Near-space Hypersonic Vehicle tracking based on Adaptive Cubature Kalman Filter Algorithm

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Abstract. Near-space hypersonic vehicle leapfrog into adjacent spaces and are capable of flying at hypersonic speeds. They have the characteristics of fast speed change and strong penetration. Fast and accurate tracking of the spacecraft’s flight path is a direct way to reduce threats. For this purpose, for the X-51A aircraft that has been successfully tested, the motion model and the corresponding radar measurement equations are established by analyzing the motion characteristics. Cubature Kalman Filter does not have good robustness and can't respond quickly. An adaptive adaptive Cubature Kalman Filter algorithm is proposed to track hyperspaced spacecraft in near-space, and the algorithm is verified by simulation. Compared with CKF, it is more advantageous in terms of operation, and at the same time, the accuracy of estimation of nonlinear systems has also been significantly improved.

1. Introduction
Near-space space is a research hotspot in the field of aerospace, specifically referring to the highest flying height of 20km above normal aviation vehicle, and this part of the sky and space space below 100km of low-orbiting satellites is a space that has not yet been fully developed and utilized by humans. Due to the low atmospheric density in the adjacent space, and the climatic conditions are located on the troposphere, between the dense atmosphere and outer space, the environment is very stable. From the point of view of the unique location near the space, although there are still many technical difficulties in the development of the Near Space Hypersonic Vehicle (NSHV) [6][7], it has extraordinary strategic significance. Therefore, near space may serve as a new channel for a new type of strategic weapon to quickly attack long-range threats, and it is also opening up new forms of combat.

The accuracy of the filtering algorithm relies more on the judgment of the statistical properties of noise, the uncertainty of the statistical properties of the noise and the abrupt change of the characteristics, which will greatly affect the filtering effect. Moreover, most of the improved filtering algorithms nowadays also set the noise characteristics as ideal Gaussian white noise, which lacks the judgment of the filter effect in the case of abrupt change. The adaptive filtering algorithm for this type of problem has also been rapidly developed. Cubature Kalman Filter [4][5] is a new type of Gaussian filtering method proposed by Arasaratnam in recent years to achieve better nonlinear filtering requirements. The spherical-radial integration criterion is used to select sample points. UKF [2][3] and CKF have similar characteristics. Since Cubature Kalman Filter will select the sampling points on the hypersphere, there is no weight is negative, all have the same weight value Compared to several Kalman filters, Cubature Kalman Filter has a higher accuracy.

In this paper, using the innovation sequence and residual sequence, the actual characteristics of noise are modified online. The improved adaptive Cubature Kalman filter algorithm is used in the tracking of
near-space hypersonic vehicle. In the simulation experiment, in system noise characteristics are unknown, the filtering accuracy has been significantly improved, better than Cubature Kalman Filter.

2. Establishment of system model

2.1 Radar observation model
Radar observation coordinate system selected local Cartesian coordinates coordinate system. Take the east, north, and north of the radar center as the three-axis $x$, $y$, and $z$ directions of the coordinate system, and take $x_c$, $y_c$, and $z_c$ as the observations of the target on the radar. The distance $t$, the altitude angle $\alpha$, and the azimuth angle $\beta$ can be measured, where $v_t$, $v_a$, and $v_\beta$ are measurement noises, and the observation equation can be established as follows:

$$t = (x_c + y_c + z_c)^2 + v_t$$

(1)

$$\alpha = \arctan \left(\frac{z_c}{(x_c^2 + y_c^2)^{1/2}}\right) + v_a$$

(2)

$$\beta = \arctan \left(\frac{y_c}{x_c}\right) + v_\beta$$

(3)

2.2 Near-space Vehicle Sports model
Because the speed of spacecraft near to reach Mach 5-25, the impact of the non-spherical disturbances, revolutions and rotations of the Earth is ignored. The motion model is based on Earth Centered Inertial. This paper uses the American test X51A flight characteristics to carry out simulation studies According to known experimental results, the flight trajectory of the approaching spacecraft is divided into two phases: scramjet punching boosting phase and no-power glide phase, scramjet stamping boosting phase dynamic equation Build according to Sanger trajectory:

$$\frac{dh(t)}{dt} = V(t) \sin \gamma(t)$$

(4)

$$\frac{dV(t)}{dt} = \frac{P(t) \cos \phi(t) - D(t) - mg \sin \gamma(t)}{m}$$

(5)

$$\frac{d\gamma(t)}{dt} = \frac{(P(t) \sin \phi(t) + L(t)) \cos \delta - g \cos \gamma(t)}{V(t)} + \frac{V(t) \cos \gamma(t)}{R + h(t)}$$

(6)

Among them: $h(t)$ is the height; $V(t)$ is the speed of the aircraft; $\gamma(t)$ is the trajectory inclination; $\phi(t)$ is attack angle; $P(t)$ is the target thrust; $D(t)$ is the resistance; $L(t)$ is lift; $R = 6371km$ is the radius of the Earth; $m$ is the mass of the aircraft; $\delta$ is the roll angle of the track. For the unpowered gliding phase of an approaching spacecraft, the thrust $P(t)$ of the target is set to zero, and the corresponding dynamic equation can be obtained.

