Precision electroweak tests of the standard model

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Abstract. CKM Unitarity in the Standard Model predicts $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$. Experiments currently give 0.9999(6). The outstanding agreement constrains “new physics” effects at the tree and quantum loop level. Examples considered are: exotic muon decays, heavy quark or lepton mixing, high scale induced 4 fermion operators (e.g. excited W* bosons from extra dimensions) and additional Z' gauge bosons. Also, combining $K_{\mu2}$ decays and CKM unitarity gives a charged Higgs mass bound $m_{H^\pm} \gtrsim 5 \tan \beta$ GeV. Constraints from other precisely measured electroweak observables are also discussed.

Unitarity of the bare (unrenormalized) Cabibbo-Kobayashi-Maskawa (CKM) $3 \times 3$ quark mixing matrix $V_{ij}$, $i = u, c, t$ and $j = d, s, b$ implies the orthonormal tree level relations

$$\sum_i V_{ij}^0 V_{ik}^\dagger = \sum_i V_{ji}^0 V_{ki}^\dagger = \delta_{jk} \quad (1)$$

Standard Model quantum loop effects are important and corrected for such that eq. 1 continues to hold at the renormalized level [2]. That prescription generally involves normalization of all charged current semileptonic amplitudes relative to the Fermi constant

$$G_\mu = 1.1663788(7) \times 10^{-5} \text{ GeV}^{-2} \quad (2)$$

generated from the precisely determined (recently improved) muon lifetime [3]

$$\tau_\mu = \Gamma^{-1}(\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu(\gamma)) = 2.1969803(22) \times 10^{-6} \text{ sec} \quad (3)$$

measured to about 1 ppm. In all processes, Standard Model SU(3)$_C \times$SU(2)$_L \times U(1)_Y$ radiative corrections are explicitly accounted for [4].

Of particular interest here is the first row constraint

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad (4)$$

An experimental deviation from that prediction would be evidence for “new physics” beyond Standard Model expectations in the form of tree or loop level contributions to muon decay and/or the semileptonic processes from which the $V_{ij}$ are extracted. Of course, if eq. (4) is respected at a high level of certainty, it implies useful constraints on various “new physics” scenarios.

In the case of $V_{ud}$, its value is extremely well determined from superallowed nuclear beta decays ($0^+ \rightarrow 0^+$ transitions) which are theoretically pristine to a high level of certainty [5]. A recent survey finds [6]...
where the first error stems from experiment and nuclear theory while the second is due to radiative corrections [7]. The radiative corrections to $|V_{ud}|^2$ are rather large \cite{5,7} $\sim 3.6\%$ but relatively free of strong interaction uncertainties $\pm 0.04\%$. The good agreement among the many different nuclei surveyed \cite{6} over a large range in $Z$, suggests that nuclear theory uncertainties are fairly represented by the error on $V_{ud}$ (see however \cite{8}).

It appears unlikely that $V_{ud}$ will shift much beyond the range in eq. (5). Nevertheless, a more definitive determination, without nuclear uncertainites, would be welcome and is in principle possible using (planned) precise measurements of the neutron lifetime, $\tau_n$, and the decay asymmetry parameter, $g_A \equiv G_A/G_V$, via \cite{7,9}

$$|V_{ud}|^2 = \frac{4908.7(1.9)\text{sec}}{\tau_n(1 + 3g_A^2)}$$

(6)

Note, one can turn eq. (6) into a determination of $\tau_n$ via $|V_{ud}|$ and $g_A$. Then, using $g_A = 1.2750(9)$ favored by the most recent experiments \cite{10} rather than the PDG \cite{11} average of 1.2695(29) along with $|V_{ud}| = 0.97425(22)$ suggests $\tau_n = 880.0(1.1)\text{s}$, well below the PDG \cite{11} average of 885.7(8)s and close to a relatively recent high precision measurement \cite{12} of 878.5(8)s. Such a significant downward shift in $\tau_n$ and increase in $g_A$, if confirmed, would have important implications for many aspects of nuclear and particle physics such as: the proton spin distribution, muon capture, Bjorken sum rule, neutrino flux rates and cross-sections etc. It would have a major impact \cite{13} on primordial nuclear abundance predictions within big bang nucleosynthesis along with their implication for sterile neutrinos and other hypothetical light new particles.

