On Time and Space Double-slit Experiments

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Abstract

Time double-slit experiments have been achieved and presented as complementary to spatial double-slit experiments, providing further confirmation of the wave-particle duality. Numerical solutions of the free particle time dependent Schrödinger equation have been presented as explanation of the experimental results. To be considered as exhibiting "interference in time" has been objected to on the basis that the standard non relativistic quantum theory does not have the property of coherence in time. In this note the theoretical and experimental results are derived in a schematic but analytic solution of the TDSE with appropriate initial boundary conditions. The time evolution at a fixed position is shown to exhibit an oscillating transient behavior. The particular boundary conditions are justified by the experimental setups that actually result in having only a single electron at any given time in the double-slit arrangement; and consequently achieve the construction of double peak single electron wave packets, whose spreading gives rise to the time behavior noted. The progressive complementarity of "which-path" ("which-time") information and "space interference" ("oscillating time transient") pattern build up is also exhibited.
I. INTRODUCTION

In recent years, time electron double-slit experiments \(^{1,2}\) have been achieved and presented as complementary to electron spatial double-slit experiments \(^{3}\), providing a further confirmation of the wave-particle duality. A very important feature is that the experimental conditions allow asserting that only one electron is present at any time. The fringe patterns follow from the progressive accumulation of single particle events.

Interference from a double-slit allowed Thomas Young to demonstrate the wave nature of light over two centuries ago. \(^{4}\) However the explanation by Einstein of the photoelectric effect in 1905, on the basis of Planck’s energy quantum hypothesis, followed by the Compton light scattering experiment in 1923, brought forward evidence of a corpuscular behavior of light. On the other hand, the daring assumption in 1924 by Louis de Broglie to conversely associate a wave to matter was corroborated in 1927 by the Davisson-Germer experiment of the diffraction of electrons by crystals. \(^{5}\) Since then the wave-particle duality in nature and the interpretation of quantum mechanics have been the subject of extensive discussions and research.

The particle two-slit arrangement figured prominently as a thought experiment in the exchange between Bohr and Einstein in the Solvay meetings of 1927 and 1930 \(^{6}\) on the complementarity of the wave and particle aspects. As expressed by Bohr, an attempt to detect through which slit the particle goes excludes the development of an interference pattern. Einstein, on the other hand, tried to override such assertion. As stated by Feynman, the double-slit set up “... has in it the heart of quantum mechanics. In reality it contains the only mystery” \(^{7}\).

The two-slit arrangement is included in most quantum mechanics textbooks to illustrate the consequence of the de Broglie hypothesis \(^{7,8}\). Its experimental realization, however, had to wait more than 50 years but has already involved electrons \(^{3,9}\), neutrons \(^{10}\), atoms \(^{11}\) and molecules \(^{12}\). Most striking is the one carried out by Akira Tonomura and coworkers \(^{3}\) where the buildup of the fringe pattern is achieved by the accumulation of time spaced successive single electron impacts. The development of cavity quantum electrodynamics (CQED) has been recently applied to carry out more sophisticated “which path” experiments \(^{13}\).

The development of double-slit experiments in the time domain brings out a new facet to the wave-particle duality (\(^{12,2}\) and references therein). The role of the slits is played by
rapidly successive time windows of very short duration. Also to be noted is that the question of diffraction in time had been raised a long time before\textsuperscript{14} and confirmed experimentally only recently.\textsuperscript{15} This is also included below, as in all these cases, either an analytical solution\textsuperscript{14} or numerical integrations\textsuperscript{1,2} of the time dependent Schrödinger equation (TDSE) have been presented as the explanation of the results.

