Modeling, analysis and PID controller implementation on suspension system for quarter vehicle model

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INTRODUCTION

Thermal Only one-fourth of the car is taken into account in the quarter vehicle model and the ride quality is evaluated on the grounds of only vertical acceleration, which is the acceleration of the sprung mass. Depending on their degree of freedom (DOF), different kinds of quarter car designs are used. Suspension systems are the vehicle's most significant component influencing passenger ride comfort and the vehicle's road holding ability, which is essential for ride safety [1]. The suspension system is a mechanism that distinguishes the body of the car from the wheel physically. A suspension system's primary role is to reduce the vehicle body's vertical acceleration that is passed on to passengers, thereby contributing to the convenience of the trip. It also has to maintain the tyres in touch with the highway, helping to manage the car [2]. Suspension systems are primarily categorized into three types, i.e. passive, semi-active and active suspension systems, based on operating methods to enhance car comfort, vehicle safety, minimize road damage and general performance [3]. Figure 1 indicates a vehicle model of two DOF quarters.

ABSTRACT – The aim of this research is to acquire a mathematical model for the quarter vehicle model’s passive and active suspension system and to build an active and passive suspension control for a quarter-vehicle model. The passive design of the suspension scheme is a compromise between vehicle handling capacity and passenger ride comfort. Passive suspension provides one of these two circumstances that conflict. Current car suspension systems use passive components only by using a coefficient of spring and damping with a fixed coefficient and two degrees of freedom. In the design and manufacture of cars, passenger comfort is a very significant parameter. The objective of the active suspension system is to enhance both the comfort of the ride and the handling of the highway by directly regulating the actuators of suspension power. In the active suspension system, the Proportional Integral Differential Controller (PID) method is applied. Various kinds of highway profiles and controllers such as P, PD, PI, and PID controllers evaluate the efficiency of the active suspension system comparing it with passive suspension scheme. This efficiency will be determined by using the MATLAB and SIMULINK to perform computer simulations.

Passive suspension is generally an elderly standard system with fixed parameter uncontrolled springs and dampers and no internet feedback is used [4, 5]. It is a significant job to design an outstanding suspension system with optimum vibration efficiency under various road circumstances. Over the years, it has been suggested that passive and active suspension systems optimize car performance [6, 7]. On the other side, active suspension systems that improve the quality of the ride use extra energy to provide a response-dependent damper [8, 9]. Using sensor data linked to the vehicle [10],
a feedback control law determines these additional forces. In active suspension schemes, an actuator is placed perpendicular to the suspension buildings between the wheel and the car's frame. By pulling the car down or pushing it up, the actuator utilizes the suspension room to suppress its vibrations owing to highway irregularities. Active suspensions, however, usually involve a big supply of energy, which is the primary drawback that prevents the extensive use of this method in practice. Semi-active suspension consumes less energy than active suspensions.

Traditional car suspension designs were a compromise between three competing demands: road handling, cargo handling, and humane body comfort [11]. Decent drive comfort needs a soft suspension but applied sensitivity to changes in loads. No rigid or soft suspension setting is required for good handling. The conventional passive suspension has a constant spring and damper coefficient[12]. The suspension system should compromise between these two issues because of these conflicting requirements, as shown in Figure 2 [13].

![Figure 2. Damping compromise for passive dampers [14]](image)

Specifically for electrorheological (ER) dampers and magneto-rheological (MR) dampers are accessible in exercise. In engineering applications, semi-active suspensions are more practical than ever. Many scientists have been studying with MR dampers the active suspension system for vehicle suspensions[15,16]. Many control methods have been suggested in recent years, such as skyhook, surface hook and hybrid control [17,18], H infinity control [19,20], sliding mode control model [21], adaptive control described in [10, 22-24], fuzzy control in [25, 26], and ideal control created by [26].

