Analysis of Forced Convection heat transfer in laminar flow through a compact pipe filled with Nanofluids using CFD

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Abstract. In this paper, a Numerical investigation of developing laminar flows with forced convection through a compact circular pipe has been done using water-Al2O3 nano-fluid. Uniform heat flux (UHF) and the steady-state has been maintained at the wall in each model. Throughout the numerical experiments, Nano-fluid models were made by Alumina volume fraction and were processed under the Re=1050. A single-phase fluid model was determined by nano-fluid thermal and physical properties calculation, whereas the Two-phase model, i.e., granular mixture model was defined in 100nm diameter. The results show that the Al2O3 volume fraction increases, the heat transfer rate and Nusselt number increases. All the numerical simulations were developed in ANSYS FLUENT. The result displays the rise of thermal transfer from the volume fraction concentration.

Keywords: compact circular pipe, Nano-fluids, volume fraction, heat transfer rate, Nusselt number.

1. Introduction

To improve the energy transfer efficiency of heat exchangers, researchers across the globe are not only focusing on intensifying the structure of heat exchangers but improving the heat transfer fluid itself. The thermal conductivity can be increased by a higher volume fraction of solid particles. This was proven by Maxwell [1]. In modern times the particle diameter is smaller than 100nm which is known as ‘Nano-sized’. These smaller sized particles strengthen the performance of heat exchangers [2]. Besides, nanoparticles are extremely stable and they do not settle down under passive conditions thereby makes suspension of particles easier. Therefore, nanofluids behave as a perfect fluid to get maximum heat transfer [3]. However, there is no clear understanding of the heat transfer mechanism with nanofluids due to the lack of agreement with various research groups [4]. Due to this, the nanofluid development has been interrupted.

In this research work, a homogeneous flow mixture model [5] of Nanofluids was assumed where the nanofluid particles are moving with constant velocity. The main reason for this study is to observe the rise in the heat transfer coefficient in a compact pipe and compare it with a single and multiphase model. Aluminium oxide mixed with water nanoparticle material is chosen with diameter 100nm and volume fractions less than 5%. By employing the limit of the 2-D model, the implicit effects such as gravity and buoyancy are neglected in this study.
2. Mathematical modelling

Figure 1(a). Actual model of Geometry.

Figure 1(b). Part of the mesh with a Computation cell with a 0.96 ratio.

2.1. Geometry and Boundary Conditions

Figure 1.a shows the actual model of geometry and Fig 1.b shows a fragment of 2-D mesh. Meshing was created with edge sizing method, with the bias factor of 3 on the vertical axis. The number of elements creates are 100000 and the number of nodes is 102051 and Quad type mesh is created. Simulations were done to study the near-wall behaviour of nanoparticles, velocity distribution, and drag coefficient near the walls of the pipe. In order to study these parameters, a relatively concentrated mesh is generated near the ends of the wall, thus the overall mesh becomes non-uniform. In the present study, the model is assumed to be an axisymmetrical analysis problem. Laminar Flow conditions are considered while keeping both velocity and temperature constant at the inlet. It is also assumed that uniform heat flux of 5000 W/m² is induced at the pipe wall. The tube is composed of water and aluminium oxide nanofluid and its diameter and length are 5mm and 550mm respectively. The upper horizontal line is a uniform heat flux wall while the bottom line is an axis that makes a 3-D pipe model at the axisymmetric environment. These fluids enter with uniform temperature and axial velocity profiles.

2.2. Governing Equations

For the observation of the mixture model the single-phase model is inspected as described. Below equations is the mathematical formulation of the single and continuous phase of the mixture model. [5-7]

Energy equation:

$$
\text{div}(\rho \vec{V} C_p T) = \text{div}(k \text{grad} T) + S_e
$$

(1)
Conservation of mass:
\[
\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m u_m) = 0
\] (2)

Mass-averaged velocity \( u_m \) is
\[
u = \frac{\sum_{k=1}^{n} \alpha_k \rho_k u_k}{\rho_m}
\] (3)

