Heat and Mass Transfer on MHD Flow of an incompressible fluid past an infinite vertical porous plate

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Abstract

We have considered the MHD flow of an incompressible fluid with heat and mass transfer over a vertical plate in the presence of magnetic field with Soret and Dufour effects, chemical reaction and a convective heat exchange at the surface with the surrounding has been studied. The similarity solution is used to transform the system of partial differential equations and an efficient numerical technique is implemented to solve the reduced system by using the Runge-Kutta fourth order method with shooting technique. The results are presented graphically and the conclusion is drawn that the flow field and other quantities of physical interest are significantly influenced by the flow parameters.

Key words: Vertical plate, convective boundary condition, chemical reaction, heat and mass transfer, magnetic field, similarity solution.

Nomenclature:

\[ u, v \] velocity components along x-and y- axes respectively
\[ x, y \] Cartesian coordinates along x-,y-axes respectively
\[ Ha \] Magnetic field parameter
\[ Gr \] Thermal Grashof number
\[ Gc \] Solutal Grashof number
\[ Du \] Dufour number
$Sr$  Soret number
$Ec$  Eckert number
$Bi$  Convective heat transfer parameter
$Pr$  Prandtl number
$Sc$  Schmidt number
$kr^2$  Chemical reaction rate constant
$Re$  Reynolds number
$T$  fluid temperature
$C$  fluid concentration
$\kappa$  the thermal conductivity
$D_m$  mass diffusivity
$B_0$  Magnetic induction
$g$  gravitational acceleration
$C_{\infty}$  free stream concentration
$K'$  permeability of the porous medium
$f$  dimensionless stream function
$C_f$  skin-friction coefficient
$h$  plate heat transfer coefficient
$L$  plate characteristic length
$C_w$  species concentration at the plate surface

**Greek Letters**

$v$  kinematics viscosity
$\rho$  density
$\sigma$  Electric conductivity
$\beta_T$  Thermal expansions
$\beta_c$  Concentration
$\alpha$  Thermal diffusivity
$\theta$  Dimensionless temperature $\{=(T-T_\infty)/(T_w-T_\infty)\}$
$\phi$  Dimensionless concentration $\{=(C-C_\infty)/(C_w-C_\infty)\}$
$\eta$  Similarity variable
$\psi$  Stream function
$\lambda$  plate surface concentration exponent
1. Introduction

