Wormholes without exotic matter in Einstein-Cartan theory

K. A. Bronnikov\textsuperscript{a,b,c,}\textsuperscript{1} and A. M. Galiakhmetov\textsuperscript{d,}\textsuperscript{2}

\textsuperscript{1}VNIIMS, Ozyornaya ul. 46, Moscow 119361, Russia
\textsuperscript{2}Peoples’ Friendship University of Russia, ul. Miklukho-Maklaya 6, Moscow 117198, Russia
\textsuperscript{3}National Research Nuclear University “MEPhI” (Moscow Engineering Physics Institute), Moscow, Russia
\textsuperscript{4}Donetsk National Technical University, ul. Kirova 51, 84646, Gorlovka, Ukraine

We study the possible existence of static traversable wormholes without invoking exotic matter in the framework of the Einstein–Cartan theory. A family of exact static, spherically symmetric wormhole solutions with an arbitrary throat radius, with flat or AdS asymptotic behavior, has been obtained with sources in the form of two noninteracting scalar fields with nonzero potentials. Both scalar fields are canonical (that is, satisfy the weak energy condition), one is minimally and the other nonminimally coupled to gravity, and the latter is a source of torsion.

1 Introduction

Recent cosmic observations favor an isotropic, spatially flat Universe, which is at present expanding with acceleration. Establishment of the origin of this acceleration has become one of the most important problems in cosmology and even in theoretical physics as a whole. Different theoretical models trying to explain it have been put forward (see, e.g., the reviews [1–4] and references therein). In the framework of general relativity (GR), this expansion may be ascribed to a source in the form of an unknown substance with a large negative pressure, called dark energy (DE), and its most popular model agreeing with observations is the cosmological constant. Many models have been suggested in alternative theories of gravity, in particular, in the Einstein–Cartan theory (ECT), see, e.g., [5–7]. This theory [8–11] is an extension of GR to a space-time with torsion, and it reduces to GR if the torsion vanishes.

The ECT is the simplest version of the Poincaré gauge theory of gravity (PGTG), in which the torsion is not dynamic because the gravitational action is proportional to the curvature scalar of Riemann–Cartan space–time. In this sense, the ECT is a degenerate gauge theory [11–14]. This shortcoming is absent in the full PGTG since its gravitational Lagrangian includes invariants quadratic in the curvature and torsion tensors. Nevertheless, the ECT is a viable theory of gravity whose observational predictions are in agreement with the classical tests of GR, and it significantly differs from GR only at very high densities of matter [11,15,16].

The interest in the ECT has recently grown in connection with the fact that torsion arises naturally in supergravity [17–19], Kaluza–Klein [20–22] and superstring [23–25] theories. f(R) gravity with torsion has been developed [26,27] as one of the simplest extensions of the ECT. In [27] it has been demonstrated that, in f(R) gravity, torsion can be a geometric source of accelerated expansion.

Some cosmological models in ECT have turned out to be nonsingular, with a cosmological singularity replaced by a bounce [7], while a similar result is achieved in GR only by invoking “exotic” sources, violating the energy conditions respected by usual matter, above all, the weak and null energy conditions (WEC and NEC). It is well known that in GR such a violation is a necessary condition for the existence of traversable wormholes [28]. Wormholes as possible time machines, or “tunnels” between universes or distant parts of the same universe, are a subject of particular interest, for reviews see [29–31] and references therein.

Well known are wormhole solutions of GR with minimally coupled phantom scalar fields (those with a wrong sign of kinetic energy), with or with-
out electric or magnetic charges as well as potentials, see [32–35] and references therein, while with normal fields having positive kinetic energy wormholes are impossible. For scalar fields with nonminimal couplings the situation is more subtle: even with normal fields there exist some special wormhole solutions (see [32, 36] for solutions with conformal coupling and [37–39] for other couplings). However, in all such cases the wormhole solutions inevitably contain regions where the effective gravitational constant becomes negative, that is, the gravitational field itself becomes a phantom [40, 41]. Moreover, all such configurations, whose existence is connected with the phenomenon of conformal continuation [40, 42], turn out to be unstable under radial perturbations [39, 43, 44].

An aspect of interest for scalar fields with nonminimal coupling is that the existence of a throat as a local property of space-time does not yet mean that the configuration as a whole is a wormhole. It has been shown, in particular, that in the Brans-Dicke scalar-tensor theory of gravity, throats in static, spherically symmetric solutions can exist with any value of the coupling constant \( \omega \) whereas wormholes as global entities only exist in the “phantom range” \( \omega < -3/2 \) [45].

