Research on the Method of Transforming Vague to Fuzzy Value

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Abstract. In the study of the transformation of Vague sets to Fuzzy sets, the key is to judge the tendency of the Vague degree. To determine the tendency of Vague degree, it is necessary to make decision on the information of the uncertainty. Therefore, the transformation of the Vague set to the Fuzzy set can be regarded as the decision maker's risk decision under the uncertain conditions. Risk preference of decision-makers is a key decision parameter when decision makers make risk decision. In this paper, the weight of risk-income weighted weight is introduced into the transformation model. Risk preference of decision-makers is well reflected in the transformation results. The membership degree of the Fuzzy set is decomposed into two parts: the true membership value of the Vague set, the value of the risk and the value of the tendencies. The results show that it is necessary and meaningful to consider the risk preference of the decision-makers. The thinking mode of dealing with this kind of problem can be changed from "point" thinking to "face" thinking. The fuzziness and risk preference of decision-makers are presented to the decision-makers in the form of a two-dimensional decision map. The transformation model is to help decision makers to fully understand the facts. The role of the transformation model is to assist decision makers in making scientific decisions rather than making decisions for decision makers.

1. Introduction
In the real world, because the concept itself is not clear, the affiliation of the object to the collection is not clear. In order to describe and deal with such ambiguity problems, Zadeh first proposed the use of membership functions to express ambiguity. Thus it broke through Cantor's classical set theory and laid the foundation for fuzzy theory (Zadeh, 1965). The element not only belongs to or does not belong to a set, but also reflects the hesitancy between the belonging and the non-belonging. In the voting model, for example, there were 10 votes, of which 4 were supported, 2 were opposed, and 4 abstained. Fuzzy sets are not effective in dealing with such problems. In order to solve the problem that the fuzzy set cannot express and deal with such fuzzy information, Gau et al. proposed the Vague set theory (Gau, 1993). Gau and Buehree introduced the concept of affirmative membership degree and negative membership degree by analyzing the characteristics of fuzzy sets, and expressed the degree of membership in the form of interval values. Its core idea is to use the concepts of affirmative membership, negative membership, and ambiguity to describe the three aspects of support, opposition, and uncertainty. Vague set theory has obvious advantages over Cantor set and Fuzzy set in expressing ambiguity and uncertainty. However, the unknown degree theory of Vague sets is not very mature. The method of transforming Vague set into Fuzzy set is constructed. By using the maturity theory of Fuzzy set, we calculate the unknown degree of Vague set and carry out similar reasoning (Burillo P, Bustince H, 1996; Szmidt E, Kacprzyk Kacprzyk, 2001). ZHANG Xiao-xia et al (2011) constructed weighted arithmetic and weighted geometric integration operators of interval Vague values, defined the membership degree of interval Vague sets, and constructed an effective sorting method for interval...
Vague values. WEI Bo et al (2012) has constructed a simplex geometry representation method that transforms Vague sets into fuzzy sets, which effectively solves the problem of geometric interpretation in the transformation method or model from Vague set to fuzzy set. WANG Wan-jun et al (2012) proposed a new biased method for transforming fuzzy values into Vague values. The essence of Vague values converted to fuzzy values is a biased tendency to support the unascertained tendency in Vague values. LUO Jun (2011), in order to better understand the process of transforming a Vague set into a fuzzy set, transforming the Vague set into a fuzzy set is considered as a final round of voting without abandoning the voting process, thereby constructing an effect function to express support. WANG Wanjun(2013) proposed a partial contact method based on Vague value converted to fuzzy value.

At present, the existing method of transforming Vague sets into Fuzzy sets does not consider risk preference when analyzing the ambiguity. The decision risk preference under non-deterministic conditions is a very important decision factor that must be considered. The conversion model that has been constructed has rarely studied this issue. A common mode of thinking is to determine a certain value from ambiguity information, and the rules for determining this value usually have no objective theoretical basis or strict mathematical proof as the basis. This makes the correctness and effectiveness of the transformation model difficult to be universally recognized. At present, different methods and standards for obtaining the “determined value” from different documents are extremely inconsistent. It is necessary to question the rationality of these transformation methods. However, merely questioning the construction rule system of individual transformation models cannot fundamentally promote the study of this issue. This paper proposes that the methodology for solving this problem will change from a "point" of thinking to a "face" of thinking. The ambiguity and decision maker risk preference are presented to the decision maker in the form of a two-dimensional decision matrix map. The transformation model is to help decision-makers to understand the facts scientifically and comprehensively and assist decision-makers in decision-making, not to make decisions for decision-makers. The transformation model constructed in this paper has greater reference for decision makers to make effective decisions. The structure of this paper is as follows: First, analyze the advantages and disadvantages of the existing conversion methods. Then from the perspective of risk decision-making, the problem of transforming Vague sets into Fuzzy sets is studied, and a double weighted transformation operator is constructed. The transformation model constructed in this paper can consider the subjective risk appetite of decision makers and objective ambiguity of information.

