Two-dimensional multicomponent Abelian-Higgs lattice models

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(Dated: December 4, 2019)

We study the two-dimensional lattice multicomponent Abelian-Higgs model, which is a lattice compact U(1) gauge theory coupled with an N-component complex scalar field, characterized by a global SU(N) symmetry. In agreement with the Mermin-Wagner theorem, the model has only a disordered phase at finite temperature and a critical behavior is only observed in the zero-temperature limit. The universal features are investigated by numerical analyses of the finite-size scaling behavior in the zero-temperature limit. The results show that the renormalization-group flow of the 2D lattice N-component Abelian-Higgs model is asymptotically controlled by the infinite gauge-coupling fixed point, associated with the universality class of the 2D CPN−1 field theory.

I. INTRODUCTION

Models of complex scalar matter fields with abelian and nonabelian gauge symmetries effectively emerge in several interesting systems, such as superconductors and superfluids, quantum Hall states, quantum SU(N) antiferromagnets, unconventional quantum phase transitions, etc., see, e.g., Refs. [1–12] and references therein. Among the paradigmatic models considered, an important role is played by the multicomponent lattice Abelian-Higgs (AH) model or lattice scalar electrodynamics, which is a lattice U(1) gauge theory coupled with an N-component complex scalar field, characterized by a global SU(N) symmetry. Its phase diagram has been thoroughly investigated in three dimensions, considering both the compact and noncompact formulation of electrodynamics, see, e.g., Refs. [13–27]. On the other hand, multicomponent lattice AH systems have been much less considered in two dimensions. Beside their theoretical interest—their study allows us to deepen our understanding of statistical field theories with gauge symmetries—2D lattice AH models may turn out to be relevant for actual physical systems as well, as they may emerge in particular regimes of low-dimensional condensed-matter systems, for example in systems of ultracold atoms in optical lattices [28].

A lattice formulation of the AH model on a square lattice is obtained by considering N-dimensional complex unit vector \( \mathbf{z}_x \) defined on the sites of the lattice (they satisfy \( \mathbf{z}_x \cdot \mathbf{z}_x = 1 \)), U(1) link variables \( \lambda_{x,\mu} \equiv e^{i\varphi_{x,\mu}} \), and the Hamiltonian

\[
H = -J N \sum_{x,\mu} (\mathbf{z}_x \cdot \lambda_{x,\mu} \mathbf{z}_{x+\hat{\mu}} + \text{c.c.}) \tag{1}
\]

\[
- \gamma \sum_{x,\mu>\nu} (\lambda_{x,\mu} \lambda_{x+\hat{\mu},\nu} \lambda_{x+\hat{\nu},\mu} \lambda_{x,\nu} + \text{c.c.}) ,
\]

where \( \hat{\mu} = \hat{1}, \hat{2} \) are unit vectors along the lattice directions, the first sum runs over all lattice links, while the second one runs over all plaquettes. Note that we use here the standard Wilson compact formulation of electrodynamics. The partition function of the system reads

\[
Z = \sum_{\{\lambda\}} e^{-\beta H}, \quad \beta \equiv 1/T. \tag{2}
\]

We set \( J = 1 \) without loss of generality (the explicit factor of \( N \) in the first term of the Hamiltonian is a standard choice, useful when one considers the large-\( N \) limit). One can easily verify that the AH model (1) is invariant under a local U(1) gauge symmetry \( \mathbf{z}_x \to e^{i\lambda_x} \mathbf{z}_x \), \( \lambda_{x,\mu} \to e^{i\lambda_x} \lambda_{x,\mu} e^{-i\lambda_x} \), and under a global SU(N) symmetry \( \mathbf{z}_x \to U \mathbf{z}_x \) with \( U \in \text{SU}(N) \). The parameter \( \gamma \) plays the role of inverse gauge coupling.

