Modelling Solar Cells with Thermal Phenomena Taken into Account

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Abstract. The paper is devoted to modelling properties of solar cells. The authors’ electrothermal model of such cells is described. This model takes into account the influence of temperature on its characteristics. Some results of calculations and measurements of selected solar cells are presented and discussed. The good agreement between the results of calculations and measurements was obtained, which proves the correctness of the elaborated model.

1. Introduction
In recent years, more and more attention is paid to renewable energy sources. To this class of energy sources belong e.g. photovoltaic systems [1 - 4]. The main components of these systems are solar cells, in which radiation energy (for example solar radiation) is converted into electrical energy. The solar cell comprises a p-n junction, whose properties strongly depend on temperature [5, 6].

In the design and analysis of electronic circuits the computer software is commonly used. Nowadays the most popular is program SPICE [7, 8]. For example, the book [9] is dedicated to the problem of modelling photovoltaic systems using SPICE. This book presents a method of modelling particular components of a photovoltaic system - from solar cells to inverters and batteries. Unfortunately, the models presented in the cited book do not take into account many important physical phenomena, such as self-heating, or they describe some phenomena in a highly simplified manner, e.g. the influence of changes in the ambient temperature on the characteristics of a solar cell.

In turn, in the paper [8] the model of the photovoltaic module dedicated for the SPICE software is proposed. The equations describing characteristics of the module take into account the influence of temperature on its characteristics. In this model the temperature is a parameter and it does not change during the analysis, therefore it can be interpreted as the ambient temperature. In the considered model the dependence of the photocurrent on the power density of the radiation is also skipped.

As it can be seen in papers [1, 2, 9, 10], the solar cell temperature during its operation can be significantly higher than the ambient temperature due to self-heating or heating by the light source [11 - 13]. The increasing temperature of the solar cell could shorten its lifetime and reduce the value of the voltage at the terminals of solar cells [9].

In the paper [14] the electrothermal model of the solar cell for the SPICE software is proposed. This model takes into account self-heating phenomena, but it skips an increase of temperature of this element resulting from the exchange of a part of the energy of radiation on the solar cell area into heat.

In this paper, which is an extended version of the paper [14], an electrothermal model of solar cell for SPICE is proposed. This model takes into account the influence of the photovoltaic effect and drawbacks of thermal phenomena on the characteristics of this device. The presented model is
elaborated and verified for silicon monocrystalline solar cells.

In the second section, the elaborated model is described, whereas in the third section the selected results of measurements and calculations are presented.

2. Model description

The worked out by the authors electrothermal model of the solar cell takes into account photovoltaic phenomena, self-heating and a temperature increase resulting from the thermal radiation falling on the surface of this solar cell. The network representation of the elaborated model is shown in figure 1.

![Figure 1. Network representation of the electrothermal model of a solar cell.](image)

The elaborated model consists of two parts: the electro-optical model (EOM) and the thermal model (TM). In the EOM, the controlled current source $G_1$ represents the photo current described by the following formula

$$I_{FO} = P \cdot S \cdot \eta \cdot x \cdot \left[ 1 + \alpha_T \cdot (T_j - T_0) \right]$$

where $P$ denotes the power density of the lighting radiation, $S$ - active area of the solar cell, $\eta$ – efficiency of photovoltaic conversion, $x$ - the coefficient equal to 1 $V^{-1}$, $\alpha_T$ – temperature coefficient of changes of the photo current, $T_j$ – internal temperature of the solar cell, $T_0$ - reference temperature.

In turn, the controlled current source $G_2$ represents the current of the p-n junction described by the classical Shockley’s model of the form

$$I_1 = J_0 \left( \frac{T_j}{T_0} \right)^2 \cdot \exp \left( - \frac{U_{go}}{n \cdot h \cdot T_j} \right) \cdot S \cdot \exp \left( \frac{u_D}{n \cdot h \cdot T_j} \right) - 1$$

where $J_0$ denotes the parameter of the saturation current density, $n$ – emission factor of the p-n junction, $U_{go}$ – voltage corresponding to the band-gap of silicon, $u_D$ – junction voltage, and $h$ is the quotient of the Boltzmann constant through the electron charge.

