Dynamics superposition mechanism analysis of the overlapped tunnel groups during the shield tunnel undercrossing

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Abstract: Studies on superposition mechanism analysis of the old–new overlapped tunnel systems during the undercrossing process are still exploratory. Based on the viscoelastic-plastic theory of the underground structure and nonlinear evolution model of tunnel deformation and instability, the nonlinear dynamic evolution model, energy evolution model, and failure criterion of the three-dimensional (3D) tunnel system are established. The temporal and spatial superposition mechanism of the old–new overlapped tunnel system during the undercrossing process is quantitatively analyzed. The spatial expansibility and temporal continuity of the superposition mechanism are also verified. The intersecting line formula and its expansion formula of the cylinder and cone calculate the position, boundary, and volume of the superposition area. Based on the disturbed state concept theory, the disturbed function during the new shield tunnel undercrossing is theoretically derived. This study is significant for the quantitative analysis and control of the overlapped tunnel stability during the undercrossing process.

Keywords: overlapped tunnel; shield tunnel undercrossing; dynamics; superposition mechanism

1. Introduction

Applying the overlapped tunnel is different from the mature technology for the single-hole and parallel tunnel [1], which has multiple connectivity and short-distance undercrossing. Additionally, with complex geological conditions, terrible load conditions, and sensitive surroundings, the multiple-factor superposition effect is significant. Especially, it is challenging to reveal the superposition mechanism of the old–new overlapped tunnel groups during the undercrossing process, and it is still exploratory. Studies on the interaction between short-distance tunnels involve theoretical and empirical analysis, such as numerical simulation, field monitoring, modeling, and simulation tests. There are also achievements [2-6]. Fotieva and Sheinin (1966) discussed the interaction of two adjacent tunnels in the elastic soil by Laplace Conversion. Afterward, various empirical formulas of stability analysis were proposed. Recently, researchers discussed the tunnel interaction from various aspects. These empirical formulas have certain practicability but have limitations. The most widely used methods of numerical simulation are finite element, finite difference, and infinite element methods. Their simulation results reflect the rationality of modeling and its parameters [7-11]. Sun et al. (2002) simulated the design and construction processes of the short-distance overlapped tunnel system in the
Shanghai Pearl Line Project, and it was well-performed. These methods simulated tunnel constructions, and some quantitative and semi-quantitative results were obtained. However, the fine-scale simulation of the construction process must be extended further.

The field monitoring, modeling, and simulation tests are combined with the actual project [12-15]. Some qualitative results have been obtained through practical experiments of the interactive motivation between tunnels, but the costs are high.

This paper establishes a three-dimensional (3D) nonlinear dynamic evolution model and an energy evolution model of the tunnel system and provides the corresponding criteria based on the viscoelastic-plastic theory of underground structure and the nonlinear evolution model of tunnel deformation and instability. Therefore, the spatial-temporal superposition mechanism of the old–new overlapped tunnel groups during the undercrossing process is analyzed systematically and quantitatively. With the intersection line formula and its expansion of the cylinder and cone, the location, boundary, and volume of the coupled area are calculated. Based on the disturbed state concept (DSC) theory, the disturbed function of the new shield tunnel undercrossing process is derived, providing innovative procedures for stability analysis and control of the new overlapped tunnel during the undercrossing process.

2. 3D nonlinear dynamic model during a shield tunnel undercrossing

The process of a new shield tunnel undercrossing a well-built tunnel refers to continue moving the shield tunnel face forward. The complexity of the non-axisymmetric sectional geometry and initial geo-stress load can be overcome by taking a circular tunnel as an example to simulate the process of nonlinear shield tunnel undercrossing. This study simplified the longitudinal rectangular tunnel face as a semicircle, overcoming the stress and displacement distribution problems caused by the longitudinal rectangular tunnel face, keeping consistent with previous research on tunnel viscoelastic-plastic theory [16-17], and simplifying the calculation and analysis (Figure 1).

To unify the influence area of the longitudinal semicircular and rectangular tunnel face, it should be

\[ S_3 = S_1 + S_2 \] (Figure 1). The center position of the semicircular depends on the distance between the center and the rectangular tunnel face (d). Thus,

\[ d = \frac{\pi r}{4} \approx 0.7854r \] (1)

Consequently, the radius of the plastic zone of the longitudinal semicircular tunnel face equals the radius of the plastic zone of the tunnel cross-section. Figure 2 shows the longitudinal influence zone of shield tunneling.
Figure 2. The influence partition of shield tunneling

The tunnel face continues moving forward at a certain speed. Therefore, the face center is also moving, and its 3D expression is

\[ \mathbf{O}(x, y, z) = \mathbf{O}(x, y, z) + \mathbf{v}(x, y, z) \cdot t, \]  

(2)

where, \( \mathbf{O}(x, y, z) \) is the initial position of the face center, \( \mathbf{O}'(x, y, z) \) is the position of the face center at time \( t \), \( \mathbf{v}(x, y, z) \) is the tunneling speed, and \( t \) is the time.

