M_{\text{SUSY}}) = \frac{g^2 + g'^2}{4} \cos^2 \beta + \frac{\lambda^2}{2} \sin^2 2\beta + \delta \lambda_H |_{\text{tree}} + \delta \lambda_H |_{\text{loop}},$$

where $\delta \lambda_H |_{\text{tree}}$ is the threshold correction coming from tree-level exchange of the singlet scalar, and $\delta \lambda_H |_{\text{loop}}$ is from the loops involving the stops:

$$\delta \lambda_H |_{\text{tree}} \simeq -\frac{\lambda^2 (2 \mu_{\text{eff}} - A_\lambda \sin 2\beta)^2}{2 m_S^2},$$

$$\delta \lambda_H |_{\text{loop}} \simeq \frac{3 y_t^4}{8 \pi^2} \left( X_t - \frac{X_t^2}{12} \right),$$

where $X_t$ is the stop mass.

4To obtain the couplings of the SM-like Higgs boson more precisely, one needs to know the mixing between the SM-like Higgs boson and singlet scalar. To this end, one can replace the scalar part of (3.1) by

$$-L_{\text{eff}} |_{\text{scalar}} = \frac{\lambda_H - \delta \lambda_H |_{\text{tree}}}{2} (|H|^2 - v^2)^2 + (m_S^2 + \lambda^2 v^2)|S - v_S|^2$$

$$- \left\{ \frac{\lambda (2 \mu_{\text{eff}} - A_\lambda \sin 2\beta)}{2} (|H|^2 - v^2)(S - v_S) + \text{c.c.} \right\},$$

and $\mu_{\text{eff}}$ by $\lambda S$. The small mixing with the singlet scalar reduces the couplings of the Higgs boson $h$. The reduced couplings can be obtained by taking the replacement

$$h \rightarrow \left( 1 - \frac{\lambda v |2 \mu_{\text{eff}} - A_\lambda \sin 2\beta|}{m_S^2} \right) h.$$
where \( X_t = (A_t - \mu_{\text{eff}} \cot \beta)^2 / M_{\text{SUSY}}^2 \) with \( A_t \) being the \( A \)-parameter for \( H_u \bar{t}_R \bar{Q}_L \).

The physical mass of the CP-even neutral Higgs boson \( h \) can be obtained using the relation \( m_h^2 = 2 \lambda H v^2 \). For this, we need \( \lambda \) renormalized at the weak scale. In the effective theory, a low energy value of \( \lambda_H \) is determined by the renormalization group (RG) running equation:

\[
\mu \frac{d \lambda_H}{d \mu} = \frac{1}{16 \pi^2} \left( 12 \lambda_H^2 + 4(3 y_t^2 + y_u^2 + y_d^2 - 3 A) \lambda_H + 3 B - 12 y_t^4 - 4(y_u^2 + y_d^2)^2 \right),
\]

with the parameters \( A \) and \( B \) defined by

\[
4A = 3g^2 + g'^2, \quad 4B = 3g^4 + 2g^2g' + g'^4.
\]

Here one should note that the mixing between the neutral Higgs boson and singlet scalar would slightly modify the running equations.

To see the qualitative properties of the Higgs mass, we make an approximation taking into account that the dominant effects on the RG running come from the term \( y_t^4 \), and also from the terms \( y_{u,d}^4 \) if \( \lambda \) is not small. The Higgs boson mass is found to be approximately given by

\[
m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3m_t^2}{4\pi^2 v^2} \left( \ln \left( \frac{M_{\text{SUSY}}^2}{m_t^2} \right) + X_t - \frac{X_t^2}{12} \right) + M_Z^2 \frac{2\lambda^2}{g^2 + g'^2} \left( \sin^2 2\beta \left( \frac{2\mu_{\text{eff}} - A_\lambda \sin 2\beta}{m_S^2} \right)^2 + \frac{\lambda}{4\pi^2} \ln \left( \frac{M_{\text{SUSY}}^2}{\mu_{\text{eff}}^2} \right) \right),
\]

for \( m_t = y_t v \) and \( \mu_{\text{eff}} \gg m_h \). The first line is the well-known result for the Higgs boson mass in the MSSM \([27, 28]\). On the other hand, those in the second line correspond to the additional contributions arising due to \( S \), i.e. as a consequence of the extra Higgs quartic coupling \( \lambda^2|H_u H_d|^2 \), the mixing between the singlet scalar and neutral Higgs boson, and the singlino Yukawa interactions affecting the running of the Higgs quartic coupling at low energy scales. The last two contributions are approximately estimated as

\[
\delta m_h |_{\text{mix}} \approx -10 \left( \frac{130 \text{GeV}}{m_h} \right) \left( \frac{30}{m_S^2/\mu_{\text{eff}}^2} \right) \left( \frac{1 - A_\lambda \sin 2\beta}{2\mu_{\text{eff}}} \right)^2 \left( \frac{\lambda}{0.8} \right)^2 \text{GeV}, \quad (3.9)
\]

\[
\delta m_h |_{\text{rad}} \approx 4.1 \left( \frac{130 \text{GeV}}{m_h} \right) \left( \frac{\ln(M_{\text{SUSY}}^2/\mu_{\text{eff}}^2)}{\ln 30} \right) \left( \frac{\lambda}{0.8} \right)^4 \text{GeV}. \quad (3.10)
\]

A negative contribution to \( m_h \) from the mixing with the singlet scalar is present in any NMSSM model. In the PQ-NMSSM, \( \mu_{\text{eff}}^2 \ll m_S^2 \) leads to a large suppression of this effect for \( A_\lambda \lesssim \mu_{\text{eff}} \). For \( A_\lambda \gg 2\mu_{\text{eff}} \), small mixing can still be obtained at some values of \( \tan \beta \). One should also note that there is a PQ-NMSSM specific contribution \( \delta m_h |_{\text{rad}} \) arising because the PQ symmetry makes the lightest neutralino get a relatively small mass.\(^5\) This positive contribution is insensitive to \( \tan \beta \), and becomes important for a small value of \( \mu_{\text{eff}}/M_{\text{SUSY}} \) contrary to \( \delta m_h |_{\text{mix}} \).

