SU(2) WZW D-branes and quantized worldvolume
U(1) flux on $S^2$

Alexander KLING*, Maximilian KREUZER† and Jian-Ge ZHOU‡

Institut für Theoretische Physik, Technische Universität Wien, Wiedner Hauptstraße 8–10, A-1040 Wien, AUSTRIA

Abstract

We discuss possible D-brane configurations on SU(2) group manifolds in the sigma model approach. When we turn the boundary conditions of the spacetime fields into the boundary gluing conditions of chiral currents, we find that for all D-branes except the spherical D2-branes, the gluing matrices $R^a_b$ depend on the fields, so the chiral Kac-Moody symmetry is broken, but conformal symmetry is maintained. Matching the spherical D2-branes derived from the sigma model with those from the boundary state approach we obtain a $U(1)$ field strength that is consistent with flux quantization.

*e-mail: kling@hep.itp.tuwien.ac.at
†e-mail: kreuzer@hep.itp.tuwien.ac.at
‡e-mail: jgzhou@hep.itp.tuwien.ac.at
1 Introduction

In recent years there has been much interest in the study of D-branes on group manifolds (see for instance [1–7]). String theory on group manifolds is governed by a WZW model, which has two distinct descriptions: the current algebra and the sigma model realization. Since the WZW model is a typical example of an exact string background, whose CFT is known explicitly, one approach to find possible D-brane configurations is to impose gluing conditions on the chiral currents $J^a(z)$ and $\bar{J}^a(\bar{z})$ in terms of which the CFT is defined. Actually the boundary state approach has been applied widely to find D-brane configurations on group manifolds [2–4, 6–9]. In [4] it was found that the D-branes in WZW models associated with the gluing condition $J^a = -\bar{J}^a$ along the boundary are configurations of quantized conjugacy classes. As the gluing conditions in the boundary state approach are defined on the chiral currents rather than on the spacetime fields there is a lack of an obvious geometric interpretation of WZW boundary states and in particular of the corresponding D-brane configurations. Since the WZW model provides also an example of a string background with a sigma model description, which allows a complementary study of the D-brane configurations, it is interesting to compare the D-brane configurations obtained from the sigma model realization with those from the boundary state approach (CFT) in order to see how they match with each other.

The other motivation for this work was to see the quantization of the worldvolume $U(1)$ flux on the spherical D2-brane. In [10,11] it was suggested that the $U(1)$ worldvolume flux $\int F$ rather than that of $\int [(2\pi\alpha')^{-1}B + F]$ should be quantized. In [10,11], the quantization problem was mainly discussed from the Born-Infeld theory, so it is quite interesting to see whether we can study it from the worldsheet perspective. Since the $U(1)$ gauge field appears in the action of the sigma model, we wonder what the D-brane configurations constructed from the sigma model approach has to say about this problem.

Motivated by the above, in this paper we study WZW D-branes on the group manifold of $SU(2)$ from the sigma model point of view and compare the results with the boundary state approach. Our strategy is that we turn the boundary conditions of the spacetime fields into gluing condition of the chiral current at the boundary and try to adjust the $U(1)$ gauge field to make the gluing matrices field independent in order to check chiral Kac-Moody symmetry. For the spherical D2-branes we find that in order to keep the infinite-dimensional symmetry of the current algebra, the $U(1)$ worldvolume gauge field strength has to take the form $F = -\frac{\kappa}{2\pi}\psi_0\epsilon_2$, where $\kappa$ is the integer level of the associated current algebra and $\psi_0$ describes the radius of the spherical D2-branes. Imposing the quantization condition on the flux $\int F$ that follows from a consistent definition of the sigma model action we will recover the quantization of the brane positions $\psi_0^{(n)} = n\pi/\kappa$ [10,11]. For other D-branes we find it impossible to adjust the $U(1)$ gauge field to make the gluing matrices $R^a_b$ field independent (the gluing matrix is defined by the gluing condition $J^a(z) + R^a_b\bar{J}^b(\bar{z}) = 0$ at the boundary). The dependence of the gluing matrices $R^a_b$ on the spacetime fields certainly breaks the chiral Kac-Moody symmetry, but we
find that conformal invariance is maintained for these D-branes, at least in our classical approximation. The straightforward extension our result to twisted conjugacy classes is briefly discussed in the last section.

