THREE KEY TESTS TO GRAVITY

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Three presumably unrelated open questions concerning gravity and the structure of the Universe are here discussed: 1) To which extent is Lorentz invariance an exact symmetry? 2) What is the equation of state of the Universe? 3) What is the origin of the so-called Pioneer anomaly?

1 Lorentz Symmetry

Lorentz invariance is one of the most well established symmetries of physics and is a basic ingredient in all known physical theories. However, recently, there has been evidence that this symmetry may be broken in at least three different phenomena:

i) Observation of ultra-high energy cosmic rays (UHECRs) with energies beyond the so-called Greisen-Zatsepin-Kuzmin (GZK) cutoff, \( E_{\text{GZK}} \approx 4 \times 10^{19} \text{eV} \). Breaking of Lorentz invariance may explain these events as it implies that resonant scattering reactions with photons of the Cosmic Microwave Background Radiation (CMBR), e.g. \( p + \gamma_{2.73K} \rightarrow \Delta_{1232} \), are suppressed. Astrophysical solutions to this paradox are possible and require identifying energetically viable sources within \( D_{\text{Source}} \lesssim 50 - 100 \text{Mpc} \), so that the traveling time of the emitted particles is shorter than the attenuation time due to particle photoproduction on the CMBR. Within a volume of radius 50 – 100 Mpc around us only neutron stars, active galactic nuclei, gamma-ray bursts and cluster of galaxies are feasible acceleration sites. However, problems related with the lack of spatial correlations between observed UHECRs and candidate sources, and the mismatch of fluxes has rendered these alternatives untenable so far (see e.g. Ref. [10] and references therein).

ii) Events involving gamma radiation with energies beyond 20 TeV from distant sources such as Markarian 421 (\( z = 0.031 \)) and Markarian 501 (\( z = 0.033, D \approx 157 \text{Mpc} \)) blazars. These observations can be explained via the breaking of Lorentz symmetry as otherwise, due to
pair creation, there should exist a strong attenuation of fluxes beyond 100 Mpc by the diffuse extragalactic background of infrared photons.\textsuperscript{12}

iii) Studies of the evolution of air showers produced by ultra high-energy hadronic particles suggest that pions live longer than expected.\textsuperscript{13}

Of course, breaking of Lorentz symmetry may lead to other threshold effects related with pair creation with asymmetric momenta, photon stability, alternative Čerenkov effects, etc.\textsuperscript{14,15}

From the theoretical point of view, work in the context of string/M-theory shows that Lorentz symmetry may be spontaneously broken by nontrivial solutions in string field theory,\textsuperscript{16} a feature that is also found in loop quantum gravity,\textsuperscript{17–18} in noncommutative field theories,\textsuperscript{19,20} and in quantum gravity inspired spacetime foam scenarios.\textsuperscript{21} A related issue arising in noncommutative field theories concerns the breaking of translational symmetry by interactions, leading to a potential solution for the abovementioned astrophysical problems.\textsuperscript{22} The interactions resulting from the breaking of Lorentz symmetry may lead to striking implications at low-energy,\textsuperscript{23,24,25,26} and as well as to the breaking of CPT symmetry.\textsuperscript{27} The breaking of CPT allows, on its hand, for a thermodynamical mechanism for baryogenesis.\textsuperscript{28} An extension of the Standard Model (SM) that incorporates violations of Lorentz and CPT symmetries has already been built.\textsuperscript{29}

On quite general terms, attempts to explain the three discussed paradoxes rely on deformations of the relativistic dispersion relation for a particle species $a$, such as

$$E_a^2 = p_a^2 c_a^2 + m_a^2 c_a^4 + F(E_a, p_a, m_a, c_a) \ ,$$

where $c_a$ is the maximal attainable velocity for particle $a$ and $F$ is a function of $c_a$ and of the relevant kinematic variables.

