The New Fuzzy Analytical Hierarchy Process with Interval Type-2 Trapezoidal Fuzzy Sets and Its Application

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ABSTRACT

The degree of type-1 fuzzy sets membership function cannot express the linguistic variable of a complex problem. The type-2 fuzzy sets as a problem solver such that more fuzziness for constructing membership functions can be handled. Recently, many multi-criteria decision making (MCDM) methods have been expanded using type-2 fuzzy sets. Analytical Hierarchy Process (AHP) is one of the well-known MCDM that can take into account multiple and conflicting criteria at the same time. Our goal is to develop an interval type-2 trapezoidal fuzzy AHP through the new proposed ranking i.e. the modified total integral value. Based on the illustrative examples for trapezoidal type-2 fuzzy sets, the new proposed ranking has a well-performance in ranking. Furthermore, we apply the new trapezoidal type-2 fuzzy AHP to a supplier selection problem. Based on the results of the application, the new fuzzy AHP has the same ranking results as the existing fuzzy AHP.

1. Introduction

In a decision support system on conventional multi-criteria decision making (MCDM) problems, the perceptions of experts called linguistic variables are represented by crisp numbers. The judgment of perception is strongly influenced by the subjectivity factor of an expert in decision making. However, several real problems, the judgment’s expert that is fuzziness and vagueness can be an impact on the accuracy of data. Evaluation on criteria is subjective and qualitative, it is very difficult for decision makers to represent it in numbers [1]. The conventional MCDM approach is considered less effective to overcome the fuzziness and vagueness value of linguistic variables [2]. Analytical Hierarchy Process (AHP) was introduced by Saaty [3] is well-known MCDM, having structured multi-criteria technique in organising and analysing complex decisions including conflicting criteria. The fuzzy AHP technique is rated as an advanced analysis method that is developed from conventional AHP is capable to overcome fuzziness and vagueness in the decision maker’s judgment [4].

Based on the references, linguistic variables of fuzzy AHP are represented by type-1 fuzzy sets had been discussed. Triangular Fuzzy AHP based logarithmic least squares method...
to calculate fuzzy eigenvectors [5]. Extended analysis of triangular fuzzy AHP [6,7]. Fuzzy Programming method based geometrical representation of the prioritisation process to enhanced consistency of pairwise comparison matrix in fuzzy AHP [8]. Direct fuzzification eigenvector of fuzzy AHP uses Lambda-Max method [9]. Fuzzy geometric row means to improve the consistency of pairwise comparison matrices [10]. Group fuzzy preference programming based on combines the group synthesis and prioritisation on fuzzy AHP [11]. Eigenvector method to calculate interval fuzzy weight from pairwise comparison matrices of trapezoidal and triangular fuzzy AHP [12]. A logarithmic fuzzy preference programming to compute the fuzzy weight of fuzzy AHP [13]. Consistency analysis and fuzzy priority on pairwise comparison matrices of triangular fuzzy AHP use fuzzy arithmetic operations based on transitivity equations where this equation reflects consistency by using geometric means and modal mean [14]. Fuzzy AHP based eigenvector to improve consistency of pairwise comparison matrix and expected values to decision making [15,16].

Normally, the degree of type-1 fuzzy sets (T1FS) membership function is evaluated by crisp numbers. However, for various complexity problems, the degree of membership function will be difficult to express the linguistic variables of the problem. Therefore, the concept of type-2 fuzzy sets (T2FS) is extended from T1FS was introduced by Zadeh [17] can be handled. T2FS is very compatible of the problems that is difficult to evaluate by the degree of T1FS membership function, such that T2FS can be expressed the different of linguistic the expert’s judgment [18]. The main difference between T1FS and T2FS lies in the dimension of membership function value, where T1FS is two dimensions, while T2FS is three dimensions [19]. The new three dimensions provide an additional degree of membership function which allows for direct expression of fuzziness [20].

Furthermore, the interval type-2 fuzzy sets (IT2FS) is a special case of T2FS and has been implemented in fuzzy MCDM [19,21–27], especially MCDM problems. The modified of Buckley’s fuzzy AHP with pairwise comparison matrix represented by IT2FS [28]. Best Non-fuzzy Performance (BNP) method to rank IT2FS of pairwise comparison matrices and was applied on the modified of Buckley’s fuzzy AHP [20]. The modified of Van Laarhoven and Pedrycz’s fuzzy AHP with interval type-2 trapezoidal fuzzy number (IT2TrFN) and was applied in new product development project screening [29]. The application of Kahraman et al.’s fuzzy AHP with IT2TrFN to analysed business project priorities [30], for event log-based fraud ranking [31] and for green supplier selection with case studies of household appliance manufacturers [32]. Buckley’s fuzzy AHP is based on Z-numbers to evaluate social sustainable development factors [33]. In addition, many researchers have combined IT2FS fuzzy AHP with other MCDM methods [34], such as the combination of Kahraman et al.’s fuzzy AHP and fuzzy TOPSIS for decision making in maritime transport engineering [35], to evaluate the quality of life in 28 EU countries and other 6 countries [36] and to determine the main priority in distribution [37]. The combination of the modified of Buckley’s fuzzy AHP based balanced score card and fuzzy TOPSIS to strategy selection problem [38]. Buckley’s fuzzy AHP is based on Abbasimehr et al.’s ranking and fuzzy VIKOR to evaluate online review [39].

The main research contribution is the development of type-1 trapezoidal fuzzy AHP was proposed by Prascevi and Prascevi [16] to be interval type-2 fuzzy AHP with linguistic expert’s judgment represented by IT2TrFN [20]. Furthermore, the new proposed ranking based left, right and total integral value is presented on the ranking process. Numerical examples are provided to illustrate the capability of the new proposed ranking and a case study is provided to demonstrate the new interval type-2 fuzzy AHP.
The paper is ordered as follows: Section 2 is interval type-2 trapezoidal fuzzy sets, and the ranking methods for interval type-2 trapezoidal fuzzy sets are discussed in Section 3. Then, in Section 4, the new interval type-2 trapezoidal fuzzy AHP is presented. Section 5 describes in practice of the new interval type-2 trapezoidal fuzzy AHP. The conclusion is provided in Section 6.

2. Interval Type-2 Trapezoidal Fuzzy Sets

This section provides an overview of the concept of interval type-2 fuzzy sets (IT2FS) as follows [40]:

Let \( \tilde{K} \) is a type 2 fuzzy set (T2FS) with membership function denoted by \( \mu_{\tilde{K}}(x, a) \) where \( x \in X \) and \( a \in J_x \subseteq [0, 1] \). Therefore, \( \tilde{K} \) can be defined as follows [17]:

\[
\tilde{K} = \{(x, a) \mid \forall x \in X, \forall a \in J_x \subseteq [0, 1], 0 \leq \mu_{\tilde{K}} \leq 1\}
\]  

(1)

where \( J_x \) is the primary level membership function on \( x \) and \( \mu_{\tilde{K}}(x, a) \) for \( x \in X, a \in J_x \) is a secondary or second level membership function. When the membership function of \( \tilde{K} \) is continue, so \( \tilde{K} \) can be represented as follows:

\[
\tilde{K} = \int_{x \in X} \left( \int_{a \in J_x} \frac{\mu_{\tilde{K}}(x, a)}{a} \right) dx
\]  

(2)

where \( \int \int \) denoted by union \( \forall x, a \) in feasible area and the membership function is discrete, it notation can be changed with \( \sum \).

Based on Equation (1), \( \tilde{K} \) is T2FS with support R (real number). Both of them are normal and convex. So, support \( \tilde{K} \) or supp(\( \tilde{K} \)) can be defined as follows [41]:

\[
\text{supp}(\tilde{K}) = \{(x, a) \mid \forall x \in X, \forall a \in J_x \subseteq [0, 1]\}
\]  

(3)

From Equation (3), \( \tilde{K} \) for all \( \mu_{\tilde{K}}(x, a) = 1 \) can be defined as IT2FS, such that Equation (2) can be changed to membership function of IT2FS \( \tilde{K} \) which is represented as follows:

\[
\tilde{K} = \int_{x \in X} \int_{a \in J_x} \frac{1}{(x, a)} dx = \int_{x \in X} \left( \int_{a \in J_x} \frac{1}{a} \right) dx
\]  

(4)

For simplicity, given \( \tilde{K} \) is IT2FS which is limited by two T1FS are \( \tilde{K}_L \) is LMF (lower membership function) and \( \tilde{K}_U \) is UMF (upper membership function). The membership function graph of interval type-2 trapezoidal fuzzy set (IT2TrS) can be seen in Figure 1 with FOU (footprint of uncertainty) is uncertainty in primary membership function of IT2TrS \( \tilde{K} \) consists of a restricted area. FOU can be denoted by FOU = \( \bigcup_{x \in X} J_x \).

The generalised interval type-2 trapezoidal fuzzy number (GIT2TrFN) \( \tilde{K} \) define \([k^L_1, k^U_1, k^L_2, k^U_2]\). The value LMF is equal to \( \omega^L_1, \omega^L_2 \in [0, 1] \) in \([k^L_1, k^L_2]\). Whereas, the value of UMF is equal to \( \omega^U_1, \omega^U_2 \in [0, 1] \) in \([k^U_1, k^U_2]\). Therefore, GIT2TrFN \( \tilde{K} \) is denoted by \( \tilde{K} =

\[
\tilde{K} = [k^L_1, k^U_1, k^L_2, k^U_2]
\]
Figure 1. Interval type-2 trapezoidal fuzzy set \( \tilde{K} = (\tilde{K}^L, \tilde{K}^U) \) \[40].

\((\tilde{K}^L, \tilde{K}^U) = ((k_1^L, k_2^L, k_3^L, k_4^L; \omega_1^L, \omega_1^U), (k_1^U, k_2^U, k_3^U, k_4^U; \omega_1^U, \omega_1^U))\) and its membership function as follows

\[
\mu_{\tilde{K}^L}(x) = \begin{cases} 
\mu_{\tilde{K}^L_1}(x) = \frac{\omega_1^L}{k_1^L} & k_1^L \leq x < k_2^L \\
\mu_{\tilde{K}^L_2}(x) = (\omega_2^L - \omega_1^L) \frac{(x - k_2^L)}{(k_2^L - k_1^L)} + \omega_1^L & k_2^L \leq x \leq k_3^L \\
\mu_{\tilde{K}^L_3}(x) = \frac{\omega_2^L}{k_4^L - k_3^L} & k_3^L < x \leq k_4^L \\
\mu_{\tilde{K}^L_4}(x) = 0 & x \leq k_1^L, x \geq k_4^L 
\end{cases}
\]

\[
\mu_{\tilde{K}^U}(x) = \begin{cases} 
\mu_{\tilde{K}^U_1}(x) = \frac{\omega_1^U}{k_1^U} & k_1^U \leq x < k_2^U \\
\mu_{\tilde{K}^U_2}(x) = (\omega_2^U - \omega_1^U) \frac{(x - k_2^U)}{(k_2^U - k_1^U)} + \omega_1^U & k_2^U \leq x \leq k_3^U \\
\mu_{\tilde{K}^U_3}(x) = \frac{\omega_2^U}{k_4^U - k_3^U} & k_3^U < x \leq k_4^U \\
\mu_{\tilde{K}^U_4}(x) = 0 & x \leq k_1^U, x \geq k_4^U 
\end{cases}
\]

For special case, if \(\omega_1^L = \omega_1^U = 1\) and \(\omega_2^L = \omega_2^U = \omega_3^U = 1\) then GIT2TrFN is called generalised interval type 2 trapezoidal flat fuzzy number and perfect IT2TrFN \[42\].

