Investigation of Changing Air Temperatures in Cross-Tilt Inclined Cracks

M N Zherlykina, Y A Vorob’eva, E E Burak
Department of Housing and Communal Services, Voronezh State Technical University, 84, 20-letiya Oktjabrya St., Voronezh 394000, Russia

E-mail: cccp38@yandex.ru

Abstract. The process of heat transfer through an inclined crack is considered with relation to its air convection and thermal conductivity. To solve the problem, the Galeckin projection method is used according to which the solution of the heat equation in a moving medium is sought by solving a one-dimensional problem. The graphic dependence of temperature distribution and air velocity in the inclined crack is obtained as well as the nature of the deviation of the temperature profile and velocity from the central line.

Aging of buildings contributes to the appearance of damages, cracks and cracks in the enclosures, penetration of moisture into the structure, increased infiltration of outside air, which causes a decrease in the thermal protection properties of fences, deterioration of the microclimate of the premises and, as a consequence, increased energy for its heating. Consider the wear of the outer wall in the form of a crack passing in an arbitrary direction.

When determining the air temperatures in the gap, it is assumed that heat transfer in the gap occurs due to thermal conduction and air convection; Radiant heat transfer in the gap is not taken into account. The process of free convection in a crack that has an arbitrary orientation with respect to the gravity vector is a variant of the interaction of temperature and air velocity. Consider a crack in the outer wall, passing in an arbitrary direction. The equation of thermal conductivity in a driving medium can be represented as follows:

\[
\frac{\partial t}{\partial \tau} + U \frac{\partial t}{\partial x} + V \frac{\partial t}{\partial y} + W \frac{\partial t}{\partial z} = a \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right),
\]

where \( t \) – air temperature in the gap, \( K \); \( \tau \) - time, s; \( a \) - thermal diffusivity of air in the gap, \( m^2/s \); \( U, V, W \) – velocity components along the coordinate axes, m/s, determined from [1-5]

Since the linear dimensions of the crack (length and depth) are much greater than the width of its opening in the transverse direction \( \delta \gg r, L \gg r \), the temperature change will be taken only along the \( x, y \) axes:

\[
\frac{\partial t}{\partial \tau} + U \frac{\partial t}{\partial x} + V \frac{\partial t}{\partial y} = a \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \right)
\]
We now turn to the dimensionless form of writing Equation (3) due to a proper choice of scale. The boundary conditions for the relative temperature are taken at $x = 0 \ \theta = (t_n - t) / (t_n - t^\nu) = 0$, at $x = \delta \ \theta =1$. The slope of the crack to the gravity vector is expressed in terms of the slope angle $\varphi$. The geometric diagram of the problem is shown in Figure 1.

By changing the angle $\varphi$ (Figure 1), it is possible to take into account the effect of the crack inclination on the temperature and velocity profiles inside the crack:

$$\vec{n} = \vec{i} \cdot \cos \varphi + \vec{j} \cdot \sin \varphi$$

(4)

where $\vec{i}$, $\vec{j}$ - unit vectors in the direction of the $x$ and $y$ axes.

The solution for the temperature distribution in the crack will be found using projection methods. The most common of projection methods are the Galerkin methods, according to which the solution of equation (3) is sought both in solving a one-dimensional problem in the form:

$$\theta_{ax} = \sum_{j=1}^{N} \bar{\theta}_j(t) \phi_j(x, y)$$

(5)

where $\{ \bar{\theta}_j(t) \}$ - coefficients to be determined; $\{ \phi_j(x, y) \}$- be some chosen system of functions that vanish on the boundary of the domain of definition, called the test function.

The coefficients $\{ \bar{\theta}_j \}$ are found from the Petrov-Galerkin scheme from the system of equations [6,7]:

$$(L(\theta a), \psi k(x, y)) = 0, \ k = 1, ..., N$$

(6)

where the parentheses denote the scalar product $L(\theta a)$ and $\psi k(x, y)$, i.e. Integral over the domain $D$; $\psi k(x, y) = \phi_j(x, y)$ - verification function.

