Area laws and entanglement distillability of thermal states

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We study the entanglement distillability properties of thermal states of many-body systems. Following the ideas presented in [D. Cavalcanti et al., arXiv:0705.3762], we first discuss the appearance of bound entanglement in those systems satisfying an entanglement area law. Then, we extend these results to other topologies, not necessarily satisfying an entanglement area law. We also study whether bound entanglement survives in the macroscopic limit of an infinite number of particles.

I. INTRODUCTION

Quantum Information Theory was born as a framework capable of describing the physics behind the processing, storage, and exchange of information under the rules of Quantum Mechanics [1]. Since it uses a very general and rather abstract approach, we do not need to talk about specific systems, interactions, or Hamiltonians, but instead we can play with qubits, channels, logic gates, etc. Beyond information purposes, this abstraction has also been proven very useful when studying relevant questions in other subareas of physics, such as Condensed Matter [2], Statistical Mechanics [3], Quantum Optics [4], or Astrophysics [5].

In a recent work [6], we have discussed the entanglement distillability properties of thermal states of some quantum many-body models with local interactions. We have shown the existence of a temperature range for which no pure-state entanglement can be distilled from the system despite being entangled. This type of irreversible quantum correlations is also known as bound entanglement [7]. This result, which can be valid for systems of arbitrary size, was connected to the so called entanglement-area laws, a typical feature of these systems that relates the entanglement of two distinguished regions to the area between them [8]. In the present contribution, we extend these ideas by considering two ways of addressing area laws and discussing the role they play in the appearance of bound entanglement. We also show that bound entanglement can appear in systems with different topologies, not necessarily fulfilling an entanglement area law.

Let us start by saying few words about the main subjects considered here: distillability and area laws. It is well known that some entangled mixed states have the property of being distillable. This means that if one considers many copies of such a state, it is possible to purify the entanglement into a smaller number of entangled pure states using local operations and classical communication (LOCC) [8,10]. Although all two-qubit and one-qubit-one-qutrit states are distillable, there are states in higher dimension for which no LOCC operation can purify entanglement [2]. These states are called bound entangled. Specifically, in the case of a multipartite system an entangled state of $n$ parties is bound entangled whenever the $n$ parties cannot distill any pure-state entanglement out of it by LOCC.

The first criterion able to detect bound entanglement was given by the Peres criterion [11]: all distillable states have a non-positive partial transposition. Thus, finding an entangled state with positive partial transposition (PPT) guarantees non-distillability. In this paper we will use a quantitative version of this criterion, namely the negativity $E_N(\rho)$ [12], to analyze the distillability properties of thermal states. The negativity is defined as follows:

$$E_N(\rho) = \sum_{\lambda_i < 0} |\lambda_i|,$$

being $\lambda_i$ the eigenvalues of $\rho^{T_A}$, the partial transposition of $\rho$ with respect to a given part $A$ of the system. In other words, $E_N(\rho)$ is given by the sum of the absolute values of the negative eigenvalues of $\rho^{T_A}$. So if the negativity of a state is zero, its partial transposition is positive, and the system is non-distillable (either separable or bound entangled), i.e.,

$$E_N(\rho) = 0 \Rightarrow \rho \text{ is non-distillable},$$

$$E_N(\rho) > 0 \Rightarrow \rho \text{ is entangled}.$$

Another useful and related quantity used throughout this paper is the logarithmic negativity $E_l(\rho)$, given by $E_l(\rho) = \log_2(1 + E_N(\rho))$. Clearly the implications above also apply to $E_l(\rho)$.

Consider a system in a pure state and a bipartition of it into two complementary subsystems. It is well known that for bipartite pure states the entropy of one of the reduced systems uniquely measures its entanglement [13,14]. As in Thermodynamics, one may expect that the entropy increases with the volume of the reduced state, but curiously this is not the case for the ground state of many models considered so far [2,11,16,17,18]. In fact, it is instead seen that the entanglement between the two complementary regions scales at most as their boundary area (in non-critical situations), a behavior that is generally named entanglement-area law. The above mentioned works approach entanglement-area laws in a variety of ways. Here, we will in particular focus on two different approaches. On one hand, we will address the entanglement-area relation by keeping fixed the size
of the system while changing the geometry of the bipartition. For example, in the case in which the distinguished subsystem is contiguous and the total system is considered in the macroscopic limit such an approach is usually named as block entropy. When a system obeys an area law under this approach we will say that an area law of type I is fulfilled. On the other hand, one can consider also the opposite approach. Namely, how the entanglement scales when a given partition is kept fixed while changing the size of the system. An example of such an approach is given, again for contiguous partitions, in Ref. [1], where an area law in the half-half partition (i.e., when a contiguous group composed by half of the particles belongs to the first subsystem and the other half to the second one) is established. In this case we will say that an area law of type II is obeyed. We consider both alternatives here, showing in which sense they are not equivalent and how they help in enlightening the appearance of bound entanglement.

