Universal and Wide Shear Zones in Granular Bulk Flow

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We present experiments on slow granular flows in a modified (split-bottomed) Couette geometry in which wide and tunable shear zones are created away from the sidewalls. For increasing layer heights, the zones grow wider (apparently without bound) and evolve towards the inner cylinder according to a simple, particle-independent scaling law. After rescaling, the velocity profiles across the zones fall onto a universal master curve given by an error function. We study the shear zones also inside the material as a function of both their local height and the total layer height.

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Slowly sheared granular matter does not flow homogeneously like a liquid. Instead, granulates form rigid, solidlike regions separated by narrow shear bands where the material yields and flows [1–6]. Shear localization is ubiquitous in granular flow—think of geological faults and soil fractures, avalanches, pipe flows, and silo discharges [2,7–14].

Despite their crucial importance, granular shear flows are still poorly understood, in part because shear localization itself remains enigmatic [1–4]. On the one hand, shear bands have a typical thickness of five to ten grain diameters and such steep gradients are difficult to capture by continuum theories [1–14]. On the other hand, the experimental handles for probing shear localization are limited. For example, studies in Couette cells always show the formation of a narrow shear band near the inner cylinder, irrespective of dimensionality, driving rate, or geometry [5,6,15–18]: Shear banding is very robust.

In this Letter, we introduce a general experimental protocol that can yield very wide shear zones away from the sidewalls. We modify a Couette cell by splitting its bottom at radius $R_s$. The resulting concentric rings are attached to the stationary inner and rotating outer cylinder, respectively, and the cell is filled with grains up to height $H$ (Fig. 1). When driving the system, a shear zone is found to propagate from the slip position $R_s$ towards the surface where we measure the average grain velocities. Note that our strategy differs from previous works, which were carried out for large filling heights and smooth bottoms so as to minimize the effect of the bottom boundary [5,16,17]. Here we take advantage of gravity and “drive the system from the bottom.”

Our main findings are the following: (i) For large $H$, a regime of wall-localized shear band near the inner cylinder is recovered [5,16,17], but for intermediate $H$, we observe shear zones of tunable width away from the boundaries. This Letter focuses on describing these “bulk” shear zones. (ii) The angular velocity profiles $\omega(r)$ of bulk shear zones fall onto a universal master curve which is best fitted by an error function. These profiles are therefore fully characterized by two parameters only: their center position $R_c$ and width $W$. A concise presentation of these results has appeared in [19]. (iii) $R_c$ and $W$ depend on $H$, $R_s$, and particle properties in specific manners. The center of the shear zone $R_c$ evolves to the inner cylinder with increasing $H$ in a particle-independent manner. The shear zone width $W$ grows continuously with $H$ and depends on particle size and shape, but not on the slip radius $R_s$. (iv) For a given height inside the material, the width and position of the shear zones depend on the height of the free surface $H$.

Setup.—A sketch of our split-bottomed Couette cell is shown in Fig. 1(a). When the inner cylinder of the “Couette” geometry is removed we obtain the “disk” geometry [Fig. 1(b)]. Different sets of bottom rings allow us to vary $R_s$ from 45 to 95 mm. Grains, similar to those used in the bulk, are glued to the sidewalls and bottom rings to obtain rough boundaries. We studied spherical glass beads of size distributions 0.25–0.42 mm (I), 0.5–0.8 mm (II), 1–1.2 mm (III), and 2–2.4 mm (IV), and irregularly shaped plastic flakes (1.0–1.6 mm) (V), aluminum oxide beads (1.5–2 mm) (VI), and coarse sand (1.2–2.4 mm) (VII). After filling the cell, an adjustable

![FIG. 1.](image-url)
blade flattens the surface at the desired height. The outer cylinder and its comoving ring are then rotated. A Pulnix TM-6710 8-bit CCD camera records 2000-frame movies of the resulting flow at the top surface at a rate of 120 frame/s with pixel resolution 100 µm.

The flow rapidly (~1 s) reaches a stationary state where it is purely in the azimuthal direction, so that the surface velocities are a function of the radial coordinate only [5,6,15–17]. We checked that these velocities are proportional to the driving rate Ω [5,16,18,19] for 0.16 < Ω < 1.5 rad/s, and subsequently fix Ω at 0.16 rad/s. We thus focus on the velocity profile ω(r), the dimensionless ratio of the average angular velocity, and Ω. We measure ω(r) with high radial resolution by particle image velocimetry, i.e., by determining the averaged angular correlation function as function of r of two temporally separated frames. Unless noted otherwise, the time separation between frames is around 0.3 s.

