Clockwork origin of neutrino mixings

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The clockwork mechanism provides a natural way to obtain hierarchical masses and couplings in a theory. We propose a clockwork model which has nine clockwork generations. In this model, the candidates of the origin of the neutrino mixings is nine Yukawa mass elements $Y^{a\beta}$ which connect neutrinos and clockwork fermions, nine clockwork mass ratios $q_{a\beta}$ and nine numbers of clockwork fermions $n_{a\beta}$, where $a, \beta = 1, 2, 3$. Assuming $|Y^{a\beta}| = 1$, the neutrino mixings are origin from pure clockwork sector. We show that the observed neutrino mixings are exactly obtained from a clockwork model in the case of $q_{a\beta}$ origin scenario. In the $n_{a\beta}$ origin scenario, the correct order of magnitude of the observed neutrino mixings are obtained from a clockwork model.

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I. INTRODUCTION

One of the outstanding problems in the particle physics is the origin of the neutrino masses and mixings [1]. There are theoretical mechanisms to generate tiny neutrino masses, such as seesaw mechanisms [2, 3], radiative mechanisms [4, 13] and scotogenic model [14]. On the other hand, the neutrino mixings have been studied under assumptions of the existence of underlying flavor symmetries in the theories [15–17].

Recently, a new mechanism, the clockwork mechanism [18], attracts attention. The clockwork mechanism provides a new way to obtain hierarchical masses and couplings in a theory. In a series of the gears in a clock, large (small) movement of the gear in one side of the series can generate a small (large) movement of the gear in the opposite side. In the theories based on the clockwork mechanisms, large number of fields, so-called clockwork gears, are introduced. The zero mode state of the clockwork gears $\psi_R^{(0)}$ in the one side of the series of the clockwork gears connect to the gear in the opposite side $\psi_R$ via intermediate gears. We obtain the following relation

$$\psi_R \sim \frac{1}{q^n} \psi_R^{(0)},$$

where $q$ ($q > 1$) denotes the mass ratio of the gears and $n$ denotes the number of gears [19, 20]. Even if the mass ratio $q$ is not so hierarchical, e.g. $q = 1.5, q = 2.0$, etc, a large suppression factor $1/q^n$ for large $n$ may provide a small coupling or mass for $\psi_R$ in the model. The applications of the clockwork mechanism have been extensively studied in the literature, e.g., for axion [21, 30], for inflation [31, 32], for dark matter [33, 37], for muon $g - 2$ [38], for string theory [34, 41], for gravity [42, 43], for charged fermion masses and mixings [19] and for quark masses and mixings [44].

The applications of the clockwork mechanism for the neutrino sector have been studied for tiny neutrino masses [20, 45, 46] and for their masses and mixings [47].

Ibarra, et. al. In this model, the neutrino mass $m_\nu^{a\beta}$ is obtained as

$$m_\nu^{a\beta} = f(Y^{a\beta}, q_{a\beta}, n_{a\beta}),$$

where $a, \beta = 1, 2, 3$ and $\beta \geq 2$ for observed three neutrino generations, $Y^{a\beta}$ denotes the Yukawa coupling (which connects the standard model sector to the clockwork sector), $q_{a\beta}$ is the clockwork mass ratio and $n_{a\beta}$ is number of clockwork fields in the $\beta$-th clockwork generation. The main role of the clockwork sector, e.g., $q_{a\beta}$ and $n_{a\beta}$, is genesis of the tiny neutrino masses. On the other hand, the mixings of the neutrinos are originated from the Yukawa couplings.

In this paper, we extend the clockwork model proposed by Ibarra et. al. [47] to propose a clockwork model which has nine clockwork generations. In the extended model, only three clockwork generations can couple with one generation of the standard model lepton doublet, other three clockwork generations can only couple with other one generation of the lepton doublet and the remaining three clockwork generations can only couple with the remaining one generation of the lepton doublet. The final expression of neutrino mass is obtained as a function of the $Y^{a\beta}$, $q_{a\beta}$ and $n_{a\beta}$:

$$m_\nu^{a\beta} = f(Y^{a\beta}, q_{a\beta}, n_{a\beta}),$$

where $a, \beta = 1, 2, 3$. In this model, not only the Yukawa coupling $Y^{a\beta}$ but also $q_{a\beta}$ and $n_{a\beta}$ can be origin of the neutrino mixings. Indeed, we will show that a model with the democratic Yukawa matrix $|Y^{a\beta}| = 1$ is consistent with the observed neutrino masses and mixings. In this case, the mixings of the neutrinos are originated from the clockwork fields instead of the Yukawa couplings.

