Magnetically assisted self-injection and radiation generation for plasma-based acceleration

J Vieira¹, J L Martins¹, V B Pathak¹, R A Fonseca¹,², W B Mori³,⁴ and L O Silva¹

¹ GoLP/Instituto de Plasmas e Fusão Nuclear-Laboratório Associado, Instituto Superior Técnico, 1049-001 Lisboa, Portugal
² ISCTE-IUL, Lisbon University Institute, Portugal
³ Department of Electrical Engineering, University of California, Los Angeles, California 90095, USA
⁴ Department of Physics and Astronomy, University of California, Los Angeles, CA 90095, USA
E-mail: jorge.vieira@ist.utl.pt

Received 25 June 2012, in final form 5 September 2012
Published 21 November 2012
Online at stacks.iop.org/PPCF/54/124044

Abstract
It is shown through analytical modeling and numerical simulations that external magnetic fields can relax the self-trapping thresholds in plasma-based accelerators. In addition, the transverse location where self-trapping occurs can be selected by adequate choice of the spatial profile of the external magnetic field. We also find that magnetic-field assisted self-injection can lead to the emission of betatron radiation at well-defined frequencies. This controlled injection technique could be explored using state-of-the-art magnetic fields in current/next generation plasma/laser wakefield accelerator experiments.

1. Introduction
Plasma-based accelerators (PBAs) use high intensity laser pulses [1], with intensities above \( I \sim 10^{18} \text{ W cm}^{-2} \), or highly charged particle bunch drivers [2], with more than \( 10^{10} \) charged particles, to excite ultra-relativistic plasma waves. The ideal plasma density to maximize charge and energy gain depends on the nature of the driver (i.e., lepton, hadron or laser pulse), typically ranging between \( n_0 = 10^{14} \) and \( n_0 = 10^{19} \text{ cm}^{-3} \). At these plasma densities, charged particle bunches can be accelerated by plasma wakefields to 1–100 GeV in 1–100 cm [3].

The proof-of-principle of PBA is firmly demonstrated [4, 5]. Currently, connection with applications [6] is an essential step to further improve this technology. To this end, fine control over the properties of the accelerated electrons is required. Several techniques were proposed to control self-trapping. Control over charge and energy of accelerated bunches can be reached using plasma ramps [7], counter propagating lasers [8], ionization mechanisms [9], and resorting to non-linear optical effects such as self-focusing [10].

A novel technique using transverse magnetic fields to relax self-injection thresholds has been recently proposed [11]. The use of external magnetic fields in plasma acceleration was first proposed to extend the acceleration distances in plasma accelerators in the surfatron model [12]. The role of external magnetic fields in PBA was also explored in [13], and the use of longitudinal magnetic fields to enhance the self-injected charge in laser wakefield acceleration was investigated in [14].

This paper presents a detailed derivation of the self-trapping threshold condition in the presence of external fields. Using the particle-in-cell (PIC) code Osiris [15], it is shown that magnetic injection can be used to generate single or multiple off-axis self-injected bunches with well-defined radial injection positions. Using the post-processing radiation code JRad [16] it is demonstrated that these electrons may emit betatron radiation at well-defined frequencies close to the undulator regime. This paper is structured as follows. Section 2 describes an analytical trapping condition in the presence of external fields. In section 3, 3D PIC simulation results are employed to analyze the relevant
physical mechanisms of magnetic-field assisted self-injection. The use of different B-field geometries to control transverse features of magnetically injected electrons is described in section 4. Section 5 shows that magnetically assisted injection can lead to the emission of clearly defined betatron radiation harmonics for the first time in PBAs. Conclusions are stated in section 6.