2 Parametrization of the sigma model action and chiral currents

We start with the $SU(2)$ WZW action with gauge bundles $A$ defined on the brane subspaces of the group manifold $[1]$:

$$
S = \frac{\kappa}{8\pi} \int_\Sigma \text{tr}(g^{-1}\partial_+ gg^{-1}\partial_- g) + \frac{1}{2\pi\alpha'} \int_\Sigma g^* B + \int_{\partial\Sigma} g^* A,
$$

where we use the string normalization of target space fields. For world sheets with boundary the WZ part of the action is defined in terms of the field strengths $H = dB$ and $F = dA$ by employing a closed auxiliary surface $\Sigma' = \partial M$ that is the union of $\Sigma$ with $n$ disks $D^{(i)}$ if the boundary $\partial \Sigma$ has $n$ connected components, and by extending $g$ to the 3-dimensional manifold $M$:

$$
S_{WZ} = \frac{\kappa}{12\pi} \int_M \text{tr}(g^{-1}dg)^3 - \sum_{i=1}^n \frac{1}{2\pi\alpha'} \int_{D^{(i)}} g^*(B + 2\pi\alpha' F).
$$

The $U(1)$ field strength $F = dA$ is defined on the D-brane submanifold (i.e. at the allowed positions of the boundaries of the embedded world sheet; in case of several branes we have to introduce an independent gauge connection $A$ for every brane and the respective field strength has to be used for each disk). Moreover, we need to choose a gauge where $B$ is not singular on the brane.

Independence of all quantum amplitudes of the various choices involved in this definition implies that the level $\kappa$ and all integrals $\int_{S^2} F/2\pi$ of the respective field strengths $F$ over any 2-spheres $S^2$ embedded in the branes have to be integers $[1,12]$: If we close some component of the boundary of $\Sigma$ with two different disks $D$ and $D'$, then the difference in the action is $\frac{1}{2\pi\alpha'}((\int_{M'} - \int_M)H - (\int_{D'} - \int_D)(B + 2\pi\alpha' F))$. Since $H = dB$ globally on the respective brane, $B$ drops out and we are left with an integral of $F$ over the 2-sphere $D \cup D'$.

We can also think about the action $[1]$ in the following way: The gauge transformation $B \rightarrow B + dA$ leads to surface terms that can only be compensated if we introduce a gauge field $A$ that transforms as $A \rightarrow A - 2\pi\alpha' \Lambda$ at the boundary. The gauge symmetry $A \rightarrow A + d\lambda$ just corresponds to the trivial part of the reducible $\Lambda$-transformation that leaves $B$ invariant. Hence only the field strength $H$ is physical outside the branes, whereas at the allowed positions of the boundary of the world sheet $F + B/2\pi\alpha'$ also becomes observable.
To proceed we choose the parametrization
\[
g = \begin{pmatrix}
\cos \psi - i \sin \psi \sin \theta \sin \phi & - \sin \psi \sin \theta \cos \phi - i \sin \psi \cos \theta \\
\sin \psi \sin \theta \cos \phi - i \sin \psi \cos \theta & \cos \psi + i \sin \psi \sin \theta \sin \phi
\end{pmatrix}
\] (3)
of the group manifold with
\[
0 \leq \psi \leq \pi, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq 2\pi.
\] (4)

In these coordinates the metric and the NS three-form field are given by
\[
ds^2 = \kappa \alpha'[d\psi^2 + \sin^2 \psi (d\theta^2 + \sin^2 \theta d\phi^2)]
\] (5)
\[
H = \frac{1}{6} \kappa \alpha' \text{tr}(g^{-1} dg)^3 = 2 \kappa \alpha' \sin^2 \psi \sin \theta d\psi \wedge d\theta \wedge d\phi
\] (6)
and the action turns into
\[
S = \frac{1}{2 \pi \alpha'} \int d\tau d\sigma \left\{ \frac{1}{2} \eta^{\alpha \beta} \kappa \alpha' \left( \partial_\alpha \psi \partial_\beta \psi + \sin^2 \psi \partial_\alpha \theta \partial_\beta \theta + \sin^2 \theta \sin^2 \phi \partial_\alpha \phi \partial_\beta \phi \right) \\
+ B_{\theta \phi} \left( \partial_\alpha \theta \partial_\beta \phi - \partial_\alpha \phi \partial_\beta \theta \right) + B_{\psi \phi} \left( \partial_\alpha \psi \partial_\beta \phi - \partial_\alpha \phi \partial_\beta \psi \right) \right\} + \int_{\partial \Sigma} g^* A
\] (7)
where \( \eta^{\alpha \beta} = \text{diag}(-1, 1) \).