Next, the arithmetic operation of GIT2TrFN is given as follows \[25,42,43\]:
Definition 2.1: Given \( \tilde{K} = (\tilde{K}^L, \tilde{K}^U) = ((k_1^L, k_2^L, k_3^L, k_4^L, \sigma_{1K}^{L}, \sigma_{2K}^{L}), (k_1^U, k_2^U, k_3^U, k_4^U, \sigma_{1K}^{U}, \sigma_{2K}^{U})) \)
and \( \tilde{M} = (\tilde{M}^L, \tilde{M}^U) = ((m_1^L, m_2^L, m_3^L, m_4^L, \sigma_{1M}^{L}, \sigma_{2M}^{L}), (m_1^U, m_2^U, m_3^U, m_4^U, \sigma_{1M}^{U}, \sigma_{2M}^{U})) \) are
GIT2TrFN and \( c \) is crisp number. Some arithmetic operations on GIT2TrFN explained as follows:

**Addition:**

\[
\tilde{K} \oplus \tilde{M} = (\tilde{K}^L \oplus \tilde{M}^L, \tilde{K}^U \oplus \tilde{M}^U) = \\
[(k_1^L + m_1^L, k_2^L + m_2^L, k_3^L + m_3^L, k_4^L + m_4^L; \min\{\sigma_{1K}^{L}, \sigma_{1M}^{L}\}, \min\{\sigma_{2K}^{L}, \sigma_{2M}^{L}\}), \\
(k_1^U + m_1^U, k_2^U + m_2^U, k_3^U + m_3^U, k_4^U + m_4^U; \min\{\sigma_{1K}^{U}, \sigma_{1M}^{U}\}, \min\{\sigma_{2K}^{U}, \sigma_{2M}^{U}\})]
\]

**Subtraction:**

\[
\tilde{K} \ominus \tilde{M} = (\tilde{K}^L \ominus \tilde{M}^L, \tilde{K}^U \ominus \tilde{M}^U) = \\
[(k_1^L - m_1^L, k_2^L - m_2^L, k_3^L - m_3^L, k_4^L - m_4^L; \min\{\sigma_{1K}^{L}, \sigma_{1M}^{L}\}, \min\{\sigma_{2K}^{L}, \sigma_{2M}^{L}\}), \\
(k_1^U - m_1^U, k_2^U - m_2^U, k_3^U - m_3^U, k_4^U - m_4^U; \min\{\sigma_{1K}^{U}, \sigma_{1M}^{U}\}, \min\{\sigma_{2K}^{U}, \sigma_{2M}^{U}\})]
\]

**Multiplication:**

\[
\tilde{K} \otimes \tilde{M} = [(o_1^L, o_2^L, o_3^L, o_4^L), (o_1^U, o_2^U, o_3^U, o_4^U)]
\]

where

\[
\begin{align*}
o_1^L &= \min\{k_1^L \times m_1^L, k_1^L \times m_4^L, k_4^L \times m_1^L, k_4^L \times m_4^L\} \\
o_2^L &= \min\{k_2^L \times m_2^L, k_2^L \times m_3^L, k_3^L \times m_3^L, k_3^L \times m_2^L\} \\
o_3^L &= \max\{k_2^L \times m_2^L, k_2^L \times m_3^L, k_3^L \times m_2^L, k_3^L \times m_3^L\} \\
o_4^L &= \max\{k_4^L \times m_4^L, k_4^L \times m_4^L, k_3^L \times m_4^L, k_3^L \times m_4^L\} \\
\sigma_{1O}^L &= \min\{\sigma_{1K}^{L}, \sigma_{1M}^{L}\}, \sigma_{2O}^L = \min\{\sigma_{2K}^{L}, \sigma_{2M}^{L}\} \\
o_1^U &= \min\{k_1^U \times m_1^U, k_1^U \times m_4^U, k_4^U \times m_1^U, k_4^U \times m_4^U\} \\
o_2^U &= \min\{k_2^U \times m_2^U, k_2^U \times m_3^U, k_3^U \times m_3^U, k_3^U \times m_2^U\} \\
o_3^U &= \max\{k_2^U \times m_2^U, k_2^U \times m_3^U, k_3^U \times m_2^U, k_3^U \times m_3^U\} \\
o_4^U &= \max\{k_4^U \times m_4^U, k_4^U \times m_4^U, k_3^U \times m_4^U, k_3^U \times m_4^U\} \\
\sigma_{1O}^U &= \min\{\sigma_{1K}^{U}, \sigma_{1M}^{U}\}, \sigma_{2O}^U = \min\{\sigma_{2K}^{U}, \sigma_{2M}^{U}\}
\end{align*}
\]

Unique case, if \( \tilde{K}^L, \tilde{K}^U, \tilde{M}^L \) and \( \tilde{M}^U \) are flat generalised interval type-2 trapezoidal fuzzy number, then

\[
\tilde{K} \otimes \tilde{M} = (\tilde{K}^L \otimes \tilde{M}^L, \tilde{K}^U \otimes \tilde{M}^U) = \\
[(k_1^L \times m_1^L, k_2^L \times m_2^L, k_3^L \times m_3^L, k_4^L \times m_4^L; \min\{\sigma_{1K}^{L}, \sigma_{1M}^{L}\}, \min\{\sigma_{2K}^{L}, \sigma_{2M}^{L}\}), \\
(k_1^U \times m_1^U, k_2^U \times m_2^U, k_3^U \times m_3^U, k_4^U \times m_4^U; \min\{\sigma_{1K}^{U}, \sigma_{1M}^{U}\}, \min\{\sigma_{2K}^{U}, \sigma_{2M}^{U}\})]
\]

**Scalar multiplication:**
if $c \geq 0$ then
\[
c\tilde{K} = (cK^L, cK^U) = [(ck^L_1, ck^L_2, ck^L_3, ck^L_4, \omega^L_{1K}, \omega^L_{2K}), (ck^U_1, ck^U_2, ck^U_3, ck^U_4, \omega^U_{1K}, \omega^U_{2K})]
\]
if $c \leq 0$ then
\[
c\tilde{K} = (cK^L, cK^U) = [(ck^L_4, ck^L_3, ck^L_2, ck^L_1, \omega^L_{1K}, \omega^L_{2K}), (ck^U_4, ck^U_3, ck^U_2, ck^U_1, \omega^U_{1K}, \omega^U_{2K})]
\]

Example 2.1: Let three perfect GI\(\tilde{T}2\)TrFN as follows
\[
\tilde{A} = [(0.3, 0.6, 1, 1.1; 0.4, 0.3), (0.1, 0.2, 0.8, 1.3; 0.6, 0.8)]
\]
\[
\tilde{B} = [(0.2, 0.25, 0.54, 1.2; 0.8, 0.96), (0.07, 0.16, 0.57, 2; 0.9, 0.97)]
\]
Based on Definition 2.1 we get that,

(1) \(\tilde{A} \oplus \tilde{B} = ((0.5, 0.85, 1.54, 2.3; 0.4, 0.3), (0.17, 0.36, 1.37, 3.3; 0.6, 0.8))\)
(2) \(\tilde{A} \ominus \tilde{B} = ((-0.9, 0.06, 0.75, 0.85; 0.4, 0.3), (-1.9, -0.37, 0.64, 1.23; 0.6, 0.8))\)
(3) \(\tilde{A} \otimes \tilde{B} = ((0.06, 0.15, 0.54, 1.32; 0.4, 0.3), (0.007, 0.032, 0.456, 2.6; 0.6, 0.8))\)
\[
k = 0.5 > 0
\]
(4) \(c\tilde{A} = ((-0.55, -0.5, -0.3, -0.15; 0.4, 0.3), (-0.65, -0.4, -0.1, -0.05; 0.6, 0.8))\)
\[
k = -0.5 < 0
\]

3. Ranking Methods for Interval Type-2 Trapezoidal Fuzzy Sets

The ranking process of interval type-2 trapezoidal fuzzy sets is divided into two steps. First step, interval type-2 trapezoidal fuzzy sets as determined as type-1 trapezoidal fuzzy sets by using reduction process. Second step, the ranking methods of type-1 trapezoidal fuzzy sets use to find equality of interval type-2 trapezoidal fuzzy sets [18]. In the following, we present several ranking methods that are already in the references as well as the new proposed ranking.

3.1. Centroid of a Type-2 Fuzzy Set

Centroid \(c_{R\tilde{K}}(x)\) of T2FS is union from all of centroid \(n_{\tilde{K}}\) embedded T1FS \(K_e[CR\tilde{K}(K_e)]\) [18].
\[
c_{R\tilde{K}} = \bigcup_{\forall K_e} c_{R\tilde{K}}(K_e) = \{c_{R\tilde{K}}(\tilde{K}), \ldots, c_{R\tilde{K}}(\tilde{K})\} = [c_{R\tilde{K}}(\tilde{K}), c_{R\tilde{K}}(\tilde{K})]
\]

where
\[
c_{R\tilde{K}}(\tilde{K}) = \min(\text{CR}(K_e)) = \min_{\forall \lambda_e \in \{\mu_{\tilde{K}}(x_i), \varphi_{\tilde{K}}(x_i)\}} \sum_{i=1}^{N} x_i \lambda_i \sum_{i=1}^{N} \lambda_i
\]
\[ CR_\text{f}(\tilde{K}) = \max_{\forall K_e}(CR(K_e)) = \max_{\lambda \in [\mu_{\tilde{K}}(x_i), \overline{\mu}_{\tilde{K}}(x_i)]} \frac{\sum_{i=1}^{N} x_i\lambda_i}{\sum_{i=1}^{N} \lambda_i} \quad (7) \]

Karnik and Mendel [18] presented the iterative algorithm to compute Equations (6) and (7). The iterative algorithm can be shown in Algorithm 1.

