Flow past a cylinder in diluted polymer solutions

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Abstract. Putting a small amount of polymer into the water is one of the most important ways to control turbulent flow passively. We here investigate the flow past a cylinder at a broad range of both Reynolds numbers and Weissenberg numbers by direct numerical simulation in two dimensions. The models used for Newtonian fluid or dilute polymer in solution are respectively Navier-Stokes and Oldroyd-B models. The obstacles are taken into account using the volume penalization method. The robust finite difference code keeps numerical stability well at high Weissenberg numbers. The cylinder drag may enhance or reduce in the two dimensional Reynolds number-Weissenberg number space. The flow features have a strong link to the drag behavior. Furthermore the elastic instability occurs and is captured when Weissenberg number is high; and the energy balance of the system is checked.

1. Introduction

Drag reduction is one of the most attractive topic in fluid mechanics. Since Toms reported that a small amount of polymer can reduce the flow drag (Toms, 1949), many investigations and discussions have been carried out to understand drag reduction, turbulent flow and viscoelastic fluid. Most of the works focus on the turbulent drag reduction mechanism and on the drag reduction ability of an inner flow, for which the pressure difference drag is absent. For the flows around a bluff body, the pressure difference brings a significant part of the drag forces. So the polymer may play different roles in drag behavior according to the flow. Hence these two kinds of flow on the one hand have the features of themselves and on the other hand have some links between each other, for example drag enhancement of the cylinder at small Reynolds number in polymer solution (François et al., 2008) may shed some light on decreasing drag reduction ability of the polymer for the rough channel wall (Virk, 1970). On the other hand the large bluff body like ship hulls can be thought as a channel wall locally, hence, the so-called maximum drag reduction (MDR) of the channel by polymer additives will guide us for minimizing the drag of complex bluff body.

However, the flow past a bluff body in polymer solutions received less attention. In fact, both drag enhancement and drag reduction of the cylinder were reported at present (James & Acosta, 1970; Verhelst & Nieuwstadt, 2004; François et al., 2008; Oliveira & Miranda, 2005; Richter et al., 2010). The drag behavior of a cylinder in polymer solutions was not clear until a recent numerical work by Xiong et al. (2010). Their numerical results show that the drag on a cylinder can be enhanced for low Reynolds numbers but can be reduced for higher ones.
Richter et al. (2010) performed a three-dimensional simulation at $Re = 300$, their work shows the polymer reduces the drag at this Reynolds number and suppresses the three-dimensional Newtonian spanwise instability.

In this paper, we study the polymer solutions flow past a cylinder in a wide parameters space in order to know the exact drag behavior of the cylinder. Many numerical simulations for viscoelastic fluids governed by Oldroyd B model are conducted to plot the drag map of the flow around a cylinder, so that the definitive drag behavior can be found. At the same time, the main flow features which are hand-in-hand with the drag behavior of the cylinder are studied. At high Reynolds number, we found that the polymer can make the wake stable and instable again. The mechanism of the drag reduction, drag enhancement and the elastic instability are discussed.

2. Governing equations and numerical method

Our computational domain is shown in Figure 1. The Viscoelastic fluid flows into the domain from an inlet boundary $\Gamma_i$ with a velocity profile of Poiseuille flow. The channel is formed by two parallel no-slip wall ($\Gamma_w$). A cylinder with diameter of 0.1 is placed in the center of the channel. This will produce a small blockage ratio with 0.1 which reduces the effect of the channel wall (Huang & Feng, 1995). The governing equations of a non-dimensional unsteady incompressible flow with an Oldroyd-B model in the domain $\Omega \times (0, T)$ read:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} + u \frac{\beta}{Re} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{1 - \beta}{Re \cdot Wi} \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} \right), \tag{2}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} + v \frac{\beta}{Re} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{1 - \beta}{Re \cdot Wi} \left( \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} \right), \tag{3}
\]

