Baryogenesis from Symmetry Principle

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Abstract

In this work, a formalism based on symmetry which allows one to express asymmetries of all the particles in terms of conserved charges is developed. The manifestation of symmetry allows one to easily determine the viability of a baryogenesis scenario and also to identify the different roles played by the symmetry. This formalism is then applied to the standard model and its supersymmetric extension, which constitute two important foundations for constructing models of baryogenesis.
I. INTRODUCTION

The evidences that we live in a matter-dominated Universe are very well-established \cite{1}. While the amount of antimatter is negligible today, the amount of matter (i.e. baryon) of the Universe has been determined with great precision by two independent methods. From the measurement of deuterium abundance originated from Big Bang Nucleosynthesis (BBN) when the Universe was about a second old (with temperature $T_{BBN} \sim \text{MeV}$), Ref. \cite{2} quotes the baryon density normalized to entropic density as $10^{11} Y_{BBN}^B = 8.57 \pm 0.18$. From the measurement of temperature anisotropy in the cosmic microwave background radiation imprinted by acoustic oscillation of photon-baryon plasma when the Universe was about 380000 years old ($T_{CMB} \sim 0.3 \text{eV}$), Planck satellite gives $10^{11} Y_{CMB}^B = 8.66 \pm 0.06$ \cite{3}. The impressive agreement between the two measurements is a striking confirmation of the standard cosmological model.

In order to account for the cosmic baryon asymmetry, baryogenesis must be at work before the onset of BBN. Although the Standard Model (SM) of particle physics (and cosmology) contains all the three ingredients for baryogenesis: baryon number violation, $C$ and $CP$ violation, and the out-of-equilibrium condition \cite{4}, it eventually fails and new physics is called for \cite{5}. Clearly these ingredients are necessary but not sufficient. Moreover, the early Universe is filled with particles of different types that interact with each other at various rates, rendering it a daunting task to analyze them. In this work, I would like to advocate the use of symmetry as an organizing principle to analyze such a system. In particular, I will show that by identifying the symmetries of a system, one can relate the asymmetries of all the particles to the corresponding conserved charges without having to take into account details of how those particles interact.\footnote{It should be stressed immediately that the symmetries do not have to be exact. If a symmetry is approximate, the corresponding charge will be quasi-conserved with its evolution described by nonequilibrium formalism like Boltzmann equation. In other words, the description of the system boils down to identifying \textit{only} the interactions related to approximate symmetries.} This should not come as a surprise since symmetry dictates physics: when we specify a symmetry and how particles transform under it, the interactions are automatically fixed. I will first review the formalism in Sec. \textsection II Then the roles of $U(1)$ symmetries are clarified in Sec. \textsection III In Sec. \textsection IV and \textsection V respectively, I will apply this formalism to the SM and its supersymmetric extension as they form important bases for constructing models of baryogenesis. Finally I conclude in Sec. \textsection VI.
II. FORMALISM

Here I will review the formalism that we will use in this work. For a system with $s$ number of symmetries labeled $U(1)_x$ and consisting of $r \geq s$ distinct types of complex particles labeled $i$ (i.e. not self-conjugate like real scalar or Majorana fermion) with corresponding chemical potentials $\mu_i$ and charges $q^x_i$ under $U(1)_x$, the most general solution is given by

$$\mu_i = \sum_x C_x q^x_i,$$

where $C_x$ is some real constant corresponding to $U(1)_x$. It is apparent that Eq. (1) is the solution for chemical equilibrium conditions for any possible in-equilibrium interactions since by definition, the interactions necessarily preserve the symmetry. Note that symmetry discussed in this work always refers to $U(1)$ which characterizes the charge asymmetry between particles and antiparticles. The $U(1)_x$ can be exact (like gauge symmetry) or approximate (due to small couplings, and/or suppression by mass scale and/or temperature effects). The diagonal generators of a nonabelian group do not contribute as long as the group is not broken [6]. For instance one does not need to consider conservation of third component of weak isospin $T_3$ before electroweak (EW) phase transition.