\(^5\)See also \([10]\) for a similar discussion in a singlet extension of the MSSM having a relatively light neutralino. However, in our situation, the LEP bound on the invisible Z-boson decay width excludes large values of \( \lambda \) at large \( \tan \beta \).
Figure 1: The mass of the SM-like Higgs boson for $M_{\text{SUSY}} = 1.5$ TeV in the PQ-NMSSM. Here $\lambda$ is taken to be the maximum value satisfying the bound $\Gamma_{Z \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0} \lesssim 2$ MeV. The solid curve in the left panel is the upper bound on $m_h$ for $\mu_{\text{eff}} = 240$ GeV (green) and $\mu_{\text{eff}} = 420$ GeV (blue), which is obtained for no mixing case $\delta m_h|_{\text{mix}} = 0$. The gray line is the MSSM value of $m_h$. In the left panel, the dashed lines are for $X_t = 0$ while the solid ones for $X_t = 6$. Meanwhile, the right panel shows $m_h$ for $X_t = 6$, $\mu_{\text{eff}} = 420$ GeV and a given value of $A_\lambda$: the black curve is for $A_\lambda = 1.2 M_{\text{SUSY}}$ while the red one for $A_\lambda = 2.4 M_{\text{SUSY}}$.

In Fig. 1 we show the upper bound on $m_h$ in the PQ-NMSSM, which is obtained taking the maximum value of $\lambda$ allowed by the constraint (2.20) from the $Z$ invisible decay. Here we have solved the RG equation to get $\lambda_H$ at the weak scale, and restricted $\lambda$ to be less than unity as would be necessary to maintain its perturbativity up to $M_{\text{GUT}}$. In the left panel, the value of $m_h$ is shown for $\delta m_h|_{\text{mix}} = 0$. While the MSSM generates $m_h \simeq 115$ GeV (128 GeV) at $\tan \beta \gtrsim 10$ for $X_t = 0$ (6) and $M_{\text{SUSY}} = 1.5$ TeV, the additional contribution from $\lambda$ can lead to $m_h$ larger than 115 GeV also at low $\tan \beta$ as in other NMSSM models. The loops of stops involving $A_t$ can further increase $m_h$. The maximum comes at $X_t = 6$. It is also important to note that, when $\mu_{\text{eff}} \gtrsim 400$ GeV, $m_h$ can be raised by a few GeV from the MSSM value even at large $\tan \beta$ owing to the PQ-NMSSM specific contribution $\delta m_h|_{\text{rad}}$. On the other hand, the right panel shows the value of $m_h$ for a given value of $A_\lambda$. For $A_\lambda \gtrsim 2 \mu_{\text{eff}}$, the mixing effect is suppressed only at some limited region of $\tan \beta$. In the figure, we consider values of $\tan \beta$ giving $(2 \mu_{\text{eff}} - A_\lambda \sin 2\beta)^2/m_S^2$ less than 0.1.

3.2 Phenomenological aspects

Since there appears a light neutralino as a consequence of the PQ symmetry, the Higgs boson $h$ can invisibly decay into pairs of the lightest neutralino. This process is mediated by the Yukawa interaction $h\tilde{\chi}_1^0\tilde{\chi}_1^0$, which is generated due to the mixing between $\tilde{S}$ and $\tilde{H}_{u,d}^0$, and has a coupling given by

$$y_{h\tilde{\chi}_1^0\tilde{\chi}_1^0} = -\sqrt{2}\lambda \left( N_1^u \tilde{H}_{u}^0 \sin \beta + N_1^d \tilde{H}_{d}^0 \cos \beta \right) \approx \frac{\sqrt{2}\lambda^2 v \sin 2\beta}{\mu_{\text{eff}}}, \quad (3.11)$$

where the approximation is valid for small $\lambda v/\mu_{\text{eff}}$. The above coupling becomes negligible at large $\tan \beta$. If dominates, such non-standard invisible decay would make the Higgs
discovery at hadron colliders much more difficult.

Recent LHC data have excluded the Higgs boson with SM properties in the mass range between 141 GeV and 476 GeV at the 95% confidence level \[23\]. For \( h \) with mass lighter than 141 GeV, the main processes for its decay are \( h \to bb \) and \( h \to WW^* \), \( ZZ^* \) \[30\]. The Higgs boson \( h \) in the PQ-NMSSM, which would have a small singlet component for \( \mu_{\text{eff}}^2/m_h^2 \ll 1 \), can decay through a non-standard mode \( h \to \tilde{\chi}_1^0 \tilde{\chi}_1^0 \) \[17\]. If it is kinematically accessible, the process \( h \to \tilde{\chi}_1^0 \tilde{\chi}_1^0 \) takes place with the relative decay strength

\[
\frac{\Gamma_{h \to \tilde{\chi}_1^0 \tilde{\chi}_1^0}}{\Gamma_{h \to bb}} \approx \frac{1}{3} \left( \frac{y_{h \tilde{\chi}_1^0 \tilde{\chi}_1^0}}{m_h/v} \right)^2 \approx 128 \left( \frac{\lambda}{0.8} \right)^4 \left( \frac{\sin 2\beta}{0.6} \right)^2 \left( \frac{200\text{GeV}}{\mu_{\text{eff}}} \right)^2, \tag{3.12}
\]

\[
\frac{\Gamma_{h \to \tilde{\chi}_1^0 \tilde{\chi}_1^0}}{3\Gamma_{h \to WW^*}} \approx 32\pi^2 \left( \frac{y_{h \tilde{\chi}_1^0 \tilde{\chi}_1^0}}{g^2} \right)^2 \approx 440 \left( \frac{0.3}{R(x)} \right) \left( \frac{\lambda}{0.8} \right)^4 \left( \frac{\sin 2\beta}{0.6} \right)^2 \left( \frac{200\text{GeV}}{\mu_{\text{eff}}} \right)^2, \tag{3.13}
\]

where we have ignored the masses of the final states, and \( R(x) \) is defined by

\[
R(x) = \frac{3(1 - 8x + 20x^2)}{(4x - 1)^{1/2}} \arccos \left( \frac{3x - 1}{2x^{3/2}} \right) - \frac{1 - x}{2x} (2 - 13x + 47x^2) - \frac{3}{2} (1 - 6x + 4x^2) \ln x, \tag{3.14}
\]

with \( x = M_W^2/m_h^2 \). The decay rate for the process \( h \to ZZ^* \) is similar to \( \Gamma_{h \to WW^*} \). When \( h \) has a sizable singlet component, the Higgs decay width for each process is modified, but the ratio between decay widths remains the same up to small correction arising due to that \( y_{h\tilde{\chi}_1^0\tilde{\chi}_1^0} \) receives contribution not only from \( H^0\tilde{H}_u^0\tilde{S} \) but also from \( S\tilde{H}_d^0\tilde{H}_d^0 \). The Higgs invisible decay to neutralinos would not dominate the SM decay processes either if \( \lambda \) is strong enough to make \( \tilde{\chi}_1^0 \) heavier than \( m_h/2 \):

\[
\lambda \gtrsim 0.85 \left( \frac{m_h}{130\text{GeV}} \right)^{1/2} \left( \frac{\mu_{\text{eff}}}{200\text{GeV}} \right)^{1/2} \left( \frac{0.6}{\sin 2\beta} \right)^{1/2}, \tag{3.15}
\]

or if \( \lambda \) is small enough to suppress the Yukawa coupling of \( \tilde{\chi}_1^0 \) to the Higgs boson: \( \text{Br}(h \to \tilde{\chi}_1^0\tilde{\chi}_1^0) \) is less than 0.5 for

\[
\lambda \lesssim 0.27 \left( \frac{\mu_{\text{eff}}}{200\text{GeV}} \right)^{1/2} \left( \frac{0.6}{\sin 2\beta} \right)^{1/2}. \tag{3.16}
\]

