Mathematical models and methods of assisting state subsidy distribution at the regional level

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Abstract. One of the most common forms of state support in the world is subsidization. By providing direct financial support to businesses, local authorities get an opportunity to set certain performance targets. Successful achievement of such targets depends not only on the amount of the budgetary allocations, but also on the distribution mechanisms adopted by the regional authorities. Analysis of the existing mechanisms of subsidies distribution in Russian regions shows that in most cases the choice of subsidy calculation formula and its parameters depends on the experts' subjective opinion. The authors offer a new approach to assisting subsidy distribution at the regional level, which is based on mathematical models and methods, allowing to evaluate the influence of subsidy distribution on the region's social and economic development. The results of calculations were discussed with the regional administration representatives who confirmed their significance for decision-making in the sphere of state control.

1. Introduction

One of the essential strategic goals of any region is to provide a decent standard of living to the population. To achieve this goal, it is necessary to deal with a number of socio-economic issues existing in a certain region, the most urgent of which are high unemployment, low per capita income, slow growth of gross regional product etc.

We have studied the current situation in a number of Russian regions and the history of their development, and our analysis shows that there are not only social, but also economic reasons for most problems. These problems can be solved by way of creating effective mechanisms of cooperation between the authorities and the local enterprises. By providing financial support to businesses regional authorities can encourage enterprises’ management to take an active part in tackling the most important socio-economic issues.

State support comprises various economic, legal and political measures aimed at promotion of entrepreneurship. Both, economic agents and public authorities, can benefit from that. One of the most common forms of state support in the world is subsidization [1–3].

By providing direct financial support to businesses, local authorities get an opportunity to set certain productivity and effectiveness goals and to control their achievement. As for
companies’ economic performance indicators that have special significance for most regions, we should mention profit, number of employees, output and average monthly nominal pay.

For regional authorities a successful result of economic support of business is the achievement of the set social and economic performance goals at the regional level. These goals are normally set for the following performance indicators: gross regional product, production index, average monthly nominal pay, number of profit-making enterprises, number of jobs, unemployment level etc. The target values for such performance indicators are reflected in the strategy and programs for social and economic development of the region.

In Russia subsidies are normally distributed at the federal, as well as regional level. At the federal level experts determine the general order of giving out subsidies, the industries to be supported and the amount of federal funds to be provided. Regional authorities, in their turn, provide for the order and concrete procedures of subsidies allocation. It must be stressed that the successful achievement of the set objectives depends not only on the amount of the budgetary allocations, but also on the distribution mechanisms adopted by the regional authorities.

The analysis of the existing mechanisms of subsidies distribution in Russian regions shows that in most cases the choice of subsidy calculation formula and its parameters depends on the experts’ subjective opinion. It has been mentioned by researchers that the decisions taken are often not sufficiently grounded and highly subjective. This allows for information manipulation. Besides, little attention is paid to specific characteristics of the supported enterprises. As a result, budgetary funds are misspent at all levels [4, 5]. Therefore, it is crucial to develop scientifically grounded approaches to elaboration of subsidies allocation procedures in order to improve their objectivity and effectiveness.

The complexity of this task is associated with the specific aspects of the region as a sophisticated economic and social system. Among these aspects we should mention the variety of elements and connections, the unique character of each enterprise, the complexity of enterprises’ behavior and the conflict between economic concerns of entrepreneurs and social concerns of the regional authorities, the lack of statistics available, high level on uncertainty etc. Mathematical models and methods are effective instruments helping to take decisions concerning such systems’ management.

The aim of this research is to develop a subsidy distribution algorithm that will assist regional authorities in elaboration of the procedures of financial support provision. The algorithm will be based on mathematical models and methods allowing to:

– assess (on the basis of the statistical data available) the influence of the possible variants of the subsidy’s distribution on the performance indicators of each enterprise having applied for financial support;

– assess the influence of the possible variants of the subsidy distribution on the regional development indicators;

– make well-grounded choice of the subsidy distribution rule with the aim of achieving the best possible social and economic results in the region’s development;

– determine the target values of the enterprises’ effectiveness indicators.

