Field theoretic interpretations of interacting dark energy scenarios and recent observations

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Cosmological models describing the non-gravitational interaction between dark matter and dark energy are based on some phenomenological choices of the interaction rates between dark matter and dark energy. There is no such guiding rule to select such rates of interaction. In the present work we show that various phenomenological models of the interaction rates might have a strong field theoretical ground. We explicitly derive several well known interaction functions between dark matter and dark energy under some special conditions and finally constrain them using the latest cosmic microwave background observations from final Planck legacy release together with baryon acoustic oscillations distance measurements. Our analyses report that one of the interacting functions is able to alleviate the $H_0$ tension. We also perform a Bayesian evidence analyses for all the models with reference to the $\Lambda$CDM model. From the Bayesian evidence analyses, although the reference scenario is preferred over the interacting scenarios, however, we found that two interacting models are close to the reference $\Lambda$CDM model.

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1. INTRODUCTION

Observational evidences from various astronomical sources suggest that a non-zero interaction in the dark sectors, i.e., between dark matter (DM) and dark energy (DE) is allowed [1, 2], and consequently, a mild deviation from the non-interacting $\Lambda$-cosmology is expected. Although within 1$\sigma$ confidence level one may recover $\Lambda$CDM model, but however, the null-interaction is not yet confirmed. The question arises why should we consider the interaction between DM and DE? The answer could be given in different ways. Since the nature and evolution of DM and DE are not known to us then there is no justification to avoid the possibility of mutual interaction between these dark sectors. In fact, the interaction in the dark sector is a promising approach which solves the coincidence problem [3, 4, 5, 6], and it was motivated to solve the cosmological constant problem [8]. Investigations by several investigators in the last couple of years explored some more interesting properties of the interacting DE models [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50].

It has been shown that the interaction between DM and DE can solve the tension on the present Hubble constant value, $H_0$, appearing from its local and global measurements [51, 52, 53, 54, 55, 56] and also the tension in the amplitude of the matter power spectrum, $\sigma_8$, by different observations [57, 58]. However, although the models in such theory are phenomenologically motivated, but nevertheless, from the particle physics point of view, the interaction between DM and DE is a natural phenomenon because any two fields (DM field and DE field) can interact with each other. In the last several years, several people have studied the DM and DE dynamics with different choices for the interaction function relating the energy densities of the dark sectors. Depending on the choice of the function, the interaction becomes linear or non-linear in the energy densities of the dark sectors.

Mathematically and physically as well, we have no rigid theoretical bounds for the mentioned interaction functions. If the universe contains $n$ matter components with the energy momentum tensors $T_{\mu\nu}^i$, $i = 1, \ldots, n$, such that either all or some of the energy components interact with each other, then the energy conservation condition

$$\nabla^\mu \sum_{i=1}^n T_{\mu\nu}^i = 0$$

is fulfilled only for all matter, but not for every $i$-th component. So we can add and subtract any interaction function $Q_\nu \equiv Q_{ij}^\nu$ for $i$-th and $j$-th components:

$$\nabla^\mu T_{\mu\nu}^{ij} = Q_\nu, \quad \nabla^\mu T_{\mu\nu}^{ji} = -Q_\nu. \quad (1)$$

The investigators mentioned earlier (see again [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43]) worked with different phenomenological variants of the interaction function $Q$. In particular, in the interactive models [3, 8, 59, 60, 61] the DE component was described as a scalar field $\phi$, that takes part in the phenomenologically constructed interaction with the standard cold DM (an ideal fluid with zero pressure).

In this paper we suggest a variant of motivation to consider the interaction term $Q$ in different linear and...
non-linear forms. Our approach includes a symmetric description of both DM and DE as two scalar fields $\phi_1$ and $\phi_2$, where they may interact via their common potential $V(\phi_1, \phi_2)$. It is widely known that scalar fields can simulate cosmological evolution (see reviews [62, 63]). Models with two scalar fields were suggested and studied in Refs. [64, 65, 66, 67, 68], however, the authors’ interest was not concentrated on the possibilities to describe interaction of dark components.

In this approach we suppose, that the interaction function $Q$ can be deduced from the (more fundamental) potential $V(\phi_1, \phi_2)$. The connection between $V$ and $Q$ is rather complicated in general, in particular, the linear dependence of $Q$ on densities (the linear interaction) is not the most obvious result of this approach. In any case we have a degree of freedom on a certain level: when we choose the potential $V$, or the interaction term $Q$.

We organize the work as follows. In section 2 we give the details of the mathematical formulation of an interacting universe and describe, how different forms of the interaction function $Q$ can be deduced from the common potential of scalar fields. In section 3 we list the observational data to analyse the models and consequently present the results of various observational analyses. Finally, we summarize the main findings of the work in section 4.

2. INTERACTING DARK ENERGY: A FIELD THEORETIC DESCRIPTION

We consider a cosmological scenario where two heavy dark fluids in the universe, namely, the DM and DE non-gravitationally interact with other. The other components, namely the baryons and radiation do not take part in the interaction. To describe such interacting universe, as usual we assume the homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker line element given by

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - \kappa r^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right].$$

Here, $a(t)$ is the scale factor of this FLRW universe and $\kappa$ is the curvature sign of the universe. The curvature sign may describe three different geometries of the universe, namely, flat ($\kappa = 0$), open ($\kappa = -1$) and closed ($\kappa = 1$). Since most of the observational estimates prefer a flat geometry of the universe, see for instance [63, 70], henceforth, we fix the spatial flatness of the universe in this work. Thus, in such a prescribed geometric structure of the universe one can write down the Einstein’s field equations as

$$H^2 = \frac{8\pi G}{3} \sum_i \rho_i,$$

$$\dot{H} = -4\pi G \sum_i (p_i + \rho_i),$$

where an overhead dot in any quantity denotes its cosmic time differentiation; $H = \dot{a}/a$ is the Hubble rate of this FLRW universe; $(\rho_i, p_i)$ respectively refer to the energy density and pressure of the $i$-th fluid. Precisely, $\rho_r, \rho_b, \rho_c, \rho_x$ are respectively the energy densities of radiation, baryons, DM and a DE fluid. Similarly, $p_r, p_b, p_c$ and $p_x$ are respectively the pressure components of radiation, baryons, DM and DE. Since radiation and baryons do not take part in the interaction, thus they follow the standard evolution equations while the conservation equations of the interacting DM and DE follow,

$$\dot{\rho}_c + 3H(\rho_c + p_c) = -Q,$$  \hspace{1cm} (5)

$$\dot{\rho}_x + 3H(\rho_x + p_x) = Q.$$  \hspace{1cm} (6)

