Strangeness in the baryon ground states

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Abstract

We compute the strangeness content of the baryon ground states based on an analysis of recent lattice simulations of the BMW, PACS, LHPC and HSC groups for the pion-mass dependence of the baryon masses. Our results rely on the relativistic chiral Lagrangian and large-$N_c$ sum rule estimates of the counter terms relevant for the baryon masses at $N^3$LO. A partial summation is implied by the use of physical baryon and meson masses in the one-loop contributions to the baryon self energies. A simultaneous description of the lattice results of the BMW, LHPC, PACS and HSC groups is achieved. We predict the pion- and strangeness sigma terms and the pion-mass dependence of the octet and decuplet ground states at different strange quark masses.

1 Introduction

The study of the quark-mass dependence of hadron masses is of great importance to unravel the intricate QCD dynamics at low energies. It provides a bridge between effective field theory approaches and lattice simulations. By now, various unquenched three flavor simulations for the pion-mass dependence of the baryon ground state masses are available \cite{1,2,3,4,5,6,7,8}. Less well studied is the kaon-mass dependence of the hadron masses.

The purpose of this Letter is to present detailed predictions on the strangeness content of the baryon octet and decuplet ground states. This is possible using the available lattice data on the pion-mass dependence of the baryon masses. Given a systematic analysis based on the chiral Lagrangian, the strange-quark mass dependence of the baryon masses can be calculated. Such results scrutinize the consistency of the chiral extrapolation approach and lattice simulations of the baryon masses. Variations of the baryon masses along suitable pathes in the pion-kaon mass plane are important to improve the determination of the low-energy parameters of QCD as encoded into the chiral Lagrangian.
Our work is based on the relativistic chiral Lagrangian with baryon octet and decuplet fields where effects at $N^3$LO (next-to-next-to-next-leading order) are considered systematically. The details of the approach are published in [9,10,11]. The chiral extrapolation of baryon masses with strangeness content is controversially discussed in the literature [10,3,12,13,11,8]. A straightforward application of chiral perturbation theory appears futile (see e.g. [3,12,8]). A crucial element of our scheme is the use of physical baryon masses in the one-loop contribution to the baryon self energies. Furthermore, the low-energy constants required at $N^3$LO are estimated by sum rules that follow from QCD in the limit of a large number of colors ($N_c$) [14,11]. Our approach was successfully tested against the available lattice data on the nucleon and omega masses of the BMW group [5]. Based on a fit to the BMW data, we obtained results that are in qualitative agreement with the predictions of the HSC group [4]. We challenge our approach further against recent lattice data from the PACS and LHPC groups [3,8], which assumed strange-quark masses significantly larger than the physical one. Furthermore, we aim at a quantitative description of the HSC data taking into account their slightly too small strange quark mass.

Since we obtain a simultaneous and quantitative description of the lattice data of the BMW, PACS, LHPC and HSC groups, we find it justified to present detailed results on the strange-quark mass dependence of all members of the baryon octet and decuplet states. In particular, we confront our parameter set against recent analyses of the BMW and QCDSF-UKQCD groups on the pion- and strangeness-sigma terms of the baryon octet states [7,15,16].

2 Chiral extrapolation of baryon masses

We consider the chiral extrapolation of the baryon masses to unphysical quark masses. Assuming exact isospin symmetry, the hadron masses are functions of $m_u = m_d \equiv m_q$ and $m_s$. The dependence on the light quark masses may be traded against a dependence on the pion and kaon masses. It is conventional to display the baryon masses as a function of the squared pion mass at fixed strange quark mass. The 'physical' strange quark mass is determined such that at the physical pion mass the empirical kaon mass is reproduced. Throughout this work we follow our approach as documented in [9,10,11]. In particular, we assume a quark-mass dependence of the pion and kaon masses as predicted by $\chi$PT at the next-to-leading order with parameters as recalled in [11]. The baryon masses are computed at $N^3$LO where we use physical baryon and meson masses in the one-loop contributions to the baryon self energies and assume systematically large-$N_c$ sum rules for the parameter set.