The WZW model has conserved chiral currents (with \( \partial_\pm = \partial_\tau \pm \partial_\sigma \)),
\[
J = -\partial_+ g g^{-1}, \quad \bar{J} = g^{-1} \partial_- g
\] (8)
Inserting the parametrization (3) into (8) we have
\[
J^a = -\bar{e}^a_{\mu} \partial_+ X^\mu, \quad \bar{J}^a = e^a_{\mu} \partial_- X^\mu
\] (9)
\[
\bar{e}^a_{\mu} = -
\begin{pmatrix}
\cos \theta & - \sin \psi \cos \psi \sin \theta & \sin^2 \psi \sin^2 \theta \\
\sin \theta \cos \phi & \sin \psi \cos \psi \cos \theta \cos \phi & - \sin \psi \cos \psi \sin \theta \sin \phi \\
\sin \theta \sin \phi & \sin \psi \cos \psi \sin \theta \sin \phi & \sin \psi \cos \psi \sin \theta \cos \phi \\
+ \sin^2 \psi \cos \phi & - \sin^2 \psi \sin \theta \cos \phi
\end{pmatrix}
\] (10)
\[
e^a_{\mu} =
\begin{pmatrix}
- \cos \theta & \sin \psi \cos \psi \sin \theta & \sin^2 \psi \sin^2 \theta \\
- \sin \theta \cos \phi & - \sin \psi \cos \psi \cos \theta \cos \phi & \sin \psi \cos \psi \sin \theta \sin \phi \\
- \sin \theta \sin \phi & - \sin \psi \cos \psi \sin \theta \sin \phi & \sin \psi \cos \psi \sin \theta \cos \phi \\
+ \sin^2 \psi \cos \phi & - \sin^2 \psi \sin \theta \cos \phi
\end{pmatrix}
\] (11)
where $X^1 = \psi, X^2 = \theta, X^3 = \phi$, and $J^a, \bar{J}^a$ are defined by

$$J^a = \frac{i}{2} \text{tr} (\sigma^a \partial_+ gg^{-1}), \quad \bar{J}^a = -\frac{i}{2} \text{tr} (\sigma^a g^{-1} \partial_- g)$$

and $\sigma^a$ are the Pauli matrices. The vielbein matrices $e$ and $\bar{e}$ satisfy $e^T e = \bar{e}^T \bar{e} = G$, where $G$ is the metric

$$G_{\mu\nu} = \text{diag}(1, \sin^2 \psi, \sin^2 \psi \sin^2 \theta).$$

3 D-brane configurations constructed in the sigma model approach

When we vary the action (7) we get the equations of motion. In addition we work out the boundary conditions from which we can construct possible D-brane configurations. We first consider the solution

$$(B + 2\pi\alpha' F)_{\theta\phi} = \kappa\alpha' (\psi - \frac{\sin 2\psi}{2} + f) \sin \theta$$

to $H = dB + 2\pi\alpha' F$ with constant $f$ and with all other components vanishing, which is suggested by the symmetry of our choice of coordinates. The 2-form $\sin \theta d\theta d\phi$ is singular at $\psi = 0$ and at $\psi = \pi$, which suggests to associate the term proportional to $f$ with the gauge field strength $F$. The remaining $B$ field is then regular everywhere except at the point $\psi = \pi$.

At the present stage of our discussion $f$ is an undetermined parameter. We can formally extend the domain of $F$ and effectively obtain a family of choices for the $B$ field. Obviously $f \to f - \pi$ then shifts the $B$ field into another gauge with a singularity in a single point, but this time at $\psi = 0$, the unit element of the group (at the same time the flux $\int F/2\pi$ through a sphere at fixed $\psi = \psi_0$ is shifted by the integer $-\kappa$). It turns out that $f$ can not be fixed by the leading order condition of conformal invariance at the boundary given in (3)

$$\partial_\mu [\sqrt{G} G^\mu\nu G^\rho\sigma (B + 2\pi\alpha' F)_{\nu\rho}] = 0$$

where the metric is given by (13).