For some particular values of \( \gamma \) the AH model is equivalent to simpler models. In the limit \( \gamma \to \infty \), the gauge link variables are all equal to one modulo gauge transformations, and the AH model becomes equivalent to the standard \( O(n) \) vector model with \( n = 2N \). For \( \gamma = 0 \), corresponding to the infinite gauge-coupling limit, the AH model (1) is a particular lattice formulation of the CPN−1 model [29–34]. 2D CPN−1 models have been much studied in the literature, both analytically and numerically, because they provide a theoretical laboratory to understand some of the mechanisms of quantum field theories of fundamental interactions. In particular, they share some notable features with quantum chromodynamics (QCD), the theory of the hadronic strong interactions, such as the asymptotic freedom and the so-called \( \theta \) dependence related to topology, see, e.g., Refs. [35–38].

For both \( \gamma = 0 \) and \( \gamma = \infty \), the lattice model shows a universal critical behavior for \( \beta \to \infty \). In this limit, the correlation length \( \xi \) increases exponentially, \( \xi \sim e^{c\beta} \). Of course, the critical behaviors for \( \gamma = 0 \) and \( \gamma \to \infty \) differ, belonging to the universality class of the 2D CPN−1 model for \( \gamma = 0 \) and to that of the 2D O(2N) vector model for \( \gamma \to \infty \).

In this paper we investigate the phase diagram of 2D multicomponent lattice AH models for generic values of the gauge parameter \( \gamma \), studying their critical behavior in the zero-temperature limit. In three dimensions the AH model shows two phases, separated by a transition where the global SU(N) symmetry is broken and the gauge-
condenses. In two dimensions, according to the Mermin-Wagner theorem [39], such a condensation cannot occur. Thus, we expect the existence of a unique disordered phase. A critical behavior only occurs for $\beta \to \infty$, as it happens in the 2D CP$^{N-1}$ model.

To investigate the critical behavior of 2D lattice AH models in the zero-temperature limit, we present finite-size scaling (FSS) analyses of numerical results obtained by Monte Carlo (MC) simulations. The asymptotic low-temperature behavior for generic finite values of $\gamma$ turns out to be independent of $\gamma$, at least for $\gamma$ not too negative. Therefore, it belongs to the 2D universality class of the 2D CP$^{N-1}$ model. For any $\gamma$ we observe an exponentially increase of the correlation length, the same universal FSS curves, and the same dimensionless renormalization-group (RG) invariant combinations of observables in the thermodynamic limit. In the language of RG theory, the gauge coupling flows toward the infinite gauge-coupling limit ($\gamma \to 0$), which represents the stable fixed point of the RG flow. In the zero gauge-coupling limit ($\gamma \to \infty$), the asymptotic large-$\beta$ behavior is expected to change into that of the 2D O($2N$) $\sigma$ model, which corresponds to an unstable fixed point of the RG flow.

The paper is organized as follows. In Sec. II we describe the FSS framework which we employ to investigate the low-temperature critical behavior. In Sec. III we discuss two particular limits of the lattice AH model: the infinite and zero gauge-coupling limits. In Sec. IV we discuss the numerical results for $N = 2$ and $N = 10$. Finally, in Sec. V we draw our conclusions.

II. FINITE-SIZE SCALING

In this paper we investigate the nature of the asymptotic large-$\beta$ behavior of the lattice multicomponent AH model for different values of $\gamma$. For this purpose we consider AH models on a square lattice of linear size $L$ with periodic boundary conditions.

We mostly focus on correlations of the gauge-invariant local matrix variable $Q^a_x$ defined in Eq. (3), which is hermitean and traceless. Its two-point correlation function is defined as

$$G(x - y) = \langle \text{Tr} Q_x Q_y \rangle,$$

where the translation invariance of the system has been taken into account. The susceptibility and the correlation length are defined as $\chi = \sum_x G(x)$ and

$$\xi^2 \equiv \frac{1}{4 \sin^2 (\pi / L)} \frac{\tilde{G}(0) - \tilde{G}(p_m)}{\tilde{G}(p_m)},$$

where $\tilde{G}(p) = \sum_x e^{ip \cdot x} G(x)$ is the Fourier transform of $G(x)$, and $p_m = (2\pi / L, 0)$. We also consider the Binder parameter defined as

$$U = \frac{\langle \mu_2^2 \rangle}{\langle \mu_2 \rangle^2}, \quad \mu_2 = \sum_{x,y} \text{Tr} Q_x Q_y.$$

