Effect of mechanical mode - qubit interaction on perfect optical non-reciprocity in double cavity system

Tarun Kumar¹, D. S. Gosain¹ and Aranya B Bhattacherjee²

¹Department of Physics, Ramjas College, University of Delhi, Delhi-110007, India, ²Department of Physics, Birla Institute of Technology and Science, Pilani, Hyderabad Campus, Telangana State - 500078, India.

Abstract. Quantum devices exhibiting optical non-reciprocity are indispensable for quantum information processing and communication technology. In this paper we propose a double cavity optomechanical system where the movable mirror of the cavity is linearly coupled with a qubit placed inside the cavity. We study the effect of this mechanical mode - qubit linear interaction on time reversal symmetry i.e, optical non-reciprocity and prove that mechanical mode - qubit interaction can be used as a new tool to control optical non-reciprocity.

1. Introduction
Nonreciprocal optical devices have played an important role in several applications such as quantum sensing and communications and hence have attracted the attention of researchers. Optical non-reciprocity is related to time reversal symmetry breaking, if source and detectors are exchanged [1,2,3]. Optical circulators [4] and isolators [5] are the common examples of such devices. These devices find vast applications in the field of topological photonics [6,7], advanced optical communication and quantum information processing [8,9], since such devices can suppress spurious modes and unwanted signals. Lorentz reciprocity theorem describes reciprocity in optics. Therefore, to achieve optical nonreciprocal devices, one has to break Lorentz reciprocity condition [10,11].

The conventional approaches to break reciprocity is to use magneto-optical materials such as ferrites [12,13,14,15,16,17] or by using transistors biased by static electric currents [18,19,20,21]. But both of the conventional approaches have disadvantages to study optical reciprocity. Former devices are very bulky, costly and are non-amenable to integrated circuits because ferrite crystal lattices are incompatible with semiconductor materials. Whereas later devices show poor noise characterization and inherent non linearity leads to undesirable applications when very weak or very strong signals are used. Due to the above-mentioned limitations of conventional devices to produce optical non-reciprocity, a quest for new devices leads to the development of magnetless nonreciprocal systems. Alternative approaches to realize optical non-reciprocity include the use of spatio-temporal modulation [22,23,24,25], use of nonlinear processes to construct a nonlinear optical isolation [26,27,28].

In the past few years, systems related to cavity optomechanics have been studied extensively [29,30,31,32,33,34,35,36]. Cavity opto-mechanics is the study of radiation pressure induced interaction of optical modes with mechanical modes [37,38,39,40,41,42]. This system has influenced new regimes such as ultrahigh precision measurement [43], Gravitational wave detection interferometers [44,45], optomechanically induced transparencies [46,47,48,49,50,51], atomic force microscopes [52,53], quantum entanglement [54,55,56]. Recently various schemes have also been proposed based on optomechanical systems to realize non-reciprocity [57,58,59,60,61,62,63].

In this paper, we consider a two-cavity optomechanical system where the movable mirror of the cavity is linearly coupled with a qubit placed inside the cavity. We study the effect of this mechanical mode - qubit linear interaction on time reversal symmetry i.e, optical non-reciprocity. We prove that mechanical mode - qubit interaction can be used as a new tool to control optical non-reciprocity.
2. Theoretical Framework

In this framework we consider an optical cavity, containing a mechanical membrane at the middle (figure 1). Two geometrically distinct optical modes ‘a’ and ‘b’ are present inside the cavity. The two optical modes have same frequency $\omega_c$ but different photon decay rates $\kappa_1$ and $\kappa_2$ respectively. Operator ‘c’ describes the mechanical mode associated with the membrane with decay rate $\gamma_b$. The two-cavity system is driven by two strong external optical fields from left as well as right side with frequency $\omega_c$. In addition, the system is probed by two probe fields having same frequency $\omega_p$ from the left and right fixed mirrors. The amplitudes of coupling fields are $\varepsilon_c$ and $\varepsilon_p$ whereas that of probe fields are $\varepsilon_L$ and $\varepsilon_R$. The Hamiltonian of the system in the rotating wave frame of coupling frequency $\omega_c$ is given by (with $\hbar = 1$)

