Hamiltonian of Tensionless Strings with Tensor Central Charge Coordinates

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Abstract

A new class of twistor-like string models in four-dimensional space-time extended by the addition of six tensorial central charge (TCC) coordinates $z_{mn}$ is studied.

The Hamiltonian of tensionless string in the extended space-time is derived and its symmetries are investigated. We establish that the string constraints reduce the number of independent TCC coordinates $z_{mn}$ to one real effective coordinate which composes an effective 5-dimensional target space together with the $x^m$ coordinates. We construct the P.B. algebra of the first class constraints and discover that it coincides with the P.B. algebra of tensionless strings. The Lorentz covariant antisymmetric Dirac $\hat{C}$-matrix of the P.B. of the second class constraints is constructed and its algebraic structure is further presented.

1 Introduction

The supersymmetry algebra with tensor central charges \cite{1, 2, 3} is relevant for the brane sector \cite{4, 5, 6} of M/string theory \cite{7, 8, 9}. It is consistent with an extension of space time by tensorial central charge (TCC) \cite{10, 11, 12, 13}. A wide of relevant models were constructed in \cite{14, 15, 16} where the coordinates $z_{m_1m_2...m_p}$ are considered as new independent degrees of freedom corresponding to the p-form TCC generators $Z_{m_1m_2...m_p}$. A central charge carried by the BPS brane/string preserving 1/2 of the N=1 supersymmetry \cite{17} appears in QCD \cite{18, 19} due to spontaneous breakdown of discrete chiral symmetry \cite{20} and it is associated with a domain wall created by the gluino condensate. The TCC extension of the superparticle \cite{21, 22} and superbrane \cite{23, 24} models has led to new solutions which define the fraction of spontaneously broken supersymmetries. In \cite{25} combinations of momentum and domain-wall charges were used to characterize the BPS state spectrum of preserved fractions of $D = 4$ $N = 1$ supersymmetry. The interesting physical and mathematical foundations for the central extension of superspace were considered in \cite{26}, where a connection between the topological charges and TCC coordinates was analysed.
All the above points to the relevance of studying the dynamical role of the TCC coordinates in string/brane models. To this end new models for strings moving in $D = 4$ space-time extended by six real coordinates $z_{mn}$ corresponding to the TCC charges $Z_{mn}$ were constructed in [27]. The action integrals in these models generalize a twistor like formulations [28] for the Nambu-Goto and tensionless string actions. The suggested models have a natural supersymmetric generalization and may be considered as a bosonic sector of tensile or tensionless superstrings moving in the extended space-time. Keeping in mind the mechanisms for the generation of tension [32, 33, 35, 28] induced by the interaction of tensionless strings [36, 37, 38] and branes [39] or D-branes [40] with additional coordinates or fields one could expect a similar effect in the presence of the TCC coordinates.

Therefore, in [27] the action of tensionless string minimally extended by the introduction of the term linear in the derivatives of TCC coordinates was chosen for study. It was shown that inclusion of the $z_{mn}$ coordinates lifts the light-like character of the tensionless string worldsheet and removes the degeneracy of the worldsheet metric. This could be treated as a hint of string tension being generated. A particular set of solutions for the system of the string equations and integrability conditions found in [27] describes the string motion free of transverse oscillations in the $x-$directions and a wave process in the $z-$directions. These particular solutions do not capture full effects due to the TCC coordinates.

However, these effects have to be visible at the level of the generalized Virasoro algebra for the local symmetry generators of the model. To derive this algebra we need to construct the Hamiltonian formalism and that is the objective of the present paper.

By solving this problem the Hamiltonian and the constraints of the tensionless string in the extended space-time are constructed. After a covariant separation of the constraints into first and second classes we find that only 10 phase space variables of 36 are independent. This corresponds to the string moving in an effective $(4 + 1)-$dimensional target space instead of the primary $(4 + 6)-$dimensional space-time. As described in section 5 we expect this effective target space to be $AdS_5$. The P.B. algebra for the first class constraints is isomorphic to the corresponding P.B. algebra for tensionless string. This algebra has a structure similar to that of the contracted algebra of rotations of (Anti) de Sitter space. However, the second class constraints in the model under question differ from the corresponding constraints for the tensionless string and therefore encode the physical effects of the TCC coordinate. The second class constraints will deform the original P.B. algebra of the first class constraint into the Dirac bracket algebra and restore the correct equations of motion. Then the D.B. algebra of the first class constraints together with the string equations will define the structure of the string world sheet and the effective 5-dimensional target-space. (Equivalently, one can use a formalism where the second class constraints are converted into first class constraints in an extended phase space [41, 42, 43].)

We construct the Lorentz covariant antisymmetric Dirac $\hat{C}$-matrix of the P.B. of the second class constraints displaying its algebraic structure. It may be presented in a condensed form as a 9x9 complex matrix.
2 Strings with tensor central charge coordinates

To describe the string dynamics we start from a twistor-like representation of the ten-
sile/tensionless string action \[28\]

\[ S = \kappa \int (p_{mn} \, dx^m \wedge dx^n + \Lambda), \]  

where the local bivector \( p_{mn}(\tau, \sigma) \) is composed of the local Newman-Penrose dyads at-
tached to the worldsheet and the \( \Lambda \)–term fixes the orthonormality constraint for the
spinorial dyads (or twistor like variables).

For the case of tensionless string \( p_{mn}(\tau, \sigma) \) should be a null bivector defined by the
condition

\[ p_{mn} p^{mn} = 0, \quad \eta_{mn} = (- + + +), \]  

which implies the general solution for \( p_{mn}(\tau, \sigma) \) in the form of a bilinear covariant

\[ p_{mn}(\tau, \sigma) = i \bar{U} \gamma_{mn} U = 2i [u^\alpha (\sigma_{mn})_\beta^\alpha u^\beta + \bar{u}_{\dot{\alpha}} (\bar{\sigma}_{mn})_{\dot{\beta}}^\dot{\alpha} \bar{u}^{\dot{\beta}}], \]  

where \( U_a \) is a Majorana bispinor

\[ U_a = \left( \begin{array}{c} u_\alpha \\ \bar{u}_{\dot{\alpha}} \end{array} \right), \quad \gamma_{mn} = \frac{1}{2} [\gamma_m, \gamma_n], \]

\[ \sigma_{mn} = \frac{1}{4} (\sigma_m \bar{\sigma}_n - \sigma_n \bar{\sigma}_m). \]  

