A New Concept of Information Based on Heisenberg's Uncertainty Principle and Its Experimental Verification

Frank Z. Wang (✉ frankwang@ieee.org)
University of Kent

Research Article

Keywords: Information theory, Heisenberg's uncertainty principle, Landauer bound, information carrier, Brownian particle, Newton's cradle, reversible computing, quantum computing, Plank constant

DOI: https://doi.org/10.21203/rs.3.rs-654447/v1

License: ©️ This work is licensed under a Creative Commons Attribution 4.0 International License. Read Full License
A New Concept of Information based on Heisenberg's Uncertainty Principle and Its Experimental Verification

Frank Z. Wang

ABSTRACT This study is the first use of Heisenberg’s energy-time uncertainty principle to define information quantitatively from a measuring perspective: the smallest error in any measurement is a bit of information, i.e., \( 1 \text{ (bit)} = \frac{2\Delta E \Delta t}{\hbar} \). If the input energy equals the Landauer bound, the time needed to write a bit of information is \( 1.75 \times 10^{-14} \text{ s} \). Newton's cradle was used to experimentally verify the information-energy-mass equivalences deduced from the aforementioned concept. It was observed that the energy input during the creation of a bit of (binary) information is stored in the information carrier in the form of the doubled momentum or the doubled “momentum mass” (mass in motion) in both classical position-based and modern orientation-based information storage. Furthermore, the experiments verified our new definition of information in the sense that the higher the energy input is, the shorter the time needed to write a bit of information is. Our study may help understand the fundamental concept of information and the deep physics behind it.

INDEX TERMS Information theory, Heisenberg's uncertainty principle, Landauer bound, information carrier, Brownian particle, Newton's cradle, reversible computing, quantum computing, Planck constant.

I. INTRODUCTION

In information theory, information can be interpreted as the resolution of uncertainty in the sense that it answers the question of "What an entity is" [1][2]. Uncertainty is inversely proportional to the probability of occurrence of an event: more information is required to resolve their uncertainty for more uncertain events [2]. A bit of information is “that which reduces uncertainty by half” [3].

In quantum computing, Heisenberg’s uncertainty principle states that the more precisely the position of a Brownian particle is determined, the less precisely its momentum can be predicted from the initial conditions, and vice versa [4]. This prescription is different from classical Newtonian physics, which holds all variables of particles to be measurable to an arbitrary accuracy given sufficiently precise equipment. Heisenberg’s principle also applies to the energy and time, in that one cannot measure the energy of a particle precisely in a finite amount of time.

In this study, we used Heisenberg’s energy-time uncertainty principle to define a bit of information quantitatively from a measuring perspective as follows:

\[
1 \text{ (bit)} = 2\Delta E\Delta t / \hbar,
\]

where \( \Delta E \) is the standard deviation in the energy, \( \Delta t \) is the standard deviation in the time, and \( \hbar = 1.05 \times 10^{-34} \text{ J} \cdot \text{s} \) is the reduced Planck constant or the Dirac constant. Obviously, \( \Delta E \Delta t / \hbar \) is unitless, which does not violate the definition of information in units.

FIGURE 1. Heisenberg’s energy-time uncertainty principle is used to define information quantitatively from a measuring perspective: the smallest error in measuring the energy and time of a Brownian particle is a bit of information. This new concept mathematically defines the energy–time cost of an information manipulation: the higher the energy input is, the shorter the time needed to write a bit of information is and vice versa.

We defined information to be interchangeable with energy over time, where the smallest error (the fundamental limit to the accuracy) in measuring a pair of prescribed physical
attributes of a particle is a bit of information, as shown in Fig.1.

The principle presented in Eq.1 corresponds to stating that the higher the energy input is, the shorter the time needed to write a bit of information is and vice versa.

The energy term in Eq.1 can be expressed by the Landauer bound $E = k_B T \ln 2$, where $k_B$ is the Boltzmann constant and $T$ is the temperature [5]. This bound has been verified using an experiment, in which a particle is trapped in a double-well potential [6].