In our previous work [11] we adjusted the parameter set to the physical masses of the baryon octet and decuplet states and to the results for the pion-mass
dependence of the nucleon and omega masses as predicted by the BMW group. An accurate reproduction of the physical baryon masses and lattice data was achieved. All parameters, except of the symmetry preserving counter terms relevant at $N^3$LO, were considered. Given the large-$N_c$ sum rules of [14], there are 5 independent parameters only, which were all set to zero. The latter have a rather minor effect on the baryon masses and can be determined only with very precise lattice data. For details of the fit strategy and the definition of the various parameters we refer to [11]. The parameters of that Fit 0 are recalled in the 2nd column of Tab. 1. From the 16 parameters shown in Tab. 1 eight parameters were used to recover the exact physical baryon masses. Two further parameter combinations were found to be redundant. That left six free parameters only to reproduce the eight lattice data of the BMW group.

While the results of the BMW group are for a physical strange quark mass, the results of the PACS and LHPC groups rest on significantly larger strange quark masses. The HSC group uses strange quark masses that are slightly smaller than the physical one. Given a lattice data point with specified pion and kaon mass, we determine the corresponding quark masses, which are then used in the computation of the baryon masses. In the following, we consider three scenarios leading to three distinct fit strategies. Like in our previous works [17,9,10,11], we use $F \simeq 0.45$ and $D \simeq 0.80$ together with the values

| Parameter | Fit 0 | Fit 1 | Fit 2 | Fit 3* |
|-----------|-------|-------|-------|--------|
| $\bar{M}_{[8]}$ [GeV] | 0.9111 | 0.8095 | 0.8004 | 0.8951 |
| $\bar{M}_{[10]}$ [GeV] | 1.0938 | 0.9508 | 1.1570 | 1.0961 |
| $\bar{b}_0$ [GeV$^{-1}$] | -0.9086 | -0.9449 | -0.9722 | -0.8559 |
| $\bar{b}_D$ [GeV$^{-1}$] | 0.5674 | 0.4451 | 0.4778 | 0.4623 |
| $\bar{b}_F$ [GeV$^{-1}$] | -0.5880 | -0.4799 | -0.5197 | -0.4959 |
| $\bar{d}_0$ [GeV$^{-1}$] | -0.2300 | -0.3902 | -0.1247 | -0.2082 |
| $\bar{d}_D$ [GeV$^{-1}$] | -0.3617 | -0.4044 | -0.4258 | -0.4009 |
| $\bar{c}_0$ [GeV$^{-3}$] | 0.0176 | 0.0108 | 0.0101 | 0.0165 |
| $\bar{c}_4$ [GeV$^{-3}$] | -0.1659 | 0.3031 | 0.1273 | 0.3341 |
| $\bar{c}_5$ [GeV$^{-3}$] | -0.3320 | -0.7264 | -0.5005 | -0.7647 |
| $\bar{c}_6$ [GeV$^{-3}$] | -1.2366 | -1.4208 | -1.2841 | -1.5495 |
| $\bar{c}_4$ [GeV$^{-3}$] | -0.2520 | -0.2089 | -0.5068 | -0.3618 |
| $\bar{\zeta}_0$ [GeV$^{-2}$] | 1.1279 | 0.9520 | 0.8778 | 1.0921 |
| $\bar{\zeta}_D$ [GeV$^{-2}$] | 0.2848 | 0.3360 | 0.3432 | 0.2928 |
| $\bar{\zeta}_F$ [GeV$^{-2}$] | -0.2221 | -0.2973 | -0.3068 | -0.2338 |
| $\bar{\xi}_0$ [GeV$^{-2}$] | 1.1964 | 1.0558 | 1.0425 | 1.1753 |

