Twisted graphene bilayers provide a versatile platform to engineer metamaterials with novel emergent properties by exploiting the resulting geometric moiré superlattice. Such superlattices are known to host bulk valley currents at tiny angles ($\alpha \approx 0.3^\circ$) and flat bands at magic angles ($\alpha \approx 1^\circ$). We show that tuning the twist angle to $\alpha^* \approx 0.8^\circ$ generates flat bands away from charge neutrality with a triangular superlattice periodicity. When doped with $\pm 6$ electrons per moiré cell, these bands are half-filled and electronic interactions produce a symmetry-broken ground state (Stoner instability) with spin-polarized regions that order ferromagnetically. Application of an interlayer electric field breaks inversion symmetry and introduces valley-dependent dispersion that quenches the magnetic order. With these results, we propose a solid-state platform that realizes electrically tunable strong correlations.

Controllably engineering quantum states of matter is one of the leading goals of modern physics. This basic idea has been realized in a plethora of platforms ranging from cold-atom setups [1–3] to atom-by-atom deposited solids [4, 5]. In recent years, the discovery of graphene has opened numerous new avenues [6], including the possibility of stacking two-dimensional crystals and forming so-called ‘van der Waals’ materials [7], which allow to engineer exotic states [7–11]. Tuning the relative angle $\alpha$ between graphene layers [12] and applying electric potentials $V$ across the layers [11, 13] have played central roles in purposely designing the physical properties of such systems. In this Letter, we combine these two ideas in a new parameter regime: (i) we identify an angle $\alpha$ that generates flat bands with strong correlations leading to a magnetic instability, while (ii) an electric bias $V$ across the layers reintroduces dispersion and thus allows to dynamically tune the magnetic response of the bilayer system.

Stacking graphene layers at a finite relative angle $\alpha$ produces a moiré superlattice [see Fig. 1(a)] with properties that are sensitive to the twist angle [14–16]. The spectrum of the superlattice is composed of graphene bands that are folded back to the mini-Brillouin zone where they bundle into separate groups; these can be tuned to become flat at small twist angles $\alpha$. Such weakly twisted bilayer graphene then provides a versatile platform to explore strongly correlated physics. Much work has focused on the so-called magic angle $\alpha = 1.1^\circ$ producing two flat bands near charge neutrality (each fourfold degenerate) with strong correlations [9, 17–20] and superconductivity [10, 21, 22] appearing under weak doping. The question arises, whether other angles and bands can be used in engineering novel properties. Here, we show that tuning the twist to the angle $\alpha^* \approx 0.8^\circ$ flattens the bands above and below the ones near charge neutrality. We find that doping these bands to half-filling with $\pm 6$ electrons per triangular supercell produces a correlated state with ferromagnetic order.

Such flat bands have been termed ‘pseudo-Landau levels’ (PLLs) [23–25]; they can be understood as the result of artificial gauge fields that arise from nonuniform strain in graphene monolayers [26–28] or twist-induced strain between layers [29]. Proper tuning of such strain leads to hopping phases that interfere destructively, localizing the states and producing flat bands [23, 24]. Similar flat-band modes have been proposed in other twisted multilayer systems [30], such as tiny-angle graphene bilayer [31], graphene trilayers [32], graphene bi-bilayers [33, 34], and dichalcogenide multilayers [35].

An alternative way to engineer the electronic states in multilayers of Dirac materials is to apply an interlayer bias $V$ to induce valley Berry curvature [13]. The latter results from breaking inversion symmetry, that induces topology through (compensating) valley fluxes [11, 36]. Such valley fluxes and associated valley currents are particularly pronounced at small twist angles and cause dispersive splittings between bands [36]. This motivates the idea of engineering the bandwidth and correlations via electrical bias and geometric twist. Here, we combine twist-induced emergent flat bands at small twist $\alpha$ and an interlayer bias $V$ in order to manipulate the strong-correlation physics in a flat-band material with valley-topological properties.