Below we will assume that DM is the cold DM (or, pressureless DM), and thus, $p_c = 0$. The interactive term $Q$ in Eqs. (5), (6) is usually factorized by the Hubble rate $H$ and can depend on the densities $\rho_c, \rho_x$, pressures $p_c, p_x$, other parameters [3, 8, 59, 60, 61] (see also the classification in Ref. [1]). We can choose a function $Q(H, \rho_c, \rho_x, \ldots)$ in many variants: different possibilities of this choice and a more fundamental approach to deduce $Q$ may be illustrated in the following scheme.

We generalize the traditional scalar field simulation of DE [3, 8, 59, 60] and suppose that both DM and DE fluids are described correspondingly as two real scalar fields $\phi_1$ and $\phi_2$. Their interaction is naturally managed by a common potential $V(\phi_1, \phi_2)$ in the action [64, 65, 66, 67]

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{\epsilon_1}{2} (\nabla \phi_1)^2 - \frac{\epsilon_2}{2} (\nabla \phi_2)^2 - V(\phi_1, \phi_2) \right] + S^m. \hspace{1cm} (7)$$

Here, $R = g^{\mu\nu} R_{\mu\nu}$ is the Ricci scalar, the factors $\epsilon_j = \pm 1$ determine quintessential or phantom nature of a field, $(\nabla \phi_j)^2 = g^{\mu\nu} \partial_\mu \phi_j \partial_\nu \phi_j$, the term $S^m$ describes the remaining matter (baryons, radiation).

The action (7) is symmetric with respect to DM $\phi_1$ and DE $\phi_2$. This form is convenient to generate an interaction of these fluids, however, it does not coincide with the widely used approach [3, 8, 59, 60, 61, 62, 63], where a scalar field $\phi$ describes only the DE. For DM one can find some form of a scalar field description in mimetic models [71, 72] (also see [73, 74, 75, 76, 77]), but in the action (7) the field $\phi_1$ is not connected with conformal degrees of freedom of any auxiliary metric.

If we vary the action (7) over $g^{\mu\nu}$, $\phi_1$ and $\phi_2$, we deduce
the dynamical equations

\[
R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = 8\pi G \left[ \sum_{j=1}^{2} \epsilon_j \left( \partial_\mu \phi_j \partial_\nu \phi_j - \frac{1}{2} (\nabla \phi_j)^2 g_{\mu\nu} \right) - V g_{\mu\nu} + T^m_{\mu\nu} \right],
\]

\[
\nabla^\mu \nabla_\mu \phi_j = \epsilon_j \frac{\partial V}{\partial \phi_j}, \quad j = 1, 2.
\]

The covariant divergence of the equation (8) leads to the energy conservation equation \( \nabla^\mu T^m_{\mu\nu} = 0 \) for baryons and radiation, because the terms with \( \phi_j \) vanish as the consequence of Eqs. (9).

For the FLRW universe (2) in its flat case \( \kappa = 0 \) the equations (8) may be reduced to the form (3), (4), but with the following new content of the total density and pressure:

\[
\rho_{\text{tot}} = \frac{\epsilon_1}{2} \dot{\phi}_1^2 + \frac{\epsilon_2}{2} \dot{\phi}_2^2 + V(\phi_1, \phi_2) + \rho_b + \rho_r,
\]

\[
\rho_{\text{tot}} = \frac{\epsilon_1}{2} \dot{\phi}_1^2 + \frac{\epsilon_2}{2} \dot{\phi}_2^2 - V(\phi_1, \phi_2) + p_b + p_r.
\]

The scalar field equations (9) for the FLRW universe (2) take the similar form

\[
\ddot{\phi}_1 + 3H \dot{\phi}_1 = -\epsilon_1 \frac{\partial V}{\partial \phi_1},
\]

\[
\ddot{\phi}_2 + 3H \dot{\phi}_2 = -\epsilon_2 \frac{\partial V}{\partial \phi_2},
\]

but they describe interaction of the fluids \( \phi_1 \) and \( \phi_2 \), if the potential \( V(\phi_1, \phi_2) \) is not equal to a sum \( V(\phi_1) + V(\phi_2) \).

This interaction between \( \phi_1 \) and \( \phi_2 \) can be rewritten and represented in the form (3), (4), if we divide the common potential \( V(\phi_1, \phi_2) \) into two parts (introducing an additional degree of freedom with this division)

\[
V(\phi_1, \phi_2) = V_1(\phi_1, \phi_2) + V_2(\phi_1, \phi_2)
\]

and determine the densities and pressures of the dark components:

\[
\rho_1 = \frac{\epsilon_1}{2} \dot{\phi}_1^2 + V_1, \quad p_1 = \frac{\epsilon_1}{2} \dot{\phi}_1^2 - V_1,
\]

\[
\rho_2 = \frac{\epsilon_2}{2} \dot{\phi}_2^2 + V_2, \quad p_2 = \frac{\epsilon_2}{2} \dot{\phi}_2^2 - V_2.
\]

In these notations the dynamical equations (11), (12) for 2 scalar fields will take the form (4), (6):

\[
\dot{\rho}_1 + 3H(\rho_1 + p_1) = -Q,
\]

\[
\dot{\rho}_2 + 3H(\rho_2 + p_2) = Q,
\]

with the interacting term

\[
Q = \dot{\phi}_1 \frac{\partial V_2}{\partial \phi_1} - \dot{\phi}_2 \frac{\partial V_1}{\partial \phi_2}.
\]

Obviously, this interaction term \( Q \) equals zero in the case of non-interacting decomposing potential [65]

\[
V(\phi_1, \phi_2) = V_1(\phi_1) + V_2(\phi_2).
\]

In the general case (13) we have the mentioned degree of freedom, when \( V \) is divided into \( V_1 \) and \( V_2 \). However, this degree of freedom is a form of gauge transformations and does not change the model behavior: if we redefine \( V_1 \) to \( V_1 = V_1 + \delta V(\phi_1, \phi_2) \) (and \( V_2 \) to \( V_2 = V_2 - \delta V \)), we will obtain the correspondent redefinition of \( \rho_1 \) and \( Q \), in particular, \( Q \to Q = Q - \frac{d}{dt} \delta V \). But the dynamical equations (3), (11), (12) and observable manifestations will remain just the same.