The theoretical part of our research was based on the scientific works dedicated to the problems of economic mechanisms theory (E. Maskin, R. Myerson and others [6, 7]); the research of market participants’ motives and behavior (G. Stigler, J. Tirole, V. L. Tambovtsev, A. Y. Shastiko and others [8, 9]); works on development of the optimal mechanisms for Russian State regulation system (A. A. Yakovlev, F. T. Aleskerov, V. Z. Mazloyev and many others [4, 10, 11]).

The possibilities of mathematical methods application for state support optimization have been widely discussed and investigated from different points of view. In this respect we need to mention the works by V. N. Burkov, D. A. Novikov and K. A. Bagrinovsky dedicated to the development of resource planning and distribution mechanisms at various management
levels [12, 13]; research by A. A. Yakushev concerning the problem of optimizing the mechanism for providing state financial support to small businesses by way of a probability classification model application [14]; works by G. A. Ougolnitsky, in which the author examines general hierarchical models for regional management [15] and many others. The main ideas and models of cooperation between the regional authorities and enterprises in the sphere of social and economic development indicators elaboration were discussed in the article by I. V. Goroshko and Y. V. Bondarenko [16].

The approach to the problem of subsidies distribution optimization proposed in this paper is characterized by special attention paid to the specificity of regional management and the peculiarities of the enterprises existing in the region. For each enterprise a separate optimization model for rational distribution of own funds and subsidies received is created, with due regard to the enterprise’s individual characteristics. The models’ parameters are determined on the basis of statistics available for the authorities of the region. The models allow regional authorities to calculate, with a certain level of accuracy, the anticipated values of economic performance indicators (output, profit, human resources, fixed assets cost) for each separate variant of subsidy distribution. To make the final decision concerning the procedure of subsidy distribution, the influence of the enterprises’ performance on social and economic indicators of the region’s development is evaluated. The main criterion of the optimal choice is maximization of the weighted total of the region’s development indicators’ standardized values.

2. Materials and methods

2.1. General description of the approach

To provide a formalized description of the problem of subsidies’ distribution, we need to consider a subsystem of the region’s socioeconomic system, which is comprised of the following elements:

1) regional enterprises performing their economic activities in the territory of the region;
2) regional government.

Each enterprise is associated with a governing body (administration of the enterprise), which is interested in getting maximum possible profit in the market of a certain region.

The regional government (regional authorities or regional administration) is responsible for handling strategic and tactical issues and determining the course of the region’s development that will ensure decent living standards for the people of the region. The key indicators of social and economic development of the region are prescribed in the development strategy and programs as target indicators. The regional government sets target values for each target indicator.

The success in achieving each of the prescribed target values depends on a number of factors. We will further consider social and economic indicators that are determined by the results of certain economic entities’ performance. Let us suppose that in order to achieve target values of performance indicators the authorities have taken a decision to provide state support in the form of a subsidy to certain entities.

For each subsidy, the regional authorities determine its recipient’s performance indicators and set their target values. The subsidy is given to an enterprise at the beginning of the year, while the results achieved are analyzed at the end of the year. Let us introduce the following notation:

- $B$ — the budget allocations for a subsidy provided in the current financial period (year) in the amount of $B^f$ from the federal budget and $B^r$ from the regional budget, $B = B^f + B^r$;
- $M$ — the number of enterprises eligible for the subsidy allocation ($m = 1, \ldots, M$);
- $\overline{R}$ — the set of social and economic indicators of the region’s development depending on the results of the enterprises’ economic activities;
- $J$ — the number of indicators of $\overline{R}$ set, ($j = 1, \ldots, J$);
• \( \bar{R}_j \) — target values of the region’s development indicators to be achieved during this period (year), \( j = 1, \ldots, J \);

• \( N \) — the number of the enterprises’ performance indicators determining the region’s development indicators \( n = 1, \ldots, N \);

• \( \bar{N} \) — the set of the enterprises’ performance indicators used for effectiveness evaluation;

• \( (s^m)^0 = ((s^m_1)^0, (s^m_2)^0, \ldots, (s^m_N)^0) \) — indicators of enterprise \( m \) performance in the last accounting period, the values of which are taken by the authorities from the statistical reports and accounting statements \( (m = 1, \ldots, M) \).