**II. POSITION-BASED INFORMATION STORAGE**

The Landauer bound can be illustrated by an information carrier (an electron or another particle) moving upwards and downwards between impenetrable barriers at the two ends of a one-dimensional nanotube [Fig.2(a)]. Assuming collisions are nearly or totally elastic, the carrier loses no kinetic energy in a collision and thus has the same speed after the collision as before the collision. According to the impulse-momentum theorem [7], the average force, $F_{avg}$, between the two collisions is defined such that

$$\Delta p = F_{avg}\Delta t = F_{avg} \frac{2x}{v_0} = 2m_0 v_0,$$

$$F_{avg} = \frac{m_0 v_0^2}{x}.$$  

For ideal gases (molecules, ions, etc.), macroscopic phenomena, such as temperature, can be explained in terms of the classical mechanics of microscopic particles [8][9][10]. According to the equipartition theorem, each classical degree of freedom of a freely moving particle has an average kinetic energy

$$E_k = \frac{1}{2} m_0 v_0^2 = k_B T/2,$$  

where $m_0$ is the mass of the information carrier, $v_0$ is the velocity of the information carrier, and $k_B$ is Boltzmann's constant [11]. In choosing an information carrier, one should bear in mind that, at a fixed temperature, the speed of a particle increases with the decreased mass.

Therefore, the work an information carrier does on a frictionless piston during information erasure [Fig.2(c)] is

$$W = \int_{L/2}^{L} F_{avg} dx = \int_{L/2}^{L} \frac{m_0 v_0^2}{x} dx = k_B T \ln 2.$$  

This result is the Landauer bound, which has a value of approximately $3 \times 10^{-21}$ J at room temperature (300 K) [5][6].

From Eq.1, we can now use the Landauer bound to calculate the time needed to write a bit of information as follows:

$$\Delta t = \frac{h}{2\Delta E} = \frac{h}{2 \times 3 \times 10^{-21} J} = 1.75 \times 10^{-14} \text{s}.$$  

This calculation result agrees reasonably with the picosecond timescale demonstrated by ultrafast magnetization reversal [12].

FIGURE 2. Position-based information storage. In (a), a free state (without any written information) exists, in which the information carrier is only reciprocating inside a one-dimensional tube without remaining in either half of the tube, where $v_0$ and $-v_0$ denote the velocities before and after a collision, respectively. In (b), a bit of information is created by adding a partition to confine the information carrier in the desired half of the tube, in harmony with the observation "a bit of information reduces uncertainty by half" [3]. In (c), the partition becomes a frictionless piston and the information carrier can push it to do useful work in exchange for the loss of the written information (that is, the half of the tube that is occupied). According to the impulse-momentum theorem [7], the average force, $F_{avg}$, between the two collisions is defined such that $F_{avg}\Delta t = F_{avg} \frac{2x}{v_0} = 2m_0 v_0$. Interestingly, the writing operation in (b) could be performed by a "Maxwell's demon" [8][9][10] that consumes energy to observe the position of the information carrier and insert/release the partition, where the consumed energy equals the work exerted for information erasure in (c). The compressed ideal gas (as a bit of written information) can be modelled by an elastic spring, for which the elastic potential energy is zero in the free state at $x = L$ and reaches a maximum at $x = L/2$. The information erasure in (c) can be illustrated by the release of a compressed spring.

Theoretically, a room-temperature computer could be operated at a rate of 57 Tera-bits/s according to Eq.6. As a matter of fact, a modern computer uses millions of times as much energy as the Landauer bound [13] so that the theoretical time needed to write a bit of information could be much shorter than the value given by Eq.6, as shown in Fig.3.

If the write time is as short as the Planck time ($5.39 \times 10^{-44}$ s) that is the shortest validly measurable time length [14], the corresponding energy needed to write a bit of information is

$$\Delta E = \frac{h}{2\Delta t} = \frac{1.05 \times 10^{-34} J s}{2 \times 5.39 \times 10^{-44} s} = 9.7 \times 10^8 J,$$  

which is 29 orders of magnitude larger than the Landauer bound.

As shown in Fig.2, a compressed ideal gas (as a bit of written information) can be approximated by an elastic spring with a (constant) elasticity coefficient $K$. Under the application of a force $F$, the contraction of the spring is described by Hooke’s law: $F = K \times \text{contraction}$ [15]. For a spring with a free length $L$, we can use Eq.3 to estimate the elasticity coefficient $K$ at $x = L/2$ as follows:

$$K = \frac{F_{avg}(x=L/2)}{L/2} = \frac{m_0 v_0^2}{(L/2)^2}.$$  

2
Heisenberg’s uncertainty principle can be used to define information quantitatively from a measuring perspective:

$$1 \text{ (bit)} = 2\Delta E \Delta t / \hbar,$$

from which the time needed to write a bit of information is calculated depending on the input energy. If the input energy equals the Landauer bound, the write time is

$$1.75 \times 10^{-14} \text{ s}.$$

If the write time equals the Planck time ($5.39 \times 10^{-44} \text{ s}$), the write energy is

$$9.7 \times 10^{8} \text{ J}.$$

The elastic potential energy stored in this spring is

$$E_{\text{elastic}} = \frac{1}{2} K (L - x)^2 = \frac{m_0 v_0^2}{2(2L)^2} (L - x)^2. \quad (9)$$

