Entanglement, measurement, and conditional evolution of the Kondo singlet interacting with a mesoscopic detector

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Abstract. We investigate various aspects of the Kondo singlet in a quantum dot (QD) electrostatically coupled to a mesoscopic detector. The two subsystems are represented by an entangled state between the Kondo singlet and the charge-dependent detector state. We show that the phase-coherence of the Kondo singlet is destroyed in a way that is sensitive to the charge-state information restored both in the magnitude and in the phase of the scattering coefficients of the detector. We also introduce the notion of the ‘conditional evolution’ of the Kondo singlet under projective measurement on the detector. Our study reveals that the state of the composite system is disentangled upon this measurement. The Kondo singlet evolves into a particular state with a fixed number of electrons in the QD. Its relaxation time is shown to be sensitive only to the QD-charge dependence of the transmission probability in the detector, which implies that the phase information is erased in this conditional evolution process. We discuss implications of our observations in view of the possible experimental realization.

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1. Introduction

Quantum interference and its suppression caused by interactions with external degrees of freedom have been central subjects of mesoscopic physics for more than a decade (see e.g. [1]). These subjects deal with the transition from quantum to classical phenomena in mesoscopic scales. In particular, ‘which-path’ (WP) detection in mesoscopic quantum interferometers provides an ideal playground for studying the complementarity (which is often identified with ‘wave-particle duality’) in quantum theory. Experiments on the controlled dephasing have been performed in mesoscopic structures based on quantum dots (QDs) [2]–[4]. A prototype experimental set-up for this kind of study [2] is as follows. Coherent transmission of electrons is monitored by using an Aharonov–Bohm (AB) interferometer with a QD inserted in one of the interferometer’s arms [5, 6]. A mesoscopic detector is electrostatically coupled to the QD. Because of electrostatic interactions, the electron state in the detector depends on the charge state of the QD, which results in a quantum correlation (i.e. ‘entanglement’) between the QD and the detector. The AB oscillation of the conductance through the interferometer is suppressed because of the WP information transferred to the detector. This ‘measurement-induced dephasing’ is controlled through the voltage applied across the mesoscopic detector. The controlled dephasing experiments were also carried out without an AB interferometer [3, 4], because it is possible to study the coherence by the resonant transmission through a QD. Various theoretical approaches were used to study this problem [7]–[11].

The controlled dephasing experiment was also performed in the Kondo limit of the QD [4]. A Kondo singlet is formed between the localized spin in a QD and electrons in the leads (for a review, see e.g. [12]), which gives rise to enhanced transport through the QD [13]–[19]. It was shown that a nearby quantum point contact (QPC) capacitively coupled to the QD plays a role of a ‘potential detector’ and suppresses the Kondo resonance [4]. However, characteristics of the measured suppression were very different from the theoretical prediction of [20]. The most significant deviation from the theory is that the measured suppression strength is much larger (about 30 times) than expected. Dependence on the transmission probability (T) and on the bias voltage (V) across the QPC was also inconsistent with the theoretical expectation. The analysis of the experiment [4] was based on a theory [20] of dephasing of
the Kondo resonance as a result of path detection by the QPC through the change of the transmission probability, $\Delta T$. It was pointed out that this kind of treatment does not fully take into account the WP information acquired in the detector [21]. This is because scattering of electrons at the QPC is a quantum mechanical phenomenon with complex transmission and reflection amplitudes. Therefore, in general, phase-sensitive information should also be taken into account [3, 10, 21, 22].

In this paper, first we present a theory of the entanglement of the Kondo singlet with a mesoscopic detector (section 2). The formulation is based on the variational ground state of the Kondo singlet [12, 23] correlated with the charge-dependent detector state. We then report on our investigations of dephasing in the Kondo state controlled by charge detection (section 3). Discussions in sections 2 and 3 are mainly extensions of the study in [21]. In addition, a refined model for the detector is introduced which explains the importance of the phase-sensitive WP information. In section 4, we introduce the concept of the ‘conditional evolution’ of the Kondo singlet under projective measurement on the detector. We show that the phase-sensitive information is erased and the Kondo state suffers relaxation in a way that depends on the charge sensitivity of the detector current. Our conclusion is given in section 5. Also, the relation between the scattering matrix and the parallel shift of the one-dimensional (1D) potential is derived in the appendix.