Distribution of subsidies among the entities is performed in accordance with certain rules and calculation (distribution) formulas. We will consider each rule to be a function, the arguments of which are the enterprises’ performance indicators, and the value is a nonnegative number — the amount of the subsidy provided. The formulas, as well as the parameters’ values, may differ from rule to rule.

The simplest formula for subsidies’ calculation is represented by a linear dependence:

\[
F(s) = (\alpha_f + \alpha_r) \cdot s,
\]

where \( s \) is the enterprise’s performance indicator selected for the subsidy’s calculation; \( \alpha_f, \alpha_r \) — are the rule’s parameters (the subsidy amounts provided respectively by the federal and the regional budget per unit of measure of the enterprise’s performance).

A rule can also have a more complex representation. For example, different parameters may be applied for different intervals of indicator \( s \):

\[
F(s) = \begin{cases} 
\alpha_1 \cdot s, & 0 \leq s \leq s_1, \\
\alpha_2 \cdot s, & s \geq s_1,
\end{cases}
\]

were \( \alpha_1, \alpha_2 \) are the rule’s parameters, \( s_1 \) is the threshold value of the performance indicator.

As we have already noted, the arguments of the rule are the enterprises’ performance indicators. Traditionally, in Russian regions these indicators are the amount of gross product sold and (or) intended for further processing by the recipient of the subsidy; cattle population; the amount of direct costs etc.

Naturally, not all the enterprises’ indicators are considered while distributing subsidies in compliance with a certain rule, but for notational convenience we will assume that the indicators that are not mentioned in the rules are included in the formulas with zero coefficients.

In the general case, we will assume that the region’s authorities have provided an \( H \) set of subsidy calculation rules that we will refer to as \( \Gamma \):

\[
\Gamma = \{F_1(s_1, \ldots, s_N), \ldots, F_H(s_1, \ldots, s_N)\}.
\]

The amount of the financial support \( B^m_h \) provided in compliance with the rule \( F^h \) to enterprise \( m \) depends on the value of its performance indicators for the last accounting period:

\[
B^m_h = F_h((s^m_1)^0, \ldots, (s^m_N)^0).
\]

Naturally, each of the rules of subsidy distribution must be bearable for the budget. It means that the total amount of the subsidy must not exceed the amount of the budgetary provisions:

\[
\sum_{m=1}^{M} B^m_h \leq B,
\]
Different rules of subsidies’ calculation have various effects on the performance of regional enterprises, and thus on the indicators of the region’s social and economic development. The main objective of the regional authorities is to work out such a rule for the subsidy’s calculation (permissible for the budget) $F^* \in \Gamma$, that will allow for maximum approximation of the target values of the region’s development indicators.

We will call the above mentioned problem ”the problem of subsidies’ distribution”. We propose to address this urgent management issue by way of following a seven-stage algorithm of subsidy distribution among the enterprises of the region.

**Algorithm of subsidy distribution among the enterprises of the region**

1. **Stage 1.** Specify the set of social and economic indicators depending on the results of the subsidy recipient’s performance, $\mathcal{R}$ set generation. Determine the admissible deviations from the target values.

2. **Stage 2.** Determine the set of subsidy calculation rules (1) differentiated by formulas and parameters’ values.

3. **Stage 3.** Evaluate the results of economic performance of the enterprises receiving state support in accordance with each rule of calculation.

4. **Stage 4.** For each variant of subsidy calculation, evaluate the changes in the selected indicators of the region’s development program.

5. **Stage 5.** Determine an integrated index for comparing the rules and for selecting the best possible variant. Calculate the value of the integrated index for each distribution variant.

6. **Stage 6.** Choose the rule for financial support calculation, which provides for the maximum value of the integrated index.

7. **Stage 7.** Determine target performance values for each recipient of the subsidy.

Let us contemplate the mathematical tools used at different stages of the algorithm implementation.

At the first stage, the regional government analyzes the dependence between socioeconomic indicators of the region’s development and the indicators of the subsidy recipient’s performance. $\mathcal{R}$ set is specified. For each indicator the quantitative dependency between its value and the enterprises’ performance indicators is determined:

$$R_j = R_j(s^1, s^2, \ldots, s^M), \quad j = 1, \ldots, J,$$

where $s^m = (s^m_1, \ldots, s^m_N)$ — the set of performance indicators for enterprise $m$ ($m = 1, \ldots, M$).

