Schrödinger Cat States for Quantum Information Processing

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We extensively discuss how Schrödinger cat states (superpositions of well-separated coherent states) in optical systems can be used for quantum information processing.

I. INTRODUCTION

In the early days of quantum mechanics many of its founders became very worried by some of the paradoxical predictions that emerged from thought experiments based on the new theory. Now, eighty years on, some of these early thought experiments are being experimentally realized, and more than just confirming the fundamentals of the theory they are also being recognized as the basis of 21st century technologies [1, 2]. An example is the EPR paradox, proposed by Einstein, Podolsky and Rosen in 1935 [3], which discussed the strange properties of quantum entanglement. Today, entanglement has been observed in optical [4, 5] and ion [6] systems and is recognized as a resource for many quantum information processing tasks [7].

About the same time as the EPR discussion, Schrödinger proposed his famous cat paradox [8] that highlighted the unusual consequences of extending the concept of superposition to macroscopically distinguishable objects. From a quantum optics view point, the usual paradigm is to consider superpositions of coherent states with amplitudes sufficiently different that they can be resolved using homodyne detection [9, 10]. In this chapter we discuss how, beyond their fundamental interest, these types of states can be used in quantum information processing. We then look at the problem of producing such states with the required properties.

II. QUANTUM INFORMATION PROCESSING WITH SCHRODINGER CAT STATES

A. Coherent-state qubits

We now introduce qubit systems using coherent states. A coherent state can be defined as [8, 52]

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle,$$

where $|n\rangle$ is a number state and $\alpha$ is the complex amplitude of the coherent state. The coherent state is a very useful tool in quantum optics and a laser field is considered a good approximation of it. Let us consider two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$. The two coherent states are not orthogonal to each other but their overlap $|\langle \alpha | - \alpha \rangle|^2 = e^{-4|\alpha|^2}$ decreases exponentially with $|\alpha|$. For example, when $|\alpha|$ is as small as 2, the overlap is $\approx 10^{-7}$, i.e., $|\langle \alpha | - \alpha \rangle|^2 \approx 0$. We identify the two coherent states of $\pm \alpha$ as basis states for a logical qubit as $|\alpha\rangle \rightarrow |0\rangle_L$ and $|-\alpha\rangle \rightarrow |1\rangle_L$, so that a qubit state is represented by

$$|\phi\rangle = A|0\rangle_L + B|1\rangle_L = A|\alpha\rangle + B|-\alpha\rangle.$$  

(2)

The basis states, $|\alpha\rangle$ and $|-\alpha\rangle$, can be unambiguously discriminated by a simple measurement scheme with a 50-50 beam splitter, an auxiliary coherent field of amplitude $\alpha$ and two photodetectors [22]. At the beam splitter, the qubit state $|\phi\rangle_1$ is mixed with the auxiliary state $|\alpha\rangle_2$ and results in the output

$$|\phi_R\rangle_{ab} = A|\sqrt{2}\alpha\rangle_a |0\rangle_b + B|0\rangle_a |-\sqrt{2}\alpha\rangle_b.$$  

(3)

The two photodetector are set for modes $a$ and $b$ respectively. If detector $A$ registers any photon(s) while detector $B$ does not, we know that $|\alpha\rangle$ was measured. On the contrary, if $A$ does not click while $B$ does, the measurement outcome was $|-\alpha\rangle$. Even though there is non-zero probability of failure $P_f(\phi_R) = |\langle 00|\phi_R\rangle|^2 = |A + B|^2 e^{-2\alpha^2}$ in which both of the detectors do not register a photon, the failure is known from the result whenever it occurs, and $P_f$ approaches to zero exponentially as $\alpha$ increases. Note that the detectors do not have to be highly efficient for unambiguous discrimination. Alternatively, homodyne detection can also be very efficient for the qubit readout because the overlap between the coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ would be extremely small for an appropriate value of $\alpha$.

 Alternatively, it is possible to construct an exactly orthogonal qubit basis with the equal superposition of two linear independent coherent states $|\alpha\rangle$ and $|-\alpha\rangle$. Consider the basis states

$$|e\rangle = N_+ (|\alpha\rangle + |-\alpha\rangle) \rightarrow |0\rangle_L,$$

$$|d\rangle = N_- (|\alpha\rangle - |-\alpha\rangle) \rightarrow |1\rangle_L,$$

(4)

(5)

where $N_+ = 1/\sqrt{2(1 + \exp[-2|\alpha|^2])}$. It can be simply shown that they form an orthonormal basis as $\langle e | d \rangle = 0$ and $\langle e | e \rangle = \langle d | d \rangle = 1$. The basis state $|e\rangle$ ($|d\rangle$) is called “even cat state” (“odd cat state”) because it contains only even (odd) number of photons as

$$|e\rangle = 2N_+ e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{\sqrt{(2n)!}} |2n\rangle,$$

$$|d\rangle = 2N_- e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^{2(n+1)}}{\sqrt{(2n + 1)!}} |2n + 1\rangle.$$  

(6)

(7)
The even and odd cat states can thus be discriminated by a photon parity measurement which can be represented by \( O_{\Pi} = \sum_{n=0}^{\infty}(2n)/(2n - (2n + 1))(2n + 1) \). As \( \alpha \) goes to zero, the odd cat state \(|d\rangle\) approaches a single photon state \(|1\rangle\) while the even cat state \(|e\rangle\) approaches \( |0\rangle\). No matter how small \( \alpha \) is, there is no possibility that no photon will be detected from the state \(|d\rangle\) at an ideal photodetector.

