Mathematical connections ability in solving trigonometry problems based on logical-mathematical intelligence level

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Abstract. A Mathematical connection is students’ ability in connecting various concepts, principles, and skills they have. Mathematical connections ability enables them to do analytic and synthetic reasoning. The subjects of this study are the tenth-grade students of SMA Negeri 2 Magetan who have different logical-mathematical intelligence. The aim of this study is to determine students’ mathematical connections ability based on their logical-mathematical intelligence in solving trigonometry problem. The research method applied is descriptive qualitative research. The techniques of collecting data are logical intelligence tests, mathematical connections tests, and interviews. The technique of analysing data is content analysis. The results of this study are as follows: (1) students with high logical-mathematical intelligence work smoothly by using mathematical connections ability; (2) students with medium logical-mathematical intelligence are working on problem-solving by making some errors in finding connections and relationships of various mathematical structures; and (3) students with low logical-mathematical intelligence are unable to make connections and relationships between different mathematical structures.

1. Introduction
A mathematical connections ability is one of the highest mathematical thinking abilities [1]. In mathematics learning, students are required to understand the connection between mathematical and inter-mathematical ideas with other fields of study [2]. In mathematics, at least three kinds of connections are particularly beneficial, namely connections within mathematics, across the curriculum, and with real-world contexts [3]. Mathematical connections consist of two parts, namely internal and external connections. Internal connection is to associate mathematical concepts with other mathematical concepts, while external connection links mathematical concepts with other fields of study and everyday life [4]. Mathematical connection can help students in mathematics learning activity. Mathematical connections are needed by students in the problem-solving process at each stage as an effort to find a solution based on the knowledge they have [5].

In fact, problem-solving is a basic competence in 21st-century skills [6, 7]. Mathematics learning that focuses on problem-solving can make students understand the essence of the areas under study [8, 9]. Problems in mathematics are divided into 2 types, namely problem to find and problem to proof [10]. In the problem-solving process, an individual must be able to combine appropriate operations and apply them to the solution [11]. Problem-solving steps are divided into four steps, namely understanding problems, developing problem-solving or making a resolution plan, exploring or
implementing resolutions, and reviewing problems and solutions [10, 12]. In conclusion, problem-solving in mathematics learning has three important points, namely problem solving as the main objective of mathematics learning, problem-solving as a process in mathematics learning, and problem-solving as a basic skill that must be possessed when someone learns mathematics [13].

Trigonometry is one of areas in studying mathematics in high school. Students are required to have basic competencies of trigonometry area [14]. One of the basic competencies of trigonometry area is to solve contextual problems related to trigonometry ratios in right-angled triangle [14]. In reality, trigonometry is a learning area in mathematics which very few students like it [15]. It is in line with the research that students cannot develop trigonometry concepts in solving mathematical problems [16]. Students' difficulties in working on mathematical problem-solving are influenced by several factors. Understanding in using symbols and the language, using the right steps, prerequisite facts and concepts, determining the concept that can be used, doing computation and understanding the aim of problem are the factors of students' difficulties in mathematical problem-solving [17].

There are differences in problem-solving steps for each student, due to the differences of students' logical intelligence level [18]. It is obvious that in the popular teaching of mathematics, which highly reliant on the logical-mathematical intelligence; students’ mathematical functioning is related to their logical-mathematical intelligence [19]. Logical-mathematical intelligence is students’ ability to understand the relevant concepts, solve abstract problems, and recognize patterns and sequences [20, 21]. Logical-mathematical intelligence contains three indicators, namely number or reasoning, analyze problem logically, and investigate issue scientifically [22].

Based on the results of the research, students' mathematical connections ability still has some deficiencies. This study aims to determine students’ mathematical connections ability based on logical-mathematical intelligence level in solving trigonometry problems.

2. Method

This research is a qualitative method with descriptive research type. The techniques of collecting data used are logical intelligence tests, mathematical connections tests, and interviews. This study was conducted on the even semester of the X-11 class of SMA Negeri 2 Magetan, which consists of 30 students. Students were given 2 written tests. The first test was used to divide students into three categories based on logical-mathematical intelligence, namely high, medium, and low level. Two students were selected from each group. The six subjects were given a second test in the form of trigonometry problems that are used as interview guidance to know students’ mathematical connections ability in each level.

