Superconducting resonator single-photon spectroscopy through electromagnetically induced transparency

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Investigation of intrinsic loss mechanism of superconducting resonator is a crucial task toward the study of the constituent material as well as application in quantum information process. Typical approach from transmission or reflection spectrum is however subjected to Fano-effect, which can induce systematic errors in discerning intrinsic and external losses. To avoid such requires under-coupled resonator and consequently sets a challenge when a large quality factor is expected and measurements at single-photon power levels is required. In this work, we propose and demonstrate a new approach with additional qubit coupled dispersively. Inducing electromagnetically induced transparency (EIT) in qubit spectrum, we can extract the resonator’s single-photon internal linewidth. Our work demonstrates a practical application of EIT for device spectroscopy.

Understanding loss mechanism of superconducting resonator is of importance in many perspectives. One can investigate the property of material comprising of the superconducting resonator through the loss rate [1]. It also concerns those interested in quantum information process (QIP) [2]. Bosonic mode quantum computing has been paid great attention recently thanks to the convenience in implementing quantum error correction (QEC) [3, 4]. In this scheme, harmonic system such as ion’s mechanical mode or photonic mode carry the quantum information instead of two-level quantum bit [5, 6]. Superconducting resonator is one of the most promising platform for this purpose. It goes without saying that longer lifetime of the resonator is desired to minimize the quantum errors and therefore figuring out what limits the resonator’s lifetime is highly demanded.

A typical approach to estimate the loss rate of superconducting resonator $\kappa$ is to couple the resonator to external transmission lines and measure its transmission or reflection spectrum. If one uses a reflection or side-coupled geometry, it is also possible to the total loss rate into contributions from external losses, arising from the coupling of the resonator to the transmission lines, from the internal losses intrinsic to the resonator itself. This separation is particularly important for understanding the nature and origin of the losses and ultimate potential limit of the coherence of the device.

Unfortunately, such direct spectroscopy measurements often suffer from asymmetric lineshapes, known as Fano resonances [7], which arise from interference of the signal from the resonator with signals reaching the detector via a path bypassing the device. Although the asymmetric lineshape can be reproduced by an appropriate fitting procedure, without independent open-short-load calibration of the full measurement setup, this is not sufficient to extract all parameters independently. Ultimately, because of the lack of a calibrated model of these leakage signals, it is not possible to perform the separation of the internal and external loss rates without systematic error [8].

One approach to overcome the systematic error from Fano resonances is to ensure that the resonator is highly undercoupled: $\kappa_e \ll \kappa_i$. In this case, one can approximate $Q_i \approx Q_{load}$, taking advantage of the fact that $Q_{load}$ can be extracted from Fano due to interference with leakage signals with no systematic error or unknown parameters. A disadvantage of this approach is that the signal to noise ratio of the measurement becomes very poor, resulting in very long averaging times, particularly for probing the resonator quality factor at single photon levels relevant for quantum information applications. A further challenge is that one does not know the value of $Q_i$ ahead of time, requiring the design of the coupling of the resonator to the transmission lines to be iterated until reaching the appropriate limit. Ideally, for the purpose of loss rate spectroscopy of very high Q resonators, it would be highly desirable to have a spectroscopic tool that does not suffer from systematic calibration errors, and for which the coupling of the resonator to the probe can be tuned in-situ.

Here, we present spectroscopic technique for probing the quality factor of high-Q superconducting resonators based on the principle of electrically induced transparency (EIT) [9–12] using a superconducting qubit. Taking advantage of a well characterised model of the superconducting qubit, we are able to extract the resonator quality factor from a calibrated model with no unknown parameters. The coupling of the qubit to the resonator can also be tuned in-situ using the strength of the drive signals. Our approach makes use of a ‘bad’ qubit, whose decay rate is much faster than that of the target resonator. Although EIT has been reported recently in circuit QED in several works [13–17], these reports included observations in which high coherence qubit provides a metastable state to satisfy the EIT condition. Here, we observe EIT with bad qubit condition for the first time to our best knowledge.