The most surprising point in the considered model (7) is its description of the cold DM (an ideal fluid with zero pressure) as the scalar field \( \phi_1 \). However, it is possible, if we, naturally, fix the sign \( \epsilon_1 = 1 \) and require zero value for the pressure \( p_1 \) (14):

\[
\epsilon_1 = 1, \quad p_1 = \frac{1}{2} \dot{\phi}_1^2 - V_1(\phi_1, \phi_2) = 0.
\]

In particular, this model without the DE field \( \phi_2 = 0, V_2 = 0 \) under the condition (18) recovers the Friedmann solution \( a = (t/t_0)^{2/3} \) with the following exponential potential:

\[
V_1(\phi_1) = \frac{\Lambda}{16\pi G} \sinh^2 \left[ \sqrt{6\pi G} (\phi_1 - \phi_0) \right]
\]

\[
= \frac{\Lambda}{16\pi G} \sinh^{-2} \left[ \sqrt{3\Lambda} (t - t_0) \right].
\]

Here, \( \phi_0, t_0 \) are constants of integration.

Another solution for two interacting scalar fields was obtained in Ref. [65], it describes the Big Rip singularity (at \( t = t_\star \)) with the Hubble parameter, fields

\[
H = \frac{\theta}{3} \left( \frac{1}{t} + \frac{1}{t - t_\star} \right), \quad \phi_1 = \phi_0 \log \frac{t}{t_0},
\]

\[
\phi_2 = \phi_0 \log \frac{t_\star - t}{t_0}, \quad \epsilon_2 = -1
\]

and the potential

\[
V(\phi_1, \phi_2) = \frac{\phi_0^2}{2t_0^2} \left[ (\theta - 1) e^{-2\phi_1/\phi_0} + (\theta + 1) e^{-2\phi_2/\phi_0} + 2\theta e^{-2(\phi_1 + \phi_2)/\phi_0} \right].
\]
Here, $\phi_0$, $t_0$ are constants, and $\theta = 12\pi G\phi_0^2$. The solutions (22), (23) do not satisfy the condition (18).

If we divide the term with $e^{-2} (1 + \phi_2/\phi_0)$ in the potential (23) symmetrically between $V_1$ and $V_2$ in Eq. (13), the interaction function $Q$ (16) will be

$$Q = -\frac{\phi_0^2}{2t_0} (e^{-\phi_1/\phi_0} + e^{-\phi_2/\phi_0}) e^{- \phi_1/\phi_0}.$$

It may be expressed via the Hubble parameter (22) and the densities (14) $\rho_j = \frac{1}{2} \phi_0^2 r_0^2 (e^{-\phi_1/\phi_0} + e^{-\phi_2/\phi_0}) e^{\phi_1/\phi_0}$ as follows:

$$Q = 3\xi H \frac{\rho_1 \rho_2}{\rho_1 + \rho_2}.$$  \hspace{1cm} (24)

Here $\xi = -1/\theta$. The interaction function (24) may be deduced in another way [65], if we require for the system (22), (23) do not satisfy the condition (18).

Another variant of an interacting model may be described by a potential of the type (23), however, we impose the condition (18) and choose $V_1(\phi_1)$ depending only on $\phi_1$ as follows:

$$V(\phi_1, \phi_2) = V_1(\phi_1) + V_2(\phi_1, \phi_2) = \frac{\phi_0^2}{2t_0} e^{-2\phi_1/\phi_0} + A_{\gamma_1} t_0^{\gamma_1} e^{\phi_1/\phi_0 + \gamma_2}.$$

Here, $\phi_0$, $\psi_0$, $A_2$, $t_j$, $\gamma_j$ are constants and

$$\gamma_1 + \gamma_2 = -2.$$

We consider the solution

$$\phi_1 = \phi_0 \log \frac{t}{t_1}, \quad \phi_2 = \psi_0 \log \frac{t}{t_2}, \quad H = \frac{h_0}{t}, \quad h_0 = \text{const},$$

unlike Eq. (22), it has no future singularity, but under the condition (18), that means $p_1 = 0$, we obtain

$$\rho_1 = \dot{\rho}_1 = 2V_1 = \frac{\phi_0^2}{t^2}, \quad V_2 = \frac{A_2}{t^2}.$$  \hspace{1cm} (26)

We use equations (3), (11), (12) to express the constants in Eqs. (25), (26) via dimensionless parameters $\gamma_1$ and $h_0$:

$$\phi_0^2 = \frac{\gamma_1 h_0 (3h_0 - 1)}{8\pi G (2 - 3h_0 + \gamma_1/2)},$$

$$\epsilon_2 \psi_0^2 = \frac{(2 + \gamma_1) h_0 (2 - 3h_0)}{8\pi G (2 - 3h_0 + \gamma_1/2)},$$

$$A_2 = \frac{h_0 (3h_0 - 1)(2 - 3h_0)}{8\pi G (2 - 3h_0 + \gamma_1/2)}.$$ \hspace{1cm} (27)

Note that in the case $h_0 = 2/3$ interaction vanishes and we have the potential (17), that means, $V = V_1(\phi_1) + V_2(\phi_2)$ (or $V_2 = 0$). But for $h_0 \neq 2/3$ the interaction term (16) $Q = \frac{\phi_1}{\phi_0}$ is nonzero and may be presented in the following forms:

$$Q = 3H \xi_1 \rho_1, \quad \xi_1 = \frac{2 - 3h_0}{3h_0},$$

or in the form (24) with $\xi = \frac{(2 - 3h_0 + \frac{\pi}{2})}{(3h_0 + \frac{\pi}{2})}$.

