Metal - Insulator transition in holography

Aristomenis Donos

Imperial College London

Talk at CCTP

April, 2013

Based on
arXiv:1212.2998 with S. Hartnoll
1 Introduction/Motivation

2 Devising a Metal-Insulator transition in holography

3 A concrete example

4 Summary - Plans
1. Introduction/Motivation

2. Devising a Metal-Insulator transition in holography

3. A concrete example

4. Summary - Plans
Holographic Phases

- Analyse the behavior of strongly coupled CFTs when held at finite temperature and charge density and/or in a uniform magnetic field.
  - Construct charged black hole solutions with AdS asymptotics
  - Calculate the free energies and deduce the phase diagram

- What type of thermal phases are possible?

- What kind of zero temperature ground states can we have?

- Do we find interesting new behaviour in the far IR? e.g. Lifshitz, Schrodinger, hyperscaling violating, ..., something new??

- Analyse hydrodynamics and transport
Holographic phases

Superconductivity/Superfluidity:

[Gubser], [Hartnoll, Herzog, Horowitz]

- A bulk charged scalar takes a VEV giving mass to the $U(1)$ gauge field e.g. s-wave superconductors
- Higher rank charged fields take a VEV giving rise to anisotropic superfluidity order e.g. p-wave, d-wave superconductors

Spatially modulated phases:

[Domokos, Harvey], [Nakamura, Oogiru, Park], [AD, Gauntlett, Pantelidou],
[Bergman, Jokela, Lifschytz, Lippert], [Iizuka, Kachru, Kundu, Narayan, Sircar, Trivedi]

- A neutral/charged bulk field takes a modulated VEV breaking spatial translations
- Current/Momentum density waves, CDWs, FFLO-like
Materials with charged d.o.f. can be conductors or insulators.

At the transition a bad metallic phase can appear.

Strong coupling dynamics suggested to take place → answer in AdS/CFT?
Can we carry out such “experiments” in holography?

- Describes field theories at strong coupling
- Allows for the calculation of correlators e.g. $G_{O_1 O_2}^R(\omega, k)$ by studying small perturbations around a black hole background
- Kubo’s formula for linear response allows the direct computation of the optical conductivity
  \[ \sigma = \frac{1}{i\omega} G_{J_x J_x}^R(\omega, k = 0) \]
- Use to study transport of phases of holographic matter
Consider bulk theory with metric $g_{\mu\nu}$ a cosmological constant and a $U(1)$ gauge field $A_\mu$

- Study CFT $\rightarrow$ asymptote to $AdS_4$
- Deform by chemical potential $\rightarrow$ $A_\mu \approx \mu \, dt + \cdots$
- Finite temperature $\rightarrow$ regular Killing horizon

In $D = 4$ Einstein-Maxwell theory

$$\mathcal{L}_{EM} = \sqrt{-g} \left( \frac{1}{2} R + 6 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)$$

the above translate to the AdS-RN black brane

$$ds^2_4 = -g(r) \, dt^2 + g(r)^{-1} \, dr^2 + r^2 \left( dx_1^2 + dx_2^2 \right)$$

$$A = \mu \left( 1 - \frac{r_+}{r} \right) \, dt,$$  

$$g = 2r^2 - \left( 2r_+^2 + \frac{\mu^2}{2} \right) \frac{r_+}{r} + \mu^2 \frac{r_+^2}{2r^2}$$
Holographic metals

Calculate conductivity for the resulting charged medium [Hartnoll]

- The chemical potential breaks the 1 + 2 dim Poincare group down to $T_t \times E(2)$
- The delta function $\text{Re}[\sigma(\omega)] \propto \delta(\omega)$ reflects the translational invariance of the background → have to couple the current to heavy degrees of freedom → natural by breaking translations
1 Introduction/Motivation

2 Devising a Metal-Insulator transition in holography

3 A concrete example

4 Summary - Plans
At $T = 0$ the RN-AdS black brane becomes an interpolating solution

\[ AdS_2 \times R^2 \]

Finite entropy at $T = 0$, OK for field theories at large $N$ (?)