$$
H = \Delta_c \left( a^\dagger a + b^\dagger b \right) + \omega_c c^\dagger c + g_0 \left( b^\dagger b - a^\dagger a \right) \left( c^\dagger + c \right) + J \left( a^\dagger b + b^\dagger a \right) + i \varepsilon_c \left( a^\dagger e^{i\delta t} - a e^{-i\delta t} \right) \\
+ i \varepsilon_p \left( b^\dagger e^{-i\delta t} - b e^{i\delta t} \right) + i \left( \varepsilon_c a^\dagger - \varepsilon_p a \right) + i \left( \varepsilon_p b^\dagger - \varepsilon_c b \right) + \frac{1}{2} \omega_p \sigma_z + g_0 \left( c \sigma_x + c^\dagger \sigma_\cdot \right) (1)
$$

Here $\Delta_c = \omega_c - \omega_c$ and $\delta = \omega_p - \omega_c$ are the detuning between cavity mode - coupling field and probe field - coupling field respectively. In equation one first term represents the free energies of the two

![Figure 1. Theoretical Model as described in the text](image-url)
optical modes. The second term gives the free energy of the mirror. The third term represents the cavity mode coupling interaction term, with $g_o$ being the single photon coupling constant between optical modes and mechanical oscillator. The fourth term describes the coupling between the optical cavities through the tunneling of the photons between the two cavities. Here, ‘$J$’ being the tunneling constant [65]. The tunneling parameter in solid state system (particularly in this present work) arises due to the finite probability for the light to pass through the movable mirror which is essentially is a Distributed Bragg Reflector. The fifth and the sixth terms are the probe fields and the coupling fields. The seventh terms represent the energy of the qubit and the last term coupling between the mechanical oscillator and the qubit. Also $\sigma_+$ and $\sigma_-$ are the usual Pauli operators of the qubit.

Using the Hamiltonian of the system given by equation (1), system dynamics is described by the following Langevin equations

$$
a = \left[ i \Delta - ig_o \left( c^+ + c \right) + \frac{\kappa_2}{2} \right] a + \epsilon_c + \epsilon_r \exp(-i\delta t) - iJb \tag{2}
$$

$$
b = \left[ i \Delta + ig_o \left( c^+ + c \right) + \frac{\kappa_2}{2} \right] b + \epsilon_d + \epsilon_r \exp(-i\delta t) - iJa \tag{3}
$$

$$
\dot{c} = -i\omega_m c - ig_o \left( b^* b - a^* a \right) - ig_preview_c \sigma_+ - \frac{1}{2} \gamma_c \sigma_+ \tag{4}
$$

$$
\sigma_+ = -\frac{1}{2} i \omega_m \sigma_+ + 2ig_preview_c \sigma_+ - \frac{1}{2} \gamma_\sigma \sigma_+ \tag{5}
$$

With factorization assumptions $\langle xc \rangle = \langle x \rangle \langle c \rangle, x = a, b$ and in the absence of probe fields $\epsilon_c$ and $\epsilon_r$, we can write down the steady state solutions of equations (2) - (5) as

$$
a_s = \frac{\left( \frac{\kappa_2}{2} + i\Delta_2 \right) \epsilon_c - iJ \epsilon_d}{J^2 + \left( \frac{\kappa_1}{2} + i\Delta_1 \right) \left( \frac{\kappa_2}{2} + i\Delta_2 \right)}
$$

$$
b_s = \frac{\left( \frac{\kappa_1}{2} + i\Delta_1 \right) \epsilon_d - iJ \epsilon_c}{J^2 + \left( \frac{\kappa_1}{2} + i\Delta_1 \right) \left( \frac{\kappa_2}{2} + i\Delta_2 \right)}
$$

