Superconducting proximity effect through graphene and graphite films

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Abstract. We discuss the superconducting proximity effect in graphite films (or graphene) by studying the critical current through superconductor-graphite film (or graphene)-superconductor Josephson junction. The two leads are assumed to be conventional s-wave superconductors. We especially pay attention to the dispersions of electrons in graphite films: because of the delicate band structure of graphite, the electron dispersion in the film, which undergoes the effects of various external factors such as leads, gate electrodes and sample inhomogeneities, can show a wide variety. We introduce several models for electron dispersion near Fermi energy: 1) graphene-like Fermi points, 2) semi-metal, and 3) semiconducting gap. The superconducting critical current $I_c$ through the junction is usually expressed as $I_c \propto \exp\{-d/\xi(T)\}$, where $d$ is the distance between two leads and $\xi(T)$ is the proximity length. We show that the temperature dependence of $\xi(T)$ is largely affected by the band structure of the graphite film and by examining this dependence some insight into the electronic properties of the graphite film can be obtained.

1. Introduction

Graphene films are fabricated recently \cite{1, 2} and attracting much interest because of their unique electronic properties. Their superconducting properties are also discussed theoretically and it has been shown that, in spite of peculiar band structure, supercurrent can flow through the graphene films\cite{3}. One of the present authors (AK) has fabricated ultrathin graphite films and investigated superconducting proximity effect\cite{4, 5}. Especially, it has been found that the critical supercurrent depends largely on the gate voltage and is controllable.

The key point to understand the superconducting proximity effect in the ultra-thin graphite films may be their complicated band structures. Actually, if we increase the number of atomic layers from one (graphene) to several layers, various band structure appears \cite{7} before one approaches that of graphite \cite{6}. Therefore we may expect that actual ultra-thin graphite films also have a wide variety of band structures depending on their environments, such as leads, gates and sample qualities. In order to clarify the superconducting proximity effect, it is important to know how the band structure affects the supercurrent.

The aim of this paper is to clarify the effects of band structure (electron dispersion) on the superconducting proximity effect in ultra-thin graphite films. For this purpose, we limit ourselves to the effects of electron dispersion near the Fermi energy. We assume three characteristic...
2. Calculation of free energy

2.1. Model Hamiltonian

We start from a two-dimensional (2-D) electron system on a honeycomb lattice (i.e., single layer) shown in Fig. 2 and modify the electron dispersion only near the two Fermi points, $K$ and $K'$. In actual experiment (see Fig. 1), several layers may contribute to the supercurrent, however this model may be permissible as a starting point, because of the strong two-dimensional character of the graphite.

The Hamiltonian (Bardeen-Cooper-Schrieffer model with tunneling approximation) is given by

$$
H_{L(R)} = \sum_{\vec{k},\sigma} \left\{ \sum_{\nu=1,2} \xi_{\nu}^{\vec{k},\sigma} c_{\nu,\vec{k},\sigma}^\dagger c_{\nu,\vec{k},\sigma} + \Delta \epsilon_{\nu,\vec{k}}^{\vec{k},\sigma} c_{\nu,\vec{k},\sigma}^\dagger c_{\nu,\vec{k}',\sigma} + H.c. \right\}
$$

$$
H_N = \sum_{\nu=1,2} \sum_{\vec{k},\sigma} \epsilon_{\nu,\vec{k}}^{\vec{k},\sigma} c_{\nu,\vec{k},\sigma}^\dagger c_{\nu,\vec{k},\sigma} ' H_{T(R)} = t \sum_{\nu=1,2} \sum_{\vec{k},\sigma} \epsilon_{\nu,\vec{k}}^{\vec{k},\sigma} c_{\nu,\vec{k},\sigma}^\dagger c_{\nu,\vec{k}',\sigma} + H.c. \quad (1)
$$

where $\xi_{\nu}^{\vec{k}} = |\vec{k}|^2/(2m) - \mu$ ($m$ and $\mu$ are electron mass and chemical potential in the leads, respectively), $\Delta$ the superconducting energy gap with $\phi_{\nu,\vec{k}}$ being the phase of left (right) lead, $\nu$ the index of bands ($\nu = 1$ for upper and $\nu = 2$ for lower band) and $\epsilon_{\nu,\vec{k}}^{\vec{k},\sigma}$ the electron dispersion in the graphite. The constant $t$ is the tunneling matrix element. “H.c.” is the Hermite conjugate. Here $\vec{r}_{L(R)}$ is the location of the contact. The contact is assumed to be point-like for simplicity, which, in actual experiment, is much wider and the proximity current is averaged over several lattice points. This, however, may not change the dependence of critical current on the contact separation in the long scale. In actual calculation the 1st Brillouin zone is taken as the broken hexagon in Fig. 2 (b).

