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Do Newton’s rules of reasoning guarantee truth . . . must they?

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Abstract

Newton’s *Principia* introduces four rules of reasoning for natural philosophy. Although useful, there is a concern about whether Newton’s rules guarantee truth. After redirecting the discussion from truth to validity, I show that these rules are valid insofar as they fulfill Goodman’s criteria for inductive rules and Newton’s own methodological program of experimental philosophy; provided that cross-checks are used prior to applications of rule 4 and immediately after applications of rule 2 the following activities are pursued: (1) research addressing observations that systematically deviate from theoretical idealizations and (2) applications of theory that safeguard ongoing research from proceeding down a garden path.

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1. Introduction

Book 3 of Isaac Newton’s *Mathematical principles of natural philosophy* begins by introducing four rules of reasoning for natural philosophy. As will be shown, these rules allow Newton to make valuable inductions in the early propositions of

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1 Actually, the four rules appear in the third edition. The second edition had only three, and the first edition had only two of the rules that appeared in the third edition. However, the first edition had a third rule that was dropped from later editions and for some reason Newton did not include his fifth rule in any edition, although it appears in his notes (Koyré, 1965, pp. 261–272).

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Book 3, inductions that could not have been made without rules of this sort. Nevertheless, there is a concern about whether Newton’s rules guarantee truth. I intend to show that this concern is misguided. However, against the criticism that these rules do not guarantee valid inferences, Newton’s rules are defendable insofar as they satisfy Goodman’s criteria for inductive rules and Newton’s own methodological program of experimental philosophy; provided that cross-checks are used prior to applications of rule 4 and immediately after applications of rule 2 the following activities are pursued: (1) research addressing observations that systematically deviate from theoretical idealizations and (2) applications of theory that safeguard ongoing research from proceeding down a garden path. This paper will not include a defense of Goodman’s criteria, but will argue that if his criteria are accepted, then given my defense of Newton’s rules as satisfying that criteria, Newton’s rules of reasoning are valid.

2. Newton’s rules of reasoning

Newton made numerous drafts of his rules before including them in the *Principia*. Newton’s care in stating these rules and including only certain ones in subsequent editions suggests that each rule is important. So I will dedicate some space to presenting each rule and discussing its importance to the *Principia*.

Below are Newton’s first and second rules of reasoning from the third edition of the *Principia*:

**Rule 1.** No more causes of natural things should be admitted than are both true and sufficient to explain their phenomena. (Newton, 1999, p. 794)

**Rule 2.** Therefore, the causes assigned to natural effects of the same kind must be, so far as possible, the same. (Newton, 1999, p. 795)

Rule 1 instructs us to *explicate* nature in the simplest way possible while still maintaining truth. Although Newton mentions the philosophic adage that ‘nature

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2 Koyré identifies six revisions of rule 2 between the second and third editions of the *Principia* and five revisions of rule 4 in Newton’s manuscripts before including it in the third edition (Koyré, 1965, pp. 265–271).

3 Although Bernard Cohen and Anne Whitman translate Newton as using ‘explain’ in rule 1, this translation is debatable. Newton’s rule 1 in its original Latin is the following: ‘Regula I. Causas rerum naturalium non plures admitti debere, quam quae et vera sunt et earum Phenomenis explicandis sufficiunt’ (Koyré, 1965, p. 265). Notice that Newton uses ‘explicandis’, a participle of ‘explicare’, which in the period often has the sense of the English ‘explicate’. Newton does not use ‘explanare’, which in the period was often equivalent to the English ‘explain’. This point is worth mentioning since ‘explicate’ means ‘to give a detailed analysis of’, while ‘explain’ means ‘to give the reason for or cause of’. From the wording of rule 1, Newton appears to be satisfied with detailed analyses of phenomena, regardless of whether those phenomena have been assigned a cause. If this interpretation is accurate, then it would be consistent with what Newton says in the General Scholium of the *Principia* when he mentions that he has not deduced a mechanism for gravity even though he has established gravity as the force that maintains our planetary system (Newton, 1999, p. 943).
does nothing in vain’, given Newton’s distaste for ‘hypotheses’,4 Newton’s real intent for rule 1 is to limit claims about differences in causes to ones that the empirical data force (Smith, 2002b, p. 160). In other words, rule 1 restricts theory making to information that has been gathered through experiments or other empirical observations even if only indirectly.5

Given the ‘therefore’ and the placement of rule 2 almost immediately after rule 1, rule 2 is more likely the consequent of rule 1 than a separate rule all by itself (Harper, 2002, p. 183). So, given rule 1, rule 2 assigns the same cause to the same phenomena. Newton first uses rules 1 and 2 in proposition 4 (prop. 4) for the moon test, which equates terrestrial gravity with lunar centripetal acceleration.6 (See Appendix A for a thorough discussion of the moon test.) Newton then uses rules 1 and 2 in prop. 5 to attribute the five primary planets’ centripetal acceleration to gravity.7 Newton uses the rules a final time in the scholium to prop. 5 to equate all celestial centripetal acceleration with gravity.

The importance of rules 1 and 2 in their first application is that without them Newton cannot infer that the moon’s acceleration toward the earth and terrestrial bodies’ acceleration toward the earth are in fact from identical causes. Although Newton has deduced that the value for lunar centripetal acceleration is very nearly equal to the value for terrestrial gravity—since Newton’s value in prop. 4 falls within the error bounds of Christiaan Huygens’s precise measurement of terrestrial gravity—the most that Newton has proven is that these accelerations have corresponding magnitudes and direction. Newton needs an inductive rule to instruct him that it is permissible to conclude from this similarity that these two phenomena are due to the same cause. Although rules 1 and 2 are not necessary to make this induction, they suffice.

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4 Newton clearly had distaste for hypotheses. This distaste grew between subsequent editions of the *Principia*, as evidenced by his reluctance to even use the word, except pejoratively, in the third edition. For example, Newton labels the first two rules of reasoning as ‘hypotheses’ in the first edition, but by the third edition Newton uses ‘hypotheses’ negatively in a statement following rule 4 as well as in the General Scholium when he says ‘I do not feign hypotheses’ and ‘hypotheses have no place in experimental philosophy’ (Newton, 1999, p. 943). Koyré suggests that Newton is not being contradictory here; he is merely showing how he has narrowed his definition of ‘hypotheses’. Prior to this narrowing, Newton considered hypotheses tantamount to principles. Later on, however, Newton restricts ‘hypotheses’ to ‘undemonstratable assertions’, ‘assumptions without experimental proof’, and ‘propositions not deduced from phenomena’ (Koyré, 1965, pp. 262–265).

5 I say ‘even if only indirectly’ here because, as George Smith points out, if by ‘phenomena’ Newton is referring to phenomena similar to his ‘Phenomena’ in Book 3, Newton is referring to a generalization of a collection of empirical observations made over a period of time, not individual empirical observations (Smith, 2002b, p. 160).

6 Proposition 4. ‘The moon gravitates toward the earth and by the force of gravity is always drawn back from rectilinear motion and kept in its orbit’ (Newton, 1999, p. 803).

7 Proposition 5. ‘The circumjovial planets [or satellites of Jupiter] gravitate toward Jupiter, the circumsaturnian planets [or satellites of Saturn] gravitate toward Saturn, and the circumsolar [or primary] planets gravitate toward the sun, and by the force of their gravity they are always drawn back from rectilinear motions and kept in curvilinear orbits’ (ibid., p. 805).
The second application of rules 1 and 2 serves a similar, but slightly different purpose. In prop. 5, although Newton has established in prop. 2 that the primary planets have centripetal accelerations with the same properties as the moon—namely, that they are proportional to the inverse-square of the distance from their centres of orbit in which equal areas are swept out in equal times and that they are directed toward their centres of orbit—without an inductive rule to permit an identity relation between the moon’s kind of acceleration and the kind keeping the primary planets in orbit, Newton has only shown that these accelerations have the same properties. Invoking rules 1 and 2 classifies each of these accelerations as the same kind. Rules 1 and 2 are used in the same way to designate all celestial centripetal acceleration as the gravitational kind. So, in all three applications the rules invoke identical causes; however, in the first application the cause is a particular and in the last two the cause is a kind.

