Floquet Engineering of Two Dimensional Photonic Waveguide Arrays with Cornered \( \pi \) Mode

Ma Luo

School of Optoelectronic Engineering, Guangdong Polytechnic Normal University, Guangzhou 510665, China

Floquet engineering of photonic waveguide lattice could generate \( \pi \)-mode, which is localized at the end or corner of the one or two dimensional finite arrays, respectively. In this paper, we theoretically study the Floquet engineering of two dimensional photonic waveguide lattice in three types of lattices: honeycomb lattice with Kekule distortion, breathing square lattice and breathing Kagome lattice. The Kekule distortion factor or the breathing factor in the corresponding lattice is periodically changed along the axial direction of the photonic waveguide with frequency \( \omega \). Within certain ranges of \( \omega \), the Floquet \( \pi \) mode in a gap of quasi-energy spectrum are found, which are localized at the corner of the finite two-dimensional arrays. Due to particle-hole symmetric in the model of honeycomb and square lattice, the quasi-energy level of the Floquet \( \pi \) mode is \( \pm \omega/2 \). On the other hand, for Kagome lattice, the quasi-energy level of the Floquet \( \pi \) mode is near to \( \pm 2\omega/3 \) or \( \mp 4\omega/3 \).

I. INTRODUCTION

Topological properties of physical lattice models are featured by robust edge states that exist in lower spatial dimension. Specifically, the second order topological insulator of two dimensional lattice model have robust corner modes at the corners of a finite flake \cite{1,2}. Although the lattice models have been demonstrated to host topological corner states, the experimental observation of the corner state in condensed matter physical systems is challenging \cite{3,4}, because the lattice modes require non-trivial hopping terms. The attempts to observe the topological corner states have been draw to other physical systems that mimic the topological lattice models, such as topological circuits \cite{5,6}, topological sonic crystals \cite{7} and photonic crystal \cite{8,9}.

In order to obtained the topological corner states on demand, Floquet engineering of the lattice models have been proposed \cite{10,11,12,13}. Periodical perturbation of the hopping terms and (or) the on-site potential of the lattice model effectively change the Hamiltonian, which could drive the systems into topological phase. The topological feature of the corner modes have been enriched by introducing non-Hermitian terms into the Floquet systems \cite{14,15}. One of the most interesting application of Floquet second order topological phase is to generated corner states in topological superconductor \cite{16,17,18}, so that the Floquet Majorana corner modes could be applied for quantum computing physics \cite{19,20}.

Because of the similarity between the \textit{Schrödinger} equations of quantum systems and the mode coupling equations of photonic waveguide arrays, the topological phase of two dimensional lattice models can be mimicked by finite photonic waveguide arrays \cite{21,22,23} or photonic microring resonator lattices \cite{24}. Floquet engineering of one dimensional photonic waveguide arrays have been thoroughly studied in theory and experiment. Each waveguide is periodically curved, so that the coupling strength between the nearest neighboring waveguides are periodically modified. By arraying the waveguides with alternating interval, the Floquet Su-Schrieffer-Heeger (SSH) model can be mimicked. Theoretical study found that zero modes and \( \pi \) modes, which are localized at the end of the one dimensional lattice, can be generated by the Floquet engineering \cite{25,26,27}. The localized modes have been observed in experiment \cite{28,29}. Extending to two dimensional photonic waveguide arrays, the first \cite{30,31,32,33,34} and second \cite{35} order topological insulating phases have been theoretically studied, but the experimental observation is limited to the first order topological insulating phase \cite{36,37}.

