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Abstract

We report a search for charmless hadronic decays of charged $B$ mesons to the final states $K_S^0 K_S^0 K^\pm$ and $K_S^0 K_S^0 \pi^\pm$. The results are based on a 711 fb$^{-1}$ data sample that contains $772 \times 10^6$ $B\bar{B}$ pairs, and was collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider. For $B^\pm \rightarrow K_S^0 K_S^0 K^\pm$ decays, the measured branching fraction and direct $CP$ asymmetry are $[10.64 \pm 0.49\text{(stat)} \pm 0.44\text{(syst)}] \times 10^{-6}$ and $[-0.6 \pm 3.9\text{(stat)} \pm 3.4\text{(syst)}]\%$, respectively. In the absence of a statistically significant signal for $B^\pm \rightarrow K_S^0 K_S^0 \pi^\pm$, we set the 90% confidence-level upper limit on its branching fraction at $1.14 \times 10^{-6}$.
Charged $B$-meson decays to three-body charmless hadronic final states $K_S^0 K_S^0 K^\pm$ and $K_S^0 K_S^0 \pi^\pm$ mainly proceed via the $\bar{b} \to \bar{s} \pi$ and $\bar{b} \to \bar{d} \pi$ loop transitions, respectively. Figure 1 shows the dominant Feynman diagrams that contribute to the decays. These are flavour changing neutral current transitions, which are suppressed in the standard model (SM) and hence provide a good avenue to search for physics beyond the SM \cite{1}. Further motivation, especially to study the contributions of various quasi-two-body resonances to inclusive $CP$ asymmetry, comes from the recent results on $B^\pm \to K^+ K^- K^{\pm}$, $K^+ K^- \pi^{\pm}$ and other such three-body decays \cite{2,3}. LHCb has found large inclusive asymmetries in $B^\pm \to K^+ K^- \pi^{\pm}$ and $\pi^+ \pi^- \pi^{\pm}$ decays \cite{4}, where the observed phenomena are largely in localized regions of phase space. Recently, Belle has also reported strong evidence for a large $CP$ asymmetry in the low $K^+ K^-$ invariant-mass region of $B^\pm \to K^+ K^- \pi^{\pm}$ \cite{5}.

![Feynman diagrams](image)

**FIG. 1:** Dominant Feynman diagrams that contribute to the decays $B^\pm \to K_S^0 K_S^0 K^\pm$ (left) and $B^\pm \to K_S^0 K_S^0 \pi^\pm$ (right).

The three-body decay $B^+ \to K_S^0 K_S^0 K^+$ has already been observed and subsequently studied by the Belle and BaBar Collaborations \cite{6,7,8}. Belle measured its branching fraction as $(13.4 \pm 1.9 \pm 1.5) \times 10^{-6}$ based on a small data set of 70 fb$^{-1}$ \cite{7}, while BaBar reported a branching fraction of $(10.6 \pm 0.5 \pm 0.3) \times 10^{-6}$ and an inclusive $CP$ asymmetry of $(4.1 \pm 2)\%$ using 426 fb$^{-1}$ of data \cite{8}. The quoted uncertainties are statistical and systematic, respectively. On the other hand, the decay $B^+ \to K_S^0 K_S^0 \pi^+$ has not yet been observed, with the most restrictive upper limit being available at 90\% confidence level, $\mathcal{B}(B^+ \to K_S^0 K_S^0 \pi^+) < 5.1 \times 10^{-7}$, from BaBar \cite{8}.

