I discuss developments in heavy meson spectroscopy. In particular, I consider the system of $c\bar{s}$ mesons and the puzzling state $X(3872)$, with focus on the strategies for their classification.

1 Introduction

In the last decade, many new charm and beauty hadrons have been discovered. Some of them fit the quark model scheme, others still need to be properly classified. Here I focus on $c\bar{s}$ mesons and, to introduce the topic, I describe the properties of mesons with a single heavy quark in the infinite heavy quark mass limit. Then, I turn to the state $X(3872)$ observed in the hidden charm spectrum.

Before the B-factory era, the $c\bar{s}$ spectrum consisted of the pseudoscalar $D_s(1968)$ and vector $D_s^*(2112)$ mesons, $s$-wave states of the quark model, and of the axial-vector $D_{s1}(2536)$ and tensor $D_{s2}(2573)$ mesons, $p$-wave states. In 2003, two narrow resonances were discovered: $D_{sJ}^*(2317)$ and $D_{sJ}^*(2460)$ with $J^P = 0^+, 1^+$ [1, 2]. Their identification as $c\bar{s}$ states was debated [3]; however, they have the right quantum numbers to complete the $p$-wave multiplet, and their radiative decays occur accordingly, so that their interpretation as ordinary $c\bar{s}$ mesons seems natural and now widely accepted [3–5]. Afterwards, two other $c\bar{s}$ mesons decaying to $DK$ were observed: $D_{sJ}(2860)$ [6] and $D_{sJ}(2700)$ [7], the latter with $J^P = 1^-$. Later, in [8] it was found that $D_{sJ}(2700)$ is likely the first radial excitation of $D_s^*$. In [8] also another state was observed: $D_{sJ}(3040)$. As discussed in Section 3, the predictions for the decays of $D_{sJ}(2860)$, $D_{sJ}(2700)$ and $D_{sJ}(3040)$ following from different identifications can be used for the classification [9, 10].

In Section 4, after briefly recalling some of the latest news in the spectroscopy of hidden charm and beauty mesons, I survey the properties of $X(3872)$ and study a few radiative decay modes which are useful to shed light on its structure.

2 Hadrons containing a single heavy quark $Q$

The description of mesons with a single heavy quark $Q$ is simplified in QCD in the heavy quark $m_Q \to \infty$ limit, when the spin $s_Q$ of the heavy quark and the angular momentum
$s_\ell$ of the light degrees of freedom: $s_\ell = s_T + \ell$ (with $s_T$ being the light antiquark spin and $\ell$ the orbital angular momentum of the light degrees of freedom relative to $Q$) are decoupled. Hence spin-parity $s_\ell^P$ of the light degrees of freedom is conserved in strong interactions [11] and mesons can be classified as doublets of $s_\ell^P$. Two states with $J^P = (0^-,1^-)$, denoted as $(P,P^\star)$, correspond to $\ell = 0$ (the fundamental doublet). The four states corresponding to $\ell = 1$ can be collected in two doublets, $(P_1^0,P_1^\star)$ with $s_\ell^P = \frac{1}{2}^-$ and $J^P = (0^+,1^+)$, $(P_1,P_2)$ with $s_\ell^P = \frac{3}{2}^+$ and $J^P = (1^+,2^+)$. For $\ell = 2$ the doublets have $s_\ell^P = \frac{3}{2}^-$, consisting of states with $J^P = (1^-,2^-)$, or $s_\ell^P = \frac{5}{2}^-$ with $J^P = (2^-,3^-)$ states. And so on. For each doublet, one can consider a tower of similar states corresponding to their radial excitations.

One can predict whether these states are narrow or broad. For example, strong decays of the members of the $J_{s_i}^P = (1^+,2^+)_{3/2}$ doublet to the fundamental doublet plus a light pseudoscalar meson occur in $d$-wave. Since the rate for this process is proportional to $|\vec{p}|^3$ (in general, to $|\vec{p}|^{2\ell+1}$, $p$ being the light pseudoscalar momentum and $\ell$ the angular momentum transferred in the decay), these states are expected to be narrow. On the contrary, the members of the $J_{s_i}^P = (0^+,1^+)_{1/2}$ doublet decay in $s$-wave, hence they should be broad. $D_s(1668)$, $D_s^\star(2112)$ belong to the lowest $s_\ell^P = \frac{1}{2}^-$ doublet. $D_{s1}(2536)$, $D_{s2}(2573)$ correspond to the doublet with $J_{s_1}^P = (1^+,2^+)_{3/2}$, $D_{s1}(2317)$, $D_{s1}^\star(2460)$, to that with $J_{s_1}^P = (0^+,1^+)_{1/2}$. Mixing between the two $1^+$ states is allowed at $O(1/m_Q)$; however, for non-strange charm mesons such a mixing was found to be small [12, 13].

