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To The Mathematical Modeling of Nonstationary Transonic Flows. Analytical Approach

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Abstract. The transonic mode of gas flow has significant differences in comparison with both
subsonic and supersonic regimes, combining both their general properties and specific features
inherent in each of them, the possibility of penetration of weak disturbances upstream in
subsonic flows. The behavior of small disturbances in subsonic and supersonic flows is
qualitatively different. The paper proposes new analytical model for the mathematical
modeling of unsteady transonic flows, including the effect of weak viscosity.

1. Introduction
The transonic mode of gas flow has significant differences in comparison with both
subsonic and supersonic regimes, combining both their general properties and specific features inherent
in each of them (for example, the existence of shock waves or Mach angles in a supersonic stream),
the possibility of penetration (influence) of weak disturbances upstream in subsonic flows.
The behavior of small disturbances in subsonic and supersonic flows is qualitatively
different in a supersonic flow, these disturbances maintain amplitude along the characteristics
which they propagate, in a subsonic flow, they attenuate at a distance from the source, capturing
the entire subsonic flow zone. Such a variety of phenomena in their complex combination proves
[1] that in the study of transonic flows one can hardly rely on physical intuition or on considerations
of plausibility.
The equations of stationary gas flow show (see, for example, [2]) that at the sound velocity of
the flow the content model of the flow must be nonlinear. In this regard, the opinion was formed
that the equations of flow in the transonic velocity range are essentially nonlinear and it is impossible
to carry out linearization (the main method for simplifying the equations of the model).

2. Description of the mathematical model
A small parameter in problems of transonic currents is suggested by the physical peculiarity of
the problem - the proximity of the speed of the flow to the sonic velocity. A simplified analysis of
the equations of transonic flow is based on the transonic asymptotic expansions of the considered
functions in series of small deviations of the current value of the Mach number of the flow. In the
case of a nonstationary transonic flow, such a simplification of the equations of motion was carried
out by V. Lin, E. Reissner and S. Tsien, [3], which made it possible to reduce the original system to the
only equation - Lin-Reissner-Tsien's equation (LRTs)

\[ (K_x - (\gamma + 1)\phi_x)\phi_{xx} + \phi_{yy} - 2\phi_{x,y} = 0, \quad K_x = \frac{1 - M_{\infty}^2}{\varepsilon^{2/3}}, \quad M_{\infty} = \text{Mach number}, \quad \varepsilon \ll 1. \]
The LRTs-equation became basic in the study of unsteady transonic flows [4].

The development of the theory of gas flows historically developed from the study of subsonic flows to the study of supersonic flows, since such a course of events was determined by the capabilities and needs of aviation, but the generalization of the boundary layer model to pre-break modes for the first time was formulated for a supersonic flow by V.Ya. Neyland [5]. The resulting model was called, according to its structure, triple-deck. To emphasize the difference between the triple-deck model and the usual boundary layers, the name of a non-classical (self-induced pressure) boundary layer was also used for it.

The model was extended to the transonic mode by O.S. Ryzhov [6], on this model were obtained the main results in the study of non-classical transonic boundary layers. The initial version of the model showed the non-linearity of the flow equations in the upper deck. Further analysis found [7] that, in deriving the triple-deck model equations, it is possible to choose between maintaining the nonlinearity of the equations in the upper deck and taking into account the nonstationarity of the flow in the lower deck. The latter variant was used in determining the stability of the boundary layer to small perturbations, which is a linear problem. The LRTs equation was included in the model to describe the flow in the region of a non-viscous flow (upper deck).

The wealth of solutions obtained on a three-deck model for a transonic regime, however, overshadowed a very serious problem. Later it turned out [8] that the direction of research went the way leading away from reality - the classical model was singular, but this circumstance remained ignored. The LRTS equation, which has undoubted advantages (describes both supersonic and subsonic regions of the transonic flow, its non-one-dimensional, nonstationary and nonlinear character), however, has drawbacks that do not allow the propagation of nonstationary perturbations to be correctly described: this equation is degenerate hyperbolic equation and describes the propagation of non-stationary perturbations in the flow field only partly (only upstream). In this connection, a modified model was proposed for studying the problems of the theory of non-classical transonic boundary layer [8]. The modification of the model consists in the preservation (thus arising naturally) in the LRC equation, when it is deduced from the full equations for the potential, a singular (containing a small parameter) term of the transonic expansion with the second time derivative

\( (K\alpha - (\gamma + 1)\phi_x)\phi_{xx} + \phi_{yy} - 2\phi_{xt} - \varepsilon \phi_{tt} = 0 \)

The equation obtained in this way is nondegenerate hyperbolic, its inclusion in the mathematical model of the flow gives a regular physical picture of the flow field and allows one to consider perturbations propagating in all directions.

3. Some results

Regularization of the model to describe the flow field when using the modified LRC equation is already visible from the wave front patterns from a point source of weak acoustic disturbances flown around by a transonic flow, defined by the equation [9]

\[ K\alpha t^2 + 2t(x-x_0) - \varepsilon(x-x_0)^2 = (y-y_0)^2 (1 + \varepsilon K\alpha) \]

This equation describes the propagation of perturbations in the flow field in all directions, the wave fronts from a point source of weak perturbations now have a physically justified closed form.

Further analysis of unsteady transonic flows using the modified LRC equation gives the following results. Dispersion relation obtained by studying the stability of a transonic flow, simulated by a three-deck model, for initial data

\[ u_0 = y, \ v_0 = 0, \ p_0 = const \]

and disturbances of the form

\[ f(y)\exp(\omega t + ikx) \]

has the form [5]
The difficulty analytic detection of a perturbation damper [1] becomes qualitatively more complex. It can be seen that the existence of a single neutral value for a given is an exceptional case (there are either two of them, or there is no such value at all [11]), indicating that the conditions of flow stability / instability are much more complicated than previously thought.

4. Conclusion

It also turned out that the previously considered unambiguous dependence of the perturbation frequency on the wavenumber (with the exception of a small range of small values) is not [11]. For applications, this means, in particular, that the problem of controlling the boundary layer with the help of a perturbation damper [12] becomes qualitatively more complex.

The proposed model can be effectively used in the study of other problems of non-stationary transonic aerodynamics.

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