Analysis of one dimension migration law from rainfall runoff on urban roof

Chen Weiwei

Yellow River Institute of Hydraulic Research, Key Laboratory of Yellow River Sediment Research, the Ministry of Water Resources, Zhengzhou 450003, China

Abstract: Research was taken on the hydrology and water quality process in the natural rain condition and water samples were collected and analyzed. The pollutant were included SS, COD and TN. Based on the mass balance principle, one dimension migration model was built for the rainfall runoff pollution in surface. The difference equation was developed according to the finite difference method, by applying the Newton iteration method for solving it. The simulated pollutant concentration process was in consistent with the measured value on model, and Nash-Sutcliffe coefficient was higher than 0.80. The model had better practicability, which provided evidence for effectively utilizing urban rainfall resource, non-point source pollution of making management technologies and measures, sponge city construction, and so on.

1. Introductions
The urban rainfall runoff pollution on surface was one of the main sources of urban water environment pollution. From the producing of runoff pollution and delivery process, pollutants’ motion process was more complicated because the randomness of factors such as precipitation, rainfall duration, etc. In the aspect of model building, knowledge about pollutants changing migration law on surface was not comprehensive enough, causing the simulated result uncertainty of model was higher. Since the 1960s, foreign countries in rainfall runoff pollution changing process and characteristics, the degree of impact on receiving water, rainfall erosion effect in different kinds of underlying surface condition, model building based on the big data, using 3S coupled application carried out research, and pointed out the seriousness of urban rainfall runoff pollution on surface and presented runoff pollution control management measures like LID, BMPs technology, etc.

Since the 1980s, China started to research field about urban rainfall runoff pollution and made much progress in hydrology and water quality process of rainfall runoff, runoff pollution characteristic, analysis of impact factors, and quantitative loading calculation of non-point source pollution in different kinds of underlying surface condition.

The article aiming at rainfall runoff process by separated system rainwater pipe network in Xinxiang city made experiments, according to measured data considering water quality and quantity hygrograph of rainfall pipe mouth on roof, one dimension migration model was built for pollutants to road gully on surface, then instance analysis was made which provided support for urban runoff pollutant management, quantitative loading calculation of non-point source pollution, sponge city construction, and so on.

2. Carrying out experiment about rainfall runoff on urban roof
The test point was set in building roof of Yellow River Institute of Hydraulic Research, laying pitch SBS waterproof material. Examination area was rectangle, whose length and width were 7.50, 18.05m, respectively. Longitudinal and transverse gradient were 1.0%, 0.3%, respectively. The building wall...
laid the rainfall pipe and collecting rainfall runoff used 75L PE drum in the natural condition. On the
basis of automatic weather station which was related to roof collecting data and simultaneously getting
different events rainfall hydrograph, utilizing the cumulative rainfall data then counting different time
rainfall intensity, the article made the instantaneous flow which was calculated through once sampling
time and container volume as the median flow at the sampling begin-end time, and described
different events rainfall hydrological and hydraulic process according to rainfall intensify and runoff
intensify. Collecting the water sample used 500mL polyethylene bottle, in accordance with the relevant
national standard method for environmental monitoring, pollutants included SS, COD and TN. The
sampling rate was as follows: from runoff generation within 30 minutes, every five minutes collected a
water sample. Within 30 to 60 minutes, every fifteen minutes collected a water sample. Later every
thirty minutes collected a water sample. Test dates were April 25, July 23, July 29, August 8, August 26,
and September 11, 2016.

3. Establishment and solution for one dimension migration model of runoff pollutant

3.1 Establishment of model

After rainfall runoff formation on roof, pollutants washed and driven by runoff shear stress according
to the grain size, proportion were along with water movement in some forms like suspended load, bottom
load or between them, and pollutants generated chemical reaction such as degradation, reaction
and physical process like convection, diffusion in this process, causing migration law was relatively
complex. From modeling perspective, rainfall pipe on roof followed emissions to the nearest, runoff
flow time was shorter from pipe to road, accounting for chemical action could be ignored; meanwhile,
the surface runoff and pipe flow were shallow water flow, whose convection and diffusion were
weaker, so physical process could be ignored. Assumption that pollutants equally distributed in the
width and depth direction of different section unit of fluid, was that concentration gradient were 0 in the
Y and Z direction. Only change of pollutants concentration in the X direction was taken into
account to improve operation precision.

