Phase slips dynamics in gated Ti and V all-metallic supercurrent nano-transistors: a review.

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Abstract. The effect of electrostatic gating on metallic elemental superconductors was recently demonstrated in terms of modulation of the switching current and control of the current phase relation in superconducting quantum interferometers. The latter suggests the existence of a direct connection between the macroscopic quantum phase (\(\phi\)) in a superconductor and the applied gate voltage. The measurement of the switching current cumulative probability distributions (SCCPD) is a convenient and powerful tool to analyze such relation. In particular, the comparison between the conventional Kurkijärvi–Fulton–Dunkleberger model and the gate-driven distributions give useful insights into the microscopic origin of the gating effect. In this review, we summarize the main results obtained in the analysis of the phase slip events in elemental gated superconducting weak-links in a wide range of temperatures between 20 mK and 3.5 K. Such a large temperature range demonstrates both that the gating effect is robust as the temperature increases, and that fluctuations induced by the electric field are not negligible in a wide temperature range.

1. Introduction

Although static electric fields are largely ineffective on metals due to their large free carrier density [1–8], it has been unexpectedly shown that the superconducting properties of metallic Bardeen-Cooper-Schrieffer (BCS) superconducting wires [9], Dayem bridges [10,11], and proximity superconductor-normal metal-superconductor (SNS) [12] Josephson junctions (JJ) can be manipulated via the application of a control gate voltage. The critical current (I\(_C\)) of these systems was observed to decrease as a function of an increasing control gate voltage, while no clear relation could be established with an eventual current flowing between the gate and the superconductor [13]. The gate voltage was found as well to affect the current-phase relation of superconducting quantum interference devices (SQUIDs) [14], on the one hand, through direct suppression of the critical current of a gated weak-link. On the other hand, phase shifts in the SQUID current vs flux relation were observed also for gate voltages so
small that no apparent influence on $I_C$ was observed. Such behavior was interpreted as stemming from the voltage-driven occurrence of fluctuations of the superconducting phase ($\phi$) [14]. This hypothesis has found qualitative support through the study of the evolution of the dynamics of the phase slips, i.e., local random $2\pi$ phase jumps that responsible for the superconducting-to-normal state switching [15,16]. The investigation of the switching current ($I_S$) probability distribution (SCPD) of a JJ is, indeed, a tool to access information on the phase slippage dynamics as a function of external parameters such as the temperature or externally applied fields [17–20], and allowed us to demonstrate that in metallic superconductors under the action of a gate voltage the SCPD is remarkably different from that measured as a function of the temperature. This suggests, thereby, that the gate-voltage driven suppression of $I_C$ cannot be related to a conventional thermal-like overheating of the superconductor, but rather it involves a different elusive mechanism which is yet to be identified, and fully understood [21–23].

In this manuscript, we examine in-depth the modification of the SCPD of gated all-metallic Dayem nano-bridge devices for temperatures as high as 3.75 K, i.e., in a temperature range much wider than that explored in previous analogous experiments. To this aim, we report on Ti and V gated all-metallic supercurrent nano-transistors and discuss the evolution of the switching escape rate in the presence of a control gate voltage. Our data provide another tile in the puzzle of the interaction of electric fields and the BCS condensate, and are particularly relevant, for instance, in view of the realization of electrostatically-driven phase-slip qubit based on gated metallic Josephson nanojunctions [24].

2. RCSJ model for phase slips in superconducting weak-links

The resistively capacitively shunt junction (RCSJ) model [25] is a convenient tool to analytically describe phase-slip phenomena in a JJ. It describes the behavior of a JJ under an arbitrary current bias. In this scheme, the Josephson weak-link is replaced by the parallel of a resistor $R$, a capacitor C and a non-linear current generator $I(\phi)$. For systems with a sinusoidal current-phase relation (CPR), $I(\phi,T) = I_S(T) \sin(\phi)$ [26].

