Gaussian localizable entanglement

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We investigate localization of entanglement of multimode Gaussian states into a pair of modes by local Gaussian measurements on the remaining modes and classical communication. We find that for pure states and for mixed symmetric states maximum entanglement between two modes can be localized by local homodyne detections, i.e. projections onto infinitely squeezed states. We also show that non-Gaussian measurements allow to localize more entanglement than Gaussian ones.

Quantum entanglement, the non-classical correlations exhibited by quantum systems, lies at the heart of quantum information theory. Of particular interest are multipartite entangled states which provide a resource for one-way quantum computing\(\footnote{1}\) and could form a backbone of the quantum communication network, where a part of the entangled system is located at each node of the network. For quantum communication purposes, it is of great interest to determine how much entanglement can be localized on average between two nodes of the network by performing local measurements on the states located at the remaining nodes of the network and announcing the measurement results to the two nodes. This so-called localizable entanglement has been introduced and studied in the context of quantum spin chains\(\footnote{2, 3}\).

In the present paper we investigate localization of entanglement in quantum continuous-variable systems\(\footnote{4}\). More specifically, we consider multimode Gaussian states and investigate how much entanglement can be localized between two modes by local Gaussian measurements on the other modes. We prove that for pure Gaussian states as well as for mixed symmetric Gaussian states the optimal strategy is to carry out a balanced homodyne detection (BHD) on each mode, i.e. to project it on infinitely squeezed state. Interestingly, we find that if we allow for non-Gaussian measurement strategies then we can localize more entanglement. The entanglement localization can be demonstrated experimentally with present-day technology. Various multimode entangled Gaussian states of light can be generated by combining single-mode squeezed states on an array of beam splitters\(\footnote{5}\) and highly efficient homodyne detectors are also available.

The entanglement properties of multimode Gaussian states have been investigated previously\(\footnote{6, 7}\) with particular focus on the symmetric Gaussian states invariant under arbitrary permutation of modes\(\footnote{8, 9}\).\(^\footnote{10, 11}\) These latter states could be used to establish a continuous-variable quantum teleportation network\(\footnote{12}\) where quantum teleportation occurs between two (arbitrarily chosen) modes \(A\) and \(B\) and the other parties assist the teleportation by performing local measurements \(\mathcal{M}_j\) on their modes \(C_j\) and sending the outcomes to the receiver \(B\). Adesso and Illuminati\(\footnote{13}\) determined the optimal multimode symmetric state that for a given total amount of squeezing maximizes fidelity of teleportation of coherent states from \(A\) to \(B\) and also logarithmic negativity of the effective two-mode state of \(A\) and \(B\) when each \(\mathcal{M}_j\) is balanced homodyne detection of \(p\) quadrature. Here we prove for all mixed symmetric Gaussian states that such local balanced homodyning is optimal among all (not only local) Gaussian measurements and maximizes the entanglement established between modes \(A\) and \(B\) by Gaussian measurements on modes \(C_j\).

Consider \(N\)-mode Gaussian state \(\rho_{ABC}\) shared among \(N\) parties \(A, B, C_j, j = 1, \ldots, N - 2\), with each party possessing a single mode. Parties \(C_j\) attempt to increase the entanglement between \(A\) and \(B\) by making local Gaussian measurements and communicating the measurement outcomes to \(A\) and \(B\). By a Gaussian measurement on mode \(C_j\) we mean any measurement consisting of using auxiliary modes prepared in vacuum states, passive and active linear optics (beam splitters, phase shifters and squeezers) and BHD. Any such measurement can be described by the positive operator valued measure (POVM)

\[
\Pi_j(\alpha_j) = \frac{1}{\pi} D_j(\alpha_j) \Pi_0^j D_j^\dagger(\alpha_j).
\]

Here \(D_j(\alpha_j) = \exp(\alpha_j c_j^\dagger - \alpha_j^* c_j)\) denotes the displacement operator, \(\Pi_0^j\) is a density matrix of a (generally mixed) single-mode Gaussian state with covariance matrix \(\gamma_{C_j}^M\) and zero displacement and \(\alpha_j\) is a certain linear combination of the quadrature values measured by the BHDs. In particular, homodyne detection on \(C_j\) is recovered in the limit of infinitely squeezed state \(\Pi_0^j\). A crucial feature of the Gaussian measurement is that \(\gamma_{C_j}^M\) depends only on the structure of the linear optical network but not on the measurement outcomes of the BHDs. The normalization \(\text{Tr}[\Pi_0^j] = 1\) implies that

\[
\frac{1}{\pi} \int D_j(\alpha_j) \Pi_0^j D_j^\dagger(\alpha_j) d^2\alpha_j = \mathbb{1}_j,
\]

which ensures the completeness of the POVM\(\footnote{14}\). Without loss of any generality we can assume that each \(\Pi_0^j\) is a projector onto a pure Gaussian state because any mixed Gaussian state can be expressed as a mixture of pure Gaussian states and classical mixing cannot increase entanglement.

