Isospin Effects on the Mixed Hadron-Quark Phase at High Baryon Density

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Abstract

The phase transition of hadronic to quark matter at high baryon and isospin density is analyzed. Nonlinear relativistic mean field models are used to describe hadronic matter, and the MIT bag model is adopted for quark matter. The boundaries of the mixed phase and the related critical points for symmetric and asymmetric matter are obtained. Isospin effects appear to be rather significant. The binodal transition line of the \((T, \rho_B)\) diagram is lowered to a region accessible through heavy ion collisions in the energy range of the new planned facilities, e.g. the \textit{FAIR/NICA} projects. Some observable effects are suggested, in particular an \textit{Isospin Distillation} mechanism. The dependence of the results on a suitable treatment of isospin contributions in effective QCD Lagrangian approaches, at the level of explicit isovector parts and/or quark condensates, is critically discussed.

\textit{Key words:} Nuclear Matter at High Baryon Density; Symmetry Energy; Deconfinement Transition; Critical End Point; Effective QCD Lagrangians

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1 Introduction

Several suggestions are already present about the possibility of interesting isospin effects on the transition to a mixed hadron-quark phase at high baryon density.
The weak point of those predictions is the lack of a reliable equation of state that can describe with the same confidence the two phases, hadronic and deconfined. On the other hand this also represents a strong theory motivation to work on more refined effective theories for a strong interacting matter.

For the Hadron part we use a Relativistic Mean Field (RMF) EoS ([6,7,8]) with non-linear terms and effective $\rho$--meson coupling for the isovector part, largely used to study isospin effects in relativistic heavy ion collisions [3,8]. In order to keep a smooth flow of the physics points in the discussion, details about the adopted effective nucleon-meson Lagrangians are presented in the Appendix A.
The energy density and the pressure for the quark phase are given by the MIT Bag model \[9\] (two-flavor case) and read, respectively:

\[ \epsilon = 3 \times 2 \sum_{q=u,d} \int \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m_q^2} (f_q + \bar{f}_q) + B , \]  
\[ P = 3 \times 2 \frac{3}{3} \sum_{q=u,d} \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{\sqrt{k^2 + m_q^2}} (f_q + \bar{f}_q) - B , \]  

where B denotes the bag constant (the bag pressure), taken as a rather standard value from the hadron spectra \( B = 85.7 \text{ MeV fm}^{-3} \), no density dependence, \( m_q \) are the quark masses \( (m_u = m_d = 5.5 \text{ MeV choice}) \), and \( f_q, \bar{f}_q \) represent the Fermi distribution functions for quarks and anti-quarks. The quark number density is given by

\[ \rho_i = \langle q_i^+ q_i^- \rangle = 3 \times 2 \int \frac{d^3k}{(2\pi)^3} (f_i - \bar{f}_i) , \quad i = u, d. \]  

The transition to the more repulsive quark matter will appear around the crossing points of the two EoS. We see that such crossing for symmetric matter \( (\alpha_H = \alpha_Q = 0.0) \) is located at rather high density, \( \rho_B \simeq 7 \rho_0 \), while for pure neutron matter \( (\alpha_H = \alpha_Q = 1.0) \) it is moving down to about three times \( \rho_0 \). Of course the Fig.1 represents just a simple energetic argument to support the hadron-quark transition to occur at lower baryon densities for more isospin asymmetric matter. In the rest of the paper we will rigorously consider the case of a first order phase transition in the Gibbs frame for a system with two conserved charges (baryon and isospin), in order to derive more detailed results. Since the first order phase transition presents a jump in the energy, we can expect the mixed phase to start at densities even before the crossing points of the Fig.1. The lower boundary then can be predicted at relatively low baryon densities for asymmetric matter, likely reached in relativistic heavy ion collisions. Moreover this point is certainly of interest for the structure of the crust and the inner core of neutron stars, e.g. see refs. \[10\] and the review \[12\]. The ref.\[10\] is particularly interesting since similar results are obtained with rather different hadronic approaches, the RMF and the non-relativistic Brueckner-Hartree-Fock (BHF) theory.

We finally note that the above conclusions are rather independent on the isoscalar part of the used Hadron EoS at high density, that is chosen to be rather soft in agreement with collective flow and kaon production data \[13\][14].
In the used Bag Model no gluon interactions, the $\alpha_s$-strong coupling parameter, are included. We remark that this in fact would enhance the above effect, since it represents an attractive correction for a fixed B-constant, see [15]. A reduction of the Bag-constant with increasing baryon density, as suggested by various models, see ref [10], will also go in the direction of an “earlier” (lower density) transition, as already seen in ref [2]. At variance, the presence of explicit isovector contributions in the quark phase could play an important role, as shown in the following also for other isospin properties inside the mixed phase.

