Single and pair $J/\psi$ production in the Improved Color Evaporation Model using the Parton Reggeization Approach

A.A. Chernyshev

Samara National Research University, Samara, 443086, Russia

V.A. Saleev

Samara National Research University, Samara, 443086, Russia and Joint Institute for Nuclear Research, Dubna, 141980 Russia

Abstract

In the article, we study single and pair $J/\psi$ hadroproduction in the Improved Color Evaporation Model via the Parton Reggeization Approach. The last one is based on $k_T$–factorization of hard processes in multi-Regge kinematics, the Kimber-Martin-Ryskin-Watt model for unintegrated parton distribution functions, and the effective field theory of Reggeized gluons and quarks, suggested by L.N. Lipatov. We compare contributions from the single and double parton scattering mechanisms in the pair $J/\psi$ production. The numerical calculations are realized using the Monte-Carlo event generator KaTie.

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I. INTRODUCTION

Hadroproduction of $J/\psi$-mesons is being intensively studied theoretically and experimentally for more than 50 years, after their discovery in 1974. Experimental data on single $J/\psi$ production are obtained in a wide range of the energy from $\sqrt{s} = 19$ GeV up to $\sqrt{s} = 13$ TeV [1–9]. The processes of pair production of $J/\psi$ mesons were studied in experiments at the Large Hadron Collider (LHC) by the CMS [10], ATLAS [11] and LHCb [12] collaborations at energies of 7, 8, and 13 TeV. The theoretical description of the processes of charmonium production is based on the perturbation theory of quantum chromodynamics (QCD) in the constant of strong interaction $\alpha_S(\mu)$, where the hard scale $\mu \sim m_\psi$, $m_\psi$ is the charmonium mass, and $\alpha_S \simeq 0.2$. Hadronization process of the $c\bar{c}$-pair to the charmonium is also described in terms of perturbation theory, only by the relative velocity of $c(\bar{c})$ quarks in the charmonium. It is implemented in the nonrelativistic quantum chromodynamics (NRQCD) approach [13]. In the leading approximation of the NRQCD, a quark and an antiquark are produced in the color singlet state, as assumed in the color singlet model (CSM) [14, 15]. Despite the success of NRQCD in describing the charmonium spectra and cross-sections at high energies, there are still unsolved problems: description of $\eta_c$-meson production with allowance for the octet contribution of NRQCD leads to an excess of predictions over experimental data [16]; NRQCD predicts that prompt $J/\psi$ should be produced mostly transversely polarized, experimentally this is not observed [17]. The latter ones may indicate the essential role of such nonperturbative effects that are not taken into account in the NRQCD. Alternative, but the more phenomenological approach is the Color Evaporation Model (CEM) proposed many years ago in Ref. [18, 19]. Later, the CEM was improved by Ma and Vogt [20] and now is used to describe the spectra and polarizations of $J/\psi$-mesons in the collinear parton model (CPM) [21, 22] and in the $k_T$-factorization approach [23, 24].

In the present study, we calculate the transverse momentum spectra of prompt $J/\psi$ within framework of the High Energy Factorization (HEF) or the $k_T$–factorization, which initially has been introduced as a resummation tool for $\ln(\sqrt{s}/\mu)$-enhanced corrections to the hard-scattering coefficients in the CPM, where invariant $\sqrt{s}$ referees to the total energy of the process [25–27]. We use the parton Reggeization approach (PRA) which is a version of HEF formalism, based on the modified Multi-Regge Kinematics (MRK) approximation for QCD scattering amplitudes [28–30]. The PRA is accurate both in the
collinear limit, which drives the Transverse-Momentum-Dependent (TMD) factorization [31] and in the high-energy (Multi-Regge) limit, which is important for Balitsky-Fadin-Kuraev-Lipatov (BFKL) [32–35] resummation of $\ln(\sqrt{s}/\mu)$-enhanced effects. In the PRA, we have studied successfully a heavy quarkonium production in the proton-(anti)proton collisions at the Tevatron and the LHC using NRQCD approach, see Refs. [36–39].

The paper has the following structure. In Section II, the relevant basics of the PRA formalism are outlined. The Improved Color Evaporation Model (ICEM) is shortly reviewed in Section III. In Section IV, we overview Monte-Carlo (MC) parton-level event generator KaTie [40] and the relation between calculations via the PRA and KaTie for tree-level amplitudes. In Section V, we describe the experimental data for the single prompt $J/\psi$ production at the energy range from the 19 GeV up to 13 TeV. In Section VI, we describe the experimental data for the pair prompt $J/\psi$ production at the energy of the LHC collider. Our conclusions are summarized in Section VII.

