One-loop correction to the $\gamma WW$ vertex in the $e^{-}\gamma$ collider *

JIRO KODAIRA, HIROSHI TOCHIMURA AND YOSHIKI YASUI
Department of Physics, Hiroshima University
Higashi-Hiroshima 724, JAPAN

ISAMU WATANABE
Department of Physics, Ochanomizu University
Bunkyo-ku Tokyo 112, JAPAN

Abstract
We apply the pinch technique, which is a method to construct the gauge independent off-shell Green’s functions, to the process $e^{-}\gamma \rightarrow W^{-}\nu$ to study the effects of radiative corrections to $WW\gamma$ three gauge boson vertex. The one-loop contributions to the anomalous gauge boson couplings are estimated in the standard model.

1 Introduction
Measuring the self-couplings of the electroweak gauge-bosons is one of the important topics at the next linear colliders. A general approach to examine the $WWZ$ and $WW\gamma$ vertices was discussed by K.Hagiwara et.al in terms of the effective Lagrangian. They studied the process $e^{+}e^{-} \rightarrow W^{+}W^{-}$ with generalized $ZW$ and $\gamma W$ couplings. The explicit form of the effective Lagrangian is given by,

$$
\mathcal{L}_{WWV} = ig_{V}[g_{1}^{V}(W_{\mu\nu}^{+}W_{\mu\nu}V^{\nu} - W_{\mu}^{+}V_{\nu}W_{\mu\nu}) + \kappa_{V}W_{\mu\nu}W_{\nu\nu}^{\mu} + \frac{\lambda_{V}}{m_{W}^{2}}W_{\lambda\mu}^{\nu}W_{\nu\nu}^{\nu}] + \cdots,
$$

(1)

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where \( A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \), \( V = \gamma \) or \( Z \) and \( \tilde{V}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} V^{\rho\sigma} \). In particular, \( \kappa_\gamma \) and \( \lambda_\gamma \) are related to the magnetic dipole moment \( \mu_W \) and the electric quadrupole moment \( Q_W \) as:

\[
\mu_W = \frac{e}{2m_W} (1 + \kappa_\gamma + \lambda_\gamma) \tag{2}
\]
\[
Q_W = -\frac{e}{m_W^2} (\kappa_\gamma - \lambda_\gamma) \tag{3}
\]

In the context of the standard model, \( \kappa_\gamma = 1, \lambda_\gamma = 0 \) at the tree level. A deviation from the standard model prediction for these quantities may suggest a new physics. So the radiative corrections to these quantities must be estimated before discussing new physics.

J. Papavassiliou and K. Philippides used the pinch technique to construct the one-loop gauge-invariant \( \gamma W^+ W^- \) vertex in the standard model. They applied this result to the process \( e^+ e^- \rightarrow W^+ W^- \), and estimated the contribution of the radiative correction to the anomalous couplings in the parameters \( \kappa_\gamma \) and \( \lambda_\gamma \).

Now \( e^- \gamma \) and \( \gamma \gamma \) colliders are seriously considered as the interesting options to upgrade the \( e^+ e^- \) next linear colliders. E. Yehudai investigated the effects of the anomalous couplings to the processes \( e^- \gamma \rightarrow \nu W^- \) and \( \gamma \gamma \rightarrow W^+ W^- \) in the context of the effective Lagrangian. In these papers, it has been pointed out that using the process \( e^- \gamma \rightarrow \nu W^- \) has several advantages in measuring \( W\gamma \) couplings. From refs. we get the allowed regions in the \( \kappa_\gamma - \lambda_\gamma \) plane for the experimental measurements at the next linear colliders.

This talk contains two parts of topics. First we review the S-matrix pinch technique and explain how to construct the gauge-invariant self-energy of the gauge-boson in the QCD case as a simplest example. Next we apply the pinch technique to the process \( e^- \gamma \rightarrow \nu W^- \) to study the electroweak one-loop correction to the gauge independent \( WW\gamma \) vertex. We examine the behavior of the \( WW\gamma \) vertex at the \( e^- \gamma \) collider.

