Faster-than-Nyquist Non-Orthogonal Frequency-Division Multiplexing for Visible Light Communications

Ji Zhou, Yaojun Qiao, Tiantian Zhang, Jinlong Wei, Qixiang Cheng, Qi Wang, Zhanyu Yang, Aiying Yang, and Yueming Lu

Abstract

In this paper, we propose a faster-than-Nyquist (FTN) non-orthogonal frequency-division multiplexing (NOFDM) scheme for visible light communications (VLC) where the multiplexing/demultiplexing employs the inverse fractional cosine transform (IFrCT)/FrCT. Different to the common fractional Fourier transform-based NOFDM (FrFT-NOFDM) signal, FrCT-based NOFDM (FrCT-NOFDM) signal is real-valued which can be directly applied to the VLC systems without the expensive upconversion. Thus, FrCT-NOFDM is more suitable for the cost-sensitive VLC systems. Meanwhile, under the same transmission rate, FrCT-NOFDM signal occupies smaller bandwidth compared to OFDM signal. When the bandwidth compression factor $\alpha$ is set to 0.8, 20% bandwidth saving can be obtained. Therefore, FrCT-NOFDM has higher spectral efficiency and suffers less high-frequency distortion compared to

This work was supported in part by National Natural Science Foundation of China (61427813, 61331010, 61475094, 61271192); State’s Key Project of Research and Development Plan (2016YFB0800302); BUPT Excellent Ph.D. Students Foundation; China Scholarship Council Foundation. (Corresponding author: Yaojun Qiao.)

Ji Zhou, Yaojun Qiao, and Tiantian Zhang are with the State Key Laboratory of Information Photonics and Optical Communications, School of Information and Communication Engineering, Beijing University of Posts and Telecommunications (BUPT), Beijing 100876, China (e-mail: zhouji@bupt.edu.cn and qiao@bupt.edu.cn).

Jinlong Wei and Qixiang Cheng are with the Centre for Photonic Systems, Electrical Engineering Division, Department of Engineering, University of Cambridge, Cambridge, CB3 0FA, U.K.

Qi Wang is with Electronics and Computer Science, University of Southampton, Southampton SO17 1BJ, U.K.

Zhanyu Yang is with the Department of Electrical and Computer Engineering, University of Virginia, 351 McCormick Road, Charlottesville, Virginia 22904, USA.

Aiying Yang is with the School of Optoelectronics, Beijing Institute of Technology, Beijing 100081, China.

Yueming Lu is with the Key Laboratory of Trustworthy Distributed Computing and Service, Ministry of Education, Beijing University of Posts and Telecommunications (BUPT), Beijing 100876, China.
OFDM, which benefits the bandwidth-limited VLC systems. As the simulation results show, bit error rate (BER) performance of FrCT-NOFDM with $\alpha$ of 0.9 or 0.8 is better than that of OFDM. Moreover, FrCT-NOFDM has a superior security performance. In conclusion, FrCT-NOFDM shows great potential for application in the future VLC systems.

Index Terms

Faster-than-Nyquist (FTN) signal, non-orthogonal frequency-division multiplexing (NOFDM), fractional cosine transform (FrCT), high spectral efficiency, visible light communications (VLC).

I. INTRODUCTION

Recently, visible light communications (VLC) systems have been proposed to provide high-speed network access for office, shop center, warehouse, and airplane because of many advantages such as the lost-cost front-ends, unregulated huge frequency resources, and no electromagnetic interference [1]–[4]. As a potential access option for the future 5G wireless systems, VLC systems is gaining extensive attention [5]–[7]. However, there are many obstacles for the practical VLC systems. For instance, the data rate of VLC systems is limited by the bandwidth of the light-emitting diodes (LEDs) and multipath effect [8]–[10]. The multipath effect gives rise to the frequency-selective power fading which seriously limits the effective bandwidth when the delay spread is large. How to transmit more data on the limited bandwidth is a difficult problem in VLC systems.

In order to fully utilize the limited bandwidth of VLC systems, the modulation schemes with high spectral efficiency should be employed. Orthogonal frequency-division multiplexing (OFDM) is a well-known modulation scheme with high spectral efficiency, which has been widely investigated for VLC systems [11]–[15]. As we know, VLC is an intensity-modulation/direct-detection (IM/DD) optical system. Therefore, OFDM signal for the VLC systems needs to be real-valued and unipolar. For widely-used discrete Fourier transform-based OFDM (DFT-OFDM), Hermitian symmetry should be used to generate the real-valued signal. To obtain the unipolar signal, there are two popular approaches: one is DC-biased optical OFDM (DCO-OFDM) and the other is asymmetrical clipping optical OFDM (ACO-OFDM) [16]–[18]. In DCO-OFDM, a biasing and clipping operations can be employed to make the bipolar OFDM signal unipolar. In ACO-OFDM, only odd subcarriers are utilized to transmit data, whereby the negative part of the bipolar OFDM signal is redundant. Therefore, the bipolar OFDM signal can be converted
into an unipolar signal by clipping the negative part. DCO-OFDM and ACO-OFDM have their respective advantages and disadvantages and thus we can select the suitable one depending on the application scenario.

Recently, non-orthogonal frequency-division multiplexing (NOFDM) systems have been proposed in both wireless and optical communications to further improve the spectral efficiency by compressing the subcarrier spacing [19]–[24]. In conventional NOFDM for IM/DD systems, the multiplexing usually employs inverse fractional Fourier transform (IFrFT) [19], [20], so that the subcarrier distribution in NOFDM is different from that in OFDM, thus the real-valued signal cannot be generated in FrFT-based NOFDM (FrFT-NOFDM) by employing Hermitian symmetry [23]. In general, for the complex-valued NOFDM signal, the upconversion is required to implement intensity modulation [19], [20]. There are two methods to implement the upconversion: analog method and digital method. In the analog method [19], the digital complex-valued NOFDM signal requires two-channel digital-to-analog converters (DACs) to generate the analog signal. Then, an in-phase/quadrature (I/Q) mixer with a radio frequency (RF) waveform is required to combine two-channel analog signals and up-convert to intermediate frequency. The two DACs, I/Q mixer, and RF waveform are uneconomical for the cost-sensitive VLC systems. In the digital method [20], the upconversion can be implemented by the digital signal processing which can avoid some expensive analog devices but requires the high-bandwidth DAC. In both analog method and digital method, the generated intermediate-frequency signal has higher bandwidth compared to the baseband signal, which requires the electrical, electro-optic and photoelectric devices with high electrical bandwidth. The high-bandwidth devices are expensive for the cost-sensitive VLC systems.