2. The entanglement of the Kondo singlet with a charge detector

The model system we investigate is schematically drawn in figure 1. First, to describe the Kondo singlet of the QD, we adopt the variational ground state for the impurity Anderson model [12, 23]. This variational ground state captures the essential Kondo physics in a simple but effective way. Furthermore, this approach can be easily applied for describing the entanglement of the Kondo singlet with the detector. The Hamiltonian for the QD + two electrodes + tunnelling is given by

$$H = H_L + H_R + H_D + H_T. \quad (1)$$

Figure 1. Schematic figure of the Kondo system interacting with a detector. The Kondo system is composed of a QD connected to two electrodes. For the detector we consider a two-terminal mesoscopic conductor with a single transmission channel.
The left (L) and the right (R) leads are described by the noninteracting Fermi sea as

\[ H_\alpha = \sum_{k\sigma} \varepsilon_{ak} c_{ak\sigma}^\dagger c_{ak\sigma} \quad (\alpha = \text{L}, \text{R}), \]

where \( c_{ak\sigma} \) (\( c_{ak\sigma}^\dagger \)) is an annihilation (creation) operator of an electron with energy \( \varepsilon_{ak} \), momentum \( k \), and spin \( \sigma \) on the lead \( \alpha \). The interacting QD is represented by \( H_D \) given as

\[ H_D = \sum_\sigma \varepsilon_d d_{\sigma}^\dagger d_\sigma + U n_\uparrow n_\downarrow, \]

where \( d_\sigma \) and \( d_{\sigma}^\dagger \) are the QD electron annihilation and creation operators, respectively, and \( n_\sigma = d_{\sigma}^\dagger d_\sigma \). The parameters, \( \varepsilon_d \) and \( U \), stand for the energy of the localized level and the on-site Coulomb interaction, respectively. The tunnelling Hamiltonian \( H_T \) has the form

\[ H_T = \sum_{\alpha=L,R} \sum_{k\sigma} (V_\alpha d_{\sigma}^\dagger c_{ak\sigma} + \text{h.c.}), \]

where \( V_\alpha \) is responsible for the tunnelling between the QD and the lead \( \alpha \).

In the absence of interaction between the QD and the detector, the variational ground state for the Hamiltonian \( H \) \((U \to \infty \text{ limit})\) is written as \([12, 23]\)

\[ |\Psi_G\rangle = A |0\rangle + B |1\rangle, \]

where \( |0\rangle \) denotes the Fermi sea for the leads with an empty QD state, and

\[ |1\rangle \equiv \frac{1}{\sqrt{2}} \sum_{a,\sigma, k < k_F} v_{ak} d_{\sigma}^\dagger c_{ak\sigma} |0\rangle. \]

Here \( A = \sqrt{1 - n_d} \) and \( B = \sqrt{n_d} \), with \( n_d \) being the average occupation number of the QD level and

\[ v_{ak} = \sqrt{\frac{2n_d}{1 - n_d}} \frac{V_\alpha}{E_G - \varepsilon_d + \varepsilon_{ak}}, \]

where \( E_G \) denotes the ground state energy determined by the equation

\[ E_G = 2 \sum_{a,\sigma, k < k_F} \frac{V_\alpha^2}{E_G - \varepsilon_d + \varepsilon_{ak}}. \]

The Kondo temperature \( (T_K) \), the characteristic energy scale of the system, is given as a difference between the QD level \((\varepsilon_d)\) and the ground state energy \((E_G)\): \( T_K = \varepsilon_d - E_G \).

In fact, the states \( |0\rangle \) and \( |1\rangle \) have different occupation numbers for the QD: \( n_d = 0 \) and \( n_d = 1 \), respectively. A detector (usually a mesoscopic conductor) near to the QD is able to detect the charge state, since the potential of the detector depends on the charge state of the QD. So the transmission and reflection amplitudes of the detector also depend on \( n_d \). This correlation can be described by an entangled state for the composite system as

\[ |\Psi_{\text{tot}}\rangle = A |0\rangle \otimes |\chi_0\rangle + B |1\rangle \otimes |\chi_1\rangle, \]

where \( |\chi_0\rangle \) and \( |\chi_1\rangle \) are the ground state and excited state of the detector, respectively.
where $|\chi_i\rangle$ ($i = 0 \text{ or } 1$) denotes the detector state when the Kondo system is in the state $|i\rangle$. Here, a two-terminal single-channel conductor is considered as the detector. Then, an injected electron from the left electrode of the detector can be described by the state

$$|\chi_i\rangle = r_i |r\rangle + t_i |t\rangle,$$

where $r_i$ and $t_i$ are the $i$-dependent reflection and transmission coefficients, respectively. The state $|r\rangle$ ($|t\rangle$) corresponds to the state of reflection (transmission) for an injected electron. It is important to note that the scattering coefficients are complex numbers that can be expressed as

$$r_i = |r_i| \exp(i\varphi_{ri}),$$
$$t_i = |t_i| \exp(i\varphi_{ti}),$$

and satisfy the unitarity relation $|r_i|^2 + |t_i|^2 = 1$.