The following is Newton’s third rule of reasoning:

Rule 3. Those qualities of bodies that cannot be intended and remitted [that is, qualities that cannot be increased and diminished] and that belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally. (Newton, 1999, p. 795)

William Harper refers to rule 3 as Newton’s ‘argument from phenomena’ (Harper, 2002, p. 175), and this rule does just that. If a property satisfies a certain empirical criterion, rule 3 instructs us to consider that property as universal, where the criterion is being a constant property of all bodies that are observable through experiment.

Newton explicitly invokes rule 3 only once, although he implicitly invokes it two more times. In corollary 2 of prop. 6, Newton uses rule 3—along with the results from his pendulum experiments that show a body’s weight to its mass is a constant at the earth’s surface—to make the induction that all bodies gravitate toward the

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8 Proposition 2. ‘The forces by which the primary planets are continually drawn away from rectilinear motions and are maintained in their respective orbits are directed to the sun and are inversely as the squares of their distances from its center’ (ibid., p. 802).

9 Notice that even in the first application of rules 1 and 2, Newton shows that lunar centripetal acceleration is of the same kind as terrestrial gravity, since identical kinds follow from identical particulars. My effort to distinguish Newton’s two types of applications of rules 1 and 2 is meant to highlight that Newton is ultimately engaged in something different in his second and third applications of rules 1 and 2 as compared to his first application. For example, in the second application Newton is not trying to attribute identical particular gravitational accelerations to each of the five primary planets and the moon, since that would make the centre of the earth count as the centre of planetary centripetal acceleration, which contradicts Newton’s position against geocentrism (ibid., pp. 811–819).
earth. However, since prop. 6 claims to have shown that ‘all bodies gravitate toward each of the planets’, Newton must have implicitly invoked rule 3 a second time. Probably, Newton took the generalization for the earth and rule 3 to show that prop. 6 holds. Also, in prop. 7, although Newton shows that all of the particles of any two planets gravitate toward each other, he must have implicitly invoked rule 3 in order to extend the argument to all of the particles in the universe, which as William Harper notes (2002, p. 192), could have been done since Newton’s proof included all of the planets in reach of experiments at that time.

One importance of rule 3 is that without it Newton cannot assert that all bodies near planets gravitate toward those planets in the same fashion as observed bodies near planets (for example, satellites and terrestrial bodies). However, establishing the latter is not crucial to Newton’s argument for gravity between particles since gravity between planets can be deduced from celestial centripetal acceleration being due to gravity (which I will explain later). Another importance of the rule is that the last step in the argument for universal gravity requires a generalization to all particles in the universe that only rule 3 provides. Also, notice that the first use of rule 3 is for an induction from terrestrial bodies to all bodies near the earth while the second use is for an induction from all bodies near the earth to all bodies near any planet. Thus the second induction is nested, where being ‘nested’ is having the

10 Corollary 2, Proposition 6. ‘All bodies universally that are on or near the earth are heavy [or gravitate] toward the earth, and the weights of all bodies that are equally distant from the center of the earth are as the quantities of matter in them. This is a quality of all bodies on which experiments can be performed and therefore by rule 3 is to be affirmed of all bodies universally. If the aether or any other body whatever either were entirely devoid of gravity or gravitated less in proportion to the quantity of its matter, then, since (according to the opinion of Aristotle, Descartes, and others) it does not differ from other bodies except in the form of its matter, it could by a change of its form be transmuted by degrees into a body of the same condition as those that gravitate the most in proportion to the quantity of their matter; and, on the other hand, the heaviest bodies, through taking on by degrees lose their gravity. And accordingly the weights would depend on the forms of bodies and could be altered with the forms, contrary to what has been proved in corol. 1’ (ibid., p. 809).

11 Proposition 6. ‘All bodies gravitate toward each of the planets, and at any given distance from the center of any one planet the weight of any body whatever toward that planet is proportional to the quantity of matter which the body contains’ (ibid., p. 806).

12 My point about Newton’s first implicit use of rule 3 is that he needs rule 3 to infer that all bodies gravitate toward each of the planets, not that he needs rule 3 to infer that all of the planets gravitate toward the earth or that all of the planets gravitate toward each of the planets. Given the universal scope of prop. 6 and the fact that Newton could not have performed experiments on celestial bodies beyond the scope of experiments, Newton indeed needs the inductive power of rule 3 to make prop. 6.

13 Proposition 7. ‘Gravity exists in all bodies universally and is proportional to the quantity of matter in each’ (ibid., p. 810).

14 Where ‘same fashion’ means ‘in proportion to their weight at some distance by their mass and in proportion to the inverse-square of their distance from the planet’s center’.
status of depending on other inductions.\textsuperscript{15} In effect, each successive application of rules 1, 2, and 3 lead to nested inductions.\textsuperscript{16,17}

The fourth rule of reasoning is below:

Rule 4. In experimental philosophy, propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding any contrary hypotheses, until yet other phenomena make such propositions either more exact or liable to exceptions. (Newton, 1999, p. 796)

Coined ‘the rule of conduct [for] empiricism’ by Alexandre Koyré (1965, p. 268), rule 4 does three things for natural philosophy. First, it instructs us that theories deduced from phenomena and generalized by induction always override theories that fail to do so. Second, the phrase ‘should be considered exactly or very nearly true’ instructs us to model the world as idealizations.\textsuperscript{18} Third, the clause after the last comma suggests that natural philosophy is an ongoing program of making ever better idealizations of the world. The latter two points turn natural philosophy into an exact science, not insofar as it calls for idealizations, but insofar as it suggests that phenomenal deviations from those idealizations, or theory dependent second-order phenomena,\textsuperscript{19} must be addressed in a new idealization that better approximates the world (Smith, 2002b, pp. 157–160).

Newton invokes rule 4 only once although he could have invoked it several times. In the scholium to prop. 5, Newton uses rule 4 after applying rules 1 and

\textsuperscript{15} This point is important because nested propositions carry greater risk than they would unnested—assuming all inductions carry some risk and nested propositions inherit the risk of the inductions they nest. Also, as will become apparent, nested inductions must be validated \( n \) times for \( n \) number of inductions they nest.

\textsuperscript{16} Actually, all of the inductions in Book 3 are nested simply because all of them are based on the generalized phenomena from phen. 1–6. See n. 5.

\textsuperscript{17} The second application of rules 1 & 2 is nested on the first application of these rules because the primary planets’ centripetal acceleration being due to gravity is an induction from the moon’s centripetal acceleration being due to gravity, which is an induction. The third application of rules 1 & 2 is nested on the second application of these rules because all celestial bodies’ centripetal acceleration being due to gravity is an induction from the primary planets’ centripetal acceleration being due to gravity. The second application of rule 3 is nested on the first application of this rule because all close bodies gravitating toward planets is an induction from all close bodies gravitating toward the earth, which is an induction. Although the second application of rule 3 could have been projected from the satellite test alone (which shows that the sun’s gravity is a constant at equal distances from it), I think the satellite test was merely support for the induction. I base this opinion on the fact that prop. 6 applies to ‘each of the planets’ not to ‘each of the celestial bodies’, and the satellite test is direct support for the latter clause as opposed to the former.

