Hadron diffractive scattering at ultrahigh energies, real part of the amplitude and Coulomb interaction

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On the basis of requirements of unitarity and analyticity we analyze the real and imaginary parts of the pp± scattering amplitudes at recent ultrahigh energies, 1-100 TeV. The predictions for the region $\sqrt{s} > 100$ TeV and $q^2 < 0.4$ GeV$^2$ are given supposing the black disk asymptotic regime. It turns out that the real part of the amplitude is concentrated in the impact parameter space at the border of the black disk. The interplay of hadron and Coulomb interactions is discussed in terms of the $K$-matrix function. The $pp$ diffractive scattering cross section at 7 TeV is calculated with Coulomb interaction taken into account.

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I. INTRODUCTION

Ultrahigh energy proton-proton interaction data at LHC [1-4] and the cosmic ray data [5] definitely tell us that at energies $\sqrt{s} \sim 5 - 50$ TeV the asymptotic regime is switching on for diffractive scattering processes. These data together with those of ISR [6] demonstrate the steady growth of $\sigma_{tot}$, $\sigma_{el}$ and $\sigma_{inel}$ with the energy increase, a shrinking of the diffractive cone in $d\sigma_{el}/dq^2$ and a relative supression of the real part of the scattering amplitude.

The growth of the cross sections $\sigma_{tot}$, $\sigma_{el}$, $\sigma_{inel}$ and the diffractive scattering slope is consistent with the picture of fast moving hadrons as parton clouds with an increasing transverse size. The gluonic origin of the parton cloud expilicates a slow growth of the diffractive scattering slope and a late start of the asymptotic regime: the effective mass of the soft gluon is notsmall being of the order of 800 - 1000 MeV [7, 8].

The observed growth of total cross sections at preLHC energies [9, 10] initiated studies of models with the supercritical pomeron [11-14]. The discussion of the power growth of cross sections with energy actualized the problem of $s$-channel unitarization of scattering amplitudes and the use the Glauber approach [15]. Taking into account the $s$-channel rescatterings, the power-$s$ growth of amplitudes is dampened to the $(ln s)$-type [16-18], to the limits of the Froissart bound [19]. Still, let us emphasize that exceeding it does not violate the general constraints for the scattering amplitude [20].

The black disk picture appears to be a rather natural mode for the ultrahigh energy corresponding to noncoherent parton interactions in hadron collisions. For the black disk scenario the profile function at $\sqrt{s} \gtrsim 100$ TeV gets frozen inside the disk area, $T(b) \simeq 1$ at $b < R_{black disk}$, while the increasing radius of the black disk, $R_{black disk}$, determines the total, elastic and inelastic cross sections: $\sigma_{tot} \simeq 2\pi R_{black disk}^2$, $\sigma_{el} \simeq \pi R_{black disk}^2$ and $\sigma_{inel} \simeq \pi R_{black disk}^2$. The black disk mode was intensively discussed in the last decade, see, for example, [21, 22] and references therein.

In the present paper we underline that at asymptotic energies the imaginary part of the amplitude $A_{3}(q^2, \ln s)$ turns into a generating function for the real part, $A_{R}(q^2, \ln s)$, due to unitarity and analyticity requirements. In Section 2 the real parts of the hadronic scattering amplitude are calculated for a set of energies, $\sqrt{s} = 1, 10, 10^2, ..., 10^6$ TeV, and profile functions, $T_{3}(b)$ and $T_{R}(b)$, are presented. In Section 3 we discuss a combined action of the Coulomb and hadronic interactions for the diffractive scattering region. If here the eikonal approach works, the straightforward way to take into account the interplay of these interactions is the use of the $K$-matrix function technique, thus keeping valid the unitarity condition. We present the corresponding formulae and calculate the $pp$ diffractive scattering cross section at 7 TeV with the Coulomb interaction taken into account.

II. REAL PART OF THE HADRONIC SCATTERING AMPITLUDE

The high energy scattering amplitude is dominantly imaginary, the real part of the amplitude is a next-to-leading term. So, in the pre-asymptotic region we should include into consideration of the scattering amplitude both the imaginary and real parts. The amplitude reads:

$$A(q^2, \xi) = \int d^2 b e^{ibq} T(b, \xi),$$

$$T(b, \xi) = T_{3}(b, \xi) - i T_{R}(b, \xi),$$

where $\xi = \ln s, b = |b|$. For the profile function we write:

$$T(b, \xi) = 1 - \eta(b, \xi) \exp(2i\delta(b, \xi)) = \frac{-2iK(b, \xi)}{1 - iK(b, \xi)},$$

presenting it in terms of the phase shift $\delta(b, \xi)$ and inelasticity parameter $\eta(b, \xi)$, or the $K$-matrix function $K(b, \xi)$. 