One more possible source of unusual geometries without WEC or NEC violation is a vortex gravitational field [47, 48] existing in rotating configurations. Wormholes have been found among rotating cylindrically symmetric configurations [49] without exotic matter, but the main problem with them is their non-flat asymptotic behavior which does not allow them to be interpreted as objects in the observable Universe. Torsion in the ECT is to a certain extent similar to rotation in GR, therefore it is natural to expect that wormholes can exist in it without WEC or NEC violation.

We here seek regular static, spherically symmetric configurations with normal, non-phantom scalar fields in the framework of ECT and find some examples of wormhole solutions involving a nonminimally coupled scalar field as a source of torsion.

The paper is organized as follows. In Section 2 we present the ECT equations both in the general case and for static, spherically symmetric configurations involving two scalar fields of which one is minimally and the other nonminimally coupled with space-time curvature. Section 3 is devoted to finding and analyzing the properties of a family of exact solutions, and Section 4 is a discussion.

## 2 Field equations

We start with the action

\[
S = \int \sqrt{-g} d^4x \left[ -\frac{R}{2\kappa} + \frac{m}{2} \left( \phi, k \phi^k + \xi R \phi^2 \right) - V(\phi) + \frac{n_2}{2} \psi, k \psi^k - W(\psi) \right],
\]

where \( R[\Gamma] \) is the curvature scalar obtained from the full connection \( \Gamma^k_{ij} = \{k \}_{ij} + S^k_{ij} + S^k_{ji} ; \) here \( \{k \}_{ij} \) are Christoffel symbols of the second kind for the metric \( g_{ik} ; \) \( S^k_{ij} = \Gamma^k_{ij} \) is the torsion tensor; \( \kappa = 8\pi G, G \) being the Newtonian gravitational constant; \( \phi \) and \( \psi \) are two noninteracting scalar fields with the potentials \( V(\phi) \) and \( W(\psi) \), respectively. The constants \( n_{1,2} = \pm 1 \) correspond to either usual, canonical (\( \eta = +1 \)) or phantom (\( \eta = -1 \)) scalar fields.

The metric \( g_{ik} \) has the signature \( (+ - - -) \), the Riemann and Ricci tensors are defined as

\[
R^m_{ijk} = \Gamma^m_{jk,i} - \Gamma^m_{ik,j} + \Gamma^m_{jp} \Gamma^p_{jk} - \Gamma^m_{jp} \Gamma^p_{ik}
\]

and \( R_{jk} = R^i_{ijk} \). We should note that, in the framework of ECT, a scalar field nonminimally coupled to gravity gives rise to torsion, even though the scalar field has zero spin. It follows from [1] that the torsion can interact with a scalar field only through its trace: \( S_i = S^k_{ik} \) (see [46]). Hence, the curvature scalar \( R[\Gamma] = g^{ik} R_{jk} \) can be presented in the form [46]

\[
R[\Gamma] = R[\{\}] + 4\nabla_k S^k - (8/3) S_k S^k,
\]

where \( R[\{\}] \) is the Riemannian part of the curvature built from the Christoffel symbols; \( \nabla_k \) is the covariant derivative of Riemannian space.

Varying the action with the Lagrangian \( \{\} \) in \( g_{ij}, S_k, \phi \) and \( \psi \), we obtain the following set of equations:

\[
G_{ij}[\{\}] = \kappa (T_{ij}[\phi] + T_{ij}[\psi]) + \Lambda_{ij},
\]

\[
S^k = \frac{3}{2} \xi \Psi \phi \phi^k,
\]

\[
\Box \phi - \xi \phi R[\Gamma] + n_1 dV / d\phi = 0,
\]

\[
\Box \psi + n_2 dW / d\psi = 0,
\]
where

\[ T_{ij}[\phi] = \eta_1 \left\{ \phi_j \phi_i - \frac{1}{2} [\phi_m \phi^m + \xi R(\{ \})] \phi^2 - 2 \eta V(\phi) \right\} g_{ij} + \xi \left[ -4 S_i \nabla_j + 2 g_{ij} S^m \nabla_m \right] \]

\[ - \nabla_i \nabla_j + g_{ij} \Box + R_{ij}(\{ \}) - \Lambda_{ij} \phi^2 \right\}, \tag{7} \]

\[ T_{ij}[\psi] = \eta_2 \left( \psi_i \psi_j - \frac{1}{2} \psi_m \psi^m g_{ij} \right) + W(\psi) g_{ij}, \tag{8} \]

\[ \Lambda_{ij} = \frac{8}{3} S_i S_j - \frac{4}{3} S_k S^k g_{ij}. \tag{9} \]

Here \( \Box \) is the d’Alembertian operator of Riemannian space, and we denote \( \Psi = \kappa (\eta_1 - \xi \kappa \phi^2)^{-1} \).

