Atypical Fractional Quantum Hall Effect in Graphene at Filling Factor $\nu_G = 1/3$

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We study, with the help of exact diagonalization calculations, a four-component trial wave function that may be relevant for the recently observed graphene fractional quantum Hall state at a filling factor $\nu_G = 1/3$. Although it is adiabatically connected to a 1/3 Laughlin state in the upper spin branch, with SU(2) valley-isospin ferromagnetic ordering and a completely filled lower spin branch, it reveals physical properties beyond such a state that is the natural ground state for a large Zeeman effect. Most saliently, it possesses at experimentally relevant values of the Zeeman gap low-energy spin-flip excitations that may be unveiled in inelastic light-scattering experiments.

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The recent observation of the fractional quantum Hall effect (FQHE) in graphene\cite{12} has proven the relevance of Coulomb interactions in this novel two-dimensional (2D) electron system, in agreement with theoretical expectations \cite{3,8}. The most pronounced state is the one found when the ratio $\nu_G = n_{el}/n_B$ between the electronic density $n_{el}$ and that of the flux quanta $n_B = eB/h$ is $\nu_G = 1/3$. Although this state is reminiscent, at first sight, of the prominent 1/3 state observed in semiconductor heterostructures\cite{3}, which is described to great accuracy by the Laughlin state\cite{10}, several questions arise when taking fully into account the four-component structure of graphene, due to its four-fold spin-valley degeneracy. Whereas first numerical approaches\cite{3} considered the physical spin to be frozen by the Zeeman effect and concentrated on the valley-isospin degree of freedom in a two-component system, a four-component approach\cite{8} seems to be more appropriate in view of the rather small energy scale associated with the Zeeman effect $\Delta_Z$, when compared to the leading energy scale of the Coulomb interaction, $e^2/\epsilon l_B$ at the magnetic length $l_B = \sqrt{\hbar c/eB}$. Indeed for a $q$-factor of 2\cite{11}, one obtains $\Delta_Z/(e^2/\epsilon l_B) \sim 0.002 \sqrt{B/[1]} \times \epsilon$, where $\epsilon$ is the relative dielectric constant.

A further complication arises in graphene, as compared to the 2D electron gas in semiconductor heterostructures, when one considers the definition of the filling factor $\nu_G$, which is proportional to the carrier density $n_{el}$. In graphene, the carrier density vanishes at the Dirac point, where the spectrum is particle-hole symmetric. In the presence of a magnetic field, a four-fold degenerate zero-energy Landau level (LL) is formed that happens to be half-filled when $n_{el} = 0$ and thus $\nu_G = 0$ – the situation at $\nu_G = 0$ is therefore more reminiscent of a filling factor of $\nu = 2$ in a usual four-component quantum Hall system\cite{8}, and the observed FQHE at $\nu_G = 1/3$ corresponds to a situation where two of the four spin-valley subbranches are completely filled and a third one $1/3$-filled ($\nu = 2 + 1/3$). As a consequence the observed FQHE is not a simple Laughlin state with an SU(4)-spin-valley ferromagnetic ordering, which would arise at $\nu_G = -2 + 1/3$ (or by particle-hole symmetry, at $\nu_G = 2 - 1/3$)\cite{6,8}. A natural candidate for large values of the Zeeman gap would then be a valley-SU(2)-ferromagnetic Laughlin state $\Psi_{1/2}^{SU(2)}$ in the spin-down branch of the zero-energy LL similar to the usual 1/3 physics. In this scenario, both states $K$ and $K'$ are completely filled in the spin-up branch. The small relative value of the Zeeman gap, however, casts doubts on such a scenario of complete spin polarization induced by an external field, without considering a cooperative effect mediated by the Coulomb interaction.