Here, the ‘DE component’ density $\rho_2$ is proportional to $\rho_1$, but pressure $p_2$ remains nonzero

$$\rho_2 = \frac{h_0 (2 - 3h_0)(3h_0 + 1)}{8\pi G (2 - 3h_0 + \gamma_1/2)} H^2, \quad p_2 = \frac{h_0 (2 - 3h_0)}{8\pi G t^2}.$$ \hspace{1cm} (30)

The densities $\rho_2$ and $\rho_1 = \phi_0^2 / t^2$ with $\phi_0$ from Eq. (27) are positive in the following three physical cases:

(a) $h_0 > \frac{2}{3}$, $-6h_0 < \gamma_1 < 0 \Rightarrow Q < 0$;

(b) $\frac{1}{3} < h_0 < \frac{2}{3}$, $\gamma_1 > 0 \Rightarrow Q > 0$;

(c) $0 < h_0 < \frac{1}{3}$, $\max(- 6h_0, 6h_0 - 4) < \gamma_1 < 0 \Rightarrow Q > 0$.

In the cases (b) and (c) we deal only with a quintessential DE ($\epsilon_2 = 1$) and obtain $Q > 0$, but in the variant (a) we have $Q < 0$ and can construct this fluid with both signs of $\epsilon_2$.

We consider DM as cold (that means pressure-less) under the condition (18) with $p_c = 0 (p_c \equiv \rho_c)$, and DE as vacuum ($p_x = - \rho_x$). So the equations (1) and (2) for $\rho_c \equiv \rho_1$ and $\rho_x \equiv \rho_2$ take the form

$$\dot{\rho}_c + 3H \rho_c = -Q,$$

$$\dot{\rho}_x = Q,$$ \hspace{1cm} (31) \hspace{1cm} (32)

where $Q$ is the interaction function that has been already mentioned earlier. The sign of the interaction rate has a physical meaning. For positive values of the interaction rate, that means for $Q > 0$, the transfer of energy and/or momentum takes place from pressureless DM to DE while its opposite sign that means, $Q < 0$ refers to the opposite case, i.e., energy flow takes place from DE to pressureless DM. Now, for any arbitrary given interaction function $Q$, using the above conservation equations (31), (32) together with the Hubble equation (8), one can solve the evolution of $(\rho_c, \rho_x)$ either analytically or numerically. Usually, for any arbitrary interaction function, the background evolution of $(\rho_c, \rho_x)$ cannot be analytically found. For some specific interaction models, it is possible to impose some analytic structure on the background evolution of the dark sectors’ energy densities.
3. INTERACTION MODELS AND THEIR PERTURBATIONS

As already shown in section 2, the field theoretic approach returns some well known interaction models, such as the interaction model in which the interaction function is proportional to their individual energy density, namely, $Q \propto \rho_c, Q \propto \rho_x$ and another model which has a nonlinear structure involving the energy densities of the dark sectors $(Q \propto \rho_c \rho_x (\rho_c + \rho_x)^{-1})$. So, one can clearly justify that the linear combination of the energy densities of the dark components could be another feasible interaction model. Thus, in summary, we consider the interaction models shown in Table I that we plan to examine in this work.

In the following we discuss the perturbations equations of the present interaction models in a perturbed FLRW metric given by [78, 79, 80]

$$ds^2 = a^2(\tau) \left[ - (1 + 2\phi)d\tau^2 + 2\partial_i B d\tau dx^i 
+ \left( (1 - 2\psi)\delta_{ij} + 2\partial_i \partial_j E \right) dx^i dx^j \right]$$

where $\tau$ is the conformal time and the symbols, namely, $\phi, B, \psi, E$, are the gauge-dependent scalar perturbation quantities. Now, in the context of interaction, one could easily calculate the gravitational field equations for the above perturbed metric (33), see for instance [81, 82, 83, 84].

Here, we fix the synchronous gauge for our work, that means, we have $\phi = B = 0, \psi = \eta$, and $k^2 E = -h/2 - 3\eta$, where $k$ is the Fourier mode and $h, \eta$ are the metric perturbations. Let us note that $\theta = \theta_0^0$ is the volume expansion scalar of the total fluid, hence, $\theta_i$ stands for the volume expansion scalar for the $i$-th fluid. For $i = c$ we mean the cold dark matter and for $i = x$ we mean the vacuum energy. We also introduce $\delta_A = \delta \rho_A / \rho_A$ as the density perturbation for the fluid $A = (c, x)$. Now, since for all the interacting cases considered in this work, $\delta_x = 0$. Hence, the perturbations equations for cold DM, are

$$\delta_c' = -\frac{h'}{2} + \frac{aQ}{\rho_c} \delta_c,$$  

$$\theta_c' = -\mathcal{H} \theta_c,$$

where the prime ($'$) stands for the derivative with respect to the conformal time $\tau$ and $\mathcal{H}$ is the conformal Hubble rate. The first equation is known as the density perturbation and the second equation is known as the velocity perturbation. In the DM-comoving frame, the velocity for DM particles vanishes, that means, $\theta_c = 0$. In the following we prescribe the exact perturbations equations for different interaction models.

Thus, having the evolution equations of the dark sectors’ energy densities at the level of background (see the discussions at the end of section 2) and perturbations, it is possible to proceed to examine the interacting scenarios with the use of latest observational data. The next section is devoted for this purpose.

