Generation of time-bin entangled photon pairs using a single three-level emitter

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Abstract

We study single-photon emission of a classically pumped three-level A-type emitter in a high-Q cavity. In particular, generation of single-photon time-bin double-peak wave packets is shown.

1 Introduction

A source of entangled photon pairs is an essential element for quantum communication [1], linear optics quantum computing [2] and processing of quantum information protocols such as quantum teleportation [3, 4] and quantum cryptography [5, 6]. Commonly, the employed entangled states in experiments are polarization-entangled photons generated by e.g. spontaneous parametric down-conversion [7] or biexciton–exciton cascade of single semiconductor quantum dots [8, 9]. However, polarization encoding is prone to dispersion in optical fibers, which affect the polarization of the outcoupled photons. Thus, the polarization encoding is not well-suited for large distance quantum communication in real-world implementations [10].

An alternative is time-bin entangled light states [11], where quantum information is encoded in the arrival time of photons. Time-bin entangled states are more robust to the decoherence in optical fibers [12], enabling distribution of entangled photons over 300 km [13]. In these experiments spontaneous parametric down-conversion is used as a source of time-bin entangled photons [14, 15]. However, parametric down-conversion is a random process with probabilistic number of generated photon. The generation of multiple pairs reduces the accuracy and security of the quantum communication. Alternatively, schemes for generation of single pairs of time-bin entangled photons using the biexciton cascade in a quantum dot have been proposed [16]. These schemes require the preparation of the quantum dot.
into a long-lived or metastable state, which is non-trivial challenge, that prevents an experimental implementation of such schemes.

In the present article, we show a scheme for generation a single-photon qubit time-bin-encoded across superposed two spatiotemporal peaks. In an earlier work, within the frame of exact cavity quantum electrodynamics [17], we have studied the theory of generation of single-photon wave packets by means of interaction of the cavity-assisted quantized electromagnetic field with a pumped three-level Λ-type emitter [18]. In particular, the possibility to generate single-photon wave packets with requested spatiotemporal shapes has been shown. Here, we study the generation of photon wave packets of double-peak shapes—a possible implementation of time-bin entanglement. In particular, we show that by adjusting the the driving laser pulse modulation of the phase difference between the spatiotime bins of the generated single-photon state can be achieved.

2 Basic equations

We consider a single atom-like emitter that interacts with the electromagnetic field in the presence of a dispersing and absorbing dielectric medium with a spatially varying and frequency-dependent complex permittivity. We assume that only a single transition (|2⟩ ↔ |3⟩, frequency ω_{23}) is quasi resonantly coupled to a narrow-band cavity-assisted electromagnetic field (frequency ω_k), cf. Fig. 1 and that an external (classical) pump field with quasi resonant frequency ω_p and (time-dependent) Rabi frequency Ω_p(t) is applied to the |1⟩ ↔ |2⟩ transition (frequency ω_{21}), cf. Fig. 1. For the sake of simplicity of presentation, we restrict our treatment to the one-dimensional case (z axis) and assume that the resonator cavity is formed by an empty body bounded by an outcoupling fractionally transparent mirror at z = 0 and a perfectly reflecting mirror at z = −l, and (negative) z_A is the position of the emitter inside the cavity. Applying the multipolar-coupling scheme in electric dipole approximation and the rotating-wave approximation, we may write the Hamiltonian that governs the temporal evolution of the overall system, which consists of the electromagnetic field, the dielectric medium (including the dissipative degrees of freedom), and the emitter coupled to the field, in the form of (for details, see Refs. [17, 18])

\[
\hat{H} = \int dz \int_0^\infty d\omega \hbar \hat{f}^\dagger(z, \omega) \hat{f}(z, \omega) + \hbar \omega_{21} \hat{S}_{22} + \hbar \omega_{31} \hat{S}_{33} - \frac{\hbar}{2} \Omega_p(t) \left[ \hat{S}_{12} e^{-i\omega_p t} + \text{H.c.} \right] - g(t) \left[ d_{23} \hat{S}_{32} \hat{E}^{(+)}(z_A) + \text{H.c.} \right],
\]

