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Cosmological singularity theorems and splitting theorems for $N$-Bakry-Émery spacetimes.
(English) J. Math. Phys. 57, No. 2, 022504, 12 p. (2016).

Summary: We study Lorentzian manifolds with a weight function such that the $N$-Bakry-Émery tensor is bounded below. Such spacetimes arise in the physics of scalar-tensor gravitation theories, including Brans-Dicke theory, theories with Kaluza-Klein dimensional reduction, and low-energy approximations to string theory. In the “pure Bakry-Émery” $N = \infty$ case with $f$ uniformly bounded above and initial data suitably bounded, cosmological-type singularity theorems are known, as are splitting theorems which determine the geometry of timelike geodesically complete spacetimes for which the bound on the initial data is borderline violated. We extend these results in a number of ways. We are able to extend the singularity theorems to finite $N$-values $N \in (n, \infty)$ and $N \in (-\infty, 1]$. In the $N \in (n, \infty)$ case, no bound on $f$ is required, while for $N \in (-\infty, 1]$ and $N = \infty$ we are able to replace the boundedness of $f$ by a weaker condition on the integral of $f$ along future-inextendible timelike geodesics. The splitting theorems extend similarly, but when $N = 1$, the splitting is only that of a warped product for all cases considered. A similar limited loss of rigidity has been observed in a prior work on the $N$-Bakry-Émery curvature in Riemannian signature when $N = 1$ and appears to be a general feature.

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MSC:
- 83C75 Space-time singularities, cosmic censorship, etc.
- 83E30 String and superstring theories in gravitational theory
- 83E15 Kaluza-Klein and other higher-dimensional theories
- 53C21 Methods of global Riemannian geometry, including PDE methods; curvature restrictions
- 83F05 Relativistic cosmology
- 83D05 Relativistic gravitational theories other than Einstein’s, including asymmetric field theories
- 83C10 Equations of motion in general relativity and gravitational theory

Keywords:
cosmological singularity theorems; splitting theorems; $N$-Bakry-Émery spacetimes; Lorentzian manifolds; scalar-tensor gravitation; Brans-Dicke theory; Kaluza-Klein dimensional reduction; string theory; future-inextendible timelike geodesics

Full Text: DOI arXiv

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