$$
c_s = \frac{-ig_o \left( b_s \right)^2 - \left| a_s \right|^2 + ig_{\text{pert}}}{\frac{\gamma_\sigma}{2} + i\omega_m}
$$

$$
\sigma_+ = \frac{-4ig \sigma_+ c_s}{\gamma_\sigma + i\omega_q}
$$

Here $\Delta_1$ and $\Delta_2$ are the effective detuning between coupling fields and the cavity modes and defined as $\Delta_1 = \Delta_2 - g_o \left( b_s + b_s^* \right)$ and $\Delta_2 = \Delta_2 + g_o \left( b_s + b_s^* \right)$. Now to study the behaviour of quantum fluctuations of the system near its steady state, we linearize the quantum Langevin equations as $a(t) = a_s + \delta a$, $b(t) = b_s + \delta b$, $c(t) = c_s + \delta c$ and $\sigma_+(t) = \sigma_+ + \delta \sigma_+$. Using the interaction picture by
defining $\delta a \rightarrow \delta a \exp(-i\Delta t)$, $\delta b \rightarrow \delta b \exp(-i\Delta t)$, $\delta c \rightarrow \delta c \exp(-i\omega_m t)$ and $\delta \sigma \rightarrow \delta \sigma \exp(-i\frac{\omega_q}{2} t)$, the linearized quantum Langevin equations take the form

$$\dot{\delta a} = -\frac{K_a}{2} \delta a + iG_1 (\delta c^* \exp(i(\omega_m + \Delta_1) t) + \delta c \exp(-i(\omega_m - \Delta_1) t)) + \epsilon_e \exp(-i(\delta - \Delta_1) t) - iJ \delta b \exp(-i(\Delta_2 - \Delta_1) t)$$

(7)$$
\dot{\delta b} = -\frac{K_b}{2} \delta b - iG_2 e^{i\theta} (\delta c^* \exp(i(\omega_m + \Delta_2) t) + \delta c \exp(-i(\omega_m - \Delta_2) t)) + \epsilon_q \exp(-i(\delta - \Delta_2) t) - iJ \delta a \exp(i(\Delta_2 - \Delta_1) t)$$

(8)$$
\dot{\delta c} = -\frac{\gamma}{2} \delta c + iG_1 (\delta a^* \exp(i(\omega_m + \Delta_1) t) + \delta a \exp(i(\omega_m - \Delta_1) t))$$
$$-iG_2 (e^{i\theta} \delta b^* \exp(i(\omega_m + \Delta_2) t) + e^{-i\theta} \delta b \exp(i(\omega_m - \Delta_2) t)) - i g_q \delta \sigma \exp(-i\left(\frac{\omega_q}{2} - \omega_m\right) t)$$

(9)$$
\dot{\delta \sigma} = -\frac{\gamma}{2} \delta \sigma + 2i g_q \delta \sigma \delta c \exp(-i\left(\frac{\omega_q}{2} - \omega_m\right) t)$$

(10)

Where $\theta$ is the phase difference between the opto-mechanical couplings $G_i = g_0 a_i$ and $G_2 = g_0 b_i \exp(-i\theta)$, the phase difference can be controlled by varying the amplitudes of the coupling field $\epsilon_e$ and $\epsilon_q$ according to equation (6). Here $\theta$ is an important parameter to obtain optical non-reciprocity. If we assume that both the optical field drives the cavity mode at the mechanical red side band $(\Delta_1 \approx \Delta_2 \approx \omega_m, \omega_m \approx \frac{\omega_q}{2})$ and also from equation (6), it can be proved that the extra term in the detuning (in $\Delta_1$ and $\Delta_2$) i.e., $g_q \left(b_i + b_i^*\right)$ will be negligible in comparison to $\Delta_1$, since $\omega_m \ll G_i, G_2$. Therefore, we can write effective detuning $\Delta_1 = \Delta_2 = \omega_m$ and equations (7) - (10) can be approximated as