In the heavy quark limit, the various doublets are represented by effective fields: $H_a$ for $s_\ell^P = \frac{1}{2}^-(a = u,d,s$ is a light flavour index), $S_a$ and $T_a$ for $s_\ell^P = \frac{1}{2}^+$ and $s_\ell^P = \frac{3}{2}^+$, respectively; $X_a$ and $X_a^\prime$ for $s_\ell^P = \frac{3}{2}^-$ and $s_\ell^P = \frac{5}{2}^-$, respectively:

\[
H_a = \frac{1}{2} \left[ p_{a\mu} \gamma^\mu - \gamma_5 \right] \\
S_a = \frac{1}{2} \left[ p_{1a\mu} \gamma_\mu \gamma_5 - p_{0a} \right] \\
T_a^\mu = \frac{1}{2} \left\{ p^{\mu\nu}_{2a} \gamma_\nu - p^{\mu}_{1a} \sqrt{\frac{3}{2}} \gamma_5 \left[ g^{\mu\nu} - \frac{1}{3} \gamma^\nu (\gamma^\mu - \gamma_5^\mu) \right] \right\} \\
X_a^\mu = \frac{1}{2} \left\{ p^{\mu\nu}_{2a} \gamma_\nu - p^{\mu}_{1a} \sqrt{\frac{3}{2}} \gamma_5 \left[ g^{\mu\nu} - \frac{1}{3} \gamma_\nu (\gamma^\mu - \gamma_5^\mu) \right] \right\} \\
X_a^{\mu\nu} = \frac{1}{2} \left\{ p_{3a}^{\mu\nu} \gamma_\nu - p^{\mu\nu}_{a} \sqrt{\frac{5}{6}} \gamma_5 \left[ g^{\mu\nu} - \frac{1}{3} \gamma_\nu (\gamma^\mu - \gamma_5^\mu) \right] \right\} ;
\]

the various operators annihilate mesons of four-velocity $v$ (conserved in strong interactions) and contain a factor $\sqrt{m_F}$. At the leading order in the heavy quark mass and light meson momentum expansion the decays $F \to HM$ ($F = H, S, T, X, X'$ and $M$ a light pseudoscalar meson) can be described by the Lagrangian interaction terms (invariant under...
chiral and heavy-quark spin-flavour transformations) [14, 15]:

\[
\mathcal{L}_H = g \, \text{Tr} \left[ H_a H_b \gamma_\mu \gamma_5 \mathcal{A}^\mu_{ba} \right] \\
\mathcal{L}_S = h \, \text{Tr} \left[ \mathcal{H}_a S_b \gamma_\mu \gamma_5 \mathcal{A}^\mu_{ba} \right] + h.c.,
\]

(2) \[
\mathcal{L}_T = \frac{h'}{\Lambda} \text{Tr} \left[ \mathcal{H}_a T^\mu_{b \ell} (i D_\mu \mathcal{A} + i \mathcal{D} \mathcal{A}_\mu)_{ba} \gamma_5 \right] + h.c.
\]

\[
\mathcal{L}_X = \frac{k'}{\Lambda} \text{Tr} \left[ \mathcal{H}_a X^\mu_{b \ell} (i D_\mu \mathcal{A} + i \mathcal{D} \mathcal{A}_\mu)_{ba} \gamma_5 \right] + h.c.
\]

\[
\mathcal{L}_{X'} = \frac{1}{\Lambda^2} \text{Tr} \left[ \mathcal{H}_a X'^\mu_{b \ell} [k_1 \{ D_\mu, D_\nu \} \mathcal{A}_\lambda + k_2 (D_\mu D_\nu \mathcal{A}_\lambda + D_\nu D_\lambda \mathcal{A}_\mu)]_{ba} \gamma^{\lambda} \gamma_5 \right] + h.c.
\]

where \( D_{\mu ba} = -\delta_{ba} \partial_\mu + \frac{i}{2} (\bar{\xi} \delta \partial_\mu \bar{\xi} + \bar{\xi} \delta \partial_\mu \xi)_{ba}, \mathcal{A}_{\mu ba} = \frac{i}{2} (\bar{\xi} \delta \partial_\mu \bar{\xi} - \bar{\xi} \delta \partial_\mu \xi)_{ba} \) and \( \xi = \frac{e^{i \chi}}{\Lambda}. \)

\( \mathcal{M} \) is a matrix containing the light pseudoscalar meson fields (\( f_\pi = 132 \text{ MeV}, \Lambda \chi \approx 1 \text{ GeV} \) the chiral symmetry-breaking scale. \( \mathcal{L}_S, \mathcal{L}_T \) describe decays of positive parity heavy mesons with the emission of light pseudoscalar mesons in \( s \) and \( d \)-wave, respectively, \( g, h \) and \( h' \) representing effective coupling constants. \( \mathcal{L}_X, \mathcal{L}_{X'} \) describe the decays of negative parity mesons with the emission of light pseudoscalar mesons in \( p \) and \( f \)-wave with couplings \( k', k_1 \) and \( k_2 \). The structure of the Lagrangian terms for radial excitations of the doublets is the same, but the couplings \( g, h, \ldots \) have to be substituted by \( \tilde{g}, \tilde{h}, \ldots. \)

### 3 c\[\bar{s}\] mesons: The case of \( D_{sf}(2860), D_{sf}(2700) \) and \( D_{sf}(3040) \)

In 2006, BaBar observed a heavy \( c\bar{s} \) meson, \( D_{sf}(2860) \), decaying to \( D^0 K^+ \) and \( D^+ K_S \), with mass \( M = 2856.6 \pm 1.5 \pm 5.0 \text{ MeV} \) and width \( \Gamma = 47 \pm 7 \pm 10 \text{ [6].} \) Shortly after, analysing the \( D^0 K^+ \) invariant mass distribution in \( B^+ \rightarrow D^0 D^0 K^+ \) Belle Collaboration [7] found a \( j^P = 1^- \) resonance, \( D_{sf}(2710) \), with \( M = 2708 \pm 9_{-10}^{+11} \text{ MeV} \) and \( \Gamma = 108 \pm 23_{-31}^{+36} \text{ MeV} \).

