Nuclear Physics as Precision Science

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Abstract. Theoretical Nuclear Physics has entered a new era. Using the powerful machinery of chiral effective Lagrangians, the forces between two, three and four nucleons can now be calculated with unprecedented precision and with reliable uncertainties. Furthermore, Monte Carlo methods can be adopted to serve as a new and powerful approach to exactly solve problems in nuclear structure and reactions. I discuss the foundations of these new methods and provide a variety of intriguing examples. Variations of the fundamental constants of Nature can also be investigated and the consequences for the element generation in the Big Bang and in stars are considered. This sheds new light on our anthropic view of the Universe.

1. Short introduction
Nuclear physics has entered a new era, triggered by the work of Weinberg [1, 2]. The powerful machinery of chiral effective Lagrangians has given a completely new and systematic approach to nuclear forces and external currents. When combined with standard many-body techniques, these forces can also be used to address systematically topics in nuclear structure and reaction physics, see e.g. [3, 4, 5, 6, 7, 8]. In this talk, I will discuss a complementary and novel approach to investigate nuclear physics based on combining effective field theory (EFT) methods with powerful Monte Carlo simulation methods, called nuclear lattice EFT (NLEFT) or nuclear lattice simulations. As I will show, systematic and precise calculations become possible, providing also the much needed theoretical uncertainty, which is very difficult to estimate based on meson-exchange models or alike. This is important because theory much like experiment must provide uncertainties.

Before discussing these new developments, let me summarize a few basic facts about nuclear physics that underlie these developments. Atomic nuclei are self-bound systems of fermions, namely protons and neutrons, collectively called nucleons. The nucleons are bound by the strong force, that is balanced by the repulsion between the positively charged protons. Nuclei constitute non-relativistic systems that can be described by a nuclear Hamiltonian $H_{\text{nucl}} = T + V$, with $T$ the kinetic energy operator and $V$ the potential. The latter is given by a sum of terms, $V = V_{\text{NN}} + V_{\text{NNN}} + \ldots$. For small and medium mass nuclei, the two-nucleon potential $V_{\text{NN}}$ provides the largest part of the binding, but a small three-nucleon force $V_{\text{NNN}}$ is required, see e.g. the review [9]. Furthermore, it is important to realize that nuclear binding energies are much smaller than nuclear masses, at most $\sim 8.5$ MeV per nucleon in the Fe mass region. Chiral EFT allows one to systematically analyze the nuclear Hamiltonian based on the symmetries of the underlying gauge theory of the strong interactions, QCD.
2. **Effective field theory for nuclear interactions**

Let me first discuss a few new developments in the chiral EFT for nuclear forces and currents. For earlier work, I refer to the detailed review [10]. Following Weinberg, the nuclear potential is expanded in small momenta and/or pion masses making use of the power counting of chiral perturbation theory to leading order (LO), next-to-leading order (NLO) and so on. Three-nucleon forces appear at N2LO, which explains their smallness and four-nucleon forces start to contribute only at N3LO. There are two types of low-energy constants (LECs) that appear. The first set is related to pion-nucleon scattering. These LECs have recently been determined to high precision [11]. The other type of LECs that accompany the multi-nucleon contact interactions must be fixed by a fit to NN (and NNN) data. A refined analysis of the two-nucleon forces based on a combination of coordinate- and momentum space regulators at next-to-next-to-next-to-leading order (N3LO) in the expansion of the NN potential combined with a novel method to estimate the theoretical uncertainties was given in [12], for an application in few-nucleon systems, see [13]. The calculation of the forces was further refined to N4LO in [14]. At this order, no new LECs are involved and a very precise and systematically improved description of the NN phase shifts is obtained, c.f. Fig. 1. For related work on the peripheral phases, see [15]. It is obvious that a precise description with small uncertainties has been obtained. The more demanding task of working out the N3LO 3N forces based on [16, 17] and the N4LO forces from [18, 19] is presently under way by the LENPIC collaboration.

