Reinforcement Learning for Adaptive Routing

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Abstract - Reinforcement learning means learning a policy—a mapping of observations into actions—based on feedback from the environment. The learning can be viewed as browsing a set of policies while evaluating them by trial through interaction with the environment. We present an application of gradient ascent algorithm for reinforcement learning to a complex domain of packet routing in network communication and compare the performance of this algorithm to other routing methods on a benchmark problem.

I. Introduction

Successful telecommunication requires efficient resource allocation that can be achieved by developing adaptive control policies. Reinforcement learning (RL) [10], [17] presents a natural framework for the development of such policies by trial and error in the process of interaction with the environment. In this work we apply the RL algorithm to network routing. Effective network routing means selecting the optimal communication paths. It can be modeled as a multi-agent RL problem. In a sense, learning the optimal control for network routing could be thought of as learning in some traditional for RL episodic task, like maze searching or pole balancing, but repeating trials many times in parallel with interaction among trials.

Under this interpretation, an individual router is an agent which makes its routing decisions according to an individual policy. The parameters of this policy are adjusted according to some measure of the global performance of the network, while control is determined by local observations. Nodes do not have any information regarding the topology of network or their position in it. The initialization of each node, as well as the learning algorithm it follows, are identical to that of every other node and independent of the structure of the network. There is no notion of orientation in space or other semantics of actions. Our approach allows us to update the local policies while avoiding the necessity for centralized control or global knowledge of the networks structure. The only global information required by the learning algorithm is the network utility expressed as a reward signal distributed once in an epoch and dependent on the average routing time. This learning multi-agent system is biologically plausible and could be thought of as neural network in which each neuron only performs simple computations based on locally available quantities [?].

II. Domain

We test our algorithm on a domain adopted from Boyan and Littman [4]. It is a discrete time simulator of communication networks with various topologies and dynamic structure. A communication network is an abstract representation of real-life systems such as the Internet or a transport network. It consists of a homogeneous set of nodes and edges between them representing links (see Figure 1). Nodes linked to each other are called neighbors. Links may be active ("up") or inactive ("down"). Each node can be the origin or the final destination of packets, or serve as a router.

Packets are periodically introduced into the network with a uniformly random node of origin and destination. They travel to their destination node by hopping on intermediate nodes. No packets are generated being destined to the node of origin. Sending a packet down a link incurs a cost that could be thought of as time in transition. There is an added cost to waiting in the queue of a particular node in order to access the router’s computational resource—a queue delay. Both costs are assumed to be uniform throughout the network. In our experiments, each is set to be a unit cost. The level of network traffic is determined by the number of packets in the network. Once a packet reaches its destination, it is removed. If a packet has been traveling around the network for a long time it is also removed as a hopeless case. Multiple packets line up at nodes in an FIFO (first in first out) queue limited in size. The node must forward the top packet in the FIFO queue to one of its neighbors.

In the terminology of RL, the network represents the environment whose state is determined by the number and relative position of nodes, the status of links between them and the dynamics of packets. The destination of handled packets and the status of local links form the node’s observation. Each node is an agent who has a choice of actions. It decides where to send the packet according to a policy. The policy computed by our algorithm is stochastic, as opposed to deterministic, i.e. it

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sends packets bound for the same destination down different links, according to some distribution. The policy considered in our experiments does not determine whether or not to accept a packet (admission control), how many packets to accept from each neighbor, or which packets should be assigned priority.

The node updates the parameters of its policy based on the reward. The reward comes in the form of a signal distributed through the network by acknowledgment packets once a packet has reached its final destination. The reward depends on the total delivery time for the packet. We measure the performance of the algorithm by the average delivery time for packets once the system has settled on a policy (ordinate axes on figure 2). We denote by \[ \mathbb{E} \{ r(t, h) \} \] the expected cumulative discounted (by a factor of the temperature \( \Xi \)) reward that can be written as

\[
V(\theta) = \sum_{t=1}^{\infty} \gamma^t \sum_{h \in \mathcal{H}_t} \Pr(h | \theta) r(t, h).
\]

However, in the spirit of reinforcement learning, we assume no knowledge of a world model that would allow the agent to calculate \( \Pr(h | \theta) \), so we must retreat to stochastic gradient ascent instead. We sample from the distribution of histories by interacting with the environment, and calculate during each trial an estimate of the gradient, accumulating the quantities:

\[
\gamma^t r(t, h) \sum_{\tau=1}^{t} \frac{\partial \ln \Pr(a(t, h) | h^t-1, \theta)}{\partial \theta_{oa}}.
\]

Applying this algorithm in a network of connected controllers basically constitutes the algorithm of routing by distributed gradient ascent policy search (GAPS).

