Worldline approach to Sudakov-type form factors in non-Abelian
gauge theories

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Abstract

We calculate Sudakov-type form factors for isolated spin-1/2 particles (fermions) entering non-Abelian gauge-field systems. We consider both the on- and the off-mass-shell case using a methodology which rests on a worldline casting of field theories. The simplicity and utility of our approach derives from the fact that we are in a position to make, a priori, a more transparent separation (factorization), with respect to a given scale, between short- and long-distance physics than diagramatic methods.

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1. Introduction

Sudakov effects enter prominently calculations of form factors and structure functions within Quantum ChromoDynamics (QCD) in connection with the application of the renormalization group \cite{1,2}. In particular, to establish the validity of perturbation theory in exclusive processes, requires resummation of double-flow Sudakov logarithms to render factorization sound and isolate them from single-flow logarithms due to the renormalization group (for a comprehensive review, see, for example, \cite{7}).

More recently, it was shown that within a modified factorization scheme, which retains transverse degrees of freedom, the appearance of Sudakov form factors for bound quarks suppresses large-distance contributions that invalidate a consistent perturbative calculation of exclusive processes \cite{8,12}. The physical basis for this suppression is that the typical interquark transverse distance serves as an in situ infrared (IR) cutoff for gluon radiation with larger wavelengths. On the other hand, real and virtual gluons with wavelengths smaller than these distances but still larger than $1/Q$, $Q$ being the large momentum transfer involved in the elastic scattering, do not cancel but accumulate into damping exponentials which give rise to a finite IR renormalization of the hadron wave functions \cite{8,11,13}.

The derivation of the Sudakov form factor – originally performed in \cite{14} for the electromagnetic electron form factor – within the diagrammatic context, whether for on- or off-mass-shell fermions, and whether for an Abelian or a non-Abelian gauge field system, involves an intricate and quite complex set of arguments that, to a good extent, divert attention from the physical basis on which they rest. Leading contributions to the form factor come from different integration regions over gluon momenta, each involving physics that transpires at corresponding “hard” and “soft” momentum scales and whose ultimate factorization demands an extremely technical procedure. For example, the double-flow logarithms derive from integration regions where soft and collinear boson (gluon) lines overlap, and this becomes very complicated beyond one loop. Evidently, a systematic and all-order evaluation in the running coupling constant of the soft part is required for a hard-scattering expansion to be useful, and this is by far the most demanding task involved in any Sudakov-type analysis.

One is naturally led to ask whether these technical features of the classical diagramatic approaches just mentioned are unavoidable and due to the physical nature of the problem or whether there exists an alternative framework in which the main results for Sudakov-type form factors can be derived in a more transparent and simpler way. Our point of departure is the formalism we have proposed in a series of recent works \cite{15–19}, where we have demonstrated the effectiveness of a non-diagrammatic framework for the computation of closed-form expressions for Green and vertex functions in the infrared domain. The theoretical basis of this approach is to recast gauge theories with spin-1/2 matter fields – within the diagrammatic context, whether for on- or off-mass-shell fermions, and whether for an Abelian or a non-Abelian gauge field system, involves an intricate and quite complex set of arguments that, to a good extent, divert attention from the physical basis on which they rest. Leading contributions to the form factor come from different integration regions over gluon momenta, each involving physics that transpires at corresponding “hard” and “soft” momentum scales and whose ultimate factorization demands an extremely technical procedure. For example, the double-flow logarithms derive from integration regions where soft and collinear boson (gluon) lines overlap, and this becomes very complicated beyond one loop. Evidently, a systematic and all-order evaluation in the running coupling constant of the soft part is required for a hard-scattering expansion to be useful, and this is by far the most demanding task involved in any Sudakov-type analysis.

We shall consider both the on- and off-mass-shell case for isolated spin-1/2 matter particles entering non-Abelian gauge field theories. In QCD these fermions are hypothetical
quarks corresponding to a physical situation where the quark constituents are so far apart from each other that each one radiates independently and their Sudakov form factors resemble those of free fermions. Abelian systems constitute, of course, special cases and will be referred to in the end. The justification and advantages of our approach will become obvious \textit{a posteriori}, given the calculational simplicity of our treatment and the conceptual clarity by which it attains the called for separation (factorization) between long- and short-distance physics.