Now for each $U(1)_x$, according to Noether’s theorem there is a conserved current and the corresponding conserved charge density can be constructed as

$$n_{\Delta x} = \sum_i q^x_i n_{\Delta i},$$

where $n_{\Delta i}$ is the number density asymmetry for particle $i$. To proceed we need two further assumptions. Firstly, particle $i$ is assumed to participate in fast elastic scatterings such that its phase space distribution is either Fermi-Dirac $[\exp(E_i - \mu_i)/T + 1]^{-1}$ or Bose-Einstein $[\exp(E_i - \mu_i)/T - 1]^{-1}$ for fermion or boson respectively. Secondly, there are fast inelastic scatterings for particle $i$ and its antiparticle $\bar{i}$ to gauge bosons (which have zero chemical potential) such that $\mu_{\bar{i}} = -\mu_i$. These two assumptions are justified for instance when the

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2 The formalism was first introduced by Ref. [6] to prove that the generation of hypercharge asymmetry in a preserved sector implies nonzero baryon asymmetry. See also the relevant discussion in Chapter 3.3 of Ref. [7].
particles have gauge interactions. Now Eq. (2) can be related to its chemical potential for \(\mu_i \ll T\) as follows\(^3\)

\[
n_{\Delta i} = n_i - n_{\bar{i}} = \frac{T^2}{6} g_i \zeta_i \mu_i.
\]

(3)

In the above \(g_i\) specifies the number of gauge degrees of freedom and

\[
\zeta_i \equiv \frac{6}{\pi^2} \int_{z_i}^{\infty} dx x \sqrt{x^2 - z_i^2} \frac{e^x}{(e^x \pm 1)^2},
\]

(4)

with \(z_i \equiv m_i/T\). In the relativistic limit \((T \gg m_i)\), we have \(\zeta_i = 1(2)\) for \(i\) a fermion (boson) while in the nonrelativistic limit \((T \ll m_i)\), we obtain \(\zeta_i = \frac{6}{\pi^2} z_i^2 K_2(z_i)\) with \(K_2(x)\) the modified Bessel function of type two of order two. Using Eqs. (1) and (3), Eq. (2) can be written as

\[
n_{\Delta x} = \frac{T^2}{6} \sum_y J_{xy} C_y,
\]

(5)

where we have defined the symmetric matrix \(J\) as follows

\[
J_{xy} \equiv \sum_i g_i \zeta_i q^y_i q^x_i.
\]

(6)

We can invert Eq. (5) to solve for \(C_y\) in terms of \(n_{\Delta x}\) and substituting it into Eq. (1) and then making use of Eq. (3), we obtain\(^4\)

\[
n_{\Delta i} = g_i \zeta_i \sum_{y,x} q^y_i (J^{-1})_{yx} n_{\Delta x}.
\]

(7)

Eventually one would like to relate this to baryon asymmetry i.e. the baryon charge density. By substituting Eq. (7) into Eq. (2) for baryon charge density, we have

\(^3\) The expansion in \(\mu_i/T \ll 1\) is justified as long as the number asymmetry density is much smaller than its equilibrium number density. For instance with \(n_{\Delta i}\) the order of the observed baryon asymmetry, the expansion holds when the corresponding particle mass over temperature \(m_i/T \lesssim 20\).

\(^4\) As long as \(r \geq s\) and there are no redundant symmetries, in the sense that all the symmetries are linearly independent and there is no rotation in the \(s\)-dimensional symmetry space that can make all the \(r\) distinct particles uncharged under some \(U(1)\), \(J\) always has an inverse.
\[ n_{\Delta B} = \sum_{y,x} J_{By} (J^{-1})_{yx} n_{\Delta x}. \] 

