Verification of Einstein’s Formula for Gravitational Deflection of Light Using Observations of Galactic Microlensing

A. N. Alexandrov*, V. I. Zhdanov*, and V. M. Sliusar

* Astronomical Observatory of National Taras Shevchenko University of Kyiv, Kyiv, Ukraine

Abstract—General relativity (GR) has a solid experimental base. However, the emergence of new experimental capabilities and independent observational information stimulates continuing tests of general relativity. The purpose of this work is to evaluate the potential of gravitational microlensing of distant sources on the stars of our Galaxy and to verify Einstein’s formula of gravitational refraction. This effect has been repeatedly tested in the Solar System in high-accuracy experiments with the propagation of radio waves, when the measurements are most effective for the distances from the signal trajectory to the Sun on the order of several solar radii. In the case of galactic microlensing, a quite different type of observational data and other characteristic distances are used that are determined in the high magnification events by the Einstein ring radii, which is typically of the order of 1 AU. Although the gravitational deflections of light by stars are very small and currently practically inaccessible by direct measurements, nonetheless, due to the large distances to the microlenses, the radiation flux from the source in strong microlensing events can increase several times. To verify Einstein’s formula, a more general dependence of the beam deflection angle $\alpha \propto 1/p^{1+\varepsilon}$ on its impact distance $p$ relative to the deflector is considered and, accordingly, the equations of gravitational lensing are modified. The challenge is to limit $\varepsilon$ based on observational data. The Early Warning System data obtained in 2018 within the Optical Gravitational Lensing Experiment (OGLE) (http://ogle.astrouw.edu.pl/ogle4/ews/2019/ews.html) was used. A sample of 100 light curves from the data obtained by the OGLE group in 2018 was formed. Each light curve was fitted as part of a modified model of gravitational lensing with parameter $\varepsilon$. As a result, 100 values of $\varepsilon$ and estimates of their variances were obtained. It was found that the mean value of $\varepsilon$ does not contradict GR within the limits of a one percent standard deviation. In the future, using a larger number of light curves will allow one to hope for a significant decrease in the error of $\varepsilon$ due to statistical averaging.

Keywords: gravitational light deflection, General Relativity tests, gravitational microlensing, light curves

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INTRODUCTION

General relativity (GR), which recently celebrated its centenary, has a solid experimental and observational base, which includes tests of a number of gravitational-relativistic effects with a sufficiently high accuracy [21, 1]. At present, there are no serious data that could cast doubt on the use of GR in astrophysics. However, it is natural that the emergence of new experimental possibilities stimulates the continuation of such tests. In this regard, the possibility of using gravitational lensing for testing various theories of gravity has repeatedly attracted attention (see, for example, [9–13, 7]).

The theory of gravitational lensing is based on Einstein’s formula [6] for the beam deflection angle $\alpha$ in the gravitational field of a point mass $M$, which is one of the classical prediction of GR (see, for example, [16, 4])

$$\alpha(p) = \frac{4GM}{c^2 p},$$ (1)

where $p$ is the impact parameter. One of the important research trends in this field is the study of galactic microlensing, in which a distant source is lensed by a foreground star of our Galaxy [14].
To verify Einstein’s formula, it is advisable to draw on a significant array of observational data from galactic microlensing, in particular, accumulated during the OGLE (Optical Gravitational Lensing Experiment) since 1992 [19, 20]. It is clear that the deflection of light by stars is very small compared with similar effects in the Solar System and has never been observed (nevertheless, see [5]). However, due to the huge astronomical distances, another effect comes into play here: an increase in the brightness of a distant source (a star in the Magellanic Clouds or in the Galactic Bulge) when a foreground star passes near the line of sight.

The objective of this paper is to evaluate the potential of galactic microlensing to verify the formula for the relativistic light deflection. The accuracy of such a test using one light curve of a distant source will be expected to be significantly lower than, for example, according to the results of the known tests of GR in the Solar System. But the existence of a significant and fairly homogeneous array of photometric observations [19, 20] allows us to hope for an increase in accuracy due to the data accumulation. It should be noted that, in comparison with the studies of the gravitational beam deflection and the signal delay in the solar field, the galactic microlensing data is completely independent, since it is based on a different approach and other observational material.