with ω_{31} = ω_{21} − ω_{23}. In this equation, the first term is the Hamiltonian of the field–medium system, where the fundamental bosonic fields f(z, ω) and
play the role of the canonically conjugate system variables. The second and the third terms represent the Hamiltonian of the emitter, where the $\hat{S}_{kk'}$ are the flip operators,

$$\hat{S}_{kk'} = |k\rangle\langle k'|,$$

(4)

corresponding to the $|k\rangle \leftrightarrow |k'\rangle$ transition with the frequency $\omega_{kk'}$, where $|k\rangle$ being the energy eigenstates of the emitter. Finally, the fourth term is the emitter–pump coupling energy and the last term is the emitter–field coupling energy, where

$$\hat{d}_A = \sum_{kk'} d_{kk'} \hat{S}_{kk'},$$

(5)

is the electric dipole-moment operator ($d_{kk'} = \langle k|\hat{d}_A|k'\rangle$), and the (real) time-dependent function $g(t)$ defines the (time-dependent) shape of the interaction of the emitter with the cavity field, which without loss of generality can be chosen to be normalized to unity. The operator of the medium-assisted electric field $\hat{E}(z)$ can be expressed in terms of the variables $\hat{f}(z,\omega)$ and $\hat{f}^\dagger(z,\omega)$ as follows:

$$\hat{E}(z) = \hat{E}^{(+)}(z) + \hat{E}^{(-)}(z),$$

(6)

$$\hat{E}^{(+)}(z) = \int_0^\infty d\omega \hat{E}(z,\omega), \quad \hat{E}^{(-)}(z) = [\hat{E}^{(+)}(z)]^\dagger,$$

(7)

$$\hat{E}(z,\omega) = i\sqrt{\frac{\hbar}{\varepsilon_0 \pi \omega^2}} \int d^3r' \sqrt{\varepsilon''(z',\omega)} G(z,z',\omega) \hat{f}(z',\omega).$$

(8)

In the above, $G(z,z',\omega)$ is the classical (retarded) Green tensor for the Helmholtz equation, that defines the structure of the electromagnetic field formed by the present dielectric bodies.

In what follows we assume that the emitter is initially (at time $t = 0$) prepared in the state $|1\rangle$ and the rest of the system, i.e., the part of the system that consists of the electromagnetic field and the dielectric media (i.e., the cavity), is prepared in the ground state $|\{0\}\rangle$. Since, in the case under consideration, we may approximately span the Hilbert space of the whole system by the single-excitation states, we expand the state vector of the overall system at later times $t$ ($t \geq 0$) as

$$|\psi(t)\rangle = C_1(t)|\{0\}\rangle|1\rangle + C_2(t)e^{-i\omega_{21}t}|\{0\}\rangle|2\rangle + \int dz \int_0^\infty d\omega C_3(z,\omega,t)e^{-i(\omega+\omega_{31})t}\hat{f}^\dagger(z,\omega)|\{0\}\rangle|3\rangle,$$

(9)
where $\hat{f}^\dagger(z, \omega)\{|0\rangle\}$ is an excited single-quantum state of the combined field–cavity system.

It is not difficult to prove that the Schrödinger equation for $|\psi(t)\rangle$ leads to the following system of differential equations for the probability amplitudes $C_1(t)$, $C_2(t)$ and $C_3(z, \omega, t)$:

$$\dot{C}_1 = \frac{i}{2} \Omega_p(t) e^{i\Delta_p t} C_2(t), \quad (10)$$

$$\dot{C}_2 = \frac{i}{2} \Omega_p(t) e^{-i\Delta_p t} C_1(t) - \frac{d_{23}}{\sqrt{\pi \hbar \varepsilon_0 A}} \int_0^\infty d\omega \frac{\omega^2}{c^2} \int dz \sqrt{\varepsilon''(z, \omega)} \times G(z_A, z, \omega) C_3(z, \omega, t) g(t) e^{-i(\omega - \omega_{23})t}, \quad (11)$$