2. Basic Concept Introduction

Definition 1 Let $U$ be a discourse domain. For any element $x$ in $U$, a fuzzy set $F$ in discourse domain $U$ is represented by a membership function $\mu_F : U \to [0,1]$. Then $\mu_F(x)$ denotes the degree to which the element $x$ belongs to the fuzzy set $F$, and $\mu_F(x)$ is the degree of membership of $x$ with respect to fuzzy set $F$.

Definition 2 Let $U$ be a domain of discourse. For any element $x$ in $U$, a Vague set $V$ in the domain $U$ is represented by a true membership function $t_v$ and a false membership function $f_v$. $t_v(x)$ is the lower bound of the true membership degree of $x$ derived from the evidence supporting $x$, and $f_v(x)$ is the lower bound of the false membership degree of $x$ derived from evidence against $x$. $t_v(x)$ and $f_v(x)$ constitute a pair of real numbers in interval $[0,1]$. The pair of real numbers is associated with an element in the domain $U$, namely: $t_v : U \to [0,1], f_v : U \to [0,1]$, where $t_v(x) + f_v(x) \leq 1$. The degree of membership of $x$ to Vague set $V$ is $[t_v(x), 1-f_v(x)]$, and $V(x)$ is a Vague value. Without misunderstanding, $t_v(x)$ can be abbreviated as $t_x$, and $f_v(x)$ can be abbreviated as $f_x$.

When $U$ is continuous domain,
\[ V = \int_{U} \left[ I_{x} 1 - f_{x} \right], x \in U \]

When \( U \) is a discrete domain,
\[ V = \sum_{i=1}^{t} \left[ f(x_{i}) 1 - f(x_{i}) \right], x_{i} \in U \]

\( \forall x \in X, \pi_{x} = 1 - t_{x} - f_{x} \) is the Vague degree of element \( x \) relative to Vague set \( V \). It describes the degree of enthalpy of element \( x \) belonging to Vague set \( V \). It is a measure of the uncertainty of the element \( x \) belonging to the Vague set \( V \). The larger the value of \( \pi_{x} \), the more information is unknown about element \( x \) relative to \( V \), obviously \( 0 \leq \pi_{x} \leq 1 \).

According to the definition of Vague set, the degree of membership of any element \( x \) in Vague set \( V \) is a subrange of \([0,1]\), where \( t_{x} \) is the true membership function of Vague set \( V \). It represents the degree of support \( x \) belongs to the Vague set \( V \). \( f_{x} \) is false membership function of Vague set \( V \), which indicates the supporting object \( x \) does not belong to the Vague set \( V \).

For a Vague set \( V \), when \( t_{x} + f_{x} = 1 \), the Vague set degenerates into a fuzzy set. When \( t_{x} = 1 \), \( f_{x} = 0 \), \( \pi_{x} = 0 \), Vague sets degenerate into classic Cantor sets. Therefore, the Fuzzy set is a special case of the Vague set. The Cantor set is a special case of the Fuzzy set. Vague sets are general forms of Fuzzy and Cantor sets.

Transforming criteria

The following transformation criteria should be satisfied when the Vague value is converted to a Fuzzy value\([10,11]\):

Criterion 1: \( t_{x} \leq F_{x} \leq 1 - f_{x} \)

Criterion 2: if \( t_{x} = f_{x} \), then \( k = 1/2 \)

Criterion 3: if \( t_{x} < f_{x} \), then \( 0 < k < 1/2 \)

Criterion 4: if \( t_{x} > f_{x} \), then \( 1/2 < k < 1 \)

Criterion 1 indicates the range of membership degrees after the Vague value is converted to a fuzzy value. Criterion 2 states that when support information and objection information are equal, the degree of aggression should be assigned to the same proportion of subordination and non-subordination tendencies. Criterion 3 states that when support information is greater than objection information, the degree of ambiguity assigned to affiliation tends to be greater than the value assigned to unaffiliated affiliations. Criterion 4 states that when support information is smaller than objection information, the degree of ambiguity assigned to affiliation tends to be less than the value assigned to unaffiliated affiliations.

Criterion 2, 3 and 4 are actually not very reasonable. For example, when Vague value is \( \pi_{x} = [0.05,0.96] \), obviously \( f_{x} = 0.04 < 0.05 = t_{x} \), then \( F_{x} = t_{x} + k\pi_{x} > 0.05 + \frac{1}{2} \times 0.991 = 0.5005 \). If you follow the above rules will lead to erroneous results. For example, 1000 people voted, 5 people voted for support, 4 people voted against it, and 9991 people abstained. According to criterion 4, the conversion result is \( F_{x} = 0.5005 \). The converted fuzzy set believes that at least 500 people have voted for support. In fact, only 5 out of 1,000 people voted for support. This conversion result sometimes has catastrophic consequences for decision making, so this article has modified criterion 2 to 4 to form the following two criteria:

Criterion 5: if \( t_{x} = t_{y}, f_{x} = f_{y} \), then \( F_{x} = F_{y} \)

Criterion 6: if \( t_{x} = t_{y}, f_{x} > f_{y} \), then \( F_{x} < F_{y} \).
The formula for transforming the Vague value to the Fuzzy value is prone to errors. In document [9], a method to test the transformation of Vague values into Fuzzy values is proposed. As long as the transformation method satisfies any of the following two conditions, the transformation method is not distinguishable.

