Nucleon 3D structure from double parton scattering:  
a Light-Front quark model analysis

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Abstract. Double parton distribution functions (dPDFs) represent a tool to explore the 3D  
partonic structure of the proton. They can be measured in high energy proton-proton and  
proton nucleus collisions and encode information on how partons inside a proton are correlated  
among each other. dPDFs are studied here in the valence region by means of a constituent quark  
model scenario within the relativistic Light Front approach, where two particle correlations are  
present without any additional prescription. Furthermore, a study of the QCD evolution at  
high energy scale, of the model results, has been completed in order to compare our predictions  
with future data analyses. In closing, results on the evaluation of the so called \( \sigma_{\text{eff}} \), crucial  
ingredient for the description of double parton scattering, where dPDFs can be accessed, are  
presented and discussed.

1. Introduction

The observation of multiple parton interactions (MPI), occurring in high energy hadron-hadron  
collisions, will be an important goal of the experimental and theoretical analyses of the processes  
studied, e.g., at the LHC. In particular, in these kind of events, more than one parton of a  
hadron interact with partons of the other colliding hadron. Naturally, MPI contribution to the  
total cross section is suppressed with respect to the single parton interaction. However, the  
measurement and estimate of MPI cross sections is an important challenge being MPI events  
a background for the search of new Physics. Furthermore, from a hadronic point of view, we  
are interested on MPI due to the possibility of accessing new fundamental information on the  
partonic proton structure. We focus our studies on the the double parton scattering (DPS), the  
most simple case of MPI, which can be observed, in principle, in several processes, e.g., \( WW \)  
with dilepton productions and double Drell-Yan processes (see, Refs. [1, 2, 3, 4, 5] for recent  
reviews). At the LHC, DPS, has been observed some years ago [6] and represents a background  
for the Higgs production in several channels. From a theoretical point of view, the DPS cross  
section is written in terms of a new quantity, the so called double parton distribution function  
(dPDF), \( F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) \), which describe the joint probability of finding two partons of flavors  
\( i, j = q, \bar{q}, g \) with longitudinal momentum fractions \( x_1, x_2 \) and separation \( \vec{z}_\perp \) in the transverse  
plane inside the hadron, see Ref. [7]. Here \( \mu \) is the renormalization scale. However, to date,  
being dPDFs poorly known, in order to qualitatively estimate the magnitude of the DPS cross  
section, the following approximation is usually adopted:

\[
F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) = q_i(x_1, \mu) \ q_j(x_2, \mu) \ \theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n, \quad (1)
\]
i.e., dPDFs are evaluated in a fully factorized ansatz in terms of standard one-body parton distribution functions (PDF), \( q(x) \), and \( T(\vec{z}_{i}, \mu) \), the function encoding all parton correlations in the transverse plane. The spirit of the latter assumption is to neglect all possible double parton correlations (DPCs) between the two interacting partons, being the latter almost unknown, see e.g. Ref. [8] for updates. Moreover dPDFs are non perturbative quantities in QCD so that they cannot be easily evaluated from the theory. Nevertheless, as already deeply discussed in Refs. [9, 10, 11, 12], quark model calculations of dPDFs could help to grasp the basic feature of such quantities. This strategy has been largely used in the past to study unknown distributions. However, since in this scenario dPDFs are calculated at the low hadronic scale of the model, \( \mu_{0} \sim A_{QCD} \), in order to compare the obtained outcomes with future data taken at high energy scales, \( Q > \mu_{0} \), it is then necessary to perform the perturbative QCD (pQCD) evolution of the model calculations, using the dPDF evolution equations, see Refs. [13, 14]. The idea supporting our analysis is that, thanks to this procedure, future data analyses of the DPS processes could be guided, in principle, by model calculations. In the first part of the present paper, we focus our attention on the study of the role of DPCs in dPDFs in order to verify, as a first step, the validity of the approximation Eq. (1), often used for data analyses. In particular, it is here recall that in all model calculations of dPDFs, see Refs. [9, 10, 11], the assumption Eq. (1) is violated. See also Ref. [15] for details on the violation of the factorized ansatz due to model independent relativistic effects in the calculated dPDFs. Moreover, as already pointed out, it is fundamental to realize to which extend DPCs survive at very high energy scales, where, due to the large population of partons, the role of DPCs could be less important than at the low scale of the used model. To this aim in Refs. [11, 16], DPCs in dPDFs have been studied at the energy scale of the experiments. In particular in Ref. [16] an extension of the approach used in Refs. [11] has been provided to include sea quarks and gluon degrees of freedom. Here, the most important results of the these analyses will be summarized. In the last section of the present paper, an investigation of the so called \( \sigma_{\text{eff}} \) will be discussed. Usually, indeed, DPS cross section, in processes with final state \( A + B \), is written through the following ratio (see e.g. Ref. [17]):