It is not difficult to verify that the effective scalar-torsion stress-energy tensor \( T^{(\text{eff})}_{ij}[\phi] \)

\[ T^{(\text{eff})}_{ij}[\phi] = T_{ij}[\phi] + \kappa^{-1} \Lambda_{ij}, \tag{10} \]

and the scalar stress-energy tensor \( T_{ij}[\psi] \) are separately covariantly conserved since no explicit coupling is assumed between the scalar fields:

\[ \nabla^j T^{(\text{eff})}_{ij}[\phi] = \nabla^j T_{ij}[\psi] = 0. \tag{11} \]

The general static, spherically symmetric metric can be written in the form

\[ ds^2 = A(u) dt^2 - \frac{du^2}{A(u)} - r^2(u) d\Omega^2 \tag{12} \]

in terms of the so-called quasiglobal radial coordinate \( [31] \), where \( g_{00} = A(u) \) may be called the redshift function while \( r(u) \) is the area function; \( d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \) is the linear element on a unit sphere. (As usual, the metric is only formally static: it is really static if \( A > 0 \), but it describes a Kantowski–Sachs (KS) type cosmology if \( A < 0 \), and \( u \) is then a temporal coordinate.) We also consider \( \phi = \phi(u) \) and \( \psi = \psi(u) \).

The nonvanishing components of the effective scalar-torsion stress-energy tensor are given by

\[ T^{(\text{eff})}_1[\phi] = \eta_1 \xi \phi^2 G_1^2(\{ \}) + Y + \eta_1 \left[ 2 \xi \phi \phi'' A + \xi \phi \phi' A' + (-1 + 2 \xi + 6 \xi^2 \phi^2 \Psi) \phi^2 A \right], \tag{13} \]

\[ T^{(\text{eff})}_2[\phi] = T^{(\text{eff})}_3[\phi] \]

\[ = \eta_1 \xi \phi^2 G_2^2(\{ \}) + Y + 2 \eta_1 \xi \frac{r'}{r} \phi \phi' A, \tag{14} \]

\[ T^{(\text{eff})}_0[\phi] = \eta_1 \xi \phi^2 G_0^2(\{ \}) + Y + \eta_1 \xi \phi \phi' A', \tag{15} \]

where the prime means \( d/du \) and

\[ Y = V(\phi) + \eta_1 A \left[ -2 \xi \phi \phi'' - 2 \xi \left( \frac{A'}{A} + \frac{2 r'}{r} \right) \phi \phi' + \left( \frac{1}{2} - 2 \xi - 3 \xi^2 \phi^2 \Psi \right) \phi^2 \right] \tag{16} \]

and \( G^k_k(\{ \}) \) is the Einstein tensor of Riemannian space. The stress-energy tensor of the scalar field \( \psi \) is

\[ T^k_k[\psi] = \frac{\eta_2}{2} A \psi^2 \text{diag}(1, -1, 1) + \delta_k^k W(\psi). \tag{17} \]

The scalar field equations and three independent combinations of the Einstein–Cartan equations read

\[ (1 - 6 \xi^2 \phi^2 \Psi) \left( \frac{Ar^2 \phi''}{r^2} - \frac{6 \eta_1 \xi \phi \phi^2 \Psi^2 A}{r^2} \right) \]

\[ - \eta_1 \frac{dV}{d\phi} + \xi \left[ A'' + \frac{4 \phi^2 A'}{r} + \frac{4 \phi^2 A''}{r^2} - \frac{2 A''}{r^2} - \frac{2}{r^2} \right] = 0, \tag{18} \]

\[ (Ar^2 \Psi)' = \eta_2 r^2 dW/d\psi, \tag{19} \]

\[ (Ar^2 \phi')' = -2 \eta_1 r^2 (V + W) \Psi + 2 \xi r^2 A \Psi \left[ \phi \phi'' + \phi' + 2 \left( \frac{A'}{A} + \frac{r'}{r} \right) \phi \phi' \right], \tag{20} \]

\[ 2 \frac{r''}{r} = - \eta_1 \frac{d \Psi}{d\phi} \Psi + \Psi \left[ 2 \xi \phi \phi'' \right. \]

\[ + \left. (-1 + 2 \xi + 6 \xi^2 \phi^2 \Psi) \phi^2 \right], \tag{21} \]

\[ A(r^2)'' - r^2 A'' = 2 + 2 \xi r^2 \phi \phi' \Psi \left( \frac{2 A r'}{r} - A' \right). \tag{22} \]

