Neutrino Physics

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Abstract
The theoretical and experimental bases of neutrino mass and mixing are reviewed. A brief chronological evolution of the weak interactions, the electroweak Standard Model, and neutrinos is presented. Dirac and Majorana mass terms are explained as well as models such as the seesaw mechanism. Schemes for two, three and four neutrino mixings are presented.

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11\textsuperscript{th} Mexican School of Particles and Fields
August 2004
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1 The Fermi Theory

The history of the weak interactions dates back to the discovery of radioactivity by Becquerel in 1896 [1]. In particular, $\beta$ decay, in which a nucleus emits an electron and increases its charge, apparently violated the conservation of energy (as well as momentum and, as we now understand, angular momentum). In 1931 Pauli postulated that a massless, chargeless, essentially non-interacting particle that he named the “neutron” (later renamed the neutrino by Fermi) was also emitted in the process and carried off the missing energy. Pauli’s hypothesis was verified around 1953 when the electron-type neutrino (actually the anti-neutrino $\bar{\nu}_e$) produced in a reactor was observed directly by its rescattering by Reines and Cowen. The second (muon-type) neutrino, $\nu_\mu$, associated with the $\mu$ in its interactions, was detected by its rescattering to produce a muon in 1962 by Lederman, Schwartz, and Steinberger at Brookhaven. The third charged lepton, the $\tau$, was discovered at SLAC in 1975. There was ample indirect evidence from the weak interactions of the $\tau$ that an associated neutrino, $\nu_\tau$, must exist, but it was not observed directly until 2000 at Fermilab.

In 1934, Enrico Fermi developed a theory of $\beta$ decay. The Fermi theory is loosely like QED, but of zero range (non-renormalizable) and non-diagonal (charged currents). The Hamiltonian (see Figure 1) is

$$H \sim \frac{G_F}{\sqrt{2}} J^\dagger_\mu J^\mu,$$

where $J^\mu$ is the charged current,

$$J^\dagger_\mu \sim \bar{p} \gamma_\mu n + \bar{\nu}_e \gamma_\mu e^- \quad [n \rightarrow p, \quad e^- \rightarrow \nu_e],$$

and $G_F$ is the Fermi constant, with the modern value $G_F = 1.16637(1) \times 10^{-5}$ GeV$^{-2}$. The Fermi theory was later rewritten in terms of quarks, and modified to include: $\mu^-$ and $\tau^-$ decays; strangeness changing transitions (Cabibbo); the heavy quarks $c$, $b$, and $t$; quark mixing and CP violation (the Cabibbo-Kobayashi-Maskawa matrix); vector ($V$) and axial vector ($A$) currents (i.e, $V \sim \gamma_\mu$ is replaced by $V - A \sim \gamma_\mu (1 - \gamma^5)$), which implies parity violation (Lee-Yang; Wu; Feynman-Gell-Mann); and $\nu$ mass and mixing.
The Fermi theory gives an excellent description (at tree-level) of such processes as:

- Nuclear/neutron $\beta^-$-decay: $n \rightarrow pe^-\bar{\nu}_e$.
- $\mu^-$, $\tau^-$ decays: $\mu^- \rightarrow e^-\bar{\nu}_e\nu_\mu$; $\tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau$, $\nu_\tau\bar{\nu}_e$, $\cdots$.
- $\pi$, $K$ decays: $\pi^+ \rightarrow \mu^+\nu_\mu$, $\pi^0 e^+\nu_e$; $K^+ \rightarrow \mu^+\nu_\mu$, $\pi^0 e^+\nu_e$, $\pi^+\pi^0$, $\cdots$.
- Hyperon decays: $\Lambda \rightarrow p\pi^-$, $\Sigma^- \rightarrow n\pi^-$, $\Sigma^+ \rightarrow \Lambda e^+\nu_e$, $\cdots$.
- Heavy quarks decays: $b \rightarrow c\mu^-\bar{\nu}_\mu$, $c\pi^-; t \rightarrow b\mu^+\nu_\mu$, $\cdots$.
- Neutrino scattering: $\nu_\mu e^- \rightarrow \mu^-\nu_e$, $\nu_\mu n \rightarrow \mu^- p$, $\nu_\mu n \rightarrow \mu^- X$, $\cdots$.

![Figure 2: $\nu_e - e^-$ scattering](image)

However, the Fermi theory violates unitarity at high energy. Consider the process $\nu_e e^- \rightarrow \nu_e e^-$ (see Figure 2), which is described by the effective Lagrangian,

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} J^\dagger_\mu J^\mu.$$

The amplitude has only the $L = 0$ partial wave, and the cross section, $\sigma$, is proportional to $\frac{G_F^2}{\sqrt{s}}$, where $\sqrt{s} = E_{cm}$ is the total energy in the center of mass reference frame. However, for a pure S-wave process unitarity requires $\sigma < \frac{16\pi}{s}$. Thus, for energies,

$$\frac{1}{2}E_{cm} \gtrsim \sqrt{\frac{\pi}{G_F}} \sim 500 \text{ GeV},$$

the theoretical $\sigma$ would violate unitarity. The non-unitarity of Born-approximation amplitudes is usually restored by higher order terms. However, the Fermi theory involves divergent integrals in second order, such as,

$$\int d^4k \frac{k^4 + m_e^4 - k^2}{k^2 - m_e^2 k^2},$$

which corresponds to the process in Figure 3. Moreover, it is non-renormalizable due to the dimension of the coupling $[G_F] = (\text{mass})^{-2}$. 

2
The intermediate vector boson $W^\pm$ was postulated by Yukawa in 1935 in the same paper as the meson theory, and reintroduced by Schwinger in 1957. It is a massive charged particle which mediates the weak interaction (in analogy with the photon in QED) as in Figure 4.

Assuming that the IVB is massive, in the low energy regime, $M_W \gg |Q|$, we have $G_F \sqrt{2} \sim \frac{g^2}{8M_W}$, where (in modern normalization) the coupling of the $W^\pm$ to the current is $\frac{g}{2\sqrt{2}}$. The amplitudes for processes like $\nu_e e^- \rightarrow \nu_e e^-$ are now better behaved at high energy because they are no longer pure S-wave. However, unitarity violation occurs for $E_{cm}/2 \gtrsim 500 \text{ GeV}$ for processes involving external $W^\pm$, such as $e^+ e^- \rightarrow W^+ W^-$, because of the growth of the longitudinal polarization vector $\epsilon_\mu \sim \frac{k_\mu}{M_W}$. Even though the coupling $g$ is dimensionless, the theory is still nonrenormalizable.

The divergent parts of the diagrams can be canceled if one also introduces a massive neutral boson $W^0$, with appropriate $W^0 W^+ W^-$ and $e^+ e^- W^0$ vertices. Requiring such cancellations in all processes leads to the relation $[J^-, J^+] = J^0$ for the charged and neutral currents, i.e., the couplings are those of an $SU(2)$ gauge model. However, the model is not realistic because it does not incorporate QED. In 1961 Glashow [9] developed a
gauge model with the symmetry $SU(2) \times U(1)$, in which there was the $W^\pm$ and a massive neutral $Z$ boson, as well the photon, $\gamma$, but there was no mechanism for generating the masses of the $W^\pm$ and $Z$ bosons and the fermions (bare masses would destroy renormalizability). In 1967 Weinberg [10] and Salam [11] in 1968 independently applied the idea of Higgs [12] on how massless gauge particles can acquire mass through spontaneous symmetry breaking to the $SU(2)_W \times U(1)_Y$ model\(^4\). In 1971 't Hooft and Veltman [13] proved the renormalizability of spontaneously broken gauge theories.

The $SU(2)_W \times U(1)_Y$ model worked well for leptons and predicted the existence of weak neutral current (WNC) transitions such as $\nu_e \rightarrow \nu_e$ and $e^- \rightarrow e^-$ mediated by the new $Z$ boson. However, the extension to hadrons (quarks) was problematic: the observed Cabibbo-type mixing between the $s$ and $d$ quarks in the charged current implied flavor changing neutral current (FCNC) transitions $d \leftrightarrow s$ mediated by the $Z$ because of the different transformations of the $d$ (doublet) and $s$ (singlet) under $SU(2)_W$. These were excluded by the non-observation of certain rare $K$ decays and because of the size of the $K^0 - \bar{K}^0$ mixing (for which there was also a too large contribution from a second order charged current diagram). In 1970, Glashow, Iliopoulos and Maiani [14] proposed a new quark (the charm, $c$, quark) as the $SU(2)$ partner of the $s$, avoiding the FCNC and $K^0 - \bar{K}^0$ problems. The $c$ quark was discovered in 1974 through the observation of the $J/\Psi$ meson. The (strangeness-conserving) WNC was discovered in 1973 by the Gargamelle collaboration at CERN [15] and by HPW at Fermilab [16], and was subsequently studied in great detail experimentally [17], as were the weak interactions of heavy quarks, especially the $b$. The $W$ and $Z$ were observed directly with the predicted masses in 1983 by the UA1 [20] and UA2 [21] experiments at CERN, and the $SU(2)_W \times U(1)_Y$ theory was probed at the radiative correction level in high precision experiments at LEP (CERN) and SLC (SLAC) from 1989 until $\sim 2000$ [22]. The existence of non-zero neutrino mass and mixing was firmly established by the Super-Kamiokande collaboration in 1998 [24], and intensively studied subsequently.