Notwithstanding, an objection has been raised to the claim of "interference in time" on the basis that the non relativistic quantum theory does not have the property of coherence in time.\textsuperscript{16} Indeed it is pointed out that introducing two distinct packets into the beam of an experiment at two different times would yield by construction a mixed state, for which no interference would take place. An additional argument is that this would require time to be an additional observable with a spectrum derivable from a self-adjoint operator\textsuperscript{16}, as expansion of a state vector in such a basis would provide superposition of different times. The existence of such an operator is however a long standing problem in quantum mechanics.\textsuperscript{18–20}

With respect to the diffraction in time\textsuperscript{14}, it is noted that the sudden lifting of a shutter does result in damped transient type oscillations that can be interpreted as fringes in time.

In this note it is shown, in a schematic analytic way, that both space and time double-slit experiments, as well as the time diffraction, can indeed be described by the TDSE for free particle motion with appropriate initial boundary conditions in each case. The derivations account both for the analytic structure and the numerical and experimental results. The time dependence of the space density at a fixed point exhibits in all cases an oscillatory transient type behaviour.

Finally, it is also shown that this formulation is adapted to exhibit the progressive complementarity of "which path" ("which time") information and "space interference" ("oscillating time transient") pattern build up, as has already been shown experimentally (\textsuperscript{23–25} and references therein).

\textbf{II. THE FREE PARTICLE TDSE}

The time evolution of the state vector in the free particle case is given by:

\begin{equation}
|\Psi(t)\rangle = e^{-i\hat{H}t/\hbar} |\Psi(0)\rangle = e^{-i\hat{p}^2t/2m\hbar} |\Psi(0)\rangle
\end{equation}

(1)
Introducing the space and momentum representations one has:

\[\ket{\Psi(t)} = \int d\vec{r} e^{-i\vec{p}\cdot\vec{r}/2m} \ket{\Psi(0)}\]

\[= (1/2\pi\hbar)^{3/2} \int d\vec{p} e^{-i\vec{p}\cdot\vec{r}/2m} \int d\vec{r} e^{-i\vec{p}\cdot\vec{r}/\hbar} \Psi(\vec{r}; 0)\]  

This gives the state vector at time \(t\) in terms of the initial space wave function. It then follows that the wave functions in momentum space is:

\[\Phi(\vec{p}; t) = \langle \vec{p} | \Psi(t) \rangle\]

\[= (1/2\pi\hbar)^{3/2} e^{-i\vec{p}\cdot\vec{r}/2m} \int d\vec{r} e^{-i\vec{p}\cdot\vec{r}/\hbar} \Psi(\vec{r}; 0)\]  

Its Fourier transform gives the wave function in configuration space, namely:

\[\Psi(\vec{r}; t) = \langle \vec{r} | \Psi(t) \rangle = \int d\vec{p} \langle \vec{r} | \vec{p} \rangle \langle \vec{p} | \Phi(t) \rangle\]

\[= (m/2\pi\hbar t)^{3/2} \int d\vec{r}' e^{-im(\vec{r}'-\vec{r})^2/2\hbar t} \Psi(\vec{r}'; 0)\]  

A. Space double-slit

The initial condition is taken as:

\[\Psi(\vec{r}; 0) = \delta(x) [\delta(y-a/2) + e^{-i\varphi} \delta(y+a/2)] e^{ip_0z/\hbar}\]  

corresponding to motion with initial momentum \(p_0\) in the \(z\) direction and two point slits in the \(y\) direction separated by a distance \(a\). Dirac delta functions are used for simplicity. Although mathematically they will spread instantaneously over all space, it will be seen that they do not modify the essential results that narrow Gaussians properly normalized would yield. A phase difference \(\varphi\) between the slits is introduced for generality; it would appear in a Aharonov-Bohm set up.

