One-dimensional Majorana modes can be obtained as boundary excitations of topologically nontrivial two-dimensional topological superconductors. Here, we propose instead the bottom-up creation of one-dimensional, counterpropagating, and dispersive Majorana modes as bulk excitations of a periodic chain of partially-overlapping, zero-dimensional Majorana modes in proximitized quantum nanowires via periodically-modulated magnetic fields. These dispersive one-dimensional Majorana modes can be either massive or massless. Massless Majorana modes are pseudohelical, having opposite Majorana pseudospin, and realize emergent quantum mechanical supersymmetry. The system exhibits extended supersymmetry with central extensions and with spontaneous partial breaking. We identify the massless Majorana fermions as Goldstinos, i.e., the Nambu-Goldstone fermions associated with the spontaneous breaking of supersymmetry. The experimental fingerprint of massless Majorana modes and supersymmetry is the presence of a finite zero-bias peak, which is generally not expected for Majorana modes with a finite overlap and localized at a finite distance. Moreover, slowly varying magnetic fields can realize an adiabatic Majorana pump which can be used as a dynamically probe of topological superconductivity.

Majorana fermions in high-energy physics are spin-1/2 particles that are symmetric with respect to charge conjugation symmetry, i.e., neutral fermions that coincide with their own anti-particles[1, 2]. In condensed matter, they appear as quasiparticle excitations in superconductors, where particle-hole symmetry plays the role of charge conjugation[2]. Generally, Majorana quasiparticles are topologically protected (d−1)-dimensional boundary excitations of a topologically nontrivial d-dimensional bulk. Specifically, 0-dimensional (0D) Majorana modes[3–5] correspond to the end states of 1D quantum systems with proximitized superconductivity, whereas chiral and helical 1D Majorana modes correspond to the edge states of 2D unconventional superconductors or planar superconducting heterostructures[6–21] respectively with broken or unbroken time-reversal symmetry.

Majorana quasiparticles exhibit remarkable properties such as non-abelian statistics[22, 23], conformal invariance[24], and emergent supersymmetry (SUSY)[25–34]. Besides their purely theoretical appeal, Majorana quasiparticles attracted enormous interest due to their potential technological applications in quantum computing[22, 23, 35–39]. Quite a few experiments observed signatures compatible with the presence of spatially-separated 0D Majorana modes in nanowires[40–53] and quantum chains of adatoms[54–60], and chiral 1D Majorana modes in planar heterostructures[15, 20, 21]. However, similar experimental signatures can be reproduced, e.g., by trivial Andreev bound states or by Majorana modes localized at a finite distance and with a finite overlap, known as quasi-Majorana modes (qMM)[61], in the presence of inhomogeneous potentials and disorder[62–73].

In this Letter, we propose a bottom-up approach to topological superconductivity, i.e., employing a periodic array of partially-overlapping 0D qMM to realize 1D Majorana fermions with emergent SUSY. This can be achieved, e.g., in proximitized semiconducting nanowires[40] via periodically-modulated magnetic fields[74–79]. In this regime, the topological mass gap $\mathcal{M}$ can assume alternatively positive and negative values along the wire, corresponding to topologically trivial and nontrivial segments, with partially-overlapping 0D qMM localized at their boundaries and forming a 1D periodic lattice. This results in a pair of dispersive and counterpropagating 1D Majorana modes delocalized along the whole wire and separated from the higher-energy bulk states, with a mass gap that can be tuned by externally applied fields. In the massless case, we found that these 1D Majorana modes are pseudohelical, i.e., have opposite Majorana pseudospin, and exhibit centrally extended SUSY, with a finite zero-energy density of states and zero-bias peak delocalized along the whole wire. Moreover, we identify the massless Majorana fermion with a Goldstino, i.e., the Nambu-Goldstone fermion[80] associated with spontaneously broken SUSY. As far as we know, our model provides the first condensed matter example of extended SUSY with central extensions, which plays essential roles in non-perturbative aspects of quantum field theory in...
emergent $N = 4$ quantum mechanical SUSY[30, 31] with supercharges

$$Q_1 = \sqrt{\frac{\mathcal{H}_{\text{SUSY}}}{2}} d_M (1 + P), \quad Q_2 = \sqrt{\frac{\mathcal{H}_{\text{SUSY}}}{2}} T (1 + P), \quad (4)$$

which satisfy the superalgebra \(\{Q_1, Q_2^\dagger\} = 2 \delta_{ij} \mathcal{H}_{\text{SUSY}} + Z_{ij}\), \(\{Q_1, Q_2\} = \{Q_1^\dagger, Q_2^\dagger\} = 0\), \(\{P, Q_1\} = 0\), with central charges

$$Z_{ij} = - \frac{\mathcal{H}_{\text{SUSY}}}{2} (1 + P) (-1)^{d_i d_M}, \quad Z_{22} = 0,$$

and

$$Z_{12} = \bar{Z}_{21} = \mathcal{H}_{\text{SUSY}} \{d_M (1 + P), T\}. \quad (4)$$