After the above conversion, based on the viscoelastic-plastic theory of the underground structure [18], the area at the front of the tunnel profile and the tunnel face can be divided into the plastic zone and elastic zone, and the analysis of the plastic radius equals the cross-section. Thus, the former established dynamic evolution model (ED) and energy evolution model (EE) of tunnel deformation and instability can be used to analyze the tunneling process. For both models (ED) and (EE), the basic vertical evolution model of the shield tunneling process is the same as the original pattern but with different zoning and integration range. However, the actual tunneling system should combine the horizontal and vertical sections for the 3D expression, which consists of a cylinder and hemisphere (Figure 3).

Figure 3. The schematic diagram of the 3D tunnel system during excavation

Thus, the 3D dynamical evolution model of the tunnel system can be presented as
where, \( l \) is the length of the cylinder, varying with the position of the hemisphere center, and calculated from Eq. 2. \( R \) is the radius of the hemisphere.

Similarly, the 3D energy evolution model of the tunnel system can be presented as

\[
E_{3E} = \frac{\sum \int \! \int \! \int dD_{ij} \cdot dV / \sum \int \! \int \! \int dD_{ij} \cdot dV}{\sum \int \! \int \! \int dA \cdot dV / \sum \int \! \int \! \int dA \cdot dV}.
\]  (4)

Thus, all criteria for 3D tunnel deformation and instability, including the dynamic criterion, energy criterion, and the criterion established by authors [16-17], are similar. Only the definition of the original central area from two-dimensional (2D) to 3D must be transformed. In the excavation process, the stability of the longitudinal tunnel face can be evaluated according to the author’s criteria.

3. Dynamics temporal and spatial superposition mechanism of the overlapped tunnel groups during the shield tunnel undercrossing

3.1 Dynamics spatial superposition mechanism of the overlapped tunnel groups

No matter the position of the new shield face, the change of stress and displacement between the two tunnels cause the interaction. Its performance is intuitive. The construction of a new tunnel causes some mass loss, releases the surrounding rock stress, causes rock mass loosening, and subsidence. Furthermore, the ground is disturbed, and the positive and negative excess pore water pressure can be generated, causing ground subsidence or uplift. It can be summarized as when the geo-stress varies between two tunnels, it followed strain → deformation → displacement → subsidence or uplift of the well-built tunnel → deformation of the well-built tunnel → ground subsidence or uplift, and even instability and failure.

However, the essence of the above superposition mechanism is the spatial variation in the dynamic condition and energy state resulting from the continuous exchange of substance and energy between two tunnel systems. Therefore, we can assume that (1) two tunnel systems are in the stable state before the superposition occurrence and (2) when superposition occurs, one or several relatively less stable regions can be generated, referred to as the core area. To determine the boundaries of the core area, this section introduces the sliding short–long average method and Akaike Information Criteria (AIC) [19]. Using the 3D nonlinear evolution model of the tunnel system, in the direction \( d_j \), the evolution degree \( D_{EC} \) or \( E_{EC} \) forms continuous waveform sequences \( \{D_{EC}(i)\} \) or \( \{E_{EC}(i)\} \) after superposition, and its sampling rate is \( SR \). When the time-window is changed into the distance-related bit-window, the short-distance average bit-window is \( rs \), the long-distance average bit-window is \( rl \), the detection threshold value is \( th \), and the specific formula in the direction \( d_j \) is

\[
S_{RD}^{D} = \frac{\sum_{i=(th\cdot rs \cdot SR)-1}^{th\cdot rs \cdot SR} D_{EC}(i)}{rs \cdot SR}
\]

or

\[
S_{RE}^{D} = \frac{\sum_{i=(th\cdot rs \cdot SR)-1}^{th\cdot rs \cdot SR} E_{EC}(i)}{rs \cdot SR}
\]  (5)

\[
S_{RD}^{E} = \frac{\sum_{i=(th\cdot rl \cdot SR)-1}^{th\cdot rl \cdot SR} D_{EC}(i)}{rl \cdot SR}
\]

or

\[
S_{RE}^{E} = \frac{\sum_{i=(th\cdot rl \cdot SR)-1}^{th\cdot rl \cdot SR} E_{EC}(i)}{rl \cdot SR}
\]  (6)
\[ th = \frac{SRA_D}{LRA_D} \quad \text{or} \quad th = \frac{SRA_E}{LRA_E}. \]  

Here, \( n \) is the sliding point, and its range is 1, 2, ..., \( N - r \cdot SR + 1 \), and \( N \) is the total data length. When the detection threshold is greater than the set threshold, it shows that the waveform discontinuity points emerge, and the calculation of the detection threshold \( th \) requires several tests, and its ideal value varies from 2.0–4.0. However, the sliding short–long average method can only roughly determine an approximate location of the initial point of the waveform. Therefore, the data segment \( \{D_{EC} \pm D_{EC} + r \cdot SR\} \) or \( \{E_{EC} \pm E_{EC} + r \cdot SR\} \) should be selected, and the AIC criterion can be used for accurate calculation.

\[
AIC_D(k) = k \cdot \log_{10} \left\{ \frac{1}{k} \sum_{i=1}^{k} D_{EC}(i)^2 \right\} + (N - k - 1) \cdot \log_{10} \left\{ \frac{1}{N-k} \sum_{i=k+1}^{N} D_{EC}(i)^2 \right\}
\]

\[
AIC_E(k) = k \cdot \log_{10} \left\{ \frac{1}{k} \sum_{i=1}^{k} E_{EC}(i)^2 \right\} + (N - k - 1) \cdot \log_{10} \left\{ \frac{1}{N-k} \sum_{i=k+1}^{N} E_{EC}(i)^2 \right\},
\]

where \( N \) is the tested data points; \( k = 1, 2, ..., N - 1 \).