Table 1
The parameters are adjusted to reproduce the empirical values of the physical baryon octet and decuplet masses and various lattice data as described in text.
In our first scenario we supplement the BMW data with the recent results from the LHPC group as presented in [8]. As suggested in this work, it may be advantageous to consider the following three particular mass combinations

\begin{align}
R_1 &= \frac{5}{48} \left( M_N + M_\Lambda + 3 M_\Sigma + 2 M_\Xi \right) - \frac{1}{60} \left( 4 M_\Delta + 3 M_\Sigma^* + 2 M_\Xi^* + M_\Omega \right), \\
R_3 &= \frac{5}{78} \left( 6 M_N + M_\Lambda - 3 M_\Sigma - 4 M_\Xi \right) - \frac{1}{39} \left( 2 M_\Delta - M_\Xi^* - M_\Omega \right), \\
R_4 &= \frac{1}{6} \left( M_N + M_\Lambda - 3 M_\Sigma + M_\Xi \right),
\end{align}

for which an improved analysis was provided. In Tab. 2 we recall the values for the three mass relations in (1) at three different sets of pion and kaon masses from [8]. We included the nine lattice data points into our $\chi^2$ function and obtained the parameter set shown in the third column of Tab. 1. Like for our previous results, the parameter set is a consequence of adjusting six free parameters only to the considered lattice data. The physical baryon masses are reproduced exactly. While the BMW data are still described with $\chi^2/N \simeq 0.7$, the LHPC data are described by our Fit 1 with $\chi^2/N \simeq 9.2$. We note that the rather small uncertainties in the lattice predictions, as recalled in Tab. 2 from [8], are based on a single-lattice spacing and do not yet consider effects from finite lattice volume corrections. The largest discrepancy of any of our masses as compared to the corresponding lattice result is decreasing with increasing pion masses. For the smallest pion mass, where one would expect the largest corrections from finite volume effects, the discrepancy is less than 18 MeV. If we assign an ad-hoc systematical error of the form

$$\Delta = 3 e^{-L m_\pi}/L m_\pi \, \text{GeV},$$

we find for the LHPC data $\chi^2/N \simeq 2.9$ with $L \simeq 2.5$ fm of [2]. The rough estimate (2) of finite volume effects is in line with the recent results of [18,13]. We conclude that the BMW and LHPC data sets can both be described by one parameter set. We checked that, though already our original parameter set leads to a qualitative description of the LHPC data, a combined fit improves the description of the LHPC data significantly without affecting the excellent reproduction of the BMW results. This reflects the fact that the BMW data alone do not define a steep minimum in our $\chi^2$ function.

Our second scenario considers the combined results of the BMW and PACS groups. In Tab. 3 we recall the results of the PACS group at 4 different sets of pion and kaon masses. We added the contribution of the 32 lattice points to

\[ H = 9F - 3D \text{ and } C = 2D \text{ as implied by large-$N_c$ sum rules and } f = 92.4 \text{ MeV for the chiral limit value of the pion decay constant.} \]
Table 2
Mass relations $R_1$, $R_3$ and $R_4$ in units of MeV for different values of pion and kaon masses. The lattice results are taken from [9].

| $m_\pi$ | $\frac{3}{2}R_{1\text{Fit}3}$ | $\frac{3}{2}R_{1\text{Fit}1}$ | $R_{3\text{Fit}3}$ | $R_{3\text{Fit}1}$ | $R_{4\text{Fit}3}$ | $R_{4\text{Fit}1}$ |
|---------|------------------|------------------|-----------------|-----------------|-----------------|-----------------|
| 320(2)  | 1285(6)          | -113(3)          | -41             | -40             |                 |                 |
| 640(2)  | 1270            | 1277            | -130           | -131           | -41             | -40             |
| 389(2)  | 1315(6)          | -100(2)          | -33             | -32             |                 |                 |
| 659(2)  | 1312            | 1317            | -109           | -109           | -33             | -32             |
| 557(2)  | 1454(6)          | -64(1)           | -19             | -185(1)        |                 |                 |
| 726(2)  | 1461            | 1452            | -58             | -60             | -20             | -21             |

the $\chi^2$ function and searched for its minimum. The resulting parameters are collected under Fit 2 in Tab. 1. This fit describes the PACS data with $\chi^2/N \simeq 1.4$. The quality of the description of the BMW data is still excellent with a $\chi^2/N \simeq 0.8$. It is interesting to compare the size of the various parameters of the three fits 0, 1 and 2. All parameters have quite similar values suggesting a high level of compatibility of the different lattice data sets.

