Revealing Quantum Advantage in a Quantum Network

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The assumption of source independence was used to reveal nonlocal (apart from standard Bell-CHSH scenario) nature of correlations generated in entanglement swapping experiments. In this work, we have derived a set of sufficient criteria, imposed on the states (produced by the sources) under which source independence can reveal nonbilocal nature of correlations in a quantum network. To show this, we have considered real two qubit X states thereby discussing the various utilities of assuming source independence in a quantum network.

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INTRODUCTION

The analysis of correlated statistics of measurement outcomes in a quantum network has emerged as a recent trend in the study of understanding correlations[1–5]. This sort of analysis has enriched the study of quantum theory from both theoretical as well as application point of view. From the theoretical point of view, this study led to the discovery of nonlocal nature of quantum predictions that cannot be described by any locally causal model [6]. As applications, this has revealed the potential of nonlocal nature of certain quantum states, used in quantum information technologies, specifically for private randomness generation [7, 8], quantum key distribution [9, 10], for reducing communication complexity [11], device-independent entanglement witnesses [12], device-independent quantum state estimation [13, 14]. The key feature of all these applications is the fact that quantum non locality could be used in a device independent manner. Besides, this study could also be applied in various other tasks involving quantum networks such as quantum distributed computing [2], quantum repeaters [15–17]. Entanglement swapping experiments [17] reveal nonlocal nature of certain class of quantum states (entangled states) more strongly compared to standard Bell-CHSH scenario. Motivated by this intuition, bilocal scenario was introduced in [1] and further analyzed in [2–5]. Bilocal scenario is a general scenario of quantum network involving three parties characterized by independence of two sources shared between the parties. This concept of source independence has emerged as a step towards exploiting nonlocal character of quantum correlations in a more common sense than that can be usually thought in a usual Bell scenario, thereby revealing quantum advantage in a network. For instance, visibility V > 50% is enough to reveal quantumness (non bilocality) in a bilocal network using Werner state [18] in contrast to a visibility V > 70% (for projective measurements [18]) and V > 66% (for POVM [19]), required for a Werner state to be nonlocal in usual Bell sense. In this context, one may ask the question: what would be the requirements for a quantum state to reveal non locality with a lower visibility? To be more precise, what would be the criteria to be satisfied by a quantum state to produce non bilocal correlations in a network? Our work basically discusses this query.

BILOCAL SCENARIO OF TWO-QUBIT X STATES

Consider first the bilocal experimental setup (FIG.1)[17]. There are three parties Alice(A), Bob(B) and Charlie(C) arranged in a linear pattern such that any pair of adjacent parties share a source and the two sources S1 and S2 are independent to each other. Each of these two sources S1 and S2 sends a physical system represented by λ1 and λ2 respectively. Independence of S1 and S2 guarantees independence of the two variables. All parties can perform measurements on their systems labeled by x, y, z for Alice, Bob and Charlie and they obtain outcomes denoted by a, b, c respectively. In particular, Bob might perform a joint measurement on the two systems that he receives from the two independent sources. The correlations obtained thereby take the form:

\[ p(a, b, c|x, y, z) = \int \int d\lambda_1 d\lambda_2 \rho(\lambda_1, \lambda_2) P(a|x, \lambda_1) P(b|y, \lambda_1, \lambda_2) P(c|z, \lambda_2) \] (1)

where \( \lambda_1 \) characterizes the joint state of the system produced by the source S1 and \( \lambda_2 \) for the system S2. The hidden states \( \lambda_1, \lambda_2 \) follow independent probability distribution \( \rho_1(\lambda_1) \) and \( \rho_2(\lambda_2) \) such that

\[ \rho(\lambda_1, \lambda_2) = \rho_1(\lambda_1) \rho_2(\lambda_2) \] (2)

Clearly one can consider entanglement swapping procedure (with independent sources)[1–5] as a particular case of this scenario. The correlations of the form (1) and (2) satisfy the following inequality (introduced in [2]):

\[ \sqrt{|I|} + \sqrt{|J|} \leq 1 \] (3)
The bilocal scenario([1, 2]) where three parties Alice, Bob and Charlie share two sources $S_1$ and $S_2$(characterized by the hidden states $\lambda_1$ and $\lambda_2$ respectively) which are assumed to be independent.

where $I = \frac{1}{4} \sum_{x,z=0,1} \langle A_x B^0 C_z \rangle$, $J = \frac{1}{4} \sum_{x,z=0,1} (-1)^{x+z} \langle A_x B^1 C_z \rangle$ and $\langle A_x B^0 C_z \rangle = \sum_{a,b \neq b^0 \perp c} (-1)^{a+b+\ell} P(a,b|b^1,c|x,z)$. Here $A_x$ and $C_z$ are the observables for binary inputs $x, z$ of Alice and Charlie respectively whereas $B^\ell$ denotes the observable of Bob corresponding to a single input.

For Bob there are four outputs labeled by two bits $\mathbf{b} = b^0 b^1 = 00, 01, 10, 11$.

Note that there exist some nonbilocal correlations that satisfy this inequality(3) but violation of this inequality by any correlation ensures non bilocality. Thus this inequality can be regarded as a convenient tool for testing nonbilocality and thereby for testing quantumness in a quantum network. So from the application point of view it is interesting to analyze the criteria under which the physical systems (state) sent by the sources exhibit nonbilocality. In this paper we have introduced a set of sufficient criteria for two qubit X states to give quantum advantage when used in a network(by revealing nonbilocality) thereby discussing about various advantages that one can get under bilocality assumption in a network. First we consider two copies of X state:

$$\chi_i = a_i|00\rangle\langle 00| + b_i|01\rangle\langle 01| + c_i|10\rangle\langle 10| + d_i|11\rangle\langle 11| + p_i|00\rangle\langle 11| + p_i^*|11\rangle\langle 00| + q_i|01\rangle\langle 10| + q_i^*|10\rangle\langle 01|$$

i=1,2 with $p_i$ and $q_i$ real, $a_i + b_i + c_i + d_i = 1$. $\chi_i \geq 0$ demands $p_i^2 \leq a_i d_i$ and $q_i^2 \leq b_i c_i$.