In our previous work, we proposed the first real-valued faster-than-Nyquist (FTN) NOFDM schemes which have been experimentally demonstrated in the fiber-optic communications [23], [24]. FTN signal was first proposed by Mazo in 1975, which can achieve the bandwidth narrower than the Nyquist frequency and the symbol rate faster than the Nyquist rate [25]. The real-valued FTN NOFDM requires no upconversion and has higher spectral efficiency compared to OFDM for the IM/DD systems. Under the same transmission rate, the real-valued FTN NOFDM signal occupies smaller bandwidth compared to OFDM signal, which is promising in bandwidth-limited VLC systems. In this paper, to the best of our knowledge, we first apply FTN NOFDM signal to VLC systems and study its performance in detail.

The main contributions of this paper are as follows:
A novel FTN fractional cosine transform-based NOFDM (FrCT-NOFDM) signal for VLC systems: FrCT-NOFDM signal can achieve a symbol rate faster than Nyquist rate thereby having the high spectral efficiency. Meanwhile, the FrCT-NOFDM signal is real-valued. Therefore, it is very suitable for the bandwidth-limited and cost-sensitive VLC systems.

A statistical characteristic of FrCT-NOFDM signal: We verify that FrCT-NOFDM samples can be approximated as independent identically distributed (i.i.d.) Gaussian random variables. Under this condition, the statistical characteristic of DC-biased FrCT-NOFDM is derived.

A detailed analysis for the performance of FrCT-NOFDM on the optical-wireless channel: The bit error rate (BER) performance of FrCT-NOFDM is comprehensively analyzed under the different bandwidth compression factor, root-mean-square (RMS) delay spread, DC bias, and iterative number. Meanwhile, we demonstrate that FrCT-NOFDM has the superior security performance.

The rest of this paper is organized as follows. We demonstrate the principle of FTN FrCT-NOFDM for the VLC systems in Section II. In Section III, we give the statistical characteristic of FrCT-NOFDM signal. In Section IV, we analyze the optical-wireless channel model and noise model. In Section V, we implement the simulations and give the simulation results for studying the performance of FrCT-NOFDM in the VLC systems. Finally, the paper is concluded in Section VI.

II. PRINCIPLE OF FTN FRCT-NOFDM FOR VLC SYSTEMS

In this section, we demonstrate the principle of FTN FrCT-NOFDM for VLC systems.
Fig. 2. Sketched spectra of DCT-OFDM (i.e., $\alpha = 1$) and FrCT-NOFDM when the number of subcarriers is set to 4.

A block diagram of FTN FrCT-NOFDM for VLC systems is depicted in Fig. 1. Different from the FrFT-NOFDM, the inverse FrCT (IFrCT)/FrCT algorithm is employed to realize the multiplexing/demultiplexing processing in FrCT-NOFDM. The $N$-order IFrCT and FrCT are defined as

$$x_n = \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} W_k X_k \cos \left( \frac{\pi \alpha (2n + 1) k}{2N} \right), \quad (1)$$

$$X_k = \sqrt{\frac{2}{N}} W_k \sum_{n=0}^{N-1} x_n \cos \left( \frac{\pi \alpha (2n + 1) k}{2N} \right) \quad (2)$$

where $0 \leq n \leq N - 1$, $0 \leq k \leq N - 1$,

$$W_k = \begin{cases} 1 & k = 0 \\ \frac{1}{\sqrt{2}} & k = 1, 2, \cdots, N - 1 \end{cases} \quad (3)$$

and $\alpha$ is the bandwidth compression factor. The $\alpha$ less than 1 determines the level of the bandwidth compression. IFrCT is a real-valued transform, thus if the input of IFrCT is a real-valued constellation such as $M$ pulse-amplitude modualtion ($M$-PAM), the output is a real-valued NOFDM signal. Therefore, FrCT-NOFDM needs no Hermitian symmetry to generate the real-valued signal. Moreover, for IM/DD system, the upconversion is not required in FrCT-NOFDM signal.

As a special case, Eq. (2) is the Type-II discrete cosine transform (DCT) in which the matrix is orthogonal when $\alpha$ is set to 1. The Type-II DCT is probably the most commonly used form,
which is often simply referred to as “the DCT” [26]. DCT-based OFDM (DCT-OFDM) signal for IM/DD systems can be generated by Type-II DCT and has the same BER performance compared to frequently-used DFT-OFDM signal [27], [28].

Figure 2 shows the sketched spectra of DCT-OFDM (i.e., \( \alpha = 1 \)) and FrCT-NOFDM when the number of subcarriers is set to 4. The subcarrier spacing in DCT-OFDM is equal to half of the symbol rate per subcarrier, which is half of that in DFT-OFDM. In DCT-OFDM, all the subcarriers locate on the positive frequency region and their mirror images fall on the negative frequency region. However, in DFT-OFDM, one half of subcarriers locate on the positive frequency region and the other half locate on the negative frequency region. As shown in our previous work [23], when all the subcarriers are used, the bandwidth of FrFT-NOFDM can not be compressed until its subcarrier spacing is compressed below half of that in DFT-OFDM. In the existing FrFT-NOFDM signals [19], [20], in order to compress the bandwidth once the subcarrier spacing is compressed, the subcarriers on the negative frequency region are not used to carry data, which is inefficient in terms of spectrum so that the existing FrFT-NOFDM cannot achieve the symbol rate faster than Nyquist rate. However, when all the subcarriers are valid, the bandwidth of FrCT-NOFDM can be compressed once its subcarrier spacing is smaller than that in DCT-OFDM.

