Evaluation strategy of particle swarm optimization and it’s application in pumping station system optimal operation

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Abstract. To deal with the high energy consumption issue of existing pumping station operation modes, an optimization model ensuring the flow rate and safety demands of pumping station and aiming at the minimum operating power of the system, is established based on basic Particle Swarm Optimization (PSO), which can provide theoretical support for the optimal operation of pumping station. When solving high dimension problems, PSO is easy to get into local optimum and has a slow convergence rate at the later stage of iteration. In order to overcome the shortcomings mentioned above, the Halton sequence is used to generate the initial population randomly, at the same time the inertia weight reducing along the opening downward parabola and Simulated Annealing(SA) are employed, too. The Improved Hybrid Particle Swarm Optimization (IHPSO) is implemented in Matlab. And it is applied to calculate the optimization schemes of the pumping station system of the South-to-North Water Transfer Project in China. The results show that the convergence and stability of the IHPSO are better than the PSO, and IHPSO can effectively solve pumping station optimization operation with multiple variables. The power of the optimum scheme based on IHPSO is about 7.68% less than that based on PSO, and it is decreased about 11.51% compared with the designed scheme.

Key words. Pumping station system, optimal operation, particle swarm optimization, population generation, inertia weight, simulated annealing algorithm

1. Introduction
The optimal operation of the pumping station is conducive to the rational allocation of water resources and the improvement of the utilization rate of water resources. The pumping station consumes a lot of energy during the operation, but the operating mode of most pumping stations is fixed and there is a
problem of low operating efficiency and serious energy waste in the pumping station system. Therefore, the optimal operation of the pumping station is particularly important [1].

The optimization of pumping station scheduling problem is a complex combination solving problem. From the perspective of the development history of optimization scheduling at home and abroad, intelligent algorithms are gradually being applied to the solution of pumping station optimization operation problems. Liang X, et al. [2] introduced the immune thoughts into the particle swarm algorithm, which enhanced the search precision and search range of the particle swarm. It was found that the immune particle swarm optimization algorithm can effectively solve the problem of optimal scheduling of cascade pumping stations and reduce the operating cost of pumping stations. Feng X L, et al. [3] applied the genetic algorithm, basic particle swarm optimization and simulated annealing particle swarm optimization to the large-scale pumping stations for optimal scheduling and compared them. It was found that the simulated annealing particle swarm optimization algorithm is simpler and more efficient. Wang J, et al. [4] divided the population into two parts, which are used for local search and global search respectively, and introduced a coefficient improvement update formula to effectively improve the search accuracy of the algorithm. Kiran M S [5] proposed an update rule based on normal distribution to update the position of particles and solve the problem of premature convergence of the algorithm. Considering the complexity of the optimal operation of different pumping stations, the optimization algorithm needs to be adjusted and improved in order to obtain better results.

This paper focuses on the improved PSO algorithm in terms of initial population generation method, inertia weight and algorithm optimization ability, and applies the improved algorithm to the pumping station operation optimization, to provide a reference for solving pumping station optimization problems.

2. Improved particle swarm optimization

2.1. Particle Swarm Optimization (PSO)

Particle Swarm Optimization is a population-based stochastic optimization technique proposed by Shi Y and Eberhart R C [6]. The algorithm first initializes a group of random particles and then finds the optimal solution through an updating iteration. During each iterative search, particles in the population dynamically adjust their own velocity \( v_{ij} \) and position \( x_{ij} \) by tracking individual extrema and global extrema. The individual extremum Pbest is the optimal solution found by the particle itself. The global extremum Gbest is the optimal solution currently found by the entire population. The \((t + 1)\)-th iteration of the algorithm updates the formula as follows:

\[
v_{ij}(t + 1) = \omega v_{ij}(t) + c_1 r_{1j} (t) \times (p_{ij}(t) - x_{ij}(t)) + c_2 r_{2j} (t) \times (p_{gj}(t) - x_{ij}(t))
\]

\[
x_{ij}(t + 1) = x_{ij}(t) + v_{ij}(t + 1)
\]

where \( i \) denotes the \( i \)-th particle; \( j \) denotes the particle's \( j \)-th dimension; \( \omega \) is the inertia weight, indicating how much the original velocity of the particle can be preserved; \( c_1, c_2 \) are learning factors; \( r_1, r_2 \) are two mutually independent random numbers between 0 and 1; \( v_{ij} \) represents the velocity of the particle; \( v_{max}, v_{min} \) are the maximum and minimum velocity; \( x_{ij} \) represents the position of the particle;
\( x_{\text{max}} \), \( x_{\text{min}} \) are the farthest and closest positions respectively.