This phenomenological circuital approach allows to write down the differential equation

$$I = I_S \sin(\phi) + \frac{\phi_0}{2\pi R_N} \phi + C \frac{\phi_0}{2\pi} \dot{\phi},$$

where $R_N$ is the normal-state resistance of the junction, $C$ its capacitance, $\phi_0$ is the magnetic flux quantum, and $I$ is the circulating current. Such equation is equivalent to the motion of a classical particle of mass $m_p = \frac{CN^2}{16\pi^2}$ under the action of a tilted washboard potential $U_{WB}$, as depicted in Fig. 1 [25]. $U_{WB}$ is characterized by a series of minima separated by barriers with a height expressed by [27]

$$\Delta U(I,T) = a E_J(T) \left(1 - \frac{I}{I_S(T)}\right)^b,$$

where $a$ and $b$ take into account the geometry of the systems.
The motion of a phase particle from one local minimum to the successive one leads to sharp $2\pi$ variations of $\phi$. Such events are called phase slips (PSs) and are attributed to two distinct mechanisms [18]: thermal hopping [28] and quantum tunnelling. The former requires a thermal activation energy, while the latter is only weakly dependent on temperature. The interplay of the two contributions allows us to define three different regimes divided by two crossover temperatures $T_Q < T_M$:

- for $T < T_Q$ the quantum tunnelling provides the majority of the phase slip events and the system lays in the quantum phase slip (QPS) regime [29,30];
- for intermediate temperatures, when $T_Q < T < T_M$, the thermal energy is large enough to trigger the hopping of the barrier of the washboard potential, leading the system in the thermally-activated phase slip (TAPS) regime;
- for higher temperatures, $T > T_M$, the phase slips events occur more than one at once, and the system shows the multiple phase slip (MPS) regime [31]. In contrast to QPS and TAPS, there are no analytical models for MPS.

For a particle in an equilibrium position, we can define an escape rate $\Gamma$, which evaluates the survival time of the particle in the equilibrium point. Kramers’ theory [32] provides a general formula for the escape rate in TAPS [29,33] and QPS regimes [17,34]:

$$\Gamma_{TAPS}(I, T) = \frac{L}{2\pi \xi(T) \tau_{GL}(T)} \sqrt{\frac{\Delta U(I, T)}{k_B T}} \exp \left( -\frac{\Delta U(I, T)}{k_B T} \right)$$
\[ \Gamma_{QPS}(I, T, T_{QPS}) = \frac{L}{2\pi \xi(T)\tau_{GL}(T)} \sqrt{\frac{\Delta U(I, T)}{k_B T_{QPS}}} \exp\left( -\frac{\Delta U(I, T)}{k_B T_{QPS}} \right), \]

respectively, where \( L \) is the length of the junction, \( \xi(T) \) is the coherence length, \( \tau_{GL} = \frac{\pi \hbar}{2k_B(T_C - T)} \) is the Ginzburg-Landau time constant \( [19] \) and \( T_{QPS} \) is effective temperature that allows to compare the height of the barrier \( \Delta U \) with the energy term \( k_B T_{QPS} \). Since \( \Delta U \) is a function of the weak-link bias current, the switching probability due to the occurrence of a phase slip event as a function of \( I \) can be reconstructed by acquiring a large number of \( V \) vs \( I \) characteristics. This procedure shows that the transition probability is distributed with an asymmetric bell-shaped function around a value which is conventionally defined as the critical current \( I_C \) of the weak-link.

The connection between the probability distribution \( P(I) \) and the phase slip rate \( \Gamma \) is provided by the Kurkijärvi theory (KT) \([19, 35–37]\) via the Kurkijärvi–Fulton–Dunkleberger (KFD) transformation, which for continuous and discrete distributions can be written as

\[ \Gamma(I) = P(I)\nu_I \left[ 1 - \int_0^I P(I')dI' \right]^{-1}, \quad \Gamma(I_N, T) = \frac{P(I_N, T)\nu_I}{1 - w \sum_{k=0}^N P(I_k, T)}, \]

where \( \nu_I \) is the slope of the current ramp, \( w \) is the bin size of the \( P(I,T) \) histogram, and \( P(I_k, T) \) is the switching probability in the current interval \([kw, (k+1)w]\) with \( k \in \mathbb{N} \).

It is worth to notice that the different mechanism for the escaping phenomena leads to a non-trivial behavior of the distribution width. In particular, below \( T_Q \) where the tunnelling effects are predominant, the standard deviation \( \sigma \) vs \( T \) is expected to be temperature independent because QPSs do not require a thermal activation. On the other hand, when \( T_Q < T < T_M \), \( \sigma \) is expected to grow as a function of \( T \) because of the larger available thermal energy that allows hopping events for a wider range of bias current values. Finally, for \( T > T_M \), the standard deviation is expected to decrease as a function of the temperature, as already observed in similar experiments \([18, 28, 30]\).

Such a model, although phenomenological, is very effective and allows to analyze the effect of the temperature in the dynamics of the phase slip events in mesoscopic Josephson junctions. Moreover, it is suitable to study how such dynamics is modified when the weak link is subject to the action of electrostatic gating. In the following, we will focus on this specific issue by reporting the results of experiments in which the modifications of the phase slips occurrence in nanojunctions was obtained via the application of a control gate voltage. Data are then discussed and interpreted in the framework of the conventional theory.

