Instantons and Polarized Structure Functions

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Abstract

The contribution of the quark-quark and quark-gluon interaction induced by instantons to the valence quark and proton spin-dependent structure functions \( g_1(x, Q^2) \) is estimated within the instanton liquid model for QCD vacuum. It is shown, that this interaction leads to a rather large violation of the Ellis-Jaffe sum rule.

\footnote{The talk presented at the Workshop "Deep Inelastic Scattering off Polarized Targets: Theory Meets Experiment", September 1997, DESY-Zeuthen}
1 Introduction

One of the solution of the famous ”spin crisis” (see recent review [1]), is an assumption of the large positive gluon polarization inside a nucleon [2]. Indeed, one of the NLO analyses of the polarized DIS world data on $g_1(x, Q^2)$, which was performed to extract the polarized parton densities in a nucleon, has shown some indication for a positive value of the gluon polarization [3]. However, another NLO fit leads to the conclusion that this result may be sensitive to the input shapes of the polarized parton distribution at low normalization point [4].

The different calculations of the gluon polarization inside nucleon have been performed (see [5]) and the contradiction between different approaches has been found, not only in the absolute value of the gluon polarization but even in its sign. For example it has been shown recently [8] that the instanton model for nonperturbative effects in QCD [9], [10] rules out the positive value of the gluon polarization. Therefore, it is possible that the gluon solution of the ”spin crisis” is not the case.

Another way to resolve the proton spin problem is to take into account the quark depolarization induced by nonperturbative vacuum fluctuations of the gluon fields, so-called instantons [6], [7]. The instantons describe the subbarrier transitions between different classical QCD vacuum states, that have different values of quark helicities. Therefore, account of quark interaction with instantons gives the direct way to obtain the magnitude of the quark helicity non-conservation in QCD.

In this article we estimate the contribution from quark-quark [12] and quark-gluon interaction [15] induced by instantons to the valence quark and proton $g_1(x, Q^2)$ structure functions.

2 Quark-Quark and Quark-Gluon Interaction induced by Instantons

The instanton model for QCD vacuum is widely used now in the description of the nonperturbative effects in strong interaction (see reviews [9], [10]). The existence of instantons leads to a nonperturbative quark-quark and quark-gluon interaction through the QCD vacuum, which has the following form [12], [14]:

$$
\mathcal{L}_{\text{eff}} = \int \prod_q (m_q \rho - 2\pi^2 \rho^3 \bar{q}_R(1 + \frac{i}{4} \tau^a U_{a\mu} \bar{\eta}_{a\mu} \sigma_{\mu\nu})q_L) \cdot \exp \left( - \frac{2\pi^2}{g^2} U_{a\mu} \eta_{a\mu} G_{\mu\nu}^a d\rho \right) \frac{d\rho}{d\rho} + R \leftarrow L,
$$

where $\rho$ is the instanton size, $\tau^a$ are the matrices of the $SU(2)_c$ subgroup of the $SU(3)_c$ colour group, $d\rho(\rho)$ is the density of the instantons, $d\rho$ stands for integration over the instanton orientation in colour space, $\int d\rho = 1$, $U$ is the orientation matrix of the instanton, $\bar{\eta}_{a\mu\nu}$ is the numerical t’Hooft symbol and $\sigma_{\mu\nu} = [\gamma_{\mu}, \gamma_{\nu}] / 2$.

From Eq. (1) one can obtain an effective quark-quark t’Hooft interaction [12] which in the framework of instanton liquid model reads [9], [13]

$$
\mathcal{L}^{(2)}_{\text{eff}} = F(k_1^2, k_2^2, k_3^2, k_4^2, \rho_c) \frac{4\pi^2 \rho_c^2}{3} \sum_{i \neq j} (\bar{q}_R(k_1)q_L(k_2)\bar{q}_j R(k_3)q_j L(k_4)) \cdot (1 + \frac{3}{8} (1 - \frac{3}{4} \sigma_{\mu\nu}^i \sigma_{\mu\nu}^j) \lambda_{a}^{ia} \delta_{a}^{ia} + (R \leftarrow L)),
$$

(2)
where $F(k_1^2, k_2^2, k_3^2, k_4^2, \rho_c)$ is the form factor, which is related to the Fourier transformation of the quark zero modes in instanton field, $\rho_c \approx 1.6 GeV^2$ is an average instanton size in vacuum, $i, j = u, d, s$.

Recently, it was shown that from Eq. (1) a new type of the nonperturbative quark-gluon interaction can be obtained. This interaction has the form of anomalous chromomagnetic quark-gluon interaction [15]

$$\Delta L_A = -i \mu_a \sum_q \frac{g}{2m_q^*} \bar{q} \sigma_{\mu \nu} t^a q G_{\mu \nu}^a,$$

(3)

where $m_q^* = 2\pi^2 \rho_c^2 < 0 | \bar{q}q | 0 > /3$ is a quark mass in the instanton vacuum.