3. **The framework of nuclear lattice simulations**

NLEFT is a new method to tackle the nuclear many-body problem. Euclidean space-time is represented by a discrete hyper-cubic volume, $V = L \times L \times L \times L_t$, with spatial (temporal) length $L$ ($L_t$) and corresponding lattice spacings $a$ and $a_t$, respectively. The nucleons are placed on the lattice sites, see Fig. 2 (left panel). The interactions between the nucleons are given by the same chiral EFT potentials as in the continuum, simply adapted to the lattice formulation, see e.g. [20]. The Coulomb interaction between the protons can also straightforwardly be included [21]. A finite lattice spacing entails an UV cut-off, as the maximal momentum is the given by $p_{\text{max}} = \pi/a$. For the most commonly used value of the lattice spacing, $a \simeq 2\text{ fm}$, one has $p_{\text{max}} = 314\text{ MeV}$, which corresponds to a very soft interaction. However, it can be shown that the physics in the two-nucleon system does not depend on the lattice spacing if varied from 1 to 2 fm [22]. Monte Carlo methods can then be used to numerically exactly solve the $A$-body problem for a
Figure 2. Left panel: Neutrons and protons on a space-time lattice with spatial length \( L \) and lattice spacing \( a \). Right panel: Evolution of a \(^4\)He nucleus in Euclidean time.

given set of NN and NNN interactions. A very important ingredient in these simulations is the approximate Wigner SU(4) symmetry of the nuclear interactions, that is crucial in suppressing the malicious sign oscillations that plague fermion MC studies at finite baryonic density [23, 24]. The remaining sign oscillations are caused by SU(4) non-symmetric contact terms as well as the one-pion-exchange. For more details, see the review [25].

The central object of NLEFT is the \( A \)-nucleon correlation function, \( Z_A(t) = \langle \Psi_A | \exp(-tH) | \Psi_A \rangle \), with \( t \) the Euclidean time and \( \Psi_A \) a Slater determinant of \( A \) free nucleons or a more sophisticated correlated initial/final state. From the transient energy \( E_A(t) = -d \ln Z_A(t) / dt \) one can infer the ground state energy of the \( A \)-nucleon system via \( E_A^0 = \lim_{t \to \infty} E_A(t) \). Similarly, the expectation value of any normal-ordered operator follows from \( Z^O_A(t) = \langle \Psi_A | \exp(-tH/2) O \exp(-tH/2) | \Psi_A \rangle \) in the limit of infinite Euclidean time, \( \lim_{t \to \infty}(Z^O_A(t)/Z_A(t)) = \langle \Psi_A | O | \Psi_A \rangle \). Excited state properties can also be extracted. In order to compute the low-lying excited states of a given nucleus, the Euclidean time projection method is generalized to a multi-channel calculation [26]. The Euclidean time evolution of a \(^4\)He nucleus is depicted in the right panel of Fig. 2. Initial states are either properly antisymmetrized free standing waves of four particles or more complex correlated configurations. With the help of auxiliary fields, the multi-nucleon interactions and the pion exchanges can be mapped onto insertions on a single nucleon world-line, which makes such a computation most accessible for parallel computing. One major advantage of this approach is that all possible configurations are sampled, in particular also four nucleons on one lattice site. This already lets one suspect that clustering will emerge naturally in this approach.

