Neutrino oscillations and Big Bang Nucleosynthesis*

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Abstract

We outline how relic neutrino asymmetries may be generated in the early universe via active-sterile neutrino oscillations. We discuss possible consequences for big bang nucleosynthesis, within the context of a particular 4-neutrino model.

1 Introduction

The implications of neutrino masses in cosmology and astrophysics are numerous. One fascinating effect, is that of relic neutrino asymmetries, which may be produced if there is mixing between active and sterile neutrino species [1].

Neutrino asymmetries are interesting since they affect big bang nucleosynthesis (BBN), altering the primordial helium yield. BBN is particularly sensitive to an asymmetry between the $\nu_e$ and $\overline{\nu}_e$. Additionally, since the matter potential arising from forward scattering is proportional to the asymmetry, a large asymmetry will suppress active-sterile oscillation modes which could otherwise equilibrate sterile neutrino species around the time of BBN.

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2 Relic neutrino asymmetries

First consider two-flavour mixing between an active neutrino, $\nu_\alpha$, and a sterile neutrino, $\nu_s$. For each neutrino flavour we define an asymmetry

$$L_\alpha = \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_\gamma},$$

where $n_\nu$ and $n_\gamma$ are the neutrino and photon number densities, respectively. The neutrino momentum distributions, $N(p)$, are such that

$$n = \int N(p) dp,$$

and in thermal equilibrium are just given by Fermi-Dirac distributions

$$N_{eq}(p) = \frac{1}{2\pi^2} \frac{p^2}{1 + \exp\left(\frac{p}{T} + \tilde{\mu}\right)}.$$  

Here $T$ is the temperature and $\tilde{\mu} \equiv \mu/T$, with $\mu$ being the neutrino chemical potential. In equilibrium, $L_\alpha$ is related to $\mu$ as per

$$L_\alpha \simeq -\frac{1}{24\zeta(3)} \left[ \pi^2 (\tilde{\mu}_\nu - \tilde{\mu}_\bar{\nu}) - 6(\tilde{\mu}_\nu^2 - \tilde{\mu}_\bar{\nu}^2) \ln 2 + (\tilde{\mu}_\nu^3 - \tilde{\mu}_\bar{\nu}^3) \right].$$

We wish to track $L_\alpha$ as the universe evolves, with the period of interest being the epoch between $T \sim 100\text{MeV}$ and $T \sim 1\text{MeV}$. The evolution of the neutrino ensemble will be controlled by (i) oscillations, (ii) collisions of the neutrinos with other particles, resulting in decoherence of the oscillations, and (iii) the expansion of the universe, which redshifts energies and momenta.

The matter affected mixing angle, $\theta_m$, is

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta_0}{\sin^2 2\theta_0 + (b \pm a - \cos 2\theta_0)^2},$$

where $\theta_0$ is the mixing angle in vacuum. The term $(b \pm a)$ arises from the effective matter potential $V = (b \pm a)\delta m^2/2p$, where $-/+$ corresponds to $\nu/\bar{\nu}$ and $\delta m^2$ is the mass squared difference of the two neutrino mass eigenstates. The functions $a$ and $b$ are given by

$$a(p) = -\frac{4\sqrt{2}\zeta(3)G_F T^3 L^{(\alpha)} p}{\pi^2 \delta m^2},$$

$$b(p) = \frac{2\sqrt{2}\zeta(3)G_F T^2 L^{(\alpha)} p}{\pi^2 \delta m^2}.$$
\[ b(p) = -\frac{4\sqrt{2}\zeta(3)G_FT^4A_\alpha p^2}{\pi^2\delta m^2 M_W^2}, \] (6)

where \( A_\alpha(A_{\tau,\mu}) \approx 17(4.9) \) [2]. The function \( L^{(\alpha)} \) is defined as

\[ L^{(\alpha)} = L_\alpha + L_\tau + L_\mu + L_e + \eta, \] (7)

where \( \eta \sim 10^{-10} \) is due to the baryon asymmetry. Note that the mixing angle is dependent on the size of the asymmetry, which makes the evolution of the asymmetry non-linear.