Eqs. (7) and (8) make the symmetries of the system manifest: the solutions are expressed in term of conserved charges \( n_{\Delta x} \), one for each \( U(1)_x \) symmetry. In fact \( \{ n_{\Delta x} \} \) forms the appropriate basis to describe the system. While \( q^i_x \) comprises the charges of particle \( i \) under \( U(1)_x \), \( J \) matrix embodies full information of the system (all possible interactions consistent with the symmetry are implicitly taken into account). Notice that calculating \( J \) is particularly simple and circumventing the traditional approach of having to count the number of chemical potentials and determine the chemical equilibrium conditions. It is now apparent that baryogenesis fails \( (n_{\Delta B} = 0) \) if: (I) the system does not possess any symmetry in which case \( C_x = 0 \) for all \( x \) in Eq. (10) or; (II) the system possesses only \( U(1)_x \)'s which always remain exact such that none develops an asymmetry in which case \( n_{\Delta x} = 0 \) for all \( x \).

For instance, the baryogenesis scenario proposed in Ref. [8] fails due to the following reasons. In that work, there are initially four effective symmetries: \( U(1)_{B/3-L_\alpha} (\alpha = \{1, 2, 3\}) \) and \( U(1)_{\tilde{\psi}} \). During baryogenesis, \( U(1)_{B/3-L_\alpha} \) is always conserved i.e. \( n_{\Delta (B/3-L_\alpha)} = 0 \) while a large enough \( CP \) asymmetry at the TeV scale requires fast \( U(1)_{\tilde{\psi}} \) violation i.e. \( C_{\tilde{\psi}} = 0 \). As a result, \( n_{\Delta B} = 0 \).

III. THE ROLES OF \( U(1) \) SYMMETRIES

In general, the reaction rate of a process \( \gamma \) in the early Universe is temperature-dependent \( \Gamma_\gamma(T) \). At each range of temperature \( T^* \), by comparing \( \Gamma_\gamma(T^*) \) to the expansion rate of the Universe \( H(T^*) \), we can categorize the reactions into three types [9, 10]: (i) \( \Gamma_\gamma(T^*) \gg H(T^*) \); (ii) \( \Gamma_\gamma(T^*) \ll H(T^*) \); (iii) \( \Gamma_\gamma(T^*) \sim H(T^*) \). The reactions of type (i) are fast enough to establish chemical equilibrium and are implicitly ‘resummed’ in the \( J \) matrix in Eq. (6). The reactions of type (ii) either do not occur or proceed slow enough. The former is due to exact symmetry like gauge symmetry while the latter is due to small couplings, and/or suppression by mass scale and/or temperature effects. Finally the reactions of type (iii) should be described by nonequilibrium formalism like Boltzmann equation in order to obtain quantitative prediction. In this work, the effective symmetries concern both reactions of types (ii) and (iii). In particular gauge symmetry always belongs to type (ii) and can play
an interesting role as ‘messenger’. If an approximate symmetry belongs to type (ii), it can acquire a role as a ‘messenger’ or ‘preserver’ while if it is of type (iii), it can act as ‘creator/destroyer’.

To understand the roles of $U(1)$ alluded to above, it is illuminating to group the charges as follows. Among all the charges $U = \{n_\Delta x\}$, there is a subset $U_0 = \{n_\Delta a\}$ where the net charges vanish $n_\Delta a = 0$. In this case, we can remove them from the beginning and left with $\tilde{U} = U - U_0 = \{n_\Delta m\}$ to describe the system. From Eq. (5), we have a set of linear equations $n_\Delta a = \sum_b J_{ab} C_b + \sum_m J_{am} C_m = 0$, which allows us to solve for $C_a$ in terms of $C_m$. After eliminating $C_a$, the number density asymmetry for particle $i$ can be expressed as

$$n_{\Delta i} = g_i \zeta_i \sum_{m,n} \tilde{q}^m_i \left( \tilde{J}^{-1} \right)_{mn} n_{\Delta n}. \quad (9)$$

where we have defined $\tilde{q}^m_i \equiv q^m_i - \sum_{a,b} q^{a}_{i} (J^{-1})_{ab} J_{bm}$ and $\tilde{J}_{mn} \equiv J_{mn} - \sum_{a,b} J_{ma} (J^{-1})_{ab} J_{bn}$. Substituting Eq. (9) into Eq. (2) for baryon charge density, we get

$$n_{\Delta B} = \sum_{m,n} \left[ J_{Bm} - \sum_{a,b} J_{Ba} (J^{-1})_{ab} J_{bm} \right] \left( \tilde{J}^{-1} \right)_{mn} n_{\Delta n}. \quad (10)$$

