Wave-like warp propagation in circumbinary discs – II. Application to KH 15D

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ABSTRACT
KH 15D is a protostellar binary system that shows a peculiar light curve. In order to model it, a narrow circumbinary precessing disc has been invoked, but a proper dynamical model has never been developed. In this paper, we analytically address the issue of whether such a disc can rigidly precess around KH 15D, and we relate the precessional period to the main parameters of the system. Then, we simulate the disc’s dynamics by using a 1D model developed in a companion paper, such that the warp propagates into the disc as a bending wave, which is expected to be the case for protostellar discs. The validity of such an approach has been confirmed by comparing its results with full 3D smoothed particle hydrodynamics simulations on extended discs. In the present case, we use this 1D code to model the propagation of the warp in a narrow disc. If the inner truncation radius of the disc is set by the binary tidal torques at $\sim 1$ au, we find that the disc should extend out to 6–10 au (depending on the models), and is therefore wider than previously suggested. Our simulations show that such a disc does reach an almost steady state, and then precesses as a rigid body. The disc displays a very small warp, with a tilt inclination that increases with radius in order to keep the disc in equilibrium against the binary torque. However, for such wider discs, the presence of viscosity leads to a secular decay of the tilt on a time-scale of $\approx 3000(\alpha/0.05)^{-1}$ yr, where $\alpha$ is the disc viscosity parameter. The presence of a third body (such as a planet), orbiting at roughly 10 au might simultaneously explain the outer truncation of the disc and the maintenance of the tilt for a prolonged time.

Key words: accretion, accretion discs – hydrodynamics – protoplanetary discs – circumstellar matter – stars: individual: KH 15D.

1 INTRODUCTION

It is well known that young protostars are commonly accompanied by a circumstellar disc accreting on to the central star. Moreover, the majority of stars form in binary or higher multiple systems (Ghez, Neugebauer & Matthews 1993; Simon et al. 1995; Clarke, Bonnell & Hillenbrand 2000), and circumstellar and/or circumbinary discs are likely to be present in these systems (Dutrey, Guilloteau & Simon 1994; Beust & Dutrey 2005).

Close encounters in the early stages of formation when stars are surrounded by massive discs lead to substantial disc truncation and the tidal effects of such encounters can also lead the outer parts of the disc to become strongly misaligned or warped (Heller 1995; Moeckel & Bally 2006). Bate, Lodato & Pringle (2010) have studied numerically the evolution of such a chaotic environment in star-forming regions. An observational signature of these dynamical effects, where a perturbation induces either a warp or a strong misalignment in a circumstellar disc, is the misalignment between the stellar rotation axis (determined mostly by the angular momentum of material accreted early in the star formation process) and the disc rotation axis (which might correspond to the angular momentum of material accreted later, and eventually of planets). In the case of binary systems, an analogous signature is the misalignment between the binary plane and the disc average plane. Moreover, it has been shown that even simple initial conditions with small asymmetries of the original cloud core can produce a misalignment between the two planes (Bonnell et al. 1992). Such a disc would be subject to the external torque caused by the binary potential, which would generate a warp in the disc itself. Thus, misaligned circumbinary discs are quite likely to occur in young binary systems (Chiang & Murray-Clay 2004; Akeson et al. 2007).

In a companion paper (Facchini et al. 2013, hereafter FL) we have developed a 1D numerical model describing the temporal evolution of the disc.
of the shape of a generic circumbinary disc. We have focused on
the wavelike regime (see Section 3.3), which occurs whenever \( \alpha < H/R \) (Papaloizou & Pringle 1983; Papaloizou & Lin 1995), where \( \alpha \) is the standard viscosity parameter (Shakura & Sunyaev 1973) and \( H/R \) is the aspect ratio of the disc. Instead, when \( H/R < \alpha < 1 \) the equations describing the evolution are diffusive (Pringle 1992).
The bending waves regime is therefore more suitable to describe protostellar discs, where the aspect ratio is likely to be \( \sim 0.1 \) (e.g. see Lodato 2008), while \( \alpha \leq 0.01 - 0.1 \). In FL we have tested our model by comparing our results with full 3D hydrodynamical simulations, and found a very good agreement.