3. Titanium Dayem bridge PS dynamics

Ti wires and Dayem bridges transistors are the systems in which the gating effect on metallic superconductors was originally demonstrated \([9]\). In this section, we discuss the effect of electrostatic gating on these systems in terms of its impact on the switching
Data shown in the following were acquired on a 30-nm-thick ultra high-vacuum e-beam evaporated titanium Dayem bridge (150-nm-long and 120-nm-wide) shaped with a single-step electron beam lithography of a polymethyl-methacrylate (PMMA) polymeric mask deposited on top of a sapphire (Al$_2$O$_3$) substrate. In the same evaporation, a 140-nm-wide gate electrode separated by a distance of approximately 80 nm from the constriction was deposited. A pseudo-color scanning electron micrograph (SEM) of a typical device is shown in Figure 2a.

\[ I_S(T) = I_0^S \left[ 1 - \left(\frac{T}{T_C}\right)^2 \right]^{3/2} \]

where \( I_0^S \) is the zero-temperature switching current, and \( T_C \) is the Ti film critical temperature. The fit procedure returns as parameters \( T_C \approx 348 \text{ mK} \) and \( I_0^S \approx 6.02 \mu\text{A} \) that are in agreement with similar devices values \([10]\). Similarly, the evolution of the switching current as a function of the gate voltage applied through the gate electrode is shown in Figure 2c. The curves, acquired at different bath temperatures ranging from 20 mK to 300 mK, show monotonic suppression of \( I_S \), with complete quenching for \( V_G^C \approx 34 \text{ V} \). Note the conventional widening of the plateau for which \( I_S \) is unaffected by the gate voltage as the temperature increases, in agreement with experiments performed on gated elemental superconductors \([9,12,40]\).

The effect of the temperature on the number of phase slip events was assessed by acquiring \( 10^5 \) switching current values biasing the junction with a linear current ramp in the four-probe scheme shown in Figure 2a at different temperatures ranging from 20 mK to 300 mK. \( I_S \) values were acquired with a 750 KHz bandwidth input/output analogue-to-digital/digital-to-analogue converter (ADC/DAC) board for the acquisition of the voltage drop across the junction and the generation of the bias current, respectively.
The current input signal consisted of an 8.7 Hz sawtooth wave obtained by applying a voltage signal generated by the digital board to a 1 MΩ load resistor. The resulting signal consisted of a positive linear ramp with amplitude 10 μA, and slope $\nu_l = 133$ μA/s followed by a 100 ms zero-current plateau essential for the system to cool down between two consecutive super-to-normal state transitions. $I_S$ acquisitions were combined via numerical integration to reconstruct the switching current cumulative probability distributions (SCCPDs, the S-curves), as shown in Figure 3a. These curves represent, for a given current $I$, the probability that the switching current satisfies the condition $I_S < I$.

We note that, as conventionally observed in similar systems [15], the S-curves shift to a lower value of injected current as the temperature $T$ increases. Such observation is equivalent to the decrease of the critical temperature showed in Figure 2b. The width of the SCCPDs ($\sigma$) can be estimated by computing the standard deviation of the switching current probability distribution (SCPD) that is obtained via numerical derivation of the SCCPDs. The conventional behavior of the $\sigma$ vs $T$ characteristics is

Figure 3. a. SCCPDs computed at several bath temperature ranging from 20 mK to 300 mK. $I_S$ was acquired $10^5$ times to reconstruct the SCPD. Dashed grey lines represent the fit curves obtained for QPS and TAPS regimes in the framework of the KFD theory [41]. b. $\sigma$ vs $T$ characteristic with the two crossover temperatures $T_Q \simeq 110$ mK and $T_M \simeq 150$ mK that define the transition between QPS/TAPS and TAPS/MPS regimes, respectively. c. Escape rate computed from the data in panel a with the KFD transform. They show a phase lifetime between 1 µs and 10 ms.
assessed in Figure 3b where the three different phase slip regimes can be distinguished, separated by the crossover temperatures $T_Q \simeq 110$ mK and $T_M \simeq 150$ mK:

- **QPS regime:** for $T < T_Q$, the standard deviation is constant as a function of the temperature since the quantum tunnelling process does not require activation energy.

- **TAPS regime:** for $T_Q < T < T_M$, $\sigma$ and $T$ are linearly correlated because the temperature increase provides a growing amount of thermal energy to the systems, facilitating the hopping of the potential barrier.

