Theoretical investigation of nonlinear damping and nonlinear phase shift of spin-electromagnetic waves propagating in infinite multiferroics at sub-terahertz frequencies

I A Ustinova, M A Cherkasskii, A B Ustinov and B A Kalinikos

Department of Physical Electronics and Technology, St Petersburg Electrotechnical University, 5 Popov Street, St.Petersburg 197376, Russia

E-mail: ustinovairin@yahoo.com

Abstract. The nonlinear phase shift and nonlinear damping of spin-electromagnetic waves were theoretically studied for the first time in sub-terahertz frequency range in infinite homogeneous longitudinal magnetized multiferroics. The research was based on the solution of the Ginzburg-Landau equation. It is shown that the saturation of the phase shift occurs due to the nonlinear damping if the nonlinear damping coefficients exceed $\psi_1=10^8 s^{-1}$ and $\psi_2=10^9 s^{-1}$.

Recently an interest to study new microwave structured and composite magnetic materials has been increased. One class of such materials is multiferroics which combine both electric and magnetic wave nonlinearities [1-6]. In particular, the envelope solitons of spin-electromagnetic waves (SEWs) in infinite multiferroic medium was studied theoretically [5]. A nonlinear microwave phase shifter based on a planar multiferroic composite was fabricated and investigated experimentally in [6]. A survey of the literature shows that the nonlinear properties of the SEWs are studied not enough. Some examples of the issues which are addressed by the researchers are nonlinear damping and nonlinear phase shift of the high-power spin waves propagating in thin magnetic films [7] as well as their application for development of microwave devices such as nonlinear phase shifters [8, 9], a nonlinear directional coupler [7], and a nonlinear interferometer [10].

The aim of the present work is a theoretical investigation of nonlinear damping and nonlinear phase shift of the high power SEWs propagating in longitudinally magnetized infinite multiferroic medium. The study was carried out in two stages. In the first stage, the nonlinear dispersion characteristics were numerically simulated and analyzed with the use of the nonlinear dispersion equation derived in the work [5]. In the second stage, the SEWs propagation was simulated numerically in a stable nonlinear regime. The stable nonlinear regime is due to nonlinear four-wave interaction processes for which enrichment of spectrum of microwave signal carried by SEWs does not take place. Nonlinear damping and nonlinear phase shift of SEWs belong to the four-wave processes. Therefore, numerical simulation of these processes was based on the nonlinear evolutional Ginzburg-Landau (GL) equation [8].

A dispersion law for the case of linear propagation of SEWs in longitudinally magnetized medium is well known (see, e.g., [11]). A nonlinear dispersion law was derived in [5]. The electric and magnetic wave nonlinearities were taken into account in the linear dispersion law. The relationship between the electric field vector and the variable magnetization vector was derived from Maxwell's equations. It has the following form: $\vec{e} = \omega \cdot \vec{\mu} \cdot \vec{k} \times \vec{m} \cdot (k_0^2 - k^2)^{-1}$. Substituting this expression into the
linear dispersion law and introducing the normalized variable magnetization amplitude as $|U| = |\hat{m}| \cdot (2^{1/2} \cdot M_0)^{-1}$ we get the nonlinear dispersion law of SEWs:

$$k^2 = \frac{\varepsilon_0^2 \cdot \varepsilon_L \cdot \mu_0 \cdot \omega^2 \cdot \left( \omega \pm \omega_H \pm \omega_M \cdot \left( 1 - |U|^2 \right) \right)}{\varepsilon_0 \cdot \varepsilon_L \cdot \left( \omega \pm \omega_H \right) - 2 \cdot |U|^2 \cdot \mu_0 \cdot N_e \cdot \left( \omega \pm \omega_H \pm \omega_M \cdot \left( 1 - |U|^2 \right) \right) \cdot M_0^2},$$

(1)

where $k$ is a wave number, $\omega$ is a frequency of SEWs, $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m is permittivity of vacuum, $\varepsilon_L = 19$ is relative permittivity, $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is permeability of vacuum, $\omega_H = |g| \mu_0 H$, $\omega_M = |g| \mu_0 M$, $|g|$ is module of the gyromagnetic ratio for the electron spin, $H$ is a value of the static magnetic field, $M_0$ is static magnetization in the absence of waves, $N_e = -10^{-10}$ is a coefficient of dielectric nonlinearity. In the expression (1) the upper signs correspond to waves with left circular polarization, and the bottom signs correspond to waves with right circular polarization. Hereinafter we will be interested only in waves with right circular polarization.

The expression (1) is correct only for an ideal medium in which there are no losses. In fact, the propagation of waves is inevitably accompanied by their damping. The losses were taken into account as in [8]:

$$\alpha = \frac{\omega}{c} \left( \frac{\varepsilon_L}{2} \right)^{\gamma/2} \left[ \left( \mu'_s^2 + \mu''_s^2 \right)^{\gamma/2} - \mu'_s \right],$$

(2)

where $\alpha$ is a damping constant, $\mu'_s$ and $\mu''_s$ are the real and imaginary parts of the effective scalar magnetic permeability for the circularly polarized wave, $c$ is light’s speed in vacuum.