\textsuperscript{18} This term is George Smith’s (2002b, p. 157). He uses ‘idealization’ for ‘an approximation that would hold exactly in certain specifiable circumstances’.
2. The effect of applying the rule here is to establish it as a principle that centripetal acceleration for celestial bodies is the same kind of acceleration as terrestrial gravity. Newton needs the latter to be a principle in order to establish universal gravity in a way that is not hypothetico-deductive. As we see from prop. 7, Newton’s argument for gravity between particles depends as much on his laws of motion as it does on the attraction between planets being due to gravity, since gravity between planets is a premise in the argument:

We have already proved that all planets are heavy [or gravitate] toward one another and also that the gravity toward any one planet, taken by itself, is inversely as the square of the distance of places from the center of the planet. (Newton, 1999, p. 810)

However, in order to establish gravity between planets as a principle, it has to follow from some other principle. As it happens, gravity between planets follows from celestial centripetal acceleration being one in kind with gravity. Newton provides the proof in corol. 1, prop. 5:

Therefore, there is gravity toward all planets universally. For no one doubts that Venus, Mercury, and the rest [of the planets, primary and secondary,] are bodies of the same kind as Jupiter and Saturn. And since, by the third law of motion, every attraction is mutual, Jupiter will gravitate toward all its satellites, Saturn toward all its satellites, and the earth will gravitate toward the moon, and the sun toward all the primary planets. (Newton, 1999, p. 806)

So, establishing it as a principle that gravity is the kind of acceleration keeping celestial bodies in orbit is important.

Although he did not, Newton could have invoked rule 4 after each induction, since this would have made each induction exactly or very nearly true. Newton certainly does (implicitly) invoke rule 4 after prop. 7, since in prop. 8 through prop. 13 he uses the theory of universal gravity as if it holds exactly in order to demonstrate that Copernicanism is essentially correct (Newton, 1999, pp. 811–819).

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19 The term is George Smith’s (ibid.). Theory-dependent second-order phenomena are observations that systematically deviate from a theory’s idealization. Notice that these phenomena are only observable because they depend on some theory’s idealization of the original phenomena.

20 Scholium, Proposition 5. ‘Hitherto we have called “centripetal” that force by which celestial bodies are kept in their orbits. It is now established that this force is gravity, and therefore we shall call it gravity from now on. For the cause of the centripetal force by which the moon is kept in its orbit ought to be extended to all the planets, by rules 1, 2, and 4’ (Newton, 1999, p. 806).

21 The hypothetico-deductive approach is the method of conducting experiments to test inferences from a principle in order to indirectly establish support for the principle. Confidence that the principle is true increases with each experiment that identifies inferences from the principle as true. For example, since the principle of inertia states that bodies do not change in state if no force acts on them, it follows that forces act on bodies that change in state. Since experiments show that the latter proposition is true, we can infer the principle of inertia as the contrapositive. In this way the principle is hypothetically deduced since the deduction of the principle depends on the assumption of the principle (Huygens, 1989).
3. Do Newton’s rules guarantee truth . . . must they?

The easiest case to make against Newton’s rules is against the inference that rule 2 allows. Rule 2 bluntly instructs us to assign the same cause to the same phenomena. Newton gives the following examples of how to apply rule 2:

Examples are the cause of respiration in man and beast, or of the falling of stones in Europe and America, or of the light of a kitchen fire and the sun, or of the reflection of light on our earth and the planets. (Newton, 1999, p. 795)

However, the recent epidemic of Severe Acute Respiratory Syndrome (SARS) serves as a glaring counterexample to rule 2. SARS induces the same symptoms as the common cold or influenza, but its cause is a different virus (as shown by its recalcitrance to common cold or influenza treatments).\(^{22}\) In that case, rule 2 allows for a false inference when applied to SARS and either the common cold or influenza.

The SARS argument against Newton’s rule would be of concern if the criticism were not misguided. The argument against Newton’s rule inappropriately requires truth from inductive rules instructing induction from phenomena.

Nelson Goodman identifies the old problem of induction for induction from phenomena as the difficulty of devising rules such that we know their projections will turn out to be true. Goodman replies that no set of rules would fulfill this criterion since it requires knowledge in a situation where we admittedly do not have the sought after knowledge. If we had knowledge of true projections beforehand, then we would have no need for rules that project. Instead, Goodman argues that a set of ‘inductive rules’\(^ {23}\) need only be valid. Furthermore, for Goodman (1979, p. 64), since inferences are rejected if they violate rules we are unwilling to amend and rules are amended if they yield inferences we are unwilling to accept, a set of inductive rules are valid if . . .

(1.1) No rule in the set yields an inference that we are unwilling to accept.
(1.2) Either ‘inferences we are unwilling to accept’ or ‘rules we are unwilling to amend’ is defined.

Criterion (1.1) follows from Goodman’s condition for amending rules. Criterion (1.2) is needed because (1.1) depends on either Goodman’s condition for rejecting inferences and a definition for ‘unamendable rules’ or a definition for ‘unacceptable inferences’. Since empty sets, sets of only trivial rules, and sets with conflicting unamendable rules and unacceptable inferences are possible from (1.1) and (1.2), I will add the following to Goodman’s criteria:

\(^{22}\) Also, a novel coronavirus has been identified in patients with SARS, at least 40% unique in its nucleotide sequence (Drosen et al., 2003).

\(^{23}\) From here on, I will use ‘inductive rules’ only for rules that instruct induction from phenomena. Other inductive rules, for example mathematical induction, will not be addressed.
There is at least one rule in the set.

All inferences that we are unwilling to do without follow from some rule in the set.

No inference that we are unwilling to accept follows from a rule that we are unwilling to amend.\(^24\)

‘Inferences that we are unwilling to do without’ is defined.

Also realize that given the sheer complexity of orbital motion, Newton was well aware of the difficulty in capturing the phenomenon by exact mathematical laws. This is good reason to think that Newton was aware of the fact that his rules of reasoning could lead to false inferences even before he wrote them. In a 1684 tract entitled *The motion of bodies in orbit*, Newton expresses his opinion that orbital motion is exceedingly complex:

By reason of the deviation of the Sun from the center of gravity, the centripetal force does not always tend to that immobile center, and hence the planets neither move exactly in ellipses nor revolve twice in the same orbit. There are as many orbits of a planet as it has revolutions, as in the motion of the Moon . . . But to consider simultaneously all these causes of motion and to define these motions by exact laws admitting of easy calculation exceeds, if I am not mistaken, the force of any human mind. (Newton, 1962, p. 281)

Although the initial criticism has been addressed, an immediate concern arises about Newton’s rules. Since inductive rules need only be valid, are Newton’s rules valid?

### 4. Are Newton’s rules valid?

The best criticism against the validity of rule 1 is that it pursues simplicity, which is unrelated to scientific inquiry, even on an empiricist’s account. Harper notes that some empiricists define empirical success entirely indifferent to simplicity (Harper, 2002, p. 185). Huygens provides a few of these definitions in his *Treatise on light* when he says that we best approximate the natural world when theories match observed phenomena and when theories predict new phenomena (Huygens, 1989). So, explaining or predicting phenomena should be the appropriate measures for empirical success regardless of whether those explanations or predictions are simple.

