Vibrations of dam–plate of a hydro-technical structure under seismic load

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Abstract. In present paper, the problem of the vibration of a viscoelastic dam-plate of a hydro-technical structure is investigated, based on the Kirchhoff-Love hypothesis in the geometrically nonlinear statement. This problem is reduced to a system of nonlinear ordinary integro-differential equations by using the Bubnov-Galerkin method. The resulting system with a weakly-singular Koltunov-Rzhanitsyn kernel is solved using a numerical method based on quadrature formulas. The behavior of the viscoelastic dam-plate of hydro-technical structure is studied for the wide ranges of physical, mechanical, and geometrical material parameters.

1 Introduction

When solving energy and water management problems in Uzbekistan, one of the main tasks is creating economic and reliable structures of mountain hydro-technical structures, taking into account the fact that the construction site presents a zone of high seismicity. The design of hydro-technical structures subject to potential earthquakes significantly depends on their dynamic characteristics and the vibration processes over time. Therefore, the need arises to proceed to the dynamic theory of earthquake resistance.

The intensity of structures vibrations under dynamic influences substantially depends on the degree of energy dissipation in them. It can be expected that the higher the energy dissipation in the structure, the less intense the resonant vibrations at a given level of excitation.

Theoretical description of the processes of strain during vibrations of rigid bodies and structures, taking into account internal friction, is often limited to studying the general laws of the external manifestation of the dissipation mechanism. Hypotheses and linear models of frequency-independent internal friction [1, 2] are widely used to solve the dynamics of structures. These hypotheses, reflecting the manifestation of elastic imperfections in the materials, do not describe the creep of strains and relaxation of stresses, called “hereditary properties”.

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The foundations of the modern hereditary theory of viscoelasticity, which reflects almost all the features of the quasistatic and dynamic behavior of the material, are found in classical works of Boltzmann and Volterra. The hereditary theory of viscoelasticity is more general, and it describes more accurately the mechanism of dissipation in the materials [3 - 7].

It is well known that the rheological properties of the medium significantly affect the strain process as a whole, i.e. all three of its stages: elastic, plastic and the stage of destruction. Therefore, the problems of the theory of viscoelasticity have attracted the special attention of researchers in recent years [8 - 32].

This paper is devoted to mathematical modeling and creating an algorithm for the numerical solution of dynamic problems of hereditary-deformable systems.

1.1 Statement of the problem and nonlinear equations of motion

Consider the problem of vibrations of a dam-plate made of a homogeneous viscoelastic isotropic material. The inertia forces acting on the dam-plate are generated from the motion and dam deformation, and the hydrodynamic pressure of water arises from the dam motion as a rigid body and the dam strain.

A mathematical model of the problem, relative to the transverse deflection $w_1(x, y, t)$, under certain assumptions, taking into account viscoelastic properties of the dam-plate material, is reduced to solving equations of the form

$$D(1 - R^*)V^4[w_1(x, y, t)] + \rho_1 h \frac{\partial^2(w_1 + w_0)}{\partial t^2} - \rho \frac{\partial \varphi_1}{\partial t} = 0,$$

where $w_1(x, y, t)$ is the deflection of the dam-plate; $h$ is the thickness of the dam-plate; $\rho_1$ is the density of the dam material; $\rho$ is the density of water; $\varphi_1(x, y, z, t)$ is the function of the potential of fluid flow rate arising from dam-plate strain; $\varphi_0(x, y, t)$ is the function of the potential fluid flow rate, arising from the dam motion as a rigid body; $w_0(t)$ is the law of base motion during an earthquake:

$$w_0(t) = a_0 e^{-\varepsilon_0 t} \sin \omega_0 t;$$

where $a_0$ is the initial maximum amplitude; $\varepsilon_0$ is the soil attenuation coefficient; $\omega_0$ is frequency of soil vibrations; $t$ is time. All these values are determined from the analysis of the earthquake seismogram of corresponding intensity.

2 Methods

Solution of integro-differential equations (1), satisfying the boundary conditions of the problem, is given in the form
\[ w_1(y,z,t) = \sum_{k=1}^{\infty} C_k(t)w_k(y,z), \]  

where \( C_k = C_k(t) \) is the sought for time functions; coordinate functions \( w_k(y,z) \) satisfy the boundary conditions for fixing the dam-plate edges.

