Temperature effects on the $Z_2$ symmetry breaking in the scotogenic model

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It is well known that the scotogenic model for neutrino mass generation can explain correctly the relic abundance of cold dark matter. There have been claims in the literature that an important part of the parameter space of the simplest scotogenic model can be constrained by the requirement that no $Z_2$-breaking must occur in the early universe. Here we show that this requirement does not give any constraints on the underlying parameter space at least in those parts, where we can trust perturbation theory. To demonstrate this, we have taken into account the proper decoupling of heavy degrees of freedom in both the thermal potential and in the RGE evolution.

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I. INTRODUCTION

Among the known shortcomings of the standard model (SM), probably dark matter (DM) and neutrino masses are the most important ones. While many extensions of the SM have been proposed that could either explain DM or neutrino masses, the “scotogenic” model [1] can be considered the prototype for a class of beyond SM (BSM) models that can potentially explain both phenomena at the same time.

The setup of the scotogenic model is very simple: Add to the SM (two or) three right-handed neutrinos, $N_i$, and a new scalar doublet, usually denoted by $\eta$. In order to avoid a tree-level type-I seesaw, a global $Z_2$ is added by hand. This symmetry eliminates a certain term from the scalar potential and avoids a vacuum expectation value for $\eta$, as long as certain conditions on the scalar parameters are fulfilled, see Sec. II. Under this symmetry, $N_i$ and $\eta$ are odd, while all SM particles are even. The lightest $Z_2$ odd particle, either $N_1$ or $\eta$, is then absolutely stable, providing potentially a good cold dark matter candidate. At the same time, the model is able to explain correctly the observed neutrino masses and mixing angles (for a recent global fit of all neutrino data see, for example [2]) via the one-loop diagram shown in Fig. 1.

Due to its economic structure the scotogenic model has been studied in many publications in the literature. Both DM candidates, the lightest $N_1$ [3] and the neutral component of $\eta$ [4], can give a relic density consistent with DM abundance in viable parts of parameter space, while respecting bounds from direct [5,6] and indirect [7,8] detection.

On the other hand, for any model to generate the correct dark matter abundance observed today requires the DM candidate to be stable from the time of its decoupling from the thermal bath in the early universe onwards. The scotogenic model is no exception: Breaking the $Z_2$ at this—or any later—time would lead to a rapid decay of the DM candidate and consequently, both the DM and the motivation to study the model would be lost. The authors of [9] have pointed out that when evolving the parameters of the scotogenic model to larger energies using RGEs, very often points which had a conserved $Z_2$ at low energy develop a deeper minimum at a nonzero value of the vev of $\eta$ at high energies and, thus, $Z_2$ is spontaneously broken. This is induced by a negative contribution arising from a one-loop correction to the mass of the $Z_2$-odd scalar doublet mediated by the $Z_2$-odd Majorana right-handed heavy neutrinos, $N$. It was then argued in [9], that all such points should be excluded from the parameter space for consistency.

However, the early universe is a hot thermal bath and temperature effects, while briefly discussed, were not taken into account in a consistent manner in [9]. It is well-known that the addition of thermal effects is responsible of the electroweak symmetry restoration at temperatures around 165 GeV [10,11]. One then would expect that thermal contributions should mitigate or even avoid these $Z_2$ breaking minima at high scales. In this paper, we
FIG. 1. One-loop neutrino mass diagram in the scotogenic model.

discuss how to build the effective potential of the scotogenic model [11] including one-loop thermal and nonthermal effects, as well as the corresponding resummation terms. We then improved the potential by taking into account the RGE evolution of the parameters, in order to consider the possibility that the $Z_2$ symmetry is spontaneously broken at a high energy scale, which is characterized by the temperature $T$. As a result, we find that the inclusion of thermal effects change the conclusion about the stability of the DM in the early universe. With the possible exception of parameter points close to nonperturbativity, where we cannot trust our calculation, we found no points in our scans in which the $Z_2$ is broken spontaneously. In particular, our conclusions hold in the parameter region studied in [9].

The rest of this paper is organized as follows. In Sec. II we give a description of the scotogenic model and its parameters. We also discuss briefly the tree-level stability conditions and the constraints these put on the parameter choices for the scalar potential. Section III is devoted to a discussion of the effective potential, temperature effects and the correct decoupling of the heavy degrees of freedom. In Sec. IV we discuss our numerical results. The paper then closes with a short summary.