Inserting (5) into (3), one easily obtains:

\[|\Phi(\vec{p}; t)|^2 \propto \cos^2 \left[ \frac{(p_y a/\hbar) - \varphi}{2} \right]\]  

This shows that the particle acquires momentum in the \(y\) direction with alternating maxima
and minima. Maxima are found at the values

\[(p_y a/\hbar) - \varphi]/2 = \pi n, \quad n = 0, \pm 1, \pm 2, \ldots \quad (7)\]

corresponding to angles \(\theta_n\) with the \(z\) axis such that

\[
\sin \theta_n = (p_y/p_0) = (2n\pi + \varphi)(\hbar/a p_0) \\
= (n + \varphi/2\pi)\lambda_B/a \quad (8)
\]

Here \(\lambda_B = h/p_0\) is the de Broglie wavelength of the incoming particle. When \(\varphi = 0\), there is a peak in the forward direction, \(\theta_0 = 0\), with neighboring peaks at angles \(\sin^{-1}(\pm \lambda_B/a)\). When \(\varphi \neq 0\), the interference pattern is shifted by an angle \(\theta = \sin^{-1}[(\varphi/2\pi)\lambda_B/a]\). These are well known textbook results. It is also easily seen that the interference pattern disappears if one of the holes is shut, as proven in Section III.

The time dependent wave function (4) yields now:

\[
|\Psi(r; t)|^2 \propto (2m\hbar/t)^3 \cos^2[(amy/2\hbar t) + \varphi/2] \quad (9)
\]

where \(y\) is the transverse coordinate where the interference pattern is observed. For \(y \neq 0\), besides the damping factor \(t^{-3}\), the intensity oscillates very rapidly for small \(t\) and slows down to almost constant for \(t\) large. One has therefore a transient-type pattern with a time dependent period \(\Xi(t)\) that increases with time as \(\Xi(t) = (2\pi\hbar/amy)t^2\). With \(\varphi = 0\) and \(\theta << 1\), one has as initial value \(\Xi_n(T_0) = T_0/n\) at the secondary peaks of the interference pattern at a distance \(Z\) corresponding to \(Y_n = Z\tan \theta_n \approx Z\theta_n\) and classical arrival time \(T_0 = Z/v_0 = mZ/p_0\).

Textbook presentations of the space double-slit case never include analysis of the time evolution as given here. Perhaps an experiment like that of Tonomura and coworkers could provide a confirmation. Although the electron source is continuous (field-emission electron microscope), the current density is so low as to have only one electron in the system at any time (the time of flight of a 50 keV electron to cover a source detector distance of 1.5 m. is about 11 ns, while the current density is 3000 electrons per second). This allows to record individual arrivals at the scintillation detector, which are then projected successively into the television screen as time progresses. The grouping into fringes shows that the accumulation rate of events (number of electrons per unit time) exhibits a space-dependence. But, as noted above from Eq.9, the accumulation rate also has an oscillating time-dependence. It
seems it would be a question of following the accumulation rate of successive recordings of one specific detector if the arrival times are stored in the computer.

**B. Diffraction in time**

Diffraction in time concerns the much earlier identification of the occurrence of transient effects in a dynamical description of resonance scattering. It describes the effect of the opening of a single shutter at a certain time. It is included here for completeness since it is another TDSE development that can be treated in the same schematic way.

The initial condition is now taken as:

\[
\Psi(r; 0) = \delta(x) \delta(y) \theta(-z) e^{-ip_0z/\hbar}
\]  

(10)

where \(\theta(z)\) is the Heavyside step functions. This corresponds to a plane wave with momentum \(p_0\) in the positive z direction confined to negative \(z\) until a point orifice at the origin is opened at time \(t = 0\). Inserting (10) into (4) yields for \(t \geq 0\):

\[
\Psi(r; t) = \frac{1}{2} e^{-i3\pi/2} (m/2\pi \hbar t) e^{[-im(x^2+y^2)/2\hbar t]} e^{-imz^2/2\hbar t} Y_0^2 \text{erfc}(Y_0)
\]

(11)

where erfc\((Y_0)\) is the complementary error function \((\text{erfc}=1-\text{erf})\), \(Y_0 = e^{-i\pi/4}(2\hbar t/m)^{-1/2}[z - v_0 t]\) and \(v_0 = p_0/m\). The \(z\)-dependence coincides exactly with the one dimensional wave function of Eqs.3a, 3b, 3c of Ref.12. From these equations, the ratio of the transient to the stationary current density (no shutter) at a point \(Z\) ahead of the shutter corresponding to a classical arrival time \(T = Z/v_0\), is calculated and exhibited in Fig.1 to be compared with Fig.3 of Ref.12. The signal begins to build up at \(T\) and exhibits a decaying transient behavior afterwards.

