Theoretical and Experimental Study for Queueing System with Walking Distance

Daichi Yanagisawa$^{1,2}$, Yushi Suma$^1$, Akiyasu Tomoeda$^{3,4}$, Ayako Kimura$^5$, Kazumichi Ohtsuka$^4$, and Katsuhiro Nishinari$^{4,6}$

$^1$Department of Aeronautics and Astronautics, School of Engineering, The University of Tokyo.

$^2$Research Fellow of the Japan Society for the Promotion of Science.

$^3$Meiji Institute for Advanced Study of Mathematical Sciences, Meiji University.

$^4$Research Center for Advanced Science and Technology, The University of Tokyo.

$^5$Mitsubishi Research Institute, Inc.

$^6$PRESTO, Japan Science and Technology Agency.

Japan

1. Introduction

Queueing theory has been considerably studied since Erlang started designing telephone exchange system in 1909 Erlang (1909), and many important theories have been developed Burke (1956); Jackson (1957); Kendall (1953); Little (1961). Nowadays, it is applied to study many social systems such as the internet Kasahara (2002); Mukherjee & Manna (2005), a resource management system Barabasi (2005), a vehicular traffic system Helbing et al. (2006) and a pedestrian traffic system D. Helbing & Treiber (2005).

Traffic flow and pedestrian dynamics have been studied actively by using the theory of particle systems, fluid dynamics and cellular automaton Chowdhury et al. (2000); Helbing (2001). Especially, we simulate the dynamics of cars and pedestrians efficiently by using cellular automaton models since its time and space are both discrete. Pedestrians’ movement is treated as a stochastic process in the floor field model Nishinari et al. (2004), which is a cellular automaton model for pedestrian dynamics. Then we have succeeded in calculating the total evacuation time analytically by using the floor field model, and studied the way of smooth evacuation Yanagisawa et al. (2009); Yanagisawa & Nishinari (2007).

Calculating the mean waiting time in the queueing system, whose bottleneck is a service window is similar to calculating the total evacuation time from a room, whose bottleneck is an exit. However, the former does not take into account of the effect of spatial structures, while the latter does.

According to the queueing theory, the waiting time of a fork-type queueing system is shorter than that of a parallel-type queueing system (Parallel) (Fig. 1 (a)). However, fork-type queueing system considered in the normal queueing theory (N-Fork) (Fig. 1 (b)) does not reflect the effect of the walking distances from the head of the queue to the service windows. The effect of the distances cannot be ignored in a system such as a large immigration inspection floor in an international airport since walking distances become very long.
Therefore, we have combined the queueing theory and the floor field model, and introduced the effect of delay in walking from the head of the queue to the service windows in the queueing theory. Suitable type of queueing system under various conditions is obtained by analyzing the new queueing theory. When there are plural service windows, the queueing theory indicates that a fork-type queueing system, which collects people into a single queue, is more efficient than a parallel-type queueing system, i.e., queues for each service windows. However, in our walking-distance introduced queueing theory, we find that the parallel-type queueing system is more efficient when sufficiently many people are waiting in the queues, and service time is shorter than walking time. Since the model is studied not only by simulation, but also by theoretical analysis, we have succeeded to obtain the diagram, which indicates the suitable type of queueing system according to the designing conditions.

In the fork-type queueing system, people at the head of the queue starts to move only when one of the service windows become vacant. Since this walking time reduces the efficiency, we consider two new methods for improving the efficiency of the queueing systems. First, we set the head of the queue at the center of the system to decrease the effect of walking distance. This transformation of the system decreases the walking distance to the service windows for pedestrians. Second, we have proposed to keep one person waiting at each service window when it is occupied by other person. Then the waiting person can instantaneously receive service at the window after the former person leave there.

In addition to the theoretical analysis, the experiments of the queueing system for people have been performed. We have verified that the mean waiting time of the parallel-type queueing system become smaller than that of fork-type in the congested situation, experimentally. When we have kept one person waiting at each window, the mean waiting time has dra-
matically decreased, i.e., the efficiency has improved, as we have expected from theoretical calculation.
Finally, we consider the situation where there are two kinds of people, whose service time is short and long. The analytical result says that we can decrease people's waiting time and their stress by setting up queues for each kind of people separately.

