Antihyperon Polarization in High-Energy Inclusive Reactions

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We propose a model for the antihyperon polarization in high-energy proton-nucleus inclusive reactions, based on the final-state interactions between the antihyperons and other produced particles (predominantly pions). To formulate this idea, we use the previously obtained low-energy pion-(anti-)hyperon interaction using effective chiral Lagrangians, and a hydrodynamic parametrization of the background matter, which expands and decouples at a certain freezeout temperature. 

I. INTRODUCTION

The unexpected discovery, by Bunce et al. in 1976, of large Λ polarization in the inclusive production, induced by proton-Beryllium collisions at 200 GeV, opened an interesting field of investigation. Earlier experimental data, as those on the πp, pp and Kp collisions, indicated that in general the polarization decreased with the energy and, moreover, theoretically it was hard to believe that high-energy inclusive reactions could produce significant polarization, because it was expected that a large cancellation occurred when many uncorrelated channels are superposed.

In the following years many proton-nucleus collision experiments have been performed and these experiments confirmed the Λ polarization, in the pt range of 0-2 GeV. They also showed that in the same domain the Λ polarization is very small, consistent with 0. This fact created the false idea that only the hyperons could be polarized and the antihyperons not. In 1981, Wilkinson and his collaborators measured for the first time the polarization of a charged hyperon, Σ^+, and found to be positive. In 1983, the Σ^- polarization was measured, with results consistent with the Σ^+ ones, and in 1986, the Ξ^- polarization was observed, and it was found to be negative. More recently, polarizations of hyperons produced with a Σ^- beam also have been measured.

Soon it was possible to obtain polarizations of other hyperons: in 1990 the Ξ^+ was measured, negative, and in 1993 Morelos and his collaborators measured the Σ^- polarization that is positive. These results showed that, refuting the earlier belief, the antihyperon polarizations are not always zero.

Many models have been proposed trying to explain these experimental results, such as the Lund model, the quark recombination model, and the OPER (reggeized one pion exchange), that explained the Λ polarization but were not able to calculate the Σ^- polarization. These models are based on the leading-particle effect, that means that the observed hyperon is considered as a direct product of the incoming proton after a recombination. Clearly, this mechanism can be effective only for hyperons but not for antihyperons.

Later, models based on constituent quarks and others were proposed, but a satisfactory explanation for the antihyperon polarization was not achieved. As we can see, the models are able to explain the Λ polarization (and even Υ), but in fact, all of the models predict the production of unpolarized antihyperons and this fact shows that these models must be supplemented in order to describe also the antihyperon data.

It is well known that final-state interactions are fundamental in the study of the nonleptonic hyperon decays, where the final-state strong phases must be considered. In high-energy inclusive processes, a large number of particles is produced, so we expect that something similar occur in this kind of reactions. In 1993, Y. Hama and T. Kodama used the hydrodynamical model to study the effect of final-state interactions of the produced (anti) hyperons with the background matter. It was supposed that the (anti) hyperons are produced inside a medium composed of hot hadronic matter, that is formed during the collision, and the polarization is the result of the interaction of the hyperon with the surrounding matter. In this interaction was represented by an optical potential and they showed that this mechanism could produce polarized hyperons and antihyperons. However, some important questions remained opened, namely, the origin of the potentials and why different potentials are needed for the different (anti) hyperons.

The purpose of this paper is to improve the model described in the preceding paragraph, by explicitly including some detailed microscopic interaction between the antihyperon and the surrounding thermalized medium. Considering that in this kind of collisions pions are predominant among the produced particles, as a first approximation, we will include only the pion-antihyperon interactions in the study. The observed polarization is the average effect of these interactions. An important feature is that as the antihyperons and pions are in thermal equilibrium inside this fluid,
the energy involved in the microscopic interactions is generally low. Also, since the hyperons are produced mostly as the leading particles, we shall confine ourselves only to antihyperon polarization.

A crucial factor to the polarization mechanism is the shape of the fluid rapidity distribution, $dN/da$, at the moment of emission. If $dN/da$ were constant, no polarization would be expected because, for antihyperons with rapidity $y$, the contributions coming from the $\alpha > y$ region would cancel out the ones from the $\alpha < y$ domain, as they have opposite signs. But in the relevant $\alpha$ domain for the experimentally produced antihyperons, $dN/da$ is decreasing, and then, the average polarization is dominated by the contributions from $\alpha < y$. Another remark is that, since we are interested in antihyperons which are escaping, the microscopic collisions is essentially in the forward direction.