Trigonometry problem-solving tests are about various concepts in mathematics with a target to determine students' mathematical connections ability in solving trigonometry problem-solving test. The problem-solving given are as follows:

- Problem-solving test (PST) 1
  Draw a diagram that connects the three coordinate points, point A (0,0), point B (-8,6), and point C (-8,0). Suppose \( \angle BAC = \alpha \) and \( \angle CBA = \beta \) then specify:
  a. Triangle image formed from the connection of the three points
  b. What is the value of \( \sin \alpha \cdot \cos \beta \)
  c. What is the value of \( m \), if it is known \( \cos \alpha \cdot \cos \beta = 3m \)
  d. Value \( (6 \cdot \tan \beta)^2 \)

- Problem-solving test (PST) 2:
  A cuboid of ABCD. EFGH looks like figure 1, with a height of 4 cm, a width of 3 cm, and a length of 3 times the height. If a triangle made connects points A, H, and B where \( \angle ABH = \alpha \), and \( \angle AHB = \beta \), specify,
  a. Value of \( q \) if known \( \cos \beta \cdot \tan \alpha = \frac{q}{4} \)
  b. The value of \( \cos \alpha \cdot \tan \beta \)
c. Value of x, if it is known \( \cos \alpha \cdot \sin \beta = 5x \)

![Figure 1. Cuboid ABCD. EFGH in PST 2](image)

The results of the trigonometry problems-solving test are analyzed using indicators set by the researcher that referenced to the content. The analysis stage is done after obtaining the data collection from the written data and subjects interview. Validation used in this research is the time triangulation. The researcher checks the validity of the data by conducting an interview, observation, or other techniques in different time and conditions [23]. The objectives of this study are to make problem-solving strategies by connecting different concepts/topics, and students' skills in connecting various concepts as the implementation of a problem-solving stage. While for the step of interview analysis, there are three stages, namely data reduction, data presentation, and conclusion.

3. Result

The following students were chosen based on their result of logical-mathematical intelligence test. The selected students as the research subjects are described in the following table.

| Students Initial | Score | Level  | Subject Code |
|------------------|-------|--------|--------------|
| ARS              | 75    | High   | H1           |
| MHD              | 72.5  | High   | H2           |
| ASD              | 67.5  | Medium | M1           |
| ZPN              | 62.5  | Medium | M2           |
| SHL              | 42.5  | Low    | L1           |
| AJN              | 35    | Low    | L2           |

After sampling, the researcher gives the trigonometry problem-solving test. The following is the result of the interview analysis on the problem-solving test (PST) 1 and problem-solving test (PST) 2.

3.1 Students' mathematical connections with high logical intelligence in the PST 1 and PST 2

Subjects H1 and H2 are able to understand, explain information, and know the focus of the questions in PST 1 and PST 2. Both subjects were able to plan the solution clearly and systematically. In PST 1, H1 solves the problems by relating the Cartesian coordinate concept, Pythagoras theory, and the right triangular trigonometry comparison concept. Following is an excerpted interview with subject H1.

Table 2. The Interview Result of Subject H1 in the First Interview

| Code            | Interview Result                                                                 |
|-----------------|----------------------------------------------------------------------------------|
| Researcher      | “How do you plan the solutions based on the information and questions you know from the question?” |
| Subject H1      | “The first one is drawing, after the image done, we specify the right angle vertex and then determine the value of its sin, cos by using the formula.” |
Subject H1 makes a graphic from the information which includes in the test using the information of the test given. Figure 2 is an image made by the subject H1.

