Inference in Games Without Equilibrium Restriction: An Application to Restaurant Competition in Opening Hours

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ABSTRACT
This article relaxes the Bayesian Nash equilibrium assumption in the estimation of discrete choice games with incomplete information. Instead of assuming unbiased/correct expectations, the model specifies a player’s belief about the behaviors of other players as an unrestricted unknown function. I then study the joint identification of belief and payoff functions in a game where players have different numbers of actions (e.g., \(3 \times 2\) game). This asymmetry in action sets partially identifies the payoff function of the player with more actions. Moreover, if usual exclusion restrictions are satisfied, the payoff and belief functions are point identified up to a scale, and the restriction of equilibrium beliefs is testable. Finally, under a multiplicative separability condition on payoffs, the above identification results are extended to the player with fewer actions and to games with symmetric action sets. I apply this model and its identification results to study the store hours competition between McDonald’s and Kentucky Fried Chicken in China. The null hypothesis of unbiased beliefs is rejected. If researchers incorrectly impose the equilibrium assumption, then the estimated interactive effect would be biased downward by more than 50%.

1. Introduction

Over the past two decades, econometric models of games have been actively applied to study oligopolistic competition and individuals’ social interactive behaviors. In this literature, the common practice is to assume players’ observed choices are consistent with Nash equilibrium. Under this assumption, researchers then estimate players’ utility/payoff functions and conduct counterfactual analysis.

Despite its power and usefulness in applied empirical research, the equilibrium assumption places a strong restriction of unbiased beliefs. Specifically, a player’s beliefs about other players’ behaviors correspond to other players’ actual choice probabilities, given available information. This restriction could be misspecified in many applications. First, a player may have limited ability to process information, and therefore incorrectly predicts other players’ decisions. Second, many games have multiple equilibria, which further complicates the construction of unbiased beliefs. As studied by van Huyck, Battalio and Beil (1990) and Crawford and Haller (1990), the multiplicity of equilibria could introduce strategic uncertainty such that a player is uncertain about which equilibrium strategy will be chosen by other players. Third, market conditions and government policies often vary drastically. They pose difficulties in learning other players’ behaviors through past experience. Finally, recent empirical work has shown the failure of Nash equilibrium in different types of games using both field and experimental data. A partial list includes Goeree and Holt (2001), Goldfarb and Xiao (2011), Doraszelski, Lewis, and Pakes (2018), Kashaev (2016), Aguirregabiria and Magesan (2020), and Aguirregabiria and Xie (2021).

The potential misspecification of the equilibrium condition could bias the estimates of payoff functions and counterfactual predictions. To address such an issue, this article relaxes the unbiased belief assumption. In particular, I focus on incomplete information games with independent private values and without unobserved common knowledge. This type of games has been estimated in many empirical applications, such as Brock and Durlauf (2001), Seim (2006), Augereau, Greenstein, and Rysman (2006), Bajari et al. (2010), and Gowrisankaran and Krainer (2011). In this class of games, players’ actions are independent conditional on common observables. This property allows me to focus on a player’s belief about the marginal distribution of each of other players’ actions, instead of the belief about the joint distribution of all players’ choices. Specifically, the belief is modeled as an unrestricted nonparametric function. It is estimated jointly with players’ payoff functions. Importantly, the model is agnostic about the belief formation process and allows a player’s belief to be any probability distribution over the other player’s action set. Under this framework, I study the joint identification of payoff and belief functions.

The principle of revealed preference implies that, under general conditions, researchers can infer the expected utility using data on players’ choices. However, since expected utility is a composite function of payoff and belief, it is challenging to separately identify these two functions. The first contribution addresses such a challenge in a two-player game where one player has a larger number of actions than the other player (e.g., \(3 \times 2\) game). As will be described in Section 3.2, this type of games with asymmetric action sets is prevalent. Moreover, the asymmetry provides identification power for the player with more actions. First, the payoff function is partially identified. To
the best of my knowledge, this is the first nonparametric identification result on payoff with neither equilibrium constraints nor exclusion restrictions. Second, suppose exclusion restrictions, commonly assumed in literature, are satisfied. Both payoff and belief are identified up to one scale parameter. This result further identifies the sign and lower bound of the interactive effect. Third, the restriction of equilibrium/unbiased beliefs is testable.

While the above identification results hold only for the player with more actions, the second contribution generalizes them to the other player. Existing literature commonly imposes a multiplicative separability condition on the payoff function. This condition implies that the above asymmetric feature can be constructed for any player, as long as this player has more than two actions. Therefore, with multiplicative separability, previous constructed for any player, as long as this player has more than two actions. Therefore, with multiplicative separability, previous identification results hold for both players in any $M \times N$ games, where $\min\{M, N\} > 2$. These results are also generalized to games with more than two players. Consequently, a general class of games can be identified without equilibrium restrictions. Note that since the asymmetric feature cannot be constructed for the player with two actions, these results do not apply to binary choice games. Finally, the proofs of identification results imply a constrained MLE to estimate payoff and belief functions. The unbiased belief assumption can be tested by a likelihood ratio test.

The identification results and estimation method are illustrated through an empirical application of store-hour competition between Kentucky Fried Chicken (KFC) and McDonald’s (McD) in China. Their competition is modeled as astatic game such that each chain simultaneously chooses how many of its existing outlets to open at night. In China, KFC owns more outlets than McD in most geographic markets (i.e., towns). This feature implies a larger action space of KFC, and the identification results above can be applied. The estimation results reject the null hypothesis that KFC has unbiased beliefs. For the average belief across all markets in my sample, KFC overpredicts the probability that McD will operate at night by about 10 percentage points. When the equilibrium condition is incorrectly imposed, the estimated interactive effect could be biased downward by more than 50%.

This article contributes to recent literature on players’ nonequilibrium behaviors in games. Under the concept of rationalizability proposed by Bernheim (1984) and Pearce (1984), Aradillas-Lopez and Tamer (2008) study the identification of payoffs in a 2×2 game with negative interactive effects. In contrast, the results in this article are applicable to games with a larger action space and when the sign of the interactive effect is unknown to researchers. In a framework similar to that in this article, Aguirregabiria and Magesan (2020) and Aguirregabiria and Xie (2021) showed the identification of the payoff function with equilibrium assumption in a subset of observations/games. This restriction is clearly weaker than the unbiased belief assumption in all games. In contrast, this article shows the identification power of asymmetric action sets. It partially identifies the payoff without equilibrium assumption in any game. Furthermore, the point identification of payoff can be achieved with equilibrium condition in a smaller subset of games, as compared with Aguirregabiria and Magesan (2020) and Aguirregabiria and Xie (2021).

This article’s results have an important implication on the usual two-step estimation method (Bajari et al. 2010; Bajari, Hong and Nekipelov 2013; Elickson and Misra 2011; Su 2014). This method first estimates each player’s policy function or conditional choice probability (CCP). Under the equilibrium condition, player $i$’s estimated CCP is also an estimate of player $i$’s belief. With this belief estimate, researchers could infer player $i$’s structural payoff parameters in the second step. Clearly, this two-step method crucially relies on the equilibrium assumption. Therefore, researchers should be cautious about applying this method, especially when there are some factors that could invalidate the equilibrium condition. As a comparison, this article obtains the identification results that are robust to nonequilibrium behaviors. It also derives a testable restriction of the equilibrium condition. This restriction provides an empirical guidance on whether to choose the two-step method.

The rest of this article is organized as follows. Section 2 describes the model, and Section 3 presents the identification results. The empirical application is shown in Section 4. I conclude in Section 5. Some generalizations of the model and identification results are left to the appendix (supplementary material).

2. Empirical Model

Consider a two-player static game. Players are indexed by $i \in \{1, 2\}$, and $-i$ represents the other player. Appendix A.3 (supplementary material) shows how to generalize the identification results to a game with multiple players. Each player $i$ has $(I_i + 1)$ possible choices and simultaneously chooses an action $a_i$ from her action set $A_i = \{0, 1, \ldots, I_i\}$. Players are allowed to have different action sets and different numbers of actions. The Cartesian product $A = A_1 \times A_2$ represents the space of action profiles in this game. Let $a = (a_1, a_2) \in A$ be an action profile or realized outcome of this game. Player $i$’s payoff for the action profile $a$ is

$$\pi_i(x, a, \epsilon_i) = \tilde{\Pi}_i(x, a_i, a_{-i}) + \epsilon_i(a_i),$$

where $x \in \mathbb{R}^{L_x}$ denotes a vector of state variables that affect both players’ payoffs and are common knowledge. Term $\epsilon_i(a_i)$ represents a variable that affects player $i$’s payoff of action $a_i$. It is the private information of player $i$ and unobserved by player $-i$. Therefore, it is a game of incomplete information. The payoff function $\pi_i(x, a_i, a_{-i})$ depends on $x$ and both players’ actions $(a_i, a_{-i})$. It is nonparametrically specified.

