The Deflation of $SU(3)_c$ at High Temperatures

Afsar Abbas#, Lina Paria# and Samar Abbas*

#. Institute of Physics, Bhubaneswar - 751005, Orissa, India
* Department of Physics, Utkal University, Bhubaneswar-751004, Orissa, India

Abstract

The ideas of “local” and “global” colour-singletness are not well understood within QCD. We use a group theoretical technique to project out the partition function for a system of quarks, antiquarks and gluons to a particular representation of the internal symmetry group $SU(3)_c$: colour-singlet, colour-octet and colour 27-plet at finite temperature. For high temperatures and large size it is shown that colour-singlet is degenerate with colour-octet, colour 27-plet states etc. For the composite system it is shown that $SU(3)_c$ appears to be a good symmetry only at low temperatures and at higher temperatures it gets submerged into a larger group $U(12)_q \otimes U(12)_{\bar{q}}$ (2-flavour). At high enough temperatures this conclusion is model independent. This means that a phase transition from the hadronic matter to the quark-gluon phase implies a transition from the group $SU(3)_c$ to $U(12)_q \otimes U(12)_{\bar{q}}$. Ideas of extensions beyond the standard model would have to be reviewed in the light of this result.

1 e-mail : afsar/lina/abbas@iopb.res.in
1. INTRODUCTION

Colour confinement is an experimentally well established property of QCD at temperature \( T = 0 \). Though it has not been conclusively demonstrated in QCD it is universally believed to be true. Confinement is definitely not shown to be sine qua non of the non-abelian QCD. Several model calculations indicate that indeed the 3-q and \( q\bar{q} \) colour-singlet states are more bound than for example the colour octet, decuplet, etc. representations \([1]\). However one cannot simply throw away the higher colour representations like octet as they may manifest themselves in specific situations like in the multiquark systems \([2]\) etc. Recently the colour-octet contribution has also been shown to be significant during quarkonium production in hadronic collisions \([3]\). The question we ask in this paper is what is the role of higher representations like 8-plet, 27-plet etc. for the bulk QGP at high temperatures and their possible role in the early universe QCD phase transition and also the QGP scenarios in the heavy ion collision setup. Below we shall demonstrate their significance and the new insights that they bring in.

It is believed that in QCD “transition from hadronic matter to the quark-gluon matter is a transition from local colour confinement (on the scale of 1 fm) to global colour confinement” \([4]\). It is not understood as to what maintains long-range correlations implicit in global colour-confinement, for example for sizes as large as the quark stars \([5]\). To better understand the role of the colour degree of freedom we use the colour projection technique \([6] - [14]\).

With the mathematical development of the consistent inclusion of internal symmetries in a statistical thermodynamical description of quantum gases \([6]\) the idea was applied to the colour \( SU(3)_c \) group \([7, 8]\). Therein the group theoretical projection technique was used to project out colour-singlet representation for a bulk system consisting of Quark-Gluon Plasma (QGP) at finite temperature. The requirement of imposition of colour-singletness for these systems has been found to be of great significance and much work has been done using this technique of colour projection \([9] - [14]\). Several interesting results were obtained but perhaps the most significant was that if one were to compare a colour unprojected bulk QGP system with a colour-singlet projected QGP system then important finite size corrections are introduced \([7, 8]\). These finite size corrections arising from the imposition of colour-singletness disappear as the size and/or temperature of the system increases. This was taken to mean that for large-sized QGP systems, which may have been relevant in the early universe QCD phase transition scenarios one may automatically assume global colour-singletness \([4]\) of the system without any significant modifications. This allowed for the possible existence of large size stable quark stars (which were trivially assumed to be colour-singlet \([6]\) ) in the early universe QCD phase transition \([15] - [18]\). These scenarios continue to dominate the hadronization ideas in the big bang models \([19]\). These ideas have also been extremely significant in the heavy ion collision scenarios as well \([20]\).
2. COLOUR - PROJECTION TECHNIQUE