II. PARTON REGGEIZATION APPROACH

The PRA is based on the factorization hypothesis of the HEF or $k_T$–factorization justified in the leading logarithmic approximation of the QCD at high energies [25–27]. Dependent on transverse momentum, parton distribution functions (PDF) of Reggeized quarks and gluons are calculated in the model proposed earlier by Kimber, Martin, Ryskin and Watt (KMRW) [41, 42], but with sufficient modifications [30] that will be described below. Reggeized parton amplitudes are constructed according to the Feynman rules of the L.N. Lipatov Effective Field Theory (EFT) of Reggeized gluons and quarks [43, 44]. A detailed description of the PRA can be found in Refs. [28–30], inclusion of corrections from the emission of additional partons to the leading PRA approximation was studied in the Refs. [29, 45], the development of the PRA with loop corrections was considered in the Refs [46–48].

In the PRA, the cross-section of the process $p+p \rightarrow J/\psi+X$ is related to the cross-section of the parton subprocess by the factorization formula

$$d\sigma = \sum_{i,j} \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2q_{T1}}{\pi} \Phi_i(x_1, t_1, \mu^2) \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2q_{T2}}{\pi} \Phi_j(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{PRA},$$

(1)

where $t_{1,2} = -q_{T1,2}^2$, the cross-section of the subprocess with Reggeized partons $\hat{\sigma}_{PRA}$ is expressed in terms of squared Reggeized amplitudes $|A_{PRA}|^2$ in the standard way.
The PRA hard-scattering amplitudes are gauge-invariant because the initial-state off-shell partons are considered as Reggeized partons of the gauge-invariant EFT for QCD processes in the MRK limit [43, 44]. The Feynman rules of the Lipatov EFT are written down in the Refs. [44, 49]. The easy way to use Feynman rules of Lipatov EFT is the exploration a model file ReggeQCD [29] for the FeynArts tool [50].

Unintegrated PDFs (unPDFs) in the modified KMRW model are calculated by the formula [30]

$$\Phi_i(x,t,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} T_i(t,\mu^2, x) \sum_{j=q,\bar{q},g} \int dz \frac{1}{z} P_{ij}(z) F_j(\frac{x}{z}, t) \theta(\Delta(t, \mu) - z),$$  \hspace{1cm} (2)

where $F_i(x, \mu^2_F) = x f_j(x, \mu^2_F)$. Here and below, we put factorization and renormalization scales are equal, $\mu_F = \mu_R = \mu$, and $\Delta(t, \mu^2) = \sqrt{t}/(\sqrt{\mu^2} + \sqrt{t})$ is the KMRW-cutoff function [41]. To resolve collinear divergence problem, we require that the modified unPDF $\Phi_i(x, t, \mu)$ should be satisfied exact normalization condition:

$$\int_0^{\mu^2} dt \Phi_i(x, t, \mu^2) = F_i(x, \mu^2),$$  \hspace{1cm} (3)

or

$$\Phi_i(x, t, \mu^2) = \frac{d}{dt} \left[ T_i(t, \mu^2, x) F_i(x, t) \right],$$  \hspace{1cm} (4)

where $T_i(t, \mu^2, x)$ is the Sudakov form-factor, $T_i(t = 0, \mu^2, x) = 0$ and $T_i(t = \mu^2, \mu^2, x) = 1$. The explicit form of the Sudakov form factor in the (4) was first obtained in [30]:

$$T_i(t, \mu^2, x) = \exp \left[ - \int_t^{\mu^2} dt' \frac{\alpha_s(t')}{2\pi} \left( \tau_i(t', \mu^2) + \Delta\tau_i(t', \mu^2, x) \right) \right],$$  \hspace{1cm} (5)

where

$$\tau_i(t, \mu^2) = \sum_j \int_0^1 dz \ z P_{ji}(z) \theta(\Delta(t, \mu^2) - z),$$

$$\Delta\tau_i(t, \mu^2, x) = \sum_j \int_0^1 dz \ \theta(z - \Delta(t, \mu^2)) \left[ z P_{ji}(z) - \frac{F_j(\frac{x}{z}, t)}{F_i(x, t)} P_{ij}(z) \theta(z - x) \right].$$

In contrast to the KMRW model, the Sudakov form factor [5] depends on $x$, which is necessary to preserve the exact normalization [3] for any $x$ and $\mu$. The gauge invariance of
amplitudes with Reggeized partons in the PRA guaranteed allows you to study any processes
described non-Abelian QCD structures. PRA has been successfully used for descriptions
of angular correlations in two-jet events\cite{28}, production of the charm \cite{51,52} and beauty
mesons\cite{29,53}, charmonium in the NRQCD \cite{54,55}.

