Neutron Capture by Light Nuclei at Astrophysical Energies

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Abstract—Results obtained in the studies of radiative neutron capture on light nuclei at thermal and astrophysical energies are reviewed. The capture reactions in question are part of the primordial nucleosynthesis reaction chain underlying formation and development of the Universe. The reactions are treated within the potential cluster model with orbital states of clusters classified by Young diagrams. It is demonstrated that the intercluster potentials obtained from the phase shift analysis and description of the main characteristics of bound nuclear states in cluster channels can be used for the analysis of radiative capture characteristics.

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INTRODUCTION

The continued interest in investigations of radiative neutron capture by atomic nuclei is due to the importance of this process for the studies of many fundamental properties of nuclear reactions and nuclei themselves, as well as the wide use of the capture cross section data in various nuclear physics and nuclear astrophysics applications and analysis of primordial nucleosynthesis processes in the Universe [1]. Earlier we showed a possibility [2, 3] of describing astrophysical S-factors [4, 5] of radiative charged-particle capture reactions on some light and lightest nuclei within the potential cluster model (PCM) with forbidden states (FSs) [2, 3, 6, 7]. This model includes the supermultiplet symmetry of the wave function (WF) for a cluster system with orbital states separated by Young diagrams [2, 3, 8]. The classification used for orbital states allows the structure of intercluster interactions to be analyzed, the presence and number of allowed states (ASs) and forbidden states to be determined, and thus the number of nodes of the radial wave function for the relative motion of clusters to be found [9, 10].

In this approach the intercluster interaction potentials for scattering processes are constructed on the basis of the elastic scattering phase shifts extracted from the experimental differential cross sections dur-
ing the phase shift analysis. For bound states (BSs) of light nuclei in cluster channels, potentials are constructed using a number of other criteria in addition to the scattering phase shifts. An example is the requirement that the binding energy and a few other characteristics of nuclear ground states (GSs) be reproduced, which can sometimes be the main requirement [2, 3, 6]. It is assumed that the BS arises from the cluster channel consisting of the initial particles that participate in the reaction [12, 13]. The potential cluster model is chosen for considering these cluster systems in nuclei and nuclear and thermonuclear processes at astrophysical energies [4, 14] because the probability for formation of nucleon associations, i.e., clusters, in many light nuclei and the degree of their isolation from each other are rather high, which is confirmed by a lot of experimental measurements and theoretical calculations performed by different authors in the past 50 to 60 years [12, 15, 16].

Certainly, this assumption somewhat idealizes the real situation in the nucleus, because it implies that the BS is characterized by 100% clustering of the nucleus for the entrance channel particles. The success of this potential model in describing a channel system of \( A \) nucleons in the bound state therefore depends on how high the real clustering of this nucleus in the channels of \( A_1 + A_2 \) nucleons is. At the same time, some nuclear characteristics of particular, even cluster-free, nuclei can predominantly be determined by a certain cluster channel; i.e., they can have a kind of cluster structure with a small contribution from other possible cluster configurations. In this case the one-channel cluster model allows the dominating cluster channel to be identified and its associated properties of the nuclear system to be discriminated and described [2, 3, 6, 7, 8]. Therefore, the results obtained within the one-channel model can be regarded as a test of the single-channel property of cluster configurations in those nuclei.

As the general principles of the PCM with FSs have been considered many times [2, 3, 6–10, 12, 15, 16], in the first section we briefly describe the general calculation methods used within this model. Then we consider the possibility of describing total cross sections for the \( n^2 H \) capture reaction within this model with allowance for the supermultiplet symmetry of the wave function and separation of results by orbital states by Young diagrams. Though the radiative capture reaction \( n^2 H \rightarrow ^3 H \gamma \) at astrophysical energies with the formation of an unstable tritium nucleus \( ^3 H \)-decaying to \(^3\)He is not directly included in the main thermonuclear cycles [4, 5], it can probably play a somewhat significant part in a number of Big Bang models [17]. These models assume that according to the main nuclear reaction chain of the form

\[
\begin{align*}
\text{Li} & \rightarrow \text{Be} \rightarrow \text{B} \rightarrow \text{C} \rightarrow \text{N} \rightarrow \text{O} \rightarrow \text{F} \rightarrow \text{Ne} \rightarrow \text{Mg} \rightarrow \text{Si} \rightarrow \text{P} \rightarrow \text{S} \rightarrow \text{Cl} \rightarrow \text{Ar} \\
\text{Be} & \rightarrow \text{C} \rightarrow \text{N} \rightarrow \text{O} \rightarrow \text{F} \rightarrow \text{Ne} \rightarrow \text{Mg} \rightarrow \text{Si} \rightarrow \text{P} \rightarrow \text{S} \rightarrow \text{Cl} \rightarrow \text{Ar} \\
\text{B} & \rightarrow \text{C} \rightarrow \text{N} \rightarrow \text{O} \rightarrow \text{F} \rightarrow \text{Ne} \rightarrow \text{Mg} \rightarrow \text{Si} \rightarrow \text{P} \rightarrow \text{S} \rightarrow \text{Cl} \rightarrow \text{Ar} \\
\text{C} & \rightarrow \text{N} \rightarrow \text{O} \rightarrow \text{F} \rightarrow \text{Ne} \rightarrow \text{Mg} \rightarrow \text{Si} \rightarrow \text{P} \rightarrow \text{S} \rightarrow \text{Cl} \rightarrow \text{Ar}
\end{align*}
\]

etc. [17], which includes, though not obligatorily, the \( n^2 H \) capture, primordial nucleosynthesis proceeded only at low energies which we will consider below. Note that there are other versions of the above chain [17] with a few other intermediate reactions.

Next, we consider processes of the \( n^6 \text{Li}, n^7 \text{Li}, n^{12} \text{C}, n^{13} \text{C}, n^{14} \text{C}, n^{14} \text{N}, \) and \( n^{16} \text{O} \) radiative capture within the PCM with FSs. The total cross sections of all these reactions, generally included in primordial nucleosynthesis chain (1), can be described using intercluster potentials whose parameters describe both elastic scattering phase shifts with consideration of their resonance behavior and the main characteristics of nuclear BSs. In addition, these potentials are correlated with the classification of orbital states by the Young diagrams for each cluster system.

**CALCULATION METHODS**

All the main methods for calculation of radiative capture reactions were discussed many times in [2, 3, 6–10, 13, 15, 16, 18]. Therefore, we only give their very brief description here.

**Total Capture Cross Sections**

The cluster-model total cross sections for the reactions to be considered below are given, for example, in [19] or [2, 11, 15, 18] and are written as

\[
\sigma(E) = \sum_{J_f, J_i} \sigma(NJ, J_f, J_i),
\]

where the following simple expressions are known for electric orbital \( EJ(L) \) transitions of multipolarity \( J \)[11, 15, 18] \((S_i = S_f = S):\)

\[
A_J(E, J, K) = K^\mu_j \left( \frac{Z_1}{m_1} + (-1)^j \frac{Z_2}{m_2} \right),
\]

\[
I_J(J_f, J_i) = \langle \chi_{J_i} | R | \chi_{J_f} \rangle,
\]

\[
P^2_J(EJ, J_f, J_i) = \delta_{S,J} \delta_{L,J} \left( 2J + 1 \right) \left( 2L + 1 \right) \left( 2J_f + 1 \right) \left( 2J_i + 1 \right) \times \left( 2J + 1 \right) \left( 2L + 1 \right) \left( 2J_f + 1 \right) \left( 2J_i + 1 \right)
\]

Here \( S, S_i, L, L_i, J_i \), and \( J_f \) are the spins and moments of particles in the entrance \( (i) \) and exit \( (f) \) channels, \( q \) is the wave number of the entrance channel particles, \( S_i \) and \( S_f \) are the spins of two particles in the entrance channel, \( m_1, m_2, Z_1, \) and \( Z_2 \) are the masses and charges of the entrance channel particles, \( K \) and \( J \)
are the wave number and moment of the \( \gamma \) ray in the exit channel, and \( I_f \) is the integral of the intercluster coordinate \( R \) over the wave functions for the relative motion of the clusters in the \( \chi_f \) and final \( \chi_f \) states.

A spectroscopic factor \( S_f \) of the final nuclear state is sometimes included in the above expressions for total cross sections, but in the potential cluster model used here it has always been set to unity, as in [19].

Using expressions from [20], we can obtain the following relations [6, 7] for considering the magnetic \( M1(S) \) transition determined by the spin part of the magnetic operator \( (S_f = S_f = S, I_f = I_f = I) \):

\[
P_i^2(M1, J_f, J_l) = \delta_{S,S} \delta_{I,I} \times \frac{S(S + 1)(2S + 1)(2J_f + 1)(2J_l + 1)}{2} \left\{ \begin{array}{c} S \ L \ J_f \\ J_f \ 1 \ S \end{array} \right\}^2 ,
\]

\[
A_i(M1, K) = i \frac{e \hbar}{m_\gamma c} K \sqrt{3} \left[ \mu_1^2 - \mu_2^2 \right] , \]

\[
I_f(J_f, J_l) = \langle \chi_f | \tilde{R}^J | \chi_l \rangle , \quad J = 1 ,
\]

where \( m \) is the nuclear mass, and \( \mu_1 \) and \( \mu_2 \) are the magnetic moments of the clusters with the values taken from [21].

In electromagnetic processes like radiative capture or photodisintegration, the operator of electromagnetic transitions for interaction of radiation with matter is well-known [22]. We therefore have a remarkable opportunity to clarify the forms of strong interaction between two particles in the entrance channel when they are in the continuous spectrum and between the bound states of those particles in the exit channel, i.e., the states of their discrete spectrum.

### Potentials and Wave Functions

Intercluster interaction potentials for each partial wave, i.e., for the given orbital momentum \( L \), with the point Coulomb term, which is zero in this case, can be taken in the form [2, 3, 6–8]

\[
V(r) = V_0 \exp(-\gamma r^2) .
\]

Here parameters \( V_0 \) are in units of MeV, and \( \gamma \) are in fm\(^{-2}\). They are potential parameters obtained such as to provide the best description of the elastic scattering phase shifts by considering their resonance behavior. The phase shifts are extracted by phase shift analysis from the experimental data on differential cross sections, i.e., on angular distributions of excitation functions for the continuous spectrum, or obtained from the description of the above-mentioned characteristics of BSs of nuclei under consideration, which are calculated in the cluster channels consisting of initial particles.

In some cases the Coulomb radius \( R_{\text{Coul}} \) is introduced in the Coulomb potential for charged particles, and the Coulomb part takes the form [23]

\[
V_{\text{Coul}}(r) = \begin{cases} \frac{Z_1 Z_2}{r} & r > R_{\text{Coul}} \\ \frac{Z_1 Z_2}{r^2} \left( 3 - \frac{r^2}{R_{\text{Coul}}^2} \right) & r < R_{\text{Coul}} \end{cases} .
\]

In the variational method (VM), which we use here, the wave function for the relative motion of the clusters is expanded in the nonorthogonal Gaussian basis and parameters are independently varied. The WF itself has the form [15, 16]

\[
\Phi_i(r) = \chi_i(r) = r^{\beta_i} \sum_i C_i \exp(-\alpha_i r^2) ,
\]

where \( \beta_i \) are variational parameters and \( C_i \) are expansion coefficients [24].

The behavior of the wave function for bound states, including ground states of nuclei in cluster channels at large distances, is characterized by an asymptotic constant (AC) \( C_w \), which is defined in terms of the Whittaker function [25]

\[
\chi_L(r) = \sqrt{2k_0 C_w W_{-\eta L + 1/2}(2k_0 r)} ,
\]

where \( \chi_L(R) \) is the numerical bound state wave function obtained from the solution of the Schrödinger radial equation and normalized to unity; \( W_{-\eta L + 1/2} \) is the Whittaker bound state wave function that governs the asymptotic behavior of the WF and is a solution to the same equation without the nuclear potential, i.e., at large distances \( R \); \( k_0 \) is the wave number which depends on the channel binding energy; \( \eta \) is the Coulomb parameter that is zero for reactions with neutrons; and \( L \) is the orbital momentum of the bound state.

The asymptotic constant (AC), or the asymptotic normalization coefficient (ANC), is an important nuclear characteristic. If known for nucleus \( a \) in the \( b + c \) cluster channel, it often allows determining total cross sections from the radiative capture process \( b(c, \gamma) \bar{a} \) [26]. The asymptotic constant is proportional to the nuclear vertex constant for the virtual process \( a \rightarrow b + c \), and that constant is a matrix element of the latter process on the mass shell [27].

The root-mean-square mass radius of the nucleus in the cluster model for a system of two clusters of a given size was defined as follows [11, 15]:

\[
R_m^2 = \frac{m_1}{m} \langle r_{m_1}^2 \rangle + \frac{m_2}{m} \langle r_{m_2}^2 \rangle + \frac{m_1 m_2}{m^2} I_2 ,
\]
where \( \langle r^2_m \rangle \), are squares of mass radii of clusters, for which the radii of the corresponding nuclei in a free state are taken, and \( I_2 \) is the integral of the form

\[
I_2 = \langle \chi _L (R) \rangle \langle R^2 \rangle \langle \chi _L (R) \rangle ,
\]

of the intercluster distance \( R \) over the normalized-to-unity radial wave function \( \chi _L (R) \) for the relative motion of clusters in the ground state of the nucleus with the orbital momentum \( L \).

The root-mean-square charge radius was written as

\[
R_z^2 = \frac{Z}{Z} \langle r^2_z \rangle _1 + \frac{Z}{Z} \langle r^2_z \rangle _2 + \frac{(Z/m_1^2 + Z/m_2^2)}{Zm^2} I_2 ,
\]

where \( \langle r^2_z \rangle _1,2 \) are squares of charge radii of clusters, for which again the radii of the corresponding nuclei in a free state are taken, \( Z = Z_1 + Z_2 \), and \( I_2 \) is the integral given above.

The wave function \( \chi _L (R) \) for the relative motion of clusters is a solution to the Schrödinger radial equation of the form

\[
\chi _L (R) + \left[ k^2 - V (R) - V _{\text{coul}} (R) - L (L + 1)/R^2 \right] \times \chi _L (R) = 0 .
\]

where \( V (R) \) is the intercluster nuclear potential with the dimension of \( \text{fm}^{-2} \), \( V _{\text{coul}} (R) \) is the Coulomb potential, and \( k \) is the wave number determined by the particle interaction energy \( E \),

\[
k^2 = \frac{2 \mu E}{\hbar^2} .
\]

**Numerical Calculation Methods**

Finite-difference methods (FDMs), which are a modification of methods [28] and include Coulomb interactions for charged particles, VM for solving the Schrödinger equation, and other calculation methods used to calculate nuclear characteristics, are described at length in [24]. Therefore, we only briefly touch upon the main points of general and numerical calculation methods.

In all calculations by the finite-difference and variational methods [24], in view of the asymptotic constant already found from the matching, the numerical or variational wave function was replaced with the Whittaker function [29] at the end of the asymptotic constant stabilization region, i.e., at \( \sim 10 \) to 20 fm. Numerical integration in any matrix elements was performed in the interval from 0 to 25–30 fm using the Simpson method [29], which yields good accuracies and counting rates for smooth and weakly oscillating functions, i.e., for low energies, when a few hundred steps are set for a period [24].

To perform these calculations, our computer codes based on the finite-difference method [24] for calculation of total radiative capture cross sections and characteristics of nuclear bound states were modified and rewritten in a more recent version of the FORTRAN–90 language with its wider facilities [2, 3, 6–8]. This increased the accuracy of all calculations, including calculations of the binding energy of a nucleus in the two-body channel. For example, the Wronskian-controlled accuracy of calculations of Coulomb wave functions for scattering processes and the accuracy of the search for the binding energy are at a level of \( 10^{-14} \) to \( 10^{-20} \). The real absolute accuracy of determining the binding energy by the finite-difference method for various two-body systems was \( 10^{-8} \) to \( 10^{-8} \) MeV. The Coulomb scattering wave functions themselves were calculated using the rapidly converging representation in the form of chain fractions [30], which allowed their values to be obtained with a high accuracy and in a wide range of variables within a shorter computation time [31].

The variational code [24] for finding variational WFs and nuclear binding energies in cluster channels was also rewritten in FORTRAN–90 and slightly modified, which appreciably increased the rate of searching for the minimum of the multiparameter functional that governs the binding energy of two-body systems in all nuclei under consideration [6]. This code still uses the multiparameter variational method with the expansion of the WF in the nonorthogonal variational Gaussian basis with independently varied parameters. Codes based on the multiparameter variational method and intended for performing phase shift analysis of differential cross sections and excitation functions for elastic scattering of nuclear particles were also modified [7].

In all calculations, if not stated otherwise, exact values for particle masses have been specified [21] and the constant \( \hbar^2/m_0 \) was taken to be \( 41.4686 \text{ MeV} \text{ fm}^2 \). When reactions with charged particles were considered, we represented the Coulomb parameter \( \eta = \mu Z/\eta \text{ fm} \) as \( \eta = 3.44476 \times 10^{-2} Z Z_{1/2} /q \), where \( q \) is the wave number in \( \text{fm}^{-1} \), which is determined by the energy of the interacting particles in the entrance channel. The Coulomb potential for \( R_{\text{coul}} = 0 \) was taken in the form \( V _{\text{coul}} (\text{MeV}) = 1.439975 Z Z_{1/2} /R \), where \( R \) is the relative distance between entrance channel particles in fm.

**Classification of Cluster States**

In scattering some of the lightest atomic nuclei, the states with a minimum spin turn out to be mixed by orbital Young diagrams; for example, the doublet state \( \text{p}^2 \text{H} \) [9, 12, 12] is mixed by the diagrams \{3\} and \{21\}. At the same time, these states in the discrete spectrum, e.g., the doublet \( \text{p}^2 \text{H} \) or \( \text{n}^2 \text{H} \) channel of the \( 3\text{He} \) or \( \text{H} \) nuclei, are pure, with the Young diagram \{3\} [9, 10]. Below we give a brief classification of states by orbital and spin–isospin Young diagrams using the \( \text{N}^2\text{H} \) sys-
tem as an example, and show how these results are obtained.

In the general case, possible orbital Young diagrams \(|f_l\) of a nucleus \(A|f\) consisting of two components \(A \zeta ((f_l) + A \zeta (f'_l))\) are a direct exterior product of the orbital Young diagrams of these components \(|f_l\rangle L = |f'_l\rangle L \times |f'_\rangle L\) and are determined using the Littlewood theorem [9, 10, 12]. Therefore, when the diagram (2) is used for the \(^4\text{He}\) nucleus, the possible orbital Young diagrams of the \(^2\text{H}\) system are symmetries \(|3\rangle_f\) and \(|\overline{2}\rangle_f\).

Spin–isospin diagrams are a direct interior product of the spin and isospin Young diagrams of a nucleus of \(A\) nucleons \(|f\rangle_{ST} = |f\rangle_L \otimes |f\rangle_{ST}\). Diagrams for systems of no more than eight particles are given in [32]. For the simplest \(^2\text{H}\) cluster system with the isospin \(T = 1/2\) we have \(|\overline{2}\rangle_f\), for the spin state with \(S = 1/2\) we have \(|3\rangle_f\), and at \(S = 3/2\) the diagrams are \(|3\rangle_f\) and \(|\overline{3}\rangle_f\). Upon constructing the spin–isospin Young diagram for the quartet spin state of the \(^2\text{H}\) system with \(T = 1/2\), we obtain \(|3\rangle_{fST} \otimes |\overline{2}\rangle_f\) and for the doublet spin state with \(T = 1/2\) we obtain \(|3\rangle_{fST} \otimes |\overline{2}\rangle_f\) = \(|111\rangle_f + |21\rangle_{fST} + |3\rangle_{fST}\) [32].

The total Young diagram of a nucleus is determined in a similar manner as a direct interior product of the orbital and spin–isospin diagrams \(|f\rangle_L \otimes |f\rangle_{ST}\). The full wave function of a direct system does not identically vanish under antisymmetrization unless it has an antisymmetric component \(|\overline{1}\rangle\), which occurs when conjugate \(|f\rangle_L\) and \(|f\rangle_{ST}\) are multiplied. Therefore, the diagrams \(|f\rangle_L\) conjugate to \(|f\rangle_{ST}\) are allowed in a given channel while all other orbital symmetries are forbidden because they lead to the zero full wave function of a system of particles after its antisymmetrization. It is thus obvious that only the orbital wave function with symmetry \(|\overline{2}\rangle_f\) is allowed for the \(^2\text{H}\) system in the quartet channel while the function with \(|3\rangle_f\) is forbidden because the product \(|21\rangle_{fST} \otimes |3\rangle_f\) does not lead to the antisymmetric component of the wave function. At the same time, in the doublet channel we have \(|111\rangle_{fST} \otimes |\overline{2}\rangle_f\) = \(|111\rangle_f\) and \(|21\rangle_{fST} \otimes |\overline{2}\rangle_f\) = \(|\overline{2}\rangle_f\) [32], and in both cases we obtain an antisymmetric diagram. It follows that the doublet state turn out to be mixed by the orbital Young diagrams with \(|3\rangle_f\) and \(|\overline{2}\rangle_f\).

In [9, 10] a method for separating these states by Young diagrams was proposed and it was shown that the mixed scattering phase shift could be represented as a half-sum of pure phase shifts with \(|f_l\rangle\) and \(|f'_l\rangle\)

\[ \delta^{(f_l) + (f'_l)} = 1/2(\delta^{(f_l)} + \delta^{(f'_l)}). \]

Here it is assumed that \(|f_l\rangle\) = \(|21\rangle_f\), \(|f'_l\rangle\) = \(|3\rangle_f\), and the doublet phase shifts extracted from experiment are mixed by these two diagrams. Next, it is assumed that the quartet scattering phase shift, pure in terms of the orbital Young diagram \(|21\rangle_f\), can be identified with the pure doublet \(^2\text{H}\) scattering phase shift corresponding to the same Young diagram. Then we can also find the doublet \(^2\text{H}\) phase with the diagram \(|3\rangle_f\) and use it to construct the interaction potential, pure in terms of Young diagrams, which can be further used for describing bound state characteristics [12, 15, 24]. The similar applies to the \(^3\text{H}\) and \(^3\text{He}\) systems [33].

**Phase Shift Analysis Methods**

Knowing experimental differential cross sections for elastic scattering and expressions describing them [23], we can always find a set of parameters, referred to as scattering phase shifts \(\delta_{S,L}\), which allows the behavior of these cross sections to be described with a certain accuracy. The quality of description of experimental data on the basis of a theoretical function (functional of several variables) can be estimated by the \(\chi^2\) method in the form [23]

\[ \chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left[ \frac{\sigma_i(\theta) - \sigma_i^e(\theta)}{\Delta \sigma_i^e(\theta)} \right]^2 = \frac{1}{N} \sum_{i=1}^{N} \chi_{i,ST}^2, \]

where \(\sigma\) and \(\sigma^e\) are the experimental and theoretical (i.e., calculated for some given scattering phase shifts \(\delta_{S,L}^e\)) cross sections for elastic scattering of nuclear particles at the \(i\)th scattering angle, \(\Delta \sigma^e\) is the experimental cross section error for this angle, and \(N\) is the number of measurements.

