Memory effects in response functions of driven vortex matter

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Abstract

Vortex flow in driven type II superconductors shows strong memory and history dependent effects. Here, we study a schematic microscopic model of driven vortices to propose a scenario for a broad set of these kind of phenomena ranging from “rejuvenation” and “stiffening” of the system response, to “memory” and “irreversibility” in I-V characteristics.

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An important discovery in the dynamics of vortices driven by an external current in type II superconductors is the presence of strong “memory” and history dependent effects in vortex flow (see for instance [1–6] and ref.s therein). These off-equilibrium phenomena, particularly important at low temperatures, crucially affect the overall system behaviour and, thus, have also important consequences for technological applications. Here we present a microscopic model to explain their origin and their connections with other “aging” phenomena of vortex matter such as the rearrangement of vortex domains during creep. In relation to recent experimental results [4,5], we discuss in particular the nature of “memory” effects observed in the response of the system to an external drive, i.e., the I-V characteristic. Our model explains the peculiar form of “memory” observed in vortex flow at finite $T$, which we call an “imperfect memory”. In the same framework one can explain other “anomalous” properties such as those of time dependent critical currents. The essential step is to identify the relevant time scales in the dynamics. Finally, we stress the similarities and the important differences with other “glassy” systems, such as supercooled liquids or disordered magnets (see [3,10] and ref.s therein).
We consider a schematic coarse grained model introduced to describe anomalous relaxation of vortices in superconductors, namely a system of repulsive particles wandering in a pinning landscape in presence of an external drive. The model describes several phenomena of vortex physics, ranging from a reentrant phase diagram in the \((B, T)\) plane, to the “anomalous second peak” in magnetisation loops (the “fishtail” effects), glassiness and “aging” of slow magnetic relaxation, the “anomalous creep” at very low temperatures, and many others [8].

A system of straight parallel vortex lines, corresponding to a magnetic field \(B\) along the \(z\)-axis, interacts via a potential [1]:

\[
U(r) = \frac{\phi_0^2}{2\pi \lambda'^2} [K_0(r/\lambda') - K_0(r/\xi')] ,
\]

where \(K_0\) is the MacDonald function, \(\xi\) and \(\lambda\) the correlation and penetration lengths (\(\xi' = c\xi/\sqrt{2}\), \(\lambda' = c\lambda\), \(c = (1 - B/B_c)^{-1/2}\)). The typical high vortex densities and long \(\lambda\) imply that the vortex system is strongly interacting. To make it theoretically more tractable, as proposed in [7,8], one can coarse grain in the \(xy\)-plane by introducing a square grid of lattice spacing \(l\) of the order of the London length, \(\lambda\). The number of vortices on the \(i\)-th coarse grained cell, \(n_i\), is an integer number smaller than \(N_{c2} = B_{c2}l^2/\phi_0\) [8] (\(B_{c2}\) is the upper critical field and \(\phi_0 = hc/2e\) is the flux quantum). The coarse grained interaction Hamiltonian is thus [8]:

\[
\mathcal{H} = \frac{1}{2} \sum_{ij} n_i A_{ij} n_j - \frac{1}{2} \sum_i A_{ii} n_i - \sum_i A_i^p n_i .
\]

The first two terms describe the repulsion between the vortices and their self energy, and the last the interaction with a random pinning background. For sake of simplicity, we consider the simplest version of \(\mathcal{H}\): we choose \(A_{ii} = A_0 = 1; A_{ij} = A_1 < A_0\) if \(i\) and \(j\) are nearest neighbours; \(A_{ij} = 0\) otherwise; the random pinning is delta-distributed

\[
P(A^p) = (1 - p)\delta(A^p) + p\delta(A^p - A_0^p) .
\]

In analogy with computer investigation of dynamical processes in fluids, the time evolution of the model is simulated by a Monte Carlo Kawasaki dynamics [11] on a square lattice of size \(L\) [12] at a temperature \(T\). The system is periodic in the \(y\)-direction. The two edges parallel to the \(y\)-axis are in contact with a vortex reservoir, i.e., an external magnetic field, of density \(N_{ext}\). Particles can enter and leave the system only through the reservoir. The above model, called ROM, is described in full details in [8].