III. IMPROVED COLOR EVAPORATION MODEL

The current status of the ICEM is presented in the Ref. \cite{20}. In the PRA, the initial
partons have transverse momenta, so the description of the spectra of single $J/\psi$ is already
possible at the leading order (LO) approximation in the strong interaction constant in the
parton subprocesses

$$R + R \to c + \bar{c}$$ \hspace{1cm} (6)

and

$$Q_q + \bar{Q}_q \to c + \bar{c},$$ \hspace{1cm} (7)

where $R$ is a Reggeized gluon, $Q_q(\bar{Q}_q)$ is a Reggeized quark (antiquark) and $q = u, d, s$.

In the ICEM, the cross section for the production of prompt $J/\psi$-mesons is related to the
cross section for the production of $c\bar{c}$-pairs in the single parton scattering (SPS) as follows

$$\sigma^{SPS}(p + p \to J/\psi + X) = F_\psi \times \int_{m_\psi}^{2m_D} \frac{d\sigma(p + p \to c + \bar{c} + X)}{dM} dM,$$ \hspace{1cm} (8)

where $M$ is the invariant mass of the $c\bar{c}$–pair with 4–momentum $p_{c\bar{c}}^\mu = p_c^\mu + p_{\bar{c}}^\mu$, $m_\psi$ is the mass
of the $J/\psi$–meson and $m_D$ is the mass of the lightest $D$–meson. To take into account the
kinematic effect associated with the difference between the masses of the intermediate state
and the final charmonium, the 4–momentum of $c\bar{c}$–pair and $J/\psi$–meson are related by $p_\psi^\mu = (m_\psi/M) p_{c\bar{c}}^\mu$. The universal parameter $F_\psi$ is considered as a probability of transformation
of the $c\bar{c}$–pair with invariant mass $m_\psi < M < 2m_D$ into the prompt $J/\psi$–meson.

In case of pair $J/\psi$ production via the SPS, we take into account contributions of the
following subprocesses

$$R + R \to c + \bar{c} + c + \bar{c}$$ \hspace{1cm} (9)

and

$$Q_q + \bar{Q}_q \to c + \bar{c} + c + \bar{c}.$$ \hspace{1cm} (10)
The cross section for the production of a pair of prompt $J/\psi$-mesons is related to the cross section for the production of two pairs $c\bar{c}$-quarks in the following way

$$\sigma^{\text{SPS}}(p + p \rightarrow J/\psi + J/\psi + X) = \mathcal{F}^{\psi\psi} \int_{m_\psi}^{2m_D} \int_{m_\psi}^{2m_D} \frac{d\sigma(p + p \rightarrow c_1 + \bar{c}_1 + c_2 + \bar{c}_2 + X)}{dM_1dM_2}dM_1dM_2,$$

where $M_{1,2}$ are the invariant masses of $c\bar{c}$-pairs with 4–momenta $p_{\mu c_1} = p_{\mu c_1} + p_{\mu \bar{c}_1}$ and $p_{\mu c_2} = p_{\mu c_2} + p_{\mu \bar{c}_2}$. Parameter $\mathcal{F}^{\psi\psi}$ is the probability of transformation of two pairs $c\bar{c}$ with invariant masses $m_\psi < M_{1,2} < 2m_D$ into two $J/\psi$–mesons.

In the double parton scattering (DPS) approach [56], the cross section for the production of a $J/\psi$ pair is written in terms of the cross sections for the production of single a $J/\psi$ in two independent subprocesses

$$\sigma^{\text{DPS}}(p + p \rightarrow J/\psi + J/\psi + X) = \frac{\sigma^{\text{SPS}}(p + p \rightarrow J/\psi + X_1) \times \sigma^{\text{SPS}}(p + p \rightarrow J/\psi + X_2)}{2\sigma_{\text{eff}}},$$

where the parameter $\sigma_{\text{eff}}$, which controls the contribution of the DPS mechanism is considered a free parameter. Thus, at fitting cross sections for pair $J/\psi$–meson production, we assume that the parameter $\mathcal{F}^{\psi}$ is fixed, and the parameters $\mathcal{F}^{\psi\psi}$ and $\sigma_{\text{eff}}$ are free parameters.

IV. NUMERICAL METHODS

The full gauge invariant set of Feynman diagrams of the Lipatov EFT for the subprocess (9) contains 72 diagrams. It is getting too large for analytical calculation. To proceed to the next step, we should analytically calculate squared off-shell amplitudes and perform a numerical integration using factorization formula (1) with the modified unPDFs (2). Nowadays, we can do it with the required numerical accuracy only for $2 \rightarrow 2$ [6,7] or $2 \rightarrow 3$ [29] off-shell parton subprocesses. To calculate contributions from $2 \rightarrow 4$ subprocesses with initial Reggeized partons we should apply fully numerical methods of the calculation.

