Performance Estimation of Wireless Electric Vehicle Charging System Based On Magnetic Coupling

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Abstract: In order to achieve an efficient wireless Electric Vehicle (EV) charging system in non-ideal practical scenarios, a proper design guideline has been delineated through the simulation, theoretical calculation as well as experimental investigation. It is examined that the wireless power transfer efficiency (WPTE) is invariably affected by the configuration of the charging coils (coil radius & number of turns), coupling to loss ratio, ohmic loss, radiation resistance, operating frequency, magnetic coupling as well as physical air gap between the coils. It is found that there is a certain operating regime at which maximum WPTE can be upheld. The acquired results provide a comprehensive strategic plan that can be used in EV charging system.

Keywords : Coupled mode theory, Magnetic resonance, Wireless EV charging, WPTE

I. INTRODUCTION

During this time of an Earth-wide temperature boost because of Carbon Dioxide (CO2) discharges, regional authorities everywhere throughout the world are scrambling today to execute arrangements so as to decrease CO2 outflows from their private and public vehicles [1]. Thus, there is much interest has been laid by the transportation industry towards electric vehicles to minimize the carbon emission [2-3]. Although the plug in vehicle charging system is adopted for charging vehicle but to overcome the associated difficulties as well as to improve the convenience and usability of EVs, there is a need to develop wireless power transfer system (WPTS) for charging of EVs. Many research reports [4-7] revealed that the non-radiative magnetically coupled WPTS, which illustrated the possibility to transmit power more efficiently for EV charging than the conventional inductive systems. The idea of magnetic resonance based wireless charging for EVs is recently getting much attention by industry as well as academic groups [8,9].

Nonetheless, for designing practical EV charging setup, it is not adequate to just substantiate the power transmission between the transmitting and receiving charging coils but utmost needed to surmount the associated key issues that influence the power transfer efficiency. Hence, simulation and experimental investigation have been done in the present work to delineate the design guidelines for wireless EV charging.

II. MATHEMATICAL MODEL AND ANALYSIS OF THE RESULTS

A typical MRC wireless EV charging system contains a transmitter coil that is placed at the charging station and a receiver coil which is embedded in the vehicle. Here, the power is wirelessly transferred from the transmitter coil to the receiver coil due to electromagnetic resonant inductive coupling (RIC) between both coils. By utilizing the CMT [3, 10-11], the energy trade-off between the transmitter and receiver resonating coils can be inspected by obtaining the solution of the following set of coupled first order differential equations:

\[
\frac{da_1(t)}{dt} = -(j\omega_1 + \Gamma_1)a_1(t) + jk_{11}a_2(t) + F_s(t)
\]

\[
\frac{da_2(t)}{dt} = -(j\omega_2 + \Gamma_2 + \Gamma_1)a_2(t) + jk_{21}a_1(t)
\]

Where, 
\(F_s(t)\) = sinusoidal signal applied to excite the transmitting coil.
\(a_1(t)\) = Alternating field amplitude in the transmitting coil.
\(a_2(t)\) = Alternating field amplitude induced in the receiving coil.
\(\omega_1\) = Angular resonant frequency of the transmitting coil.
\(\omega_2\) = Angular resonant frequency of the receiving coil.
\(\Gamma_1\) = Resonance width or intrinsic decay rate due to the ohmic and radiative losses in the transmitting coil.
\(\Gamma_2\) = Intrinsic decay rate in the receiving coil.
\(\Gamma_3\) = Resonance width due to electric load resistance attached to the receiving coil.