In this paper, we apply the model developed by FL to a specific case: KH 15D. Chiang & Murray-Clay (2004, hereafter CM) have described such a system as a misaligned dust laden circumbinary ring precessing around a binary system formed by two similar protostars. Such an environment is suitable in order to describe the dynamical evolution of the precessing narrow disc with our model. We will need to make several approximations and assumptions, but we will show that we are able to fully reproduce the precession of the ring, and to make many relevant dynamical considerations.

In Section 2 we summarize the observations of KH 15D and we describe the model derived by CM. In Section 3 we derive the theoretical precessional period from our 1D model and we explore the issue of rigid precession. In Section 4 we report our results and we compare them with the model by CM. Finally, in Section 5 we draw our conclusions.

2 MODELLING THE MAIN SYSTEM PARAMETERS

KH 15D is an object that presents a T-Tauri-like spectrum, and has been classified as a K pre-main-sequence star with an age of \( \sim 3 \) Myr (Herbst et al. 2010). Since 1998, astronomers realized that this object presented a peculiar light curve (Kearns & Herbst 1998), thus this star has been the object of many observations over the years. Moreover, researchers have found archival data of its light curve since 1913. This fact has given the possibility to analyse the temporal evolution of the light curve for almost 100 yr. The light curve shows some very interesting features that are reported in many papers of the last decade (Herbst et al. 2002, 2010; Winn et al. 2003; CM; Johnson & Winn 2004; Capelo et al. 2012). In order to have a detailed description, which is far from being the aim of this paper, interested readers can look at CM and Herbst et al. (2010). KH 15D undergoes very deep periodic eclipses (\( P = 48.35 \) d, \( \Delta L = 3.5 \) mag) and duration (24 d). This long duration of the minimum light suggests that this cannot be an ordinary eclipsing binary. Moreover, as reported above, the light curve undergoes a temporal evolution on a longer time-scale than the light curve itself. In particular, the eclipsing duty cycle has been increasing, and the in-eclipse light curve has shown a central reversal in brightness, which has lessened in time in terms of amplitude. Finally, from 1913 to 1951, no eclipse was observed. A model with the aim to describe such a system needs to cover all these photometric features.

A relevant question is whether KH 15D is a single or multiple stellar system. Herbst et al. (2002) and later Johnson et al. (2004) looked for evidence of a companion star via radial velocity measurements, by using highly resolved spectra from Very Large Telescope (VLT) and the High Resolution Echelle Spectrometer (HIERES) of Keck I Telescope, respectively. They both found a periodicity in the radial velocity that was compatible with the photometric one, thus pointing that the binarity of the system is one of the key features to model the light curve and its evolution. Thus, radial velocity mea-

urements suggested that KH 15D does have a companion, with almost equal mass (\( \sim 0.5 \) M\(_{\odot}\)) and temperature. Finally, Johnson et al. (2004) constrained the orbital eccentricity of the binary to \( 0.68 < e < 0.80 \).

Agol et al. (2004) described a model to reproduce KH 15D photometric variability by introducing a warped circumstellar disc. However, these models succeeded in reproducing the light curve of the system over short time-scales, but they could not explain its secular evolution. In this paper we focus on the model derived by Winn et al. (2004) and CM, where the photometric variability is caused by the partial occultation of the eccentric binary system by a narrow ring. This ring precesses rigidly on a plane that is inclined by an angle of \( \sim 0 \) with respect to the binary one, and eclipses occur whenever its ascending or descending node regresses into our line of sight towards the two stars. The observer is assumed to look at the binary plane nearly edge-on. For the geometrical details, the interested reader can look at figs 1 and 2 of CM. For our purposes, we will not discuss the uncertainties of this model in great detail, and will simply assume their derived parameters. In particular, we denote the star KH 15D with K, and its orbital companion with K’. The semimajor axis with respect to the centre of mass of the two stars, which have an equal mass \( M_K = M_{K'} = 0.5 \) M\(_{\odot}\), is \( a_K = a_{K'} = 0.13 \) au, so that the separation is \( a = 0.26 \) au. In order the light curve to be explained, a precession period of \( \sim 2770 \) yr is required. Obviously, from the measured light curve, the estimate of the precessional period is approximate, and there might be some degeneracy with other parameters (such as the disc radial extent \( \Delta R \), or its mean inclination \( \bar{l} \)).

