Radiation-assisted magnetotransport in two-dimensional electron gas systems: appearance of zero resistance states

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Abstract
Zero-resistance states (ZRS) are normally associated with superconducting and quantum Hall phases. Experimental detection of ZRS in two-dimensional electron gas (2DEG) systems irradiated by microwave (MW) radiation in a magnetic field has been quite a surprise. We develop a semiclassical transport formalism to explain the phenomena. We find a sequence of Zero-Resistance States (ZRS) inherited from the suppression of Shubnikov–de Haas (SdH) oscillations under the influence of high-frequency and large amplitude microwave radiation. Furthermore, the ZRS are well pronounced and persist up to broad intervals of magnetic field as observed in experiments on microwave illuminated 2DEG systems.

Keywords: two-dimensional electron gas, zero-resistance states, magnetoresistivity, microwave radiation, Shubnikov–de Haas oscillations

(Some figures may appear in colour only in the online journal)

I. Introduction
Magnetotransport provides important information on Fermi surface characteristics, disorder and localization mechanisms in low-dimensional electron systems. A comprehensive review of the electronic properties and electronic transport in two-dimensional electron gas (2DEG) systems was presented by Ando, Fowler and Stern [1] in early eighties. The discovery of Quantum Hall effects around the same time led to the realization that, in strong magnetic field, quantization effects lead to novel transport features [2–5]. In the late eighties, it was found that in moderately strong magnetic fields; when quantization effects are absent but cyclotron dynamics is present; interesting set of phenomena occur. In this regard, it was observed that commensurability oscillations in the magnetoresistance occur in periodically modulated 2DEG systems [6–8]. In particular, periodic oscillations in 1/B (B is the applied magnetic field) is observed in the magnetoresistance of a two-dimensional electron gas (2DEG) subjected to weak [9–14] and strong [15–17] periodic potential. In addition to magnetoresistance, magnetoplasmons in these systems have also been investigated, [18] and references therein. In a series of experiments carried out in 2001–2003, it was found that a 2DEG subjected to microwave radiation in an applied magnetic field yields even richer physics. When a high mobility 2DEG is irradiated by microwave radiation in a weak magnetic field, the longitudinal magnetoresistance exhibits giant oscillations. This was the discovery of Microwave Induced Resistance Oscillations (MIRO) [19–30]. A significant feature of these studies has been the observation of Zero-Resistance States (ZRS) in these systems; the lower order minima in MIRO go all the way to zero [31–37]. Observation of ZRS was quite a surprise; eventhough longitudinal resistance exhibits ZRS in integer quantum Hall systems but the magnetic field required here is smaller by a factor of 50. Unlike Quantum Hall phenomena, the vanishing of longitudinal magnetoresistance does not lead to quantization of Hall resistance. This led to the understanding that weak Landau quantization and weak microwave radiation can significantly alter the transport properties of a 2DEG. This discovery opened the field of
nonequilibrium transport in high Landau levels [38]. Several explanations have been put forward [23, 31, 39–42]. Most of the theoretical work relies on the combined effect of Landau quantization and applied fields on momentum relaxation due to impurity scattering with in a Landau band; alternatively experimental results are explained on the basis of redistribution of electrons in a disorder broadened Landau band due to interaction with microwaves.

The mechanism responsible for the appearance of ZRS in a 2DEG in the presence of both MW radiation and an external magnetic field is still far from settled. In this work, we will investigate whether it is possible to find an explanation of this phenomenon with in a single particle semiclassical picture. In this regard, we will focus on the effect of plane polarized electromagnetic radiation on the cyclotron motion of electrons in a 2DEG system. Cyclotron motion of electrons in a 2DEG has been extensively studied in [43–45]. We base our study on [13, 41] and extend it to include the effects of electromagnetic radiation. In particular, we find that cyclotron trajectories are significantly modified under the influence of radiation. Further, commensurability oscillations in the magnetoresistivity of 2DEG are induced by radiation. Interestingly, we find a sequence of zero-resistance states (ZRS) inherited from the suppression of Shubnikov–de Haas oscillations (SDHO) by high-frequency and large amplitude microwave radiation. The ZRS are well pronounced and persist up to broad intervals of magnetic field as observed in many experiments on microwave illuminated 2DEG [19, 32]. Moreover, the formation of ZRS strongly depends on the frequency of MW radiation and disappear at low frequencies. This fact is further confirmed by investigating a range of MW frequency where the system is completely driven to ZRS.

