The decoherence and interference of cosmological arrows of time for a de Sitter universe with quantum fluctuations

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We consider the superposition of two semiclassical solutions of the Wheeler-DeWitt equation for a de Sitter universe, describing a quantized scalar vacuum propagating in a universe that is contracting in one case and expanding in the other, each identifying a opposite cosmological arrow of time. We discuss the suppression of the interference terms between the two arrows of time due to environment-induced decoherence caused by modes of the scalar vacuum crossing the Hubble horizon. Furthermore, we quantify the effect of the interference on the expectation value of the observable field mode correlations, with respect to an observer that we identify with the spatial geometry.
I. INTRODUCTION

Resting on the assumption that the laws of Nature are fundamentally quantum, quantum cosmology is the attempt to extend the quantum formalism to the whole universe. This attempt must naturally accommodate the properties of the classical universe we observe and it should as well provide predictions about the conditions of the very early universe where a drastic divergence from the classical theory might be expected. One of the main approaches to quantum gravity and cosmology, the canonical one, starts with the quantization of the Hamiltonian formulation of general relativity, resulting in a momentum constraint enforcing the diffeomorphism invariance of the theory and a Hamiltonian constraint encoding the dynamics of the geometry [1]. Initially [2][3] the quantization was formulated in the geometrodynamical representation, were the canonical variables are the spatial metric components of the chosen foliation and the momenta conjugated to them, which are related to the extrinsic curvature. Although later introduction of different variables (Ashtekar variables and finally loop variables) helped to solve many serious technical problems [4] and lead to a promising loop quantum cosmology, in this work we will adopt the intuitive picture provided by the geometrodynamical representation.

Notwithstanding the problems of this representation, the Wheeler-DeWitt equation (the quantized form of the Hamiltonian constraint) and its solution (a complex functional of the 3-metrics and matter field configurations spanning the so-called “superspace”) can be “cured” by considering specific “minisuperspace” models, that is models obtained by dealing only with a finite number of homogeneous degrees of freedom of the whole theory and dropping all the others. The most common choice is to consider a closed Friedmann universe defined simply by a scale factor $a$ and inhabited by a homogeneous scalar field $\phi$. Then, the wave functional is usually interpreted as a true “wave function of the universe”, such as in the most notable cases of the Hartle-Hawking state [5] or the tunneling wave function by Vilenkin [6].

The construction of the Wheeler-DeWitt equation was suggested by a quantization of the Einstein-Hamilton-Jacobi equation [7] of general relativity in analogy with the construction of the Schrödinger equation from the Hamilton-Jacobi equation of classical mechanics. By its own construction, then, in the semi-classical approximation where the main contribution to the action comes only from the gravitational degrees of freedom (d.o.f.) and the non-gravitational d.o.f. can be treated as perturbations, the wave functional can be factorized by performing an expansion in orders of $M = 1/32\pi G$ [8], in analogy with the Born-Oppenheimer expansion for electrons bound to massive nuclei. This results up to the second order in a WKB-like approximation where the “semiclassical” factor depends only on the spatial geometry, with the components of the spatial metric playing the role of “slow” d.o.f. satisfying the vacuum Einstein field equations, while the “full quantum” factor depends also on the “fast” perturbative d.o.f., for which the usual time evolution in the form of the Tomonaga-Schwinger equation is naturally retrieved.

In the present work, we consider the superposition of two such solutions in the de Sitter universe, each one defining a opposite “arrow of time” [9][10] corresponding to a contracting and a expanding universe. We discuss their quantum decoherence (i.e. the suppression of the interference terms of their superposition), when they are coupled to quantum fluctuations of a scalar vacuum. In this case, the decoherence follows from the fact that the modes of these fluctuations that are beyond the Hubble scale cannot be observed. We can treat therefore these modes as a external environment and the modes inside the horizon as a probe of the spatial geometry. Decoherence of the wave function of the universe, also in relation with the arrow of time, has already been studied since works such as [12][16]. The contribution of the present work is twofold. First, we do not drop all the infinite quantum d.o.f. of the scalar vacuum and consider its second quantization, which was not done in previous works. In this case, the coupling to geometry leads to gravitational particle production [17]. Secondly, we quantify the effect of the interference of cosmological arrows of time on the expectation value of the observable field mode correlations, with respect to an observer that we identify with the spatial geometry.