There are two distinct phases in the growth of a neutrino asymmetry. The first is a collision dominated stage, which occurs at high temperature. The total collision rate of a \( \nu_e, (\nu_\mu \text{ or } \nu_\tau) \) of momentum \( p \) with particles in the background plasma is approximately

\[ \Gamma(p) \approx 4.0(2.9)\frac{p}{3.15T^5}. \] (8)

These collisions tend to destroy the coherent superposition of flavour states. In this regime, it can be shown that the growth of the neutrino asymmetry is governed by the equation

\[ \frac{dL_\alpha}{dt} \approx \frac{\pi^2}{2\zeta(3)T^3} \int dp \left( N^+ - N^- \right) \frac{s^2 \Gamma a(c - b)}{[x + (c - b + a)^2][x + (c - b - a)^2]}, \] (9)

where \( c \equiv \cos 2\theta_0, \ s \equiv \sin 2\theta_0, \)

\[ x \equiv \left[ \frac{p \Gamma(p)}{\Delta m^2} \right]^2, \ N^+ = \frac{1}{2}(N_\nu + N_\bar{\nu}). \] (10)

Eq.(9) can be derived either from a heuristic picture [1] that views collisions as measurement-like interactions, or can be extracted from the exact Quantum Kinetic Equations [3]. It is valid when the collision rate is large and the evolution is adiabatic - see [3] for details.

Note that \( dL_\alpha/dt \propto a \propto L^{(\alpha)} \), which can lead to runaway positive feedback and thus rapid growth of \( L_\alpha \). The mixing parameters for which this occurs are specified by \( |\delta m^2| \approx 10^{-4}\text{eV}^2 \) and \( \sin^2 2\theta_0 \approx 10^{-10} \), and we require lighter of the two mass eigenstates to approximately coincide with the \( \nu_e \). Typically, the asymmetry will reach values \( L_\alpha \approx O(10^{-5}) \) in this phase. Note that the sign of the asymmetry, which depends on the initial conditions, can be ambiguous [6].
The second phase is that lower temperature. Here the collisions are less important, and coherent MSW transitions drive the asymmetry to values \( L_\alpha \sim O(0.1) \) [7]. If, for example, the asymmetry is positive, the neutrino MSW resonance occurs at a much higher momentum value than that of the antineutrinos and may be neglected. The rate at which the asymmetry grows is then

\[
\frac{dL_{\nu_\alpha}}{dT} = -X \left| \frac{d}{dT} \left( \frac{p_{\text{res}}}{T} \right) \right|,
\]

(11)

where \( X \) is the difference between the number of \( \bar{\nu}_\alpha \) and \( \bar{\nu}_\beta \) at the resonance. Eq.(11) quantifies the rate at which the resonance moves through the momentum distribution, and holds when transitions are adiabatic and the resonance width is small.

3 Big bang nucleosynthesis and a four neutrino model

We wish to determine the impact of such asymmetry production on the BBN light element abundances, and in particular, on the helium yield. In general, the results are somewhat model dependent (compare, say, [7] with [8].)

The results are particularly sensitive to the size of the \( \nu_e \) asymmetry, which directly affects the neutron/proton ratio through the reaction rates

\[
\begin{align*}
\lambda(n \to p) &\simeq \lambda(n\nu_e \to p\bar{e}^-) + \lambda(ne^+ \to p\bar{\nu}_e), \\
\lambda(p \to n) &\simeq \lambda(p\bar{e}^- \to n\nu_e) + \lambda(n\nu_e \to ne^+).
\end{align*}
\]

(12)

The n/p ratio determines the helium mass fraction \( Y_P \) via the equation

\[
\frac{dY_P}{dt} = -\lambda(n \to p)Y_P + \lambda(p \to n)(2 - Y_P).
\]

(13)

For typical values of \( \delta m^2 \), one must compute the growth of the asymmetry and the effect on BBN simultaneously, as the asymmetries will not have reached their final values before the BBN epoch commences.

