Space-Time: from Indefinite at the Locality to Generality at the Infinity

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Abstract. The article presents physical and theoretical generalizations of the space-time metrics, based on invariant coordinate time transformations over the intervals of the observed periodic emission of pulsars. Synchronization of spatial reference systems by the observed passage of the wave front of the coherent radiation of a pulsar corresponds to the axiomatic matching of coordinate time scales defined by the observed rotation parameters of the pulsar with the initial coordinates of spatial reference frames within the homogeneous Galileo space of the Galaxy. As a result, all physical processes in such coordinate systems, including the observed coherent radiation of a pulsar, are indistinguishable from each of them. The estimated span of minimum recognizable on the pulsar time scale is $10^{-12} - 10^{-14}$ s in the extent from several years to several decades and then gradually decreases, reaching limit values of the order of $10^{-16} - 10^{-17}$ s in the extent compared to the lifetime of a pulsar. The relative size of the resolved interval is within approximately $10^{-19} - 10^{-29}$.

1. Introduction

Space and time are primary concepts, the essence of the form of existence of matter, in the sense that the concepts of space and time are obtained by proper generalizations that follow from the concept of space-time connections between material processes. The main idea of the theory of space-time, or the physical theory of relativity in the systematic presentation of V.A. Fock (1898-1974) [1], is precisely to establish the space-time metric so that the form of the laws of physics corresponds to this metric, which in this case, expresses the homogeneity of space-time. The same opinion was followed by A.A. Logunov (1926-2015) [2]. He highlighted the role of H. Poincaré, who, based on the principle of relativity and the preparatory work of Lorentz, discovered and formulated for all physical phenomena, everything that is the essence of the special (here in terms of physical) theory of relativity as pseudo-Euclidean space-time geometry, in which all physical processes take place.

Back in the early 20s of the last century, A.A. Fridman (1888-1925) [3] analyzed the geometrical model of 4-dimensional space, described in the Einstein’s theory just published then, from a purely mathematical point of view, essentially presented his interpretation of the relationship of time-spaces in the process of moving material points, the variety of properties of which can be expressed by a certain variety of numbers, established by the system of axioms and theorems following from these axioms. Since physical space is material space, all images of geometric space are interpreted in physical space either by material objects or by material actions with them. However, A.A. Fridman believed, we cannot produce any physical actions necessary for the experimental establishment of physical geometry in three-dimensional space, because
all these actions must be performed in time, since there is not a single physical phenomenon that can occur instantaneously. Even if it is the speed of light, then moving the material point from Andromeda and back would take many tens of thousands of years, so comparing the motion vectors at the beginning and the end of the movement trajectory would no longer have a physical meaning. If we choose a certain initial point \( M \) on the trajectory and measure the lengths of the arcs with positive numbers in one direction and negative ones to another, then by setting the number \( t \) corresponding to the arc length, we will completely determine the position of the material point \( M \) moving along the trajectory. The value of \( t \), shown by the clock of a given point \( M \), is the physical local time of point \( M \). However, for each point it will be different. There is no single universal time in relation to which, as Friedman believed, all points make one basic movement, which could be considered uniform.

Logunov [2] formulated the essence of the physical theory of relativity in a generalized form as applied to the homogeneous and isotropic Galilean space, in which all physical equations are invariant with respect to the Lorentz transformations, and the space itself at any point is orthogonal to the timelines. These properties of space-time are the conditions for the fulfillment of the fundamental physical conservation laws in an isolated system: conservation of energy, although it can be changed from one form (mechanical, kinetic, chemical, etc.) into another, conservation of linear momentum, as the product of mass and vector, conservation of angular momentum of rotating bodies. The study of various forms of matter, the laws of its movement, thus becomes at the same time the study of space and time. It was the study of electromagnetic phenomena in conjunction with the principle of relativity Poincaré led to the unification of space and time into a single four-dimensional continuum of events and allowed to establish the pseudo-Euclidean geometry of this continuum.