Given the model above, we are interested in the dynamics of the rigidly precessing ring, and we want to address the following questions.

(i) Is this precessing ring stable?
(ii) Can the ring maintain rigid precession? How large need the warp be in order to prevent differential precession?
(iii) Can such a ring stay misaligned with respect to the binary plane over a reasonable time-scale?

3 PRECESSIONAL PERIOD AND RIGID PRECESSION

3.1 Relating the precession frequency to the forcing potential

The model of Section 2 considers a narrow disc rigidly precessing around the two binary stars. First of all, we relate the precessional frequency to the forcing torque generated by the spherical asymmetry of the system. We know that \( T(R) = \Omega_{ext}(R) \times L(R) \) (Lodato & Pringle 2006; FL), where \( \Omega_{ext}(R) \) is the free precession induced at a given radius \( R \) by the binary, \( T(R) \) is the external torque density and \( L(R) \) the angular momentum of the disc per unit area. We recall that \( L = \Sigma R^2 d \Omega, \) where \( \Sigma \) defines the surface density, \( d \Omega \) the angular frequency and \( R \) the unit vector indicating the local direction of the angular momentum. If we suppose that the vector \( \Omega_{ext} \) is along the z-direction (perpendicular to the binary plane), the only interesting components of \( L \) and \( T \) are the ones along the x and y-axes. Therefore, we can rewrite the above relation in terms of a scalar equation:

\[
T(R) = \Omega_{ext}(R)L(R),
\]

where \( L(R) = \Sigma R^2 \Omega \sqrt{l_x^2 + l_y^2} \) is the modulus of the projection of \( L \) on the x–y plane. The variable \( \Omega_{ext}(R) \) is the precessional frequency at which an isolated annulus would precess if it were
subject to the external torque \( T(R) \). We now suppose that the whole disc precesses with a global precessional frequency \( \Omega_p \). By requiring that \( \Omega_p \) is the same for the whole disc (rigid precession) and by integrating equation (1) from \( R_{in} \) to \( R_{out} \) (defined as the inner and the outer edge of the disc, respectively) we obtain

\[
T_{tot} = \Omega_p L_{tot},
\]

where we have defined

\[
T_{tot} \simeq \int_{R_{in}}^{R_{out}} \Omega_p R L(R) 2\pi R \, dR
\]

and

\[
L_{tot} \simeq \int_{R_{in}}^{R_{out}} L(R) 2\pi R \, dR
\]

that are appropriate for small amplitude warps (which are expected for our ring, see below). Thus, we obtain the following relation for \( \Omega_p \):

\[
\Omega_p = \frac{\int_{R_{in}}^{R_{out}} \Omega_p R L(R) 2\pi R \, dR}{\int_{R_{in}}^{R_{out}} L(R) 2\pi R \, dR}.
\]

We then use the following relations:

\[
\Omega_{ext}(R) \propto R^{-\beta}, \quad L(R) = \Sigma R^2 \Omega \sqrt{l_x^2 + l_z^2} \propto R^{1/2-p},
\]

where we also assume that \( \Sigma \propto R^{-\beta} \). In the second relation we have considered \( \sqrt{l_x^2 + l_z^2} \approx \text{const.} \) (i.e. a small warp), a condition that is usually largely verified, especially in the case of KH 15D (see below). For an external torque due to a central binary, which is the case of KH 15D, the parameter \( s = 7/2 \). We have kept \( s \) as a generally free parameter because the same relation could also describe a disc precessing around a spinning black hole. In that case, \( s \) would be equal to 3. We scale the radial variable to the inner disc radius and define \( x = R/R_{in} \). By implementing the relations (6) in equation (5) we obtain