The resistor $R_h$ models leakage of the junction, while $R_S$ means series resistance of the solar cell in temperature $T_0$. The controlled voltage source $E_{RS}$ models the influence of temperature on series resistance of the solar cell by linear dependences of the form

$$E_{RS} = U_{RSO} \cdot \alpha_{RS} \cdot (T_j - T_0)$$

where $\alpha_{RS}$ denotes the temperature coefficient of changes of series resistance, whereas $U_{RSO}$ – the voltage on the resistor $R_s$.

The value of the internal temperature of the solar cell is calculated in the thermal model (TM). In this model, the current source $P_a$ represents power dissipated on the series resistance, on the leakage resistance and the power of radiation absorbed by the solar cell. The current of the controlled current source $P_{th}$ is described by

$$P_{th} = P \cdot a \cdot S_1 + (V_{out} - u_D) \cdot I_{out} + u_D \cdot I_1$$

where $a$ denotes the ratio of conversion of the radiation energy into heat, $S_1$ – area of the solar cell, while $V_{out}$ and $I_{out}$ are the output voltage and the output current of the solar cell, respectively. $R_{th}$ models the thermal resistance of solar cells, whereas $V_{Ta}$ represents the ambient temperature.
3. Estimation of the model parameters

The model presented in section 2 is described by means of 11 parameters. The values of these parameters are indispensable to calculate the solar cell characteristics by means of this model. In this section the manner of estimation of these parameters values is proposed. In this manner the idea of the local estimation, described among other things in the paper [15] is used.

The value of the reference temperature $T_0$ is chosen by the user and means the temperature in which the values of the remaining parameters are estimated. The area of the active part of the solar cell $S$ and the entire area of this solar cell $S_1$ are estimated on the basis of the geometrical dimensions of the suitable areas of the solar cell and formulas that are well-known from geometry.

The efficiency of the conversion $\eta$ is calculated on the basis of the measurement of the short circuit current of the solar cell $I_{SC}$ at the well-known value of the power density of radiation $P_R$. The value $\eta$ is given by the formula

$$\eta = \frac{I_{SC}}{S \cdot P_R \cdot x} \quad (5)$$

The temperature coefficient of changes of the photocurrent $\alpha_T$ can be estimated after measuring the values of the short-circuit current $I_{SC}$ and $I_{SC1}$ corresponding to the temperature values $T_0$ and $T_1$, respectively at the identical value of the power density of radiation $P_R$. The value of the coefficient $\alpha_T$ can be calculated using the following equation

$$\alpha_T = \frac{I_{SC} - I_{SC1}}{(T_0 - T_1) \cdot I_{SC}} \quad (6)$$

In order to estimate the values of parameters $J_0$, $n$, $R_S$, $\alpha_{RS}$ and $R_R$ the measurement of the characteristic $i(v)$ of the not lighted up solar cell in temperatures $T_0$ and $T_1$ is indispensable. The slope of this characteristic in the range of small voltages (typically smaller than 0.15 V) is equal to $R_R$ resistance. On the basis of the coordinates of points $A(v_1, i_1)$ and $B(v_2, i_2)$ laying on the linear section of the current-voltage characteristic $i(v)$ presented in the linear-logarithmic scale one can calculate the value of the coefficient $n$ from the formula

$$n = \frac{v_1 - v_2}{h \cdot T_0 \cdot \ln(i_1/i_2)} \quad (7)$$

and the value of the parameter $J_0$ using the formula

$$J_0 = \frac{i_1}{S} \cdot \exp\left(\frac{-v_1}{n \cdot h \cdot T_0}\right) \cdot \exp\left(\frac{U_{go}}{h \cdot T_0}\right) \quad (8)$$

The series resistance of the solar cell $R_S$ is estimated on the basis of the measurement of the coordinates of the point $C(v_3, i_3)$ laying in the range of high currents of the solar cell. In the calculations the equation of the following form is used

$$R_S = \frac{v_3 - n \cdot h \cdot T_0 \cdot \ln\left(i_3 \cdot \exp\left(-\frac{U_{go}}{h \cdot T_0}\right)\right)}{i_3} \quad (9)$$

In order to estimate the value of the coefficient $\alpha_{RS}$ one ought to estimate the values of the resistance $R_S$ for two values of temperature $T_0$ and $T_1$, and then to use the formula

$$\alpha_{RS} = \frac{R_S(T_1) - R_S(T_0)}{(T_1 - T_0) \cdot R_S(T_0)} \quad (10)$$