2. Three-point function and fermion elastic scattering

The fermion form factor within the worldline framework is given by the following connected, three-point function

\[
\mathcal{G}^{ij}_{\mu}(x,y;z) = \int_{0+}^{\infty} dT \int_{x(0) = x}^{x(T) = y} Dx(\tau) Dp(\tau) \mathcal{P} \exp \left\{ - \int_0^s d\tau [i p(\tau) \cdot \gamma + m] \right\} \Gamma_{\mu} \\
\times \mathcal{P} \exp \left\{ - \int_s^T d\tau [i p(\tau) \cdot \gamma + m] \right\} \exp \left[ i \int_0^T d\tau p(\tau) \cdot \dot{x}(\tau) \right] \\
\times \langle \mathcal{P} \exp \left[ ig \int_0^T d\tau \dot{x}(\tau) \cdot A(x(\tau)) \right] \rangle_A^{ij},
\]  

which involves two fermion lines coupled to a color-singlet current of the form \( \bar{\psi}(z) \Gamma_{\mu} \psi(z) \), and where \( < >_A \) denotes functional averaging of the bracketed quantity in the non-Abelian gauge field sector, while \( \mathcal{P} \) denotes path ordering of the exponential pertaining to the presence of \( \gamma \)-matrices and/or non-Abelian vector potentials.

Our basic computational task addresses itself to the expectation value of a Wilson line operator over paths from an initial point \( x \) to a final point \( y \) obliged to pass through the point of interaction \( z \) with the external current which injects the large momentum \( Q^2 \). It is worth pointing out that Wilson line operators facilitate the factorization process in the diagrammatic approach \[ \text{[4,6]} \], where they make their entrance through operator formalisms and pertain to lines of \textit{semi-infinite} extent. In contrast, in our case, the Wilson line operators are an integral part of the worldline casting of the field system and there is no \textit{a priori} reason for which the corresponding line segment cannot be of \textit{finite} extent. As we shall see in what follows the distinction between Wilson paths of semi-infinite and such of finite length is of crucial importance in our scheme.

For a generic non-Abelian system \( A_{\mu} \equiv A_{\mu}^b t^b \) (\( t^b \) being the group generators in the fundamental representation), a perturbative expansion is called for according to which

\[
\langle \mathcal{P} \exp \left[ ig \int_0^T d\tau \dot{x} \cdot A \right] \rangle_A = 1 + (ig)^2 \left\{ \int_0^s d\tau_1 \int_0^s d\tau_2 \theta(\tau_2 - \tau_1) \\
+ \int_s^T d\tau_1 \int_s^T d\tau_2 \theta(\tau_2 - \tau_1) + \int_0^s d\tau_1 \int_s^T d\tau_2 \right\} \dot{x}_{\mu}(\tau_1) \dot{x}_\mu(\tau_2) \\
\times \langle A_{\mu}(x(\tau_1)) A_{\nu}(x(\tau_2)) \rangle_A + \mathcal{O}(g^4).
\]
In the Feynman gauge and for a dimensionally regularized casting, the gauge field correlator reads

$$\langle A_\mu(x) A_\nu(x') \rangle^\text{reg} = \delta_{\mu\nu} C_F \frac{\mu^{4-D}}{4\pi D/2} \Gamma \left( \frac{D}{2} - 1 \right) |x - x'|^{2-D},$$

where $C_F = (N^2 - 1)/2N$ is the Casimir operator of the fundamental representation of $SU(N)$. For notational ease, the indication “reg” is omitted in the sequel.

We now isolate what amounts to contributions to Eq. (2) from a factored soft sector of the full theory by staging a calculation which adopts a straight line path going from $x$ to $z$ (for which we set $\dot{x}(\tau) = u_1$ with $0 < \tau < s$) and a second such path from $z$ to $y$ (for which we set $\dot{x}(\tau) = u_2$ with $s < \tau < T$). The no-recoil situation entailed by this restriction, except at point $z$, suggests that the active gauge-field degrees of freedom entering our computation are bound by an upper momentum scale which serves to separate “soft” from “hard” physics in our factorization scheme.