### B. Quantum teleportation

Quantum teleportation is an interesting phenomenon for demonstrating quantum theory and a useful tool in quantum information processing [2]. By quantum teleportation, an unknown quantum state is disentangled in a sending place and its perfect replica appears at a distant place via dual quantum and classical channels. The key ingredients of quantum teleportation are an entangled channel, a Bell-state measurement and appropriate unitary transformations. In what follows we shall explain how teleportation can be performed for a coherent-state qubit [17, 24].

Let us assume that Alice wants to teleport an unknown coherent-state qubit \(|\phi\rangle_a\) via a pure entangled coherent channel

\[
|\Psi_-(ab)\rangle = N_- (|\alpha\rangle_b - |\alpha\rangle_b) - |\alpha\rangle_b|\alpha\rangle_b),
\]

where \( N_- \) is the normalization factor. After sharing the quantum channel \( |\Psi_-(ab)\rangle \), Alice should perform a Bell-state measurement on her part of the quantum channel and the unknown qubit \(|\phi\rangle\) and send the outcome to Bob. The Bell-state measurement is to discriminate between the four Bell-cat states which can be defined with coherent states as [20, 21, 49, 50]

\[
\begin{align*}
|\Phi_+(ab)\rangle &= N_+ (|\alpha\rangle_b|\alpha\rangle_b + |\alpha\rangle_b|\alpha\rangle_b), \\
|\Psi_+(ab)\rangle &= N_+ (|\alpha\rangle_b|\alpha\rangle_b - |\alpha\rangle_b|\alpha\rangle_b),
\end{align*}
\]

where \( N_+ \) are normalization factors. The four Bell-cat states defined in our framework are a very good approximation of the Bell basis. These states are orthogonal to each other except \( \langle \Psi_+|\Phi_+\rangle = 1/cosh2|\alpha|^2 \), and \( \langle \Psi_+|\Psi_+\rangle \) and \( \langle \Phi_+|\Phi_+\rangle \) rapidly become orthogonal as \( |\alpha| \) grows.

A Bell-state measurement, or simply Bell measurement, is very useful in quantum information processing. It was shown that a complete Bell-state measurement on a product Hilbert space of two two-level systems is not possible using linear elements [32]. A Bell measurement scheme using linear optical elements [6] has been used to distinguish only up to two of the Bell states for teleportation [7] and dense coding [34]. However, a remarkable feature of the Bell-cat states is that each one of them can be unambiguously discriminated using only a beam splitter and photon-parity measurements [24, 24]. Suppose that the modes, \( a \) and \( b \), of the entangled state are incident on a 50-50 beam splitter. After passing the beam splitter, the Bell-cat states become

\[
\begin{align*}
|\Phi_+(ab)\rangle &\rightarrow |E\rangle_f|0\rangle_g, \\
|\Phi_-(ab)\rangle &\rightarrow |D\rangle_f|0\rangle_g, \\
|\Psi_+(ab)\rangle &\rightarrow |0\rangle_f|E\rangle_g, \\
|\Psi_-(ab)\rangle &\rightarrow |0\rangle_f|D\rangle_g,
\end{align*}
\]

where the even cat state \(|E\rangle \propto |\sqrt{2}\alpha\rangle + | - \sqrt{2}\alpha\rangle \) definitely contains an even number of photons, while the odd cat state \(|D\rangle \propto |\sqrt{2}\alpha\rangle - | - \sqrt{2}\alpha\rangle \) definitely contains an odd number of photons. By setting two photodetectors for the output modes \( f \) and \( g \) respectively to perform number parity measurement, the Bell-cat measurement can be simply achieved. For example, if an odd number of photons is detected for mode \( f \), the state \(|\Phi_-\rangle\) is measured, and if an odd number of photons is detected for mode \( g \), then \(|\Psi_-\rangle\) is measured. Even though there is non-zero probability of failure in which both of the detectors do not register a photon due to the non-zero overlap of \( |\langle 0|E\rangle|^2 = 2e^{-3|\alpha|^2}/(1 + e^{-4|\alpha|^2}) \), it is small for an appropriate choice of \( \alpha \) and the failure is known from the result whenever it occurs.

To complete the teleportation process, Bob performs a unitary transformation on his part of the quantum channel according to the measurement result sent from Alice via a classical channel. The required transformations are \( \sigma_x \) and \( \sigma_z \) on the coherent-state qubit basis, where \( \sigma_i \)'s are Pauli operators. When the measurement outcome is \(|B_1\rangle\), Bob obtains a perfect replica of the original unknown qubit without any operation. When the measurement outcome is \(|B_2\rangle\), Bob should perform \( |\alpha\rangle \leftrightarrow | - \alpha\rangle \) on his qubit. Such a phase shift by \( \pi \) can be done using a phase shifter whose action is described by \( P(\varphi) = e^{i\varphi a^\dagger a} \), where \( a \) and \( a^\dagger \) are the annihilation and creation operators. When the outcome is \(|B_3\rangle\), the transformation should be performed as \( |\alpha\rangle \rightarrow |\alpha\rangle \) and \( | - \alpha\rangle \rightarrow | - \alpha\rangle \). This transformation is more difficult but can be achieved most straightforwardly by simply teleporting the state again (locally) and repeating until the required phase shift is obtained. Therefore, both of the required unitary transformation, \( \sigma_x \) and \( \sigma_z \), can be performed by linear optics elements. When the outcome is \(|B_4\rangle\), \( \sigma_x \) and \( \sigma_z \) should be successively applied.

### C. Quantum computation

We now describe how a universal set of quantum gates can be implemented on coherent state qubits using only linear optics and photon detection, provided a supply of cat states is available as a resource. The idea was originally due to Ralph, Munro and Milburn [16] and was later expanded on by Ralph et al. [48].

