Abstract. The ignition temperature of a thermonuclear fusion reaction is found to decrease on adding a small amount of tritium ($x \sim 0.0112$) to deuterium fusion pellet. In this paper the catalytic regime for tritium has been analyzed by changing the fraction of tritium ($x$). The initial density and temperature of the pellet are also found to influence the ignition conditions in a major way. A zero dimensional three temperature model which includes the energy deposition of charged particles via small and large angle Coulomb scattering, nuclear scattering and collective plasma effects has been used. A density-temperature regime is found where internal tritium breeding occurs even on including all the radiative loss mechanisms like bremsstrahlung and inverse Compton scattering.

1. Introduction
For lower fuel temperatures ($\sim 10$ keV), the D-T reaction proceeds at a rate almost two orders of magnitude larger than that characterizing the D-D reaction [1]. Hence, in order to minimize the ignition temperature, tritium is added to deuterium fusion pellets in stoichiometric ratio (50:50). However tritium inventories in futuristic fusion reactors based on current stoichiometric DT proposals are very high, which poses a significant radiological problem [2]. Also, the breeding of tritium in external tritium blanket and its separation is quite complicated. The concept of internal tritium breeding in which a small amount of tritium in deuterium fusion pellet reduces the ignition temperature and also acts as a catalyst is very lucrative [3].

The fraction of charged particle energy that is absorbed by the ICF pellet is an important parameter determining the ignition condition. For low density and high temperature plasmas, Coulomb interactions can be approximated as small angle binary collisions [4] but for high densities and low temperatures, large angle scattering needs to be included [5]. The effect of nuclear scattering is important when the incident charged particle energy is large and for higher plasma temperatures [6].

In this paper, the variation in the thermalization range and fraction of charged particle energy deposited to the ions as a function of number density and temperature of the pellet is studied. Using the improved model of energy deposition [7], the effect of varying various pellet parameters like its density, fraction of tritium added and initial temperature on the burn fraction and tritium breeding ratio
An optimum pellet configuration is obtained which shows a high deuterium burn fraction and a catalytic regime for tritium.

2. Energy deposition of charged particles

Plasma heating by charged particles and neutrons, energy exchange between ions and electrons and radiative losses are the primary mechanisms determining the ignition conditions in a thermonuclear plasma [1]. In this section we analyze the variation of the thermalization distance and fraction of energy deposited to ions as a function of ion number density and temperature.

The total stopping power of a fast charged particle of mass \( m \), laboratory frame energy \( E = \frac{1}{2} m v^2 \) and charge \( Z_e \) which moves through a plasma with ions (mass \( m_j \), charge \( Z_{j} e \), number density \( n_j \), temperature \( T_j \)) and electrons (mass \( m_e \), charge \( e \), number density \( n_e \), temperature \( T_e \)) is

\[
\frac{dE}{dx} = A_d(E)E^{-1} + A_e(E)E^{-1} + A_N(E)E
\]

where, the subscripts d, e and N denote Coulomb scattering from ions, electrons and nuclear scattering respectively. The ionic, electronic and nuclear terms are given explicitly as

\[
A_d(E) = \sum_j -2\pi m_j Z^2 Z_j^2 e^4 \frac{m}{m_j} (F_j(y_j) \ln \Lambda_{bj} + \Theta(y_j^2) \ln[1.123(y_j)])
\]

\[
A_e(E) = -2\pi m_e Z^2 Z_e^2 e^4 \frac{4}{m_e} \frac{4 m}{mkT_e} \left( \frac{m_{j} E}{mkT_{j}} \right)^{3/2} \ln \Lambda_{be}
\]

\[
A_N(E) = -\sum_j n_j \sigma_{tj}
\]

where,

\[
F_j(y_j) = \phi(y_j) - \left(1 + \frac{m_j}{m}\right)y_j \phi'(y_j) + \frac{m_j}{mn \ln \Lambda_{bj}} \Phi(y_j)
\]

with

\[
y_j^2 = \frac{m_j E}{m kT_j}
\]

\( \Phi \) and \( \Phi' \) are the error function and its derivative respectively.

The Coulomb logarithm term is denoted by \( \ln \Lambda_b \) [8], the second term in equation (2) denotes the collective plasma effects which arise when the plasma is considered as a dielectric medium without bringing into picture its internal particle behaviour [9] whereas \( \sigma_T \) denotes the nuclear transport cross section [10].