The paper is organized as follows. In Sec. II we will point out the role that area laws play in the appearance of bound entanglement at finite temperatures. Then, in Sec. III we will focus on harmonic oscillator systems and analyze the emergence of different forms of area laws by studying different partitions and system sizes. We also consider a configuration where an area law is not seen in its simpler form and discuss the existence of bound entanglement for this case. In Sec. IV we extend our analysis to the emergence of bound entanglement also for spin-$\frac{1}{2}$ systems, obtaining very similar results as for harmonic systems. Sec. V is devoted to concluding remarks.

II. BOUND ENTANGLEMENT AND AREA LAWS

In Ref. [6] we considered systems exhibiting area laws and suggested that they are good candidates for presenting thermal bound entanglement in the macroscopic limit. This comes from the fact that when we increase the systems’ size the ground-state negativity for some partitions increases (e.g. for the even-odd cut, where even particles belong to one subsystem and odd particles to the complementary subsystem - see [17]) while for other type of partitions it saturates (e.g. in the half-half geometry). It is then natural to expect that, when temperature is added to the system, the negativity in the half-half partition vanishes for a lower temperature than in the even-odd geometry. In other words, one expects that different partitions of the system become PPT at different temperatures (that we call threshold temperatures). An important feature of the considered systems is that they are translationally invariant and then all half-half partitions are equivalent. This observation implies that when the negativities of the half-half partitions are null no pure-state entanglement can be distilled by LOCC [6]. On the other hand, there is still a temperature range where the negativity for the even-odd partition is strictly positive, which is enough to prove bound entanglement.

As already mentioned, a lot of efforts have been devoted to the study of entanglement-area laws for the ground state of various physical systems. Much less is known about equivalent laws for the case of finite temperature, where the state of the system is in a thermal mixture. Partly, this is due to the complexity characterizing the structure of entanglement for mixed states. For example, one of the known exact results concerns again the entropy of a contiguous subsystem [21]. However, for mixed states, this quantity is no longer a measure of entanglement. To the best of our knowledge, the most general result at finite temperature has been derived recently by Wolf et al. [20] and gives a bound to the mutual information between two complementary subsystems. We recall that the mutual information is a measure of the total amount of correlations, both classical and quantum. Hence, it trivially gives an upper bound to the entanglement. An important feature of such a bound is that the dependence with the temperature $T$ and the area $A$ between the complementary regions is factorized (in particular the bound scales linearly with $A$ and $T^{-1}$). On the basis of this result, one may argue that a trivial area law, i.e., in a factorized form, holds for the negativity $E_N$ whenever:

$$E_N \leq f(T)g(A),$$

where $f(T)$ and $g(A)$ are generic functions also depending on the parameters of the system. In particular, we assume that these functions do not depend on the way of partitioning the system. As said, this type of relation holds for the bound found in Ref. [20], in the case of the mutual information. In case the inequality in the previous formula becomes an equality, we refer to this form of area law as a strict area law. Notice that such a form of area law has been shown to hold for a nearest-neighbor harmonic ring at finite temperature, in the case of even-odd partition [6]. There, numerical evidences of the validity of a strict area law have been reported also for half-half partitions as well as for the analogous cases in a spin ring.

Let us now consider how a strict area law affects the existence of bound entanglement. As said, the key ingredient in the recipe above in order to show the presence of bound entanglement is that different partitions of a systems become PPT at different temperatures. As a consequence, if a strict area law of type I holds then no bound entanglement should be expected, since all partitions become PPT at the same temperature, namely when $f(T) = 0$ [22]. Notably, we have found no system showing this behavior, even considering models that exhibit the same entanglement for different partitions in the ground state, as we will report in detail in the next Sections. On the contrary we observed, for any system taken into account, that a strict area law of type I does not hold and that different partitions become PPT at different threshold temperatures (in any case, the bound given in Ref. [20] is of course not violated). This means

[Note: The rest of the text continues as per the original document.]

