Numerical analysis corresponding with experiment in compact beam simulator for heavy ion inertial fusion driver

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Abstract. Tune depression in a compact beam equipment is estimated, and numerical simulation results are compared with an experimental one for the compact beam simulator in a driver of heavy ion inertial fusion. The numerical simulation with multi-particle tracking is carried out, corresponding to the experimental condition, and the result is discussed with the experimental one. It is expected that the numerical simulation developed in this paper is useful tool to investigate the beam dynamics in the experiment with the compact beam simulator.

1. Introduction
In heavy ion inertial fusion (HIF), which is inertial confinement fusion driven by heavy ion beams, space charge dominated beam physics is critical research topic [1]. Because the beam parameters are far from those of the heavy ion beam produced by the existing particle accelerator complex [2]. The heavy ion accelerator is undoubtedly needed for HIF, however the large size is a problem issue for the HIF research and development. For this reason, compact beam equipments with electrons were proposed and developed [3, 4, 5]. Not only the experimental approaches but also the numerical analyses with multi-particle tracking were carried out according to the experimental condition [6, 7, 8].

In particular, longitudinal pulse compression is critical manipulation in the final stage of HIF driver system. The beam current is increased drastically with the extreme pulse compression. The beam parameters are changed dynamically during the pulse compression, and achieve from an emittance-dominated regime to a space-charge-dominated regime [1, 2].

In this study, the tune depression in the compact beam equipment is estimated, and the calculation results are compared with the experimental result. The multi-particle tracking based on particle-particle method is carried out, and the numerical result is discussed with the experimental one.

2. Simulating beam parameter in estimation from tune depression
The tune depression $k/k_0$, which is an index for space charge strength in the beam bunch, is given by (Eq.(4.99) in Ref.[9])
\[ k/k_0 = \sqrt{1+u^2} - u, \] (1)

where \( u \) is the dimensionless parameter (Eq.(4.92) in Ref.[9])

\[ u = \frac{K}{2k_0 \varepsilon}. \] (2)

Here \( K \) is the generalized perveance (Eq.(4.24) in Ref.[9])

\[ K = \frac{\omega_p^2 a^2}{2\beta^2 c^2}, \] (3)

and \( k_0 = |qB| / (2mc\beta\gamma) \) is the wave number (Eq.(4.86b) in Ref.[9]) and \( \varepsilon \) is the emittance (Eq.(4.72) in Ref.[9]), which is approximated as temperature in this paper assuming with a thermal equilibrium condition,

\[ \varepsilon = \frac{2a}{\beta c} \left( \frac{k_B T_e}{\gamma m} \right)^{1/2}, \] (4)

where \( a \) is the beam radius, \( \beta \) is the velocity divided by speed of light \( c \), \( q \) is the charge of particle, \( B \) is the applied magnetic flux density, \( m \) is the mass of particle, \( \gamma \) is the relativistic factor, \( k_B \) is the Boltzmann constant, \( T_e \) is the temperature, respectively. The relativistic plasma frequency is defined by \( \omega_p^2 = q^2 n / (\epsilon_0 \gamma^3 m) \) (Eq.(4.2) in Ref.[9]), where \( n \) is the number density and \( \epsilon_0 \) is the permittivity in a vacuum (the mass \( m \) is replaced by the relativistic mass \( m \), and since the electric repulsion is reduced by magnetic attraction, the potential due to the self fields is replaced by \( 1/\gamma^2 \). Finally, the nonrelativistic plasma frequency is replaced with the relativistic one by adding \( 1/\gamma^3 \).

Consequently, Eq.(2) is rewritten by

\[ u = \left| \frac{q}{B} \right| \frac{n a}{4 \epsilon_0 \gamma^2} \left( \frac{\gamma m}{k_B T_e} \right)^{1/2}. \] (5)

By using Eq.(1) with Eq.(5), the tune depression is estimated as shown in Fig. 1. In the case of this experiment, \( a = 0.5 \) mm, \( q = -1.6022 \times 10^{-19} \) C, \( \gamma = 1 \), \( T_e = 0.1 \) eV, \( m = 9.1094 \times 10^{-31} \) kg, and \( B = 11, 22, 33 \) mT are assumed, respectively. The electron beam currents are \(-265 \) \( \mu \)A for 11 mT, \(-278 \) \( \mu \)A for 22 mT, and \(-248 \) \( \mu \)A for 33 mT, as the initial condition in 100 ns pulse duration with the flat-top profile. After the pulse compression, the beam current was increased to be about 20 times in comparison with the initial one [3, 4].