The equation above can be succinctly written as $n_{\Delta B} = \sum_{m,n} \tilde{J}_{Bm} \left( \tilde{J}^{-1} \right)_{mn} n_{\Delta n}$ but it is elucidating to keep it as it is: the two terms in the square bracket of Eq. (10) represent two different types of contributions to the baryon asymmetry. The first term is the direct contribution of $\tilde{U}$ sector to the baryon asymmetry while the second term is the contribution of $\tilde{U}$ sector through $U_0$ (the messenger sector). Hence even if $\tilde{U}$ sector does not carry baryon charge $J_{Bm} = 0$, as long as it carries charges in the messenger sector $J_{bm} \neq 0$, and some baryons also carry charges in the messenger sector $J_{Ba} \neq 0$, we will have $n_{\Delta B} \neq 0$. Here $\tilde{U}$ sector can play two roles: as creator/destroyer or preserver of asymmetries depending on their rates as discussed in the beginning of this section. In short, the roles of $U(1)$ symmetries in baryogenesis can be concisely stated as follows:

1. *Creator/destroyer*: type (iii) reaction with an approximate $U(1)_m$. The dynamical violation of $U(1)_m$ results in the development of $n_{\Delta m} \neq 0$ from $n_{\Delta m} = 0$. As mentioned earlier, quantitative prediction requires one to solve dynamical equation like

\[\text{We use } a, b, \ldots \text{ to label the charges in } U_0 \text{ and } m, n, \ldots \text{ to label the charges in } \tilde{U}.\]
Boltzmann equation for $n_{\Delta m}$ and the generated asymmetry depends on the rates of creation and washout.

2. **Preserver:** type (ii) reaction with $U(1)_m$ and $n_{\Delta m} \neq 0$. The symmetry prevents the asymmetry from being washed out. The lightest electrically neutral particle in this sector can be a good (asymmetric) dark matter candidate.

3. **Messenger:** type (ii) reaction with $U(1)_a$ and $n_{\Delta a} = 0$. Further requirement is that at least some particles in $U(1)_m$ (of the preserver or the creator/destroyer) and some baryons need to be charged under $U(1)_a$ such that a nonzero asymmetry in $U(1)_m$ induces a nonzero baryon asymmetry through $U(1)_a$ conservation.

In the SM, conservations of hypercharge $U(1)_Y$ and electric charge $U(1)_Q$ ensure $n_{\Delta Y} = n_{\Delta Q} = 0$. Hence they play the role of messenger respectively before and after EW phase transition (EWPT) at $T_{\text{EWPT}}$. Eq. (10) is the generalization of the result of Ref. [6] which shows that a preserved sector which carries nonzero hypercharge asymmetry implies nonzero baryon asymmetry (set $a = b = Y$ in the second term in Eq. (10)). We can readily extend this result to post-EW-sphaleron baryogenesis scenario [11] where $U(1)_Q$ plays the role of messenger. In this case, baryon asymmetry cannot be erased by fast $B$-violating interactions as long as there is a preserved sector carrying nonzero electric charge asymmetry. Of course, phenomenological constraint will require that the electric charge asymmetry to decay away before BBN.

IV. THE STANDARD MODEL

First let us define the $U(1)_x - SU(N) - SU(N)$ mixed anomaly (coefficient) as $A_{xNN} \equiv \sum_i c_2(R) g_i q_i^x$ where $c_2(R)$ is the quadratic Casimir operator in representation $R$ of $SU(N)$ with $c_2(R) = \frac{1}{2}$ in the fundamental representation and $c_2(R) = N$ in the adjoint representation. Here $g_i$ is the degeneracy of particle $i$ of charge $q_i^x$ in a given representation. In the following, for $N = 2$, we always refer to weak $SU(2)_L$ while for $N = 3$, color $SU(3)_c$.

