Emergence of collective dynamical chirality (CDC) at mesoscopic scales plays a key role in many fundamental processes in development, health and disease. For instance, establishment of left-right asymmetry in embryonic development, one of the most intriguing biological phenomena, involves coordinated motions of many cells where the ability of cells to distinguish between left and right is evident in systems of chiral patterns formed by collective motion of identical cells confined in circular island or ring/striped-shaped micropatterns. On the other hand, CDC may also inspire new routines for fabrication of complex chiral architectures by dynamically self-assembling simple and achiral building blocks, e.g., chiral clusters of asymmetric colloidal dimers have been successfully assembled by using alternating current electric fields. Revealing how CDC arises from groups of active units is then very important for the understanding of the formation mechanism.

So far, CDC can be found in systems of active objects with individual structure chirality and/or individual dynamical chirality. For example, CDC has been reported in several experiments where vortexes were observed for microtubules, actin filaments, or sperm cells moving on a planar surface. It is believed that individual dynamical chirality might be caused by the rotation of the microtubule around its axis or by the special slender shape, while interactions between active objects help to align their moving direction. Besides, man-made catalytical nanorods or rotating disks can also be dynamically chiral in the form of swimming in circles, and hydrodynamic interaction can synchronize them to form CDC. There is also a recent study of elliptical active particles where particles can rotate in a circular confinement. For structurally chiral objects, dynamical chirality will arise when they are driven by external fields through potential landscapes, and CDC can provide an efficient method for chirality sorting. Very recently, a metastable CDC is reported in a system with achiral interaction. Since such a metastable chiral state will relax to a more stable state for a finite temperature, it is still a very attractive mystery that whether stable CDC can emerge in systems of simple particles without both individual structure chirality and dynamical chirality.

In this paper, we employ a model motivated by active motion of cells adhered on a surface in fluid environment to address such a question. The model consists of three elementary ingredients, i.e., achiral active moving, confined space for particle motion and hydrodynamic interaction (HI) between particles, to avoid other complexity such as special shape or structure chirality in real systems which perplexes us to understand the fundamental mechanism underlying formation of CDC. Remarkably, we find that CDC emerges spontaneously in the form of collective rotating for active forces larger than a critical value, near which an interesting oscillation between clockwise and anti-clockwise rotation is observed. Detailed analysis reveals that confinement and HI, along with the active motion of particles, are sufficient for the formation of CDC, while other details such as confinement shape and boundary condition are not relevant.

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Moreover, phase diagram shows that the system supports abundant collective states, e.g., two distinct states of CDC, a rotating droplet state and a rotating bubble state, are identified by a structure transition of CDC states, and interesting transition behaviors such as state reentrance from fluid-like state to rotating droplet then back to fluid-like state can also be observed. In addition, we find that the number of collective rotation is determined by the width/length ratio of the confinement. A single droplet can be found in squares, while arrays of multiple ones are observed in rectangles.

II. MODEL

We consider a system consisting of $N$ active spherical particles moving in a two-dimensional rectangle confined space of size $L \times W$, where $L$ and $W$ are the length and width, respectively. Active motion of particles is realized by exerting a constant force $f_0$ along the internal active directions, which mimics cell movement adhered on a surface. The only direct interaction between particles is exclusive volume effect taken into account by a Weeks-Chandler-Andersen potential $U(r_{ij}) = 4\epsilon \left\{ (2a/r_{ij})^{12} - (2a/r_{ij})^{6} \right\} + \epsilon$ existing only if $r_{ij} < 2\sqrt{2}a$, where $a$ is the effective repulsion radius, $r_{ij}$ the position of particle $i$, and $r_{ij}$ the distance between $i$-th and $j$-th particle. Particles can also interact with each other indirectly by long-range HI through the ambient fluid, where the force on $i$-th particle generated by the fluid is $F_{i,fl} = -\gamma r_{i} - u(r_{i}, t)$ with $u$ the fluid velocity at location $r_{i}$. In our work, $u(r_{i}, t)$ is calculated by a stochastic lattice Boltzmann method where particles are treated as point-like ones. Simulation details of the lattice Boltzmann method can be found in the supplemental information. The equations for translational motion of particles with mass $m$ are then

$$m\ddot{r}_i = f_0 n_i + F_{i,fl} - \sum_{j=1}^{N} \frac{\partial U(r_{ij})}{\partial r_{ij}} + \xi_i, \quad i = 1, \ldots, N. \quad (1)$$