Define $\pi_i(x, a_i) = \tilde{\Pi}_i(x, a_i, a_{-i} = 0)$ and $\delta_i(x, a_i, a_{-i} = 0) = \tilde{\Pi}_i(x, a_i, a_{-i} = 0) - \tilde{\Pi}_i(x, a_i, a_{-i} = 0)$. By construction, $\delta_i(x, a_i, a_{-i} = 0) = 0$. Without loss of generality, the payoff function can be written as

$$\pi_i(x, a, \epsilon_i) = \pi_i(x, a_i) + \delta_i(x, a_i, a_{-i}) \cdot \mathbb{1}(a_{-i} \neq 0) + \epsilon_i(a_i). \quad (1)$$

Throughout the article, I consider the payoff function in Equation (1) for exposition purposes. Note that it is nonparametrically specified. Following the language of de Paula and Tang (2012), $\pi_i(\cdot)$ is the base return and represents player $i$’s payoff when the other player chooses action 0. Term $\delta_i(\cdot)$ represents the interactive effect/payoff. It measures how player $i$’s payoff is affected by player $-i$’s behavior.
Assumption 1 describes the information structure of the game.

**Assumption 1.** (a) For each $i = 1, 2$, let $G_i(\cdot)$ denote the cumulative distribution function of private information $\varepsilon_i = (\varepsilon_i(0), \varepsilon_i(1), \ldots, \varepsilon_i(j_i))^\top$. $G_i(\cdot)$ is absolutely continuous with respect to the Lebesgue measure in $\mathbb{R}^{j_i+1}$. Moreover, $\varepsilon_i$ is independent across players and independent of $x$.

(b) It is common knowledge that $\varepsilon_i$ is observed by player $i$ and unobserved by player $-i$. Moreover, action set $A_i$, payoff function $\Pi_i(\cdot)$, state variables $x$, and distribution $G_i(\cdot)$ are common knowledge.

Except for preference shocks $\varepsilon_i$, all other model primitives are common knowledge. Moreover, both players know that $\varepsilon_i$ is player $i$’s private information and $\varepsilon_i \perp \!\!\!\perp x$. These restrictions imposed by Assumption 1 are standard in literature that estimates games with incomplete information. Examples include Seim (2006), Sweeting (2009), Bajari et al. (2010), de Paula and Tang (2012) and Aradillas-Lopez and Gandhi (2016), among others. Finally, the model and identification results can be generalized to the case that $G_i(\cdot)$ depends on a vector of finite-dimensional unknown parameters.

Given the game structure, define $\sigma_i(x, \varepsilon_i) : \mathbb{R}^{k_i} \times \mathbb{R}^{j_i+1} \rightarrow A_i$ as a player $i$’s strategy function that maps from all information observed by player $i$ to one of its actions in $A_i$. The focus on pure strategy is innocuous. This is because the continuous distribution of $\varepsilon_i$ implies that player $i$ is indifferent between two actions with zero probability. Given Assumption 1 (a), $\sigma_i(\cdot)$ is independent of $\sigma_{-i}(\cdot)$ conditional on $x$. It further implies that observed players’ actions are independent conditional on $x$; for instance, $\Pr(a_i, a_{-i} | x) = \Pr(a_i | x) \cdot \Pr(a_{-i} | x)$. This property allows me to focus on player $i$’s belief about the marginal distribution of $a_{-i}$, instead of the belief about the joint distribution of $(a_i, a_{-i})$. In particular, let $b_i(x)$ denote player $i$’s believed probability that player $-i$ will choose action $j$ given $x$. This belief is a nonparametric function of $x$ without any restrictions, except it is a valid probability distribution (i.e., $0 \leq b_i(x) \leq 1 \forall j$ and $\sum_{j=0}^{j_i} b_i(x) = 1$). Note that due to simultaneity and incomplete information, the belief function $b_i(x) = (b_i^0(x), b_i^1(x), \ldots, b_i^{j_i}(x))^\top$ does not depend on player $i$’s actual strategy $\sigma_i(x, \varepsilon_i)$ and private information $\varepsilon_i$. In more detail, player $-i$ does not observe $\sigma_{-i}(\cdot)$ and $\varepsilon_i$; therefore, her behavior depends on neither of them. Given this fact, player $i$ would predict player $-i$’s behavior based only on the information summarized by $x$, instead of $\sigma_{-i}(\cdot)$ and $\varepsilon_i$.

Given payoff and belief functions, player $i$’s expected payoff of action $a_i$ is

$$E[\Pi_i(x, a_i, \varepsilon_i)] = \pi_i(x, a_i) + \sum_{j=1}^{j_i} \delta_i(x, a_i, a_{-i} = j) \cdot b_i^j(x) + \varepsilon_i(a_i).$$

(2)

Let $\sigma_i^*(x, \varepsilon_i)$ denote player $i$’s optimal strategy given her belief $b_i(x)$. It is characterized by

$$\sigma_i^*(x, \varepsilon_i) = \arg\max_{a_i \in A_i} \left\{ \pi_i(x, a_i) + \sum_{j=1}^{j_i} \delta_i(x, a_i, a_{-i} = j) \cdot b_i^j(x) + \varepsilon_i(a_i) \right\}. $$

(3)

When player $-i$ correctly predicts player $i$’s optimal strategy $\sigma_i^*(x, \varepsilon_i)$, her belief equals player $i$’s conditional choice probability (CCP). Let $p_i(x) = \left( p_i^0(x), \ldots, p_i^{j_i}(x) \right)^\top$ denote a vector of player $i$’s CCPs, where $p_i^j(x)$ is her choice probability of action $j$ conditional on $x$. Given the best response function $\sigma_i^*(x, \varepsilon_i)$ defined above, the CCP takes the following form:

$$p_i^j(x) = \int \left\{ \sigma_i^*(x, \varepsilon_i) = j \right\} dG_i(\varepsilon_i).$$

(4)

For instance, if $\varepsilon_i(a_i)$ is Type I extreme value distributed and independent across actions, $p_i^j(x)$ is

$$p_i^j(x) = \frac{\exp \left\{ \pi_i(x, a_i = j) + \sum_{k=1}^{j_i} \delta_i(x, a_i = j, a_{-i} = k) \right\} - \sum_{j=0}^{j_i} \exp \left\{ \pi_i(x, a_i = l) + \sum_{k=1}^{j_i} \delta_i(x, a_i = l, a_{-i} = k) \right\} \cdot b_i^l_j(x)}{\sum_{l=0}^{j_i} \exp \left\{ \pi_i(x, a_i = l) + \sum_{k=1}^{j_i} \delta_i(x, a_i = l, a_{-i} = k) \right\} \cdot b_i^l_j(x)}.$$
expected utility, optimal strategy, and choice probability specified from Equations (2) to (4) become the following:

\[
E[\Pi_i(x, a_i, \omega, \epsilon_i)] = \pi_i(x, a_i) + \sum_{j=1}^{J} \delta_i(x, a_i, a_{-i} = j) \cdot b_i'(x, \omega) + \epsilon_i(a_i),
\]

\[
\sigma_i^*(x, \omega, \epsilon_i) = \arg\max_{a_i \in A_i} \left\{ \pi_i(x, a_i) + \sum_{j=1}^{J} \delta_i(x, a_i, a_{-i} = j) \cdot b_i'(x, \omega) + \epsilon_i(a_i) \right\},
\]

\[
p_i(x, \omega) = \int 1 \left\{ \sigma_i^*(x, \omega, \epsilon_i) = j \right\} dG_i(\epsilon_i).
\]

In addition, \(\omega\) is a discrete variable with support \(\{\omega^1, \omega^2, \ldots, \omega^b\}\) and probability mass function denoted by \(P_\omega(\cdot)\). Importantly, I allow the number of support \(L_\omega(x)\) and probability \(P_\omega(\omega|x)\) to depend on \(x\).

This extended model nests multiple equilibria as a special case, if three conditions are satisfied. First, the number of support \(L_\omega(x)\) equals the number of equilibria of the game with observable \(x\). Second, realization \(\omega^l\) indexes the \(l\)th equilibrium and \(\sigma_i^*(x, \omega^l, \epsilon_i)\) is the corresponding equilibrium strategy. Furthermore, each player \(i\) has an unbiased beliefs so that \(b_i(x, \omega^l) = p_{i-l}(x, \omega^l)\) \(\forall x, \omega^l\). Third, \(P_\omega(\cdot)\) is interpreted as an equilibrium selection mechanism; consequently, \(P_\omega(\omega|x)\) represents the probability of choosing the \(l\)th equilibrium conditional on \(x\). Note that \(P_\omega(\omega|x)\) is a nonparametric function of \(x\) without any further restrictions. Therefore, the model allows a broad class of equilibrium selection mechanisms.