2.2. Analysis of parameters

(1) Population \( M \)

The population \( M \) is an integer parameter. If \( M \) is small, the probability of the algorithm getting into a partial optimum is larger; if \( M \) is large, the algorithm's optimal search ability is good, but the convergence speed is very slow.

(2) Learning factors \( c_1 \) and \( c_2 \)

The learning factor \( c_1 \) adjusts the direction of the particle to its own best position. While the learning factor \( c_2 \) adjusts the direction of the particle to the global best position. If \( c_1 \) and \( c_2 \) are too small, the particles may be far away from the target area. If \( c_1 \) and \( c_2 \) are too large, it may suddenly fly to the target area or fly over the target area. Generally, \( c_1 \) and \( c_2 \) take a value between 1.5 and 2.

(3) Inertia weight \( \omega \)

The value of the inertia weight \( \omega \) determines how many particles in the current speed are inherited by the next generations. A larger inertia weight \( \omega \) enables the particles to have a greater speed and thus have a stronger ability to explore. In opposite, a smaller inertia weight makes the particles have a stronger development capability.

(4) Particle space initialization

A better choice of particle space initialization will greatly reduce the convergence time.

2.3. Evaluation strategy of improved PSO

(1) PSO with improved inertia weight value (IPSO\(_1\))

When the inertia weight \( \omega \) is large, it is good for global search, and the convergence speed is fast; it is advantageous to local search and improves the convergence precision of the algorithm. Therefore, Shi Y [6] introduced a method of reducing the inertia weight \( \omega \) from the maximum value to the minimum with the increase of the number of iterations, that is,

\[
\omega_i = \omega_{\text{ini}} - (\omega_{\text{ini}} - \omega_{\text{end}}) \times \frac{k}{k_{\text{max}}}
\]

Is the linear strategy the best diminishing strategy? Thus, Chen G M proposed a nonlinear reduction strategy [7]: the inertia weight decreases along the open downward parabola, that is,

\[
\omega_i = -(\omega_{\text{ini}} - \omega_{\text{end}}) \times \left(\frac{k}{k_{\text{max}}} \right)^2 + \omega_{\text{ini}}
\]

where \( k = 1, 2, 3, \ldots, k_{\text{max}} \) indicates the iteration. \( \omega_{\text{ini}} \) is the initial inertia weight value, and \( \omega_{\text{end}} \) is the inertia weight value of the evolution to the maximum iterations. For the convenience of comparison, its value is the same as linear decreasing (\( \omega_{\text{ini}} = 0.95, \omega_{\text{end}} = 0.4 \)).

(2) PSO with improved initializing method (IPSO\(_2\))

There are defects in random sequence. The generated random samples overlap each other, and there are gaps in some areas, that is, the sample cannot traverse the entire region. The Halton random
sequence is a low-difference sequence, focusing on generating uniform distribution in probability space. Instead of just using the grid to fill it up, it's essentially random. Using a clever method to "fill" the probability space, the resulting random number will be distributed to an area that has not been sampled before [8-10]. The qrandstream function in Matlab can generate Halton random sequence points set.

