A Closed-loop controller to improve the Stability of Cascaded DC/DC Converters

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Abstract: Study of the buck converter and cascaded system considering the voltage mode controller has been done. First the small signal analysis of a buck dc/dc converter is presented and its mathematical representation has been showed. Then, the cascaded converter model regarding close loop impedances and voltage gain has been studied. The controller for this converter is proposed to stabilize the performance of the plant. The effectiveness of the proposed controller has been tested on a typical buck converter.

I. INTRODUCTION

Power electronics has been a key factor in the rapid development of advanced vehicular systems such as land, sea/undersea, air, and space vehicles [1]. Real power system contains sub-systems which are designed to meet some specific goals without considering the existence of possible problems related to applying them in a network. On the other hand, when the sub-systems has been considering as a united network, instability may be expected [2][3]. The stability problem is one of the intriguing areas for researchers in academia and industry because of its high importance for the reliability of a dc network. [4]. Because of this issue, the study of stability in dc power supply distribution systems is important and received much attention among researchers [3][5][6][7].

In this paper, at first, the single buck converter with and without voltage mode controller (closed and opened loop) has been studied and small signal for them has been done. The average model of converter is derived and based on the derived transfer functions and impedances, stability issues for cascaded converter model has been studied. In this case, the whole system may be unstable when the impedance mismatching as the study of Middlebrook [8]-[13].

II. SINGLE BUCK CONVERTER

The schematic of a single buck converter is illustrated as Fig.1. The average modeling of the converter can be derived. In this model, $v_{in}$ is the small-signal of the input voltage, $d$ is the small-signal of the duty cycle, $i_{load}$ is the small signal current that comes from the output and $v_c$ is the small-signal of the output voltage. The following formulas show the model of the single buck converter:

\[
\begin{align*}
\frac{di_t}{dt} &= \frac{1}{L} \times (d \times v_{in} - v_c) \\
\frac{dv_c}{dt} &= \frac{1}{C} \times \left( i_t - \frac{1}{R} v_c - i_{load} \right) \\
i_{in} &= d \times i_t
\end{align*}
\]

where, $i_t$ and $v_c$ are considered as state variables. Respectively, input voltage, load current, and duty cycle are the inputs and $i_t$, $v_c$, and input current are applied as outputs.

\[
\begin{align*}
x^o &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]
\[ x = \begin{bmatrix} i_l \\ v_c \end{bmatrix}, \quad y = \begin{bmatrix} i_l \\ v_c \end{bmatrix}, \quad u = \begin{bmatrix} v_{in} \\ i_{load} \end{bmatrix} \]

\[ A = \begin{bmatrix} 0 & -1 \\ 1 & \frac{1}{RC} \end{bmatrix}, \quad B = \begin{bmatrix} D \\ L \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

\[ H(s) = C(SI - A)^{-1}B \]

Considering Fig. 2 and (7), the open-loop transfer functions can be derived as follow:

\[ H(s) = \frac{D(RCS + 1)}{LRCS^2 + LS + R} + \frac{R}{LRCS^2 + LS + R} \]

\[ G_{vct} = \frac{v_c(s)}{v_{in}(s)} = H_{21}(s) \]

\[ Z_{inot} = \frac{v_{in}(s)}{i_{in}(s)} = \frac{1}{H_{31}(s)} \]

\[ Z_{outot} = -\frac{v_c(s)}{i_{load}(s)} = -H_{22}(s) \]

Here a voltage controller is connected to a system which is illustrated in Fig. 3. In this situation the output voltage \( v_c \) is compared with a reference value and errors goes to a PI controller and makes the duty cycle.

![Fig. 3 Block Diagram of the Closed-Loop Buck Converter](image_url)
large input impedance and small output impedance. The bode plots show the controller help to meet these situations.