In order to classify \( D_{sf}(2860) \) and \( D_{sf}(2710) \), their strong decays were studied in [9], comparing the predictions which follow from different quantum number assignments. I summarize here the main results, starting with \( D_{sf}(2860) \). A new \( c\bar{s} \) meson decaying to \( DK \) can be either the \( j^P = 1^- \) state of the \( s^e_0 = \frac{3}{2}^- \) doublet, or the \( j^P = 3^- \) state of the \( s^e_2 = \frac{3}{2}^- \) one, in both cases with lowest radial quantum number. Otherwise \( D_{sf}(2860) \) could be a radial excitation of already observed \( c\bar{s} \) mesons: the first radial excitation of \( D^e_1 \) \( (j^P = 1^- s^e_1 = \frac{1}{2}^-) \) or of \( D_{sf}(2317) \) \( (j^P = 0^+ s^e_0 = \frac{1}{2}^+) \) or of \( D_{sf}^*(2573) \) \( (j^P = 2^+ s^e_2 = \frac{3}{2}^+) \). As for \( D_{sf}(2710) \), having \( j^P = 1^- \), it could be either the first radial excitation belonging to the \( s^e_1 = \frac{1}{2}^- \) doublet \( (D^e_0) \) or the low lying state with \( s^e_1 = \frac{3}{2}^- \) \( (D^e_1) \).

For both mesons the ratios of decay rates \( R_1 = \frac{\Gamma(D_{sf} \rightarrow D^0 K)}{\Gamma(D_{sf} \rightarrow D^+ K)}, R_2 = \frac{\Gamma(D_{sf} \rightarrow D_{sf}^*)}{\Gamma(D_{sf} \rightarrow D^0 K)} \) \( (D^*(+) K = D^*(+) K_S + D^*(0) K^+) \), obtained using eqs. (1) and (2), are useful to discriminate among the various assignments [9]. Table 1 reports such ratios in the various cases; it is interesting that they do not depend on the coupling constants, but only on the quantum numbers.
shows, among the remaining possibilities, the assignment in the Dalitz plot analysis of $B^{\Gamma}$ likely one. In this case the small width of the $D_s^{*}$ state with respect to $J/\psi$ leaves the identification of $J/\psi$ a state, which is difficult to imagine.

Among the remaining possibilities, the assignment $s^p_s = \frac{5}{2}^-$, $J^p = 2^-$, $n = 1$ seems the most likely one. In this case the small $D\bar{K}$ width is due to the kaon momentum suppression factor: $\Gamma(D_s^{\prime}\rightarrow DK) \propto q^{2}_K$. The spin partner, $D_{s2}^{*}$, has $s^p_s = \frac{5}{2}^-$, $J^p = 2^+$, decaying to $D^*K$ and not to $DK$. It would also be narrow in the $m_Q \rightarrow \infty$ limit, where the transition $D_{s2}^{*}\rightarrow D^*K$ occurs in $f$-wave. As an effect of $1/m_Q$ corrections this decay can occur in $p$-wave, so that $D_{s2}^{*}$ could be broader; hence, it is not necessary to invoke a mechanism inhibiting the production of this state with respect to $J^p = 3^-$ if $D_s^{\prime}(2860)$ has $J^p = 3^-$, it is not expected to be produced in non leptonic $B$ decays such as $B \rightarrow D\bar{D}_{s2}^{*}(2860)$. Actually, in the Dalitz plot analysis of $B^{+} \rightarrow \bar{D}^0 D^0 K^+$ no signal of $D_s^{\prime}(2860)$ was found [7].

In the latest BaBar analysis [8] $D_s^{\prime}(2860)$ has been observed decaying to $DK$ and $D^*K$ final states, hence excluding the assignment $J^p = 0^+$. However, the measurement [8]

$$\frac{BR(D_s^{\prime}(2860) \rightarrow D^*K)}{BR(D_s^{\prime}(2860) \rightarrow DK)} = 1.10 \pm 0.15_{\text{stat}} \pm 0.19_{\text{syst}}$$

leaves the identification of $D_s^{\prime}(2860)$ still an open issue. A confirmation that $D_s^{\prime}(2860)$ is a $J^p = 3^-$ state could be the detection of its non-strange partner $D_3$, also expected to be narrow, that can be produced in semileptonic and in non leptonic $B$ decays [16].

Let us now look at $D_s^{\prime}(2710)$. As Table 1 shows, $R_1$ is very different if $D_s^{\prime}(2710)$ is $D_s^{\prime}$ or

| $D_s^{\prime}(2860)$ | $R_1$ | $R_2$ |
|---------------------|-------|-------|
| $s^p_s = \frac{1}{2}^-$, $J^p = 1^-$, $n = 2$ | 1.23 | 0.27 |
| $s^p_s = \frac{1}{2}^+$, $J^p = 0^+$, $n = 2$ | 0 | 0.34 |
| $s^p_s = \frac{3}{2}^+$, $J^p = 2^+$, $n = 2$ | 0.63 | 0.19 |
| $s^p_s = \frac{3}{2}^-$, $J^p = 1^-$, $n = 1$ | 0.06 | 0.23 |
| $s^p_s = \frac{5}{2}^-$, $J^p = 3^-$, $n = 1$ | 0.39 | 0.13 |