The elements of the total POVM describing measurement on all modes \(C_j\) can be written as a product of

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the single-site elements, \( \Pi_C(\alpha) = \bigotimes_{j=1}^{N-2} \Pi_j(\alpha_j) \), where \( \alpha = (\alpha_1, \ldots, \alpha_{N-2}) \). The measurement outcome \( \alpha \) is obtained with probability density \( P(\alpha) = \text{Tr}[\mathbb{I}_{AB} \otimes \Pi_C(\alpha) \rho_{ABC}] \) and the resulting normalized density matrix of the conditional bipartite state shared by \( A \) and \( B \) reads

\[
\sigma_{AB}(\alpha) = \frac{1}{P(\alpha)} \text{Tr}_C[\mathbb{I}_{AB} \otimes \Pi_C(\alpha) \rho_{ABC}].
\]  

(3)

Let \( E[\sigma_{AB}] \) denotes a measure of entanglement of a bipartite state \( \sigma_{AB} \). We define the Gaussian localizable entanglement \( E_{L,G} \) between \( A \) and \( B \) as the maximum entanglement that can be set, on average, between two parties \( A \) and \( B \) by local Gaussian measurements performed by \( C_j \) on their modes and by communicating the measurement outcomes to \( A \) and \( B \). We have

\[
E_{L,G} = \max_{\Pi_G} \frac{1}{\text{max}_\alpha} \int_{\alpha} P(\alpha)E[\sigma_{AB}(\alpha)]d\alpha,
\]

(4)

where the maximum is taken over all local Gaussian POVMs \( \Pi_G \).

It follows from the properties of Gaussian operations and measurements [15] that for all measurement outcomes \( \alpha \) the conditioned prepared state \( \sigma_{AB}(\alpha) \) is a Gaussian state with fixed covariance matrix and varying displacement which depends linearly on \( \alpha \). The entanglement properties of a Gaussian state depend only on the covariance matrix, because the displacement can be set to zero by means of suitable local displacement operations \( D_j \). It holds that \( E[\sigma_{AB}(\alpha)] = E[\sigma_{AB}(0)] \), \( \forall \alpha \) and from the definition (1) it immediately follows that \( E_{L,G} = E[\sigma_{AB}(0)] \). In order to determine the Gaussian localizable entanglement it thus suffices to optimize over all local projections onto pure single-mode squeezed vacuum states. Generally, this optimization is still a daunting task and can be performed only numerically. However, we shall see that in case of three-mode pure states a fully analytical expression can be derived. Moreover, for general multimode pure Gaussian states we show that the optimal measurement involves homodyne detection on each mode so it suffices to optimize over the phases \( \theta_j \) which specify the quadratures being measured.

Consider a pure three-mode Gaussian state \( |\psi\rangle_{ABC} \) shared by parties \( A, B \) and \( C \) and characterized by a covariance matrix \( \gamma_{ABC} \). After projection of mode \( C \) onto a pure Gaussian state, the modes \( A \) and \( B \) will be in a pure Gaussian state \( |\phi\rangle_{AB} \) with covariance matrix

\[
\gamma_{AB} = \begin{pmatrix} \gamma_A & \delta_{AB} \\ \delta_{AB}^T & \gamma_B \end{pmatrix}.
\]

(5)

where \( \gamma_A \) (\( \gamma_B \)) denotes covariance matrix of mode \( A \) (\( B \)) and \( \delta_{AB} \) contains correlations between the quadratures of the two modes. A unique measure of entanglement of pure states is provided by the entropy of entanglement, which is the von Neumann entropy of the reduced density matrix of one party, \( E[|\phi\rangle_{AB}] = -\text{Tr}[\rho_A \log_2 \rho_A] \), where \( \rho_A = \text{Tr}_B[|\phi\rangle_{AB} \langle \phi|_{AB}] \). The von Neumann entropy of a single-mode Gaussian state with covariance matrix \( \gamma_A \) is a function of the symplectic invariant \( n_A = (\sqrt{\det \gamma_A} - 1)/2 \). Explicitly, the entropy of entanglement reads

\[
E[|\phi\rangle_{AB}] = (n_A + 1) \log_2(n_A + 1) - n_A \log_2(n_A).
\]

(6)

In order to maximize \( E[|\phi\rangle_{AB}] \) we have to maximize \( \det \gamma_A \).

