4D Einstein-Gauss-Bonnet AdS Black Holes as Heat Engine

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Motivated by studying black hole thermodynamics in a novel theory, known as 4D Einstein Gauss-Bonnet gravity. We perform a study of holographic heat engines of AdS black holes in such a theory, obtaining efficiency of a rectangular engine cycle. First, we consider the charged AdS black hole as the working substance and analyze the effective roles of electric charge and Gauss-Bonnet coupling on the heat engine efficiency. We find that they can improve the heat engine efficiency. Then, we study the heat engine of Bardeen AdS black hole and show that how magnetic charge and Gauss-Bonnet parameter affect the heat engine efficiency. Finally, by comparing engine efficiency with the Carnot efficiency, we indicate that the ratio $\eta/\eta_c$ is less than one all the time which is consistent with the thermodynamic second law.

I. INTRODUCTION

In order to understand the low-energy limit of string theory a noteworthy number of attempts in higher dimensions theory of gravity have been done (it is notable that string theory plays a major role as a candidate for the quantum gravity and also the unification of all interactions. This theory of gravity needs to higher dimensions for its mathematical consistency). Einstein-Gauss-Bonnet (EGB) gravity is an important higher dimensional generalization of Einstein gravity which first was suggested by Lanczos in 1938 \cite{1}, and then rediscovered by Lovelock in 1971 \cite{2}. EGB gravity includes string theory inspired corrections to the Einstein-Hilbert action and admits Einstein gravity as a particular case \cite{3}. The study of EGB gravity becomes very important since it provides a broader set up to explore a lot of conceptual issues related to gravity. EGB gravity includes another interesting properties such as: i) it encompasses Einstein gravity as special case, ii) this theory of gravity similar to Einstein gravity enjoys only first and second order derivatives of the metric function in the field equations. iii) it may lead to the modified Renyi entropy \cite{4}. iv) EGB gravity is free from the ghosts. v) regarding AdS/CFT correspondence, it was shown that considering EGB theory of gravity will modify entropy, electrical conductivity, shear viscosity and also thermal conductivity (see ref. \cite{5}, for more details). vi) from the cosmological point of view, EGB gravity can yield a viable inflationary era, and also can describe successfully the late-time acceleration era, \cite{6–15}.

It is notable that the case 4-dimensional of EGB gravity is special because the Euler-Gauss-Bonnet term becomes a topological invariant in which does not contribute to the equations of motion or to the gravitational dynamics. Recently, Glavan and Lin in ref. \cite{16}, have introduced a general covariant modified theory of gravity in 4-spacetime dimensions ($D = 4$) which propagates only the massless graviton and also bypasses the Lovelock’s theorem (according to the Lovelock’s theorem \cite{1, 2}, Einstein gravity (with the cosmological constant) is the unique theory of gravity if we respect to several conditions: spacetime is 4-dimensional, metricity, diffeomorphism invariance, and also the second order equations of motion). Indeed, this theory is formulated in higher dimensions (more than 4-dimensional spacetime, $D > 4$) and its action consists of the Einstein-Hilbert term with a cosmological constant, and also the Gauss-Bonnet (GB) coupling has been rescaled as $\alpha/(D - 4)$. The 4-dimensional theory is defined as the limit $D \rightarrow 4$. In this limit GB invariant gives rise to non-trivial contributions to gravitational dynamics, while preserving the number of graviton degrees of freedom and also being free from Ostrogradsky instability. This new theory is called 4D EGB gravity (see ref. \cite{16}, for more details).

4D EGB gravity enjoys the existence of several attractive properties. For example, it might resolve some singularity issues. Indeed, by considering 4D EGB gravity the static and spherically symmetric black holes the gravitational force is repulsive at small distances and thus an infalling particle never reaches the singularity. In addition, static and spherically symmetric black hole solutions in this new theory differ from the well-known Schwarzschild black hole in Einstein gravity. Compact objects and their properties in 4D EGB gravity have been studied by many authors. Some of these works are; quasinormal modes, stability, strong cosmic censorship and shadows of a black hole \cite{17, 18}, the innermost stable circular orbit and shadow \cite{19}, charged black holes in AdS spaces \cite{20}, rotating black hole shadow \cite{21},

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thermodynamics, phase transition and Joule Thomson expansion of (un)charged AdS black hole [22, 23], rotating black holes [24, 25], relativistic stars solution [26], spinning test particle orbiting around a static spherically symmetric black hole [27], thermodynamics and $P - V$ criticality of Bardeen-AdS black hole [28], the eikonal gravitational instability of asymptotically flat and (A)dS black holes [29], thermodynamic geometry of AdS black hole [31], gravitational lensing by black holes [32], Hayward black holes [33], and thin accretion disk around of black hole [34].