Expressions that describe differential cross sections are expansions of a functional \(d \sigma(\theta)/d \Omega\) in a numerical series [23], and it is necessary to find variational expansion parameters \(\delta_{S,L}^e\) that are best to describe its behavior. Since expressions for differential cross sections are usually exact [23], \(\chi^2\) should tend to a certain minimum as the number of expansion terms \(L\) increases to infinity. It is this criterion that we used for choosing the set of phase shifts leading to the minimum \(\chi^2\) that could be the global minimum of the given multiparameter variational problem [34].

We used the above-described criteria and methods for performing phase shift analysis in the \(^1\text{H}\) \(^12\text{C}\) and \(^1\text{H}\) \(^16\text{O}\) systems at low energies which are important for astrophysical applications. Expressions for differential elastic scattering cross sections that are needed for performing the phase shift analysis in the above-mentioned systems are given in [7, 23].

**Criteria for Construction of Intercluster Potentials**

Let us discuss in more detail the procedure of constructing intercluster partial potentials to be used here for a given orbital momentum \(L\). To this end we determine the criteria and sequence of finding the parameters, and indicate their error and ambiguities. The first to be found are the BS potential parameters which are uniquely fixed in binding energy, nuclear radius, and asymptotic constant in the channel under consideration at a given number of allowed and forbidden states in a given partial wave. The accuracy to which the BS
potential parameters are determined first of all on the AC accuracy, which is usually 10 to 20% because the charge radius determination accuracy is normally much higher, 3 to 5%. There are no other ambiguities in this potential because classification of states by Young diagrams allows fixing the number of BSs, forbidden or allowed in the given partial wave, which totally determines its depth while its width entirely depends on the AC value.

The intercluster potential of the nonresonance scattering is also constructed quite unambiguously using scattering phase shifts at a given number of BSs, allowed and forbidden in the partial wave under consideration. The accuracy to which parameters of this potential are determined depends first of all on the accuracy with which scattering phase shifts are extracted from the experimental data and can sometimes be as high as 20 to 30%. There are no ambiguities in this potential either, because classification by Young diagrams allows for uniquely fixing the number of BSs which totally determines its depth, while its width at a given depth is determined by the scattering phase shape.

When the nonresonance scattering potential is constructed using the data on the spectra of a nucleus in a particular channel, it is difficult to estimate the accuracy of finding its parameters even at the given number of BSs, though, hopefully, it might probably be only slightly higher than the error in the previous case. As is usually assumed for energies below 1 MeV, this potential should lead to a scattering phase shift close to zero or give a smoothly decaying phase shape because there are no resonance levels in the nuclear spectra.

In the analysis of the resonance scattering, when the partial wave at an energy no higher than 1 MeV involves a relatively low resonance with a width on the order of 10 to 50 keV, the potential is also constructed completely unambiguously at a given number of BSs. Its depth at the given number of BSs is uniquely fixed in the resonance energy of the level and its width is totally determined by the width of this resonance. The error of its parameters is usually no greater than the error in determination of the level width and amounts to 3 to 5%. This also applies to the construction of a partial potential on the basis of the scattering phase shifts and determination of its parameters on the basis of the resonance in the nuclear spectra.

As a result, none of the potentials contains ambiguities and they all permit the total cross sections for radiative capture processes to be correctly described without recourse to the spectroscopic factor $S_F$. In other words, no additional factor $S_F$ is required for considering the capture reaction within the PCM for potentials correlated in the continuous spectrum with the scattering characteristics, which allow for the resonance shape of the phase shifts, and discrete spectrum characteristics, which describe the main properties of the nuclear BSs [35].

All the effects that are present in the reaction, including the cluster configuration probability, appear to be allowed for in the construction of interaction potentials. This becomes possible because the potentials are constructed with allowance for the FS structure on the basis of the description of the observables, i.e., of the experimental characteristics of the interacting clusters in the initial state and a nuclear formed in the final state, when it is described by the cluster structure consisting of initial particles. Thus, $S_F$ is allowed for in the BS WF for the clusters defined on the basis of those potentials in the solution of the Shrödinger equation.

RADIATIVE $n^2H$ CAPTURE WITHIN THE CLUSTER MODEL

Earlier, the radiative neutron capture in a deuteron was considered [36] within the effective field theory. It was shown that it was the $M1$ transition that made the main contribution in the energy region of 40 to 140 keV, and rather good agreement could be obtained between the calculated total cross sections and their extrapolations from the database [37].

Here we will consider the possibility of describing the experimental data on the total cross sections for the radiative $n^2H$ capture at thermal ($\leq 1$ eV), astrophysical ($\leq 1$ keV), and low ($\leq 1$ MeV) energies within the potential cluster model with forbidden states and their classification by orbital Young diagrams (energy regions are very approximate). We will show that the PCM is capable of correctly describing the behavior of the experimental cross sections at energies from 10 MeV ($10 \times 10^{-3}$ eV) to 15 MeV.

The $n^2H$ capture is a mirror reaction relative to the $p^2H \rightarrow \gamma^3He$, which was considered earlier in [2, 3, 6, 38]. It is part of the thermonuclear proton–proton cycle, being its first reaction that proceeds due to electromagnetic interactions. This cycle seems to make the dominant contribution to the energy yield of nuclear reactions [39] that ensure burning of the Sun and stars in our Universe. The reaction of radiative neutron capture by a deuterium nucleus to be considered below is one of the primordial nucleosynthesis processes at the initial stage of the formation and development of our Universe.

Potential Description of the Elastic $n^2H$ Scattering

Before we begin considering the $n^2H$ system, we briefly review the results earlier obtained for the $p^2H$ scattering. Potentials of the elastic $p^2H$ scattering for each partial wave were constructed such as to describe correctly the corresponding partial phase shifts of low-energy elastic scattering [40] which turn out to be mixed by the Young diagrams $\{3\} + \{21\}$ in the doublet channel [2, 3, 6–10]. Using these representations, we
obtained the $p^2\text{H}$ interaction potentials mixed by the diagrams (3) + (21) for the scattering processes written as

$$V(r) = V_0 \exp(-\gamma r^2) + V_1 \exp(-\delta r),$$

with the parameters presented in Table 1 [18, 41].

Then pure phase shifts with the diagram (3) were separated in the spin channel and used to construct intercluster $^2S$ interaction potentials, pure in terms of the Young diagrams, for the ground state of the $^3\text{He}$ nucleus in the $p^2\text{H}$ channel. Their parameters are presented in the third row of Table 1 and in [2, 3, 8, 18, 24, 38, 41], and they allow a relatively good description of the main characteristics of the $^3\text{He}$ nucleus in the $p^2\text{H}$ channel [41].

These potentials were used to calculate the total cross sections for the radiative $p^2\text{H}$ capture and the astrophysical $S$-factors at energies no higher than 10 keV [41], though by that time we had known only the experimental data on the $S$-factor at energies above 150–200 keV [42]. Later, new experimental data for energies no higher than 2.5 keV appeared [43, 44, 45]. Their analysis showed that the earlier calculations based on the $E1$ process alone completely agree with them [41] at energies no higher than 10 keV. Thus the potential cluster model allowed not only the new data to be described but also the behavior of the astrophysical $S$-factor for the $p^2\text{H}$ capture at energies below 10 keV to be predicted. Our calculations [41] were performed in 1995, that is, before the new experimental measurements [45] carried out in 2002 and even before the results obtained in the earlier works [43, 44], which were published in 1997.

Here we will consider the radiative $n^2\text{H}$ capture at low energies using the $p^2\text{H}$ potentials obtained in [2, 8, 15, 18, 41, 46] and the same calculations methods that were tested with the $p^2\text{H}$ system [6]. The parameters of the GS potential of the $^3\text{He}$ nucleus in the $n^2\text{H}$ channel without the Coulomb interaction were slightly refined to correctly describe the tritium binding energy, which was 6.257233 MeV [47, 48]. As a result, for the parameters of the potential at $V_1 = 0$ we obtained

$$V_0 = -41.4261655 \text{ MeV and } \gamma = 0.2 \text{ fm}^{-2}. \quad (2)$$

This potential accurately reproduces the $^3\text{He}$ binding energy, giving the value $-6.257233 \text{ MeV}.$ It also gives the charge and mass radii 2.33 and 2.24 fm at the zero charge radius of the neutron, its mass radius equal to the proton radius 0.8775(51) fm, and at the deuteron radius 2.1424(21) fm [49]. The asymptotic constant determined according to [25] turned out to be 2.04(1) on the interval of 5 to 15 fm. The AC error is determined by its averaging over the given interval of distances; its values obtained in different works are presented in [25] and fall within the range of 1.82 to 2.21.

Note that the above binding energy was obtained at the accuracy $10^{-6}$ of the finite-difference method used for the energy calculation, and the higher accuracy $2 \times 10^{-9}$ allows obtaining a more accurate value $-6.257233014 \text{ MeV}.$ In addition, since the deuteron has a larger radius than that of tritium 1.755(86) fm [47], the former cannot be inside the latter in the free, i.e., undeformed, form, and the degree of its deformation, as was shown in [50], is about 30% [15]. A similar result was obtained in [51], where it was shown that the WF of the deuteron inside the triton decayed much faster than for its free state. Thus, the presence of the third particle, a neutron in this case, leads to deformation, i.e., to compression of the deuteron cluster inside the tritium nucleus. Calculations by the resonating group method (RGM) reviewed in [52] lead to approximately the same result, and the deuteron deformation is normally estimated at 20 to 40%.

The correctness of the binding energy calculations for the $^3\text{He}$ nucleus in this potential, i.e., for the bound $n^2\text{H}$ state with interaction (2), was additionally checked using the two-body variational method with the expansion of the WF in the nonorthogonal Gaussian basis and with all the parameters independently varied [6, 24]. With the Gaussian basis dimension $N = 10$ and independent variation of all parameters, the variational method yields the energy $-6.2573329999 \text{ MeV} \approx -6.257233000 \text{ MeV}.$ The asymptotic constant $C_{\infty}$ of the variational WF with the parameters as listed in Table 2 remains about the same, namely 2.05(2) at the distances of 6 to 20 fm, which does not differ from the value within the FDM, and the residuals are no larger than $10^{-11}$ [24].

It is known that the variational energy decreases with increasing dimension of the basis and gives the upper limit for the true binding energy. At the same time, the finite-difference energy rises with decreasing step and increasing number of steps [2, 3, 6, 24]. The correct binding energy in this potential can therefore be taken to be equal to the average obtained above by two methods and amounting to $-6.257233007(7) \text{ MeV}$ for

**Table 1. Double interaction potentials of the $p^2\text{H}$ system [15] ($E_{BS}$ is the bound ground state energy of the $^3\text{He}$ nucleus in the $p^2\text{H}$ channel, $E_{exp}$ is its experimental value)**

| $(^{2S+1}L \{j\})$ | $V_0$, MeV | $\gamma$, fm$^{-2}$ | $V_1$, MeV | $\delta$, fm$^{-1}$ | $E_{BS}$, MeV | $E_{exp}$, MeV |
|---------------------|-------------|-----------------|-------------|-----------------|---------------|---------------|
| $^2S \{3\} + \{21\}$ | $-55.0$     | $0.2$           | $-$         | $-$             | $-$           | $-$           |
| $^2P \{3\} + \{21\}$ | $-10.0$     | $0.16$          | $+0.6$      | $0.1$           | $-$           | $-$           |
| $^2S \{3\}$         | $-41.55562462$ | $0.2$          | $-$         | $-$             | $-5.493423$   | $-5.493423$   |
the \( n^2H \) system. Thus, the real accuracy of determining the two-body binding energy of this system in the above BS potential (2) using two different methods (VM and FDM) and two different computer codes [6, 24] is at a level of ±0.007 eV or ±7 meV.

**Total Cross Sections for the Radiative \( n^2H \) Capture**

Let us first demonstrate the effectiveness of the potential cluster model under consideration, potentials obtained from the \( p^2H \) phase shifts of the elastic scattering, and procedure of separating the pure phase and the corresponding potential of the \( ^3H \) nucleus GS (2) using its photodisintegration to the \( n^2H \) channel as an example. We considered it earlier in [41] in a wider energy interval but in less detail. In Fig. 1 the solid curve shows the results of this calculation at γ-ray energies from 6.3 to 10.5 MeV for the sum of the \( E2 \) and \( M1 \) cross sections with the above \( p^2H \) scattering potentials (see Table 1), switched-off Coulomb interaction, and GS potential (2).

The dashed curve in Fig. 1 shows the contribution from the \( M1 \) process for the breakup of the \( ^3H \) BS \( ^2S \) into the doublet \( ^2S \) wave of the \( n^2H \) scattering (Table 1), which does not noticeably contribute to the total reaction cross sections at these energies. The cross sections for the process in question turn out to be exclusively governed by the \( E1 \) transition at the \( ^3H \) GS breakup into the doublet \( ^2P \) scattering wave (Table 1). The experimental data on the total cross sections for the photodisintegration of the \(^3H \) nucleus to the \( n^2H \) channel at the energies considered are taken from [53, 54].

Here, unlike the case in [55], we will consider the results for the \( n^2H \) capture, when the negative sign of the neutron magnetic moment is taken into account. Using the parameters of the nuclear \( p^2H \) potentials for the \( ^2S \) and \( ^2P \) scattering waves from Table 1 without the Coulomb term and GS (2), we calculated total cross sections for the radiative \( n^2H \) capture in the energy range from 10 meV to 15 MeV. The results of the calculations are shown in Fig. 2 by the dot-dashed curve. It turned out that the calculated cross sections at 10 meV are slightly greater than the experimentally measured ones [58], being rather in agreement with the data [59] at 25 meV. All of the experimental data on the total cross sections for the radiative capture in the \( n^2H \) channel presented in Fig 2 are taken from [56] (points at 30, 55, and 530 keV), [57] (circles at 7 to 14 MeV) [58] (triangle at 0.01 eV), [59] (star at 0.025 eV), and [60] (square at 50 keV).

![Fig. 1. Total cross sections for the photodisintegration of the \(^3H \) nucleus to the \( n^2H \) channel. The experimental data are from [54] (△) and [53] (●). The curves are explained in the text.](image1)

![Fig. 2. Total cross sections for the \( n^2H \) radiative capture. Experimental data are from [56] (○), [57] (○), [58] (△), [59] (*), and [60] (■); (▿) and (∇) are the data from [53] and [54] after transforming them into the capture. The curves are explained in the text.](image2)
To calculate the $M1$ transition, we used the earlier obtained $p^2H$ potential for the $^2S$ scattering wave from Table 1 without the Coulomb interaction. Note that the spread of the results for phase shift extraction from the experimental data on the elastic $p^2H$ scattering [40] shown by points in Fig. 3 is as high as 10 to 20%. Therefore, even the $p^2H$ scattering potential, whose phase shift is shown in Fig. 3 by the dotted curve, is constructed on their basis with large ambiguities, and we consider the $n^2H$ system for which no phase shift analysis results in the astrophysical energy range were found at all. Further, we consider the changes that are required for the $n^2H$ potential in the $^2S$ scattering wave to allow the description of the experimental data [58]. The resulting total cross sections for the radiative capture are depicted in Fig. 2 by the solid curve.

The depth of the $^2S$ potential in the elastic $n^2H$ scattering turned out to be not much smaller than for the $p^2H$ system from Table 1

\[ V_0 = -52.0 \text{ MeV} \text{ and } \gamma = 0.2 \text{ fm}^{-2}. \]

The elastic scattering phase obtained for this potential is shown by the solid curve in Fig. 3. It is seen that the $^2S$ phase of the changed $n^2H$ potential decays at low energies much faster than the similar phase for the $p^2H$ potential from Table 1. In turn, this affects the results of calculations of totals cross sections for the $M1$ process and, as is evident from Fig. 1, this potential is quite good to describe the available data on the total cross sections at the lowest energies.

The results shown in Fig 2 demonstrate domination of the $M1$ process at energies below 1 keV. Its cross section is shown by the dashed curve, and the dotted curve in Fig. 2 is the contribution from the $E1$ transition. As is evident from Fig. 2, the $E1$ transition cross section sharply decreases and can be neglected at 0.1 keV. At the same time, above 10 keV this process dominates and entirely governs the behavior of the total cross sections, which allows a reasonable description of the available experimental data at energies from 50–100 keV up to 15 MeV.

Thus, varying the parameter of the $n^2H$ potential in the $^2S$ scattering wave by ~5%, we can reasonably describe the available experimental data at low energies. This variation in the parameters can well be due to the uncertainty of the $p^2H$ phase shifts and their absence for the elastic $n^2H$ scattering. As a result, the potential cluster model in question can be believed to correctly reproduce the experimental data on the total cross sections for the radiative $n^2H$ capture in the energy range with a difference of more than nine orders of magnitude between the energies at its ends, namely, in the range from $10^{-3}$ keV to $1.5 \times 10^4$ keV.

Thus, the calculated total cross sections for the radiative $n^2H$ capture at energies from 10 meV to 15 MeV generally agree well with the known experimental data. The potential cluster model with forbidden states and classification of orbital states of clusters by Young diagrams turned out to be able not only to describe the astrophysical $S$-factor of the $p^2H$ capture [2], but also to correctly reproduce the general behavior of the total cross sections for the $n^2H$ capture in a wide energy region. Small variations in the depth of the $^2S$ potential for this system allow an acceptable description of various experimentally measured total cross sections. These variations in parameters are quite tolerable because the $p^2H$ phase shift analysis data contain rather large ambiguities and the $n^2H$ phase shift analysis data are lacking. This all leads to an uncertainty of the parameters of the $n^2H$ interaction potentials, which, as shown above, is about 5%.

At energies from $10^{-5}$ to 0.1 keV, the calculated cross section shown by the solid curve in Fig. 2 is almost a straight line and can thus be approximated by a simple function of energy of the form

\[ \sigma_{ap}(\mu b) = \frac{1.2314}{\sqrt{E_n(\text{keV})}}. \] (3)

The reduced constant value 1.2314 $\mu$b keV$^{1/2}$ was found using one point in the total cross sections at the minimum energy of $10^{-5}$ keV. Next, we can consider the modulus of the relative deviation of the calculated theoretical cross section and its approximation by this function in the region from $10^{-5}$ to 0.1 keV

\[ M(E) = \left| \frac{[\sigma_{ap}(E) - \sigma_{\text{theor}}(E)]}{\sigma_{\text{theor}}(E)} \right|. \] (4)

It turns out that this deviation is no larger than 1.0% at energies below 100 eV. We can assume that the energy dependence of the total cross section (3) will retain its shape at lower energies as well. In this case, estimation of the cross section, for example, at the energy of 1 $\mu$eV ($10^{-6}$ eV = $10^{-9}$ keV) yields 38.9 mb.
Then the interaction potentials for the refined in the energy range from 500 to 1150 keV [63]. Later, on the basis of the new data [62], these formed in the energy range from 0.5 to 5.6 MeV in 

eration of the spin-orbit splitting was initially per-

RADIATIVE n^6Li CAPTURE

Now let us consider the possibility of describing the experimental total cross sections for the radiative n^6Li capture in the energy region from 25 meV (25 × 10^-3 eV) to 1.0–2.0 MeV within the potential cluster model with forbidden states and classification of orbital cluster states by the Young diagrams. Note that the phase shift analysis of the elastic n^6Li scattering with consideration of the spin-orbit splitting was initially performed in the energy range from 0.5 to 5.6 MeV in [61]. Later, on the basis of the new data [62], these results for the S scattering phase shifts were slightly refined in the energy range from 500 to 1150 keV [63]. Then the interaction potentials for the 2P_{3/2} ground state and 2P_{1/2} excited state of the 7Be nucleus in the p^6Li channel and the potential of the doublet 2S_{1/2} scattering wave were obtained. Those results allowed the astrophysical S-factor of the low-energy radiative p^6Li capture to be considered [2, 3, 64]. Finally, it was shown that the above approach was quite good to describe the available experimental data on the radiative p^6Li capture in a wide energy region [65, 66].

Developing these results, we consider total cross sections for the radiative n^6Li capture in the astrophysical energy region. Though this reaction is of interest for nuclear astrophysics from the standpoint of studying processes that result in formation and accumulation of lithium isotopes [17], it has been rather poorly investigated in experiments. This conclusion follows from the consideration of the experimental results stored in the databases of the Moscow State University site ([67] or EXFOR [68]). In these databases there are only data measured at 0.025 eV (25 meV) [69–72] and at three keV-range energies [73]. In addition, in [74–76] there are data on the total cross sections for the photodisintegration of the 7Li nucleus GS to the n^6Li channel, which were converted here to the capture cross sections in the region of 0.05 to 1.5 MeV. Since all these data fairly well define the general behavior of the total capture cross sections, it seems interesting to consider the possibility of their theoretical description in the energy region from 0.025 eV to 1–1.5 MeV using, as in the case of the p^6Li system [63, 64], the potential cluster model with FSs and the classification of cluster states by orbital Young diagrams [77].

Potential Description of the n^6Li Scattering

Earlier, it was pointed out [2, 6, 15, 18, 77] that two variants of potentials can exist for the 2S and 2P waves in the n^6Li system. In the former case, there are two BSs in each of these partial waves and only one of those in the 2P waves is an allowed one, corresponding to the BSs of A = 7 nuclei, while all the others are forbidden. In the latter case, each of the waves contains only one BS: in the 2S wave it is forbidden and in the 2P wave it corresponds to the allowed 2P_{3/2} and 2P_{1/2} BSs. We therefore consider both variants of the potentials for the 2S scattering states and the BS 2P levels of the 7Li nucleus in the n^6Li channel. Yet only those potentials which can provide a reasonable description of the total cross sections for the radiative n^6Li capture, elastic scattering phase shifts, and main characteristics of BSs will be considered.

Note first that, as shown in [63, 64], it is preferable to describe our results on the elastic p^6Li scattering phase shifts using the doublet 2S_{1/2} potential of the Gaussian form (at V_I = 0) with the parameters

\[ V_S = -124.0 \text{ MeV}, \quad \gamma_S = 0.15 \text{ fm}^{-2}, \quad (5) \]

which contains two forbidden bound states corresponding to the orbital Young schemes {52} and {7} [2, 8, 64]. In Fig. 4 the solid and dashed curves show respectively the 2S phase shifts of the elastic n^6Li and p^6Li scattering at low energies calculated with the above potential. Its only difference was the Coulomb interaction. The points in Fig. 4 are the results of our phase shift analysis [63] used as the p^6Li scattering phase shifts extracted from the experiment.