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that for a fixed system size, there is a temperature range for which bound entanglement is present. Now, one may wonder whether this holds also in the macroscopic limit of an infinite number of particles. In this case, it is the validity of a strict area law of type II that does give the key to a positive answer. In fact, it ensures that the threshold temperatures for each of the partitions chosen to reveal the presence of bound entanglement (e.g., even-odd and half-half partitions) stay constant as the size of the system increases. In particular, it ensures that the range of temperatures for which bound entanglement is present survives up to a macroscopic level. We will see explicitly that this is actually the case for some models consisting of harmonic chains and that the same behavior seems to be valid also for spin chains. In order to show that different scenarios may arise in other topologies, we also considered systems with a star configuration. There, we will see that a strict area law of type II is no more valid. The next sections are devoted to show explicitly the ideas explained above.

III. HARMONIC OSCILLATORS

Consider a system consisting of $N$ harmonic oscillators interacting via the following Hamiltonian:

$$H = \frac{1}{2}\left(\sum_i p_i^2 + \sum_{i,j} x_i V_{i,j} x_j\right),$$

where $x_i$ and $p_j$ represent position and momentum operators for each oscillator respectively ($i = 1, \ldots, N$).

The matrix $V$ describes both the on-site interaction (given by the diagonal elements) and the coupling between oscillators $i$ and $j$ (non-diagonal terms). This Hamiltonian is quadratic in the canonical coordinates and the oscillators are coupled through their position degrees of freedom which sets both the ground and the thermal states to be Gaussian. In this way these states are completely determined by their covariance matrix $\gamma$ defined as follows. Take the vector $S = (x_1, \ldots, x_N, p_1, \ldots, p_N)$, we then have

$$\gamma_{kl} = \text{Re} (\text{Tr}[\varrho(S_k - \bar{S}_k)(S_l - \bar{S}_l)])$$

where $\varrho$ is the density matrix of the state and $\bar{S}_k = \text{Tr}(\varrho S_k)$. If we consider the thermal state $\varrho = \exp[-H/T]/\text{Tr}[\exp[-H/T]]$ at temperature $T$, the corresponding covariance matrix is given by

$$\gamma(T) = [V^{-1/2}W(T)] \oplus [V^{1/2}W(T)],$$

where

$$W(T) = 1_N + 2[\exp(V^{1/2}/T) - 1_N]^{-1},$$

and $1_N$ denotes the $N \times N$ identity matrix. In the ground state case $W(0)$ is given by the identity matrix and so

$$\gamma(0) = V^{-1/2} \oplus V^{1/2}.$$

An analytical expression for the entanglement (quantified by the log-negativity $E_l$) between two complementary groups of oscillators, $A$ and $B$, was given in terms of the covariance matrix of the state, which can be written, in turn, only in terms of the matrix $V$. Then one gets the general formula for the log-negativity of a thermal state at temperature $T$:

$$E_l = \sum_{k=0}^{N-1} \log_2 \{\max[1, \lambda_k(Q)]\},$$

where $Q = P \omega^{+} P \omega^{-}$ and $\omega^{\pm} = W(T)^{-1}V^{\pm \frac{1}{2}}$. We denoted by $\{\lambda_k(Q)\}_{k=0}^{N-1}$ the spectrum of the matrix $Q$, whereas $P$ is an $N \times N$ diagonal matrix with the $i$-th entry given by $1$ or $-1$ depending on which group, $A$ or $B$, oscillator $i$ belongs to.

In Ref. [6] we considered the thermal states of Hamiltonian (3) with a circulant potential matrix $V$ given by

$$V = \text{circ}(1, -c, 0, \ldots, 0, -c).$$

i.e., the particles interact via nearest-neighbors interactions (see Fig. [1A]). We analyzed the entanglement behavior of the even-odd and half-half partitions while the system’s size $N$ is increased. A strict area law of type II for the log-negativity was analytically obtained for large $N$, specifically the entanglement for a given partition changes proportionally to the area $A$. The change of the system temperature just affects the rate the entanglement increases with $N$ for an even-odd partition ($A = N$ in this case) and the entanglement saturation value for the half-half partition ($A = 2$). In particular, the threshold temperatures does not depend on $A$, compatibly with a strict area law of type II.

We now investigate the area law of type I, that is for fixed $N = N_{tot}$ and varying the area by changing the geometry of the partition. Let us focus on two different ways of partitioning the chain. First, we consider the partition of the system consisting of $N_{tot} = 2^n$ particles into $2^n$ alternate blocks. The area associated to such a partition is given by $A = 2^n$. For $n_b = n$ one retrieves the even-odd partition and for $n_b = 1$ the half-half partition.
In Fig. 2 we depicted the log-negativity as a function of the area for $N_{tot} = 2^7$ and $c = 0.4$. From top to bottom the inverse temperature $\beta = 1/T$ is given by $\beta = 2.5, 2.4, 2$. The system has been partitioned into symmetric alternate blocks (see text for details).