4. OBSERVATIONAL DATA, STATISTICAL METHODOLOGY AND THE RESULTS

We describe, in this section, the observational data and the methodology for the statistical analyses of the models. We use the latest cosmic microwave background observations from Planck 2018 [85, 86] and baryon acoustic oscillations distance measurements [87, 88, 89]. To constrain the interacting scenarios, we use the cosmomc package [90, 91], a Markov chain Monte Carlo code used to extract the observational constraints. This code supports the Planck 2018 likelihood and it has a valid convergence diagnostic by Gelman-Rubin [92]. Since in all the interacting scenarios that we consider in this work, vacuum interacts with cold DM, thus, the dimension of the parameter space is seven with the following parameters:

$$\mathcal{P} = \{ \Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \log[10^{10} A_s], \xi \},$$

where the first six parameters in $\mathcal{P}$ refer to the six parameters of the $\Lambda$CDM model and the parameter $\xi$ is the coupling parameter, mentioned earlier. In Table I, we show the flat priors on various free parameters of the interacting cosmic scenarios during the statistical analysis.

4.1. IVS0

We show the observational constraints on this interacting cosmic scenario in Table III and in Fig. I for Planck 2018 alone and Planck 2018+BAO data. We include BAO with Planck 2018 in order to break the degeneracies between the parameters. From Planck 2018 data alone we find that $\xi \neq 0$ is allowed at more than 68% CL ($\xi = -0.0013^{+0.00077}_{-0.00077}$). So, a very mild interaction in the dark sector is signaled by Planck data alone, however, when BAO data are added to Planck CMB, $\xi$ becomes very small compared to its estimation from Planck CMB alone, but within 68% CL, $\xi = 0$ is consistent ($\xi = 0.00011^{+0.00040}_{-0.00040}$). The Hubble constant, $H_0$ assumes very lower value ($H_0 = 63.93^{+3.34}_{-3.34}, 95%$ CL, Planck 2018) compared to Planck’s $\Lambda$CDM based estimation [70] but with high error bars as one can see. When BAO are added to Planck 2018, $H_0$ goes up with reduced error bars ($H_0 = 67.56_{-1.35}^{+1.35}, 95\%$, Planck 2018+BAO) and becomes consistent with Planck’s $\Lambda$CDM based estimation [70]. So, as we can see that the tension on $H_0$ is not reconciled within this interacting scenario.

Regarding the estimation of $\Omega_b h_0$, we find a very strong anti correlation with $H_0$, and hence this parameter behaves accordingly with the increase or reduction of the
TABLE I: We show the interaction models that we shall study in this work. In the table we have clearly labeled the interacting scenario corresponding to some specific interaction function. Here, IVSI (i = 0, 1, 2, 3) means the Interacting Vacuum Scenario for the i-th interaction function.

| Model No. | Expression for \( Q \) | Corresponding Cosmic Scenario |
|-----------|--------------------------|------------------------------|
| Model I   | \( 3H\xi \rho_c \)       | IVS0                         |
| Model II  | \( 3H\xi \rho_x \)       | IVS1                         |
| Model III | \( 3H\xi(\rho_c + \rho_x) \) | IVS2                        |
| Model IV  | \( 3H\xi \left( \frac{\rho_c \rho_x}{\rho_c + \rho_x} \right) \) | IVS3                        |

TABLE II: Flat priors on various free parameters of the interacting scenarios have been shown.

| Parameter | Prior |
|-----------|-------|
| \( \Omega_b h^2 \) | [0.005, 0.1] |
| \( \Omega_c h^2 \) | [0.01, 0.99] |
| \( \tau \) | [0.01, 0.8] |
| \( n_s \) | [0.5, 1.5] |
| \( \log [10^{10} A_s] \) | [2, 4] |
| \( 100\theta_{MC} \) | [0.5, 10] |
| \( \xi \) | [−1, 1] |

TABLE III: Observational constraints at 68% and 95% CL on the interaction scenario driven by the interaction function \( Q = 3H\xi \rho_c \), using the CMB data from Planck 2018 and the data from BAO.

Hubble constant. An interesting observation from Fig. 1, specifically from the joint contour (\( \Omega_{m0}, \sigma_8 \)) is that, after the addition of BAO data to Planck 2018, the contour becomes vertical offering no correlation between them, while we note that for Planck 2018 data alone, the correlation between these two parameters are existing.

In summary, for this interaction model we find a very mild interaction in the dark sector which is much consistent with the non-interaction cosmology.

4.2. IVS1

The summary of the observational constraints on this interaction model has been shown in Table 1 and in Fig. 2. One can clearly see that the estimations of the coupling parameter for both Planck 2018 and Planck 2018+BAO are large compared to the previous interacting scenario IVS0. For Planck 2018, we see that at more than 68% CL, \( \xi \neq 0 \) (\( \xi = 0.132^{+0.147}_{-0.077}, 68\% \) CL), but within 95% CL, \( \xi = 0 \) is allowed. For Planck 2018+BAO, within 68% CL, \( \xi = 0 \) is consistent. So, concerning the estimations of the coupling parameter, we can safely conclude that a mild interaction is allowed within this interaction scenario.

Concerning the estimation of \( H_0 \), we find an interesting observation as follows. For Planck 2018, we find that \( H_0 \) takes a very high value with respect to the ΛCDM...
TABLE IV: Observational constraints at 68% and 95% CL on the interaction scenario driven by the interaction function $Q = 3H_0\rho_0$, using the CMB data from Planck 2018 and the data from BAO.