2 Isospin effects on the Mixed Phase

We can study in detail the isospin dependence of the transition densities [1,2,3]. The structure of the mixed phase is obtained by imposing the Gibbs conditions [16] for chemical potentials and pressure and by requiring the conservation of the total baryon and isospin densities:

$$
\begin{align*}
\mu^H_B(\rho^H_B, \rho^H_3, T) &= \mu^Q_B(\rho^Q_B, \rho^Q_3, T), \\
\mu^3_B(\rho^H_B, \rho^H_3, T) &= \mu^3_Q(\rho^Q_B, \rho^Q_3, T), \\
P^H(T)(\rho^H_B, \rho^H_3, T) &= P^Q(T)(\rho^Q_B, \rho^Q_3, T), \\
\rho_B &= (1 - \chi)\rho^H_B + \chi\rho^Q_B, \\
\rho_3 &= (1 - \chi)\rho^H_3 + \chi\rho^Q_3,
\end{align*}
$$

(4)

where $\chi$ is the fraction of quark matter in the mixed phase and $T$ is the temperature.

The consistent definitions for the densities and chemical potentials in the two phases are given by :

$$
\begin{align*}
\rho^H_B &= \rho_p + \rho_n, & \rho^H_3 &= \rho_p - \rho_n, \\
\mu^H_B &= \frac{\mu_p + \mu_n}{2}, & \mu^H_3 &= \frac{\mu_p - \mu_n}{2},
\end{align*}
$$

(5)

for the Hadron Phase and

$$
\begin{align*}
\rho^Q_B &= \frac{1}{3}(\rho_u + \rho_d), & \rho^Q_3 &= \rho_u - \rho_d, \\
\mu^Q_B &= \frac{3}{2}(\mu_u + \mu_d), & \mu^Q_3 &= \frac{\mu_u - \mu_d}{2},
\end{align*}
$$

(6)
The related asymmetry parameters are:

\[
\alpha^H \equiv -\frac{\rho_3^H}{\rho_B^H} = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}, \quad \alpha^Q \equiv -\frac{\rho_3^Q}{\rho_B^Q} = 3\frac{\rho_d - \rho_u}{\rho_d + \rho_u}. \tag{7}
\]

Nucleon and quark chemical potentials, as well as the pressures in the two phases, are directly derived from the respective EoS.

In this way we get the binodal surface which gives the phase coexistence region in the \((T, \rho_B, \rho_3)\) space. For a fixed value of the total asymmetry \(\alpha_T = -\rho_3/\rho_B\) we will study the boundaries of the mixed phase region in the \((T, \rho_B)\) plane. Since in general the charge chemical potential is related to the symmetry term of the EoS, \([8]\), \(\mu_3 = 2E_{\text{sym}}(\rho_B)\rho_3/\rho_B\), we expect critical and transition densities rather sensitive to the isovector channel in the two phases.

In the hadron sector we will use the Non-Linear Relativistic Mean Field models, \([7,8,3]\), with different structure of the isovector part, already tested to describe the isospin dependence of collective flows and meson production for heavy ion collisions at intermediate energies, \([17,18,19]\). We will refer to these different Iso-Lagrangians as: i) \(NL\), where no isovector meson is included and the symmetry term is only given by the kinetic Fermi contribution, ii) \(NL\rho\) when the interaction contribution of an isovector-vector meson is considered and finally iii) \(NL\rho\delta\) where also the contribution of an isovector-scalar meson is accounted for. See details in Appendix A and refs.\([7,8,3]\).

We will look at the effect on the hadron-quark transition of the different stiffness of the symmetry term at high baryon densities in the different parametrizations. As clearly shown in Appendix A, where a rather transparent form for the density dependence of the symmetry energy in RMF approaches is discussed, the potential part of the symmetry term will be proportional to the baryon density in the \(NL\rho\) choice and even stiffer in the \(NL\rho\delta\) case.

As already mentioned, in the quark phase we use the MIT-Bag Model, where the symmetry term is only given by the Fermi contribution. The Bag parameter \(B\) is fixed for each baryon density to a constant, rather standard, value \(B^{1/4} = 160\, MeV\), corresponding to a Bag Pressure of \(85.7\, MeV\, fm^{-3}\).

In general for each effective interactive Lagrangian we can simulate the solution of the highly non-linear system of Eqs.(4), via an iterative minimization procedure, in order to determine the binodal boundaries and the Critical End Point (CEP) \((T_c, \rho_B^B)\) of the mixed phase.

A relatively simple calculation can be performed at zero temperature. The
isospin effect (asymmetry dependence) on the Lower ($\chi = 0.0$) and Upper ($\chi = 1.0$) transition densities of the Mixed Phase are shown in Fig.2 for various choices of the Hadron EoS. The effect of a larger repulsion of the symmetry energy in the hadron sector, from $NL$ to $NL\rho$ and to $NL\rho\delta$, is clearly evident on the lower boundary with a sharp decrease of the transition density even at relatively low asymmetries.

Typical results for isospin effects on the whole binodal “surface” are presented in Fig.3 for symmetric and asymmetric matter. For the hadron part we have started from a $NL\rho$ effective Lagrangian very close to other widely used relativistic effective models, e.g. see the $GM3$ of ref.\cite{20} and the $NL3$ interaction of P.Ring and collaborators \cite{21}, which has also given good nuclear structure results, even for exotic nuclei.

