Fast swept sine cutting test for CNC lathes

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Abstract
To mitigate chatter vibration in high-speed high-precision machining, the structure of machine tools and the cutting conditions must be optimized. Therefore, it is necessary to analyze the dynamic characteristics of machine tools. Such analysis is frequently conducted with an excitation test using an impact hammer because of its convenience. However, the dynamic behavior in the cutting process is often different from that in the resting state. It is difficult to analyze machine tools during cutting because of the rotating spindle and discharged chips. Therefore, this study proposes a method called fast swept sine cutting (FSSC) for measuring dynamic behavior during the cutting process. In the method, the cutting force is the excitation force applied to the machine tool system. The sinusoidal excitation of a CNC lathe during the turning process is realized by using a workpiece that creates a sinusoidal cutting force. The proposed method allows the frequency response, including that at the resonance point, to be observed. The dynamic behavior observed using the proposed method is different from that observed using conventional impact testing. The frequency response function during the turning process is affected by the nonlinearity of the cutting system.

Keywords: Turning, Chatter, Modal analysis, Nonlinearity

1. Introduction

Multi-axis machine tools have been developed to reduce the lead time of machining (Schulz and Moriwaki, 1992; Wang et al., 2015; Lynagh et al., 2000; Lin et al., 2003; Kim et al., 2005; Altintas et al., 2011; Zhang et al., 2016; Lee et al., 2004). However, chatter vibration caused by low machine tool stiffness is a problem (Budak et al., 2009). There are several driven axes for a given machine tool, making it difficult to guarantee sufficient stiffness. Because chatter vibration decreases the lifetime of cutting tools and degrades the finished surface condition (Quintana and Ciurana, 2011), it is essential to prevent chatter vibration. The mechanism of chatter vibration has been studied. The vibration of the cutting tool before one rotation of the main spindle is projected onto the machined surface as minute waves (inner modulation); the chip thickness changes when the cutting tool cuts the workpiece including the inner modulation during the current rotation (outer modulation) (Siddhpura and Paurobally, 2012). Therefore, chatter vibration is a kind of self-excited vibration caused by the feedback of the cutting force to the cutting system. Chatter vibration can occur when the cutting chip thickness changes, e.g., due to excess cutting force and low stiffness of the machine tool, cutting tool, and workpiece. To prevent chatter vibration, it is necessary to increase stiffness. Furthermore, the phase difference between the inner modulation and outer modulation, which causes a change in the cutting chip thickness, can be minimized by tuning the spindle speed to effectively prevent chatter vibration. For both countermeasures, it is necessary to measure the dynamic stiffness accurately. In general, experimental modal analysis using impact testing (excitation) is widely used for analyzing the dynamic characteristics of machine tools (Munoa et al., 2016; Liu et al., 2016). Frequency response functions can be obtained using impact testing through the excitation of the mechanical structure by an impact hammer and the measurement of the resulting acceleration by acceleration sensors. This method can be performed easily and quickly.
However, because the cutting tool or the main spindle have to be hammered directly via physical contact, it is impossible to attach acceleration sensors to the machine tool during the cutting process and measure the dynamic behavior (Faassen et al., 2003). Therefore, frequency responses obtained with impact testing when the machine tool is stopped are used as approximate dynamic characteristics for the cutting process. Because the contact points between the cutting tool and the workpiece can be regarded as being under a high pressure force, the dynamic behavior of the machine tool during the cutting process might be different from that during rest.

In a previous study, an excitation device was installed on the outside of a lathe and the dynamic behavior was evaluated with random excitation during spindle rotation (Brussel and Peters, 1975). Although the method was not applied during the cutting process, it was found that the frequency response functions obtained during spindle rotation were different from those obtained in impact testing. Several methods that apply excitation during milling have been proposed. A method that applies random excitation by cutting the test piece to create steps for excitation using a face mill has been proposed (Cai et al., 2015). A similar method that applies random excitation by forming grooves on the test piece has been proposed (Özşahin, 2011). A method that applies sweeping, and not random, excitation during milling by increasing or decreasing the spindle speed linearly has also been suggested (Iglesias, 2016). These studies showed that the dynamic behavior during the cutting process is different from that in the resting state, and also highlighted the effects of nonlinearity, including a change in bearing status with spindle rotation.

To measure the dynamic behavior during the cutting process accurately, the cutting force itself can be used as an excitation force. The present study proposes a method called fast swept sine cutting (FSSC) for CNC lathes. This method uses the cutting force during turning as the excitation force in experimental modal analysis. When the feed per revolution is constant, the cutting force is proportional to the cutting depth. Accordingly, the sinusoidal change in cutting depth corresponds to the cutting force. In FSSC, the test piece is formed to have a sinusoidal profile in advance, and sine cutting is realized by turning the prepared test piece while increasing the spindle speed. Experiments using the proposed method confirm that the natural frequency and nonlinearity depend on the cutting force. The method is compared to conventional impact testing.