Setting aside for the moment the issue of a quantitative qualification of the above remark, let us turn our attention to the cusp occurring at the point $z$. It is a priori obvious that any singular contribution attributed to what transpires in the immediate vicinity of the cusp is independent of the geometrical form of the contours connecting $z$ with $x$ and $y$, respectively. In other words, the specific commitment we have made to straight-line paths will invariably select these singular contributions, which will thereby register as multiplicative renormalization constants for the Wilson line operator. Such renormalization factors have been originally discussed by Polyakov [26] and subsequently by other authors [27–31] in connection with Wilson loop operators. We shall show, in the framework of the worldline formalism, that the same situation occurs for open Wilson lines as well (see also [29,32]). As it will turn out, the ensuing anomalous dimensions will determine the renormalization-group evolution of the form factor and eventually, in the asymptotic limit, will produce Sudakov-type form factors supported by our worldline factorization scheme.

3. Soft contribution to the correlator

With the above remarks in place, let us proceed with the soft contribution to Eq. (2). We determine

$$\langle P \exp \left[ ig \int_0^T d\tau \dot{x} \cdot A \right] \rangle^\text{soft}_A = 1 + (ig)^2 C_F \frac{\mu^{4-D}}{4\pi D/2} \Gamma \left( \frac{D}{2} - 1 \right)$$

$$\times \left\{ |u_1|^{4-D} \int_0^s d\tau_1 \int_0^s d\tau_2 \theta(\tau_2 - \tau_1) |\tau_2 - \tau_1|^{2-D} + |u_2|^{4-D} \int_0^{T-s} d\tau_1 \int_0^{T-s} d\tau_2 \theta(\tau_2 - \tau_1) |\tau_2 - \tau_1|^{2-D} + u_1 \cdot u_2 \int_0^s d\tau_1 \int_0^{T-s} d\tau_2 |u_1\tau_1 + u_2\tau_2|^{2-D} \right\} + O(g^4).$$

(4)

Now, as long as the parameters $\tau_i$ ($i = 1, 2$) run over straight line paths of finite length, we are dealing with an off-mass-shell situation, as far as the matter particles are concerned.
Indeed, only for (semi-)infinitely extended paths will their full gauge field cloud be taken into account. Notice that such a picture is peculiar to the particle-based, worldline approach we are currently adopting. The on-mass-shell situation, corresponding to (straight line) paths of infinite length, will be taken up later on.

Choosing a frame for which \(|u_1| = |u_2| = |u|\) we find

\[
|u|^{4-D} \int_0^s d\tau_1 \int_0^s d\tau_2 \theta(\tau_2 - \tau_1) |\tau_2 - \tau_1|^{2-D} = -\frac{1}{4-D} \frac{1}{D-3} (|u|s)^{4-D}
\]  

(5)

and similarly for the second term inside the curly brackets in Eq. (4), while the third term yields

\[
u_1 \cdot u_2 \int_0^s d\tau_1 \int_0^{T-s} d\tau_2 |u_1\tau_1 + u_2\tau_2|^{2-D} = \frac{1}{(4-D)(D-3)} (|u|s)^{4-D} F_D \left( \frac{s}{T-s}, w \right) + \left[ |u| (T-s)^{4-D} \right] F_D \left( \frac{T-s}{s}, w \right),
\]  

(6)

where the relative velocity is given by \(w \equiv \frac{u_1 \cdot u_2}{|u_1||u_2|} = \frac{p_1 \cdot p_2}{|p_1||p_2|}\) and

\[
F_D(x, w) = w \left[ F \left( 1, \frac{D-2}{2}, \frac{D-1}{2}; 1-w^2 \right) - \frac{x+w}{(x^2 + 2wx + 1)^{\frac{D-2}{2}}} F \left( 1, \frac{D-2}{2}; \frac{D-1}{2}; \frac{1-w^2}{x^2 + 2wx + 1} \right) \right].
\]  

(7)

In the limit \(D \to 4\) we obtain

\[
\langle \mathcal{P} \exp \left[ ig \int_0^T d\tau \hat{x} \cdot A \right] \rangle_A^{\text{soft}} = 1 - \frac{g^2}{4\pi^2} C_F \frac{1}{D-4} \left[ \varphi(w) - 2 \right] \left[ 1 + \frac{D-4}{2} \ln \left( \frac{\pi e^{\gamma_E}}{2} \right) \right]
\]

\[- \frac{g^2}{4\pi^2} C_F \ln(|u|s) \left[ F_4 \left( \frac{s}{T-s}, w \right) - 1 \right] - \frac{g^2}{4\pi^2} C_F \ln(|u|(T-s)) \left[ F_4 \left( \frac{T-s}{s}, w \right) - 1 \right] - \frac{g^2}{4\pi^2} C_F \frac{1}{D-4} \left[ F_D \left( \frac{s}{T-s}, w \right) + F_D \left( \frac{T-s}{s}, w \right) - \varphi(w) \right],
\]  

