DEFORMATIONS, SYMMETRIES AND TOPOLOGICAL DEGREES OF FREEDOM OF THE STRING*

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Abstract

We discuss three closely related questions; i) Given a conformal field theory, how may we deform it? ii) What are the symmetries of string theory? and iii) Does string theory have free parameters? We show that there is a distinct deformation of the stress tensor for every solution to the linearised covariant equations of motion for the massless modes of the Bosonic string, and use this result to discuss the symmetries of the string. We also find an additional finite dimensional space of deformations which may correspond to free parameters of string theory, or alternatively may be interpreted as topological degrees of freedom, perhaps analogous to the isolated states found in two dimensions.

* Talk presented at the XX Meeting on Differential Geometric Methods in Theoretical Physics, June 3–7, 1991, at Baruch College, New York City.
† Work supported in part by the Department of Energy Contract Number DOE-AC02-87ER40325, TASK B
1. Introduction

Conformal field theories (with appropriate central charge) are solutions to the classical equations of motion of string theory [1], so that by studying infinitesimal deformations which preserve this conformal structure, we are examining the linearised classical equation of motion about the corresponding solution. This is an interesting problem in its own right, but it also gives us insight into the symmetry structure of string theory. We normally think of finding symmetries by looking for transformations on the fields which leave invariant the action of a theory. This requires that we solve the daunting problems of string field theory before we can address the problem of symmetry. However, there is an alternative approach; we may simply work with the equations of motion of the theory and find transformations which take one solution into another without changing the physics. A symmetry is therefore a particular case of a deformation.

Finally, we may shed some light on the question of whether string theory has free parameters. For a given conformal field theory, the interactions of all the physical states appear to be uniquely prescribed, if we want both Lorentz invariance and unitarity in space-time. Different conformal field theories are supposed to correspond simply to different background solutions of the equations of motion. This supposition has a testable consequence, that every deformation of a conformal field theory correspond to a deformation of the background value of some physical field of the string. We shall find that there are in fact deformations of the usual critical bosonic string which do not correspond to physical fields, and so may be interpreted as free parameters, or, equivalently, as isolated states or topological degrees of freedom. As such they are perhaps higher dimensional analogues of the isolated states of two-dimensional string theory which have been the focus of so much discussion at this conference [2].

2. Conformal Field Theory.

There are several equivalent definitions of a conformal field theory, of varying degrees of sophistication, but for our purposes we shall take the simplest which has the added advantage of making the algebraic properties manifest. In the parlance of physicists we shall consider a Hamiltonian formulation; that is we shall take our world-sheet to be a cylinder (twice-punctured sphere if you prefer) with an arbitrarily chosen cycle around it, parameterised by a single real coordinate $\sigma$, running from 0 to $2\pi$. A Conformal Field Theory consists of the following, defined on this cycle:

1) an algebra $\mathcal{A}$ of operator valued distributions, usually called fields
2) a representation of this algebra, and
3) two distinguished fields, $T(\sigma)$ and $\overline{T}(\sigma)$ which satisfy two mutually commuting copies of the Virasoro algebra:
\[
[T(\sigma), \sigma'(\sigma')] = \frac{-ic}{24\pi} \delta'''(\sigma - \sigma') + 2i T(\sigma') \delta'(\sigma - \sigma') - iT'(\sigma') \delta(\sigma - \sigma') \quad (1a)
\]
\[
[\overline{T}(\sigma), \sigma'(\sigma')] = \frac{ic}{24\pi} \delta'''(\sigma - \sigma') - 2i \overline{T}(\sigma') \delta'(\sigma - \sigma') + i \overline{T}'(\sigma') \delta(\sigma - \sigma') \quad (1b)
\]
\[
[T(\sigma), \overline{T}(\sigma')] = 0 \quad (1c)
\]
and which generate motion around the cycle:
\[
[L_0 - \overline{L}_0, \phi(\sigma')] = \int d\sigma [T(\sigma) - \overline{T}(\sigma), \phi(\sigma')] = -i\phi'(\sigma') \quad \forall \phi(\sigma') \in A \quad (2)
\]
(A prime may denote differentiation with respect to \( \sigma \)). These special fields are the two non-vanishing components of the energy momentum tensor of the field theory, and so include the Hamiltonian, \( H \),
\[
H = L_0 + \overline{L}_0 = \int d\sigma (T(\sigma) + \overline{T}(\sigma)) \quad (3)
\]
which may be used to define the evolution of fields off the cycle, although we shall not use this fact.