And \( \rho_m \) is the mixture density with \( \alpha_k \) volume fraction of phase \( k \)
\[
\rho_m = \sum_{k=1}^{n} \alpha_k \rho_k
\] (4)

Momentum equation for single-phase
\[
div(\rho \overline{V\overline{V}}) = -gradP + \nabla \cdot (\mu \overline{V\overline{V}}) + S_m
\] (5)

Momentum equation for multiphase
\[
\nabla \sum_{k=1}^{n} \alpha_k \rho_k u_k u_k = \nabla \cdot (\rho_m u_m u_m) + \nabla \sum_{k=1}^{n} \alpha_k \rho_k u_{mk} u_{mk}
\] (6)

In Energy Equation an assumption is made that the compression and viscous are lost by heat. The unitive effects of energy and momentum exchange are represented with source term \( S_e \) and sink term \( S_m \) with base fluid. In a single-phase model these are of zero value. The volume fraction \( \alpha \), the velocity of phase \( u_k \), diffusion velocity \( u_{mk} \) will majorly affect the momentum equation of multiphase. The drag coefficient \( C_D \), particle density \( \rho_p \), slip velocity \( u_{sp} \), and cross-sectional area of the particle \( A_p \) are used to determine the particle motion. For a single rigid spherical particle in a fluid, Clift et al equation [3] is used to study the drag force.

\[
F_D = -\frac{1}{2} A_p \rho_p C_D |u_{cp}|^2 \left[ \frac{du_{cp}}{dt} - 2 \left( \frac{\pi}{3} \rho_c \mu_c \right) \frac{ds}{\sqrt{t-s}} \right]
\] (7)

The above equation depends on small particle Reynolds numbers with particle Reynolds number
\[
C_{D,St} = \frac{24}{Re_p}
\] (8)

The below equation represents the slip velocity with \( \tau_p \) the particle relaxation time, \( \tilde{a} \) is the secondary-phase particle’s acceleration
\[
u = \frac{\tau_p (\rho_p - \rho_m)}{f_{drag}} \tilde{a}
\] (9)

The drag function is taken default by Schiller and Naumann. In this paper the Reynolds number is 1050, hence the equation chosen is as follows:
\[
f_{drag} = 0.0183 Re
\] (10)
2.3. Physical properties and Boundary conditions for Nanofluids

Boundary conditions at the inlet are defined according to fluid properties, axial velocity \( V_0 \) is determined from \( Re = 1050 \) and a constant temperature of 293K. Non-slip conditions are subjected on the tube wall with a uniform heat flux of 5000 W/m\(^2\). The single-phase model properties of nano-fluids have been calculated using elementary formulas which were advanced for conventional solid-liquid mixtures.[8]. For CFD simulations to evaluate Al2O3-water nano-fluids’ physical and thermal properties, the following equations were used.[6]

\[
\rho_{nf} = (1 - \varphi)\rho_f + \varphi \rho_p \tag{11}
\]

\[
C_{p,nf} = \frac{(1 - \varphi)(\rho C_p)_f + \varphi(\rho C_p)_p}{\rho_{nf}} \tag{12}
\]

\[
\mu_{nf} = (123\varphi^3 + 7.3\varphi + 1)\mu_f \tag{13}
\]

The temperature-dependent properties and dynamic viscosity are excluded. The above equation is applicable for particles in sphere shape with volume fraction less than 5% [9] and nanofluid thermal conductivity is calculated as shown.

\[
k_{nf} = \left[ k_s + 2k_w + 2(k_s - k_w)(1 + \beta)^3 \varphi \right] \left[ k_s + 2k_w - (k_s - k_w)(1 + \beta)^3 \varphi \right]^{-1} k_w \tag{14}
\]

In the above equation \( \beta = 0.1 \) and is defined as the ratio of nano-layer thickness to the original particle radius. The thermal and physical properties of the nano-fluid model and Al2O3 nanoparticle are shown in Table 1 and Table 2 respectively. The values in Table 1 is obtained from heat and mass transfer data handbook. The values of Table 2 are obtained by the equations (11-14). All the values of Table 1 and Table 2 were in good validation with the paper written by Kitae Yu and Cheol Park [1].