We compare the performance of our distributed GAPS algorithm to three others, as follows. “Best” is a static routing scheme based on the shortest path counting each link as a single unit of routing cost. We include this algorithm because it provides the basis for most current industry routing heuristics [3, 8]. “Bestload” performs routing according to the shortest path while taking into account queue sizes at each node. It is close to the theoretical optimum among deterministic routing algorithms even though the actual best possible routing scheme requires not simply computing the shortest path based on

**III. Algorithmic details**

Williams introduced the notion of policy search via gradient ascent for reinforcement learning in his REINFORCE algorithm [18, 19], which was generalized to a broader class of error criteria by Baird and Moore [1, 2]. The general idea is to adjust parameters in the direction of the empirically estimated gradient of the aggregate reward. We assume standard Markov decision process MDP setup [10]. Let us consider the case of a single agent interacting with a partially observable MDP (POMDP). The agent’s policy \( \mu \) is a so-called reactive policy represented by a lookup table with a value \( \theta_{oa} \) for each observation-action (destination/link) pair. The policy defines the probability of an action given past history as a continuous differentiable function of a set of parameters \( \theta \) according to a softmax rule, where \( \Xi \) is a temperature parameter:

\[
\mu(a, o, \theta) = \Pr(a(t) = a | o(t) = o, \theta) = \frac{\exp(\theta_{oa}/\Xi)}{\sum_{a'} \exp(\theta_{a'o}/\Xi)} > 0.
\]

This rule assures that for any destination \( o \) any link \( a' \) available at the node is sometimes chosen with some small probability dependent on the temperature \( \Xi \).

We denote by \( \mathcal{H}_t \) the set of all possible experience sequences \( h = \langle o(1), a(1), r(1), \ldots, o(t), a(t), r(t), o(t+1) \rangle \) of length \( t \). In order to specify that some element is a part of the history \( h \) at time \( \tau \), we write, for example, \( r(\tau, h) \) and \( a(\tau, h) \) for the \( \tau \)-th reward and action in the history \( h \). We will also use \( h^t \) to denote a prefix of the sequence \( h \in \mathcal{H}_t \) truncated at time \( \tau \leq t \);

\[
h^\tau \triangleq \langle o(1), a(1), r(1), \ldots, o(\tau), a(\tau), r(\tau), o(\tau + 1) \rangle.
\]

The value of following a policy \( \mu \) with parameters \( \theta \) is the expected cumulative discounted (by a factor of \( \gamma \in [0, 1] \)) reward that can be written as

\[
V(\theta) = \sum_{t=1}^{\infty} \gamma^t \sum_{h \in \mathcal{H}_t} \Pr(h | \theta) r(t, h).
\]

If we could calculate the derivative of \( V(\theta) \) for each \( \theta_{oa} \), it would be possible to do an exact gradient ascent on value \( V(\cdot) \) by making updates \( \Delta \theta_{oa} = \gamma \frac{\partial V}{\partial \theta_{oa}} \), for some step size \( \alpha \). Let us analyze the derivative for each weight \( \theta_{oa} \).

\[
\frac{\partial V(\theta)}{\partial \theta_{oa}} = \sum_{t=1}^{\infty} \gamma^t \sum_{h \in \mathcal{H}_t} \left[ r(t, h) \frac{\partial \Pr(h | \theta)}{\partial \theta_{oa}} \right] = \sum_{t=1}^{\infty} \gamma^t \sum_{h \in \mathcal{H}_t} \Pr(h | \theta) r(t, h) \times \sum_{\tau=1}^{t} \frac{\partial \ln \Pr(a(t, h) | h^{t-1}, \theta)}{\partial \theta_{oa}}.
\]