(8)

where \(\gamma_E\) is the Euler-Mascheroni constant and

\[
F_4(x, w) = \frac{w}{\sqrt{1-w^2}} \arctan \frac{\sqrt{1-w^2}}{w} - \frac{w}{\sqrt{1-w^2}} \arctan \frac{\sqrt{1-w^2}}{x+w},
\]  

(9)

from which it easily follows that

\[
F_4(x, w) + F_4 \left( \frac{1}{x}, w \right) = \frac{w}{\sqrt{1-w^2}} \arctan \frac{\sqrt{1-w^2}}{w} \equiv \varphi(w).
\]  

(10)
4. Effective low-energy theory

This is a good point to pause for some basic assessments. Given a full microscopic field theory, we have, by utilizing its worldline casting, isolated a sector of that theory in which the matter particles are “dressed” to a scale that makes them appear extremely heavy to the active, in this sector, gauge field degrees of freedom. In particular, the latter are in no position to create fermion-antifermion pairs nor to derail the spin-1/2 particle entities from straight-line propagation. This bona fide “soft” sector has its own high- and low-energy domains. Whereas the second domain presumably coincides with the infrared region of the full theory, given that they completely overlap, the first one displays ultraviolet (UV) divergences whose (upper) point of reference, equivalently the UV cutoff, is the aforementioned separation scale. To face these divergences we were forced to apply the usual dimensional regularization techniques. The emerging singularity structure, reflected in anomalous dimensions that will be derived shortly, is of a similar nature as the one prevailing in heavy quark effective theory (see, e.g., [33], and for some pertinent results [34]). In this light, the (arbitrary) mass scale $\mu$ can go as high as the separation point of the factored out subtheory but can certainly not exceed that of the momentum transfer involved in the form factor.

Going over to Minkowski space entails the substitution

$$w \rightarrow \frac{w}{\sqrt{1-w^2}} \arctan \frac{\sqrt{1-w^2}}{w} \rightarrow \frac{w}{\sqrt{w^2-1}} \tanh^{-1} \left( \frac{w^2-1}{w} \right).$$

Setting

$$\Gamma_{\text{cusp}} = \frac{\alpha_s}{\pi} C_F \left[ \frac{w}{\sqrt{w^2-1}} \tanh^{-1} \left( \frac{w^2-1}{w} \right) - 1 \right],$$

and

$$\Gamma_{\text{end}} = -\frac{\alpha_s}{2\pi} C_F,$$

for the corresponding anomalous dimensions of the cusp and the endpoint(s) of the contour, the coefficient $\gamma(\alpha_s, w)$ of the divergent term is expressed into the following form

$$\gamma(\alpha_s, w) = \Gamma_{\text{cusp}}(\alpha_s, w) + 2\Gamma_{\text{end}}(\alpha_s, w) = \frac{\alpha_s}{\pi} C_F \left[ \frac{w}{\sqrt{w^2-1}} \tanh^{-1} \left( \frac{\sqrt{w^2-1}}{w} \right) - 2 \right].$$

Notice the fact that these anomalous dimensions, which are exclusively associated with the factored out (soft) sector of the full theory, display explicit dependence on the momentum.

5. Off-mass-shell case

Returning to the vertex function we find, in momentum space,

$$G_\mu(p_1, p_2) = \frac{1}{p_1 \cdot \gamma - m} \Gamma_\mu \frac{1}{p_2 \cdot \gamma - m} \left\{ 1 - \frac{1}{D-4} \gamma(\alpha_s, w) - \ln \left( \frac{\mu|u|}{\lambda} \right) \gamma(\alpha_s, w) - f(\alpha_s, w) \right\} + O(g^4),$$

where $D$ is the space-time dimension.
in which \( \lambda = \frac{|m^2 - p^2|}{m} \) is an off-mass-shellness scale serving as an IR regulator, and