A few years ago, a new approach to obtaining gauge invariant amplitudes with off-shell initial state partons in scattering at high energies was proposed. The method is based on the use of spinor amplitudes formalism and recurrence relations of the BCFW type [57, 58]. In Ref. [40] was developed the Monte Carlo (MC) parton level event generator KaTie for processes at high energies with nonzero transverse momenta and virtualities. This formalism [57, 58] for numerical amplitude generation is equivalent to amplitudes built according
to Feynman rules of the Lipatov EFT at the level of tree diagrams [28, 29, 59]. At the
stage of numerical calculations, we use the MC event generator KaTie [40] for calculating
the proton-proton cross sections with contributions of all subprocesses (6), (7), (9) and (10).
The accuracy of numerical calculations for total proton-proton cross sections is equal to
0.1%.

V. SINGLE J/ψ PRODUCTION

We have performed the fit procedure for prompt J/ψ transverse momenta spectra in the
ICEM via the PRA with $\mathcal{F}^\psi$ as a free parameter and obtained a rather good agreement
between the calculations and experimental data from the energy 19.4 GeV up to 13 TeV
as it was measured by different collaborations [1–9]. The obtained results are collected in
Table I and presented in Fig.1. Thus, as $\sqrt{s}$ decreases from 13 TeV to 19 GeV, factor
$F^\psi$ increases by an order of magnitude, from about 0.02 up to 0.2. If we interpret the
parameter $\mathcal{F}^\psi$ as the probability of transformation of $c\bar{c}$-pair with invariant mass from $m_\psi$
to $2m_D$ into J/ψ-meson, its growth with decreasing energy can be explained by an increase
of the hadronization time. Energy dependence of the $\mathcal{F}^\psi$ well described by a formula

$$\mathcal{F}^\psi(\sqrt{s}) = 0.012 + 0.952(\sqrt{s})^{-0.525}.$$ (13)

The calculated transverse momentum spectra and the experimental data are presented in
Figs. 2)-(8. Grey boxes around the central lines in the Figures indicate upper and lower
limits of the cross-section obtained due to variation of the hard scale $\mu$ by the factors $\xi = 2$
or $\xi = 1/2$ around the central value of the hard scale ($\mu = \sqrt{m_\psi^2 + p_T^2}$) and the $c$-quark
mass from 1.2 to 1.4 GeV.

In contrast to the predictions obtained in the NRQCD approach, when gluon-gluon fusion
in the J/ψ hadroproduction is the dominant mechanism, the ICEM predicts a sufficiently
large contribution from the process of quark-antiquark annihilation especially at low energy,
see Fig. 9. Thus, at the energy of future proton-proton collider NICA, $\sqrt{s} \simeq 30$ GeV, the
quark-antiquark contribution may be about 30 % of the total cross section of prompt J/ψ
production.
FIG. 1: The hadronization parameter $\mathcal{F}_\psi$ as a function of proton collision energy $\sqrt{s}$. The corridor between the upper and lower lines demonstrates the uncertainty from the hard scale variation by the factor $\xi = 2$ and the $c$-quark mass from 1.2 to 1.4 GeV.