The value of the thermal resistance of the solar cell $R_{th}$ is estimated with the use of the method described in the paper [16], using the coordinates of points laying on the static characteristic of the not illuminated forward biased solar cell. The value of the parameter $a$ is estimated at the lighting of the solar cell from the light source of the well-known power density $P$ and at the small value of the
forward current of the solar cell. On the basis of the measurement of the voltage on the solar cell once the light source is switched on at the steady-state, the increase of the internal temperature of the solar cell $\Delta T$ can be estimated. The value of the parameter $a$ is calculated from the formula

$$a = \frac{\Delta T}{R_{th} \cdot P \cdot S_i}$$  \hspace{1cm} (11)

4. Investigation results

In order to verify the correctness of the elaborated model, the calculations and measurements of the selected characteristics of the solar cells operating at different temperatures and with different values of radiation intensity are performed. The measurements were made for monocrystalline silicon solar cells of different sizes. The thickness of the solar cells with the PCB, on which they were located, was equal to 1.5 mm. During the investigations these solar cells were located in the light-tight thermostat of the volume equal to about 0.25 m$^3$. The incandescent bulbs were the radiation source. The spectrum of the emitted light is close to the spectrum of the solar light [17]. Both the isothermal (without self-heating) and non-isothermal (with self-heating) characteristics of the solar cells were calculated and measured.

In order to eliminate the influence of self-heating on the result of the measurements of isothermal characteristics, the impulse method of measurements was used. The bulbs were switched on in time equal to about 5 s and then the values of the current and the voltage on the output terminals of the solar cell were measured. Between the following measurements, at least a five-minute-pause appeared to assure cooling of the solar cell to the ambient temperature. The value of the current was regulated by means of the decadal resistor, which was the load of the solar cell.

The results of isothermal calculations and measurements are presented in figures 2-5. In all figures presented in this section points mark the results of measurements, dashed lines – the results of calculations obtained with the model from the paper [9], while solid lines – the results of calculations obtained with the authors’ model.

The values of electrical and optical parameters of the solar cell Conrad 0.5V/400mA estimated with the method described in the section 3 are collected in the Table 1.

| Parameter | $S$ [m$^2$] | $S_1$ [m$^2$] | $\eta$ | $T_0$ [K] | $\alpha_T$ [1/K] | $J_0$ [A/m$^2$] |
|-----------|-------------|-------------|--------|-----------|----------------|----------------|
| Value     | 12x10$^{-4}$ | 35x10$^{-4}$ | 0.115  | 300       | 1.9x10$^{-3}$  | 4.17x10$^{-3}$  |

| Parameter | $R_s$ [Ω] | $\alpha_{rs}$ [1/K] | $R_R$ [kΩ] | $a$ |
|-----------|-----------|-----------------|-------------|-----|
| Value     | 0.3       | $3x10^{-3}$     | 10          | 0.8 |

For example, figure 2 shows the current-voltage characteristics of the solar cell Conrad 0.5V/400mA for the selected values of the ambient temperature. The presented results were obtained at illumination of the investigated solar cells with one bulb, which corresponds to the power density of radiation on the surface of the solar cell equal to 416 W/m$^2$.

As it can be seen, the output voltage of the solar cell is a decreasing function of the output current and temperature. Together with the temperature rise, the value of the short circuit current increases. Comparing the obtained results of calculations and measurements, it is visible that the good agreement of the results of calculations with the electrothermal model and measurements, ensuring the accuracy of the elaborated model, was obtained. From the calculations performed with the use of the literature model, the understated values of the output voltage were obtained in the range of low currents and high temperature.

In turn, in figure 3 the characteristics of the same solar cell obtained at illumination with two bulbs are presented. In this case the power density of radiation of the surface of the solar cell amounts to 772 W/m$^2$.

As it can be noticed, an increase in the illumination of the solar cell caused an increase in the value of its short circuit current. In this figure, it is also visible that the manner of modelling the influence of
temperature on the characteristics of the solar cell used in the literature model is incorrect. On the other hand, the electrothermal model assures the good agreement between the calculated and measured characteristics in all the considered range of changes of temperature and the output current.

**Figure 2.** Calculated and measured current-voltage characteristics of the solar cell Conrad 0.5V/400mA in the case when one bulb lights.