Phase shifts of the other shallower potential with only one FS and the parameters

\[ V_0 = -34.0 \text{ MeV}, \quad \alpha = 0.15 \text{ fm}^{-2} \quad (6) \]

are shown in Fig. 4 by the dotted curve for the p^6Li scattering and the dot-dashed curve for the n^6Li scattering. It is evident from Fig. 4 that both potentials lead to an identical description of the p^6Li phase shifts and only a slight difference in the n^6Li phase shifts at low energies. It is impossible to find a 2S scattering potential without FSs that could describe the doublet scattering phase shifts depicted in Fig. 4 because description of phase shifts and further considered radiative capture processes requires that a bound FS with the diagram {52} be present in this partial wave.
The $^2P_{3/2}$ wave potential of the $^7$Be ground state, pure in terms of the orbital symmetries with the Young scheme $[43]$ $[64]$, was constructed such as to describe the channel binding energy of the nuclear ground state as the $^p$Li system and its root-mean-square radius. Here we slightly change its depth so that it correctly reproduces the binding energy of the $^7$Li nucleus in the $^6$Li channel. In this case the pure $^2P^{[43]}$ potential $[78]$ of the $n^6$Li interaction for the $^7$Li ground state with $J^P = 3/2^-$ can be assigned the parameters

$$V_{g.s.} = -250.968085 \text{ MeV, } \gamma_{g.s.} = 0.25 \text{ fm}^{-2}. \quad (7)$$

The potential leads to the binding energy $-7.249900$ MeV, while the experimental value is $-7.2499$ MeV $[79]$, and has another forbidden state corresponding to the Young scheme $[61] [2]$. The root-mean-square charge radius proved to be 2.55 fm, and the mass radius is 2.58 fm, which generally agrees with the experimental value 2.39(3) fm $[79]$. These calculations were performed with the neutron charge radius equal to zero and mass radius equal to the proton radius and the $^6$Li radius, which is slightly larger than the $^7$Li radius and is $2.51(10)$ fm $[79, 80]$.

The asymptotic constant with potential (7) was found to be 2.45(1) on the interval of 5 to 15 fm. According to $[81]$, where various experimental data and theoretical results are collected, the AC can be $1.76(14)$ fm$^{-1/2}$. Nondimensionalizing it with $\sqrt{2k} = 1.05$, we obtain $1.68(13)$. The value given in $[82]$ is $1.890(13)$ fm$^{-1/2}$ or, in the dimensionless form, $1.800(12)$. This nondimensionalization is needed because the definition of AC used in $[81, 82]$ differs from the one used here $[25]$ by the factor $\sqrt{2k}$

$$\chi_L(r) = C \chi W_{-nL+1/2}(2k0r).$$

The parameters found for the potential of the first excited $^7$Li state with $J^P = 1/2^-$ are

$$V_{exc} = -248.935336 \text{ MeV, } \gamma_{exc} = 0.25 \text{ fm}^{-2}.$$  

This potential yields the binding energy of $-6.772300$ MeV while the experimental value is $-6.7723$ MeV $[79]$, the charge radius does not change relative to the previous results, and the AC on the interval of 5 to 15 fm is $2.33(1)$. The potential also has a forbidden bound state with the Young scheme $[61]$. Another variant of the pure $n^6$Li interaction $^2P_{3/2}$ potential of the $^7$Li GS can be represented as

$$V_{g.s.} = -75.190114 \text{ MeV, } \alpha_{g.s.} = 0.175 \text{ fm}^{-2}. \quad (8)$$

It leads to the binding energy $-7.249900$ MeV and has only one bound, allowed, state corresponding to the Young diagram $[43]$. The root-mean-square charge and mass radii are both 2.54 fm, and the AC is $2.03(1)$ on the interval of 5 to 16 fm, which is only 10 to 15% different from the results in $[81, 82]$.

The correctness of the binding energy calculation for the $^7$Li nucleus in GS potential (7) was additionally checked using the two-body variational method with independently varied parameters and expansion of the wave function in the nonorthogonal Gaussian basis $[24]$. With the basis dimension $N = 10$ and parameters independently varied, this method yielded the binding energy $-7.249898$ MeV. The asymptotic constant $C_W$ of the variational WF with the parameters presented in Table 3 retains the value 2.45(5) over distances of 5 to 15 fm, and the residuals are no larger than $10^{-11} [24]$. As was already said, the variational energy decreases with increasing basis dimension and gives the upper limit for the true binding energy, and the finite-difference energy rises with decreasing step and increasing number of steps. Consequently, an average value of $-7.249899(1)$ MeV can be taken for the correct binding energy of the $n^6$Li system in this potential. Thus, the accuracy of determining the binding energy of this two-cluster system in the above $^7$Li GS potential for the $n^6$Li channel, which is obtained within two different numerical methods (VM and FDM) using two different computer codes $[24]$ rewritten in FORTAN—90 $[6]$ is at a level of $\pm 1$ eV.

Completely identical results for the variational energy $-7.249900$ MeV, residuals no larger than $10^{-10}$, ensuing determination accuracy $<0.5$ eV and other characteristics of the $^7$Li GS in the $n^6$Li channel were obtained for potential (8). Parameters of its WF are presented in Table 4.

**Table 3. Variational parameters $\alpha_i$ and expansion coefficients $C_i$ for the WF of the $n^6$Li system GS in the $^7$Li nucleus for potential (7) (the WF normalization at distances of 0 to 25 fm is $N = 0.999999999999947$)**

| $i$ | $\alpha_i$ | $C_i$ |
|-----|-------------|-------|
| 1   | 2.468292899352664E-002 | -8.443780272416886E-004 |
| 2   | 5.659824615487678E-002 | -1.494186015886072E-002 |
| 3   | 1.229406461038807E-001 | -9.2674920265470E-002 |
| 4   | 2.513715488575826E-001 | -3.217760480874366E-001 |
| 5   | 7.328392817240388E-001 | 1.463594686074960 |
| 6   | 1.394554324801138 | 8.74468213437008E-001 |
| 7   | 1.968191404804425 | -2.564925474852117 |
| 8   | 2.224827222346167 | 3.963681316635119 |
| 9   | 2.49348282552606 | -2.317285290938208 |
| 10  | 2.835387525435829 | 4.485636531606636E-001 |

**Total Cross Sections for the Radiative $n^6$Li Capture**

Total cross sections for the radiative $n^6$Li capture were considered earlier within the folding model $[83]$, which provided reasonable agreement with the experimental data from $[73]$ for the energy range of 20 to 60 keV. Later the data from $[73]$ were considered in $[84]$ using the distorted wave approximation and a good description was obtained as well. However, in both cases only...
the energy region of 20 to 60 keV was considered and no analysis was given to the behavior of the cross sections at the lowest energies [69–72].

Treating the total cross sections for the radiative $^6\text{Li}$ capture within the PCM with FSs, we considered $E1$ transitions from the nonresonant $^2S$ and $^2D$ scattering states to the $^2P_{3/2}$ ground state and the first excited $^2P_{1/2}$ bound state of the $^7\text{Li}$ nucleus in the $^6\text{Li}$ channel. The wave function of the $^2D$ wave without spin-orbit splitting was calculated using the $^2S$ potential at $L=2$, and the expressions for the capture cross sections involved exact coefficients for the $E1$ transitions from the $^2D_{3/2}$ and $^2D_{5/2}$ scattering waves [64]. Since the ground state and excited state WFs are only insignificantly different, only GS potential (7) or (8) was used in the calculations for the WFs of both levels. This appears to be justified because we consider only the general shape of the cross sections in the energy range where the lowest energy and the highest energy differ by eight orders of magnitude. We will not consider the capture process in detail as, for example, in [85], where the possibility of describing total cross sections for the $^7\text{Li}$ photodisintegration to the $^n\text{Li}$ channel at energies of 7.3 to 8.8 MeV was treated at length.

The photodisintegration data from [74–76] were converted to the capture cross sections. They are shown in Fig. 5 by circles, open squares, and filled squares in the region up to 2.0 MeV. Since we have disintegration cross sections only for the $^7\text{Li}$ GS, the summed capture cross section to the ground and the first excited state was determined using the principle of detailed equilibrium for identical breakup cross sections

$$\sigma_c(3/2 + 1/2) = \sigma_c(3/2) + \sigma_c(1/2)$$

$$= 4A(q, K)\sigma_d(3/2) + 2A(q, K)\sigma_d(3/2),$$

where

$$\sigma_c(J_0) = (2J_0 + 1)\frac{2K^2}{q^2(2S_1 + 1)(2S_2 + 1)} \times \sigma_d(J_0) = (2J_0 + 1)A(q, K)\sigma_d(J_0).$$

The calculated total cross sections for the radiative $^6\text{Li}$ at energies from $10^{-5}$ to 1.5 MeV with potentials (5) and (7) are shown in Fig. 5 by the solid curve, and the dotted curve is the calculation with a combination of potentials (6) and (8). The corresponding calculations of the $^7\text{Li}$ GS photodisintegration to the $^n\text{Li}$ channel with potentials (5) and (7) are shown in Fig. 6.

| $i$ | $\alpha_i$ | $C_i$ |
|-----|------------|-------|
| 1   | 2.653995234178599E-002 | -9.55286397144253E-004 |
| 2   | 5.916693410819475E-002 | -1.32563299645183E-002 |
| 3   | 1.214005238211452E-002 | -8.1779360550393E-002 |
| 4   | 2.31287269355081E-002 | -1.9944048758770E-001 |
| 5   | 4.97763546319589E-001 | -1.42018482450155 |
| 6   | 5.48511902327939E-001 | 2.46191182031208 |
| 7   | 6.17356387587666E-001 | -1.53394870050848 |
| 8   | 7.39520751404922E-001 | 3.54723213275329E-001 |
| 9   | 1.00354312785149E-001 | -3.45274589300219E-002 |
| 10  | 1.50918837055481E-001 | 2.27824730025897E-003 |

**Fig. 5.** Total cross sections for the $^6\text{Li}$ radiative capture at low energies. Symbols are experimental results taken from [69–72] (●) at 25 meV), [73] (△), [74] (●), [75] (○), and [76] (□). Curves are calculations of the total cross sections with the potentials discussed in the text.

**Fig. 6.** Total cross section for the $^7\text{Li}(γ, n)^6\text{Li}$ reaction at low energies. Symbols are experimental data from [74] (●), [75] (○), and [76] (□). Curves are calculations with the potentials discussed in the text.
by the solid curve, and the calculations with potentials (6) and (8) are shown by the dotted curve. It is evident from the figures that in both cases it is possible to obtain potentials which fairly correctly reproduce the energy behavior of the total capture and photodisintegration cross sections at energies from 25 meV to 1.5 MeV [69, 76]. These interactions correlate with the elastic scattering phase shifts and fairly correctly describe some of the main characteristics of the $^7\text{Li}$ GS in the $n^6\text{Li}$ channel. The variant potential without FSs (8) reproduces the AC more correctly.

If we use the $^7\text{Li}$ GS potential without FSs that describes the AC more accurately, e.g., the one with the parameters

$$V_{gs} = -83.161074 \text{ MeV}, \alpha_{gs} = 0.2 \text{ fm}^{-2}, \quad (9)$$

and the $^3S$ scattering potential with one FS (6), we obtain the result presented in Figs. 5 and 6 by the dashed curve. In this case we have the binding energy $-7.249900 \text{ MeV}$, AC $1.85(1)$ on the interval of 5 to 13 fm, charge radius 2.54 fm, and mass radius 2.53 fm.

The results of calculating the variational energy, which was found to be $-7.249899 \text{ MeV}$, that is, determined with an accuracy of ±0.5, and other characteristics of the $^7\text{Li}$ GS in the $n^6\text{Li}$ channel for potential (9), are similar to the results obtained above with the FDM. The residuals are on the order of $10^{-10}$. The WF parameters are presented in Table 5.

Calculations of total cross sections for this potential still agree well with the data [69–72] at 25 meV because of large experimental errors, but they are slightly lower than the data at the energies from 100 keV to 1 MeV. It is evident from Fig. 6 that they are rather between the data from [74] and [75], which are shown in the figures by filled squares and open circles respectively. However, if the parameters of the $^3S$ potential are taken to be

$$V_0 = -45.0 \text{ MeV}, \alpha = 0.25 \text{ fm}^{-2}, \quad (10)$$

the calculations of the total capture and disintegration cross sections yield the results shown by the dotted curve in Figs. 5 and 6. It almost does not differ from the dotted curve that shows the results for BS potential (8) and scattering potential (6) with one BS and describes well the available experimental data. Phase shifts of potential (10) for both $^6\text{Li}$ and $n^6\text{Li}$ scattering are shown in Fig. 4 by the double dotted curve. These results show that agreement can be obtained for the descriptions of the elastic scattering phase shifts (10) and the main characteristics of the $^7\text{Li}$ BSs in the $n^6\text{Li}$ channel, including radii and AC, for the GS potential without FSs (9).

Now we turn to the description of the resonant states. Note that the resonance at 7.45 MeV [79] with the momentum $5/2^-$, which is only 0.2 MeV (c.m.s.) above the $n^6\text{Li}$ channel threshold probably belongs to the $^4P_{3/2}$ scattering wave. The possibility of the $M1$ transition during the photodisintegration of the $^7\text{Li}$ nucleus to the $n^6\text{Li}$ channel has been recently considered with allowance for this state in [85]. Generally speaking, this level can also be due to the resonance in the $^2F_{5/2}$ scattering wave, though it seems doubtful that the $F$ wave can have a resonance like this at these low energies.

The resonance at 8.75 MeV relative to the GS, which is above the $n^6\text{Li}$ channel threshold by approximately 1.5 MeV (c.m.s.) and has the width 4.7 MeV and the angular momentum $3/2^-$, can be due to the $^2P_{3/2}$ or $^4P_{3/2}$ scattering waves [79]. However, we do not consider its effect on the total cross sections for radiative capture because there are no $n^6\text{Li}$ phase shift analysis results for resonant scattering waves in this energy range. It is hardly possible to construct a potential that would lead to this wide resonance for any $P$ phase using only the data on spectra, as we did for a number of other cluster systems [2, 8, 6, 86]. The above resonances can be ultimately associated with the particular partial waves only by a thorough phase shift analysis of the $n^6\text{Li}$ elastic scattering at energies no higher than 2.0 MeV, but we failed to find results of this analysis in the literature.

As a result, it was shown that, like in the case of other light nuclei [6, 18], the potential cluster model under discussion and the intercluster potentials mentioned above allow a quite reasonable description of the radiative $n^6\text{Li}$ capture in the astrophysical energy region. Calculations of total $n^6\text{Li}$ capture cross sections on the basis of $E1$ transitions alone are in good agreement at energies of 25 meV to 1.5 MeV with the known experimental data on the capture process and with the recalculated data from the measurements of total cross sections for the two-body disintegration of the $^7\text{Li}$ nucleus to the $n^6\text{Li}$ channel. The more preferable of the two variants of intercluster potentials seems to be the potential with one BS, i.e., with an FS only

| $i$ | $\alpha_i$ | $C_i$ |
|-----|-----------|------|
| 1   | 2.665347013743804E-002 | -8.871735330500928E-004 |
| 2   | 5.94089572884596E-002  | -1.221361696319499E-002 |
| 3   | 1.21927341814190E-001  | -6.28479952239499E-002 |
| 4   | 2.34061715144998E-001  | -1.968287096274776E-001 |
| 5   | 4.75123988504844E-001  | -8.57293184508050E-001 |
| 6   | 5.48511923279393E-001  | 1.55607451398506 |
| 7   | 6.17356385785766E-001  | -1.203431194740232 |
| 8   | 7.395207514049224E-001 | 2.934610010474853E-001 |
| 9   | 1.003543127851490      | -3.090692233217297E-002 |
| 10  | 1.509188370554815      | 2.059998226181524E-003 |
in the $^2S$ scattering wave and without any FSs in the bound $^2P$ levels.

Note that this conclusion differs from our earlier conclusion drawn in [15, 18, 63, 64, 77], where we considered variants of $n^6\text{Li}$ potentials with two BSs, i.e., with two FSs in $^5S$ waves and an FS and an AS in $^2P$ waves. The current results show that it is enough to have only one BS in each partial wave, forbidden one with $\{52\}$ for $S$ states and allowed one with $\{43\}$ for doublet $P$ levels, as shown in Table 6. Intercluster potentials complying with this classification of orbital states allow all the above-considered characteristics of the $^7\text{Li}$ BS in the $^n\text{Li}$ channels, $n^6\text{Li}$ elastic scattering, and $^n\text{Li}$ radiative capture to be correctly described. It is not necessary to consider the FS with the orbital diagram (6) in the $^6\text{Li}$ nucleus, as we did earlier [15, 18, 63, 64, 77], for it does not qualitatively improve the description of the experimental data or explain any additional effects.

Thus, when classifying cluster states by Young diagrams in a system of $A = 7$ particles in the $6 + 1$ channel, we should consider only the scheme $\{42\}$ for the $^6\text{Li}$ nucleus and ignore the orbital $\{6\}$ configuration forbidden in $^4\text{Li}$. This leads to possible orbital states of the $n^6\text{Li}$ system with the diagrams $\{52\}$, $\{43\}$, and $\{421\}$, which corresponds to having an FS only in the $^2S$ wave, while the other two diagrams are compatible with the allowed $^2P$ levels, the first of which corresponds to the $^7\text{Li}$ GS in the $n^6\text{Li}$ channel [18].

In addition, as follows from Table 6, the doublet state turns out to be mixed by the schemes $\{43\}$ and $\{421\}$ and can be represented as a half-sum of pure phase shifts, as was shown earlier in [9, 10, 18]. It is usually assumed that the BS corresponds to the pure state with the diagram $\{43\}$ and the scattering phase shifts extracted from the experimental data are mixed by these diagrams [9, 18]. Therefore, all the doublet scattering potentials given above correspond to two diagrams and the BS potentials correspond to only one of them, namely, to $\{43\}$.

Since at energies from $10^{-5}$ to $\sim 100$ keV the calculated cross section is almost a straight line (see solid curve in Fig. 5), it can be approximated by a simple function in the form of Eq. (3) with the constant $246.6118 \mu\text{b keV}^{1/2}$, which was determined from one point in the cross sections at the lowest energy of $10^{-5}$ keV. The modulus of the relative deviation of the calculated theoretical cross section and its approximation by the above function (4) in the region from $10^{-3}$ to $100$ keV turns out to be smaller than 0.3%. If we assume that this shape of the total cross section dependence on energy remains unchanged at lower energies as well, the estimation of the total cross section at an energy, say, $1 \mu\text{eV} (10^{-6} \text{eV} = 10^{-9} \text{keV})$ yields the value 7.8 b. For the dotted curve in Fig. 5, the coefficient in the approximation of the cross section is 208.7136 $\mu\text{b keV}^{1/2}$, the deviation of the approximation from the calculation at energies below 100 keV is no larger than 0.9%, and the cross section at 1 $\mu\text{eV}$ is 6.6 b. For the dot-dashed curve in Fig. 5 the coefficient for Eq. (3) is 207.7438 $\mu\text{b keV}^{1/2}$.

### Table 6. Classification of states by Young diagrams in the $n^6\text{Li}$ system ($T$, $S$, and $L$ are the isospin, spin, and angular orbital momentum of the $p^6\text{Li}$ particle system; $\{\}$, $\{\}^T$, $\{\}^L$, and $\{\}^S$ are the spin, isospin, spin–isospin, and possible angular orbital Young diagrams; and $\{\}^E$ and $\{\}^F$ are the Young diagrams of the allowed and forbidden orbital states [18]. Printed in bold italics are the Young diagrams $\{\}^L$ and $\{\}^F$ conjugate to each other)

| System | $T$ | $S$ | $\{\}^T$ | $\{\}^S$ | $\{\}^L$ | $\{\}^E$ | $\{\}^F$ |
|--------|-----|-----|----------|----------|--------|-------|-------|
| $n^6\text{Li}$ | $1/2$ | $1/2$ | $\{43\}$ | $\{43\}$ | $\{52\}$ | $\{43\}$ | $\{421\}$ |
| $p^6\text{Li}$ | $3/2$ | $\{43\}$ | $\{52\}$ | $\{61\} + \{52\} + \{511\} + \{43\} + 2\{421\} + \{331\} + \{322\} + \{3211\}$ | $\{52\}$ | $\{43\}$ | $\{421\}$ |

The reaction under consideration is a mirror process of the $p^7\text{Be} \rightarrow ^7\text{B}$ capture, where the $^8\text{B}$ nucleus decays into $^8\text{Be} + e^- + \nu$ via the weak process. Neutrinos from this reaction have a relatively high energy and have been detected under the terrestrial conditions for several decades while the unstable $^8\text{Be}$ nucleus further

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**n$^7\text{Li}$ CLUSTER SYSTEM**

Let us consider the radiative $n^7\text{Li}$ capture in the energy range from 5 meV to 1.0 MeV, which is undoubtedly interesting because it belongs to the primordial nucleosynthesis chain (1) and plays a particular part in the evolution of the Universe. The $^8\text{Li}$ nucleus is stable in terms of strong interactions because it decays to $^8\text{Be}$ only due to weak forces, and thus we can assume that there is a relatively high probability for it to have the two-cluster $n^7\text{Li}$ structure and describe its characteristics using the known methods of the PCM with FSs for [6, 15, 18, 87]. It should be emphasized that from the point of view of the general concepts of this process and the $^8\text{Li}$ cluster structure we consider here the two-cluster $n^7\text{Li}$ system with the LS coupling rather than the case where the neutron is in the $p_{3/2}$ state with an admixture of $\ell_{1/2}$ relative to the $^7\text{Li}$ nucleus, as was done, for example, in [88, 89] for the $jj$ coupling.

The reaction under consideration is a mirror process of the $p^7\text{Be} \rightarrow ^7\text{B}$ capture, where the $^8\text{B}$ nucleus decays into $^8\text{Be} + e^- + \nu$ via the weak process. Neutrinos from this reaction have a relatively high energy and have been detected under the terrestrial conditions for several decades while the unstable $^8\text{Be}$ nucleus further
decays into two $\alpha$ particles. The $^8\text{Be} \rightarrow ^8\text{B} \gamma$ capture reaction is one of the final processes in the thermonuclear proton-proton cycle, which is assumed to be responsible for the burning of the Sun and most of the stars in our Universe [3, 4, 6].

Classification of the $n^7\text{Li}$ System States

Note first that the $n^7\text{Li}$ system has the isospin projection $T_z = -1$, which is only possible at the total isospin $T = 1$ [90]. Therefore, unlike the isospin-mixed $p^3\text{Li}$ with $T = 0$ and $1$ [90], this cluster system, like the $p^7\text{Be}$ structure of $^8\text{B}$ at $T_z = +1$ and $T = 1$, is isospin pure. At the same time, as in the $p^7\text{Li}$ system, the spin can take on two values $S = 1$ and $2$ and some of the $n^7\text{Li}$ system states can also be isospin mixed [6].

Now we touch upon the classification of the orbital states of the clusters in the system under consideration. Previously [13] we showed that if the diagram {7} was used for the $^7\text{Li}$ nucleus, the resulting symmetries (8) and (71) of the system of eight particles in the $l + 7$ channel turned out to be forbidden because one row cannot have more than four boxes [91, 92]. They correspond to the forbidden states with the relative angular momentum $L = 0$ and $1$, which is determined according to the Elliott rule [92].