A description of the experiment is present in Fig. 1(a). A resonator and a Transmon are dispersively coupled, whose transition frequencies are $\omega_1$ and $\omega_2$, respectively. We can couple $(g,n)$ and $(e,n-1)$ by off-resonant driving (red arrows) that creates two-photon sideband transition with coupling rate of $\Omega_{ct}$. Please note that single photon transition between these levels are forbidden in
the case of Transmon, the most ubiquitous type of qubit nowadays. In the meantime, a weak field (blue arrow) probes the qubit transition. We define the detuning between qubit and probe \( \omega_q - \omega_p \) as \( \Delta \). Also, we define \( \delta \) as \( \omega_q - \omega_t - 2\omega_d \).

When EIT condition \( |\gamma - \kappa| > \Omega_e \) satisfies, the control field \( \Omega_{\text{ct}} \) leads to a narrow transparency window in qubit transmission spectrum. This phenomena results from the interference between two different transitions, \(|g, n + 1\rangle \rightarrow |e, n\rangle\) and \(|g, n\rangle \rightarrow |e, n\rangle\). In this work, we measure the population of qubit \( \rho_{ee} \) rather than transmission \( \text{Im}[\rho_{eg}] \) for efficiency. We define \( \rho_{ee} \) for given probe frequency \( \omega_p \) as a qubit population spectrum in followings. In Fig. 1(b), we simulate the qubit population spectrum with reasonable parameters satisfying EIT condition. The detail methodology of the simulation can be found in Supplementary Information. We can also find the same features of transmission spectrum in the qubit population spectrum as well. The Lorentzian dip in the qubit population spectrum, which we call as interference dip in followings, is characterized by width \( (w) \) and minimum population \( (d) \). In linear response limit, that is \( \Omega_p \ll \Omega_{\text{ct}} \) satisfies, \( w \sim \gamma + \kappa - \sqrt{(\gamma - \kappa)^2 - \Omega_{\text{ct}}^2} \) and \( d \sim \omega_p/(\gamma + \Omega_{\text{ct}}^2) \) hold [16]. It should be remarked that although \( \Omega_p \ll \Omega_{\text{ct}} \) does not hold in the present simulation in Fig. 1, we can clearly see the interference dip and the shape of the spectrum is the same as in linear response. From numerical simulation, however, we confirm that the visibility of the interference dip decreases with excessively large probe amplitude. The width \( (w) \) and minimum population \( (d) \) of the interference dip contain information on \( \kappa \) and hence one can extract \( \kappa \) from fitting by numerical model used in Fig. 1(b).

For EIT condition, one can obtain sufficient \( \Omega_{\text{ct}} \) even with with negligible dispersive coupling between qubit and resonator. With sufficiently small \( \Omega_e \) as in Fig. 1(b), introducing dispersive coupling does not make a significant difference in qubit population spectrum. Due to such a small dispersive coupling, the effect of qubit to quality factor on the target resonator is negligible, allowing us we can probe the resonator in a non-invasive way. Broad qubit linewidth also have a vantage in finding a transition. One can detect sideband transition as long as \( \delta \leq \gamma \).

In Fig. 1(c), we present the simplified circuit diagram of the device that we used in the experiment. More detail information on the circuit design and related electronics can be found in Supplementary Information. A Transmon qubit \( \omega_q/2\pi=6.723 \) GHz, without sideband drive) is capacitively coupled to two \( \lambda/4 \) co-planar waveguide(CPW) resonators. One is target resonator \( \omega_r/2\pi=2.9 \) GHz), whose internal quality factor is of our interest. The other is readout resonator \( \omega_r/2\pi=4.07 \) GHz) to measure qubit’s population more efficiently. Both are dispersively coupled to the qubit with interaction strengths \( \chi_{qr}/2\pi=20 \) kHz and \( \chi_{qr}/2\pi=1.3 \) MHz respectively. Each resonators are inductively coupled to each different feedlines. The target resonator is extremely undercoupled on purpose \( (Q_i/Q_e \approx 100) \) and therefore \( Q_{\text{load}} \approx Q_i \) satisfies.

