Gravity–Induced Interference and Continuous Quantum Measurements.

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Abstract

Gravity–induced quantum interference is a remarkable effect that has already been confirmed experimentally, and it is a phenomenon in which quantum mechanics and gravity play simultaneously an important role. Additionally, a generalized version of this interference experiment could offer the possibility to confront against measurement outputs one of the formalisms that claim to give an explanation to the so called quantum measurement problem, namely the restricted path integral formalism. In this work we will analyze a possible extension of Colella, Overhauser, and Werner experiment and find that in the context of the restricted path integral formalism we obtain new interference terms that could be measured in an extended version of this experimental construction. These new terms not only show, as in the first experiment, that at the quantum level gravity is not a purely geometric effect, it still depends on mass, but also show that interference does depend on some parameters that appear in the restricted path integral formalism, thus offering the possibility to have a testing framework for its theoretical predictions.

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1 Introduction.

In the context of classical mechanics mass $m$ does not appear in the motion equation of a particle trajectory, gravity in classical mechanics could be then considered a purely geometric effect. But this situation does not happen in quantum theory, here mass does no longer cancel, instead it always appears in the combination $m/\hbar$ [1].

In order to detect nontrivial quantum–mechanical effects of gravity the corresponding experimental device must be able to analyze effects in which $\hbar$ appears explicitly. The free fall of an elementary particle does not allow the study of this issue, because Ehrenfest theorem (where $\hbar$ does not appear) suffices to account for it. Even the red shift effect of a photon in a gravitational field [2] does not offer this possibility, here only a frequency shift is measured, and once again $\hbar$ does not appear explicitly.

But in 1975 a neutron interferometer was used in order to detect the quantum–mechanical phase shift of neutrons caused by their interaction with Earth’s gravitational field [3]. A nearly monoenergetic beam of thermal neutrons is split in two parts, each one of these new beams follow different paths in Earth’s gravitational field, and then they are brought together. A gravity–induced phase shift emerges which depends on $(m/\hbar)^2$, being $m$ the neutron mass. It is interesting to comment that this phase shift due to gravity is seen to be verified to well within one percent. An important point in the theoretical analysis of this phase shift lies in the fact that the wave function of the thermal neutrons is much smaller than the macroscopic dimension of the loop formed by the two alternate paths, therefore the concept of classical trajectory can be used in the derivation of the expression for the phase shift [1]. At this point we may wonder how this expression has to be reformulated if we do not consider this last condition.

It would be interesting to relate this gravity–induced interference with one of the most important problems in Quantum Theory (QT), namely the so called quantum measurement problem. In the attempts to solve this very old conundrum we may find several approaches, one of them is the so called Restricted Path Integral Formalism (RPIF) [4]. This formalism explains a continuous quantum measurement with the introduction of a restriction on the integration domain of the corresponding path integral. This last condition can also be reformulated in terms of a weight functional that has to be considered in the path integral.

Let us explain this point a little bit better, and suppose that we have a particle which shows one–dimensional movement. The amplitude $A(q'', q')$ for this particle to move from the point $q'$ to the point $q''$ is called propagator. It is given by Feynman [5]
\[ A(q'', q') = \int d[q] \exp\left(\frac{i}{\hbar} S[q]\right), \] (1)

here we must integrate over all the possible trajectories \( q(t) \) and \( S[q] \) is the action of the system, which is defined as

\[ S[q] = \int_{t'}^{t''} dt L(q, \dot{q}). \] (2)

Let us now suppose that we perform a continuous measurement of the position of this particle, such that we obtain as result of this measurement process a certain output \( a(t) \). In other words, the measurement process gives the value \( a(t) \) for the coordinate \( q(t) \) at each time \( t \), and this output has associated a certain error \( \Delta a \), which is determined by the experimental resolution of the measuring device. The amplitude \( A_a(q'', q') \) can be now thought of as a probability amplitude for the continuous measurement process to give the result \( a(t) \). Taking the square modulus of this amplitude allows us to find the probability density for different measurement outputs.

Clearly, the integration in the Feynman path–integral should be restricted to those trajectories that match with the experimental output. RPIF says that this condition can be introduced by means of a weight functional \( \omega_a[q] \) [4]. This means that expression (1) becomes now under a continuous measurement process

\[ A_a = \int d[q] \omega_a[q] \exp(iS[q]). \] (3)

The more probable the trajectory \( [q] \) is, according to the output \( a \), the bigger that \( \omega_a[q] \) becomes [4]. This means that the value of \( \omega_a[q] \) is approximately one for all trajectories \( [q] \) that agree with the measurement output \( a \) and it is almost 0 for those that do not match with the result of the experiment. Clearly, the weight functional contains all the information about the interaction between measuring device and measured system.