II. THE SCOTOGENIC MODEL

This section gives a brief description of the scotogenic model and a discussion of its scalar potential. The model is a simple extension of the SM, to which three copies of right-handed neutrinos, $N \propto (1, 1, 0)$, and a new scalar doublet, $\eta \propto (1, 2, 1/2)$, are added. Here, $(x, y, z)$ indicate quantum numbers under the SM group in the usual order $SU(3)_C \times SU(2)_L \times U(1)_Y$. $N$, and $\eta$ are assumed odd under a new $Z_2$ symmetry. The Lagrangian of the model then contains the terms:

$$\mathcal{L} \supset -(Y_N)_{ab} L_a \bar{N}_b + \frac{1}{2} M_N^2 N_a N_a - \mathcal{V}.$$  

Here, the fermions are described in terms of 2-component spinors and $H$ is the usual SM Higgs doublet. Note that the $Z_2$ symmetry eliminates terms such as $m_{12}^2 H^\dagger H$. We will assume in the following that $\lambda_5$ is real, as this does not affect any of our findings.

Neutrino masses arise at one-loop order as shown in Fig. 1. The $3 \times 3$ Majorana neutrino mass matrix from this diagram is given by

$$\langle m_{SC} \rangle_{ab} = \sum_{i=1}^{3} \frac{Y_{Ni} Y_{Ni\beta}}{2(4\pi)^2} M_{Ni} \left[ \frac{m_R^2}{m_R^2 - M_{N_i}} \log \left( \frac{m_R^2}{M_{N_i}^2} \right) - \frac{m_i^2}{m_i^2 - M_{N_i}^2} \log \left( \frac{m_i^2}{M_{N_i}^2} \right) \right] \equiv \frac{1}{32\pi^2} (Y_N^T M Y_N)_{ab},$$  

where $m_{R,L}^2$ are the masses of the neutral scalar and pseudoscalar components of $\eta$:

$$m_R^2 = \mu^2 + \frac{1}{2} \lambda_L v_1^2, \quad m_L^2 = \mu^2 + \frac{1}{2} \lambda_S v_1^2,$$  

with $v_1 = \sqrt{2} \langle H \rangle$, $\lambda_{L,S} = \lambda_3 + \lambda_4 \pm \lambda_5$. Equation (3) is of a form which allows to use a minimally modified version [12] of the Casa-Ibarra parametrization [13] to fit all neutrino data. Since such fits have been discussed many times in the literature, we will not repeat any details here. We stress the importance of the term proportional to $\lambda_5$ in (2). In the limit $\lambda_5 \to 0$ the model conserves lepton number and the Majorana neutrino mass diagram vanishes identically. Since this corresponds to an enhanced symmetry, a small value of $\lambda_5$ is technically natural and somewhat small values of $\lambda_5$ are usually used in the neutrino mass fit, allowing the Yukawas to be even order $O(1)$.

We are interested in the question whether $Z_2$ breaking points can appear during the thermal evolution of the universe, which for temperature $T = 0$ would nevertheless appear to be $Z_2$ symmetric. For this we have to take into account the thermal contributions to the scalar potential as discussed in the subsequent section. These corrections can be expressed in terms of tree-level masses for which we give here the corresponding formulas allowing for a potential spontaneous $Z_2$ breaking. We decompose the scalar doublets as follows

$$\lambda_5 = 0 \Rightarrow m_R^2 = m_L^2,$$  

In more technical terms, one sees from (4) that $\lambda_5 = 0$ implies $m_R^2 = m_L^2$, which in turn gives $M = 0$.  

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where we explicitly allow for a possible nonzero vacuum expectation value for the \( Z_2 \)-odd field \( \eta \). We have slightly abused notation here, as in case of a \( Z_2 \) breaking the would-be Goldstone fields are an admixture of \( G^+ \) and \( \eta \), as well as an admixture of \( G^0 \) and \( \eta_i \).