A universal single qubit quantum gate element can be constructed from the following sequence of gates: Hadamard (\( H \)); rotation about the Z-axis by angle \( \theta \) (\( R(\theta) \)); Hadamard (\( H \)) and; rotation about the Z-axis by angle \( \phi \) (\( R(\phi) \)). If the two qubit gate, control sign
(CS), is also available then universal processing is possible (See Fig. 1). We now describe how these gates can be implemented. We will assume that deterministic single qubit measurements can be made in the computational basis, $|\alpha\rangle$ , $|\alpha\rangle$ and the phase superposition basis $|\alpha\rangle \pm \exp[i\epsilon]|-\alpha\rangle$. As described in the previous section, computational basis measurements can be achieved using either homodyne or photon counting techniques. The phase superposition basis can be measured using photon counting in a Dolinar receiver type arrangement [13, 55].

The simplest case is for $\epsilon = 0$ where we need to differentiate only between odd or even photon numbers in direct detection. We also assume we can make two qubit Bell-measurements and, more generally, perform teleportation, as described in the previous section.

**Hadamard Gate:** The Hadamard gate ($H$) can be defined by its effect on the computational states: $H |\alpha\rangle = |\alpha\rangle + |\alpha\rangle$ and $H |-\alpha\rangle = |-\alpha\rangle + |-\alpha\rangle$ where for convenience we have dropped normalization factors. One way to achieve this gate is to use the resource state $|HR\rangle = |\alpha\rangle + |\alpha\rangle - |\alpha\rangle - |\alpha\rangle$ and $|HR\rangle = |\alpha\rangle + |\alpha\rangle$ and $|HR\rangle = |\alpha\rangle + |\alpha\rangle$. This state can be produced non-deterministically from cat state resources, as will be described shortly. It is straightforward to show that if a Bell-state measurement is made between an arbitrary qubit state $|\sigma\rangle$ and one of the modes of $|HR\rangle$ then the remaining mode is projected into the state $H |\sigma\rangle$, where dependent on the outcome of the Bell measurement a bit-flip correction, a phase-flip correction, or both may be necessary.

**Phase Rotation Gate:** The phase rotation gate ($R(\theta)$) can be defined by its effect on the computational states: $R(\theta) |\alpha\rangle = \exp[i\theta]|\alpha\rangle$ and $R(\theta) |-\alpha\rangle = \exp[-i\theta]|-\alpha\rangle$. One way to achieve this gate is the following: The arbitrary qubit, $\mu |\alpha\rangle + \nu |-\alpha\rangle$ is split on a 50:50 beamsplitter giving the two mode state: $\mu |\alpha/\sqrt{2}\rangle |\alpha/\sqrt{2}\rangle + \nu |-\alpha/\sqrt{2}\rangle |-\alpha/\sqrt{2}\rangle$. One of the modes is then measured in the phase superposition basis $|\alpha/\sqrt{2}\rangle \pm \exp[-2i\theta]|-\alpha/\sqrt{2}\rangle$, thus projecting the other mode into the state $\mu \exp[i\theta]|\alpha/\sqrt{2}\rangle \pm \nu \exp[-i\theta]|-\alpha/\sqrt{2}\rangle$. The amplitude decrease can be corrected by teleportation in the following way [13]. The asymmetric Bell state entanglement, $|\alpha/\sqrt{2}\rangle |\alpha\rangle + |-\alpha/\sqrt{2}\rangle |-\alpha\rangle$ is produced by splitting the cat state $|\sqrt{3}/2\alpha\rangle + |-\sqrt{3}/2\alpha\rangle$ on a 1/3 : 2/3 beamsplitter. Teleportation is then carried out with the Bell state measurement being performed between the matching $|\alpha/\sqrt{2}\rangle$ modes and the teleported state ending up on the $|\alpha\rangle$ mode. Dependent on the outcome of the phase basis measurement and the Bell-measurement a bit-flip correction, a phase-flip correction, or both may be necessary.

**Control Sign Gate:** The control-sign gate (CS) can be defined by its effect on the two qubit computational states: $CS |\alpha\rangle |\alpha\rangle = |\alpha\rangle |\alpha\rangle$; $CS |\alpha\rangle |-\alpha\rangle = |-\alpha\rangle |\alpha\rangle$; $CS |-\alpha\rangle |\alpha\rangle = |\alpha\rangle |-\alpha\rangle$ and; $CS |-\alpha\rangle |-\alpha\rangle = |-\alpha\rangle |-\alpha\rangle$. One way to achieve this gate is the following: The two arbitrary qubits, $\mu |\alpha/\sqrt{2}\rangle |\alpha/\sqrt{2}\rangle + \nu |-\alpha/\sqrt{2}\rangle |-\alpha/\sqrt{2}\rangle$ and $\gamma |\alpha/\sqrt{2}\rangle |\alpha/\sqrt{2}\rangle + \delta |-\alpha/\sqrt{2}\rangle |-\alpha/\sqrt{2}\rangle$. A Hadamard gate is then performed on the second mode of the first qubit giving the state $\mu |\alpha/\sqrt{2}\rangle (|\alpha/\sqrt{2}\rangle + |-\alpha/\sqrt{2}\rangle) + \nu |-\alpha/\sqrt{2}\rangle (|\alpha/\sqrt{2}\rangle - |-\alpha/\sqrt{2}\rangle)$. If a Bell-measurement is then carried out between the second mode of the first qubit and one of the modes of the second qubit a CS gate will be achieved. The amplitude reduction can be corrected as before using teleportation. Dependent on the outcome of the various Bell-measurements, bit-flip corrections, phase-flip corrections, or both may be necessary.

