Effect of magnetic field configuration on Faraday MHD accelerator

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Abstract. This works aims at the study of a Faraday electric accelerator with the plasma of argon used as the working gas, which is pre-ionized by nuclear energy. The electron density of plasma is above $10^{20}/m^3$. Different configurations of magnetic field are numerical studied to obtain the influences on the plasma ionization and flowing characteristics. A relatively higher velocity and electron number density are obtained in monotonically decreasing magnetic field generated by magnet with a shorter length. When the plasma flows through the magnetic field generated by 75-mm magnets, the outflow velocity is 2040 m/s, which is higher than 2016 m/s when the plasma flows through the magnetic field generated by 100-mm magnets.

1. Introduction
The concept of electric propulsion has been gaining increasing attentions since it was brought up by Tsiolkovsky in 1903 and Goddard in 1906, respectively [1]. The principle of electric propulsion systems is the acceleration of propellant by using electrical fields, currents, and magnetic fields [2]. Four electric propulsion technologies are mainly studied today including electro-thermal [3], electromagnetic, electro-static and the mixed type. Acceleration of charged ions using static electric fields [4, 5] is categorized into electrostatic acceleration. Current carrying quasi-neutral plasma using electromagnetic Lorentz forces [6, 7] is named electromagnetic acceleration. There are so far two major concerns on the studies of the above-mentioned electromagnetic, electrostatic and mixed type propulsion technologies. One of the concerns is the way and effectiveness to realize the ionization of working gas. The other critical issue is the ejection of ionized plasma by using the electrical power. The duration of voyage in deep space exploration is required to be as short as possible due to voyage influence, survival needs, safety and endurance of the astronauts. For effective operation of the manned deep space exploration and orbit manoeuvring, it is necessary to minimize the fuel load and increase load capacity. Furthermore, high thrust propulsion is required to shorten the time of orbit manoeuvring and voyage. The thrust of most current electric propulsion technologies is infinitesimal, the value of which ranges from tens to hundreds of mN. To alleviate the contradiction, high-power electric propulsion is gaining increasing interests for shortening the duration of orbit manoeuvring and space travel, as well as carrying enough propellant for voyage.
With electrostatic-type propulsion, Hall effect thrusters are being constantly improved and developed towards high power and thrust. Under the condition of low magnetic field strength and 200-500 V discharge potential, the testing of NASA-457Mv2 stationary plasma thruster showed the achievement of the maximum thrust of 2.5 N with a maximum power of 50 kW [8]. A configuration of nesting multiple discharge channel was developed by NASA and investigated by groups of Jacobson [9] and McVey [10]. The proof of concept work on the X2 NHT (Nested Hall thruster) showed that such a configuration met or exceeded the performance of conventional single-channel thrusters [11]. A 100-kW class laboratory-model NHT known as the X3 was researched by University of Michigan in order to extend the NHT concept to higher operating powers with a wider throttling range [12].

Magnetoplasma dynamic thrusters (MPDT) produce thrust through the acceleration of ionized propellant species using electromagnetic force. The propellant, which initially may be a gas, liquid, or solid state, is ionized by the current arc in MPDT. Paccani et al. discussed the operation and performance of a quasi-steady, ablative MPDT with four different polymer propellants [7]. The highest thrust and thrust-to-power ratio were provided by Tyflon, whereas Hyflon offered the lowest values of these working parameters. Choueiri et al. [13] carried out the measurements of thrust for various mass flow rates for propellant gas, such as argon, xenon, hydrogen and deuterium. Corresponding to the mass flow rate of 0.5 to 6 g/s, the thrust can be obtained between 5 and 120 N. Miyazaki et al. [14] conducted experiments for thrust performance evaluation at an input power of 1 MW. The best performance of stable operating condition is obtained for an Argon propellant with a mass flow rate at 1.8 g/s.

To summarize, many experts have widely carried out research on thrusters propulsion performance under different acceleration principles, various channel structures and sorts of working fluids. However, most of those studies are realized under a uniform magnetic field or a single magnetic field configuration. For performance optimization, this paper mainly discussed the effect of different magnetic field configurations on a Faraday MHD electromagnetic accelerator. Noble gas Argon is used as the working medium assumed to be heated and pre-ionized by the nuclear energy before flowing into the channel and being accelerated by the Lorenz force through the applied electric field and magnetic field.

2. Numerical model

Fig. 1 shows schematic of the simulation domain with the channel active length at 200 mm. Ten pairs of electrodes with width of 10 mm are placed on the right and left surface of the channel. Channel height and width increase along the flow direction with a divergence angle of 1 degree. Inlet width and height are 20 mm and the outlet ones are 27 mm. The magnetic field is generated by magnets integrated on the upper and lower sides. The length of the magnet is $M_x$, the width is $M_y$ and the distance between the magnets is $H$.