Herein, $\xi_i(t)$ denotes the fluctuation force satisfying the fluctuation-dissipation relation $< \xi_i(t)\xi_j(t') >= 2\gamma k_B T \delta(t-t')\delta_{ij}$, where $k_B$ is the Boltzmann constant, $T$ denotes the temperature, and $n_i = (\cos \theta_i, \sin \theta_i)$ is the direction of active force. Besides, the angle $\theta_i$ is steered towards the direction of total force on $i$-th particle by the rule similar to the one in Ref.\[39\]

$$\dot{\theta}_i = (1/\tau_0)(\text{arg}(\vec{r}_i) - \theta_i) + \zeta_i \quad (2)$$

where $\text{arg}(\vec{r}_i)$ is the angle of total force vector, and $\zeta_i(t)$ is the rotational fluctuation satisfying $< \zeta_i(t)\zeta_i(t') >= 3k_B T \delta(t-t')\delta_{ij}/(2a^2\gamma)$. Such a steering rule is motivated by the fact that cells that respond to mechanical forces\[10\] \[40\] \[41\], which is also consistent with reported positive feedback regulation of front-rear cell polarity by actual cell displacements\[42\] \[43\]. The resistance time $\tau$ for active orientation measures the ability of particles resisting such external steering. For a small $\tau$, active particles will yield to the steering very fast, while for a large enough $\tau$ they tend to keep their internal random behavior similar to the conventional ones without steering rule\[44\] \[45\]. We rescale the density, time and length by the particle density, the simulation time step of the lattice Boltzmann method, and the grid length of the lattice, respectively. We fix $k_B T = 10^{-7}$, $\epsilon = 5 \times 10^{-4}$, $a = 0.75$, and $\gamma = (32/3)\alpha\nu\rho$ with fluid viscosity $\nu = 0.1$ and density $\rho = 1$, for which the system will reach a fluid-like state in the absence of active force. The resistance time, active force, number of particles and size of the space are $\tau = 1$, $f_0 = 1.25 \times 10^{-5}$, $N = 2500$ and $L = W = 100$ (corresponding to a volume fraction about 0.442), if not otherwise stated. The interaction between active particles and confined boundary is realized by bounce-back rule.

III. RESULT

To begin, we investigate how the collective motion of particles depends on the magnitude $f_0$ of the active force. For a small active force, e.g., $f_0 = 8 \times 10^{-7}$, the system is still fluid-like where particles move randomly. Quite interestingly, a CDC state emerges spontaneously if the active force becomes large enough. A typical snapshot of such a state for $f_0 = 1.25 \times 10^{-5}$ is presented in Fig.\[1\](a) where all the particles rotate collectively around the center of the space $r_c = (L/2, W/2)$. To quantitatively characterize CDC of the system, we define an order parameter

$$q = \frac{1}{N} \sum_{i=1}^{N} \varphi_i. \quad (3)$$

Here, $\varphi_i = \omega_i/|\omega_i|$ denotes the “dynamical-chirality spin” of particle $i$, where $\omega_i = (r_{i} - r_c) \times \dot{r}_i/|r_{i} - r_c|^2$ is the angular velocity of $i$-th particle relative to $r_c$. Note that $\varphi_i$ equals to 1 for anti-clockwise rotation and −1 for clockwise one. Time-dependencies of $q$ for $f_0 = 8 \times 10^{-7}$ and $1.25 \times 10^{-5}$ are plotted in Fig.\[4\](b). It can be observed that $q$ fluctuates around a fixed value after a quick relaxation, indicating that the system can finally reach a stable steady state. Time-averaged $q$ equals 0 for the fluid-like state, and is of a negative (or positive) value for the CDC state with collective clockwise (or anticlockwise) rotation.