We get the background potential by expanding (2):

\[
V_0(v_1, v_2) = \frac{1}{2} \mu_1^2 v_1^2 + \frac{1}{2} \mu_2^2 v_2^2 + \frac{1}{8} \lambda_1 v_1^4 + \frac{1}{8} \lambda_2 v_2^4 + \frac{1}{4} \lambda_L v_1^2 v_2^2. 
\]

(6)

We briefly summarize here the tree-level conditions leading to the correct electroweak symmetry breaking while respecting the \( Z_2 \) symmetry, for details see, e.g., [14] and references therein. There are in principle four possible minima:

1. \( v_1^2 = v_2^2 = 0 \);
2. \( v_1^2 = -2\mu_1^2 v_1, \quad v_2^2 = 0 \);
3. \( v_1^2 = 0, \quad v_2^2 = -2\mu_2^2 v_2 \);
4. \( v_1^2 = -2\mu_1^2\lambda_1 - \mu_2^2\lambda_2 v_2^2, \quad v_2^2 = -2\mu_2^2\lambda_1 - \mu_1^2\lambda_2 v_1^2 \).

The corresponding minimum should fulfill the conditions: \( v_{1,2}^2 \geq 0 \), the Hessian should be positive, i.e., \( \text{det} \left[ \frac{\partial^2 V_0}{\partial v_1 \partial v_2} \right] > 0 \).

The potential at minimum 2 should be the lowest of the four: Taking \( \lambda_1, \lambda_2 > 0 \), as required to avoid unbounded from below directions, one finds beside \( \mu_1^2 < 0 \) the conditions,

\[
\lambda_1 < \lambda_L \frac{\mu_1^2}{\mu_2^2} \quad \text{if} \quad \mu_2^2 < 0 \quad \text{and} \quad \lambda_L > 0, 
\]

(7a)

\[
\lambda_1 > \lambda_L \frac{\mu_1^2}{\mu_2^2} \quad \text{if} \quad \mu_2^2 > 0 \quad \text{and} \quad \lambda_L < 0. 
\]

(7b)

Moreover, minimum 2 is always the global one if \( \lambda_L > 0 \) and \( \mu_2^2 > 0 \). The requirement, that the potential is bounded from below also implies [16],

\[
0 < \sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 - |\lambda_5| 
\]

(8a)

\[
0 < \lambda_3 + \sqrt{\lambda_1 \lambda_2} 
\]

(8b)

It has been shown in [9] that due to the evolution of the renormalization group equations (RGEs) the parameters get changed such that the desired \( Z_2 \) gets spontaneously broken at high energy scales. The most relevant RGE in this context is the one for \( \mu_2^2 \):

\[
Q \frac{d\mu_2^2}{dQ} = \frac{1}{16\pi^2} \left[ \left( 6\lambda_2 - \frac{3}{2} \frac{\mu_2^2}{\lambda_1} + 3g_2^2 \right) + 2 \text{tr} (Y_N^+ Y_N) \right] \mu_2^2 
\]

\[
+ 2(2\lambda_3 + \lambda_4)\mu_2^2 - 4 \sum_{i=1}^3 M_{N_i}^2 (Y_N^+ Y_{N,i}) 
\]

(9)

where \( Q \) denotes the renormalization scale. In particular, the last contribution proportional to \( M_{N_i}^2 \) can drive \( \mu_2^2 \) to negative values if the corresponding Yukawa couplings are sizable and the larger the ratios \( M_{N_i}^2/M_{N_j}^2 \) are.

We will investigate to which extent this implies that the \( Z_2 \) symmetry could be broken at high energies during the thermal evolution of the Universe. For this, one has to take into account the thermal contributions to the potential and check if there are indeed parameter points where the \( Z_2 \) breaking minimum could potentially arise at a high temperature \( T \). In such a minimum, also \( v_2 \) would be nonzero and we give here the corresponding tree-level mass matrices from which we calculated the input to the one-loop contributions discussed in the next section. The mass matrices for the CP-odd, -even, and charge scalars in the basis \( (H, \eta, v) \) are given by

\[
\mathcal{M}_R^2 = \begin{pmatrix}
\mu_1^2 + \frac{3}{2} \lambda_1 v_1^2 + \frac{1}{2} \lambda_L v_2^2 & \lambda_1 v_1 v_2 \\
\lambda_1 v_1 v_2 & \mu_2^2 + \frac{3}{2} \lambda_2 v_2^2 + \frac{1}{2} \lambda_L v_1^2
\end{pmatrix},
\]