In the symmetric matter case the mixed phase is evaluated from the simpler Maxwell conditions. The results are shown in Fig.4 for the same hadron and quark EoS’s as in Fig.3 at temperatures $T=0$, 50 and 80 MeV. The equal chemical potential densities (intersection of the dotted line in the lower panel) must correspond to the equal pressure densities of the upper panels. We nicely see: i) at $T=0$ MeV the mixed phase is centered around $\rho/\rho_0 \simeq 7.0$, exactly the $\alpha = 0$ crossing point of Fig.1, confirming our energetic argument about the transition location; ii) the size of the mixed phase is shrinking with tem-
Fig. 3. Binodal surface for symmetric ($\alpha = 0.0$) and asymmetric ($\alpha = 0.2$) matter. Hadron EoS from $NL\rho$ interaction. Quark EoS: MIT bag model with $B^{1/3}=160$ MeV. The grey region corresponds to the conditions reached in Heavy Ion Collisions simulations at few AGeV beam energies, see Section 3.

Fig. 4. Maxwell construction for symmetric ($\alpha = 0.0$) matter at temperatures $T=0$, 50 and 80 MeV. For Pressure and Chemical Potentials we use: Hadron EoS (black curves) from $NL\rho$ interaction; Quark EoS (grey curves) from MIT bag model with $B^{1/3}=160$ MeV.
perature, it is very narrow at \( T = 50 \text{ MeV} \) and finally at \( T = 80 \text{ MeV} \) we cannot have anymore a first order transition. In fact the Critical End Point is found at \( T_c \simeq 58 \text{ MeV}, \rho_c/\rho_0 \simeq 3.8, P_c \simeq 120 \text{MeV/fm}^3 \) and \( \mu_c \simeq 1130 \text{MeV}, \) see also Fig. 3. The result is dependent on the choice of the Bag constant, with increasing of the critical temperature with the Bag value due to the reduction of the pressure in the quark phase, while the chemical potentials are not affected. This point will not be further discussed since here the main focus is on the isospin dependence of the mixed phase at lower temperatures, that can be probed in heavy ion collisions at intermediate energies.

As expected, the lower boundary of the mixed phase is mostly affected by isospin effects. In spite of the relatively small total asymmetry, \( \alpha = 0.2 \), we clearly observe in Fig. 3 a shift to the left of the first transition boundary, in particular at low temperature, and an indication of a relatively “earlier” Critical End Point, around \(( T_c = 50 - 55 \text{ MeV}, \rho_c/\rho_0 = 2.5 - 2.8 )\).

Actually we see from Fig. 3 that for asymmetric matter, we are not able to explicitly reach the CEP of the binodal surface. Since this feature appears also in the following figures we must add a general comment. From the present results we cannot exclude a CEP at higher temperature and smaller baryon density. However we have also to mention some numerical problems. The solution of the Gibbs conditions, the highly non-linear system of Eqs.(4), is found through an iterative multi-parameter minimization procedure (the Newton-Raphson method). When we are close to the Critical End Point, for each \( \chi \)-concentration the baryon and isospin densities of the hadron and quark phases become very similar and we start to have problems in finding a definite minimum. We are working on improving the numerical accuracy but in any case the trends towards the CEP are always quite clearly observed, as a severe reduction of the mixed phase.

In the following we will concentrate on properties of the mixed phase mostly located at high density and relatively low temperature, well described in the calculation and within the reach of heavy ion collisions in the few AGeV range, see Section 3.

2.1 Inside the Mixed Phase of Asymmetric Matter

For \( \alpha = 0.2 \) asymmetric matter, in the Figs. 5, 6 we show also the \(( T, \rho_B )\) curves inside the Mixed Phase corresponding to a 20\% and 50\% presence of the quark component \(( \chi = 0.2, 0.5 )\), evaluated respectively with the two choices, \( NL\rho \) and \( NL\rho\delta \), of the symmetry interaction in the hadron sector. We note, as also expected from Fig. 2, that in the more repulsive \( NL\rho\delta \) case the lower boundary is much shifted to the left. However this effect is not so
Fig. 5. Asymmetric $\alpha = 0.2$ matter. Binodal surface and $(T, \rho_B)$ curves for various quark concentrations ($\chi = 0.2, 0.5$) in the mixed phase. Quark EoS: MIT Bag model with $B^{1/4} = 160$ MeV. Hadron EoS: NL\rho\delta Effective Interaction.

Fig. 6. As in Fig.5 for the $NL\rho\delta$ Effective Interaction in the Hadron sector.

evident for the curve corresponding to a 20% quark concentration, and almost absent for the 50% case. The conclusion seems to be that for a stiffer symmetry term in a heavy-ion collision at intermediate energies during the compression stage we can have more chance to probe the mixed phase, although in a region with small weight of the quark component.
Fig. 7. Asymmetric $\alpha = 0.2$ matter. Binodal surface and $(T, \rho_B^H, \rho_B^Q)$ curves for various quark concentrations in the mixed phase. Quark EoS: MIT Bag model with $B^{1/4} = 160$ MeV. Hadron EoS: NLρ Effective Interaction.

Fig. 8. As in Fig 7 for the NL$\rho\delta$ Effective Interaction in the Hadron sector.

In fact from the solution of the system Eq.(4) we get the baryon densities $\rho_B^H, \rho_B^Q$ in the two phases for any $\chi$ value. In the Figs. 7, 8 we present the results for the same weights 20%, 50% of the quark phase of the previous figures. The quark phase appears always with larger baryon density, even for the lowest value of the concentration.