When the diagram {43} is taken for the $^7\text{Li}$ nucleus, the $n^7\text{Li}$, $p^7\text{Be}$ ($T = 1$) or $p^7\text{Li}$, $n^7\text{Be}$ ($T = 0$, 1) systems contain in the triplet spin channel the levels with the Young scheme {43} forbidden in the $^3\text{P}$ waves and in the $^3\text{S}_1$ wave at the WF symmetry {44} and the allowed $^3\text{P}$ state with the spatial diagram {431}. Thus, $n^7\text{Li}$ potentials in the triplet spin state should have the forbidden bound $^3\text{S}_1$ state with the diagram {44} for the scattering processes to be considered below, and the forbidden and allowed bound states with the Young diagrams {53} and {431} in the $^3\text{P}$ waves, of which the latter state corresponds to the bound ground $^3\text{P}_2$ state of $^8\text{Li}$ in the $n^7\text{Li}$ channel.

For the spin $S = 2$ there are no allowed symmetries and thus bound allowed levels in the $n^7\text{Li}$ system at any orbital momentum $L$ [13, 32]. Therefore, the potential of the $^5\text{S}_0$ scattering wave can also have the bound FS with the diagram {44}, and in $^5\text{P}$ scattering waves the potential has FSs with the diagrams {53} and {431}, the latter of which can be in the continuous spectrum, and the potential thus has only one bound FS with the diagram {53}. This conclusion does not seem to be unambiguous, and a variant with two bound FSs with the diagrams {53} and {431} is possible for the $^5\text{P}$ scattering potentials.

Both allowed Young schemes {7} and {43} for the ground state of the $^7\text{Li}$ nucleus can be treated as the third variant, because they both are present in FSs and ASs of this nucleus in the $^4\text{He}$ configuration [13, 15]. Then the classification of the levels will be slightly different, the number of FSs increases, and in each partial wave with $L = 0$ and $1$ there is an extra forbidden bound level. It is this classification that we earlier considered [6, 8, 13] and that is given in Table 7. The results for the first variant, when only the orbital diagram {7} is used in $^7\text{Li}$, are printed in italics in the last four columns and are separated from the others by a solid line.

Since it was earlier shown in the $n^6\text{Li}$ system that only the allowed diagram {42} can be used for $^6\text{Li}$ with its forbidden configuration [6] ignored, we will further treat the second variant of the structure of FSs and potentials with the diagram {43} allowed in the $^7\text{Li}$ nucleus as the main variant of the classification of FSs and ASs in this system. We will thus assume that potentials of the $^3,5\text{S}$ scattering waves, which we need for considering electromagnetic $E1$ transitions to the $^8\text{Li}$ GS at the $n^7\text{Li}$ capture, have bound forbidden states with the diagram {44}. The potential of the $^5\text{P}_1$ resonant scattering wave at 0.25 MeV, which allowed considering the $M1$ transition to the $^8\text{Li}$ GS, can have one, with {53}, or two, with {53} and {431}, bound forbidden states. The potential of the $^5\text{P}_3$ GS in the $n^7\text{Li}$ channel, which is a mixture of two $^3\text{P}_2$ and $^5\text{P}_2$ states, has one forbidden bound state with {53} and one allowed bound state with {431} corresponding to the GS at the binding energy $-2.03239$ MeV [90].

Potential Description of the Elastic $n^7\text{Li}$ Scattering

We failed to find data on phase shifts of the elastic $n^7\text{Li}$ scattering at astrophysical energies [67, 68]. We will therefore construct potentials of the $n^7\text{Li}$ scattering by analogy with the $p^7\text{Li}$ scattering [13, 38] using the data on the spectra of the $^8\text{Li}$ levels [90], which are shown in Fig. 7 together with the similar spectra of the $^8\text{Be}$ and $^8\text{B}$ nuclei. The spectra are presented so as to approximately match the $2^+$ levels, which are the $^8\text{Li}$ and $^8\text{B}$ grounds states stable in terms of nuclear interactions, decaying only via the $\beta$ process.
The earlier considered \([13, 38]\) bound state of the \(p^7\text{Li}\) system with \(J^pT = 0^+0^+\) \([90]\), which corresponds to the \(8^\text{Be}\) ground state, can be formed only in the triplet spin channel with \(L = 1\) due to the addition rules and turns out to be spin-pure \(3^p\text{Li}\) state with \(T = 0\) \([90]\) (see Fig. 7). Therefore, in the description of the electromagnetic transitions, all of the potentials earlier obtained for this system \([6, 8, 13]\) corresponded to the triplet spin state with a certain number of ASs and FSs. All electromagnetic transitions proceeded between different levels in the triplet spin state that had an allowed Young diagram and thus an allowed bound state corresponding to the ground state of the \(8^\text{Be}\) nucleus in the \(p^7\text{Li}\) channel (see Table 7).

In particular, we considered \(E1\) transitions between the \(3^S_1\) scattering state \((T = 0\) and \(1\) isospin-mixed) and the bound \(3^P_1\) ground state with \(T = 0\) and the \(M1\) transition between the resonant \(3^P_1\) wave \((T = 1)\) and the \(8^\text{Be}\) GS, which all proceed with a change of the isospin. Note that some of the scattering states, e.g., the \(3^S_1\) wave, turn out to be isospin mixed with \(T = 0, 1\) because in the \(p^7\text{Li}\) system \(T_c = 0\). Therefore, only part of the potential with \(T = 1\) was actually obtained for the \(3^S_1\) wave \([6, 8, 13]\), and the \(3^P_1\) wave and its potential have a phase resonance, i.e., a resonance level of the \(8^\text{Be}\) nucleus for which the value \(T = 1\) was experimentally found, and this state turns out to be isospin pure. These two processes allowed almost all the experimental data on the astrophysical \(S\)-factor of the \(p^7\text{Li}\) capture to be described. It can be assumed that all these transitions proceed with an isospin change, i.e., the condition \(ΔT = 1\) is fulfilled \([6, 8, 13]\).

In this case, the bound state of the \(n^7\text{Li}\) system with \(J^pT = 2^+1^+\) corresponding to the \(8^\text{Li}\) GS can form at \(S = 1\) and 2 with the orbital momentum \(L = 1\) and is a mixture of \(3^P_2\) and \(\bar{3}^P_2\) states. Though there are no ASs in the \(S = 2\) channel, as follows from \([8, 6, 13]\) and the above classification (see Table 7), it should be assumed that the GS WF has an admixture of the \(3^P_2\) wave, which is necessary for further consideration of the \(M1\) transition from the \(5^P_3\) resonance in the \(n^7\text{Li}\) scattering to the \(3^P_2\) component of the \(8^\text{Li}\) GS WF.

The resonance with \(J^pT = 3^+1^+\) in the \(8^\text{Li}\) spectrum (see Fig. 7) corresponds to the \(3^P_1\) phase of the elastic \(n^7\text{Li}\) scattering at an energy of 0.22 MeV (c.m.s.) or 0.25 MeV (lab. syst.) above the \(n^7\text{Li}\) threshold \([90]\). This \(3^P_1\) resonant state of the \(8^\text{Li}\) nucleus can form only at the total spin of the \(n^7\text{Li}\) system \(S = 2\). Further, to construct the potential corresponding to this elastic \(n^7\text{Li}\) scattering phase resonance, we will use data on the spectra of the \(8^\text{Li}\) levels and widths of these states \([90]\). Certainly, the state with \(J^pT = 3^+1^+\) can also be formed by the triplet \(3^F_3\) configuration of the \(n^7\text{Li}\) system, and there will be a resonance in the \(3^F_3\) phase shift.

| Systems | \(T\) | \(S\) | \(\langle J_T \rangle\) | \(\langle J_S \rangle\) | \(\langle \gamma_{ST} = \langle J_S \otimes J_T \rangle \rangle\) | \(\langle J_L \rangle\) | \(\langle \gamma_{AS} \rangle\) | \(\langle \gamma_{FS} \rangle\) |
|---------|-------|-------|------------------|------------------|--------------------------|----------------|----------------|----------------|
| \(p^7\text{Li} n^7\text{Be}\) | 0 | 1 | \((44)\) | \((53)\) | \((71) + (611) + (53) + (521) + (431) + (4211) + (332) + (3221)\) | \((8)\) | \((71)\) | \((53)\) |
| | 2 | \((44)\) | \((62) + (521) + (44) + (431) + (422) + (3311)\) | \((8)\) | \((71)\) | \((53)\) |
| \(p^7\text{Be} n^7\text{Li}\) | 1 | 1 | \((53)\) | \((53)\) | \((8) + 2(62) + 7(71) + (611) + (53) + (44) + 2(521) + (5111) + (44) + (332) + 2(431) + 2(422) + (4211) + (3311) + (3221)\) | \((8)\) | \((71)\) | \((53)\) |
| | 2 | \((53)\) | \((71) + (62) + (611) + 2(53) + 2(521) + 2(431) + (422) + (4211) + (332)\) | \((8)\) | \((71)\) | \((53)\) |

Table 7. Classification of states by Young diagrams \([32]\) in the \(n^7\text{Li}\) system \((T, S, L)\) are the isospin, spin, and angular orbital momentum of the \(n^7\text{Li}\) particle system; \(\langle J_S \rangle, \langle J_T \rangle, \langle J_{ST} \rangle, \text{ and } \langle J_L \rangle\) are the spin, isospin, spin—isospin, and possible angular orbital Young diagrams; \(\langle J_{AS} \rangle\) and \(\langle J_{FS} \rangle\) are the Young diagrams of the allowed and forbidden orbital states \([18]\).
of the $n^7\text{Li}$ elastic scattering. In this case it is not necessary to assume that the $^8\text{Li}$ GS in the $n^7\text{Li}$ channel has an admixture of the $^3P_2$ state and it will be enough to consider the $^3P_2$ configuration alone. However, all of the earlier phase analyses [93, 94, 95] and similar results for other similar cluster systems [6, 7] indicate that the presence of a resonance in the $^3P_3$ phase of the $n^7\text{Li}$ scattering at this low energy appears to be rather doubtful.

The state with $F^T=1^+1$ is due to $S=1$ and 2, and $L=1$ is the $^3S_1$ level in the $n^7\text{Li}$ channel and turns out to be bound at the energy 0.9808 MeV relative to the $^8\text{Li}$ GS or $-1.05149$ MeV relative to the $n^7\text{Li}$ channel threshold [90]. Next, we consider $E1$ transitions to this level from the triplet and quintet $S$ scattering waves. All further results will therefore refer to the $^7\text{Li}(n, \gamma_0)^8\text{Li}$ and $^7\text{Li}(n, \gamma_1)^8\text{Li}$ reactions and the sum of their cross sections. By analogy with the $^7\text{Li}$ scattering and on the basis of the data [90], we assume that the $^3S_1$ and $^5S_2$ phase shifts in the region below 1 MeV are practically zero. This is confirmed by the absence of negative-parity resonance levels at these energies in the $^7\text{Li}$ spectra.

Since in the $^7\text{Li}$ system [13] we considered variants of potentials with two FSs, now for the sake of comparison we will also use potentials with a different number of FSs in all partial scattering waves needed for the radiative capture calculations. We will first obtain the $S$ and $P$ potentials with two FSs (according to the above results) as the third variant of the AS and FS classification (see Table 7), and then we will consider variants with one FS (second classification variant) and zero FS, i.e., with no FS at all in each partial wave.

The practically zero phase shift for the $^3S_1$ and $^5S_2$ low-energy scattering waves can be obtained with the parameters

$$V_S = -145.5\ \text{MeV and } \gamma_S = 0.15\ \text{fm}^{-2}. \quad (24)$$

We consider this variant of the potential because an analogous potential was used for considering the $^7\text{Li}$ scattering in the $^3S_1$ state [8, 13]; it contains two bound FSs with $\{8\}$ and $\{44\}$, as follows from the third variant of the above classification of states. The zero phase shift can also be obtained with the potential

$$V_S = -50.5\ \text{MeV and } \gamma_S = 0.15\ \text{fm}^{-2}, \quad (25)$$

which has only one bound FS with $\{44\}$ for the second classification variant and no FSs at the zero depth of potential (1), i.e., at $V_0 = 0$ for both $S$ scattering waves.

The near-zero $S$ phase shifts in both spin channels can be obtained using other variants of the Gaussian potential parameters as well. In this sense, parameters of this interaction cannot be uniquely fixed, and other combinations of $V_0$ and $\gamma$ are possible for parameters (24) and (25) with a different number of FSs. However, it will be shown below that it is the near-zero values of the scattering phase shifts obtained with these interactions, rather than various combinations of the parameters $V_0$ and $\gamma$ and the number of FSs, that are most important for the description of the total cross sections for the radiative capture.

The resonant $^5P_3$ phase shift of the $n^7\text{Li}$ elastic scattering can be described by the Gaussian potential with the following parameters

$$V_p = -4967.45\ \text{MeV and } \gamma_p = 3.0\ \text{fm}^{-2}. \quad (26)$$

This potential has two bound forbidden states, which can be associated with the diagrams $\{53\}$ and $\{431\}$ for the second variant of FS classification if the FS with the scheme $\{431\}$ is taken to be bound. The results of calculating the $^5P_3$ scattering phase shift are shown in Fig. 8 by the dotted curve. The resonance is seen to be at the energy of 254 keV (lab. syst.), which coincides with the experimental value 254(3) keV [90] at the width of 37 keV (c.m.s.).

The parameters of the potential with one bound FS with $\{53\}$, which also corresponds to the second classification variant provided that the FS with $\{431\}$ is in the continuous spectrum, are

$$V_p = -2059.75\ \text{MeV and } \gamma_p = 2.5\ \text{fm}^{-2}. \quad (27)$$

The results of calculation of the phase shifts are shown in Fig. 8 by the dashed curve. The resonance is seen to be at the energy of 254 keV. The width of the $^5P_3$ resonance turns out to be 35 keV (c.m.s.) while the experimental values are 35(5) keV or 33(6) keV (c.m.s.) according to different data from [90].

The parameters of the potential without bound FSs, which will be considered additionally, are

$$V_p = -425.15\ \text{MeV and } \gamma_p = 1.5\ \text{fm}^{-2}. \quad (28)$$

The results of calculating the $^5P_3$ phase shift with these parameters are shown in Fig. 8 by the solid curve. It is seen that the resonance is at the energy of 255 keV.
and its width is 34 keV (c.m.s.). It should be stressed that parameters of this potential with a given number of FSs are uniquely determined from the energy and width of the resonance, and the width of the \(^2P_3\) resonance is defined by the expression

\[
\Gamma_{\text{c.m.}} = 2 (d\delta/dE_{\text{c.m.}})^{-1}.
\]

Since we will concentrate on the second variant of the cluster classification for the nuclear BS with one FS, the potential of the bound \(^{3+5}P_2\) state of the \(^n\)Li system, which corresponds to this state of the \(^8\)Li nucleus in the cluster channel under consideration, can be assigned the parameters

\[
V_0 = -429.383779 \text{ MeV and } \gamma = 0.5 \text{ fm}^{-2}. \quad (29)
\]

Apart from the allowed BS with \(\{43\}\), which corresponds to the ground state of the \(^8\)Li nucleus, this \(^{3+5}P_2\) potential has a bound FS with \(\{53\}\). The binding energy \(-2.0322900\) MeV exactly coinciding with the experimental value \([90]\), the charge radius 2.38 fm, and the mass radius 2.45 fm are obtained with this potential. It seems that the root-mean-square charge radius of the \(^8\)Li nucleus should not be noticeably larger than the radius of the \(^7\)Li nucleus, which is 2.35(10) fm \([90]\). Therefore, the above value of the root-mean-square radius for the \(^n\)Li channel of the \(^8\)Li GS is quite reasonable. The neutron charge radius was taken to be zero, as usual, and its mass radius was set equal to the corresponding proton radius 0.8775(51) fm \([21]\).

The asymptotic constant of this GS potential turned out to be \(C_w = 0.78(1)\). The error of the asymptotic constant is found by averaging on the interval of 4 to 20 fm, where its value is relatively stable. The width of GS potential (29) was chosen such as to describe correctly the nuclear charge radius and the AC, and its depth was chosen such as to reproduce the binding energy at the required number of FSs and ASs. As a result, its parameters are also fixed quite uniquely.

For the sake of comparison, we cite the AC for the \(^n\)Li system \(C(\left|\Psi_2\right|) = 0.62 \text{ fm}^{-1/2}\) obtained from the analysis of the experimental data \([89]\), which is nondimensionalized with \(\sqrt{2k} = 0.767\) to 0.81. This value agrees with the result obtained for the above GS potential of \(^8\)Li in the \(^n\)Li channel. In \([131]\) the value 0.78 fm\(^{-1/2}\) is obtained, which yields the nondimensionalized value 1.02; in \([81]\) the value 0.74 fm\(^{-1/2}\) is obtained, which leads to 0.96; and in \([82]\) the value 0.59 fm\(^{-1/2}\) is given for \(C(\left|\Psi_3\right|)\) and 0.28 fm\(^{-1/2}\) for \(C(\left|\Psi_2\right|)\), which gives 0.77 and 0.36 after nondimensionalization.

The parameters obtained for the potential of the first excited state (FES) with \(J^T = 1^+\) are

\[
V_0 = -422.126824 \text{ MeV and } \gamma = 0.5 \text{ fm}^{-2}. \quad (30)
\]

Here the allowed BS with \(\{43\}\) corresponds to the first excited state of the \(^8\)Li nucleus at 0.9808 MeV. In addition, this \(^{3+5}P_1\) potential has an FS with \(\{53\}\) in total conformity with the second variant of the orbital state classification. The calculation using the FDM \([24]\) with these parameters yielded the binding energy \(-1.052490\) MeV with an accuracy of \(10^{-6}\) MeV, which totally coincides with the experimental value \([90]\), the charge radius 2.39 fm, and the mass radius 2.52 fm.

The asymptotic constant of this potential turned out to be \(C_w = 0.59(1)\). The AC error is determined by its averaging on the interval of 4 to 25 fm, where the asymptotic constant remains relatively stable.

The binding energy calculation was additionally checked using the two-body variational method with the cluster WF for the relative motion of the \(^n\)Li system expanded in the nonorthogonal Gaussian basis and with the parameters independently varied \([15, 16]\). With the basis dimension \(N = 10\), the energy \(-2.0322896\) MeV was obtained with the given variant of GS potential (29). Residuals are on the order of \(10^{-14}\), the asymptotic constant on the interval of 5 to 20 fm turned out to be 0.78(1), and the charge radius does not differ from the previous FDM results. The expansion parameters of the VM-obtained radial wave function for the \(^8\)Li GS in the cluster \(^n\)Li channel are presented in Table 8.

The average two-body binding energy in this potential can thus be taken to be \(-2.0322898(2)\) MeV. In other words, the accuracy of determining the \(^8\)Li binding energy in the cluster \(^n\)Li channel with GS potential (29) using two different numerical methods (FDM and VM) and two different computer codes is at a level of \(\pm 0.2\) eV.

The VM-obtained first excited state energy is \(-1.051488\) MeV with the residuals \(10^{-14}\). All other characteristics do not differ from those obtained by the FDM. The WF expansion parameters are presented in Table 9. The average energy can be taken to be \(-1.051489(1)\) MeV, i.e., the error of its calculation with this potential is 1 eV, which agrees with the specified FDM accuracy of \(10^{-16}\) MeV.

Radiative Capture \(^7\)Li(\(n, \gamma\))\(^8\)Li

Note that the \(E1\) transition in the \(^n\)Li system during the radiation capture seems to be first considered in \([96]\), where it was shown that total cross sections in the nonresonance energy region could be correctly described within the single-particle model with the Woods—Saxon potential adapted to the energies of the \(^7\)Li levels. Later this process was considered within the direct capture model in \([88]\). Recent investigations \([17]\) also provide a reasonable description of total capture cross sections on the basis of the \(E1\) process with their resonant behavior ignored.

As for the resonance in the total cross sections at 0.25 MeV, the first attempt to describe it was made, to our knowledge, in \([97]\) on the basis of the generator coordinate method. In addition, reasonable descrip-
tions of these radiative capture cross sections in the resonance energy region have very recently been obtained using model-independent methods [98]. Below we will show that similar results can also be obtained within the potential cluster model when this resonance is described on the basis of the $M_1$ transition from the $^5P_3$ scattering wave, which has a resonance at this energy, to the $^3P_2$ component of the WF for the $^8$Li GS in the $n^7$Li channel.

Here, unlike the case in our previous work [99], we consider the calculation where the neutron magnetic moment is taken into account, which does not lead to a change in the shape and magnitude of all intercluster potentials. When considering the electromagnetic processes in the $^7$Li($n, \gamma$)$^8$Li reaction, we will take into account, as previously for the $p^7$Li capture [13], the $E1$ transition from the nonresonant $^3S_1$ scattering wave to the triplet $^3P_2$ component of the $^8$Li GS WF. But unlike the case in the $p^7$Li system, there is an additional transition from the quintet $^5S_2$ scattering wave to the quintet $^5P_2$ component of the $^8$Li GS WF. And, as was already mentioned, we also consider the $M1$ transition from the $^5P_2$ resonant scattering wave with the $J^\pi T = 3^+1$ level (see Fig. 7) to the quintet $^5P_2$ component of the $^8$Li GS WF. Within the model under discussion it is impossible to separate explicitly the $^5P_2$ and $^3P_2$ components of the GS WF, and we will therefore perform calculations using the spin–mixed function $P_2$, which is obtained with the BS potential for parameters (29).

Dealing with transitions to the first excited $^3 + ^5P_1$ state, we considered the $E1$ process from both $^3 + ^5S$ scattering waves. Then the total capture cross section with allowance for all electromagnetic transitions considered in the $n^7$Li capture can be written as

$$
\sigma(E1 + M1) = \sigma(E1, ^3S_1 \rightarrow ^3P_2) + \sigma(E1, ^5S_2 \rightarrow ^5P_2) + \sigma(M1, ^5S_3 \rightarrow ^5P_2) + \sigma(E1, ^3S_1 \rightarrow ^3P_2) + \sigma(E1, ^5S_2 \rightarrow ^5P_2) + \sigma(M1, ^5S_3 \rightarrow ^5P_2).
$$

and

$$
\sigma_i(E1) = \sigma(E1, ^3S_1 \rightarrow ^3P_1) + \sigma(E1, ^5S_2 \rightarrow ^5P_1).
$$

The results of these calculations were compared with the experimentally measured total capture reaction cross sections in the energy range from 5 meV to 1.0 MeV [100–105].