FIG. 2: The transverse momentum spectra of prompt $J/\psi$ at the different ranges of rapidities. The data are from LHCb collaboration at the $\sqrt{s} = 13$ TeV. [9]
| Collaboration | Energy       | Rapidity | Transverse momentum | $\mathcal{F}^\psi$ |
|---------------|--------------|----------|---------------------|------------------|
| NA3:          | $\sqrt{s} = 19.4$ GeV | $y > 0$  | $p_T \in [0, 5]$ GeV | $0.213^{+0.008}_{-0.008}$ |
| AFS:          | $\sqrt{s} = 30$ GeV | $|y| < 0.5$ | $p_T \in [0, 5]$ GeV | $0.201^{+0.026}_{-0.026}$ |
|               | $\sqrt{s} = 53$ GeV | $|y| < 0.5$ | $p_T \in [0, 7]$ GeV | $0.121^{+0.012}_{-0.012}$ |
| PHENIX:       | $\sqrt{s} = 200$ GeV | $|y| < 0.35$ | $p_T \in [0, 9]$ GeV | $0.102^{+0.033}_{-0.033}$ |
| CDF:          | $\sqrt{s} = 1.96$ TeV | $|y| < 0.6$ | $p_T \in [0, 20]$ GeV | $0.044^{+0.018}_{-0.018}$ |
| LHCb:         | $\sqrt{s} = 5$ TeV | $2.0 < y < 2.5$ | $p_T \in [0, 14]$ GeV | $0.025^{+0.007}_{-0.007}$ |
| ALICE:        | $\sqrt{s} = 7$ TeV | $|y| < 0.9$ | $p_T \in [0, 7]$ GeV | $0.037^{+0.007}_{-0.007}$ |
| ATLAS:        | $\sqrt{s} = 7$ TeV | $|y| < 0.75$ | $p_T \in [7, 70]$ GeV | $0.013^{+0.002}_{-0.001}$ |
|               |              |           | $0.75 < |y| < 1.50$ | $p_T \in [5, 70]$ GeV | $0.007^{+0.001}_{-0.001}$ |
|               |              |           | $1.5 < |y| < 2.0$ | $p_T \in [1, 30]$ GeV | $0.011^{+0.001}_{-0.001}$ |
|               |              |           | $2.0 < |y| < 2.4$ | $p_T \in [5, 30]$ GeV | $0.009^{+0.001}_{-0.001}$ |
| CMS:          | $\sqrt{s} = 7$ TeV | $|y| < 0.9$ | $p_T \in [8, 70]$ GeV | $0.005^{+0.001}_{-0.001}$ |
|               |              |           | $0.9 < |y| < 1.2$ | $p_T \in [8, 45]$ GeV | $0.007^{+0.001}_{-0.002}$ |
|               |              |           | $1.2 < |y| < 1.6$ | $p_T \in [6.5, 45]$ GeV | $0.007^{+0.002}_{-0.002}$ |
|               |              |           | $1.6 < |y| < 2.1$ | $p_T \in [6.5, 30]$ GeV | $0.009^{+0.002}_{-0.002}$ |
|               |              |           | $2.1 < |y| < 2.4$ | $p_T \in [5.5, 30]$ GeV | $0.009^{+0.002}_{-0.002}$ |
| LHCb:         | $\sqrt{s} = 13$ TeV | $2.0 < y < 2.5$ | $p_T \in [0, 14]$ GeV | $0.021^{+0.004}_{-0.004}$ |
|               |              |           | $2.5 < y < 3.0$ | $p_T \in [0, 14]$ GeV | $0.022^{+0.004}_{-0.004}$ |
|               |              |           | $3.0 < y < 3.5$ | $p_T \in [0, 14]$ GeV | $0.021^{+0.004}_{-0.004}$ |
|               |              |           | $3.5 < y < 4.0$ | $p_T \in [0, 14]$ GeV | $0.018^{+0.005}_{-0.005}$ |
FIG. 3: The transverse momentum spectra of prompt $J/\psi$ at the different ranges of rapidities. The data are from ATLAS collaboration at the $\sqrt{s} = 8$ TeV [7].

FIG. 4: The transverse momentum spectra of prompt $J/\psi$ at the different ranges of rapidities. The data are from CMS collaboration at the $\sqrt{s} = 8$ TeV [8].
FIG. 5: The transverse momentum spectra of prompt $J/\psi$. In the left panel, the data are from ALICE collaboration at the $\sqrt{s} = 7$ TeV [6]. In the right panel, the data are from LHCb collaboration at the $\sqrt{s} = 5$ TeV [5].

FIG. 6: The transverse momentum spectra of prompt $J/\psi$. In the left panel, the data are from CDF collaboration at the $\sqrt{s} = 1.8$ TeV [4]. In the right panel, the data are from PHENIX collaboration at the $\sqrt{s} = 0.2$ TeV [3].

FIG. 7: The transverse momentum spectra of prompt $J/\psi$. In the left panel, the data are from AFS collaboration, at the $\sqrt{s} = 30$ GeV [2]. In the right panel, the data are from AFS collaboration at the $\sqrt{s} = 53$ GeV [2].
FIG. 8: The transverse momentum spectrum of prompt \( J/\psi \). The data are from NA3 collaboration at the \( \sqrt{s} = 19.4 \) GeV [1].

FIG. 9: The relative contributions of the parton subprocess (6) and (7) in the prompt \( J/\psi \) production as a function of energy obtained in the ICEM via the PRA.
VI. PAIR $J/\psi$ PRODUCTION

In Ref. [60], the pair $J/\psi$ production was studied in the next to leading order approximation of the collinear parton model. The authors assumed

$$F_{\psi\psi} = (F_{\psi})^2$$

(14)

and found that the contribution of the SPS production mechanism is negligible and the experimental data can only be described by the DPS mechanism. In our opinion, the relation (14) is valid only in the case of the dominant role of the fragmentation approximation for the production of the $J/\psi$ pair. However, the fragmentation mechanism of $J/\psi$ production becomes dominant for $p_T^{\psi} \geq 15$ GeV, i.e. at much larger transverse momentum of the $J/\psi$ than at which the measurements [10–12] were made.