(1) For any \( a \in [0,1] \), \( V_x = [0,a] \), then \( F_x = 0 \).

(2) For any \( a \in (0,1] \), \( V_x = [a,1] \), then \( F_x = 1 \).

3. Construction of Transformation Model based on Risk Preference

The membership value of element \( X \) belonging to Vague set \( V \) is in interval \( C \). Which specific value is based on the existing information about the element \( X \) and the set \( V \) cannot be accurately determined. Decision-makers estimate their own risk-return preferences and obtain the following risk-benefit trade-offs, as shown in Table (1).

| Risk scale | Risk - Earnings Trade Weights |
|-----------|------------------------------|
| \( R_2 = \{ r_{12}, r_{22}^2 \} \) | \( W_2 = \{ A_{12}, A_{22}^2 \}, \) where, \( A_{12} + A_{22}^2 = 1 \) |
| \( R_3 = \{ r_{13}, r_{23}, r_{33} \} \) | \( W_3 = \{ A_{13}, A_{23}, A_{33} \}, \) where, \( A_{13} + A_{23} + A_{33} = 1 \) |
| \( \vdots \) | \( \vdots \) |
| \( R_p = \{ r_{1p}, r_{2p}, \ldots, r_{pp} \} \) | \( W_p = \{ A_{1p}, A_{2p}, \ldots, A_{pp} \}, \) where, \( A_{1p} + A_{2p} + \ldots + A_{pp} = 1 \) |

The larger the index value of the risk scale, the higher the risk. The greater the risk-benefit trade-off weights, the more the policy makers prefer the corresponding risk-return portfolio. The value of the value \( p \) is determined by the decision maker based on subjective and objective conditions.

\[ \forall x \in U, \] the membership degree of element \( x \) to Vague set \( V \) is in the interval \([t_x, 1 - f_x]\), so the membership degree of element \( x \) to the fuzzy set \( F \) is also in interval \([t_x, 1 - f_x]\). Using a line segment of length \( \frac{M}{P} \) to divide the interval \([0,1 - t_x - f_x]\), a series of segmentation points \( Q \) (including the right endpoint of the interval) are obtained, and the number of segmentation points is recorded as \( k[x] \), and these segmentation points are respectively denoted as \( y_i, i = 1,2, \ldots, k[x] \). The formula for calculating the value of risk in value \( F_x \) is:

\[ R(x) = \sum_{i=1}^{k[x]} \lambda^k[y_i] \]

Whether the element \( x \) belongs to the Vague set \( V \) has certain degree of hesitation \( F_x \). This kind of hesitant degree tends to support the element \( x \) belonging to the Vague set \( V \), which is affected by the following two factors: the promotion of support degree \( t_x \) and the resistance effect of opposing degree \( f_x \). The formula for calculating the value of tendencies is:

\[ P(x) = \frac{t_x}{t_x + f_x} \]

When \( t_x = 0 \) and \( f_x = 0 \), there is no tendency to support and oppose any. So \( P(x) = 0 \), the value of \( F_x \) has only \( R_x \). From the above analysis, the transformation of the Vague set \( V \) to the Fuzzy set \( F \) can be achieved by the following double weighted transformation operators.
4. Numerical Analysis

The decision-maker's weight value for the value of the risk and the value of the trend value is obtained. The method described in the third part of the paper is compared with the method constructed in this paper.

![Figure 1. Method 1](image1)

![Figure 2. Method 2](image2)

![Figure 3. Method 3](image3)

From the results of the transformation in Figure 5, it can be seen that since the results estimated in the method 1-3 are different, even the difference is still very obvious. This shows that the idea of
model building is defective. The transformation model constructed in this paper deals with the hesitation of the Vague set by introducing the risk preference of the decision-maker. From the result of transformation, it can be seen that the method constructed in this paper can reflect the influence of the degree of support and opposition to the hesitancy tendency. However, when decision-makers have different risk preferences, the final conversion results should also be different, which is consistent with the common sense. From Figure 5, it can be seen that the conversion result of curve F1 indicates that the decision maker is risk aversion. The result of the conversion of curve F2 indicates that the decision maker is risk-neutral. The result of the transformation of curve F3 indicates that the decision maker is risk appetite. The transformation results of method 1, method 2 and method 3 constructed in the previous literature are basically located in the area enclosed by curves F3 and F1, and they are only special cases under specific risk preferences. Therefore, the method constructed in this paper is more reasonable.

![Figure 5. Comparison of Conversion Results](image)

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