\begin{algorithm}
\textbf{Algorithm 1:} The centroid reduction
\begin{enumerate}
\item Set \( \lambda_i = \frac{(\mu_{\tilde{K}}(x_i) + \overline{\mu}_{\tilde{K}}(x_i))}{2} \) for \( i = 1, \ldots, N \) and calculate
\[ CR_\text{f}'(\tilde{K}) = CR(\lambda_1, \ldots, \lambda_N) = \frac{\sum_{i=1}^{N} x_i\lambda_i}{\sum_{i=1}^{N} \lambda_i} \quad \text{by using Equation (7)}. \]
\[ CR_\text{r}'(\tilde{K}) = CR(\lambda_1, \ldots, \lambda_N) = \frac{\sum_{i=1}^{N} x_i\lambda_i}{\sum_{i=1}^{N} \lambda_i} \quad \text{by using Equation (8)}. \]
\item Find \( c(1 \leq c \leq N - 1) \) such that \( x_c \leq CR_\text{f}'(\tilde{K}) \leq x_{c+1} \) and \( x_c \leq CR_\text{r}'(\tilde{K}) \leq x_{c+1} \)
\item Set \( \lambda = \mu_{\tilde{K}}(x_i) \) for \( i \leq c \) and \( \lambda = \overline{\mu}_{\tilde{K}}(x_i) \) for \( i > c \) and calculate \( CR_\text{r}(c) \) by using Equation (9) and \( \lambda = \mu_{\tilde{K}}(x_i) \) for \( i \leq c \) and \( \lambda = \overline{\mu}_{\tilde{K}}(x_i) \) for \( i > c \) and calculate \( CR_\text{f}(c) \) by using Equation (9).
\[ CR_\text{f}(\tilde{K}) = \frac{\sum_{i=1}^{c} x_i\mu_{\tilde{K}}(x_i) + \sum_{i=c+1}^{N} x_i\overline{\mu}_{\tilde{K}}(x_i)}{\sum_{i=1}^{c} \mu_{\tilde{K}}(x_i) + \sum_{i=c+1}^{N} \overline{\mu}_{\tilde{K}}(x_i)} \quad (8) \]
\[ CR_\text{r}(\tilde{K}) = \frac{\sum_{i=1}^{c} x_i\mu_{\tilde{K}}(x_i) + \sum_{i=c+1}^{N} x_i\overline{\mu}_{\tilde{K}}(x_i)}{\sum_{i=1}^{c} \mu_{\tilde{K}}(x_i) + \sum_{i=c+1}^{N} \overline{\mu}_{\tilde{K}}(x_i)} \quad (9) \]
\item Check \( CR_\text{f}(c) = CR_\text{f}'(\tilde{K}) \) and \( CR_\text{r}(c) = CR_\text{r}'(\tilde{K}) \). If yes, stop. \( CR_\text{r}(c) \) is minimum value of \( CR_\text{f}(\tilde{K}) \) and \( CR_\text{f}(c) \) is maximum value of \( CR_\text{r}(\tilde{K}) \). If no, go to step 5.
\item Set \( CR_\text{f}'(\tilde{K}) = CR_\text{f}(c) \) and \( CR_\text{r}'(\tilde{K}) = CR_\text{r}(c) \) and go to step 2.
\end{enumerate}
\end{algorithm}

3.2. Indices of a Type-2 Fuzzy Set

Niewiadomski et al. [44] presented proposed optimistic, moderate, pessimistic and weighted average indices which obtain different points of view for the type reduction of interval type-2 fuzzy sets. If \( \tilde{K} \) is an interval-valued fuzzy set in the universe \( X \). The indices reduction of optimistic from \( \tilde{K} \) denoted by \( TR_{\text{opt}}(\tilde{K}) \), moderate from \( \tilde{K} \) denoted by \( TR_{\text{mode}}(\tilde{K}) \) and pessimistic from \( \tilde{K} \) denoted by \( TR_{\text{pess}}(\tilde{K}) \) and weighted average from \( \tilde{K} \) denoted by \( TR_{\text{wae}}(\tilde{K}) \).

\[ TR_{\text{opt}}(\tilde{K}) = \mu_{\tilde{K}}(x_i), \quad x \in X \quad (10) \]
\[ TR_{\text{pess}}(\tilde{K}) = \overline{\mu}_{\tilde{K}}(x_i), \quad x \in X \quad (11) \]
\[ TR_{\text{mode}}(\tilde{K}) = \frac{(\mu_{\tilde{K}}(x_i) + \overline{\mu}_{\tilde{K}}(x_i))}{2}, \quad x \in X \quad (12) \]
\[ TR_{\text{wae}}(\tilde{K}) = \omega_1 \mu_{\tilde{K}}(x_i) + \omega_2 \overline{\mu}_{\tilde{K}}(x_i), \quad x \in X \quad (13) \]

where \( \omega_1 + \omega_2 = 1, \omega_1, \omega_2 \in [0, 1] \).
3.3. Lee and Chen’s Ranking Value of Interval Type-2 Fuzzy Sets

Lee and Chen [45] proposed the concept of ranking values GIT2TrFS. Given \( \bar{K} = (\bar{K}_L, \bar{K}_U) = ((k_1^L, k_2^L, k_3^L, k_4^L; \omega_{1K}^L, \omega_{2K}^L), (k_1^U, k_2^U, k_3^U, k_4^U; \omega_{1K}^U, \omega_{2K}^U)) \) is GIT2TrFN, the ranking values that is denoted by \( \text{Rank}(\bar{K}) \) of \( \bar{K} \) is defined as follows:

\[
\text{Rank}(\bar{K}) = Q_1(\bar{K}_U) + Q_1(\bar{K}_L) + Q_2(\bar{K}_U) + Q_2(\bar{K}_L) + Q_3(\bar{K}_U) + Q_3(\bar{K}_L)
\]

\[
- \frac{1}{4} (P_1(\bar{K}_U) + P_1(\bar{K}_L) + P_2(\bar{K}_U) + P_2(\bar{K}_L) + P_3(\bar{K}_U) + P_3(\bar{K}_L)
\]

\[
+ P_4(\bar{K}_U) + P_4(\bar{K}_L) + \omega_1(\bar{K}_U) + \omega_1(\bar{K}_L) + \omega_2(\bar{K}_U) + \omega_2(\bar{K}_L)
\]

(14)

where \( Q_p(\bar{K}) \) is the average of elements \( k_i^d \) and \( k_i^d+1 \) with \( Q_p(\bar{K}) = \frac{(k_i^d + k_i^d+1)}{2} \), \( 1 \leq p \leq 3 \), \( P_q(\bar{K}) \) is standard deviation of the elements \( k_1^d, k_2^d, k_3^d, k_4^d, P_q(\bar{K}) = \sqrt{\frac{1}{2} \sum_{c=1}^{q+1} (k_i^d - \frac{1}{2} \sum_{c=1}^{q+1} k_i^d)^2} \), \( 1 \leq q \leq 3 \), \( P_4(\bar{K}) \) is is membership degree of the element \( k_{f+1}^d \) in the trapezoidal membership function \( \bar{K} \), \( 1 \leq f \leq 2 \) and \( j \in \{U, L\} \).

3.4. Chen and Lee’s Likelihood Ranking

Chen and Lee [46] presented the following ranking method for type-2 fuzzy sets. They first calculate the likelihood of \( K_s^U \geq K_f^U \) by Equation (14)

\[
p(K_s^U \geq K_f^U) = \max \left( 1 - \max \left( \frac{\sum_{c=1}^{4} \max((k_i^c - k_i^{1c}), 0) + \sum_{c=1}^{2} \max((\omega_c(K_s^U) - \omega_c(K_f^U)), 0)}{\sum_{c=1}^{4} |k_i^U - k_i^{1U}| + \sum_{c=1}^{2} |\omega_c(K_s^U) - \omega_c(K_f^U)|} \right), 0 \right)
\]

(15)

Next, the Chen and Lee’s likelihood ranking values for upper and lower membership functions are given by Equations (15) and (16), respectively:

\[
\text{Rank}(\bar{K}_U) = \frac{1}{n(n-1)} \left( \sum_{c=1}^{n} p(K_s^U \geq K_f^U) + \frac{n}{2} - 1 \right)
\]

(16)

and

\[
\text{Rank}(\bar{K}_L) = \frac{1}{n(n-1)} \left( \sum_{c=1}^{n} p(K_s^L \geq K_f^L) + \frac{n}{2} - 1 \right)
\]

(17)

and lastly, the Chen and Lee’s likelihood ranking values of \( \bar{K} \) be computed by Equation (18)

\[
\text{Rank}(\bar{K}) = \frac{1}{2} (\text{Rank}(\bar{K}_U) + \text{Rank}(\bar{K}_L))
\]

(18)

3.5. Kahraman et al.’s Ranking

Kahraman et al. [20] was adjusted the centre of area method Best Non-fuzzy Performance value for ranking IT2TrFS. They presented the ranked interval type-2 trapezoidal fuzzy set
(RTrT) approach as follows:

\[
RTrT = \frac{1}{2} \left[ \left( \frac{(k_4^U - k_1^U)}{4} + \left(\omega_1^U k_2^U - k_4^U \right) + \left(\omega_2^U k_3^U - k_4^U \right) \right) + k_4^U \right] + \left( \frac{(k_4^L - k_1^L)}{4} + \left(\omega_1^L k_2^L - k_2^L \right) + \left(\omega_2^L k_3^L - k_2^L \right) \right) + k_4^L
\]  \tag{19}

3.6. The New Proposed Ranking

The modified total integral value is proposed to rank GIT2TrFN. The new proposed ranking uses the novel left and right of integral value as follows.

Given \( \tilde{K} = (\tilde{K}^L, \tilde{K}^U) = ((k_1^L, k_2^L, k_3^L; \omega_1^L, \omega_2^L), (k_1^U, k_2^U, k_3^U; \omega_1^U, \omega_2^U)) \) is GIT2TrFN with membership function on Equations (5) and (6). The novel left and right of integral value ranking consist of two membership functions based on LMF and UMF in accordance with the graph of GIT2TrFN membership function in Figure 2. The novel left and right of integral value based on LMF as follows.

\[
s_{Lr}(\tilde{K}^L) = \omega_2^L (k_2^L - x_min) - \int_{k_1^L}^{k_2^L} \mu_{k_1}^L (x) = \omega_2^L \frac{(x - k_1^L)}{(k_2^L - k_1^L)} dx
\]

\[
= \omega_2^L (k_2^L - x_min) - \frac{\omega_2^L}{2} (k_2^L - k_1^L)
\] \tag{20}

\[
s_{Rg}(\tilde{K}^L) = \omega_1^L (k_3^L - x_min) + \int_{k_2^L}^{k_3^L} \mu_{k_2}^L (x) dx + \int_{k_3^L}^{k_4^L} \mu_{k_3}^L (x) dx
\]

\[
= \omega_1^L (k_3^L - x_min) + (\omega_1^L - \omega_1^L) \int_{k_3^L}^{k_4^L} \frac{(x - k_2^L)}{(k_4^L - k_3^L)} dx
\]

\[
+ \omega_1^L \int_{k_2^L}^{k_3^L} 1 dx + \omega_1^L \int_{k_3^L}^{k_4^L} \frac{(k_4^L - x)}{(k_4^L - k_3^L)} dx
\]

\[
= \frac{1}{2} [\omega_1^L (3k_3^L - k_2^L - 2x_min) + \omega_2^L (k_2^L + k_4^L)]
\] \tag{21}

where \( x_min = \inf P, \inf P = \bigcup_{i=1}^{4} P_i, P_i = \{x/\mu_{k_i}^L > 0\}, \omega_1^L = \sup_{x} \mu_{k_1}^L \) and \( s_{Lr}^L, s_{Rg}^L \geq 0 \)

