NMR imaging analogue of the individual qubit operations in superconducting flux-qubit chains

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Abstract. A solid-state quantum computer conventionally requires the control lines for each qubit to operate individually. This makes quantum circuits complicated, especially in multi-qubit systems. Here we propose a simple single-qubit operation in a superconducting flux-qubit chain without control lines in an analogy of the Nuclear Magnetic Resonance (NMR) imaging technique. A superconducting flux qubit can be regarded as an artificial atom with spin 1/2 that can be manipulated by an external magnetic field. To identify the location of each qubit in the qubit chain, a static field gradient is introduced along the qubit chain. This magnetic field gradient causes differences in the Zeeman frequencies of each separate qubit. This allows a large number of distinct qubits to be individually addressed. Single-qubit operations are then possible when the external electromagnetic frequency is adjusted to the resulting energy-level separation of the target qubit.

1. Introduction

The quantum computer has been attracting increasing attention over the last decade because its unrivaled power exceeds that of its classical counterpart as regards solving certain problems. A quantum computer requires two kinds of quantum gates, i.e., a single-qubit gate and a two-qubit gate. In a quantum computer based on solid-state devices, quantum circuits consisting of control lines designed for performing such gate operations become complicated. This is particularly true for multi-qubit systems capable of practical quantum computation. Thus their controllability generally deteriorates, leading to decoherence, which disturbs quantum-mechanical coherent temporal evolution. A superconducting qubit family based on the Josephson effect is one candidate for a quantum computer because of its scalability and relatively long decoherence time [1]. However, the above difficulties are inevitable even in superconducting systems.

In this paper, we theoretically investigate single-qubit operations without individual control lines in superconducting flux qubits [2]. In certain candidates such as Nuclear Magnetic Resonance (NMR) system [3], no external electrodes can be introduced to control individual qubit states in principle. In such a system, a static field gradient is introduced along the qubit chain to identify each qubit location in the qubit chain. This magnetic field gradient causes differences in the Zeeman frequencies of each separate qubit. This allows a large number of distinct qubits to be individually addressed. This is called the NMR imaging technique [4]. Here we apply NMR imaging techniques to superconducting flux qubits.
2. System: a zigzag flux-qubit chain

The system that we are considering is composed of a Josephson transmission line and a zigzag chain of bipartite superconducting flux qubits with an alternating arrangement, i.e., odd-numbered qubits function as logical bits, while their nearest neighbors, namely even-numbered qubits, function as switches between logical bits (see Fig. 1(a)) [5]. For simplicity, the Zeeman frequencies of all logical qubits are assumed to be the same i.e., \( \epsilon_{2n+1} = \hbar \omega_1 \) (n= positive integer) and very different from those of switch qubits \( \epsilon_{2n} = \hbar \omega_2 \neq \hbar \omega_1 \). The Hamiltonian is expressed as

\[
H = H_1 + H_s + H_{int} = \sum_{n=1}^{\infty} \frac{\hbar \omega_1}{2} \sigma_{2n-1}^z + \frac{\hbar \omega_2}{2} \sigma_{2n}^z + g \sum_{n=1}^{\infty} \left( \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y \right),
\]

where \( \sigma_{n}^{x,y,z} \) denotes the Pauli matrix in the \( n \)th qubit bases \(|0\>_n\) and \(|1\>_n\). The first and second terms in Eq. (1) represent the logical-qubit and switch-qubit energies, respectively. The last term describes capacitive coupling with the coupling constant \( g \) between the logical qubit and the switch qubit. The logical qubits are initially decoupled from each other in this system since \( |\omega_1 - \omega_2| \gg g/\hbar \). The coupling between two logical qubits is introduced through a third (switch) qubit controlled by a fluxon motion, i.e., the qubit-qubit interaction is turned on when the fluxon induces an energy level shift equal to the energy level separation of the logical qubit, so as to resonate energetically among three qubits. During the resonance, the state vector \(|\Psi(t)\rangle\) evolves through the relation \(|\Psi(t)\rangle = U(t)|\Psi(0)\rangle\) with \( U \) being a time-translational operator. The evolution makes two-qubit operation possible [5].

3. Single-qubit operation

Now let us consider individual qubit operations without individual control lines in the zigzag flux-qubit chain. Rabi oscillations are widely employed to manipulate single qubits, i.e., coherent temporal oscillations of the population inversion in a qubit are driven by an external electromagnetic field with a frequency close to the energy level separation, and lead to coherent superpositions of the states. In the zigzag flux-qubit chain, we assume that all logical qubits have same energy level separations. This makes individual single-qubit operations by the uniform alternating bias flux \( \Phi_0 \cos (\omega t + \theta) \) impossible, where \( \Phi_0 \) and \( \omega \) are the oscillation amplitude and the frequency, respectively. \( \theta \) is the initial phase. To identify the target qubit for manipulation, we apply the NMR imaging technique to our zigzag flux-qubit chain. In NMR imaging, a static field gradient is introduced along the qubit chain. This magnetic field gradient causes a bias flux for each separate qubit of a different magnitude \( \Phi_n^d \) and differences in the Zeeman frequencies. This identifies each qubit location in the qubit chain. Thus, a large number of distinct qubits

