Chemical reaction, radiation and activation energy effects on MHD buoyancy induced nanofluid flow past a vertical surface

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Abstract. This paper explores the effects of thermal radiation, buoyancy force, chemical reaction, and activation energy on magnetohydrodynamic (MHD) nanofluid flow past a stretching vertical surface. The resulting nonlinear momentum, energy, solute, and nanoparticle concentration boundary layer equations are simplified by the transformation of similarity. The transformed equations solved numerically by using the shooting technique. For various related parameters, the corresponding results to the dimensionless velocity, temperature, solute, nanoparticle concentration profiles, Skin friction, local Nusselt number, local Sherwood number, and local nanoparticle Sherwood number are illustrated graphically. It is found that the temperature, and nanoparticle concentration profiles increase on increasing thermal radiation and temperature difference parameters. With the increase of the regular buoyancy parameters, the local Nusselt number decreases on increasing the fitting rate constant, Biot number, and thermal radiation parameters.

Keywords. Chemical reaction; Activation energy; MHD; Buoyancy force; Nanofluid; Thermal radiation.

1. Introduction

The impact of chemical reaction on MHD buoyancy induced nanofluid flow is one of the important applications in many industrial processes, such as chemical coating of flat plates, ejection of polymer and hot rolling observed by Ibrahim and Makinde [1]. The pure water or air in nature has not existed and there might be distant mass in the water or air. These mixtures within the substances cause chemical reactions studied by Jabeen et al. [2]. Niranjan et. al. [3] investigated the chemical reaction effect in MHD mixed convection flow over a plate in a porous medium. They found that the chemical reaction parameter increases on increasing the velocity and concentration profiles in generative case but decreasing in the destructive case. Mallikarjuna et al. [4] analyzed the impact of chemical reaction on heat transfer and flow of a viscous nanofluid in a variable porous medium along a vertical cone. Hayat et al. [5] investigated the chemical reaction and double stratification of Williamson nanoliquid by a stretched sheet in the MHD stagnation point flow. Ramzan and Bilal [6] found that the effect of chemical reaction and magnetic field in the three-dimensional flow behavior of an elastic-viscous nanofluid over a stretching sheet. In porous medium Sivasankaran et al. [7] studied the impact of slip, chemical reaction, radiation on mixed convection, and convective boundary conditions of viscous fluid flow over a plate. They observed that the rate of mass transfer increases and heat transfer decreases on increasing the chemical reaction parameter.

The process of transfer of mass along with the chemical reaction and activation energy having wide applications in food processing, water mechanics, geothermal reservoirs and oil emulsions, chemical engineering, etc. Dhlamini et al. [8] analyzed the effects of activation energy and chemical reaction with convective boundary
condition in mixed convective nanofluid flow. They found that the thermophoresis increases on increasing the chemical reaction parameter. Makinde et al. [9] numerically investigated the natural convection unsteady flow under the influence of activation energy with nth order reaction. In mixed convection flow, the exothermic/endothemic reaction with activation energy has been proposed by Maleque [10]. Hayat et al. [11] examined the characteristics of activation energy and exponential dependent heat source in Carreau fluid flow with cross-diffusion effect. Awad et al. [12] used the modified Arrhenius function to study the rotating flow of binary fluid through a deformed impulsive surface. Makinde and Gnaneswara Reddy [13] examined the effect of velocity slip on peristaltic flow of electrically conducting casson fluid in a channel embedded with a porous medium. They observed that the velocity and temperature profiles decrease on increasing the magnetic field and casson fluid parameters. Anuradha and Yegammal [14] studied binary chemical reaction with activation energy in nano liquid radiative hydromagnetic flow over a vertical plate. Nanofluid is a colloidal solution, which comprises nanoparticles of size ranges from 1–100 nm and that are uniformly distributed in the base fluid. Generally, nanofluids have been consuming in several industrial and engineering works. Zaib et al. [15] examined the physical characteristics of Newtonian Carreau fluid immersed within the nanofluid near a point of stagnation towards a thermal radiation stretching sheet. The explicit numerical analysis is given by Isa et al.[16] for a mixed convection MHD flow of Casson fluid over an infinitely permeable shrinking sheet. Ganga et al. [17] investigated that with increasing values of magnetic parameter, aligned angle, nanosolid volume fraction parameter and slip parameters, the nanofluid temperature increases. Ullah et al. [18] discovered the impact of radiation, MHD flow in Marangoni convection nanoliquid. They observed that the temperature enhanced on increasing the radiation and heat generation/absorption parameters. Sivasankaran and Narrein [19] examined the nanofluids could lead to substantial thermal enhancement with a lower pressure drop compared to water. Alsaadi et al. [20] explored the generation of entropy in nonlinear convective mixed flow of nanofluid in porous space affected by thermal radiation and activation energy.