4. Results from nuclear lattice simulations

4.1. General remarks

Before discussing results obtained using NLEFT, a few general remarks are in order. It is important to realize that nuclear structure and reactions dynamics should be treated on the same footing. This has important implications for the simulations. While originally all LECs have been determined in few-nucleon systems, which has led to a number of intriguing results, it was realized later that nucleus-nucleus collisions should also be used for determining some LECs as this appears to be advantageous in pinning down more precisely the three- and higher-body forces. Furthermore, the framework of nuclear lattice simulations could only be established as a novel quantum many-body method since one was able to solve problems that before could not be mastered in the well established schemes like e.g. the no-core-shell model.
Most results in NLEFT have been obtained with an NNLO action that involved a Gaussian smearing of the two LO contact interactions, with the smearing parameter fixed from the average S-wave np effective range. The canonical lattice had a coarse lattice spacing of $a = 1.97$ fm and $L \approx 10\ldots16$ fm depending on the nucleus or system under investigation. For such a coarse lattice, the NLO and NNLO corrections can be treated as perturbations, in particular, the contribution from the two-pion exchange can be absorbed in the LECs of the 4N operators. At this order, one has 11 LECs related to np, nn and pp scattering as well as two 3N LECs. The 2N LECs were determined from fits to np phase shifts using the spherical wall method [27] and its refinement [28] as well as the nn and pp scattering lengths. The 3N LECs were determined from a fit to the triton binding energy and the spin-doublet neutron-deuteron scattering phase shift. The first non-trivial prediction is then the $^3$He-$^3$H binding energy difference [29, 21], see the first entry in Tab. 1. The overall improvement in the algorithms over the years allowed to substantially decrease the uncertainty in the ground state energies of alpha-cluster nuclei as listed in Tab. 1, see [30, 31].

| E [MeV] | $^3$He-$^3$H | $^4$He | $^8$Be | $^{12}$C | $^{16}$O | $^{20}$Ne | $^{24}$Mg | $^{28}$Si |
|-------|-------------|-------|-------|-------|-------|-------|-------|-------|
| NLEFT | 0.78(5) | -28.3(6) | -55.2(2) | -92.3(3) | -131.1(1) | -166.1(1) | -198.2(2) | -234.3(3) |
| Exp.  | 0.76 | -28.3 | -56.5 | -92.2 | -127.6 | -160.6 | -198.3 | -236.5 |

Table 1. Ground state energies at NNLO. The improvement in the simulations is shown in the much decreased uncertainty of the heavier nuclei based on the new algorithm compared to the older calculations for the lighter nuclei up to carbon [29, 21].

Similarly, excited states can be computed with similar accuracy. In Fig. 3 the LO calculation of the first two $0^+$ states in $^{12}$C is shown, starting from various initial states (plane waves and alpha cluster states) [32]. One set of these initial states directly gives the ground state (left panel), whereas the other set first traces out the first excitation with the same quantum numbers as shown by the intermediate plateau (right panel). This is the famous Hoyle state [33]. The thermalization of the various initial states with growing Euclidean time to almost the same energy gives a handle on the systematic uncertainties inherent to the simulations. For more details, see e.g. [31].

Using this framework, a number of interesting results has been obtained, such as the first *ab initio* calculation of the Hoyle state in $^{12}$C [26, 32], the study of the triple-alpha process under variations of some fundamental constants [34, 35], the calculation of the ground state energies of the alpha-cluster nuclei up to $^{28}$Si with an accuracy of about 1% [30], an *ab initio* calculation of the spectrum and structure of $^{16}$O [36], and the first ever microscopic calculation of alpha-alpha scattering [37]. However, the employed NNLO action works well for alpha-type nuclei, but is less precise for other systems. Therefore, new forms of smearing including also the pion-exchange as well as a non-local distribution of lattice creation and annihilation operators have been employed to gain further insight. Based on these improved LO actions, it was found that nuclear physics is near a quantum phase transition from a Bose gas to the nuclear liquid, where first alpha-cluster nuclei are formed [38]. Furthermore, isotopic chains from H to O could be calculated and new insights into nuclear clustering was obtained recently, including also a new algorithm that for the first time allows to calculate density distributions in nuclei and the corresponding form factors [39]. Some selected topics from this rich spectrum of results will be discussed in what follows. Most of these results have been obtained on supercomputers like JUGENE and JUQUEEN at the Forschungszentrum Jülich. The CPU scaling is approximately quadratic in atomic number, so nuclei up to $A \approx 30$ have been investigated. Going to larger nuclei requires more fine-tuned actions to suppress the remaining sign oscillations.
Figure 3. Results for the lowest $0^+$ states in $^{12}$C at LO. The left panel shows the results using various initial states, each of which approaches the ground state energy with increasing Euclidean time $t$. The right panel shows the results using other initial states. These trace out an intermediate plateau at an energy $\sim 7$ MeV above the ground state. Note that the respective energies are extracted from the large $t$ behaviour, and not from any plateau as often done in lattice QCD.