To determine the universal features of the asymptotic behavior of the AH model, we use a FSS approach [40–43]. At finite-temperature continuous transitions the FSS limit is obtained by taking $\beta \to \beta_c$ and $L \to \infty$ keeping $X \equiv (\beta - \beta_c) L^{1/\nu}$ fixed, where $\beta_c$ is the inverse critical temperature and $\nu$ is the correlation-length exponent. Any RG invariant quantity $R$, such as the ratio

$$R_X \equiv \xi / L,$$

and the Binder parameter $U$, is expected to asymptotically behave as $R(\beta, L) = f_R(X) + O(L^{-\omega})$, where $\omega$ is a universal exponent. The scaling function $f_R(X)$ is universal apart from a trivial normalization of its argument; it only depends on the shape of the lattice and on the boundary conditions. Since $R_X$ is generally monotonic, we can also write [41–44],

$$R(\beta, L) = F_R(R_X) + O(L^{-\omega}),$$

where $F_R$ is a universal scaling function. Eq. (8) is particularly convenient, as it allows a direct check of universality, without the need of tuning any parameter. Moreover, it applies directly, without any change, to two-dimensional asymptotically free models, like the 2D CP$^{N-1}$ model and the 2D O($N$) nonlinear $\sigma$ model [29], in which a critical behavior is only obtained in the limit $\beta \to \infty$, see Refs. [46–50] and references therein. In this case, scaling corrections decay as $L^{-2 \log p} L$, where $p$ cannot be determined in perturbation theory (see Ref. [49] for a discussion in the O($N$) model).

In the following, we will consider the finite-size behavior of the Binder parameter $U$, which varies between [20]

$$\lim_{R_X \to 0} U = \frac{N^2 + 1}{N^2 - 1}, \quad \lim_{R_X \to \infty} U = 1.$$
III. INFINITE AND ZERO GAUGE COUPLING

In this section we discuss two particular limits of the lattice AH model: the infinite and zero gauge-coupling limits, corresponding to $\gamma = 0$ and $\gamma \to \infty$ respectively.

For $\gamma = 0$, the AH model corresponds to a lattice formulation of the CP$^{N-1}$ model. Indeed, for $\gamma = 0$ one can integrate out the link variables $\lambda_{x,\mu}$ in the partition function, obtaining

$$Z = \sum_{\{z\}} \prod_{x,\mu} I_0(2\beta N|\bar{z}_x \cdot z_{x+\hat{\mu}}|),$$

where $I_0(x)$ is a modified Bessel function. The corresponding effective Hamiltonian reads

$$H_{\text{eff}} = -\beta^{-1} \sum_{x,\mu} \ln I_0(2\beta N|\bar{z}_x \cdot z_{x+\hat{\mu}}|).$$

Taking into account that $I_0(x) = 1 + x^2/4 + O(x^4)$, one recovers the CP$^{N-1}$ model in the continuum limit, i.e., the quantum field theory defined as

$$Z = \int [dz] \exp \left[ -\int dx \mathcal{L}(z) \right],$$

$$\mathcal{L} = \frac{1}{2g} D_\mu \bar{z} \cdot D_\mu z, \quad D_\mu = \partial_\mu + iA_\mu,$$

where $A_\mu = i\bar{z} \cdot \partial_\mu z$ is a composite gauge field. For $N = 2$ the CP$^1$ field theory is locally isomorphic to the O(3) non-linear $\sigma$ model. The mapping is obtained by identifying the three-component real vector $s^a_x$ with the combination $\sum_{ij} \bar{s}^a_i \sigma^a_{ij} s^j_x$, where $a = 1, 2, 3$ and $\sigma^a$ are the Pauli matrices.

In two dimensions, the lattice CP$^{N-1}$ model is asymptotically free, a property it shares with QCD, the theory of strong interactions. It does not show a finite-temperature transition, but nonetheless it shows a universal behavior for $\beta \to \infty$. In this limit the correlation length behaves as (see, e.g., Ref. [33])

$$\xi = A(2\pi\beta)^{-2/N} e^{2\pi\beta [1 + O(\beta^{-1})]},$$

where $A$ is a model-dependent constant.