The content of this work is the following. In the next section, the previously developed low energy pion-hyperon interactions [22, 23, 24], based on chiral Lagrangians, are shortly reported. In Sec. III these results are used to calculate the polarizations in high-energy inclusive processes. In Sec. IV the results are shown and, finally, conclusions are drawn in Sec. V.

II. LOW-ENERGY PION-HYPERON INTERACTIONS

In this section, the basic question to be answered is how to describe the low energy $\pi Y$ interactions in a reliable way. The path to be followed is to study the $\pi Y$ interactions and then to use the CPT invariance (neglecting the possible effects of $CP$ violation) to determine the amplitudes in the $\pi Y$ scattering. We will study the $\pi \Lambda$, $\pi \Sigma$ and $\pi \Xi$ interactions, and then, the quantities of interest that are cross sections and polarizations, will be calculated. The $Y$ polarization, for example, is just the $Y$ one, with the opposite sign. The cross sections for charge conjugate reactions are the same.

Unfortunately, one must remark that this kind of interaction is not very well studied experimentally, the first measurement of phase shifts that has been made, was the difference $\delta_{\pi} - \delta_{\rho}$ in the $\pi \Lambda$ interaction for $\sqrt{s} = m_\Xi$, by the Fermilab E756 experiment [25], and improved last year by the HyperCP experiment [26], but still with reasonable experimental uncertainties.

So, the option is to describe these interactions with a model. Some models [29, 33] have been proposed to study the $\pi \Lambda$ phase shifts, but these values are not accurately determined yet. On the other hand, the low energy $\pi N$ interactions can be described with a high degree of accuracy, with models based on chiral symmetry [27, 28]. So, in this work we will use a model [22, 24] which is based on effective chiral Lagrangians, that considers baryons, spin $3/2$ resonances, rho mesons and the $\sigma$ term. An important feature of the model is the inclusion of resonances in the intermediate states. In the $\pi^+ P$ scattering ($3/2$ isospin channel) the $\Delta^{++}(1232)$ dominates the total cross section at low energies and is also very important in the other isospin channels. In the $\pi Y$ case, we expect that the same behavior occurs. The Lagrangians to be considered are

$$\mathcal{L}_{\pi BB'} = \frac{g_a}{2 f_\pi} \left[A' \gamma_\mu \gamma_5 T^a B \right] \partial^\mu \phi_a + \text{h.c.}$$  \hspace{2cm} (1)

$$\mathcal{L}_{\pi BR} = \frac{g_\pi}{2 f_\pi} \left[\bar{R}^i \gamma_\mu \left( Z + \frac{1}{2} \gamma_\mu \gamma_5 \right) T^a B \right] \partial^\mu \phi_a + \text{h.c.}$$  \hspace{2cm} (2)

$$\mathcal{L}_{B \rho B'} = \frac{g_\pi}{2} \left[\bar{B} \gamma_\mu T^a B \right] \partial^\mu \phi_a + \frac{g_\pi}{2} \left[\bar{B} \left( \frac{\mu_B - \mu_B}{4 m_B} \right) i \sigma_{\mu \nu} T^a B \right] \partial^\nu \phi_a + \text{h.c.}$$  \hspace{2cm} (3)

$$\mathcal{L}_{\rho \pi \pi} = \gamma_0 \bar{\rho}_\mu (\phi \times \partial^\mu \phi) - \frac{\gamma_0}{4 m_\rho^2} (\partial_\mu \rho_{\nu} - \partial_\nu \rho_{\mu}) (\partial^\mu \phi \times \partial^\nu \phi) + \text{h.c.}$$  \hspace{2cm} (4)

where $B$, $R_B$, $\phi$, $\rho$ are the baryon, the spin $3/2$ resonance, the pion and the rho fields with masses $m_B$, $m_R$, $\mu$, and $m_\rho$, respectively, $\mu_B$ is the baryon magnetic moment, $T^a$ are the isospin matrices and $Z$ is a parameter representing the possibility of the off-shell-resonance having spin $1/2$.