Thus, \( \{AIC_D(k)\}_{\min} \) or \( \{AIC_E(k)\}_{\min} \) are the initial points of waveform discontinuity, i.e., the boundary points toward \( d_j \) in the core area. Similarly, its boundary points can be calculated in each direction and can be connected as the boundary of the core area (Figure 4).

Additionally, according to the nonlinear analysis of tunnel deformation and instability [16-17], the core area feature can be summarized:

(1) for the dynamic aspect, the \( \left( \frac{\partial D}{\partial \tau} \right)_{\text{max}} \) zone must be in the core area. If the zone of the dynamical evolution degree \( D_{EC} \geq 0.5 + \Delta \) (after superposition), it must fall in the core area.

(2) for the energy aspect, the \( \left( \frac{\partial E}{\partial \tau} \right)_{\text{max}} \) zone must be in the core area. If the zone of the energy evolution degree \( E_{EC} < 1 \) (after superposition), it must fall in the core area.

The interaction (superposition) of the two tunnel systems generates the core area. Through continuous superposition, the state of the core area starts to change, and the change is transmitted to the surrounding rock of the core area, affecting the tunnel support structure. Simultaneously, the change in the tunnel support structure will affect the surrounding rock and is transmitted to the core area. This
interaction continues until a new stable state is reached. This superposition mechanism can be summarized as follows: ① the superposition ② the core area emerges ③ the status variation of the core area (stress, displacement) ④ the variation is transmitted to the surrounding soil and rock ⑤ the deformation of the soil and rock ⑥ the state variation of the support structure (stress, displacement). The superposition mechanism is explained below.

① The superposition: two tunnel systems emerge to affect each other, and superposition occurs.

② The core area emerges: according to the dynamic criterion and the energy criterion for tunnel failure, in the core area, after superposition, the correlation dimension $D_c$ and the entropy $S_e$ ($K$ entropy) of the new system are greater than the maximum value of the original system, initially $D_c > D_{max}$, $S_e > S_{max}$; or the correlation dimension $D_c$ and the entropy $S_e$ ($K$ entropy) of the new system are smaller than the maximum value of the original system, initially $D_c < D_{min}$, $S_e < S_{min}$.

③ The state variation of the core area refers to the continuous superposition and variation of the stress $\sigma$ and displacement $U$ between two tunnel systems, and the continuous substance and energy exchange between the core area and its surrounding.

④ The variation is transmitted to the surrounding soil and rock: the transmission of stress $\sigma$ and displacement $U$ from the core area to its surrounding is the superposition of the stress $\sigma$ and displacement $U$, which equals the state delivery. Thus, the variation of stress $\sigma$ and displacement $U$ shows the substance and energy exchange.

⑤ The deformation of the soil and rock: the variation of the displacement $U$ reflects the variation of the soil-rock state, and this variation is transmitted out continuously, causing the deformation of the surrounding soil-rock.

⑥ The state variation of the supporting structure: the state of the core area is transmitted to the supporting structure through the soil-rock media, causing a variation in stress $\sigma$ and displacement $U$.

When the supporting structure is changing, the substance and mass exchange is also conducting with the surrounding rock. The entire process is simultaneously conducted in the opposite direction and it is overlapped with the positive process.

This exchange of substance and energy from the core area to its surrounding indicates the transmission of the state from the core area to its surrounding, and the same in the other area. According to the nonlinear evolution model of the tunnel system, it is an axisymmetric system. Thus, the state is transferred regardless of the direction during the undercrossing. Without considering the large disturbance and fluctuation in the system, the exchange of substance and energy can be expressed as

Substance exchange:
$$\Delta M = \iiint_{D_{max} > S_{max} \text{ or } D_{min} < S_{max}} \rho \cdot dV$$

(9)

Energy conversion:
$$\Delta E = \iiint_{D_{max} > S_{max} \text{ or } D_{min} < S_{min}} \Delta \sigma \cdot \Delta U \cdot dV$$

(10)

However, when a large disturbance causes a huge fluctuation in the system, the exchange will not correlate with the principles, potentially causing the concentrated energy to release and substance loss. Also, this mechanism is the fluctuation and association of a new system, generated by the superposition of two tunnel systems, also causing energy and substance flow. When the substance changes and energy converts, in the surrounding of the core area $D_c > D_{max}$, $S_e > S_{max}$ or $D_c < D_{min}$, $S_e < S_{min}$ will happen. This is the expansion of the core area and it determines the stability of the overall system (without disturbance or the disturbance is not the principal impact). Similarly, the expansion area can be described as ① the superposition ② the core area emerges ③ the new core area forms ④ the new stable system forms. If the expansion of the core area stays in a certain range, the internal structure of the new system can maintain a similar state with the original system. If
the fluctuation exceeds the range, the new system will lose its stability and changes into a new state. Consequently, the superposition of the overlapped tunnel groups can expand spatially during the undercrossing, and the core area is the source of the new system failure.