Notice the style of argument of this criticism. It argues that if phenomena are explained or predicted, then we have empirical success. It does not restrict empirical success to these criteria. In effect, we could accomplish empirical success by other criteria, even in the absence of prediction or explanation. I am not proposing that

\(^{24}\) This criterion is helpful if both ‘inferences we are unwilling to accept’ and ‘rules we are unwilling to amend’ are defined.
Newton thought empirical success is indifferent to prediction or explanation or that none of Newton’s inspiration for this rule was due to a belief that nature is essentially simple. I am suggesting that Newton had a valid empirical reason to include this rule besides a mere appeal to simplicity. Given Newton’s intent of establishing experimental philosophy as a new way of doing natural philosophy, this rule should be viewed as an instruction to limit theories to the empirical data supporting them and to discourage ‘hypotheses’. Newton’s opinion of experimental philosophy is clear from the penultimate paragraph of the General Scholium:

I have not as yet been able to deduce from phenomena the reason for these properties of gravity, and I do not feign hypotheses. For whatever is not deduced from the phenomena must be called a hypothesis; and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy. In this experimental philosophy, propositions are deduced from the phenomena and are made general by induction. (Newton, 1999, p. 943)

Newton’s views agree with this defense of rule 1 as we see from his comments in the first paragraph after rule 3:

Certainly idle fancies ought not to be fabricated recklessly against the evidence of experiments, nor should we depart from the analogy of nature, since nature is always simple and ever consonant with itself. (Ibid., p. 795)

This explanation of rule 1 becomes even more plausible once we note the difficulty that seventeenth-century philosophers had with marshalling evidence for a phenomenon that supported only one explanation of that phenomenon. For instance, data that supported the Copernican system for the planets also supported the Tychonic system (Gingerich, 1993, pp. 30–31). In that case, it makes sense to devise a rule that restricts theory construction to only those propositions that are supported by empirical evidence (since the task is difficult enough even before ‘hypotheses’ are proposed). Having a rule that does this is the first step toward winnowing one overriding explanation for a phenomenon out of a consortium of possible explanations. Therefore, despite Newton’s preoccupation with simplicity, rule 1 is justified by its importance to the methodology of Newton’s experimental philosophy.

The validity of rule 2 is questionable in the following methodological sense: beyond the fact that the rule occasionally allows for false inferences is the fact that the rule systematically misleads under certain specifiable circumstances. For instance, when the empirical data supporting the phenomena are inadequate or when the phenomena are exceedingly complex, rule 2 should be abandoned. Abandoning the rule under these circumstances serves to safeguard a theory from pro-

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25 Interpret ‘hypotheses’ in Newton’s sense of the word.

26 Smith (2002b, p. 171) notes that Newton wrote about the ease of adapting data to disparate theories in a 1670s letter responding to objections in his publications on optics.
ceeding down a garden path, perhaps involving fictional physical kinds (since assigning phenomena to the same cause designates the cause as a kind). Furthermore, given the complexity of planetary motion or the fact that the empirical data were not precise enough in the derivation of universal gravity, while rule 2 may be justifiable in other circumstances, certainly it is not justifiable for Newton’s purposes. As a result, rule 2 is invalid not because it is misleading under some circumstances, but because it is misleading under the very circumstances for which Newton intends to use it.

First, the concern over the applicability of rule 2 should not depend on the complexity of the phenomena. One reason why the rule is useful is because it allows for an idealization that is testable with theory-mediated measurements, which is a method of handling complex phenomena. For example, since the first application of rule 2 leads to the idealization that lunar centripetal acceleration is due to the same cause as terrestrial gravity, and since from prop. 3 lunar centripetal acceleration varies as the inverse-square of the distance from the earth’s centre, the theory mediates the measurement that terrestrial gravity varies as the inverse-square of the distance from the earth’s centre. With this new measurement for terrestrial gravity, experiments can be done to test whether terrestrial gravity actually varies in the way that the theory prescribes. Varying values for terrestrial gravity may be predicted in the following way.

Assuming that the earth is spherical, terrestrial gravity is proportional to the inverse-square of the radius to the earth’s surface. This makes the ratio of terrestrial gravity at some height \( h \) from the earth’s surface, \( g' \), to terrestrial gravity at the earth’s surface, \( g \), equal to the ratio of the square of the earth’s radius \( r \) to the

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27 This term is George Smith’s. A garden path happens when science proceeds in the wrong direction. The length of a garden path is the amount of science that will have to be discarded once we discover that science has been going in the wrong direction.

28 For example, Smith argues that Newton’s data were not precise enough to establish that planetary centripetal forces are directed toward the focus of an ellipse as opposed to the centre, which would have suggested a force that varies as the distance instead of the inverse-square of the distance, according to prop. 10 of Book 1 (Smith, 2002a, p. 196).

29 The term is George Smith’s. Theory-mediated measurements are quantifications of magnitude or direction that are attainable only because a theory provides the measures. For example, given Newton’s measure for motive force, \( F_m \propto m (\Delta v / \Delta t) \), Newton’s measure for gravitational force, \( F_g \propto M m / r^2 \), Newton’s identification of celestial centripetal force as the celestial motive force, \( F_c = F_m \), and Newton’s theorem that celestial centripetal force is the gravitational force, \( F_c = F_g \), Newton’s theory mediates the direct proportionality of \( F_m \) to \( F_g \), \( M m / r^2 \propto m (\Delta v / \Delta t) \). Add to this the idealization that celestial orbits are circular, and Newton’s theory mediates a relative measure for celestial masses, \( M_1 / M_2 \propto (r_1^3 T_1^2) / (r_2^3 T_2^2) \), where \( T \) denotes periods.

30 Proposition 3. ‘The force by which the moon is maintained in its orbit is directed toward the earth and is inversely as the square of the distance of its places from the center of the earth’ (Newton, 1999, p. 802).

31 Actually, lunar centripetal acceleration varying as the inverse square of the distance from earth’s centre is already an idealization, since Newton’s value for the lunar inverse power of the distance is \( 2^2 / 243 \) even after correcting for solar perturbation (ibid., pp. 545 & 803).
square of the quantity of \( r \) plus \( h \).

\[
g' = \frac{r^2}{(r + h)^2} \tag{2.1}
\]

Substituting the product of \( r \) and a fraction \( k \) for the variable \( h \), executing some operations, then re-substituting the quotient of \( h \) and \( r \) for \( k \), provides a function for \( g' \) dependent on \( r \). But since the circumference of the lunar orbit \( C \) was available in the seventeenth century (Newton, 1999, p. 804) and the distance between the earth and moon’s centres was estimated to be \( 60r \) (ibid., pp. 803–804), a value for \( r \) could have been calculated. Substituting the value for \( r \), we find a simple function for \( g' \) dependent only on \( h \).

\[
g' = \frac{[1/(120rh/C + 1)^2]}{g} \tag{2.2}
\]

So, from (2.2), depending on how precise our measurement of \( g \) is, we can drop objects over finite distances at some \( h \) in order to detect a discrepancy between \( g \) and \( g' \); where precision for \( g \) is inversely related to the minimum \( h \) required for detecting a discrepancy between \( g \) and \( g' \).32

The worry about following a garden path due to complex phenomena may be addressed with cross-checks,33 or in other words, some method for checking the inductive inferences made by the theory (for example, by deriving the same inference using different sets of assumptions).34 So, for these two reasons, rule 2 should be viewed as a pragmatic tool for handling complex phenomena. However, the second point is more worrisome.

Since the empirical data Newton had available were not precise enough to rule out problematic alternative derivations—like celestial centripetal acceleration varying as the distance (see note 28)—the point is well taken about an application of rule 2 being systematically misleading. However, this criticism does not invalidate rule 2, it only cautions the application of the rule under this circumstance. Nevertheless, admitting that rule 2 systematically misleads under the very circumstance that Newton uses it is an issue that should be addressed. But let me put this issue aside for the moment in order to address the validity of the remaining two rules.

As for rule 3, its validity should not be attacked on the basis that it embodies the new riddle of induction,35 which is the difficulty of describing valid projection.

---

32 The accuracy of this prediction depends on the extent to which a sphere represents the earth, a circle represents the lunar orbit, and \( 60r \) represents the lunar orbit’s radius. The worse the spherical representation, the more \( g \) varies. The more \( g \) varies, the less accurate our prediction of \( g' \). Also, the worse the spherical representation or the less circular the lunar orbit or the less reliable \( 60r \) is as the lunar orbit’s radius, the worse \( C/2\pi \) approximates \( r \). Also, notice how precise our measurement of \( g \) needs to be in order to detect miniscule discrepancies in acceleration at high altitudes. For example, experiments would have to be done at an altitude of 332 Paris feet just in order to detect a 0.2% discrepancy between \( g \) and \( g' \).