The first case to be considered is the The $\pi \Lambda$ interaction. Since $\Lambda$ has isospin 0, the scattering amplitude $T_{\pi \Lambda}$ has the general form

$$T_{\pi \Lambda}^{ba} = \bar{\pi}(p') \left\{ A + \frac{(k + k')}{2} B \right\} \delta_{ab} u(p) + \text{h.c.}$$  \hspace{2cm} (5)

where $p_\mu$ and $p_\mu'$ are the initial and final 4-momenta of $\Lambda$, $k_\mu$ and $k'_\mu$ are those of the pion. Indices $a$ and $b$ indicate the initial and final isospin states of the pion. Fig. 1 shows the relevant diagrams, where we have omitted the crossed diagrams, although included in the calculations. To calculate the diagrams from Fig. 1, we must use the Lagrangians (1) and (2), that in this case are
\[ \mathcal{L}_{\pi\Lambda\Sigma} = \frac{g_{\Lambda\pi\Sigma}}{2m_\Lambda} \left\{ \sum_{\mu} \gamma_\mu \gamma_5 \tau_\Lambda \right\} \partial^\mu \phi + h.c. \]  \hspace{1cm} (6)

\[ \mathcal{L}_{\pi\Sigma\sigma} = g_{\Lambda \pi \Sigma \sigma} \left\{ \sum_\mu \left[ g_{\mu\nu} - \left( Z + \frac{1}{2} \right) \gamma_\mu \gamma_\nu \right] \tau_\Lambda \right\} \partial^\nu \phi + h.c. \]  \hspace{1cm} (7)

The interaction with the intermediate resonance Σ∗, shown in Fig. 1b, is

\[ A_{\Sigma^*} = \frac{g_{\Lambda \pi \Sigma^*}}{3m_\Lambda} \left\{ \frac{\nu}{\nu_r^2 - \nu^2} \hat{A} - \frac{m_\Sigma^* + m_\Lambda m_{\Sigma^*}}{m_{\Sigma^*}^2} (2m_{\Sigma^*}^2 + m_\Lambda m_{\Sigma^*} - m_\Sigma^* - 2\mu^2) \\
+ 4m_\Lambda \left[ (m_\Lambda + m_{\Sigma^*})Z + (2m_{\Sigma^*} + m_\Lambda)Z^2 \right] k.k' \right\}, \]

\[ B_{\Sigma^*} = \frac{g_{\Lambda \pi \Sigma^*}}{3m_\Lambda} \left\{ \frac{\nu}{\nu_r^2 - \nu^2} \hat{B} - \frac{8m_\Lambda^2 \nu Z^2}{m_{\Sigma^*}^2} \right\}, \]  \hspace{1cm} (9)

with

\[ \hat{A} = \frac{(m_{\Sigma^*} + m_\Lambda)^2 - \mu^2}{2m_{\Sigma^*}^2} \left\{ 2m_{\Sigma^*}^2 - 2m_\Lambda^2 - 2m_{\Sigma^*} m_\Lambda \right\} \\
- 2m_{\Sigma^*}^2 m_{\Sigma^*} + \mu^2 (m_\Lambda - m_{\Sigma^*}) \right\} + \frac{3}{2} (m_\Lambda + m_{\Sigma^*}) t, \]

\[ \hat{B} = \frac{1}{2m_{\Sigma^*}^2} \left[ (m_{\Sigma^*}^2 - m_\Lambda^2)^2 - 2m_\Lambda m_{\Sigma^*} (m_{\Sigma^*} + m_\Lambda)^2 \\
+ 6\mu^2 m_\Lambda (m_{\Sigma^*} + m_\Lambda) - 2\mu^2 (m_{\Sigma^*} + m_\Lambda)^2 + \mu^4 \right] + \frac{3}{2} t, \]  \hspace{1cm} (10)

where \( \nu \) and \( \nu_r \) are defined in the Appendix A. Note that eq. (10) shows small differences when compared with the one shown in [22], where there was a mistake. The correct expression is presented here.

In addition to these contributions, the σ term (Fig. 1c) was also taken into account. In [33], [34], in the study of \( \pi N \) interactions, it was included just as a parametrization

\[ A_\sigma = a + bt, \]
\[ B_\sigma = a, \]  \hspace{1cm} (11)

where \( a = 1.05 \mu^{-1} \) and \( b = -0.80 \mu^{-3} \) are constants. Recent works [27], [35] consider that the σ term may be understood in terms of the exchange of two pions, in loop diagrams, as it is shown in Fig 2.

So, these ideas lead to a σ contribution of the form

\[ A_\sigma = \sigma_\Sigma + \sigma_{\Sigma^*}, \]
\[ B_\sigma = 0, \]  \hspace{1cm} (12)
FIG. 2: The scalar form factor receives contributions from tree interactions (white blob) and triangle diagrams with spin $1/2$ and $3/2$ intermediate states.

where $\sigma_\Sigma$ and $\sigma_{\Sigma^*}$ have been calculated in [23].

