Contact interaction in the nonrelativistic pion absorption operator

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Abstract

Chirally invariant Lagrangian containing nucleon, pion and chiral scalar fields is considered. A consistent scheme is proposed for obtaining the nonrelativistic operator of pion absorption by a $NN$-pair taking into account the chiral scalar exchange. An important role of the contact interaction is pointed out.

Keywords: chiral Lagrangian, pion absorption

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1. Taking into consideration a scalar meson in the effective chiral Lagrangian has led to a successful description of the three-nucleon interaction and pion production in the $NN$-collisions near threshold [1,2]. It was shown [1] that cross sections of the reactions $pp \rightarrow pp\pi^0$, $pp \rightarrow d\pi^+$ and $pp \rightarrow pn\pi^+$ near threshold can be explained taking into account a contact interaction described by the $\pi SNN$-vertex ($S$ being a chiral scalar-isoscalar meson using as a model approximation for the two-pion exchange between nucleons).

The Lagrangian describing interaction of the nucleon, pion and chiral scalar fields can be presented [2] as follows

$$L_{psN} = \overline{N}i\gamma_\mu \partial^\mu - g\overline{N}(\sigma + i\gamma_5 \tau \varphi)N - \frac{g_s}{f_\pi} \overline{SN}(\sigma + i\gamma_5 \tau \varphi)N + L_0 + L_{sb},$$

(1)

$$L_{sb} = f_\pi \mu^2 \sigma = f_\pi \mu^2 \sqrt{f_\pi^2 - \varphi^2},$$

$$L_0 = \frac{1}{2}(\partial_\mu S \partial^\mu S - m_s^2 S^2) + U(S) + \frac{1}{2}(\partial_\mu \varphi \partial^\mu \varphi + \partial_\mu \sigma \partial^\mu \sigma),$$

$N$, $\varphi$ and $S$ being nucleon, pion and chiral scalar fields, $L_{sb}$ being a part of the Lagrangian breaking the chiral symmetry, $f_\pi$ — the pion decay constant, $g$ and $g_s$ — $\pi NN$ and $SNN$ coupling constants; $m = gf_\pi$ is a nucleon mass, while $\mu$ and $m_s$ are pion and chiral scalar masses respectively; $U(S)$ describes the self-interactions of the scalar field. Lagrangian (1) describing the $\pi NN$-interaction is chirally invariant except for $L_{sb}$: at the same time it is nonlinear since pion fields are transformed in a nonlinear way by chiral transformations while nucleon fields of this Lagrangian are transformed linearly.

Let us apply a unitary transformation $u(i\gamma_5 \tau \varphi)$ to the nucleon fields of Lagrangian (1):

$$N = u(i\gamma_5 \tau \varphi) \psi, \quad u(\sigma + i\gamma_5 \tau \varphi) u = f_\pi.$$

It is easy to show that

$$u = \sqrt{\frac{f_\pi + \sigma}{2f_\pi}} \left( 1 - i\gamma_5 \frac{\tau \varphi}{f_\pi + \sigma} \right).$$

(2)

As a result instead of pseudoscalar $\pi NN$-coupling we obtain pseudovector coupling and Lagrangian (1) can be presented as follows

$$L_{pv} = \overline{\psi} i\gamma_\mu \mathcal{D}^\mu \psi - m \overline{\psi} \psi + \frac{g}{2m} \overline{\psi} \gamma_\mu \gamma_5 i\tau \mathcal{D}_\mu \varphi \psi - g_s \overline{\psi} \tau \varphi \psi + L_0' + L_{sb},$$

(3)

$$L_0' = \frac{1}{2}(\partial_\mu S \partial^\mu S - m_s^2 S^2) + U(S) + \frac{1}{2}(\mathcal{D}^\mu \varphi)^2,$$

$\mathcal{D}^\mu \varphi$ and $\mathcal{D}^\mu \psi$ being covariant derivatives of the pion and nucleon fields:

$$\mathcal{D}^\mu \varphi = \partial^\mu \varphi - \frac{1}{\sigma + f_\pi} \partial^\mu \sigma \varphi, \quad \mathcal{D}_\mu \psi = \left[ \partial_\mu + i \frac{1}{f_\pi(\sigma + f_\pi)} \tau (\varphi \times \partial^\mu \varphi) \right] \psi.$$

Since the transformation connecting $\psi$ and $N$ is a canonical one, corresponding $T$-matrix elements calculated using Lagrangians (1) and (3) will be the same for particles on the mass shell. Of course, both Lagrangians satisfy PCAC condition

$$\partial^\mu A_\mu^\alpha = f_\pi \mu^2 \varphi^\alpha,$$
\( A_\mu^{a} \) being a component of the axial current calculated by use of (1) or (3).