The calculation was carried out for the homogeneous multiferroic medium having parameters corresponding to Al-substituted barium hexaferrite BaAl$_2$Fe$_{10}$O$_{19}$ [12]: saturation magnetization $M_0 = 147$ kA/m, the uniaxial magnetocrystalline anisotropy field $H_A = 2677$ kA/m.

Figure 1. The spectrum and the losses of SEWs. Solid curves correspond to the spectrum of linear SEWs. Dashed curves correspond to the spectrum of nonlinear SEWs for $|U|^2 = 0.4$. Dash-dotted curve shows SEW dissipation.
The linear and nonlinear dispersion laws and the dependence of losses for waves in the hexaferrite are shown in figure 1. The “operating point” for further calculations was chosen to be in the region of strong dispersion. It has the following parameters: \( k_0 = 7550 \text{ rad/m}, f_0 = 7.6 \times 10^{10} \text{ Hz} \). The losses were \( \alpha = 0.957 \) at this point. Note, that for the hexaferrite one can find a point of intersection of linear and nonlinear dispersion laws at \( k = 10025 \text{ rad/m} \) and \( f = 1.2 \times 10^{11} \text{ Hz} \). This corresponds to the change in the type of nonlinearity. Spectrum exists in the sub-terahertz frequency range.

Further, the theoretical investigation of nonlinear properties of the SEWs in a stable regime of propagation was carried out. It was based on the dispersion law with the use of the method of envelopes [13]. The nonlinear GL equation was obtained under this method [7]:

\[
\frac{dU}{dt} + V_g \left( \frac{dU}{dz} \right) - i \left( \frac{D}{2} \right) \left( \frac{d^2U}{dz^2} \right) + \left( \nu_1 + i \cdot N \right) |U|^2 \cdot U + \nu_2 |U|^4 \cdot U = -\eta \cdot U \tag{3}
\]

It has the following coefficients: \( V_g = \partial \omega / \partial k \) is group velocity, \( N = \partial \omega / \partial |U|^2 \) is nonlinear self-interaction coefficient, \( D = d^2 \omega / dk^2 \) is dispersion coefficient, \( \eta \) is linear damping coefficient (which equals \( \alpha \)), \( \nu_1 \) and \( \nu_2 \) are cubic and quintic damping coefficients. Note that the nonlinear coefficient is a function of both magnetic and electric field components of the SEWs in the model.

We assume that the coefficient D equals zero for the continuous wave signal, as well as for the pulsed signal when the microwave pulses have a relatively long duration (more than 1 \( \mu \)s). In this case the time-independent analytic solution of GL equation (3) for the wave envelope amplitude \( U(z) \) is described by the transcendental equations below. Introducing the notation \( G = 4\eta \nu_2 - \nu_1^2 \), these equations can be written in the following form:

\[
\frac{z\eta}{V_g} = -\ln \left[ \frac{U(z)}{U_0} \right] + \frac{1}{4} \ln \left[ \frac{\nu_2 U^2(z) + \nu_1 U^2(z) + \eta}{\nu_2 U^4_0 + \nu_1 U^4_0 + \eta} \right] + \frac{\nu_1}{2G^{1/2}} \left[ \tan^{-1} \left( \frac{2\nu_2 U^2(z) + \nu_1}{G^{1/2}} \right) - \tan^{-1} \left( \frac{2\nu_2 U^2_0 + \nu_1}{G^{1/2}} \right) \right] \tag{4}
\]

for \( 4\eta \nu_2 > \nu_1^2 \),

\[
\frac{z\eta}{V_g} = -\ln \left[ \frac{U(z)}{U_0} \right] + \frac{1}{4} \ln \left[ \frac{\nu_2 U^2(z) + \nu_1 U^2(z) + \eta}{\nu_2 U^4_0 + \nu_1 U^4_0 + \eta} \right] - \frac{\nu_1}{4(-G)^{1/2}} \ln \left[ \frac{2\nu_2 U^2(z) + \nu_1 + (-G)^{1/2} \left( 2\nu_2 U^2_0 + \nu_1 + (-G)^{1/2} \right) \right] = \frac{1}{4(-G)^{1/2}} \ln \left[ \frac{2\nu_2 U^2(z) + \nu_1 - (-G)^{1/2} \left( 2\nu_2 U^2_0 + \nu_1 + (-G)^{1/2} \right) \right] \tag{5}
\]

for \( 4\eta \nu_2 < \nu_1^2 \), and

\[
\frac{z\eta}{V_g} = \frac{1}{2} \ln \left[ \frac{U^2_0 (\nu_2 U^2(z) + 2\eta)}{U^2(z) (\nu_2 U^2_0 + 2\eta)} \right] - \frac{\eta}{\nu_1 U^2(z) + 2\eta} + \frac{\eta}{\nu_1 U^2_0 + 2\eta} \tag{6}
\]

for \( 4\eta \nu_2 = \nu_1^2 \). In this equations \( U_0 \) is the amplitude of SEWs at \( z=0 \). The solution of the GL equation for phase shift has the following form:
\[
\phi_{NL}(U) = -\left( N \, \frac{1}{V_g} \right) \left[ z(U) \cdot U^2 + \int_{U}^{U_f} z(U') \cdot 2U' \, dU' \right].
\]

The function \(z(U)\) is defined by relations (4)-(6).