2. Fast swept sine cutting test

Figure 1 shows a schematic diagram of the proposed method. Figure 2 shows the cross-sectional profile of a test piece. The cutting force is turned into the excitation input for the frequency response function of the CNC lathe by cutting a cylindrical shaft workpiece with a sinusoidal profile with a constant feed per revolution $f$ [mm/rev] and linearly increasing the spindle speed $N$ [min$^{-1}$]. The test piece profile $C$ is defined in Eq. (1), where $A$ is the amplitude of the sinusoid on the surface, $n$ is the number of sinusoids per revolution, $\theta$ is the rotation angle in Fig. 2, and $(x, y)$ is the position vector on the sinusoidal profile.

$$
C = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r + A \sin n \theta \\ \cos \theta \end{pmatrix} \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}.
$$

(1)

In this study, the test piece is generated by milling with a ball endmill. The center trajectory of the ball endmill $E$ to form $C$ is described as:

$$
E = \begin{pmatrix} r + A \sin(n \theta) \\ \cos \theta \end{pmatrix} + \frac{R}{\sqrt{B^2 + D^2}} \begin{pmatrix} \sin \theta + D \cos \theta \\ -B \cos \theta + D \sin \theta \end{pmatrix},
$$

(2)

where

$$
B = \frac{dx}{d\theta} = An \cos(n \theta) \cos \theta + [r + A \sin(n \theta)] (- \sin \theta),
$$

$$
D = \frac{dy}{d\theta} = An \cos(n \theta) \sin \theta + [r + A \sin(n \theta)] \cos \theta.
$$

The frequency response function is measured as the acceleration. The input to the system is the cutting force and the output is the acceleration. The acceleration can be measured directly using acceleration sensors mounted on the tool holder, but the cutting force cannot be measured directly by cutting force dynamometers. Because the cutting force dynamometers are part of the dynamic system during the cutting process, the proper frequency function cannot be obtained. Therefore, cutting theory is adopted for calculating the cutting force. Frequency response functions are obtained by applying a fast Fourier transform to the measured acceleration and the calculated cutting force.
The cutting force is calculated using Kronenberg’s expression (Kronenberg, 1966; Kuljanic et al., 2002). The cutting force is defined as \( P \) [N], which is expressed in terms of the specific cutting resistance \( k_s \) [N/mm\(^2\)] and the cross-sectional area of the cutting chip \( q \) [mm\(^2\)]:

\[
P = k_s q,
\]

\[
k_s = C_{ks} \left( \frac{G}{5} \right)^{g_s} q^{z_s},
\]

where \( C_{ks} \) is a constant determined by the workpiece material and the rake angle of the cutting tool, \( G \) is the ratio of the cutting depth to the feed rate, and \( g_s \) and \( z_s \) are determined by the workpiece material. Because the cutting force is proportional to both the cutting depth and the feed rate, it is proportional to only the cutting depth when the feed rate is fixed. Therefore, a sinusoidal excitation force is obtained by cutting the sinusoidal profile under a constant \( f_r \).

Finally, the excitation frequency \( f \) is proportional to \( N \) and \( n \):

\[
f = \frac{N \cdot n}{60}.
\]

Accordingly, \( f \) and \( n \) are determined by considering the sweep range of \( f \) and the maximum spindle speed.

### 3. Experimental

#### 3.1 Experimental setup for FSSC

To determine the excitation frequency range for the proposed FSSC method, conventional impact testing was carried out on the tip of the cutting tool of the CNC lathe (see Fig. 4(a)). The sweep range of \( f \) was set to 900-1300 Hz because the peak frequency of the main compliance was detected at approximately 1100 Hz. When a sweep excitation method is used to obtain the frequency response function, the maximum sweep speed is limited by the following equation (ISO 5348, 1998):

\[
\frac{df}{dt} \left( \frac{4}{Q^2} \right) = \frac{54 f_r^2}{Q^2} = 216 f_r^2 \zeta_r^2,
\]

where \( \zeta_r \) is the damping ratio and \( Q \) is the Q value. \( \zeta_r \) is calculated as 0.012 from Fig. 4, and thus the sweep speed must be less than 1920 Hz/s. In addition, \( f_r \) is selected so as to ensure that the threading process is unaffected by the cutter mark before one rotation to prevent chatter vibration. Test pieces with various sinusoidal profile amplitudes (0.08, 0.14, 0.17, and 0.20 mm) were prepared to verify the effect of the cutting force. The nominal cutting depth was obtained by adding a static cutting depth of 0.1 mm to \( A \). Carbon steel (S55C) was adopted for the test pieces. \( N \) was determined from the set excitation frequency range of 900-1300 Hz and \( n \) using Eq. (2), after \( n \) and the base radius \( r \) for the test piece were determined considering the maximum rotation speed of the spindle, allowable cutting speed of the turning chip, and allowable sweep speed.