3 Aspects of the $SU(2)_W \times U(1)_Y$ Theory

It is convenient to define the left (right) chiral projections of a fermion field $\Psi$ by $\Psi_{L(R)} \equiv \frac{1}{2} (1 \mp \gamma^5) \Psi$. $\Psi_{L(R)}$ coincide with states of negative (positive) helicity for a massless fermion, while chirality and helicity differ in amplitudes by terms of $O(m/E)$ for relativistic fermions of mass $m$ and energy $E$. In the electroweak Standard Model (SM) [1, 25] the left ($L$) and right ($R$) chiral fermions are assigned to transform respectively as doublets and singlets of $SU(2)_W$ in order to incorporate the observed $V - A$ structure of the weak charged current. The left-chiral quark and lepton doublets are

\[
\begin{align*}
q^0_{mL} &= \begin{pmatrix} u^0_m \\ d^0_m \end{pmatrix}_L \\
\nu^0_{mL} &= \begin{pmatrix} \nu^0_m \\ e^{-0}_m \end{pmatrix}_L,
\end{align*}
\]

\(^4\)The subscripts $W$ and $Y$ refer to weak and hypercharge, respectively.
where the 0 superscript refers to weak eigenstates, i.e., fields associated to weak transitions, \( m = 1, \ldots, F \) is the family index, and \( F \) is the number of families. The right-chiral singlets are \( u^0_{mR}, d^0_{mR}, e^0_{mR}, (\nu^0_{mR}) \). The weak eigenstates will in general be related to the mass eigenstate fields by unitary transformations. The quark color indices \( \alpha = r, g, b \) have been suppressed (e.g., \( u^0_{m\alpha R} \)) to simplify the notation. The fermionic part of the weak Lagrangian is

\[
\mathcal{L}_F = \sum_{m=1}^{F} \left( \bar{d}^0_{mL} i D^0_{mL} + \bar{u}^0_{mL} i D^0_{mL} + \bar{e}^0_{mL} i D^0_{mL} + \bar{d}^0_{mR} i D^0_{mR} + \bar{e}^0_{mR} i D^0_{mR} \right).
\]

The assignment of the chiral \( L \) and \( R \) fields to different representations leads to parity violation, which is maximal for \( SU(2)_W \), and also implies that bare mass terms are forbidden by the gauge symmetry. The gauge covariant derivatives are given by

\[
\begin{align*}
D_\mu q^0_{mL} &= \left( \partial_\mu + \frac{ig}{2} \tau^j W^j_\mu + \frac{ig'}{6} B_\mu \right) q^0_{mL}, \\
D_\mu l^0_{mL} &= \left( \partial_\mu + \frac{ig}{2} \tau^j W^j_\mu - \frac{ig'}{2} B_\mu \right) l^0_{mL}, \\
D_\mu u^0_{mR} &= \left( \partial_\mu + ig' B_\mu \right) u^0_{mR}, \\
D_\mu d^0_{mR} &= \left( \partial_\mu - i g' B_\mu \right) d^0_{mR}, \\
D_\mu e^0_{mR} &= \left( \partial_\mu - ig B_\mu \right) e^0_{mR},
\end{align*}
\]

where \( g \) and \( g' \) are respectively the \( SU(2)_W \) and \( U(1)_Y \) gauge couplings, while \( W^i, i = 1,3, \) and \( B \) are the (massless) gauge bosons. The weak hypercharge \( (U(1)_Y) \) assignments are determined by \( Y = Q - T^3 \). The right-handed fields do not couple to the \( SU(2)_W \) terms. One can easily extend (4) and (5) by the addition of SM-singlet right-chiral neutrino fields \( \nu^0_{mR} \), with \( D_\mu \nu^0_{mR} = \partial_\mu \nu^0_{mR} \).

The Higgs mechanism may be used to give masses to the gauge bosons and chiral fermions. In particular, let us introduce a complex doublet of scalar fields,

\[
\phi = \left( \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right),
\]

and assume that the potential for \( \phi \) is such that the neutral component acquires a vacuum expectation value (VEV)\(^5\). Then (in unitary gauge)

\[
\phi \rightarrow \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ \nu + H \end{array} \right),
\]

where \( \nu = \sqrt{2} \langle \phi^0 \rangle = 246 \text{ GeV} \), and \( H \) is the physical Higgs scalar. Three of the original four \( SU(2)_W \times U(1)_Y \) gauge bosons will become massive and one, the photon, remains massless.

\(^5\)For a single Higgs doublet one can always perform a gauge rotation so that only the neutral component acquires an expectation value.
If we rewrite the Lagrangian in the new vacuum, the scalar kinetic energy terms take the form
\[
(D_\mu \phi)^\dagger D^\mu \phi \sim \frac{1}{2} \begin{pmatrix} 0 & \nu \\ \mu & 0 \end{pmatrix} \left[ \frac{g}{2} \tau^i W^i_\mu + \frac{g'}{2} B_\mu \right] \begin{pmatrix} 0 \\ \nu \end{pmatrix}
\]
\[
\rightarrow M_W^2 W^+ W^- + \frac{M_Z^2}{2} Z^\mu Z_\mu,
\]
where the gauge interaction and kinetic energy terms of the physical $H$ particle have been omitted. The mass eigenstate gauge bosons are
\[
W^\pm = \frac{\sqrt{2}}{2} (W^1 \mp iW^2),
\]
\[
Z = -\sin \theta_W B + \cos \theta_W W^3;
\]
\[
A = \cos \theta_W B + \sin \theta_W W^3,
\]
where the weak angle is defined by $\tan \theta_W \equiv g'/g$. The masses of the gauge bosons are predicted to be
\[
M_W = \frac{g\nu}{2}, \quad M_Z = \sqrt{g^2 + g'^2 \nu^2}, \quad M_A = 0.
\]
$W^\pm$ are the IVB of the charged current, $A = \gamma$ corresponds to the photon, and the $Z$ is a massive neutral boson predicted by the theory, which mediates the new neutral current interaction. The Goldstone scalars transform into the longitudinal components of the $W^\pm$ and $Z$.

![Diagram](image)

Figure 5: Diagram for $\nu_e d \rightarrow e^- u$ and its low energy limit.

A typical weak process is shown in the Figure 5. The propagator of the $W$ boson goes like $\frac{1}{Q^2 M_W^2}$. For $|Q^2| \ll M_W^2$, $\frac{1}{Q^2 M_W^2} \rightarrow -\frac{1}{M_W^4}$, which leads to the effective zero-range four-Fermi interaction in Eq. (2), with $\frac{G_F}{\sqrt{2}} \sim \frac{g^2}{sM_W}$ as in the IVB theory. The Fermi constant is determined from the lifetime of the muon, $\tau_\mu^{-1} \sim \frac{G_F^2 m_\mu^2}{192\pi^4}$, yielding the electroweak scale
\[
\nu = 2M_W/g \approx (\sqrt{2}G_F)^{-1/2} \approx 246 \text{ GeV}.
\]
Comparing the $A$ coupling to QED, one finds $g = \frac{e}{\sin \theta_W}$, where $e$ is the electric charge of the positron.

Similarly, the Yukawa couplings of the Higgs doublet to the fermions introduce fermion mass matrices when $\phi$ is replaced by its vacuum expectation value.
3.1 The Weak Charged Current

The major quantitative tests of the SM involve gauge interactions and the properties of the gauge bosons. The charged current interactions of the Fermi theory are incorporated into the Standard Model and made renormalizable. The $W$-fermion interaction (Figure 6) is given by

$$\mathcal{L} = -\frac{g}{2\sqrt{2}} \left( J_W^\mu W^-_\mu + J_W^{\mu\dagger} W^+_\mu \right),$$

where the weak charge-raising current is

$$J_W^{\mu\dagger} = \sum_{m=1}^{F} \left[ \bar{\nu}_m \gamma^\mu (1 - \gamma^5) e_m^0 + \bar{u}_m \gamma^\mu (1 - \gamma^5) d_m^0 \right]$$

$$= \left( \bar{\nu}_e \  \bar{\nu}_\mu \  \bar{\nu}_\tau \right) \gamma^\mu (1 - \gamma^5) \left( \begin{array}{c} e^- \\ \mu^- \\ \tau^- \end{array} \right) + \left( \bar{u} \  \bar{c} \  \bar{t} \right) \gamma^\mu (1 - \gamma^5) V_{CKM} \left( \begin{array}{c} d \\ s \\ b \end{array} \right),$$

and where we have specialized to $F = 3$ families in the second line and introduced the fields $e, \mu, \tau; u, c, t; d, s, b$ of definite mass (mass eigenstates). We are ignoring neutrino masses for now and simply define $\nu_e, \nu_\mu$ and $\nu_\tau$ as the weak partners of the $e, \mu$ and $\tau$. The pure $V - A$ form ensures maximal P and C violation, while CP is conserved except for phases in $V_{CKM}$. We define the vectors,

$$u_L \equiv \left( \begin{array}{c} u_L \\ c_L \\ t_L \end{array} \right), \quad u^0_L \equiv \left( \begin{array}{c} u^0_L \\ c^0_L \\ t^0_L \end{array} \right),$$

and the unitary transformations from the weak basis to the mass basis,

$$u^0_L = A^u_L u_L, \quad d^0_L = A^d_L d_L, \quad e^0_L = A^e_L e_L,$$

with $A^{u\dagger}_L A^u_L = A^{d\dagger}_L A^d_L = I$, etc. In terms of these the unitary quark mixing matrix, due to the mismatch of the unitary transformations in the $u$ and $d$ sectors, is

$$V_{CKM} = A^{u\dagger}_L A^d_L = \left( \begin{array}{ccc} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{array} \right),$$