![Figure 1. Schematic of simulation domain.](image-url)
Pure argon is used as working gas in the simulations. The non-equilibrium argon plasma in the channel consists of ions, electrons and atoms under a two-temperature model [15, 16] which is described by the combination of state equations and Maxwell equations. Naver-Stokes equations include Lorentz force term and energy equation with energy exchange term are solved by Large Eddy simulation (LES) method. Non-slip and constant-temperature wall condition is used. Segmented Faraday connection is used for the electrodes. The intensity of electrical field is 5000 V/m between the anode and cathode. Table 1 shows the working conditions of the seed-free argon plasma used in the simulations. The verification of the simulation code has been accomplished in our previous work [17].

| Table 1. Conditions used in simulations. |
|-----------------------------------------|
| Working gas | Argon |
| Total inflow pressure | 0.64 mPa |
| Total inflow temperature | 6701 K |
| Inlet electron temperature | 8000 K |
| Magnetic flux density | 0.5-2.1 T |

2.1. The equations for heavy particles

a. Continuity equation:

Continuity equation can be written as follows

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \]  \hspace{1cm} (1)

where \( \rho \) is gas density, \( \mathbf{u} \) is the flow velocity.

b. Momentum equation:

\[ \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \mathbf{\tau} \]  \hspace{1cm} (2)

The notation \( \mathbf{J} \) is the current density, \( \mathbf{B} \) is the magnetic flux density, \( p \) is the static pressure, \( \mathbf{\tau} \) is the viscous stress.

c. Energy equation:

\[ \frac{\partial E}{\partial t} + \nabla \cdot (E_i + p) \mathbf{u} = \mathbf{J} \cdot \mathbf{E} - \mathbf{\nabla} \cdot \mathbf{q} + \nabla \cdot (\mathbf{\tau} \mathbf{u}) \]  \hspace{1cm} (3)

where \( \mathbf{E} \) is the electric field, \( E_i \) is the total energy, \( c_v \) is the constant volume specific heat, \( T \) is the static temperature, \( \mathbf{q} \) is the conductive heat flux.

2.2. The equations for ions and electrons

a. Conservation of ion number density:

\[ \frac{\partial n_i^+}{\partial t} + \nabla \cdot n_i^+ \mathbf{u} = n_i^+ = k_r n_e n_i - k_r n_e n_i^+ \]  \hspace{1cm} (4)

where \( n_i^+ \) is the number density of argon ions, \( n_e \) is the number density of electron, \( n_i \) is the number density of argon atom. The three-body recombination rate coefficient \( k_r \) is determined by

\[ k_r = k_{rh} k_m / (k_{rh} + k_m) \]  \hspace{1cm} [16], where \( k_{rh} = 1.09 \times 10^{-20} T_e^{-9/2} \) and \( k_m = 3.33 \times 10^{-44} \left( \frac{135300}{T_e} + 2 \right) \exp \left( \frac{47800}{T_e} \right) \).

The ionization rate coefficient \( k_f \) is derived on the basis of the principle of detailed balance with the Saha equilibrium: \( k_f n_i n_e = k_i n_e \) \( k_e \), \( k_f = k_f \left( \frac{2 \pi m_e k T_e}{h^2} \right)^{3/2} \exp \left( \frac{-\phi_i}{k T_e} \right) \), where \( \phi_i \) is the statistical
weight of the ground state of the ion, \( g_0 \) is the statistical weight of the ground state of the neutral argon atom, \( \varepsilon_i \) is ionization potential of argon atom, the subscripts \( e \) and \( i \) and the superscript + denote the electrons, neutral atoms and ionized particles, respectively.

b. Energy equation for electron:

\[
\frac{\partial U_e}{\partial t} + \nabla \cdot (U_e \mathbf{u}_e) = \left[ \frac{|\mathbf{J}_e^f|}{\sigma} - p_e \nabla \cdot \mathbf{u}_e - 3n_e k(T_e - T) \sum_{i} m_i \mathbf{v}_{i} \right] \tag{5}
\]

where, \( U_e \) is the electron energy, \( u_e \) is the velocity of electrons, \( m_e \) is the mass of electron, \( p_e \) is the electron pressure, \( k \) is Boltzmann constant, \( T_e \) is electron temperature, \( \varepsilon_i \) is the ionization energy. The LES-Smagorinsky model is applied as turbulence model to solve the equations of (1), (2) and (3) for the description of the fluid flow.