II. SEMICLASSICAL APPROXIMATION OF GEOMETRODYNAMICS IN DE SITTER UNIVERSE

We will briefly review the semiclassical approximation of geometrodynamics. As introduced before, in this formulation of quantum cosmology the state of the universe is described by a wave functional $\Psi = \Psi[\sigma, \phi]$ defined on superspace of 3-metrics $\sigma(x)$ and matter fields configurations $\phi(x)$ on the associated 3-surfaces. This representation of the wave functional is a solution of the Wheeler-DeWitt equation in its geometrodynamical form (integration over space is implied as usual)

$$\left(-\frac{\hbar^2}{2M} \nabla^A \nabla_A - 2M \sqrt{\sigma} \left(\frac{3}{2} - 2 \Lambda \right) + \mathcal{H}_\phi \right) \Psi = 0, \quad \nabla^A \nabla_A := G^{AB} \frac{\delta}{\delta \sigma^A} \frac{\delta}{\delta \sigma^B}. \quad (1)$$

$\Lambda$ is the cosmological constant, and $G_{AB}$ is the 3-dimensional Einstein-Hilbert action, with $\sigma$ being the 3-volume. The Hamiltonian $\mathcal{H}_\phi$ is the Hamiltonian constraint encoding the dynamics of the geometry [1]. Initially [2][3] the quantization was formulated in the geometrodynamical representation, were the canonical variables are the spatial metric components of the chosen foliation and the momenta conjugated to them, which are related to the extrinsic curvature. Although later introduction of different variables (Ashtekar variables and finally loop variables) helped to solve many serious technical problems [4] and lead to a promising loop quantum cosmology, in this work we will adopt the intuitive picture provided by the geometrodynamical representation.

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Here \( G^{AB} \equiv G^{ijkl} = \sqrt{\sigma} \left( \sigma^{ik} \sigma^{jl} + \sigma^{il} \sigma^{jk} - 2\sigma^{ij} \sigma^{kl} \right) / 2 \) is the de Witt supermetric (latin capital letters representing pairs of spatial indexes and \( \sigma \) being the determinant of the spatial metric), \( ^{(3)}R \) is the Ricci scalar of the spatial metric, \( \Lambda \) is the cosmological constant and \( M = 1/32\pi G = (M_p/\hbar)^2 \) with \( M_p \) being the reduced Planck mass (in the following we will set \( \hbar = c = 1 \)), while \( \mathcal{H}_\phi = \mathcal{H}_\phi[\sigma, \phi] \) is the Hamiltonian density for the matter fields (in the following we will have only one matter scalar field for simplicity). While technical features such as the absence of a clear scalar product for the wavefunctional and the divergences that may appear while handling the functional derivatives make the Wheeler-DeWitt not well-behaved in its general form (see e.g. \([11]\) for the factor-ordering problem for the product of local operators acting at the same space point), the equation is well-defined within proper cosmological minisuperspace models such as the one we consider.

In the oscillatory region of the wave function, associated to classically allowed solutions, one may consider the ansatz \([15]\)

\[
\Psi[\sigma, \phi] = C[\sigma] \exp \left( i M S[\sigma] \right) \chi[\sigma, \phi]
\]

where \( C[\sigma] \) is a slowly varying prefactor and \( \chi[\sigma, \phi] \) represents small perturbations of order \((M)^0\) to the action \( M S[\sigma] \). For this ansatz, at order \((M)^{-1}\) the Wheeler-DeWitt equation gives the Einstein-Hamilton-Jacobi equation for the action \( S[\sigma] \)