Expanding (1) and (3) up to terms of the order of \( (\varphi/f_\pi)^2 \) one obtains

\[
\mathcal{L}_{ps} = \overline{N(i \cdot \beta - m)N} + \frac{1}{2} \left( (\partial_\mu \varphi)^2 - \mu^2 \varphi^2 \right) - g \overline{N} i \gamma_5 \tau \varphi N + \frac{g}{2f_\pi} \overline{N} N \varphi^2 - \frac{g_s}{f_\pi} \overline{N} \left( f_\pi - \varphi^2 + i\gamma_5 \tau \varphi \right) N + \frac{1}{2} (\partial_\mu S \partial_\mu S - m_s^2 S^2) + U(S),
\]

(4)

\[
\mathcal{L}_{p} = \overline{\psi} (i \cdot \beta - m) \psi + \frac{1}{2} \left( (\partial_\mu \varphi)^2 - \mu^2 \varphi^2 \right) + \frac{g}{2m} \overline{\psi} \gamma_\mu \gamma_5 \tau \psi \partial_\mu \varphi - \frac{1}{(2f_\pi)^2} \overline{\psi} \gamma_\mu \tau \psi (\varphi \times \partial_\mu \varphi) - g_s \overline{S} \psi \psi + U(S) + \frac{1}{2} (\partial_\mu S \partial_\mu S - m_s^2 S^2). \]

(5)

To describe the \( \pi N \rightarrow \pi N \) amplitude one has to use only parts of Lagrangians (4) and (5) responsible for the interaction in \( \pi N \) sector, namely

\[
\mathcal{L}_{ps}^{\pi N} = - \frac{g_s}{2} \overline{N} i \gamma_5 \tau \varphi N + \frac{g}{2f_\pi} \overline{N} N \varphi^2,
\]

(6)

\[
\mathcal{L}_{p}^{\pi N} = \frac{g}{2m} \overline{\psi} \gamma_\mu \gamma_5 \tau \psi \partial_\mu \varphi - \frac{1}{(2f_\pi)^2} \overline{\psi} \gamma_\mu \tau \psi (\varphi \times \partial_\mu \varphi).
\]

(7)

The \( \pi N \) scattering amplitude calculated to the second order of the perturbation theory (using Lagrangians (6) and (7)) may be presented by diagrams of Fig.1. Calculations using (6) and (7) give the same result as it was expected (see e.g. [3]).

Another consequence of the Lagrangians (4) and (5) equivalency is the amplitude \( \pi N \rightarrow SN \) equality being calculated for diagrams of Fig.2 using the interaction Lagrangians

\[
\mathcal{L}_{p} = \frac{g}{2m} \overline{\psi} \gamma_\mu \gamma_5 \tau \psi \partial_\mu \varphi - g_s \overline{S} \psi \psi,
\]

(8)

\[
\mathcal{L}_{ps} = - \frac{g_s}{f_\pi} \overline{N} (f_\pi + i\gamma_5 \tau \varphi) N - g \overline{N} i \gamma_5 \tau \varphi N.
\]

(9)

The \( \pi N \rightarrow SN \) amplitude calculated using Lagrangian (8) (pseudovector coupling) and diagrams of Fig.2 is as follows

\[
T = i \frac{g}{2m} g_s \overline{u}(p') \tau_\alpha \left( \frac{p_d + m}{p_d^2 - m^2} \gamma_5 + \frac{p_x + m}{p_x^2 - m^2} \right) u(p),
\]

(10)

\( p_d = p + q, \ p_x = p' - q, \ \tau_\alpha \) is a component of the isospin operator \( \tau \).