Since three variants of potentials were obtained for each partial scattering wave, we will not describe in detail the results for each these combinations and give the ultimate, probably best, calculation result for the total cross section of the radiative $n^7$Li capture to the GS at an energy below 1 MeV (lab. syst.) with allowance for all transitions, which is shown in Fig. 9 by the dot-dashed curve. The value of the cross section at the resonance is about 59 $\mu$b. These results were obtained for GS potential (29), $S$ scattering waves in the triplet

| $i$ | $\alpha_i$ | $C_i$ |
|-----|------------|-------|
| 1   | 2.111922863906128E-001 | -1.32720117117602E-001 |
| 2   | 1.05488904037163E-001  | -4.625421860118692E-002 |
| 3   | 9.25117926861837E-003   | -1.875176936729967E-004 |
| 4   | 2.236449875501786E-002  | -2.434284188136483E-003 |
| 5   | 4.990617934603718E-002  | -2.108830320871615E-002 |
| 6   | 3.84914298848459E-001   | -2.613687472261875E-001 |
| 7   | 5.453825421384008E-001  | -2.108302320871615E-001 |
| 8   | 1.163891769476509       | 1.438162032150163       |
| 9   | 1.716851806191120       | 1.42657649534997        |
| 10  | 2.4953876008367         | 1.792643814712334E-001  |

Table 8. Coefficients and parameters of expansion of the radial wave function for the $^8$Li ground state in the $n^7$Li channel in the nonorthogonal basis [15, 16] (the wave function normalization coefficient on the interval 0–25 fm is $N = 9.99999839217028E–001$)

| $i$ | $\alpha_i$ | $C_i$ |
|-----|------------|-------|
| 1   | 2.03486983989546E-001 | -1.26895422420545E-001 |
| 2   | 9.605255016688968E-002 | -4.502984818616291E-002 |
| 3   | 6.47302760802913E-003  | -2.029700124120304E-004 |
| 4   | 1.74388096986541E-002  | -2.3084389772190E-003 |
| 5   | 4.24148102854091E-002  | -1.167539819061673E-002 |
| 6   | 3.943411589808715E-001  | -2.876208138367455E-001 |
| 7   | 5.758070107927670E-001  | -1.30719768138061E-001 |
| 8   | 1.148526246366072       | 1.33502364621784        |
| 9   | 1.7062962490575450      | 1.303208908841006       |
| 10  | 2.491484117851039       | 1.558051077479201E-001  |

Table 9. Coefficients and parameters of expansion of the radial wave function for the first excited state of the $^8$Li nucleus in the $n^7$Li channel in the nonorthogonal basis [15, 16] (the wave function normalization coefficient on the interval 0–25 fm is $N = 9.999907842436313E–001$)
and quintet states with parameters (27), and potential of the \( ^5P_3 \) resonant scattering wave with parameters (27). The dashed curve is the cross section corresponding to the sum of the \( E1 \) transitions from the \( ^3S_1 \) and \( ^5S_2 \) waves to the GS, the dotted curve is the cross section for the \( M1 \) transition between the \( ^3P_2 \) scattering states and the \( ^8Li \) GS in the \( n^7Li \) channel with the cross section resonance value about 47 \( \mu b \). The solid curve shows the total cross sections with allowance for all transitions to the GS and FES, which amount to 62 \( \mu b \) at the resonance. Potential (30) and the same \( ^3+^5S \) scattering wave potential (25) were used for the FES.

Figure 10 shows in more detail the shape and magnitude of the calculated capture cross sections after summing up all contributions for these variants of the potentials at the energies from 1 meV to 150 keV. Designations of the curves are the same as in the previous figure. It is evident from these results that the potential cluster model involving potentials with one FS in the \( S \) and \( P \) waves is quite capable of describing the available experimental data in the maximum wide energy range from 5 meV to 1.0 MeV. Note that in [104], the results from which are shown in Fig. 10 by open circles, only cross sections for the capture to the \( ^8Li \) GS were measured. In [102] both the cross sections for the capture to the GS (filled squares) and the total cross sections with transitions to the GS and FES (open upside-down triangles) were measured.

The variant of the \( ^3P_2 \) wave scattering potential with two FSs increases cross sections at the resonance by approximately 5%. Thus, the above-mentioned nonuniqueness of the number of FSs in this potential does not affect the results. However, calculation of cross sections for this interaction without FSs leads to a decrease in the total cross sections for transitions to the GS and FES at the resonance energy to about 48 \( \mu b \) with the cross section for the \( M1 \) transition being 33 \( \mu b \). This result generally improves the quality of the description of the available experimental total cross section data [101] for this energy region.

But the data from [101] have the maximum of 37 \( \mu b \) at 245 keV, while the resonance is at 255 keV and there are not data on it. It is therefore impossible to evaluate the experimental cross sections at the resonance energy and find out which variant of the potential describes it better. Variants of \( S \) wave scattering potentials with two FSs or without FSs practically do not affect the cross section calculation results. Even the parameters of the \( S \) potentials with a depth of 100 MeV and a width of 0.3 fm\(^{-2} \), i.e., twice as different as the initial one, which also lead to zero scattering phase shifts, only slightly affect the ultimate results.

Thus, the potentials of the \( ^8Li \) GS in the \( n^7Li \) channel with one FS (29) and the corresponding scattering potentials (25) and (27) allow a reasonable description of the available experimental data on total cross sections for radiative capture in the entire energy range considered, whose upper and lower limits differ by almost nine orders of magnitude. Presence or absence of FSs in the \( S \) wave scattering potentials is almost of no importance; it is only zero \((0^\circ \pm 2^\circ)\) values of scattering phase shifts that are important. Small variations in the calculated values of the capture cross sections do not allow us to draw an unambiguous conclusion about the number of bound FSs (1 or 2) for the \( ^5P_3 \).
scattering potential in the resonance energy region. Scarcity of experimental data does not allow any definite conclusions to be drawn even about the number of bound FSs (0 or 1, 2) for this $^5P_3$ potential at the resonance energy.

In general, however, the results allow us to believe that the $S$ and $P$ potentials can comply with the second variant of the FS classification and contain one FS, while the GS $P$ wave also includes an AS at the nuclear binding energy in the channel under consideration. As a result, using the described potentials with forbidden states adjusted for the elastic cluster scattering phase shifts and characteristics of $^3$Li BSs, we can correctly describe the available experimental data for the radiative $n^7$Li capture in a wide energy region.

By analogy with the previous systems and since the calculated cross section at energies from 1 meV to 100 keV is almost a straight line (see Fig. 10, solid curve), it can be approximated by a simple function in the form of Eq. (3). The constant 265.7381 $\mu$keV$^{3/2}$ is obtained here from one point in the cross sections at the minimum energy of 1 meV. As in previous cases, we can consider the modulus of the relative motion of the calculated theoretical cross section and its approximation by Eq. (3) as a function of the energy in the region from $10^{-6}$ to 100 keV. We then find out that this value is no larger than 1.0% at energies below 100 keV.

If we assume again that this shape of the total cross section dependence on energy (3) will be retained at lower energies as well, we can estimate the cross section at the energy of 1 $\mu$eV ($10^{-6}$ keV = 10$^{-9}$ keV) and obtain 8.4 b. For the dot-dashed curve in Fig. 10, the coefficient in the approximation of the cross section is 210.538 $\mu$keV$^{3/2}$ and the cross section at 1 $\mu$eV is 6.7 b.

**RADIATIVE CAPTURE IN THE $n^{12}$C AND $^{13}$C SYSTEMS**

The neutron capture to the GS and three ESs of the $^{12}$C nucleus was considered within the direct capture model [106–108] and a good description was obtained for the available experimental data in the energy region from 20 to 200 keV. In [109] the energy dependence of the total $n^{12}$C capture cross sections for transitions from different partial scattering waves was demonstrated within the direct capture model as well. In addition, the possibility of describing the $n^{12}$C capture at energies from 20 to 600 keV was investigated in [110] using the generalized optical model. As to the neutron capture in the $^{13}$C nucleus, the total capture cross sections to the GS and five ESs of the $^{14}$C nucleus were considered within the direct capture model in [111], and general agreement with the experimental data [112] was obtained in the energy range from 25 to 60 keV.

To consider radiative capture in heavier nuclei within the cluster model, we will examine the possibility of describing the experimental data for the total $n^{12}$C and $n^{13}$C capture cross sections in the energy region from 25 meV to 1.0 MeV. The $n^{12}$C and $n^{13}$C capture at thermal and astrophysical energies are reactions within the main thermonuclear reaction chain of primordial nucleosynthesis (1) [17]. The available experimental data on total cross sections, e.g., for the $n^{12}$C reaction, are given in [109, 113–116], which can be found in the databases [67, 68]. Though they do not cover the entire energy range, they give a general idea of the behavior of radiative capture cross sections in a wide energy range. It is therefore interesting to find out whether these cross sections can be described within the PCM with FSs, as was earlier done for the radiative $p^{12}$C and $p^{13}$C capture [3, 6, 117, 118]. Note that the recent phase shift analysis of the experimental data including new data on differential cross sections for the elastic $p^{12}$C and $p^{13}$C scattering at astrophysical energies [3, 93, 94] has allowed fully unique potentials of the $p^{12}$C and $p^{13}$C interactions to be constructed on the basis of the measured phase shifts. They generally should not significantly differ from the similar potentials of the $n^{12}$C and $n^{13}$C scattering and bound states of the $^{12}$C nucleus in the $n^{12}$C channel and $^{14}$C in the $n^{13}$C channel.

**Total Cross Sections for the $n^{12}$C Capture**

In these calculations of the radiative $n^{12}$C capture, we consider the $E1$ transition that is due to the orbital part of the electric operator $Q_{el}(L)$ [15]. In the $n^{12}$C $\rightarrow$ $^{13}$C$^+$ process this transition can occur between the doublet $^2S_1/2$ scattering state and the ground $^2P_{1/2}$ bound state of the $^{13}$C nucleus in the $n^{12}$C channel. Further, we consider $E1$ transitions not only to the ground state of the $^{13}$C nucleus, i.e., a reaction like $^{12}$C$(n, \gamma_0)^{13}$C, but also to the first three excited levels. Two-body interaction potentials are constructed, as usual, on the basis of the description of the elastic $n^{12}$C scattering phase shifts in the entrance channel and a reasonable reproduction of the main characteristics of the $^{13}$C BSs in the cluster $n^{12}$C configuration for the exit channel [6, 86].

Classification of orbital cluster states by Young diagrams for the $n^{12}$C and $p^{13}$C systems was considered in [117], where we showed that allowed orbital Young diagrams for a complete system of 13 nucleons had the form $\{1\} \times \{444\} = \{544\} + \{4444\}$ [9, 12, 92]. The former is compatible with the orbital momentum $L = 0$, 2 and is forbidden because there cannot be five nucleons in the $s$ shell. The latter diagram is allowed and compatible with the orbital momentum $L = 1$, determined according to the Elliott rules [92]. The state with $L = 1$ corresponds to the allowed bound ground state of the $^{13}$C nucleus in the $n^{12}$C channel with the momentum and isospin $J^T = 1/2^-, 1/2$. Thus, in the $^2S_1/2$ wave potential there should be an forbidden BS, and the $^2P$ wave has only the state allowed in the $n^{12}$C channel at the binding energy $-4.94635$ MeV [119].

However, since we do not have complete tables of Young diagram products for systems of more than
eight particles [32], which we earlier used for similar calculations [6, 15, 86], the above results should only be treated as qualitative estimates of possible orbital symmetries in the ground state of the $^{13}C$ nucleus for the $n^{12}C$ channel. At the same time, it is this very classification that allowed a reasonable explanation to be obtained for the available experimental data on the radiative $p^{12}C$ capture [117]. We will therefore consider the $n^{12}C$ system using the above orbital symmetry classification of states, which leads us to a definite number of FSs and ASs in partial interaction potentials and thus allows their depth to be definitely fixed.

Earlier [2, 3, 6, 8, 117] the $^{2}S_{1/2}$ wave potential for the $p^{12}C$ scattering has been constructed such that the corresponding partial elastic-scattering phase shift that has a pronounced resonance at 0.42 MeV was correctly described. The $n^{13}C$ system, according to [119], does not have resonances at energies up to 1.9 MeV, and its $^{2}S_{1/2}$ phase shift should therefore be relatively smooth in this energy region. We failed to find any earlier performed phase shift analysis of the elastic $n^{12}C$ scattering at an energy below 1.0–1.5 MeV [67, 68], though its results should noticeably differ from those of the analysis for the $p^{12}C$ scattering [117].

To determine the exact behavior of the $^{2}S_{1/2}$ phase shift that is important for the calculations, we therefore performed the phase shift analysis of the elastic $n^{12}C$ scattering at astrophysical energies, namely, at 50 keV to 1.0 MeV [95]. Differential cross sections for the elastic $n^{12}C$ scattering in the energy range from 0.05 to 2.3 MeV was experimentally measured in [120]. The results of our analysis for the $^{2}S_{1/2}$ elastic scattering phase shift are presented in Fig. 11a by filled points. Note that since the $^{2}S$ phase shift has a forbidden bound state, its values in Fig. 11a begin with 180° [12, 18].

Now we turn to the description of the results for the GS potentials and then return to the phase shift analysis and scattering potentials. The potential of the $^{13}C$ ground state in the $n^{12}C$ channel for the $^{2}P_{1/2}$ wave without FSs was constructed on the basis of our results obtained earlier for the $p^{12}C$ system [117]. This potential should correctly reproduce the binding energy of the $^{13}C$ nucleus in the $n^{12}C$ channel, which is $-4.94635$ MeV [119]. The value taken for the $^{12}C$ charge and mass radii is 2.472 (15) fm [119], the charge radius of the neutron is taken to be zero, and its mass radius is set equal to the proton radius of 0.877(5) fm [21].

Using the earlier obtained results for the $p^{12}C$ channel of the $^{13}N$ nucleus, we find the $n^{12}C$ system parameters

$$V_{g.s.} = -135.685683 \text{ MeV, } \gamma_{g.s.} = 0.425 \text{ fm}^{-2}. \quad (11)$$

The potential yields the binding energy $-4.946350$ MeV at the FDM accuracy of $10^{-6}$ MeV, root-mean-square charge radius $R_{ch} = 2.48$ fm, and mass radius $R_{m} = 2.46$ fm. The AC on the interval of 5 to 16 fm turned out to be 0.99(1). The reduced AC error is determined by the averaging of the AC over the above distance interval.

Note that according to [81], where a lot of AC results are cited, the value of this constant is 1.54(3) fm$^{-1/2}$, which is nondimensionalized with $\sqrt{2k} = 0.971$ to 1.59(3), and in [25] its nondimensionalized value is 1.60(3). Note again that this nondimensionalization is needed because in those works the definition of the asymptotic constant differed from ours by the factor $\sqrt{2k}$.

For the sake of comparison we give another variant of the $n^{12}C$ potential of $^{13}C$ GS in the $n^{12}C$ channel with the parameters

$$V_{g.s.} = -72.173484 \text{ MeV, } \gamma_{g.s.} = 0.2 \text{ fm}^{-2}. \quad (12)$$
With the FDM accuracy of $10^{-8}$ MeV, it yields the binding energy $-4.94635034$ MeV and the charge radius 2.48 fm, the root-mean-square radius proves to be slightly larger, $R_m = 2.50$ fm, and the AC is $1.52(1)$ on the interval of 5 to 18 fm, which better agrees with the results in [25, 81]. The shape of the wave function for this $^2S_{1/2}$ potential is shown in Fig. 11b by the solid curve.

The binding energy calculations were additionally checked using the variational method [24], which yielded the energy $-4.94635032$ MeV on the grid of dimension $N = 10$ with independent variation of parameters for GS potential (12). The asymptotic constant $C_\infty$ of the variational WF with the parameters listed in Table 10 is at a level of 1.52(2) at distances of 5 to 15 fm, and the residuals are no larger than $10^{-12}$ [24]. The charge radius does not differ from that obtained in previous FDM calculations.

As was already said, since the variational energy decreases with increasing dimension of the basis and gives the upper limit of the true binding energy while the finite-dimensional energy increases with decreasing step and increasing number of steps, the correct binding energy in this potential can be taken to be $-4.94635033(1)$ MeV. As a result, the accuracy of determining the two-body binding energy based on two different mathematical methods and calculated within two different computer codes is at a level of $\pm 10 \times 10^{-9}$ MeV = $\pm 10$ MeV.

Returning to the description of the scattering processes, we note that if for the potential of the $^2S_{1/2}$ wave in the $^{12}$C scattering we take the parameters earlier obtained for the $^p$C system

$$V_S = -102.05 \text{ MeV}, \gamma_S = 0.195 \text{ fm}^{-2},$$

which do not lead to a resonance with a change in the Coulomb potential, as the dashed curve shows in Fig. 11a, we obtain total cross sections for the radiative capture that are several orders of magnitude lower that the experimentally observed ones at all energies from 25 meV to 1.0 MeV.

Now let us use the potential that describes well the $^2S$ phase of the elastic $^{12}$C scattering. The phase shift calculation results are shown in Fig. 11a by the solid curve, and the parameters are as follows [95]

$$V_S = -98.57558 \text{ MeV}, \gamma_S = 0.2 \text{ fm}^{-2}. \quad (13)$$

The wave function of this potential at an energy of 10 keV is shown in Fig. 11b by the dashed curve.

Parameters of potential (13) are given with a high accuracy for correct description of the energy of the bound state in the $^2S_{1/2}$ wave, which is $-1.856907$ MeV deep relative to the $^{12}$C channel threshold. When Coulomb interaction is switched off in the initial $^2S_{1/2}$ potential of the $^p$C scattering that correctly describes the above-threshold resonance at 0.42 MeV, this state becomes bound. Now apart from the forbidden state, the potential also has an allowed bound state corresponding to the first excited state (ES) of the $^{13}$C nucleus at an energy of 3.089 MeV with $J^P = 1/2^+$ relative to its ground state. This potential correctly describes not only the elastic scattering phase, but also, as seen below, the main characteristics of the BS in this partial wave.

The results of calculating total capture cross sections for this scattering potential and GS interaction (11) are shown by the dashed curve in Fig. 12a. The calculated cross sections lie twice as low as the data at 25 meV from [59, 113], and for the energies from 20 to 200 keV they are noticeably lower than the data from [100, 115, 116]. Let us compare them with the results obtained with scattering potential (13) and GS interaction (12), which describes the ACs shown in Fig. 12a by the dotted curve. The dot-dashed curve is the calculation of the total cross sections for the transition from the $^2D_{3/2}$ wave to the $^2P_{1/2}$ GS. For the $^2D_{3/2}$ wave, potential (13) for $L = 2$ was used, and the solid curve is the sum of these transitions. They are seen to lead to a correct description of the total cross sections obtained in different experimental investigations at energies beginning with 25 meV and up to 550 keV.

We emphasize that these results are obtained for potentials (12) and (13), which are generally correlated with the $^{12}$C GS characteristics and phase shifts of the $^{12}$C elastic scattering at low energies and have no adjustable parameters. Thus, the combination of the parameters used to describe the main characteristics of the discrete and continuous spectrum for the $^{12}$C system, including ACs, allows the available data on the experimental cross sections for the radiative $^{12}$C capture to the $^{13}$C GS to be well reproduced at energies from 25 meV to 550 keV, i.e., in the energy region of seven orders of magnitude.

Turning to the consideration of transitions to excited states, we note that in [131] the AC for the first excited $1/2^+$ state of the $^{12}$C nucleus in the $^{12}$C channel turned out to be 1.61 fm$^{-1/2}$, which is nondimen-

| $i$ | $B_i$ | $C_i$ |
|-----|-------|-------|
| 1   | 1.500426018861289E-002 | 1.223469853688857E-004 |
| 2   | 1.002841633851088E-001 | 3.05327391493124E-002 |
| 3   | 1.981842450457470E-001 | 1.1517430485543E-001 |
| 4   | 3.011361231511710E-002 | 1.8980773407565E-003 |
| 5   | 1.460253375610869E-001 | 2.604340601242970E-002 |
| 6   | 5.11529009073104E-002 | 9.24576920991236E-003 |
| 7   | 9.74205708542415E-002 | 1.2382077902581E-003 |
| 8   | 3.220854607507809E-001 | 1.870518591470587E-001 |
| 9   | 8.80198523907210E-002 | 7.19573163787223E-003 |
| 10  | 5.612447142811238E-002 | 1.05028601397638E-002 |
sionalized with $\sqrt{2k} = 0.76$ to 2.12. At the same time, in one of the works [17] the value 1.84(16) fm$^{-1/2}$ is obtained for the first excited state, which comes to 2.42(17) after nondimensionalization. In this case we can consider the $E1$ transition from the scattering $P_{1/2}$ and $P_{3/2}$ to the excited $S_{1/2}$ state, which is bound in the $n^{12}$C channel. Since the $P$ wave has no FSs and spectra of the $^{13}$C spectra have no resonances of negative parity at an energy below 1 MeV, we can simply set $P$ wave potentials to zero, and since the width of the ES potential only slightly affects the root-mean-square nuclear radii, we construct this potential such that it approximately reproduces the above AK.

As a result, for the excited BS in the $S_{1/2}$ wave with FSs we will use potential (13), which leads to the binding energy $-1.856840$ MeV at the FDM accuracy of $10^{-6}$ MeV, charge radius $2.47$ fm, mass radius $2.44$ fm, and AC $0.30(1)$ at distances of 2 to 24 fm. The potential does not have FSs and correctly reproduces the AC [131].

Calculations of total cross sections for the $n^{12}$C capture from the $2S_{1/2}$ scattering state with potential (13) to the bound $3/2^-$ level are shown by the solid curve in Fig. 12c, which also presents experimental data from [100, 113, 115, 116]. It is seen that in this case the approach used also yields a reasonable description of the total cross sections for the transition to the second ES of the $^{13}$C nucleus. The intercluster potentials are still correlated with the scattering phase shifts and correctly reproduce the main characteristics, including the AC, of the BS under consideration, which is the second ES of the $^{13}$C nucleus.

To consider the transition from the $P_{3/2}$ scattering wave to the third excited $D_{3/2}$ state, bound in the $n^{12}$C channel, at the energy $-1.09254$ MeV relative to the $n^{12}$C channel threshold, which is the third ES of the $^{13}$C nucleus, we cite the AC values. In [131] the value 0.11 fm$^{-1/2}$ was obtained, and in [17] the value 0.15(1) fm$^{-1/2}$ was proposed, which are nondimensionalized with $\sqrt{2k} = 0.665$ to 0.16 and 0.23. For the $P_{3/2}$ wave we continue using the zero potential, and for the potential associated with one FS in the $D_{3/5}$ state channel under consideration we use the same geometry as for the $^{13}$C GS (12). For the third ES potential we then have

$$V_{D} = -263.174386 \text{ MeV}, \gamma_{D} = 0.2 \text{ fm}^{-2},$$
which gives the binding energy $-1.092540$ MeV at the FDM accuracy of $10^{-6}$ MeV, charge radius 2.49 fm, mass radius 2.61 fm, and AC 0.25(1) at distances of 6 to 25 fm. The potential has an FS and on the whole correctly reproduces the magnitude of the AC. Calculations of the total cross sections for the capture to 25 fm. The potential has an FS and on the whole correctly reproduces the magnitude of the AC. Calculations of the total cross sections for the capture to distances of 6 fm. The modulus of the relative deviation of the calculated theoretical cross section and its approximation by function (4) in the energy region of $10^{-5}$ to $10$ keV is smaller than 1.0%. Assuming that the shape of the total cross section dependence on energy will be retained at even lower energies as well, we can obtain an estimate of the cross section at 1 μeV ($10^{-6}$ eV = $10^{-9}$ keV), which is 402.5 mb.

