Ensemble kalman filter for crude oil price estimation

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Abstract. Oil has become the most important sector for the world economic sector. Since 1983, oil has been the main focus of economists after the known impact of oil prices on the economy of the United States after the Second World War. Oil has also been the key role to a world economy although its nature changes over time. The relationship between capital markets and commodities is one of the most challenging problems for investors. Turmoil in one market can affect other market price indexes. Crude Oil Prices are influenced by political conditions and weather-related factors, which can create an unexpected shift in influencing supply and demand. Oil price volatility can be resolved by estimating world crude oil prices so that economists can predict when world oil prices fall or rise and set policies in the purchase and use of crude oil. Estimates are made because a problem can normally be resolved using previous information or data related to the problem. The Kalman filter is a method of estimating the state variables from a discrete linear dynamic system that minimizes estimated covariance errors. The objective of this study is to estimate the price of crude oil using the Kalman Filter (KF) and Ensemble Kalman Filter (EnKF) method. The simulation results show that the EnKF method has a high accuracy of less than 2% and KF method has accuracy of less than 8%.

1. Introduction
Oil has become the most important sector in the world economy. Since the 1980s, oil has been the main focus of economists after the impact of oil prices on the economy of the United States after the Second World War was observed. Oil has also become the key to the world economy even though its nature has changed from time to time. Unclear oil prices on the market affect the turnover of the industrial sector which then results in changes in the Gross Domestic Product (GDP) and inflation [1]. The relationship between commodities and the capital market is one of the most challenging problems for investors. Fluctuations in one market can affect other market price indices. Crude Oil Prices are influenced by political conditions and weather-related factors, which can create an unexpected shift in supply and demand that affects the value of the Oil Volatility Index (OVX). OVX is an index of offers given for Crude oil stock options listed in the Chicago Board Options Exchange (CBOE). Understanding OVX is very important, because its existence can lead to uncertainty in prices in all economic sectors and can cause economic stability or instability for exporting and importing countries. OVX can provide a description of the risks to producers and consumers whose existence is dependent on oil [2].

Economic instability affects all sectors of life, so estimation or prediction is required to reduce such instability. Many studies on estimation are carried out in all scientific fields, including estimation of stock prices [3], estimation of water level in the boiler steam drum [4], estimation of AUV trajectory [5,6], and estimation of missile trajectories [7]. Regarding oil price estimation, to reduce the instability of oil prices, estimating world crude oil prices can be taken into consideration so that economists can predict when world oil prices will fall or rise and determine policies in the purchase and use of crude oil. Therefore, this paper will apply the method of estimating world crude oil prices with Ensemble Kalman Filter (EnKF) to determine government policies related to world crude oil.
purpose of this paper is to develop and compare estimation methods for world crude oil prices by using the Kalman Filter (KF) and Ensemble Kalman Filter (EnKF) methods for estimating crude oil prices.

2. Kalman Filter (KF)
The algorithm of Kalman Filter (KF) can be seen [8]:

1. Model system and measurement model.
\[
x_{k+1} = A_k x_k + B_k u_k + G_k w_k \\
z_k = H_k x_k + v_k \\
x_0 \sim N(\bar{x}_0, P_{x_0}) \quad ; \quad w_k \sim N(0, Q_k) \quad ; \quad v_k \sim N(0, R_k)
\]

2. Initialization
\[
\hat{x}_0 = x_0 \\
P_0 = P_{x_0}
\]

3. Time Update
Estimation: \( \hat{x}_{k+1} = A_k \hat{x}_k + B_k u_k \)
Error covariance: \( P_k^- = A_k P_k^- A_k^T + G_k Q_k G_k^T \)

4. Measurement Update
Kalman gain: \( K_{k+1} = P_k^- H_{k+1}^T (H_{k+1} P_k^- H_{k+1}^T + R_{k+1})^{-1} \)
Estimation: \( \hat{x}_{k+1} = \hat{x}_{k+1}^- + K_{k+1} (z_{k+1} - H_{k+1} \hat{x}_{k+1}^-) \)
Error covariance: \( P_{k+1}^- = [I - K_{k+1} H_{k+1}] P_{k+1}^- \)

3. Ensemble Kalman Filter (EnKF)
The algorithm Ensemble Kalman Filter (EnKF) can be seen [9]:

1. Model system and measurement model
\[
x_{k+1} = f(x_k, u_k) + w_k \\
z_k = H_k x_k + v_k \\
w_k \sim N(0, Q_k) \quad , \quad v_k \sim N(0, R_k)
\]