For example, Coleman and Glashow\textsuperscript{30} consider, to explain the observation of cosmic rays beyond the GZK cutoff, that each particle has its own maximal attainable velocity and $F = 0$. A tiny difference between the maximal attainable velocities, $c_p - c_\Delta \equiv \epsilon_{p\Delta} \simeq 1.7 \times 10^{-25} c$, can explain the events beyond the GZK cutoff.\textsuperscript{31} This bound on Lorentz symmetry is about three orders of magnitude more stringent than the experimental one, $\delta < 3 \times 10^{-22}$.

Function $F$ arising from the Lorentz violating extension of the SM fermionic sector discussed in Ref.\textsuperscript{29} is shown to be\textsuperscript{32}

$$F = -2c_{00} E^2 \pm 2d_{00} E p \ ,$$

with $c_a = c$ for all particles. Coefficients $c_{00}$ and $d_{00}$ are time-like and flavour-dependent, while the plus/minus sign concerns the difference in the chirality of particles.

It has been pointed out that some quantum gravity, stringy inspired and noncommutative field theory models lead to a modification of the dispersion relation that is cubic in the momentum,\textsuperscript{23,24,25,26,31}

$$F = -k_a \frac{p_a^3}{M} \ ,$$

where $k_a$ is a constant and $M$ a mass scale. A deformation of this type can potentially explain the three discussed paradoxes.\textsuperscript{13,23,24,25,26,31} On any account, it is clear that further observational and theoretical work is still required in order to settle to which extent Lorentz symmetry is an exact symmetry. Implications for gravity can be manifold. We shall discuss in Section 3 an example.
2 Generalized Chaplygin Gas Model

It has recently been suggested that evidence for a dark energy component to the total energy density of the Universe as inferred from Type Ia Supernova observations might be accounted for by a change in the equation of state of the background fluid, rather than by a cosmological constant or the dynamics of a scalar field rolling down a potential. In the context of Friedmann-Robertson-Walker cosmology, one considers an exotic background fluid, the generalized Chaplygin gas (GCG), which is described by the equation of state

\[ p_{\text{ch}} = -\frac{A}{\rho_{\text{ch}}^\alpha}, \]  

where \( \alpha \) is a constant in the range \( 0 < \alpha \leq 1 \) (the Chaplygin gas corresponds to the case \( \alpha = 1 \), and clearly \( \alpha = 0 \) to the \( \Lambda \)CDM model) and \( A \) a positive constant. Inserting this equation of state into the relativistic energy conservation equation, leads to an energy density that evolves as

\[ \rho_{\text{ch}} = \left( A + \frac{B}{a^{3(1+\alpha)}} \right)^{-\frac{1}{\alpha}}, \]

where \( a \) is the scale-factor of the Universe and \( B \) an integration constant. It is remarkable that the model naturally interpolates between a universe dominated by dust at early time and a De Sitter one at late time, with an intermediate phase described by a cosmological constant plus matter with a “soft” equation of state, \( p = \alpha \rho \) (\( \alpha \neq 1 \)). Notice that the chosen interval for \( \alpha \) ensures that the sound velocity \( (c_s^2 = \alpha A/\rho_{\text{ch}}^{1+\alpha}) \) does not exceed, in the “soft” equation of state phase, the velocity of light. Furthermore, as pointed out in Ref. [34], it is only for \( 0 < \alpha \leq 1 \) that the analysis of the evolution of energy density fluctuations makes sense. It is also shown that the GCG can be described by a complex scalar field with a generalized Born-Infeld action. It is clear that the GCG is a potential candidate for explaining the observed accelerated expansion of the Universe as it leads to an asymptotic phase where the equation of state is dominated by a cosmological constant, \( 8\pi G A^{1/1+\alpha} \). It has also been shown that the model admits, under conditions, an inhomogeneous generalization which can be regarded as a unification of dark matter and dark energy \( [34,35] \) without conflict with standard structure formation scenarios \( [34,35,36] \). Thus the GCG model is an interesting alternative to models where the accelerated expansion of the Universe is due to an uncanceled cosmological constant (see [42] and references therein) or a rolling scalar field as in quintessence models with one or two scalar fields \( [39] \).

