Threshold effects in P-wave bottom-strange mesons

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Using a nonrelativistic constituent quark model in which the degrees of freedom are quark-antiquark and meson-meson components, we have recently shown that the $D^{±}K$ thresholds play an important role in lowering the mass of the physical $D_{s0}^*(2317)$ and $D_{s1}(2460)$ states. This observation is also supported by other theoretical approaches such as lattice-regularised QCD or chiral unitary theory in coupled channels. Herein, we extend our computation to the lowest $P$-wave $B_s$ mesons, taking into account the corresponding $J^P = 0^+, 1^+$ and $2^+$ bottom-strange states predicted by the naive quark model and the $B K$ and $B^* K$ thresholds. We assume that mixing with $B_{s1}^{(*)}$ and isospin-violating decays to $B_{s0}^{(*)} \pi$ are negligible. This computation is important because there is no experimental data in the $b \bar{s}$ sector for the equivalent $j^P = 1/2^+ (D_{s0}^*(2317), D_{s1}(2460))$ heavy-quark multiplet and, as it has been seen in the $c \bar{s}$ sector, the naive theoretical result can be wrong by more than 100 MeV. Our calculation allows to introduce the coupling with the $D$-wave $B^* K$ channel and to compute the probabilities associated with the different Fock components of the physical state.

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I. INTRODUCTION

Despite many of the $B_s$ states should be accessible by the $B$-factories (CLEO, BaBar, Belle) and also by the proton–anti-proton colliders (CDF and D0), much of the $b \bar{s}$ excitation spectrum remains to be observed. Only the ground $S$-wave spin-singlet and spin-triplet states ($B_s$ and $B_s^*$) and the orbitally excited $B_{s1}(5830)$ and $B_{s2}^*(5840)$ mesons are presently well established [1]. It is expected that the situation will change in the near future thanks to the LHCb experiment and to the future high-luminosity flavour and $p \bar{p}$ facilities.

In the heavy quark limit ($m_Q \rightarrow \infty$), flavour and spin symmetries hold and the open-flavoured heavy mesons can be classified in doublets. This is because in the heavy quark limit the spin $s_Q$ of the heavy quark and the total angular momentum of the light quark $j_q$ decouple and are separately conserved in strong interaction processes [2]. Therefore, $Q\bar{q}$ mesons can be classified according to the value of $j_q$, and can be collected in doublets; the two states of each doublet are spin partners with total spin $J = j_q \pm \frac{1}{2}$ and parity $P = (-1)^{\ell+1}$, with $\ell$ the orbital angular momentum of the light degrees of freedom such as $j_q = \ell + s_q$ ($s_q$ is the light antiquark spin).

The well established $B_{s1}(5830)$ and $B_{s2}^*(5840)$ states belong to the $j^P = 3/2^+$ doublet and therefore the unambiguous experimental determination of the $b \bar{s}$ $j^P = 1/2^+$ doublet is of particular interest because, following the predictions of heavy quark symmetry(HQS), they should be almost degenerated and broad. However, we know that the experimental values of the masses and widths of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons, which belong to the same multiplet but in the $c \bar{s}$ sector, do not accommodate into the theoretical estimates.