Here, we analyse the system in the zero-energy LL with the help of exact-diagonalization (ED) calculations for relativistic electrons in the spherical geometry with SU(4) symmetry that interact via the Coulomb interaction\cite{4,3,8}. We show that already for a very small Zeeman effect, one may obtain a FQHE at $\nu_G = 1/3$ in graphene. This state may be described in terms of a four-component Halperin wave function $\Psi_{2+1/3}^{SU(4)}$ which is adiabatically connected to the valley-SU(2)-ferromagnetic state in the upper spin branch. The latter is the natural ground state for a large Zeeman splitting. Most saliently, in spite of this adiabatic connection, the low-energy excitations in an intermediate range of the Zeeman splitting are different from those of the simple SU(2) Laughlin state. In addition to the charge-density-wave mode with its characteristic magneto-roton minimum and the valley-isospin wave, which is the Goldstone mode as associated with the spontaneous valley-isospin breaking in the spin-down branch, we find a low-energy spin-flip mode with a gap that depends linearly on the Zeeman splitting. These modes may be experimentally accessible in inelastic light-scattering measurements that have revealed similar modes in conventional quantum Hall systems in GaAs heterostructures\cite{12,13}. That electrons in graphene reside at the sample surface makes this novel 2D electron system even better adapted to optical mea-
measurements than the latter.

In order to describe the FQHE state at \( \nu_G = 1/3 \), which corresponds to a filling factor of \( \nu = 2 + 1/3 \) when counted from the bottom of the central \( n = 0 \) LL, we investigate the trial four-component wave function

\[
\Psi^{SU(4)}_{2+1/3} = \prod_{\xi=K,K'} \prod_{i<j} \left( z_i^\sigma \xi - z_j^\sigma \xi \right)^3 \prod_{i,j} \left( z_i^{\downarrow} K - z_j^{\downarrow} K' \right)^3 \times \prod_{\xi=K,K'} \prod_{i<j} \left( z_i^\sigma \xi - z_j^\sigma \xi \right),
\]

where \( z_j^\sigma \xi \) denotes the complex coordinate of the \( j \)-th electron in the spin-valley subbranch \( \sigma, \xi \) (\( \sigma = \uparrow \) or \( \downarrow \) and \( \xi = K \) or \( K' \)). We have omitted an ubiquitous Gaussian factor in the expression. Notice that, in the absence of a symmetry-breaking field, the wave function (1) is not a good trial state because the Coulomb interaction potential respects the SU(4)-spin-valley symmetry [11], whereas the wave function (1) is not an eigenstate of the SU(4)-Casimir operators.

This is indeed corroborated by our ED calculations for \( N = 17 \) particles with \( N_B = 6 \) flux quanta threading the sphere, the relation between \( N \) and \( N_B \) being \( N_B = (3/7)N - 9/7 \) for the state (1) [12], which yields the required filling \( \nu = 7/3 \) in the thermodynamic limit. The ground state is then found in spin sectors different from that, \( 2S_z = 11 \), expected for the state (1). A simple manner to stabilize the state (1) is to use appropriate pseudo-potentials [14] that break the SU(4)-spin-valley symmetry in the interaction potential. However, surprisingly, this trial state becomes the ground state also when the SU(4) symmetry is broken by an external field – e.g. a very small value of the Zeeman effect \( \Delta Z \), the ground state is found in the maximally spin-polarized sector \( 2S_z = 11, \) red diamonds). The inset shows a zoom on the region for small values of \( \Delta_Z \).

![FIG. 1:](image1.png) Energy spectrum for \( N = 17 \) electrons at a filling factor \( \nu_G = 1/3 \) \( (\nu = 2 + 1/3) \), as a function of \( \Delta_Z \), obtained from ED calculations of the Coulomb interaction on the sphere \( (N_B = 6) \) with implemented SU(4) symmetry. Above \( \Delta_Z^1 \approx 0.01e^2/\epsilon B \), the ground state is found in the maximally spin-polarized sector \( 2S_z = 11, \) red diamonds). The inset shows a zoom on the region for small values of \( \Delta_Z \).

![FIG. 2:](image2.png) Classification of the excitations of the \( \Psi^{SU(4)}_{1/3} \). The excitations of a one-component Laughlin state are found in the same spin-valley sector (C), whereas the Goldstone mode due to the broken SU(2) valley-isospin symmetry in the spin-\( \downarrow \) branch is an insospin-wave mode (ISW). In addition to these conventional modes, the four-component state (1) possesses a spin-flip (SF) mode.