The novel total integral value of LMF with index of optimistic or \( \alpha \in [0, 1] \) based Equations (19) and (20) as follows.

\[
s_{Lr}^\alpha(\tilde{K}^L) = \alpha s_{Rg}^L(\tilde{K}^L) + (1 - \alpha)s_{Lr}^L(\tilde{K}^L)
\]

\[
= \alpha \left[ \frac{\omega_1^L}{2} (3k_3^L - k_1^L) + \frac{\omega_2^L}{2} (k_4^L - k_2^L) \right] + \frac{\omega_1^L}{2} (k_2^L - k_1^L) - x_min
\] \tag{22}

Meanwhile, the novel left and right of integral value based on UMF as follows.

\[
s_{Lr}(\tilde{K}^U) = \omega_1^U (k_2^U - x_min) - \int_{k_1^U}^{k_2^U} \mu_{k_1}^U (x) = \omega_1^U \frac{(x - k_1^U)}{(k_2^U - k_1^U)} dx
\]
Figure 2. The generalised interval type-2 trapezoidal fuzzy number [42].

\[ S_{\alpha T}(\tilde{K}^U) = \alpha S_{\text{rg}}(\tilde{K}^U) + (1 - \alpha) S_{\text{lf}}(\tilde{K}^U) \]

where \( x_{\text{min}} = \inf P, \inf P = \bigcup_{i=1}^{q} P_i, P_i = \{x/\mu_{K_i}^U > 0\}, \sigma_{\text{lf}}^U = \sup_x \mu_{K_i}^U \) and \( S_{\text{lf}}, S_{\text{rg}} \geq 0 \)
Furthermore, given $\tilde{K} = (\tilde{K}^L, \tilde{K}^U) = ((k_1^L, k_2^L, k_3^L, k_4^L, \nu_1^L, \nu_2^L), (k_1^U, k_2^U, k_3^U, k_4^U, \nu_1^U, \nu_2^U))$ and $\tilde{M} = (\tilde{M}^L, \tilde{M}^U) = ((m_1^L, m_2^L, m_3^L, m_4^L, \nu_1^L, \nu_2^L), (m_1^U, m_2^U, m_3^U, m_4^U, \nu_1^U, \nu_2^U))$. Then, Algorithm 2 can be used to compare $\tilde{K}$ and $\tilde{M}$.

**Algorithm 2**: The modified total integral value

| Step | Description |
|------|-------------|
| 1    | Determine $s_T^\alpha (\tilde{K}^L)$ via Equation (22) and $s_T^\alpha (\tilde{K}^U)$ via Equation (25) |
| 2    | Determine the average of novel total integral value $\bar{s}_T (\tilde{K})$ via Equation (26) |
| 3    | Finding the ranking of $\tilde{K}$ and $\tilde{M}$ using the following properties:  
   (a) if $\tilde{K} > \tilde{M}$ then $\bar{s}_T (\tilde{K}) > \bar{s}_T (\tilde{M})$  
   (b) if $\tilde{K} < \tilde{M}$ then $\bar{s}_T (\tilde{K}) < \bar{s}_T (\tilde{M})$  
   (c) if $\tilde{K} = \tilde{M}$ then $\bar{s}_T (\tilde{K}) = \bar{s}_T (\tilde{M})$ |

In the following four GIT2TrFN cases with different height are given in Figure 3 and by using the modified integral value in Algorithm 2 with moderate value of optimistic index or $\alpha = 0.5$ can be found $\bar{s}_T^{3a} (\tilde{K}) = 25, 9125$, $\bar{s}_T^{3b} (\tilde{K}) = 23, 2875$, $\bar{s}_T^{3c} (\tilde{K}) = 23, 5375$ and $\bar{s}_T^{3d} (\tilde{K}) = 25, 1625$. So, the new proposed ranking gives ranking of $3a > 3d > 3c > 3b$ of GIT2TrFN, whereas the existing rankings give ranking $3a = 3b = 3c = 3d$ [18], $3a = 3b = 3c = 3d$

![Figure 3](image-url)  
**Figure 3.** Illustrative numerical examples of GIT2TrFN [20].
Table 1. The comparison result of ranking methods.

| Ranking methods                  | Figure 3(a) | Figure 3(b) | Figure 3(c) | Figure 3(d) |
|----------------------------------|-------------|-------------|-------------|-------------|
| Karnik and Mendel [18]           | 24.2154(1)  | 24.2154(1)  | 24.2154(1)  | 24.2154(1)  |
| Niewiadomski et al. [44]         | 1.425(1)    | 1.425(1)    | 1.425(1)    | 1.425(1)    |
| Lee and Chen’s ranking [45]      | 156.7(2)    | 161.5(1)    | 154.6(3)    | 152.3(4)    |
| Chen and Lee’s likelihood ranking [46] | 0.25(2)    | 0.272(1)    | 0.253(3)    | 0.218(4)    |
| Kahraman et al. [20]             | 25.24(2/3)  | 26.21(1)    | 25.34(2/3)  | 23.46(4)    |
| New proposed ranking             | 25.9125(1)  | 23.2875(4)  | 23.5375(3)  | 25.1625(2)  |

From the results of these ranking, the new proposed ranking succeeded to rank numerical examples of GIT2TrFN as with Lee and Chen’s ranking [45] and Chen and Lee’s likelihood ranking[46], while proposed ranking who is proposed by Karnik and Mendel [18], Niewiadomski et al. [44], and Kahraman et al. [20] failed to rank numerical examples of GIT2TrFN. In detail, the comparison ranking result of the methods can be seen in Table 1.

4. The New Interval Type-2 Trapezoidal Fuzzy AHP

In this section, in order to the concept of interval type-2 trapezoidal fuzzy AHP can be easily understood, we will first give a brief overview of type-1 trapezoidal fuzzy AHP and its algorithms. Then, the new interval type-2 trapezoidal fuzzy AHP is presented.

4.1. Type-1 Trapezoidal Fuzzy AHP

Sometimes, subjective and qualitative decision criteria are uncertain, so that the decision makers find it difficult to express the strengths of preference and demonstrating the exact pairwise comparison. So, the crisp number is not suitable for representing the uncertainty of pairwise comparison. Therefore, the judgment of the decision maker or experts team is uncertain and imprecise. It will be much better to represent the value of pairwise comparison matrices as fuzzy numbers instead of crisp numbers. Due to the existing complexity and uncertainty of problems in the real life. Therefore, it is impossible to make decisions that can provide the exact judgment according to the problem [43].

To overcome these shortcomings due to the crisp number, the fuzzy AHP was developed to solve MCDM. Prascevic and Prascevic [16] developed a type-1 trapezoidal fuzzy AHP

Table 2. Linguistic scale type-1 trapezoidal fuzzy number for weight matrix [6].

| T1FN  | Reciprocal T1FN | Linguistic scale |
|-------|-----------------|------------------|
| Crisp | Fuzzy           | Crisp            | Fuzzy           |                  |
| 1     | (1, 1, 1, 1)    | $1^{-1}$         | (1, 1, 1, 1)    | Equally significant |
| 3     | (2, 2, 1, 1)    | $3^{-1}$         | (1, 1, 1, 1)    | Slightly significant |
| 5     | (4, 4, 4, 4)    | $5^{-1}$         | (1, 1, 1, 1)    | Very significant  |
| 7     | (6, 6, 6, 6)    | $7^{-1}$         | (1, 1, 1, 1)    | Greatly significant |
| 9     | (8, 8, 8, 8)    | $9^{-1}$         | (1, 1, 1, 1)    | Absolutely significant |
Algorithm 3: Type-1 trapezoidal fuzzy AHP

Step 1: Defined hierarchical structure of problems such as the goals to be achieved, the criteria that are owned (sub criteria, if any) and alternatives to achieve goals.

Step 2: Input Data: the number of criteria-\( n \), the number of alternative-\( m \), fuzzy comparison of alternative \( \tilde{A}_l^{(j)} \) still have related of criteria \( C_t \), \( j = 1, 2, \ldots, n \)

\[
\tilde{A}_l^{(j)} = \begin{pmatrix}
\tilde{a}_{12}^{-1} & 1 & \ldots & \tilde{a}_{1m}^{-1} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{a}_{lm}^{-1} & (\tilde{a}_{2m}^{-1})^{-1} & \ldots & 1
\end{pmatrix}
\]

fuzzy pairwise comparison matrix of criteria \( \tilde{C}_t = (C_t, C_t, C_m, C_t) \) by

\[
\begin{align*}
C_l & = \begin{pmatrix}
1 & ct_{12,1} & \ldots & ct_{1k,1} \\
ct_{12,2} & 1 & \ldots & ct_{2k,1} \\
\vdots & \vdots & \ddots & \vdots \\
ct_{1k,2} & ct_{2k,2} & \ldots & 1
\end{pmatrix} & C_m & = \begin{pmatrix}
1 & ct_{12,m} & \ldots & ct_{1k,m} \\
ct_{12,n} & 1 & \ldots & ct_{2k,m} \\
\vdots & \vdots & \ddots & \vdots \\
ct_{1k,n} & ct_{2k,n} & \ldots & 1
\end{pmatrix} \\
C_n & = \begin{pmatrix}
1 & ct_{12,n} & \ldots & ct_{1k,n} \\
ct_{12,m} & 1 & \ldots & ct_{2k,n} \\
\vdots & \vdots & \ddots & \vdots \\
ct_{1k,m} & ct_{2k,m} & \ldots & 1
\end{pmatrix} & C_u & = \begin{pmatrix}
1 & ct_{12,u} & \ldots & ct_{1k,u} \\
ct_{12,l} & 1 & \ldots & ct_{2k,u} \\
\vdots & \vdots & \ddots & \vdots \\
ct_{1k,l} & ct_{2k,l} & \ldots & 1
\end{pmatrix}
\end{align*}
\]

Step 3: Test the consistency of ratio (CR) from type-1 trapezoidal fuzzy of pairwise comparison matrix for \( \tilde{C}_t \) and \( \tilde{A}_l^{(j)} \), \( j = 1, 2, \ldots, n \)

