Asymptotic properties of high-$p_T$ particle production in hadron-hadron, hadron-nucleus and nucleus-nucleus collisions at high energies

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Abstract

The concept of $z$-scaling reflecting the general features of internal particle substructure, constituent interaction and mechanism of real particle formation is reviewed. Experimental data on the cross sections obtained at ISR, SpS and Tevatron are used in the analysis. The properties of data $z$-presentation, the energy and angular independencies, the power law, $A$- and $F$-dependencies, are discussed. It is argued that the properties reflect the fundamental symmetries such as self-similarity, locality, fractality and scale-relativity. The use of $z$-scaling to search for new physics phenomena in hadron-hadron, hadron-nucleus and nucleus-nucleus collisions is suggested. The violation of $z$-scaling characterized by the change of the fractal dimension is considered as a new and complimentary signature of nuclear phase transition.

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1 Introduction

Asymptotic properties of particle formation are believed to reveal themselves more clearly at high energy $\sqrt{s}$ and transverse momenta. It is considered also that partons produced in hard scattering retain information about primary collision during hadronization. The mechanism of particle formation can be modified by nuclear environment and can be sensitive to the phase transition as well. Therefore, the features of single inclusive particle spectra of hadron-hadron, hadron-nucleus and nucleus-nucleus collisions are of interest to search for new physics phenomena, for example the phase transition of nuclear matter, at extremal conditions and a quantitative test of theory.

One of the methods to study the properties of particle formation in nuclear matter is to search for the violation of known regularities, e.g. the Bjorken, Feynman scaling laws and quark counting rules [1, 2], established in elementary collisions ($l-p, p-p, \bar{p}-p$, etc.).

In the report the general concept of the new scaling, $z$-scaling, [3, 4, 5, 6] is reviewed. The scaling reflects the properties of particle formation over a high $p_T$ range in $h-h$, $h-A$ and $A-A$ collisions at high energies.

The scaling was proposed in [3] to describe the feature of charged hadron produced in $p-p$ and $\bar{p}-p$ collisions. The idea of the scaling was developed for the analysis of direct photon production in $p-p$ [7], $\bar{p}-p$ [8] and $p-A$ [8] collisions. The scaling properties of jet production in $\bar{p}-p$ and $p-p$ collisions were analyzed in [10]. The scaling features of $\pi^0$-meson production in $p-p$, $p-A$ and $A-A$ collisions were established in [11, 12] and charged and neutral hadrons produced in $\pi-p$ and $\pi-A$ collisions were studied in [14]. The $z$-scaling relevance to the fractal structure of space-time itself was discussed in [13, 15].

The general concept of the scaling is based on the fundamental principles of self-similarity, locality, fractality and scale-relativity. The first one reflects the dropping of certain dimensional quantities or parameters out of the physical picture of the interactions. The second principle concludes that the momentum-energy conservation law is locally valid for the interacting constituents. The fractality principle says that both the structure of interacting particles and their formation are self-similar revealing properties of fractals at any scale. The fourth one, the scale relativity principle, states that the self-similar and fractal substructures in the interaction obey relativistic principle concerning various scales [13, 16].

As shown in [3, 4, 5, 6], the $z$-presentation of experimental data can be obtained using the experimental quantities. They are the inclusive cross section $E d^3s/dq^3$ and the multiplicity density of charged particles $\rho(s, \eta)$. The scaling function $\psi(z)$ is found to be independent of the center-of-mass energy $\sqrt{s}$ and the angle $\theta$ of the produced particle over a wide kinematic range. The function $\psi(z)$ describes the probability density to form a particle with a formation length $z$. The scaling variable $z = z_0 e^{-\delta}$ reveals the property of a fractal measure, where $\epsilon$ is the scale resolution, and $\delta$ was interpreted as the fractal dimension in the particle formation process. It was shown [8] that, in the framework of $z$-presentation, the A-dependence of high $q_T$ hadron production is described by the function $\alpha$ depending on the single parameter, the atomic weight A. The existence of the scaling and its properties is assumed to reflect the fundamental features of particle structure, constituent interaction and particle production such as self-similarity, locality, fractality and scale-relativity.
2 Z-scaling