\[
\sigma_{\text{DPS}}^{A+B} = \frac{m}{2} \frac{\sigma_{\text{SPS}}^{A} \sigma_{\text{SPS}}^{B}}{\sigma_{\text{eff}}}, \tag{2}
\]

where \( m \) is combinatorial factor depending on the final states \( A \) and \( B \) (\( m = 1 \) for \( A = B \) or \( m = 2 \) for \( A \neq B \)) and \( \sigma_{\text{SPS}}^{A(B)} \) is the single parton scattering cross section with final state \( A(B) \). Expressing the \( \sigma_{\text{DPS}} \) cross section in Eq. (2) in terms of product of \( \sigma_{\text{SPS}} \), one assumes that, as a first approximation, double parton distributions can be written as in Eq. (1). The present knowledge on DPS cross sections has been condensed in the experimental and model dependent extraction of \( \sigma_{\text{eff}} \) [17, 18, 19, 20, 21, 22, 23, 24]. To date, \( \sigma_{\text{eff}} \simeq 15 \text{ mb} \), compatible, within errors, with a constant, irrespective of centre-of-mass energy of the hadronic collisions, the final state and \( x_{i} \). Let us stress again that the latter result is obtained considering the fully uncorrelated scenario shown in Eq. (1) where all DPCs are neglected. Also in this case non perturbative methods have been used to study \( \sigma_{\text{eff}} \), through the calculation of dPDFs, in order to characterize “signals” of DPCs in \( \sigma_{\text{eff}} \). On top of that, in Ref. [25], the possible dependence of the latter quantity on the longitudinal momentum fraction carried by the interacting partons has been addressed. Recent results on this topic will be here discussed, including also new intriguing outcomes addressed in Refs. [15, 26]. In Ref. [15] the contribution of relativistic effects to \( \sigma_{\text{eff}} \) have been estimated by means of the Light-Front approach (LF), already used for the evaluation of dPDFs in Ref. [11]. Furthermore, in Ref. [26], \( \sigma_{\text{eff}} \) has been calculated in a AdS/QCD framework, showing, also here, a strong \( x_{i} \) dependence of \( \sigma_{\text{eff}} \) in the valence region.
2. Calculation of dPDFs within LF CQMs

In this section, details of the calculations of dPDFs, within the LF approach (see Refs. [27, 28] for fundamental details), will be discussed verifying if the factorized ansatz Eq. (1) would work in this relativistic scenario. In this framework, among all positive features of the LF approach, let us remark that one can obtain a fully Poincaré covariant description of relativistic strongly interacting systems with a fixed number of on-shell constituents, LF boosts and the “plus” components of momenta ($a^+ = a_0 + a_3$) are kinematical operators. Furthermore, being the LF hypersurface, the one where the initial conditions of the system are fixed, tangent to the light-cone, the kinematics of DIS processes is naturally obtained. For these reasons, this strategy has been adopted in the past for the evaluation unknown distributions, see e.g. Refs. [29, 30, 31].

All the details on the present calculations are deeply discussed and reported in Ref. [11] and will not be repeated here. The final expression of the dPDF in momentum space for two unpolarized quarks of flavor $u$, reads:

$$F_{uu}^{λ_1,λ_2}(x_1, x_2, k_1^⊥) = 2(\sqrt{3})^3 \int \prod_{i=1}^{3} d\bar{k}_i \sum_{λ_i/τ_i} \delta \left( \sum_{i=1}^{3} \bar{k}_i \right) ψ^* \left( \bar{k}_1 - \frac{\bar{k}_1}{2}, \bar{k}_2 + \frac{\bar{k}_1}{2}, \bar{k}_3; \{λ_f, τ_i\} \right) \times \Psi \left( \bar{k}_1, \bar{k}_2, \bar{k}_3; \{λ_f, τ_i\} \right) \delta \left( x_1 - \frac{k_1^+}{P^+} \right) \delta \left( x_2 - \frac{k_2^+}{P^+} \right),$$

being $k_i^+$ the relative transverse momentum of one of the parton in the amplitude and its complex conjugate and $\bar{k}_i$ the momentum of the $i$– quark.