The orthogonality relation for the associated characters \(\chi(p, q)\) of the \((p, q)\) multiplet of the group \(SU(3)_c\) with the measure function \(\zeta(\phi, \psi)\) is

\[
\int_{SU(3)_c} \, d\phi \, d\psi \, \zeta(\phi, \psi) \chi^*(p, q)(\phi, \psi) \chi(p', q')(\phi, \psi) = \delta_{p \, p'} \, \delta_{q \, q'}
\]

Let us now introduce the generating function \(Z_G\) as

\[
Z_G(T, V, \phi, \psi) = \sum_{p, q} \frac{Z(p, q)}{d(p, q)} \chi(p, q)(\phi, \psi)
\]

with

\[
Z(p, q) = tr_{(p, q)} \left[ \exp \left( -\beta \hat{H}_0 \right) \right]
\]

\(Z(p, q)\) is the canonical partition function. The many-particle states which belong to a given multiplet \((p, q)\) are used in the statistical trace with the free hamiltonian \(\hat{H}_0\), \(d(p, q)\) is its dimensionality and \(\beta\) is the inverse of the temperature \(T\). The projected partition function \(Z_{(p, q)}\) can be obtained by using the orthogonality relation for the characters. Hence the projected partition function for any representation \((p, q)\) is

\[
Z_{(p, q)} = d(p, q) \int_{SU(3)_c} \, d\phi \, d\psi \, \zeta(\phi, \psi) \chi^*(p, q)(\phi, \psi) \, Z_G(T, V, \phi, \psi)
\]

The characters of the different representations are as follows:

\[
\chi(1, 0) = \exp(2i\psi/3) + 2 \exp(-i\psi/3) \cos(\phi/2)
\]

\[
\chi(0, 1) = \chi^*_1(1, 0)
\]

\[
\chi(1, 1) = 2 + 2 \left[ \cos\phi + \cos(\phi/2 + \psi) + \cos(-\phi/2 + \psi) \right]
\]

\[
\chi(2, 2) = 2 + 2 \left[ \cos\phi + \cos(3\phi/2)\cos(\phi/2) \right] + 2 \left( 1 + 2 \cos\phi \right) \left\{ \cos(\phi/2 + \psi) + \cos(-\phi/2 + \psi) + \cos2\psi + (1/2) \cos\phi \right\}
\]

The expressions of the generating function used in (4) is

\[
Z_G(T, V, \phi, \psi) = tr \left[ \exp(-\beta \hat{H}_0 + i\phi \hat{I}_z + i\psi \hat{Y}) \right]
\]

where \(\hat{I}_z\) and \(\hat{Y}\) are the diagonal generators of the maximal abelian subgroup of \(SU(3)_c\). Our plasma consists of light spin 1/2 (anti) quarks in the (anti) triplet representation \((0, 1)\) and \((1, 0)\) respectively, and massless spin one gluons in the octet representation \((1,1)\). Note that the
non-interacting hamiltonian $\hat{H}_0$ is diagonal in the occupation-number representation. In the same representation one can write the charge operators $\hat{I}_z$ and $\hat{Y}$ as linear combinations of particle-number operators. Hence $Z^G$ can be easily calculated in the occupation-number representation. With an imaginary ‘chemical potential’ this is just like a grand canonical partition function for free fermions and bosons. One obtains

\[
Z_{\text{quark}}^G = \prod_{q=l,m,n} \prod_k [1 + \exp(-\beta \epsilon_k - i\alpha_q)] [1 + \exp(-\beta \epsilon_k + i\alpha_q)]
\]

(10)

\[
Z_{\text{glue}}^G = \prod_{g=\mu,\nu,\rho,\sigma} \prod_k [1 - \exp(-\beta \epsilon_k + i\alpha_g)]^{-1} [1 - \exp(-\beta \epsilon_k - i\alpha_g)]^{-1}
\]

(11)