In this document, the PID controller is considered to run excellent control of the automobile suspension. The strategy proposed has more benefits than the standard passive strategy [28, 29]. In the Simulink environment, modeling and simulation are performed and a sophisticated controller has been launched. Proportional Integral Differential controller. Simulink is a versatile interface capable of easily managing various types of controllers [16, 30]. The current article assesses the effectiveness of semi-active suspension control when applying two-degree vehicle freedom suspension. A set system parameter simulates dynamic response with highway disturbances. Further, implement the PID controller and performance enhancement analyzes conducted for such a system [31, 32].

**MATHEMATICAL MODELING**

Modeling, particularly in the field of engineering control, is a very significant component. In particular, before concentrating on real design manufacturing, the scientists and technicians will generate the system model's motion equation. Mathematical modeling is developed to depict and define the current model or model to be built in mathematical language. The investigator has followed several phases in the modeling method to ensure the accuracy of the simulation assessment consequence. The mathematical modeling for the active suspension system of a two-degree liberty quarter vehicle body is carried out using fundamental mechanical legislation. The following observations [3] were taken into consideration in modeling the suspension scheme.

- The suspension system here is regarded to be a two-degree system of liberty and is presumed to be a linear or roughly linear system for quarter vehicles.
- Some negligible forces are overlooked to reduce the complexity of the scheme owing to the low-intensity impact of these forces. These are therefore left out for the model of the system [32].
- Tyre material is regarded to have both damping and stiffness properties.

**Passive Suspension for Quarter Vehicle Model**

Practically all suspensions on heavy-duty cars such as trucks and semi-trailers in use today in the globe are passive. The passive suspension system utilizes passive components such as tyres, connections, springs of coils and leaves, donuts of rubber, stops of bumps and hydraulic dampers [34, 35]. Generally, springs and dampers are put in these structures between the vehicle's sprung and unsprung masses. These suffer from the severe constraints of momentarily storing or dissipating energy at a steady pace. The forces they produce also rely on the relative local movements. Their capacities are therefore essentially restricted, but these systems with fixed suspension configurations must deal with a broad range of road and loading circumstances. These suspensions do not need an external power supply. Simple, cheap and reliable
are these systems [35]. The hydraulic actuator is operated for use in an active suspension system. Proportional and integrator gains PI controller was used for force tracked control of the hydraulic actuator. Their study findings show that with an appropriate force tracking error, the hydraulic actuator can get the real force close to the target force. To reduce the effects of road profile and vibration on the quality of the car chassis, a PID state feedback regulator was used [37 - 39]. It can be seen from the simulation results that the limited state feedback controller shows significant advancement in rising body acceleration magnitude and settling time, vertical body momentum and vertical displacement suspension. About rim displacement, it is noted that while for the active suspension system the magnitude of the vertical wheel displacement is slightly worse than the passive system, the tire friction settling time of the active system is less than passive. Using the system's undamped natural frequency, some of the path differences are implemented.

![Figure 3. A quarter-vehicle model for passive suspension system [38]](image)

The movement equation will be as follows: applying the 2nd law of Newton to the two mass and the 3rd law of Newton, interaction for the two mass.

\[
M_s\ddot{z}_s + b_s(\dot{z}_s - \dot{z}_u) + K_s(z_s - z_u) = 0 \quad (1a)
\]

\[
\ddot{z}_s = \frac{1}{M_s}[b_s(\dot{z}_u - \dot{z}_s) - K_s(z_s - z_u)] \quad (1b)
\]

For the unsprung mass

\[
M_u\ddot{z}_u - b_s(\dot{z}_s - \dot{z}_u) - K_s(z_s - z_u) + b_t(\dot{z}_u - \dot{z}_r) + K_t(z_u - z_r) = 0 \quad (2a)
\]

\[
\ddot{z}_u = \frac{1}{M_u}[b_u(\dot{z}_u - \dot{z}_s) + K_s(z_s - z_u) - b_t(\dot{z}_u - \dot{z}_r) - K_t(z_u - z_r)] \quad (2b)
\]

which \(M_s\), \(M_u\), \(K_s\), \(K_t\), \(b_s\), and \(b_t\) indicate the sprung and unsprung elements mass, stiffness, and damping frequency, respectively. \(Z_r\) represents the route variants and the numbers \(z_s\) and \(z_u\) are the displacements of the body and wheel respectively.

