Visual, Symbolic, and Verbal Mathematics Representation Abilities in Junior High School’s Students

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Abstract: The purpose of this research is to describe the ability of mathematical representation that includes visual, symbolic, and verbal representations of eight grade of junior high school students in solving math problems. This research is a qualitative descriptive study. The instrument that is used in this research is the main instrument (researcher) and supporting instruments (test and interviews). The test is given to 31 students of eight grade B of Junior High School 2 Ngawi. The research subject consisted of two students who provided more information to be studied more deeply. Subjects were selected based on the purpose of sampling technique, namely based on the results of students’ written answers which showed more than one representation ability for each number. The results showed that students were able to use various kinds of representations in solving the problems given, although there were still errors in the calculations. Students can fulfill their visual representation skills well, namely presenting information in the image form. Students are less able to find the ability of symbolic representation, which is marked by formula errors and calculation errors. Students can fulfill verbal representation well, namely writing statements in written sentences.

1. Introduction

One of the mathematics skills that is important for students learning is the ability to represent mathematics, because this ability serves as a role for students in understanding of mathematics [1]. Mathematical representation is included in one of the objectives of mathematics [2]. Besides that, mathematical representation is also included in one of the indicators of proficiency tested in PISA [3]. It shows that the ability of mathematical representation is very important for students to have. This statement is in accordance with the statement that the ability of mathematical representation is very important because it can help students organize their thinking when solving problems so that this ability also affects student learning outcomes [4].

The ability of representation is a person's ability to present mathematical ideas that include translating mathematical problems or ideas into interpretations in the form of images, mathematical equations, or words [5]. Mathematical representation is a mathematical ability in the form of expressing mathematical ideas (problems, statements, definitions, etc.) in various ways [6]. These expressions can be in the form of pictures, tables, graphs, diagrams, symbols, and words.

The fact is, the student's representation ability is still low, it is based on the 2018 PISA study results which have decreased compared to 2015, the value has decreased from a score of 386 to 379 [7]. Indonesia is in a low position, based on the results of the 2015 TIMSS (Trends in International Mathematics and Science Study) study, Indonesia is ranked 44th out of 49 countries with an average score of 397 out of the average International score of 500[8]. The lack of students' mathematical
representation ability was also strengthened by the results of research conducted by Rahmah which stated that there were students who made mistakes in the answering of the questions given. These students' mistakes such as understanding the meaning of the question’s errors, not knowing the formula, and calculation errors [9].

Mathematical representations are divided into two parts, namely external representations (real world) and internal representations (thoughts) [10]. The ability of internal representation is related to mental or thought processes that occur in a person's brain so that these processes cannot be observed directly. The ability of external representation is related to how individuals solve problems which can be in the form of images, diagrams, words, and symbols that allow it to be observed directly. When individuals think to solve a given problem, mental processes related to internal representations occur. After the individual writes or expresses the results of his thoughts in other forms such as visual, symbolic, or verbal, it can be said that the individual shows the ability of external representation [11].

External representation is related to visual representations, mathematical expressions or equations, and descriptions or statements [12]. Students who have good representation skills can easily translate their problems into visual representations (tables, pictures, graphs), symbolic representations (formulas, symbols), and verbal representations (word or written) [10]. Students can think in different ways even though the subject is the same. There are not a few explanations that help a student but it doesn’t work for another, the difference can be due to the way of students thinking that is more inclined towards visuals, words, or diagrams [13].

Not a few students remember easily if those are in pictures rather than words [14]. Visual representation allows students to learn how to read displays visually to build skills in problem-solving [15]. The use of symbolic representations by students in solving problems allows the teacher to see the stages of students' arguments and can find out the difficulties or obstacles of students to develop conceptual understanding [16]. Student explanations often use their own language and are represented verbally, where students learn to articulate their reasons by presenting their thoughts. Students in high grades must be able to present mathematical arguments in written (verbal) form [17].

The student's ability to understand the problems in real contexts is not as good as when students are faced with math problems that are not related to the real context, in that case, students think about how to solve the problem by translating student's understanding to the form of mathematical symbols, rewriting the contents of the problem with words- he said himself, even not high-class people often experience to the same problem [18].

When students are faced with a problem, students need to observe and find the certain patterns or rules and students must have good representation skills to be able to articulate a problem into various forms, but the fact is there are many students experiencing failure in terms of these representations' skills [10].

Based on this description, this article explains how the student's ability of visual, symbolic, and verbal representations solve math problems.