Generalization of Logunov, based on the postulate of the invariance of physical laws regarding the transition from one method of digitalization to another and the choice of spatial reference systems, is fully consistent with the postulate of materiality on which the causality principle is based. According to this postulate, two different-time phenomena cannot be, with the help of digitalization, obeying the postulate of reality, reduced to simultaneity. Thus, cause and effect with one method of digitization remain cause and effect also with any other method of introducing spatial coordinates.

The paper discusses the physical and theoretical generalizations of the time-space metrics, based on invariant coordinate time transformations, set at intervals of the observed periodic radiation of neutron stars - pulsars, in spatial reference systems. The metric equivalence of the pulsar radiation intervals, expressed analytically by the observed rotation parameters of the pulsar, and dynamic ephemeris time, based on the laws of motion of the Solar system’s celestial bodies in the space of the International Spatial Reference System (ICRS), is shown. A generalization of the physical principle of Galilean relativity to the entire galactic space, in which the front of electromagnetic waves of periodic radiation of a pulsar freely propagates, is substantiated.

2. On indefiniteness of inhomogeneous space-time

The equations of motion, field equations, or any other laws of physics expressed in any coordinates in accordance with the rules of the Lagrange transform are obtained by the same operations — the compilation of partial derivatives with respect to coordinates and velocities and time differentiation. The form of equations in the transition from one coordinate system to another can be obtained by direct transformation, this is a formal mathematical question, not expressing any law of nature and not related to the physical content of the equations. Galileo’s principle of relativity, on the contrary, is a physical principle related to the physical content of the equations, allowing for experimental verification. According to this principle, any phenomenon in one system can be compared with the same phenomenon in another system, and
this possibility is a decisive condition for the applicability of the Galilean principle of relativity. It involves the physical transfer of phenomena from one system to another.

The possibility of physical transfer means a special kind of invariance, which can be called physical invariance. This is the invariance of all functions describing a given physical process with respect to a combined transformation consisting of transforming coordinates and changing initial and limiting conditions (the transfer itself). The formulas of such a transformation are linear, and, like the Galilean transformation formulas, leave invariant coordinate expressions for the distance and for the time interval. For infinitely close points and an infinitely small time interval, such an invariant expression in Galilean coordinates takes the form:

$$ds^2 = c^2dt^2 - (dx^2 + dy^2 + dz^2)$$  \hspace{1cm} (1)

In contrast to inertial systems, in which the invariance of physical equations is determined by the complete homogeneity of four-dimensional space and time, in Einstein’s theory, where space-time is heterogeneous and its metric depends on the processes occurring in it, the covariance conditions of the equations are satisfied only in the local region. Equations are valid only if gravity is neglected [4]. As a result, Einstein actually had to abandon the physical theory of relativity of Galileo-Lorentz, in which space-time is homogeneous, and inertial reference systems are equal, while the theory of space curvature is attributed to the property of its heterogeneity. And, as a result, this led to the abandonment of the fundamental laws of conservation of energy-momentum [2].

The properties of the field, according to Einstein, are due to the properties of space and time, taken in his theory as a starting point, as a postulate. The field will no longer be an external field that exists in space with pre-set properties, as in a homogeneous space. Rejection of the theory of homogeneous time-space leads to the assumption that it forms a four-dimensional manifold with an indefinite metric, which is generalized for the square of the interval, expressed by a set of coefficients of the metric tensor. In the case of a homogeneous space, the metric tensor satisfies a system of second-order equations representing the condition for the metric tensor to be reducible to the form (1).

