Special features of design and calculation for structures made of anisotropic fiberglass

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Abstract. In recent years composite materials find wide application in various fields of engineering, because they have a number of advantages over other structural materials. A variety of composites’ physical and mechanical properties (especially anisotropy) requires an improvement of existing calculation methods and creation of new ones for structural elements made of these materials. This is an important task which will contribute to their wider use. In this paper some famous criteria of anisotropic materials are examined, and their advantages and disadvantages are discussed. The authors of the paper suggest new variants of strength criteria for anisotropic materials. These new criteria are based on new mechanical characteristics which are more convenient for experimental obtaining. Also new criteria use separate form of writing for each quadrant of the stress plain.

1. Introduction
Composite materials, primarily fiberglass, have their special place among all the materials used today in construction. They have the most attractive prospect because of their physical and mechanical characteristics. For example strength of fiberglass (glass-reinforced plastics or GRP) is comparable to strength of steel, their thermal conductivity is equal to the one of wood, and its ease is similar to the ease of PVC. Composite materials combine the best qualities of traditional materials but without their disadvantages. A large class of structures which have been usually made of metal and concrete, aluminium and PVC-plastic (polyvinylchloride) can be successfully and efficiently transferred to the use of fiberglass.

2. Basic properties of fiberglass plastics
Fiberglass is a composite material consisting of two main components which are polymer (binder) and glass fibers (filler). Various combinations of these components allow adjusting the properties of glass-reinforced plastics in a wide range. These materials have a lot of excellent properties such as: high specific strength, relatively low weight, low thermal conductivity, high chemical stability and damping capacity, the ability to withstand high temperatures for a short time, high workability, high fatigue strength and others [1]. Fiberglass is not subjected to corrosion and rot, do not require painting; some of them pass up to 90% of light and 70% of ultraviolet rays. Strength of some fiberglass exceeds
strength of the best structural steels and aluminium alloys, and it is much more than the one concrete and reinforced concrete.

3. Application of fiberglass in construction
Fiberglass gives the opportunity to create structures of almost any shape. It is efficient to use fiberglass plastics in different types of building structures: covering plates, shells, domes in translucent constructions, in lightweight long-span structures, in special radio transparent structures, as well as in the construction on subsiding soil or in seismic areas.

In spatial elements the relatively low modulus of elasticity of fiberglass plastics is compensated by the increased stiffness and stability of dimensional systems as well as by the use of rational cross sections. Spans of spatial constructions made of fiberglass can reach 50 meters with a small material consumption and low weight of structures.

It is very perspective to use fiberglass for highly loaded elements and structures, and for those which are operating under impact shock loads.

Also fiberglass is used for manufacturing pipes. Main advantages of such pipes in comparison with metal ones are their high strength, resistance to corrosion from action of gases, salt solutions, acids and alkalis. Such pipes are lightweight (4 - 5 times less than weight of similar steel pipes), have low thermal conductivity. They are used in plumbing, in oil industry, as a lining shaft, for the manufacture of rod systems for building structures.

An important advantage of fiberglass plastics is the ability to use them for effective multi-layered structures such as sandwich plates and shells. In such structures the outer layers are made of fiberglass plastics, and inner layers are made of foamed polystyrene or sotoplast or goffered aggregates. The combination of layers having different properties makes it possible to provide a reliable structure for work in adverse environmental conditions. These structures have relatively low weight and good heat insulation, electrical insulation and sound insulation properties. Multi-layer constructions provide structure’s mass reduction by 12 - 40% with set limits on the strength and stiffness. At the same load the bearing capacity of panels with composites is 10 - 65% lower than that of panels with metal skins. The use of such panels makes it possible to reduce the load on the supporting structures and therefore decrease the weight of trusses, pillars and foundations, as well as reduce the cost of transportation, installation, operation, which provides additional savings.

The most widely varied opportunities of fiberglass plastics are disclosed in public buildings and facilities, allowing the creation of original designs, as fiberglass combines both structural and protecting functions, and sometimes lighting function. Dimensional fiberglass structures are used for erection of pavilions coverings, sports facilities, indoor markets and others.

In the manufacture of the composites may be used various techniques (extrusion, contact molding, extrusion, casting, winding).