We have recently argued in Ref. [3] that the disagreement between theory and experiment for the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons can be explained as a delicate compromise between the bare masses and the coupling with their respective $DK$ and $D^* K$ thresholds. The study of Ref. [3] taught us that the traditional quark model do not reproduce the masses and widths of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons. When, following the proposal of Ref. [4], we include one-loop corrections to the one-gluon exchange potential as derived by Gupta et al. [7], these $\alpha_s^2$ corrections affect basically the $0^+$ $D_{s0}^*(2317)$, bringing this state closer to the $DK$ threshold. However, the same correction leaves the $D_{s1}(2460)$ almost degenerated with the $D_{s1}(2536)$. It is the coupling with the $DK$ and $D^* K$ channels which lowers the bare masses of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons to the observed experimental values. Since their masses are below their respective $DK$ and $D^* K$ thresholds, their open-flavoured strong decays are forbidden and thus the states are narrower than expected theoretically. Similar conclusions were drawn by dynamical coupled-channel approaches [8, 9] and by lattice-regularised QCD com-
In this work we closely follow Ref. [5] and address the mass-shifts, due to the $B^{(*)}K$ thresholds, of the lowest lying $P$-wave $b\bar{s}$ states with total spin and parity quantum numbers $J^P = 0^+, 1^+, 2^+$. A first lattice-regularised QCD study of the lowest-lying $P$-wave bottom-strange mesons in which $B^{(*)}K$ operators are explicitly incorporated in the interpolator basis has been recently released [12]. We shall mainly compare our results with the ones obtained by them, but a number of phenomenological model and effective field theory (EFT) mass determinations of the same states can be found in Tables I and II.

The first conclusion one can obtain from such tables is that the naive quark models, including the recent calculation of Godfrey et al., predict masses above the $BK$ and $B^*K$ thresholds, respectively. The authors of Ref. [18] use a nonrelativistic quark model which include $\alpha_s^2$-corrections. Taken a set of parameters fitted to the charm sector, they succeed to bring the $0^+ b\bar{s}$ state below the $BK$ threshold. To break the degeneration between the $B_{s1}^*$ and $B_{s1}$ mesons, they adjust a mixing angle that also place the $B_{s1}^*$ below the $B^*K$ threshold. The result of Bardeen et al. is calculated using a set of parameters fitted to the $D_{s0}^*(2317)$.

The second conclusion one can make is that the results of lattice computations, which take explicitly $B^{(*)}K$ operators in the interpolator basis, and the estimates of EFTs, that study dynamically generated bound states from the $B^{(*)}K$ scattering, are 80 MeV lower than the quark model predictions.

Despite the significant progress made by lattice calculations incorporating open-flavoured thresholds, two main drawbacks remain: i) the thresholds are added only as $S$-wave channels and ii) no statement can be made about the probabilities of the different Fock components in the physical state. Our approach solves these two issues and allows e.g. to introduce the coupling with the $D$-wave $B^0 K$ channel in the $1^+$ $b\bar{s}$ sector and to compute the amount of $B^{(*)}K$ component in the meson.

Our theoretical framework is a nonrelativistic constituent quark model in which quark-antiquark and meson-meson degrees of freedom are incorporated. The constituent quark model (CQM) was proposed in Ref. [26] (see references [27] and [28] for reviews).

In order to keep the predictive power of the formalism we do not change any model parameter. Moreover, it is worth to emphasize here that the quark model has been applied to a wide range of hadronic observables, e.g. Refs. [29–36], and thus the model parameters are completely constrained.

The manuscript is arranged as follows. In Sec. II, we describe briefly the main properties of our theoretical formalism. Section III is devoted to present our results for the lowest lying $P$-wave $B_s$ states with total spin and parity quantum numbers $J^P = 0^+, 1^+, 2^+$. We finish summarizing and giving some conclusions in Sec. IV.

## II. CONSTITUENT QUARK MODEL

Constituent light quark masses and Goldstone-boson exchanges, which are consequences of dynamical chiral symmetry breaking in Quantum Chromodynamics (QCD), together with the perturbative one-gluon exchange and a nonperturbative confining interaction are the main pieces of our constituent quark model [26, 28].

A simple Lagrangian invariant under chiral transformations can be written in the following form [37]

$$\mathcal{L} = \bar{\psi}(i \gamma \cdot \partial - M(q^2)U^\gamma) \psi,$$

where $M(q^2)$ is the dynamical (constituent) quark mass and $U^\gamma = e^{i\lambda \cdot \phi \gamma / f}$. Is the matrix of Goldstone-boson exchanges.
fields that can be expanded as
\[ U^{\gamma 5} = 1 + \frac{i}{f_{\pi}} \gamma_5 \lambda^a \pi^a - \frac{1}{2f_{\pi}^2} \pi^a \pi^a + \ldots \]  
(2)

The first term of the expansion generates the constituent quark mass while the second gives rise to a one-boson exchange interaction between quarks. The main contribution of the third term comes from the two-pion exchange which has been simulated by means of a scalar-meson exchange potential.