2. Trapping conditions in the presence of external fields

The dynamics of the electrons in the fields created by an intense laser in the blowout regime can be described using Hamiltonian dynamics [9]. A general trapping condition in the presence of external fields can be found by examining the evolution of the Hamiltonian of plasma electrons in the co-moving frame, \((x = x, y = y, \xi = v_\phi t - z, s = z)\), given by \(H = H - v_\phi P_\parallel\), where \((x, y)\) are the transverse coordinates, \(z\) is the distance, \(v_\phi\) is the wake phase velocity (determined by the driver group velocity), \(P_\parallel\) is the longitudinal canonical momentum, \(H = \sqrt{m_e^2c^2 + (P + eA)^2} - e\phi\) is the Hamiltonian of a charged particle in the presence of electric and magnetic fields, \(m_e\) and \(e\) are the electron mass and charge, \(c\) is the speed of light, \(A\) and \(\phi\) are the vector and scalar potentials, and \(P = p - eA\), where \(P\) and \(p\) are the canonical and linear momenta, respectively. Normalized units will be used henceforth unless explicitly stated. Mass and charges are normalized to \(m_{ei}\) and \(e\), respectively, velocity \(v\) to \(c\), time to \(\omega_e = \sqrt{4\pi n_0e^2/m_e}\), momentum to \(m_e c\), and density to the background plasma density \(n_0\). Vector and scalar potentials are normalized to \(e/m_e c^2\) and \(e/m_e c\), respectively. Magnetic fields \((B)\) are normalized to \(\omega_e / \omega_p\), where \(\omega_p = eB/m_e c\) is the cyclotron frequency.

In order to derive a trapping threshold condition in the presence of external fields, we consider first the expression for the temporal evolution of \(H\):

\[
\frac{dH}{dt} = (v_\phi - v_i) \frac{dH}{ds} = \frac{\partial H}{\partial s} = \left[ v_\phi \frac{\partial A}{\partial s} - \frac{\partial \phi}{\partial s} \right].
\]

Integration over the particle trajectory yields

\[
H_f - H_i = \int_{dr} \frac{dH}{dt} = \int_{v_\phi - v_i} \frac{dH}{dr}.
\]

where the subscripts ‘i’ and ‘f’ refer to the initial and final (trapped) electron positions. The integration is performed along the electron trajectory, and \(dr = d\xi / (v_\phi - v_i)\). Combining equation (1) with equation (2) gives

\[
H_f - H_i = \int d\xi \left[ v_\phi \frac{\partial A}{\partial \xi} - \frac{\partial \phi}{\partial \xi} \right].
\]

Using the definition for \(H\), and considering that initially electrons are at rest (i.e. \(p_i = 0\)):

\[
H_f - H_i = \gamma_f - v_\phi P_f - 1 - (\phi_f - v_\phi A_f) - (\phi_i - v_\phi A_i),
\]

with \(\phi = \phi^{pl} + \phi^{ext}\), \(A_i = A^{pl}_i + A^{ext}_i\), and where the superscripts ‘pl’ and ‘ext’ refer to the plasma and external fields, respectively, and \(\gamma = (1 - v^2)^{-1/2}\) is the relativistic factor. Defining the wake potential \(\psi = \phi - v_\phi A_i\), \(\Delta \psi = \psi_f - \psi_i\), and assuming that for trapping the longitudinal velocity of the electron must reach the velocity of the wake, i.e. \(v_z = v_\phi\), equation (4) readily becomes

\[
H_f - H_i = \frac{\gamma}{\gamma_f} - 1 - \Delta \psi^{pl} - \Delta \psi^{ext},
\]

where \(\gamma_0 = (1 - v_\phi^2)^{-1/2}\) is the gamma factor of the plasma wave. Using equation (3) to express \(H_f - H_i\) leads to the trapping condition [11]

\[
1 + \Delta \psi^{pl} = \frac{\gamma}{\gamma_f} - \int \frac{dH}{ds} d\xi - \Delta \psi^{ext}.\]

Equation (6) is a general trapping condition in the presence of external fields, and valid beyond the range of validity of the quasi-static approximation [17]. Analytical solutions to equation (6), however, are not yet known because the calculation of \(\Delta \psi^{pl}\) and \(\int dH / ds d\xi \) requires accurate prediction of the particle trajectories and field structures at the back of the bubble, where the applicability of the standard analytical models [18] is limited.