One must remark that considering the diagrams of Fig. 1, the resulting amplitude will be real and consequently the $S$ matrix will not be unitary. So, the unitarization of the amplitudes must be done. Observing that the scattering matrix may be expressed as

$$M_{ba} = \frac{T_{ba}}{8\pi\sqrt{s}} = f_1 + \frac{(\bar{\sigma} \cdot \bar{k})(\sigma \cdot k)}{kk'} f_2$$  \hspace{1cm} (13)

the partial wave decomposition can be made with

$$a_{l+} = \frac{1}{2} \int_{-1}^{1} \left[ P_l(x) f_1(x) + P_{l\pm 1}(x) f_2(x) \right]$$  \hspace{1cm} (14)

and the unitarization of these amplitudes, can be made if we reinterpret them as elements of $K$ matrix and write

$$a_{l+}^U = \frac{a_{l+}}{1 - ik a_{l+}}$$  \hspace{1cm} (15)

The phase-shifts are then computed as

$$\delta_{l\pm} = t g^{-1}(k a_{l\pm})$$  \hspace{1cm} (16)

and details on this calculation are shown in [22], [24].

The masses and magnetic moments that we used may be found in [36]. The $\Lambda\pi\Sigma$ coupling constant is $g_{\Lambda\pi\Sigma}=12.92$ (see Appendix A). The resonance coupling constants were calculated with the comparison of the calculated amplitude and the Breit-Wigner one, with the data from [36] as it was done in [22]. The same procedure was adopted to calculate the other $Y\pi Y^*$ couplings. The obtained value is $g_{\Lambda\pi\Sigma^*}$ is $9.38$ GeV$^{-1}$. With these results, we can calculate $\sigma_t$, $d\sigma/d\Omega$ and $P$, that will be needed in the next section.

FIG. 3: Total $\pi\Lambda$ cross section.
Observing Fig. 3, one can see a peak in the $\Sigma^*(1385)$ mass, what shows that the resonance dominates the total cross section at low energies.

In the case of $\pi\Sigma$ interaction, the particles have isospin 1, so the composed system can have isospin 2, 1 or 0. For this reason, the scattering amplitude in this case has the general form

$$T_{\alpha\gamma,\beta\delta} = \langle \pi_\gamma \Sigma_\delta | T | \pi_\alpha \Sigma_\beta \rangle$$

where $\alpha, \beta, \gamma$ and $\delta$ are isospin indices of the particles.

The diagrams that we consider for the $\pi\Sigma$ interactions are shown in Fig. 5. The interactions with the intermediate resonance $\Sigma^*(1385)$ could also be included, but its decay branching ratio to the $\pi\Sigma$ channel is only 11%, what may be considered as a correction, and then we will neglect it.

The amplitudes of eq. (17) for the diagrams of Fig. 5 have been calculated in [22], and the $\sigma$ contribution is given by

$$B_\sigma = \sigma_\Lambda + \sigma_\Sigma + \sigma_{\Lambda^*},$$
$$A_\sigma = A'_\sigma = B'_\sigma = C_\sigma = B''_\sigma = 0,$$  \hspace{1cm} (18)

where $\sigma_\Lambda$, $\sigma_\Sigma$ and $\sigma_{\Lambda^*}$ are taken from [23].

The calculated total cross sections with $\Sigma^+$ in the final states are shown in Fig. 6. The resonance, that in this case is $\Lambda^*(1405)$, is still important, but now, the peak is not so high than it is in the other cases (less than 30 mb). It appears in the reactions where the 0 isospin state is important. The polarizations for these reactions are shown in Fig. 7.

Now, let us turn our attention to the $\pi\Xi$ interaction. This case is very similar to the $\pi N$ scattering, because $\Xi$ has isospin 1/2 (as the nucleon) and the main difference is that the resonance of interest $\Xi^*(1533)$ has isospin $I=1/2$.
FIG. 6: $\pi\Sigma$ cross sections for channels with $\Sigma^+$ in the final state.

(instead of $I=3/2$ as $\Delta(1232)$). Then, the scattering amplitude $T_{ba}^{\pi\Xi}$ has the general form

$$T_{ba}^{\pi\Xi} = \frac{n(p')}{n(p)} \left\{ [A^+ + \frac{(k + k')}{2} B^+] \delta_{ba} ight. + \left. [A^- + \frac{(k + k')}{2} B^- i \epsilon_{bac} \tau^c] u(\vec{p}') \right\}.$$  \hfill (19)

The calculated $A^\pm$ and $B^\pm$ for the diagrams of Fig. 8 may be found in [22],[24].

In Fig. 9 we can see that one more time the resonance is very important, now the $\Xi(1533)$ resonance contribution dominates three of the reactions.