We consider the case of a single agent interacting with a partially observable MDP (POMDP). The agent’s policy \( \mu \) is a so-called reactive policy represented by a lookup table with a value \( \theta_{oa} \) for each observation-action (destination/link) pair. The policy defines the probability of an action given past history as a continuous differentiable function of a set of parameters \( \theta \) according to a softmax rule, where \( \Xi \) is a temperature parameter:

\[
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\[
\frac{\partial V(\theta)}{\partial \theta_{oa}} = \sum_{t=1}^{\infty} \gamma^t \sum_{h \in \mathcal{H}_t} \left[ r(t, h) \frac{\partial \Pr(h | \theta)}{\partial \theta_{oa}} \right] = \sum_{t=1}^{\infty} \gamma^t \sum_{h \in \mathcal{H}_t} \Pr(h | \theta) r(t, h) \times \sum_{\tau=1}^{t} \frac{\partial \ln \Pr(a(t, h) | h^{t-1}, \theta)}{\partial \theta_{oa}}.
\]

However, in the spirit of reinforcement learning, we assume no knowledge of a world model that would allow the agent to calculate \( \Pr(h | \theta) \), so we must retreat to stochastic gradient ascent instead. We sample from the distribution of histories by interacting with the environment, and calculate during each trial an estimate of the gradient, accumulating the quantities:

\[
\gamma^t r(t, h) \sum_{\tau=1}^{t} \frac{\partial \ln \mu(a(o, o, \theta))}{\partial \theta_{oa}} \text{ for all } t.
\]

For a particular policy architecture, this can be readily translated into a gradient ascent algorithm guaranteed to converge to a local optimum \( \theta^* \) of \( V(\theta) \). Under our chosen policy encoding we get:

\[
\frac{\partial \ln \mu(a(o, o, \theta))}{\partial \theta_{oa'}} = \begin{cases} 
0 & \text{if } o' \neq o, \\
\frac{1}{2} \mu(a', o, \theta) & \text{if } o' = o, a' \neq a, \\
\frac{1}{2} \left[ 1 - \mu(a, o, \theta) \right] & \text{if } o' = o, a' = a.
\end{cases}
\]

Applying this algorithm in a network of connected controllers basically constitutes the algorithm of routing by distributed gradient ascent policy search (GAPS).
network loads, but also analyzing how loads change in time according to routing decisions. Since calculating the shortest path at every single step of the simulation would be prohibitively costly in terms of computational resources, we implemented “Bestload” by readjusting the routing policy only after a notable change of loads in the network. We consider 50 successfully delivered packets to constitute a notable load change. Finally, “Q-routing” is a distributed RL algorithm applied specifically to this domain by Littman and Boyan [4]. While our algorithm is stochastic and performs policy search, Q-routing is a deterministic, value search algorithm. Note that our implementation of the network routing simulation is based on the software Littman and Boyan used to test Q-routing. Even so, the results of our simulation of “Q-routing” and “Best” on the “6x6” network differ slightly from Littman and Boyan’s due to certain modifications in traffic modeling conventions. For instance, we consider a packet delivered and ready for removal only after it has passed through the queue of the destination node and accessed its computational resources, and not merely when the packet is successfully routed to the destination node by an immediate neighbor, as in the original simulation.

We undertake the comparison between GAPS and the aforementioned algorithms with one important caveat. The GAPS algorithm explores the class of stochastic policies while all other methods pick deterministic routing policies. Consequently, it is natural to expect GAPS to be superior for certain types of network topologies and loads, where the optimal policy is stochastic. Later, we show that our experiments confirm this expectation.

We implement the distributed GAPS in POMDP. In particular, we represent each router as a POMDP, where the state contains the sizes of all queues, destinations of all packets, state of links (up or down); the environment state transition function is a law of the dynamics of network traffic; an observation o consists of the destination of the packet; an action a corresponds to sending the packet down a link to an adjacent node; and finally, the reward signal is the average number of packets delivered per unit of time. Each agent is using a GAPS RL algorithm to move parameterization values down the gradient of the average reward. It has been shown [14] that an application of distributed GAPS causes the system as a whole to converge to local optimum under stationarity assumptions. This algorithm is essentially the one described in chapter 3 and developed in chapter 5 of Peshkin’s dissertation [13].

Policies were initialized in two different ways: randomly and based on shortest paths. We tried initialization with random policy uniformly chosen over parameter space. With such initialization results are very sensitive to the learning rate. High learning rate often causes the network to stick to local optima in combined policy space, with very poor performance. Low learning rate results in a slow convergence. What constitutes high or low learning rate depends on the specifics of each network and we did not find any satisfactory heuristics to set it. Obviously, such features as average number of hops necessary to deliver a packet under the optimal policy as well as learning speed crucially depend on the particular characteristics of each network such as number of nodes, connectivity and modularity.