$$\dot{\delta a} = -\frac{K_a}{2} \delta a + iG_1 (\delta c^* + \epsilon_e e^{-i\omega_m t} - iJ \delta b$$

$$\dot{\delta b} = -\frac{K_b}{2} \delta b - iG_2 e^{i\theta} (\delta c^* + \epsilon_q e^{-i\omega_m t} - iJ \delta a$$

$$\dot{\delta c} = -\frac{\gamma}{2} \delta c + iG_1 \delta a - iG_2 e^{i\theta} \delta b - i g_q \delta \sigma$$

$$\dot{\delta \sigma} = -\frac{\gamma}{2} \delta \sigma + 2i g_q \delta \sigma \delta c$$

(11)

Here $x = \delta - \omega_m$. If the damping rates of both the cavities are equal $K_1 = K_2$, then we can write $G_1 = G_2 = G$ [65]. Here the parameter $G$ is the effective optomechanical coupling generated due to the radiation pressure of the cavity field. Here By assuming $\delta y = \delta y_x \exp(-ixt) + \delta y_c \exp(ixt)$, $y = a, b, c, \sigma$, we can obtain.
To achieve perfect optical non-reciprocity, transmission amplitudes

\[ \delta a = \frac{2(4G^2 + \gamma'_d \kappa')e_x + (8G^2 e^{-i\theta} - 4iJ')e_y}{8G^2 \kappa + (4J^2 + \kappa_s)\gamma'_d - 16iG^2 J \cos(\theta)} \]  

\[ \delta b = \frac{2(4G^2 + \gamma'_d \kappa')e_x + (8G^2 e^{-i\theta} - 4iJ')e_y}{8G^2 \kappa + (4J^2 + \kappa_s)\gamma'_d - 16iG^2 J \cos(\theta)} \]  

\[ \delta c = \frac{2(4G^2 + \gamma'_d \kappa')e_x + (8G^2 e^{-i\theta} - 4iJ')e_y}{8G^2 \kappa + (4J^2 + \kappa_s)\gamma'_d - 16iG^2 J \cos(\theta)} \]  

Here \( \kappa_s = \kappa - 2i\kappa \), \( \gamma'_d = \gamma - 2i\kappa - \frac{8g^2 \sigma}{\gamma_d} \). Using the input output formalism of cavity optomechanics [66], we can derive the output fields \( \epsilon_{\text{out,\,R}} \) and \( \epsilon_{\text{out,\,L}} \) from the relations

\[ \epsilon_{\text{out,\,L}} + i e^{i\delta} = \sqrt{k} \delta a \]

\[ \epsilon_{\text{out,\,R}} + e^{i\delta} = \sqrt{k} \delta b \]  

Here \( \epsilon_{\text{L,R}} = \frac{\epsilon_{\text{L,R}}}{\sqrt{k}} \). Under the assumption \( \delta y = \delta y, \exp(-i\theta) + \delta y, \exp(i\theta) \), \( y = a, b, c, \sigma_+ \), the output fields take the form

\[ \epsilon_{\text{L,R}} = \sqrt{k} \delta a - \frac{\epsilon_{\text{L}}}{\sqrt{k}} \]  

\[ \epsilon_{\text{L,R}} = \sqrt{k} \delta b - \frac{\epsilon_{\text{L}}}{\sqrt{k}} \]  

and \( \epsilon_{\text{L,L}} = \epsilon_{\text{R,L}} = 0 \).