**Resource State:** The resource state $|HR\rangle$ can be produced in the following way. Consider the beamsplitter interaction given by the unitary transformation

$$U_{ab} = \exp[i\theta/2(ab + a^*b)]$$

where $a$ and $b$ are the annihilation operators corresponding to two coherent state quibits $|\gamma\rangle_a$ and $|\beta\rangle_b$, with $\gamma$ and $\beta$ taking values of $-\alpha$ or $\alpha$. It is well known that the output state produced by such an interaction is

$$U_{ab}|\gamma\rangle_a|\beta\rangle_b = |\cos \theta/2 \gamma + i \sin \theta/2 \beta\rangle_a |\cos \theta/2 \beta + i \sin \theta/2 \gamma\rangle_b$$

where $\cos^2 \theta/2 (\sin^2 \theta/2)$ is the reflectivity (transmissivity) of the beamsplitter. Suppose two cat states are fed into the beamsplitter and both output beams are then teleported, the output state will be:

$$e^{-\theta^2/4} (e^{i\theta a^2} |\alpha\rangle_a |\alpha\rangle_b \pm e^{-i\theta a^2} |\alpha\rangle_a |\alpha\rangle_b \pm e^{-\theta a^2} |\alpha\rangle_a |\alpha\rangle_b \pm e^{i\theta a^2} |\alpha\rangle_a |\alpha\rangle_b)$$

where the $\pm$ signs depend on the outcome of the Bell measurements. If we choose $\phi = 2\theta a^2 = \pi/2$ then the resulting state is easily shown to be locally equivalent to $|HR\rangle$ (related by phase rotations). Preparation of this state is non-deterministic because of non-unit overlap between the state of Eq. 13 and the Bell states used in the teleporter. As a result the teleporter can fail by recording photons at both outputs in the Bell-measurement. The probability of success is $e^{-\theta^2/2}$. For $\alpha = 2$ this is about 92% probability of success.

**Correction of Phase-flips:** After each gate we have noted that bit flip and/or phase flip corrections may be

\[ H \rightarrow R(\theta) \rightarrow H \rightarrow \phi \]

FIG. 1: A set of Hadamard ($H$) gates, rotations ($R$) about the Z-axis and control sign (CS) gates can provide universal gate operations.
necessary since our gate operations are based on the teleportation protocol. As discussed in the previous section, bit flips can be easily implemented using a phase shifter, \( P(\pi) \), while phase-flips are more expensive. We now argue that in fact only active correction of bit-flips is necessary. This is because phase-flips commute with the phase rotation gate and the control sign gate but are converted into bit flips by the Hadamard gate. This suggests the following strategy: After each gate operation any bit-flips are corrected whilst phase-flips are noted. After the next Hadamard gate the phase flips are converted to bit-flips which are then corrected and any new phase-flips are noted. By following this strategy only bit-flips need to be corrected actively, with, at worst, some final phase-flips needing to be corrected in the final step of the circuit.

**D. Entanglement purification for Bell-cat states**

It is not possible to perfectly isolate a quantum state from its environment. A quantum state inevitably loses its quantum coherence in a dissipative environment. This process is called decoherence and has been known as the main obstacle to the physical implementation of quantum information processing. Quantum error correction [11, 18, 45] and entanglement purification [9, 23] have been studied for quantum information processing using cat states to overcome this problem. Here we discuss an entanglement purification technique.

An entanglement purification for entangled coherent states (Bell-cat states) have been studied by several authors [8, 29]. It has been found that certain types of mixed states including the Werner-type mixed states composed of the Bell-cat states can be purified by simple linear optics elements and inefficient detectors [27]. The other types of mixed states need to be transformed to the Werner type states by local operations. This scheme performs amplification of the Bell-cat states simultaneously with entanglement purification. This is an important observation because Bell-cat states of large amplitudes are preferred for quantum information processing while their generation is hard. A similar technique is employed to generate single-mode large cat states [32].

We first explain the purification-amplification protocol for entangled coherent states by a simple example and then apply it to a realistic situation [23]. Let us suppose that Alice and Bob want to distill entangled coherent states \( |\Phi_+\rangle \) from a type of ensemble

\[
\rho_{ab} = F|\Phi_+\rangle\langle \Phi_+| + G|\Psi_+\rangle\langle \Psi_+|,
\]

where \( F + G \approx 1 \) for \( |\alpha| \gg 1 \). We shall assume this condition, \( |\alpha| \gg 1 \), for simplicity. The purification-amplification process can be simply accomplished by performing the process shown in Fig. 2. Alice and Bob choose two pairs from the ensemble which are represented by the following density operator

\[
\rho_{ab}\rho_{a'b'} = F^2|\Phi_+\rangle\langle \Phi_+| \otimes |\Phi_+\rangle\langle \Phi_+| + F(1-F)|\Phi_+\rangle\langle \Phi_+| \otimes |\Psi_+\rangle\langle \Psi_+| + F(1-F)|\Psi_+\rangle\langle \Psi_+| \otimes |\Phi_+\rangle\langle \Phi_+| + (1-F)^2|\Psi_+\rangle\langle \Psi_+| \otimes |\Psi_+\rangle\langle \Psi_+|.
\]

The fields of modes \( a \) and \( a' \) are in Alice’s possession while \( b \) and \( b' \) in Bob’s. In Fig. 2(a), we show that Alice’s action to purify the mixed entangled state. The same is conducted by Bob on his fields of \( b \) and \( b' \).