The idea of z-scaling \[3\] is based on the assumptions that inclusive particle distribution of the process (1) at high energies can be described in terms of the corresponding kinematic characteristics

\[ P_1 + P_2 \rightarrow q + X \]  

of the exclusive sub-process \[17\] written in the symbolic form (2)

\[(x_1M_1) + (x_2M_2) \rightarrow m_1 + (x_1M_1 + x_2M_2 + m_2)\]  

and that the scaling function depending on a single variable \(z\) exists and is expressed via the dynamic quantities, invariant inclusive cross section \(E d^3 \sigma / dq^3\) of the process (1) and particle multiplicity density \(\rho(s, \eta)\). The kinematic quantities of the process (1) are \(P_1, P_2, q\) and \(M_1, M_2, m_1\), the momenta and masses of the colliding objects (hadron, nuclei) and inclusive particles, respectively. The parameter \(m_2\) is introduced to satisfy the internal conservation laws (for isospin, baryon number, and strangeness). The \(x_1\) and \(x_2\) are the scale-invariant fractions of the incoming four-momenta \(P_1\) and \(P_2\). They determine the minimum energy, which is necessary for the production of the secondary particle with the mass \(m_1\) and the four-momentum \(q\).

2.1 Fractions \(x_1\) and \(x_2\)

The elementary parton-parton collision is considered as a binary sub-process which satisfies the condition

\[(x_1P_1 + x_2P_2 - q)^2 = (x_1M_1 + x_2M_2 + m_2)^2.\]  

The equation reflects minimum recoil mass hypothesis in the elementary sub-process. To connect kinematic and structural characteristics of the interaction, the coefficient \(\Omega\) is introduced. It is chosen in the form

\[\Omega(x_1, x_2) = m(1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2},\]  

where \(m\) is a mass constant and \(\delta_1\) and \(\delta_2\) are factors relating to the fractal structure of the colliding objects \[5\]. The fractions \(x_1\) and \(x_2\) are determined to maximize the value of \(\Omega(x_1, x_2)\), simultaneously fulfilling condition (3)

\[d\Omega(x_1, x_2)/dx_1|_{x_2=x_2(x_1)} = 0.\]  

The variables \(x_1, x_2\) are equal to unity along the phase space limit and cover the full phase space accessible at any energy.

2.2 Scaling function \(\psi(z)\) and variable \(z\)

The scaling function \(\psi(z)\) is written in the form \[5\]

\[\psi(z) = \frac{\pi s_A}{\rho_A(s, \eta)\sigma_{inel}} J^{-1} E \frac{d\sigma}{dq^3}.\]  

Here \(\sigma_{inel}\) is the inelastic cross section, \(s_A \simeq s \cdot A\) and \(s\) are the center-of-mass energy squared of the corresponding \(h - A\) and \(h - N\) systems, \(A\) is the atomic weight and
$\rho_A(s, \eta)$ is the average particle multiplicity density. The factor $J$ is the known function of the kinematic variables, the momenta and masses of the colliding and produced particles [3].

We would like to emphasize that the function $\psi(z)$ depends on a single scaling variable $z$. The existence of such a solution is not evident in advance.

The expression (6) relates the differential cross section for the production of the inclusive particle $m_1$ and the average particle multiplicity density $\rho_A(s, \eta)$ with the scaling function $\psi(z)$. The function is normalized as

$$\int_{z_{\text{min}}}^{\infty} \psi(z) dz = 1.$$ (7)

The equation allows us to give the physical meaning of the function $\psi$ as a probability density to form a particle with the corresponding value of the variable $z$.

In accordance with the ansatz suggested in [3] the variable $z$ is taken in the form (8) as a simple physically meaningful variable reflecting self-similarity and fractality as a general pattern of hadron production at high energies

$$z = \frac{\sqrt{s_\perp}}{\Omega \cdot \rho_A(s)}. \quad (8)$$

Here $\sqrt{s_\perp}$ is the minimal transverse energy of colliding constituents necessary to produce a real hadron in the reaction (1). The factor $\Omega$ is given by (4) and $\rho_A(s) = \rho_A(s, \eta)|_{\eta=0}$. The transverse energy consists of two parts which represent the transverse energy of the inclusive particle and its recoil. The form of $z$ determines its variation range $(0, \infty)$. These values are scale independent and kinematically accessible at any energy.

One of the features of the procedure to construct $\psi(z)$ and $z$ described above is the joint use of the experimental quantities characterizing hard ($E d^3\sigma / d^3q^3$) and soft ($\rho_A(s, \eta)$) processes of particle interactions. Therefore, there is a real problem for a theoretical description of $z$-scaling in the framework of perturbative QCD. We would like to note that $z$-construction is not direct mathematical consequence of parton model of strong interaction but it is a new self-similarity pattern motivated by parton-parton and string-like scenarios of particle interactions.