Through these examples we can see that a strict area law of type I does not hold. However, notice that even if the entanglement does not vary strictly proportionally to the area of the subsystem, it still increases with the area. The details of such a behavior of course depend on many factors, such as the way the system is partitioned and the entanglement measure. Nevertheless a general feature seems to be independent of the partitioning: the more the interaction bonds intercepted by the partition the more the entanglement across it.

Summarizing, we have seen that in the case of a harmonic nearest-neighbor ring a strict area law of type I is violated, allowing for the presence of bound entanglement, whereas a strict area law of type II is valid, ensuring that bound entanglement survives for large systems. In the next section, we show that a similar behavior is also found for spin rings.

Let us now turn to a system in which a strict area law of type II is not valid either. As anticipated, we studied a different topology, namely a star configuration (see Fig. 1B). The system is described by the Hamiltonian with potential matrix given by $V_{ii} = 1 + (N-1)c$, $V_{ii} = 1 + c$, $V_{ij} = -c$ and $V_{ij} = 0$ otherwise ($2 \leq i \leq N, c > 0$), i.e., all the oscillators are equally connected to a central one. Clearly, translational symmetry does not hold anymore. The area law of type I is violated also in this case, allowing to find a temperature range in which the state is bound entangled. In Fig. 3 we depicted how the threshold temperatures for which the log-negativity is zero, $T_{th}^{\text{star}}$ (central particle) and $T_{th}^{\text{star}}$ (half-half partition) and $T_{th}^{\text{half}}$ (central particle versus the outer ones), vary with $N$. In the region between these two curves, bound entangled states are present. Nonetheless this is not enough to guarantee that bound entanglement survives for a large number of particles. First, notice that the range of temperature $T_{th}^{\text{star}}$ is no longer constant with $N$, namely, a strict area law of type II is not valid in this case. Second, our numerical calculations suggest that the entanglement between the central particle and the rest (which is the largest for this configuration) goes to zero as $N \rightarrow \infty$. Actually, an analytical expression for the log-negativity can be guessed for $T = 0$. In this case, being the state pure, the information about the entanglement is completely given by the reduced covariance matrix $\gamma_{\text{red}}$ of the central particle. The latter is simply given by the elements of $\gamma(0) = V^{-1/2} \otimes V^{1/2}$ corresponding to the central particle itself. By calculating explicitly $\gamma_{\text{red}}$ for a small number of particles $N$, one can recognize the following structure:

$$\gamma_{\text{red}} = \left( \frac{1}{N} + \frac{N-1}{N \sqrt{1 + Nc}} \right) \otimes \left( \frac{1}{N} + \frac{N-1}{N \sqrt{1 + Nc}} \right).$$

The negativity $E_N$ between the central particle and the rest is now simply given by

$$E_N = \max[0, \frac{1 - \nu}{\nu}],$$

where $\nu = \sqrt{\Delta - \sqrt{\Delta - 1}}$ and $\Delta$ is the determinant of $\gamma_{\text{red}}$ (see, e.g., Ref. [22]). Assuming now that the structure given in Eq. (9) holds for a generic $N$ we can extrapolate the behavior of the negativity in the macroscopic

![Fig. 2: Log-negativity as a function of the area in the case of nearest-neighbor harmonic ring of fixed size $N_{tot} = 2^7$ and $c = 0.4$. From top to bottom the inverse temperature $\beta = 1/T$ is given by $\beta = 2.5, 2.4, 2$. The system has been partitioned into symmetric alternate blocks (see text for details).](image1)

![Fig. 3: Same as Fig. 2, but partitioning the system in a non symmetric way (see text for details). The inverse temperature $\beta = 1/T$ is given by $\beta = 1.87, 1.865, 1.863$, from top to bottom.](image2)
bound entanglement is present in the macroscopic limit. In fact, since \( \Delta \to 1 \) in the limit \( N \to \infty \), the negativity itself goes to zero. Considering now the generic case at \( T > 0 \), it is then reasonable to expect that the log-negativity goes to zero too for a large number of particles. As said, our numerical calculations confirms this intuition (see also the inset in Fig. 4). As a consequence, the system is fully PPT in the macroscopic limit. Such a configuration gives then a non-trivial example for which the system is fully PPT in the macroscopic limit. Such a delimitation is present in the macroscopic limit [22].

### IV. SPIN SYSTEMS

The scope of this section is to extend the previous analysis to spin systems. We concentrate on the thermal state of systems composed by \( N \) spin-\( \frac{1}{2} \) particles, interacting with the Hamiltonian

\[
H = -\sum_{<i,j>} (\sigma^x_i \sigma^x_j + \sigma^y_i \sigma^y_j) + \hbar \sum_{i=1}^{N} \sigma^z_i, \tag{11}
\]

where the pairs of indices \( i \) and \( j \) over which we sum define the topology of the system.