This formalism has been employed in several situations, i.e., the analysis of the response of a gravitational wave antenna of Weber type [4], the measuring process of a gravitational wave in a laser–interferometer [6], or even to explain the emergence of the classical concept of time [7].

But even though there are already some theoretical predictions that could render a framework which could allow us to confront RPIF against experimental outputs [8], it is also true that more results are needed in this direction.

The idea in this work is to derive some results using RPIF in the context of a generalized gravity–induced interference scenario. We will consider a possible generalization of Colella, Overhauser, and Werner (COW) experiment, namely the condition...
around the size of the wave packets (they must be smaller than the dimensions of
the loop of the two alternate paths) will not be introduced anymore, i.e., we can not
analyze this interference process using the classical trajectories of the split beams.
Each one of these split beams will also be subject to the continuous monitoring
of its vertical coordinate, additionally we introduce the possibility that the whole
experimental device could be mounted on an accelerated (but not rotating) coordinate
system.

The interference pattern under these conditions will show a dependence not only
on the mass of the respective particles, i.e., once again at the quantum level gravity is
not a purely geometric effect, but it also depends on some parameters that appear in
RPIF and therefore renders the possibility of confronting its theoretical predictions
with a generalized version of COW experimental construction. Noninertial effects
also emerge.

2 Generalized Gravity–Induced Interference.

Let us consider the case of a particle with mass $m$ located in a region where the Earth’s
gravitational field can be considered homogeneous, i.e., the gravitational acceleration
$g$ is a constant. Then the motion equation along the vertical direction $Z$ with respect
to an inertial reference frame is given by

$$m\ddot{Z} + mg = 0.$$  \(4\)

Now we observe this particle using a reference frame which has an acceleration
along the vertical direction with respect to our previous inertial coordinate equal to
$g[f(t) - 1]$, where $f(t)$ is an arbitrary function of time. Denoting by $z$ the vertical
coordinate on this accelerated reference system we obtain as motion equation the
following expression

$$m\ddot{z} + mgf(t) = 0,$$  \(5\)

and the corresponding Lagrangian is [9]
At this point we introduce a generalized version of COW experiment [3]. A beam of particles is split into two parts and then brought together. Each one of these two split beams follows a different path along the vertical direction $z$. We do not assume that the wave packets are much smaller than the dimension of the loop formed by the alternate paths. We also monitor continuously the $z$ coordinate of each one of these two beams, and the whole experimental construction is at rest with respect to this accelerated reference frame that we have just introduced.

Under these conditions the associated restricted path integral for each one of the beams is

$$U_{[c(t)]}(z_2, z_1) = \int_\Omega \omega_{[c(t)]}[z(\tau)]d[z(\tau)]exp(iS[z(\tau)]/\hbar), \quad (7)$$

$$S = \int_{\tau''}^{\tau'} \frac{1}{2} m(\dot{z})^2 - mgf(t)z \, d\tau. \quad (8)$$

Here $z_1$ and $z_2$ are the endpoints of the motion along the vertical direction, $\omega_{[c(t)]}[z(\tau)]$ is the corresponding weight functional which contains all the information concerning the measuring apparatus, $c(t)$ the resulting measured trajectory, and $\Omega$ the set of all functions such that $z(\tau') = z_1$ and $z(\tau'') = z_2$. Of course, each beam has its particular $c(t)$. The propagator contains also a contribution coming from $x$ and $y$ but it plays no role in this analysis.

At this point, in order to obtain theoretical predictions, we must choose a particular expression for $\omega_{[c(t)]}[z(\tau)]$. We know that the results coming from a Heaveside weight functional [10] and those coming from a gaussian one [11] coincide up to the order of magnitude. This last result allows us to consider as our weight functional a gaussian expression. But a sounder justification of this choice comes from the fact that there are measuring processes in which the weight functional has precisely a gaussian form [12]. In consequence we could think about a measuring device whose weight functional is very close to a gaussian behavior.

