In this paper, we consider the distributed power tracking and energy balancing problem of a general energy storage system subject to unreliable switching communication network. In order to deal with the uncertainty of the network topology, a distributed observer-based approach has been proposed. First, an adaptive distributed observer is employed to recover the reference power for the entire energy storage system for each energy storage unit. Second, based on the estimated reference power, a certainty equivalent control law is synthesized to simultaneously achieve power tracking and energy balancing. Numerical simulations are provided to validate the proposed control approach.

1. Introduction

Facing the more and more severe global warming problem due to the release of greenhouse gases generated by using fossil fuels, the legacy grid is experiencing a rapid transition to the future smart grid, which involves deeply with the renewable energy sources such as solar, wind, tidal, and so on. As an indispensable part of the smart grid, energy storage system plays a key role to deal with the intermittency of the renewable energy sources by absorbing superfluous energy during off-peak time and releasing stored energy during the peak time [1–5]. There are many types of energy storage systems, such as flywheel energy storage system [6], supercapacitor energy storage system [7], battery energy storage system [8], and hybrid energy storage system [9], just to name a few. Each type of energy storage system has its own cons and pros from the perspective of power density, energy density, lift-time, maintenance, environmental impact, power quality, etc.

There have thus far been various energy management strategies and control approaches for energy storage systems. The model predictive control [10–12] could ensure robust performance for nonlinear systems and has the ability to work at a relatively low switching frequency, but it cannot deal with uncertain system parameters. By offering minimal steady state error at changing frequencies, repetitive control is one popular control method to be employed by energy storage unit. While, system stability is hard to be guaranteed under repetitive control [13–15]. The $H_\infty$ control proves to be robust against system uncertainties and can offer minimal tracking errors. Nevertheless, it comes with the price of slow system response and complicated mathematical treatment [16–18]. Like the $H_\infty$ control, the sliding mode control is also good at tackling uncertain system dynamics and parameters, which, however, is less appealing due to the chattering issues [19–21].

The control configuration for the energy storage system consisting of multiple energy storage units can be roughly categorized into three groups, namely, the centralized control, the decentralized control, and the distributed control. For the centralized control, all the commands and control decisions are made by the central controller, which thus only applies to small-scale energy storage systems and is not scalable. The decentralized control usually adopted the idea of the conventional droop control, which is easy to implement, but would result in poor voltage performance [22, 23]. To overcome this issue, many other variants of droop control have been developed, such as modified droop control featuring quick transient response [24, 25], combined droop control featuring equal power sharing [26, 27],
networked droop control featuring efficient active power allocation [28], and hierarchical droop control featuring compromise voltage and frequency regulation [29]. The distributed control relies on the infrastructure of the communication network, which can be made very sparse in contrast to the centralized control. On one hand, the information will be spread over the communication network in a distributed way in the sense that each energy storage unit can only obtain information of its neighbors over the communication network. On the other hand, the control decisions will be made locally by each energy storage unit. With the significant growth of the multiagent theory through the past two decades, the consensus algorithm-based control has found many applications for the energy management of microgrids. In Reference [30], a distributed cooperative control scheme was proposed to achieve secondary regulation of an islanded microgrid by taking use of feedback linearization. Later, by adopting the distributed observer approach, Reference [31] considered energy management problem of both the VCVSI and CCVSI interfaced microgrid. In Reference [32], by feeding the local and neighboring state-of-charge information to the controller, a consensus-based distributed integral control approach was proposed to achieve state-of-charge synchronization for battery packs. Reference [33] developed a semiconsensus mechanism for a hybrid energy storage system targeting for multiple control objectives, including power dispatch, bus voltage regulation, and state-of-charge balancing. The energy management problem for a hybrid energy storage system featuring a cascaded multiport converter was investigated in Reference [34]. To balance the power output and maintain the bus voltage, a consensus-based method was developed and validated by hardware-in-loop experiments. Reference [35] focused on small-scale energy storage system, which not only achieved frequency regulation but also fulfilled state-of-charge balancing by adopting a finite time consensus scheme for a leader-follower network.