In the limit $\gamma \to \infty$, the gauge link variables are all equal to one, modulo gauge transformations. Therefore, the lattice AH model can be exactly mapped onto a lattice O($n$) vector model with $n = 2N$, with standard Hamiltonian

$$H_{O(n)} = -\sum_{x,\mu} s^x_x \cdot s^x_{x+\hat{\mu}},$$

where $s^x_x$ are $n$-component real vectors satisfying $s^x_x \cdot s^x_x = 1$. We recall that in two dimensions, also O($n$) vector models are asymptotically free: A critical behavior is only obtained for $\beta \to \infty$. In this limit, the correlation length increases exponentially [29]:

$$\xi = B(2\pi\beta)^{1/(n-2)} e^{2\pi\beta},$$

where $B$ is a model-dependent constant and $\beta = \beta/(n - 2)$. For the lattice O($n$) vector model, one can define a second-moment correlation using Eq. 5 and the two-point function

$$G_{O(n)}(x, y) \equiv \langle s^x_x \cdot s^y_y \rangle.$$  \hspace{1cm} (16)

The Binder parameter is defined analogously as

$$U_{O(n)} = \frac{\langle m^2 \rangle}{\langle m^2 \rangle^2}, \quad m^2 = \sum_{x,y} G_{O(n)}(x, y).$$  \hspace{1cm} (17)

Although the AH model is mapped onto the O($n$) model for $\gamma \to \infty$, one should note that the mapping between AH and O($n$) observables is not trivial. The correlation length and the Binder parameter defined using Eqs. (16) and (17) in the O($n$) model do not correspond to those defined in the AH model. The correct correspondence is discussed in Ref. [27], where the same issue was discussed in three dimensions.

IV. NUMERICAL RESULTS IN THE ZERO-TEMPERATURE LIMIT

To identify the universal features of the large-$\beta$ behavior of the AH model, we have performed MC simulations for $N = 2$ and 10 and several values of $\gamma$. The linearity of Hamiltonian 11 with respect to each lattice variable allows us to employ an overrelaxed algorithm for the updating of the lattice configurations. It consists in a stochastic mixing of microcanonical and standard Metropolis updates. To update each lattice variable, we randomly choose either a standard Metropolis update, which ensures ergodicity, or a microcanonical move, which is more efficient than the Metropolis one but does not change the energy. On average, we perform three/four microcanonical updates for every Metropolis proposal. In the Metropolis update, changes are tuned so that the acceptance is 1/3. The same algorithm was used in Ref. [27] for 3D systems.

A. The lattice AH model with $N = 2$

We first consider the $N = 2$ AH model. We will show that the critical behavior in the zero-temperature limit belongs to the universality class of the 2D O(3) non-linear $\sigma$ model independently of $\gamma$, at least for $\gamma$ not too negative.

To begin with, we present results for $\gamma = 0$, i.e., for the CP$^1$ model. As discussed in Sec. III the CP$^1$ lattice model can be mapped onto a lattice O(3) $\sigma$ model, although different from the standard one reported in Eq. (14), see e.g. Ref. [33]. It is easy to verify that, under this mapping, the two-point function, the second-moment correlation length $\xi$, and the Binder parameter $U$ of the CP$^1$ model, defined in Eqs. (4), (5) and (6), respectively, correspond to those of the O(3) vector model...
FIG. 1: Estimates of $U$ versus $R_\xi$ for the CP$^1$ model ($N = 2$ AH model with $\gamma = 0$), up to $L = 144$, and for the standard lattice O(3) $\sigma$ model (up to $L = 100$). The data appear to converge to the same FSS curve. The horizontal dashed line corresponds to the asymptotic value $U = 5/3$ for $R_\xi \to 0$. The full line represents an interpolation of the O(3) data, which provides the universal FSS curve with an accuracy of few per mille for $R_\xi \lesssim 0.8$ [51].

FIG. 2: Plot of $U$ versus $R_\xi$ for $\gamma \to \infty$, as computed in the O(4) vector model. The horizontal dashed line corresponds to the asymptotic value $U = 5/3$ for $R_\xi \to 0$. For comparison we also report the O(3)/CP$^1$ FSS curve ($\gamma = 0$), obtained from the interpolation [51] of the O(3) data shown in Fig. 1.