We theoretically studied photonic waveguide arrays in three types of lattice structure, which host localized corner modes. Because the structure of the proposed systems is relatively simple, they could be more feasible for experimental implementation. The photonic waveguides are arrayed in honeycomb lattice with Kekule distortion, breathing square lattice (which is similar to coupling multiple parallel 1D SSH chains with alternating strength), or breathing Kagome lattice. For the lattice arrays of straight waveguide, the static models of the three types of lattice have different properties. For the honeycomb lattice with static Kekule distortion, the two Dirac cones mix with each other, which opens a band gap. For the breathing square lattice, if the SSH lattice in each chain is topological, an isolated edge band is generated by coupling the end states of the 1D SSH chains. For the breathing Kagome lattice, the model could be in the second order topological insulating phase with zero energy corner states, depending of the breathing factor. As the waveguides being periodically curved, the coupling strength between neighboring waveguide is periodically change along the axial direction. Effectively, the zero energy corner
state, and the isolated edge band are modified; the corner states with quasi-energy being $\pi$, $\pm 2\pi/3$ or $\pm 4\pi/3$ appear.

The paper is organized as follows: In Sec. II, the lattice structure and modulation scheme of the waveguide are described; the theoretical method is described. In Sec. III, the numerical results of the waveguide arrays in three types of lattice are presented and discussed. In Sec. IV, the discussion and conclusion is given.

II. THEORETICAL MODEL

A. Structure of the finite waveguide array in three types of lattices

The structure of the finite waveguide array in three types of lattices are plotted in Fig. 1. In Fig. 1(a), the waveguide are array in honeycomb lattice with O-type Kekule distortion. In each primitive unit cell, six waveguides are array in a hexagon. The distance between the center of the hexagon and each waveguide is designated as $d(z)$, which is a periodical function of the axial coordinate $z$. We assumed that the function of the distance takes the form $d(z) = d_0 + d_1 + d_2 \sin(2\pi z/L)$, where $d_0$ is one third of the lattice constant, $d_1$ is the static Kekule distortion factor, and $d_2$ is the dynamic Kekule distortion factor. The two basis vectors of the primitive unit cell are $\pm \frac{1}{\sqrt{3}} d_0 \hat{x} + \frac{\sqrt{3}}{2} d_0 \hat{y}$. The finite array is consisted of $N$ primitive unit cells along the direction of both basis vectors. The distance between two nearest neighboring waveguide in the same unit cell is $d(z)$; that between two nearest neighboring waveguide in the adjacent unit cells is $3d_0 - 2d(z)$.

The structure of waveguide arrays in breathing square lattice is plotted in Fig. 1(b). In each primitive unit cell, four waveguide are arrayed in a square. The distance between each waveguide and the corresponding center of the unit cell, $d(z)$, is indicated by the dashed (blue) lines, which is periodic about the axial coordinate $z$.

The structure of waveguide arrays in breathing Kagome lattice is plotted in Fig. 1(c). In each primitive unit cell are $2\sqrt{3}d_0 \hat{x}$ and $\sqrt{3}d_0 \hat{y}$. The distance between two nearest neighboring waveguide in the same unit cell is $\sqrt{3}d(z)$; that between two nearest neighboring waveguide in the adjacent unit cells is $2\sqrt{3}d_0 - \sqrt{3}d(z)$. The finite waveguide array contains $N$ unit cells in $2\sqrt{3}d_0 \hat{x}$ direction and $N - \ell$ unit cells in $\sqrt{3}d_0 \hat{x} + 3d_0 \hat{y}$ direction, with $\ell$ being the index along the first direction.