In the nearest-neighbor configuration (see Fig. 4A) we proceeded as we did for Fig. 3 and progressively increase the boundary area between two regions, starting in the half-half partition and changing particles from one partition to the other. In this situation, for a chain with \( N = 10 \), we can observe (Fig. 5) again the presence of bound entanglement, since we have that some partitions are entangled while others are PPT at the same temperature. This again means that a strict area law of type I is not valid. On the other hand the validity of an area law of type II has been numerically shown up to twelve particles in Ref. [6], in analogy to what we have seen for the harmonic ring. This features strongly support the existence of bound entanglement in the macroscopic limit also for spin rings.

Let us now move to a system with a star configuration as in Fig. 1B, and consider the negativity corresponding to partially transpose either the middle particle or one of the outer particles. A remarkable feature of this system is that the ground state entanglement for both partitions is the same for any fixed \( N \), actually these partitions are both maximally entangled. This can be easily seen by recalling the explicit expression of the ground state given in Ref. [24]. One may then wonder whether such a behavior holds also at non-zero temperature, which would suggest that a strict area law of type I is valid. Our calculations show that this is not the case. Again, different partitions become PPT at different temperatures, as can be seen in Figs. 6 and 7. Notice that the central particle now becomes PPT at lower temperatures with respect to the external ones. Nevertheless, one can conclude the presence of bound entanglement also in this case by computing the threshold temperature for a half-half partition.

Another interesting feature of this system is that, by symmetry reasons, the entanglement between the middle particle and the external spins is independent of the system size (see Fig. 6). This is due to the fact that all the eigenstates of this system are of the form [24]:

\[
\frac{1}{\sqrt{2}} (|0 \rangle |\alpha_{m,j} \rangle \pm |1 \rangle |\alpha'_{m,j} \rangle) \tag{12}
\]

where the first ket correspond to the central particle, whereas the second one to the external particles. The
states $|\alpha_{m,j}\rangle$ and $|\alpha'_{m,j}\rangle$ are orthonormal eigenvectors of a high dimensional fictitious spin. The key point here is that the partial transpositions with respect to the central particle do not change the structure of these eigenvectors and this is true for any $N$. As a consequence, once expanded the thermal state in the eigenbasis above, one can see that $T_{th}^{co}$ does not depend on $N$. In other words a strict area law of type II holds for this partition. Notice however that this behavior does not hold in the case of other partitions, as we can see in Fig. 7 for the case of one external particle with respect to the rest. In particular, the threshold temperature for this partition increases with the system size. This fact, considering that the threshold temperature is size-independent for the middle particle partition, suggests that the temperature gap for which bound entanglement appears between the two partitions increases with the system size. Recall that this gap appears constant between the half-half and even-odd partitions in systems with nearest-neighbor interaction.

We can also check the negativity corresponding to transpose one of the external particles, for different systems sizes at a fixed temperature. In this scenario, one can see a peculiar behavior: for low temperatures the entanglement for the central vs. the rest partition can decrease as the system increases, but for higher temperature the opposite holds. The crossing temperature appears clearly non-trivial, between 2.2 and 2.4, see the inset of Fig. 7. Again the entanglement in different partitions do not vanish at the same temperature, allowing the presence of bound entanglement.

Before ending this section, let us stress that our numerical calculations are restricted to small number of particles due to computational hardness. Although not shown here, similar results can be found for other types of interactions, e.g. using Heisenberg-type hamiltonians.

To conclude, we have extended the results of Ref. [6] and considered different ways of studying the entanglement distillability properties of thermal states of many-body systems. We have considered systems of harmonic oscillators and spin-one-half particles in a chain and star topology. In general, our results show that a strict entanglement area law of type I (when changing the partitions for a fixed system size) is not fulfilled. Since the different partitions become PPT at different temperatures, bound entanglement appears for a temperature range in a natural way. Concerning the preservation of this range of temperatures when the system size is increased, we pointed out that a different approach to area laws should be addressed. In particular, an entanglement area law of type II (when changing the system size) is then useful to prove the presence of bound entanglement in the macroscopic limit of an infinite number of particles.

**Acknowledgments**

This work is supported by the EU QAP project, the Spanish MEC, under FIS2004-05639 and Consolider-Ingenio QOIT projects, and a “Juan de la Cierva” grant, the Generalitat de Catalunya, and the Università di Milano under grant “Borse di perfezionamento all’estero”.

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