- **MPS regime:** for $T > T_M$, the width of the transition decreases, as already observed in previous systems.

The escape rate $\Gamma(I,T)$ is another relevant quantity to be extracted from SCCPDs. It provides an estimate of the phase lifetime in the Josephson nanojunctions [42, 43], and ranges between 1 $\mu$s ($\Gamma \sim 10^6$ Hz) and 10 ms ($\Gamma \sim 10^2$ Hz). The values obtained for the escape rate are in agreement with the literature data [18, 19, 44] performed on superconducting weak-links.

![Figure 4](image.png)

**Figure 4.** a. SCCPDs acquired at several gate voltages ranging from 2 V to 30 V. b. $\sigma$ vs $T$ characteristic with the two crossover voltages $V_E$ and $V_Q$ that define the transition between QPS/EAPS and EAPS/MPS regimes, respectively. c. Escape rates computed from the data in panel a. They shows a phase lifetime between 10 $\mu$s and 100 ms.
The effect of electrostatic gating on the shape of SCCPDs is shown in Figure 4a. The curves were acquired at 20 mK for several gate voltage values ranging from 2 V to 30 V. The shape of the S-curves is substantially modified by gating. Indeed, for $V_G < 8$ V the curves are almost identical to the zero-gate one, but a non-zero probability transition for low current values appears for $8 \text{ V} < V_G < 14$ V. In the range between $14$ V $< V_G < 24$ V the curves widens significantly due to the increase of the phase slip events. Finally, at higher gate voltages ($V_G > 24$ V), the S-curve narrows.

The evolution of the shape of SCCPDs is plotted in Figure 4b with the $\sigma$ vs $V_G$ characteristic. Notably, a region of constant standard deviation is observed, stating a minor contribution of gate effect to the phase slips occurrence for low $V_G$ values. In analogy with the thermal case, such evolution seems to be equivalent to the QPS regime. By increasing $V_G$ the standard deviation grows up to a value of about 200 nA. This region is defined as an electrically-activated phase slip (EAPS) regime due to the increase of gate-induced phase slips. At higher gate voltage, $\sigma$ decreases and saturates to approximately 75 nA. Despite resembling the MPS regime, such behavior cannot be ascribed to conventional heating effects. The evolution of $\sigma$ vs $V_G$ defines two crossover gate voltages $V_Q \simeq 8$ V and $V_E \simeq 21$ V that represent the transition between QPS/EAPS and EAPS/MPS regimes, respectively.

It is worth to emphasize that the lifetime of the phase particle, represented in Figure 4c as $\Gamma$, in the gate-driven regimes ranges between 10 $\mu$s and 100 ms and on average is around one order of magnitude smaller than the thermal rates. This indicates a possible action of the electrostatic gating in driving the superconductor far from equilibrium.

Figure 5. a. $\sigma$ vs $I_C$ curves as a function of temperature at $V_G = 0$ (violet to orange), and vs gate voltage at 20 mK (blue to yellow). b. $I_C$-matched S-curves, brown and orange distributions were acquired for $V_G = 0$ at selected temperatures whereas blue and green characteristics were measured at $T = 20$ mK for different gate voltage values.

The differences between thermal and electric S-curves are highlighted by comparing the characteristics with the same critical current 2.2, 2.8, 4.0 $\mu$A acquired at $V_G = 0$ V (brown to orange) and $T = 20$ mK (blue to yellow). Figure 5b highlight the differences
in the shape of the distributions and in particular the larger width of the gate-induced
ones. Such a behavior is probably due to a gate-driven strong non-equilibrium state
induced in the superconducting junction. Also, the direct comparison between the
\( \sigma(I_s(V_G)) \) and the \( \sigma(I_s(T)) \) characteristics (reported in Figure 5b) displays a value of
the standard deviation of the gate-driven curve larger by one order of magnitude in the
first case. We emphasize that such a strong deviation from the thermal case prevents
to exploit the conventional theory \[19\] to extract an effective electronic temperature in
the weak link, which should be much higher than the Ti critical temperature. A result,
therefore, completely devoid of physical meaning. This observation suggests that the
gate action affects deeply the system phase dynamics resulting in an increased switching
probability in a wider bias current range with respect to the corresponding thermal case.
This behavior is compatible with the picture of gate-induced phase fluctuations in the
Dayem bridge \[14\], and with the formation of a glassy phase configuration along the
supercurrent path \[23\].