Fig. 4  Bode diagram of voltage gain

Fig. 5  Bode diagram of input impedance

Fig. 6.  Bode diagram of output impedance

Fig. 7 Designing of controller for the converter

The matlab tool box is used to design the PI controller. The PI controller has following parameters:

\[
K_p = 0.0093602 \\
K_i = 275.3
\]

III. CASCADE CONVERTER

From section II, it can be derived that single buck converter is stable and controller helps the system to be more stable. In this section two buck converters are connected together as Fig. 8.

Fig. 8 Cascaded converter systems

The transfer functions of the cascaded model can be derived regarding the block diagram Fig. 9 as follow

\[
v_{c1}(s) = v_{in2}(s) = H_{21} v_{in1}(s) + H_{22} i_{in2}(s) \\
\quad + H_{23} G_e(s) (v_{ref}(s) - v_{in2}(s)) \quad \text{(16)}
\]

\[
v_{in2}(1 + H_{23} G_e) = H_{21} v_{in1}(s) + H_{22} i_{in2}(s) \quad \text{(17)}
\]

\[
v_{c2}(s) = H'_{21} v_{in2}(s) + H'_{23} G'_e(s) (-v_{c2}(s)) \quad \text{(18)}
\]
Fig. 9. closed-loop cascaded converter

\[ v_{c2}(1 + H'_{23} G'_{c(s)}) = H'_{21} v_{in2(s)} \]  
(19)

\[ i_{in2}(s) = \frac{H'_{31}}{G_{vge}t2} v_{c2} - H'_{33} G'_{c} v_{c2} \]  
(20)

\[ i_{in2}(s) = \left[ \frac{1}{G_{vg}} Z_{inc1}(s)^{-1} \right] v_{c2} \]  
(21)

\[ v_{in2}(1 + H_{23} G_{c}) = H_{21} v_{in1(s)} + H_{22} \left[ \frac{1}{G_{vg}} Z_{inc1}(s)^{-1} \right] v_{c2} \]  
(22)

\[ G_{vgt} = \frac{v_{c2}}{v_{in1}} = \frac{G_{vge}t2 \times G_{vge}t1}{1 + \frac{Z_{outc11}}{Z_{inc12}}} \]  
(23)

Regarding the Middlebrook study, for having stable system following criterion should be considered:

\[ \frac{Z_{outc11}}{Z_{inc12}} < 1 \]  
(25)

**IV. CASE STUDY**

In this section, the buck converter and cascade system are simulated by MATLAB/PLECS software. Table I shows the parameter which are used in simulation.

![Fig. 10. output voltage of first buck converter](image)

| Parameter of case study | \( V_{in1} \rightarrow V_{in2} \) | 100 v | L₁ | 0.167 mH |
|-------------------------|-------------------------------|-------|----|----------|
| C₁                      | 3.75 µF                       |       | R₁ | 5        |
| L₂                      | 0.003 mH                      |       | C₂ | 23.44 µF |
| R₂                      | 0.8                           |       |    |          |

In first scenario, each converter has been simulated separately to show the operation of the buck converter. Then the cascaded converter has been studied. It is worth noting that in second scenario R₁ is ignored.

Fig. 10, 11, and 12 show the output voltage of the first, second and cascaded converters respectively.
Based on the mentioned systems, parameter of voltage controller are as below

| Table II | The parameters of the controller |
|---------|----------------------------------|
| $K_{p1}$ | 0.0093602                       | $K_{i1}$ | 275.3     |
| $K_{p2}$ | 0.01956                         | $K_{i2}$ | 537.4     |

System is stable and controller helps them to work more reliable. But if two buck converters form cascaded converter we cannot expect to have stable system, even they are stable separately.

V. Conclusion

The aim of this report was study of stability of buck converters. When the converters work separately,
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[13] A. Parizad, A. H. Khazali and M. Kalantar, "Siting and sizing of distributed generation through Harmony Search Algorithm for improve voltage profile and reduction of THD and losses," CCECE 2010, Calgary, AB, 2010, pp. 1-7