tan $\beta$ Determination From Heavy Higgs Boson Production at Linear Colliders

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Abstract

We study the production at future $e^+e^-$ linear colliders of the heavy neutral Higgs bosons $H$ and $A$ of the minimal supersymmetric standard model in association with top and bottom quarks. The cross sections have a strong dependence on the parameter $\tan \beta$, and thus provide a good way to determine it. At a linear collider with $\sqrt{s} = 0.5 - 1$ TeV and expected integrated luminosities, we find significant sensitivities for determining $\tan \beta$. In the Supergravity scenario, the sensitivity is particularly strong for $\tan \beta \gtrsim 10$, reaching a 15% or better measurement. In the general MSSM scenario, the interplay between the $4b$ and $4t$ channels results in a good determination for $\tan \beta \lesssim 10$, while the sensitivity is weakened for higher values of $\tan \beta$. 

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I. INTRODUCTION

One of the most promising avenues for physics beyond the standard model (SM) is supersymmetry (SUSY) [1], since it can provide a fundamental understanding of electroweak symmetry breaking (EWSB) and it allows unification of the electroweak and strong interactions at a grand unified scale [2]. Because of its great theoretical attraction, extensive phenomenological work continues to explore the ways for discovery and precision study of supersymmetric particles at present and future colliders.

Most of these investigations are directed to the Minimal Supersymmetric standard model (MSSM), which has the minimal new particle content [1]. The MSSM contains two Higgs doublets which develop vacuum expectation values $\langle H_1 \rangle = v_1/\sqrt{2}$ and $\langle H_2 \rangle = v_2/\sqrt{2}$ that break the SU(2) $\times$ U(1) gauge symmetry spontaneously [3]. There are 5 physical Higgs bosons in the MSSM: two CP-even states $h$ and $H$, a CP-odd state $A$, and two charged states $H^\pm$; the lightest Higgs boson is $h$.

The ratio $v_2/v_1 = \tan \beta$ is a critical parameter of the MSSM: It characterizes the relative fraction that the two Higgs doublets contribute to the EWSB, and it enters all sectors of the theory. The interactions of both the SUSY particles and the Higgs bosons depend on $\tan \beta$, and the relations of SUSY particle masses to the soft symmetry breaking parameters of supersymmetry involve $\tan \beta$ [3]. A measurement of $\tan \beta$ from one sector will thereby allow predictions or tests in other sectors. The renormalization group evolution of the Yukawa couplings from the unification scale to the electroweak scale are sensitive to the value of $\tan \beta$. The large top quark mass can naturally be explained with $m_b - m_\tau$ unification as a quasi-infrared fixed point of the top Yukawa coupling if $\tan \beta \approx 1.8$ or $\tan \beta \approx 56$ [4]. The possibility of SO(10) Yukawa unification requires the high $\tan \beta$ solution [5]. The predicted mass of the lightest SUSY Higgs boson also depends on $\tan \beta$, with $m_h \sim 105$ GeV at $\tan \beta \approx 1.8$ and $m_h \sim 120$ GeV at $\tan \beta \gtrsim 20$ [3].

Because of the significance of $\tan \beta$ for the theory and phenomenology of the MSSM, it is important to find processes in which $\tan \beta$ can be best determined. Some regions
of the MSSM parameter space have been excluded at LEP2 [7] due to the lower bound on the lightest Higgs boson mass ($m_h$), particularly at low $\tan \beta$ near 1. Much of the parameter space remains to be explored at the upgraded Tevatron [8,9], the LHC [10–12], the future linear colliders [13–18] and muon colliders [19]. The $\tan \beta$ constraints that may be obtained from $m_h$ via radiative corrections [6], or from precision electroweak measurements [20], or from SUSY particle production usually depend also on other SUSY parameters. Furthermore, measurements of $\sin \beta$ or $\cos \beta$ via other SUSY processes without directly involving Higgs bosons do not accurately determine large $\tan \beta$ values [17]. For general SUSY Higgs phenomenology, we refer the readers to reviews [21].