Can we expect some signatures related to the subsequent hadronization in the following expansion?

An interesting possibility is coming from the study of the asymmetry $\alpha^Q$ in the quark phase. In fact since the symmetry energy is rather different in the two
Fig. 9. Quark asymmetry in the mixed phase vs. the quark concentration for asymmetric matter with $T = 0$ and $\alpha = 0.2$. $NL\rho$ and $NL\rho\delta$ Effective Hadron Interactions are considered. Quark EoS: MIT bag model with $B^{1/4} = 160$ MeV.

phases we can expect an Isospin Distillation (or Fractionation), very similar to the one observed in the Liquid-Gas transition in dilute nuclear matter \cite{22,23,8}, this time with the larger isospin content in the higher density quark phase.

In Fig.9 we show the asymmetry $\alpha^Q$ in the quark phase as a function of the quark concentration $\chi$ for the case with global asymmetry $\alpha = 0.2$ (zero temperature). The calculation is performed with the two choices of the symmetry term in the hadron sector. We see an impressive increase of the quark asymmetry when we approach the lower boundary of the mixed phase, even to values larger than one, likely just for numerical accuracy \cite{21}. Of course the quark asymmetry recovers the global value 0.2 at the upper boundary $\chi = 1$. A simple algebraic calculation allows to evaluate the corresponding asymmetries of the hadron phase. In fact from the charge conservation we have that for any $\chi$-mixture the global asymmetry $\alpha$ is given by:

$$\alpha \equiv -\frac{\rho_3}{\rho_B} = \frac{(1 - \chi)\alpha^H}{(1 - \chi) + \chi \frac{\rho^Q_B}{\rho_B}} + \frac{\chi\alpha^Q}{(1 - \chi)\frac{\rho^H_B}{\rho_B} + \chi}$$

(8)

For any $\chi$, from the calculated $\alpha^Q$ of Fig.9 and the $\rho^H_B, \rho^Q_B$ of Figs.7, 8 we can get the correspondent asymmetry of the hadron phase $\alpha^H$. For a 20% quark concentration we have an $\alpha^Q/\alpha^H$ ratio around 5 for $NL\rho$ and around 20 for $NL\rho\delta$, more repulsive in the isovector channel. It is also interesting to compare the isospin content $N/Z$ of the high density region expected from transport simulations without the Hadron-Quark transition and the effective $N/Z$ of the quark phase in a 20% concentration. In the case of $Au + Au$
(initial $N/Z = 1.5$) central collisions at 1AGeV in pure hadronic simulations we get in the high density phase a reduced $N/Z \sim 1.2 - 1.25$ (respectively with $NL\rho \delta - NL\rho$ interactions) due to the fast neutron emission $18, 19$. The corresponding isospin content of the quark phases is much larger, $N/Z = 3$ for $NL\rho$ and $N/Z = 5.7$ for $NL\rho \delta$. This is the neutron trapping effect discussed in the Section 3. We could expect a signal of such large asymmetries, coupled to a larger baryon density in the quark phase, in the subsequent hadronization.

We finally remark that at higher temperature and smaller baryon chemical potential (ultrarelativistic collisions) the isospin effects discussed here are expected to vanish $25$, even if other physics can enter the game and charge asymmetry effects are predicted also at $\mu_B = 0$ and $T \simeq 170$ MeV $26, 27$.

*Isospin in Effective Quark Models*

All the above results will be very sensitive to the explicit inclusion of isovector interactions in effective non-perturbative QCD models in regions at high baryon chemical potentials. Unfortunately few attempts have been worked out for two main reasons: i) the difficulties of Lattice-QCD calculations at high baryon densities; ii) the main interest on the QGP phase transition at high temperature and small baryon chemical potentials, as probed in the expanding fireball of ultrarelativistic heavy ion collisions. A first approach can be supplied by a two-flavor Nambu-Jona Lasinio ($NJL$) model $28$, which in fact describes the chiral restoration but not the deconfinement dynamics. The isospin asymmetry can be included in a flavor-mixing picture $29, 30$, corresponding to different couplings to the $(u,d)$ quark-antiquark condensates. As a consequence we can have now a dependence of the constituent mass of a given flavor to both quark condensates. We devote the Appendix B to a detailed study of this isospin effects in the $NJL$ chiral dynamics.

Due to the scalar nature of the interacting part of the corresponding Lagrangians only the quark effective mass dynamics will be affected. In the “realistic” small mixing case, see also $29, 31$, we get a definite $M_u^* > M_d^*$ splitting at high baryon density (before the chiral restoration).

All that can indicate a more fundamental confirmation of the $m_p^* > m_n^*$ splitting in the hadron phase, as suggested by the effective $QHD$ model with the isovector scalar $\delta$ coupling, see $7, 8$. However such isospin mixing effect results in a very small variation of the symmetry energy in the quark phase, related only to the Fermi kinetic contribution. Moreover we remind that confinement is still missing in this $NJL$ mean field approach. In any case there are extensive suggestions about a favored chiral symmetry restoration in systems with large neutron excess $33$. 


