OBSERVATIONS ON THE HOLOGRAPHIC DUALS OF 4-D EXTREMAL BLACK HOLES

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ABSTRACT

We argue that extremal black holes of N=4 Poincaré supergravity coupled to conformal matter have a quantum-exact dual 5-d description in the maximally supersymmetric extension of the Randall-Sundrum theory. This dual is a classical, static supergravity solution describing a string with both Neveu-Schwarz and Ramond charge with respect to different antisymmetric tensor doublets. We also discuss the issue of the singularity present in a class of such supergravity solutions found by Cvetič, Lü and Pope.
1 Introduction

The 5-d Randall-Sundrum (RS) compactifications \cite{1, 2} have a holographic interpretation in terms of a dual 4-d theory consisting of dynamical gravity coupled to a strongly-interacting field theory (see \cite{3} and references therein; see also \cite{4, 5}). Its supersymmetric extension has been studied by many authors. In particular, the findings in \cite{6, 7, 8} will have a direct bearing on this paper. The N=4 extension of the model in \cite{2}, usually called RS2, allows for compelling quantitative checks due to non-renormalization theorems \cite{9, 5}. An interesting problem in this context is to find the 5-d configuration dual to a 4-d Schwarzschild black hole on the UV brane. The linearized analysis of \cite{10} has not been extended to an exact all-order solution, in spite of various attempts in the literature.

Recently, an intriguing reason for this failure has been proposed by Emparan, Fabbri and Kaloper \cite{11}, who have pointed out that the 5-d configuration dual to the Schwarzschild black hole must also reproduce its Hawking radiation \textit{at the classical level}, so that it cannot be stationary, since the 4-d black hole necessarily radiates away in an asymptotically Minkowsky background.

In any extension of the RS2 scenario with $N \geq 2$ supersymmetry, the 4-d dual possesses absolutely stable extremal black holes, that preserve some of the supersymmetry and do not Hawking-radiate. This suggests the possibility that such 4-d black holes can be represented in 5-d AdS space by classical, \textit{static} configurations. Indeed, solutions were found in \cite{7} in the N=2 case, and in \cite{8} in the N=4 case. They are strings ending on the UV brane, and extending in a straight line from the brane to the AdS$_5$ horizon \cite{1}. Their drawback is that they always have null singularities on the AdS$_5$ horizon. In this paper, we use general properties of maximally extended supersymmetry to argue that, whether or not the solutions in \cite{7, 8} are unique, \textit{any} other solution corresponding to an extremal 4-d black hole of N=4 Poincaré supergravity must describe a string ending on the UV brane. and it must also be quantum-mechanically stable.

In the next Section, we briefly recall some relevant aspects of 4-d, N=4 Poincaré supergravity, and its extremal black holes. In Section 3 we identify the duals of these black holes in N=8, AdS$_5$ supergravity \cite{12, 13}. In particular, we point out that the duals are strings carrying charges under some of the 12 antisymmetric forms of the 5-d gauged supergravity, which are doublets of $SL(2, R)$ and in the 6 of $SU(4) \sim SO(6)$. The fact that the antisymmetric forms are doublets of $SL(2, R)$ means that they define charges of both Ramond and Neveu-Schwarz type. If the string carries a single charge, it is 1/2 supersymmetric; otherwise, it is 1/4 supersymmetric. This property has a nice holographic counterpart. The antisymmetric forms are dual to the 6 graviphotons of the 4-d Poincaré theory, and 4-d black holes can carry both electric and magnetic

\footnote{By AdS$_5$ horizon we mean, of course, the horizon of the Poincaré coordinate patch.}
charges under these $U(1)$s. Black holes carrying only one type of charges are 1/2 BPS, while those with both electric and magnetic charge (under different $U(1)$s) are 1/4 BPS \cite{14,15}. This fact is yet another proof that the $SL(2)$ electric-magnetic duality in 4-d, N=4 supergravity \cite{16} maps into the $SL(2)$ symmetry of type IIB superstrings. We conclude in Section 4 with some remarks on the puzzle arising from the explicit form of the known 5-d supergravity metrics associated with BPS black holes.

2 N=4, 4-d Poincaré Supergravity and Its Extremal Black Holes

N=4 Poincaré Supergravity in four dimensions \cite{17} has two types of multiplets: gravitational, and matter.

The gravitational multiplet contains 16 bosons plus 16 fermions:

$$g_{\mu\nu}, \quad A_{\mu}^{[AB]}, \quad S, \quad \psi_{\mu\alpha}^{A}, \quad \chi_{\alpha}.$$

(1)

Here $A, B = 1,..4$ are $SU(4)$ indices. Notice that $[AB] \sim \Lambda$, where $\Lambda = 1,..6$ is a vector index of $SO(6) \sim SU(4)$. $S = i \exp(-2\phi) + a$, $\phi$ is the string dilaton, and $a$ is the RR 10-d axion.