**Active Suspension for Quarter Vehicle Model**

An active suspension system consists of one or more actuators that can exert independent suspension force(s) to improve riding characteristics [39]. The active suspension system differs in its ability to pump and store energy from a conventional passive suspension into the system as well as dissipate it. Active suspensions are more costly and take up more space than a passive suspension counterpart and use more power. Besides that, such a system requires frequent maintenance which raises the operating costs of the device.

It can be categorized into two categories based on their vehicle mounting [6], weak bridge, and rigid suspension. Weak suspension systems are defined by a spring and damper series actuator. The damper passively monitors the motion of the wheel so that its role can be controlled by body movement. Therefore, such a type of suspension system can be used to increase the ride’s comfort. An actuator parallel to the damper and the spring is a high bandwidth or rigid suspension system. Since the actuator connects the unsprung mass of the leg, it is possible to control the motion of the wheel hop as well as the movement of the head. Therefore, the high bandwidth or rigid suspension system will improve both ride comfort and ride handling considerably. Therefore, almost all work on active suspension systems requires high suspension systems of bandwidth. A hydraulic, pneumatic, or electrical actuator can be used for the active suspension [40]. A hydraulic suspension is a hydraulic or pneumatic actuator, a damper and a mechanical spring [8]. The actuator force is controlled by an electronic controller whose methodology. The vehicle engine drives a hydraulic pump to supply the hydraulic suspension power to the actuator, creating oscillation-damping forces between the vehicle flow mass and the vehicle unsprung mass. To control the force of the actuator, the low-power electromagnetic actuator operated by the control unit with the electrical converter operates a hydraulic valve.
An electromagnetic active suspension consists of a mechanical spring and an electromagnetic actuator, all parts operating mechanically in parallel. The electromagnetic actuator’s natural control versatility contributes to a significant improvement in the suspension behavior, as the active suspension can generate the active control force to rapidly absorb road shocks, eliminate roll and pitch motions and improve safety and comfort [8]. In contrast, the electrical active suspension’s other possible value is that the electromagnetic actuator can function as a generator. This feature makes energy recovery when the actuator generates the damping force from the suspension. Thus, the energy consumption of the vehicle is reduced [3]. An automotive suspension block diagram using a linear electromagnetic actuator. The complex and expensive hydraulic power supply is now replaced by an electrical generator feeding a battery [8]. The hydraulic valve and actuator were replaced by the machine. The main component of the rim suspension system is an electrical actuator that is controlled through a power electronics converter by the control system. In this case, the actuator and power electronics have to be bigger, but the process is modest because it has fewer tools and motorized components. Since it has no hydraulic devices, this is an oil-free system. At the same time, the energy stored in the electromagnetic active suspension can be fed back to the battery via the electrical converter when the electromagnetic actuator acts as a generator. One with active force sources is an active suspension. Thus the mechanical design of an active suspension, as shown in Figure 4, is identical to a passive one. However, by changing the damper’s features, some control of the damping coefficient is accomplished. An active suspension can usually be electrically turned remotely to either soften the suspension or stiffen it. Its coefficient of damping can be continually or discontinuously altered.

STATE SPACE FORMULATION

The system can be posed in state-space form by selecting the state variable as follows:

\[ y_1 = Z_s - Z_u; \quad y_2 = \dot{Z}_s; \quad y_3 = Z_u - Z_r; \quad y_4 = \dot{Z}_u \] (5)

