Energy transport analysis in flow of Carreau nanofluid inspired by variable thermal conductivity and zero mass flux conditions

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Abstract
The presence of nanometric particles in the base fluids lead to form nanofluids. Nanofluids are prominent due to their astonishing features in thermally conducting flows and in the development of electronic and mechanical devices. Based on these motivations, we have designed our article to investigate the thermal conduction features in the free and forced convection flow of unsteady Carreau nanofluid due to stretching cylinder with the effects of variable magnetic field. Moreover, the transport of thermal energy in the flow is properly examined by including the impacts of variable thermal conductivity and nonuniform heat rise/fall. Furthermore, the transport of solutal energy in the flow of nanofluid is encountered under the influences of activation energy and binary chemical reactions. A momentous feature of this study is to employ the zero-mass flux condition at the wall of the cylinder. A section of this study is proposed for mathematical modelling of the current problem. Moreover, the impacts of involved physical constraints are explored by employing an efficient numerical technique namely bvp4c. The features of all physical constraints on flow, thermal and solutal curves are illustrated in the form of graphs and discussed with reasonable physical arguments in discussion section of the article. The core findings of this study are mentioned in the section of closing remarks. The core upshot of the current study is that the nanoparticles concentration rate of nanofluid depicts ascending trend for escalating values of activation energy constraint. A significant upsurge in the coefficients of skin friction and Nusselt number is detected with an escalation in the constraints of buoyancy and thermophoresis forces, respectively. The references regarding this article are also provided at the end.

Keywords
Carreau nanofluid, variable thermal conductivity, non-uniform heat source/sink, zero mass flux conditions, binary chemical reaction

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Introduction
Magnetohydrodynamic (MHD) flows deals with the flows of fluids under the consideration of magnetic forces. According to Rossow, the concern in this field was initiated in 1918, when a discovery of electromagnetic pump was came into being due to which a Hartmann number was familiarised which defines in terms of viscous and electromagnetic forces. In October 1942, Alfven revealed that when a conducting fluid continuously experience the effects of uniform magnetic force then every fluid motion produces a force known

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as electromotive force which is utilised in the manufacturing of electric current. It is also exposed that one type of this force is used in production of current whereas, other kind engenders the effect of Lorentz force. Alfven\textsuperscript{2} reported in his study that these currents produce mechanical forces which lead to alter the state of flow of the fluid. The implementation of MHD are stated in different areas including developments of MHD pumps, magnetic generators, advancement of fusion reactors, metallurgy section, development of MHD flow metres and metals dispersion process. The problems regarding conduction of liquids with magnetic field are also encountered in the field of Geophysics. MHD convection flows have significant importance in aeronautics, stellar and planetary magnetospheres and in electrical engineering. Also, it is worthy to state that the effectiveness of MHD in flow equation can only be assured of such flows whose magnetic Reynolds number is very small. The characteristics of viscous Reynolds fluid are examined by Liron and Wilhelm\textsuperscript{3} by incorporating the influence of magnetic field in transverse direction of the flow and in the other technical systems. After that, Makinde\textsuperscript{4} disclosed the impacts of Lorentz force in the stagnant flow of fluid near a surface by considering the free and force convection effects. He came to know that the flow curves of the fluid behave in descending manner for escalation extent of magnetic force constraint. Moreover, The MHD mixed convection fluid flow was investigated by Pal et al.\textsuperscript{5} under the influences of thermal radiation towards a stretching and shrinking geometry. Later, Govardhan et al.\textsuperscript{6} explored the magnetic features in the flow of micropolar fluid due to a stretching surface. Additionally, Waqas et al.\textsuperscript{7} studied the impacts of Magnetohydrodynamic (MHD) on free and forced convected flow of micropolar fluid towards non-linear stretching geometries. Recently, Daniel et al.\textsuperscript{8} examined the electrically conducted flow of nanofluid corresponding to Magnetohydrodynamics and mixed convection. Soomro et al.\textsuperscript{9} presented a MHD mixed convective flow model and considered the thermal velocity slip effects. Sharma et al.\textsuperscript{10} explored the impact of Lorentz force on mixed convective flow in view of heat generation/absorption along a stretching surface. Recently Iqbal et al.\textsuperscript{11} scrutinised the Burgers nanofluid flow with the impacts of magnetic field and chemical reactions. The reported that the thermal contours of the magneto viscoelastic fluid significantly rise for improving the scales of magnetic parameter. Furthermore, Khan et al.\textsuperscript{12} developed a new model to explore the characteristics of MHD Burgers fluid flow due to stretching cylinder near a stagnation point. They disclosed that the nanoparticles concentration rate of magnetic fluid augmented by for escalating amount of magnetic force constraint.