Here, \(\mathcal{H}_{\text{SUSY}} = \mathcal{H}_{\text{eff}} + 2h |v| (\text{with } h > 1)\) is the many-body Hamiltonian having nonnegative energy levels, \(P = \langle -i \rangle^N \prod_{j=1}^N \gamma_{\alpha_j} \gamma_{B_j}\) the fermion parity, and \(T\) the translation defined by \(T \gamma_{\alpha_j} T^\dagger = \gamma_{B_j}, T \gamma_{B_j} T^\dagger = \gamma_{\alpha_j + 1 (\text{mod } N)}\), which satisfies \(\{T, P\} = 0\) and \([T, \mathcal{H}_{\text{SUSY}}] = 0\) for \(|v| = |w|\). All many-body eigenstates, including the groundstate, have superpartners with opposite parity. Thus, the Witten index is zero, and SUSY is spontaneously broken[25]. Precisely, the supersymmetry \(Q_1\) is unbroken whereas \(Q_2\) is spontaneously broken: The $N = 4$ superalgebra \((Q_1, Q_2)\) is spontaneously broken down into the $N = 2$ superalgebra \(Q_1\). This mandates the presence of a Goldstino[80], which we identify with the massless Majorana fermion. The zero mass gap is protected by SUSY, i.e., the gap closes if and only if the Hamiltonian exhibits SUSY. This is also true in the case of disorder and broken translational symmetry[31]: The mass gap in the presence of several 0D qMM closes if and only if the Hamiltonian exhibits SUSY.

The Hamiltonian in Eq. (1) may describe the low-energy effective theory of a 1D topological superconductor with spatially-modulated fields. Specifically, we consider a semiconductor nanowire with Rashba spin-orbit coupling and coated with a conventional superconductor[86], as in Fig. 1. A periodically-modulated magnetic field is induced by an array of nanomagnets[74–77] with magnetic moments parallel to the $z$-axis. In addition, we consider a uniform applied field $B_{\text{app}}$ in the $z$-direction, which can be used to control the lengths of the nontrivial and trivial segments. The wire is described by the Hamiltonian

$$H = \left(\frac{\mathbf{p}^2}{2m} + \frac{\alpha}{\hbar} \sigma_y p - \mu(x) + \mathbf{b}(x) \cdot \sigma\right) \tau_z + \frac{\Delta(x)}{2} \text{sgn} \tau_x, \quad (5)$$

where $\sigma$ and $\tau$ are the vectors of Pauli matrices in spin and particle-hole space, $m$ the effective mass, $\alpha$ the spin-orbit coupling, $\mu(x)$ the chemical potential, $\mathbf{b}(x) = (g/2) \mu_B \mathbf{B}(x)$ the Zeeman field, and $\Delta(x)$ the proximization-induced superconducting pairing. We require the wavelength $\lambda$ of the periodically-modulated field to be comparable with the Majorana localization length, which is $\xi_M \approx (b_z / E_{SO}) \alpha / \Delta$ and $\alpha / \Delta$ respectively for $E_{SO} = m^2 \hbar^2 / 2 \Delta$ and $\Delta > \Delta$ (weak and strong spin-orbit coupling regimes)[87–89]. In the accompanying work[84] we discuss other examples of nanostructures[90–93] which may realize similar physics.