\[
\frac{1}{2} \nabla A S \nabla A S + V = 0, \quad V = -2\sqrt{\sigma} \left( ^{(3)}R - 2\Lambda \right),
\]

as well as the current conservation equation for the density \( C^2 \)

\[
\nabla A S \nabla A C + \frac{C}{2} \nabla A \nabla A S = 0
\]

with geometrodynamical momentum

\[
\pi^A \equiv M \nabla A S
\]

congruent to \( \sigma^A \) and proportional to the extrinsic curvature of the geometry. At order unity \((M)^0\), the Wheeler-DeWitt equation gives (see e.g. \([18]\) for a formal derivation)

\[
i \nabla A S \nabla A \chi - \mathcal{H}_\phi \chi = 0
\]

where integration over space is still implied. It is convenient to introduce the vector tangent to the set of solutions determined by \([3]\),

\[
\frac{\delta}{\delta t} := \nabla A S \nabla A
\]

where the affine parameter along the trajectories, \( t[\sigma_{ij}(x)] \), is found to correspond to usual time and usually referred to as “WKB” or “bubble” time \( t \). Then \([6]\) corresponds to the Tomonaga-Schwinger equation for the matter field, \( \mathcal{H}_\phi \) being the Hamiltonian density of interaction between geometrodynamical and matter degrees of freedom.

In the present case, we are interested in a spatially flat FLRW universe. It is convenient to write the 3-metric in the form \( \sigma_{ij} = \sigma^2 \delta_{ij} \), which gives the EHJ equation \([3]\) in the form \([8]\)

\[
- \frac{3}{16} \sqrt{\sigma} \left( \frac{\delta S}{\delta \sqrt{\sigma}} \right)^2 + \frac{1}{2} \delta_{ik} \delta_{jl} \frac{\delta S}{\delta \sigma_{ij}} \frac{\delta S}{\delta \sigma_{kl}} - 2\sqrt{\sigma} \left( ^{(3)}R - 2\Lambda \right) = 0.
\]

For the de Sitter universe with spatially flat slicing, we have \(^{(3)}R = 0\) and \( S = S(\sqrt{\sigma}) \) if we neglect the contribution of the tensor mode \( \tilde{\sigma}_{jk} \), which gives

\[
S = S_{\pm} = \pm 8\sqrt{\frac{\Lambda \sigma}{3}},
\]

where we have fixed the spatial volume \( V_0 = \int d^3 x \) to unity and the sign ambiguity is due to the invariance under time inversion of the Einstein-Hamilton-Jacobi equation, the minus sign corresponding to the arrow of time of a expanding universe and the plus sign corresponding to the arrow of time of a contracting one. Inserted in \([3]\), \([9]\) gives a constant prefactor \( C \) for the de Sitter universe and the WKB time it defines through \([7]\) for an expanding universe \( (S_{-}) \) is

\[
\frac{\delta}{\delta t} = - \frac{3}{8} \sqrt{\sigma} \frac{\delta S_{-}}{\delta \sqrt{\sigma}} \frac{\delta}{\delta \sqrt{\sigma}} = \mp \sqrt{3\sigma \Lambda} \frac{\delta}{\delta \sqrt{\sigma}}.
\]
One can check that (8) is equivalent to the vacuum Friedman equation and that the WKB time (10) is the Friedman time providing the usual expression for the expansion scale factor \( a(t) = \exp(\sqrt{3} \frac{H}{\alpha}) \), with Hubble constant given by \( H = \sqrt{\frac{\Lambda}{3}} \). The chosen metric is

\[
 ds^2 = -dt^2 + a^2(t)dx^2.
\]  

which, introducing the conformal time \( \eta \in (-\infty, 0) \) as \( d\eta = dt/\alpha \), can also be written in the conformal form

\[
 ds^2 = a^2(\eta)(-d\eta^2 + dx^2), \quad a(\eta) = -1/\alpha \eta.
\]  