The class of X states is of particular interest as it includes many well know class of states such as Bell diagonal states, Werner states(a one-parameter family of states which encompasses both separable and entangled states[18]), etc [20]. In fact, any two qubit state can be converted to a X state by passing it through a noisy channel[21]. Besides, density matrix structure of two spin X states are encountered in many physical scenarios and can also be achieved in various experiments [23–29]. For instance, this class of states was encountered in [22] while analyzing entanglement of an atom in a quantized electromagnetic field. They constitute the spectra of all the systems with odd-even symmetry, for example, in the Ising and the XY models[30, 31]. X states were also studied in condensed matter systems and in various other fields of quantum mechanics[32–36]. The evolution of entanglement in X states(subjected to spontaneous emission) was analyzed in [37] where it was further shown that some forms of X states remain invariant under general decoherence [37] whereas some disentangle within finite time. In [38], the author showed that the “sudden death of entanglement” of this class of states can be increased, decreased or averted with the aid of local operations. Thus X state emerges as an important two qubit class of states. Here we analyze the restrictions to be imposed on the state parameters of two copies of an X state to generate nonbilocal nature of correlations when used in a quantum network. For that, we consider Bob to perform complete Bell state measurement on his particles. For Alice and Charlie we consider simple von Neumann measurements $M_j^A$ and $M_j^C$ respectively:

$$M_j^A = M_j^C = \cos \theta_j^a \hat{\sigma}_z + \sin \theta_j^a \hat{\sigma}_x$$

where $\theta_j^a = (-1)^j \frac{\Pi}{4} - \frac{\Pi}{8}$, and $\mu \in [0, 1]$, $j \in \{0, 1\}$, $\sigma_x, \sigma_z$ denote the Pauli matrices.

Under these measurement settings Eq.(3) takes the form:

$$\sqrt{\prod_{i=1}^{2} (a_i - b_i - c_i + d_i)^2} + 2 \prod_{i=1}^{2} (p_i + q_i)^2 \leq \sqrt{1 + \cos \frac{\Pi}{4} \mu} \leq 2.$$ (6)

Suppose, $\rho$, denotes a two qubit state; $\rho = \frac{1}{2} \sum_{i_1, i_2=0}^2 t_{i_1 i_2} \sigma_{i_1} \otimes \sigma_{i_2}$ where $\sigma_{i_k}^\dagger$, denotes the identity operator in the Hilbert space of qubit k and $\sigma_{i_k}$, are the Pauli operators along three perpendicular directions, $i_k = 1, 2, 3$.

$$M(\rho) = t_{11}^2 + t_{22}^2$$ (7)

where $t_{11}^2$ and $t_{22}^2$ are the two largest eigen values of $T_\rho^A T_\rho^C$, where $T_\rho^T_\rho$ is the transpose of the correlation tensor $T_\rho(formed by the real coefficients $t_{i_1 i_2} = Tr[\rho (\sigma_{i_1} \otimes \sigma_{i_2})]$). By Horodecki criteria [39], a state is nonlocal in the sense of Bell-CHSH, if $M(\rho) > 1$. Here,
Let for each $i$, $M(\chi_i) = \text{Max}_k\{\sigma_{i,k}\}$, where

\begin{align*}
\sigma_{i,1} &= 8(p_i^2 + q_i^2) \\
\sigma_{i,2} &= (a_i - b_i - c_i + d_i)^2 + 4(p_i + q_i)^2 \\
\sigma_{i,3} &= (a_i - b_i - c_i + d_i)^2 + 4(p_i - q_i)^2
\end{align*}

\(i = 1, 2\) such that:

\begin{align*}
\sigma_{i,1} &= 1 - \epsilon_i \\
\sigma_{i,2} &= 1 - \delta_i \\
\sigma_{i,3} &= 1 - \xi_i
\end{align*}

\(i = 1, 2\) for two copies of X state $\chi_1$ and $\chi_2$. If $\chi_i$ is local then each of $\epsilon_i$, $\delta_i$ and $\xi_i$ must lie in [0,1] whereas if nonlocal then at least one of these three variables must lie in [-1,0). If source $S_i$ emits $\chi_i$ $(i = 1, 2)$ then bilocality equation takes the form (for proof, see Appendix):

\[
\sqrt[4]{\prod_{i=1}^{2}(1 - \delta_i + \epsilon_i - \xi_i) + \prod_{i=1}^{2}(1 - \delta_i - \epsilon_i + \xi_i)} \leq \frac{4}{\sqrt{2(1 + \cos \frac{\mu}{2})}} \tag{12}
\]

(iii) Depending on the possible values of the state parameters, $\delta_i$, $\epsilon_i$ and $\xi_i$ get further restricted as:

1. $F_i \leq \sqrt{2 + |G_i - H_i|}$ when $(p_i, q_i, p_i - q_i)$ are of same sign and $a_i - c_i > 0$ or $(p_i, q_i, q_i - p_i)$ are of same sign but $p_i - q_i$ is of opposite sign and $a_i - c_i < 0$.

2. $H_i \leq \sqrt{2 - (F_i + H_i)}$ when $(p_i, q_i, p_i - q_i)$ are of same sign and $a_i - c_i < 0$ or $(p_i, q_i, q_i - p_i)$ are of same sign but $p_i - q_i$ is of opposite sign and $a_i - c_i > 0$ or $(q_i, p_i + q_i, q_i)$ are of same sign but $p_i$ is of opposite sign and $a_i + c_i < 0$.

3. $G_i \leq \sqrt{2 + |F_i - H_i|}$ when $(p_i, q_i + q_i, q_i)$ are of same sign but $q_i$ is of opposite sign and $a_i - c_i > 0$ or $(q_i, q_i + p_i, q_i)$ are of same sign but $p_i$ is of opposite sign and $a_i + c_i < 0$.

where $F_i = \sqrt{1 - \epsilon_i + \xi_i - \delta_i}$, $G_i = \sqrt{1 - \epsilon_i - \xi_i + \delta_i}$ and $H_i = \sqrt{1 + \epsilon_i - \xi_i - \delta_i}$.