Materials containing the magnitude of 1–100 nm are designated as the nanomaterials or nanoparticles. These types of particles were earliest studied by Masuda et al.\textsuperscript{13} under the influence of thermal conductivity. Afterward, investigation of such type of particles was done by a researcher named as Choi\textsuperscript{14} and he used these nano particles in the base fluid and generalised the mixture as nanofluid. After this, many researchers performed their investigations on nanofluids as nanofluids have magnificent thermophysical properties. Additionally, Nanofluids have considerable particle implementations in the field of applied mathematics and engineering sciences like, materials processing, heat exchangers, thermal cooling, electronic devices, cooling processes and in the mathematical modelling of different convective thermal energy transport phenomena. Furthermore, the characteristics of nano fluids are examined by Buongiorno,\textsuperscript{15} who presented the two-phase model known as Buongiorno two phase model for dealing nanofluids. In this model he examined the features of nanofluids by employing the process of thermophoresis and Brownian diffusion. The investigation of boundary layer flow of nanofluids over a vertical plate was performed by Kuznetsov and Nield.\textsuperscript{16} They examined the nanofluid features by employing the model of Buongiorno. They developed analytical solutions of the problem. The theoretical investigation of the nanofluids in the boundary layer flow caused by a stretching geometry was also done by Khan and Pop.\textsuperscript{17} Turkyilmazoglu\textsuperscript{18} developed the thermal conduction model for nanofluid flow by considering the slip mechanism. Hsiao\textsuperscript{19} investigated the mixed convection flow of nanofluids by employing the impacts of multimedia. He analysed that the temperature curves rise for larger Brownian motion constraint. Hsiao\textsuperscript{20} examined the slip flow of nanofluid on a stretching surface by considering magnetic field effects. Hsiao\textsuperscript{21} explored the flow of Carreau nanofluid and employed parameters control method for energy extrusion system. Turkyilmazoglu\textsuperscript{22} employed the Buongiorno model in an axisymmetric channel flow of nanofluid. Turkyilmazoglu\textsuperscript{23} employed the single phase nanofluids model to study the cooling of jets. Khan et al.\textsuperscript{24} investigated the peristaltic flow of nanofluids in a channel by incorporating magnetic field effects. Muhammad et al.\textsuperscript{25} explored the flow of Carreau nanofluid past a wedge geometry by incorporating the effects of Bioconvection. Ahmad et al.\textsuperscript{26} considered the variable viscosity to explore the entropy generation of hybrid nanofluid. Ahmed et al.\textsuperscript{27} presented a model to explore the thermal features of Maxwell nanofluid past a stretching surface by considering the effect of Joule heating. They reported that the thermal curves of Maxwell nanofluid augment by varying the mount of Eckert number. Some more recent studies on nanofluids can be found in Vasudevan et al. and Tariq et al.\textsuperscript{28,29}
In the present paper, numerical solutions of unsteady mixed convection boundary layer flow of Carreau nanofluid past a stretching cylinder are developed. Additionally, the effect of Lorentz force is also taken into account to examine the flow features of nanofluid. Thermal energy transport is studied by employing the influences of thermal conductivity and non-uniform heat source sink in this article. Moreover, the effects of activation energy and binary chemical reactions are considered to examine the characteristics of solutal transport. Additionally, the effect of Lorentz force is also taken into account to examine the flow features of nanofluid. Impacts of all physical parameters are explored by employing bvp4c a numerical technique in MATLAB program. Results are depicted in the form of graphical representation and discussed with reasonable physical judgements.

Development of mathematical model

In this study an unsteady incompressible flow of Carreau fluid over an inclined permeable stretching cylinder which makes an angle $\chi$ with horizontal axis is examined. The influence of mixed convection over the flow and heat transfer is investigated. We further consider the variable thermal conductivity and zero mass flux conditions on the surface of the cylinder. Buongiorno’s two phase model consisting of the Brownian motion and thermophoresis diffusion is also incorporated. To examine the flow analysis, we considered the cylindrical coordinates $(x, \theta, r)$ in such an arrangement that cylinder is stretched in $x$ direction while $r$ axis is normal to the stretching (see Figure 1). A non-uniform magnetic field applied in such a way that cylinder is normal to it and the fluid is supposed to be electrically conducting with strength $B(t) = \frac{B_0}{\sqrt{1-ct^2}}$