With the choice (14) we can read off the boundary condition by varying the action

---

1 Two other simple choices are $(B + 2\pi\alpha' F)_{\phi\theta} = 2\kappa\alpha' (\phi \sin^2 \psi \sin \theta + f')$ and $(B + 2\pi\alpha' F)_{\psi\phi} = 2\kappa\alpha' (\sin^2 \psi \cos \theta + f'')$ with all other components vanishing in both cases. For $f \neq 0$ they are, however, singular at $\theta = 0 \mod \pi$ for all $\psi$ and therefore not useful for $D2$ branes.
(4) and we find

\[
\frac{\delta \psi \partial_{\sigma} \psi}{\partial \Sigma} = 0
\]

\[
\delta \theta \left( \sin^2 \psi \partial_{\sigma} \theta - (\psi - \frac{\sin 2\psi}{2} + f) \sin \theta \partial_{\tau} \phi \right) \bigg|_{\partial \Sigma} = 0
\]

\[
\delta \phi \left( \sin^2 \psi \sin^2 \theta \partial_{\sigma} \phi + (\psi - \frac{\sin 2\psi}{2} + f) \sin \theta \partial_{\tau} \theta \right) \bigg|_{\partial \Sigma} = 0
\]

By exploiting (16) we can look for D-brane configurations of various dimensions by considering the following simple boundary conditions.

**D0-brane:**

\[
\psi \bigg|_{\partial \Sigma} = \psi_0, \quad \theta \bigg|_{\partial \Sigma} = \theta_0, \quad \phi \bigg|_{\partial \Sigma} = \phi_0
\]

(17)

**D1-branes:**

\[
\psi \bigg|_{\partial \Sigma} = \psi_0, \quad \theta \bigg|_{\partial \Sigma} = \theta_0, \quad \partial_{\sigma} \phi \bigg|_{\partial \Sigma} = 0
\]

(18)

\[
\psi \bigg|_{\partial \Sigma} = \psi_0, \quad \partial_{\sigma} \theta \bigg|_{\partial \Sigma} = 0, \quad \phi \bigg|_{\partial \Sigma} = \phi_0
\]

(19)

\[
\partial_{\sigma} \psi \bigg|_{\partial \Sigma} = 0, \quad \theta \bigg|_{\partial \Sigma} = \theta_0, \quad \phi \bigg|_{\partial \Sigma} = \phi_0
\]

(20)

**Spherical D2-brane**

\[
\psi \bigg|_{\partial \Sigma} = \psi_0
\]

\[
\left( \sin^2 \psi \partial_{\sigma} \theta - (\psi - \frac{\sin 2\psi}{2} + f) \sin \theta \partial_{\tau} \phi \right) \bigg|_{\partial \Sigma} = 0
\]

\[
\left( \sin^2 \psi \sin^2 \theta \partial_{\sigma} \phi + (\psi - \frac{\sin 2\psi}{2} + f) \sin \theta \partial_{\tau} \theta \right) \bigg|_{\partial \Sigma} = 0
\]

(21)

where \(\psi_0, \theta_0, \phi_0\) are arbitrary constants. The last two D1-brane candidates still have to be closed by the antipodal halves of the respective circles but we have to drop them from our considerations anyway because our ansatz for \(B + F\) is singular at their location (this is not a big loss, however, because these configurations can be obtained from global rotations of the group manifold, as we will discuss below). Replacing the Dirichlet boundary condition in (21) by \(\partial_{\sigma} \psi \big|_{\partial \Sigma} = 0\) we formally get a D3-brane, but this is inconsistent because we cannot have a B-field without singularity on the group manifold.

---

\(^2\)When \(\psi_0 = 0\) and \(\pi\), the spherical D2-branes reduce to D0-branes, and the D0-branes described by (17) can be derived from the D0-branes of \(\psi_0 = 0\) and \(\pi\) by an inner automorphism.
4 Comparison of the D-brane configurations between two approaches and quantized $U(1)$ worldvolume flux on $S^2$

Now we compare the D-brane configurations derived from the above sigma model with those from the boundary state approach. To do so, we construct the gluing condition $J^a(z) + R^a_bJ^b(z)|_{\partial\Sigma} = 0$ from the boundary condition of the spacetime fields $\psi, \theta, \phi$ for various D-brane configurations. We try to adjust the undetermined parameter $f$ to see whether we can get spacetime field independent gluing matrices $R^a_b$ in order to check the infinite-dimensional symmetry of the current algebra.