Finite spectral weight at $\omega = 0$, $k \neq 0$

Einstein-Maxwell-Scalar theories have solutions conformal to $AdS_2 \times \mathbb{R}^2$ with zero entropy and finite spectral weight at $\omega = 0$, $k \neq 0$ [Charmousis, Gouteraux, Kim, Kiritsis, Meyer], [Hartnoll, Shoughoulian]
In order to maintain the same low and high energy physics we need to have a UV-IR benign lattice:

- Introduce a UV relevant deformation $\mathcal{O}(x)$
- The IR operator should be irrelevant with respect to $AdS_2$
- Solve the PDEs and show that the IR remains $AdS_2 \times \mathbb{R}^2$

Natural choice for the lattice operators is to have a non-uniform chemical potential in a spatial direction:

$$\mu(x) = \mu_0 + A_0 \cos(k_Lx)$$

Can check that only involves irrelevant $AdS_2$ operators in Einstein-Maxwell theory with dimensions $\Delta_i(k_L, \mu_0) > 1/2$. 
Calculating the conductivity with respect to the background deformed by the lattice resolves the low $\omega$ delta function to a Drude peak.

- The resistivity $\rho = \text{Re}[\sigma(0)]^{-1}$ is a power law $\rho \propto T^{\Delta(k_L,\mu)}$ for small $T$ [Hartnoll, Hofman]

- Mid-infrared power law $|\sigma| \propto \omega^{-2/3} + C$ [Horowitz, Santos, Tong]
In Einstein-Maxwell the picture inferred from the $AdS_2$ spectrum $\Delta_i(k_L, \mu_0) > 1/2$ is

In order to flow to an insulator, localization effects have to be important.

The translation breaking lattice has to break the translations of the $E(2)$ group in the IR. One way is if $\Delta_i(k_L, \mu_0) < 1/2$ for some values of $k_L$. 
Technical simplification

We consider theories deformed by chemical potential $\mu$ and a lattice. To avoid having to solve PDEs:

- Consider $1 + 3$ dimensional CFTs $\Rightarrow$ 5 bulk dimensions
- Uniform potential $\mu$ breaks the Poincare group to $T_t \times E(3)$, we still need to break translations of $E(3)$
- To reduce the problem to ODEs keep the constant bulk radius slices homogeneous

$\Rightarrow$ Keep $\partial_{x_2}, \partial_{x_3}$ and $p^{-1}\partial_{x_1} + (x_2\partial_{x_3} - x_3\partial_{x_2})$

Give a helical structure to preserve a Bianchi $VII_0$ subgroup
The model

Consider model with a metric $g_{\mu\nu}$, a $U(1)$ gauge field $A_\mu$ and a (not too massive) 1-form $B_\mu$.

$$S = \int d^5x \sqrt{-g} \left( R + 12 - \frac{1}{4} F_{ab} F^{ab} - \frac{1}{4} W_{ab} W^{ab} - \frac{m^2}{2} B_a B^a \right)$$

$$- \frac{\kappa}{2} \int B \wedge F \wedge W$$

- $A_\mu$ used to deform by a uniform chemical potential
- $B_\mu$ used to introduce helical “lattice”
- CS coupling $\kappa$ helps to flow to an insulating geometry
- Will consider $m^2 = 0$
Making the consistent ansatz

\[ A = a(r)dt , \quad B = w(r)\omega_2 , \]
\[ ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + e^{2v_1(r)}\omega_1^2 + e^{2v_2(r)}\omega_2^2 + e^{2v_3(r)}\omega_3^2 \]

with the left-invariant Killing one-forms

\[ \omega_1 = dx_1 , \quad \omega_2 + i\omega_3 = e^{ipx_1}(dx_2 + idx_3) \]

yields a non-linear system of ODEs for the radial functions.

Basic ingredients for a metallic state are captured:

- **AdS\(_5\)** with \( a = w = 0, \ U = r^2 \) and \( v_i = \ln r \)
- **AdS\(_2 \times \mathbb{R}^3\)** with \( w = 0, \ a = 2\sqrt{6}r, \ U = 12r^2 \) and \( v_i = 0 \)
Close to the $AdS_5$ the boundary conditions are fixed by the deformation parameters $\mu$, $\lambda$ and $p$

\[
a = \mu + \frac{\nu}{r^2} + \cdots, \quad w = \lambda + \frac{\beta - \lambda \frac{p^2}{2} \log r}{r^2} + \cdots,
\]
\[
U = r^2 - \frac{\epsilon/3 + p^2 \frac{\lambda^2}{6} \log r}{r^2} + \cdots
\]
\[
\nu_i = \log r + \frac{g_i + s_i \frac{\lambda^2 p^2}{24} \log r}{r^4} + \cdots
\]

- In the IR of the geometry we impose boundary conditions for the existence of a regular black horizon at temperature $T$
- What are the possible zero temperature IR behaviors?
- Always $AdS_2 \times \mathbb{R}^3$ with varying $\lambda$ and $p$?
Instructive to find the spectrum of operators on $AdS_2 \times \mathbb{R}^3$. Perturb the background as

$$U = 12 r^2 (1 + \varepsilon u_1 r^\delta), \quad v_i = v_o (1 + \varepsilon v_i r^\delta), \quad a = 2\sqrt{6} r (1 + \varepsilon a_1 r^\delta)$$
$$w = \varepsilon w_1 r^\delta$$

for small $\varepsilon$ and find the values for $\delta$.