Considering black holes as thermodynamic systems in the extended phase space, it is natural to assume them as heat engines. In fact, the mechanical term $P dV$ in the first law provides the possibility of calculating the mechanical work and in result the efficiency of these heat engines. A heat engine is defined as a closed path in the $P - V$ diagram which works between two reservoirs with temperatures $T_H$ (high temperature) and $T_C$ (low temperature). During the working process, the heat engine absorbs an amount of heat $Q_H$ from the warm reservoir. Some of this thermal energy is converted into work ($W$) and some heat ($Q_C$) is usually returned to the cold reservoir (see Fig. 1 for more details). The efficiency of the heat engine is defined by

$$
\eta = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}.
$$

(1)

The heat engine’s efficiency depends on the equation of state of the black hole and the paths forming the heat cycle in the $P - V$ diagram. There are different classical cycles which the Carnot cycle is the simplest cycle that can be considered. This cycle involves a pair of isotherms at temperatures $T_H$ and $T_C$ where the system absorbs some heat during an isothermal expansion and loses some of this thermal energy during an isothermal compression. These two temperatures connect to each other with adiabatic paths. The Carnot efficiency is determined as

$$
\eta_c = 1 - \frac{T_C}{T_H},
$$

(2)

this the maximum efficiency any heat engine can have and any higher efficiency would violate the second law of thermodynamics. For the static black holes, the thermodynamic volume $V$ and the entropy $S$ are only a function of event horizon $r_+$. In fact, they are dependent on each other. So, heat capacity equals to zero at constant volume ($C_V = 0$). The vanishing of $C_V$ results into "isochore equals adiabat". In other words, Carnot and Stirling cycles coincide (see left panel of Fig. 2). Therefore an explicit expression for $C_P$ would suggest that one can define a new engine cycle in $P - V$ plane, as a rectangle (see right panel of Fig. 2) which is composed of two isobars (paths of $1 \rightarrow 2$ and $3 \rightarrow 4$) and two isochores (paths of $2 \rightarrow 3$ and $4 \rightarrow 1$). We can calculate the work done along the heat cycle as

$$
W = \oint P dV = W_{1\rightarrow2} + W_{2\rightarrow3} + W_{3\rightarrow4} + W_{4\rightarrow1}
= W_{1\rightarrow2} + W_{3\rightarrow4} = P_1 (V_2 - V_1) + P_4 (V_4 - V_3),
$$

(3)
in the above equation the work done along paths of $2 \to 3$ and $4 \to 1$ are zero ($W_{2\to3} = W_{4\to1} = 0$).

The upper isobar will give the heat input as

$$Q_H = \int_{T_1}^{T_2} C_P \left( P_1, T \right) dT = \int_{T_1}^{T_2} C_P \left( P_1, T \right) \left( \frac{\partial T}{\partial S} \right) dS$$

$$= \int_{S_1}^{S_2} T dS = \int_{H_1}^{H_2} dS = M_2 - M_1. \quad (4)$$

In 2014, Johnson in ref. [35], calculated the efficiency of heat engine for a black hole. Using the concepts introduced by Johnson, the efficiency of heat engines for other types of black holes such as GB black holes [36], Born–Infeld AdS [37], dilatonic Born-Infeld black holes [38], rotating black holes [39], charged AdS black holes [40], Kerr AdS and dyonic black holes [41], BTZ [42], polytropic black holes [43], black holes in conformal gravity [44], black holes in massive gravity [45], benchmarking black hole [46], accelerating AdS black holes [47], black holes in gravity’s rainbow [48], charged accelerating AdS black holes [49], non-linear charged AdS black holes [50], charged rotating accelerating AdS black holes [51], and Hayward-AdS black hole [52].

This paper is organized as follows. In section 2, we would like to compute the efficiency of charged AdS black hole in 4D EGB gravity and study how the GB coupling and electric charge affect the efficiency of the black hole heat engine, and then compare it to the Carnot efficiency. In section 3, we discuss the effect of GB coupling and magnetic charge on the heat engine efficiency of Bardeen AdS black hole in such gravity. The last section is devoted to conclusions.

![P-V diagram of thermodynamic cycles for the heat engine: Left panel: Carnot engine, which for static black holes is also a Stirling engine. Right panel: the rectangle cycle.](image)

**FIG. 2:** $P$-$V$ diagram of thermodynamic cycles for the heat engine: Left panel: Carnot engine, which for static black holes is also a Stirling engine. Right panel: the rectangle cycle.