![Figure 1. Schematics of a flux qubit chain.](image-url)
could be individually addressed. The Hamiltonian for this NMR imaging technique is given by

\[ H_{\text{ctrl}} = \sum_m \frac{\hbar \delta \omega_m}{2} \sigma_m^z + \sum_m \frac{\hbar \Omega}{2} (\sigma_m^x \cos(\omega t + \theta) + \sigma_m^y \sin(\omega t + \theta)) \]  

(2)

where the suffix \( m \) takes only odd positive numbers. The Rabi frequency \( \Omega \) is defined as

\[ \hbar \Omega = |I_0| \Phi^a \]

where \( I_0 \) is the off-diagonal matrix element of the persistent current operator of the flux qubit. The first term in Eq. (2) represents a qubit energy shift \( \delta \omega_m \) caused by the magnetic field gradient applied to the zigzag flux-qubit chain. Thus each logical qubit in the chain possesses a different resonance frequency. The biased resonance frequency is expressed as

\[ \omega'_m = \omega_1 + \delta \omega_m = \sqrt{\epsilon_m^2 + \Delta^2} \]  

(3)

where \( \Delta \) is the tunneling splitting between the persistent current states and \( \epsilon_m \) is the energy bias, which is approximately proportional to the bias flux at the \( m \)th flux qubit \( \Phi_m^a \). For a linear magnetic-field gradient, \( \omega'_m \) is shown in Fig. 2(b). The second term in Eq. (2) describes an external alternating magnetic field applied to the zigzag flux-qubit chain in order to produce the Rabi oscillations at a target qubit with \( \omega = \omega'_m \). In the interaction picture, a state vector for each logical qubit evolves followed by \( |\phi(\tau)\rangle_m = U_m(\tau)|\phi(0)\rangle_m \) where the time-translational operator \( U_m(\tau) \) for the \( m \)th logical qubit is given as

\[ U_m(\tau) = e^{-i\frac{(\omega_m-\omega)\tau}{2}}\sigma_m^z \exp \left[ -i \left\{ \frac{(\omega_m - \omega)\tau}{2} \sigma_m^z + \frac{\Omega \tau}{2} \left( \sigma_m^x \cos \theta + \sigma_m^y \sin \theta \right) \right\} \right]. \]  

(4)

The target flux qubit, say the \( j \)th qubit, can easily be extracted from a large number of identical qubits in the chain if \( \omega = \omega_j \) is set. In this case, the translational operator \( U_j(\tau) \) is given as

\[ U_j(\tau) = e^{-i\frac{\delta \omega_j \tau}{2}}\sigma_j^z e^{-i\frac{\Omega \tau}{2} \left( \sigma_j^x \cos \theta + \sigma_j^y \sin \theta \right)}, \]  

(5)

otherwise, all the qubits except for the target bit hold the nonresonance condition \( |\omega_k - \omega_j| \gg \Omega \), resulting in the expression

\[ U_{k \neq j}(\tau) \sim e^{-i\frac{(\omega_m-\omega)\tau}{2}}\sigma_k^z e^{-i\frac{(\omega_j-\omega)\tau}{2}}\sigma_j^z = e^{-i\frac{\delta \omega_k \tau}{2}}\sigma_k^z. \]  

(6)

4. Refocusing scheme

As shown in Eqs. (5) and (6), undesired phase differences \( \delta \omega_j \tau \) and \( \delta \omega_k \tau \) are generated in each flux qubit during the Rabi oscillations under the magnetic field gradient. These additional phases result in the extra phase operation defined as \( \exp \left[ -i \delta \omega_m \tau / 2 \right] \). To remove these undesired phase operations, we also apply refocusing techniques [3] that are well established in the NMR research field as follows:
After this refocusing sequence, the translational operators for $k \neq j$ in Eq. (6) are canceled i.e., $U_{k \neq j}(2\tau) \sim 1$. Meanwhile, translational operator for the $j$-th qubit (5) yields

$$U_j(2\tau) = e^{-i\Omega \theta} \left( \sigma_x^j \cos \theta + \sigma_y^j \sin \theta \right) = \begin{pmatrix} \cos \left( \frac{\Omega \theta}{2} \right) & -ie^{-i\theta} \sin \left( \frac{\Omega \theta}{2} \right) \\ -ie^{-i\theta} \sin \left( \frac{\Omega \theta}{2} \right) & \cos \left( \frac{\Omega \theta}{2} \right) \end{pmatrix}. \quad (7)$$

This operation is simply a Rabi rotating gate, which is a universal single qubit gate operation [6].

5. Summary

We have proposed single flux-qubit operations analogous to the NMR imaging technique. This allows us to manipulate an arbitrary single flux qubit in a zigzag flux-qubit chain without using individual control lines, and then greatly simplify quantum solid-state circuits. It is known that any quantum algorithm is decomposed into this Rabi rotating gate and a two-qubit gate, such as the controlled NOT gate. Thus, a solid-state quantum computer without individual control lines is made possible by using this single-qubit operation technique and the fluxon-controlled two-qubit interaction technique [5].

Acknowledgments

This work was supported in part by a Grant-in-Aid for Scientific Research (18540352 and 195836) from the Ministry of Education Culture, Sports, Science and Technology of Japan.

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