**X states with maximally mixed marginals:** This is a subclass of two qubit X states which are characterized by having maximally mixed subsystems: $\rho^A = \frac{I}{d_A}$ and $\rho^B = \frac{I}{d_B}$ with $d_{A(B)} = \text{dim}(H^{A(B)})$ as the dimension of the local Hilbert space $H^{A(B)}$. This class of states[41] can
be written (in terms of the Pauli matrices) as:

\[
\tau = I \otimes I + \sum_{i=1}^{3} c_i \sigma_i^A \otimes \sigma_i^B,
\]

where, \( I \) is the identity matrix and \( c = (c_1, c_2, c_3) \) is a real vector with \( c_i \in [-1, 1] \). Clearly, this subclass belongs to \( S \). When two copies \( \tau_1 \) and \( \tau_2 \) are used in a network then nonbilocality correlations are produced if the parameters of these two copies of \( \alpha \) states satisfy the inequality:

\[
\sqrt{\prod_{j=1}^{2} c_j^2} + \sqrt{\prod_{j=1}^{2} c_j^2} > \frac{1}{2}
\]

where \( c_1 \) and \( c_3 \) are the parameters of the copies \( \tau_1 \) and \( \tau_2 \) respectively. A particular subclass of \( X \) states with maximally mixed marginals is the one parameter family of \( \alpha \) states [42]:

\[
\rho(\alpha) = \begin{pmatrix}
\frac{1}{2} & 0 & 0 & \frac{1}{2} \\
0 & \frac{1}{2} - i\alpha & 0 & 0 \\
0 & 0 & \frac{1}{2} + i\alpha & 0 \\
\frac{1}{2} & 0 & 0 & \frac{1}{2}
\end{pmatrix}
\]

with \( \alpha \in [0, 1] \). For this family Eq.(12) takes the form:

\[
\sqrt{\prod_{i=1}^{2} (2\alpha_i - 1)} + \sqrt{\prod_{i=1}^{2} \alpha_i} > \sqrt{2} \quad (13)
\]

\( \alpha_i \) denotes the parameter of the \( i \)-th copy \((i = 1, 2)\) of the \( \alpha \) state. Here \( a_i - c_i = \alpha_i - \frac{1}{2}, p_i = \frac{1}{2} - q_i = 0 \). Hence \( p_i \pm q_i \) and \( p_i \) are of the same sign. Also \( \epsilon_i = 1 - 2\alpha_i^2, \delta_i = \xi_i = 1 - \alpha_i^2 - (2\alpha_i - 1)^2 \). Clearly, criteria (i) and (ii) are satisfied by \( \alpha \). Depending on the signature of \( a_i - c_i \), some restrictions imposed on \( \alpha_i \) by criteria (iii).

If \( a_i - c_i > 0 \), i.e., \( \alpha_i > \frac{1}{2} \) then \( \alpha_i \) restricted by criteria (1) and (3) of (iii), whereas if \( a_i - c_i < 0 \) then criteria (2) restricts the range of \( \alpha_i \). However, one can easily check that no further restriction is imposed on the state parameter other than it should lie in the range \([\frac{1}{2}, 1]\) in the former case, whereas the range should be \([0, \frac{1}{2}]\) in the latter one. Finally, under the restriction imposed by Eq.(13) the two copies of \( \alpha \) state can generate nonbilocality only if \( \alpha > \frac{1}{2} \) (see FIG.2).

**Criteria for maximal violation of (12):** It becomes clear from the symmetry of \( \epsilon_i \) and \( \xi_i \) (12) that the subclass \( S \) of \( X \) states violates the nonbilocality inequality (12) maximally in the \( \xi_i = \epsilon_i, \quad (i = 1, 2) \) plane. For \( \xi_i = \epsilon_i, \quad (i = 1, 2) \) Eq.(12) becomes:

\[
\prod_{i=1}^{2} (1 - \delta_i) \leq \frac{4}{(1 + \cos \mu)^2} \quad (14)
\]

It becomes clear from Eq.(14) that with the measurement settings (Eq.5), violation of Eq.(14) in a network is due to \( \delta_i \) \((i = 1, 2)\) and hence the locality of the local copy and nonlocal nature of the nonlocal copy are required to (due to \( \delta_1 \) and \( \delta_2 \) respectively) violate this inequality. It is clear from Eq.(14) that when \( \delta_1 < 0 \) and \( \delta_1 > 0 \), if \( \mu \) increases then \( \delta_1 \) must decrease. Hence if one copy is local then amount of locality (value of Bell-CHSH operator) due to \( \delta_1 \) must increase (See FIG.3). We now impose restrictions on the variables \( \epsilon_i (= \xi_i) \), and \( \delta_i (i = 1, 2) \) of local copy and nonlocal copy separately which in turn restrict the parameters of the two copies(local and nonlocal) of the state produced by the two sources \( S_i \) \((i = 1, 2)\) simultaneously. These restricted states suffice to generate nonbilocality in a network. An obvious restriction is imposed on \( \delta_1 \) and \( \delta_2 \) (see FIG.3) through the inequality:

\[
\prod_{i=1}^{2} (1 - \delta_i) > \frac{4}{(1 + \cos \mu)^2} \quad (15)
\]

For further restrictions, we start with the subclass \( S \) of \( X \) state under the above assumption. When \( \chi_1 \) local and
\( \chi_2 \) nonlocal, the restrictions are (for proof, see the Appendix):

\( \text{C1 : } 0 \leq \delta_1 \leq \epsilon_1 \leq \frac{4\sqrt{2(1-\delta_1^2)+5(\delta_1-1)}}{2} \), when \((p_1, q_1, p_1 - q_1)\) are of same sign and \(a_1 = c_1 > 0\) or, \((p_1, q_1)\) are of same sign but \(p_1 - q_1\) is of opposite sign and \(a_1 = c_1 < 0\).

\( \text{C2 : } 0 \leq \epsilon_1 \leq \delta_1 \), when \((p_1, p_1 + q_1)\) are of same sign but \(q_1\) is of opposite sign and \(a_1 = c_1 > 0\) or, \((q_1, p_1 + q_1)\) are of same sign but \(p_1 + q_1\) is of opposite sign and \(a_1 = c_1 < 0\).