2.3. Governing equations for electrical-magnetic

a. Generalized Ohm’s law:

\[
\mathbf{J} + \beta \frac{\partial \mathbf{B}}{\partial t} \mathbf{J} \times \mathbf{B} = \sigma (\mathbf{E} + \mathbf{u} \times \mathbf{B}), \quad \sigma = \frac{e^2 n_e}{m_e v_{ch}} \tag{6}
\]

In the equation, \( \mathbf{v}_{ch} = \sum_{h} n_h Q_{eh} C_e \) is the average momentum transfer collision frequency for an electron \( e \) with a heavy particle \( h \), where \( Q_{eh} \) is the energy-averaged momentum transfer cross section, \( C_e = \frac{8kT_e}{\pi m_e} \) is the mean electron velocity for a Maxwellian distribution and \( \sigma \) is the electrical conductivity.

b. Maxwell equations:

\[
\nabla \times \mathbf{E} = \beta J \frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = -\frac{\partial \mathbf{D}}{\partial t}, \quad \nabla \cdot \mathbf{D} = \rho_e \tag{7}
\]

Since the charge is assumed as neutrality and magnetic Reynolds number is quite small, the Maxwell equations are simplified as follows:

\[
\nabla \times \mathbf{E} = 0, \quad \nabla \cdot \mathbf{J} = 0 \tag{8}
\]

In equation (8), \( \mathbf{E} = -\nabla \Phi \) is the electric field, where \( \Phi \) is the electrical potential.

c. Equations of external magnetic field

Distribution of external magnetic field \( \mathbf{B}_0 = (B_{0x}, B_{0y}, B_{0z}) \) is governed by Biot-Savart law:

\[
B_{0x} = \gamma \sum_{k=1}^{1} \sum_{i=1}^{1} \sum_{j=1}^{1} (ijk) \tanh \left[ \frac{y-0.5jM_y}{r(i,j,k)} \right] \tag{9}
\]

\[
B_{0y} = \gamma \sum_{k=1}^{1} \sum_{i=1}^{1} \sum_{j=1}^{1} (ijk) \tanh \left[ \frac{y-0.5iM_z}{r(i,j,k)} \right] \tag{10}
\]

\[
B_{0z} = \gamma \sum_{k=1}^{1} \sum_{i=1}^{1} \sum_{j=1}^{1} (ijk) \left[ -\arctan \left( \frac{(x-0.5iM_x)(y-0.5jM_y)}{(z-0.5kH)r(i,j,k)} \right) \right] \tag{11}
\]

\[
r(i,j,k) = \left[ (x-0.5iM_x)^2 + (y-0.5jM_y)^2 + (z-0.5kH)^2 \right]^{1/2}
\]
Equations (4) and (5) are solved by means of the user defined functions code. The elliptic equations for the potential derived from equations (6) and (7) are solved by MHD model of Fluent. The set of the above equations is solved numerically by means of the first-order upwind scheme.

3. Results and discussions

The induced electric potential $U = u_x B_{0y} d$ ( $B_{0z}$ is the magnetic flux density, $d$ is the distance of electrodes) is produced by the conductive flow under the magnetic field. As both $B_{0z}$ and $d$ are constant, the induced electric potential increases with the velocity. Hence, the induced potential in the region with higher exit velocity may exceed the external potential and lead to deceleration of the flow. Appropriate reduction of magnetic flux density in higher velocity regions may alleviate the contradiction. The electromagnetic acceleration motion is numerically studied in this paper under different magnetic field intensities.

![Figure 2. Distribution of external magnetic field along the center line of X direction.](image1)

![Figure 3. Distribution of current density of y direction along the center line of X direction.](image2)

The distribution of the magnetic flux density along the center line of the channel is shown in Fig. 2. The solid line represents the magnetic field generated by a short magnet ($M_x=75$ mm). The dashed line represents the magnetic field generated by a long magnet ($M_x=100$ mm). For convenience, we define the simulation under the short magnets condition as case 1 and the simulation under the long magnets condition as case 2. Both of the external magnetic fields are confined to the 15-160 mm region.

3.1. The characteristics of the plasma

From equation (6), the current density of $y$ direction is obtained as $J_y = \sigma (E_y - u_x B_{0y})$. Even the velocity increases, the current density $J_y$ is still positive in case 1 due to the insignificant induced potential $u_x B_{0y}$ in the decay magnetic field. The intensity of the magnetic field in case 2 exhibits small changes. On the contrary, the electric field $u_x B_{0y}$ produced by the induced potential $U = u_x B_{0y} d$ is higher than $E_y$ generated by the external potential, which leads to a negative value of the current density (from 85 mm to 160 mm in Fig 3). The distribution of electric power $P_g = J \cdot E$ along the center line of the mainstream has the same trend with the current density. Available electric power in case 1 is higher than that in case 2. Therefore, the energy for ionization in case 2 is less than that in case 1.

