Generalized Statistics Framework for Rate Distortion Theory

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A statistical physics analysis for the variational problem encountered in rate distortion (RD) theory and the information bottleneck (IB) method, derived within the generalized nonextensive statistics framework of Tsallis, is presented [1]. The governing equations of the alternate minimization algorithm for the generalized RD model are

\[ p(\tilde{x} | x) = \frac{p(\tilde{x}) \exp q \left( -\beta^* d(x, \tilde{x}) \right)}{\tilde{Z}(x, \beta^*)} , \beta^* = \frac{\beta}{\mathfrak{Z}_{RD}} , \tilde{Z}(x, \beta^*) = \mathfrak{Z}_{RD}^{(1-q)} , \]

\[ \mathfrak{Z}_{RD} = q \sum_x p(\tilde{x}) \left( \frac{p(\tilde{x}|x)}{p(x)} \right)^q + (q - 1) \beta \langle d(x, \tilde{x}) \rangle_{p(\tilde{x}|x=x)} \] and, \[ p(\tilde{x}) = \sum_x p(x) p(\tilde{x} | x) . \]

(1)

Here, \( \beta^* \in [0, 1] \), and, the nonextensivity parameter satisfies \( 0 < q < 1 \). The nonextensive RD curves possesses a lower threshold for the minimum compression information in the distortion-compression plane, as compared to equivalent RD models derived within the Boltzmann-Gibbs-Shannon (B-G-S) framework. The equations governing the EM-like algorithm for the generalized IB method, for the Markov relation \( \tilde{X} \leftrightarrow X \leftrightarrow Y \), are

\[ p(\tilde{x}|x) = \frac{p(\tilde{x})}{\tilde{Z}(x, \beta_{IB}^*)} \exp q^* \left\{ -\beta_{IB}^* I_q [p(y|x) \| p(y|\tilde{x})] \right\} , \beta_{IB}^* = \frac{\beta}{\mathfrak{Z}_{IB}} , \tilde{Z}(x, \beta_{IB}^*) = \mathfrak{Z}_{IB}^{1-q}, \]

\[ \mathfrak{Z}_{IB} = \sum_x p(\tilde{x}) \left( \frac{p(\tilde{x}|x)}{p(x)} \right)^q + (q - 1) \beta \left( \sum_y p(y|x) \ln_q \left( \frac{p(y|x)}{p(y|\tilde{x})} \right) \right)_{p(\tilde{x}|x=x)} \]

\[ p(\tilde{x}) = \sum_x p(x) p(\tilde{x} | x) \] and, \[ p(y | \tilde{x}) = \frac{1}{p(\tilde{x})} \sum_x p(\tilde{x} | x) p(x, y) . \]

(2)

Here, \( \beta_{IB}^* \in [0, 1] \). The qualitative enhancement of the generalized IB method vis-\-à-\-vis an equivalent B-G-S model are demonstrated by the relevance-compression curves. A generalized Bregman RD (GBRD) model is presented [2]. A Tsallis-Bregman lower bound for the RD function is derived. A computational strategy to implement the GBRD model is described.

References

[1] R. C. Venkatesan. Generalized Statistics Framework for Rate Distortion Theory. *Physica A*, To Appear, 2007. Preprint available at http://arxiv.org/abs/cond-mat/0611567.

[2] R. C. Venkatesan. Generalized Statistics Framework for Rate Distortion Theory with Bregman Divergences. Preprint available at http://arxiv.org/abs/cond-mat/0701218.

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