3. Identification

This section first assumes no unobserved heterogeneity in the belief function and establishes the identification results with asymmetric action sets. The intuition is straightforward. Recall that player \(i\) has a number of \((J_i + 1)\) actions while player \(-i\) has \((J_{-i} + 1)\). Player \(i's\) observed CCPs identify the difference of any two actions' expected utilities. It then constitutes \(I_i\) restrictions. In contrast, player \(-i\)'s belief is a \(J_{-i}-\)simplex over player \(-i\)'s action set. When player \(i\) has more actions (i.e., \(J_i > J_{-i}\)), the model generates more restrictions than belief unknowns. Therefore, these belief unknowns can be differentiated out and what remains is a relationship between unknown payoffs and observed choice probabilities. This establishes the identified set of the payoff function for the player with more actions. Moreover, this identified set can be further shrunk with the usual exclusion restrictions. Section 3.3 generalizes these identification results to each player with a multiplicative separability restriction on the payoff function. Section 3.4 discusses identification when unobserved heterogeneity is introduced into the belief function. Recall that this extension nests multiple equilibria. All proofs are left to the appendix (supplementary material).

3.1. Conditions on the Data-Generating Process

Suppose researchers have access to a dataset about the same two players that play \(M\) independent games (e.g., one game in each of \(M\) isolated markets). In each game/observation indexed by \(m\), both players and econometricians observe realizations of the state variables \(x_m\). Moreover, each player \(i\) observes her own payoff shock \(\epsilon_{i,m}\). Researchers cannot observe player \(i's\) private information but know its probability distribution \(G_i(\cdot)\). Given \(x_m\), each player forms a belief and chooses her optimal action.

Assumption 2. A player forms the same belief for any two observations with the same \(x\). That is, for \(m \neq m'\) but \(x_m = x_{m'} = x\), we have \(b_{i,m}(x) = b_{i,m'}(x) = b_i(x)\).

Assumption 2 is an implication of the model described in Section 2, where belief is defined as a nonparametric function of \(x\). The consistency comes from \(M \rightarrow \infty\). In this situation, \(\hat{\pi}_i(x_m)\) can be consistently estimated. Consequently, for illustration purposes, \(p_i(x)\) is assumed to be known by researchers for every realization of \(x\). Researchers’ objective is to identify player \(i's\) base return \(\pi_i(x, a_i)\), interactive effect \(\delta_i(x, a_i, a_{-i})\) and belief function \(b_i(x)\) using the data described above.

It is known in the discrete choice literature that only differences in payoffs are identified. Therefore, a normalization is required to achieve identification, as summarized in Assumption 3. With this normalization, the payoff of action \(a_i\) is interpreted as the payoff differences between \(a_i\) and base action 0.

Assumption 3. For player \(i = 1, 2\), the payoff of action 0 is normalized to zero. That is, \(\pi_i(x, a_i = 0) = 0\) and \(\delta_i(x, a_i = 0, a_{-i}) = 0\) \(\forall x, a_{-i}\).

This article considers the regular case where the interactive effect is nonzero for any action \(a_i\) other than the base action 0. For instance, \(\delta_i(a_i, a_{-i}) \neq 0\) \(\forall a_i \neq 0\). This restriction is innocuous from the perspective of identification. Suppose instead \(\delta_i(a_i, a_{-i}) = 0\) for some \(a_i \neq 0\), then this action’s expected utility solely depends on its base return. Equivalently, player \(i's\) evaluation of action \(a_i\) does not depend on her belief. Therefore, the data will directly identify the base return \(\pi_i(a_i)\) without the knowledge of player \(i's\) belief, as shown in Equation (5).

This article’s identification results hold for any \(x \in \mathbb{R}^{k+}\). Therefore, it is suppressed for notation simplicity. Following Hotz and Miller (1993), Assumptions 1 and 3 imply that there is a one-to-one mapping \(F_i(\cdot) : \mathbb{R}^{k+} \rightarrow \mathbb{R}^{k+}\) from player \(i's\) CCPs to her expected payoffs. Specifically, let \(F_i(\cdot)\) be the inverse of the integral function defined by Equation (4); we then have the following equation for any action \(a_i = k\):

\[
\pi_i(a_i = k) + \sum_{j=1}^{J_i} \delta_i(a_i = k, a_{-i} = j) \cdot b_i' = F_i(k, p_i) \quad \forall 0 \leq k \leq J_i,
\]

(5)

where \(F_i^k(\cdot)\) denotes the \(k\)th element of the inverse function. Note that \(F_i^0(\cdot) = 0\) based on the normalization stated in Assumption 3. Given that \(G_i(\cdot)\) is known by researchers, \(F_i(\cdot)\) is also known. For instance, if \(\epsilon_i(a_i)\) is independently Type I extreme value distributed, we have the mapping \(F_i^k(\cdot) = \log \left(\frac{\tilde{F}_i^k(\cdot)}{\tilde{F}_i^{k-1}(\cdot)}\right)\).

Under the equilibrium condition stated in Remark 1, player \(i's\) belief \(b_i'\) corresponds to the other player’s choice proba-
bility \( p^j_{-i} \). With this property, researchers could first estimate \( p^j_{-i} \) consistently. In the second step, researchers replace \( b^j_i \) in Equation (5) by the first-step estimate \( \hat{b}^j_i \), and then estimate player \( i \)'s payoffs. When an additional exclusion restriction is satisfied, this two-step method achieves the consistent estimation of the payoff function. However, as described in the Introduction, this method crucially depends on the equilibrium assumption. When such an assumption is difficult to justify, researchers should be cautious about applying the above two-step method. In contrast, this article achieves identification results that are robust to nonequilibrium behaviors. Moreover, the testable implication of the equilibrium condition—derived in this article—also guides empirical researchers on when to apply the usual two-step method. In a similar vein, the above discussion also applies to structural models in other settings.

For instance, in single-agent dynamic models, researchers first estimate the transition probability of state variables and/or the agent’s policy function. These estimates allow researchers to calculate the agent's value function in the second step, by either numerical iteration (Rust 1987) or simulation (Hotz et al. 1994). In addition, Bajari, Benkard and Levin (2007) apply a similar idea to dynamic games. All these methods require unbounded rationality in the sense that agents can correctly predict the transition probability of state variables, their own future behaviors, and other players' strategies. Consequently, researchers should also be cautious about applying these methods when agents are only boundedly rational. Recently, the unbounded rationality assumption has been relaxed in both single-agent dynamic models (An, Hu and Xiao 2021) and dynamic games (Aguirregabiria and Magesan 2020).

### 3.2. Identification With Asymmetric Action Spaces

This subsection exploits the identification power provided by the asymmetric action sets (i.e., \( J_i > J_{-i} \)). For the sake of brevity, in the main text, I present the results when \( J_{-i} = 1 \) so that player \(-i\) has binary choice set \([0,1]\). Results when \( J_{-i} > 1 \) are similar and presented in Appendix A.2 (supplementary material).

**Proposition 1.** Suppose Assumptions 1–3 hold and \( J_i > J_{-i} = 1 \) (i.e., player \(-i\) has binary choice set \([0,1]\)); then for any two choice alternatives \( j, k \neq 0 \), the sharp identified set of player \( i \)'s payoffs \( \pi_i(\cdot) \) and \( \delta_i(\cdot) \) is given by the set of values that satisfies the following restriction:

\[
0 \leq \frac{F^j_i(p_i) - \pi_i(a_i = j)}{\delta_i(a_i = j, a_{-i} = 1)} = \frac{F^k_i(p_i) - \pi_i(a_i = k)}{\delta_i(a_i = k, a_{-i} = 1)} \leq 1
\]

**Proposition 1** characterizes the sharp identified set of payoff functions when player \( i \) has more actions than the other player. To the best of my knowledge, this is the first nonparametric identification result on the payoff function without imposing either equilibrium constraints or exclusion restrictions. To see the informativeness of this identified set, consider that player \( i \) has three choices \([0,1,2]\). There exist four payoff unknowns: \( \pi_i(a_i = j) \) and \( \delta_i(a_i = j, a_{-i} = 1) \) for \( j = 1,2 \). **Proposition 1** restricts these four unknowns to lie on a subset of the three-dimensional hyperplane, instead of being any point in the four-dimensional space.

In practice, the identified set by **Proposition 1** could be wide and provide limited information. However, it leads to the main result in this article that studies the joint identification power of asymmetric action spaces and exclusion restrictions. **Assumption 4** states the conditions on exclusion restrictions that are commonly imposed in the existing literature.

**Assumption 4.** (a) [Player specific payoff shifter]: For each player \( i \), there exists a variable \( z_i \in \mathbb{R} \) that affects only player \( i \)'s payoff; moreover, \( z_i \) has exogenous variation over its support.

(b) [Interactive effect shifter]: There exists a variable \( s \in \mathbb{R} \) that affects each player's interactive effect \( \delta_i(\cdot) \) but not the base return \( \pi_i(\cdot) \); moreover, \( s \) has exogenous variation over its support.

