Dissipation for $Z$-channels in fission, alpha-decay and cluster emission

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Abstract. The dissipated energy in fission, cluster and alpha decay was estimated microscopically by using the time dependent pairing equations. The single particle level schemes were determined in the framework of the superasymmetric two center shell model. A strong dependence of the dissipated energy as function of the mass asymmetry is evidenced. The dissipated energy is much lower for alpha decay and cluster emission than for fission.

1. Introduction
As mentioned earlier in Ref. [1], the motion of any real macroscopic physical system is not only managed by conservative forces, but also by frictional ones. Therefore, in dynamical treatments we need to take into account the dissipative forces that produce an irreversible flow of energy or angular momentum from the macroscopic degrees of freedom to the intrinsic ones. The concept of nuclear friction can be employed if we use collective coordinates and microscopic ones. A such ensemble can be found in the case when the Schrodinger equation is solved for microscopic potentials that vary in time. In nuclear physics, an interesting case could be that of nucleons that are moving in the Nilsson potential.

In the following, we will use the time dependent pairing equations [2, 3] to determine the dissipated energy for the fission, for the alpha decay and for the $^{14}$C emission of $^{236}$U. The dissipated energy will be determined for predefined families of shapes that start from one spherical nucleus and ends as two separated fragments. A nuclear shape parametrization given by two intersected spheres of different radii will be used. The variations of the single particle energies is obtained with the Nilsson two center shell model [4, 5, 6]. Only the two degrees of freedom, namely, the elongation and to the mass asymmetry are used. Such calculations were realized in the past only for three fission channels [7]. We will investigate systematically the dissipated energy for all even-even partitions in the $Z$-channels.

2. The model
In our investigation, the time dependent pairing equations is used to evaluate the dissipation. The calculations are based on solutions given by the superasymmetric two center shell model.
2.1. The two center shell model
Investigating the fission in a wide range of mass asymmetries, the two-center shell models allow
a continuous description of the changes of nuclear shapes from one initial nucleus to two final
fragments. It is a generalization of the one center Nilsson model. The two-center shell-model
Hamiltonian is obtained from the two-center oscillator \( V_{2c} \) by adding the spin orbit interaction
term \( V_{ls} \) and a correcting term \( V_{l^2} \) [5].

\[
H = T + V_{2c} + V_{ls} + V_{l^2}
\]  

where \( T \) is the kinetic energy. The general expressions of these terms can be found in Refs.
[4, 6]. The Nilsson parameters coupling constants are taken from previous works [8].

2.2. Dissipated energy
The microscopic description of the dynamics of a many nucleon system has been extensively
investigated with the time dependent Hartree-Fork method [11]. However, in this method,
the residual interactions are neglected. That leads to the unpleasant feature that a system
even moving infinitely slowly could not end up in its ground state. In order to avoid all these
difficulties one could use the pairing residual interaction. We shall start from the variational
principle taking the following energy functional

\[
\delta L = \delta < \varphi | H - i\hbar \frac{\partial}{\partial t} - \lambda N | \varphi > ,
\]

and we assume the many-body state as a BCS seniority zero wave function

\[
| \varphi(t) > = \prod_l (u_l(t) + v_l(t)a^+_l a^+_l)|0 > .
\]

where \(|0 > \) denotes the vacuum state. \( a^+_l \) and \( a^+_l \) are creation operators of the coupled states
\( l \) and \( l \), and \( u_l \) and \( v_l \) are vacancy and occupation amplitudes, respectively. \( H \) is the time
dependent many-body Hamiltonian with pairing residual interactions

\[
H(t) = \sum_{k>0} \epsilon_k(t)(a^+_k a_k + a^+_k a^+_k) - G \sum_{k,l>0} a^+_k a^+_k a^+_l a^+_l.
\]

and the particle number operator is \( N \). By performing the variation of the Lagrangian in a way
similar as in [9, 10], the next system of coupled differential equations are obtained.

\[
i\hbar \dot{\rho}_l = \kappa_l \Delta* - \kappa_l^* \Delta,
\]

\[
i\hbar \dot{\kappa}_l = (2\rho_l - 1)\Delta + 2\kappa_l(\epsilon_l(t) - \lambda(t)) - 2G\rho_l\kappa_l .
\]

where \( \Delta = G\sum_k \kappa_k \) is the pairing gap, \( \kappa_k = u_k v_k \) are pairing moment components, \( \rho_k = |v_k|^2 \)
are single-particle densities. In our formula, the sum over pairs generally runs over index \( k \).