Matter, which in what follows play no significant role, is in N=4, Lie-algebra valued Yang-Mills multiplets. In the solutions we will discuss, these fields are set consistently to zero.

The N=4 pure supergravity black holes are the axion-dilaton black holes studied in \cite{14,15,18,19}. In particular, 1/4 BPS black holes have nonzero entropy, given by the Bekenstein-Hawking formula, and their axion-dilaton scalar evolves towards a fixed value at the horizon \cite{20,22,23}. In a generic BPS black hole background of pure N=4 supergravity, the central charge (complex) $SO(6)$ vector is

$$Z_{\Lambda} = e^{K/2}(q_{\Lambda} - Sp_{\Lambda}),$$

(2)

where $q_{\Lambda}, p_{\Lambda}$ are the asymptotic charges coupled to the 6 graviphotons. $K = -\log(i\bar{S} - iS)$ is the Kähler potential of the $SU(1,1)/U(1)$ axion-dilaton sigma model. $Z_{\Lambda}$ changes by an overall phase under the $SL(2,\mathbb{Z})$ symmetry of type IIB 10-d supergravity.

The eigenvalues $Z_{1}, Z_{2}$ of the central-charge matrix are given by \cite{23,24}

$$|Z_{1,2}|^2 = \frac{1}{2}(|Z_{1}|^2 + |Z_{2}|^2) \pm \frac{1}{4} \sqrt{p^2 q^2 - (p \cdot q)^2},$$

$$|Z_{1}|^2 + |Z_{2}|^2 = \frac{1}{2} Z_{\Lambda} Z^{\Lambda} = \frac{1}{4} [e^{2\phi} q^2 + e^{-2\phi} p^2 + e^{2\phi} (a^2 p^2 - 2ap \cdot q)]$$

(3)

The modulus $|Z_{1}|$ is the ADM mass (in Planck units) of the extremal black hole. If $p^2 q^2 = (p \cdot q)^2$, $|Z_{1}| = |Z_{2}|$, and the resulting configuration is 1/2 BPS and has zero entropy. If $p^2 q^2 > (p \cdot q)^2$, \ldots
the resulting configuration is $1/4$ BPS, and it has a nonzero entropy given by the attractor equation \[20, 21, 22, 23\]

\[ S = \frac{1}{4} A = \pi |Z_1|^2, \]  

where $Z_1$ is the value of $Z_1$ at the stationary point

\[ \frac{\partial}{\partial a} Z_1 = \frac{\partial}{\partial \phi} Z_1 = 0. \]  

These equations fix the value of the dilaton and the axion to

\[ a^* = \frac{p \cdot q}{p^2}, \quad e^{-2\phi^*} = \frac{1}{\sqrt{p^2 q^2 - (q \cdot p)^2}}. \]  

Substituting these values into Eq. (4), we arrive at

\[ S = \frac{1}{2} \pi \sqrt{p^2 q^2 - (q \cdot p)^2}. \]  

We note that by an $SO(6)$ rotation, followed by an $SL(2, R)$ transformation, we can set

$q_\Lambda = (q, 0, 0, ..., 0), \quad p_\Lambda = (0, p, 0, ..., 0)$,

which gives $\exp(-2\phi^*) = |q/p|, \quad a^* = 0$. Then, we can re-write the equation for $|Z_1|$ as

\[ |Z_1| = \frac{1}{2 \sqrt{2}} (e^\phi |q| + e^{-\phi} |p|). \]

### 3 The 5-d Duals of 4-d Extremal Black Holes

The $1/2$ and $1/4$ BPS black holes we have briefly described here are extremal, so that they do not decay by Hawking radiation. Since they also possess some residual supersymmetry, they belong to short multiplets of $N=4$, and are thus completely stable, both classically and quantum mechanically. Therefore, their 5-d duals must be classical, static configurations of $AdS_5$ supergravity.

To identify these configuration, we must first understand which 5-d fields give rise to the graviphotons. The crucial observation is that they cannot be the $SU(4)$ 5-d gauge fields, since they have no zero mode, and, moreover, they do not transform under the $SU(4)$ symmetry as the 4-d graviphotons. As we noticed previously, the graviphotons of 4-d Poincaré supergravity belong to the 6 of $SO(6)$. Moreover, their corresponding electric and magnetic fields are doublets under $SL(2, R)$. These are precisely the quantum numbers of the 12 antisymmetric tensors of the gauged $N=8$ supergravity in $AdS_5$\[3\]. It is therefore natural to identify the 4-d graviphotons

\[3\]The antisymmetric tensors obey first-order equations, so that they can be naturally identified with 4-d field strengths, as it is required by the $SL(2, R)$ symmetry \[4\].
with the zero modes of these antisymmetric tensors. This identification is indeed the correct one, as shown in [7, 8] for the N=4, AdS$_5$ reduction, and in [8] for the N=8 reduction.