Figure 6: $W$-fermion interaction
called the Cabibbo-Kobayashi-Maskawa (CKM) matrix \cite{26, 27}. After removing the unobservable $q_L$ phases, the $V_{CKM}$ matrix involves three angles and one CP-violating phase. As described previously, the low energy limit of the charged current interaction reproduces the Fermi theory (with the improvement that radiative corrections can be calculated because of the renormalizability of the theory), which successfully describes weak decays and neutrino scattering processes \cite{6}. In particular, extensive studies, especially in $B$ meson decays, have been done to test the unitarity of $V_{CKM}$ as a probe of new physics and to test the origin of CP violation \cite{28, 29}. However, the CP-violating phase in $V_{CKM}$ is not enough to explain baryogenesis (the cosmological asymmetry between matter and antimatter), and therefore we need additional sources of CP violation\footnote{Possibilities include electroweak baryogenesis \cite{30} in the supersymmetric extension of the SM, especially if there are additional scalar singlets, or leptogenesis \cite{31} in theories with a heavy Majorana neutrino.}

The mixing of the third quark family with the first two is small, and it is sometimes a good zeroth-order approximation to ignore it and consider an effective $F = 2$ theory. Then $V_{CKM}$ is replaced by the Cabibbo matrix

$$V = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix},$$

\begin{equation} (14) \end{equation}

which depends on only one parameter, the Cabibbo angle, $\sin \theta_c \approx 0.22$. In this case there is no CP violation because all the phases can be reabsorbed in the fields.

In addition to reproducing the Fermi theory, the charged current propagator of the SM has been probed at HERA in $e^\pm p \leftrightarrow \not{\nu} e X$ at high energy. The theory has also been tested at the loop level in processes such as $K^0 - \bar{K}^0$ and $B - \bar{B}$ mixing, and in the calculation of (finite) radiative corrections, which are necessary for the consistency of such processes as $\beta$ and $\mu$ decay.

\textbf{3.2 The Weak Neutral Current}

The weak neutral current was predicted by the $SU(2)_W \times U(1)_Y$ model and the Lagrangian for it is

$$\mathcal{L} = -\sqrt{g^2 + g'^2} J_Z^\mu \left( -\sin \theta_W B_\mu + \cos \theta_W W_\mu^3 \right)$$

\begin{equation} (15) \end{equation}

where we have used $\cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$. The neutral current is given by

$$J_Z^\mu = \sum_m \left[ \bar{u}_{mL} \gamma^\mu u_{mL} - \bar{d}_{mL} \gamma^\mu d_{mL} + \bar{\nu}_{mL} \gamma^\mu \nu_{mL} - \bar{e}_{mL} \gamma^\mu e_{mL} \right] - 2 \sin^2 \theta_W J_Q^\mu,$$

\begin{equation} (16) \end{equation}

where $J_Q^\mu = \sum_i q_i \bar{\psi}_i \gamma^\mu \psi_i$ is the electromagnetic current. $J_Z^\mu$ is flavor-diagonal in the Standard Model like the electromagnetic current, i.e., the weak neutral current has the same
form in the weak and the mass bases, because the fields which mix have the same assignments under the \( SU(2)_W \times U(1)_Y \) gauge group. This was the reason for introducing the GIM mechanism, discussed in section 2, into the theory. The neutral current has two contributions; one is purely \( V - A \), while the other is proportional to the electromagnetic current and is purely vector. Therefore parity is violated but not maximally.

The neutral current interaction of fermions in the low momentum limit is given by

\[
\mathcal{L}_{\text{NC}}^{\text{eff}} = -\frac{G_F}{\sqrt{2}} J^{\mu}_Z J_{Z\mu}. \tag{17}
\]

The coefficients for the charged and neutral current interactions are the same because

\[
\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{g^2 + g'^2}{8M_Z^2}. \tag{18}
\]

Since its discovery, the weak neutral current has been tested and extensively studied in many processes [6, 7, 19, 23, 32, 33, 34]. These include pure weak processes such as \( \nu e \rightarrow \nu e \), \( \nu N \rightarrow \nu N \), and \( \nu N \rightarrow \nu X \); weak-electromagnetic interference, including parity violating asymmetries in processes like polarized \( e^+D \rightarrow eX \), atomic parity violation, and \( e^+e^- \) annihilation above and below the \( Z \)-pole; and high precision (e.g., 0.1%) \( Z \)-pole reactions at CERN and SLAC.

## 4 Neutrino Preliminaries

Neutrinos are a unique probe of many aspects of physics and astrophysics on scales ranging from \( 10^{-33} \) to \( 10^{+28} \) cm [35]. Decays and scattering processes involving neutrinos have been powerful tests of many aspects of particle physics, and have been important in establishing the Fermi theory and the SM (at \( \sim 1\% \)), searching for new physics at the TeV scale, as a probe of the structure of hadrons, and as a test of QCD. Small neutrino masses are sensitive to new physics at scales ranging from a TeV up to grand unification and superstring scales. Similarly, neutrinos are important for the physics of the Sun, stars, core-collapse supernovae, the origins of cosmic rays, the large scale structure of the universe, big bang nucleosynthesis, and possibly baryogenesis.
4.1 Weyl Spinors

A Weyl two-component spinor is the minimal fermionic degree of freedom. A left-chiral Weyl spinor $\Psi_L$ satisfies

$$P_L \Psi_L = \Psi_L, \quad P_R \Psi_L = 0,$$

where $P_{L(R)} = \frac{1 \pm \gamma^5}{2}$, $A$ annihilates $L$ particles or creates $R$ antiparticles. Similarly, a right-chiral spinor $\Psi_R$ satisfies $P_R \Psi_R = \Psi_R, \quad P_L \Psi_R = 0$. A Weyl spinor can exist by itself (e.g., the $\nu_L$ in the SM), or can be considered a projection of a 4-component Dirac spinor $\Psi = \Psi_L + \Psi_R$ (e.g., $e^- = e_L + e_R$). A $\Psi_L$ field is related by CP or CPT to a right-chiral antiparticle spinor $\Psi_R^c$ ($\Psi_L^{\text{CP,CPT}} \leftrightarrow \Psi_R^c$), where $\Psi_R^c$ annihilates $R$ antiparticles or creates $L$ particles and $P_R \Psi_R^c = \Psi_R^c$. Under CP, for example,

$$\Psi_L(\vec{x}, t) \rightarrow \gamma^0 \Psi_R^c(-\vec{x}, t).$$

$\Psi_R^c$ is essentially the adjoint of $\Psi_L$:

$$\Psi_R^c = C \Psi_L^T = C \gamma_0^T (\Psi_L^\dagger)^T, \quad \Psi_L = C \Psi_R^T,$$

where $T$ represents the transpose and $C$ is the charge conjugation operator, defined by $C \gamma_\mu C^{-1} = -\gamma^T_\mu$. In the Pauli-Dirac representation $C = i \gamma^2 \gamma^0$. The free Weyl field is

$$\Psi_L(x) = \int \frac{d^3 \vec{p}}{\sqrt{(2\pi)^3 2E_p}} \left[ a_L(\vec{p}) u_L(\vec{p}) e^{-i \vec{p} \cdot \vec{x}} + b_R^\dagger(\vec{p}) v_R(\vec{p}) e^{i \vec{p} \cdot \vec{x}} \right] \text{ (no sum over spin)}.$$

When two Weyl spinors are present, we can use either $(\Psi_L$ and $\Psi_R)$ or $(\Psi_L$ and $\Psi_R^c)$ to define the theory. In the Standard Model, for example, baryon ($B$) and lepton ($L$) numbers are conserved perturbatively, and it is convenient to work with, e.g., $u_L$ and $u_R$ and similarly for the other fermionic fields. Their CPT partners are $u_R^c \leftrightarrow u_L$ and $u_L^c \leftrightarrow u_R$. In some extensions of the SM, such as supersymmetry or grand unification, it is more convenient to work with $L$ spinors $\Psi_L$ and $\Psi_L^c$.

4.2 Active and Sterile Neutrinos

Active (a.k.a. ordinary or doublet) neutrinos are left-handed neutrinos which transform as $SU(2)$ doublets with a charged lepton partner, and which therefore have normal weak interactions. The $L$ doublets and their right-handed partners are

$$\left( \begin{array}{c} \nu_e \\ e^- \end{array} \right)_L \overset{\text{CPT}}{\leftrightarrow} \left( \begin{array}{c} e^+ \\ \nu_e^c \end{array} \right)_R.$$

---

7 Which is called the particle and which the antiparticle is a matter of convenience.

8 Thus, in our notation the subscript $L$ ($R$) always refers to a left (right) chiral spinor, independent of whether it is for a particle or antiparticle.
Sterile (a.k.a. singlet or “right-handed”) neutrinos, which are present in most extensions of the SM, are $SU(2)$ singlets. They do not interact except by mixing, Yukawa interactions, or beyond the SM (BSM) interactions. It is convenient to denote the right-chiral spinor as $N_R$ and its conjugate as $N_L^c$.

$$N_R \xrightarrow{\text{CPT}} N_L^c.$$  \hfill (23)

5 Neutrino Mass Models

Most extensions of the Standard Model lead to nonzero neutrino mass at some level. There are many models of neutrino mass. Here, only a brief survey of the principal classes is given. For more detail see [50].