In the frame of our model, the difference

\[
E = E' - E_0
\]

behaves as a dissipated energy [2]. Here \( E' \) is the energy of the system obtained within solutions
of the time dependent pairing equations and \( E_0 \) is the BCS energy. These equations were
recently used in the study of fission [12, 13, 14, 15]
3. Results

Three nuclear decay modes are treated in an unified manner. A nuclear shape parametrization given by two intersected spheres of different radii is used. We need to know the variation of the mass asymmetry as function of the elongation for each nuclear decay mode. Information about this dependence can be found in Refs. [16, 17] where it was shown that in the case of very large mass asymmetries, that is, for alpha decay and cluster emission, the radius of the light fragment must be approximately constant \( R_2 = 1.16 \times \frac{A_2^{1/3}}{A^{1/3}} \) fm. In the case of fission the light fragment emerges with a radius that gives approximately a constant emitted volume. \( A_1 \) and \( A_2 \) denote the mass numbers of the parent and of the light fission fragment, respectively.

Figure 1. Family nuclear shapes, from the spherical configuration up to \( \alpha \) emission. The distances between the center of the fragments in fm are marked on the plot.

Figure 2. Same as Fig. 1 for the \( ^{14}\text{C} \) emission.

Figure 3. Same as Fig. 1 for fission.

In the case of the alpha decay and of the \( ^{14}\text{C} \) emission, the shapes are displayed in Figs. 1 and 2, respectively. The emitted fragment starts to appear at an elongation \( R = 1.16 \times ((A - A_2)^{1/3} - A_2^{1/3}) \) fm. A different situation appears in the case of fission for which the shapes are plotted in Fig. 3. In this last case, a deviation from the spherical shape can be observed from the beginning of the process, that is \( R \approx 0 \). The neutron and proton level schemes are obtained with the superasymmetric two-center shell model improved in Ref. [6]. The fact that the nucleus begins to deforms at larger values of \( R \) is reflected by the variations of single particle levels schemes in the case of alpha decay and carbon emission as evidenced in Figs. 4 and 5. In the case of fission, a different situation appears, as displayed in Fig. 6. In the three mentioned cases, the level scheme behave as a Nilsson diagram for prolate deformation.

Having the level schemes for the three decay modes it is possible to compute the dissipation during the disintegrations with the time dependent pairing equations. The velocity of the internuclear distance was fixed at \( \frac{\partial R}{\partial t} = 10^6 \text{ m/s} \), that gives approximately \( 10^{-21} \text{ s} \) as the time to tunnel the barrier [6]. This is considered as a characteristic time for scission.

Several \( Z \)-channels were tested in the case of \( ^{236}\text{U} \) fission. We used the partitions that give a maximum fission yield as given by existing compilations [18]. In this context we simulated the even-even complementary fragments like: \((^{118}\text{Pd}, ^{118}\text{Pd})\), \((^{114}\text{Ru}, ^{122}\text{Cd})\), \((^{106}\text{Mo}, ^{130}\text{Sn})\), \((^{100}\text{Zr}, ^{132}\text{Te})\), \((^{96}\text{Sr}, ^{140}\text{Xe})\), \((^{92}\text{Kr}, ^{144}\text{Ba})\), \((^{86}\text{Se}, ^{150}\text{Ce})\), \((^{82}\text{Ge}, ^{154}\text{Nd})\), \((^{78}\text{Zn}, ^{158}\text{Sm})\), \((^{72}\text{Ni}, ^{164}\text{Gd})\) and the last ones \((^{68}\text{Fe}, ^{168}\text{Dy})\).

In Fig. 7, the total dissipated energy in fission as function of of the charge number \( Z \) of the light fragment and the elongation \( R \) is represented. It is important to note that the dissipated energy at scission surpasses in general 7 MeV. For alpha decay and cluster emission,
the dissipated energies as function of the elongation are represented in Figs. 8 and 9. At scission the total dissipated energy amounts to 3 MeV for $\alpha$ decay and it is lower than 4.5 MeV for cluster emission. The dissipated energy reflect differences in the structure of these three decay modes.

The supersymmetric fission proceeds through adiabatic states, without dissipation, while in the fission at low energy a considerable amount of intrinsic excitation is produced. Therefore, the alpha decay and the cluster emission can be considered as cold rearrangement processes \cite{19, 20, 21, 22}. It is a confirmation of cold rearrangement hypothesis used in the prediction of
cluster decay [23]. The differences in dissipated energy have their origin in the variations of the single particle energies. As observed in the level schemes, the degree of rearrangement of the levels is much lower for superasymmetric fission than for fission.

Our study treats the alpha decay within fission models as in Ref. [24]. In general this decay modes is analysed with preformation models [25, 26] or liquid drop ones [27].

Acknowledgments
One of us (N.S.S.) wishes to express her love and gratitude to her beloved families especially to her mother; for their understanding and endless love, through the duration of her studies.

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Figure 8. Dissipated energy as function of elongation in $\alpha$-emission.

Figure 9. Dissipated energy as function of elongation in $^{14}$C emission.