The canonical proton wave function $ψ^{cf}$ is embedded in the function $Ψ$ here above, which can be written as follows:

$$Ψ(\bar{k}_1, \bar{k}_2, \bar{k}_3; \{λ_f, τ_i\}) = \prod_{i=1}^{3} \sum_{λ_i} D^{1/2}_{λ_i λ_f} (R_{cf}(\bar{k}_i)) \psi^{cf}(\{\bar{k}_i, λ_i, τ_i\}), \quad (3)$$

where $λ_i^c$ and $τ_i$ are the canonical parton helicity and the isospin, respectively. Here the short notation $\{α_i\} = α_1, α_2, α_3$ is introduced. In Eq. (3), the Melosh operators, which allow to rotate the LF spin into the canonical one are introduced:

$$D_i = D^{1/2}_{μ λ} (R_{cf}(\bar{k}_i)) = ⟨μ | m + x_i M_0 - i \vec{σ}_i \cdot (\vec{z}_i × \vec{k}_i^⊥) \sqrt{(m + x_i M_0)^2 + \vec{k}_i^⊥^2} | λ ⟩, \quad (4)$$

being $M_0 = \sum_i \sqrt{m^2 + k_i^2}$ the total free energy mass of the system and $μ$ and $λ$ generic canonical spins. In this scenario, being $M_0$, appearing in delta functions in Eq. (3), a kinematical quantity, dPDFs are well defined in the $x_1 + x_2 < 1$ region, at variance with what happens in the instant form calculations of PDFs and dPDFs (see, e.g., Ref. [10]). Let us remark that the main ingredient in Eq. (3) is the canonical proton wave function which has been calculated by means of a relativistic CQM, the so called hyper-central CQM described in Ref. [32]. The choice of this model is motivated by its simplicity and capability to basically reproduce the spectrum of light-hadrons. Since actually for dPDFs there are not yet available data, model calculations of these quantities could provide essential information on their relevant features. Before discussing in the next section the results of the calculations of dPDFs, one should notice that in principle the role of the model independent Melosh rotations can be important. Indeed, from Eq. (4), one can realize that such operator con introduce, in the calculation of dPDFs, non trivial correlations between $x_i$ and $k_i^⊥$. As deeply discussed in Ref. [15], in order to taste possible effects of these objects, it is instructive to study the following quantity:
\[ DD^I(\vec{k}_\perp, x_1, x_2, \vec{k}_{1\perp} = 0, \vec{k}_{2\perp} = 0) = \langle SU(6)|D^I_1D^I_2D^I_3|SU(6) \rangle, \]

being \( \vec{k}_{i\perp} \) the intrinsic transverse component of the \( i \) quark momentum. In the latter quantity use has been made of the commonly adopted SU(6) symmetry in order to evaluate the spin part of the proton wave function. In this scenario, the quantity (5) is rather model independent. Here we consider two fast partons (FF) with \( x_1 = 0.2, x_2 = 0.3 \) and one slow and one fast parton (SF) with \( x_1 = 0.04, x_2 = 0.3 \) and two slow partons (SS) with \( x_1 = 0.04, x_2 = 0.03 \). The calculation of Eq. (5) in these kinematic regions is presented in Fig. (1) where one may identify three distinct regions as a function of \( k_\perp \). For \( k_\perp \to 0 \) the Melosh’s in all kinematic configurations reduce to unity. In an intermediate region of \( k_\perp \) the curves show a dip whose depth depends on the chosen kinematic configuration and at larger \( k_\perp \) the curves flattens out with different asymptotics. This complicated pattern, generated by Melosh’s rotations, affects the calculation of dPDFs, which, in general, are distributions evaluated also at \( k_\perp \neq 0 \).

![Figure 1](image_url)  
**Figure 1.** The quantity Eq. (5), as function of \( k_T = k_\perp \), evaluated in different regions of \( x_1 \) and \( x_2 \) with \( k_{1\perp} = k_{2\perp} = 0 \).