There are four possibilities for the fields of \( a \) and \( a' \) incident onto the beam splitter (BS1), which gives the output (In the following, only the cat part for a component of the mixed state is shown to describe the action
of the apparatuses
\begin{equation}
|\alpha\rangle_a|\alpha\rangle_{a'} \rightarrow \sqrt{2}\alpha_f \langle 0|_{f'},
\end{equation}
\begin{equation}
|\alpha\rangle_a - |\alpha\rangle_{a'} \rightarrow |0|_f \sqrt{2}\alpha_f',
\end{equation}
\begin{equation}
| - \alpha\rangle_a|\alpha\rangle_{a'} \rightarrow |0|_f| - \sqrt{2}\alpha_f',
\end{equation}
\begin{equation}
| - \alpha\rangle_a| - \alpha\rangle_{a'} \rightarrow | - \sqrt{2}\alpha_f)(0|_{f'}.
\end{equation}

In the boxed apparatus P1, Alice checks if modes a and a' were in the same state by counting photons at the photodetectors A1 and A2. If both modes a and a' are in |\alpha⟩ or | - \alpha⟩, f' is in the vacuum, in which case the output field of the beam splitter BS2 is |\alpha⟩, | - \alpha⟩11,12. Otherwise, the output field is either |2\alpha⟩(11,12) or |2\alpha⟩(11,12). When both the photodetectors A1 and A2 register any photon(s), Alice and Bob are sure that the two modes a and a' were in the same state but when either A1 or A2 does not register a photon, a and a' were likely in different states. The remaining pair is selected only when Alice and Bob's all four detectors click together. Of course, there is a probability not to register a photon even though the two modes were in the same state, which is due to the nonzero overlap of |0⟩\sqrt{2}\alpha⟩. Note that inefficiency of the detectors does not degrade the the quality of the distilled entangled coherent states but decreases the success probability.

It can be simply shown that the second and third terms of Eq. (10) are always discarded by the action of P1 and Bob's apparatus same as P1. For example, at the output ports of BS1 and Bob's beam splitter corresponding to BS1, |Φ+⟩ab|Ψ+⟩a'b' becomes

\begin{equation}
|Φ+⟩ab|Ψ+⟩a'b' \rightarrow N^2_{Φ+}(\sqrt{2}\alpha, 0, 0, \sqrt{2}\alpha) + |0, \sqrt{2}\alpha, \sqrt{2}\alpha, 0)|0, |0, \sqrt{2}\alpha, \sqrt{2}\alpha, 0) \rangle (f_f f'_{g'}),
\end{equation}

where g and g' are the output field modes from Bob's beam splitter corresponding to BS1. The fields of modes f' and g' can never be in |0⟩ at the same time; at least, one of the four detectors of Alice and Bob must not click. The third term of Eq. (10) can be shown to lead to the same result by the same analysis.

For the cases of the first and fourth terms in Eq. (10), all four detectors may register photon(s). After the beam splitter BS1, the ket of (|Φ−⟩|Φ−⟩)ab (|Φ−⟩|Φ−⟩)ab of Eq. (10) becomes

\begin{equation}
|Φ−⟩ab|Φ−⟩a'b' \rightarrow |Φ'⟩fg |0, 0⟩_{f'g'} - |0, 0⟩_{fg} |Φ'⟩_{f'g'},
\end{equation}

where |Φ'⟩ = N′_Φ(\sqrt{2}\alpha, \sqrt{2}\alpha) + | - \sqrt{2}\alpha, - \sqrt{2}\alpha⟩ with the normalization factor N′_Φ. The normalization factor in the right hand side of Eq. (22) is omitted. The first term is reduced to (|Φ'⟩|Φ'⟩)fg after (|0, 0⟩|0, 0⟩)_{f'g'} is measured out by Alice and Bob's P1's. Similarly, the fourth term of Eq. (10) yields (|Ψ'⟩|Ψ'⟩)fg, where |Ψ'⟩ is defined in the same way as |Φ'⟩, after (|0, 0⟩|0, 0⟩)_{f'g'} is measured. Thus the density matrix for the field of modes f and g conditioned on simultaneous measurement of photons at all four photodetectors is

\begin{equation}
\rho_{fg} = F'(|Φ'⟩|Φ'⟩ + (1 - F')|Ψ'⟩|Ψ'⟩),
\end{equation}

where F' = F^2/(F^2 + (1 - F)^2), and F' is always larger than F for any F > 1/2. By reiterating this process, Alice and Bob can distill some maximally entangled states |Φ⟩ of a large amplitude asymptotically. Of course, a sufficiently large ensemble and initial fidelity F > 1/2 are required for successful purification [2].

We now apply our scheme to a realistic example in a dissipative environment. When the entangled coherent channel |Φ⟩ is embedded in a vacuum, the channel decoheres and becomes a mixed state of its density operator ρab(τ), where τ stands for the decoherence time.

By solving the master equation [42]

\begin{equation}
\frac{\partial}{\partial \tau} \tilde{ρ} = \tilde{J} \hat{ρ} + \hat{L} \rho;
\end{equation}

\begin{equation}
\tilde{J} = \frac{\gamma}{2} \sum |a_i\rangle\langle a_i|- |a_i\rangle\langle a_i|\rho_{ab} + \rho_{ab} |a_i\rangle\langle a_i|
\end{equation}

where γ is the energy decay rate, the mixed state ρab(τ) can be straightforwardly obtained as

\begin{equation}
\rho_{ab}(\tau) = \tilde{N}(\tau) \left\{ |t\alpha⟩|t\alpha⟩ + | - t\alpha⟩| - t\alpha⟩ \right\} - \Gamma (|t\alpha⟩| - t\alpha⟩ + | - t\alpha⟩|t\alpha⟩),
\end{equation}

where \( |± t\alpha⟩ = \pm |t\alpha⟩ \pm | - t\alpha⟩ \pm | - t\alpha⟩ \pm |t\alpha⟩ \), t = e^{-\gamma/2}, \Gamma = \exp[-4(1 - t^2)|\alpha|^2], and \( \tilde{N}(\tau) \) is the normalization factor.