![Figure 2](image)

**Figure 2. The Photograph of Subject H1’s Writing Test Answer in The First Activity**

Based on figure 2, subject H1 does the calculation in solving the given problem correctly. Students are able to make the connection between the concept of coordinate system and Pythagoras theory in solving the problem. After finding all the required information, students are able to make mathematical connections between the triangle information (triangle edges, and corners) with the knowledge of the trigonometry comparison. In PST 2, H1 solves using a bound of mathematical connections ability. She connects the diagonal knowledge of the field and the diagonal of the space with the trigonometry knowledge and subsequently shows the mathematics calculation correctly. Figure 3 is the results of the H1’s work on the second question:

![Figure 3](image)

**Figure 3. The Photograph of Subject H1’s Writing Test Answer in The Second Activity**

From figure 3(a), Subject H1 shows AH side length calculations using Pythagoras theory. Subject H1 understands the diagonal concept in the field, so she can seek length side of AH. In addition form figure 3(b), after obtaining the length of AH, the subject searches the length of HB which is the diagonal space of ABCD. EFGH cuboid by using Pythagoras theory concept. Then she links the knowledge about the sides and angles of the triangle ABH to determine the value of the trigonometry comparison.
In PST 1, Subject H2 explains that to solve the problem, the information of coordinate points is used to create images. It can determine the trigonometry comparison value. Table 3 is an excerpted interview with subject H2:

**Table 3. The Interview Result of Subject H2 in the First Activity**

| Code | Interview Result |
|------|------------------|
| Researcher | “Please explain the relationship of information contained in the item to the question given!” |
| Subject H2 | “These coordinate points are used for drawing, then after the drawing is complete, it can just take the next step.” |
| Researcher | “Explain how do you mean by that next step?” |
| Subject H2 | “After we get the triangle image, we find all sides, then used to find the value of sin, cos, and tan (by formula).” |

Figure 4 shows that subject H2 does calculations correctly and precisely in solving the given problem. Based on Figure 4(a), subject H2 is able to create a relationship between the image concept and the Pythagoras formula to find the right-angled triangle side skew. On figure 4(b), the H2 is able to use the trigonometry comparison formula and do algebra calculation appropriately.

![Figure 4](image)

**Figure 4.** The Photograph of Subject H2’s Writing Test Answer in the First Activity

Subject H2 solves PST 2 well. Just like subject H1, subject H2 connects the diagonal concept of the field, and the space diagonal to find the sides of the triangle ABH. He explains that the steps to solve the problem are by connecting knowledge of the diagonal field with Pythagoras theory, as well as looking for the length of the BH side which is one of the diagonal spaces of the cuboid. Based on the calculation, H2 uses the comparison formula of right-angled triangle trigonometry. Having obtained sin, cos, and tan values from α and β angles, then the subjects perform an appropriate algebra operation.

3.2 Students’ mathematical connections with medium logical intelligence in the PST 1 and PST 2

The results of the interview with the subjects who have medium logical-mathematical intelligences show that the subjects M1 and M2 know the supporting information and questions on the problem. Subject M1 says that to solve the problem, it needs to connect various knowledge. The required knowledge is drawing concept on the Cartesian coordinate field, and Pythagoras theory to
find the side skew, then the formula of trigonometry comparison. In the problem-solving process, M1 makes an error in drawing the Cartesian coordinate field. Figure 5 is the answer of the M1.

Figure 5. The Photograph of Subject M1’s Writing Test Answer in The First Activity

Subject M1 makes a drawing error on the Cartesian coordinate field. Based on the work of the M1 subject, it shows that she is fluent in relating the various mathematical concepts to the plan, but the final outcome is less appropriate. In PST 2, M1 relates knowledge of the diagonal field and the diagonal of space to find the lengths of the AH, and BH side of the triangle ABH appropriately. Then, she connects the information from the triangle ABH with the concept of right triangular trigonometry comparison. However, she makes an error in determining the value of \( \tan \alpha \) and \( \cos \alpha \). As a result, it causes miscalculation of problems 1a, and 1b. The student’s error is shown in figure 6 below:

Figure 6. The Photograph of Subject M1’s Writing Test in the Second Activity

The subject makes an error in determining the trigonometry comparison value of \( \tan \alpha \). It should be a value of

\[
\tan \alpha = \frac{5}{12}
\]

Similarly, the value of \( \cos \alpha \) should be
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\[
\cos \alpha = \frac{12}{13}
\]

In general, subject M2 understands and plans the problem either in PST 1 or PST 2 by mentioning the supporting information and focusing on the question. At the problem-solving stage, subject M2 draws in the coordinate field precisely. She is able to make the relationship between the information in the problem and the drawing concept on the coordinate.