For tensile string the bivector \( p_{mn}(\tau, \sigma) \) may be presented as a sum of two null bivectors \( p_{mn}^{(+)} \) and \( p_{mn}^{(-)} \) \[28\]

\[ p_{mn} = p_{mn}^{(+)} + p_{mn}^{(-)} = i [U \gamma_{mn} U + V \gamma_{mn} V], \]  

where \( V_a = \left( \begin{array}{c} v_\alpha \\ \bar{v}_{\dot{\alpha}} \end{array} \right) \) is the second component of Newman-Penrose dyads \((u_\alpha(\tau, \sigma), v_\alpha(\tau, \sigma))\)

\[ u^\alpha v_\alpha = 1, \quad u^\alpha u_\alpha = v^\alpha v_\alpha = 0 \]  

and the \( \Lambda(\tau, \sigma) \)–term is

\[ \Lambda(\tau, \sigma) = \lambda (u^\alpha v_\alpha - 1) - \bar{\lambda} (\bar{u}^{\dot{\alpha}} \bar{v}_{\dot{\alpha}} - 1). \]  

The action \[(\ref{eq:action})\] may be rewritten in an equivalent spinor form

\[ S = i\kappa \int [p^{ab} \, dx_a \wedge dx_b C^{ed} + \Lambda], \]  

where \( C^{ed} = (\gamma^0)^{ed} \) is the charge conjugation matrix in the Majorana representation and
\( p^{ab} \) is a symmetric local spin-tensor. In the general case \( p^{ab} \) may be presented as a bilinear
combination of the Majorana bispinors \( U_a \) and \( V_a \).

\[ p^{ab} = \alpha U^a U^b + \beta V^a V^b + \varphi (U^a V^b + U^b V^a) \]  

with arbitrary coefficients \( \alpha, \beta \) and \( \varphi \).
The representation (8) includes an interesting object - the differential 2-form of the worldsheet area element $\xi_{ab}$ in the spinor representation

$$\xi_{ab} = \xi_{ba} = C^{cd} dx_{ae} \wedge dx_{db},$$

where

$$dx_{ab} = (\gamma_m C^{-1})_{ab} dx^m.$$  

Unlike of the vector representation for the area element $dx_m \wedge dx_n$ the the spinor representation $\xi_{ab}$ is a symmetric spin-tensor 2-form under permutations of the spinor indices $a$ and $b$.

To include the real antisymmetric central charge coordinates $z_{mn}$ we note that they may be presented by a symmetric real spin-tensor $z_{ab}$

$$z_{ab} = i z_{mn} (\gamma^{mn} C^{-1})_{ab},$$

Then following [21] we replace the world vector $x_{ab}$ by a more general spin-tensor $Y_{ab}$

$$x_{ab} \longrightarrow Y_{ab} = x^m (\gamma_m C^{-1})_{ab} + i z_{mn} (\gamma^{mn} C^{-1})_{ab}. $$

It was remarked in [27] that the differential $dY_{ab}$ (13) may be used as a building block for the construction of a generalized differential area element $\Xi_{ab}$

$$\Xi_{ab} \longrightarrow \Xi_{ab} = dY_{al} \wedge dY_{bl}. $$

As a result of the extension (14) the string action (8) is also generalized to the form,

$$S = i \kappa \int (p^{ab} \Xi_{ab} + \Lambda ) = i \kappa \int (p^{ab} dY_{ae} \wedge dY_{db} C^{cd} + \Lambda ). $$

proposed in [27], to include the TCC coordinates $z_{mn}$. The modification of the area element $dx_m \wedge dx_n$ induced by the contribution of $z_{lm}$ becomes more apparent after the substitution of $Y_{ab}$ (13) into (14) which gives the following representation for the generalized area element $\Xi_{ab}$

$$\Xi_{a}^{b} = ( dx_{m} \wedge dx_{n} - 8dz_{ml} \wedge dz_{n}^{l} ) (\gamma^{mn})_{a}^{b} - 4i dx^{l} \wedge dz_{lm} (\gamma^{m})_{a}^{b}. $$

To derive the representation (16) the standard relations for the $\gamma -$ matrices

$$[\gamma^{m_{1} n_{2}}, \gamma^{m}] = 2 (\eta^{m_{1} m} \gamma^{n_{2}} - \eta^{n_{2} m} \gamma^{m_{1}}),$$

$$[\gamma^{m_{1} n_{2}}, \gamma^{m_{1} n_{2}}] = 2 (\eta^{m_{1} m_{1}} \gamma^{n_{2} n_{2}} - \eta^{n_{1} n_{2}} \gamma^{m_{1} m_{2}} + \eta^{m_{1} n_{2}} \gamma^{n_{1} m_{2}} - \eta^{m_{2} n_{2}} \gamma^{n_{1} m_{1}} )$$

have been used.

As a result the generalized action $S$ (15) takes the following form

$$S = i \kappa \int \{ ( dx_{m} \wedge dx_{n} - 8dz_{ml} \wedge dz_{n}^{l} ) \gamma^{mn} - 4i dx^{l} \wedge dz_{lm} \gamma^{m} )_{a}^{b} p^{a} b + \Lambda \} .$$

In (19) $x^m$ and $z^{mn}$ appear on equal footing, and we would ultimately want to study solutions to this model. In a first investigation, however, we consider the minimally extended action obtained by omitting the term quadratic in $z$

$$S = i \kappa \int \{ ( dx_{m} \wedge dx_{n} \gamma^{mn} - 4i dx^{l} \wedge dz_{lm} \gamma^{m} )_{a}^{b} p^{a} b + \Lambda \}. (20)$$
The model (20) was treated in [27] by considering a tensionless string minimally extended by the tensor central charge coordinates. This case corresponds to $\Lambda = 0$ and a spin-tensor $p^{ab}$ of the form [28]

$$p^{ab} = U^a U^b.$$ 

The minimally extended null string action then takes the form

$$S = i \int U^a (dx_m \wedge dx_n \gamma^{mn} - 4i dx^l \wedge dz_{lm} \gamma^m) \frac{1}{2} U^b,$$

where the constant $\kappa$ is in a redefinition of $x_m$ and $z_{lm}$ making all variables in (21) dimensionless.

In [27] a Lagrangian treatment of the model (21) was given. In view of the linear character of $S$ (21) in the $\tau$ world-sheet derivatives $\dot{x}_m$ and $\dot{z}_{lm}$ this model is characterized by a non-trivial set of constraints and its quantization requires an investigation of these constraints and construction of the corresponding Hamiltonian mechanics.

We shall investigate this problem below.