Table 1: Predicted ratios $R_1$ and $R_2$ (see text for definitions) for the various assignment of quantum numbers to $D_s^{\prime}(2860)$ and $D_s^{\prime}(2710)$. | $D_s^{\prime}(2710)$ | $R_1$ | $R_2$ |
|---------------------|-------|-------|
| $s^p_s = \frac{1}{2}^-$, $J^p = 1^-$, $n = 2$ | 0.91 | 0.20 |
| $s^p_s = \frac{3}{2}^-$, $J^p = 1^-$, $n = 1$ | 0.043 | 0.163 |
\( D_{s1} \). Comparing the results in that Table with the BaBar measurement [8]:

\[
\frac{BR(D_{sJ}(2710) \to D^s K)}{BR(D_{sJ}(2710) \to DK)} = 0.91 \pm 0.13_{\text{stat}} \pm 0.12_{\text{syst}}
\]

allows to conclude that \( D_{sJ}(2710) \) is most likely \( D_{s1}^{*} \), the first radial excitation of \( D_{sJ}^{*}(2112) \).

From the computed widths, assuming that \( \Gamma(D_{sJ}(2710)) \) is saturated by the considered modes and identifying \( D_{sJ}(2710) \) with \( D_{sJ}^{*} \), the coupling \( g \), analogous to \( g \) in (2) when \( H \) is the doublet of the \( n = 2 \) radial excitations, can be determined \( g = 0.26 \pm 0.05 \), a value similar to those obtained for analogous effective couplings [17]. This result for \( g \) can provide information about \( D_{sJ}^{*} \), the spin partner of \( D_{sJ}(2710) \) having \( J^P = 0^- \); it is the first radial excitation of \( D_s \) and can decay to \( D^{*0}K^+\), \( D^{*+}K^0_{S(L)} \), \( D^{*0}_s \eta \). In the heavy quark limit, these partners are degenerate. Using the result for \( g \) one predicts \( \Gamma(D_{sJ}^{*}) = (70 \pm 30) \) MeV.

Identifying \( D_{sJ}(2700) \) with \( D_{sJ}^{*} \), its charmed non strange partners are \( D^{*+} \) and \( D^{*0} \), the radial excitations of \( D^{*+0} \). Their masses can be fixed to 2600 \( \pm \) 50 MeV assuming that \( D_{sJ}(2700) \) is heavier by an amount of the size of the strange quark mass. \( D^{*0} \) can decay to \( D^{*0} \to D^\pi \), \( D_2 K \), \( D_s K \), \( D^{*+} \), \( D^{*0} \), \( D_s K \) so that the previous result for \( g \) gives \( \Gamma(D^{*+0}(0)) = (128 \pm 61) \) MeV. Noticeably, studying \( D^{+}\pi^- \), \( D^{0}\pi^+ \), \( D^{*+}\pi^- \) systems, BaBar found four new charmed non strange mesons [18] and, among these, the state \( D^s(2600) \) likely to be identified with \( D^{*0} \) (the non strange partner of \( D_{sJ}(2700) \)), and the state \( D(2550) \) 0 likely to be the spin partner of \( D^*(2600) \), corresponding to the first radial excitation of the \( D \) meson. Comparison of the measured widths \( \Gamma(D^s(2600)) = 93 \pm 6 \pm 13 \) MeV, \( \Gamma(D(2550)) = 130 \pm 12 \pm 13 \) MeV with the prediction for \( \Gamma(D^{*+0}(0)) \) supports the proposed identification.

In [8] another broad structure was observed, \( D_{sJ}(3040) \), with \( M = 3044 \pm 8_{\text{stat}} \pm 30_{\text{syst}} \) MeV and \( \Gamma = 239 \pm 35_{\text{stat}} \pm 48_{\text{syst}} \) MeV. \( D_{sJ}(3040) \) decays to \( D^s K \) and not to \( DK \), hence it has unnatural parity: \( J^P = 1^+, 2^-, 3^+, \ldots \). The lightest not yet observed states with such quantum numbers are the two \( J^P = 2^- \) states belonging to the doublets with \( s_t = 3/2 \) and \( s_l = 5/2 \) denoted as \( D_{s2} \) and \( D_{s2}^{*} \), respectively. The identification with the radial excitations with \( n = 2 \), \( J^P = 1^+ \), and \( s_t = 1/2 \) (the meson \( D_{s1}^{*} \)) or \( s_l = 3/2 \) (the meson \( D_{s1} \)) is also possible. Notice that, if the identification of \( D_{sJ}(2860) \) as the \( J^P = 3^-_{5/2} \) meson were experimentally confirmed, this would disfavor the assignment of \( D_{sJ}(3040) \) to its spin partner \( D_{s2}^{*} \) with \( J^P = 2^-_{5/2} \), since a mass inversion in a spin doublet seems unlikely. For a similar reason, one would also disfavor the identification of \( D_{sJ}(3040) \) with \( D_{s2} \), although in that case the two mesons would belong to different doublets. The strong decays of \( D_{sJ}(3040) \) to a charmed meson and a light pseudoscalar one can be evaluated using the effective Lagrangians in Eq.(2). In particular, one can compute the ratio \( R_1 = \frac{\Gamma(D_{sJ}(3040) \to D_{sJ}^{*}K)}{\Gamma(D_{sJ}(3040) \to DK)} \)

\( (D^*K = D^{*0}K^+ + D^{*+}K^0_{S(L)}) \), with results collected in Table 2 [10]. The spread among them is useful to discriminate among the assignments, in particular between \( D_{s1}^{*} \) and \( D_{s2}^{*} \).