The subcarrier spacing of DCT-OFDM is equal to \( 1/2T \) where \( T \) denotes the time duration of one DCT-OFDM symbol [29]–[32]. Due to the compression of subcarrier spacing, the subcarrier spacing of FrCT-NOFDM is equal to \( \alpha/2T \). When the number of subcarriers, \( N \), is large enough, the bandwidth of FrCT-NOFDM can be defined as

\[
B = \frac{\alpha(N - 1)}{2T} + \frac{1}{T} \approx \frac{\alpha N}{2T}. \tag{4}
\]

The bandwidth of FrCT-NOFDM decreases with the decreasing of the \( \alpha \). When \( \alpha \) is set to 0.8, 20% bandwidth saving is obtained. Therefore, FrCT-NOFDM has higher spectral efficiency compared to DCT-OFDM. As we know, Nyquist rate is equal to the twice of \( B \),

\[
R_N = \frac{\alpha N}{T}. \tag{5}
\]

The symbol rate of FrCT-NOFDM is equal to

\[
R_S = \frac{N}{T}. \tag{6}
\]

The symbol rate of FrCT-NOFDM is 25% faster than the Nyquist rate when the \( \alpha \) is set to 0.8. Therefore, FrCT-NOFDM can be considered as an FTN signal.
III. STATISTICAL CHARACTERISTIC OF FRCT-NOFDM SIGNAL

As we know, DCT-OFDM signal has the approximately Gaussian distribution [27]. In this section, we give analysis of the statistical characteristic for the FrCT-NOFDM signal.

FrCT-NOFDM signal can be considered as a sum of a group of independent identically distributed (i.i.d.) samples on the condition that the size of FrCT is large enough. On the basis of the central limit theorem, FrCT-NOFDM signal has the approximately Gaussian distribution with zero mean. Therefore, we can model the FrCT-NOFDM signal $x_n$ using i.i.d. Gaussian process with the following probability density function (PDF),

$$pdf(x) = N(x; 0, E[|x_n|^2]) \quad (7)$$

with

$$N(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (8)$$

where $\mu$ and $\sigma^2$ are the mean and variance, respectively, of the Gaussian distribution. As the solid lines in Fig. 3 depict, the statistical characteristic of FrCT-NOFDM signal is simulated when the size of FrCT is set to 256. FrCT-NOFDM signal is normalized such that its variance is equal to one. The dash lines in Fig. 3 depict the theoretical results of PDF and cumulative distribution function (CDF) of the standard normal distribution. Obviously, the simulated results of PDF and CDF agree well with the theoretical results.
Figure 4 reveals the complementary cumulative distribution function (CCDF) of peak-to-average power ratio (PAPR) for DCT-OFDM (i.e., $\alpha = 1$) and FrCT-OFDM when the subcarrier number is set to 256. The CCDF curves of FrCT-OFDM agree well with that of DCT-OFDM, thus FrCT-NOFDM has the same PAPR with DCT-OFDM. Similar to OFDM signal, high peak-to-average power ratio (PAPR) is a serious problem in FrCT-NOFDM.

For VLC systems, the transmitted signal should be real-valued and unipolar. FrCT-NOFDM signal is real-valued but bipolar. A biasing and clipping operations can be used to make the bipolar FrCT-NOFDM signal unipolar. Without loss of generality, we employ the single-sided clipping operation in VLC systems. The generated DC-biased FrCT-NOFDM signal can be defined as

$$ s_n = \begin{cases} 
    x_n + B_{DC}, & x_n \geq -B_{DC} \\
    0, & x_n < -B_{DC} 
\end{cases} $$

(9)

where $B_{DC}$ is DC bias which is related to the power of $x_n$,

$$ B_{DC} = k \sqrt{E\{x_n^2\}}. $$

(10)

The size of DC bias can be defined as the power ratio of $s_n$ to $x_n$ in dB, i.e. $10 \times \log_{10}(k^2 + 1)$ dB.
The PDF of the DC-biased FrCT-NOFDM signal $s_n$ is defined as

$$\text{pdf}_{s}(x) = \begin{cases} N(x; B_{DC}, \sigma^2), & x \geq 0 \\ Q\left(\frac{B_{DC}}{\sigma}\right)\delta(x), & x = 0 \end{cases}$$

(11)

where $\delta(x)$ is the Dirac delta function with an unit impulse at $x$ only and the well-known $Q$ function is defined as the integral over the PDF of the standard normal distribution,

$$Q(v) = \int_{v}^{\infty} N(\tau; 0, 1)d\tau = 1 - \Phi(v).$$

(12)

In general, the electrical power $P_e$ is proportional to $E[s_n^2]$. Without loss of generality, we define $P_e = E[s_n^2]$. Thus, the electrical power $P_e$ of DC-biased FrCT-FOFDM is given by

$$P_e = E[s_n^2] = \int_{-\infty}^{\infty} x^2 \cdot \text{pdf}_{s}(x)dx = \sigma^2\Phi\left(\frac{B_{DC}}{\sigma}\right) + B_{DC}^2\Phi\left(\frac{B_{DC}}{\sigma}\right) + \frac{\sigma B_{DC}}{\sqrt{2\pi}} e^{-\frac{B_{DC}^2}{2\sigma^2}}.$$  

(13)

As Eq. (13) shows, the electrical power of DC-biased FrCT-NOFDM signal contains three parts: the power of the useful signal, DC bias, and clipping noise. The clipping operation induces the distortions including the attenuation of $x_n$ and clipping noise. Therefore, the useful signal can be written as $\eta \times x_n$ where $\eta$ is the attenuation factor, which is equal to $\sqrt{\Phi\left(\frac{B_{DC}}{\sigma}\right)}$. With the increasing of $B_{DC}$, the attenuation $\eta$ approaches to 1 and the power of clipping noise decreases to 0, but the power of DC bias increases. Therefore, when $B_{DC}$ is sufficiently large, Eq. (13) is simplified to

$$P_e = \sigma^2 + B_{DC}^2.$$  

(14)

A large $B_{DC}$ can be used to eliminate the clipping-induced distortion, but it is inefficient in terms of the signal power because $B_{DC}$ cannot carry any information. For application in VLC systems, we need to employ a suitable $B_{DC}$ to make bipolar FrCT-NOFDM unipolar.