First of all, we review the setup of the pair $J/\psi$ measurements:

- LHCb, $\sqrt{s} = 13$ TeV, $2.0 < y^{\psi} < 4.5$, $p_T^{\psi} < 10.0$ GeV.
- ATLAS (REG-I), $\sqrt{s} = 8$ TeV, $|y^{\psi_1}| < 2.10$, $p_T^{\psi_1} > 8.5$ GeV, $|y^{\psi_2}| < 1.05$, $p_T^{\psi_2} > 8.5$ GeV, where $p_T^{\psi_2} < p_T^{\psi_1}$.
- ATLAS (REG-II), $\sqrt{s} = 8$ TeV, $|y^{\psi_1}| < 2.10$, $p_T^{\psi_1} > 8.5$ GeV, $1.05 < |y^{\psi_2}| < 2.10$, $p_T^{\psi_2} > 8.5$ GeV, where $p_T^{\psi_2} < p_T^{\psi_1}$.
- CMS (REG-I), $\sqrt{s} = 7$ TeV, $|y^{\psi}| < 1.20$, $p_T^{\psi} > 6.5$ GeV.
- CMS (REG-II), $\sqrt{s} = 7$ TeV, $1.20 < |y^{\psi}| < 1.43$, $p_T^{\psi} \in (6.5 \rightarrow 4.5)$ GeV.
- CMS (REG-III), $\sqrt{s} = 7$ TeV, $1.43 < |y^{\psi}| < 2.20$, $p_T^{\psi} > 4.5$ GeV.

In case of pair $J/\psi$ production, we took into account the contributions of the SPS and the DPS production mechanisms. Parameters $F_{\psi\psi}$ and $\sigma_{\text{eff}}$ obtained by the separate fits of production cross sections for different experiments are shown as a contour plot in the Fig.10. Two curves for each experiment ($k = \text{LHCb, ATLAS, CMS}$) correspond $x_k = \pm 1$, where

$$x_k = \frac{\sigma_{\text{exp}}^k - \sigma_{\text{theor}}^k}{\Delta \sigma_{\text{exp}}^k}.$$  

We find that there is a common region of parameters $F_{\psi\psi}$ and $\sigma_{\text{eff}}$ for all experiments. If we collect all experimental data into one set for fit, we find more strong conditions in a plane of
FIG. 10: Regions of the parameters $F^{\psi\psi}$ and $\sigma_{\text{eff}}$ in the ICEM for pair $J/\psi$ production, obtained as a result of data fitting. The relevant pairs of isolines correspond to $x_k = \pm 1.0$ for different experiments.

these two parameters, which are shown as a contour plot in Fig. 11. The isolines correspond to the numerical values of the parameter $x = 1.0, 1.5$ and $2.0$, where

$$x = \sum_{k=1}^{n} \frac{|\sigma_{k}^{\text{exp}} - \sigma_{k}^{\text{theor}}|}{\Delta \sigma_{k}^{\text{exp}}}$$

and the sum is taken over all cross-sections of three experiments: CMS [10], ATLAS [11] and LHCb [12]. The best description of the data, when $x < 1.0$, is reached in the parameter domain $0.021 < F^{\psi\psi} < 0.023$ and $10.75 < \sigma_{\text{eff}} < 11.2$ mb. In fact, at the LHC energies one has $F^{\psi\psi} \simeq F^{\psi}$. The optimal obtained value for $\sigma_{\text{eff}}$ is in a good agreement with the estimates obtained early in other studies [10, 12].

To demonstrate agreement between our calculations in the ICEM via the PRA and experimental data for pair $J/\psi$ production, we plot in the Figs. 12)-(14 different spectra, which have been obtained with $F^{\psi} = 0.02$, $F^{\psi\psi} = 0.02$ and $\sigma_{\text{eff}} = 11$ mb. It is interesting compare the ratio of the SPS and DPS contributions, $R = \sigma_{SPS}^{\psi\psi} / \sigma_{DPS}^{\psi\psi}$, to the pair $J/\psi$ production cross sections with the above mentioned values of $F^{\psi}$, $F^{\psi\psi}$ and $\sigma_{\text{eff}}$: for the LHCb data ($\sqrt{s} = 13$ TeV) – $R \simeq 0.2$, for the CMS data ($\sqrt{s} = 7$ TeV) – $R \simeq 0.5$, but for the ATLAS data ($\sqrt{s} = 8$ TeV) – $R \simeq 1.5$. In such a way, the DPS production mechanism is a dominant source of $J/\psi$ pairs only when the both $J/\psi$ are produced in the forward region of rapidity, as it is measured by LHCb Collaboration.
\[ x = \sum_k \frac{\vert \sigma_{exp}^k - \sigma_{theor}^k \vert}{\Delta \sigma_{exp}^k} \]

\[ F_{\psi \psi} \]

\[ x = 1.0, 1.5 \text{ and } 2.0. \]

**VII. CONCLUSIONS**

We obtain a quite satisfactory description for the single prompt $J/\psi$ $p_T$–spectra and cross sections in the ICEM using the PRA at the wide range of the collision energy. The obtained values of the hadronization parameter $F_{\psi}$ depend on energy, and such dependence can be approximated by the formula $F_{\psi}(\sqrt{s}) = 0.012 + 0.952(\sqrt{s})^{-0.525}$. The exact physical interpretation of such energy dependence needs special analysis.