Let us transform the amplitude (10) by means of the identities

\[
S_F(p_d) \ \gamma_5 u(p) = [1 + 2m S_F(p_d)] \gamma_5 u(p), \ \ u(p') \ \gamma_5 S_F(p_x) = u(p') \gamma_5 [1 + 2m S_F(p_x)],
\]

where \( S_F(p) = (\not{p} + m)/(p^2 - m^2) \) is a nucleon propagator, \( u(p) \) is a Dirac free spinor. Such a transformation leads to the following expression for amplitude \( T \):
which is identical to the expression one obtains in the calculation of the amplitudes of Fig. 2 using Lagrangian (9) (see also [2]).

2. Normally the pion creation operator for processes \( NN \rightarrow NN\pi\) is being calculated starting from an effective Lagrangian describing interaction of pion, nucleon and other fields. As a rule in such an approach influence of \( NN \) interaction on the precise form of creation operator is not being taken into consideration. At the same time the pion creation operator depends on the nucleon potential [4–6] when one uses a nonrelativistic equation to describe nuclear dynamics. Partial conservation of axial current (PCAC) also connects concrete form of the pion creation (absorption) operator with nonrelativistic dynamics of nucleon interaction [7]. The precise form of such an operator for a nonrelativistic case may be obtained starting from relativistic \( T \)-matrix, and the final result may turn out to be Galilean-invariant [8].

Let us consider the pion absorption amplitude on a nucleon caused by the scalar-isoscalar interaction described by Lagrangian as followed

\[
\mathcal{L}_{\text{int}} = -ig \mathcal{N} \gamma_5 \tau \varphi N - V_s \mathcal{N} N.
\]

This amplitude will correspond to diagrams \( a \) and \( b \) of Fig. 2 with a virtual \( S \)-meson and will look as two first terms of (11):

\[
T_s = ig v_s(k) \overline{\pi}(p') \tau_\alpha (S_F(p_d) \gamma_5 + \gamma_5 S_F(p_x)) u(p),
\]

\( v_s(k) \) being a vertex function, i.e. Fourier transform of the potential \( V_s \). To find the nonrelativistic limit of eq. (12) let us divide the nucleon propagator \( S_F \) into positive and negative frequency parts [1]:

\[
S_F(p) = \frac{m}{E_p} \left[ \frac{\Lambda^+(p)}{p_0 - E_p} + \frac{\Lambda^-(p)}{p_0 + E_p} \right] = S_F^+(p) + S_F^-(p),
\]

\[
S_F^+(p) = \frac{1}{2E_p} \left[ \frac{\hat{p} + m}{p_0 - E_p - \gamma_0} \right], \quad S_F^-(p) = \frac{1}{2E_p} \left[ \gamma_0 - \frac{\hat{p} + m}{p_0 + E_p} \right], \quad E_p^2 = p^2 + m^2,
\]

\( \Lambda^+(p) \) and \( \Lambda^-(p) \) being projection operators for positive and negative energy solutions of Dirac equation. Taking advantage of an expression describing nonrelativistic limit of \( \gamma_5 \)-matrix, namely

\[
\overline{\pi}(p_d) \gamma_5 u(p) \approx \frac{1}{2m} \left[ \sigma(p - p_d) + \sigma(p_d + p) \frac{\varepsilon_{pd} - \varepsilon_p}{4m} \right],
\]

\( \varepsilon_p \) being kinetic energy of the nucleon, one can obtain an expression for the amplitude corresponding to diagram \( a \) of Fig. 2:

\[
T_a \approx i \frac{g}{2m} v_s(k) \chi_f^+ \left[ \frac{-\sigma q + \omega \sigma(p_d + p)/4m}{\varepsilon_p - \varepsilon_p - \varepsilon_{pd}} - \frac{\sigma(p_d + p)}{4m} + \frac{\sigma(p + p' + q)}{2m} \right] \chi_i,
\]

(13)
χ being a two-component spinor, ω is the pion energy.