33 This term is George Smith’s.

34 Although it may be anachronistic to mention J. J. Thomson, Thomson used this exact method of deriving the same inference using different sets of assumptions to defend his hypothesis that cathode rays were streams of corpuscles. Thomson measured a stable value for \( m/e \), corpuscular mass divided by corpuscular charge, through both a thermodynamic and an electrostatic experiment (Smith, 2001, p. 40).

35 This term is Nelson Goodman’s (1979, Ch. 3).
Since this riddle permeates any inductive rule, the best way to question the validity of rule 3 is to question unique aspects of it. In Newton’s day, explicitly including the phrase ‘on which experiments can be made’ was unique (Newton, 1999, p. 795). However, immediately the question arises: what counts as an experiment? Peculiarly enough, nowhere in the *Principia* does Newton discuss the parameters of an acceptable experiment, which seems odd since Newton is promoting experimental philosophy. In order to capture the strength of this criticism, let me use an analogy. Instead of its actual wording, imagine rule 3 stated, ‘on which blibbles can be made’. However, this instruction only raises the question: what is a blibble? In that case, questioning what counts as an experiment is not absurd when questioning the validity of rule 3.

Although I agree that a vague rule is invalid, it is not clear that Newton’s audience cannot extrapolate the parameters of an acceptable experiment by reading the *Principia*. Examples are the pendulum experiments Newton presents in the scholium to the laws of motion and prop. 6 of Book 3. However, from reading prop. 6, we see a new type of experiment that Newton is willing to count as acceptable: *vicarious investigations*. In Newton’s satellite test, he uses theory-mediated measurements to investigate whether the sun emits an equal gravitational field on Jupiter and its satellites.

Newton begins by assuming that at some distance $r$ from the sun, the ratio of one of Jupiter’s satellite’s weight to mass is greater than the ratio of Jupiter’s weight to mass. He designates the first ratio as the acceleration $d$ and the second as the acceleration $e$.

\[
\frac{W_s}{m_s} > \frac{W_J}{m_J} \tag{3.1}
\]

\[
d = \frac{W_s}{m_s} \tag{3.2}
\]

\[
e = \frac{W_J}{m_J} \tag{3.3}
\]

Using the theory-mediated measure for the force of gravity and since weight is the force of gravity, weight is proportional to the inverse square of the distance. Rearranging some terms shows that the ratio of the separation between Jupiter’s satellite and the sun to the separation between Jupiter and the sun is as the square root of $d$ to the square root of $e$.

\[
d : e :: 1/r_{SS}^2 : 1/r_{JS}^2 \tag{3.4}
\]

\[
d : e :: r_{JS}^2 : r_{SS}^2 \tag{3.5}
\]

\[
\sqrt{d} : \sqrt{e :: r_{JS} : r_{SS}} \tag{3.6}
\]

Then Newton supposes that at some distance Jupiter’s satellite’s acceleration is

---

35 This term is Nelson Goodman’s (1979, Ch. 3).

36 Although Newton does not use ‘experimental philosophy’ in every edition of the *Principia*, he does use the term in the third edition. In that case, it is safe to assume that Newton is explicitly promoting experimental philosophy by the third edition.

37 This term is Sylvain Bromberger’s (1992, p. 118).
greater than Jupiter’s by one thousandth of Jupiter’s. (The point here is to suppose a miniscule difference in accelerations.)

\[ d = (1/1000)e + e \]  

(3.7)

In that case, the separation between Jupiter’s satellite and Jupiter’s distances from the sun would be two thousandths of the separation between Jupiter and the sun.

\[ r_{JS} - r_{S} = r_{JS} \left( \sqrt{1000} / \sqrt{1001} \right) \approx (1/2000)r_{JS} \]  

(3.8)

Since this value is a fifth of the distance between Jupiter and its outermost satellite, it would be easily observed. Since such a discrepancy has not been observed, the sun’s gravity must be equal at equal distances from the sun. In other words, the ratio of weight (as a function of distance) to mass is a constant.38

\[ W_r / m = \text{constant} \]  

(3.9)

Since the satellite test provides a constant quality of bodies, if the satellite test exhausts the vicarious investigations that can be made, we could use rule 3 to make an inductive inference.39 Newton’s rule 3 is valid because the *Principia* provides clear examples as to what counts as an acceptable experiment.

Looking at rule 4, we see a glaring problem with respect to its validity. Newton employs the word ‘phenomena’ twice in order to refer to the standard on which we should base our inductions. However, as I have already mentioned, ‘phenomena’ as Newton uses it refer to generalizations from empirical observations, not empirical observations themselves (see note 5). Examples are the six propositions that comprise Newton’s ‘Phenomena’ chapter in Book 3. The question now becomes whether phenomena are deduced or induced from the data. If phenomena are induced, then rule 4 is nested on an induction whose rule for construction has not been explicitly provided.

The best way to determine if phenomena are induced and if so, what rule is used to construct them, is to step through one of Newton’s phenomena constructions. Phenomenon 1 is below:

Phenomenon 1. The circumjovial planets [or satellites of Jupiter], by radii drawn to the center of Jupiter, describe areas proportional to the times, and their periodic times—the fixed stars being at rest—are as the \( \frac{3}{2} \) powers of their distances from that center. (Newton, 1999, p. 797)

---

38 Notice that the reasoning here only shows that (3.9) holds to high approximation, since a difference in acceleration much smaller than one thousandth of \( e \) would produce an undetectable discrepancy in the separation between Jupiter and its outermost satellite.

39 Newton performs other vicarious investigations as well. In prop. 4, Newton carries out the moon test to measure lunar centripetal acceleration. Also, in prop. 12 and prop. 13, Newton combines the measure for the force of gravity with the measure for motive force to calculate masses and prove that the centre of our planetary system is its centre of gravity.
Newton presents data collected from Borelli, Townly, and Cassini representing the periods of Jupiter’s four known satellites and their distances from the centre of Jupiter. Each satellite’s period and mean distance is in Table 1:

| Satellite | Period (in days) | Mean distance (in Jupiter radii) |
|-----------|-----------------|---------------------------------|
| 1         | 1.769           | 5.46                            |
| 2         | 3.551           | 8.61                            |
| 3         | 7.155           | 13.71                           |
| 4         | 16.689          | 24.42                           |

If a number $x$ with exponent $y$ equals another number $z$, then the logarithm of $z$ to base $x$ equals $y$. Since the logarithm of $z$ to base $x$ equals the quotient of the logarithm of $z$ to base ten divided by the logarithm of $x$ to base ten, $y$ equals that quotient. Finding the exponential relation between the satellites’ distances to their periods merely requires that we represent periods $T$ by $z$ and distances $r$ by $x$. A manipulation shows that $y$ is the slope $m$ of the logarithm of the period to base ten as a function of the logarithm of the distance to base ten.

$$\log T = m \log r$$  \hspace{1cm} (4)

Since the equation is linear, the slope is found by finding the quotient of differences in $\log T$ divided by differences in $\log r$. Three such slopes are found for each of the three intervals. Their values are in Table 2.

| Interval | Slope |
|----------|-------|
| 1        | 1.48  |
| 2        | 1.44  |
| 3        | 1.38  |

Notice that (4) is an approximation since $T$ is proportional to $r^m$, not equal to it. So, the equation should be, $\log k + \log T = m \log r$, where $k$ is a constant that holds to a high approximation for Jupiter–satellite interactions. Notice that (4) is a better approximation the closer $\log k$ is to zero. Also, note that constructing an equation by proportions yields the following: $\log(T_s/T_y) = (m_s/m_y) \log (r_s/r_y)$. But then, an approximation is still needed since the slope is merely a proportion and cannot provide a power rule value. At best, such an equation could have informed Newton of how precise the power rule value is. Furthermore, even if one could construct an equation that is a better approximation than (4), the point of this exercise is to address the underdetermination in deducing the three-halves power rule, which can be made using any of a number of equations since measurement error will exist regardless of the equation one uses.
From Table 2 we see that the mean slope is 1.43, which is 0.07 from three halves. From this close approximation, Newton concludes that the periods of Jupiter’s satellites are as the three halves power of their distances.