Basic phenomenology.—Figure 2 illustrates the main features of these velocity profiles. For shallow layers, a collapse on a universal curve which is extremely well can be observed. Thus, in the disk geometry, i.e., by determining the averaged angular correlation function as function of r of two temporally separated frames. Unless noted otherwise, the time separation between frames is around 0.3 s.

Universal velocity profiles.—Figure 3 illustrates our main result: After proper rescaling, all bulk profiles collapse on a universal curve which is extremely well fitted by an error function:

$$\omega(r) = 1/2 + 1/2 \operatorname{erf}(r - R_c)/W$$  \hspace{1cm} (1)

A residue analysis comparing the fit to Eq. (1) to an alternative fit to an hyperbolic tangent shows that the fit to the error function is always better [Fig. 3(b) and 3(c)]. By repeating this procedure for the other particle mixtures, we establish the general superiority of Eq. (1): particle shape does not influence the functional form of the velocity profiles. The robust form of ω(r) contrasts with the particle dependence found for wall-localized shear bands [5]. For these, the vicinity of the wall causes layering, in particular, for monodisperse mixtures. Apparently such layering effects play no role for our bulk shear zones. Accurate measurement of the tail of the velocity profile [Fig. 3(d)] further validate Eq. (1), and rule out an exponential tail of the velocity profile here. The strain rate is therefore Gaussian, and the shear zones are completely determined by their centers Rc and widths W.

What limits the universal regime? Apart from wall localization (see Fig. 2), we find that in the disk geometry ω(r) starts to deviate from Eq. (1) when H exceeds ~Rc/2. The symmetry of the velocity profile, easily detectable by a simple χ² test, is then weakly broken [21]. In the following, we focus on the functional dependencies of Rc and W on the parameters Rc, H, and particle type for the universal profiles given by Eq. (1).

Shear zone position.—Remarkably, the shear zone center evolution with height Rc(H), turns out to be independent of the grain properties [Fig. 4(a)]. Therefore, the only relevant length scales for Rc are the geometric scales H and Rc. The dimensionless displacement of the shear zone, (Rc − Rc)/Rc, should thus be a function of the

![FIG. 2. Main features of the normalized angular surface velocity in the Couette geometry for 0.3 mm glass beads (mixture I) and Rc = 85 mm. (a) ω(r) for a range of equidistant heights h = 3, 6, ..., 53 mm (right to left). (b) Contour plots of ω(r), where the symbols correspond to, from left to right, ω = 0.1, 0.25, 0.5, 0.75, and 0.9. The curve indicates the strain rate maximum and shows the rapid qualitative change of the profiles when the inner cylinder is approached.](image-url)
dimensionless height \((H/R_s)\) only. The simple relation
\[
(R_s - R_c)/R_s = (H/R_s)^{5/2}
\]
fits the data well [Fig. 4(a)] [22]. To check Eq. (2) we have varied \(R_s\) over a substantial range. Only the presence of the inner cylinder limits the range of \(R_s\). We find no differences between bulk velocity profiles measured with or without the inner cylinder—**Bulk shear zones are insensitive to the presence of the sidewalls.** So, we subsequently switched to the disk geometry [Fig. 1(b)], and obtained an excellent agreement between \(R_c\) and Eq. (2) over the range \(45 < R_c < 95\) mm [Fig. 4(b)].

**Shear zone width.**—The width of the shear zones depends on the particle size and type [Figs. 5(a)–5(c)], but not on \(R_c\) [Fig. 5(d)]. First of all, \(W\) grows with \(H\) and increases for larger particles [Figs. 5(a) and 5(b)]. The data shown in Figs. 5(a) and 5(b) can be made to collapse when plotted as \(W/d\) vs \(H/d\) (not shown), where \(d\) denotes the grain size. Grain shape and type also influence \(W(H)\): irregular particles display narrower zones than spherical ones of similar diameter [Fig. 5(c)]. Finally, for the universal velocity profiles, \(W\) is independent of \(R_c\) [Fig. 5(d)]. We therefore conclude that the relevant length scale for \(W\) is given by the grain properties.

The evolution of the velocity profiles from a step function at the bottom to an error function at the surface, is reminiscent of a diffusive process along the vertical axis. However, \(W\) grows faster than \(\sqrt{H}\) as diffusion would suggest, but slower than \(H\). Intriguingly, we obtain the best fit for \(W \propto H^{2/3}\) over the limited range where we have reliable data. We cannot, however, rule out other functional dependencies such as a crossover from square-root to linear behavior.

