The application of a priori structural information based regularization in image reconstruction in Magnetic Induction Tomography

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Abstract. Magnetic induction tomography (MIT) is a non-invasive contactless modality that could be capable of imaging the conductivity distribution of biological tissues. In this paper we consider the possibility of using absolute MIT voltage measurements for monitoring the progress of a peripheral hemorrhagic stroke in a human brain. The pathology is modelled as a local blood accumulation in the white matter. The solution of the MIT inverse problem is nonlinear and ill-posed and hence requires the use of a regularisation method. In this paper, we describe the construction and present the performance of a regularisation matrix based on a priori structural information of the head tissues obtained from a very recent MRI scan. The method takes the MRI scan as an initial state of the stroke and constructs a learning set containing the possible conductivity distributions of the current state of the stroke. This data is used to calculate an approximation of the covariance matrix and then a subspace is constructed using principal component analysis (PCA). It is shown by simulations the method is capable of producing a representative reconstruction of a stroke compared to smoothing Tikhonov regularization in a simplified model of the head.

1. Introduction

Magnetic induction tomography (MIT) is a low frequency electromagnetic modality, which aims to reconstruct the conductivity changes from coupled field measurements taken by inductive sensors [1]-[2]. MIT is a subject of research for medical clinical applications. One particular application of interest is imaging local blood accumulation in the brain also termed stroke. In this paper we show that absolute imaging for detecting the stroke using standard smoothing operators is a difficult task since the implemented a-priori information is not representative of the complex structure of the head. However, using images taken by MRI or CT, a regularization matrix can built which carries useful structural information about the head. It is shown this information makes the MIT inverse problem less under-determined and can make MIT useful for post diagnosis monitoring.

2. MIT system and forward problem

In order to generate the simulated measurement data and calculate the induced voltages, a custom eddy current software developed at the University of Manchester was employed [3]. With a pre-computed primary magnetic vector potential $A_0$ in empty space, the package solves the eddy current problem in the target and calculates the induced voltages in the receiver coils. The incident field $A_0$ is

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obtained via a simulation of the MIT system in a commercial FE package (COMSOL) (Figure 1b). As can be seen, the MIT system model comprises of 16 pairs of exciter and receiver coils arranged in two circles of radii 141.5mm and 131.5mm respectively. Given a conductivity distribution (Figure 1a) excitation coils are sequentially driven with a time varying sinusoidal unit current at a frequency of 10 MHz, and 256 computed induced voltages are collected for 16 excitations. In an attempt to increase the number of independent measurements, additional planes of data are computed by displacing the coil array vertically along the z direction in 20 mm steps.

![Figure 1. MIT system with a) Simulated Target (Head and stroke), and b) coil array](image)

3. Theory

For conductivity image reconstruction the non-linear MIT inverse problem is solved in an iterative scheme. In this process, the sensitivity matrix \( J \in \mathbb{R}^{m \times n} \) is updated in every iteration. The formulation used for computing the sensitivity is an extension of the Gezelowitz theorem to the general electromagnetic case. For optimisation, the damping Gauss Newton method is employed, where the corresponding iteration is described by the following equations:

\[
(J^T J + \lambda_k R)d_{\text{dGN}} = -J^T (F(\sigma_k) - D) - \lambda_k R \sigma_k
\]

\[
\sigma_{k+1} = \sigma_k + d_{\text{dGN}}
\]

where the regularization parameter \( \lambda_k \) is updated in a damping process identical to the Levenberg Marquardt method, and the a-priori information about the conductivity distribution is implemented via the regularisation matrix \( R \). Here, \( D \) is the simulated measurement data, \( F(\sigma_k) \) is the computed forward solution vector and \( \sigma_k \) is the absolute conductivity vector at the current iterate \( k \).