The amplitude $T_x$ of the pion absorption by the nucleon corresponding to diagram $b$ of Fig. 2 to the same approximation is as follows:

$$T_x \approx i g \frac{v_s(k)}{2m} \chi^\dagger \tau \left[ \frac{-\sigma q + \omega \sigma(p' + p_x)/4m}{\varepsilon_{p'} - \omega - \varepsilon_{px}} + \frac{\sigma(p_x + p')}{4m} - \frac{\sigma(p + p' - q)}{2m} \right] \chi_i. \quad (14)$$

Taking into consideration that $\varepsilon_{p'} = \varepsilon_p + \omega$ one can see that the first terms of the expressions (13) and (14) correspond to a matrix element of the interaction operator

$$H(x) = -\frac{g}{2m} \sigma \nabla \varphi_\alpha - \frac{g}{8m^2} \{ \sigma p, \dot{\varphi}_\alpha \},$$

$$\dot{\varphi}_\alpha = -i \omega \varphi_\alpha, \quad \varphi_\alpha = \tau_\alpha \exp(iq x),$$

$\{A, B\}$ is an anticommutator. This matrix element is taken between nonrelativistic nucleon wave functions calculated to the first approximation concerning the potential $V_s$. The rests of expressions (13) and (14) are so-called contact terms presented by Fig. 3.

The contribution of contact terms to the absorption amplitude is equal to

$$T_c = -i g \frac{v_s(k)}{2m} \chi^\dagger \tau \sigma(p - p' - q) \frac{2m}{2m} \chi_i, \quad (15)$$

and interaction $H_c(x)$ corresponding to (15) in the coordinate space has a form:

$$H_c(x) = \frac{g}{4m^2} \sigma (\varphi_\alpha \nabla V_s + 2V_s \nabla \varphi_\alpha);$$

$$T_c = \int \exp(-ip' x) H_c(x) \exp(px) dx.$$

Total contribution of $H(x) + H_c(x)$ to the pion absorption operator coincides with leading terms of the expression for nonrelativistic pseudoscalar $\pi N$ interaction obtained by means of Foldy–Wouthuysen transformation for a nucleon moving in a scalar potential [6]. It is possible to show that a similar procedure leads to the same result for leading terms in the case of pseudoscalar $\pi N$ interaction for a nucleon moving in a potential being transformed as the forth component of a four-vector.

While considering processes of pion absorption (or creation) by a nucleon pair a static propagator of exchange is often used. Such a propagator takes into account nucleon correlations and this is quite reasonable when particles heavier than the pion are involved in the process under consideration. Considering the $S$-scalar contribution, e.g. for reaction $pp \rightarrow d\pi^+$ near threshold, the authors of [1] took into account $Z$-diagrams for the $S$-scalar exchange between nucleons, and also the contact term in the $SN \rightarrow \pi N$ amplitude as it follows from Lagrangian (9). The contribution of the latter turned out to be more than one order bigger than the contribution of $Z$-diagrams.

Note that in case of the $S$-scalar exchange one should take into account a contribution of the absorption (or creation) operator arising in the nonrelativistic description of the internucleonic interaction. As it is seen from (15) and (11), near threshold, where the pion
momentum is small, the contribution of leading contact \( \pi SNN \) term must be diminished by factor 1/2 since

\[
\pi(p') \gamma_5 \frac{u(p)}{m} \approx \frac{\sigma(p - p')}{2m^2},
\]

while the contribution of \( T_c \) is proportional to \( -\sigma(p - p')/4m^2 \).

It seems interesting to estimate a contribution of the chiral scalar from the term

\[
-g_s \sqrt{N} \left( f_\pi - \frac{\varphi^2}{2f_\pi} + i\tau \varphi \gamma_5 \right) N / f_\pi
\]

of Lagrangian (4) to the amplitudes of reactions \( NN \rightarrow NN\pi\pi \) near threshold where as it was shown e.g. in [9] it is necessary to take into account excitation of Roper \( N_{11} \)-resonance.
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FIGURE CAPTIONS

Fig. 1. $\pi N$ scattering amplitudes.

Fig. 2. $\pi N \to SN$ amplitudes: a, b — for pseudovector interaction, a, b, c — for pseudoscalar interaction.

Fig. 3. Contact terms of the pion absorption operator.
Fig. 1

Fig. 2

Fig. 3