Both mechanisms, SPS and DPS, for the pair $J/\psi$ production have been considered. We show the assumption $F_{\psi \psi} = F_{\psi} \times F_{\psi}$ is not correct in the ICEM, and we find $F_{\psi \psi} \simeq F_{\psi}$ at the high energy.

The data for the pair $J/\psi$ production cross sections at the energy range $7 - 13$ TeV can be fitted self-consistently with two free parameters $F_{\psi \psi}$ and $\sigma_{eff}$. We have found the best fit with $F_{\psi \psi} \simeq 0.02$ and $\sigma_{eff} \simeq 11.0$ mb, when parameter $F_{\psi}$ is fixed independently in the study of the single $J/\psi$ production. We find the dominant role of the DPS mechanism only in the case of forward pair $J/\psi$ production. At the central region of $J/\psi$ rapidities, both mechanisms contribute approximately equally.
FIG. 12: Different spectra of pair $J/\psi$ production on $m_{\psi\psi}, |\Delta y_{\psi\psi}|, p_T^{\psi\psi}, A_T^{\psi\psi}$ and $|\Delta \phi_{\psi\psi}|$. The data are from LHCb collaboration [12].

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FIG. 13: Different spectra of pair $J/\psi$ production on $m_{\psi\psi}$ and $p_T^{\psi\psi}$ for central and forward rapidity regions. The data are from ATLAS collaboration [11].