To retrieve a general trapping threshold in the absence of external fields, and in the conditions where the quasi-static approximation is valid, it should be considered \(\Delta \psi^{ext} = 0\), and \(\int dH / ds d\xi = 0\) in equation (6). For an ultra-relativistic plasma wave with \(v_\phi \to \infty\) trapping occurs when 1 + \(\Delta \psi^{pl}\) = 0, or, equivalently, \(\Delta \psi^{pl} = -1\). Generally, this condition can only be met at the back of the plasma wave, in regions of maximum accelerating fields, where \(\psi^{pl}\) is minimum and approaches \(\psi^{pl} = -1\) [9]. In the presence of static external fields the trapping condition becomes \(1 + \Delta \psi^{pl} = -\Delta \psi^{ext}\). The trapping thresholds are relaxed because they can be met when \(\Delta \psi^{pl}\) is larger than \(-1\) provided that \(\Delta \psi^{ext} > 0\). In other words, trapping may occur for lower values of peak accelerating gradients. Moreover, trapping may be suppressed if \(\Delta \psi^{ext} < 0\).

Trapping can also be relaxed (or suppressed) when the external fields vary spatially in \(z\) because of the contribution of finite \(\int dH / ds d\xi \neq 0\) to equation (6). If the profile of the external fields profile leads to \(\int dH / ds d\xi > 0\) along the electron trajectory then trapping is facilitated (suppressed) [10, 11]. Physically, the fact that \(\int dH / ds d\xi > 0\) is typically associated with the reduction of the wake phase velocity through the accordion effect, thus facilitating self-injection [10, 11, 19].

3. Magnetically controlled self-injection in LWFA and PWFA

To investigate controlled self-trapping in the presence of external static magnetic fields we present in this section 3D PIC simulations of laser (LWFA) and plasma (PWFA) wakefield accelerators using the PIC code Osiris. Figure 1 illustrates the evolution, and highlights the key mechanisms of injection assisted by external \(B\)-fields in the LWFA. The simulation window moves at the speed of light, with dimensions of
Figure 1. 3D osiris simulation results illustrating the magnetic self-injection mechanisms. (a), (c) and (e) show the electron plasma density in gray, the self-trapped particles in blue, and the laser pulse envelope in red at t = 110/ωp, t = 126/ωp and t = 159/ωp. (b), (d) and (f) show the corresponding ξ phase space. The magnetic field leads to off-axis self-injection in a narrow angular region. The inset in (f) represents the transverse momentum phase space of the self-injected electron bunch residing within the bubble. The B-field profile is schematically represented on the top of the figure. The laser driver moves from left to right as indicated by the arrow.

24 × 24 × 12 (c/ωp)3, divided into 480 × 480 × 1200 cells with 1 × 1 × 2 electrons per cell in the (x, y, z) directions, respectively. The plasma ions are immobile. A linearly polarized laser pulse with central frequency ω0/ωp = 20 was used, with a peak vector potential of a0 = 3, a duration ωpFWHM = 2√πa0, and a transverse spot size matched to the pulse duration such that W0 = cFWHM [3]. The plasma density is of the form n = n0(z)(1 + Δn/r2) for r < √10 c/ωp and n = 0 for r > √10c/ωp with Δn = Δnc = 4/W0c2 (i.e. the normalized matching condition given by Δnc = 4/(πωp2c0 3), where r0c = e2/mec2 is the classical electron radius) being the linear guiding condition in the normalized units, and where n0(z) is a linear function of z which increases from n0 = 0 to n0 = 1 for 50 c/ωp to ensure a smooth vacuum-plasma transition. The channel guides the front of the laser thereby minimizing the evolution of the bubble. A static external B-field pointing in the positive y-direction was used. At the point where the plasma density reaches its maximum value, the external field rises with Bext = ωc/ωp = 0.6 sin2[πz/(2Lramp)] + Φ1, with Lramp = 10c/ωp. It is constant and equal to Bext = 0.6 for Lcrit = 40 c/ωp, and then drops back to zero with Bext = 0.6 sin2[πz/(2Lramp)] + Φ2. Moreover, Φ1 and Φ2 are phases chosen to guarantee the continuity of the external B-field profile. Qualitatively, the longitudinal profile of the magnetic field thus resembles that of [20].