The paper is organized as follows. In Sec. III we present brief review of the neutrinos masses and mixings, and the fermion clockwork mechanisms. In Sec. IV we propose a clockwork model which has the origin of the neutrino mixings in the pure clockwork sector. Sec. V is devoted to a summary.

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II. BRIEF REVIEWS

A. Observed neutrino masses and mixings

The simple clockwork model of fermions yields the Dirac neutrinos [18]. Although the models of the Majorana neutrinos with the clockwork mechanisms are discussed [20, 47], we assume that the neutrinos are Dirac particles for simplicity.

The neutrino mass matrix
\[ m_\nu = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}, \]  
(4)
satisfies the following relation [48]
\[ m_\nu m_\nu^\dagger = U_{\text{PMNS}} \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U_{\text{PMNS}}^\dagger, \]  
(5)
where \( m_1, m_2 \) and \( m_3 \) denote the neutrino mass eigenstates and
\[ U_{\text{PMNS}} = \begin{pmatrix} c_{12} c_{13} s_{12} c_{13} & s_{12} c_{13} & c_{13} \\ -s_{12} c_{23} - s_{12} s_{23} s_{13} & c_{12} c_{23} - s_{12} s_{23} s_{13} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} & -c_{12} s_{23} - s_{12} c_{23} s_{13} & c_{23} c_{13} \end{pmatrix}, \]  
(6)
denotes the mixing matrix [49]. We use abbreviation \( c_{ij} = \cos \theta_{ij} \) and \( s_{ij} = \sin \theta_{ij} \) (\( i,j = 1,2,3 \)) and ignore the CP violating phase.

Although the neutrino mass ordering (either the normal mass ordering \( m_1 < m_2 < m_3 \) or the inverted mass ordering \( m_3 < m_1 < m_2 \)) is unsolved problems, a global analysis shows that the preference for the normal mass ordering is mostly due to neutrino oscillation measurements [50, 51]. We assume the normal mass ordering. The best-fit values of the squared mass differences \( \Delta m^2_{ij} = m^2_i - m^2_j \) and the mixing angles for normal mass ordering are estimated as [52]
\[ \Delta m^2_{21}/(10^{-5}\text{eV}^2) = 7.50 \ (7.03 - 8.09), \]
\[ \Delta m^2_{31}/(10^{-3}\text{eV}^2) = 2.524 \ (2.407 - 2.643), \]
\[ \theta_{12}/^\circ = 33.56 \ (31.38 - 35.99), \]
\[ \theta_{23}/^\circ = 41.6 \ (38.4 - 52.8), \]
\[ \theta_{13}/^\circ = 8.46 \ (7.99 - 8.90), \]  
(7)
where the parentheses denotes 3\( \sigma \) region. With the best-fit values, the neutrino mass matrix is to be
\[ m_\nu = \begin{pmatrix} 0.824 m_1 & 0.547 m_2 & 0.147 m_3 \\ -0.495 m_1 & 0.569 m_2 & 0.657 m_3 \\ 0.275 m_1 & -0.614 m_2 & 0.740 m_3 \end{pmatrix}, \]  
(8)
where
\[ m_2 \approx \sqrt{7.50 \times 10^{-5} + m_1^2} \text{ eV}, \]
\[ m_3 \approx \sqrt{2.524 \times 10^{-3} + m_1^2} \text{ eV}. \]  
(9)