We should also mention that the formulas (3) can either be used for calculations or reflect the dependency between the indicators in the form of regression equations.

For each $R_j$ indicator the regional administration has determined the admissible deviation from the target value $\Delta_j \geq 0$. If an increase in $R_j$ indicator value is considered advantageous for the region, the admissible range of values is $Z_j = \left[\bar{R}_j - \Delta_j, +\infty\right)$. Conversely, when decrease in the indicator value is considered advantageous, the admissible range of values is $Z_j = \left(-\infty, \bar{R}_j + \Delta_j\right]$.

At the second stage of the algorithm implementation the set of possible rules for the subsidy calculation is determined. The formulas for the rules are developed by the experts from the regional administration on the basis of general requirements and recommendations. We suggest that one of the following strategies is to be used for determination of the rules’ parameters variants:
• experts’ participation. In this case experts from the regional government determine the possible parameters for subsidy distribution rules on the basis of their own experience and expertise;

• parameters’ generation. According to this strategy, each rule parameter is considered to be a random variable, the values of which are evenly distributed within the admissible variations interval. Different variants of random variable realization provide for different values of the parameters;

• mixed strategy combining the first two options.

The result of the second stage implementation must be a set of possible subsidy calculation rules:

\[ \Gamma = \{ F_1(s_1, \ldots, s_N), \ldots, F_H(s_1, \ldots, s_N) \} \].

The third stage will be carried out with the assistance of a mathematical tool — a model of rational funds distribution by the enterprise’s management. The mathematical model is created on the basis of the statistical data concerning the performance of each subsidy recipient, which is available to the regional administration. This model provides the regional authorities an opportunity to calculate the values of the enterprise’s performance indicators which can be achieved through rational distribution of own funds and subsidies received. In this context rationality is understood as the natural urge to get maximum possible profit.

In order to provide a general representation of the model, let us consider the example of enterprise \( m \). We will assume that at the beginning of the financial period (year) the enterprise’s management has own funds in the amount of \( B_m \) and, besides, the enterprise receives a subsidy, in compliance with rule \( h \), in the amount of \( B^h_m \).

The funds available are distributed by the enterprise’s management between current production and expansion (development) programs, which causes changes in the values of economic performance indicators. Each case of funds distribution can be represented as a manipulated value \( u_m(B_m, B^h_m) \).

The model of rational funds distribution by the enterprise’s management has to do with determining the management strategy that will provide for the maximum possible profit:

\[ \pi^m(s^m, u_m(B_m, B_m^h)) \rightarrow \max. \]

On condition that

\[ s_n = \varphi^m_n \left( (s^m)^0, u_m(B_m, B^h_m) \right), \quad n = 1, \ldots, N, \]  \hspace{1cm} (4)

where \( \pi^m(\cdot) \) — the enterprise’s profit, \( s^m = (s^m_1, \ldots, s^m_N) \) — the set of economic performance indicators (variable models); \( \varphi^m_n \) — the rule of changing the value of indicator \( n \) for enterprise \( m \).

The model described above (4) is an example of an optimization problem. The solution to this problem is represented by the optimal values of enterprise \( m \) performance indicators:

\[ \tilde{s}^m_h = \left( (s^m_1)^h, \ldots, (s^m_n)^h \right). \]

After the third stage for each subsidy recipient the values of performance indicators that can be achieved through rational distribution of funds are determined with regard for each subsidy calculation rule \( F_h \) (where \( h = 1, \ldots, H \)):

\[ \tilde{s}_h = \left( \frac{1}{s_h^1}, \frac{2}{s_h^2}, \ldots, \frac{M}{s_h^M} \right), \quad h = 1, \ldots, H. \]  \hspace{1cm} (5)
We suggest that these values should be considered as the basis for the assessment of the influence that each of the subsidy distribution rules has over the enterprises’ performance indicators.

