THE LONGITUDINAL STRUCTURE FUNCTION $F_L$ AT SMALL $X$

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We present a set of formulae to extract the longitudinal deep inelastic structure function $F_L$ from the transverse structure function $F_2$ and its derivative $dF_2/d\ln Q^2$ at small $x$. Using $F_2$ HERA data we obtain $F_L$ in the range $10^{-4} \leq x \leq 10^{-2}$ at $Q^2 = 20$ GeV$^2$.

We present here the values of $F_L(x, Q^2)$ at small $x$, extracted from experimental HERA data\footnote{In the time of preparing this article, the H1 collaboration presented\textsuperscript{7} the first (preliminary) measurement of $F_L$ at small $x$}, using the method of replacement of the Mellin convolution by ordinary products\footnote{We use PDF multiplied by $x$ and neglect the nonsinglet quark distribution at small $x$.}. By analogy with the case of the gluon distribution function (see\footnote{p.\textsuperscript{7}} and its references) it is possible to obtain the relation between $F_L(x, Q^2)$, $F_2(x, Q^2)$ and $dF(x, Q^2)/d\ln Q^2$ at small $x$. Thus, the small $x$ behaviour of the SF $F_L(x, Q^2)$ can be extracted directly from the measured values of $F_2(x, Q^2)$ and its derivative without a cumbersome procedure\footnote{p.\textsuperscript{6}}. These extracted values of $F_L$ may be well considered as \textit{new small $x$ "experimental data"}. When experimental data for $F_L$ at small $x$ become available with a good accuracy\footnote{p.\textsuperscript{6}} a violation of the relation will be an indication of the importance of other effects as higher twist contribution and/or of non-perturbative QCD dynamics at small $x$.

We follow the notation of our previous work in ref.\footnote{p.\textsuperscript{6}}. The singlet quark $s(x, Q^2_0)$ and gluon $g(x, Q^2_0)$ parton distribution functions (PDF)\footnote{p.\textsuperscript{6}} at some $Q^2_0$ are parameterized by (see, for example\footnote{p.\textsuperscript{6}})

$$p(x, Q^2_0) = A_p x^{-\delta}(1 - x)^{\nu_p}(1 + \epsilon_p \sqrt{x} + \gamma_p x) \quad \text{(hereafter } p = s, g) \quad (1)$$

Further, we restrict the analysis to the case of large $\delta$ values (i.e. $x^{-\delta} \gg 1$) which correspond to BFKL pomeron which is supported by HERA data.
more complete analysis may be found in [8], where we took into account also the case \( \delta \sim 0 \) corresponding to the standard pomeron.

Assuming the Regge-like behaviour for the gluon distribution and \( F_2(x, Q^2) \):

\[
g(x, Q^2) = x^{-\delta} \tilde{g}(x, Q^2), \quad F_2(x, Q^2) = x^{-\delta} \tilde{s}(x, Q^2),
\]

we obtain the following equation for the \( Q^2 \) derivative of the SF \( F_2 \):

\[
\frac{dF_2(x, Q^2)}{d\ln Q^2} = -\frac{1}{2} x^{-\delta} \sum_{p=s, g} \left( r_{1+\delta}^{g}(\alpha) \tilde{p}(0, Q^2) + r_{1+\delta}^{s}(\alpha) x \tilde{p}'(0, Q^2) + O(x^2) \right)
\]

\[
F_L(x, Q^2) = x^{-\delta} \sum_{p=s, g} \left( r_{1+\delta}^{L}(\alpha) \tilde{p}(0, Q^2) + r_{1+\delta}^{L}(\alpha) x \tilde{p}'(0, Q^2) + O(x^2) \right) (2)
\]

where \( r_{1+\delta}^{g}(\alpha) \) and \( r_{1+\delta}^{L}(\alpha) \) are the combinations (see [8]) of the anomalous dimensions of Wilson operators and Wilson coefficients of the \( \eta \) "moment" (i.e., the corresponding variables extended from integer values of argument to non-integer ones). With accuracy of \( O(x^{2-\delta}, \alpha x^{1-\delta}) \) (see [4], [8]), we have for Eq. (2)

\[
F_L(x, Q^2) = -\xi^\delta \left[ 2 r_{1+\delta}^{Lg} \frac{dF_2(x\xi, Q^2)}{d\ln Q^2} + \left( r_{1+\delta}^{Ls} - \frac{r_{1+\delta}^{Lg}}{r_{1+\delta}^{sg}} s \right) F_2(x\xi, Q^2) \right] + O(x^{2-\delta}, \alpha x^{1-\delta}) \quad (3)
\]

Using NLO approximation of \( r_{1+\delta}^{g} \) and \( r_{1+\delta}^{L} \) for concrete values of \( \delta = 0.5 \) and \( \delta = 0.3 \) we obtain (for \( f=4 \) and \( \overline{MS} \) scheme):

\[
F_L(x, Q^2) = \frac{0.87}{1 + 22.9\alpha} \left[ \frac{dF_2(0.70x, Q^2)}{d\ln Q^2} + 4.17\alpha F_2(0.70x, Q^2) \right] + O(\alpha^2, x^{2-\delta}, \alpha x^{1-\delta}) \quad \text{if} \quad \delta = 0.5 \quad (4)
\]

\[
F_L(x, Q^2) = \frac{0.84}{1 + 59.3\alpha} \left[ \frac{dF_2(0.48x, Q^2)}{d\ln Q^2} + 3.59\alpha F_2(0.48x, Q^2) \right] + O(\alpha^2, x^{2-\delta}, \alpha x^{1-\delta}) \quad \text{if} \quad \delta = 0.3 \quad (5)
\]

With the help of Eq. (3) we have extracted the longitudinal SF \( F_L(x, Q^2) \) from HERA data, using the slopes \( dF_2/d\ln Q^2 \) of H1 and ZEUS HERA data. Fig. 1 shows the extracted values of the longitudinal SF and the QCD prediction from set MRS(G). The agreement is excellent. There is also a
relative good agreement with a recent experimental H1 point for $F_L$, if one takes into account the systematic error.

In summary, we have presented Eqs. (3)-(5) for the extraction of the longitudinal SF $F_L$ at small $x$ from the $F_2$ and its $Q^2$ derivative. These equations provide the possibility of the non-direct determination of $F_L$. This is important since the direct extraction of $F_L$ from experimental data is a cumbersome procedure. Moreover, the fulfillment of Eqs. (3)-(5) in DIS experimental data can be used as a cross-check of perturbative QCD at small values of $x$.

Figure 1: The extracted longitudinal SF $F_L$ (see text for details)

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