If all fields are uniform, the Hamiltonian above reduces to the Oreg-Lutchyn minimal model[4, 5]. Assuming the magnetic field to be in the $xz$-plane, the nontrivial phase is realized

$$\mathcal{H}_{\text{eff}} = \sum_k [c_k^\dagger, c_{-k}] \cdot \mathbf{H}(k) \cdot \mathbf{\tau} \cdot [c_{-k}, c_k],$$

up to a constant term, with $\mathbf{H}(k) = (0, vsin k, vcosk - w)$, and $\mathbf{\tau}$ the vector of Pauli matrices. The energy dispersion is $E_k = |\mathbf{H}(k)| = \sqrt{w^2 + v^2 - 2v \cos k}$ with a topological mass gap $\Delta_{\text{eff}} = |w| - |v|$. In the continuum limit (and assuming $vw > 0$), the Hamiltonian coincides with a 1D Dirac equation $H = tv \tau_y + (mv^2 - \frac{v}{2k^2}) \tau_x$ with mass gap $\Delta_{\text{eff}} = v - w$ and a quadratic correction in the momentum. The covariant form is obtained by multiplying the Hamiltonian by $\tau_x$. The Dirac equation is topologically trivial or nontrivial respectively for $vw < 0$ and $vw > 0$, i.e., for $\Delta_{\text{eff}} > 0$ and $\Delta_{\text{eff}} < 0$[85].

In the massless case $|v| = |w|$ the zero-energy eigenstates are doubly degenerate at gapless points and described by the fermionic operator $d_M = (\gamma_A + i \gamma_B) / 2$ and its hermitian conjugate $d_M^\dagger$, where the nonlocal Majorana operators are $\gamma_A = (1 / \sqrt{N}) \sum_j \gamma_{\alpha_j}$ and $\gamma_B = (1 / \sqrt{N}) \sum_j \gamma_{B_j}$. The gapless state $v = \pm w$ separates two topologically inequivalent phases described by the topological invariant $\mathcal{M}_{\text{eff}} = \pm 1$ where $\Delta_{\text{eff}} = |w| - |v|$. The Hamiltonian also exhibits 0D Majorana end modes in the nontrivial phase $|v| > |w|$ in the case of open boundary conditions. The two end states are $\gamma_L \propto \sum_j (w/v) \gamma_{\alpha_j}$ and $\gamma_R \propto \sum_j (w/v)^N + 1 \gamma_{B_j}$ localized at the opposite ends of the chain with localization length $\xi_{\text{eff}} = 1 / |\log|w/v||$.

The massless Majorana fields $|v| = |w|$ describe a 1D free Majorana fermion in a 1+1D conformal field theory[24], which coincides with a pair of counterpropagating 1D Majorana modes. In this case, the expectation values of the Majorana pseudospin

$$\langle \tau/2 \rangle = \frac{|\mathbf{H}(k)|}{2E_k},$$

have opposite directions for the two modes near the gapless point, being $\langle \tau \rangle = \text{sgn} (vsin k)$ at $k \to 0$, $\pi$ for $v = \pm w$, respectively (see accompanying work[84]). Consequently, the two modes form a pseudohelical pair, and elastic backscattering is suppressed. In the massless case, Eq. (1) exhibits high energy physics[81–83]. Note also that zero-bias peaks are generally not expected in the presence of several 0D qMM localized at finite distance.

We consider a bipartite 1D lattice of $2N$ 0D Majorana modes

$$\mathcal{H}_{\text{eff}} = \sum_{j=1}^N (w \gamma_{A_j} \gamma_{B_j} + v \gamma_{B_j} \gamma_{A_j + 1}),$$

where $\gamma_{A_j}, \gamma_{B_j}$ are the Majorana operators corresponding to a single Dirac operator $c_j = (\gamma_{B_j} + \gamma_{A_j})/2$ per unit cell, and with $w, v \in \mathbb{R}$. This model is a special case of the Kitaev chain model[3] if $\mu = 2w$ and $t = \Delta = -v$ (see accompanying work[84]). In momentum space we get

$$\mathcal{H}_{\text{eff}} = \sum_k [c_k^\dagger, c_{-k}] \cdot \mathbf{H}(k) \cdot \mathbf{\tau} \cdot [c_{-k}, c_k],$$

for all fields are uniform, the Hamiltonian above reduces to the Oreg-Lutchyn minimal model[4, 5]. Assuming the magnetic field to be in the $xz$-plane, the nontrivial phase is realized
One has M gap pairing is not uniform, one can define a local Majorana mass phases, and M nal field b