The fourth stage has to do with the analysis and quantitative assessment of the changes in the region’s development indicators resulting from the changes in the enterprises’ performance indicators. The assessment is to be made separately for each variant of the subsidy’s distribution:

\[ R^h_j = R_j \left( \tilde{s}^1_h, \tilde{s}^2_h, \ldots, \tilde{s}^M_h \right), \quad j = 1, \ldots, J, \quad h = 1, \ldots, H. \]  

The subsidies’ calculation rules where each \( R^h_j \) indicator value belongs to acceptable range \( Z_j \) are chosen for further consideration. The set of indices used in these rules will be denoted by \( H' \):

\[ H' = \left\{ h : R^h_j \in Z_j, \quad j = 1, \ldots, J \right\}. \]

To make the final choice of the subsidy distribution rule you need to work out an integrated index allowing to evaluate the degree of conformity to the target values of the region development indicators:

\[ \Phi(R_1, R_2, \ldots, R_J) = \sum_{j=1}^{J} \lambda_j \cdot R^0_j(s^1, s^2, \ldots, s^M), \]  

where \( \lambda_j \) — is the priority index of the region development indicator \( j \) determined by the regional administration experts \( \left( \lambda_j \geq 0, \sum_{j=1}^{J} \lambda_j = 1 \right) \); \( R^0_j(s^1, s^2, \ldots, s^M) \) — the standardized value of indicator \( j \).

If an increase in \( j \) indicator value is considered advantageous for the region, its standardized value is calculated by the following formula: \( R^0_j(s^1, s^2, \ldots, s^M) = \frac{R_j(s^1, s^2, \ldots, s^M)}{R^*}. \) Conversely, when decrease in the indicator value is considered advantageous, the standardized value is calculated as follows: \( R^0_j(s^1, s^2, \ldots, s^M) = \frac{R^*}{R_j(s^1, s^2, \ldots, s^M)}. \)

The region’s authorities have to choose such a subsidy distribution rule \( F_{h^*} \in \Gamma \), where the integrated index \( \Phi \) acquires the maximum possible value:

\[ \Phi \left( R^h_1, \ldots, R^h_J \right) = \max_{h \in H'} \Phi \left( R^h_1, \ldots, R^h_J \right). \]

Now from the optimal values of the enterprises’ performance indicators calculated for \( F_{h^*} \) rule in accordance with model (4) we need to choose those indicators’ values that are used for effectiveness evaluation (included in \( \tilde{N} \) set). We suggest that these values should be considered target values.

The results of the algorithm application are the following:

- subsidy distribution rule \( F_{h^*} \) provides for maximum approximation of the region’s development indicators’ values to the target levels;
- target performance levels of each subsidy recipient are calculated for \( F_{h^*} \) rule in compliance with the models (4).

It has to be noted that the results of the algorithm application are auxiliary tools used for decision making. The experts working for regional authorities can adjust them as needed.
2.2. Refinement of the model of rational funds distribution by the enterprise management

For practical realization of the algorithm of subsidy distribution among the regional enterprises, the third-stage models (4) have to be refined. Each model’s restrictions are based on the results of research concerning the problems of Russian regions’ development, which has allowed to determine the key enterprises’ performance indicators. Among these indicators we need to mention: profit, output, average monthly nominal pay, number of employees and fixed assets.

For convenience, when mentioning enterprise $m$, we are going to omit hereinafter designation $m$. Let us introduce the following notation for the selected performance indices:

- $L$ — number of employees (human resources); $\omega$ — average annual nominal pay;
- $K$ — fixed assets;
- $\pi$ — the enterprise’s profit.

The funds available are comprised of own and borrowed funds in the amount of $B_0$ as well as the subsidy $B_h$, calculated in compliance with $F_h$ rule.

The parameters regulated by the management of the enterprise are:

- $y$ — output;
- $\delta$ — the share of money intended for fixed assets acquisition;
- $\nu$ — the share of money intended for creation of new jobs.

The model of rational funds distribution by the enterprise’s management will look like this:

$$
\pi \rightarrow \text{max}.
$$

The following restrictions must be taken into account:

- the enterprise’s manufacturing capability:
  $$
y \leq f(K, \omega \cdot L);
$$

- allocation of own and donated capital for fixed assets acquisition:
  $$
  K = (1 - \gamma)K_0 + \delta(B_0 + B_h);
$$

- allocation of own and donated capital for additional human resources:
  $$
  L = L_0 + \nu(B_0 + B_h)/\omega';
$$

- generation of current period profit:
  $$
  \pi = y - (b \cdot y + \omega \cdot L/(1 - \rho) + \gamma \cdot K_0);
$$

- financial restrictions:
  $$
  (\delta + \nu) \cdot (B_0 + B_h) + b \cdot y + \omega \cdot L/(1 - \rho) + \gamma \cdot K \leq B_0 + B_h;
$$

- restrictions on variables: $\delta, \nu, y \geq 0$.