It should be noted that the scalar field equation [18] follows from [19]–[22].

Noteworthy, in the Einstein–Cartan equations [18]–[22] the terms induced by torsion contain the factor \( \xi^2 \), i.e., they exist due to nonminimal coupling of the \( \phi \) field with space-time curvature. It is this nonminimal coupling that makes ECT solutions different from those of GR, and in particular, as we shall see, it makes possible to obtain wormhole solutions with non-phantom scalar fields \( \phi \) and \( \psi \) (that is, with \( \eta_1 = \eta_2 = 1 \)).

### 3 Exact solution

Let us now try to find an exact solution to the Einstein–Cartan equations with canonical (normal)
scalar fields: \( \eta_1 = \eta_2 = +1 \). Since finding solutions with given nonzero scalar field potentials is a hard problem even in the simpler case of GR, let us use the inverse problem method, by analogy with \cite{34,35}, that is, consider the potentials as unknowns to be found but specify some other functions in accord with the desired properties of the solution. Since there are two potentials \( V(\phi) \) and \( W(\psi) \), we can choose two such functions.

First, let us choose the function \( r(u) \) in a way suitable for a wormhole solution:

\[
r(u) = \sqrt{u^2 + b^2} = b\sqrt{x^2 + 1},
\]

where \( x = u/b \), and \( b \) is an arbitrary constant (the throat radius). Evidently, \( r' = 0 \) and \( r'' > 0 \) at \( u = 0 \), as required.

Second, we take the expression for \( \phi(x) \) in the form

\[
\phi(x) = (\kappa \xi (x^2 + 1))^{-1/2},
\]

then Eq. \((21)\) gives

\[
\frac{2}{b^2(x^2 + 1)^2} = -\frac{\kappa(x^2 + 1)}{x^2} \psi'^2 + \frac{6\xi - 1}{\xi b^2(x^2 + 1)^2} + \frac{4}{b^2x^2(x^2 + 1)^2}.
\]

Eq. \((25)\) is solved for \( \xi = 1/4 \) and

\[
\psi'^2 = \frac{4}{\kappa b^2(x^2 + 1)^2},
\]

whence

\[
\psi(x) = \psi_0 \pm 2x[\kappa(x^2 + 1)]^{-1/2}.
\]

Now, taking into account \((23)\) and \((24)\), we can obtain the metric function \( A(x) \) from \((22)\). It is convenient to do that by bringing \((22)\) to the form

\[
B'' + 2B' \frac{2x^2 + 1}{x(x^2 + 1)} + \frac{2}{(x^2 + 1)^2} = 0
\]

in the dimensionless variables \( x \) and \( B = A/(x^2 + 1) \); the prime means here \( d/dx \). As a result, we obtain

\[
A(x) = (x^2 + 1) \left[ B_0 - \frac{C_1}{x} - C_1 \arctan x - 2 \frac{\arctan x}{x} - \arctan^2 x \right],
\]

where \( C_1 \) and \( B_0 \) are integration constants. Since \( A(x) \) should be regular at all \( x \) including \( x = 0 \), we put \( C_1 = 0 \). It means that our family of regular solutions contains only metrics symmetric with respect to the throat \( x = 0 \).

The asymptotic expression for \( A(x) \) as \( x \to \pm \infty \) can be presented as

\[
A(x) \simeq \left( B_0 - \frac{\pi^2}{4} \right)x^2 + 1 + B_0 - \frac{\pi^2}{4} + \frac{\pi}{3x}.
\]

Thus the following cases are distinguished:

(i) \( B_0 > \pi^2/4 \). The solution describes a traversable wormholes with two AdS asymptotic regions (AdS-AdS);

(ii) \( B_0 = \pi^2/4 \). The solution describes a twice asymptotically flat (M–M) traversable wormhole.