The possibility of describing dark energy via the GCG has led to great interest in constraining the model using observational data, particularly those arising from Type Ia Supernovae and gravitational lensing statistics. Furthermore, it has been shown more recently that the positions of peaks and troughs of the CMBR power spectrum as arising from WMAP and BOOMERanG data, allow constraining a sizeable portion of the parameter space of the GCG model \( [13,14] \) (see also Refs. [45,46]). Indeed, bounds on the locations of the first two peaks and the first trough, from WMAP measurements of the CMBR temperature angular power spectrum

\[
\ell_{p_1} = 220.1 \pm 0.8, \\
\ell_{p_2} = 456 \pm 10, \\
\ell_{d_1} = 411.7 \pm 3.5, 
\]

(6)
and the location of the third peak, from BOOMERanG measurements

\[ \ell_{ps} = 825^{+10}_{-13}, \]

do imply in nontrivial constraints on the parameters of the GCC model.

These constraints can be summarized as follows. The Chaplygin gas model, \( \alpha = 1 \), is incompatible with the data and so are models with \( \alpha \gtrsim 0.6 \). For \( \alpha = 0.6 \), consistency with data requires for the spectral tilt, \( n_s > 0.97 \). Actually, ΛCDM model barely fits the data for \( n_s \simeq 1 \) (notice that WMAP data yield \( n_s = 0.99 \pm 0.04 \)) and \( h \gtrsim 0.71 \) is required \( (H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}) \); for lower values of \( n_s \), ΛCDM and the GCG model give similar results. Moreover, one finds that data favours \( \alpha \simeq 0.3 \) GCG models.

These results are compatible with the ones of Ref. [43] using BOMERanG data for the third peak and Archeops data for the first peak, as well as Type Ia Supernovae bounds, namely \( 0.2 \lesssim \alpha \lesssim 0.6 \), \( 0.81 \lesssim A_s \lesssim 0.85 \), where \( A_s \equiv A/\rho_{ch0} \).

As pointed out in Ref. [43], considering, besides CMBR and Supernovae data, bounds arising from faraway quasar sources lead to more stringent bounds on the parameters of the GCG model. Bounds arising from Type Ia Supernovae data, which suggest that \( 0.6 \lesssim A_s \lesssim 0.85 \), are consistent with results of Ref. [44] for \( n_s = 1 \) and \( h = 0.71 \).

It worth mentioning that future Supernovae data together with statistics of gravitational lensing surveys may further constrain the parameter space of the GCG model, although such studies are very dependent on the considered fiducial models.

The interested reader is referred to Refs. [43,44] for technical details and to Ref. [50] for a brief review on the GCG model.

3 Pioneer Anomaly

Studies of radiometric data from the Pioneer 10/11, Galileo and Ulysses have revealed the existence of an anomalous acceleration on all four spacecraft, inbound to the Sun and with a constant magnitude of \( a_A \simeq (8.5 \pm 1.3) \times 10^{-10} \text{ ms}^{-2} \). Attempts to explain this anomaly as a result of poor accounting of thermal and mechanical effects, as well as errors in the tracking algorithms, have shown to be unsuccessful (see however Ref. [52]). Since the two Pioneer spacecraft follow approximate opposite hyperbolic trajectories away from the Solar System, while Galileo and Ulysses describe closed orbits, and from the fact that the three designs are geometrically distinct, it seems plausible to assume that this anomaly involves new physics.

Many proposals have been advanced to explain this anomaly: a Yukawa-like or higher order corrections to the Newtonian potential, interaction of the spacecraft with a long-range scalar field, determined by an external source term proportional to the Newtonian potential to mention just a few. It is interesting that in higher-curvature theories of gravity where the gravitational coupling is asymptotically free, a feature that has attracted some attention in the context of the dark matter problem, the gravitational coupling is stronger on large scales, thus implying, in principle, to a Pioneer-like anomalous acceleration.

As an alternative explanation, we exploit the implications of the idea that the Pioneer anomaly reflects the bimetric nature of spacetime in the Solar System. This proposal, although not fully consistent (see below), is based on the bimetric theory of gravity put forward long ago by Rosen.