Magneto-hydrodynamic (MHD) boundary layers with heat and mass transfer over a flat surface are found in many engineering and geophysical applications such as geothermal reservoirs, thermal insulation, cooling of nuclear reactors. Many chemical engineering processes like metallurgical and polymer extrusion processes involve cooling of a molten liquid being stretched into a cooling system; the fluid mechanical properties of the penultimate product depend mainly on the cooling liquid used and the rate of stretching. Some polymer fluids like Polyethylene oxide and polysobutylene solution in cetane, having better electromagnetic properties, are normally used as cooling liquid as their flow can be regulated by external magnetic fields in order to improve the quality of the final product. Bejan and Khair\(^3\) investigated the free convection boundary layer flow in a porous medium owing to combined heat and mass transfer. Lai and Kulacki\(^9\) used the series expansion method to investigate coupled heat and mass transfer in natural convection from a sphere in a porous medium. The suction and blowing effects on free convection coupled heat and mass transfer over a vertical plate in a saturated porous medium were studied by Raptis \textit{et al.}\(^{15}\) and Lai and Kulacki\(^{10}\) respectively. The effect of thermal radiation on heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field was reported in Makinde and Ogulu\(^{12}\). Mikinde\(^{11}\) studied the similarity solution of hydromagnetic heat and mass transfer over a vertical plate with a convective surface boundary condition. The paper demonstrates that a similarity solution is possible if the convective heat transfer associated with the hot fluid on the lower surface of the plate is proportional to the inverse square root of the axial distance. Recently Gnaneswar Reddy and Bhaskar Reddy\(^5\) reported Soret and Dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation. In all these studies Soret / Dufour effects are assumed to be negligible. Such effects are significant when density differences exist in the flow regime. For example when species are introduced at a surface in fluid domain, with different (lower) density than the surrounding fluid, both Soret and Dufour effects can be significant. Also, when heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more intricate nature. It has been found that an energy flux can be generated not only by temperature gradients but by composition gradients as well. The energy flux caused by a composition gradient is called the Dufour or diffusion-thermo effect. On the other hand, mass fluxes can also be created by temperature gradients and this is called the Soret or thermal-diffusion effect. The thermal-diffusion (Soret) effect, for instance, has been utilized for isotope separation, and in mixture between gases with very light molecular weight (H\(_2\), He) and of medium molecular weight (N\(_2\), air), the diffusion-thermo (Dufour) effect was found to be of a considerable magnitude such that it cannot be ignored (Eckert and Drake\(^4\)). In view of the importance of these above mentioned effects, Dursunkaya and Worek\(^4\) studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams\(^8\) presented the same effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Recently, Anghel \textit{et al.}\(^2\) investigated the Dufour and Soret effects on free convection boundary layer flow over a vertical surface embedded in a porous medium. Very recently, Postelnicu\(^{14}\) studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Alam \textit{et al.}\(^1\) have investigated the Dufour and Soret Effects on Unsteady MHD Free Convection and Mass Transfer Flow past a Vertical Porous Plate in a Porous Medium. Shariful Alam \textit{et al.}\(^{16}\) has presented the Local Similarity Solutions for Unsteady MHD free Convection and Mass Transfer Flow Past an Impulsively Started Vertical Porous Plate with Dufour and Soret Effects. The Dufour and Soret effects on unsteady MHD Convective Heat and Mass Transfer Flow due to a Rotating Disk was analyzed by Maleque\(^{13}\). However, the interaction of Soret and Dufour effects on steady...
MHD free convection flow in a porous medium with viscous dissipation has received little attention. Recently, Krishna and Swarnalathamma discussed the peristaltic MHD flow of an incompressible and electrically conducting Williamson fluid in a symmetric planar channel with heat and mass transfer under the effect of inclined magnetic field. Swarnalathamma and Krishna discussed the theoretical and computational study of peristaltic hemodynamic flow of couple stress fluids through a porous medium under the influence of magnetic field with wall slip condition. Veera Krishna and M.G. Reddy discussed MHD free convective rotating flow of visco-elastic fluid past an infinite vertical oscillating plate. Veera Krishna and G.S. Reddy discussed unsteady MHD convective flow of second grade fluid through a porous medium in a Rotating parallel plate channel with temperature dependent source.

Keeping the above mentioned facts, in this paper we have discussed to analyze Soret and Dufour Effects on Similarity solution of hydro magnetic heat and mass transfer over a vertical plate with a convective surface boundary condition and chemical reaction. The governing equations are transformed by using similarity transformation and the resultant dimensionless equations are solved numerically using the Runge-Kutta method with shooting technique. The effects of various governing parameters on the velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are shown in figures and tables and analyzed in detail.

2. Formulation and Solution of the Problem:

Let us consider a steady, laminar, hydro magnetic coupled heat and mass transfer by mixed convection flow over a vertical plate. The fluid is assumed to be Newtonian, electrically conducting and its property variations due to temperature and chemical species concentration are limited to fluid density. The density variation and the effects of the buoyancy are taken into account in the momentum equation (Boussinesq’s approximation). In addition, there is no applied electric field and all of the Hall effects and Joule heating are neglected. Since the magnetic Reynolds number is very small for most fluid used in industrial applications, we assumed that the induced magnetic field is negligible. The physical configuration of the problem is as shown in Figure 1.

![Figure 1. Physical configuration of the problem](image)

Let the x-axis be taken along the direction of plate and y-axis normal to it. If $u, v, T$ and $C$ are the fluid $x$-component of velocity, $y$-component of velocity, temperature and concentration respectively, then under the Boussinesq and boundary-layer approximations, the governing equations for this problem can be written as:
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

(1)

\[ \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \beta_v^2 (u - U_\infty)}{\rho} + g \beta_T (T - T_\infty) + g \beta_C (C - C_\infty) \]  

(2)

\[ \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_p c_r} \frac{\partial^2 C}{\partial y^2} + \frac{\nu}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 \]  

(3)

\[ \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2} + \nu r^2 (C - C_\infty) \]  

(4)

The boundary conditions at the plate surface and for into the cold fluid may be written as

\[ u(x,0) = v(x,0) = 0, \quad \frac{\partial T}{\partial y}(x,0) = h[T - T_w(x,0)], \quad C_w(x,0) = A x^4 + C_\infty, \]  

\[ u(x,\infty) = U_\infty, T(x,\infty) = T_\infty, C(x,\infty) = C_\infty. \]  