Here we have naively estimated the value of \( \lambda \) required for \( m_{\tilde{\chi}_1^0} > m_h/2 \) by taking the leading term in \( (2.14) \), which is expanded in powers of \( \lambda^2 v^2/\mu_{\text{eff}}^2 \). It is interesting to see that, in a low \( \tan \beta \) region, the Higgs boson decays mainly through the invisible channel \( h \to \tilde{\chi}_1^0\tilde{\chi}_1^0 \) for \( \lambda \gtrsim 0.4 \) and \( \mu_{\text{eff}} \lesssim 400 \text{ GeV} \). A large \( \mu_{\text{eff}} \) can weaken this decay mode, but would lead to large mixing between \( H^0 \) and \( S \). On the other hand, for \( \tan \beta \gtrsim 10 \), the constraint from the invisible \( Z \)-boson decay \( (2.20) \) requires \( \lambda \lesssim 0.36 \times (\mu_{\text{eff}}/200\text{GeV}) \). Thus, in this case, \( \text{Br}(h \to \tilde{\chi}_1^0\tilde{\chi}_1^0) \) cannot be larger than 0.5 if \( \mu_{\text{eff}} \) is smaller than about 360 GeV.

Fig. 2 shows the branching ratio for the invisible Higgs decay \( h \to \tilde{\chi}_1^0\tilde{\chi}_1^0 \) in the \((\tan \beta, \lambda)\) plane. In the figure, the yellow region is excluded by the experimental bound \( \Gamma_{\tilde{\chi}_1^0\tilde{\chi}_1^0} \approx 2 \)}
Figure 2: The branching ratio of non-standard mode $h \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ for $\mu_{\text{eff}} = 240$ GeV (left) and $\mu_{\text{eff}} = 420$ GeV (right). The dashed red line is the contour for $\text{Br}(h \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0)$. We also show a contour plot for $m_{\tilde{\chi}_1^0}$ larger than $M_Z/2$, which is given in blue. The yellow region is excluded by the experimental bound on the invisible Z decay width. Meanwhile, the black contour shows the value of $m_h$ obtained for $\delta m_{h|\text{mix}} = 0$ in the case with $M_{\text{SUSY}} = 1.5$ TeV and $X_t = 6$. The Higgs boson has a mass larger than 115 GeV above the dashed black line. In the figure, masses are given in the GeV unit.

Figure 3: The parameter region where $Z \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ is kinematically forbidden. The blue contour shows the value of $m_{\tilde{\chi}_1^0}$ for $\mu_{\text{eff}} = 120$ GeV. We also show the value of $m_h$ by a black contour for the case with $M_{\text{SUSY}} = 1$ TeV, $X_t = 6$ (left) and $X_t = 0$ (right). The masses are given in the GeV unit. The dashed red line is a contour for $\text{Br}(h \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0)$. Above the solid red line, the lightest neutralino obtains a mass larger than $m_h/2$. Here we have taken $\mu_{\text{eff}} = 240$ GeV for the left plot, and $\mu_{\text{eff}} = 420$ GeV for the right plot. Since there is an extra contribution to $m_h$ from $\lambda$ as (3.8), the Higgs mass can be raised above 115 GeV at low $\tan \beta$. Notice also that the contribution $\delta m_{h|\text{rad}}$ raises $m_h$ by a few GeV even at $\tan \beta \gtrsim 10$ compared to the MSSM value that corresponds to the $\lambda = 0$ case. The process $h \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ can be the dominant mode of the Higgs decay at $\tan \beta \lesssim 10$, but is suppressed at large $\tan \beta$. 
On the other hand, for small $\mu_{\text{eff}}$ less than $\lambda v$, the lightest neutralino is a sizable mixture of $\tilde{S}$ and $\tilde{H}^0_{u,d}$ at low $\tan \beta$. In this case, it can acquire a mass larger than $M_Z/2$ so that the invisible decay $Z \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$ is kinematically forbidden. For some parameter region, it is possible for $\tilde{\chi}_1^0$ to get a mass even larger than $m_h/2$. Otherwise, $h$ would decay dominantly through the invisible process $h \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$ at low $\tan \beta$ because $\tilde{\chi}_1^0$ has a sizable Higgsino component. One should also note that the contribution from $\lambda$ can raise $m_h$ well above 115 GeV for $M_{\text{SUSY}} \lesssim 1$ TeV at $\tan \beta \lesssim 3$. In Fig. 3, we show the region of $\lambda$ and $\tan \beta$ where $\tilde{\chi}_1^0$ has a mass larger than $M_Z/2$. The Higgs boson mass is also shown for the case with $M_{\text{SUSY}} = 1$ TeV.

4. UV completion

When one considers GUT models to UV complete the PQ-NMSSM, an important issue is how the GUT partners of the MSSM Higgs doublets, which also carry a PQ charge, acquire heavy masses. We point out that the PQ-NMSSM can emerge as a low energy effective theory of a missing-partner model for supersymmetric SU(5) GUT. The missing-partner model has been considered to explain a large mass splitting of the SU(2) doublet partners, and also three SU(5) singlets: $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$. The idea is to introduce higher dimensional representations that contain Higgs triplets but no doublets. Then, if a mass term $\lambda v$ is kinematically forbidden. For some parameter region, it is possible for $\tilde{\chi}_1^0$ to get a mass even larger than $m_h/2$. Otherwise, $h$ would decay dominantly through the invisible process $h \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$ at low $\tan \beta$ because $\tilde{\chi}_1^0$ has a sizable Higgsino component. One should also note that the contribution from $\lambda$ can raise $m_h$ well above 115 GeV for $M_{\text{SUSY}} \lesssim 1$ TeV at $\tan \beta \lesssim 3$. In Fig. 3, we show the region of $\lambda$ and $\tan \beta$ where $\tilde{\chi}_1^0$ has a mass larger than $M_Z/2$. The Higgs boson mass is also shown for the case with $M_{\text{SUSY}} = 1$ TeV.