When $x = L$, $E_{\text{elastic}} = 0$, which corresponds to the free state of the spring; when $x = L/2$, $E_{\text{elastic}} = \frac{1}{2} m_0 v_0^2$, which indicates that the elastic potential energy reaches a maximum and originates from the kinetic energy of the information carrier. The calculated potential energy ($\frac{1}{2} m_0 v_0^2$) stored in this imaginary spring is slightly different from the Landauer bound ($k_B T \ln 2 = m_0 v_0^2 \ln 2$) because the elastic coefficient is actually not constant for a (contracted) ideal gas.

To investigate the mass effect of the information carrier, the Landauer bound in Eq.5 can be rewritten in terms of the mass as follows:

$$\Delta E_{\text{min}} = k_B T \ln 2 = m_0 v_0^2 \ln 2. \quad (10)$$

As the translational motion of a particle has three degrees of freedom, the average translational kinetic energy of a freely moving particle in a system with temperature $T$ will is

$$E_k = \frac{1}{2} m_0 v_0^2 = \frac{3k_B T}{2}. \quad (11)$$

Considering that the average molar mass of dry air is

$$28.97 \text{ g/mol} = \frac{28.07 \times 10^{-3} \text{ kg}}{6.022 \times 10^{23}} = 4.81 \times 10^{-26} \text{ kg},$$

we obtain

$$v_0 = \sqrt{\frac{3k_B T}{m_0}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \text{ m}^2 \text{kg} \text{s}^{-2} \text{K}^{-1} \times 300 \text{ K}}{4.81 \times 10^{-26} \text{kg}}} \approx 509 \text{ m/s}. \quad (12)$$

This velocity is larger than the speed of sound (340 m/s), but much smaller than the speed of light. For this reason, Einstein’s energy-mass formula $E = mc^2$ [16] should not be used to convert energy to mass in the Landauer bound [17]. Relativistic effects do not need to be considered in this classical thermodynamic system.

Actually, the two aforementioned equivalences form an information-energy-mass triangle (Fig.4), which shows that information is ultimately physical in terms of requiring energy to create/manipulate it as well as a carrier (of a mass $m_0$) to carry it.

III. EXPERIMENTS

Newton’s cradle [18] was used to experimentally verify the aforementioned information-energy-mass equivalences. As shown in Fig.5, a typical Newton’s cradle consists of identical metal balls suspended in a metal frame such that the balls just touch each other at rest. A ball at one end (equivalent to the frictionless piston in Fig.2) is lifted and released, thereby striking a stationary ball (equivalent to the information carrier in Fig.2), through which a force is transmitted that pushes the ball at the other end upward.
An experiment is conducted on the free state without any written information. The one-dimensional tube shown in Fig.2(a) is simulated here. The "tube" is 6 cm long, and a heavy hammer is used at one end to trap a ball as an information carrier. This information ball is only reciprocating inside the simulated tube, which reproduces the behaviour of a trapped reciprocating ideal gas particle in the Landauer bound calculation [5].

Fig.6 shows an experiment conducted on the free state without any written information. A one-dimensional tube with a trapped information ball is simulated. The kinetic energy of the information ball can be measured by the swing height $H_{\text{swing}}$ of a test ball resulting from a collision between the two balls that brings the information ball to a complete stop, whereby all its kinetic energy is fully converted into the potential energy of the test ball, i.e., $m_0 g H_{\text{swing}} = \frac{1}{2} m_0 v_0^2$, where $g$ is the gravitational constant.