### 3. Current- and phase-sensitive dephasing

### 3.1. The reduced density matrix for the Kondo system

Dephasing of the Kondo singlet takes place when an observer (detector) monitors the Kondo system (which is a part of the composite system). This can be described in terms of the reduced density matrix approach. Before interaction of the two subsystems, the density matrix of the Kondo singlet is given as $\rho_0 = |\Psi_G\rangle\langle\Psi_G|$. Upon a single scattering event with the detector, the reduced density matrix $\rho$ of the Kondo system is given as

$$\rho = \text{Tr}_{\text{det}}(|\Psi_{\text{tot}}\rangle\langle\Psi_{\text{tot}}|),$$

where $\text{Tr}_{\text{det}}(\cdots)$ denotes a trace over the detector degree of freedom.

It is found that the diagonal elements of $\rho$ do not change upon scattering at the detector. On the other hand, the off-diagonal elements are modified by

$$\rho_{01} = \lambda \rho_{01}^0,$$

where

$$\lambda = \langle\chi_1 | \chi_0 \rangle = r_0 r_1^* + t_0 t_1^*.$$}

This quantity represents the information of the charge state in the QD transferred to the detector. That is, $\lambda = 0$ implies that the two states are orthogonal. Thus, a complete charge-state information is transferred to the detector. For $|\lambda| = 1$, the two charge states are identical which means that the detector does not obtain any information on the QD state. Dephasing of the Kondo singlet takes place for $|\lambda| < 1$. It is obvious from equation (15) that the dephasing is associated not only with the current sensitivity but also with the phase sensitivity of the scattering coefficients of the detector.
3.2. The time evolution of the density matrix in the weak measurement limit

We consider the weak continuous measurement limit where the scattering through the detector takes place on a timescale much shorter than the relevant timescales in the Kondo singlet. In our case, \( \Delta t \ll t_d \), where \( \Delta t = \hbar/2eV \) denotes the average time between two successive scattering events with \( V \) being the voltage applied across the detector. The parameter, \( t_d \), is the dephasing time of the Kondo singlet. This assumption allows us to use the Markov approximation that neglects the memory effect in the detector. Then after the scattering of \( n \) electrons through the detector the off-diagonal component of the density matrix is given as

\[
\rho_{01}(t) = \lambda^n \rho_{01}(0),
\]

where \( t = n \Delta t \). Note that the time evolution of the density matrix here is written in the Heisenberg picture in order to eliminate the less important dynamical phase factor. This equation can be rewritten in the form

\[
\rho_{01}(t) = e^{(i\Delta \epsilon - \Gamma_d)t} \rho_{01}(0),
\]

where the two parameters, \( \Delta \epsilon \) and \( \Gamma_d \), represent the phase shift and the dephasing rate, respectively, caused by the detection processes. One can find that

\[
\Delta \epsilon = \frac{2eV}{h} \arg \lambda,
\]

\[
\Gamma_d = \frac{1}{t_d} = -\frac{2eV}{h} \log |\lambda|.
\]

On the other hand, the diagonal terms are independent of time. This implies that no relaxation takes place in the Kondo singlet.

In the weak measurement limit, \( \lambda \sim 1 \), \( \Gamma_d \) and \( \Delta \epsilon \) can be expressed in terms of the changes in the magnitude and phase of the scattering amplitudes of the detector as

\[
\Gamma_d = \Gamma_T + \Gamma_\phi,
\]

where

\[
\Gamma_T = \frac{eV}{h} \frac{(\Delta T)^2}{4T_0(1 - T_0)},
\]

\[
\Gamma_\phi = \frac{eV}{h} T_0(1 - T_0)(\Delta \phi)^2,
\]

and

\[
\Delta \epsilon = \frac{eV}{\pi} (1 - T_0) \Delta \phi_t + \frac{eV}{\pi} T_0 \Delta \phi_r.
\]