3.1. Regularisation

The inverse conductivity solution was calculated with two different regularization methods presented below:

a) 2nd Order Laplacian smoothing operator \( R_L = L^T L \), where \( L \) (Neighboring matrix) is a discrete difference operator between neighboring conductivity voxels. Typically, \( L_{ij} = -1 \) for \( i \neq j \) when two elements are neighbors, and \( L_{ij} = -\sum_{j \neq i} L_{ij} \)

b) Structural Subspace regularization matrix (SSRM) \( R_{\text{SSRM}} \). A learning set of expectable conductivity distributions is constructed (i.e. \( \sigma_i, i = 1, ..., Q \)) near which the true conductivity distribution is assumed to be. By assuming the relative probabilities for these estimated conductivity distributions are uniform an approximation for the covariance matrix \( C_{\sigma} \) can be calculated as: \( C_{\sigma} = Q^{-1} \sigma_i \sigma_i^T \). Then, the eigenvalues \( \lambda_i \) and the
corresponding eigenvectors \( \omega_i \) are computed from \( C_\sigma \). A subspace \( S_\omega \) is constructed by taking the \( M \) first eigenvectors corresponding to the largest singular values (i.e. \( S_\omega = \text{span}\{\omega_m | 1 \leq m \leq M\}, M \ll Q \)). The orthogonal projector onto the subspace \( S_\omega \) can be written as \( P = WW^T \), where \( W = (\omega_1, \ldots, \omega_M) \in \mathbb{R}^{N \times M} \). The regularization matrix is constructed as \( R_{SSRM} = L^T L \), where \( L = I - WW^T \). For more details on the method please see [4]

4. Computational experiments

The non-linear image reconstruction was tested with a model approximating the shape of the head. A spherical layer crudely approximating the cerebral spinal fluid (CSF) was added together with a rectangular perturbation simulating the abnormality Figure 1a). The head was meshed into 9012 cubic voxels of 7.5mm\(^3\) resolution which were distributed as (white matter: 7900, CSF: 1008, stroke region: 104) elements. The internal tissues were assigned conductivities as (white matter: 0.16, CSF: 1.5, stroke region: 1.1) Sm\(^{-1}\). Note the CSF conductivity has been reduced to 1.5 Sm\(^{-1}\) from the value reported in the literature [5] of 2 Sm\(^{-1}\) in order to compensate for the increased thickness caused by mesh resolution. 3D conductivity maps were reconstructed from 11 planes of measurements, which amount to 2816 independent data. The inverse problem was calculated with a-priori information implemented using the aforementioned techniques namely: 2\(^{nd}\) order Laplacian operator and SSRM. When using this latter, we assumed the initial state of the perturbation is given by Figure 2 a). The actual state of the perturbation to be reconstructed at the current time is shown in Figure 2 c). A learning set of 21 possible conductivity distributions was constructed to simulate different states of the perturbation randomly chosen between the minimal and maximal states Figure 2 a) and b). Vertical slices of reconstructed images using these regularization methods are shown in Figure 2 e) and d).

Figure 2. Case of head with two tissues (white matter, CSF) and stroke region (blood). a) Minimal and b) maximal state of the stroke. c) True image. Reconstructed conductivity using d) 2\(^{nd}\) Order Laplacian Smoothing matrix, e) SSRM matrix, from noise free data

To further test the SSRM method’s robustness, an extra tissue simulating the SCALP (skin and muscle) was modeled with conductivity of 0.6 Sm\(^{-1}\). We attempted to reconstruct two different states
of the stroke region as shown by Figure 3 a) and b). In case a) this time, a learning set of 84 conductivity estimates was constructed which simulates 21 different states and 4 different conductivities of the stroke region. In case b), Gaussian noise with 1% of the mean voltage data was added to the simulated measurement data. Reconstructed images are shown below:

![Reconstructed Images](image)

Figure 3. Case of head with three tissues (SCALP, CSF:, white matter:) and stroke (blood). a)-b) True images. c) reconstructed image from 84 conductivity and state estimates-d) reconstructed image from noise contaminated data (SNR=40dB)

5. Conclusion
The paper presents the synthesis of regularisation operator based on structural prior information of the head tissues. It was demonstrated with simulations on a head model with three tissues the stroke region can recovered using absolute imaging provided that the prior information containing the distribution of the head tissues is accurate.

6. Reference:
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