5.1 Fermion Masses

Mass terms convert a spinor of one chirality into one of the opposite chirality,

$$-\mathcal{L} \sim m(\Psi_{2L}\Psi_{1R} + \Psi_{1R}\Psi_{2L}).$$  \hfill (24)

An interpretation is that a massless fermion has the same helicity (chirality) in all frames of reference, while that of a massive particle depends on the reference frame and therefore can be flipped.

5.2 Dirac Mass

A Dirac mass (Figure 8) connects two distinct Weyl neutrinos$^9$ $\nu_L$ and $N_R$. It is given by

$$-\mathcal{L}_D = m_D(\bar{\nu}_L N_R + \bar{N}_R \nu_L) = m_D \bar{\nu} \nu.$$  \hfill (25)

Thus, $\Psi_{1R} \neq \Psi_{2R}$ and we have two distinct Weyl spinors. These may be combined to form a Dirac field $\nu \equiv \nu_L + N_R$ with four components: $\nu_L$, $\nu_R^c$, $N_R$ and $N^c_L$. This can be generalized to three or more families.

Figure 8: Dirac mass term

This mass term allows a conserved lepton number $L$, which implies no mixing between $\nu_L$ and $N^c_L$, or between $\nu_R^c$ and $N_R$. However, it violates weak isospin $I_W$ by $\Delta I_W = \frac{1}{2}$. A Dirac mass can be generated by the Higgs mechanism, as in Figure 9. The Dirac neutrino

$^9$These are usually active and sterile. However, there are variant forms involving two distinct active (Zeldovich-Konopinski-Mahmoud) or two distinct sterile neutrinos.
masses $m_D = h_\nu \langle \phi^0 \rangle = h_\nu \frac{\nu}{\sqrt{2}}$ are analogous to the quark and charged lepton masses. The upper bound on the neutrino mass, $m_\nu \lesssim 1$ eV, implies a Yukawa coupling of the neutrino to the Higgs, $h_\nu < 10^{-11}$, which is extremely small in comparison with the coupling for the top quark, $h_t = O(1)$, or for the electron $h_e^- \sim 10^{-5}$.

5.3 Majorana Mass

A Majorana mass term describes a transition between a left-handed neutrino and its CPT-conjugate right-handed antineutrino, as shown in Figure 10. It can be viewed as the annihilation or creation of two neutrinos, and therefore violates lepton number by two units, $\Delta L = 2$. A Majorana mass term has the form

$$-L = \frac{1}{2} m_T (\bar{\nu}_L \nu_R^c + \bar{\nu}_R^c \nu_L) = \frac{m_T}{2} (\bar{\nu}_L C \nu_L^T + h.c) = \frac{1}{2} m_T \bar{\nu} \nu,$$

(26)

where $\nu = \nu_L + \nu_R^c$ is a self-conjugate two-component (Majorana) field satisfying $\nu = \nu^c = C \nu^T$. A Majorana mass for an active neutrino violates weak isospin by one unit, $\Delta I_W = 1$, and can be generated either by the VEV of a Higgs triplet (see Figure 11) or by a higher-dimensional operator involving two Higgs doublets (which could be generated, for example, in a seesaw model). A Majorana $\nu$ is its own antiparticle and can mediate neutrinoless double beta decay ($\beta\beta_0\nu$) [55, 56], in which two neutrons turn into two protons and two electrons, violating lepton number by two units, as shown in Figure 12.

A sterile neutrino can also have a Majorana mass term of the form,

$$-L = \frac{m_S}{2} [\bar{N}_L^c N_R + \bar{N}_R N_L^c].$$

(27)

In this case, weak isospin is conserved, $\Delta I_W = 0$, and thus $m_S$ can be generated by the VEV of a Higgs singlet\(^10\).

\(^10m_S\) could also be generated in principle by a bare mass, but this is usually forbidden by additional

---

Figure 9: Yukawa coupling to the Higgs field

Figure 10: A Majorana mass term
5.4 Mixed models

If active and sterile neutrinos are both present, one can have both Dirac and Majorana mass terms simultaneously. For one family, the Lagrangian has the form

\[-\mathcal{L} = \frac{1}{2} \begin{pmatrix} \bar{\nu}^0_L & \bar{N}^0_L \end{pmatrix} \begin{pmatrix} m_T & m_D \\ m_D & m_S \end{pmatrix} \begin{pmatrix} \nu^0_R \\ N^0_R \end{pmatrix} + \text{h.c.},\]

(28)

where 0 refers to weak eigenstates and the masses are

\[m_T : |\Delta L| = 2, \quad \Delta I_W = 1 \quad (\text{Majorana}),\]
\[m_D : |\Delta L| = 0, \quad \Delta I_W = \frac{1}{2} \quad (\text{Dirac}),\]
\[m_S : |\Delta L| = 2, \quad \Delta I_W = 0 \quad (\text{Majorana}).\]

Diagonalizing the matrix in (28) yields two Majorana mass eigenvalues and two Majorana mass eigenstates, \(\nu_i = \nu^c_i + \nu^{e_i}_i = \nu^c_i\), with \(i = 1, 2\). The weak and mass bases are related by the unitary transformations

\[\begin{pmatrix} \nu_1^L \\ \nu_2^L \end{pmatrix} = U_L^{\nu \dagger} \begin{pmatrix} \nu^0_L \\ N^0_L \end{pmatrix}, \quad \begin{pmatrix} \nu^c_1^R \\ \nu^c_2^R \end{pmatrix} = U_R^{\nu \dagger} \begin{pmatrix} \nu^0_R \\ N^0_R \end{pmatrix}.\]

(29)

\(U_L\) and \(U_R\) are generally different for Dirac mass matrices, which need not be Hermitian. However, the general \(2 \times 2\) neutrino mass matrix in (28) is symmetric because of (21). In our phase convention, in which \(\nu^c_i = C \nu_i \nu^t\), this implies \(U_L^{\nu} = U_R^{\nu \ast}\).
5.5 Special Cases

(a) Majorana: \( m_D = 0 \) in (28) corresponds to the pure Majorana case: the mass matrix is diagonal, with \( m_1 = m_T, m_2 = m_S, \) and

\[
\begin{align*}
\nu_{1L} &= \nu_L^0, \\
\nu_{2L} &= N_L^{0c}, \\
\nu_{1R} &= \nu_R^{0c}, \\
\nu_{2R} &= N_R^0.
\end{align*}
\]

(b) Dirac: \( m_T = m_S = 0 \) is the Dirac limit. There are formally two Majorana mass eigenstates, with eigenvalues \( m_1 = m_D \) and \( m_2 = -m_D \) and eigenstates

\[
\begin{align*}
\nu_{1L} &= \frac{1}{\sqrt{2}}(\nu_L^0 + N_L^{0c}), \\
\nu_{2L} &= \frac{1}{\sqrt{2}}(\nu_L^0 - N_L^{0c}), \\
\nu_{1R} &= \frac{1}{\sqrt{2}}(\nu_R^{0c} + N_R^0), \\
\nu_{2R} &= \frac{1}{\sqrt{2}}(\nu_R^{0c} - N_R^0).
\end{align*}
\]

Note that \( \nu_{1,2} \) are degenerate in the sense that \( |m_1| = |m_2| \). To recover the Dirac limit, expand the mass term

\[
-L = \frac{m_D}{2}(\bar{\nu}_{1L}\nu_{1R}^c - \bar{\nu}_{2L}\nu_{2R}^c) + h.c.
\]

which clearly conserves lepton number (i.e., there is no \( \nu_L^0 - N_L^{0c} \) or \( \nu_R^{0c} - N_R^0 \) mixing). Thus, a Dirac neutrino can be thought of as two Majorana neutrinos, with maximal (45°) mixing and with equal and opposite masses. This interpretation is useful in considering the Dirac limit of general models.

(c) Seesaw: \( m_T = 0 \) and \( m_S \gg m_D \) (e.g., \( m_D = O(m_u, m_e, m_d) \) and \( m_S = O(M_X) \), where \( M_X \sim 10^{14} \text{ GeV} \)) is the seesaw limit [57, 58, 59], with eigenstates and eigenvalues

\[
\begin{align*}
\nu_{1L} &\sim \nu_L^0 - \frac{m_D}{m_S} N_L^{0c} \sim \nu_L^0, \\
\nu_{2L} &\sim \frac{m_D}{m_S} \nu_L^0 + N_L^{0c} \sim N_L^{0c},
\end{align*}
\]

- \( m_1 = \frac{m_D}{m_S} \ll m_D, \)

\[
\nu_{2L} \sim \frac{m_D}{m_S} \nu_L^0 + N_L^{0c} \sim N_L^{0c}, \quad m_2 \sim m_S.
\]

The seesaw mechanism is illustrated in Figure 13.