3 Tensionless string with central charge coordinates.

The primary constraints

To study the constraints and Hamiltonian dynamics of $S$ (21) we start from its Weyl representation [27]

$$S = i \int (u^a dx_{\alpha\lambda} \wedge \tilde{d} x^{\tilde{\lambda} \beta} u_{\beta} + \bar{u}_a d\tilde{z}^{\tilde{\alpha} \lambda} \wedge dx_{\lambda\beta} \tilde{u}^{\beta})$$

$$+ 2(u_a dz^{\alpha\beta} \wedge dx_{\beta\lambda} \tilde{u}^{\lambda} - \bar{u}_a d\tilde{z}^{\tilde{\alpha} \beta} \wedge u^{\lambda} dx_{\lambda\beta}),$$

where

$$\tilde{\sigma}_{mn} = \frac{1}{2} d\tilde{x}_{\lambda\beta} \wedge dx_{\lambda\beta},$$

and the antihermitian 2-form $\Omega_{\alpha\beta}$

$$\Omega_{\alpha\beta} \equiv -8i (dz_{ml} \wedge dx^l)(\sigma^{mn})_{\alpha\beta} = 2[(dz_\lambda \wedge dx_{\lambda\beta} + dx_{\alpha\lambda} d\tilde{z}_{\beta}],$$

where

$$z_{m_1 m_2} = \frac{i}{4}[z^{\alpha} (\sigma_{m_1 m_2})_{\beta} + \tilde{z}_{\alpha} \tilde{(\tilde{\sigma}_{m_1 m_2})_{\beta}}]$$

(26)
Note that $S (22)$ may be equivalently presented in the compact form

$$S = 2i \int (u^\alpha \Sigma_\alpha^\beta u_\beta + \bar{u}_\alpha \bar{\Sigma}_\alpha^\beta \bar{u}_\beta^\dagger + \frac{1}{2} u^\alpha \Omega_{\alpha\beta} \bar{u}_\beta^\dagger). \quad (27)$$

For the Hamiltonian description we need the canonical momentum densities

$$P^M \equiv (P^\alpha_{\dot{\alpha}}, \pi_{\alpha\beta}, \bar{\pi}_{\dot{\alpha} \dot{\beta}}, P^\alpha_{\alpha}, \bar{P}^\alpha_{\alpha}, \bar{P}^\alpha_{\alpha}) = \frac{\partial L}{\partial \dot{q}_M} \quad (28)$$

which are conjugate to the target space coordinates

$$q_M = (x_{\alpha \dot{\alpha}}, z_{\alpha \beta}, \bar{z}_{\dot{\alpha} \dot{\beta}}, u_\alpha, \bar{u}_\alpha, v_\alpha, \bar{v}_\alpha) \quad (29)$$

in the Poisson bracket

$$\{P^M(\sigma), q_N(\sigma')\}_{P.B.} = \delta^M_N \delta(\sigma - \sigma') \quad (30)$$

As far as the action (22) is linear in the proper time derivative $\dot{x}_{\alpha \dot{\alpha}}, \dot{z}_{\alpha \beta}, \dot{\bar{z}}_{\dot{\alpha} \dot{\beta}}$ the definition (28) leads to the primary constraints $\Phi$

$$\Phi_{\alpha \dot{\alpha}} \equiv (P^\alpha_{\dot{\alpha}} - \Delta^\alpha_{\dot{\alpha}}) = 0, \quad (31)$$

where

$$\Delta^\alpha_{\dot{\alpha}} \equiv 2i (u^\alpha r_{\alpha} - r^\alpha \bar{u}_\alpha^\dagger), \quad r^\alpha \equiv (\bar{u} \bar{x}')^\alpha + (u z')^\alpha, \quad (32)$$

and $\Psi$

$$\Psi_{\alpha \beta} \equiv \pi_{\alpha \beta} + \Delta_{\dot{\alpha} \dot{\beta}} = 0, \quad (33)$$

where

$$\Delta_{\dot{\alpha} \dot{\beta}} \equiv 2i [u^\alpha (\bar{u} \bar{x}')^\beta + u^\beta (\bar{u} \bar{x}')^\alpha]. \quad (34)$$

These constraints have to be added by the primary constraints for the dyads $u^\alpha$ and $v^\alpha$

$$P^\alpha_u = 0, \quad P^\alpha_v = 0, \quad (35)$$

together with their complex conjugate (c.c.).

The standard definition of the canonical Hamiltonian density

$$\mathcal{H}_0 = P^M \dot{q}_M + \frac{1}{2} \pi_{\alpha \beta} \dot{z}_{\alpha \beta} + \frac{1}{2} \bar{\pi}_{\dot{\alpha} \dot{\beta}} \dot{\bar{z}}_{\dot{\alpha} \dot{\beta}} - \mathcal{L}, \quad (36)$$

where

$$q_M \equiv (x_{\alpha \dot{\alpha}}, u_\alpha, \bar{u}_\alpha, v_\alpha, \bar{v}_\alpha), \quad (37)$$

is consistent with the Poisson brackets definition (30) and generates the Hamiltonian equations of motion

$$\frac{df}{d\tau} \equiv \dot{f}(\tau, \sigma) = \int d\sigma' \{\mathcal{H}_0(\sigma'), f(\tau, \sigma)\}_{P.B.}. \quad (38)$$
In the considered case we have from the definitions (28) and (36) that
\[ H_0 = 0 \quad (39) \]
and the evolution of the string is described by the generalized Hamiltonian
\[ H = H_1 + H_2, \]
\[ H_1 \equiv a_{\alpha\dot{\alpha}} \Phi^{\alpha\dot{\alpha}} + b_{\alpha\beta} \Psi^{\alpha\beta} + \bar{b}_{\dot{\alpha}\dot{\beta}} \bar{\Psi}^{\dot{\alpha}\dot{\beta}}, \]
\[ H_2 = [\mu_{\alpha} P^\alpha_u + \varrho_{\alpha} P^\alpha_v + \lambda (u^\alpha v_{\alpha} - 1)] + \text{c.c.} \quad (40) \]

Before we embark on the analysis of the consistency of the constraints, we remark on some interesting consequences of (31) and (33), e.g.,
\[ P_{\dot{\alpha}\alpha} x_{\alpha} = 2 \frac{i}{\lambda} \pi_{\alpha\beta} \pi_{\dot{\alpha}\dot{\beta}} = 4 \left( u^\alpha \bar{u}^\alpha \right)^2, \]
\[ \pi_{\alpha\beta} \pi_{\dot{\alpha}\dot{\beta}} = 4 i \left( u^\alpha \bar{u}^\alpha \right), \]
\[ \pi^{\alpha\beta} \pi_{\dot{\alpha}\dot{\beta}} = 4 \left( u^\alpha \bar{u}^\alpha \right), \]
It follows from (41), (42) and (43) that the string constraints reduce to the tensionless string constraints \[ P_{\dot{\alpha}\alpha} x_{\alpha} \big|_{z=\pi=0} = 0, \quad P_{\dot{\alpha}\alpha} x_{\alpha} \big|_{z=\pi=0} = 0, \]
if the TCC coordinates \( z_{\alpha\beta} \) and their momenta \( \pi^{\alpha\beta} \) are equal zero. It shows a nontrivial contribution of the TCC coordinates in the constraint algebra and the null string dynamics. Moreover, the constraint (42) may be presented in a slightly different form
\[ P_{\dot{\alpha}\alpha} x_{\alpha} = \frac{1}{2} \left( \pi_{\alpha\beta} \pi_{\dot{\alpha}\dot{\beta}} + \bar{\pi}^{\dot{\alpha}\dot{\beta}} \bar{\pi}_{\dot{\alpha}\dot{\beta}} \right) \quad (46) \]
using (44). The constraint (46) together with the constraints (35) are equivalent to to the first class constraints \[ T_{\pm} \]
\[ T_{\pm} \equiv P_{\dot{\alpha}\alpha} x_{\alpha} + \frac{1}{2} \left( \pi_{\alpha\beta} \pi_{\dot{\alpha}\dot{\beta}} + \bar{\pi}^{\dot{\alpha}\dot{\beta}} \bar{\pi}_{\dot{\alpha}\dot{\beta}} \right) \]
\[ \pm \left[ (u^\alpha P^\alpha_v + \bar{u}^\alpha \bar{P}^\alpha_v) + (v^\alpha P^\alpha_u + \bar{v}^\alpha \bar{P}^\alpha_u) \right] = 0, \]
where \( T_{\pm} \) describe the world sheet diffeomorphisms corresponding to \( \sigma \)-shifts (modulo the constraints (35) for the sign \( - \) in (47)).

