Research Article

Statistical Assessment of Simplified CPTU-Based Hydraulic Conductivity Curves

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1. Introduction

Empirical correlations become significant to estimate soil parameters during construction projects, especially due to the lack of test equipment or cost, limited time for testing, etc. [1–4]. In this perspective, there are some attempts available to obtain in situ hydraulic conductivity for engineering application. As a labor-saving and cost-effective approach, gradually, instead of a conventional test, the piezocone penetration test (CPTU) has been employed widely in geotechnical engineering to determine the mechanical properties of the soil, including the hydraulic conductivity [5–9]. The methodology to assess hydraulic conductivity of soils from piezocene soundings can be divided into three major categories, the dissipation test method [10, 11], the soil behavior index method [6], and theoretical analysis method, wherein the last one has explicit equations to interpret the penetration process based on Darcy’s law and the cavity expansion theory [12–18]. In this method, Chai et al.’s method [16] modified from Elsworth and Lee’s method [14, 15] can be regarded as the classical and well-known approach in terms of a bilinear line defined by \(K_D - B_qQ_t\), for undrained/drained soils. Yet, it increases difficulties in distinguishing different soils. And the large variability of the intersection point also hinders the accuracy, simplicity, and operability of the piecewise functions. In addition, a dividing line may be selected with a certain degree of subjectivity [17], and the points in undrained and drained soils may be located in the adverse zone, which may lead us to doubt the necessity of two intersecting lines to distinguish the undrained and drained soils.

Hitherto, an attempt for a uniform and relatively simple curve can be more suitable and accessible from a utilitarian point of view. Herein, three types of uniform curves, arc, parabola, and ellipse, are proposed to estimate \(B_qQ_t\) from piezocene soundings in USA, Japan, and China more universally and precisely. And then, these three curves are compared with a classical bilinear line using statistical methodology.
2. Methodology

2.1. Classical Theoretical Method. Based on numerous CPTU data and practice, Elsworth and Lee [14] proposed an explicit method (short for Elsworth’s method) to calculate the hydraulic conductivity from the piezocone sounding records:

\[
K_D = \frac{4k\sigma_v}{\gamma_w U},
\]

or

\[
k = \frac{\gamma_w K_D U}{4k\sigma_v},
\]

in which, \(B_q\) and \(Q_t\) are defined as [19]

\[
B_q = \frac{u_2 - u_0}{q_t - \sigma_v},
\]

\[
Q_t = \frac{q_t - \sigma_v}{\sigma_v}.
\]

Then, Elsworth and Lee [14, 15] suggested a more reasonable correction using curve fitting adjustment:

\[
K_D = \frac{0.62}{(B_q Q_t)^{1.6}}.
\]

In place of a spherical surface flow of pore water, assuming a half-spherical surface flow covers the tip of the cone, Chai et al. [16] presented a classical modified approach (short for Chai’s method), in terms of a bilinear relation (see Figure 1):

\[
K_D' = \begin{cases} 
\frac{1}{B_q Q_t}, & B_q Q_t < 0.45, \\
0.044, & B_q Q_t > 0.45.
\end{cases}
\]

2.2. Empirical Expressions. From observation of the piezocone penetration test results from Elsworth and Lee [15] and Chai et al. [16] (see Figure 1), it is proposed that the dividing line is \(B_q Q_t = 0.45\), which coincides with Equation (11) presented by Chai et al. [16]. The dotted line \(K_D = 0.62/(B_q Q_t)^{1.61}\) and solid line \(K_D = 0.044/(B_q Q_t)^{4.91}\) are from Elsworth’s method [14] and Chai’s method [16], respectively, and the dotted line \(K_D' = 1/B_q Q_t\) pertains to Elsworth’s method and Chai’s method that are in forms of bilinear correction. It is obvious that Chai’s method [16] agrees with the scatter points better than Elsworth’s method; however, the dividing line \(B_q Q_t = 0.45\) which distinguished the undrained and drained soils seems to be factitious and fluctuant. If this argument is accepted, a uniform curve that neglects the different soils and is less complicated can provide an accurate and simple correlation. Hence, the fitting curves of the arc, parabola,
and ellipse are presented in terms of dash-dotted line, dotted line, and heavy solid line shown in Figure 1. Through curve fitting, the "best-fit" expressions for the arc, parabola, and ellipse to the data collected by Elsworth and Lee [15] are given by

\[
\log(K_D) + 3.8 + \log\left(\frac{B_q Q_i}{C_0/C_1}\right) + 9.9 = 10.3, \tag{6}
\]

\[
\log\left(\frac{B_q Q_i}{C_0/C_1}\right) = 0.06 \log^2(K_D) - 0.42 \log(K_D) - 0.3, \tag{7}
\]

\[
\frac{(\log(K_D) + 5)^2}{9.4^2} + \frac{(\log\left(\frac{B_q Q_i}{C_0/C_1}\right) + 5)^2}{5.5^2} = 1. \tag{8}
\]

The three curves seem complex in terms of logarithm, yet they are simple in the process because the x-axis and y-axis are also on double logarithmic scales. It is noted that there are three, three, and two variables in Equations (6) through (8), separately, for the reason that the factor of 5 in Equation (8) is constant which may be derived from the logarithm of the minimum of x-axis and y-axis. Owing to concentration of the data from [16], all four curves conform well to the data, which are represented as rings in Figure 1.