More generally starting from the QCD Lagrangian one can arrive to an effective color current-current interaction where an expansion in various components can provide isovector contributions, \cite{34}.

In this respect we remark another interesting “indirect” isospin effect, i.e. not directly related to isovector terms in the effective Lagrangian, related to the presence of quark condensates due to the attractive gluon interaction. We note that just a color-pairing effect in the two-flavor system (the 2SC phase \cite{35}) would imply a stiffer symmetry energy in the quark EoS since we have a larger attraction when the densities of up and down quarks are equal. A first study of the high density hadron-quark transition including such gluon correlation in the Bag model has been presented very recently in \cite{36}. Now the symmetry energy difference between hadron and quark phases is partially reduced, at least at low temperatures, and consequently also the isospin effects discussed in detail in this work will be less robust, although still present. An interesting point is that in any case the quark phase is more bound due to the attractive gluon contribution. Hence the transition to the mixed phase will still appear at relatively low baryon densities, now for an “isoscalar” mechanism, within the reach of “low energy” heavy ion collisions, i.e. in the range of few AGeV. As an intuitive picture we can refer again to the Fig.1. Essentially the difference between the $\alpha_Q = 0.0$ and $\alpha_Q = 1$ curves is increasing but meanwhile both are decreasing.

With increasing temperature the color pairing effect will be in general reduced, as confirmed in \cite{37} in an extended $NJL$ calculation, and so isospin effects, as discussed before, will be more relevant. All that, as well as the location of the Critical-End-Point, is naturally related to the used value of the superconducting gap, opening new stimulating perspectives. In this sense new experiments on mixed phase properties observed with isospin asymmetric heavy ion collisions, as suggested in the final section, will be extremely important.

3 Perspectives and suggested observables

Based on the qualitative argument of the Introduction and on more detailed calculations in a first order phase transition scheme, we have predicted rather “robust” isospin effects on the hadron-quark transition at high baryon densities, not depending on details of the EoS parametrizations in the hadron and quark phases.

Our results seem to indicate a specific region where the onset of the mixed phase should be mainly located: $2 < \rho_B/\rho_0 < 4, T \leq 50-60 \ MeV$, for realistic asymmetries $\alpha \sim 0.2-0.3$. A key question is if such a region of the phase space can be explored by means of Heavy-Ion-Collisions. In refs. \cite{23} it is shown that
even collisions of stable nuclei at intermediate energies \((E/A \sim 1 - 2 \text{ GeV})\) make available the pertinent \((T, \rho_B, \alpha)\) region where the phase transition is expected to occur.

In this respect we can refer to the reaction \(^{238}\text{U} + ^{238}\text{U}\) (average isospin asymmetry \(\alpha = 0.22\)) at 1 \(\text{AGeV}\) that has been investigated in ref.\cite{2}, using a consistent Relativistic Mean Field approach with the same interactions, for a semicentral impact parameter \(b = 7 \text{ fm}\), chosen just to increase the neutron excess in the interacting region. The evolution of momentum distribution and baryon/isospin densities in a space cell located in the c.m. of the system has been also studied. After about 10 \(\text{fm/c}\) a local equilibration is achieved still in the compressed phase, before the fast expansion. We have a unique Fermi distribution and from a simple fit the “local” temperature can be evaluated. A rather exotic nuclear matter is formed in a transient time of the order of \(10 - 20 \text{ fm/c}\), with baryon density around \(3 - 4\rho_0\), temperature \(50 - 60 \text{ MeV}\), and isospin asymmetry between 0.2 and 0.3, likely inside the estimated mixed phase region. We note that high intensity \(^{238}\text{U}\) beams in this energy range will be available in the first stage of the FAIR facility \cite{4,38} and also at JINR-Dubna in the Nuclotron first step of the NICA project \cite{25}.

Which are the observable effects to look at if we enter and/or cross the mixed phase?

As already stressed, a first expectation will be the Isospin Distillation effect, a kind of neutron trapping in the quark phase, supported by statistical fluctuations \cite{2} as well as by a symmetry energy difference in the two phases, as discussed in Section 2.1. In fact while in the pure hadron matter (neutron-rich) at high density we have a large neutron potential repulsion (in both \(NL\rho, NL\rho\delta\) cases), in the quark phase the \(d\)-quarks see a much smaller symmetry repulsion only due to the kinetic contribution from the Fermi gas. As a consequence while in a pure hadronic phase neutrons are quickly emitted or “transformed” in protons by inelastic collisions \cite{19}, when the mixed phase starts forming, neutrons are kept in the interacting system, in the quark phase which is also at larger baryon density, up to the subsequent hadronization in the expansion stage \cite{3}. Observables related to such neutron “trapping” could be an inversion in the trend of the formation of neutron rich clusters, an enhancement of the production of isospin-rich nucleon resonances and subsequent decays, i.e. an increase of \(\pi^-/\pi^+, K^0/K^+\) yield ratios for reaction products coming from high density regions, that could be selected looking at large transverse momenta, corresponding to a large radial flow.