For CR of \( \tilde{C}_t \) by solving fuzzy eigenvalues problem equation as follows

\[
\tilde{C}_t \otimes \tilde{\omega} = \tilde{\lambda} \otimes \tilde{\omega} \quad (27)
\]

where \( \tilde{\omega} = (\omega_l, \omega_m, \omega_n, \omega_u); \ \tilde{\lambda} = (\tilde{\lambda}_l, \tilde{\lambda}_m, \tilde{\lambda}_n, \tilde{\lambda}_u) \) by using expected value was presented by Prascevic and Prascevic? Equation (42) divided to be four equation systems

\[
\begin{align*}
\tilde{C}_l \otimes \omega_l & = \tilde{\lambda}_l \otimes \omega_l \\
\tilde{C}_m \otimes \omega_m & = \tilde{\lambda}_m \otimes \omega_m \\
\tilde{C}_n \otimes \omega_n & = \tilde{\lambda}_n \otimes \omega_n \\
\tilde{C}_u \otimes \omega_u & = \tilde{\lambda}_u \otimes \omega_u 
\end{align*} \quad (28)
\]
where

\[
\begin{align*}
\bar{C}_l &= 2C_l + C_m \\
\bar{C}_m &= C_l + 2C_m \\
\bar{C}_n &= 2C_n + C_m \\
\bar{C}_u &= C_u + 2C_t
\end{align*}
\]

\[
\omega_l = [\omega_{1,l}, \omega_{2,l}, \ldots, \omega_{n,l}]^T
\]

\[
\omega_m = [\omega_{1,m}, \omega_{2,m}, \ldots, \omega_{n,m}]^T
\]

\[
\omega_n = [\omega_{1,n}, \omega_{2,n}, \ldots, \omega_{n,n}]^T
\]

\[
\omega_u = [\omega_{1,u}, \omega_{2,u}, \ldots, \omega_{n,u}]^T
\]

\[
\begin{align*}
\bar{\lambda}_l &= 2\lambda_l + \lambda_m \\
\bar{\lambda}_m &= \lambda_m + 2\lambda_l \\
\bar{\lambda}_n &= 2\lambda_n + \lambda_n \\
\bar{\lambda}_u &= \lambda_u + 2\lambda_n
\end{align*}
\]

\[
\begin{align*}
\lambda_l &= \frac{(2\tilde{\lambda}_l - \lambda_m)}{3} \\
\lambda_m &= \frac{(2\tilde{\lambda}_m - \lambda_l)}{3} \\
\lambda_n &= \frac{(2\tilde{\lambda}_n - \lambda_u)}{3} \\
\lambda_u &= \frac{(2\tilde{\lambda}_u - \lambda_n)}{3}
\end{align*}
\]

Step 3: calculate the type-1 trapezoidal fuzzy priority weights of criteria

\[
\tilde{\bar{w}} = (\tilde{w}_l, \tilde{w}_m, \tilde{w}_n, \tilde{w}_u)
\]

as follows

\[
\begin{align*}
\tilde{w}_l &= \frac{\omega_l}{\lambda_l} \\
\tilde{w}_m &= \frac{\omega_m}{\lambda_m} \\
\tilde{w}_n &= \frac{\omega_n}{\lambda_n} \\
\tilde{w}_u &= \frac{\omega_u}{\lambda_u}
\end{align*}
\]

(29)

calculate the index of consistency (CI) uses formulation as follows,

\[
CI = \frac{\lambda_{\max} - n}{n - 1}
\]

(30)

\[
CR = \frac{CI}{RI}
\]

(31)

where \(\lambda_{\max}\) the maximum eigenvalue of crisp matrix

\[
C_t = \frac{C_m + C_t}{2}
\]

RI is random consistency

If \(CR > 0.1\) then repeat Step 1 until Step 3 such that this condition is satisfied (\(CR \leq 0.1\)).

For CR of \(\tilde{A}_l^{(j)}, j = 1, 2, \ldots, n\) by expressed \(\tilde{A}_l^{(j)}\) be \(\tilde{A}_l^{(j)} \cdot \tilde{A}_m^{(j)} \cdot \tilde{A}_n^{(j)} \cdot \tilde{A}_u^{(j)}\) based on criteria \(\bar{C}_t\) to solve eigenvector equation as follows
\[ \tilde{A}^{(j)} \otimes \tilde{\rho}^{(j)} = \tilde{\lambda}^{(j)} \otimes \tilde{\rho}^{(j)} \] (32)

the finding type-1 trapezoidal fuzzy principal eigenvalues
\[ \tilde{\lambda}^{(j)} = (\lambda_1^{(j)}, \lambda_m^{(j)}, \lambda_n^{(j)}, \lambda_u^{(j)}) \], fuzzy eigenvector \[ \tilde{\rho}^{(j)} = (\rho_1^{(j)}, \rho_m^{(j)}, \rho_n^{(j)}, \rho_u^{(j)}) \], index of consistency \( CI^{(j)} \), and consistency of ratio \( CR^{(j)} \) by Equations (46) and (47).

If \( CR^{(j)} > 0.1 \), then repeat Step 1 until Step 3 such that this condition is satisfied \( (CR^{(j)} \leq 0.1) \).

Step 4: Determine the fuzzy weight vector of alternatives \[ \tilde{\rho}^{(j)} = (\tilde{\rho}_1^{(j)}, \tilde{\rho}_m^{(j)}, \tilde{\rho}_n^{(j)}, \tilde{\rho}_u^{(j)}) \] by formulation as follows

\[
\begin{align*}
\tilde{\rho}_1^{(j)} &= \frac{\rho_1^{(j)}}{\lambda_1^{(j)}} \\
\tilde{\rho}_m^{(j)} &= \frac{\rho_m^{(j)}}{\lambda_m^{(j)}} \\
\tilde{\rho}_n^{(j)} &= \frac{\rho_n^{(j)}}{\lambda_n^{(j)}} \\
\tilde{\rho}_u^{(j)} &= \frac{\rho_u^{(j)}}{\lambda_u^{(j)}}
\end{align*}
\] (33)

Step 5: Calculate type-1 trapezoidal fuzzy global weight \[ \tilde{g}_i = (g_{i,l}, g_{i,m}, g_{i,n}, g_{i,u}) \] of alternatives as follows:

\[
\begin{align*}
g_{i,l} &= \tilde{\rho}_l \tilde{\omega}_l = (g_{1,l}, g_{2,l}, \ldots, g_{m,l})^T \\
g_{i,m} &= \tilde{\rho}_m \tilde{\omega}_m = (g_{1,m}, g_{2,m}, \ldots, g_{m,m})^T \\
g_{i,n} &= \tilde{\rho}_n \tilde{\omega}_n = (g_{1,n}, g_{2,n}, \ldots, g_{m,n})^T \\
g_{i,u} &= \tilde{\rho}_u \tilde{\omega}_u = (g_{1,u}, g_{2,u}, \ldots, g_{m,u})^T
\end{align*}
\] (34)

where

\[
\begin{align*}
\tilde{\rho}_l &= (\tilde{\rho}_l^{(1)}, \tilde{\rho}_l^{(2)}, \ldots, \tilde{\rho}_l^{(n)}) \\
\tilde{\rho}_m &= (\tilde{\rho}_m^{(1)}, \tilde{\rho}_m^{(2)}, \ldots, \tilde{\rho}_m^{(n)}) \\
\tilde{\rho}_n &= (\tilde{\rho}_n^{(1)}, \tilde{\rho}_n^{(2)}, \ldots, \tilde{\rho}_n^{(n)}) \\
\tilde{\rho}_u &= (\tilde{\rho}_u^{(1)}, \tilde{\rho}_u^{(2)}, \ldots, \tilde{\rho}_u^{(n)})
\end{align*}
\] (35)
step 5: and
\[
\bar{\omega}_l = (\bar{\omega}_{1,l}, \bar{\omega}_{2,l}, \ldots, \bar{\omega}_{n,l})^T \\
\bar{\omega}_m = (\bar{\omega}_{1,m}, \bar{\omega}_{2,m}, \ldots, \bar{\omega}_{n,m})^T \\
\bar{\omega}_n = (\bar{\omega}_{1,n}, \bar{\omega}_{2,n}, \ldots, \bar{\omega}_{n,n})^T \\
\bar{\omega}_u = (\bar{\omega}_{1,u}, \bar{\omega}_{2,u}, \ldots, \bar{\omega}_{n,u})^T
\]

(36)

Step 6: Ranked of alternatives $A_l(i = 1, 2, \ldots, m)$ based on expected value ($g_{i,e}$), standard deviation ($\sigma_i$) and coefficient of variation ($CV_i$) as follows

\[
g_{i,e} = \frac{g_{i,u}^2 + g_{i,n}g_{i,u} + g_{i,n}^2 - g_{i,l}^2 + g_{i,l}g_{i,m} - g_{i,m}^2}{3(g_{i,u} - g_{i,l} + g_{i,n} - g_{i,m})},
\]

(37)

\[
\sigma_i = \sqrt{\frac{g_{i,u}(g_{i,u}^2 + g_{i,u}g_{i,n} + g_{i,n}^2) + g_{i,n}^2}{6(g_{i,u} - g_{i,l} + g_{i,n} - g_{i,m})} - \frac{(g_{i,u})^2}{(g_{i,e})^2}}
\]

(38)

\[
CV_i = \frac{\sigma_i}{g_{i,e}}, \quad i = 1, 2, \ldots, m
\]

(39)

\[
g_{i,e} = g_{i,c} \quad y_c = \frac{g_{i,u} + 2(g_{i,n} - g_{i,m}) - g_{i,l}}{3(g_{i,u} - g_{i,l} + g_{i,n} - g_{i,m})}
\]

(40)

\[
R(\bar{g}) = \sqrt{g_{i,c}^2 + y_c^2}
\]

(41)

AHP based on eigenvalues and expected value. The linguistic value of fuzzy AHP into type-1 trapezoidal fuzzy number on the criteria and alternatives in shown Table 2. The steps of Prascevic and Prascevic’s fuzzy AHP are given in Algorithm 3.

### 4.2. The New Interval Type-2 Trapezoidal Fuzzy AHP

In this section, Prascevic and Prascevic’s type-1 trapezoidal fuzzy AHP method will be modified by using interval type-2 fuzzy sets. The linguistic value of fuzzy AHP into interval type-2 trapezoidal fuzzy number on the criteria and alternatives in shown Table 3. In detail, the steps of this fuzzy AHP method are explained in Algorithm 4.

### 5. In Practice

Based on the case study presented by Kahraman et al. [20] regarding the problem of selecting a supplier with two alternatives (SP1 and SP2) and four criteria, namely price (PC), quality (QY), delivery (DV) and capacity (CP) with the hierarchy structure presented in Figure 4 and the abbreviations of the linguistic variables are expressed in Table 3.

We apply the steps of the proposed trapezoidal interval type-2 fuzzy AHP in Algorithm 4 as follows.
**Algorithm 4**: The new interval type-2 trapezoidal fuzzy AHP

Step 1: Defined hierarchical structure of problems such as the goals to be achieved, the criteria that are owned (sub criteria, if any) and alternatives to achieve goals.