Self-injection is absent from the regions where the B-field rises. In these regions, electrons traveling backward (vz < 0) near the back of the bubble feel an increasing v × Bext force that rotates electrons anti-clockwise thereby locally decreasing (increasing) the blowout radius for x > 0 (x < 0). Then, as the B-field rises, the local wake phase velocity at the back of the bubble increases (decreases) for x > 0 (x < 0). For x > 0, vφ is superluminal, f dψ dξ < 0, and self-injection cannot occur. For x < 0 trapping is precluded because electrons reach the axis in regions where the plasma focusing and accelerating fields are unable to focus and trap electrons. Thus, although for x < 0 f dψ dξ > 0, we have that f dψ dξ + Δψext < 0.

Self-trapping occurs in the uniform regions of the external magnetic field where x > 0. For x > 0, electrons rotating anti-clockwise reach the axis in regions of maximum focusing and accelerating fields with larger p⊥ and can be trapped. For x < 0, electrons reach regions of the axis (where focusing and accelerating fields are lower) with lower p⊥, and are lost to the surrounding plasma. A threshold B-field for injection may be retrieved in the limit where ωp → ∞. Neglecting the plasma fields (Δψpl = 0), and noting that the external longitudinal vector potential Aψext = −B⊥x is consistent with the considered magnetic field, leads to a simplified trapping condition Δψext = −B⊥Δx = 0, where Δx = x−x0 ≃ −r0, where r0 is the blowout radius and where it was considered that the initial (final) trapped electron radial position is x = r0 (x = 0). It shows that injection is facilitated in the region where Δx < 0 is minimum. As a consequence, injection occurs off-axis (for x > 0), and in a well defined azimuthal region defined by −B⊥r0 sin θ = 1, where θ is the angle between the plane of the electron trajectory with the B-field [11]. Note, however, that this trapping threshold condition overestimates the threshold B-field for self-injection because it neglects the plasma fields.

There is an upper B⊥ value, given by ωp/ωp ≲ 1, beyond which injection may be suppressed in regions where the B-field is flat. The later condition ensures that the plasma wakefields are nearly undisturbed by the external fields. Simulations
then showed that when $\omega_c/\omega_p \gg 1$ there is a suppression of the wakefields that prevents injection. Hence trapping can be relaxed in the regions of uniform $B$-fields provided that $1/r_b \lesssim B \lesssim 1$ or, equivalently $170/r_b(10\mu m) \lesssim B(T) \lesssim 32\sqrt{n_0}(10^{19} cm^{-3})$.

The above-mentioned upper $B$-field limit for injection is absent from the down-ramp regions, where a stronger self-injection burst occurs for $x > 0$. Injection occurs within the same angular and radial region as in the uniform $B$-field section (figure 1(c)). For $x > 0$, when the $B$-field lowers $r_b$ increases, $\nu_p$ lowers and $\int d\xi \sqrt{d\xi^2} > 0$, facilitating injection. For $x < 0$, $\nu_p > 1$, trapping is suppressed. The resulting phase space at $t = 126c/\omega_p$ is shown in figure 1(d).

After the magnetized plasma region, the magnetically injected electron bunch is clearly detached from the back of the bubble, leading to the generation of a quasi-monoenergetic electron bunch. The magnetic injected electron bunch right after the $B$-field is shown at $t = 159c/\omega_p$ in figure 1(e), and the corresponding phase space in figure 1(f). The inset of figure 1(f) shows the transverse phase space of the magnetically injected electron bunch residing within the blowout region. The asymmetrical distribution results from the fact that the injection process occurs off-axis. At this location, the emittance of the beam is on the order of $1\pi$ mm mrad in both transverse directions. Although comparison of beam emittance with a similar scenario without the $B$-field is not meaningful because without the field the amount of self-injected charge is much smaller. However, the measured beam emittance is at the same values or lower than typical emittances of LWFAs.