3. Perfect Optical Non-reciprocity

To achieve perfect optical non-reciprocity, transmission amplitudes \( T_{L\rightarrow R} \) (TLR) and \( T_{R\rightarrow L} \) (TRL) must satisfy

\[ T_{L\rightarrow R} = \left| \frac{\epsilon_{\text{out,\,R}}}{\epsilon_{\text{out,\,L}}} \right| = 1, T_{R\rightarrow L} = \left| \frac{\epsilon_{\text{out,\,L}}}{\epsilon_{\text{out,\,R}}} \right| = 0 \]  

\[ T_{L\rightarrow R} = \left| \frac{\epsilon_{\text{out,\,L}}}{\epsilon_{\text{out,\,R}}} \right| = 0, T_{R\rightarrow L} = \left| \frac{\epsilon_{\text{out,\,R}}}{\epsilon_{\text{out,\,L}}} \right| = 1 \]  

Equations (17) and (18) implies that input signal can be transmitted completely from one side to the other side but the reverse is not true hence above two equations represent two directions of isolations. Here we discuss the case of equation (17) in details since equation (18) gives identical result. Combining results of equations (12), (13) and (14) with that of equations (15) and (16), the two transmitted light fields can be written as
It is evident from equations (19) and (20) that the two output fields are identical and hence shows optical reciprocity when $\theta = n\pi$ (n being an integer). However, when $\theta \neq n\pi$, the system also shows optical non-reciprocity. It is also evident from eqns. (19) and (20) that the reason of optical non-reciprocity is the quantum interference between linear coupling interaction ‘J’ and the optomechanical interaction ‘G’.

4. Result
In this section, we consider two cases. In first case, we study, in detail, optical non-reciprocity for $\theta = -\pi / 2$ analytically. In second case, we study optical non-reciprocity for any $\theta$ numerically. Case I: Let the nonreciprocal phase difference $\theta = -\pi / 2$, the two output fields of equations (19) and (20) becomes

\[
\frac{E_{in}}{E_{out}} = \frac{4\kappa \left(2G^2 e^{i\theta} - iJ \gamma \gamma' \right)}{8G^2 \kappa + \left(4J^2 + \kappa_i^2\right) \gamma \gamma' + 16iG^2 J \cos(\theta)} \quad (19)
\]

\[
\frac{E_{out}}{E_{in}} = \frac{4\kappa \left(2G^2 e^{-i\theta} - iJ \gamma \gamma' \right)}{8G^2 \kappa + \left(4J^2 + \kappa_i^2\right) \gamma \gamma' + 16iG^2 J \cos(\theta)} \quad (20)
\]

Equations (18) and (19) can be used to obtain the conditions of perfect non-reciprocity and are given by

\[
\frac{TLR}{E_{in}} = \frac{4\kappa \left(2G^2 + J \gamma \gamma' \right)}{8G^2 \kappa + \left(4J^2 + \kappa_i^2\right) \gamma \gamma'} \quad (21)
\]

\[
\frac{TRL}{E_{out}} = \frac{4\kappa \left(2G^2 - J \gamma \gamma' \right)}{8G^2 \kappa + \left(4J^2 + \kappa_i^2\right) \gamma \gamma'} \quad (22)
\]
In fig. 2 we plot the transmission amplitudes $T_{L\rightarrow R}$ and $T_{R\rightarrow L}$ as a function of normalized detuning $x/\kappa$ for different coupling constant $g_q(0,0.1)$. We observe that the system shows optical non-reciprocity near $x=0$. However, when $g_q = 0$, we get perfect non-reciprocity, but with $g_q \neq 0$, the response is not perfect but is sharp at $x=0$.

Now we consider case 2, where we study non-reciprocity for any $\theta$. For simplicity we consider two decay rates i.e, photon decay rate is equal to the mechanical decay rate $\kappa = \gamma$, equations (19) and (20) becomes