(5)

The velocity components \( u \) and \( v \) are respectively obtained as follows:

\[ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \]  

(6)

A similarity solution of Equations (1)-(6) is obtained by defying an independent variable \( \eta \) and a dependent variable 'f' in terms of the stream function \( \Psi \) as

\[ \eta = \sqrt[3]{\frac{U_\infty}{u_x}}, \quad \psi = \sqrt{u_x U_\infty} f(\eta). \]  

(7)

The dimensionless temperature and concentration are given as

\[ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \]  

(8)

Where \( T_w \) is the temperature of the hot fluid at the left surface of the plate. Substituting the equations (6)-(8) in to Equations (1)-(5), we obtain

\[ f'' + \frac{1}{2} ff'' - Ha(f' -1) + Gr \theta + Gc \phi = 0 \]  

(9)

\[ \theta'' + \frac{1}{2} Pr f \theta' + \frac{1}{2} Pr Du \phi'' + \frac{1}{2} Pr Ec f' n^2 = 0 \]  

(10)
\[ \phi^{''} + \frac{1}{2} Scf \phi^{'} + \frac{1}{2} ScSr \theta^{''} - \kappa r^2 Sc \phi = 0 \]  
(11)

\[ f(0) = 0, f^{'}(0) = 0, \theta^{'}(0) = Bi [\theta(0) - 1], \phi(0) = 0 \]  
(12)

\[ f^{''}(\infty) = 1, \theta(\infty) = \phi(\infty) = 0. \]  
(13)

Where the prime symbol represents the derivative with respect to \( \eta \)

Where, \( Ha = \sigma B_0^2 x / \rho U_\infty \) is Magnetic field parameter (Hartmann number), \( Gr = (g \beta_r (T_w - T_\infty) x) / U_\infty^2 \) is the Thermal Grashof number, \( Gc = (g \beta_c (C_w - C_\infty) x) / U_\infty^2 \) is the Solutal Grashof number, \( Du = (D_w k_r (C_w - C_\infty)) / (c, c_p (T_w - T_\infty)) \) is Dufour number, \( Sr = (D_w k_r (T_w - T_\infty)) / (\nu T_m (C_w - C_\infty)) \) is Soret number, \( Ec = (U_\infty^2 / c_p (T_w - T_\infty)) \) is Eckert number, \( Bi = (h / k) \sqrt{\nu x / U_\infty} \) is Convective heat transfer parameter, \( Pr = \nu / \alpha \) is Prandtl number, \( Sc = \nu / D_m \) is Schmidt number, \( Kr = \kappa r^2 2x / U_\infty \) is Chemical reaction rate constant \( Re = Ut / \nu \) is the Reynolds number

It is noteworthy that the local parameters \( Bi, Ha, Gr \) and \( Gc \) in Equations (9)-(13) are functions of \( x \). However, in order to have a similarity solution all the parameters \( Bi, Ha, Gr, Gc \), \( Du, Sr, Ec \) must be constant and we therefore assume

\[ h = c x^{-\frac{1}{2}}, \sigma = a x^{-1}, \beta_r = b x^{-1} \text{ and } \beta_c = d x^{-1} \]  
(14)

Where a, b, c, d are constants, Other physical quantities of interest in this problem such as the skin friction parameter \( C_f = 2(Re)^{-\frac{1}{2}} f^{''}(0) \), the plate surface temperature \( \theta(0) \), Nusselt number \( Nu = -(Re)^2 \theta^{'}(0) \)

and the Sherwood number \( Sh = -(Re)^2 \phi^{'}(0) \) can be easily computed. For local similarity case, integration over the entire plate is necessary to obtain the total skin friction, total heat and mass transfer rate.

The set of coupled non-linear governing boundary layer equations (9)-(11) together with the boundary conditions (12&13) are solved numerically by using the Runge-Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential Equations (9)-(11) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain et al.7). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size \( \Delta \eta = 0.05 \) is used to obtain the numerical solution with decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and the Sherwood number, which are respectively proportional to \( f^{''}(0), -\theta^{'}(0) \) and \( -\phi^{'}(0) \), are also sorted out and their numerical values are presented in a tabular form.