(10)

\[
\mathcal{M}_S^2 = \begin{pmatrix}
\mu_1^2 + \frac{1}{2} \lambda_1 v_1^2 + \frac{1}{2} \lambda_S v_2^2 & \lambda_S v_1 v_2 \\
\lambda_S v_1 v_2 & \mu_2^2 + \frac{1}{2} \lambda_2 v_2^2 + \frac{1}{2} \lambda_S v_1^2
\end{pmatrix},
\]

(11)

\[
\mathcal{M}_Z^2 = \begin{pmatrix}
\mu_1^2 + \frac{1}{2} \lambda_1 v_1^2 + \frac{1}{2} \lambda_3 v_2^2 & \frac{1}{2} (\lambda_4 + \lambda_5) v_1 v_2 \\
\frac{1}{2} (\lambda_4 + \lambda_5) v_1 v_2 & \mu_2^2 + \frac{1}{2} \lambda_2 v_2^2 + \frac{1}{2} \lambda_3 v_1^2
\end{pmatrix},
\]

(12)

where the lightest eigenvalues of \( \mathcal{M}_R^2 \) and \( \mathcal{M}_Z^2 \) need to be zero when evaluated at any of the minima, since they correspond to the Goldstone states of electroweak symmetry breaking. It is worth noticing that in the unbroken \( Z_2 \) phase \( \lambda_4 - |\lambda_5| < 0 \) when \( N \) is heavier than all scalars to avoid an electrically charged stable dark matter candidate. The masses of the electroweak gauge bosons are given by

\[
m_W^2 = \frac{1}{4} g_L^2 (v_1^2 + v_2^2), \quad m_Z^2 = \frac{1}{4} (g_1^2 + g_2^2) (v_1^2 + v_2^2), \quad m_T^2 = 0.
\]

The standard model charged fermion masses are simply calculated from the SM Higgs Yukawa couplings
as, \( m_2^2 = \frac{1}{2} Y_2^2 v_1^2 \) whereas for \( v_2 \neq 0 \) a tree-level type-I seesaw contribution to the neutrino masses appears, given by:

\[
M_v = \begin{pmatrix} 0 & m_D \\ m_D & M_N \end{pmatrix} \text{ with } m_D = \frac{1}{\sqrt{2}} Y_N v_2.
\]

### III. EFFECTIVE POTENTIAL, TEMPERATURE EFFECTS AND DECOUPLING OF HEAVY DEGREES OF FREEDOM

We will combine the one-loop RGEs with the effective potential and include also the thermal contributions. The region of parameter space, where potentially a \( \text{potential and include also the thermal contributions.} \) 

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\text{\noindent \small \text{III. EFFECTIVE POTENTIAL, TEMPERATURE EFFECTS AND DECOUPLING OF HEAVY DEGREES OF FREEDOM}}
\]

We will briefly discuss this after presenting the thermal contributions. In the following we will neglect the contributions from SM fermions except for the top quark, as they do not play any role here.

The effective potential at one-loop and zero temperature is given by \( V_{\text{eff}} = V_0 + V_1 \), where the temperature-independent one-loop correction to the potential is the Coleman-Weinberg potential is given by,

\[
V_1 = \frac{1}{64\pi^2} \sum_i m_i^2 \left( \ln \frac{m_i^2}{Q^2} - c_i \right). \tag{13}
\]

Here, \( m_i^2 \) are the field dependent mass squares of the various particles and the \( n_i \) count the degrees of freedom and the sign is positive for bosons and negative for fermions: \( n_Z = 3, n_W = 6, n_\tau = 2, n_{\nu_l} = 1, n_{\tau^c} = 2, n_t = -12 \) and \( n_{\nu_R} = -2. \) \( S^0 \) and \( S^\pm \) refers to any neutral or charged scalar. We take the Landau gauge (\( \xi = 0 \)) and work in the \( \overline{\text{MS}} \) scheme [20,21]. Thus, one has \( c_i = 3/2 \) in case of scalars and fermions and \( c_i = 5/6 \) for the vector bosons.