Thus we have that

$$\omega_{[c(t)]}[z(\tau)] = exp\{-\frac{2}{T\Delta c^2} \int_{\tau'}^{\tau''} [z(\tau) - c(\tau)]^2 d\tau\}. \quad (9)$$

$$L = \frac{1}{2} m(\dot{z})^2 - mgf(t)z + \frac{1}{2} m\left((\dot{x})^2 + (\dot{y})^2\right). \quad (6)$$
Here \( T = \tau'' - \tau' \) and \( \Delta c \) represents the error in the position measuring. The path integral of each beam is then

\[
U_{\{c(t)\}}(z_2, z_1) = \int_{\Omega} d[z(\tau)] \exp\left\{ \frac{i}{\hbar} \int_{\tau'}^{\tau''} \left[ \frac{1}{2} m(\dot{z})^2 - mg f(t) z \right] d\tau \right\} \times \\
\exp\left\{ -\frac{2}{T\Delta c^2} \int_{\tau'}^{\tau''} [z(\tau) - c(\tau)]^2 d\tau \right\}. \tag{10}
\]

This path integral is easily calculated \([4]\)

\[
U_{\{c(t)\}}(z_2, z_1) = \exp\left( -2<\frac{c^2}{\Delta c^2}> \right) \sqrt{\frac{mw}{2\pi i\hbar \sin(wT)}} \exp\left( \frac{i}{\hbar} S \right). \tag{11}
\]

Here \( \exp\left( -2<\frac{c^2}{\Delta c^2}> \right) = \frac{1}{T} \int_{\tau'}^{\tau''} c^2(t) dt \), and \( S \) is the classical action of a driven complex harmonic oscillator \([13]\) defined by \( \frac{1}{2} m(\dot{z})^2 - \frac{w^2}{2} z^2 + F(t)z = 0 \), where \( F(t) = -mg f(t) - i\frac{4\hbar}{T\Delta c} c(t) \) and \( w = \sqrt{-i\frac{4\hbar}{mT\Delta c}} \).

\[
S = \frac{mw}{2\sin(wT)} \left\{ \left[ z_1^2 + z_2^2 \right] \cos(wT) - 2z_1z_2 \right. \\
+ \frac{2z_2}{mw} \int_{\tau'}^{\tau''} F(t) \sin[w(t - \tau')] dt \\
- \frac{2z_1}{mw} \int_{\tau'}^{\tau''} F(t) \sin[w(t - \tau'')] dt \\
- \frac{2}{(mw)^2} \int_{\tau'}^{\tau''} dt \int_{\tau'}^{t} ds F(t) \times \\
\left. \left[ \sin[w(\tau'' - t)] F(s) \sin[w(s - \tau')] \right] \right\}. \tag{12}
\]
3 Interference Terms.

The resulting interference pattern at point $z_2$, which appears when the two beams are recombined is given by

$$|U_{[a(t)]}(z_2, z_1) + U_{[b(t)]}(z_2, z_1)|^2. \quad (13)$$

Here we denote the measurement outputs for the $z$ coordinate of the beams by $a(t)$ and $b(t)$. If we re-write the actions of the two complex oscillators as $S_{[a(t)]}^{(1)} + iS_{[a(t)]}^{(2)}$ and $S_{[b(t)]}^{(1)} + iS_{[b(t)]}^{(2)}$, where $S_{[a(t)]}^{(1)}, S_{[b(t)]}^{(1)}, S_{[a(t)]}^{(2)},$ and $S_{[b(t)]}^{(2)}$ are real functions, then the interference term becomes

$$I = \cos \left( \frac{1}{\hbar} [S_{[a(t)]}^{(1)} - S_{[b(t)]}^{(1)}] \right). \quad (14)$$

After a lengthy calculation we find that we may regroup the terms in $S_{[a(t)]}^{(1)} - S_{[b(t)]}^{(1)}$ in five different contributions ($S_{[a(t)]}^{(1)} - S_{[b(t)]}^{(1)} = I_1 + I_2 + I_3 + I_4 + I_5$). The first two components are

$$I_1 = [z_1^2 + z_2^2] \sqrt{\frac{m}{2\hbar T}} \left( \frac{1 - e^{-4\theta} + 2e^{-2\theta} \sin(2\theta)}{1 + e^{-4\theta} - 2e^{-2\theta} \cos(2\theta)} \Delta a \right)$$

$$- \left( \frac{1 - e^{-4\rho} + 2e^{-2\rho} \sin(2\rho)}{1 + e^{-4\rho} - 2e^{-2\rho} \cos(2\rho)} \Delta b \right), \quad (15)$$

$$I_2 = -\sqrt{\frac{8m}{\hbar T}} z_1 z_2 \left( \frac{(1 - e^{-2\theta}) \cos(\theta) + (1 + e^{-2\theta}) \sin(\theta)}{e^\theta[1 + e^{-4\theta} - 2e^{-2\theta} \cos(2\theta)]} \Delta a \right)$$

$$- \left( \frac{(1 - e^{-2\rho}) \cos(\rho) + (1 + e^{-2\rho}) \sin(\rho)}{e^\rho[1 + e^{-4\rho} - 2e^{-2\rho} \cos(2\rho)]} \Delta b \right). \quad (16)$$