**Assumption 4** (a) requires a variable \( z_i \) that affects player \( i \)'s payoff but not player \(-i\). This player specific payoff shifter is commonly assumed in the literature. As shown by Bajari et al. (2010) and Aradillas-Lopez (2010), without such an exclusion restriction, players' payoffs are nonidentified even when equilibrium constraints are imposed.

**Assumption 4** (b) requires that the interactive effect shifter \( s \) does not affect player \( i \)'s base return; for instance, it has zero impact on player \( i \)'s payoff if player \(-i\) chooses a particular action, denoted by action 0. Even though such an exclusion restriction is ignored in literature on identification of games, it is imposed in many existing studies. For instance, in my empirical application of store-hour competition, a plausible candidate for \( s \) is the distance between KFC and McD in a market. If McD does not operate through the night, then such a distance would have no impact on KFC's night profit. In contrast, the interactive effect would be affected by this distance, since an opponent of closer proximity may have a larger impact than one that is further away. This type of horizontal differentiation created by distance has been studied in empirical games by Seim (2006), Zhu and Singh (2009), and Rennhoff and Owens (2012). Moreover, in an entry game with network effect, Ciliberto and Tamer (2009) specify the competitor’s characteristics in surrounding markets as the interactive effect shifter. This type of network effect is prevalent in many industries such as retail, fast food and banking. All previous articles introduce variable \( s \) as a plausible model specification, instead of an exclusion restriction to facilitate identification and estimation. In contrast, this article formally discusses the role of such a variable in identification without equilibrium assumption.

With the exclusion restrictions stated in **Assumption 4**, player \( i \)'s payoff function is then

\[
\Pi_i(z_i, s_i, a_i, e_i) = \pi_i(z_i, a_i) + \delta_i(z_i, s, a_i) \cdot 1(a_{-i} \neq 0) + s_i(a_i).
\]

(6)

The belief and CCP are functions of all common information state variables. For instance, \( p^j_i(z_i, s_i, a_i, e_i) \) and \( b^j_i(z_i, s_i, a_i, e_i) \). The framework described in Section 2 fits such an extension with exclusion restrictions.

**Proposition 2.** (a) Under conditions met in **Proposition 1** and **Assumption 4** (a), \( \delta_i(z_i, a_i = j, a_{-i} = 1) = \frac{\delta_i(z_i, a_i = k, a_{-i} = 1)}{\delta_i(z_i, a_i = j, a_{-i} = 1)} \) is identified for any two
actions \( j, k \neq 0 \) if there exist at least two realizations of \( z_{-i} \), say \( z_{1,i} \) and \( z_{2,i} \), such that \( p_i(z_{1,i}, z_{1,i}^* ) \neq p_i(z_{2,i}, z_{2,i}^* ) \).

(b) Suppose further Assumption 4 (b) holds and there exist at least two realizations of \( s \), say \( s^1 \) and \( s^2 \), such that \[ \frac{\delta_i(z_{1,i}, a_i)|a_{-i}=1}{\delta_i(z_{2,i}, a_i)|a_{-i}=1} \neq \frac{\delta_i(z_{1,i}, a_i)|a_{-i}=1}{\delta_i(z_{2,i}, a_i)|a_{-i}=1} \], then \( \pi_i(z_{1}, a_i) \) and \( \delta_i(z_{2}, s, a_i, a_{-i}=1) = 1 \times b_i^1(z_i, z_{-i}, s) \) are point identified \( \forall a_i \in A_i \) and \( \forall z_{-i}, z_{-i}, s \).

Proposition 2 establishes the main results in this article. First, point (a) identifies the interactive effect ratio \( \delta_i(z_{1,i}, a_i)|a_{-i}=1 \) and sheds light on a player’s choice incentive. In coordination games, it measures which of player \( i \)'s actions is more sensitive to the other player’s behavior. Intuitively, a player has the incentive to choose a sensitive action to exploit positive spillover effects. In contrast, in an entry/expansion game, an incentive for a firm to open an additional store is to alleviate the negative impact of other firms; such an incentive is quantified by the interactive effect ratio, as it represents how the negative impact is attenuated of other firms; such an incentive is quantified by the interactive effect ratio, as it represents how the negative impact is attenuated.

The subsection above obtains identification results only for one action could be due to government regulation, or this action is a conservative measure of the other player’s impact. Moreover, with the interactive effect ratio identified by Proposition 2, the sign of \( \delta_i(z_{1,i}, a_i)|a_{-i}=1 \) determines the strategic nature of the game. Suppose we have estimated \[ \frac{\delta_i(z_{1,i}, a_i)|a_{-i}=1}{\delta_i(z_{2,i}, a_i)|a_{-i}=1} > 1 \] for all \( j > k \), the players’ actions are strategic substitutes if \( \delta_i(z_{1,i}, a_i)|a_{-i}=1 \) are strategic complements if \( \delta_i(z_{1,i}, a_i)|a_{-i}=1 \). Determining the strategic nature is the main empirical question in many studies, such as Sweeting (2009), de Paula and Tang (2012), and Shen and Xiao (2014). It is also one of the central questions in my empirical application.

Proposition 3(b) states that the ratio of player \( i \)'s beliefs at any two realizations of \( z_{-i} \), is identified. It measures how player \( i \) adjusts her beliefs when the other player’s specific payoff shifter varies. It also provides a testable implication of unbiased beliefs; under such a condition, player \( i \)'s adjustment of her beliefs should equal the actual change of player \(-i\)'s CCPs. Finally, Proposition 3(c) establishes that the payoff and belief functions are identified up to player \( i \)'s belief at just one realization of \( z_{-i} \). This result can be used to construct a likelihood ratio test of unbiased beliefs. Section 4 describes the procedure.

### 3.2.1. Some Examples of Games with Asymmetric Action Spaces

Even though existing literature mainly focuses on games with symmetric choice sets, there exist many applications with asymmetric action spaces. For instance, consider the competition (e.g., in price, quantity, or quality) among multi-product or multi-store firms. When decision is made at the product or store level; then different numbers of products or stores, owned by different firms, generate the asymmetry in action spaces. This type of games has been studied by Gowrisankaran and Krainer (2011) who focus on the banks’ adoption decisions about ATMs at the branch level. My empirical application also shares this feature. In addition, different incumbency statuses could suggest asymmetric choice sets. In an entry/expansion game, the incumbent chooses one of the following three options: (expand, maintain current level, exit); while potential entrant only chooses between enter and not enter. This type of games was studied by Ericson and Pakes (1995). Finally, players could have symmetric action sets in nature, but it is common knowledge that one player would not choose one particular action. This exclusion of one action could be due to government regulation, or this action is strictly dominated. In literature, the existence of strictly dominated action has been exploited to identify the payoff function (Tamer 2003; Bajari, Hong, and Ryan 2010).

### 3.3. Identification with Multiplicative Separability

The subsection above obtains identification results only for the player with more actions, while it leaves the player with fewer actions nonidentified. This subsection generalizes the results to each player in a general class of games, regardless of symmetric or asymmetric action spaces. In many empirical applications, researchers assume that the interactive effect is multiplicatively separable between a player’s own action and her opponent’s behavior. Under this condition, each player’s belief is summarized by a one-dimensional sufficient statistic. It implies a reduction in the dimension of player \( i \)'s beliefs from \( J-i \) to one. As a result, each player’s identification problem has the asymmetric feature described in Section 3.2, regardless of the number of actions available to her opponent. Consequently, most of the identification results hold for both players. Appendix A.3 (supplementary material) generalizes the results to games with multiple players.
This subsection assumes \( \min\{f_1, f_2\} > 1 \) so that each player has more than two choices. There is no further restriction imposed on the choice sets. For the sake of caution, since the asymmetric feature can be constructed only for the player with more than two actions, the identification results cannot be generalized to binary-choice games. Assumption 5 states the condition of multiplicative separability. It is commonly imposed in the literature, such as Augereau, Greenstein, and Rysman (2006), Nishida (2014), and Aradillas-Lopez and Gandhi (2016).

**Assumption 5.** \( \delta_i(a_i, a_{-i}) = \delta_i(a_i, a_{-i} = 1) \cdot \eta_i(a_{-i}) \) with \( \eta_i(a_{-i} = 1) = 1 \).

First note that \( \delta_i(\cdot) \) and \( \eta_i(\cdot) \) are nonparametric functions of state variables \( (x, z_i, s) \); they are suppressed for the sake of brevity. It is convenient to describe Assumption 5 in an entry game, where action \( j \) is interpreted as opening \( j \) stores. In this context, \( \delta_i(a_i, a_{-i} = 1) \) measures the effect of player \( -i \)'s first store on player \( i \)'s profit. It is nonparametrically specified without any restrictions. Player \( -i \) may cause a larger impact when it opens more stores and the proportional change of this additional impact is captured by \( \eta_i(a_{-i}) \). This proportional change is restricted to be independent of player \( i \)'s action, but is allowed to depend on \( a_{-i} \), nonparametrically. There are many situations when such an assumption holds true. The simplest case is that each store opened by player \( -i \) has the same impact on player \( i \). It suggests \( \eta_i(a_{-i}) = a_{-i} \), as assumed in Augereau, Greenstein, and Rysman (2006). A slightly complicated case is when each additional store opened by player \( -i \) has a diminishing marginal impact, but each of player \( i \)'s stores suffers the same impact. This is the condition imposed by Nishida (2014) as he assumes \( \eta_i(a_{-i}) = \log(1 + a_{-i}) \). In addition, with a linear demand function, Assumption 5 also holds in a Cournot or differentiated Bertrand model. In particular, each firm's profit is multiplicatively separable between its own price/quantity and the other firm’s choices. The store entry decisions can be seen as a discretized version of Cournot competition. Finally, the same restriction is also imposed in Aradillas-Lopez and Gandhi (2016).