The system is prepared by zero field cooling and then increasing \(N_{ext}\) at constant rate.
up to the working value (here, $N_{ext} = 10$). Then we monitor the system relaxation after applying a drive, $I$ (due to the Lorentz force), in the $y$-direction. As in similar driven lattice gases [13], the effect of the drive is simulated by introducing a bias in the Metropolis coupling of the system to the thermal bath: a particle can jump to a neighbouring site with a probability $\min\{1, \exp[-(\Delta H - \epsilon I)/T]\}$. Here, $\Delta H$ is the change in $H$ after the jump and $\epsilon = \pm 1$ for a particle trying to hop along or opposite to the direction of the drive and $\epsilon = 0$ if orthogonal jumps occur. A drive $I$ generates a voltage $V$ [14]:

$$V(t) = \langle v_a(t) \rangle$$

where $v_a(t) = \frac{1}{2\Delta t} \int_{t-\Delta t}^{t+\Delta t} v(t') dt'$ is an average vortex “velocity” in a small interval around the time $t$. We consider such an average to improve the statistics on $V(t)$ and choose $\Delta t$ accordingly. Here, $t$ is the Monte Carlo time (measured in units of single attempted update per degree of freedom), $v(t) = \frac{1}{L} \sum_i v_i(t)$ is the instantaneous flow “velocity”, $v_i(t) = \pm 1$ if the vortex $i$ at time $t$ moves along or opposite to the direction of the drive $I$ and $v_i = 0$ otherwise. The data presented below are averaged over up to 3072 realizations of the pinning background.

To characterise the properties of history dependent effects, we consider a striking manifestation of “memory” observed in experiments where the drive is cyclically changed [5]. A drive $I$ is applied to the system and, after a time $t_1$, abruptly changed to a new value $I_1$; finally, after waiting a time $t_2$, the previous $I$ is restored and the system evolves for a further $t_3$ (see lower inset of Fig.1). The measured $V(t)$ is shown in the main panel of Fig.1 for $T = 0.1$. A first observation is that after the switch to $I_1$ the system seems to abruptly reinitiate its relaxation approximately as if it has always been at $I_1$ (see for example the dashed curve in Fig.1), a phenomenon known as “rejuvenation” in thermal cycling of spin-glasses and other glassy systems [10]. The more surprising fact is, however, that for $I_1$ small enough (say $I_1 \ll I^*$, $I^*$ to be quantitatively defined below) when the value $I$ of the drive is restored the voltage relaxation seems to restart from where it was at $t_1$, i.e., where it stopped before the application of $I_1$ (see Fig.1). Actually, if one “cuts” the evolution dur-
ing $t_2$ and “glues” together those during $t_1$ and $t_3$, an *almost* perfect matching is observed (see upper inset of Fig.1). What is happening during $t_2$ is that the system is trapped in some metastable states, but *not* completely frozen as shown by a small magnetic, as well as voltage, relaxation. These non trivial “memory” effects are experimentally found in vortex matter \[5\] and glassy systems \[10\]. We call them a form of “imperfect memory”, because they tend to disappear when the time spent at $I_1$ becomes too long or, equivalently (as explained below) when, for a given $t_2$, $I_1$ becomes too high, as clearly shown in the inset of Fig.1. These “rejuvenation and memory” effects are very general in off-equilibrium “glassy” dynamics and can be found well above the ideal glass transition point, $T_c$ \[8\], whenever the observation time scales are shorter than the intrinsic relaxation scale $\tau_V$ (to be defined below).

We now turn to some additional aspects concerning the time dependent properties of the current-voltage characteristic. As in real experiments on vortex matter \[5\], we let the system undergo a current step of hight $I_0$ for a time $t_0$ before starting to record the I-V by ramping $I$, as sketched in the inset of Fig.2. Fig.2 shows (for $T = 0.1$) that the I-V depends on the waiting time $t_0$. The system response is “aging”: the longer $t_0$ the smaller the response, a phenomenon known as “stiffening” in glass formers \[9,10\]. These effects are manifested in a violation of time translation invariance of the two times correlation functions, which have dynamical scaling properties \[8\] analogous to those of glass formers \[9\].

These simulations also reproduce the experimentally found time dependence of the critical current \[5\]. Usually, one defines an effective critical current, $I_{c}^{eff}$, as the point where $V$ becomes larger than a given threshold (say $V_{thr} = 10^{-5}$ in our case): one then finds that $I_{c}^{eff}$ is $t_0$ and $I_0$ dependent (like in experiments \[5\] $I_{c}^{eff}$ is slowly increasing with $t_0$, see Fig.2).