Flat-band magnetism in twisted bilayer graphene has been discussed before with a focus on the bands near charge neutrality and manipulation of the magnetic state by interlayer bias [37–39] and by pressure [39]. Rather than manipulating the 0th PLL near charge degeneracy, we focus on the ±1 PLLs that become optimally flat at the smaller angle $\alpha^* \approx 0.8^\circ$ where valley currents are more pronounced. Different from the 0th PLL bands that are described by an effective honeycomb lattice and flat Dirac cones, the ±1 PLLs are described by a triangular lattice. We study the delocalization effect of the bias-induced local valley fluxes and find that these act against the subtle spatial interference that generates the original flat bands. The bias-induced dispersion then reduces the correlations and thus the ferromagnetic (quasi-)order in the material.
FIG. 1. Tunability of the single-particle bilayer graphene properties as a function of the twist angle. (a) Stacking two honeycomb lattices (light/dark blue) with small twist angle $\alpha$ leads to a hexagonal moiré superlattice. The long-range structure has a moiré unit cell (opaque overlay) with varying local stacking order AA/AB/BA, where A and B correspond to the two atomic sites in each of the stacked graphene unit cells. (b) Density of states (DOS) as a function of twist angle $\alpha$, showing peaks ascribed to emergent flat bands associated with three main pseudo-Landau levels (PLLs). Grey dotted lines denote the position of Fermi levels at half-filled ±1-PLL bands: The two 0-PLL bands near charge neutrality hold 4 electrons each; similarly, each of the ±1-PLL bands holds 4 electrons and thus is half-filled at a doping level of ±6 electrons per moiré cell. At the flat-band angle $\alpha^* = 0.8^\circ$, the bandwidth of the ±1-PLLs becomes minimal. (c) Local density of states (LDOS) in the −1-PLL, showing the emergence of an effective triangular lattice of states localized around the AA regions. Dashed lines mark effective hopping amplitudes in an effective triangular superlattice Hamiltonian, see text. (d)-(f) Band structures for $\alpha < \alpha^*$, $\alpha = \alpha^*$ and $\alpha > \alpha^*$ illustrating how the ±1-PLL bands evolve to generate flat bands at $\alpha^*$ (marked in blue). (f) The 0-PLL flattens at the magic angle $[15, 16]$. Note that there is a gap to the ±1-PLLs (small in this plotted scale).

leading to electrically tunable magnetism. We thus arrive at a new platform where correlations and topology can be controlled at the same time.

Below, we start from the real-space bilayer tight-binding model and find flat bands at an angle $\alpha^* = 0.8^\circ$; the states in these ±1 PLLs are strongly localized on the triangular superlattice of the moiré structure. We include local interactions on a mean-field level and find the magnetic instability. In a third step, we include the interlayer bias and map out the local Berry curvature in real-space that introduces band dispersion and consequently reduces the magnetic order.

Twisted bilayer graphene is formed by stacking two honeycomb lattices of carbon with a small twist angle $\alpha$, resulting in a moiré structure of (approximate) periodicity $L_M$ that grows inversely with $\alpha$, see Fig. 1(a). This supercell can exceed the lattice constant $a$ in size by 1–2 orders of magnitude and features regions with well-defined local stacking orders AA, AB and BA, with A and B denoting the inequivalent atomic sites in each hexagonal unit cell. We model the twisted bilayer with a real-space tight-binding Hamiltonian

$$H_0 = \sum_{(i,j),s} \epsilon_{i,s}^\dagger c_{j,s}^\dagger c_{j,s} + \sum_{i,j,s} t_{ij}^\dagger c_{i,s}^\dagger c_{j,s} - \sum_{i,s} \mu_i c_{i,s}^\dagger c_{i,s},$$