FIG. 3: Plot of $U$ versus $R_\xi$ for the $N = 2$ AH model for several lattice sizes and values of $\gamma$ in the range $[-2, 4]$. The data appear to approach the universal curve associated with the O(3) universality class (full line, $\gamma = 0$), obtained by interpolating [51] the O(3) results shown in Fig. 1.

defined in Eqs. (16) and (17). We thus expect the large-$\beta$ critical behavior of the AH model for $\gamma = 0$ to have the same universal features as that of the O(3) $\sigma$ model. In Fig. 1 we report the Binder parameter $U$ versus $R_\xi$, up to $L = 144$. These results are compared with those obtained by cluster [52] simulations of the standard O(3) $\sigma$ model [14] up to $L = 100$. As expected, the data of $U$ versus $R_\xi$ for the two lattice models converge towards the same scaling curve with increasing $L$, confirming that they belong to the same 2D universality class.

In the limit $\gamma \to \infty$, the $N = 2$ AH model becomes equivalent to the standard lattice O(4) vector model defined in Eq. (14). Therefore, for $\gamma \to \infty$ the asymptotic large-$\beta$ behavior should correspond to that of the O(4) vector model. In Fig. 2 we show the AH Binder parameter defined in Eq. (10) as computed in the O(4) model (as already mentioned, this is not the usual O(4) Binder parameter, see Ref. [27] for details). The curves for $\gamma = 0$ and $\gamma = \infty$ are clearly different, although they both converge to $U = 5/3$ and $U = 1$ for $R_\xi \to 0$ and $R_\xi \to \infty$, respectively.

We now consider the lattice AH model for finite nonzero values of $\gamma$. Results for $\gamma \in [-2, 4]$ are shown in Fig. 3. In all cases the data of $U$ versus $R_\xi$ appear to approach the curve of the O(3) vector model. These results show that in a wide interval of values of $\gamma$ around zero, the $N = 2$ lattice AH model has the same critical behavior as the O(3) vector model. Differences decay rapidly with $L$, consistently with the expected $O(L^{-2})$ approach.

Notice that the universality of the curves of $U$ versus $R_\xi$ for different values of $\gamma$ is not trivial, since the dependence on $\beta$ significantly changes when changing $\gamma$, as demonstrated in Fig. 4. In the language of asymptotically free theories, see, e.g., Ref. [33], the 2D lattice AH models with different values of $\gamma$ are variant actions of the 2D CP$^{N-1}$ model with $\Lambda$ parameters that significantly depend on $\gamma$.

On the basis of the previous results, we conjecture that the $N = 2$ AH model has the same critical behavior as the O(3) vector model for any positive finite $\gamma$. The same conjecture may not hold for all negative values. Indeed, for $\gamma$ large and negative one would obtain a fully frustrated model, which might have a critical behavior distinctly different from that of a ferromagnetic model. This indeed occurs in the fully frustrated XY model [52], which corresponds to the $N = 1$ AH model in the $\gamma \to -\infty$ limit. In any case, our results show that, at least for $\gamma > -2$, the behavior is the same as that for $\gamma \geq 0$. In
the lattice AH model in the limit

$$U$$

with

$$U$$

FIG. 4: Estimates of $$U$$ versus $$\beta$$ for the $$N = 2$$ AH model for several values of $$\gamma$$, in the range $$-1 \leq \gamma \leq 4$$. Results for

$$L = 50$$.

$$N = 2$$

FIG. 5: Estimates of $$U$$ versus $$R_\xi$$ for the $$N = 2$$ AH model with $$\gamma = 8, 12$$. Data are compared with the FSS curves of the lattice AH model in the limit $$\gamma \to \infty$$ (dashed line) and $$\gamma = 0$$ (full line). The data for the smallest lattice sizes are close to the $$\gamma \to \infty$$ FSS curve, then they approach the $$\gamma = 0$$ curve with increasing $$L$$.

Since the AH model becomes equivalent to the O(4) vector model for $$\gamma \to \infty$$, we expect significant crossover phenomena for large values of $$\gamma$$. This is confirmed by the MC data for $$\gamma = 8$$ and 12 reported in Fig. 5. They provide evidence of the predicted crossover behavior. For small values of $$L$$ the data are close to the O(4) ($$\gamma = \infty$$) scaling curve, and then move systematically towards the asymptotic O(3) curve.