B. Fermionic clockwork mechanism

In the clockwork sector, there are \( n \) left-handed chiral fermions: \( \psi_{Li}, (i = 0,1,\cdots,n-1) \) and \( n+1 \) right-handed chiral fermions: \( \psi_{Ri}, (i = 0,1,\cdots,n) \). The clockwork Lagrangian is [18, 32, 47]
\[ \mathcal{L}_{\text{cw}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{nearest}} + \mathcal{L}_{\text{mass}}, \]  
(10)
where \( \mathcal{L}_{\text{kin}} \) denotes the kinetic term for clockwork fermions,
\[ \mathcal{L}_{\text{nearest}} = -\sum_{i=0}^{n-1} (m_i \bar{\psi}_{Li} \psi_{Ri} - m_i' \bar{\psi}_{Li} \psi_{Ri+1} + \text{h.c.}) \],
(11)
denotes the nearest neighbor interaction term and
\[ \mathcal{L}_{\text{mass}} = -\frac{1}{2} \sum_{i=0}^{n-1} M_{Li} \bar{\psi}_{Li} \psi_{Li} - \frac{1}{2} \sum_{i=0}^{n} M_{Ri} \bar{\psi}_{Ri} \psi_{Ri}, \]  
(12)
denotes the Majorana mass term. For simplicity, we take the universal Dirac mass assumption: \( m_i = m, m_i' = m q \) and the universal Majorana mass assumption: \( M_{Li} = M_{Ri} = m q \) for all [47]. The nearest neighbor interaction term can be written in the following simple form
\[ \mathcal{L}_{\text{nearest}} = -\frac{1}{2} (\bar{\Psi} \mathcal{M} \Psi + \text{h.c.}), \]  
(13)
where
\[ \Psi = (\psi_{L0}, \psi_{L1}, \cdots, \psi_{Ln-1}, \psi_{R0}^c, \psi_{R1}^c, \cdots, \psi_{Rn}^c), \]  
(14)
and
\[ \mathcal{M} = m \begin{pmatrix} \bar{q} & 0 & \cdots & 0 & 1 & -q & \cdots & 0 \\ 0 & \bar{q} & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \bar{q} & 0 & 0 & \cdots & -q \\ 1 & 0 & \cdots & 0 & \bar{q} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -q & 0 & 0 & \cdots & \bar{q} \end{pmatrix}. \]  
(15)
The eigenvalues of the \((2n+1) \times (2n+1)\) matrix \( \mathcal{M} \) are obtained as [36]
\[ m_0 = m q, \]
\[ m_k = m \left( \bar{q} - \sqrt{\lambda_k} \right), \ k = 1,\cdots,n, \]
\[ m_{n+k} = m \left( \bar{q} + \sqrt{\lambda_k} \right), \ k = 1,\cdots,n, \]  
(16)
where
\[ \lambda_k = q^2 + 1 - 2q \cos \frac{k \pi}{n+1}. \]  
(17)
The interaction eigenstates $\Psi_i$ and mass eigenstates, denoted by $\chi_i$, are related each other by the unitary transformation $\Psi_i = \sum_j U_{ij} \chi_j$, where $U$ is the following $(2n + 1) \times (2n + 1)$ unitary matrix

$$U = \left( \begin{array}{cc} \tilde{\Omega} & \frac{1}{\sqrt{2}} U_L \sqrt{\frac{1}{2}} ; \sqrt{\frac{1}{2}} U_R \end{array} \right),$$

with

$$\tilde{\Omega}_i = 0, \quad i = 1, \ldots, n,$$

$$(\tilde{u}_R)i = \frac{1}{q^i} \frac{\sqrt{q^i - 1}}{q^i - q^{2n}}, \quad i = 0, \ldots, n,$$

$$(U_L)_{ij} = \sqrt{\frac{2}{n+1}} \sin \frac{ij\pi}{n+1}, \quad i, j = 1, \ldots, n,$$

$$(U_R)_{ij} = \sqrt{\frac{2}{(n+1)\lambda_j}} \left( q \sin \frac{ij\pi}{n+1} - \sin \frac{(i+j)\pi}{n+1} \right), \quad i = 0, \ldots, n, \quad j = 1, \ldots, n. \quad (19)$$