The SM Yukawa sector is described by

$$-\mathcal{L}_Y = (y_u)_{\alpha\beta} \bar{Q}_\alpha \epsilon H^* U_\beta + (y_d)_{\alpha\beta} \bar{Q}_\alpha HD_\beta + (y_e)_{\alpha\beta} \bar{\ell}_\alpha HE_\beta + \text{H.c.},$$  \hspace{1cm} (11)
Figure 1. Symmetries of the SM in the early Universe in between BBN and grand unified theory scale ($T_{\text{GUT}} \sim 10^{16}$ GeV). $T_{\text{EWsp}}^- < T < T_{\text{EWsp}}^+$ is the range of temperature where EW sphalerons are in thermal equilibrium. At very high temperature $T \gtrsim T_{\text{EWsp}}^+$, some of the interactions due to quark intergeneration mixing can become ineffective, resulting in baryon flavor conservation $U(1)_{B_{\alpha}}$. While $U(1)_Y$ and $U(1)_Q$ are gauge symmetries which have to be exact; $U(1)_{B_{(\alpha)}}$, $U(1)_{L_{\alpha}}$ and $U(1)_{B/3-L_{\alpha}}$ are global symmetries which can be broken dynamically, providing the avenue for baryogenesis.

where $\alpha, \beta = \{1, 2, 3\}$ are fermion family indices and the $SU(2)_L$ contraction is shown explicitly with antisymmetric tensor $\epsilon_{01} = -\epsilon_{10} = 1$ and $\epsilon_{ii} = 0$ while the $SU(3)_c$ contraction is left implicit. In Eq. (11), $Q_{\alpha}$, $\ell_{\alpha}$, $H$ are respectively the left-handed quark, left-handed lepton and Higgs $SU(2)_L$ doublets while the right-handed quark and lepton singlets are $U_1 = u$, $D_1 = d$, $E_1 = e$ and so on. Besides $U(1)_Y$ or $U(1)_Q$, it is well-known that there are baryon $U(1)_B$ and lepton flavors $U(1)_{L_{\alpha}}$ as accidental symmetries. The relevant $U(1)$ charges are listed in Table I. $U(1)_Y$ and $U(1)_Q$ as gauge symmetries are ensured to be anomaly-free. On the other hand, $U(1)_B$ and $U(1)_{L_{\alpha}}$ both have $SU(2)_L$ mixed anomaly with $A_{B22} = A_{L_{\alpha}22} = \frac{N_f}{2}$ where $N_f$ is the number of fermion family ($N_f = 3$ in the SM). As a result of the anomaly, $B$ and $L_{\alpha}$ are both violated by sphaleron-mediated dimension-6 operator $O_{\text{EWsp}} = \sum_{\alpha} (QQQ\ell)_{\alpha}$ (here onwards, these interactions will be referred to as EW sphalerons). Nonetheless, one can form an anomaly-free charge combination $U(1)_{(B-L)_{\alpha}}$ respected by $O_{\text{EWsp}}$. Although exponentially suppressed today, the EW sphalerons are in thermal equilibrium in the temperature range $T_{\text{EWsp}}^- < T < T_{\text{EWsp}}^+$ with $T_{\text{EWsp}}^- \sim 100$
Table I. The list of SM fields, their $U(1)$ charges $q_i^{x}$ and gauge degrees of freedom $g_i$ with fermion family index $\alpha$. Here $N_H - 1$ is number of extra pairs of Higgses $H'$ with the assumption that they maintain chemical equilibrium with the SM Higgs $H$.

