A memristor random circuit breaker model accounting for stimulus thermal accumulation

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Abstract: Random circuit breaker (RCB) model is a powerful tool to investigate the formation and rupture processes of conductive filaments which occur in unipolar memristor devices. However, the existing RCB models do not integrate the time and thermal parameters, which downgrades significantly the model accuracy in emulating the dynamics of conductive filaments under pulse stimulus. Meanwhile, current research lacks detailed discussions about the above-mentioned problems. In this paper, a SPICE-based optimized RCB model is proposed to explore the unipolar resistive switching characteristics of memristor devices under pulse stimulus. Compared with the original RCB model, the set and reset transitions of each breaker in the proposed model are assumed to be dominated by the thermal heat, which introduces time variable by cascading the thermal equivalent circuits on the original main breaker network and thus allows to explore the filament formation and rupture dynamics along with time. It can simulate all unipolar memristor device operations realized by the original RCB model and is accurate enough to interpret effectively the novel features arising from pulse stimulus such as nonlinear current-voltage relation, multi-states storage capacity, etc.

Keywords: memristor, RRAM, random circuit breaker model, thermal effect

Classification: Electron devices, circuits and modules

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1 Introduction

Resistive Random Access Memory (RRAM) based on transition metal oxide, also known as memristor [1], is significantly attractive for future non-volatile memory applications due to its excellent features such as extremely scaling prospects, fast switching and multi-state memory storage capabilities [2, 3]. Although the underlying physical mechanism for the resistive change is still under debate, the formation and rupture of conductive filaments composed of oxygen vacancies and/or metal precipitates are widely acknowledged to be responsible for the resistive switching dynamics [2, 4]. In order to explore the filament formation/rupture characteristics, several filamentary models have been proposed [5, 6, 7]. Among them, a random circuit breaker (RCB) model [5] was proposed to explain the reversible resistive switching (RS) in the TiO₂ based RRAM and have demonstrated its validity on explaining the RS phenomena, e.g. wide distribution of SET/RESET voltage [8], stochastic switching [9], resistive state variability [10, 11].
The pulse-based electrical characterization is an important method to explore the underlying switching mechanism of RRAM devices. For example, Pickett et al. [12] studied the resistive switching dynamics by using a pulse-based electrical state-test protocol and then built a physical memristor model. Salaoru et al. [13] applied pulse stimulus on TiO2-based RRAM devices to study the coexistence behaviour of memristor and memcapacitor. Stephan et al. [14] explored the ultra nonlinear switching behaviour between the resistive state change and the applied pulse width/amplitude. Meanwhile, there arises numerous new RRAM features from the pulsing characterization process such as non-linear current-voltage relation [15], multi-states storage capacities [10, 15], statistical behaviours of the programming threshold voltage [15], voltage-time dilemma during the resistive state tuning [2, 13]. However, the evolution of filament formation and rupture under voltage pulse stimulation cannot be effectively interpreted by the original RCB model due to the lack of time information within it.

Previous literatures have demonstrated that Joule heating is the key factor during the filament rupture process (reset) within the device state changing from low resistance state (LRS) to high resistance state (HRS) in the unipolar switching devices [2, 4]. Although Joule heating also plays an important role in bipolar switching devices, this manuscript only focus on the unipolar switching phenomena. The simulation of reset process with heat effect has been covered by the models in Refs. [6, 7], while the study of thermal effect in the electroforming/set process is still missing. Actually, Joule heating has been widely found accompanying the electroforming/set process [4, 16, 17]. Karg et al. [16] found the working Cr-doped SrTiO3 cells can achieve several hundred degrees centigrade through the infrared thermal micrograph, which proves a large amount of Joule heating generation during the switching. Strachan et al. [17] probed the electrode-TiO2 interface of electroformed Pt/TiO2/Pt RRAM devices and found a threshold temperature of 350°C above which the amorphous TiO2 would be crystallized to anatase structure to form the localized conductive channels. Kim et al. [4] found the volcano-like morphological change occurring in Pt/TiO2/Pt electroforming process and deduced that the thermally driven phase-change mechanism could not be excluded in the RRAM devices. Based on the observations above, it is reasonable to assume the set transition from HRS to LRS may also be dominated by thermal heat.