4. UV completion

When one considers GUT models to UV complete the PQ-NMSSM, an important issue is how the GUT partners of the MSSM Higgs doublets, which also carry a PQ charge, acquire heavy masses. We point out that the PQ-NMSSM can emerge as a low energy effective theory of a missing-partner model for supersymmetric SU(5) GUT. The missing-partner model has been considered to explain a large mass splitting of the SU(2) doublet and color triplet Higgses. The idea is to introduce higher dimensional representations that contain Higgs triplets but no doublets. Then, if a mass term $H_5 \bar{H}_5$ for $5 + \bar{5}$ Higgs multiplets is absent in the superpotential, the doublet-triplet splitting can be achieved without fine-tuning from the interactions of $H_5$ and $\bar{H}_5$ with the higher dimensional Higgs multiplets. This is possible because the superpotential is not renormalized in perturbation theory.

The Higgs sector of the original model consists of the chiral multiplets, $\Sigma(75)$, $\theta(50) + \bar{\theta}(\bar{50})$ and $H(5) + \bar{H}(\bar{5})$. To incorporate the PQ symmetry without spoiling the missing-partner mechanism, we modify the model by introducing three pairs of $50 + \bar{50}$ and $5 + \bar{5}$ multiplets, and also three SU(5) singlets:

$$
\Sigma(75), \quad \theta_i(50) + \bar{\theta}_i(\bar{50}), \quad H_i(5) + \bar{H}_i(\bar{5}), \quad X_i(1),
$$

where $i = (1, 2, 3)$, and $U(1)_{\text{PQ}}$ charges are assigned as

$$
\Sigma(0), \quad X_1(q), \quad X_2(-3q), \quad X_3(2q),
$$

$$
\theta_1(-p) + \bar{\theta}_1(p), \quad \theta_2(-3q - p) + \bar{\theta}_2(3q + p), \quad \theta_3(-q - p) + \bar{\theta}_3(q + p),
$$

$$
H_1(p) + \bar{H}_1(-q - p), \quad H_2(3q + p) + \bar{H}_2(-p), \quad H_3(q + p) + \bar{H}_3(-3q - p).
$$

This model seems similar to the minimal model with $U(1)_{\text{PQ}}$ considered in [5], but it turns out that more than two pairs of chiral multiplets are needed to obtain the PQ-NMSSM as

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6A 5 dimensional SU(5) unified theory can also yield the PQ-NMSSM when compactified on $S^1/(Z_2 \times Z_2')$ orbifold [11, 12]. For instance, one can introduce a pair of Higgs hypermultiplet $H + \bar{H}$ which form $5 + \bar{5}$ representation of SU(5) and carry a PQ charge $-1$. Then, doublet-triplet splitting is achieved taking the orbifold projection such that $H$ (\bar{H}) has a SU(2) doublet Higgs chiral multiplet transforming as $(+,-)$ under $Z_2 \times Z_2'$ and a triplet Higgs with $(+,-)$. This also leads to the terms [2] for the PQ-NMSSM below the compactification scale.
an effective theory below $M_{\text{GUT}}$. The missing-partner mechanism is implemented by the following PQ-invariant superpotential terms

$$ W = \frac{1}{2} M \text{Tr}(\Sigma^2) + \frac{1}{3} a \text{Tr}(\Sigma^3) + b_i \theta_i \Sigma H_i + c_i \bar{\theta}_i \Sigma \bar{H}_{i+1} + \tilde{M}_i \theta_i \bar{\theta}_i, \quad (4.3) $$

with the identification $\bar{H}_4 = \bar{H}_1$. As in the original model, the vacuum expectation value of $\Sigma$ breaks SU(5) to the SM gauge groups, and gives rise to the triplet mass terms. This becomes clear after integrating out the heavy triplets in $\theta_i + \bar{\theta}_i$, which leads to

$$ W_{\text{eff}} = M_i^c H_i \bar{H}_{i+1}^c, \quad (4.4) $$

where $M_i^c \sim M_{\text{GUT}}^2 / \tilde{M}_i$, and $H_i^c$ denotes the color-triplet from $H_i$. Because the 50 representation does not contain Higgs doublets, no doublet mass terms are generated from the superpotential (4.3), and the three pairs of doublet Higgses remain massless. Mass terms for these doublet Higgses arise from

$$ W = \lambda_i X_i H_i \bar{H}_i + \frac{\xi}{M_{Pl}} X_1^3 X_2, \quad (4.5) $$

where we have chosen a basis of $X_3$ such that $X_1^2 \bar{H}_3 \bar{H}_3$ is removed in the superpotential. Including soft SUSY breaking terms for the gauge singlet scalars

$$ -L_{\text{soft}} = m_{X_i}^2 |X_i|^2 + \left( A_i \frac{\xi}{M_{Pl}} X_1^3 X_2 + \text{h.c.} \right), \quad (4.6) $$

the second term in the above superpotential fixes $X_{1,2}$ at

$$ |X_{1,2}|^2 \approx \frac{M_{Pl} (-m_{X_1}^2)^{1/2}}{\xi} \sim \frac{M_{Pl} M_{\text{SUSY}}}{\xi}, \quad (4.7) $$

thereby leading to that $U(1)_{\text{PQ}}$ is spontaneously broken around $10^{11}$ GeV for $\xi \sim 1$, and there appears the axion which is a mixture of $\text{arg}(X_{1,2})$. Here we have assumed $m_{X_1}^2 < 0$. Hence, the doublet Higgses in $H_{1,2} + \bar{H}_{1,2}$ obtain large masses from the vacuum expectation value of $X_{1,2}$, respectively. On the other hand, the doublet Higgses in $H_3 + \bar{H}_3$ remain massless until $X_3$ acquires a nonzero vacuum expectation value.

Notice that the PQ charge assignment (4.2) allows direct mass terms $H_i \bar{H}_{i+1}$ and the Yukawa term $X_1 X_2 X_3$ in the renormalizable superpotential. These terms should be absent in order for the missing-partner mechanism to work and for only one pair of Higgs doublets to remain light. Once we do not put these superpotential terms, radiative corrections will not change the situation owing to supersymmetry.\(^7\) It is also important to note that the model possesses two global $U(1)$ symmetries associated with the independent charges $p$ and $q$. To eliminate one of them, as was considered in [8], we introduce three right-handed neutrino multiplets $N(1)$ that implement the conventional see-saw mechanism through the

\(^7\)One can assign a different PQ charge to $X_2$ and the Higgs multiplets to forbid a superpotential term $X_1 X_2 X_3$. Then, other mechanism is needed to fix the PQ breaking scale because the term $X_1^2 X_2$ in the scalar potential (4.6) is not allowed.
superpotential terms \( NLH_u + X_i NN \) with \( i = 1 \) or 2. The PQ charges are then fixed as \( 5p = -8q \) when the Majorana masses for \( N \) arise from \( X_1 NN \), and \( 5p = -12q \) if one instead chooses \( X_2 NN \).