In the heavy quark sector chiral symmetry is explicitly broken and Goldstone-boson exchanges do not appear. However, it constrains the model parameters through the light-meson phenomenology and provides a natural way to incorporate the pion exchange interaction in the molecular dynamics.

The one-gluon exchange (OGE) potential is generated from the vertex Lagrangian
\[ \mathcal{L}_{qgq} = i\sqrt{4\pi} a_2(\gamma \psi \gamma_5 G_c^\mu \lambda^\mu \psi), \]
where \( \lambda^\mu \) are the \( SU(3) \) colour matrices and \( G_c^\mu \) is the gluon field. The resulting potential contains central, tensor and spin-orbit contributions.

To improve the description of the open-flavour mesons, we follow the proposal of Ref. \[1\] and include one-loop corrections to the OGE potential as derived by Gupta et al. \[2\]. These corrections show a spin-dependent term which affects only mesons with different flavour quarks.

It is well known that multi-gluon exchanges produce an attractive linearly rising potential proportional to the distance between infinite-heavy quarks. However, sea quarks are also important ingredients of the strong interaction dynamics that contribute to the screening of the rising potential at low momenta and eventually to the breaking of the quark-antiquark binding string.

Our model tries to mimic this behaviour using the following expression:
\[ V_{\text{CON}}(\vec{r}) = [-a_c(1 - e^{-\mu_c r}) + \Delta] (\vec{X}_c, \vec{X}_{\bar{c}}), \]
(4)
where \( a_c \) and \( \mu_c \) are model parameters.

Explicit expressions for all the potentials and the value of the model parameters can be found in Ref. \[26\], updated in Ref. \[41\]. Meson eigenenergies and eigenstates are obtained by solving the Schrödinger equation using the Gaussian Expansion Method \[41\].

The quark-antiquark bound state can be strongly influenced by nearby multiquark channels. In this work, we follow Refs. \[35\], \[36\] to study this effect in the spectrum of the bottom-strange mesons and thus we need to assume that the hadronic state is given by
\[ |\Psi\rangle = \sum_\alpha c_\alpha |\psi_\alpha\rangle + \sum_\beta \chi_\beta(P)\phi_A\phi_B\beta, \]
(5)
where \( |\psi_\alpha\rangle \) are \( b\bar{s} \) eigenstates of the two-body Hamiltonian, \( \phi_M \) are wave functions associated with the \( A \) and \( B \) mesons, \( |\phi_A\phi_B\beta\rangle \) is the two meson state with \( \beta \) quantum numbers coupled to total \( J^P \) quantum numbers and \( \chi_\beta(P) \) is the relative wave function between the two mesons in the molecule. To derive the meson-meson interaction from the quark-antiquark interaction we use the Resonating Group Method (RGM) \[43\].

The coupling between the quark-antiquark and meson-meson sectors requires the creation of a light quark pair. The operator associated with this process should describe also the open-flavour meson strong decays and is given by
\[ T = -\sqrt{3} \sum_{\mu,\nu} \int d^4p \bar{d}p_\mu \bar{p}_\nu \delta(3) (\tilde{p}_\mu + \tilde{p}_\nu) \frac{g_8}{2m} \sqrt{2\pi} \]
\[ \times \left[ Y_1 \left( \tilde{p}_\mu - \tilde{p}_\nu \right) \otimes \left( \begin{array}{c} 1 \\ 2 \\ 1 \end{array} \right) \right] a_\mu(p_\mu) b_\nu(p_\nu), \]
(6)
where \( \mu \) (\( \nu \)) are the spin, flavour and colour quantum numbers of the created quark (antiquark). The spin of the quark and antiquark is coupled to one. The \( Y_{lm}(\vec{p}) = \hat{P} Y_{lm}(\hat{P}) \) is the solid harmonic defined in function of the spherical harmonic. We fix the relation of \( g_8 \) with the dimensionless constant giving the strength of the quark-antiquark pair creation from the vacuum as \( \gamma = g_8/m \), being \( m \) the mass of the created quark (antiquark).