For an energy storage system, there are two fundamental control objectives. The first one is the energy storage system as a single entity should track its reference power arranged possibly by some upper level control. The second one is to balance the energy level of all the energy storage units. Till now, there have been fruitful results for the state-of-charge balancing for battery energy storage systems on both the cell and pack level, such as References [36–38]. On the contrary, for general energy storage system, there have been few results. By utilizing the simultaneous eigenvalue placement technique, Reference [39] has considered the absolute energy balancing problem for a general energy storage system based on leaderless consensus algorithm. Later, the concept of state-of-energy was proposed by Reference [40], where a distributed control approach was proposed to solve the power tracking and energy balancing problem simultaneously. However, in Reference [40], the communication network is assumed to be static and reliable, which might not be realistic in practice due to constant communication links failure and restoration.

In this paper, we further consider the distributed power tracking and energy balancing problem of a general energy storage system subject to unreliable switching communication network. In order to deal with the uncertainty of the network topology, a distributed observer-based approach has been proposed. First, an adaptive distributed observer is employed to recover the reference power for the entire energy storage system for each energy storage unit. Second, based on the estimated reference power, a certainty equivalent control law is synthesized to simultaneously achieve power tracking and energy balancing. By rigorous Lyapunov analysis, it is proven that the proposed control approach can work effectively in the presence of switching network topology.

The main contributions of this paper are twofold:

(i) In contrast to the existing results of References [36–39], both the energy balancing and the power tracking control objectives are considered in this paper. By doing so, the power capacity of the energy storage system can remain maximal for all the time, and thus, the energy storage system can always be fully functional for grid supporting.

(ii) Different from the result of Reference [40], the communication network considered in this paper is allowed to switch among different connected subgraphs. On one hand, the communication network might not be always reliable in practice, which might lead to time-varying network topologies. On the other hand, the communication network can be designed in an intermittent manner so as to lower the communication burden. In this sense, the result of this paper is more practical than that of Reference [40].

2. Graph Notation

A graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is defined by a node set $\mathcal{V} = \{1, \ldots, N\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. For $i, j = 1, 2, \ldots, N$, $i \neq j$, $(i, j) \in \mathcal{E}$ means there exists an edge in $\mathcal{E}$ from node $i$ to node $j$. If $(i, j) \in \mathcal{E}$, then node $i$ is called a neighbor of node $j$. Let $\mathcal{N}_i = \{j, (j, i) \in \mathcal{E}\}$ denote the neighbor set of node $i$. If $(i, j) \in \mathcal{E}$ if and only if $(j, i) \in \mathcal{E}$, then the edge $(i, j)$ is called undirected. If all the edges of a graph are undirected, then the graph is called undirected. If $\mathcal{G}$ contains a set of edges of the form $(i_1, i_2), (i_2, i_3), \ldots, (i_k, i_{k+1})$, then the set $\{i_1, i_2, i_3, \ldots, i_k, i_{k+1}\}$ is called a path of $\mathcal{G}$ from node $i_1$ to node $i_k+1$, and node $i_{k+1}$ is said to be reachable from node $i_1$. A graph $\mathcal{G}$ is said to contain a spanning tree if there exists a node in $\mathcal{G}$ such that all the other nodes are reachable from it, and this node is called the root of the spanning tree. Given a set of $m$ graphs $\mathcal{G}_k = (\mathcal{V}, \mathcal{E}_k), k = 1, \ldots, m$, the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $\mathcal{E} = \cup_{k=1}^m \mathcal{E}_k$ is called the union of $\mathcal{G}_k$, denoted by $\mathcal{G} = \cup_{k=1}^m \mathcal{G}_k$.