B. Tight binding model

The mode coupling theory is applied to describe the waveguide arrays. Only the coupling between the nearest neighboring waveguide is considered in the model. The waveguide are assumed to be single mode cylindrical waveguide, whose electric field exponentially decay at the transversal direction outside of the dielectric rod. The coupling strength is exponentially dependent on the distance between the adjacent waveguide $d$, which is given as $c(d) = c_0 e^{-(d-d_0)/\delta}$. $\delta$ is a normalization factor that is proportional to the transversal decay factor of the electric field of guiding mode in each waveguide. $c_0$ is the coupling strength as the distance between two waveguide being $d_0$. $d_0$ is the distance between nearest neighboring waveguide for the pristine lattice (with $d_1 = d_2 = 0$). $\delta$ and $d_0$ are proportional to $d(z)$ and $d_0$, respectively. For honeycomb lattice with Kekule distortion, $d$ between two nearest neighboring sites in the same unit cell is equal to $d(z)$. For the breathing square lat-
tice, $\tilde{d}$ between two nearest neighboring sites in the same unit cell is equal to $\sqrt{2}d(z)$. For the breath Kagome lattice, $\tilde{d}$ between two nearest neighboring sites in the same unit cell is equal to $\sqrt{3}d(z)$. In the following, the proportionality coefficient is absorbed in the coefficient $\delta$ for each types of lattice. The mode coupling equations are equivalent to the Schrödinger equations, with the time being replaced by the axial coordinate $z$, and the hopping terms in the Hamiltonian being replace by the coupling strength. Thus, the Hamiltonian is given as $H = \sum_{i,j} c_{i,j}$ with the summation covering the nearest neighboring sites, and $c_{i,j}$ being the coupling strength between the $i$-th and $j$-th waveguide. Since the coupling strength is periodic function of $z$ with period $L$, the Hamiltonian can be expanded as Fourier series of axially oscillating Hamiltonian with frequency being $n\omega$ where $\omega = 2\pi/L$, and $n$ is a series of integer. Specifically, the coupling strength is expanded as

$$c[d(z)] = c_0 e^{-\frac{d_1 + d_2 \sin(2\pi z)}{L}} = \sum_n c_n e^{\frac{2\pi n z}{L}}$$

, where

$$c_n = \int_0^L c_0 e^{-\frac{d_1 + d_2 \sin(2\pi z)}{L}} e^{i2\pi nz} dz$$

Thus, the Hamiltonian is expanded as $H(z) = \sum_{n=-\infty}^{+\infty} H_n e^{i2\pi n z}$. According to the Floquet theorem, the periodically varying Hamiltonian can be transformed into effective Floquet Hamiltonian $H_F$, which is axially-independent (independent of $z$). The effective Hamiltonian is consisted of diagonal block, which are $H_n$ with diagonal element being shifted by $n\omega$ for the $n$-th replica; non-diagonal block at the $n$-th row and $m$-th column as $H_{n-m}$. Diagonalization of the effective Hamiltonian gives the quasi-energy level $\varepsilon$ and the corresponding eigen-state $|\psi\rangle = \sum_n \varphi_{i,n} e^{i\varepsilon n z}$, where $\varphi_{i,n}$ is the amplitude at the $i$-th waveguide of the $n$-th replica. For high frequency axially oscillation, the Floquet replica can be truncated as $|\psi\rangle = \sum_{n=\text{Max}}^{n_{\text{Max}}+\text{Max}} \varphi_{i,n}$. In our numerical simulation, $n_{\text{Max}} = 2$ is applied, which have high accuracy.

III. NUMERICAL RESULT

The numerical result of waveguide arrays in three types of lattice structure is summarized in the following three subsections.

A. Honeycomb lattice with Kekule distortion

If the waveguide are arrayed in honeycomb lattice with Kekule distortion, the bulk energy band is gapped. The zigzag nanoribbons have two nearly flat bands near to zero energy, but the armchair nanoribbons remain gapped. We considered the nanoflake with armchair edge and four corners, including two zigzag corners and two armchair corners. When the static Kekule factor $d_1$ is positive, the coupling strength of intra-unit cell is weaker than that of inter-unit cell. A zero energy corner state appear at each zigzag corner. On the other hand, if $d_1$ is negative, there is not zero energy corner state.