If the sinusoidal signal described as
\[ F_1(t) = A_1 e^{-j\omega t} \] (3)

Then the solutions of the linear differential equations (1) and (2) will be
\[ a_1(t) = A_1 e^{-j\omega t} \] (4)
\[ a_2(t) = A_2 e^{-j\omega t} \] (5)

Putting the above values in equation (2), it can be obtained as given below:
\[ j\omega A_2 e^{-j\omega t} = (j\omega + \Gamma_2 + \Gamma_k) A_2 e^{-j\omega t} - jk_{12} A_1 e^{-j\omega t} \]
\[ A_2 e^{-j\omega t} (j\omega + j\omega + \Gamma_2 + \Gamma_2) = jk_{12} A_1 e^{-j\omega t} \]
\[ \frac{A_2}{A_1} = \frac{jk_{12}}{\Gamma_2 + \Gamma_2} \] (6)

or
\[ \frac{A_2}{A_1} = \frac{\Gamma_2 + \Gamma_2}{jk_{12}} \] (7)

If \( \Gamma_1 \) is the energy contained in the transmitter coil and \( \Gamma_2 \) is the energy contained in the receiver coil, then by the help of transmitter coil power is observed \( (P_1) \) and received by the receiver coil \( (P_2) \), after that the power is transferred to the electric load resistance which is connected to the receiver coil \( (P_L) \) will be as follows:
\[ P_1 = 2\Gamma_1 |A_1|^2 \] (8)
\[ P_2 = 2\Gamma_2 |A_2|^2 \] (9)
\[ P_L = 2\Gamma_L |A_L|^2 \] (10)

Hence the PTE of the WPT system will be calculated as follows:
\[ \eta_{12} = \frac{P_2}{P_1 + P_2 + P_L} \]
\[ \Rightarrow \eta_{12} = \frac{2\Gamma_2 |A_2|^2}{2\Gamma_1 |A_1|^2 + 2\Gamma_2 |A_2|^2 + 2\Gamma_L |A_L|^2} \]
\[ \Rightarrow \eta_{12} = \frac{\Gamma_2}{\Gamma_1 + \Gamma_2 + \Gamma_L} \] (11)

Substituting the value of \( A_2/A_1 \) from equation (7) in equation (11) it can be obtained as:
\[ \eta_{12} = \frac{\Gamma_2}{\Gamma_1 + \Gamma_2 + \Gamma_L} \frac{k_{12}^2}{1 + \frac{k_{12}^2}{\Gamma_1 \Gamma_L}} \] (12)

In order to maximize the WPT efficiency, an optimal value of \( \frac{\Gamma_1}{\Gamma_2} \) should be chosen that can be obtained from the derivative of the power transfer efficiency \( \eta_{12} \) expression which is given in equation (11). It is expressed as follow:
\[ \frac{\Gamma_1}{\Gamma_2} = \sqrt{1 + \frac{k_{12}^2}{\Gamma_1 \Gamma_L}} \] (13)

Putting the value of \( \frac{\Gamma_1}{\Gamma_2} \) obtained from equation (13) in the equation (12), the power transfer efficiency is found to be
\[ \eta_{12} = \frac{\Gamma_2}{\Gamma_1 + \Gamma_2 + \Gamma_L} \frac{k_{12}^2}{1 + \frac{k_{12}^2}{\Gamma_1 \Gamma_L} + 1 + \frac{k_{12}^2}{\Gamma_1 \Gamma_L}} \] (14)

For identical transmitter and receiver coils;
\[ \Gamma_1 = \Gamma_2 = \Gamma \]
\[ k_{12} = k \]

By substituting the above two conditions in equation (14), the WPTE of the system are calculated as follows [4]:
\[ \eta = \frac{k^2}{\Gamma^2} \sqrt{1 + \frac{k^2}{\Gamma^2}} \left( 1 + \frac{k^2}{\Gamma^2} \right)^{-1} \] (15)

The equation (15) unveils that the system is operates in a strongly coupled region i.e. \( k/\Gamma \gg 1 \) because of the \( k/\Gamma \) is very large. Therefore, the WPTE will be maximum.