Figure 13: The seesaw mechanism, in which a light active neutrino mixes with a very heavy Majorana neutrino.
(d) **Pseudo-Dirac:** This is a perturbation on the Dirac case, with $m_T$, $m_S \ll m_D$. There is a small lepton number violation, and a small splitting between the magnitudes of the mass eigenvalues. For example, $m_T = \epsilon$, $m_S = 0$ leads to $|m_{1,2}| = m_D \pm \epsilon/2$.

(e) **Mixing:** The general case in which $m_D$ and $m_S$ (and/or $m_T$) are both small and comparable leads to non-degenerate Majorana mass eigenvalues and significant ordinary–sterile ($\nu^c_L - N^0_L$) mixing (as would be suggested if the LSND results are confirmed). Only this and the pseudo-Dirac cases allow such mixings.

As we have seen, the fermion mass eigenvalues can be negative or even complex. However, only the magnitude is relevant for most purposes, and in fact fermion masses can always be made real and positive by chiral transformations (e.g., redefining the phases of the $\nu^c_{iR}$). However, the relative signs (or phases) may reappear in the $\beta\beta_{0\nu}$ amplitude or new (beyond the SM) interactions.

### 5.6 Extension to Three Families

It is straightforward to generalize (28) to three or more families, or even to the case of different numbers of active and sterile neutrinos. For $F = 3$ families, the Lagrangian is

$$-L = \frac{1}{2} \begin{pmatrix} \bar{\nu}_L^0 & \bar{N}_{L}^{0c} \end{pmatrix} \begin{pmatrix} m_T & m_D \\ m_D^T & m_S \end{pmatrix} \begin{pmatrix} \nu_R^0 \\ N_R^0 \end{pmatrix} + h.c., \quad (31)$$

where $\nu_L^0$ and $N_{L}^{0c}$ are three-component vectors

$$\nu_L^0 = \begin{pmatrix} \nu_{1L}^0 \\ \nu_{2L}^0 \\ \nu_{3L}^0 \end{pmatrix}, \quad N_{L}^{0c} = \begin{pmatrix} N_{1L}^{0c} \\ N_{2L}^{0c} \\ N_{3L}^{0c} \end{pmatrix}, \quad (32)$$

and similarly for $\nu_R^0$ and $N_{R}^{0}$. $m_S$, $m_D$ and $m_T$ are $3 \times 3$ matrices, where $m_S = m_S^T$ and $m_T = m_T^T$ because of (21). There are six Majorana mass eigenvalues and eigenvectors. The transformation to go from the weak to the mass basis is given by

$$\nu_L = U_L^{\nu} \begin{pmatrix} \nu_L^0 \\ N_{L}^{0c} \end{pmatrix}, \quad (33)$$

where $U_L^{\nu}$ is a $6 \times 6$ unitary matrix and $\nu_L$ is a six-component vector. The analogous transformation for the $R$ fields involves $U_R = U_L^{*}$ because the $6 \times 6$ Majorana mass matrix is necessarily symmetric.

First consider the case in which there are only Dirac masses, which is analogous to the quarks and charged leptons. The mass Lagrangian is

$$-L = \bar{\nu}_L^0 m_D N_{R}^{0} + h.c., \quad (34)$$

where $m_D$ need not be Hermitian. We may change from the weak to the mass basis with a unitary transformation,

$$-L = \bar{\nu}_L m_D N_R, \quad (35)$$
where \( \hat{m}_D \) is the diagonal matrix of eigenvalues, \( \hat{m}_D = U^\nu_L m_D U^\nu_L = \text{diag}(m_1, \ldots, m_3) \), and the eigenvectors are

\[
\nu_L = U^\nu_L \nu_L^0, \quad N_R = U^\nu_R N_R^0.
\]  

(36)

If \( m_D \) is not Hermitian, \( U^\nu_{L,R} \) can be found by diagonalizing the Hermitian matrices \( m_D m_D^\dagger \) and \( m_D^\dagger m_D \),

\[
U^\nu_L m_D m_D^\dagger U^\nu_L = U^\nu_R m_D m_D^\dagger U^\nu_R = |\hat{m}_D|^2 = \text{diag}(|m_1|^2 \cdots |m_3|^2).
\]  

(37)

The \( U^\nu_{L,R} \) are not unique. If a given \( U^\nu_L \) satisfies (37), then so does

\[
\hat{U}^\nu_L = U^\nu_L K^\nu_L,
\]  

(38)

where \( K^\nu_L \) is a diagonal phase matrix\(^{11} \), \( K^\nu_L = \text{diag}(e^{i\alpha_1} e^{i\alpha_2} e^{i\alpha_3}) \). This of course corresponds to the freedom to redefine the phases of the mass eigenstate fields. Similar statements apply to \( U^\nu_R \) and to the unitary transformations for the quark and charged lepton fields.

The weak charged current is given by

\[
J_W^\mu = 2 \bar{e}_L^0 \gamma^\mu \nu_L^0 = 2 \bar{e}_L \gamma^\mu U_L^\nu e_L \nu_L,
\]  

(39)

and \( U^\nu_L U^\nu_L \) is the lepton mixing matrix \( V_{PMNS} \). A general unitary \( 3 \times 3 \) matrix involves 9 parameters: 3 mixing angles and 6 phases. However, five of the phases in \( V_{PMNS} \) (and similarly in \( V_{CKM} \)) are unobservable in the Dirac case because they depend on the relative phases of the left-handed neutrino and charged lepton fields, i.e., they can be removed by an appropriate choice of \( K^\nu_L \) and the analogous \( K^\nu_R \), so there is only one physical CP-violating phase. The analogous \( K_R^{ \nu,e} \) in \( U_R^{ \nu,e} \) can then be chosen to make the mass eigenvalues real and positive. The \( K_R \) phases are unobservable unless there are new BSM interactions involving the \( R \) fields.

The diagonalization of a Majorana mass matrix is similar, except for the constraint \( U^\nu_L = \hat{U}^\nu_R \). This implies that \( K^\nu_L \) and \( K^\nu_R \) cannot be chosen independently (because the \( L \) and \( R \) mass eigenstates are adjoints of each other). One must then use the freedom in \( K^\nu_L \) to make the mass eigenvalues real and positive, so that one cannot remove as many phases from the leptonic mixing matrix. For three light mass eigenstates (e.g., for three active neutrinos with \( m_T \neq 0, m_D = 0 \); or for the seesaw limit of the \( 6 \times 6 \) case), this implies that there are two additional “Majorana” phases, i.e., \( V_{PMNS} = \hat{V}_{PMNS} \times \text{diag}(e^{i\beta_1} e^{i\beta_2} 1) \), where \( \hat{V}_{PMNS} \) has the canonical form with one phase, and \( \beta_{1,2} \) are CP-violating relative phases associated with the Majorana neutrinos. \( \beta_{1,2} \) do not affect neutrino oscillations, but can in principle affect \( \beta \beta_{0e} \) and new interactions.

Let us conclude this section with a few comments.

\(^{11}\)There is more freedom if there are degenerate eigenvalues.

\(^{12}\)This is for Maki, Nakagawa, and Sakata [60] in 1962 and Pontecorvo [61] in 1968.
• The LSND oscillation results would, if confirmed, strongly suggest the existence of very light sterile neutrinos which mix with active neutrinos of the same chirality [62]. The pure Majorana and pure Dirac cases do not allow any such mixing (sterile neutrinos are not even required in the Majorana case). The seesaw limit only has very heavy sterile neutrinos and negligible mixing. Only the general and pseudo-Dirac limits allow significant ordinary-sterile mixing, but in these cases one must find an explanation for two very small types of masses.

• There is no distinction between Dirac and Majorana neutrinos except by their masses (or by new interactions). As the masses go to zero, the active components reduce to standard active Weyl spinors in both case. There are additional sterile Weyl spinors in the massless limit of the Dirac case, but these decouple from the other particles.

• One can ignore $V_{PMNS}$ in processes for which the neutrino masses are too small to be relevant. In that limit, the neutrino masses are effectively degenerate (with vanishing mass) and one can simply work in the weak basis.

5.7 Models of Neutrino Mass

There are an enormous number of models of neutrino mass [50].

Models to generate Majorana masses are most popular, because no Standard Model gauge symmetry forbids them. (However, models descended from underlying string constructions may be more restrictive because of the underlying symmetries and selection rules [63].) Models constructed to yield small Majorana masses include: the ordinary (type I) seesaw [57, 58, 59], often combined with additional family and grand unification symmetries; models with heavy Higgs triplets (type II seesaw) [59, 64, 65]; TeV (extended) seesaws [66], with $m_{\nu} \sim m^{p+1}/M^p$, $p > 1$, e.g., with $M$ in the TeV range; radiative masses (i.e., generated by loops) [67]; supersymmetry with $R$-parity violation, which may include loop effects; supersymmetry with mass generation by terms in the Kähler potential; anarchy (random entries in the mass matrices); and large extra dimensions (LED), possibly combined with one of the above.

Small Dirac masses may be due to: higher dimensional operators (HDO) in intermediate scale models (e.g., associated with an extended $U(1)'$ gauge symmetry [68] or supersymmetry breaking); large intersection areas in intersecting brane models or large extra dimensions, from volume suppression if $N_R$ propagates in the bulk [69].

Simultaneous small Dirac and Majorana masses, as motivated by LSND, may be due, e.g., to HDO [70] or to sterile neutrinos from a “mirror world” [71].

