Characteristic of student's false concessive failure on fractions concept

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Abstract. False concessive failure of students is students construction failure in achieving goals, where the process is false, the results obtained is true. This failure occurs because the schemes are not connected to each other, so it cannot form a new scheme. This research is a qualitative research that aim to describe the characteristics of students’ false concessive failure on fractional concepts. This fractional concept is a fraction of the same part of the whole. To be able to solve the concept of this fraction, students must possess the concept of partition, the concept of addition and the concept of multiplication of the fraction.

1. Introduction
At elementary school, fractions was introduced symbolically [1], as (1) regular fractions (such as: $\frac{1}{2}$, $\frac{1}{4}$, etc.), (2) decimal fractions (such as: 0.25, 0.50, 0.75, etc.), and (3) percentage fractions (such as: 5, 10, 12%, etc.). Actually, according to [2], teachers may use three approaches to model a fraction to elementary school students, namely: (1) area (e.g., 1 3 of a garden), (2) length (e.g., 3 4 of an inch), and (3) set or quantity (e.g., 1 2 of the class).

Concept construction is usually begun with actions of viewing the objects. Study indicated that partitions of circle (Figure 1) is very effective in helping students to construct the concept of fraction [3]. Using the part-whole construct is effective starting point for building meaning of fractions [4]. This enable the students to graph the whole-part relationship and the relative size concept in the concept of fraction.

Figure 1. Model Circle

Understanding of a mathematical concept is the result of construction or reconstruction of mathematical objects [5]. Construction or reconstruction is done through activities of actions, processes, and objects of mathematics organized in a scheme to solve mathematical problems. These
actions, processes, objects and schemes are called APOS. [6] investigated Avi’s errors in constructing the meaning of $\frac{3}{5}$, where she represented the fraction as in Figure 2.

![Figure 2. Avi errors in fractional construction $\frac{3}{5}$](image)

She interpreted the numerator of $\frac{3}{5}$ as three circles, and divided each of the three circles into five equal parts to represent its denominator. Based on APOS theory [5], actions done to understand the concept of fraction was inadequate. Therefore, she was unable to internalize the concept properly, and come up with a wrong object. Her conception of a fraction was different with the concept of fraction owned by mathematician. Limited intervention is needed to reconstruct Avi’s error [7].

Another case of fractions [8] describes fractions $\frac{3}{6} = \frac{2}{4}$ based on the consequences arising from student actions [9]. Students divide quantity into 6 equal parts and choose 3 of the 6 parts leads to the same effect as dividing the quantity into 4 equal parts and selecting 2 of the 4 parts. The fraction doesn’t have similar value, the student has a failure in determining a fraction worth $\frac{3}{6}$. In this case the student-connected schemes are integrated by a larger scheme. Condition that influences the larger scheme is said to be the complete dominant. The complete dominant sense in biology according to Mendel is occurs when one allele – or “version” – of a gene completely masks another. The trait that is expressed and described as being “dominant” over the trait that is not expressed. Recessive is relating to or denoting heritable characteristics controlled by genes that are expressed in offspring only when inherited from both parent, i.e., when not masked by a dominant characteristic inherited from one parent. Intermediate (semi-dominant) is the case if the two versions affect each other to produce intermediate versions.

Then these two understandings in biology are made analogies to the dominant schemes, recessive schemes, and intermediate schemes. Where the dominant scheme is the connected schemes (schemes that students have with schemes in the face of problems) are integrated by larger schemes (schemes on the problem). Recessive schemes are student-linked schemes that are not integrated by larger schemes (schemes on the problem), but are integrated by the initial scheme of the problems encountered. Intermediate schemes are interconnected linked schemes resulting in a combination of integrated schemes.

The success of constructing fractional concepts by students is evident from the work they do in the absence of errors in the process and performing the steps incorrectly solving the problem on the outcome. However, not all fractional concept constructs performed by students succeed, there are times when the fractional concept construction is done fail. This failure occurs because the schemes are not connected to each other, so it can not form a new scheme. This inherent linkage leads to misuse of the strategy, ignoring the error in the process of obtaining a solution, judging the wrong solution is right, failure in doing so causing deadlock or failure in changing the problem so that it is not by the concept structure.

The success and failure in constructing the fractional concept is an analogy of true-false logic in conjunction. According to Big Indonesian Dictionary Success is the success of achieving the visible goals of the process and its results are correct. Process is a series of actions, manufacture, or processing that produce the product. Results are something that is made (made, made) by business. While failure is a failure to achieve goals. Failures occur with 3 possibilities, namely (1) on the true process and the result is false, (2) on the process is false and the result is true, and (3) in the process is false and the result is also false.
The image of success and failure in constructing the fractional concept of this right-false logic analogy can be seen below.