### 3. Data

A collection data of seven sites in the Yangtze Delta that lie on the eastern part of China is shown in Figure 2: Map of the CPTU sites.

Table 1: Soil properties and description of sites.

| Site name                        | Major soil layer       | Depth range (m) | Site name                        | Major soil layer       | Depth range (m) |
|----------------------------------|------------------------|-----------------|----------------------------------|------------------------|-----------------|
| Yushan station (Suzhou)          | Clay                   | 1.0–4.8         | Xinghu road station              | Silty sand             | 12.0–18.5       |
|                                  | Clay-rich silt         | 5.5–7.0         |                                 | Clay                   | 2.0–11.8        |
|                                  | Silty clay             | 18.6–22.4       |                                 | Silty sand             | 19.5–28.6       |
| Hongzhuang station (Suzhou)      | Clay                   | 0.8–3.8         |                                 | Clay                   | 1.7–4.9         |
|                                  | Silty clay             | 5.5–13.6        | Zuhui road station (Suzhou)      | Silt                   | 11.0–16.8       |
|                                  | Silty sand-rich        | 21.2–28.0       |                                 | Silty clay             | 16–22.6         |
|                                  | silty clay             |                 |                                 |                        |                 |
| Jiangbei work well (Nanjing)     | Silty clay             | 2.3             |                                 | Silty clay             | 1.2–4.1         |
|                                  | Muddy clay             | 8–10            | The fourth Yangtze              | Silty sand             | 6.2–9.8         |
|                                  | Clay                   | 16–18           | river bridge (Nanjing)           | Silty sand             | 16.5–18.4       |
| The Yangtze bridge at Taizhou (Taizhou) | Muddy clay       | 0.8–1.8         |                                 |                        |                 |
|                                  | Silty clay             | 3.1–13.4        |                                 |                        |                 |
|                                  | Silty sand             | 18.1–28.6       |                                 |                        |                 |
A summary description of soils in these sites is presented in Table 1. Typical profiles of CPTU measurements, including cone tip resistance \( q_t \), side friction resistance \( f_s \), and pore water pressure \( u_2 \) versus depth recorded at Hongzhuang station in Suzhou, are indicated in Figure 3.

![Figure 3: Typical CPTU soundings recorded at Hongzhuang station, Suzhou.](image)

Figure 4: Relationship between measured nondimensional hydraulic conductivity \( K_D \) and \( B_qQ_t \) (data from this paper).

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![Figure 4: Relationship between measured nondimensional hydraulic conductivity \( K_D \) and \( B_qQ_t \) (data from this paper).](image)
Predicted $B_{\text{Q}_i}$ vs. Measured $B_{\text{Q}_i}$

(a) $\log(B_{\text{Q}_i,\text{Fit}}) = 1.52 \log(B_{\text{Q}_i,\text{me}})$
$R^2 = 0.86$
Number of data points = 59

(b) $\log(B_{\text{Q}_i,\text{Fit}}) = 1.0871 \log(B_{\text{Q}_i,\text{me}})$
$R^2 = 0.9001$
Number of data points = 59

(c) $\log(B_{\text{Q}_i,\text{Fit}}) = 1.088 \log(B_{\text{Q}_i,\text{me}})$
$R^2 = 0.9002$
Number of data points = 59

Figure 5: Continued.
The applied CPTU device is produced by Vertek-Hogentogler & Co. of USA. The equipment is a versatile piezocone system fabricated with 60°-tapered, 10 cm² tip area cone which provided measurements of \( q_t, f_s \), and \( u_2 \) with a 5 mm thick porous filter located just behind the cone tip. The penetration rate in this study was 20 mm/s.

### 4. Analysis

#### 4.1. Qualitative Analysis

For the data proposed in this paper, four curves are revealed in Figure 4. Figure 4 shows that the bilinear correction gives worse performance, while the others make slight difference obviously.

#### 4.2. Quantitative Analysis

There is little doubt that qualitative analysis provides a visualized way to compare these curves; however, this does not give quantitative accuracy. Hence, five measures of effectiveness (MOEs) are used to further assess the validity of the abovementioned curves. RMSE is the square root of the average of the squared, which is given as

\[
\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (B_c - B_m)^2}. \tag{9}
\]

The ratio \( K \) of the estimated \( B_q Q_t \) to measured \( B_q Q_t \) can be defined by

\[
K = \frac{B_c}{B_m}. \tag{10}
\]

The ranking index (RI) proposed by Briaud and Tucker [20] and the ranking distance (RD) proposed by Cherubini and Giasi [21] are other methods of expressing an overall judgment, which are expressed as

\[
\text{RI} = \mu_{\ln (K)} + \sigma_{\ln (K)}, \tag{11}
\]

\[
\text{RD} = \sqrt{(1 - \mu_K)^2 + \sigma_K^2}. \tag{12}
\]

For correlation equations where the precision, indicated by the standard deviation and mean value, and the accuracy are similar, RD gives a better result than RI, while for those that are either very accurate or very precise, RI gives the best result [2, 4]. RD has been used by numerous investigators [2, 4, 22–24] to evaluate the performance of empirical equations.