In the overwhelming majority of events, the microlensing mass and the radiation source can be regarded as pointlike. In this paper, we consider a microlensing model wherein the contributions of other objects (for example, planets or star-companions of binaries) can be neglected, and the relative motion of a point source and a point mass can be considered as inertial. To test the formula for the gravitational deflection of light, we use open data from the early warning system of the fourth stage of OGLE-IV for 2018 (http://ogle.astrouw.edu.pl/ogle4/ews/ews.html).

**BASIC RELATIONS**

In the theory of gravitational lensing, the key role is played by the so-called lens mapping, which relates the angular position of the source \( \mathbf{y} \) on the celestial sphere with the angular position \( \mathbf{x} \) of its image (see, e.g., [16, 4]). In the case of a point mass \( M \) (the so-called Schwarzschild lens), normalized lens mapping can be represented in the following form:

\[
y = \mathbf{x} \left( 1 - \frac{R_E^2}{|\mathbf{x}|^2} \right), \quad R_E = \left[ \frac{4GM_D_{LS}}{c^2 D_S D_L} \left( c^2 D_S D_L - 1 \right) \right]^{1/2};
\]

where \( R_E \) is the angular radius of the Einstein ring, \( D_S \) and \( D_L \) are the distances from the observer to the source and to the lens, \( D_{LS} \) is the distance from the lens to the source; and \( \mathbf{y} \) and \( \mathbf{x} \) can be considered as two-dimensional vectors, respectively, in the planes of the source and lens, touching the unit sphere. The microlens position is selected as the coordinate’s origin.

To verify formula (1), we modify formula (2) as follows:

\[
y = \mathbf{x} F(\xi, \varepsilon), \quad \xi = R_0/r, \quad r = |\mathbf{x}|.
\]

where \( R_0 \) is an analogue of \( R_E \) (in angular measure) and the dimensionless parameter \( \varepsilon \) describes the deviation of the right-hand side of (2) from expression of GR \( F(\xi,0) = 1 - \xi^2 \). Taking into account the available experiments (see, for example, reviews in [21, 1]), it is natural to assume that the indicated deviation and parameter \( \varepsilon \) are small. Our task is to obtain constraints on the deviation parameter \( \varepsilon \) for the selected function \( F(\xi, \varepsilon) \) (and, possibly, a reliable estimate of the deviation, if any).

The magnification factor of an individual image of a point source located at a point \( \mathbf{y} \) is (see, for example, [7])

\[
K(r, \varepsilon) = \det \left| \frac{\partial \mathbf{y}_j}{\partial \mathbf{x}_i} \right|^{-1} = \left| F \left[ \frac{d}{dr} (rF) \right] \right|^{-1}, \quad F \equiv F(R_0/r, \varepsilon),
\]

where \( \mathbf{x} \) should be found from the lens equation (3); \( r = |\mathbf{x}| \). For \( \varepsilon = 0 \), Eq. (3) is equivalent to Eq. (2), which has, for \( r \neq 0 \), two solutions corresponding to two images. If \( F(\xi, \varepsilon) \) is a smooth function of its variables at \( r \neq 0 \), then the number of images does not change at small \( \varepsilon \), that is, we again have two images \( \mathbf{x}_1(y, \varepsilon) \) and \( \mathbf{x}_2(y, \varepsilon) \). Since, in the case of galactic microlensing, different images of a distant source are not optically separated, fitting the light curves requires the total amplification factor of both images

\[
K_{tot}(y, \varepsilon) = K(\mathbf{x}_1(y, \varepsilon), \varepsilon) + K(\mathbf{x}_2(y, \varepsilon), \varepsilon).
\]
Taking into account the circular symmetry, it can be seen from Eqs. (2) and (3) that \( K_{\text{rot}}(y, \varepsilon) \) depends only on the modulus \( y = |y| \) but not on the direction of the vector \( y \). With rectilinear uniform movement of the microlens relative to the source

\[
y(t) = Y(t, p, V, t_0) = \sqrt{p^2 + V^2(t - t_0)^2},
\]

where \( p \) is the impact distance, \( V \) is the speed, and \( t_0 \) is the moment of maximum brightness.