2. Initialization
Generate \( N \) ensemble as the first guess \( \bar{x}_0 \)
\[
\bar{x}_{0,i} = [x_{0,1}, x_{0,2}, \ldots, x_{0,N}] \\
The first value: \( \hat{x}_0 = \frac{1}{N} \sum_{i=1}^{N} \bar{x}_{0,i} \)

3. Time Update
\[
\hat{x}_{k,i}^- = f(\hat{x}_{k-1,i}, u_{k-1,j}) + w_{k,i} \quad \text{where} \quad w_{k,i} \sim N(0, Q_k)
\]
Estimation: \( \hat{x}_k^- = \frac{1}{N} \sum_{i=1}^{N} \hat{x}_{k,i}^- \)
Error covariance:
\[
P_k^- = \frac{1}{N-1} \sum_{i=1}^{N} (\hat{x}_{k,i}^- - \hat{x}_k^-)(\hat{x}_{k,i}^- - \hat{x}_k^-)^T
\]

4. Measurement Update
\[
z_{k,i} = H_k x_{k,i} + v_{k,i} \quad \text{where} \quad v_{k,i} \sim N(0, R_k)
\]
Kalman gain: \( K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \)
Estimation: \( \hat{x}_{k,i} = \hat{x}_{k,i}^- + K_k (z_{k,i} - H_k \hat{x}_{k,i}^-) \)
\[ x_k = 1 \sum_{i=1}^{N} \tilde{x}_{k,i} \]  
\[ \text{Error covariance } P_k = [I - K_k H] P_k^- \]  
(22)  
(23)

To evaluate of estimation result accuracy from KF and EnKF algorithm, can be show with calculate Root Mean Square Error (RMSE) [10].

\[ \text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (x_{\text{abs},i}(k) - x_{\text{model},i}(k))^2}{n}} \]  
(24)

With

- \( x_{\text{abs},i}(k) \) = observation data
- \( x_{\text{model},i}(k) \) = model data
- \( n \) = iteration

### 4. Computational Result

This simulation of the application of the KF and EnKF algorithms to the crude oil functions obtained from Mathematical software simulation showed the oil crude data in Table 1. The simulation results were evaluated and compared to the established oil crude functions, and the crude oil price functions in equation (25) are as follows:

\[ f(x) = 50,2x^2 - 822,4x + 7942 \]
\[ f'(x) = 100,4x - 822,4 \]  
(25)

Due to the system requires discretation, so the crude oil functions model in equation (25) are discreted using the finite difference method. The change of state variables in respect to the time is approximated by forward scheme of finite difference. Then we get the following.

\[ f' = \frac{df}{dt} \approx \frac{f_{k+1} - f_k}{\Delta t} \]  
(26)

from equations (23) and (25), the modified the oil crude functions model in (26) is obtained as follows:

\[ f_{k+1} = (100,4x_k - 822,4)\Delta \]

Data of the World Crude Oil Prices are as follows:

| No | Trade Date     | Price |
|----|----------------|-------|
| 1  | 3-Jan-2017     | 52.33 |
| 2  | 3-Jan-2017     | 53.26 |
| 3  | 5-Jan-2017     | 53.76 |
| 4  | 6-Jan-2017     | 53.99 |
| 5  | 9-Jan-2017     | 51.96 |
| 6  | 10-Jan-2017    | 50.82 |
| 7  | 11-Jan-2017    | 52.25 |
| 8  | 12-Jan-2017    | 53.01 |
| 9  | 13-Jan-2017    | 52.37 |
| 10 | 17-Jan-2017    | 52.48 |
| 11 | 18-Jan-2017    | 51.08 |
| 12 | 20-Jan-2017    | 52.42 |
| 13 | 23-Jan-2017    | 52.75 |
| 14 | 24-Jan-2017    | 53.18 |
|   | Date       | Value  |
|---|------------|--------|
| 15| 25-Jan-2017| 52.75  |
| 16| 26-Jan-2017| 53.78  |
| 17| 27-Jan-2017| 53.17  |
| 18| 30-Jan-2017| 52.63  |
| 19| 31-Jan-2017| 52.81  |
| 20| 1-Feb-2017  | 53.88  |
| 21| 2-Feb-2017  | 53.54  |
| 22| 3-Feb-2017  | 53.83  |
| 23| 6-Feb-2017  | 53.01  |
| 24| 7-Feb-2017  | 52.17  |
| 25| 8-Feb-2017  | 52.34  |
| 26| 9-Feb-2017  | 53.00  |
| 27| 10-Feb-2017 | 53.86  |
| 28| 13-Feb-2017 | 52.93  |
| 29| 14-Feb-2017 | 53.20  |
| 30| 15-Feb-2017 | 53.11  |
| 31| 16-Feb-2017 | 53.36  |
| 32| 17-Feb-2017 | 53.40  |
| 33| 21-Feb-2017 | 54.06  |
| 34| 22-Feb-2017 | 53.59  |
| 35| 23-Feb-2017 | 54.45  |
| 36| 24-Feb-2017 | 53.99  |
| 37| 27-Feb-2017 | 54.05  |
| 38| 28-Feb-2017 | 54.01  |
| 39| 1-Mar-2017  | 53.83  |
| 40| 2-Mar-2017  | 52.61  |
| 41| 3-Mar-2017  | 53.33  |
| 42| 6-Mar-2017  | 53.20  |
| 43| 7-Mar-2017  | 53.14  |
| 44| 10-Mar-2017 | 48.49  |
| 45| 15-Mar-2017 | 48.86  |
| 46| 17-Mar-2017 | 48.78  |
| 47| 20-Mar-2017 | 48.22  |
| 48| 22-Mar-2017 | 48.04  |
| 49| 27-Mar-2017 | 47.73  |
| 50| 30-Mar-2017 | 50.35  |
| 51| 3-Apr-2017  | 50.24  |
| 52| 4-Apr-2017  | 51.03  |
| 53| 5-Apr-2017  | 51.15  |
| 54| 6-Apr-2017  | 51.70  |
| 55| 7-Apr-2017  | 52.24  |
| 56| 10-Apr-2017 | 53.08  |
| 57| 11-Apr-2017 | 53.40  |
| 58| 12-Apr-2017 | 53.11  |
| 59| 13-Apr-2017 | 53.18  |
| 60| 14-Apr-2017 | 52.65  |
In this study a simulation was carried out by applying the KF and EnKF algorithms to the function of crude oil. The simulation results were evaluated by comparing the real conditions in the field with those of the results of KF and EnKF estimates. This simulation used $\Delta t = 0.1$ and 300 iterations and generated 100, 200 and 300 ensembles. Figure 1 is a comparison of the estimated results of KF and those of EnKF which generated 100 ensembles. Figure 2 is the result of the simulation of the KF and EnKF methods by generating 200 ensembles. Figure 3 is a simulation of the KF and EnKF methods by generating 300 ensembles.

Figure 1. Estimation of crude oil prices using KF and EnKF method with 100 ensembles

Figure 2. Estimation of crude oil prices using KF and EnKF method with 200 ensembles
Figure 3. Estimation of crude oil price using KF and EnKF method with 300 ensembles

Figure 1 shows that the estimated crude oil price has a pattern that is almost the same as the price of real crude oil, where the estimated crude oil price using the EnKF method has high accuracy with an error of less than 2%, and RMSE of 0.26335. However, the estimation results using the KF method have a considerable error of around 8% with RMSE of 1.5082. In Figure 2 and Figure 3, it appears that the EnKF method has higher accuracy than the KF method, where the accuracy of the EnKF method is around 99%, while the KF method is around 90%. The EnKF method has higher accuracy than the KF method due to an number of ensembles generated. In Table 2, it appears that the EnKF method by generating 300 ensembles has higher accuracy than that by generating 100 and 200 ensembles, because in this case the number of ensembles generated also affects accuracy.

Table 2. Comparison of the values of RMSE using Ensemble Kalman Filter based on the generated number of 100, 200 dan 300 Ensemble.

|                | 100 Ensemble | 200 Ensemble | 300 Ensemble |
|----------------|--------------|--------------|--------------|
| crude oil prices | 0.26335      | 0.25987      | 0.24582      |
| Simulation Time  | 3.3123 s     | 5.9334 s     | 7.6231 s     |

In Table 3, it appears that the EnKF has much higher accuracy than the KF either with 100, 200 or 300 ensembles. However when viewed from the length of computation, the KF is faster due to the absence of generating a number of ensembles.