Relative error (RE) [23–32] is the proportion of the absolute difference between the measured and the estimated to the measured hydraulic conductivity, which is given as

\[
\text{RE} = \frac{|B_c - B_m|}{B_m}. \tag{13}
\]

The lower the RMSE, RE, RI, and RD values are, the better the performance the curve gives.

Figure 5 and Table 2 reveal that the bilinear line proposed a worst correlation with the determination coefficient \( R^2 \) value of 0.86 between the measured and predicted \( B_q Q_t \) values, whereas the ellipse curve gave the best performance \( (R^2 = 0.9005) \).

![Figure 5: Measured versus predicted $B_q Q_t$ values for different curves: (a) bilinear, (b) arc, (c) parabola, and (d) ellipse (data from this paper, China).](image)

Log($B_q Q_{t,p}$) = 1.0764log($B_q Q_{t,m}$)

\( R^2 = 0.9005 \)

Number of data points = 59
5. Discussion

A summary of the results of the RMSE, \( K \), RE, RI, and RD analyses for soil data from this paper is revealed in Table 2. Regarding RMSE, the bilinear curve (RMSE = 1.234) gave the best prediction unexpectedly. For the general overestimation (\( K > 1 \) or RE > 0) of \( B_q Q_t \), all four curves had below 50% of the \( K \) values greater than 1. In terms of accuracy, the ellipse curve, proposed in this paper, with two variables gave the most accurate evaluation of \( B_q Q_t \), with a mean \( K \) of 0.667, yet the bilinear curve gave the worst performance with a mean \( K \) of 0.463. In terms of precision determined by SD of \( K \), the bilinear curve gave the most precise prediction with a mean SD of 0.432. Yet, in terms of precision determined by SD of RE, the parabola curve gave the most precise prediction with a mean SD of 0.282. With respect to RI, the best performance was delivered by the arc curve proposed in this study (RI = 1.408). In terms of RD, the most efficient curve was the ellipse curve (RD = 0.580) proposed in this paper. In these MOEs, RD is a better parameter for comparing the suitability of the different curves [2]. In addition, only the ellipse curve involves two variables; the other curves use three variables. Hence, the overall best curve was the ellipse (RD = 0.580).

6. Conclusions

The existing classical bilinear relation defined by \( K_D = B_q Q_t \), for undrained and drained soils, may be selected with a certain degree of subjectivity, and several undrained/drained points occur in opposite sites. A uniform and relatively simple curve including an arc, parabola, or ellipse correlation can be more suitable and accessible. A database in the Yangtze Delta region has been collated to assess the three curves compared with the bilinear line. On the basis of abundant analyses executed in this study, the following conclusions can be obtained:

Regarding RMSE, the bilinear curve (RMSE = 1.234) gave the best prediction unexpectedly. For the general overestimation (\( K > 1 \) or RE > 0) of \( B_q Q_t \), all four curves had below 50% of the \( K \) values greater than 1. In terms of accuracy, SD, RD, and RI, the arc, parabola, and ellipse curves that ignored distinction of the undrained and drained soils and took out the intersecting line could provide expressions for simplification and gave better performance than the bilinear line. Considering the consistency of measured and predicted \( B_q Q_t \) values, the overall best curve was the ellipse (RD = 0.580) with two variables. This is expected to improve the application scope and simplicity.

Data Availability

Data supporting the results of my study were generated during the study.

Conflicts of Interest

The authors declare no conflict of interest.

Authors’ Contributions

Mingfei Zhang provided the main ideas and data processing of the whole paper. Liyuan Tong did the experiments and provided guidance for ideas and data processing.

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Table 2: Results of the RMSE, \( K \), RE, RI, and RD (data from the present paper, China).

| Curve  | RMSE  | \( > 1 \) (%) | \( K \) | \( \sigma \) | \( > 0 \) (%) | \( RE \) | \( \sigma \) | RI | RD |
|--------|-------|--------------|-------|--------|--------------|-------|--------|-----|-----|
| The bilinear | 1.234 | 0.119 | 0.463 | 0.432 | 0.119 | -0.537 | 0.305 | 2.692 | 0.689 |
| Arc    | 1.330 | 0.153 | 0.656 | 0.469 | 0.153 | -0.344 | 0.288 | 1.507 | 0.581 |
| Parabola | 1.342 | 0.186 | 0.656 | 0.474 | 0.186 | -0.344 | 0.282 | 1.559 | 0.586 |
| Ellipse | 1.327 | 0.203 | 0.667 | 0.475 | 0.203 | -0.333 | 0.290 | 1.408 | 0.580 |
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