### 3.2 Dynamics temporal superposition mechanism of the overlapped tunnel groups

When the new shield tunnel crosses the old one, the undercrossing process can be divided into several stages.

1. **The initial superposition**
   When a new shield tunnel system starts to couple with the old system, additional stress could cause subsidence of the outside area of the rock-soil sliding surface toward excavation and could lead to the deformation and displacement of the well-built tunnel. This happens because of the decline of the groundwater level and the stress release of the excavation face.

2. **The superposition in front of the tunnel face**
   When the tunnel face approaches the bottom of the well-built tunnel, the soil-rock stress between tunnels will increase or release, causing the plastic deformation, which could result in the subsidence or uplift of the tunnel. This happens because of the squeezing or loosening of the soil and rock in front of the tunnel face.

3. **The superposition during the tunnel face crossing**
   When the tunnel face reaches the bottom of the well-built tunnel, the direct disturbance of excavation is the principal influencing factor of the interaction between the two tunnel systems. Because of the different tunnel excavation methods and tunneling parameters, the stress state of the tunnel face is different. They can strongly disturb the well-built tunnel system.

4. **The superposition behind the tunnel face**
   When the tunnel face has passed the well-built tunnel, the release of stress and decrease in density after undercrossing continue to cause plastic deformation.

5. **The subsequent superposition**
   For an extended period after the two tunnels crossing, the previous deformation continues, which mainly comes from the soil-rock creep of plastic deformation and the new tunnel undercrossing.

To illustrate this superposition mechanism more clearly, Figure 5 explains the shield tunneling as an example.

**Figure 5.** The temporal superposition of the overlapped tunnel groups during shield tunneling (modified from Ref. [20])
The essence of the temporal superposition mechanism is still the substance and energy exchange between two tunnel systems. As the new shield tunnel progresses, external energy is input continuously to the well-built system, and substance and energy are removed. During the energy conversion, energy is stored as recoverable elastic energy because of the distinct rock and soil properties, and others dissipate unrecoverable (plastic deformation, heat loss). The substance and energy flows continue until it arrives at a new equilibrium and form a new system. The exchange rate closely relates to the geological condition and excavation rate. If the excavation is too fast, it could cause a fast superposition of stress and displacement, strong disturbance on the tunnel face, high soil pressure, concentrated substance and energy flow, and high fluctuation, increasing the stability risk. If the excavation is too slow, it extends the disturbance duration and it could cause creep of the surrounding condition. Additionally, it prolongs the duration of the substance and energy flows in the unstable state, the fluctuation duration of the undercrossing, and the duration of stability control. Thus, the superposition of the overlapped tunnel groups can continue temporally, and the continuity of the substance and energy flows are essential factors.

From the qualitative analysis above, combined with the 3D nonlinear evolution model of the tunnel system, toward the new shield tunnel undercrossing, the normal plane at the longitudinal direction of the well-built tunnel is taken as the boundary, and the old tunnel and its influence zone are divided into the new shield tunnel entering and leaving the side. Given this, a coordinate is established for the superposition mechanism analysis (Figure 6).

![Figure 6](image)

**Figure 6.** The coordinates for the temporal superposition mechanism analysis of the overlapped tunnel groups

Thus, the evolution degree of the well-built tunnel $D_e$ (or $E_e$) after the superposition $D_{ec} = D_e$ (or $E_{ec} = E_e$) is discussed here. In the coordinate, the equation for the superposition emerging when the new shield tunnel face reaches the influence boundary on the entering side of the influence scope of the old tunnel is

$$\frac{\partial D_e}{\partial r} > 0 \quad \text{or} \quad \frac{\partial E_e}{\partial r} > 0$$

$$\frac{\partial^2 D_e}{\partial r^2} > 0 \quad \frac{\partial^2 E_e}{\partial r^2} > 0$$

The equation for the superposition ending when it leaves the influence boundary on the leaving side of the influence scope of the old tunnel is

$$\frac{\partial D_e}{\partial r} > 0 \quad \text{or} \quad \frac{\partial E_e}{\partial r} > 0$$
\[
\begin{align*}
\frac{\partial D_k}{\partial r} < 0 & \quad \text{or} \quad \frac{\partial^2 E_k}{\partial t^2} < 0 \\
\frac{\partial^2 D_k}{\partial r^2} < 0 & \quad \frac{\partial^2 E_k}{\partial t^2} < 0
\end{align*}
\] (12)

where the influence boundary of the tunnel is its stability analysis range, that is, the influence scope of the old tunnel. Generally, it is no more than the tunnel depth, or the influence range of the underground excavation is \(6r_i\).

Table 1 and Table 2, respectively, present the evolution of the superposition process on the entering and leaving side when the overlapped tunnel system remains stable.