Let us consider the definition of the variable $z = \sqrt{s_\perp} / (\Omega \rho_A)$ more closely and clarify its physical meaning. The value $\sqrt{s_\perp}$ is the minimal transverse energy of colliding constituents necessary to produce a real hadron in the reaction (1). It is assumed that two point-like and massless elementary constituents interact with each other in the initial state and convert into real hadrons in the final state. The conversion is not instant process and is called hadronization or particle formation. The microscopic space-time picture of the hadronization is not understood enough at present time. We assume that number of hadrons produced in the hard interaction of constituents is proportional to $\rho_A$. Therefore the value $\sqrt{s_\perp} / \rho_A$ corresponds to the energy density per one hadron produced in the sub-process. The factor $\Omega$ is relative number of all initial configurations containing the constituents which carry the momentum fractions $x_1$ and $x_2$. This factor thus represents a tension in the considered sub-system with respect to the whole system. Taking into account the qualitative scenario of hadron formation as a conversion of a point-like constituent into a real hadron we interpreted the variable $z$ as particle formation length.
2.3 Fractality

Fractality in particle and nuclear physics concerns the internal structure of particles, their interactions and formation of real particle. It is manifested by their self-similarity on any scale. This general principle reflects the existence of power law dependencies of the corresponding quantities. In our case the quantity $\Omega$, given by (4), connects the kinematic and fractal characteristics of the interaction and is described by the power law. As it will be shown below the scaling function $\psi(z)$ reveals the power behavior in the asymptotic region too. The factors $\delta_1$ and $\delta_2$ are fractal dimensions of the colliding objects. The fractal structure itself is defined by the structure of the interacting constituents, which is not an elementary one either. In this scheme, high energy hadron-hadron, hadron-nucleus and nucleus-nucleus interactions are considered as interactions of fractals.

In the case of collisions of asymmetric objects, the approximation for the measure $\Omega$ is written as

$$\Omega = (1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2} = (1 - \bar{x}_1)^{\delta_1}(1 - \bar{x}_2)^{\delta_2}. \quad (9)$$

The equation shows a correlation between the fraction $x_i$ and the fractal dimension $\delta_i$. (The scale transformation can be chosen so that $\bar{\delta}_1 = \bar{\delta}_2$.) Thus, the measure is an invariant under simultaneity of the scale transformation of Lorenz invariants $x_i$ and multiplicative transformation of $\delta_i$.

Taking into account the scale transformation of $x_i$ and $\delta_i$ similar to (9) the measure of the interaction $\Omega$ can be written as

$$\Omega = V^\delta, \quad (10)$$

where $\delta$ is the coefficient (fractal dimension) describing the intrinsic structure of the interaction constituents revealed at high energies. The factor $V$ is part of the full phase-space of fractions $\{x_1, x_2\}$ corresponding to such parton-parton collisions in which the inclusive particle can be produced. The scaling variable $z$ can be written in the form

$$z = z_0 \cdot V^{-\delta} \quad (11)$$

where $z_0 = \sqrt{s_\perp}/\rho_A(s)$. The variable $z$ has character of a fractal measure. The fractal property of the collision reveals itself so that only the part of all multiple scattering corresponding to the phase space $V^\delta$ produces the inclusive particle.

3 Properties of $z$-scaling

In this section we discuss properties of the $z$-scaling for particles ($\pi^{\pm,0}, K^{\pm,\circ}, \bar{p}, \gamma, \text{jet}$) produced in $h-h$, $h-A$ and $A-A$ collisions. They are the energy and angular independence of data $z$-presentation, the power law of the scaling function at very high-$p_T$, $A$- and $F$-dependence of $z$-scaling. All properties are asymptotic ones because they reveal themselves at extreme conditions (high $\sqrt{s}$ and $p_T$). Numerous experimental data on inclusive cross sections at high-$p_T$ obtained at U70 [13, 30], ISR [20-27, 31-35], [44, 55-57], SpS [36, 37, 42, 43, 48, 49, 53, 54], and Tevatron [18, 29, 38, 39, 40, 41, 45, 46, 47, 50, 51] were used in the analysis.

3.1 Energy independence

The energy independence of data $z$-presentation means that the scaling function $\psi(z)$ has the same shape for different $\sqrt{s}$ over a wide $p_T$ range.
Figures 1(a)-4(a) show the dependence of the cross section of the \( p - p \) and \( \pi - p \) interactions on transverse momentum \( q_T \) at \( \sqrt{s} = 11 - 62 \text{ GeV} \) and a produced angle of \( 90^0 \). We would like to note that the data cover a wide transverse momentum range, \( q_T = 1 - 14 \text{ GeV}/c \).

Some features of the hadron spectra should be stressed. The first one is the strong dependence of the cross section on energy \( \sqrt{s} \). The second feature is a tendency that the difference between hadron yields increases with the transverse momentum and the energy \( \sqrt{s} \). The third one is a non-exponential behavior of the spectra at \( q_T > 1 \text{ GeV}/c \).