Momentum modes which are depleted due to the oscillations, are refilled by

\footnote{Ref.[4] disputed the production of large asymmetries. However, this issue has been resolved, with [4] shown to be in error due to the use of an approximation outside its region of validity [5].}
scattering and annihilation processes as per

\[ \frac{d}{dt} \left( \frac{N_{\text{actual}}}{N_0} \right) = \Gamma(p) \left( \frac{N_{\text{eq}}}{N_0} - \frac{N_{\text{actual}}}{N_0} \right), \]  

(14)

where \( N_{\text{eq}} \) is the equilibrium distribution in Eq.(3), and \( N_0 \equiv N_{\text{eq}}|_{\tilde{\mu} = 0} \).

The example we consider is a particular “2+2” style 4-neutrino model, consisting of a maximally mixed \( \nu_\mu - \nu_\tau \) pair, separated by a mass gap from \( \nu_e - \nu_s \). The oscillations modes most important in the evolution of the various flavour neutrino asymmetries will be the ones with the largest \( \delta m^2 \). Specifically, the \( \nu_{\mu,\tau} - \nu_s \) oscillation modes will create large \( L_\mu \) and \( L_\tau \) asymmetries, and the \( \nu_{\mu,\tau} - \nu_e \) oscillation modes will transfer a small amount of this asymmetry to the electron neutrinos. See [8] for further details.

Numerically integrating the appropriate generalisations of eq.(11), we find that the magnitude of the final lepton asymmetries generated are

\[ |L_\mu|/h = |L_\tau|/h \simeq 0.16, \]
\[ |L_e|/h \simeq 6.7 \times 10^{-3}, \]  

(15)

for \( \delta m^2 \gtrsim 4\text{eV} \), where \( h = T^{3}_{\nu}/T^{3}_\gamma \).

Integrating the BBN reaction rates, eq. (12), the direct effect of the \( \nu_e \) asymmetry on the \( n/p \) ratio, and hence \( Y_P \), is determined. We obtain \( \delta Y_P \simeq \pm 0.0023 \), corresponding to an effective change in the number of neutrinos of \( \delta N^{\text{eff}}_{\nu_e} \simeq \pm 0.19 \). In addition, there is a small direct change to the energy density, leading overall to

\[ \delta N^{\text{eff}}_\nu \simeq -0.3, \text{ positive asymmetry}, \]
\[ \delta N^{\text{eff}}_{\nu_e} \simeq +0.1, \text{ negative asymmetry}, \]  

(16)

for \( \delta m^2 \gtrsim 3\text{eV}^2 \).

The value of \( N^{\text{eff}}_{\nu} \) obtained in this way must then be compared with that calculated from the observed light abundances [9]. It is not yet clear whether the observed helium and deuterium abundances can be accommodated within the standard BBN scenario with \( N^{\text{eff}}_{\nu} = 3 \), and it is possible that a small \( \nu_e \) asymmetry such as that discussed here would help to provide a better fit [10]. Recent measurement of the cosmic microwave background radiation (CMBR) anisotropy also provides a determination of the number of neutrinos in the early universe [11], although the constraints are, as yet, not as strong as those arising from BBN.\(^2\) However, future measurements are expected to pin down

\(^2\) Note, however, that the effective number of neutrinos extracted from the CMBR
the effective neutrino number very precisely \cite{12} and thus will be of great use in constraining models which feature the mixing of active and sterile neutrinos.

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\underbar{and BBN data are not necessarily the same parameter. The two determinations are complementary, since the BBN quantity depends on flavour through its sensitivity to the $\nu_e$ asymmetry, while the CMBR measures the total energy density residing in neutrinos (at a much later epoch of the universe.)}