**C. Time double-slit**

Consider a superposition at a time \(t\) of two state vectors evolving under the same conditions from different initial times \(t_1\) and \(t_2\) with \(t_1 < t_2\). Then, as the unitary evolution operator
$U(t, t')$ satisfies the relation $U(t, t') = U(t, t'')U(t'', t')$:

$$|\Psi(t)\rangle = U(t, t_1) |\Phi(t_1)\rangle + U(t, t_2) |\Theta(t_2)\rangle = U(t, t_2) U(t_2, t_1) |\Phi(t_1)\rangle + U(t, t_2) |\Theta(t_2)\rangle$$

Thus:

$$|\Psi(t)\rangle = U(t, t_2) \{|\Phi(t_2)\rangle + |\Theta(t_2)\rangle\}$$

Then $|\Psi(t)\rangle$ is given by the evolution from the superposition at the same instant $t_2$. On this basis, with $\Phi = \Theta = \Psi$, the initial condition ($t_2 = 0$) for the time double-slit is taken as:

$$\Psi(r; 0) = \delta(x) \delta(y) \times [\delta(z - a/2) + e^{-i\varphi} \delta(z + a/2)] e^{imz/\hbar}$$

(12)

It consists of two pulses moving in the $z$ direction with velocity $v = p_0/m$, separated by a distance $a = (p_0/m)\tau$ where $\tau$ is the time delay between pulses. A phase shift is introduced that may be related to the pulse creation mechanism. Inserting (12) into (3) one obtains:

$$|\Phi(p; t)|^2 \propto \cos^2 \left[ \frac{(p_z - p_0)a/\hbar - \varphi}{2} \right] = \cos^2 \left[ \frac{(p_z - p_0)(p_0\tau/m\hbar) - \varphi}{2} \right]$$

(13)

with alternating maxima and minima. The peaks occur at momenta $p_z = p_n$ such that

$$p_n = p_0 + (m\hbar/p_0\tau)(2\pi n + \varphi), \quad n = 0, \pm 1, \pm 2, \ldots$$

(14)

In terms of energy one has peaks at

$$E_n = p_n^2/2m = E_0 + \hbar^2/[2\pi n + \varphi] + \frac{\hbar^2}{4E_0\tau^2}[2\pi n + \varphi]^2$$

where $E_0 = p_0^2/2m$. Thus, neglecting the second term, the separation $\delta E$ between consecutive peaks is given by $2\pi\hbar/\tau = \hbar/\tau$, as exhibited in Fig.1b of Ref.2. More precisely $\delta E \geq \hbar/\tau$ or $\tau \times \delta E \geq \hbar$. There is thus a complementary relation of the time delay with the energy interference pattern.
The time evolution of the space wave function (4) yields:

\[
|\Psi(r; t)|^2 \propto \left(\frac{2m\pi\hbar}{t}\right)^3 \cos^2\left[\frac{p_0\tau}{2m\hbar}(p_0 - mz/t) + \varphi/2\right]
\]

\[
= \left(\frac{2m\pi\hbar}{t}\right)^3 \cos^2\left[\frac{E_0\tau}{\hbar}(1 - \sqrt{mc^2/2E_0(z/ct)}) + \varphi/2\right]
\]  (15)

This exhibits the spread of the original wave packets (in this case infinite because of the delta pulses) that gives rise to a fringe type pattern, as well as the overall damping with time to conserve probability. At a fixed position \(z\) it consists of a damped oscillation (the \(t^{-3}\) factor) with a period increasing with time, namely \(\Xi(t) = (\pi\hbar/E_0\tau)(p_0/mz)t^2\). It oscillates very rapidly for small \(t\) and flattens down to almost constant for \(t\) large. It is a transient response.\(^{17}\) For finite width peaks, the transient response would begin to be detected at a point \(Z\) at the classical time the pulse gets there, namely \(T_z = Z/v_0 = mZ/p_0\), as in the case of the shutter and in Ref.2. The period of oscillation would then have the value \(\Xi(T_z) = (\pi\hbar/E_0\tau)T_z\), increasing thereafter from this value. This behavior is exhibited in Fig.2, where the position \(Z\) and classical arrival time \(T\) correspond to those of Fig.1e of Ref.2.