4.2. Ab initio calculation of alpha-alpha scattering

Let us now consider $\alpha-\alpha$ scattering as a prototypical nuclear reaction. This is related to the facts that processes involving $\alpha$-type nuclei comprise a major part of stellar nucleosynthesis, and control the production of certain elements in stars. Also, ab initio calculations of scattering and reactions suffer from exponential or factorial scaling with the number of nucleons in the clusters, so therefore it was not possible so far to perform an ab initio calculation of $\alpha-\alpha$ scattering. It is therefore a challenging task to use the lattice to tackle such type of processes. We note that on the lattice one only has discrete energy levels, and therefore a direct calculation of scattering processes appears impossible. This hurdle can be overcome by the so-called adiabatic projection method, that splits the problem of the calculation of scattering and inelastic reactions into two parts. First, using the Euclidean time projection method, one constructs an low-energy cluster Hamiltonian, called the adiabatic Hamiltonian. In the second step, one then computes scattering phase shifts or reaction amplitudes using this adiabatic Hamiltonian. The method was developed and refined in [40, 41, 42, 43, 44] and resembles in the methodology the Hamiltonian matrix approach combining the no-core-shell model with the resonating group method, see e.g. [45, 46, 47]. In more detail, the construction a low-energy effective theory for clusters proceeds as follows: One uses initial states as a direct product of two clusters located on the lattice, parameterized by the relative separation between the clusters, as shown in the left panel of Fig. 4

$$|\vec{R}\rangle = \sum_{\vec{r}} |\vec{r} + \vec{R}\rangle \otimes \vec{r}. $$

These are projected in Euclidean time with the chiral EFT Hamiltonian $H$, $|\vec{R}\rangle_{\tau} = \exp(-H\tau)|\vec{R}\rangle$. These so-called dressed cluster states include all possible interaction effects such a polarizations as well as deformations and, of course, the Pauli principle. The adiabatic Hamiltonian is then given by $[H_{\tau}]_{\vec{R}\vec{R}} = \tau \langle \vec{R}\vert H\vert \vec{R}\rangle_{\tau}$. In general, this Hamiltonian needs to be normalized, which requires left and right multiplication with the corresponding norm matrices. What concerns the strong interactions, it can be shown that asymptotically, the adiabatic Hamiltonian is nothing but the free Hamiltonian for two clusters, eventually supplemented by infinite-range interactions as the Coulomb one. The underlying simulations can be simplified considerably by employing the so-called radial Hamiltonian based on the lattice version of
angular momentum projection and binning the lattice points in rings of a given width. Further, the long-range Coulomb interaction can also be included exactly. For that, one performs first simulations in small box with a volume \( L'^3 \sim (16 \text{ fm})^3 \), with all interactions switched on. This is the supplemented by a second set of simulations in a large box with a volume of about \( L^3 \sim (120 \text{ fm})^3 \), where the strong interactions are turned off and the long-range Coulomb interaction is included by imposing Coulomb boundary conditions on a spherical wall with radius \( R_W \approx 40 \text{ fm} \), see the right panel of Fig. 4. In that way, all effects of the strong and the electromagnetic interactions are included.