Here, \( T_0 = |t_0|^2 \) (\( T_1 = |t_1|^2 \)) is the transmission probability in the absence (presence) of an extra electron in the QD. \( \Delta T \equiv |t_0|^2 - |t_1|^2 = |r_1|^2 - |r_0|^2 \) represents the change in the transmission probability. The phase shift \( \Delta \phi \) is given by \( \Delta \phi = \Delta \phi_t - \Delta \phi_r \), where \( \Delta \phi_t \) (\( \Delta \phi_r \)) is the change in the transmission (reflection) amplitude resulting from the different charge states: \( \Delta \phi_t = \phi_{t0} - \phi_{t1}, \Delta \phi_r = \phi_{r0} - \phi_{r1} \).
3.3. Dephasing and the Kondo-assisted transport

The effect of dephasing can be investigated through electron transport in the Kondo system. The best way for studying the dephasing would be to compose a two-path AB interferometer with a Kondo-correlated QD inserted in one arm of the interferometer [17] (figure 2). In this case, the total transmission probability \( T_{AB} \) through the interferometer is given as

\[
T_{AB} = \left| t_{ref} + t_{QD} e^{i\theta} \right|^2 = |t_{ref}|^2 + |t_{QD}|^2 + 2|t_{ref}||t_{QD}| \cos \theta, \tag{24}
\]

where \( t_{ref} \) and \( t_{QD} \) stand for the transmission amplitudes through the reference arm and through the QD, respectively. The relative phase shift \( \theta \) is controlled by the external AB flux \( \Phi \) as

\[
\theta = \frac{2\pi e \Phi}{\hbar c} + \text{const.}
\]

The magnitude of the AB oscillation in equation (24), denoted by \( \mathcal{V}_{AB} \), is given as

\[
\mathcal{V}_{AB} = 2|t_{ref}||t_{QD}|. \tag{25}
\]

The measurement-induced dephasing is expected to reduce \( |t_{QD}| \). For a more quantitative study, we use the following relationship between \( t_{QD} \) and Green’s function for the QD, \( G_d(\omega) \), at the Fermi energy (\( \omega = 0 \)) as \([24]\)

\[
t_{QD} = -2i\sqrt{\Gamma_L\Gamma_R} G_d(0), \tag{26}
\]

where \( \Gamma_L(\Gamma_R) \) is the tunnelling rate of an electron between the QD and the left (right) electrode. Green’s function for the mixed state described by the reduced density matrix \( \rho \) (see the previous section) is defined by

\[
G_d(\omega) = -i \int_0^\infty dt \ e^{i\omega t} \text{Tr} \left[ \rho(t) \{ d_\sigma(t), d_\sigma^\dagger \} \right]. \tag{27}
\]
where \([ \cdots, \cdots ]_s\) denotes the anti-commutator. Green’s function can be evaluated using a similar method to that used in [12, 23]. We need to use the equations of motion for various Green’s functions and truncate higher order terms of \(1/N_s\) with \(N_s\) being the spin degeneracy. Neglecting the incoherent background and the energy shift \(\Delta \epsilon\), we obtain the following expression in the Kondo limit \((n_d \sim 1)\)

\[
G_d(\omega) \simeq \frac{(1 - n_d)}{\omega - T_K + i\Gamma_d}.
\]

Therefore we find that the magnitude of the AB oscillation is reduced by the dephasing when the voltage \(V\) is applied in the detector by the factor

\[
\frac{V_{AB}(V)}{V_{AB}(V = 0)} = \sqrt{\frac{T_K^2}{T_K^2 + \Gamma_d^2}}.
\]

Note that the \(V\)-dependence of \(V_{AB}\) comes through the relation (19).

Alternatively, one can study the dephasing of the Kondo singlet through direct transport through the QD without interferometry (figure 1). The phase coherence of the Kondo state appears in the resonant transport through the double-barriers which is an electronic analogue of the Fabry–Perot interferometer (see e.g. [25]). The experiment carried out in [4] used this geometry. In this case, the conductance is proportional to \(|t_{QD}|^2\), leading to the suppression of the conductance at a finite detector bias \(V\), by the factor

\[
g(V) = \frac{T_K^2}{T_K^2 + \Gamma_d^2}.
\]

### 3.4. The dephasing rate and symmetric versus asymmetric charge responses

From our discussion, it is obvious that the Kondo resonance is reduced by the charge detection of the QD through the coherent scattering at the detector. This coherent scattering is described by the complex transmission and reflection coefficients. In the weak measurement limit, the dephasing rate is given by the phase-sensitive \(\Delta \phi\) as well as the current-sensitive \(\Delta T\) detection.