**Figure 3.** Calculated and measured current-voltage characteristics of the solar cell Conrad 0.5V/400mA in the case when two bulbs light.

In figure 4 – the dependences of the output power on the load current for the considered solar cell are presented.

**Figure 4.** Calculated and measured dependences of the output power on the load current for the solar cell Conrad 0.5V/400mA.
As it should be anticipated, while lighting the solar cell with two bulbs, almost twice higher maximum values of the output power were obtained. The temperature increase from 25 to 108 °C causes a fall in the value of this power by even about 40% and an increase of the maximum value of the output current even by over 10%.

In figure 5 the characteristics of two monocrystalline solar cells of the different surface obtained from the measurements and calculations performed with the use of the electrothermal model are compared. The solar cell Conrad 0.5V/400mA has dimensions 76x46 mm, whereas the solar cell 0.5V/100mA - 46x26 mm. As it can be noticed, for both the considered solar cells the good agreement between the calculated and measured characteristics was obtained in a wide range of temperature changes. However, the achievement of this agreement demanded taking into account the fact, that the quotient of the areas S of both the investigated solar cells was not equal to the quotient of the total areas of these elements, but it was equal to the quotient of the nominal values of the output current of the considered solar cells. The observed relation results from the fact that only some part of the surface of the investigated solar cells is the active area, and the banks of the element do not participate in generation of the photocurrent.

![Image of Figure 5](image1)

**Figure 5.** Calculated and measured current-voltage characteristics of the solar cells Conrad 0.5V/400mA and Conrad 0.5V/100mA.

In figure 6 the calculated and measured characteristics of the solar cell Conrad 0.5V/400 obtained for the not lighted up device at different values of the ambient temperature are presented. As one can notice, together with the temperature rise the shift of the considered characteristics in the left direction is observed, similarly as it is observed for p-n or m-s junctions [5]. The temperature coefficient of changes of the forward voltage equals -1.9 mV/K.

![Image of Figure 6](image2)

**Figure 6.** Calculated and measured dark current-voltage characteristics of the solar cells Conrad 0.5V/400mA.
In figure 7 the influence of the cooling conditions of the solar cell on its current-voltage characteristics is illustrated. In this figure the curve a denotes the characteristic corresponding to ideal cooling of the investigated device (the isothermal measurement), the curve b - the characteristic obtained in the steady-state for the solar cell situated in the closed light-tight chamber of the volume equal approx. to 0.25 m$^3$, whereas the curve c - the characteristic obtained in the steady-state for the solar cell situated in the open chamber. In all the cases the solar cell is lighted up by two bulbs of the nominal power 75 W, which corresponds to the power density of radiation equal to 772 W/m$^2$. In the calculations, the results of which are shown in figure 7, the following values of thermal resistance of the solar cell are used: $R_{th} = 0$ for the isothermal characteristics, $R_{th} = 40.3$ K/W for the solar cell situated in the closed chamber and $R_{th} = 14.3$ K/W for the solar cell situated in the open chamber.

![Figure 7. Calculated and measured current-voltage characteristics of the solar cells Conrad 0.5V/400 mA obtained at different cooling conditions.](image)

As can be observed, worsening of the cooling conditions (the increase in value $R_{th}$) results in a decrease in the value of the voltage on the output of the solar cell and with an increase in the value of the short circuit current. It is good to notice that as a result of self-heating and absorption of the heat generated by the light source, the change in the course of characteristics of the solar cell has the identical character as changes in the course of these characteristics due to the change of the value of the ambient temperature. The values of the internal temperature of the solar cell obtained from the calculations for the device situated in the open chamber amount to about 55°C, and for the device situated in the closed chamber – up to 111°C.

5. Conclusions

In the paper the form of the electrothermal model of the solar cell is proposed. This model is dedicated to be implemented in the form of the subcircuit for the SPICE software. It takes into account self-heating phenomena and heating resulting from the exchange of energy of the part of the absorbed radiation into heat and changes of the ambient temperature. The presented manner of estimation of the value of model parameters is very simple to realize.

The presented in the previous section initial results of the experimental verification of the worked out model confirm its usefulness in modelling characteristics of solar cells over a wide range of changes in the output current of the solar cell, the ambient temperature, cooling conditions and active areas of the solar cell. The presented model assures the better agreement between the results of calculations and measurements than the classical literature model.

6. References

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