There are also many “texture” models [54], involving specific guesses about the form of the $3 \times 3$ neutrino mass matrix or the Dirac and Majorana matrices entering seesaw models. These are often studied in connection with models also involving quark and charged lepton mass matrices, such as grand unification (GUTs), family symmetries, or left-right symmetry.
6 Laboratory and Astrophysical Constraints on Neutrino Counting and Mass

6.1 Laboratory Limits

The most precise measurement of the number of light \((m_\nu < M_Z/2)\) active neutrino types and therefore the number of associated fermion families comes from the invisible \(Z\) width \(\Gamma_{\text{inv}}\), obtained by subtracting the observed width into quarks and charged leptons from the total width from the lineshape. The number of effective neutrinos \(N_\nu\) is given by

\[
N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_l} \left(\frac{\Gamma_l}{\Gamma_\nu}\right)_{SM},
\]

where \((\Gamma_l/\Gamma_\nu)_{SM}\) is the SM expression for the ratio of widths into a charged lepton and a single active neutrino, introduced to reduce the model dependence. The experimental value is \(N_\nu = 2.984 \pm 0.008 \) [7], excluding the possibility of a fourth family unless the neutrino is very heavy. Other unobserved particles from \(Z\) decay would also give a positive contribution to \(N_\nu\). For example, the decay \(Z \rightarrow MS\) in models with spontaneous lepton number violation and a Higgs triplet [72], where \(M\) is a Goldstone boson (Majoron) and \(S\) is a light scalar, would yield a contribution of 2 to \(N_\nu\) and is therefore excluded.

Kinematic laboratory measurements on neutrino masses are relatively weak [73]. For the \(\tau\)-neutrino the most stringent limit is \(m_{\nu_\tau} < 18.2\) MeV, which comes from the decay channel \(\tau \rightarrow 5\pi + \nu_\tau\). For the \(\mu\)-neutrino the most sensitive measurement gives \(m_{\nu_\mu} < 0.19\) MeV from \(\pi \rightarrow \mu\nu_\mu\) decay\(^{13}\). The analysis of tritium beta decay at low energies gives a limit on the mass of the \(e\)-neutrino of \(m_{\nu_e} \sim 2.8\) eV. Including mixing, this should be interpreted as a limit on \(m_\beta \equiv \sqrt{\sum_i |V_{ei}|^2 m_i^2}\), where \(V = V_{PMNS}\). The latter should be improved in the future to a sensitivity of around 0.2 eV by the KATRIN experiment.

For Majorana masses, the amplitude for neutrinoless double beta decay [55, 56] is \(A \sim A_{\text{nuc}}m_{\beta\beta}\), where \(A_{\text{nuc}}\) contains the nuclear matrix element and \(m_{\beta\beta} \equiv \sum_i (V_{ei})^2 m_i\) is the effective Majorana mass in the presence of mixing between light Majorana neutrinos\(^{14}\). \(m_{\beta\beta}\) is just the \((1,1)\) element of \(m_T\) or of the effective Majorana mass matrix in a seesaw model. It involves the square of \(V_{ei}\) rather than the absolute square, and is therefore sensitive in principle to the Majorana phases (though this is difficult in practice). The expression allows for the possibility of cancellations between different mass eigenstates\(^{15}\), and in fact shows why \(\beta\beta_0\) vanishes for a Dirac neutrino, which can be viewed as two canceling Majorana neutrinos. At present, \(|m_{\beta\beta}| < (0.35 - 1)\) eV, where the range is from the theoretical uncertainty in \(A_{\text{nuc}}\). Members of one experiment claimed an observation of \(\beta\beta_0\), which corresponds to \(m_{\beta\beta} \sim 0.39\) eV, but this has not been confirmed. Future experiments should be sensitive to \(m_{\beta\beta} \sim (0.1 - 0.2)\) eV.

\(^{13}\)The limits on \(m_{\nu_\tau}\) and \(m_{\nu_\mu}\) are now obsolete because of oscillation and cosmological constraints.

\(^{14}\)There can also be other BSM contributions to the decay, such as heavy Majorana neutrinos or \(R\)-parity violating effects in supersymmetry.

\(^{15}\)In the convention for the field phases that the masses are real and positive the cancellations are due to the phases in \(V_{ei}\). Alternatively, one can use the convention that the mass eigenvalues may be negative or complex.
6.2 Cosmological Constraints

The light nuclides $^4\text{He}$, $^3\text{He}$, $^7\text{Li}$ were synthesized in the first thousand seconds in the early evolution the universe [44, 74], corresponding to temperatures from 1 MeV to 50 keV. At temperatures above the freezeout temperature $T_f \sim$ few MeV, the neutron to proton ratio was kept in equilibrium by the reactions

\[
\begin{align*}
 n + \nu_e & \leftrightarrow p + e^- , \\
 n + e^+ & \leftrightarrow p + \bar{\nu}_e .
\end{align*}
\]

(41)

For $T < T_f$ their rates became slow compared to the expansion rate of the universe, and the neutron to proton ratio, $n/p$, froze at a constant (except for neutron decay) value $\frac{n}{p} = \exp(-\frac{m_n - m_p}{T_f})$. Most of the neutrons were incorporated into $^4\text{He}$, so that the primordial abundance (relative to hydrogen) can be predicted\(^{16}\) in terms of $T_f$. $T_f$ is predicted by comparing the reaction rate for the processes in Eq. (41), $\Gamma \sim G_F^2 T_f^5$, and the Hubble expansion rate, $H = \frac{1}{\sqrt{g^* T^2}}$, where $M_{\text{Pl}}$ is the Planck scale. Therefore, $T_f \sim g_*^{1/6}$, where $g_*$ counts the number of relativistic particle species, determining the energy density in radiation. It is given by $g_* = g_B + \frac{1}{\pi^2} g_F$, where $g_F = 10 + 2\Delta N'_{\nu}$, and where the 10 is due to $3\nu + 3\bar{\nu} + 2$ helicities each of $e^\pm$. $\Delta N'_{\nu}$ is the effective number of additional neutrinos present at $T \gtrsim T_f$. It includes new active neutrinos with masses\(^{17}\) $\lesssim 1$ MeV, and also light sterile neutrinos, which could be produced by mixing with active neutrinos for a wide range of mixing angles\(^{18}\). It does not include the right-handed sterile components of light Dirac neutrinos, which would not have been produced in significant numbers unless they couple to BSM interactions. The prediction of the primordial mass abundance of $^4\text{He}$ relative to $H$ is $\sim 24\%$ for $\Delta N'_{\nu} = 0$. There is considerable uncertainty in the observational value, but most estimates yield $\Delta N'_{\nu} < 0.1 - 1$.

Neutrinos with masses in the eV range would contribute hot dark matter to the universe [44]. They would close the universe, $\Omega_\nu = 1$, for the sum of masses of the light neutrinos (including sterile neutrinos, weighted by their abundance relative to active neutrinos) $\Sigma \equiv \Sigma_i |m_i| \sim 35$ eV. However, even though some 30% of the energy density is believed to be in the form dark matter, most of it should be cold (such as weakly interacting massive particles) rather than hot (neutrinos), because the latter cannot explain the formation of smaller scale structures during the lifetime of the universe. The Wilkinson Microwave Anisotropy Probe (WMAP), together with the Sloan Digital Sky Survey (SDSS), the Lyman alpha forest ($\text{Ly} \alpha$), and other observations, leads to strong constraints on $\Sigma \lesssim 1$ eV [75], with the most stringent claimed limit of 0.42 eV [76]. Using future Planck data, it may be possible to extend the sensitivity down to $0.05 - 0.1$ eV.

\(^{16}\)There is also a weak dependence on the baryon density relative to photons, which is determined independently by the $D$ abundance and by the cosmic microwave background (CMB) anisotropies.

\(^{17}\)There would be an enhanced contribution from a $\nu_\tau$ in the 1-20 MeV range, which was once important in constraining its mass, or a reduced contribution from $\nu_\tau$ decay.

\(^{18}\)The effects of light sterile neutrinos can be avoided in variant scenarios or compensated by a large $\nu_e - \bar{\nu}_e$ asymmetry [49].
close to the minimum value $0.05 \text{ eV} \sim \sqrt{|\Delta m^2_{\text{atm}}|}$ allowed by the neutrino oscillation data. However, there are significant theoretical uncertainties.

### 6.3 Neutrino Oscillations

Neutrino oscillations can occur due to the mismatch between weak and mass eigenstates, and are analogous to the time evolution of a quantum system which is not in an energy eigenstate, or a classical coupled oscillator in which one starts with an excitation that is not a normal mode.

