Comparisons of magnetic charge and axial charge meson cloud distributions in the PCQM

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The meson cloud distributions in r-space are extracted from the nucleon electromagnetic and axial form factors which are derived in the perturbative chiral quark model. The theoretical results indicate that the magnetic charge and axial charge distributions of the three-quark core have the similar distributions in r-space, the magnetic charge distributions of the meson cloud and three-quark core are more or less in the same region and peak at distances of around 0.4 fm, which is in good agreement with the finding of works in the framework of chiral perturbation theory, but the axial charge meson cloud distributes mainly inside the three-quark core.

The meson cloud of the nucleon, undoubtedly, plays a relevant role in the study of low energy electroweak properties of the nucleon. The meson cloud model, where the nucleon is considered as a system of three valence quarks surrounded by a meson cloud, has recently been employed to study the generalized parton distribution, nucleon electroweak form factors, nucleon strangeness, etc. In refs 26–28, meson cloud contributions to the neutron charge form factor have been studied and discussed in the meson cloud model, while the effects of the meson cloud on electromagnetic transitions have been estimated in refs 29–31. In our previous works, the electromagnetic and axial form factors as well as electroweak properties of octet baryons have been studied in the perturbative chiral quark model (PCQM) in the low energy region $Q^2 \leq 1 \text{GeV}^2$. The theoretical results in the PCQM with the predetermined quark wave functions are in good agreement with the experimental data and lattice QCD values. In addition, ref. 33 reveals that the meson cloud plays an important role in the axial charge of octet baryons, contributing 30–40% to the total values, and the similar effects have been also observed in other frameworks.

The investigation of the size or length scale of the meson cloud distribution inside the nucleon is interesting and important since it may help us to understand the internal structure of nucleon intuitively. In ref. 36, the meson cloud distribution has been extracted from the nucleon EM form factors in the constituent quark model. The $\pi$-meson cloud distribution is found very long ranged, ~2 fm (see Fig. 1 dashed curve), and is interpreted as the result of a pion cloud around the bare nucleon. Contrary to ref. 36, however, a much more confined $\pi$-meson cloud distribution of the nucleon EM form factors in r-space has been derived in the chiral perturbation theory (ChPT)37, 38. The results in refs 37 and 38 reveal that the $\pi$-meson cloud distributions peak around $r = 0.3$ fm and fall off smoothly with increasing the distance as shown in Fig. 2. Similar results have been also obtained in the chiral soliton model39, 40. The results in refs 37–40 may indicate that there is no structure at larger distances. In this work, we attempt to quantitatively study and define the r-space meson cloud distribution inside the nucleon in the framework of the PCQM.

**Perturbative chiral quark model**

In the framework of the PCQM, baryons are considered as the bound states of three relativistic valence quarks moving in a central potential with $V_{\text{eff}}(r) = S(r) + \gamma^0 V(r)$, while a cloud of pseudoscalar mesons, as the sea-quark excitations, is introduced for chiral symmetry requirements, and the interactions between quarks and mesons are achieved by the nonlinear $\sigma$ model in the PCQM. The Weinberg-type Lagrangian of the PCQM under an unitary chiral rotation16, 17 is derived as,

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\[ \mathcal{L}^W(x) = \mathcal{L}_d(x) + \mathcal{L}^V_d(x) + m(\pi^0), \]

\[ \mathcal{L}_d(x) = \bar{\psi}(x) \left[ i\gamma^\mu D_\mu(x) - S(r) \right] \psi(x) - \frac{1}{2} \Phi(x)(\Box + M_2^2) \Phi(x), \]

\[ \mathcal{L}^V_d(x) = \frac{1}{2F} \partial_\mu \Phi_i(x) \Phi_\mu(x) \gamma^\mu \gamma^5 \lambda_i \psi(x) + \frac{f_{ik}}{4F^2} \Phi_i(x) \partial_\mu \Phi_j(x) \gamma^{ij} \lambda_i \psi(x), \]

where \( f_{ik} \) are the totally antisymmetric structure constant of \( SU(3) \), the pion decay constant \( F = 88 \text{ MeV} \) in the chiral limit, \( \Phi_i \) are the octet meson fields, and \( \psi(x) \) is the triplet of the \( u, d \), and \( s \) quark fields taking the form

\[ \psi(x) = \begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix}. \]