However, an *induction* is needed to conclude the three halves power relation since there are higher-order values for the power of the distances that are closer to 1.43 (Harper, 2002, p. 196). For example, seven fifths is 0.03 from 1.43 and ten sevenths is only 0.00143 from the value. Furthermore, since Newton must have made an induction, in order to validate the induction, he must have used either an already mentioned inductive rule or an unmentioned inductive rule. An appeal to rule 1 does not work since seven fifths, ten sevenths, and three halves are all simple fractions and are all equally supported by the data (since the imprecision in data exceeds these fractions). An appeal to rule 2 or rule 3 is unhelpful since they do not apply. Invoking rule 4 is unhelpful because this rule presupposes that the induction has already been made. Hence, unless Newton is invoking a rule that he has not mentioned, his phenomena are invalidly inferred. Nesting a rule on an invalid induction makes the rule invalid. But, could Newton have had a valid reason to make an induction to the three halves power rule even if no inductive rule applied?

Given the difficulty of marshalling evidence sufficient to infer phenomena, Newton could have chosen the three halves power rule for the relation of periods to distances *provided that* the part of his theory developed so far could produce a sufficient cross-check of the relation. Newton’s theory before Book 3 can do such a cross-check, and the results of the cross-check support a three-halves power relation and fail to support a seven-fifths or a ten-sevenths power relation.

Corollary 7, prop. 4, Book 1 provides the following universal measure for centripetal ‘accelerative force’ from the power rule of a body’s period to distance:

\[ T \propto r^s \iff F_a \propto r^{1-2s} \quad (5.1) \]

Now, (5.1) can be used to measure the competing centripetal accelerative forces from the competing power rules for celestial periods to distances.

\[ T \propto r^{3/2} \iff F_a \propto 1/r^2 \quad (5.2) \]
\[ T \propto r^{7/5} \iff F_a \propto 1/r^{12/5} \quad (5.3) \]
\[ T \propto r^{10/7} \iff F_a \propto 1/r^{17/7} \quad (5.4) \]

The cross-check arises from definition 7 when Newton defines centripetal accelerative force as varying directly with change in velocity and inversely with change in time (Newton, 1999, p. 407). Equating these different measures of the same force

---

41 My point here is that the three halves power relation cannot be deduced from the data. In that case, Newton must have made an induction to the three halves power relation, although this induction could have restricted generalizations to the time period over which orbital data were available to Newton. So I am neutral as to whether Newton’s phenomena are constrained by time periods or temporally open-ended.

42 Newton’s accelerative force is equivalent to acceleration (Stein, 2002, p. 285).
we find the following relation:

$$\frac{1}{r^q} \propto \frac{\Delta v}{\Delta t}$$  \hspace{1cm} (5.5)

Since celestial orbits are observed to be very nearly circular, we may approximate change in velocity over change in time by the circumference of a circle divided by the square of the period. With a few manipulations, the following proportion arises:

$$T \propto r^{(1+q)/2}$$  \hspace{1cm} (5.6)

Substituting our competing values for the inverse $1 – 2s$ of the distance for $q$ serves to check the relation of period to distance, since (5.1) and (5.6) rest on different assumptions. The following shows the results of the check:

$$T \propto r^{3/2} \text{ if } q = 2$$  \hspace{1cm} (5.7)

$$T \propto r^{17/10} \text{ if } q = \frac{12}{5}$$  \hspace{1cm} (5.8)

$$T \propto r^{24/7} \text{ if } q = \frac{17}{7}$$  \hspace{1cm} (5.9)

Since the relation in (5.7) is equal to the relation in (5.2), but the relations in (5.8) and (5.9) are not equal to the relations in (5.3) and (5.4), respectively, the check shows that the three halves power rule is the valid induction to make.

Since the cross-check assumes celestial orbits are circular, the check does not rule out higher-order power rules that better characterize less circular orbits. In this sense, the reliability of the check depends on the reliability of observational measurements that celestial orbits are at least very nearly circular. If Newton had that sort of data, then his induction to the three halves power rule in phen. 1 was valid. Similar arguments follow for Newton’s construction of phen. 2 through phen. 6. So, nesting rule 4 does not have to be invalid, since rule 1 could apply or other valid reasons for the induction on which rule 4 is nested could be proposed (for example, if a proposition passes a cross-check).\(^{43}\)

\(^{43}\) Another example of a cross-check for the three halves power relation involves using Newton’s precession theorem. In corol. 1, prop. 45 of Book 1, Newton develops a measure for the motion of orbital apsides that are very nearly circular. He finds the following relation of apsidal precession to the proportionality rule governing an orbit’s centripetal accelerative force: $p^\prime$ precession iff $F_a \propto r^{(360/360+p)^2-3}$. Given (5.1) and the precession theorem, we find the following cross-check relation: $T \propto r^p$ iff $p^\prime$ precession. In order to find a function for $p$ we set the proportionality rule for the centripetal accelerative force in (5.1) equal to the proportionality rule in the precession theorem. Doing so provides the function: $p = (360 / \sqrt{(4 - 2s)}) - 360$. Substituting values for $s$ shows that when $s$ equals $3/2$, $7/5$, or $10/7$, $p$ equals $0$, $-31^{1/3}$, or $-23^{1/4}$, respectively. The result is that if the power relation between period and distance is $s$, then the orbital aphelions of Jupiter’s satellites are precessing at $p^\prime$ per revolution. Since apsidal precession of magnitudes as great as $31^{1/3}$ or $23^{1/4}$ were observable, but were not observed, the three halves power relation was the valid induction to make. Although the precession theorem was derived for orbits that are very nearly circular, William Harper (2002), p. 181 points out that the theorem is adaptable for elliptical orbits. Since the precession theorem does not depend on the eccentricity of the orbit, this cross-check is arguably stronger than the one presented in (5.1) through (5.9).
5. Validating rule 2

Although the other rules have been validated, still there remains the lingering issue that challenges the validity of rule 2, namely, that the rule can mislead if the data are not precise enough to dismiss problematic alternative derivations. Although the explanation is complex, there are two reasons why rule 2 is valid despite the fact that it misleads in this way.

First, if the theory constructed in part by rule 2 proves successful in ongoing research, then the justification for rule 2 increases directly with that success. Since, as mentioned before, rule 2 allows for idealizations, explicating identified theory-dependent second-order phenomena with new idealizations lends increasing support to rule 2, since any new idealization depends in part on rule 2 if the original idealization depends in part on rule 2. Also, the process of developing increasingly better idealizations of the world reduces the amount of adjustments that a theory will need to make in order to explicate the world. In this sense, each successful explication of theory-dependent second-order phenomena with theory developed in part by rule 2 not only increases support for rule 2, but decreases the number of instances in which rule 2 can mislead. So, developing new idealizations and reducing the amount of future explications for theory-dependent second-order phenomena effectively increase the validity of rule 2 (Smith, 2002b, pp. 161–162). In Book 3, Newton’s attempt to explicate the inequalities in lunar motion is an example of how to validate rule 2 in this way (Newton, 1999, pp. 832–874).