4. Vanadium Dayem bridge PS dynamics

The experiment performed on Ti weak-links allowed us to study the phase slips dynamics
in a temperature range \( T < 300 \) mK where thermal fluctuations and the electron-phonon
coupling in the superconductor are strongly suppressed. To assess the influence of the
gate-induced phase slip events in a different temperature range the same analysis has
to be performed on devices based on different elemental superconductors with a higher
critical temperature. This is the case of gate-controllable vanadium (V) Dayem bridges,
in which the higher \( T_C \) allowed to probe the SCCPD up to bath temperatures above 2
K.

The vanadium gated device, shown in Figure 6a, consists of a 60-nm-tick, 160-
nm-long, 90-nm-wide constriction aligned with a 70-nm-far, and 120-nm-wide side-gate.
The metal deposition was performed on a silicon/silicon-dioxide (Si/SiO\(_2\)) substrate at
a rate of 0.36 nm/s in an ultra-high vacuum e-beam evaporator with a base pressure
of \( \sim 10^{-11} \) Torr \[45–51\]. Figure 6a shows the pseudo-color SEM of a representative
V Josephson weak-link along with the four-probes biasing scheme used for the low-
temperature characterization \[52\].

The device shows a normal-state resistance of about 110 \( \Omega \) and switching and
retrapping currents of 1.4 mA and 0.35 mA, respectively, at a temperature of 2 K. The
evolution of the switching current as a function of the bath temperature, shown in Figure
6b, follows Bardeen’s law (red dotted line in Figure 6b). The fit procedure yielded a
critical temperature \( T_C \approx 3.62 \) K and \( I^0_S \approx 2.2 \) mA, consistently with the expectation
for vanadium-based devices \[45,47,49–51\].

The evolution of the switching current as a function of the gate voltage applied
through the gate electrode is shown in Figure 6c. The curves, acquired at different
bath temperatures from 2 K to 3.3 K, show a monotonic suppression of \( I_S \), with a
complete quenching of the critical current occurring for \( V_G^C \approx 34 \) V. Also in V devices
Figure 6. a. Pseudo-color SEM image of a representative vanadium gated (orange) Dayem bridge (purple) with the four-probes bias scheme used for the low temperature characterization. b. $I_S$ vs $T$ characteristic fitted with the conventional Bardeen’s formula [38] (red dotted line). c. $I_S$ vs $V_G$ characteristics measured at different bath temperatures ranging between 2 K and 3.3 K. The switching current values were computed by averaging over 50 acquisitions.

the widening of the $I_S$ plateau with temperature was observed [9–13,40].

Figure 7. a. SCCPDs acquired at several bath temperatures from 3 K to 3.5 K. The temperature range is compatible with the MPS regime [19]. b. SCCPDs acquired at several gate voltages from 0 to 15 V at 3.0 K. The curves show starkly different shape due to gate-induced phase slips that widen the zero-to-one probability transition.

We focus now on the SCCPD behavior as a function of bath temperature and the
gate voltage. In particular, we analyze the S-curves with bias current smaller than 330 µA. Notably, the gate-induced characteristics present a somewhat wider transition than the thermal ones in the same range of switching current. Such a characteristic, already observed in experiments on Ti weak-links, is present in a temperature interval as large as $T > 3$ K, i.e., where thermal coupling with the lattice phonons and thermal fluctuations are much larger than in Ti devices \cite{41}. This proves that the gating action can dramatically increase the occurrence of phase slips events also in presence of an efficient coupling with the phononic bath near the film critical temperature.

5. Conclusion

In this review, we resumed the results of experiments performed on Ti and V gated superconducting mesoscopic weak links. They shed light on the interplay between the conventional gating and the dynamics of phase slips in elemental superconductors Dayem bridges. Firstly, in Ti-based devices, we showed the impact of the temperature on the shape of the S-curves in a regime where both the electron-phonon coupling and thermal fluctuations were strongly suppressed. At the same time, the gate-driven characteristics present a wider zero-to-one transition for the same value of the critical current compared with the thermal curves. Such SCCPDs comparison is further evidence that the effect of electrostatic gating cannot be simply interpreted as a trivial overheating of the junction via current injection or Joule heating.

Secondly, we performed a similar experiment in a V-based device that presents a critical temperature larger by more than one order of magnitude than the Ti Dayem bridges. The higher critical temperature allowed us to explore a temperature regime where both thermal fluctuations and the electron-phonon coupling are exponentially larger compared to devices operating at sub-kelvin temperatures. Yet, also in this regime, the comparison between thermal- and gate-driven- cumulative switching current probability distributions confirms the results of the previous experiments, highlighting that electrostatic gating affects the dynamics of the phase slips in a starkly different way with respect to temperature.

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