Due to the lack of integration of time and thermal parameters, the existing RCB models [5, 6, 7, 11] are quite insufficient to emulate the evolution dynamics of filament under pulsing during the set transition, rendering a significant degeneration of the model accuracy. To improve the RCB model accuracy and expand the applications of the original RCB model, this paper integrates the time and thermal variables into the optimized model by introducing the thermal-driven switching rule and equivalent thermal circuits. Compared with the other RCB models [5, 6, 7, 11], both the set and reset transitions of the circuit breaker is assumed to be controlled by its local temperature. This assumption makes the optimized model capable to simulate not only the general features of RRAM such as electroforming, set and reset operations, but also explain effectively the new RRAM features arising from the pulse operations.
In this manuscript, section 2 describes the working principles of the proposed model. Then in section 3, we firstly show the model validity on simulating the forming, set, reset operations, then we employ this model to study the statistical features of the threshold voltage and the phenomenon of voltage-time dilemma during device resistance modifying process.

2 Thermal-driven RCB model

Fig. 1. Illustration of the working principle in the thermal-driven RCB model. (a) Main breaker network of the model. Blue: off-state circuit breaker with value of $r_h$; Red: on-state circuit breaker with value of $r_L$. (b) Heat generation and dissipation demonstration for each circuit breaker. $T_{loc}$: local temperature of the breaker. (c) Thermal equivalent circuit deduced from Eq. 1. (d) Thermal-driven switching rules. When $T_{loc} > T_{set}$, the $r_h$ of the off-state breaker switches to $r_l$; while if $T_{loc}$ of the on-state breaker is above $T_{reset}$, the $r_l$ of the on-state breaker changes to $r_h$.

Fig. 1 depicts the schematic sketch of the proposed model. We divide the thin film into a two-dimensional resistor network consisting of $M \times N$ circuit breaker cells. Each circuit breaker has two metastable value of $r_h$ and $r_l$ ($r_h \gg r_l$), marked in blue and red color, respectively.

In the original RCB model [5], the switching of the circuit breaker is assumed to be only dependent on the absolute value of voltage drop across each breaker, denoted as $\Delta v$. When $\Delta v > V_{on}$, the off-state breaker switches to on-state. In contrast, the breaker with on-state will switch to off-state when $\Delta v > V_{off}$. The $\Delta v$ value of one breaker depends on its location, the whole on-state breakers distribution and the external voltage bias value [5]. This kind of switching rule is termed as the voltage-driven rule in the manuscript. In the proposed model, we assume the breaker state switching is totally controlled by the Joule heating, which is called thermal-driven switching rule [6].

As shown in Fig. 1(a), the external voltage pulse stimulus $V_{ext}$ is applied on top electrode (TE) while bottom electrode (BE) is grounded. During biasing, the local temperature ($T_{loc}$) of each breaker will increase due to the joule heating effect of flowing current, while the $T_{loc}$ will decrease because of the heat conduction.
dissipation to the surrounding space. Accordingly, the $T_{loc}$ change is the competing result of Joule heating and thermal dissipation. The competing process is illustrated in Fig. 1(b). To simplify the analysis and keep consistent with results in Ref. [6], we assume the heat only dissipates to a thermal bath with a constant temperature $T_b$, while the heat interaction among breakers is out of the consideration. The time-dependent local temperature $T_{loc}$ obeys the thermal equation [6]:

$$\frac{\Delta v^2}{r} = i^2 r = C_{th} \frac{dT_{loc}}{dt} + \frac{T_{loc} - T_b}{R_{th}}$$

where $C_{th}$, $R_{th}$, $r$ are the model parameters of thermal capacitor, thermal resistor and circuit breaker resistance, respectively. The equivalent thermal circuits are shown in Fig. 1(c) where the thermal source is represented by a controlled current source with value decided by the Joule heating power $i^2 r$ or $\Delta v^2 / r$. By adopting the thermal circuit, the breaker local temperature and voltage can be obtained in a single step other than two separate steps performed in Ref. [6]. Fig. 1(d) shows the thermal-driven switching rule with two threshold values $T_{set}$ and $T_{reset}$ ($T_{reset} > T_{set}$). In specific, the circuit breaker will switch from $r_h$ to $r_l$ when $T_{loc} > T_{set}$, while switch the opposite direction when $T_{loc} > T_{reset}$.