It is also worth discussing the behavior of gauge-invariant correlations of the vector variable $$z_x$$, such as

$$\langle z_x \cdot z_y \prod_{\ell \in \mathcal{C}} \lambda_\ell \rangle$$

where the product extends over the link variables that belong to a lattice path $$\mathcal{C}$$ connecting points $$x$$ and $$y$$. In particular, we consider correlations between points that belong to lattice straight lines. We select a generic lattice direction $$\hat{v}$$ and define the vector correlation function

$$G_v(\ell, \beta) = \frac{1}{L^2} \sum_x \text{Re} \left( \langle z_x \cdot z_y \prod_{n=0}^{\ell-1} \lambda_{x+n\hat{v},y} \rangle \right),$$  

(18)

where $$y = x + \ell \hat{v}$$ and all coordinates should be taken modulo $$L$$, because of the periodic boundary conditions.

Since the AH model is controlled by the stable infinite gauge-coupling fixed point, belonging to the 2D O(3) vector universality class.

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(18)

where $$y = x + \ell \hat{v}$$ and all coordinates should be taken modulo $$L$$, because of the periodic boundary conditions. We average over all lattice sites $$x$$ exploiting the translation invariance of systems with periodic boundary conditions. Note that the correlation length $$\xi$$ is not periodic, i.e., it does not satisfy $$G_v(\ell + L, \beta) = G_v(\ell, \beta)$$, because of the string of gauge fields between the spatial points.

For sufficiently large lattice sizes and relatively small distances $$\ell$$, $$G_v$$ has an approximately exponentially decaying behavior. By assuming that $$G_v \sim e^{-\ell/\xi}$$, one can define a vector length scale as

$$\xi_{v,X} = L X/2 \ln[|G_v(LX/2, \beta)/G_v(LX, \beta)|],$$  

(19)

where $$0 \leq X < 1$$ is a parameter. We have measured $$\xi_{v,X}$$ for $$X = 1/4$$ (other values of $$X$$ give similar results) and $$\gamma = 0$$. At variance with what happens with $$\xi$$, $$\xi_{v,X}$$ shows very small size corrections. The asymptotic infinite-volume results are obtained on quite small lattices, even when $$\xi$$ is still of the order of $$L$$. Apparently, gauge modes and gauge-invariant modes encoded in $$\xi$$ are little correlated. The results, reported in Fig. 6, are definitely consistent with a simple power-law behavior: apparently they are compatible with $$\xi_v \sim \beta^2$$. Therefore, for $$\beta \to \infty$$, $$\xi_v$$ increases much slower than $$\xi$$, which instead increases exponentially, see Fig. 6. These results imply

$$\xi_v/\xi \to 0$$  

(20)
exponentially, for $\beta \to \infty$. Therefore, vector and gauge modes are irrelevant in the continuum limit, explaining why $\gamma$ is an irrelevant parameter in the theory, i.e., why it does not change the universality class of the model.

B. The lattice AH model with $N = 10$

We now consider the lattice AH model with $N = 10$. Fig. 7 shows $U$ versus $R_{\xi}$ for the model with $\gamma = 0$, which is a particular lattice formulation of the CP$^9$ model. The data converge to a universal FSS curve. At variance with the CP$^1$ model, the FSS curve is nonmonotonic. It shows a pronounced peak, which is related to the presence of bound states in the large-$N$ limit, whose physical size increases as $N^{1/3}$, as already noted in Ref. [54].

As in the $N = 2$ case, the numerical results show that the asymptotic FSS curve is independent of $\gamma$. This is clearly shown by the data reported in Fig. 8. The data fall on top of the CP$^9$ results for all values of $\gamma$ in $[-2, 8]$. Note that this agreement is not trivial, as the $\beta$ dependence of $U$ varies significantly when changing $\gamma$. We note again that larger scaling corrections are observed with increasing $\gamma$. But this is again expected, due to the fact that in the large-$\gamma$ limit the model becomes equivalent to a lattice O(20) vector model.

These results show that the RG flow of the 2D $N = 10$ lattice AH model is generally controlled by the stable infinite gauge-coupling fixed point, corresponding to the 2D CP$^9$ theory, as it occurs for $N = 2$. For large values of $\gamma$, crossover phenomena are again expected, due to the unstable zero gauge-coupling fixed point corresponding to a 2D O(20) vector model.