After passing to stellar magnitudes, we have

\[
m(t, \varepsilon, p, V, t_0, C) = -2.5 \log \left( K_{\text{rot}}(Y(t, p, V, t_0), \varepsilon) \right) + C,
\]

where \( C \) is a constant associated with the source brightness in the absence of lensing and with the choice of the brightness standard.

The problem is reduced to minimizing the function

\[
H(\varepsilon, p, V, t_0, C) = \sum_{i=1}^{N} W_i \left( m_i + 2.5 \log \left( K_{\text{rot}}(Y(t_i, p, V, t_0), \varepsilon) \right) - C \right)^2;
\]

as a result, we determine the parameters \( p, V, t_0, C, \varepsilon \). Here, \( N \) is the number of points on the light curve, \( m_i \) are the observed magnitudes corresponding to the time moments \( t_i \), \( W_i \) are the weight coefficients determined in accordance with the observation conditions, \( i = 1, \ldots, N \). We considered two options for fitting: (1) with the same weights, (2) with \( W_i \propto \Delta m_i^{-2} \), where \( \Delta m_i \) are the estimates of errors in individual measurements taken from the OGLE database. A similar procedure can be carried out with data on the dependence of the radiation flux on time with the appropriate recalculation of the weights.

**CHOICE OF LENS MAPPING MODEL**

Below we restrict ourselves to such a modification of formula (1) for the deflection angle

\[
\alpha(p) = \left( \frac{4GM}{c^2 p} \right)^{1+\varepsilon}.
\]

Hence follows the lens equation with the function \( F(\xi, \varepsilon) \) appearing in the lens mapping formula (3):

\[
F(\xi, \varepsilon) = 1 - \xi^{2+\varepsilon}, \quad \xi \equiv R_0/r, \quad R_0^{2+\varepsilon} = R_E^{2+\varepsilon} \left( \frac{4GM}{c^2 D_L} \right)^{\varepsilon}.
\]

It should be noted that the standard parametrized post-Newtonian (PPN) formalism, which is often used when discussing GR tests, cannot be directly applied to formula (8), taking into account its nonanalyticity in the variable \( \xi \).

Equation (3), corresponding to (7), in terms of variables \( x, y \) on the celestial sphere has the following form:

\[
y = x \left[ 1 - \left( \frac{R_0}{r} \right)^a \right], \quad a = 2 + \varepsilon.
\]

Hence, we obtain

\[
y = r \left[ 1 - \left( \frac{R_0}{r} \right)^a \right], \quad y = |y|.
\]

An elementary analysis shows that, for small \( \varepsilon \), for any \( y > 0 \), there are two solutions to Eq. (8) with respect to \( r \): one solution for \( r_1 \in (0, R_0) \) and the second solution for \( r_2 > R_0 \), which turns into \( r = y \) for large \( y \). These solutions are easily determined by numerical methods. Accordingly, for solutions of Eq. (9), at \( y \neq 0 \), we have

\[
x_1 = -\mathbf{n}r_1, \quad x_2 = \mathbf{n}r_2, \quad \mathbf{n} = y/|y|.
\]
The magnification factor of a separate image \((4)\) takes the form

\[
K(r, \varepsilon) = \left[1 - \left(\frac{R_0}{r}\right)^a\right]^{-1} \left[1 + (a - 1)\left(\frac{R_0}{r}\right)^a\right]^{-1}.
\]  

(11)

The total magnification factor \(K_{\text{tot}}(y, \varepsilon)\) is given by the sum (5) of the magnification factors of two images, the coordinates of which \(x_1(y, \varepsilon)\) and \(x_2(y, \varepsilon)\) are solutions to Eq. (9).

In [2], we proposed to use the expansion in terms of \(\varepsilon\), which was implemented for a specific form (8) of the function \(F\). In the linear approximation on \(\varepsilon\), analytical expressions were obtained for \(\varepsilon\), \(\varepsilon_{xy}\), and the magnification factor (11). This approach makes it possible to explicitly write the correction to the light curve due to the presence of \(\varepsilon\). It can be useful when comparing different options for a function \(F\).