3. Results and Discussions

Numerical results are reported in the tables (1-2). The prandtl number was taken to be \( Pr=0.72 \) which corresponds to air, the value of Schmidt number (\( Sc \)) were chosen to be \( Sc=0.24, 0.62, 0.78, 2.62 \), representing...
diffusing chemical species of most common interest in air like $H_2$, $H_2O$, $NH_3$ and Propyl Benzene respectively. Attention is focused on positive value of the buoyancy parameters that is, Grashof number $Gr > 0$ (which corresponds to the cooling problem) and solutal Grashof number $Gc > 0$ (which indicates that the chemical species concentration in the free stream region is less than the concentration at the boundary surface). In order to benchmark our numerical results, we have compared the plate surface temperature $\theta(0)$ and the local heat transfer rate at the plate surface $\theta'(0)$ in the absence of both magnetic field and buoyancy forces for various values of $Bi$ with those of Gnaneswar Reddy and Bhaskar Reddy and found them in agreement as demonstrated in table 1. From table 2, it is important to note that the local skin friction together with the local heat and mass transfer rate at the plate surface increases with increasing intensity of buoyancy forces ($Gr$, $Gc$), magnetic field ($Ha$), convective heat change parameter ($Bi$), Eckert number ($Ec$), Dufour number ($Du$) and Soret number ($Sr$). However, an increase in the Schmidt number ($Sc$) and chemical parameter ($Kr$) causes a decrease in both skin friction and surface heat transfer rate and an increase in the surface mass transfer rate.

The effects of various parameters on velocity profiles in the boundary layer are depicted in Figures (1-9). It is observed from Figures (1-9), that the velocity starts from a zero value at the plate surface and increase to the free stream value far away from the plate surface satisfying the far field boundary condition for all parameter values. In Figure 1 the effect of increasing the magnetic field strength on the momentum boundary layer thickness is illustrated. It is now a well established fact that the magnetic field presents a damping effect on the velocity field by creating drag force that opposes the fluid motion, causing the velocity to decrease. However, in this case an increase in the ($Ha$) only slightly slows down the motion of the fluid away from the vertical plate surface towards the free stream velocity, while the fluid velocity near the vertical plate surface increases. Similar trend of slight increase in the fluid velocity near the vertical plate is observed with an increase in convective heat transfer parameter ($Bi$). Figures (3, 4, 7, 8 & 9) have shown the variation of the boundary-layer velocity with the buoyancy forces parameters ($Gr$, $Gc$), Eckert number ($Ec$), Dufour number ($Du$) and Soret number ($Sr$). In the above cases an upward acceleration of the fluid in the vicinity of the vertical wall is observed with increasing intensity of buoyancy forces. Further downstream of the fluid motion decelerates to the free stream velocity. Figure 5 and 6 shows a slight decrease in the fluid velocity with an increase in the Schmidt number ($Sc$) and chemical reaction parameter ($Kr$).

Generally, the fluid temperature attains its maximum value at the plate surface and decreases exponentially to the free stream zero value away from the plate satisfying the boundary condition. This is observed in Figures 10-19. From these figures, it is interesting to note that the thermal boundary layer thickness decreases with an increase in the intensity of magnetic field $Ha$, the buoyancy forces ($Gr$, $Gc$), permeability parameter ($Pr$) and Soret number ($Sr$). Moreover, the fluid temperature increases with an increase in the Schmidt number ($Sc$), the convective heat exchange at the plate surface ($Bi$), chemical reaction parameter ($Kr$), Eckert number ($Ec$) and Dufour number ($Du$) leading to an increase in thermal boundary layer thickness.

Figures (20-27) depict chemical species concentration profiles against span wise coordinate $\zeta$ for varying values physical parameters in the boundary layer. The species concentration is highest at the plate surface and decrease to zero far away from the plate satisfying the boundary condition. From these figures, it is noteworthy that the concentration boundary layer thickness decreases with an increase in the magnetic field intensity $Ha$, the buoyancy forces ($Gr$, $Gc$), Schmidt number ($Sc$), Eckert number ($Ec$), Dufour number ($Du$) and chemical reaction parameter ($Kr$) and Moreover, the fluid concentration increases with an increase in the Soret number ($Sr$) leading to an increase in thermal boundary layer thickness.
Fig. 1: Variation of the velocity component $f'$ with $Ha$ for $Pr=0.72$, $Sc=0.62$, $Gr=Gc=Bi=0.1$, $Kr=0.5$, $Du=0.2$, $Sr=1.0$, $Ec=0.01$.