The Higgs couplings of $H, A, H^\pm$ to heavy quarks are given by

$$
\begin{align*}
A\bar{t}t & : -\frac{g_m t}{2m_W} \cot \beta \gamma_5 \\
H\bar{t}t & : -\frac{ig_m t \sin \alpha}{2m_W \sin \beta} \approx \frac{ig_m t}{2m_W} \cot \beta \\
H\bar{b}b & : -\frac{igm_b \cos \alpha}{2m_W \cos \beta} \approx -\frac{igm_b}{2m_W} \tan \beta \\
H^+\bar{t}b & : \frac{ig V_{td}}{2\sqrt{2}m_W} \left[(m_b \tan \beta + m_t \cot \beta) + (m_b \tan \beta - m_t \cot \beta)\gamma_5\right],
\end{align*}
$$

where the decoupling limit $M_A \gg M_Z$ has been assumed for the approximate forms of $H\bar{t}t$, $H\bar{b}b$. In this limit, the lightest Higgs boson $h$ becomes SM-like and its couplings are insensitive to SUSY parameters. For the $H, A, H^\pm$ Higgs bosons, $\tan \beta$ is essentially the unique parameter for Higgs-heavy quark couplings. This suggests that studies of the associated production of the Higgs bosons and heavy quarks may effectively probe the $\tan \beta$ parameter.

Heavy Higgs boson production at future $e^+e^-$ colliders was discussed in Ref. [15]. In a recent study Feng and Moroi [16] evaluated the prospects for determining $\tan \beta$ in $e^+e^-$ collisions with $\sqrt{s} = 0.5$ and 1 TeV c.m. energy via the processes

$$
e^+e^- \rightarrow Zh, \ AH, \ t\bar{b}H^- \ and \ \bar{t}bH^+.
$$

The primary channel in their study involves the $H^+\bar{t}b$ coupling. They found that the strong dependence of heavy Higgs branching fractions on $\tan \beta$ allows stringent constraints to be placed for moderate $\tan \beta$ [16] in the MSSM. In the present paper, we report results of
a complementary study of the associated production of a neutral Higgs boson and heavy quarks

\[ e^+e^- \to H\bar{t}t, H\bar{b}b, A\bar{t}t, \text{ and } A\bar{b}b. \] (5)

These processes involve \( \bar{t}t \) and \( \bar{b}b \) production separately and are thereby expected to be complementary for low and high values of \( \tan\beta \). We study the sensitivity to probe \( \tan\beta \) in two scenarios: the minimal Supergravity model (mSUGRA) and the MSSM.

The paper is organized as follows: We present the Higgs decay branching fractions and the cross sections for the associated production of the Higgs bosons and heavy quarks in Sec. II. We analyze the sensitivity to determine the value of \( \tan\beta \) at future linear colliders in Sec. III. We discuss our results, make some general remarks and conclude in Sec. IV.

II. NEUTRAL HIGGS PRODUCTION

A. Input Parameters

The mass matrix of the CP-even Higgs bosons of the MSSM is given by

\[ M^2 = \begin{pmatrix} m_A^2 \sin^2 \beta + m_Z^2 \cos^2 \beta & -(m_A^2 + m_Z^2) \sin \beta \cos \beta \\ -(m_A^2 + m_Z^2) \sin \beta \cos \beta & m_A^2 \cos^2 \beta + m_Z^2 \sin^2 \beta \end{pmatrix} + \Delta M^2, \] (6)

where \( \Delta M^2 \) represents the radiative corrections. At tree level, the input parameters are \( m_A \) and \( \tan\beta \), and the phenomenology is relatively easy to analyze. The radiative corrections may be substantial in this CP-even sector and they are dependent on other SUSY parameters, especially on the masses and couplings/mixings of the top quark and heavy scalar quarks (squarks) [6]. In a general MSSM analysis the parameters required as input are

\[ m_Q, \ m_U, \ m_D, \ M_1, \ M_2, \ A_U, \ A_D \text{ and } \mu, \] (7)

where \( m_Q \) is the soft SUSY breaking mass parameter of left-handed stop (where only the heavy third generation parameters are relevant), \( m_U \) (\( m_D \)) the SUSY breaking mass parameter of right-handed stop (sbottom), \( M_1, M_2 \) the gaugino masses, \( A_U \) (\( A_D \)) the stop trilinear
soft breaking term, and $\mu$ the Higgs mixing parameter. The large parameter space involved makes phenomenological studies difficult. On the other hand, once a precision measurement is made in the Higgs sector in future collider experiments, we would expect to learn more about the SUSY sector due to the radiative relations among the physical SUSY masses. Instead of exploring the large space of the MSSM soft parameters, we focus on the following two scenarios for illustration.