4. Anisotropy of the physical and mechanical properties of fiberglass
It is a very important feature of fiberglass plastics made on the basis of glass fabrics or directed glass strands. Anisotropy is a dependence of the material’s characteristics of the angle between the directions of the reinforcement and operating stresses. Elastic and strength properties of fiberglass (including their long-term strength, creep and others) have the feature of anisotropy. Anisotropy of strength means that the fiberglass tensile strength, compressive or shear strength depends on the orientation of the glass fibers in the sample. In other words, anisotropy means the difference of properties in different directions. Anisotropy of fiberglass plastics can be a technological, i.e. appearing in the process of manufacturing of elements, and a constructive i.e. defined during designing of details.

A study of anisotropy of fiberglass’s elastic and strength properties has a great theoretical and practical interest [2]. Using the correct choice of the anisotropy’s character in fiberglass it is possible to create lightweight structures of equal strength. This is a very important advantage of GRP compared to many other construction materials. It is possible to use the material in a structure in an optimal way by changing the order of layers and direction for stacking threads in the layer. The strongest materials can
be obtained if they are reinforced by straight threads that have been laid with pretension. Depending on the location and type of fiber, there are various types of reinforced plastics. The most advantageous in terms of improving the strength is location of reinforcement in the direction of the greatest tensile forces. However, in most cases, the best option of reinforcement type can be find only by calculations. When studying the anisotropy it is convenient to use methods of the tensor calculus.

The topical is the question of the strength calculation for GRP structural elements under complex stress state, taking into account the materials’ features of mechanical properties. The real physical model of a composite’s work in a structure is complicated, it is necessary to choose a proper approximating model to simplify practical calculations of strength and stability. Currently, a phenomenological approach is widespread in the mechanics of composites, when the composite material is considered as a continuous and homogeneous anisotropic medium with averaged macro stresses and strains.

5. Strength criteria for anisotropic materials

For practical calculations of elements made of anisotropic materials it is necessary to use any criterion of strength for anisotropic materials [3], [4]. The basic characteristics of the material included in the strength criteria, should be obtained from experiments in simple forms of strain-stress condition.

There are quite a lot of phenomenological strength criteria for anisotropic materials, which have different graphic interpretation in the stresses space. Common approach to the description of a strength surface does not exist, so the choice of this or that criterion is always a relatively subjective task. The particular form of expression taken as a strength criterion for a material can be verified only experimentally. The best known criteria are the following phenomenological strength criteria for anisotropic materials: ones named by Mieses - Hill, Zakharov - Malmeyster, Goldenblat – Kopnov, etc.

Despite the large number of strength criteria for anisotropic materials, their application in practice of design is rather limited for several reasons:
1) None of the known strength criteria is universal, i.e. they all have certain limits of applicability. The most universal criterion from this point of view is Goldenblat - Kopnov criterion.  
2) Analytic expressions of some criteria of strength are rather cumbersome and contain many basic material strength characteristics (ultimate tensile strength, compression and shear ones).  
3) It is technically difficult to determine experimentally some parameters of the ultimate strength (i.e. ultimate shear strength in directions at angle 45° with respect to the axes of anisotropy).

The task of creating of strength criteria for anisotropic materials (such as reinforced plastics) has been solved in two ways: by generalization of the well-known strength criteria of isotropic materials and by development of essentially new criteria.

V. Kopnov and S. Shambina suggested several new variants of known strength criteria without the mentioned above drawbacks. It was suggested to use new basic strength constants of materials. Also the authors proposed new approach to some known strength criteria; it was suggested to write these criteria separately for each quadrant of stress plane.

Let us consider in more details how the new modifications Mieses- Hill criterion for anisotropic materials were obtained. This criterion has been get on the basis of the plasticity criterion proposed by Mieses for metals. For the case of plane stress state Mieses- Hill criterion for anisotropic materials is the following:

\[
\frac{\sigma_{11}^2}{\sigma_{T1}^2} + \frac{\sigma_{22}^2}{\sigma_{T2}^2} + \frac{\sigma_{12}^2}{\tau_{T12}^2} = \left(\frac{1}{\sigma_{T1}^2} + \frac{1}{\sigma_{T2}^2} + \frac{1}{\sigma_{T3}^2}\right) \cdot \sigma_{11} \cdot \sigma_{22} = 1.
\]

where \(\sigma_{T1}, \sigma_{T2}\) - the limits of material yield stress in the material’s first and second main directions; \(\tau_{T12}\) - the limit of material yield stress in pure shear; \(\sigma_{11}, \sigma_{22}, \sigma_{12}\) - the values of the current stresses.