One of the properties of the Galerkin method is the possibility of decreasing the order of the highest derivative appearing in the weak form of the equation (6) in question by transferring this order to the derivative of the verification function $\psi k(x, y)$. Applying the Green's theorem to reduce the order of the highest derivative appearing in (6), substituting the expansion (5) into (6) and changing the order of summation and integration, we obtain a system of linear algebraic equations for finding the coefficients $\bar{\theta}_j$:
If we take into account the form of the boundary conditions for the given problem (Figure 1), then all the inner products in equation (7) are surface integrals:

\[
\left( \frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial x} \right) = \int \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial x} \, dx \cdot dy
\]  

(8)

The main difficulty in solving equation (7) is related to the selection of a suitable form of functions \( \psi_k(x, y) \). Trial solutions can be determined independently in each of the elements using the local coordinate system \((\tilde{x}, \tilde{y})\) (Figure 2).

When using Lagrangian rectangular elements (Figure 2), two-dimensional test functions can be represented in the form of products of one-dimensional test functions:

\[
\psi_k(x, y) = (\phi_k^x(\tilde{x}) + \alpha x \gamma_k^x(\tilde{x})) (\phi_k^y(\tilde{y}) + \alpha y \gamma_k^y(\tilde{y})), \quad k = 1, \ldots, N
\]  

(9)

\[
\phi_k^x(\tilde{x}) = 0.5(1 - \tilde{x})
\]  

(10)

\[
\phi_k^y(\tilde{y}) = 0.5(1 - \tilde{y})
\]  

(11)

\[
\gamma_k^x(\tilde{x}) = \begin{cases} 
-3\tilde{x}(1 - \tilde{x}) & \text{in the element } [k, k+1] \\
3\tilde{x}(1 - \tilde{x}) & \text{in the element } [k-1, k] 
\end{cases}
\]  

(12)
Substituting expressions (10) ... (13) into equation (9), we obtain verification functions for 1 ... 4 knots, respectively. The degree of introduction of differences upstream is now regulated in each element by four parameters \( \alpha \). To determine the parameter \( \alpha_{ij} \), one can use the formula [114]:

\[
\alpha_{ij} = \text{cth} \beta - 1/ \beta
\]

where \( \beta = 0,25 \ h \ (u_i \cos \varphi + v_j \sin \varphi)/\alpha \); \( h \) - step in the direction \( i \) and \( j \); \( u_i, v_j \) - velocity vectors at nodes \( i \) and \( j \).

After calculating the terms of equation (7), the parameters \( \alpha_x, \alpha_y \) allow to regulate the degree of introduction of differences with upstream shift, i.e. these two parameters are associated with the k-th node. The resulting algebraic equations are very cumbersome, but can be conveniently represented using test functions on a homogeneous rectangular grid (\( \Delta x = \Delta y = h \)).

The results of calculations of the dimensionless temperature, \( \theta \) are presented in table 1. The time step \( \Delta \tau = 0.02 \) was used.

| \( \tau \) | \( N=2 \) | \( N=3 \) | \( N=4 \) |
|---|---|---|---|
| 0  | 1.5 | 1.5 | 1.5 |
| 0.02 | 1.32103 | 1.32103 | 1.32103 |
| 0.04 | 1.1741 | 1.1741 | 1.1741 |
| 0.06 | 1.05345 | 1.05345 | 1.05345 |
| 0.08 | 0.9544 | 0.9544 | 0.9544 |
| 0.10 | 0.87308 | 0.87308 | 0.87308 |
| 0.12 | 0.80631 | 0.80631 | 0.80631 |
| 0.14 | 0.75149 | 0.75149 | 0.75149 |
| 0.16 | 0.70648 | 0.70648 | 0.70648 |
| 0.18 | 0.66953 | 0.66953 | 0.66953 |
| 0.20 | 0.63919 | 0.63919 | 0.63919 |

The time step is sufficiently small that its mean-square errors given in table 1 turn out to be due only to spatial approximation. Figure 3 is a graphical depiction of the dependence of the dimensionless temperature \( \theta \) in a node located at the point \( (0.5; 0.5) \) from the dimensionless time \( \tau \).