Step 2: Input Data: the number of criteria-\(n\), the number of alternative-\(m\), fuzzy comparison of alternative \(\tilde{A}_l^{(j)}\) still have related of criteria \(\tilde{C}_t_j, j = 1, 2, \ldots, n\)

\[
\tilde{A}_l^{(j)} = \begin{bmatrix}
\tilde{A}_l^1 & \tilde{A}_l^2 & \ldots & \tilde{A}_l^m
\end{bmatrix}
\]

where

\[
\tilde{a}_l = [(a_l^{(1)1}, a_l^{(1)2}, a_l^{(1)3}, a_l^{(1)4}, \sigma_1(a_l^{(1)2}), \sigma_2(a_l^{(1)3}), \ldots, a_l^{(m)1}, a_l^{(m)2}, a_l^{(m)3}, a_l^{(m)4}, \sigma_1(a_l^{(m)2}), \sigma_2(a_l^{(m)3})]
\]

\[
\tilde{a}_l^{-1} = \left[\begin{array}{ccc}
\frac{1}{a_l^{(1)4}}, \frac{1}{a_l^{(1)3}}, \frac{1}{a_l^{(1)2}}, \frac{1}{a_l^{(1)1}}; \sigma_1(a_l^{(1)2}), \sigma_2(a_l^{(1)3}), \\
\frac{1}{a_l^{(2)4}}, \frac{1}{a_l^{(2)3}}, \frac{1}{a_l^{(2)2}}, \frac{1}{a_l^{(2)1}}; \sigma_1(a_l^{(2)2}), \sigma_2(a_l^{(2)3}), \\
\ldots & \ldots & \ldots & \ldots
\end{array}\right]
\]

fuzzy pairwise comparison matrix of criteria \(\tilde{C}_t = (\tilde{C}_t_l, \tilde{C}_t_m, \tilde{C}_t_n, \tilde{C}_t_u)\) by

\[
\tilde{C}_t_l = \begin{pmatrix}
1 & \tilde{c}_{12,l} & \ldots & \tilde{c}_{1k,l} \\
\tilde{c}_{12,u} & 1 & \ldots & \tilde{c}_{2k,l} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{c}_{1k,u} & \tilde{c}_{2k,u} & \ldots & 1
\end{pmatrix}
\]

\[
\tilde{C}_t_m = \begin{pmatrix}
1 & \tilde{c}_{12,m} & \ldots & \tilde{c}_{1k,m} \\
\tilde{c}_{12,n} & 1 & \ldots & \tilde{c}_{2k,m} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{c}_{1k,n} & \tilde{c}_{2k,n} & \ldots & 1
\end{pmatrix}
\]

\[
\tilde{C}_t_n = \begin{pmatrix}
1 & \tilde{c}_{12,n} & \ldots & \tilde{c}_{1k,n} \\
\tilde{c}_{12,m} & 1 & \ldots & \tilde{c}_{2k,n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{c}_{1k,m} & \tilde{c}_{2k,m} & \ldots & 1
\end{pmatrix}
\]

\[
\tilde{C}_t_u = \begin{pmatrix}
1 & \tilde{c}_{12,u} & \ldots & \tilde{c}_{1k,u} \\
\tilde{c}_{12,l} & 1 & \ldots & \tilde{c}_{2k,u} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{c}_{1k,l} & \tilde{c}_{2k,l} & \ldots & 1
\end{pmatrix}
\]

where

\[
\tilde{C}_t = [(c_t^{(1)1}, c_t^{(1)2}, c_t^{(1)3}, c_t^{(1)4}, \sigma_1(c_t^{(1)2}), \sigma_2(c_t^{(1)3}), \ldots, c_t^{(m)1}, c_t^{(m)2}, c_t^{(m)3}, c_t^{(m)4}, \sigma_1(c_t^{(m)2}), \sigma_2(c_t^{(m)3})]
\]

\[
\tilde{C}_t^{-1} = \left[\begin{array}{ccc}
\frac{1}{c_t^{(1)4}}, \frac{1}{c_t^{(1)3}}, \frac{1}{c_t^{(1)2}}, \frac{1}{c_t^{(1)1}}; \sigma_1(c_t^{(1)2}), \sigma_2(c_t^{(1)3}), \\
\frac{1}{c_t^{(2)4}}, \frac{1}{c_t^{(2)3}}, \frac{1}{c_t^{(2)2}}, \frac{1}{c_t^{(2)1}}; \sigma_1(c_t^{(2)2}), \sigma_2(c_t^{(2)3}), \\
\ldots & \ldots & \ldots & \ldots
\end{array}\right]
\]
Step 3: Test the consistency of ratio (CR) from interval type-2 trapezoidal fuzzy pairwise comparison matrices for $C_t$ and $\tilde{A}^{(j)}$, $j = 1, 2, \ldots, n$

For CR of $C_t$ by solving fuzzy eigenvalues problem equation as follows

$$\tilde{C}_t^z \otimes \tilde{\omega}^z = \tilde{\lambda}^z \otimes \tilde{\omega}^z$$

(42)

where $\tilde{\omega}^z = (\tilde{\omega}_l^z, \tilde{\omega}_m^z, \tilde{\omega}_n^z, \tilde{\omega}_u^z); \tilde{\lambda}^z = (\tilde{\lambda}_l^z, \tilde{\lambda}_m^z, \tilde{\lambda}_n^z, \tilde{\lambda}_u^z); z \in \{U, L\}$;

by using expected value was presented by Prascevic and Prascevic [16]

Equation (42) divided to be four equation systems

$$\tilde{C}_t^z_l \otimes \tilde{\omega}_l^z = \tilde{\lambda}_l^z \otimes \tilde{\omega}_l^z$$

$$\tilde{C}_t^z_m \otimes \tilde{\omega}_m^z = \tilde{\lambda}_m^z \otimes \tilde{\omega}_m^z$$

$$\tilde{C}_t^z_n \otimes \tilde{\omega}_n^z = \tilde{\lambda}_n^z \otimes \tilde{\omega}_n^z$$

$$\tilde{C}_t^z_u \otimes \tilde{\omega}_u^z = \tilde{\lambda}_u^z \otimes \tilde{\omega}_u^z$$

(43)

where

$$\tilde{C}_t^z_l = 2\tilde{C}_l^z + \tilde{C}_m^z$$

$$\tilde{C}_t^z_m = \tilde{C}_l^z + 2\tilde{C}_m^z$$

$$\tilde{C}_t^z_n = 2\tilde{C}_n^z + \tilde{C}_l^z$$

$$\tilde{C}_t^z_u = \tilde{C}_u^z + 2\tilde{C}_l^z$$

$$\tilde{\omega}_l^z = [\tilde{\omega}_l^z, \tilde{\omega}_m^z, \tilde{\omega}_n^z, \tilde{\omega}_u^z]^T$$

$$\tilde{\omega}_m^z = [\tilde{\omega}_m^z, \tilde{\omega}_l^z, \tilde{\omega}_n^z, \tilde{\omega}_u^z]^T$$

$$\tilde{\omega}_n^z = [\tilde{\omega}_n^z, \tilde{\omega}_m^z, \tilde{\omega}_l^z, \tilde{\omega}_u^z]^T$$

$$\tilde{\omega}_u^z = [\tilde{\omega}_u^z, \tilde{\omega}_n^z, \tilde{\omega}_m^z, \tilde{\omega}_l^z]^T$$

$$\tilde{\lambda}_l^z = \frac{(2\tilde{\lambda}_l^z - \tilde{\lambda}_m^z)}{3}$$

$$\tilde{\lambda}_m^z = \frac{(2\tilde{\lambda}_m^z - \tilde{\lambda}_l^z)}{3}$$

$$\tilde{\lambda}_n^z = \frac{(2\tilde{\lambda}_n^z - \tilde{\lambda}_u^z)}{3}$$

$$\tilde{\lambda}_u^z = \frac{(2\tilde{\lambda}_u^z - \tilde{\lambda}_n^z)}{3}$$

calculate the interval type-2 trapezoidal fuzzy priority weights of criteria

$$\tilde{\mathbf{w}} = (\tilde{\mathbf{w}}_l, \tilde{\mathbf{w}}_m, \tilde{\mathbf{w}}_n, \tilde{\mathbf{w}}_u)$$

as follows

$$\tilde{\mathbf{w}}_l = \frac{\tilde{\omega}_l^z}{\tilde{\lambda}_l^z}$$

$$\tilde{\mathbf{w}}_m = \frac{\tilde{\omega}_m^z}{\tilde{\lambda}_m^z}$$

$$\tilde{\mathbf{w}}_n = \frac{\tilde{\omega}_n^z}{\tilde{\lambda}_n^z}$$

$$\tilde{\mathbf{w}}_u = \frac{\tilde{\omega}_u^z}{\tilde{\lambda}_u^z}$$
Step 4:

\[
\tilde{w}_n^z = \frac{\tilde{\omega}_n^z}{\tilde{\lambda}_n^z}
\]
\[
\tilde{w}_u^z = \frac{\tilde{\omega}_u^z}{\tilde{\lambda}_u^z}
\]

(44)

calculate the index of consistency (CI) uses formulation as follows,

\[
CI = \frac{\lambda_{\text{max}} - n}{n - 1}
\]

(45)

\[
CR = \frac{CI}{RI}
\]

(46)

where \(\lambda_{\text{max}}\) the maximum eigenvalue of crisp matrix by using the new proposed ranking in Algorithm 2, RI is random consistency. If CR > 0.1 then repeat Step 1 until Step 3 such that this condition is satisfied (CR \(\leq 0.1\)).

For CR of \(\tilde{\lambda}^{(j)}\), \(j = 1, 2, \ldots, n\) by expressed \(\tilde{\lambda}^{(j)}\) be \(\tilde{\lambda}^{(j)}_1, \tilde{\lambda}^{(j)}_m, \tilde{\lambda}^{(j)}_u\) based on criteria \(\tilde{Ct}\) to solve eigenvector equation as follows

\[
\tilde{\lambda}^{(j)}_1 \otimes \tilde{\rho}^{(j)}_1 = \tilde{\lambda}^{(j)}_m \otimes \tilde{\rho}^{(j)}_m
\]

(47)

the finding interval type-2 trapezoidal fuzzy principal eigenvalues

\(\tilde{\lambda}^{(j)}_{\text{max}} = (\tilde{\lambda}^{(j)}_1, \tilde{\lambda}^{(j)}_m, \tilde{\lambda}^{(j)}_n, \tilde{\lambda}^{(j)}_u)\), fuzzy eigenvector \(\tilde{\rho}^{(j)}_{\text{max}} = (\tilde{\rho}^{(j)}_1, \tilde{\rho}^{(j)}_m, \tilde{\rho}^{(j)}_n, \tilde{\rho}^{(j)}_u)\), index of consistency \(CI^{(j,z)}\), and consistency of ratio \(CR^{(j,z)}\) by Equations (46) and (47). if CR\(^{(j,z)} > 0.1\) then repeat Step 1 until Step 3 such that this condition is satisfied (CR\(^{(j,z)} \leq 0.1\)).