The effect of radiation on the flow of MHD and heat transfer in technological and industrial areas occurs at very high temperatures, and knowledge of the transfer of heat radiation becomes important for the design of specific devices. In manufacturing industries, the heat radiative flow and mass transfer play an important role in the designing of aircraft, space vehicles, gas turbines, nuclear power plants, satellites, energy utilization, and numerous agricultural applications. Ganesh et al. [21] studied numerically and analytically the impact of radiation over a horizontal stretching sheet on the flow of nanofluids. Dar [22] and Kho et al. [23] analyzed numerically radiation effects in Casson nanofluid on MHD flow heat and mass transfer over a stretching sheet. Ullah et al. [24] observed the radiation parameter and nanoparticles volumetric fraction are qualitatively similar for the temperature. In the presence of suction, Daniel et al. [25] addressed the combined effects of chemical reaction, thermal radiation, Joule Heating and viscous dissipation were discussed in a two-dimensional natural convection flow of an electrical MHD nanofluid. Hamid et al.[26] and Chandrakala and Raj [27] studied the influence of radiation on MHD flow through an impulsively induced infinite vertical plate. The effect of thermal radiation on transient MHD flow with mass transfer through an impulsively vertical plate was proposed by Ahmed and Sarmah [28]. Mukhopadhyay [29] studied the thermal radiation effects on mixed convection flow and transfer of heat over the porous stretching surface. Hayat et al. [30] discussed the Marangoni thermosolutal convective flow of nanoliquid under the effect of space-dependent exponential thermal radiation and internal heat source.

Several natural and forced flows are influenced by magnetic fields, for example, during the pumping, stirring, and heating process. Sheikholeslami et al. [31] studied the effect of radiative heat transfer on MHD non-Darcy nanofluid. The heat transfer on the MHD boundary layer flow over a stretching sheet and they found that
fluid velocity decreases when increasing the magnetic field parameter was studied in ref. [32-34]. Chutia and Deka [35] studied an unsteady and MHD Couette flow of incompressible, electrically conducting, viscous fluid between two infinitely long porous parallel plates in the presence of a transverse magnetic field. Devi et al. [36] examined the impact of heat and mass transfer on the MHD flow of an incompressible and radiating fluid through an exponentially stretched sheet, and they found that the fluid is thicker due to magnetic parameter. Under convective boundary surface conditions, Makinde and Olanrewaju [37] studied the effect of buoyancy forces over a vertical plate on thermal boundary layers. Ramzan et al. [38] discovered the velocity field decreases to increase buoyancy ratio parameter values. The convective flow of magneto-nanofluid is reported numerically, under the influence of chemical reaction and activation energy reported by Mustafa et al. [39].

Inspired by all the above works, here we have attempted a numerical study of anticipating by all these above mentioned works. The main objectives of this paper to explore the effects of thermal radiation, buoyancy force, chemical reaction and activation energy on the two-dimensional magnetohydrodynamic (MHD) nanofluid flow past a stretching vertical surface. To best of our knowledge, these combined effects on MHD buoyancy induced nanofluid were not carried in the past. Using similarity transformation, the nonlinear governing equations are transformed into a system of ordinary differential equations. Then these transformed equations are solved numerically by using Shooting technique. The impact of physical parameters are discussed through graphically.

2. Mathematical formulation

The steady, incompressible, laminar, MHD flow of an electrical conducting nanofluid along with a vertically stretchable sheet. In this problem, the combined influence of chemical reaction and activation energy are taken into account. The velocity components $u$ and $v$ are denoted along $x$ and $y$ directions respectively, where $x$ - axis coordinates extended along the surface and $y$ - axis coordinates extended along the normal to it (Fig. 1).