For our model we use the following notation: $f(K, \omega \cdot L)$ — production function; $\gamma$ — rate of fixed assets depreciation; $\rho$ — payroll tax; $b$ — manufacturing costs per one unit; $K_0, L_0$ — fixed assets and number of employees at the beginning of the year, respectively; $\omega'$ — cost of workplace creation.
The model provided has been worked out for an optimization problem with variables \( y, \delta, \nu \). The target function is linear, while the system of restrictions contains an inequation providing for the enterprises’ manufacturing capabilities, with a non-linear production function. The problem with a non-linear (and, in the general case, non-convex) production function should be solved by Sobol method that was described in [17].

For each subsidy distribution formula the ideal solution provides for such values \( \tilde{y}_h, \tilde{\delta}_h, \tilde{\nu}_h \) that will allow to calculate the enterprise’s profit \( \tilde{\pi}_h \), the number of employees \( \tilde{L}_h \), fixed assets \( \tilde{K}_h \).

2.3. Results and discussion

To apply the subsidy distribution algorithm towards the companies of the region, a software package has been developed in C# environment. The package includes: a database containing information on a number of enterprises from Voronezh Region, a set of computation programs providing solutions for non-linear optimization problems and a procedure of generating parameters for subsidy calculation rules. We have tested this algorithm on training and real-life tasks, and verified the models’ validity.

Let us consider, for example, a subsidy provided in Voronezh Region to agricultural companies (other than private farms) for partial compensation of costs associated with prevention of African swine fever epizooty.

The target indices of the development program and their target values are shown in Table 1.

| Targets of the development program | Target values (2017) | Calculation formulas | Indices |
|-----------------------------------|----------------------|----------------------|---------|
| Pork production (live weight/meat on hoof) | 130.5 ths. tons | \( R_1 = \sum_{m=1}^{M} y^m, \) where \( y^m \) is pork production at enterprise \( m \) | Pork production |
| Creating new vacancies 100 | 100 staff positions | \( R_2 = \sum_{m=1}^{M} \Delta L^m, \) where \( \Delta L^m \) is the number of jobs created at enterprise \( m \) | The number of jobs created |
| Maintenance of index wages | 19 867 rub/month | \( R_3 = \frac{1}{12} \cdot \mu \sum_{m=1}^{M} \Phi^m, \) where \( R_3 \) is average monthly nominal pay, \( \Phi^m \) — annual payroll budget of enterprise \( m \), \( \mu \) — average number of employees per year at \( M \) enterprises | Average annual pay, number of employees |
In the Government Decree of Voronezh Region No 132 dated 15 February 2017 the following order of subsidy calculation is provided for:

\[ F = \alpha \cdot y^0, \]  

where \( F \) is the subsidy amount paid to the recipient (rub); \( \alpha \) — the subsidy rate; \( y^0 \) — the amount of pork sold by the subsidy recipient (in 2016).

The subsidy rate approved by the Voronezh region government in 2017 was \( \alpha = 674 \text{ rubles/ton} \).

Table 2 contains a fragment of a chart showing agricultural enterprises from Voronezh Region, the amounts of pork sold in 2016 and the subsidy amounts for 2017.

| Agricultural enterprises                  | Amount of pork sold (thousands of tons) in the year 2016 | Subsidy, ths. rub |
|-------------------------------------------|--------------------------------------------------------|-------------------|
| "APK Agroeco"                             | 21.2                                                   | 14 288.8          |
| "Agroresource-Voronezh"                   | 2.4                                                   | 1 617.6           |
| "SGC"                                     | 6.7                                                   | 4 515.8           |
| "Vishnevsky"                              | 4.8                                                   | 3 235.2           |
| "Alfa"                                    | 3.8                                                   | 2 561.2           |
| "Agricultural Enterprise Moskovskoe"      | 3.6                                                   | 2 426.4           |
| "IX Pyatiletka"                           | 2.5                                                   | 1 685.0           |
| "Kalacheevsky Meat Plant"                 | 2.4                                                   | 1 617.6           |
|                                            | ...                                                   |                   |
| **Total**                                 |                                                        | **48 750**        |