A time signal $\sigma(t) : [0, +\infty) \longrightarrow \mathcal{S} = \{1, \ldots, s\}$ for some positive integer $s$ is called a piecewise constant switching signal with dwell time $\tau$ for some $\tau > 0$ if there exists a time sequence $\{t_k, k = 0, 1, 2, \ldots\}$ satisfying, $t_0 = 0$; for any positive integer $k$, $t_k - t_{k-1} \geq \tau$; $\sigma(t) = p$, $p \in \mathcal{S}$, for all $t \in [t_{k-1}, t_k)$. Given a node set $\mathcal{V} = \{1, \ldots, N\}$ and a piecewise constant switching signal $\sigma(t)$, define a switching
graph $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)})$ where $\mathcal{E}_{\sigma(t)} \subseteq \mathcal{V} \times \mathcal{V}$ for all $t \geq 0$. For a switching graph, let $\mathcal{N}_i(t)$ denote the neighbor set of node $i$ at time instant $t$. Associated with a switching graph $\mathcal{G}_{\sigma(t)}$, the matrix $\mathcal{A}_{\sigma(t)} = [a_{ij}(t)] \in \mathbb{R}^{N \times N}$ is called a time-varying weighted adjacency matrix of $\mathcal{G}_{\sigma(t)}$ if $a_{ii}(t) = 0$; $a_{ij}(t) > 0 \iff (j, i) \in \mathcal{E}_{\sigma(t)}$; and $a_{ij}(t) = 0$ otherwise. Let $\mathcal{L}_{\sigma(t)} = [l_{ij}(t)] \in \mathbb{R}^{N \times N}$ be such that $l_{ii}(t) = \sum_{j=1}^{N} a_{ij}(t)$ and $l_{ij}(t) = -a_{ij}(t)$ if $i \neq j$. Then, $\mathcal{L}_{\sigma(t)}$ is called the Laplacian of $\mathcal{G}_{\sigma(t)}$ associated with $\mathcal{A}_{\sigma(t)}$.

### 3. Problem Statement

In this paper, we consider an energy storage system consisting of $M$ energy storage units. Suppose the energy capacities of all the energy storage units are the same, denoted by $C_{ei}$. In the meantime, we let $C_{ei}(t)$ denote the remaining energy of the $i$-th energy storage unit at time instant $t$. Therefore, the state-of-energy $\chi_i(t)$ of the $i$-th energy storage unit can be defined by

$$\chi_i(t) = \frac{C_{ei}(t)}{C_{ei}}. \quad (1)$$

The relationship between energy and power is given by

$$\dot{\chi}_i(t) = -C_{ei} \varphi_i(t), \quad (2)$$

where $\varphi_i(t)$ denotes the power output of the $i$-th energy storage unit. In particular, $\varphi_i(t) > 0$ means the energy storage unit is releasing energy to the grid, and $\varphi_i(t) < 0$ means the energy storage unit is absorbing energy from the grid.

Then, by (1) and (2), we can obtain the dynamics of the state-of-energy of the $i$-th energy storage unit as follows:

$$\dot{\chi}_i(t) = -C_{ei} \varphi_i(t). \quad (3)$$

In terms of $\varphi_i(t)$, the total power output of the entire energy storage system can be obtained by

$$\varphi_{\text{ess}}(t) = \sum_{i=1}^{M} \varphi_i(t). \quad (4)$$

In addition, we define the following reference generator:

$$\dot{r}_0(t) = R_0 r_0(t), \quad (5)$$

where $r_0(t) \in \mathbb{R}^n$ is the state of the reference generator and $\varphi_{\text{ess}}(t) \in \mathbb{R}$ denotes the reference power output for the entire energy storage system. $R_0$ and $W_0$ are constant matrices, which satisfy the following assumptions.

**Assumption 1.** All the eigenvalues of $R_0$ have nonpositive real parts.

**Remark 1.** Assumption 1 does not lose any generality from the practical point of view since exponential divergent signals can hardly be used in practice. On the other hand, under Assumption 1, we allow the reference power signal to be the combination of multitone sinusoidal signals with polynomial signals, which can model a large class of reference signals.

Let switching graph $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)})$ model the unreliable communication network for the energy storage system as well as the reference generator, where $\mathcal{V} = \{0, 1, \ldots, M\}$ and $\mathcal{E}_{\sigma(t)} \subseteq \mathcal{V} \times \mathcal{V}$. The node $0$ represents the reference generator, and the node $i$ represents the $i$-th energy storage unit. For $i = 0, 1, \ldots, M$, $j = 1, \ldots, M$, $(i, j) \in \mathcal{E}_{\sigma(t)}$ if and only if the $j$-th energy storage unit can get message from the reference generator or the $i$-th energy storage unit at time instant $t$. In addition, let $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)})$ be defined by $\mathcal{V} = \{1, \ldots, M\}$ and $\mathcal{E}_{\sigma(t)} = \mathcal{E}_{\sigma(t)} \cap \{\mathcal{V} \times \mathcal{V}\}$. Moreover, define $\mathcal{G}_{\sigma(t)} = [a_{ij}(t)] \in \mathbb{R}^{(M+1) \times (M+1)}$ as the weighted adjacency matrix of $\mathcal{G}_{\sigma(t)}$, and let $\mathcal{L}_{\sigma(t)}$ be the Laplacian of $\mathcal{G}_{\sigma(t)}$.