Two cases with $d_1$ being positive and negative are calculated and plotted in Fig. 2(a,b) and (c,d), respectively. The amplitude of the dynamic Kekule factor $d_2$ is the same for both cases. When $d_1$ is positive, the zero energy corner states are eliminated by the dynamic modulation, when the frequency of the modulation is in certain region. For the particular case in Fig. 2(a), with $\omega/c_0 \in [1.60, 2.15]$ or $\omega/c_0 \in [2.55, 2.95]$, the zero energy corner states are eliminated. Within the region of $\omega/c_0 \in [2.15, 2.55]$ or $\omega/c_0 \in [2.95, 3.7]$, the bulk state is gapless at zero energy. When $\omega/c_0 > 3.7$, the zero energy corner state appears again. Within the region

FIG. 2: Quasi-energy spectrum of the Kekule distorted hexagonal lattice in Fig. 1(a) with $N=11$ as a function of $\omega/c_0$. The static Kekule distortion factors for the systems in the top and bottom rows are $d_1 = 0.1d_0$ and $d_1 = -0.1d_0$, respectively. The oscillating Kekule distortion factors for both systems are $d_2 = 0.15d_0$. The figures in the right column are the zoom in view of the spectrum with $\pi$ modes in the save system as the corresponding figure in the left column. The band of the $\pi$ mode are marked by red (thick) lines. The band of the corner mode at the armchair corner are marked by blue (thick) lines. (e) and (f) are the spatial distribution of the mode amplitude of the $\pi$ modes and the armchair corner mode, respectively.
\(\omega/c_0 \in [4.05, 5.70]\), a Floquet energy gap of bulk states is open around the quasi-energy level \(\omega/2\). The \(\pi\) mode appear within this energy gap, as shown by the red line in Fig. 2(a) and (b). The spatial distribution of the magnitude of a typical \(\pi\) mode is plotted in Fig. 2(c), which shows that the maximum of the mode magnitude is at the two sites that are nearest neighboring to the zigzag termination. With quasi-energy being above and below the \(\pi\) mode, there are two bands of edge states. Beyond the two bands lay two bands of armchair corner states, which is indicated by the red lines. The spatial distribution of the magnitude of a typical armchair corner state is plotted in Fig. 2(f). When the static Kekule factor \(d_1\) flips sign, the quasi-energy band structure is shown in Fig. 2(c) and (d). The zero energy corner state does not appear with varying \(\omega/c_0\). The \(\pi\) mode appear within the region of \(\omega/c_0 \in [2.30, 2.85]\), as shown by the red line. The two bands of armchair corner states lay between the quasi-energy level \(\omega/2\) and the other bands of quasi-energy level, as shown by the blue lines. In summary, the zero and \(\pi\) energy corner state at the zigzag terminations can be controlled by the Floquet engineering.

**B. Breathing square lattice**

For the waveguide arrays in breathing square lattice, the simulation results are summarized in Fig. 3. Each individual chain along \(x\) or \(y\) direction is an SSH model. As \(d_1\) being positive, the SSH model is topological with two zero energy states that are localized at both ends. When the chains are coupled to the neighboring chain to form the breathing square lattice, four bands of bulk states are form. Assuming that the static intra-cell and inter-cell coupling strength are \(c_{01} = c_0 e^{-d_1/b}\) and \(c_{02} = c_0 e^{d_1/b}\), the band structure of the four bands are \(E = \pm \sqrt{2} \sqrt[4]{c_0^2 e^{-d_1/b} + c_0 e^{d_1/b}} + c_{0x} \text{ with } c_{0x} = c_{01}^2 + c_{02}^2 + 2c_{01} c_{02} \cos k_x, c_{0y} = c_{01}^2 + c_{02}^2 + 2c_{01} c_{02} \cos k_y\) and \(c_{0xy} = c_{01}^2 + c_{02}^2 + c_{01} c_{02} (\cos k_x + \cos k_y), k_x(y) \in [-\pi, \pi]\) being the Bloch phase along the \(x(y)\) direction. Two bands (designated as band II and III) touch each other at high symmetric points, which form gapless energy spectrum around zero energy; two other bands (designated as band I and IV) are gapped from the former two bands with higher and lower energy level. The end state at the end of each SSH chain is coupled with the end states at the neighboring chains with alternating strength. The coupling end states form edge bands, which mimic a one dimensional edge SSH model along each edge. The edge SSH model have two bands (designated as SSH edge states), whose energy spectrum lay within the gap between band III and IV (or band I and II). The propagating SSH edge states in each edge are bounded back and forth between two corners, which form standing waves with quasi-continue energy spectrum in a finite flake. As the dynamic breathing factor \(d_2\) being turned on, the SSH edge states of different Floquet replica couple to each other. When \(\omega\) is near to the energy difference between the two bands of the SSH edge states, the coupling between the 0-th and 1-st Floquet replica is large. In this case, a band gap around the quasi-energy level \(\omega/2\) is opened. Within each Floquet dynamic gap, the effective model of the driven SSH edge states is topological. The topological number of the neighboring edges are opposite, due to the shift of half period along the length direction of the edges. Thus, there are two corner states at each corner with quasi-energy level being \(\omega/2\). The edge of the breathing square lattice has four corners, so that eight localized corner states appear at the quasi-energy level \(\omega/2\), as indicated by the red line in Fig. 3(a). The spatial distribution of magnitude of the corner mode is plotted in Fig. 3(c). The maximum magnitude is located at two lattice sites that is nearest neighbor to the corner termination. In the opposite case when \(d_1\) is negative, the SSH model of each individual chain is topologically trivial. Thus, the SSH edge states does not exist, and then the \(\pi\) mode does not exist either, as shown in Fig. 3(b).