\( \text{C3 : } 0 \leq \epsilon_2 \leq \delta_2 \leq \frac{4\sqrt{2(1-\delta_2^2)+5(\delta_2-1)}}{2} \), when \((p_2, q_2, p_2 - q_2)\) are of same sign and \(a_2 = c_2 > 0\) or, \((p_2, q_2)\) are of same sign but \(p_2 - q_2\) is of opposite sign and \(a_2 = c_2 < 0\).

\( \text{C4 : } 0 \leq \delta_2 \leq \delta_2 \leq \frac{4\sqrt{2(1-\delta_2^2)+5(\delta_2-1)}}{2} \), when \((p_2, p_2 + q_2)\) are of same sign but \(q_2\) is of opposite sign and \(a_2 = c_2 > 0\) or, \((q_2, p_2 + q_2)\) are of same sign but \(p_2 + q_2\) is of opposite sign and \(a_2 = c_2 < 0\).

with \(\delta_1 \in [0, 1/2]\) and \(\delta_2 \in (-1, 0]\).

When \(\chi_i(i = 1, 2)\) both nonlocal: In this case, sign of \(\delta_1\) gives rise to two subcases. If \(\delta_1 \in [0, 1/2]\) then the restrictions on \(\chi_1\) gets modified as

\[
\frac{\delta_1 - 1}{2} \leq \epsilon_1 < 0,
\]

when \((p_1, p_1 + q_1)\) are of same sign but \(q_1\) is of opposite sign and \(a_1 = c_1 > 0\) or, \((q_1, p_1 + q_1)\) are of same sign but \(p_1 + q_1\) is of opposite sign and \(a_1 = c_1 < 0\), whereas that on \(\chi_2\) remain same. If \(\delta_1 < 0\) then the restrictions on \(\epsilon_1\) and \(\delta_1\) are the same, as those imposed on \(\epsilon_2\) and \(\delta_2\).

**Simplification of criteria for Bell Mixture:** Now we consider Bell mixture separately:

\[
\beta_i = w_1^i |\psi^-\rangle + w_2^i |\psi^+\rangle + w_3^i |\phi^-angle + w_4^i |\phi^+\rangle
\]

where \(i = 1, 2\), \(|\psi^-\rangle, |\psi^+\rangle, |\phi^-\rangle, |\phi^+\rangle\) are the four Bell states (in standard notation), \(0 \leq w_j^i \leq 1\) and \(\sum_{i=1}^{4} w_j^i = 1\), \(i = 1, 2\), \(j = 1, 2, 3, 4\). Clearly Bell mixture belongs to the subclass \(S\) of X states. For this class of states Eq.(3) takes the form:

\[
\left( \prod_{i=1}^{2} (w_1^i - w_2^i + w_3^i - w_4^i) \right) + \sqrt{\frac{2}{\prod_{i=1}^{2} (w_1^i - w_2^i + w_3^i - w_4^i)}} \frac{1 + \cos \frac{\pi}{4}}{4} \leq 2.
\]

**When \(\beta_i(i = 1, 2)\) both nonlocal:** If \(\delta_1 \in [0, 1/2]\) then just as for the X state, only the restrictions on \(\beta_1\) modified as:

\[
\frac{\delta_1 - 1}{2} \leq \epsilon_1 < 0 \quad \text{if (all of } A_1, B_1 \text{ and } C_1 < \frac{1}{2}) \text{ or, (only one of } A_1, B_1 \text{ or } C_1 < \frac{1}{2} \text{)}.
\]

If \(\delta_1 < 0\) then the restrictions are same for both the nonlocal copies \(\beta_i(i = 1, 2)\).

It is already pointed in [2] that the assumption of source independence in a network helps in analyzing quantumness in the network. Besides, it also enhances the study of nonlocality of the correlations produced due to joint measurements. The set of sufficient criteria of nonbilocality, as discussed above, characterizes the parameters of X states to some extent so that they can be used in a network to demonstrate quantumness. Such type of characterization may be helpful in developing a better insight regarding the study of nonlocal feature of correlations which in turn can be applied in performing various information processing tasks such as private randomness generation, quantum key distribution, reducing communication complexity, etc. At this point it will now be
interesting to discuss the various advantages of source independence assumption in a network.

Advantage Of Bilocality Assumption In A Network

As already discussed violation of Eq.(12) is a convenient tool to demonstrate nonlocal nature of correlations (which may not be nonlocal in Bell-CHSH sense) in a quantum network. So after analyzing the constraints(imposed on the states emitted by the sources $S_i$) that suffice to produce nonbilocial correlations, we would like to present the region where a local correlation violates inequality (12) thereby revealing its nonbilocial nature. Now a necessary condition for the tripartite correlations to be local [43] is that the corresponding bipartite correlations shared between Alice and Charlie(conditioned on a particular output of Bob) must satisfy CHSH inequality [40]. Assuming that Bob obtains $|\phi^-=\rangle$ for($\epsilon_i = \xi_i (i=1,2)$), the CHSH polynomial is given by:

$$|CHSH| = \sqrt{\prod_{i=1}^{2} (1 - \delta_i)(\cos \frac{\mu}{4} + \sin \frac{\mu}{4})} \text{ or } 0. \tag{18}$$

Clearly with the measurement settings (5), $|CHSH| \leq 2$ always. This in turn points out that restrictions imposed on the parameters of the states(produced by the sources $S_i(i=1,2)$) to generate nonbilocial suffice to produce quantum advantage in the network (see FIG.4).

Resistance to noise: Another way to realise the advantage of bilocality assumption is to quantify the resistance to noise of the nonbilocial correlations. For that we consider an entanglement swapping scenario where each of two independent sources $S_i$ produces a noisy two qubit state:

$$\omega_i = \alpha_i |\psi^-\rangle\langle \psi^-| + (1 - \alpha_i) \frac{1}{4}, \text{ (where } \alpha_i \in [0,1](i=1,2) \tag{19})$$

Here $\alpha_i$ denotes the visibility of $\omega_i (i = 1,2)$ which is a measure of the resistance to noise given by the state.