where $\epsilon_{i,s}^{(\pm)}$ destroys (creates) an electron at site $r_i = (x_i, y_i, z_i)$ in one of the layers ($z_i = \pm d/2$) with spin $s = \uparrow, \downarrow$ and $\mu$ is the chemical potential; $t$ is the nearest-neighbor hopping within each layer and $t_{ij}^\dagger = t_{\perp}(|z_i - z_j|^2/|r_i - r_j|^2)e^{-|r_i - r_j|^2/d^2}$ is the twist-angle dependent hopping $[40]$ from $r_i$ to $r_j$ with amplitude $t_{\perp} \simeq 0.12t$, range $\ell \simeq 0.13a$, and the interlayer distance $d \simeq 1.4a$. We utilize a scaling relation that brings the low-energy physics of small angles $\alpha$ to larger ones by appropriately increasing the interlayer hopping amplitude $t_{\perp} [31, 37, 41–43]$. This allows us to perform our analysis at moiré unit cells that are small enough for numerical treatment.

The twist $\alpha$ effectively creates a nonuniform interlayer hopping with a corresponding gauge field $[29, 44]$. This, in turn, leads to a destructive interference that generates our ±1 PLL flat bands that naturally lend themselves for strong-correlation physics. In Fig. 1(b), we present the low-energy density of states (DOS) of the bilayer as a function of its twist angle. Three main PLL bands are indexed with $-1, 0, 1$, which become flat at the marked specific angles. In this work, we are interested in the ±1 bands with states that are localized around the AA regions of the superlattice, see Fig. 1(c). At negative (positive) energies, as a function of the twist angle $\alpha$, the targeted band evolves from having a negative (positive) effective mass to a positive (negative) one, see Figs. 1(d)-(f). At our “flat-band” angle $\alpha^* \approx 0.8^\circ$ [Fig. 1(e)], we achieve maximal isolation of the ±1 PLL bands in energy. They hold up to ±4 electrons per moiré unit cell and doping with ±6 electrons corresponds to their half-filling. Note that the two 0 PLL bands hold ±4 electrons each and become flat and nearly-degenerate at the magic angle $\alpha \approx 1.1^\circ$ $[14–16, 45–48]$.

The ±1-PLL states arrange in a triangular lattice of “flower”-shaped Wannier orbitals centered around the AA regions, see Fig. 1(c). The effective low-energy physics
change splitting is associated with a net magnetic moment of approximately $2 \mu_B$ per moiré unit cell (obtained by integrating over the mildly-antiferromagnetic texture) aligned ferromagnetically between cells. Given the macrostructure of the system, this yields a triangular superlattice of local magnetic moments, see Fig. 2(b).

The emergence of magnetism in this flat-band angle regime is analogous to observations of magnetic instabilities at other angles and dopings [37, 38], in particular, in magic-angle superlattices [54–56]. Unique to our regime is the sensitivity of the ±1-PLL bands to an interlayer voltage-bias: as elaborated below, the latter modifies the electronic spectrum of the bilayer and hence allows for the tuning of the aforementioned magnetic order.

We now discuss the implications of an interlayer voltage-bias as described by the Hamiltonian $H = H_0 + H_V$ (at $U = 0$) with

$$H_V = \frac{V}{2} \sum_{i,s} \text{sgn}(z_i) c_{i,s}^\dagger c_{i,s}$$

where $V$ is the interlayer bias and $\text{sgn}(z_i) = \pm 1$ for the top and bottom layers, respectively. This bias lowers the symmetry from $C_{3v}$ to $C_3$ by breaking the inversion symmetry between the layers. Finite $V$ broadens and eventually washes out the singularity in the density of states [Fig. 3(a)] by introducing a dispersive valley-splitting into the band structure [Fig. 3(b)]. The substantial impact of the bias $V$ on these flat bands is in stark contrast to the magic-angle regime, where it was found that a small bias has a negligible effect [21].