The decohered state \( \rho_{ab}(\tau) \) may be represented by the dynamic Bell-cat states defined as follows:

\begin{equation}
|Φ_{±}⟩_{ab} = \tilde{N}_{±}(|t\alpha⟩|t\alpha⟩ + | - t\alpha⟩| - t\alpha⟩), \end{equation}

\begin{equation}
|Ψ_{±}⟩_{ab} = \tilde{N}_{±}(|t\alpha⟩| - t\alpha⟩ + | - t\alpha⟩|t\alpha⟩),
\end{equation}

where \( \tilde{N}_{±} = 2(1 + e^{-2t^2}|\alpha|^2})^{-1/2}. The decohered state is then

\begin{equation}
\rho_{ab}(\tau) = \tilde{N}(\tau) \left\{ |Φ−⟩|Φ−⟩ + |Φ−⟩|Φ−⟩ + \frac{(1 - \Gamma)}{N^2} |Φ−⟩|Φ−⟩ \right\}

\equiv F(\tau)|Φ−⟩|Φ−⟩ + (1 - F(\tau))|Φ−⟩|Φ−⟩ \end{equation}

where, regardless of the decay time τ, |Φ−⟩ is maximally entangled and |Φ−⟩ and |Φ−⟩ are orthogonal to each
other. The decohered state (28) is not in the same form as Eq. (13) so that we need some bilateral unitary transformations before the purification scheme is applied. A Hadamard gate $H$ for coherent-state qubits can be used to transform the state (28) into a distillable form

$$ H_\alpha H_\beta \rho_{ab} (\tau) H_b^\dagger H_a^\dagger = F(\tau) |\Psi^+\rangle \langle \Psi^+ | (1 - F(\tau)) |\Phi^+\rangle \langle \Phi^+ |, $$

which is now in the same form as Eq. (16).

The ensemble of state (28) can be purified successfully only when $F(\tau)$ is larger than 1/2. Because $F(\tau)$ is obtained as

$$ F(\tau) = \frac{N_+^2 (1 + \Gamma)}{N_+^2 (1 + \Gamma) - N_-^2 (1 - \Gamma)}, $$

it is found that purification is successful when the decoherence time $\gamma \tau < \ln 2$ regardless of $\alpha$. This result is in agreement with the decay time until which teleportation can be successfully performed via an entangled coherent state shown in Ref. [24].

### III. Production of Schrödinger cat states

A key requirement of quantum information processing with cat states is the generation of cat states in free-propagating optical fields. This has been known to be extremely demanding using current technology because strong nonlinearity [60] or precise photon counting measurements [12, 54] are necessarily required. However, very recently, there has been remarkable progress which may enables one to generate free-propagating cat states without strong nonlinearity or photon counting measurements. For example, it was shown that cat states of reasonably large amplitudes can be produced with simple linear optics elements and single photons [32]. Relatively small nonlinearity was shown to be still useful with conditioning homodyne detection [27] or with a single photon interacting with a coherent state [37] to generate cat states. It was shown that a deterministic cat-state source can be obtained using a single-atom cavity [58]. A recent experimental progress [59] could be directly improved by the cat-amplification scheme in Ref. [32] to generate a cat state of a larger amplitude and higher fidelity. The above proposals have now brought the generation of free-propagating cat states of $\alpha \approx 2$ within reach of current technology. Electromagnetically induced transparency (EIT) has also been studied as a method to obtain a giant Kerr nonlinearity [19, 31, 51], and there has been an improved suggestion to generate cat states with it [41].

In what follows, some of these suggestions will be briefly covered.

#### A. Schemes using linear optics elements

Since it is extremely hard to generate cat states using $\chi^{(3)}$ nonlinearity, some alternative methods have been studied based upon conditional measurements [12, 54]. A crucial drawback of these schemes is that a highly efficient photon counting measurement, which is extremely demanding in current technology, is necessary. However, it was shown recently that a free-propagating optical cat state can be generated with a single photon source and simple optical operations without efficient photon detection [32]. This suggestion contains two main points:

- An arbitrarily large cat state can be produced out of arbitrarily small cat states using the simple experimental set-up depicted in Fig. 3.
- A small odd cat state with $\alpha \leq 1.2$ is very well approximated by a squeezed single photon, $S(s)|1\rangle$, where $S(s)$ is the squeezing operator with the squeezing parameter $s$ and $|1\rangle$ is the single-photon state.

Firstly, the cat-amplification process summarized as follows. Suppose two small cat states, $|\alpha_\phi \psi\rangle$ and $|\alpha_\beta \beta\rangle$, with amplitudes $\alpha$ and $\beta$. The reflectivity $r$ and transmittivity $t$ of BS1 are set to $r = \beta/\sqrt{\alpha^2 + \beta^2}$ and $t = \alpha/\sqrt{\alpha^2 + \beta^2}$, where the action of the beam splitter is represented by $\hat{B}_{a,b}(r,t)|\alpha_\phi \psi\rangle = |\alpha_\phi + r\beta\rangle_f - r\alpha_\phi + t\beta\rangle_g$. The other beam splitter BS2 is a 50:50 beam splitter ($r = t = 1/\sqrt{2}$) regardless of the conditions and the amplitude $\gamma$ of the auxiliary coherent field is determined as $\gamma = 2\alpha\beta/\sqrt{\alpha^2 + \beta^2}$. The resulting state for mode $f$ then becomes $|\text{cat}_\phi \psi\rangle$.

