A characteristic plot of pomeron-exchanged processes in diffractive DIS

Zhang Yang

_Institut für theoretische Physik, FU Berlin, Arnimallee 14, 14195 Berlin, Germany_

Abstract

The dependence of the fractal behaviors of the pomeron induced system in deep inelastic lepton-nucleon scattering upon the diffractive kinematic variables is found rather robust and not sensitive to the distinct parameterization of the pomeron flux factor and structure function. A feasible experimental test of the phenomenological pomeron-exchanged model based on the fractal measurement in DESY ep collider HERA is proposed.
Diffractive processes in high energy collisions have long been described by using the phenomenology of Regge theory\textsuperscript{1} by the $t$-channel exchange of mesons and, at high energy, by the leading vacuum singularity, i.e. the pomeron\textsuperscript{2}. Many calculations based on the pomeron theory has been pursued concerning with the collective aspects of the diffractive processes, such as cross section of hard diffraction\textsuperscript{3–5}, the distribution of large rapidity gap\textsuperscript{6,7}, jet production\textsuperscript{5,8,9} in hard diffractive processes etc.. But because of the ignorance for the nature of the pomeron and its reaction mechanisms, there exists different kinds of approaches and parameterizations of pomeron in current Regge theory. The calculation results in the above-mentioned literature were found very sensitive to the distinct parameterization of the pomeron. So the experimental data concerning those features can constantly be conformed by pomeron model with tuning all these unknown parameterizations, although the parameterizations are quite different for different respects of data\textsuperscript{†}, which makes it rather difficult to conclude whether pomeron theory indeed works in the diffractive process. In this respect, it is nature to ask the following questions: Is there a way to test and justify the pomeron exchange model by using current diffractive experiment equipment (e.g. DESY $ep$ collider HERA) while the criterion to the measurements do not depend upon concrete parameterization of pomeron? If yes, what is the characteristic behavior of the pomeron exchanged model in the expected experimental measurements?

Having in mind that the fractal and fluctuation pattern of the multiparticle production reveals the nature of the correlations of the spatial-temperature evolutions in both levels of

\textsuperscript{†}For example, the earlier data of UA8 Collaboration\textsuperscript{8} about jet production in CERN $SppS$-Collider dictated a soft gluon content of pomeron, while newer and more comprehensive data of UA8 Collaboration\textsuperscript{8} required a hard pomeron content; the analysis of structure function by ZEUS Collaboration\textsuperscript{10} in DESY $ep$ collider HERA needed both hard and soft component of the pomeron; the recent study of structure function of QCD evolution by H1 collaboration\textsuperscript{11} favored a rather peculiar one-hard-gluon pomeron.
parton and hadron and is, therefore, sensitive to the interact dynamics of the high-energy process, it has been proposed to investigate the fractal behavior of the diffractively produced system by calculating the scaled factorial moments of the multihadronic final state. In this note, we find the dependence of the fractal behavior of the pomeron induced system upon the diffractive kinematic variables is rather robust and not sensitive to the different parameterization of the phenomenology pomeron model in the deep inelastic lepton-nucleon scattering (DIS). So the characteristic plot about fractal behavior of the diffractively produced system can be considered as a clear experimental test of the pomeron exchanged model by DESY ep collider HERA.

The fractal (or intermittency) behavior of the diffractively produced system in DIS (and also hadron-hadron collider) can be extracted by measuring the \( q \)-order scaled factorial moments (FMs) of the final-state hadrons excluding the intact proton from the incident beam, which are defined by

\[
F_q(\delta x) = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m(n_m - 1)\ldots(n_m - q + 1) \rangle}{\langle n_m \rangle^q}, \tag{1}
\]

where, \( x \) is some phase space variable of the multihadronic final-state, e.g. (pseudo-)rapidity, the scale of phase space \( \delta x = \Delta x/M \) is the bin width for a \( M \)-partition of the region \( \Delta x \) in consideration, \( n_m \) is the multiplicity of diffractively produced hadrons in the \( m \)th bin, \( \langle \cdots \rangle \) denotes the vertical averaging with the different events for a fixed scale \( \delta x \). Since the factorial moments \( F_q \) can rule out the statistic noise around probability \( p_m \) for a particle to be produced in the small phase space of final state, and associated directly with the scaled probability moments