(3) PSO with Simulated Annealing (IPSO$_3$)
When solving the optimization problem with constraint conditions, the update strategy of the feasibility principle is adopted in the particle swarm optimization. Infeasible solutions face enormous selection pressure and are difficult to be retained in the population. In order to avoid premature and early convergence of the algorithm effectively, the literatures of [11-14] introduce the idea of Simulated Annealing (SA) to basic particle swarm optimization and enhance the global optimization ability of the algorithm. The earliest idea of the SA was proposed by Metropolis in 1953, it was successfully introduced to the combinational optimization area by Kirkpatrick and others in 1983[15]. The main idea of the algorithm is in the minimizing target $f(x)$, $x$ is the current solution and $x'$ is the new solution generated by the $x$-neighborhood. Make $\Delta E=f(x')-f(x)$, then the probability that $x'$ instead of $x$ to become a new solution is $p_a=\min\{1, \exp(-\Delta E/T)\}$, where $T$ is the temperature parameter. Obviously, if $f(x')<f(x)$, $p_a=1$, the algorithm is given a good solution by probability 1; if $f(x')>f(x)$, the $p_a$ is a value between 0 and 1, it means that the algorithm accepts a new solution with a certain probability, so that the algorithm can generate a sudden jump behavior, which can effectively prevent the search process from falling into a local minimum and improve the convergence speed and accuracy of the algorithm.

2.4. Improved Hybrid Particle Swarm Optimization
The single improvement strategies proposed above make the algorithm have different effects in terms of calculation speed, accuracy, convergence, and stability. However, it cannot make the algorithm perform well in all aspects. Therefore, it is necessary to comprehensively consider each strategy of the improved characteristics and make adjustments with other improvement strategies to furtherly improve the performance of the algorithm and ensure the practicality of the algorithm. For the problem of optimal operation of pumping stations, this paper proposes that the above mentioned three improvement strategies should be adjusted at the same time to make the algorithm more stable and accurate.

3. Operation Optimization Model Establishment and Solution for Pumping Station System

3.1. Establishment of Operation Optimization Model for Pumping Station System
During the running of pumping station, under the premise that the flow rate meeting the requirements and the pumping station operating safely, to minimize the total power of the pumping station system, a mathematic model for optimal operation of the pumping station system was established.

(1)Objective function
\[ P = \min \sum_{i=1}^{m} \left( \frac{\rho g Q_i H_{zi} \cdot n_i}{1000 \cdot \eta_{zi} \eta_{dr} \eta_{moti}} + P_{\text{uni}} + \Delta P_{\text{re}} + \Delta P_{\text{si}} \right) \]  

(5)

(2) Constraint conditions

\[
\begin{align*}
\sum_{i=1}^{m} Q_i \cdot n_i &= Q_z \\
Q_{\text{min}} &\leq Q_i \leq Q_{\text{max}} \\
\alpha_{\text{min}} &\leq \alpha_i \leq \alpha_{\text{max}} \\
0 &\leq n_i \leq N_i
\end{align*}
\]

where \( i \) is the number of pumping station; \( m \) is the quantity of pumping stations; \( H_{zi} \) is the pump assembly head for the \( i \)-th pumping station, \( m^3/s \); \( \alpha_i \) is the pump blade angle of the \( i \)-th pumping station, (°); \( n_i \) is the number of running pumps for the \( i \)-th pumping station; \( N \) is the number of installed pumps for the \( i \)-th pumping station; \( \eta_{zi} \) is the pump assembly efficiency for the \( i \)-th station; \( \eta_{\text{moti}} \) is the motor efficiency for the \( i \)-th station; \( \eta_{\text{dr}} \) is the driving efficiency of the \( i \)-th pumping station, when driving directly \( \eta_{\text{dr}} = 1.0 \); \( Q_{\text{max}} \) is the maximum flow rate allowed for a single pump in the \( i \)-th pumping station, \( m^3/s \); \( Q_{\text{min}} \) is the minimum flow rate allowed for a single pump in the \( i \)-th pumping station, \( m^3/s \); \( P_{\text{uni}} \) is the power consumption of the auxiliary equipment, kW; \( \Delta P_{\text{re}} \) is the energy loss in power transmission, kW; \( \Delta P_{\text{si}} \) is the energy loss of the transformer, kW.

### 3.2. Solution Process of Operation Optimization Model for Pumping Station System

The operation optimization model for pumping station system is a nonlinear optimization model with equality and inequality constraints at the same time. In this paper, PSO is used to solve the problem. The optimal process is as follows:

- **Step 1**: the position and speed of \( M \) particles are initially generated, that is, \( M \) running schemes are initially generated and each scheme includes two variables: the number of running \( n \) and the single machine flow rate \( q \);

- **Step 2**: evaluating all running schemes in the population, assign the current running schemes to the individual extremum Pbest and assign the best scheme in Pbest to the global extremum Gbest.