Here, in this experimental and simulation study, both the circular spiral coils are of similar dimension (outer radius \( R_1 = R_2 = R \); Inductance \( L_1 = L_2 = L \)). The coupling coefficient between the transmitter and receiver circular spiral coils can be deliberated from the given equation [12-13]:
\[ k = \frac{\pi M}{\sqrt{L_1 L_2}} = \frac{\pi M}{L} \] (16)

Where \( M \) is the total mutual inductance between the coils and \( f \) is the operating frequency of the system. The mutual inductance \( (M) \) in-between the two circular coils can be determined as follows [13-17]:
\[ M_{12} = \mu_0 R \left[ \frac{2}{x} - \frac{x}{x} \right] K(x) - \frac{2}{x} E(x) \] (17)

Where
\[ x = \frac{\pi M}{\sqrt{4R^2 + D^2}} \]

In this case, \( K(x) \) and \( E(x) \) are known as the complete elliptic integrals of the 1st and 2nd kind. The separation distance between two resonant coils is \( D \) and the free space permeability is free space \( \mu_0 \). For
the two coils, each with a finite number of turns, the total mutual inductance can be calculated by summing over the entire loop pairing between the coils as follows:

$$M = \sum_{i=1}^{N} \sum_{j=1}^{N} M_{ij}$$  

(18)

The approximated mutual inductance between the two identical circular spiral coils is found as a function of the coil dimensions, number of coil turns and physical separation distance which can be interpreted as follows [12]:

$$M \simeq \frac{\mu_0 N^2 R^4}{2(R^2 + D^2)^{3/2}}$$  

(19)

Here, $N$ is the number of turns of the coils, $\mu_0$ is the permeability of free space and $D$ is the separation distance between the two resonant coils.

As we know the WPT system based on magnetic resonance coupling method, the energy can be dissipated through ohmic and radiative losses and from the transmitter coil to the receiver coil in shorter interval the energy is transferred. Thus the PTE of the system is maximum.

The intrinsic loss rate ($\Gamma$) for radiative resistance ($R_{\text{radiative}}$) and ohmic resistance ($R_{\text{ohmic}}$) of the system will be represented as given below [3,14]:

$$\Gamma = \frac{R_{\text{ohmic}} + R_{\text{radiative}}}{2L}$$  

(20)

Where ohmic resistance ($R_{\text{ohmic}}$) and radiative resistance ($R_{\text{radiative}}$) for the circular spiral coil which is made of a copper wire (i.e. electrically conducting) conductivity ($\sigma$), total length of the coil ($l$) and radius of the cross sectional area ($a$) with inductance ($L$), outer radius ($R$) and $N$ is total number of turns are as follows:

$$R_{\text{ohmic}} = \frac{RN}{2a} \sqrt{\frac{\mu_0 \sigma}{\epsilon_0}}$$  

(21)

$$R_{\text{radiative}} = \sqrt{\frac{\mu_0}{\epsilon_0} \left( \frac{4\pi^4 N^2}{3} \left( \frac{R}{c} \right)^4 \right)}$$  

(22)

Where, the permittivity of free space is $\epsilon_0$ and $c$ is the speed of light. Thus, from the above equations (16-22), the $k_f$ is found to be

$$k_f = \frac{2\pi M}{\left( R_{\text{ohmic}} + R_{\text{radiative}} \right)}$$  

(23)

or

$$k_f = \frac{\mu_0 \pi^2 N R^2}{\left( R^2 + D^2 \right)^{3/2}} \left[ \frac{1}{2a} \sqrt{\frac{\mu_0 \sigma}{\epsilon_0}} + \sqrt{\frac{\mu_0}{\epsilon_0} \left( \frac{4\pi^4 N^2}{3} \left( \frac{f}{c} \right)^4 \right)} \right]$$  

(24)

From the PTE equation (15) and the coupling to loss ratio equation (24), it can be illustrated that the WPTE of the system depends on various factors such as electromagnetic coupling coefficient, mutual inductance between two resonant coils, operating resonant frequency ($f$), separation distance between the coils ($D$), coil dimensions, and coil material property.