All the potentials considered do not have adjustable parameters to vary in the calculations of the total cross sections. All the parameters are preset on the basis of the BS and scattering characteristics.

Since the calculated cross section at energies of $10^{-5}$ to $10$ keV is almost a straight line (solid curve in Fig. 12a), it can be approximated in the low-energy region by a simple function in the form of Eq. (3) with the constant 12.7292 μb keV$^{1/2}$, which is determined on the basis of one point in the cross sections at the minimum energy of $10^{-3}$ keV. The modulus of the relative deviation of the calculated theoretical cross section and its approximation by function (4) in the energy region of $10^{-5}$ to $10$ keV is smaller than 1.0%. Assuming that this shape of the total cross section dependence on energy will be retained at even lower energies as well, we can obtain an estimate of the cross section at 1 μeV ($10^{-6}$ eV = $10^{-9}$ keV), which is 402.5 mb.

Total Cross Sections for the n$^{13}$C Capture

Earlier we considered the classification of orbital states by Young diagrams for the p$^{13}$C, and thus n$^{13}$C, system in [118]. Recall that for the p$^{13}$C system within the 1p shell we obtain $|1⟩ \times \{4441\} \rightarrow \{5444\} + \{4442\}$ [92]. The first of the above Young diagrams is compatible with the orbital momentum $L = 1$ and is forbidden because there cannot be five nucleons in the s shell, and the second diagram is allowed and consistent with the orbital momenta 0 and 2 [92]. Thus, confining ourselves to the lowest partial waves alone, we can state that in the $^3S_1$ wave potential there is no forbidden state but one can be a bound AS, while the $^3P_0$ wave has a forbidden state and an allowed state at the binding energy of the n$^{13}$C systems in the $^{14}$C nucleus, which is $-8.1765$ MeV [119]. This state corresponds to the ground state of the $^{14}$C nucleus in the n$^{13}$C channel at $F^r = 0^+$ because the momentum and the isospin of the $^{13}$C nucleus are $F^r, T = 1/2^-$, 1/2 at $T_z = -1/2$. Note that the isospin projection in the n$^{13}$C system is $-1$ so that the total isospin is unity, and this is the second isospin-pure cluster system with the maximum isospin out of those which we considered earlier [2, 3, 6, 8, 118].

Next, we considered the E1 transition which is possible in the n$^{13}$C $\rightarrow$ $^{14}$Cγ capture between the triplet $^3S_1$ and $^3D_1$ scattering states and the bound ground $^3P_0$ state of the $^{14}$C nucleus in the n$^{13}$C channel. To calculate total cross sections for the radiative capture, the nuclear component of the intercluster potential of the n$^{13}$C interaction is, as usual, represented as a Gaussian without the Coulomb term. The $^3S_1$ scattering potential for $L = 2$ was used to calculate $^3D_1$ waves. For the $^3S_1$ wave potential without FSs we first used the values

![Figure 12c](image1.png) Total cross sections for the low-energy n$^{12}$C radiative capture to the second excited 3/2$^-$ level of the $^{13}$C nucleus. Symbols are experimental data from [100] (●), [113] (○), [116] (■), and [115] (△). Curves are calculations using different potentials with the parameters that are given in the text.

![Figure 12d](image2.png) Total cross sections for the low-energy n$^{12}$C radiative capture to the third excited 5/2$^+$ level of the $^{13}$C nucleus. Symbols are experimental data from [100] (●), [116] (■), [115] (△), and [122] (▲). Curves are calculations using different potentials with the parameters that are given in the text.
obtained in the $p^{13}$C scattering [94, 118] with the parameters

$$V_S = -265.4 \text{ MeV}, \gamma_S = 3.0 \text{ fm}^{-2}. \quad (14)$$

The results of calculating the $^3S_1$ elastic scattering phase shift with this $p^{13}$C potential without the Coulomb interaction, i.e., for the $n^{13}$C system, are shown in Fig. 13a by the dashed curve. Now they lack the resonant character [94] and are a smoothly decaying function of energy. Since the potential does not have an FS, its phase begins with zero degree.

The one-FS potential of the bound triplet $^3P_0$ state should correctly reproduce the binding energy of the $^{14}$C ground state with $J^\pi = 0^+$ in the $n^{13}$C channel and reasonably describe the root-mean-square radius of the $^{14}$C nucleus, the experimental value of which is 2.4962(19) fm [119]. The ultimately obtained parameters

$$V_{g.s.} = -399.713125 \text{ MeV}, \gamma_{g.s.} = 0.45 \text{ fm}^{-2}. \quad (15)$$

are a modified variant of the potential of the bound $p^{13}$C state in the $^{14}$N nucleus.

The potential leads to the binding energy $-8.176500$ MeV at the FDM accuracy of $10^{-6}$, root-mean-square charge radius 2.47 fm, and mass radius 2.47 fm. The neutron and $^{13}$C radii have the same values as in the previous section. The dimensionless value of the asymptotic constant [25] on the interval of 4 to 12 fm is 1.85(1). The error of the constant is determined, as usual, by its averaging over the above interval. Note that in [123], the value obtained for the AC is 1.81(26) fm$^{-1/2}$, which is nondimensionalized with $\sqrt{2k} = 1.02$ to 1.77(25).

The GS energy calculation was additionally checked using the two-body variational method [24], which yielded the value $-8.176498$ MeV on the grid with the dimension $N = 10$ and with independently varied parameters of BS potential (15). Parameters of the variational radial WF are presented in Table 11. Residuals are no larger than $10^{-11}$ [24]. The charge radius and AC do not differ from the FDM-calculated ones. As was already said, the average FDM–VM value $-8.176499(1)$ MeV can be taken for the correct binding energy in this potential; i.e., the accuracy of determining the $^{14}$C binding energy in the $n^{13}$C channel using two methods and two different computer codes for potential (15) is at a level of $\pm 1$ eV.

Now we turn to our calculations of total cross sections for the radiative capture. Note that experimental data available for the total $n^{13}$C capture cross sections [112, 113, 124–126] are obtained using the database [67]; those falling within the energy range of 25 meV to 100 keV are shown in Fig. 14a. As previously in [118], we calculated total cross sections using only the $E1$ transition from the nonresonant $^3S_1$ and $^3D_2$ scattering waves with potential (14) to the bound triplet $^3P_0$ state of the $n^{13}$C clusters in the $^{14}$C nucleus for potential (15).

However, the total cross sections for the radiative $n^{13}$C capture to the ground state of the $^{14}$C nucleus calculated with the above potentials are almost an order of magnitude larger than the cross sections obtained in [113, 126] at 25 meV and much larger than the cross sections for the energy range of 10 to 100 keV in [112]. To describe the available data correctly, the $^3S_1$ scattering wave potential should be slightly changed and its depth should be taken to be $-215.770460$ MeV without changing the geometry. The phase shift calculated with this modified potential is shown in Fig. 13a, and the total cross sections are shown by solid curves in Fig. 14a.
This potential allows the position of the $^3S_1$ level with $J^\pi = 1^-$, bound in the $n^{13}C$ channel but excited at 6.0938 MeV, to be correctly described and leads to the binding energy $-2.08270$ MeV relative to the $n^{13}C$ channel threshold, charge and mass radii 2.47 fm, and AC $1.13(1)$ on the interval of 2 to 22 fm. The situation is similar to that for the previous system, when the above-threshold resonance state in the $p^{13}C$ system becomes the bound $^3S_1$ state after the Coulomb interaction is switched off. It is seen that a change of less than 20% in the depth of the potential allows describing experimental data in the range from 25 meV to 100 keV. A slightly gentler decrease in the cross sections at 0.5 to 1.0 MeV is due to the contribution from the $E1$ transition from the $^3D_1$ scattering wave, which can be noticed only in this energy range.

It should be emphasized that, unlike the case in the consideration of the $n^{12}C$ system, we failed to find independent results for the AC of the first ES in the bound $^3S_1$ wave. The scattering potential used can therefore have some ambiguity in its parameters. Basically, there can be another set of parameters for this potential that correctly describes characteristics of the bound state, in particular binding energy and total capture cross sections, but leads to a slightly different AC.

Our evaluation of the cross section for the $E2$ transition from the $^3P_2$ resonant scattering wave with $J^\pi = 2^+$ at 153 keV (lab. syst.) to the $^{14}C$ GS in the resonance energy region led to the value about 1% of the $E1$ cross section. However, there can be the $E1$ transition from the $^3P_2$ resonant scattering wave of the $^3S_1$, FES of the $^{14}C$ with the energy 6.0938 MeV at $J^\pi = 1^-$, for which the potential has already been obtained. The cross section for this process must have a narrow resonance, well amounting to 1 to 2 mb.

To perform these calculations, we first find the $^3P_2$ resonant wave potential without FSs, which has the parameters

$$V_p = -10719.336 \text{ MeV}, \gamma_p = 40.0 \text{ fm}^{-2}.$$  

This potential leads to the resonance energy 153 keV (lab. syst.) and the width 3.7 keV (c.m.s.). Its phase is shown in Fig. 13b by the solid curve. Parameters for the $^3P_2$ resonant wave potential with a bound FS are found to be

$$V_p = -46634.035 \text{ MeV}, \gamma_p = 60.0 \text{ fm}^{-2}.$$  

It leads to a resonance at 153 keV with a width of 4.0 keV. The scattering phase obtained with this potential is shown in Fig. 13b by the dashed curve. This potential agrees with the above classification by Young diagrams if the AS in this partial wave is assumed not to be bound.

In both cases the phase of the potentials at the resonance energy is $90.0(5)$ degrees and the widths of the potentials well agree with the experimental data, being within their error interval of $3.4(0.7)$ keV, as are the resonance energies $152.9(1.4)$ keV [119]. The parameters of the potential are uniquely determined in both cases as well if the number of FSs is specified, though their values are rather exotic. The depth of the potential determines the position of the resonance and its width specifies the calculated width of this resonance.

Now we give the results of the calculation for the $E1$ transition with these two potentials. The results obtained with the $^3S_1$ and $^3P_2$ wave potentials without FSs are shown in Fig. 14b by the dotted curve. The dashed curve is the calculations for the total cross sec-
tions for the transition to the GS shown in Fig. 14a by the solid curve. The solid curve in Fig. 14b is the sum of these cross sections. At the resonance energy the cross sections obtained by summing up all contributions amount to 1.42 mb. This value can be used for a comparison with possible future measurements of total capture cross sections in the resonance region.

Experimental data for the cross sections after summing up all contributions with the transition to the GS and all ESs are given in [112, 127, 128] and shown in Fig. 14b by filled squares and open triangles at 30 [127] and 40 keV [128].

The results of similar calculations for the $^3P_2$ wave scattering potential with FSs do not practically differ from those in Fig. 14b, except that at the resonance energy the cross sections have a smaller value of 1.40 mb, which can be caused by a slightly larger resonance width for the potential with a bound FS. Thus, the results of both calculations practically do not depend on the presence of the bound FS in the $^3P_2$ resonant potential. They completely depend on how correctly the resonance energy of the level and first of all its width are reproduced.

We failed to find results of any elastic $n^{13}C$ scattering phase shift analysis or experimental data on differential cross sections for elastic scattering at energies below 1.0 MeV. The available measured data for energies above 1.26 MeV [129] were obtained with a large energy step, which does not yet allow the phase shift analysis to be performed, as we earlier did for the $p^{13}C$, $n^{12}C$, or $p^{13}C$ scattering at energies below 1.0 MeV [93–95]. Note that about 30 measurements of the differential cross sections were performed in different works at four scattering angles for the $p^{13}C$ elastic scattering in the region of the resonance at 0.55 MeV and its width 23(1) keV. These detailed data allowed us to accurately reproduce the shape of the resonance at $J^P = 1^–$ and 8.06 MeV relative to the $^{14}N$ GS or 0.551(1) relative to the $p^{13}C$ channel threshold in our phase shift analysis [94].

In this case, because of a large energy interval of cross section measurements, the available differential cross sections for the $n^{13}C$ scattering at energies above 1.26 MeV [129] do not allow for revealing of the shape of the resonance with $J^P = 1^–$ at 9.8 MeV relative to the GS or 1.75 MeV relative $n^{13}C$ threshold, whose characteristics are presented in Table 17 in [119], to say nothing of the resonance with $J^P = 2^+$ at 153 keV above the threshold and with the width 3.4 keV (c.m.s.). It is therefore impossible to obtain the $^3S_1$ scattering phase shift from the experimental data on differential cross sections at low energies and to construct the scattering potential on its basis. However, using a number of other criteria for the construction of the intercluster potentials, we can find their parameters that lead to a reasonable description of the available experimental data.

Thus, the $n^{13}C$ system BS interaction quite reasonably reproduces the main characteristics of the $^{13}C$ GS, as was earlier obtained for the $p^{13}C$ channel in the $^{14}N$ nucleus [118]. However, lack of results for the AC makes it impossible to draw any definite and ultimate

| Table 12. Variational parameters and expansion coefficients for the radial WF in the $^{14}C$ system for the $^{15}C$ GS potential (the WF normalization on the interval of 0 to 30 fm is $N = 9.999975198490593E–001$) |
|---|---|---|
| $I$ | $\alpha_i$ | $C_i$ |
| 1 | 4.400254682811078E-003 | $–1.911899202003393E–003$ |
| 2 | 1.08005384380744E-002 | $–1.812966082174555E–002$ |
| 3 | 2.56423603223276E-002 | $–5.66744071622722E–002$ |
| 4 | 5.87194660420144E-002 | $–1.1000367735067E–001$ |
| 5 | 1.272854180415382E-001 | $–1.6894912977194E–001$ |
| 6 | 2.72236794080272E–001 | $–8.92756832717325E–002$ |
| 7 | 3.45131298413449E–001 | $4.97617053854868E–001$ |
| 8 | 5.153383537797701E–001 | $4.660987945089151E–001$ |
| 9 | 7.10517147157914E–001 | $1.38970908932875E–001$ |
| 10 | 1.048194029972669 | $5.16744702889906E–003$ |
conclusions about the parameters of the potential in the $3S_1$ wave. Except for this, the situation is similar to that for the $n^{12}$C system, when the $S$ phase variations in the scattering were not limited to the variations in the Coulomb interaction alone. The correct $S$ phase shift of the $n^{12}$C scattering obtained in the phase shift analysis and shown in Fig. 11a by filled circles had slightly lower values than the calculated phase for the $p^{12}$C potential with the Coulomb interaction switched off (see dotted curve in Fig. 11a).

Thus, almost all potentials in the discussed approach are constructed uniquely, except the $3S_1$ potential, whose parameters depend primarily on the AC of this bound state, and its value is not known so far. However, as in some previous cases, a single potential can be constructed in the $3S_1$ wave for a particular bound state with a given momentum and for scattering processes in the corresponding partial wave. It is then used to calculate continuous and discrete spectrum characteristics of the states of the clusters under consideration, including total radiative capture cross sections.

In conclusion, note again that since the calculated cross section at energies from 25 meV to 10 keV is almost a straight line, it can also be approximated by a simple function in the form of Eq. (3) with the constant 4.6003 μb keV$^{-1/2}$. It was determined, as usual, from one point in the calculated cross sections at the minimum energy of 10 meV. Modulus $M(E)$ (4) of the relative deviation of the calculated theoretical cross section and its approximation by function (3) in the region from 25 meV to 10 keV is no larger than 0.4%. As previously discussed, we can assume that this shape of the total cross section dependence on energy (3) will be retained at lower energies as well, and the estimation of the cross section, e.g., at 1 μeV (10$^{-6}$ eV = 10$^{-2}$ keV) yields 145.5 mb.

**CAPTURE IN THE $n^{14}$C AND $n^{14}$N SYSTEMS**

Now let us consider the $n^{14}$C and $n^{14}$N capture at energies of 23 keV to 1.0 MeV, though only the former appears in the primordial nucleosynthesis chain (1) which has ultimately given rise to the Sun, stars, and our entire Universe [4]. Its cross section smoothly decreases with decreasing energy and obeys the $E$ law. The cross section for the other reaction as a function of the energy in the thermal region increases as $1/E$. Thus, the processes under consideration, $14$C($n$, $\gamma$)$^{15}$C and $14$N($n$, $\gamma$)$^{15}$N, demonstrate two types of behavior of capture cross sections with variations in energy.

Though the $n^{14}$N $\rightarrow$ $^{15}$N capture reaction does not belong to any primordial nucleosynthesis reaction chain or thermonuclear cycles [4, 14], it results in formation and accumulation of $^{15}$N nuclei. Thus, this reaction is complementary to the $14$C($n$, $\gamma$)$^{15}$C($\beta^-$)$^{15}$N reaction appearing in (1) and increases the number of $^{15}$N nuclei participating in reactions of synthesis of.

### Table 13. Variational parameters and expansion coefficients for the radial WF in the $n^{14}$N system for $^{15}$N GS potential (18) (the WF normalization on the interval of 0 to 30 fm is $N = 1.000000000000001$)

| $i$  | $\alpha_i$ | $C_i$                        |
|------|------------|------------------------------|
| 1    | 2.763758363387135E-002 | –3.542736101468866E-004 |
| 2    | 5.252886535294879E-002  | –6.584466560462019E-003 |
| 3    | 9.444729283481111E-002  | –4.679747747384075E-002 |
| 4    | 1.088660404093062E-001  | 1.318491526218144E-002   |
| 5    | 1.503486884800729E-001  | –1.087314408835770E-001 |
| 6    | 2.226413018972464E-001  | –8.992256982354141E-002 |
| 7    | 2.684402252313877E-001  | –5.7459859099638E-003    |
| 8    | 3.736191607656845E-001  | –2.83446397890703E-002   |
| 9    | 7.499707281247036E-001  | –1.30228976420581E-004   |
| 10   | 6.009438691088970      | 2.06048984551562E-006    |

### Table 14. Variational parameters and expansion coefficients for the radial WF in the $n^{16}$O system for $5/2^+$ ground state potential (21) (the WF normalization on the interval of 0 to 30 fm is $N = 9.9999999999603E–001$)

| $i$  | $\alpha_i$ | $C_i$                        |
|------|------------|------------------------------|
| 1    | 2.970820484267648E-002 | 5.99989848526680E-002 |
| 2    | 1.355376641105716E-002 | –5.268187781652860E-006 |
| 3    | 2.971441871730051E-002 | –6.024647870785407E-002 |
| 4    | 6.553466412237838E-002 | –3.087819670804185E-003 |
| 5    | 1.253814313181326E-001 | –1.90646276257192E-002 |
| 6    | 2.156627509028788E-001 | –6.58529656252988E-002 |
| 7    | 3.393826502547065E-001 | –1.06636861923295E-001 |
| 8    | 5.166966410860497E-001 | –4.62770949895152E-002 |
| 9    | 1.063579836670607   | –1.11668337203532E-003 |
| 10   | 1.639614546923715   | 2.44047137526920E-004    |

### Table 15. Variational parameters and expansion coefficients for the radial WF in the $n^{16}$O system for first excited $1/2^+$ state potential (22) (the WF normalization on the interval of 0 to 30 fm is $N = 9.999999975230215E–001$)

| $i$  | $\alpha_i$ | $C_i$                        |
|------|------------|------------------------------|
| 1    | 1.268144327251019E-002 | 7.01423164869143E-003 |
| 2    | 4.193709029136675E-003 | 4.794229104701904E-005 |
| 3    | 2.881642596445157E-002 | 5.67776979981124E-002   |
| 4    | 6.245243687002310E-002 | 1.5703099321747E-001   |
| 5    | 1.259974114760052E-001 | 2.619817762571221E-002 |
| 6    | 2.163927868868810E-001 | –4.8335194604935E-002       |
| 7    | 3.38330162751630E-001  | –7.52945579180532E-002       |
| 8    | 5.187698913796229E-001  | –4.06251868776323E-001       |
| 9    | 1.062316903143099      | –5.10691880411997E-003       |
| 10   | 1.867671209905880      | 4.23549037636246E-004       |
The cross sections for these reactions are comparable only at an energy about 100 keV, being at a level of 10$\mu$b. Therefore, the neutron capture by the $^{14}$C nucleus plays a large part in primordial nucleosynthesis, i.e., at high particle interaction energies.

### Classification of States and Potentials of the $n^{14}$C Scattering

Turning to the analysis of total cross sections for the $n^{14}$C and $n^{14}$N capture with the formation of the $^{15}$C and $^{15}$N GSs, we note that we discussed the classification of orbital states of the $^{14}$C nucleus in the $n^{13}$C system or the $^{14}$N nucleus in the $p^{13}$C channel by the Young diagrams in [118, 130] and in the previous section. However, since we do not have complete tables of Young diagram products for systems of more than eight particles [32], which we earlier used for similar calculations [38], the results obtained below should be regarded as only a qualitative estimate of possible orbital symmetries in the ground states of the $^{15}$C and $^{15}$N nuclei in the channels under consideration. At the same time, it is a similar classification that allowed quite reasonable explanation of the available experimental data on the radiative $n^{13}$C [130] and $p^{13}$C captures [118]. Here, therefore, we will also use the classification of cluster states by orbital symmetries, which leads to a certain number of FSs and ASs in partial intercluster potentials, and thus to a certain number of nodes in the wave function for the relative motion of clusters.

We assume that for $^{14}$C we can take the orbital Young diagram $\{4442\}$, and for the $n^{14}$C system within the $1p$ shell we thus have $\{1\} \times \{4442\} \rightarrow \{5442\} + \{4443\}$ [92]. The former of the diagrams is compatible with the orbital momentum $L = 0$ and 2 and is forbidden because there cannot be five nucleons in the $s$ shell. The latter diagram is allowed and compatible with the orbital momentum $L = 1$ [92]. Thus, confining ourselves to only the lowest partial waves with the orbital momentum $L = 0$ and 1, we can say that for the $n^{14}$C system the $2S_{1/2}$ wave potential has a forbidden bound state and the $2P_{1/2}$ wave has only an AS at the $n^{14}$C system binding energy $-1.21809$ MeV [119]. At the same time, elastic $2P$ scattering wave potentials do not have FSs. For the $n^{14}$C system, the $2S_{1/2}$ scattering wave potential has a forbidden bound state and the $3P_{1/2}$ wave has only an AS at the $n^{14}$C system binding energy $-10.8333$ MeV [119].