2. Method
The type of this research is a qualitative descriptive study that produces descriptive data in the description form of the students' mathematical representations ability in solving quadrilateral material math problems. The instrument that is used in this research is the main instrument (researcher) and supporting instruments (test and interviews). The test is given to thirty-one students of class VIII B of SMP Negeri 2 Ngawi so that the results of the students' answers are analyzed according to the indicators of mathematical representation. Based on the student's answers results, the answers from two students were selected that provided a lot of information so that it could be studied more deeply. Two students were selected based on the purpose of sampling technique for interviews. The result of the analysis from the answer sheets and interviews of the two students were compared to determine the validity of the data.

The test consists of three questions, each of them is based on an indicator of mathematical representation. The questions given have been validated by experts and it is stated that these questions
can be used to measure the mathematical representations ability. The first problem allows the students to answer with visual mathematical representation indicators, which is presenting information in the image form. The second problem allows the students to answer with symbolic mathematical representations indicators, namely solving problems using symbolic representations, such as mathematical models and mathematical equations. The third question allows the students to answer with verbal representation indicators ability, which is writing down the solution through written sentences.

3. Result and Discussions
The following shows the results of the research along with a discussion, mathematical representation abilities which include the visual, symbolic, and verbal representation abilities of junior high school students.

3.1. Visual representation ability
From the 31 students who were given the test, 15 students were able to meet the visual representation indicators well, namely being able to draw garden illustrations correctly along with their measurements. The remaining 16 students were able to draw garden illustrations correctly but did not include the size of the area or include sizes but were not quite right. The results of this study are in accordance with the research conducted by Wijaya that most students who are given the test are able to draw well, although there are still some students who are correct in drawing but are still confused in making explanatory symbols or numbers [4]. The following questions are given for the ability of visual mathematical representation.

Question 1: A city park in the form of a rectangle with an area of 8m×6m. If a center point is taken on each side of the garden, then the center point is connected to another midpoint, a rhombus-shaped childrens playground will be formed and surrounded by a triangular flower garden area. Draw an illustration of the park along with the size of each area.

**Figure 1.** Question number 1

The ability of students to visual representations can be seen in students’ answers to question number 1 below.

**Figure 2.** The answer result of R1
Figure 3. The answer result of R2

In Figure 1, R1 is able to fulfil a visual representation by drawing a garden illustration correctly and its size. R1 also shows another representation ability, namely verbal representation. R1 rewrites the statement on the question such as the statement is known and asked for the answer, and R1 clarifies the size of the park by writing over the description of the size verbally. In Figure 2, R2 can fill the ability of visual representation by drawing garden illustrations well. Each of them is given a visual and verbal explanation. Unlike the answer by R1, R2 calculates the area of each park area. Based on this, R2 shows another ability, namely the ability of symbolic representation. In this answer it can be observed that R2 is incorrect in writing the diagonal length of the rhombus, where R2 inserts the hypotenuse of the triangle into the diagonal of the rhombus. This shows that the symbolic representation ability of R2 is still lacking.

The summary of the interview results with R1 regarding the steps in working of question number 1 as follows: 1) drawing a rectangular garden; 2) drawing a rhombus at the specified midpoints; 3) find the size of the triangle (flower garden), which is obtained from the half-length and half-width of the garden; 4) find the hypotenuse of a triangle using the Pythagorean formula and the result is the size of the fourth side of the rhombus. Based on the interview results, R1 was able to explain well the steps in working on question number 1 and according to the written answer by R1. A good explanation by R1 is included in the verbal representation ability, so it can be said that R1 fulfils the verbal representation ability well. The summary of the interview results with R2 stated that the error was due to R2 noticing the image incorrectly, so that it incorrectly entered the diagonal length of the rhombus with the hypotenuse of the triangle.

The results of analysis of written answers and summary interviews with R1 in accordance with the results of research by Sahendra which states that students have high representation skills if they use more than one representation to solve a problem [19], where students can use different representations in solving these problems [20]. The results of analysis of written answers and summary interviews with R2 in accordance with the results of research by Rahmah which states that students who can use all forms of mathematical representation does not guarantee the correctness of problem solving [8]. The proof is that the R2 can use two forms of mathematical representation but made mistakes on the math calculations.

3.2. Symbolic representation ability

From the 31 students who were given the test, there were 10 students who perfectly fulfilled the symbolic representation indicators by writing down mathematical symbols, formulas, and calculations correctly. As many as 11 students did not find the indicators of mathematical representation, among them there were students who did not write mathematical symbols and formulas correctly even though they were accompanied by correct calculations. A total of 10 students did not write formulas and did not calculate
correctly. This is accordance with the research conducted by Wijaya that there are students who lack the ability to work on questions related to symbolic representations, this is indicated by the wrong results of student work and is accompanied by a messy problem-solving arrangement, not even a few students also only write the final answer without calculation [4]. This indicates that although there are students who fulfill symbolic representations well, there are also students who lack the ability to represent symbolic representations. The following questions are given for the ability of symbolic mathematical representation.