We present herein an improved measurement of the branching fraction and direct $CP$ asymmetry of the decay $B^+ \to K_S^0 K_S^0 K^+$ as well as a search for the decay $B^+ \to K_S^0 K_S^0 \pi^+$ based on the full $\Upsilon(4S)$ data sample, containing $772 \times 10^6 B\bar{B}$ pairs, collected with the Belle detector \cite{9} at the KEKB asymmetric-energy $e^+e^-$ (3.5 on 8.0 GeV) collider \cite{10}. The direct $CP$ asymmetry in the former case is given by

$$\mathcal{A}_{CP} = \frac{N(B^- \to K_S^0 K_S^0 K^-) - N(B^+ \to K_S^0 K_S^0 K^+)}{N(B^- \to K_S^0 K_S^0 K^-) + N(B^+ \to K_S^0 K_S^0 K^+)},$$

where $N$ is the signal yield obtained for the corresponding mode. The principal detector components used in the study are: a silicon vertex detector, a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and a CsI(Tl) crystal electromagnetic calorimeter (ECL). All these components are located inside a 1.5 T solenoidal magnetic field.

To reconstruct $B^+ \to K_S^0 K_S^0 h^+$ decay candidates, we combine a pair of $K_S^0$ mesons with a charged kaon or pion. Each charged track candidate must have a distance of closest
approach with respect to the interaction point (IP) of less than 0.2 cm in the transverse $r-\phi$ plane and less than 5.0 cm along the $z$ axis. Here, the $z$ axis is the direction opposite the $e^+$ beam. Charged kaons and pions are identified based on a likelihood ratio $\mathcal{R}_{K/\pi} = \mathcal{L}_K / (\mathcal{L}_K + \mathcal{L}_\pi)$, where $\mathcal{L}_K$ and $\mathcal{L}_\pi$ denote the individual likelihood for kaons and pions, respectively, calculated using specific ionization in the CDC and information from the ACC and the TOF. A requirement, $\mathcal{R}_{K/\pi} > 0.6$, is applied to select the kaon candidates; track candidates failing it are classified as pions. The efficiency for kaon (pion) identification is 86% (91%) with a pion (kaon) misidentification rate of about 14% (9%).

The $K^0_S$ candidates are reconstructed from pairs of oppositely charged tracks, both treated as pions, and are identified with a neural network (NN) [11]. The NN uses the following seven input variables: the $K^0_S$ momentum in the laboratory frame, the distance along the $z$ axis between the two track helices at their closest approach, the $K^0_S$ flight length in the transverse plane, the angle between the $K^0_S$ momentum and the vector joining the IP to the $K^0_S$ decay vertex, the angle between the pion momentum and the laboratory frame direction in the $K^0_S$ rest frame, the distances of closest approach in the transverse plane between the IP and the two pion helices, and the total number of hits (in the CDC and SVD) for each pion track. We also require that the reconstructed invariant mass be between 491 and 505 MeV/c$^2$, corresponding to $\pm 3\sigma$ around the nominal $K^0_S$ mass [12].

$B$ meson candidates are identified using two kinematic variables: beam-energy constrained mass, $M_{bc} = \sqrt{E_{beam}^2/c^4 - \sum_i \vec{p}_i/c^2}$, and energy difference, $\Delta E = \sum_i E_i - E_{beam}$, where $E_{beam}$ is the beam energy, and $\vec{p}_i$ and $E_i$ are the momentum and energy, respectively, of the $i$-th daughter of the reconstructed $B$ candidate in the center-of-mass (CM) frame. We retain events with $5.271\, \text{GeV}/c^2 < M_{bc} < 5.287\, \text{GeV}/c^2$ and $-0.10\, \text{GeV} < \Delta E < 0.15\, \text{GeV}$ for further analysis. The $M_{bc}$ requirement corresponds to approximately $\pm 3\sigma$ around the nominal $B^+$ mass [12]. We apply a looser ($-6\sigma, +9\sigma$) requirement on $\Delta E$ as it is used in the fitter (described below). The average number of $B$ candidates found per event is 1.13 (1.49) for $B^+ \to K^0_SK^0_SK^+$ ($K^0_SK^0_S\pi^+$). In events with multiple $B$ candidates, we choose the one with the lowest $\chi^2$ value obtained from a $B$ vertex fit. This criterion selects the correct $B$-meson candidate in 75% (63%) of MC events for $B^+ \to K^0_SK^0_SK^+$ ($K^0_SK^0_S\pi^+$).