The smallest visibility( $\alpha_i$) for which $\omega_i$ is local(in standard CHSH scenario) is called the local visibility threshold($V_i$)[12]). The smallest visibility for which the correlation produced in the network is bilocal is called the bilocal visibility threshold($V_{biocl}$). Analogously $V_{loc}$ denotes the smallest visibility for the correlations produced in the network to be local. Clearly quantum advantage is obtained if $V_{biocl} < V$ or $V_{biocl} < V_{loc}$ or both. $V_{biocl} < V_{loc}$ corresponds to generation of local but nonbilocial correlations in the network which has been already discussed above. The visibility of bilocal and local correlations produced in a network using the noisy states[19]are determined by $\prod_{i=1}^{2} \alpha_i[2]$. With the measurement settings(Eq.5) $V_{biocl} = \frac{1}{2}, V_{loc} = 1$ (for $\mu = 0[2]$ whereas for different measurement settings(which gives maximal violation of Bell-CHSH operator[15]), $V = 1\sqrt{2}$. Clearly $V_{biocl} < V_{loc}$ and $V_{biocl} < V$. Now it will be interesting to investigate the constraints on the states(19) produced by the sources that suffice to give quantum advantage:

- If $\omega_1$ is local and $\omega_2$ nonlocal: Nonbilocial correlations are produced for: $\phi_1 - \phi_2 + \sqrt{2}\phi_1\phi_2 < 0$ where $\alpha_1 = \frac{1}{\sqrt{2}} + \phi_1$, $\alpha_2 = \frac{1}{\sqrt{2}} + \phi_2$, $\phi_1$ and $\phi_2$ can thus be interpreted as measure of locality of $\omega_1$ and nonlocality of $\omega_2$ respectively that suffice to generate nonbilocial correlations in a network using $\omega_1$ and $\omega_2$. It is clear from FIG.5 that $\phi_1 \in (0,0.21)$ which in turn implies that the local copy must be entangled.

- If both $\omega_1$ and $\omega_2$ are nonlocal then required criteria is given by: $\phi_1 + \phi_2 + \sqrt{2}\phi_1\phi_2 > 0$ where $\alpha_1 = \frac{1}{\sqrt{2}} + \phi_1$, $\alpha_2 = \frac{1}{\sqrt{2}} + \phi_2$. Nonbilocial versus hidden nonlocality: Recently a subclass of X state has been considered for revealing hidden nonlocality [44].

$$g_i = |\alpha_i| |\psi^=\rangle\langle \psi^=| + (1 - \alpha_i) |0\rangle\langle 0| \frac{1}{2}, 0 \leq \alpha_i \leq 1 \text{ (i = 1,2) \tag{20}}$$

A local model of this class of states (which is entangled for $\alpha_i > 0$) exists for $V = |\alpha_i| \leq \frac{1}{2}$. However this state is local for $V \leq \frac{1}{\sqrt{2}}$ (Horodecki criteria). This class exhibits hidden nonlocality (in Bell-CHSH sense) for $V > 0(44)$. Now let two copies of this subclass are used in a network(characterized by source independence). The correlations generated in the network are nonbilocial for $V_{biocl} = \prod_{i=1}^{2} \alpha_i > \frac{1}{2}$. However the correlations produced in the network are local as $V_{loc} = \prod_{i=1}^{2} \alpha_i \leq 1$. Hence nonlocal character (apart from the usual sense) can be generated without performing sequential measurements for a restricted range of visibilities of the two copies which is the same as that for the class of states(19) (see FIG. 5). Clearly at least one of the two copies must be nonlocal and if $g_2$ be maximum nonlocal then the visibility ($\alpha_1$) of the local copy ($g_1$) must be at least 0.5.
With these restrictions on the two copies used in the network, maximum quantum advantage \((V_{\text{biloc}} < V)\) is obtained for \(\mu = 0\). (Eq.5).

**Conclusion:** In [1] the authors showed that there exists quantum advantage in a network under the assumption of source independence. ‘Quantum advantage’ refers to revelation of nonlocal character of correlations (generated in a network) apart from the standard CHSH sense. Here we have analyzed the restrictions which when imposed on the states generated by the sources in a network (characterized by source independence), suffice to produce quantum advantage. For this, we have provided a set of sufficient criteria for two qubit X states thereby obtaining corresponding region of quantum advantage. From the discussion so far it can be concluded that violation of Eq.\((3)\) acts as a convenient tool to demonstrate quantum advantage in a network using two qubit states. It is clear from the restrictions on the class of Werner states \((19)\) that both the copies of the state used in the network (with the Von-Neumann measurement settings \((5)\)) must be entangled so as to give quantum advantage via the violation of this inequality. But Eq.\((3)\) is only a sufficient condition to get nonbilocal correlations \([2]\). Again in [1] an example of a separable state is given, which when used along with a singlet, generate nonbilocal correlations. So at this point it will be interesting to analyze whether it is possible to generate nonbilocal correlations in a network using only separable states. Besides, one can also modify this inequality or consider any other measurement settings such that violation of this inequality \((3)\) serves as a necessary condition to give quantum advantage. Apart from this it will also be interesting to study relaxation of physical constraints such as measurement independence, determinism in this context.

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It is clear from Eq.(21,22,23) that
\[ a_i - c_i = \sqrt{1 - \delta_i} \cos \theta_i, \quad p_i + q_i = \sqrt{1 - \delta_i} \sin \theta_i. \]

Using these expressions of \( p_i \) and \( q_i \) in Eq.(21) we get:
\[ \cos \theta_i = \pm \sqrt{1 - \delta_i - (1 - \xi_i) \cos^2 \theta_i + \sqrt{1 - \delta_i} \sin \theta_i}, \]
\[ \sin \theta_i = \pm \sqrt{1 - \delta_i - \epsilon_i + \xi_i}. \]

Hence \( a_i - c_i = \sqrt{1 - \delta_i - \epsilon_i} \cos \theta_i \) and \( p_i + q_i = \sqrt{1 - \delta_i - \epsilon_i} \sin \theta_i \).

Using these relations, Eq.(25) gets modified to Eq.(12).