Fig.7 shows an experiment conducted on the information creation/erasure. The hammer is now moved towards the test ball to confine the info ball in the desired half of the tube (with a length reduced from 6 cm to 3 cm). The test ball is released from the same height as that in Fig.6 in order to give the information ball the same velocity $v_0$, which implies that the temperature $T$ remains unchanged since $E_k = \frac{1}{2} m_0 v_0^2 = k_B T/2$. The corresponding travel time of the information ball in one reciprocation cycle ranges from 0.7 s to 0.8 s, which is half that (0.4 s to 0.6 s) in the experiment conducted on the free state shown in Fig.6. That is, the momentum of the information ball is doubled within the same time period as that of the free state. An important conclusion of this study is that the energy input during the creation of a bit of (binary) information is converted into the doubled momentum or the doubled momentum mass of the information ball. In (c)-(d), the information ball exerts useful work in pushing the test ball (analogous to the piston in Fig.2) to release irreversible heat to the environment as a result of losing that written information (that is, the side of the partition that is occupied). This so-called "information engine" is illustrated in Fig.8, where the information erasure is an irreversible manipulation of the created information that increases the entropy. Another way of interpreting the Landauer bound is that, if the information is "burnt", the "Maxwell’s demon" that “created” information in Fig.2(b) or the observer who created "information" in Fig.7(a-b) loses the ability to extract work from the system.
The comparison of Fig. 6 and Fig. 7 indicates that the swing height $H_{\text{swing}}$ of the test ball in (d) after the operation is nearly the same (within the acceptable error tolerance), which implies that the kinetic energy of the information ball remains the same across the two cases. It is obvious that the travel time of the information ball in one reciprocation cycle is halved during the information creation, which implies that the impulse of the information ball doubles for the same time period as that of the free state. This result is obtained because the impulse is time-specific (the time interval must be specified to determine the corresponding impulse value) according to the definition of the impulse $\Delta p = F \cdot t$. That is, the energy input during the creation of a bit of (binary) information is converted into the doubled momentum or the doubled “momentum mass” (mass in motion) $m_p = 2m_0$ of the information ball because $\Delta p = F \cdot t = 2m_0v_0 = m_pv_0$ and the velocity $v_0$ is a constant.

FIGURE 9. An experiment is conducted on the free state without any written information. The (Brownian motion) velocity of the information carrier doubles, which implies that the temperature $T$ increases by 4 times since $E_k = \frac{1}{2}m_0v_0^2 = k_B T/2$. 

FIGURE 8. An information “burning” engine. The information erasure is an irreversible manipulation of the created information, i.e., the “Maxwell’s demon” or the observer that “created” the information loses the ability to extract work from the system after the information is “burnt”.

(a) The test ball is lifted at $4H_{\text{swing}}$ and released; 

(b) It strikes the info ball, transmitting $2v_0$; 

(c) The info ball is bounced back from a stopper and strikes back the test ball; 

(d) The test ball swings at $4H_{\text{swing}}$. 
FIGURE 10. An experiment is conducted on the information creation/erasure. The (Brownian motion) velocity of the information carrier doubles, which implies that the temperature $T$ increases by 4 times since 

$$
\frac{\mathbb{E}}{k} = \frac{1}{2}m_0v_0^2 = kBT/2.
$$

The corresponding travel time of the information ball in one reciprocation cycle ranges from 29.086 s to 29.131 s, which is still roughly half that (0.967 s to 1.067 s) in the experiment conducted on the free state shown in Fig.9. It is demonstrated again that the momentum of the information ball is doubled within the same time period as that of the free state. Furthermore, it verifies our new definition of information ($\text{1 (bit) = } 2\Delta \mathbb{E} \Delta t/\hbar$) in the sense that the higher the energy input is, the shorter the time needed to write a bit of information is.

As elaborated above, the position of a Brownian particle is used to store information in a classical thermodynamic system (Fig.2). A spin can be used as a modern computing paradigm to replace a charge for information storage, allowing for faster, low-energy operations [19]. For example, an electron has a charge and a spin that are inseparable.

Fig.11 shows the Stern–Gerlach experiment demonstrating the deflection of silver atoms with nonzero magnetic moments by a magnetic field gradient [20]. The screen reveals discrete points of accumulation, rather than a continuous distribution, resulting from the quantized spin. Historically, this experiment is a seminal benchmark experiment of quantum physics providing evidence for the reality of angular-momentum quantization in all atomic-scale systems [20].

IV. ORIENTATION-BASED INFORMATION STORAGE

As shown in Fig.11, the energy of flipping a spin in a magnetic field $B$ can be expressed as

$$
\Delta \mathbb{E}_1 = g\mu_B B = 2\mu_B B,
$$

where $\mu_B$ is the Bohr magneton and the value of the electron spin $g$-factor is roughly equal to 2.

In this “nonclassical” information system, the magnetic field $B$ (as an environmental parameter) is analogous to the temperature $T$ (as another environmental parameter) in the Landauer bound, which determines a new energy bound.