**Table 1.** Properties of nano-fluids at \( Re=1050 \) and 293K

| \( \Phi (\%) \) | Density (kg/m\(^3\)) | Viscosity (kg/ms) | Inlet Velocity (m/s) | Heat conductivity (W/mK) | Specific heat (J/kgK) |
|-----------------|-----------------------|-------------------|----------------------|-------------------------|-----------------------|
| 0               | 998.245               | 9.97e-04          | 0.2336               | 0.5972                  | 4182.04               |
| 1               | 1027.017              | 1.024e-03         | 0.2325               | 0.6203                  | 4053.25               |
| 2               | 1055.835              | 1.049e-03         | 0.2317               | 0.6437                  | 3931.452              |
| 3               | 1084.644              | 1.072e-03         | 0.2309               | 0.6679                  | 3816.163              |
| 4               | 1113.471              | 1.097e-03         | 0.2302               | 0.6925                  | 3706.842              |

**Table 2.** Al2O3 nanoparticle properties at the temperature 293K

| Chemical formula | Density (kg/m\(^3\)) | Spherical diameter (nm) | Heat conductivity (W/mK) | Specific heat (J/kgK) |
|------------------|-----------------------|-------------------------|--------------------------|-----------------------|
| Al2O3            | 3882                  | 100                     | 0.5973                   | 4182.34               |
2.4. Numerical approach

To solve the present problem commercial computational fluid dynamic software FLUENT 20.1 is used. The control volume approach is used to solve the governing equation (1) (2) and (3). For the convective and diffusive terms, the second-order upwind method is employed. SIMPLE (Semi Implicit Method for Pressure Linked Equations) in single-phase model was employed in coupling pressure and velocity because SIMPLE algorithm takes more time to converge due to the high refinement of the mesh. Unlike single-phase, multi-phase requires an alternative to pressure-based and density-based segregated algorithms, hence Coupled algorithm was used with SIMPLE type pressure-velocity coupling. The convergence time is the major difference between SIMPLE and Coupled algorithm [6] and it is also noted that in a multi-phase Couple algorithm has converged much faster than SIMPLE. As the velocity distribution of flow remains constant, the fully developed region is seen on laminar flow. In the current study, according to the equation, (15) after 0.307m fully developed region appears [12]

\[
L_{laminar} = 0.065 \cdot Re \cdot D
\]  

(15)

Figure 2 shows the velocity distributions where the overall rise in the velocity profile is observed because of the overall rise in outlet temperature as compared to Case 1. Fig. 2a represents the profiles of axial velocity at several locations and Fig. 2b shows for different volume fractions. This implies that at the exit section model nano-fluid motion is fully developed.[7]

![Figure 2](image)

**Figure 2.** (a) Profiles of axial velocity at different locations (b) Profiles of axial velocity at a point for different volume fractions for Re=1050, q=5000 W/m².

In the present study, after comparing Nusselt number to Churchill-Ozoe calculation equation (16)[14] and Siegel Sparrow-Hallman equation (17)[15] and also comparing to common Nusselt number
equation (18) graph is plotted as shown in Figure. 3 where Pr is the Prandtl number and x* is a dimensionless axial position for thermally developing flows.

\[ Nu = 5.364[1 + (0.70.0x^*)^{-109}^{-3/10}] - 1 \]  

\[ Nu = \frac{2}{\left[ \frac{t_w - t_0}{q_0 / k} \right] \left( \frac{4x / r_0}{Re Pr} \right)} \]  

\[ Nu = \frac{h(x)D}{k_{inlet}} \]  

The heat transfer coefficient \( h(x) \) is defined as

\[ h(x) = \frac{q}{T_w(x) - T_b(x)} \]  

Where \( T_w(x) \) and \( T_b(x) \) are the solid surface temperature at the wall and mean bulk temperature respectively. And the average Nusselt number is defined as

\[ Nu_{av} = \frac{h_{av} \cdot D}{k_{inlet}} \]