Second, applying the theory developed in part from rule 2 to solve problems that could expose limitations of the theory is a safeguard that reduces the risk involved with rule 2. Said otherwise, relentlessly using a theory as soon as it is developed exposes the risky features of the theory (Smith, 2002b, p. 165). For example, since variation of terrestrial gravity is developed in part from rule 2—see my derivation of (2.2)—then immediately engaging in experiments to confirm the variation of terrestrial gravity safeguards our use of the rule.

Newton demonstrates the theory application safeguard by solving six problems presented after his work on lunar motion (Newton, 1999, pp. 874–938). For example, in problems 21 and 22 Newton applies gravitational theory to account for the trajectory of comets. In particular, Newton uses the theory to account for the periodicity of Halley’s comet, which recurs every seventy-five to seventy-seven years. Since the third edition of the Principia was published in 1726, and the last sighting of Halley’s comet before the third edition was in 1682, this particular safeguard could have been substantiated as early as thirty-one years after the Principia’s publication; which could have reduced the length of a garden path involving rule 2 to a matter of three decades. In that case, as long as theory application safeguards are pursued immediately after theory construction, then in a theory depending in part on rule 2, the length of a garden path involving rule 2 should be appreciably reduced.

These theory application safeguards should also be narrowly tailored to expose the risk arising from rule 2, since rules 1 and 3 have risk as well. However, since rule 1 always accompanies rule 2 in Book 3, the safeguard should be narrowly
tailored to expose the risk involved in rules 1 and 2, but to exclude the risk involved in rule 3. Confirming the variation of terrestrial gravity is a good example of a narrowly tailored safeguard that does so. Also, notice that my argument for the validity of rule 2 is forward looking, insofar as the justification of the rule comes after the rule has been applied. But this is why explicating theory-dependent second-order phenomena and initiating theory application safeguards must be done immediately after theory construction. Successful ongoing research validates rule 2 only if the risk of using the rule propels ongoing research.

6. Conclusion

Earlier, plausibility for inductions from phenomena was restricted to the validity of those inductions; where validity was established with Goodman’s criteria.44 Addressing the best criticisms against the validity of each rule of reasoning serves to demonstrate that no rule yields an inference that we are unwilling to accept, where ‘unacceptable inferences’ was defined in each criticism. Since Goodman’s criteria are fulfilled, each rule of reasoning is valid—provided that the appropriate supporting data are available for rule 4 as well as explications for theory-dependent second-order phenomena and theory application safeguards are pursued for rule 2. Validating Newton’s rules does not mean that these are the only valid rules for natural or even empirical philosophy. All it means is that these rules suffice for reasoning validly from phenomena to inductions in natural philosophy.

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Appendix A. Newton’s moon test

Introduction

Given the importance of Newton’s moon test to Book 3, I will step through the test in order to distinguish Newton’s deduction to identical values for lunar

44 See (1.1) through (1.6).
centripetal acceleration and terrestrial gravity from his induction to the same cause for these values. Newton’s moon test is provided after prop. 4:

Let us assume a mean distance [of the moon from the earth] of 60 semi-diameters in the syzygies; and also let us assume that a revolution of the moon with respect to the fixed stars is completed in 27 days, 7 hours, 43 minutes, as has been established by astronomers; and that the circumference of the [lunar orbit] is 123,249,600 Paris feet, according to the measurements made by the French. If now the moon is imagined to be deprived of all its motion and to be let fall so that it will descend to the earth with all that force urging it by which it is kept in orbit, then in the space of one minute, it will by falling describe $151/12$ Paris feet. This is determined by a calculation carried out either by using prop. 36 of book 1 or (which comes to the same thing) by using corol. 9 to prop. 4 of book 1. For the versed sine of the arc which the moon would describe in one minute of time by its mean motion at a distance of 60 semi-diameters of the earth is roughly $151/12$ Paris feet ... And therefore that force by which the moon is kept in its orbit, in descending from the moon’s orbit to the surface of the earth, comes out equal to the force of gravity here on earth, and so (by rules 1 and 2) is that very force which we generally call gravity.

(Newton, 1999, p. 804)

First, we must understand Newton’s terminology, namely, what he means by ‘syzygies’ and ‘versed sine’.

The term ‘syzygy’ is from the Greek ‘suzugia’ which means ‘union’. The lunar syzygy is either of two points on the lunar orbit where the sun, moon, and earth can lie in a straight line. The importance of syzygies lies in the fact that only at the syzygies could astronomers in Newton’s day extrapolate information about the distance between the earth and the moon. Furthermore, if the lunar orbit is approximately circular, then information about the earth–moon distance at a syzygy can be extended to any earth-moon distance on the lunar orbit, making the syzygial earth–moon distance an estimation of the lunar orbit’s semidiameter.

The term ‘versed sine’ is an antiquated mathematical term, best understood with an example. See Fig. 1.

Let $B$ represent the centre of a circle, $AC$ an arc, and $BC$ and $AB$ radii. Let $Aa \perp BC$. Then, $Ba$ is the cosine of $AC$, $aC$ is the versed sine of $AC$, and if $ABC$
equals $\theta$, $\theta$ equals $AC / BC$. Also, $\cos \theta = Ba / AB$.\textsuperscript{45} These terms are important because Newton intends to show that if the moon travels an arc over some time $t$ while orbiting the earth, which is at the centre of its orbit, then if the moon did not have any inertial velocity, it would travel toward the earth in proportion to the versed sine of that arc over the same time $t$.

Use Fig. 2 for the deduction to follow.

The deduction

Let $C$ represent the orbit’s circumference, $T$ the moon’s period, $r$ the earth’s radius, $v$ the moon’s velocity, and $D$ the distance between the moon’s and the earth’s centres. Assume the earth and the moon are spheres, the earth’s gravity is concentrated at its centre, the moon’s orbit is uniform and circular, the quotient of $D$ divided by the sum of $r$ plus the moon’s radius is $D$, and that $D$ is sixty times the magnitude of $r$.

Since ...  

\[
C = 2\pi r \quad \text{and} \quad r = C / 2\pi \\
D = 60r \\
v = 2\pi D / T
\]

It follows that ...

\[
v = 2\pi (60r) / T \\
= 2\pi(60C) / 2\pi T \\
= 60C / T
\]

For $\Delta t = 1$ unit of time, it follows that ...

\[
v \propto \Delta d / 1 \\
\propto \Delta d \\
\propto \text{arc length}
\]

\textsuperscript{45} Cohen (1999), pp. 305–306.
Since \ldots
\[ \theta = \text{arc length/radius} \]
It follows that \ldots
\[ \theta = \frac{60C}{(T \times D)} \]
\[ = \frac{60C}{T(60r)} \]
\[ = \frac{60C(2\pi)}{T(60C)} \]
\[ = \frac{2\pi}{T} \]
Since \ldots
\[ \cos \theta = \frac{\text{cosine}}{\text{radius}} \]
It follows that \ldots
\[ \cos(\frac{2\pi}{T}) = \frac{\text{cosine}}{D} \]
\[ = \frac{\text{cosine}}{60r} \]
\[ = \left( \frac{\text{cosine} \times 2\pi}{60C} \right) \]
And \ldots
\[ \text{cosine} = \frac{(60C \times \cos(\frac{2\pi}{T}))}{2\pi} \]
Since \ldots
\[ \text{versed sine} = \text{radius} - \text{cosine} \]
It follows that \ldots
\[ \text{versed sine} = D - \left( \frac{60C \times \cos(\frac{2\pi}{T})}{2\pi} \right) \]
\[ = 60r - \left( \frac{60C \times \cos(\frac{2\pi}{T})}{2\pi} \right) \]
\[ = \frac{60C}{2\pi} - \left( \frac{60C \times \cos(\frac{2\pi}{T})}{2\pi} \right) \]
\[ = \left( \frac{60C - (60C \times \cos(\frac{2\pi}{T}))}{2\pi} \right) \]
\[ = \frac{60C(1 - \cos(\frac{2\pi}{T}))}{2\pi} \]
Since \ldots
\[ C = 123,249,600 \text{ Paris feet} \quad \& \quad T = 39,343 \ldots \]
It follows that \ldots
\[ \text{versed sine} = 60(123,249,600 \text{ Paris feet})\left(1 - \cos(\frac{2\pi}{(39,343)})\right) \]
\[ = 15.008 \text{ Paris feet} \]

Since the moon’s orbit around the earth is perturbed by an acceleration toward the sun, Newton calculates that any value for the moon’s acceleration toward the earth
should be corrected by $1 / 178.725$ of that value. So, $(15.008 \text{ Pft} \times 1 / 178.725) + 15.008 \text{ Pft} = 15.092$ Paris feet.