Step 4: Determine the fuzzy weight vector of alternatives \(\tilde{\rho}^{(j)} = (\tilde{\rho}^{(j)}_1, \tilde{\rho}^{(j)}_m, \tilde{\rho}^{(j)}_n, \tilde{\rho}^{(j)}_u)\) by formulation as follows

\[
\tilde{\rho}^{(j)}_1 = \frac{\tilde{\rho}^{(j)}_1}{\tilde{\lambda}^{(j)}_1}
\]
\[
\tilde{\rho}^{(j)}_m = \frac{\tilde{\rho}^{(j)}_m}{\tilde{\lambda}^{(j)}_m}
\]
\[
\tilde{\rho}^{(j)}_n = \frac{\tilde{\rho}^{(j)}_n}{\tilde{\lambda}^{(j)}_n}
\]
\[
\tilde{\rho}^{(j)}_u = \frac{\tilde{\rho}^{(j)}_u}{\tilde{\lambda}^{(j)}_u}
\]

(48)
Step 5: Calculate interval type-2 trapezoidal fuzzy global weight \( \tilde{g}_i = (\tilde{g}_{i,u}, \tilde{g}_{i,m}, \tilde{g}_{i,l}) \) of alternatives as follows:

\[
\begin{align*}
\tilde{g}_{i,j} &= \tilde{\rho}_i \tilde{\omega}_j = (\tilde{g}_{1,j}^z, \tilde{g}_{2,j}^z, \ldots, \tilde{g}_{m,j}^z)^T \\
\tilde{g}_{i,m} &= \tilde{\rho}_m \tilde{\omega}_m = (\tilde{g}_{1,m}^z, \tilde{g}_{2,m}^z, \ldots, \tilde{g}_{m,m}^z)^T \\
\tilde{g}_{i,n} &= \tilde{\rho}_n \tilde{\omega}_n = (\tilde{g}_{1,n}^z, \tilde{g}_{2,n}^z, \ldots, \tilde{g}_{m,n}^z)^T \\
\tilde{g}_{i,u} &= \tilde{\rho}_u \tilde{\omega}_u = (\tilde{g}_{1,u}^z, \tilde{g}_{2,u}^z, \ldots, \tilde{g}_{m,u}^z)^T
\end{align*}
\]

where

\[
\begin{align*}
\tilde{\rho}_i &= \left(\tilde{\rho}_i^{(1,z)} \tilde{\rho}_i^{(2,z)} \ldots \tilde{\rho}_i^{(n,z)}\right) \\
\tilde{\omega}_j &= \left(\tilde{\omega}_j^{(1,z)} \tilde{\omega}_j^{(2,z)} \ldots \tilde{\omega}_j^{(n,z)}\right)
\end{align*}
\]

and

\[
\begin{align*}
\tilde{\omega}_n &= \left(\tilde{\omega}_n^{(1,z)} \tilde{\omega}_n^{(2,z)} \ldots \tilde{\omega}_n^{(n,z)}\right)^T \\
\tilde{\omega}_u &= \left(\tilde{\omega}_u^{(1,z)} \tilde{\omega}_u^{(2,z)} \ldots \tilde{\omega}_u^{(n,z)}\right)^T
\end{align*}
\]

Step 6: Ranked of alternatives \( A_l(i = 1, 2, \ldots, m) \) based mean value \((g_{i,e})_l\), and coefficient of variation \((CV)_l\) for uniform distribution [47]. Not only that, ranking of alternatives also according to the distance \( R(\tilde{g}_i) \) of the original point and centroid point.

\[
\begin{align*}
g_{i,e}^z &= \frac{g_{i,u}^2 + g_{i,m}^2 - g_{i,l}^2}{3(g_{i,u} - g_{i,l} + g_{i,m})}, \quad z \in \{U, L\} \\
g_{i,e}^{average} &= \frac{g_{i,u} + g_{i,m}}{2}, \\
\sigma_i^z &= \sqrt{\frac{g_{i,u}(g_{i,u}^2 + g_{i,u}g_{i,m} + g_{i,m}^2) + g_{i,m}^3 - g_{i,l}^3 - g_{i,l}(g_{i,u}^2 + g_{i,u}g_{i,m} + g_{i,m}^2)}{6(g_{i,u} - g_{i,l} + g_{i,m})} - (g_{i,e}^U)^2} \\
\sigma_i^{average} &= \frac{\sigma_i^U + \sigma_i^L}{2},
\end{align*}
\]
Step 6:

\[ CV_i = \frac{\sigma_i^{\text{average}}}{g_{i,e}}, \quad i = 1, 2, \ldots, m \]  
(56)

\[ g_{i,e}^z = g_{i,c}^z \quad y_c^z = \frac{g_{i,u} + 2(g_{i,n} - g_{i,m}) - g_{i,l}}{3(g_{i,u} - g_{i,l} + g_{i,n} - g_{i,m})} \quad z \in \{U, L\} 
\]  
(57)

\[ y_c^{\text{average}} = \frac{y_c^U + y_c^L}{2}, \]  
(58)

\[ R(\tilde{g}) = \sqrt{(g_{i,c}^{\text{average}})^2 + (y_c^{\text{average}})^2} \]  
(59)

---

**Figure 4.** Illustrative the hierarchy of supplier selection problem [20]

**Step 1.** Creation of the pairwise comparison matrices. Tables 4 and 5 present criteria and alternatives of pairwise comparison matrices using linguistic variables term.

**Step 2.** Consistency ratio test of pairwise comparison matrices for criterion (\(\tilde{C}_t\)) by using Equations (36) and (37) are obtained

\[
\begin{align*}
\tilde{C}_U &= \begin{pmatrix} 3 & 0.44 & 10 & 16 \\ 10 & 3 & 16 & 16 \\ 0.44 & 0.34 & 3 & 4 \\ 0.34 & 0.34 & 0.64 & 3 \end{pmatrix} \\
\tilde{\omega}_U &= \begin{pmatrix} 0.2680 \\ 0.6009 \\ 0.0796 \\ 0.0515 \end{pmatrix} \\
\tilde{\lambda}_U &= 3.172667 \\
\tilde{C}_L &= \begin{pmatrix} 3 & 0.45 & 10.6 & 16.6 \\ 10.6 & 3 & 16.6 & 16.6 \\ 0.45 & 0.34 & 3 & 4.6 \\ 0.34 & 0.34 & 0.78 & 3 \end{pmatrix} \\
\tilde{\omega}_L &= \begin{pmatrix} 0.2666 \\ 0.6014 \\ 0.0808 \\ 0.0512 \end{pmatrix} \\
\tilde{\lambda}_L &= 3.283579
\end{align*}
\]
Table 3. Linguistic scale generalised interval type-2 trapezoidal fuzzy number for weight matrix [20].

| Crisp | Fuzzy | Reciprocal Crisp | Fuzzy | Linguistic scale |
|-------|-------|------------------|-------|-----------------|
| \( \tilde{1} \) | \((1, 1, 1, 1; 1, 1) \) | \( \tilde{1}^{-1} \) | \((1, 1, 1, 1; 1, 1) \) | Equally significant |
| \( \tilde{3} \) | \((1, 2, 3.5, 5.5; 1, 1) \) | \( \tilde{3}^{-1} \) | \((1, 2, 3.5, 5.5; 1, 1) \) | Slightly significant |
| \( \tilde{5} \) | \((3, 4, 6, 7; 1, 1) \) | \( \tilde{5}^{-1} \) | \((3, 4, 6, 7; 1, 1) \) | Very significant |
| \( \tilde{7} \) | \((5, 6, 8, 9; 1, 1) \) | \( \tilde{7}^{-1} \) | \((5, 6, 8, 9; 1, 1) \) | Greatly significant |
| \( \tilde{9} \) | \((7, 8, 9, 9.6; 1, 1) \) | \( \tilde{9}^{-1} \) | \((7, 8, 9, 9.6; 1, 1) \) | Absolutely significant |

Table 4. The pairwise comparison matrix of criteria.

| Criteria | PC | QY | DV | CP |
|----------|----|----|----|----|
| PC       | \( \tilde{1} \) | \( \tilde{5}^{-1} \) | \( \tilde{5} \) | \( \tilde{7} \) |
| QY       | \( \tilde{5}^{-1} \) | \( \tilde{1} \) | \( \tilde{7}^{-1} \) | \( \tilde{7} \) |
| DV       | \( \tilde{5}^{-1} \) | \( \tilde{7}^{-1} \) | \( \tilde{1} \) | \( \tilde{3} \) |
| CP       | \( \tilde{7}^{-1} \) | \( \tilde{7}^{-1} \) | \( \tilde{3}^{-1} \) | \( \tilde{1} \) |

Table 5. The pairwise comparison matrix of alternatives.

| w.r.t QY | SP-1 | SP-2 | w.r.t PC | SP-1 | SP-2 |
|----------|------|------|----------|------|------|
| SP-1     | \( \tilde{1} \) | \( \tilde{3}^{-1} \) | SP-1    | \( \tilde{1} \) | \( \tilde{3} \) |
| SP-2     | \( \tilde{3} \) | \( \tilde{1} \) | SP-2    | \( \tilde{3}^{-1} \) | \( \tilde{1} \) |
| w.r.t DV | SP-1 | SP-2 | w.r.t CP | SP-1 | SP-2 |
| SP-1     | \( \tilde{1} \) | \( \tilde{3} \) | SP-1    | \( \tilde{1} \) | \( \tilde{9} \) |
| SP-2     | \( \tilde{3}^{-1} \) | \( \tilde{1} \) | SP-2    | \( \tilde{9} \) | \( \tilde{1} \) |

\[
\tilde{C}_{m}^{U} = \begin{pmatrix} 3 & 0.46 & 11 & 17 \\ 11 & 3 & 17 & 17 \\ 0.46 & 0.35 & 3 & 5 \\ 0.35 & 0.35 & 0.75 & 3 \end{pmatrix} \quad \tilde{\omega}_{m}^{U} = \begin{pmatrix} 0.2661 \\ 0.6010 \\ 0.0822 \\ 0.0507 \end{pmatrix} \quad \tilde{\lambda}_{m}^{U} = 3.744298
\]

\[
\tilde{C}_{m}^{L} = \begin{pmatrix} 3 & 0.48 & 11.6 & 17.6 \\ 11.6 & 3 & 17.6 & 17.6 \\ 0.48 & 0.35 & 3 & 5.6 \\ 0.35 & 0.35 & 0.86 & 3 \end{pmatrix} \quad \tilde{\omega}_{m}^{L} = \begin{pmatrix} 0.2655 \\ 0.6009 \\ 0.0834 \\ 0.0503 \end{pmatrix} \quad \tilde{\lambda}_{m}^{L} = 3.857224
\]
Calculate fuzzy weight priority of criteria \( \tilde{\omega} \) by using Equation (39) are resulted

\[
\begin{align*}
\tilde{\omega}_U^n &= \begin{pmatrix} 0.8503 \\ 1.9065 \\ 0.2523 \\ 0.1634 \end{pmatrix}, & \tilde{\omega}_L^n &= \begin{pmatrix} 0.8753 \\ 1.9747 \\ 0.2652 \\ 0.1682 \end{pmatrix}, & \tilde{\lambda}_U^n &= \begin{pmatrix} 1.3407 \\ 2.9135 \\ 0.4949 \\ 0.2706 \end{pmatrix}, & \tilde{\lambda}_L^n &= \begin{pmatrix} 1.2978 \\ 2.8394 \\ 0.4536 \\ 0.2596 \end{pmatrix} \\
\tilde{\omega}_U^u &= \begin{pmatrix} 0.9962 \\ 2.2503 \\ 0.3077 \\ 0.1899 \end{pmatrix}, & \tilde{\omega}_L^u &= \begin{pmatrix} 1.0240 \\ 2.3178 \\ 0.3215 \\ 0.1938 \end{pmatrix}, & \tilde{\lambda}_U^u &= \begin{pmatrix} 1.6651 \\ 3.5814 \\ 0.6489 \\ 0.3529 \end{pmatrix}, & \tilde{\lambda}_L^u &= \begin{pmatrix} 1.5572 \\ 3.3577 \\ 0.5547 \\ 0.3198 \end{pmatrix}
\end{align*}
\]

Obvious that using Algorithm 2, \( \lambda_{\text{max}} = 4.3 \) and by Equations (45) and (46) with \( n = 4 \) is resulted \( CR = 0.1 \) such that pairwise comparison matrix of \( \tilde{C}_r \) is consistent.