The total Lagrangian of the standard model with clockwork sector reads

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{cw} + \mathcal{L}_{int},$$

where $\mathcal{L}_{SM}$ is the standard model Lagrangian and $\mathcal{L}_{int}$ describes the interactions between the standard model sector and the clockwork sector. We assume that the last site of the clockwork fields only couples to the left-handed neutrinos (left-handed lepton doublets) in the standard model [8]

$$\mathcal{L}_{int} = -Y \tilde{H} L \psi_{Rn},$$

where $L$ denotes the left-handed lepton doublet, $\tilde{H} = v \tau_2 H^*$ ($H$ denotes the standard model Higgs doublet) and $Y$ denotes Yukawa matrix. In the terms of the mass eigenstates, we have

$$\mathcal{L}_{int} = -Y \tilde{H} L U_{nk} \chi_k = -\sum_{k=0}^{2n} Y_k \tilde{H} \chi_k,$$

where

$$Y_0 = Y(U_R)n = Y \frac{\sqrt{q^2 - 1}}{q^2 - q^{2n}},$$

$$Y_k = Y_{n+k} = \frac{1}{\sqrt{2}} Y(U_R)nk$$

$$= Y \sqrt{\frac{1}{(n+1)\lambda_j}} \left( q \sin \frac{nk\pi}{n+1} \right), \quad k = 1, \ldots, n.,$$

Now we generalize the above setup to three leptonic generations and $N$ clockwork generations. The nearest neighbor interaction term for $N$ clockwork generations is

$$\mathcal{L}_{\text{nearest}} = -\sum_{k=0}^{2n} \left( m_{\alpha \beta} \bar{\psi}^\alpha L \psi_{Rk} - m_{\alpha \beta} \bar{\psi}^\alpha L \psi_{Rk+1} + \text{h.c.} \right),$$

where $\alpha, \beta = 1, \ldots N$. For simplicity, we assume $m_{\alpha \beta} = m_d^{\alpha \beta}$, $m_{\alpha \beta} = m_q^{\alpha \beta}$ and $M_L^{\alpha \beta} = M_{Rk}^{\alpha \beta} = m_d^{\alpha \beta} = 0$ [47]. The nearest neighbor interaction term can be

$$\mathcal{L}_{\text{nearest}} = -\frac{1}{2} (\bar{\Psi} a^\alpha M^{\alpha \beta} \Psi^\beta + \text{h.c.}),$$

where

$$\Psi^\alpha = (\psi_{L0}^\alpha, \psi_{L1}^\alpha, \ldots, \psi_{L_{N-1}}^\alpha, \psi_{R0}^\alpha, \psi_{R1}^\alpha, \ldots, \psi_{RN}^\alpha).$$

In the terms of the mass eigenstates $\chi^\alpha = \sum_{\beta} U^{\alpha \beta} \chi^\beta$, the interactions between the left-handed neutrinos and clockwork fields can be written as

$$\mathcal{L}_{\text{int}} = -\sum_{k=0}^{2n} Y_k^{\alpha \beta} \bar{L} a^\alpha \tilde{H} \chi^\beta_k,$$

where $a = 1, 2, 3$. We define new fields $N_L^\alpha = (\nu_L^1, N_L^1, \ldots, N_L^N)$ and $N_R^\alpha = (N_R^0, N_R^1, \ldots, N_R^N)$

where

$$N_{Lk} = \frac{1}{\sqrt{2}} (-\chi_k^L + \chi_k^L n), \quad k = 1, \ldots, n,$$

$$N_{Rk} = \frac{1}{\sqrt{2}} (\chi_k^R + \chi_k^R n), \quad k = 0, \ldots, n. \quad (28)$$

for $\alpha = 1, \ldots, N$. The nearest neighbor interaction term can be cast as:

$$\mathcal{L}_{\text{nearest}} = -N_L^{2} \nu Y_{nk}^{\alpha \beta} + \text{h.c.},$$

and we have the following interaction Lagrangian

$$\mathcal{L}_{\text{int}} = -\sum_{k=0}^{n} Y_k^{\alpha \beta} \bar{L} a^\alpha \tilde{H} \chi^\beta_{nk},$$

with $Y_k^{\alpha \beta} = Y^{\alpha \beta} U_{nk}$ for the Dirac neutrinos. After electroweak symmetry breaking, the neutrino mass matrix is to be

$$m_{\nu} = \begin{pmatrix}
N_{R0}^\beta & N_{R1}^\beta & N_{R2}^\beta & \cdots & N_{Rn}^\beta \\