Man gap \( \Delta_Z \). Above the critical value \( \Delta_Z^1 \), the ground state is found in the maximally-polarized spin sector that corresponds to the state (1), whereas the excited state with the lowest energy is in the same spin sector, \( 2S_z = 11 \), only above a second value \( \Delta_Z^2 \approx 0.03e^2/\epsilon B \). For values of the Zeeman gap \( \Delta_Z^2 \leq \Delta_Z \leq \Delta_Z^1 \), the excited state with lowest energy is found in the spin sector \( 2S_z = 9 \). Above \( \Delta_Z^2 \), however, the energy cost of this spin-flip excitation (SF, see Fig. 2) is larger than the lowest-lying excitation in the fully polarized sector \( 2S_z = 11 \) (C in Fig. 2).

These results suggest that the state (1) may have physical properties beyond the simple 1/3-Laughlin state in the spin-\( \downarrow \) branch, in the form of coherent spin-flip excitations in an intermediate range of Zeeman gaps. In order to test this scenario in more detail, we have investigated the two-component wave function

\[
\Psi^{SU(2)}_{1+1/3} = \prod_{i<j} \left( z_i^\downarrow - z_j^\downarrow \right)^3 \prod_{i,j} \left( z_i^\uparrow - z_j^\uparrow \right),
\]

which would be a candidate in a two-component quantum
Halls system, such as a conventional 2D electron gas in a GaAs heterostructure, at a filling factor $\nu = 1 + 1/3$. It is insofar related to the four-component wave function $^{11}$ as it describes the same physical situation if the valley-isospin degree of freedom for spin-$\downarrow$ electrons is neglected. The novel wave function $^{2}$ therefore does not reveal any valley-isospin-wave mode (ISW, see Fig. 2) that is the Goldstone mode of the spontaneously broken valley-SU(2) symmetry in the spin-$\downarrow$ branch and that may eventually become gapped if one takes into account a possible valley splitting. In contrast to its four-component analogue $^{11}$, the two-component wave function $^{2}$ allows for a more detailed study of different system sizes in ED with an implemented SU(2) symmetry. Indeed, our ED calculations with an implemented SU(4) symmetry allowed only for one single system size ($N = 17$ particles, only $N_z = 3$ populate the upper spin branch), in which case the subspace with maximal spin polarization ($2S_z = 11$) is of dimension one such that the overlap with the wave function $^{11}$ is trivially 1.

Figure 3(a) shows the energy spectrum for $N = 22$ particles and $N_B = 15$ flux quanta, in the different spin sectors, obtained by ED of the SU(2) Coulomb interaction potential in the lowest LL. The spectrum is in qualitative agreement with that obtained for the four-component system at $\nu = 2 + 1/3$ (Fig. 1) – because the wave function $^{2}$ is not an eigenstate of the SU(2)-symmetric Coulomb potential, it does not describe the ground state at $\Delta_Z = 0$, where one obtains a three-fold degenerate state (with $2S_z = 0, \pm 2$), but in a compressible sector ($L \neq 0$). As for the four-component case, a small symmetry-breaking Zeeman gap $\Delta_Z^1 \simeq 0.01 e^2/\epsilon l_B$ suffices to stabilize a state with maximal spin polarization ($2S_z = 10$ and $N_z = 6$), which has an overlap of 99% with the wave function $^{2}$ $^{28}$, and the lowest-lying excited state in an intermediate range of the Zeeman gap, $\Delta_Z^1 \leq \Delta_Z \leq \Delta_Z^2 \simeq 0.08 e^2/\epsilon l_B$, involves a spin flip as it is found in the spin sector $2S_z = 8$.

It has been argued that, for vanishing Zeeman splitting, the state at $\nu = 1 + 1/3$ should be a spin-singlet composite-fermion (CF) state with reversed flux attachment $^{20}$. Hund’s rule, according to which the system chooses a maximally polarized spin inside each energy level, would then predict an unpolarized state because $\nu = 2/3$ corresponds to a completely filled lowest CF-LL $^{20}$, but the same rule favors a completely polarized state if applied to the original electron coordinates. Our results indicate that already for a very small Zeeman splitting, a completely polarized state is favored that satisfies the electronic instead of the CF version of Hund’s rule. Notice, however, that a direct numerical comparison between both states, CF spin singlet and state $^{2}$, is problematic in the spherical geometry because the spin-singlet state has a different flux-particle-number relation, $N_B = (3/4)N - 1$, than the polarized state $^{2}$, $N_B = (3/4)N - 3/2$. We find for $N = 20$ and $N_B = 14$ (results not shown) that the ground state is indeed a singlet at low Zeeman splittings, but it is maximally polarized above a value of $\Delta_Z/(e^2/\epsilon l_B) \sim 0.003$, which is on the same order of magnitude as $\Delta_Z^1$.