(iii) \( B_0 < \pi^2/4 \). We obtain configurations with two de Sitter asymptotics (dS–dS), which contain static regions only if \( B_0 > 2 \). If \( B_0 < 2 \), the solution describes a pure KS cosmology interpolating between two dS states; if \( B_0 = 2 \), then two KS epochs are separated by a double horizon.

Plots of \( B(x) = A(x)/(x^2 + 1) \) with different \( B_0 \) are shown in Fig. 1.

In the twice asymptotically flat case (ii), comparing \((29)\) with the Schwarzschild expression \( A = 1 - 2Gm/r \) and taking into account that \( r \approx u = bx \) at large \( x \), we find the Schwarzschild mass \( m \):

\[
m = \frac{\pi b}{6G} = \frac{\pi}{6} m_{\text{pl}} \frac{b}{l_{\text{pl}}},
\]

Figure 1: Plots of \( B(x) \). Curves \( \lambda 1-\lambda 6 \) correspond to \( B_0 = 3, 2.6, \pi^2/4, 2.2, 2.0, 1.8 \).
where $m_{pl} = 1/\sqrt{G}$ and $l_{pl} = \sqrt{G}$ are the Planck mass and length, respectively. It is easy to estimate that if we suppose that the wormhole is large enough for transportation purposes, say, $b = 10$ m, then the mass will be of the order $1.5 \cdot 10^{31}$ g, which makes about 1/130 of the Sun’s mass. The gravitational field in such a wormhole will be evidently too strong for a human being to survive.

Let us find other quantities characterizing the solution. Choosing in (27), without loss of generality, the plus sign and $\psi_0 = 0$, we obtain for $W(x)$ from (19):

$$W(x) = W_0 + (8\kappa b^2(x^2 + 1)^2)^{-1} \left[ 8B_0x^2(2 + x^2) + 25x^2(5 + 4x^2) - 2(75x^2 + 125x^2 + 32) \arctan x \right] - \left( 75x^4 + 150x^2 + 67 \right) \arctan^2 x, \quad (31)$$

where $W_0$ is an integration constant. Like $B(x)$, the function $W(x)$ is even. Plots of $W(x)/8\kappa b^2 \to W(x)$ with $W_0 = 0$ are shown in Fig. 2. Note that, in accord with the wormhole symmetry, the function $W(x)$ is even.

In terms of $\psi$ the potential $W$ is

$$W(\psi) = W_0 + \frac{1}{128\kappa b^2} \left[ 8B_0\psi^2(8 - \psi^2) - 25\psi^4 + 500\psi^2 + 8(\psi^4 - 8\psi^2 - 134) \arctan^2 \psi \right. + 4(9\psi^4 - 122\psi^2 - 256) \sqrt{4 - \psi^2} \left. \frac{\arctan \psi}{\psi} \right], \quad (32)$$

where $\psi$ stands for $\sqrt{\kappa \phi}$.

From equation (20) one derives $V(x)$:

$$V(x) = \frac{1}{8\kappa b^2(x^2 + 1)^2} \left[ -8\kappa b^2 W_0 - 8B_0(4x^4 + 6x^2 + 1) - 100x^4 - 101x^2 + 16 + 2(99x^4 + 149x^2 + 32) \arctan x \right] + (99x^4 + 182x^2 + 75) \arctan^2 x]. \quad (33)$$

Plots of $V(x)/8\kappa b^2 \to V(x)$ with $W_0 = 0$ are shown in Fig. 3.

In terms of $\phi$ we obtain

$$V(\phi) = \frac{1}{8\kappa b^2} \left[ -8\kappa b^2 W_0 + 8B_0(\chi^4 + 2\chi^2 - 4) + 17\chi^4 + 99\chi^2 - 100 - 2(18\chi^4 + 49\chi^2 - 99) \arctan U \right] - \left( 8\chi^4 + 16\chi^2 - 99 \right) \arctan^2 U]. \quad (34)$$

where $\chi = \sqrt{\kappa \xi} \phi$, and $U = \sqrt{1 - \chi^2/\chi}$.

