Crossover from Fulde-Ferrell state to Larkin-Ovchinnikov state in cold fermion gases

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Abstract. We study an effect of rotation on the two component Fermi superfluid gases with population imbalance in a toroidal trap. We investigate how the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state is changed by the rotation using the Bogoliubov-de Gennes equation in the quasi-one-dimensional regime. We find that two novel FFLO phases, i.e., the half quantum vortex state and the intermediate state of Fulde-Ferrell (FF) state and Larkin-Ovchinnikov (LO) state, are stabilized by the rotation. The phase diagram for the FF state, LO state, intermediate state, and half quantum vortex state is shown in $T$-$\Phi$ plane. We show the order parameter of these states.

1. Introduction
Since the superfluidity in the two component Fermi gases with population imbalance was realized by MIT and Rice groups [1, 2], various researches have been conducted [3]. One of the investigating motivations in this system is the realization of Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state [4, 5]. Recent experiment found an evidence for the FFLO superfluid state in the elongated harmonic trap [6], and attracts much attention in a variety of fields.

Most of theoretical works on the FFLO state are focused on the two phases. One is the Fulde-Ferrell (FF) state which has the order parameter $\Delta(r) \propto e^{i q \cdot r}$, and the other is the Larkin-Ovchinnikov (LO) state in which $\Delta(r) \propto \cos(q \cdot r)$. The LO state is regarded as a mixture of two FF states with opposite momentum $\Delta(r) \propto e^{i q \cdot r}$ and $\Delta(r) \propto e^{-i q \cdot r}$. It has been shown that the LO state is stable against the FF state in most cases [7]. However, various novel FFLO states may manifest themselves in cold atomic gases. This idea triggers the studies of the effects of rotation, trap potential, and atom species. For example, the angular FFLO (A-FFLO) state in which the order parameter changes its sign along the angular direction is stable in a toroidal trap [8]. Hence, it is interesting to study the superfluid phases in the toroidal trap. In this paper, we investigate the FFLO phases induced by the rotation. It is expected that those studies will be tested by the experiments. Such controllable experiment is an advantage of cold atom gases.

We show that the FF state is stabilized in the imbalanced gas near the superfluid critical temperature $T_c$, and the phase transition to the “intermediate state” between the FF and LO states occurs with decreasing the temperature. Furthermore, the “half quantum vortex state” becomes stable in a certain parameter range. This phase leads to the half quantized flux of mass and the unusual Little-Parks oscillation of $T_c$. We elucidate the superfluid order parameter of these FFLO states.
2. Formulation

We study rotating two component Fermi gases in a toroidal trap, in which atoms are loaded on a quasi-one-dimensional ring [9, 10]. We assume that the angular velocity is perpendicular to the plane of ring, leading to $\Omega = \Omega \hat{z}$. With use of the mean field approximation, the Hamiltonian is approximately described by the following one-dimensional Hamiltonian:

$$H_m = -t \sum_{j,\sigma} \left( e^{i \Phi} \hat{c}_{j+1,\sigma}^\dagger \hat{c}_{j,\sigma} + \text{h.c.} \right) - \sum_{j,\sigma} \mu_\sigma \hat{n}_{j,\sigma} + \sum_j \langle \Delta_j \rangle^2 \hat{c}_{j,\uparrow}^\dagger \hat{c}_{j,\downarrow} + \text{h.c.} + \sum_j \frac{|\Delta_j|^2}{|U|}, \tag{1}$$

where $\sigma = \uparrow, \downarrow$ describe two hyperfine states, $\hat{n}_{j,\sigma} = \hat{c}_{j,\sigma}^\dagger \hat{c}_{j,\sigma}$ is the number operator, $U$ is the coupling constant of attractive interaction, $\mu_\sigma$ is the chemical potential for $\sigma$ particles, and $\Delta_j \equiv -|U| \langle \hat{c}_{j,\uparrow} \hat{c}_{j,\downarrow} \rangle$ is the order parameter. The mass of atoms $m$ is related to $t = 1/2M$, and the phase $\Phi = R\Omega / 2t$ denotes the Peierls phase, where $R$ is the radius of ring. We take the unit $\hbar = k_B = 1$ and $t = 1$. We focus on the weak coupling BCS region, since the mean field Bogoliubov-de Gennes (BdG) equation is appropriate there. The mean field Hamiltonian is diagonalized with use of the Bogoliubov transformation, $\hat{c}_{j,\nu} = \sum_{\nu'} u_{\nu,\nu'} \hat{c}_{j,\nu'}$, $\hat{c}_{j,\nu} = \sum_{\nu''} u_{\nu',\nu''}^\dagger \hat{c}_{j,\nu''}$, where $\hat{c}_{j,\nu}$ and $\hat{c}_{j,\nu'}$ are the creation and annihilation operators of quasiparticles having the energy $E_{\nu'}$, respectively. The wave function of quasiparticles $(u_{\nu,\nu'}^\dagger, u_{\nu',\nu})$ satisfies the BdG equation