**Table 1.** The evolution mechanism of each point (area) on the entering side during the superposition process.

| No. | The superposition process | Dynamic evolution mechanism | Energy evolution mechanism |
|-----|---------------------------|-----------------------------|---------------------------|
| 1   | Before superposition      | \(\frac{\partial D_k}{\partial r} < 0\) | \(\frac{\partial E_k}{\partial r} < 0\) |
|     |                           | \(\frac{\partial^2 D_k}{\partial r^2} \leq 0\) | \(\frac{\partial^2 E_k}{\partial t^2} \leq 0\) |
| 2   | Emergence of superposition| \(\frac{\partial D_k}{\partial r} > 0\) | \(\frac{\partial E_k}{\partial r} > 0\) |
|     |                           | \(\frac{\partial^2 D_k}{\partial r^2} > 0\) | \(\frac{\partial^2 E_k}{\partial t^2} > 0\) |
| 3   | Development of superposition | \(\frac{\partial D_k}{\partial r} \geq 0\) | \(\frac{\partial E_k}{\partial r} \geq 0\) |
|     |                           | \(\frac{\partial D_k}{\partial t} \geq 0\) | \(\frac{\partial E_k}{\partial t} \geq 0\) |
| 4   | End of superposition      | \(\frac{\partial D_k}{\partial r} > 0\) | \(\frac{\partial E_k}{\partial r} > 0\) |
|     |                           | \(\frac{\partial^2 D_k}{\partial r^2} < 0\) | \(\frac{\partial^2 E_k}{\partial t^2} < 0\) |
| 5   | After superposition       | \(\frac{\partial D_k}{\partial r} < 0\) | \(\frac{\partial E_k}{\partial r} < 0\) |
|     |                           | \(\frac{\partial^2 D_k}{\partial r^2} \leq 0\) | \(\frac{\partial^2 E_k}{\partial t^2} \leq 0\) |

**Table 2.** The evolution mechanism of each point (area) on the leaving side during the superposition process.

| No. | The superposition process | Dynamic evolution mechanism | Energy evolution mechanism |
|-----|---------------------------|-----------------------------|---------------------------|
| 1   | Before superposition      | \(\frac{\partial D_k}{\partial r} > 0\) | \(\frac{\partial E_k}{\partial r} > 0\) |
|     |                           | \(\frac{\partial^2 D_k}{\partial r^2} \leq 0\) | \(\frac{\partial^2 E_k}{\partial t^2} \leq 0\) |
| 2   | Emergence of superposition | \(\frac{\partial D_k}{\partial r} < 0\) | \(\frac{\partial E_k}{\partial r} < 0\) |
|     |                           | \(\frac{\partial^2 D_k}{\partial r^2} > 0\) | \(\frac{\partial^2 E_k}{\partial t^2} > 0\) |
Both Table 1 and Table 2 show that the superposition of overlapped tunnel groups can continue temporally, and the continuity of the substance and energy flows is the driving source.

4. The dynamics superposition zone of the overlapped tunnel groups during the shield tunnel undercrossing

When the new shield tunnel overlaps with the well-built one, the overlapped area between the influence zones of the two tunnel systems (the superposition zone) includes two parts, namely the tunnel superposition zone and the front of tunnel face superposition zone. Here, they are discussed, respectively.

4.1 The tunnel superposition zone

Mathematically, the geometry of the overlapped tunnel system is the case of multiple connected components (Figure 7). If the overlapped tunnel system and its surrounding rock system are simplified as two cylinders, they are intersected. The intersecting part is the superposition zone of these overlapped tunnels. The superposition zone can be calculated by first calculating the intersecting line (Figure 8). With the intersecting line, the shape and position of the overlapped tunnels can be expressed completely, and the volume of the intersection part can be calculated.

![Figure 7. The schematic diagram of the overlapped tunnel group model](image-url)
Figure 8. The schematic diagram of the intersection of the overlapped tunnel group

Figure 9 and Figure 10 introduce a coordinate system to calculate the intersection line of two cylinders by applying the intersection and expansion formulas of the cylinder and cone [21]. By considering the location and intersection patterns, it can be discussed as full intersection and mutual intersection.

Figure 9. The schematic diagram of the intersection of two cylinders in a coordinate system
Figure 10. The schematic diagram of the intersection position of two cylinders in a coordinate system

(1) Full intersection
Full intersection means that every plain line of Cylinder I intersects a corresponding line of Cylinder II. By the plain line method, the intersection line can be expressed as

\[
\begin{align*}
X &= \frac{1}{\sin \beta} (R_1 \cos \beta \sin \theta - R_1 \sin \varphi) \\
Y &= R_2 \cos \theta \\
Z &= R_2 \sin \theta 
\end{align*}
\]

(13)

where \( \varphi \) is \( 0 - 360^\circ \), \( \beta \) is the angle between the projection of Axis II on the XOZ plane and the \( x \)-axis, \( \theta \) is the angle between the projection of Axis II on the YOZ plane and the \( y \)-axis, \( a \) is the distance between two cylinder axes, and \( R_1, R_2 \) are the outer diameters of Cylinder I and Cylinder II, respectively.

\[
\begin{align*}
\sin \theta &= \frac{1}{R_2} \sqrt{R_2^2 - (R_1 \cos \varphi + a)^2} \\
\cos \theta &= \frac{1}{R_2} (R_1 \cos \varphi + a)
\end{align*}
\]