It now becomes apparent that the missing-partner model with the superpotential terms (4.3) and (4.5) leads to the PQ-NMSSM. The doublet Higgses in \( H_3 + \bar{H}_3 \) correspond to the ordinary MSSM Higgses, while \( X_1, 2, 3 \) play the role of \( X \) and \( S \), respectively. In the model, the higher dimensional operators

\[
\mathcal{L} = \int d^4 \theta \left( \frac{\kappa_1 X_1^2 X_3}{M_{Pl}} + \frac{\kappa_2 X_1 X_2 X_3}{M_{Pl}} \right) + \text{h.c.}
\]

(4.8)
can generate an effective tadpole term for \( X_3 \) as

\[
W_{\text{eff}} = \tilde{m}_0^2 X_3,
\]

(4.9)
where \( \tilde{m}_0^2 \sim M_{\text{SUSY}}^2 \) for \( \kappa_{1,2} \sim 1 \) and \( \xi \sim 1 \). Then, \( X_3 \) is naturally expected to get a vacuum expectation value around the weak scale for \( M_{\text{SUSY}} \sim 1 \text{ TeV} \). However, if PQ-breaking mass terms \( \theta_i \bar{\theta}_j \) with \( i \neq j \) are present, the loops of heavy triplets would generate large tadpoles for \( X_3 \). This implies that \( U(1)_{\text{PQ}} \) is crucial to avoid the tadpole problem. The PQ symmetry plays an important role also in suppressing dangerous higher dimensional operators leading to too rapid proton decays. The triplet Higgses mediate dimension 5 operators violating the baryon number \([33]\), which carry a nonzero PQ charge and therefore are further suppressed by a small factor \( X_1 X_2 / (M_1^c M_2^c) \sim M_{\text{SUSY}} M_{Pl} / (M_1^c M_2^c) \) compared to those in the minimal SU(5) GUT model.

Let us finally discuss the difference from the model of \([8]\). That model contains two pairs of Higgs doublets \( H_f + \bar{H}_f \) and \( H'_f + \bar{H}'_f \) which are vector-like also under \( U(1)_{\text{PQ}} \). One pair of them becomes heavy through \( PH_f \bar{H}_f \) for \( P \) being a \( U(1)_{\text{PQ}} \) breaking gauge singlet field, and the other remains light. If one introduces an additional singlet \( P' \) having a term \( P' H'_f \bar{H}_f \) in the superpotential, \( P' \) necessarily carries a PQ charge such that \( PP' \) is invariant under \( U(1)_{\text{PQ}} \) transformations. Thus, even if one omits \( PP' \) in the superpotential, a Kähler potential term \( PP' \) would induce too large tadpole term for \( P' \). This makes it difficult for \( P' \) to play the role of \( S \) in the PQ-NMSSM.

5. Conclusions

Extended to incorporate the PQ mechanism solving the strong CP problem, the NMSSM becomes compatible with the grand unification since the PQ symmetry forbids large tadpoles for the SM singlet \( S \) to be generated from loops of heavy fields coupling to \( S \). Another important property of the PQ-NMSSM is that all the mass parameters are determined by the SUSY breaking scale \( M_{\text{SUSY}} \) and \( F_a^2 / M_{Pl} \) with \( F_a \) being the axion decay constant. Thus, the electroweak symmetry breaking is naturally achieved at the correct scale. Furthermore, the model can avoid the domain wall problem in the presence of the PQ messengers.

An important consequence of the PQ symmetry is that the lightest neutralino is singlino-like with a small Higgsino admixture, and is relatively light compared to other
sparticles. The Higgsino component is determined by the coupling of $S$ to the Higgs doublets, which is constrained by the LEP bound on the invisible $Z$-boson decay width. This constraint becomes severe at large $\tan \beta$. Meanwhile, the SM-like Higgs boson decays mainly through the conventional decay modes at large $\tan \beta$ and in a portion of parameter space for small values of $\tan \beta$. The decay of the Higgs boson into a pair of the lightest neutralino can be the main mode at low $\tan \beta$, for which case the Higgs search at colliders will be modified. Also important is that the SM-like Higgs mass receives an additional positive contribution from the loops involving the singlino Yukawa coupling. This PQ-NMSSM specific contribution can lead to a significant increase of the Higgs boson mass by a few GeV even at large $\tan \beta$ compared to the MSSM.

We found that the PQ-NMSSM is realized as a low energy effective theory of a missing-partner model for supersymmetric SU(5) GUT with the PQ symmetry, which solves the doublet-triplet splitting problem and the proton decay problem. It is interesting to note that such a UV completion achieves the relation $F_a \sim \sqrt{M_{\text{SUSY}} M_{\text{Pl}}}$. Hence, all the mass parameters of the resulting PQ-NMSSM have values of the order of $M_{\text{SUSY}}$.

Note added

After submitting the manuscript, the ATLAS and CMS collaborations at the LHC reported their updated results in the Higgs search [34], which may indicate a SM-like Higgs boson with mass around 125 GeV. To explain a 125 GeV Higgs mass within the MSSM, we need large stop mixing or heavy stops with mass larger than about 10 TeV. The PQ-NMSSM improves the situation because the Higgs mass receives an additional positive contribution, which can be of a few GeV even at large $\tan \beta$.

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A. Mass matrix

In this appendix, we present the tree-level mass matrices for the neutral scalar fields. After rotating the upper left $2 \times 2$ submatrix of the mass matrix for the CP even scalars, one
obtains

\[
\begin{align*}
(M_H^2)_{11} &= M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta; \\
(M_H^2)_{22} &= \frac{2b_{\text{eff}}}{\sin 2\beta} + (M_Z^2 - \lambda^2 v^2) \sin^2 2\beta; \\
(M_H^2)_{33} &= m_S^2 + \lambda^2 v^2; \\
(M_H^2)_{12,21} &= \frac{1}{2} (M_Z^2 - \lambda^2 v^2) \sin 4\beta; \\
(M_H^2)_{13,31} &= \lambda v (\mu_{\text{eff}} - A_\lambda \sin 2\beta); \\
(M_H^2)_{23,32} &= \lambda v A_\lambda \cos 2\beta.
\end{align*}
\]

The mass matrix for the pseudoscalar fields is given by

\[
M_A^2 = \begin{pmatrix} b_{\text{eff}} \cot \beta & b_{\text{eff}} & \lambda v A_\lambda \cos \beta \\ b_{\text{eff}} & b_{\text{eff}} \tan \beta & \lambda v A_\lambda \sin \beta \\ \lambda v A_\lambda \cos \beta & \lambda v A_\lambda \sin \beta & m_S^2 + \lambda^2 v^2 \end{pmatrix}.
\]

It is easy to see that there are one massless mode, which is absorbed into gauge boson, and two massive CP odd scalars:

\[
M_{A_{1,2}}^2 = \frac{b_{\text{eff}}}{\sin 2\beta} + \frac{1}{2} (m_S^2 + \lambda^2 v^2) \pm \sqrt{\left( \frac{b_{\text{eff}}}{\sin 2\beta} - \frac{1}{2} (m_S^2 + \lambda^2 v^2) \right)^2 + A_\lambda^2 \lambda^2 v^2}.
\]

Using the stationary condition (2.7), one can find that \( M_{A_1} = 0 \) if \( \kappa = 0 \).