\[
\Omega_p = \Omega_{ext}(R) \int_{x_{out}}^{1} x^{3/2-p-1} \, dx \int_{x_{in}}^{1} x^{3/2-p} \, dx,
\]

where \( x_{out} = R_{out}/R_{in} \). In reasonable set-ups, the condition \( 0 < p < S/2 \) is verified, and since we know that \( s \geq 3 \), we have that \( s + p > 5/2 \). Therefore we obtain

\[
\Omega_p = \Omega_{ext}(R) \frac{1 - x_{out}^{5/2-p}}{x_{out}^{5/2-p} - 1} \frac{(5/2 - p)}{(s + p - 5/2)}.
\]

In the case where \( R_{out}/R_{in} \gg 1 \), by knowing that the last factor on the right-hand side (rhs), that depends on \( s \) and \( p \), is \( \approx O(1) \), we can approximate relation (8) to the following one:

\[
\Omega_p \approx \Omega_{ext}(R) \left( \frac{R_{out}}{R_{in}} \right)^{\frac{p-1}{2}}.
\]

Equation (9) contains two factors. The first indicates that the precession is forced by the external torque at the inner edge. The second is due to the fact that most of the angular momentum lies in the outer region of the disc [since \( M(R) \propto R^{2-p} \)], therefore, the larger the outer radius, the longer the precessional period.

In order to have an estimate of the free precessional frequency, we need the forcing potential of the binary case. Therefore we focus on the case where \( s = 7/2 \). In FL we have deduced it as a time-independent gravitational potential, in which the forcing term depends on the non-spherical symmetry of the gravitational source. The approximation is to spread the mass of the two stars on two

## 3.2 Conditions to maintain rigid precession

Hitherto we have deduced the precessional frequency of the disc by assuming that it can precess as a rigid body. In order to do so, we have mostly used the parameters defining the gravitational potential. Now we address the issue of whether a disc, and in particular KH 15D, can precess rigidly. Therefore now we focus on the properties of the disc. We have already defined the surface density profile. We then know that the sound speed is related to the disc scale height by \( H = c_s/\Omega \), and we will assume that \( c_s \propto R^{-3/4} \) throughout the paper. We set \( H_{in}/R_{in} = 0.1 \) and \( \alpha = 0.05 \).

Papaloizou & Terquem (1995) and Larwood & Papaloizou (1997) developed the arguments introduced by Papaloizou & Lin (1995), and deduced that the disc can rigidly precess when

\[
\Omega_p < \frac{c_s}{\Delta R}.
\]

where \( \Delta R = (R_{out} - R_{in}) \). The sound crossing time for the disc thus needs to be shorter than the inverse of the precession frequency. In other words, in order to precess rigidly, the disc needs to communicate the precessional frequency to the outer radii, since the precessional frequency is mostly fixed by the external torque at the inner edge where the external torque is stronger. From equation (13) we obtain

\[
\frac{\Delta R}{R} < \frac{H}{\Omega \Omega_p},
\]

which evaluated at the outer edge becomes, in a slightly different form:

\[
\frac{\Delta R}{R_{out}} < \frac{H_{out} \Omega_{out}}{R_{out} \Omega_p}.
\]

There is one more requirement that the disc has to verify. In the presence of viscosity, the disc will secularly align with the binary plane. We thus have to require that such alignment time is longer.
than the precessional period $t_{\text{align}} > 1/\Omega_{p}$, as well as the time since the tilt was initiated. We briefly address this issue in Section 3.3, where we summarize the equations that describe the bending waves propagation in a disc, and that intrinsically regulate the dissipation time-scale of the misalignment.