\nu_{L0} & \nu_{L1} & \nu_{L2} & \cdots & \nu_{LN} \\

N_{L0} & 0 & 0 & \cdots & 0 \\

N_{L1} & 0 & 0 & \cdots & 0 \\

\vdots & \vdots & \vdots & \ddots & \vdots \\

N_{Ln} & 0 & 0 & \cdots & 0 \\
\end{pmatrix},$$

where $v = 246/\sqrt{2}$ GeV denotes the vacuum expectation value of the standard model Higgs field and $M_k^\beta$ denotes the mass of the $k$-th clockwork fields for the Dirac pair $(N_L^k, N_{Rk}^\beta)$.

Assuming $M_k^\beta \gg v Y_{0}^{\alpha \beta}$, the active neutrino masses are obtained as

$$m_{\nu}^{\alpha \beta} = v Y_{0}^{\alpha \beta} = v Y_{0}^{\alpha \beta} \sqrt{\frac{q_{3}^2 - 1}{q_{3}^2 - q_{3}^2 - 2n^2}},$$

where $q_3$ and $n_3$ denote the clockwork mass ratio and the number of clockwork fermions in the $\beta$-th clockwork generation, respectively.
III. ORIGIN OF NEUTRINO MIXINGS

A. Model

We extend the clockwork model proposed by Ibarra et al., to propose a new clockwork model which has nine generations in the clockwork sector. We assume that only three clockwork generations can couple with one generation of the standard model lepton doublet, other three clockwork generations can only couple with other one generation of the lepton doublet and the remaining three clockwork generations can only couple with the remaining one generation of the lepton doublet.

Under these assumptions, the interaction Lagrangian, in terms of \( N_R \), for three leptonic generations and nine clockwork generations is

\[
\mathcal{L}_{\text{int}} = -\sum_{k} Y^\alpha_{\beta} \tilde{L}^a \tilde{H}_0 N^\beta_{Rk},
\]

where \( a = 1, 2, 3, \beta = 1, \cdots, 9 \),

\[
Y_{\beta}^{\alpha} = \begin{cases} 0, \text{ others}, & \beta = 1, 2, 3, a = 1 \\
4, 5, 6, & a = 2, \\
7, 8, 9, & a = 3 \end{cases},
\]

or equivalently,

\[
Y_{\beta}^{\alpha} = \begin{pmatrix} ** & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & ** & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & ** & 0 & 0 \end{pmatrix},
\]

where * denotes nonzero values. If we assign the lepton number to the clockwork sector as shown in TABLE I and assume the lepton number is conserved in the interactions of \( Y_{\beta}^{\alpha} \tilde{L}^a \tilde{H}_0 N^\beta_{Rk} \), we obtain the configuration in Eq. (34).

The interaction Lagrangian reads:

\[
\mathcal{L}_{\text{int}} = \begin{cases} 
-\tilde{H}_0 \tilde{L}^1 \left( \sum_{k} Y_{11}^{11} N_{1k}^1 + \sum_{k} Y_{12}^{12} N_{1k}^2 + \sum_{k} Y_{13}^{13} N_{1k}^3 \right) \\
-\tilde{H}_0 \tilde{L}^2 \left( \sum_{k} Y_{24}^{24} N_{2k}^4 + \sum_{k} Y_{25}^{25} N_{2k}^5 + \sum_{k} Y_{26}^{26} N_{2k}^6 \right) \\
-\tilde{H}_0 \tilde{L}^3 \left( \sum_{k} Y_{37}^{37} N_{3k}^7 + \sum_{k} Y_{38}^{38} N_{3k}^8 + \sum_{k} Y_{39}^{39} N_{3k}^9 \right). 
\end{cases}
\]