The quark field \( \psi(x) \) could be expanded in

\[ \psi(x) = \sum_\alpha \left( b_\alpha u_\alpha(x) e^{i\chi_\alpha} + d_\alpha^\dagger v_\alpha(x) e^{i\xi_\alpha} \right), \]

where \( b_\alpha \) and \( d_\alpha^\dagger \) are the single quark annihilation and antiquark creation operators. The ground state quark wave function \( u_\alpha(x) \) may, in general, be expressed as

\[ u_\alpha(x) = \begin{pmatrix} g(r) \\ i\sigma \cdot \xi(f(r)) \end{pmatrix} \chi_\alpha x_\alpha x_\alpha. \]
where $\chi_s^i$, $\chi_f^i$, and $\chi_c^i$ are the spin, flavor, and color quark wave functions, respectively.

The calculation technique in the PCQM is based on the Gell-Mann and Low theorem, in which the expectation value of an operator $\hat{O}$ can be calculated from

$$
\langle \hat{O} \rangle = \frac{\langle \phi_0 \rangle}{\langle \phi_0 \rangle} \sum_{n=0}^{\infty} \int d^4 x_1 \cdots \int d^4 x_n \langle L_{\mu}^{\nu} \rangle(x_1) \cdots \langle L_{\mu}^{\nu} \rangle(x_n) \hat{O} | \phi_0 \rangle^2
$$

where the state vector $| \phi_0 \rangle$ corresponds to the unperturbed three-quark states projected onto the respective baryon states, which are constructed in the framework of the SU(6) spin-flavor and SU(3) color symmetry. The subscript $c$ in Eq. (7) refers to contributions from connected graphs only. $L_{\mu}^{\nu}$ is the quark-meson interaction Lagrangian as given in Eq. (3). The Feynman diagrams contributing to the electromagnetic and axial form factor of octet baryons up to the one-loop order are shown in upper and lower panel of Fig. 3, respectively.

In our previous works, the ground state quark wave functions have been determined by fitting the PCQM theoretical result of the proton charge form factor $G_E^p(Q^2)$ to the experimental data, and the electromagnetic and axial form factors as well as electroweak properties of octet baryons in low energy region ($Q^2 \leq 1$ GeV$^2$) have been studied in the PCQM based on the predetermined quark wave functions. As the results shown in Fig. 4, the EM and axial form factors are in good agreement with the experimental data up to $Q^2 = 1$ GeV$^2$ based on the predetermined quark wave functions. Meanwhile, the nucleon magnetic moment $\mu_p = 2.735 \pm 0.121$ and axial charge $g_A = 1.301 \pm 0.230$, which are the magnetic and axial form factors in zero-recoil, differ from the experimental data by only 2%, and are also consistent with the lattice QCD values. It is noted that there is no any free parameter in the numerical calculations. Thus one may indicate that the PCQM is credible for the low energy region $Q^2 \leq 1$ GeV$^2$, and able to quantitatively study and evaluate the $r$-space meson cloud distribution in the framework of the PCQM. More details could be found in refs 32 and 33.

**Magnetic and axial charge distributions of meson cloud.** Following our previous works, we present in Fig. 5 the $Q^2$-dependence of proton magnetic and nucleon axial form factors separately in leading order (LO) diagram and loop Feynman diagrams. This division is the PCQM dependent, and the LO diagram is attributed to 3q-core and the loop Feymann diagrams could be interpreted as the effects of the pion cloud. The PCQM results shown in Fig. 5 clearly reveal that the LO diagram results in a dipole-like form factor and dominates the form factor, while the meson cloud leads to a flat contribution to the magnetic and axial form factors. The flat contribution may indicate that the meson cloud of the nucleon may distribute mainly in a very small region.

In general, the form factor $F(q^2)$ is the Fourier transformation of charge distribution in $r$-space and takes the form,

$$
F(q^2) = \frac{1}{(2\pi)^3} \int \rho(\bar{r}) e^{-i\bar{q}\cdot\bar{r}} d^3\bar{r},
$$

where $\rho(\bar{r})$ is the charge density, and $\bar{q}^2$ is the three-momentum transfer. If $F(q^2)$ has been determined, in principle, the charge distribution $\rho(r)$ could be derived by the inverse Fourier transformation,

$$
\rho(r) = \frac{1}{(2\pi)^3} \int F(q^2) e^{i\bar{q}\cdot\bar{r}} d^3\bar{q}.
$$

In this work, we extract, based on the inverse Fourier transformation Eq. (9), the magnetic charge and axial charge meson cloud distributions of the nucleon from the EM and axial form factors as shown in Fig. 5.