In order to gain further insight into the nature of the low-lying excitations, we have calculated the spectrum [Fig. 3(b)] at an intermediate value of the Zeeman gap, $\Delta_Z = 0.05 e^2/\epsilon l_B$, where spin-flip excitations are expected to be relevant. The spectrum is now plotted as a function of the angular momentum in order to make apparent possible low-energy collective excitations of the incompressible state $^{2}$. Within the charge sector with no change in the spin polarization, one observes in Fig. 3(b) the usual magneto-roton branch (red diamonds) $^{21}$ which arises from gapped density-wave excitations $^{22}$ and which is a prominent feature of Laughlin-type physics. However, another mode is apparent in Fig. 3(b) that indicates the presence of collective excitations beyond the usual one-component Laughlin state and that
is precisely a spin-flip excitation (blue squares). This mode, which is the lowest-energy excitation in the low-$L$ regime, is well separated from the high-energy part of the excitation spectrum, such that it is likely to be a true collective mode. Notice that in the large-$L$ limit, the magneto-roton branch has a lower energy, and one may thus conjecture that the activation gap, i.e. the energy to create a well-separated quasiparticle-quasihole pair at large values of $L$, does not involve a spin-flip excitation, but is governed by one-component Laughlin physics.

The relevance of collective spin-flip excitations in an intermediate Zeeman-gap range may eventually be tested experimentally in inelastic light-scattering experiments that are capable of probing collective excitations at finite wave vectors $\mathbf{L}$. Indeed, these experiments probe characteristic parts of the dispersion relation that show an enhanced density of states (such as at its minima and maxima). Because the spin-flip mode in Fig. 3(b) is almost flat at low angular momenta $L$ that correspond to small wave vectors, one may expect an enhanced peak in such inelastic light-scattering measurements, at energies in the $0.1 e^2/\epsilon_l B$ range (roughly half of the energy of the magneto-roton minimum, for the particular choice $\Delta_Z = 0.05 e^2/\epsilon_l B$). As one may see in Fig. 3 the spin-flip excitation scales linearly with the Zeeman gap, such that the associated peak is expected to scale linearly with the magnetic field as well, whereas that of the usual magneto-roton would scale as $\sqrt{B}$ [see Fig. 3(b)]. The observation of such a linear $B$-field dependence of the light-scattering peak would be clear evidence for the relevance of spin-flip, beyond the properties of the Laughlin liquid, of the $\nu_G = 1/3$ state in graphene.

In conclusion, we have shown, within ED calculations for a four- and a two-component system on the sphere, how a FQHE can arise in graphene at $\nu_G = 1/3$ even at very small values of a spin-valley symmetry-breaking Zeeman field. Although the leading energy scale is given by the SU(4)-invariant Coulomb interaction, a small Zeeman gap $\Delta_Z/\epsilon_l \sim 0.01$ is sufficient to fully polarize the electronic spin and thus to stabilize the state $\mathbf{L}$ which we have identified as being responsible for the observed graphene FQHE [1,2]. In spite of its reminiscence with the Laughlin state, novel collective excitations that are inherent to the four-component character of graphene determine the low-energy spectrum at intermediate values of the Zeeman gap, $\Delta_Z \lesssim \Delta_Z^Z \lesssim \Delta_Z^Z$, that correspond to the experimental situation in which the FQHE has been observed. In order to gain further insight into the nature of these spin-flip excitations, which may be visible in inelastic light-scattering experiments, we have performed ED calculations in an analogous two-component quantum Hall system at a filling factor $\nu = 1 + 1/3$ that corresponds to a completely filled spin-$\uparrow$ and a one-third filled spin-$\downarrow$ branch. The spin-flip excitation is well separated from the high-energy part of the energy spectrum thus indicating its collective nature, in addition to the usual magneto-roton branch that determines the low-energy spectrum in the large angular-momentum regime.

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