Hybrid Quarkonia with High Statistics from NRQCD

UKQCD Collaboration - T. Manke, I.T. Drummond, R.R. Horgan, H.P. Shanahan *

aDAMTP, Silver Street, Cambridge CB3 9EW, ENGLAND

We have studied the $O(mv^6)$ effects in NRQCD on the spectrum of heavy quarkonia and compare our results for different lattices (quenched and dynamical). We also report on an investigation into hybrid states within the framework of NRQCD. This suggests that the lowest lying hybrid is around the $B^*B$ threshold and 3 standard deviations above the $B\bar{B}$.

1. INTRODUCTION

NRQCD has proved to be very successful when applied to heavy quark spectroscopy as relativistic effects can be taken into account systematically. In particular the low lying experimental spectrum can be reproduced directly from $\mathcal{L}_{\text{NRQCD}}$. Because NRQCD allows very fast and accurate measurements it is also possible to obtain reliable results for the spin structure which reveals the quark content in the confining regime. In a previous paper [1] we reported on a study of higher order corrections to the spin splittings. Here we will analyse these results more carefully on different lattices including some preliminary studies on unquenched configurations. It is very appealing to reveal also the gluon content in the non-perturbative regime of QCD. To this end we couple the magnetic field to the quark-antiquark pair. This hybrid state will allow quantum numbers which are not permitted in the ordinary quark model. There are many predictions for the masses of such hybrids from phenomenological models [2,3] and from lattice calculations in the static limit [4]. Here we go beyond the static approximation and employ NRQCD to treat the quarks dynamically. In Section 2 we present our Lagrangian and the results from a detailed analysis of the systematic errors in NRQCD. In Section 3 we report on hybrid results from NRQCD.

2. SYSTEMATIC ERRORS

For our investigation into the systematic errors of NRQCD we use the evolution equation

$G_{t+1} = K_{t+1} U_t^\dagger(x) K_t \left(1 - a\delta H\right) G_{t \geq 1}$,

$G_{t=1} = K_{t=1} U_t^\dagger(x) K_{t=0} S(x)$,

$K_t = \left(1 - \frac{aH_0}{2n}\right)^n$, $H_0 = -\frac{\Delta^2}{2m_b}$.

(1)

The source term, $S(x)$, depends on the operator to be propagated and $\delta H$ is given in [1] including terms up to $O(mv^6)$. The stability parameter $n$ depends on the quark mass $m_b$. We also reduced the lattice artefacts by redefining the lattice derivatives and the electromagnetic field tensor in Equation (1). All links are tadpole improved and we use the tree-level coefficients for all operators. For our analysis with lower accuracy in the velocity expansion we use the same evolution equation as in [5]. As opposed to [5] we employed gauge invariant operators throughout. We constructed the meson operators with definite $J^{PC}$ from extended link variables to improve the overlap with the ground state [1].

2.1. $O(mv^4)$ vs. $O(mv^6)$

For $\Upsilon$ the relativistic corrections to the spin splittings are expected at the 10% level from power counting arguments. In Figure 1 we show our results for the hyperfine structure, $^3S_1 - ^1S_0$. We see a clear effect in this very accurate quantity and the change of 13(3)% compares well with this expectation. For charmonium the results are less encouraging and we see 51(2)% effects which are beyond the naïve expectations from NRQCD.

*Talk presented by T. Manke. Supported in part by EU grant ERBCHB6-CT94-0523 and EPSRC 94007885. Our calculations were performed at the HPCF at Cambridge University.
Similarly, there is also a reduction of the fine structure, \(3P_J - \bar{P}\), away from the experimental values. This is a 34(14)% effect and it is still consistent with our expectations. But we note that the fine structure ratio is now consistent with experiment, \(R_{fs} = 0.56(19)\). This indicates that the inclusion of our new terms is important to reproduce the internal structure of the P-wave triplet accurately. \(3P_J - \bar{P}\).

Figure 1. Hyperfine Structure in \(\Upsilon\). The values around \(a^2 \approx 0.02 \text{ fm}^2\) are shifted for clarity.

2.2. \(\beta = 5.7\) vs. \(\beta = 6.0\)

There will be lattice artefacts from discretisation errors which are not removed at the tree level. In Figure 1 and 3 all results are plotted against \(a^2\), determined from the \(1P - 1S\) splitting. We see that our new evolution equation has reduced the lattice spacing dependence in the spin structure significantly. The remaining effects can be interpreted as being \(O(\alpha a^2)\) \([5]\). A different tadpole prescription may reduce these discretisation errors further \([5]\). Comparing our results to those in \([8]\) we see that going from \(u_{0P} \rightarrow u_{0L}\) changes the hyperfine splitting by the same magnitude as the change from \(O(mv^4) \rightarrow O(mv^6)\) but in the opposite direction. We expect a similar behaviour for the P-state triplet.

2.3. \(N_f = 0\) vs. \(N_f = 2\)

We have also used preliminary data from unquenched configurations at \(\beta = 5.2\) with 2 dynamical flavours \([9]\). Our values with \(\kappa_{\text{sea}} = 0.139\) are directly comparable to the results at \(\beta = 5.7, N_f = 0\). For this sea quark mass we see a very weak dependence on the number of flavours. The systematic errors discussed above have certainly a larger effect. Therefore unquenching effects should be studied for spin-independent quantities where spin corrections cannot obscure the analysis. However, those quantities have a bigger statistical errors and the observed effects are small as well \([8]\).

3. HEAVY HYBRIDS

From phenomenological models one expects the lowest hybrid state to come from a magnetic gluon \(J^P = 1^+\) which couples to \(QQ\) in an octet.
representation. The simplest lattice operators for such a state are

$$H_{ij}^\pm(x) = \chi_\mp^+(x)\sigma_i B_j \Psi(x)$$

(2)

and the corresponding spin-singlet without $\sigma_i$. On a lattice they belong to the representations $A^{++}, T_1^{++}, T_2^{++}, E^{++}, T_1^{--}$. This list also includes the exotic hybrid $1^{--}$. As a first step in establishing the masses of the lowest lying hybrid states we employed the leading order NRQCD correct to $O(mv^2)$. At this level of accuracy all these states are degenerate in energy. The optimisation of the overlap with the hybrid state is more elaborate and requires most of the CPU time. We use a fuzzing algorithm for the link variables from which we construct the extended operators with correct $J^{PC}$. We calculated propagators for several combinations of operators with different extent in the sink and source. After averaging over 20,000 sources from 500 configurations at $\beta = 6.0$ we obtained a very clear signal. We show the correlated fit results in Figure 3. To illustrate this result we compare the spin independent spectrum from NRQCD with the experimental values in Figure 4. The hybrid state is our prediction and it is approximately 1.64(16) GeV above the $\Upsilon$. A more careful analysis is needed to study the systematic errors. In particular the finite volume errors are of concern because such states will be rather broad on the lattice we used ($16^3 \times 48, a^{-1} = 2.44(4)$ GeV). Future work will also include the spin-dependent terms in NRQCD to study the splittings within the hybrid multiplets.

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