The magnetic interaction between the two spin-1/2 valence electrons across a separation ($2.18\sim2.76 \text{ } \mu m$) was measured [21]. According to our calculation, one electron applies a magnetic field $B = 8.82 \times 10^{-14} T$ to the other electron (across a separation of 2.76 $\mu m$), which is much smaller than $B \approx 0.1 T$, $\frac{\partial B}{\partial z} = 1 \times 10^3 T/m$ in the Stern–Gerlach experiment [20]. Accordingly, the energy to flip a spin via the spin-spin interaction in the presence of the magnetic field $B$ is $\Delta E = 2\mu_B B = 1.64 \times 10^{-36} J$. This energy bound is 15 orders of magnitude lower than the Landauer bound ($3 \times 10^{-21} J$) [19]. The energy used to retain the defined spin state must still be higher than the Landauer bond to keep the electron at one side of the potential well. At the readout or
erasure stage, there is no need to move the electron from one side of the potential well to the other side, which is different from the energy based on an electron’s position. In either case, we cannot separate the (internal, intrinsic) spin and charge of an electron [19].

An information-energy-mass triangle for spin information storage is shown in Fig.12. The mass of an information carrier remains an important and necessary apex of this triangle in terms of carrying information in a carrier. The energy input during the information creation is still converted into the doubled momentum mass of the information carrier.

As summarized in Fig.14, data storage can be categorized into two types: (classical) position-based and (modern) orientation-based. As illustrated in Table 1, a specific information carrier is needed for each type of data storage depending on the chosen physical feature.

![An information-energy-mass triangle for modern orientation-based information storage.](image)

**TABLE 1** A specific information carrier is needed for each type of data storage. Since the velocity is much smaller than the speed of light, relativistic effects do not need to be considered unless those particles travelling (fully/nearly) at the speed of light (such as photons and neutrinos) are used.

| Info carrier            | Mass $m_0$ | Velocity of Brownian motion $v_0$ at R. T. | Position-based | Orientation-based |
|-------------------------|------------|--------------------------------------------|---------------|------------------|
| Electron                | $9.1 \times 10^{-31}$ kg |                                    | ✓             | ✓               |
| Hydrogen atom           | 1 g/mol    | 2741 m/s                                   | ✓             | ✓               |
| Air molecule [11]       | 29 g/mol   | 509 m/s                                    | ✓             | ✓               |
| Sr$^+$ ion as a spin carrier [21] | 88 g/mol | 292 m/s                                    | ✓             | ✓               |
| Polystyrene bead (C$_6$H$_5$) manipulated by a Szilárd-type Maxwell demon [10] | 104 g/mol | 268 m/s                                    | ✓             | ✓               |
| Ag atom in Stern–Gerlach experiment [20] | 109 g/mol | 263 m/s                                    | ✓             | ✓               |
| Fe$_{50}^+$ molecular magnet as a collective S=10 (20$\mu_B$) giant spin carrier [23] | 448 g/mol | 130 m/s                                    | ✓             | ✓               |
| Rydberg atom [24]       |            |                                            |               |                  |

Regardless of whether a bit of information is position-based or orientation-based, the energy barrier (the Landauer bound) of the bistable potential well needs to be overcome to create/write the bit from scratch. For position-based data storage, a rewriting, (destructive) readout or erasure operation still needs to move the information carrier between the two stable states. For orientation-based data storage, a readout or erasure operation of spin information does not need to move the information carrier (electron) from one side of the potential well to the other side. Although the energy of flipping a spin is much lower than the energy based on an electron’s position, we cannot separate the (internal, intrinsic) spin from its carrier, and the mass $m_0$ of an information carrier still needs to be considered in our energy calculation in a classical thermodynamic way, as illustrated in Fig.14.

Newton’s cradle was used to experimentally verify the deduced information-energy-mass equivalences. These experiments vividly demonstrate that the energy (to halve the reciprocating motion distance of the information carrier) input during the creation of a bit of (binary) information is stored in the information carrier in the form of the doubled momentum or the doubled “momentum mass” (mass in...
motion). During the information erasure, the stored energy was found to release irreversible heat to the environment as a result of losing that written information. Furthermore, the experiments verified our new definition of information (1 bit = \((2\Delta E \Delta t)/\hbar\)) in the sense that the higher the energy input is, the shorter the time needed to write a bit of information is.

