Stability investigation of a nonlinear hydraulic actuator in terms of wave processes

V V Syrkin¹, Y F Galuza¹, I N Kvasov¹, I A Abramova²
¹Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia
²Omsk Tank-Automotive Engineering Institute, 119, st. 14th Town, Omsk, 644098, Russia

Abstract. In technological machines for various purposes, hydraulic actuators are widely used, which are constantly being improved in terms of the characteristics of power, speed and accuracy. These parameters must be stable, which requires making certain changes in the design of these actuators that allow them to withstand the operating modes of technological machines in the limits search. The authors consider results of the stability investigation of a nonlinear hydraulic actuator under the action of wave processes in its lines. They propose mathematical models of the wave process, which to some extent reflect its influence on the movement of the hydraulic motor in the form of a damping effect. For a nonlinear hydraulic actuator under the action of wave processes, a mathematical model and the results of the study of this hydraulic actuator are presented in the form of conditions, under which the stability of its operation is ensured.

1. Introduction
In hydraulic actuators of technological machines equipped with sufficiently long lines, wave processes are observed, the influence of which increases with an increase in their power and speed. The magnitude of the overpressure caused by wave phenomena, with a rapid change in the flow rate of the working fluid, and its elasticity in the pipeline can be determined analytically or graphically [1, 2]. There are studies of the influence of wave processes on the dynamic processes of a hydraulic actuator [4], which present mathematical models of wave processes in hydraulic actuators and their influence on the parameters of hydraulic systems of technological machines.

2. Problem Statement
Well-known studies are quite complex mathematical models that are represented by partial differential equations, which makes it difficult to use the results of these studies in engineering calculations. In this regard, the following tasks are set:
– to simplify the modeling of wave processes in hydraulic actuators;
– to analyze the obtained mathematical models;
– to investigate the influence of wave processes in nonlinear hydraulic actuators on the stability of their operation.

3. Theory
The effect of the reflected pressure wave can be taken into account under one of the following assumptions.
In the cavity of the hydraulic engine the reflected pressure wave $\Delta p_{ref}$ comes without loss, with time delay $\Delta t = \frac{2L}{a}$, where $L$ is the length of the pipeline; $a$ is the speed of the sound in the pipeline (the first model of the wave process).

The reflected pressure wave has a plane leading edge and its effect on the value of the wave pressure in the cavity of the hydraulic motor is reduced to a periodic decrease in the $\Delta p_{ref}(t)$ to zero at times $t_i = i \frac{2L}{a}$, where $i$ is an integer.

Studies have shown that the wave process during braking has a damping effect on the movement of the hydraulic motor, and the magnitude of the damping effect depends on the ratio $\frac{\Delta t}{T}$, where $T$ is the period of the natural undamped oscillations of the hydraulic actuator.

The damping effect of the wave process can be estimated by the amount of energy $U_w$ of the wave process spent on braking or accelerating the hydraulic motor during $t_2 - t_1$ and determined by the expression

$$U_w = \int_{t_1}^{t_2} \Delta p_w F V dt,$$

(1)

where $\Delta p_w$ is the pressure generated by the wave process in the cavity of the hydraulic motor, taking into account the forward and reflected waves; $F$ is the effective area of the hydraulic motor; $V$ is the speed of the hydraulic motor. It is obvious from the expression (1) that at negative energy $U_w$ there is a damping of vibrations, and at positive energy – an amplification.

The periodic motion of the hydraulic motor piston under the initial conditions $x_0 = 0$, $Dx_0 = v_0$ ($D$ is the differentiation operator) changes according to the law

$$x = A \sin \omega t,$$

(2)

where $\omega$ is an angular frequency, $A$ – amplitude.

For the first model of the wave process, the expression for energy

$$U_w = a \rho F \left[ \int_{t_1}^{t_2} [(V_0 - Dx)_f - 2(V_0 - Dx)_{ref}] Dx dt, \right.$$

(3)

where $\rho$ is the density of the working fluid; the indices «ref» and «f» refer to the forward and reflected pressure waves, and the pressure phase $a \rho (V_0 - Dx)_{ref}$ is shifted relative to the phase $a \rho (V_0 - Dx)_f$ for a time $\Delta t = \frac{2L}{a}$.

The calculation of the dependence of the increment $\Delta U_w$ energy wave process for one period of movement of the piston, the ratio $\frac{\Delta t}{T}$ is shown in Fig. 1. Curve 2 corresponds to the first model of the wave process. From the graph, it is obvious that in the interval $0.15 < \frac{\Delta t}{T} < 0.85$ the wave process creates a damping effect, which reaches its maximum value at $\frac{\Delta t}{T} = 0.5$. 