The pre-ionization electron number density of gas at the inlet is about $1 \times 10^{20}/m^3$. The electron number density shown in Fig. 4 changes between $0.975 \times 10^{20}$ and $1.03 \times 10^{20}$/m$^3$ in two cases. The electron number density in case 2 is smaller than that in case 1 due to lack of the ionization energy. From Fig. 5, the electric conductivity varies from 13 to 106 /ohm-m in two cases. With no magnetic field, case 1
and case 2 share the same trend of electric conductivity. But the distinction of electric conductivity is significant in the electromagnetic region, where the value is smaller in case 1 than that in case 2.

![Figure 4](image1.png) **Figure 4.** Distribution of electron number density along the center line of X direction.

![Figure 5](image2.png) **Figure 5.** Distribution of electric conductivity along the center line in X direction.

### 3.2. The characteristics of the flow

The acceleration of mainstream velocity \( u_x \) mainly depends on the generated Lorentz force. Meanwhile, the intensity of the current density \( J_y \) mainly varies with \( u_x \). Lorentz force \( F_x = J_y B_0 \) determines the distribution of electric conductivity of the flow, which plays an important role to the distribution of current density. In addition, Lorentz force \( F_x \) varies with the current density \( J_y \). Fig 6 is the contour of current density in two cases. With the increasing of velocity in the channel in both cases, current density in case 2 declines to be negative in the range of 85 mm to 160 mm, Fig 6 (b). The corresponding Lorentz force in this range becomes negative in case 2 (Fig 7(b)), which stands for reverse direction to the flow. As the value of \( B_0 \) in case 1 decreases monotonically from 1.98 to 0.75 T, the current density stays in positive although the velocity increases, hence, the Lorentz force is still with the same direction of flow from Fig 7(a).

![Figure 6](image3.png) **Figure 6.** Contour of current density at Z=0 plane: (a) case 1, (b) case 2.

![Figure 7](image4.png) **Figure 7.** Contour and vector of Lorentz force at Z=0 plane: (a) case 1, (b) case 2.

Fig 8 shows that Lorentz force stays in positive due to the positive value of the current density \( J_y \) along the center line of X direction in case 1. The Lorentz force declines to be negative at the location of 85 mm in case 2. However, the Lorentz force is consistently positive near the electrodes areas because of the injection of electric energy. It is indicated that the negative current cannot form a path
out of the electrode although Lorentz force is in opposite direction of the flow in case 2 resulted by greater value of the external potential than the induced potential. Therefore, the plasma in case 2 is ultimately accelerated.

![Figure 8. Distribution of Lorentz force in x direction along the center line of X direction.](image)

![Figure 9. Contour and streamlines of electric current at X=140 mm plane: case 2.](image)

Fig 9 shows the Eddy streamlines of the electric currents which is formed by the induced potential suppressed by external potential. As previously described, electric currents generated by the external potentials still flow in and out of the electrodes and detour past the eddy of electric currents. As a result, the plasma in case 2 is still accelerated, but the acceleration effect is weakened by the induced potential, which offsets part of the external electric energy. The acceleration effect of two cases can be observed in Fig 10. Obvious separation of two lines appears at the location of 85 mm. The velocity at the outlet are 2040 m/s in case 1 and 2016 m/s in case 2, respectively.

![Figure 10. Distribution of velocity in X direction along the center line.](image)

4. Summary
Influences of different magnetic field configurations on Faraday MHD accelerator are numerically studied in this paper. The noble argon plasma is used as working gas and its electron number density is about $1 \times 10^{20}/m^3$ at the inlet. Ionization and acceleration characteristics are mainly discussed under two magnetic fields, which are generated by long and short length of magnets, respectively. Both the applied external electric fields are 5000 V/m. The ionization and acceleration effects are weakened due to the offset of larger induced potential $u \times B$ appeared in the magnetic field generated by long magnets. The electron number density varies between $0.975 \times 10^{20}/m^3$ and $1 \times 10^{20}/m^3$ at magnetic field generated by 100 mm length magnets. The electron number density is kept over than $1 \times 10^{20}/m^3$ at magnetic field generated by 75-mm magnets. With 75-mm magnets, the exit velocity is 2040 m/s, which is higher than 2016 m/s in field generated by 100-mm magnet. Further research will be carried
out to apply different external potentials under different magnetic field configurations in order to optimize acceleration effect.

5. References

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