From the operator in Eq. \[6\], we define the transition potential \( h_{\beta\alpha}(P) \) within the \( \mathcal{P}_0 \) model as \[36\]
\[ \langle \phi_A\phi_B\beta|T|\psi_\alpha\rangle = P h_{\beta\alpha}(P) \delta(3) (\vec{P}) \]
(7)
where \( P \) is the two-meson relative momentum.

The usual version of the \( \mathcal{P}_0 \) model gives vertices that are too hard specially when we work at high momenta. Following the suggestion of Ref. \[44\], we use a momentum dependent form factor to truncate the vertex as
\[ h_{\beta\alpha}(P) \rightarrow h_{\beta\alpha}(P) \times e^{-P^2/2\Lambda^2}, \]
(8)
where \( \Lambda = 0.84 \) GeV is the value used herein.

Adding the coupling with bottom-strange states we end up with the coupled-channels equations
\[ c_\alpha M_\alpha + \sum_\beta \int h_{\alpha\beta}(P) \chi_\beta(P) P^2 dP = E c_\alpha, \]
\[ \sum_\beta \int H_{\gamma\beta}(P',P) \chi_\beta(P) P^2 dP + \sum_\alpha h_{\beta\alpha}(P') c_\alpha = E \chi_{\beta'}(P'), \]
(9)
where \( M_\alpha \) are the masses of the bare \( b\bar{s} \) mesons and \( H_{\gamma\beta} \) is the RGM Hamiltonian for the two-meson states obtained from the \( q\bar{q} \) interaction. Solving the coupling with the \( b\bar{s} \) states, we arrive to a Schrödinger-type equation
\[ \sum_\beta \int (H_{\gamma\beta}(P',P) + V_{\gamma\beta}(P',P)) \times \chi_{\beta'}(P') P^2 dP = E \chi_{\beta'}(P'), \]
(10)
TABLE III. Masses, in MeV, of the low-lying $P$-wave bottom-strange mesons predicted by the constituent quark model ($\alpha_s$) and those including one-loop corrections to the one-gluon exchange potential ($\alpha_s^2$). For completeness, our predictions for the 0$^-$ and 1$^-$ states that belong to the $j_{q\bar{q}}^P = 1/2^-$ doublet are included. Experimental data are taken from Ref. [1].

| State | $J^P$ | Mass | Width | $P[qq^1(sP_0)]$ | $P[BK(S - wave)]$ |
|-------|-------|------|-------|-----------------|-------------------|
| $B_{s0}$ | 0$^+$ | $5851$ | $5801$ | $38.8$ |
| $B_{s2}$ | $1^+$ | $5883$ | $5858$ | $38.8$ |
| $B_{s1}$ | (5830) | 0.5841 | 0.5850 | 0.5828 ± 0.41 |
| $B_{s2}'$ | (5840) | 2$^+$ | $5856$ | $5867$ | 5839.98 ± 0.20 |

TABLE IV. Masses, widths and probabilities of the different Fock components for the states found in the $J^P = 0^+$ $b\bar{s}$ sector.

where

$$V_{\beta\beta}^{\text{eff}}(P', P; E) = \sum_\alpha \frac{h_{\beta\alpha}(P')h_{\alpha\beta}(P)}{E - M_\alpha}.$$  \hspace{1cm} (11)

III. RESULTS

The heavy-quark doublet $j_{q\bar{q}}^P = 3/2^+$ is well established in the PDG, with the $B_{s1}(5830)$ and $B_{s2}(5840)$ mesons belonging to this doublet. Table III shows the predicted masses within the naive quark model; one can see our results taking into account the one-gluon exchange potential ($\alpha_s$) and including its one-loop corrections ($\alpha_s^2$). In both cases our values are slightly higher than the experimental figures but compatible.