where $B_0$ is a constant. Also $T_w$ and $C_w$ are temperature and concentration of the fluid and is kept constant at the surface of the cylinder. Finally, the stretching velocity of the cylinder is taken to be $u_w(x, t) = \frac{\alpha_1}{T_w(x) + ct}$, where $\alpha$ and $c$ are constants. The thermal conductivity of the fluid can be considered by the following expression

$$k(T) = K_\infty \left[ 1 + \varepsilon (T - T_\infty) \right]$$

Here $\varepsilon$ shows the variable thermal conductivity parameter, $K_\infty$ the thermal conductivity of the fluid far away from the surface of the cylinder and $\Delta T = T_w - T_\infty$ is fluid temperature difference.

Under these assumptions the governing equations for the Carreau nanofluid can be written as\textsuperscript{10-32}

\textbf{Continuity:}

$$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0. \quad (1)$$

\textbf{Momentum equation:}

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \nu \frac{\partial^2 u}{\partial r^2} \left[ 1 + \Gamma^2 \left( \frac{\partial u}{\partial r} \right)^2 \right]^{\frac{1}{2}} - \frac{\sigma^2 B^2(t)}{\rho} u$$

$$+ \nu (n-1) \left( \frac{\partial u}{\partial r} \right)^2 \left[ 1 + \Gamma^2 \left( \frac{\partial u}{\partial r} \right)^2 \right]^{\frac{1}{2}} + g(\beta T - T_\infty) + \beta_c(T - T_\infty) \cos \chi. \quad (2)$$

\textbf{Energy equation:}

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = \frac{1}{\rho c_p} \frac{1}{r} \left[ \frac{\partial}{\partial r} k(T) \frac{\partial T}{\partial r} \right]$$

$$+ \tau \left[ \frac{\partial C}{\partial r} \frac{\partial T}{\partial r} + \frac{D_T}{T_w} \left( \frac{\partial T}{\partial r} \right)^2 \right]$$

$$+ \frac{\alpha u_w}{x} \left[ A^*(T_w - T_w) f' + B^*(T_w - T_w) \right]. \quad (3)$$

\textbf{Concentration equation:}

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial r} = D_b \frac{\partial}{\partial r} \left( r \frac{\partial C}{\partial r} \right)$$

$$+ \frac{D_T}{T_w} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right)$$

$$+ k_-^2 C \left( \frac{T}{T_w} \right)^n \exp \left( \frac{-E_a}{kT} \right). \quad (4)$$

The physical realistic boundary conditions are...
The stream function is given by 
\[ \psi = \frac{u^2}{2 \nu}, \]
and equations (2)–(4) take the following form:
\[ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0, \]
\[ u = u_0(x,t), \quad v = 0, \quad T = T_0, \quad D_B \frac{\partial C}{\partial r} + \frac{D_T}{T_0} \frac{\partial T}{\partial r} = 0 \text{ at } r = R, \]
\[ u = 0, \quad T = T_\infty, \quad C = C_\infty \text{ as } r \to \infty. \]
We have the similarity transformations
\[ \eta = \frac{r^2 - R^2}{2R}, \quad \psi = (u/v) \eta^{1/2} \frac{R f(\eta)}{\eta}, \]
\[ \theta(\eta) = \frac{T - T_\infty}{T_\infty - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_\infty - C_\infty}. \]
The stream function \( \psi(r,x) \) is given by \( u = \frac{1}{r} \frac{\partial \psi}{\partial \eta} \) and \( v = -\frac{1}{r} \frac{\partial \psi}{\partial \eta} \), so that the continuity equation (1) is automatically satisfied.