We prove that it is optimal to measure a suitably chosen quadrature of mode \( C \), \( x_{C,\theta} = (e^{-\theta} + e^{\theta} \hat{a})/\sqrt{2} \). Consider a bipartite splitting \( AB|C \) of the pure state \( |\psi\rangle_{ABC} \). It has been shown [14] that there exist unitary Gaussian transformations \( U_{AB}^r \) and \( U_{C}^r \) acting on modes \( AB \) and \( C \), respectively, which transform the three-mode pure Gaussian state \( |\psi\rangle_{ABC} \) into a product of a two-mode squeezed vacuum state in modes \( A \) and \( C \) and a vacuum state in mode \( B \), \( U_{AB}^r \otimes U_{C}^r |\psi\rangle_{ABC} = |\lambda\rangle_{AC} |0\rangle_B \). Here \( |\lambda\rangle_{AC} = \sqrt{1 - \lambda^2} \sum_{n=0}^{\infty} \lambda^n |n\rangle_A |n\rangle_C \), \( \lambda^2 = (\sqrt{\det \gamma_C} - 1)/(\sqrt{\det \gamma_C} + 1) \), \( \gamma_C \) is CM of mode \( C \) prior to measurement and \( |n\rangle \) denotes the \( n \)-photon Fock state. The situation is illustrated in Fig. 1. The transformation \( U_{C} \) can be absorbed into the Gaussian measurement on mode \( C \). Projection of one part of the two-mode squeezed vacuum state onto single-mode squeezed state with covariance matrix \( \gamma_C \) is \( W(\theta)V(\nu)W^T(\theta) \), where

\[
W(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}, \quad V(\nu) = \begin{pmatrix} e^{2\nu} & 0 \\ 0 & e^{-2\nu} \end{pmatrix},
\]

(7)

prepares the mode \( A \) in a similar pure single-mode squeezed vacuum state \( |s;\theta\rangle \) with covariance matrix \( \gamma_A = W(-\theta)V(s)W^T(-\theta) \), where

\[
e^{2s} = \frac{1 - \lambda^2 + (1 + \lambda^2)e^{2\nu}}{1 + \lambda^2 + (1 - \lambda^2)e^{2\nu}}.
\]

(8)

The resulting two-mode Gaussian state of modes \( A \) and \( B \) can be obtained from the product Gaussian state of modes \( A \) and \( B \) by the action of \( U_{AB} \), \( |\phi\rangle_{AB} = U_{AB}|s;\theta\rangle_A |0\rangle_B \). Let \( S_{AB} \) denotes the symplectic matrix corresponding to the unitary \( U_{AB} \) which in the Heisenberg picture governs the linear transformation of quadrature operators. We decompose \( S_{AB} \) with respect to the \( A|B \) splitting,

\[
S_{AB} = \begin{pmatrix} S_{AA} & T_{AB} \\ T_{BA} & S_{BB} \end{pmatrix}.
\]

(9)

The covariance matrix \( \gamma_{AB} \) of the state \( |\phi\rangle_{AB} \) can be expressed as \( \gamma_{AB} = S_{AB}(\gamma_A \oplus \mathbb{I}_B)S_{AB}^T \), where the identity

\[
FIG. 1: Decomposition of pure three-mode Gaussian state.
matrix $\mathbb{I}_2$ represents the covariance matrix of vacuum state. After a straightforward calculation we arrive at
\[ \gamma_A = S_{AA}^T \gamma_A S_{AA}^T + T_{AB}^T T_{AB}. \] To calculate $\gamma_A$ which we want to maximize we use the formula for determinant of a sum of two $2 \times 2$ symmetric matrices $X$ and $Y$,
\[ \det(X + Y) = \det X + \det Y + \text{Tr}[XRY^T], \quad (10) \]
where $R = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Taking into account that determinant of a covariance matrix of a pure Gaussian state is equal to unity, $\det \gamma_A = 1$, we arrive at
\[ \det \gamma_A = (\det S_{AA})^2 + (\det T_{AB})^2 + \text{Tr}[\hat{\gamma}_A M], \quad (11) \]
where $M = S_{AA}^T R T_{AB}^T T_{AB}^T R^T S_{AA}$ is a symmetric positive semidefinite matrix. $M$ can be diagonalized by orthogonal rotation, $W(\theta_0) MW^T(\theta_0) = \text{diag}(M_{xx}, M_{pp})$. Since we optimize over all phases $\theta$ we can, without loss of any generality, make the substitution $\theta \rightarrow \theta - \theta_0$ and assume that $M$ is diagonal. The nontrivial part of $\det \gamma_A$ which should be maximized then reads
\[ e^{2s}(M_{xx} \cos^2 \theta + M_{pp} \sin^2 \theta) + e^{-2s}(M_{xx} \sin^2 \theta + M_{pp} \cos^2 \theta), \quad (12) \]