Since \( Vir \times Vir \subset A \), and any subalgebra acts on its parent through commutation (the adjoint action), the elements of \( A \) will themselves be grouped into representations of \( Vir \times Vir \). It is therefore natural to define \textbf{Primary Fields of dimension} \((d, \overline{d})\), which transform simply under the adjoint action, by
\[
[T(\sigma), \Phi(d, \overline{d})(\sigma')] = id\Phi(d, \overline{d})(\sigma') \delta'(\sigma - \sigma') - (i/\sqrt{2}) \partial \Phi(d, \overline{d})(\sigma') \delta(\sigma - \sigma')
\]
\[
[\overline{T}(\sigma), \Phi(d, \overline{d})(\sigma')] = -i\overline{d}\Phi(d, \overline{d})(\sigma') \delta'(\sigma - \sigma') - (i/\sqrt{2}) \overline{\partial} \Phi(d, \overline{d})(\sigma') \delta(\sigma - \sigma') \quad (4)
\]
The symbols \( \partial \) and \( \overline{\partial} \) indicate differentiation with respect to the light-cone coordinates \( x^\pm = (\sigma \pm \tau)/\sqrt{2} \), and so take us off the space-like cycle. From our algebraic point of view, we should think of the symbol \( \partial \phi(\sigma') \) as meaning \( i\sqrt{2}[L_0, \phi(\sigma')] \) for any field \( \phi(\sigma') \in A \), with a similar meaning for \( \overline{\partial} \). The definition of primary field, Eq. 4, is thus an empty tautology for the zero modes of the energy momentum tensor, but is non-trivial for the others.

To conclude this section, we will try to make clearer what is meant by the belief that conformal field theories are solutions of the classical equations of motion of the string. Consider a string moving in some space-time with metric \( G \), then a natural choice for the two-dimensional field theory to describe this situation is
\[
T(\sigma) = \frac{1}{2} G^{\mu\nu}(X) \partial X_\mu \partial X_\nu(\sigma)
\]
\[
\overline{T}(\sigma) = \frac{1}{2} G^{\mu\nu}(X) \overline{\partial} X_\mu \overline{\partial} X_\nu(\sigma)
\]
where the \( X \) are scalar fields which can be thought of as coordinates for the string in space-time, and
\[
\partial X_\mu(\sigma) = \frac{1}{\sqrt{2}} (\pi_\mu(\sigma) + G_{\mu\nu}(X) X^{\nu'}(\sigma)) \quad \overline{\partial} X_\mu(\sigma) = \frac{1}{\sqrt{2}} (\pi_\mu(\sigma) - G_{\mu\nu}(X) X^{\nu'}(\sigma))
\]
\[ (6) \]
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and \( \pi(\sigma) \) is the momentum conjugate to \( X \); the only non-vanishing bracket among the \( X \) and \( \pi \) is

\[
[\pi_\mu(\sigma), X^\nu(\sigma')] = -i \delta_\mu^\nu \delta(\sigma - \sigma')
\]