The simulation is implemented via the cooperation of Matlab R2013b and LTspice V4.22. Matlab firstly writes scripts to build the main circuit breaker network and the attached thermal circuit network. Then Matlab applies the pulse stimulus and calls the LTspice to calculate node voltage and cell current based on Kirchhoff’s current and voltage laws. Finally, the simulation data is collected by Matlab and the thermal-driven switching rule is checked for each breaker. It is noted that LTspice works in transient simulation mode, which facilitates to obtain the time-dependent $T_{loc}/$node voltage/node current distribution.

3 Result and discussion

We employ $50 \times 20$ lattice to verify the proposed model functionality. The evolution of conductive filaments during the test sequence of electroform - reset - set is illustrated in Fig. 2. During the simulation, we use the staircase pulses which increase the amplitude in steps of 0.1 V with fixed pulse duration time $pw$ of 1 ms. Other model parameters are listed in the caption of Fig. 2(b). Initially, 1% on-state circuit breakers are uniformly random distributed inside the lattice with four on-state breakers piled up vertically to form the filament preferable generation path. From Fig. 2(a) to Fig. 2(b), the electroforming procedure is implemented by connecting sets of on-state breakers to establish a penetrated filament. In this case, the resistance of the network is quite low and the device enters LRS. Compared Fig. 2(b) with Fig. 2(c), the RESET procedure is completed by breaking some on-state breakers. Those breakers have the largest current and consequently become the hottest locations in the network. Finally, the penetrated filaments are restored above the resident broken filaments in the previous RESET procedures. This SET process is shown in Fig. 2(d). The Fig. 2(e) and Fig. 2(f) show the corresponding $I-V$ and $R-V$ curve during the three procedures, respectively.

It is clear from Fig. 2(e) that the simulated electroforming voltage is much larger (4.9 V) than the set voltage $V_{set}$ (2.3 V). The similar result can also be
obtained by employing the original RCB model [5], which demonstrates the optimized model is accurate enough to reproduce the basic behaviours of RRAM devices. Besides, the optimized model can emulate the non-linear $I-V$ relation which is more consistent with the practical devices characteristics [14, 15]. The non-linear characteristics is more clearly observed in $R-V$ domain as shown by the Electroforming and SET curves in Fig. 2(f). It is worth noting that this feature cannot be attained from the original RCB model. Another advance of the optimize feature is the capability to simulate the multi-state behavior of RRAM devices. Here we take the Electroforming/SET process in Fig. 2(f) as an example. When $V_{ext}$ is small, the $T_{loc}$ for any breaker is too small to trig a switching event due to the low flowing current. Accordingly, the network resistance stabilizes at the HRS. After $V_{ext}$ exceeds a threshold voltage $V_{threshold}$, several intermediate resistance states are observed. That means we can use a signal pulse or sets of pulses to tune the device resistance. In comparison, the electroforming $R-V$ curve employing the original RCB model [5] is depicted in Fig. 2(f) (the blue dash line) where an abrupt drop from HRS to LRS without intermediate states is observed.