| Parameters | Planck 2018 | Planck 2018+BAO |
|------------|-------------|-----------------|
| $\Omega_m h^2$ | 0.0687$^{+0.0244}_{-0.0677}$ | 0.0996$^{+0.0225}_{-0.0383}$ |
| $\Omega_b h^2$ | 0.02230$^{+0.00030}_{-0.00029}$ | 0.02233$^{+0.00032}_{-0.00027}$ |
| $\Omega_{MC}$ | 1.04409$^{+0.00054}_{-0.00049}$ | 1.04188$^{+0.00023}_{-0.00027}$ |
| $\tau$ | 0.055$^{+0.015}_{-0.015}$ | 0.055$^{+0.016}_{-0.016}$ |
| $n_s$ | 0.9723$^{+0.0083}_{-0.0081}$ | 0.9734$^{+0.0097}_{-0.0098}$ |
| $\ln(10^{10} A_s)$ | 3.055$^{+0.010}_{-0.010}$ | 3.057$^{+0.012}_{-0.012}$ |
| $\xi$ | 0.132$^{+0.0093}_{-0.0097}$ | 0.059$^{+0.011}_{-0.010}$ |
| $\Omega_{m0}$ | 0.191$^{+0.015}_{-0.016}$ | 0.261$^{+0.016}_{-0.016}$ |
| $H_0$ | 70.84$^{+3.50}_{-5.94}$ | 68.82$^{+1.30}_{-2.77}$ |

Based Planck’s estimation [70] having in addition significantly high error bars that enables us to reach its local estimation [83], and thus, within 68% CL, the tension on $H_0$ is resolved. When BAO data are added to Planck 2018, $H_0$ is lowered with reduced error bars, however, due to slightly increased estimated value of $H_0$ having slightly high error bars ($H_0 = 68.82^{+2.77}_{-2.64}$ at 95% CL, Planck 2018+BAO) compared to $\Lambda$CDM based Planck [70], the tension on $H_0$ is mildly alleviated due to the high error bars.

Thus, in summary, this interaction model has the ability to alleviate the $H_0$ tension offering a mild evidence of an interaction in the dark sector which is more pronounced for Planck 2018 alone.

4.3. IVS2

We show the observational constraints for this interacting scenario in Table V and in Fig. 3. In a similar fashion we concentrate on the key parameters $\xi$ and $H_0$ for this model. Note that the observational constraints for this model are almost similar to the IVS0 scenario. Thus, similar to IVS0 scenario, for this interaction model, an evidence of a non-null interaction at more than 68% CL is favored ($\xi = -0.0013^{+0.00085}_{-0.00081}$, 68% CL, Planck 2018) for Planck 2018 data. While for Planck 2018+BAO, $\xi = 0$ is consistent within 68% CL.

Concerning the estimations of $H_0$ for both Planck 2018 and Planck 2018+BAO, we find that for Planck 2018, it takes lower values with high error bars unlike to what we find in $\Lambda$CDM based estimation [70]. Thus, the tension on $H_0$ remains true for Planck 2018 data. However, when BAO data are added to Planck 2018, $H_0$ slightly goes up with reduced error bars, but effectively the $H_0$ tension is not alleviated.

4.4. IVS3

Finally, we consider the last interaction model in this series. Let us note that it is a nonlinear interaction model in the energy densities of the dark sectors’ components unlike the previous three interaction models which are linear in the energy densities of DM and DE. So, concerning its structure, it has certain interest in this context. In a similar way we summarize the observational constraints for this model in Table V and in Fig. 4.

Concerning the observational constraints on the coupling parameter $\xi$, we notice that for both Planck 2018 and Planck 2018+BAO, $\xi = 0$ is consistent within 68% CL. For Planck 2018 alone: $\xi = 0.012^{+0.240}_{-0.408}$ (68% CL)}

![FIG. 2: One dimensional marginalized posterior distributions of some selective parameters and two-dimensional joint contours of various combinations of the model parameters for IVS1 scenario have been displayed.](image-url)
and for Planck 2018+BAO: $\xi = 0.062^{+0.069}_{-0.067}$ (68% CL). However, both the datasets allow the nonzero values of the coupling parameter. So, the possibility of interaction in the dark sector through this coupling function is equally probable.

From the constraints of $H_0$, we see that Planck 2018 alone estimates a lower $H_0$ with very high error bars ($H_0 = 66.34^{+6.33}_{-6.13}$, 68% CL), and due to this, as one can see, within 68% CL, it almost reaches the local estimation of $H_0$ [93], and thus, the tension on $H_0$ is alleviated. Note that the alleviation of the tension is purely due to the error bars. However, when BAO data are added to Planck 2018, $H_0$ goes up but its error bars are reduced significantly compared to the error bars for Planck 2018, and eventually, the tension is not solved. We can say that the tension on $H_0$ is slightly weakened.

Therefore, in conclusion, within this interaction model, a mild coupling between DM and DE is supported by the observational data. Additionally, the model is also able to alleviate the $H_0$ tension due to its large error bars.

### 4.5. Model comparisons

In the previous subsections we have described the observational constraints on the prescribed IVS scenarios. In this section we aim to compare the models through their observational direction as well as we also perform a Bayesian evidence analysis in order to test the observational viability of the models with respect to some reference model. Since $\Lambda$CDM is the ideal choice to compare the interacting cosmological models under consideration, therefore, for Bayesian evidence analysis, we set $\Lambda$CDM as the base/reference model.

In the first half of this section we compare the models focusing on their observational constraints as well as their effects on the large scale of the universe. In the second half of this subsection we provide the Bayesian evidence analysis for all the models with respect to the base $\Lambda$CDM model.

In Fig. 5, we show the whisker graphs for the coupling parameter $\xi$ of all the interacting scenarios, namely, IVS2: Planck 2018 and IVS2: Planck 2018+BAO.