It is interesting to consider another current cycling experiment which outlines the concurrent presence of irreversibility and memory effects. The I-V is measured by ramping $I$ up to some value $I_{max}$. Then $I$ is ramped back to zero, but at a given value $I_w$ the system is let to evolve for a long time $t_w$. Finally, $I$ is ramped up again (see inset of Fig.3). The resulting *irreversible* $V(I)$ is shown in Fig.3. For $I > I_w$ the decreasing branch of the plot (empty
circles) slightly deviates from the increasing one (filled circles), showing the appearance of irreversibility. This is even more apparent after $t_w$: for $I < I_w$ the two paths are clearly different. Interestingly, upon increasing $I$ again (filled triangles), $V(I)$ doesn’t match the first increasing branch, but the latest, the decreasing one: in this sense there is coexistence of memory and irreversibility. Also very interesting is that by repeating the cycle with a new $I_w$ (squares), the system approximately follows the same branches. This non-reversible behaviour is also found in other glassy systems [10]. However, spin glasses, for instance, show the presence of the so called chaos effects [10]. The chaos effect is absent in our system as it is also in other ordinary glass formers [10]. This kind of interplay between irreversibility and memory can be checked experimentally in superconductors and thereby assess the present scenario.

The above experiments outline a set of very important “history” dependent phenomena ("rejuvenation", “imperfect memory”, “stiffening” and “irreversibility” of response, etc...). A crucial theoretical step to understand them is the identification of the characteristic time scales of the driven dynamics. This we now discuss.

Upon applying to the system a small drive, $I$, its response, $V$, relaxes following a pattern with two very different parts: at first a rapidly changing non-linear response is seen, later followed by a very slow decrease towards stationarity (see $V(t)$ in Fig.[4] for $T = 1$ and $I \in \{1, 2, 3\}$). For instance, for $I = 3$ in a time interval $\Delta t \simeq 2 \cdot 10^{-1}$, $V$ leaps from about zero to $\Delta V_i \sim 2 \cdot 10^{-3}$, corresponding to a rate $r_i = \Delta V_i / \Delta t \sim 10^{-2}$. This is to be compared with the rate of the subsequent slow relaxation from, say, $t = 2 \cdot 10^{-1}$ to $t = 10^4$, $r_f \sim -10^{-7}$: $r_i$ and $r_f$ differ of 5 orders of magnitude.

In agreement with experimental findings [11-13], the slow relaxation of $V(t)$ has a characteristic double step structure, which asymptotically can be well fitted by stretched exponentials [14]: $V(t) \propto \exp(-t/\tau_V)^\beta$. The above long time fit defines the characteristic asymptotic scale, $\tau_V$, of relaxation. The exponent $\beta$ and $\tau_V$ are a function of $I$, $T$ and $N_{ext}$ (see inset Fig.[4]): in particular $\tau_V(I)$ decreases with $I$ and seems to approach a finite plateau for $I < I^*$, with $I^* \approx O(1)$. In this sense, the presence of a drive $I$ makes the approach to
stationarity faster and has an effect similar to an increase in $T$. A reasonable fit for $\tau_V(I)$ is: $\tau_V(I) = (\tau_0^V - \tau^\infty_V)/[1 + (I/3I^*)^a] + \tau^\infty_V$, with $\tau_0^V = 2590$, $\tau^\infty_V = 1720$, $I^* = 0.86$ and $a = 2$.

The outlined properties of $\tau_V$ clearly explain the history dependent effects in the experiments previously considered. For instance, the “imperfect memory”, discussed in Fig.1, is caused by the presence of a long, but finite, scale $\tau_V$ in the problem: for a given $I_1$ the system seems to be frozen whenever observed on times scales smaller than $\tau_V(I_1)$. Thus, if $t_2$ is short enough ($t_2 < \tau_V(I_1)$) the system preserves a strong “memory” of its state at $t_1$. The weakening of such a “memory” found for higher currents $I_1$ in Fig.1 is also a consequence of the strong decrease of $\tau_V(I)$ with $I$. The phenomenon of “rejuvenation” (see Fig.1) is, in turn, a consequence of the presence of the extremely fast first part of relaxation found in $V(t)$ upon applying a drive and of the above long term memory. The existence of the slow part in the $V(t)$ relaxation also affects the “stiffening” of the response in the I-V of Fig.2, which is due to the non-stationarity of the vortex flow on scales smaller that $\tau_V$. Actually, in Fig.2, for a given $I$ the value of $V$ on the different curves corresponds to the system being probed at different stages of its non-stationary evolution (this also outlines that the proper definition of $I_c$ is the asymptotic one given later on). Finally, in brief, the fact that $\tau_V(I)$ is smaller at high currents, $I$, and larger at small $I$ (and $T$), is responsible for the surprisingly concomitant effects of irreversibility and memory of Fig.3.