The results shown in Fig. 6 are the LO and meson cloud contributions to the proton magnetic form factor $\rho_E^p(r)$ (left panel) and the nucleon axial form factor $\rho_A^N(r)$ (right panel) in $r$-space derived by Eq. (9). It is clear that the magnetic and axial charge attributed to 3q-core (LO) show a distribution ranging 2 fm, while the meson cloud effects (Loop) are much smaller at distances beyond 1 fm which is in accordance with the finding of ref. 37 as shown in Fig. 2. But it is much smaller than the result of ref. 36 about 2.5 fm (see Fig. 1), in which the proton is thought of as virtual neutron-positively charged pion pair. As results shown in the left panel of Fig. 6, the peaks...
Figure 4. The results of EM and axial form factors in the PCQM taken from refs 32, 33.

Figure 5. Leading order (solid) and loop (dashed) contributions to the proton magnetic (left panel) form factor and neutron axial (right panel) form factor.

Figure 6. Comparisons between the LO and meson cloud distributions for proton magnetic (left panel) and axial (right panel) form factors in $r$-space.
of the $\rho_M^M(r)$ of the 3q-core and the meson cloud are almost in the same region, but the peak of the loop diagrams contributions to the $\rho_M^N(r)$ as presented in the right panel of Fig. 6 are in a clearly smaller region than the one of 3q-core, which may indicate that the axial charge meson cloud distributes mainly inside the three-quark core.

Furthermore, we compare the 3q-core contributions to the $\rho_M^M(r)$ and $\rho_M^N(r)$ in $r$-space as presented in the left panel of Fig. 7. It is found that the $\rho_M^M(r)_{\text{LO}}$ and $\rho_M^N(r)_{\text{LO}}$ show a similar $r$-dependence, which may reveal that the magnetic and axial charge distributions of the constituent quarks are the same. The meson cloud contributing to $\rho_M^M(r)$ and $\rho_M^N(r)$ in the right panel of Fig. 7 show that the axial charge distribution of the meson cloud $\rho_M^N(r)_{\text{Loop}}$ is narrower and the peak is closer to the origin. We also found that the magnetic charge distribution $\rho_M^M(r)_{\text{Loop}}$ in the right panel of Fig. 7 present a significant peak around $r \approx 0.4 \text{ fm}$ and fall off smoothly when the distance increases. To compare with the right panel of Fig. 2, the $\rho_M^M(r)_{\text{Loop}}$ distribution in the PCQM turns out to be similar to the ChPT finding of refs 37, 38.

### Summary

In this work, we quantitatively study and evaluate the electromagnetic and axial form factors of nucleon distributions in $r$-space in the framework of the PCQM. The PCQM-dependent results are separated into the 3q-core contributions and the meson cloud effects, and the results are in good agreement with the ChPT results of refs 37, 38. In summary, one may conclude that the similar $r$-dependence of the magnetic and axial form factors resulted from the LO diagrams show a distribution ranging 2 fm and may reveal that the magnetic charge and axial charge distributions of the constituent quarks are the same. The meson cloud effects to the magnetic and axial charge distributions are much smaller at distances beyond 1 fm which is in accordance with the finding of refs 37, 38. Meanwhile, the magnetic charge distributions of the 3q-core and the meson cloud are more or less in the same region and peak at distances of around 0.4 fm, quite consistent with the earlier determinations of ChPT in refs 37, 38, while the peak of the loop diagrams contributions to the $\rho_M^M(r)$ are in a clearly smaller region than the one of 3q-core, which may indicate that the axial charge meson cloud distributes mainly inside the 3q-core.

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**Figure 7.** Comparisons between the magnetic and axial distributions in $r$-space for the LO (left panel) and loop (right panel) diagrams.
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Author Contributions
X.L. carried out the study. Z.L. and K.K. offered helpful suggestions. A.L. and Y.Y. supervised the project. All authors discussed the results and commented on the manuscript.

Additional Information
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