\[
Q^2 = -(p_1 - p_2)^2 = -2p^2 + 2p_1 \cdot p_2, \tag{15}
\]
or, equivalently,

\[
\frac{p_1 \cdot p_2}{|p|^2} = w = 1 + \frac{Q^2}{2|p|^2}, \tag{16}
\]

where \( p_1 \) and \( p_2 \) are, respectively, the four-momenta of the initial and final quark with mass \( m \). Finally, the last term in the curly brackets on the rhs of Eq. (14) is given by

\[
f(\alpha_s, w) = \lim_{D \to 4} \left[ 2F_D(1,w) - \varphi(w) \right] \frac{1}{D-4} \frac{\alpha_s}{\pi}, \tag{17}
\]

and is actually finite.

Now, the reference mass scale \( \mu \) can range from a minimum value \( \mu_{\text{min}} \) all the way up to \( \mu_{\text{max}} \sim |Q| \). Soft contributions transpiring below \( \mu_{\text{min}} \) correspond to genuine infrared effects which are to be summed over in any physical expression via the instructions of the KLN theorem [35]. We consequently demand that for \( \mu = \mu_{\text{min}} \) the form

\[
G_\mu^{(0)} = \frac{1}{p_1 \cdot \gamma - m} \Gamma_\mu \frac{1}{p_2 \cdot \gamma - m} \tag{18}
\]
of the free vertex function is obtained. We are thus led to the equation

\[
\ln \left( \frac{\mu_{\text{min}} |u|}{\lambda} \right) \gamma(\alpha_s, w) + f(\alpha_s, w) = 0, \tag{19}
\]

which is easily solvable in the limit \( \frac{Q^2}{p^2} \to \infty \). We have, in this case,

\[
\gamma(\alpha_s, w) \simeq \frac{\alpha_s}{\pi} C_F \ln \frac{Q^2}{p^2}, \quad f(\alpha_s, w) \simeq \frac{\alpha_s}{4\pi} C_F \ln^2 \frac{Q^2}{p^2}, \tag{20}
\]

whereupon we determine

\[
\mu_{\text{min}} = \frac{|m^2 - p^2|}{(Q^2 p^2)^{1/4}}. \tag{21}
\]

Similar results have been obtained by Ivanov, Korchemsky and Radyushkin [36–38] (see also [39]).

6. On-mass-shell case

The on-mass-shell case is reached in the limit where the lengths of the two paths on either side of the cusp go to infinity while setting, at the same time, \( |u_i| = 1, i = 1, 2 \). Finally, the
mass \( \lambda \) assigned to the gauge fields replaces the off-mass-shellness as the appropriate scale to provide protection against infrared divergences.

Taking into account that \( |u| = 1 \), we obtain

\[
    u^2 \mu^{4-D} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2 + \lambda^2} \int_0^\infty d\tau_1 \int_0^\infty d\tau_2 \ e^{-ik(ut_1 - ut_2)} = -\Gamma \left( 2 - \frac{D}{2} \right) \frac{1}{(4\pi)^{D/2}} \left( \frac{1}{\lambda} \right)^{4-D}
\]

and

\[
    u_1 \cdot u_2 \mu^{4-D} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2 + \lambda^2} \int_0^\infty d\tau_1 \int_0^\infty d\tau_2 \ e^{-ik(u_1\tau_1 + u_2\tau_2)}
    = 2\Gamma \left( 2 - \frac{D}{2} \right) \frac{1}{(4\pi)^{D/2}} \left( \frac{\mu}{\lambda} \right)^{4-D} \frac{w}{\sqrt{1 - w^2}} \text{arctan} \left( \frac{\sqrt{1 - w^2}}{w} \right),
\]

so that the renormalized expression for the vertex function assumes the form

\[
    G^R_{\mu}(p_1, p_2) = \frac{1}{p_1 \cdot \gamma - m} \frac{1}{p_2 \cdot \gamma - m} \left[ 1 - \Gamma_{\text{cusp}}(\alpha_s, w) \ln \frac{\mu}{\lambda} \right],
\]

in which the superscript \( \text{ms} \) stands for “on mass shell”.