Now let us consider the radiative capture in the $n^{14}$C system at energies from 20 keV to $\sim 1.0$ MeV, for which there are experimental data cited in the data-

### Table 16. Characteristics of nuclei and cluster systems and references to the works where radiative capture processes with their participation were investigated

| No. | Nucleus ($J^\pi$, $T$) | Cluster channel | $T_z$ | $T$ | Ref. |
|-----|------------------------|-----------------|------|-----|------|
| 1.  | $^3$He(1/2, +1/2) | $p^2$H | +1/2 + 0 = +1/2 | 1/2 | [2, 38] |
| 2.  | $^3$He(1/2, -1/2) | $n^2$H | -1/2 + 0 = -1/2 | 1/2 | [55] |
| 3.  | $^3$He(0, +0) | $p^3$H | +1/2 - 1/2 = 0 | 0 + 1 | [2, 6] |
| 4.  | $^6$Li(1, +0) | $^2$H$^4$He | 0 + 0 = 0 | 0 | [2, 6] |
| 5.  | $^7$Li(3/2, -1/2) | $^3$H$^4$He | -1/2 + 0 = -1/2 | 1/2 | [2, 6] |
| 6.  | $^7$Be(3/2, -1/2) | $^3$He$^4$He | +1/2 + 0 = +1/2 | 1/2 | [2, 6] |
| 7.  | $^7$Li(3/2, +1/2) | $\rho^6$Li | +1/2 + 0 = +1/2 | 1/2 | [157] |
| 8.  | $^7$Be(0, +0) | $\rho^7$Li | +1/2 - 1/2 = 0 | 0 + 1 | [13] |
| 9.  | $^8$Li(2, +1) | $n^7$Li | -1/2 - 1/2 = -1 | 1 | [99] |
| 10. | $^{10}$B(3, 0) | $p^9$Be | +1/2 - 1/2 = 0 | 0 + 1 | [2, 6] |
| 11. | $^{10}$Be(0, +1) | $n^9$Be | -1/2 - 1/2 = -1 | 1 | [158] |
| 12. | $^{13}$N(1/2, -1/2) | $p^{12}$C | +1/2 + 0 = +1/2 | 1/2 | [117] |
| 13. | $^{13}$C(1/2, -1/2) | $n^{12}$C | -1/2 + 0 = -1/2 | 1/2 | [130] |
| 14. | $^{14}$N(1/2, -1) | $p^{13}$C | +1/2 - 1/2 = 0 | 0 + 1 | [118] |
| 15. | $^{14}$C(0, +1) | $n^{13}$C | -1/2 - 1/2 = -1 | 1 | [130] |
| 16. | $^{15}$C(1/2, 3/2) | $n^{14}$C | -1/2 - 1 = -3/2 | 3/2 | [159] |
| 17. | $^{15}$N(1/2, -1/2) | $n^{14}$N | -1/2 + 0 = -1/2 | 1/2 | [159] |
| 18. | $^{16}$N(2, -1) | $n^{15}$N | -1/2 + 1/2 = -1 | 1 | [160] |
| 19. | $^{16}$O(0, +0) | $^4$He$^{12}$C | 0 + 0 = 0 | 0 | [161] |
| 20. | $^{17}$O(5/2, +1/2) | $n^{16}$O | -1/2 + 0 = -1/2 | 1/2 | This work |
base [67]. Constructing the $^2P_{1/2}$ wave potential for $n^{14}C$ scattering without the FS, we bear in mind that the $J^p = 1/2^+$ resonance occurs in the spectra of the $^{15}C$ nucleus only at 3.103(4) MeV relative to the GS, or about 1.9 MeV (c.m.s.) above the $n^{14}C$ channel threshold, and its width is $\approx 40$ keV [119]. Since we consider only the energy range below 1.0 MeV, the $^2P_{1/2}$ scattering phase in this energy region can be assumed to be zero. Consequently, the depth of the potential $V_0$ can also be set to zero because it has no forbidden states. It is this variant of the entrance $n^{14}C$ channel interaction that we are going to use in our calculations. The same applies to the $^2P_{3/2}$ Scattering wave potential because the corresponding resonance of the $^{13}C$ nucleus is at an even higher energy of 4.66 MeV [119].

The bound $^2S_{1/2}$ state potential with one FS should correctly reproduce the binding energy of the $^{15}C$ ground state with $J^p = 1/2^+$ in the $n^{14}C$ channel at $-1.21809$ MeV [119], reasonably describe the root-mean-square radius of the $^{15}C$ nucleus (which appears to be not much larger than the radius 2.4962(19) fm of the $^{14}C$ nucleus [119]), and reproduce the AC in the $n^{14}C$ channel that was earlier obtained in independent investigations. This $^2S_{1/2}$ state is associated with the $^{13}C$ GS in the $n^{14}C$ channel because $^{14}C$ has the momentum $F = 0^+$ [119]. To fulfill the above conditions, we propose the variant of the $n^{13}C$-channel $^{15}C$ GS potential with one FS and parameters

$$V_{g.s.} = -93.581266 \text{ MeV}, \gamma_{g.s.} = 0.2 \text{ fm}^{-2},$$

which leads to the AC of 1.85(1) on the interval of 7 to 27 fm and gives the binding energy $-1.2180900 \text{ MeV}$ at the FDM accuracy of $10^{-7}$ MeV, charge radius 2.52 fm, and mass radius 2.73 fm. The charge radius of the neutron was taken to be zero and its mass radius was set equal to the proton radius [21]. The error of the constant was determined by its averaging over the above distance interval.

In [131] the value given for the AC with a reference to [132] is 1.13 fm$^{-1/2}$, which is nondimensionalized with $\sqrt{2k} = 0.686$ to 1.65. The AC values are reviewed in detail in one of the most recent works on determination of ACs from the characteristics of various reactions [133]. It is shown that these values fall within the range of 1.22(6) fm$^{-1/2}$ to 1.37(5) fm$^{-1/2}$, or 1.8 to 2.0 when nondimensionalized, of which the recommended value is 1.87(13) in complete agreement with the value that we obtained.

Calculations of the $^{15}C$ GS energy with the above potential were additionally checked using the variational method [24], which yielded the energy $-1.2180898 \text{ MeV}$ on the grid of dimension $N = 10$ with independently varied parameters for BS potential (16). Parameters of the variational radial WF are presented in Table 12. Residuals are no larger than $10^{-12}$ [24]. The charge radius and the AC in the region of 5 to 25 fm do not differ from those obtained in the above FDM calculations.

As was already said, the average value $-1.2180899(1) \text{ MeV}$ from the FDM and VM calculations can be taken for the correct binding energy in this potential. Thus, the accuracy of determining the two-body binding energy of the $^{13}C$ GS in the $n^{14}C$ channel for potential (16) using two different computer codes, each based on different numerical methods, is at a level of ±0.1 eV.

**Total Cross Sections for the $n^{14}C$ Capture**

Before starting to describe the results of our calculations, we note that the available experimental data on the total radiative $n^{14}C$ capture cross sections [134–138] obtained with the aid of the database [67] show large ambiguities in these cross sections measured by different authors. For example, at an energy of 23 keV [135, 137] the cross sections are different by a factor of 3, and various data for the energy range of 100 to 1000 keV show a factor of 3 to 4 uncertainty [134, 136–138]. The experimental data from the above works for the energy range of 23 keV to 1.0 MeV are presented in Figs. 15 and 16.

In this system we consider only the capture to the $^{13}C$ GS because it was shown in [139] that the contribution from the radiative capture with the transition to the first excited state with $J^p = 5/2^+$ was 25 to 30 times smaller and could be ignored in view of the errors and uncertainties in the total cross section measurements. As earlier in [118, 130], we described total cross sections considering the $E1$ transition from the $^2P_{1/2}$ and $^2P_{3/2}$ scattering waves, nonresonant at energies below 1.0 MeV, with the zero-depth potential without FSs, i.e., with zero scattering phase shifts, to the bound $^2S_{1/2}$ ground state of the $n^{14}C$ clusters in the $^{15}C$
The results of calculating total $n^{14}$C capture cross sections with GS potential (16) at energies below 1.0 MeV are shown in Figs. 15 and 16 by solid curves; in Fig. 16 they begin at 1 eV. The results of the calculations lie between the known experimental data for the energy region from 23 keV to 1.0 MeV and seem to agree best with the results [137] obtained in the late 2000s.

It is evident from the figures that total capture cross sections entirely depend on the shape of the potential for the $^{15}$C ground state in the $n^{14}$C channel because, at the energies under consideration, the entrance-channel $^{2}P$ potentials without FSs can be simply set equal to zero. Thus, the BS potential allowing a reasonable description of the available experimental data on total radiative capture cross sections also leads to a correct description of the main GS characteristics, namely, the binding energy, charge radius, and AC.

Note that if we use the $^{15}$C GS potential without FSs with the parameters, say,

$$V_{g.s.} = -19.994029 \text{ MeV}, \gamma_{g.s.} = 0.2 \text{ fm}^{-2},$$

which leads to the binding energy $-1.218090 \text{ MeV}$, AC 1.46(1) on the interval of 5 to 30 fm, charge radius 2.51 fm, and mass radius 2.63 fm, we obtain the calculated total cross sections shown in Fig. 16 by the dashed curve running well below all the experimental data.

To allow the results that on the whole correctly describe the experimental data, the $^{15}$C GS potential without FSs must have the parameters

$$V_{g.s.} = -4.593639 \text{ MeV}, \gamma_{g.s.} = 0.02 \text{ fm}^{-2},$$

which leads to a very large width and small depth of the interaction. The accordingly calculated cross sections are shown in Fig. 16 by the dotted curve, which practically coincides with the solid curve. This potential leads to the charge radius 2.53 fm and mass radius 2.92 fm, and its AC on the interval of 15 to 30 fm turns out to be 3.24(1), which noticeably differs from the results for potential (16) and the findings in [131–133].

Since at energies from 1 eV to 1 keV the calculated cross section is nearly a straight line (solid curve in Fig. 16), it can be approximated by a simple function of the form

$$\sigma_{n}(\mu b) = 0.7822 \sqrt{E_{c.m.}(\text{keV})}.$$  

The reduced constant value 0.7822 $\mu b$ keV$^{-1/2}$ was determined from one point at the minimum energy of 1 eV (c.m.s.). Then it turned out that the modulus $M(E)$ of the relative deviations of calculated theoretical cross section (4) and its approximation by the function at energies below 1 keV is no larger than 0.4%. If we assume that this shape of the total cross section dependence on energy will be retained at even lower energies as well, we can evaluate the cross section, obtaining, e.g., at an energy of 1 meV ($10^{-3}$ eV = 10$^{-6}$ keV), the value $0.78 \times 10^{-3} \mu b$.

Since total cross sections for all processes of radiative neutron capture by light nuclei that we considered were measured at energies beginning with the region of 5 to 25 meV [130], it would be interesting to measure cross sections for this capture reaction at energies of 1 eV to 1 keV, for which the above formula predicts values of 0.025 to 0.782 $\mu b$.

### Potentials of the $n^{14}$N Scattering

Since the $^{14}$N nucleus has $^{2}F$, $T = 1^{+}$, 0 [119], the ground state of the $^{14}$N nucleus with $^{2}F$, $T = 1/2^{+}$, 1/2 can be represented as a mixture of the doublet $^{2}P_{1/2}$ and quartet $^{4}P_{1/2}$ states. Therefore, the transitions to be considered below as the main ones will be the $E1$ transitions from the doublet $^{2}S_{1/2}$ and quartet $^{4}S_{1/2}$ scattering waves with one bound FS, which are nonresonant at energies up to 0.5 – 0.6 MeV, to the GS, $^{2}S_{1/2} + ^{4}S_{1/2} \rightarrow ^{2+4}P_{1/2}^*$; i.e., the following total capture reaction cross sections will be calculated:

$$\sigma_{0}(E1) = \sigma(E1, ^{2}S_{1/2} \rightarrow ^{2}P_{1/2}^*) + \sigma(E1, ^{4}S_{1/2} \rightarrow ^{4}P_{1/2}^*).$$

Thus, as in the case of the $n^{7}$Li capture, we consider transitions to the doublet and quartet components of the GS WF, which do not differ in this approach and correspond to the BS in one potential. We limit ourselves to the transitions to BSs with the minimum $J^{*} = 1/2^{±}$. At the energy of 9.2221 MeV the spectrum of the $^{15}$N nucleus features an excited $^{2}F = 1/2^{−}$-level that is bound in the $n^{14}$N channel at $-1.6112 \text{ MeV}$. We can therefore consider additional transitions $^{2}S_{1/2} +$
\[ ^2S_{1/2} + ^4S_{3/2} \rightarrow ^2P^+_1, \text{ and the corresponding total cross sections} \]

\[
\sigma(E1) = \sigma(E1, ^2S_{1/2} \rightarrow ^2P^+_1) \\
+ \sigma(E1, ^4S_{3/2} \rightarrow ^4P^+_1).
\]

In addition, we will consider possible transitions from the \( P_{1/2} \) and \( P_{3/2} \) scattering states to the second, seventh, and ninth excited states of the \( ^{15}\text{N} \) nucleus with \( J = 1/2^+ \) at the energies 5.298822, 8.3126, and 9.04971 MeV, which are bound in the \( n^{14}\text{N} \) channel and which we can refer to the doublet \( ^2S_{1/2} \) wave. We will consider the \( ^2+4P_{1/2} \) scattering state, which has a resonance at 492.6(0.65) keV with a width of \( \sim 8(3) \) keV (see, for example, Table 15.4 in [119], a resonance at 11.2928(7) MeV). The nearby resonant state at 430(5) keV with \( J \geq 3/2 \), width about 3 keV (energy 11.235(5) MeV in Table 15.4 [119]), and so far unknown parity [119] will be ignored. Potentials of \( ^2+4P_{1/2} \) waves will be taken to be zero for they do not have FSs and resonances below 1.0 MeV. Then we can analyze processes like \( ^2P_{1/2} + ^2P_{3/2} \rightarrow ^2P^+_1 \) and write total cross sections as

\[
\sigma(E1) = \sigma(E1, ^2P_{1/2} \rightarrow ^2S^1_{1/2}) + \sigma(E1, ^2P_{3/2} \rightarrow ^2S^1_{1/2}) \\
+ \sigma(E1, ^4P_{1/2} \rightarrow ^2S^1_{1/2}) + \sigma(E1, ^4P_{3/2} \rightarrow ^2S^1_{1/2}) \\
+ \sigma(E1, ^2P_{1/2} \rightarrow ^2S^3_{1/2}) + \sigma(E1, ^2P_{3/2} \rightarrow ^2S^3_{1/2}) \\
+ \sigma(E1, ^4P_{1/2} \rightarrow ^2S^1_{1/2}) + \sigma(E1, ^4P_{3/2} \rightarrow ^2S^1_{1/2}).
\]

Proceeding to construct potentials of all these states, we note that the doublet bound \( ^2S_{1/2} \) levels with one bound FS have distinctly different binding energies, and interaction potentials will be obtained for each of them. Potentials of the \( ^2S \) and \( ^4S \) scattering waves with one bound FS should also be explicitly different, but we failed to construct potentials that allow for resonances in those waves. Note that the first resonance in the \( ^2S_{1/2} \) wave is at 11.4376(0.7) MeV, \( J^* = 1/2^+ \), or 0.639(5) MeV (c.m.s.) above the threshold of the \( n^{14}\text{N} \) channel, width 34 keV (lab. syst.), and in the \( ^4S_{3/2} \) it is at 11.763(3) MeV, \( J^* = 3/2^+ \), or 0.998(5) MeV above the threshold of the \( n^{14}\text{N} \) channel, neutron width \( \sim 45 \text{ keV} \) (lab. syst.) [119]. We will therefore assume that they lead to near-zero scattering phase shifts and use identical parameters of potentials for them; i.e., these resonances are ignored. The \( ^2+4P \) scattering states or bound \( P \) levels are mixed in spin because the total momentum \( J^* = 1/2^- \) or \( J^* = 3/2^- \) can be obtained for both the \( ^2P \) and \( ^4P \) waves. Therefore, the potentials of the \( ^2+4P \) states are further constructed for the states with the total momentum \( J \), and they are then turn out to be mixed in spin.

As a result, parameters for the potentials of the doublet \( ^2S_{1/2} \) and quartet \( ^4S_{3/2} \) scattering waves with one bound FS were chosen such as to ignore of resonances in the given energy region

\[
V_S = -19.0 \text{ MeV, } \gamma_S = 0.06 \text{ fm}^{-2},
\]

and the calculations of the \( ^2S_{1/2} \) and \( ^4S_{3/2} \) phase shifts with this potential at energies up to 1.0 MeV yield the values in the region of \( 0 \pm 2^\circ \).

The parameters used for the \( ^2+4P_{1/2} \) wave potential without FSs and with a resonance at 493 keV are

\[
V_p = -13328.317 \text{ MeV, } \gamma_p = 50.0 \text{ fm}^{-2},
\]

and they lead to the resonance energy 493 keV at the level width 18.2 keV, which is slightly larger than the measured value [119]. Note that obtaining the correct level width would require an even narrower potential while the parameter of the width is already unusually large.

The potential of the bound \( ^2+4P_{1/2} \) state with FSs should correctly reproduce the binding energy of the \( ^{15}\text{N} \) GS in the \( n^{14}\text{N} \) channel at \(-10.8333 \text{ MeV} \) [119] and reasonably describe the root-mean-square \( ^{15}\text{N} \) radius, the experimental value of which is 2.612(9) fm [119], while the experimental \( ^{14}\text{N} \) radius is 2.560(11) fm [67]. As a result, the parameters obtained for the potential of the \( ^{15}\text{N} \) GS in the \( n^{14}\text{N} \) channel without FSs were

\[
V_{g.s} = -55.442290 \text{ MeV, } \gamma_{g.s} = 0.1 \text{ fm}^{-2}.
\]

The potential leads to the binding energy \(-10.83330001 \text{ MeV} \) at the FDM accuracy of \( 10^{-8} \text{ MeV} \), root-mean-square charge radius 2.57 fm, and mass radius 2.59 fm. The asymptotic constant nondimensionalized on the interval of 7 to 13 fm was found to be 4.94(1). In [81] its value is 5.69(7) fm\(^{-1/2} \), which comes to 4.81(6) after nondimensionalization with \( \sqrt{2/k} = 1.184 \).

The GS energy calculation was additionally checked using the variational method, which yielded the value 10.83330000 MeV on the grid of dimension even as small as \( N = 10 \) with independently varied parameters of potential (18). The parameters of the variational radial WF are presented in Table 13. The residuals are no larger than \( 10^{-8} \). The AC in the region of 7 to 14 fm turned out to be 4.9(1), and the charge radius does not differ from the FDM-calculated one.

As a result, the average value \(-10.83330005(5) \text{ MeV} \) can be taken for the correct binding energy in this potential. Thus, the accuracy of determining the binding energy of the \( ^{15}\text{N} \) nucleus for intercluster potential (18) obtained by two methods (FDM and VM) and using two different computer codes can be written as \( \pm 5 \times 10^{-9} \text{ MeV} = \pm 5 \text{ meV} \), which agrees with the specified FDM accuracy.

Parameters of the potential for the \( ^{15}\text{N} \) ES at an energy of \(-1.6112 \text{ MeV} \) in the \( n^{14}\text{N} \) channel with the angular momentum \( J^* = 1/2^- \) coinciding with the GS momentum are.
The potential leads to the binding energy $-1.783620$ MeV, charge radius 2.58 fm, mass radius 2.82 fm, and AC 2.78(1) on the distance interval of 8 to 27 fm.

**Total Cross Sections for the n$^{14}$N Capture**

Before describing the results of our calculations we note that all experimental data on the total cross sections for the radiative n$^{14}$N capture that we used in our work were taken from the database [67], and the data themselves are reported in [69, 140–142]. They were obtained for the energy region of 25 meV to 65 keV and are shown in Fig. 17, together with our results of calculating total cross sections for the radiative n$^{14}$N capture to the bound $2^+_1$ and $2^+_3$ states of the $^{15}$N nucleus with the above-mentioned potentials at energies below 1.0 MeV. The total cross sections at an energy of 25 meV obtained in different works fall within the interval of 77 to 80 mb. They are shown in Fig. 17 by one point. The most recent result for this energy is 80.3(6) mb [141].

The dashed curve in the nonresonance region in Fig. 17 is the calculation of the cross sections for the transition from the S scattering waves (16) to the GS (18), the dot-dashed curve is the calculation for the transition to the excited $2^+_3$ state with potential (19), and the dot-double-dash curve is their sum. The resonant component of the cross sections for the $^2P_{1/2}$ scattering potential in the form of (17) and the zero potential of the $^2P_{3/2}$ wave in the continuum is governed by transitions to the first, second, and third bound $^2S_{1/2}$ states. These cross sections are shown by the dashed, dotted, and dot-dashed curves; the solid curve in the resonance region, i.e., at ~5 to 1000 keV, is their sum. The other solid curve shows the cross sections after summing up all the contributions in the entire energy region from $10^{-5}$ to 1.0 MeV.

It is evident from Fig. 17 that our calculations fairly well describe the data on the capture cross sections at 25 meV but do not reproduce measurements at 65 keV [142], as shown by the open circle. This is probably because the transitions from the $^2S_{1/2}$ and $^4S_{1/2}$ resonant scattering waves with a relatively large width to the GS and the first excited $^2^+^4P_{1/2}$ state with $J^p=1/2^-$ are ignored.

If, for the sake of comparison, we use the $^2S_{1/2}$ and $^4S_{1/2}$ scattering potentials with zero phase shifts and a zero depth, which have no FSs, i.e., do not agree with the above FS and AS classification by Young diagrams, the cross sections calculated with GS potential (18) and ES potential (19) are more than one order of magnitude higher than all the experimental data.

It is evident from the results that with the above-considered combination of potentials (16) and (18), it is well possible to obtain consistent descriptions of the total cross sections for the radiative capture at the lowest energy and the BS characteristics, including AC, of...
the $^{15}$N nucleus in the $n^{14}$N channel. In other words, if the parameters of the potential of the $^{15}$N GS in the $n^{14}$N channel are fixed on the basis of the correct description of the $^{15}$N characteristics, including the AC, then, based on the FS and AS classification by Young diagrams, we can find $^{2+4}S_{1/2}$ scattering potentials such that they will allow a correct description to be obtained both for the elastic scattering phase shifts close to zero and for the total cross sections of the radiative $n^{14}$N capture at 25 meV. Further measurements of total cross sections at other energies would more clearly reveal the quality of description of these cross sections with the potentials with FSs within the model under consideration.

Since at energies from 10 meV to 10 keV the calculated cross sections shown in Fig. 17 by the solid curve are close to a straight line, they can be approximated by a function of energy in the form of Eq. (3). The constant value 406.4817 µb keV$^{1/2}$ was determined from one point in the cross sections at the minimum energy of 10 meV (lab. syst.). Modulus (4) of the relative deviation of the calculated theoretical cross section and its approximation by function (3) in the region up to 10 keV is no larger than 0.9%. Evaluation of the cross section, for example, at an energy of 1 µeV ($10^{-6} \text{ eV} = 10^{-9} \text{ keV}$) yields the value about 12.8 b.