Fig. 2. Model operation illustration. (a) Initial state with four on-state circuit breakers piled on the top while others are evenly distributed. (b) Electroformed state with sets of on-state breakers connected together. (c) RESET state with some circuit breakers broken near the TE. (d) SET state. Off-state breakers near the TE re-link again. (e) Current vs voltage during three procedures above. (f) Corresponding resistance change as the function of applied voltages. The blue dash curve indicates the electroforming process simulated by the original RCB model [5]. For all panels, thermal RCB model parameters are listed: defect density $p = 1\%$, $M = 50$, $N = 20$, $r_b = 100$, $r_i = 1$, $R_{th} = 1.515 \text{ M}\Omega$, $C_{th} = 0.0173 \text{ pf}$, $T_{set} = 350$, $T_{reset} = 931.5$, $pw = 1 \text{ ms}$. For the original RCB model, $V_{set} = 0.2 \text{ V}$, other parameters are set the same with the optimized RCB model.
To gain further insight into the working principle of the optimized RCB model, four snapshots during the electroforming process are extracted out to illustrate the dynamic evolution of conductive filaments. The results are depicted in Fig. 3(a)–(d). Each snapshot is attained after the pulse with value shown above each figure is applied and the switching rules are executed. It can be seen that the percolation of breaker cluster is an avalanche process. In the beginning, when the stimulus is relatively smaller but above the $V_{\text{threshold}}$, the filament grows slowly as shown from Fig. 3(a) to 3(b). Then, as the pulse amplitude continues increasing, an abrupt increase in the number of on-state breakers occurs, which is obvious by comparing Fig. 3(b) with Fig. 3(c). The abrupt change can be attributed to two main factors. On one hand, as the length of connected filament increases, the number of off-state breaker ahead the longest filament is reduced. Consequently, the average voltage $\Delta v$ on each reminding off-state breaker increases. On the other hand, higher pulse amplitude in the later stage of pulsing brings higher average $\Delta v$. Because of the relation that higher $\Delta v$ generates more heat during the same given stress time, it is more likely for a large number of reminding off-state breakers to reach the threshold temperature simultaneously as the pulsing process continues. This accumulation of pulsing effects and the significant acceleration of resistive change by larger voltage have been reported in real $\text{TiO}_2$-based RRAM device [15, 18], which in turn demonstrates the proposed model validity.

Fig. 3(e) and (f) present the internal voltage and temperature distribution after 4.6 V pulse is applied on the breaker network of Fig. 3(a) but before the switching rule is checked. This allows to find out how the switching rule works. It is noted that a high temperature region shown with yellow color in Fig. 3(f) lays below the
longest filament. Meanwhile, the voltage in the corresponding region in Fig. 3(e) is relatively higher than other region. The temperature in the region A in Fig. 3(f) is too low to trig any switching event, which is correlated with the theoretical hypothesis that the filament formation is a self-limited process [19]. In specific, the longest filament serves a shunt function to the adjacent horizontal shorter filaments, or in other words, the short filament grow will be suppressed by the adjacent longer filament. In Fig. 3(e) and (f), the dashed rectangles mark the locations where the set switching happens. It is noted that the pattern where the off-state breakers with highest temperature also have high voltage drop $\Delta V$, implying an equivalent relation between the thermal-driven and voltage-driven switching rules. Therefore, the proposed thermal RCB model is capable to implement all the functions of the original RCB model.

![Threshold voltage simulations.](image)

**Fig. 4.** Threshold voltage simulations. (a)(b): two circuit breaker configurations with same initial resistance. (c) resulting $R_V$ curves during SET process. Inset: applied $V_{ext}$ waves. (d) $V_{threshold}$ stochastic behaviour for 20 times simulations. The same number of on-state breakers are randomly distributed in the lattice for each simulation. (e) $V_{threshold}$ dependence on pulse width based on Case I. $\tau$ is the thermal constant. For all panels, defect density $p = 10\%$. All other parameters are the same with Fig. 2.

We also find a threshold voltage $V_{threshold}$ existing in Fig. 2(f) during the electroforming and SET process below which the network resistance remains unchanged. The threshold voltage is generally considered as a definite value which is uncorrelated to the device initial states and types of external electrical stimulus in other memristor device models [20, 21]. However, in our $TiO_2$ solid-state RRAM device, we find the threshold voltage is pulse-duration-dependent with sorts of stochastic features [15]. In specific, longer pulse duration is associated to smaller threshold voltage. For each pulse duration, the threshold voltage lies in a certain range rather than a definite value. This behaviour can be well interpreted by the
proposed RCB model. Firstly, regarding the $V_{\text{threshold}}$ stochastic behaviour emulation, we employ two on-state breaker configurations as shown in Fig. 4(a) and (b) where the same percentage (10%) of on-state breakers are equally randomly distributed inside the breaker lattice. After applying same pulses sequence depicted in the inset of Fig. 4(c), the HRS in both cases are switched into LRS. The resulting $R_V$ curves are also shown in Fig. 4(c). Although the difference of the initial resistance for two cases is negligible, they experience dissimilar switching dynamics. The $V_{\text{threshold}}$ for case-I is 3.6 V, slightly smaller than that of case-II (3.8 V). After repeating the same simulation for 20 times, the statistics of $V_{\text{threshold}}$ are depicted in Fig. 4(d) and it is clear that $V_{\text{threshold}}$ ranges between 3.0 V and 4.0 V. It is worthy to point out that $V_{\text{threshold}}$ can be significantly concentrated if the filament preferable generation path, similar with Fig. 2(a), exists in the initial filament distribution [5]. Regarding the pulse-duration-dependent feature, the $V_{\text{threshold}}$ simulation result is depicted in Fig. 4(e) where the stair-case ramp pulse stimulus with $pw$ varying from 0.01$t$ to 10$t$ is applied on the Case-I breaker network. The $\tau$ ($=R_{th}C_{th}$) is the thermal constant for the thermal circuit in Fig. 1(c). It can be seen that a clear voltage-duration trade-off is observed during the simulation. We also observe that $V_{\text{threshold}}$ is delayed faster than exponentially with the increase of pulse width before it reaching the thermal constant $\tau$, which is in accordance with the practical RRAM device test [15].