![Figure 3. Grid validation employing Churchill and Ozoe, Siegel-Sparrow-Hallman](image)

### 3. Results

Forced convective heat transfer of \( \text{Al}_2\text{O}_3 \)-water in a circular tube under laminar flow has been studied in this work through various numerical simulations. The simulations were carried out by employing the single-phase and mixture model of \( \phi = 1\% \) to \( \phi = 4\% \), \( Re = 1050 \), and \( q = 5000W/m^2 \) with a constant state. Spherical particles of 100nm size have been considered in all cases. An increase in particle concentration as shown by the dimensionless temperature results in thermal development acceleration of the fluid at the various axial location along the radius, the same has been reported in (fig 4 (a) \( \phi = 0\% \) and b \( \phi = 4\% \)). A higher slope of relative local heat transfer coefficient \( h_r \) for \( \phi = 4\% \) as shown in fig
5(a) indicated that the motion is thermally developed for both base fluid and the nanofluid. Fig 5(a) shows the increase in relative local heat transfer coefficient with axial location and the rise in volume fraction when the fluid properties are constant. Correlation between the wall and bulk temperature is higher in the model with a narrow hydrodynamic radius. Hence the difference between the wall and the bulk temperature decreases and the heat transfer coefficient increase with constant heat flux as shown in Fig 5(b).

**Figure 4.** Dimensionless temperatures at several locations for (a) φ=0% (b) φ=4%

**Figure 5.** (a) Increase in volume fraction and increment in nano-fluid heat transfer coefficient for constant properties along the axis of the tube. (b) Heat transfer coefficient for each volume fraction.

Investigation of temperature profiles at profiles X/L = 1 for Re=1050 and q=5000W/m² due to the effect of particle presence is done in Fig 6. The temperature is strongly dependent on φ concentration. The wall temperature decreases because of the temperature difference between the surface and the axis decrease by higher forced convection heat transfer as the multi-phase concentration increases. In the case of a single-phase, due to the absence of particles, the heat transfer between the wall and the inner surface is not remarkable. In Fig 6, the existence of a uniform temperature fluid can be seen for both the phases and becomes more visible for a higher value of (φ at r/r₀=0.6) *. It can also be observed that an average temperature increment of 1.3K from 293K only occurs on single-phase models.
Figure 6. Comparison of temperatures along tube radius at x/L = 0.99 for several concentrations at constant properties.

Table 3. Heat transfer coefficient for Re=1050 and q=W/m² and investigation models

| Φ (%) | $h_{av}$ (W/mK) | $h_{nf}/h_{bf}$ | $Nu_{av}$ | $Nu_{nf}/Nu_{bf}$ |
|-------|-----------------|-----------------|-----------|-------------------|
| 0     | 862.129         | 1               | 6.5       | 1                 |
| 1     | 1233.986        | 1.431           | 8.95      | 1.377             |
| 2     | 1593.468        | 1.847           | 11.1      | 1.707             |
| 3     | 1955.125        | 2.266           | 13.2      | 2.027             |
| 4     | 2302.034        | 2.696           | 14.9      | 2.302             |

4. Conclusion
Table 3 shows the average heat transfer coefficient and the Nusselt number concerning Φ. It can be inferred that the total heat transfer rates and the Nusselt number increase relative to the concentration of nanoparticle volume fraction Φ. It was found that the presence of nanoparticles significantly enhanced the Heat Transfer, reduced the thermal resistance and increased the heat removal capacity [16]. The values of $h_{nf}/h_{bf}$ that is the ratio of heat transfer coefficient of nanofluid to heat transfer coefficient of base fluid are always greater than 1 hence confirming the nanofluid beneficial nanofluids in thermal properties with respect to the base fluid. Heat transfer enhancement is directly proportional to particle volume. The gradual increase in the Nusselt number can be observed with an increase in the volume fraction of nanofluid particles. The volume concentration and charge volume of nanoparticles increase thermal efficiency and significantly reduced the thermal resistance of the heat pipe when compared to the base fluid.[17]

The advent of CFD software could provide a fair and agreeable result from experimental correlation as shown in this study.[18]
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