$$\sum_{\nu'} \left[ H_{j,\nu'} \delta_{\nu,\nu'} - \Delta_j \delta_{\nu,\nu'} \right] \begin{bmatrix} u_{\nu,\nu'}^\dagger \\ u_{\nu',\nu} \end{bmatrix} = E_{\nu} \begin{bmatrix} u_{\nu,\nu'}^\dagger \\ u_{\nu',\nu} \end{bmatrix}, \tag{2}$$

where $H_{j,\nu} = -te^{i \Phi} \delta_{j-1,\nu} - te^{-i \Phi} \delta_{j+1,\nu} - \mu_\nu \delta_{\nu,\nu}$. The self-consistent equation for the order parameter is obtained as $\Delta_j = -|U| \sum_{\nu} u_{\nu,\nu'}^\dagger u_{\nu',\nu} f(E_{\nu})$, where $f(E_{\nu})$ is the Fermi distribution function. Several self-consistent solutions corresponding to the metastable states are obtained. We determine the stable phase by comparing the free energy of those states. The free energy is evaluated to be

$$F = \sum_{\nu} E_{\nu} f(E_{\nu}) + \sum_j \left[ \frac{|\Delta_j|^2}{|U|} - \mu_\nu \right] + T \sum_{\nu} \{ f(-E_{\nu}) \log[f(-E_{\nu})] + f(E_{\nu}) \log[f(E_{\nu})] \}. \tag{3}$$

We solve BdG equations for a fixed chemical potential $\mu_\sigma$. We define the “magnetic field” $h = (\mu_\uparrow - \mu_\downarrow)/2$ as the difference of chemical potential between $\uparrow$ and $\downarrow$ particles.

3. Numerical Results

We set $|U|/t = 1.5$ and assume the chemical potential $(\mu = (\mu_\uparrow + \mu_\downarrow)/2 = -0.8)$ leading to $\langle \hat{n}_{j,\sigma} \rangle \leq 0.4$ so that the lattice Hamiltonian appropriately describes the continuum gas. The following results are obtained for the number of lattice sites $N_L = 200$ with imposing the periodic boundary condition.

In all our results the order parameter $\Delta_j$ is approximately described as $\Delta_j = \Delta_+ e^{iq_j + j} + \Delta_- e^{-i q_j - j}$, where the center-of-mass momentum of Cooper pairs is $q_{\pm} = \pm 2\pi m_{\pm}/N_L$ with $m_{\pm}$ being an integer. The FF state is described by a set of order parameter $(\Delta_+ + \Delta_-) \propto (1, 0)$ or $(\Delta_+ - \Delta_-) \propto (0, 1)$, while $(\Delta_+ + \Delta_-) \propto (1, \pm 1)$ with $m_+ = m_- = m$ in the LO state.

Figure 1 shows the phase diagram in the $T - \Phi$ plane for a field $h = 0.129$ where the LO state with $m_+ = m_- = 4$ is stable at rest. We denote the superfluid critical temperature $T_c(h, \Phi/\Phi_0)$ as a function of the magnetic field $h$ and the normalized phase factor $\Phi/\Phi_0$, where $\Phi_0 = 2\pi / N_L$. We see several intriguing features in Fig. 1. First, the FF state with $\Delta_+ = 0$ or $\Delta_- = 0$ is stabilized near the critical temperature in the rotating gas with $\Phi/\Phi_0 \neq 0$, because the degeneracy of two FF states is lifted by the rotation. The critical temperature is increased by the rotation since one of the FF states is stabilized.
The solid and dashed lines show the second and first order transition lines, respectively. The integers \( m_{\pm} \) denote the momentum of Cooper pairs \( q_{\pm} = \Phi_0 m_{\pm} \), and \( \Delta_{\pm} \) are the amplitude of order parameter for \( m_{\pm} \), respectively. The phase diagram is periodic for \( \Phi \) with the period \( \Phi_0 \). The FFLO phases with \( (m_+, m_-) = (5, 4) \) and \( (6, 3) \) are regarded as the half quantum vortex state.