![Diagram of construction of fractional concepts](image)

**Figure 3.** Success and Failure of Fraction Concrete Construction. Sustainability: The process is true, the result is false (shown with a black arrow). Failure: 1. The process is true, the result is false (shown with a pink arrow), 2. The process is false, the result is true (indicated by a red arrow), 3. The process is false, the result is false (shown with a blue arrow).

Figure 3 shows that the success happened when the process and the result is done right. Avi’s failure and fraction case \( \frac{3}{6} = \frac{2}{4} \) including in the third failure possibility. So it is necessary to do research on the failure of the students only on the false process and the results are true. This failure is examined based on the actions, processes and objects the student does not fit the scheme, and does not reach the goal, but he knows the result. The subject cannot perform the correct actions, processes and objects in getting results.

This failure is called a false concessive failure. Where the researcher analogizes the notion of failure, concessive conjunction and false of the Big Indonesian Dictionary is the failure to achieve the objective, the clause stating the condition or condition contrary to something stated in the main clause, and as the original/actually. This definition, used to analogize from failure, concessiveness and falsehood to the student's false concessive failure. The notion of failure is seen as a failure of construction by students in achieving objectives that are scientifically appropriate. While the concessive is seen as the wrong process and the correct result on the construction. While false is seen as a construction that looks like a goal. This false describes the student can know the result correctly, but can not do the process to get the results.

From the analogy of the definition is summarized globally, the notion of student's false concessive failure is the failure of the constructions made by students in achieving goals, where the process is done false and the results obtained in true. This research aims at how to describe the characteristics of failure on the wrong process, the result is correct?

2. Methods
This research is a qualitative research [4]. Subject selection began with a preliminary study, communication with the subject teacher, and homeroom teacher. The selected research subjects were 10 students consisting of classes V and VI from two schools at Malang, one private and one public.
The subjects were selected from 40 students based on their works in solving routine problems. A total of 25 students who indicated failure in solving the routine problems were then given a written problem and asked to solve it through thinking aloud.

The students were given a square ABCD. Four smaller squares with one in red shading (called a unit square), 2 medium-sized rectangles (one shaded in blue) and 2 square jaws were located inside the square. The students were expected to be able to determine the corresponding fractions of each colored region, and represented it in other units namely quadrilateral, circle, and equilateral triangle. Finally, the students were asked to represent fraction made by the combination of red and blue regions into targeted quadrilateral, circle, and equilateral triangle.

Based on students’ work, three categories of failure are emerged, namely: (1) the process is correct but the result is wrong, (2) the process is wrong but the result is correct, and (3) wrong process and wrong result. There are 10 students in the first category, 10 students in the second category, and 5 students in the last category.

3. Results and discussion

Based on the analogy of the biologists on the genes / properties of the findings [6,8,9], false concessive failure can be categorized into two categories: (1) dominant schemes, (2) recessive schemes, and (3) intermediate schemes. The following will feature reviews for each category of false concessive failure.

3.1 Dominant scheme

The dominant scheme [10] is characterized by simply describing the surface properties of the task / problem and tends to use standard procedures (the procedure of finding fractions on a square ABCD and rectangle) even though the procedure is absolutely unusable to the problem at hand.

An example of dominant scheme students’ work is presented in figure 4 below.

From the Figure 4, it can be seen that in the first step in the presentation of the second image (circle) it occurs the perception of the circle is cut into pieces so it can form the same part as in the ABCD square and rectangle. This can be seen also in the equilateral triangle images cut into two equal parts, then the piece on the right is placed on the top of the piece on the left. The cut on the circle was noticed by the students not the same, the process was continued to produce 16 sections. Students attempt to link the knowledge scheme they have with the knowledge scheme to the problem. this resulted in the focus of attention out of the fractional concept as the same part of the whole.

Then in step three in the presentation of the second image (on the circle) the students begin to change the shape of the piece. This answer model is presented in the figure below.
In this situation the student changes the shape of the piece to the circle, then makes the cut shape on the circle the same as the rectangle. Students build their own knowledge from their own experience with the environment. The process of forming new knowledge (in circular presentation) is believed to be the result of a series of processes (ACT-Process-Object-Schema, known as APOS). APOS [6] is a construction theory about how one learns a mathematical concept. When the student action in the cut is repeated (reflection occurs), then the process of the student has been encapsulated into an object. Then the object is decomposed (de-encapsulate) as to where it came from. Next the students are echoed based on actions, processes, objects and other schemes that are interconnected so as to form a cut frame on the circle inside the mind the same shape with a rectangle.