If such kinetic selection of particles from the mixed phase can really be successful also other potential signatures would become available. One is related to the general softening of the matter, due to the contribution of more degrees of freedom, that should show up in the damping of collective flows \cite{39}.
The azimuthal distributions (elliptic flows) will be particularly affected since particles mostly retain their high transverse momenta escaping along directions orthogonal to the reaction plane without suffering much rescattering processes. Thus a further signature could be the observation, for the selected particles, of the onset of a quark-number scaling of the elliptic flow: a property of hadronization by quark coalescence that has been predicted and observed at RHIC energies, i.e. for the transition at $\mu_B = 0$ [40].

We note that all the above results, on the Binodal Boundaries of the mixed phase and on the Isospin Distillation are sensitive to the symmetry term in the hadron sector, although the main isospin effects are present for all the parametrizations of the isovector interaction. At variance, for the quark sector the lack of explicit isovector terms could strongly affect the location of the phase transition in asymmetric matter and the related expected observables.

In conclusion the aim of this work is twofold:

- To stimulate new experiments on isospin effects in heavy ion collisions at intermediate energies (in a few $AGeV$ range) with attention to the isospin content of produced particles and to elliptic flow properties, in particular for high-$p_t$ selections.
- To stimulate more refined models of effective Lagrangians for non-perturbative QCD, where isovector channels are consistently accounted for and/or gluon correlations, leading to diquark condensates, can induce symmetry energy effects.

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A Equation of state for hadronic matter

Nonlinear (NL) relativistic mean field model with constant couplings

A Lagrangian density with “minimal” meson channels and non-linear terms is used. The nuclear interaction is mediated by two isoscalar, the scalar $\sigma$
and the vector $\omega$, and two isovector, the scalar $\delta$ and the vector $\rho$, mesons. Non linear terms are considered only for the $\sigma$ contribution to account for the correct compressibility around saturation. Constant nucleon-meson couplings are used, chosen to reproduce the saturation properties and to represent a reasonable average of the density dependence predicted by Relativistic Brueckner-Hartree-Fock calculations [41, 42], see details in refs. [7, 8].

\[ L = \bar{\psi} [\gamma_{\mu} \partial^{\mu} - (M - g_\sigma \sigma - g_\delta \vec{\tau} \cdot \vec{\delta}) - g_\omega \gamma_{\mu} \omega^{\mu} - g_\rho \gamma^{\mu} \vec{\tau} \cdot \vec{b}^{\mu}] \psi + \frac{1}{2} (\partial_{\mu} \sigma \partial^{\mu} \sigma - m_\sigma^2 \sigma^2) - U(\phi) + \frac{1}{2} m_\omega^2 \omega^{\mu} \omega_{\mu} + \frac{1}{2} m_\rho^2 \vec{b}^{\mu} \cdot \vec{b}^{\mu} + \frac{1}{2} (\partial_{\mu} \vec{\tau} \cdot \partial^{\mu} \vec{\tau} - m_\delta^2 \vec{\delta}^2) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \vec{G}_{\mu\nu} \vec{G}^{\mu\nu}, \] (A.1)

where $F_{\mu\nu} \equiv \partial_{[\mu} \omega_{\nu]} - \partial_{[\mu} \omega_{\nu]} - \partial_{\nu} \vec{b}^{\mu} - \partial_{\mu} \vec{b}^{\nu}$, and the $U(\sigma)$ is the nonlinear potential of $\sigma$ meson: $U(\sigma) = \frac{1}{3} a \sigma^3 + \frac{1}{4} b \sigma^4$.

The EoS for nuclear matter at finite temperature in the mean-field approximation (MFA) is given by

\[ \epsilon = 2 \sum_{i=n,p} \int \frac{d^3k}{(2\pi)^3} E_i^*(k)(f_i(k) + \bar{f}_i(k)) + \frac{1}{2} m_\sigma^2 \sigma^2 + U(\sigma) + \frac{1}{2} m_\omega^2 \rho_\omega^2 + \frac{1}{2} m_\rho^2 \rho_\rho^2 + \frac{1}{2} m_\delta^2 \rho_\delta^2, \] (A.2)

\[ P = \frac{2}{3} \sum_{i=n,p} \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{E_i^*(k)} (f_i(k) + \bar{f}_i(k)) - \frac{1}{2} m_\sigma^2 \phi^2 - U(\phi) + \frac{1}{2} m_\omega^2 \rho_\omega^2 + \frac{1}{2} m_\rho^2 \rho_\rho^2 + \frac{1}{2} m_\delta^2 \rho_\delta^2, \] (A.3)

where $E_i^* = \sqrt{k^2 + M_i^*}$. The nucleon effective masses are defined as

\[ M_i^* = M - g_\sigma \phi \mp g_\delta \delta_3 \quad (- \text{proton}, + \text{neutron}). \] (A.4)