As one can see on the left panel of Fig. (2), being the ratio \( r_2 \), Eq. (6), different from the unity in all the kinematical range, also the factorized form of dPDFs in terms of the product of single PDFs is not supported by the present approach. In closing, at the hadronic scale, in all model calculations of dPDFs (see also Refs. [9, 10]), the assumption Eq. (1) is violated due to DPCs.

3. Results of the calculations of dPDFs at the hadronic scale

In this section the main results of the calculations of dPDFs at the hadronic scale, \( \mu_R^2 \sim 0.1 \text{ GeV}^2 \), will be presented. In particular, as already mentioned, here the emphasis of the analysis is focused on testing the validity of the approximation Eq. (1). As already discussed in the previous section, in general, Melosh rotations introduce correlations between \( k_\perp \) and \( x_1, x_2 \) so that, as also found in Ref. [11], at the hadronic scale such factorization is violated in all model calculations of dPDFs within the LF approach, see Ref. [15]. Furthermore, in order to study the validity of the approximations Eq. (1), in the \( x_1 \) and \( x_2 \) dependence of dPDFs, the following ratio have been evaluated:

\[ r_2 = \frac{2uu(x_1, x_2, k_\perp = 0)}{u(x_1)u(x_2)}, \]

As one can see in the right panel of Fig. (2), being the ratio \( r_2 \), Eq. (6), different from the unity in all the kinematical range, also the factorized form of dPDFs in terms of the product of single PDFs is not supported by the present approach. In closing, at the hadronic scale, in all model calculations of dPDFs (see also Refs. [9, 10]), the assumption Eq. (1) is violated due to DPCs.

4. pQCD evolution of the calculated dPDFs

A fundamental point, discussed e.g. in Ref. [11], is the analysis of the pQCD evolution of model calculations of dPDFs. This procedure is essential to relate CQM predictions with present and future experimental analyses. For the moment being, the pQCD evolution of dPDFs is known only for the longitudinal momentum dependence, which means \( k_\perp = 0 \), and using the same energy scale for both the acting partons. In these case, the evolution equations are obtained as a proper generalization of the usual Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) ones
(see Refs. [13, 14] for details), defined for the evolution of PDFs. In the first part of this section, results will be shown for the non-singlet sector. In particular, the ratio \( r_2 \), Eq. (6), has been calculated using the dPDFs evaluated at a generic high energy scale, e.g., \( Q^2 = 10 \text{ GeV}^2 \), using pQCD evolution. As one can see on the right panel of Fig. 2, for small values of \( x_i \), e.g., close to the LHC kinematics, \( r_2 \sim 1 \). This means that dynamical correlations, for valence quarks, are suppressed after the evolution. Nevertheless, in order to complete this analysis, in Ref. [16], new ratios sensitive to longitudinal correlations have been defined and calculated at high energy scale including perturbative and non perturbative degrees of freedom. To this aim let us generalize the expression of \( r_2 \) for different partonic species:

\[
\text{ratio}_{ab} = \frac{F_{ab}(x_1, x_2 = 0.01, k_\perp = 0; Q^2)}{a(x_1; Q^2)b(x_2 = 0.01; Q^2)} \; ;
\]

where here \( a, b = q, \bar{q}, g \), \( a(x; Q^2) \) and \( b(x; Q^2) \) are the single PDFs for two given partons of flavor \( a \) and \( b \) and \( Q^2 = 250 \text{ GeV}^2 \). In such ratio, dPDFs in the numerator and PDFs in the denominator evolve by means of different evolution equations. Due to this feature, \( \text{ratio}_{ab} \) is sensitive to non perturbative correlations, encoded in the proton wave function used to calculate dPDFs and PDFs and to the perturbative ones due to the difference in the evolution equations of dPDFs and PDFs. In order to disentangle these two different contributions to understand which among them could affect the dPDF evaluations, the following ratios have been defined and calculated within the LF CQM approach:

\[
\text{ratio}^P_{ab} = \frac{F_{ab}(x_1, x_2 = 0.01, k_\perp = 0; Q^2)}{a(x_1; Q^2)b(x_2 = 0.01; Q^2)} \; ; \quad \text{ratio}^{NP}_{ab} = \frac{F_{ab}(x_1, x_2 = 0.01, k_\perp = 0; Q^2)}{F_{ab}(x_1, x_2 = 0.01, k_\perp = 0; Q^2)} \; ;
\]

where here:

\[
F_{ab}(x_1, x_2 = 0.01, k_\perp = 0; Q^2) = \left[ a(x_1; Q^2)b(x_2 = 0.01; Q^2) \right]^{dPDF\text{evolution}}
\]