The mass of the $B_{s0}'$ state obtained using the naive quark model and without the one-loop spin corrections to the OGE potential is 5851 MeV, which is compatible with the quark model predictions shown in Table I but, nevertheless, is much higher than the average value predicted by lattice and EFT approaches. The mass associated to the $B_{s0}'$ state is sensitive to the $\alpha_s^2$-corrections of the OGE potential. This effect brings down its mass about 50 MeV but still is incompatible with the lattice and EFT results. The mass-shift due to the $\alpha_s^2$-corrections allows the 0$^+$ state to be closer to the $BK$ threshold. This makes the $BK$ coupling a relevant dynamical mechanism in the formation of the $B_{s0}'$ bound state. When we couple the 0$^+$ $b\bar{s}$ ground state with the $BK$ threshold the mass goes down towards 5741 MeV (Table IV), in good agreement with lattice and EFT estimations.

We turn now to discuss the probabilities of the different Fock components in the physical state. The lattice-regularised QCD study of Ref. [12] is only able to remark that both quark-antiquark and meson-meson lattice interpolating fields have non-vanishing overlaps with the physical state. Our wave function probabilities are given in Table IV which reflects that the $B_{s0}'$ meson is mostly of quark-antiquark nature. This is in agreement with the fact that lattice-regularised QCD computations observe this state even with only $q\bar{q}$ interpolators [19].

In our model the probability of the $BK$ state depends basically on three quantities: the bare meson mass, the $\Lambda P_0$ coupling constant and the residual $BK$ interaction. Obviously, as neither of the three are observables, they can take different values depending on the dynamics, making the results, and hence the $BK$ probability, model dependent. No model parameters have been changed from our study of the $D_{s0}^*(2317)$ meson in Ref. [2]. Moreover, the above mentioned quantities have been constrained in previous works by reproducing other physical observables like strong decays [43] (the $\Lambda P_0$ coupling constant), bottomonium spectrum (the bare mass) [5] and $NN$ and $p\bar{p}$ interactions (the $BK$ residual interaction) [16, 17].

The scattering length is sensitive to the $BK$ compositeness in the $B_{s0}'$ wave function. Therefore, to complete the analysis of the $J^P = 0^+ b\bar{s}$ sector, we have calculated the value of the scattering length from the value of the $T$-matrix at threshold, obtaining $a_{BK} = -1.18$ fm. This value is compatible with the one reported by Ref. [12], that is, $a_{BK} = (-0.85 \pm 0.10)$ fm, where the difference may originate from the binding energies predicted in each study.

In addition to the $B_{s0}'$ state below the $BK$ threshold, we find a resonance with mass 5.88 GeV and width 300 MeV which is denoted as $B_{s0}''$ in Table IV. The resonance is a $\sim 84\%$ $b\bar{s}$ state which comes from the residual bare pole, that does not disappear but it is dressed with the $BK$ interaction and quickly moves into the complex plane. This resonance can be potentially observed in the $BK$ channel, but its large width is a handicap for the experiments.

The quark model including $\alpha_s^2$-corrections to the OGE potential predicts that the states corresponding to the $B_{s1}'$ and $B_{s1}(5830)$ mesons are almost degenerated, with masses close to the experimentally observed mass of the $B_{s1}(5830)$. This $B_{s1}'$ result goes in the same line than the ones predicted by other phenomenological models (see Tables I and II). However, the average of phenomenological model predictions, including our naive one, is around 80 MeV higher than lattice studies and EFT estimations.