### B. Global structure of the Higgs potential

In the PQ-NMSSM, where the Higgs sector is extended to include the singlet \( S \), the Higgs potential may develop another minimum away from the weak scale. The model parameters are constrained to avoid such a minimum since it would generally appear at a field value similar to or larger than \( M_{\text{SUSY}} \) and thus be deeper than the electroweak vacuum. For the case with \( m_S^2 > 0 \) and \( \lambda \ll 1 \), we shall show that the region \( \lambda^2 (|H_u^0|^2 + |H_d^0|^2) \sim M_{\text{SUSY}}^2 \) somewhat near the \( D \)-flat direction is potentially dangerous for much of the parameter space. This argues that the condition to avoid a deeper minimum can approximately be examined by looking at the shape of the potential along the \( D \)-flat direction.

After integrating out \( S \) by the minimization condition, the Higgs potential reads

\[
V = \left( \frac{g^2 + g'^2}{2} \cos^2 2\theta + \lambda^2 \sin^2 2\theta \right) \phi^4 - \frac{(A_\lambda \lambda \phi^2 \sin 2\theta + B_\lambda m_0^2)^2}{2\lambda^2 \phi^2 + m_S^2} + 2 \left( m_{H_u}^2 \sin^2 \theta + m_{H_d}^2 \cos^2 \theta - \lambda m_0^2 \sin 2\theta \right) \phi^2 + \text{constant},
\]

where \( |H_u^0| = \sqrt{2}\phi \sin \theta \) and \( |H_d^0| = \sqrt{2}\phi \cos \theta \) with \( 0 \leq \theta < \pi/2 \) and \( 0 \leq \phi \). At very large values of \( \phi \), the first term becomes dominant and lifts the potential along the \( \phi \)-direction.

It is also straightforward to see that \( \partial_\phi V = 0 \) when \( \phi = 0 \) or when

\[
\left( \sin^2 2\theta + \frac{g^2 + g'^2}{2\lambda^2} \cos^2 2\theta \right) \left( 2\lambda^2 \phi^2 + m_S^2 \right)^3 - k m_S^2 \left( 2\lambda^2 \phi^2 + m_S^2 \right)^2
\]

\[
+ 2 \left( \frac{A_\lambda m_0^2}{2} \sin 2\theta - \lambda B_\lambda m_0^2 \right)^2 = 0,
\]

where
where \( k \) is a function of \( \theta \),
\[
k = \left(1 + \frac{A^2}{2m^2_S}\right) \sin^2 2\theta + \frac{g^2 + g'^2}{2\lambda^2} \cos^2 2\theta - 2m^2_{H_u} \sin^2 \theta + m^2_{H_d} \cos^2 \theta - \lambda m^2_0 \sin 2\theta \] (B.3)

The above relation shows that the potential can have at most one local minimum at \( \phi \neq 0 \) along the \( \phi \)-direction for a given \( \theta \), which would appear at \( 2\lambda^2 \phi^2 + m^2_S \sim M^2_{\text{SUSY}} \). On the other hand, along the angular direction, the slope of the potential vanishes when
\[
\sin 2\theta = -\left(\frac{r \phi^2}{\Lambda^2_1} \sin 2\theta + \frac{\Lambda^2_2}{\Lambda^2_1}\right) \cos 2\theta,
\] (B.4)
for which
\[
\partial^2_\theta V = 4 \left(-\cos 2\theta + \frac{\Lambda^2_2}{\Lambda^2_1} \sin 2\theta\right) \Lambda^2_1 \phi^2 \tan^2 2\theta.
\] (B.5)

Here \( r \) is defined by
\[
r = g^2 + g'^2 - 2 \left(1 - \frac{A^2}{2\lambda^2 \phi^2 + m^2_S}\right) \lambda^2,
\] (B.6)
and \( \Lambda^2_{1,2} \) are given by
\[
\Lambda^2_1 = m^2_{H_d} - m^2_{H_u},
\Lambda^2_2 = 2 \left(1 + \frac{A \lambda B_\kappa}{2\lambda^2 \phi^2 + m^2_S}\right) \lambda m^2_0,
\] (B.7)
both of which are generally of \( O(M^2_{\text{SUSY}}) \), and positive. For \( r > 0 \), one can find (i) \( \partial_\theta V = 0 \) can have a solution at \( \tan \theta > 1 \) with a positive curvature \( \partial^2_\theta V > 0 \), implying that there is only one minimum along the angular direction for a given \( \phi \), and (ii) for large values of \( \phi \), \( \phi^2 \gg \Lambda^2_1 \sim M^2_{\text{SUSY}} \), a minimum along the angular direction is located near \( \tan \theta = 1 \), i.e. near the \( D \)-flat direction.

Let us examine further the case with \( r > 0 \) and \( m^2_S \sim M^2_{\text{SUSY}} \). Note that \( r \) is positive at \( 2\lambda^2 \phi^2 + m^2_S \sim M^2_{\text{SUSY}} \) if \( \lambda \lesssim 0.5 \) for small \( A \lambda \), and if \( \lambda \lesssim 1 \) for \( A \lambda \sim M_{\text{SUSY}} \) and \( B_\kappa \sim M_{\text{SUSY}} \). At the electroweak vacuum, which lies at \( \phi^2 \sim M^2_W \ll M^2_{\text{SUSY}} \) and \( \theta = \beta \), the condition (B.4) gives
\[
\frac{\Lambda^2_2}{\Lambda^2_1} \approx -\tan 2\beta.
\] (B.8)

The extremum condition (B.4) can then be written
\[
\left(1 - \frac{r \phi^2 \tan^2 \theta - 1}{\Lambda^2_1 \tan^2 \theta + 1}\right) \tan 2\theta \approx \tan 2\beta,
\] (B.9)
for \( \phi^2 \lesssim M^2_{\text{SUSY}} \). This tells that a minimum along the angular direction arises at \( 1 < \tan \theta < \tan \beta \) for a given \( \phi \), and approaches the \( D \)-flat direction, \( \tan \theta = 1 \), as \( \phi \) increases. Thus it is useful to first analyze the potential along the \( D \)-flat direction though the actual another minimum, if exists, appears somewhat away from the \( D \)-flat direction.

