Q^2 dependence of chiral-odd twist-3 distribution e(x, Q^2)

Y. Koike and N. Nishiyama

Graduate School of Science and Technology, Niigata University, Niigata 950-21, Japan

ABSTRACT

We discuss the Q^2 dependence of the chiral-odd twist-3 distribution e(x, Q^2). The anomalous dimension matrix for the corresponding twist-3 operators is calculated in the one-loop level. This study completes the calculation of the anomalous dimension matrices for all the twist-3 distributions together with the known results for the other twist-3 distributions g_2(x, Q^2) and h_L(x, Q^2). We also have confirmed that in the large N_c limit the Q^2-evolution of e(x, Q^2) is wholly governed by the lowest eigenvalue of the anomalous dimension matrix which takes a very simple analytic form as in the case of g_2 and h_L.

1. Introduction

The nucleon has three twist-3 distributions g_2, h_L and e [1]. g_2 is chiral-even and the other two are chiral-odd. e is spin-independent and the other two are spin-dependent. Compared to e, g_2 and h_L have more chance to be measured experimentally since they become leading contribution to proper asymmetries in the polarized deep inelastic scattering (DIS) and Drell-Yan process, respectively. Their Q^2 evolution has been studied in [2,3] for g_2 and in [4,5] for h_L.

Similarly to the distribution functions, there are three twist-3 fragmentation functions which describes hadronization processes of partons in semi-inclusive processes [6], ĝ_2, ĥ_L and ě. (Their naming is parallel to the corresponding distribution functions.) In the inclusive pion production in the transversely polarized DIS, the chiral-odd fragmentation function ě of the pion appears as a leading contribution together with h_1 (twist-2) of the nucleon. Although the Q^2 evolution of the twist-2 fragmentation functions is known to be obtained from that of the corresponding distributions in the one-loop level (Gribov-Lipatov reciprocity [7]), no such relation is known for the higher twist fragmentation functions.

In our recent work [8], we have investigated the Q^2 evolution of e(x, Q^2). Theoretically, this completed the calculation of the anomalous dimension matrices of all the twist-3 distributions. Phenomenologically, we expect it will shed light on the Q^2 evolution of ě(z, Q^2), anticipating the day when their relation is clarified. This talk is a brief summary of [8].

2. Q^2 evolution of e(x, Q^2)

The chiral-odd twist-3 distribution function e(x, Q^2) is defined by the relation [1]

\[ \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P|\bar{\psi}(0)\psi(\lambda n)|Q|P \rangle = 2Me(x, Q^2), \]  

(1)
where \(|P\rangle\) is the nucleon (mass \(M\)) state with momentum \(P\). Two light-like vectors \(p\) and \(n\) defined by the relation, \(P = p + \frac{M^2}{2}n\), \(p^2 = n^2 = 0\), \(p \cdot n = 1\), specify the Lorentz frame of the system. Gauge-link operators are implicit in (1). Taking the moments of (1) with respect to \(x\), one can express the moments of \(e(x, Q^2)\) in terms of the nucleon matrix elements of the twist-3 operators \(V^{\mu_1 \cdots \mu_n}\) (at \(m_q = 0\)):

\[
\mathcal{M}_n[\langle e(Q^2) \rangle] = e_n(Q^2),
\]

\[
\langle PS|V^{\mu_1 \mu_2 \cdots \mu_n}|PS\rangle = 2e_nM(P^{\mu_1}P^{\mu_2} \cdots P^{\mu_n} - \text{traces}),
\]

\[
V^{\mu_1 \cdots \mu_n} = \sum_{l=2}^{n} U_l^{\mu_1 \cdots \mu_n} + E^{\mu_1 \cdots \mu_n},
\]

\[
U_l^{\mu_1 \cdots \mu_n} = \frac{1}{2}S_n\bar{\psi}\gamma^\mu_1iD^{\mu_2} \cdots gG^{\mu_n}iD^{\mu_n}\psi - \text{traces},
\]

\[
E^{\mu_1 \cdots \mu_n} = \frac{1}{2}S_n[\bar{\psi}iD^\gamma_1iD^{\mu_2} \cdots iD^{\mu_n}\psi + \bar{\psi}\gamma^{\mu_1}iD^{\mu_2} \cdots iD^{\mu_n}\bar{\psi}] - \text{traces},
\]

where \(\mathcal{M}_n[\langle e(Q^2) \rangle]\equiv \int_1^1 dx\, x^n e(x, Q^2)\) and the covariant derivative \(D^\mu = \partial^\mu - igA^\mu\) restores the gauge invariance.

