Proton polarizability contribution to hydrogen Lamb shift

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Abstract

The correction to the hydrogen Lamb shift due to the proton electric and magnetic polarizabilities is expressed analytically through their static values, which are known from experiment. The numerical value of the correction is $-71 \pm 11 \pm 7$ Hz.

High experimental precision attained in the hydrogen and deuterium spectroscopy (see, e.g., [1]) stimulates considerable theoretical activity in this field. In particular, the deuteron polarizability contribution to the Lamb shift in deuterium was calculated in Refs. [2 – 9]. An estimate of the proton polarizability contribution to the Lamb shift in hydrogen was made in Ref. [2]. A special feature of the corrections obtained in Refs. [2 – 9] is that they contain logarithm of the ratio of a typical nuclear excitation energy to the electron mass, $\ln \bar{E}/m_e$. In fact, the contribution of the nuclear electric polarizability to the Lamb shift had been obtained with the logarithmic accuracy in Ref. [10] for an arbitrary nucleus.

In the present note we consider the problem of the proton polarizability correction to the Lamb shift in hydrogen. The typical excitation energy for the proton $\bar{E}_p \sim 300$ MeV is large as compared to other nuclei (to say nothing of the deuteron). So, the logarithm $\ln \bar{E}/m_e$ is not just a mere theoretical parameter, it is truly large, about $6 - 7$, which makes the logarithmic approximation quite meaningful quantitatively. As distinct from Ref. [10], we take into account in our final formula not only the electric polarizability $\bar{\alpha}$, but as well the magnetic one $\bar{\beta}$ (though it does not very much influence the result numerically).

In our calculation we follow closely the approach of Ref. [7]. In particular, we use the gauge $A_0 = 0$ for virtual photons, so that the only nonvanishing components of the photon propagator are $D_{lm} = d_{lm}/k^2$, $d_{lm} = \delta_{lm} - k_i k_m/\omega^2$ $(i, m = 1, 2, 3)$. The electron-proton forward scattering amplitude, we are interested in, is

$$T = 4\pi i\alpha \int \frac{d^4k}{(2\pi)^4} D_{im} D_{jn} \frac{\gamma_i(\hat{l} - \hat{k} + m_e)\gamma_j}{k^2 - 2lk} M_{mn}.$$  \hspace{1cm} (1)

Here $l_\mu = (m_e, 0, 0, 0)$ is the electron momentum. The nuclear-spin independent Compton forward scattering amplitude, which is of interest to us, can be written as

$$M = \bar{\alpha}(\omega^2, k^2)E^*E + \bar{\beta}(\omega^2, k^2)B^*B = M_{mn}\epsilon_m\epsilon_n^*,$$  \hspace{1cm} (2)
where $\bar{\alpha}$ and $\bar{\beta}$ are the nuclear electric and magnetic polarizabilities, respectively. The structure $\gamma_i(l-\hat{k}+m_e)\gamma_j$ in (1) reduces to $-\omega \delta_{ij}$. Perhaps, the most convenient succession of integrating expression (1) is as follows: the Wick rotation; transforming the integral over the Euclidean $\omega$ to the interval $(0, \infty)$; the substitution $k \to k\omega$. Then the integration over $\omega$ is easily performed with the logarithmic accuracy:

$$\int_0^\infty \frac{d\omega^2}{\omega^2 + 4m_e^2/(1+k^2)^2} \left[ (3 + 2k^2 + k^4)\bar{\alpha}(-\omega^2, -\omega^2 k^2) - 2k^2 \bar{\beta}(-\omega^2, -\omega^2 k^2) \right]$$

$$= \left[ (3 + 2k^2 + k^4)\bar{\alpha}(0) - 2k^2 \bar{\beta}(0) \right] \ln \frac{E^2}{m_e^2}. \tag{3}$$

The crucial point is that within the logarithmic approximation both polarizabilities $\bar{\alpha}$ and $\bar{\beta}$ in the lhs can be taken at $\omega = 0$, $k^2 = 0$. The final integration over $d^3k$ is trivial.

The resulting effective operator of the electron-proton interaction (equal to $-T$) can be written in the coordinate representation as

$$V = -\alpha m_e \left[ 5\bar{\alpha}(0) - \bar{\beta}(0) \right] \ln \frac{E}{m_e} \delta(r). \tag{4}$$

This expression applies within the logarithmic accuracy for arbitrary nuclei. As it was mentioned above, it differs from the result obtained earlier in Ref. [10] by the account for the magnetic polarizability $-\bar{\beta}(0)$ only.

As refers to hydrogen, the experimental data on the proton polarizabilities, which follow from the Compton scattering, can be summarized as follows [11]:

$$\bar{\alpha}_p(0) + \bar{\beta}_p(0) = (14.2 \pm 0.5) \times 10^{-4} \text{ fm}^3;$$
$$\bar{\alpha}_p(0) - \bar{\beta}_p(0) = (10.0 \pm 1.8) \times 10^{-4} \text{ fm}^3. \tag{5}$$

Now,

$$5 \bar{\alpha}_p(0) - 2 \bar{\beta}_p(0) = 2 \left[ \bar{\alpha}_p(0) + \bar{\beta}_p(0) \right] + 3 \left[ \bar{\alpha}_p(0) - \bar{\beta}_p(0) \right] = (58.4 \pm 5.3) \times 10^{-4} \text{ fm}^3. \tag{6}$$

The errors are added in quadratures.

Finally, at $E_p \sim 300$ MeV the proton polarizability correction to the hydrogen 1S state is

$$-71 \pm 11 \pm 7 \text{ Hz.} \tag{7}$$

Here the first error is that of the logarithmic approximation, which we estimate as 15%. The second one originates from the values of the polarizabilities.

The corresponding estimate presented in Ref. [10] differs from our result by the factor at $\bar{\alpha}_p(0)$ (2 instead of 5) and by the absence of $\bar{\beta}_p(0)$.

The obtained correction (7) is not so far away from the accuracy expected soon in the measurements of the isotope shift between deuterium and hydrogen 1S−2S transitions.

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References

[1] K. Pachucki, D. Leibfried, M. Weitz, A. Huber, W. König, and T.W. Hänsch, J.Phys. B 29, 177 (1993).

[2] K. Pachucki, D. Leibfried, and T.W. Hänsch, Phys. Rev. A 48, R1 (1993).

[3] K. Pachucki, M. Weitz, and T.W. Hänsch, Phys. Rev. A 49, 2255 (1994).

[4] Yang Li and R. Rosenfelder, Phys.Lett. B 319, 7 (1993); ibid, 333, 564 (1994).

[5] W. Leidemann, and R. Rosenfelder, Phys. Rev. C 51, 427 (1995).

[6] J. Martorell, D.W.L. Sprung, and D.C. Zheng, Phys. Rev. C 51, 1127 (1995).

[7] A.I. Milstein, S.S. Petrosyan, and I.B. Khriplovich, Zh.Eksp.Teor.Fiz. 109, 1146, 1996 [Sov.Phys. JETP 82, 616 (1996)].

[8] J.L. Friar, and G.L. Payne, nucl-th/9702019; Phys. Rev. C, in press (1997).

[9] J.L. Friar, and G.L. Payne, nucl-th/9704032.

[10] J. Bernabéu and T.E.O. Ericson, Z.Phys. A 309, 213 (1983).

[11] B.E. MacGibbon, G. Garino, M.A. Lucas, A.M. Nathan, G. Feldman, and B. Dolbinkin, Phys. Rev. C 52, 2097 (1995).