Next we consider the Bell mixture(16). Again from Eq.(8) and Eq.(11, 12, 13) we get:
\[ (1 - 2(w_1^4 + w_2^4))^2 + (1 - 2(w_2^4 + w_3^4))^2 = 1 - \epsilon_i \] (26)
\[ (1 - 2(w_1^4 + w_3^4))^2 + (1 - 2(w_2^4 + w_3^4))^2 = 1 - \delta_i \] (27)
\[ (1 - 2(w_1^4 + w_2^4))^2 + (1 - 2(w_2^4 + w_3^4))^2 = 1 - \xi_i \] (28)

It is clear from the above three equations that Eq.(24) holds here. Then under the same procedure as that for the subclass of X state, Eq.(17) gets modified to 12. Now Eq.(12) can be written as:
\[ \left( \prod_{i=1}^{2} |P_i - Q_i| + \prod_{i=1}^{2} |P_i + Q_i| \right) \sqrt{2(1 + \cos \mu \frac{\Pi^2}{4})} \leq 4 \] (29)
where \( P_i = 1 - \delta_i \) and \( Q_i = \xi_i - \epsilon_i. \) Clearly \( P_i \geq 0. \)

If \( \delta_i \geq 0 \) then \( P_i \in [0,1] \) and if \( \delta_i < 0 \) then \( P_i \in [1,2]. \)

Eq.(24) implies \( P_i > |Q_i|(i = 1, 2). \) Now we consider \( \chi_i (i = 1, 2) \) to be both local. In that case maximum value of \( \left( \prod_{i=1}^{2} |P_i - Q_i| + \prod_{i=1}^{2} |P_i + Q_i| \right) \sqrt{2(1 + \cos \mu \frac{\Pi^2}{4})} \) is 4. Hence the correlations produced in a network using two local copies \( \chi_i \) cannot violate Eq.(12). Again let \( \delta_2 < 0 \) and \( \delta_1 \geq \frac{1}{2}. \) Then the maximum value of \( \left( \prod_{i=1}^{2} |P_i - Q_i| + \prod_{i=1}^{2} |P_i + Q_i| \right) \sqrt{2(1 + \cos \mu \frac{\Pi^2}{4})} \) is 4. Hence in this case also Eq.(12) cannot be violated. So if \( \delta_1 > 0(i = 1, 2) \) or if one of the \( \delta_i (i = 1, 2) \) say \( \delta_2 < 0 \) and the other one \( \delta_1 \geq \frac{1}{2} \) or both then Eq.(12) cannot be violated.

Proof of the sufficient criterion. To prove the sufficient criterion, first we introduce the following variables \( A', B', C', D' \) and \( E' \) such that \( A' = 2\sqrt{2}p_i, B' = 2\sqrt{2}q_i, C' = 2(p_i + q_i), D' = 2(p_i - q_i), E' = (a_i - b_i - c_i + d_i) \) \((i = 1, 2). \) Then Eq.(10,11,12) can be written in the form
\[ (A')^2 + (B')^2 = 1 - \epsilon_i \] (30)
\[ (E')^2 + (C')^2 = 1 - \delta_i \] (31)
\[(E^i)^2 + (D^i)^2 = 1 - \zeta_i, \quad (32)\]

Now the variables defined above and Eq.(11,12,13) immediately implies \((C^i)^2 = 1 + \frac{\zeta_i - c_i - a_i}{2}, \quad (D^i)^2 = 1 - \frac{\zeta_i - c_i + a_i}{2}\)
and \((E^i)^2 = 1 - \zeta_i + c_i - a_i\). State conditions impose the constraints \(p_i^2 \leq a_i - b_i, \quad q_i^2 \leq b_i - c_i\). Since \(a_i = d_i, \quad b_i = c_i\) (by our assumption), \(2(a_i + c_i) = 1\) and \(E^i = 2(a_i - c_i)\), the above constraints reduces to
\[
-\left(1 + \frac{E^i}{\sqrt{2}}\right) \leq A^i \leq \frac{1 + E^i}{\sqrt{2}} \quad (33)
\]
and
\[
-\left(1 - \frac{E^i}{\sqrt{2}}\right) \leq B^i \leq \frac{1 - E^i}{\sqrt{2}} \quad (34)
\]
Again \(A^i = \frac{C^i + D^i}{\sqrt{2}}\) and \(B^i = \frac{C^i - D^i}{\sqrt{2}}\), so the above equations get modified as
\[
-\left(1 + \frac{E^i}{\sqrt{2}}\right) \leq \frac{C^i + D^i}{\sqrt{2}} \leq \frac{1 + E^i}{\sqrt{2}} \quad (35)
\]
and
\[
-\left(1 - \frac{E^i}{\sqrt{2}}\right) \leq \frac{C^i - D^i}{\sqrt{2}} \leq \frac{1 - E^i}{\sqrt{2}} \quad (36)
\]
There are 16 possible different cases corresponding to the different sign of \(A^i, B^i, C^i, D^i\) and \(E^i\) which ultimately give rise to the following four pairs of inequalities:

- **1a.** \[0 \leq \frac{\sqrt{1 + \zeta_i - c_i - a_i} + \sqrt{1 - \zeta_i - c_i + a_i}}{2} \leq \frac{\sqrt{1 + \zeta_i - c_i - a_i} - \sqrt{1 - \zeta_i - c_i - a_i}}{2} \leq 1\]

  when \((p_i, q_i, p_i - q_i)\) are of same sign and \(a_i - c_i > 0\) or \((p_i, q_i, p_i - q_i)\) are of opposite sign and \(a_i - c_i < 0\).

- **1b.** \[0 \leq \frac{\sqrt{1 + \zeta_i - c_i - a_i} - \sqrt{1 - \zeta_i - c_i + a_i}}{2} \leq \frac{\sqrt{1 + \zeta_i - c_i - a_i} + \sqrt{1 - \zeta_i - c_i - a_i}}{2} \leq 1\]

  when \((p_i, q_i, p_i - q_i)\) and \((p_i, q_i, p_i - q_i)\) are of opposite sign and \(a_i - c_i < 0\) or \((p_i, q_i, p_i - q_i)\) are of same sign and \(a_i - c_i > 0\).