The phase-sensitive contribution to dephasing \(\Gamma_{\phi}\) was not taken into account in the experimental report of [4]. The much stronger dephasing rate than expected in the theory in [20] which takes only \(\Gamma_T\) into account suggests a large contribution from the phase-sensitive dephasing, i.e. \(\Gamma_{\phi} \gg \Gamma_T\). One of the authors (KK) has pointed out previously [21] that phase-sensitive dephasing might be dominant in a generic situation if the asymmetry in the charge sensitivity of the detector potential is taken into account. Here, we provide a refined version of the detector model for demonstrating this behaviour. First, \(\Delta \phi = 0\) if the detector potential and its variation resulting from an extra QD electron have inversion symmetries [26, 27] and thus the phase-sensitive contribution vanishes\(^2\). However, in reality, there is no reason to believe that the response of the detector potential to the QD charge should be symmetric. We take into account the asymmetric as well as the symmetric variation of the detector potential. The potential profile

\(^2\) An exception can be found in a detector with resonant transmission, where an abrupt phase change in scattering phases contributes to dephasing. See [28].
Figure 3. A model for the detector potentials $V_i(x)$ depending on the charge state $i (\in \{0, 1\})$ of the QD. The symmetric and the asymmetric responses to an extra electron of the QD are taken into account via $\delta V_0$ and $\delta x$, respectively.

$V_i(x)$ depends on the charge state of the QD $i \in 0, 1$. We use a 1D inverse harmonic potential for the detector as (see figure 3)

\[ V_0(x) = V_0 - \frac{1}{2}m\omega_x^2x^2, \]

\[ V_1(x) = V_0 + \delta V_0 - \frac{1}{2}m\omega_x^2(x - \delta x)^2. \]

The parameter $\delta V_0$ corresponds to the symmetric component of the charge sensitivity. Its asymmetry is accounted for by the parallel shift $\delta x$ of the potential profile. The transmission probability $T_i (i \in 0, 1)$ can be exactly calculated in this model [29]. We find that

\[ T_0 = \frac{1}{1 + \exp(-2\pi\varepsilon_0)}, \]

\[ T_1 = T_0 - \Delta T = \frac{1}{1 + \exp(-2\pi(\varepsilon_0 - \delta v_0))}, \]

where the dimensionless variables $\varepsilon_0$ and $\delta v_0$ are defined by

\[ \varepsilon_0 = \frac{E - V_0}{\hbar\omega_x}, \quad \delta v_0 = \frac{\delta V_0}{\hbar\omega_x}. \]
In the weak measurement limit ($\delta v_0 \ll 1$), we find that

$$\Delta T \simeq 2\pi T_0 (1 - T_0) \delta v_0.$$  \hfill (36)

$\delta V_0$, the symmetric component of the potential response, does not contribute to the phase-sensitive dephasing [26]–[28]. Then the phase sensitivity of the detector is purely given by the parallel shift $\delta x$ of the potential as (see appendix)

$$\Delta \phi = 2k_F \delta x,$$  \hfill (37)

where $k_F$ denotes the Fermi wavevector. Therefore, for the potential profile used in equation (32), the dephasing rates are given by

$$\Gamma_T = \frac{\pi^2 e V}{h} T_0 (1 - T_0) (\delta v_0)^2,$$  \hfill (38)

$$\Gamma_\phi = 4 \frac{e V}{h} T_0 (1 - T_0) (k_F \delta x)^2.$$  \hfill (39)

In the case where the asymmetric response of the potential is comparable to the symmetric response, the following relation will be satisfied:

$$\frac{\delta V_0}{\hbar \omega_x} \sim \left( \frac{\delta x}{x_0} \right)^2,$$

where $x_0 \equiv \sqrt{\hbar / m \omega_x}$ characterizes the length scale of the detector potential. This relation implies that the changes of the energy scales resulting from the symmetric and the asymmetric responses are comparable. With this condition, we get

$$\Gamma_\phi = 4 \frac{e V}{h} T_0 (1 - T_0) (k_F x_0)^2 \delta v_0.$$  \hfill (40)

In the experiment of [4], the Fermi wavelength ($\lambda_F = 2\pi / k_F$) and the length scale of the detector potential are about $\lambda_F \sim 44$ nm and $x_0 \sim \mathcal{O}(20$ nm), respectively. Therefore, $k_F \delta x \sim k_F x_0 \sqrt{\delta v_0} \sim 3 \sqrt{\delta v_0}$ and we find that

$$\frac{\Gamma_\phi}{\Gamma_T} \sim (\delta v_0)^{-1}.$$  \hfill (41)

Because $\delta v_0 \ll 1$ in our description, the condition $\Gamma_\phi \gg \Gamma_T$ can be achieved if the asymmetric response in the detector potential is not negligible. This conclusion can also be understood as follows. The transmission probability is affected across the region $|x| \lesssim x_0$, while the phase is affected through a relatively wide region, so that the phase-sensitive detection is more effective. This leads to a large contribution of phase-sensitive dephasing and can be a natural explanation for the anomalously large dephasing rate observed in [4].