Here the single-particle energies are given as $\epsilon_k$. For $(1, 0)$, $(0, 1)$ and $(1, 1)$ multiplets, the eigenvalues of $\hat{I}_z$ and $\hat{Y}$ gives the expression for different angles as:

\[
\alpha_l = (1/2) \phi + (1/3) \psi, \quad \alpha_m = (-1/2) \phi + (1/3) \psi, \quad \alpha_n = (-2/3) \psi
\]

\[
\alpha_\mu = \alpha_l - \alpha_m, \quad \alpha_\nu = \alpha_m - \alpha_n, \quad \alpha_\rho = \alpha_l - \alpha_n, \quad \alpha_\sigma = 0
\]

(12)

(13)

We neglect the masses of the light quarks. At large volume the spectrum of single particle becomes a quasi-continuous one and $\Sigma \rightarrow V/(2\pi)^3 \int d^3p...$ Then one gets

\[
Z^G(T, V, \phi, \psi) = Z_{\text{quark}}^G(T, V, \phi, \psi) Z_{\text{glue}}^G(T, V, \phi, \psi)
\]

(14)

This then enables us to obtain the partition function for any representation ie. $Z_{(p, q)}$. One may thus obtain any thermodynamical quantity of interest for a particular representation. For example the energy

\[
E_{(p, q)} = T^2 \frac{\partial}{\partial T} \ln Z_{(p, q)}.
\]

(15)

3. RESULTS

Work was done earlier by several groups to impose colour-singletness on the system [3, 4]. Note that in these calculations perturbative interactions had been neglected. But this may not be a bad approximation especially at high temperatures. The most dramatic consequence of the colour interaction is to cause global colour-confinement of quarks and gluons and this is automatically taken care of by restricting the partition function to colour-singlet states [4]. This perhaps may be taking care of a major chunk of non-perturbative aspect of QCD interaction. It was found that

\[
E_{(0, 0)} = E_0 + E_{\text{corr}}
\]

(16)
where $E_0$ was the unprojected energy (ie. with no colour restriction whatsoever) given by

$$E_0 = 3 \ a_q \ V \ T^4$$

(17)

with $a_q = (37 \pi^2/90)$ and $E_{\text{corr}}$ was the correction introduced due to the imposition of colour-singletness. They found that $E_{\text{corr}}$ was significant only for the finite size i.e. when $TV^{1/3}$ was small ($< 2$) and vanished when $TV^{1/3}$ became large ($> 2$). This would mean that colour-singletness restriction only affects for size say $\sim 1.0$ fm for $T = 160 \ MeV$ while for large size and higher temperatures one need not perform explicit colour projection calculation because the consequent corrections are negligible therein $[8, 9]$. But below we shall show that this is not the whole story.

In this paper we project out different representations like octet $(1, 1)$, 27-plet $(2, 2)$ etc. on these QGP. The idea is that for ground state one knows that the singlet state is bound and the higher representations are expelled to infinite energies $[1]$. Also for the ground states the role of the higher representations is also quite well studied $[2, 3]$. The point to be emphasized is that the role of global colour-singletness at high temperatures is only an assumption and has never been explicitly demonstrated even in a model calculations. Here we would like to study the basis of this assumption and also the role, if any, of higher representations like octet, 27-plet etc.

Let us look at octet, 27-plet etc. projection. For simplicity we take $\mu = 0$ case with 2 flavours. We plot in Fig. 1

$$D_{(p, q)}^{\text{eff}} = E_{(p, q)}/E_0 = 1 + E_{(p, q)}^{\text{corr}}/E_0$$

(18)

Also shown is $D_{(0, 0)}^{\text{eff}}$ as obtained earlier by other groups $[3]$.

4. DISCUSSION

It is interesting to note from Fig. 1 that for large values of $TV^{1/3}$ all representations ; singlet, octet, 27-plet etc are degenerate with the unprojected energy. For sufficiently large QGP and sufficiently high temperatures all the energies of all the representations are found to be degenerate. There is nothing which favours the colour-singlet representation over the colour-octet at high temperatures. Note that this result could directly be seen from the expression $Z_{(p, q)}$ (eq.(3)) which for the continuum approximation for sufficiently large volume and temperature becomes independent of representation. Hence energies for each colour representation are degenerate with each other for large $TV^{1/3}$. As the neglect of perturbative interactions is justified at high enough temperatures, this important result seems to be quite model independent.