Consider a plate with the following boundary conditions:

1. Edges \( z = \pm \alpha \) are freely supported

\[ w_1 = 0, \frac{\partial w_1}{\partial \alpha} + \mu \frac{\partial^2 w_1}{\partial \zeta^2} = 0 \]  

2. Edge \( y = 0 \) is rigidly fixed,

\[ w_1 = 0, \frac{\partial w_1}{\partial \alpha} = 0 \]  

3. Edge \( y = b \) is free.

\[ \frac{\partial w_1}{\partial \gamma^2} + \mu \frac{\partial w_1}{\partial \zeta^2} = 0, \frac{\partial^3 w_1}{\partial \gamma^3} + (2-\mu)\frac{\partial^3 w_1}{\partial \zeta \partial \gamma^2} \]

Functions \( w_k(y,z) \) satisfying these boundary conditions of fixation are taken in the form

\[ w_k(y,z) = V_k(y)H_k(z), \]

where

\[ H_k(z) = \cos \frac{k\pi z}{2a}; \]

\[ V_k(y) = V_{1k}(y) + E_k V_{2k}(y). \]

Here

\[ E_k = \frac{1}{V_{2k}^*(b)} \left[ V_{1k}^*(b) - \left( \frac{k\pi}{2a} \right)^2 V_{1k}(b) \mu \right], \]

\[ V_{1k}(y) = \frac{1}{k} \sin \frac{k\pi y}{2b} - \frac{1}{k+2} \sin \frac{(k+2)\pi y}{2b}; V_{2k}(y) = \cos \frac{k\pi y}{2b} - \cos \frac{(k+2)\pi y}{b}, k = 1,3,5,... \]

So, the expression for function \( W_k(y,z) \), is determined and has the form.
\( w_k(y,z) = \cos \frac{k \pi y}{2a} \int \frac{1}{R} \sin \frac{k \pi y}{2b} - \frac{1}{k + 2} \sin \frac{(k + 2) \pi y}{2b} + E_k \left( \cos \frac{k \pi y}{b} - \cos \frac{(k + 2) \pi y}{b} \right) \).  \tag{5} 

Substituting (5) and (3) into equation (1) and applying the Bubnov-Galerkin procedure to determine the unknowns \( C_k = C_k(t), k = 1,3,5,... \), the following system of integro-differential equations is obtained:

\[
\sum_{k=1,3,...}^{\infty} \left[ L_{mk} \ddot{C}_k(t) + \omega^2 (1 - R^2) M_{mk} C_k(t) \right] + a_0 \omega^2 N_m(t) = 0 . \tag{6} 
\]

\[ C_k(0) = C_{0k}, \quad \dot{C}_k(0) = \dot{C}_{0k}, \quad k,m = 1,3,5,... \]

Here

\[
L_{mk} = \frac{1}{ab} \left\{ \int_{0-a}^{b-a} \int_{-a}^{a} w_k(y,z) w_m(y,z) dy dz + \frac{2}{k \pi \rho} \frac{1}{h} \times \int_{0-a}^{b-a} \int_{-a}^{a} (z-a) w_k(y,z) \cos \gamma_k y dy dz \right\} 
\]

\[
M_{mk} = \frac{b^3}{a \pi^4} \int_{0-a}^{b-a} \int \nabla^4 w_k(y,z) w_m(y,z) dy dz ; 
\]

\[
N_m(t) = \frac{1}{aba_0 \omega^2} \int_{0-a}^{b-a} \int \dot{\Phi}_0(t) - \frac{\rho}{\rho_1} \frac{1}{h} \left[ \frac{\partial \Phi_0}{\partial t} + \frac{1}{2} \left( \frac{\partial \Phi_0}{\partial x} \right)^2 \right] + \frac{1}{2} \left( \frac{\partial \Phi_0}{\partial y} \right)^2 \bigg|_{x=w_0(t)} \right\} w_m(y,z) dy dz . 
\]

where \( \omega \) is the frequency of the basic mode of vibration: \( \omega = \sqrt{\frac{D}{\rho_1 h}} \left( \frac{\pi}{b} \right)^4 \)

In calculations, the Koltunov–Rzhanitsyn kernel is used:

\[ R(t) = A t^{\alpha-1} \exp(-\beta t), \quad A, \beta > 0, \quad 0 < \alpha < 1. \]

Integration of the system of equations (6), obtained based on numerous approximations of deflections, was performed using a numerical method based on the use of quadrature formulas [4, 6, 7, 21].
3 Results and discussion

In figure 1 the graphs of curves $w\left(\frac{1}{2}, \frac{1}{2}, t\right)$ for various values of rheological parameter $A$ are presented. An analysis of results shows that in the initial point in time, the solutions of elastic and viscoelastic problems differ little from each other. Over time, the vibrations at $A = 0$ occur closer to the harmonic law, and with increasing $A$, the amplitude and frequency of vibrations decrease significantly.