Two assumptions regarding the communication graphs $\mathcal{G}_{\sigma(t)}$ and $\mathcal{E}_{\sigma(t)}$ are listed as follows.

**Assumption 2.** The switching graph $\mathcal{G}_{\sigma(t)}$ is undirected and connected for all $t \geq 0$.

**Assumption 3.** There exists a subsequence $\{\alpha_k; k = 0, 1, 2, \ldots\}$ of $\{k = 0, 1, 2, \ldots\}$ satisfying $t_{\alpha_{k+1}} - t_{\alpha_k} < \gamma_\alpha$ for some $\gamma_\alpha > 0$, such that every node $i, i = 1, \ldots, N$, is reachable from node 0 in the union graph $\bigcup_{k=1}^{\infty} \mathcal{E}_{\sigma(t)}$.

**Remark 2.** Assumption 2 requires bidirectional communication between energy storage units, which can be implemented in practice through wired communication network. Moreover, it is further required that though the unreliable communication network topology can be switching, the connectivity of the network topology should be guaranteed. Together with Assumption 2, Assumption 3 means for the energy storage system, the information obtained from the reference generator, though not necessarily available for all the time being, should at least be frequently available with the maximal time interval less than $\gamma_\alpha$.

Now, we are ready to state the problem considered in this paper as follows.

**Problem 1.** Given systems (3), (5), and the unreliable communication network $\mathcal{G}_{\sigma(t)}$, for $i = 1, \ldots, M$, design $\varphi_i(t)$ such that

$$\lim_{t \to \infty} (\varphi_{\text{ess}}(t) - \varphi_i(t) = 0, \quad (6)$$

and

$$\lim_{t \to \infty} (\chi_i(t) - \chi_j(t)) = 0, \quad i, j = 1, \ldots, M. \quad (7)$$

**Remark 3.** The control objective (6) means that the differences between the power output of the energy storage system and its reference should be regulated to zero asymptotically, and the control objective (7) means that the energy levels of all the energy storage units should keep balanced with respect to each other.
4. Main Result

In this section, we will solve Problem 1 by a distributed control law, which consists of two parts. First, an adaptive distributed observer is employed to recover the reference power for the entire energy storage system for each energy storage unit. Second, based on the estimated reference power, a certainty equivalent control law is synthesized to simultaneously achieve power tracking and energy balancing.

To begin with, we introduce the adaptive distributed observer as follows. For \( i = 1, \ldots, N \), design

\[
\dot{R}_i(t) = \mu_R \sum_{j=0}^{M} a_{ij}(t) (R_j(t) - R_i(t)),
\]

\[
\dot{W}_i(t) = \mu_W \sum_{j=0}^{M} a_{ij}(t) (W_j(t) - W_i(t)),
\]

\[
\dot{r}_i(t) = R_i(t)r_i(t) + \mu_r \sum_{j=0}^{M} a_{ij}(t) (r_j(t) - r_i(t)),
\]

\[
\varphi_{\text{ess},i}(t) = W_i(t)r_i(t),
\]

where \( R_i(t) \in \mathbb{R}^{n \times n} \), \( W_i(t) \in \mathbb{R}^{1 \times n} \), \( r_i(t) \in \mathbb{R}^{n} \), and \( \varphi_{\text{ess},i}(t) \in \mathbb{R} \) are the estimate of \( R_0, W_0, r_0(t) \), and \( \varphi_{\text{ess}}(t) \), respectively. \( \mu_R, \mu_W, \mu_r \geq 0 \) are the observer gains.

For \( i = 1, \ldots, N \), define

\[
\dot{\hat{R}}_i(t) = R_i(t) - R_0,
\]

\[
\dot{\hat{W}}_i(t) = W_i(t) - W_0,
\]

\[
\dot{\hat{r}}_i(t) = r_i(t) - r_0(t),
\]

\[
\varphi_{\text{ess}}(t) = \varphi_{\text{ess},i}(t) - \varphi_{\text{ess}}(t).
\]

Then, by Theorem 4.8 of Reference [41], it follows that, under Assumptions 1–3, all \( \dot{\hat{R}}_i(t), \dot{\hat{W}}_i(t), \dot{\hat{r}}_i(t) \), and \( \varphi_{\text{ess}}(t) \) approach to zero exponentially for any \( \mu_R, \mu_W, \mu_r \geq 0 \). In general, the larger the observer gains are selected, the faster the estimation can be realized. However, fast convergence of the distributed observer may come at the price of large overshoot for transient response.