In the next Sections we will construct the remaining first and second class constraints using the Dirac’s selfconsistency procedure.
4 The first class constraints

The conservation conditions of the constraints (3), (31), (33), (35) and their c.c.

\[ \frac{d\phi}{d\tau} \equiv \dot{\phi}(\tau, \sigma) = \int d\sigma' \{ H(\sigma'), \phi(\tau, \sigma) \}_{P.B.} \approx 0, \]  

(48)

where the symbol \( \approx \) means the Dirac’s weak equality, must either restrict the Lagrange multipliers in \( H \) (40) or produce new secondary constraints. The conservation of the constraint (3) leads to the condition

\[ \varrho_\alpha u^\alpha - \mu_\alpha v^\alpha = 0, \]  

(49)

which has the general solution

\[ \mu_\alpha = hu_\alpha + gv_\alpha, \quad \varrho_\alpha = -hv_\alpha + \nu v_\alpha. \]  

(50)

Analogously, the conservation condition for the constraint \( P_\alpha^\gamma = 0 \) (35) gives

\[ \lambda = 0. \]  

(51)

As a result \( H_2 \) in (40) takes the form

\[ H_2 = [h(u_\alpha P^\alpha_u - v_\alpha P^\alpha_v) + gv_\alpha P^\alpha_u + \nu u_\alpha P^\alpha_v] + \text{c.c.} \]  

(52)

It is easy to check that

\[ W \equiv u_\alpha P^\alpha_v = 0 \]  

(53)

and its c.c. are the first class constraints generating the gauge symmetry of the action by the complex shifts

\[ \delta v_\alpha = \delta \nu u_\alpha, \quad \delta u_\alpha = 0. \]  

(54)

To check the conservation of the constraint \( P^\alpha_u = 0 \) we note that

\[
\begin{align*}
\{ P^\gamma_\alpha(\sigma), \Phi^{\alpha\hat{\alpha}}(\sigma') \}_{P.B.} &= -2i[u^\alpha \bar{x}^\hat{\alpha} x^\gamma - \bar{u}^\hat{\alpha} z^\hat{\alpha} \alpha \gamma + \varepsilon^{\alpha\gamma}(\bar{\bar{x}}^\hat{\alpha} u + \bar{u} z^\hat{\alpha})\hat{\alpha}]\delta(\sigma - \sigma'), \\
\{ P^\gamma_\alpha(\sigma), \Psi^{\alpha\beta}(\sigma') \}_{P.B.} &= 2i[\varepsilon^{\alpha\gamma}(u \bar{u})^{\beta} + \varepsilon^{\gamma\beta}(u \bar{u})^{\alpha}]\delta(\sigma - \sigma'), \\
\{ P^\gamma_\alpha(\sigma), \bar{\Psi}^{\hat{\alpha}\hat{\beta}}(\sigma') \}_{P.B.} &= -2i[u^\hat{\alpha} \bar{x}^\hat{\alpha} \gamma + \bar{u} \bar{z}^\hat{\alpha} \gamma]\delta(\sigma - \sigma').
\end{align*}
\]  

(55)

Then we find the following equation

\[ \dot{P}^\gamma_\alpha = 2i [u(ax' - x'\bar{a}) - (z'a - 2bx')\bar{u} - \bar{u}(z'\bar{a} - 2\bar{b}x')]\gamma \approx 0 \]  

(56)

which does not involve the Lagrange multipliers \( h, g \) and \( \nu \). It follows from (47) that Eqs. (54) have as a particular solution

\[ a_{aa} = a_0 x'_{aa}, \quad b_{\alpha\beta} = b_0 z'_{\alpha\beta}, \quad a_0 = \bar{a}_0, \quad b_0 = \bar{b}_0. \]  

(57)

In fact, substituting (57) into (56) transforms the latter into the equation

\[ \dot{P}^\gamma_\alpha = -2i (a_0 - 2b_0)(z' x' \bar{u} + \bar{u} z' x')^\gamma \approx 0, \]  

(58)

which has the expected solution

\[ b_0 = \frac{1}{2} a_0. \]  

(59)
To find the second solution of (56) and the solutions for the Lagrange multipliers $h, g$ and $\nu$ from $\mathcal{H}_2$ we consider the consistency conditions for $\Phi^{\alpha\dot{\alpha}}$ and $\Psi^{\alpha\beta}$.

To this end we note that

$$\{\Phi^{\alpha\dot{\alpha}}(\sigma), \Phi^{\beta\dot{\beta}}(\sigma')\}_{P.B.} = 2i[(\varepsilon^{\alpha\beta} \ddot{u}^{\dot{\alpha}} \ddot{u}^{\dot{\beta}} - u^{\alpha} u^{\dot{\beta}} \varepsilon^{\dot{\alpha}\dot{\beta}})\big|_{\sigma'} - (\sigma' \to \sigma)] \partial_{\sigma} \delta(\sigma - \sigma'),$$

$$\{\Psi^{\alpha\beta}(\sigma), \Psi^{\gamma\delta}(\sigma')\}_{P.B.} = \{\Psi^{\alpha\beta}(\sigma), \Psi^{\dot{\gamma}\dot{\delta}}(\sigma')\}_{P.B.} = 0 \tag{60}$$

and

$$\{\Phi^{\alpha\dot{\alpha}}(\sigma), \Psi^{\beta\gamma}(\sigma')\}_{P.B.} = -2i[(\varepsilon^{\alpha\beta} u^\gamma + \varepsilon^{\alpha\gamma} u^\beta) \ddot{\alpha} \big|_{\sigma} - (\sigma' \to \sigma)] \partial_{\sigma} \delta(\sigma - \sigma'). \tag{61}$$