GeV and $T_{EWsp}^+ \sim 10^{12}$ GeV \cite{13, 14}. For most of the epoch in the early Universe, quark intergeneration mixing violates baryon flavors and hence the exact symmetries are instead $U(1)_{\Delta_\alpha}$ with $\Delta_\alpha \equiv B/3 - L_\alpha$.\footnote{In the SM, in the absence of right-handed neutrinos, lepton flavors are conserved. In beyond the SM scenario, fast lepton flavor violation could occur see for e.g. \cite{15}.} Outside this temperature window, $B$ and $L_\alpha$ are effectively conserved. In the SM before EWPT, $U(1)_{\Delta_\alpha}$ can act as both creator and preserver while $U(1)_Y$ is a messenger. After EWPT, the role of $U(1)_{\Delta_\alpha}$ is taken over by $U(1)_B$ while the messenger becomes $U(1)_Q$. More often than not, when one considers scenario beyond the SM, there are new symmetries (see Sec. V) which can play the roles of $U(1)$ discussed before. The symmetries of the SM in the early Universe is summarized in Fig. 1.

A. Some specific cases

Let us define the vectors $q_i^T \equiv (q_i^{\Delta_\alpha}, q_i^Y)$ and $n^T \equiv (n_{\Delta_\alpha}, n_{\Delta Y})$. Consider first the temperature regime $T \sim 10^4$ GeV when all Yukawa interactions are in thermal equilibrium and all particles are relativistic. In this case, the $J$ matrix is easily determined from Eq. (6) to be (here we express in its inverse)

$$J^{-1} = \frac{1}{3 (198 + 39 N_H)}$$
where \( N_H - 1 \) is number of extra pairs of Higgses \( H' \) with the assumption that they maintain chemical equilibrium with the SM Higgs \( H \). Using the matrix above, \( n_{\Delta i} \) can be expressed in terms of conserved charge densities through Eq. (7). In particular, setting \( N_H = 1 \) and \( n_{\Delta Y} = 0 \), we obtain lepton asymmetries \( n_{\Delta \ell} \) and Higgs asymmetries \( n_{\Delta H} \) in terms of \( n_{\Delta \alpha} \) which are in agreement with Ref. [16]. We can further consider cases at higher temperature when \( e \) Yukawa interactions are out-of-equilibrium in which we gain a chiral \( U(1)_e \). Formally we can create another conserved charge \( n_{\Delta e} \) and determine \( J \) which is now a 5 \( \times \) 5 matrix. However if we were to take \( n_{\Delta e} = 0 \), in practice, we can just set \( \zeta_e = 0 \) from the beginning to exclude its contribution. Next consider the case when both \( u \) and \( d \) Yukawa interactions are out-of-equilibrium. Since both \( U(1)_u \) and \( U(1)_d \) have \( SU(3)_c \) mixed anomaly, no effective symmetry is gained. Nevertheless \( u \) and \( d \) are now indistinguishable under QCD sphalerons and we simply have to set \( q_u^Y = q_d^Y = \frac{1}{2} \left( \frac{2}{3} - \frac{1}{3} \right) = \frac{1}{6} \). It is straightforward to consider further cases and the results of Refs. [16, 17] are verified.

### B. Relation between \( B \) and \( B - L \)

An important quantity for high scale baryogenesis (occurs at \( T > T_{\text{EWsp}}^- \)) is the relation between the \( B \) and \( B - L \) charge densities during the time when EW sphalerons freeze out. For simplicity, we will assume that all particles are relativistic although particle decoupling effects [18, 19] can be straightforwardly taken into account by considering generic form of \( \zeta_i \) as in Eq. (4). Assuming \( T_{\text{EWsp}}^- > T_{\text{EWPT}} \), the conserved charges are \( n_{\Delta Y} \) and \( n_{\Delta(B - L)} \equiv \sum_{\alpha} n_{\Delta \alpha} \). By defining the vectors \( q_i^T \equiv (q_i^{B-L}, q_i^Y) \) and \( n^T \equiv (n_{\Delta(B - L)}, n_{\Delta Y}) \) and keeping the number of fermion family as \( N_f \) with \( N_H \) pairs of Higgses, we obtain the following

\[
J^{-1} = \frac{1}{N_f (22N_f + 13N_H)} \begin{pmatrix}
10N_f + 3N_H & -8N_f \\
-8N_f & 13N_f
\end{pmatrix}.
\]