The mass of \( D_{sJ}(3040) \) is large enough to allow decays to \( (D^0_{sJ}, D^1_{J}) K, (D_1, D_2) K \) and \( D^{*0}_{sJ} \eta \), with different features in the four cases. Other allowed modes are into \( DK^* \) or \( D_{sJ} \), which can be described using an approach based on effective Lagrangian terms [19]. The results
obtained in the four possible identifications are collected in Table 2 [10], from which some conclusions can be drawn. The determination of the wave in which a particular decay proceeds is useful to predict a hierarchy among the widths of the states in the four cases. Consequently, the two \( J^P = 1^+ \) are expected to be broader than the two \( J^P = 2^+ \) states, hence it is likely that \( D_{s1}(3040) \) should be identified with one of such two axial-vector mesons. These can be distinguished since the widths to the \( DK^* \) and \( D_s\phi \) decay modes are larger for \( D_{s1}' \) than for \( D_{s1} \). Finally, although less probable, the identification with \( D_{s2} \) can be discarded/confirmed studying the \( D_{s2}K s^- \) wave final state.

### 4 Heavy quarkonium and the intriguing case of \( X(3872) \)

Besides the new charmed mesons, new heavy quarkonium or charmonium-like states were observed. Some have been classified as standard quarkonia: the charmonia \( h_c \) [20], \( \eta_c(2S) \) [21], \( \chi_{c2}(2P) \) [22], and, in the beauty case, the \( \eta_b(1S) \) [23], \( h_b(1P) \) [24,25] and \( h_b(2P) \) [25]. Others are still awaiting for the right interpretation, since not only their quantum numbers are not established, but even their \( Q\bar{Q} \) structure is questioned [26]. Among these, the charged \( Z(4430)^- \) state seen by Belle Collaboration in \( B \to Z^-K \), decaying to \( \psi(2S)\pi^- \), \( \chi_{c1}\pi^- \) [27]. The minimal quark content of this state would be \( c\bar{c}u\bar{d} \), identifying it necessarily as an exotic state. Search for \( Z^- \) was performed by BaBar, but no signal was found [28]. Later on, Belle found other charmonium-like charged \( Z \) states [29] and, more recently, also bottomonium-like \( Z_b(10610) \) and \( Z_b(10650) \) states decaying to \( Y(nS)\pi^\pm \) (n=1,2,3) and \( h_b(mP)\pi^\pm \) (m=1,2) [30]. These states require confirmation, too.

Here I focus on the state \( X(3872) \), discovered in 2003 by Belle Collaboration in \( B^\pm \to K^\pm X \to K^\pm J/\psi\pi^\pm\pi^- \) decays [31] and confirmed by BaBar [32], CDF [33] and D0 [34] Collaborations. The PDG resonance parameters are: \( M(X) = 3871.57 \pm 0.25 \) MeV and \( \Gamma(X) < 2.3 \) MeV (90\%/ C.L.) [35]. Looking at the \( J/\psi\pi^\pm\pi^- \) channel, no charged partners were found [36]. The mode \( X \to J/\psi\gamma \) allows to fix charge conjugation of \( X \) to \( \overline{C} = \overline{1} \).
Moreover, a $D^0\bar D^0\pi^0$ enhancement in $B \to D^0\bar D^0\pi^0 K$ decay was reported [37] with 
\[ \frac{B(X\to J/\psi\pi^+\pi^-)}{B(X\to J/\psi\pi^+\pi^-)} = 9 \pm 4, \]
hence $X$ mainly decays into final states with open charm mesons.

These measurements, though not fully consistent with the charmonium interpretation (as far as the mass of $X$ is concerned), do not contradict it. However, the observation of $X \to J/\psi\pi^+\pi^-\pi^0$ with the measurement 
\[ \frac{B(X\to J/\psi\pi^+\pi^-\pi^0)}{B(X\to J/\psi\pi^+\pi^-)} = 1.0 \pm 0.4 \pm 0.3 \] [38] implies, considering the two modes as induced by $\rho^0$ and $\omega$ intermediate states, isospin violation.

The three pion decay is also important to fix the spin-parity of $X$. While the angular analysis in $X \to J/\psi\pi^+\pi^-$ favours $J^P = 1^+$, studies of the three pion distribution in $X \to J/\psi\omega \to J/\psi\pi\pi\pi$ are more favourable to $J^P = 2^-$ [39]. Hence, if $X$ is a $c\bar c$ state it can be either the first radial excitation of $\chi_{c1}$, $\chi_{c1}'$, or the state $\eta_{c2}$ having $J^{PC} = 2^{-+}$.