### III. Decoherence of Different Arrows of Time

We will consider the interference between the two different WKB branches associated to opposite sign of actions, taking a real solution of the vacuum Wheeler-DeWitt equation in the oscillatory regime

\[
 \Psi(0)(a) \propto e^{iMS_+}(a) + e^{iMS_-}(a),
\]  

which corresponds to a superposition of a contracting and an expanding universe. The generalization of the following results to other choices is straightforward. For simplicity of notation we make explicit only the dependency of the action \( S \) on the conformal factor although it is a function of its time derivative as well. We consider then a perturbation of the action represented by the wave functional of some non-gravitational field modes \( \chi(\varphi_k, \varphi_{-k}) \), the precise form of which we will derive later. The total wave functional will become

\[
 \Psi(0)(a) \rightarrow \Psi(a; \varphi_k, \varphi_{-k}) = \frac{1}{\sqrt{2}} \left( e^{iMS_+}(a) \chi^*(a; \varphi_k, \varphi_{-k}) + e^{iMS_-}(a) \chi(a; \varphi_k, \varphi_{-k}) \right)
\]  

since in the contracting and expanding branches the wave functionals of the perturbations are solutions of the Tonomaga-Schwinger equation and its conjugate, respectively. These perturbations can be used in a sense as a probe of the gravitational field to which they are coupled.

In addition to interference, we will take into account a mechanism for the decoherence of the two arrows of time. A classic example of quantum decoherence is provided in the context of the widely discussed problem of measurement in quantum mechanics. We do not intend to give here a full review nor to focus on the measurement problem, for which classic example of quantum decoherence is provided in the context of the widely discussed problem of measurement in the gravitational field to which they are coupled.

While decoherence does not explain how the apparatus collapses on a specific reading, it does provide an explanation for the transition from quantum amplitudes to classical probabilities for its outcomes by adding a environment \( \mathcal{E} \) represented in a nearly orthonormal basis \( \{ |e_i \rangle \} \). By doing so, the evolution of the initial tripartite state will be

\[
 \rho_{S,A} = \sum_{i,j} c_i c_j^* |s_i \rangle \langle s_j| \otimes |R \rangle \langle R| \rightarrow \sum_{i,j} c_i c_j^* |s_i \rangle \langle s_j| \otimes |a_i \rangle \langle a_j|. \tag{15}
\]  

What is usually identified as the measurement problem is the so called “problem of definite outcomes”, which questions why the apparatus is in practice found in a specific reading \( |a_i \rangle \) among all possible ones. (An other aspect of the measurement problem consists in the ambiguity in the freedom we have to chose the representations \( |a_i \rangle \) and \( |s_i \rangle \), which also finds its place in the treatment.)

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\[
 \rho_{S,AE} = \sum_{i,j} c_i c_j^* |s_i \rangle \langle s_j| \otimes |R \rangle \langle R| \otimes |E \rangle \langle E| \rightarrow \sum_{i,j} c_i c_j^* |s_i \rangle \langle s_j| \otimes |a_i \rangle \langle a_j| \otimes |e_i \rangle \langle e_j|. \tag{16}
\]  

The important observation at this point is that the environmental degrees of freedom are not accessible to the observer of the system, so that the system will be effectively described by the reduced density matrix

\[
 \rho'_{S,A} = \text{Tr}_E \{ \rho_{S,AE} \} = \sum_{i,j} c_i c_j^* |s_i \rangle \langle s_j| \otimes |a_i \rangle \langle a_j| \langle e_i| e_j \rangle \tag{17}
\]
obtained by tracing out the environmental degrees of freedom. The decay of the off-diagonal elements associated to interference is due to the vanishing of $\langle e_i | e_j \rangle$.

The same principle of environment-induced decoherence can be applied to the arrows of time, where the measured system can be identified with the de Sitter geometry while the measuring apparatus and the environment can be identified respectively with field modes inside and outside the Hubble horizon, the latter being inaccessible to the observer. We will consider as a simple example the perturbations described by a free massless scalar perturbations $\phi$ of a vacuum minimally coupled to the metric of the pure de Sitter universe. This could represent scalar perturbations of the metric or perturbations of a non-dynamical inflaton field, although we will continue to refer to it as matter or non-gravitational d.o.f. 