If the imaginary part of the amplitude dominates, the next-to-leading terms include real part due to analyticity requirement. Namely, we should take into account contributions both of the $s$-channel and the $u$-channel. Suggesting $\sigma_{tot}(pp) = \sigma_{tot}(p\bar{p})$ at $s \to \infty$ we write:

$$\frac{1}{2} \left[ A(q^2, \ln s) + A(q^2, \ln(-s)) \right] \simeq A(q^2, \ln s) + \frac{\partial A(q^2, \ln s)}{\partial (\ln s)} \cdot \left(-\frac{i\pi}{2}\right)$$

$$\simeq A_\Im(q^2, \ln s) + \frac{\partial A_\Im(q^2, \ln s)}{\partial (\ln s)} \cdot \left(-\frac{i\pi}{2}\right),$$

where $\ln(-s) = \ln s - i\pi$. It means the amplitude $A_\Im(q^2, \ln s)$ is the generating function for $A_\Re(q^2, \ln s)$. Analogously we write for the profile function:

$$T_\Re(b, \ln s) \simeq \frac{\pi}{2} \frac{\partial T_\Im(b, \ln s)}{\partial (\ln s)}.$$

The usual notation reads $T_\Re(b, \xi)/T_\Im(b, \xi) = \rho(b, \xi)$, therefore the total and elastic cross sections are written as:

$$T(b, \xi) = (1 + i\rho)T_\Im(b, \xi),$$

$$\sigma_{tot} = 2 \int d^2 b T_\Im(b, \xi),$$

$$4\pi \frac{d\sigma_{el}}{dq^2} = (1 + \rho^2)A_\Re^2(q^2).$$

Taking into account that $\rho^2$ is small, $\rho^2 \sim 0.01$, one can approximate:

$$\left| A_\Im(q^2, \xi) \right| \simeq 2\pi^2 \sqrt{\left| \frac{d\sigma_{el}}{dq^2} \right|},$$

that make possible direct calculations of the real part of the scattering amplitude, $A_\Re(q^2, \xi)$, on the basis of the energy dependence of the diffractive scattering cross section.
FIG. 3: Imaginary and real parts of the $K$-matrix function, $K^\Im(b, \xi)$ and $K^\Re(b, \xi)$, at energies $\sqrt{s} = 1, 10, 10^2, ..., 10^6$ TeV.

A. Calculation of the real part of the scattering amplitude

Using Eq. (6) we calculate $A_R(q^2, \xi)$ and, correspondingly, the profile functions $T_R(b, \xi)$ and $T_S(b, \xi)$ see Figs. [1] [2]. The figure [1] for $T_S(b, \xi)$ is taken from [27] where a comparison with data is given as well. Calculations at $\sqrt{s} \gtrsim 100$ TeV are done suggesting the black disk mode as a realization of the asymptotic regime.

An advantage of the $K$-matrix function technique is the separation of the two-particle rescattering states which turn out to be mass-on-shell for leading terms of the amplitude [28]. Following Eq. (2), we write:

$$-iK(b, \xi) = \frac{T(b, \xi)}{2 - T(b, \xi)} \equiv K_\Im(b, \xi) - iK_\Re(b, \xi). \quad (7)$$

The functions $K_\Re(b, \xi)$ and $K_\Im(b, \xi)$ for $\sqrt{s} = 1, 10, 10^2, ..., 10^6$ TeV are shown in Fig. 3.

B. Eikonal approach for scattering amplitude at ultrahigh energies and the Feynman diagram technique

For the $pp$-scattering amplitude $A_{pp\rightarrow pp}(pp_{\text{in}} \rightarrow pp_{\text{out}})$ the reproducing integral reads:

$$A_{pp\rightarrow pp}(pp_{\text{in}} \rightarrow pp_{\text{out}}) = K_{pp\rightarrow pp}(pp_{\text{in}} \rightarrow pp_{\text{out}})$$

$$+ \int \frac{d^3k_{2'}}{(2\pi)^3} A_{2\rightarrow 2}(pp_{\text{in}} \rightarrow 1'2')$$

$$\times \frac{K_{pp\rightarrow pp}(1'2' \rightarrow pp_{\text{out}})}{(m^2 - k_{1'}^2 - i0)(m^2 - k_{2'}^2 - i0)}, \quad (8)$$

to be definite we consider proton-proton scattering. Here $K_{pp\rightarrow pp}$ is the block without two-particle states thus being up to factor the $K$-matrix function, indices $(1', 2')$ refer to protons in the intermediate state.