To obtain a global picture for how CDC emerges as $f_0$ increases, magnitude of time-averaged $q$,

$$Q = \left| \lim_{t_0 \to \infty} \frac{1}{t_0} \int_{0}^{t_0} q(t) dt \right| \quad (4)$$

as a function of $f_0$ is drawn in Fig.\[1\](c). For spherical active particles, there is also another parameter to
measure the activity of particles, i.e., the Péclét number $Pe = |f_0|a/(k_B T)$. To be comparison, the corresponding value of $Pe$ are also shown in the top axis in Fig.1(c). Clearly, a continuous-like transition from fluid-like state to CDC state induced by particle activity is observed: $Q$ is nearly zero for small forces and quickly increases to be nearly 1 for $f_0$ larger than a threshold $f_c \approx 1.9 \times 10^{-6}$. By defining the standard deviation as $\sigma_Q = \sqrt{(1/t_0) \int_0^{t_0} [q(t) - \bar{q}]^2 dt}$ where $\bar{q}$ is the mean value of $q(t)$, $\sigma_Q$ exhibits a clear-cut peak as shown in the top inset of Fig.1(c). The presence of $f_c$ may be understood by the following observations. By taking a close look at the top-right and bottom-left corner in Fig.1(a), it can be seen that collective rotation can lead to accumulation of particles near the boundary at those corners. The accumulation will in return provide an obstacle for particles to rotate collectively and consequently an effective barrier for emergence of CDC.

Remarkably, we also observe an interesting oscillation of CDC. As depicted in Fig.1(b), particles rotate periodically between clockwise and anti-clockwise for an active force slightly smaller than $f_c$, e.g., $f_0 = 1.8 \times 10^{-6}$, whose typical snapshots are shown in the bottom-left and bottom-right insets of Fig.1(c). The formation of CDC oscillation may be due to competition of the onset process of CDC away from $q = 0$ and decay process towards $q = 0$. In Fig.1(d), rates of these two processes are presented by absolute values of the time-series slope for $f_0 = 1.67 \times 10^{-6}$, where the onset rate is about 4.92 while the decay one is 5.56. When the active force $f_0$ approaches the threshold $f_c$, for example $f_0 = 1.8 \times 10^{-6}$, the onset process is accelerated to be of a slope 13.1, in the meanwhile, the decay one decreases to 2.406 (Fig.1(e)). The observation implies that CDC is hard to onset and easy to decay for small active forces, and the decay rate may approach 0 and only emergence of CDC can be observed for large enough active forces. Thus, an appropriate active force can lead to CDC oscillation. To elucidate more clearly the detailed mechanism for the formation of CDC oscillation, a follow-up study may needed.

To identify the region of CDC oscillation, the time-averaged magnitude of $q$, $Q' = \lim_{t \to \infty} (1/t_0) \int_0^{t_0} |q(t)| dt$, is presented in Fig.1(c). It can be found that $Q'$ is overlapped with $Q$ very well for small or large active forces. In the CDC region where active force is in the range $1.55 \times 10^{-6} < f_0 < f_c$, $Q = 0$ indicates that there is no time-averaged CDC, while $Q'$ is larger than zero obviously, demonstrating that particles do rotate collectively for a given time. The standard deviation of $Q'$, $\sigma_{Q'} = \sqrt{(1/t_0) \int_0^{t_0} [\bar{q}(t) - |\bar{q}|]^2 dt}$ where $|\bar{q}|$ is the mean value of $|q(t)|$, is also plotted in the top inset of Fig.1(c), which shows a peak at $f_0 \simeq 1.9 \times 10^{-6}$ as same as the one of $Q$, demonstrating that chirality oscillation is not a new dynamical phase of the system.

