The $d^*$ dibaryon in the extended quark-delocalization, 
color-screening model

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Abstract

The quark-delocalization, color-screening model (QDCSM), extended by inclusion of a one-pion-exchange (OPE) tail, is applied to the study of the deuteron and the $d^*$ dibaryon. The results show that the properties of the deuteron (an extended object) are well reproduced, greatly improving the agreement with experimental data as compared to our previous study (without OPE). At the same time, the mass and decay width of the $d^*$ (a compact object) are, as expected, not altered significantly.
I. INTRODUCTION

Quantum Chromodynamics (QCD) is believed to be the fundamental theory of the strong interactions. High energy phenomena can be described very well by using its fundamental degrees of freedom: quarks and gluons. However the direct use of QCD for low energy hadronic interactions, for example the nucleon-nucleon ($NN$) interaction, is still impossible because of the nonperturbative complications of QCD. Quark models are therefore a useful phenomenological tool. Which of the models or which effective degrees of freedom represent the physics of QCD better remains an open question.

The traditional meson-exchange model [1–4] describes the $NN$ scattering data quantitatively very well, where the effective degrees of freedom are nucleons and mesons. The intermediate- and long-range parts of the $NN$ interaction are attributed to two-pion-exchange, usually parameterized in terms of a $\sigma$ meson, and one-pion-exchange (OPE). The short-range part is either parameterized by a repulsive core or regularized by means of vertex form factors. Such parameterizations are difficult to extend to the study of new phenomena, such as multiquark systems.

In light of its success in describing the properties of hadrons, the constituent quark model (CQM) [5], where the effective degrees of freedom are constituent quarks and gluons, has been extended to the study of the $NN$ interaction. The short-range repulsive core is successfully reproduced by a combination of the quark Pauli exclusion principle and the color hyperfine interaction. On the other hand, the intermediate- and long-range part of the $NN$ interaction cannot be accounted for in the CQM. Meson-exchange has to be invoked again; this leads to ”hybrid” models [6–8]. However, the quantitative agreement with experimental data is not as good as for traditional meson-exchange models.

Recently a new approach, the Goldstone-boson-exchange model [9], where the effective degrees of freedom are constituent quarks and Goldstone bosons, has appeared. It appears to give a rather good description of the baryon spectrum and has also been applied to the $NN$ interaction [10].
Another quark model approach, which is closer in spirit to the original CQM, the quark-delocalization, color-screening model (QDCSM) [11], has been developed with the aim of understanding the well known similarities between nuclear and molecular forces despite the obvious energy and length scale differences. The model has been applied to baryon-baryon interactions [11–15] and dibaryons [16–18]. Quantitative agreement with the experimental data on \( NN \) and \( YN \) (hyperon) scattering has been obtained [15].

Although the intermediate range attraction of the \( NN \) interaction is reproduced in the model, the long range tail is missing, similar to the case of the CQM. For example, the deuteron, a highly extended object, is not well reproduced [19]. An increased value for the color screening parameter in the QDCSM can generate enough attraction to bind the deuteron, but the radius and D-wave mixing thus obtained are both too small. Also, the attraction in \( NN \) scattering is a little bit too strong. From these effects, from the parallel to the CQM and from consideration of the coordinate space behavior of the adiabatic \( NN \) potential obtained in the model [15], we concluded that the long range part of the \( NN \) interaction was what was missing. Additionally, the most convincing result of spontaneously broken chiral symmetry is the small mass of the pion and its coupling to the nucleon. Pion exchange between nucleons is also uniquely well established by partial wave analyses of \( NN \) scattering data [20].

This paper reports a study of some effects of adding OPE to the QDCSM. This is carried out with the inclusion of a short-distance coordinate space cutoff in order to avoid, or at least, to minimize, double counting of the intermediate to short range part of meson exchange already accounted for in the model by delocalization of the quark wave functions. We first recalculate the deuteron, then apply this same addition to the \( d^* \) dibaryon. Another important application is that to \( NN \) scattering, but this is left to a future publication. Sect.II gives a brief description of the model Hamiltonian, wave functions and calculation method. The results and a discussion are presented in Sect.III.
II. MODEL HAMILTONIAN, WAVE FUNCTIONS AND CALCULATION METHOD

The details of the QDCSM can be found in Refs. [11,16] and the resonating-group calculation method (RGM) has been presented in Refs. [19,21]. Here we present only the model Hamiltonian, wave functions and the necessary equations used in the current calculation.