The aforementioned conclusions on the information-energy-mass equivalences may help understand the fundamental concept of information and the deep physics behind it. It may also arouse considerable interest in reversible computing, in which no information is erased to reversible computing, in which no information is erased to release irreversible heat to the environment. According to Koomey’s law [25], the increase in the irreversible computing, in which no information is erased to release irreversible heat to the environment. According to Koomey’s law [25], the increase in the computational energy consumption based on the Landauer bound will come to a halt by 2050.

REFERENCES

[1] A short overview is found in: Luciano Floridi (2010). Information - A Very Short Introduction. Oxford University Press. ISBN 978-0-19-160954-1.

[2] https://en.wikipedia.org/wiki/Information, accessed 15 June, 2021.

[3] DT&SC 4-5: Information Theory Primer, 2015, University of California, Online Course, https://www.youtube.com/watch?v=9qanHTredVE&list=PLljBSCvWCU3rNm46D3R85fM0hrJuAlg&index=42, accessed 15 June, 2021.

[4] W. Heisenberg, "Uber den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik", Zeitschrift für Physik, 43 (3–4): 172–198, 1927.

[5] R. Landauer, “Irreversibility and Heat Generation in the Computing Process”, IBM J. Res. Develop. 5, 183–191, 1961.

[6] A. Be’rut, A. Arakelyan, A. Petrosyan, S. Ciliberto, R. Dillenschneider & E. Lutz, “Experimental Verification of Landauer’s Principle Linking Information and Thermodynamics”, Nature, Vol.483, 187, 2012.

[7] Clarke, Chapter 9: Impulse and Momentum, SMU PHYS1100.1, fall 2008.

[8] R. Clausius, The Mechanical Theory of Heat – with its Applications to the Steam Engine and to Physical Properties of Bodies. London: John van Voorst, 1867.

[9] L. Szilárd, “On the Decrease in Entropy in a Thermodynamic System by the Intervention of Intelligent Beings” (Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen), Zeitschrift fur Physik, 53, 840-56, 1929.

[10] S. Toyabe, T. Sagawa, M. Ueda, E. Muneyuki & M. Sano, “Experimental demonstration of information-to-energy conversion and validation of the generalized Jarzynski equality”, Nature Physics, 14 Nov, 2010.

[11] https://en.wikipedia.org/wiki/Entropy, accessed 15 June, 2021.

[12] Yang, Y. et al. “Ultrafast magnetization reversal by picosecond electrical pulses”. Sci. Adv. 3, 1603117–1603122, 2017.

[13] S. Moore, "Landauer Limit Demonstrated". IEEE Spectrum, 2012.

[14] M. Planck, The Theory of Heat Radiation. Masius, M. (transl.) (2nd ed.). P. Blakiston's Son & Co. OL 7154661M, 2014.

[15] R. Hooke, De Potentia Recessitivae, or of Spring. Explaining the Power of Springing Bodies, London, 1678.

[16] A. Einstein, "Ich die Trägheit eines Körpers von seinem Energieinhalt abhängig?", Annalen der Physik 18: 639–643, 1905.

[17] M. Vopson, “The mass-energy-information equivalence principle”, AIP Advances 9, 095206, 2019.

[18] https://en.wikipedia.org/wiki/Newton%27s_cradle, accessed 15 June, 2021.

[19] F. Z. Wang, “Can We Break the Landauer Bound in Spin-Based Electronics?”. IEEE Access 9, pp. 67109-67116, 2021.

[20] W. Gerlach, O. Stern, "Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld". Zeitschrift für Physik. 9 (1): 349–352, 1922.

[21] S. Kotler, N. Akerman, N. Navon, Y. Glickman & R. Ozeri, “Measurement of the Magnetic Interaction Between Two Bound Electrons of Two Separate Ions”, pp.376-380, Nature, Vol.510, Issue.7505, 2014.

[22] M. Planck, The Theory of Heat Radiation. translated by Masius, M. P. Blakiston’s Sons & Co., 1914.

[23] R. Gaudenzi, E. Burzuri, S. Maegawa, H. S. J. van der Zant, and F. Luis, “QuantumLandauer erasure with amolecular nanomagnet,'’, Nature Phys., Vol. 14, No. 6, pp. 565568, Jun. 2018.

[24] Rydberg Atom. https://en.wikipedia.org/wiki/Rydberg_atom, Accessed: June23 , 2021.

[25] J. Koomey, S. Berard, M. Sanchez, H. Wong, "Implications of Historical Trends in the Electrical Efficiency of Computing", IEEE Annals of the History of Computing, 33 (3): 46–54, 2010.