(2) Mutual intersection
Mutual intersection means that when \( \varphi \) varies within a certain range, the plain lines of Cylinder I do not intersect with any plain lines of Cylinder II, whereas when the intersection happens, there will be two \( \theta \) values corresponding with each \( \varphi \). Similarly, the formula of the intersection line can be expressed as

\[
\begin{align*}
X &= \frac{1}{\sin \beta} (R_2 \cos \beta \sin \theta - R_1 \sin \varphi) \\
Y &= R_2 \cos \theta \\
Z &= R_2 \sin \theta 
\end{align*}
\]

(14)

where,

\[
\begin{align*}
\sin \theta &= \frac{1}{R_1} \sqrt{R_1^2 - (R_1 \cos \varphi + a)^2} \\
\cos \theta &= \frac{1}{R_1} (R_1 \cos \varphi + a) \\
\sin \theta &= -\frac{1}{R_2} \sqrt{R_2^2 - (R_2 \cos \varphi + a)^2} \\
\cos \theta &= \frac{1}{R_2} (R_2 \cos \varphi + a)
\end{align*}
\]
According to the values of $\sin \theta$ and $\cos \theta$, the variation range of $\theta$ can be calculated. $\varphi \in [\varphi_1, \varphi_2]$, $\varphi_1, \varphi_2$ are the roots of $R^2_1 - (R_1 \cos \varphi + a)^2 = 0$, and $\varphi_2 > \varphi_1$.

Furthermore, the outer surface of the two cylinders can be described by the following formulas:

$$\text{Cylinder I: } y^2 + z^2 = R^2_1 \quad \text{and}$$

$$\text{Cylinder II: } (x-k_{xy}y)^2 + (z+a-k_{zy}y)^2 = R^2_2 \,, \quad (15)$$

where $k_{xy}$ is the slope of projection of Axis II on the plane XOY and $k_{zy}$ is the slope of projection of Axis II on the plane ZOY.

From Eq. 13–16, the equations for the curved surface, surrounded by the intersection line of two outer cylindrical surfaces, are recorded as $S_{11}(x, y, z) = 0$ and $S_{21}(x, y, z) = 0$, and simplified as $z_{11} = S'_{11}(x, y)$ and $z_{21} = S'_{21}(x, y)$, respectively. If the projection of the intersection line on the plain XOY is $I_{xy1}$, the volume of the intersection part of two cylinders can be expressed as

$$V_{121} = \int_{I_{xy1}} |z_{11} - z_{21}| dxdy \quad (17)$$

![Figure 11. The intersection part of two cylinders](image)

Consequently, by dividing the tunnel system into different segments, the volume of the superposition area of the two tunnels can be calculated using Eq. 17.

### 4.2. The front of tunnel face superposition zone

Based on the 3D tunnel system shown in Figure 1, the tunnel surface can be simplified as a hemispherical body. Therefore, the superposition area in front of the tunnel can be simplified as the intersection area of a cylinder and hemisphere. By transforming the coordinates, the intersection line between the cylinder and hemisphere (Figure 12) can be expressed as

$$\begin{cases} X = \pm R \sin \alpha \\
Y = R \sin \alpha \\
Z = \pm R \sqrt{1 - \sin^2 \alpha} \end{cases} \quad (18)$$
where \( \sin \alpha = \frac{a - R_z}{R_1 - R_z} \) and other parameters are the same as in Eq. 13.

**Figure 12.** The schematic diagram of the intersection between a cylinder and a hemisphere in the coordinate system

As in 4.1, the hemisphere surface integrating with Cylinder I and the outer cylindrical surface of Cylinder II can be expressed as

The hemisphere surface: \( x = R^2_1 - y^2 - z^2 \) \hspace{1cm} (19)

The cylindrical surface: \( (x - k_{xy} y)^2 + (z + a - k_{zy} y)^2 = R^2_2 \) \hspace{1cm} (20)

From Eq. 18–20, the equations for the curved surface surrounded by the intersection line of the hemisphere surface and cylindrical surface are recorded as \( S_{12}(x, y, z) = 0 \) and \( S_{22}(x, y, z) = 0 \), and simplified as \( z_{12} = S'_{12}(x, y) \) and \( z_{22} = S'_{22}(x, y) \), respectively. If the projection of the intersection line on the plain XOY is \( I_{xy2} \), the volume of the intersection part of the cylinder and hemisphere can be expressed as

\[
V_{122} = \iiint_{I_{xy2}} |z_{12} - z_{22}| \, dx \, dy
\]

\hspace{1cm} (21)

**Figure 13.** The intersection part of the cylinder and hemisphere
Consequently, by dividing the tunnel system into different segments, the volume of the front superposition area of the tunnel face between the well-built tunnel and new shield tunnel can be calculated from Eq. 21.