\[
\frac{\partial \sigma_{xx}}{\partial t} + u \frac{\partial \sigma_{xx}}{\partial x} + v \frac{\partial \sigma_{xx}}{\partial y} + \frac{\sigma_{xx}}{K} + \frac{\sigma_{xx} - 1}{Wi} - 2\sigma_{xx} \frac{\partial u}{\partial x} + 2\sigma_{xy} \frac{\partial u}{\partial y} + \kappa \left( \frac{\partial^2 \sigma_{xx}}{\partial x^2} + \frac{\partial^2 \sigma_{xx}}{\partial y^2} \right), \tag{4}
\]

\[
\frac{\partial \sigma_{xy}}{\partial t} + u \frac{\partial \sigma_{xy}}{\partial x} + v \frac{\partial \sigma_{xy}}{\partial y} + \frac{\sigma_{xy}}{K} + \frac{\sigma_{xy}}{Wi} = \sigma_{xx} \frac{\partial v}{\partial x} + \sigma_{yy} \frac{\partial u}{\partial y} + \kappa \left( \frac{\partial^2 \sigma_{xy}}{\partial x^2} + \frac{\partial^2 \sigma_{xy}}{\partial y^2} \right), \tag{5}
\]

\[
\frac{\partial \sigma_{yy}}{\partial t} + u \frac{\partial \sigma_{yy}}{\partial x} + v \frac{\partial \sigma_{yy}}{\partial y} + \frac{\sigma_{yy}}{K} + \frac{\sigma_{yy} - 1}{Wi} = 2\sigma_{yy} \frac{\partial v}{\partial y} + 2\sigma_{xy} \frac{\partial v}{\partial x} + \kappa \left( \frac{\partial^2 \sigma_{yy}}{\partial x^2} + \frac{\partial^2 \sigma_{yy}}{\partial y^2} \right). \tag{6}
\]
where \( u \) and \( v \) are the velocity, \( p \) the local non-dimensional pressure, and \((x, y)\) represents the space vector in Cartesian coordinates, here our domain is placed \( 0 \leq x \leq 4 \) and \( 0 \leq y \leq 1 \). \( \sigma_{ij} \) is a symmetrical conformation tensor of the polymer molecular that gives three additional unknowns in two dimensions. \( \beta \) denotes the ratio of the solvent viscosity to that of the polymer solutions, it is 0.99 for the diluted polymer solutions in this paper. The flow rate in the channel is dimensionless to 1, and the pressure is set to 0 on the outlet of the channel. Here \( Wi \) and \( Re \) are Weissenberg number and Reynolds number respectively. The last term of the Oldroyd B constitutive equation is an artificial diffusion term in order to enhance the stability of the system, \( \kappa \) is reasonable small to maintain the original flow behavior (R. Sureshkumar & Beris, 1995).

The cylinder is built by an additional term, which is the last term of the left hand part in the momentum and constitutive equations, \( K \) is set to \( 10^{16} \) in the fluid domain (\( \Omega_f \)) and \( 10^{-7} \) in the cylinder (\( \Omega_s \)). The boundary condition for the polymer conformation tensor is obtained by the analytical solution from the constitutive model; an artificial non-reflecting boundary is derived for the outlet (\( \Gamma_o \)) of the channel and can be found in Bruneau & Fabrie (1996) in detailed. The simulations are performed on a fine \( 2048 \times 512 \) mesh by means of multigrid method.

3. Results and discussion

3.1. The cylinder drag behavior

The drag coefficient of the cylinder is defined by \( Cd = \frac{2F_x}{D\overline{U}^2} \), here \( F_x \) is the mean drag force of the cylinder, \( \overline{U} \) is the mean velocity and \( D \) is the diameter of the cylinder. In our results, we defined the Reynolds number and Weissenberg number by \( Re = \frac{UD}{\eta} \) and \( Wi = \frac{\tau U}{R} \), where \( \eta \) and \( \tau \) are the kinetic viscosity and relaxation time of the polymer solutions respectively.