The Hamiltonian for the 3-quark system is the same as the usual potential model. For the six-quark system, it is assumed to be

\[ H_6 = \sum_{i=1}^{6} (m_i + \frac{p_i^2}{2m_i}) - T_{CM} + \sum_{i<j=1}^{6} \left( V_{ij}^c + V_{ij}^G + V_{ij}^\pi \right), \]

\[ V_{ij}^G = \alpha_s \frac{\mathbf{\lambda}_i \cdot \mathbf{\lambda}_j}{4} \left[ \frac{1}{r_{ij}} - \frac{\pi \delta(\mathbf{r})}{m_i m_j} \left( 1 + \frac{2}{3} \mathbf{\sigma}_i \cdot \mathbf{\sigma}_j \right) + \frac{1}{4m_i m_j} \left( \frac{3(\mathbf{\sigma}_i \cdot \mathbf{r})(\mathbf{\sigma}_j \cdot \mathbf{r})}{r^5} - \frac{\mathbf{\sigma}_i \cdot \mathbf{\sigma}_j}{r^3} \right) \right], \]

\[ V_{ij}^\pi = \theta(r - r_0) f_{\pi NN}^2 \frac{1}{r_{ij}} e^{-m_{\pi} r} \]

\[ \times \left[ \frac{1}{3} \mathbf{\sigma}_i \cdot \mathbf{\sigma}_j + \frac{3(\mathbf{\sigma}_i \cdot \mathbf{r})(\mathbf{\sigma}_j \cdot \mathbf{r})}{r^2} - \mathbf{\sigma}_i \cdot \mathbf{\sigma}_j \right] \left( \frac{1}{(\mu_{\pi} r)^2} + \frac{1}{\mu_{\pi} r} + \frac{1}{3} \right), \]

\[ V_{ij}^c = -a_c \mathbf{\lambda}_i \cdot \mathbf{\lambda}_j \begin{cases} \frac{r_{ij}^2}{1 - e^{-\mu_{\pi} r}} & \text{if } i, j \text{ occur in the same baryon orbit,} \\ \frac{1}{1 - e^{-\mu_{\pi} r}} & \text{if } i, j \text{ occur in different baryon orbits,} \end{cases} \]

\[ \theta(r - r_0) = \begin{cases} 0 & r < r_0, \\ 1 & \text{otherwise}, \end{cases} \]

where all the symbols have their usual meaning, and the confinement potential \( V_{ij}^c \) has been discussed in Refs. [18,19].

The quark-pion coupling constant \( f_{\pi NN} \) can be obtained from the nucleon-pion coupling constant \( f_{NN\pi} \) by using the equivalence of the quark and nucleon pictures of the NN interaction: If the separation between two nucleons is large, then their interaction energy can be well described by a Yukawa potential; the quark description of the same separation should lead to the same potential. For example, for the (IS)=(01) NN channel, (with the nucleon taken as a point particle,) the Yukawa potential at separation R, is

\[ V_{NN}^{\pi N} = -f_{NN\pi}^2 \frac{1}{R} e^{-\mu_{\pi} R}. \]

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Taking the nucleon as a $3q$ system, for sufficiently large separation, $R$, we can express the potential between the two $3q$ systems as

$$V_{NN}^{\pi q} = \langle [N(123)N(456)]^{IS} | V_{14}^{\pi} | [N(123)N(456)]^{IS} \rangle = -\frac{25}{9} f_{qq}^2 \frac{1}{R} e^{-\mu_R \epsilon_R \mu_R^2 b^2/2},$$

(3)

since all of the exchange terms tend to zero at large $R$. The quantity $b (\sim 0.6 \text{ fm})$ is the size parameter characterizing the quark wave functions in a nucleon; see Refs. [18,19] and below.