The effect of the hydrodynamic pressure of water on the dam behavior was investigated.

Figure 2 shows the graphs of curves $w\left(\frac{1}{2}, \frac{1}{2}, t\right)$ for various values of the parameter $\rho/\rho_1$. The results obtained here show that in the initial point in time, the curves almost coincide, and over time they differ significantly from each other. An analysis shows that with an increase in the parameter value $\rho/\rho_1$, the amplitude of dam-plate vibrations decreases. So, an account for hydrodynamic pressure of water leads to a decrease in the vibration amplitude, and the frequency of vibrations does not change significantly.

Figure 3 shows the graphs of curves $w\left(\frac{1}{2}, \frac{1}{2}, t\right)$ for various values of the parameter $\lambda$. With increasing values of $\lambda$, the amplitude of vibrations decreases and a phase shift to the right is observed.

Figure 4 shows the graphs of curves $w\left(\frac{1}{2}, \frac{1}{2}, t\right)$ for various values of the rheological parameter $\alpha$. An analysis of results shows that an increase in the value of this parameter leads to an increase in the amplitude and frequency of vibrations.

Figure 5 shows the graphs of curves $w\left(\frac{1}{2}, \frac{1}{2}, t\right)$ for various values of the rheological parameter $\beta$. An analysis of results shows that taking parameter $\beta$ into account does not significantly affect the amplitude and frequency of vibrations of a dam-plate.

When calculating the deflection value by formula (1), the first five harmonics were held ($N=5$). The calculations showed that a further increase in term number does not significantly affect the amplitude of dam-plate vibrations (figures 6 and 7).
Fig 1. $\alpha = 0.25; \beta = 0.05; \lambda = 1; \mu = 0.3; \rho / \rho_1 = 1/2.4; A = 0(1); 0.05(2); 0.1(3)$.

Fig 2. $A = 0.05; \alpha = 0.25; \beta = 0.05; \lambda = 1; \mu = 0.3; \rho / \rho_1 = 0(1); 1/5(2); 1/2.4(3)$. 
Fig 3. $A = 0.05; \alpha = 0.25; \beta = 0.05; \mu = 0.3; \rho / \rho_i = 1/2.4; \lambda = 1(1); 1/5(2); 2(3)$.

Fig 4. $A = 0.05; \beta = 0.05; \lambda = 1; \mu = 0.3; \rho / \rho_i = 1/2.4; \alpha = 0.25(1); 0.5(2); 0.75(3)$.

Fig 5. $A = 0.05; \alpha = 0.25; \lambda = 1; \mu = 0.3; \rho / \rho_i = 1/2.4; \beta = 0.05(1); 0.075(2); 0.1(3)$.
Fig 6. $A = 0.05; \alpha = 0.25; \lambda = 1; \mu = 0.3; \rho / \rho_1 = 1/2.4; \beta = 0.05; N = 1(1); 3(2); 5(3)$.

Fig 7. $A = 0.05; \alpha = 0.25; \lambda = 1; \mu = 0.3; \rho / \rho_1 = 1/2.4; \beta = 0.05; N = 5(1); 7(2)$.

4 Conclusions

Mathematical models of the dynamics problems of a dam-plate of constant and variable thickness are constructed taking into account:
- viscoelastic properties of the material;
- inertia forces arising from the dam-plate motion as a rigid body and from its deformation.

Based on the Bubnov-Galerkin method in combination with a numerical method based on the use of quadrature formulas:
- the methods have been developed for solving interconnected integro-differential equations of Volterra type;
- a computational algorithm has been developed that allows studying the vibration problems of a dam-plate of constant thickness, taking into account the viscoelastic properties of the material.