**II. CHARGED ADS BLACK HOLE IN 4D EINSTEIN-GAUSS-BONNET GRAVITY AS HOLOGRAPHIC HEAT ENGINE**

In this section, we first introduce the thermodynamics of charged AdS black hole in 4D EGB gravity by reviewing ref. [53, 54].

The metric of such black holes have the following form

$$ds^2 = -f\left(r\right)dt^2 + \frac{1}{f\left(r\right)}dr^2 + r^2d\Omega_{D-2}^2, \quad (5)$$

where under the limit $D \to 4$, $f\left(r\right)$ is

$$f\left(r\right) = 1 + \frac{r^2}{2\alpha} \left( 1 \pm \sqrt{1 + 4\alpha \left( \frac{2M}{r^3} - \frac{Q^2}{r^4} - \frac{1}{l^2} \right)} \right), \quad (6)$$

where $Q$ and $\alpha$ are the electric charge of the black hole and GB coupling, respectively. The AdS length $l$ is related to the cosmological constant as $\Lambda = -\frac{3}{l^2}$. The mass of the black hole ($M$) is obtained by using the condition $f\left(r_+\right) = 0$,

$$M = \frac{r_+}{2} \left( 1 + \frac{r^2}{l^2} + \frac{Q^2 + \alpha}{r_+^2} \right). \quad (7)$$
Hawking temperature related to surface gravity $\kappa$ on event horizon is calculated as

$$T = \frac{\kappa}{2\pi} = \frac{f'(r_+)}{4\pi} = \frac{3r_+^2}{4\pi l^2 (r_+^2 + 2\alpha)} + \frac{r_+^2 - Q^2 - \alpha}{4\pi r_+ (r_+^2 + 2\alpha)}. \quad (8)$$

The black hole entropy is given by,

$$S = \int_{r_0}^{r_+} \frac{dM}{T} = \pi r_+^2 + 2\alpha \log r_+. \quad (9)$$

In the extended phase space, the cosmological constant corresponds to thermodynamic pressure with $\Lambda = -8\pi P$, and its conjugate variable corresponds to thermodynamic volume with

$$V = \left( \frac{\partial M}{\partial P} \right)_{S,Q,\alpha} = \frac{4}{3}\pi r_+^3. \quad (10)$$

Now, we are going to investigate holographic heat engine for this solution. Using Eq. (3), the useful work is obtained as

$$W = \frac{4}{3}\pi (P_1 - P_4) \exp \left( \frac{-6\pi \alpha \text{LambertW} \left( \frac{S/2^\alpha}{2\alpha} \right) - 3S}{4\pi \alpha} \right) \bigg|_{S_2}^{S_1}, \quad (11)$$

and $Q_H$ is calculated as

$$Q_H = \frac{\chi_1^1}{6} \left( 3 + 8\pi P_1 \chi_2 + \frac{3(\alpha + Q_2)}{\chi_2} \right) \bigg|_{S_2}^{S_1}, \quad (12)$$

where

$$\chi_n = \exp \left( \frac{-2n\pi \alpha \text{LambertW} \left( \frac{S/2^\alpha}{2\alpha} \right) - nS}{4\pi \alpha} \right). \quad (13)$$

By substituting Eqs. (11) and (12) in Eq. (1), one can obtain the engine’s efficiency. To calculate Carnot efficiency, we consider the $T_H$ and $T_C$ in our cycle correspond to $T_2$ and $T_4$, respectively. So, this efficiency is

$$\eta_c = 1 - \frac{T(P_4, S_1)}{T(P_3, S_2)} = 1 - \frac{y_1 (y_2 + 2\alpha)(8\pi P_4 x_4 + x_2 - Q^2 - \alpha)}{x_1 (x_2 + 2\alpha)(8\pi P_4 y_4 + y_2 - Q^2 - \alpha)}, \quad (14)$$

where

$$x_n = \exp \left( \frac{-2n\pi \alpha \text{LambertW} \left( \frac{S/2^\alpha}{2\alpha} \right) - nS_1}{4\pi \alpha} \right),$$

$$y_n = \exp \left( \frac{-2n\pi \alpha \text{LambertW} \left( \frac{S/2^\alpha}{2\alpha} \right) - nS_2}{4\pi \alpha} \right). \quad (15)$$