3. Adaptive Cubature Kalman Filter

3.1 Cubature Kalman Filter
Cubature Kalman Filter using spherical-ra-dial cubature rule, The selected sample points are evenly distributed on the hypersphere. Therefore, each sample point has the same positive weight value. The volume points are continuously transmitted to implement filtering, and each filter is updated in two steps: time update and measurement update. Discrete-time nonlinear system is provided as follows:

$$x_{k+1} = F(x_k) + w_k$$

(7)
\[ z_{k+1} = H(x_{k+1}) + v_{k+1} \] (8)

\( x_k \) denotes the state vector of the system at time \( k \), \( z_k \) denotes the measurement vector of the system, used to represent the characteristic parameters such as position, velocity, and acceleration of the target at the moment, assuming that the process noise \( w_k \) and the measurement noise \( v_k \) are mutually independent Gaussian white noise, i.e. \( w_k : (0, R_k) \), \( v_k : (0, Q_k) \).

Cubature Kalman Filter algorithm needs to calculate the volume point, and then uses 2n volume points to perform weighted summation to approximate the Gaussian integral. To solve the integral problem using the volume integral criterion for solving an arbitrary distribution function \( f(x) \), it can be expressed as:

\[
\int f(x)N(x;u,P) \approx \sum_{i=1}^{2n} \omega_i f(\mu + \sqrt{P} \xi_i) \\
\xi_i = \sqrt{n}[L]
\] (9)

\( \omega_i \) denotes the weight of the volume point, and \( x_i \) denotes the set of transmitted volume points.

Assuming that the posterior probability density function \( P(x_{k+1}) = N(\hat{x}_{k+1}, P_{k+1}) \) at time \( k+1 \) is known, the algorithm steps are as follows:

**Step1 Time update**

\[ S_{k\mid k-1} = \text{chol}(P_{k\mid k-1}) \] (10)

\[ x_{k\mid k-1}^j = S_{k\mid k-1} \xi_i + \hat{x}_{k\mid k-1} \] (11)

\[ x_{k\mid k-1} = F(x_{k\mid k-1}) \] (12)

\[ \hat{x}_{k+1} = \sum_{i=1}^{2n} \omega_i * x_{k\mid k-1}^i \] (13)

\[ P_{x_{k+1}} = \sum_{i=1}^{2n} \omega_i x_{k\mid k-1}^i (x_{k\mid k-1}^i)^	op + \hat{x}_{k+1} (\hat{x}_{k+1})^	op + R_k \] (14)

**Step2 Measurement update**

\[ S_{k\mid k} = \text{chol}(P_{k\mid k}) \] (15)

\[ x_{k\mid k}^i = S_{k\mid k} \xi_i + \hat{x}_{k\mid k} \] (16)

\[ z_{k\mid k}^i = H(x_{k\mid k}^i) \] (17)

\[ \hat{z}_{k+1} = \sum_{i=1}^{2n} \omega_i * z_{k\mid k}^i \] (18)

\[ P_{z_{k+1}} = H(\hat{z}_{k+1}) P_{x_{k\mid k}} H^T (\hat{z}_{k+1}) + Q_k \] (19)

\[ P_{x_{k+1}} = \sum_{i=1}^{2n} \omega_i x_{k\mid k}^i (z_{k\mid k}^i)^	op + \hat{x}_{k+1} (\hat{z}_{k+1})^	op \] (20)

**Step3 Calculate the gain matrix, state update value, and covariance update value:**
\[ k_k = P_{xz,k|x,k-1} \cdot (P_{xz,k|x,k-1})^T \]  

\[ \hat{x}_{i,i} = \hat{x}_{i,i-1} + k_k (z_k - \hat{z}_{i,i-1}) \]  

\[ P_{4i,k} = P_{k|x,k-1} - k_k P_{xz,k|x,k-1} k_k^T \]  

3.2 Adaptive Cubature Kalman Filter

According to the requirements of Cubature Kalman Filter, known system noise variance matrix and measurement noise variance matrix are needed. Due to the complexity of the external environment, it is difficult to accurately determine the measured noise characteristics, and even if it is determined, it cannot be adjusted according to the environment. Therefore, this paper proposes a new adaptive Kalman filtering algorithm based on the covariance matching principle, using the new information and the residual sequence to adaptively improve the statistical properties of the noise, to adapt to the interference caused by the external environment, the filtering accuracy and improve the stability. Define the innovation sequence and residual sequence separately as follows:

\[ \delta_k = z_{k+1} - H(\hat{x}_{i,i-1}) \]  

\[ \eta_k = z_{k+1} - H(\hat{x}_{i,i}) \]  