The value of the quark anomalous chromomagnetic moment in the liquid instanton model is

$$\mu_a = -\frac{f \pi}{2\alpha_s},$$

(4)

where $f = n_c \pi^2 \rho_c^4$ is the so-called packing fraction of instantons in vacuum. The value of $n_c$ is connected with the value of the gluon condensate by the formula

$$n_c = \langle 0 | \alpha_s G_{\mu \nu} G_{\mu \nu}^a | 0 \rangle /16 \pi \approx 7.5 \times 10^{-4} GeV^4.$$

(5)

The following estimate for the value of the anomalous quark chromomagnetic moment has been obtained for $\rho_c = 1.6 GeV^{-1}$ in [15]

$$\mu_a = -0.2.$$

(6)

The principal difference between the instanton induced interaction (1), (2) and the perturbative quark-gluon vertex is the large quark helicity flip at instanton vertex, namely $\Delta \Sigma = -2N_f$. Therefore, this interaction can be responsible for a rather strong violation of Ellis-Jaffe sum rule [16] for the first moment of the spin-dependent structure function $g_1^q(x, Q^2)$.

### 3 Instanton Contribution to Valence Quark and Proton Structure Functions

The graphs which give the contribution to the structure functions from quark-quark interaction (2) are presented in Fig.1.

We calculate the contribution of this interaction to the valence quark spin-dependent structure function $g_1^q(x, Q^2)$ by using the projection of the imaginary part of the forward Compton scattering amplitude $T_{\mu \nu}$

$$g_1^q(x, Q^2) = -i \epsilon_{\mu \nu \rho \sigma} p^\rho q^\sigma \text{Im} T_{\mu \nu} / p.q,$$

(7)

where $p$ is the momentum of the valence quark in nucleon. The straightforward calculation of the contribution of the diagram given by Fig.1b leads to the result

$$g_1^{q,q}(x, Q^2) = \frac{e_q^2 \rho_c^4}{128} \int_0^{(x - 1)/4x} dk_1^2 \int_{-k_2^2}^{-\infty} dk_2^2 (k_1^2 + k_2^2(1 - x)) F^2(\frac{|k|\rho_c}{2}),$$

(8)

The calculation has shown that the contribution of the diagram in Fig.1a is very small and therefore can be neglected. The terms which are proportional to the product of the colour matrix $t^a$ in (2) have $1/N_c$ suppression factor and give the very small contribution as well.
Figure 1: The instanton contribution to the quark distribution functions: a) the contribution to the sea quark distribution function; b) the contribution to the valence quark distribution function. The label $I$ ($\bar{I}$) denotes instanton (antiinstanton).

where

$$F(z) = z \frac{d}{dz} [I_0(z)K_0(z) - I_1(z)K_1(z)],$$

(9)

and $x = Q^2/2p.q$.

The graph which is responsible for the contribution from the quark-gluon chromomagnetic interaction (3) to structure functions is presented in Fig.2. The calculation of the contribution of diagram in Fig.2 leads to the result

$$g_{1g}^q(x, Q^2) = -\frac{e_q^2}{8} |\mu_a| \rho_c^2 \int_0^{Q^2/(1-x)} \frac{d k_{\perp}^2}{(1 - \frac{4xk_{\perp}^2}{(1-x)Q^2})},$$

(10)

where $k_{\perp}^2 = k_{\perp}^2/(1-x)$, and the relation $\alpha_s \mu_c^2/m_a^2 \rho_c^2 = 3\pi|\mu_a|/8$ has been used.

The very interesting feature of the instanton contributions (8) and (10) is their specific $Q^2$ dependence. At small $Q^2 \ll 1/\rho_c^2$ they are proportional to $Q^2$ and for $Q^2 \gg 1/\rho_c^2$ they are constant. Therefore $Q^2$ dependence of the instanton contribution to polarized structure functions should be different from $\text{Log}(Q^2/\Lambda^2)$ evolution of the perturbative gluon corrections to DIS structure functions. The fundamental reason for this feature is the quark spin-flip induced by instanton vertex, which gives the extra powers of $k_{\perp}$ to the matrix element for forward Compton scattering amplitude. As result, at $Q^2 = 0$ the instanton contribution to $g_1(x, Q^2)$ is zero.