$B_0$ is the intensity of a uniform magnetic field is applied normal to the stretchable sheet. Let us assume that, the sheet surface is stretched by the linear velocity $u_w = ax$ in the vertical direction, where $a > 0$ indicates the stretching rate. Let us consider that at ambient state surface temperature $T_w$ is greater than the fluid temperature.

The governing equations may be written as

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma^2 B_0^2}{\rho_f} u + \frac{1}{\rho_f} \left[(1-C_\alpha) \frac{\rho_{f\alpha} \beta (T-T_\alpha) - (\rho_p - \rho_{f\alpha}) (C-C_\alpha)}{\beta} \right] g \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \frac{\alpha}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{D_b}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 + \alpha \frac{\partial q_r}{\partial y} + \beta K_i \frac{T}{T_\infty} \exp \left[ \frac{-E_a}{KT} \right] (S-S_\infty) \tag{3}
\]

\[
u \frac{\partial S}{\partial x} + \nu \frac{\partial S}{\partial y} = \frac{D_b}{C_p} \frac{\partial^2 S}{\partial y^2} - K_i \frac{T}{T_\infty} \left( \frac{T}{T_\infty} \right)^n \exp \left[ \frac{-E_a}{KT} \right] \tag{4}
\]

\[
u \frac{\partial C}{\partial x} + \nu \frac{\partial C}{\partial y} = D_b \frac{\partial^2 C}{\partial y^2} + \frac{D_t}{T_\infty} \frac{\partial^2 T}{\partial y^2} \tag{5}
\]
Where \( \alpha = \frac{k}{(\rho c)_f} \) and \( \tau = \frac{(\rho c)_p}{(\rho c)_f} \).

Subject to the boundary conditions are,

\[
\begin{align*}
\text{u} &= u_w(x) = cx, \quad \text{v} = 0, \quad D_x \frac{\partial C}{\partial y} + D_x \frac{\partial T}{\partial y} = 0, \quad -k \frac{\partial T}{\partial y} = h_f \left( T_f - T \right), \quad S = S_w \text{ at } y = 0 \\
\text{u} &\to 0, \quad T \to T_\infty, \quad C \to C_\infty, \quad S \to S_\infty \text{ at } y \to \infty
\end{align*}
\] (6)

Where \( \text{u} \) and \( \text{v} \) are the velocity components in the \((x, y)\) axes, respectively. The temperature of the fluid is \( T \), the kinematic viscosity of a fluid is \( \nu = \frac{\mu}{\rho} \) and the thermal conductivity is \( k \). The radiative heat flux \( q_r \) is given by

\[
q_r = -\frac{4\sigma^*}{3K'} \frac{\partial T^4}{\partial y}
\] (7)

The governing equations (1)-(5) are transformed into the ordinary differential equations by using the equations (8)-(10), as follows

\[
f'' - f' + f - M f' + \lambda \left( \theta - N r \phi \right) = 0
\] (11)
\[
\theta^*(1 + \frac{4}{3} \frac{Rd}{Pr} f \theta' + Nt Pr \theta' \phi' + Nt Pr \theta^2 + Pr \sigma Nc \left[1 + \theta \delta\right] \exp \left(\frac{-E}{1 + \theta \delta}\right) = 0 \quad (12)
\]

\[
\chi'' + Scf \chi' - Sc \sigma \chi' \left(1 + \theta \delta\right)^\eta \exp \left(\frac{-E}{1 + \theta \delta}\right) = 0 \quad (13)
\]

\[
\phi'' + Scf \phi' + \left(\frac{Nt}{Nb}\right) \theta'' = 0 \quad (14)
\]

The corresponding boundary conditions, equation (6), are

\[
f'(0) = 1, \quad f'(0) = 0, \quad \phi'(0) = -\left(\frac{Nt}{Nb}\right) \theta'(0), \quad \theta'(0) = -Bi \left[1 - \theta(0)\right], \quad \chi(0) = 1 \quad \text{at} \quad \eta \to 0
\]

\[
f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0, \quad \chi(\infty) = 0 \quad \text{as} \quad \eta \to \infty
\]