On the other hand, in the case with \( r < 0 \), the potential is minimized along the angular direction at \( \tan \theta > \tan \beta \) or at \( \tan \theta < 1 \) for \( \phi^2 \gtrsim M^2_{\text{SUSY}} \). In the former case, making the potential develop no other minimum in the region near the \( D \)-flat direction is not enough to guarantee the absence of a deeper minimum.
C. RG running equations

For the low energy effective theory (3.1) below $M_{\text{SUSY}}$, the RG running equations for the Yukawa couplings read

$$8\pi^2 \frac{d y_t^2}{d\mu} = \left( \frac{9}{2} y_t^2 + y_u^2 + y_d^2 - \frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 \right) y_t^2,$$

$$8\pi^2 \frac{d y_u^2}{d\mu} = \left( 3y_t^2 + \frac{5}{2} y_u^2 + 4y_d^2 - \frac{9}{20} g_1^2 - \frac{9}{4} g_2^2 \right) y_u^2,$$

$$8\pi^2 \frac{d y_d^2}{d\mu} = \left( 3y_t^2 + 4y_u^2 + \frac{5}{2} y_d^2 - \frac{9}{20} g_1^2 - \frac{9}{4} g_2^2 \right) y_d^2,$$ (C.1)

at the one-loop, and those for SM gauge couplings are

$$8\pi^2 \frac{d g_1}{d\mu} = -\frac{103}{30} - \frac{1}{10} \theta(\mu - m_h) - \frac{17}{30} \theta(\mu - m_t) - \frac{2}{5} \theta(\mu - \mu_{\text{eff}}),$$

$$8\pi^2 \frac{d g_2}{d\mu} = \frac{13}{3} - \frac{1}{6} \theta(\mu - m_h) - \theta(\mu - m_t) - \frac{2}{3} \theta(\mu - \mu_{\text{eff}}),$$

$$8\pi^2 \frac{d g_3}{d\mu} = 8 - \theta(\mu - m_t),$$ (C.2)

with $g_1 = \sqrt{5/3}g'$ and $g_2 = g$. Here $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x < 0$.

References

[1] For a review of the NMSSM, see M. Maniatis, “The Next-to-Minimal Supersymmetric extension of the Standard Model reviewed,” Int. J. Mod. Phys. A25, 3505-3602 (2010). [arXiv:0906.0777 [hep-ph]]; U. Ellwanger, C. Hugonie, A. M. Teixeira, “The Next-to-Minimal Supersymmetric Standard Model,” Phys. Rept. 496, 1-77 (2010). [arXiv:0910.1785 [hep-ph]].

[2] H. P. Nilles, M. Srednicki and D. Wyler, “Constraints On The Stability Of Mass Hierarchies In Supergravity,” Phys. Lett. B 124, 337 (1983); A. B. Lahanas, “Light Singlet, Gauge Hierarchy And Supergravity,” Phys. Lett. B 124, 341 (1983); U. Ellwanger, “Nonrenormalizable Interactions From Supergravity, Quantum Corrections And Effective Low-Energy Theories,” Phys. Lett. B 133, 187 (1983); H. P. Nilles and N. Polonsky, “Gravitational divergences as a mediator of supersymmetry breaking,” Phys. Lett. B 412, 69 (1997) [hep-ph/9707249].

[3] J. Bagger, E. Poppitz, “Destabilizing divergences in supergravity coupled supersymmetric theories,” Phys. Rev. Lett. 71, 2380-2382 (1993). [hep-ph/9307317]; V. Jain, “On destabilizing divergences in supergravity models,” Phys. Lett. B351, 481-486 (1995). [arXiv:hep-ph/9407382 [hep-ph]]; J. Bagger, E. Poppitz, L. Randall, “Destabilizing divergences in supergravity theories at two loops,” Nucl. Phys. B455, 59-82 (1995). [hep-ph/9505244].

[4] A. Vilenkin, “Cosmic Strings And Domain Walls,” Phys. Rept. 121, 263 (1985).

[5] S. A. Abel, S. Sarkar, P. L. White, “On the cosmological domain wall problem for the minimally extended supersymmetric standard model,” Nucl. Phys. B454, 663-684 (1995). [hep-ph/9506359].
[6] R. D. Peccei and H. R. Quinn, “CP Conservation In The Presence Of Instantons,” Phys. Rev. Lett. 38, 1440 (1977); “Constraints Imposed By CP Conservation In The Presence Of Instantons,” Phys. Rev. D 16, 1791 (1977).

[7] A. Masiero, D. V. Nanopoulos, K. Tamvakis and T. Yanagida, “Naturally Massless Higgs Doublets In Supersymmetric SU(5),” Phys. Lett. B 115, 380 (1982).

[8] J. Hisano, T. Moroi, K. Tobe and T. Yanagida, “Suppression of proton decay in the missing partner model for supersymmetric SU(5) GUT,” Phys. Lett. B 342, 138 (1995) [arXiv:hep-ph/9406417].

[9] H. M. Lee, S. Raby, M. Ratz, G. G. Ross, R. Schieren, K. Schmidt-Hoberg and P. K. S. Vaudrevange, “Discrete R symmetries for the MSSM and its singlet extensions,” Nucl. Phys. B 850 (2011) 1 [arXiv:1102.3595 [hep-ph]]; U. Ellwanger, G. Espitalier-Noel and C. Hugonie, “Naturalness and Fine Tuning in the NMSSM: Implications of Early LHC Results,” arXiv:1107.2472 [hep-ph]; G. G. Ross and K. Schmidt-Hoberg, “The fine-tuning and phenomenology of the generalised NMSSM,” arXiv:1108.1284 [hep-ph]; A. Delgado, C. Kolda and A. de la Puente, “Solving the Hierarchy Problem with a Light Singlet and Supersymmetric Mass Terms,” arXiv:1111.4008 [hep-ph].

[10] K. Nakayama, N. Yokozaki and K. Yonekura, “Relaxing the Higgs mass bound in singlet extensions of the MSSM,” JHEP 1111, 021 (2011) [arXiv:1108.4338 [hep-ph]].

