Investigation of students’ metacognitive failures in mathematical problem solving based on metacognitive behavior

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Abstract. This article aims to trace the students’ metacognitive failures in mathematical problem solving based on their metacognitive behaviors. There are three types of metacognitive failure in mathematical problem solving: metacognitive blindness, metacognitive vandalism and metacognitive mirage. From 20 students taken as research subjects, they were grouped into 2 groups. The first group is students who perform 2 times on the completion of mathematical problem and the second group is students who perform 3 times on the completion of mathematical problem. Research subjects were taken by a person representing each group. Data were collected by using a task sheet where the subject worked with think aloud then followed by interviewing based on the subject’s work. The findings in this study indicates that students who perform the completion of mathematical problem for 2 times experienced 6 times successful of metacognitive behavior and 6 times failures of metacognitive behaviors with the tendency to metacognitive vandalism. While students who perform the completion of mathematical problem for 3 times experienced 5 times successful of metacognitive behavior and 13 times failures of metacognitive behavior with the tendency to metacognitive vandalism and metacognitive mirage.

1. Introduction
Problem solving is very important in mathematics learning because it improves students’ ability to apply the accurate completion strategies to obtain more precise answers from a problem [1]. There are a lot of benefits of problem solving and one of them is to improve students’ ability in obtaining a more accurate answer from the problem given [1]. Regarding to the importance of problem solving, then it is incorporated into the curriculum [2]. In Mathematics Department, Faculty of Teacher Training and Education, Jambi University, problem solving is one of the learning materials that should be taught in Kapita Selekta I course [3]. Problem solving can be done by using high-level thinking process involving mental processes and students’ metacognitive.

Metacognitive is a very important role in problem solving [4,5,6]. Metacognitive is defined as "thinking about what is thought". Metacognitive is not only mean knowing but better understanding on how one knows what is known [7]. There are 3 metacognitive activities which are metacognitive awareness (MA), metacognitive evaluation (ME) and metacognitive regulation (MR) [8,9]. Furthermore, according to [9], metacognitive awareness includes 5 metacognitive behaviors which are thought of: 1) what is already known, 2) whether there is similar problem has ever been solved before,
3) whether there is previous problem solving which has been done that very helpful at the moment, 4) Knowing what must be done, and 5) knowing the type of the present problem. Metacognitive evaluation includes 5 metacognitive behaviors of recalling: 1) what I will do to solve the problem, 2) whether what I was doing work well or not, 3) I check the answers which has already done, 4) whether the problem solving was correct or not, and 5) I thought I cannot solve the problem. Metacognition regulation includes 4 metacognitive behaviors: 1) making a plan of completion, 2) making different ways to solve problems, 3) making the next plan for solving the problem, and 4) changing the way of problem solving.

Although metacognitive is very important in problem solving but there is still a metacognitive failure which exists in mathematical problem solving. Some researchers find metacognitive failures in mathematical problem solving, such as the failure of students’ metacognitive on mathematical problem solving based on the framework of student’s assimilation and accommodation [10]. Furthermore, the other researcher find the identification of various forms of metacognitive failures by highlighting the ways in which verbal interactions can affect each student’s thought [11].

There are 3 metacognitive failures namely metacognitive blindness, metacognitive vandalism and metacognitive mirage [11]. Metacognitive blindness is a metacognitive failure which occurs if the student does not notice that something is wrong during the problem-solving. Metacognitive vandalism is a metacognitive failure which occurs when students change the problem by applying improper conceptual structures to overcome the deadlock. In addition, metacognitive mirage is a metacognitive failure that occurs if the student does not have difficulty in solving the problem but leaves the right strategy and changes the correct calculation.

Metacognitive failure based on metacognitive behavior can be observed from several indicators during the mathematical problem-solving process. Indicators of metacognitive behavior in metacognitive blindness are where the students: 1) maintain inappropriate mathematical problem-solving strategies, 2) ignore the calculation errors, 3) do procedural errors, 4) make an error in understanding what is known from the problem 5) unable in linking an understanding with another understanding. Indicators of metacognitive behavior on metacognitive vandalism are when students: 1) change the condition of the problem according to their knowledge that they think acceptable, 2) add other information to the problem, 3) have difficulty to separate mathematical knowledge with other knowledge, 4) change the answer to be wrong, so that more important solutions are lost, 5) explore the idea too far and return to the original calculation. Indicators of metacognitive behavior on metacognitive mirage are where student: 1) do not make judgments about the truth of their work, 2) do not know whether he or she has solved the problem correctly and 3) is being confused to see if he has determined the correct way to solve the problem.

Based on the results of researchers’ observation in Program Studi Pendidikan Matematika FKIP Universitas Jambi, on January 2018, researchers found 85% of students had done metacognitive activity in problem solving but still failed in their metacognitive activities based on metacognitive behavior. Regarding to the description above, this study is aimed to trace students’ metacognitive failure based on metacognitive behavior.

