The internal rotation of the GW Vir star PG0112+200 through the eyes of asteroseismology

A. H. Córsico¹, L. G. Althaus¹, S. D. Kawaler², M. M. Miller Bertolami¹, and E. García–Berro³,⁴

¹Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque s/n, (1900) La Plata, Argentina
²Department of Physics and Astronomy, Iowa State University, 12 Physics Hall, Ames, IA 50011, U.S.A.
³Departament de Física Aplicada, Universitat Politècnica de Catalunya, c/Esteve Terrades 5, 08860 Castelldefels, Spain
⁴Institute for Space Studies of Catalonia, c/Gran Capità 2-4, Edif. Nexus 104, 08034 Barcelona, Spain

Abstract. We investigate the internal rotation profile of the GW Vir (PG1159-type) star PG 0122+200 by employing an asteroseismological model that closely reproduces the observed periods of this star. We adopt a forward approach and two inversion methods based on the rotational splitting of the pulsation frequencies to explore the properties of the rotation of PG 0122+200. We found evidence for differential rotation in this star.

1. Introduction

In recent years, asteroseismology has become a powerful tool to unravel the internal structure of oscillating stars, in particular, pulsating white dwarfs and pre-white dwarfs (Winget & Kepler 2008, Althaus et al. 2010). The approach is suitable also to place constraints on the stellar rotation. In fact, stellar rotation removes the intrinsic mode degeneracy of a nonradial g-mode characterized by an harmonic degree ℓ and a radial order k. As a result, each pulsation frequency is split into multiplets of \((2\ell + 1)\) frequencies specified by different values of the azimuthal index \(m\), with \(m = 0, \pm 1, \ldots, \pm \ell\) (Unno et al. 1989). If the rotation rate is slow compared with the pulsation frequencies, the frequency separation between each component of the multiplet is proportional to the rotation velocity of the star (\(\Omega\)). In this work, we perform a detailed asteroseismological study aimed at placing constraints on the internal rotation of the GW Vir star PG 0122+200, using the best existing evolutionary and seismic model for this star.

2. PG 0122+200 and the asteroseismological model

PG 0122+200 has \(T_{\text{eff}} = 80\,000 \pm 4\,000\) K and \(\log g = 7.5 \pm 0.5\) (Dreizler & Heber 1998). This pulsating PG1159 star currently defines the locus of the low-luminosity red
edge of the GW Vir instability strip (Althaus et al. 2010). In this paper, we employ the high-quality observational data on PG 0122+200 gathered by Fu et al. (2007), consisting of 23 frequencies corresponding to modes with $\ell = 1$ that include seven rotational triplets ($m = -1,0,+1$) and two isolated frequencies with (probably) $m = 0$ — see Table 5 of Fu et al. (2007). The theoretical frequency splittings were assessed employing the non-rotating asteroseismological model of PG 0122+200 derived by Córtsico et al. (2007). This model reproduces the $m = 0$ observed periods with an average of the period differences (theoretical vs. observed) of $\lesssim 0.9$ s. Employing an asteroseismological model represents a substantial improvement over previous works of this kind — see, e.g., Charpinet et al. (2009), for the case of PG 1159−035.

3. The forward approach

This method has been described in detail in Charpinet et al. (2009). The theoretical frequency splittings are obtained varying the assumed rotation profile and then they are compared with the observed ones until a best global match is found. The theoretical splittings are computed using the expressions resulting from the perturbative theory to first order in $\Omega$, that assumes that the pulsating star rotates with a period much longer than any of its pulsation periods (Unno et al. 1989. The goodness of the match between theoretical ($\delta \nu^T_i$) and observed ($\delta \nu^O_i$) rotational splittings is described using a quality function defined as:

$$
\chi^2 = \frac{1}{N_s} \sum_{i=1}^{N_s} \frac{1}{\sigma_i^2} (\delta \nu^T_i - \delta \nu^O_i)^2.
$$

Each difference is weighted with the inverse of the squared standard uncertainty ($\sigma_i$) of the observed splittings, which is derived from the uncertainties in the frequencies given in Fu et al. (2007).