Table 3. Comparison the values of RMSE by the application of the KF and EnKF based on 100, 200 and 300 Ensembles.

|                | KF vs EnKF 100 ensemble | KF vs EnKF 200 ensemble | KF vs EnKF 300 ensemble |
|----------------|--------------------------|--------------------------|--------------------------|
| crude oil prices | KF 1.5082 EnKF 0.26335  | KF 2.1866 EnKF 0.25987   | KF 2.6543 EnKF 0.24582   |
| Simulation Time  | KF 3.2891 s EnKF 3.3123 s | KF 5.865 s EnKF 5.9334 s | KF 7.4691 s EnKF 7.6231 s |
In general the KF and EnKF method can be effectively used as a method to estimate crude oil prices with fairly good accuracy. The method EnKF has higher accuracy than the KF method, because there is a process of generating a number of ensembles at the stage of prediction and correction so as to make more accurate estimates. But the weakness of the EnKF method takes longer computation time than the KF method.

5. Conclusion
Based on the results of the simulation analysis, in general the KF and EnKF method can be effectively used as a method to estimate crude oil prices with fairly good accuracy. It appears that the EnKF has much higher accuracy than the KF either with 100, 200 or 300 ensembles. It could be concluded that the KF and EnKF methods could be applied to estimate crude oil functions with high accuracy. The resulting errors were less than 2% for EnKF method and less than 8% for KF method. Regarding the accuracy result in the implementation of both methods, it was observed that the EnKF method gave more accurate crude oil price estimation result than the KF method did.

Open problem. How to implemented Fuzzy Kalman Filter (FKF) and Unscented Kalman Filter (UKF) for estimation of crude oil price.

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References
[1] Gokmenoglua K, Azina V, Taspinara N 2015 The Relationship between Industrial Production, GDP, Inflation and Oil Price: The Case of Turkey Procedia Economics and Finance 25 pp 497 – 503.
[2] Pindyck R 2003 Volatility in Natural Gas and Oil Markets Cambridge: Massachusetts Institute of Technology.
[3] Karya D F, Puspandam K and Herlambang T 2018 Stock Price Estimation Using Ensemble Kalman Filter Square Root Methods Journal of Physics: Conf. Series 1008 012026.
[4] Herlambang T, Mufarrikoh Z, Fidita, D F Rahmania D 2018 Estimation Of Water Level And Steam Temperature In Steam Drum Boiler Using Ensemble Kalman Filter Square Root (EnKF-SR) Journal of Physics: Conf. Series 1008 012026.
[5] Ermayanti E Aprilini E Nurhadi H and Herlambang T 2015 Estimate and Control Position Autonomous Underwater Vehicle Based on Determined Trajectory using Fuzzy Kalman Filter Method International Conference on Advance Mechatronics, Intelligent Manufacture, and Industrial Automation (ICAMIMIA)-IEEE Surabaya Indonesia.
[6] Herlambang T, Nurhadi H and Subchan 2018 Trajectory Estimation of UNUSAITS AUV Based on Dynamical System with Ensemble Kalman Filter Square Root for Building Platform of Navigation and Guidance Control System Proc. of Design of UNUSAITS AUV Motion Control Using Sliding PID (SPID) IEEE ICETASIA 2018 Surakarta Indonesia.
[7] Herlambang T 2017 Design of a Navigation and Guidance System of Missile with Trajectory Estimation Using Ensemble Kalman Filter Square Root (EnKF-SR) International Conference on Computer Applications and Information Processing Technology (CAIPT)-IEEE Bali Indonesia.
[8] Kalman RE 1960 A New Approach to Linear Filtering and Prediction Problems ASME Journal of Basic Engineering vol 82 pp 35-45.
[9] Subchan, Herlambang T and Nurhadi H 2018 Navigation and Guidance System of Autonomous Underwater Vehicle with Trajectory Estimation of Nonlinear Modeling using Ensemble Kalman Filter Proc. of Design of UNUSAITS AUV Motion Control Using Sliding PID (SPID) IEEE, Surakarta, Indonesia.
[10] Herlambang T Djamiko EB and Nurhadi H 2015 Ensemble Kalman Filter with a Square Root Scheme (EnKF-SR) for Trajectory Estimation of AUV SEGOROGNI ITS *International Review of Mechanical Engineering* IREME Journal vol. 9 pp 553-560 ISSN 1970 – 8734.