For fixed \(t\), the space probability density (15) exhibits maxima at

\[
\frac{p_0\tau}{2m\hbar}(p_0 - mz/t) = \frac{p_0^2\tau}{2m\hbar}(1 - mz/p_0t)
\]

\[
= 2\pi n - \varphi/2
\]  (16)

or, equivalently at

\[
[(E_0\tau/\hbar)(1 - \sqrt{mc^2/2E_0 z/ct})] = 2\pi n - \varphi/2, \quad n = 0, \pm 1, \pm 2, \ldots
\]  (17)

where \(E_0 = p_0^2/2m\), \(v_0 = p_0/m\) and \(c\) the velocity of light.

The above expressions can now be compared with the experimental and calculated results of Refs.2 and 3, where measurements are made in the energy domain and related to the time delay between wave packet peaks and their time evolution. As stated in Ref.2, the cosine function oscillates, at a given time \(t\) and for a given \(\varphi\), with variations of:

- momentum \(p_0\) (and thus energy \(E_0 = p_0^2/2m\)), for \(\tau\) and \(z\) fixed;
- delay time \(\tau\), for \(p_0\) and \(z\) fixed;
- distance \(z\) from the origin, for \(p_0\) and \(\tau\) fixed.
Furthermore, there is a displacement in the $z$ direction with time given by

$$z = v_0 t = (p_0/m)t = (2E_0/m)^{1/2}t$$

(18)

reflecting the conservation of momentum in the free particle motion.

The dependence on the incident energy in Eq.17 allows comparison with the measured energy spectra in Fig.2 of Ref.1, as well as the exchange of maxima and minima with a $\pi/2$ change in phase. As shown following Eq.14 above, the separation between consecutive maxima is $\delta E \approx \hbar/\tau$, that for $\tau = 2$ fs yields $\delta E \approx 2$ eV. In an interval of 14 eV, one then expects seven peaks, in agreement with the experimental results in Fig.2 and the numerical simulation in Fig.3 of that paper.

For an electron energy $E_0 = 0.3$ eV and a time delay between pulses $\tau = 120$ fs, Eq.18 yields $z = 113$ nm at $t = 350$ fs, $z = 293$ nm at $t = 900$ fs and $z = 1626$ nm at $t = 5000$ fs, values that roughly correspond to the calculated wave fronts in Figs.1(c,d,e) of Ref 2. Also for $\tau = 96$ fs, $\hbar/\tau = 43$ meV, in agreement with the calculated energy peak separation in Fig.1b and the experimental one in Fig.3d of this reference.

It is easily shown (Section III) that the oscillation disappears if one of the temporal slits in (12) is suppressed, in agreement with the results of Ref.1 when only one temporal slit is generated.

### III. COMPLEMENTARITY IN WAVE-PARTICLE DUALITY

Complementary weights can be assigned to each slit in the space double-slit case by taking the initial wave function to be:

$$\Psi(\mathbf{r}; 0) = \delta(x) \left[ \alpha \delta(y - a/2) + (1 - \alpha) e^{-i\varphi} \delta(y + a/2) \right]$$

$$\times \delta(z) e^{i p_0 z/\hbar}$$

(19)

where $\alpha$ varies between 0 and 1. The values 0 and 1 correspond to having only one slit open, while intermediate values correspond to partially blocking one while opening the other. Substitution in Eq.3 yields:

$$|\Phi(\mathbf{p}; t)|^2 \propto \alpha^2 + (1 - \alpha)^2 + 2\alpha(1 - \alpha) \cos[(p_y a/\hbar) - \varphi]$$

$$= (2\alpha - 1)^2 + 4\alpha(1 - \alpha) \cos^2\left[\left(\frac{p_y a/\hbar - \varphi}{2}\right)\right]$$