If the number of primitive unit cell along \(x\) and \(y\) direction is an integer plus \(\frac{1}{2}\) (the number of lattice sites in the 1D chain is odd number), the numerical result of
the quasi-energy spectrum is shown in Fig. 3(c) and (d). If a chain of SSH model has odd number of sites, one of the end always host a topological end state, because the nearest neighboring bonding to the terminations at the two ends have different strength. If the bulk state corresponding to the left end is trivial, then that corresponding to the right end is topological. Similarly, if the breathing square lattice has odd number of lattice sites in each individual chain along $x$ and $y$ direction, two of the four edges have SSH edge states. Between the two edges with SSH edge states, two corner states with quasi-energy level $\omega/2$ appear. Between the trivial and topological edges, only one corner state with quasi-energy level $\omega/2$ appears. Thus, there are four corner states in total. The energy spectrum with the corner states are indicated by the red lines in Fig. 3(c) and (d). The spatial distribution of the magnitude of corner states are plotted in Fig. 3(f) for the systems in Fig. 3(c). In this case, the left and bottom edges are topological, so that two corner states are localized at the left-bottom corner, and one corner state are localized at each of left-top and right-bottom corners.

C. Breathing Kagome lattice

For the waveguide arrays in breathing Kagome lattice, the simulation results are summarized in Fig. 4. The static Kagome lattice with $d_1 > 0$ and $d_2 = 0$ is in the second order topological insulator phase, which has zero energy corner states [2]. As the dynamic breathing factor $d_2$ being turned on, the zero energy states remain being robust, as shown in Fig. 4(a). In addition, other corner states appear within the dynamic gap, whose energy level are marked by the thick (red) lines. The spatial distribution of magnitude of a typical corner mode is plotted in Fig. 4(d). The dynamic corner states only appear near to the energy levels $\frac{1}{4}\omega$ and $-\frac{3}{4}\omega$. If $d_1$ is zero, the bulk state of the static systems are gapless at zero energy. The dynamic breathing opens Floquet band gap around the energy levels $\frac{1}{4}\omega$ and $-\frac{3}{4}\omega$, which also have corner states, as shown in Fig. 4(b). The static Kagome lattice with $d_1 < 0$ and $d_2 = 0$ is topologically trivial. The dynamic breathing with lower frequency open Floquet band gaps around the energy levels $\frac{1}{2}\omega$ and $-\frac{1}{2}\omega$. As the frequency of the dynamic breathing being larger, another two Floquet band gaps are opened around the energy levels $\frac{3}{4}\omega$ and $-\frac{3}{4}\omega$. The Floquet band gaps at both of the two frequency range have in-gap corner states, as shown in Fig. 4(c). For all cases in the breathing Kagome lattice, the energy level of the corner state within one Floquet gap are slightly dependent on the frequency of the dynamic breathing, which is due to the absence of the particle-hole symmetric in the model Hamiltonian.