In order to study the effects of electric charge and GB coupling on the heat engine efficiency, we have plotted Fig. (3). The left panel of Fig. (3), shows variation of efficiency $\eta$ versus GB parameter ($\alpha$) for different values of the electric charge with fixed pressure ($P_1, P_4$) and entropy ($S_1, S_2$). From this figure, one can observe that the behavior of the efficiency is crucially dependent on the electric charge and GB coupling. As we see, $\eta$ is an increasing function...
of the electric charge. For small values of $Q$, the efficiency is a monotonously increasing function with the growth of $\alpha$. Whereas, for large $Q$, the efficiency curve has a global minimum value. This reveals the fact that there is a certain value of the GB parameter at which the heat engine of the black hole works at the lowest efficiency. The right panel of Fig. 3 exhibits the ratio between the efficiency and the Carnot efficiency ($\frac{\eta}{\eta_c}$) versus $\alpha$ for different values of $Q$. From this figure, it is evident that the efficiency is close to the Carnot efficiency for large values of $\alpha$. For large electric charges, the ratio $\frac{\eta}{\eta_c}$ monotonically increases as the GB parameter increases. But for small values, as we see this ratio decreases rapidly firstly and reaches a minimum value, then the efficiency grows and approaches the Carnot efficiency by increasing $\alpha$. In order to see the effect of pressure on the heat engine efficiency, we have plotted $\eta$ versus entropy $S_2$ for different values of pressure difference $\Delta P$ with fixed $Q$ and $\alpha$ in Fig. 4. We observe that increasing the pressure makes the increasing of the efficiency (see Figs. 4a and 4c). As we see, depending the values of the GB parameter and electric charge, increasing entropy $S_2$ leads to the increasing or decreasing of the heat engine efficiency. By looking at bold lines in two left panels of Fig. 4, one can find that for black holes with large electric charge or stronger GB coupling, the efficiency decreases with the growth of the entropy $S_2$ (corresponding to volume $V_2$). This shows that the heat engine efficiency gets smaller when volume difference between the small black hole ($V_1$) and large black hole ($V_2$) becomes bigger. Whereas, opposite behavior will be observed for small electric charge and weak GB coupling (see thin lines of Figs. 4a and 4c). From two right panels of Fig. 4, we see that the ratio $\frac{\eta}{\eta_c}$ is a monotonously decreasing function of the entropy $S_2$. In the limit of that the entropy $S_2$ goes to the infinity, this ratio reaches a constant value. Also, these two figures shows that the efficiency would get closer and closer to the Carnot efficiency when volume difference between small and large black holes gets smaller.

III. BARDEEN ADS BLACK HOLE IN 4D EINSTEIN-GAUSS-BONNET GRAVITY AS HOLOGRAPHIC HEAT ENGINE

In this section we investigate the case of the EGB Bardeen AdS black hole. We begin by reviewing the thermodynamics of this black hole in ref. [55]. The solution for a spherical Bardeen AdS black hole in 4-dimensions reads

$$ds^2 = -f(r) dt^2 + \frac{1}{f(r)} dr^2 + r^2 d\Omega_2^2,$$

where $f(r)$ is

$$f(r) = 1 + \frac{r^2}{2\alpha} \left( 1 + \sqrt{1 + 4\alpha \left( \frac{2M}{(r^2 + q^2)^2} - \frac{1}{l^2} \right)} \right),$$

where $q$ is the magnetic monopole charge. The mass of the black hole in terms of its horizon radius is expressed as

$$M = \frac{(r_+^2 + q^2)^{\frac{3}{2}}}{2l^2 r_+} \left( r_+^2 (r_+^2 + l^2) + \alpha l^2 \right).$$
FIG. 4: Variation of $\eta$ and $\frac{\eta}{\eta_c}$ versus $S_2$ for $S_1 = 1$, $\Delta P = 0.54$ (continuous line), $\Delta P = 0.52$ (dashed line) and $\Delta P = 0.5$ (dotted line). Up panels for $\alpha = 0.1$, $Q = 1$ (bold lines) and $Q = 0.2$ (thin lines). Down panels for $Q = 0.2$, $\alpha = 1$ (bold lines) and $\alpha = 0.1$ (thin lines).

The Hawking temperature for this solution is obtained as

$$T = \frac{3r_+^6 + r_+^2 l^2 (r_+^2 - \alpha)}{4\pi r_+ l^2 (r_+^2 + 2\alpha)} - \frac{2q^2 l^2 (r_+^2 + 2\alpha)}{4\pi r_+ l^2 (r_+^2 + 2\alpha)}.$$ (19)

The entropy, pressure and thermodynamic volume are given by

$$S = \pi r_+^2 + 2\pi \alpha \ln r_+^2, \quad \& \quad P = \frac{3}{8\pi l^2}, \quad \& \quad V = \frac{4\pi}{3} r_+^3.$$ (20)