We apply the sideband drive directly to Josephson junction of the circuit through the direct drive line (red arrow in Fig. 1(c)). Qubit probe \( \Omega_{pr} \) is applied through the feedline coupled to the readout resonator (right blue arrow). In addition, we have direct probe to the target resonator (left blue arrow). We use total four microwave sources in the experiment. Each of them is used for qubit probe, qubit readout, sideband drive and direct
resonator probe respectively. In order to avoid measurement induced broadening in qubit population spectrum, we perform the measurement in pulsed form. We apply a 20-μs long probe pulse shortly followed by a 200-ns long readout pulse. Consequently, qubit population spectrum is unaffected by the readout photons in the readout resonator.

In Fig. 2(a), we measure the interference dip in the qubit population spectrum, which results from a sideband transition between the qubit and the target resonator. Sideband drive frequency \( \omega_d \) is swept around \( 2\omega_d = \omega_q - \omega_c \) and we find \( \delta=0 \) when \( \omega_d/2\pi=1.945 \) GHz. Interference dip is conspicuously identified in the comparison to the spectrum without sideband transition in Fig. 2(b). Probe amplitude \( \Omega_p \) and sideband drive amplitude \( \Omega_d \) in Fig. 2 are \( 2\pi \times 264 \) kHz and \( 2\pi \times 1.08 \) GHz respectively. The calibration method is provided in Supplementary Information for both.

The value we chose is compromising between high contrast of interference dip and proper measurement time. Sideband amplitude \( \Omega_d \) is also determined considering several aspects. First of all, we need to have \( \Omega_d \sim \kappa \) to extract efficiently the \( \kappa \) from numerical fitting of the interference dip. Secondly, we need to avoid heating up the fridge by high power driving and it also limits the maximum power we can apply. It is turned that \( \Omega_d \sim 2\pi \times 1 \) GHz results in \( \Omega_d = 2\pi \sim 100 \) kHz with our device. This is a desirable quantity since the expected \( \kappa \) is around \( 2\pi \times 20 \) kHz according to the normal reflection spectrum through direct probe channel. Also no reference of fridge heating is observed till we increase \( \Omega_d \) more than \( 2\pi \times 1 \) GHz.

In addition to the process of EIT, observations similar to those in Figure 2 can also arise from the process of Ather-Towns splitting (ATS)[18]. Whereas EIT is the result of the quantum interference between transition induced by two fields (Control and probe), ATS arises from the result of electromagnetic pumping that results in a dressed normal mode splitting of the two modes in the rotating frame of the pump. In order to distinguish EIT from ATS, one can numerically fit the data by given simplified model in linear response limit. [19] The system is in EIT regime can be modeled by,

\[
\rho_{ee,\text{EIT}} = \frac{C^2}{\delta^2 + \gamma^2} - \frac{C^2}{\delta^2 + \gamma^2} \quad (1)
\]

When the system is in the ATS regime, \( \rho_{ee} \) in linear response limit is

\[
\rho_{ee,\text{ATS}} = \frac{C^2}{(\delta - \delta_0)^2 + \gamma} + \frac{C^2}{(\delta + \delta_0)^2 + \gamma}. \quad (2)
\]

All the parameters in above expressions are free fitting parameters. We perform a numerical fit with two different fitting models above corresponding each phenomena. The results are given in Fig. 3(c). The data is taken with the same condition in (b) but different probe frequency range and step. While EIT model Eq. 1 shows excellent agreement with the data (solid line), ATS model Eq. 2 fails to nicely explain the data. Accordingly, the fact that the system is in the EIT regime is clearly demonstrated. It is noticeable that EIT model is still applicable to the data even when \( \Omega_p \ll \Omega_d \) does not hold.