From the above expression, it is clear that the classic symmetry relation, $f_{qq\pi} = \frac{3}{5} f_{NN\pi}$, holds except for a small correction (since $\mu_\pi b \sim 0.4$) due to the finite size of the nucleon in the quark description.

After introducing generator coordinates to expand the relative motion wave function and including the wave function for the center-of-mass motion[1], the ansatz for the two-cluster wave function used in the RGM can be written as

$$\Psi_{6q} = A \sum_k \sum_{i=1}^{n} \sum_{L_k=0,2} C_{k,i,L_k} \int \frac{d\Omega_{S_i}}{4\pi} \prod_{\alpha=1}^{3} \psi_\alpha(S_i, \epsilon) \prod_{\beta=4}^{6} \psi_\beta(-S_i, \epsilon)$$

$$\left[ [\eta_{I_1,k_1}(B_{1k})\eta_{I_2,k_2}(B_{2k})]^{IS} Y_{L_k}(\tilde{S}_i) \right]^J [\chi_c(B_1)\chi_c(B_2)]^{[\sigma]},$$

(4)

where $k$ is the channel index. For example, for the deuteron, we have $k = 1, \ldots, 5$, corresponding to the channels $NN \ S = 1 \ L = 0$, $\Delta \Delta \ S = 1 \ L = 0$, $\Delta \Delta \ S = 3 \ L = 2$, $NN \ S = 1 \ L = 2$, and $\Delta \Delta \ S = 1 \ L = 2$. Also,

$$\psi_\alpha(S_i, \epsilon) = \left( \phi_\alpha(S_i) + \epsilon \phi_\alpha(-S_i) \right) / N(\epsilon),$$

$$\psi_\beta(-S_i, \epsilon) = \left( \phi_\beta(-S_i) + \epsilon \phi_\beta(S_i) \right) / N(\epsilon),$$

$$N(\epsilon) = \sqrt{1 + \epsilon^2 + 2\epsilon e^{S_i^2/4b^2}}.$$

$$\phi_\alpha(S_i) = \left( \frac{1}{\pi b^2} \right)^{3/4} e^{-\frac{1}{2b^2}(r_\alpha - \vec{S}_i/2)^2}$$

$$\phi_\beta(-S_i) = \left( \frac{1}{\pi b^2} \right)^{3/4} e^{-\frac{1}{2b^2}(r_\beta + \vec{S}_i/2)^2}.$$

\footnote{For details, see Refs. [19,21].}
are the delocalized single-particle wave functions used in QDCSM. The delocalization parameter, \( \epsilon \), is determined by the six-quark dynamics.

With the above ansatz, Eq. (4), the RGM equation becomes an algebraic eigenvalue equation,

\[
\sum_{j,k,L} C_{j,k,L} H_{j,k}^{L,L'} = E \sum_{j} C_{j,k,L} N_{i,j}^{L,L'}
\]

where \( N_{i,j}^{L,L'} \) and \( H_{i,j}^{L,L'} \) are the (Eq. (4)) wave function overlaps and Hamiltonian matrix elements (without the summation over \( L' \)), respectively. By solving the generalized eigenproblem, we obtain the energies of the 6-quark systems and their corresponding wave functions.

The partial width for \( d^* \) decay into the \( NN \) D-wave state is obtained by using “Fermi’s Golden Rule”,

\[
\Gamma = \frac{1}{\mathcal{I}} \sum_{M_{J_i},M_{J_f}} \frac{1}{(2\pi)^2} \int p^2 dp \, d\Omega \, \delta(E_f - E_i) |M|^2
\]

\[
= \frac{1}{\mathcal{I}} \sum_{M_{J_i},M_{J_f}} \frac{1}{32\pi^2 m_{d^*}} \sqrt{m_{d^*}^2 - 4m_N^2} \int |M|^2 d\Omega,
\]

where \( M_{J_i} \) and \( M_{J_f} \) are the spin projections of the initial and final states. The nonrelativistic transition matrix element, \( M \), includes the effect of the relative motion wave function between the two final state nucleons,

\[
M = \langle d^* | H_I | \Psi_{N_1} \Psi_{N_2} \rangle^{LS} e^{i\vec{p} \cdot \vec{R}},
\]

where \( \vec{R} \) is the relative motion coordinate of the two clusters of quarks (nucleons) and \( p = \frac{1}{2} \sqrt{m_{d^*}^2 - 4m_N^2} \) is the available relative momentum between the nucleons as determined by the energy conserving \( \delta \)-function in Eq. (7). The interaction Hamiltonian, \( H_I \), is comprised of the tensor parts of OGE and OPE.