The fraction of energy deposited to ions and the thermalization distance are obtained from the total stopping power, i.e., equation (1).

The increase in the fraction of charged particle energy deposited to the ions leads to higher fusion gains [11] as energy deposited to the electrons lead to radiation losses from the plasma. Increase in plasma temperature results in an increase in nuclear scattering leading to higher energy deposition to
the ions. Also, the effect of large angle Coulomb scattering (which leads to energy deposition mainly to the ions) increases with increasing density. In figure 1(a), the fraction of charged particle (deuteron) energy deposited to the ions in deuterium plasma as a function of plasma temperature and number density is shown. In figure 1(b), the thermalization distance of a 3.5 MeV deuteron in deuterium plasma as a function of plasma density and temperature is plotted. Deuteron deposits more energy in a denser and colder plasma showing a reduction in the thermalization distance.

![Figure 1: (a) Fraction of charged particle (deuteron) energy deposited to the ions in deuterium plasma as a function of plasma temperature and logarithm of the number density. (b) Thermalization distance of a 3.5 MeV deuteron in deuterium plasma as a function of plasma temperature and logarithm of the number density.](image)

3. Internal Tritium breeding

The zero dimensional simulation model of thermonuclear fusion in a DT fusion pellet considers the rate of decay or buildup of the six nuclides (D, T, He$^3$, He$^4$, p, n) participating in the 4 reactions: D-D (neutron channel), D-D (proton channel), D-T and D-He$^3$. Energy balance equations for ions, electrons and radiation, within the three-temperature model, including all the energy exchange processes, determine the time dependent temperatures. Finally, the hydrodynamic disassembly of the pellet determines the extent of burn [7]. The reactions predominantly occurring are

\[
\begin{align*}
&D_1^2 + D_1^2 \rightarrow He_2^3 + n_0^1 + 3.269 \text{ MeV} \\
&D_1^2 + D_1^2 \rightarrow T_1^3 + p_1^1 + 4.033 \text{ MeV} \\
&D_1^2 + T_1^3 \rightarrow He_2^4 + n_0^1 + 17.589 \text{ MeV} \\
&D_1^2 + He_2^3 \rightarrow He_2^4 + p_1^1 + 18.353 \text{ MeV}
\end{align*}
\]

The total number of particles, $N_k$, of species $k$ is governed by the equation

\[
\frac{dN_k}{dt} = \sum_j a_k^j N_{j(1)} N_{j(2)} \langle \sigma v \rangle_j \frac{1}{V}
\]

where $V$ is the volume of the heated plasma, $\langle \sigma v \rangle_j$ is the Maxwell averaged reaction rate [12] of reaction $j$ and $a_k^j$ is the number of particles of the species $k$ created or destroyed in the reaction $j$. The equation of energy balance for ions and electrons are
\[
\frac{3}{2} \frac{d}{dt} (N_i T_i) = \sum_j \sum_k f_k^j w_k^j E_j N_{j(1)} N_{j(2)} < \sigma v > \frac{1}{V} - \frac{P_{ie}}{V} - N_i T_i 4 \pi R^2(t) c_s \frac{1}{V} 
\] 
\[ (8) \]

and
\[
\frac{3}{2} \frac{d}{dt} (N_e T_e) = \sum_j \sum_k (1 - f_k^j) w_k^j E_j N_{j(1)} N_{j(2)} < \sigma v > \frac{1}{V} + \frac{P_{ie}}{V} - \frac{P_B}{V} - P_C 
\]
\[ - N_e T_e 4 \pi R^2(t) c_s \frac{1}{V} \]
\[ (9) \]

with \( T_i \) and \( T_e \) denoting the ion and electron temperatures, \( N_e \) the number of electrons, \( R(t) \) the pellet radius, \( c_s \) the sound speed, \( E_j \) the energy yield of reaction \( j \), \( w_k^j \) the fraction of \( E_j \) carried by the product \( k \) and \( f_k^j \) is the fraction of the energy of the product \( k \) created in the reaction \( j \) that is deposited to the plasma ions. \( N_i \) is the total number of ions. \( P_{ie}, P_B \) and \( P_C \) are the ion-electron energy exchange term, bremsstrahlung loss term and the inverse Compton scattering term [7]. The rate equation for radiation temperature is
\[
\frac{dT_r}{dt} = \frac{P_B}{V} + P_C - R_{\text{loss}} 
\] 
\[ (10) \]

where \( R_{\text{loss}} \) is the photon energy loss rate [13].