The latter quantity is calculated by evolving the product of PDFs by means of dPDF evolution. Due to this feature, the numerator and denominator of \( \text{ratio}^P_{ab} \) evolve differently in pQCD while the input at the initial scale is the same, so that \( \text{ratio}^P_{ab} \) is sensitive to perturbative correlations. On the contrary, since in ratio \( \text{ratio}^{NP}_{ab} \) the numerator and denominator evolve within the same scheme, but using different input at the initial scale, it is sensitive to non perturbative correlations. Due to these features, the ratio \( \text{ratio}_{ab} \), shown in Fig. 3 (left panel) for

\[
\text{Figure 2.} \quad \text{Left panel: the ratio } r_2, \text{ Eq. (6) evaluated at the scale } \mu_0^2. \text{ Right panel: same of the left panel but at the scale } Q^2 = 10 \text{ GeV}^2.
\]
two $a = b = g$, is particularly emblematic. The full ratio $\text{ratio}_{ab}$, Eq. (7), (dashed line), clearly influenced by both perturbative (dot-dashed line) and non-perturbative (continuous line) effects, is compared with those where perturbative and non-perturbative correlations are disentangled, contributing to the behavior of $g - g$ dPDFs at low values of $x_1$ and $x_2$. As one can see in the left panel of Fig. 3, for the gluon-gluon distribution such components coherently interfere. Here we report results only for gluons, being the highest partonic component at LHC kinematics. Furthermore, in order to show how much these conclusions do not depend on the used model, a semi factorized ansatz has been adopted in order to include non perturbative see quarks and gluons at a given initial scale. All details of the procedure are discussed in Ref. [16]. We assume that at a given initial scale $Q_0 > \mu_0$ one has:

$$\begin{align*}
F_{uu}(x_1, x_2, k_\perp = 0, Q_0^2) & \sim F_{uv\mu v}(x_1, x_2, k_\perp = 0, Q_0^2) + \left\{ u_V(x_1, Q_0^2)\bar{u}(x_2, Q_0^2) + \bar{u}(x_1, Q_0^2)\bar{u}(x_2, Q_0^2) \right\} (1 - x_1 - x_2) \theta(1 - x_1 - x_2),
\end{align*}$$

(10)

where in the above expression, $F_{uv\mu v}(x_1, x_2, k_\perp = 0, Q_0^2)$ and $u_V(x_1, Q_0^2)$ are the dPDFs and PDFs respectively calculated by means of the LF CQM approach obtained evolving the same quantities from $\mu_0^2$ to $Q_0^2$ and $\bar{u}(x_2, Q_0^2)$ is the PDF evaluated through to the MSTW2008 parametrization [33], see Ref. [16] for further details. In particular, within this choice, $Q_0^2 = 1$ GeV$^2$. As one can see in the right panel of Fig. 3, where $\text{ratio}_{gg}$ has been calculated starting the evolution from $\mu_0^2$ and $Q_0^2$, conclusions arisen from the precedent analysis, see left panel of Fig. 3, are confirmed. For gluon-gluon the correlations induced at low-$x$ still contain a specific sign of the correlations introduced in the valence sector and this is due to the presence of the valence component in the quark-singlet sector in the evolution procedure. The strength of the correlations seems to become smaller but they are still sizable and should not be neglected.