The paper is organized as follows: in section II, the model Hamiltonian of our system which is a 2DEG of the presence of a perpendicular magnetic field and microwave radiation is introduced. The eigenstates of the system in the absence of radiation are determined. The investigation of cyclotron motion of electrons in the presence of both MW radiation and an external magnetic field is formulated in the framework of Heisenberg equation of motion technique and the semiclassical formalism is derived from the full quantum description. Moreover, the influence of microwave radiation on the magnetic field-assisted dynamics is studied in detail.

Section III is dedicated to the formulation of magnetoresistivity by finding the enhancement in diffusion coefficients using the drift velocity of electrons.

In section IV the results based on our model are discussed. The different limiting cases are analyzed in detail. Finally, conclusions are drawn in section V.

II. Model Hamiltonian

The single particle Hamiltonian of an electron in a 2DEG system (in the xy plane) in the presence of electromagnetic (MW) radiation polarized along the x – direction in the plane subjected to a perpendicular magnetic field is given by

\[ \hat{H} = \frac{\hat{\mathbf{p}}^2}{2m^*} + e\mathbf{A} \cdot \mathbf{v}, \]

where \( \hat{\mathbf{p}} = (\hat{p}_x, \hat{p}_y) = (\hat{\mathbf{p}} + e \mathbf{A}) \) is the two-component kinetic momentum with the canonical momentum operator \( \hat{\mathbf{p}} \), and \( \mathbf{A} \) is the vector potential given by \( \mathbf{A} = (0, xB, 0) \) in the Landau gauge. Moreover, \( m^* \) is the effective mass of electron in 2DEG. The second term in the above Hamiltonian represents interaction of the electromagnetic wave (MW) with the electron. The constant \( E_0 \) is the amplitude of the electric field of the electromagnetic wave and \( \omega \) is its angular frequency. In the above Hamiltonian, we have neglected the spatial variation in the electric field of the wave [46]. This approximation is reasonable if we consider the MW with wavelength larger than the diameter of the cyclotron orbit \( d_c = 2k_y = 2k_F l \), with \( k_F \) being the Fermi momentum and \( l = \sqrt{\mu eB} \) the magnetic length.

In the absence of MW, the normalized eigenstates of the system are given by

\[ \psi_{nk}(x, y) = \frac{1}{\sqrt{L_y}} e^{-ik_y y} \varphi_n(x), \]

where \( L_y \) is the length of the sample in the y-dimension. The functions \( \varphi_n(x) \) represents the eigenstates of harmonic oscillator with guiding centre at \( x_0 \) described by

\[ \varphi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} e^{-\frac{(x-x_0)^2}{2}} H_n \left( \frac{x-x_0}{l} \right), \]

where \( H_n(x) \) is the nth-order Hermite polynomial, \( x_0 = \frac{\hat{\mathbf{p}}_x}{\mu} \) is the centre of cyclotron orbit. In the above expression \( n = 0, \pm 1, \pm 2, \ldots \) characterizes the Landau levels and \( k_y \) is the electron wave number with the translational invariance in the y direction. The quantum number \( k_y \) is conveniently determined by periodic boundary condition as

\[ k_y = \frac{2\pi}{L_y n}. \]

The maximum value of \( n \) can be specified by the condition that the centre of the cyclotron orbit should be within the sample: \( 0 < x_0 < L_x \), where \( L_x \) is the dimension of the sample in the x-dimension. Alternatively

\[ |k_y| < \frac{L_x}{l}, \]

II.A. Quantum description of cyclotron motion in a 2DEG

In this section, we analyze the cyclotron motion of a charged particle in a 2DEG illuminated by MW within a quantum mechanical approach. The time evolution of the cyclotron trajectories is determined using Heisenberg equation of motion. As a consequence, the time evolution of the position operator \( \hat{r} \) in Heisenberg picture reads

\[ \frac{d\hat{r}}{dt} = i \hbar [\hat{H}, \hat{r}], \]