Here we have that $\theta = \sqrt{\frac{2\pi \hbar T}{m \Delta a^2}}$ and $\rho = \sqrt{\frac{2\pi \hbar T}{m \Delta b^2}}$. $I_1$ and $I_2$ show clearly, once again, that at the quantum level gravity is not a purely geometric effect. Indeed, here we have the combination $m/\hbar$, as in the usual case [1, 3]. But these contributions to
the interference pattern depend also on $\Delta a$ and on $\Delta b$. This fact means that if we measure the $z$ coordinate of the beams using two different devices (the corresponding measuring errors are not the same, i.e., $\Delta a \neq \Delta b$), then this difference will render a nonvanishing contribution to the interference pattern. This last result could allow us to test the theoretical predictions of RPIF against the measurements outputs coming from a generalized version of COW experiment.

The second pair of interference terms is

$$I_3 = -\frac{m}{\hbar} \frac{z_1 e^{-\theta}}{1 + e^{-4\theta} - 2e^{-2\theta} \cos(2\theta)} \left\{ (1 - e^{-2\theta}) \cos(\theta) \times \right.$$ \[
\int_{\tau'}^{\tau''} \frac{4\hbar}{m T \Delta a^2} a(t) \sin(\gamma/\sqrt{2})(1 + e^{-2\gamma/\sqrt{2}}) 
- g f(t) \cos(\gamma/\sqrt{2})(1 - e^{-2\gamma/\sqrt{2}}) \right\} dt 
- (1 + e^{-2\theta}) \sin(\theta) \times \]
\[
\int_{\tau'}^{\tau''} \frac{4\hbar}{m T \Delta a^2} a(t) \cos(\gamma/\sqrt{2})(1 - e^{-2\gamma/\sqrt{2}}) 
+ g f(t) \sin(\gamma/\sqrt{2})(1 + e^{-2\gamma/\sqrt{2}}) \right\} dt 
\]
\[
+ \frac{m}{\hbar} \frac{z_1 e^{-\rho}}{1 + e^{-4\rho} - 2e^{-2\rho} \cos(2\rho)} \left\{ (1 - e^{-2\rho}) \cos(\rho) \times \right.$$ \[
\int_{\tau'}^{\tau''} \frac{4\hbar}{m T \Delta a^2} a(t) \sin(\gamma/\sqrt{2})(1 + e^{-2\gamma/\sqrt{2}}) 
- g f(t) \cos(\gamma/\sqrt{2})(1 - e^{-2\gamma/\sqrt{2}}) \right\} dt 
- (1 + e^{-2\rho}) \sin(\rho) \times \]
\[
\int_{\tau'}^{\tau''} \frac{4\hbar}{m T \Delta a^2} a(t) \cos(\gamma/\sqrt{2})(1 - e^{-2\gamma/\sqrt{2}}) 
+ g f(t) \sin(\gamma/\sqrt{2})(1 + e^{-2\gamma/\sqrt{2}}) \right\} dt \right\}, (17)
\]