Let us consider the scattering amplitude in the c.m. system where we write for the initial protons: $p_1 = (p_0, p_\perp, p_z) = (p + m^2/2p_0, 0, p)$ and $p_2 = (p + m^2/2p_0, 0, -p)$. For intermediate and final state protons we have:

$$k_1' \perp + k_2' \perp = 0, \quad k_1 \perp + k_2 \perp = 0.$$ 

$q_2^2 \equiv (q_2 - k_2')^2 \approx -(k_{2\perp} - k_{2\perp}')^2, \quad (9)$$

where $k_1, k_2$ refer to momenta of outgoing protons.

At ultrahigh energies the $K$-matrix function is dominantly imaginary for the black disk and resonant disk modes [27, 29]. That means the dominance of the mass-on-shell contribution in the loop diagrams. For the rescattering diagrams this is realized in the replacement:

$$\left[(m^2 - k_{1'}^2 - i0)(m^2 - k_{2'}^2 - i0)\right]^{-1} \rightarrow -2\pi^2\delta(m^2 - k_{1'}^2)\delta(m^2 - k_{2'}^2)$$

$$= -2\pi^2\delta(k_{1'}^{(+)}k_{1'}^{(-)} - (m^2 + k_{2\perp}^2))$$

$$\times \delta(k_{2'}^{(+)}k_{2'}^{(-)} - (m^2 + k_{2\perp}'^2)), \quad (10)$$

where $k^{(+)} = k_0 + k_2$, $k^{(-)} = k_0 - k_2$. Then the right-hand side of Eq. (8) reads:

$$A_{2\rightarrow 2}(k_{2\perp}, \xi) = K_{2\rightarrow 2}(k_{2\perp}, \xi)$$

$$+ \int \frac{d^3k_{2\perp}}{(2\pi)^3} A_{2\rightarrow 2}(k_{2\perp}, \xi) \frac{i}{4s} K_{2\rightarrow 2}((k_{2\perp} - k_{2\perp}')^2, \xi), \quad (11)$$

where $K_{2\rightarrow 2}(k_{1\perp}, \xi)/(4s) = K(k_{1\perp}, \xi)$ is the $K$-matrix function in momentum representation.
C. Impact parameter presentation

The Fourier transform gives the K-matrix function in the impact parameter space:

\[
\frac{1}{4\pi} K_{2\to2} (k^2_1, \xi) = \int d^2b \exp(i\mathbf{k}b) K(b, \xi),
\]
\[
i \frac{1}{4\pi} A_{2\to2} (k^2_1, \xi) = \int d^2b \exp(i\mathbf{k}b) a(b, \xi),
\]

Equation (12) in the impact parameter space is written as:

\[
a(b, \xi) = iK(b, \xi) + a(b, \xi) iK(b, \xi).
\]

Thus, we have the formula of the eikonal approach:

\[
a(b, \xi) = \frac{iK(b, \xi)}{1 - iK(b, \xi)}.
\]

The function \( K(b, \xi) \) depends on the energy and realizes effectively the interaction which manifests itself in the shrinking of diffractive cones with the energy increase.

III. DIFFRACTIVE SCATTERING AMPLITUDE AT ULTRAHIGH ENERGIES AND COULOMB INTERACTION

A. Interplay of hadronic and Coulomb interactions in the \( K \)-matrix function technique

We consider two types of scattering amplitudes and corresponding profile functions: the amplitude with combined interaction taken into account, \( A^{C+H}(q^2, \xi) \) and \( T^{C+H}(b, \xi) \), and that with switched-off the Coulomb interaction, following to [11] we use for them notation \( A(q^2, \xi) \) and \( T(b, \xi) \). For the combined interaction profile function we write:

\[
T^{C+H}(b, \xi) = \frac{-2iK^{C+H}(b, \xi)}{1 - iK^{C+H}(b, \xi)} = \frac{-2i(K^C(b) + K(b, \xi))}{1 - i(K^C(b) + K(b, \xi))},
\]

and the Coulomb interaction is written as:

\[
A^C(q^2) = \pm i f_1(q^2) \frac{4\pi\alpha}{q^2 + \lambda^2} f_2(q^2),
\]

\[
-2i K^C(b) = \pm i \int \frac{d^2q}{(2\pi)^2} e^{iqb} f_1(q^2) \frac{4\pi\alpha}{q^2 + \lambda^2} f_2(q^2).
\]