Figure 3 shows the setup inside the CNC lathe for FSSC testing. Figure 4 shows the results of impact testing at points a, b, and c in Fig. 3. The main compliance of the test piece at point c was approximately 0.2 \( \mu \)m/N at around 1000 Hz. A similar value was observed at the tool tip. In FSSC, it is desirable for the main compliance of the test piece not to be
close to that of the cutting tool because there is a possibility that the dynamic behavior will cause mutual forces to be exerted at the contact point in the cutting process. Nevertheless, these test pieces were used in this study. The effect is discussed in the next section. The measurement of acceleration in FSSC was carried out at point b (rear of the tool holder) because it is difficult to measure acceleration at the tool tip (point a).

3.2 Results of FSSC testing

A photograph of a test piece after FSSC testing is shown in Fig. 5. The experimental data obtained from the test when \( A = 0.08 \) mm are shown in Fig. 6. The sweep of the spindle speed had a certain degree of linearity and the cutter mark had no overlap and a constant pitch, as shown in Fig. 5 and Fig. 6(b). Figure 6(a) shows a comparison of the measured acceleration and calculated cutting force based on Eq. (3), and the measured spindle speed (Fig. 6(b)) in the time domain. Dynamic behavior was observed for the acceleration output under a steady cutting force. As mentioned above, actual force measurement using dynamometers cannot be performed because the dynamometer must be included in the vibration system. However, the observed cutting force is shown in Fig. 6(c) for reference. Although the force fluctuates depending

![Fig. 3 Experimental setup and position for pilot impulse testing](image1)

![Fig. 4 Results of pilot impulse testing](image2)

| Table 1 Experimental conditions |
|---------------------------------|
| Test piece                      |
| Base radius, \( r \) [mm]       | 15                                 |
| Amplitude of sinusoid, \( A \) [mm] | 0.08, 0.14, 0.17, 0.20 |
| Number of sinusoids, \( n \)     | 12                                 |
| Material                        | S55C                               |
| Cutting conditions              |
| Static cutting depth [mm]       | 0.1                                 |
| Feed per revolution, \( f_r \) [mm/rev] | 0.7                                 |
| Spindle speed, \( N \) [min\(^{-1}\)] | 2696-3951                          |
| Cutting speed [m/min]           | 50-400                              |
| Excitation frequency, \( f \) [Hz] | 899-1317                           |

![Fig. 5 Photograph and enlarged view of test piece after FSSC testing](image3)
on the dynamic behavior of the machine tool, the validity of the approximated steady cutting force is confirmed.

Fig. 6 Plots of FSSC test results in time domain ($A = 0.08$ mm)

Fig. 7 Comparison of compliance between impulse and FSSC testing
Moreover, the relationship between the calculated cutting force and the cutting depth is shown in Fig. 6(d). The cutting force in Fig. 6(c) is the data offset by the static force (approximately 165 N) from the data in Fig. 6(d) to apply the Fourier transform.

Figure 7 shows a comparison of the frequency response functions obtained with FSSC testing and conventional impulse testing. From Fig. 7(a), it can be seen that the resonance peaks (both compliances and frequencies) in the FSSC test results are lower than those for impulse testing. Specifically, the FSSC results for the test piece at \( A = 0.08 \) mm are 1036 Hz and 0.04 \( \mu \)m/N (other amplitudes have similar values; 1010 Hz and 0.02 \( \mu \)m/N) and the impulse test results are 1110 Hz and 0.048 \( \mu \)m/N. Because the force in FSSC testing is estimated from a calculation based on Kronenberg’s expression, comparing compliances between impulse testing and FSSC testing requires caution. An evaluation of the FSSC results for amplitudes of \( A = 0.08 \) to 0.20 mm was conducted. The compliances for \( A = 0.14 \) to 0.20 mm are half of that for \( A = 0.08 \) mm. In addition, the frequencies of the resonance peaks can be compared between impulse testing and FSSC test results because the frequencies are directly based on accelerometer measurements. From the above, it can be concluded that the frequency of the resonance peak (1110 Hz) obtained by FSSC testing is at most about 10% lower than that obtained by impulse testing. The compliance for the resonance peak is also lower (extent depends on cutting depth); however, compliances cannot be compared between impulse testing and FSSC testing.