The wrong action is done on the dominant of this scheme, that is to observe and understand the image being the focus and apply it to the presentation of other images. While in the process done in determining the shape of the cut on the presentation of the image is limited only to the reference image rectangle. The specified object will be different on each piece.

From the error of action, these processes and objects are recognized by students and students to understand that the fractions in the circle are the same as the fractions on the square ABCD and the rectangle (this shows the result), but are unable to elaborate. (global process fails).

When viewed from the work of this subject, apparent false concessive failure is caused by two things, namely; (1) the connected scheme is integrated by a larger scheme, and (2) the attachment to the known starting image. It tends to work procedure by looking at the procedure at the beginning. [11]

3.2 Recessive scheme
Recessive schemes are characterized by the perception of cutting the image presented into equal parts to the shape of the presentation image. In a formal mathematics, images are not the final settlement of a problem. Graphic images or sketches are used only for early identification of possible mathematical models or appropriate problem-solving strategies. In the picture below, students perform a fractional analysis of value but not applied to the presentation of the image. The deductions done still see as many pieces in the previous stages.

Figure 5. Continued the work of the dominant scheme

Figure 6. The work of students from the recessive scheme
Fractions on the blue in the ABCD square are solved using a fractional analysis of worth. Students connect schemes that belong to a fractional scheme with a valuable fractional scheme. Then the cutting indicates that when the student action in the cut is repeated (reflection occurs for different cuts in each presentation of the image according to the shape of the image), then the process of the student has been encapsulated into an object. Then the object is decomposed (de-encapsulate) as to where it came from. Next the students are echoed based on actions, processes, objects and other schemes that are interconnected so as to form a cut frame on rectangles, circles, and equilateral triangles in the same mind shape with the presentation on each image. Action performed, ie observe and understand the image with the focus of each form and apply it to the presentation of each image. The process done in determining the cut shape on the presentation of the image is limited only to the reference image respectively. The specified object will be different on each piece.

When viewed from the work of this subject, apparent false concessive failure is caused by two things, namely; (1) the connected scheme is integrated with the initial scheme, and (2) the attachment to the analysis of each known image. It tends to work by procedure and analysis, because it depends on the characteristics of the task or problem to be solved. [12]

3.3 Intermediate scheme

Intermediate schemes are characterized by the perception of cutting the image presented into equal parts with the combined shape of the known image with the presentation image.

Students connect the schemes they have in the form of a fractional scheme on a rectangle with an equilateral triangle fractional scheme. Then the deductions performed indicate that when the student action in the cut is repeated (reflection occurs for different cuts in each presentation of the composite image, ie the shape of the image that is referred to with the image on the problem), then the process of the student has been encapsulated (encapsulated) into an object. Then the object is decomposed (de-encapsulate) as to where it came from. Next the students are echoed based on actions, processes, objects and other interconnected schemes that form a cut frame on the equilateral triangle in mind in the form of a combination of rectangular and equilateral triangular representation forms. Action performed, ie observe and understand the image with the focus of each form and apply it to the combined presentation, the initial image with the problem. The process done in defining the shape of the pieces on the presentation of the image is limited to the combined reference of the focus image and the problem. The specified object will be different on the cut form.

When viewed from the work of this subject, apparent false concessive failure is caused by two things, namely; (1) the connected scheme is integrated with the combined initial scheme and problem scheme, and (2) the attachment to the initial drawing analysis with the problem being worked on.

The three characters of the false concessive failure of the students on this fractional concept can be corrected by limited intervention [1] by the teacher at the time the student made a mistake.
4. Conclusion
The student's false concessive failure on this fractional concept can be classified into 3 categories, namely: dominant schemes, recessive schemes and intermediate schemes. The scheme's dominance is characterized by the use of unsuitable standard procedures resulting in linked schemes being integrated by larger schemes and elaborating information on issues that have attachments to the initial image of a known problem.

Recessive schemes are characterized by the use of procedures with analysis that result in linked schemes integrated with the initial scheme, and the translation of the information on the problem has an attachment to the analysis of each known representation image. Intermediate schemes are characterized by the use of procedures with analyses that result in integrated integrated schemes forming a combined scheme and elaboration of the problem in the problem on the image analysis by incorporating the initial scheme with the scheme on the problem.

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