External magnetic fields also relax the self-trapping thresholds in the PWFA. Figure 2 shows results from a 3D simulation of a magnetized PWFA. A 30 GeV electron bunch was considered with density profile given by $n_b = n_0 \exp(-x^2/(2\sigma^2)) \exp(-z^2/(2\sigma_z^2))$, with $\sigma_\perp = 0.17c/\omega_p$, $\sigma_z = 1.95c/\omega_p$, and $n_b/n_0 = 19$. These parameters ensure that $r_b$ is similar to the magnetized LWFA investigated above. The simulation window dimensions are $24 \times 24 \times 24 (c/\omega_p)^3$, and it is divided into $480 \times 480 \times 640$ cells with $1 \times 1 \times 2$ electrons per cell in the $(x, y, z)$ directions, respectively. The magnetic-field profile is similar to the LWFA case.

The accelerating structures are similar for the LWFA and PWFA parameters described above since the blowout radius is similar for both cases. However, as shown by equation (6), self-injection thresholds are harder to meet in the PWFA than in the LWFA because $\gamma_{\text{B-PWFA}} \approx 60 \times 10^3 \gg \gamma_{\text{B-LWFA}} \approx 20$. In contrast to the LWFA scenario, injection is then absent in the PWFA in the uniform regions of the $B$-field, where the larger $p_z$ at the back of the bubble for $x > 0$, associated with the additional electrons $v \times B$ anti-clockwise rotation, is still below the required for injection. Magnetic self-injection occurs only in the $B$-field down-ramp (figures 2(b) and (c)), where the injection mechanism is similar to that ascribed to the LWFA. In general, stronger self-injection bursts occur in the $B$-field down-ramp for both LWFA and PWFA.

The amount of self-injected charge can be tuned by changing the $B$-field amplitude. The inset of figure 2(c) shows the spectra of the self-injected charge in the first plasma bucket using $B_{ext} = 0.6$ (red curve), $B_{ext} = 0.2$ (green curve), and $B_{ext} = 0.0$ (blue curve). The amount of trapped charge is negligible in the unmagnetized scenario, and it is roughly eight times larger for $B_{ext} = 0.6$ than for $B_{ext} = 0.2$ (notice that the plot is logarithmic in the vertical $y$-direction). These results show that higher $B$-field amplitudes increase the total amount of injected charge.

Because of beam-loading [21], higher amounts of self-injected charge lead to lower accelerating gradients. Consequently, the maximum energy that can be achieved is lower for self-injected bunches with higher charges. This is consistent with the inset of figure 2(c) which shows that self-injected bunches with lower charges reach higher energies.

The inset of figure 2(c) also shows that the energy spread of the magnetically injected electrons are on the order of 100%. Due to the short duration of the self-injected bunch in comparison with the plasma wavelength, which guarantees uniform acceleration throughout the entire bunch length, the relative energy spread would decrease as the beam accelerates. Moreover, the energy spread would further narrow down near the dephasing length due to the bunch phase-space rotation [22].
For these parameters the threshold magnetic field for self-injection is $B_{\text{ext}}^\text{th} \gtrsim 0.2$. To connect these simulations with actual experimental conditions, we take $n_0 = 10^{17} \text{ cm}^{-3}$ for which the electron beam and plasma parameters match those available at SLAC [5] with $\sigma_z = 50.4 \mu\text{m}$, $\sigma_r = 84 \mu\text{m}$ and a total number of $3 \times 10^{10}$ electrons. For these parameters, $B_{\text{ext}}^\text{th} = 0.2$ corresponds to 20 T. These magnetic fields could be produced with state-of-the-art magnetic-field generation techniques [20, 25]. By tune further the plasma parameters controlled injection with magnetic fields as low as 5 T can also be achieved (see section 5).