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Figure 1. Dependence of the energy increment $\Delta U_w$ of the wave process over the period of oscillations on the ratio $\Delta t/T$ at the stroke of the piston $A=2.66$ cm and the frequency $\omega=75$ s$^{-1}$

When using the second wave process model

$$U_w = a p F \sum_{i=1}^{i \leq i} \int \left[ \int D^2 x d t - \int \left[ D^2 x d t \right] d x d t \right], \quad (4)$$

The dependence $\Delta U_w(\Delta t/T)$ for the second model of the wave process is shown in Fig. 1 (curve 1). The maximum damping effect occurs at $\Delta t/T = 0.75$.

The amount of energy that can be used for damping of the hydraulic motor when accounting for the second model, the wave process is less than in the first model, but the values area of the $\Delta t/T$ is wider and equal to the interval 0 to 1.0. It should be noted that qualitative agreement between the curves 1 and 2 is in the interval $0.15 < \Delta t/T < 0.85$.

When using axial-piston hydraulic motors as actuating devices of technological machines, the operation of the hydraulic actuator is significantly influenced by some features inherent in this type of hydraulic motors. When studying the stability of the hydraulic actuator analytically, it is necessary to take into account friction losses in the motor and the impact of the wave processes in the pipelines if they are significantly long.

A mathematical model of this kind of hydraulic actuator can be represented as follows:

$$\dot{Q}_n - \delta_1 p_1 - K_1 s p_1 - q\Omega = 0;$$

$$q\Omega - \delta_2 p_2 - K_2 s p_2 = 0;$$

$$q(p_1 - p_2) - \varepsilon(p_1 + p_2) - (b - K_r)\Omega + (\Omega_0 - \Omega)a p q - J s \Omega = 0,$$  \(5\)

where $p_1$, $p_2$ are the pressures in the discharge and drain pipelines; $\delta_1$, $\delta_2$, $K_1$, $K_2$ are the leakage and compliance coefficients for the discharge and return pipelines, respectively; $q$ is the working volume of the hydraulic motor; $\Omega$ is the angular velocity; $(b - K_r)$ is the coefficient that takes into account damping ($b$) and negative resistance ($K_r$); $\Omega$ is the proportionality coefficient between the frictional moment and the total pressure in the cavities of the hydraulic motor; $a$ is sound speed in the discharge pipe; $p$ is the density of the working fluid; $\Omega$ is the initial angular velocity of the hydraulic motor shaft; $s$ is the differentiation operator; $J$ is the moment of inertia of the rotating parts of the hydraulic motor.
Taking into account the harmonic linearization, the coefficient of friction is transformed as $\varepsilon(\Omega) = \varepsilon(A) = \frac{4\varepsilon_0}{\pi A}$, where $A$ is the amplitude of periodic speed changes.

The general equation of the hydraulic actuator model is reduced to the form

$$ JK_1 K_3 s^3 \Omega + \{J(K_1 \delta_2 + K_2 \delta_1) + K_1 K_2 [c(p_{10} + p_{20}) + (b - K_F + apq)]\} \times $$

$$ \times s^3 \Omega + [q^2(K_1 K_2) + J_2 \delta_2 + (\delta_1 K_2 + \delta_2 K_1)](s - K_F + apq) + c \times $$

$$ \times (p_{10} + p_{20})]\} s \Omega + \{q^2(\delta_1 + \delta_2) [c(p_{10} + p_{20}) (b - K_F + apq)] + $$

$$ + q^2(\delta_1 + \delta_2 + \delta_2 \delta_1) [c(p_{10} + p_{20}) (b - K_F + apq)]\} \Omega = $$

$$ = qQ_0 (\delta_2 + K_2 s) + \Omega_0 apq (\delta_1 + K_1 s) (\delta_2 + K_2 s). $$

