In the framework of the baryogenesis via leptogenesis mechanism we study the link between the amount of baryon asymmetry and neutrino mass matrices. In particular, neglecting phases, we find that if the Dirac neutrino mass matrix is related to the up quark mass matrix, the baryon asymmetry is about three orders smaller than the required value. If the Dirac neutrino mass matrix is related to the down quark or the charged lepton mass matrix, the baryon asymmetry is about two orders smaller than the required value. In order to get a sufficient amount of baryon asymmetry we need a more moderate hierarchy in the Dirac neutrino mass matrix.
I. INTRODUCTION

The origin and amount of baryon asymmetry in our universe depends on both cosmological features and particle-physics properties. Sakharov [1] discovered that three conditions must be realized in order to obtain a baryon asymmetry: baryon number ($B$) violation, charge conjugation ($C$) and combined charge conjugation and parity ($CP$) violations, and finally out-of-equilibrium dynamics. The first two conditions come out from particle physics, while the third one is usually provided by the expansion of the universe.

The typical particle-physics theory in which these conditions can be realized is the grand unified theory [2], where $B$ violation is a key signature. However, this approach has some shortcomings. In fact, electroweak sphaleron processes [3] could wash out a previously created baryon asymmetry [4]. Moreover, unified theories are not yet confirmed by proton decay.

Another approach is based on the production of the baryon asymmetry at the electroweak scale by sphaleron processes [3]. However, the standard model realization fails, due to the lower mass limit of the Higgs boson, and the supersymmetric version is in the corner of the parameter space [4].

Then, a good alternative is the baryogenesis via leptogenesis mechanism [5,6], based on the out-of-equilibrium decay of heavy right-handed Majorana neutrinos, which generates a lepton asymmetry to be partially converted into a baryon asymmetry by the sphaleron processes. Indeed, the existence of very heavy neutrinos can account for the smallness of effective neutrino mass by means of the seesaw mechanism [7].

In this paper we explore the link between the amount of baryon asymmetry generated in the baryogenesis via leptogenesis mechanism and quark-lepton mass matrices. We assume large mixing of solar neutrinos and relate the Dirac neutrino mass matrix to the up quark or down quark mass matrix.

The present article is similar to an update of Ref. [8]. In fact, we use double (atmospheric and solar) large lepton mixing, which is favoured by recent data,
instead of single (atmospheric) large lepton mixing, and quark mass matrices from
the recent Ref. [9], instead of the mass matrices from Ref. [10].

In section II the baryogenesis via leptogenesis is introduced. In section III the
link between baryon asymmetry and mass matrices is studied, and finally, in section
IV, we give our conclusion.

II. BARYOGENESIS VIA LEPTOGENESIS

A baryon asymmetry can be generated from a lepton asymmetry, due to elec-
troweak sphaleron processes [5]. The lepton asymmetry is produced by the decay
of heavy right-handed neutrinos, which are Majorana particles and therefore their
mass terms violate lepton number ($L$). The sphalerons, which violate $B + L$ but
conserv $B - L$, convert part of this lepton asymmetry into a baryon asymmetry.
The $CP$ violation is present because of complex Yukawa couplings of right-handed
(singlet) neutrinos with the left-handed lepton doublet and the Higgs doublet. These
interactions generate the decay of heavy neutrinos and also the Dirac neutrino mass
matrix $M_{\nu}$ through the vacuum expectation value (VEV) of the Higgs doublet.

The baryon asymmetry can be defined as [11]

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} = \frac{n_B - n_{\bar{\nu}}} {7n_\gamma} = \frac{\eta} {7},$$

(1)

where $n_B$, $n_{\bar{B}}$, $n_\gamma$ are number densities, $s$ is the entropy density, and $\eta$ is the baryon-
to-photon ratio. The master formula for the calculation of the baryon asymmetry
in the baryogenesis via leptogenesis mechanism is given by

$$Y_B \simeq \frac{1}{2} \frac{d \epsilon_1}{g^*},$$

(2)

where

$$\epsilon_1 \simeq \frac{3}{16\pi} \left[ \frac{\text{Im}[(y^T_D y_D)_{12}]^2}{(y^T_D y_D)_{11}} M_1 \right] M_2 + \frac{\text{Im}[(y^T_D y_D)_{13}]^2}{(y^T_D y_D)_{11}} M_3 \right]$$