The expression for the squared trace of the torsion $S^2 = S_kS^k$ has the form

$$S^2 = \frac{9A(x)}{b^2 x^2(x^2 + 1)^2}. \quad (35)$$

Thus $S^i$ is a spacelike vector for $A > 0$ and a timelike one for $A < 0$, i.e., in a KS cosmology. The torsion turns out to be singular at the throat $x = 0$. On the other hand, at large $x$ it decays by the laws

$$S^2 \left|_{x \to \pm \infty} \sim x^{-4} \right. \to 0 \quad \text{for } B_0 \neq \pi^2/4, \quad S^2 \left|_{x \to \pm \infty} \sim x^{-6} \right. \to 0 \quad \text{for } B_0 = \pi^2/4. \quad (36)$$
4 Concluding remarks

We have found a family of exact static, spherically symmetric solutions with non-phantom scalar fields, describing asymptotically flat or AdS wormholes. Its existence proves that not only wormhole throats but also wormholes as global configurations are possible in the ECT due to nonzero space-time torsion, without NEC or WEC violation. In our view, it is a result of interest despite the special nature of this solution (in particular, a special value of the nonminimal coupling constant, $\xi = 1/4$) and some unpleasant physical properties of such wormholes: see the relation $\langle 30 \rangle$ between their Schwarzschild masses and throat radii, making the enormous wormhole gravity so unfavorable for life. Let us recall for comparison that in GR with a phantom scalar field a wormhole may have zero mass combined with a throat of arbitrary size $\langle 32 \rangle$. Such solutions are “force-free”, or ultrastatic, in the sense that $g_{00} \equiv 1$ in the whole space. On the other hand, if we try to substitute $A = g_{00} \equiv 1$ to Eqs. $\langle 18 \rangle$-$\langle 22 \rangle$ using the ansatz $\langle 23 \rangle$, it follows from $\langle 22 \rangle$ that either $\xi = 0$ or $\phi = \text{const}$, which means that there is no torsion $[\text{see } \langle 4 \rangle]$, and wormhole solutions are impossible. Though, one cannot exclude their existence with $r(x)$ other than $\langle 23 \rangle$.

Another problem with the above family of solutions is the divergent torsion at the throat $x = 0$, although the metric, the scalar fields and their potentials are perfectly regular in the whole spacetime. One can show, however, that this singularity is not a generic property of such solutions but is an artifact of the method used for solving the equations. Indeed, the singularity in $\langle 35 \rangle$ is obtained due to the ansatz $\langle 24 \rangle$ for the scalar $\phi$, which leads to $\Psi = \kappa(x^2 + 1)x^{-1/2}$. Quite evidently, choosing a slightly different $\phi$ (let us say, $\phi = [\kappa \xi(x^2 + a^2)]^{-1/2}$ with $a$ slightly smaller than 1) will make the quantities $\Psi$ and $S^2$ finite and regular at all $x$, preserving the qualitative properties of our solution, but the results would not take such a simple and observable form.

References

[1] T. Padmanabham, Phys. Rep. 380, 335 (2003).
[2] P.J.E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 599 (2003).
[3] V. Sahni and A.A. Starobinsky, Int. J. Mod. Phys. D 15, 2105 (2006).
[4] M. Li, X.-D. Li, S. Wang, and Y. Wand, Commun. Theor. Phys. 56, 525 (2011); astro-ph/1103.5870(CO).
[5] A.M. Galiakhmetov, Grav. Cosmol. 13, 217 (2007).
[6] A.M. Galiakhmetov, Class. Quantum Grav. 27, 055008 (2010).
[7] A.M. Galiakhmetov, Class. Quantum Grav. 28, 105013 (2011).
[8] E. Cartan, Ann. Ec. Norm. Suppl. 40, 325 (1923).
[9] E. Cartan, Ann. Ec. Norm. Suppl. 41, 1 (1924).
[10] E. Cartan, Ann. Ec. Norm. Suppl. 42, 17 (1925).
[11] F.W. Hehl and Yu.N. Obukhov, Ann. Fond. Louis Broglie 32, 157 (2007); gr-qc/0711.1535.
[12] F.W. Hehl, P. von der Heyde, G.D. Kerlik, and J.M. Nester, Rev. Mod. Phys. 48, 393 (1976).
[13] V.N. Ponomarev, A.O. Barvinsky, and Yu.N. Obukhov, Geometrodynamics Methods and Gauge Approach in the Theory of Gravity (Energoatomizdat, Moscow, 1985, in Russian).
[14] A.V. Minkevich and A.S. Garkun, Class. Quantum Grav. 23, 4237 (2006); gr-qc/0512130.
[15] A. Trautman, Einstein-Cartan theory, in: Encyclopedia of Mathematical Physics, Eds. J.-P. Françoise, G.L. Naber, and S.T. Tsou (Elsevier, Oxford, 2006), p. 189.
[16] P. Baekler and F.W. Hehl, Class. Quantum Grav. 28, 215017 (2011).
[17] M.A.J. Vandyck, Class. Quantum Grav. 4, 683 (1987).
[18] S.D. Odintsov, Europhys. Lett. 8, 309 (1989).
[19] I.L. Buchbinder and S.D. Odintsov, Europhys. Lett. 8, 595 (1989).
[20] M.W. Kalinowski, Acta phys. austr. 23, 641 (1981).
[21] G. German, Class. Quantum Grav. 2, 455 (1982).
[22] Yu.S. Vladimirov and A.D. Popov, Vestnik Mosk. Univ., Fiz., Astron. 4, 28 (1988).
[23] K. Akdeniz, A. Kızılersü, and E. Rızaoglu, Phys. Lett. B 215, 81 (1988).
[24] P. Kiusuk, Gen. Relativ. Gravit. 21, 185 (1989).
[25] W.M. Baker, Class. Quantum Grav. 7, 717 (1990).
[26] S. Capozziello, R. Cianci, C. Stornaiolo and S. Vignolo, Class. Quantum Grav. 24, 6417 (2007); gr-qc/0708.3038.
[27] S. Capozziello, R. Cianci, C. Stornaiolo and S. Vignolo, Phys. Scr. 78, 065010 (2008); gr-qc/0810.2549.