**mSUGRA**

Motivated by the mSUGRA model and the requirements of radiatively generated electroweak symmetry breaking (EWSB), we relate the soft SUSY breaking parameters to the common scalar, fermion and trilinear parameters $m_0, m_{1/2}$ and $A_0$ at the grand unified scale. For specific choices of $\tan \beta$ the results depend on the sign of $\mu$. The $\mu > 0$ sign is less constrained by $b \to s\gamma$ decay [22] and we adopt this convention in our analysis.

We make use of the ISAJET package [23] to determine the SUSY masses and couplings from the GUT scale input parameters. The Higgs mass eigenvalues are among the outputs of this program. These values agree with the corresponding results from the code of Ref. [24] to a precision $\lesssim 0.3\%$. The soft-supersymmetry-breaking parameters are evolved according to renormalization group (RG) equations [4,25,26]. For our illustrations we make the parameter choice

$$m_0 = 250 \text{ GeV}, \quad m_{1/2} = 150 \text{ GeV}, \quad A_0 = -300 \text{ GeV},$$

along with the positive sign of $\mu$. The magnitude of $\mu$ is fixed in terms of $M_Z$ through the radiately generated EWSB. For three representative $\tan \beta$ values, the mass eigenvalues of Higgs bosons and SUSY soft-breaking terms are listed in Table I. For charginos and neutralinos, we list only the masses of the lightest ones. In fact, our choice of the above parameters is somewhat conservative in exploring the SUSY Higgs sector. A large $m_0$ results
in heavy $H, A, H^\pm$. Consequently it leads to the “decoupling limit” \cite{27} so that the lightest Higgs boson $h$ becomes SM-like and thus insensitive to the SUSY parameters.

**MSSM**

We also perform the same study in the MSSM scenario, in which $\tan \beta$ as well as the masses of the Higgs bosons (determined by $m_A$) are all free parameters to explore. The choice of other input parameters is as follows

$$\mu = 272 \text{ GeV}, \quad m_Q = 356 \text{ GeV}, \quad m_U = 273 \text{ GeV},$$

$$m_D = 400 \text{ GeV}, \quad A_U = -369 \text{ GeV}, \quad A_D = -672 \text{ GeV}$$

$$m_{\chi^\pm} = 111 \text{ GeV}, \quad m_{\chi^0} = 59 \text{ GeV}.$$  \hspace{1cm} (10)

(11)

(12)

These soft SUSY breaking parameters are similar to mSUGRA parameters with $\tan \beta \approx 15$. In particular we study two cases with $m_A = 200 \text{ GeV}$ and $400 \text{ GeV}$, while $m_H$ is nearly degenerate with $m_A$. These choices represent the kinematical situations for $A, H$ to be below and above $t\bar{t}$ threshold.

**B. Branching Fractions**

We use the program provided in Ref. \cite{24} for obtaining the branching fractions for the Higgs boson decay. In this program all kinematically allowed decay channels in MSSM are included and RG improved values of Higgs masses and couplings with the main NLO corrections \cite{3} are implemented.
In Fig. 1 we plot the branching fraction of the decays (a) for $A$ and (b) for $H$ versus $\tan \beta$ in mSUGRA. As $\tan \beta$ increases, the branching fractions of $A$ and $H$ decay into $t\bar{t}$ drop rapidly and the decays into $b\bar{b}$ increase dramatically. The branching fractions into chargino and neutralino pairs peak at intermediate values $\tan \beta \sim 5$ and can be as large as 30%. Branching fractions of $H$ decay into $hh$ and $WW$ are also shown in Fig. 1(b) for comparison. With this strong dependence of the branching fractions on $\tan \beta$, we expect neutral Higgs production channels to be useful in determining the value of $\tan \beta$. In particular, it is interesting to note the complementarity between $t\bar{t}$ and $b\bar{b}$ modes for small and large values of $\tan \beta$.