Of course, there is no exact transformation of a four-dimensional manifold with an indefinite metric in the form (1) in Galilean coordinates, but since Newton’s field is characterized by the potential of $U(x, y, z)$ in a homogeneous space, and by Einstein it is a metric tensor, it is permitted to expect that one approximately expresses itself through the other [4]. In fact, when using coordinates that differ little from Galilean, an approximate solution of the Einstein equations, corresponding to the mass distribution with the Newtonian potential $U(x, y, z)$, leads to the following expression for the square of the interval:

$$ds^2 = (c^2 - 2U)dt^2 - \left(1 + \frac{2U}{c^2}\right)(dx^2 + dy^2 + dz^2)$$  \hspace{1cm} (2)

Comparison of this formula with the formula (1) for a completely homogeneous four-dimensional space and time, which is obtained from the formula (2) under the condition $U(x, y, z) = 0$, gives a visual idea of the effect on the metric, i.e. on the properties of space and time. Unlike expression (1), the coefficients in the expression (2) are not constant, but they depend on the potential of gravity, and, moreover, differ in magnitude in the temporal and spatial parts. In the theory of inhomogeneous space-time with a metric that depends on the processes occurring in it, the mathematical condition for the covariance of equations, performed only in a local area, is not a sufficient condition for physical relativity. As a result, the transition from local space-time to final values violates the invariance of lengths and time intervals, as well as the principle of simultaneity of identical events and, accordingly, the principle of causality in the sequence of observed events.
The identification and generalization of the basic laws governing the physical processes in various metric systems of four-dimensional space-time cannot do without direct long-term observations of the movement of the systems of material masses distributed over sufficiently large lengths, such as the Solar System or the Galaxy. The combination of mathematical equations and physical measurements of the motion of celestial bodies and the propagation of electromagnetic waves provides the basis for a general classification of events with respect to their sequence in time for all inertial reference systems.

3. The laws of motion and the ephemeris of celestial bodies
The laws of motion of the planets of the Solar System, discovered by Kepler (1571-1630) according to the direct observations of moving planets, later found their equivalent expression in the form of kinematic solutions of differential equations in the Newton form. It is not difficult, in particular, to verify the exact mathematical correspondence of the first Kepler law of the motion of planets along an ellipse, in one of whose foci the Sun is located, and the laws of Newton’s mechanics. Indeed, the Newton’s Law of Gravitation binds the force per unit mass \( U \) with the distance \( r \) as

\[
U = \frac{-GM}{r^2} = -GMU^2,
\]

where \( G \) is the universal gravitational constant, \( M \) is the mass of the star. As a result, we obtain a differential equation

\[
\frac{d^2 x}{dt^2} = -k^2(1 + m) \frac{x}{r^3} + \frac{\partial R}{\partial x},
\]

\[
\frac{d^2 y}{dt^2} = -k^2(1 + m) \frac{y}{r^3} + \frac{\partial R}{\partial y},
\]

\[
\frac{d^2 z}{dt^2} = -k^2(1 + m) \frac{z}{r^3} + \frac{\partial R}{\partial z},
\]

Replacing \( U \) with \( 1/r \) and believing \( \theta_0 = 0 \), we have

\[
r = \frac{1}{U} = \frac{l^2}{GM} \frac{1}{1 + e \cos \theta}.
\]

As a result, we obtain the equation of a conic section with eccentricity \( e \) and the origin of the coordinate system in one of the foci. Thus, the first law of Kepler directly follows from the first and second laws of Newton.

The main function of ephemeris astronomy, based on general dynamic theories of celestial mechanics and observational astrometry, is to develop and implement methods for direct measurement and prediction of the positions of celestial bodies, topocentric distances, and refinement of the parameters of theories of planetary motions using these measurements [5]. Building a theory of the movement of large planets in the Solar system in the general field of the Sun and planets, described by the law of the world in the form of Newton, is reduced to the problem of determining the translational movement of large planets in heliocentric orbits based on solving differential Lagrange-Gauss equations of disturbed motion taking into account the perturbing gravitational influence of other planets:

\[
R = \sum_i k^2 m_i \left( \frac{1}{r_{m,m_i}} - \frac{x x_i + y y_i + z z_i}{r_i^3} \right)
\]

Expression (4) characterizes the disturbing influence of planets with masses \( m_i \) on the movement of a planet with mass \( m \). The perturbation function \( R \) takes into account the effect on the motion of the investigated celestial body of disturbing factors that deflect the real orbit from the unperturbed Kepler orbit, in which the body would move if there was only one center of gravity. Since the masses of the major planets \( m_i \) are small in comparison with the mass of
the Sun, the task is reduced to calculating perturbations in the elliptical (Keplerian) movements of the planets, determining changes in the osculating orbital elements that are in contact with the real body trajectory.