At the same time, the parameters $\Omega$ and $p_1 + p_2$ for small oscillations can be represented as $\Omega = \Omega_0 - \Delta \Omega$ and $p_1 + p_2 = (p_{10} + \Delta p_1) + (p_{20} + \Delta p_2)$, where $\Delta \Omega$, $\Delta p_1$ and $\Delta p_2$ are the increment of velocity and pressure. Excluding the products $\Delta \Omega \Delta p_1$ and $\Delta \Omega \Delta p_2$ as second-order quantities of the cavity, we can present a mathematical model of the hydraulic actuator in the form

$$ JK_1 k_3 s^3 \Delta \Omega + \{J(\sigma_1 k_2 + \sigma_2 k_1) + k_1 k_2 \left[ \frac{4\varepsilon_0}{\pi A} (p_{10} + p_{20}) - (b - K_F + apq) \right] \} s^2 \Delta \Omega + $$

$$ + \{J(\sigma_1 k_2 + \sigma_2 k_1) \left[ \frac{4\varepsilon_0}{\pi A} (p_{10} + p_{20}) - (b - K_F + apq) \right] + \frac{4\varepsilon_0}{\pi A} \Omega_0 q(k_1 - k_2) + q^2(k_1 + k_2)] s \Delta \Omega + $$

$$ + \{\sigma_1 \sigma_2 \left[ \frac{4\varepsilon_0}{\pi A} (p_{10} + p_{20}) - (b - K_F + apq) \right] + \frac{4\varepsilon_0}{\pi A} \Omega_0 q(\sigma_1 - \sigma_2) + q^2(\sigma_1 - \sigma_2)] \Delta \Omega = 0. $$

The characteristic equation for the hydraulic actuator model under consideration is expressed as follows

$$ B_3 s^3 \Omega + (B_2 + c \alpha)s^2 \Omega + (B_1 + c \beta)s \Omega + (B_0 + c \gamma) \Omega = 0, $$

where $B_3$ is the coefficient of $s^2 \Omega$ or $s^2 \Delta \Omega$; $B_2 + c \alpha$ is the coefficient of $s \Omega$ or $s \Delta \Omega$; $B_1 + c \beta$ is the coefficient of $s \Omega$ or $s \Delta \Omega$; $B_0 + c \gamma$ is coefficient at $\Omega$ or $\Delta \Omega$, $c = \frac{4\varepsilon_0}{\pi A}$.

To study the stability of third-order systems, we use the Hurwitz stability criterion

$$(B_2 + c \alpha)(B_1 + c \beta) \geq B_3 (B_0 + c \gamma).$$

The equation of the stability boundary takes the form

$$ \alpha \beta c^2 + (\alpha B_1 + \beta B_2 - \gamma B_3) c + (B_1 B_2 - B_0 B_3) = 0. $$

Taking into account a number of assumptions, the stability condition is reduced to the form

$$ c \geq - \frac{B_1 B_2 - B_0 B_3}{\alpha B_1 + \beta B_2 - \gamma B_3}. $$

Figure 2 shows the limits of the stability of the hydraulic actuator in the plane of its parameters, the analysis of which shows that the stable operation of the hydraulic actuator can be provided with its parameters corresponding to the Hurwitz stability criterion.
Figure 2. The stability regions of the hydraulic actuator in the plane of its parameters – at $k_1 = 0.08 \text{ cm}^2/\text{N}$ (1–6); $k_2 = 0.08 \text{ cm}^2/\text{N}$ (1, 4); $k_2 = 120 \text{ mm}^2/\text{N}$ (2, 3, 5, 6) and $p_{10} + p_{20} = 5 \text{ MPa}$ (1, 2, 3); $p_{10} + p_{20} = 10 \text{ MPa}$ (4, 5, 6), curves 1, 3, 4, 6 are constructed according to the equation (5), curves 2, 5 – according to the equation (6).

4. Conclusions
Model of friction process in the hydraulic motor, depending on the total initial pressure ($p_{10} + p_{20}$) in the cavities of the hydraulic motor, suggests that the increase of friction (increasing the ratio $\varepsilon$) facilitates the transition of a hydraulic actuator in the range of stable operation. The linearization of the friction characteristic (Fig. 2) $\Omega_c(p_1 + p_2)$ under the assumption $p_1 + p_2 \approx p_{10} + p_{20}$ gives a wider stability range (curves 3, 6) than the linearization under the assumption $\Delta\Omega_\Delta p = 0$ (curves 2, 5), so the calculations should be performed by the second type of linearization in order to increase the stability margin. The increase in flexibility of the return pipeline leads to the shift of the region of stability to the left (curves 1, 4), and with increasing initial pressure ($p_{10} + p_{20}$) the area of sustainable modes is extended (for example, curves 4, 5, 6). When you increase the slope of the curve of the “negative” friction $K_F(\Omega)$ to maintain a steady state it is necessary to increase the pressure in the cavities of the hydraulic motor, or use motors with a high friction coefficient $\varepsilon$, considering that the latter leads to a decrease in the efficiency of the hydraulic actuator.

5. References
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