By utilising equation (7) in equations (1)–(4) with
\[ \frac{\partial}{\partial x} \text{ and } \frac{\partial}{\partial y}, \]
we know that equation (1) satisfied automatically and equations (2)–(4) take the following form:
\[ (1 + 2\gamma \eta) \left[ 1 + nWe^2(f''(y)) \right] \left[ 1 + We^2(f''(y)) \right] f'' + \frac{f'''}{f'} - \frac{f''}{f'} + \frac{2}{2} f' \]
\[ - \frac{A}{2} f'' - Mf' + R_i(\theta - \phi) \cos \chi = 0, \]
\[ (1 + 2\gamma \eta) \left[ \theta'' + \frac{2}{2} \left( \theta'' + \frac{2}{2} \right) \right] + 2\gamma \theta' + \frac{Prf f'}{Pr} \frac{A}{2} \theta' \]
\[ + Pr(1 + 2\gamma \eta) \left( N_b \theta' \phi' + N_i \theta^2 \right) \]
\[ + Prf f' + \frac{A}{2} \theta' = 0, \]
\[ (1 + 2\gamma \eta) \frac{\phi''}{\phi'} + 2\gamma \phi' + Scf \phi' \]
\[ + \frac{N_b}{N_i} \left[ (1 + 2\gamma \eta) \theta'' + 2\gamma \theta' \right] Scf \frac{\eta}{2} \phi' \]
\[ - Scf \left( 1 + \delta \frac{\phi''}{\phi} \right) \phi \exp \left( - \frac{E}{1 + \delta \phi} \right) = 0, \]
and transformed boundary conditions are:
\[ f(0) = 0, f'(0) = 1, \quad \theta(0) = 1, \quad N_b \phi' + N_i \theta' = 0, \]
\[ f'(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0. \]
Here, the local Weissenberg number \( We \), curvature parameter \( \gamma \), space dependent heat source parameter \( Q' \), time dependent heat source parameter \( Q \), magnetic parameter \( M \), buoyancy parameter \( R_i \), thermophoresis diffusion \( N_i \), Brownian diffusion \( N_b \), Prandtl number, buoyancy force ratio parameter \( N_f \), Schmidt number \( Sc \), unsteadiness parameter \( A \), reaction rate parameter \( \sigma \), activation energy parameter \( E \) and temperature difference parameter \( \delta \) are defined as follows:

\[ We = \left( \frac{a^3 \rho^2 R^2}{(1 - c \rho)^2 R^2} \right)^{1/2}, \gamma = \left( \frac{\nu(1 - \rho)c}{aR^2} \right)^{1/2}, \]
\[ Q' = \frac{A^*}{\nu (p_c p)} = \frac{Q}{\nu (p_c p)}, \]
\[ M = \left( \frac{\alpha^* B^0}{\rho a} \right)^{1/2}, R_i = \frac{g \beta_c (T_w - T_\infty) z}{u_w}, \]
\[ (Gr)_c = \frac{g \beta_c (T_w - T_\infty) z^2}{\nu^2}, A = \frac{c}{a}, \]
\[ (Gr)_c = \frac{g \beta_c (C_w - C_\infty) z^2}{\nu^2}, N_r = (Gr)_c, \]
\[ N_i = \frac{\tau D_c (T_w - T_\infty)}{\nu T_w}, \quad Pr = \frac{\mu c_p}{k}, \]
\[ N_b = \frac{\tau D_b (C_w - C_\infty)}{\nu D_B}, \quad Sc = \frac{\nu}{D_B}, \quad \sigma = \frac{k_e^2}{a}, \quad E = \frac{E_a}{k T_w}. \]
The skin friction and heat and mass transport coefficients are given by:
\[ C_f = \frac{\tau_{x-w}}{\rho \mu^w}, \quad Nu = \frac{q_w(x-w)}{k (T_w - T_\infty)}, \quad Sh = \frac{q_m(x-w)}{D_B (C_w - C_\infty)}. \]

\[ \tau_{x-w} = \mu_0 \frac{\partial \psi}{\partial x} \left[ 1 + Pr^2 \left( \frac{\partial \psi}{\partial x} \right)^2 \right]^{1/2}, \]
\[ q_w = -k (\partial T / \partial x), \quad q_m = -D_B (\partial \phi / \partial x). \]
The dimensionless surface drag, heat and mass transfer rates takes the form:
\[ Re \hat{C}_f = f''(0) \left[ 1 + We^2 f''(0)^2 \right], \]
\[ Re^{-2} Nu = -\theta'(0), \quad Re^{-2} Sh h = -\phi'(0), \]
where \( Re = xu_w / \nu \) defines the local Reynolds number.