4. Simultaneous generation of multiple self-injected electron bunches

The transverse location where self-trapping is relaxed can be selected by adequate choice of the profile of the external magnetic field. As an example, figure 3 shows the results from a 2D slab geometry simulation using a magnetic field which reverses sign at $x = 0$ (this is equivalent to an azimuthal $B$-field profile in cylindrical symmetry). In this case, the magnetic field points outside (inside) the simulation plane for $x > 0$ ($x < 0$). The 2D simulations use a simulation box that moves at $c$ with dimensions $12 \times 32 (c/\omega_p)^2$, and is divided into $640 \times 3000$ cells with 3 x 3 electrons per cell in the ($x$, $\xi$) directions, respectively. The laser pulse and plasma channel parameters are similar to those of the 3D LWFA simulation (see figure 1). The amplitude of the external $B$-field is $B_{\text{ext}}^x = \omega_c/\omega_p = 0.6\sin^2[\pi z/(2L^\text{amp}) + \Phi_1]x/|x|$, with $L^\text{amp} = 10c/\omega_p$, it is constant and equal to $B_{\text{ext}}^x = 0.6$ for $L^\text{flat} = 50 c/\omega_p$ and drops back to zero with $B_{\text{ext}}^\text{out} = 0.6\sin^2[\pi z/(2L^\text{amp}) + \Phi_2]x/|x|$, where the choice of $\Phi_1$ and $\Phi_2$ ensures the continuity of the $B$-field longitudinal profile.

Figure 3(a) shows the magnetically injected electrons in the regions where the $B$-field is uniform. Two off-axis injection bursts occur at well-defined transverse positions in the flat $B$-field regions. The two bunches are then injected symmetrically close to $x = 0$. An additional and stronger self-injection burst occurs at the $B$-field down-ramp (figure 3(b)). After the magnetized plasma region, the two self-injected electron bunches continuously accelerate in the wakefield (figure 3(c)). Note that figure 3(c) refers to the early propagation of the electron bunch, much shorter than the dephasing length. Similarly, the propagation distance is much smaller than the betatron period of oscillation. The physical mechanisms under which self-injection occurs in the present configuration are identical to those presented in sections 2 and 3.

Interestingly, figure 3 reveals that injection occurs in a highly spatially localized region. Off-axis injection from well-defined radial and azimuthal regions was observed in section 3 in 3D simulations. Generally, however, this effect is more noticeable in 2D slab geometry simulations than in 3D. These results also suggest that ring-like electron bunches could be obtained in 3D. This could be advantageous for radiation generation purposes because bunch particles would perform betatron oscillations with similar amplitudes.

5. Emission of betatron radiation at well-defined frequencies

Typical synchrotron radiation experiments in plasma accelerators reveal that radiation emission occurs in the wiggler regime. The wiggler regime enables emission of x-rays with broad spectra [23]. This contrasts with the undulator regime, where radiation is emitted at well-defined harmonics. Although not yet attained experimentally, the undulator regime provides ideal conditions for radiation amplification, being critical for the realization of a ion-channel plasma-based laser [24]. This section illustrates how could magnetically injected electrons emit betatron radiation at well-defined frequencies, closer to the undulator regime.

The PWFA beam and plasma simulation parameters, presented in section 3, were tuned in order to lower the required magnetic field for injection, such that it could be more easily reached experimentally, and in order to lower the amplitude of the betatron oscillations in comparison with the plasma skin depth, such that distinguishable betatron radiation harmonics
could be emitted. Systematic 3D parameter scans then showed that the threshold magnetic field for injection is 5.5 T at \( n_0 = 10^{15} \text{ cm}^{-3} \). At this plasma density, \( L^{\text{flat}} = 40 \text{ cm}/\omega_p = 6.8 \text{ mm} \), and \( L^{\text{ramp}} = 10 \text{ cm}/\omega_p = 1.68 \text{ mm} \), and the maximum \( B \)-field amplitude is \( B_0^{\text{max}} = 0.55 \omega_c/\omega_p \). These parameters are within current technological reach [20, 25]. Simulations used a simulation box with 12 \( \times \) 16 \( \times \) 480 \( \times \) 480 \( \times \) 640 cells with 2 \( \times \) 2 \( \times \) 1 particles per cell for the electron beam and background plasma.

Self-trapping occurs off-axis at the \( B \)-field down-ramp. Injection is localized radially and azimuthally, enabling the bunch to perform synchronized betatron oscillations (figure 4(a)). The transverse \( x \)-axis shift of the electron trajectories results from the electron beam driver deflection when traversing the magnetized plasma region. The deflection angle is small and could be corrected by adding additional magnetized plasma regions with alternating \( B \)-fields along the propagation direction [11].