Rescaling this physical field as $\varphi = a \phi$, the perturbation of the geometrical action $MS$ is determined by the Lagrangian

$$L = \frac{1}{2} \int d^3x \left( \left( \varphi' - \frac{a'}{a} \varphi \right)^2 - (\partial_i \varphi)^2 \right)$$  \hspace{1cm} (18)

(where $' = \partial/\partial\eta$), which gives equations of motion of the form

$$\varphi'' - \left( \frac{\partial^2}{a^2} + \frac{a''}{a} \right) \varphi = 0.$$  \hspace{1cm} (19)

We introduce the Fourier mode decompositions of the field and its conjugate momentum as

$$\varphi(\eta, x) = \int \frac{d^3k}{(2\pi)^3/2} \varphi_k(\eta) e^{ik\cdot x}, \quad p(\eta, x) = \int \frac{d^3k}{(2\pi)^3/2} p_k(\eta) e^{ik\cdot x},$$  \hspace{1cm} (20)

where $p_k = \varphi_k^{\ast} - H_c \varphi_k$, $H_c = a'/a = aH$ being the conformal Hubble constant. Following the usual quantization procedure, we express the associated operators in terms of the time-dependent creation and annihilation operators $\hat{\phi}_k(\eta)$ and $\hat{\phi}_k(\eta)$

$$\hat{\varphi}_k = \frac{1}{\sqrt{2k}} (\hat{c}_k + \hat{c}_k^\dagger), \quad \hat{p}_k = -i \sqrt{\frac{k}{2}} (\hat{c}_k - \hat{c}_k^\dagger),$$  \hspace{1cm} (21)

so that the Hamiltonian $\hat{H}_\varphi = \int dx^3 \hat{H}_\varphi$ can be expressed as

$$\hat{H}_\varphi(\eta) = \int_{(R_+)^3} d^3k \left[ \frac{k}{2} \left( \hat{c}_k^\dagger \hat{c}_k + \hat{c}_k \hat{c}_k^\dagger \right) + iH_c \left( \hat{c}_k \hat{c}_k^\dagger - \hat{c}_k \hat{c}_k \right) \right],$$  \hspace{1cm} (22)

where $k = |k|$. The first term of (22) corresponds to the usual free time-dependent Hamiltonian while the second term provides the coupling of the gravitational and non-gravitational d.o.f. and determines the production of “particles” through squeezing of the initial vacuum state due to the expansion (squeezing along the field quadrature) or contraction (squeezing along the momentum quadrature) of the universe. The strength of the coupling is determined by the conformal Hubble constant $H_c$.

The Heisenberg equations of motion for the operators $\hat{c}_k$ and $\hat{c}_k^\dagger$ for the two modes are

$$\hat{c}_k' = i \left[ \hat{H}_\varphi, \hat{c}_k \right] = -ik \hat{c}_k + \frac{a'}{a} \hat{c}_k^\dagger, \quad \hat{c}_k^\dagger' = i \left[ \hat{H}_\varphi, \hat{c}_k^\dagger \right] = i k \hat{c}_k^\dagger + \frac{a'}{a} \hat{c}_k. \hspace{1cm} (23)$$

The operators at conformal time $\eta_0$ are related to those at a later time $\eta > \eta_0$ by a Bogolyubov transformation

$$\begin{pmatrix} \hat{c}_k(\eta) \\ \hat{c}_k(\eta) \end{pmatrix} = \begin{pmatrix} \alpha_k(\eta) & \beta_k(\eta) \\ \beta_k^*(\eta) & \alpha_k^*(\eta) \end{pmatrix} \begin{pmatrix} \hat{c}_k(\eta_0) \\ \hat{c}_k(\eta_0) \end{pmatrix},$$  \hspace{1cm} (24)