### 3.3 Equations of warp propagation

The dynamics of warped discs has been discussed in several papers. When $H/R < \alpha < 1$, the evolution of the warp is described by diffusive equations (Papaloizou & Pringle 1983; Pringle 1992; Ogilvie 1999), whereas when $\alpha < H/R$ the evolution is tracked by wave equations (Papaloizou & Lin 1995; Lubow & Ogilvie 2000, 2001). As discussed in Section 1, protostellar systems are more likely to be in the bending wave regime, since usually $\alpha < H/R$. In this paper, as we have done in FL, we use the formulation given by Lubow & Ogilvie (2000). Under the assumption that $\partial, \Sigma \approx 0$, the linearized equations are

$$\Sigma R^2 \frac{\partial l}{\partial t} = \frac{1}{R} \frac{\partial G}{\partial R} + T,$$

(15)

and

$$\frac{\partial G}{\partial t} + \left( \frac{\lambda^2 - \Omega^2}{\Omega^2} \right) \Omega \times G + \alpha \Omega G = \Sigma R^2 \frac{\partial l}{4 \partial R},$$

(16)

where $T$ is the external torque expressed in equation (10), $\Omega$ is the vertical oscillation frequency, $\lambda$ is the epicyclic frequency and $2\pi G$ is the internal torque. The explicit expressions for the relevant frequencies $\Omega$, $\Omega_{p}$, and $\kappa$ can be found e.g. see FL. Note that the third term of the rhs of equation (16) dissipates the waves through an exponential factor (Bate et al. 2000). This term indicates that for linear bending waves the damping occurs on a time-scale $t_{\text{damp}} \approx 1/(\alpha \Omega)$. Even in the case in which the disc precesses as a rigid body, and the tilt reaches a steady state solution (a stationary wave), viscosity still dissipates energy, and the disc tends to get to the lower energy state, which is the alignment with respect to the binary plane. For such a rigidly precessing disc, Papaloizou & Terquem (1995), Larwood et al. (1996) and Bate et al. (2000) estimate that the alignment time-scale $t_{\text{align}}$ is of the order of the viscous time-scale $t_{\nu} = R_{\nu}^{2}/\nu$, where $\nu$ is the kinematic viscosity ($\nu = \alpha \epsilon H$). More precisely, Bate et al. (2000) (see also Lubow & Ogilvie 2001) derived an expression for $t_{\text{align}}$:

$$t_{\text{align}} = \frac{1}{\alpha \Omega_{p}} \left( \frac{H_{\text{in}}}{R_{\text{in}}} \right)^{2} \left( \frac{\Omega(R_{\text{out}})}{\Omega_{p}} \right).$$

(17)

This expression has been derived for the cases where $R_{\text{out}}/R_{\text{in}} > 1$. Note that in equations (15) and (16) dissipation, and therefore alignment, is automatically implemented via the viscous term.

With these equations we have described the disc as a series of annuli that can interact by pressure and viscous forces, and can both evolve in the inclination angle and precess. We can solve them by using the ring code described in FL, which is a similar version of the 1D code used by Lubow, Ogilvie & Pringle (2002) to simulate a disc affected by a Lense–Thirring external torque.

### 4 RESULTS

The discussion we have gone through so far is quite general. In this section we discuss more specifically the case of KH 15D, and compare our results with the ones suggested by CM.

### Table 1. Estimates of the most relevant quantities for the two models.

| Parameter       | $p = 0.5$ | $p = 1$ |
|-----------------|-----------|---------|
| $R_{\text{in}}$ (au) | 1.00      | 1.00    |
| $R_{\text{out}}$ (au) | 6.71      | 9.00    |
| $T_{p}$ (yr) analytical | 2770      | 2770    |
| $T_{p}$ (yr) numerical | 2670      | 2680    |
| $\Delta t/f$ | 0.012 | 0.040 |
| $t_{\text{align}}$ (yr) analytical | 14 051 | 9045 |
| $t_{\text{align}}$ (yr) numerical | 4744 | 3012 |

4.1 Analytical results

By using equation (12), and implementing KH 15D’s parameter in it, we obtain an estimate for the precessional period:

$$T_{p} = 78.9 \left( \frac{p+1}{5/2-p} \right) \left( \frac{x_{\text{out}} - p - 1}{1 - x_{\text{out}}^{p-1}} \right) \text{yr},$$