One considers a region of spacetime endowed, besides the dynamical metric \( g_{\mu\nu} \), with a non-dynamic metric \( \eta_{\mu\nu} \), that has signature +2, is Riemann flat or has a constant curvature. The background metric \( \eta_{\mu\nu} \) is static and asymptotically \( g_{\mu\nu} + n_{\mu\nu} \rightarrow n_{\mu\nu} \).

In this theory the equation for the dynamical gravitational field is given by.
\begin{eqnarray}
\Box g_{\mu\nu} - g^{\alpha\beta} \eta_{\alpha\beta} g_{\mu\nu} = -16\pi G (g/\eta)^{1/2} (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T) ,
\end{eqnarray}

where \( T_{\mu\nu} \) is the trace of the energy-momentum of matter and \( \Box \) is the D’Alembertian operator with respect to \( \eta_{\mu\nu} \). One can always choose coordinates in which \( (\eta_{\mu\nu}) = \text{diag}(-1, 1, 1, 1) \), and \( (g_{\mu\nu}) = \text{diag}(-c_0, c_1, c_1, c_1) \), where \( c_0 \) and \( c_1 \) are parameters that may vary on a Hubble \( H^{-1} \) timescale.

Rosen’s bimetric theory explicitly breaks Lorentz invariance. This can be understood by resorting to the Parametrized Post-Newtonian (PPN) formalism, a systematic expansion of first-order \( 1/c^2 \) terms in the Newtonian gravitational potential and related quantities. It turns out that all metric theories of gravity can be classified according to ten PPN parameters: \( \gamma, \beta, \xi, \alpha_1, \alpha_2, \alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4 \). These are the linear coefficients of each possible first-order term (generated from rest mass, energy, pressure and velocity), and relate a particular theory with fundamental aspects of physics: conservation of linear and angular momentum, preferred-frame and preferred-location effects, nonlinearity and space-curvature per unit mass, etc.

Einstein’s General Relativity, the most successful theory up to date, exhibits a set of PPN parameters where \( \beta = \gamma = 1 \) and the remaining ones are equal to zero. Rosen’s bimetric theory has \( \beta = \gamma = 1 \), but a nonvanishing \( \alpha_2 = c_0/c_1 - 1 \) coefficient. This indicates that the theory is semiconservative, i.e. angular momentum is not conserved, and that it exhibits preferred-frame effects, meaning that Lorentz symmetry is broken and the Strong Equivalence Principle does not hold.

By linearizing Eq. (8) in the vacuum, one obtains the wave equation for weak gravitational waves, whose solution corresponds to a wave propagating with speed \( c_g = \sqrt{c_1/c_0} \). Thus, \( \alpha_2 \) measures the difference in the propagation velocities (measured by an observer at rest in the Universe rest frame) between electromagnetic and gravitational waves. A rigorous study of deviations between the Sun’s spin axis and the ecliptic does lead to the experimental constraint \( |\alpha_2| < 1.2 \times 10^{-7} \).

Due to their small mass, the abovementioned spacecraft can be regarded test particles, and their acceleration can be obtained by computing the timelike geodesics of the full metric, \( h_{\mu\nu} = \eta_{\mu\nu} + g_{\mu\nu} \):

\begin{eqnarray}
\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0 .
\end{eqnarray}

In the Newtonian limit and for \( v \ll c \), one obtains for a diagonal metric \( h_{\mu\nu} \) the acceleration

\begin{eqnarray}
a^i = -\Gamma^i_{00} = \frac{1}{2} h^{ij} \partial_j h_{00} .
\end{eqnarray}

In the case of a very weak central gravitational field, such as in the Solar System, the background metric is not flat, but given (in Cartesian coordinates with the Sun in the origin) by \( (g_{\mu\nu}) = \text{diag}(-1 - 2U, 1, 1, 1) \), where \( U(r) = -GM/\xi - C/r \) is the gravitational potential. Substitution into Eq. (10) yields

\begin{eqnarray}
a_i = -(1 + c_1)^{-1} \left[ \partial_i U + \frac{1}{2} \partial_j c_0 \right] ,
\end{eqnarray}

from which one can identify the radial anomalous component of the acceleration:

\begin{eqnarray}
\vec{a}_A = c_1 \vec{\nabla}_r U - \frac{1 - c_1}{2} \vec{\nabla}_r c_0 .
\end{eqnarray}

