Students’ reflective abstraction of middle school in reconstructing quadratic equation concept based on high mathematical ability

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Abstract. A quadratic equation is one of the mathematics concepts that must be learned in secondary school. Students have many difficulties in solving quadratic equations. One of them is experiencing a process skill error. Students need new abstractions and processes to shape experiences to distinguish whether experience falls into student abstraction or beyond student abstraction. Reflective abstraction is students' understanding of mathematics through problems that reflect and reconstruct mental structures, then the scheme is used to solve problems. The reflective abstraction mechanism that can be applied to metal structures is the APOS theory, namely: Actions, Processes, Objects, and Schemes to construct new mental structures. The process of mental reconstruction of each student is different, so for the formation of mathematical concepts, it needs mathematical abilities. In other words, if someone has good mental reconstruction abilities, hopefully, his concept formation is also good. This research conducted using a qualitative approach. The subject of this research was selected from grade 9 junior high school students in Surabaya. Students must answer two tests, namely tests of mathematical abilities and reflective abstraction tests. Analysis and categorization of scores were carried out on both tests. Then two students who have high mathematical ability was selected as subject. Semi-structured interviews is conducted individually for each subject. The purpose of this study is to find out the reflective abstraction of students that have reconstructing concept of quadratic equations in terms of high mathematical ability. The results of the study indicate that the subject can solve the problem according to the stages of the APOS.

1. Introduction

The material of quadratic equations is one of the mathematical concepts that must be studied by all students during the learning process in secondary education. Studying mathematical material, one of is of quadratic equations requires understanding a good concept, it will make students have more experience in combining and connecting their abilities. There are still difficulties that students make in solving quadratic equations, namely errors of transformation and process skills [1]. Because there are still difficulties in resolving quadratic equations, students need a mathematical concept.

Concepts are abstract ideas that can be used to classify the similarities of ideas, events, or objects that are related or have similarities [2]. The concept is formed from the process of abstraction and classification, where abstraction is a conscious activity of thinking the similarity of ideas of students in
classifying an object based on the similarity of the nature of new experiences formed and classification is gathering experiences that have similarities from the results of abstraction [3].

The abstract is a consolidated construction that can be used to create a new construction [4]. There are three types of abstractions, one of them is reflective abstraction [5]. Reflective abstraction is a construction used to reach existing structures to build new structures. Reflective abstraction is an action that reflects or feedbacks students' knowledge and understands various structured problem-solving strategies [6]. Based on above, reflective abstractions can form new concepts of students by identifying problems, then looking for solutions using various appropriate rules in finding solutions from lower stages to higher stages which refer to students' mental structures.

The abstraction mechanism that can be applied to the mental structure is should write the abbreviation of APOS for the first time [7]. Action is the cognitive structure of students in transforming objects learned as an external part in completing operations. The process is the cognitive structure of students who carry out the same operations as the actions but fully students carry out these operations as part of internal construction. The object is the cognitive structure of students who realize the stages of action and process as a totality (one unit) transformation, and actually, build transformations explicitly so it can be said that students have reconstructed the process into objects. Schema is a collection of actions, processes, and objects to form a skeletal framework of students in solving problems related to the concepts learned. APOS theory is a theory of how mathematical concepts and mental constructs can be learned. Students who solve problems by finding new concepts can be said to do construction. Construction is a reference for the emergence of new structures through constructive actions [4]. Reconstruction of different concepts in students depends on the level of reflective abstraction of students that produces new experiences or concepts. The reconstruction process of each student is different, in reconstruction or building mathematical concepts there is a need for mathematical abilities.

The mathematical ability has a significant influence on the thought process in reconstructing mathematical models to solve problems [8]. Having significant mathematical ability on student performance in understanding and finding problem-solving, is the same as constructing mathematical models [9]. In other words, if someone has a good concept of reconstruction ability, it is expected that his concept formation is also good.

Based on the description above, to find out the reflective abstraction that students have about reconstructing the concept of quadratic equations it is necessary to do research. This study can provide an empirical description of the mental structure of students in the quadratic equation material.

2. Method
The research was qualitative research with a type of case study research. This is in line with what was conveyed [10], that is, case studies are suitable when researchers want to answer descriptive questions or explanatory questions. The research subjects were two students in junior high school Surabaya. The two students are those who have high mathematical abilities and can answer the questions given by the researcher. Both subjects were asked to solve problems about mathematical models that must be changed in the form of individual quadratic equations.

The problems in question are as follows.