Here \( \alpha = 1/137 \); the upper/lower signs refer to the same/opposite charges of the colliding particles. The cutting parameter \( \lambda \), which removes infrared divergency, can be put to zero in the final result for \( A^{C+H}(q^2, \xi) \). Colliding hadron form factors, \( f_1(q^2) \) and \( f_2(q^2) \), guarantee the convergency of the integrals at \( q^2 \to \infty \); for the \( pp^\pm \)-collisions we use:

\[
f_1(q^2) = f_2(q^2) = \frac{1}{(1 + \frac{q^2}{4\pi\alpha GeV^2})^2}.
\]

B. Numerical calculations

In Fig. [11] we show \( K^C(b) \) for \( \lambda = 0.1 \) GeV and 0.01 GeV. The inclusion of the Coulomb interaction into consideration of hadron diffractive scattering does not
change the imaginary part of the $K$-matrix function, $K_{3}^{H+C}(b, \xi) = K_{3}(b, \xi)$. The real parts of the $K$-matrix functions $K_{3}^{H+C}(b, \xi) = K_{3}(b, \xi) + K^{C}(b)$ for different $\lambda$ are shown in Fig. 5.

Imaginary and real parts of the profile functions, $T^{H+C}(b, \xi)$ for the hadronic region, $b < 25 \text{ GeV}^{-1}$, are shown in Fig. 6; considerable perturbations are seen in the real part.

With these $\lambda$’s we calculate at $\sqrt{s} = 7 \text{ TeV}$ the profile function $T^{C+H}(b, \xi_{LHC})$ and the corresponding
amplitude $A^{C+H}(q^2, \xi_{LHC})$. The determination of the hadronic amplitude, $A_\Im(q^2, \xi_{LHC})$, is performed in terms of two versions:

1) with a direct application of the approximation (6) to the TOTEM data [1],

2) using the results of the Dakhno-Nikonov model [21, 26].

The description of the data for $d\sigma_{el}(q^2, \xi_{LHC})/dq^2$ in terms of these two versions is shown in Fig. 7: here Figs. 7a,b refer to the version (1) and Figs. 7c,d correspond to the version (2).

The Dakhno-Nikonov model gives a somewhat worse description of the $d\sigma_{el}(q^2, \xi_{LHC})/dq^2$ at 7 TeV than that using Eq. (9). This is not surprising because the model describes the data in a broad energy interval, 0.5-50 TeV [27], and the model parameters are responsible for a complete set of the data.

The specificity of the $K$-matrix function treating the amplitudes with the Coulomb interaction is the use of the determination: $K^C(b) = t g^C(s, b)$. Another determination was applied in [33]: $K^C(b) = \delta^C(s, b)$.

IV. CONCLUSION

On the basis of requirements of analyticity we calculate the leading terms of the real part of the $pp$-scattering amplitude for diffractive interactions at ultrahigh energies, $\sqrt{s} > 1$ TeV, and small momenta transferred, $q^2 < 0.4$ GeV$^2$. We do not include into consideration the region with larger $q^2$: in the region of larger momenta transferred mechanisms with conventional Pomeron as well as short-range non-Pomeron interactions are possible (for example, see Refs. [36–39]) but here we concentrate our attention on peripheral interactions.

In the region of the diffractive scattering cone the imaginary part of the amplitude prevails over the real part $A_R(q^2, \xi)/A_\Im(q^2, \xi) \sim 1/\xi$. The unitarity and analyticity requirements give unambiguously the leading term of the real part. We calculate $A_R(q^2, \xi)$ at $q^2 \leq 0.4$ GeV$^2$ supposing for energies $\sqrt{s} > 100$ TeV the black disk mode.

Presently we have a number of papers devoted to the asymptotic behavior of diffractive amplitudes at ultrahigh energies, see for example [40][41][42] and references therein; in these papers, however, the analyticity condition related to the $u$-channel is disregarded.

The interplay of the hadronic and Coulomb interac-
tions at very small $q$. Such functions allow to incorporate the Coulomb interaction terms into the scattering amplitude straightforwardly, keeping the unitarity condition. We present corresponding formulae and perform calculations for the black disk mode.

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