Remark 4. The dynamic compensator (8) is distributed since it only relies on neighboring information over the communication network. It is called an adaptive distributed observer of the reference generator (5) due to the following two reasons. First, it does not take the system matrices of the reference generator as prior knowledge but instead estimates all these matrices adaptively, and this explains why it is named “adaptive”. Second, it recovers all the information of the reference generator, and in this sense it is called an observer of the reference generator.

Next, we show the local certainty equivalent control law as follows. For \( i = 1, \ldots, N \), design

\[
\varphi_i(t) = -\mu_p \sum_{j=1}^{M} a_{ij}(t) (\chi_j(t) - \chi_i(t)) + \frac{\varphi_{\text{ess},i}(t)}{N},
\]

where \( \mu_p > 0 \) is the control gain.

We have the following main results.

Theorem 1. Given systems (3), (5), and the unreliable communication network \( \mathcal{G}(\sigma(t)) \) under Assumptions 1–3, Problem 1 is solvable by the control law composed of (8) and (10) for any \( \mu_R, \mu_W, \mu_r, \mu_p > 0 \).

Proof: Substituting (10) to (3) gives

\[
\dot{x}_i(t) = C_{\text{el}}^{-1} \left( \mu_p \sum_{j=1}^{M} a_{ij}(t) (x_j(t) - x_i(t)) - \frac{\varphi_{\text{ess},i}(t)}{N} \right).
\]

Define \( \bar{\mu}_p = C_{\text{el}}^{-1}\mu_p \) and \( \bar{\mu}_p = C_{\text{el}}^{-1}/N \). Then we have

\[
\dot{x}_i(t) = \bar{\mu}_p \sum_{j=1}^{M} a_{ij}(t) (x_j(t) - x_i(t)) - \bar{\mu}_p \varphi_{\text{ess},i}(t)
\]

\[
= \bar{\mu}_p \sum_{j=1}^{M} a_{ij}(t) (x_j(t) - x_i(t)) - \bar{\mu}_p \varphi_{\text{ess},i}(t) - \bar{\mu}_p \varphi_{\text{ess}}(t).
\]

Let \( \mu_i \in \mathbb{R}^{n_i}, \ i = 1, \ldots, M \), \( \text{col}(a_1, \ldots, a_M) = [a_1^T, \ldots, a_M^T]^T \chi(t) = \text{col}(x_1(t), \ldots, x_M(t)) \) and \( \varphi_{\text{ess}}(t) = \text{col}(\varphi_{\text{ess},1}(t), \ldots, \varphi_{\text{ess},M}(t)) \). Then we have

\[
\dot{x}(t) = -\bar{\mu}_p \mathcal{L}_{\sigma(t)}(\chi(t)) - \bar{\mu}_p \varphi_{\text{ess}}(t) + 1_M - \bar{\mu}_p \varphi_{\text{ess}}(t),
\]

where \( 1_M = \text{col}(1, \ldots, 1) \in \mathbb{R}^{M} \).

Define \( Q = [Q_1, Q_r] \) where \( Q_1 = 1_M^{\dagger} \sqrt{M} \) and \( Q_r \in \mathbb{R}^{M \times (M-1)} \) is selected such that \( Q \) is orthogonal. Then, \( Q^{-1} = Q^T \). Furthermore, since

\[
Q^T Q = \begin{bmatrix} Q_1^T & Q_r^T \\ Q_1 & Q_r \\ Q_r^T & Q_r & Q_r \end{bmatrix} = \begin{bmatrix} I_M & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.
\]

it follows that \( Q_1^T Q_1 = 0 \). Therefore,

\[
Q^{-1} 1_M = \begin{bmatrix} Q_1^T \\ Q_r^T \end{bmatrix} 1_M = \begin{bmatrix} \sqrt{M} \\ 0 \\ \vdots \\ 0 \end{bmatrix}.
\]