These considerations led us to a different way of initializing controllers. Namely, we begin by computing shortest path and set controllers to route most of the traffic down the shortest path, while occasionally sending a packet to explore an alternative link. We call this “ε-greedy routing”. In our experiments, ε is set to .01. We believe that this parameter would not qualitatively change the outcome of our experiments since it only influences exploratory behaviour in the beginning.

The exploration capacity of the algorithm is regulated in a different way as well. Both temperature and learning rate are simply kept constant both for considerations of simplicity and for maintaining the controllers’ ability to adjust to changes in the network, such as links failure. However, our experiments indicate that having a schedule for reducing learning rate after a key initial period of learning would improve performance. Alternatively, it would be interesting to explore different learning rates for the routing parameters on one hand, and the encoding of topological features on the other.

IV. Empirical results

We compared the routing algorithms on several networks with various number of nodes and degrees of connectivity and modularity, including 116-node “LATA” telephone network. On all networks, the GAPS algorithm performed comparably or better than other routing algorithms. To illustrate the principal differences in the behavior of algorithms and the key advantages of distributed GAPS, we concentrate on the analysis of two routing problems on networks which differ in a single link location.

Figure 1(left) presents the irregular 6x6 grid network topology used by Boyan and Littman [4] in their experiments. The network consists of two well connected components with a bottleneck of traffic falling on two bridging links. The resulting dependence of network performance on the load is depicted in figure 2(left). All graphs represent performance after the policy has converged, averaged over five runs. We tested the network on loads ranging from .5 to 3.5, to compare with the results ob-
tained by Littman and Boyan. The load corresponds to the value of the parameter of Poisson arrival process for the average number of packets injected per time unit. On this network topology, GAPS is slightly inferior to other algorithms on lower loads, but does at least as well as Bestload on higher loads, outperforming both Q-routing and Best. The slightly inferior performance on low loads is due to exploratory behaviour of GAPS — some fraction of packets is always sent down random link.

To illustrate the difference between the algorithms more explicitly, we altered the network by moving just one link from connecting nodes 32 and 33, to connecting nodes 20 and 27 as illustrated by figure 1. Right. Since node 20 obviously represents a bottleneck in this configuration, the optimal routing policy is bound to be stochastic. The resulting dependence of network performance on the load is presented in figure 2. Right. GAPS is clearly superior to other algorithms at high loads. It even outperforms “Bestload” that has all the global information in choosing a policy, but is bound to deterministic policies.

Notice how the deterministic algorithms get frustrated at much lower loads in this network configuration than in the previous one since from their perspective, the bridge between highly connected components gets twice thinner (compare left and right of Figure 2).

The GAPS algorithm successfully adapts to changes in the network configuration. Under increased load, the preferred route from the left part of the network to the right becomes evenly split between the two “bridges” at node 20. By using link 20 – 27, the algorithm has to pay a penalty of making a few extra hops compared to link 20 – 21, but as the size of the queue at node 21 grows, this penalty becomes negligible compared to the waiting time. Exploratory behavior helps GAPS discover when links go down and adjust the policy accordingly. We have experimented with giving each router a few bits of memory in finite state controller [13, 17] but found that this does not improve performance and slows down the learning somewhat.

V. Related Work

The application of machine learning techniques to the domain of telecommunications is a rapidly growing area. The bulk of problems fit into the category of resource allocation, e.g. bandwidth allocation, network routing, call admission control (CAC) and power management. RL appears promising in attacking all of these problems, separately or simultaneously.

Marbach, Mihatsch and Tsitsiklis [11] have applied an actor-critic (value-search) algorithm to address resource allocation within communication networks by tackling both routing and call admission control. They adopt a decompositional approach, representing the network as consisting of link processes, each with its own differential reward. Unfortunately, the empirical results even on small networks, 4 and 16 nodes, show little advantage over heuristic techniques.

Carlström [7] introduces another RL strategy based on decomposition called predictive gain scheduling. The control problem of admission control is decomposed into a time-series prediction of near-future call arrival rates and precomputation of control policies for Poisson call arrival processes. This approach results in faster learning without performance loss. Online convergence rate increases 50 times on a simulated link with capacity 24 units/sec.

Generally speaking, value-search algorithms have been more extensively investigated than policy search ones in the domain of communications. Value-search (Q-learning) algorithms have arrived at promising results.
Boyan and Littman’s [4] algorithm - Q-routing, proves superior to non-adaptive techniques based on shortest path, and robust with respect to dynamic variations in the simulation on a variety of network topology, including an irregular 6 × 6 grid and 116-node LATA phone network. It regulates the trade-off between the number of nodes a packet has to traverse and the possibility of congestion.