Consistency ratio test of pairwise comparison matrices for alternatives \( A^j, j = PC, QY, DV, CP \) by solving Equation (47).

For \( J = PC \), so

\[
\begin{align*}
\tilde{\rho}_{PC, U} &= \begin{pmatrix} 0.673 \\ 0.722 \\ 0.722 \\ 0.673 \end{pmatrix}, & \tilde{\rho}_{PC, L} &= \begin{pmatrix} 0.676 \\ 0.724 \\ 0.724 \\ 0.676 \end{pmatrix} \\
\tilde{\lambda}_{PC, U} &= (1.430, 1.756, 2.323, 3.365), & \tilde{\lambda}_{PC, L} &= (1.535, 1.829, 2.206, 2.875)
\end{align*}
\]
\[
\begin{bmatrix}
0.187 & 0.171 & 0.171 & 0.187 \\
0.812 & 0.828 & 0.828 & 0.812
\end{bmatrix}
\]

For \( J = QY \), so
\[
\begin{bmatrix}
0.183 & 0.169 & 0.169 & 0.183 \\
0.816 & 0.830 & 0.830 & 0.816
\end{bmatrix}
\]

\[
\tilde{\lambda}_{QY,U} = (1.655, 1.816, 2.224, 2.529) \quad \tilde{\lambda}_{QY,L} = (1.691, 1.850, 2.164, 2.442)
\]

\[
\begin{bmatrix}
0.673 & 0.722 & 0.722 & 0.673 \\
0.326 & 0.277 & 0.277 & 0.326
\end{bmatrix}
\]

For \( J = DV \), so
\[
\begin{bmatrix}
0.673 & 0.724 & 0.724 & 0.673 \\
0.326 & 0.275 & 0.275 & 0.326
\end{bmatrix}
\]

\[
\tilde{\lambda}_{DV,U} = (1.430, 1.756, 2.323, 3.365) \quad \tilde{\lambda}_{DV,L} = (1.535, 1.829, 2.206, 2.875)
\]

\[
\begin{bmatrix}
0.109 & 0.105 & 0.105 & 0.109 \\
0.890 & 0.894 & 0.894 & 0.890
\end{bmatrix}
\]

For \( J = CP \), so
\[
\begin{bmatrix}
0.111 & 0.106 & 0.106 & 0.111 \\
0.888 & 0.893 & 0.893 & 0.888
\end{bmatrix}
\]

\[
\tilde{\lambda}_{CP,U} = (1.853, 1.942, 2.060, 2.171) \quad \tilde{\lambda}_{CP,L} = (1.884, 1.976, 2.024, 2.130)
\]

Clear that \( n = 2 \) such that \( R^I = 0 \). Hence, all of \( CR^I, j = PC, QY, DV, CP \) are 0 such that all of \( A^I, j = PC, QY, DV, CP \) pairwise comparison matrices are consistent.

**Step 3.** Determine fuzzy weight vector of \( A^I, j = PC, QY, DV, CP \) by using Equation (48) are obtained

\[
\begin{bmatrix}
0.962 & 1.268 & 1.677 & 2.264 \\
0.467 & 0.487 & 0.645 & 1.100
\end{bmatrix}
\]

If \( j = PC \) then
\[
\begin{bmatrix}
1.039 & 1.325 & 1.598 & 1.945 \\
0.496 & 0.503 & 0.607 & 0.930
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.310 & 0.311 & 0.381 & 0.474 \\
1.345 & 1.505 & 1.843 & 2.055
\end{bmatrix}
\]

If \( j = QY \) then
\[
\begin{bmatrix}
0.311 & 0.314 & 0.369 & 0.447 \\
1.380 & 1.536 & 1.795 & 1.994
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.962 & 1.268 & 1.677 & 2.264 \\
0.467 & 0.487 & 0.645 & 1.100
\end{bmatrix}
\]

If \( j = PC \) then
\[
\begin{bmatrix}
1.039 & 1.325 & 1.598 & 1.945 \\
0.496 & 0.503 & 0.607 & 0.930
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.203 & 0.205 & 0.217 & 0.238 \\
1.650 & 1.737 & 1.843 & 1.933
\end{bmatrix}
\]

If \( j = PC \) then
\[
\begin{bmatrix}
0.207 & 0.210 & 0.215 & 0.234 \\
1.667 & 1.766 & 1.808 & 1.896
\end{bmatrix}
\]
Step 4. Calculate fuzzy weight global \( \tilde{g}^z, z \in \{U, L\} \) by using Equations (43), (44) and (45) are obtained if

\[
\tilde{\omega}_U^U = \begin{pmatrix} 0.850 \\ 1.063 \\ 0.265 \\ 0.168 \end{pmatrix} \quad \tilde{\omega}_L^U = \begin{pmatrix} 0.310 \\ 0.962 \\ 0.203 \\ 0.130 \end{pmatrix} \quad \tilde{\rho}_U^U = \begin{pmatrix} 0.310 \\ 0.962 \\ 0.203 \\ 0.130 \end{pmatrix} \quad \tilde{\rho}_L^U = \begin{pmatrix} 0.962 \\ 0.203 \\ 0.130 \end{pmatrix}
\]

\( \tilde{g}_U^U = 2.375 \), \( \tilde{g}_L^U = 2.422 \).

If

\[
\tilde{\omega}_U^L = \begin{pmatrix} 0.875 \\ 1.974 \\ 0.265 \\ 0.168 \end{pmatrix} \quad \tilde{\omega}_L^L = \begin{pmatrix} 0.311 \\ 1.039 \\ 0.311 \\ 1.039 \end{pmatrix} \quad \tilde{\rho}_U^L = \begin{pmatrix} 0.311 \\ 1.039 \\ 0.311 \\ 1.039 \end{pmatrix} \quad \tilde{\rho}_L^L = \begin{pmatrix} 1.039 \\ 0.311 \\ 1.039 \\ 0.311 \end{pmatrix}
\]

then

\( \tilde{g}_U^L = 2.634 \), \( \tilde{g}_L^L = 2.603 \).

Step 5. Calculate ranking of alternatives according to mean value \( \tilde{g}_{\text{average}}^e \) by using Equation (53) and Coefficient of variance \( \text{CV}_{\text{average}}^e \) by using Equation (46). Moreover, ranking of alternatives based on the distance \( R(\tilde{g}) \) by using Equation (59). The results of ranking for alternatives can be seen in Tables 6 and 7. Based on these tables, supplier 2 is selected with the best ranking according to the mean value, the coefficient of variance and the distance of the original point to the centroid point.

In order to compare our results with Prascevic and Prascevic’s type-1 fuzzy AHP, we solve the same problem using type-1 fuzzy sets. The comparison ranking Prascevic and Prascevic’s type-1 fuzzy AHP and our type-2 fuzzy AHP can be seen in Table 8.
Table 6. The result of ranking for alternatives based on mean value and STD.

| Rank | Al | Mean value $g_{ie}$ | Al | Coefficient of var. CV$_i$(%) | Standard deviat. $\sigma_i$ |
|------|----|---------------------|----|------------------------------|-----------------------------|
| 2    | SP-1 | 5.449               | SP-2 | 1.081                       | 5.895                       |
| 1    | SP-2 | 7.291               | SP-1 | 0.681                       | 4.965                       |

Table 7. The result of ranking for alternatives based on the distance.

| Rank | Al | Mean value $g_{ie}$ | $\gamma_{ci}$ | $R(\bar{g})$ |
|------|----|---------------------|----------------|-------------|
| 2    | SP-1 | 5.449               | 0.422          | 2.423       |
| 1    | SP-2 | 7.291               | 0.420          | 2.776       |

Table 8. The comparison ranking of type-1 and type-2 fuzzy AHP.

| Al | Mean value | Type-1 Rank | Type-2 Rank | Type-1 Rank | Type-2 Rank |
|----|------------|-------------|-------------|-------------|-------------|
| SP-1 | 5.673 | 1           | 5.499       | 2           | 2.470       | 1           | 2.423       | 2           |
| SP-2 | 4.980 | 2           | 7.291       | 1           | 2.320       | 2           | 2.776       | 1           |

Table 9. The comparison ranking of existing and the new of type-2 trapezoidal fuzzy AHP.

| Al | Kahraman et al.’s fuzzy AHP | The new type-2 fuzzy AHP | Rank |
|----|------------------------------|--------------------------|------|
| SP-1 | 0.3841                      | 2.423                    | 2    |
| SP-2 | 0.6158                      | 2.776                    | 1    |

Both methods give different ranking results. Type-1 and type-2 fuzzy AHP use pairwise comparison matrices under fuzziness. But, type-2 fuzzy AHP sustains us to construct membership functions with the longest flexibility. Moreover, We also compare the type-2 fuzzy AHP proposed with type-2 fuzzy AHP was proposed by Kahraman et al. [20] shown in Table 9 and this table shows that the ranking result by Kahraman et al.’s fuzzy AHP equal to the new type-2 fuzzy AHP.

6. Conclusion

Type-1 fuzzy sets have difficulties when constructing complex membership functions due to the many vagueness and fuzziness of the expert team’s judgment. Type-2 fuzzy set as a problem solver. The modification of Prascaevic and Prascaevic’s type-1 trapezoidal fuzzy AHP to be interval type-2 trapezoidal fuzzy AHP has been developed and discussed. Based on the study case provided, the ranking value produced by the new interval type-2 trapezoidal fuzzy AHP is equal to ranking value of Kahraman et al.’s fuzzy AHP and is different from the ranking value Prascaevic and Prascaevic’s type-1 trapezoidal fuzzy AHP. Furthermore, there are limited number of ranking methods for trapezoidal interval type-2 fuzzy sets in the references. The modified total integral value is proposed and compared with the existing ranking methods. Based on the four numerical examples, the new proposed ranking is superior to three existing ranking methods and is equal to two existing methods.
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