## 2 Pinch technique

Using the pinch technique we get the gauge-invariant off-shell Green’s functions. In this section we briefly review the S-matrix pinch technique. We show the outline to construct
the gauge-invariant two-point function of a gauge boson in the QCD case, as a simplest example. The case of spontaneously symmetry broken theories was discussed in refs. [3].

Now we consider the two-body scattering of two test quarks. The S-matrix element $T$ is gauge independent to any order in the perturbation theory. We can decompose the S-matrix element $T$ in the form,

$$T(t, s, m_1, m_2) = A(t) + B(t, m_1, m_2) + C(t, s, m_1, m_2),$$

(4)

where $m_1$ and $m_2$ are masses of the test quarks, $t$ and $s$ are the Mandelstam variables,

$$t = -(p_1' - p_1)^2 = -q^2 \quad \text{and} \quad s = (p_1 + p_2)^2.$$

(5)

The functions $A$, $B$ and $C$ are evidently gauge independent. From the different kinematical structure of these functions, we have possibility to define new Green’s function consulting eq.(4). For example, the gauge-independent function $A(t)$ can be considered as the new two-point function.(fig.1)

The explicit calculation at the one-loop level is as follows. It is noted that the gauge dependence of the self-energy part fig.2(a) is canceled by the propagator-like contribution from fig.2(b) and fig.2(c) which we call “pinch part”. For simplicity, we choose the Feynman-t’Hooft gauge ($\xi = 1$) in the following calculation. In this gauge, there are no pinch parts from the box diagrams fig.2(c). The three gluon vertex $\Gamma_{\alpha\mu\beta}$ is given by, (in the following, we omit the group factors.)

$$\Gamma_{\alpha\mu\beta} = \Gamma_{\alpha\mu\beta}^F + \Gamma_{\alpha\mu\beta}^P$$

(6)

with

$$\Gamma_{\alpha\mu\beta}^F = -(2k + q)_\mu g_{\alpha\beta} + 2q_\alpha g_{\mu\beta} - 2q_\beta g_{\alpha\mu},$$

$$\Gamma_{\alpha\mu\beta}^P = k_\alpha g_{\mu\beta} + (k + q)_\beta g_{\alpha\mu}.$$

(7)

(8)

From the simple Ward identity,

$$\gamma_\alpha k^{\alpha} = (\not{p} + \not{k}) - \not{p}$$

(9)
and using the equation of motion for the on-shell quarks ($\not{p}\psi(p) = 0$), we note that the contribution from the $\Gamma^P$ looks like fig.3(b). Then the decomposition of the vertex $\Gamma$ into $\Gamma^F$ and $\Gamma^P$ corresponds to that of the contribution fig.3(a) into the two type parts fig.3(b) and fig.3(c). We redefine the modified two-point function by adding the pinch parts from the vertex contributions fig.3(b) to the conventional two-point function. We get the new gauge-invariant self-energy at the one-loop level by,

$$\hat{\Pi}(q)_{\mu\nu} = \left[ g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] \hat{\Pi}(q)$$

with

$$\hat{\Pi}(q) = -bg^2q^2\ln(-q^2/\mu^2).$$

Here $b$ is the coefficient of $\mathcal{O}(g^3)$ in $\beta(g)$-function,

$$\beta(g) = -bg^3 - \sum_{N=1}^{\infty} b_N g^{2N+3}$$

and $b = 11N/48\pi^2$ for SU(N) Yang-Mills theory.

We can construct the gauge independent n-point Green’s functions by following just the same procedure.

### 3 One-loop correction to the $\gamma WW$ vertex

In ref.[4] E.Yehudai pointed out the advantages of the process $e^-\gamma \rightarrow \nu W^-$ in measuring $W\gamma$ couplings in the context of the effective Lagrangian. In this section, we applied the pinch technique to the process $e^-\gamma \rightarrow \nu W^-$ to construct the gauge-invariant $WW\gamma$ vertex. Here we omit the details of the calculation and we only show Feynman graphs contributing to $WW\gamma$ vertex. Diagrams fig.4(a) and fig.4(b) are the tree level contributions, fig.4(c) and fig.4(d) are the conventional self-energies and vertex corrections. Fig.4(e) is the pinch part from the $e^-\nu W^-$ vertex and fig.4(f) is the contribution from the box diagram to the $WW\gamma$ vertex.