On the other hand, the peculiar features of $X$ suggested the conjecture that it is not a charmonium state. In particular, the coincidence between its mass and the $D^{*0}\bar D^0$ mass: $M(D^{*0}\bar D^0) = 3871.2 \pm 1.0$ MeV, inspired the proposal that $X(3872)$ could be a molecule [40], a bound state of $D^{*0}$ and $\bar D^0$ with small binding energy [41], an interpretation that would account for a few properties of $X(3872)$. For example, if the wave function of $X(3872)$ has various hadronic components [42] one could explain why this state seems not to have definite isospin. However, the molecular binding mechanism still needs to be clearly identified, while the description of $X(3872)$ as a charmonium state presents alternative arguments to the molecular description [43, 44]. Concerning the isospin violation, to correctly interpret the large ratio 
\[ \frac{B(X\to J/\psi\pi^+\pi^-\pi^0)}{B(X\to J/\psi\pi^+\pi^-)} \], one has to consider that phase space effects in two and three pion modes are very different and it turns out that the isospin violating amplitude is 20% of the isospin conserving one [45]: 
\[ \frac{B(X\to J/\psi\rho)}{B(X\to J/\psi\omega)} \simeq 0.2. \]

I focus on two studies of $X$ decays. The first one [46] compares the charmonium versus the molecular interpretation, discussing the argument that, if $X(3872)$ is a $DD^*$ molecule the decay $X \to D^0\bar D^0\gamma$ should be dominant with respect to $X \to D^+D^-\gamma$, such decays being mainly due to the decays of its meson components [42]. In order to discuss whether this is true, in [46] the ratio $R = \frac{\Gamma(X\to D^0D^-\gamma)}{\Gamma(X\to D^0\bar D^0\gamma)}$ has been computed assuming that $X(3872)$ is an ordinary $J^{PC} = 1^{++}$ charmonium state.

The transition $X(3872) \to D\bar D\gamma$ can be studied assuming that the radiative decay amplitude is dominated by polar diagrams with $D^*$ and the $\psi(3770)$ mesons as intermediate...
states nearest to their mass shell (fig.1). These amplitudes can be expressed in terms of two unknown quantities: the coupling constant \( \hat{g}_1 \) governing the \( XDD^* (D\bar{D}') \) matrix elements, and the one appearing in the \( X\psi(3770)\gamma \) matrix element. For the matrix element \( XDD^* (D\bar{D}') \) one can use a formalism suitable to describe the interaction of the heavy charmonium with the doublet \( H \) in (1) [47]. In the multiplet:

\[
\rho^{(Q\bar{Q})}\mu = \left( \frac{1 + \hat{g}_1}{2} \right) \left( \chi_2^{\mu\alpha} \gamma_\alpha + \frac{1}{\sqrt{2}} e^{\mu\alpha\beta\gamma} \eta_\alpha \gamma_\beta \chi_1 \gamma_\gamma + \frac{1}{\sqrt{3}} (\gamma^\mu - \eta^\mu) \chi_0 + h^\mu_1 \gamma_5 \right) \left( \frac{1 - \hat{g}_1}{2} \right)
\]

the fields \( \chi_2, \chi_1, \chi_0 \) correspond to the spin triplet with \( J^{PC} = 2^{++}, 1^{++}, 0^{++} \), respectively, while the spin singlet \( h_1 \) has \( J^{PC} = 1^{--} \). If \( X(3872) = \chi_1' \), it is described by \( \chi_1 \). The strong interaction with the \( D \) and \( D^* \) mesons can be described by the effective Lagrangian [48]

\[
\mathcal{L}_1 = ig_1 \text{Tr} \left[ \rho^{(Q\bar{Q})}\mu \mathcal{P}_{1a} \gamma_\mu \mathcal{P}_{2a} \right] + h.c.
\]

Using (4) the couplings \( XDD^* \) which enter in the second and the third diagrams in fig.1, can be expressed in terms of the dimensionless coupling constant \( \hat{g}_1 = g_1 \sqrt{m_D} \). Notice that, due to isospin symmetry, the couplings of the meson \( X \) to charged and neutral \( D \) are equal, at odds with the molecular description where \( X \) mainly couples to neutral \( D \).

The matrix element \( < D (k_1) \gamma (k, \bar{e}) | D^* (p_1, \xi) > = i e c' e^{\alpha\beta\gamma\delta} \tilde{\xi}_\alpha \bar{e}_\beta \gamma_{\gamma\delta} p_1 \tau_0 \) is also required. The parameter \( c' \) accounts for the coupling of the photon to both the charm and the light quark and can be fixed from data on radiative \( D^{*\pm} \) decays [35].

To compute the first diagram in fig.1 the matrix element \( < \psi(3770) (q, \eta) \gamma (k, \bar{e}) | X (p, e) > = i e c e^{\alpha\beta\mu\nu} \tilde{\xi}_\alpha \bar{e}_\beta \tilde{\eta}_{\mu\nu} k_\tau \) is needed; \( c \) is an unknown parameter. On the other hand, the coupling \( \psi(3770)D\bar{D} \) can be fixed from experiment to \( g_{\psi D\bar{D}} = 25.7 \pm 1.5 \).

Putting all the ingredients together one obtains the ratio \( R = \frac{\Gamma(X \rightarrow D^{*+} D^- \gamma)}{\Gamma(X \rightarrow D^0 D^\prime \gamma)} \), plotted in fig.2 [46] versus \( \frac{1}{\hat{g}_1} \), showing that the radiative \( X \) decay into charged \( D \) mesons is always suppressed with respect to the mode with neutral \( D \) and in any case \( R < 0.7 \). Moreover, for small values of \( \frac{1}{\hat{g}_1} \) the ratio \( R \) is tiny, so that this is not peculiar of a molecular structure of \( X(3872) \).

\( \hat{g}_1 \) enters also in the mode \( X(3872) \rightarrow D^0 D^0 \pi^0 \) that can be considered as induced by intermediate \( D^* \) states. The amplitude depends on the coupling constant \( D^* D \pi \), proportional to the constant \( g \) in eq. (2). Using data on \( D^{*\pm} \) decays to \( D\pi \) [35], one can derive \( g = 0.64 \pm 0.07 \). This allows to constrain \( \hat{g}_1 < 4.5 \) from the upper bound \( \Gamma(X \rightarrow D^0 D^0 \pi^0) < \Gamma(X(3872)) < 2.3 \) MeV. Hence, a value of \( \hat{g}_1 \) of the typical size of the hadronic couplings can reproduce the small width of \( X(3872) \).