So, now we have the $\pi\Lambda$, $\pi\Sigma$ and $\pi\Xi$ scattering amplitudes, that are the basic elements to calculate the polarization in high-energy processes.

III. POLARIZATION IN INCLUSIVE PROCESSES

In this section, it will be shown how to calculate the $\bar{Y}$ polarization in high energy processes. As it was said, we consider that during the collision, a system composed of expanding hot hadronic matter is formed. This system may be considered to be a fluid, composed of elements, that expands according to a Gaussian law [37],[38]

$$\frac{1}{\text{cho}_{\alpha} \text{sho}_{\alpha}} \frac{d^2 \sigma}{d\alpha d\phi} = A \left[ e^{-\beta(\alpha-\alpha_0)^2} + e^{-\beta(\alpha+\alpha_0)^2} \right] e^{-\beta_\alpha \alpha^2},$$ \hfill (20)

where $\alpha$ and $\alpha_0$ are the longitudinal and transverse rapidities of these elements of fluid, $\phi$ is the azimuthal angle and $\beta$, $\beta_\alpha$ and $\alpha_0$ are parameters that determine the shape of this distribution. The values of these parameters depend on the participants of the collision (proton, beryllium, ...) and on their energies.

The pions and hyperons are produced thermally inside these fluid elements, with initial momenta (relative to the fluid element) $\vec{p'}_\pi$ (pions) and $\vec{p'}_\Lambda$ (hyperons) and energies $E'_{\pi_0}$, $E'_{\Lambda_0}$, obeying the statistical distributions

$$F_{BE}(\pi'_{0,\pi}) \propto \frac{1}{\exp(E'_{\pi_0}) - 1},$$ \hfill (21)

$$F_{FD}(\Lambda'_{0,\Lambda}) \propto \frac{1}{\exp(E'_{\Lambda_0}) + 1}.$$ \hfill (22)

The hyperon emerges with final momentum $\vec{p'}$, energy $E'$ and polarization $\vec{P'}$. 


FIG. 7: Polarizations in the $\pi \Sigma$ Interaction.
The produced hyperons interact, and as the relative energy between the $\pi$ and the $Y$ is small, the amplitudes obtained in the last section may be used to calculate the polarization $\vec{P}'$ and $d\sigma/dt$. Then the average polarization may be calculated by the expression

$$\langle \vec{P} \rangle = \frac{\int \left\{ \left( \frac{d\sigma}{dt} \right) R_1 \right\} \cdots \left\{ \left( \frac{d\sigma}{dt} \right) R_N \right\} \mathcal{G} d\tau}{\int \left\{ \left( \frac{d\sigma}{dt} \right) R_1 \right\} \cdots \left\{ \left( \frac{d\sigma}{dt} \right) R_N \right\} \mathcal{G} d\tau} ,$$

(23)

that considers the elastic and with charge exchange $R_i$ reactions, in the interactions of the hyperons with $\pi^+$, $\pi^-$ and $\pi^0$.

The factor $\mathcal{G}$ that appears in eq. (23) contains the statistical weighs of the production of the particles and the ones relative to the expansion of the fluid, and can be written as

$$\mathcal{G} = \frac{(d^2 N/d\alpha d\sigma) \Lambda_0^2 \pi_0^2}{(\exp \left( \frac{E_0'}{T} \right) - 1) \left( \exp \left( \frac{E_0'}{T} \right) + 1 \right)} \times \delta \left( E_0' + E_{\pi_0}' - E' - \sqrt{m_\pi^2 + (m_0' + \Lambda_0' - \Lambda')^2} \right) ,$$

(24)

and the eight dimension integration element is

$$d\tau = d\alpha \, d\sigma_1 \, d\Lambda_0' \, d\pi_0' .$$

(25)

In order to calculate the average polarization (23), we must know the parameters $\beta$, $\beta_0$ and $\alpha_0$ of the rapidity distribution (20) of the fluid elements and if this expression is a good choice. It is possible to answer these questions if we use the experimental data of the rapidity distributions of the pions produced in proton-proton collisions (that is a collision similar to the $p$–Be). In the hydrodynamical framework, the longitudinal rapidity distribution of the produced pions is given by

$$\frac{d\sigma}{dy} = \int \frac{d\sigma}{d\alpha}(\alpha) \frac{d\sigma}{dy}(y - \alpha) \, d\alpha ,$$

(26)
that is a convolution of the rest-frame rapidity distribution $d\sigma/dy'$, and the fluid rapidity distribution, $d\sigma/d\alpha$.