The small blowout radius \( (r_b \approx 1.5 \text{ cm}/\omega_p) \) ensures that the betatron amplitudes of oscillation \( r_p \) are much smaller than the plasma skin depth \( (r_p \approx 0.06 \text{ cm}/\omega_p) \). The corresponding radiation strength parameter \( \alpha_\beta = \gamma K_\beta r_p k_p \) distribution, where \( K_\beta = 1/\sqrt{2\gamma} \) is the normalized betatron frequency, and \( k_p \) is the plasma wavenumber, is shown in the inset of figure 4(a). It shows that a significant portion (83\%) of the electrons radiate with \( \alpha_\beta < 1 \), an indication that single harmonics could be distinguishable in the emitted radiation spectrum. To retrieve the radiation spectrum, a random sample of the self-injected electrons was post-processed using the radiation code JRad [16]. Figures 4(b) and (c) show the radiation spectrum in the transverse central lines of a virtual detector placed at a distance 5100 \( c/\omega_p \) from the exit of the plasma. The detector lies on the \( x-y \) plane.

The asymmetries in the \( x \)-direction depicted in figure 4(b) result from the tilt of the magnetically injected electron trajectories. Figures 4(b) and (c) reveal that radiation is emitted at well-defined frequencies, which are particularly clear at larger angles, i.e. for larger \( |x| \). The width of each harmonic present in figure 4(b) is larger than that expected in an idealized scenario, where radiation would be purely emitted in the undulator regime. This widening is due to the spread on the \( \alpha_\beta \) distribution (through \( \gamma \) and \( r_\beta \) spreads) and also because some electrons radiate with strength parameters which are larger than unity \( \alpha_\beta \gtrsim 1 \).

For an electron bunch with constant relativistic factor \( \gamma \), and constant \( r_\beta \) in a pure ion-channel, the frequency of the betatron radiation harmonics emitted in the undulator regime is given by [26]

\[
\omega_n = 2n\gamma^2 K_\beta \left( 1 + \alpha_\beta^2/2 \right) \cos \theta + 2\gamma^2 (1 - \cos \theta),
\]

where \( n \) corresponds to the \( n \)-th emitted harmonic, and \( \theta \) is the angle between the velocity vector of the electron and the point in the detector. Radiation collected on-axis only exhibits odd-harmonics. To compare the predictions of equation (7) with simulation results we computed the particles trajectories average \( \langle \gamma \rangle = 400 \) and \( \langle r_\beta \rangle = 0.06 \). This yields \( \alpha_\beta \approx 0.7 \), consistent with the inset of figure 4. The analytical prediction equation (7) is shown by the dashed lines in figures 4(b) and (c).

Equation (7) is in good agreement with the simulation results especially for larger values of \( |x| \). Discrepancies are due to the fact that the beam trajectories are tilted, and that \( \alpha_\beta, r_\beta \) and \( \gamma \) vary in time and for each electron.

6. Conclusions

In conclusion, we explored further a recent controlled injection technique that uses transverse, static magnetic fields to tailor transverse properties of self-injection. This scheme leads to off-axis self-injection in well-defined radial and azimuthal regions. A configuration consisting of a section of transversely uniform magnetized plasma yielding off-axis self-injection was investigated. It was shown that simultaneous self-injection of electron bunches could be achieved using transversely...
non-uniform fields. This work also suggests that a series of magnetized regions could be used to produce a temporal sequence of electron bunches. Moreover, multiple spatially separated electrons could be produced simultaneously with transversely non-uniform $B$-fields. We showed that this technique could be used to produce electron bunches capable to emit betatron radiation at well-defined frequencies with current technology.

**Acknowledgments**

This work was partially supported by FCT (Portugal) through the grants SFRH/BPD/71166/2010, PTDC/FIS/111720/2009 and CERN/FP/116388/2010 by the European community through LaserLab-Europe/Charpac EC FP7 Contract No 228464. The simulations were performed at the IST Cluster, at Jaguar supercomputer under INCITE and on the JuGENE supercomputer.