Since antisymmetric fields couple to strings, instead of point particles, it is obvious that the 5-d configuration dual to extremal black holes must be strings. The ones corresponding to regular, nonzero-entropy black holes must carry charge under at least two antisymmetric forms, one of NS and the other of R type.

The form of the N=8 Poincaré superalgebra, and its possible AdS$_5$ extension, also implies that the 5-d strings are in the 6 of SO(6) [25]. To see this, we write the relevant part of the 4-d super-Poincaré algebra, namely the anticommutator of two same-chirality supercharges

$$\{Q^A_{\alpha}, Q^B_{\beta}\} = \epsilon_{\alpha\beta} Z^{[AB]}, \quad \alpha, \beta = 1, 2.$$  (9)

This expression corresponds to the 5-d anticommutator

$$\{Q^A_{\alpha}, Q^B_{\beta}\} = (\gamma_{\mu} C)_{ab} Z^{[\mu AB]} , \quad a, b = 1, .., 4, \quad \mu = 0, .., 4.$$  (10)

This equation shows as promised that 5-d strings are in the antisymmetric representation of SU(4).

Supergravity solutions corresponding to 5-d string have been found in [8]. In that paper, it was shown that it is possible to lift the SL(2, Z) black holes of N=4 Poincaré supergravity to full solution of the equations of motion of N=8, AdS$_5$ supergravity (indeed, to solutions of 10-d type IIB supergravity). The significance of those solutions has been questioned, since they exhibit null singularities at the AdS$_5$ horizon.

What is clear from our analysis of the R-symmetry of the 4-d Poincaré theory, is that any 5-d configuration corresponding to a BPS black hole must be a string, perpendicular to the UV brane. The fact that these configurations have a source extending into the 5-d bulk may seem strange; nevertheless, it does not contradict any property of the holographic duality, and the R-symmetry properties of the graviphotons require it to be so. Before discussing further the explicit solutions of refs. [7, 8], we must notice that Eq. (5) can be exactly reinterpreted as the extremization of the 5-d string tension [26], since, schematically

$$L \times \text{String Tension in 5d} \sim \text{BH Mass in 4-d.}$$  (11)

Here $L$ is the radius of curvature of AdS$_5$.

In order to have 1/4 BPS states it is essential that the 5-d two-forms are nonsinglets under the R-symmetry. Therefore, unlike the case of 10 dimensions [27], $(p, q)$-strings can also be 1/4 BPS, instead of just 1/2 BPS.

We conclude this Section by pointing out that unlike 5-d Poincaré supergravity, AdS$_5$ supergravity does not allow for duality transformations exchanging antisymmetric forms with vectors.
The Singularity at the IR Horizon

The known black strings solutions corresponding to 4-d, 1/4 BPS black holes have 5-d metric and dilaton given by

$$ds^2 = \frac{L^2}{z^2} \left[ dz^2 - (H_1 H_2)^{-1} dt^2 + H_1 H_2 (dr^2 + r^2 d\Omega_2^2) \right],$$

$$e^{-2\phi} = \frac{H_1}{H_2}, \quad H_1 = \frac{1}{g_s} + \frac{p}{r}, \quad H_2 = g_s + \frac{q}{r}. \tag{12}$$

Here $d\Omega_2^2$ is the standard metric element on the unit-radius 2-sphere, and $g_s$ is the string coupling constant, i.e. the asymptotic value of $\exp(\phi)$ away from the string.

This metric has a null singularity at $z = \infty$. Since we argued in the previous Sections that all 1/4 BPS black holes have stable, classical $AdS_5$ duals describing strings extending along the coordinate $z$ through the horizon at $z = \infty$, we are faced with two possibilities: either there exist more complicated string solutions besides Eq. (12), or the null singularity at $z = \infty$ can be made meaningful in the context of the holographic duality.

Since Eq. (12) admits Killing spinors, it seems at least plausible that it is unique. As we have argued, since the 5-d solution describes a string, it must have anyway a cylindrical horizon, extending all the way to $z = \infty$. The possibility exists, therefore, that whatever the 5-d duals of 1/4 BPS black holes are, they do have a naked singularity at $z = \infty$. That this is not a disaster may be argued using the holographic duality. In the 4-d picture, indeed, the null singularity describes a pathology arising only in the extreme infrared region of the theory. Because of the UV/IR duality, $z = \infty$ corresponds to infinite wavelength. This means that the singularity may simply reflect a pathology unaccessible to any local observer. We may also recall that even worse naked singularities do arise in $N=8, AdS_5$ supergravity solutions describing RG flows to non-conformal field theories (see for instance [28, 29]). These singularities reflect the existence of physics beyond the supergravity approximation, rather than a pathology of the RG flow itself. Indeed, in some cases, their resolution is explicitly known [30, 31].

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