(This definition of \( \partial X \) is consistent with the one given above). If our bracket is the Poisson bracket, then the \( T \) and \( \overline{T} \) defined in Eq. 5 satisfy \( Vir \times Vir \) for all choices of \( G \). However, if, as we want, they are defined as normal ordered (with respect to the Fourier modes of \( X \) and \( \pi \)) products of fields, and the bracket is a commutator, then they satisfy \( Vir \times Vir \) only when \( G \) satisfies certain conditions which look something like the Einstein equations of motion. Since the spectrum of the bosonic string includes a state which has all the properties of a disturbance of the space-time metric (a, “graviton”), this condition of conformal invariance is naturally interpreted as an equation of motion for this physical field. It is one of the goals of the work described in this talk to clarify this relationship between equations of motion for space-time fields and conformal field theories, and so to generalise it to include the full, infinite set of space-time fields.

3. Deformations of Conformal Field Theories.

3.1 The Deformation Equations.

Having defined a conformal field theory in the previous section, we may now consider making an infinitesimal deformation which preserves the axioms listed above. We may in principle deform a conformal field theory through any of its elements, \( \text{viz.} \) the algebra \( \mathcal{A} \), the distinguished fields \( T(\sigma) \) and \( \overline{T}(\sigma) \) or even the representation (deforming the cycle should make no difference). However, physicists usually have a canonical choice for all elements of a theory except its Hamiltonian, and so we shall consider only changes in the fields \( T(\sigma) \) and \( \overline{T}(\sigma) \). We are thus interested in deforming the embedding \( Vir \times Vir \subset \mathcal{A} \). The more general problem of deforming a morphism of algebras has been discussed in the mathematical literature [3].

We must preserve \( Vir \times Vir \), including the value of the central charge, \( c \), and the fact that \( L_0 - \overline{L}_0 \) generates translations. This last fact means that \( L_0 - \overline{L}_0 \) may deform at most by a central element, and will generally be invariant. To first order, then, \( \delta T(\sigma) \) and \( \delta \overline{T}(\sigma) \) must satisfy

\[
\begin{align*}
[\delta T(\sigma), T(\sigma')] + [T(\sigma), \delta T(\sigma')] &= 2i \delta T(\sigma')\delta'(\sigma - \sigma') - i \delta T'(\sigma')\delta(\sigma - \sigma') \quad (8a)

[\delta \overline{T}(\sigma), \overline{T}(\sigma')] + [\overline{T}(\sigma), \delta \overline{T}(\sigma')] &= -2i \delta \overline{T}(\sigma')\delta'(\sigma - \sigma') + i \delta \overline{T}'(\sigma')\delta(\sigma - \sigma') \quad (8b)

[\delta T(\sigma), \overline{T}(\sigma')] + [T(\sigma), \delta \overline{T}(\sigma')] &= 0 \quad (8c)
\end{align*}
\]

We shall refer to Eq. 8 as the deformation equations.

3.2 Canonical Deformations.

Let \( \Phi_{(1,1)}(\sigma) \) be a primary field of dimension \( (1,1) \), then the deformation equations 8 are satisfied by

\[
\delta T(\sigma) = \delta \overline{T}(\sigma) = \Phi_{(1,1)}(\sigma)
\]
This result [4] follows from the definition of a primary field, Eq. 4, and we shall call such deformations canonical. An alternative, and world-sheet covariant, discussion of canonical deformations has been given by Campbell, Nelson and Wong [5]. Note that such a deformation mixes left and right moving sectors, so that it is not sufficient to consider just one sector. Also, since \( \delta T(\sigma) = \delta \overline{T}(\sigma) \), \( T(\sigma) - \overline{T}(\sigma) \) is an invariant of the canonical deformation class, and its zero mode therefore continues to generate translations, satisfying Eq. 2.

Canonical deformations have a number of features which indicate that they are indeed the way we should, “turn on,” a space-time field:

1) They agree with our preconceptions of how massless fields appear in the energy momentum tensor. Varying the space-time metric \( G \) in Eq. 5, including the implicit dependence made explicit in Eq. 6, yields a canonical deformation.