It can be readily seen that, if \( c_0 \) and \( c_1 \) are homogeneous in space, the derived anomalous acceleration is not constant, which seems to contradict the observation. Therefore, assuming...
that these parameters depend on the distance to the Sun, the simplest choice consistent with an homogeneous $\alpha_2$ is:

$$c_0 = A r, \quad c_1 = B r,$$

with $A, B > 0$ so that the resulting anomalous acceleration is inbound. Clearly, from the bound on $\alpha_2$, $|A/B - 1| < 4 \times 10^{-7}$ it follows that $A \approx B$. Notice that $A$ cannot be exactly equal to $B$, since this implies that $\alpha_2 = 0$, meaning that General Relativity is recovered, as there is a coordinate transformation which pulls the metric $g_{\mu \nu}$ back to the asymptotically flat metric $\eta_{\mu \nu}$.

Substituting Ansatz Eq. (13) into Eq. (12), leads to

$$a_A = -\frac{A}{2} \left[ 1 - \frac{B \left( 2C - Br \right)}{Ar} \right] \approx -\frac{A}{2} \left[ 1 - \frac{2C}{r} - Ar \right].$$

In geometric units ($c = G = 1$), $|a_A| \approx 8.5 \times 10^{-10} \text{ms}^{-2} = 1.5 \times 10^{-15} \text{AU}^{-1}$, from which follows that $A = 3 \times 10^{-15} \text{AU}^{-1}$ as $C \equiv GM_\odot = 10^{-8} \text{AU}$.

Hence, in this proposal an hypothetical dedicated probe to confirm the Pioneer anomaly, as suggested in Refs. [63,64], would not need to venture into too deep space to detect such an anomalous acceleration, but just to a distance where it is measurable against the regular acceleration and the solar radiative pressure, actually approximately from Jupiter onwards. Unfortunately, as it stands, Rosen’s bimetric theory, is inconsistent, as it predicts emission of dipolar gravitational radiation from gravitationally bound systems, which is incompatible with data from the binary PSR 1913+16, or no gravitational radiation at all. Possible alternatives will be presented elsewhere.

In any case, it seems fair to conclude that if the Pioneer anomaly turns out to be a real physical effect it may be an important clue on new gravitational physics. Unfortunately, given that Pioneer 10 has, since last February, fallen beyond detectability, most likely only through a dedicated mission the desired confirmation will be achieved.

Acknowledgments

This contribution is based on work developed with various collaborators and friends. I would like to mention their names and thank them explicitly: Maria Bento, Carla Carvalho, Don Colladay, Luis Guisado, Alan Kostelecký, Clovis de Matos, Pedro Martins, David Mota, Jorge Páramos, Robertus Potting, Nuno Santos, Anjan Sen, Pedro Silva and Martin Tajmar. I would like also to express my gratitude to Serge Reynaud who suggested me to participate in the Rencontres de Moriond, Gravitational Waves and Experimental Gravity, and to Jacques Dumarchez for the patience, support and for the organization of such a pleasant and profitable meeting.