### PROCESSING OF LIGHT CURVES

The data of the OGLE project [19, 20] are used, in addition to studies of microlensing itself, by a number of observational programs, in particular, to identify and classify variable stars, identify dwarf novae, search for exoplanets, and study the Magellanic Clouds. In this paper, we used data from the early warning system of the OGLE project for 2018, from which a sample of one hundred events of strong microlensing were visually formed (Table 1). The sampling criteria were a sufficient increase in brightness at maximum, the accuracy of photometric observations, and the number of points on the light curve during microlensing as well as the absence of clear signs indicating a companion of comparable mass or a planet (see, for example, [3, 8, 18]) and the absence of parallax effects [15, 17].

The procedure for processing the light curve with number \(j\) \((j = 1, \ldots, K, K\) is the sample size\) was as follows. At the first stage, the parameters were fitted in the standard model of the Schwarzschild lens [16] \((\varepsilon = 0)\). In doing so, we found the zeroth approximation parameters \(p_j^{(0)}, V_j^{(0)}, t_{0j}, C_j^{(0)}\) for fitting the \(j\)th light curve. At the second stage, these parameters served as initial values for the procedure of minimizing the function (6). As a result, the minimum residual value (residual sum of squares) was determined

\[
\text{RSS}_j = \min \{H_j(\varepsilon, p, V, t_{0}, C)\},
\]  

(12)

and the values of the fitting parameters \(\varepsilon_j, p_j, V_j, t_{0j}, C_j\).

We fitted the light curves using three methods, which mainly differ in the choice of weights.

1. In the first approach, the OGLE data, given in magnitudes, were converted to radiation fluxes; to solve Eq. (9) and calculate the magnification factor (11), a linear approximation by \(\varepsilon\) [2] was used; the weights were taken as equal to each other.

2. In the next two methods, we performed calculations directly in magnitude, and solved Eq. (10) numerically.

3. For each light curve, the weights of the individual observations were considered as equal to each other. Note that, in this case, unequal weights correspond to equal \(W_j\) in case one.

**Table 1.** List of used light curves for microlensing events from the OGLE database (OGLE-2018-BLG-XXXX)

|   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|
| 0003 | 0097 | 0182 | 0247 | 0304 | 0409 | 0448 | 0541 | 0596 | 0655 |
| 0009 | 0098 | 0187 | 0249 | 0305 | 0410 | 0493 | 0545 | 0612 | 0658 |
| 0012 | 0100 | 0193 | 0261 | 0315 | 0411 | 0506 | 0566 | 0613 | 0660 |
| 0028 | 0105 | 0197 | 0263 | 0317 | 0418 | 0512 | 0567 | 0615 | 0662 |
| 0031 | 0127 | 0198 | 0287 | 0339 | 0419 | 0514 | 0574 | 0618 | 0663 |
| 0034 | 0137 | 0199 | 0288 | 0346 | 0421 | 0515 | 0575 | 0622 | 0672 |
| 0035 | 0142 | 0205 | 0292 | 0367 | 0423 | 0516 | 0576 | 0629 | 0681 |
| 0038 | 0149 | 0216 | 0293 | 0371 | 0436 | 0518 | 0579 | 0633 | 0711 |
| 0075 | 0156 | 0232 | 0300 | 0393 | 0437 | 0536 | 0581 | 0639 | 0713 |
| 0077 | 0171 | 0236 | 0303 | 0403 | 0447 | 0538 | 0589 | 0644 | 0727 |

The magnification factor of a separate image \((4)\) takes the form

\[
K(r, \varepsilon) = \left[1 - \left(\frac{R_0}{r}\right)^a\right]^{-1} \left[1 + (a - 1)\left(\frac{R_0}{r}\right)^a\right]^{-1}.
\]  

(11)
The choice of weights in Eq. (6) for each light curve was made according to \( W_j \propto \Delta m_i^{-2} \), where \( \Delta m_i \) are the estimates of errors in individual measurements from the OGLE database.

After calculating \( \varepsilon_j \) for \( K \) light curves of the sample, we determined the weighted average \( \langle \varepsilon \rangle = \sum_{j=1}^{K} w_j \varepsilon_j \) and its root mean square (standard) deviation \( \sigma_{\langle \varepsilon \rangle} \); the weights \( w_j \) were chosen as follows: \( w_j \propto 1/RSS \) and \( \sum_{j=1}^{K} w_j = 1 \). Such a choice of weights speeds up the calculations, but, strictly speaking, corresponds to linear models in the least squares method, while the contribution of nonlinearity is possible in our case. Therefore, we compared it directly with the choice of weights according to \( w_j \propto (\Delta \varepsilon_j)^{-2} \), where \( (\Delta \varepsilon_j)^2 \) is the marginal estimate of the variance obtained using the Monte Carlo simulation with the \( j \)th light curve in the framework of method two. Comparison of the two approaches to the choice of weights showed their satisfactory agreement.