Note that the quarks and gluons are non-interacting in our model. While this is justified at high temperatures, the neglect of interactions may not be justified at low temperatures. However
as one of the most dramatic effect of colour interaction is ensuring colour-singletness on the bulk system it is already taken care of in our model [4]. So perhaps a significant portion of the non-perturbative effect may already be there in our model calculations. From Fig. 1 note that for small $TV^{1/3}$ values the octet and the 27-plet energies shoots up. Though gauge interactions are believed to be essential to show confinement in QCD, what we note here is that our projection technique at even low temperatures is able to discriminate between the singlet and the octet states etc.

The $\mu \neq 0$ case may be significant for giving finer details [21], however, we do not expect any significant change in our basic results [3]. Fig. 2 gives $\mu = 0$ result for zero flavour, 2-flavour and 3-flavour for (0, 0), (1, 1) representations.

We have found the degeneracy of the states for large $TV^{1/3}$ for each colour representation for $\mu = 0$ to be true for 0, 2, 3 flavours etc. So it is independent of number of flavours. We find it occurs for the $\mu \neq 0$ case also [21]. The fact that the colour singlet representation gets favoured over the octet representation etc. at low temperature, group theoretically it indicates that for composite systems $SU(3)_c$ is a good symmetry only for small temperatures and above the QCD phase transition temperature (actually $TV^{1/3}$) gets submerged into a larger group. We would like to emphasize that this is definitely true at high enough temperatures. This larger group is $U(18)_q \otimes U(18)_{\bar{q}}$ for 3-flavours, $U(12)_q \otimes U(12)_{\bar{q}}$ for 2-flavours, $U(6)_q \otimes U(6)_{\bar{q}}$ for 0-flavours. The subgroup structure for say the 2-flavour case is

$$U(12)_q \otimes U(12)_{\bar{q}} \supset SU(12)_{csf} \supset SU(6)_{cs} \otimes SU(2)_f \supset SU(3)_c \otimes SU(2)_s \otimes SU(2)_f$$

(19)

where the subscripts 'csf' stand for colour, spin and flavour respectively. It is only below the QCD phase transition temperature that $SU(3)_c$ becomes a good symmetry for the composite system. For the composite system, at high temperatures $SU(3)_c$ does not remain the relevant symmetry and gets submerged into $U(12)_q \otimes U(12)_{\bar{q}}$ (for 2-flavours). As transition from hadronic matter to quark-gluon matter is a transition from “local” colour confinment to “global” colour confinement [4], in view of our result it also implies going from the group $SU(3)_c$ to $U(12)_q \otimes U(12)_{\bar{q}}$.

We would like to remind the reader that in the quark model in the static limit it was found fruitful to look at $SU(3)_f$ as submerged in $SU(6)_{fs}$ as $SU(6)_{fs} \supset SU(3)_f \otimes SU(2)_s$. In the same spirit we have found here that at high temperatures and/or large size the $SU(3)_c$ gets deflated and submerged in a larger group e.g. $U(12)_q \otimes U(12)_{\bar{q}}$. We would like to emphasize that at sufficiently high temperatures and/or large size this conclusion is model independent.

Note that the currently favoured scenario for the early universe above the QCD and the electroweak phase transition temperatures, the unbroken group is believed to be $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. Our work in this paper casts doubt in this view. Globally it is not simply $SU(3)_c$ but $U(12)_q \otimes U(12)_{\bar{q}}$ in the early universe. Ideas of extensions beyond the standard model have to be reviewed in the light of this result.
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Figure Captions

Fig. 1 : $D_{eff}$ (see text) for the colour representations singlet, octet and 27-plet (with two flavours) as a function of $TV^{1/3}/hc$

Fig. 2 : $D_{eff}$ for the colour representations singlet and octet as a function $TV^{1/3}/hc$ for different number of flavours ; 0, 2 and 3.
Deff vs. $(TV^{1/3})/hc$ for Singlet, Octet, and 27-plet.