In our third scenario we consider the combined lattice results of the BMW, LHPC and PACS groups. The minimum of our $\chi^2$ function is obtained with the parameter set of Fit 3 in Tab. 1. Again, the physical masses are reproduced exactly. The minimum of the $\chi^2$ function is reached by the variation of six free parameters only. This fit describes the LHPC data with $\chi^2/N \simeq 3.9$, the PACS data with $\chi^2/N \simeq 1.7$ and the BMW data with $\chi^2/N \simeq 0.7$. Detailed results are included in Tab. 2 and 3. We find this an encouraging result giving us high confidence in the extracted low-energy constants as collected in Tab. 1.

In Fig. 1 we show the pion-mass dependence of the baryon masses as implied by the three parameter sets of Tab. 1. The solid lines follow with parameter set 3, the dotted lines with set 2 and the dashed lines with set 1. The results are confronted against the lattice data from the BMW and HSC groups. The spread in the three lines is comfortably small. We recall that the HSC data rely on a slightly too small strange quark mass. Therefore, the HSC data points should not be compared quantitatively with any of the lines in Fig. 1. Our three line types assume a physical strange quark mass as required by a comparison with the BMW data points. In order to provide a quantitative comparison, we compute the baryon masses for the pion and kaon masses as assumed by the HSC group. The filled circles show our results based on the parameter set 3. The distance of the filled circles form the solid lines in Fig. 1 measures the importance of taking the precise physical strange quark mass in
Fig. 1. Baryon masses as a function of the squared pion mass at the physical strange quark mass as explained in the text.
| $m_\pi$ | $M_N^{\text{lat}}$ | $M_N^{\text{Fit2}}$ | $M_\Lambda^{\text{lat}}$ | $M_\Lambda^{\text{Fit2}}$ | $M_\Sigma^{\text{lat}}$ | $M_\Sigma^{\text{Fit2}}$ | $M_\Xi^{\text{lat}}$ | $M_\Xi^{\text{Fit2}}$ |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 155(6) | 929(78)         | 1136(21)        | 1215(21)        | 1389(7)         | 1408 1402    |                  |                  |                  |
| 552(2) | 949 955         | 1163 1164       | 1260 1264       |                  | 1443(10)     |                  |                  |                  |
| 295(3) | 1090(19)        | 1250(14)        | 1311(15)        |                  | 1446 1439    |                  |                  |                  |
| 592(2) | 1059 1073       | 1240 1245       | 1317 1320       |                  | 1446 1439    |                  |                  |                  |
| 410(2) | 1211(11)        | 1346(8)         | 1396(8)         |                  | 1498(7)      |                  |                  |                  |
| 633(1) | 1195 1215       | 1332 1341       | 1390 1392       |                  | 1489 1484    |                  |                  |                  |
| 383(3) | 1156(15)        | 1270(9)         | 1312(10)        |                  | 1404(7)      |                  |                  |                  |
| 580(2) | 1156 1170       | 1274 1282       | 1323 1329       |                  | 1412 1411    |                  |                  |                  |
|        |                 |                 |                 |                 | 1486 1486    |                  |                  |                  |
| 155(6) | 1257(82)        | 1493(30)        | 1637(15)        |                  | 1766(7)      |                  |                  |                  |
| 552(2) | 1245 1241       | 1433 1428       | 1616 1606       |                  | 1782 1776    |                  |                  |                  |
| 295(3) | 1396(20)        | 1539(15)        | 1678(13)        |                  | 1809(11)     |                  |                  |                  |
| 592(2) | 1362 1364       | 1524 1520       | 1678 1668       |                  | 1819 1808    |                  |                  |                  |
| 410(2) | 1508(14)        | 1619(9)         | 1727(9)         |                  | 1834(8)      |                  |                  |                  |
| 633(1) | 1497 1501       | 1615 1616       | 1737 1728       |                  | 1846 1838    |                  |                  |                  |
| 383(3) | 1460(19)        | 1550(16)        | 1643(13)        |                  | 1735(11)     |                  |                  |                  |
| 580(2) | 1463 1461       | 1565 1561       | 1667 1658       |                  | 1762 1752    |                  |                  |                  |

Table 3
Baryon masses in units of MeV. The lattice results are taken from [3].

the computation of the baryon masses. Most prominent is the effect in the pionmass dependence of the omega mass, where the filled circles are reasonably close to the predictions of the HSC group. The HSC data set is described with a $\chi^2/N \simeq 1.0$. Assuming parameter set 2, we obtain an almost as good description with $\chi^2/N \simeq 1.1$. The parameter set 1 leads to a significantly worse description with $\chi^2/N \simeq 3.5$.