Using (60) and (61) we find

$$\frac{1}{2i} \dot{\Psi}^{\beta\gamma} = [(u^{\beta}(\ddot{u} a)^\gamma + u^{\gamma}(\ddot{u} a)^\beta) + (\beta \to \gamma)] + [(\zeta^\beta (u \ddot{x})^\gamma + u^\gamma (\ddot{x}^\beta)^\gamma) + (\beta \to \gamma)], \tag{62}$$

where

$$\zeta^\alpha = hu^\alpha + gv^\alpha. \tag{63}$$

After multiplication (62) by $u_\beta u_\gamma$ we find

$$g = (u^{\beta} u_\beta) \frac{(\ddot{u} a u)}{(u \ddot{x} u)}. \tag{64}$$

Moreover, the $u'_\alpha$ expansion in the base dyades $u_\alpha$ and $v_\alpha$

$$u'_\alpha = \varphi u_\alpha + \chi v_\alpha \tag{65}$$

transforms equations (62) and (64) into the equations

$$u^{\beta}[(\varphi + \bar{\varphi})(\ddot{u} a)^\gamma + (h + \bar{h})(\ddot{u} x)^\gamma + \bar{\chi}(\ddot{v} a)^\gamma + g(\ddot{v} x)^\gamma] + u^{\gamma}[\chi(\ddot{u} a)^\gamma + g(\ddot{u} x)^\gamma] + \{\beta \to \gamma\} = 0 \tag{66}$$

and

$$g = -\chi \frac{(\ddot{u} a u)}{(u \ddot{x} u)}. \tag{67}$$

In the first place we note that the particular solution (57) for $\ddot{\alpha}^{\dot{\alpha}}$

$$\ddot{\alpha}^{\dot{\alpha}} = a_0 \ddot{x}^{\dot{\alpha}}, \quad a_0 = \bar{a}_0 \tag{68}$$

have to be a solution of Eqs. (56) and (57). After a substitution of (68) into these equations they are reduced to the relations

$$(a_0 \varphi + h) + (\bar{a}_0 \bar{\varphi} + \bar{h}), \quad g = -a_0 \chi \tag{69}$$

or equivalently to

$$h = -a_0 \varphi + i\theta_0, \quad g = -a_0 \chi, \quad \varphi = (u^{\alpha} v_\alpha), \quad \chi = -(u^{\alpha} u_\alpha), \tag{70}$$
where \( \theta_0 \) is an arbitrary real function. The expression for \( \mathcal{H}_2 \) (52) corresponding to the solution (70) takes the form

\[
\mathcal{H}_2 = \left[ -a_0 (u'_\alpha P_u^\alpha + v'_\alpha P_v^\alpha) + \tilde{\nu} u_\alpha P_v^\alpha + i\theta_0 (u_\alpha P_u^\alpha - v_\alpha P_v^\alpha) \right] + c.c.,
\]

(71)

where \( \tilde{\nu} \equiv \nu + (v'^\alpha v_\alpha) \) and the relation

\[
u_\alpha v_\beta - v_\alpha u_\beta = \varepsilon_{\alpha\beta}
\]

(72)

was used. The expression for \( \mathcal{H}_2 \) given in (71) and that for \( \mathcal{H}_1 \) in (40) shows that the Lagrangian multiplier \( a_0 \) corresponds to the first class constraints \( T \). (which equals \( T \) modulo the constraints \( P_u^\alpha \approx 0 \) and \( P_v^\alpha \approx 0 \)). The last \( \theta_0 \)-term in (71) shows the invariance of \( \Psi^{\alpha\beta} \) under the transformations

\[
u'_\alpha = e^{i\Theta(\tau,\sigma)} u_\alpha, \quad v'_\alpha = e^{-i\Theta(\tau,\sigma)} v_\alpha.
\]

(73)

However, the transformations (73) is not a symmetry of the action (22) and consequently we have to choose the integration “constant” \( \theta_0 \) in the solution (70) vanish

\[
\theta_0(\tau,\sigma) = 0.
\]

(74)

We conclude that the solution (70) generates the first class constraint

\[
T \equiv P^{\alpha\dot{\alpha}} x_{\alpha\dot{\alpha}} + \frac{1}{2}(\pi^{\alpha\beta} \dot{z}_{\alpha\beta} + \pi^{\dot{\alpha}\dot{\beta}} \dot{z}_{\dot{\alpha}\dot{\beta}})
+ [(u'_\alpha P_u^\alpha + \bar{u}'_{\bar{\alpha}} \bar{P}_u^\bar{\alpha}) + (v'_\alpha P_v^\alpha + \bar{v}'_{\bar{\alpha}} \bar{P}_v^\bar{\alpha})] \approx 0
\]

(75)

which is one of the generators of the generalized Virasoro algebra corresponding to the \( \sigma \)-reparametrization of the string world sheet.

The second generator of this algebra corresponds to the second solution of Eqs. (66) and (67)

\[
\bar{a}^{\dot{\alpha}} = a_1 u^\alpha \bar{u}^{\dot{\alpha}}, \quad a_1 = \bar{a}_1
\]

(76)

which restricts the corresponding Lagrange multipliers \( g, h \) and \( \chi \) in \( \mathcal{H}_2 \) (52) to be

\[
g = 0, \quad h = i\theta_1, \quad \chi = 0,
\]

(77)

where \( \theta_1(\tau,\sigma) \) is an arbitrary real function. Due to the general \( u'_\alpha \) expansion (53) we conclude that the solution (77) implies a secondary constraint

\[
u'^\alpha u_\alpha = 0.
\]

(78)

It is easy to check that this constraint is preserved by the Hamiltonian (40) because

\[
g = 0
\]

(79)

for the both solutions (70) and (77). The constraint (78) is a second class constraint.