Using the same NNLO Hamiltonian as for the studies of the spectrum and structure of \(^{12}\text{C}\) and \(^{16}\text{O}\), the S- and D-wave phase shifts have been computed in [37], as shown in Fig. 5. At LO in the employed counting, the Coulomb interaction is not included, so both the S- and D-wave phase shifts are off the data. This is visibly improved at NLO and further at NNLO for the D-wave. The small NNLO corrections in the S-wave are due to the coarse lattice spacing. Overall, one finds a good description of the scattering data. In the S-wave, we find a bound state corresponding to \(^8\text{Be}\) that is bound by \(-0.11(1) \text{ MeV}\), whereas in nature this nucleus is unbound by \(+0.09 \text{ MeV}\). This deviation of about 200 keV reflects the precision of the calculation. In the D-wave at NNLO, the resonance parameters are \( E_{R}^{\text{NNLO}} = 3.27(12) \text{ MeV} \) and \( \Gamma_{R}^{\text{NNLO}} = 2.09(16) \text{ MeV} \), not far off the empirical data of 2.92(18) MeV and 1.35(50) MeV, respectively. May be the most significant result of this study is the fact that the computational time scales quadratically with the number of nucleons in the two clusters, \( t_{\text{CPU}} \sim (A_1 + A_2)^2 \), with \( A_i \) the number of nucleons in cluster \( i \) (\( i = 1, 2 \)). This means that the computational time for the so-called holy grail of nuclear astrophysics, the radiative alpha capture on \(^{12}\text{C}\) at stellar energies (given by the Gamov peak), \( \alpha + ^{12}\text{C} \rightarrow ^{16}\text{O} + \gamma \), is in reach, requiring only 8 times as much CPU time as the computation of elastic \( \alpha-\alpha \) scattering (twice the number of nucleons and two channels). Before doing that, however, the chiral forces should be worked out to N3LO so as to reach the required accuracy.

4.3. New insights into nuclear clustering

Clustering in nuclei is an old but ever fascinating topic, introduced by Wheeler in 1937 in this seminal paper on “Molecular Viewpoints in Nuclear Structure” [49]. The most prominent type of
As already mentioned above, alpha-clustering emerges naturally in NLEFT and a number of intriguing results on alpha-type nuclei and clustering have already been obtained, such as the first ab initio calculation of the Hoyle state or the observation that nuclear physics is close to a quantum phase transition from a Bose gas of $\alpha$’s to a nuclear liquid for $\alpha$-type nuclei. However, when adding extra neutrons and/or protons, the precision of the calculations quickly deteriorates due to the remaining sign oscillations. To overcome this, a new LO action with smeared SU(4) local and non-local symmetric contact interactions as well as smeared one-pion exchange was constructed in [39]. The non-local smearing distributes any nucleon creation and/or annihilation operator over the six neighboring lattice sites as depicted in the left panel of Fig. 6, $a^{(t)}_{\text{NL}}(n) = a^{(t)}(n) + s_{\text{NL}} \sum_{n'} a^{(t)}(n')$, where $s_{\text{NL}}$ is a real parameter, and the notation $\sum_{n'}$ represents the summation over nearest-neighbor lattice sites of the site $n$. While this smearing was originally designed to just suppress the remaining sign oscillations when extra neutrons and/or protons are added to alpha-type nuclei, it turned out to work much better. For that, consider a LO action that is SU(4) symmetric with local and non-local smearing as well as smeared one-pion exchange. This action has three LECs, the strength of the SU(4)-symmetric contact term, the parameter related to the local smearing and the above-mentioned $s_{\text{NL}}$. Fitting these to the average np S-wave scattering lengths and effective ranges and also the $\alpha-\alpha$ S-wave scattering length, one can predict the isotopes chain from hydrogen to oxygen as shown in the right panel of Fig. 6. These have an accuracy of 0.7 MeV per nucleon or better. This is quite amazing given this highly simplified LO action. Clearly, NLO effects (and higher orders) need to be accounted for to achieve e.g. a better description of the $^1S_0$ np phase.