**IV. OPTICAL-WIRELESS CHANNEL MODEL AND NOISE MODEL**

In this section, we analyze the optical-wireless channel model and noise model for VLC systems.

The optical-wireless channel can be modeled as a linear baseband system. The received signal can be given as

$$r(t) = h(t) \ast x(t) + n(t)$$

(15)
where $h(t)$ is the channel impulse response, $x(t)$ is the transmitted signal, $n(t)$ is the noise component, and $\ast$ denotes the convolution operation.

The channel impulse response $h(t)$ can be defined as

$$h(t) = \sum_{n=0}^{N_D} h_n \delta(t - n\Delta\tau) \quad (16)$$

where $N_D$ is the number of paths, $h_n$ is the channel coefficient and $n\Delta\tau$ is the delay of the $n^{th}$ path.

In general, the optical-wireless channel models fall into two categories: directed and non-directed (i.e. diffused) models [33]–[35]. In the directed model, line of sight (LOS) plays the major role. The directed model can be appropriately considered as an additive white Gaussian noise (AWGN) channel model. In the non-directed model, the optical power propagates along various paths with different lengths, which causes the multipath effect. In this paper, we employ the non-directed model to investigate the influence of the multipath effect on FrCT-NOFDM. The multipath effect is usually caused by two ways: one is the multiple-reflection light and the other is the single-reflection light. The exponential-decay (ED) and ceiling-bounce (CB) models were proposed to model both multiple reflection light and single reflection light [33]. The channel impulse response of the ED model can be defined as

$$h_{ed}(t) = \frac{1}{2D} e^{-\frac{t}{2D}} u(t) \quad (17)$$

where $D$ is the RMS delay spread of the multiple reflections and $u(t)$ is the unit step function.

The channel impulse response of the CB model is given by

$$h_{cb}(t) = \frac{6a^6}{(t + a)^7} u(t) \quad (18)$$

where $a = 12\sqrt{\frac{11}{13}}D$.

Figure 5 shows the transfer function of the ED and CB models. When the RMS delay spread is set to 1.5 ns, the 3-dB bandwidth of ED model and CB model is 33.7 MHz and 31.4 MHz, respectively. Therefore, the 3-dB bandwidth of the ED model is slightly wider than that of the CB model for the same RMS delay spread. The 3-dB bandwidth of the CB model is 20.9 MHz, 11.7 MHz, and 8.3 MHz when the RMS delay spread is set to 3 ns, 6 ns, and 9 ns, respectively. The 3-dB bandwidth of the CB model decreases with the increasing of the RMS delay spread. The multipath effect causes the frequency-selective power fading which seriously limits the effective bandwidth. For VLC systems, there are many other complex channel models which are
derived from the non-directed model by considering many other conditions such as the position of LED and photodiode (PD) and the field of view of the PD [36]–[38]. In this paper, we aim to investigate the performance of FrCT-NOFDM signal influenced by the multipath effect. Without loss of generality, we can employ the CB model to achieve the aim in the simulation.

In VLC systems, there are two dominant noise components: photon noise and receiver circuit thermal noise [34], [35]. They are both independent of the transmitted signal and can be modeled as white Gaussian distribution. Therefore, we can model the total noise $n(t)$ as the Gaussian and signal-independent distribution in the simulation.

V. SIMULATION SETUPS AND RESULTS

In this section, we will present the simulations to validate the performance of FrCT-NOFDM for VLC systems.

A. Simulation Setups

In this section, we will introduce the simulation setups and the key algorithms for eliminating the distortions.

For VLC systems, the simulation of FrCT-NOFDM is implemented based on the block diagram shown in Fig. [1]. The encoding block mainly consists of the real constellation mapper, 256-point
IFrCT, and cyclic prefix (CP) addition. In our simulation, the modulated constellation employs the 2-PAM. The CP is 1/16 of the symbol duration, in which 16 samples are used. In one frame, 256 FrCT-NOFDM symbols and 10 training symbols are transmitted. Eight frames are used to calculate the BER. A suitable DC bias is required to make the FrCT-NOFDM signal unipolar. The transmission rate of the generated FrCT-NOFDM signal is set to 100 Mbit/s.

As shown in Section IV, the optical-wireless channel employs the CB model and the adding noise is Gaussian and signal-independent in the simulation. Fig. 6 shows the spectra for the DCT-OFDM and FrCT-NOFDM signals with the transmission rate of 100 Mbit/s after the optical-wireless channel. The dark line denotes the transfer function of CB model with 3-ns RMS delay spread. After the optical-wireless channel, the signals suffer the high-frequency distortion. The bandwidth of DCT-OFDM signal is equal to 50 MHz, which is half of the transmission rate. The bandwidth of FrCT-NOFDM signal is compressed due to the compression of subcarrier spacing, thus it is smaller than that of DCT-OFDM. The bandwidth of FrCT-NOFDM is 45 MHz, 40 MHz, and 35 MHz when the $\alpha$ is set to 0.9, 0.8, and 0.7, respectively. Therefore, FrCT-NOFDM occupies smaller bandwidth and thus achieves higher spectral efficiency compared to DCT-OFDM.