\begin{align}
TLR &= \frac{\mathcal{E}_{\text{out}}}{\mathcal{E}_{\text{in}}} = \frac{4\gamma\left(2G^2e^{\theta/2} - iJ\gamma'_{dz}\right)}{8G^2\gamma_x + \left(4J^2 + \gamma_x^2\right)\gamma'_{dz} + 16iG^2J\cos(\theta)} \tag{24}
\\
TRL &= \frac{\mathcal{E}_{\text{out}}}{\mathcal{E}_{\text{in}}} = \frac{4\gamma\left(2G^2e^{\theta/2} - iJ\gamma'_{dz}\right)}{8G^2\gamma_x + \left(4J^2 + \gamma_x^2\right)\gamma'_{dz} + 16iG^2J\cos(\theta)} \tag{25}
\end{align}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Transmission amplitudes TLR and TRL are plotted as a function of normalized detuning $x/\kappa$ and $\theta = 3\pi/4$ for different coupling constant $g_q$: (a) $g_q / 2\pi = 0.1$ (b) $g_q / 2\pi = 1$ and (c) $g_q / 2\pi = 2$. The other parameters used are: $\gamma / 2\pi = 1$, $\gamma_d / 2\pi = 0.01$, $\sigma_x / 2\pi = -1$, $J = 0.75$.}
\end{figure}
The equations (24) and (25) are plotted for perfect optical non-reciprocity numerically. In fig. 3, we plot TLR and TRL for $\theta = 3\pi / 4$. We find that when the mirror qubit coupling ($g_q$) is small, we observe optical non-reciprocity (fig. 3a), however if the value of the coupling ($g_q$) is increased, we lose optical non-reciprocity (fig. 3b) and the two transmission amplitudes (TLR and TRL) overlaps (fig. 3c). The mechanical mode as is evident from the results appears to be a new handle to coherently control the non-reciprocity in double cavity optomechanical system. In comparison to ref. [65], our work is the first work that considers interaction between mechanical mode and a qubit to study non-reciprocity and its application to quantum devices.

5. Conclusion
In conclusion, we have shown ways to achieve perfect optical non-reciprocity in a hybrid two-cavity opto-mechanical system in the presence of a qubit. The qubit-mechanical mode coupling is found to coherently manipulate the non-reciprocity. For $\theta = -\pi / 2$, when $g_q = 0$, we get perfect non-reciprocity with a wide width in the transmission spectrum but as $g_q \neq 0$, the width becomes very narrow. Such a narrow width transmission spectrum is extremely suitable for application in sensitive "all optical switching" devices. When $\theta = 3\pi / 4$, the transmission spectrum becomes asymmetric with a not so perfect non-reciprocity. With a proper choice of $\theta$ and $g_q$, one can attain perfectly symmetric non-reciprocity and the qubit-mechanical mode appears as a new handle to control the non-reciprocity.