- **2a.** \[0 \leq \frac{\sqrt{1 + \zeta_i - c_i - a_i} - \sqrt{1 - \zeta_i - c_i - a_i}}{2} \leq \frac{\sqrt{1 + \zeta_i - c_i + a_i} + \sqrt{1 - \zeta_i - c_i + a_i}}{2} \leq \frac{5(\delta_1 - 1) + 4\sqrt{2(1 - \delta_1)}}{2} \quad (41)\]

  Hence, for \(\delta_1 \leq \frac{1}{2}, \quad 37\) and Eq.(41) implies

  \[\epsilon_i \leq \min\{\frac{5(\delta_1 - 1) + 4\sqrt{2(1 - \delta_1)}}{2}, \quad \frac{1 + \delta_1 - 2\epsilon_1}{2}\} \quad (42)\]

  But \(\frac{5(\delta_1 - 1) + 4\sqrt{2(1 - \delta_1)}}{2} \leq \frac{1 + \delta_1 - 2\epsilon_1}{2}\). Hence for a local copy \(0 \leq \delta_1 \leq \epsilon_i \leq \frac{5(\delta_1 - 1) + 4\sqrt{2(1 - \delta_1)}}{2}\) when \(\delta_1 \leq \frac{1}{2}\). Following the same procedure as for 1a and 1b, for the last pair (i.e 4a and 4b) we have \(0 \leq \epsilon_i \leq \delta_1\). Now we consider the pair (2a, 2b); similarly as in the previous cases, from the second inequality of (2b) we get

  \[
  \sqrt{1 + \delta_1 - 2\epsilon_1} \leq 1 - \sqrt{2(1 - \delta_1)}. \quad (43)
  \]

  The above inequality is satisfied only when \(\delta_1 \geq \frac{1}{2}\). Again from Eq.(14) it is clear that the inequality holds...
only when $\delta_1 < \frac{1}{2}$. Hence nonbilocial correlation cannot be obtained if one uses a local state whose parameters are restricted by this pair of inequalities (2a and 2b) as in a network with the measurement settings (Eq. (5)) and (non)bilocial inequality considered here. Same analysis holds good for the pair (3a) and (3b).

**Different cases of the nonlocal copy ($\chi_2$):** Here, $\delta_2$ and/or $\epsilon \in [-1, 0]$. But as already discussed before, when $\chi_1$ is local ($\delta_1 > 0$) then if $\delta_2 > 0$, Eq. (12) cannot be violated. Hence $\delta_2 < 0$ is a sufficient condition for violation of Eq. (12). For (1a,1b) case, first inequality of (1a) imposes obvious restriction. Again positivity of $(1 + \delta_2 - 2\epsilon)$ and second inequality of (1a) i.e.,

$$\frac{\| i \|}{\| i \|^2} \leq \frac{\| i \|^2}{\| i \|^2}$$

together give:

$$\frac{\| i \|^2}{\| i \|^2} \leq \frac{\| i \|^2}{\| i \|^2} < \frac{\| i \|^2}{\| i \|^2}.$$  

(44)

Also from first and second inequality of (2b) implies

$$\frac{\| i \|^2}{\| i \|^2} \leq \frac{\| i \|^2}{\| i \|^2} < \frac{\| i \|^2}{\| i \|^2}.$$  

(45)

It is easy to check that the restriction imposed on $\epsilon_2$ in Eq. (45) is more stronger than in Eq. (44). Hence, Eq. (44) and Eq. (45) together implies $\delta_2 \leq \epsilon_2 \leq \frac{\| i \|^2}{\| i \|^2}$ for $\delta_2 < 0$. Following the same procedure as in the previous case, for the last case (4a, 4b) we have

$$\frac{\| i \|^2}{\| i \|^2} \leq \frac{\| i \|^2}{\| i \|^2} < \frac{\| i \|^2}{\| i \|^2}.$$  

(46)

and

$$\frac{\| i \|^2}{\| i \|^2} \leq \frac{\| i \|^2}{\| i \|^2} < \frac{\| i \|^2}{\| i \|^2}.$$  

(47)

respectively. But the last Eq. (47) holds only when $\delta_2 > \frac{1}{2}$ and since $\delta_2 \leq \epsilon_2$, So Eq. (46) and Eq. (47) simultaneously hold only when $\frac{1}{2} < \delta_2 < \epsilon_2$. But this is not possible for a nonlocal copy.

Similarly as in local copy nonbilocial correlation cannot be obtained when one uses (2a, 2b) and (3a, 3b) as a nonlocal copy. Hence in order to get nonbilocial correlation (1a, 1b) and (4a, 4b) are the only valid pairs of inequalities to impose restrictions on both local and nonlocal copies ($\chi_1$ and $\chi_2$ respectively) which along with restriction given by Eq. (14) suffice to violate Eq. (12). Next we consider both $\chi_i (i = 1, 2)$ to be nonlocal. Firstly we put another restriction on $\chi_1$ by considering $\delta_1 > 0$. Under these restrictions (4a, 4b) is the only valid pair of inequality which imposes the restriction: $\frac{\| i \|^2}{\| i \|^2} \leq \epsilon_1 < 0$. But if $\delta_1 < 0$, then we get the same restrictions as that for $\delta_2$ and $\epsilon_2$.

Now let us consider Bell mixture. In this case we introduce the following variables ($A^i \chi_1$, $B^i \chi_2$ and $C^i \chi_2$), such that ($A^i \chi_1^2 = (1 - 2w_1 - 2w_1^2)^2$, $B^i \chi_2^2 = (1 - 2w_2 - 2w_2^2)^2$ and $C^i \chi_2^2 = (1 - 2w_1 - 2w_1^2)^2$ ($i = 1, 2$). Then Eq. (10,11,12) can be written in the form

$$\frac{\| i \|^2}{\| i \|^2}^2 + \frac{\| i \|^2}{\| i \|^2}^2 = 1 - \epsilon_1 \quad (48)$$

$$\frac{\| i \|^2}{\| i \|^2}^2 + \frac{\| i \|^2}{\| i \|^2}^2 = 1 - \delta_i \quad (49)$$

$$\frac{\| i \|^2}{\| i \|^2}^2 + \frac{\| i \|^2}{\| i \|^2}^2 = 1 - \xi_i \quad (50)$$

Using our assumption $\xi_i = \epsilon_1$, from the above three equation one has ($A^i \chi_1^2 = \frac{1 - \delta_1}{2}$, $B^i \chi_2^2 = \frac{1 - \delta_1}{2}$ and $C^i \chi_2^2 = \frac{1 + \delta_1 - 2\epsilon_1}{2}$. There are 8 possible different cases corresponding to different sign of $A^i \chi_1$, $B^i \chi_2$ and $C^i \chi_2$.