[11] E. Bertuzzo and M. Farina, “Higgs boson signals in lambda-SUSY with a Scale Invariant Superpotential,” arXiv:1105.5389 [hep-ph]; M. Almarashi and S. Moretti, “Reinforcing the no-lose theorem for NMSSM Higgs discovery at the LHC,” Phys. Rev. D 84, 035009 (2011) [arXiv:1106.1599 [hep-ph]]; U. Ellwanger, “Higgs Bosons in the Next-to-Minimal Supersymmetric Standard Model at the LHC,” arXiv:1108.0157 [hep-ph]; M. M. Almarashi and S. Moretti, “LHC Signals of a Heavy CP-even Higgs Boson in the NMSSM via Decays into a Z and a Light CP-odd Higgs State,” arXiv:1109.1735 [hep-ph].

[12] K. Hamaguchi, K. Nakayama and N. Yokozaki, “NMSSM in gauge-mediated SUSY breaking without domain wall problem,” arXiv:1107.4760 [hep-ph].

[13] J. -J. Cao, K. -i. Hikasa, W. Wang and J. M. Yang, “Light dark matter in NMSSM and implication on Higgs phenomenology,” Phys. Lett. B 703, 292 (2011) [arXiv:1104.1754 [hep-ph]]; M. Carena, N. R. Shah and C. E. M. Wagner, “Light Dark Matter and the Electroweak Phase Transition in the NMSSM,” arXiv:1110.4378 [hep-ph].

[14] For a review, see J. E. Kim, “Light Pseudoscalars, Particle Physics and Cosmology,” Phys. Rept. 150, 1 (1987); H. Y. Cheng, “The Strong CP Problem Revisited,” Phys. Rept. 158, 1 (1988); J. E. Kim and G. Carosi, “Axions and the Strong CP Problem,” Rev. Mod. Phys. 82, 557 (2010) [arXiv:0807.3125 [hep-ph]].

[15] C. Panagiotakopoulos and K. Tamvakis, “New minimal extension of MSSM,” Phys. Lett. B 469, 145 (1999) [hep-ph/9908351].

[16] C. Panagiotakopoulos, A. Pilaftsis, “Higgs scalars in the minimal nonminimal supersymmetric standard model,” Phys. Rev. D63, 055003 (2001). [hep-ph/0008268].

[17] A. Dedes, C. Hugonie, S. Moretti and K. Tamvakis, “Phenomenology of a new minimal supersymmetric extension of the standard model,” Phys. Rev. D 63, 055009 (2001) [arXiv:hep-ph/0009125].

[18] A. Menon, D. E. Morrissey and C. E. M. Wagner, “Electroweak baryogenesis and dark matter in the nMSSM,” Phys. Rev. D 70 (2004) 035005 [arXiv:hep-ph/0404184].
[19] C. Balazs, M. S. Carena, A. Freitas and C. E. M. Wagner, “Phenomenology of the nMSSM from colliders to cosmology,” JHEP 0706, 066 (2007) [arXiv:0705.0431 [hep-ph]]; J. Cao, H. E. Logan and J. M. Yang, “Experimental constraints on nMSSM and implications on its phenomenology,” Phys. Rev. D 79, 091701 (2009) [arXiv:0901.1437 [hep-ph]].

[20] P. Fayet, “Supergauge Invariant Extension Of The Higgs Mechanism And A Model For The Electron And Its Neutrino,” Nucl. Phys. B 90, 104 (1975).

[21] J. E. Kim and H. P. Nilles, “The Mu Problem And The Strong CP Problem,” Phys. Lett. B 138, 150 (1984).

[22] K. Blum, C. Delaunay and Y. Hochberg, “Vacuum (Meta)Stability Beyond the MSSM,” Phys. Rev. D 80, 075004 (2009) [arXiv:0905.1701 [hep-ph]].

[23] S. Kasuya, M. Kawasaki and T. Yanagida, “Domain wall problem of axion and isocurvature fluctuations in chaotic inflation models,” Phys. Lett. B 415, 117 (1997) [hep-ph/9709202].

[24] H. P. Nilles, “Supersymmetry, Supergravity And Particle Physics,” Phys. Rept. 110, 1 (1984); H. E. Haber and G. L. Kane, “The Search For Supersymmetry: Probing Physics Beyond The Standard Model,” Phys. Rept. 117, 75 (1985).

[25] K. Nakamura et al. [Particle Data Group], “Review of particle physics,” J. Phys. G 37, 075021 (2010).

[26] M. Masip, R. Munoz-Tapia and A. Pomarol, “Limits on the mass of the lightest Higgs in supersymmetric models,” Phys. Rev. D 57, R5340 (1998) [hep-ph/9801437].

[27] Y. Okada, M. Yamaguchi and T. Yanagida, “Upper bound of the lightest Higgs boson mass in the minimal supersymmetric standard model,” Prog. Theor. Phys. 85, 1 (1991); “Renormalization Group Analysis On The Higgs Mass In The Softly Broken Supersymmetric Standard Model,” Phys. Lett. B 262, 54 (1991).

[28] J. R. Ellis, G. Ridolfi and F. Zwirner, “Radiative corrections to the masses of supersymmetric Higgs bosons,” Phys. Lett. B 257, 83 (1991); H. E. Haber and R. Hempfling, “Can the mass of the lightest Higgs boson of the minimal supersymmetric model be larger than m(Z)?”, Phys. Rev. Lett. 66, 1815 (1991).

[29] The ATLAS and CMS collaborations, ATLAS-CONF-2011-157 and CMS PAS HIG-11-023 (November, 2011).

[30] G. Pocsik and T. Torma, “On The Decays Of Heavy Higgs Bosons,” Z. Phys. C 6, 1 (1980); T. G. Rizzo, “Decays Of Heavy Higgs Bosons,” Phys. Rev. D 22 (1980) 722; W. Y. Keung and W. J. Marciano, “Higgs Scalar Decays: H → W+ X,” Phys. Rev. D 30, 248 (1984).

[31] Y. Kawamura, “Triplet doublet splitting, proton stability and extra dimension,” Prog. Theor. Phys. 105, 999 (2001) [hep-ph/0012125].

[32] L. J. Hall and Y. Nomura, “Gauge unification in higher dimensions,” Phys. Rev. D 64, 055003 (2001) [arXiv:hep-ph/0103125].

[33] N. Sakai and T. Yanagida, “Proton Decay In A Class Of Supersymmetric Grand Unified Models,” Nucl. Phys. B 197, 533 (1982); S. Weinberg, “Supersymmetry At Ordinary Energies. 1. Masses And Conservation Laws,” Phys. Rev. D 26, 287 (1982).

[34] The ATLAS and CMS collaborations, ATLAS-CONF-2011-163 and CMS-PAS-HIG-11-032 (December, 2011).