We couple the two $1^+$ $b\bar{s}$ states associated with the $B_{s1}'$ and $B_{s1}(5830)$ mesons with the $B^*K$ threshold. Following lattice criteria, we couple first the $B^*K$
channel in an S-wave. One can see in Table V that the state associated with the \( B'_s \) meson goes down in the spectrum and it is located below \( B^*K \) threshold. Our value, 5793 MeV, is compatible within errors with lattice data and with most of the EFT estimates. The state associated with the \( B_1(5830) \) meson is almost insensitive to the \( B^*K \) S-wave coupling. The mass predicted in this case is \( m_{B_1(5830)} = 5850 \) MeV, which is the same than the one obtained using quark model without coupling. This is because the \( B_1(5830) \) state is the \( J^P = 1^+ \) member of the \( J^P = 3/2^+ \) doublet predicted by HQS and thus it couples mostly in a \( D \)-wave to the \( B^*K \) threshold. It is important to highlight that the lattice computation of Ref. [12] does not take into account the coupling of the \( B_1(5830) \) meson to the \( B^*K \) threshold either in \( S \)- or \( D \)-wave. This is because the coupling of the \( B^*K \) in \( D \)-wave is not easy to implement and, moreover, they assume that the coupling to \( B^*K \) in \( S \)-wave is small.

The coupling in \( D \)-wave of the \( B^*K \) threshold is trivially implemented in our approach. Table V shows that the state associated with the \( B'_s \) meson experience a negligible modification. This is understandable because it is almost the \([1/2, 1^+]\) eigenstate of HQS. The state associated with \( B_1(5830) \) meson suffers a moderate mass-shift with a final mass of 5833 MeV. This value compares nicely with the one reported in Ref. [12]: \( 5831 \pm 10 \) MeV, but the coupling in \( D \)-wave of the \( B^*K \) threshold in the lattice result needs to be incorporated in order to make a stronger statement. In any case, our result for the \( B_1(5830) \) is compatible with other theoretical approaches and with the experimental value reported in PDG.

Table V shows the probabilities of the different Fock components in the physical \( B'_s \) and \( B_1(5830) \) states. When the \( B^*K \) threshold is coupled in an \( S \)-wave, the meson-meson component is 44\% for the \( B'_s \) and 50\% for the \( B_1(5830) \). It is also interesting to point out that the relation between the quark-antiquark partial wave components is close to the predictions of HQS, being the \( B'_s \) meson a dominant \( j_q = 1/2 \) state (81\%) and the \( B_1(5830) \) a \( j_q = 3/2 \) state (83\%). One can also see in Table V that the coupling of the \( B^*K \) threshold in \( D \)-wave has very little effect for the formation of the \( B'_s \) meson, keeping the probabilities pretty much the same. However, the \( D \)-wave coupling of the \( B^*K \) channel is relatively important in the formation of the \( B_1(5830) \) with a contribution to its physical wave function of about 37\%. Moreover, in the case of the \( B_1(5830) \), the distribution of the meson-meson component in \( S \)- and \( D \)-wave channels does not affect very much the probabilities of the quark-antiquark components, being still dominant the \( j_q = 3/2 \) HQS state.

As in the \( J^P = 1^+ \) sector, we can predict the value of the scattering length for the \( S \)-wave \( B^*K \) channel, \( a^{B^*K} = -1.35 \) fm, and find that is slightly higher but compatible with the \( a^{B^*K} = -0.97 \pm 0.16 \) fm reported by Ref. [12]. Again, the different mass for the \( B'_s \) predicted in each study can account for the difference in the predicted scattering lengths.

In the \( J^P = 1^+ \) sector we also find a broad resonance above the \( B^*K \) threshold. It is originated from the \( j_q = 1/2^+ \) component of the bare \( b\bar{s} \) pole, which is largely dressed with the \( B^*K \) \( S \)-wave interaction. This resonance, labelled \( B'_s \) in Table V, is located at the mass region of 5.94 GeV and has a width of 271 MeV, which complicates but not forbids its possible experimental observation.