It should be noted that our discussion on the large phase-sensitive dephasing is not restricted to the Kondo limit. However, in the Kondo limit, electrons in the Kondo cloud may interact with the electron in the detector. We expect that this interaction rarely affects the transmission
probability. But it contributes to the phase-sensitive dephasing. Interactions with the Kondo cloud is expected to increase the asymmetric response of the detector potential. This argument could explain why the phase-sensitive dephasing is more pronounced in the Kondo limit than in the Coulomb blockade limit.

We also briefly remark on the $T_0$-dependence of $\Gamma_1$. For the simple model of the detector considered here, $\Gamma_1$ is expected to be proportional to the partition noise \( \propto T_0(1 - T_0) \) of the ideal single-channel detector. However, the experimental $\Gamma_1$–$T_0$ curve shows a double peak behaviour \[4\] in contrast to the theoretical model. This qualitative discrepancy might be related to the so called ‘0.7 anomaly’ \[30\] where the shot noise is also suppressed \[31\] or to the charge screening effect \[11, 32\]. This issue requires more careful experimental and theoretical analysis on the correlation between the dephasing of the Kondo state and the shot noise of the detector.

4. The conditional evolution of the Kondo state

In this section, we investigate the time evolution of the Kondo singlet conditioned on the observation of a particular measurement on the detector. The conditional dynamics of a state are obtained by an operation on a part of the system that corresponds to a specific classical outcome of measurement and renormalizing the reduced wavefunction so that it has a total probability of one (see e.g. \[33\]). In the case of a two-terminal mesoscopic detector, there are two possible outcomes of measurement on the detector, that is, transmission and reflection, for each of the injected electrons \[34\]. These measurement processes are described by the operators $\hat{M}_t$ and $\hat{M}_r$ defined as

\[
\hat{M}_t = \frac{|t\rangle\langle t|}{\sqrt{\langle \Psi_\text{tot}|t\rangle\langle t|\Psi_\text{tot}\rangle}}, \quad \hat{M}_r = \frac{|r\rangle\langle r|}{\sqrt{\langle \Psi_\text{tot}|r\rangle\langle r|\Psi_\text{tot}\rangle}}.
\]

(42)

Upon a measurement $\hat{M}_t$ the state $|\Psi_\text{tot}\rangle$ of equation (9) is reduced to $|\Psi^t\rangle$ as

\[
|\Psi^t\rangle = \hat{M}_t|\Psi_\text{tot}\rangle = (A'|0\rangle + B'|1\rangle) \otimes |t\rangle,
\]

(43)

where

\[
A' = \frac{A_0}{\sqrt{|A|^2T_0 + |B|^2T_1}}, \quad B' = \frac{B_1}{\sqrt{|A|^2T_0 + |B|^2T_1}}.
\]

(44)

Unlike the state $|\Psi_\text{tot}\rangle$, the state $|\Psi^t\rangle$ of equation (43) is not entangled but expressed as a product state of the Kondo system and the detector. That is, under the projective measurement of the detector, the two subsystems are disentangled. This is because the measurement $\hat{M}_t$ selects one of the two possible outcomes of the detector and the detector electron is collapsed on to the state $|t\rangle$. This means that we can describe the Kondo state under measurement $\hat{M}_t$ by a pure state $A'|0\rangle + B'|1\rangle$. Therefore, under a continuous weak measurement, we can write the conditional evolution of the Kondo singlet as

\[
|\Psi^t_{\text{G}}(t)\rangle = A(t)|0\rangle + B(t)|1\rangle,
\]

(45)

where the time evolution of the amplitudes $A(t)$ and $B(t)$ satisfy the relations (upon a mean time interval $\Delta t \equiv \hbar/2eVT_0$ between successive transmissions of electrons in the detector)
\[ A(t + \Delta_T) = \frac{t_0}{\sqrt{|A(t)|^2 T_0 + |B(t)|^2 T_1}} A(t), \]  
\[ B(t + \Delta_T) = \frac{t_1}{\sqrt{|A(t)|^2 T_0 + |B(t)|^2 T_1}} B(t), \]

or simply one can find that
\[ \frac{B(t + \Delta_T)}{A(t + \Delta_T)} = \frac{t_1}{t_0} \frac{B(t)}{A(t)}. \]