(3)

is a $CP$ violating asymmetry, $d$ is a dilution factor to be discussed in the following,
and $g^* \simeq 100$ counts the light degrees of freedom in the theory (see for example
Ref. [12] and references therein). The $CP$ violating asymmetry comes out from the interference between the tree level and one loop graphs in the out-of-equilibrium decay of the lightest heavy neutrino. This lightest neutrino is in equilibrium during the decays of the two heavier ones, washing out the lepton asymmetry generated by them. The Yukawa matrices $y_\nu$ are given by $M_\nu/v$, where $v \simeq m_t$ is the VEV of the Higgs doublet. Matrices $y_D$ are obtained by $y_D = y_\nu U_R$, where the unitary matrix $U_R$ diagonalizes the mass matrix of heavy neutrinos, $M_R$, with three eigenvalues $M_1 < M_2 < M_3$. The factor $1/2$ in Eqn.(2) indicates the part of the lepton asymmetry which is converted into a baryon asymmetry [13].

The dilution factor should be calculated by solving the Boltzmann equations of the system. It includes the effect of the decay width of the lightest heavy neutrino, as well as the wash out effect of lepton number violating scatterings. In Ref. [14] it is shown that the dilution depends mostly on the mass parameter

$$\tilde{m}_1 = \frac{(M_D^\dagger M_D)_{11}}{M_1},$$

with $M_D = y_D v$, and for high $\tilde{m}_1$ some dependence on $M_1$ also appears. Minor dilution, $d$ of order $10^{-1}$, is obtained for $10^{-5} < \tilde{m}_1 < 10^{-2}$ eV. If $\tilde{m}_1$ is too low, the Yukawa couplings are too small to produce a sufficient number of heavy neutrinos at high temperature, while if $\tilde{m}_1$ is too large, the wash out effect is too strong and destroys the generated asymmetry. See also the discussion in Ref. [15]. In the present paper, emphasis is given to the possible dependence of the baryon asymmetry on neutrino mass matrices, without a detailed study of the dilution.

### III. Leptogenesis and Mass Matrices

In this section we explore the link between the baryon asymmetry and lepton mass matrices. We use symmetric phenomenologically allowed forms for the quark mass matrices with five and four texture zeros [3,4], and get neutrino mass matrices by relating them to the up quark mass matrix $M_u$ or the down quark mass matrix $M_d$. The charged lepton mass matrix $M_e$ is always related to the down quark mass
matrix. This is suggested by unified and left-right models \cite{17,18}. As a matter of fact, the baryogenesis via leptogenesis mechanism is active also within such theories (see the review \cite{19} and the two papers in Ref. \cite{20}). However, we may also assume the quark-lepton mass relations as phenomenological inputs within the standard model with heavy right-handed neutrinos, in such a way to avoid possible problems with proton decay \cite{21}.

We consider only the large mixing solution to the solar neutrino problem, which is favoured by recent analyses. In Ref. \cite{22} it is shown that the double large lepton mixing with the hierarchy $m_1 \ll m_2 \ll m_3$ for the light (effective) Majorana neutrinos leads to the approximate democratic form

$$M_L^{-1} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{1}{m_1},$$

where $M_L$ is the effective neutrino mass matrix. We obtain the mass matrix of heavy neutrinos by means of the inverse seesaw formula

$$M_R \simeq M_\nu M_L^{-1} M_\nu.$$

In this way, $M_R$ is obtained by following a kind of bottom-up approach. The Dirac neutrino mass matrix comes from a theoretical or phenomenological hint, and then the heavy neutrino mass matrix is inferred, through the inverse seesaw formula, from the effective neutrino mass matrix.

Charged fermion masses are hierarchical, according to

$$m_u/m_c \sim m_c/m_t \sim \lambda^4$$

$$m_d/m_s \sim m_s/m_b \sim \lambda^2$$

$$m_e/m_\mu \sim m_\mu/m_\tau \sim \lambda^2,$$

5
where $\lambda = 0.22$ is the Cabibbo parameter. Hence, the charged lepton and down quark mass hierarchies are similar. We insert these hierarchies into the quark mass matrices of Ref. [9] and then obtain six approximate forms for the Dirac neutrino mass matrix:

$$M_{\nu}^I \sim \begin{pmatrix} 0 & 0 & \lambda^4 \\ 0 & \lambda^4 & 0 \\ \lambda^4 & 0 & 1 \end{pmatrix} m_t, (10)$$

$$M_{\nu}^{II} \sim \begin{pmatrix} 0 & 0 & \lambda^4 \\ 0 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^4 & 1 \end{pmatrix} m_t, (11)$$

$$M_{\nu}^{III} \sim \begin{pmatrix} 0 & \lambda^8 & \lambda^4 \\ \lambda^8 & \lambda^4 & 0 \\ \lambda^4 & 0 & 1 \end{pmatrix} m_t, (12)$$