The value of $V_{us}$ can be obtained from $K_{\ell 3}$ decays ($K \to \pi\ell\nu$, $\ell = e, \mu$). Flavianet \cite{14} global fits to the neutral and charged $K_{\ell 3}$ decay rates give

$$f_+(0)|V_{us}| = 0.2163(5)$$

(7)

where the form factor $f_+(0)$ deviates from 1 due to small second order SU(3) breaking. Recent lattice gauge theory studies \cite{14} find

$$f_+(0) = 0.960(5)$$

(8)

From that result and eq. (7), one obtains

$$|V_{us}| = 0.2253(13) \quad K_{\ell 3} \text{ decays}$$

(9)

A second independent determination \cite{15} of $V_{us}$ with similar sensitivity comes from experimental measurements of $\Gamma(K^+ \to \mu^+\nu)/\Gamma(\pi^+ \to \mu^+\nu)$ combined with a lattice gauge theory calculation of $f_K/f_\pi$. Currently one finds \cite{14} $|V_{us}|/|V_{ud}| = 0.2758(5)f_\pi/f_K$. Employing $f_K/f_\pi = 1.193(6)$ from ref \cite{14} leads to

$$|V_{us}| = 0.2252(13) \quad K_{\mu2} \text{ decay}$$

(10)

That value is consistent with eq. (9). Together they give the average

$$|V_{us}| = 0.2253(9) \quad K \text{ Decays Average}$$

(11)

which I use in the subsequent discussion. Note that the error in $|V_{us}|$ for both approaches is dominated by lattice gauge theory uncertainties which will hopefully be reduced in the near future.
Using (the rather negligible) \(|V_{ub}|^2 \simeq 1.5 \times 10^{-5}\) in conjunction with eqs. (5) and (11) leads to
\[
|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999(4)V_{ud}(4)V_{us} = 0.9999(6)
\]  
(12) 

The outstanding agreement with unitarity provides an impressive confirmation of Standard Model radiative corrections \([5, 7]\) (at about the 60 sigma level!). It can be used to constrain “new physics” effects which, if present, would manifest themselves as an apparent deviation from 1, \(i.e.\) what would appear to be a breakdown of unitarity.

I will give several examples of the utility eq. (12) provides for constraining “new physics”. Each case is considered in isolation, \(i.e.\) it is assumed that there are no accidental cancellations.

i) Exotic Muon Decays

If the muon can undergo decay modes beyond the Standard Model \(\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu\) and its radiative extensions, those exotic decays shorten the muon lifetime. That would mean that the “real” Fermi constant, \(G_F\), is actually smaller than the value in eq. (2) and we should be finding
\[
|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - BR(\text{exotic muon decays})
\]  
(13) 

A unitarity sum below 1 could be interpreted as possible evidence for such decays. Alternatively, eq. (12) provides at (one-sided) 95% CL
\[
BR(\text{exotic muon decays}) < 0.001
\]  
(14) 

That is, of course, not competitive with, for example, the direct bound \(BR(\mu^+ \rightarrow e^+\gamma)1 \times 10^{-11}\) \([11]\). However, for decays such as \(\mu^+ \rightarrow e^+\bar{\nu}_e\nu_\mu\) (wrong neutrinos), eq. (14) is about a factor of 10 better than the direct constraint \([11]\) \(BR(\mu^+ \rightarrow e^+\bar{\nu}_e\nu_\mu) < 0.012\). That constraint is useful for possible future neutrino factories where the neutrino beams originate from muon decays. If such a decay were to exist, it would provide a background to neutrino oscillations.