To better understand the mechanism that lifts the valley degeneracy, we analyze the spatial profile of the frequency-resolved valley Berry curvature [57]

$$\frac{\partial \Omega_v(r)}{\partial \omega} = \int_{\mathcal{BZ}} \frac{d^2 k}{(2\pi)^2} \epsilon_{\alpha\beta} \langle r | G_v(\partial_{\epsilon,\alpha} G_v^{-1})(\partial_{\epsilon,\beta} G_v) | r \rangle,$$

where $\int_{\mathcal{BZ}} d^2 k \cdots$ denotes integration over the Brillouin zone and $\epsilon_{\alpha\beta}$ is the Levi-Civita tensor. We denote $G_v = [\omega - H(k) + i0^+]^{-1} P_v$ as the Green’s function of the Bloch Hamiltonian $H(k) = H_0(k) + H_V(k)$, and $P_v$ is the valley polarization operator [31, 58, 59] that weighs the states with $\pm 1$ depending on which graphene valley $(K, K')$ they originate from. This allows us to find the valley-Chern number through integration over energies and over the unit cell, i.e., $C_v = \int_{\mathcal{BZ}} d^2 k \int_{-\omega}^\omega d\omega \partial_\omega \Omega_v(r)$, which only takes nonzero values if time-reversal or inversion symmetry are broken. By breaking inversion symmetry, the interlayer bias $V$ produces a finite valley-Chern number with the spatial texture in the Berry curvature as shown in Fig. 3(c), featuring alternating signs between AB/BA-stacked regions [13, 60, 61]. These, in turn, can be interpreted as a fake staggered valley magnetic field. At smaller
angles $\alpha < \alpha^*$, this spatial texture in the Berry curvature sharpens up and eventually leads to the valley helical networks [36] recently observed in experiments [11].

If we regard the formation of flat bands in Fig. 1(e) as the result of fine-tuned destructive interference between neighboring Wannier orbitals [15, 16], we can explain how the dispersive valley-splitting in Fig. 3(b) arises: the emergent valley-chiral magnetic field induced by the interlayer bias alters the relative phases between the Wannier orbitals, and creates additional valley-splittings. This effect can be captured by the effective triangular superlattice Hamiltonian via the inclusion of Peierls phases in the hoppings [43]. These phases correspond to a finite magnetic flux per triangle that averages to zero over one unit cell, see Fig. 3(c), reminiscent of Haldane’s model for Chern insulators [62]. As the two valleys are time-reversal-symmetric partners these fluxes have opposite sign, thus canceling one another locally. To summarize, the interlayer bias in the full model takes the role of the valley-magnetic flux per plaquette in the low-energy model.

We can now analyze how the interlayer bias affects the interaction-induced correlated state by considering the Hamiltonian $H = H_0 + H_D + H_V$. We expect that the interlayer-bias-induced band dispersion modifies the magnetic state discussed in Fig. 2. Indeed, as shown in Fig. 3(d), the ground state magnetization is substantially quenched even for moderate bias ($V \approx 100$ meV). Hence, the interlayer bias serves as an external control for the correlated state at our flat-band-angle. We once more emphasize that this feature is in striking contrast to magic-angle graphene, where a small interlayer bias does not substantially change the correlated state [21].

There are three interesting avenues for further investigations beyond the scope of this work. First, we emphasize that our analysis does not take into account possible lattice relaxations [63, 64], which may impact the specific angle at which the targeted band becomes sufficiently flat. Second, the existence of twist-angle disorder [65] is likely to modify the width of the targeted band and uniform twist angles may be required in order to observe the correlated state – similarly to the phenomenology shown at the magic angle [22]. Third, the triangular nature of the superlattice suggests that a spin-spiral state may exist that is energetically competitive with the ferromagnetic configuration discussed above. Indeed, we performed our self-consistent mean-field analysis for several candidates and found indications for a 120° spin-spiral state (maximally antiferromagnetic) that competes with the collinear order. This observation leads us to conclude that the system is profoundly frustrated, with sizable antiferromagnetic exchange coupling to second-neighbor moiré cells. Since such triangular lattices have been proposed to give rise to spin-liquid phases [66], further investigations into our proposed flat-band system might unveil a nontrivial realization of such physics.