V. CONCLUSIONS

We have investigated the critical behavior of the 2D multicomponent lattice AH model with Hamiltonian (1). It represents a 2D lattice U(1) gauge theory coupled to an $N$-component complex scalar field. Beside the U(1) gauge invariance, it is invariant under a global SU($N$) symmetry. In agreement with the Mermin-Wagner theorem, it does not show an ordered phase where the global SU($N$) symmetry is spontaneously broken. Therefore, no finite-temperature transition occurs and, for any fixed $\gamma$, a critical behavior is only observed for $\beta \to \infty$.

To investigate the critical behavior of the model, we have performed simulations for $N = 2$ and $N = 10$ and studied the FSS behavior of RG invariant observables for different values of $\gamma$. We find that the asymptotic small-temperature behavior for generic finite values of $\gamma$ is independent of $\gamma$ (at least for $\gamma$ not too negative). Therefore, the 2D AH model belongs to the universality class of the 2D CP$^N$ model, obtained for $\gamma = 0$. Therefore, FSS curves and dimensionless RG invariant combinations of observables in the thermodynamic and in the FSS limit should be independent of $\gamma$. Moreover, $\xi$ should always be...
FIG. 9: Estimates of $U$ versus $R_{\xi}$ for the lattice $\text{CP}^2$ model obtained by setting $\gamma = 0$ in the $N = 3$ AH model. The horizontal dashed line shows the asymptotic value $U = 5/4$ for $R_{\xi} \to 0$.

FIG. 10: Estimates of $U$ versus $R_{\xi}$ for the lattice $\text{CP}^3$ model ($\gamma = 0$). The horizontal dashed line corresponds the asymptotic value $U = 17/15$ for $R_{\xi} \to 0$.

crease as $e^{c/T}$. We expect this scenario to occur for any $N \geq 2$. For instance, for any $\gamma$ the Binder parameter should have the same FSS curve as in the lattice $\text{CP}^{N-1}$ models. Thus, for $N = 3$ and $N = 4$, the universal behavior should be that shown in Figs. 9 and 10, where we report results for $\gamma = 0$.

The independence of the critical behavior on $\gamma$ is related to the subleading behavior of the gauge modes. For $\beta \to \infty$ also vector and gauge correlations order, but in a much slower fashion. In particular, in the infinite-volume limit $\xi_c/\xi \to 0$ as $\beta \to \infty$, indicating that the critical behavior is completely determined by the modes encoded in the gauge-invariant quantity $Q^{ab}$.

Summarizing, our numerical results show that the continuum 2D $\text{CP}^{N-1}$ field theory provides the stable fixed point of the RG flow of the 2D lattice AH model for any finite, not too negative $\gamma$. Note that, in the limit $\gamma \to \infty$, the AH model becomes equivalent to the $O(2N)$ $\sigma$ model. Therefore, in this limit we expect significant crossover phenomena, controlled by the unstable $O(2N)$ fixed point.

It is worth noting that the behavior of the 2D AH model presents some analogies with that of its 3D counterpart. Indeed, also in three dimensions the nature of the transition and the critical behavior, along the line separating the two phases, is the same for any finite $\gamma$. We mention that a similar behavior also emerges in multiflavor scalar chromodynamics, i.e., in lattice nonabelian gauge theories with multicomponent scalar fields.

As already mentioned in the introduction, 2D $\text{CP}^{N-1}$ models present nontrivial topological properties similar to those of QCD. Issues related to topology have been largely investigated, both analytically and numerically, see, e.g., Refs. [33, 38, 56–63]. The 2D multicomponent lattice AH model is expected to share the same topological properties of $\text{CP}^{N-1}$ models. In this paper we do not pursue these issues further, although we believe that they are worth being investigated within the general lattice AH model.

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$$f(x) = 1.66661 - (0.05343 x + 15.49447 x^2)(1 - e^{-10x}) - 1.72086 x^2 + 62.76441 x^3 - 97.24352 x^4 + 71.99411 x^5 - 20.96845 x^6$$

valid for $x \in [0, 0.8]$. The error is smaller than 0.5%.

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