The article used unit water whose length was Δx as the modeling object. Based on the mass
balance principle, pollutants quality change equaled with access and discharge in unit of the fluid in the
Δt time. This was expressed in equation (1):
\[
C(t + \Delta t) \cdot y(t + \Delta t) \cdot \Delta x - C(t) \cdot y(t) \cdot \Delta x = [Q(x) \cdot C(x) \cdot \Delta t - Q(x + \Delta x) \cdot C(x + \Delta x) \cdot \Delta t] - \Delta P \Delta x
\]
(1)

Where, \( C(t) \), \( C(t + \Delta t) \) was the pollutants concentration before and after the \( \Delta t \), respectively,
mg/L; \( y(t) \), \( y(t + \Delta t) \) was the sectional area of single wide water before and after the \( \Delta t \), respectively,
m²; \( Q(x) \). \( Q(x + \Delta x) \) was the quantity of flow running into or running out of the \( \Delta x \), respectively,
m³/s; \( C(x) \). \( C(x + \Delta x) \) was the pollutants concentration running into or running out of the \( \Delta x \), respectively,
mg/L; \( \Delta P \Delta x \) was the quality of pollutants which are washed into fluid in the unit surface area, mg/s·m². Both ends of equation (1) divided \( \Delta x \Delta t \), conclusion was equation (2):
\[
\frac{\partial yC}{\partial t} = - \frac{\partial (QC)}{\partial x} - \frac{\partial P}{\partial t}
\]
(2)

Equation (2) was one dimension migration model of rainfall runoff pollutants on surface. On
account of different affected degree of runoff pollution in the road rain gate, lacking for monitoring
data of pollutants deposition grain size, quantity in pipe. Therefore, ignoring pollution resulting from
deposition in research process, then neglecting increase of pollution \( - \frac{\partial P}{\partial t} \) along the flow direction,
equation (2) can be simplified equation (3):
\[
\frac{\partial (yC)}{\partial t} + \frac{\partial (QC)}{\partial x} = 0
\]
(3)

Equation (3) was after simplified as one dimension migration model of rainfall runoff pollutants on
surface.

3.2 Solution of model
According to the finite difference method’s theory, model could be developed numerical analysis, derivative approximation formula of specific point was applied to derivative of partial differential in solving area; the difference equation was built and solved by the Newton iteration method, thus approximate value could be acquired.

3.2.1 Establishment of the difference equation. For solving equation (2), generally selecting two equidistant parallel lines as gridlines, equation (4) were described as follows:

\[
\begin{align*}
x &= x_0 + j \Delta x \quad i = 0, 1, 2 \\
t &= t_0 + j \Delta t \quad j = 0, 1, 2
\end{align*}
\]

Where, \( \Delta x \) was the distance increment, \( \Delta t \) was the time increment, \( i, j \) were the site numbers at the intersection of distance and time, and this study only counted approximate value in grid intersection point. Equation (2) was developed by the first-order backward difference quotient then got equation (5) and equation (6):

\[
\frac{\partial Q}{\partial x} = \frac{(Q_{i,j} - Q_{i,j+1})}{\Delta x}
\]

Where, the boundary condition \( Q_{0,j} \) was 0, the initial condition \( Q_{i,0} \) was 0.

\[
\frac{\partial P}{\partial t} = \frac{(P_{i,j} - P_{i,j-1})}{\Delta t}
\]

Where, the boundary condition \( P_{0,j} \) equaled to \( P(t) \), the initial condition \( P_{i,0} \) equaled to \( P \), \( P \) was the average quality of pollutants in sunny day accumulated in study area, g/m². Let

\[
\frac{\partial (yC)}{\partial t} = \frac{(y_{i,j}C_{i,j}) - (y_{i,j+1}C_{i,j+1})}{\Delta t}
\]

\[
\frac{\partial (QC)}{\partial x} = \frac{(Q_{i,j}C_{i,j}) - (Q_{i,j+1}C_{i,j+1})}{\Delta x}
\]

Let equation (6), equation (7), equation (8) substituted into equation (2),

\[
\frac{(y_{i,j}C) - (y_{i,j+1}C_{i,j+1})}{\Delta t} + \frac{Q_{i,j}C_{i,j} - Q_{i,j+1}C_{i,j+1}}{\Delta x} + \frac{P_{i,j} - P_{i,j-1}}{\Delta t} = 0
\]

Where, the boundary condition \( C_{0,j} \) equaled to 0, the initial condition \( C_{i,0} \) equaled to 0. In equation (9), \( C_{i,j} \) was the runoff pollutant concentration in \( x \) point at \( t_j \) time, mg/L, \( y_{i,j} \), \( Q_{i,j} \), \( (P_{i,j} - P_{i,j-1}) / \Delta t \) all could be got by monitoring test data or solving.