![FIG. 3: One dimensional marginalized posterior distributions of some selective parameters and two-dimensional joint contours of various combinations of the model parameters for IVS2 scenario have been displayed.](image)

![FIG. 4: One dimensional marginalized posterior distributions of some selective parameters and two-dimensional joint contours of various combinations of the model parameters for IVS3 scenario have been displayed.](image)

| Parameters | Planck 2018 | Planck 2018+BAO |
|------------|-------------|-----------------|
| $\Omega_{c} h^2$ | 0.1204$^{+0.069}_{-0.067}$ | 0.1109$^{+0.024}_{-0.022}$ |
| $\Omega_{b} h^2$ | 0.0230$^{+0.069}_{-0.067}$ | 0.0223$^{+0.024}_{-0.022}$ |
| 1000 $\mu$C | 1.04078$^{+0.0166}_{-0.0159}$ | 1.04120$^{+0.0060}_{-0.0014}$ |
| $\tau$ | 0.054$^{+0.016}_{-0.016}$ | 0.055$^{+0.007}_{-0.015}$ |
| $\sigma_8$ | 0.9721$^{+0.016}_{-0.016}$ | 0.9734$^{+0.004}_{-0.008}$ |
| $\ln(10^{10} A_s)$ | 3.055$^{+0.016}_{-0.016}$ | 3.056$^{+0.016}_{-0.016}$ |
| $\xi$ | 0.012$^{+0.008}_{-0.007}$ | 0.012$^{+0.008}_{-0.007}$ |
| $\Omega_{m0}$ | 0.349$^{+0.016}_{-0.016}$ | 0.288$^{+0.016}_{-0.016}$ |
| $H_0$ | 66.34$^{+10.78}_{-13.13}$ | 68.29$^{+1.2}_{-1.4}$ |

| TABLE VI: Observational constraints at 68% and 95% CL on the interaction scenario driven by the interaction function $Q = 3H(\xi p_c p_x)/(p_c + p_x)$, using the CMB data from Planck 2018 and the data from BAO. |

![TABLE VI: Observational constraints at 68% and 95% CL on the interaction scenario driven by the interaction function $Q = 3H(\xi p_c p_x)/(p_c + p_x)$, using the CMB data from Planck 2018 and the data from BAO.](image)
IVS0, IVS1, IVS2 and IVS3, considering the analyses Planck 2018 and Planck 2018+BAO. Let us note that for IVS0 and IVS2, the region of $\xi$ coincides with the vertical line representing $\xi = 0$. The reason for such an overlap is that, for both IVS0 and IVS2, the constraints on $\xi$ are very close to zero (see Tables 3 and 4 for this purpose). That is why in the right graph of Fig. 5 we have separately shown the whisker plot for IVS0 and IVS2. Now, from the left graph, one can safely conclude that models IVS1 and IVS3 assume similar constraints, although for IVS3, $\xi < 0$ is allowed. While the constraints from IVS0 and IVS2 are almost same. This is pretty clear from the right graph of Fig. 5.

In order to understand how the present IVS models could be effective in alleviating/solving the $H_0$ tension, in Fig. 6, we show the whisker graph for $H_0$ (at 68% CL) for Planck 2018 and Planck 2018+BAO. The grey vertical band corresponds to $H_0$ estimated by the Planck 2018 release [70] and the pale blue vertical band denotes the $H_0$ estimation by SH0ES collaboration [93]. One can clearly see from Fig. 6 that IVS0 remains unable to alleviate the $H_0$ tension while the remaining three IVS models assume higher values of $H_0$ for Planck 2018 alone. However, among IVS1, IVS2, and IVS3, from the analyses, IVS1 seems more sound to alleviate the $H_0$ tension. Finally, one might be interested to look at the 3D scattered plots for the IVS models shown in Fig. 7 and 8. From the scattered plots displayed in Fig. 7 and 8 one can understand the behaviour of the coupling parameter, $\xi$, with higher and lower values of the Hubble constant, $H_0$. For both Planck 2018 and Planck 2018+BAO, as we can see, for higher values of $H_0$, $\xi$ assumes (although mildly) positive values, indicating an energy transfer from pressure-less DM to DE. For lower values of $H_0$, exactly opposite scenario is confirmed. The scattered plots actually give a nice statistical comparisons between the models.

We also display in Fig. 9 the temperature anisotropy in the CMB TT spectra for all the IVS models using some specific values of the dimensionless coupling parameter $\xi$. In the upper panel of Fig. 9 we show the CMB TT spectra for a specific positive value of $\xi = 0.05$ and in the lower plot of Fig. 9 we display the same physical quantity for negative value of the coupling parameter, that means, $\xi = -0.05$. For comparison purpose, we have also included the non-interacting $\Lambda$CDM scenario ($\xi = 0$). One can quickly realize that IVS0 and IVS2 are quite different compared to IVS1 and IVS3. Let us describe the physics behind the plots. From the upper plot...
one can understand this phenomena from the evolution of the matter-radiation equality for all the IVS models. It is clear that if we add an interaction in the dark sector, the evolution of the CDM sector will not follow its usual evolution which is \( \rho_c \propto a^{-3} \), hence, the evolution of the matter sector, \( \Omega_m \) (= \( \Omega_c + \Omega_b \)) that includes CDM and baryons, will
On the other hand, for \( \xi > 0 \) (upper plot) and \( \xi < 0 \) (lower plot). Here, \( \Omega_m \) denotes the total matter sector that means cold dark matter plus baryons, that means \( \Omega_m = \Omega_c + \Omega_b \). The horizontal line denotes \( \Omega_m = \Omega_c \), that means where matter density becomes equal to the radiation density. To draw both the plots, we set the following values of the parameters: \( \Omega_{c0} = 0.28, \Omega_{c0} = 0.68, \Omega_{b0} = 0.0001, \) and \( \Omega_{b0} = 1 - \Omega_{c0} - \Omega_{b0} - \Omega_{x0} = 0.0399 \).

We also investigate the effects of the IVS models in the matter power spectrum. In Fig. 11 we show the matter power spectrum for all the IVS models for two specific values of the coupling parameter, namely, \( \xi > 0 \) (upper panel of Fig. 11) and \( \xi < 0 \) (lower panel of Fig. 11). We again find that the behaviour of IVS0 and IVS2 are completely different (in fact, violent) compared to the other IVS models. To understand the behaviour of various IVS models compared to the no-interaction scenario, we have considered the matter power spectrum for the non-interacting \( \Lambda \text{CDM} \) model. From the upper plot of Fig. 11 we see that the amplitude of the matter power spectrum for all IVS models increases compared to the \( \xi = 0 \) case. The significant increase in the matter power spectrum is transparent for IVS0 and IVS2 while for other two IVS models, it is quite difficult to understand the changes in the matter power spectrum from the non-interacting scenario (\( \xi = 0 \)). The enhancement in the matter power spectrum is for the earlier matter-radiation equality, see the upper plot of Fig. 10. The reverse situation occurs for \( \xi < 0 \) (see the lower plot of Fig. 11). In this case the matter power spectrum are suppressed and this again corresponds to the late matter-radiation equality as shown in the lower plot of Fig. 10.

