Mathematical modeling of a water-in-oil emulsion droplet behavior under the microwave impact

Y I Fatkhullina¹, A A Musin¹, L A Kovaleva¹ and I S Akhatov¹,²

¹ Bashkir State University, Center for Micro and Nanoscale Dynamics of Dispersed Systems, 32 Zaki Validi Street, Ufa 450074, Russia
² North Dakota State University, Department of Mechanical Engineering, Fargo, ND, USA

E-mail: liana-kovaleva@yandex.ru

Abstract. The problem of microwave (MW) electromagnetic radiation impact on a single water-in-oil droplet is considered. The system of heat equations within the droplet and in the surrounding liquid, incompressible Navier-Stokes equations within the droplet and in the surrounding liquid, and equation of state are considered. The formulated problem is solved numerically using TDMA (Tri-diagonal-matrix algorithm), SIMPLE algorithm and VOF method (volume of fluid method for the dynamics of free boundaries) in Euler coordinates. The results in the form of the dependence of the temperature within the droplet and in the surrounding liquid on the time of microwave impact and streamlines thermal convection are represented; dependence of the velocity of droplet’s moving on the power of the microwave impact is shown. The obtained results can help to establish criteria for the efficient applicable of the microwave method for the water-in-oil emulsions destruction.

Introduction

The formation of high stability of water-in-oil emulsions is one of the negative factors in extracting and processing of oil, their preparation and transportation, as well as liquidation/recycling of oil-sludge barns. The use of conventional techniques to destroy the emulsions yields no positive results. The use of microwave radiation is one of the perspective method for the destruction of water-in-oil emulsions [1-3].

The emulsions are regarded as a heterogeneous system comprising two immiscible liquids such as oil and water, with one dispersed in the other in the presence of surfactants [4]. Besides, tiny solid rock particles (clay, quartz, salts, etc.) suspended in that system can also act as emulsion stabilizers [5]. High stability of water-in-oil emulsions is conditioned by the oil high-molecular polar components which cover water droplets forming the so called armor envelope, and that prevents the coalescence of the droplets [6,7].

To study the MW actions on the water droplet surrounded by a dielectric liquid the system of heat and Navier-Stokes equations within the droplet and in the surrounding liquid is considered. These
equations are based on fundamental laws of motion of multi-phase medium. Formulation of the problem.
It is assumed that the water droplet with radius \( r_0 \) surrounded by a hydrocarbon liquid in a
gravitational and MW fields. Spherical droplet is in the center of a cylindrical vessel with the height \( h \)
and the radius \( r_1 \) (fig.1.). The surface tension effect is modeling. In addition, it is considered that the
infinitely thin of armor envelope is formed on the surface of the droplet. It is assumed, that droplet
doesn’t deform and keeps spherical shape in all time of influence. The inertial and ponderomotive
forces are neglected. It's taken into account that the dissipation of the microwave energy is within a
water droplet only. The motion of each component of the system is described by a system of thermal
convection equations in linear Boussinesq approximation [8]. The law of conservation of energy and
the conditions of dynamic equilibrium are executed on the interface.
TDMA (Tri-diagonal-matrix algorithm), SIMPLE algorithm and VOF method (volume of fluid
method for the dynamics of free boundaries) were used for the numerical solution of this problem [9].
The surface tension converts to the bulk force using continuum surface force (CSF) model. The model
interprets surface tension as a continuous, three-dimensional effect across an interface, rather than as a
boundary value condition on the interface [10].
The direction and magnitude of bulk force are determined by the density gradient and curvature of the
surface. In view of the assumptions made about the safety of a spherical droplet bulk forces are always
directed to the center of the droplet and the curvature of the surface of the droplet is constant and equal
to \( 2/r_0 \).
Electromagnetic field energy is converted on heating of the water, and is modeled as a distributed heat
source [2], magnitude of which is determined from the expression:
\[
q_0 = \frac{\varphi \varepsilon_0 \varepsilon' \tan \delta}{2} |E|^2
\]
where \( \varphi \) is the frequency of electromagnetic radiation; \( \varepsilon_0 \) is the dielectric constant; \( \varepsilon' \) is the
dielectric permittivity; \( \tan \delta \) is the dielectric loss tangent; \( E \) is the electrical intensity.
Then the dimensionless system of equations of thermal convection is considered in the following
dimensionless form:
\[
\frac{\partial \rho u}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho u u) + \frac{\partial}{\partial z} (\rho v u) = - \frac{\partial P}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \eta \frac{\partial u}{\partial r} \right)
\]
\[
\frac{\partial \rho v}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho u v) + \frac{\partial}{\partial z} (\rho v v) = - \frac{\partial P}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \eta \frac{\partial v}{\partial r} \right)
\]
\[
\frac{\partial \rho c T}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho c u T) + \frac{\partial}{\partial z} (\rho c v T) = \frac{1}{Pr} \frac{1}{r} \frac{\partial}{\partial r} \left( r k \frac{\partial T}{\partial r} \right)
\]
\[
\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho u) + \frac{\partial \rho v}{\partial z} = 0,
\]
volume expansion; C is the concentration (the volume fraction); \( q \) is the dimensionless density of distributed heat sources. Dimensionless numbers are defined as follows:

\[
\Pr = \frac{c \rho_0}{k_0} \quad \text{is the Prandtl number}
\]

\[
Gr = \frac{Bg \rho_0^3 T_0 \rho_0}{\eta_0^2} \quad \text{is the Grashof number}
\]

\[
Os = \frac{r_0^2 q_0}{T_0 k_0} \quad \text{is the Ostrogradskii number}
\]

\[
We = \frac{P_0 \rho_0^2 r_0}{\sigma} \quad \text{is the Weber number}
\]

\[
Eu = \frac{P_0}{\rho_0 v_0^2} \quad \text{is the Euler number}
\]

\[
Re = \frac{v_0 r_0 P_0}{\eta_0} \quad \text{is the Reynolds number}
\]

Due to the symmetry the problem was solved in a cylindrical coordinate system \((r, h, \varphi)\) with the origin at the center of the drop (Fig. 1). Coordinate axes are oriented so that the direction of the \(z\)-axis coincides with the direction of the vector of electrical intensity \( \mathbf{E} = (0, E_z, 0) \) and the vector of gravitational acceleration \( \mathbf{g} = (0, g_z, 0) \). Thus, there is no dependence on the coordinate \( \varphi \).

![Figure 1. The computational domain scheme.](image)

Accordingly the boundary conditions can be written as follows:

1. \( T(r, z, t = 0) = T_0 \)
2. \( u, v(r, z, t = 0) = 0 \)
3. \( \frac{\partial T(0, z, t)}{\partial r} = 0, \frac{\partial T(R_{\text{max}}, z, t)}{\partial r} = 0, \frac{\partial T(r, Z_{\text{max}}, t)}{\partial z} = 0, \frac{\partial T(r, 0, t)}{\partial z} = 0 \)
4. \( u|_{r=0} = 0, u, v|_{r=R_{\text{max}}} = 0, u, v|_{Z=0} = 0, u, v|_{Z=Z_{\text{max}}} = 0 \)
5. \( \frac{\partial v}{\partial r}|_{r=0} = 0 \)
Calculation results
Profiles of temperature along the equator and the poles in the droplet and surrounding liquid in case \( q = 5 \times 10^7 \text{ W/m}^3 \) and temperature coefficient \( \gamma = 1.5 \text{ 1/K} \) are shown in fig. 2-3.

![Figure 2](image1.png)  ![Figure 3](image2.png)

**Figure 2.** Profiles of temperature along the equator (dashed line) and the poles (solid line) in the droplet and surrounding liquid in case \( t = 10 \text{ sec} \).

**Figure 3.** Profiles of temperature along the equator (dashed line) and the poles (solid line) in the droplet and surrounding liquid in case \( t = 40 \text{ sec} \).