(18)

where we estimate $R_{\text{in}}$ as the radius at which the binary potential tidally truncates the disc. From Artymowicz & Lubow (1994) we know that for binary systems with such a high eccentricity $R_{\text{in}} \approx 4a$. Since $\alpha \approx 0.26$ au, we set $R_{\text{in}} = 1$ au. Note that this value is much lower than the one assigned by CM in their work. We can now derive an estimate for $R_{\text{out}}$ from equation (18), by setting $T_{p} = 2770$ yr. We set the parameter $p$ to two possible values: $p = 0.5$ and 1. We obtain $R_{\text{out}} = 6.71$ and 9.00 au, respectively (see Table 1). Both values are larger than the one used by CM in their numerical model.

More important, we have checked that the disc verifies the condition expressed by relation (14), with the parameters of both models. Such discs can therefore precess rigidly.

4.2 Numerical results

We run simulations for the two models reported in Table 1. We set $\alpha = 0.05$, a typical value for protostellar discs that ensures that the warp propagates as a bending wave. We have also performed simulations with a different viscosity, in order to explore the dependence of the alignment time-scale on $\alpha$. We also adopt the following choice for the remaining parameters: $H_{\text{in}}/R_{\text{in}} = 0.1, c_s \propto R^{-3/4}, \eta = M_{K} M_{K}/M = 1/4$ and $\alpha = 0.26$ au.

We assume a zero-torque boundary condition for the horizontal torques both at the inner and at the outer radius. As initial condition we set a warped untwisted disc. The warp initially propagates inwards and outwards in a wave-like fashion (Fig. 1). When the two waves reach the edges of the disc, they bounce back into the disc. This process continues until the tilt reaches a quasi-steady state, which corresponds to a stationary wave. The disc tends to an almost flat, misaligned steady state (shown in Fig. 2), with almost no twist. From Paper I, we expect a small twist to be present for a viscous disc, and as predicted its amplitude increases with the value of $\alpha$. The disc gets to its steady state in a time $\sim 4000 \Omega_{\text{in}}^{-1}$, after which it starts precessing regularly. If we observe the $x$ and $y$ components of the tilt, we observe a sinusoidal dependence on time (Fig. 3).
Figure 1. From top left- to bottom right-hand panel: KH 15D’s tilt evolution of an initial warped untwisted disc. In this case, the disc extents from $R_{\text{in}} = 1$ au to $R_{\text{out}} = 6.71$ au ($p = 0.5$). The time values are expressed in dynamical time units $\Omega_0^{-1}$, where $\Omega_0 = \Omega(R = 1)$.

The disc dynamics, the precessional period and the tilt final state do not depend on the initial shape of the warp. We have run other simulations with different initial conditions (a flat misaligned disc and a warped disc with inclination decreasing outwards), and we obtain the same results. We conclude that the disc does precess as a rigid body, with an almost flat, untwisted shape. This result confirms the analytical considerations we have made in Section 4.1. From Fig. 3, we also notice clearly that the tilt declines exponentially with time, ...
under the effect of viscosity. We discuss this process more in detail below.

From the simulations we obtain a precessional period that is in a very good agreement with the theoretical prediction in both models (see Table 1).