We also measure single-photon level \( \kappa \) by normal reflection spectrum in Fig. 3(b) to verify the above result. Fano resonance is also considered in the fitting process. From the fitting, \( \kappa/2\pi \approx 20.3 \pm 1.5 \) kHz and \( \Omega_{ct} = 112 \pm 0.5 \) kHz are extracted. When \( \Delta = 0 \), photon number in the target resonator is approximately 0.3 based on the master equation solution with extracted parameters.

The EIT spectroscopy technique presented here is extensible to many target resonators as long as a qubit is coupled to all of them with proper coupling strength.
Typically, for the spectroscopy of multiple resonators on a chip requires a circuit design with a long feedline such that all the resonators are properly coupled to the feedline. Such a structure could induce some slotline mode and limit scalability of the design. For our method, such a long feedline is not necessary as one need to feed probe and readout pulse only to the qubit, providing a relatively simple measurement technique for spectroscopy of multiple resonators on a chip. In addition, in contrast to direct reflection measurements, EIT spectroscopy does not suffer from Fano resonances, which make the interpretation of fits to extract quality factors difficult.

To summarize, we demonstrate electromagnetically induced transparency in a dispersively coupled qubit-resonator system with bad qubit and good resonator configuration. From interference dip in the qubit spectrum, we obtain single-photon linewidth of the resonator. We validate our result using an independent measurement of the resonator linewidth in a highly-undercoupled reflection measurement, finding a linewidth in agreement with that derived from the EIT model, suggesting that the EIT spectroscopy method presented here is promising for non-invasive spectroscopy of high-Q superconducting resonators.

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The data that support the findings of this study are available from the corresponding author upon reasonable request.
Supplementary Information

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I. QUANTUM INTERFERENCE BASED SPECTROSCOPY FITTING MODEL

The effective Hamiltonian of the reduced system in Fig. 1(b) is given by

\[
\hat{H} = \frac{\omega_q}{2} \hat{\sigma}_z + \omega_c \hat{a} \hat{a} + 2 \chi_{qc} \hat{\sigma}_z \hat{a} \hat{a} + \frac{\Omega_c}{2} \left( \hat{a} \hat{\sigma}_+ e^{-2i \omega_d t} + \hat{a} \hat{\sigma}_- e^{2i \omega_d t} \right) + \frac{\Omega_p}{2} \left( \hat{\sigma}_+ e^{-i\omega_p t} + \hat{\sigma}_- e^{i\omega_p t} \right),
\]

where \( \hat{\sigma}_z \) represents the two-level qubit Hamiltonian and \( \hat{\sigma}_\pm \) are rising and lowering operators of the qubit state. \( 2 \chi_{qc} \) is dispersive shift between qubit and resonator. By applying following time-dependent unitary transform,

\[
\hat{U} = \exp \left[ i \left( \omega_p - 2 \omega_d \right) t \hat{a} \hat{a} + i \omega_p t \hat{\sigma}_z \right].
\]

the Hamiltonian can be simplified to,

\[
\hat{H}' = \left( \frac{\omega_q - \omega_p}{2} \right) \hat{\sigma}_z + \left( \omega_c + 2 \omega_d - \omega_p \right) \hat{a} \hat{a} + 2 \chi_{qc} \hat{\sigma}_z \hat{a} \hat{a} + \frac{\Omega_c}{2} \left( \hat{a} \hat{\sigma}_+ + \hat{a} \hat{\sigma}_- \right) \]

Here, we can define \( \Delta = \omega_q - \omega_p \) and \( \delta = \omega_q - \omega_c - 2 \omega_d \). Both \( \Delta \) and \( \delta \) are identical to those in Fig. 1(b). For \( \delta = 0 \), the Hamiltonian then becomes,

\[
\hat{H}_{model} = \frac{\Delta}{2} \hat{\sigma}_z + \Delta \hat{a} \hat{a} + 2 \chi_{qc} \hat{\sigma}_z \hat{a} \hat{a} + \frac{\Omega_c}{2} \left( \hat{a} \hat{\sigma}_+ + \hat{a} \hat{\sigma}_- \right) \]