In the remaining part of this section we wish to explore how to further improve the determination of the low-energy constants. This is a crucial issue in view of the additional five symmetry conserving parameters that were not considered so far. We feel that an inclusion of those five parameters in our $\chi^2$ function would not be significant at this stage. The total $\chi^2/N \simeq 1.6$ computed for all lattice points considered here is too close to one. A more detailed estimate of
systematic lattice effects like finite volume effects or the influence of smaller lattice spacings would be necessary. Given the small spread in our predictions for the pion-mass dependence of the baryon masses at physical strange quark mass, we find it useful to consider the flavor symmetric extrapolation with \( m_q = m_s \), where the baryons form a degenerate octet and decuplet. In Fig. 2 the octet and decuplet masses are shown as a function of the pion mass. In the limit of vanishing pion masses the figure predicts the three-flavor chiral limit value of the baryon masses. These values are of crucial importance in any effective field theory approach to hadron physics involving strangeness. We suggest that QCD lattice collaborations perform simulations for such a flavor symmetric case. Even results at semi-heavy pion masses would be very useful to further constrain the flavor singlet parameters of the chiral Lagrangian.

We close the discussion of the chiral extrapolation by showing in Fig. 3 our predictions for the pion-mass dependence of the baryon masses at an unphysical strange quark mass. At the physical pion mass our particular choice implies a kaon mass of 400 MeV. Our results suggest that such lattice simulations, in particular of any the decuplet masses, would be quite discriminative and therefore extremely useful in the path of establishing QCD’s low energy parameters.

Fig. 2. Pion-mass extrapolation of the baryon octet and decuplet masses in the flavor symmetric limit.
Fig. 3. Baryon masses as a function of the squared pion mass at an unphysical strange quark mass as explained in the text.
3 Pion and strangeness baryon sigma terms

The pion- and strangeness baryon sigma terms play an important role in various physical systems. For instance, the pion-nucleon sigma term is of greatest relevance in the determination of the density dependence of the quark condensate at low baryon densities and therefore provides a first estimate of the critical baryon density at which chiral symmetry may be restored (see e.g. [19]). Similarly, the kaon-nucleon sigma terms are key parameters for the determination of a possible kaon condensate in dense nuclear matter [20].

Assuming exact isospin symmetry with $m_u = m_d \equiv m_q$, the pion-nucleon sigma term reads

$$\sigma_{\pi N} = m_q \langle N(p) | \bar{q} q | N(p) \rangle = m_q \frac{\partial}{\partial m_q} m_N. \quad (3)$$

The pion-sigma terms of the remaining baryon states are defined analogously to (3). As indicated in (3), the matrix elements of the scalar quark operator are accessible via the derivative of the nucleon mass with respect to the light quark mass $m_q$. This follows directly from the Feynman-Hellman theorem. The strangeness sigma term $\sigma_{sN}$ of the nucleon is determined by the derivative of the nucleon mass with respect to the strange quark mass $m_s$:

$$\sigma_{sN} = m_s \langle N(p) | \bar{s} s | N(p) \rangle = m_s \frac{\partial}{\partial m_s} m_N. \quad (4)$$

In Tab. 4 we present our predictions for the pion- and strangeness sigma terms

|     | Fit 1 | Fit 2 | Fit 3 |
|-----|-------|-------|-------|
| $\sigma_{\pi N}$ | 39(4)$^{+18}_{-7}$ | 31(3)(4) | 59(2)(17) |
| $\sigma_{s N}$ | 67(27)$^{+55}_{-47}$ | 71(34)(59) | -4(23)(25) |
| $\sigma_{\pi \Lambda}$ | 29(3)$^{+11}_{-5}$ | 24(3)(4) | 39(1)(10) |
| $\sigma_{s \Lambda}$ | 180(26)$^{+48}_{-77}$ | 247(34)(69) | 126(26)(35) |
| $\sigma_{\pi \Sigma}$ | 28(3)$^{+19}_{-3}$ | 21(3)(3) | 26(2)(5) |
| $\sigma_{s \Sigma}$ | 245(29)$^{+50}_{-72}$ | 336(34)(69) | 159(27)(45) |
| $\sigma_{\pi \Xi}$ | 16(2)$^{+8}_{-3}$ | 16(3)(4) | 13(2)(1) |
| $\sigma_{s \Xi}$ | 312(32)$^{+72}_{-77}$ | 468(35)(59) | 267(31)(50) |