Thus, here the main component of the motion is explicitly highlighted, which is a mathematical description of the motion of idealized bodies and can be expressed within purely kinematic form in accordance with equations (3), without considering the causes of motion (mass, forces generating it, etc.), using the concepts of space and time only. Such a motion, being the translational one, is a generalization of both the rectilinear and the rotational motion of a system of material points of an absolutely rigid body. Mathematically, the translational motion in its final result is equivalent to parallel transfer — a particular case of motion in which all points of space move in the same direction by the same distance. In the general case, it can be considered as a set of turns which are not completed rotations. This implies that the rectilinear motion is a turn around the center of a tank that is infinitely distant from the body. Since the perturbing forces are random and small compared with the attraction of the central body, on larger time scales in real orbits the main, stable component of the motion, determined by the Kepler-Newton laws, is not sensitive to the random effects of the perturbing forces.

Studying the stability of the motion of celestial bodies, A.M. Molchanov [6] expressed in the form of a hypothesis that he found the full resonance of the entire Solar System - compatibility of the orbital times (or average angular velocities) of the planets around the Sun, or satellites of the planet around it or the planets (satellites) around its axis:

\[ n_1 \omega_1 + n_2 \omega_2 + \ldots + n_k \omega_k = 0, \tag{5} \]

here \( \omega_1, \omega_2, \ldots, \omega_k \) are frequency of revolution (or average angular velocity) of the corresponding celestial bodies. According to the hypothesis, evolutionary mature oscillatory systems are inevitably resonant, and their state is determined, like quantum systems, by a set of integers. The resonance of the orbits, according to this hypothesis, is provided by small dissipative forces: tidal, inhibiting from interstellar dust matter and others. Dissipative forces, although very small, so their contribution to the displacement of any orbit is orders of magnitude smaller than weak disturbances due to the interactions of the planets and is beyond the limits of the observed accuracy, but acting for millions of years or more, they bring the movements of the planets to stationary resonant orbits, which in the main component, according to Kepler’s laws, are determined by the field of the Sun - the main, sharply dominant body of the system. The perturbation theory showed that because of the small interactions of the planets between them, small ripples are superimposed on their movements, and the influence of the planets through their interaction leads only to some jitter from the orbits about the ideal trajectories prescribed by Newton, in accordance with Kepler’s laws.

Thus, after Newton formulated the Law of Gravity in 1687, and then explained the first Kepler’s law, celestial mechanics became a quantitative mathematical science. The results of the further development of ideas about the nature of the motion of celestial bodies led to the fact that celestial mechanics, following geometry, became a purely axiomatic science and reduced to just a few equations of motion. The construction of the axioms of any phenomenon, as concludes Molchanov, inevitably raises the question of the limits of applicability of the mathematical model. Here we are talking about exceptional initial conditions, everywhere tightly connected and extending to infinity of space and time.

4. Coherence of the initial conditions of position and motion in space-time coordinate systems

The study of the dynamics of celestial bodies and astronomical phenomena is directly related to the measurement of time. Among the three fundamental units of mechanics, time has special properties. In contrast to the definition of scalar values of mass and length, to determine
the time, in addition to the scalar value, you also need to introduce the concept of time scale, which is of fundamental importance in astronomical research in the field of celestial mechanics. Astronomy deals with the issues of measuring and storing time, establishing a one-to-one correspondence between the measure of time and the observed astronomical (and, more recently, physical) phenomenon, which may be associated with some periodic and discretely counting process.