We observe that the difference between the on- and the off-mass-shell case, as far as anomalous dimensions are concerned, is the extra contribution \( \Gamma_{\text{end}} \) to the latter. On physical grounds this is understood as follows. Finite worldline segments, corresponding to off-mass-shell situations, entail sudden accelerations or decelerations of the matter particles which take place at the open ends. It is this collinear emission of gauge modes that gives rise to the additional \( \Gamma_{\text{end}} \) contribution which is present no matter how minutely off-mass-shell the matter particles may are.

7. Renormalization group evolution

We now turn our attention to the renormalization group equation which furnishes the evolution of the form factor in the soft sector of the full theory. For the on-mass-shell case it reads as follows (S stands for “soft”)

\[
    \left\{ \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + \Gamma_{\text{cusp}}(w, g) \right\} F_S \left( w, \frac{\mu^2}{\lambda^2} \right) = 0.
\]

In the limit \( w \to \infty \) we obtain

\[
    \Gamma_{\text{cusp}}(w, g) \simeq \ln \left( \frac{Q^2}{p^2} \right) \Gamma_{\text{cusp}}(g), \quad \Gamma_{\text{cusp}}(g) = \frac{g^2}{4\pi^2} C_F + \mathcal{O}(g^4)
\]

in accordance with the results found previously in [37, 38].

As already mentioned, the above renormalization group scheme operates strictly within the “soft” sector which has been factorized with respect to the full microscopic theory. This
means that the arbitrary mass \( \mu \) runs from a minimum scale, below which KLN physics takes over, to a maximum scale which serves as the “roof” for the soft sector. We now set, according to the standard procedure (recall that \( \alpha_s = g^2/4\pi \) is the running coupling constant),

\[
\alpha(\mu^2) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)},
\]

where \( \beta_0 = \frac{11}{3} C_A - \frac{2}{3} n_F, \ C_A = (N_c + 1)/2N_c \) being the Casimir operator of the adjoint representation of \( SU(3) \), and \( \Lambda \equiv \Lambda_{QCD} \) is the characteristic scale of QCD, entering via dimensional transmutation.

Imposing the boundary condition \( F_S(w,1) = 1 \) we obtain in the leading logarithm approximation, once we have adopted a frame in which \( |p| = |p_i|, \ i = 1,2, \)

\[
F_S \left( \frac{Q^2}{p^2}, \frac{\mu^2}{\lambda^2} \right) = \exp \left[ -\ln \left( \frac{Q^2}{p^2} \right) \int_{\lambda^2}^{\mu^2} \frac{dt}{2t} \Gamma_{\text{cusp}}(g(t)) \right]
= \exp \left[ -\frac{C_F}{2\pi} \frac{4\pi}{\beta_0} \ln \left( \frac{Q^2}{p^2} \right) \ln \left( \frac{\ln \mu^2}{\ln \Lambda^2} \right) \right], \quad (28)
\]

where \( \mu \) now denotes the separation point between the soft and the hard sectors of the theory.

The fact that the overall form factor \( F \), which factorizes into a “hard” (H) and a “soft” (S) part according to \( F = F_H \otimes F_S \), should be independent from the separation scale \( \mu \), along with the relation \( \frac{\partial}{\partial \ln Q^2} \ln F_S = -\int_{\lambda^2}^{\mu^2} \frac{dt}{2t} \Gamma_{\text{cusp}}(g(t)) \), which can be surmised from Eq. (28), leads to the following solution in the asymptotic regime \( Q^2/\lambda^2 \to \infty \)

\[
F \left( Q^2, \lambda^2 \right) = \exp \left\{ -\frac{C_F}{2\pi} \frac{4\pi}{\beta_0} \ln \left( \frac{Q^2}{\Lambda^2} \right) \ln \left[ \frac{\ln (Q^2/\Lambda^2)}{\ln (\lambda^2/\Lambda^2)} \right] + \frac{C_F}{2\pi} \frac{4\pi}{\beta_0} \ln \left( \frac{Q^2}{\lambda^2} \right) \right\}. \quad (29)
\]