(20)
which clearly shows the fading of the interference pattern as \( \alpha \) approaches either 0 or 1. For \( \alpha = 1/2 \), Eq.6 is recovered, which corresponds to maximum visibility of the interference pattern. This progressive complementarity has been recently confirmed experimentally by placing a movable mask in front of a double slit to control the transmissions through the individual slits.\(^{25}\)

An entirely similar result is obtained when applied to the time double-slit case, where the extreme \( \alpha \)-values 0 and 1 correspond to having only a single pulse and consequently no oscillating transient pattern.\(^{2}\)

**IV. CONCLUSIONS**

It has been shown in an analytic schematic way that the free particle TDSE does give rise to the calculated and observed space and time double-slit results, as well as the diffraction in time, when initial boundary conditions appropriate to the experimental or theoretical setup are considered. Moshinsky’s ”diffraction in time” actually corresponds to the fact that the transient behavior generated by the shutter opening has the appearance of a Fresnel pattern.\(^{14,16,17}\) It is seen here that the time evolution of the wave function in the time and space double-slit experiments exhibits also an oscillatory transient response. The reservation raised in Ref.16 is related to whether states corresponding to different times can give rise to a coherent superposition, and thus generate cross terms in the probability density in some representation, to be interpreted as interference terms; or alternatively, they can only give rise to a mixture. It indeed can be shown that two states distinguished in some way (e.g, by a particular quantum number) do not allow a superposition but only a mixed state.\(^{21}\) As an example, the construction of a beam inserting at one time electrons *with spin up* and at another time electrons *with spin down* is presented in Ref.16. Although being correct, it does not apply here.

The success of the numerical solutions of the TDSE in Refs.1 and 2, as well as of the analytic developments presented here, revolves on whether the experimental setups\(^{1,2}\) actually result in having only a single electron at any given time in the double-slit arrangement. In the space double-slit experiment this is achieved by a very low flux (”the average interval of successive electrons is 1.5 m. In addition, the length of the electron wave packet is as short as \( \sim 1 \mu \text{m} \)”\(^{3,22}\) In the time double-slit experiments, photoionization is induced by two
time-delayed femtosecond laser pulses ("So far the free interfering electrons are originating neither from double ionization of one atom nor from single ionization of different atoms"\(^2\)) or by phase stabilized few-cycle laser pulses of femtosecond duration that open one to two windows (slits) of attosecond duration ("The temporal slits leading to electrons of given final momentum are spaced by approximately the optical period"\(^1\)). It is then claimed that the wave packets thus generated have to be considered as one double peaked free electron wave packet, as has been assumed in this paper. Indeed one is considering a single wave packet amplitude that happens to have initially two peaks that do not overlap; in the plane perpendicular to the direction of motion in the case of the space double slit; in the direction of motion in the case of the time double slit. These peaks spread in time and will eventually overlap and cause the probability density to oscillate as a function of time at any space location. This is supported by the results of the experiments and constitutes an extraordinary technical achievement.

Finally, it is also shown (Section III) that the progressive closing of one of the space slits (or time slits) results in the progressive disappearance of the interference pattern as the "which-path" ("which-time") information is affirmed. This is in agreement with experiments that have indeed revealed the possibility of partial fringe visibility and partial which-path information (\(^{23-25}\) and references therein).

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V. **BIBLIOGRAPHY**

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Figure Captions

Figure 1: Ratio of the transient to the stationary current at a point $Z = v_0T$, where $T$ is the classical arrival time.

Figure 2: Transient response at $Z=1626$ nm from the classical time of arrival $T(5000$ fs) to $2T$ ($E_0 = 0.3$ eV, $\tau = 120$ fs)