FIG. 4: Quasi-energy spectrum of the breathing kagome lattice in Fig. 1(c) with $N = 10$, as a function of $\omega/\omega_0$. The static breathing factor is $d_1 = 0.2d_0/\sqrt{3}$ for (a), $d_1 = 0$ for (b), and $d_1 = -0.2d_0/\sqrt{3}$ for (c). The oscillating breathing factors for both systems are $d_2 = 0.2d_0/\sqrt{3}$. The band of the $\pm\pi/3$ and $\pm2\pi/3$ mode are marked by red (thick) lines. (d) is the spatial distribution of the mode amplitude of a typical $\pm\pi/3$ or $\pm2\pi/3$ modes.

IV. DISCUSSION AND CONCLUSION

The numerical results show that the dynamic model of the three types of photonic waveguide arrays have Floquet corner states with energy levels being $\pm\frac{1}{2}\omega$ or $\pm\frac{1}{3}\omega$. For the two models in honeycomb lattice with dynamic Kekule distortion and Kagome lattice with dynamic breathing, the dynamic coupling generates Floquet gap directly from the bulk states. As a result, for arbitrary $d_1$, the dynamic coupling with nonzero $d_2$ and appropriate frequency could always open the Floquet band gap. On the other hand, for the model in square lattice with dynamic breathing, the Floquet band gap is generated from the band of the SSH edge states. For the case that each 1D chain are trivial SSH lattice, the SSH edge states is absent, so that the Floquet band gap is also absent.

Comparing the efficiency of the three lattice models, the breathing Kagome lattice has the fewest lattice site in one unit cell, which result in the highest localization. By comparing the spatial distribution of magnitude in Fig. 2(e,f), 3(e,f) and Fig. 4(d), the corner states in the honeycomb lattice with Kekule distortion and in the breathing square lattice are mainly localized at the two lattice
sites neighboring to the corner termination; those in the breathing Kagome lattice are mainly localized at the terminating lattice site at the corner. Thus, the breathing Kagome lattice is the most suitable for the application that require higher localization. On the other hand, the energy levels of the corner states in the breathing Kagome lattice are not fixed to a certain value, but slightly varying around $\pm \frac{1}{(2)} \omega$. Thus, the other two lattice models are more suitable for the application that require higher accuracy of the energy level.

In conclusion, the Floquet corner states in the photonic waveguide arrays can be engineered by the periodic modification of the coupling strength between neighboring waveguides. The three type of lattice structures could have corner states at the outer corner, which is intersection of two outer edges (edge between the lattice structure and vacuum). As comparison, the corner states in the photonic second-order topological insulator are usually at the corner which is intersection of two inner edges (edge between a topological lattice and a trivial lattice). The lattice modes with Floquet corner state could be further engineering as photonic crystal fiber, whose optical mode is localized at the outer corner.

Acknowledgments

This project is supported by the startup grant at Guangdong Polytechnic Normal University (Grant No. 2021SDKYA117) and the National Natural Science Foundation of China (Grant No. 11704419).