The decoding block mainly consists of the CP removal, channel equalization, 256-point FrCT, iterative detection (ID) algorithm, and constellation demapper. The channel estimation and
frequency-domain equalization are implemented to compensate the channel distortion. The non-orthogonal subcarriers give rise to the inter-carrier interference (ICI) which seriously deteriorates the BER performance. The ID algorithm can be employed to eliminate the ICI.

Figure 7 depicts the block diagram of channel estimation and frequency-domain equalization for FrCT-NOFDM. In the channel estimation, the training symbols are employed to estimate the channel characteristic. The received training symbols are sent to the DFT module to obtain its frequency-domain symbols $T'$. The channel matrix $H$ can be calculated by $T'/T$ where $T$ is the transmitted training symbols in the frequency domain. In the frequency-domain equalization, the received payload symbols are firstly transformed from the time domain to the frequency domain. The frequency-domain payload symbols $X'$ are divided by the channel matrix $H$ to realize the equalization. After equalization, the output $X''$ are sent to IDFT module. The output of IDFT module is the time-domain FrCT-NOFDM signal after equalization.

Algorithm 1 depicts the detailed processing of the ID algorithm for FrCT-NOFDM with 2-PAM constellation. Firstly, we calculate the correlation matrix $C$, in which the elements $C_{l,m}$ are the values of cross-correlation representing interference between subcarriers $l$ and $m$. Then, the iteration can be implemented by three steps. The first step is eliminating the ICI by

$$S_i = R - (C - e)S_{i-1}$$  \hspace{1cm} (19)

where $i$ denotes the $i^{th}$ iteration. The second step is the decision operation. The signals falling on the certainty regions (i.e., the value of the signal is larger than $d$ or smaller than $-d$) can be mapped to the corresponding constellation points. The last step is updating the decision level by $d = 1 - i/I$ where $I$ is the total iterative number. Finally, the recovered symbol $S$ can be obtained after the iteration. For the higher-order constellation, the ID algorithm is similar to the 2-PAM constellation except the decision operation, but its performance decreases due to the
Algorithm 1 ID algorithm for FrCT-NOFDM.

**Input:** Received symbol : $R$; Compression factor : $\alpha$; Iterative number : $I$; Identity matrix : $e$.

**Output:** Recovered symbol : $S$.

1: \textbf{for} $l = 0$; $l < N$; $l$ ++ \textbf{do} & Correlation matrix $C$
2: \hspace{1em} \textbf{for} $m = 0$; $m < N$; $m$ ++ \textbf{do}
3: \hspace{2em} $C_{l, m} = \frac{1}{N} \sum_{k=0}^{N-1} \cos\left(\frac{\alpha \pi}{2N} \frac{(2k+1)l}{2N}\right) \times \cos\left(\frac{\alpha \pi}{N} \frac{(2k+1)m}{N}\right)$.
4: \hspace{2em} \textbf{end for}
5: \hspace{1em} \textbf{end for}
6: Initialization : $S_0 = 0$, $d = 1$ & Beginning of iteration
7: \textbf{for} $i = 1$; $i \leq I$; $i$ ++ \textbf{do}
8: \hspace{1em} $S_i = R - (C - e)S_{i-1}$ & Eliminating the ICI
9: \hspace{2em} \textbf{if} $S_i > d$ \textbf{then} & Signal decision
10: \hspace{3em} $S_i = 1$
11: \hspace{2em} \textbf{else if} $S_i < -d$ \textbf{then}
12: \hspace{3em} $S_i = -1$
13: \hspace{2em} \textbf{else}
14: \hspace{3em} $S_i = S_i$
15: \hspace{2em} \textbf{end if}
16: \hspace{1em} $d = 1 - i/I$ & Updating $d$
17: \hspace{1em} \textbf{end for}
18: $S = S_I$
19: Return $S$

smaller Euclidean distance. Therefore, the more effective ICI cancellation algorithm is required for the high-order constellations such as hybrid ID fixed sphere decoder (ID-FSD) [39]. Some coded schemes have good performance on resisting the ICI such as the convolutional code with Bahl-Cocke-Jelinek-Raviv (BCJR) decoding [40].

**B. Simulation Results**

In this section, we give the simulation results of FrCT-NOFDM for the VLC systems. The BER performance of FrCT-NOFDM is comprehensively analyzed under different $\alpha$, RMS delay
spread, DC bias, and iterative number. Moreover, we demonstrate that FrCT-NOFDM has the superior security performance.

Figure 8 shows BER versus the SNR for DCT-OFDM and FrCT-NOFDM signals. The RMS delay spread is set to 3 ns, the DC bias is set to 7 dB, and the iterative number of ID algorithm is set to 20. When the SNR is less than 22 dB, FrCT-NOFDM with $\alpha$ of 0.9 has almost the same BER performance compared to that with $\alpha$ of 0.8. Their BER can achieve the 7% forward error correction (FEC) limit at the SNR of 22 dB. In DCT-OFDM, the BER can achieve the 7% FEC limit at the SNR of 24.2 dB. When $\alpha$ is set to 0.9 or 0.8, FrCT-NOFDM exhibits an improvement in the SNR of 2.2 dB compared to DCT-OFDM. As shown in Fig. 6, the power of high-frequency part in DCT-OFDM signal is seriously declined, but the high-frequency part in FrCT-NOFDM is empty due to the compression of bandwidth. Therefore, DCT-OFDM suffers more high-frequency distortion and has worse BER performance compared to FrCT-NOFDM with $\alpha$ of 0.9 and 0.8.

After ID algorithm, the residual ICI in FrCT-NOFDM increases with the decreasing of $\alpha$. As Fig. 8 depicts, when the SNR is larger than 22 dB, the BER of FrCT-NOFDM with $\alpha$ of 0.8 is higher than that of FrCT-NOFDM with $\alpha$ of 0.9. This is because the residual ICI in FrCT-NOFDM with $\alpha$ of 0.8 is larger than that in FrCT-NOFDM with $\alpha$ of 0.9, and with the
increasing of SNR, the residual ICI turns into the major distortion. Due to the large residual ICI, FrCT-NOFDM with $\alpha$ of 0.7 can not achieve the 7% FEC limit at the SNR of 30 dB.