Step1 Real-time adaptive measurement noise:

\[ Q_k = P_{zz,k|x,k-1} - \frac{2n}{N} \sum_{i=1}^{N} \omega_i z^T_{i,k|x,k-1} \]  

Using the window estimation method from the innovation vector can get the innovation covariance matrix \( N \) is the window size:

\[ E[\delta^T_k \delta^T_k] = \frac{1}{N} \sum_{i=1}^{N} \delta^T_{k,i} \delta^T_{k,i} = P_{zz,k|x,k-1} \]  

The same residual vector covariance matrix is:

\[ E[\eta^T_k \eta^T_k] = \frac{1}{N} \sum_{i=1}^{N} \eta^T_{k,i} \eta^T_{k,i} \]  

Substituting (27) into (26) yields an estimate of the covariance matrix \( Q_k \) of the measured noise:

\[ Q_k = \frac{1}{N} \sum_{i=1}^{N} \delta^T_{k,i} \delta_{k,i} - \frac{2n}{N} \sum_{i=1}^{N} \omega_i z^T_{i,k|x,k-1} \]  

Step2 Real-time adaptive process noise:

The difference between the innovation sequence and the residual sequence is available:

\[ E[\delta - \eta] = E[\delta^T \delta^T] - E[\eta^T \eta^T] = H(\hat{x}_{i,i} - \hat{x}_{i,i}) \]  

\[ H \hat{r} H^T = \frac{1}{N} \sum_{i=1}^{N} \eta_{k,i} \eta_{k,i}^T + \frac{1}{N} \sum_{i=1}^{N} \delta_{k,i} \delta_{k,i}^T - H(P_{i,i} + P_{i,i}) H^T \]  

Substituting (27), (28) and (14) into the above equation:
4. Simulation experiment and result analysis

4.1 Experimental Design
In order to better verify the effectiveness of the algorithm, IMM estimator was used for experimental verification. The flight trajectory is shown in figure 1.

![Figure 1 Near-space Hypersonic Vehicle flight trajectory](image)

4.2 Simulation condition settings
Assume that the initial geographic position of near-space hypersonic vehicle is [75° N, 25° E, 16km]. The initial velocity is $V_0 = Ma_5$, the initial mass is 4200 kg, the initial heading angle is 60°, the initial trajectory inclination angle is 0°, and the initial attack angle is 0°. Under the above conditions, Monte Carlo simulation was performed 100 times.

In order to better verify the effectiveness of the algorithm, this algorithm is compared with the standard CKF algorithm, and the standard root mean square error (RMSE) is introduced for comparison. The position root mean square error is now defined as follows:

$$RESM^p_k = \left[ \frac{1}{M} \sum_{i=1}^{M} (x_i - \hat{x}_i)^2 + \frac{1}{M} \sum_{i=1}^{M} (y_i - \hat{y}_i)^2 \right]^{1/2}, k = 1, 2, 3, \cdots M$$  \hspace{1cm} (33)

Where $M$ is the number of Monte Carlo simulations. $(x_k, y_k), (\hat{x}_k, \hat{y}_k)$ correspond to the real component and position estimation component of the target at the first time. Similarly, the root mean square error of the speed can be obtained.

4.3 Simulation
The measurement noise characteristics are set as steady-state noise. In the entire simulation process, the actual noise covariance is set to twice the initial value, and the simulation is compared using Adaptive Cubature Kalman Filter algorithm and Cubature Kalman Filter algorithm. The root mean square error of the position and the root mean square error of the speed are shown in Figure 2 and Figure 3, respectively.

![Figure 2. Root mean square error of speed](image)

![Figure 3. Root mean square error of position](image)
From the analysis of the uncertainty of the noise characteristics, it can be seen from Figure 2 and Figure 3 that the measurement noise has a greater impact on the accuracy of the filter. The near-space hypersonic vehicle has higher maneuverability and strong mutation ability, which directly leads to the instability of the filtering algorithm. The algorithm used in this paper can analyze the measurement noise situation to change the measurement noise covariance of the system itself, adapt to the noise caused by the noise characteristics of the filter, effectively reduce the overall error caused by tracking the aircraft, and have good stability. ACKF is more practical than Cubature Kalman Filter algorithm and has better results.

5. Conclusion
In order to solve the problem that there may be unknown noise characteristics in near-space hypersonic vehicle, a corresponding flight trajectory is established and Adaptive Cubature Kalman Filter algorithm is established. The system noise characteristics can be adjusted online, and the divergence of the filter is suppressed. ACKF can effectively improve the tracking accuracy of high maneuvering targets. The experimental results are compared with each other. This algorithm has faster processing speed and strong adaptability. It is applied to the tracking of hypersonic vehicles in the near-space.

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