The second analysis that I discuss also aims at shedding light on the structure of \( X(3872) \) through the calculation of its radiative decay rates to \( J/\psi \gamma \) and \( \psi(2S)\gamma \) assuming that it is the state \( \chi_1' \) [49] and using an effective Lagrangian approach which exploits spin symmetry for heavy \( Q\bar{Q} \) states [50]. Unlike the heavy-light \( Q\bar{Q} \) mesons, in heavy quarkonia
The effective Lagrangian describing radiative transitions among members of the system can help in understanding charmonium. However, at a qualitative level, bottomonium system can help in understanding charmonium.

Identifying $X$ with the state $\chi'$, it belongs to the multiplet with $L = 1$ introduced in (3). $J/\psi$ and $\psi(2S)$ are described by the $J^P = 1^-$ $H_1$ component of the doublet:

$$J = \frac{1 + \gamma^5}{2} H_1^{\mu} \gamma_\mu - H_0 \gamma_5 \frac{1 - \gamma^5}{2}.$$  

The effective Lagrangian describing radiative transitions among members of the $P$ wave and of the $S$ wave multiplets has been derived in [50]:

$$\mathcal{L}_n^{P\rightarrow mS} = \delta_Q^{nPmS} \text{Tr} \left[ J(mS)J_\mu(nP) \right] \nu_F F^{\mu\nu} + \text{h.c.}.$$  

$F^{\mu\nu}$ the electromagnetic field strength tensor. Hence, a single constant $\delta_Q^{nPmS}$ describes all the transitions among the members of the $nP$ multiplet and those of the $mS$ one.

I consider the ratios $R_{J}^{(b)} = \frac{\Gamma(\chi_1(2P) \rightarrow Y(1S) \gamma)}{\Gamma(\chi_1(2P) \rightarrow Y(1S) \gamma)}$, proportional to $R_{\delta}^{(b)} = \frac{\delta^{PmS}}{\delta_{\delta}^{PS}} (J = 0, 1, 2)$. From the measured branching ratios of $\chi_1(2P) \rightarrow Y(1S) \gamma, Y(2S) \gamma$ [35], the average value can be obtained: $R_{\delta}^{(b)} = 8.8 \pm 0.7$. It is reasonable that, even though the couplings might be different in the beauty and the charm cases, their ratios stay stable. Therefore, using the result for $R_{\delta}^{(b)}$ in the case of $\chi'_1$ decays, I get:

$$R_1^{(c)} = \frac{\Gamma(\chi_1(2P) \rightarrow \psi(2S) \gamma)}{\Gamma(\chi_1(2P) \rightarrow \psi(1S) \gamma)} = 1.64 \pm 0.25.$$  

Figure 2: Ratio of $X \rightarrow D^+D^-\gamma$ to $X \rightarrow D^0\bar{D}^0\gamma$ decay widths versus the ratio of parameters $c/\hat{g}_1$. There is no heavy flavour symmetry [51], hence it would not be possible to exploit data on charmonium to obtain quantitative information on bottomonium or viceversa. However, their results for $R$ different in the beauty and the charm cases, their ratios stay stable. Therefore, using the result for $R_{\delta}^{(b)}$ in the case of $\chi'_1$ decays, I get:
In [52] the following ratio has been measured ¹:

\[ R_X = \frac{\Gamma(X(3872) \rightarrow \psi(2S) \gamma)}{\Gamma(X(3872) \rightarrow \psi(1S) \gamma)} = 3.5 \pm 1.4. \]  

In view of the underlying approximation, one can conclude that the experimental value in (8) and the theoretical prediction (7) are close enough to consider plausible the identification \( X(3872) = \chi_{c1}(2P) \), in contrast to the composite scenarios, in which the mode \( X(3872) \rightarrow \psi(2S) \gamma \) is suppressed compared to \( X(3872) \rightarrow \psi(1S) \gamma \) [43, 54].

5 Conclusions

In the last decade, many predicted charm and beauty mesons have been discovered, along with many unexpected ones. In the case of \( D_{sJ} \) mesons, the analysis of their decay modes allows to classify them as ordinary \( c\bar{s} \) states, although the identification of \( D_{sJ}(2860) \) is still under scrutiny.

The case of hidden charm and beauty mesons is more complicated. As for \( X(3872) \), two analyses of the radiative decays of \( X \) show that the charmonium interpretation seems to be a likely one, although experimentally it is still unclear whether its spin-parity is \( J^P = 1^+ \) or \( J^P = 2^- \).

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References

[1] B. Aubert et al., Phys. Rev. Lett. 90, 242001 (2003).
[2] D. Besson et al., Phys. Rev. D68, 032002 (2003) [Erratum-ibid. D75, 119908 (2007)].
[3] P. Colangelo, F. De Fazio and R. Ferrandes, Mod. Phys. Lett. A19, 2083 (2004).
[4] P. Colangelo and F. De Fazio, Phys. Lett. B570, 180 (2003).
[5] P. Colangelo, F. De Fazio and A. Ozpineci, Phys. Rev. D72, 074004 (2005).
[6] B. Aubert et al., Phys. Rev. Lett. 97, 222001 (2006).
[7] J. Brodzicka et al., Phys. Rev. Lett. 100, 092001 (2008).