- **1.** $w_1^1 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_2^1 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_3^1 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}$ when $A_i < \frac{1}{2}, B_i < \frac{1}{2}$ and $C_i > \frac{1}{2}$.
- **2.** $w_1^2 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_2^2 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_3^2 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}$ when $A_i > \frac{1}{2}, B_i < \frac{1}{2}$ and $C_i < \frac{1}{2}$.
- **3.** $w_1^3 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_2^3 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_3^3 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}$ when $A_i < \frac{1}{2}, B_i > \frac{1}{2}$ and $C_i < \frac{1}{2}$.
- **4.** $w_1^4 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_2^4 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_3^4 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}$ when $A_i > \frac{1}{2}, B_i > \frac{1}{2}$ and $C_i > \frac{1}{2}$.
- **5.** $w_1^5 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_2^5 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_3^5 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}$ when $A_i > \frac{1}{2}, B_i < \frac{1}{2}$ and $C_i < \frac{1}{2}$.
- **6.** $w_1^6 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_2^6 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_3^6 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}$ when $A_i > \frac{1}{2}, B_i > \frac{1}{2}$ and $C_i > \frac{1}{2}$.
- **7.** $w_1^7 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_2^7 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_3^7 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}$ when $A_i > \frac{1}{2}, B_i < \frac{1}{2}$ and $C_i > \frac{1}{2}$.
- **8.** $w_1^8 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_2^8 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}, \quad w_3^8 = \frac{i + \sqrt{1 - 2s_1 + s_1^2}}{4}$ when $A_i < \frac{1}{2}, B_i > \frac{1}{2}$ and $C_i < \frac{1}{2}$. 


Now, throughout the discussion below, we consider $\beta_2$ as the nonlocal copy. Now for the other copy $\beta_1$, first we consider it to be local. 

**Different cases of the local copy ($\beta_1$)** under the assumption of $\xi_1 = \epsilon_1$. Clearly, in case (1), $0 \leq w_1^i \leq 1$, $0 \leq w_2^i \leq 1$ and $w_3^i \leq 1$. Now the positivity of $w_3^i$ implies

$$\sqrt{1 - 2 \epsilon_i + \delta_i} \leq 1 - \sqrt{2}(1 - \delta_i). \quad (51)$$

The above relation holds only when $\delta_i \geq \frac{1}{2}$. But the bilocal inequality given in Eq.(14) cannot be violated for any $\delta_i \geq \frac{1}{2}$ (FIG.3). Hence case 1 fails to violate the bilocal inequality. Following the same procedure it can be shown that cases 2, 3 and 4 also fail to violate the bilocal inequality. Now we consider case 5, clearly $w_1^i \leq 1$, $w_2^i \leq 1$ and $w_3^i \leq 1$. Now the positivity of $w_1^i$ (and/or, $w_2^i$) and $w_3^i$ implies

$$\epsilon_i \geq \frac{\delta_i - 1}{2} \quad (52)$$

and

$$\sqrt{1 - 2 \epsilon_i + \delta_i} \geq \sqrt{2}(1 - \delta_i) - 1 \quad (53)$$

respectively. If $\delta_i < \frac{1}{2}$, then both side of 53 is positive. Now by squaring both side of the inequality we have

$$\epsilon_i \leq \frac{4 \sqrt{2(1 - \delta_i)} + 5(\delta_i - 1)}{2}. \quad (54)$$

The constraints $0 \leq w_1^i + w_2^i + w_3^i \leq 1$ is obvious in this case. Already we have used the constraints $w_1^i + w_2^i + w_3^i = 1$ to derive Eq. (51). Hence violation of bilocal inequality is possible in this case when

$$\frac{\delta_i - 1}{2} \leq \epsilon_i \leq \frac{4 \sqrt{2(1 - \delta_i)} + 5(\delta_i - 1)}{2} \quad (55)$$

and

$$\prod_{i=1}^{2}(1 - \delta_i) \leq \frac{4}{(1 + \cos \mu)} \quad (56)$$

Since $\delta_i$ and $\epsilon_i \in [0, 1]$ for a local copy and $\frac{\delta_i - 1}{2} \leq 0$ for any value of $\delta_i \in [-1, 1]$, hence for a local copy the range of $\epsilon_1$ becomes

$$0 \leq \epsilon_1 \leq \frac{4 \sqrt{2(1 - \delta_1)} + 5(\delta_1 - 1)}{2} \quad (57)$$

But for a nonlocal copy $\epsilon_2$ can be negative, hence the range of $\epsilon_2$ remain same as in Eq.(55). Similarly it can be shown that correlations produced in a 'bilocal network' exhibit nonlocal character (apart from Bell-CHSH) in case (6, 7, 8) when the parameters satisfy the relation Eq.(57) for local copy, Eq.(55) for a nonlocal copy and the inequality given in Eq.(56). Next we consider both $\beta_i (i = 1, 2)$ to be nonlocal. Firstly we put further restriction on $\beta_1$ by considering $\delta_1 > 0$. Under this restriction it is easily seen that cases (5),(6), (7) and (8) are the only valid ones to violate Eq.(14) Following similar procedure as above it can be shown that the bound on $\epsilon_1$ gets modified: $\frac{\delta_i - 1}{2} \leq \epsilon_1 < 0$. If there is no such restriction on $\delta_1$ of the nonlocal copy $\beta_1$ then we get the same restriction on $\epsilon_1$ and $\delta_1$ as that on $\epsilon_2$ and $\delta_2$. 