![FIG. 3: A schematic of the non-deterministic cat-amplification process. See text for details.](image)

The success probability $P_{\phi,\psi}(\alpha, \beta)$ for a single iteration is

$$ P_{\phi,\psi}(\alpha, \beta) = \frac{(1 - e^{-\frac{\alpha^2 + \beta^2}{\alpha^2 + \beta^2}})^2 [1 + \cos(\phi + \psi) e^{-2(\alpha^2 + \beta^2)}]}{2(1 + \cos \phi e^{-2\alpha^2})(1 + \cos \delta e^{-2\beta^2})}, $$

which approaches 1/2 as the amplitudes of initial cat states becomes large. Note that the probabilities depend
to obtain a cat state of high quality. Highly efficient mode matching of a single photon from parametric down conversion and a weak coherent state from an attenuated laser beam at a beam splitter has been experimentally demonstrated using optical fibers [48]. Such techniques could be employed for the implementation of our scheme. The success probability will rapidly drop down and the required resources will exponentially increase as the number of steps increases. However, if quantum optical memory is available, one can considerably boost up the success probability by holding the resulting states for every step [20].

B. Schemes using cavity quantum electrodynamics

Cavity quantum electrodynamics (QED) has been studied to enhance nonlinear effects to generate macroscopic superpositions [50]. Some success has been reported in creating such superposition states within high Q cavities in the microwave [6] and optical [32] domains. Simplified versions of cavity QED schemes have been developed for deterministic generation of cat states in a cavity [17]. While this method is relatively effective to generate cat states in cavity, most of the schemes suggested for quantum information processing with coherent states require free propagating cat states.

Recently, a method was proposed to generated free propagating cat states by using cavity-assisted interactions of coherent optical pulses [58]. This suggestion employs an atom of three relevant levels trapped in an optical cavity with a coherent-state pulse $|\alpha\rangle$ incident onto the cavity. One of the atomic level, $|e\rangle$, is its excited state, and the other two levels, $|g_0\rangle$ and $|g_1\rangle$, are in the ground state with different hyperfine spins. The transition from $|g_1\rangle$ to $|e\rangle$ is resonantly coupled to a cavity mode while $|g_0\rangle$ is decoupled from the cavity mode. In such a preparation, if the trapped atom was prepared in state $|g_0\rangle$, the input field becomes $|-\alpha\rangle$ after a resonant reflection as the input pulse is resonant with the bare cavity mode. On the other hand, if the atom was prepared in state $|g_1\rangle$, it remains $|\alpha\rangle$ due to a strong atom-cavity coupling. Therefore, if the trapped atom was prepared in a superposition state such as $(|g_0\rangle + |g_1\rangle)/\sqrt{2}$, the reflected field becomes an entangled state $(|g_0\rangle|\alpha\rangle + |g_1\rangle|-\alpha\rangle)/\sqrt{2}$, which can be projected to a single mode cat state by a measurement on a superposed basis $|g_0\rangle \pm |g_1\rangle$. An advantage of this scheme is weak dependence on dipole coupling, but wave front distortion due to differences between resonant and non-resonant interactions could be a problem in a real experiment. This work [58] concludes that a cat state with a quite large amplitude ($\alpha \approx 3.4$) could be generated in this way with a 90% fidelity using current technology.
C. Schemes using weak nonlinearity

There has been a suggestion to use relatively weak nonlinearity with beam splitting with a vacuum and conditioning by homodyne detection to generate cat states \(27\). As beam splitting with a vacuum and homodyne measurement can be highly efficient in quantum optics laboratories, this shows that relatively weak nonlinearity can still be useful to generate cat states.

The Hamiltonian of a single-mode Kerr nonlinear medium is \( H_{NL} = \omega a a + \lambda (a^\dagger a)^2 \), where \( a \) and \( a^\dagger \) are annihilation and creation operators, \( \omega \) is the energy level splitting for the harmonic-oscillator part of the Hamiltonian and \( \lambda \) is the strength of the Kerr nonlinearity \(60\).

Under the influence of the nonlinear interaction the initial coherent state \( |\alpha\rangle \) evolves to the following state at time \( \tau = \pi/\lambda N \) \(30\):

\[
|\psi_N\rangle = \sum_{n=1}^{N} C_{n,N} | - \alpha e^{i 2 n \pi/N} \rangle, \tag{30}
\]

where

\[
C_{n,N} = \frac{1}{N} \sum_{k=0}^{N-1} (-1)^k \exp[-i \frac{\pi k}{N} (2n-k)]. \tag{31}
\]

The length \( L \) of the nonlinear cell corresponding to \( \tau \) is \( L = v \pi/2 \lambda N \), where \( v \) is the velocity of light. For \( N = 2 \), we obtain a desired cat state of relative phase \( \varphi = \pi/2 \). We again emphasize the nonlinear coupling \( \lambda \) is typically very small such that \( N = 2 \) cannot be obtained in a length limit where the decoherence effect can be neglected.