Figures 1(b)-4(b) show \( z \)-presentation of the same data sets. Taking into account the experimental errors we can conclude that the scaling function \( \psi(z) \) demonstrates energy independence over a wide energy and transverse momentum range at \( \theta_{cm}^{hN} \approx 90^0 \).

Figures 5-8 show the energy dependence of data \( q_T \) - and \( z \) -presentation for direct-\( \gamma \) and \( \text{jet} \) production in \( p - p \) and \( \bar{p} - p \) collisions. Experimental data on cross sections were obtained at ISR \([31]-[35], [44]\), SpS \([36, 37]\) and Tevatron \([38, 39, 40, 41, 45, 46]\).

One can see that all data sets reveal the property of the energy independence of \( \psi(z) \) in \( z \)-presentation.

### 3.2 Angular independence

The angular independence of data \( z \)-presentation means that the scaling function \( \psi(z) \) has the same shape for different values of an angle \( \theta \) of produced particle over a wide \( p_T \) range and \( \sqrt{s} \). Taking into account the energy independence of \( \psi(z) \) it will be enough to verify the property at some \( \sqrt{s} \).

To analyze the angular dependence of the scaling function \( \psi(z) \) we use some data sets. The first one obtained at ISR \([24, 26]\) includes the results of measurements of the invariant cross section \( E d^3\sigma/dq^3 \) at \( \sqrt{s} = 23 \text{ GeV} \) over a momentum and angular ranges of \( q_T = 1.2 - 3. \text{ GeV}/c \) and \( \theta_{cm}^{pp} = 15^0 - 90^0 \), respectively. A strong dependence of the cross section on the angle of the produced \( \pi^0 \)-meson was experimentally found. The second one is the D0 data \([41]\) for direct \( \gamma \)'s produced in \( \bar{p} - p \) collisions at \( \sqrt{s} = 1800 \text{ GeV} \) and the rapidity range \( \eta = 0.0 - 2.5 \). The D0 data on invariant cross sections of jets production in \( \bar{p} - p \) collisions at \( \sqrt{s} = 1800 \text{ GeV} \) and \( \eta = 0. - 3. \) are taken from \([47]\).

Figures 9(a)-11(a) show the dependence of the cross section of \( \pi^0 \)-meson, direct-\( \gamma \) and \( \text{jet} \) production in \( p - p \) and \( \bar{p} - p \) collisions on transverse momentum at fixed \( \sqrt{s} \) and for different rapidity intervals.

Figures 9(b)-11(b) demonstrate \( z \)-presentation of the same data sets. The obtained results show that the function \( \psi(z) \) is independent of the angle \( \theta \) over a wide range at ISR and Tevatron energies. This is the experimental confirmation of the angular scaling of data \( z \)-presentation.

### 3.3 Power law

Here, we discuss a new feature of data \( z \)-presentation for \( \pi^0 \)-meson, direct-\( \gamma \) and \( \text{jet} \) production. This is the power law of the scaling function, \( \psi(z) \sim z^{-\beta} \).

As seen from Figures 2(b), 5(b)-8(b), 10(b) and 11(b) the data sets demonstrate a linear \( z \)-dependence of \( \psi(z) \) on the log-log scale at high \( z \). The quantity \( \beta \) is the slope parameter.

Taking into account the accuracy of the available experimental data, we can conclude that the behavior of \( \psi(z) \) for \( \pi^0 \)-mesons produced in \( p - p \) collisions reveals a power dependence and the value of the slope parameter \( \beta_{pp} \) is independent of the energy \( \sqrt{s} \).
over a wide range of high transverse momentum. The mean values of the slope parameter for $\pi^0$-meson production in $p-p$ and $\bar{p}-p$ were found to be 7.30 and 5.75, respectively.

The values of the slope parameter for direct $\gamma$ and jet production in $p-p$ and $\bar{p}-p$ collisions were found to be different ones so that $\beta_{pp} > \beta_{\bar{p}p}$.

The mean values of $\beta_{pp}^\gamma$ and $\beta_{\bar{p}p}^\gamma$ are found to be 5.91 and 5.48, respectively. Direct photons are mainly produced in $p-p$ and $\bar{p}-p$ collisions through the Compton and annihilation processes, respectively. This fact can be the main reason of different values of the slope parameters $\beta_{pp}^\gamma$ and $\beta_{\bar{p}p}^\gamma$.