¹Belle Collaboration has recently provided an upper limit for the Ratio \( R_X < 2.1 \) (at 90% C.L.) [53].
[8] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D80, 092003 (2009).

[9] P. Colangelo et al., Phys. Lett. B642, 48 (2006); Phys. Rev. D77, 014012 (2008).

[10] P. Colangelo and F. De Fazio, Phys. Rev. D81, 094001 (2010).

[11] For a review see: M. Neubert, Phys. Rept. 245, 259 (1994).

[12] K. Abe et al. [Belle Collaboration], Phys. Rev. D69, 112002 (2004).

[13] P. Colangelo, F. De Fazio and R. Ferrandes, Phys. Lett. B634, 235 (2006).

[14] M. B. Wise, Phys. Rev. D45, R2188 (1992); G. Burdman et al., Phys. Lett. B280, 287 (1992); P. Cho, Phys. Lett. B285 145 (1992); H. Y. Cheng et al., Phys. Rev. D46, 1148 (1992).

[15] R. Casalbuoni et al., Phys. Lett. B299, 139 (1993).

[16] P. Colangelo, F. De Fazio and G. Nardulli, Phys. Lett. B478, 408 (2000).

[17] V. M. Belyaev et al., Phys. Rev. D51, 6177 (1995); P. Colangelo et al., Phys. Lett. B334, 175 (1994); Phys. Lett. B339, 151 (1994); Phys. Rev. D52, 6422 (1995); Eur. Phys. J. C4, 503 (1998); Phys. Lett. B532, 193 (2002).

[18] P. del Amo Sanchez et al. [The BABAR Collaboration], Phys. Rev. D82, 111101 (2010).

[19] R. Casalbuoni et al., Phys. Lett. B 292, 371 (1992).

[20] J. L. Rosner et al. [CLEO Collaboration], Phys. Rev. Lett. 95, 102003 (2005).

[21] S. K. Choi et al. [BELLE collaboration], Phys. Rev. Lett. 89, 102001 (2002) [Erratum-ibid. 89, 129901 (2002)]; B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 92, 142002 (2004); D. M. Asner et al. [CLEO Collaboration], Phys. Rev. Lett. 92, 142001 (2004).

[22] S. Uehara et al. [Belle Collaboration], Phys. Rev. Lett. 96, 082003 (2006).

[23] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 101, 071801 (2008).

[24] J. P. Lees [The BABAR Collaboration], arXiv:1102.4565 [hep-ex].

[25] I. Adachi et al. [Belle Collaboration], arXiv:1103.3419 [hep-ex].

[26] For a review: N. Brambilla et al., Eur. Phys. J. C71, 1534 (2011).

[27] S. K. Choi et al. [BELLE Collaboration], Phys. Rev. Lett. 100, 142001 (2008).

[28] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 79, 112001 (2009).

[29] R. Mizuk et al. [Belle Collaboration], Phys. Rev. D 78, 072004 (2008).

[30] Belle Collaboration, arXiv:1105.4583 [hep-ex].
[31] S. K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262001 (2003).
[32] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D71, 071103 (2005).
[33] D. Acosta et al. [CDF II Collaboration], Phys. Rev. Lett. 93, 072001 (2004).
[34] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 93, 162002 (2004).
[35] K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010).
[36] B. Aubert et al. [BaBar Collaboration], Phys. Rev. D71, 031501 (2005).
[37] G. Gokhroo et al., Phys. Rev. Lett. 97, 162002 (2006).
[38] K. Abe et al. [Belle Collaboration], Phys. Rev. Lett. 94, 182002 (2005).
[39] P. del Amo Sanchez et al. [BABAR Collaboration], Phys. Rev D 82, 011101 (2010).
[40] M. B. Voloshin and L. B. Okun, JETP Lett. 23, 333 (1976).
[41] See M. B. Voloshin, Prog. Part. Nucl. Phys. 61, 455 (2008) and references therein.
[42] M. B. Voloshin, Int. J. Mod. Phys. A21, 1239 (2006).
[43] T. Barnes and S. Godfrey, Phys. Rev. D69, 054008 (2004).
[44] T. Barnes et al., Phys. Rev. D72, 054026 (2005); E. J. Eichten et al., Phys. Rev. D69, 094019 (2004); Phys. Rev. D73, 014014 (2006) [Erratum-ibid. D73, 079903 (2006)].
[45] M. Suzuki, Phys. Rev. D72, 114013 (2005).
[46] P. Colangelo, F. De Fazio and S. Nicotri, Phys. Lett. B650, 166 (2007).
[47] R. Casalbuoni et al., Phys. Rept. 281, 145 (1997).
[48] P. Colangelo, F. De Fazio and T. N. Pham, Phys. Rev. D69, 054023 (2004).
[49] F. De Fazio, Phys. Rev. D79, 054015 (2009) [Erratum-ibid. D83, 099901 (2011)].
[50] R. Casalbuoni et al., Phys. Lett. B302, 95 (1993).
[51] B. A. Thacker and G. P. Lepage, Phys. Rev. D43, 196 (1991).
[52] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 102, 132001 (2009).
[53] V. Bhardwaj et al. [Belle Collaboration], arXiv:1105.0177 [hep-ex].
[54] E. S. Swanson, Phys. Rept. 429, 243 (2006).