\[
C_q = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle p_m^q \rangle}{\langle p_m \rangle^q}, \tag{2}
\]

it is clear that \( F_q \) would saturate to some constant with decreasing \( \delta x \) to the some typical size \( \delta x_0 \) (e.g. the correlations length from resonance decays) if there were no dynamic fluctuation in the multiparticle producing process and the probability distribution of the phase-space densities were smooth, especially in small scale of the phase space. The manifestation of the
Fractality and intermittency in high energy multiparticle production refers to the anomalous scaling behavior of \( F_{q}(\delta x) \sim (\delta x)^{-\phi_{q}} \sim M^{\phi_{q}} \), as \( M \to \infty, \delta x \to 0 \).

The \( q \)-order intermittency index \( \phi_{q} \) can be connected with the anomalous fractal dimension \( d_{q} \) of rank \( q \) of spatial-temporal evolution of high energy collisions as:

\[
d_{q} = \frac{\phi_{q}}{(q - 1)}
\]

Pomeron exchange in Regge theory has been used to describe successfully the main features of the high energy elastic and diffractive process. While waiting for the experimental measurements of DESY \( ep \) collider HERA (and also hadron-hadron collider) for the above-mentioned fractal behavior of the diffractively produced system, let us now take a closer look at what we can learn from the current pomeron exchange theory.

The deep inelastic diffractive lepton-nucleon scattering has been well modeled by several Monte Carlo generators such as POMPYT and RAPGAP, which are based on the assumption that the process can be considered as taking place in two steps. Firstly, a beam hadron emits a pomeron (\( P \)) with longitudinal momentum fraction \( x_{P} \); in the second step this pomeron interacts with the other beam lepton in a large momentum transfer process between the basic constituents of the pomeron and the lepton. This pomeron factorization allows the diffractive hard scattering cross-section to be written as:

\[
\frac{d^{4}\sigma(ep \to e + p + X)}{dx_{P}dt\beta dQ^{2}} = f_{P}(t, x_{P}) \frac{d^{2}\sigma(eP \to e + X)}{d\beta dQ^{2}}
\]

where \(-Q^{2}\) and \( t \) are the mass-squared of virtual photon and exchanged pomeron respectively, and \( \beta = x_{B}/x_{P} \) the momentum fraction of parton in pomeron. The first factor in Eq. (5) is the pomeron flux, i.e. probability of emitting a pomeron from the proton. For lack of the understanding for the reaction mechanism of the pomeron, there exist different approaches for the parameterization of the pomeron flux:

In standard Regge theory with a supercritical pomeron trajectory, \( \alpha_{P}(t) = 1 + \epsilon + \alpha_{P}^{t}t \), the pomeron flux was given as.
\[ f_{pP}(t, x_P) = \frac{\beta^2_{pP}(t)}{16\pi^2} x_P^{1-2\alpha_{P}(t)}, \]  

(6)

where the coupling of the pomeron to the proton, \( \beta_{pP}(t) \) was parametrized as \( \beta_{pP}(t) = 6.3e^{3.25t} \) according to the \( t \)-dependence of cross section in higher \( x_P \)-range of ISR data. By approximating the cross section of proton and pomeron with a constant \( \sigma_{pP} = 1\text{mb} \), Ingelman and Schlein compared the factorization expression of hard diffractive \( p\bar{p} \) process with the single diffractive differential cross section at the SPS collider. They obtained the pomeron flux in a proton as

\[ f_{pP}(t, x_P) = \frac{1}{2.3 x_P} (6.38e^{-8|t|} + 0.424e^{-3|t|}). \]  

(7)

Alternatively, Donnachie and Landshoff assumed the pomeron-photon analogy in the couples to quark in nucleon and gave the pomeron flux as

\[ f_{pP}(t, x_P) = \frac{9\beta^2_{qP}}{4\pi^2} [F_1(t)]^2 x_P^{1-2\alpha_{P}(t)} \]  

(8)

with coupling constant of the pomeron to a quark \( \beta^2_{qP} = 3.24\text{GeV}^{-2} \) and the elastic form factor

\[ F_1(t) = \frac{4m^2_{P} - 2.79t}{4m^2_{P} - t} \cdot \frac{1}{(1-t/0.71)^2}. \]  

(9)