For the following comparison, we need the explicit expressions for $J^a$ and $\bar{J}^a$. Using (9)-(11) we rewrite them as

\begin{align}
J^1 &= \cos \theta \partial_r \psi + \cos \theta \partial_{\sigma} \psi - \sin \psi \cos \psi \sin \theta \partial_{\rho} \theta + \sin^2 \psi \sin^2 \theta \partial_{r} \phi \\
    &\quad + \sin \theta (\sin^2 \psi \sin \theta \partial_{\sigma} \phi - \sin \psi \cos \psi \partial_{\rho} \theta) \\
J^2 &= \sin \theta \cos \phi \partial_r \psi + \sin \theta \cos \phi \partial_{\sigma} \psi - \sin^2 \psi \sin \phi \partial_r \theta + \sin \psi \cos \psi \cos \theta \cos \phi \partial_{\sigma} \theta \\
    &\quad - \sin^2 \psi \sin \theta \cos \phi \partial_{r} \phi - \sin \psi \cos \psi \sin \theta \sin \phi \partial_{\sigma} \phi \\
    &\quad - \sin \phi (\sin^2 \psi \partial_r \theta + \sin \psi \cos \psi \sin \theta \partial_{r} \phi) \\
    &\quad - \cos \theta \cos \phi (\sin^2 \psi \sin \theta \partial_{\sigma} \phi - \sin \psi \cos \psi \partial_{r} \theta) \\
J^3 &= \sin \theta \sin \phi \partial_r \psi + \sin \theta \sin \phi \partial_{\sigma} \psi + \sin^2 \psi \cos \phi \partial_r \theta + \sin \psi \cos \psi \cos \theta \sin \phi \partial_{\sigma} \theta \\
    &\quad - \sin^2 \psi \sin \theta \cos \phi \partial_{r} \phi + \sin \psi \cos \psi \sin \theta \cos \phi \partial_{\sigma} \phi \\
    &\quad + \cos \phi (\sin^2 \psi \partial_r \theta + \sin \psi \cos \psi \sin \theta \partial_{r} \phi) \\
    &\quad - \cos \theta \sin \phi (\sin^2 \psi \sin \theta \partial_{\sigma} \phi - \sin \psi \cos \psi \partial_{r} \theta) \\
\end{align}

\begin{align}
\bar{J}^1 &= -\cos \theta \partial_r \bar{\psi} + \cos \theta \partial_{\rho} \bar{\psi} - \sin \psi \cos \psi \sin \theta \partial_{\rho} \theta + \sin^2 \psi \sin^2 \theta \partial_{r} \phi \\
    &\quad - \sin \theta (\sin^2 \psi \sin \theta \partial_{\sigma} \phi - \sin \psi \cos \psi \partial_{\rho} \theta) \\
\bar{J}^2 &= -\sin \theta \cos \phi \partial_r \bar{\psi} + \sin \theta \cos \phi \partial_{\rho} \bar{\psi} - \sin^2 \psi \sin \phi \partial_r \theta + \sin \psi \cos \psi \cos \theta \cos \phi \partial_{\rho} \theta \\
    &\quad - \sin^2 \psi \sin \theta \cos \phi \partial_{r} \phi - \sin \psi \cos \psi \sin \theta \sin \phi \partial_{\rho} \phi \\
    &\quad + \sin \phi (\sin^2 \psi \partial_r \theta + \sin \psi \cos \psi \sin \theta \partial_{r} \phi) \\
    &\quad + \cos \theta \cos \phi (\sin^2 \psi \sin \theta \partial_{\rho} \phi - \sin \psi \cos \psi \partial_{r} \theta) \\
\bar{J}^3 &= -\sin \theta \sin \phi \partial_r \bar{\psi} + \sin \theta \sin \phi \partial_{\rho} \bar{\psi} + \sin^2 \psi \cos \phi \partial_r \theta + \sin \psi \cos \psi \cos \theta \sin \phi \partial_{\rho} \theta \\
    &\quad - \sin^2 \psi \sin \theta \cos \phi \partial_{r} \phi + \sin \psi \cos \psi \sin \theta \cos \phi \partial_{\rho} \phi \\
    &\quad - \cos \phi (\sin^2 \psi \partial_r \theta + \sin \psi \cos \psi \sin \theta \partial_{r} \phi) \\
    &\quad + \cos \theta \sin \phi (\sin^2 \psi \sin \theta \partial_{\rho} \phi - \sin \psi \cos \psi \partial_{r} \theta) \\
\end{align}