Similar considerations applied to the off-mass-shell case lead to the following asymptotic behavior, as \( Q^2/M^2 \to \infty \), for the form factor \( (M^2 \equiv -p_1^2 = -p_2^2) \)

\[
F \simeq \exp \left\{ -\int_{M^2}^{Q^2} \frac{dt}{2t} \ln \left( \frac{Q^2}{t} \right) \Gamma_{\text{cusp}}(g(t)) - \int_{\frac{M^2}{Q^2}}^{1} \frac{dt}{2t} \ln \left( \frac{t Q^2}{M^2} \right) \Gamma_{\text{cusp}}(g(t)) \right\}
= \exp \left\{ -\frac{C_F}{2\pi} \frac{4\pi}{\beta_0} \ln \left( \frac{Q^2}{\Lambda^2} \right) \ln \left[ \frac{\ln (Q^2/\Lambda^2)}{\ln (M^2/\Lambda^2)} \right] + \frac{C_F}{2\pi} \frac{4\pi}{\beta_0} \ln \left( \frac{Q^2 \Lambda^2}{M^4} \right) \ln \left[ \frac{\ln (Q^2 \Lambda^2/M^4)}{\ln (\Lambda^2/M^2)} \right] \right\}. \quad (30)
\]

Two basic readjustments, with respect to the on-mass-shell case, are entailed here. First, the lower limit of integration is given by the mass scale defined by Eq. (21), meaning that \( \lambda \) in Eq. (28) is replaced by \( \mu_{\text{min}} \). In this connection, let us observe that to incorporate this adjustment into the diagrammatic analysis, Korchemsky (second paper of [3]) follows Fishbane and Sullivan [10] (see also [37]) to introduce an extra, “infrared” component \( F_{IR} \) to
his factorization formula, in addition to the “soft”, “jet” (collinear subgraphs), and “hard” terms. As already noted, our factorization procedure encompasses all non-hard contributions into the soft sector of the full theory. Second, an additional term enters the anomalous dimension which, however, is not momentum-dependent, (cf. Eq. (13), and thereby gives a much weaker contribution to the form factor in the asymptotic limit.

8. Abelian case

To adjust the above results to the Abelian case, we proceed as follows. We first observe, focusing on the on-mass-shell case, that

$$\ln \frac{Q^2/\Lambda^2}{\ln \lambda^2/\Lambda^2} \approx 1 + \frac{\beta_0}{4\pi} \alpha_s(Q^2) \ln \frac{Q^2}{\lambda^2}. \quad (31)$$

The above relation along with Eq. (27), when inserted into Eq. (29), give, in the limit $Q^2/\lambda^2 \to \infty$,

$$F \simeq \exp \left\{ \frac{C_F}{2\pi} \left( \frac{4\pi}{\beta_0} \right)^2 \frac{1}{\alpha_s} \left[ \frac{\beta_0}{4\pi} \ln \frac{Q^2}{\lambda^2} - \frac{1}{2} \alpha_s \left( \frac{\beta_0}{4\pi} \right)^2 \ln^2 \frac{Q^2}{\lambda^2} \right] - \frac{C_F}{2\pi} \beta_0 \ln \frac{Q^2}{\lambda^2} \right\}, \quad (32)$$

which coincides with the well-known result, established in [1,41,42]. In reference to QED, the familiar result obtained by Jackiw [43] (see also [40,44–46])

$$F_{ms} = e^{-\frac{g^2}{16\pi} \ln^2 \frac{Q^2}{\lambda^2}}. \quad (33)$$

is recovered. Similar considerations produce the off-mass-shell result

$$F_{off-ms} = e^{-\frac{g^2}{8\pi} \ln^2 \left( \frac{Q^2}{M^2} \right)}, \quad (34)$$

which was what Sudakov obtained to begin with [14]. Note the extra factor of 2 entering the exponent, as compared to Eq. (33); it comes from the combined contribution of the two terms entering the exponent in Eq. (30) and underlines the significance of the lower limit $\mu_{\min}$ associated with the off-mass-shell case.

9. Concluding remarks

In conclusion, the present approach to the derivation of Sudakov-type form factors for spin-1/2 (isolated) matter fields in a non-Abelian gauge field theory has avoided to face intricate issues in a diagramatic context, such as the role of jet subdiagrams and how they communicate with soft and hard parts, or how the operator formalism comes to the aid of field theory, etc. The clean way by which a soft sector of the full theory can be factorized and, basically, determine the form factor structure is, we believe, a notable accomplishment of the worldline scheme. The next step, of course, is to assess the merits of our approach to exclusive hadronic processes like hadron form factors or hadron-hadron elastic scattering.
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