\(U_l\) contains \(G_{\mu\nu}\) explicitly, which indicates that \(e(x)\) represents the quark-gluon correlations in the nucleon. \(E\) is the EOM (equation of motion) operator which vanishes by use of the QCD equation-of-motion. Although the physical matrix elements of EOM operators vanish, one needs to take into account the mixing with \(E\) to carry out the renormalization of \(U_l\), as is discussed in [2,4] in the context of the renormalization of \(g_2\) and \(h_L\). From (4) one sees that \(U_l\) appears in the form of

\[
R^{\mu_1 \cdots \mu_n}_{n,l} = U^{\mu_1 \cdots \mu_n}_{n-l+2} + U_l^{\mu_1 \cdots \mu_n}, \quad (l = 2, \ldots, \left[\frac{n}{2}\right] + 1).
\]

By this combination, \(R_{n,l}\) has a definite charge conjugation \((-1)^n\). Note the similarity between the present \(\{U_l, R_{n,l}\}\) and \(\{\theta_l, R_{n,l}\}\) which appeared in \(h_L\) [1,4]. In fact the presence of \(\gamma_5\) in \(\theta_l\) is the mere difference from \(U_l\). \(R_{n,l}\) of \(h_L\) is defined as \(\theta_{n-l+2} - \theta_l\) and has a charge conjugation \((-1)^{n+l+1}\) which is opposite to the above \(R_{n,l}\) in (7). We also recall that \(e(x)\) does not mix with the gluon-distribution owing to the chiral-odd nature.

For the renormalization of \(e(x, Q^2)\), we choose \(R_{n,l}\) \((l = 2, \ldots, \left[\frac{n}{2}\right] + 1)\), \(E\) as a basis of the operators. As in the case of \(g_2\) and \(h_L\), we eventually obtained the renormalization constants \(Z_{ij}\) among \(R_{n,l}, N\) and \(E\) in the following matrix form:

\[
\begin{pmatrix}
R^B_{n,l} \\
E^B_n
\end{pmatrix} = \begin{pmatrix}
Z_{ln}(\mu) & Z_{lE}(\mu) \\
0 & Z_{EE}(\mu)
\end{pmatrix} \begin{pmatrix}
R_{n,m}(\mu) \\
E_{n}(\mu)
\end{pmatrix}, \quad (l, m = 2, \ldots, \left[\frac{n}{2}\right] + 1).
\]

This \(Z_{ij}\) gives the anomalous dimension matrix for \(\{R_{n,l}, E\}\).

3. Large-\(N_c\) limit
In [3,5], it has been proved that all the (nonsinglet) twist-3 distributions \( g_2, h_L \) and \( e \) obey a simple GLAP equation similarly to the twist-2 distributions in the \( N_c \to \infty \) limit. In this limit, \( Q^2 \) evolution of these distributions is completely determined by the lowest eigenvalue of the anomalous dimension matrix which has a simple analytic form. For \( e \), this can be also checked using the obtained result for the mixing matrix \( Z_{ij} \). At \( N_c \to \infty \), i.e., \( C_F \to N_c/2 \), the lowest eigenvalue of the anomalous dimension matrix for \( R_{n,l} \) is given by (ignoring the factor \( g^2/8\pi^2 \))

\[
\gamma^e_n = 2N_c \left( S_n - \frac{1}{4} - \frac{1}{2(n+1)} \right).
\]

(9)

Furthermore the \( Q^2 \) evolution of \( e \) becomes

\[
\mathcal{M}_n \left[ e(Q^2) \right] = L^{\gamma^e_n/b_0} \mathcal{M}_n \left[ e(\mu^2) \right].
\]

(10)

As an example we compare the results for \( n = 4 \). The exact \( Q^2 \) evolution is

\[
\mathcal{M}_4 \left[ e(Q^2) \right] = (0.983a_{4,2}(\mu) + 0.520a_{4,3}(\mu)) \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{9.59/b_0} + (0.017a_{4,2}(\mu) - 0.020a_{4,3}(\mu)) \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{15.3/b_0},
\]

(11)

and the leading \( N_c \) evolution is

\[
\mathcal{M}_4 \left[ e(Q^2) \right] = \left( a_{4,2}(\mu) + \frac{1}{2}a_{4,3}(\mu) \right) \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{10.4/b_0},
\]

(12)

where \( a_{n,l} \) is defined by

\[
\langle P|R^{\mu_1 \ldots \mu_n}(\mu^2)P \rangle = 2a_{n,l}(\mu^2)MS_n(P^{\mu_1} \ldots P^{\mu_n} - \text{traces}),
\]

(13)

and \( b_0 = \frac{14}{3}N_c - \frac{2}{3}N_f \). One sees clearly that the coefficients in the second term of (11) are small (i.e., \( 1/N_c^2 \) suppressed) and the anomalous dimension in (12) is close to the smaller one in (11).

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