To summarize, we have shown that electronic correlations can arise in twisted graphene bilayers at fillings of 6 electrons/holes per moiré unit cell. This correlated regime is shown to appear at angles around 0.8° at doping levels of ±6 electrons per moiré unit cell, for which the chemical potential falls into one of the ±1-pseudo-Landau levels. For that regime, we show that interactions promote the formation of local magnetic moments in the moiré supercells arranging on a triangular lattice. Furthermore, we have shown that the interlayer bias can be used to control the magnetic instability. The origin of this tunability was
demonstrated to be related to the control over an effective staggered valley-magnetic field in the heterostructure, that modifies fine-tuned interferences in the superlattice states, thereby substantially affecting the low-energy dispersion. Our results put forward a new regime in twisted graphene multilayers hosting correlations that result in a magnetic instability that is highly tunable with weak interlayer biases.

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Supplementary material for
“Electrically Tunable Flat Bands and Magnetism in Twisted Bilayer Graphene”

T. M. R. Wolf, J. L. Lado, G. Blatter, and O. Zilberberg

Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland
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In this supplemental material, we provide additional details on (i) methods used for the analysis of the microscopic model of the twisted bilayer (section I) and (ii) the effective (phenomenological) low-energy model describing the triangular lattice of localized AA Wannier orbital, as well as its effectiveness in capturing the effects observed in the main text (section II). Specifically, using (ii), we discuss how the emergence of flat bands \( \pm 1 \)-PLLS can be understood to be a result of destructive interference (i.e., suppressed hopping amplitudes) and how the interlayer bias (interpreted as Haldane-like valley-flux texture in the moiré supercell) detunes the interference and induces valley-dependent dispersion.

I. METHODS FOR ANALYZING THE MICROSCOPIC TIGHT-BINDING MODEL

In the main text, we model the non-interacting twisted bilayer with a real-space tight-binding Hamiltonian [see Eqs. (1) and (3) in the main text]. The spectrum of the twisted bilayer system results from backfolding and hybridizing the graphene bands of both layers in a mini-Brillouin zone. In the current work, the spectrum is obtained using exact diagonalization. Due to computational resource bounds, this exact approach limits how large we can choose our moiré unit cells. To reduce the size of these orbitals to effective atomic sites and describe their coupling using a tight-binding model from both valley degrees of freedom of the bilayer, see Fig. S1(a). We can, therefore, coarse grain the electron in the respective Wannier orbital in AA region \( \eta \), \( \gamma_1 \) and \( \gamma_2 \) are real nearest and next-nearest hopping amplitudes respecting \( C_{3v} \) symmetry.

\[
H_{\text{eff}} = \sum_{\langle \eta \eta' \rangle, \sigma} \gamma_1 \, d_{\eta \sigma}^\dagger d_{\eta' \sigma} + \sum_{\langle \eta \eta' \rangle, \sigma} \gamma_2 \, d_{\eta \sigma}^\dagger d_{\eta' \sigma}, \tag{S1}
\]

where \( \sigma = (s, \nu) \) labels the four orbitals (spin \( s = \uparrow, \downarrow \) and valley \( \nu = \pm \)), and \( d_{\eta \sigma}^\dagger \) destroys (creates) an electron in the respective Wannier orbital in AA region \( \eta \).
A. Flat band through destructive interference

Generally, the effective hopping in the low-energy model (S1) is controlled by the twist angle $\alpha$ and the interlayer hopping $t_\perp$ in the microscopic model, i.e., $\gamma_i \equiv \gamma_i(\alpha, t_\perp)$. In Fig. S2, we fit the effective hopping amplitudes $\gamma_1$ and $\gamma_2$ such that we can phenomenologically reproduce the $-1$-PLL band dispersion in Figs. 1(d)–(f).