It has been reported that a voltage-time dilemma between fast switching under write process and long retention under read voltage stress [2] occurs in many transition metal oxide RRAM devices, e.g. TiO$_2$ [15], SrTiO$_3$ [14], HfO$_2$ [22]. The proposed model can provide an alternative viewpoint to interpret this phenomenon. To be consistent with the experiment data of SrTiO$_3$ [14], we simulate the SET process on Case-I breaker distribution in Fig. 4(a) with $pw$ ranging from 0.01$t$ to 10$t$.
10r and pulse voltage ranging from 3 V to 6.5 V. Simulated results are shown in Fig. 5. It is noted that to attain the same resistance change, simulation cases under narrow \( pw \) require higher stimulus voltage than that with wider \( pw \) case. For an example, case A and B in Fig. 5 attains nearly the same resistance change, while the applied potential with narrow \( pw \) (0.01r) is nearly two times larger (5.95 V) than pulse (3.65 V) with wider \( pw (r) \), exhibiting enormously high non-linearity of switching kinetics. The voltage-time dilemma is mainly attributed to the \( T_{loc} \) dependence on the competition of heating generation and heating dissipation [6].

Considering the hottest breaker point, Eq. 1 becomes:

\[
\frac{dT_{loc}}{T_{loc} - T_b} = (\tau_1 - \tau_2)dt
\]

where \( \tau_1 = \Delta v^2/\rho h/[C_{th}(T_{loc} - T_b)] \) and \( \tau_2 = 1/(R_{th}C_{th}) \), and the initial value of \( T_{loc} \) is \( T_b \). The first and second terms come from Joule heating and heat dissipation process respectively. When \( V_{ext} \) is small enough so that \( \tau_1 < \tau_2 \), \( T_{loc} \) cannot reach \( T_{set} \) as pulse width increases. In this case, the device state cannot be changed. If \( V_{ext} \) is big enough to cause \( \tau_1 > \tau_2 \), \( T_{loc} \) increases as time passes. When \( T_{loc} \) reaches \( T_{set} \), the off-state breaker changes to on-state. Because of the inverse relation between \( \Delta v \) and stress time \( dt \), the stress time under high write voltage is much smaller than that in low write voltage. The analysis implies that a small read voltage can be used to sense the resistance of RRAM devices without disturbance to the internal state, while a higher voltage can be used in the write process to obtain much faster switching speed.

### 4 Conclusion

In this manuscript, a SPICE-based optimized RCB model comprising equivalent thermal circuits has been proposed. The model assumes both the set and reset transitions of one circuit breaker are dominated by the local temperature. After analysis of the temperature and voltage distribution inside the network, we find the highest local voltage region also possess the highest temperature. In this case, the switching rules of voltage-driven and thermal-driven can be regarded equivalent. Simulated results demonstrate the proposed model can achieve all the functions of original RCB model. In addition, the RS features arising from pulsebased electrical characterization process such as nonlinear current-voltage relation, multi-states storage capacity, statistical behaviours of the programming threshold voltage, and the voltage-time dilemma are explored by the proposed model. We conclude that the stochastic feature of threshold voltage comes from the difference of the on-state breaker distribution and the voltage-time dilemma results from the competition of thermal heat generation and dissipation. Future work will focus on studying the effect of time interval between two consecutive pulses and model validation with more RRAM device experiment data.

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