Figure 9 shows the BER against the SNR for different RMS delay spread of the optical-wireless channel. The $\alpha$ of FrCT-NOFDM is set to 0.9, the DC bias is set to 7 dB and the iterative number of ID algorithm is set to 20. The BER performance deteriorates with the increasing of the RMS delay spread due to the decreasing of the channel bandwidth. When RMS delay spread is set to 9 ns, the bandwidth of CB channel is only 8.3 MHz which is insufficient for the signal with 45-MHz bandwidth. Under this condition, BER can only achieve the 7% FEC limit when SNR is 30 dB. Next, the performance comparison between DCT-OFDM and FrCT-NOFDM is given under the different RMS delay spread.

Figure 10 depicts the required SNR at the 7% FEC limit against the RMS delay spread for DCT-OFDM and FrCT-NOFDM with $\alpha$ of 0.9. When RMS delay spread is set to 1.5 ns, the required SNR for DCT-OFDM signal is about 4.4-dB higher than that for FrCT-NOFDM signal. As shown in Fig. 5 when the RMS delay spread is set to 1.5 ns, the transfer function of the CB model is fading fast while the bandwidth is larger than 45 MHz, looking like a cliff. This fast high-frequency power fading seriously degrades the performance of the subcarriers between 45 MHz and 50 MHz in DCT-OFDM. However, there is no subcarrier between 45 MHz and
50 MHz in FrCT-NOFDM with $\alpha$ of 0.9, thus it is almost not influenced by that fast high-frequency power fading. When RMS delay spread increases, the high-frequency power fading becomes smooth. Therefore, the difference of the required SNR between DCT-OFDM and FrCT-NOFDM is no longer so large. The required SNR in FrCT-NOFDM is about 2 dB smaller than that in DCT-OFDM when RMS delay spread is equal or greater than 3 ns.

Figure 11 shows the BER versus the SNR for different DC bias in FrCT-NOFDM when $\alpha$ is set to 0.9. The iterative number of ID algorithm is set to 20 and the RMS delay spread is set to 3 ns. Compared to FrCT-NOFDM with the DC bias of 4 dB, FrCT-NOFDM with the DC bias of 7 dB requires about 2 dB more SNR to achieve the 7% FEC limit. Compared to FrCT-NOFDM with the DC bias of 7 dB, FrCT-NOFDM with the DC bias of 10 dB needs about 3 dB more SNR to achieve the 7% FEC limit. As shown in Eq. (13), FrCT-NOFDM suffers the clipping distortion which decreases with the increasing of DC bias. If the DC bias is large enough, there is almost no clipping noise in FrCT-NOFDM and its power equals to the power of useful signal plus the power of DC bias as shown in Eq. (14). Therefore, when the DC bias is large enough, the difference between the required SNR of FrCT-NOFDM is equal to the difference between their DC bias power. When the DC bias is 4 dB, the signal still suffers the clipping noise, thus the difference between the required SNR of FrCT-NOFDM with DC bias of 4 dB and 7 dB is
smaller than 3 dB. However, while DC bias is larger than 7 dB, there is little clipping noise. The difference between the required SNR of FrCT-NOFDM with DC bias of 7 dB and 10 dB is almost equal to 3 dB. The simulation result coincides with the theoretical analysis.

Figure 12 shows the BER against the SNR for different iterative numbers of the ID algorithm.
The $\alpha$ of FrCT-NOFDM is set to 0.8, the RMS delay spread is set to 3 ns, and the DC bias is set to 7 dB. When the ID algorithm is not employed (i.e., the iterative number is set to 0), the BER cannot achieve the 7% FEC limit although the SNR is set to 30 dB. This is because the ICI seriously degrades the BER performance of FrCT-NOFDM. With the increasing of the iterative number of ID algorithm, the BER performance is markedly improved. Therefore, ID algorithm can effectively eliminate the ICI for FrCT-NOFDM. However, the complexity of ID algorithm increases with the iterative number. The performance of ID algorithm will no longer be improved obviously with the increasing of the iterative number while the iterative number is large. Therefore, we should choose the suitable iterative number by synthetically considering the effect and complexity of ID algorithm.

Figure 13 shows the BER versus $\Delta \alpha$ for FrCT-NOFDM signal when $\alpha$ at the transmitter is set to 0.9 and SNR is set to 28 dB. The $\Delta \alpha$ denotes the deviation of $\alpha$ between the transmitter and receiver. When $\Delta \alpha$ is larger than 0.001, the BER performance will be significantly deteriorated (BER $> 7\%$ FEC limit). In other word, the data can only be recovered accurately when $\Delta \alpha$ is less than 0.001. This is because both FrCT and ID algorithm at the receiver need an accurate $\alpha$. Therefore, $\alpha$ can be used as an encryption key for the security communications. If the $\alpha$ can
not be known accurately, the transmitted data cannot be accurately recovered at the receiver. It reveals that FrCT-NOFDM system has the superior security performance for application in the security VLC systems.

VI. CONCLUSION

This paper proposed the FTN FrCT-NOFDM signal for VLC systems. Compared to FrFT-NOFDM signal, FrCT-NOFDM signal is real-valued which can be directly applied to the VLC systems without upconversion. Thus, FrCT-NOFDM signal is more suitable for cost-sensitive VLC systems. Meanwhile, FrCT-NOFDM signal can be categorized as an FTN signal which has higher spectral efficiency than Nyquist signal such as OFDM signal. Under the same transmission rate, FrCT-NOFDM signal occupies smaller bandwidth compared to OFDM signal. When bandwidth compression factor $\alpha$ is set to 0.8, the 20% bandwidth saving can be obtained. By this way, FrCT-NOFDM signal suffers less high-frequency distortion, which is suited to the bandwidth-limited VLC systems.