2. Walking-Distance introduced Queueing Theory

2.1 Distance introduced Fork-type Queueing System: D-Fork
In a parallel-type queueing system (Parallel) (Fig. 1 (a)), people wait just behind the former person, so that there is no delay in walking. While, in a fork-type queueing system, people take some time to walk from the head of the queue to the service windows. However, the walking time is not taken into account in a fork-type queueing system in the normal queueing theory (N-Fork) (Fig. 1 (b)), and people move from the head of the queue to the service windows instantaneously. Therefore, we consider D-Fork as in Fig. 2 by representing the walking distance using cellular automaton. The gray cells are window cells, and the numbers described in there are window numbers. The white cells are passage cells. Note that the letter “C” described in the passage cells represents the common passage cells. For example, both persons who are going to the window 3 and 4 pass the common passage cells. People sometimes cannot go forward in the common passage cells since there is a possibility that other people stand in front of them. The place that people are waiting, which is not divided into cells, is a queue. \( s \in \mathbb{N}, \lambda \in [0, \infty), \) and \( \mu \in [0, \infty) \) represent the number of service windows, the arrival rate, and the service rate, respectively. \( a \) and \( b \) represent the length of the passage, and \( k \) is the interval length between two service windows. The distance from the head of the queue to the service window \( n \in [1, s] \) is described as \( d_n = a + b + k(n - 1) \). Fig. 2 represents the case \( s = 4, a = 2, b = 2, \) and \( k = 2 \). Service windows have two states: vacant and occupied. When a person at the head of the queue decides to move to the vacant service window \( n \), it changes into occupied state. The person proceed to the service window by one cell with the rate \( p \in [0, \infty) \) as the asymmetric simple exclusion process. A service starts when the person arrives at the service window, and after it finishes the state of the service window changes into vacant state.

2.2 Update Rules
The simulation of walking-distance introduced queueing systems consists of the following five steps per unit time step.

1. If there is at least one vacant service window and one person in the queue, and the first cell of the passage is vacant, then the person decide to proceed to a vacant service window which is the nearest to the head of the queue, and the state of the service window become occupied.

2. Add one person to the queue with the probability \( \lambda \Delta t \), where \( \Delta t \) is the length of the unit time step.

3. Proceed each person in the passage cells to his/her service windows with the probability \( p \Delta t \) if there is not other person at their proceeding cell.

4. Remove people at the service windows and change their states into vacant state with the probability \( \mu \Delta t \).

5. If 1. takes into practice, proceed the person at the head of the queue to the first passage cell with the probability \( p \Delta t \).
2.3 Stationary Equations
We define the sum of the walking time and the service time at service window $n$ as a throughput time $\tau_n$ and its reciprocal as a throughput rate $\mu_n$. Here, we calculate the mean throughput rate $\bar{\mu}_n$ when $n$ service windows are occupied, and obtain stationary equations of D-Fork. We suppose that all passage cells are vacant by mean field approximation. Then, the mean value of the throughput time $E(\tau_n)$ is described as follows.

$$E(\tau_n) = \frac{1}{\mu} + \frac{a + b + k(n - 1)}{p}. \quad (1)$$

The throughput rate $\mu_n$ is obtained as

$$\mu_n = \frac{1}{E(\tau_n)} = \frac{1}{\frac{1}{\mu} + \frac{a + b}{p} + \frac{k(n - 1)}{p}} = \frac{\mu}{1 + \alpha + 2\beta(n - 1)}, \quad (2)$$

where

$$\alpha = \frac{\mu(a + b)}{p}, \quad \beta = \frac{k\mu}{2p}. \quad (3)$$

In the case $2\beta(n - 1)/(1 + \alpha) \ll 1$, we calculate the mean throughput rate $\bar{\mu}_n$ as

$$\bar{\mu}_n = \frac{1}{n} \sum_{i=1}^{n} \mu_i \approx \frac{\mu}{1 + \alpha + \beta(n - 1)}. \quad (4)$$
By using (4) the stationary equations are described as follows:

\[
\begin{align*}
\lambda P_0 &= \tilde{\mu}_1 P_1 \\
\lambda P_{n-1} + (n + 1)\tilde{\mu}_{n+1} P_{n+1} &= (\lambda + n\tilde{\mu}_n)P_n \quad (1 \leq n \leq s - 1) \\
\lambda P_{n-1} + s\tilde{\mu}_s P_{n+1} &= (\lambda + s\tilde{\mu}_s)P_n \quad (n \geq s).
\end{align*}
\]

(5)

We obtain the mean waiting time \( W_q \) by solving (5) analytically. In the case \( \alpha = \beta = 0 \), we have the stationary equations of \( M/M/s \) Bolch et al. (1998) from (5), thus \( \alpha \) and \( \beta \) represent the effect of walking time.