This relation gives the time evolution of the ratio between the two coefficients as
\[ \frac{B(t)}{A(t)} = \exp \left[ (-\Gamma_{\text{rel}}/2 + i\eta)t \right], \]

where
\[ \Gamma_{\text{rel}} = \frac{-2eVT_0}{h} \log \left( 1 - \frac{\Delta T}{T_0} \right), \]
\[ \eta = \frac{2eVT_0}{h} \Delta \phi_t. \]

Therefore the relative probability of the two states \(|B(t)|^2/|A(t)|^2\) goes to zero at \(t \rightarrow \infty\) as
\[ \frac{|B(t)|^2}{|A(t)|^2} = \exp (-\Gamma_{\text{rel}} t). \]

This result implies that the Kondo state evolves into the state \(|0\rangle\) at \(t \rightarrow \infty\) with its relaxation rate \(\Gamma_{\text{rel}}\). In the weak measurement limit \(\Delta T/T_0\) must be much smaller than unity and the relaxation rate is simplified as
\[ \Gamma_{\text{rel}} \simeq \frac{2eV}{h} \Delta T. \]

Several interesting observations can be made from equations (52) and (53). First, the relaxation rate is sensitive only to the change of the transmission probability. It is independent of the phase shift. This is in strong contrast with the dephasing rate \(\Gamma_d\) of equations (20)–(22) where the charge-state information is contained both in the change of the transmission probability and the phase shift. In other words, the measurement \(\hat{M}_t\) on the state \(|\Psi_{\text{tot}}\rangle\) washes out part of the charge-state information encoded in the phase shift. Indeed, for a detector sensitive only to the scattering phases (that is, for \(\Delta T = 0\)), the Kondo singlet of equation (45) remains unchanged (aside from the phase factor \(e^{i\eta t}\)). In this case the phase coherence of the Kondo singlet is fully preserved. In fact, this corresponds to the quantum erasure of the charge-state information by a particular measurement (\(\hat{M}_t\) in our case) on the detector [35]. The time evolution of the Kondo State is given by
\[ A(t) = \frac{1}{\sqrt{1 + \frac{2eV}{h} \Delta T}} \left[ 1 - \frac{2eV}{h} \Delta T \right] A(t), \]
\[ B(t) = \frac{1}{\sqrt{1 + \frac{2eV}{h} \Delta T}} \left[ 1 - \frac{2eV}{h} \Delta T \right] B(t). \]
singlet, conditioned on the measurement $M_t$, does not show any relaxation if the detector current is not sensitive to the charge state of the QD.

In an experiment, this conditional evolution and relaxation of the Kondo singlet can be investigated by correlating the transport of the electron through the QD and detection of electron at the output lead of the detector. For instance, let us consider an AB interferometer with a QD embedded in one of its arms and a detector nearby the QD as discussed in section 3 (figure 2). The Kondo-resonant transport under the measurement $M_t$ can be studied through the zero-frequency cross-correlation measurement between the two output leads, one from the interferometer and the other from the detector (see e.g. [35]). In this case, the interference in the cross-correlation will be reduced in proportion to the relaxation rate $\Gamma_{rel}$. As discussed above, the suppression of the interference is related to the charge sensitivity of the detector in transmission probability. The phase-sensitive information would not affect the visibility in the joint detection of the electrons at the two output electrodes.

We also point out that the cross-correlation measurement is able to resolve the anomaly observed in a controlled dephasing experiment of the Kondo-correlated QD [4]. The experimental results show unusually larger than expected dephasing rate with theory [20] based on ‘current-sensitive’ dephasing which is equivalent to $\Gamma_T$ in equation (21). As we have shown in section 3.4, the phase-sensitive contribution of dephasing can be dominant (that is, $\Gamma_\phi \gg \Gamma_T$) by taking into account asymmetry in the potential response. The relaxation rate $\Gamma_{rel}$ does not contain the phase shift of the scattering coefficients. Therefore, by measuring the conditional count on the Kondo-resonant transmission, one can extract the value $\Delta T$. Therefore, by combining the cross-correlation and the usual current measurement, we can get the two different contributions of dephasing $\Gamma_T$ and $\Gamma_\phi$. This would be a direct way to confirm the theoretical prediction on the importance of the contribution of asymmetry in the detector.