We will show the results of the numerical computations below. We put the Higgs mass $M_H = 100$ GeV and Top mass $M_t = 174$ GeV. In fig.5, we show the dependence
on the beam polarization, with the beam energy being $\sqrt{S} = 300$ GeV. The cross section for the process $e_L^{-}\gamma(\pm) \rightarrow \nu W^-$ are plotted in fig.5(a). The indices $(\pm)$ on $\gamma$ mean the polarizations of $\gamma$ beams. We plot the ratios of the tree level cross section and the cross section including the modified $WW\gamma$ vertex in fig.5(b). We define the ratio $R$ by,

$$R = \frac{\sigma_{PT} - \sigma_{tree}}{\sigma_{tree}} \times 100$$

where $\sigma_{PT}$ is the cross section which include the vertex correction using pinch technique and $\sigma_{tree}$ is the cross section at the tree level. We note that the size of the vertex correction is very sensitive to the beam polarizations. We plot the energy dependence of the unpolarized cross section in fig.6(a), and ratios $R$ in fig.6(b) at $\sqrt{s} = 300$ and 500 GeV. In fig.7, we plot (a)$\Delta \kappa_\gamma \equiv \kappa_\gamma - 1$ and (b)$\lambda_\gamma$ for the unpolarized case at $\sqrt{s} = 300$ and 500 GeV. Since we have calculated the one-loop effects in the context of the standard model, these quantities depend on $\theta(t)$.

4 Summary

The pinch technique is an algorithm to construct the gauge invariant off-shell Green’s function in the perturbation theory. Using the pinch technique, we can estimate the contribution of the one-loop correction to $WW\gamma$ and $WWZ$ vertices with the gauge invariance being kept.

We applied the pinch technique to the process $e^-\gamma \rightarrow \nu W^-$, and estimated the radiative corrections to the $WW\gamma$ vertex. It is noted that the vertex corrections are sensitive to the beam polarization. One-loop contribution to $\Delta \kappa_\gamma$ is $O(10^{-2})$ and $\lambda_\gamma$ is $O(10^{-3})$ at $\sqrt{s} = 300 \sim 500$ GeV $e^-\gamma$ collider.

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Figure 4: One loop diagrams for $e^-\gamma \rightarrow \nu W^-$ process using the pinch technique.
Figure 1: Two-point function

Figure 2: One loop diagrams for the two quark scattering.

Figure 3: Contribution of the $\Gamma^F$ and $\Gamma^P$
Figure 5: Dependence on the beam polarizations. (a) Cross section included the $WW\gamma$ vertex corrections for the process $e^{-}\gamma_{(-)} \rightarrow \nu W^{-}$ (solid line) and $e^{-}\gamma_{(+)} \rightarrow \nu W^{-}$ (dashed line). (b) The plots of the ratio $R$. Solid line is the $e^{-}\gamma_{(-)}$ collision and dashed line is $e^{-}\gamma_{(+)}$ collision.
Figure 6: Energy dependence of the cross sections. (a)$e^-\gamma \rightarrow \nu W^-$ cross section including the vertex corrections in the unpolarized case. Solid line is at $\sqrt{S} = 300$ GeV and dashed line is at $\sqrt{S} = 500$ GeV. (b) The plots of the ratio $R$. Solid line is at $\sqrt{S} = 300$ GeV, dashed line is at $\sqrt{S} = 500$ GeV.
Figure 7: Contribution of the one-loop correction to the (a) $\Delta \kappa_\gamma$ and (b) $\lambda_\gamma$. Solid line is at $\sqrt{S} = 300$ GeV and dashed line is at $\sqrt{S} = 500$ GeV.