Although she understands the concept, she finds some difficulties in several parts. In PST 1 after the image is drawn, she cannot relate the knowledge of right-angled triangle with the trigonometry comparison concept of right-angled triangle. She does not know that the next step is to look for the oblique side of the right-angled triangle. Subject M2 uses the trigonometry comparison concept of the special angle because she finds difficulty in determining the formula used to solve the given problem.

Following figures are the results of the student’s work:

![Figure 7](image1.png)

**Figure 7:** The Photograph of Subject M2’s Writing Test Answer in the First Activity

Based on figure 7(a), M2 subject is able to make the relationship between the information in the problem and the drawing concept on the coordinate field. In figure 7(b), M2 subject shows a calculation to find the length of the AB side of the ABC triangle. However, she makes an error concept to determine the trigonometry comparison value in the question.

Besides, in PST 2, subject M2 makes an error concept to determine the trigonometry comparison value in the question. When solving the problem, she is careless in doing the calculation so that the final answer is not appropriate. At the problem-solving stage, subject M2 makes some errors. Figure 8 shows that subject M2 still makes some errors. From figure 8(a), the subject of M2 shows the triangle side ABH calculation by using the information on the ABCD.EFGH cuboid correctly, whereas in figure 8(b), subject M2 makes error calculation because it is not complete in writing item 1a.

The subject writes item 1a with,

\[
\cos \beta \tan \alpha = ...
\]

It should be,

\[
\cos \beta \tan \alpha = \frac{q}{4}
\]

The second error made by the subject is in answering question c. M2 makes an error in the count operation.
3.3 Students’ mathematical connections with low logical intelligence in the PST 1 and PST 2
The subjects with low logical intelligence, L1 and L2 are able to know the supporting information and focus on problem-solving given. Subject L1 has difficulty in planning problem-solving either in PST 1 or PST 2. She cannot make the connections between the knowledge in the problem and the question. In PST 1, subject L1 does not know what mathematical concepts and topics used after drawing the Cartesian coordinate. She stated that after finishing drawing on the field the next step is to find the area of the angle. Table 4 is an interview result with subject L1 in PST 1.

Table 4. The Interview Result of Subject L1 in the First Activity

| Code       | Interview Result                                                                 |
|------------|----------------------------------------------------------------------------------|
| Researcher | “What is the relationship of the information on the problem to the question in that problem?” |
| Subject L1 | “Firstly, we are told to draw based on this coordinate point,”                   |
| Researcher | “Then what next?”                                                                |
| Subject L1 | “After drawing, then determining the angle”                                      |
| Researcher | “The point is to determine how big the angle is?”                                |
| Subject L1 | “From the image formed we determine the angle to answer questions 1b, 1c, and 1d” |

In the problem-solving step, the subject makes an error in drawing the coordinate field. She is not able to make the relationship of the various mathematical knowledge to solve the trigonometry problem. Subject L1 states that in solving the problem, she only guesses it in determining the completion formula. Figures 9 are the results of the L1 subject's work. Based on figure 9(a), it shows that L1 does not know the relationship between the supporting information and the questions presented in the problem. Meanwhile in the figure 9(b), the subject makes an error in using the concept to answer the question.
In PST 2, L1 says that in solving this second problem, she only uses trigonometry area, and the subject also does not understand the trigonometry area. Subject L1 solves this second problem by using the concept of special angular trigonometry. Here is an excerpt interview with the subject.