Aside from the phase-sensitive contribution to dephasing, $\Gamma_\phi$, it is interesting to note that $\Gamma_{rel} \neq \Gamma_T$. Let us consider a system with perfect inversion symmetry (thus $\Gamma_\phi = 0$) in the detector. For the weak continuous measurement considered in our study, we can find that $\Gamma_{rel} \gg \Gamma_T$. In other words, the interference in the Kondo-assisted transmission under the measurement $M_t$ is reduced much faster than in the case without the measurement. This can be regarded as an interesting manifestation of the nonlocality of quantum theory.

So far in this section we have discussed conditional evolution under the measurement $M_t$. The same kind of investigation can be done for the measurement $M_r$. One can find that the Kondo singlet under this measurement (denoted by $|\Psi_r(t)\rangle$) evolves into the state $|1\rangle$ as

$$|\Psi_r(t)\rangle = \tilde{A}(t)|0\rangle + \tilde{B}(t)|1\rangle,$$

where the two coefficients $\tilde{A}(t)$ and $\tilde{B}(t)$ satisfy the relation

$$\frac{|\tilde{B}(t)|^2}{|\tilde{A}(t)|^2} = \exp(\Gamma_{rel} t)$$

with its relaxation rate $\Gamma_{rel}$ being equivalent to the one obtained in equation (53):

$$\Gamma_{rel} \simeq \frac{2eV}{\hbar} (|r_1|^2 - |r_0|^2) = \frac{2eV}{\hbar} \Delta T.$$

An important point from our observation is that the present charge detection process should not be considered as an irreversible phase randomization which may be present because of some
uncontrollable degrees of freedom. As described above, cross-correlation measurements can be used to recover the interference and therefore clarify that it cannot be attributed to irreversible phase randomization. Another kind of interferometer + detector set-up has also been investigated that is able to confirm this point of view [36].

5. Conclusion

We have described the Kondo singlet in a QD entangled with a mesoscopic charge detector. Without any ‘measurement’ on the detector, the ‘coherence’ of the Kondo singlet is reduced. The dephasing rate is sensitive to the charge-state information encoded both in the magnitude and in the phase of the scattering coefficients of the detector. A detector model is introduced to account for the two different contributions of dephasing and to provide a possible solution to a recent experimental puzzle [4]. In the case that projective measurements are performed on the detector electrodes, the Kondo singlet is disentangled from the detector state. In this case, the Kondo singlet evolves into a particular state with a fixed number of electrons in the QD. Its relaxation rate is shown to be sensitive only to the QD-charge dependence of the transmission probability in the detector. This implies that the phase information is erased in the conditional evolution process. This kind of relaxation can be investigated by a cross-correlation measurement on the two output electrodes, one from the Kondo system and the other from the detector.

Appendix. Change of scattering coefficients by the shift of the 1D potential

Here we discuss the relation between the scattering matrix and the translation of a 1D potential, and derive equation (37). First, let us consider an arbitrary 1D potential $V = V(x)$ that leads to the corresponding scattering matrix

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}. \quad (A.1)$$

Then the wavefunction $\Psi(x)$ for the electron initially injected from $x \to -\infty$ can be written as (at the asymptotic region)

$$\Psi(x) = \begin{cases} e^{ikx} + re^{-ikx} & (x \ll 0), \\ te^{ikx} & (x \gg 0), \end{cases} \quad (A.2)$$

where $k$ is the wavenumber of the electron.

Now assume that the potential is shifted by $\delta x$, that is, the potential is given as $V = V(\bar{x})$ where $\bar{x} \equiv x - \delta x$. It is obvious that the wavefunction in the shifted potential $\Psi(\bar{x})$ has the same form as $\Psi(x)$ given in equation (A.2). That is

$$\Psi(\bar{x}) = \begin{cases} e^{ik\bar{x}} + re^{-ik\bar{x}} & (x \ll 0), \\ te^{ik\bar{x}} & (x \gg 0), \end{cases} \quad (A.3)$$

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Using the original coordinate $x$ instead of $\bar{x}$, one can find that

$$
\bar{\Psi}(x) = e^{-i\delta x} \begin{cases} 
  e^{ikx} + \bar{r}e^{-ikx} & (x \ll 0), \\
  -e^{ikx} & (x \gg 0),
\end{cases}
$$

where $\bar{t} = t$ and $\bar{r} = re^{2i\delta x}$.

In other words, the transmission amplitude is invariant under translation of the potential, but the reflection amplitude suffers phase shift of $2k\delta x$. Applying the result derived here for discussion of phase-sensitive dephasing in section 3.4, we get equation (37).

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