It is established that an account for viscoelastic properties of the dam-plate material leads to a decrease in the amplitudes and frequencies of vibrations.
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References

1. Panovko Ya.G. Internal friction under vibrations of elastic systems (M.: Fizmatgiz). (1960)
2. Sorokin E S, To the theory of internal resistance under vibrations of elastic systems (M.: Gosstroyizdat) p.131. (1960).
3. Arutyunyan N K and Kolmanovsky V B, The creep theory of inhomogeneous hereditary aging media (M.: Nauka). (1983).
4. Badalov F B, Methods for Solving Integral and Integro-differential Equations of the Hereditary Theory of Viscoelasticity (Tashkent: Mekhnat). (1987).
5. Pobedrya B E, The mechanics of composite materials (M: Moscow State University Publishing House) p.336. (1984).
6. Badalov F B, Khudayarov B A and Abdukarimov A, Effect of the hereditary kernel on the solution of linear and nonlinear dynamic problems of hereditary deformable systems, Journal of Machinery Manufacture and Reliability 36, pp 328-335, (2007)
7. Badalov F B, Eshmatov Kh and Yusupov M, Some Methods of Solution of the Systems of Integro-differential Equations in Problems of Viscoelasticity, Applied Mathematics and Mechanics 51 (5) pp 867-871 (1987)
8. Khudayarov B A, Komilova K M and Turaev F Z Numerical study of the effect of viscoelastic properties of the material and bases on vibration fatigue of pipelines conveying pulsating fluid flow. Engineering Failure Analysis, 115, 104635. (2020).
9. Khudayarov B A, Turaev F, Zhuvonov Q, Vahabov V, Kucharov O and Kholturaev Kh, Oscillation modeling of viscoelastic elements of thin-walled structures. IOP Conference Series: Materials Science and Engineering. 883 (1), 012188, (2020).
10. Turaev F, Khudayarov B, Kucharov O, Rakhatmullaev A, Zhuvonov K and Gulomov O, Dynamic stability of thin-walled structure elements considering hereditary and inhomogeneous properties of the material. IOP Conference Series: Materials Science and Engineering. 883 (1), 012187. (2020).
11. Khudayarov B, Turaev F, Vakhobov V, Gulamov O and Shodiyyev S Dynamic stability and vibrations of thin-walled structures considering heredity properties of the material. IOP Conference Series: Materials Science and Engineering. 869 (5),052021. (2020)
12. Khudayarov B A, Komilova Kh M and Turaev F Zh 2020 Dynamic analysis of the suspended composite pipelines conveying pulsating fluid, Journal of Natural Gas Science and Engineering 75 103148. https://doi.org/10.1016/j.jngse.2020.103148. (2020)
13. Khudayarov B A, Komilova Kh M, Turaev F Zh and Aliyarov J A, Numerical simulation of vibration of composite pipelines conveying fluids with account for lumped masses, International Journal of Pressure Vessels and Piping 179 104034. https://doi.org/10.1016/j.ijpvp.2019.104034. (2020)
14. Khudayarov B A, Ruzmetov K Sh, Turaev F Zh, Vaxobov V V, Hidoyatova M A, Mirzaev S S and Abdikarimov R, Numerical modeling of nonlinear vibrations of viscoelastic shallow shells Engineering Solid Mechanics 2020 8 (3) pp 199-204. (2020)
15. Khudayarov B A, Komilova Kh M and Turaev F Z, Numerical Simulation of Vibration of Composite Pipelines Conveying Pulsating Fluid, International Journal of Applied Mechanics 2019 11 (9) 1950090. https://doi.org/10.1142/S175882511950090X. (2019)
16. Khudayarov B A and Komilova Kh M, *Vibration and dynamic stability of composite pipelines conveying a two-phase fluid flows*, Engineering Failure Analysis 2019 104 500-512. https://doi.org/10.1016/j.engfailanal.2019.06.025. (2019)

17. Khudayarov B A, Komilova Kh M and Turaev F Zh *The effect of two-parameter Pasternak foundations on the oscillations of composite pipelines conveying gas-containing fluids*, International Journal of Pressure Vessels and Piping 176 103946. https://doi.org/10.1016/j.ijpvp.2019.103946 (2020)

18. Khudayarov B A and Komilova Kh M, *Numerical modeling of vibrations of viscoelastic pipelines conveying two-phase slug flow*, Vestnik Tomskogo Gosudarstvennogo Universiteta. Matematika i Mekhanika 2019 61 pp 95-110. DOI: https://doi.org/10.17223/19988621/61/9. (2019).