Now let us first consider the spherical D2-brane characterized by (21). In the boundary state approach the spherical D2-brane is described by the gluing condition [3]

\begin{equation}
J^a = \bar{J}^a
\end{equation}
at the boundary $\partial \Sigma$. Here we have turned the gluing condition for the spherical D2-brane in the boundary state approach, which uses the closed string picture, into the open string picture. Comparing $J^a$ and $\bar{J}^a$ we find that consistency of the boundary conditions (21) with the gluing condition (24) in the boundary state approach, requires:

$$\psi_0 - \frac{\sin 2\psi_0}{2} + f = -\sin \psi_0 \cos \psi_0,$$

which results in

$$f = -\psi_0.$$  

In [4] it was shown that the D-brane configurations in the WZW model associated with the gluing condition $J^a = -\bar{J}^a$ (in the closed string picture) are the conjugacy classes, and in the case of $SU(2)$ group the D-brane configurations are spherical D2-branes, which are described by the boundary conditions (21) in the sigma model approach. Imposing the quantization of the $U(1)$ worldvolume flux $\int F/2\pi = f\kappa/\pi$ that follows from the definition of the action we thus recover the quantization $\psi_0^{(n)} = \frac{n\pi}{\kappa}$ of the brane positions.

For D0- and D1-brane configurations the gluing condition can be written as

$$J^a + R^a_b \bar{J}^b = 0 \quad (27)$$

with

$$R = \bar{e}ye^{-1},$$

where the vielbein matrices $e$ and $\bar{e}$ are defined in [10, 11] and the matrix $y$ is defined by

$$\partial_+ X^\mu = -y'^\nu \partial_- X'^\nu.$$  

(29)

For the D0-branes $y = \text{diag}(1, 1, 1)$, so that $R$ corresponds to the inner automorphism that translates the brane to the unit at $\psi = 0$ [6]. For D1-branes at constant $\psi$ and $\theta$, on the other hand, we find $y = \text{diag}(1, 1, -1)$.

Since there is no place to put the magnetic field strength that could balance the tension on a D1-brane worldvolume, the D1-brane configurations are believed to be unstable. Except for the case of spherical D2-branes, and trivially for the D0 branes, the gluing matrices $R^a_b$ depend on the target space position, which indicates that the chiral Kac-Moody symmetry is broken. For the $SU(2)$ group manifold, the energy-momentum tensor is $T(z) = \frac{1}{\kappa+2}J^a J^a$. Since $R^T R = 1$, we have $T(z) = \bar{T}(\bar{z})$ at the boundary, so that conformal invariance is preserved even though the chiral Kac-Moody symmetry is broken [3].

3There should be a minus sign difference between open and closed string picture [6].

4For example, let us consider $J^1 = \bar{J}^\bar{1}$, the first line of $J^1$ is equal to that of $\bar{J}^\bar{1}$ with the help of the first equation in (21), but the second line differs a minus sign. To get $J^1 = \bar{J}^\bar{1}$ at the boundary $\partial \Sigma$, we must demand $(\sin^2 \psi \sin \theta \partial_\phi - \sin \psi \cos \psi \partial_\theta)|_{\partial \Sigma} = 0$. When we exploit the third equation in (21), we obtain (25).
5 Summary and discussion

We have investigated possible D-brane configurations from the sigma model point of view. In order to see what the counterparts of these D-branes are in the boundary state approach, we turned the boundary conditions of the spacetime fields into gluing conditions of the chiral currents at the boundary. We have shown that except for spherical D2-brane configurations the gluing matrices for all other D-brane configurations depend on the spacetime fields. For the spherical D2-branes we have seen that the configurations derived from the sigma model do not match those from the boundary state approach automatically. If we demand that they coincide with each other, the $U(1)$ worldvolume flux $\int F$ has to be quantized as has been conjectured in [10], and as it indeed follows from the ambiguity in the definition of the action.

Since the group manifold is $O(4)$ symmetric, which manifests itself in the global symmetry of the action $g \rightarrow \lambda g \rho$ under left- and right-multiplication with constant group elements, it is clear that there should also be (stable) spherical D2-branes that are not centered around the unit element. Our coordinates are, of course, not very convenient for the discussion of these objects, but it is obvious that our results carry over to that situation and that they are related to the (inner) automorphisms of the current algebra that were discussed in [6, 8]. Indeed, since $(rgr^{-1})h\rho^{-1} = r\lambda(hr^{-1})\lambda^{-1} = \rho(hr^{-1})\rho^{-1}r$ with $\lambda = g\rho^{-1}$ and $\rho = rgr^{-1}$ the twisted conjugacy class defined by $h$ and the inner automorphism corresponding to $r$ is just the sphere through $hr^{-1}$ centered around $r$. In the exact CFT treatment it turns out that, at small levels [9], the brane positions are somewhat smeared out. It would be interesting to find out what the fate of the apparently unstable D1 branes is after quantum corrections are taken into account.