In our simulation the distribution of the throughput time is gamma distribution. We approximate it as exponential distribution in this calculation, however, when \( \beta \) is small the results from the exponential distribution approximated well to those from gamma distribution.

3. Comparison of a Parallel Queue and a Fork Queue

We compare the mean waiting time \( W_q \) of Parallel (Fig. 1 (a)), N-Fork (Fig. 1 (b)), and D-Fork (Fig. 2 (a)). Figure 3 (a) show \( W_q \) against the utilization \( \rho(= \lambda/(s\mu)) \). The results of analysis agree with those of the simulation very well. We see that \( W_q \) of N-Fork is smaller than that of Parallel and D-Fork in \( 0 \leq \rho < 1 \). There is a possibility that more than one person is waiting in one queue and no one is in the other queue in Parallel (\( s \geq 2 \)), however there is no vacant service window in N-Fork when people are waiting in the system. This is the reason why \( W_q \) of N-Fork is always smaller than that of Parallel. Since N-Fork does not take into account of the effect of the walking distances, i.e. \( \beta = 0 \), it is obvious that \( W_q \) of N-Fork is smaller than that of D-Fork. The N-Fork is the most efficient of the three; however, it is an ideal system and does not exist in reality. By focusing on the curves of Parallel and D-Fork, we can clearly observe the crossing of them. This means that when the utilization \( \rho \) is small, i.e., there are not sufficiently many people in the system; we should form D-Fork to decrease the waiting time. On the contrary, when the utilization \( \rho \) is large, i.e., there are many people in the system, we should form Parallel. When \( \beta \) become large, the crossing point move to...
Fig. 4. Schematic views of walking-distance introduced queueing systems \((s = 4)\). (a) D-Fork-Center. The head of the queue is at the center of the system. (b) D-Fork-Wait. People can wait at the cells, which are described as “W”.

the left. The strong effect of the walking distances extend the suitable \(\rho\) region for Parallel. This agrees with our intuition, since D-Fork is influenced by the distances but Parallel does not. The reversal phenomenon of \(W_q\) is obtained for the first time by introducing the effect of distance.

Figure 3 (b) shows the type of queueing system, which minimize \(W_q\) against \(\rho\) and \(\beta\) in the case \(s = 4\). This figure is useful for designing queueing systems. The curves divide the \(\rho - \beta\) plane into three regions. In the lower left region \(W_q\) of D-Fork \((s = 4)\) is the smallest, and in the upper right region \(W_q\) of Parallel is the smallest. Surprisingly, \(W_q\) of D-Fork \((s = 2) \times 2\) (Fig. 2 (b)) is the smallest in the middle region. This indicates that the choice of the type of queueing systems is not only Parallel and D-Fork, but also a combination of them. According to (3), \(\beta\) represents the ratio of walking time and service time. Therefore, D-Fork is suitable when service time is much longer than walking time. The value of \(\beta\) is small in most D-Fork in reality, however, in large queueing system such as an immigration inspection floor in the international airport, we should divide the large D-Fork into the several small D-Forks to decrease the effect of the walking distances.

4. Methods for Shortening Waiting Time in D-Fork

4.1 Set the Head of the Queue at the Center: D-Fork-Center

The head of the queue is usually set at the end of the system since we can efficiently use the space for the queue. However, people have to walk a long distance to the farthest window as in Fig. 2. Thus, we propose to set the head of the queue at the center (D-Fork-Center) as in Fig. 4 (a). Then, the mean throughput rate is described as follows:

\[
\hat{\mu}_n \approx \frac{\mu}{1 + \alpha + \frac{\beta}{2} \left( n + \frac{1 + (-1)^{n+1}}{2n} \right)} \tag{6}
\]
Comparing (4) and (6), we see that the coefficient of \( n \) in (6) is half of that in (4). Therefore, the effect of the walking distance approximately becomes half if we set the head of the queue at the center. The mean waiting time is approximately calculated by replacing \( \beta \) of D-Fork with \( \beta/2 \) in Sec. 4.3. Note, that the (6) represents the \( \hat{\mu}_n \) in the case that both \( s \) and \( k \) are even number. The mathematical formulation of the other cases are described in Ref. Yanagisawa et al. (2008).