The maximum of this function should be found in the interval $s \in [-s_{\text{max}}, s_{\text{max}}]$, where $e^{2s_{\text{max}}} = (1+\lambda^2)/(1-\lambda^2)$, c.f. Eq. (8). The maximum squeezing $\pm s_{\text{max}}$ is obtained if mode $C$ is projected onto the eigenstate of quadrature operator using homodyne detection. It is straightforward to check that for any $\theta$ expression (12) is maximized if $s = s_{\text{max}}$ or $s = -s_{\text{max}}$. This proves that the optimal Gaussian measurement on $C$ which maximizes the entanglement between $A$ and $B$ is homodyne detection. This remains valid even if we take into account the squeezing operation $U_C$ which is included in this measurement. The only difference is that instead of quadrature $q_C$ we measure quadrature $x_{C,0}$ defined as
\[ U_C q_C U_C^\dagger = a_x x_C + a_p p_C = \sqrt{a_x^2 + a_p^2} x_{C,0}. \]
The optimal phase $\theta_{opt}$ specifying the quadrature $x_{C,0}$ can be determined analytically by solving quadratic equation for $\tan \theta_{opt}$ [13]. Alternatively, the three-mode pure state can be transformed by local unitary Gaussian operations to a standard form, where all correlations between amplitude quadratures $x_j$ and phase quadratures $p_k$ vanish [10]. In this case it is optimal to measure either $x_C$ or $p_C$.

We will next argue that for an arbitrary multimode pure Gaussian state $|\psi\rangle_{ABC}$ the maximum $E_{L,G}$ between $A$ and $B$ is obtained if each party $C_j$ performs homodyne detection of some quadrature $x_{C_j,0}$. The proof is based on the reduction argument. Consider the multipartite state illustrated schematically in Fig. 2. Suppose that all parties $C_k$ except for $C_j$ perform their local projections on pure Gaussian states described by POMVs [11]. This prepares the three-mode system $ABC_j$ in a pure Gaussian state with a fixed covariance matrix that does not depend on the measurement outcomes $\alpha_k$ of $C_k$, see Fig. 2. For a three-mode pure Gaussian state we proved that $E_{L,G}$ is maximized if $C_j$ makes a balanced homodyne detection of appropriately chosen quadrature. We can thus see that, irrespective of the measurements carried out by $C_k$, the optimal choice for $C_j$ is balanced homodyning. Note that $C_j$ even does not need to learn the measurement outcomes $\alpha_k$ of the other parties $C_k \neq j$. It suffices if $A$ and $B$ receive from all parties $C_l$ the data $\alpha_l$. By local displacement operations they can then deterministically compensate for the resulting displacements of their modes which are linear in $\alpha_l$. The above argument can be applied to any $C_k$ which proves that the optimal measurement strategy must consist of balanced homodyning on each mode $C_k$.

We now consider general mixed symmetric Gaussian states [8, 9] which are invariant under arbitrary permutation of modes. This implies that the covariance matrix has a highly symmetric form,
\[ \gamma_{\text{sym}} = \begin{pmatrix} \beta & \epsilon & \cdots & \epsilon \\ \epsilon & \beta & \cdots & \epsilon \\ \vdots & \epsilon & \cdots & \epsilon \\ \epsilon & \cdots & \cdots & \beta \end{pmatrix}, \quad (13) \]
where $\beta$ and $\epsilon$ denote symmetric $2 \times 2$ matrices. By means of local canonical transformations it is possible to simultaneously diagonalize both $\beta$ and $\epsilon$ [11] so without loss of any generality we may assume that $\beta = \text{diag}(\beta_1, \beta_2)$ and $\epsilon = \text{diag}(\epsilon_1, \epsilon_2)$. We choose the logarithmic negativity $E_N$ as the measure of entanglement. We prove that maximum entanglement between two modes (labeled $A$ and $B$) can be localized by homodyne detection of either $x$ or $p$ on all remaining $N-2$ modes $C_j$. Consider the bipartite $2 \times (N-2)$ splitting $AB|C$. By means of unitary Gaussian transformation on modes $C_j$ which can be physically realized by interference on an array of $N-3$ unbalanced beam splitters all modes $C_k, k \neq 1, 2$ can be decoupled from $A, B$ and $C_1$ thereby effectively reducing the problem to three-mode case [11]. Similarly, interference of $A$ with $B$ on a balanced beam splitter decouples $B$ from $A$ and $C_1$. In this way we obtain the equivalent representation shown in Fig. 4 where $U_{AB}$ represents mixing on a balanced beam splitter, mode $B$ is initially in a mixed state with $\gamma_{B,\text{in}} = \beta - \epsilon$ while the initial CM of modes $A$ and $C_1$ reads
\[ \gamma_{AC_1,\text{in}} = \begin{pmatrix} \beta + \epsilon & \sqrt{2(N-2)} \epsilon \\ \sqrt{2(N-2)} \epsilon & \beta + (N-3) \epsilon \end{pmatrix}. \quad (14) \]