Thus, from the behaviour of the IVS models presented in the CMB TT and matter power spectra shown respectively in Fig. 9 and 11 it is clearly pronounced that definitely change from its usual evolution, and hence the matter-radiation equality will alter. If one looks at the upper plot of Fig. 10 it is clear that for all IVS models, the matter-radiation equality happens earlier for \( \xi > 0 \) compared to the non-interacting \( \Lambda \text{CDM} \) model. Due to earlier matter-radiation equality, the sound horizon is decreased, hence, for the present IVS models, the first peak in the CMB TT spectrum is decreased. For IVS0 and IVS2, the matter-radiation equality happens much earlier compared to IVS1 and IVS3, and this has been encoded in the CMB TT spectrum in terms of significant reduction of the first peak compared to other two IVS models. On the other hand, for \( \xi < 0 \), exactly the opposite scenario happens in the CMB TT spectrum (see the lower panel of Fig. 9) and this behaviour become clear when one looks at the corresponding matter-radiation equality presented in the lower plot of Fig. 10.
models IVS0 and IVS2 are significantly different from the rest two IVS models, namely, IVS1 and IVS3, and additionally, they are very far from the non-interacting ΛCDM model which is only detected through the analysis of formation of structure of the universe.

Finally, we perform the Bayesian evidence analysis for a better understanding on the models with respect to some reference model. To calculate the evidences we use the MCEvidence [94,95], a cosmological code for computing the evidences of the interacting scenarios (also see [96,97] for detailed descriptions). To quantify the observational support of the models, we use the revised Jeffrey’s scale through different values of ln $B_{ij}$. The strength of evidence of the underlying model ($M_j$) with respect to the reference ΛCDM scenario ($M_i$) is characterized as follows [98]: (i) for $0 \leq \ln B_{ij} < 1$, a weak evidence, (ii) for $1 \leq \ln B_{ij} < 3$, a Definite/Positive evidence; (iii) for $3 \leq \ln B_{ij} < 5$, a strong evidence, and (iv) for $\ln B_{ij} \geq 5$, a very strong evidence for the reference ΛCDM model (“ii”) against the underlying model (here the interacting scenario) is considered. In Table VII we have summarized the values of $\ln B_{ij}$. From Table VII we find that ΛCDM is favored by the observational data over all the IVS scenarios, but the models, namely, IVS1 and IVS3 are relatively close to ΛCDM compared to the remaining two IVS models (IVS0 and IVS2). So, the models IVS1 and IVS3 have some importance according to the data.

5. CONCLUDING REMARKS

Interacting DM – DE models have gained potential interest for explaining various cosmological puzzles beginning from the cosmic coincidence problem to the $H_0$ tension. It has been almost 20 years as of now, interacting models have been investigated by various investigators. The interacting models are entirely dependent on the interaction function, $Q$, that determines the rate of energy transfer between the dark sectors DM and DE. Despite of a lot of investigations in this context, a fundamental question – what should be the possible functional form of $Q$ – is still unknown to the cosmological community. Since the nature of DM and DE are unknown, it is very difficult to extract the exact functional form for the interaction function. Thus, the easiest approach followed from earlier to present time, is to assume some phenomenological functions for $Q$ and then to test them using the available cosmological data. The lack of a definite mechanism to construct the interaction functions raises questions over the interaction models. This motivated us to investigate the interaction models from the field theoretical arguments with an aim to search for a valid route to find out the models that are widely used. Our answer is affirmative in this direction. We have shown that various linear and nonlinear interaction functions that have been widely examined in the past and present, can be derived. This is the main essence of this work and probably this is the first time in the literature where we show the exact derivations of some very well known interaction models having a solid field theoretic ground.

We then examine the interaction models using CMB data from Planck 2018 final release and with the BAO data. The inclusion of BAO to CMB is motivated to

| Dataset      | Model | $\ln B_{ij}$ |
|--------------|-------|--------------|
| Planck 2018  | IVS0  | 4.2          |
| Planck 2018+BAO | IVS0 | 6.7          |
| Planck 2018  | IVS1  | 1.0          |
| Planck 2018+BAO | IVS1 | 1.8          |
| Planck 2018  | IVS2  | 2.8          |
| Planck 2018+BAO | IVS2 | 6.7          |
| Planck 2018  | IVS3  | 0.3          |
| Planck 2018+BAO | IVS3 | 1.4          |

TABLE VII: Summary of $\ln B_{ij}$ values computed for the ΛCDM model with respect to IVS0, IVS1, IVS2 and IVS3.
break the degeneracies in some parameters that may exist during the analysis with CMB data alone. The results are summarized in Tables IV, V and VI. Our analyses show that although both Planck 2018 and Planck 2018+BAO mildly allow a non-zero interaction in the dark sector but $\xi = 0$ seems to be the most consistent picture. We also find that the second interaction model, namely, IVS1 is the most promising candidate to alleviate the $H_0$ tension in an effective way. The models have been investigated further through their effects on the CMB TT and matter power spectrum. Such an analysis is really important because this offers more insights on the models. Our analyses clearly depict that IVS0 and IVS2 are different compared to other models. We found that presence of an interaction in the dark sector alters the matter-radiation equality and hence this effects are encoded in the CMB TT and matter power spectrum. We notice that IVS1 and IVS3 are relatively close to the $\Lambda$CDM model.

Finally, we perform a qualitative comparisons between the IVS models through the observational constraints and the Bayesian evidence analysis with respect to the reference $\Lambda$CDM scenario. We find that IVS0 and IVS2 behave similarly, on the other hand, IVS1 and IVS3 behave similarly, but the last two models have essential advantages when we make the Bayesian evidence analysis. However, it is true that the $\Lambda$CDM scenario is still preferred over the IVS models.

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