The solutions (58) and (77) have also to be the solutions of the equation

\[
\dot{\Phi}^{\alpha\dot{\alpha}}(\tau,\sigma) = \int d\sigma' \left\{ \mathcal{H}(\sigma'), \Phi^{\alpha\dot{\alpha}}(\tau,\sigma) \right\}_{P.B.} \approx 0,
\]

(80)
Using the P.B. (60) and (61) we find
\[ \{H_1 + H_2, \Phi^{\dagger \alpha}\}_{P.B.} = -2i[u^{\alpha}(\bar{a}u)^{\dagger} + u^{\alpha}(\bar{a}u')^{\dagger}] + 2(u^{\alpha}(\bar{b}u)^{\dagger} + u^{\alpha}(\bar{b}u')^{\dagger}) + \zeta^{\alpha}(\bar{x}'u)^{\dagger} + u^{\alpha}(\bar{x}'\zeta)^{\dagger} + \zeta^{\alpha}(\bar{z}'u)^{\dagger} + u^{\alpha}(\bar{z}'\zeta)^{\dagger} - c.c. \approx 0. \] (81)

The substitution of the expansion (82)
\[ \zeta_{\alpha} = h u_{\alpha}, \quad u'_{\alpha} = \varphi u_{\alpha} \]
into Eqs. (81) and using the solution (79) leads to
\[ \{2[\varphi(\bar{a}u)^{\dagger} + h(\bar{x}'u)^{\dagger}] + [2(\varphi + \bar{\varphi})(\bar{b}u)^{\dagger} + (h + \bar{h})(\bar{z}'u)^{\dagger}]\} - c.c \approx 0. \] (83)
The substitution of (82) and (83) into (83) yields the equation
\[ \{2(\alpha \varphi + h)(x'u)^{\dagger} + [\alpha (\varphi + \bar{\varphi}) + (h + \bar{h})(z'u)^{\dagger}]\} - c.c \approx 0 \]
which is satisfied due to the relations (70) and (74).

The substitution of the second solution (76) and (86) into Eq. (83) gives the equation
\[ [h(\bar{x}'u)^{\dagger} + (\varphi + \bar{\varphi})(\bar{b}u)^{\dagger}] - c.c \approx 0 \]
which has the following solution for the Lagrange multipliers \( h \) and \( b_{\alpha \beta} \)
\[ h = 0, \quad b_{\alpha \beta} = b_1 u_{\alpha} u_{\beta}. \] (86)

The next step is the substitution of the solutions (76) and (86) into Eq. (56) which transform it into the final equation
\[ a_1[(ux'\bar{u}) + (\bar{u}z'\bar{u})] = 2b_1(ux'\bar{u}). \] (87)
As long as \((ux'\bar{u}) \neq 0\), Eq. (76) allows us to express the Lagrange multiplier \( b_1 \) as a function of \( a_1 \)
\[ b_1 = \frac{a_1}{2}[1 + (\bar{u}z'\bar{u})/(ux'\bar{u})], \quad \bar{b}_1 = \frac{a_1}{2}[1 + (uz'\bar{u})/(ux'\bar{u})]. \] (88)

Using Eqs. (7), (77), (78), (82) and the expression for \( H_2 \) we find the Hamiltonian density (40) corresponding to the second solution of the selfconsistency conditions
\[ H = a_1\{u_\alpha \bar{u}_\alpha \Phi^{\dagger \alpha} + \frac{1}{2}[1 + (\bar{u}z'\bar{u})/(ux'\bar{u})]u_\alpha u_\beta \Psi^\alpha \beta \]
\[ + \frac{1}{2}[1 + (uz'\bar{u})/(ux'\bar{u})]\bar{u}_\alpha \bar{u}_\beta \Psi^{\dagger \alpha \beta} + [i \theta_1 (u_\alpha P^\alpha_u - v_\alpha P^\alpha_v) + c.c.]. \] (89)

In just the same way as above we resume that the integration “constant” \( \theta_1 \) should vanish
\[ \theta_1(\tau, \sigma) = 0, \] (90)

because it corresponds to the transformations (73) which are not symmetries of the action (24). Then \( \mathcal{H} \) in (89) yields the second Virasoro generator
\[ Q \equiv P^{\dagger \alpha} u_\alpha \bar{u}_\alpha + \frac{1}{2}[1 + (\bar{u}z'\bar{u})/(ux'\bar{u})]u_\alpha \bar{u}_\beta + \frac{1}{2}[1 + (uz'\bar{u})/(ux'\bar{u})]\bar{u}_\alpha \bar{u}_\beta \approx 0. \] (91)

This result completes the Dirac procedure for the construction of the first class constraints \( W \) (eqn. (73)), \( T \) (eqn. (75)), \( Q \) (eqn. (71)).

11
5 Hamiltonian and algebra of the first class constraints

In the previous Section we found four real first class constraints: two real constraints $T$ and $Q$ and one complex constraint $W$

\[ T \equiv P^{\bar{\beta}} x_{\bar{a} \dot{\alpha}} + \frac{1}{2} (\pi^{\alpha \beta} z_{\alpha \dot{\beta}} + \bar{\pi}^{\bar{\alpha} \bar{\beta}} \bar{z}_{\bar{\alpha} \bar{\beta}}) + (u_a P^a + \nu' P^v) + (\bar{u}_{\bar{a}} \bar{P}^{\bar{a}} + \bar{v}' \bar{P}^{\bar{v}}) \approx 0, \]

\[ Q \equiv P^{\bar{\alpha}} u_{\alpha} \bar{u}_{\bar{\alpha}} + \frac{1}{2} [1 + \left( \frac{\bar{u} \bar{z} \bar{u}}{u x' \bar{u}} \right)] \pi^{\alpha \beta} u_{\alpha} u_{\beta} + \frac{1}{2} [1 + \left( \frac{u' \bar{x}' \bar{u}}{u x' \bar{u}} \right)] \bar{\pi}^{\bar{\alpha} \bar{\beta}} \bar{u}_{\alpha} \bar{u}_{\beta} \approx 0, \]

\[ W \equiv (u_a P^a) \approx 0, \quad \bar{W} \equiv (\bar{u}_{\bar{a}} \bar{P}^{\bar{a}}) \approx 0. \] (92)

The Hamiltonian density $\mathcal{H}$ is correspondingly

\[ \mathcal{H} = a_0 T + a_1 Q + \nu W + \bar{\nu} \bar{W} \approx 0. \] (93)

If we use the following condensed notation for the constraints $Q$, $W$ and $\bar{W}$

\[ \mathcal{K}_I \equiv (Q, W, \bar{W}), \quad (I = 1, 2, 3), \] (94)

we find that the P.B. algebra of the first class constraints $T$ and $\mathcal{K}_I$ takes the compact form

\[ \{ T(\sigma), T(\sigma') \}_{P.B.} = [T(\sigma) + T(\sigma')] \partial_{\sigma'}(\sigma - \sigma'), \]

\[ \{ T(\sigma), \mathcal{K}_I(\sigma') \}_{P.B.} = \mathcal{K}_I(\sigma) \partial_{\sigma'}(\sigma - \sigma'), \]

\[ \{ \mathcal{K}_I(\sigma), \mathcal{K}_I(\sigma') \}_{P.B.} = 0. \] (95)

The algebra coincides with the corresponding algebra for tensionless string \cite{31}

\[ \{ T_0(\sigma), T_0(\sigma') \}_{P.B.} = [T_0(\sigma) + T_0(\sigma')] \partial_{\sigma'}(\sigma - \sigma'), \]

\[ \{ T_0(\sigma), \mathcal{K}_{0I}(\sigma') \}_{P.B.} = \mathcal{K}_{0I}(\sigma) \partial_{\sigma'}(\sigma - \sigma'), \]

\[ \{ \mathcal{K}_{0I}(\sigma), \mathcal{K}_{0I}(\sigma') \}_{P.B.} = 0, \] (96)

where

\[ T_0 \equiv T|_{z=\pi=0}, \quad \mathcal{K}_{0I} \equiv \mathcal{K}_I|_{z=\pi=0} \] (97)

So, we conclude that the minimal inclusion of the TCC coordinates $z_{\alpha \bar{\beta}}$ has no effects on the level of P.B. algebra for the first class constraint. To find these effects we have to make the transition to the Dirac brackets which take into account the second class constraints for the considered action \cite{27}. These constraints differ from the second class constraint of tensionless string because of the TCC coordinates $z_{\alpha \bar{\beta}}$ presence.