Astronomical systems, measuring ephemeris time, based on the phenomenon of the daily rotation of the Earth by astronomical observations of the passage of stars through the local celestial meridian, can be determined again for any time points in the past from existing archival observations of this kind in previous epochs. Ephemeris time scales correspond to the fundamental laws of Newton’s dynamics, are determined by the gravitational theory of the Earth’s motion in orbit around the Sun, developed by Newcomb, and agreed upon by the criterion of the best approximation of their difference over the entire duration of observations of the motion of celestial bodies.

Ephemeris of celestial bodies of the Solar system and the corresponding ephemeris time scales is used, among other observations, for high-precision timing of the periodic radiation of radio pulsars in regular observations at the BSA LPI Radio Telescope of the Pushchino Radio Astronomy Observatory. The estimated difference of Earth (geocentric) TT time and dynamic time reduced in EPM 2010 ephemeris and dynamic time in the barycenter of the TDB Solar system in the calculated interval of 1880-2020 does not exceed 1.6 ns [7]. This value can be interpreted as the upper limit of the instrument uncertainty of pulsar timing by modern astrometric means.

Used as a local clock the atomic time system, based on the process of electromagnetic radiation or energy absorbed during quantum transitions in atoms and molecules, provides only the frequency standard: it determines only a unit of time, but does not keep a continuous count of units required to determine the interval from any initial epoch in the past. Astronomical definitions of them are essential for the expression of epochs and reference points of time to them, since there are no artificial clocks that could continue their work indefinitely like celestial movements [8].

For an objective perception of time in space, the measurement of time in astronomy is based on the time scale, defined as outlined in [5]:

1) some material system with a continuous and stable movement and representing a certain measurable parameter \( P \), changing with time;

2) a theory giving the values of this parameter \( P \) in a function of the independent variable \( t \), called as time. The function \( P = f(t) \) should be unambiguous and allow for the distinction between its particular values. Then the time it can be expressed in terms of the measured values of the determining parameter \( P \) in the form \( t = \varphi(P) \). The scale, thus determined makes it possible to order events in the sense of “earlier” or “later” with respect to a certain time \( t_0 \), depending on whether there is \( t_0 > t \) or \( t_0 < t \), if it is the point in time at which the events occurred. Thus, when measuring time, it is necessary to strictly distinguish the point in time (or the epoch of the event) from the time interval (or time scale interval).

The problem of the materialization of time scales for measuring the fourth space-time coordinate is that in the dynamic equations of motion the time parameter \( t \) is just a calculated mathematical argument and is not an observable and directly measurable quantity that could be directly compared with the calculated values. It is clear that the solution of such problems cannot be limited only to locally selected points in space, it implies a transition to space as a whole, down to infinity, taking into account the orthogonality of time to such space, which is the same for all points of this space. The implementation of the principle of simultaneity of observed events, confirmed by the coordinate time scales, corresponds to the synchronous condition in inertial systems and, consequently, the equivalence of all physical processes for an observer in any
of them. It is obvious that local reference systems do not initially have such capabilities. Within the Solar system, the metric of the geometric world is determined by ephemeris which are the numerical solutions of the equations of motion of celestial bodies with respect to selected points — the barycenter of the Solar System and geocenter oriented in the space of the International Space System ICRS. The transition from local points in the equations of motion of celestial bodies in infinite physical space includes as a necessary condition the fulfillment of the principle of simultaneity in any inertial coordinate system within the entire space [9].