(a) The drag coefficient of the cylinder

(b) The drag phase diagram

Figure 2: The drag coefficient of the cylinder and the drag phase diagram

The drag of the cylinder is shown in the figure 2a, for the small Reynolds numbers flow, the drag of the cylinder can either maintain the Newtonian drag or enhance when the Weissenberg number is high. But When the Reynolds number is high, the Newtonian drag part shortens, then there is a significant drag reduction for intermediate Weissenberg numbers, and for high Weissenberg numbers the drag enhances like for the small Reynolds number flows. So, as the Reynolds number increases, the drag changes from two regimes to three regimes in the
Weissenberg number space. The drag phase diagram plotted in the figure 2b shows the drag behavior of the cylinder in the $Re - Wi$ space. We can defined two critical Weissenberg numbers $Wi_1$ and $Wi_2$ according to the drag behavior. The drag behaves the Newtonian drag when the Weissenberg number is less than $Wi_1$ and the drag enhances when the Weissenberg number is larger than $Wi_2$. The superposition of $Wi_1$ and $Wi_2$ at small Reynolds numbers suggests the drag will not be reduced, and the drag reduction obtained between $Wi_1$ and $Wi_2$ corresponds to the fact that the vortex shedding still occurs at these Weissenberg numbers. In figure 2a, we can observe that the maximum drag reduction increases as the Reynolds number increases, until it reduces about 50% of drag reduction.

![Vorticity snapshots at $Re = 200$](image)

The drag of the cylinder is obviously affected by the flow in the vicinity of the cylinder. In figure 3, the vorticity snapshot is plotted at $Re = 200$. One can observe that the flow is significantly modified by the addition of polymers so that at $Wi = 5.0$ the first pair of vortices downstream is pushed away by the effect of the polymer, the pressure well is pushed far away from the cylinder, thus the pressure difference is drastically reduced and so is the drag coefficient. The elongated vortices shedded from the cylinder make the vortex street is very different from the Newtonian case. The modification of the vortex street and the reduction of the shedding frequency are in agreement with the early soap film experiments by Cressman et al. (2001).

To check the mean flow variation, we can refer to the figures 4 and 5. In figure 4, the mean streamwise velocity is plotted at five different positions (One can obtain the position by the no-slip boundary condition). On the inlet boundary, there is a Poiseuille flow for the different Weissenberg number cases; while these velocity profiles are very different in the cylinder wake. Three kinds of behavior of velocity profiles can be seen which correspond to the three zones of the drag diagram. When the Weissenberg number is less than $Wi_1$, the velocity profiles are almost the same than for the Newtonian fluid, which has the maximum velocity located on the centerline of the channel, the local velocity profile behaves like a Gaussian function. In the drag reduction zone, the feature of the maximum velocity on the centerline degenerates. For the drag enhancement part, an obviously low velocity zone exists in the wake. These features disappear along the streamwise as the distance with the cylinder increases due to viscous dissipation and elastic relaxation. The flow will certainly recover the Newtonian behavior at the wake far away from the cylinder enough.

In figure 5, we compared the pressure and streamwise velocity profiles on the centerline for two different Reynolds numbers which have different drag regimes. For $Re = 20$, both the pressure and the streamwise velocity are the same than those of the Newtonian fluid for $Wi = 2$. As the Weissenberg number increases, the pressure difference between the stagnation point and the rear of the cylinder increases, it indicates the drag enhancement. The recirculation point moves toward the cylinder as seen on the streamwise velocity. On the contrary the pressure difference first reduces as the Weissenberg number increases for $Re = 200$, which results in the drag reduction, then after the Weissenberg number exceeds $Wi_2$, both the pressure difference and the recirculation length reverse, so the drag of the cylinder enhances.
Figure 4: Mean streamwise velocity profiles for different positions at $Re = 100$.

(a) $Re = 20$

(b) $Re = 200$

Figure 5: Mean pressure and streamwise velocity profiles on the centerline ($y = 0.5$), $X$ in the figures is the streamwise coordinate, $X_c$ is the the position of the cylinder center and $R$ is the radius of the cylinder.