In the large $Q^2$ limit and small $x$ the contribution to $g_{1g}^q(x)$ from quark-quark interaction (Fig.1b) has the anomalous $1/x^2$ behavior. This behavior comes from the point-like instanton vertex (4), (2) which leads to very fast growth of the $\bar{q}q$ cross section induced by instantons with increasing the energy $S = (p - k)^2$. At high $S$ the contribution of instantons should be small because the perturbative QCD should work in this region. The imaginary part of the lower part of the diagram Fig.1b is proportional to a sum of the imaginary parts of the correlators of the pseudoscalar $j = \bar{q}\gamma_5 q$ and scalar $j = \bar{q}q$ currents ($s = \sqrt{S}$) (see the structure of Lagrangian $\mathcal{L}$).
that have been analyzed in QCD sum rule approach, taking into account direct instanton contribution \[17,9\]. Therefore, the threshold value for the energy \(S_0\) for cut-off of the instanton contribution should approximately equal to the value of duality interval in QCD sum rule for nonet of the pseudoscalar and scalar mesons which is \(S_0 \approx 2 GeV^2\) \[7\].

The final formula for the contribution from quark-quark interaction \(g_1^q(x)\) in Bjorken limit \(Q^2 \to \infty\) can be written in the following form

\[
g_1^q(x, Q^2) = \frac{e_q^2 \rho_c^4}{128} \int_0^{S_0} dS \int_{\frac{S_0}{s_a}}^{\infty} dk^2 S(xS - k^2(1 - x)) \frac{k^4 F_s^2(\frac{k \rho_c^2}{2})}{k^4},
\]

(12)

Due to a cut-off in \(S\), the \(1/x\) divergence is absent now and instanton contribution to first moment of \(g_1(x)\) is finite.

In the same limit the contribution to \(g_1^q(x)\), due to quark-gluon chromomagnetic interaction, reads

\[
g_1^q(x) = -\frac{e_q^2}{4} |\mu_a|.
\]

(13)

The sign of both contributions is negative and comes from the negative quark polarization induced by instantons inside proton.

To calculate the contribution to proton structure function, the simple convolution model for structure function has been used

\[
g_1^p(x) = \sum_q \int_1^1 \frac{dy}{y} g_1^q\left(\frac{x}{y}\right) \Delta q_V(y),
\]

(14)

where \(\Delta q_V(y)\) are the initial valence quark polarizations, which were taken in the form

\[
\Delta u_V(x) = 3.7(1 - x)^3, \quad \Delta d_V(x) = -1.3(1 - x)^3,
\]

(15)
and normalized to the experimental data on the weak decay coupling constants of hyperons

\[ g_A^3 = \Delta u_V - \Delta d_V = 1.25; \quad g_A^8 = \Delta u_V + \Delta d_V = 0.6. \]  \hspace{1cm} (16)

In Fig.3 the result of the calculation of the contribution from quark-quark interaction induced by instanton to \( g_1^p(x) \) in the region of Bjorken variable \( x > 0.0001 \) is presented.

\[ \int_0^1 dx g_1^{p,q}(x) = -0.007. \]  \hspace{1cm} (17)

This value is approximately one fourth of the observed violation of the Ellis-Jaffe sum rule [18]. The remaining part of the violation can be related to the contribution from the anomalous quark-gluon interaction induced by instantons.

In Fig.4 the result of the calculation of the contribution to \( g_1^p(x) \) structure function which comes from the quark-gluon chromomagnetic interaction induced by instantons is shown. This contribution is also negative and has a more hard \( x \) dependence than the contribution from the quark-quark interaction. Therefore by taking into account the quark-gluon chromomagnetic interaction induced by instantons we can explain the decrease of the \( g_1^p(x) \) structure function at large Bjorken variable \( x \).

The contribution to the first moment of \( g_1^p \) from this interaction is

\[ \int_0^1 dx g_1^{p,g}(x) = -0.019. \]  \hspace{1cm} (18)
The total contribution from both interactions is $\delta I_{\text{inst}}^p = -0.026$. This number one can compare with the modern experimental data on the value of the violation of the Ellis-Jaffe sum rule $\delta I_{\text{exp}}^p = -0.02 \div -0.04$ [18]. Thus, by taking into account the accuracy of the available experimental data and some ambiguities in the extrapolations of the $g_1(x)$ to very low $x$ region in the current experiments, we can conclude that the instanton model gives a rather good description of the the Ellis-Jaffe sum rule violation for proton. The prediction of the instanton model for the neutron $g_1$ structure function is very sensitive to the details of the violation of the $SU(6)$ symmetry for the valence quark distribution function and will be discussed elsewhere.

4 Summary

In summary, the instanton induced quark-quark and quark-gluon interaction leads to a large negative contribution to the proton spin-dependent structure function $g_1^p(x, Q^2)$. This contribution allows us to explain the observed violation of the Ellis-Jaffe sum rule.

Acknowledgements

The author is thankful to M.Anselmino, J.Blümlein, A.E.Dorokhov, A.V.Efremov, R.L.Jaffe, E.Leader, E.Reya, and T.Morii for helpful discussions.

This work was supported in part by the Heisenberg-Landau program and by the Russian Foundation for Fundamental Research (RFFR) 96-02-18096.
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