![FIG. S1](image1.png)

**FIG. S1.** Sketch of the effective superlattice for Wannier states localized near AA stacked regions (black circles) in the moiré structure (a) without interlayer bias and (b) and with finite interlayer bias $V > 0$. The effective physics of the $\pm 1$ PLL bands is captured by nearest- and next-nearest-neighbor hopping amplitudes $\gamma_1$ and $\gamma_2$. The interlayer bias induces complex phases in $\gamma_1$ and $\gamma_2$ that encode the emergent valley magnetic flux texture piercing the triangular unit cells (red/$\bigcirc$, and blue/$\bigtriangledown$ in- and out-of-plane flux, respectively).

![FIG. S2](image2.png)

**FIG. S2.** Band dispersion of the effective model (S1) (dotted red) such that it fits the $-1$-PLL band dispersion of the twisted bilayer graphene (pale blue) for different twist angles (a) $\alpha < \alpha^*$, (b) $\alpha = \alpha^*$, and (c) $\alpha > \alpha^*$ [cf. Figs. 1(d)–(f)]. We see that the effective hopping parameter $\gamma_1$ of the dispersive band in (a) and (c) switches sign while $\gamma_2$ remains approximately constant. This, in turn, generates the negative and positive effective mass of the band. At the transition (b), $\gamma_1 \sim 0$ and the band flattens.
FIG. S3. Band dispersion of the effective triangular model (S1) once complex hopping amplitudes (S2) have been incorporated in order to account for the valley-flux. The effective band dispersion was fitted to that of the −1-PLL band of twisted bilayer graphene (pale colors) for different voltage-bias (a) $V = 0$ and (b) $V = 0.07t$ [cf. Fig. 3(b)]. As discussed in the main text, the emergent valley-magnetic fluxes (that are induced by the interlayer bias) lift the (approximate) valley degeneracy.

**B. Interlayer voltage bias as valley-flux**

As discussed in the main text, a voltage-bias between the twisted graphene layers breaks inversion symmetry and consequently lifts the degeneracy between the two valleys of the material. Our analysis of the valley Berry curvature revealed a fake valley (or ‘chiral’) magnetic field that we can incorporate into our single-site four-orbital triangular hopping model (S1) via a Peierl’s substitution with opposing fluxes between the two valleys, i.e.,

$$\gamma_j \rightarrow \gamma_j e^{i\nu 2\pi \phi_j},$$  \hspace{1cm} (S2)

where $\nu = \pm$ is again the valley degree of freedom. The phases should be compatible with the Haldane-like valley flux pattern in Fig. 3(c), i.e., $0 < \phi_1 < 1$ and $0 \leq \phi_2 \ll 1$.

In Fig. S3, we show that by selecting the appropriate hopping parameters $\gamma_j$ and phases $\phi_j$ reproduces the interlayer bias-induced valley splitting. The parameters of panel (a) are the same as in Fig. S2(b) and those for panel (b) have phases $\phi_1 = 0.75$ and $\phi_2 = 0.1$.

**III. ELECTRONIC INTERACTIONS IN TWISTED MULTILAYER GRAPHENE**

**A. Hubbard versus screened Coulomb interactions**

In twisted graphene multilayers, the long-ranged part of the Coulomb interaction is known to enhance the correlated gaps that emerge in flat-band regimes [5, 6]. In what follows, we show this enhancement by comparing the band dispersion of the (self-consistent) mean-field Hamiltonians for (a) a local Hubbard interaction with (b) that of (screened) Coulomb interaction. To that end, let us consider the more generic interaction