References

1. N. Hayashida et al., (AGASA Collab.), *Phys. Rev. Lett.* **73**, 3491 (1994);
M. Takeda et al., (AGASA Collab.), *Phys. Rev. Lett.* **81**, 1163 (1998);
D.J. Bird et al., (Fly’s Eye Collab.), *Phys. Rev. Lett.* **71**, 3401 (1993); *Ap. J.* **424**, 491 (1994); **441**, 144 (1995);
M.A. Lawrence, R.J.O. Reid, A.A. Watson (Haverah Park Collab.), *J. Phys.* **G17**, 733 (1991);
N.N. Efimov et al., (Yakutsk Collab.), ICRR Symposium on Astrophysical Aspects of the Most Energetic Cosmic Rays, eds. N. Nagano, F. Takahara (World Scientific, 1991).
2. K. Greisen, *Phys. Rev. Lett.* **16**, 748 (1966);
G.T. Zatsepin, V.A. Kuzmin, *JETP Lett.* **41**, 78 (1966).
3. H. Sato, T. Tati, *Progr. Theor. Phys.* **47**, 1788 (1972).
4. S. Coleman, S.L. Glashow, *Phys. Lett.* B405, 249 (1997); *Phys. Rev.* D59, 116008 (1999).
5. O. Bertolami, C.S. Carvalho, *Phys. Rev.* D61, 103002 (2000).
6. F.W. Stecker, *Phys. Rev. Lett.* 11, 1016 (1968).
7. C.T. Hill, D. Schramm, T. Walker, *Phys. Rev.* D36, 1007 (1987).
8. A.M. Hillas, *Ann. Rev. Astron. Astrophys.* 22, 425 (1984).
9. J.W. Cronin, *Rev. Mod. Phys.* 71, S165 (1999).
10. O. Bertolami, *Gen. Relativity and Gravitation* 34, 707 (2002).
11. F.W. Stecker, *Phys. Rev. Lett.* 11, 1016 (1968).
12. C.T. Hill, D. Schramm, T. Walker, *Phys. Rev.* D36, 1007 (1987).
13. A.M. Hillas, *Ann. Rev. Astron. Astrophys.* 22, 425 (1984).
14. J.W. Cronin, *Rev. Mod. Phys.* 71, S165 (1999).
15. O. Bertolami, *Gen. Relativity and Gravitation* 34, 707 (2002).
16. F.W. Stecker, *Phys. Rev.* D61, 103002 (2000).
17. C.T. Hill, D. Schramm, T. Walker, *Phys. Rev.* D36, 1007 (1987).
18. A.M. Hillas, *Ann. Rev. Astron. Astrophys.* 22, 425 (1984).
19. J.W. Cronin, *Rev. Mod. Phys.* 71, S165 (1999).
20. O. Bertolami, *Gen. Relativity and Gravitation* 34, 707 (2002).
21. F.W. Stecker, *Phys. Rev.* D61, 103002 (2000).
22. C.T. Hill, D. Schramm, T. Walker, *Phys. Rev.* D36, 1007 (1987).
23. A.M. Hillas, *Ann. Rev. Astron. Astrophys.* 22, 425 (1984).
24. J.W. Cronin, *Rev. Mod. Phys.* 71, S165 (1999).
25. O. Bertolami, *Gen. Relativity and Gravitation* 34, 707 (2002).
26. F.W. Stecker, *Phys. Rev.* D61, 103002 (2000).
27. C.T. Hill, D. Schramm, T. Walker, *Phys. Rev.* D36, 1007 (1987).
28. A.M. Hillas, *Ann. Rev. Astron. Astrophys.* 22, 425 (1984).
29. J.W. Cronin, *Rev. Mod. Phys.* 71, S165 (1999).
30. O. Bertolami, *Gen. Relativity and Gravitation* 34, 707 (2002).
36. J.C. Fabris, S.B.V. Gonçalves, P.E. de Souza, *Gen. Relativity and Gravitation* **34**, 53 (2002).
37. M.C. Bento, O. Bertolami, *Gen. Relativity and Gravitation* **31**, 1461 (1999);
   M.C. Bento, O. Bertolami, P.T. Silva, *Phys. Lett.* **B498**, 62 (2001).
38. M. Bronstein, *Phys. Zeit. Sowjet Union* **3**, 73 (1933);
   O. Bertolami, *Il Nuovo Cimento* **93B**, 36 (1986);
   Fortschr. Physik **34**, 829 (1986);