- **Step 3**: updating all running schemes;

- **Step 4**: calculating each scheme’s daily operating costs of pumping station system in the population;

- **Step 5**: comparing the daily operating costs of pumping station system of the current running schemes with the cost of the Pbest, if the cost of the current running scheme is less, then the current running scheme is used to update the Pbest;

- **Step 6**: comparing the cost of the Pbest with it of the Gbest, if the cost of the Pbest is less, then the Pbest is used to update the Gbest;

- **Step 7**: if the termination condition is satisfied, outputting the Gbest with its daily operating costs of pumping station system and stopping the algorithm, otherwise, turning back to step 3.
4. Case study

4.1. Optimization Model Establishment and Solution

Taking the case of a parallel pumping station system of the East Route of China’s South-to-North Water Transfer Project, when the pumping station head is 4.5m, and the total pumping flow rate is 400 m³/s, we will determine the number of running pumps in each pumping station system, as well as the blade angles and the total operating power. In this calculation case, the performance parameters of the parallel pumping station are shown in Table 1.

Table 1. Performance parameters of pumping units.

| Station name | Pump Type   | Installed sets | Single-machine flow rate $Q$/m³.s⁻¹ | Total flow rate $Q_T$/m³.s⁻¹ | Single-machine capacity $P$/kW | Total installed capacity $P_T$/kW | Designed head $H$/m |
|--------------|-------------|----------------|-----------------------------------|-----------------------------|--------------------------------|---------------------------------|-------------------|
| 1st          | 1.75ZLQ-7   | 8              | 10                                | 80                          | 1000                           | 8000                            | 7.8               |
| 2nd          | 1.75ZLQ-7   | 8              | 10                                | 80                          | 1000                           | 8000                            | 7.8               |
| 3rd          | 2000ZLQ13.5-7.8 | 10            | 13.5                              | 135                         | 1600                           | 16000                           | 7.8               |
| 4th          | 2900ZLQ30-7.8 | 7             | 30                                | 210                         | 3400                           | 23800                           | 7.8               |

Due to the 1st and 2nd pumping station have same performance, and there is almost no difference in auxiliary equipment. Therefore, the 1st and 2nd pumping station can be regarded as a pumping station with 16 installed stations, according to the mathematical model of pumping station system optimal operation, the number of pumping station in equation (5) and equation (6) is 3.

The IHPSO is applied to the calculation of the above optimization model. Compute a program based on MATLAB. The algorithm runs $a$ times independently, if the results are consistent, the algorithm has convergence consistency; otherwise, the minimum value of the results is used as the optimal solution of the algorithm and calculating the average value and standard deviation value of the results. Among them, the smaller the average value is, the better the average performance of the algorithm search; the smaller the relative deviation is, the smaller the degree of the algorithm is to deviate from the mean solution, and the better the convergence is.

After a lot of trial, when the population of the algorithm is 400, the maximum iteration is 350 and the times of the program runs independently is 50, the algorithm has better stability and convergence. Using PSO, IPSO₁, IPSO₂, IPSO₃ and IHPSO to solve separately, and comparing result with that of the design scheme.

4.2. Result and analysis

Table 2 shows the optimal solution, average solution, standard deviation and relative deviation obtained by different algorithms running 50 times independently. Compared with PSO algorithm, the energy consumption obtained by the IPSO₁ and IPSO₂ algorithms is 0.02% and 0.05% less than the PSO algorithm respectively. The relative deviation of the IPSO₃ algorithm is reduced by 0.74%, and the convergence is improved significantly. However, the accuracy, convergence and stability of the
three single improved particle swarm algorithms are not uniform, there are differences in the results obtained by running the same algorithm independently for 50 times. So, the single improved particle swarm algorithm still has obvious deficiency. The IHPSO algorithm improves the inertia weight of the algorithm, initializes the method, and introduces the simulated annealing algorithm. The optimal solution, average solution, and relative deviation calculated are all significantly less than the PSO algorithm, therefore, the average performance of the IHPSO algorithm is better, and the degree to which the algorithm deviates from the average solution is small and convergence is good. Table 3 shows the optimized operation solution obtained by IHPSO algorithm.