Let \( \mathcal{J}_{\sigma(t)} = Q_1^T \mathcal{L}_{\sigma(t)} Q_r \). Since \( \mathcal{L}_{\sigma(t)} 1_N = 0 \) and \( \mathcal{L}_{\sigma(t)} \) is symmetric for all \( t \geq 0 \), we have

\[
Q^T \mathcal{L}_{\sigma(t)} Q = \begin{bmatrix} Q_1^T & Q_r^T \\ Q_1 & Q_r \end{bmatrix} \begin{bmatrix} \mathcal{L}_{\sigma(t)} 1_N, \mathcal{L}_{\sigma(t)} Q_1, \mathcal{L}_{\sigma(t)} Q_r \end{bmatrix} = \begin{bmatrix} Q_1^T & 0 \\ Q_r^T & Q_r \end{bmatrix} \begin{bmatrix} 0 \mathcal{L}_{\sigma(t)} Q_1, \mathcal{L}_{\sigma(t)} Q_r \end{bmatrix} = \begin{bmatrix} 0 0 \\ 0 Q_r \end{bmatrix}.
\]

Then, under Assumption 2, by Theorem A.1 of Reference [41], \( \mathcal{J}_{\sigma(t)} \) is symmetric and all the eigenvalues of \( \mathcal{J}_{\sigma(t)} \) are positive for all \( t \geq 0 \). Let \( \chi(t) = Q^{-1} \chi(t) \). Then we have
system (13) can be decomposed into the following two subsystems:

\[
\begin{align*}
\dot{\tilde{\chi}}(t) &= -\overline{\mu}_{\phi} Q^{-1} \mathcal{L}_{\phi(t)} \chi(t) - \overline{\mu}_{\phi} \psi_{\text{ess}}(t) Q^{-1} M - \overline{\mu}_{\phi} Q^{-1} \psi_{\text{ess}}'(t) \\
&= -\overline{\mu}_{\phi} Q^T \mathcal{L}_{\phi(t)} \psi(t) - \overline{\mu}_{\phi} \psi_{\text{ess}}'(t) Q^{-1} M - \overline{\zeta}(t),
\end{align*}
\]

where \(\overline{\zeta}(t) = \overline{\mu}_{\phi} Q^{-1} \psi_{\text{ess}}'(t)\).

Let

\[
\begin{align*}
\tilde{\chi}(t) &= \text{col} (\chi_{ss}(t), \chi_{tr}(t)), \\
\overline{\zeta}(t) &= \text{col} (\overline{\chi}_{ss}(t), \overline{\psi}_{ss}(t)),
\end{align*}
\]

where \(\chi_{ss}(t), \overline{\chi}_{ss}(t) \in \mathbb{R}\) and \(\chi_{tr}(t), \overline{\chi}_{tr}(t) \in \mathbb{R}^{M-1}\). Then system (13) can be decomposed into the following two subsystems:

\[
\begin{align*}
\dot{\chi}_{ss}(t) &= -\overline{\mu}_{\phi} \psi_{ss}(t) \sqrt{M} - \overline{\chi}_{ss}(t), \\
\dot{\chi}_{tr}(t) &= -\overline{\mu}_{\phi} \mathcal{J}_{\phi(t)} \chi_{tr}(t) - \overline{\chi}_{tr}(t).
\end{align*}
\]  

Associate with system (19b), define the following auxiliary system:

\[
\dot{\theta}(t) = -\overline{\mu}_{\phi} \mathcal{J}_{\phi(t)} \theta(t),
\]

and design the following Lyapunov function:

\[
V(t) = \frac{1}{2} \theta(t)^T \theta(t).
\]

Then, along system (20), it follows that

\[
\dot{V}(t) = -\overline{\mu}_{\phi} \theta(t)^T \mathcal{J}_{\phi(t)} \theta(t) \leq -\overline{\mu}_{\phi} J_{\text{min}} \theta(t)^T \theta(t) = -2\overline{\mu}_{\phi} J_{\text{min}} V(t),
\]

where \(J_{\text{min}}\) is the smallest eigenvalue of \(\mathcal{J}_{\phi(t)}\) for all \(t \geq 0\). By (22), \(V(t)\) tends to zero exponentially, and so is \(\theta(t)\) of system (20), that is, the origin of system (20) is exponentially stable. As a result, since \(\overline{\chi}_{ss}(t)\) tends to zero exponentially, by Lemma 2.8 of [41], it follows that \(\chi_{ss}(t)\) of system (19b) also tends to zero exponentially. In consequence, by noting that