Setting \( n_{\Delta Y} = 0 \), we have from Eq. (5)
\[ i = U_{\alpha,L} D_{\alpha,L} U_{\alpha} D_{\alpha} \nu_{\alpha,L} E_{\alpha,L} E_{\alpha} W^+ H^+ \]

\[
q_i^\Delta = \frac{1}{2} \frac{1}{5} \frac{1}{5} \frac{1}{5} \frac{1}{5} -1 -1 -1 0 0 \\
q_i^Q = \frac{2}{3} \frac{2}{3} \frac{2}{3} -\frac{2}{3} -\frac{2}{3} 0 -1 -1 1 1 \\
q_i^B = \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} 0 0 0 0 0 \\
q_i^L = 0 0 0 0 0 1 1 1 0 0 \\
g_i = 3 3 3 3 1 1 1 3 N_H - 1
\]

Table II. Similar to Table I but for field components after EWPT where we use subscript ‘L’ to denote the left-handed fields which participate in weak interaction.

\[ n_{\Delta B} = \frac{4 (2 N_f + N_H)}{22 N_f + 13 N_H} n_{\Delta (B-L)}. \] \hspace{1cm} (14)

On the other hand, assuming \( T_{EWsp}^\ast < T_{EWPT} \), we need to consider the components of \( SU(2)_L \) doublets and use \( Q \) in place of \( Y \) as in Table II. Doing so we obtain

\[ J^{-1} = \frac{1}{2 N_f \left[ 24 N_f + 13 (2 + N_H) \right]} \begin{pmatrix}
2 (6 + 8 N_f + 3 N_H) & -8 N_f \\
-8 N_f & 13 N_f
\end{pmatrix}. \] \hspace{1cm} (15)

Now setting \( n_{\Delta Q} = 0 \), we have from Eq. (8)

\[ n_{\Delta B} = \frac{4 (2 + 2 N_f + N_H)}{24 N_f + 13 (2 + N_H)} n_{\Delta (B-L)}. \] \hspace{1cm} (16)

The results above agree with Ref. [20] albeit obtained from simpler derivation based on symmetry principle.

V. THE MINIMAL SUPERSYMMETRIC STANDARD MODEL

Here we consider a well-motivated extension to the SM which is the minimal supersymmetric SM (MSSM). The MSSM superpotential is given by

\[ W = \mu_H H_u \epsilon H_d + (y_u)_{\alpha \beta} Q_{\alpha} \epsilon H_u U^c_{\beta} + (y_d)_{\alpha \beta} Q_{\alpha} \epsilon H_d D^c_{\beta} + (y_e)_{\alpha \beta} \ell_{\alpha} \epsilon H_d E^c_{\beta}, \] \hspace{1cm} (17)
where all the fields above stand for left-chiral superfields. One observes that the superpotential has an $R$ symmetry $U(1)_R$ for e.g. with $q^R (H_d) = q^R (\ell_\alpha) = q^R (U^c_\alpha) = -q^R (E^c_\alpha) = 2$ and the rest of the fields having zero charges.\footnote{Note that the $R$-symmetry is preserved also with $R$-parity violating terms as well as in supersymmetric type-I seesaw with right-handed neutrino chiral superfields $N^c_i$ having $R (N^c_i) = 0$.} The $R$ symmetry has mixed anomalies $A_{R33} = 3 - N_f$ and $A_{R22} = 2 - N_f$ where $N_f$ is the number of fermion family. With $N_f = 3$, there is only $A_{R22} = -1$ anomaly. Thus one can form an anomaly-free charge combination as follows