**Solution methodology**

The set of self-similar equations (8)–(10) along with assisting boundary conditions (11) and (12) has been tackled numerically through bert method. The final system is reduced into set of first order ordinary differential equations and alters into initial value problem as
\[ f = Y_1, f' = Y_2, f'' = Y_3, \theta = Y_4, \theta' = Y_5, \phi = Y_6, \phi' = Y_7. \]
Table 1. A comparison of skin friction $Re^{1/2}C_f$ with published literature\textsuperscript{30,33,34} when $n = 1$, $We = 0$, $M = 0$, $A = 0$, $Ri = 0$, $Nr = 0$, for distinct $\gamma$.

| $\gamma$ | Rangi and Ahmad\textsuperscript{33} | Poply et al.\textsuperscript{34} | Hashim et al.\textsuperscript{30} | Present study |
|----------|----------------------------------|----------------------------------|-------------------------------|---------------|
| 0.0      | -1.0000                          | -1.0000                          | -1.0000                      | -1.0000       |
| 0.25     | -1.094378                        | -1.094373                       | -1.094375                    | -1.094375     |
| 0.5      | -1.188715                        | -1.188727                       | -1.188731                    | -1.188731     |
| 0.75     | -1.281833                        | -1.281819                       | -1.281822                    | -1.281822     |
| 1.0      | -1.459308                        | -1.363865                       | -1.453373                    | -1.453377     |

$$\begin{pmatrix} Y_1' \\ Y_2' \\ Y_3' \\ Y_4' \\ Y_5' \\ Y_6' \\ Y_7' \end{pmatrix} = \begin{pmatrix} Y_2 \\ Y_3 - \frac{Y_1 Y_3 + A (Y_2 + \frac{Y_3}{2}) + M^2 Y_3 - Ri Y_3 - Nr Y_3 \cos \Psi}{(1 + 2 n \eta)[(1 + n We Y_1^2)(1 + We Y_3^2)]} \\ -\frac{2 \gamma Y_3 - \frac{2 \gamma Y_3 + 1}{2 \gamma} + 2 \gamma Y_3 - Pr Y_3 - Pr(1 + 2 \gamma) Y_3 (N_b Y_3 + N_r Y_3^2) + Pr A \Omega Y_3 - Pr(Q Y_2 + Q Y_3)}{2 \gamma Y_3 + 1 + 2 \gamma Y_3 + \gamma} \\ -\frac{2 \gamma Y_3 - \frac{2 \gamma Y_3 + 1}{2 \gamma} + 2 \gamma Y_3 - Sc \Omega Y_3 + Sc \gamma Y_3 (1 + \delta Y_3) \gamma Y_3 \exp \left( \frac{d}{2 \gamma} \right)}{1 + 2 \gamma Y_3} \end{pmatrix}, \quad (16)$$

with initial conditions:

$$\begin{pmatrix} Y_1(0) \\ Y_2(0) \\ Y_3(0) \\ Y_4(0) \\ Y_5(0) \\ N_b Y_6(0) + N_r Y_5(0) \\ Y_6(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}. \quad (17)$$

Validation of numerical code

To inspect the accuracy of numerical bvp4c scheme, the values of pertinent scales of skin friction coefficient $Re^{1/2}C_f$ have been calculated for varying curvature parameter ($\gamma$) for the case of steady flow ($i.e A = 0$) of Newtonian fluid ($n = 1$ or $We = 0$) and in the absence of magnetic and buoyancy force parameters ($M = 0$&$Ri = 0$) in Table 1. From table, it has been observed that the data generated by the bvp4c code and those reported by Rangi and Ahmad,\textsuperscript{33} Poply et al.\textsuperscript{34} and Hashim et al.\textsuperscript{30} are identical to many extents which shows the validity of the present numerical scheme.

Results and discussion

The exact solutions of nonlinear ordinary differential equations (8)–(10) with corresponding boundary conditions (11) and (12) cannot be obtain due the presence of highly non linear terms. We have employed the numerical scheme bvp4c to overcome this problem and solved the differential equations numerically. We analysed the impact of several physical constraints present in the model against flow $f'(\eta)$, thermal $\theta(\eta)$ and concentration $\varphi(\eta)$ distributions of Carreau nanofluid and illustrated the graphical results in Figures 2 to 21. We adjusted the default magnitudes for important physical constraints such as $A = 0.2$, $M = 1.0$, $We = 3.0$, $Ri = 0.6$, $Nr = 0.8$, $\gamma = 1.0$, $N_b = 0.4$, $N_r = 0.6$, $Sc = 3.0$, $E = 2.0$, $Pr = 4.5$, $\delta = 0.5$, $\sigma = 1.0$, $Q = 1.0$, $Q' = 0.8$, $n^* = 0.5$ and $n = 0.5$ in overall computations. We discuss the features of velocity, thermal and solutal distribution of Carreau nanofluid for different physical parameters in following three sub sections.