$$M_{\nu}^{IV} \sim \begin{pmatrix} 0 & \lambda^6 & \lambda^8 \\ \lambda^6 & 0 & \lambda^2 \\ \lambda^8 & \lambda^2 & 1 \end{pmatrix} m_t, (13)$$

$$M_{\nu}^{V} \sim \begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ 0 & \lambda^4 & 1 \end{pmatrix} m_t, (14)$$

$$M_{\nu}^{VI} \sim \begin{pmatrix} 0 & \lambda^3 & 0 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix} m_b. (15)$$
The charged lepton mass matrix is in the form (15) and gives negligible lepton mixing. The matrix (10) contains three texture zeros. The other matrices contain only two texture zeros. We adopt such matrices to calculate the baryon asymmetry, according to the formulas (2),(3). Since we are interested in the general trend, without considering in detail the \( CP \) violating phases, we drop the imaginary part in Eqn.(3). The required value for \( Y_B \), from primordial nucleosynthesis, is in the range \( 10^{-11} - 10^{-10} \) (see for example the report [23]).

For the Dirac matrices \( M^{I-III}_\nu \) we get

\[
M_R \sim \begin{pmatrix}
\lambda^8 & \lambda^8 & \lambda^4 \\
\lambda^8 & \lambda^8 & \lambda^4 \\
\lambda^4 & \lambda^4 & 1
\end{pmatrix} \frac{m_t^2}{m_1},
\]

and

\[
U_R \sim \begin{pmatrix}
1 & 1 & \lambda^4 \\
-1 & 1 & \lambda^4 \\
\lambda^4 & -\lambda^4 & 1
\end{pmatrix},
\]

and \( y_D^i y_D = y_R \), with \( y_R = M_R/v_R \). The parameter \( v_R \simeq M_3 \) is the VEV of the singlet Higgs field which generates the heavy neutrino mass (although it can be generated as bare Majorana mass term). Note that in the 1-2 sector of \( M_R \) all entries are of the same order in \( \lambda \). The eigenvalues of \( M_R \) are \( M_3 \simeq m_t^2/m_1, M_2 \simeq \lambda^8 M_3, \) and \( M_1 \simeq 0 \) or \( M_1 \sim M_2 \). Here \( M_1 \sim 0 \) means that \( M_1 \) is much suppressed with respect to \( M_2 \), and \( M_1 \sim M_2 \) means that they may differ by about one order in \( \lambda \).

In the first case \( \epsilon_1 \) is suppressed and, as a consequence, \( Y_B \) is much smaller than the required value. In the other case

\[
\epsilon_1 \simeq \frac{3}{16\pi} \left( \frac{\lambda^{16}}{\lambda^8} \cdot 1 + \frac{\lambda^8}{\lambda^8} \cdot \lambda^8 \right) \sim 10^{-7},
\]

and \( \tilde{m}_1 \simeq m_1 \), so that \( Y_B \sim 10^{-11} \). A sufficient level of baryon asymmetry can be obtained, but \( m_1 \) has not to be less than \( 10^{-5} \) eV, whereas we know that it is less than \( 10^{-2} \) eV. A more precise expression for \( M_\nu \), keeping the democratic form (5) valid, leads to the case \( M_1 \sim 0 \) and hence to a suppression of \( Y_B \).
For the Dirac matrix $M^{IV}_L$ we have

$$M_R \sim \begin{pmatrix} \lambda^{12} & \lambda^8 & \lambda^6 \\ \lambda^8 & \lambda^4 & \lambda^2 \\ \lambda^6 & \lambda^2 & 1 \end{pmatrix} \frac{m_t^2}{m_1},$$

with eigenvalues $M_3 \simeq m_t^2/m_1$, $M_2 \simeq \lambda^4 M_3$ and $M_1 \simeq \lambda^{12} M_3$. We have also

$$U_R \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^6 \\ -\lambda^4 & 1 & \lambda^2 \\ \lambda^6 & -\lambda^2 & 1 \end{pmatrix},$$

and again $y_D^\dagger y_D = y_R$. This relation is due to the democratic form (5). The CP violating asymmetry is

$$\epsilon_1 \simeq \frac{3}{16\pi} \left( \frac{\lambda^{16}}{\lambda^{12}} \cdot \lambda^8 + \frac{\lambda^{12}}{\lambda^{12}} \cdot \lambda^{12} \right) \sim 10^{-10},$$

and $\bar{m}_1 \simeq m_1$, so that $Y_B \sim 10^{-14}$. This is a too small value.