where $\alpha_k$ and $\beta_k^*$ are the Bogolyubov coefficients satisfying $|\alpha_k|^2 - |\beta_k|^2 = 1$, with the initial conditions $\alpha_k(\eta_0) = 1$ and $\beta_k(\eta_0) = 0$ (here and in the following we use the notation $k = |k|$). The time evolution of these coefficients is obtained straightforwardly from (23)

$$\alpha'_k = -i k \alpha_k + \frac{a'}{a} \beta_k^*, \quad \beta'_k = -i k \beta_k + \frac{a'}{a} \alpha_k^*,$$  \hspace{1cm} (25)

and in the case of de Sitter universe with vacuum fixed at $\eta_0 \to -\infty$ they are given by

$$\alpha_k = \left( 1 - \frac{i}{2k\eta} \right) e^{-i k \eta}, \quad \beta_k = \frac{i}{2k\eta} e^{i k \eta},$$  \hspace{1cm} (26)
In general, the vacuum state can be defined as the eigenstate of the annihilation operator at time $\eta_0$

$$\hat{c}_k(\eta_0)|0_k\rangle_{\text{in}} = 0.$$  (27)

Introducing the Schrödinger picture of the state at $\eta$, the transformation (25) gives the vacuum condition as

$$(\alpha^+_k(\eta)\hat{c}_k - \beta_k(\eta)\hat{c}_k^\dagger)|0_k\rangle_{\text{out}} = 0$$  (28)

In the basis that diagonalizes $\{\hat{\varphi}_k, \hat{\varphi}_{-k}\}$, the vacuum condition (28) together with (21) provides a Gaussian wave function for each mode of the out-state

$$\chi_k(a; \varphi_k, \varphi_{-k}) = (\varphi_k, \varphi_{-k})_{\text{out}} = \left(\frac{2}{\pi} \text{Re}(\xi_k)\right)^{\frac{1}{2}} \exp(-\xi_k \varphi_k \varphi_{-k}),$$  (29)

$$\xi_k = \left. \frac{1}{k \alpha^+_k + \beta_k} \right|_0 = \frac{k}{1 - iHc/k}.$$  

Notice that for $H \to 0$ these will reduce to the Gaussian vacuum fluctuations of Minkowski spacetime.

The ket vector describing the universe will therefore be

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left( |S_\pi\rangle \otimes \bigotimes_{\nu=0}^a |0^+_{\nu}\rangle_{\text{out}} \otimes \bigotimes_{\nu=a}^\infty |0^-_{\nu}\rangle_{\text{out}} \right) + \frac{1}{\sqrt{2}} \left( |S_-\rangle \otimes \bigotimes_{\nu=0}^a |0^+_{\nu}\rangle_{\text{out}} \otimes \bigotimes_{\nu=a}^\infty |0^-_{\nu}\rangle_{\text{out}} \right)$$  (30)

where we have used the notation $|\sigma|S_\pm \rangle \sim e^{iMS_\pm(a)}$.

For a given $a$, the reduced density matrix $\rho_R$ obtained by tracing out the environment (i.e. field modes with $k \leq H_c$) will be

$$\rho_R = \frac{1}{2} \left( |S_\pi\rangle \langle S_\pi| \otimes \bigotimes_{\nu=a}^\infty |0^+_{\nu}\rangle_{\text{out}} \langle 0^+_{\nu}| + |S_-\rangle \langle S_-| \otimes \bigotimes_{\nu=a}^\infty |0^-_{\nu}\rangle_{\text{out}} \langle 0^-_{\nu}| \right)$$  (31)

$$+ \frac{1}{2} \left( |S_\pi\rangle \langle S_-| \otimes \bigotimes_{\nu=a}^\infty |0^+_{\nu}\rangle_{\text{out}} \langle 0^-_{\nu}| + |S_-\rangle \langle S_\pi| \otimes \bigotimes_{\nu=a}^\infty |0^-_{\nu}\rangle_{\text{out}} \langle 0^+_{\nu}| \times \Delta \right)$$  (32)