4.2 Keep One Person Waiting at the Window: D-Fork-Wait

The walking distance in D-Fork is essentially problematic, since it delays the start of services, i.e., people have to walk the passage before they start to receive the service. Thus, we propose to keep one person waiting at the service window. We call a queueing system which this method is applied to as D-Fork-Wait (Fig. 4 (b)). Since people are waiting just next to the service windows, they can receive service instantaneously when their former people leave there. The delay in walking is almost removed by this method, i.e., the effect of walking distance does not need to be considered. Therefore, the mean waiting time is approximately calculated by the expression for N-Fork.

4.3 Approximated Calculation and Simulation

Figure 5 show the mean waiting time \( W_q \) in D-Fork, D-Fork-Center, and D-Fork-Wait. We see that \( W_q \) in D-Fork-Center is always smaller than that in D-Fork in \( 0 \leq \rho < 1 \). This result verifies that we can decrease \( W_q \) by setting the head of the queue at the center. We also find that \( W_q \) in D-Fork-Wait is the smallest of the three since it almost completely removes the effect of walking time by keep one person waiting at the window. The well correspondence between the results of the theoretical analysis and simulation in \( \rho \leq 0.85 \) also verifies our assumption that D-Fork-Wait becomes close to N-Fork. However, in the large-\( \rho \) region, the result of the simulation becomes larger than that of the theoretical analysis. In D-Fork-Wait, there is a possibility that 2\( s \) people are in the window cells or passage cells, while \( s \) people is the maximum in the other cases. Thus, when \( \rho \) is large, people in the common passage cells cannot often proceed since other people are in front of them. This jam in the common
passage cells, which is not considered in the theoretical calculation, increase the waiting time in D-Fork-Wait when \( \rho \) is large.

5. Experiments

We have performed the experiments to examine the two results in the former section. 1. There is a case that \( W_q \) in Parallel becomes smaller than that in D-Fork. 2. We can decrease \( W_q \) by keeping one person waiting at the window. We made the queueing system as in Fig. 1 (a), 2, and 4 (b), whose parameters are \( s = 4, a = 1 [m], b = 0.5 [m], k = 3 [m], \lambda = 188/600 \) [persons/sec], and \( \mu = 1/8 \) [persons/sec]. Figure 6 (a) is the snap shot of the experiment. Participants of the experiments enter the system and line up in the queue when the staff says to do so. They proceed to the windows and receive service. After that they wait at the starting position until the staff let him/her enter the system again. We put 188 people in 600 [sec] in one experiment. Note, that the arrival was random while the service was deterministic for simplicity. According to the Pollaczek-Khintchine formula Bolch et al. (1998), \( W_q \) becomes small when the service is deterministic. Since this effect acts on the all kinds of the queueing systems in the same way, the results are not critically influenced by deterministic service. Therefore, we can examine the result of the theoretical analysis and simulations by these experiments. Figure 6 (b) shows the result of the experiments. We see that \( W_q \) in Parallel is smaller than that in D-Fork. This result verifies our theoretical analysis and simulation by using the walking-distance introduced queueing theory. The reversal of \( W_q \) between Parallel and D-Fork is observed experimentally for the first time in this paper. We also find that \( W_q \) becomes dramatically small in D-Fork-Wait. This new result indicates that the method “Keep one person waiting” is an effective way to shorten the waiting time empirically.