The second factor in Eq. (5), i.e. lepton-pomeron hard cross section, can be calculated in the way that

\[ \frac{d^2\sigma(eP \rightarrow e + X)}{d\beta dQ^2} = \int d\beta G(\beta) \frac{d^2\sigma_{\text{hard}}(e + \text{parton} \rightarrow e + X)}{d\beta dQ^2}, \]  

(10)

if we could figure out the density \( G(\beta) \) of the quarks and gluons with fraction \( \beta \) of the pomeron momentum, needless to say, hard scattering cross section \( d\sigma_{\text{hard}}(e + \text{parton} \rightarrow e + X) \) is to be computed according to standard QED and the perturbative QCD correction. But the pomeron structure function is still a main uncertainty in pomeron model and it is even unknown whether the pomeron consist mainly of gluons or of quark, although measurements of hard diffractive scattering have been performed in both lepton-hadron and
hadron-hadron collider (see Footnote †). Two extreme gluon densities has been extensively used in Ref.† and many other literature, i.e.

\[ \beta G(\beta) = 6\beta(1 - \beta), \]  

\[ \beta G(\beta) = 6(1 - \beta)^5. \]  

(11)  

(12)  

The first one corresponds to the “unrealistically hard” gluon distribution, where two gluons share the pomeron momentum; in latter case, the gluons in the pomeron are as soft as that in the proton. Taking into account the information from Regge theory in \( \beta \rightarrow 0 \) and the power counting rules in \( \beta \rightarrow 1 \), Berger et al. parametrized the gluon distribution in the pomeron as

\[ \beta G(\beta) = (0.18 + 5.46\beta)(1 - \beta). \]  

(13)  

In above three parametrizations of structure function of pomeron, the normalization of all parton distributions are, by default, chosen to fulfill the momentum sum rule

\[ \int_0^1 d\beta \beta G(\beta) = 1. \]  

(14)  

But it is not clear that this relation must hold for the pomeron which is a virtual exchanged object that need not behave as a normal hadron state. In the approach of Donnachie and Landshoff, the dominating quark density was

\[ \beta G(\beta) = \frac{1}{3} C\pi\beta(1 - \beta), \quad \text{with } C = 0.23, \]  

(15)  

the flavors summation and the momentum integral of which is a factor 7.5 lower than the normalization in Eq. (14).

In addition to these theoretical uncertainties there is also a uncertainty in the \( Q^2 \) evolution of the parton densities of the pomeron. Numerical calculations using ordinary QCD evolution equation (Altarelli-Parisi or DGLAP), and GLR-MQ equation in which the inverse recombination processes of partons has been taken into account, turned out that
the $Q^2$ evolution of the pomeron structure function can be very much different depending upon whether the non-linear recombination term of the QCD evolution equation is included or not. Furthermore, depending upon the initial parton distribution at a given momentum scale which is unknown, the size of nonlinear term may become too large for the QCD evolution equation to be reliable without further, but also unknown, correction. By assuming both leading and subleading Regge trajectory with a flux akin to Eq. (6), a fit according to the NLO DGLAP evolution equations to HERA data\textsuperscript{11} of $F_2^{D(3)}(x_P, \beta, Q^2)$ has favored a rather peculiar “one-hard-gluon” distribution for the pomeron (see also Footnote \textsuperscript{†}). Since what we try to pursue in this note is to find out whether and in which range the fractal behavior of pomeron induced system depend on varieties of different parameterization of pomeron, we leave the possible anomalous scaling behavior in the QCD evolution processes to the further discussion\textsuperscript{26}.

In typical kinematic region of hard diffractive processes of DESY ep collider HERA (say, i.e. $M_X > 1.1$GeV, and $x_P < 0.1$), majority properties of the diffractive events can be well reproduced by RAPGAP generator (see, e.g.\textsuperscript{6,7,9,11}). In the following intermittency analysis of the lepton-nucleon diffractive process, we use RAPGAP generator\textsuperscript{7} to simulate the pomeron exchange processes, in which the virtual photon ($\gamma^*$) will interact directly with a parton constituent of the pomeron for a chosen pomeron flux and structure function. In addition to the $O(\alpha_{em})$ quark-parton model diagram ($\gamma^*q \rightarrow q$), the photon-gluon fusion ($\gamma^*g \rightarrow q\bar{q}$) and QCD-Compton ($\gamma^*q \rightarrow qg$) processes are generated according to the $O(\alpha_{em}\alpha_s)$ matrix elements. Higher order QCD corrections are provided by the colour dipole model as implemented in (ARIADNE)$\textsuperscript{27}$, and the hadronization is performed using the JETSET$\textsuperscript{28}$. The QED radiative processes are included via an interface to the program HERACLES$\textsuperscript{29}$.