where $\Delta$ in the last two terms is the damping factor (analogous to $\langle e_i|e_j \rangle$ in (17) that determines the decoherence of the arrows of time as a function of the scale factor $a$

$$\Delta(a) = \prod_0^a \left( \langle 0^+_k|0^+_k \rangle_{\text{out}} \right)^{d^3\nu} = \prod_0^a \left( \frac{\text{Re}(\xi_k(a))}{\xi_k(a)} \right)^{d^3\nu} = \prod_0^a \left( \frac{1}{1 + ia/\nu} \right)^{d^3\nu} = e^{i(n-2\pi)a^3/a^3},$$  (33)

where $n = \pi(4 + \ln(4) - \pi)/3 \approx 2.35$ and we have used $\prod_a^b f(x)dx = \exp \left( \log \left( \prod_a^b f(x)dx \right) \right) = \exp \left( \int_a^b \log f(x)dx \right)$ for the integral product over the dimensionless index $x$. Notice also that using the spherical symmetry of the problem, we have $\int_{\mathbb{R}}^3 d^3\nu = 4\pi \int_{\mathbb{R}} \nu^2 d\nu$. The plot for $\Delta(\log(a))$ is shown in Fig[1]. The associated decoherence time (time corresponding to a damping factor of $1/e$) is of the order of one tenth of the Hubble time

$$t_d = \frac{1}{3H} \log \left[ (1 + n)/n \right] \simeq 0.12/H.$$  (34)
IV. OBSERVATION

We consider now the observable effect of the interference of the two arrows of time in the field mode correlations, which would lead to observable results in the power spectrum. We will focus here on a specific mode $k$ and drop all other modes, that are now unobserved but in principle observable. Besides the problem of measurement, what becomes particularly relevant in quantum cosmology is the problem of the observer: since quantum cosmology is concerned with the total state of the universe, the question arises whether the observer should be included in this state and, if so, what the meaning of the wave function of the universe is. Discussion of this fundamental issue in transposing the tools and concepts of quantum mechanics to the study of the whole universe goes beyond the purpose of the present work, but one comment is of order if we want to discuss the possible effect of the interference. In quantum mechanics one usually implicitly assumes the presence of a classical and external observer behind the scenes, separated from the observed system as well as from the measuring apparatus and/or the environment. This observer does not “enter the equations” explicitly. Its implied existence may be seen in the derivation of the Tomonaga-Schwinger equation [0], where it can be identified with the 3-geometry and survives only in the form of the affine (time) parameter along the local worldline of the “laboratory”. In this sense, one may say that the observer is identified with the laboratory’s spatial geometry and clock. In the ansatz (2), such observers is specified by a single WKB branch, but in the general solution the observer will be in a superposition of branches. Unlike the one of non-relativistic quantum mechanics, this observer is in principle quantum and is internal to the wave functional $\Psi$. The semiclassical approximation then may be seen as the analogous of a large mass limit of the observer, which is not affected by the observed system but can still be found in a quantum superposition of two clocks, each ticking in a different time direction.

While we discuss observation from this tentative point of view, the interpretation of the wave functional of the universe and its observer deserves to be considered more accurately, possibly along the lines of a suitable formulation of quantum reference frames [20–22] or [23–25] for recent works) or other relational approaches to quantum observations. Also, works on the concept of “evolution without evolution” such as outlined in [25] and [27] may be of help in clarifying the issue with reference to the emergence of time. As far as we are concerned here, with what seems a natural extension from the non-relativistic case to the cosmological case, we identify the observer with what we previously referred to as the “system” and take it to be in the superposition (13). The state $\hat{\rho}_{R(\varphi)}$ of the observable matter field for such observer, up to a proper normalization constant $N$, is obtained by projecting $\rho_R$ over such superposition, i.e.