3.2. High Weissenberg number flow instabilities

As the Weissenberg number grows, the elasticity of the fluid increases, the elastic fluid will affect the vortices in different scales. Amarouchene & Kellay (2002) and De Angelis et al. (2005) have
confirmed that the polymer affects the fluctuation at different scales by soap film experiment and numerical simulation respectively, but few report on flow instabilities. The main reason is that most studies haven’t observed the stable flow at high Weissenberg numbers. The elastic instability is now becoming a more and more important topic for the flow of polymer solutions. In our extensive simulations, three kinds of flow instabilities at high Weissenberg numbers are observed at different Reynolds numbers ranges. Two of them occur at small Reynolds numbers and can be found in Oliveira & Miranda (2005); James & Acosta (1970); Xiong (2010). We here only check the wake flow of a single cylinder at high Reynolds number.

At high Reynolds numbers, the shear instability is inhibited as the Weissenberg number increases like in the experimental observations (Cadot & Lebey, 1999). As shown in the figure 6 where the vorticity snapshot at different Weissenberg numbers at $Re = 5000$ are plotted. One can observe that the polymer first inhibits and even extinguish the wake instabilities, then the Kelvin Helmholtz like instability is obtained at higher Weissenberg numbers. When the Weissenberg number is small, the first pair of the vortices at the rear of the cylinder elongates, like for low Reynolds number. The vortices in the wake even disappear when the Weissenberg number exceeds $Wi = 4.0$ in our computational domain. But the Kelvin’s ‘cat’s-eye’ pattern of the vorticity appears in the free shear layer at $Wi = 5.0$. It suggests us to check the shear rate in the wake. As shown in figure 7a, a low-velocity zone formed at high Weissenberg numbers. However the shear rate does not increase any more at $Wi = 5.0$, but reduces. So the shear rate is not the only reason to induce instability in the viscoelastic wake flow. Indeed, the viscous shear stress is not dominant any more at $Wi = 4.0$, a simple calculation indicates the most part of the shear stress come from elastic-induced shear stress as shown in figure 7b. As the Weissenberg number increases, $\sigma_{xy}$ still increases at $Wi = 5.0$, the decaying elastic stress in the wake should induce the instability. As we can expect, the instability occurs
also at lower Reynolds number but higher Weissenberg number in the soap film experiment (Xiong, 2010).

Figure 7: The mean streamwise velocity and the non-diagonal element of the conformation tensor at $x = 2.0$ for $Re = 5000$

The elasticity of the polymer is paid more and more attention as it can be associated to the drag reduction mechanism even though the physical link is under debate at present (Tabor & De Gennes, 1987; Sreenivasan & White, 2000; De Angelis et al., 2005). The momentum equation and constitutive equation in the governing equations can be transferred to the kinetic and elastic energy equations which read

\[
\langle \frac{DE_k}{Dt} \rangle = -\langle \epsilon_v \rangle - \langle \epsilon_p \rangle + \langle P_f \rangle, \tag{7}
\]

\[
\langle \frac{DE_p}{Dt} \rangle = \langle \epsilon_p \rangle - \frac{E_p}{\tau}. \tag{8}
\]

Where $\langle \cdot \rangle = \int \cdot dV$ and $V$ is the total volume of the computational domain. The above two equations provide the energy transfer relation between the polymer and the flow. For our study object, the statistic steadiness yields the left hand side is zero. The sum of the two equations gives the energy balance relation. The forcing power $\langle P_f \rangle$ is consumed by the viscous dissipation and the relaxation of the elastic energy of the polymer. The forcing power is mainly composed from the driving work due to the pressure difference and the kinetic energy flux between the inlet and the outlet boundary. The rate of viscous dissipation $\langle \epsilon_v \rangle = \beta \langle |w|^2 \rangle / Re$ can be obtained from the vorticity of the fluid, and the elastic stress power $\langle \epsilon_p \rangle = \frac{1-\beta}{Re \cdot Wi} \left( \sigma_{xx} \frac{\partial u}{\partial x} + \sigma_{xy} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \sigma_{yy} \frac{\partial v}{\partial y} \right)$ extracts kinetic energy and serves the relaxation of the polymer. The elastic energy (Min et al., 2003) of Oldroyd B model is calculated by $\langle E_p \rangle = \frac{1}{2} \frac{1-\beta}{Re \cdot Wi} \left( \sigma_{xx} + \sigma_{yy} - 2 \right)$. 