In [5] it was suggested to express the coefficients of Mieses-Hill criterion through the limits of tensile (compressive) strength and the limit of shear on the site situated at the angle of 45° with the main direction of the material (\(\tau_{\alpha 45}\)) and the limit of shear on the sites coinciding with the main directions of the material (\(\tau_{\alpha 12}\)).
For the case of plane stress state this criterion has the form:

\[
\frac{\sigma_{11}^2}{\sigma_{11}^2} + \frac{\sigma_{22}^2}{\sigma_{22}^2} + \left( \frac{1}{\sigma_{11}} + \frac{1}{\sigma_{22}} - \frac{1}{\tau_{45}} \right) \cdot \sigma_{11} \cdot \sigma_{22} + \frac{\sigma_{12}^2}{\tau_{12}^2} = 1.
\] (2)

The possibility of using Mieses plasticity criterion as a strength criterion for anisotropic materials was studied in [5]. For materials with strong anisotropy this criterion is not consistent with the results of experiments, whereas this criterion satisfactorily approximates the results of experiments for materials with wear anisotropy. Obviously, this criterion should only be used to anisotropic materials having the same tensile and compressive strength.

Mieses criterion (Eq. 2) was first recorded in a tensor form in [4]:

\[
\Pi_{ikmn} \cdot \sigma_{ik} \cdot \sigma_{mn} = 1.
\] (3)

In the main coordinate system in the case of plane stress state the criterion (Eq. 3) contains 4 tensor components of strength:

\[
\Pi_{1111} \cdot \sigma_{11}^2 + \Pi_{2222} \cdot \sigma_{22}^2 + 2 \cdot \Pi_{1122} \cdot \sigma_{12}^2 + 4 \cdot \Pi_{1212} \cdot \sigma_{12}^2 = 1.
\] (4)

To express the components \( \Pi_{ikmn} \) through elementary limit states (tension, compression and shear), two cases of uniaxial tension were considered: 1) \( \sigma_{11} = \sigma_{45} \); \( \sigma_{22} = 0 \); \( \sigma_{12} = 0 \); 2) \( \sigma_{22} = \sigma_{45} \); \( \sigma_{11} = 0 \); \( \sigma_{12} = 0 \). Substituting them in Eq. 4 we can get the following formulas for \( \Pi_{1111} \) and \( \Pi_{2222} \):

\[
\Pi_{1111} = \frac{1}{\sigma_{11}^2}; \quad \Pi_{2222} = \frac{1}{\sigma_{22}^2}.
\] (5)

After considering the shear state ( \( \sigma_{11} = 0 \); \( \sigma_{22} = 0 \); \( \sigma_{12} = \tau_{45} \) ) and substituting these relations into Eq. 4, we can obtain a formula for determination of component \( \Pi_{1212} \):

\[
\Pi_{1212} = \frac{1}{4 \cdot \tau_{45}^2}.
\] (6)

To obtain expressions for the component \( \Pi_{1122} \) it is necessary to consider a state of pure shear ( \( \sigma_{11} = \tau_{45} \); \( \sigma_{22} = -\tau_{45} \); \( \sigma_{12} = 0 \) ) and substitute these relations to Eq. 4:

\[
\Pi_{1122} = \frac{1}{2} \cdot \left( \frac{1}{\sigma_{11}^2} + \frac{1}{\sigma_{22}^2} - \frac{1}{\tau_{45}^2} \right).
\] (7)

It should be kept in mind that even small changes in value of \( \Pi_{1122} \) can significantly affect the type of strength line. Since there is not enough experimental data on the composite strength under complex stress state even for well-known composites, so it is difficult to find the value of component \( \Pi_{1122} \) that could be reliable to use in calculations of structures made of these materials. Some ways to resolve this problem will be discussed below.