Wolpert, Tumer and Frank [20] construct a formalism for the so-called Collective Intelligence (COIN) neural net applied to Internet traffic routing. The approach involves automatically initializing and updating the local utility functions of individual RL agents (nodes) from the global utility and observed local dynamics. Their simulation outperforms a Full Knowledge Shortest Path Algorithm on a sample network of seven nodes. Coin networks employ a method similar in spirit to the research presented here. They rely on a distributed RL algorithm that converges on local optima without endowing each agent node with explicit knowledge of network topology. However, COIN differs from our approach in requiring the introduction of preliminary structure into the network by dividing it into semi-autonomous neighborhoods that share a local utility function and encourage cooperation. In contrast, all the nodes in our network update their algorithms directly from the global reward.

The work presented in this paper focuses on packet routing using policy search. It resembles the work of Tao, Baxter and Weaver [12] who apply a policy-gradient algorithm to induce cooperation among the nodes of a packet switched network in order to minimize the average packet delay. While their algorithm performs well in several network types, it takes many (tens of millions) trials to converge on a network of just a few nodes.

Applying reinforcement learning to communication often involves optimizing performance with respect to multiple criteria. For a recent discussion on this challenging issue see Shelton [13]. In the context of wireless communication it was addressed by Brown [5] who considers the problem of finding a power management policy that simultaneously maximizes the revenue earned by providing communication while minimizing battery usage. The problem is defined as a stochastic shortest path with discounted infinite horizon, where discount factor varies to model power loss. This approach resulted in significant (50%) improvement in power usage.

Gelenbe et al. [9] also compute the reward as a weighted combination of the probability of packet loss and packet delay. The packets themselves are agents controlling routing and flow control in a Cognitive Packet Network. They split packets into three types: ”smart”, ”dumb” and ”acknowledgment”. A small number of smart packets learn the most efficient ways of navigating through the network, dumb packets simply follow the route taken by the smart packets, while acknowledgment packets travel on the inverse route of smart packets to provide source routing information to dumb packets. The division between smart and dumb packet is an explicit representation of the explore/exploit dilemma. Smart
packet allow the network to adapt to structural changes while the dumb packets exploit the relative stability between those changes. Promising results are obtained both on a simulation network of 100 nodes and on a physical network of 6 computers.

Subramanian, Druschel and Chen [15] adopt an approach from ant colonies that is very similar in spirit. The individual hosts in their network keep routing tables with the associated costs of sending a packet to other hosts (such as which routers it has to traverse and how expensive they are). These tables are periodically updated by "ants"-messages whose function is to assess the cost of traversing links between hosts. The ants are directed probabilistically along available paths. They inform the hosts along the way of the costs associated with their travel. The hosts use this information to alter their routing tables according to an update rule. There are two types of ants. Regular ants use the routing tables of the hosts to alter the probability of being directed along a certain path. After a number of trials, all regular ants on the same mission start using the same routes. Their function is to allow the host tables to converge on the correct cost figure in case the network is stable. Uniform ants take any path with equal probability. They are the ones who continue exploring the network and assure successful adaptation to changes in link status or link cost.

VI. Discussion

Admittedly, the simulation of network routing process presented here is far from being realistic. A more realistic model could include such factors as non-homogeneous networks with regard to links bandwidth and routing nodes buffer size limits, collisions of packets, packet ordering constraints, various costs associated with say, particular links chosen from commercial versus government subnetworks, minimal Quality of Service requirements. Introducing priorities for individual packets brings up yet another set of optimization issues. However, the learning algorithm we applied shows promise in handling adaptive telecommunication protocol and there are several obvious ways to develop this research. Incorporating domain knowledge into controller structure is one such direction. It would involve classifying nodes into subnetworks and routing packets in a hierarchical fashion. One step further down this line is employing learning algorithms for routing in ad-hoc networks. Ad-hoc networks are networks where nodes are being dynamically introduced and terminated from the system, as well as existing active nodes are moving about, loosing some connections and establishing new ones. Under the realistic assumption that physical variations is the network are slower than traffic routing and evolution, adaptive routing protocol should definitely outperform any heuristic pre-defined routines. We are currently pursuing this line of research.

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