Consider a system in which there are only two neutrino flavors, e.g., $\nu_e$ and $\nu_\mu$. Then

$$|
u_e\rangle = |\nu_1\rangle \cos \theta + |\nu_2\rangle \sin \theta,$$

$$|
u_\mu\rangle = -|\nu_1\rangle \sin \theta + |\nu_2\rangle \cos \theta,$$

(42)

where $\theta$ is the neutrino mixing angle. Suppose that at initial time, $t = 0$, we create a pure weak eigenstate (as is typically the case), such as $\nu_\mu$ from the decay $\pi^+ \rightarrow \mu^+ \nu_\mu$, i.e., $|\nu(0)\rangle = |\nu_\mu\rangle$. Since $|\nu_\mu\rangle$ is a superposition of mass eigenstates, each of which propagates with its own time dependence, then at a later time the state may have oscillated into $|\nu_e\rangle$, which can be identified by its interaction, e.g., $\nu_e n \rightarrow e^- p$. To quantify this, after a time $t$ the state $|\nu(0)\rangle$ will have evolved into

$$|\nu(t)\rangle = -|\nu_1\rangle \sin \theta e^{-iE_1 t} + |\nu_2\rangle \cos \theta e^{-iE_2 t},$$

(43)

where $E_1 = \sqrt{p^2 + m_1^2} \sim p + \frac{m_2^2}{2p}$ and $E_2 = \sqrt{p^2 + m_2^2} \sim p + \frac{m_2^2}{2p}$, and we have assumed that the neutrino is highly relativistic, $p \gg m_{1,2}$, as is typically the case. After traveling a distance $L$ the oscillation probability, i.e., the probability for the neutrino to interact as a $\nu_e$, is

$$P_{\nu_\mu \rightarrow \nu_e}(L) = \frac{\left| \langle \nu_e | \nu(t) \rangle \right|^2}{\sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) = \sin^2 2\theta \sin^2 \left( \frac{1.27 \Delta m^2 (\text{eV}^2) L (\text{km})}{E (\text{GeV})} \right)},$$

(44)

where $L \sim t$, $E = p$ and $\Delta m^2 = m_2^2 - m_1^2$. The probability for the state to remain a $\nu_\mu$ is $P_{\nu_\mu \rightarrow \nu_\mu}(L) = 1 - P_{\nu_\mu \rightarrow \nu_e}(L)$.

There are two types of oscillation searches:

1. Appearance experiments, in which one looks for the appearance of a new flavor, e.g., of $\nu_e$ or $\nu_\tau$ (i.e., of $e^-$ or $\tau^-$) in an initially pure $\nu_\mu$ beam.

2. Disappearance experiments, in which one looks for a change in, e.g., the $\nu_\mu$ flux as a function of $L$ and $E$.

Even with more than two types of neutrino, it is often a good approximation to use the two neutrino formalism in the analysis of a given experiment. However, a more precise or
general analysis should take all three neutrinos into account\textsuperscript{19}. The general lepton mixing for three families contains three angles $\theta_{12}$, $\theta_{13}$, and $\theta_{23}$; one Dirac CP-violating phase $\delta$; and two Majorana phases $\beta_1, \beta_2$. (The latter do not enter the oscillation formulae.) In the basis in which the charged leptons are mass eigenstates, the neutrino mass eigenstates $\nu_i$ and the weak eigenstates $\nu_a = (\nu_e, \nu_\mu, \nu_\tau)$ are related by a unitary transformation $\nu_a = V_{ai}\nu_i$, where $V$ is the $3 \times 3$ lepton mixing matrix $V_{PMNS}$, which is parametrized as

$$V_{PMNS} = \begin{bmatrix}
 c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
 -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
 s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}\end{bmatrix} \times \text{diag}(e^{i\beta_1/2}, e^{i\beta_2/2}, 1),$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. The oscillation probability for $\nu_a \rightarrow \nu_b$ after a distance $L$ is then

$$P_{\nu_a \rightarrow \nu_b}(L) = \sum_{a \neq b} |V_{ai}V_{bj}|^2 + \text{Re} \sum_{i \neq j} V_{ai}V_{bi}^*V_{aj}^*V_{bj}e^{-i\Delta_{ij}L/2E},$$

where $\Delta_{ij} = m_i^2 - m_j^2$.

## 7 Atmospheric Neutrinos

Many experiments have searched for neutrino oscillations at reactors, accelerators, and from astrophysical sources \cite{77}. Although the first indications of an effect involved the Solar neutrinos, the first unambiguous evidence came from the oscillations of atmospheric neutrinos. Atmospheric neutrinos are the product of pion and muon decays, which are produced in the upper layers of the atmosphere due to the interaction of primary cosmic rays. The data from the Kamiokande and Super-Kamiokande water Čerenkov detectors indicated the disappearance of $\nu_\mu$-neutrinos. This was first seen in the ratio of the $\nu_\mu/\nu_e$ fluxes, and later confirmed by the zenith angle distribution of $\nu_\mu$ events. Results from other experiments such as MACRO and Soudan confirm the results, as does the recent long-baseline K2K experiment involving neutrinos produced at the KEK lab and observed in the Super-Kamiokande detector (Figure 14). The details of the observations have now established that the dominant effect is indeed oscillations of the $\nu_\mu$ into\textsuperscript{20} $\nu_\tau$, with near-maximal mixing ($\sin^2 2\theta_{23} > 0.92$), and the difference between the mass squares of the two mass eigenstates of order $|\Delta m^2_{\text{atm}}| \sim 2 \times 10^{-3}$ eV.

\textsuperscript{19}If there are light sterile neutrinos which mix with the active neutrinos one should generalize to include their effects as well.

\textsuperscript{20}Oscillations into $\nu_e$ or a sterile neutrino as the dominant effect are excluded by the Super-Kamiokande data and reactor experiments.
Figure 14: Contours enclosing the 90% confidence regions for two-flavor ($\nu_\mu \leftrightarrow \nu_\tau$) neutrino oscillation parameters for the K2K results, compared to results from Super-Kamiokande atmospheric neutrinos (from [78]).

8 Solar Neutrinos

The first indications [79] of neutrino oscillations were from the low number of $\nu_e$ events observed in the Davis Solar neutrino chlorine experiment, compared with the predictions of the Standard Solar Model (SSM) [82], shown in Figure 15. The deficit was later confirmed (and that the observed events do indeed come from the Sun) in the Kamiokande and Super-Kamiokande water Čerenkov experiments. These observed the high energy $^8B$ neutrinos by the reaction $\nu e^- \rightarrow \nu e^-$, for which the cross section for oscillated $\nu_\mu,\tau$ by the neutral current is about 1/6 that for $\nu_e$. The gallium experiments GALLEX, SAGE, and (later) GNO were sensitive to the entire Solar spectrum and also showed a deficit. Comparing the depletions in the various types of experiments, which constituted a crude measurement of the spectrum, showed that neutrino oscillations of $\nu_e$ into $\nu_{\mu,\tau}$ or possibly a sterile neutrino were strongly favored over any plausible astrophysical uncertainty in explaining the results [84]. Combining with information on the spectrum and time dependence of the signal (which could vary between day and night due to matter effects in the Earth) zeroed in on two favored regions for the oscillation parameters, one with small mixing angles (SMA) similar to the quark mixing, and one with large mixing angles (LMA), and with $\Delta m^2 \sim 10^{-5} - 10^{-4}$ eV$^2$. The LMA solution allowed oscillations to $\nu_{\mu,\tau}$ but not to sterile neutrinos as the primary process (they were differentiated by the sensitivity to $\nu_{\mu,\tau}$ in the water Čerenkov experiments), while the SMA solution allowed both$^{21}$.

The situation was clarified by the Sudbury Neutrino Observatory (SNO) experiment [85].

$^{21}$Big bang nucleosynthesis also disfavored the LMA solution for sterile neutrinos.
Flux at 1 AU (cm$^{-2}$s$^{-1}$MeV$^{-1}$) [for lines, cm$^{-2}$s$^{-1}$]

![Figure 15: Total predicted flux vs. energy of Solar neutrinos in the Standard Solar Model. The ranges of sensitivity of the various experiments is also shown, the SNO range being similar to SuperK (from [83]).](image)

SNO uses heavy water $D_2O$, and can measure three reactions,

$$\nu_e + D \rightarrow e^- + p + p,$$

$$\nu_{e,\mu,\tau} + D \rightarrow \nu_{e,\mu,\tau} + p + n,$$

$$\nu_{e,\mu,\tau} + e^- \rightarrow \nu_{e,\mu,\tau} + e^-.$$ 

In the first (charged current) reaction, the deuteron breakup can be initiated only by the $\nu_e$, while the second (neutral current) reaction can be initiated by neutrinos of all the active flavors with equal cross section. SNO could therefore determine the $\nu_e$ flux arriving at the detector and the total flux into active neutrinos (which would equal the initially produced $\nu_e$ flux if there are no oscillations into sterile neutrinos) separately, as in Figure 16. SNO observed that the total flux is around three times that of $\nu_e$, establishing that oscillations indeed take place, and that the total flux agrees well with the predictions of the SSM. They also verified the LMA solution and that the Solar mixing angle $\theta_{12}$ is large but not maximal, i.e., $\sin^2 2\theta_{12} \sim 0.8$.