Now we introduce the following dimensionless quantities.

\[
\lambda = \frac{g \beta (1 - C) (T_w - T_e)}{C^2 x}, \quad Gr_x = \frac{g \beta (1 - C) (T_w - T_e) x^3}{v^2}, \quad Re_x = \frac{u x}{v}
\]

\[
M = \frac{\sigma f^2}{\rho f c}, \quad Nr = \frac{(\rho_p - \rho f c) C}{\rho f c (1 - C) (T_w - T_e)}, \quad Pr = \frac{v f}{\alpha f}, \quad Nb = \frac{\tau D f C}{v f}
\]

\[Nt = \frac{\tau D f (T_w - T_e)}{T_e v f}, \quad Sc = \frac{v f}{D f}, \quad \sigma = \frac{k_f}{c}, \quad E = \frac{E_a}{KT_e}
\]

\[\delta = \frac{T_w - T_e}{T_e}, \quad Nc = \frac{\beta (S - S_e)}{T_w - T_e}, \quad Rd = \frac{4 \sigma^* T_e^3}{k K'}
\]

The physical quantities describing the skin friction \(C_f\), the Local Nusselt number \(Nu_x\), the local Sherwood number \(Sh_x\), and the local nanoparticle Sherwood number \(Nn_x\) are shown below.

\[
C_f = \frac{\tau_w}{\rho U_w^2}, \quad Nu_x = \frac{x q_w}{k (T - T_e)}, \quad Sh_x = \frac{x q_s}{D_s (S - S_e)}, \quad Nn_x = \frac{x q_n}{D_b (C - C_e)} \quad (17)
\]

Where \(\tau_w = \left[ \frac{\partial u}{\partial y} \right]_{y = 0}\), \(q_w = -k \left[ \frac{\partial T}{\partial y} \right]_{y = 0}\), \(q_s = -D_s \left[ \frac{\partial S}{\partial y} \right]_{y = 0}\), \(q_n = -D_b \left[ \frac{\partial C}{\partial y} \right]_{y = 0}\)

Using the non-dimensional functions and similarity variable given in eq. (9), we obtain

\[
C_f (Re_x)^{\frac{1}{2}} = f''(0), \quad \frac{Nu_x}{(Re_x)^{\frac{1}{2}}} = -\theta'(0), \quad \frac{Sh_x}{(Re_x)^{\frac{1}{2}}} = -\chi'(0), \quad \frac{Nn_x}{(Re_x)^{\frac{1}{2}}} = -\phi'(0) \quad (18)
\]

Where \(Re_x = \frac{x U_w(x)}{v}\) defined as the local Reynolds’s number.
3. Results and discussion

The combined influence of chemical reaction, activation energy, thermal radiation, buoyancy force on magnetohydrodynamic (MHD) nanofluid flow over a stretching vertical surface is considered in this study. By using appropriate boundary conditions, the governing problem is solved numerically through the MATLAB bvp4c. The present numerical results are compared with the results of Mustafa et al. [39] for dissimilar values and they are exposed in the Table 1. This provides sureness in our numerical results to be conveyed subsequently. The effect of local Nusselt number \(-\theta'(0)\), thermophoresis parameter \(\mathcal{Nt}\) and mixed convection parameter \(\mathcal{A}\) are tabulated. It is observed that the Prandtl number and mixed convection parameters are increases and the thermophoresis parameter reduces on increasing the local Nusselt number.