**Table 1.** The numerical solution of the problem of non-stationary heat transfer in a crack.

**Figure 3.** Dependence of the dimensionless temperature \( \theta \) in the node located at the point \( (0.5, 0.5) \) from the dimensionless time, \( \tau \).

With increasing \( \tau \) (Figure 3), the perturbation is damped, so that the linear temperature distribution remains stationary and equation (3) can be written both for the steady convective diffusion:
The solutions of equation (15) were constructed on a homogeneous grid $\Delta x = \Delta y = 0.1$ for a value of $a = \frac{\sqrt{u^2 + v^2}}{v} = 500$. The distributions of the dimensionless temperature at $x = 0.5$ and $\varphi = 21.8^\circ$ are presented in Table 2.

**Table 2.** Variation of the dimensionless temperature at $x = 0.5$.

| $y$  | $\alpha x = \alpha y = 1$ | Finite differences, shift (15) |
|------|--------------------------|-------------------------------|
| 0    | 0                        | 0                             |
| 0,1  | 0                        | 0,03                          |
| 0,2  | -0,02                    | 0,06                          |
| 0,3  | -0,03                    | 0,13                          |
| 0,4  | 0,14                     | 0,25                          |
| 0,5  | 0,52                     | 0,44                          |
| 0,6  | 0,87                     | 0,67                          |
| 0,7  | 1,01                     | 0,89                          |
| 0,8  | 1,01                     | 0,99                          |
| 0,9  | 1,0                      | 1,0                           |
| 1,0  | 1,0                      | 1,0                           |

The result for the temperature distribution in an inclined slit along the centerline of the CD (CABD area see Figure 1) is shown in Figure 4.

![Figure 4](image.png)

**Figure 4.** Temperature and velocity distribution on the CD line by the Petrov-Galerkin method.

According to Figure 4, it is seen that in the inclined crack there is a deviation of the temperature profile and velocity from the central line.

The obtained results of calculation of temperature fields obtained convergence with practical research barriers made of concrete with a through slot opening width from 1 to 3 mm [7]. In the first stage of calculation of temperature fields the resulting distribution of temperature along the axis of the gap, characterizing the possibility of heating the enclosure the flow of outdoor air. To obtain the final temperature field taking into account air filtration through the crack perform a second calculation step of confirming the offset of the temperature field in the building envelope due to the formation of cracks or crevices.
References

[1] Lykov A V 1967 *Theory of heat conduction* (Moscow: Higher education. School) p 600

[2] Vorob’eva Y A 2006 *Influence of the aging process of enclosing structures and engineering systems of residential buildings on the microclimate of premises* (Voronezh) p 18

[3] Vorob’eva Y A 2013 Theoretical studies of the change in the air permeability of enclosing structures in the development of cracks *Actual problems of urban construction* (Penza: PGUAS) pp 131–134

[4] Tabunschikov Y A, Lame Y D and Matrosov Y A 1986 *Thermal protection enclosing structures of buildings and constructions* (Moscow: Stroiizdat) p 373

[5] Samarskii A A 2003 *Numerical methods. The solution of problems of convection-diffusion* (Moscow: URSS) p 248

[6] Fletcher K 1988 *Numerical methods on the basis of Galerkin's method* (Moscow: Izd-vo Mir) p 352

[7] Vorob’eva Y A 2010 Analysis of the reduction in the thermal efficiency of the enclosing structures of residential buildings with various wear and tear *Scientific herald of the Voronezh State Architectural and Construction University. Series: High technologies. Ecology* (Voronezh) pp 54–56