The pion momentum distribution with respect to the fluid element (considering that they are produced thermally) is

$$
\frac{d\sigma}{d\vec{p}_\pi'} = \frac{1}{E_{\pi}' d\vec{y}' d\vec{p}_{t}'} \frac{d\sigma}{dy'} = \frac{1}{e^{E_{\pi}'/T} - 1}$$

(27)

that after some manipulations gives

$$
\frac{d\sigma}{dy'} \sim Ce^{-\beta'y'}
$$

(28)

with $\beta' \sim 0.98$ for the temperature $T \sim m_{\pi}$.

The distribution $d\sigma/d\alpha$ may be obtained from eq. (20), and will be a sum of Gaussian functions

$$
\frac{d\sigma}{d\alpha} = A'[e^{-\beta'(\alpha-\alpha_0)^2} + e^{-\beta(\alpha+\alpha_0)^2}]
$$

(29)
inserting this expression in (26), integrating and fitting the parameters $A'$, $\beta$ and $\alpha_0$, we can compare the results with the experimental data from ISR [39]. The results are shown in Fig. 11.

The best fits for $d\sigma/dy$ are obtained with the parameters $A' = 0.45$, $\alpha_0 = 1.8$ and $\beta = 2.0$ for 491 GeV ($\sqrt{s_{NN}}=31$ GeV) and $A' = 0.58$, $\alpha_0 = 1.9$ and $\beta = 1.5$ for 1030 GeV ($\sqrt{s_{NN}}=44$ GeV) incident protons. Our real interest is for 330, 400 and 800 GeV incident protons, that are the values for which there are antihyperon polarization data. The respective parameters may be obtained by interpolation. Observing Fig. 11 one can see that the $\beta$ parameters may vary from 1.0 to 3.0 without changing significantly the accord with the experimental data. So in the calculation of the average polarization, we will consider the parameter $\beta$ inside this range. One must remark that for heavier targets, the value of the parameters may change, as it can be seen in Fig. 12, where $\beta=4.0$ is reasonable for $p−Ar$ collisions ($m_{Ar}=39.95$) and 2.0 for $pp$ collisions.

The transverse rapidity parameter $\beta_t$ will be assumed to be $\beta_t = 6.6$ for 800 GeV, 7.5 for 400 GeV and 8.0 for 330 GeV.

The reactions $R_i$ to be considered to calculate the $\Lambda$ polarization with eq. (23) are

\[
\begin{align*}
\pi^+\Lambda &\rightarrow \pi^+\Lambda \\
\pi^-\Lambda &\rightarrow \pi^-\Lambda \\
\pi^0\Lambda &\rightarrow \pi^0\Lambda
\end{align*}
\]

The reactions $R_i$ to be considered to calculate the $\bar{\Lambda}$ polarization with eq. (23) are

\[
\begin{align*}
\pi^+\bar{\Lambda} &\rightarrow \pi^+\bar{\Lambda} \\
\pi^-\bar{\Lambda} &\rightarrow \pi^-\bar{\Lambda} \\
\pi^0\bar{\Lambda} &\rightarrow \pi^0\bar{\Lambda}
\end{align*}
\]
To calculate the $\Sigma^-$ polarization, we consider six reactions (elastic and charge exchange), that are the charge conjugate of the ones shown in Fig. 6:

\begin{align*}
\pi^- \Sigma^- &\to \pi^- \Sigma^- \\
\pi^0 \Sigma^- &\to \pi^0 \Sigma^- \\
\pi^+ \Sigma^- &\to \pi^+ \Sigma^- \\
\pi^- \Sigma^+ &\to \pi^- \Sigma^+ \\
\pi^- \Sigma^- &\to \pi^0 \Sigma^- \\
\pi^0 \Sigma^+ &\to \pi^+ \Sigma^- \\
\end{align*}

(31)

It is important to remark that even at low energies, the exchange reactions $\pi \Lambda \leftrightarrow \pi \Sigma$ may occur. However, observing the fact that the resonance diagrams dominate the cross sections in this region, and the facts that the $\Lambda^*$ resonances may not be formed in $\pi \Lambda$ interactions, and that $g_{\Sigma^* \Sigma^*} < < g_{\Lambda \pi \Sigma^*}$ (considering $\Sigma^*(1385)$) we expect that the amplitude of the $\pi \Lambda \to \pi \Sigma$ reactions are not larger than 20% of the ones for the $\pi \Sigma \to \pi \Sigma$, so, in a first approximation this kind of reactions may be neglected.