References

[1] Eslam Khalaf, Phys. Rev. B 97, 205136(2018).
[2] Motohiko Ezawa, Phys. Rev. Lett. 120, 026801(2018).
[3] Motohiko Ezawa, Phys. Rev. B 98, 045125(2018).
[4] Guido van Miert and Carmine Ortix, Phys. Rev. B 98, 081110(R)(2018).
[5] Akishi Matsugatani and Haruki Watanabe, Phys. Rev. B 98, 205129(2018).
[6] S. Franca, J. van den Brink, and I. C. Fulga, Phys. Rev. B 98, 201114(R)(2018).
[7] Weixuan Zhang, Deyuan Zou, Qingsong Pei, Wenjing He, Jiacheng Bao, Honjun Sun, and Xiangdong Zhang, Phys. Rev. Lett. 126, 146802(2021).
[8] Yuanfeng Xu, Zhida Song, Zhijun Wang, Hongming Weng, and Xi Dai, Phys. Rev. Lett. 122, 256402(2019).
[9] Huanhuan Yang, Z.-X. Li, Yuanyuan Liu, Yunshan Cao, and Peng Yan, Phys. Rev. Research 2, 022028(R)(2020).
[10] Huahui Qiu, Meng Xiao, Fan Zhang, and Chunyin Qiu, Phys. Rev. Lett. 127, 146601(2021).
[11] Bi-Ye Xie, Hong-Fei Wang, Hai-Xiao Wang, Xue-Yi Zhu, Jian-Hua Jiang, Ming-Hui Lu, and Yan-Feng Chen, Phys. Rev. B 98, 205147(2018).
[12] Bi-Ye Xie, Guang-Xu Su, Hong-Fei Wang, Hai Su, Xiao-Peng Sheng, Peng Zhan, Ming-Hui Lu, Zhen-Lin Wang, and Yan-Feng Chen, Phys. Rev. Lett. 122, 233903(2019).
[13] Xiao-Dong Chen, Wei-Min Deng, Fu-Long Shi, Fu-Li Zhao, Min Chen, and Jian-Wen Dong, Phys. Rev. Lett. 122, 233902(2019).
[14] Tanay Nag, Vladimir Juricić, and Bitan Roy, Phys. Rev. B 103, 115308(2021).
[15] Jiaxin Pan and Longwen Zhou, Phys. Rev. Research 2, 043431(2020).
[16] Tomoki Ozawa, Hannah M. Price, Alberto Amo, Nathan Goldman, Mohammad Hafezi, Ling Lu, Mikael C. Rechtsman, David Schuster, Jonathan Simon, Oded Zilberberg, and Iacopo Carusotto, Rev. Mod. Phys. 91, 015006(2019).
[17] Raditya Weda Bomantara, Longwen Zhou, Jiaxin Pan, and Jiangbin Gong, Phys. Rev. B 99, 045441(2019).
[18] Hong Wu, Bao-Qin Wang, and Jun-Hong An, Phys. Rev. B 103, L041115(2021).
[19] Jiaxin Pan and Longwen Zhou, Phys. Rev. B 102, 094305(2020).
[20] Tanay Nag, Vladimir Juricić, and Bitan Roy, Phys. Rev. Research 1, 032045(R)(2019).
[21] Ranjani Seshadri, Anirban Dutta, and Diptiman Sen, Phys. Rev. B 100, 115403(2019).
[22] Martin Rodriguez-Vega, Abhishek Kumar, and Babak Seradjeh, Phys. Rev. B 100, 085138(2019).
[23] Raditya Weda Bomantara, Luz M. Carreño, and Jiangbin Gong, Phys. Rev. B 103, L041402(2021).
[24] Akishi Matsugatani and Haruki Watanabe, Phys. Rev. B 101, 085401(2020).
[25] Arnob Kumar Ghosh, Tanay Nag, and Arijit Saha, Phys. Rev. B 101, 085424(2021).
[26] Raditya Weda Bomantara, Phys. Rev. Research 2, 033495(2020).
[27] Kirill Plekhanov, Manisha Thakurathi, Daniel Loss, and Jelena Klinovaja, Phys. Rev. Research 1, 032013(R)(2019).