By chosing the pomeron flux factor $f_{\rho\rho}(t, x_P)$ and the pomeron structure function $G(\beta)$ as shown in Eq. (6) and Eq. (11) respectively, we generate 100,000 MC events, and calculate the second-order factorial moments in 3-dimensional ($\eta, p_\perp, \phi$) phase space, where the pseudorapidity $\eta$, transverse momentum $p_\perp$ and the azimuthal angle $\phi$ are defined with
respect to the sphericity axis of the event. The cumulative variables $X$ translated from $x = (\eta, p_\perp, \phi)$, i.e.

$$X(x) = \int_{x_{\text{min}}}^{x} \rho(x) dx / \int_{x_{\text{min}}}^{x_{\text{max}}} \rho(x) dx,$$

were used to rule out the enhancement of FMs from a non-uniform inclusive spectrum $\rho(x)$ of the final produced particles. The obtained result of second-order FM versus the decreasing scale of the phase space is shown in Fig.1(a) in double logarithm. There exists obviously anomalous scaling behavior in the pomeron induced interaction, so we fit the points in Fig. 1(a) to Eq. (3) with least square method and obtain the intermittency index $\phi_2 = 0.428 \pm 0.003$. In Fig. 1, we have also shown the Monte Carlo result of the second-order factorial moments for different kinds of the parameterization of the pomeron flux factor and structure function. In Fig. 1(a), (b), and (c), we keep the pomeron structure function $G(\beta)$ fixed but vary the pomeron flux factor $f_pP(t, x_p)$ as Eq. (6), (7), and (8) respectively. The fractal behaviors of the pomeron induced system keep almost unchanged for different flux factor. On the contrary we keep the pomeron flux factor fixed in Fig.1(a), (d), (e), and (f), but vary the pomeron structure function as Eq. (11), (12), (13), and (15) respectively. For a given pomeron flux, the fractal behaviors become weaker when the pomeron become softer. In Fig. 1(d) the parton distribution is as soft as that in proton, the intermittency index is smallest, which is understandable since if the hard parton in pomeron is involved it is more possible to evoke jets and then the anomalous short-range correlation in the final-state so that the intermittency index increases, and vice versa.

It is of the special interest to investigate the dependence of the fractal behavior of the pomeron induced system upon the diffractive kinematic variables. We generate 500,000 events by RAPGAP Monte Carlo generator, and divided the whole sample into 10 subsamples according to the diffractive kinematic variables, e.g. $x_B$. For each subsample, we calculate the second order scaled FM and the intermittency index $\phi_2$, and to see how the fractal behavior of the pomeron induced multihadronic final states depends upon the considered kinematic variable. In Fig. 2 is shown the dependence of the second order in-
termittency index \( \phi_2 \) on the different diffractive kinematic variables. Since it is well known that the gluon density increase sharply as \( x_B \) decreases in small-\( x_B \) region, the MC result from pomeron model in Fig. 2(a) means that the anomalous fractal dimension \( d_2 \) of the diffractively produced system decrease with increasing gluon density, which is not inconceivable if one take into account the fact (see, e.g.\(^{13,15}\)) that the effect of superposition of fractal systems can remarkably weaken the intermittency of whole system. Obvious dependence of \( \phi_2 \) on pomeron momentum fraction \( x_P \) of a hadron and parton momentum fraction \( \beta \) of a pomeron as shown in Fig. 2(b) and (c) implies that, the intermittency calculated here can not be only referred to the hadronization processes and there should be substantial correlations between the fractal calculation and pomeron dynamics. In Fig. 2(a) and (b), the intermittency index \( \phi_2 \) is less than 0 for the lower \( x_B \) and \( x_P \), which can be imputed to the constraint of the momentum conservation in the high energy process.\(^{32}\) Since the Leading Proton Spectrometer (LPS) has been used in ZEUS detector to detect protons scattered at very small angles (say, \( \leq 1 \) mrad), which make it possible to measure precisely the square of the four-momentum transfer \( t \) at the proton vertex, we also showed in Fig. 2(e) the \( t \)-dependence of second order intermittency index in the \( t \)-region of LPS detector, i.e. \( 0.07 < -t < 0.4 \text{GeV}^2 \). To be different from the results of other kinematic variables, the fractal index for the \( \gamma^* \Pi \) system doesn’t depend upon the \( t \).