The Nyquist diagram in Fig. 7(b) shows that all FSSC test results have distorted profiles, whereas the impulse test results have a nearly ideal circular profile, which indicates the presence of a nonlinear element for FSSC. There is a possibility that the resonance peak at around 1000 Hz that is the natural frequency on the test piece exposes in the FSSC results. However, in that case, because resonance peaks for both the cutting tool and the test piece should appear in the FSSC results, it is considered that the peak frequencies associated with the cutting tool side declined by the effect of the nonlinear element.

4. Discussion

As described above, it is assumed that a nonlinear element that depends on the cutting force exists in the cutting system because both the compliances and frequencies of the resonance peaks decrease with increasing cutting force. This phenomenon strongly resembles that for a vibration system that includes stiffness, mass, a viscous dumper, and coulomb friction (as a nonlinear element). For example, a reduction in the resonance peak with increasing amplitude of the excitation force applied to the mechanical system, including bolt joints, has been reported (Jalali et al., 2007; Asamadian et al., 2007). It is considered that nonlinearity is generated even in the general turning setup, because a friction force that depends on the amplitude of the external dynamic force is generated at the bolts between the turret and the holder, and between the holder and the tool, due to tightening pressure. Therefore, it is assumed that the reduction in the resonance peaks in FSSC testing is due to the effect of friction when a significant cutting force is applied as the excitation force to the vibration system at the contact point between the cutting tool and the test piece, at the connection part of the cutting tool and at other locations. This assumption will be verified in future work. This paper confirms only that the frequency response functions obtained in FSSC testing include nonlinearity.

To verify the nonlinearity in the vibration system, a Hilbert transform was applied to the measured frequency response functions (Fieldman, 2011). When the vibration system is linear, because the frequency response function is symmetric in the Nyquist diagram, there is no difference between the Fourier transform and the Hilbert transform. However, if the vibration system is influenced strongly by nonlinearity, because the Hilbert transform becomes asymmetric, the results of the two transforms do not match. Thus, the Hilbert transform is useful for verifying nonlinearity. The equations for the Hilbert transform are (Simon and Tomlinson, 1983)

\[
H_R(\omega_c) = \frac{2}{\pi} PV \int_0^\infty \frac{\omega G_I(\omega)}{\omega^2 - \omega_c^2} d\omega,
\]

\[
H_I(\omega_c) = \frac{2\omega_c}{\pi} PV \int_0^\infty \frac{G_R(\omega)}{\omega^2 - \omega_c^2} d\omega,
\]

where \( H_R \) and \( H_I \) are the real and imaginary parts of the Hilbert transform, respectively, \( G_R \) and \( G_I \) are the real and imaginary parts of the measured mobility, respectively, and \( PV \) is the Cauchy principal value for the integral. Furthermore, moment integration is defined for quantifying the difference between the Fourier transform and the Hilbert transform:

\[
ER = \frac{\int_{\omega_1}^{\omega_2} H(\omega)d\omega}{\int_{\omega_1}^{\omega_2} G(\omega)d\omega},
\]

where \( ER \) is the energy ratio. Figure 8 shows the results of the Hilbert transform. As shown in Fig. 8(a), the difference between the Fourier transform and the Hilbert transform is small for impulse testing. Therefore, the effect of nonlinearity is weak. In contrast, the difference between these transforms is large for FSSC testing (Fig. 8(b)), indicating the presence of a nonlinear element. Figure 9 shows a comparison of the deviation of moment integration from the results of the
Hilbert transform for the two testing methods. The deviation for FSSC testing is generally larger than that for impulse testing. There is an increasing tendency from shallow to deep cutting depths (the exception is for $A = 0.17$ mm). Therefore, the nonlinearity depends on the cutting force.

5. Conclusion

This study proposed a method called FSSC for testing the dynamic behavior of a CNC lathe during the cutting process. To apply a sinusoidal excitation to a vibration system, a sinusoidal test piece was cut at a linearly increasing spindle speed. Using the proposed method, we obtained the following results from a comparison with conventional impulse testing.

1. The compliance and frequency of resonance peaks for FSSC testing were lower than those for impulse testing.
2. Analysis using the Hilbert transform identified a large difference between impulse testing and FSSC testing. In FSSC testing, a nonlinear element was identified during the cutting process.
3. Because the nonlinearity of the dynamic behavior depends on the cutting depth (i.e., excitation force), it is assumed that the nonlinearity is caused by friction between the cutting tool and the test piece, at attachment surfaces of the cutting tool and at other locations.
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