Using this action, one can also obtain deeper insight into nuclear clustering. For that, define as probes of alpha clusters the quantities $\rho_4 = \sum_n : \rho^4(n) / 4! :$ and $\rho_3 = \sum_n : \rho^3(n) / 3! :$. Here, $\rho_4$ couples to the center of the $\alpha$-cluster while $\rho_3$ gets contributions from a wider portion of the alpha-particle wave function and thus these can be used for “measuring” cluster properties. Note that $\rho_3$ and $\rho_4$ depend on the regulator, the lattice spacing $a$, but not on the nucleus. However, the ratios $\rho_3 / \rho_3,\alpha$ and $\rho_4 / \rho_4,\alpha$ are free of short-distance ambiguities. If properly defined, the effective number of alpha clusters should be greater than or equal to $N_\alpha$. A value equal to

| $\delta_0$ (degrees) | $\delta_2$ (degrees) |
|----------------------|----------------------|
| 0                    | 0                    |
| 2                    | 4                    |
| 4                    | 8                    |
| 6                    | 10                   |
| 8                    | 12                   |
| 10                   | 12                   |
| 12                   | 12                   |

**Figure 5.** Left panel: S-wave $\alpha-\alpha$ phase shift $\delta_0$. Right panel: D-wave $\alpha-\alpha$ phase shift $\delta_2$. Shown are the NLEFT LO (green triangles), NLO (blue circles) and NNLO (red squares) results. The data (black triangles with error bars) are from [48].
**Figure 6.** Left panel: Two-dimensional illustration of the non-local smearing of a nucleon creation/annihilation operator over the neighboring lattice sites. Right panel: Ground state energies versus number of nucleons $A$ for the hydrogen, helium, beryllium, carbon, and oxygen isotopes (NLEFT: squares with error bars, experiment: circles). The errors are one-standard deviation error bars associated with the stochastic errors and the extrapolation to an infinite number of time steps.

$N_\alpha$ indicates that the alpha clusters are behaving as indivisible objects, and the nucleus can be regarded as a compound fluid of alpha particles and neutrons. If the effective number is significantly greater than $N_\alpha$, then the description in terms of individual alpha clusters breaks down and the system behaves more as a nuclear liquid of protons and neutrons. The behavior is shown in the left panel of Fig. 7, where it is seen that for the oxygen isotope chain the entanglement between the clusters leads to expectation values of $\rho_3/\rho_{3,\alpha}$ and $\rho_4/\rho_{4,\alpha}$ much larger than 4. This shows that the transition from cluster-like states in light systems to nuclear liquid-like states in heavier systems should not be viewed as a simple suppression of multi-nucleon short-distance correlations, but rather as an increasing entanglement of the nucleons involved in the multi-nucleon correlations.

Another important development of Ref. [39] was the formulation of the so-called pinhole algorithm, see the right panel of Fig. 7. In general, auxiliary field quantum MC calculations involve states that are superpositions of many different center-of-mass positions, so a direct calculation of density distributions of nucleons in a nucleus is not possible. This can be overcome by inserting a screen with pinholes with spin and isospin labels that allows nucleons...
with corresponding spin and isospin to pass. In that way, one measures the $A$-body density operator $\rho_{i_1,j_1,\cdots, i_A,j_A}(n_1,\cdots,n_A) = \rho_{i_1,j_1} \cdots \rho_{i_A,j_A}$. MC sampling of the amplitude $A_{i_1,j_1,\cdots, i_A,j_A}(n_1,\ldots,n_A,L_i) = |\psi(\tau/2)|\rho_{i_1,j_1,\cdots, i_A,j_A}(n_1,\ldots,n_A)|\psi(\tau/2)|$ then allows to measure the proton and neutron densities as well as more complicated two-, three- or higher-body correlations of nucleons within a given nucleus. This is because the pinhole sheet allows one to determine the center-of-mass (cm) of a given nucleus given simply by the minimal distance to all nucleons. Further, the resolution of this method is $a/A$ because the cm position $r_{cm}$ is an integer $n_{cm}$ times $a/A$. Results for the proton and neutron distributions in the isotopes $^{12,14,16}$C are shown in the left panel of Fig. 8. The proton size of $r^p_{cm} = 0.84$ fm is accounted for and asymptotic properties for the volume dependence of N-body bound states [54] have been used. Upon Fourier-transformation of these densities, one can obtain the corresponding elastic form factor. This is shown in the right panel of Fig. 8 for $^{12}$C. Given the simplicity of the underlying Hamiltonian, the agreement is quite satisfactory. This paves the way for detailed nuclear structure studies.