The values of the slope parameter found for jet and dijet production in $\bar{p}-p$ and jet production in $p-p$ collision differ considerably each other. It give us possibility to study the features of these different processes in the same approach. The mean values of $\beta$ for jet and dijet production in $\bar{p}-p$ were found to be $5.3 \pm 0.2$ and $4.75 \pm 0.05$, respectively. The mean value of $\beta_{pp}^{jet}$ found from the $p-p$ data is equal to $5.92 \pm 0.17$. Single jets are mainly produced in softer environment than dijets. The fact can be responsible for the different values of the slope parameters $\beta_{pp}^{jet}$ and $\beta_{\bar{p}p}^{jet}$. Note that the value of $\beta_{pp}^{jet}$ is constant with the high accuracy and independent of rapidity intervals and the energy $\sqrt{s}$ for the separate data sets (see Ref. [12]).

Thus we can conclude based on the obtained results that behavior of $\psi(z)$ for $\pi^0$-meson, direct-$\gamma$ and jet production at high $z$ reveals the power dependence with high accuracy. The value of the slope parameter is independent of the colliding energy $\sqrt{s}$ and the angle of produced particles over a wide range of high transverse momentum. The existence of the power law, $\psi(z) \sim z^{-\beta}$, means, from our point of view, that the mechanism of particle formation reveals fractal behavior.

### 3.4 A-dependence

A study of $A$-dependence of particle production in $h-A$ and $A-A$ collisions is traditionally connected with nuclear matter influence on particle formation. The difference between the cross sections of particle production on free and bound nucleons is normally considered as an indication of unusual physics phenomena like EMC-effect $J/\psi$-suppression and Cronin effect [13].

A-dependence of $z$-scaling for particle production in $p-A$ collisions was studied in [8]. It was established $z$-scaling for every nuclei ($A = D-Pb$) and type of produced particles ($\pi^{\pm,0}, K^{\pm,0}, \bar{p}$). The symmetry transformation of the scaling function $\psi(z)$ and variable $z$ under the scale transformation $z \to \alpha_A z$, $\psi \to \alpha_A^{-1} \psi$ was suggested to compare the scaling function for different nuclei. It was found that $\alpha$ depends on the atomic number only and can be parameterized by the formula $\alpha(A) = 0.9 A^{0.15}$ [8].

Figures 12(a) and 13(a) demonstrate the spectra of $\pi^+$- and $\pi^0$-mesons produced in proton-nucleus collisions. The $z$-presentations of the same data are shown in Figures 12(b) and 13(b).

Similar results was obtained for particle production in $\pi^- - A$ collisions too [14]. The dependence of the inclusive cross section of $K^-$-mesons on transverse momentum and the corresponding scaling function versus $z$ at $p_{lab} = 200$ and 300 GeV/c are shown in Figures 14(a) and 14(b), respectively.

The interesting results are obtained under the comparison of the scaling function of direct-$\gamma$ production in $p-p$ and $p-Be$ collisions. The experimental data on the inclusive cross section as a function of transverse momentum are shown in Figure 15(a).

The shape of the scaling functions (Figure 15(b)) is found to be a linear one on log-
log scale for both cases. The value of the slope parameter $\beta_{pBe}^\gamma$ is equal to 5.97 and $\beta_{pBe}^\gamma \simeq \beta_{pp}^\gamma$. The fact means that the nuclear matter changes the probability of photon formation with different formation length $z$ and do not change the fractal dimension of the mechanism of photon formation (photon ”dressing”). The nuclear effect can be described by the ratio $R_{A/p}(z,A)$ of the scaling functions of nucleus and proton. One can see from Figure 15(b) that the ratio is practically independent of $z$. The function $R_{A/p}$ can depend on atomic number only. The experimental verification of the statement should be performed. Taking into account an experimental accuracy of data used in the analysis, the obtained results show that the fractal dimension $\delta$ and the slope parameter $\beta$ is independent of $A$. Direct $\gamma$ is considered to be a good probe to study the Quark Gluon Plasma (QGP) formation. Therefore the experimental investigations of $A$-dependence of $z$-scaling for direct photons produced in hadron-nucleus collisions at RHIC and LHC energies are very important to obtain any indications on nuclear phase transition.

It is assumed that high energy-density nucleon matter produced in heavy-ion collisions could give an direct indication of phase transition to new state of nuclear matter, QGP. High-$p_T$ $\pi^0$-meson spectra should be sensitive to the transition [52]. Recently, the WA80 [53] and WA98 [54] Collaborations have measured the $\pi^0$-meson spectra of $S-S$, $S-Au$ and $Pb-Pb$ collisions at $p_{lab} = 200 \text{ AGeV}/c$ and 158 $\text{ AGeV}/c$, respectively. Therefore, we compare $p_T$-dependence of the cross sections for proton-nucleus and nucleus-nucleus collisions using the available experimental data [50, 53, 54]. In our analysis, we use the data for high-$p_T$ $\pi^0$ production in light-ion ($d-d$, $\alpha-\alpha$) collisions obtained at ISR [52, 55, 57]. The cross sections were measured for $p_T > 2 \text{ GeV}/c$ at an angle of about 90° and $\sqrt{s} = 26$ and 31 GeV.