Under Assumption 5, player \( i \)'s expected payoff for action \( a_i \), defined in Equation (2), becomes

\[
E\left[\Pi_i(a_i, \varepsilon_i)\right] = \pi_i(a_i) + \sum_{j=1}^{J_{-i}} \delta_i(a_i, a_{-i} = 1) \cdot \eta_i(a_{-i} = j) \\
\cdot b_j^i + \varepsilon_i(a_i) \\
= \pi_i(a_i) + \delta_i(a_i, a_{-i} = 1) \cdot \left[ \sum_{j=1}^{J_{-i}} \eta_i(a_{-i} = j) \cdot b_j^i \right] \\
+ \varepsilon_i(a_i) \\
= \pi_i(a_i) + \delta_i(a_i, a_{-i} = 1) \cdot \eta_i + \varepsilon_i(a_i),
\]

(7)

where \( \eta_i = \sum_{j=1}^{J_{-i}} \eta_i(a_{-i} = j) \cdot b_j^i \) represents player \( i \)'s subjective expectation of the value \( \eta_i(a_{-i} = j) \). In the previous subsection, if \( J_{-i} = 1 \), then player \( i \)'s expected payoff of action \( a_i \) is

\[
E\left[\Pi_i(a_i, \varepsilon_i)\right] = \pi_i(a_i) + \delta_i(a_i, a_{-i} = 1) \cdot b_1^i + \varepsilon_i(a_i).
\]

It is easy to see that if we treat \( \tilde{\eta}_i \) in Equation (7) as being analogous to \( b_1^i \) in the above equation, then these two equations would share the same structure. As a consequence, most of the identification results in Propositions 1 to 3 hold true. They are summarized in the following corollary.

**Corollary 1.** (a) Under Assumptions 1 to 3 and Assumption 5, for any two actions \( j, k \neq 0 \), the identified set of player \( i \)'s payoffs \( \pi_i(\cdot) \) and \( \delta_i(\cdot) \) is given by the set of values that satisfies the following restriction:

\[
\frac{F_i^j(p_i) - \pi_i(a_i = j)}{\delta_i(a_i = j, a_{-i})} = \frac{F_i^k(p_i) - \pi_i(a_i = k)}{\delta_i(a_i = k, a_{-i})}.
\]

(b) Furthermore, suppose researchers observe player specific payoff shifter \( z_i \) as in Assumption 4 (a) and \( p_i(z_i, z_{j, -i}^{z_i}) \neq p_i(z_i, z_{j, -i}^{z_{j, -i}}) \) for some \( z_{j, -i}^{z_i}, z_{j, -i}^{z_{j, -i}} \); then \( \delta_i(z_i, a_{-i} = 1, a_{-i} = 0) \) is identified.

(c) In addition, suppose researchers further observe interactive effect shifter \( s_i \) as in Assumption 4 (b) and \( \eta_i(z_i, a_{-i} = 1, a_{-i} = 0) \) varies with \( s_i \); then \( \pi_i(\cdot) \) and \( \delta_i(a_i, a_{-i} = 1) \cdot \eta_i(\cdot) \) are identified.

(d) If \( \delta_i(a_i, a_{-i} = 1) \cdot \eta_i(\cdot) \neq 0, \) then it implies that \( \tilde{\eta}_i(z_i^{1_a}, z_i^{2_{a_i}}) \) is identified for any two values of \( z_i^{1_a}, z_i^{2_{a_i}} \). It also implies a testable restriction of the equilibrium condition: \( \tilde{\eta}_i(z_i^{1_a}) \cdot \tilde{\eta}_i(z_i^{2_{a_i}}) = \tilde{\eta}_i^{eqm}(z_i^{1_a}) \cdot \tilde{\eta}_i^{eqm}(z_i^{2_{a_i}}) \), where \( \tilde{\eta}_i^{eqm}(\cdot) \) is the expectation of \( \eta_i(\cdot) \) under equilibrium beliefs.

(e) If \( \delta_i(a_i, a_{-i} = 1) \cdot \eta_i(\cdot) \neq 0, \) then it implies the point identification of \( \delta_i(\cdot) \) up to the value of \( \tilde{\eta}_i(z_i) \) at only one realization of \( z_i \).

Though most of the results in Section 3.2 are generalized to both players by Corollary 1, there are two differences worth comments. First, \( \eta_i(\cdot) \) measures player \( i \)'s subjective expectation and is not a probability distribution as \( b_i(\cdot) \). Therefore, even though \( \delta_i(\cdot) \cdot \tilde{\eta}_i(\cdot) \) is identified, the sign and lower bound of \( \delta_i(\cdot) \) are undetermined unless some restrictions are imposed on \( \eta_i(\cdot) \). For instance, researchers normally assume that \( \eta_i(\cdot) > 0 \) and such a condition identifies the sign of the interactive effect. Another common restriction is that the marginal impact of \( a_{-i} \) on \( \eta_i(\cdot) \) is nonnegative and nonincreasing, for instance, \( 0 \leq \eta_i(a_{-i} = 1) - \eta_i(a_{-i}) \leq \eta_i(a_{-i} = 1 - \eta_i(a_{-i} = 1) \). Such a restriction identifies the lower bound of \( \delta_i(\cdot) \) by letting \( b_i^{1_a} = 1 \) and \( \eta_i(a_{-i} = 1) = j \). The second difference is established by Corollary 1 (d). It identifies the change of subjective expectations \( \tilde{\eta}_i(\cdot) \), but leaves the change of beliefs nonidentified. The reason is that any mean-preserving transformation of beliefs would maintain the same expectation and predict the same choice probability. Similarly, the testable restriction of equilibrium essentially tests whether player \( i \)'s subjective expectation is correct. It could be the case that this player has an incorrect belief which is a mean-preserving transformation of the equilibrium one. As described above, this incorrect belief is indistinguishable from the equilibrium one as both specifications predict the same choice probability.
3.4. Identification with Unobserved Heterogeneity in Belief Functions

As shown in Section 2.1, introducing an unobserved heterogeneity into the belief function could accommodate multiple equilibria. This subsection discusses the identification under this extended model. First, the proofs of all above identification results are constructive. Each identified objective is written as a known function of players’ CCPs. Therefore, with unobserved heterogeneity $\omega$ in the belief function, all identification results would still hold if the choice probability $p(x, \omega)$ can be identified. Unlike Section 3.1, the identification of $p(x, \omega)$ is a nontrivial task as $\omega$ is unobserved to researchers. However, the existence of $\omega$ implies that players’ actions are correlated conditional on $x$. Aguirregabiria and Mira (2019) showed that such a correlation provides identification power and established the point identification of $p(x, \omega)$ in games with more than two players. Even though the main text of this article focuses on a two-player game, the results have been generalized to games with more than two players, as shown in Appendix A.3 (supplementary material). Therefore, in these multi-player games, all identification results still hold with unobserved heterogeneity in beliefs. In contrast, this unobserved heterogeneity cannot be accommodated in a two-player game as $p(x, \omega)$ is not point identified.

4. Empirical Application

4.1. Motivation and Data Description

This section studies China’s Western-style fast food industry. This industry is a duopoly of KFC and McD. In contrast with Western countries, these two firms serve similar products and are considered as close substitutes in China. Moreover, KFC has a leading role. It operated 4952 outlets at the beginning of 2016 while McD owned 2231 stores. These two global giants have been competing in China for 30 years and in many dimensions. In particular, this article focuses on their competition through store-hour decisions. Specifically, the local manager decides the operating hours for each store in his/her local market. This decision is based on each store’s bi-annual review and will be fixed until the next review. At the time of decision, a manager is unlikely to perfectly predict the competitor’s review results. This institutional fact motivates a static game of incomplete information. In each local market, each firm simultaneously decides how many of its current stores to operate at night. Due to its dominant role, KFC owns more stores than McD in many local markets. This feature generates the asymmetric action sets. Therefore, the identification results in Section 3, especially Propositions 2 and 3, can be applied.