$$I_4 = \frac{m}{\hbar} \frac{z_2 e^{-\theta}}{1 + e^{-4\theta} - 2e^{-2\theta} \cos(2\theta)} \left\{ (1 - e^{-2\theta}) \cos(\theta) \times \right.$$ \[
\int_{\tau'}^{\tau''} \frac{4\hbar}{m T \Delta a^2} a(t) \sin(\gamma/\sqrt{2})(1 + e^{-2\gamma/\sqrt{2}}) 
- g f(t) \cos(\gamma/\sqrt{2})(1 - e^{-2\gamma/\sqrt{2}}) \right\} dt 
- (1 + e^{-2\rho}) \sin(\rho) \times \]
\[
\int_{\tau'}^{\tau''} \frac{4\hbar}{m T \Delta a^2} a(t) \cos(\gamma/\sqrt{2})(1 - e^{-2\gamma/\sqrt{2}}) 
+ g f(t) \sin(\gamma/\sqrt{2})(1 + e^{-2\gamma/\sqrt{2}}) \right\} dt \right\}.$$
\[
\int_{\tau'}^{\tau''} e^{\mu / \sqrt{2}} \frac{4\hbar}{mT\Delta a^2} a(t) \cos(\mu / \sqrt{2})(1 - e^{-2\mu / \sqrt{2}}) \\
+ gf(t) \sin(\mu / \sqrt{2})(1 + e^{-2\mu / \sqrt{2}}) \, dt \bigg] \\
- \frac{m}{\hbar} \frac{z_2e^{-\rho}}{1 + e^{-4\rho} - 2e^{-2\rho} \cos(2\rho)} \{(1 - e^{-2\rho})\cos(\rho) \times \\
\int_{\tau'}^{\tau''} e^{\nu / \sqrt{2}} \frac{4\hbar}{mT\Delta b^2} b(t) \sin(\nu / \sqrt{2})(1 - e^{-2\nu / \sqrt{2}}) \, dt \\
- g f(t) \cos(\nu / \sqrt{2})(1 - e^{-2\nu / \sqrt{2}}) \bigg] \\
- (1 + e^{-2\rho}) \sin(\rho) \times \\
\int_{\tau'}^{\tau''} e^{\nu' / \sqrt{2}} \frac{4\hbar}{mT\Delta b^2} b(t) \cos(\nu / \sqrt{2})(1 - e^{-2\nu / \sqrt{2}}) \\
+ gf(t) \sin(\nu / \sqrt{2})(1 + e^{-2\nu / \sqrt{2}}) \, dt \bigg}. 
\] (18)

In these last two expressions we have introduced the following definitions
\[
\gamma = \sqrt{\frac{4\hbar}{mT\Delta a^2}}(t - \tau''), \quad \Gamma = \sqrt{\frac{4\hbar}{mT\Delta b^2}}(t - \tau'), \quad \mu = \sqrt{\frac{4\hbar}{mT\Delta a^2}}(t - \tau'), \quad \nu = \sqrt{\frac{4\hbar}{mT\Delta b^2}}(t - \tau').
\]
Once again we may see that the factor \(m/\hbar\) appears in scene. The effects of performing the experiment on a noninertial reference frame emerge in these two expressions, i.e., \(f(t)\) is present in both of them. If the alternate paths are not the same, then an additional term contributes to the interference pattern, i.e., \(a(t)\) and \(b(t)\) appear explicitly. Of course, as in \(I_1\) and \(I_2\), we have a dependence on the measuring errors \(\Delta a\) and \(\Delta b\), a difference in the measuring errors is an interference source.

The presence of \(f(t)\) and the fact that \(a(t) \neq b(t)\) render a second possibility to confront RPIF theoretical predictions against a generalized COW experiment.