6. References
[1] Deak L and Fulop T., "Reciprocity in quantum, electromagnetic and other wave scattering," Annals of Physics, vol. 327, no. 4, pp. 1050(1077), 2012.
[2] Feng L, Ayache M, Huang J, Xu Y, Lu M, Chen Y, Fainman F, and Scherer A, Nonreciprocal light propagation in a silicon photonic circuit," Science, vol. 333, no. 6043, pp. 729(733), 2011.
[3] Kamal A, Clarke J, and Devoret M, Noiseless non-reciprocity in a parametric active device," Nature Physics, vol. 7, no. 4, p. 311, 2011.
[4] Scheucher M, Hilico A, Will E, Volz J, and Rauschenbeutel A, Quantum optical circulator controlled by a single chirally coupled atom," Science, p. aa12118, 2016.
[5] Sayrin C, Junge C, Mitsch R, Albrecht B, OShea D, Schneeweiss P, Volz J, and Rauschenbeutel A, Nanophotonic optical isolator controlled by the internal state of cold atoms," Phy. Rev. X, vol. 5, no. 4, p. 041036, 2015.
[6] Tian L and Li Z, Nonreciprocal quantum-state conversion between microwave and optical photons," Phy. Rev. A, vol. 96, no. 1, p. 013808, 2017.
[7] Metelmann A and Clerk A, Nonreciprocal quantum interactions and devices via autonomous feedforward," Phy. Rev. A, vol. 95, no. 1, p. 013837, 2017.
[8] Cira J, Zoller P, Kimble H J, and Mabuchi H, Phys. Rev. Lett. 78, 3221 (1997).
[9] Lodahl P, Mahmoodian S, Stobbe S, Rauschenbeutel A, Schneeweiss A P, Volz J, Pichler H, and Zoller P, Nature 541, 473 (2017).
[10] Fan S, Baets R, Petrov A, Yu Z, Joannopoulos JD, Freude W, Melloni A, Popovic M, Vanwolleghem M, Jalas M D, et al., Science 335, 38 (2012).
[11] Jalas D, Petrov A, Eich M, Freude W, Fan S, Yu Z, Baets R, Popovic M, Melloni A, Joannopoulos JD, et al., Nature Photonics 7, 579 (2013).
[12] Aplte I J and Carson J W, Appl. Opt. 3, 544 (1964).
[13] Shirasaki M and Asama K, Appl. Opt. 21, 4296 (1982).
[14] Sato T, Sun J, Kasahara R, and Kawakami S, Opt. Lett. 24, 1337 (1999).
[15] Bi L, Hu J, Jiang P, Kim D H, Dionne G F, Kimerling L C, and Ross C A, Nat. Photonics 5, 758 (2011).
[16] Shalaby M, Peccianti M, Ozturk Y, and Morandotti R, Nat. Commun. 4, 1558 (2013).
[17] Tamagnone M, Moldovan C, Poumirol J, Kuzmenko A B, Ionescu A M, Mosig J R, and Perruisseau-carrier J, Nat. Commun. 7, 1 (2016).
[18] Tanaka S, Shimomura S N, and Ohtake K, Proc. IEEE 53, 260 (1965).
[19] Kodera T, Sounas D L, and Caloz C, Appl. Phys. Lett. 99, 31114 (2011).
[20] Kodera T, Sounas D L, and Caloz C, IEEE Trans. Microw. Theory Techn. 61, 1030 (2013).
[21] Wang Z, Wang Z, Wang J, Zhang B, Huangfu J, Joannopoulos JD, Seljačić M, and Ran L, Proc. Natl. Acad. Sci. U. S. A. 109, 15194 (2012).
Yu Z and Fan S, *Nature photonics* 3, 91 (2009).

Kang M S, Butsch A, and Russell P S J, *Nature Photonics* 5, 549 (2011).

Estep N A, Sounas D L, Soric J, and Alu A, *Nature Physics* 10, 923 (2014).

Sounas D L, Caloz C, and A. Alu, *Nature communications* 4, 2407 (2013).

Fan L, Wang J, Varghese I T, Shen H, Niu B, Xuan, Y, Weiner A M, and Qi M, *Science* 335, 447 (2012).

Bender N, Factor S, Bodyfelt J D, Ramezani H, Christodoulides D N, Ellis F M, and Kottos T, *Phys. Rev. Lett.* 110, 234101 (2013).

Chang L, Jiang X, Hua S, Yang C, Wen J, Jiang L, Li G, Wang G, and Xiao M, *Nature photonics* 8, 524 (2014).

Aspelmeyer M, Kippenberg T and Marquardt F, (Springer, Verlag, Berlin, Heidelberg, 2014).

Hansch T W and Schawlow A L, *Optics Comm.* 13, 68 (1975).

Gigan S et al., *Nature* 444, 67 (2006). [9] O. Arcizet et al., *Nature* 444, 71 (2006).

Höhberger-Metzger C and Karrai K, *Nature* 432, 1002 (2004).

Corbitt T et al., *Phys. Rev. Letts.* 98, 150802 (2007).

Corbitt and Mavalvala N, J. Opt. B: Quantum Semi-class. Opt. 6, S675 (2004).

Chu S, Hollberg L, Bjorkholm JE, Cable A and Ashkin A, *Phys. Rev. Letts.* 55, 48 (1985).