In the case of the MW influence the field energy dissipates directly within the droplet. Along the equator the droplet heats irregularly, one can observe two characteristic peaks. This is due to the fact that the convective flow (fig. 4) carries heating, thereby the center of droplet is heated slower than the edge (fig. 5). Also, both poles of the droplet are heated differently too. Surrounding liquid is heated primarily from the top this is due to the fact that the droplet falls down (fig. 5). Graph of droplet velocity depending on the time is shown in fig. 6.

![Figure 4](image3.png)  ![Figure 5](image4.png)

**Figure 4.** Streamlines in the droplet and the surrounding liquid under and MW heating.

**Figure 5.** Distribution of temperature inside and outside the water droplet in case \( t = 40 \text{ sec} \).
Figure 6. Droplet velocity depending on the time

Droplet velocity is established by the time \( t = 36 \) sec.

Profiles of temperature along the equator and the poles in the droplet and surrounding liquid in case \( q=5 \times 10^7 \) W/m\(^3\) and temperature coefficient \( \gamma = 0.8 \) 1/K are shown in fig. 7-8.

Figure 7. Profiles of temperature along the equator (dashed line) and the poles (solid line) in the droplet and surrounding liquid in case \( t=10 \) sec.

Figure 8. Profiles of temperature along the equator (dashed line) and the poles (solid line) in the droplet and surrounding liquid in case \( t=100 \) sec.

The droplet of water is heated by the previous case, but over time the heated region on top of the drop is increased significantly (fig. 7-8). This is due to the fact that in this case, the droplet doesn’t fall, so the convective flow (fig. 9) carries the heated liquid from bottom to top. This is clearly seen in fig. 10.
It should also be noted that the maximum temperature doesn’t exceed 75 degrees and doesn’t change with time due to temperature stabilization.

Conclusions

The numerical results show, that the singularity heating of the droplet and droplet movement in the liquid are different at various temperature coefficients. In case of decrease of the temperature coefficient one can see a more intense heating of the surrounding liquid with motionless droplet. When $\gamma=1.5$ 1/K the droplet up down with established velocity $V=0.00271$ m/sec. The influence of the temperature coefficient is caused by the fact that the increasing of the temperature coefficient leads to more intense heating the medium, reducing its viscosity, as a result of which droplets fall downwards under the action of gravity. Streamlines in both cases are the same, but fluid moves clockwise to the right and counter-clockwise to the left.

Acknowledgments

This research is supported by the Ministry of Education and Science of the Russian Federation (government contract No.3.1251.2014/K and grant No. 11.G34.31.0040) and the RFBR grant № 14-01-97005.

References

[1] Kovaleva L A , Zinnatullin R , Minnigalimov R Z 2009 Russia. J. Oil-field business. 5 54
[2] Zinnatullin R R , Kovaleva L A , Minnigalimov R Z 2009 Russia. J. Oil facilities. 7 75
[3] Noik C, Chen J, and Dalmazzone C 2006 SPE 103808 12
[4] Leal-Calderon F , Bibette J , Schmitt V 2007 Springer 228
[5] Gamal M , Mohamed A M O and Zekri A Y 2005 J. Petroleum Science and Engin. 16 209
[6] Kovaleva L A , Minnigalimov R Z and Zinnatullin R R 2011 J. Energy & Fuels 8 3731
[7] Kovaleva L, Musin A, Zinnatullin R, Akhatov I S 2011 *Proceedings of ASME 2011 International Mechanical Engineering Congress & Exposition, IMECE2011-62935* (Denver, Colorado, USA)

[8] Boussinesq J 1903 *Theorie analytique de la chaleur* (Gauthier-Villars, Paris vol 3)

[9] Patankar C V 1980 *Numerical Heat Transfer and Fluid Flow*. (McGraw-Hill, New York)

[10] Brackbill U, Kothe D B, and Zemach C 1992 *J. Comp. Physics* **21** 335