$$\dot{C}_3(z, \omega, t) = \frac{d_{23}^*}{\sqrt{\pi \hbar \varepsilon_0 A}} \frac{\omega^2}{c^2} \sqrt{\varepsilon''(z, \omega)} G^*(z_A, z, \omega) C_2(t) g(t) e^{i(\omega - \omega_{23})t}, \quad (12)$$

where $A$ is the area of the coupling mirror of the cavity, and $\Delta_p = \omega_p - \omega_{21}$ is the detuning of the pump frequency from the $|1\rangle \leftrightarrow |2\rangle$ transition frequency. The Green function $G(z, z', \omega)$ determines the spectral response of the resonator cavity. In particular, its poles define quasidiscrete set of lines, where we assume that the $k$th mode of the cavity with the complex frequency

$$\tilde{\omega}_k = \omega_k - \frac{1}{2} i \Gamma_k, \quad (13)$$

is quasi-resonantly coupled to the transition $|2\rangle \leftrightarrow |3\rangle$ with the transition frequency $\omega_{23}$ (cf. Fig. 1). Then, substituting the formal solutions to Eqs. (10)
and (12) [with the initial condition $C_1(0) = 1$, $C_2(0) = 0$ and $C_3(z, \omega, 0) = 0$] into Eq. (11), we can derive the integro-differential equation

$$\dot{C}_2 = \frac{i}{2} \Omega_p(t) e^{-i \Delta_p t} + \int_0^t dt' K(t, t') C_2(t'),$$

(14)

where the kernel function $K(t, t')$ reads

$$K(t, t') = -\frac{1}{4} \Omega_p(t) \Omega_p(t') e^{-i \Delta_p (t-t')} - \frac{1}{4} \alpha_k \tilde{\omega}_k g(t) g(t') e^{-i(\Delta_k - i \Gamma_k / 2)(t-t')},$$

(15)

with $\Delta_k = \omega_k - \omega_{23}$ and

$$\alpha_k = \frac{4 |d_{23}|^2}{\hbar \epsilon_0 A l} \sin^2 (\omega_k z_A / c).$$

(16)

From Eq. (14) together with Eq. (15) we can conclude that $R_k \equiv \sqrt{\alpha_k \omega_k}$ can be regarded as vacuum Rabi frequency of emitter–cavity interaction.

### 3 Single-photon Generation and wave-packet shape

Following Ref. [17], it can be shown that when the Hilbert space of the system is effectively spanned by a single excitation, the Wigner function of the quantum state of the excited outgoing wave packet can be derived to be

$$W(\alpha, t) = [1 - \eta(t)] W(0)(\alpha) + \eta(t) W(1)(\alpha),$$

(17)

with $W(0)(\alpha)$ and $W(1)(\alpha)$ being the Wigner functions of the vacuum state and the one-photon Fock state. As we see, the excited outgoing mode is basically prepared in a mixed state of a one-photon Fock state and the vacuum state, due to unavoidable existence of unwanted losses, where $\eta(t)$ can be regarded as being the efficiency to prepare the excited outgoing wave packet in a one-photon Fock state:

$$\eta(t) = \int_0^\infty d\omega |F(\omega, t)|^2 \simeq \int_{-\infty}^\infty d\omega |F(\omega, t)|^2,$$

(18)

with

$$F(\omega, t) = \frac{d_{23}}{\sqrt{\pi \hbar \epsilon_0 A l}} \sqrt{\frac{c \omega^2}{\omega c^2}} \int_0^t dt' G^*(0^+, z_A, \omega) C_2^*(t') g(t') e^{i\omega(1-t')} e^{i\omega_{23} t'} e^{i\omega_{31} t'},$$

(19)

where $0^+$ indicates the position $z = 0$ outside the cavity. The excited outgoing wave packet (mid-frequency $\omega_k$) is characterized by the mode function

$$F_1(\omega, t) = \frac{F(\omega, t)}{\sqrt{\eta(t)}}$$

(20)
and spatiotemporal shape
\[ \phi(z, t) = \frac{1}{2} \int_0^\infty d\omega \sqrt{\frac{\hbar \omega}{\varepsilon_0 c \pi A}} e^{-i\omega z/c} F_1(\omega, t). \] (21)