   M. Ozer, M.O. Taha, *Nucl. Phys.* **B287**, 776 (1987);
   B. Ratra, P.J.E. Peebles, *Phys. Rev.* **D37**, 3406 (1988);
   *Ap. J. Lett.* **325**, 117 (1988);
   C. Wetterich, *Nucl. Phys.* **B302**, 668 (1988);
   R.R. Caldwell, R. Dave, P.J. Steinhardt, *Phys. Rev. Lett.* **80**, 1582 (1998);
   I. Zlatev, L. Wang, P.J. Steinhardt, *Phys. Rev. Lett.* **82**, 986 (1999);
   J.P. Uzan, *Phys. Rev.* **D59**, 123510 (1999);
   T. Chiba, *Phys. Rev.* **D60**, 083508 (1999);
   L. Amendola, *Phys. Rev.* **D60**, 043501 (1999);
   O. Bertolami, P.J. Martins, *Phys. Rev.* **D61**, 064007 (2000);
   N. Banerjee, D. Pavón, *Phys. Rev.* **D63**, 043504 (2001);
   A.A. Sen, S. Sen, S. Sethi, *Phys. Rev.* **D63**, 107501 (2001).
39. Y. Fujii, *Phys. Rev.* **D61**, 023504 (2000);
   A. Masiero, M. Pietroni and F. Rosati, *Phys. Rev.* **D61**, 023504 (2000);
   M.C. Bento, O. Bertolami, N.C. Santos, *Phys. Rev.* **D65**, 067301 (2002).
40. J.C. Fabris, S.B.V. Gonçalves, P.E. de Souza, astro-ph/0207430;
   A. Dev, J.S. Alcaniz, D. Jain, astro-ph/0209379;
   J.S. Alcaniz, D. Jain, A. Dev, astro-ph/0210476.
41. M. Makler, S.Q. de Oliveira, I. Waga, astro-ph/0209486.
42. P.T. Silva, O. Bertolami, astro-ph/0303353.
43. M. C. Bento, O. Bertolami, A. A. Sen, *Phys. Rev.* **D67**, 063003 (2003).
44. M. C. Bento, O. Bertolami, A. A. Sen, astro-ph/0303538.
45. D. Carturan, F. Finelli, astro-ph/0211626.
46. L. Amendola, F. Finelli, C. Burigana, D. Carturan, astro-ph/0304325.
47. D.N. Spergel et al., astro-ph/0302207; L. Page et al., astro-ph/0302220.
48. P. Bernardis et al., *Ap. J.* **564**, 559 (2002).
49. A. Benoit et al., astro-ph/0210306.
50. M. C. Bento, O. Bertolami, A. A. Sen, *Gen. Relativity and Gravitation* to appear, astro-ph/0305086.
51. J.D. Anderson, P.A. Laing, E.L. Lau, A.S. Liu, M.M. Nieto, S.G. Turyshhev, *Phys. Rev.* **D65**, 082004 (2002).
52. L.K. Scheffer, astro-ph/0107092.
53. J.D. Anderson, P.A. Laing, E.L. Lau, A.S. Liu, M.M. Nieto and S.G. Turyshhev, *Phys. Rev. Lett.* **81**, 2858 (1998).
54. S. Calcchi Novati, S. Capozziello, G. Lambiase, *Grav. Cosmol.* **6**, 173 (2000).
55. J.P. Mbelek, M. Lachieze-Rey, gr-qc/9910105.
56. T. Goldman, J. Pérez-Mercader, F. Cooper, M. Martin-Nieto, *Phys. Lett.* **B281**, 219 (1992).
57. O. Bertolami, J.M. Mourão, J. Pérez-Mercader, *Phys. Lett.* **B311**, 27 (1993).
58. O. Bertolami, J. Garcia-Bellido, *Int. J. Mod. Phys.* **D5**, 363 (1996).
59. O. Bertolami, J. Páramos, DF/IST-1.2003 in preparation.
60. N. Rosen, *Gen. Relativity and Gravitation* **4**, 435 (1973).
61. C.M. Will, “Theory and Experiment in Gravitational Physics”, C.M. Will (Cambridge U. P., 1993).
62. K. Nordtvedt, *Ap. J.* **320**, 871 (1987).
63. O. Bertolami, M. Tajmar. gr-qc/0207123
64. J.D. Anderson, M.M. Nieto, S.G. Turyshev. gr-qc/0205059
65. C.M. Will, D.M. Eardley, Ap. J. Lett. 212, L91 (1977).
66. N. Rosen, Ap. J. 221, 284 (1978).