Table 5. The Interview Result of Subject L1 in Second Activity

| Code               | Interview Result                                                                                                                                 |
|--------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| Researcher         | “Do you think the information on this question is enough to answer the question?                                                                 |
| Subject L1         | “Not yet (not enough)”                                                                                                                        |
| Researcher         | “Not yet, why is that? Please explain!”                                                                                                        |
| Subject L1         | “Because I do not know sin, cos and tan formula”                                                                                             |
| Researcher         | “Researcher: If you do not know, how can you solve this problem?”                                                                             |
| Subject L1         | “This uses the cos sin formula in physics lessons”                                                                                             |
| Researcher         | “What is a special angular trigonometry formula?, then how can you set cos 75° with half root three?”                                           |
| Subject L1         | “I just guessed the answer (inconsequential).”                                                                                               |

Figure 9. The Photograph of Subject L1’s Writing Test Answer in the First Activity

Figure 10. The Photograph of subject L1’s Writing Test Answer in the Second Activity
Figure 10 shows that subject L1 cannot relate the existing knowledge to the problem with other mathematical concepts. L1 uses the knowledge that is recalled by her such as the concept of special angular trigonometry.

Just like subject L1, subject L2 also has difficulty in solving the problem. She makes many errors in PST 1 and PST 2. She cannot determine what concepts will be used to solve the given problem. In PST 1, subject L2 cannot draw the Cartesian coordinates appropriately. She also cannot make the relation of various mathematical concepts to solve the problem. The following is the interview result of the subject L2.

**Table 6. The Interview Result of Subject L2 in the First Activity**

| Code       | Interview Result                                                                 |
|------------|-----------------------------------------------------------------------------------|
| Researcher | “Do you think this triangle image is right?”                                      |
| Subject L2 | “Yes (but subject looks hesitant with the answer)”                                |
| Researcher | “In addition to drawing this triangle, what other concepts do you use in solving this problem?” |
| Subject L2 | “Hehhe ... .. (Subject L2 shakes his head / indicates his ignorance)”             |
| Researcher | “Do not know huh? Then how can you do this?”                                      |
| Subject L2 | “Logic... (only alleged subject L2 that the concept used is ordinary multiplication)” |

Figure 11 is the result of student’s work in solving the problem. Figure 11(a) shows that the student cannot understand the concept of drawing on the Cartesian coordinate field. Meanwhile figure 11(b) shows that the student is working on the given problem by doing algebra multiplication without the final result.

![Figure 11(a)](image1)

![Figure 11(b)](image2)

**Figure 11. The Photograph of Subject L2’s Writing Test Answer in the First Activity**

Just like in PST 1, subject L2 also makes an error in determining the concept used to solve the second problem. In completing the second problem, L2 uses the length, width, and height of the cuboid to determine the final result. She does not understand the proper completion step.

Figure 12 is the results of the L2's work in solving the second trigonometry problem. Figure 12(a) shows that subject L2 cannot make the exact completion step. She uses the information about the length, width, and height of the cuboid to determine the sin, cos, and tan values of α and β angles. L2 says that if the solution remembered is a comparison, she forgets which formula to use. Figure 12(b) also shows the same steps by the L2 subject in solving the second trigonometry problem.
4. Discussion
From the results of the research above, subjects with high logical intelligence do mathematical connections from the information contained in the problem with other mathematical knowledge. Subjects in the medium logical intelligence, at the first step, they connect various mathematical concepts well, but they make errors in the process of coordinate concepts and calculation of algebra. Subjects with low categories cannot make mathematical connections so that they are false in determining the concepts used to solve problem-solving. In addition, the low logical intelligence, subjects do not seem to understand the concept of trigonometry comparison correctly. In the low logical intelligence, students cannot make connections between mathematical concepts correctly [23]. D.K Zaenab states that students’ mathematical connection ability with contextual learning will be better than conventional [24]. Based on that statement, choosing the right method of learning can improve students’ mathematical connections ability.

5. Conclusion
This research has one problem formulation that is to know mathematical connection ability in trigonometry problem viewed from 3 categories of logical-mathematical intelligence. After the research and the discussions are completed, the researchers draw some conclusions. It can be concluded that from two trigonometry problems given, students with high logical intelligence are able to create connections or relationships of various mathematical concepts. They can plan and do problem-solving appropriately step by step. Then, the students in the medium logical intelligence are able to make relationships of various mathematical concepts. However, they do some errors in the calculations and understanding the concept in planning and problem-solving stage. At least not the last, subjects with low logical intelligence, they do not understand the concepts used to solve the problem. In every problem-solving, the subjects do mistakes. They also can not make connections from various mathematical concepts in solving trigonometry problems. It becomes very important for teachers to use appropriate learning methods according to the mathematical problems faced by the students such as doing exercises about trigonometry problem-solving on a regular basis.