In the MSSM scenario, for the case of $m_A = 200$ GeV, Figure 2 shows the branching fraction of the decays of $A$ and $H$ versus $\tan \beta$. Note that the $t\bar{t}$ channel is not open. For the case of $m_A = 400$ GeV, the corresponding branching fractions are shown in Fig. 3. We see that Figs. 1 and 3 are alike due to similar kinematical thresholds.
C. Cross Sections and Final State Signature

As a representative example of the processes in Eq. (5), the tree-level Feynman diagrams for $e^+e^- \rightarrow Ht\bar{t}$ are shown in Fig. 4. For the other processes, we simply need to replace $H$ with $A$, or/and $t\bar{t}$ with $b\bar{b}$. The last diagram in Fig. 4 involving the $ZZH$ coupling is unique to the process which has $H$ in the final state. We have included both the diagrams of Higgs radiation off a heavy quark ($Ht\bar{t}$) and Higgs decay ($HA \rightarrow Ht\bar{t}$). It is important to note that the $H$ ($A$) decay processes are sensitive to $\tan \beta$ only when the branching fractions vary rapidly. The $H, A \rightarrow bb$ branching fractions gradually approach unity at large $\tan \beta$, and the dependence on $\tan \beta$ is thus reduced here. On the other hand, diagrams with $H$ ($A$) radiation off a heavy quark typically have a quadratic dependence on $\tan \beta$, and are thus quite sensitive to $\tan \beta$.

Figure 5 shows the calculated total cross sections of the processes $e^+e^- \rightarrow A(H)t\bar{t}, A(H)b\bar{b}$ versus the center of mass energy ($\sqrt{s}$) for $\tan \beta = 3$ and 30 in the mSUGRA scenario. The cross sections for $Ab\bar{b}, At\bar{t}$ can be typically of $0.1-10$ fb for this range of $\tan \beta$ at linear collider energies of $0.5-2$ TeV. The maximum rate is reached at a c.m. energy.
FIG. 3. MSSM: Leading branching fractions of decays with $m_A = 400$ GeV (a) of $A$ and (b) of $H$ versus $\tan \beta$.

about 300 GeV or so above the $At\bar{t}$ threshold. Note the different mass thresholds in this figure for the two values of $\tan \beta$, as given by the masses in Table I. For the heavy Higgs bosons under consideration, we concentrate on a collider energy $\sqrt{s} \sim 1$ TeV. We plot the cross sections versus $\tan \beta$ in Fig. 6 again in the mSUGRA scenario. At low $\tan \beta$ the associated production of $A$ with $t\bar{t}$ is dominant but this channel is greatly suppressed at large $\tan \beta$ values. On the other hand the production of $A$ in association with $b\bar{b}$ is small at low

FIG. 4. Feynman diagrams contributing to $e^+e^- \to Ht\bar{t}$. The diagrams for $e^+e^- \to At\bar{t}$ are similar, except that the last diagram above is absent.
tan $\beta$ and increases rapidly with tan $\beta$. Figures 5 and 6 show that associated production of $H$ with $tt$ or $b\bar{b}$ has similar characteristics to $A$ production. Figure 7 shows the cross sections similar to Fig. 6 but in the MSSM scenario for cases: $m_A = 200$ GeV at $\sqrt{s} = 500$ GeV (solid) and 400 GeV at $\sqrt{s} = 1$ TeV (dashes).

Concerning the final state signature with the $A(H)$ decays, we notice that at low tan $\beta$, both the production cross section for $A\bar{t}t$ ($H\bar{t}t$) and the branching fraction for $A$ ($H$) decay into $t\bar{t}$ are large as a result of the typical $(\cot \beta)^4$ enhancement. The $e^+e^- \rightarrow t\bar{t}t\bar{t}$ signal is dominant at low tan $\beta$ but at high tan $\beta$, $e^+e^- \rightarrow b\bar{b}b\bar{b}$ becomes dominant because of the $(\tan \beta)^4$ enhancement. For intermediate values of tan $\beta \sim 5$, the SUSY decay modes, such as $A, H \rightarrow \chi^+\chi^-$ and $\chi^0\chi^0$ can be more important. We show in Fig. 8(a) the total cross sections at $\sqrt{s} = 1$ TeV versus tan $\beta$ including the different final states. The complementarity of the three final states in different range of tan $\beta$ can be seen in this figure. Figure 8(b) again shows the contribution of these final states for tan $\beta$ values where they are most important: $4b$ for tan $\beta = 30$, $b\bar{b}\chi^0\chi^0$ or $b\bar{b}\chi^0\chi^0$ for tan $\beta = 10$, and $4t$ for tan $\beta = 3$. The $4b$ standard
FIG. 6. mSUGRA: Total Higgs production cross sections versus $\tan \beta$ at $\sqrt{s} = 1$ TeV (a) for $e^+e^- \rightarrow A\bar{t}t$ (dashes) and $A\bar{b}b$ (solid) and (b) for $e^+e^- \rightarrow H\bar{t}t$ (dashes) and $H\bar{b}b$ (solid).

model background is also shown by the dot-dashed curve.