The dominant background is from the $e^+e^- \to q\bar{q} \,(q = u,d,s,c)$ continuum process. To suppress it, observables based on the event shape topology are utilized. The event shape in the CM frame is expected to be spherical for $BB$ events, in contrast to jet-like for continuum events. We employ another NN [11] to combine the following six input variables: the Fisher discriminant formed from 16 modified Fox-Wolfram moments [13], the cosine of the angle between the $B$ momentum and the $z$ axis, the cosine of the angle between the $B$ thrust and the $z$ axis, the cosine of the angle between the thrust axis of the $B$ candidate and that of the rest of the event, the ratio of the second to the zeroth order Fox-Wolfram moments, and the vertex separation along the $z$ axis between the $B$ candidate and the remaining tracks. The first five quantities are calculated in the CM frame. The NN training and optimization are performed with signal and $q\bar{q}$ Monte Carlo (MC) simulated events. The signal MC sample is generated with the EvtGen program [14] assuming a three-body phase space. We require the NN output ($C_{NB}$) to be greater than $-0.2$ to substantially reduce the continuum background. The relative signal efficiency due to this requirement is approximately 91%, whereas the achieved continuum suppression is close to 84% for both decays. The remainder of the $C_{NB}$ distribution strongly peaks near 1.0 for signal, making it difficult to model it.
with an analytic function. However, its transformed variable

$$C'_{NB} = \log \left[ \frac{C_{NB} - C_{NB,\text{min}}}{C_{NB,\text{max}} - C_{NB}} \right],$$

where $C_{NB,\text{min}} = -0.2$ and $C_{NB,\text{max}} \approx 1.0$, has a Gaussian-like distribution.

The background due to $B$ decays mediated via the dominant $b \to c$ transition is studied with an MC sample comprising such decays. The resulting $\Delta E$ and $M_{bc}$ distributions are found to strongly peak in the signal region for both $B^+ \to K_s^0 K_S^0 K^+$ and $K_S^0 K_S^0 \pi^+$ decays. For $B^+ \to K_S^0 K_S^0 K^+$, the peaking background predominantly stems from $B^+ \to D^0 K^+$ with $D^0 \to K_S^0 K_S^0$ and $B^+ \to \chi_c(1P) K^+$ with $\chi_c(1P) \to K_S^0 K_S^0$. To suppress these backgrounds, we exclude candidates for which $M_{K_S^0 K_S^0}$ lies in the ranges of $[1.85, 1.88]$ GeV/$c^2$ and $[3.38, 3.45]$ GeV/$c^2$ corresponding to about $\pm 3 \sigma$ window around the nominal $D^0$ and $\chi_c(1P)$ mass $[12]$, respectively. On the other hand, in case of $B^+ \to K_S^0 K_S^0 \pi^+$, the peaking background largely arises from $B^+ \to D^0 \pi^+$ with $D^0 \to K_S^0 K_S^0$. To suppress this background, we exclude candidates for which $M_{K_S^0 K_S^0}$ lies in the aforementioned $D^0$ mass window.