The actual amount of the budgetary provisions for subsidy in 2017 is 48 750 thousand rubles. All in all, 84 thousand tons of live weight pork was sold by the regional enterprises in 2016. The figures show that the total amount of subsidy calculated by formula (8) must be 56 616 thousand rubles, but due to the lack of funds some of the enterprises were refused financial support. For organizational reasons, most of these enterprises are usually small companies whose role in the region’s development is undoubted. Moreover, outbreaks of African swine fever are especially common in small farms.

In this calculation experiment we used type (8) rules with different rate values. We made calculations on the basis of the statistical data collected for 45 large- and medium-scale hog-raising farms of Voronezh region. Some of the calculations results are presented in Table 3.

Let us analyze the results of our calculations. It must be noted that the target value of the average monthly nominal pay was calculated with allowance for the models’ restrictions.

The subsidy rate \( \alpha = 750 \) is not effective enough and cannot be recommended for further consideration.

The first variant offered (implying the current subsidy rate) provides for substantial support offered to important market players, which is likely to maximize the output growth. At the same time, the number of new jobs is not so impressive.

If the subsidy rate is lowered, more small to medium-scale businesses will be eligible for financial support, which will allow them to expand and to create new jobs.
Table 3. Some results of the calculation experiment.

| Subsidy rate, rub/ton | Calculated values of target indicators |
|-----------------------|---------------------------------------|
|                       | Pork production, tons ($R_1$) | Number of new vacancies, staff positions ($R_2$) |
| 674                   | 94 531.4                     | 67                           |
| 590                   | 92 008.6                     | 75                           |
| 750                   | 87 989.4                     | 62                           |
| 550                   | 89 001.2                     | 77                           |

Let us develop an integrated criterion for choosing the subsidy distribution rule:

$$
\Phi(R_1, R_2) = 0.3 \cdot R_1/130500 + 0.7 \cdot R_2/100.
$$

The value of the integrated criterion for the rate $\alpha = 674$ is 0.686 points, while for the rate $\alpha = 590$ it is 0.736 points, while for $\alpha = 550$ it is 0.743 points. In accordance with the chosen integrated criterion, the following rule for subsidy calculation may be offered:

$$
F = 550 \cdot y^0.
$$

It must be noted that due to the importance of the problem under consideration, the results of the calculations must be verified and, if needed, amended by the government experts.

When we discussed this approach with the government representatives, the following advantages were mentioned:

1) substantial assistance in generating an expert opinion concerning subsidy distribution rules;
2) cutting down the time needed for taking a decision, and improving judgement at the same time;
3) the ability to carry out calculation experiments with different rules of subsidy distribution.

The main disadvantage of the method is associated with the necessity to keep a database. In future the method could be improved by way of developing instruments that would allow to consider various typical situations. We are also going to introduce our software product.

3. Conclusion

In this article we offer a new approach to assisting subsidy distribution at the regional level, which is based on mathematical models and methods. The algorithm proposed allows:

- to work out a rule for subsidy calculation, which will allow the region’s development indices to come as close as possible to the target values;
- determine the target values of performance and effectiveness for each subsidy recipient.

The algorithm proposed is based upon a mathematical model of rational finds distribution by the enterprise’s management. The restrictions of the models reflect the basic production dependencies, while the parameters are unique for each enterprise. The model provides an opportunity to calculate such values of performance indicators for each case of subsidy allocation, which can be reached on condition of rational funds distribution. The experiment with different variants of subsidy calculation rules provides an opportunity to make a justified choice of the rule and target values of the enterprises’ performance indices. The use of statistics available for the region’s authorities ensures practical implementation of the approach proposed.
The calculation experiments carried out prove that it is not only necessary, but also possible to make regional subsidy distribution mechanisms more objective and justified. Discussion of the approach and the experiment results with the regional administration shows its practical significance. The basic opportunities for further development of the method are now being considered.

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