6. New variants of strength criteria for anisotropic materials

In the terms of strength tensor components Eq. 5, Eq. 6, Eq. 7 there is no difference between the tensile and compressive strength. Therefore this criterion in such form can be used only for anisotropic materials with the same meanings of tensile and compressive strength in each direction ( \( \sigma_{i1}^+ = \sigma_{i1}^- = \sigma_{i1} \) ) and with equal two shear strength on areas located at an angle of 45° to the main direction of the material ( \( \tau_{45}^+ = \tau_{45}^- = \tau_{45} \) ). There are not so many of such materials available.

V. Kopnov and S. Shambina [6] suggested extending Mieses-Hill criterion for anisotropic materials with varying tensile and compressive strength. It can be achieved through using of this criterion separately for each quadrant of the plane stresses.

Also it is possible to use new material’s strength characteristics instead of \( \tau_{45} \) [7]. For example, the following characteristics can be used: limits of strength under uniform of biaxial stretching
\( \sigma_{11}^{+} \) and uniform biaxial compression \( \sigma_{22}^{-} \) (with stresses ratio \( \sigma_{11}/\sigma_{22} = 1 \)) or limits of strength under biaxial stretching \( \sigma_{11}^{+} \) or biaxial compression \( \sigma_{22}^{-} \) (with a stress ratio \( \sigma_{11}/\sigma_{22} = 1:2 \)). This criterion was written separately for each quadrant of the plane stresses with new strength characteristic in the following form:

1) for quadrant \( \sigma_{11} > 0; \sigma_{22} > 0 \) (biaxial stretching):

\[
\left( \frac{\sigma_{11}}{\sigma_{11}^{*}} \right)^2 + \left( \frac{\sigma_{22}}{\sigma_{22}^{*}} \right)^2 + \frac{1}{2} \left[ \left( \frac{1}{\sigma_{11}^{*}} \right)^2 - \left( \frac{2}{\sigma_{22}^{*}} \right)^2 - \left( \frac{1}{\sigma_{22}^{*}} \right)^2 \right] \cdot \sigma_{11} \cdot \sigma_{22} = 1; \tag{8}
\]

2) for quadrant \( \sigma_{11} < 0; \sigma_{22} < 0 \) (biaxial compression):

\[
\left( \frac{\sigma_{11}}{\sigma_{11}^{*}} \right)^2 + \left( \frac{\sigma_{22}}{\sigma_{22}^{*}} \right)^2 + \frac{1}{2} \left[ \left( \frac{2}{\sigma_{11}^{*}} \right)^2 - \left( \frac{1}{\sigma_{22}^{*}} \right)^2 - \left( \frac{1}{\sigma_{22}^{*}} \right)^2 \right] \cdot \sigma_{11} \cdot \sigma_{22} = 1; \tag{9}
\]

3) for quadrant \( \sigma_{11} > 0; \sigma_{22} < 0 \):

\[
\left( \frac{\sigma_{11}}{\sigma_{11}^{+}} \right)^2 + \left( \frac{\sigma_{22}}{\sigma_{22}^{*}} \right)^2 + \frac{1}{2} \left[ \left( \frac{2}{\sigma_{11}^{+}} \right)^2 - \left( \frac{1}{\sigma_{22}^{*}} \right)^2 - \left( \frac{1}{\tau_{12}^{045}} \right)^2 \right] \cdot \sigma_{11} \cdot \sigma_{22} = 1; \tag{10}
\]

4) for quadrant \( \sigma_{11} < 0; \sigma_{22} > 0 \):

\[
\left( \frac{\sigma_{11}}{\sigma_{11}^{*}} \right)^2 + \left( \frac{\sigma_{22}}{\sigma_{22}^{*}} \right)^2 + \frac{1}{2} \left[ \left( \frac{1}{\sigma_{11}^{*}} \right)^2 + \left( \frac{1}{\sigma_{22}^{*}} \right)^2 - \left( \frac{1}{\tau_{12}^{045}} \right)^2 \right] \cdot \sigma_{11} \cdot \sigma_{22} = 1. \tag{11}
\]

7. Conclusions

Knowledge of mechanical characteristics for anisotropic materials and methods of calculation for structures made of these materials gives to the engineers the perfect opportunity to create reliable constructions. In this article some new variants of a well-known strength criterion were suggested. These new criteria are based on using new strength characteristics have been obtained with the help of writing the criterion separately for each of the four quadrants. This new criterion gives the opportunity to obtain proper results for calculations of structures made of a wider range of anisotropic materials.

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