The last contribution to the interference term is
\[
I_5 = \frac{m}{\hbar} \frac{T e^{-\theta}}{\theta \left[1 + e^{-4\theta} - 2e^{-2\theta} \cos(2\theta)\right]} \left(1 - e^{-2\theta}\right) \cos(\theta) \\
- (1 + e^{-2\theta}) \sin(\theta) \times \int_{\tau'}^{\tau''} \int_{\tau'}^t \frac{e^{(\sigma + \epsilon)/2}}{2} \left[g^2(f(t) + f(s)) \\
- \left(\frac{4\hbar}{mT\Delta a^2}\right)^2(a(t) + a(s))\right] \cos\left(\frac{\epsilon - \sigma}{\sqrt{2}}\right) \times \\
(e^{-\sqrt{2} \epsilon} + e^{\sqrt{2} \epsilon}) - \cos\left(\frac{\epsilon + \sigma}{\sqrt{2}}\right)(1 + e^{-\sqrt{2}(\sigma + \epsilon)}) \right]
\]
\[
+ \frac{4h}{mT\Delta a^2}g[f(t)a(s) + f(s)a(t)][\sin(e + \sigma)\sqrt{\frac{e}{2}}(1 - e^{-\sqrt{2}(\sigma+\epsilon)})
+ (-e^{-\sqrt{2}\sigma} + e^{-\sqrt{2}\epsilon})\sin(\frac{e - \sigma}{\sqrt{2}})]dtds + [(1 - e^{-2\rho})\cos(\theta)
+ (1 + e^{-2\rho})\sin(\theta)] \int_{t''}^{t'} \int_{t'}^{t} \frac{e^{(\alpha+\beta)/2}}{2} \left(g^2(f(t) + f(s)) + \sin(\frac{e + \sigma}{\sqrt{2}})(1 - e^{-\sqrt{2}(\sigma+\epsilon)})\right)
- \frac{4h}{mT\Delta a^2^2}(a(t) + a(s))[\sin(\frac{e - \sigma}{\sqrt{2}})(-e^{-\sqrt{2}\sigma} + e^{-\sqrt{2}\epsilon})
+ \sin(\frac{e + \sigma}{\sqrt{2}})(1 - e^{-\sqrt{2}(\sigma+\epsilon)})]
- \frac{4h}{mT\Delta a^2^2}g[f(t)a(s) + f(s)a(t)][\cos(\frac{e - \sigma}{\sqrt{2}})(1 + e^{-\sqrt{2}(\sigma+\epsilon)})
+ (e^{-\sqrt{2}\sigma} + e^{-\sqrt{2}\epsilon})\cos(\frac{e - \sigma}{\sqrt{2}})]dtds \}
- \frac{m}{h\rho}[1 + e^{-4\rho} - 2e^{-2\rho}\cos(2\rho)] \left[-[(1 - e^{-2\rho})\cos(\rho)
- (1 + e^{-2\rho})\sin(\rho)] \int_{t''}^{t'} \int_{t'}^{t} \frac{e^{(\alpha+\beta)/2}}{2} \left(g^2(f(t) + f(s)) - (e^{-\sqrt{2}\alpha} + e^{-\sqrt{2}\beta}) - \cos(\frac{\beta + \alpha}{\sqrt{2}})(1 + e^{-\sqrt{2}(\alpha+\beta)})\right)
+ \frac{4h}{mT\Delta b^2^2}g[f(t)b(s) + f(s)b(t)][\sin(\frac{\beta - \alpha}{\sqrt{2}})(1 - e^{-\sqrt{2}(\alpha+\beta)})
+ (-e^{-\sqrt{2}\alpha} + e^{-\sqrt{2}\beta})\sin(\frac{\beta - \alpha}{\sqrt{2}})]dtds + [(1 - e^{-2\rho})\cos(\rho)
+ (1 + e^{-2\rho})\sin(\rho)] \int_{t''}^{t'} \int_{t'}^{t} \frac{e^{(\alpha+\beta)/2}}{2} \left(g^2(f(t) + f(s)) - (e^{-\sqrt{2}\alpha} + e^{-\sqrt{2}\beta})\right)
+ \sin(\frac{\beta + \alpha}{\sqrt{2}})(1 - e^{-\sqrt{2}(\alpha+\beta)})\right)\}
- \frac{4h}{mT\Delta b^2^2}g[f(t)b(s) + f(s)b(t)][\cos(\frac{\beta - \alpha}{\sqrt{2}})(1 + e^{-\sqrt{2}(\alpha+\beta)})
+ (e^{-\sqrt{2}\alpha} + e^{-\sqrt{2}\beta})\cos(\frac{\beta - \alpha}{\sqrt{2}})]dtds \}. \tag{19}
\]
In this last expression we have defined $\epsilon = \sqrt{\frac{4\hbar}{mT\Delta a^2}}(\tau'' - t)$, $\sigma = \sqrt{\frac{4\hbar}{mT\Delta a^2}}(s - \tau')$, $\alpha = \sqrt{\frac{4\hbar}{mT\Delta b^2}}(s - \tau')$, and finally $\beta = \sqrt{\frac{4\hbar}{mT\Delta b^2}}(\tau'' - t)$. Once again the term $m/\hbar$ emerges, as well as the dependence on the measuring errors $\Delta b$ and $\Delta a$ and on the alternate paths $a(t)$ and $b(t)$. Here a new feature appears, namely there are terms in $I_5$ which result from the multiplication of $f(t)$ and the alternate paths.

4 Conclusions.

We have analyzed, in the context of RPIF, a possible generalization of COW gravity–induced interference experiment in which the vertical coordinate of the two involved split beams is continuously measured. Mass emerges in the interference terms once again in the form $m/\hbar$, i.e., gravity is at the quantum level, as was already shown in COW experiment, not a purely geometric effect. We have also seen that interference depends not only on the alternate paths of the split beams, as in the original experiment, but also on the measuring error of the experimental devices. In other words, if we measure the vertical coordinate of the beams with experimental devices that have different measuring errors, then this difference will be an interference source.

It must also be mentioned that using Hawking’s approach no quantum gravity effect appears in connection with the here discussed problem [14].

Summing up, from this analysis we could conclude that RPIF theoretical predictions could be confronted against the measurement outputs of a generalized version of COW gravity–induced interference experiment.

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