$$\overline{R} \equiv R + \frac{2}{3c_{BL}} (c_B B + c_L L),$$

with $c_{BL} \equiv c_B + c_L$ any number. Notice that $\overline{R}$ is exactly conserved by Eq. (17). Further setting $\mu_H = 0$, we gain an anomalous global $U(1)_{PQ}$ for e.g. with $-q^{PQ} (Q_\alpha) = q^{PQ} (\ell_\alpha) = q^{PQ} (H_u) = q^{PQ} (H_d) = 1$, $q^{PQ} (E^c_\alpha) = -2$ and the rest of the fields having zero charges. One can verify that $U(1)_{PQ}$ is anomalous with $A_{PQ33} = -N_f$ and $A_{PQ22} = -N_f + N_H$. With $N_f = 3$ and $N_H = 1$, the $A_{PQ22}$ anomaly-free charge combination is

$$\overline{P} \equiv \frac{3}{4} c_{BL} PQ + c_B B + c_L L.$$

In order to cancel the $A_{PQ33}$ anomaly, we need another mixed $SU(3)_c$ anomalous symmetry. For instance, when the $u$ Yukawa interactions are out-of-equilibrium, we gain an anomalous chiral symmetry $U(1)_{u^c}$ with $A_{u^c33} = q^{u^c}/2$. The anomaly-free charge combination is

$$\overline{\chi_{u^c}} \equiv \overline{P} + \frac{9}{2} c_{BL} u^c / q^{u^c}.$$

The $U(1)$ charges of the superfields are listed in Table III. The anomalous $U(1)_R$ and $U(1)_{PQ}$ discussed above were first studied in Ref. [21] and were shown to be effective at $T \gtrsim 10^7$ GeV when the interactions mediated by weak scale $\mu_H$, soft trilinear couplings and gaugino masses are out-of-equilibrium. While these symmetries impart only order of one effects in the standard supersymmetric leptogenesis [9], it significantly enhances the efficiency of soft leptogenesis [10].
Table III. The $U(1)$ charges of left-handed chiral superfields. All gauginos $\tilde{G}, \tilde{W}$ and $\tilde{B}$ have both $R$ and $\bar{R}$ charges equal 1. Since all fermions in chiral superfields have $R$ charges one less than that of bosons i.e. $R(\text{fermion}) = R(\text{boson}) - 1$, the differences between number density asymmetries of bosons and fermions are equal to that of gauginos.

Finally it should be remarked that $c_B$ and $c_L$ can be chosen at will depending on the baryogenesis model under consideration. For instance, considering a model which violates lepton number through $O_L = (\ell_\alpha \epsilon H_u)^2$, we can choose $c_B = -5c_L/3$ such that $\bar{R}$ and $\bar{P}$ are conserved by $O_L$. As another example, considering a model which violates baryon number through $O_B = U^c_\alpha D^c_\beta D^c_\delta$, a good choice is $c_B = 0$ and $c_L \neq 0$ such that $\bar{R}$ and $\bar{P}$ are conserved by $O_B$. Choosing $c_B = c_L$, the results obtained are in disagreement with Ref. [21] due to sign error of gaugino chemical potential in their Eq.(3.3).

VI. CONCLUSIONS

The use of symmetry principle in analyzing the early Universe system allows all the particle asymmetries to be expressed in terms of conserved charges corresponding to the symmetries. These charges form the appropriate basis to describe the system. Besides its simplicity i.e. without having to resort to details of how the particles interact, this method

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8 For instance we have $n_{\Delta B} = 6N_f (2n_{\Delta Q} + n_{\Delta \tilde{G}})$ instead of $n_{\Delta B} = 2 [6N_f n_{\Delta Q} - (4N_f - 9) n_{\Delta \tilde{G}}]$. Since the derivation here is solely based on symmetries of the system, this mistake will not occur.
serves as a powerful tool in accessing the viability of a baryogenesis scenario. In addition, the roles of $U(1)$ symmetries as creator/destroyer, preserver or messenger become apparent, rendering it easier to construct interesting models of baryogenesis.

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