The law of propagation of electromagnetic disturbances in free space directly characterizes the properties of space and time. If we consider not the steady-state electromagnetic field, but the propagation of a sudden disturbance, then the most characteristic is the law of propagation of the wave front, that is, the law of motion of the surface separating the region already filled with electromagnetic disturbance from that region to which the disturbance has not yet reached. For the electromagnetic field, this form of the law follows directly from the Maxwell-Lorentz equation:

\[
\frac{1}{c^2} \left( \frac{d\omega}{dt} \right)^2 - \left[ \left( \frac{d\omega}{dx} \right)^2 + \left( \frac{d\omega}{dy} \right)^2 + \left( \frac{d\omega}{dz} \right)^2 \right] = 0
\]  

Equation (6), which is not related to the specifics of light waves and the electromagnetic field, is quite general: it refers to the extremely rapid propagation of a wave front of any nature. The equation expresses the properties of space and time mathematically, in the form of the following postulates [4]:

1) every point of the wave surface (wave front) moves in the direction normal to the surface with a constant speed equal to the speed of light;
2) from this follows the straightness of the beam, defined as the locus of the indicated points;
3) the equation reflects the fact that the geometry of space is Euclidean, and the space is homogeneous;
4) the equation contains a statement about the independence of the speed of light on the speed of the source.

It follows that the concept of an inertial reference system can now be generalized by including in its definition, in addition to the first Newton’s law, also the law of propagation of the wave front of an electromagnetic field. Thus, a generalized formulation of the Galilean principle of relativity is obtained, according to which all physical processes in inertial reference systems proceed in the same way, regardless of whether the system is stationary or it is in a state of uniform and rectilinear motion that is also generalized to translational motion. From this follows the absolute nature of time: the simultaneity of two events, even remote from each other, is an absolute concept that is independent of any additional conditions.

According to observations of electromagnetic radiation of pulsars in the radio frequency range, we established the property of the temporal and spatial coherence of periodic radiation of neutron stars, expressed by stable reproduction of the period and the intervals of the observed pulsed radiation, regardless of the epoch, duration and location of the observations, due to the monotonic slowing of rotation from the relatively weak, but continuously occurring dissipative losses of rotational energy of neutron stars [10].

The principal difference between the resonant dissipative celestial systems, whether planetary systems or rotating neutron stars, is a sufficiently high level of stored kinetic energy on the stage of their formation, which allows maintaining the stability of periodic spin-orbital motions for many years. It can be expected that a pulsar with an energy reserve that is many orders of magnitude greater than the energy of the spin-orbit resonance motions of the planets of the solar system is a unique galactic object with pronounced spin-resonance properties with a gradual monotonic deceleration of the angular velocity of rotation, whose periodic radiation in the form of an electromagnetic front (radio) wave propagates within galactic space, can fulfill the
role of a material system possessing continuous and stable rotation and representing a certain measurable parameter $P$ as a function of the independent variable of time $t$.

The timing data of pulsars B1919+21, B0809+74, B0834+06, B0329+54, B1822-09, B0531+21, J1509+5531, observed at the radio telescope BSA FIAN during 1978-2018, confirm the steady patterns of periodic pulsar radiation, expressed through the observed radiation parameters pulsars. Indeed, the numerical value of the observed period $P(t)$ is expressed as a function of the independent time variable $t$ on the observed rotation parameters — the period value $P_0$ for the initial epoch and the derivatives of the period $\dot{P}$, $\ddot{P}$:

$$P(t) = P_0 \pm \left( \dot{P} \cdot t + \frac{1}{2} \ddot{P} \cdot t^2 \right).$$

The deceleration index of all pulsars is within the numerical invariant $n = -(0.9 \pm 0.2)$, which indicates the monotony of the deceleration of rotation ($\dot{P} > 0, \ddot{P} > 0$) [11]. From the observed for about two decades and the predicted stability of the parameters of the rotation of the pulsars and, accordingly, the pulsar time scales based on them in the future, it can be concluded that the spatial and temporal parts fully express the expression (1.2), and the periodic radiation intervals calculated from the observed $P, \dot{P}, \ddot{P}$ parameters are analytical time scales within the time interval of about $10^6-10^7$ years typical for neutron stars.