A change of the shape of the $p_T$ spectra could evidence a modification of the mechanism of particle formation in nuclear matter.

Figure 16 shows dependence of inclusive cross section of $\pi^0$-meson produced in nucleus-nucleus collisions on transverse momentum and the results of $z$-presentation of the same data sets. We found that the dependence $\alpha$ on the atomic weight for $AA$ collisions can be parameterized by the formula $\alpha(A) = 0.93A^{0.32}$ [12]. In the case the fractal dimension $\delta$ is equal to 0.5. Thus the available experimental data [48–51], [53–57] give no strong indications of the scaling violation over the range $z = 4 - 20$.

New data on high-$p_T$ cross sections of $\pi^0$-mesons produced in $Au-Au$ collisions at RHIC are presented by PHENIX Collaboration in [58]. The pion spectra for different centralities are shown in Figure 18 (a). We compared the scaling function for minimum bias data set with function found at ISR and SpS energies. The obtained results are shown in Figure 18(b). Note that the dramatic change of the value of the fractal dimension $\delta$ was found. The scaling function at ISR and SpS energies corresponds to $\delta = 0.5$ whereas at RHIC energies the value of $\delta$ is found to be 3.5. This is indication that nuclear environment changes essentially the mechanism of particle formation. The increase of fractal dimension $\delta$ means that mechanism of multiple scattering play an important role of particle formation in nuclear medium created at RHIC energies. We suggest to use the change of the fractal dimension (“$\delta$-jump”) as signature of nuclear matter transition. Therefore it is of interest to investigate the energy dependence of fractal dimension $\delta(s)$ and determine the shape of the dependence.

3.5 F-dependence

As we mention above the physics meaning of the scaling function $\psi(z)$ is the proba-
bility density to produce particle with formation length $z$. The existence of the scaling is the confirmation of self-similarity at different scales, regulated by the energy $\sqrt{s}$ and transverse momentum $p_T$. The power law, $\psi(z) \sim z^{-\beta}$, observed at very high-$p_T$ range is characterized by the slope parameter $\beta$. It was found the independence of the parameter on $\sqrt{s}$ over a wide transverse momentum range. Therefore it is of interest to study the dependence of the slope parameter $\beta$ on type of produced particle $(\pi^{\pm,0}, K^{\pm}, \bar{p})$. The dependence is named as the $F(lavor)$-dependence of $z$-scaling.

Figure 17(a) shows the scaling functions for produced particles with different flavor content. The symmetry transformation of the scaling function $\psi \rightarrow \alpha_{F}^{-1}\psi$ and the variable $z \rightarrow \alpha_{F}z$ was used to compare the scaling function for different particles. The results give us the indication on the existence of the universal asymptotic of $\psi(z)$ at high $z$ for different type of particles, $\pi^{\pm,0}, K^{\pm}, \bar{p}$. The property of data $z$-presentation reflects new feature of particle formation, the flavor independence of the scaling function in the asymptotic region. The verification of the property for other particles, $J/\psi, \Upsilon, D, B, W^{\pm}$ and $Z^{0}$, will be possible at RHIC and LHC and it is important for understanding of the mechanism of particle formation.

4 $z - p_T$ plot

To know the kinematic region where the $z$-scaling can be violated is very important for modelling of the experiment. The $z - p_T$ plot allows us to determine the high transverse momentum range interesting for such investigations.

Figure 17(b) shows the $z - p_T$ plot for $p - Be$ interaction at $\sqrt{s} = 31 - 500$ GeV. As seen from Figure 17(a) the scaling function $\psi(z)$ is measured up to $z \simeq 20$. Therefore the kinematic ranges interesting for searching new physics phenomena will be $p_T > 8, 12, 20$ and $24$ GeV/$c$ at $\sqrt{s} = 31, 63, 200$ and $500$ GeV, respectively.

5 Conclusions

Analysis of numerous experimental data on high-$p_T$ particle production in hadron-hadron, hadron-nucleus and nucleus-nucleus collisions obtained at ISR, SpS and Tevatron in the framework of $z$-scaling concept was presented. The general scheme of data $z$-presentation for different processes was formulated.