FIG. 2: Reduction of $N$-mode pure Gaussian state to three-mode pure Gaussian state by local projections on modes $C_k$.
A eigenvalue of CM of partially transposed state of modes (measurements on modes C) max(0), amount of entanglement between modes min[eig(\lambda)], 

Projection of C_1 onto state with CM \tilde{\gamma}_C prepares mode A in state with CM \gamma_{A,in} = \beta + \epsilon - 2(N - 2)\epsilon|\beta + (N - 3)\epsilon + \gamma_C|^{-1}\epsilon. The logarithmic negativity \text{E}_N = \max(0, -\log_2 \mu) where \mu is the minimum symplectic eigenvalue of CM of partially transposed state of modes A and B \[16\]. After some algebra one finds that \mu^2 = \min[eig(\gamma_{A,in} R \gamma_{B,in} R^T)] where eig(A) denotes eigenvalues of a matrix A. In order to maximize \text{E}_N we have to minimize \mu over all admissible \tilde{\gamma}_C. This can be done analytically and one can prove \[15\] that it is optimal to measure either the x or p quadrature of C_1, depending on the relation between \epsilon_1 and \epsilon_2. This optimal measurement is a joint measurement on the original modes C_j that can be performed locally by measuring either x or p quadrature of each mode and then properly averaging the results. \mu^2 is particularly simple for \( N = 3 \) when it reads \mu^2 = (b - \epsilon_1)(b - \epsilon_2)(1 + 2\min(\epsilon_1, \epsilon_2))/b.

Finally, we show that if we allow also non-Gaussian measurements on modes C then we can localize a higher amount of entanglement between modes A and B than one can localize by Gaussian measurements. This is yet another example of the well known fact that non-Gaussian operations are optimal for certain tasks such as cloning \[17\] or partial estimation of coherent states \[18\]. We demonstrate the superiority of non-Gaussian measurements on a particular illustrative example. Consider the state preparation scheme shown in Fig. 3(a). The three-mode state is generated from a two-mode squeezed vacuum state |\lambda\rangle_{BC} in modes B and C by mixing mode B on a balanced beam splitter BS with mode A which is initially in a vacuum state.

The optimal Gaussian measurement on C which maximizes entanglement between A and B is a homodyne detection and the Gaussian localizable entanglement \text{E}_{L,G} can be evaluated from formula \[8\] with \( n_A = \frac{1}{\sqrt{2}} \sqrt{1 - \lambda^2} - \frac{1}{2} \). Suppose now that we would measure the number of photons \( n \) in mode C. With probability \( p_n = (1 - \lambda^2)\lambda^{2n} \) we would prepare in mode B \( n \)-photon Fock state |n\rangle, which then impinges on a balanced beam splitter BS, c.f. Fig. 3(a). The resulting state of A and B expressed in the Fock state basis reads, \(|\psi_n\rangle_{AB} = \frac{1}{2^n} \sum_{k=0}^{n} \sqrt{\binom{n}{k}} |n - k\rangle_{AB} \). The entropy of entanglement of \(|\psi_n\rangle_{AB}\) is given by

\[ S_n = -\frac{1}{2^n} \sum_{k=0}^{n} \binom{n}{k} \log_2 \left[\frac{1}{2^n} \binom{n}{k}\right] \tag{15} \]

and has to be evaluated numerically. The average entanglement between A and B is then \text{E}_{L,NG} = \sum_{n=0}^{\infty} p_n S_n. Both \text{E}_{L,G} and \text{E}_{L,NG} are plotted in Fig. 3(b) as a function of the squeezing parameter \( \lambda \). We can see that \text{E}_{L,NG} > \text{E}_{L,G} so the non-Gaussian measurement strategy outperforms the best Gaussian one.

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