Figure 2. (a) LDOS at zero energy of a nanowire in a periodically-modulated magnetic field, as a function of the position and the external field b applied in the z-direction, calculated with periodic boundary conditions. The peaks of the LDOS indicate the presence of 1D Majorana modes, corresponding to a periodic lattice of overlapping 0D qMM localized at the boundaries between trivial (M > 0) and nontrivial (M = 0) segments. (b) Energy spectra with dispersive 1D Majorana modes (highlighted) below the particle-hole gap. The dispersion becomes gapless (massless) when the overlaps between localized 0D qMM across the nontrivial and trivial segments become equal when b = bSUSY. (c) and (d) Same as before, but with open boundary conditions. The wire becomes nontrivial after the closing of the particle-hole gap for b > bSUSY and exhibits 0D Majorana modes localized at its opposite ends with diverging density (out of scale). (e) Local Majorana mass M = \sqrt{\mu(x)^2 + \Delta(x)^2} - |b(x)| and its nodes M = 0 (continuous line) as a function of the position and applied field. (f) Dispersion of the 1D Majorana modes as a function of the applied field.

for \( b^2 > \mu^2 + \Delta^2 \).[4, 5] The topological invariant coincides with the sign of the Majorana mass gap M = \sqrt{\mu^2 + \Delta^2} - |b|.

One has M > 0 and M < 0 for the trivial and nontrivial phases, and M = 0 at the closing of the particle-hole gap. If the chemical potential, Zeeman field, or superconducting pairing is not uniform, one can define a local Majorana mass gap M(x) = \sqrt{\mu(x)^2 + \Delta(x)^2} - |b(x)| which may be alternatively positive and negative values along the wire. In this case, segments with M > 0 and M < 0 are trivial and nontrivial with a local topological invariant \( \mathcal{P}(x) = \text{sgn} M(x) \).

Hence, 0D qMM localize at the boundaries between trivial and nontrivial segments at the nodes of the Majorana mass gap M(x) = 0 (see Fig. 1) with localization length \( \xi_M \) and mutual distance \( L_{AB} \) and \( L_{BA} \). If the wavelength \( \lambda \) of the periodically-modulated field is comparable with the Majorana localization length \( \xi_M \), the 0D qMM \( \gamma_{A1} \) and \( \gamma_{B1} \) have finite overlaps \( \sim e^{-L_{AB}/\xi_M} \) and \( \sim e^{-L_{BA}/\xi_M} \) and realize a periodic bi-partite 1D lattice. Hence, projecting onto the subspace of Majorana operators, one obtains the effective low-energy Hamiltonian in Eq. (1), where the coupling parameters \( v, w \) coincide with the Hamiltonian matrix elements between contiguous 0D qMM separated by trivial and nontrivial segments, respectively. For \( |v| = |w| \), the overlaps between contiguous 0D qMM across trivial and nontrivial segments become equal, and the 1D Majorana modes become massless (gapless). We calculate the magnetic field B\( _{\text{ext}} \) of the nanomagnets array via the finite-element method with ONELAB[94]. We then discretize the Hamiltonian and take realistic values for the model parameters[41, 68], assuming uniform chemical potential and \( \Delta \propto 1 - B^2/B_{\text{c}}^2 \), where B\( _{\text{c}} \) is the critical field (see accompanying work[84]).

Figures 2(a) and (b) show the local density of states (LDOS) at zero energy \( g(x) = \frac{1}{2} \text{Im} \sum_{n} |\Psi_n(x)|^2 / (\Omega^2 - E_n) \) (here, \( E_n \) and \( \Psi_n(x) \) is the energy and Nambu spinor of each eigenstate) with finite broadening \( \Gamma \approx \Delta \) (to simulate the experimental conditions), and energy spectra as a function of the applied magnetic field, in the case of periodic boundary conditions. The LDOS shows the presence of a periodic lattice of 0D qMM with finite overlap, localized at the boundaries between trivial (M > 0) and nontrivial (M < 0) segments [see also Fig. 2(e)]. This lattice corresponds to dispersive 1D Majorana modes with energy below the particle-hole gap and separated from the higher-energy bulk states, highlighted in Fig. 2(b), and with a Majorana mass gap equal to \( M_{\text{eff}} = \max |w| - |v| \). When the overlaps \( v, w \) between 0D qMM across the nontrivial and trivial segments become equal when \( b = b_{\text{SUSY}} \) (which, in first approximation, occurs when \( L_{AB} \approx L_{BA} \)), the periodic lattice becomes invariant up to translations \( T \) [see Eq. (4)]. Hence, the wire exhibits SUSY and the dispersion becomes gapless (massless) with maximum LDOS at zero energy.