Even though the main purpose of this empirical exercise is to illustrate the identification results, the estimation results can answer several empirical questions. First, do firms have biased expectations of their competitors’ behaviors? If so, what are the features of biased beliefs and how do they affect the estimated payoffs? Second, is the decision of store hours a strategic substitute or complement? Understanding this strategic nature has important policy implications in many retail industries, who face strong regulations that restrict stores from operating at night or during the weekend. In this context, a central policy debate is whether to deregulate these restrictions. As shown by Klemperer and Padilla (1997), the strategic nature of store hours is the key to this policy debate. Specifically, if store hours are strategic complements, the deregulation policy would cause longer operating hours, higher consumer surplus, but lower firm’s profits, as compared with strategic substitutes. Unfortunately, there is little empirical evidence on this issue, and the estimation results in this article fill such a gap.

To answer the above questions, I obtain the store-level information, such as address and 24-hr service, from each chain’s official website. Following the seminal work by Bresnahan and Reiss (1990, 1991), this article defines a market as each geographically isolated small town. The exclusion of markets in big cities is to avoid the potential interconnection among markets. Furthermore, to exploit asymmetric action spaces, I focus on markets where KFC owns no fewer stores than McD. Table 1 presents the joint distribution of the number of stores per market for these two chains. The competition in markets where KFC owns more stores than McD is modeled as a $3 \times 2$ game. In particular, KFC has three choices: open zero, one, or more than one 24-hour store while McD chooses whether to operate at night or not. This asymmetry can be used to estimate KFC’s payoffs and beliefs. Unfortunately, there are only seven markets where McD owns more stores. These few observations are insufficient to estimate McD’s belief; therefore, the analysis focuses on KFC. Finally, the sample contains markets where McD owns zero stores. In these markets, KFC faces no competition. Its optimal decision depends only on the base return (i.e., night monopoly profit) but not on its belief. Therefore, the existence of these markets facilitates the identification of the base return.

Local control variables are obtained from different sources, including China Data Online, National Geophysical Data Center, and Baidu map. Table 2 presents the control variables and their summary statistics. It also emphasizes two exclusion restrictions that facilitate the identification. Variable $z_i$ is the market’s distance to firm $i$’s nearest distribution center. This distance directly affects chain $i$’s operation cost but has no direct impact on the other firm’s profit. It acts as the player specific payoff shifter. The identification power of $z_i$ has been exploited by Holmes (2011). In the context of entry by Wal-Mart, he finds that a closer distribution center could substantially save the firm’s delivery costs. The other variable $s$ is the distance between two chains in the same market. This distance only affects KFC’s night profit if McD operates at night. It then serves as the interactive effect shifter. Finally, Appendix A.4 (supplementary material) presents a reduced form analysis. It

| KFC Stores | 0 | 1 | Total |
|------------|---|---|------|
| 1          | 390 | 34 | 424  |
| 2          | 107 | 42 | 149  |
| 3          | 36  | 32 | 68   |
| 4          | 23  | 17 | 40   |
| Total      | 556 | 125| 681  |

Table 1. Distribution of the number of markets by the number of KFC and McD stores.
justifies the exclusion restrictions and a model of incomplete information game.

### 4.2. Estimation Procedures and Results

Since the sample includes both asymmetric action sets and exclusion restrictions, it is ideal to illustrate the main identification results, established by Propositions 2 and 3. Consistent with notations in Section 2, $x$ denotes all control variables in Table 2, except for $z_1$ and $s$. Vector $\tilde{x} = (x, z_{KFC}, z_{McD}, s)$ then denotes all common observables. Moreover, $e_i$ is assumed to follow iid Type I extreme value distribution. Therefore, each player’s CCP takes the Logit form.

Even though the identification results require no parametric assumptions on payoff and belief functions, a nonparametric specification would be imprecisely estimated given my limited sample size. Therefore, this section considers a parametric model with a linear payoff function of KFC:

$$
\pi_{KFC}(x, z_{KFC}, a_{KFC} = j) = \left( x', \log(z_{KFC}) \right) \cdot \theta^{\text{Base}},
$$

$$
\delta_{KFC}(x, s, a_{KFC} = j, a_{McD} = 1) = \left( x', \log(1 + s) \right) \cdot \theta^{\text{Int}}, \quad (8)
$$

where $\theta^{\text{Base}}$ and $\theta^{\text{Int}}$ represent parameters in the base return and interactive effect, respectively.

Since $e_i$ follows iid Type I extreme value distribution, both McD’s CCP of operating one 24-hr store and KFC’s belief take the Logit form. In particular, KFC’s belief is specified as follows:

$$
b^1_{KFC}(\tilde{x}) = \frac{\exp \left[ \log \left( p^1_{McD}(\tilde{x}) \right) + (z_{McD}, z_{McD} \cdot \tilde{x}) \theta^{\text{Bias}} \right]}{1 + \exp \left[ \log \left( p^1_{McD}(\tilde{x}) \right) + (z_{McD}, z_{McD} \cdot \tilde{x}) \theta^{\text{Bias}} \right]}, \quad (9)
$$

Note that $\log \left( p^1_{McD}(\tilde{x}) \right)$ is the inverse function of the Logit formula for McD’s actual CCP. With this specification, $\theta^{\text{Bias}}$ is then interpreted as the belief bias parameter. When $\theta^{\text{Bias}} = 0$, Equation (9) turns to $b^1_{KFC}(\tilde{x}) = p^1_{McD}(\tilde{x})$ so that KFC has unbiased beliefs about McD’s decisions. In contrast, $\theta^{\text{Bias}} \neq 0$ implies that KFC has biased beliefs. The sign and magnitude of $\theta^{\text{Bias}}$ then inform the direction and magnitude of the bias. Note that the belief bias parameters can be identified only for $z_{McD}$ and its interaction with market observable $\tilde{x}$, as suggested by Proposition 3. In addition, Equation (9) imposes one restriction that $p^1_{KFC}(\tilde{x}) = p^1_{McD}(\tilde{x})$ if $z_{McD} = 0$, regardless of the value of $\theta^{\text{Bias}}$. As shown in Proposition 3, this unbiased belief restriction at only one value of $z_{McD}$ is sufficient for point identification of KFC’s payoff and belief. Importantly, Propositions 2 and 3 suggest that even though such a restriction is misspecified, it still achieves the consistent estimation of base return, unbiased belief test, sign and ratio of the interactive effect. Cautiously, it could bias the actual magnitude of the interactive payoff.

As this empirical application does not estimate McD’s payoff and belief, its choice probability is treated as a nuisance parameter and takes the following form:

$$
p^1_{McD}(\tilde{x}) = \frac{\exp \left[ f(\tilde{x}, \theta_{McD}) \right]}{1 + \exp \left[ f(\tilde{x}, \theta_{McD}) \right]}, \quad (10)
$$

where $f(\cdot)$ is a polynomial function. This specification would well approximate McD’s CCP with high enough orders of polynomials. Due to limited sample size, I consider the second order approximation.

Denote $\theta = (\theta^{\text{Base}}, \theta^{\text{Int}}, \theta^{\text{Bias}}_{\text{McD}})$ as the vector of all unknown parameters. Given KFC’s payoff function by Equation (8) and belief function by Equation (9), its choice probability $p^1_{KFC}(\tilde{x}, \theta)$ is well-defined. Consequently, $\theta$ can be estimated by maximizing the following log-likelihood function:

$$
LL = \max_\theta \sum_{m=1}^M \sum_{j=0}^2 \mathbb{1}(a_{KFC,m} = j) \log \left( p^1_{KFC}(\tilde{x}_m, \theta) \right) + \sum_{j=0}^1 \mathbb{1}(a_{McD,m} = j) \log \left( p^1_{McD}(\tilde{x}_m, \theta_{McD}) \right). \quad (11)
$$

As described above, the equilibrium/unbiased belief assumption is nested in this specification by restricting $\theta^{\text{Bias}} = 0$. Therefore, the payoff parameters, under the equilibrium condition, can be

### Table 2. Summary statistics on local markets.

| Variable       | Definition                                      | Mean   | Std. Dev. | Min   | Max   |
|----------------|-------------------------------------------------|--------|-----------|-------|-------|
| **Market Characteristics** | | | | | |
| Income         | GDP Per Capita, 10,000 RMB                      | 4.83   | 3.37      | 0.59  | 31.57 |
| Pop            | Population, 100,000                             | 7.15   | 3.94      | 0.28  | 28.5  |
| Center         | Dummy, =1 if market located at city center     | 0.20   | 0.40      | 0     | 1     |
| Light          | Night Light Density                             | 56.51  | 6.76      | 28    | 63    |
| KFCDist        | Average Distance between KFC’s Stores, km      | 0.51   | 0.86      | 0     | 6.10  |
| Cinemas        | Number of Cinemas                               | 2.82   | 1.87      | 1     | 14    |
| KFCStores      | Number of KFC’s Stores                          | 1.59   | 0.89      | 1     | 4     |
| McDStores      | Number of McD’s Stores                          | 0.18   | 0.39      | 0     | 1     |
| KFC24h         | Number of KFC’s 24-Hour Stores                  | 0.56   | 0.71      | 0     | 3     |
| McD24h         | Number of McD’s 24-Hour Stores                  | 0.38   | 0.49      | 0     | 1     |
| **Exclusion Restrictions** | | | | | |
| $z_{KFC}$      | Distance to Nearest KFC’s Distribution Center, 100 km | 2.12   | 1.36      | 0.09  | 10.16 |
| $z_{McD}$      | Distance to Nearest McD’s Distribution Center, 100 km | 2.69   | 1.76      | 0.31  | 9.91  |
| $s$            | Distance between Two Chains’ Centroids, km      | 0.80   | 0.75      | 0.01  | 3.73  |
| Observations   |                                                 | 681    |           |       |       |