**Figure 4. Question number 2**

The ability of a student's symbolic representation can be seen in the students' answers to question number 2 below.

In Figure 3, R1 can find the symbolic representation indicators, namely problem solving using symbolic representations, such as mathematical models, mathematical equations, and performing mathematical calculations. Students can write the formula for the kite area, the formula for the rectangle area, and the formula for the circumference of the kites correctly. Students are able to find the components needed to find the kite area and do the calculations correctly. In Figure 4, R2 is able to fulfill symbolic representations well, namely problem solving using symbolic representations, such as modelling mathematics and performing mathematical calculations. Students can write the formula for the kite area, the formula for the rectangle area, and the formula for the circumference of the kites correctly. Students are able to find the components needed to find the kite area and do the calculations correctly. R2 shows the ability of visual representation, namely redrawing the illustration given.

The summary of the results of the interviews with students regarding the steps taken by the students in working on question number 2 as follows: 1) calculating the length of one of the diagonals of the kite which is none other than one side of the rectangle using the known rectangle area; 2) calculate the circumference and the kite area.
The results of analysis of written answers and summary interviews with R1 and R2 in accordance with the results of research by Sahendra which states that students have high representation skills if they use more than one representation to solve a problem [19], where students can use different representations in solving these problems [20]. The results of analysis of written answers and summary interviews with R2 in accordance with the results of research by Rahmah which states that there are students who draw back illustrations of the problems given which are very helpful for students to present information in another form of representation [9], like R2 which redraws the picture given to make it easier to work on the problem.

3.3. Verbal representation ability
From the 31 students who took the test, 18 students were able to answer correctly with correct reasons. As many as 4 students were able to answer correctly without any reasons, and 9 students did calculations without verbal answers and reasons. The following questions are given for the ability of verbal mathematical representation.

A room measuring 6m×8m. Originally the room was floored with square tiles measuring 30cm×30cm. This tile will be replaced with a new tile that is twice the size of the original tile. What happens to the number of tiles after replacing? Explain your reasons.

**Figure 7.** Question number 3

The ability of student’s verbal representation can be seen in the students' answers to question number 3 below.

**Figure 8.** The answer result of R1

**Figure 9.** The answer result of R2
In Figure 5, R1 can fulfil verbal representations well, that is, the students can provide answers with correct reasons. Summary of interviews with R1, that R1 worked on these questions with reason, namely the wider the size of the tiles, the fewer tiles will be required. In Figure 6, R2 can meet the indicators of verbal representation ability, namely writing completion through written sentences. R2 can answer that the number of tiles is reduced compared to the original number of tiles, and it can explain why the number of tiles is less than before. Based on the answers, it can be observed that R2 shows the symbolic representation ability. It is indicated by the mathematical calculations carried out by R2. The mathematical calculation is not correct, where R2 includes the unit area and does the calculation incorrectly. It shows that even though R2 fulfils verbal representation skills well, its symbolic representation skills are still lacking.

It was found in the problem that the size of the new tile was twice the size of the old tile, while not a few students wrote that the area of the new tile was twice the area of the original tile. This indicates that students do not understand well what is meant by the questions. Errors in understanding these questions result in errors in calculations. The following research is conducted by Sukmaningingthias and Hadi which have stated that students who have difficulty analyzing and representing mathematical problems will have difficulty solving the problems that are given [20].

4. Conclusion
The result showed that of the 31 students who were given the test, 48.38% students were able to meet the visual representation ability indicators well, while 51.62% students did not meet the indicators of visual representation skills well. As many 32.25% students were able to meet the symbolic representation ability indicators well, while 67.75% students did not meet the indicators of symbolic representation skills well. As many 58.06% students were able to meet the verbal representation ability indicators well, while 41.94% students did not meet the indicators of verbal representation skills well.

The results showed that students can fulfil their visual representation skills well, namely presenting information in the image form. Students are less able to find the ability of symbolic representation, which is marked by formula errors and calculation errors. Students can fulfil verbal representation well, namely writing statements in written sentences. In the future teachers are expected to gives questions in various forms of representation so that students are accustomed on questions working with various representations, and give freedom to students to work on questions in their way, so that students' representation skills are more honed.

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