Fig. 2: Variation of the velocity component $f'$ with $Bi$ for $Pr=0.72$, $Sc=0.62$, $Gr=Gc=Ha=0.1$, $Kr=0.5$, $Du=0.2$, $Sr=1.0$, $Ec=0.01$.

Fig. 3: Variation of the velocity component $f'$ with $Gr$ for $Pr=0.72$, $Sc=0.62$, $Gc=Ha=Bi=0.1$, $Kr=0.5$, $Du=0.2$, $Sr=1.0$, $Ec=0.01$.

Fig. 4: Variation of the velocity component $f'$ with $Gc$ for $Pr=0.72$, $Sc=0.62$, $Gr=Ha=Bi=0.1$, $Kr=0.5$, $Du=0.2$, $Sr=1.0$, $Ec=0.01$.

Fig. 5: Variation of the temperature with $Sc$ for $Pr=0.72$, $Gr=Gc=Ha=Bi=0.1$, $Kr=0.5$, $Du=0.2$, $Sr=1.0$, $Ec=0.01$.

Fig. 6: Variation of the velocity component $f'$ with $Kr$ for $Pr=0.72$, $Sc=0.62$, $Gr=Gc=Ha=Bi=0.1$, $Du=0.2$, $Sr=1.0$, $Ec=0.01$. 

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Fig. 7: Variation of the velocity component $f'$ with $Ec$ for $Pr=0.72$, $Sc=0.62$, $Gr=Ge=Ha=Bi=0.1$, $Kr=0.5$, $Du=0.2$, $Sr=1$.

Fig. 8: Variation of the velocity component $f'$ with $Du$ for $Pr=0.72$, $Sc=0.62$, $Gr=Ge=Ha=Bi=0.1$, $Kr=0.5$, $Sr=1.0$, $Ec=0.01$.

Fig. 9: Variation of the velocity component $f'$ with $Sr$ for $Pr=0.72$, $Sc=0.62$, $Gr=Ge=Ha=Bi=0.1$, $Kr=0.5$, $Du=0.2$, $Ec=0.01$.

Fig. 10: Variation of the temperature $\theta$ with $Ha$ for $Pr=0.72$, $Sc=0.62$, $Ge=Ge=Ha=Bi=0.1$, $Kr=0.5$, $Du=0.2$, $Sr=1.0$, $Ec=0.01$.

Fig. 11: Variation of the temperature $\theta$ with $Bi$ for $Pr=0.72$, $Sc=0.62$, $Gr=Ge=Ha=0.1$, $Kr=0.5$, $Du=0.2$, $Sr=1.0$, $Ec=0.01$.

Fig. 12: Variation of the temperature $\theta$ with $Gr$ for $Pr=0.72$, $Sc=0.62$, $Ge=Ge=Ha=Bi=0.1$, $Kr=0.5$, $Du=0.2$, $Sr=1.0$, $Ec=0.01$. 

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Fig. 13: Variation of the temperature $\theta$ with $Gc$ for $Pr=0.72, Sc=0.62, Gr=Bi=Ha=0.1$, $Kr=0.5, Du=0.2, Sr=1.0, Ec=0.01$

Fig. 14: Variation of the temperature $\theta$ with $Sc$ for $Pr=0.72, Gr=Bi=Gc=Ha=0.1$, $Kr=0.5, Du=0.2, Sr=1.0, Ec=0.01$

Fig. 15: Variation of the temperature $\theta$ with $Kr$ for $Pr=0.72, Sc=0.62, Gr=Bi=Ha=Gc=0.1$, $Du=0.2, Sr=1.0, Ec=0.01$

Fig. 16: Variation of the temperature $\theta$ with $Ec$ for $Pr=0.72, Sc=0.62, Gr=Bi=Ha=Gc=0.1$, $Kr=0.5, Du=0.2, Sr=1.0, Ec=0.01$

Fig. 17: Variation of the temperature $\theta$ with $Gc$ for $Pr=0.72, Sc=0.62, Gr=Bi=Ha=0.1$, $Kr=0.5, Sr=1.0, Ec=0.01$

Fig. 18: Variation of the temperature $\theta$ with $Sr$ for $Pr=0.72, Sc=0.62, Gr=Bi=Gc=Ha=0.1$, $Kr=0.5, Du=0.2, Sr=1.0, Ec=0.01$. 

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Fig. 19: Variation of the temperature $\vartheta$ with $Pr$ for $Sc=0.62, Gr=Bi=Ha=Gc=0.1, Kr=0.5, Du=0.2, Sr=1.0, Ec=0.01$.