Table 2. Comparison of optimization performance of different algorithms.

| Pump assembly head $H_z$/m | Method | System power $P$/kW |
|---------------------------|--------|---------------------|
|                           |        | Optimal solution    | Average solution | Standard deviation | Relative deviation | Daily operating cost/10^4 RMB |
|                           |        | $P$                 | $P$              | $P$               | $P$                | $P$                      |
| PSO                       | 28200.999 | 28622.527 | 421.528 | 1.47% | 40.61 |
| IPSO_1                    | 28191.612 | 28610.544 | 418.932 | 1.46% | 40.60 |
| IPSO_2                    | 28189.829 | 28680.005 | 490.176 | 1.71% | 40.59 |
| IPSO_3                    | 28260.848 | 28468.047 | 207.200 | 0.73% | 40.70 |
| IHPSO                     | 26036.581 | 26257.724 | 221.143 | 0.84% | 37.49 |

When the pumping station runs at pump design blade angle (0°) and a certain head, the pumping unit performance curve can be used to solve the corresponding single-pumping flow rate and pump assembly efficiency. When determining the operating schemes, high-efficiency pumping station is priority of operation. Considering the actual operation, the number of pump units rounds up to an integer. Under the same condition, the pumping station system operation optimization of design scheme and actual scheme are shown in Table 4. Actually, the flow rate of actual scheme is 425 m$^3$/s, which slightly larger than the required flow rate. The meanings of the symbols in the table remain the same as Table 3.

Table 3. Optimal operation scheme based on IHPSO algorithm.

| Pump assembly head $H_z$/m | Method | 1$^{st}$, 2$^{nd}$ station | 3$^{rd}$ station | 4$^{th}$ station |
|---------------------------|--------|---------------------------|-----------------|-----------------|
|                           |        | Number of running pumps n/set | Blade angles $\alpha^\circ$ | Number of running pumps n/set | Blade angles $\alpha^\circ$ | Number of running pumps n/set | Blade angles $\alpha^\circ$ | Power $P$/kW |
|                           |        |                           |                 |                 |                 |                 |                 |                 |
| 4.5                       | IHPSO  | 14                        | -3.961          | 0               | —               | 7               | -3.108          | 26036.581 |

Comparing the pumping station system’ daily average operating costs of each flow rate obtained by different algorithms, with electricity price is 0.6 yuan/kW·h. And the results shows in table 5, where $F_0$ and $F_1$ indicate the pumping station system’ daily average operating cost per flow rate based on actual
scheme and design scheme $F_2$ and $F_3$ indicate the cost obtained by the PSO and IHPSO respectively. It can be seen that the daily average operating cost obtained by PSO and IHPSO is reduced by 4.15% and 11.52% respectively compared with the cost obtained from the design scheme. At the same time, the cost of the IHPSO is 15.10% lesser compared with the actual scheme. In addition, the cost of IHPSO algorithm is saved 7.68% more than that of PSO algorithm.

Table 4. Optimal operation schemes of design scheme and actual scheme.

| Pump assembly head $H_z$/m | Method       | 1st, 2nd station | 3rd station | 4th station |
|----------------------------|--------------|------------------|--------------|-------------|
|                            |              | Number of running pumps n/set | Blade angles $\alpha$ (°) | Number of running pumps n/set | Blade angles $\alpha$ (°) | Number of running pumps n/set | Blade angles $\alpha$ (°) | Power $P$/kW |
| 4.5                        | Design scheme | 16               | 0            | 0           | 6           | 0           | 29422.400 |
|                            | Actual scheme | 14               | -3.3         | 10          | -4.9        | 4           | 32584.500 |

Table 5. Comparison of the daily average operating cost per flow rate based on different methods.