Assuming \( M_\nu^3 >> vY_0^{\alpha \beta} \), after electroweak symmetry breaking, the neutrino masses are to be

\[
m_{\beta}^{\alpha} = vY_0^{\alpha \beta} \frac{q_{\beta}^2 - 1}{q_{\beta}^2 - q_{\beta}^{2n_3}}.
\]
where

\[ m_{\nu}^{a\beta} = \frac{\nu Y^{a\beta}}{q_{a\beta}} \sqrt{\frac{q_{a\beta}^2 - 1}{q_{a\beta}^2 - q_{a\beta}^{-2n_{a\beta}}}}, \]

\[ a = 1, 2, 3, \quad \beta = 1, 2, 3. \] \hspace{1cm} (46)

There are the following four possible origin of the neutrino mixings in this model:

(a) Yukawa matrix $Y^{a\beta}$ \cite{47},

(b) clockwork mass ratio in the $a\beta$-th generations $q_{a\beta}$,

(c) number of clockwork fermions in the $a\beta$-th generations $n_{a\beta}$,

(d) others (both of $Y^{a\beta}$ and $q_{a\beta}$, etc).

We assume a democratic form of the Yukawa matrix

\[ |Y^{a\beta}| = 1, \] \hspace{1cm} (47)

more concretely,

\[ Y^{a\beta} = \begin{pmatrix}
\text{sign}(m_{\nu}^{11}) & \text{sign}(m_{\nu}^{12}) & \text{sign}(m_{\nu}^{13}) \\
\text{sign}(m_{\nu}^{21}) & \text{sign}(m_{\nu}^{22}) & \text{sign}(m_{\nu}^{23}) \\
\text{sign}(m_{\nu}^{31}) & \text{sign}(m_{\nu}^{32}) & \text{sign}(m_{\nu}^{33})
\end{pmatrix}. \] \hspace{1cm} (48)

In this case, the neutrino mass $m_{\nu}^{a\beta}$ only depends on the clockwork mass ratio $q_{a\beta}$ and number of clockwork fermions $n_{a\beta}$:

\[ |m_{\nu}^{a\beta}| = \frac{\nu}{q_{a\beta}} \sqrt{\frac{q_{a\beta}^2 - 1}{q_{a\beta}^2 - q_{a\beta}^{-2n_{a\beta}}}}, \] \hspace{1cm} (49)

and the case (b), (c) and (d) are relevant for possible origin of the neutrino mixing. In what follows, we study the cases (b), (c) and (d).

### B. $q_{a\beta}$ origin with universal $n$

First, we assume that the number of clockwork fermions is common for all clockwork generations. In this case, the origin of the neutrino mixings is the clockwork mass ratios $q_{a\beta}$ (case (b) in Sec.III A). For example, if we take the following universal number of clockwork fermions

\[ n = n_{a\beta} = 50, \] \hspace{1cm} (50)

for all $a$ and $\beta$, the following clockwork mass ratios

\[ \begin{pmatrix}
q_{11} & q_{12} & q_{13} \\
q_{21} & q_{22} & q_{23} \\
q_{31} & q_{32} & q_{33}
\end{pmatrix} = \begin{pmatrix}
1.841 & 1.845 & 1.844 \\
1.860 & 1.844 & 1.789 \\
1.882 & 1.841 & 1.785
\end{pmatrix}, \] \hspace{1cm} (51)

yield the best-fit values of the squared mass differences and the mixing angles in Eq.1 for $m_1 = 0.01 \text{eV}$.

**FIG. 1**: The magnitude of the clockwork mass ratio $q_{a\beta}$ for the best-fit values of the squared mass differences and the mixing angles under the normal mass ordering condition, where $n$ denotes the universal number of fermions for all clockwork generations ($n = 30$ in the upper panel and $n = 50$ in the lower panel).

Figure 1 shows the magnitude of the clockwork mass ratio $q_{a\beta}$ for the best-fit values of the squared mass differences and the mixing angles under the normal mass ordering condition, where $n$ denotes the universal number of fermions for all clockwork generations ($n = 30$ in the upper panel and $n = 50$ in the lower panel). The upper limit of $m_1 \leq 0.03$ is obtained from the observed data $m_\nu < 0.120$ by the Planck collaboration \cite{53}.