In the MSSM scenario, similar curves for the $4t$ and $4b$ final state signals are shown in Fig. 9 for two cases: $m_A = 200$ GeV at $\sqrt{s} = 0.5$ TeV and $m_A = 400$ GeV at $\sqrt{s} = 1$ TeV.

D. Background

The most robust channels, $b\bar{b}b\bar{b}$ and $t\bar{t}t\bar{t}$, from neutral Higgs production have rather small SM backgrounds. The SM expectation for $e^+e^- \rightarrow b\bar{b}b\bar{b}$ production is shown in Fig. 8(b). The cross section decreases with increasing $\sqrt{s}$ as $(1/\sqrt{s})^2$. At $\sqrt{s} = 1$ TeV, the $4b$ background is only 0.1 fb, much smaller than the signal rate at large $\tan \beta$. The SM cross section for $e^+e^- \rightarrow t\bar{t}t\bar{t}$ is smaller than $10^{-3}$ fb at $\sqrt{s} = 1$ TeV and thus is negligible. The SM $4b$ background at 500 GeV is about 3.7 fb. We take this background into consideration when we calculate the limits at $\sqrt{s} = 500$ GeV. Since the SM backgrounds are small relative to the signals of interest, we do not need to impose sophisticated kinematical cuts and the signal rates are thereby better preserved. The final states involving the gauginos may have rather large SM backgrounds from $b\bar{b}$, $t\bar{t}$, gauge boson production. We neglect those channels in
III. ANALYSES AND RESULTS

As the parameter $\tan \beta$ is varied from small to intermediate to large values, the dominant Higgs signal comes from the three channels $tt\bar{t}$, $b\bar{b}\chi\chi$, $b\bar{b}b\bar{b}$, respectively. Since the sizes of the signal cross sections depend sensitively on $\tan \beta$, a determination of $\tan \beta$ should be possible throughout $\tan \beta$ ranges where there are substantial signal event rates. In our analyses, we employ the $tt\bar{t}$ signal at low $\tan \beta$ and the $b\bar{b}b\bar{b}$ signal at large $\tan \beta$. For the intermediate $\tan \beta$ values, we combine these two channels. We do not include the channels with gaugino final state in our consideration since the signatures would depend upon other SUSY parameters such as the slepton and squark masses. We thus regard the results of our analyses to be conservative.

We consider a $\sqrt{s} = 1$ TeV collider with three integrated luminosities of 50, 100 and 500 fb$^{-1}$. After applying the geometrical cut.
FIG. 8. mSUGRA: Total cross sections with different final states including $A, H$ decays (a) versus $\tan \beta$ at $\sqrt{s} = 1$ TeV, and (b) versus $\sqrt{s}$ for representative values of $\tan \beta = 3, 10$ and 30. The SM expectation of $4b$ production is also included for comparison.

$$\cos(\theta_b) < 0.9 \quad (13)$$

to the $4b$ signals, the total cross section is reduced to 23%, which we take as the geometrical efficiency. Because of the low background cross section, we only need low purity of $b$-tagging; we assume a $b$-tagging efficiency, $\epsilon_b \approx 65\% \quad [28]$. Since $b$-quark flavors are conserved in the production process, we can relax the requirement to tag only three $b$-quarks, as is a standard practice. Then the efficiency of detecting $3b$ in a $4b$ sample is $4\epsilon_b^3 - 3\epsilon_b^4 \approx 56\%$. For the $4t$ channel, although the event kinematics would be more involved, the distinctive event topology compared to the SM multi-jet backgrounds should allow a clear signal separation. Nonetheless, we still require the identification of at least three $b$ quarks. At a given $\tan \beta$ value, we multiply the total cross section of $4t$ or $4b$ channel with the geometrical efficiency, the $b$-tagging efficiency, and the integrated luminosity to get the signal event rate $N_S$. The statistical standard deviation is $\sigma = \sqrt{N_S}$. In the presence of SM backgrounds, we similarly determine the background event rate $N_B$. We then take the conservative estimate for the signal fluctuation.
FIG. 9. MSSM: Total cross sections with 4t and 4b final states including $A, H$ decays versus tan $\beta$. The solid curve is for $m_A = 200$ GeV at $\sqrt{s} = 500$ GeV; the dashes are for $m_A = 400$ GeV at $\sqrt{s} = 1$ TeV.