![Figure 2. Illustration of $f'(\eta)$ for varying $We$.](image-url)
Flow features of Carreau nanofluid

Figures 2 to 6 illustrate the impact of $We$, $A$, $M$, $N_r$ and $R_i$ on flow distribution of Carreau nanofluid for the case of sheet ($\gamma = 0$) and the case of cylinder ($\gamma = 1$). Figure 2 is portrayed to disclose the flow characteristics of Carreau nanofluid against Weissenberg number ($We$). From this graphical representation we observed that the flow curves of nanofluid depict descending trend for escalating values of $We$. Physically, Weissenberg number is define in terms of specific process time and relaxation time and we know that the intensifying values of relaxation time leads to increase the resistance between fluid particles. Hence, larger Weissenberg number leads to increase the viscosity of the nanofluid and consequently the flow distribution of the fluid depicts decreasing trend. The influence of unsteadiness parameter $A$ towards the velocity distribution has been depicted in Figure 3. From this graph it is disclosed that the flow curves and velocity boundary layer thickness of the fluid depressed by growing the values of unsteadiness parameter $A$. These conclusions make sense physically, because the stretching coefficient $a$ and the unsteadiness parameter $A$ are inversely proportional to each other. Thus, for larger values of $A$, there will be decrease in stretching parameter $a$. As a consequence, the velocity of the fluid depreciated. Figure 5 exposes the buoyancy force ratio constraint ($N_r$) influences on the performance of flow distribution of Carreau nanofluid. It is assessed that the flow curves of the nanofluid decline for higher extent of $N_r$. Figure 6 displays the features of velocity distribution for the influence of buoyancy force parameter ($R_i$). It reveals that the flow contours and the velocity layer thickness of Carreau nanofluid improve by augmented values of $R_i$. As the buoyancy force has prevalent effects over viscous forces for greater values of $R_i$. Hence, the mixed convection parameter is responsible to make the fluid flow more fast, due to this reason the flow of the fluid increases.

Thermal features of Carreau nanofluid

Figures 7 to 15 are being sketched to inspect the thermal feature for different values of $R_i$, $Q$, $Q'$, $M$, $A$, $e$, $N_r$, $N_s$, for both cases of sheet ($\gamma = 0$) and cylinder ($\gamma = 1$). The impacts of buoyancy force parameter ($R_i$) on thermal distributions are depicted in Figure 7. From this figure it is explored that the thermal curves of the fluid and temperature boundary layer...
thickness depreciated with an augmentation in the extent of \( R_i \). To disclose the characteristics of temperature distribution for heat generation parameter \( Q \) Figure 8 is plotted. It is scrutinised that higher values of \( Q > 0 \) causes augmentation in temperature profile and the associated boundary layer thickness also improved. While, the opposite situation is observed in heat absorption case \( Q < 0 \). It physically make sense that, obviously when more amount of heat is generated then temperature profile boost up and when heat is absorbed then temperature profile is depreciated. Figure 9 irradiated the ramification of space dependent heat rise constraint \( Q' \). Rise in thermal distribution of nanofluid is noticed for varying amount of \( Q' \). On the other hand, thermal boundary layer become thicker for augmented scales of \( Q' \). It is due to the reason that heat source parameter generates more heat which leads to rise the temperature distribution of fluid. Figure 10 portrayed the characteristics of magnetic parameter \( M \) for temperature distributions. In fact, from plots we analysed that higher magnetic field effects intensifies the field temperature. Additionally, the thermal boundary layer thickness is also improved by intensifying magnetic field effects. basically, these findings are identical to the physical happening that when magnetic effects slow down the velocity of the fluid then temperature of the field boost up. Hence, Magnetic field effects can be taken into account to control the flow...
characteristics. Figure 11 ensures the behaviour of unsteadiness parameter $A$ regarding temperature field. It is experienced that the temperature of the field diminishes by an variation in the amount of $A$. Figure 12 is designed to enlighten the thermal features of Carreau fluid for developing scales of thermal conductivity parameter $\epsilon$. Increasing behaviour of temperature curves is noted within the boundary layer for higher extent of $\epsilon$. Additionally, the temperature boundary layer thickness is also improved for growing mount of $\epsilon$. It is due to the fact that the Prandtl no ($Pr$) depreciates for intensifying scales of thermal conductivity constraint. Consequently, the temperature of the fluid grows up. Figure 13 envisioned to picture the behaviour of buoyancy force ratio parameter $Nr$ on temperature profiles. From this figure it is concluded that enhancement in the buoyancy force parameter produce augmentation in temperature curves of nanofluid. While, temperature boundary layer become thinner by growing values of $Nr$. Figure 14 exposed the enactment of thermophoresis parameter $Nt$ against temperature distribution. From Figure 14, it is detected that the thermal contours of the fluid as well as the temperature layer thickness is improved with an intensification in the thermophoresis constraint $Nt$. These findings relates with physically phenomenon of thermophoresis that is, the particles which are heated moves from hot regions towards cold regions and consequently the fluid temperature grows up. Figure 15
discloses the characteristics of temperature field against the Prandtl number \( Pr \). Depreciation in temperature is noticed for intensifying effects of \( Pr \). Additionally, thermal layer of fluid also become thinner for the higher amount of Prandtl number. It is due to the fact that Prandtl number depends upon thermal diffusivity which become weaker for larger Prandtl number. Obviously, weaker thermal diffusivity results in falloff to the transport of thermal energy in the flow of nanofluid.