We note for completeness that (13) depends on the choice of the renormalization scale \( Q \). To minimize its effect one should use the improved effective potential by using the renormalization group, i.e.,

\[
\left[ Q \frac{\partial}{\partial Q} + \beta_i \frac{\partial}{\partial \lambda_i} - \gamma_m m_i^2 \frac{\partial}{\partial m_i} - \gamma_{\nu, v_i} v_i \frac{\partial}{\partial v_i} \right] V_{\text{eff}}^{(0)}(Q, \lambda_i, m_i^2, v_i) = 0. \tag{14}
\]

The solution is given by \( V_{\text{eff}}^{(0)}(Q, \lambda_i, m_i^2, v_i) = V_{\text{eff}}^{(0)}(Q, \lambda_i(Q), m_i^2(Q), v_i(Q)) \), with \( \lambda_i(Q) \), \( m_i^2(Q) \) and \( v_i(Q) \) the running parameters [22,23].

The effective potential including finite temperature effects is defined as \( V_{\text{eff}} = V_0 + V_1 + V_T \) at one-loop level. Here, the leading order temperature-dependent correction is given by,

\[
V_T = \frac{T^4}{2\pi^2} \left[ \sum_{i \text{bosons}} n_i J_B(m_i^2/T^2) + \sum_{i \text{fermions}} n_i J_F(m_i^2/T^2) \right], \tag{15}
\]

where the functions \( J_B(y^2) \) come from the evaluation of the thermal loop with the corresponding statistic for bosons or fermions and they are defined as,

\[
J_B(y^2) = \int_0^\infty dxx^2 \ln \left[ 1 \pm e^{-\sqrt{x^2+y^2}} \right], \tag{16}
\]

see, e.g., [21] and references therein.

One should take the thermal mass \( m(v_i, T) \) to avoid infrared divergences for \( T \gg m \). The thermal mass is only important for the zero Matsubara mode [21], so only boson masses have to be modified according to \( m_i^2(v_i, T) \rightarrow \Pi_i^2(v_i, T) \equiv m_i^2 + k_i T^2 \).

\[
\Pi_\omega^2 = m_\omega^2 + 2g_\omega^2 T^2, \tag{17}
\]

\[
\Pi_{\mu_i}^2 = \mu_i^2 + \left[ \frac{1}{8} g_i^2 + \frac{1}{16} (g_i^0 + g_i^2) \right] + \left[ \frac{1}{4} \lambda_1 + \frac{1}{6} \lambda_3 \right] + \frac{1}{12} \lambda_4 + \frac{1}{4} m_i^2 \right] T^2, \tag{18}
\]

\[
\Pi_{\nu_i}^2 = \mu_i^2 + \left[ \frac{1}{8} g_i^2 + \frac{1}{16} (g_i^0 + g_i^2) \right] + \left[ \frac{1}{4} \lambda_2 + \frac{1}{6} \lambda_3 + \frac{1}{12} \lambda_4 \right] + \frac{1}{24} \text{Tr}(Y_i^0 Y_i^0) \right] T^2, \tag{19}
\]

\[
\Pi_{s_i}^2 = \frac{1}{2} [m_i^2 + \Delta m_i^2, T^2] + (g_{s_i}^0 + g_{s_i}^2) T^2, \tag{20}
\]

\[
\Pi_{t_i}^2 = \frac{1}{2} [m_i^2 - \Delta m_i^2, T^2] + (g_{t_i}^0 + g_{t_i}^2) T^2, \tag{21}
\]

with \( \Delta^2 = m_i^2 + (g_{t_i}^0 - g_{s_i}^0)^2 (4 T^2 + v_i^2 + v_i^2) T^2 \). Note, that in case of the gauge bosons only the longitudinal components get a thermal contribution from the zero Matsubara mode. The method described so far is known as the Parwani method [24]. Alternatively, one could use the so-called Arnold-Espinosa method [25] which is based on a resummation of daisy-diagrams [25]. Both approaches have been compared for example in [26], where it has been found that they lead to very similar numerical results in the context of singlet extensions of the SM, whereas the results of [27] indicate that the Arnold-Espinosa method leads to stronger...
constraints on the parameter space in the context of two Higgs doublets models.