2) (1,1) primary fields are in natural correspondence with the physical states of string theory, being the vertex operators which create asymptotic physical states and describe their scattering. This means that canonical deformations have a straightforward interpretation as changes in space-time fields.

3) Canonical deformations work for massive states just as well as they do for massless, and avoid certain ambiguities and pathologies which may be implicit in other approaches.

Note that the third of these virtues seems to involve some small revision of the standard lore on the relationship between sigma models and strings: in particular, the, “standard,” sigma model, containing only terms of naive dimension two, not only puts the massive fields on shell, but also puts them equal to zero.

Appealing though they are, canonical deformations have a significant drawback; they correspond to turning on space-time fields in a particular gauge. This is most easily seen in an example. For simplicity, consider a conformal field theory of free scalars, defined by the energy momentum tensor of Eq. 5 with \( G \) the standard flat Minkowski metric. A short calculation soon shows that primary (1,1) fields of naive dimension two are of the form

\[
\Phi_{(1,1)} = H^{\mu\nu}(X) \partial X_\mu \overline{\partial X_\nu}
\]

where the coefficient functions \( H \) must satisfy certain conditions, if we are working in the quantum case where all fields are understood to be normal ordered and the bracket is a commutator

\[
\square H^{\mu\nu}(X) = 0 \quad (11)
\]

\[
\partial_\mu H^{\mu\nu}(X) = \partial_\nu H^{\mu\nu}(X) = 0 \quad (12)
\]

The first of these is an equation of motion, something that we would expect to arise in making a conformal deformation, but Eq. 12 is a gauge condition (of course, (11) is only the correct equation of motion when this gauge condition holds). As we shall explain in the next section, this gauge condition is a serious nuisance when we come to the problem of symmetry, and motivated us to explore [6] more general solutions of the deformation equations (see an alternative approach in [7]).
4. Symmetries.

A symmetry is a change in the space-time fields which does not change the physics (i.e., all masses and S-matrix elements are unchanged). Since the physics is determined by the conformal field theory corresponding to the field configuration in question, a transformation on the fields will be a symmetry if the corresponding conformal field theories are isomorphic. From our definition of a conformal field theory in section 2, it is clear what we mean by such an isomorphism; there must exist an isomorphism of the two operator algebras, \( \rho: A_1 \rightarrow A_2 \) which maps energy momentum tensors on to one another. In particular, if \( \rho \) is an automorphism such that 
\[ \rho(T_\Phi(\sigma)) = T_{\Phi + \delta \Phi}(\sigma), \]
then \( \Phi \rightarrow \Phi + \delta \Phi \) is a symmetry transformation. Here \( \Phi \) is a space-time field configuration, and as such indexes the energy momentum tensors of conformal field theories; for example, in Eq. 5, all space-time fields \( \Phi \) are zero except for the metric \( G \).

Even inner automorphisms are interesting in this context, and appear to give rise to the gauge symmetries of string theory. In this talk we shall restrict ourselves to inner automorphisms, and so we are interested in infinitesimal operators \( h \in A \) such that
\[ i[h, T_\Phi(\sigma)] = T_{\Phi + \delta \Phi}(\sigma) - T_\Phi(\sigma) \]  
(13)
(Changing all operators by their commutator with a fixed infinitesimal operator, \( h \), is an algebra isomorphism by virtue of the Jacobi identity; this is the infinitesimal version of a similarity transformation). The right hand side of Eq. 13 is a deformation, which is one reason for being interested in the subject. There is a large class of operators \( h \) which make the right hand side of Eq. 13 a canonical deformation:

Let \( h \) be the sum of zero modes of (1,0) and (0,1) primary fields, then

\( h \) generates a canonical deformation.

The proof of this statement \([4]\) is a straightforward application of the definition of a primary field, Eq. 4, and the Jacobi identity.

