Tree-level Scattering Amplitude in de Sitter Space

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Abstract

In previous papers [1, 2], it was proved that a covariant quantization of the minimally coupled scalar field in de Sitter space is achieved through addition of the negative norm states. This causal approach which eliminates the infrared divergence, was generalized further to the calculation of the graviton propagator in de Sitter space [3] and one-loop effective action for scalar field in a general curved space-time [4]. This method gives a natural renormalization of the above problems. Pursuing this approach, in the present paper the tree-level scattering amplitudes of the scalar field, with one graviton exchange, has been calculated in de Sitter space. It is shown that the infrared divergence disappears and the theory automatically reaches a renormalized solution of the problem.

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1 Introduction

Positivity condition stemming from the concept of probability, constitutes one of the principles of the axiomatic QFT [5]. Application of this principle i.e. the positivity condition in the gauge QFT (GQFT), in Minkowski space, results in breaking of the Lorentz covariance and the appearance of the infrared divergence. It has been shown that a causal and covariant GQFT [6] can be obtained through an indefinite metric quantization (i.e. maintaining the negative norm states: NNS). As a result the positivity condition must be eliminated in GQFT in Minkowski space. The general properties of this quantization, i.e. the indefinite metric Fock quantization, were studied by Mintchev [7].

The same problems of the GQFT appear in the quantization of the free massless minimally coupled scalar field in de Sitter (dS) space-time (i.e. appearance of the infrared divergence [8] and the breaking of the dS covariance [9]). It has been shown that the presence of negative norm states are indispensable for a fully covariant quantization of the minimally coupled scalar field in de Sitter space [1, 2]. This type of NNS is different from of the NNS in the GQFT. So we call this construction a “Krein QFT” (KQFT). This approach in the free field quantization, while leaving the physical content of the theory unchanged, removes the infrared and the ultraviolet divergences. Following this pattern the normal ordering procedure is rendered useless since the vacuum energy remains convergent [2]. The infrared divergence in the two-point function disappears as well [10]. The crucial element in the KQFT is the presence of negative frequency solutions, which are indispensable for preservation of the full covariance of the problem. It has also been discussed that the application of this method to the free “massive” scalar field in de Sitter space results in an automatic and covariant renormalization of the vacuum energy [2], as well.

The linear quantum gravity in dS space can be constructed from a projector tensor and a massless minimally coupled scalar field [11, 12]. However, in so doing the above problems appear again [13, 14]. It is also shown that the infrared divergence or the pathological behavior of the graviton propagator is gauge dependent and therefore should not appear in an effective way as a physical quantity [3, 11, 15, 16, 17, 18]. An explicit construction of the covariant quantization of the traceless rank-2 “massless” tensor field in dS space (linear covariant quantum gravity) has been thoroughly studied [11, 12]. The main ingredient of this construction is the presence of two different types of NNS. The first one is similar to those non physical states, dual to gauge states, which appear in GQFT in Minkowski space. The other is due to the consideration of negative frequency solutions in order to preserve de Sitter covariance. This is similar to the case of free massless minimally coupled scalar field in dS space. This construction allows us to avoid the infrared divergence, and enables us to obtain a covariant two point function [3]. It should be noted that by adopting KQFT framework, various Green functions for a scalar field, in a general curved space-time, appear to be convergent in the ultraviolet limit. A natural renormalized one-loop effective action has been also obtained [4].

The appearance of the infrared divergence and the necessity of the inclusion of the NNS for preservation of the covariance of the problem constitutes the common denominator between GQFT in Minkowski space and the calculation of massless minimally coupled scalar field in dS space. In GQFT, however, a local symmetry is present, where as in the dS space, a global symmetry is the central elements of the problem. The infrared divergence disappears in the dS space case. In other words there are two different types of NNS, one is due to the local gauge
invariance (sign of the metric) and the other is due to the global gauge invariance (negative frequency solution). The important question which deserves to be considered is: *Is it possible to generalize the free KQFT in dS space, to the interaction field?*

By extending the KQFT method to the interacting quantum field in Minkowski space-time ($\lambda\phi^4$ theory,) a natural renormalization to the one-loop approximation has been achieved [19]. A very interesting consequence is that the Schwinger commutator function, the retarded and the advanced Green functions, are one and the same in both formalisms. Another remarkable result is that the vacuum energy, automatically reduces to zero [19].

In this paper the indefinite metric quantization *i.e.* the presence of negative frequency states, as auxiliary unphysical states, have been included for calculation of the tree-level scattering amplitudes of the scalar field with one graviton exchange in the de Sitter space. It is shown that the previous divergence [20], disappears and the theory is automatically renormalized.

### 2 Tree-level scattering amplitudes

Recalling the previous calculation of the tree-level scattering amplitudes of two scalar particles in the one-graviton-exchange approximation [20]. An expression of the following form must be calculated

$$
M = \int T^{\mu\nu}(x)G_{\mu\nu,\mu'\nu'}(x, x')dV_xdV_{x'},
$$

(1)

where $T^{\mu\nu}$ is a conserved, physical energy-momentum tensor of the scalar field and $G_{\mu\nu,\mu'\nu'}$ is the graviton two-point function. By explicit computation, it was shown that the only growing part of $M$, for large $z$, comes from the transverse and traceless part of the graviton two-point function [20],

$$
G_{TT}^{\mu\nu,\mu'\nu'}(x, x') \sim \ln(1 - z)D^{\mu\nu,\mu'\nu'} \text{ for large } z,
$$

(2)

where $D$ is a maximally symmetric bi-tensor.