Especially, we calculate the intermittency index shown in Fig. 2 using different kinds of pomeron parameterization. We denote the different shapes of points in Fig. 2 for the different kinds of the pomeron parameterization, just in the same way as that in Fig. 1. In conventional investigation, the pomeron theory has been used to compare with the data about cross section of hard diffraction\(^{3-5}\), the rapidity distribution of large rapidity gap\(^{6,7}\), and jet rapidity distribution\(^{8,9}\) and jet shape\(^{10}\) etc., where the results of the pomeron model in such quantities concerning with the collective nature of diffractive process were found very sensitive to the parameterization of the pomeron, and the experimental data in different aspects preferred different kinds of parameterization\(^{8,9}\) (see also Footnote \(^{†}\)). It is remarkable that the dependence of the intermittency index, which concerned with the
inherent scaling behaviors of diffractive processes, upon the diffractive kinematic variables in this implementation of the pomeron exchanged model are rather robust and almost the same for the different parameterization of the pomeron flux and structure function!

In conclusion, we have given an outline of the mainly uncertainty of the pomeron exchanged model on the distribution functions for finding partons in a pomeron and for finding pomeron in a hadron, which is confronted in the conventional hard diffractive calculation (see, e.g. Refs. [3, 4, 21]). By the intermittency analysis according to Monte Carlo implementation of RAPGAP we present a characteristic plot for the pomeron exchange model, which is independent of aboved-mentioned uncertainty. So in order to test and justify the pomeron theory to the diffractive processes, it is urgent and feasible to check this characteristic fractal plot in DESY $ep$ collider HERA. And the substantial revision would be necessary in the manner in which we have treated diffraction by using the current pomeron theory if it should turn out that experimental measurements differ drastically from this characteristic plot presented here.

**Acknowledgements**

I would like to thank T. Meng, R. Rittel, and K. Tabelow for their hospitality, H. Jung for correspondence, and the Alexander von Humboldt Stiftung for financial support.
REFERENCES

1. T. Regge, Nuov. Cim. 14, 951 (1959), *ibid.* 18, 947 (1960).

2. G. Chew and S. Frautschi, Phys. Rev. Lett. 7, 294 (1961); G. Chew, S. Frautschi and S. Mandelstam, Phys. Rev. 126, 1202 (1962).

3. G. Ingelman and P. Schlein, Phys. Lett. B 152, 256 (1985).

4. H. Fritzsch and K. Streng, Phys. Lett. B 164, 391 (1985).

5. E. Berger, J. Collins, D. Soper and G. Sterman, Nucl. Phys. B 286, 704 (1987).

6. H1 Collaboration, T. Ahmed et al., Nucl. Phys. B 429, 477 (1994).

7. H. Jung, Comput. Phys. Commun. 86, 147 (1995).

8. UA8 Collaboration, R. Bonino, et al., Phys. Lett. B 211, 239 (1988), A. Brandt, et al., *ibid.* B 297, 417 (1992).

9. H1 Collaboration, P. Marage, in: Proceeding of DIS 97, Chicago, edited by J. Repond and D. Krakauer, (AIP 1997), p. 570; ZEUS Collaboration, J. Terron, *ibid.*, p. 574; J. Hernandez, *ibid.*, p. 592.

10. M. Derrick et al., ZEUS Coll., Z. Phys. C 68, 569 (1995).

11. C. Adloff et al., H1 Coll., Z. Phys. C 76, 613 (1997);

12. A. Bialas and K. Peschanski, Nucl. Phys. B 273, 703 (1986); B 308, 857 (1988).

13. For a recent review of fractal (intermittency) in high energy physics, see e.g., E.A. De Wolf, I.M. Dremin and W. Kittel, Phys. Rep. 270, 1 (1996).