![Figure 8](image_url)

**Figure 8.** Left panel: Proton and neutron densities for the ground states of $^{12,14,16}$C versus radial distance for MC data with $L_t = 9, 11, 13, 15$ time steps. The experimental data are from [53]. Right panel: Elastic form factor of $^{12}$C. Data: black open symbols.

### 4.4. Fine-tunings and the multiverse

In nuclear physics, we observe a number of so-called fine-tunings, for some reviews and recent works, see e.g. [56, 57, 58, 59, 60]. A prominent example is the lightest nucleus, the deuteron. It is bound by a mere 2 MeV, just one percent of its total mass. Also, the aforementioned Hoyle state must be very closely placed to the triple-alpha threshold, in nature the energy difference is just 380 keV, much less than typical nuclear excitation energies of a few MeV. This close proximity is required so that in hot, old stars a sufficient amount of carbon and also oxygen is generated [33]. It is therefore natural to ask how much the fundamental parameters of the Standard Model can be detuned so that this resonance condition is no longer viable? First, however, we must find out what the relevant parameters are. Nuclear binding is a delicate balance between the attractive strong and the repulsive electromagnetic interactions. The latter are given in terms of Sommerfeld’s fine-structure constant, $\alpha_{EM} \simeq 1/137$. As concerns the strong interaction, the strong coupling constant $\alpha_S$ is intimately tied to the nucleon mass because of dimensional transmutation, and therefore the small light quark masses $m_u, m_d$ are the relevant parameters that control nuclear binding. This appears at first counter-intuitive, as the major part of the nucleon mass is given by gluon field energy by means of the trace anomaly, while
Figure 9. Left panel: Schematic drawing of the quark mass dependence in the two-nucleon force. Right panel: “Survivability bands” for carbon-oxygen based life due to 0.5% (broad outer band), 1% (medium band) and 5% (narrow inner band) changes in $m_q$ in terms of the input parameters $\tilde{A}_s$ and $\tilde{A}_t$. The black cross denotes the results of the N2LO analysis from [66].

the light quark contribution to the nucleon mass is given by the so-called pion-nucleon $\sigma$-term, $\sigma_{\pi N} = 59(3)$ MeV [55]. However, the quark mass values of a few MeV (which are, of course, scale- and scheme-dependent) are of the same size as the nuclear binding energy per nucleon, $E/A$, so that these are the pertinent strong interaction parameters. To be more precise, the rate of the triple-alpha process is given by $r_{3\alpha} \sim \Gamma_\gamma \exp(-\Delta E/kT)$, with $k$ the Boltzmann constant, $T$ the temperature, $\Gamma_\gamma$ the width of the Hoyle state and $\Delta E = E^{*2}_{12} - 3E_\alpha = 379.47(18)$ keV, with $E^*$ the energy of the Hoyle state. The question now is how much $\Delta E$ be changed so that there is still enough $^{12}\text{C}$ and $^{16}\text{O}$ produced in the stars? This was answered in a calculation of the element generation in stars by varying $\Delta E$ but no other parameter. It turned out that the allowed variation is $\delta|\Delta E| \lesssim 100$ keV [61, 62], which does not appear to be any form of fine-tuning. However, one still has to make the connection to the fundamental parameters of the Standard Model. While it can be done for the electromagnetic interactions in cluster-type models as used e.g. in [61], the variation of the quark masses requires a more microscopic framework as provided by chiral EFT. This is depicted for the quark mass dependence of the LO NN force in the left panel of Fig. 9. Here, quark mass and pion mass dependence can be used synonymously, as the Gell-Mann–Oakes–Renner relation, $M_\pi^2 \sim m_u + m_d$, is fulfilled to better than 94% in QCD [63]. As can be seen from this figure, there are explicit (through the pion propagator) and implicit (through the pion-nucleon coupling, the nucleon mass and the four-nucleon couplings) pion mass dependences. All this can be accounted for systematically and precisely using chiral EFT. Coming back to the triple alpha-process, nuclear lattice simulations are the appropriate tool to study its dependence on the fundamental parameters, for details see [34, 35]. For that, one has to translate the condition $\delta|\Delta E| \lesssim 100$ keV into a constraint for the quark masses (and similarly for the fine-structure constant). For the quark masses, it reads (for fixed $\alpha_{EM}$)