Fig. 20: Variation of the concentration $\phi$ with $Ha$ for $Pr=0.72, Sc=0.62, Gr=Gc=Bi=0.1, Kr=0.5, Du=0.2, Sr=1.0, Ec=0.01$.

Fig. 21: Variation of the concentration $\phi$ with $Gr$ for $Pr=0.72, Sc=0.62, Ha=Gc=Bi=0.1, Kr=0.5, Du=0.2, Sr=1.0, Ec=0.01$.

Fig. 22: Variation of the concentration $\phi$ with $Gc$ for $Pr=0.72, Sc=0.62, Gr=Ha=Bi=0.1, Kr=0.5, Du=0.2, Sr=1.0, Ec=0.01$.

Fig. 23: Variation of the concentration $\phi$ with $Sc$ for $Pr=0.72, Kr=0.5, Gr=Gc=Ha=Bi=0.1, Du=0.2, Sr=1.0, Ec=0.01$.

Fig. 24: Variation of the concentration $\phi$ with $Kr$ for $Pr=0.72, Du=0.2, Sc=0.62, Gr=Gc=Bi=Ha=0.1, Sr=1.0, Ec=0.01$. 

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Fig. 25: Variation of the concentration $\phi$ with $Ec$ for $Pr=0.72$, $Kr=0.5$, $Sc=0.62$, $Gr=Gc=Bi=Ha=0.1$, $Du=0.2$, $Sr=1.0$.

Fig. 26: Variation of the concentration $\phi$ with $Du$ for $Pr=0.72$, $Kr=0.5$, $Sc=0.62$, $Gr=Gc=Bi=Ha=0.1$, $Sr=1.0$, $Ec=0.01$.

Fig. 27: Variation of the concentration $\phi$ with $Sr$ for $Pr=0.72$, $Sc=0.62$, $Gr=Gc=Bi=Ha=0.1$, $Kr=0.5$, $Du=0.2$, $Ec=0.01$.

| $Bi$ | $Gr$ | $Gc$ | $Ha$ | $Sc$ | $Sr$ | $Kr$ | $Du$ | $Ec$ | $C_f$ | $Nu$ | $Sh$ |
|------|------|------|------|------|------|------|------|------|------|------|------|
| 0.1  | 0.1  | 0.1  | 0.1  | 0.62 | 1    | 0.5  | 0.2  | 0.01 | 0.596611 | 0.075537 | 0.381552 |
| 1    | 0.1  | 0.1  | 0.1  | 0.62 | 1    | 0.5  | 0.2  | 0.01 | 0.649006 | 0.253486 | 0.384407 |
| 10   | 0.1  | 0.1  | 0.1  | 0.62 | 1    | 0.5  | 0.2  | 0.01 | 0.671671 | 0.332795 | 0.385626 |
| 0.1  | 0.5  | 0.1  | 0.1  | 0.62 | 1    | 0.5  | 0.2  | 0.01 | 0.700125 | 0.075922 | 0.387268 |
| 0.1  | 1    | 0.1  | 0.1  | 0.62 | 1    | 0.5  | 0.2  | 0.01 | 0.821246 | 0.076328 | 0.393721 |
| 0.1  | 0.1  | 0.5  | 0.1  | 0.62 | 1    | 0.5  | 0.2  | 0.01 | 0.994141 | 0.076825 | 0.402332 |
| 0.1  | 0.1  | 1    | 0.1  | 0.62 | 1    | 0.5  | 0.2  | 0.01 | 1.444356 | 0.077844 | 0.423252 |
| 0.1  | 0.1  | 0.1  | 0.4  | 0.62 | 1    | 0.5  | 0.2  | 0.01 | 0.813629 | 0.076167 | 0.391165 |
| 0.1  | 0.1  | 0.1  | 0.6  | 0.62 | 1    | 0.5  | 0.2  | 0.01 | 0.930447 | 0.076430 | 0.395548 |
| 0.1  | 0.1  | 0.1  | 0.1  | 0.78 | 1    | 0.5  | 0.2  | 0.01 | 0.594611 | 0.075216 | 0.402517 |
| 0.1  | 0.1  | 0.1  | 0.2  | 1    | 0.5  | 0.2  | 0.01 | 0.584549 | 0.073423 | 0.520237 |
| 0.1  | 0.1  | 0.1  | 0.1  | 0.62 | 2    | 0.5  | 0.2  | 0.01 | 0.598834 | 0.075883 | 0.358936 |
| 0.1  | 0.1  | 0.1  | 0.1  | 0.62 | 3    | 0.5  | 0.2  | 0.01 | 0.600445 | 0.076128 | 0.342961 |
| 0.1  | 0.1  | 0.1  | 0.1  | 0.62 | 1    | 1    | 0.2  | 0.01 | 0.588073 | 0.073054 | 0.544738 |
| 0.1  | 0.1  | 0.1  | 0.1  | 0.62 | 1    | 1.5  | 0.2  | 0.01 | 0.579232 | 0.069785 | 0.757191 |
| 0.1  | 0.1  | 0.1  | 0.1  | 0.62 | 1    | 0.5  | 0.4  | 0.01 | 0.599633 | 0.073623 | 0.381748 |
| 0.1  | 0.1  | 0.1  | 0.1  | 0.62 | 1    | 0.5  | 0.6  | 0.01 | 0.602658 | 0.071702 | 0.381944 |
| 0.1  | 0.1  | 0.1  | 0.1  | 0.62 | 1    | 0.5  | 0.2  | 0.02 | 0.596901 | 0.075318 | 0.381569 |
| 0.1  | 0.1  | 0.1  | 0.1  | 0.62 | 1    | 0.5  | 0.2  | 0.03 | 0.597481 | 0.074882 | 0.381603 |
Table 2. Skin-friction coefficient $C_f$, Nusselt number $Nu$ and Sherwood number $Sh$