\[
\hat{\rho}_{R(\varphi)} \propto (|S_+| + |S_-|) \hat{\rho}_R (|S_+| + |S_-|) \\
\propto \left( \bigotimes_{\nu=a} \langle 0_k^\varphi \rangle_{\text{out}} \langle 0_k | + \bigotimes_{\nu=a} \langle 0_k \rangle_{\text{out}} \langle 0_k | \right) \\
+ \left( \bigotimes_{\nu=a} \langle 0_k^\varphi \rangle_{\text{out}} \langle 0_k | \times \Delta^* + \bigotimes_{\nu=a} \langle 0_k \rangle_{\text{out}} \langle 0_k | \times \Delta^* \right). \tag{35}
\]

\[
\Delta = \Delta \left( \langle a | \right) \tag{36}
\]
The expectation value for the correlations of a given field mode \( k \geq H_c \) is then given by two contributions, 
\[
\langle \hat{\phi}_k \hat{\phi}_{-k} \rangle = \langle \hat{\phi}_k \hat{\phi}_{-k} \rangle_{\text{no}} + \langle \hat{\phi}_k \hat{\phi}_{-k} \rangle_{\text{int}},
\]
where
\[
\langle \hat{\phi}_k \hat{\phi}_{-k} \rangle_{\text{no}} \propto \text{Tr} \left[ \frac{1}{\text{Re}[\xi_k]} \right] = \frac{1}{k} \left( 1 + \frac{H_c^2}{k^2} \right)
\]
is the usual contribution from the terms that survive decoherence, while
\[
\langle \hat{\phi}_k \hat{\phi}_{-k} \rangle_{\text{int}} \propto \text{Re} \left[ \Delta \frac{1 + iH_c/k}{1 - iH_c/k} \right]
\]
is the contribution coming from the interference of the arrows of time. The normalization constant \( N \) for the diagonal elements of \( \hat{\rho}_R(\varphi) \) associated to a given value of scale factor is
\[
N(a)^{-1} = 2 \left( 1 + \text{Re}[\xi_k(a)] \right) \text{Re} \left[ \Delta(a) \right]
\]
which gives normalized correlations (Fig.2)
\[
\langle \hat{\phi}_k \hat{\phi}_{-k} \rangle = \frac{1}{2k} \left( 1 + (H_c/k)^2 \right) + \text{Re}[\Delta(a)]
\]
Using \( \varphi_{-k}(\eta) = \varphi_k^*(\eta) \) and therefore \( \varphi_{+k}(\eta) \varphi_{-k}(\eta) = |\varphi_k(\eta)|^2 \), the probability distribution for the outcome of measurement of the power spectrum \( |\varphi_k|^2 \in \{0, \infty\} \) is given simply by
\[
P(|\varphi_k|^2) \propto |\chi_k(a; |\varphi_k|^2)|^2 + \text{Re} \left[ \chi_k^2(a; |\varphi_k|^2) \Delta \right],
\]
which describes how the interference between arrows of time is expected to affect the usual power spectrum outcome distribution (the first term in (41)) at very early times (that is, early compared to the Hubble time).

![Graph](image_url)

**FIG. 2:** Plot of the field mode correlations for different values of length scales (in units of Hubble radius) as a function of the e-folding (i.e. time in units of Hubble time), superposed to their usual value when interference is not taken into account.

### V. CONCLUSIONS

We have discussed the interference of two different orientations for the cosmological arrow of time identified in the expanding and contracting modes of a de Sitter universe in the semiclassical approximation of the Wheeler-DeWitt equation, as well as their decoherence due to quantum fluctuations of a massless scalar vacuum subject to gravitational particle production. In principle, the interference of the expanding and contracting modes could be observed in the
field mode correlations and power spectrum at very early times before the decoherence becomes strong. Whether
this simple model can be applied to the inflationary de Sitter phase of our universe or other scenarios where the
interference of geometries comes into play is left as a question for future inquiries. We have provided a tentative
reason for identifying the observer with the spatial geometry and therefore the decoherence refers to the decoherence
of the observer itself, which is coupled to the scalar vacuum. In any case, besides the details of the decoherence
mechanism, a better understanding of the role of the observer in quantum cosmology in future works is auspicated.

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