Fig-2 and Fig-3 depicts the effect of various values of regular buoyancy parameter \(\mathcal{Nc}\) on velocity profile \(\left( f'(\eta) \right) \), solute concentration profile \(\left( \chi(\eta) \right) \). Fig-2 represents the increment of the velocity of the fluid with regular buoyancy parameter and gradual decrement in the thickness of the boundary layer. From Fig-3 we noticed that if the value of \(\mathcal{Nc}\) increases on decreasing the value of solute concentration profile and the boundary layer thickness decreases gradually. Fig-4 to Fig-6 indicates the influence of diverse values of thermal radiation \(\mathcal{Rd}\) on velocity profile \(\left( f'(\eta) \right) \), temperature profile \(\left( \theta(\eta) \right) \) and nanoparticle concentration profile respectively. Here velocity, temperature and fluid concentration are increases with increasing the thermal radiation parameter and produces a thinner boundary layer with gradual decrement. From thermal radiation, we can control the fluid temperature because the temperature is very sensitive for thermal radiation. It means that heat flux is more at the surface. Fig-7 represent the impact of Biot number \(\mathcal{Bi}\) on temperature profile. It shows that the temperature of the fluid increases and the boundary layer decreases gradually on increasing the values of \(\mathcal{Bi}\). An increase of Biot number correlates to a low Brownian movement, which contributes to quick dissemination of concentration at the surface. Fig-8 and Fig-9 represents the effect of different values of temperature difference parameter \(\left( \delta' \right) \) on temperature profile and nanoparticle concentration profile respectively. Fig-8 and Fig-9 illustrate that the temperature and nanoparticle concentration profile increases on increasing the values of \(\left( \delta' \right) \), and the thickness of the boundary layer decreases gradually. The impact of various values of the Schmidt number \(\left( \mathcal{Sc} \right) \) on the solute concentration profile is represented in Fig-10. It is observed that the Schmidt number increases on decreasing the solute concentration and rapid decrement in corresponding boundary layer thickness. Fig-11 represents the influence of various values of non-dimensional activation energy \(\left( \mathcal{E} \right) \) on the velocity profile. It can be perceived that the fluid velocity decreases with increasing \(\mathcal{E}\) and the thickness of the boundary layer decreases gradually.

Fig-12 displays the Skin friction increases on increasing the values of regular buoyancy parameter \(\mathcal{Nc}\) and fitted rate constant \(\mathcal{n}\) i.e., friction on the vertical surface is increased. Fig-13 illustrates the local Nusselt number for varying values of regular buoyancy parameter \(\mathcal{Nc}\), against the fitted rate constant. From the
stretching boundary, it is clear that heat flux is inversely proportional to the regular buoyancy force. Moreover, 
\(-\theta'(0)\) decays in a nonlinear fashion when \(' n'\) is increased. For the increment of regular buoyancy parameter 
\((Nc)\) and fitted rate constant the local nanoparticle Sherwood numbers increase and the layer thickness also 
increases as shown Fig-14. It indicates the effectiveness of mass convection at the surface. Fig-15 sketched to 
Skin friction with regular buoyancy parameter and Biot number. It is observed that the vertical surface friction 
increases with \(' Nc'\), but decreases with \(' Bi'\) and layer thickness decreases gradually. Fig-16 depicts the reduced 
heat transfer rate with increased regular buoyancy parameter and increased thermal flux with Biot number.

The local Sherwood number decreases on increasing the Biot number as shown in Fig-17. The flow of 
heat transfer is decreased with increasing regular buoyancy parameter, but increases with thermal radiation, and 
layer thickness is decreased, which are represented in Fig-18. Increment of local nanoparticle Sherwood number 
with regular buoyancy parameter clearly shown in Fig-19. But with increased thermal radiation, the local nanoparticle Sherwood number decreases.

4. Conclusion

The collective impact of chemical reaction, activation energy, thermal radiation, buoyancy force on magneto-
hydrodynamic (MHD) nanofluid flow over a stretching vertical surface is described numerically. From this investigation, we detect the following:

- The velocity profile at the vertical surface increases on increasing the regular buoyancy parameter and thermal radiation, but reduces with activation energy.
- The temperature of the fluid is raised with thermal radiation, Biot number and temperature difference parameter.
- Nanoparticle concentration of the fluid is increases on increasing the thermal radiation and temperature difference parameter.
- The fluid solute concentration reduces on increasing of regular buoyancy parameter and Schmidt number.
- Skin friction of the fluid at the vertical surface increases with regular buoyancy parameter and fitted rate constant, i.e. velocity of the fluid is decreasing. But Skin friction reduces with increasing the Biot number and thermal radiation, i.e. fluid velocity is increasing.