19. Khudayarov B, Turaev F and Kucharov O, *Computer simulation of oscillatory processes of viscoelastic elements of thin-walled structures in a gas flow*, E3S Web of Conferences 2019 97 06008. https://doi.org/10.1051/e3sconf/20199706008. (2019).

20. Khudayarov B A, *Modeling of supersonic nonlinear flutter of plates on a visco-elastic foundation*. Advances in aircraft and spacecraft science 6 (3) pp 257-272. https://doi.org/10.12989/aas.2019.6.3.257. (2019).

21. Khudayarov B A and Turaev F Zh, *Mathematical Simulation of Nonlinear Oscillations of Viscoelastic Pipelines Conveying Fluid*, Applied Mathematical Modelling 66 pp 662-679. https://doi.org/10.1016/j.apm.2018.10.008, (2019).

22. Khudayarov B A and Komilova K M, Numerical simulation of vibrations of viscoelastic pipelines conveying two-phase medium in a slug flow regime. Bulletin of Tomsk State University, Mathematics and Mekanika. (61), pp. 95-110. (2019).

23. Khudayarov B A and Turaev F Zh *Nonlinear vibrations of fluid transporting pipelines on a viscoelastic foundation*, Magazine of Civil Engineering 86 (2). pp. 30-45. DOI: 10.18720/MCE.86.4. (2019).

24. Khudayarov B A and Turaev F Zh, *Nonlinear supersonic flutter for the viscoelastic orthotropic cylindrical shells in supersonic flow*, Aerospace Science and Technology, 84 pp 120-130. doi: 10.1016/j.ast.2018.08.044. (2019)

25. Khudayarov B A, *Flutter of a viscoelastic plate in a supersonic gas flow*. International Applied Mechanics. 46 (4), pp 455-460. (2010).

26. Khudayarov B A and Bandurin N G, *Numerical investigation of nonlinear vibrations of viscoelastic plates and cylindrical panels in a gas flow*. Journal of Applied Mechanics and Technical Physics. 48 (2), pp. 279-284. (2007)

27. Khudayarov B A *Flutter analysis of viscoelastic sandwich plate in supersonic flow*. American Society of Mechanical Engineers, Applied Mechanics Division, AMD 256, pp. 11-17. (2005).

28. Khudayarov B A *Numerical analysis of the nonlinear flutter of viscoelastic plates*. International Applied Mechanics. 41 (5), pp. 538-542. (2005).

29. Khudayarov B A, *Behavior of viscoelastic three-layered structures in a gas flow*. Problems of machine building and reliability of machines. (6), pp. 87-90. (2004).

30. Filippov, I. G. and Kudainazarov, K. *Refinement of equations describing longitudinal-radial vibrations of a circular cylindrical viscoelastic shell*. Soviet Applied Mechanics, 26 (2), 161–168. doi:10.1007/bf00887110. (1990).

31. Filippov, I. G. &Kudainazarov, K. *General transverse vibrations equations for a circular cylindrical viscoelastic shell*. Soviet Applied Mechanics, 26 (4), pp 351–357. doi:10.1007/bf00887127. (1990).
32. Filippov, I. G. and Kudainazarov, K. Boundary-value problems of longitudinal vibrations of circular cylindrical shells. International Applied Mechanics, 34(12), 1204–1210. doi:10.1007/bf02700874. (1998).

33. Abdullayev, A.A., Ergashev, T.G. Poincare-tricomi problem for the equation of a mixed elliptico-hyperbolic type of second kind. Vestnik Tomskogo Gosudarstvennogo Universiteta, Matematika i Mekhanika, (65), pp. 5–21, (2020).

34. DOI 10.17223/19988621/65/1

35. Islomov B. I. Abdullayev A.A. On a problem for an elliptic type equation of the second kind with a conormal and integral condition. Nanosystems: Physics, Chemistry, Mathematics, 9 (3), pp. 307-318, DOI 10.17586/22208054201893307318. (2018)

36. T. K. Yuldashev, B. I. Islomov, A. A. Abdullaev. On solvability of a Poincare-Tricomi Type Problem for an Elliptic-hyperbolic Equation of the Second Kind. Lobachevskii Journal of Mathematics, 42, (3), pp. 662–674. (2021)

37. DOI: 10.1134/S1995080221030239