4.3 The superposition zone during undercrossing

Accordingly, both the hemisphere and Cylinder I can be unified to calculate the superposition zone when the new shield tunnel under-crosses the well-built tunnel. In Figure 14, the center of the hemisphere is the reference point of the new shield tunnel system, and its initial coordinate is assumed as \( (y_0, 0, 0) \). The tunneling speed \( v(x, y, z) \) in Eq. 2 can be simplified as \( v(y) \), and the coordinates of the new shield tunnel system move forward with speed \( v(y) \) along the \( x \)-axis during the undercrossing. Thus, the coordinates of the hemisphere center are \( (y_0 + v \cdot t, 0, 0) \). The coordinates of the hemisphere and Cylinder I can be unified by adding \( (y_0 + v \cdot t, 0, 0) \) to the coordinates of the front tunnel face of the new shield tunnel system.

\[
V_{12}^T = V_{121} + V_{122}.
\]  

**Figure 14.** The superposition when the new shield tunnel crosses the well-built tunnel in the coordinate

The volume of the superposition zone of the overlapped tunnel group (Figure 15) can be expressed as

\[
V_{12}^T = V_{121} + V_{122}.
\]  

**Figure 15.** The superposition zone of the overlapped tunnel system during the undercrossing
Three types of superposition (intersection) states exist.

1. If only the front of the tunnel face of the new shield tunnel couples with the well-built tunnel system, the volume of the superposition area is

\[ V_{12}^{T} = V_{121}^{T} \]

2. When the new shield tunnel crosses and couples with the well-built tunnel system, the volume of the superposition area is

\[ V_{12}^{T} = V_{121}^{T} + V_{122}^{T} \]

3. When the new shield tunnel has crossed through the well-built tunnel, and the front of the tunnel face will no longer couple with the well-built tunnel, the volume of the superposition area is

\[ V_{12}^{T} = V_{122}^{T}. \]

Thus, the superposition area can be calculated from Eq. 21 and used to describe the evolution process of the superposition area of the overlapped tunnel system during the undercrossing.

5. The disturbed function of the shield tunnel in the superposition area

The disturbance of the new shield tunnel crossing is a crucial factor for the formation of the superposition area. Thus, combined with the DSC theory [22-23], this section presents the disturbed function of the shield tunnel crossing.

From the DSC theory, the behavior of the relatively intact (RI) state and the full adjustment (FA) state (so-called reference state) can express the observed behavior of the rock material units. The elastic, elastoplastic, and other models can express the RI state, and it can be assumed as a continuous medium with elastic and inelastic strain and stress, corresponding to the initial state of the material. The critical state or other models can express the FA state, and it corresponds to an extension of cracks or pores until its final failure, which arrives at a constant state of energy dissipation. The disturbance can be defined by the disturbed degree \( D \), representing the relationship among the observed response (the actual behavior of the material), the initial response (RI response), and the critical response (FA response). It can be expressed by RI or FA state material (Figure 16).

![Figure 16. The disturbed state theory diagram (modified from [22] and [23]).](image)

During the crossing process, mechanical extrusion and loose, loading and unloading, and friction decreased the strength of the soil and rock. With different degrees of disturbance, the decrease of the strength is different. According to the expansion of the spherical cavity with the equilibrium differential equation, the Tresca and boundary condition [24], the disturbed degree is assumed as a
logarithmic decrement in the radial direction. Based on the former DSC theory, the disturbed degree function \( D \) is introduced, and \( S_u \) is the disturbed strength. In the plastic zone, it can be defined as

\[
S_u = S_{u0}(1 - D).
\]

The disturbed degree \( D \) function in the radial direction is

\[
D(r) = (1 - \frac{1}{S_i}) \ln \frac{R_p}{r} / \ln \frac{R_p}{R_0},
\]

where \( S_i \) is the sensitivity, \( S_i = \frac{S_{u0}}{S_u} \), and \( S_{u0} \) and \( S_u \) are the in-situ strength of the soil and the strength of the complete disturbed soil, the residual strength. \( R_p \) is the plastic zone radius, and \( R_0 \) is the tunnel radius.

From the viscoelastic-plastic theory of underground structure, in the elastic zone, the disturbance directly affects the displacement \( u \) by \( \sigma \). Parts of the elastic zone can change into plastic after disturbance. Thus, the disturbed degree function can be expanded to the elastic zone

\[
D(r) = (1 - \frac{1}{S_i}) \ln \frac{R_p + R_e}{r} / \ln \frac{R_p + R_e}{R_p},
\]

where \( R_e \) is the calculated radius of the elastic zone.

In the plastic zone, the disturbance influences the displacement \( u \) by the increment of pore water pressure.

According to the Hankel Function \[25\],

\[
\Delta p = \Delta \sigma_{oct} + \alpha^* \Delta \tau_{oct},
\]

where \( \alpha^* = \frac{\sqrt{2}}{2}(3A_f - 1) \) is the Hankel pore water pressure coefficient, \( A_f \) is the excess pore water pressure coefficient

\[
\Delta \sigma_{oct} = \frac{1}{3}(\Delta \sigma_e + \Delta \sigma_o + \Delta \sigma_p)
\]

and

\[
\Delta \tau_{oct} = \frac{1}{3} \sqrt{3(\sigma_e - \sigma_o)^2 + (\sigma_o - \sigma_p)^2 + (\sigma_p - \sigma_e)^2}.
\]

According to the derivation,

\[
\Delta p = \frac{2}{3} S_{u0} \cdot D + \frac{\sqrt{2}}{3} S_{u0} \cdot \alpha^* \sqrt{3 + 4D^2 - 6D}.
\]

The above disturbance is mainly caused by mass loss. However, it could also be caused by the vibration load of the train in the well-built tunnel, the positive thrust during the construction of the shield tunnel, and the friction between the shield shell and surrounding soil. Compared with tunnel underpass, the disturbance of the train vibration load is small and can be ignored. However, additional stresses because of normal thrust and friction should not be ignored \[26-29\]. The disturbance intensity and range can be calculated according to the specific situation, and the disturbance function can be calculated using superposition with the mass loss disturbance.