Another way to illustrate the above constraint is to extract the Fermi constant from nuclear, \(K\) and \(B\) decays assuming the validity of CKM unitarity without employing muon decay. Semileptonic decays then give
\[
G_{\text{CKM}}^F = 1.166309(350) \times 10^{-5}\text{GeV}^{-2} \quad \text{CKM Unitarity}
\]  
(15) 

which is in fact the second best determination of \(G_F\), after eq. (2). The comparison between \(G_\mu\) in eq. (2) and \(G_{\text{CKM}}^F\) in eq. (15) is providing the constraints on “new physics”, if it affects them differently. So far, they are equal to within errors.

ii) Heavy Quarks and Leptons

As a second example, consider the case of new heavy quarks or leptons that couple to the ordinary 3 generations of fermions via mixing \([2]\). For a generic heavy charge \(-1/3\) \(D\) quark from a 4th generation, mirror fermions, \(\text{SU}(2)_L\) singlets etc., one finds at the one-sided 95% CL
\[
|V_{uD}| \leq 0.03
\]  
(16) 

Considering that \(|V_{ub}| \simeq 0.004\), such an indirect constraint appears not to be very stringent but it can be useful in some models to rule out large loop induced effects from mixing. In the case of heavy neutrinos with \(m_N > m_\mu\), one finds similarly
\[
|V_{\ell N}| \leq 0.03, \quad \ell = e, \mu
\]  
(17)
iii) Four Fermion Operators

If there are induced dim. 6 four fermion operators of the form
\[ \pm i \frac{2\pi}{\Lambda^2} \bar{u} \gamma_{\mu} d \bar{e} \gamma^\mu \nu_e \] (18)
where \( \Lambda \) is a high effective mass scale due to compositeness, leptoquarks, excited \( W^* \) bosons (e.g. extra dimensions) or even heavy loop effects, they will interfere with the Standard Model beta decay amplitudes and give
\[ G_F^{\text{CKM}} = G_\mu \left( 1 \pm \sqrt{2} \pi G_\mu \Lambda^2 \right) \] (19)
One finds at 90%CL
\[ \Lambda \gtrsim 30 \text{ TeV} \]
Similar constraints apply to new 4 fermion lepton operators that contribute to \( \mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \). Of course, in some cases there can be some cancellation between semileptonic and purely leptonic effects and a reduced bound results.

The high scale bounds in eq. (19) apply most directly to compositeness because no coupling suppression was assumed. For leptoquarks, \( W^* \) bosons etc. the bounds should be about an order of magnitude smaller due to weak couplings. A \( m_{W^*} \) bound of about 4-6 TeV results if we assume it affects leptonic and semileptonic decays very differently; but that assumption may not be valid and may need to be relaxed (see below). In the case of new loop effects, those bounds should be further reduced by another order of magnitude. For example, I next consider the effect of heavy \( Z' \) bosons in loops that enter muon and charged current semileptonic decays differently where a bound of about 400 GeV is obtained.

iv) Additional \( Z' \) Gauge Bosons

As my next example, I consider the existence of additional \( Z' \) bosons that influence unitarity at the loop level by affecting muon and semi-leptonic beta decays differently. Such a possibility was considered by Alberto Sirlin and myself [16] about 24 years ago. In general, we found that the unitarity sum was predicted to be greater than one in most scenarios. In fact, one expects
\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + 0.01 \lambda \ell n \frac{X}{(X-1)} \]
\[ X = \frac{m_{Z'}^2}{m_W^2} \] (20)
where \( \lambda \) is a model dependent quantity of \( O(1) \). It can have either sign, but generally \( \lambda > 0 \).

In the case of SO(10) grand unification \( Z' = Z_\chi \) with \( \lambda \simeq 0.5 \), one finds at one-sided 90% CL
\[ m_{Z_\chi} \gtrsim 400 \text{GeV} \] (21)
That bound is somewhat smaller than tree level bounds on \( Z' \) bosons from atomic parity violation and polarized Moller scattering [17] as well as the direct collider search bounds \( m_{Z_\chi} > 1 \text{ TeV} \).