Since the centripetal force that holds the moon in orbit varies as the inverse of the square-distance, $F_c \propto 1 / r^2$, the centripetal force at earth is to the centripetal force at the moon as $1 / r^2$ is to $1 / (60r)^2$. In that case, the centripetal force at earth is $60^2$ times the centripetal force at the moon. Since force is proportional to change in distance, a change in distance on earth due to the centripetal force is $60^2$ times a change in distance at the moon due to the centripetal force. So, if the moon falls $d$ feet at the moon, it would fall $60^2 d$ feet at earth. However, if the moon falls $d$ feet in 1 minute at the moon, that does not mean it would fall $1/60$ of $60^2 d$ feet in 1 second on earth.

Rather, objects fall toward the earth such that the change in distance is proportional to the square of the change in time, $\Delta d \propto (\Delta t)^2$. (This is one of Galileo’s discoveries in projectile motion.) Now, since $\Delta d \propto (\Delta t)^2$ on earth, it follows that $\Delta d_1 / (\Delta t_1)^2 = \Delta d_2 / (\Delta t_2)^2$, where subscripts of letters denote intervals. Since the moon would fall $60^2 \times 15.008$ Paris feet in 60 seconds on earth, the following equation solves the distance the moon would fall in 1 second on earth: $(60^2 \times 15.092 \text{ Pft}) / (60 \text{ s})^2 = X \text{ Pft} / (1\text{s})^2$. Solving for $X$ yields...

$$X \text{ Pft} = (60^2 \times 15.092 \text{ Pft})(1\text{s})^2 / (60 \text{ s})^2 = (60^2)(15.008 \text{ Pft})(1^2)(\text{s})^2 / (60^2)(\text{s})^2 = 15.092 \text{ Pft}$$

Hence, the moon would drop 15.092 Paris feet in 1 second on earth.

Christiaan Huygens found that objects on earth drop $15.096 \pm 0.01$ Paris feet in 1 second. In fact, Huygens discovered that the value for gravitational acceleration is proportional to the distance an object falls in one second divided by half the length of a pendulum beating in Paris, which is 1.52966 Paris feet. So, $15.092 \text{ Pft} / 1.52966 \text{ Pft} = 9.8662$, which is close to Huygens’s value $9.8689 \pm 0.01$. (Notice that any value $\pm 0.01$ of 9.86 is also $\pm 0.01$ of $\pi^2$.)

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46 Newton contentiously derives this correction from corol. 2, prop. 45, in Book 1 (Newton, 1999, p. 545).

47 Proposition 2, Third Day. ‘The spaces described by a body falling from rest with a uniformly accelerated motion are to each other as the squares of the time-intervals employed in traversing these distances’ (Galilei, 1991, p. 174).

48 Although Newton’s value for lunar centripetal acceleration is an exact number—since $D$, $T$, and $C$ are exact assumptions—and is not presented with error bounds, the value is actually only as precise as $D$, $T$, and $C$ are reliable. For example, Harper estimates $\pm 0.429$ error for Newton’s value just by using all six of the reported values for $D$ in prop. 4 (Harper, 2002, p. 182). Also, the agreement between Newton’s value for lunar centripetal acceleration and Huygens’s value for terrestrial gravity varies with different exact assumptions. For instance, Newton recalculates lunar centripetal acceleration in corol. 7, prop. 37 with a different value for $D$ and finds that the acceleration is 0.133% greater than the value calculated in prop. 4, which makes it outside of Huygens’s error bounds (Newton, 1999, pp. 803 & 879).
The Induction

Since $9.8662 = 9.8689 \pm 0.01$, nature is simple from rule 1, equivalent phenomena are from equivalent causes from rule 2, & 9.8689 $\pm$ 0.01 is from earth’s gravity, it follows that 9.8662 is from earth’s gravity.

Conclusion

So, Newton’s moon test deduces that the centripetal acceleration keeping the moon in orbit has the same magnitude as the acceleration of objects toward earth. Newton then makes an induction that those two accelerations are actually the same acceleration.\(^{49}\) Hence, the moon is kept in orbit by earth’s gravity. The moon test is important because it was the first explicit use of the rules of reasoning, it served as the foundation for Newton’s argument for universal gravity, and it served as the foundation for Newton’s argument that the Copernican system of the planets is essentially correct.

References

Bromberger, S. (1992). *On what we know we don’t know: Explanation, theory, linguistics, and how questions shape them*. Chicago: University of Chicago Press.

Cohen, I. B. (1999). *A guide to Newton’s ‘Principia’*. Berkeley: University of California Press.

Drosen, C., et al. (2003). Identification of a novel coronavirus in patients with severe acute respiratory syndrome. *The New England Journal of Medicine*, 348, 1967–1976.

Galilei, G. (1991). *Dialogues concerning two new sciences*. (H. Crew, & A. de Salvio, Trans.). Buffalo: Prometheus Books.

Gingerich, O. (1993). *The eye of heaven*. New York: American Institute of Physics.

Goodman, N. (1979). *Fact, fiction, and forecast* (4th ed.). Cambridge: Harvard University Press. (First published 1954)

Harper, W. (2002). Newton’s argument for universal gravitation. In I. B. Cohen, & G. E. Smith (Eds.), *The Cambridge companion to Newton* (pp. 174–201). Cambridge: Cambridge University Press.

Huygens, C. (1989). *Treatise on light*. In M. R. Matthews (Ed.), *Scientific background to modern philosophy* (pp. 124–132). Indianapolis: Hackett.

Koyré, A. (1965). *Newtonian studies*. Cambridge: Harvard University Press.

Newton, I. (1962). The motion of bodies in orbit. In A. R. Hall, & M. B. Hall (Eds.), *Unpublished scientific papers of Isaac Newton* (pp. 239–292). Cambridge: Cambridge University Press.

Newton, I. (1999). *The ‘Principia’, mathematical principles of natural philosophy: A new translation*. (I. B. Cohen & A. Whitman, Trans.). Berkeley: University of California Press.

Smith, G. (2001). J. J. Thomson and the electron, 1897-1899. In J. Z. Buchwald, & A. Warwick (Eds.), *Histories of the electron: The birth of microphysics* (pp. 22–76). Cambridge: MIT Press.

Smith, G. (2002a). From the phenomenon of the ellipse to an inverse-square force: Why not? In D. Malament (Ed.), *Reading natural philosophy: Essays in the history and philosophy of science and mathematics to honor Howard Stein on his 70th birthday* (pp. 31–70). La Salle: Open Court.

Smith, G. (2002b). The methodology of the Principia. In I. B. Cohen, & G. E. Smith (Eds.), *The Cambridge companion to Newton* (pp. 138–173). Cambridge: Cambridge University Press.

Stein, H. (2002). Newton’s metaphysics. In I. B. Cohen, & G. E. Smith (Eds.), *The Cambridge companion to Newton* (pp. 256–307). Cambridge: Cambridge University Press.

\(^{49}\) In prop. 3 Newton has already proved that lunar centripetal acceleration has the same direction as terrestrial gravity, namely, toward the centre of the earth.