Despite its simplicity, this is a very interesting class of symmetries. It includes the familiar general coordinate and two-form gauge invariances, generated by the, “obvious,” such currents \( \xi^\mu(X) \partial X_\mu \) and \( \zeta^\mu(X) \bar{\chi} X_\mu \), as well as an infinite class of higher symmetries generated by currents which classically would have higher dimension, such as \( \psi^{\mu\nu\lambda}(X) \partial X_\mu \partial X_\nu \bar{\chi} X_\lambda \). In each case, these currents are primary fields of dimension (1,0) or (0,1) only if the parameters of the transformations, \( \xi, \zeta, \psi \ldots \), satisfy certain differential constraints.

Differential constraints on parameters are quite familiar from field theory. They are nothing to do with the equations of motion for the fields, but arise from the demand that a transformation preserve a gauge condition. Since canonical deformations are associated with with gauge conditions, as in Eq. 12, it is inevitable that gauge transformations corresponding to canonical deformations have differential constraints on their parameters. Correspondingly, if we wish to lift these constraints we must find a more general set of deformations, unaccompanied by such gauge conditions.
Exhibiting the explicit transformations on the space-time fields is possible only for those conformal field theories over which we have good computational control, such as free bosons. Nevertheless, with such examples as guides, it is possible to draw certain conclusions about this class of symmetries [4]. They are all gauge symmetries, and most or all of the states of the string are the corresponding gauge fields. The higher symmetries are spontaneously broken (so that the massive states are so by virtue of the Higgs mechanism), and mix states at different mass levels. It is also possible to argue that the symmetric solutions of string theory should correspond to topological world-sheet field theories, as suggested by Witten.

However, to exhibit the claims of the previous paragraph more concretely, and to use these insights about symmetry to do physics, we need more. We must understand precisely what the set of generators is for a wider class of conformal field theories, and we must relax the differential constraints on the parameters of the transformations, alluded to above (for the higher symmetries, these constraints exclude the global transformations which encode so much of the physics). We shall address the second of these problems and, in so doing, learn something about the first [6]. As we argued above, if we are to relax the differential constraints, we must first understand more than canonical deformations, and that will be the subject of the next section.

5. Beyond Canonical Deformations.

Can we find fully gauge covariant deformations, or does conformal invariance necessarily come accompanied with a gauge condition? We shall consider the simplest possible case [6], that of turning on massless fields of the bosonic string about flat twenty-six dimensional space-time. Thus our starting conformal field theory is twenty-six free bosons, defined by Eq. 5, with \( G \) the canonical Minkowski metric.

We shall attempt to solve the deformation equations, Eq. 8, but will take a more general \( \text{ansätz} \) than usual, the most general operator of naive dimension two:

\[
\delta T = H^{\nu\lambda}(X) \partial X_\nu \bar{\partial} X_\lambda + A^{\nu\lambda}(X) \partial X_\nu \partial X_\lambda \\
+ B^{\nu\lambda}(X) \bar{\partial} X_\nu \bar{\partial} X_\lambda + C^{\nu}(X) \partial^2 X_\nu + D^{\lambda}(X) \bar{\partial}^2 X_\lambda
\]

\[
\delta \overline{T} = \overline{H}^{\nu\lambda}(X) \overline{\partial} X_\nu \bar{\partial} X_\lambda + \overline{A}^{\nu\lambda}(X) \overline{\partial} X_\nu \bar{\partial} X_\lambda \\
+ \overline{B}^{\nu\lambda}(X) \overline{\partial} X_\nu \partial X_\lambda + \overline{C}^{\nu}(X) \bar{\partial}^2 X_\nu + \overline{D}^{\lambda}(X) \partial^2 X_\lambda
\]