After straightforward calculations the above equation of motion yields
\[ \dot{r}(t) = \dot{r}(0) + \frac{\hat{p}_x(0)}{m^* \omega_x} \left( 1 - e^{-i\omega_x t + \varphi_0} \right) + \frac{eE_0}{m^*} \frac{e^{-i\omega c t + \varphi_0}}{\omega_c^2 - \omega^2} \]

\[ = \frac{eE_0}{m^* (\omega_c^2 - \omega^2)} \left[ \cos(\omega t + \omega \varphi_0) - i\frac{\omega}{\omega_c} \sin(\omega t + \omega \varphi_0) \right]. \tag{7} \]

where \( \omega_c = \sqrt{\frac{|eB|}{m_e}} \) is the cyclotron frequency and \( \dot{r}(0) \) specify the initial coordinates of the centre of the cyclotron orbit which commute with the Hamiltonian of the system and consequently remains constant in time. The constant phase \( \varphi_0 \) locates the initial position of the particle in the cyclotron orbit. Equation (7) reveals that the cyclotron trajectories are significantly affected by the microwave (MW). In order to analyze the MW-assisted dynamics of the particle in a magnetic field we need to evaluate the expectation values of the time dependent position operators. Using the complex notations \( \hat{r} = \hat{x} - i \hat{y} \) and \( \hat{p} = \hat{p}_x - i \hat{p}_y \), the x-component of the cyclotron motion can be expressed as

\[ \dot{x}(t) = \dot{x}(0) + \frac{\hat{p}_x(0)}{m^* \omega_x} \sin(\omega_x t + \varphi_0) + \frac{\hat{p}_x(0)}{m^* \omega_x} [\cos(\omega_x t + \varphi_0) - 1] \]

\[ + \frac{eE_0}{m^* (\omega_x^2 - \omega^2)} \cos(\omega t + \omega \varphi_0) \]

\[ - \frac{eE_0}{m^* (\omega_x^2 - \omega^2)} \cos(\omega t + \omega \varphi_0). \tag{8} \]

Similarly, the y-component of the cyclotron motion can be described in the form

\[ \dot{y}(t) = \dot{y}(0) - \frac{\hat{p}_y(0)}{m^* \omega_y} [\cos(\omega_y t + \varphi_0) - 1] + \frac{\hat{p}_y(0)}{m^* \omega_y} \sin(\omega_y t + \varphi_0) \]

\[ + \frac{eE_0}{m^* (\omega_y^2 - \omega^2)} \sin(\omega_y t + \omega \varphi_0) \]

\[ - \frac{eE_0}{m^* (\omega_y^2 - \omega^2)} \sin(\omega_y t + \omega \varphi_0). \tag{9} \]

II.B. Semiclassical formulation of cyclotron motion

Equations (8) and (9) give the full quantum mechanical description of the cyclotron motion in two dimensional electron systems in the presence of an external perpendicular magnetic field when the system is irradiated by MW. However, in order to obtain analytic results we develop a semiclassical formalism. In this regard, we are interested in the classical expectation values of the cyclotron trajectories which are obtained by replacing the operators by their corresponding classical variables. The analytic expressions for the electron dynamics in external magnetic field and MW radiation can be obtained in the semiclassical regime specified by the criterion, \( k_F d \gg 1 \), where \( k_F \) is the Fermi wave vector given by \( k_F = \sqrt{2\pi n_e} \) with \( n_e \) being the electron density. Semiclassical results can be obtained from quantum mechanical results by ignoring the quantum fluctuations of the operators corresponding to dynamical variables. In our case, one can derive the semiclassical results from the full quantum mechanical equations (8) and (9) by treating the operators \( \hat{x}(t), \hat{y}(t), \hat{\varphi}_x, \) and \( \hat{\varphi}_y \) as classical variables. Consequently, in the semiclassical limit expectation value of the x-component of the position operator can be expressed as

\[ x(t) = x(0) + \kappa_s k^2 [\cos(\omega_c t + \varphi_0) - 1] + \frac{eE_0}{m^* (\omega_c^2 - \omega^2)} \cos(\omega t + \omega \varphi_0) \]

\[ - \frac{eE_0}{m^* (\omega_c^2 - \omega^2)} \cos(\omega t + \omega \varphi_0). \tag{10} \]

In a similar way, the expectation value of the y-component of the position operator can be written as

\[ y(t) = y(0) - \frac{eE_0}{m^* (\omega_y^2 - \omega^2)} \sin(\omega_y t + \omega \varphi_0) \]

\[ - \frac{eE_0}{m^* (\omega_y^2 - \omega^2)} \sin(\omega_y t + \omega \varphi_0). \tag{11} \]

where \( k_y \) is the y-component of the electron wave vector given by equation (4) and \( k_y \) is its x-component which is determined by the relation, \( k_y = \frac{\sqrt{k_F^2 - k_x^2}}{k_y} \).