5. Coordinate transfer of pulsar time scales in galactic space

In order to obtain a physical pulsar time scale at any point with given space coordinates, associated with the ephemeris of celestial bodies, it is sufficient to extend the topocentric pulsar time scale obtained analytically from observations of a pulsar on a radio telescope from the observation point to this space point. Formed from observations with a radio telescope analytical coordinate pulsar time scale, it extents to any coordinate system with a center given by three local geodetic coordinates of a certain material point, the position of which in the process of translational motion is determined by the ephemeris of celestial bodies within the Solar system or galactic space. This proves that the periodic radiation from the galactic source of the pulsar determines the local coordinate pulsar time scale by form-invariant equations in any inertial Galilean coordinate system [12]. Thus, by transferring the analytical pulsar time scale, obtained directly from the observed wave front of the radio telescope, the spatial reference systems are synchronized by the coherent radiation of the pulsar.

The rules for the transfer of coordinate pulsar time scales follow from the maximum, that is, in general, the homogeneity of the Galilean space, according to V.A. Fok [1], in which:

i: All points and moments of time are equal, which corresponds to the transformation consisting in shifting the origin $(x_0, y_0, z_0)$ and the start of the time count, determined by the period $P_0$ at the epoch of the initial observed pulsar radiation event at this point $(x_0, y_0, z_0)$:

ii: All directions are equal, which corresponds to the transformation consisting in the rotation of the coordinate axes, containing three parameters (three angles) and signifying the orthogonality of space to the time lines, the principle of simultaneity in all points of space as a whole with respect to the origin point $(x_0, y_0, z_0)$ and, accordingly, the epoch of the initial radiation event measured on the pulsar time scale:

iii: All inertial systems are equal. Equality of inertial systems corresponds to the transformation, consisting in the transition from one reference system to another, moving progressively relative to the original. This transformation contains three parameters (the three components of relative velocity), which do not affect the magnitude of the observed radiation parameters of the pulsar $P_0, \dot{P}, \ddot{P}$ and, accordingly, the invariance (equivalence) of analytical pulsar time scales for coordinate transformations.

Thus, with coordinate transformations, the physical parameters $P_0, \dot{P}, \ddot{P}$ of the observed process of periodic pulsar radiation for coinciding epochs of local time, expressed analytically through the observed rotation parameters, are the same in any inertial system. The formulas of
coordinate transformations of pulsar time scales in inertial systems have the following form:

\[ P(t) = P_0(x_0, y_0, z_0) \pm \left( \dot{P} \cdot t + \frac{1}{2} \ddot{P} \cdot t^2 \right); \quad t = P_0(x_0, y_0, z_0)N; \quad -\infty < N < \infty. \]  

(7)

\[ PT(N) = P_0N + \frac{1}{2}P_0\dot{P}N^2 + \frac{1}{6}(P_0^2\ddot{P} - 2P_0\dot{P}^2)N^3. \]  

(8)

Here: \((x_0, y_0, z_0)\) are coordinates of a material point - the origin of the coordinates of the inertial system thus selected; \(P_0(x_0, y_0, z_0)\) is the initial value of the observed period of radiation of the pulsar in this a point on the epoch of an arbitrarily chosen initial event, from which the coordinate time is measured on a pulsar time scale.

Thus, the initial period value \(P_0(x_0, y_0, z_0)\) is the only transfer parameter of the coordinate time scale \(PT(N)\) from one inertial system to another. It sets the origin of the local coordinate time in an inertial system, the beginning of which is given by the initial coordinates \((x_0, y_0, z_0)\). Pulsar time scale \(PT(N)\) determines the local coordinate time, the same at any point in space, corresponding to the Lorentz coordinate transformations, and as the observed physical process of wave front propagation in free space, synchronizing inertial coordinate systems, is indistinguishable from each of them [12].

Figure 1. The local uncertainty in measuring intervals of pulsar time and corresponding length of galactic space.