[28] D. Hochberg and M. Visser, Phys. Rev. D 56, 4745 (1997); gr-qc/9704082.

[29] M. Visser, Lorentzian Wormholes: from Einstein to Hawking (AIP, Woodbury, 1995).

[30] F.S.N. Lobo, Exotic solutions in general relativity: traversable wormholes and “warp drive” space-times. Arxiv: 0710.4474.

[31] K.A. Bronnikov and S.G. Rubin, Black Holes, Cosmology and Extra Dimensions (World Scientific, 2012).

[32] K.A. Bronnikov, Acta Phys. Polon., B4, 251 (1973).

[33] H. Ellis, J. Math. Phys. 14, 104 (1973)

[34] K.A. Bronnikov and J.C. Fabris, Phys. Rev. Lett. 96, 251101 (2006); gr-qc/0511109.

[35] S.V. Bolokhov, K.A. Bronnikov, and M.V. Skvortsova, Class. Quantum Grav. 29, 245006 (2012).

[36] C. Barceló and M. Visser, Phys. Lett. B. 466, 127 (1999); gr-qc/0908029.

[37] C. Barceló and M. Visser, Class. Quantum Grav. 17, 3843 (2000).

[38] K.A. Bronnikov, Grav. Cosmol. 2, 221–226 (1996).

[39] K.A. Bronnikov and S. Grinyok, Festschrift in honour of Prof. Mario Novello., Rio de Janeiro, 2002; gr-qc/0205131.

[40] K.A. Bronnikov, J. Math. Phys. 43, 6096–6115 (2002); gr-qc/0204001.

[41] K.A. Bronnikov and A.A. Starobinsky, Pis’ma v ZhETF 85, 1, 3-8 (2007); JETP Lett. 85, 1, 1-5 (2007); gr-qc/0612032.

[42] K.A. Bronnikov, Acta Phys. Polon. B 32, 3571–3592 (2001); gr-qc/0110125.

[43] K.A. Bronnikov and S.V. Grinyok, Grav. Cosmol. 7, 297–300 (2001); gr-qc/0201083.

[44] K.A. Bronnikov and S.V. Grinyok, Grav. Cosmol. 10, 237–244 (2004); gr-qc/0411063.

[45] K.A. Bronnikov, M.V. Skvortsova and A.A. Starobinsky, Grav. Cosmol. 16, 216–222 (2010); ArXiv: 1005.3262.

[46] V.G. Krechet and D.V. Sadovnikov, Grav. Cosmol. 3, 133 (1997).

[47] V.G. Krechet and D.V. Sadovnikov, Grav. Cosmol. 13, 269 (2007).

[48] V.G. Krechet and D.V. Sadovnikov, Grav. Cosmol. 15, 337 (2009).

[49] K.A. Bronnikov, V.G. Krechet, and José P.S. Lemos, Phys. Rev. D 87, 084060 (2013); ArXiv: 1303.2993.