Second, the “half quantum vortex state” is stabilized by the rotation. Figure 1 shows that the LO state with \( (m_+, m_-) = (4, 4) \) changes to other phase \( (m_+, m_-) = (5, 4) \), as the rotation increases. The latter phase is regarded as the half quantum vortex state since the order parameter is approximately described as \( \Delta_j \propto \exp[i(\Phi_0 \times 0.5)j] \cos[(\Phi_0 \times 4.5)j] \). The exponential provides the phase \( \pi \) for a circuit along the ring, which corresponds to a half quantized vortex. Another phase \( \pi \) required for the uniqueness of order parameter is given by the odd number of phase changes in the sinusoidal function. Thus, the order parameter of the half quantum vortex state is regarded as the product of those in the FF and LO states with an unusual period. Owing to the nucleation of half quantum vortex, the \( T_c \) shows the unusual Little-Parks oscillation with the period being a half of the conventional Little-Parks oscillation.

Third, we point out another phase in Fig. 1, that is the mixture of LO state and FF state. For instance, the FFLO state with \( (m_+, m_-) = (4, 4) \) is not a pure LO state in the sense that the amplitude \( \Delta_+ \), for \( m_+ = 4 \) is not equivalent to \( \Delta_- \) for \( m_- = 4 \). The temperature dependence of the ratio \( \delta \equiv \Delta_+ / \Delta_- \) at \( h = 0.129 \) and \( \Phi / \Phi_0 = 0.06 \) is shown in Fig. 2, where the superfluid phase is divided into three regions, (I), (II), and (III). In the low temperature region (I), the ratio \( \delta \) is almost unity corresponding to the LO state, while the FF state with \( \delta = 0 \) is stabilized in the high temperature region (III). Interestingly, the intermediate ratio \( 0 < \delta < 1 \) is obtained in a wide intermediate temperature range (II). Thus, the “intermediate state” between the FF and LO states is stabilized by the rotation, where the order parameter is obtained as \( \Delta_j \propto \delta \exp[i(\Phi_0 m_+)j] + \exp[-i(\Phi_0 m_-)j] \).

We here turn to the order parameter. Figure 3 shows that the spatial profile of the phase of order parameter in the FF, LO, intermediate, and half quantum vortex states. As shown in Fig. 3(a), the phase of the LO state is zero or \( \pm \pi \). On the other hand, the FF state has the phase between \( -\pi \) and \( \pi \). In case of the half quantum vortex state, the exponential of the order parameter has the phase from 0 to \( \pi \), while the phase of the sinusoidal function is zero or \( \pm \pi \). Hence the phase of the half quantum vortex state behave as shown in Fig. 3(b). As increasing the temperature, the phase of LO state and half quantum vortex state smoothly changes to that of the FF state via the intermediate state.
Figure 3. Spatial profile of the phase of order parameter at $h = 0.129$. (a) $\Phi/\Phi_0 = 0.06$. The red-dotted, green-solid, and blue-dashed lines represent the data for $T/T_c(0.129, 0) = 0$ (LO state), $T/T_c(0.129, 0) = 0.909$ (intermediate state), and $T/T_c(0.129, 0) = 1.00$ (FF state), respectively. (b) $\Phi/\Phi_0 = 0.2$, and the other parameters are the same as Fig. 3(a). The half quantum vortex state is stabilized at $T/T_c(0.129, 0) = 0$ in (b).

4. Conclusion
We have studied effects of rotation on the FFLO state of imbalanced Fermi gases loaded on a toroidal trap. We found several novel phase transitions induced by the rotation. First, the FF state is stabilized in the rotating gases near the critical temperature. Second, the FF state changes to the intermediate state through the second order phase transition. As decreasing the temperature, the crossover occurs from the intermediate state to the LO state. Third, we showed a sequence of quantum phase transitions with increasing the rotation. The FFLO state with an integer vortex changes to the half quantum vortex state. The flux of mass is half-quantized in the half quantum vortex state, as we will show elsewhere. Thus, the observation of the half quantized flux would be a clear evidence for the FFLO state in cold Fermi gases.

Acknowledgments
This work was supported by a Grant-in-Aid for Scientific Research on Innovative Areas “Heavy Electrons” (No. 21102506) from MEXT, Japan. It was also supported by a Grant-in-Aid for Young Scientists (B) (No. 20740187) from JSPS. Numerical computation in this work was partly carried out at the Yukawa Institute Computer Facility.

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