$$H_U = \sum_{i,j} V(|r_i - r_j|) n_{i\uparrow} n_{j\downarrow},$$  \hspace{1cm} (S3)
FIG. S4. Band dispersion in presence of electronic interactions [see Eq. (S3)] for (a) onsite Hubbard repulsion $U_0 = 2t$ ($V_0 = 0$) and for (b) reduced Hubbard repulsion $U_0 = 0.5t$ but in presence of screened Coulomb interactions ($V_0 = 0.37t$ and $\lambda^{-1} = a$, where $a$ is the inter-atom distance); obtained from self-consistent mean-field iteration. Near the chemical potential, the two band dispersions are qualitatively very similar, showing that the long-ranged part of the interaction effectively renormalizes the onsite Hubbard repulsion. Note that we used a twist angle $\alpha_{n1}$ with $n = 8$.

where $n_{i,s} = c_{i,s}^\dagger c_{i,s}$ is the local density operator at each position $r_i$ and $V(r)$ is the two-particle interaction. The latter shall contain two contributions: an onsite (Hubbard) repulsion $U_0$ and a long-ranged Coulomb potential with screening length $\lambda^{-1}$, i.e.,

$$V(r) = \begin{cases} 
U_0, & \text{for } r = 0 \\
V_0 \frac{e^{-\lambda(r-a)}}{r} & \text{for } r \neq 0,
\end{cases} \quad (S4)$$

where $a$ the carbon-carbon distance. Note that $V_0 = 0$ recovers the Hubbard Hamiltonian used in the main text.

As in the main text, we perform a mean-field decoupling in Eq. (S3), i.e., $n_{i\uparrow}n_{i\downarrow} \approx \langle n_{i\uparrow} \rangle \langle n_{i\downarrow} \rangle$ for $\alpha \rightarrow \alpha' = \lambda \alpha$ with $t_\perp \rightarrow t'_\perp = \lambda t_\perp$. In what follows, we deduce the scaling of the interaction strength $U_0$ from the charging energy $U_{\text{eff}} = U_0/N$ of the low-energy states [5]. Here, $N$ is the number of atoms per moiré cell and grows inversely with $\alpha^2$ (it is proportional to the area of the unit cell). To keep the low-energy physics unchanged, we have to keep the ratio of typical bandwidths and charging energy fixed when performing the rescaling:

$$U_0/t_\perp = U'_0/t'_\perp \quad \text{for} \quad \alpha \rightarrow \alpha' = \lambda \alpha, \quad t_\perp \rightarrow t'_\perp = \lambda t_\perp, \quad N \rightarrow N' = \lambda^{-2} N. \quad (S5)$$

B. Rescaling the interaction strength along with the unit cell

As explained above (see section I), we rescaled our unit cells and interlayer hopping amplitudes towards larger angles, i.e. $\alpha \rightarrow \alpha' = \lambda \alpha$ with $t_\perp \rightarrow t'_\perp = \lambda t_\perp$. In what follows, we deduce the scaling of the interaction strength $U_0$ from the charging energy $U_{\text{eff}} = U_0/N$ of the low-energy states [5]. Here, $N$ is the number of atoms per moiré cell and grows inversely with $\alpha^2$ (it is proportional to the area of the unit cell). To keep the low-energy physics unchanged, we have to keep the ratio of typical bandwidths and charging energy fixed when performing the rescaling:
This yields the rescaled Hubbard interaction $U'_0 = \lambda^{-1} U_0$.

For the parameters of our system, this would imply using a value $U_0 \approx 0.6t$ for $n = 11$ for the realistic interaction term of Eq. (S3), together with the long-range part of the Coulomb interaction. We note that performing non-local mean-field calculations is substantially more computationally demanding than local ones. For this reason, in the main manuscript we have captured the effect of the non-local term by a renormalized local Hubbard Hamiltonian, namely taking $U_0 = 2t$ and $V = 0$ in Eq. (S3), as shown in Fig. S4(a) (see section III.A). This choice of parameters yields a physical charging energy of 30 meV at $\alpha \approx 0.8^\circ$, similar to charging energies at the magic angle.

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