For the final pulse compression in HIF driver, the tune depression is estimated in the range from 0.9 (before pulse compression) to 0.16 (after pulse compression) [10]. In comparisons with Fig. 1, it is expected that the compact beam simulator covers the space charge strength during the pulse compression in the final stage of HIF driver.

### 3. Numerical simulation corresponding to experimental condition

Figure 2 shows the computational box for the numerical simulation corresponding to the experimental condition [3, 4]. In the compact beam equipment, the electrons are emitted from the thermal cathode [3, 4], and are transported through the solenoid transport line after the modulation gap. At the gap, the longitudinal velocity of electrons is modulated with the voltage produced by induction modulators for the drift compression. The behavior of electron beam is
assumed with non-relativistic regime, because the kinetic energy of electrons is 2.8 keV at the maximum.

According to the above conditions, we carried out the multi-particle tracking simulations to confirm the beam dynamics in the compact beam simulator. Molecular Dynamics (MD) simulation [11], which represent a so-called Particle–Particle (PP) method [12], is applied to the calculation of particle dynamics in three-dimensional space [8]. The time-dependent particle position \( \mathbf{x}_i = \{x_i(t), y_i(t), z_i(t)\} \) for \( i \) th super particle is obtained by

\[
d\mathbf{x}_i/dt = \mathbf{v}_i,
\]

where the particle velocity \( \mathbf{v}_i \) for \( i \) th super particle is temporally changed by the external applied electric and magnetic fields and the interaction with \( j \) th super particle, and is calculated by

\[
m_i \frac{d\mathbf{v}_i}{dt} = q_i \mathbf{E}_{ext} + \frac{q_i q_j}{4\pi \varepsilon_0} \sum_{j \neq i} \frac{\mathbf{x}_i - \mathbf{x}_j}{((\mathbf{x}_i - \mathbf{x}_j)^2 + s^2)^{3/2}} + q_i \mathbf{v}_i \times \mathbf{B}_{ext},
\]

where \( m_i \) is the mass of \( i \) th super particle, \( q_i \) is the charge of \( i \) th super particle, \( \varepsilon_0 \) is the permittivity in vacuum, \( \mathbf{E}_{ext} \) and \( \mathbf{B}_{ext} \) are the external applied electric field and the external applied magnetic flux density, respectively. Here \( s \) is the softening parameter [13], and \( s = 2r_e \) is applied in this study, where \( r_e \) is the classical electron radius. The applied electric field \( \mathbf{E}_{ext} \) is produced by the modulation voltage, and is only given in the modulation gap placed at the entrance of transport line.

Figure 3 shows the beam current waveform at \( z = 1.93 \) m after the modulation gap. Figure 4 shows the compression ratio, which is defined by the ratio of the beam current at each time to the initial one, at \( z = 1.93 \) m after the modulation gap. In the experiment with the compact beam simulator, the beam current was measured by using the Faraday cup [3, 4]. The initial beam current is \(-265 \) \( \mu \)A, and the applied magnetic flux density is \( 11 \) mT in the solenoid transport.
The beam current was compressed as highest as 25 times in comparison to the initial one as shown in Fig. 4. As shown in Figs. 3 and 4, the numerical results reproduce the experimental one. Consequently, it is implied that the numerical simulation developed in this paper is useful to discuss the beam behavior in the compact beam equipment.

Figure 3. Beam current waveform at transport distance of 1.93 m after modulation gap. Red-full circles indicate experimental results, and solid line shows numerical result.

Figure 4. Compression ratio at transport distance of 1.93 m after modulation gap. Red-full circles indicate experimental results, and solid line shows numerical result.

4. Conclusion
The tune depression in the compact beam equipment was estimated, and the calculation results were compared with the experimental result in the compact beam simulator for the final stage of HIF driver system. The numerical simulation with the multi-particle tracking was carried out, and the calculation result reproduced the experimental result. It was expected that the numerical simulation developed in this paper is useful tool to investigate the beam behavior in the compact beam equipment. The HIF system is designed to clear the beam behavior by using the approaches with the compact beam equipment and the numerical simulation.

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