**Below the surface.**—So far we have only discussed observations of the surface flow. To get some insight into the 3D bulk flow structure, we put patterns of lines of colored tracer particles at given \(H_b\) [Fig. 6(a)]. More material is carefully added so that the line pattern is buried under a given amount of grains \((H > H_b)\). We then rotate the system for a short period \((\sim 8\) s), and recover the deformed line pattern by carefully removing the upper layers of grains [Fig. 6(b)]. Comparing the snapshots of the deformed pattern to the initial one allows for the determination of the velocity profiles in the 3D bulk of the material [Fig. 6(c)]. We have checked that transient effects are limited, that the measurements reproduce well and that there is no significant motion in the vertical direction.

The position of the shear zones in the 3D bulk are presented in Fig. 6(d). Clearly, the evolution of \(R_c\) with \(H_b\) inside the material depends on the total amount of matter, as given by \(H\): the more material added, the more the shear zone shifts towards the center. This observation is confirmed by recent theory [23], numerics [24], and MRI measurements [25]. The widths of the 3D shear zones are more difficult to measure accurately, but a clear trend can be identified: shear zones become wider when more matter is added on top [Fig. 6(e)].

**Outlook.**—In this Letter we have presented a simple experimental protocol in which wide and tunable shear zones can be generated in a variety of granular materials. Perhaps the biggest surprise is the robust and remarkably simple form of the velocity profiles—for granular...
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[1] J. Duran, Sand, Powders and Grains (Eyrolles, Paris, 1997).
[2] R. Nedderman, Statics and Kinematics of Granular Materials (Cambridge University Press, Cambridge, United Kingdom, 1992).
[3] H. M. Jaeger, S. R. Nagel, and R. P. Behringer, Rev. Mod. Phys. 68, 1259 (1996).
[4] H. M. Jaeger and S. R. Nagel, Science 255, 1523 (1992).
[5] D. M. Mueth, et al. Nature (London) 406, 385 (2000).
[6] R. R. Hartley and R. P. Behringer, Nature (London) 421, 928 (2003).
[7] D. R. Scott, Nature (London) 381, 592 (1996).
[8] M. Oda and H. Kazama, Géotechnique 48, 465 (1998).
[9] J. Bridgewater, Géotechnique 30, 533 (1980).
[10] H. B Mulhiaus and I. Vardoulakis, Géotechnique 37, 271 (1987).
[11] T. S. Komatsu, S. Inagaki, N. Nakagawa, and S. Nasuno, Phys. Rev. Lett. 86, 1757 (2001).
[12] A. Daerr and S. Douady, Nature (London) 399, 241 (1999).
[13] O. Pouliquen and R. Gutfraind, Phys. Rev. E 53, 552 (1996).
[14] R. M. Nedderman and C. Laohakul, Powder Technol. 25, 91 (1980).
[15] D. Howell, R. P. Behringer, and C. Veje, Phys. Rev. Lett. 82, 5241 (1999).
[16] W. Losert, L. Bocquet, T. C. Lubensky, and J. P. Gollub, Phys. Rev. Lett. 85, 1428 (2000).
[17] W. Losert and G. Kwon, Adv. Complex Syst. 4, 369 (2001).
[18] M. Lützel, S. Luding, H. J. Herrmann, D. W. Howell, and R. P. Behringer, Eur. Phys. J. E 11, 325 (2003).
[19] D. Fenistein and M. van Hecke, Nature (London) 425, 256 (2003).
[20] For an essentially 2D flow, i.e., stratified and far away from the bottom, the shear stresses increase for decreasing radius [18]. Presumably, for deep enough layers a related mechanism pulls the shear zones inward.
[21] Note that an asymmetric correction to Eq. (1) also occurs when one assumes that the shear rate \( \gamma(r) \) is strictly symmetric (Gaussian), and calculates \( \omega(r) = \int \gamma(r) dr \). This correction to Eq. (1) is, however, negligibly small for our values of \( H \) and \( R_s \), so we have ignored it here. More importantly, such correction grows continuously with \( H \), rather than suddenly appearing at \( H = R_s/2 \).
[22] We found some indications that strongly elliptical grains (birdseed/not presented here) deviate from Eq. (2).
[23] T. Unger, J. Török, J. Kertész, and D. E. Wolf, cond-mat/04041143.
[24] S. Luding (private communications).
[25] P. Umbanhowar (private communications).
[26] D. Bonamy, F. Daviaud, L. Laurent, M. Bonetti, and J. P. Bouchaud, Phys. Rev. Lett. 89, 034301 (2002).