[28] Raditya Weda Bomantara and Jiangbin Gong, Phys. Rev. B 101, 085401(2020).
[29] S. Franca, J. van den Brink, and I. C. Fulga, Phys. Rev. B 101, 085424(2020).
[30] Arnob Kumar Ghosh, Tanay Nag, and Arijit Saha, Phys. Rev. B 103, 045441(2019).
[31] Martin Rodriguez-Vega, Abhishek Kumar, and Babak Seradjeh, Phys. Rev. B 100, 085138(2019).
[32] Raditya Weda Bomantara, Phys. Rev. B 103, L041115(2021).
[33] Arnob Kumar Ghosh, Tanay Nag, and Arijit Saha, Phys. Rev. B 103, 045441(2019).
[34] Yuanfeng Xu, Zhida Song, Zhijun Wang, Hongming Weng, and Xi Dai, Phys. Rev. Lett. 126, 146802(2021).
[35] Haochen Wang, Peng Zhan, Ming-Hui Lu, Zhen-Lin Wang, and Yan-Feng Chen, Phys. Rev. Lett. 122, 233903(2019).
[36] Weiwei Zhu, Y. D. Chong, and Jiangbin Gong, Phys. Rev. B 98, 205114(R)(2018).
[37] Mark J. Ablowitz and Justin T. Cole, Phys. Rev. A 99, 033495(2020).
[38] Daniel Leykam, M. C. Rechtsman, and Y. D. Chong, Phys. Rev. E 67, 036601(2003).
[39] Tomoki Ozawa, Hannah M. Price, Alberto Amo, Nathan Goldman, Mohammad Hafezi, Ling Lu, Mikael C. Rechtsman, David Schuster, Jonathan Simon, Oded Zilberberg, and Iacopo Carusotto, Rev. Mod. Phys. 91, 015006(2019).
[40] Ruixing Zhang and Zhi-Cheng Yang, Phys. Rev. B 103, 115308(2020).
[41] Swati Chaudhary, Arbel Haim, Yang Peng, and Gil Refael, Phys. Rev. Research 2, 043431(2020).
[42] Arnob Kumar Ghosh, Ganesh C. Paul, and Arijit Saha, Phys. Rev. B 101, 235403(2020).
[43] Biao Huang and W. Vincent Liu, Phys. Rev. Lett. 124, 216601(2020).
[44] Haiping Hu, Biao Huang, Erhai Zhao, and W. Vincent Liu, Phys. Rev. Lett. 124, 057001(2020).
[45] Bo Zhu, Honghua Zhong, Yongguan Ke, Xiaozhi Qin, An-
[38] J. Petráček and V. Kuzmiak, Phys. Rev. A 101, 033805 (2020).
[39] Yiming Pan and Bing Wang, Phys. Rev. Research 2, 043239 (2020).
[40] A. Bisianov, A. Muniz, U. Peschel, and O. A. Egorov, Phys. Rev. A 102, 053511 (2020).
[41] Shengjie Wu, Wange Song, Shenglun Gao, Yuxin Chen, Shining Zhu, and Tao Li, Phys. Rev. Research 3, 023211 (2021).
[42] Lu Qi, Yan Xing, Xue-Dong Zhao, Shitian Liu, Shou Zhang, Shi Hu, and Hong-Fu Wang, Phys. Rev. B 103, 085129 (2021).
[43] Qingqing Cheng, Yiming Pan, Qianjin Wang, Tao Li, and Shining Zhu, Laser Photonics Rev. 9, No. 4, 392-398 (2015).
[44] Qingqing Cheng, Yiming Pan, Huaiqiang Wang, Chaoshi Zhang, Dong Yu, Avi Gover, Haijun Zhang, Tao Li, Lei Zhou, and Shining Zhu, Phys. Rev. Lett. 122, 173901 (2019).
[45] Sebabrata Mukherjee, Alexander Spracklen, Manuel Valiente, Erika Andersson, Patrik Öhberg, Nathan Goldman and Robert R. Thomson, Nat. Commun. 8, 13918 (2017).
[46] Lukas J. Maczewsky, Julia M. Zeumer, Stefan Nolte and Alexander Szameit, Nat. Commun. 8, 13756 (2017).