Eq.(21) shows that the D2-brane sphere should be a fuzzy sphere. Indeed, there have been some discussions of noncommutative geometry on the spherical D2-branes with B-fields [13, 14]. Especially in [13] the low-energy effective action on the fuzzy $S^2$ was proposed and it would be interesting to see whether there exists a similar Seiberg-Witten map [15] on the fuzzy sphere, and if so, how the nonlinear $\Lambda$-symmetry in noncommutative geometry is realized as in [16]. As we know, among all examples in AdS/CFT correspondence, the boundary theory of $AdS_2 \times S^2$ is most poorly understood, see [17] for references. In [18] it was argued that besides the fuzzy $S^2$ there is also a fuzzy $AdS_2$. It would be interesting to see whether there is a way to study the fuzzy $AdS_2$ in the context of WZW models.

Acknowledgement

We would like to thank A.Y. Alekseev, N. Ishibashi, C. Schweigert, V. Schomerus and S. Stanciu for helpful discussions. This work is supported in part by the Austrian Research Funds FWF under grants Nr. P13125-TPH and Nr. M535-TPH.
References

[1] C. Klimiccek, P. Severa, Open strings and D-branes in WZNW models, Nucl. Phys. B 488 (1997) 653, hep-th/9609112

[2] M. Kato, T. Okada, D-branes on group manifolds, Nucl. Phys. B 499 (1997) 583, hep-th/9612148

[3] S. Stanciu, A. Tseytlin, D-branes in curved spacetime: Nappi-Witten background, JHEP 06 (1998) 010, hep-th/9805006

[4] A. Yu Alekseev, V. Schomerus, D-branes in the WZW model, Phys. Rev. D60 (1999) 061901, hep-th/9812193

[5] K. Gawedski, Conformal field theory: A case study, hep-th/9904143

[6] S. Stanciu, D-branes in group manifolds, JHEP 01 (2000) 025, hep-th/9909163

[7] J. M. Figueroa-O’Farrill, S. Stanciu, D-branes in AdS$_3 \times S^3 \times S^3 \times S^1$, JHEP 0004 (2000) 005, hep-th/0001199

[8] L. Birke, J. Fuchs, C. Schweigert, Symmetry breaking boundary conditions and WZW orbifolds, hep-th/9905038

[9] G. Felder, J. Fröhlich, J. Fuchs, C. Schweigert, The geometry of WZW branes, J.Geom.Phys. 34 (2000) 162, hep-th/9909030

[10] C. Bachas, M. R. Douglas, C. Schweigert, Flux stabilization of D-branes, hep-th/0003037

[11] W. Taylor, D2-branes in B-fields, hep-th/0004141

[12] J. Pawelczyk, SU(2) WZW D-branes and their noncommutative geometry from DBI action, hep-th/0003057

[13] A. Y. Alekseev, A. Recknagel, V. Schomerus, Brane dynamics in background fluxes and noncommutative geometry, JHEP 0005 (2000) 010, hep-th/0003187

[14] M. Li, Fuzzy gravitons from uncertain spacetime, hep-th/0003173

[15] N. Seiberg, E. Witten, String theory and noncommutative geometry, JHEP 09 (1999) 032, hep-th/9908142

[16] M. Kreuzer, J.-G. Zhou, A-Symmetry and background independence of the noncommutative gauge theory on $R^n$, JHEP 01 (2000) 011, hep-th/9912174

[17] J.-G. Zhou, Super-$0$-branes and GS superstring actions on $AdS_2 \times S^2$, Nucl. Phys. B 559 (1999) 92, hep-th/9906013; M. Kreuzer, J.-G. Zhou, Killing gauge for the $0$-brane on $AdS_2 \times S^2$ coset superspace, Phys. Lett. B 472 (2000) 309, hep-th/9910067

[18] P. M. Ho, M. Li, Fuzzy spheres in AdS/CFT correspondence and holography from noncommutativity, hep-th/0004072