5. Calculation of $\sigma_{\text{eff}}$

As previously mentioned, an important quantity, relevant for the experimental analyses of DPS is the effective cross section, $\sigma_{\text{eff}}$ whose expression in terms of PDFs and dPDFs, has been presented in Ref. [25]. In order to discuss the main features of $\sigma_{\text{eff}}$, we restrict the analysis to the zero rapidity region $y = 0$ and show results, within the LF approach, in the left panel of Fig. 4. Here the latter quantity has been calculated at $Q^2 = 250$ GeV$^2$ for a sea and a valence quarks. It is worth to notice that the three old experimental extractions of $\sigma_{\text{eff}}$ from data, Refs. [34, 35, 36], lie in the obtained range of values of the calculated $\sigma_{\text{eff}}$. It is fundamental to remark that, at the hadronic scale, as demonstrated in Ref. [15], if Melosh rotations would neglected, the average value of $\sigma_{\text{eff}}$ would change by a factor 2, making the
impact of relativistic effects in $\sigma_{eff}$ very strong. In the same figure, as one can see, DPCs generate a strong $x_i$ dependence of $\sigma_{eff}$ in the valence region. Such feature, as already discussed, is related to the combined effect of pQCD evolution correlations and non perturbative ones. In order to provide a fully model independent analysis of the $x$ dependence of $\sigma_{eff}$, in Ref. [26], such quantity has been calculated through an evaluation of dPDFs based on the AdS/QCD correspondence, see Refs. [37, 38, 39] for fundamental details. In particular the authors found that, at the hadronic scale, the $x$ dependence of $\sigma_{eff}$ is comparable with the one found within the LF approach. Moreover, in order to emphasize such feature, in the right panel of Fig. 4, the ratio $\sigma_{eff}(x_1, x_2, \mu_0^2)/\sigma_{eff}(x_1 = 10^{-3}, x_2, \mu_0^2)$ has been plotted. As one can see, such ratio strongly depends on $x_2$ in the valence region, at the variance of the case in which longitudinal correlations were neglected.

6. Conclusions

In this work, dPDFs have been calculated within a fully Poincaré covariant constituent quark model approach, the Light-Front one, fulfilling the essential properties of dPDFs, see Ref. [11]. The main goal of this first analysis was to establish the role of DPCs in dPDFs. In particular, the factorization of dPDFs, at the hadronic scale of the model, in the $x_1 - x_2$ and $(x_1, x_2) - k_\perp$ dependences is violated, as already found in previous analyses c.f. Refs. [9, 10]. In order to compare our results with actual and future experimental studies, the pQCD evolution of the calculated dPDFs has been deeply investigated as discussed in Refs. [11, 16] where, at very high energy scales, like the experimental ones, it was found that longitudinal correlations may survive also at small values of $x_i$. Furthermore, as also explained in Ref. [16], where also non perturbative degrees of freedom have been taken into account in the analysis, there are different source of longitudinal correlations, the perturbative one, induced by the pQCD evolution scheme and the non perturbative ones due to the model used to describe the proton wave function, used to calculate the dPDFs. For example, in the gluon gluon case, such correlations coherently interfere in the small $x_i$ region at very high energy scales making them sizable. However, since dPDFs can not be directly measured in DPS, within this model calculation approach, the so called $\sigma_{eff}$, a quantity experimentally studied to estimate the DPS cross section, has been calculated. The latter one could be very important from the theoretical and experimental point of view to obtain information on partonic structure of the proton being sensitive to DPCs. In particular, signals of DPCs in $\sigma_{eff}$, have been found in its $x_i$ dependence in the valence region also at high energy scales, for details see Ref. [25]. Furthermore $\sigma_{eff}$ has been also calculated within the AdS/QCD correspondence and also in this case, an important $x_i$ dependence has been found, in particular

Figure 4. Left panel: $\sigma_{eff}(x_1, x_2, Q^2)$ for the values of $x_1, x_2$ measured in Ref. [34] evaluated for a valence quark and a sea quark. Right panel: The ratio $\sigma_{eff}(x_1, x_2, \mu_0^2)/\sigma_{eff}(x_1 = 10^{-3}, x_2, \mu_0^2)$ as a function of as a function of $x_1$ at fixed $x_2 = 0.001, 0.01, 0.1, 0.2$. 
in the valence region. Since, the role of correlations is crucial to grasp new information on the partonic structure of the proton, relativistic effects in the calculation of dPDFs have been studied in order to identify model independent correlations in the \( (x_1, x_2) - k_\perp \) dependence of dPDFs. In particular, as deeply discussed in Ref. [15], the role of Melosh rotations, essential feature of the LF approach, introduce non negligible correlations in such dependence. Furthermore new studies for the extraction of the proton dPDFs from \( pA \) collisions are under investigations, see Ref. [40]. We conclude that the investigation of dPDFs, also trough the analysis of \( \sigma_{eff} \), could unveil new and interesting information on the 3D partonic structure of the proton. I thank Sergio Scopetta, Marco Traini, Vicente Vento and Federico Alberto Ceccopieri for a nice and fruitful collaboration on this subject. This work was supported in part by the Mineco under contract FPA2013-47443-C2-1-P and SEV-2014-0398.

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