Figure 1 shows in logarithmic scale the magnitude of the uncertainty in measuring the emission intervals and, accordingly, the distances \(L\) in selected local points of space using pulsar time scales expressed analytically through the observed rotation parameters of the pulsar. According to the physical time intervals \(PT\) determined by the pulsar rotation parameters \(P_0, \dot{P}, \ddot{P}\) at the initial epoch of the selected radiation event, the local coordinate systems establish a numerical correspondence between the spatial \(L\) and the time \(PT\) of the components of the four-dimensional space-time interval (1). At the same time, the physical characteristics of the accuracy of the measured values are estimated at any allowable length within the galactic space of the order of \(10^5\) light years and duration within the estimated time interval of \(10^6–10^7\) years typical for neutron stars.

As follows from Figure 1, the estimated uncertainty of local measurements of \(PT\) intervals is in the range from 1 ns to \(10^{-5}\) ns, and the corresponding uncertainty in measuring length is in the range from 0.3 cm to \(3 \cdot 10^{-5}\) cm, with a tendency to gradually reduce errors as the duration of measurements increases. The spatial and temporal components of the interval (1) show mutual consistency over the entire temporal extension, which indicates the absence of non-uniformity of four-dimensional time-space, the constancy of the velocity of propagation of
the neutron star coherent radiation wave front. The estimates obtained are summarized the physical data of timing from observations of pulsars conducted on the BSA LPI radio telescope in an interval close to almost two decades (40 years or more) in combination with the data of the coordinate time intervals for the predicted rotation parameters consistent with the observed intervals.

Figure 2. Estimated measure of the achievable resolution of the pulsar time scale within galactic space.

Figure 2 shows the estimated logarithmic value of the pulsar time scale resolution $dPT$ in absolute and relative terms, within an interval compared to the lifetime of the pulsar. From Figure 2 it follows that with an increase in the duration of the interval of parametric approximation of intervals, a monotonous increase in the resolution of space-time measurements on the pulsar time scale occurs both in absolute terms due to a gradual increase in the accuracy of the approximation of the observed intervals according to the observed rotation parameters, and due to an increase in the length of the series approximated coherent radiation intervals.

The estimated span $dPT$ of minimum recognizable on the pulsar time scale is $10^{-12}–10^{-14}$ s in the extent from several years to several decades and then gradually decreases, reaching limit values of the order of $10^{-16}–10^{-17}$ s in the extent compared to the lifetime of a pulsar. The relative size of the resolved interval is within approximately $10^{-19}–10^{-29}$.

6. Conclusion

The study of the dynamics of celestial bodies and astronomical phenomena is directly related to the measurements of time. For an objective perception of time in space, the measurement of time is based on the pulsar time scales $PT$, associated with a continuous and measurable rotation period $P$ of galactic neutron stars, changing with time, so that variable $PT$ it can be expressed in the form $PT = f(P)$. According to observations of electromagnetic radiation of pulsars in the radio frequency range, we established the property of the temporal and spatial coherence of periodic radiation of neutron stars, expressed by stable reproduction of the period and the intervals of the observed pulsed radiation, regardless of the epoch, duration and location of the observations, due to the monotonic slowing of rotation from the dissipative loss of rotation energy neutron stars [10].

In order to obtain a physical pulsar time scale at any point with given space coordinates, associated with the ephemeris of celestial bodies, it is sufficient to extend the topocentric pulsar time scale obtained analytically from the timing of a pulsar in any coordinate system with a center given by three local geodetic coordinates of a certain material point of space $(x_0, y_0, z_0)$.
the position of which is determined by the ephemeris of celestial bodies within the solar system or galactic space. Thus, by transferring the analytical pulsar time scale, obtained directly from the observed wave front of the radio telescope, the spatial reference systems are synchronized by the coherent radiation of the pulsar [11].

Thus, the initial period value \( P_0(x_0, y_0, z_0) \) is the only transfer parameter of the coordinate scale \( PT(N) \) from one inertial system to another. It sets the origin of the local coordinate time in an inertial system, the same at any point in space, corresponding to the observed physical process of wave front propagation in free space, synchronizing inertial coordinate systems, which are indistinguishable from each of them [12].

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