The fraction of neutron energy deposited is obtained using a multi-group neutron slowing down model taking into account the escape probability from the pellet [7]. Cross-sections for neutron interaction are taken from Barrett and MacFarlane [14].

Figure 2: (a) Tritium breeding ratio versus time for DTx pellets having different initial pellet densities. (b) Deuterium burn fraction as a function of the pellet density.

Using the above described zero dimensional three temperature model which considers all the energy deposition mechanisms like small and large angle Coulomb scattering, nuclear scattering and
collective plasma effects, the effect of varying various pellet parameters like its density, fraction of tritium added and initial temperature on the burn fraction and tritium breeding ratio is studied.

3.1. Effect of pellet density on tritium breeding ratio and deuterium burn fraction
The initial density of the DT pellet determines the burn fraction of deuterium and also the breeding ratio of tritium. For the purpose of simulation we consider a pellet of radius 25 μm, initial ion and electron temperature 10 keV, initial radiation temperature 1 keV and tritium fraction x=0.0112. If the initial pellet density is less than 4000 gm/cc, tritium breeding is not possible i.e., the amount of tritium left in the pellet after the burn is less than the amount we started with [figure 2(a)]. However, above a density of 4000 gm/cc, tritium acts as a catalyst and results in more efficient deuterium burning [figure 2(b)]. The deuterium burn fraction is defined as $f_b = (N_{D,\text{initial}} - N_{D,\text{final}})/N_{D,\text{initial}}$, where $N_{D,\text{initial}}$ is the total number of deuterons in the pellet initially and $N_{D,\text{final}}$ is the number left in the pellet after the burn. The burn fraction steeply decreases with decreasing density as depicted in figure 2(b).

3.2. Effect of initial temperature of ions and electrons
Below a certain initial temperature of the pellet (both ions and electrons are assumed to be at the same temperature initially), the losses exceed the energy production and the pellet does not burn. The rate of fusion reactions decrease on decreasing the initial plasma temperature within the pellet and this leads to slow burnup and reduced production of tritium [figure 3(a)]. A pellet of radius 25 μm, density 5000 gm/cc, initial radiation temperature 1 keV and tritium fraction x=0.0112 is considered. Below 4 keV, the reactions become so slow that within the pellet disassembly time the rate of fusion reactions remains negligibly small. As a result the deuterium burn fraction is also found to decrease steeply on reducing the initial temperature below 6 keV [figure 3(b)].

![Figure 3: (a) Tritium breeding ratio versus time for DT₃ pellets having different initial plasma temperatures. (b) Deuterium burn fraction as a function of the initial plasma temperatures.](image)

3.3. Effect of tritium fraction (x) in the pellet
For a pellet of radius 25 μm, density 5000 gm/cc, initial ion and electron temperature 10 keV and initial radiation temperature 1 keV, as the fraction of tritium (x) in the pellet is decreased below 0.005, it is no more able to ignite the deuterium so that the burn becomes slower. Also the deuterium burn fraction keeps on decreasing as the tritium content is decreased. However, it is also observed that increasing the tritium fraction beyond 0.03 does not increase the burn fraction any further and the initial amount of tritium in the pellet is also not replenished [figure 4(a) and (b)].
Finally, from the above studies on the various pellet parameters like its density, initial temperature and fraction of tritium added, we conclude that for sufficient burning of the pellet and for tritium to behave as a catalyst, the following pellet configuration is necessary:

- the initial pellet density $\geq 3500\text{gm/cc}$
- initial plasma temperature $\geq 4\text{ keV}$
- fraction of tritium added lies between 0.005 and 0.02 i.e., $0.005 \leq x \leq 0.02$.

Figure 4: (a) Tritium breeding ratio versus time for DT$_x$ pellets having different initial tritium fraction ($x$). (b) Deuterium burn fraction as a function of the initial tritium fraction ($x$).

4. Conclusion
In this paper, the dependence of the thermalization distance and fraction of charged particle energy deposited to the ions on the density and temperature of the fully ionized plasma is obtained by considering energy deposition via small and large angle Coulomb scattering, nuclear scattering and collective plasma effects. This improved model of energy deposition has been used to obtain an optimum pellet configuration in terms of initial pellet temperature, density and tritium fraction. In this regime, tritium acts as a catalyst, helps in reducing the ignition temperature and the deuterium in the pellet burns sufficiently before the pellet disassembles.

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