6. Simultaneous Arrival of Many People

In the former sections, we discuss the queueing systems, whose distribution of interarrival time is exponential distribution, i.e. random arrival. Here, we consider the situation that many people arrive at the same time, such as arrivals of people who alight from trains and airplanes.
Welcome

Both kind of people are

Long-People

Only

Short-People

Only

(a) (S-a)

Mix (s=4)

(b) (S-b)

Separate (s_S=1, s_L=3)

(c) (S-c)

Separate (s_S=1, s_L=3) → Long (s_L=4)

Fig. 7. Schematic views of queueing systems. (a) (S-a) Mix queueing system (s = 4). (b) (S-b) Separate queueing system (s_S = 1, s_L = 3). (c) (S-c) Separate queueing system (s_S = 1, s_L = 3) → Queueing system for LP (s_L = 4).

Fig. 8. (a) The probability distribution of the total throughput time $T_{total}$. (b) Number of people in the queueing system against time step $t$. The value of plot S-b and S-c decrease dramatically, when people start to be given services. This is because SPs get out from the system quickly by avoiding the disturbance by LPs.

The total throughput time $T_{total}$ that all people finish leaving the system is calculated and studied to decrease it. We consider the system, which has four service windows. There are two kinds of people whose service time are short (SP) and long (LP). The service rate of SP and the number of service windows only for SP are denoted as $\mu_S$ and $s_S$, respectively. Similarly, those of LP are described as $\mu_L$ and $s_L$. We have three strategies as follows:

(S-a) Mix (s = 4) (Fig. 7 (a))
Both SPs and LPs use the same windows.

(S-b) Separate (s_S = 1, s_L = 3) (Fig. 7 (b))
SPs use the window only for SP and LPs use the windows only for LP.

(S-c) Separate (s_S = 1, s_L = 3) → Long (s_L = 4) (Fig. 7 (c))
First same as (b), but after all SPs have left, the window, which was only for SP, is open for LP.
The distribution of the \( T_{total} \) when 50 SPs and 50 LPs arrive at the same time is described in Fig. 8 (a). We see that the mean of \( T_{total} \) of S-c is the smallest and that of S-b is the largest. When S-b or S-c is adopted, the queue of the SP is not affected by the distances. Thus SPs leave the system quickly. After all SPs have left, all four service windows are used efficiently in S-c, however, one window is not used in S-b. This makes S-c the best and S-b the worst.

We also discuss the stress of waiting people by Fig. 8 (b), which describes the number of people waiting in the queue against the time step \( t \). People suffer from a stress when they are waiting in the queue and do not when they leave the system. Therefore, an area surrounded by the axes and the curves represents the sum of all people’s stress until they left. Clearly, we observe that \((\text{Area of S-c}) < (\text{Area of S-b}) < (\text{Area of S-a})\). It is interesting that when we compare \( T_{total} \), S-a is better than S-b, however, comparing the stress, the result is opposite. We also find that S-c is the best strategy from the both point of views: \( T_{total} \) and the stress.

If the arrival of people is random, it is difficult to adopt S-c since we cannot find out when to change the type of the queueing system. However, if we have information about people’s arrival, we can decrease both waiting time and stress of people by adapting the type of the queueing system into proper one.

When we apply this study to the real system, the service rates are estimated by the data from the measurement.

7. Conclusion

We have introduced the effect of walking distance from the head of the queue to the service windows and shown that the performance of a parallel-type queueing system is better than that of a fork-type queueing system when there are sufficiently many people in the system. The effectiveness of the two new methods is also studied. The mean waiting time becomes small when we set the head of the queue at the center or keep one person waiting at each service window in a fork-type queueing system since the effect of walking distance decreases. It also turns out that dynamical change of a queueing system decrease both waiting time and stress of people when two kinds of people, whose service time are short and long, come into the system at the same time.

We would like to emphasize that our study is based on the theoretical analysis, simulations, and experiments. Therefore, the results in this paper are reliable enough to apply to the queueing systems in the real world. It is an important future work to study the effect of the costs raised by the two methods. When we set the head of the queue at the center, the space for waiting people decreases, and at least one clerk is needed at the head of the queue to keep one person waiting at each window.

Acknowledgement

We thank Dai Nippon Printing Co., Ltd. in Japan for the assistance of the experiment, which is described in Sec. 5. This work is financially supported by Japan Society for the Promotion of Science and Japan Science and Technology Agency.

8. References

Barabasi, A.-L. (2005). The origin of bursts and heavy tails in human dynamics, *Nature* **435**: 207–211.