Note also that the structure of the P.B. algebra \cite{15} resembles the structure of the contracted rotation algebra in (anti) de Sitter space with the splitted “fifth” coordinate and the space radius $R$ going to infinity. In view of an universal character of the $T$ generator we do not expect that the two first P.B. in the algebra \cite{15} will change after the transition to the Dirac bracket. But, we may expect the appearance of a non-zero contribution into the right hand side of the Dirac bracket originated from the last P.B. in \cite{27}. If so, that scenario gives a hint that the introduction of TCC coordinates restors a finite value of $R$ without breaking the conformal symmetry of the string action. Below we derive the second class constraints and describe their algebraic structure. We will show that the introduction of the TCC coordinates $z_{mn}$ effectively adds only one coordinate to the $x_m$ coordinates.
6 The second class constraints

For the covariant splitting of the second class constraints from the constraints (31), (33), (35) note that the constraints $T$ and $Q$ are the projections of $\Phi^{\dot{\alpha} \dot{\alpha}}$ onto the 4-vectors $u_{\dot{\alpha}} u_{\dot{\alpha}}$ and $x'_{\dot{\alpha} \dot{\alpha}}$. So, taking into account that $\Phi^{\dot{\alpha} \dot{\alpha}}$ is a 4-vector it is useful to introduce a local moving frame composed of the real 4-vectors $n^{(+)}_{\dot{\alpha} \dot{\alpha}} = u_{\dot{\alpha}} u_{\dot{\alpha}}$, $n^{(-)}_{\dot{\alpha} \dot{\alpha}} = v_{\dot{\alpha}} v_{\dot{\alpha}}$, $m^{(+)}_{\dot{\alpha} \dot{\alpha}} = u_{\dot{\alpha}} v_{\dot{\alpha}} + v_{\dot{\alpha}} u_{\dot{\alpha}}$, $m^{(-)}_{\dot{\alpha} \dot{\alpha}} = i (u_{\dot{\alpha}} v_{\dot{\alpha}} + v_{\dot{\alpha}} u_{\dot{\alpha}})$ (98)

attached to the string world sheet. The 4-vector $\Phi^{\dot{\alpha} \dot{\alpha}}$ may then be expanded in this 4-vector basis (98). Taking into account the condition $(ux'\bar{u}) \neq 0$, which shows that 4-vector $x'_{\dot{\alpha} \dot{\alpha}}$ has a nonzero projection onto the light-like basis 4-vector $n^{(-)}_{\dot{\alpha} \dot{\alpha}}$, one can choose the projections

$$M^{(\pm)} \equiv \Phi^{\dot{\alpha} \dot{\alpha}} m_{\dot{\alpha} \dot{\alpha}}^{(\pm)} = 0$$

(99)
to be the second class constraints. The primary constraints $\Phi^{\dot{\alpha} \dot{\alpha}}$ then split covariantly into two real first class constraints $T, Q$ and two real second class constraints $M^{\pm}$

$$\Phi^{\dot{\alpha} \dot{\alpha}} \Rightarrow (T, Q) \oplus (M^{(+)}, M^{(-)})$$

(100)

Taking into account the P.B. relations for $\Phi^{\dot{\alpha} \dot{\alpha}}$ (60) together with the “orthonormality” conditions

$$n^{(+)}_{\dot{\alpha} \dot{\alpha}} n^{(+)}_{\dot{\alpha} \dot{\alpha}} = 0, \quad n^{(-)}_{\dot{\alpha} \dot{\alpha}} n^{(+)}_{\dot{\alpha} \dot{\alpha}} = 1,$$

$$m^{(+)}_{\dot{\alpha} \dot{\alpha}} m^{(\pm)}_{\dot{\alpha} \dot{\alpha}} = -2, \quad m^{(-)}_{\dot{\alpha} \dot{\alpha}} m^{(+)_{\dot{\alpha} \dot{\alpha}} = 0,}$$

$$n^{(\pm)}_{\dot{\alpha} \dot{\alpha}} m^{(\pm)}_{\dot{\alpha} \dot{\alpha}} = 0, \quad m^{(+)}_{\dot{\alpha} \dot{\alpha}} m^{(+)}_{\dot{\alpha} \dot{\alpha}} = 0,$$

(101)

it is easy to find that the $M^{(\pm)}$-constraints have zero P.B.

$$\{M^{(\pm)}, M^{(\pm)}\}_{P.B.} = 0, \quad \{M^{(+)}, M^{(-)}\}_{P.B.} = 0$$

(102)

It is suitable to introduce one complex constraint $M$ instead of two real $M^{(\pm)}$-constraints

$$M \equiv \frac{1}{2} (M^{(+) + iM^{(-)}); \quad \bar{M} \equiv \frac{1}{2} (M^{(+) - iM^{(-)})}$$

(103)

forming the abelian P.B. algebra

$$\{M, M\}_{P.B.} = 0, \quad \{M, \bar{M}\}_{P.B.} = 0.$$ 

(104)

As a result the primary constraint $\Phi^{\dot{\alpha} \dot{\alpha}}$ (60) are splitted into

$$\Phi^{\dot{\alpha} \dot{\alpha}} \Rightarrow (T, Q) \oplus (M, \bar{M}).$$

(105)

As to the three complex constraints $\Psi^{\alpha \beta}$ they are already presented in the abelian form

$$\{\Psi^{\alpha \beta}, \bar{\Psi}^{\gamma \delta}\}_{P.B.} = 0, \quad \{\Psi^{\alpha \beta}, \bar{\Psi}^{\gamma \delta}\}_{P.B.} = 0.$$ 