One may be wondering what are the key ingredients that lead to the above interesting observations. In fact, we find that besides the active driving, the long rang HI and space confinement are two other necessary conditions for emergence of CDC as well as the CDC-oscillation. To show this, we have performed parallel simulations with the same parameter settings as above but with the HI turned off by using Brownian dynamics where diffusion coefficient of a free-diffusion particle is ensured to be the same. The obtained $Q$ without HI is plotted in Fig.1(c) to be compared with the one with HI. Clearly, there is no CDC can be observed for all range of parameters. We also repeat similar simulations for collective motion of particles without confinement, and no CDC is found, too. What’s more, other rules of interaction between active particles and confined boundary such as reflecting rule are also tested, and our findings are not sensitive to.
FIG. 2: (a) Dependence of order parameters $Q$ and $\Psi_6$ on resistance time $\tau$. (b) Standard deviation of order parameters as functions of $\tau$. (c) Typical snapshots of crystal-like state for $\tau = 10^8$ and (d) rotating bubble state for $\tau = 10^4$. The locally averaged velocities of particles normalized by the maximal one are presented by red arrows.

the boundary condition. In short, HI and confinement, along with activity of particles are the three key factors for the emergence of CDC. Notice that, active particles we used here are force monopoles to mimic cell movement adhered on a surface. For microswimmers suspended in fluid, they form at least force dipoles. We have repeated similar simulations for microswimmers, and no CDC was observed for the parameters presented here.

It is noted that, dynamics of active particles may change dramatically for different resistance time for active direction, thus, we now try to figure out how $\tau$ affects collective motion of active particles. In Fig.2(a), $Q$ as a function of $\tau$ is plotted by fixing $f_0 = 1.25 \times 10^{-5}$, where a transition between CDC and an achiral state can also be found. A typical snapshot of the achiral state is presented in Fig.2(c). Different to the fluid-like state, the achiral state observed here is crystal-like where particles are arranged in hexagonal ordering. The ordering can be measured by

$$\Psi_6 = \frac{1}{N} \sum_{m=1}^{N} \frac{1}{N_m} \sum_{l=1}^{N_m} \exp(6i\theta_{ml}),$$

where $i$ is the imaginary unit, $\theta_{ml}$ is the angle between an arbitrary reference axis and the displacement vector between particles $m$ and $l$, and the sum runs over the nearest $N_m = 6$ particles within a cutoff radius of 2.6$a$ from particle $m$ (for particles adjacent to the confined boundary, only $N_m = 4$ neighbors are needed to form hexagonal ordering). In Fig.2(a), $\Psi_6$ increases from a value near 0 to about 1 as $\tau$ increases, indicating a structure transition when collective motion of particles changes CDC state to crystal-like state.

FIG. 3: (a) Phase diagram on $f_0 - \tau$ plane. There is a triple-point-like point $(f_{tp}, \tau_{tp})$ above which no CDC can be formed for $\tau > \tau_{tp}$, and a state reentrance from fluid-like state to rotating droplet state then back to fluid-like state can be found as indicated by the gray arrow. (b) The color plot for $Q$ and the contour plot for $\Psi_6$ in the same parameter region as in (a). The color from gray to white indicates the value of $Q$ from 0.01 to 1 and lines with labels present corresponding values of $\Psi_6$.

If one takes a closer look at the dependence of $\Psi_6$ on $\tau$, it can be found that there seems to be a shoulder before the transition happens. For more detailed information, standard deviations $\sigma$ of $Q$ and $\Psi_6$ are given in Fig.2(b).

As expected, peaks are observed for both $\sigma_Q$ and $\sigma_{\Psi_6}$ at $\tau_c \approx 7 \times 10^4$, corresponding to the transition of both chirality and structure from CDC state to crystal-like state. It is quite interesting that there is also another peak of $\sigma_{\Psi_6}$ at $\tau_s \approx 2 \times 10^3$ where no peaks of $\sigma_Q$ are found, i.e., the CDC state undergoes a structure transition without loss of chirality. By comparison between the snapshot for $\tau = 1 < \tau_s$ in Fig.1(b) and a typical snapshot presented in Fig.2(d) for $\tau_s < \tau = 10^4 < \tau_c$, we can mark these two states of CDC as rotating droplet for the former and rotating bubble for the later.