We used very simple functional forms for $\Omega(r)$, and in particular, linear differential rotation profiles, $\Omega(r) = (\Omega_s - \Omega_c) r + \Omega_c$, where $\Omega_s$ and $\Omega_c$ are the rotation rates at the stellar surface and center, respectively. We performed forward computations varying the parameters $\Omega_s$ and $\Omega_c$ in the range $0 - 20 \mu$Hz. We found that there exists a unique, well-localized best-fit solution at $\Omega_c = 10.62 \pm 1.8 \mu$Hz and $\Omega_s = 4.41 \pm 1.1 \mu$Hz. The quoted uncertainties are derived by means of an error analysis in which we assume that the uncertainties of the observed frequency splittings have a Gaussian distribution with a standard deviation of $\sim 0.08 \mu$Hz. The existence of this solution suggests that the central regions of PG 0122+200 could be rotating more than twice faster than the surface. These results disagree with those of Córtsico & Althaus (2010), that indicate that the surface of PG 0122+200 could be spinning faster than the core. The discrepancy has its origin in the fact that, in that preliminary work, the fits of the rotational splittings were made without weighting the terms of the sum in the quality function, and thus, the impact of the different uncertainties of the observational data on the final result was neglected.

Vauclair et al. (2011) have found that seven oscillation frequencies of PG 0122+200 are experiencing changes over time, with much larger amplitudes and shorter time scales than those expected by cooling. We have redone our forward computations by considering these frequency drifts. We have arrived at the conclusion that, even with the uncertainties produced by the changes of some frequencies of PG 0122+200 over time, rigid rotation can be discarded.
4. Other inversion methods

We have gone beyond the forward approach, and employed another different, independent method to explore the rotation profile of PG 0122+200. This is the Regularized Least Squares (RLS) fitting technique — see Kawaler et al. (1999) for details. We first tested the reliability of our RLS scheme by employing this technique on “synthetic” frequency splittings generated with the asteroseismological model of PG 0122+200 through the forward approach. Specifically, we considered rotational splittings corresponding to consecutive $\ell = 1$ $g$-modes with $k = 1, \cdots, 40$. The regularization matrix corresponded to the smoothing of the second derivative of $\Omega(r)$. In all of the cases examined so far, the inversions were able to recover the input rotation profile we used to compute the synthetic splittings, provided that an adequate range of values of the regularization parameter $\lambda$ was adopted. Fig. 1 illustrates the particular case of a linearly increasing rotation profile. Next, we applied the RLS method to infer the internal rotation profile of PG 0122+200. We have employed the seven observed (averaged) $\ell = 1$ splittings. In Fig. 2 we show the inverted rotation profiles. For very small values of $\lambda$, the inverted profiles exhibit strong variations that lack physical meaning. However, as the value of $\lambda$ is increased, the inverted solution gradually stabilizes. The resulting rotation profile (corresponding to $\lambda \gtrsim 10^{-2}$) consists of an almost linearly decreasing rotation rate with $\Omega_c \sim 10.75 \mu$Hz and $\Omega_s \sim 4.58 \mu$Hz, in excellent agreement with the results of the forward approach. An analysis of errors similar to the one performed for the forward approach leads to the conclusion that even with the inclusion of reasonable uncertainties in the observed splittings, the rotation of PG 0122+200 is faster at the central regions than at the surface. Specifically, we found $\Omega_c = 10.75 \pm 2.4 \mu$Hz and $\Omega_s = 4.58 \pm 1.7 \mu$Hz.

Finally, we have also made rotational inversions onto a fixed functional basis. The method is called “function fitting” — for details, see Kawaler et al. (1999). Specifically, we explored linear functional forms for $\Omega(r)$. The optimal values we found are $\Omega_c = \ldots$
10.74 ± 2.9 μHz and $Ω_0 = 4.57 ± 1.8 μHz$, in excellent agreement with the RLS fits and also with the forward approach.

5. Conclusions

The three methods employed in this work to explore the internal rotation of PG 0122+200 suggest that it is differential, with the central regions rotating more than twice faster than stellar surface. This constitutes the first asteroseismic evidence of differential rotation with depth for an evolved star.

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