2. Method
This study was conducted on January 2018 at the students of Mathematics Department, Faculty of Teacher Training and Education, Jambi University. Qualitative research method was applied in the study. There were 20 students taken as research subjects and they were grouped into two groups. The first group is students who perform two times on the completion of mathematical problem and the second group is students who perform three times. Research subjects were taken by choosing one student of each group to represent the two groups. Data were collected by using a task sheet that the subject worked with think aloud then followed by interviewing based on the work of the subject. The duties and interview guidelines used had been validated by validators. Research data were analyzed by applying several steps: (1) transcribing the results of think aloud and students’ interviews, (2) conducting data reduction by creating an abstraction, (3) encoding every thinking process done by student, (4) describing
structure of students’ thinking process, (5) analyzing what happened during the study, and (6) drawing conclusions.

3. Result and discussion
This section will describe the process of changing the metacognitive behavior at the first subject (S1) and the second subject (S2) in each completion.

3.1. Exposure data of S1

3.1.1. S1’s metacognitive behavior on the first problem completion of mathematical problems. S1 thought the average velocity of a person climbing a hill was 1.5 km, arriving at the top of the hill he took a break with sufficient time, the average velocity of going down from the hill was 3 times faster than the average velocity of climbing the hill, the time needed to climb up and go down the hill was 6 hours, determine the distance from the foothill to the top of the hill (A11) was a successful metacognitive behavior. S1 thought when climbing the hill was a half of the total time (E11) which was the failure of metacognitive behavior. In this case, S1 changed the condition of the problem in accordance with the knowledge that S1 thought applicable. Thus, S1 experienced metacognitive vandalism. Further S1 thought that the distance of climbing the hill was the average velocity multiplied by the time (A31) and it is a successful metacognitive behavior.

In determining the distance, S1 thought that the distance of climbing the hill was the average velocity of climbing the hill multiplied by the time needed to climb the hill and the distance of going down from the hill was the average velocity of going down from the hill multiplied by the time of going down the hill (E21). Thus, it categorized as a successful metacognitive behavior. By using the time of climbing and going down the hill which was 3 hours, S1 thought that to get the distance of climbing the hill was 4.5 km and the distance of going down from the hill was 13.5 km (R11) where it was a failure of metacognitive behavior. In this case, S1 also changed the condition of the problem in accordance with the knowledge that S1 thought was applicable so that S1 experienced metacognitive vandalism. Next S1 thought that the results obtained were unacceptable because the climbing distance was not equal to the distance of going down from the hill (E41) where it was a successful metacognitive. Next, changes in S1’s metacognitive behavior on the first completion can be seen in figure 1.

3.1.2. S1’s metacognitive behavior on the second completion of mathematical problems. At the second completion, S1 thought the average velocity of climbing the hill, the average velocity of going down from the hill, the time used to climb and go down the hill, where the question being asked was the distance from the foothill to the top of the hill (A12) and it is a successful metacognitive behavior. S1 thought again there were 3 activities conducted by using 6 hours which were the activities of climbing the hills, taking a rest on the top of the hill, and going down from the hill (E13). In this case S1 added another information on the problem, so S1 experienced metacognitive vandalism. Next S1 thought that the results obtained were unacceptable because the climbing distance was not equal to the distance of going down from the hill (E41) where it was a successful metacognitive. Next, changes in S2’s metacognitive behavior on the second completion can be seen in figure 2.

Based on the description above, it can be seen that A11, A21, A31, E21 and E41 are successful metacognitive behavior of S1, while E11, E13, E23, E31, R11 and R21 are a failure of student’s metacognitive behavior of S1.
3.2. Exposure data of S2

3.2.1. S2’s metacognitive behavior the first problem solving of mathematical problems. S2 thought the average velocity of a person climbing a hill was 1.5 km, when he arrived at the top of the hill, he took a rest with sufficient time, the average velocity of going down from the hill was 3 times faster than the average velocity of climbing the hill, the time needed to climb up and go down the hill was 6 hours, and the question was to determine the distance from the foothill to the top of the hill (A11) where it was a successful metacognitive behavior. S2 drew a sketch of what had been understood by S2 (A32) where it was a successful metacognitive behavior. Next S1 thought the time needed to climb and go down the hill was 6 hours, while the time to take a rest at the top of the hill was for X hours (E11). In this case S2 adds another information to the problem, so S2 has metacognitive vandalism.

Next S2 thought that there was a relationship of mathematical problems with the existing problems in physics (A21) where it was a successful metacognitive behavior. S2 thought using the formula of total average velocity obtained by summing the average total velocity with the average velocity of going down from the hill (R11). In this case, S2 does not make a judgment about the truth of his work, so S2 has metacognitive mirage. S2 thought to determine the distance from the foothill to the top of the hill but S2 does not use the physics formula (E22). In this case the S2 was difficult to associate mathematical knowledge with another knowledge, so that S2 underwent the metacognitive vandalism. S2 thought again to determine the distance of climbing the hill which was equal to the distance of going down from the hill so that S2 divided by two with the distance of climbing the hill with the distance of going down the hill (R12). In this case S2 changed the condition of the problem in accordance with the knowledge that S2 thought applicable, so that S2 had metacognitive vandalism. Next, changes in S2’s metacognitive behavior of S2 can be seen in figure 3.