The pulse function is defined as

\[
h_p(t) = \begin{cases} 
1 & t_i \leq t \leq t_j \\
0 & \text{other} \end{cases}
\]

where the duration of the disturbance \( t_j - t_i \) depends on the disturbance of the specific construction, and \( h_p(t) \) will be 1 with disturbance and 0 without disturbance.
Therefore, from the above discussion, if the disturbance is introduced from \( t = t_i \), the disturbed function of the overlapped tunnel system during undercrossing (axial symmetry without \( \theta \) variation) is

\[
f_T(r, \theta, t) = \begin{cases} 
- \sigma(t = t_i) \cdot D(r) \cdot h_T(t) & \text{elastic zone} \\
\Delta p \cdot h_T(t) & \text{plastic zone}
\end{cases}
\]  \quad (29)

6. Engineering application

This section uses the overlapped tunnel Tianheanyi Road, which is the cross between Guangzhou Metro Line One and the transportation system of Zhujiang Newtown, as an example for the stability analysis. The right and left branch of the transportation system twice crossed underneath the operating Metro Line One successfully in September 2007 and June 2008. The minimum distance between the transportation system and Metro Line One is 2.275 m. The geological condition of the cross-section includes miscellaneous soil, silty clay layer, conglomerate layer, argillaceous siltstone, which was strengthened before the tunnel undercrossing.

In total 22 monitoring sections (Figure 17) and 110 monitoring points (Figure 18) exist in the up and down two lines at the cross-section of the Metro Line One. Monitoring was conducted for 21 months, and the frequency was from once per hour to four times per hour.

![Figure 17. The plan of the monitoring cross-section [29]](image)

![Figure 18. The point set on the monitoring cross-section [29]](image)
Note: The code rules require the line number plus monitoring section number plus monitoring point number. For example, XA1 is monitoring point 1 of section A in the downline, and SA1 is monitoring point 1 of section A in the upper line.

From the three-dimensional nonlinear dynamic model of shield tunnel undercrossing, combined with dynamic space-time superposition mechanism analysis, the chaotic and energy dissipation characteristics of the entire process of the transportation system undercrossing Metro Line 1 are discussed. The progress of a new tunnel undercrossing the well-built tunnel can be divided into five periods: before first across (BFA), after first across (AFA), before second across (BSA), after second across (ASA), and none across (NA). The variation in the correlation dimension D2 (Figure 19) and entropy K2 (Figure 20) of the 22 cross-sections in polar coordinates were calculated. The relative positions of the monitoring points were set at the actual monitoring sites.

![Figure 19. The correlation dimension of SC cross-section](image1)

![Figure 20. The entropy K2 of SC cross-section](image2)

Accordingly, during the shield tunnel undercrossing, both the correlation dimension D2 and entropy K2 of Metro Line One are small. The D2 is a decimal level and K2 tends to 0 but is larger than 0. It shows the uncertainty of the entire system, but it is low. Considering the noise, train vibration load, and short operation period, the system is in a stable development stage. From the analysis of dynamic temporal and spatial superposition mechanisms, grouting reinforcement input high entropy into the system before tunnel crossing, destroying the orderly evolution of shield tunnel crossing. The grouting layer consumes and absorbs the energy generated by superposition and reduces the strain energy difference caused by shield tunneling disturbance, preventing system failure. Consequently, Metro Line One was in the stable state during the shield tunnel undercrossing, which is consistent with the monitored results. It indicates that the method for analyzing the dynamic superposition of tunnel groups under shield tunnel undercrossing is reliable and rational, and valuable for actual engineering design and construction.

7. Conclusions

(1) The nonlinear dynamic evolution and energy evolution models and failure criterion of the 3D tunnel system are established, and the temporal and spatial superposition mechanism of the old–new
overlapped tunnel system during the undercrossing is quantitatively analyzed. It reveals the spatial expansibility and temporal continuity of the superposition mechanism during the undercrossing, and that its driving source is the substance and energy flows in these two tunnel systems. It also provides a new way to essentially reveal the superposition mechanism of the overlapped tunnel groups during the shield tunnel undercrossing.

(2) The location, boundary, and volume of the superposition area and the variation of its disturbance function when the new shield tunnel crosses the well-built tunnel are discussed. It reveals that the superposition range of the overlapped tunnel groups mainly depends on the geological conditions and disturbance intensity of the shield tunnel. It is also the foundation of the quantitative analysis of overlapped tunnel stability during the undercrossing process.

(3) By actual engineering project application, the grouting is effective to consume and absorb the energy produced by the superposition of two overlapped tunnel systems, reducing the strain energy difference caused by the shield disturbance. It also shows the reliability and rationality of the method for analyzing the dynamic superposition of tunnel groups under shield tunnel undercrossing and is valuable for actual engineering design and construction.

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