The distributions of events by values \( \varepsilon_j \) in the sample, obtained by three methods, are presented in the form of histograms in Fig. 1. Here, \( p(\varepsilon) \) denotes the proportion of those events for which the value of the variable falls into the corresponding bin. It turned out that, according to method 1, 97% of events fall into the interval \([-0.242, 0.259]\); according to method 2, 98% of events fall into the interval \([-0.250, 0.263]\); according to method 3, 99% of events fall into the interval \([-0.257, 0.248]\).

Statistical estimates obtained in three ways are presented in Table 2, wherein the values of the median, mean \( \langle \varepsilon \rangle \), and standard deviation \( \sigma_{\langle \varepsilon \rangle} \) for the distributions obtained by the three described methods are shown.

As we can see, the errors in individual measurements can be relatively large, but the value \( \langle \varepsilon \rangle \) has the order \((1-2)\sigma_{\langle \varepsilon \rangle}\), that is, it is statistically insignificant.

**DISCUSSION**

The purpose of our work was to evaluate the possibility of using the light curves observed in the events of galactic microlensing to test Einstein’s formula for the gravitational deflection of light. As we can see from Table 2, the root-mean-square error \( \sigma_{\langle \varepsilon \rangle} \sim 10^{-2} \) and \( \langle \varepsilon \rangle \) is approximately the same order of magnitude in all three considered approaches. Therefore, deviations from zero should be considered statistically insignificant. This is another confirmation of Einstein’s formula, which indicates an alternative method for GR testing.

**Table 2.** Median, mean, and standard deviation of the distributions \( \varepsilon_j \) obtained by the three fitting methods.

| Method | Median | \( \langle \varepsilon \rangle \) | \( \sigma_{\langle \varepsilon \rangle} \) |
|--------|--------|----------------|----------------|
| 1      | 0.0053 | 0.0066         | 0.0085         |
| 2      | 0.0079 | 0.015          | 0.0087         |
| 3      | 0.0003 | –0.0036        | 0.011          |
In order to discuss our results, we note that they were obtained on observational data, completely independent of experiments in the Solar System. In this paper, we limited ourselves to considering the specific simplest modification (9) of the lens mapping, which is reduced to a change in the power-law dependence on the impact parameter. At the same time, the use of light curves makes it possible to consider other forms of lens mapping in order to verify general relativity.

Note that, in the effects of strong galactic microlensing with $D_s \gg D_L$, the typical angular distance between the source and the microlens is on the order of $R_E = 1.4 \times 10^{-8} (M/M_\odot)^{1/2} (D/\text{kpc})^{1/2}$, which corresponds to the impact parameter of the order of several astronomical units. This observational situation and the nature of the data are fundamentally different from experiments in the Solar System wherein the main information on the propagation of light and radio signals were obtained from observations with impact parameters on the order of several $R_\odot$. Bearing in mind the known estimates of the PPN-parameter $\gamma$, we can assume that the best relative error of experiments on measuring the gravitational beam deflection in the Solar System is on the order of $10^{-3}$ [21, 1]. However, if we recalculate it for impact distances close to the parameters of galactic microlensing, there will be a loss of accuracy by two orders of magnitude. Nevertheless, this accuracy is an order of magnitude higher than our estimates. Therefore, the question arises on how to improve the obtained estimate within the framework of the problem under consideration. The obvious answer is to bring more data into the analysis. In this paper, we inspected about 700 first events from the list of OGLE 2018 data to obtain a sample of 100 eligible events. Without even turning to future observations, we note that the fourth phase of the OGLE project already contains data for approximately 16000 microlensing events (the previous phases are much less). These data, according to our estimates, make it possible to increase the sample in 2018 data alone by 23 times and, accordingly, decrease $\sigma_{\theta_{\rm e}}$ by approximately five times. Thus, the discussed approach to the verification of GR has good prospects.

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