2.2. Calculation of free energy

Here we introduce the Green’s function by $f_{L(R)}(\vec{k}, i\omega_n) = \Delta \epsilon_{\nu,\vec{k}}^{\vec{k},\sigma}/(|\omega_n + \xi_{\nu}^{\vec{k},\sigma}| + \Delta^2)$, $g_{\nu,\vec{k}}^{\nu,\vec{k},\sigma}(\vec{k}, i\omega_n) = -(i\omega_n - \epsilon_{\nu,\vec{k}}^{\vec{k},\sigma})^{-1}$, where $\sigma$ is spin index and $\omega_n = \pi i(2n + 1)T$ with $T$ being temperature. Employing the perturbation expansion with respect to $t$, we obtain the formula for free energy corresponding to superconducting coupling between leads as

$$
F_J = 2t^4 T \text{Re} \left[ \sum_{\omega_n} \sum_{\nu,\nu'=1,2} \sum_{\vec{k},\vec{k}'\vec{k},\vec{k}} e^{-i(\vec{k} - \vec{k}')(\vec{r}_R - \vec{r}_L)} \right]
$$

$$
\times f_R(\vec{k}, i\omega_n) g_{\nu,\vec{k}}^{\nu,\vec{k},\sigma}(\vec{k}, i\omega_n) g_{\nu',\vec{k}'}^{\nu',\vec{k}',\sigma}(\vec{R}', -i\omega_n) f_L(\vec{k}, i\omega_n)
$$

$$
\simeq -2t^4 T (N_F S)^2 \text{Re} \left[ \frac{(\pi \Delta)^2 e^{-i(\phi_{\vec{k} - \vec{k}'} - \phi_{\vec{k}})}}{(\pi T)^2 + \Delta^2} \left\{ |J(i\pi T)|^2 + |J(-i\pi T)|^2 \right\} \right]
$$

(2)

where $S$ and $N_F$ are the 2-D area of the lead and density of state at Fermi energy, respectively, $\vec{d} = \vec{r}_R - \vec{r}_L$ and $J(z) = \sum_{\vec{k},\nu} e^{-i\vec{k} \cdot \vec{d}}/(z - \epsilon_{\nu,\vec{k}}^{\vec{k},\sigma})$. In the following, we study the $\vec{d}$ dependence of the critical current, which, as we can see from Eq. (2), comes from that of $|J(i\pi T)|^2 = |J(-i\pi T)|^2$ and derive the behavior of the proximity length $\xi(T)$ as a function of $T$. Although we have neglected higher order terms in $t$ and terms corresponding to $n \leq -2$ and $1 \leq n$ in the summation over $\omega_n$,
these terms are not important in the present context, since they give rise to corrections which decay faster than the Eq. (2) as a function of $|\vec{d}|$. We have used the Green function for infinite system instead of taking the boundary condition of graphite system seriously. Therefore, the validity of this approximation is limited to $|\vec{d}| \ll a$, where $a$ is the lattice spacing of the graphite. This condition is applicable for realistic systems.

2.3. Proximity length for various electron dispersions

We introduce three characteristic models for the electron dispersion $\epsilon^\nu_k$. 

2.3.1. Fermi-point case As in the case of graphene, we consider the case where upper band touches lower band at $K$ and $K'$. Denoting the vector from $\Gamma$ to $K$ ($K'$) by $\vec{K}$ ($\vec{K}'$), $\epsilon^\nu_k$ is expressed as,

$$
\epsilon^\nu_{K+i} = C|\vec{l}|, \quad \epsilon^\nu_{K'+i} = -C|\vec{l}|, \quad \epsilon^\nu_{K+i}' = C|\vec{l}|, \quad \epsilon^\nu_{K'+i}' = -C|\vec{l}|
$$

(3)

where $C$ is a constant $\mu \sim C/a$. In this case the integral in $J(i\pi T)$ can be estimated analytically and we obtain $|J(i\pi T)|^2 \propto K_0(\pi T|\vec{d}|/C)^2 \sim \exp(-2\pi T|\vec{d}|/C) (|\vec{d}| \rightarrow \infty)$ where $K_0(z)$ is the modified Bessel function. In fig.3, we present the numerical estimation of $J(i\pi T)$.

2.3.2. Semi-metal case We set

$$
\epsilon^\nu_{K+i} = C'|\vec{l}|^2, \quad \epsilon^\nu_{K'+i} = -C'|\vec{l}|^2, \quad \epsilon^\nu_{K+i}' = C'|\vec{l}|^2, \quad \epsilon^\nu_{K'+i}' = -C'|\vec{l}|^2.
$$

(4)

where $C'$ is a constant, $\mu \sim C'a^2$. Here $|J(i\pi T)|^2$ is fitted by $\exp(-d/\xi(T))$ as shown in fig.4(a).

In this case we can roughly see $\xi(T) \propto 1/T$ behavior as in fig.4(b).
2.3.3. Semiconducting case We set

\[ \epsilon_{\nu}^{\nu=1} = C'|\vec{l}|^2 + D, \quad \epsilon_{\nu}^{\nu=2} = -(C'|\vec{l}|^2 + D), \quad \epsilon_{\nu}^{\nu=1} = C''|\vec{l}|^2 + D, \quad \epsilon_{\nu}^{\nu=2} = -(C''|\vec{l}|^2 + D), \]  

(5)

where \( D \) is the energy gap. In this case, \( \xi(T)^{-1} \) shows a concave structure, although it dose not diverge as \( T \to 0 \), as one can see from fig.5 (\( Da^2/C = 1 \) is taken).

3. Summary We have estimated the strength of the superconducting proximity effect in graphite (or graphene) film, assuming three possible band structures (electron dispersions). It has been shown that temperature dependence of the proximity length \( \xi(T) \) reflects the band structure and can be a good probe for electronic state in the film.

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