3.2.2 Calculation method and material. The model calculation software adopted Visual Basic6.0, Excel 2007, between them, data switching made Excel as ActiveX part, and call statement that “Dim ExApp As Excel.Application”, “Set ExApp equal to Create object” set up cite for object of ActiveX part. One dimension migration model of the runoff pollutant was mainly solution to water quality change process of different runoff collection points, whose result not only was numerical solution of the previous section, but also input value of the second part, the result included rainfall runoff flow on surface, pollutants washed and migration transmission. In calculation time step was a minute and distance step was 0.5 meter. Parameters such as rainfall pipe mouth quantity process, pavement quantity of flow process, pollutants concentration change process, pollutants initial value, scouring coefficient and length, width, gradient of pollutants evolution process in sink flow adopted the test data or achievement.
4. Instance analyses

4.1 Pollutant process simulation of rainfall pipe mouth
SS as target, water quality process of rainfall pipe mouth was simulated on April 25 and shown in figure 1. From figure 1 simulation precision of model was higher for SS and model could reflect water quality change process in rainfall pipe mouth. The main reason was that factors affecting the simulation precision reduced to minimum, considering roof surface was impervious and rainfall pipe mouth was the runoff collection point on roof.

![Figure 1 Simulation of concentration change process of SS in rainfall pipe mouth on April 25](image1)

4.2 Pollutant process simulation on road
Simulation of water quality change process made SS as target and calculated on September 11, conclusion was shown in figure 2. From figure 2 simulation precision of model was higher for SS and model also reflected water quality change process on road surface. The main reason was that rainfall runoff directly entered into road from roof to pipe mouth, resulting in influence factor decreased to bottom. Meanwhile, road was located in working section, where the roughness coefficient was lower, influence degree was weaken for runoff migration.

![Figure 2 Simulation of concentration change process of SS on road on September 11](image2)

4.3 Rationality analysis of simulated result
Nash-Sutcliffe coefficient (NSC) was used to evaluate the calculated result. When calculated value equaled to measured value, NSC equaled to 1, thus the simulated result was best, NSC was...
between 0 and 1, the larger \( NSC \) was, the better calculated value and measured value matched. The results showed that \( NSC \) was 0.84, 0.81 on April 25, on September 11, both of them were higher than 0.80, which indicated that the simulated result was better.

5. Conclusions
Based on the mass balance principle, pollutant migration law was summarized from rainfall pipe mouth on roof to the road rain gate; one dimension migration model was built for rainfall runoff pollution in surface. The difference equation was developed according to the finite difference method, by applying the Newton iteration method for solving it. The simulated pollutant concentration process was in consistent with the measured value on model, and Nash-Sutcliffe coefficient was higher than 0.80. Pollution formation process of rainfall runoff was more complicated, and research for the migration law needed a large number of the scene monitoring data, therefore, carrying out hydrology and water quality process test monitoring of rainfall runoff was necessary, contributing to revising and improving the existing result.

Acknowledgement
This research is supported by a grant from: The Central Public-interest Scientific Institution Basal Research Fund (HKY-JBYW-2016-46) and the National Natural Science Funds (51379085). Deeply grateful!

References
[1] Wang Baoshan 2011 Migration law of rainfall runoff pollutants on urban[D] Xi’an University of Architecture and Technology
[2] Ren Nanqi, Feng Yujie and Chen Wei, etc 2012 Pollutants transformation law and resource theory and technology of urban water system Beijing Science Press
[3] Pan Anjun, Zhang Shuhan and Chen Jiangang, etc 2010 Utilization technology research and application of urban rainfall China Water& Power Press
[4] Chen Weiwei, Zhang Huimin and Huang Fugui, etc 2011 Analysis of rainfall runoff hydrology and water quality characteristics on urban roof Journal of water resources and water engineering. 22(3) 86-88
[5] Li Liqing and Yin Chengqing 2009 Migration process and source analysis of rainfall runoff pollution in urban combined sewer system Journal of Environmental science. 30(2) 368-375