In figure 1 we have shown the dynamics of the electronic classical orbit evaluated in terms of the expectation values of the cyclotron trajectories in the basis described by equation (2). It is evident from this figure that the cyclotron trajectories are strongly modified by the microwave radiation. This modification in the cyclotron orbit depends on the amplitude of MW electric field which specifies the coupling of the radiation to the electronic degrees of freedom.

In order to independently investigate the effect of MW frequency on cyclotron motion, we plot the results in figure 2. The comparison of thick red curve and thin blue curve shows that larger shift in guiding center takes place at higher frequency. In summary, a shift is produced in the guiding center of the electronic cyclotron orbit under the effect of MW radiation which in turn affects the transport properties of the system.

III. Magnetoresistivity

We adopt the semiclassical approach for evaluating magnetoresistivity developed by Beenakker [13] and Kennett [41]. In order to simplify the analysis, we take the electric dipole moment \( \mu = e\mathbf{t} \) of the electron and the electric field \( E_0 \) of the microwave radiation to be parallel polarized. As a result, Lorentz force is experienced by the electron that causes drift \( E_0 \times B \) of the guiding center of the cyclotron orbit in the transverse direction. The drift velocity of the electron guiding center in the transverse direction can be described to the lowest order of the MW radiation field as

\[ v_y(t) \approx \frac{E_0}{B} \cos[qx(t) - \omega t], \quad v_x(t) = 0, \tag{12} \]
where $x(t)$ is the instantaneous position of the electron in cyclotron motion and $q$ is the wave number of the MW. Due to the transverse velocity, the diffusion coefficient tensor $D_{yy}$ in the transverse direction is enhanced. This diffusion coefficient can be determined from the autocorrelation function of the electron velocities by taking average over all the particle trajectories and scattering events

$$D_{yy} = \int_0^\infty \int_0^{2\pi} \int_0^{2\pi} \frac{d\phi_0}{2\pi} \frac{d\xi}{2\pi} v_y(t)v_y(0),$$

(13)

where $\xi = q\tau(0)$. Once the transverse diffusion tensor $D_{yy}$ is known, one can find the longitudinal resistivity tensor $\rho_{xx}$ using Einstein diffusion relation [13]

$$\rho_{xx} = \frac{D_{yy}}{D_0},$$

(14)

where $\rho_0$ is the Drude resistivity in zero magnetic field and $D_0$ represents the unperturbed diffusion coefficient. For a 2DEG the diffusion coefficient in the presence of magnetic field is given by $D_0 = r^2/2\tau$ with $\tau = \nu_F/\omega_c$ being the classical cyclotron radius and $\tau$ the transport relaxation time. Using equations (10), (12)–(14) and the Bessel function identities\(^3\) one can find

$$\rho_{xx} = \frac{\tau e E_0 \nu_F}{2\nu_F} \left[ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} J_n^2(qk^2) J_m^2(2qk^2) \frac{e q \nu_F}{m(\omega_c^2 - \omega^2)} \right] \frac{\mu_0}{1 + [\omega(1+s) - \omega_c(n+m+k)]^2 \tau^2},$$

(15)

where $\nu_F$ is the Fermi velocity, $e$ is the Fermi energy of the electron and $J_n(x)$ is the $n$-th order Bessel function of the first kind.

\(^3\) Bessel function identities: $e^{\nu e^{i\phi}} = \sum_{n=-\infty}^{\infty} J_n(e^{i\phi}) e^{i\nu n}$. 

Figure 1. Semiclassical cyclotron orbit dynamics of the particle, (a) without microwave radiations (MWs), whereas (b), (c) and (d) with MWs. The $x$- and $y$- coordinates are measured in $\mu$m. The experimentally relevant set of parameters used are: the frequency of MW is $f = 25$ GHz, charge carrier density is $n_e = 65 \times 10^{14}$ m$^{-2}$ and the effective mass of the electron is $m_e = 0.068 m_0$. Length of the system is $L_x = 6$ mm and its width is $L_y = 6$ mm. The amplitude of MW electric field is $E_0 = 2 \times 10^4$ V m$^{-1}$ for (b) and $E_0 = 4.5 \times 10^4$ V m$^{-1}$ for (c) and (d). The external magnetic field is $B = 0.08$ T. The initial coordinates and phase are $x(0) = 0$, $y(0) = 0$, and $\phi_0 = 0$, respectively. In (d) we have demonstrated the time evolution of the cyclotron orbit where the time is measured in units of nano second. The simulation time is always $t = 10 \pi/\omega$. 