We presented the simulations to investigate the performance of FrCT-NOFDM. When the RMS delay spread is set to 3 ns (i.e., the channel bandwidth is 20.9 MHz) and the transmission rate is set to 100 Mbit/s, FrCT-NOFDM with $\alpha$ of 0.9 or 0.8 exhibits an improvement in the SNR of 2.2 dB compared to DCT-OFDM. This is because FrCT-NOFDM signal suffers less high-frequency distortion. Moreover, FrCT-NOFDM has the superior security performance for application in the security VLC systems. It is worth noting that the residual ICI after ID algorithm is still a critical problem for the high-order constellations. The more effective algorithm need to be investigated to solve this problem in the future work. In conclusion, FrCT-NOFDM shows great potential for application in the future VLC systems.

REFERENCES

[1] M. Biagi, S. Pergolini, and A. Vegni, “LAST: A Framework to Localize, Access, Schedule, and Transmit in Indoor VLC Systems,” *J. Lightw. Technol.*, vol. 33, no. 9, pp. 1872–1887, May 2015.

[2] H. Elgala, R. Mesleh, and H. Haas, “Indoor optical wireless communication: Potential and state-of-the-art,” *IEEE Commun. Mag.*, vol. 49, no. 9, pp. 56–62, Sep. 2011.

[3] T. Gross, S. Mangold, and S. Schmid, “Software-Centric VLC Networking for the IoT,” in *Proc. IEEE Photonics Society Summer Topical Meeting Series*, Newport Beach, CA, USA, July 2016, pp. 62–63.

[4] N. Chi, H. Haas, M. Kavehrad, T. Little, and X. Huang, “Visible light communications: demand factors, benefits and opportunities [Guest Editorial],” *IEEE Wireless Communications*, vol. 22, no. 2, pp. 5–7, April 2015.
[5] S. Wu, H. Wang, and C. Youn, “Visible light communications for 5G wireless networking systems: from fixed to mobile communications,” IEEE Network, vol. 28, no. 6, pp. 41–45, Nov. 2014.

[6] M. Rahaim and T. Little, “Toward practical integration of dual-use VLC within 5G networks,” IEEE Wireless Communications, vol. 22, no. 4, pp. 97–103, Aug. 2015.

[7] M. Ayyash, H. Elgala, A. Khreishah, V. Jungnickel, T. Little, S. Shao, M. Rahaim, D. Schulz, J. Hilt, and R. Freund, “Coexistence of WiFi and LiFi toward 5G: concepts, opportunities, and challenges,” IEEE Commun. Mag., vol. 54, no. 2, pp. 64–71, Feb. 2016.

[8] Z. Ghassemlooy, S. Arnon, M. Uysal, Z. Xu, and J. Cheng, “Emerging Optical Wireless Communications-Advances and Challenges,” IEEE J. Sel. Areas Commun., vol. 33, no. 9, pp. 1738–1749, Sep. 2015.

[9] N. Chi, Y. Wang, Y. Wang, X. Huang, and X. Lu, “Ultra-high-speed single red-green-blue light-emitting diode-based visible light communication system utilizing advanced modulation formats,” Chinese Optics Letters, vol. 12, no. 1, pp. 0106051–0106054, Jan. 2014.

[10] M. Biagi, T. Borogovac, T. Little, “Adaptive Receiver for Indoor Visible Light Communications,” J. Lightw. Technol., vol. 31, no. 23, pp. 3676–3686, Dec. 2013.

[11] D. Tsonev, S. Videv, and H. Haas, “Unlocking spectral efficiency in intensity modulation and direct detection systems,” IEEE J. Sel. Areas Commun., vol. 33, no. 9, pp. 1758–1770, Sep. 2015.

[12] H. Elgala and T. Little, “SEE-OFDM: Spectral and energy efficient OFDM for optical IM/DD systems,” in Proc. IEEE 25th Annu. Int. Symp. Pers., Indoor, Mobile Radio Commun., Washington, DC, USA, Sep. 2014, pp. 851–855.

[13] Q. Wang, C. Qian, X. Guo, Z. Wang, D. Cunningham, and I. White, “Layered ACO-OFDM for intensity-modulated direct-detection optical wireless transmission,” Opt. Express, vol. 23, no. 9, pp. 12382–12393, May 2015.

[14] Q. Wang, Z. Wang, and L. Dai, “Asymmetrical Hybrid Optical OFDM for Visible Light Communications with Dimming Control,” IEEE Photon. Tech. Lett., vol. 27, no. 9, pp. 974–977, Feb. 2015.

[15] M. Mossaad, S. Hranilovic, and L. Lampe, “Visible Light Communications Using OFDM and Multiple LEDs,” IEEE Trans. Commun., vol. 63, no. 11, pp. 4304–4313, Nov. 2015.

[16] J. Armstrong and B. J. C. Schmidt, “Comparison of Asymmetrically Clipped Optical OFDM and DC-Biased Optical OFDM in AWGN,” IEEE Commun. Lett., vol. 12, no. 5, pp. 343–345, May 2008.

[17] S. Dissanayake and J. Armstrong, “Comparison of ACO-OFDM, DCO-OFDM and ADO-OFDM in IM/DD Systems,” J. Lightw. Technol., vol. 31, no. 7, pp. 1063–1072, April 2013.

[18] J. Zhou, Y. Yan, Z. Cai, Y. Qiao, and Y. Ji, “A Cost-Effective and Efficient Scheme for Optical OFDM in Short-Range IM/DD Systems,” IEEE Photon. Tech. Lett., vol. 26, no. 13, pp. 1372–1374, July 2014.

[19] I. Darwazeh, T. Xu, T. Gui, Y. Bao, and Z. Li, “Optical SEFDM System: Bandwidth Saving Using Non-Orthogonal Sub-Carriers,” IEEE Photon. Tech. Lett., vol. 26, no. 4, pp. 352–355, Dec. 2013.