There are a few background modes that contribute in the $M_{bc}$ signal region but have the $\Delta E$ peak shifted from zero on the positive (negative) side for $B^+ \to K_S^0 K_S^0 K^+$ $(K_S^0 K_S^0 \pi^+)$. The so-called “feed-across background” modes, mostly arising due to $K^- \pi$ misidentification, are identified with a $B \bar{B}$ MC sample in which one of the $B$ mesons decays via $b \to u, d, s$ transitions. The feed-across background includes contribution from $B \to K_S^0 K_S^0 \pi$ $(K_S^0 K_S^0 K)$ in $B^+ \to K_S^0 K_S^0 K^+$ $(K_S^0 K_S^0 K^+)$ The events that remain after removing the signal and feed-across components comprise the “combinatorial background.” After all selection requirements, the efficiency for correctly reconstructed signal events ($\epsilon_{\text{rec}}$) is 24% (28%) for $B^+ \to K_S^0 K_S^0 K^+$ $(K_S^0 K_S^0 \pi^+)$. The fraction of misreconstructed signal events ($f_{\text{SCF}}$) is 0.45% (1.05%) for $B^+ \to K_S^0 K_S^0 K^+$ $(K_S^0 K_S^0 \pi^+)$. As $f_{\text{SCF}}$ represents a small fraction of the signal events for both decays, we consider it as a part of signal. Note that $\epsilon_{\text{rec}}$ and $f_{\text{SCF}}$ are determined with an MC simulation in which decays are generated assuming a three-body phase space.

The signal yield and $A_{CP}$ are obtained with an unbinned extended maximum likelihood fit to the two-dimensional distributions of $\Delta E$ and $C'_{NB}$. We define a probability density function (PDF) for each event category $j$ (signal, $q\bar{q}$, combinatorial $B \bar{B}$, and feed-across backgrounds) as

$$P_j^i = \frac{1}{2} (1 - q^{i} \cdot A_{CP,j}) \times P_j(\Delta E^i) \times P_j(C'_{NB}^i),$$

where $i$ denotes the event index, $q^i$ is the charge of the $B$ candidate in the event ($\pm 1$ for $B^\pm$), $P_j$ is the PDF corresponding to the component $j$. Since the correlation between $\Delta E$ and $C'_{NB}$ is found to be negligible, the product of two individual PDFs is a good approximation for the total PDF. We apply a tight requirement on $M_{bc}$ instead of including it in the fitter as it exhibits large correlation with $\Delta E$ for signal and feed-across components. The extended likelihood function is

$$\mathcal{L} = \frac{e^{-\sum n_j}}{N!} \prod_i \left[ \sum_j n_j P_j^i \right],$$

where $n_j$ is the yield of the event category $j$ and $N$ is the total number of events. To account for crossfeed between the $B \to K_S^0 K_S^0 K$ and $B \to K_S^0 K_S^0 \pi$ channels, they are simultaneously
fitted, with the $B \rightarrow K_s^0 K_s^0 K$ signal yield in the correctly reconstructed sample determining the normalization of the crossfeed in the $B \rightarrow K_s^0 K_s^0 \pi$ fit region, and vice versa.

Table I lists the PDF shapes used to model $\Delta E$ and $C'_{NB}$ distributions for various event categories for $B \rightarrow K_s^0 K_s^0 K$. For $B \rightarrow K_s^0 K_s^0 \pi$, we use similar PDF shapes except for the feed-across background component, where we use a sum of a Gaussian, asymmetric Gaussian and first order polynomial to parametrize $\Delta E$, and a sum of Gaussian and asymmetric Gaussian functions to parametrize $C'_{NB}$. For $B \rightarrow K_s^0 K_s^0 K$, the yields for all event categories except for that of the combinatorial $BB$ background are allowed to vary in the fit. The latter yield is fixed to the MC value as it is found to be correlated with the continuum background yield. For $B \rightarrow K_s^0 K_s^0 \pi$, the yields for all event categories are allowed to vary. For both $B \rightarrow K_s^0 K_s^0 K$ and $K_s^0 K_s^0 \pi$, the following PDF shape parameters of the continuum background are floated: the slope of the first order polynomial used for $\Delta E$, and one of the means and widths of the Gaussian functions used to model $C'_{NB}$. The PDF shapes for signal and other background components are fixed to the corresponding MC expectations. Shared parameters in the simultaneous fit are the signal yields of $K_s^0 K_s^0 K$ and $K_s^0 K_s^0 \pi$. The ratio of the $K_s^0 K_s^0 K$ feed-across to the signal $K_s^0 K_s^0 \pi$ yield is floated, whereas the ratio of the $K_s^0 K_s^0 \pi$ feed-across to the signal $K_s^0 K_s^0 K$ yield is fixed in the fitter. This is because the latter contribution is small. We correct the signal $\Delta E$ and $C'_{NB}$ PDF shapes for possible data-MC differences, according to the values obtained with a large-statistics control sample of $B \rightarrow D^0(K_s^0 \pi^\pm \pi^-)\pi$. The same correction factors are also applied for the feed-across background component of $B \rightarrow K_s^0 K_s^0 \pi$. The stability of the two-dimensional simultaneous fit is checked via ensemble tests using both PDF-sampled and simulated MC events.