In the $\Xi^+$ case, four reactions are considered,

\begin{align*}
\pi^- \Xi^+ &\to \pi^- \Xi^+ \\
\pi^0 \Xi^+ &\to \pi^0 \Xi^+ \\
\pi^+ \Xi^+ &\to \pi^+ \Xi^+ \\
\pi^+ \Xi^+ &\to \pi^- \Xi^+ .
\end{align*}

(32)

IV. RESULTS

Integrating eq. (23) with the Monte Carlo method, we obtained the $\Lambda$, $\Sigma^-$ and $\Xi^+$ polarizations that are shown in Fig. 13-19.

Fig. 13 shows the $\Lambda$ as a function of $p_t$, produced from a 400 GeV proton beam and for the production angles $\theta_L = 7.5$ and 20 mrad compared with the data from [4], [5] We can see that the model gives $P_{\Lambda} \sim 0$ in the considered $p_t$ region, what is consistent with the experimental results. This fact may be justified observing Fig. 4, in an average calculation the small contributions of the microscopic processes are almost totally canceled.

![Graph](image)

**FIG. 13:** $\Lambda$ polarization compared with the experimental data of [4] and [5] (proton beam).

In Fig. 14 the calculated polarization is now compared with the data obtained with a 330 GeV $\Sigma^-$ beam [3], and the prediction $P_{\Lambda} \sim 0$ is still consistent with the data.

The calculated $\Xi^+$ polarization can be seen in Fig. 15. The resultant polarization is negative and with $\beta \sim 2$, the results are in accord with the experimental data from the E761 experiment [11], where $\theta_L = 2.4$ mrad and $p_L = 800$ GeV.
FIG. 14: $\Lambda$ Polarization for $\beta = 2$, compared with the WA89 experimental data [9], obtained with a 330 GeV $\Sigma^-$ beam and at $<x_F> = 0.11$.

FIG. 15: $\Xi^+$ polarization, for some values of $\beta$ compared with the E761 data [10] (production angle of 2.4 mrad and incoming beam of 800 GeV).

FIG. 16: $\Sigma^-$ polarization compared with the data of [11] (for $<x_F> = 0.5$).
FIG. 17: $\Lambda$ polarization, for $\beta = 2$ and some production angles

FIG. 18: $\Xi^+$ polarization, for $\beta = 2$ and some production angles

FIG. 19: $\Sigma^-$ polarization for $\beta = 3$ and some production angles
The resultant $\Sigma^-$ polarization is positive, but with magnitude smaller than the one of the experimental data \cite{11} (Fig. 16). Considering that the experiment is for $p$-Cu collisions one must expect a value of $\beta$ between 2.0 and 4.0 (see Fig. 12). The production angle $\theta_L$ is between 0.9 mrad and 2.9 mrad, and we made the calculations in this region, considering the average value of $x_F$ in the experiment, that is 0.5 \cite{11}. In this case, we expect that some effects must be included in order to improve the accord with the experimental data.

Fig. 17-19 shows the $\Lambda$, $\Sigma^-$ and $\Xi^+$ polarization dependence with the production angle $\theta_L$. Observing the figures, we can conclude that the antihyperon polarization is a small angle effect, increasing $\theta_L$ the polarization vanishes. The model predicts $P_Y \sim 0$ for $\theta_L > 50$ mrad. This fact explains why the antihyperon polarization was found experimentally in angles as small as few mrad.

V. DISCUSSION OF THE RESULTS

In this paper, we proposed a model in order to explain the antihyperon polarization data. It is an indirect mechanism, the antihyperon is produced unpolarized, and then becomes polarized by the final-state interactions. This model supplements the direct mechanisms, that describe the hyperon polarization, and are based on the leading particle effect \cite{12}-\cite{15}. We expect that the proposed mechanism also produces hyperons, but when compared with the direct mechanisms, it will be just a correction.

The final $\Lambda$ and $\Xi^+$ polarizations agree quite well with the experimental data. The $\Sigma^-$ polarization is also in accord with the data, but in order to improve the agreement, some effects that we neglected during the calculation may be taken into account. The $\pi \Lambda \rightarrow \pi \Sigma$, for example were not included, and they may contribute to include the final $\Sigma^-$ polarization. This kind of reaction will be included in future works. The $\rho YY'$ coupling is another object that must be considered, the current value for these couplings is far from being well established yet.

As we can see, the model shows that the $Y^-$ polarization is a small angle effect, our prediction is that $P_{Y^-}$ is very small for $\theta_L > 50$ mrad. Another result is that $P_Y$ does not depend much on the incident particle ($p$, $\Sigma$ ...). It may only change when the parametrization of $d\sigma/d\alpha$ changes, what would occur specially in $AA$ collisions.