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Table 1: Comparison of wall slope of temperature $-\theta'(0)$ values with Mustafa et al. [39] for $M = Nt = 0.5$, $\delta = 1$ and $Sc = 5$.

| Pr | $\dot{N}t$ | $E$ | $\sigma$ | $n$ | $\lambda$ | Mustafa et al. [39] | Present study  |
|----|----------|----|--------|----|---------|-------------------|---------------|
| 2  | 0.5      | 1  | 1      | 0.5| 0.5     | 0.706605          | 0.706831      |
| 4  |          |    |        |    |         | 0.935952          | 0.936056      |
| 7  |          |    |        |    |         | 1.132787          | 1.132900      |
| 10 |          |    |        |    |         | 1.257476          | 1.257032      |
| 5  | 0.1      | 1  | 1      | 0.5| 0.5     | 1.426267          | 1.426012      |
| 5  | 0.5      |    |        |    |         | 1.013939          | 1.038594      |
| 5  | 0.7      |    |        |    |         | 0.846943          | 0.846593      |
| 5  | 1.0      |    |        |    |         | 0.649940          | 0.649752      |
| 10 | 0.5      | 1  | 2      | 0.5| 0.0     | 1.032281          | 1.032085      |
| 0.5|          |    |        |    |         | 1.056704          | 1.056294      |
| 3.0|          |    |        |    |         | 1.154539          | 1.154956      |
| 5.0|          |    |        |    |         | 1.215937          | 1.216012      |

Figure 1. Schematic diagram of the problem
Figure 2. Velocity profile for various values of regular buoyancy parameter $Nc$, when $M = 0.1, \lambda = -1$, $Pr = 7, Rd = 0.3, E = 2, Nb = Nr = Nt = n = 0.5, \delta = \sigma = Bi = Sc = 1$.

Figure 3. Solute concentration profile for various values of regular buoyancy parameter $Nc$, when $Nr = 0.1$, $Pr = 7, Rd = 0.3, E = 2, \lambda = Nb = Nt = n = 0.5, \delta = \sigma = M = Bi = Sc = 1$.

Figure 4. Velocity profile for various values of thermal radiation parameter $Rd$, when $Nc = 0.1, Pr = 7$, $E = Nb = Nr = Nt = 0.5, \delta = \lambda = \sigma = M = Bi = Sc = n = 1$.
Figure 5. Temperature profile for various values of thermal radiation parameter $Rd$, when $Nc = 0.2$, $Pr = 7, \sigma = Nr = 0.1, \lambda = Nb = Nt = n = 0.5, \delta = E = M = Bi = Sc = 1$.

Figure 6. Nanoparticle concentration profile for various values of thermal radiation parameter $Rd$, when $Nc = 0.2$, $Pr = 7, \sigma = Nr = 0.1, \lambda = Nb = Nt = n = 0.5, \delta = E = M = Bi = Sc = 1$.

Figure 7. Temperature profile for various values of Biot number $Bi$, when $Nc = 0.2$, $Rd = 0.3$, $Pr = 7$, $\sigma = Nr = 0.1, \delta = E = M = Sc = 1, \lambda = Nb = Nt = n = 0.5$. 

Figure 8. Temperature profile for various values of temperature difference parameter $\tilde{\delta}$, when $Pr = 5$, $Nc = Nr = Rd = 0.1, \lambda = Nb = Nt = 0.5, \sigma = E = M = Bi = Sc = n = 1$.

Figure 9. Nanoparticle concentration profile for various values of temperature difference parameter $\tilde{\delta}$, when $Pr = 5, Nc = Nr = Rd = 0.1, Nb = Nt = \lambda = 0.5, \sigma = E = M = Bi = Sc = n = 1$.

Figure 10. Solute concentration profile for various values of Schmidt number $Sc$, when $Pr = 5, Nr = Nc = Rd = 0.1, \lambda = Nb = Nt = 0.5, \sigma = E = M = Bi = n = 1$. 
Figure 11. Velocity profile for various values of non-dimensional activation energy $E$, when $\text{Pr} = 7$, $\text{Pr} = 0.1, \text{Rd} = 0.3, \lambda = \text{Nb} = \text{Nr} = n = 0.5, \delta = \sigma = M = Bi = Sc = 1$.