$$\sigma = \sqrt{N_S + N_B}. \quad (14)$$

For a 95% confidence level (C.L.) cross section measurement, the range for the number of events is taken to be $N_S \pm 1.96\sigma$. The corresponding bounds on the signal cross sections can be translated into allowed ranges $\Delta \tan \beta$ given by

$$\Delta \tan \beta = \tan \beta_{\pm} - \tan \beta, \quad (15)$$

where $\tan \beta$ is determined from $N_S$ and $\tan \beta_{\pm}$ is determined from $N_S \pm 1.96\sqrt{N_S + N_B}$.

We first consider the mSUGRA scenario at a $\sqrt{s} = 1$ TeV linear collider. We combine both $A$ and $H$ channels. In Fig. [10], the 95% C.L. constraints on $\Delta \tan \beta$ for 50 fb$^{-1}$ (solid), 100 fb$^{-1}$ (dashes) and 500 fb$^{-1}$ (dotted) are shown versus tan $\beta$. We find encouraging results for the tan $\beta$ determination. For instance, with a luminosity of 100 fb$^{-1}$, $|\Delta \tan \beta| \approx 3$ or better can be reached at the low value of tan $\beta$, mainly via the 4t channel. At the high value of tan $\beta$, $|\Delta \tan \beta| \approx 5$ can be reached, mainly via the 4b channel, which is better than 15% accuracy. The slightly more difficult region is tan $\beta \approx 6 - 7$, where the 4t and 4b channels
both yield smaller contributions. We expect that the inclusion of the chargino channels would improve the determination. Nevertheless, a good determination has been seen for the whole tan β range of interest.

![Graph showing determination of tan β](image)

**FIG. 10.** mSUGRA: Determination of tan β at $\sqrt{s} = 1$ TeV combining both $A$ and $H$ channels; 95% C.L. constraints on the tan β values are shown for 50 fb$^{-1}$ (solid), 100 fb$^{-1}$ (dashes) and 500 fb$^{-1}$ (dotted).

We next consider the MSSM Scenario. For the case with $m_A = 200$ GeV, the $A, H \to t\bar{t}$ decay channel is closed and we only make use of the processes with $4b$ in the final state. With the lower Higgs masses, it is sufficient to consider a linear collider with $\sqrt{s} = 500$ GeV. The 95% C.L. constraints on the tan β determination are show in Fig. 11(a) for 100 fb$^{-1}$ (solid), 200 fb$^{-1}$ (dashes) and 500 fb$^{-1}$ (dotted). In Fig. 11(b) we compare our result for 100 fb$^{-1}$ (solid) with that obtained by Feng and Moroi (dot-dashed) [16] and they are comparable.

For the case with $m_A = 400$ GeV, the constraints on tan β values are shown in Fig. 12, similar to the previous figure. Since the 4t channel is available in this case, the determination at low tan β is significantly improved. For most values of tan β, in particular higher values, we get more stringent constraints than the results in [16], indicating the potential of better determination on tan β via the neutral $H$ and $A$ channels under consideration. We list our tan β constraints in Table II based on a 100 fb$^{-1}$ integrated luminosity and compare with the
values that we estimate from the results by Feng and Moroi [16], where a different statistical procedure of $\chi^2$ was adopted. The results are largely comparable, but our constraints are somewhat tighter, especially for higher values of $\tan \beta$ as already seen in Fig. 12(b).

FIG. 11. MSSM: Determination of $\tan \beta$ for $m_A = 200$ GeV at $\sqrt{s} = 500$ GeV; (a) 95% C.L. constraints on the $\tan \beta$ values for 100 fb$^{-1}$ (solid), 200 fb$^{-1}$ (dashes) and 500 fb$^{-1}$ (dotted), (b) Comparison of 95% C.L. constraints on $\tan \beta$ for 100 fb$^{-1}$ of our result (solid) with that obtained from Ref. [16] (dot-dashed).