The origin of these time dependent properties of the driven flow, and in turn those of I-V’s, traces back to the concurrent vortex creep and reorganisation of vortex domains. In fact, both with or without an external drive, the system evolves in presence of a Bean like profile (see Fig.5) which in turn relaxes following stretched exponentials with a characteristic time scale $\tau_M$. An important discovery is that $\tau_M(I)$ and $\tau_V(I)$ are approximately proportional, as shown in Fig.5. This outlines that the strong non-linear, non-stationary voltage relaxation is structurally related to the reorganisation of vortices during the creep (a fact confirmed by recent experiments). By assuming that the system is composed of evolving vortex domains characterised by drive dependent (free) energy barriers which grow with the sizes of the domains, a hierarchy of time scales naturally arises. The simultane-
ous presence of both long time scales ("aging modes") and short time scales is a simple mechanism able to describe the above phenomena (see [10]).

Finally, let us notice that the existence at $T = 1$ of a finite $\tau_V$ limit for $I \to 0$ (see Fig.4) implies that the flux flow can be activated for any finite $I$. This shows that the system’s asymptotic (i.e., for $t \to \infty$) critical current, $I_c$, is zero. It is possible that the asymptotic value of $I_c$ can be non-zero for real three dimensional systems if vortex line cutting is negligible. The above also points out that a true divergence in $\tau_V(I)$, i.e., a true asymptotic finite $I_c$, can be found only for $T < T_c$, $T_c$ being the ideal glass transition point [1,8], where by definition $\tau_M(T)|_{I=0}$ diverges [8].

The few key “experiments” we discussed in the context of our schematic microscopic model depict a new clear scenario for “aging” and “memory” effects in driven vortex matter. In the present framework a broad range of phenomena can be understood (from response “stiffening” and “rejuvenation”, to time and history dependent critical currents, $I_{eff}^c$). We stress that most of these features can also be found well above the ideal glass transition point $T_c$. We have shown how creep and response functions in driven media are related. Theoretically intriguing is the observation of similar behaviours found in vortex matter and others glass formers such as random magnets and supercooled liquids.
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FIG. 1. At $T = 0.1$ for a drive $I = 1$, the voltage, $V(t)$, is plotted as a function of time. As shown in the lower inset, after a time lag $t_1$, the drive is abruptly changed to $I_1$ for a time $t_2$ and finally it is set back to its previous value. When $I$ is switched to $I_1$ the system seems to “rejuvenate”: it suddenly restarts its relaxation along the path it would have had if $I = I_1$ at all times (consider the continuous and dashed bold curves, corresponding to $I = I_1 = 1$ and $I = I_1 = 0.8$, plotted for comparison). By restoring $I$ after $t_2$, the system shows a strong form of “memory”: if $t_2$ and $I_1$ are small enough (see text) the relaxation of $V(t)$ restarts where it was at $t_1$. However, if $t_2$ and $I_1$ are too large, this is not the case, as shown in the upper inset. In this sense, the above is an “imperfect memory”.
FIG. 2. The I-V obtained at $T = 0.1$ by ramping $I$ after keeping the system in presence of a drive $I_0 = 1$ for a time $t_0$ as shown in the inset. The response, $V$, is “aging” (i.e., depends on $t_0$) and, more specifically, *stiffening*: it is smaller the longer $t_0$. 
FIG. 3. The I-V is measured at $T = 0.1$ during cycles of $I$ (see also the inset): $I$ is at first increased up to $I_{\text{max}}$ (filled circles); along the descending branch of the cycle (empty circles), when $I = I_w$ (in the main panel the $I_w$'s, for two cycles, are located by the arrows) the drive is kept fixed for a time $t_w = 10^4$ and then the cycle restarted; finally, $I$ is ramped up again (filled triangles). For $I > I_w$, the first increasing ramp and the decreasing one (resp. filled and empty circles) do not completely match, showing irreversibility in the I-V. After waiting $t_w$ at $I_w$, a much larger separation is seen. However, by raising $I$ again (filled triangles) a strong memory is observed: the system doesn’t follow the first branch (filled circles), but the decreasing one (empty circles). Furthermore, in a cycle with a lower $I_w$ (squares), the same branches are found.
FIG. 4. The time evolution of the response function, $V(t)$, for the shown values of the drive $I$ (at $T = 1$ and $N_{ext} = 10$). In the asymptotic regime $V(t)$ is well fitted with: $V(t) \propto \exp[-(t/\tau_V)^\beta]$.

Inset The characteristic scale of relaxation, $\tau_V(I)$, as a function of $I$. For $I \to 0$, $\tau_V(I)$ seems to saturate to a finite value which implies $I_c = 0$. 
FIG. 5. **Left** Vortex driven flow and creep are strictly related. Actually, the characteristic time scales of voltage relaxation, $\tau_V(I)$, and of magnetic creep, $\tau_M(I)$ (shown for $T = 1$), are approximately proportional. **Right** The Bean field profile, $N(x)$, present in the system during relaxation is shown for the plotted values of $I$ and $t$. 