Now we are going to calculate efficiency and Carnot efficiency for heat engine of such a black hole. The work down is obtained as

$$W = \frac{4}{3} \pi (P_1 - P_4) \exp \left( -\frac{6\pi \alpha \text{LambertW}(\frac{8\pi}{2\alpha})}{4\pi \alpha} - 3S \right) \bigg|_{S_2}^{S_1},$$ (21)

and $Q_H$ is calculated as

$$Q_H = \frac{(\chi_2 + q^2)^2}{6\chi_4} \left( 8\pi P_1 \chi_4 + 3 (\alpha + \chi_2) \right) \bigg|_{S_2}^{S_1},$$ (22)

the Carnot efficiency is given by

$$\eta_c = 1 - \frac{T(P_4, S_1)}{T(P_1, S_2)} = 1 - \frac{y_1 (y_2 + 2\alpha) (y_2 + q^2) \left[ 8\pi P_1 x_6 + x_4 - \alpha x_2 - 2q^2 (2\alpha + x_2) \right]}{x_1 (x_2 + 2\alpha) (x_2 + q^2) \left[ 8\pi P_1 y_6 + y_4 - \alpha y_2 - 2q^2 (2\alpha + y_2) \right]}.$$ (23)
In order to study how magnetic charge, GB coupling and pressure affect the efficiency, we have plotted efficiency and its difference from the Carnot efficiency versus $S_2$ in Fig. 5. As we see from up panels of Fig. 5, $\eta$ and $\frac{\partial \eta}{\partial q}$ are a decreasing function of magnetic charge $q$. For small magnetic charges, $\eta$ ($\frac{\partial \eta}{\partial q}$) monotonically increases (decrease) as the entropy $S_2$ grows and then tends to a constant value. But for large values of $q$, $\eta$ and $\frac{\partial \eta}{\partial q}$ decrease rapidly at first and reaches a minimum value, then they increase with growth of the entropy $S_2$. This means that the heat engine of the black hole works at the lowest efficiency for a certain value of the entropy $S_2$. By looking at Fig. 5, one can find that increasing GB coupling makes the increasing of the efficiency. Also, this figure shows that for black holes with stronger (weak) GB coupling, $\eta$ is a monotonously decreasing (increasing) function of the entropy $S_2$. But when volume difference between small and large black holes becomes very big, the efficiency obtains a certain value for all values of GB parameter. As we see from Fig. 5, for small volume difference, the ratio $\frac{\partial \eta}{\partial q}$ is an increasing function of $\alpha$. But for large volume difference, the ratio $\frac{\partial \eta}{\partial q}$ decreases by increasing $\alpha$. This shows that the efficiency approaches to Carnot efficiency for very small volume difference and large GB parameter. Regarding the effect of pressure on the efficiency, from down panels of Fig. 5 one can observe that the corresponding black hole will have the larger efficiency with increase of pressure.

### IV. CONCLUSION

In this paper, we have considered 4D EGB AdS black holes as a working substance and studied the holographic heat engine by taking a rectangle heat cycle in the $P - V$ plot. First, we investigated the concept of the heat engine for the charged EGB-AdS black hole and showed that the electric charge, GB coupling and thermodynamic pressure affect significantly the efficiency of the black hole heat engine. We observed that the efficiency will increase with growth of electric charge and pressure. Regarding the effect of GB coupling, we noticed that in presence of a weak electric field, the efficiency of black hole is an increasing function of GB parameter. Whereas, for black holes with stronger electric charge, the efficiency will have a global minimum for a certain value of this parameter. In other words, there is a finite value of the GB coupling at which the heat engine of this black hole works at the lowest efficiency. Also, we found that the heat engine efficiency will approach to Carnot efficiency if volume difference between small and large black holes becomes very small.

Then we have constructed a heat engine for Bardeen EGB-AdS black holes and studied the effect of magnetic charge, GB coupling and pressure on the heat engine efficiency. We noticed that the efficiency is an increasing (a decreasing) function of GB coupling and pressure (magnetic charge). Then, we have compared our efficiency results with the Carnot efficiency. We saw that black holes with weak magnetic charge and strong GB coupling will have efficiency close to Carnot efficiency provided that volume difference between small and large black holes becomes very small. Also, we found that the ratio $\frac{\partial \eta}{\partial q}$ versus entropy $S_2$ is less than one all the time which is consistent with the thermodynamic second law.

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FIG. 5: Variation of $\eta$ and $\frac{\Delta P}{P_0}$ versus $S_2$ for $P_1 = 1$, $S_1 = 1$ and different values of $q$ (up panels), different values of $\alpha$ (middle panels) and different values of $\Delta P$ (down panels).
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