$$\left|0.571(14)\tilde{A}_s + 0.934(11)\tilde{A}_t - 0.069(6)\right| \frac{\delta m_q}{m_q} < 0.0015,$$

with $m_q = (m_u + m_d)/2$ the average light quark mass (as strong isospin breaking plays no role here), $\tilde{A}_{s,t} \equiv \partial a_{s,t}/\partial M_\pi|_{M_{\pi}^{phys}}$, where $a_s, a_t$ denotes the singlet and the triplet NN scattering length, respectively. Independently of the precise values of these two quantities, it can be shown that the various fine-tunings in the triple-alpha process (the closeness of the $^8\text{Be}$ binding energy to the $2\alpha$ threshold and the closeness of the Hoyle state to the $3\alpha$ threshold) are indeed correlated. This had been speculated before [64] but could only be worked out precisely using NLEFT. Bounds on $\tilde{A}_{s,t}$ had been obtained earlier based on resonance saturation of 4N operators [65] in [66] (see also [67]) as shown by the black cross in the right panel of Fig. 9. The fairly
large uncertainty can eventually be overcome using lattice QCD to calculate these quantities. In the plane of $\bar{A}_s - \bar{A}_t$, varying the quark mass leads to diagonal bands whose widths depend on the assumed variations. This is shown for variations of $\delta m_q/m_q$ of 0.5, 1 and 5% by the three different bands. Clearly, a smaller variation leads to a broader band. If one focuses on the central value of $\bar{A}_{s,t}$, one finds that variations of $2 - 3\%$ of $m_q$ are allowed so that the abovementioned condition is fulfilled. The large uncertainties in $\bar{A}_{s,t}$ do not allow for a more precise statement. For $\alpha_{EM}$, no such uncertainties are present and it can be stated with certainty that it can be varied by at most 2.5%. Also, no other bounds are found if one varies both the quark masses and the fine-structure constant at the same time. This is clearly a stronger fine-tuning as for $|\Delta E|$ and its consequences for our anthropic view of the Universe are discussed in [68]. Lattice QCD can be used to tighten the bounds on $\bar{A}_{s,t}$, for the state-of-the-art see [69].

5. Summary and outlook

Let me briefly summarize the main messages of this talk:

- Chiral EFT for nuclear forces provides a precise framework for 2N and 3N forces with small uncertainties. It can also be formulated for varying strong and electromagnetic forces, which is a necessary requirement to study fine-tunings in nuclear physics.
- Nuclear lattice simulations are a new quantum many-body approach that is based on the successful continuum nuclear chiral EFT. Already a number of intriguing results have been obtained based on NLEFT. In particular clustering emerges naturally and $\alpha$-cluster nuclei are well described. Further, with an improved chiral action based on non-local smearing, neutron- and proton-rich nuclei can also be studied. With the invention of the pinhole algorithm, the calculation of charge densities and form factors has become possible. Furthermore, fine-tuning in nuclear reactions can be studied.
- Various bridges to lattice QCD studies need to be explored, in particular in pinning down some of the LECs related to multi-nucleon forces or the quark mass dependence of multi-nucleon operators.
- Finally, it must be said that many open issues in nuclear structure and reaction physics can now be addressed in a truly quantitative manner. For example, the “holy grail” of nuclear astrophysics [70], the \textit{ab initio calculation} of the reaction $^4\text{He}+^{12}\text{C} \to ^{16}\text{O} + \gamma$, is in reach.

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