| Pump assembly head $H_z$/m | Actual scheme | Design Scheme | PSO | IHPSO | Cost Comparison /% |
|----------------------------|---------------|---------------|-----|-------|---------------------|
|                            | $F_0$         | $F_1$         | $F_2$ | $F_3$ | $(F_1/F_2)$ /$F_1$ | $(F_1/F_3)$ /$F_1$ | $(F_2/F_3)$ /$F_2$ | $(F_0/F_3)$ /$F_0$ |
| 4.5                        | 1104.04       | 1059.21       | 1015.24 | 937.32 | 4.15               | 11.52               | 7.68               | 15.10               |

In summary, compared with the PSO algorithm, the IHPSO algorithm solves the problem of optimal operation of pumping station system, with the average cost of the solution saved by 7.68%. And the operating cost of the pumping station can be effectively reduced. The algorithm has good stability and convergence, and also has better applicability.

5. Conclusion

Based on the established model for optimal operation of pumping stations, according to the basic theory and characteristics of PSO, the algorithm is adjusted from the aspects of the initial population of the algorithm, the value of the inertia weight, and the global search ability of the algorithm, to find a hybrid improved PSO algorithm more suitable for solving pumping station optimization problem. Taking a pumping station system of the East Route of the South-to-North Water Diversion Project in China as an example, when the head is 4.5m, and assembly flow rate is $400m^3/s$. The IHPSO algorithm obtains 7.68% improvement in cost savings compared with the PSO, 11.52% and 15.10% lesser compared with the design scheme and actual scheme respectively. Thus, IHPSO algorithm is more suitable for solving pumping station optimization problems.

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Reference

[1] Sang G Q 2012 Research on operation and control optimization of cascade pumping station water-delivery system based on dynamic balance. Shandong University.

[2] Liang X, Liu M Q, Liu Z Y, Wu Y W and Yan H 2013 Optimum dispatching of multistage pumping station based on mixed particle swarm optimization Engineering Journal of Wuhan University 46 (04): 536-9

[3] Feng X L, Qiu B Y, Yang X L, Shen J and Pei B 2011 Optimal methods and its application of large pumping station operation Journal of Drainage and Irrigation Machinery Engineering 29 (02): 127-32

[4] Wang J and Li H 2017 Particle Swarm Optimization with Enhanced Global Search and Local Search Journal of Intelligent Systems 26(3): 421-32

[5] Kiran M S 2017 Particle swarm optimization with a new update mechanism Applied Soft Computing 60: 670-8

[6] Shi Y and Eberhart R C 1999 Empirical Study of Particle Swarm Optimization Proceedings of the 1999 Congress on Evolutionary Computation (3): 1945~50

[7] Chen G M, Jia J Y and Han Q 2006 Study on the strategy of decreasing inertia weight in particle swarm optimization algorithm Journal of Xi’an Jiao Tong University 40 (1): 53-6

[8] Zhang X Y, Dong Z C and Ma H L 2017 Study on Optimization Operation of Xiaolangdi Reservoir Based on Improved Multi-objective Genetic Algorithm Water Resources and Power (1): 65-8

[9] Xu L and Zhang L 2013 Quantum-behaved particle swarm optimization based on sobol sequence solving economic power dispatch Journal of Southwest China Normal University (Natural Science Edition) 38 (04): 98-101

[10] Zhao Y B, Wen X L and Xu Y X 2015 Cylindricity error inspection and evaluation based on CMM and QPA Chinese mechanical engineering, 26 (18): 2432-36

[11] Wang Y, Cao J, Zhang F Y 2017 Improved chaotic particle swarm optimization algorithm based on simulated annealing Journal of Inner Mongolia University of Technology: Natural Science Edition 36 (3): 173-7

[12] Gao Y and Xie S L 2004 Particle swarm optimization algorithms based on simulated annealing Computer Engineering and Application (1):47-50

[13] Gao S, Yang J Y, Wu X J, et al. 2005 Particle swarm optimization based on the ideal of the simulated annealing algorithm Computer Application and Software 22 (1):103-4, 80

[14] Wang H Q and Cao C X 2005 Parallel particle swarm optimization based on simulated annealing Control and Decision 20 (5):500-4

[15] Kirkpatric S, Gelatt C D and Vecchi M P 1983 Optimization by simulated annealing Science 220:671-80