*Doubled the square of a number plus twelve times that number minus thirty-two is equal to zero. Determine the variable, determine the mathematical model in the form of a quadratic equation, then specify the set of solutions!*

*Fifteen times the square of a number plus sixty times numbers minus four hundred eighty equals zero. Determine the variable, determine the mathematical model in the form of a quadratic equation, then specify the set of solutions!*

Besides, to represent his thoughts the subject also provides written answers. During the work process, researchers conduct unstructured interviews to dig deeper into the subject's thinking. So, the data obtained in the form of written answers as well as recording the results of interviews obtained later than
analyzed qualitatively through the process of transcription, reduction, classification, interpretation, and presentation of data. Assessment will be carried out according to the indicators in Table 1.

### Table 1. Indicator of reflective abstraction.

| Structure | Indicator |
|-----------|-----------|
| Action    | a. Determine variables. |
| Process   | a. Determine mathematical models in the form of quadratic equations. |
| Object    | a. Determine the set of resolution from the mathematical model that has been made. |
| Looking back | a. Linking actions, processes, and objects, to form or build or make a schema.  
  b. Using schemes that have been formed through the stages of action, processes, and object that have been in the minds of students to solve more complex problems. |

### 3. Result and Discussion

#### 3.1. Result

To obtain an overview of the mental structure possessed, the presentation of results will be carried out for each research subject on each point of the problem to be given. Then the first subject and the second subject will be referred to as T1 and T2. And for researchers themselves symbolized by P.

**Analysis of subject 1**

The solution to problem 1 and problems 2 in figure 1 and figure 2 respectively.

**Figure 1.** The solution to problem 1.

**Figure 2.** The solution to problem 2.

Subject T1 shows the results on problem 1, where the subject can directly determine the quadratic equation then determine the variable.

P: Which ones are the variables from this mathematical model?

T1: That number. Then I suppose with x

P: How do you change this problem in quadratic equations?

T1: I understand the problem first, right there is "a number", I say a number x
P: How many solutions are there in quadratic equations? Can all the equations be done that way? Why!
T1: There are three. Maybe. Because if you can do it with the ABC formula, it can be sure with all three methods that you can
P: Why did you finish using the ABC formula?
T1: I use the ABC formula because I just enter a, b, and c into the formula.

The T1 subject fulfills the stages of the reflective abstraction indicator. Subject forms directly into the equation, then to the action stage. The reason the subject forms directly into quadratic equations is so that it is easy to determine variables. Subject 1 prefers to solve quadratic equations using ABC formula should be consistent, because the subject more easily determines the values of A, B and C and directly enters into the ABC formula. At the action stage, the subject can determine the variable with x. At the process, the stage he can determine in quadratic equations. In the object stage, students determine the set of solutions using the ABC formula.

Problem 2 is, given during interviews which aim to show a more complex scheme. From the results of problem-solving 2, the subject can be said to have a more complex scheme because he can solve the same problem as problem 1.

Analysis of subject 2
The solution to problem 1 and problem 2 in figure 3 and figure 4.

Subject T2 shows the results on problem 1, where the subject can directly determine the quadratic equation then determine the variable.
P: Which ones are the variables from this mathematical model?
T2: That number. Then I suppose with x
P: How do you change this problem in quadratic equations?
T2: I understand the problem first, right there there is "a number", I say a number x
P: How many solutions are there in quadratic equations? Can all the equations be done that way? Why!
T2: There are three. Maybe. Because if you can do it with the ABC formula, it can be sure with all three methods that you can
P: Why did you finish using the factoring formula?
T2: I use the factoring formula, in my opinion, is faster factoring, but my first step is to divide so that the equation is easier when factored.
The T2 subject fulfils the stages of the reflective abstraction indicator. At the action stage, the subject can determine the variable with $x$. At the process stage, he can be determined in the squared equation. At the object stage, the student determines the set of solutions by using factoring.

Problem 2 is given during interviews which aim to show a more complex scheme. From the results of problem-solving 2, the subject can be said to have a more complex scheme because he can solve the same problem with problem 1.

4. Conclusion
The use of sections to divide the text of the paper is optional and left as a decision for the author. Where the author wishes to divide the paper into sections the formatting shown in table 2 should be used. The results of the study show that the reflective abstraction of students in reconstructing the quadratic equation can solve the problem in different ways. In solving the problem of quadratic equations student T1 at the object stage can solve the equation using the ABC formula, students choose the ABC formula because the formula is easier and feels quite fast in solving quadratic equations. While in solving the problem of quadratic equations students T2 at the object stage can solve the equation using the factoring formula, selecting the factoring formula because students feel the formula is easy but still need steps to simplify the equation to be factored. Although in different ways both subjects have met reflective abstraction indicators. Next, to bring up the scheme stage, students are given advanced questions with different numbers but the operation is the same. Students (T1 and T2) can solve advanced problem problems with the same answers, thus students (T1 and T2) can be said to fulfil reflective abstractions even with more complex questions.

Because research is only limited to reflective abstraction in reconstructing the concept of quadratic equations, further research is expected to obtain a more complete picture by adding other concepts. Similar research can also be done for other mathematical concepts.

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