The WPTE characteristics w.r.t $N$ of the coil for a fixed coil size is depicted in Fig. 1. It is observed that the optimum frequency range corresponding to the strongly coupled region varies with the variation of $N$ of the coil. This may be due to the change in intrinsic loss rate that means change in ohmic loss and radiative loss with respect to frequency for different turns of a fixed coil size. The frequency characteristics of ohmic loss and radiative loss for different turns of the coil are depicted in Fig. 2 and 3, respectively. It is observed that radiative loss and ohmic loss both are increased with the $N$ for a fixed coil size. Therefore, the effect of the $N$ on the operating frequency has to be taken care off for build an efficient WPT system. Therefore, it can be concluded that for an efficient WPT system, to operate in a particular frequency region, size of the coil has to be considered consequently.
In Fig. 4 shows how the coil size effect on the PTE and its results are depicted that the operating frequency region for maximum PTE varies with the coil radius. Hence the strongly coupled region varies with the coil radius. Moreover for larger coil size the resonant frequency band of optimum PTE moves towards low frequency zone. This is reinforced by the fact that both the radiative and ohmic losses are increases with the coil radius (R) in comparison to the coupling coefficient of the system.

The WPTE characteristics w.r.t operating frequency at selected vertical separation gap between the coils is displayed in Fig. 5. It can be noticed that the maximum PTE is obtained for a particular frequency range called optimum frequency band and the $\eta$ declines crucially outside that frequency range. This occurs due to both the transmitter and receiver coils that are magnetically coupled with one another at that range of resonant frequency. Also from Fig. 6, it is depicted that at the optimum resonant frequency, the $\eta$ is very large.

It illuminates that the PTE relies on the operating frequency range of MRC based wireless power transfer system. Therefore it can be concluded that individual system will have a particular operating frequency range (strongly coupled region; $\eta/T >> 1$) for which maximum PTE can be achieved.

It also can be presumed from these above results that the physical spacing between the coils has an effect on the PTE. The optimum frequency region for maximum PTE varies with the physical separation gap between the coils. The PTE trims down for larger physical separation distance between the coils. It can be observed that there exists a wider optimum frequency range for smaller physical separation gap between the coils and it becomes narrower for higher vertical separation distance. This can be due to the decrease of mutual inductance and increased coupling coefficient between both the coils. It explicates to consider an optimum frequency band corresponding to their physical separation distance for designing an efficient WPT system for EV charging.

### III. ELECTROMAGNETIC (EM) SIMULATION MODEL AND ANALYSIS OF RESULTS

Using computer simulation Technology (CST), the electromagnetic simulation is done. From the simulation model (as depicted in Fig. 7) it has been noticed that the PTE is influenced by the coil size, turns of the coil, physical air gap and frequency of operation as analyzed by mathematical model. The results are depicted in Fig. 8. The observed magnetic field pattern (as illustrated in Fig. 9) reveals the effective magnetic coupling between both the coils entailing the efficient wireless power transfer. The revealed results provide the design directives for practical EV charging setup.
WPT system [9].

IV. EXPERIMENTAL INVESTIGATION

Based on the effect of analyzed operating parameters, a practical wireless power transfer system is designed, as illustrated in Fig. 10. In the design, the transmitting coil is excited with ac source at a specific resonant frequency. The magnetic coupling enables the resonant receiver coil to capture the magnetic field placed over a distance apart. The induced receiver voltage is given to the load at the receiver side. The experimental frequency dependent PTE is depicted in Fig. 11. It has been affirmed that maximum efficiency is achieved at resonant frequency. It elucidates that there is a certain magnetically coupled operating regime depending on the coil configuration, physical air gap and operating frequency for which maximum $\eta$ can be upheld for a practical wireless EV-charging system.

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V. CONCLUSION

In this work, the consequence of design parameters on the performance of WPTS used for EV charging has been analyzed with the help of coupled mode theory. A fundamental mathematical model as well as electromagnetic simulation model has been developed to delineate the performance of design parameters on power transfer process. Based on the analysis, a practical WPTS has also been setup to experimentally investigate its performance. Both the theoretical and experimental results depict the design guidelines for practical EV charging under non-ideal usage scenarios.
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