Bolch, G., Greiner, S., de Meer, H. & Trivedi, K. (1998). *Queueing Networks and Markov Chains*, A Wiley-Interscience Publication, U.S.A.
Burke, P. J. (1956). The output of a queueing system, *Operations Research* **4**(6): 699–704.

Chowdhury, D., Santen, L., & Schadschneider, A. (2000). *Phys. Rep.* **329**: 199.

D. Helbing, R. J. & Treiber, M. (2005). Analytical investigation of oscillations in intersecting flows of pedestrian and vehicle traffic, *Phys. Rev. E* **72**: 046130.

Erlang, A. K. (1909). The theory of probabilities and telephone conversations, *Nyt. Tidsskr. Mat. Ser. B* **20**: 33–39.

Helbing, D. (2001). Traffic and related self-driven many-particle systems, *Rev. Mod. Phys.* **73**: 1067–1141.

Helbing, D., Treiber, M. & Kesting, A. (2006). Understanding interarrival and interdeparture time statistics from interactions in queueing systems, *Physica A* **363**: 62–72.

Jackson, J. K. (1957). Networks of waiting lines, *Operations Research* **5**(4): 518–521.

Kasahara, S. (2002). Towards queueing theory for the internet design, *TECHNICAL REPORT OF IEICE* **101**(649): 25–30. (in Japanese).

Kendall, D. G. (1953). Stochastic processes occurring in the theory of queues and their analysis by the method of the imbedded markov chain, *Ann. Math. Statist.* **24**(3): 338–354.

Little, J. C. D. (1961). A proof for the queueing formula \( l = \lambda w \), *Operations Research* **9**(3): 383–387.

Mukherjee, G. & Manna, S. S. (2005). Phase transition in a directed traffic flow network, *Phys. Rev. E* **71**: 066108.

Nishinari, K., Kirchner, A., Namazi, A., Schadschneider, A. & Nonmembers (2004). *IEICE Trans. Inf. Syst.* **E87-D**: 726.

Yanagisawa, D., Kimura, A., Tomoeda, A., Nishi, R., Suma, Y., Ohtsuka, K. & Nishinari, K. (2009). Introduction of frictional and turning function for pedestrian outflow with an obstacle, *Phys. Rev. E* **80**: 036110.

Yanagisawa, D. & Nishinari, K. (2007). Mean-field theory for pedestrian outflow through an exit, *Phys. Rev. E* **76**: 061117.

Yanagisawa, D., Tomoeda, A., Kimura, A. & Nishinari, K. (2008). Analysis on queueing systems by walking-distance introduced queueing theory, *Journal of JSIAM* **18**(4): 507–534. (in Japanese).
Nowadays robotics is one of the most dynamic fields of scientific researches. The shift of robotics researches from manufacturing to services applications is clear. During the last decades interest in studying climbing and walking robots has been increased. This increasing interest has been in many areas that most important ones of them are: mechanics, electronics, medical engineering, cybernetics, controls, and computers. Today’s climbing and walking robots are a combination of manipulative, perceptive, communicative, and cognitive abilities and they are capable of performing many tasks in industrial and non-industrial environments. Surveillance, planetary exploration, emergence rescue operations, reconnaissance, petrochemical applications, construction, entertainment, personal services, intervention in severe environments, transportation, medical and etc are some applications from a very diverse application fields of climbing and walking robots. By great progress in this area of robotics it is anticipated that next generation climbing and walking robots will enhance lives and will change the way the human works, thinks and makes decisions. This book presents the state of the art achievements, recent developments, applications and future challenges of climbing and walking robots. These are presented in 24 chapters by authors throughtot the world The book serves as a reference especially for the researchers who are interested in mobile robots. It also is useful for industrial engineers and graduate students in advanced study.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:

Daichi Yanagisawa, Yushi Suma, Akiyasu Tomoeda, Ayako Kimura, Kazumichi Ohtsuka and Katsuhiro Nishinari (2010). Theoretical and Experimental Study for Queueing System with Walking Distance, Climbing and Walking Robots, Behnam Miripour (Ed.), ISBN: 978-953-307-030-8, InTech, Available from: http://www.intechopen.com/books/climbing-and-walking-robots/theoretical-and-experimental-study-for-queueing-system-with-walking-distance