Acknowledgments

We would like to thank M.R. Robilotta for the discussions about the hadron interactions. This work was supported by FAPESP.
APPENDIX A: BASIC FORMALISM

In this paper \( p \) and \( p' \) are the initial and final hyperon 4-momenta, \( k \) and \( k' \) are the initial and final pion 4-momenta, so the Mandelstam variables are

\[
\begin{align*}
  s &= (p + k)^2 = (p' + k')^2 \\
  t &= (p - p')^2 = (k - k')^2 \\
  u &= (p' - k)^2 = (p - k')^2.
\end{align*}
\]

With these variables, we can define

\[
\begin{align*}
  \nu &= \frac{s - u}{4m} \\
  \nu_0 &= \frac{2m^2 - t}{4m} \\
  \nu_r &= \frac{m^2 - m^2 - k.k'}{2m}.
\end{align*}
\]

where \( m, m_r \) and \( m_\pi \) are, respectively, the hyperon mass, the resonance mass and the pion mass. The scattering amplitude for an isospin \( I \) state is

\[
T_I = \mp(p') \left\{ A^I + \frac{(k + k')}{2}B^I \right\} u(p),
\]

where \( A_I \) and \( B_I \) are calculated using the Feynman diagrams. So the scattering matrix is

\[
M^p_{ba} = \frac{T^p_{ba}}{8\pi\sqrt{s}} = f_I(\theta) + \hat{\sigma}.\hat{n}g_I(\theta) = f^I_1 + \frac{(\hat{\sigma}.\hat{k})}{kk'}f^I_2,
\]

with

\[
\begin{align*}
  f^I_1(\theta) &= \frac{(E + m)}{8\pi\sqrt{s}} [A_I + (\sqrt{s} - m)B_I] , \\
  f^I_2(\theta) &= \frac{(E - m)}{8\pi\sqrt{s}} [-A_I + (\sqrt{s} + m)B_I] ,
\end{align*}
\]

where \( E \) is the hyperon energy. The partial-wave decomposition is done with

\[
a_{l_{\pm}} = \frac{1}{2} \int_{-1}^{1} [P_l(x)f_1(x) + P_{l_{\pm}}(x)f_{2}(x)] dx .
\]

In our calculation (tree level) \( a_{l_{\pm}} \) is real. With the unitarization, as explained in Section III, we obtain

\[
a_{l_{\pm}}^U = \frac{1}{2ik} \left[ e^{2i\delta_{l_{\pm}}} - 1 \right] = \frac{e^{i\delta_{l_{\pm}}}}{k} \sin(\delta_{l_{\pm}}) \to a_{l_{\pm}} .
\]

These complex amplitudes are used to calculate

\[
\begin{align*}
  f(\theta) &= \sum_{l=0}^{\infty} [(l + 1)a_{l+} + la_{l-}] P_l(x) , \\
  g(\theta) &= i \sum_{l=1}^{\infty} [a_{l+} - a_{l-}] P^{(1)}_l(x) .
\end{align*}
\]

We have, then, in the center-of-mass frame,

\[
\begin{align*}
  \frac{dq}{dl} &= |f|^2 + |g|^2 , \\
  \frac{d\sigma}{d\Omega} &= \frac{\pi}{k^2} [ |f|^2 + |g|^2 ] , \\
  \vec{P} &= -2i \text{Im}(fg) , \\
  \sigma_l &= 4\pi \sum_{l} [ (l+1)|a_{l+}|^2 + l|a_{l-}|^2] .
\end{align*}
\]
The $Y\pi Y'$ coupling constants may be obtained considering the SU(3) relations \[41\]

\begin{align*}
    g_{\pi NN} &= g \\
    g_{\Lambda\pi\Sigma} &= \frac{2}{\sqrt{3}} g\alpha \\
    g_{\Sigma\pi\Sigma} &= 2g(1 - \alpha) \\
    g_{\Xi\pi\Xi} &= g(1 - 2\alpha) \, ,
\end{align*}

(A19)

considering the current value $g_{\pi NN} = 13.40$ and $g_{\Lambda\pi\Sigma} = 12.92$, obtained in \[42\] from the study of the hypernuclei, we have $\alpha = 0.83$, and the other coupling constants are determined. Similar relations hold for the $Y\rho Y'$ couplings. The $Y\pi Y^*$ couplings are calculated with the procedure described in \[22\].
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