| Pr  | Ec  | Du  | Sc  | Sr  | Previous work [6] | Present work |
|-----|-----|-----|-----|-----|-------------------|--------------|
|     |     |     |     |     | $C_f$          | $Nu$         | $Sh$        | $C_f$          | $Nu$         | $Sh$        |
| 0.71| 0.01| 0.2 | 0.6 | 1   | 0.823025        | 0.861862     | 0.436223   | 0.823225       | 0.861996     | 0.438769   |
| 1   | 0.01| 0.2 | 0.6 | 1   | 0.753718        | 1.108723     | 0.290841   | 0.753458       | 1.108611     | 0.294998   |
| 0.71| 0.02| 0.2 | 0.6 | 1   | 0.824066        | 0.859063     | 0.437788   | 0.825714       | 0.859385     | 0.439263   |
| 0.71| 0.01| 0.4 | 0.6 | 1   | 0.840539        | 0.829245     | 0.457354   | 0.840980       | 0.829253     | 0.450044   |
| 0.71| 0.01| 0.2 | 0.78| 1   | 0.773150        | 0.846942     | 0.499496   | 0.774987       | 0.846364     | 0.497626   |
| 0.71| 0.01| 0.2 | 0.6 | 2   | 1.008445        | 0.933033     | 0.293132   | 1.008444       | 0.933535     | 0.291731   |

1. Conclusions

A comparison with previously published work is performed and good agreement between the results is obtained. The conclusions are made as the following.

1. The effect of increasing strength of the magnetic field on the momentum boundary layer thickness that the magnetic field presents a damping effect on the velocity field by creating drag force that opposes the fluid motion, causing the velocity to decrease.

2. Similar trend of slight increase in the fluid velocity near the vertical plate is observed with an increase in convective heat transfer parameter. the variation of the boundary-layer velocity with the buoyancy forces parameters, Eckert number, Dufour number and Soret number.

3. An upward acceleration of the fluid in the vicinity of the vertical wall is observed with increasing intensity of buoyancy forces.

4. Downstream of the fluid motion decelerates to the free stream velocity.

5. A slight decrease in the fluid velocity with an increase in the Schmidt number and chemical reaction parameter.

6. The thermal boundary layer thickness decreases with an increase in the intensity of magnetic field, the buoyancy forces, permeability parameter and Soret number.

7. The fluid temperature increases with an increase in the Schmidt number, the convective heat exchange at the plate surface, chemical reaction parameter, Eckert number and Dufour number leading to an increase in thermal boundary layer thickness.

8. The species concentration is highest at the plate surface and decreases to zero far away from the plate satisfying the boundary condition.

9. the local skin-friction coefficient, local heat and mass transfer rate at the plate surface increases with an increase in intensity of magnetic field, buoyancy forces, convective heat exchange parameter, Soret number, Eckert number, Dufour number and chemical reaction parameter.

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