Because $q_{a3}$ ($a = 1, 2, 3$) depends on $m_{\nu}^{a3} \propto m_3$ and $m_4 = \sqrt{2.524 \times 10^{-3} + m_1^2} \sim \sqrt{2.524 \times 10^{-3}} \text{ eV}$ for $m_1 \leq 0.03$, the magnitude of $q_{13}$, $q_{23}$ and $q_{33}$ are almost independent of the rightest neutrino mass $m_1$ as we see in FIG.1.

### C. $n_{a\beta}$ origin with universal $q$

Second, we assume that the clockwork mass ratio is common for all clockwork generations. In this case, the origin of the neutrino mixings is the number of the clockwork fermions $n_{a\beta}$ (case (c) in Sec.III A). For example, if we take the following universal clockwork mass ratio

\[ q = q_{a\beta} = 2.01, \] \hspace{1cm} (52)
for all $a$ and $\beta$, the following numbers of clockwork fermions

$$
\begin{pmatrix}
n_{11} & n_{12} & n_{13} \\
n_{21} & n_{22} & n_{23} \\
n_{31} & n_{32} & n_{33}
\end{pmatrix} =
\begin{pmatrix}
44 & 44 & 44 \\
45 & 44 & 42 \\
46 & 44 & 42
\end{pmatrix},
$$

(53)
yield

$$
\begin{align*}
\Delta m^2_{21} &= 8.23 \times 10^{-5} \text{eV}^2, \\
\Delta m^2_{31} &= 1.53 \times 10^{-3} \text{eV}^2, \\
\theta_{12} &= 40.4^\circ, \\
\theta_{23} &= 45.5^\circ, \\
\theta_{13} &= 10.2^\circ.
\end{align*}
$$

(54)

Although these predicted values (excepted with $\theta_{23}$) are out of range of the $3 \sigma$ region in Eq.(7), the order of the magnitude of these values are consistent with the observed data.

We should perform more general parameter search with various set of the universal clockwork mass ratio and number of clockwork fermions $\{q, n_{a\beta}\}$; however, this is a numerical challenging task. For example, there are $\sim 10^{21}$ loops in the code to perform a numerical search for $q = 1.01, 1.02, \cdots, 4.00$, $n_{a\beta} = 10, 11, \cdots, 100$ and $m_1 = 0.001, 0.002, \cdots, 0.03$ eV for only the best-fit values of the neutrino parameters. In this paper, we abort such a full parameter search and only show some examples of the parameter set which are consistent with neutrino observations.

### D. $n_{a\beta}$ origin with quasi-universal $q$

If we relax the universal $q$ requirement and allow the existence of the small perturbations of the clockwork mass ratios, $\Delta q$ ($\Delta q \ll q$), we can obtain the collect neutrino mass parameters within the $n_{a\beta}$ origin scenario of the neutrino mixings (case (c)) with small correction of the universal clockwork mass ratio in Sec IIIA. For example, if we take the following quasi-universal clockwork mass ratios,

$$
\begin{pmatrix}
q_{11} & q_{12} & q_{13} \\
q_{21} & q_{22} & q_{23} \\
q_{31} & q_{32} & q_{33}
\end{pmatrix} = q + \Delta q
= 2.01 \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix}
+ \begin{pmatrix}
0.01 & 0 & -0.002 \\
-0.02 & 0.005 & -0.008 \\
-0.011 & 0 & -0.014
\end{pmatrix},
$$

(55)

the following number of clockwork fermions, same as Eq.(53),

$$
\begin{pmatrix}
n_{11} & n_{12} & n_{13} \\
n_{21} & n_{22} & n_{23} \\
n_{31} & n_{32} & n_{33}
\end{pmatrix} =
\begin{pmatrix}
44 & 44 & 44 \\
45 & 44 & 42 \\
46 & 44 & 42
\end{pmatrix},
$$

(56)
yield

$$
\begin{align*}
\Delta m^2_{21} &= 7.29 \times 10^{-5} \text{eV}^2, \\
\Delta m^2_{31} &= 2.46 \times 10^{-3} \text{eV}^2, \\
\theta_{12} &= 31.7^\circ, \\
\theta_{23} &= 41.6^\circ, \\
\theta_{13} &= 8.08^\circ.
\end{align*}
$$

(57)

These predicted values are consistent with the observed data in Eq.(7).