The field equations in the relativistic mean field (RMF) approach are
\[ \sigma = -\frac{a}{m_{\sigma}^2} \sigma^2 - \frac{b}{m_{\sigma}^2} \sigma^3 + \frac{g_\sigma}{m_{\sigma}^2} (\rho_{sp} + \rho_{sn}) , \quad (A.5) \]

\[ \omega_0 = \frac{g_\omega}{m_{\omega}^2} \rho , \quad (A.6) \]

\[ b_0 = \frac{g_\rho}{m_{\rho}^2} \rho_3 , \quad (A.7) \]

\[ \delta_3 = \frac{g_\delta}{m_{\delta}^2} (\rho_{sp} - \rho_{sn}) , \quad (A.8) \]

with the baryon density \( \rho \equiv \rho_H^H = \rho_p + \rho_n \) and \( \rho_3^H = \rho_p - \rho_n \), \( \rho_{sp} \) and \( \rho_{sn} \) are the scalar densities for proton and neutron, respectively. The \( f_i(k) \) and \( \bar{f}_i(k) \) in Eqs. (A.2, (A.3) are the fermion and antifermion distribution functions for protons and neutrons (\( i = p, n \)):

\[ f_i(k) = \frac{1}{1 + \exp\{ (E_i^* (k) - \mu_i^*) / T \} } , \quad (A.9) \]

\[ \bar{f}_i(k) = \frac{1}{1 + \exp\{ (E_i^* (k) + \mu_i^*) / T \} } . \quad (A.10) \]

where the effective chemical potential \( \mu_i^* \) is determined by the nucleon density \( \rho_i \)

\[ \rho_i = 2 \int \frac{d^3k}{(2\pi)^3} (f_i(k) - \bar{f}_i(k)) , \quad (A.11) \]

while the scalar density \( \rho_{s,i} \), which gives the coupling to the scalar fields is given by

\[ \rho_{s,i} = 2 \int \frac{d^3k}{(2\pi)^3} \frac{M_i^*}{E_i^*} (f_i(k) + \bar{f}_i(k)) , \quad (A.12) \]

note the \( M_i^* / E_i^* \) quenching factor at high baryon density.

The \( \mu_i^* \) are related to the chemical potentials \( \mu_i = \partial \epsilon / \partial \rho_i \) in terms of the vector meson mean fields by the equation

\[ \mu_p = \mu_p^* + \frac{g_{\omega}^2}{m_{\omega}^2} \rho_B + \frac{g_{\rho}^2}{m_{\rho}^2} \rho_3 \]

\[ \mu_n = \mu_n^* + \frac{g_{\omega}^2}{m_{\omega}^2} \rho_B - \frac{g_{\rho}^2}{m_{\rho}^2} \rho_3 . \quad (A.13) \]
The baryon and isospin chemical potentials in the hadron phase can be expressed in terms of the \((p,n)\) ones as

\[
\mu_B^H = \frac{\mu_p + \mu_n}{2}, \quad \mu_3^H = \frac{\mu_p - \mu_n}{2}. \tag{A.14}
\]

In presence of the coupling to the two isovector \(\rho, \delta\)-meson fields, the expression for the symmetry energy has a simple transparent form, see \[7,43,8\]:

\[
E_{\text{sym}}(\rho) = \frac{1}{6} \frac{k_F^2}{E_F^*} + \frac{1}{2} \left[ f_\rho - f_\delta \left( \frac{M^*}{E_F^*} \right)^2 \right] \rho, \tag{A.15}
\]

where \(M^* = M - g_\sigma \sigma\) and \(E_F^* = \sqrt{k_F^2 + M^{*2}}\).

Now we easily see that in the \(NL\rho\delta\) choice we have a large increase of the symmetry energy at high baryon densities. The potential symmetry term is given by the combination \(\left[ f_\rho - f_\delta \left( \frac{M^*}{E_F^*} \right)^2 \right] \rho\) of the repulsive vector \(\rho\) and attractive scalar \(\delta\) isovector couplings. Thus, when the \(\delta\) is included we have to increase the \(\rho\)-meso coupling in order to reproduce the same asymmetry parameter \(a_4\) at saturation. The net effect will be a stiffer symmetry energy at higher baryon densities due to the \(M^*/E_F^*\) quenching of the attractive part.

**Parameter determination**

The coupling constants are fixed from good saturation properties and from Dirac-Brueckner estimations, see the detailed discussions in refs.\[7,43,8\]. The isoscalar part of the EoS is chosen to be rather soft at high densities, see \[44\], in order to satisfy the experimental constraints from collective flows and kaon production in intermediate energy heavy ion collisions \[13,14\].

The coupling constants, \(f_i \equiv g_i^2/m_i^2, \ i = \sigma, \omega, \rho, \delta\), and the two parameters of the \(\sigma\) self-interacting terms : \(A \equiv a/g_\sigma^3\) and \(B \equiv b/g_\sigma^4\) are reported in Table 1. The \(\sigma\) mass is fixed at \(550 \text{ MeV}\). The corresponding properties of nuclear matter are listed in Table 2. Here the binding energy is defined \(E/A = \epsilon/\rho - M\).