- They come in pairs $\delta^{(i)}_{\pm}(p) = -1/2 \pm \nu^{(i)}(p)$, for stable solutions $\nu^{(i)}(p) \geq 0$

- To “shoot out” from $AdS_2 \times \mathbb{R}^3$ we need to use to modes with $\delta^{(i)}_{\pm}(p) \geq 0$

- If one of the modes is relevant $\delta^{(i)}_{\pm}(p) < 0$, a flow cannot exist.
This is where the CS coupling $\kappa$ plays a central role

i) For $\kappa = 0$ we have $\delta^{(i)}(p) \geq 0$ for all $p$

ii) For $0 < \kappa \leq 1/\sqrt{2}$ we have a mode with $\delta^{(i)}(p) < 0$ for a range of $p$

iii) For $\kappa > 1/\sqrt{2}$ the mode can have complex $\delta^{(i)}(p)$ indicating a phase transition

Option iii) leads to spontaneous breaking of translations, we will go with option ii) so that $AdS_2 \times \mathbb{R}^3$ is dynamically stable.
Black holes

- Good indication that for $0 < \kappa \leq \frac{1}{\sqrt{2}}$ we could flow down to something which is not $AdS_2 \times \mathbb{R}^3$.
- Look at low temperature black holes and vary $p$.

Plot entropy density $s$ vs $p$ for $T/\mu = 10^{-3}, 10^{-4}, 10^{-5}$ and fixed lattice strength.
- Jump in the entropy indicates phase transition.
It turns out there is a symmetry breaking solution. At leading order in small $r$

$$ds_5^2 = -r^2 dt^2 + r^{-2} dr^2 + r^{-2/3} \omega_1^2 + r^{4/3} \omega_2^2 + r^{2/3} \omega_3^2$$

$$A = 0, \quad B = w_0 \omega_2$$

Strong effect from the lattice in the IR leads to a ground state which breaks translations.

It is a solution of Einstein-Maxwell, the CS coupling helps connect to $AdS_5$ in the UV.

It doesn’t have any relevant mode.

At low temperatures it leads to entropy $s \propto T^{2/3}$. 

Aristomenis Donos

Metal - Insulator transition in holography
Domain walls

- Construct solutions at strictly zero temperature

- Left solid part: Domain wall with insulating IR
- Right solid part: Domain wall with $AdS_2 \times \mathbb{R}^3$
- Dashed part: Low temperature black hole (numerically hard to get the domain walls)
- Left: AC conductivity for black holes with metallic $\text{AdS}_2 \times \mathbb{R}^3$ IR. At low $\omega$ agreement with Drude peak

- Right: AC conductivity for black holes with insulating IR. At low omega $\text{Re}[\sigma(\omega)] \propto \omega^{4/3}$

- Opposite temperature dependence of DC conductivity $\rho = \text{Re}[\sigma(0)]^{-1}$ on $T$
Resistivity

Plotting resistivity vs temperature for different lattice momenta \( p \)

- Red: Metallic phase, resistivity increases as a power law
  \[ \rho \propto T^\Delta(p) \]
- Blue: Insulating phase, resistivity decreases as a power law
  \[ \rho \propto T^{-4/3} \]
For a range of $0 < \kappa < 0.57$ there is a third scaling solution which has relevant or unstable modes and can mediate the transition. It can be either continuous of first order. [Hartnoll, Huijse]

For $\kappa > 0.57$ this solution doesn’t exist in the model and the transition is infinite order.
1 Introduction/Motivation

2 Devising a Metal-Insulator transition in holography

3 A concrete example

4 Summary - Plans
Summary - Plans

- Constructed black holes with broken translations using ODEs
- AC conductivity with a Drude peak
- Interaction driven metal-insulator transition
- It can be embedded in string theory through $D = 5$ minimal gauged SUGRA
- Explore the phase diagram more thoroughly
- Consider other theories with neutral scalars ($N = 4^+$ Roman’s theory)
- Construct insulating geometries with a gap
- Construct Mott insulators? → necessarily solve PDEs