Figure 12. Skin friction for various values of $N_c$ and $n$, when $\text{Pr} = 7, \text{Nr} = 0.1, \text{Rd} = 0.3, \text{Nb} = N_t = \lambda = 0.5, \delta = \sigma = E = M = Bi = Sc = 1$.

Figure 13. Local Nusselt number for various values of $N_c$ and $n$, when $\text{Pr} = 7, \text{Nr} = 0.1, \text{Rd} = 0.3, \lambda = \text{Nb} = N_t = 0.5, \delta = \sigma = E = M = Bi = Sc = 1$. 
Figure 14. Local Nanoparticle Sherwood number for various values of $Nc$ and $n$, when $Pr = 7$, $Nr = 0.1$, $Rd = 0.3$, $\lambda = Nb = Nt = 0.5$, $\delta = \sigma = E = M = Bi = Sc = 1$.

Figure 15. Skin friction for various values of $Nc$ and $Bi$, when $Pr = 5$, $Nr = 0.1$, $Rd = 0.3$, $\lambda = Nb = Nt = 0.5$, $\delta = \sigma = E = M = Sc = n = 1$.

Figure 16. Local Nusselt number for various values of $Nc$ and $Bi$, when $Pr = 5$, $Nr = 0.1$, $Rd = 0.3$, $\lambda = Nb = Nt = 0.5$, $\delta = \sigma = E = M = Sc = n = 1$. 
Figure 17. Local Sherwood number for various values of $N_c$ and $Bi$, when $Pr = 5, Nr = 0.1, Rd = 0.3, \lambda = Nb = Nt = 0.5, \delta = \sigma = E = M = Sc = n = 1$.

Figure 18. Local Nusselt number for various values of $N_c$ and $Rd$, when $Pr = 5, \sigma = Nr = 0.1, \lambda = Nb = Nt = 0.5, \delta = E = M = Bi = Sc = n = 1$.

Figure 19. Local Nanoparticle Sherwood number for various values of $N_c$ and $Rd$, when $Pr = 5, \sigma = Nr = 0.1, \lambda = Nb = Nt = 0.5, \delta = E = M = Bi = Sc = n = 1$. 
Nomenclature

$B_0$ - Strength of magnetic field

$C$ - Concentration

$D_B$ - Brownian diffusion coefficient

$D_T$ - Thermophoretic diffusion coefficient

$E$ - Non-dimensional activation energy

$E_a$ - Activation energy

$f'$ - Dimensionless velocity

$Gr_i$ - Local Grashof number

$g$ - Gravitational acceleration

$K$ - Boltzmann constant

$K'$ - Mean absorption coefficient

$k$ - Thermal conductivity

$k_r^2$ - Reaction rate

$M$ - Magnetic field parameter

$Nb$ - Brownian diffusion parameter

$Nc$ - Regular buoyancy parameter

$Nr$ - Buoyancy ratio parameter

$Nt$ - Thermophoresis parameter

$n$ - Fitted rate constant

$Pr$ - Prandtl number of base fluid

$q_r$ - Radiative heat flux

$Rd$ - Thermal radiation parameter

$Re_x$ - Local Reynolds number

$Sc$ - Schmidt number

$T$ - Temperature

Greek Symbols

$\lambda$ - Mixed convection parameter

$\sigma$ - Dimensionless reaction rate

$\delta$ - Temperature difference parameter
\( \nu \) - Kinematic viscosity
\( \sigma_e \) - Electrical conductivity
\( \sigma^* \) - Stefan-Boltzmann constant
\( \rho_f \) - Density of the fluid
\( \beta \) - Coefficient of thermal expansion
\( \rho_p \) - Nanoparticle density
\( \alpha \) - Thermal diffusivity of the base fluid
\( \tau \) - Ratio of the effective heat capacity of the nanoparticle material and the heat capacity of the fluid
\( (\rho c)_f \) - Heat capacity of the base fluid
\( (\rho c)_p \) - Effective heat capacity of the nanoparticle material
\( \theta \) - Dimensionless temperature
\( \chi \) - Dimensionless solute concentration
\( \phi \) - Dimensionless nanoparticle concentration

Subscripts
\( w \) - Condition at a wall
\( \infty \) - Condition at free stream

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