The renormalization scale $Q$ should be chosen such, that large logarithms are avoided in the calculation of the loop corrected effective potential in (13). We use here the \bar{MS} scheme where one has to decouple heavy states by hand. In view of the fact that a potential $Z_2$ breaking requires $M^2_{N_i}/\mu_2^2 \geq 1$, we have chosen the following procedure: assuming that the additional scalars have masses in the range of a few hundred GeV we decouple the right-handed neutrinos if their mass is above max (1 TeV, $T$). The idea here is that right-handed neutrinos will only contribute if the temperature $T$ is sufficiently high. The renormalization scale is then chosen to be:

$$Q = \sqrt[3]{\prod_{k=1}^n m_k^2(v_i, T)},$$

(22)

where $3 \leq n \leq 6$ depending on how many neutrinos decoupled, for instance $n = 3$ if all of them are decoupled and 6 if none is decoupled. In the RGE evolution, as well as in the calculation of the effective potential including the thermal effects, the right-handed neutrinos are included if their masses $m_{N_i} \leq Q$, otherwise they are decoupled. We have checked that the results presented in the next section remain the same if we replace this geometric mean for $Q$ by the maximum of the masses which are taken into account.

IV. NUMERICAL RESULTS

As already explained in the previous sections, the improvement of the RGEs to the effective potential implies that the potential only depends on four scales that can be related: the renormalization scale $Q$, the temperature $T$ and the background fields values $v_1$ and $v_2$. To keep the logarithmic corrections under control, we considered the renormalization scale as a function of the thermal mass (22). Then, for each set of input values, we numerically minimized the effective potential in terms of $v_1$ and $v_2$ for different values of the temperature ranging from 0 to $10^{16}$ GeV, building our way back in time in the evolution of the universe. The mass spectrum and the one-loop RGEs are numerically computed by a modified version of SPHENO [28,29] and SARA [30,31]. The minimization of the thermal effective potential is done using MINUIT [32]. Certain checks and cuts are applied during the procedure regarding unboundedness from below and nonperturbativity. The initial values should generate a valid electroweak minimum, i.e., fulfill the conditions (7). Also, we only considered points that satisfy the bounded from below conditions (8) at every step of the RGE running. One should be especially cautious about this point, as the well-known bounded from below conditions of the scotogenic or the IDM model are just necessary conditions, but not sufficient. Then, we checked possible nonperturbative couplings, as depending on the initial values at low scale, the RGE running can drive the couplings outside the perturbative regime. Our computation of the effective potential cannot be trusted in such cases, so we omitted these points.

In Fig. 2 we show the behavior of the $v_1$ and $v_2$ minima for a benchmark point as a function of the energy scale. We consider three different RGE-improved potentials: tree-level potential only, one-loop zero-temperature potential and thermal effective potential. For this point, $\mu_2^2$ runs into negative values around $10^5-10^6$ GeV (9). We see that when considering only the tree-level potential the vev of $\eta$ becomes nonzero around this scale, when the condition (7a) fails. This contrasts with the behavior of the one-loop potential or the thermal potential, where no Z2-breaking minimum is found. Once the correct treatment of the potential with the effective one-loop improved potential is taken into account, the spontaneous breaking of $Z_2$ arising from the RGEs disappear. For completeness, in the last panel we show a zoom of the region of interest for the last potential just to compare with the previous case. As stated in [10,11] the symmetry restoration should occur at temperatures around 150–170 GeV, which is what we found here. This leads to a merging of the global and local minima at $v_{1,2} = 0$.

We scanned over the input BSM parameters $(Y_N)_{ij}, M_{N_i}, \mu_2$ and $\lambda_{2,3,4}$ searching for a $Z_2$ breaking minima at any step in the thermal evolution. For simplicity, we considered the mass hierarchy $M_{N_i} = (1, 1.5, 2) \times M_n$, as well as all the entries of the diagonal Yukawa matrix $Y_N$ to be the same and equal to $Y_n$. We randomly generate $5 \times 10^4$ points in logarithmic scale in the ranges,

$$\begin{align*}
|\lambda_{2,3,4}| : [10^{-5}, 1] \\
|Y_n| : [10^{-5}, 1] \\
\mu_2 : [10^2, 10^4] \text{ GeV} \\
M_n : [10^2, 10^4] \text{ GeV}
\end{align*}
$$