**Note:** Statistics for GDPPerCapita, 10,000 RMB and $s$ and McD24h are calculated conditionally on the existence of McD.
estimated by the following constrained MLE:

\[
\begin{align*}
LL^{\text{Eqm}} &= \max_{\theta} \sum_{m=1}^{M} \left[ 2 \sum_{j=0}^{2} \mathbb{I}(s_{KFC,m} = j) \log (p'_{KFC}(x_{m}; \theta)) \right. \\
&\left. + \sum_{j=0}^{2} \mathbb{I}(s_{McD,m} = j) \log (p'_{McD}(x_{m}; \theta_{\text{McD}})) \right],
\end{align*}
\]

s.t. \( \theta^{\text{Bias}} = 0. \) \hspace{1cm} (12)

Equations (11) and (12) naturally imply the standard likelihood ratio test of the unbiased belief assumption. The test statistic is \( 2 \times (LL - LL^{\text{Eqm}}) \) and the degree of freedom is the dimension of \( \theta^{\text{Bias}} \). Equivalently, it is a test of the null hypothesis that \( \theta^{\text{Bias}} = 0 \).

Table 3 presents the estimates of belief bias parameters \( \theta^{\text{Bias}} \).

Due to limited sample size and for the sake of precise estimation, the belief bias is specified to depend only on \( z_{\text{McD}} \) and its interactions with Income and KFCDist. First, the null hypothesis that \( \theta^{\text{Bias}} = 0 \) is rejected at the 5% significance level. It suggests that KFC has biased beliefs. Second, these estimates further provide information on the direction and magnitude of the belief bias. In McDs’s perspective, a longer distance to its distribution center causes a higher delivery cost and a lower profit. Therefore, a higher \( z_{\text{McD}} \) would reduce McD’s operating hours. KFC correctly predicts this direction but over-predicts the impact of a longer distance, as reflected by a negative significant coefficient on \( z_{\text{McD}} \).

In addition, the actual impact of \( z_{\text{McD}} \) on McD is alleviated in a market with higher Income. KFC also correctly predicts the direction but over-predicts such an alleviation effect, as reflected by a positive significant coefficient on \( z_{\text{McD}} \times \text{Income} \).

Table A.2 in Appendix A.4 (supplementary material) presents the estimates of \( \hat{\theta}_{\text{Eqm}} \) in McD’s CCP. These estimates, together with \( \hat{\theta}_{\text{Bias}} \) in Table 3, allow us to calculate KFC’s (biased) beliefs in each market in the sample. These beliefs are highly dispersed. In the most pessimistic market, KFC under-predicts the probability that McD will operate at night by about 34 percentage points. As a comparison, in the most optimistic market, KFC over-predicts such a probability by about 91 percentage points. On average across all markets, KFC’s belief is biased upward by about 10 percentage points.

To better understand the direction and magnitude of KFC’s biased beliefs, Figure 1 plots the estimated impacts of \( z_{\text{McD}} \) on both McD’s CCPs (dotted lines) and KFC’s beliefs (solid lines) for different deciles of Income. In poor markets shown in the top left graph, KFC over-estimates the negative effect of the higher cost caused by larger \( z_{\text{McD}} \), as reflected by a steeper line of KFC’s beliefs than the one of McD’s CCPs. In addition, the impact of \( z_{\text{McD}} \) on McD’s operating hours is attenuated as Income rises. This feature is indicated by the increasing flatter lines of McD’s CCPs, moving from the top left graph to the bottom right one. KFC knows this attenuation effect but over-predicts its magnitude. Specifically, in rich markets shown on the bottom right graph, KFC incorrectly believes that the high profitability in these markets would completely compensate the negative impact of \( z_{\text{McD}} \). It implies a horizontal line of KFC’s beliefs. In addition, as shown on the off-diagonal graphs, KFC’s beliefs are close to McD’s actual choice probabilities and unbiased belief is a good approximation in these markets with a medium level of Income. Finally, Appendix A.4 (supplementary material) conducts a reduced form analysis. It studies the relationship between KFC’s biased belief and market characteristics. In summary, in markets that are richer (i.e., higher Income) and further from each chain’s distribution center (i.e., higher \( z_{\text{KFC}} \) and \( z_{\text{McD}} \)), KFC tends to over-predict the probability that McD will operate at night. In contrast, KFC under-predicts such a probability in markets where KFC owns more stores (i.e., higher KFCStores).

Table 4 presents the estimates of KFC’s payoff parameters.

It compares the estimates under potentially biased beliefs (column titled “Unknown belief”) with the ones under the unbiased belief restriction (column titled “Eqm belief”). The top panel shows the results of the interactive payoff parameters \( \theta^{\text{Int}} \). For the sake of precise estimation, the interactive effect is specified to depend only on \( \log(1 + s) \), Center, and their interactions. In this specification, Center is a dummy variable that captures the location of the market. It equals 1 if the market is at the city center and equals 0 if it is located at a satellite town. The estimates under column “Unknown Belief” suggest that the strategic nature of store hours depends on the location of the market. Specifically, store hours are strategic complements in the city center while they are strategic substitutes in a satellite town. These results further indicate that extending operations to overnight service is vertically differentiated as a quality measure, especially in the city center. Intuitively, the best response to a competitor’s quality improvement is to increase one’s own quality, indicating a strategic complement. In contrast, the decision of store hours is horizontally differentiated in a satellite town and suggests a strategic substitute. Distinguishing the strategic nature has some implications on the deregulation policy that lifts restrictions on store hours. First, compared with that in a satellite town, the strategic complement at the city center generates a positive feedback between each chain’s business hours. Consequently, the deregulation policy would cause longer store hours. Second, the additional quality aspect of store hours implies a higher consumer welfare.

Figure 2 plots the estimated interactive effect as a function of \( s \) under both the equilibrium constraint (dotted line) and the unknown belief specification (solid line). In an average market in a satellite town, the estimated interactive effect is -7.474 when the model allows biased beliefs. This effect is actually sizable. Given the estimated coefficient on \( \log(\text{Income}) \) in the base return, it requires about six times increase of Income to compensate this negative interactive effect (i.e., 7.474/1.225). In my sample, this effect is roughly equivalent to moving from the
poorest market (Income = 0.59) to the median one (Income = 3.97). In contrast, when the equilibrium condition is falsely imposed, the estimated interactive effect in an average satellite town is only −3.352. This corresponds to an attenuation bias that is more than 50%. A similar comparison also applies to an average market in the city center (6.008 v.s. 4.658) where the attenuation bias is about 20%. Finally, the estimates under the equilibrium constraints suggest a highly insignificant interactive effect in satellite towns. In contrast, under the unknown belief specification, the strategic substitutability is significant at the 5% level. In literature, this type of attenuation bias caused by biased beliefs has also been found by Aguirregabiria and Magesan (2020).

The bottom panel of Table 4 shows the estimates of the base return parameters $\theta_{\text{Base}}$. As expected, KFC’s base return is higher in markets with higher Income, higher number of own stores (i.e., KFCStores), a closer distribution center (i.e., lower $z_{\text{KFC}}$), and more active night life (measured by Light and Cinemas). Consistent with Holmes (2011), the distance to distribution center substantially affects a chain’s delivery cost and profit. Specifically, the impact of 1% increase of $z_{\text{KFC}}$ on KFC’s profit is equivalent to about 0.8% decrease of Income (i.e., 0.956/1.225). Interestingly, the estimates of $\theta_{\text{Base}}$ are stable under both the unknown belief specification and the equilibrium restriction. This is in contrast with the estimates of $\theta_{\text{Int}}$ where incorrectly imposing the unbiased belief assumption causes a considerable attenuation bias. This comparison has a simple explanation. By definition, base return is the part of KFC’s payoff that is independent of McD’s behavior. Intuitively, we can construct an equation to identify $\theta_{\text{Base}}$, and this equation does not depend on KFC’s belief. The proof of Proposition 2 formally shows this result. Consequently, the estimates of $\theta_{\text{Base}}$ are robust to different belief specifications. As a comparison, it is the multiplication of interactive effect and KFC’s belief that affects KFC’s decision. Therefore, a misspecification of the belief function could seriously affect the estimate of $\theta_{\text{Int}}$.