Table 4
Pion- and strangeness sigma terms of the baryon octet states in units of MeV.
of the baryon octet states. They are compared with two recent lattice determinations [7,15]. Our values for the sigma terms are in reasonable agreement with the lattice results. We find it encouraging that our values are compatible with both lattice groups within one sigma deviation in almost all cases. In particular, we obtain a rather small value for the pion-nucleon sigma term, which is within reach of the seminal result $\sigma_{\pi N} = 45 \pm 8$ MeV of Gasser, Leutwyler and Sainio in [21]. The size of the pion-nucleon term can be determined from the pion-nucleon scattering data. It requires a subtle subthreshold extrapolation of the scattering data. Despite the long history of the sigma-term physics, the precise determination is still highly controversial (for one of the first reviews see e.g. [22]). Such a result is also consistent with the recent analysis of the QCDSF collaboration [16], which suggests a value $\sigma_{\pi N} = 38 \pm 12$ MeV. Our best estimate for the strangeness sigma term of the nucleon with $\sigma_{sN} \simeq 2$ MeV is compatible with the currently most precise lattice prediction $\sigma_{sN} = 12^{+23}_{-16}$ MeV in [16].

In Tab. 4 we recall also the results of a chiral extrapolation attempt of the recent PACS data by Camalich et al. [12]. The analysis is based on the baryon masses truncated at $N^2$LO with phenomenologically adjusted values for the meson-baryon coupling constants. For almost all sigma terms we find significant differences to our results. This may reflect the significantly much less accurate reproduction of the PACS data and the physical baryon masses in [12].

We turn to the sigma terms of the decuplet states, for which our predictions are compiled in Tab. 5. Again, we compare our values with the ones obtained in [12]. Like in the case of the baryon octet states there are significant differences.

|                | [12]     | Fit 1 | Fit 2 | Fit 3 |
|----------------|----------|-------|-------|-------|
| $\sigma_{\pi \Delta}$ | 55(4)(18) | 37    | 32    | 33    |
| $\sigma_{\pi \Sigma^+}$  | 39(3)(13) | 31    | 26    | 26    |
| $\sigma_{\pi \Xi^+}$  | 22(3)(7)  | 22    | 15    | 17    |
| $\sigma_{\pi \Omega}$   | 5(2)(1)   | 14    | 7     | 9     |
| $\sigma_{s \Delta}$     | 56(24)(1) | 82    | 0     | 28    |
| $\sigma_{s \Sigma^+}$   | 160(28)(7)| 255   | 167   | 200   |
| $\sigma_{s \Xi^+}$      | 274(32)(9)| 425   | 320   | 371   |
| $\sigma_{s \Omega}$     | 360(34)(26)| 559  | 460   | 503   |

Table 5
Pion and strangeness sigma terms of the baryon decuplet states in units of MeV.
4 Summary and outlook

In this work we presented the first simultaneous and quantitative description of the baryon octet and decuplet masses as computed by four different lattice groups, BMW, LHPC, PACS and HSC. Using the chiral Lagrangian at N^3LO we obtained a universal parameter set that leads to a quantitative reproduction of the baryon masses and the lattice simulation data at different pairs of pion and kaon masses. While the physical masses are reproduced exactly, the lattice data are successfully fitted with six free parameters only. The total $\chi^2/N \simeq 1.6$ of all considered lattice data is amazingly small. This suggests a high level of compatibility of the different lattice data sets. Based on this result, we predicted the pion and strangeness sigma terms of all baryon ground states. Assuming different lattice data as input, we find rather small variations in our predictions only. In particular, we obtain a quite small pion-nucleon sigma term of about 32 ± 2 MeV and an even smaller strangeness sigma term of the nucleon of about 2 MeV for our best parameter set.