References
[1] NCTM 2000 Principles and Standards for School Mathematics. Resston, VA: author
[2] Heris H, Ujang R S, and Utari S 2014 Mathematical Connection Ability and Self-Confidence International Journal of education 8 p 2-3
[3] Ruspiani 2000 Kemampuan Siswa dalam Melakukan Koneksi Matematika. Tesis Jurusan Matematika (Bandung: UPi)
[4] Satriawati Gusni and Kurniawati Lia 2008 Jurnal Matematika dan Pendidikan Matematika 3 p 97
[5] Schoenfeld A H 1982 “Heuristic in Classroom”. Problem solving in School Mathematics. NCTM
[6] P21 2007b 21st Century Curriculum and Instruction (Washington DC: Partnership for 21st
Century Skills

[7] Wilson J W, Fernandez M L, and Hadaway N 1993 Mathematical Problem Solving In Problem Solving Research Ideas for the Classroom: High School Mathematics, ed Wilson (New York: Macmillan)

[8] Olkun S and Toluk S 2002 Textbooks, Word Problems, and Student Success on Addition and Subtraction International Journal for Mathematics Teaching and Learning 11 p 162-70

[9] Rizvi N F 2004 Perspective Teacher’s Ability to Pose Word Problem International Journal for Mathematics Teaching & Learning 10 p 79 -88

[10] Polya G 1973 How To Solve it. (New Jersey: Princenton University Press)

[11] Bernardo A B 1999 Overcoming Obstacles in Understanding and Solving Word Problem in Mathematics Educational psychology: An international Journal of Experimental Educational Psychology 19 p144-63

[12] Santrock J W 2006 Educational Psychology, Second Edition. (New York: Mc. Graw Hill)

[13] Branca N A 1980 Problem Solving as a Goal, Process, and Basic Skill In Problem Solving in School Mathematics 1980 Yearbook of the National Council of Teachers of Mathematics, ed S Krulik and R. E. Reys (Reston, VA: National Council of Teachers of Mathematics)

[14] Permendikbud Tahun 2016 Nomor 24

[15] Gur Hulya 2009 Trygonometri Learning. Researchgate

[16] Arfani M A K 2016 Keefektifan Pembelajaran Trigonometri di Kelas X SMA Menggunakan Pendekatan Saintifik Dengan Model Pembelajaran Cooperative Learning Tipe Think Pair Share Ditinjau Dari Prestasi Belajar Siswa

[17] Ruhyana 2016 Jurnal Computech & Bisnis 10 p 106-18

[18] Ike P E 2017 Proc: SI MaNis (Seminar Integrasi Matematika dan Nilai Islami) 1 p 617-21

[19] Niroo Mohammad 2012 The Effect of Gardner Theory Application on Mathematical / Logical Intelligence and Student’s Mathematical Functioning Relationship. Procedia: social and behavioral sciences 47 pp. 2169-75

[20] Gardner H 2007 Five Minds for The Future. (Jakarta : Gramedia)

[21] Bellanca J 2009. 200+ Active Learning Strategies and Projects for Engaging Students Multiple Intelligences. (California: Corwin Press)

[22] Fetaji B and Fetaji M. 2009 E-Learning Indicator : A Multi-Dimensional Model for Planning and Evaluating e-Learning Software Solution. Electroic Journal of e-Learning. 7 p 1-28

[23] Sugiyono 2012 Metode Penelitian Pendidikan Pendekatan Kuantitatif, Kualitatif, dan R&D. (Bandung: ALFABETA)

[24] Nila K, Budiyono, and Teguh W 2013 Electronic Journal Program Studi Pendidikan Matematika. 2 p 44-5

[25] Zaenab D K 2010 Pengaruh Pembelajaran Kontekstual Terhadap Kemampuan Koneksi Matematik Siswa. (Jakarta: UIN Syarif Hidayatullah)