### III. RESULTS AND DISCUSSION

Our model parameters are given in Table I. They have been fixed by matching baryon properties, except for the color screening parameter (\( \mu \) in the confining potential in Eq. (4))
which has been determined by matching the mass of the deuteron. We have examined the values 0.6 fm and 1.0 fm for the short-range cutoff of OPE. For each cutoff, the model parameters were readjusted to best match all of the available data. In all cases, the contribution of OPE to the baryon mass is not large because of the short-range cutoff. Clearly this is model dependent, as the mass of baryon comes mainly from OPE in the GBE model [9], and there is no net contribution from OPE in the hybrid model of Fujiwara [8].

Table I. Model parameters and calculated results for deuteron and $d^*$. 

|                  | $r_0 = 0.6$ fm | $r_0 = 1.0$ fm | without OPE |
|------------------|----------------|----------------|-------------|
|                  | deuteron $d^*$ | deuteron $d^*$ | deuteron $d^*$ |
| $m$ (MeV)        | 313            | 313            | 313         |
| $b$ (fm)         | 0.6010         | 0.6021         | 0.6034      |
| $a_c$ (MeV fm$^{-2}$) | 25.40          | 25.02          | 25.13       |
| $\alpha_s$       | 1.573          | 1.550          | 1.543       |
| $\mu$ (fm$^{-2}$) | 0.75           | 0.95           | 1.50        |
| mass (MeV)       | 1876           | 2186           | 1876        |
|                  | 2165           | 2165           | 2116        |
| $\sqrt{\langle r^2 \rangle}$ (fm) | 2.1           | 1.9            | 1.5         |
|                  | 1.3            | 1.3            | 1.2         |
| $P_D$            | 5.2%           | 4.5%           | 0.2%        |
| decay width (MeV) | 7.92           | 5.76           | 4.02        |

For comparison, the results of our earlier calculation [13] without OPE is also included in Table I. Clearly, the “deuteron” obtained there is not the physically correct one, due to its small size and negligible D-wave mixing, although it has the correct binding energy. By adding OPE with a cutoff, our results are significantly improved; the deuteron is now well reproduced with either cutoff scale. Our calculations show that as the cutoff decreases, the deuteron size and D-wave mixing both increase. However, one cannot decrease the cutoff to near zero as the D-wave mixing becomes too strong and the size too large. This is presumably due to the double counting with the quark delocalization effect already having accounted for the shorter distance contributions of (more off-shell) pion exchange. From
the overall good agreement, it seems that it may correct to conclude that the quark de-
localization and color screening mechanism also works well in the short and intermediate
ranges to replace the phenomenological short-range repulsive core and $\sigma$-meson or double
pion exchange contributions of more conventional models.

However, the case of the $d^*$ is quite different. The OPE changes the mass of the $d^*$ by
only a few percent and increases the $NN$ decay width by less than 4 MeV. These results
were to be expected because of the high degree of compactness of $d^*$, which can be seen from
its size, and the short-range cutoff of OPE. The combination of these two effects significantly
reduces the contribution available from OPE to the $d^*$.

In summary, we find that the deuteron can be well described in the extended QDCSM.
The quark delocalization and color screening mechanism can account for the short-range re-
pulsive core and most of the intermediate-range attraction, while the missing long-range tail
can be economically incorporated by OPE with a short-range cutoff. With new parameters
thus fixed, the properties of the $d^*$ dibaryon are minimally affected.

This research is supported by the National Science Foundation of China, the Fok Ying-
dung Educational Fund, the Natural Science Foundation of Jiangsu Province and the U.S.
Department of Energy under contract W-7405-ENG-36.
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