**References**

[1] Tajima T and Dawson J M 1979 Phys. Rev. Lett. 43 267  
Pukhov A and Meyer ter Vehn J 2002 Appl. Phys. B 74 355

[2] Chen P, Dawson J M, Huff R and Katsouleas T 1985 Phys. Rev. Lett. 54 693

[3] Lu W, Tzoufras M, Joshi C, Tsung F S, Mori W B, Vieira J, Fonseca R A and Silva L O 2007 Phys. Rev. ST Accel. Beams 10 061301

[4] Esarey E and Sprangle P 1996 IEEE Trans. Plasma Sci. 24 252  
Caldwell A, Lotov K, Pukhov A and Simon F 2009 Nature Phys. 5 363  
Martins S F, Fonseca R A, Lu W, Mori W B and Silva L O 2010 Nature Phys. 6 311  
Vieira J, Fang Y, Mori W B, Silva L O and Muggli P 2012 Phys. Plasmas 19 061050

[5] Leemans W P, Nagler B, Gonsalves A, Tóth Cs, Nakamura K, Geddes C G R, Esarey E, Schroeder C B and Hooker S 2006 Nature Phys. 2 696  
Kneip S et al 2009 Phys. Rev. Lett. 103 035002  
Froula D H et al 2009 Phys. Rev. Lett. 103 215006

[6] Blumenfeld I et al 2007 Nature 445 741

[7] Geddes C G R, Nakamura K, Plateau G R, Tóth Cs, Cormier-Michel E, Esarey E, Schroeder C B, Cary J R and Leemans W P 2008 Phys. Rev. Lett. 100 215004

[8] Faure J, Rechatin C, Norin A, Lifshitz A, Glinec Y and Malka V 2006 Nature 444 737

[9] Pak A, Marsh K A, Martins S F, Lu W, Mori W B and Joshi C 2010 Phys. Rev. Lett. 104 025203  
Oz E et al 2007 Phys. Rev. Lett. 98 084801

[10] Kalmykov S, Yi S A, Khudik V and Shvets G 2009 Phys. Rev. Lett. 103 135004

[11] Vieira J, Martins S F, Pathak V B, Fonseca R A, Mori W B and Silva L O 2011 Phys. Rev. Lett. 106 225001

[12] Katsouleas T and Dawson J M 1983 Phys. Rev. Lett. 51 392

[13] Pukhov A and Meyer ter Vehn J 1996 Phys. Rev. Lett. 76 2495  
Ren C and Mori W B 2004 Phys. Plasmas 11 1978  
Zhidkov A, Hosokai T, Masuda S, Oishi Y, Juji T and Kodama R 2012 arXiv:1202.3828v1

[14] Hosokai T et al 2006 Phys. Rev. Lett. 97 075004  
Hosokai T et al 2010 Appl. Phys. Lett. 96 121501

[15] Fonseca R A et al 2002 Proceedings of ICCS 2002. Lecture Notes in Computer Science ed P M A Sloot, C J K Tan and J J Dongara vol 2331 (New York: Springer)

[16] Martins J L, Martins S F and Silva L O 2009 Proc. SPIE 7359 73590V

[17] Sprangle P, Esarey E and Ting A 1990 Phys. Rev. Lett. 64 2011

[18] Lu W, Tzoufras M, Zhou M, Mori W B and Katsouleas T 2006 Phys. Rev. Lett. 96 165002

[19] Katsouleas T 1986 Phys. Rev. A 33 2056

[20] Pollock B B, Froula D H, Davis P F, Ross J S, Fulkerson S, Bower J, Satariano J, Price D, Krushelnick K and Glenzer S H 2010 Phys. Rev. Lett. 105 065001

[21] Tzoufras M, Tsung F S, Huang C, Mori W B, Katsouleas T, Vieira J, Fonseca R A and Silva L O 2008 Phys. Rev. Lett. 101 145002

[22] Tsung F S, Narang R, Joshi C, Fonseca R A and Silva L O 2004 Phys. Rev. Lett. 93 185002

[23] Kneip S et al 2010 Nature Phys. 6 980–3

[24] Whittum D H, Sessler A M and Dawson J M 1990 Phys. Rev. Lett. 64 2511

[25] Kumada M et al 2003 Proc. of PAC 1993–1995

[26] Esarey E, Shadwick B A, Catravas P and Leemans W P 2002 Phys. Rev. E 65 056505