With the numerical simulations, we are able to address another relevant issue: the amplitude of the warp. We know that a warp has to be present, in order for internal pressure torques to compensate the differential external torque generated by the central binary. As shown in Fig. 2, however, the amplitude of the warp is very small in both models and we report them in Table 1 in terms of $\Delta I/\bar{I}$, where $\Delta I = I(R_{\text{out}}) - I(R_{\text{in}})$. This is due to the fact that the external torque is not strong enough to induce a large warp. Obviously, the closer is $R_{\text{in}}$ to the stars, the larger is the warp. Our amplitudes are significantly smaller than the ones predicted by CM, where they deduce the value of $\Delta I/\bar{I}$ from an empirical relation (cf. equation 4 in their paper). Moreover, they propose that the inclination scales with radius as $I \propto R^{-2}$ in a disc dominated by a thermal pressure. This is unphysical. The inclination must increase with radius, since the external torque is much stronger at the inner edge. Finally, we have found that the amplitude of the warp depends very weakly on $\alpha$. The main difference when changing $\alpha$ is the time-scale in which the tilt reaches a steady state: when $\alpha$ is low, it takes a longer time to reach it, as expected. The expected amplitude of the warp for discs around circumbinary discs has been recently computed also by Foucart & Lai (2013). Our results are in broad agreement with theirs. Applying their estimates to the case at hand, we get $\Delta I/\bar{I} \approx 0.03(0.02)$ for the case $p = 0.5(1)$. While the order of magnitude is the same as that obtained in our simulations, their precise values differ by a factor of 2–3, and we do not see the strong dependence on $\alpha$ predicted by their results. Moreover, the dependence on the parameter $p$ appears to be different (which might be due to the fact that we impose a specific profile for the sound speed, while they assume it to depend on $p$ as well). Finally, note that the calculations by Foucart & Lai (2013) apply to a steady disc extending to infinity, while our ring is narrow radially and is not steady, but rigidly precessing.

We have answered the first two questions listed at the end of Section 1. We now address the third issue: how long is the alignment time-scale $t_{\text{align}}$ with respect to the precession period of such a disc? We have evaluated the alignment time by fitting the exponential decay of the tilt over a long time (see Fig. 3) with a function of the form $\exp(-t/t_{\text{align}})$. The fit is very accurate and we obtain
\[ t_{\text{align}} = 4474 \text{ and } 3012 \text{ yr (for } p = 0.5 \text{ and 1, respectively)}. \]

Thus, with \( \alpha = 0.05 \) (the value assumed in these simulations), the disc aligns with the binary within 1–2 precessional periods. However, we have verified that \( t_{\text{align}} \) is inversely proportional to \( \alpha \), as predicted by equation (17). For example, if we consider the model with \( p = 1 \), we can express the alignment time-scale as a function of \( \alpha \) only:

\[ t_{\text{align}} \approx 3000 \left( \frac{0.05}{\alpha} \right) \text{ yr.} \quad (19) \]

In order to maintain a non-zero inclination for a significant number of precessional times, we require \( \alpha \lesssim 0.005 \), a relatively low value, that may be compatible with a rather evolved protostellar disc, that is expected to be cold such that the magnetorotational instability might not provide a large stress.

Note that our numerically derived values of \( t_{\text{align}} \) are of the same order of those estimated through equation (17), from which we would obtain \( t_{\text{align}} = 14051 \) and 9045 yr (for \( p = 0.5 \) and 1, respectively) when \( \alpha = 0.05 \), a factor of 3 larger than what measured from the simulations.

### 4.3 Chiang and Murray-Clay model

In this paper most of the work is based on the observational model by CM. Given the parameters values known from the observations (reported at the end of Section 2), CM develop two models for the disc extent. A first one, where the differential precession is prevented by internal pressure, and a second one, where it is prevented by the disc self-gravity. In this section, we briefly compare the first of these two models with the ones developed in this paper. CM assume a disc inner edge of 2.25 au, and an outer edge of 3.75 au. They use these two parameters in order to have a precessional period of \( \approx 2770 \) yr (cf. equation 1 of their paper) and a narrow disc. Note that the choice of \( R_{in} = 2.25 \) au is arbitrary. We have performed numerical simulations with such parameters for the disc, and we obtain a precessional period of 3300 yr (which is also confirmed by the analytical formula of equation 18). Thus, also in this case the precessional period is in rough agreement with what required observationally, although in order to have such a narrow width, one is forced to fix the inner edge of the disc further out than the expected truncation radius due to the inner eccentric binary. Such a narrow disc model, on the other hand, has the property of decaying much more slowly than an extended one, thus not requiring a very low viscosity in order to be maintained long enough. In particular, we measure numerically a decay time of the order of \( 10^5 \) yr.

Finally, we also measure the warp amplitude needed to balance the binary torques and find that with these parameters it is as small as \( \Delta I/I \approx 10^{-4} \), two orders of magnitude smaller than that estimated by CM.