**Solutal features of Carreau nanofluid**

Figure 16 highlights the influence of fitted rate constant \( n^* \) against the performance of nanoparticles concentration rate. The concentration profile \( \phi(\eta) \) improves as fitted rate constant varies its range from \( n = 0.2 \) to 0.6. Figure 17, exhibits the impacts of unsteadiness parameter \( A \) on concentration distribution. This plot indicates that nanoparticles concentration distribution rises and associated boundary layer thickness reduces for growing values of \( A \). The aspects of temperature difference parameter \( \delta \) for concentration of Carreau nanofluid with shear thinning/thickening characteristics are demonstrated in Figure 18. There is a diminishing trend being observed for escalating amount of temperature difference constraint. Hence, larger \( \delta \) exhibits that the concentration curves and solutal thickness of the layer decay for augmented
magnitude $\delta$. Influence of activation energy against solutal curves of Carreau nanofluid is illustrated in Figure 19. From the figure it is analysed that the term $\exp\left(-\frac{E}{kT}\right)$ increases for larger estimation of activation energy constraint which leads to depreciate the concentration rate of nanoparticles the flow. Figure 20 shows the characteristics of $\varphi(\eta)$ with the variation of $\sigma$. It is observed that the solutal curves of the Carreau nanofluid depict diminution trend for rising magnitude of destructive chemical reaction constraint ($\sigma>0$). Actually, some amount of solutal energy destroys due to increase in destructive chemical reaction constraint and hence solutal rate depreciates. Figure 21 depicted the effects of Schmidt number ($Sc$) for the concentration distribution of gases hydrogen, helium, water vapours and oxygen. The estimation of $Sc$ are taken from 0.1 to 0.5 and it is concluded that concentration of the fluid depreciated by increase in Schmidt number. These findings are identical to the physical phenomenon, because Schmidt number is the ratio of mass diffusivity and momentum diffusivity (kinematic viscosity). It is the dimensionless number which describes liquid flows. Increase in $Sc$ means decrease in molecular diffusivity which consequently results in reduction of solutal energy transport. Therefore, the concentration of species is lesser for growing values of $Sc$. Hence presence

**Figure 18.** Illustration of $\varphi(\eta)$ for varying $\delta$.

**Figure 19.** Illustration of $\varphi(\eta)$ for varying $E$.

**Figure 20.** Illustration of $\varphi(\eta)$ for varying $\sigma$.

**Figure 21.** Illustration of $\varphi(\eta)$ for varying $Sc$. 
of Schmidt number in the system explicitly modifies concentration profile throughout the region.

**Behaviour of skin friction and Nusselt number profiles**

The computation of numerous physical quantities like, $Re^{1/2}C_f$ and $Re^{-1/2}Nu$ are assessed for numerous values of different parameters are depicted via Figures 22 to 25. The difference in skin friction ($Re^{1/2}C_f$) incorporated with magnetic constraint ($M$) for growing magnitude of buoyancy force constraint ($R_i$) have been demonstrated in Figure 22. A significant upsurge in the skin friction coefficient is noticed for escalated buoyancy force parameter. This shows that the fiction factor is improving for higher $R_i$. The variation in $Re^{1/2}C_f$ against $M$ for higher values of $We$ is depicted in Figure 23. From this graph a significant rise is seen in the skin friction coefficient for larger amount of $We$. It is clearly noticed that shear stress at the boundary is increasing for larger $We$. The dimensionless Nusselt number ($Re^{-1/2}Nu$) associated with thermal conductivity parameter ($e$) is plotted in Figure 24 for varying scales of Prandtl number ($Pr$). It is investigated that the magnitude of Nusselt number is raised with an intensification in the extent Prandtl number. This situation occurs because of the reason that larger Prandtl number leads to decline the thermal boundary layer thickness and consequently an improvement in the heat transport rate is noted. Further, Figure 25 we portray the heat transport coefficient $Re^{-1/2}C_0u_0$ against heat generation parameter for different values of thermophoresis parameter $N_t$. The enhancement in the magnitude of Nusselt number is seen for varying scales of $N_t$.