Figure 2 shows $\Delta E$ and $C'_{NB}$ projections of the fit to $B^+$ and $B^-$ samples separately for $B \rightarrow K_s^0 K_s^0 K$ and overall fit for $B \rightarrow K_s^0 K_s^0 \pi$. We determine the branching fraction as,

$$B(B^+ \rightarrow K_s^0 K_s^0 h^+) = \frac{N_{sig}}{\epsilon \times N_{BB} \times [B(K_s^0 \rightarrow \pi \pi)]^2} \quad (5)$$

where, $N_{sig}$, $\epsilon$ and $N_{BB}$ are the signal yield, corrected reconstruction efficiency and total number of $BB$ pairs, respectively. For $B^+ \rightarrow K_s^0 K_s^0 \pi^+$, we obtain a signal yield of $69 \pm 26$, where the error is statistical only. The inclusive branching fraction for $B^+ \rightarrow K_s^0 K_s^0 \pi^+$ is $(0.70 \pm 0.26 \pm 0.07) \times 10^{-6}$, where the first uncertainty is statistical and the second is systematic. Its signal significance is estimated as $\sqrt{2 \log(L_0/L_{max})}$, where $L_0$ and $L_{max}$ are the likelihood value with the signal yield set to zero and for the nominal case, respectively. Including systematic uncertainties (described below), we determine the significance to be 2.6 standard deviations ($\sigma$). In view of the significance being less than $3\sigma$, we set an upper

| Event category | $\Delta E$ | $C'_{NB}$ |
|----------------|-----------|-----------|
| Signal         | 3 G+Poly1 | G+AG      |
| Continuum $q\overline{q}$ | Poly1 | 2 G |
| Combinatorial $B\overline{B}$ | Poly1 | 2 G |
| Feed-across    | G+Poly1  | G         |
FIG. 2: (colour online). Projections of two-dimensional simultaneous fit to $\Delta E$ for $C_{NB}' > 0.0$ and $C_{NB}'$ for $|\Delta E| < 50 \text{MeV}$. Points with error bars are the data, solid blue curves are the total PDF, long dashed red curve is the signal component, dashed green curve is the continuum $q\bar{q}$ background, dotted magenta curve is the combinatorial $B\bar{B}$ background and dash-dotted cyan curve is the feed-across background.

limit (UL) on the branching fraction of $B \to K_S^0 K_S^0 \pi$. For this purpose we convolve the likelihood with a Gaussian function of width equal to the systematic error. Assuming a flat prior we set an UL of $1.14 \times 10^{-6}$ at 90% confidence level. Our limit is somewhat looser than that of BaBar [8] owing to our comparatively larger signal yield.