### 5 CONCLUSIONS

In this paper, we have developed the precessing disc model, introduced by CM, to describe the peculiar eclipsing binary system KH 15D. In particular, we have improved on previous analytical estimates of the disc fundamental parameters, such as its radial extent and the magnitude of the warp. We assume, following CM, that the light curve of the system is well described by a model where a stellar binary is surrounded by an inclined disc, rigidly precessing with a period of about 2770 yr. We fix the inner edge of the disc to the expected tidal truncation radius due to the binary, at \( \approx 1 \) au and find that, in order to reproduce the required precession time-scale, the disc needs to be wider than previously assumed, extending out to 5–10 au, depending on the slope of the disc surface density profile.

We test our model predictions by using the bending wave time-dependent formalism introduced by Lubow & Ogilvie (2000). We find indeed that the disc rapidly attains a rigidly precessing shape, with precession time-scales matching our analytical predictions. In such a steady precessing configuration, the disc needs to be warped, in order to counteract the binary torques. However, the warp amplitude \( \Delta I/I \) across the disc radial extent is much smaller than the one predicted by the formulae by CM for the corresponding parameters, and is of the order of \( \approx 0.01 \). Note that CM had incorrectly attributed a ‘reverse’ warp (where the disc inclination with respect to the binary decreases outwards) in the case of a warp propagating through pressure, rather than via self-gravity. Here we demonstrate that also in this case the disc is more aligned with the binary in its inner parts and less in the outer parts, in agreement with what indicated by recent observations (Capelo et al. 2012).

With such a wider disc, the issue of its long-term maintenance arises, as wider discs tend to align more rapidly with the binary plane. Our time-dependent calculations, that also include a dissipative viscous term, allow us to compute such decay rate. We find that the analytical formula for the decay rate derived by Bate et al. (2000) in the case of a tidally confined circumprimary disc works well also in our case. If \( \alpha = 0.05 \), the disc aligns with the binary within 1–2 precessional periods, depending on the model parameters. However, we have verified that the decay rate is inversely proportional to \( \alpha \), so that with \( \alpha \lesssim 0.005 \) (which might be not unreasonable for a cold and evolved protostellar disc such as the one surrounding KH 15D) a non-zero inclination can be maintained for a reasonable time. If the decay rate could be measured through its effect on the long-term light curve of the system, this would give a constraint on the magnitude of \( \alpha \). This kind of modelling shows that one can use warped discs effectively in order to put interesting constraints on the magnitude of the disc viscosity based on observed systems (see also King et al. 2013).

We have also tested numerically the model parameters introduced by CM. Such models are all characterized by a larger inner radius of the disc with respect to what expected based on the tidal truncation from the binary torques, and by a smaller radial extent with respect to ours. Nevertheless, such models do reproduce the correct precession period, but we find that the required warp amplitude is much smaller (by two orders of magnitude) than what inferred by CM. Such narrow rings, on the other hand, are able to maintain a non-zero inclination for much longer than our extended discs.

Our model is still simplified and approximate in many respects. First of all, note that the precessional time-scale of 2770 yr is not uniquely fixed by the observations, and might well range between 2000 and 3000 yr. This would reflect in a relatively modest change on the long-term light curve of the system, this would give a constraint on the magnitude of \( \alpha \). This kind of modelling shows that one can use warped discs effectively in order to put interesting constraints on the magnitude of the disc viscosity based on observed systems (see also King et al. 2013).

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The most important limitation of our model is the assumption that the binary orbit is circular, while we know that its eccentricity might range over 0.68 < \( e < 0.80 \). The effects of an eccentric orbit might be to push further back the inner edge of the disc. A detailed examination of such effects would require full 3D numerical simulations.

Finally, as in the case of the models by CM, an outstanding issue is the origin of such a narrow disc. Most likely the circumbinary disc is being confined by a third body (such a planetary companion or a very low mass star). Also the effects of such purported body on the long-term evolution of the system need to be explored more thoroughly.
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