The scaling function $\psi(z)$ and scaling variable $z$ are expressed via the experimental quantities, the invariant inclusive cross section $E d^{3}\sigma/dq^{3}$ and the multiplicity density of charged particles $\rho_{A}(s, \eta)$. The physics interpretation the scaling function $\psi$ as a probability density to produce a particle with the formation length $z$ is argued. The quantity $z$ has the property of the fractal measure and $\delta$ is the fractal dimension describing the intrinsic structure of the interaction constituents revealed at high energies. The fractal dimensions of nucleon $\delta_{N}$, pion $\delta_{\pi}$ and nuclei $\delta_{A}$ were found. They satisfy the relation $\delta_{\pi} < \delta_{N} << \delta_{A}$.

It was shown that the properties of $z$-scaling, the energy and angular independence, the power law $\psi(z) \sim z^{-\beta}$, $A$- and $F$-dependence are confirmed by the numerous experimental data obtained by different Collaborations at ISR, SpS and Tevatron.

The new data on $\pi^{0}$-meson spectra in $Au - Au$ collisions obtained at RHIC were analyzed. The dramatic change of the fractal dimension ("$\delta$-jump") was found. The results is considered as one of the confirmation on creation of new state of nuclear matter.

Thus, the obtained results show that data $z$-presentation demonstrates general
properties of the particle production mechanism such as self-similarity, locality, scale-
relativity and fractality. The properties reveal themselves both in $h - h$, $h - A$ and
$A - A$ collisions and reflect the features of elementary constituent formation.

The violation of $z$-scaling due to the change of the value of the fractal dimension $\delta$
is suggested to search for new physics phenomena such as quark compositeness, new
type of interactions, nuclear phase transition, fractal structure of space-time in hadron-
hadron, hadron-nucleus and nucleus-nucleus collisions at SpS, RHIC, Tevatron, HERA
and LHC.

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Figure 1. (a) The inclusive differential cross sections for the $\pi^+$-mesons produced in $p-p$ collisions at $p_{lab} = 70, 200, 300$ and 400 GeV/c and $\theta_{cm}^p \approx 90^0$ as functions of the transverse momentum $q_T$. (b) The corresponding scaling function $\psi(z)$. Solid lines are obtained by fitting of the data at $p_{lab} = 70, 400$. Experimental data are taken from [18, 19].

Figure 2. (a) The dependence of the inclusive cross section of $\pi^0$-meson production on transverse momentum $q_T$ in $pp$ collisions at $\sqrt{s} = 23, 30, 53$ and 62 GeV and an angle $\theta_{cm}^p$ of $90^0$. The experimental data on the cross section are taken from [20, 21, 23, 24, 27]. (b) The corresponding scaling function $\psi(z)$. 
Figure 3. (a) The inclusive differential cross sections for $K^-$-mesons produced in $p-p$ collisions at $p_{lab} = 70, 200, 300$ and $400$ GeV/c and $\theta_{cm}^{pp} \simeq 90^0$ as functions of the transverse momentum $q_T$. Solid lines are obtained by fitting of the data at $p_{lab} = 70$, and $400$ GeV/c. Experimental data are taken from [18, 19]. (b) The corresponding scaling function $\psi(z)$.

Figure 4. (a) Dependence of the inclusive cross section of $\pi^-$-meson production on transverse momentum $q_T$ at $p_{lab} = 40, 200$ and $300$ GeV/c and $\theta_{cm}^{pp} \simeq 90^0$ in $\pi^- - p$ collisions. Experimental data are taken from [29, 30]. (b) The corresponding scaling function $\psi(z)$. 
Figure 5. (a) Dependence of the inclusive cross section of direct photon production in \( pp \) collisions on \( q_T \) at energy \( \sqrt{s} = 23 \) and 63 GeV and pseudorapidity \( \eta \approx 0 \). Experimental data on the cross section \( \circ \) - WA70, \( \bullet \) - R806, \( \circ \) - R807, \( \triangle \) - R108, \( \square \) - R110 are used. (b) The corresponding scaling function \( H(z) \).

Figure 6. (a) Inclusive differential cross section for prompt \( \gamma \)-production in \( \bar{p}p \) collisions as a function of transverse momentum \( q_T \) for different c.m.s. energies \( \sqrt{s} = 24, 630 \) and 1800 GeV and at a produced angle \( \theta = 90^0 \). Experimental data are taken from \( \star \) - UA2, \( \circ \) - UA6, \( \circ \) - CDF and \( \triangle \) - D0. (b) Corresponding scaling function \( H(z) \).
Figure 7. (a) The dependence of the inclusive cross section of jet production in $pp$ collisions at $\sqrt{s} = 38.8, 45$ and 63 GeV/c and central rapidity range on transverse momentum $q_T$. Experimental data of the cross sections obtained by the AFS and E557 Collaborations are taken from [44, 45]. (b) The corresponding scaling function $\psi(z)$.