**Concluding remarks**

The notable features of current article are mentioned below

- It is noticed that the solutal curves depreciate for escalating scales of activation energy constraint.
- An augmentation in the constraint of temperature difference leads to decline the solutal rate of nanoparticles.
- A significant enhancement in the skin friction coefficient is detected for varying magnitude of Weissenberg number.
The thermal contours of nanofluid expose the ascending nature for growing magnitude of thermal conductivity constraint.

The coefficient of skin friction depicts enhancing nature of curves for developing magnitude of buoyancy force parameter while it exposes the opposite trend for varying scales of Weissenberg number.

A significant development is presumed in the rate of heat transport at the wall against the progressive values of Prandtl number and thermophoresis force constraint.

The solutal rate of nanoparticles reports the ascending nature for larger estimation of activation energy constraint.

Authors’ contribution
Zahoor Iqbal: Formulated the mathematical model. Masood Khan: Developed the code for numerical solutions. Aamir Hamid: Analyse the whole problem. Awais Ahmed: Help in physical discussion of the outcomes.

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**Appendix**

**Notation**

**Nomenclature**

| Symbol | Description                                      | Symbol | Description                                      |
|--------|--------------------------------------------------|--------|--------------------------------------------------|
| $u, v$ | Velocity components                             | $Pr$  | Prandtl number                                   |
| $x, r$ | Cylindrical coordinates                         | $C_f$ | Skin friction coefficient                        |
| $T$    | Fluid temperature                               | $Re$  | Local Reynolds number                            |
| $T_w$  | Surface temperature                             | $q_w$ | Surface heat flux                                 |
| $T_a$  | Ambient temperature                             | $\Gamma'$ | Relaxation time                                |
| $C$    | Fluid Concentration                             | $p$   | Fluid density                                    |
| $C_w$  | Nanoparticles concentration at surface          | $\mu$ | Dynamic viscosity                                |
| $C_a$  | Ambient concentration                           | $\mu_0$ | Zero shear viscosity                            |
| $D_B$  | Brownian diffusion coefficient                  | $\gamma$ | Magnitude of deformation rate                    |
| $D_T$  | Thermophoresis diffusion coefficient            | $\nu$ | Kinematic viscosity                              |
| $u_{sw}$ | Stretching sheet velocity                      | $\psi$ | Stream function                                  |
| $k$    | Thermal conductivity                            | $\tau_w$ | Surface shear stress                             |
| $M$    | Magnetic parameter                              | $\theta$ | Dimensionless temperature                       |
| $f'$   | Dimensionless fluid velocity component          | $\phi$ | Dimensionless concentration                      |
| $N_b$  | Brownian motion parameter                       | $\eta$ | Dimensionless similarity variable                |
| $N_t$  | Thermodiffusion parameter                       | $(\rho c_p)_b$ | Effective heat capacity of nanoparticles |
| $\alpha$ | Thermal diffusivity                          | $(\rho c_p)_f$ | Heat capacity of base fluid                     |
| $c_p$  | Specific heat                                   | $q$   | Heat flux                                        |
| $We$   | local Weissenberg number                        | $\tau$ | Parameter defined by ratio $(\rho c_p)_b/(\rho c_p)_f$ |
| $Sc$   | Schmidt number                                  | $A^*, B^*$ | Constants                                        |
| $k'$   | Boltzmann constant                              | $\gamma$ | Curvature parameter                              |
| $a, c$ | Constants                                       | $\delta$ | Temperature difference parameter                 |
| $s_r$  | Reaction rate parameter                         | $E$   | Activation energy                                |
| $n^*$  | Fitted rate constant                            | $Nu$  | Nusselt number                                   |
| $\beta_1$ | Fluid relaxation time parameter                  | $Q^*$ | Space dependent heat source/sink parameter      |
| $\beta_3$ | Fluid retardation time parameter                | $Q$   | Time dependent heat source/sink parameter       |
| $\gamma_1$ | Thermal Biot number                         |       |                                                  |