RADIATIVE CAPTURE IN THE $n^{16}$O SYSTEM

The next process to be considered is $^{16}$O$(n, \gamma)^{17}$O, which enters into the primordial nucleosynthesis chain (1) and is of special interest because it is a reaction on the last $1p$-shell nucleus with the formation of the $^{17}$O nucleus, which is beyond this shell. It is assumed that the $^{17}$O BS is determined by the cluster channel consisting of initial particles that participate in the reaction, i.e., of $n^{16}$O clusters. Before analyzing total $n^{16}$O capture cross sections, we consider classification of orbital $n^{16}$O system states by Young diagrams. The Young diagram corresponding to the bound ground state of the $^{16}$O nucleus is $\{4444\}$ [2, 18, 117]. Therefore, for the $n^{16}$O system we have $\{1\} \times \{4444\} \rightarrow \{5444\} + \{44441\}$ [92]. The first of the diagrams obtained is compatible with the orbital momentum $L = 0$ and is forbidden because there cannot be five nucleons in the $s$ shell. The second diagram is allowed and compatible with the orbital momentum $L = 1$ [92].

Thus, in the potential of the $^{2}S_{1/2}$ wave, which corresponds to the first excited state of the $^{17}$O nucleus in the $n^{16}$O channel and scattering states of these clusters, there is a forbidden bound state, and the $^{3}P$ scattering wave does not contain FSs, but the allowed state can be in both the continuous and the discrete spectrum. The ground state of the $^{17}$O nucleus in the $n^{16}$O channel at an energy of $-4.1436 \text{ MeV}$ [143] belongs to the $^{2}D_{3/2}$ wave and does not have BSs either. However, as was already said, we do not have complete tables of Young diagram products for a system of more than eight particles [32], which we used earlier for similar calculations [2, 3, 6, 8, 15]. Therefore, the above result should be regarded as only a qualitative estimate of possible orbital symmetries in the bound states of the $^{17}$O nucleus for the $n^{16}$O channel.

Phase shifts and Potentials of the $n^{16}$O Scattering

Calculation of radiative capture cross sections within the PCM requires knowing potentials of the $n^{16}$O elastic scattering in the $^{2}S_{1/2}$, $^{2}P_{1/2}$, $^{2}D_{3/2}$, and $^{2}D_{5/2}$ waves and interactions of the ground $^{2}D_{5/2}$ and first excited yet bound $^{3}S_{1/2}$ state of the $^{17}$O nucleus in the $n^{16}$O channel. It is for the transition to these BSs that the experimental data on total radiative capture cross sections are available [144].

As was mentioned, scattering potentials are constructed on the basis of the elastic scattering phase shifts obtained at energies above 1.1 MeV in [145, 146]. For the energy region of 0.2 to 0.7 MeV, there are results of the phase shift analysis [147] based on the measurements of the differential cross sections for the elastic $n^{16}$O scattering [148] in the region of the resonance at 0.433 MeV [143]. Later, new experimental data [149] on the excitation function at energies from 0.5 to 6.2 MeV appeared in the EXFOR database [68]. As far as we know, they have never been used in phase shift analyses at energies in the $^{2}D_{3/2}$ resonance region of 1.0 MeV [143].

We used the data [149] for performing the phase shift analysis and extracting the shape of the phase in the $^{2}D_{3/2}$ wave of the $n^{16}$O scattering. The relevant excitation functions at $40^\circ$ (lab. syst.) [149] are shown in Fig. 18 by open circles in the energy range from 0.75 to 1.25 MeV (lab syst.). Experimental errors, which
amount to 25% at some points, are not shown in the figure. They would make it overcrowded because, in our analysis, we used more than 500 data points for the cross sections at different energies from the excitation functions obtained in [149].

Note that below 0.7 to 0.8 MeV the data [149] show dramatically increasing ambiguity, but the energy region shown in Fig. 18 is quite enough to extract the \( ^2D_{3/2} \) scattering phase shift. Ambiguities are relatively small in this region, and these data can well be used for the phase shift analysis. Earlier, we already performed similar phase shift analyses of the \(^n\)\(^{12}\)C [95], \(^p\)\(^{12}\)C [93], \(^p\)\(^6\)Li [63], and \(^p\)\(^{13}\)C systems [94, 150], mainly at astrophysical energies.

The method that we used to search for phase shifts in the elastic scattering of particles with spin 1/2+0 is detailed in [7], the main expressions can be found in [7, 23], and the results of our analysis for the elastic \(^n\)\(^{16}\)O scattering in the energy region from 0.75 to 1.25 MeV are shown in Fig. 19 by open circles. Filled squares in Fig. 19 are the results of the phase shift analysis from [145] obtained at energies above 1.1 MeV, and triangles are the results of the analysis from [147].

Since only one point is considered in the cross sections for each energy, \( \chi^2 \) has an average value \( 4.7 \times 10^{-3} \), while the maximum partial value is \( \chi_i^2 = 0.6 \) at the energy of 999.5 keV. Description of cross sections in excitation functions [149], at least at energies up to 1.2 to 1.25 MeV, does not require considering the \( ^2S_{1/2} \) scattering phase because it does not change the value of \( \chi^2 \) and thus can be set to zero.

The \( ^2D_{3/2} \) phase obtained from the phase shift analysis can be described using a simple Gaussian potential without FSs with the parameters

\[
V_D = -95.797 \text{ MeV}, \gamma_D = 0.17 \text{ fm}^{-2},
\]

which leads to the resonance energy 1000 keV at the phase \( 90.0(0.1)^\circ \) with the level width 88 keV (lab. syst.) or 83 keV (c.m.s.). At the same time, the width in Table 17.17 [149] is 96 keV (c.m.s.) or 102 keV (lab. syst.) at the level energy \( 1000 \pm 2 \) keV (lab. syst.).

The energy dependence of the \( ^2P_{3/2} \) phase of potential (20) is shown in Fig. 19 by the solid curve. This potential describes well the behavior of the scattering phase in the region of the resonance and agrees with the previous extractions of the scattering phase shift [145, 147]. The shape of the cross sections in the excitation functions, which is calculated with the \( ^2D_{3/2} \) phase shift of potential (20) at zero values of other phase shifts, is shown in Fig. 18 by the solid curve.

We will further consider total cross sections for radiation capture with \( E1 \) transitions from the \( ^2P_{3/2} \) resonance at 433 keV in the \(^n\)\(^{16}\)O scattering to the ground and first excited states of the \(^{17}\)O nucleus. In constructing the \( ^2P_{3/2} \) scattering potential, we not only used the data on the position and width of this level from [143] (see Table 17.17), but also the results of the phase shift analysis from [147] shown by triangles in Fig. 20. We finally found that the description of the \( ^2P_{3/2} \) resonant scattering phase at 433(2) keV (lab. syst.) requires the potential without FSs with the parameters

\[
V_p = -1583.545 \text{ MeV}, \gamma_p = 6.0 \text{ fm}^{-2},
\]
which leads to the level width 44 keV (c.m.s.), or 47 keV (lab. syst.), at the resonance of 433 keV (lab. syst.), i.e., its phase at this energy is 90.0(0.2)°. The total phase shift dependence on the energy in the region of the resonance is shown in Fig. 20 by the solid curve. For the potentials of the $^2P_{1/2}$ and $^2D_{5/2}$ scattering waves, we used $V_O = 0$ MeV, i.e., zero scattering phase shifts, because the levels with $J = 1/2^-$ and $5/2^+$ are not observed in the $^{17}$O spectra below 1.0 to 1.3 MeV and the potentials in question do not have FSs.

Note again that the potential is uniquely constructed from the known energy of the resonance level in the $^{17}$O spectra (see Table 17.17 [143]) and its width. It is impossible to find any other parameters $V_O$ and $\gamma$ which could correctly reproduce the resonance energy and width of the level at the specified number of FSs, which is zero in this case. The depth of this potential uniquely determined the position of the resonance, i.e., the resonance energy of the level, and its width specifies the width of this resonant state.

Since we consider electromagnetic transitions to the bound ground state with $J^P$, $T = 5/2^-$, $1/2$ at the energy of $-4.1436$ MeV and the first excited state with $J^P = 1/2^+$ at $-3.2729$ MeV of the $^{17}$O nucleus in the $^{16}$O channel [143], we need interaction potentials of $^{16}$O clusters in the BS. The widths of these potentials are fixed such as to correctly describe the binding energy of the level and its charge radius in the $^{17}$O nucleus, which is 2.6932(75) fm in the GS [151]. Then the asymptotic constants of the $^{16}$O channel calculated with them are compared with other data.

As a result, the parameters found for the FS-free $^2D_{5/2}$ potential of the $^{17}$O GS in the $^{16}$O channel are

$$V_{D0} = -102.2656782 \text{ MeV, } \gamma_{D0} = 0.15 \text{ fm}^{-2}.$$  

with which the binding energy $-4.1436000$ MeV at the accuracy of $10^{-7}$ MeV, charge radius $2.71$ fm, and mass radius $2.73$ fm were obtained, and the AC on the interval of 6 to 16 fm turned out to be $C_W = 0.75(1)$ [25]. The charge radius of the neutron was taken to be zero, its mass radius was set equal to the proton mass radius $0.8775(51)$ fm [49], and the value used for the $^{16}$O charge radius was $2.710(15)$ FM [143]. In [131] the GS AC was found to be 0.9 fm$^{-1/2}$, which comes to 0.96 after nondimensionalization using $\sqrt{2k} = 0.933$. In [25] the value 0.77(8) is given for the GS, which in view of the errors exactly agrees with the above value.

The parameters obtained for the $^2S_{1/2}$ potential of the first excited state of the $^{17}$O nucleus in the $^{16}$O channel with one FS are

$$V_{S1} = -81.746753 \text{ MeV, } \gamma_{S1} = 0.15 \text{ fm}^{-2}.$$  

They lead to the binding energy $-3.2729000$ MeV relative to the $^{16}$O channel threshold or $0.8707$ MeV relative to the $^{17}$O GS at the accuracy of $10^{-7}$ MeV, charge radius $2.71$ fm, mass radius $2.80$ fm, and AC on the interval of 6 to 17 fm $C_W = 3.09(1)$. In [131] the AC for this level was found to be $3.01 \text{ fm}^{-1/2}$, which comes to 3.22 after nondimensionalization using $\sqrt{2k} = 0.934$. As is seen, the asymptotic constants agree with an accuracy of ~4%.

The calculations of the binding energy were additionally checked using the variational method [24], which yielded the GS energy $-4.1435998$ MeV on the grid of dimension $N = 10$ with the parameters of potential (22) independently varied. Parameters of the variational WF are presented in Table 14. Residuals are no larger than $10^{-8}$ [24]. The charge radius and the AC on the interval of 6 to 16 fm do not differ from those calculated within the FDM.

As was already said, the average value $-4.1435999(1)$ MeV can be taken for the correct binding energy in this potential. Thus, the accuracy of determining the binding energy by two methods using different computer codes is at a level of $\pm 0.1$ eV in full agreement with the binding energy search error of $10^{-7}$ MeV specified in the FDM code.

The energy obtained for the first BS on the grid of dimension $N = 10$ with independently varied parameters of potential (23) is $-3.2728998$ MeV. Parameters of the variational WF are presented in Table 15. Residuals are no larger than $10^{-10}$ [24]. The charge radius and AC on the interval of 6 to 20 fm do not differ from those obtained in the FDM calculations. Here again we can take the average value $-3.2728999(1)$ MeV for the correct binding energy because the accuracy of the energy determination by two methods using two different computer codes is also at a level of $\pm 0.1$ eV = $\pm 100$ meV.

Total Cross Sections for Radiative Capture

The radiative $^{16}$O capture reaction was earlier considered within the direct capture model in [108], where the possibility of describing the available experimental data [144] in the region of 20 to 280 keV was shown. Later, on the basis of the folding model [152], it was demonstrated that it was possible to describe the available experimental data [144] in the energy region of 20 to 60 keV. Then in [153] a correct description of the total cross sections in the region of 20 to 280 keV was obtained using the generator coordinate method (GCM) by considering the $E1$ transition alone. Finally, in [154] the experimental total cross sections at energies from 25 meV [155] to 280 keV [144], i.e., to the resonance at 433 keV corresponding to the $^2P_{3/2}$ wave of the $^{16}$O scattering [143], were on the whole correctly reproduced using the GCM and the microscopic $R$-matrix method involving $E1$ and $M1$ processes, and the possible behavior of the cross sections in the region of the $^2D_{5/2}$ resonance was predicted. However, energies in the region near the $^2P_{3/2}$ resonance have not been considered so far, though rather new experimental data [83] have been obtained at the corresponding energies from 160 to 560 keV.
from the resonant $2^D_{3/2}$ wave to the first excited $2^S_{1/2}$ level, $M1$ from the nonresonant $2^D_{3/2}$ scattering wave to the ground $2^D_{1/2}$ state, etc., turned out to be two to three orders of magnitude smaller. The measurements in the region of 20 to 280 keV [144] made it possible to separately consider transitions to the GS as well as to the first ES. In addition, the measurements [144] revealed domination of the capture to the first ES over the capture to the GS of the $^{17}$O nucleus.

Our calculations of the total cross sections for $M1$ and $E1$ transitions to the GS with potentials (20)–(22) are shown by the solid curve in Fig. 21, together with the experimental data [83, 144]. The cross sections for the transition from the $2^D_{3/2}$ scattering wave at energies below 1.3 MeV, i.e., in the region of the $2^D_{3/2}$ resonance, are shown by the dashed curve, and the cross sections for the $E1$ transition from the $2^P_{3/2}$ scattering wave to the GS are shown by the dotted curve. It is evident from Fig. 21 that the calculations with the $M1$ and $E1$ transitions quite reasonably reproduce the experimentally measured total cross sections [144] for the capture to GS of the $^{17}$O nucleus, which smoothly decrease with decreasing energy. The potentials of the $2^P_{3/2}$ and $2^D_{3/2}$ scattering waves and the bound $2^D_{3/2}$ state of the $n^{16}$O system of the clusters that have no FSs were constructed on the basis of simple assumptions that the scattering potential matches the scattering phase and the BS potential matches the main $^{17}$O BS characteristics (binding energy, charged radius, AC). In Fig. 21, filled squares are the measurements of the total cross sections in the resonance region of 160 to 560 keV from [83], where combined data for transitions to the GS and the first excited state seem to be presented.

Our calculations of the cross sections for the $E1$ transitions from the $2^P_{3/2}$ and $2^P_{1/2}$ scattering waves to the first ES of the $^{17}$O nucleus in the energy region from 10 keV to 1.3 MeV, neglecting the second resonance at 1312 keV in the $2^P_{3/2}$ wave [143], are shown in Fig. 22 by the solid curve. The filled circles in the figure are the measurements of the cross sections [144] for the transition to the first excited $1/2^+$ level in the energy region of 20 to 280 keV and the filled squares are the measurements of the total cross sections from [83]. It is evident from the figure that measurements [83] agree better with earlier experimental data for transitions to the first ES [144] and are reasonably described in our calculations at 20 to 560 keV.

It is further evident from the above-discussed classification of FSs by Young diagrams that there can be an allowed BS in the $P$ wave. Therefore, by analogy with the earlier considered $n^{12}$C system [156], we can assume that the second excited state with $J^P = 1/2^-$ at 3.055 MeV relative to the GS can belong to the $2^P_{1/2}$ wave and be the scattering potential BS. Then we should admit the presence of a bound AS in this partial

Total cross sections for other possible transitions, e.g., $E2$ from the resonant $2^D_{3/2}$ wave to the first excited $2^S_{1/2}$ level, $M1$ from the nonresonant $2^D_{3/2}$ scattering wave to the ground $2^D_{1/2}$ state, etc., turned out to be two to three orders of magnitude smaller. The measurements in the region of 20 to 280 keV [144] made it possible to separately consider transitions to the GS as well as to the first ES. In addition, the measurements [144] revealed domination of the capture to the first ES over the capture to the GS of the $^{17}$O nucleus.
wave, which should lead as previously to the zero scattering phase shift.

In this case we take, for example, the $^2P_{1/2}$ potential parameters to be slightly deeper than for the interaction with $J^P = 3/2^-$.

$$V_{1/2} = -1593.43535 \text{ MeV}, \gamma_{1/2} = 6.0 \text{ fm}^{-2}. \quad (21)$$

This potential yields near-zero phase shifts in the region below 1 MeV and leads to the binding energy $-1.08824$ MeV relative to the $n^{16}O$ threshold at the FDM accuracy of $10^{-5}$ MeV. The charge radius of the $^{17}O$ nucleus in the second excited $1/2^-$ state is 2.70 fm, the mass radius is 2.65 fm, and the AC is 0.22 on the interval of 2 to 18 fm. The calculation of the total cross sections for the $E1$ capture $^2P_{1/2} + ^2P_{3/2} \rightarrow ^2S_{1/2}$ with this potential and potential (23) yields the results almost coinciding with the previous ones shown in Fig. 22 by the dashed curve.

We failed to find the AC for the second excited state at 3.055 MeV with $J^P = 1/2^-$, and the comparison with the AC obtained above is therefore impossible. To make the parameters of this scattering potential less ambiguous, we consider its variant with a wider interaction well

$$V_{1/2} = -270.71124 \text{ MeV}, \gamma_{1/2} = 1.0 \text{ fm}^{-2},$$

which also leads to the near-zero phase shifts, binding energy $-1.08824$ MeV at the FDM accuracy of $10^{-5}$ MeV, charge radius 2.70 fm, mass radius 2.69 fm, and AC 0.39 on the interval of 3 to 23 fm, which is nearly twice as large as the results of the previous potential. The total capture cross sections obtained with this $1/2^-$ wave potential are shown in Fig. 22 by the dotted curve. It is evident from these results that the cross section is not strongly dependent on whether there is a bound allowed state in the $1/2^-$ wave if the width of the potential is comparable with the interaction width in the $^2P_{3/2}$ wave and falls within the range of 1.0 to 6.0 fm, while the AC values fall within the range of 0.2 to 0.4. This transition cannot thus be used for uniquely choosing the form of the interaction potential in the $^2P_{1/2}$ scattering wave and finding out whether it has the allowed bound state with $J = 1/2^-$. The summed cross section for the $E1$ transitions to the $5/2^+$ GS and the first excited $1/2^+$ state is shown in Fig. 23. It is seen that the calculations reproduce quite well the data [144] while the measurements [83] are in general slightly below the calculated curve. Note that we do not consider possible $M1$ transitions at the lowest energies, which can slightly increase the total cross sections at the minimum energy shown in Fig. 23. When the $M1$ transition in the thermal energy region is considered in the description of the cross sections for the capture in the given cluster system, the above criteria are not enough to determine uniquely the $S$ potential. As will be seen below, its parameters have to be slightly modified for a better description of the experimental data [155] at the lowest energy of 25 meV.

Turning to the consideration of the low-energy region, we note that below 100 eV the capture cross section is entirely determined by the $M1$ transition from the $^2S_{1/2}$ scattering wave to the first excited $^3S_{1/2}$ state of the $^{17}O$ nucleus. As follows from the above classification of cluster states, the FS-containing potential of the $n^{16}O$ $^2S_{1/2}$ scattering wave must lead at the energies under consideration to an almost zero scattering phase, but since it has a FS, its depth cannot be zero. The shape of this potential was refined only for the correct description of the cross section at 25 meV, and its parameters turned out to be

$$V_S = -10.0 \text{ MeV}, \gamma_S = 0.03 \text{ fm}^{-2}. \quad (24)$$

The results of calculating total cross sections with allowance for the $M1$ transition at 10 meV to 1.0 MeV are shown in Fig. 24 by the solid curve. The triangles at 25 meV show the results of the experimental measurements [155], which fall within the interval of 150 to 200 µb. The dashed curve in Fig. 24 shows the cross sections for the $M1$ process alone, and the dotted curve shows the cross sections for the $E1$ transition at energies no higher than 10 meV. It is evident from the figure that the $E1$ cross section sharply decreases to become almost three times as small as the $M1$ transition cross section at 100 eV.

It should be emphasized that only the $^2S_{1/2}$ scattering potential with an FS allows a correct description of the total capture cross sections at 25 meV. The potential without an FS, whatever its parameters, is unable to reproduce correctly the behavior of the total cross sections at this energy. Namely, the $^2S_{1/2}$ scattering

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**Fig. 23.** Total cross sections for the $n^{16}O$ radiative capture of $E1$ multipolarity to the $5/2^+$ GS and the first excited $1/2^+$ state of the $^{17}O$ nucleus. The experimental data are from [144] (●) and [83] (■). The solid curve is our calculation of the total cross section with the potentials given in the text.
Thus, the above-discussed methods for constructing interaction potentials of clusters that comply with the classification of states by Young diagrams allow a generally correct description of the experimental data on the total radiative capture cross sections at energies from 20 to 560 keV. We obtained the $^2S_{1/2}$ scattering wave potential with an FS and a near-zero phase shift, which allows the behavior of the experimental cross sections at the lowest energies to be correctly described. It is shown that the description of the low-energy cross sections is only possible if the potential has a FS.

In the future, new experiments should be conducted to measure total capture cross sections at 1 eV to 1 keV, for which calculations predict a particular behavior with a smooth minimum at 0.4 keV and a value on the order of 3 mb (Fig. 24). In addition, a definite value of the second maximum in the cross sections is also obtained around 1.0 MeV, i.e., in the region of the $^2D_{3/2}$ resonance (Fig. 21). In both cases our results slightly differ from those obtained in [154], and probably only new experimental measurements can eliminate the discrepancy.

Since at energies from 10 meV to approximately 10 eV the calculated cross section shown in Fig. 24 by the solid curve is almost a straight line, it can be approximated by a simple function in the form of Eq. (3). The constant value 1.0362 μb keV$^{1/2}$ was found from one point in the cross sections at the minimum energy of 10$^{-5}$ keV. We can consider the modulus of the relative deviation of the calculated theoretical cross section and its approximation by function (4) in the energy region from 10 meV to 10 eV. At energies below 10 eV, this deviation is at a level of 2.5%, and below 1 eV it is no larger than 0.5%. We can well assume that this shape of the total cross section dependence on the energy will be retained at even lower energies as well, and evaluation of the cross section, e.g., at 1 μeV (10$^{-6}$ eV = 10$^{-9}$ keV), yields 32.8 mb.

**CONCLUSIONS**

Thus, in all the cases considered it appears to be generally possible to correlate the descriptions of the elastic scattering (scattering phase shifts), main characteristics of nuclear BSs (binding energy, root-mean-square radius, AC), and total cross sections for radiative capture of neutral particles using unified variants of intercluster potentials for each reaction [7, 55, 130].

In conclusion, we would like to point out that we have considered as many as 21 cluster systems within the potential cluster model with the classification of orbital states by Young diagrams [6, 86] and managed to describe reasonably well the characteristics of the corresponding radiative capture of nucleons or light clusters by predominantly 1$p$-shell nuclei [9, 18]. Properties of these nuclei, some of the characteristics, and cluster channels considered are listed in Table 16.

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