Another important constraint results from using the \( K_{\mu 2} \) value of \( V_{us} \) in eq. (10) together with eq. (5) such that
\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999(7) \quad K_{\mu 2} \] (22)
which is consistent with 1. However, in a 2 Higgs doublet model with a charged Higgs scalar, \( H^\pm \), one would expect [15]
\[ 1 - 2|V_{us}|^2 \tan^2 \beta (m_{K^+}^2 - m_{Z^+}^2)/m_{H^\pm}^2 \] (23)
with $\tan \beta = v_2/v_1$, ratio of vacuum expectation values. That implies, at one-sided 90% CL

$$m_{H^\pm} \gtrsim 5 \tan \beta \, \text{GeV}$$

(24)

which is quite prohibitive, particularly for large $\tan \beta \approx 40$.

In addition, interesting unitarity constraints can be placed on supersymmetry [18] where SUSY loops affect muon and semileptonic decays differently. Again, one expects constraints up to mass scales of $O(500 \, \text{GeV})$, depending on the degree of cancellation between squark and slepton effects.

In the future, the unitarity constraint could improve from $\pm 0.0006$ to $\pm 0.0004$ if $f_+(0)$ and $f_K/f_\pi$ errors as well as uncertainties from radiative corrections can be reduced.

As an added comment, let me again mention that eqs. (2) and (15) represent our two best measurements of the Fermi constant. Their agreement reinforces the validity of using $G_\mu$ to normalize electroweak charged and neutral current amplitudes in other precision searches for "new physics". In fact, either $G_\mu$ or $G_{F\text{CKM}}$ could be used without much loss of sensitivity, since all other experiments are currently less precise than both. For example, one of the next best determinations of the Fermi constant (which is insensitive to $m_t$) comes from [19]

$$G_{F(2)}^{(2)} = \frac{\pi \alpha}{\sqrt{2} m_W^2 \sin^2 \theta_W (m_Z)_{\overline{\text{MS}}}} (1 - \Delta r (m_Z)_{\overline{\text{MS}}})$$

(25)

where

$$\alpha^{-1} = 137.035999084(51)$$

(26a)

$$m_W = 80.398(25) \, \text{GeV}$$

(26b)

$$\sin^2 \theta_W (m_Z)_{\overline{\text{MS}}} = 0.23125(16)$$

(26c)

$$\Delta r (m_Z)_{\overline{\text{MS}}} = 0.0696(2)$$

(26d)

One finds

$$G_{F(2)}^{(2)} = 1.165629(1100) \times 10^{-5} \, \text{GeV}^{-2}$$

(27)

with an uncertainty about 1400 times larger than $G_\mu$ and about 3 times larger than $G_{F\text{CKM}}$. The value in eq. (27) is, nevertheless, very useful for constraining "new physics" that affects it differently than $G_\mu$ or $G_{F\text{CKM}}$. Perhaps the two best examples are the $S$ parameter [20]

$$S \simeq \frac{1}{6\pi} N_D$$

(28)

which depends on the number of new heavy SU(2)$_L$ doublets (e.g. $N_D = 4$ in the case of a 4th generation) and a generic $W^*$ Kaluza-Klein excitation associated with extra dimensions [19] that has the same quark and lepton couplings. Either would contribute to $G_\mu$ or $G_{F\text{CKM}}$ but not to $G_{F(2)}^{(2)}$. Therefore, one has the relation

$$G_\mu \simeq G_{F\text{CKM}} \simeq G_{F(2)}^{(2)} (1 + 0.0085 S + \mathcal{O}(1) \frac{m_{W^*}^2}{m_{W^*}^2})$$

(29)

The good agreement among all three Fermi constants then suggests $m_{W^*} \gtrsim 2 \sim 3 \, \text{TeV}$ and $S \simeq 0.1 \pm 0.1$ (consistent with zero). Those constraints are similar to what is obtained from global fits to all electroweak data. Taken at face value they suggest any "new physics" near the TeV scale that we hope to unveil at the LHC is hiding itself quite well from us in precision low energy data. It will be interesting to see what the LHC finds.
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