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IV. Results and discussions

In this section, we illustrate the results of our model and discuss the various features arising in magnetotransport, in particular, the formation of zero-resistance states. We use the parameters set that is in the experimentally relevant range given in [25, 32, 33]. In figure 3, we demonstrate the resistivity ratio given by equation (15) as a function of $B/B_f$, where $B_f = 2\pi fm^*/le$.

A detailed analysis of this plot reveals that in the static limit ($\omega \to 0$) the magnetoresistivity exhibits the usual Shubnikov–de Haas oscillations (SdHO), see the black dashed curve in figure 3. This effect arises from Landau level quantization in the magnetic field. This feature of the system can be described by writing equation (15) in the form

$$\rho_{\text{as}} \approx \frac{16\pi^2 (\omega_c^2 - \omega^2)^2}{\pi^4 q^4 k^2 \lambda^2} \sum_{n,m=-\infty}^{\infty} \frac{j_n^2(q k, l^2 f^2)}{1 + (n + m)^2 \omega_c^2 r^2},$$

which in the asymptotic limit closely resembles the Weiss Oscillations in the diffusion contribution to the resistivity [6, 7, 11, 47]. The scenario changes and interesting features appear when the system is illuminated by high-frequency microwave radiation. The Shubnikov–de Haas oscillations (SdHO) are suppressed and even the resistivity of the system vanishes within certain intervals of the magnetic field. The states responsible for zero resistivity are known as zero-resistance states (ZRS). The mechanism of suppressed resistivity and the consequent zero-resistance states can be better understood by the following analytic analysis of the above equation: the suppression of SdHO is enhanced by the interference effects between periodically oscillating functions. In order to understand this mechanism we consider the asymptotic behavior of the Bessel function in the limits, $q k_1 l^2$, $q k_2 l^2$, $\frac{eE_0 q}{m(\omega_c^2 - \omega^2)} \gg 1$.

Under the above approximation equation (15) can be recast into the form ($m \neq 0$)

$$\rho_{\text{as}} \approx \frac{16\pi^2 (\omega_c^2 - \omega^2)^2}{\pi^4 q^4 k^2 \lambda^2} \sum_{n,m,k,s} \frac{k_2 k_3}{(1 + [\omega(1 + s) - \omega c(n + m + k)^2 r^2])} \cos^2\left(\frac{q k_2 l^2 - \frac{m}{2} - \frac{\pi}{2}}{k_2 k_3} \cos^2\left(2q k_2 l^2 - \frac{m}{2} - \frac{\pi}{2}\right) \right) \times \cos^2\left[\frac{eE_0 q f}{m^*(\omega_c^2 - \omega^2)} \frac{s n}{2} + \frac{\pi}{4}\right],$$

Minima in the resistivity can be obtained if at least one of the following conditions is fulfilled:

$$q k_2 l^2 \approx \frac{\pi}{2}\left(n + \frac{3}{2}\right) \frac{\pi}{2}\left(n - \frac{1}{2}\right) \text{or } 2q k_2 l^2$$

$$\approx \frac{\pi}{2}\left(m - \frac{1}{2}\right) \frac{\pi}{2}\left(m + \frac{3}{2}\right)$$

$$\text{or } \frac{eE_0 q f}{m^*(\omega_c^2 - \omega^2)} \approx \frac{\pi}{2}\left(k + \frac{3}{2}\right) \div \frac{\pi}{2}\left(k - \frac{1}{2}\right).$$

Due to the alternating behavior of the integers $n$, $m$, $s$ and $k$, the above conditions can often be satisfied. That is why the zero-resistance states are very pronounced and persist up to broader intervals of the magnetic field compared to the one pointed out in [39–41]. Furthermore, the most pronounced ZRS (right side of figure 3) occurs at about $4/9 B_f$, which is in good agreement with the occurrence of ZRS observed in [32]. The next ZRS

Figure 2. Cyclotron orbit dynamics of the electron under the influence of microwave radiations for two different frequencies. The amplitude of SW electric field is $E_0 = 4.5 \times 10^3$ Vm$^{-1}$ and the other parameters are the same as used in figure 1.

