Localized magnetostatic modes as guiding waves in ferromagnetic nanostripes

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Abstract. We show that magnetostatic modes with amplitude localized in different regions of the sample occur in ferromagnetic nanorods and ferromagnetic nanostructures along the direction of the magnetic field. The modes in question have amplitude localized in two regions symmetric with respect to the center of the sample (bulk-dead modes) or in its central part (comb modes), or spread across the whole sample (bulk modes). Having established that the localization of these magnetostatic modes varies with their frequency, we demonstrate that these localized modes can be used for guiding magnetostatic waves along a stripe.

1. Introduction
In ferromagnetic materials of linear dimensions comparable with the wavelength of spin waves the wave vector becomes quantized, which implies a discrete energy spectrum [1, 2]. However, even in homogeneously magnetized samples their finite size causes uncompensated magnetic dipoles to occur at the surface. Producing a demagnetization field oriented in the direction opposite to that of the applied magnetic field, these uncompensated magnetic dipoles cause the resultant magnetic field inside the sample to be inhomogeneous, with profile depending on the sample geometry. The determination of the demagnetizing field is a non-trivial problem even in the case of homogeneously magnetized samples [3, 4, 5], and its occurrence leading to complex magnetostatic mode spectra [6, 7, 8]. These effects, among others, contribute to a complex and still unelucidated mechanism of formation of magnetostatic spectrum in finite samples [9, 10].

In this paper we determine numerically magnetostatic excitations in ferromagnetic nanorods of square cross section and in nanostructures in magnetic field saturating the magnetization along the rod axis and perpendicular to the stripe length, respectively. We demonstrate that magnetostatic modes with various localization properties occur in both samples. In the case of the nanostructure, these localized modes can be used for guiding magnetostatic waves in different channels along the sample.

2. Magnetostatic modes in nanorods and nanostructures
Let us consider a system of magnetic moments $\mu(\mathbf{r}) = M(\mathbf{r}) a^3$ (where $M$ is the magnetization vector) regularly disposed in sites $\mathbf{r}$ of a simple cubic crystal lattice with lattice constant $a$. The system has the shape of a stripe of length $2L_x a$ along the $x$-axis and a rectangular base in the $(y, z)$ plane with dimensions $L_y a$ and $(N - 1)a$ along the $y$ and $z$ axes, respectively (see Fig.
The magnetic field $\mathbf{h}_R$ produced by all the magnetic moments in a site $\mathbf{R} \equiv (0,0,n)$, indicated by the plane number $n$, on the central axis (i.e. on the z-axis) reads:

$$h_n = \frac{1}{4\pi} \sum_{r \neq \mathbf{R}} \frac{3(\mathbf{r} - \mathbf{R}) \cdot (\mathbf{r} - \mathbf{R}) \cdot \mathbf{r} - \mathbf{R})}{|\mathbf{r} - \mathbf{R}|^6},$$

where the above sum involves all the sites except the reference point (i.e. $\mathbf{r} \equiv \mathbf{R}$). We will consider samples in static magnetic field $H_0$ applied along the z-axis and strong enough to saturate the magnetization (Fig. 1(a)). Assuming the magnetic wave propagation to be allowed in the z direction only (i.e. uniform precession in the $(x,y)$-plane), we shall only consider the magnetic field along the z-axis. The magnetization vector in this case can be regarded as a superposition of the static magnetization $M_S$ (uniform and parallel to the z-axis) and the dynamic magnetization $\mathbf{m}_n = (m_x, m_y)$ (lying in the $(x,y)$ plane): $\mathbf{M}_R = M_S \hat{z} + \mathbf{m}_n$. Similarly, also the dipolar field $\mathbf{h}_n$ can be resolved into two components: the static component $h_n^s$ parallel to the z-axis and the dynamic component $h_n^d$ lying in the $(x,y)$ plane.

The magnetization dynamics (in the linear approximation and with exchange interactions neglected) along the central axis will be calculated from Landau-Lifshitz equation:

$$-i \omega \mathbf{m}_n = \gamma \mu_0 \hat{z} \times \left[ M_S h_n^d - \mathbf{m}_n (H_0 + h_n^s) \right],$$

where $\omega$ is the magnetostatic mode frequency, $\gamma \approx 1.760 \times 10^{11} \text{ s}^{-1} \text{T}^{-1}$ is the electron gyromagnetic ratio, and $\mu_0$ denotes the magnetic permeability of vacuum. In our model the lattice constant will affect the frequency of the magnetostatic modes by the value of the static magnetic field $\Omega$. This is visualized in Fig. 1(b), where absolute values of dynamic magnetization ($m_n^d = m_x + im_y$) of magnetostatic modes along the central axis are depicted at corresponding frequency levels. The frequency is presented on the reduced frequency scale: $\Omega \equiv \frac{4\pi \omega}{(\gamma \mu_0 M_S)}$, and the constant magnetic field has the reduced value $\Omega_H \equiv 4\pi H_0/M_S = 5.23$ ($M_S = 0.8 \times 10^6 \text{ Am}^{-1}$). The dimensions of the nanorod are: $2L_xa = 2L_ya = 20a$ and $(N-1)a = 200\times a$, it corresponds to 88641 lattice point in the nanograin.