The tensors \( H^{\nu\lambda}, \ldots, \overline{D}^{\lambda} \) are initially taken to be completely independent. This \( \text{ansätz} \) is then substituted into the deformation equations, Eq. 8. Some tedious but straightforward manipulation reduces these conditions, after an appropriate redefinition, to the following:

\[
\delta T(\sigma) = K^{\nu\lambda} \partial X_\nu \bar{\partial} X_\lambda + (\partial - \bar{\partial}) [C^{\nu} \partial X_\nu - D^{\lambda} \bar{\partial} X_\lambda] \\
\delta \overline{T}(\sigma) = K^{\nu\lambda} \partial X_\nu \bar{\partial} X_\lambda - (\partial - \bar{\partial}) [\overline{C}^{\nu} \overline{\partial} X_\nu - \overline{D}^{\lambda} \bar{\partial} X_\lambda]
\]
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Note that $\delta T$ and $\delta \overline{T}$ differ only by a derivative with respect to $\sigma$, so that $L_0 - \overline{L}_0$ is invariant, as argued in section 3.1. The quantities $C, \overline{C}, D, \overline{D}$ are given in terms of $K$, which we interpret as the sum of the graviton and two-form, by

\begin{align}
\partial_\nu C^{\nu} &= 0 \\
D^\lambda &= -\frac{1}{2} \partial_\mu K^{\mu \lambda} \\
\overline{D}^\lambda &= -\frac{1}{2} \partial_\mu K^{\lambda \mu} \\
\partial^\lambda C^{\nu} &= \frac{1}{2} \Box K^{\nu \lambda} - \frac{1}{2} \partial^{\nu} \partial_\mu K^{\mu \lambda} \\
\partial^\lambda \overline{C}^{\nu} &= \frac{1}{2} \Box K^{\lambda \nu} - \frac{1}{2} \partial^{\nu} \partial_\mu K^{\lambda \mu}
\end{align}

At first sight there is no equation of motion for the physical field $K$, and the dilaton is nowhere to be seen. However Eqs. 19 and 20 cannot be solved for $C$ and $\overline{C}$ for arbitrary $K$. There is an integrability condition which $K$ must satisfy which turns out to yield both an equation of motion and the dilaton:

\[ \Box K^{\nu \lambda} - \partial^\nu \partial_\mu K^{\mu \lambda} - \partial^\lambda \partial_\mu K^{\nu \mu} + \partial^\nu \partial^\lambda K^{\mu \mu} = \partial^\nu \partial^\lambda \phi + \alpha^{\nu \lambda} \]

for some scalar function $\phi$, which we identify as the dilaton. Eq. 16 yields the dilaton equation of motion, $\Box \phi = 0$.

We have thus achieved our goal of finding deformations which are associated with a covariant equation of motion and no gauge condition. We get exactly the expected linearised equations, except for the last term on the right hand side of Eq. 21. $\alpha$ is antisymmetric in its indices and constant. This term therefore indicates that there is a finite dimensional space of additional deformations, over and above those associated with turning on the physical fields.

What are we to make of these additional deformations? Some caution is appropriate [6], since our calculations are only to first order in the deformation, but there is nonetheless a natural interpretation of these parameters $\alpha$. With lowered indices, $\alpha$ is a harmonic two-form, and so may be integrated over compact two-dimensional submanifolds of space-time. This integration may be pulled back to the world-sheet to yield

\[ S_\alpha = \int \alpha_{\mu \nu} dX^\mu dX^\nu \]

which, since $\alpha$ is closed, is invariant under deformations of the map $X$. In the language of non-linear sigma models, $S_\alpha$ is proportional to an instanton number, and when added to the usual action (a "$\theta$-term") will usually have physical consequences.

In particular, if $S_\alpha$ is added to the usual action with a coefficient proportional to $ln \epsilon$ (where $\epsilon$ is, say, the parameter of dimensional regularisation), then it will clearly amend the condition for conformal invariance [8]. The equations of motion will then be affected in just the way we see in Eq. 21. It is worthy of note that, at least in some cases, the topological charge density does have just such an ultraviolet divergence at one loop [9].
Given this interpretation of these additional deformations, we may quite reasonably refer to $\alpha$ either as free parameters or topological degrees of freedom of string theory. Their description as $\theta$-terms is very similar to that given by other speakers at this conference of the isolated states found in two-dimensional string theory [2].