For $B^+ \to K_S^0 K_S^0 K^+$, we perform the fit in seven bins of $M_{K_S^0 K_S^0}$ to incorporate contribu-
tions from possible two-body intermediate resonances. Efficiency, signal yield, differential branching fraction, and $A_{CP}$ thus obtained are listed in Table II. Figure 3 shows the branching fraction and $A_{CP}$ plotted as a function of $M_{K_S^0K_S^0}$. We observe an excess of events around 1.5 GeV/c$^2$, whereas no significant evidence for $CP$ asymmetry is found in any of the bins. The inclusive branching fraction obtained by integrating the differential branching fractions over the entire $M_{K_S^0K_S^0}$ range is

$$B(B^+ \rightarrow K_S^0K_S^0K^+) = (10.64 \pm 0.49 \pm 0.44) \times 10^{-6},$$

where the first uncertainty is statistical and the second is systematic. Similarly, the inclusive $A_{CP}$ over the full $M_{K_S^0K_S^0}$ range is

$$A_{CP}(B \rightarrow K_S^0K_S^0K) = (-0.6 \pm 3.9 \pm 3.4)\%.$$  

This is obtained by weighting the $A_{CP}$ value in each bin with the fitted yield divided by the detection efficiency in that bin. As the statistical uncertainties are bin-independent, their total contribution is a quadratic sum. On the other hand, for the systematic uncertainties, the total contribution from the bin-correlated sources is taken as a linear sum while that from the bin-uncorrelated sources is determined as a quadratic sum. The results are in agreement with BaBar [6], where they had reported an overall $A_{CP}$ consistent with zero, and the presence of intermediate resonances $f_0(1500)$ and $f_2'(1525)$ in the aforementioned invariant-mass region.

TABLE II: Signal yield, efficiency, differential branching fraction, and $A_{CP}$ for each $M_{K_S^0K_S^0}$ bins.

| $M_{K_S^0K_S^0}$ (GeV/c$^2$) | Yield | Eff. (%) | $\Delta B \times 10^{-6}$ | $A_{CP}$ (%) |
|-----------------------------|--------|----------|-------------------------|-------------|
| <1.1                        | 98 ± 11| 24.0 ± 0.4| 1.14 ± 0.13 ± 0.06      | -3.2 ± 11.0 ± 3.0 |
| 1.1-1.3                     | 145 ± 14| 23.4 ± 0.2| 1.74 ± 0.17 ± 0.07      | -4.4 ± 9.1 ± 3.1 |
| 1.3-1.6                     | 250 ± 18| 22.9 ± 0.1| 3.06 ± 0.23 ± 0.12      | +6.1 ± 6.8 ± 3.6 |
| 1.6-2.0                     | 122 ± 13| 21.8 ± 0.1| 1.56 ± 0.17 ± 0.06      | +16.0 ± 10.0 ± 4.0 |
| 2.0-2.3                     | 103 ± 12| 24.1 ± 0.1| 1.20 ± 0.14 ± 0.05      | -1.8 ± 11.0 ± 2.9 |
| 2.3-2.7                     | 92 ± 12 | 25.2 ± 0.1| 1.02 ± 0.13 ± 0.04      | -2.0 ± 12.0 ± 3.2 |
| > 2.7                       | 86 ± 15 | 26.3 ± 0.0| 0.91 ± 0.16 ± 0.04      | -31.2 ± 17.0 ± 4.2 |
Major sources of systematic uncertainties on the branching fractions are same for both $B^+ \to K^0_SK^0_SK^+$ and $K^0_SK^0_S\pi^+$ decays. These are listed along with their contributions in Tables III and IV. We use partially reconstructed $D^{*+} \to D^0(K^0_S\pi^+\pi^-)\pi^+$ decays to assign the systematic uncertainty due to charged-track reconstruction (0.35% per track). The $D^{*+} \to D^0(K^-\pi^+)\pi^+$ control sample is used to determine the systematic uncertainty due to the $R_{K/\pi}$ requirement. The uncertainty due to the total number of $B\bar{B}$ pairs is 1.37%. The uncertainties due to the $M_{bc}$ and continuum suppression criteria are estimated using a control sample of $B^+ \to D^0(K^0_S\pi^-\pi^+)\pi^+$ decays. The uncertainty arising due to the $K^0_S \to \pi^+\pi^-$ reconstruction is estimated from $D^0 \to K^0_SK^0_S$ analysis [15]. Potential fit bias is checked by performing an ensemble test comprising 1000 pseudo-experiments, where the signal component is embedded from the corresponding MC samples and PDF shapes are used to generate the dataset for other event categories. The uncertainties due to signal PDF shape parameters are estimated by varying the correction factors (discussed earlier) by $\pm 1\sigma$ of their error. Similarly, the uncertainties due to background PDF shape parameters are calculated by varying all fixed parameters by $\pm 1\sigma$. We evaluate the uncertainty due to the fixed yields of combinatorial backgrounds by varying it up and down by its statistical error. The uncertainties due to the dependence of PDF shapes on $M_{K^0_SK^0_S}$ are evaluated in each $M_{K^0_SK^0_S}$ bin and propagated to the branching fraction measurement. The total systematic uncertainty is calculated by summing all these contributions in quadrature.
TABLE III: Systematic uncertainties in the branching fraction for \( B \to K^0_S K^0_S \pi \).