\[
\chi(t) = Q \overline{\chi}(t) = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} \overline{\chi}_{ss}(t) \\ \overline{\chi}_{tr}(t) \end{bmatrix} = Q_1 \overline{\chi}_{ss}(t) + Q_2 \overline{\chi}_{tr}(t),
\]

it concludes that

\[
\lim_{t \to \infty} \left( \chi(t) - \frac{1}{\sqrt{M}} \overline{\chi}(t) \right) \equiv 0,
\]

that is,

\[
\lim_{t \to \infty} \left( \chi_i(t) - \chi_j(t) \right) = 0, \quad i, j = 1, \ldots, N, i \neq j.
\]
Figure 3: Continued.
In this section, we will use a numerical example to illustrate the proposed control method. Consider an energy storage system consisting of 6 energy storage units. Suppose the reference generator is given by $\omega = 1/3600 \text{rad/s}$ and thus $\varphi_{\text{ess}}(t) = 6 \times 10^3 \sin((1/3600)t) \text{kW}$.

Suppose the control gains are selected to be $\mu_p = \mu_W = \mu_r = 10$ and $\mu_q = 10^5$. Note that $\mu_q$ should be sufficiently large so that the energy balancing objective can be achieved timely.

The initial state-of-energy of all the energy storage units takes values in $[0.85, 0.92]$.

The initial values of the control laws take values in $[0, 0.5]$.

In the following, we will conduct the simulations for two cases.

5. Numerical Simulations

In this section, we will use a numerical example to illustrate the proposed control method. Consider an energy storage system consisting of 6 energy storage units. Suppose the energy capacity of each energy storage unit is 10 kWh.

The unreliable switching communication network is shown by Figure 1. To be more specific, the communication network is assumed to switch among four subgraphs $G_1, G_2, G_3,$ and $G_4$ periodically every $T_c$ sec. In the simulation, suppose $T_c = 10$. It can be easily verified that Assumptions 2 and 3 are satisfied. From these four subgraphs, it can also be noticed that the information of the reference generator is only available to the energy storage system for half of the time.

Suppose the reference generator is given by

$$\varphi_{\text{ess}}(t) = \mu_p \sum_{i=1}^{M} a_{ij}(t) \left( x_j(t) - x_i(t) \right)$$

where $\omega > 0$. Clearly, Assumption 1 is satisfied. In the simulation, we let $\omega = 1/3600 \text{rad/s}$ and thus $\varphi_{\text{ess}}(t) = 6 \times 10^3 \sin((1/3600)t) \text{kW}$.

Suppose the control gains are selected to be $\mu_p = \mu_W = \mu_r = 10$ and $\mu_q = 10^5$. Note that $\mu_q$ should be sufficiently large so that the energy balancing objective can be achieved timely.

The initial state-of-energy of all the energy storage units takes values in $[0.85, 0.92]$.

The initial values of the control laws take values in $[0, 0.5]$.

In the following, we will conduct the simulations for two cases.
hours. While, Figure 3(c) shows that the power tracking is fulfilled very soon after initialization. Figure 3(e) shows the power outputs of all the energy storage units. Due to the switching of the communication network, there are oscillations in the transient response of the power outputs.

5.2. Case 2: Power Saturation. In this case, we consider the practical scenario that the power output of the energy storage unit is subject to saturation. Specifically, we assume the power output of each energy storage unit is limited to ±3 kv. The simulation results are shown by Figure 3. It can be seen from Figures 3(b) and 3(d) that the power output saturation does not affect much the performance on energy balancing, but it takes a little bit longer time to achieve power tracking. Moreover, Figure 3(f) shows that all the power outputs are within the specified limits.

6. Conclusion

In this paper, the dual objectives of power tracking and energy balancing for a general energy storage system are considered and solved by a distributed control law. To make the proposed control law robust against unreliable communication links, the distributed observer approach has been adopted, which together with the local certainty equivalent control law solves the dual objective problem over the unreliable switching communication network. In the future, it would be interesting to further extend this work for a jointly connected communication network without assuming that the information exchange among energy storage units is undirected. Moreover, it would also be meaningful to consider the case of energy storage units with heterogenous power capacities.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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