**Table 1.** Parameter set.
| Parameter Set | NLρ | NLρδ |
|---------------|-----|------|
| $f_\sigma$ (fm$^2$) | 10.32924 | 10.32924 |
| $f_\omega$ (fm$^2$) | 5.42341 | 5.42341 |
| $f_\rho$ (fm$^2$) | 0.94999 | 3.1500 |
| $f_\delta$ (fm$^2$) | 0.000 | 2.500 |
| $A$ (fm$^{-1}$) | 0.03302 | 0.03302 |
| $B$ | -0.00483 | -0.00483 |

Table 2. Saturation properties of nuclear matter.

\[
\begin{align*}
\rho_0 \text{ (fm}^{-3}\text{)} & \quad 0.16 \\
E/A (\text{MeV}) & \quad -16.0 \\
K (\text{MeV}) & \quad 240.0 \\
E_{\text{sym}} (\text{MeV}) & \quad 31.3 \\
M^*/M & \quad 0.75
\end{align*}
\]

We finally note that these Lagrangians have been already used for flow [17], pion production [18], isospin tracer [45] and kaon production [19] calculations for relativistic heavy ion collisions with an overall good agreement to data.

\section*{B Nambu-Iona Lasinio model for asymmetric matter}

From the above discussion it appears (extremely) important to include the Isospin degree of freedom in any effective QCD dynamics. A first approach can be supplied by a two-flavor Nambu-Jona Lasinio model where the isospin asymmetry can be included in a flavor-mixing picture [29,30]. The lagrangian is given by

\begin{equation}
L = L_0 + L_1 + L_2,
\end{equation}

with $L_0$ the free part

\[L_0 = \bar{\psi}(i \not{\partial} - m)\psi,
\]

and the two different interaction part given by
\[ L_1 = G_1 \left\{ (\bar{\psi} \psi)^2 + (\bar{\psi} \tau \psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 + (\bar{\psi} i \gamma_5 \tau \psi)^2 \right\} \]
\[ L_2 = G_2 \left\{ (\bar{\psi} \psi)^2 - (\bar{\psi} \tau \psi)^2 - (\bar{\psi} i \gamma_5 \psi)^2 + (\bar{\psi} i \gamma_5 \tau \psi)^2 \right\}. \] \tag{B.2}

In the mean field approximation the new Gap Equations are \( M_i = m_i - 4G_1 \Phi_i - 4G_2 \Phi_j, \ i \neq j, (u, d) \), where the \( \Phi_{u,d} = <\bar{u}u>, <\bar{d}d> \) are the two (negative) condensates which are given by

\[ \Phi_f = -2N_c \int \frac{d^3 p}{(2\pi)^3} \frac{M_f}{E_{p,f}} \left\{ 1 - f^-(T,\mu_f) - f^+(T,\mu_f) \right\}. \] \tag{B.3}

and \( m_{u,d} = m \) the (equal) current masses.

Introducing explicitly a flavor mixing, i.e. the dependence of the constituent mass of a given flavor to both condensate, via \( G_1 = (1 - \beta)G_0, G_2 = \beta G_0 \) we have the coupled equations

\[ M_u = m - 4G_0 \Phi_u + 4\beta G_0 (\Phi_u - \Phi_d), \]
\[ M_d = m - 4G_0 \Phi_u + 4(1 - \beta)G_0 (\Phi_u - \Phi_d). \] \tag{B.4}

For \( \beta = 1/2 \) we have back the usual NJL \( (M_u = M_d) \), while small/large mixing is for \( \beta \Rightarrow 0/\beta \Rightarrow 1 \) respectively. The value of \( \beta \) has a consequence on the structure of the phase diagram in the region of low temperatures and high chemical potential. In fact as shown in \[29,30\] for \( \beta = 0 \) there are two distinct phase transitions for the up quarks and for the down quarks, but for this value the interaction is symmetric under \( U_A(1) \) transformations and it is unrealistic. While for \( \beta \geq 0.1 \) the \( U_A(1) \) symmetry becomes explicitly broken and there is only a single first order phase transition. Realistic estimations of \( \beta \) fitting the physical \( \eta \)-meson mass give a value of \( \beta \approx 0.11 \) \[29,31\].

In neutron rich matter \( |\Phi_d| \) decreases more rapidly due to the larger \( \rho_d \) and so \( (\Phi_u - \Phi_d) < 0 \). In the “realistic” small mixing case we will get a definite \( M_u > M_d \) splitting at high baryon density (before the chiral restoration). This expectation is confirmed by a full calculation of the coupled gap equations with standard parameters \[31,32\]. All that can indicate a more fundamental confirmation of the \( m_\pi^* > m_\sigma^* \) splitting in the hadron phase, as suggested by the effective \( QHD \) model with the isovector scalar \( \delta \) coupling, see \[7,8\].

However such isospin mixing effect results in a very small variation of the symmetry energy in the quark phase, still related only to the Fermi kinetic contribution. In fact this represents just a very first step towards a more complete treatment of isovector contributions in effective quark models, of large interest for the discussion of the phase transition at high densities.
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In principle there is nothing wrong with $\alpha^Q > 1$ evaluations (although we note that for pure "neutron" matter the quark phase asymmetry should be 1, since $\rho_d = 2\rho_u$). Indeed for low $\chi$-values, $\chi < 0.1$, very small quark concentrations, we can get $\alpha^Q$ values slightly larger than 1. However we are cautious about these results since we can expect also some numerical problems. In fact for very small $\chi$ values the weight of the $\alpha^Q$ contribution in the minimization procedure is expected not too relevant, as we can clearly see from the Eq. (8). In any case the important point is that this is not affecting the general discussion about the isospin distillation.

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