3.2.2. S2’s metacognitive behavior on the second completion of the mathematical problem. On the second completion, S2 thought that someone needed time to take rest on the top of the hill for X hours (A32). In this case, S2 adds another information to the problem, so S2 has metacognitive failure which is metacognitive vandalism. S2 thought that the total distance obtained by multiplying the average total velocity with the time reduced with X hours (R23). In this case, S2 did not know whether he solves the problem correctly, so that S2’s metacognitive failure is metacognitive blindnes. S2 also thought that the distance from the foothill to the top of the hill was 16.5 km (R31). In this case, S2 does not know whether he has completed the problem correctly or wrong, so that S2 metacognitive failure is metacognitive mirage. S2 thought...
again if the rest time at the top of the hill was 40 minutes then the total distance was 32 km (E32). In this case, S2 maintains an inappropriate problem-solving strategy, so that S2 metacognitive failure is metacognitive blindness. S2 thought that the distance from the foothill to the top of the hill was 16 km which was obtained from the total distance divided by 2 (R32). In this case S2 does not know whether he has solved the problem correctly, so that S2’s metacognitive failure is a metacognitive mirage. S2 thought that the rest time at the top of the hill was longer then the shorter of the distance from the foothill to the top of the hill (E41). In this case, S2 does not make an assessment of the truth of his work, so that S2 metacognitive failure is metacognitive mirage. Next, changes in S2’s metacognitive behavior can be seen in figure 4.

![Figure 4](image4.png)

Description: 
- : successful behavior,
- : metacognitive blindness,
- : metacognitive vandalism,
- : the direction of changes in metacognitive behavior

**Figure 4.** Changes in S2’s metacognitive behavior on the second problem solving.

3.2.3. S2’s metacognitive behavior on the third completion of the mathematical problem. On the third completion, S2 thought the essentials of the given problem (A12) which is a successful metacognitive behavior. S2 though the proportion between the average velocities of climbing the hill with average velocity of going down from the hill was 1: 3 (E24) which is a successful metacognitive behavior. Next S2 thought the distance from the foothill to the top of the hill by multiplying the total velocity with the proportion of the average velocity of climbing the hill and the average velocity of going down from the hill (R41). In this case, S2 changes the condition of the problem according to his knowledge that he considered applicable, so that S2’s metacognitive failure is metacognitive vandalism.

Finally, S2 thought to do calculation of the total distance by using the formula of the total time equal the total distance divided by the average proportion of velocity value with the final average velocity (E25). In this case, S2 changes the answer to be wrong and the important solution becomes lost, so S2 metacognitive failure is metacognitive vandalism. Next, changes in S2’s metacognitive behavior on the third completion can be seen in figure 5.

![Figure 5](image5.png)

Description: 
- : successful behavior,
- : metacognitive vandalism,
- : the direction of changes in metacognitive behavior

**Figure 5.** Changes in S2’s metacognitive behavior on the third problem solving.
Based on the description above, it can be seen A11, A12, A21, A31, and E24 are successful metacognitive behavior of S2, while A32, E11, E22, E25, E31, E32, E41, R11, R12, R23, R31, R32, and R41 are failures of S2’s metacognitive behavior.

This study deals with metacognitive failure, in which the students’ metacognitive behavior results in improper problem solving. The results obtained identify the various forms of metacognitive failure of students in problem solving caused by changes in metacognitive behavior. Student’s metacognitive behavior was observed based on metacognitive activity developed from metacognitive cards [9]. The study used a qualitative approach which studies students’ metacognitive activities in solving math problems. The result of the analysis shows that students’ metacognitive failure on problem solving is caused by two things: the students do not understand that the problem given is different from the problems they have previously solved and the students often add information to facilitate the solving of mathematical problems. Metacognitive behavior of students can provide information to lecturers to encourage successful students in solving mathematical problems which are very important in learning mathematics [11].

4. Conclusion
From this study, it can be concluded that students who perform 2 times completion on mathematical problem experienced 12 times metacognitive behavior which consists of 6 times being successful in metacognitive behavior and 6 times being failed in metacognitive behavior with students’ tendency to metacognitive failures in solving mathematical problem is metacognitive vandalism. Then, Students who perform 3 times completion on mathematical problem experienced 18 times metacognitive behavior which consist of 5 times being successful in metacognitive behaviors and 13 times being failed in metacognitive behavior with the tendency to metacognitive failure of math problem solving is metacognitive vandalism and metacognitive mirage.

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