A typical kinetic energy balance is plotted in figure 8a for $Wi = 3.0$. The changing rate of kinetic energy has a statistic zero sum, the input energy balance with the negative dissipation of the viscosity and elasticity. Especially the turbulent fluctuation kinetic energy $k_e = 0.5(u'^2 + v'^2)$ and the viscous dissipation versus the Weissenberg number are plotted in figure 8b for $Re = 5000$. The fluctuation kinetic energy decreases monotonously as the Weissenberg number increases, however the variation of the viscous dissipation is comparatively small and even increases at high Weissenberg number. More physical quantities are listed in the table 1. As the Weissenberg number increases, the fluctuation kinetic energy decreases, and the elastic energy and the elastic stress power increases proportionally. Indeed the ratio of the two in related to the Weissenberg number, our results show a good agreement of their balance.

![Figure 8: The total energy of the domain at $Re = 5000$](image)

(a) The total energy balance at $Wi = 3.0$

(b) $k_e$ and $\epsilon_v$

Table 1: Summary of the principal global quantities at $Re = 5000$.

| $Wi$ | $\langle \epsilon_v \rangle$ | $\langle \epsilon_p \rangle$ | $\langle k_e \rangle$ | $\langle E_p \rangle$ | $\langle P_f \rangle$ |
|------|-----------------|----------------|----------------|----------------|----------------|
| 0    | 0.0622          | 0              | 0.4585         | 0              | 0.062          |
| 1    | 0.0274          | 0.0419         | 0.1973         | 0.0021         | 0.070          |
| 2    | 0.0184          | 0.0691         | 0.0759         | 0.0069         | 0.087          |
| 3    | 0.0162          | 0.0866         | 0.0293         | 0.0130         | 0.103          |
| 4    | 0.0210          | 0.1310         | 0.0072         | 0.0257         | 0.155          |
| 5    | 0.0211          | 0.1789         | 0.0022         | 0.0409         | 0.179          |

4. Conclusions

Numerical simulation have become an important tool to understand the intriguing polymer drag reduction phenomenon. A large number of DNS of viscoelastic flows in a channel with a fix cylinder is conducted to draw the drag map of the cylinder in an adequate $Re - Wi$ parameter space. Numerical results show the drag of the cylinder does not change for the low Weissenberg numbers, and enhances for the high Weissenberg numbers. When Reynolds number is high enough to induce vortex shedding for the Newtonian flow, there is a drag reduction zone...
at intermediate Weissenberg numbers. The drag reduction and drag enhancement zones are represented in a phase diagram.

The different drag behavior of the cylinder can also be identified by its flow features, when the flow behavior is the same than for the Newtonian flow, then the drag is equal to the Newtonian drag. When the first pair of the vortices at the rear of the cylinder elongates, and make the mean recirculation zone increases, the drag of the cylinder reduces. For the drag enhancement flow, the recirculation zone of the cylinder disappears, and there is a comparatively steady low-velocity zone in the cylinder wake. At high Reynolds numbers, the polymer can first make the wake regular at a quite high Weissenberg number and can induce the Kelvin-Helmholtz like instability at a higher Weissenberg number. The energy balance relation shows the budget of the fluctuation kinetic energy reduced in the presence of polymers, a considerable part of energy drains from polymer relaxation. As the Weissenberg number increases, the fluctuation kinetic energy decreases and the elastic energy increases.

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