The dynamics of the system is then given by following Lindbald equation,

\[
\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} \left[ \hat{H}_{model}(t), \hat{\rho}(t) \right] + \frac{\gamma}{2} D[\hat{\sigma}_-] \rho + \frac{\kappa}{2} D[\hat{a}] \rho,
\]
where $D[\hat{O}\hat{\rho}] = 2\hat{O}\hat{\rho}\hat{O}^\dagger - \hat{O}^\dagger\hat{O}\hat{\rho} - \hat{\rho}\hat{O}^\dagger\hat{O}$. $\kappa$ and $\gamma$ is the decay rate of target resonator and qubit respectively. Here, we assume that pure dephasing rate of qubit is zero. This is a reasonable assumption since we employ single-junction fixed frequency qubit and therefore qubit’s transition frequency is insensitive to magnetic flux noise, known to be the primary source of dephasing in Transmon qubits. This is also experimentally confirmed by the observation that $T_2 \simeq 2T_1$ in separate time domain measurements. For steady state solution $d\hat{\rho}_{ss}/dt=0$, one can obtain steady state qubit population by tracing out the resonator state, $\hat{\rho}_{ee} = \text{Tr}_{\text{res}}[\hat{\rho}_{ss}]$.

II. EXPERIMENTAL SETUP.

![Diagram of the device and related electronics used in the experiment.](image)

In Fig. 1, we depict the device and related electronics in the experiment. A qubit (red dashed box) is coupled to two co-planar waveguide (CPW) resonators. Both resonators are inductively coupled to two separate feedlines. Total four microwave sources ($V_{1-4}$) are used in the experiment, used for qubit driving, qubit readout, direct resonator probing and sideband driving respectively.

III. PROBE AMPLITUDE CALIBRATION.

We apply a 60-ns long Gaussian pulse with a width of $\sigma=15$ ns at the qubit resonant frequency through the source $V_1$, which is followed by a 200-ns long readout pulse from the same source. Rabi oscillation as sweeping the peak voltage of the pulse envelop $V_{\text{peak}}$ is depicted in Fig. 2. The phase $\theta$ of this oscillation is given by $\theta = \Omega_{\text{peak}} \int_{-2\sigma}^{2\sigma} \exp[-t^2/(2\sigma^2)]dt$. 

For $\theta = \pi$, $\Omega_{\text{peak}} = 2\pi \times 13.94$ MHz and $V_{\text{peak}} = 0.54$ a.u. This yields a conversion factor $\Omega_{\text{peak}}/V_{\text{peak}} = 25.81$ MHz/a.u. If the probe field frequency is near the qubit transition frequency, then probe amplitude $\Omega_p$ is then readily calibrated out of $V_p$ using conversion factor acquired from above.

![Graph](image)

**FIG. 2.** Qubit probe amplitude calibration. A Gaussian Rabi pulse applied through the readout resonator at the qubit resonant frequency. The peak voltage of the pulse measured at room temperature is converted to probe amplitude (red line) in an unit of angular frequency based on the phase of oscillation in homodyne readout signal (black dot and blue line).

**IV. SIDEBAND DRIVE AMPLITUDE CALIBRATION.**

The sideband drive induces not only sideband transition but also shifts the qubit transition frequency. A significant frequency shift is observed when $\Omega_{cl} \approx 2\pi \times 100$ kHz (Fig. 3). We have calibrated $\Omega_d$ is $2\pi \times 1.08$ GHz based on AC stark shift and Bloch-Siegert shift [1]. Similar shifts have also been accounted for in finding qubit spectrum and determining the frequency of sideband drive.

[1] M. Gely et. al, Science. 68, 127 (2019).
FIG. 3. Sideband drive amplitude calibration. The amount of the frequency shift is 12.6 MHz is observed when we have $\Omega_{ct} \approx 2\pi \times 100 \text{ kHz}$. 