When the applied field increases above the threshold \( b_{\text{NT}} \), such that \( |b(x)| > \sqrt{\mu(x)^2 + \Delta(x)^2} \) \( \forall x \), the Majorana mass becomes negative on the whole wire and the trivial segments disappear. Conversely, when the applied field decreases below the threshold \( b_{\text{T}} \), such that \( |b(x)| < \sqrt{\mu(x)^2 + \Delta(x)^2} \) \( \forall x \), the Majorana mass becomes positive on the whole wire and the nontrivial segments disappear. In these two cases, the 0D qMM at the ends of the trivial (or nontrivial) segments fuse into finite-energy Andreev-like fermionic modes. The continuous crossover between Majorana and Andreev-like modes is realized by increasing the overlaps between contiguous 0D qMM at the ends of either the trivial or nontrivial segments, such that \( |v| > |w| \) or \( |w| > |v| \), without closing the particle-hole gap. This crossover also occurs when the wavelength \( \lambda \) becomes smaller than the Majorana localization length \( \lambda \lesssim \xi_M \). This results in larger overlaps between contiguous 0D qMM fusing into fermionic Andreev-like modes (see accompanying work[84]). In the opposite regime \( \lambda \gg \xi_M \) one has \( v, w \rightarrow 0 \), which corresponds to decoupled qMM with flat dispersion \( E_k \approx 0 \).
In the case of open boundary conditions. For sliding harmonic field $\propto \pi x/\lambda$ direction $\theta$, the local Majorana mass gap becomes $\mathcal{M}(x) = -(b_{nm} a_{nm}/h) \sin(2\pi x/\lambda + \theta)$ which has equally-spaced nodes at $x_n/\lambda = \theta/2\pi + n/2$. Slowly varying the applied field direction $\theta$ induces the adiabatic sliding of the 1D lattice of 0D qMM, corresponding to the pumping of one 0D quasi-Majorana state every half-turn $\theta \rightarrow \theta + \pi$ and one full fermionic state every full turn $\theta \rightarrow \theta + 2\pi$ of the applied field direction. In the dynamic Floquet regime, this induces a finite “Majorana current” through the wire. In the case of periodic and closed boundary conditions, a half-turn of the applied field direction corresponds to the translation $T$ of 0D qMM entering the definition of the supercharges in Eq. (4).

To obtain an approximately harmonic field, we apply a field $B_\theta = B_\theta - (B_{nm})$, with $(B_{nm})$ equal to the average of the field $B_{nm}$ along the wire, and where $B_\theta$ rotates in the $x\hat{z}$-plane forming an angle $\theta$ with the $x$-axis. Figure 3(d) shows the intensity of the total magnetic field (superposition of the applied and nanomagnets fields) as a function of $\theta$. Figure 3(a) shows the evolution of the LDOS when the applied field direction $\theta$ turns around in the $x\hat{z}$-plane. As the field rotates, 0D qMM slide along the wire, resulting in an adiabatic pumping of 0D qMM, as shown in Fig. 3(a). 0D qMM translate by $\pi$ at each half-turn $\theta \rightarrow \theta + \pi$. Figure 3(b) shows the energy of the dispersive 1D Majorana modes below the gap, which corresponds to the sliding of the 0D quasi-Majorana lattice. For reference, Fig. 3(c) shows the local Majorana mass $\mathcal{M}$ and its nodes $\mathcal{M} = 0$ as a function of $\theta$.

To experimentally realize our proposal, the wire must be much longer than the field periodicity, which must be comparable with the Majorana localization length, i.e., $L \gg \lambda \equiv \xi_{\mathcal{M}} \equiv \alpha/\Delta$, as discussed in the accompanying work[84]. Moreover, variations of the gate and spin-orbit coupling fields must be negligible at length scales larger than $\lambda$, to guarantee an unbroken translational invariance at the mesoscopic level. Conversely, the physics described here is not affected by perturbations having a length scale shorter than $\lambda$ (e.g., disorder).

Concluding, we proposed the realization of dispersive 1D Majorana fermions in nanowires via spatially-modulated magnetic fields. The massless 1D Majorana fermions are the Nambu-Goldstone fermions (Goldstinos) associated with the spontaneous partial breaking of SU(4). Their experimental signatures are the finite LDOS at zero energy (zero-bias peak) delocalized on the whole length of the wire. This has to be contrasted with zero-bias peaks of 0D Majorana end modes, localized only at the ends of the wire, and to the general case of 0D qMM, whose energy is lifted by their finite overlap. We finally showed how to realize an adiabatic Majorana pump by varying the applied magnetic field direction, which induces a sliding lattice of 0D qMM with quantized transport of one quasi-Majorana per a half cycle. The manipulation of Majorana modes via spatially-modulated fields may lead to the realization of alternative non-abelian braiding protocols.
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