Only quark-antiquark operators were used in the lattice study of the \( B'_s \) meson. They obtained a value of \( 5835 \pm 13 \) MeV for the mass. This is in qualitative agreement with experiment and with our naive quark model prediction, confirming that this state can be described well within the \( b\bar{s} \) picture.

Finally, let us clarify that we have omitted the possible coupling of the states to the \( B'^{(+)\pi} \) channels since they violate isospin and thus the effect should be subleading. We also neglect effects coming from the \( B'(\pi)\eta \) coupling, partially motivated by the threshold lying \( O(140 \) MeV) above the \( B'^{(+)\pi} \) threshold. Similar criteria has been followed by the lattice analysis of Ref. [12].

### IV. EPILOGUE

Within the formalism of a nonrelativistic constituent quark model in which quark-antiquark and meson-meson...
components are incorporated, we have performed a coupled-channel computation taking into account the $J^P = 0^+, 1^+$ and $2^+$ bottom-strange states predicted by the naive quark model and the $BK$ and $B^\ast K$ thresholds. Our method allows to introduce the coupling with the $D$-wave $B^\ast K$ channel and to compute the probabilities associated with the different Fock components of the physical state, features which cannot be addressed nowadays by other theoretical approaches.

Our study has been motivated by the fact that there are no experimental evidences of the $b\bar{s}$ mesons which belong to the doublet, $j^P = 1/2^+$ and, as it has been seen in the $c\bar{s}$ sector [3], the naive theoretical result can be wrong by more than 100 MeV. In order to keep the predictive power of the formalism we do not change any parameter of the calculation in Ref. [3]. Moreover, it is worth to emphasize again that the quark model has been applied to a wide range of hadronic observables and thus the model parameters are completely constrained.

The level assigned to the $B_{s0}^\ast$ meson within the naive quark model is much higher than the ones predicted by lattice and EFT approaches. However, it has been shown that the value is compatible with other phenomenological model predictions. The one-loop corrections to the OGE potential brings down this level and locates it slightly above the $BK$ threshold. This makes the coupling with the nearby threshold to acquire an important dynamical role. When coupling, the level is down-shifted again towards the average mass obtained by lattice and EFT formalisms. We predict a probability of around $40\%$ for the $BK$ component of the $B_{s0}^\ast$ wave function. Lattice QCD can only state that both quark-antiquark and meson-meson operators have important overlaps with the physical state.

The $B_{s1}^\ast$ and $B_{s1}(5830)$ mesons appear almost degenerated using the naive quark model that includes the one-loop corrections to the OGE potential. We have coupled the two $1^+$ $b\bar{s}$ states associated with the $B_{s1}^\ast$ and $B_{s1}(5830)$ mesons with the $B^\ast K$ threshold. When coupling the $B^\ast K$ channel in a $S$-wave, the $B_{s1}^\ast$ state goes down in the spectrum and it is located below $B^\ast K$ threshold with a mass compatible with lattice and EFT predictions. The $B_{s1}(5830)$ meson is almost insensitive to this coupling because it is the $|3/2, 1^+\rangle$ state predicted by HQS and thus couples mostly in a $D$-wave to the $B^\ast K$.

When such coupling is included the state associated with $B_{s1}(5830)$ meson suffers a moderate mass-shift and it is in very good agreement with other theoretical approaches and with the value reported in PDG. We observe that the meson-meson component is around $50\%$ for both $B_{s1}^\ast$ and $B_{s1}(5830)$ mesons, taking the quark-antiquark partial waves the other $50\%$.

It is worth mentioning that, while finishing the preparation of this work, a new study was published [48], supporting the idea that the $B_{s0}^\ast$ and the $B_{s1}^\ast$ should be in the mass region of $5720$ and $5770$ MeV, respectively. Such values are compatible with other EFT predictions and with the results presented in this work.

Finally, the mass of the $B_{s2}^\ast(5840)$ meson is predicted reasonably well within our quark model approach taking into account only quark-antiquark degrees-of-freedom. The same conclusion has been drawn by lattice-regularised QCD computations.

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