Figure 2. Group velocity (a) and nonlinear coefficient (b) as a function of frequency

Figure 3. Squared amplitude (a) and nonlinear phase shift (b) of SEW as a function of propagation path. Squared amplitude (c) and nonlinear phase shift (d) at \(z=0.03\) as a function of squared amplitude at \(z=0\).
The results of the numerical simulation of group velocity and nonlinear coefficient as a function of frequency are shown in figure 2. It is seen that \( V_g = 4.6 \times 10^7 \text{ m/c} \) and \( N = 9.3 \times 10^{10} \text{ s}^{-1} \) correspond to frequency \( f_0 = 76 \text{ GHz} \).

Figure 3 shows the typical dependences of \( U^2(z), \phi_{NL}(z), U^2(U_{\phi 0}), \phi_{NL}(U_{\phi 0}) \) calculated by equations (4)-(7). Solid lines are drawn for nonlinear coefficients \( v_1=0 \text{ s}^{-1}, v_2=0 \text{ s}^{-1} \), dashed lines are drawn for \( v_1=6 \times 10^9 \text{ s}^{-1}, v_2=0 \text{ s}^{-1} \), dash-dotted lines are drawn for \( v_1=6 \times 10^9 \text{ s}^{-1}, v_2=9 \times 10^9 \text{ s}^{-1} \). Characteristics of \( U^2(U_{\phi 0}) \) and \( \phi_{NL}(U_{\phi 0}) \) were calculated for \( z=0.03 \text{ m} \). As is seen from the figure 3 (b) the nonlinear phase shift as a function of distance demonstrates the saturation even for the case of absence of the nonlinear damping. Occurrence of the nonlinear damping (see figure 3 (a)) reduces the saturation level. Figure 3 (c) shows saturation of wave amplitude with an increase in excitation amplitude \( U_0 \) in the case of nonlinear damping. Similar saturation takes place for nonlinear phase shift shown in Figure 3 (d). Detailed analysis of the theoretical data obtained for various values of \( v_1 \) and \( v_2 \) shows that the saturation of the phase shift occurs due to the nonlinear damping if the nonlinear damping coefficients exceed \( v_1=10^8 \text{ s}^{-1} \) and \( v_2=10^9 \text{ s}^{-1} \).

In conclusion, the results of our investigation show that the presence of nonlinear damping leads to a strong decrease in amplitude and to saturation in the nonlinear phase shift of SEWs. The features of the nonlinear damping of the SEWs obtained in this work could be used for creating of new sub-terahertz multiferroic devices. Thus, the examined materials are perspective for using in devices such as a phase shifter and a power limiter.

Acknowledgement
The work was supported in part by the Russian Foundation for Basic Research, grant of President of Russian Federation, and by Ministry of Education and Science of Russian Federation (Project "Goszadanie").

References
[1] Vaz C A F 2012 J.Phys.: Condens. Matter. 24 333201
[2] Grishin S V, Beginin E N, Morozova M A, Sharaevskii Yu P and Nikitov S A 2014 J. Appl. Phys. 115 053908
[3] Burdin D A, Chashin D V, Ekonomov N A, Fetisov L Y, Fetisov Y K, Sreenivasulub G and Srinivasan G 2014 Journal of Magnetism and Magnetic Materials. 358-359 98
[4] Sadovnikov A V, Bublikov K V, Beginin E N and Nikitov N A 2014 Journal of Communications Technology and Electronics. 59 914
[5] Cherkasskii M A and Kalinikos B A 2013 Tech. Phys. Lett. 39 182
[6] Ustinov A B, Kalinikos B A and Srinivasan G 2014 Appl. Phys. Lett. 104 052911
[7] Ustinov A B and Kalinikos B A 2006 Appl. Phys. Lett. 89 172511
[8] Ustinov A B and Kalinikos B A 2008 Appl. Phys. Lett. 93 102504
[9] Kuanr B K, Anderson N R, Celinski Z J and Camley R E 2015 IEEE Magnetics Letters 6 Article № 3500304
[10] Ustinov A B and Kalinikos B A 2007 Appl. Phys. Lett. 90 252510
[11] Lax B and Button K J 1962 Microwave ferrites and ferrimagnetics (New York: McGraw-Hill)
[12] Ustinov A B, Tatarenko A S, Srinivasan G and Balbashov A M 2009 J. Appl. Phys. 105 023908
[13] Karpman V I 1975 Nonlinear waves in dispersive media (London: Pergamon Press)