[20] Y. Wang et al., “SEFDM Based Spectrum Compressed VLC System Using RLS Time-domain Channel Estimation and ID-FSD Hybrid Decoder ;” in Proc. 42nd European Conference and Exhibition on Optical Communications , Düsseldorf, Germany, Sep. 2016, pp. 827–829.

[21] T. Xu and I. Darwazeh, “A soft detector for spectrally efficient systems with non-orthogonal overlapped sub-carriers,” IEEE Commun. Lett., vol. 18, no. 10, pp. 1847–1850, Oct. 2014.

[22] D. Nopchinda, T. Xu, R. Maher, B. Thomsen, and I. Darwazeh, “Dual polarization coherent optical spectrally efficient frequency division multiplexing,” IEEE Photon. Tech. Lett., vol. 28, no. 1, pp. 83–86, Jan. 2016.

[23] J. Zhou, Y. Qiao, Z. Yang and E. Sun, “Faster-than-Nyquist non-orthogonal frequency-division multiplexing based on fractional Hartley transform,” Opt. Lett., vol. 41, no. 19, pp. 4488–4491, Oct. 2016.
[24] J. Zhou, Y. Qiao, Z. Yang, M. Guo, and X. Tang, “Capacity limit for faster-than-Nyquist non-orthogonal frequency-division multiplexing signals,” arXiv, preprint arXiv:1611.10309, Nov. 2016.
[25] J. E. Mazo, “Faster-than-nyquist signaling,” The Bell System Technical Journal, vol. 54, no. 8, pp. 1451–1462, Oct. 1975.
[26] N. Ahmed, T. Natarajan, and K. R. RAO, “Discrete Cosine Transform,” IEEE Trans. Computers, vol. C-23, no. 1, pp. 90–93, Jan. 1974.
[27] J. Zhou, Y. Qiao, Z. Cai, and Y. Ji, “Asymmetrically clipped optical fast OFDM based on discrete cosine transform for IM/DD systems,” J. Lightw. Technol., vol. 33, no. 9, pp. 1920–1927, May 2015.
[28] J. Zhou, Y. Qiao, T. Zhang, E. Sun, M. Guo, Z. Zhang, X. Tang, and F. Xu, “FOFDM Based on Discrete Cosine Transform forIntensity-Modulated and Direct-Detected Systems,” J. Lightw. Technol., vol. 34, no. 16, pp. 3717–3725, Aug. 2016.
[29] P. Tan and N. C. Beaulieu, “A Comparison of DCT-Based OFDM and DFT-Based OFDM in Frequency Offset and Fading Channels,” IEEE Trans. Commun., vol. 54, no. 11, pp. 2113–2125, Nov. 2006.
[30] J. Zhao and A. D. Ellis, “A Novel Optical Fast OFDM with Reduced Channel Spacing Equal to Half of the Symbol Rate Per Carrier,” in Proc. the Opt. Fiber Commun. Conf. Exhib./Nat. Fiber Opt. Eng. Conf., San Diego, CA, USA, Mar. 2010, Paper OMR1.
[31] E. Giacoumidis, S. Ibrahim, J. Zhao, J. Tang, I. Tomkos, and A. D. Ellis, “Experimental demonstration of cost-effective intensity-modulation and direct-detection optical fast-OFDM over 40km SMF transmission,” in Proc. the Opt. Fiber Commun. Conf. Exhib./Nat. Fiber Opt. Eng. Conf., Los Angeles, CA, USA, Mar. 2012, Paper JW2A.65.
[32] E. Giacoumidis, A. Tsokanos, C. Mouchos, G. Zardas, C. Alves, J. L. Wei, J. M. Tang, C. Gosset, Y. Jaouën, and I. Tomkos, “Extensive Comparisons of Optical Fast-OFDM and Conventional Optical OFDM for Local and Access Networks,” Journal of Optical Communications and Networking, vol. 4, no. 10, pp. 724–733, Oct. 2012.
[33] J. B. Carruthers and J. M. Kahn, “Modeling of Nondirected Wireless Infrared Channels,” IEEE Tran. commun., vol. 45, no. 10, pp. 1260–1268, Oct. 1997.
[34] J. M. Kahn and J. R. Barry, “Wireless Infrared Communications,” Proceedings of the IEEE, vol. 85, no. 2, pp. 265–298, Feb. 1997.
[35] V. Jungnickel, V. Pohl, S. Nonnig, and C. Helmolt, “A Physical Model of the Wireless Infrared Communication Channel,” IEEE J. Sel. Areas Commun., vol. 20, no. 3, pp. 631–640, April 2002.
[36] C. Chen, D. Basnayaka, and H. Haas, “Non-line-of-sight Channel Impulse Response Characterisation in Visible Light Communications,” in Proc. IEEE International Conference on Communications (ICC), Kuala Lumpur, Malaysia, May 2016, pp. 1-6.
[37] P. Chvojka, S. Zvanovec, P. Haigh, and Z. Ghassemloopy,“Channel Characteristics of Visible Light Communications Within Dynamic Indoor Environment,” J. Lightw. Technol., vol. 33, no. 9, pp. 1719–1725, May 2015.
[38] K. Lee, H. Park, and J. Barry, “Indoor Channel Characteristics for Visible Light Communications,” IEEE Commun. Lett., vol. 15, no. 2, pp. 217–219, Feb. 2011.
[39] T. Xu, R. C Grammenos, F. Marvasti, and I. Darwazeh, “An Improved Fixed Sphere Decoder Employing Soft Decision for the Detection of Non-orthogonal Signals,” IEEE Commun. Lett., vol. 17, no. 10, pp. 1964–1967, Oct. 2013.
[40] J. B. Anderson, F. Rusek, and V. Owall, “Faster-Than-Nyquist Signaling,” Proceedings of the IEEE, vol. 101, no. 8, pp. 1817–1830, Aug. 2013.