(23)

Note that $\lambda_3$ is not among the input parameters, as the neutrino mass (3) can be used to fix its value. All input parameters are given at the scale $Q = 160$ GeV. $[m_i, \langle m_i \rangle]$. As $\lambda_3$ and $\lambda_4$ can be negative, the sign is also randomized. In the RGE evolution the top-Yukawa coupling $Y_t$ and the strong coupling $g_3$ are numerically important, in particular for $\lambda_1$. We have set their values at $Q = 160$ GeV to $Y_t = 0.93761$ and $g_3 = 1.16787$. Dimensionless parameters are conservatively taken up to 1, as even for this value the running will drive the parameter to the nonperturbative

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4We indeed found that from the set of randomly generated input values, around a 0.05% led to an unbounded from below potential even passing the conditions (8). Of course, such points were excluded from our result.
in our case, at a very low scale. \( \mu_2 \) should not be very large to maximize the possibility of getting a \( Z_2 \) breaking minimum, but cannot be very low, as there are already constraints to the mass of \( \eta \) around 100 GeV [33]. Regarding \( M_n \), naively one would expect that its effect gets larger the larger it is. However, a large mass means that it gets effective at higher temperatures implying that it contributes only at a later stage to the RGE evolution. We are thus interested in making this value large to drive \( \mu_2 \) faster toward negative values when doing the running, see (9), but not too large so right-handed neutrinos couple at sufficiently low energies before the thermal contribution becomes dominant. For each set of input values we computed the RGE-improved effective thermal potential and minimized it with respect to \( v_1 \) and \( v_2 \) for different temperatures. Finally, from the list of minima for each temperature, we search for \( Z_2 \) symmetry breaking minima, i.e., \( |v_2| > 0 \) (actually, larger than the numerical uncertainty of MINUIT). We have considered here temperatures \( T \) up to \( 10^{16} \) GeV in the cases where all couplings remain perturbative. We have stopped at lower temperatures if the modulus of one of the quartic couplings exceeds \( \sqrt{4\pi} \) as latest at this stage perturbation theory breaks down. As can be seen in Fig. 3, we found no \( Z_2 \) breaking points in the parameter space considered.

Figure 3 also shows, for comparison, the points which break \( Z_2 \) symmetry according to [9]. The black dots are points within our scan that would be considered to be \( Z_2 \) symmetry breaking in [9], but not in our approach, using
the whole thermal effective potential (see Fig. 2 and explanations therein). Note that in [9] the authors evaluated and minimized only the tree-level potential at different steps of the RGE running, finding that the RGEs can drive $\mu_2^2$ toward large negative values generating a $Z_2$ breaking minimum. This behavior is clear in Fig. 3, the negative contribution to the running of $\mu_2^2$ is proportional to $M_n^2$ and $Y_n^2$, so the black points appear toward larger values of $M_n$ and $Y_n$, and of course lower initial values of $\mu_2$.

V. CONCLUSIONS

In this work we have studied the effects of temperature on the $Z_2$ symmetry breaking in the scotogenic model. It had previously been shown [9] that in the running of the parameters of the scotogenic model to high energy, the mass squared parameter for the inert doublet turns negative in sizable parts of parameter space. It was concluded then in [9] that such points should be excluded from the parameter space, since dark matter would be lost due to the spontaneous breaking of the $Z_2$ symmetry by the vacuum expectation value of $\eta$.

Here, however, we have shown that this conclusion was premature. For a more realistic calculation, we have constructed the effective potential of the scotogenic model, taking special care on the inclusion of thermal effects. We then take into account the RGE evolution of parameters, including the correct decoupling of the right-handed neutrinos in case that they are heavy. Our numerical scans over the parameter space then show, that no spontaneous $Z_2$ breaking occurs in the parts of parameter space studied in [9]. Our scans are quite general, but since our numerical programs fail for points very close to nonperturbativity, we cannot make any claims for that part of parameter space. Models with such large quartic couplings have been investigated for example in [34,35] in the context of two-step electroweak phase transitions. With this possible caveat, however, our main result is that considering the complete one-loop contribution to the effective potential with the thermal corrections changes the conclusion about the stability of dark matter in the early universe.

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