FIG. 14: Different spectra of pair $J/\psi$ production on $m_{\psi\psi}$, $|\Delta \phi^{\psi\psi}|$ and $p_T^{\psi\psi}$. The data are from CMS collaboration [10].
[1] J. Badier et al. (NA3), Z. Phys. C 20, 101 (1983).
[2] C. Kourkoumelis et al., Phys. Lett. B 91, 481 (1980).
[3] A. Adare et al. (PHENIX), Phys. Rev. Lett. 98, 232002 (2007), hep-ex/0611020.
[4] D. Acosta et al. (CDF), Phys. Rev. D 71, 032001 (2005), hep-ex/0412071.
[5] R. Aaij et al. (LHCb), JHEP 11, 181 (2021), 2109.00220.
[6] B. Abelev et al. (ALICE), JHEP 11, 065 (2012), 1205.5880.
[7] G. Aad et al. (ATLAS), Nucl. Phys. B 850, 387 (2011), 1104.3038.
[8] S. Chatrchyan et al. (CMS), JHEP 02, 011 (2012), 1111.1557.
[9] R. Aaij et al. (LHCb), JHEP 10, 172 (2015), [Erratum: JHEP 05, 063 (2017)], 1509.00771.
[10] V. Khachatryan et al. (CMS), JHEP 09, 094 (2014), 1406.0484.
[11] M. Aaboud et al. (ATLAS), Eur. Phys. J. C 77, 76 (2017), 1612.02950.
[12] R. Aaij et al. (LHCb), JHEP 06, 047 (2017), [Erratum: JHEP 10, 068 (2017)], 1612.07451.
[13] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D 51, 1125 (1995), [Erratum: Phys.Rev.D 55, 5853 (1997)], hep-ph/9407339.
[14] R. Baier and R. Ruckl, Z. Phys. C 19, 251 (1983).
[15] E. L. Berger and D. L. Jones, Phys. Rev. D 23, 1521 (1981).
[16] M. Butenschoen, Z.-G. He, and B. A. Kniehl, EPJ Web Conf. 137, 06009 (2017).
[17] Z.-B. Kang, Y.-Q. Ma, J.-W. Qiu, and G. Sterman, Phys. Rev. D 91, 014030 (2015), 1411.2456.
[18] H. Fritzsch, Phys. Lett. B 67, 217 (1977).
[19] F. Halzen, Phys. Lett. B 69, 105 (1977).
[20] Y.-Q. Ma and R. Vogt, Phys. Rev. D 94, 114029 (2016), 1609.06042.
[21] V. Cheung and R. Vogt, Phys. Rev. D 95, 074021 (2017), 1702.07809.
[22] V. Cheung and R. Vogt, Phys. Rev. D 104, 094026 (2021), 2102.09118.
[23] V. Cheung and R. Vogt, Phys. Rev. D 98, 114029 (2018), 1808.02909.
[24] R. Maciu la, A. Szczurek, and A. Cisek, Phys. Rev. D 99, 054014 (2019), 1810.08063.
[25] J. C. Collins and R. K. Ellis, Nucl. Phys. B 360, 3 (1991).
[26] S. Catani and F. Hautmann, Nucl. Phys. B 427, 475 (1994), hep-ph/9405388.
[27] L. V. Gribov, E. M. Levin, and M. G. Ryskin, Phys. Rept. 100, 1 (1983).
[28] M. A. Nefedov, V. A. Saleev, and A. V. Shipilova, Phys. Rev. D 87, 094030 (2013), 1304.3549.
[29] A. Karpishkov, M. Nefedov, and V. Saleev, EPJ Web Conf. 158, 03010 (2017).
[30] M. A. Nefedov and V. A. Saleev, Phys. Rev. D 102, 114018 (2020), 2009.13188.
[31] J. Collins, Foundations of perturbative QCD, vol. 32 (Cambridge University Press, 2013), ISBN 978-1-107-64525-7, 978-1-107-64525-7, 978-0-521-85533-4, 978-1-139-09782-6.
[32] L. N. Lipatov, Sov. J. Nucl. Phys. 23, 338 (1976).
[33] E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Sov. Phys. JETP 44, 443 (1976).
[34] E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Sov. Phys. JETP 45, 199 (1977).
[35] I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. 28, 822 (1978).
[36] B. A. Kniehl, D. V. Vasin, and V. A. Saleev, Phys. Rev. D 73, 074022 (2006), hep-ph/0602179.
[37] B. A. Kniehl, V. A. Saleev, and D. V. Vasin, Phys. Rev. D 74, 014024 (2006), hep-ph/0607254.
[38] V. A. Saleev, M. A. Nefedov, and A. V. Shipilova, Phys. Rev. D 85, 074013 (2012), 1201.3464.
[39] B. A. Kniehl, M. A. Nefedov, and V. A. Saleev, Phys. Rev. D 94, 054007 (2016), 1606.01079.
[40] A. van Hameren, Comput. Phys. Commun. 224, 371 (2018), 1611.00680.
[41] M. A. Kimber, A. D. Martin, and M. G. Ryskin, Phys. Rev. D 63, 114027 (2001), hep-ph/0101348.
[42] G. Watt, A. D. Martin, and M. G. Ryskin, Eur. Phys. J. C 31, 73 (2003), hep-ph/0306169.
[43] L. N. Lipatov, Nucl. Phys. B 452, 369 (1995), hep-ph/9502308.
[44] L. N. Lipatov and M. I. Vyazovsky, Nucl. Phys. B 597, 399 (2001), hep-ph/0009340.
[45] M. A. Nefedov and V. A. Saleev, Phys. Part. Nucl. 51, 714 (2020).
[46] M. Nefedov and V. Saleev, Mod. Phys. Lett. A 32, 1750207 (2017), 1709.06246.
[47] M. A. Nefedov, JHEP 08, 055 (2020), 2003.02194.
[48] M. A. Nefedov, Nucl. Phys. B 946, 114715 (2019), 1902.11030.
[49] E. N. Antonov, L. N. Lipatov, E. A. Kuraev, and I. O. Cherednikov, Nucl. Phys. B 721, 111 (2005), hep-ph/0411185.
[50] T. Hahn, Comput. Phys. Commun. 140, 418 (2001), hep-ph/0012260.
[51] R. Maciula, V. A. Saleev, A. V. Shipilova, and A. Szczurek, Phys. Lett. B 758, 458 (2016), 1601.06981.
[52] A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, and A. V. Shipilova, Int. J. Mod. Phys. A 30, 1550023 (2015), 1411.7672.
[53] A. Karpishkov, V. Saleev, and A. Shipilova (2016), [Erratum: Phys.Rev.D 94, 114012 (2016)], 1610.04975.
[54] V. A. Saleev, M. A. Nefedov, and A. V. Shipilova, Phys. Rev. D 85, 074013 (2012), 1201.3464.
[55] Z.-G. He, B. A. Kniehl, M. A. Nefedov, and V. A. Saleev, Phys. Rev. Lett. 123, 162002 (2019), 1906.08979.
[56] M. G. Ryskin and A. M. Snigirev, Phys. Rev. D 83, 114047 (2011), 1103.3495.
[57] A. van Hameren, P. Kotko, and K. Kutak, JHEP 01, 078 (2013), 1211.0961.
[58] A. van Hameren, K. Kutak, and T. Salwa, Phys. Lett. B 727, 226 (2013), 1308.2861.
[59] K. Kutak, R. Maciula, M. Serino, A. Szczurek, and A. van Hameren, JHEP 04, 175 (2016), 1602.06814.
[60] J.-P. Lansberg, H.-S. Shao, N. Yamanaka, Y.-J. Zhang, and C. Noüs, Phys. Lett. B 807, 135559 (2020), 2004.14345.