Figure 8. (a) The dependence of the inclusive cross section of jet production in $\bar{p}p$ collisions at $\sqrt{s} = 546, 630$ and 1800 GeV/c and central rapidity range on transverse momentum $q_T$. Experimental data of the cross sections obtained by CDF Collaboration are taken from [46]. (b) The corresponding scaling function $\psi(z)$. 
Figure 9. (a) The dependence of the inclusive cross section of $\pi^0$-meson production in $p-p$ collisions at $\sqrt{s} = 23\,\text{GeV}$ and an angle $\theta = 15^0 - 90^0$ on transverse momentum $q_T$. The experimental data on the cross sections are taken from [24]. (b) The corresponding scaling function $\psi(z)$.

Figure 10. (a) Dependence of the inclusive cross section of direct photon production in $\bar{p}-p$ collisions on momentum $q_T$ for different pseudorapidity intervals at energy $\sqrt{s} = 1800\,\text{GeV}$. Experimental data on the cross sections obtained by the D0 Collaboration are taken from [41]. (b) The corresponding function $H(z)$. 
Figure 11. (a) The dependence of the inclusive cross section of jet production in $\bar{p} - p$ collisions at $\sqrt{s} = 1800$ GeV/c and different rapidity intervals on transverse momentum $q_T$. Experimental data of the cross section obtained by D0 Collaboration are taken from [47]. (b) The corresponding scaling function $\psi(z)$.

Figure 12. (a) Inclusive differential cross section for the $\pi^+$-mesons produced in $p - A$ interactions at $p_{lab} = 70, 400$ GeV/c and in the central region, $\theta_{cm} \simeq 90^0$ as a function of the transverse momentum $q_T$. Solid and dashed lines are obtained by fitting of the data for $D, W$ and $D, Pb$, respectively. Experimental data are taken from [18, 19]. (b) The corresponding scaling function $\psi(z)$. 
Figure 13. (a) Dependence of the inclusive cross section of $\pi^0$-meson production on transverse momentum $q_T$ in $p - A$ collisions at $\sqrt{s} = 19 - 38$ GeV. Experimental data are taken from [48]-[51]. (b) The corresponding scaling function $\psi(z)$.

Figure 14. (a) Dependence of the inclusive cross section of $K^-$-meson production on transverse momentum $q_T$ in $\pi^- - A$ collisions at $p_{lab} = 200, 300$ GeV/c. Experimental data are taken from [23]. (b) The corresponding scaling function $\psi(z)$. 
Figure 15. (a) Dependence of the inclusive cross section of direct photon production on transverse momentum $q_T$ in $pp$ and $pBe$ collisions. The experimental data on cross section: ◊ - WA70 [31], + - R806 [32], ○ - R807 [33], △,⋆ - E706 [51]. (b) The corresponding scaling function $H(z)$. Solid lines are obtained by fitting the function taken in the form $H(z) = a_1/z^{a_2}$.

Figure 16. (a) Dependence of the inclusive cross section of $\pi^0$-meson production on transverse momentum $q_T$ in minimum bias nucleus-nucleus collisions at $\sqrt{s} = 17 - 31$ GeV. Experimental data are taken from [49], [53]–[57]. (b) The corresponding scaling function $\psi(z)$. 
Figure 17. (a) Scaling function $\psi(z)$ for $\pi^{\pm, 0}, K^{\pm}, \bar{p}$ particles produced in $p–Be$ collisions at $p_{LAB} = 400, 530$ and $800$ GeV/c. Experimental data are taken from [18, 51]. (b) Dependence of the variable $z$ of $\pi^0$-mesons produced in $p–Be$ collisions on transverse momentum $q_T$ at different energy $\sqrt{s}$ and $\theta_{NN} \simeq 90^\circ$. Points, $\times - 31$ GeV, $\diamond - 63$ GeV, $\circ - 200$ GeV, $+ - 500$ GeV, are the calculated results.

Figure 18. (a) Spectra of $\pi^0$-meson production in $Au–Au$ collisions at $\sqrt{s_{NN}} = 130$ GeV. Experimental data are obtained by PHENIX Collaboration at RHIC [58]. (b) Scaling function $\psi(z)$ of $\pi^0$-mesons produced in nucleus-nucleus collisions at ISR, SpS and RHIC ($\ast$). The solid line ($-$) are obtained by fitting of experimental data taken from [49], [53]-[57].