In order to explore fully how parameters affect particles’ collective motion, a phase diagram in $f_0 - \tau$ plane is obtained by extensive simulations (Fig.3(a)). Several interesting remarks can be made. Firstly, there is a triple-point-like point located at $(f_{tp}, \tau_{tp})$ where CDC state meets both fluid-like and crystal-like state. For $\tau < \tau_{tp}$, CDC can arise spontaneously, while for larger $\tau$ only fluid-like state and crystal-like state can be observed. Notice that the steering rule in Eq.2 can be neglected for large enough $\tau$, the observation is in good agreement with findings reported in literature [14] [15]. Secondly, structure transition between the two states of CDC, rotating droplet and rotating bubble, occurs only for $f_0 > f_{tp}$, below which rotating droplet is the sole CDC state. Lastly, the system can also support other interesting state transi-
tion behaviors. As indicated by the gray arrow in Fig[3] a state reentrance can be found as $\tau$ increases for $f_0 < f_{tp}$, i.e., particles are firstly fluid-like for small $\tau$s then change to be of CDC for intermediate $\tau$s, then back to be fluid-like when $\tau$ is large enough. The corresponding color plot for $Q$ and contour plot for $\Psi_6$ are also presented in Fig[3](b). It can be found that, as $\tau$ passes by $\tau_{tp}$, transition from the fluid/crystalline to the chiral state results in a rapid increase in $Q$ for large $f_0$ as already shown in Fig[2]. Moreover for small $f_0$ the change is much more subdued, indicating that the two fluid-like areas are connected for very small $f_0$. When $f_0$ crossing $f_{tp}$, i.e. the system goes from the fluid to the crystalline state, we see that the $\Psi_6$ increases rapidly, showing a crystallisation/melting transition due to activity. Furthermore, the rotating droplet has a higher $\Psi_6$ than the rotating droplet, which is probably due to the walls facilitating a hexagonal order.

At last, effects of confinement shape on CDC are also considered. A typical snapshot for collective motion of particles in a circular confined space with diameter 100 is shown in Fig[4](a). Similarly, CDC emerges for the same parameters as in the square one, indicating that formation of CDC is not sensitive to the confinement shape. Dynamics of particles in confinement with different length/width ratio is also investigated. Typical snapshots for $L \times W = 160 \times 80$ and $240 \times 80$ are presented in Fig[4](b) and Fig[4](c), respectively, where number of particles is set to be $N = 3200$ for the former and $N = 4800$ for the later to keep the volume fraction unchanged. Interestingly, array of vortexes with opposite chirality for adjacent ones is observed, and the number of vortexes seems to be proportional to the length/width ratio.

IV. CONCLUSION

In summary, it was revealed that active motion, space confinement and hydrodynamic interaction are sufficient for the emergence of CDC in a system of active particles without individual structure chirality and dynamical chirality. CDC states were found to be formed via a chirality transition from other achiral state such as fluid-like state or crystal-like state, while they can also undergo a structure transition to form two distinct states, i.e., a rotating droplet and a rotating bubble. Phase diagram showed that CDC formation is controlled by the active force and the resistance time for particles to maintain their internal motion. More interestingly, CDC oscillation can also be supported by the system. Formation of CDC oscillation may be due to a competition between the onset process of CDC away from $q = 0$ and decay process towards $q = 0$. Since emergence of CDC underlies many formation processes of chiral structures, our finding may inspire experimental studies to explore new routines for fabrication of complex chiral architectures by simple and achiral units, and shed light on the understanding of chirality formation in other complex systems such as establishment of left-right asymmetry in embryonic development.

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