The Solar neutrino conversions are actually not due just to the vacuum oscillations as the neutrinos propagate from the Sun to the Earth. One must also take into account the coherent forward scattering [86, 87] of the neutrinos from matter in the Sun (and in the Earth at night), which introduces the analog of an index of refraction. This is of order $G_F^2$ rather than $G_F^2$, and it distinguishes $\nu_e$, $\nu_{\mu,\tau}$, $\bar{\nu}_e$, $\bar{\nu}_{\mu,\tau}$, and sterile neutrinos $\nu_s$ because of their different weak interactions. The propagation equation in electrically neutral matter
Figure 16: Fluxes of $^8B$ Solar neutrinos deduced from the SNO charged current (CC), neutral current (NC) and electron scattering (ES) reactions, and the prediction of the Standard Solar Model (from [85]).

is

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_{\mu,\tau,s} \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{2E} \cos 2\theta + \sqrt{2} G_F n \\ \frac{\Delta m^2}{2E} \sin 2\theta \end{pmatrix} \begin{pmatrix} \frac{\Delta m^2}{2E} \cos 2\theta - \sqrt{2} G_F n \\ \frac{\Delta m^2}{2E} \sin 2\theta \end{pmatrix},$$

where

$$n = \begin{cases} n_e & \text{for } \nu_e \rightarrow \nu_{\mu,\tau}, \\ n_e - \frac{1}{2} n_n & \text{for } \nu_e \rightarrow \nu_s, \end{cases}$$

and where $n_e(n_n)$ is the number density of electrons (neutrons). This reduces to the vacuum oscillation case (with parameters $\theta$ and $\Delta m^2$) for $n = 0$. Under the right conditions, the matter effect can greatly enhance the transitions. In particular, if the Mikheyev-Smirnov-Wolfenstein (MSW) resonance condition $\frac{\Delta m^2}{2E} \cos 2\theta = \sqrt{2} G_F n$ is satisfied, the diagonal elements vanish and even small vacuum mixing angles lead to a maximal effective mixing angle. In practice $n$ decreases as the neutrinos propagate from the core of the Sun where they are produced to the outside. For the LMA parameters, the higher energy Solar neutrinos all encounter a resonance layer. It is hoped that a future precise experiment sensitive to the lower energy $pp$ neutrinos will observe the transition between the vacuum and MSW dominated regimes.

Recently, the KamLAND long-baseline reactor experiment [88] in Japan has beautifully confirmed the LMA solution, free of any astrophysical uncertainties, by observing a depletion and spectral distortion of $\bar{\nu}_e$ produced by power reactors located $O(100)$ km away. The combination of KamLAND and the Solar experiments also limits the amount of oscillations into sterile neutrinos that could accompany the dominant oscillations into $\nu_{\mu,\tau}$. 

24
9 Neutrino Oscillation Patterns

The neutrino oscillation data is sensitive only to the mass-squared differences. The atmospheric and Solar neutrino mass-squares are given by

\[ |\Delta m^2_{32}| \equiv |\Delta m^2_{\text{atm}}| \sim 2 \times 10^{-3} \text{ eV}^2, \]
\[ \Delta m^2_{21} \equiv \Delta m^2_{\text{solar}} \sim 8 \times 10^{-5} \text{ eV}^2. \]

Vacuum oscillations depend only on the magnitude of \( \Delta m^2 \), so the sign of \( \Delta m^2_{\text{atm}} \) is not known, while MSW (matter) effects establish that \( \Delta m^2_{\text{solar}} > 0 \). The respective mixing angles are \( \sin^2 \theta_{23} > 0.92 \) (90%), consistent with maximal; and \( \sin^2 \theta_{12} \sim 0.8 \), i.e., \( \tan^2 \theta_{12} = 0.40_{-0.07}^{+0.09} \), which is large but not maximal. On the other hand, the third angle is small, \( \sin^2 \theta_{13} < 0.03 \) (90%) from short-baseline (\( \sim 1 \text{ km} \)) reactor disappearance limits, especially CHOOZ [73].

The LSND experiment [89] at Los Alamos has claimed evidence for oscillations, especially \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \), with \( |\Delta m^2_{\text{LSND}}| \gtrsim 1 \text{ eV}^2 \) and small mixing. This has not been confirmed by the KARMEN experiment, but there is a small parameter region allowed by both. The Fermilab MiniBooNE experiment is currently running, and should be able to confirm or exclude the LSND results. If LSND is confirmed, the most likely explanation would be that there is mixing of a fourth neutrino with the three known ones. This would have to be sterile because the invisible \( Z \) width excludes a fourth light active neutrino.

9.1 Three Neutrino Patterns

If the LSND results are not confirmed, then the remaining data can be described by oscillations amongst the three light active neutrinos. Let us choose a phase convention in which the mass eigenvalues are real and positive, and label the states such that \( m_1 < m_2 \) is responsible for the Solar oscillations, while the atmospheric oscillations are due to the 32 mass-squared difference. Since neither the sign of \( \Delta m^2_{32} \) nor the absolute scale of the masses is known, there are several possible patterns for the masses. These include:

1. The normal (or ordinary) hierarchy, i.e., \( m_1 \ll m_2 \ll m_3 \), as in the left diagram in Figure 17. In this case, \( m_3 \simeq \sqrt{\Delta m^2_{\text{atm}}} \sim 0.04 - 0.05 \text{ eV}, \) \( m_2 \simeq \sqrt{\Delta m^2_{\text{solar}}} \sim 0.009 \text{ eV}, \) and \( m_1 \sim 0. \)

2. The inverted (or quasi-degenerate) hierarchy, i.e., \( m_1 \simeq m_2 \simeq \sqrt{\Delta m^2_{\text{atm}}} \sim 0.04 - 0.05 \text{ eV} \gg m_3, \) as in the right diagram in Figure 17.

3. The degenerate case, i.e., \( m_1 \simeq m_2 \simeq m_3, \) with small splittings responsible for the oscillations.

Of course, these are only limiting cases. One can interpolate smoothly from cases (1) and (2) to the degenerate case by increasing the mass of the lightest neutrino.
Figure 17: Three neutrino mass-squared patterns. The $\nu_e$ fraction of each eigenstate is in green (light gray in black & white printing), the $\nu_\mu$ fraction is indicated in red (dark gray), and the $\nu_\tau$ fraction in dark blue (black). The left (right) figure is the normal (inverted) hierarchy.

### 9.2 Four Neutrino Patterns

If the LSND data is confirmed, then there is most likely at least one light sterile neutrino. In the case of four neutrinos the mass-squared difference for LSND is $\Delta m_{LSND}^2 \sim 1$ eV$^2$. In the 2+2 pattern in Figure 18 the sterile component must be mixed in significantly with either the Solar or atmospheric pair. However, it is well established that neither the Solar nor the atmospheric oscillations are predominantly into sterile states, and this scheme is excluded. In the 3+1 schemes a predominantly sterile state is separated from the three predominantly active states. This is consistent with the Solar and atmospheric data, but excluded when reactor and accelerator disappearance limits are incorporated [90, 91]. However, some 5 $\nu$ (i.e., 3+2) patterns involving mass splittings around 1 eV$^2$ and 20 eV$^2$ are more successful [92]. All of these run into cosmological difficulties, although there are some (highly speculative/creative) loopholes [49].

Figure 18: The 2 + 2 (left) and 3 + 1 (right) four neutrino mass-squared patterns. One can invert the sign of $\Delta m_{LSND}^2$ and (for the 3 + 1 case) $\Delta m_{atm}^2$. 
10 Conclusions

Non-zero neutrino mass is the first necessary extension of the Standard Model. Most extensions of the SM predict non-zero neutrino masses at some level, so it is difficult to determine their origin. Many of the promising mechanisms involve very short distance scales, e.g., associated with grand unification or string theories. There are many unanswered questions. These include:

- Are the neutrinos Dirac or Majorana? Majorana masses, especially if associated with a seesaw or Higgs triplet, would allow the possibility of leptogenesis, i.e., that the observed baryon asymmetry was initially generated as a lepton asymmetry associated with neutrinos, and later converted to a baryon (and lepton) asymmetry by nonperturbative electroweak effects. The observation of neutrinoless double beta decay would establish Majorana masses (or at least \( L \) violation), but foreseeable experiments will only be sensitive to the inverted or degenerate spectra. If the neutrinos are Dirac, this would suggest that additional TeV scale symmetries or string symmetries/selection rules are forbidding Majorana mass terms.

- What is the absolute mass scale (with implications for cosmology)? This is very difficult, but ordinary and double beta decay experiments, as well as future CMB experiments, may be able to establish the scale.

- Is the hierarchy ordinary or inverted? Possibilities include matter effects in future long-baseline experiments or in the observation of a future supernova, the observation of \( \beta\beta_{0\nu} \) if the neutrinos are Majorana, and possibly effects in future Solar neutrino experiments.

- What is \( \theta_{13} \)? This is especially important because the observation of leptonic CP violation requires a non-zero \( \theta_{13} \). This will be addressed in a program of future reactor and long-baseline experiments, and possibly at a dedicated neutrino factory (from a muon storage ring).

- Why are the mixings large, while those of the quarks are small? For example, the simplest grand unified theory seesaws would lead to comparable mixings, although this can be evaded in more complicated constructions.

- If the LSND result is confirmed, it will suggest mixing between ordinary and sterile neutrinos, presenting a serious challenge both to particle physics and cosmology, or imply something even more bizarre, such as CPT violation.

- Are there any new \( \nu \) interactions or anomalous properties such as large magnetic moments? Most such ideas are excluded as the dominant effect for the Solar and atmospheric neutrinos, but could still appear as subleading effects.

Answering these questions and unraveling the origin of the masses is therefore an important and exciting probe of new particle physics. Future Solar neutrino experiments,
observations of the neutrinos from a future core-collapse supernova, and observation of high energy neutrinos in large underground/underwater experiments would also be significant probes not only of the neutrino properties but also the underlying astrophysics.

Acknowledgements

This work was supported in part by CONACYT (México) contract 42026–F, by DGAPA–UNAM contract PAPIIT IN112902, and by the U.S. Department of Energy under Grant No. DOE-EY-76-02-3071.
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