(106)
and are good candidates for the next set of the second class constraints. These $\Psi$-constraints may be unified with the two complex constraints (6) and (78)

$$u^\alpha v_\alpha = 1, \quad u'^\alpha u_\alpha = 0.$$  \hspace{1cm} (107)

to form a larger set including five complex second class constraints $Y^A$ ($A = 1, 2, 3, 4, 5$)

$$Y^A \equiv (\Psi^{\alpha\beta}, u^\alpha v_\alpha - 1, u'^\alpha u_\alpha) = 0,$$

$$\bar{Y}^A \equiv (\bar{\Psi}^{\dot{\alpha}\dot{\beta}}, \bar{u}^{\dot{\alpha}} \bar{v}_{\dot{\alpha}} - 1, \bar{u}'^{\dot{\alpha}} \bar{v}_{\dot{\alpha}}) = 0.$$  \hspace{1cm} (108)

which have zero P.B. between themselves

$$\{Y^A, Y^B\}_{P.B.} = 0, \quad \{Y^A, \bar{Y}^B\}_{P.B.} = 0.$$  \hspace{1cm} (109)

At last, going over to the constraints (35) ($P^\alpha_u = P^\alpha_v = 0$) and observing that the projection $P^\alpha_v u_\alpha$ and its complex conjugate form the first class constraints $W, \bar{W}$ (53) one can choose the complex projection

$$P^\alpha_v v_\alpha = 0$$  \hspace{1cm} (110)

together with the two complex constraints

$$P^\alpha_u = 0$$  \hspace{1cm} (111)

as a new set of the three complex second class constraints $\Xi^\Lambda$ ($\Lambda = 1, 2, 3$)

$$\Xi^\Lambda \equiv (P^\alpha_u, P^\beta_v v_\beta) = 0,$$

$$\bar{\Xi}^\Lambda \equiv (\bar{P}^\dot{\alpha}_u, \bar{P}^\dot{\beta}_v v^\dot{\beta}) = 0.$$  \hspace{1cm} (112)

The three complex constraints $\Xi^\Lambda$ and $\bar{\Xi}^\Lambda$ have zero P.B. between themselves

$$\{\Xi^\Lambda, \Xi^\Sigma\}_{P.B.} = 0, \quad \{\Xi^\Lambda, \bar{\Xi}^\Sigma\}_{P.B.} = 0.$$  \hspace{1cm} (113)

We have thus, that the total set of second class constraints following from the action (22) may be divided into three complex abelian subsets:

the abelian $\textbf{M}$-subset including one complex constraint

$$\textbf{M} \equiv (M, \bar{M}),$$

$$\{\textbf{M}, \textbf{M}\}_{P.B.} = 0;$$  \hspace{1cm} (114)

the abelian $\textbf{Y}^\textbf{A}$-subset including five complex constraint

$$\textbf{Y}^\textbf{A} \equiv (Y^A, \bar{Y}^A),$$

$$\{\textbf{Y}^\textbf{A}, \textbf{Y}^\textbf{B}\}_{P.B.} = 0;$$  \hspace{1cm} (115)

the abelian $\textbf{Ξ}^\textbf{A}$-subset including three complex constraint

$$\textbf{Ξ}^\textbf{A} \equiv (\Xi^\Lambda, \bar{\Xi}^\Lambda),$$

$$\{\textbf{Ξ}^\textbf{A}, \textbf{Ξ}^\Sigma\}_{P.B.} = 0.$$  \hspace{1cm} (116)
Having calculated the P.B. between these subsets

\[
\begin{align*}
\{Y^A, M\}_\text{P.B.} &= G^A, \\
\{Y^A, \Xi^A\}_\text{P.B.} &= F^{AA}, \\
\{\Xi^A, M\}_\text{P.B.} &= H^A.
\end{align*}
\]  

one can present the antisymmetric real 18x18 Dirac’s \( \hat{C} \)-matrix constructed of the P.B. of the second class constraints in the compact form of an 9x9 antisymmetric complex matrix

\[
\hat{C} = \begin{pmatrix}
0 & F^{AA} & G^A \\
-F^{AA}^T & 0 & H^A \\
-G^A^T & -H^A^T & 0
\end{pmatrix}
\]  

This \( \hat{C} \)-matrix is then used to construct the Dirac bracket

\[
\{f(\sigma), g(\sigma')\}_\text{D.B.} = \{f(\sigma), g(\sigma')\}_\text{P.B.} - \sum \int \{f(\sigma), \bullet\}_\text{P.B.} (\hat{C}^{-1})^{**} \{\bullet, g(\sigma')\}_\text{P.B.}
\]  

and to derive the correct Hamiltonian equation of motion defined by this Dirac bracket

\[
\dot{f}(\tau, \sigma) = \int d\sigma' \{\mathcal{H}(\sigma'), f(\tau, \sigma)\}_\text{D.B.}
\]  

and the D.B. algebra resulting from the P.B. algebra

7 Conclusion

Here we constructed the Hamiltonian and studied the constraint structure in a new model of strings in a four dimensional space-time extended by the addition of six real TCC coordinates \( z_{mn} \). A covariant separation of these constraints into first and second classes has been described. We found that the 20 real primary (31), (33), (35) and 2 secondary (78) constraints are covariantly splitted into 4 real first class constraints (92) and 18 real second class constraints (103), (108) and (112)

\[
20|\text{primary constr.} + 2|\text{secondary constr.} = 4|\text{first class constr.} + 18|\text{second class constr.}
\]  

The total number of the phase variables (28) and (29) in the model is 36 = (18 + 18). Recalling that the 4 first class constraints kill 8 phase variables, we conclude that the number of independent physical phase variables is 10 = 36 − (4 + 4 + 18). These 10 phase space variables correspond to 5 off shell physical degree of freedom. So, we conclude that the TCC coordinates \( z_{mn} \) together with the dyads \( u_\alpha \) and \( v_\alpha \) contribute in fact only one real off shell physical coordinate into addition to the 4 real world coordinates \( x_{\alpha\dot{\alpha}} \). Accordingly, the described string moves in an effective 5-dimensional space-time.

The problem is now how to construct this 5-dimensional space-time.

With this in mind we constructed the P.B. algebra for the first class constraints. It has a structure similar to that of the contracted rotation algebra of (Anti) de Sitter space-time. We found that this P.B. algebra coincides with the correspondent algebra for tensionless strings and that discovered an essential role for the second class constraints which codes
the physical effects associated with the TCC coordinates. A formulation in terms of the Dirac bracket algebra (or an equivalent construction \[11,12,13\]) is needed to derive the covariant string dynamics in the effective 5-dimensional space-time.

Having the Dirac bracket constructed we are able to study the Dirac bracket algebra for the first class constraints and to describe the effective 5-dimensional space-time associated with the the TCC coordinates $z_{mn}$. We expect this effective space-time to be $AdS_5$. Now we are in a progress to realize this goal.

After completion of this work D. Polyakov informed us that the addition of a 5-form vertex operator corresponding to a brane-like state into the NSR superstring action, effectively curves the $D = 10$ space-time to that of $AdS_5 \times S^5$ \[45\]. This vertex operator contains the world sheet NSR fermions and plays the role of cosmological-like term.

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