For the Dirac matrix $M^{IV}_L$ we have

$$M_R \sim \begin{pmatrix} \lambda^{12} & \lambda^{10} & \lambda^6 \\ \lambda^{10} & \lambda^8 & \lambda^2 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix} \frac{m_t^2}{m_1},$$

with eigenvalues $M_3 \simeq m_t^2/m_1$, $M_2 \simeq \lambda^8 M_3$ and $M_1 \simeq \lambda^{12} M_3$. We have also

$$U_R \sim \begin{pmatrix} 1 & \lambda^2 & \lambda^6 \\ -\lambda^2 & 1 & \lambda^4 \\ \lambda^6 & -\lambda^4 & 1 \end{pmatrix},$$

and again $y_D^\dagger y_D = y_R$. The CP violating asymmetry is

$$\epsilon_1 \simeq \frac{3}{16\pi} \left( \frac{\lambda^{20}}{\lambda^{12}} \cdot \lambda^4 + \frac{\lambda^{12}}{\lambda^{12}} \cdot \lambda^{12} \right) \sim 10^{-10},$$

and $\bar{m}_1 \simeq m_1$, so that $Y_B \sim 10^{-14}$. Again this is a too small value.
For the Dirac matrix $M_{\nu I}^{VI}$ we find

$$\epsilon_1 \simeq \frac{3}{16\pi} \left( \frac{m_b}{m_t} \right)^2 \left( \frac{\lambda^{10}}{\lambda^6} \cdot \lambda^2 + \frac{\lambda^6}{\lambda^6} \cdot \lambda^6 \right) \sim 10^{-9},$$

(25)

$\tilde{m}_1 \simeq m_1$ and $Y_B \sim 10^{-13}$. Even in this case the baryon asymmetry is too low. Note that the matrix $M_{\nu I}^{VI}$ can be obtained from $M_{\nu}^{V}$ dividing powers of $\lambda$ by two and replacing $m_t$ with $m_b$. The factor $(m_b/m_t)^2$ comes out from the different scale of Yukawa couplings. It erases almost all the enhancement effect of the less pronounced hierarchy of Yukawa couplings and of heavy neutrino masses. Instead, the dilution factor is similar in any case, because of the relation $y_D^T y_D \simeq y_R$. Dividing again powers of $\lambda$ by two, we find $\epsilon_1 \sim 10^{-7}$ and a sufficient amount of baryon asymmetry $Y_B \sim 10^{-11}$.

The previous results are valid in the nonsupersymmetric model. However, in the supersymmetric model the amount of baryon asymmetry is only slightly enhanced. Model V with $M_R$ given by Eqn.(22) is similar to the one considered in Ref. [24], based on the $U(2)$ horizontal symmetry, where a different approximation for the dilution factor was used. We confirm the results of the numerical analysis in Ref. [12].

The relation $\tilde{m}_1 \simeq m_1$ has been found, under some general circumstances, for the bilarge lepton mixing, in Ref. [25]. With respect to that paper, we find $\epsilon_1$ and $Y_B$ smaller by about three orders. This happens because assumptions A1 and A3, but not A2, of Ref. [25] are fulfilled by matrices $M_{\nu}^{I-V}$. In particular, some mixing angles in the unitary matrix that diagonalizes the Dirac neutrino mass matrix are much smaller than the corresponding ratios of Dirac neutrino masses, $s_{ij} \ll \sqrt{m_{\nu_i}/m_{\nu_j}}$.

**IV. CONCLUSION**

In this paper we have studied the impact of the form of the Dirac neutrino mass matrix $M_{\nu}$ on the generation of the baryon asymmetry through leptogenesis. Assuming a full hierarchical spectrum of Majorana neutrinos, out of five $M_{\nu}$ related to $M_u$ none of them generates a sufficient amount of baryon asymmetry. Even the
matrix $M_{\nu}^{VI}$, related to $M_d$ (or $M_e$), is not able to produce the required value of the asymmetry. If $M_1 \sim M_2$ the asymmetry is enhanced, so that $M_{\nu}^{I-III}$ can produce enough asymmetry. This case is not realized if a more precise expression for the Dirac neutrino mass matrix is used. Therefore, we conclude that, in the present context, both the quark-lepton symmetry ($M_{\nu} \sim M_u$) and the up-down symmetry ($M_{\nu} \sim M_e$) for mass matrices are not reliable for the baryogenesis via leptogenesis mechanism. We stress that, with a more moderate hierarchy in $M_{\nu}$, we can get a sufficient amount of baryon asymmetry.

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