IV. DISCUSSION AND CONCLUSION

For the mSUGRA scenario, $\tan \beta$ is essentially the only variable after fixing the other soft SUSY breaking parameters as in Eq. (9). The masses of the $H, A$ Higgs bosons decrease as $\tan \beta$ increases. Thus the corresponding Higgs branching fractions and the production cross sections at a given energy increase with $\tan \beta$, especially for large values. This leads to possible accurate determinations of $\tan \beta$ in mSUGRA (Fig. 10), for high values in particular. In contrast, for the MSSM scenario the masses of $A$ and $H$ are independent of $\tan \beta$, and are kept fixed in the analyses. At large $\tan \beta$ values the decay branching fractions and the
FIG. 12. MSSM: Determination of $\tan\beta$ for $m_A = 400$ GeV at $\sqrt{s} = 1$ TeV; (a) 95% C.L. constraints on the $\tan\beta$ values for 100 fb$^{-1}$ (solid), 200 fb$^{-1}$ (dashes) and 500 fb$^{-1}$ (dotted), (b) Comparison for 95% C.L. constraints on $\tan\beta$ for 100 fb$^{-1}$ of our result (solid) with that obtained from Ref. [16] (dot-dashed).

production cross sections of $A$ or $H$ with $b\bar{b}$ reach a plateau in the MSSM. Consequently the determination of $\tan\beta$ in that range is less effective.

There are other processes by which $\tan\beta$ may also be constrained: (i) Chargino pair production in $e^+e^-$ collisions can provide good measurements on $\tan\beta$ for low $\tan\beta$ values [13,18]; (ii) $\tilde{\tau}_L - \tilde{\tau}_R$ mixing can be a sensitive probe of $\tan\beta$ [14]; (iii) Gaugino production in $e\gamma$ collisions may provide information on $\tan\beta$ [17]; (iv) Kinematical distributions from the decay products of SUSY particles can be used to determine the $\tan\beta$ value [11]; (v) The magnetic dipole moment of the muon may be useful for $\tan\beta \gtrsim 20$ if slepton masses $m_{\tilde{l}} \lesssim 300$ GeV [29]; (vi) The branching fractions of $H, A \to \tau\bar{\tau}$ may be useful to set a lower bound $\tan\beta \gtrsim 10$ [10,30]. The alternative methods in (i)-(iv) probe either $\sin\beta$ or $\cos\beta$; thus the sensitivity to $\tan\beta$ is degraded at high values of $\tan\beta$. On the other hand, methods (v) and (vi) are only effective for high values of $\tan\beta$. In contrast, Higgs boson production processes under consideration and the $t\bar{b}H^\pm$ process discussed in Ref. [16] are direct probes
Tan β

This analysis Feng and Moroi

| tan β | 2.4 < tan β < 3.6 | tan β < 5.2 |
|-------|-------------------|--------------|
|       | 4.3 < tan β < 6.3 | 3.0 < tan β < 6.0 |
| 10    | 6.2 < tan β < 12.7 | 6.5 < tan β |
| 20    | 14 < tan β < 32   | 7.5 < tan β < 90 |
| 30    | 18 < tan β < 80   | 8.0 < tan β |

TABLE II. Constraints on values of tan β by 95% C.L. statistical measurement on the cross sections combining both A and H channels in the MSSM scenario; the results of Feng and Moroi shown here are estimated from the curves in Ref. [16] based on the t\bar{b}H^± process.

In summary, we studied heavy neutral Higgs boson production in the minimal super-symmetric theories at a linear collider with \( \sqrt{s} = 0.5 - 1 \) TeV with the expected integrated luminosities of 50 – 500 fb\(^{-1}\). The cross sections have a strong dependence on the fundamental supersymmetry parameter tan β, and thus provide a good way to determine it. We considered the 4b and 4t final states which are sensitive and complementary in determining tan β. In the Supergravity scenario, the sensitivity is particularly good for tan β > ∼10 in comparison with other methods, reaching a 15% or better determination in a 95% C.L. cross section measurement. In the general MSSM scenario, the interplay between the 4b and 4t channels results in a good determination for tan β < ∼10 (see Table II). For higher values of tan β the sensitivity is weakened. The accuracy of tan β determination is generally sufficient to distinguish theories with a low value (≈ 2) from a high value (> 30) and thus to provide information on testing certain GUTs scenarios.

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