4.3. Estimation With Symmetric Action Spaces

The above subsection illustrates the identification results with asymmetric action spaces. This subsection further shows how to apply the identification results, established in Section 3.3, to games with symmetric action sets. First, for these identification results to hold, each player has to have more than two actions (e.g., $3 \times 3$ game). However, this requirement is satisfied in only 21 markets, where both chains own more than 1 outlet. Since the sample size is smaller than the number of parameters, neither payoff nor belief can be estimated. As a compromise, this subsection performs a simulation exercise to illustrate the identification power in the finite sample.

The simulated data are generated through two steps. First, for each observation in the original sample, I fix the market characteristics $\mathbf{x}$ but assume that both KFC and McD own two outlets. This procedure extends McD’s action space and generates a sample of $3 \times 3$ games. Second, in these hypothetical games, each chain’s choice is simulated using the estimates in Section 4.2. With this simulated sample of symmetric games, I then estimate payoffs and beliefs by a similar method described in Section 4.2. Details about simulation and estimation procedures are left to Appendix A.5 (supplementary material).

Table 5 presents the averages and standard deviations of the estimates of belief parameters $\theta_{\text{Bias}}$ among 500 simulated datasets. These parameters are precisely estimated with small biases. The standard deviations are also small, as compared
Table 4. Estimates of KFC’s payoff parameters, $\theta_{\text{Int}}$ and $\theta_{\text{Base}}$.

| Interactive payoff parameters: $\theta_{\text{Int}}$ | EQM belief | Unknown belief |
|-------------------------------------------------|------------|----------------|
| 1 - Center                                       | 1.001      | -2.834         |
|                                                 | (5.665)    | (6.416)        |
| Center                                          | 6.528      | 10.044*        |
|                                                 | (5.910)    | (5.842)        |
| $\log(1 + s) \times (1 - \text{Center})$       | 2.339      | -1.008         |
|                                                 | (6.164)    | (7.208)        |
| $\log(1 + s) \times \text{Center}$              | -6.737     | -10.239*       |
|                                                 | (5.451)    | (5.629)        |

| Base return parameters: $\theta_{\text{Base}}$ |
|-----------------------------------------------|
| $\log(\text{Income})$                        | 0.749***   |
|                                              | (0.245)    |
| $\log(\text{Pop})$                           | 0.211      |
|                                              | (0.221)    |
| $\log(z_{\text{KFC}})$                       | -0.178     |
|                                              | (0.152)    |
| $\mathbb{1}(\text{KFC stores} = 2)$          | 1.696***   |
|                                              | (0.462)    |
| $\mathbb{1}(\text{KFC stores} = 3)$          | 2.209***   |
|                                              | (0.644)    |
| $\mathbb{1}(\text{KFC stores} = 4)$          | 3.052***   |
|                                              | (1.026)    |
| $\log(1 + \text{KFCDist})$                   | -0.810     |
|                                              | (0.522)    |
| Center                                        | -5.603     |
|                                              | (5.029)    |
| Light                                         | 0.052***   |
|                                              | (0.020)    |
| Cinemas                                       | 0.169**    |
|                                              | (0.078)    |

| Log-likelihood                              |
|---------------------------------------------|
| -434,942                                    |

| Observations                                |
|---------------------------------------------|
| 681                                         |

The top panel of Table 6 shows the results of the interactive payoff parameters $\theta_{\text{Int}}$. It also compares the estimates under the unknown belief specification with the ones under the equilibrium restriction. First, when beliefs are allowed to be biased, the sign and ratio of the interactive effect can be reliably estimated. Second, there is an upward bias of the actual magnitude. Such a bias could be due to the nonlinear feature of the model. Since the interactive effect and belief are multiplied together, a small downward bias on belief (i.e., toward zero) could transmit to an extremely large upward bias on the interactive effect. In contrast, a same size upward bias on belief has a limited impact on the interactive effect due to a zero lower bound for its absolute value. This feature causes an upward bias of the magnitude of the interactive effect in the finite sample. As a comparison, the belief estimates suffer less from this problem as the model imposes both an upper and lower bound (i.e., $0 \leq b_j \leq 1$). Finally, the upward bias of the interactive effect is much smaller than the downward bias caused by incorrectly imposing the equilibrium condition, as shown in the column titled “EQM belief.” At least for the estimates found in this empirical application, the misspecification of the equilibrium condition incurs a higher
cost. It illustrates the flavor of the identification results in the finite sample.

The bottom panel of Table 6 presents the results of the base return parameters $\theta^{\text{Base}}$. In particular, these parameters are reliably estimated under both the unknown belief specification and the equilibrium constraint. As explained in Section 4.2 and shown in the proof of Proposition 2, a player's base return can be identified without knowing this player’s belief. Therefore, the estimates of $\theta^{\text{Base}}$ are robust to different specifications of beliefs.

| Interactive Payoff Parameters: $\theta^{\text{Int}}$ | Values in simulation | Unknown belief | EQM belief |
|--------------------------------------------------|----------------------|----------------|------------|
| $1 - \text{Center}$                              | $-1.123$             | $-1.082$       | $-0.427$   |
| Center                                           | $8.366$              | $(10.399)$     | $(3.523)$  |
| $\log{(1 + s)} \times (1 - \text{Center})$      | $1.508$              | $2.021$        | $1.173$    |
| $\log{(1 + s)} \times \text{Center}$            | $-7.605$             | $-9.722$       | $-2.213$   |

| Base Return Parameters: $\theta^{\text{Base}}$ | Values in simulation | Unknown belief | EQM belief |
|------------------------------------------------|----------------------|----------------|------------|
| $\log{\text{Income}}$                         | $0.721$              | $0.775$        | $1.302$    |
| $\log{\text{Pop}}$                            | $0.232$              | $0.217$        | $0.214$    |
| $\log{\text{Pop}}$                            | $(0.375)$            | $(0.377)$      | $(0.377)$  |
| $\log{\text{PC}}$                             | $-0.211$             | $-0.215$       | $-0.123$   |
| $\log{(1 + \text{KFCDist})}$                  | $-0.906$             | $-0.971$       | $-1.294$   |
| $\log{(1 + \text{KFCDist})}$                  | $(0.490)$            | $(0.456)$      | $(0.456)$  |
| Center                                          | $-6.009$             | $-6.392$       | $-6.365$   |
| Light                                           | $0.053$              | $0.055$        | $0.047$    |
| Cinemas                                         | $0.159$              | $0.174$        | $0.163$    |

| Values in simulation | Unknown belief | EQM belief |
|----------------------|----------------|------------|
| $0.721$              | $0.775$        | $1.302$    |
| $0.232$              | $0.217$        | $0.214$    |
| $-0.211$             | $-0.215$       | $-0.123$   |
| $-0.906$             | $-0.971$       | $-1.294$   |
| $-6.009$             | $-6.392$       | $-6.365$   |
| $0.053$              | $0.055$        | $0.047$    |
| $0.159$              | $0.174$        | $0.163$    |

| Values in simulation | Unknown belief | EQM belief |
|----------------------|----------------|------------|
| $1.225$              | $1.313$        | $1.128$    |
| $0.646$              | $0.640$        | $0.665$    |
| $-0.956$             | $-1.005$       | $-0.795$   |
| $-0.910$             | $-0.978$       | $-0.898$   |
| $-5.746$             | $-6.295$       | $-5.425$   |
| $0.131$              | $0.138$        | $0.127$    |
| $0.197$              | $0.211$        | $0.144$    |

| Values in simulation | Unknown belief | EQM belief |
|----------------------|----------------|------------|
| $0.762$              | $0.772$        | $(0.772)$  |
| $0.055$              | $0.047$        | $(0.047)$  |
| $0.023$              | $(0.023)$      | $(0.023)$  |
| $0.120$              | $(0.102)$      | $(0.102)$  |
| $0.100$              | $(0.100)$      | $(0.100)$  |

NOTE: In these simulated datasets, KFC owns two stores in all markets. Therefore, variable KFC stores have no variation and is excluded from KFC’s payoff function in this exercise.

The identification problem that allows both a flexible information structure and biased beliefs is an important area for future research.

5. Conclusion

This article studies the identification of an incomplete information game without the constraint of Bayesian Nash equilibrium. The econometric model specifies a player’s belief as an unknown function and allows it to be any probability distribution over the other player’s action set. When players have different numbers of actions, this asymmetric feature partially identifies the payoff of the player with more actions. When the usual exclusion restrictions are satisfied, the payoff and belief are identified up to a scale. There is also a testable implication of the equilibrium restriction. Finally, these identification results are extended to both players with a multiplicatively separable condition on the interactive effect.

The identification results in this article rely on an informational assumption such that private information is independent across players. In practice, the existence of unobserved heterogeneity and potential correlation among private information could invalidate this assumption. Under the equilibrium framework, Grieco (2014), Aguirregabiria and Mira (2019), and Magnolli and Roncoroni (2019) relaxed the informational assumptions and showed the identification of payoff parameters.

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