14. Zhang Yang, Phys. Rev. D 57, R1327 (1998);

15. P. Lipa and B. Buschbeck, Phys. Lett. B 223, 465 (1989); R. C. Hwa, Phys. Rev. D 41, 1456 (1990); Zhang Yang, Liu Lianshou and Wu Yuanfang, Z. Phys. C 71, 499 (1996).
16. See, e.g. P. Collins, *An Introduction to Regge Theory and High-Energy Physics*, (Cambridge University Press, Cambridge, 1977); K. Goulianos, Phys. Rep. 101, 169 (1983).

17. P. Bruni and G. Ingelman, DESY-93-187, in: Proceedings of the Rutherford Physics Conference on HEP, Marseilles, 1993, Edited by J. Carr and M. Perrottet (Editions Frontieres, Singapore, 1994), p. 595.

18. R. D. Field and G. Fox, Nucl. Phys. B 80, 367 (1974); A. B. Kaidalov and A. K. Ter-Martirosyan, Nucl. Phys. B 75, 471 (1974).

19. M. Albrow et al., Nucl. Phys. B 108, 1 (1976).

20. M. Bozzo et al., UA4 Coll., Phys. Lett. B 136, 217 (1984).

21. A. Donnachie and P. V. Landshoff, Nucl. Phys. B 244, 322 (1984); Phys. Lett. B 191, 309 (1987).

22. S. Nussinov, Phys. Rev. Lett. 34, 1286 (1975); Phys. Rev. D 14, 246 (1976); F. E. Low, Phys. Rev. D12, 163 (1975).

23. G. Ingelman and K. Prytz, Z. Phys. C 58, 285 (1993).

24. Yu. L. Dokshitzer, JETP 46, 641 (1977); V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. 15, 438 (1972); G. Altarelli and G. Parisi, Nucl. Phys. B 126, 298 (1977).

25. L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rep. 100, 1 (1983); A. H. Mueller and J. Qiu, Nucl. Phys. B 268, 427 (1986).

26. Zhang Yang, (in preparation).

27. L. Lönnblad, Comput. Phys. Commun. 71, 15 (1992).

28. T. Sjostrand, Comput. Phys. Commun. 39, 347 (1986).

29. A. Kwiatkowski, H. Spiesberger and H. Möhring, Comput. Phys. Commun. 69, 155 (1992);
30. A. Bialas and M. Gazdzicki, Phys. Lett. B 252, 483 (1990); W. Ochs, Z. Phys. C 50, 339 (1991).

31. See e.g. ZEUS Collaboration, M. Derrick et al., Phys. Lett. B 345, 576 (1995); H1 Collaboration, S. Aid et al., ibid. B 354, 494 (1995).

32. Liu Lianshou, Zhang Yang and Deng Yue, Z. Phys. C 73, 535 (1997).
FIGURES

Fig. 1. The second-order scaled factorial moments $F_2$ getting from MC simulation of RAPGAP generator versus the number $M$ of subintervals of 3-dimensional $(\eta, p_\perp, \phi)$ phase space in log-log plot, and the intermittency index $\phi_2$ correspondingly. The different kinds of points denote different parameterization of the pomeron flux factor $f_{pP}(t, x_P)$ and structure function $G(\beta)$, i.e. $f_{pP}(t, x_P)$ keeps fixed as Eq. (11) but $G(\beta)$ varies as Eq. (6), (7), and (8) in Figs. (a), (b), and (c) respectively; on the contrary, $G(\beta)$ keeps fixed as Eq. (13) but $f_{pP}(t, x_P)$ varies as Eq. (11), (12), (13), and (15) in Figs. (a), (d), (e), and (f) respectively.

Fig. 2. The dependence of second-order intermittency index $\phi_2$ in RAPGAP Monte Carlo implementation upon different kinematic variables. The different shapes of points denote different parameterization of pomeron flux factors and the structure functions in the same way as that in Fig. 1.
Fig. 1

\( \phi_2 = 0.428 \pm 0.003 \)

\( \phi_2 = 0.449 \pm 0.003 \)

\( \phi_2 = 0.462 \pm 0.003 \)

\( \phi_2 = 0.278 \pm 0.002 \)

\( \phi_2 = 0.409 \pm 0.002 \)

\( \phi_2 = 0.449 \pm 0.003 \)
Fig. 2