For simplicity, a conformally coupled scalar field was considered

$$
\phi(x) = \sum_k a_k f_k(t) e^{i\vec{k} \cdot \vec{x}} + C.C.,
$$

(3)

where $f_k(t) = \sqrt{\frac{2|\vec{k}|}{4\pi^2t^2\sin t|\vec{k}|}}$. A particular coordinate system is chosen

$$
ds^2 = -\frac{1}{t^2}dt^2 + \frac{1}{t^2}d\vec{x}^2,
$$

(4)

in which $z = 1 + \frac{1}{4t^2}[(t - t')^2 - (\vec{x} - \vec{x}')^2]$. For the S-channel diagram the only term, which depend on the scattering angle $\vec{k} \cdot \vec{k}'$, gives

$$
M^{\theta\text{-dep.part}} \sim -8(\vec{k} \cdot \vec{k}')^2 \int dV_xdV_{x'}t^2t'^2 f_k^2(t)f_{k'}^2(t')
$$

$$
\ln(1 - z) \left(1 + \frac{4}{3} \frac{(\vec{x} - \vec{x}')^2}{(\vec{x} - \vec{x}')^2 - (t - t')^2} + \frac{8}{15} \left[\frac{(\vec{x} - \vec{x}')^2}{(\vec{x} - \vec{x}')^2 - (t - t')^2}\right]^2\right),
$$

(5)
where \( dV_x = \frac{1}{2} dt d^3x \). This term has a \( \cos^2 \theta \) dependence characteristic of a spin-two exchange, but due to the \( \ln(1 - z) \) element, it appears to be divergent.

At this stage, the new method of field quantization is implemented. In this method the field operator (3) is defined by

\[
\phi(x) = \frac{1}{\sqrt{2}} \sum_k a_k f_k(t) e^{i\vec{k}.\vec{x}} + b_k f_k^*(t) e^{-i\vec{k}.\vec{x}} + C.C.,
\]

(6)

where

\[
[a(\vec{k}), a^\dagger(\vec{k}')] = \delta(\vec{k} - \vec{k}'), \quad a^\dagger(\vec{k}) | 0 > = | 1_\vec{k} > = | \text{one-particle state} >,
\]

\[
[b(\vec{k}), b^\dagger(\vec{k}')] = -\delta(\vec{k} - \vec{k}'), \quad b^\dagger(\vec{k}) | 0 > = | \bar{1}_\vec{k} > = | \text{unphysical state} >
\]

\[
\begin{align*}
\langle \text{unphysical state} | \text{physical state} \rangle &= 0; \quad a(\vec{k}) | \text{physical state} \rangle > = 0, \\
\end{align*}
\]

(7)

By using the above relations it can be shown that the stress tensor for the physical state is not affected by the negative norm state ([2] section 6). Therefore, the only departure from previous method appears in the graviton two-point function. In this case the logarithmic term \( \ln(1 - z) \) must be replaced by [3, 10, 12]

\[
\text{const. } \epsilon(X^0 - X'^0) \theta(z - 1),
\]

(8)

where

\[
X^0 = \sinh t + \frac{1}{2} e^t |\vec{x}|^2, \quad \epsilon(X^0 - X'^0) = \begin{cases} 
1 & X^0 > X'^0 \\
0 & X^0 = X'^0 \\
-1 & X^0 < X'^0,
\end{cases}
\]

(9)

and \( \theta \) is the Heaviside step function. This expression (eq.(8)) can also be obtained by the use of the fact that the Krein-Feynman propagator is the real part of the previous Feynman propagator (eq.(13a) of [21].) By the application of the equation (8) in the integral (5), we obtain

\[
\mathcal{M}^\theta \text{-dep.part} \sim \text{const. } (\vec{k}, \vec{k}')^2,
\]

which is free of any infrared divergence.

By the very same procedure, it can also be shown that the effective action, in the one-loop approximation, for the large-distance behavior, is convergent. This result reaffirms the Iliopoulos’s calculation [13].

3 Conclusion

The appearance of singularities in physical quantities in QFT, is a manifestation of presence of anomalies in QFT. For eliminating these anomalies, the normal ordering procedure and the renormalization techniques are applied to QFT.

The covariant quantization of the free minimally coupled scalar field in de Sitter space in absence of the positivity condition, and its consequences has been studied [2]. In this case without changing the physical content of the theory, an automatically renormalized results was obtained. Following this procedure, the covariant principle in dS space has been conserved as
well. In the interaction case, the tree-level scattering amplitudes of the scalar field, with one graviton exchange in de Sitter space, has been calculated by this method. It is shown that the previous infrared divergence disappears, and the theory is automatically renormalized as well.

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