Finally, inserting Eq. (20) together with Eq. (19) into Eq. (21), we find the spatiotemporal shape of the excited outgoing wave packet outside the cavity as (in the following, for the sake of simplicity, we assume \( \omega_{31} = 0 \))
\[ \phi(z, t) = \frac{R_k}{2} \sqrt{\frac{\hbar \omega_k \gamma_{\text{rad}}}{2\varepsilon_0 c A \eta(t)}} \int_0^{t-z/c} dt' C_2^* (t') g(t') e^{-i(\Delta_k + i\Gamma_k/2)t'} e^{i\omega_k^* (t-z/c)}, \] (22)

where \( \gamma_{\text{rad}} \) describes the wanted radiative losses due to transmission of the radiation through the fractionally transparent mirror.

4 Generation of time-bin wave packets

As we can see from Eq. (22) together with Eq. (14) the shape of single-photon outgoing wave packet can be changed by changing the rates and shapes of the emitter-cavity interaction and/or the driving pulse. In the context of time-bin entangled states of light, a question of interest is the generation of single-photon wave packets with a double-peak spatiotemporal structure, where every of these peaks have probability to carry the photon. Thus, let us assume that the desired double-peak spatiotemporal shape of the single-photon outgoing wave packet is given by a function \( \phi(z, T) \) at some time \( T \), where the condition \( T \gg \Gamma_{k}^{-1} \) ensures that the wave packet has almost completely left the cavity. From Eq. (22) we find that
\[ C_2(t) = \frac{2}{R_k g(t)} \sqrt{\frac{2\varepsilon_0 c A \eta(T)}{\hbar \omega_k \gamma_{\text{rad}}}} e^{-i\omega_2 t} \left\{ \frac{d\phi[c(T-t), T]}{dt} - i\omega_k^* \phi[c(T-t), T] \right\}. \] (23)

Further, inserting Eq. (15) into Eq. (14) we see that
\[ D(t) = f(t) + \int_0^t dt' f(t) f(t') C_2(t'), \] (24)

where \( D(t) \) is defined by
\[ D(t) \equiv \dot{C}_2(t) + \frac{R_k^2}{4} \int_0^t dt' C_2(t') g(t') e^{-i(\Delta_k - i\Gamma_k/2)(t-t')}, \] (25)

and \( f(t) \) is related to the shape of the pump pulse \( \Omega_p(t) \) according to
\[ f(t) = \frac{i}{2} \Omega_p(t) e^{-i\Delta_p t}. \] (26)
Differentiation of Eq. (24) with respect to $t$ then yields the following differential equation for $f(t)$:

$$D(t) \dot{f}(t) + C_2(t) f^3(t) - \dot{D}(t) f(t) = 0. \quad (27)$$

Hence, for the desired double-peak spatio-temporal shape $\phi(z, T)$ of the single-photon outgoing wave packet, the solution of the differential equation (27) yields the sought shape of the driving pulse.

In Fig. 2(a) we illustrate the shape of the driving pulse required for the generation of a symmetric double-peak wave packet in the case, when the emitter-cavity interaction is constant. Assuming equal probabilities for finding the photon in every of the time bins, this corresponds to a quantum superposition state of the form $|\Psi\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)$, where $|01\rangle$ and $|10\rangle$ describe finding the photon in the first and second bin, accordingly. In Fig. 2(b) we present the case, when a phase difference equal $\pi$ between the first and the second peaks is established.

Finally, the proposed method allows the generation of double-peak wavepackets of even more complex structure. Let us have a look at Fig. 3, which illustrates the generation of an asymmetric two-peak single-photon wave packet the shape of which resembles the monument called ”Tatik-Papik” (”Grandma-Grandpa”).

In summary, we have proposed a scheme for generation of single-photon time-bin double-peak wave packets based on resonant interaction of a three-level $\Lambda$-type emitter in a high-$Q$ cavity. We have shown, that control over relative phase of the individual time-bins is possible. Importantly, the results illustrate the ability to encode large amount of information within a single photon.
Figure 3: Shape of the driving pulse (solid red line) required to generate single-photon outgoing wave packets whose shape (dotted red line) resembles the "Tatik-Papik" monument.

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