Figure 3. Magnetoresistivity of GaAs based two dimensional electron gas (2DEG). The dashed black curve represents magnetoresistivity in static limit ($\omega \to 0$), whereas the blue curve denotes the dynamical magnetoresistivity in the presence of microwave radiation. The wavelength of MW radiation is $\lambda = 4$ μm, the amplitude of MW electric field is $E_0 = 4.3 \times 10^5$ Vm$^{-1}$, whereas its frequency is $f = 75$ GHz. The other parameters are the same as used in figure 1.
in our model is near \(\frac{4}{13} B_f\) and so on. In summary, the theoretical model presented in this paper predicts the dynamics of a particle to be composed of the product of many harmonics. When all these harmonics are in phase, the drift is enhanced. However, if at least any two harmonics become out of phase, they cancel the effects of each other and the resistivity takes a minimum value. Moreover, in the situation investigated here, the domain of oscillations for in phase/out of phase harmonics is broader due to the alternating behavior of the Bessel functions. Based on the result of magnetoresistivity given by equation (15), our model predicts different regimes: (i) For weak enough coupling of electromagnetic wave to electron in the limit \(\omega^2 - \omega_0^2 \gg \frac{eE}{m}\) and \(qk_BI^2\), \(2qk_BI^2 \ll 1\), the commensurability oscillations of the resistivity are significantly suppressed. That is why the zero-resistance states are very pronounced at large magnetic field. (iii) At low magnetic field the commensurability oscillations in the magnetoresistivity of the system are suppressed because the oscillating Bessel functions average out to a constant. (iv) At low frequency of MW radiation, the system exhibits SdH oscillations in the resistivity, see figure 4. The same results are approached in the high magnetic field limit discussed above.

Moreover, the expression given by equation (15) reveals that the system shows a sequence of resonances at those values of the magnetic field which can fulfill the condition, \(\omega(1 + s) \approx (n + m + k)\omega_0\). In this case the expression for dynamical resistivity is described in the form

\[
\rho_{xx} \approx \left( \frac{\tau eE_0I}{2e}\right)^2 J_2^2 \left[ \frac{eE_{0q}}{m^*(\omega_0^2 - \omega^2)} \right] \sum_{n,m,k} J^2_0(qk_BI^2) \times J^2_0(2qk_BI^2) J^2_0 \left[ \frac{eE_{0q}}{m^*(\omega_0^2 - \omega^2)} \right] \delta_{n+m+k,0}. \tag{19}
\]

Note that the static case \((\omega = 0)\) is dominated by the terms \(n = m = k = s = 0\) in the sums, whereas all other values of these integers contribute to the dynamic case. In the regime of intermediate microwave radiation frequency range, \(1\ll\omega_T\ll\omega_f\) and strong magnetic field, the classical oscillations take the form

\[
\rho_{xx} \approx \left( \frac{eE_0I}{2e\omega}\right)^2 \left[ J^2_0(qk_BI^2) J^2_0 \left[ \frac{eE_{0q}}{m^*(\omega_0^2 - \omega^2)} \right] \right]. \tag{20}
\]

Figure 4 demonstrates the microwave radiation frequency-dependence of \(\rho_{xx}/\rho_0\) oscillations. It shows that the mechanism...
of suppression of SdHO strongly depends on the microwave radiation frequency. We see that high frequency MW can efficiently drive the system to zero-resistance state as observed in experiments [25, 32, 33]. This is also obvious from figure 6 where \( \rho_{\text{xx}} \) vanishes when the MW frequency is sufficiently large. Hence in this regime the system resides in a zero-resistance state.

V. Summary and conclusions

In conclusion, we have analyzed a semiclassical theory of magnetotransport in two dimensional electron gas (2DEG) systems irradiated by microwaves. In this regard, we have discussed radiation-assisted dynamics of a charged particle in an external magnetic field. In the presence of a perpendicular magnetic field the resistivity of the system shows Shubnikov–de Haas oscillations (SdHO). However, if the system is illuminated by high-frequency microwave radiation, the SdHO are suppressed and consequently the resistivity of the system vanishes in some intervals of the magnetic field which are associated with zero-resistance states (ZRS). Moreover, a detailed investigation of parametric regimes where zero resistance states in our system can be observed has been performed. Furthermore, experimental relevance with MW illuminated 2DEG systems has been established.

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