In order to further diminish the residual uncertainties in the low-energy constants of QCD, we presented predictions for the pion-mass dependence of the baryon masses in the flavor symmetric limit and for a somewhat reduced strange quark mass. Future lattice simulations along those pathes in the pion-kaon plane would help to consolidate our parameter set and may lead to the determination of the additional five flavor symmetric counter terms that were not considered yet. The latter play a decisive role in meson-baryon scattering processes at next-to-leading order already.

References

[1] C. Aubin, et al., Light hadrons with improved staggered quarks: Approaching the continuum limit, Phys. Rev. D70 (2004) 094505.

[2] A. Walker-Loud, et al., Light hadron spectroscopy using domain wall valence quarks on an Asqtad sea, Phys. Rev. D79 (2009) 054502.

[3] S. Aoki, et al., 2+1 Flavor Lattice QCD toward the Physical Point, Phys. Rev. D79 (2009) 034503.

[4] H.-W. Lin, et al., First results from 2+1 dynamical quark flavors on an anisotropic lattice: light-hadron spectroscopy and setting the strange-quark mass, Phys. Rev. D79 (2009) 034502.

[5] S. Durr, et al., Ab-Initio Determination of Light Hadron Masses, Science 322 (2008) 1224–1227.

[6] C. Alexandrou, et al., Low-lying baryon spectrum with two dynamical twisted mass fermions, Phys.Rev. D80 (2009) 114503.
[7] S. Durr, et al., Sigma term and strangeness content of octet baryons, Phys. Rev. D85 (2012) 014509.

[8] A. Walker-Loud, Evidence for non-analytic light quark mass dependence in the baryon spectrum, arXiv:1112.2658 [hep-lat].

[9] A. Semke, M. F. M. Lutz, Baryon self energies in the chiral loop expansion, Nucl. Phys. A778 (2006) 153–180.

[10] A. Semke, M. F. M. Lutz, On the possibility of a discontinuous quark-mass dependence of baryon octet and decuplet masses, Nucl. Phys. A789 (2007) 251–259.

[11] A. Semke, M. F. M. Lutz, Quark-mass dependence of the baryon ground-state masses, Phys. Rev. D85 (2012) 034001–034012.

[12] J. Martin Camalich, L. S. Geng, M. J. Vicente Vacas, The lowest-lying baryon masses in covariant SU(3)-flavor chiral perturbation theory, Phys. Rev. D82 (2010) 074504.

[13] L.-s. Geng, X.-l. Ren, J. Martin-Camalich, W. Weise, Finite-volume effects on octet-baryon masses in covariant baryon chiral perturbation theory, Phys. Rev. D84 (2011) 074024.

[14] M. F. M. Lutz, A. Semke, Large-Nc operator analysis of 2-body meson-baryon counterterms in the chiral Lagrangian, Phys. Rev. D83 (2011) 034008.

[15] R. Horsley, et al., Hyperon sigma terms for 2+1 quark flavours, arXiv:1110.4971 [hep-lat].

[16] G. S. Bali, et al., The strange and light quark contributions to the nucleon mass from Lattice QCD, arXiv:1111.1600 [hep-lat].

[17] M. F. M. Lutz, E. E. Kolomeitsev, Relativistic chiral SU(3) symmetry, large $N_c$ sum rules and meson-baryon scattering, Nucl. Phys. A700 (2002) 193–308.

[18] M. Procura, B. U. Musch, T. Wollenweber, T. R. Hemmert, W. Weise, Nucleon mass: From lattice QCD to the chiral limit, Phys. Rev. D73 (2006) 114510.

[19] M. F. M. Lutz, B. Friman, C. Appel, Saturation from nuclear pion dynamics, Phys. Lett. B474 (2000) 7–14.

[20] D. Kaplan, A. Nelson, Strange Goings on in Dense Nucleonic Matter, Phys.Lett. B175 (1986) 57–63.

[21] J. Gasser, H. Leutwyler, M. Sainio, Sigma term update, Phys.Lett. B253 (1991) 252–259.

[22] E. Reya, Chiral symmetry breaking and meson - nucleon sigma commutators: A Review, Rev. Mod. Phys. 46 (1974) 545–580.