The occurrence of these four groups of modes is caused by the inhomogeneity of the internal magnetic field due to dipolar interactions. Represented in Fig. 1(b) by the thick gray solid line, the internal magnetic field $H(n)$ along the central axis ($n$ numbering the discrete planes of magnetic moments perpendicular to the central axis) takes on maximum values, close to the value of external magnetic field $\Omega_H$, in the central part of the rod; as we move towards the ends of the nanorod, $H(n)$ decreases to reach minimum values at the surface and rise steeply beyond it. This results in the formation of peripheral wells in the internal magnetic field profile; it is in these peripheral wells that low-frequency magnetostatic modes with low number $l$ (surface modes and bulk-dead modes) are localized (see Fig. 1(b)). All modes from those groups have double degenerated frequency, due to zero amplitude in the middle of nanorod. Characterized by rapid amplitude oscillations in the central part of the rod and vanishing amplitude at its extremities, comb modes have nondegenerated frequencies, near the maximum value of $H(n)$. Modes of frequencies above the internal field value are harmonic modes with nonzero amplitude throughout the length of the rod.
Figure 1. (a) Example of the considered $2L_x a \times 2L_y a \times (N-1)a$ nanostripe ($a$ denoting the lattice constant). Magnetostatic waves are assumed to form standing waves only along the $z$-axis and to be uniform along the $x$ and $y$ axes. The applied magnetic field $H_0$ aligns all magnetic moments in the direction of the $z$-axis. Planes perpendicular to the central axis (i.e. to the $z$-axis) are numbered with successive integers $n$. (b) Absolute values of dynamic magnetization ($|m^+(n)|$) of magnetostatic modes (numbered $l$) along the central axis in an elongated nanorod of square cross section ($2L_x a = 2L_y a = 20a$ and $(N-1)a = 200a$), depicted on the reduced frequency scale. The bold line represents the profile of local field $H(n)$ along the central axis.

Figure 2. (a) Local magnetic field $H(n)$ along the central axis in ferromagnetic stripe, plotted versus stripe length $2L_x$ with lateral dimensions fixed at $2L_y = 20$ and $(N-1) = 200$. (b) Pattern of the waveguiding magnetostatic modes parallel to the magnetic field in the ferromagnetic nanostripe, perpendicular to its length.

Let us extend our model from nanorods to nanostripes by increasing $L_x \in (10, 250)$ at constant values of $2L_y = 20$ and $(N-1) = 200$. The excited magnetostatic modes get elliptical polarization, but if $L_x$ is large enough we can still assume the waves are homogeneous in the plane perpendicular to the magnetic field. Increasing the nanostripe length $L_x$ proves to only cause a slight reduction of the internal magnetic field along the central axis, see Fig. 2(a). As a result, the structure of magnetic modes is conserved as a whole (as was verified in calculations), and the relations visualized in Fig. 1(b), with the occurrence of bulk-dead, comb and bulk modes, still apply in the asymmetric case with only slight decreasing of the respective frequencies.

If the value of $L_x$ is large with respect to $L_y$ the stripe can be regarded as a thin waveguide for magnetostatic waves propagating along the $x$-axis. Magnetostatic waves will propagate along the waveguide with amplitude proportional to $\exp(ik_x x)$. For small wave vectors $k_x$ the amplitude can be assumed to be homogeneous along the big parts of the stripe length and throughout its thickness, due to the thinness of the sample (very small $L_y$). Finally, our previous assumptions are met, and the transmission of magnetostatic waves can be considered to be carried out by
discrete modes excited perpendicularly to the direction of propagation, i.e. by the bulk-dead, comb or bulk modes described above and visualized in Fig. 2(b).

By changing the frequency of a magnetostatic wave entering the waveguide we can change the character of the waveguiding mode, and consequently, the transmission channel. Low-frequency magnetostatic waves will be transmitted with the help of bulk-dead modes in two regions symmetric with respect to the center of the sample. These two transmission channels will progressively become closer to each other with increasing frequency to eventually merge, forming a single channel in the central part of the stripe in which waves of higher frequency will be transmitted by comb modes. Finally, high-frequency magnetostatic waves will be transmitted by bulk modes with amplitude spread across the whole width of the stripe. This scenario will happen only in materials with small enough value of the exchange constant to leave unchanged spectrum of magnetostatic modes (for detailed description role of the exchange see [11]). Our results can be of use for the explanation of the experimental results obtained by Brillouin light scattering by Demidov et al. [12], which indicate a homogeneous ferromagnetic stripe can act as a waveguide for spin waves, with a frequency-dependent number of transmission channels: two channels for low-frequency waves and only one channel for high-frequency ones. In this experiment spin waves were excited by transmitting a microwave current through the antennae placed above the ferromagnetic stripe.

3. Conclusions
We have demonstrated that bulk-dead, comb and bulk magnetostatic modes excited along the direction of magnetic field can be used for transmitting magnetostatic waves in a ferromagnetic nanostripe acting as a waveguide for spin waves. The necessary condition to be fulfilled is the sample being thin enough for the excited magnetostatic waves to be nearly homogeneous throughout its thickness. This property can underlie the experimentally observed dependence of the transmission channel on the frequency of magnetostatic waves propagating in the direction perpendicular to the magnetic field [12]. Our results indicate the same effect will occur also in samples of much smaller dimensions.

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