### E. effective $n_{a\beta}$ origin with universal $q$

Finally, we show an alternative way to obtain the correct neutrino mixings with the $n_{a\beta}$ origin scenario for a universal clockwork mass ratio $q$.

Although the number of the clockwork fermions in the $a\beta$-th clockwork generation $n_{a\beta}$ should be a real integer number, we relax this requirement (small correction of case (c) in Sec IIIA). In this case, for example, if we take the following universal clockwork mass ratio

$$
q = q_{a\beta} = 2,
$$

(58)

for all $a$ and $\beta$, the following effective numbers of clock-
work fermions

\[
\begin{pmatrix}
n_{11} & n_{12} & n_{13} \\
n_{21} & n_{22} & n_{23} \\
n_{31} & n_{32} & n_{33}
\end{pmatrix} = \begin{pmatrix}
44.06 & 44.24 & 44.18 \\
44.79 & 44.19 & 42.03 \\
45.64 & 44.08 & 41.85
\end{pmatrix}, \quad (59)
\]

yield the best-fit values of the squared mass differences and the mixing angles in Eq. (1) for \( m_1 = 0.01 \text{eV} \).

Figure 2 shows the magnitude of the effective number of clockwork fermions \( n_{a\beta} \) for the best-fit values of the squared mass differences and the mixing angles under the normal mass ordering condition, where \( q \) denotes universal clockwork mass ratio (\( q = 2.0 \) in the upper panel and \( q = 2.5 \) in the lower panel).

### IV. SUMMARY

We have proposed a clockwork model which has nine clockwork generations. Only three clockwork generations can couple with one generation of the standard model lepton doublet, other three clockwork generations can only couple with another generation of the lepton doublet and the remaining three clockwork generations can only couple with the remaining one generation of the lepton doublet. Under these assumptions, the neutrino masses depend on the nine Yukawa matrix elements \( Y^{a\beta} \), nine clockwork mass ratios \( q_{a\beta} \) and nine numbers of clockwork fermions \( n_{a\beta} \). In this model, the candidates of the origins of the neutrino mixings are \( Y^{a\beta}, q_{a\beta} \) and \( n_{a\beta} \). We have assumed \( |Y^{a\beta}| = 1 \), thus the Yukawa coupling is not main origin of the neutrino mixings. The main origin of the neutrino mixing is in the clockwork sector, \( q_{a\beta} \) and \( n_{a\beta} \), in this model.

We have shown that the observed neutrino mixings are exactly obtained with a clockwork model in the case of \( q_{a\beta} \) origin scenario. In the \( n_{a\beta} \) origin scenario, although the predicted values (excepted with \( \theta_{23} \)) are out of range of the 3\( \sigma \) region, the correct order of magnitude of the observed neutrino mixings are obtained from a clockwork model. To obtain the neutrino parameters within 3\( \sigma \) region in the \( n_{a\beta} \) origin scenario, it is suggested that some modification schemes should be employed, such as the quasi-universal \( q \) or the effective \( n_{a\beta} \).

Finally, we would like comment on the collider phenomenology. In the model proposed in this paper, the neutrino masses are to be small via zero mode interactions of clockwork fermions: however, in general, unsuppressed effects at low energy phenomena, such as an unobserved lepton flavor violating decay \( \mu \to e\gamma \), are allowed. The upper bound of the lepton flavor violating processes yield constraints on the mass scale of the clockwork fermions. Ibarra et.al. shown that the clockwork fermions must be larger than \( \sim 40 \text{ TeV} \) in order to evade the experimental constraints [17]. In this paper, we have assumed that the clockwork fermions are heavy enough to consistent with collider phenomenology.

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