| Source                                | Relative uncertainty in \( B \) (%) |
|---------------------------------------|------------------------------------|
| Tracking                              | 0.35                               |
| Particle identification               | 0.80                               |
| Number of \( B\bar{B} \) pairs       | 1.37                               |
| Continuum suppression                 | 0.34                               |
| Requirement on \( M_{bc} \)          | 0.03                               |
| \( K^0_S \) reconstruction efficiency | 3.25                               |
| Fit bias                              | 1.86                               |
| Signal PDF                            | +2.50, −0.00                       |
| Combinatorial \( B\bar{B} \) PDF     | +2.42, −1.78                       |
| Feed-across PDF                       | +6.47, −9.56                       |
| Fixed yields                          | 0.00                               |

Systematic uncertainties on \( A_{CP} \) are listed in Table IV. The systematic errors due to the signal and background modeling are estimated with the same procedure as done for the branching fraction. Uncertainties due to intrinsic detector bias on charged particle detection is evaluated from the \( A_{CP} \) value obtained using a data sample of 89.4 fb\(^{-1}\) recorded 60 MeV below the \( \Upsilon(4S) \) resonance. We obtain an asymmetry of \((-2.7 \pm 2.0)\)% for this dataset, from which we take the absolute value of the central shift (2.7%) as the uncertainty due to detector asymmetry. The uncertainties due to the dependence of PDF shapes on \( M_{K^0_S K^0_S} \) are evaluated in each \( M_{K^0_S K^0_S} \) bin and propagated to the \( A_{CP} \) value.
In summary, we have reported measurements of the suppressed decays $B^+ \to K^0_3 K^0_S K^+$ and $B^+ \to K^0_3 K^0_S \pi^+$ using the full $\Upsilon(4S)$ data sample collected with the Belle detector. We perform a two-dimensional simultaneous fit to extract the signal yields of both decays. We report a 90% upper limit on the branching fraction of $1.14 \times 10^{-6}$ for the decay $B^+ \to K^0_3 K^0_S \pi^+$. We also report the branching fraction and $A_{CP}$ as a function of $M_{K^0_3 K^0_S}$ for $B^+ \to K^0_3 K^0_S K^+$. We observe an excess of events at low $M_{K^0_3 K^0_S}$ region, likely caused by the two-body intermediate resonances reported by BaBar [6]. An amplitude analysis with more data is needed to further elucidate the nature of these resonances. The measured inclusive branching fraction and direct $CP$ asymmetry are $\mathcal{B}(B^+ \to K^0_3 K^0_S K^+) = (10.64 \pm 0.49 \pm 0.44) \times 10^{-6}$ and $A_{CP} = (-0.6 \pm 3.9 \pm 3.4)^\%$, respectively. These supersede Belle’s earlier measurements [7] and constitute the most precise results to date.
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