6. Symmetries Redux.

Finally, let us set $\alpha = 0$ and return to the question of symmetries. With the apparently covariant set of deformations found in the previous section, we can now find the full set of unbroken gauge transformations as inner automorphisms. It is straightforward to see that

$$h = \int d\sigma \left( \xi^\mu(X) \partial \! X_\mu + \zeta^\mu(X) \bar{\partial} X_\mu \right),$$

(23)

generates deformations of the type given in Eq. 15, and that the fields transform in the conventional way. Note that we have achieved our goal of eliminating the differential constraints on the parameters $\xi$ and $\eta$.

These explicit calculations need to be extended in two ways; we would like to understand the higher symmetries, and we would like to know how the generators of symmetry deform, if at all, with the conformal field theory. To do this let us first understand why the operators of Eq. 23 had to generate a symmetry. The argument has two parts: i) turning on the space-time fields without restricting their gauge corresponded to the most general conformal deformation by world-sheet fields, and ii) commuting the free energy-momentum tensor with the $h$ of Eq. 23 produces a deformation of this form (by conservation of naive dimension). Hence it has to be possible to pull back the inner automorphism generated by $h$ to a symmetry transformation on the space-time fields.

It is very tempting to generalise this argument to the massive fields and the higher symmetries. That is, we might conjecture that arbitrary space-time fields are turned on, without imposing any restriction as to gauge, by considering an ansatz which is the obvious generalisation of Eq. 14 to the appropriate naive dimension, and demanding that it be a conformal deformation, satisfying Eq. 8. Then a deformation of this form would necessarily be generated by the corresponding dimensional generalisation of the generator in Eq. 23. The problem is that this symmetry appears to be larger than we need; working out some examples [6] it is easy to see that $h$ contains many more degrees of freedom than there are gauge conditions to relax. It therefore seems likely that the most general conformal deformation of higher naive dimension contains not just the physical space-time fields, but also unphysical auxiliary fields, which are pure gauge artifacts. This is very much akin to the situation which arises in the superspace formulation of supersymmetric gauge theories, where there exist auxiliary gauge artifacts over and above those needed to account for the halving of fermionic degrees of freedom in going on shell. If we are willing to live with these auxiliary fields, then we have answered the question
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of which operators generate the higher symmetries. On the other hand, if we wish
to restrict ourselves to the physical, propagating fields with non-trivial dynamics
(“Wess-Zumino gauge”), then it remains to be determined what the appropriate
restrictions on the generators are.

What happens as when we consider a theory other than that of free bosons?
In the discussion of section four, the generators were associated with primary fields,
which deform with the conformal field theory and so are not known explicitly over
the whole deformation class. However the two preceeding paragraphs contain no
mention of primary fields. The only place where a specific property of the free theory
was used was an occasional appeal to conservation of naive dimension, but this was
only a convenience which allowed us to discuss one mass-level at a time, and may
be dispensed with. It would seem, then, that deforming the set of operators which
generate canonical deformations is complicated because of the need to preserve a
gauge condition, rather than to ensure an interpretation as a transformation on
space-time fields. Space-time fields (including auxiliaries) are turned on with the
most general solution of the deformation equations in terms of world-sheet fields,
and so symmetries are generated by all operators which commute with the generator
of translations, $L_0 - T_0$. Since this operator is an invariant of the deformation class,
so are the symmetry generators. In summary:

Symmetries are generated by the centraliser of $L_0 - T_0$, and this set
is invariant over the whole deformation class.

Acknowledgements.

M.E. would like to thank Burt Ovrut for collaboration on some of the early
work described here, and S. Catto and A. Rocha for the invitation to talk at a very
stimulating conference.

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