Wineland D J, Drullinger R E and Walls F L, *Phys. Rev. Letts.* 40, 1639 (1978).

Aspelmeyer M, Kippenberg T and Marquardt F, *Rev. Mod. Phys.* 86, 1391 (2014).

Aspelmeyer M, Meystre P and Schwab K, *Phys. Today* 65, 29 (2012).

Marquardt F and Girvin S, *Physics* 2, 40 (2009).

Kippenberg T J and Vahala K J, *Science* 321, 1172 (2008).

Hu Y W, Xiao Y F, Liu Y C and Gong Q, *Front. Phys.* 8, 475 (2013).

Zhang K, Zhou L, Gong G and Zhang W, *Front. Phys.* 6, 237 (2011).

Teufel J D, Donner T, Castellanos-Beltran M A, Harlow J W and Lehnhert K W, *Nat. Nanotechnol.* 4, 820 (2009).

Caves C M, *Phys. Rev. Lett.* 45, 75 (1980).

Loudon R, *Phys. Rev. Lett.* 47, 815 (1981).

Weis S, Riviere R, Delglise S, Gavartin E, Arcizet O, Schliesser A, Kippenberg T J, *Science* 330, 1520 (2010).

Ma J, You C, Si L, Xiong H, Li J, Yang X and Wu Y, *Sci. Rep.* 5, 11278 (2015).

Agarwal G S and Huang S, *Phys. Rev. A* 81, 041803 (2010).

P. Tassin, L. Zhang, R. Zhao, A. Jain, T. Koschny, and C. M. Soukoulis, *Phys. Rev. Lett.* 109, 187401 (2012).

Safavi-Naeini A H, Alegre T P, Chan J, Eichenfield M, Winger M, Lin Q, Hill JT, Chang D E and Painter O, *Nature* (London) 472, 69 (2011).

Ma P C, Zhang J Q, Xiao Y, Feng M, and Zhang Z M, *Phys. Rev. A* 90, 043825 (2014).

Mertz J, Marti O, Mlynek J, *Appl. Phys. Lett.* 62, 2344 (1993).

Milburn G J, Jacobs K, and Walls D F, *Phys. Rev. A* 50, 5256 (1994).

Hofer S G, Wieczorek W, Aspelmeyer M, and Hammerer K, *Phys. Rev. A* 84, 052327(2011).

Wang Y D, Chesi S, and Clerk A, *Phys. Rev. A* 91, 013807 (2015).

Aggarwal N, Debnath K, Mahajan S, Bhattacharjee A B and Man Mohan, *Int. J. Quantum Inform.* 12, 1450024 (2014).

Manipatruni S, Robinson J T, and Lipson M, *Phys. Rev. Lett.* 102, 213903 (2009).

Hafezi M and Rabl P, *Optics express* 20, 7672 (2012).

Ruesink F, Miří M A, Alu A, and Verhagen E, *Nature communications* 7, 13662 (2016).

Xu X W and Li Y, *Phys. Rev. A* 91, 053854 (2015).

Fang K, Luo J, Metelmann A, M. Matheny M H, Marquardt F, Clerk A A, and Painter *Nature Physics* 13, 465 (2017).

Ruesink F, Miří M A, Alu A, and Verhagen E, *Nature communications* 7, 13662 (2016).

Yan X, Lu H, Gao F, Yang L, *Frontiers of Physics*, 14(5), 52601 (2019)

Zhao L, Li X, Lu H and Tian X, *Commun. Theor. Phys.* 71 1011–1016 (2019)

Wu S C, Qin L G, Jing J, Yan T M, Lu J and Wang Z Y, *Phys. Rev. A*, 98, 013807 (2018)

Walls D F and Milburn G J, *Quantum Optics* (Springer-Verlag, Berlin, 1994)

6. Acknowledgement
Tarun Kumar and D. S. Gosain acknowledge Ramjas College, University of Delhi for providing research facility.