If \( \lambda \) is not as large as required to generate the cat state, the state \(30\) with \( N > 2 \) may be obtained by choosing an appropriate interaction time. From the state \(30\), it is required to remove all the other coherent component states except two coherent states of a \( \pi \) phase difference. First, it is assumed that the state \(30\) is incident on a 50-50 beam splitter with the vacuum onto the other input of the beam splitter. The initial coherent amplitude \( \alpha \) is taken to be real for simplicity. The state \(30\) with initial amplitude \( \alpha \) after passing through the beam splitter becomes

\[
|\psi_N\rangle = \sum_{n=1}^{N} C_{n,N} | - \alpha e^{i 2 n \pi/N} \rangle - \alpha e^{i 2 n \pi/N} \sqrt{2}, \tag{32}
\]

where all \( |C_{n,N}| \)'s have the same value. The real part of the coherent amplitude in the state \(32\) is then measured by homodyne detection in order to produce the cat state in the other path. By the measurement result, the state is reduced to

\[
|\psi^{(1)} \rangle_N = \sum_{n=1}^{N} C^{(1)}_{n,N}(\alpha_n) | - \alpha e^{i 2 n \pi/N} \rangle, \tag{33}
\]

where \( C^{(1)}_{n,N}(\alpha_n) = N \omega \sum_{n=1}^{N} C_{n,N} (X - \alpha e^{i 2 n \pi/N} \sqrt{2}) \) with \( N \) the normalization factor and \( |X\rangle \) the eigenstate of \( \hat{X} = (a + a^\dagger)/\sqrt{2} \). After the homodyne measurement, the state is selected when the measurement result is in certain values. If coefficients \( |C^{(1)}_{n,N}(\alpha_n)| \) and \( |C^{(1)}_{n,N}(\alpha_n)| \) in Eq. \(33\) have the same nonzero value and all the other \( |C^{(1)}_{n,N}(\alpha_n)| \)'s are zero, then the state becomes a desired cat state. Suppose \( N = 4k \) where \( k \) is a positive integer number. If \( X = 0 \) is measured in this case, the coefficients \( |C^{(1)}_{n,N}(\alpha_n)| \)'s will be the largest when \( n = N/4 \) and \( n = 3N/4 \), and become smaller as \( n \) is far from these two points. The coefficients can be close to zero for all the other \( n \)'s for an appropriately large \( \alpha \) so that the resulting state may become a cat state of high fidelity. Using this technique, one may observe a conspicuous signature of a cat state even with a 1/100 times weaker nonlinearity compared with the currently required level \(25\).

In particular, this approach can be useful to produce a cat state with a significantly large amplitude such as \( \alpha \geq 10 \).

Another scheme \(37\) proposes for linear optics quantum computation \(22\) uses weak cross-Kerr nonlinearity of the interaction Hamiltonian \( H = \hbar \chi a_1^\dagger a_2 a_2^\dagger \) to generate a cat state. The interaction between a coherent state, \( |\alpha\rangle \), and a single-photon qubit, e.g., \( \alpha(0)_1 + (1)_1 \), described as

\[
U_K|\psi_1\rangle_1 |\alpha\rangle_2 = e^{i H_K t/\hbar} \frac{1}{\sqrt{2}} (|0\rangle_1 + |1\rangle_1)|\alpha\rangle_2 \tag{34}
\]

where \( |0\rangle \) (\( |1\rangle \)) is the vacuum (single-photon) state, \( \alpha \) is the amplitude of the coherent state, and \( \theta = \chi t \) with the interaction time \( t \). If \( \theta = \pi \) and one measures out mode 1 on a superposed basis \( (|0\rangle \pm |1\rangle)/\sqrt{2} \), a macroscopic superposition state (so-called Schrödinger cat state), \( (|\alpha\rangle \pm |\alpha\rangle)/\sqrt{2} \). Using dual rail logic instead of the superposition between the single photon and the vacuum, the measurement on the superposed basis can be simply realized with a beam splitter and two photodetectors \(17\). It is again extremely difficult to obtain \( \theta = \pi \) using currently available nonlinear media. However, simply by increasing the amplitude \( \alpha \), one can gain an arbitrarily large separation between \( \alpha \) and \( \alpha e^{i \theta} \) within an arbitrarily short interaction time. It is possible to transform the state of the form of \( |\alpha\rangle \pm |\alpha\rangle \) into the symmetric form of \( |\alpha\rangle \pm |\alpha\rangle \) by the displacement operation which can be simply performed using a beam splitter with the transmission coefficient close to one and a strong coherent state being injected into the other input port. Therefore, weak cross-Kerr nonlinearity can also be useful to generate a cat state with a single photon, strong coherent states, beam splitters and two photodetectors. Remarkably, it can be shown that this approach can also reduce decoherence effects by increasing the initial amplitude \( \alpha \), which is also true for Ref. \(25\). It should also
be noted that the detectors and the single photon source in Ref. [17] which can be directly combined with Ref. [37] do not have to be efficient for a conditional generation of a cat state because these factors only degrade the success probability to be less than unity.

IV. CONCLUSION

We have discussed quantum processing tasks using qubits based on coherent states and Schrödinger cat states as resources. We have shown that a universal set of processing tasks can be achieved using only linear optics, feedforward and photon counting. This is a similar result to that of Knill Laflamme and Milburn for single photon qubits [27]. However, far fewer operations per gate are needed in the coherent state scheme and shortcuts are available for certain tasks. On the other hand these advantages are not useful unless a good way of producing cat states can be found. Thus we have spent some time discussing various proposals, both linear and non-linear, for producing cat states. We believe the near term prospects for demonstrating small travelling-wave cat states and basic processing tasks based on them are good. Whether coherent state qubits or single photon qubits will prove better for larger scale quantum optical processing in the long run remains an open question.
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