Now the system’s state equation can be written as:

\[ \dot{y} = A[y] + B[f] + D[r] \] (6)

where

\[ \{Z\} = [Z_s \ Z_u \ Z_r \ Z_u]_T \] is the state vector,

\[ \{f\} = F_t \] is the control force,

\[ \{r\} = \{Z_r\} \] is the road input vector,

\[ A, B \] and \[ D \] are invariant coefficients.
PID CONTROL EXECUTION

A Proportional Integral Differential controller is a generic feedback system used commonly in engineering control systems for control loops. A PID controller analyses an "error" value as the distinction between the variable of a measured process and the setpoint required. By changing the process control inputs, the controller tries to minimize the mistake. The PID controller's algorithm has three distinct continuous parameters and is sometimes referred to as a three-term control: proportional, integral, and dependent, called P, I, and D. Simply put, these values can be interpreted as time: P is dependent on the current error, I on past error accumulation, and D is a forecast of future errors based on the current rate of change. The weighted sum of these three operations is used to change the technique using a control component such as a control valve condition or damper. The PID block is available in the Sim Mechanics and Simulink toolbox library, which can be added to the model of the suspension.

\[ U_c(t) = K_p e(t) + \frac{K_p}{T_i} \int_0^t e(t) dt + K_p T_d \frac{de(t)}{dt} \]  

(7)

where \( K_i = \frac{K_p}{T_i} \) is the integral gain and \( K_d = K_p T_d \) is the derivative gain, \( U_c \) is the input, \( K_p \) is the proportional gain, and \( T_i \) and \( T_d \) are the integral and derivative time constants of the Proportional Integral Differential controller respectively.

Figure 5. PID controller [28]

Figure 5. demonstrates the PID controller's block diagram. The \( U_c \) signal passes through the controller that calculates the error signal derivative and integral [44]. The \( U_c \) signal is sent to the plant or system as acquired from Eq. (7) and the output of the process variable is achieved. The signal is then sent to the sensor to confirm that a mistake in the scheme has happened.

BLOCK DIAGRAM

Passive Suspension System

The limitations of the Passive Suspension System Simple Passenger Quarter Vehicle Model \( M_s = 241.5 \text{ kg} \), \( M_u = 41.5 \text{ kg} \), \( K_s = 6000 \text{ N/m} \), \( K_t = 14000 \text{ N/m} \), \( b_s = 300 \text{ N-s/m} \), \( b_t = 1500 \text{ N-s/m} \) [45].

Figure 6. Passive suspension model
The constraints of an active suspension system are \( M_s = 241.5 \text{ kg} \), \( M_u = 41.5 \text{ kg} \), \( K_s = 6000 \text{ N/m} \), \( K_t = 14000 \text{ N/m} \), \( b_s = 300 \text{ N-s/m} \), \( b_t = 1500 \text{ N-s/m} \) [45]. The integral constant \( k_i = 0.2 \), derivative constant \( k_d = 0.6 \), and proportionality constants \( k_p = 0.001 \).

RESULTS AND DISCUSSION

MATLAB / SIMULINK software is used to simulate the quarter vehicle’s mathematical model. Where the scheme input is expected to be traffic disruption. In terms of linear displacement, the efficacy of the suspension scheme is also referred to as suspension travel. The objective is to acquire a small value of amplitude for suspension travel, and it should also be fast to attain a stable state. The input of the scheme is assumed to be three normal types of traffic disturbance. It is presumed that highway profile 1 and highway profile 2 are one bump that is shown in figure 8 and figure 9 below where the amplitude of the bump is indicated by ‘a’. A frequency of 0.1 HZ was described for the sinusoidal bump,

\[
 \omega(t) = \begin{cases} 
 a \left[ u(t - 5) - u(t - 10) \right] \sin(0.2\pi t), & 5 \leq t \leq 10 \\
 0 & \text{Otherwise} 
\end{cases}
\]

Where, \((a = 0.1 \text{m (road bump height)})\)

Performance under two types of excitement on the highway, i.e. the built suspension system. Jerk and random input are evaluated by computer simulation.
Figure 9. Road profile 2

Figure 10. Active versus passive system travel with road profile 1

Figure 11. Active versus passive system travel with road profile 1

Figure 12. Active versus passive system travel with road profile 1
Figure 13. Active versus passive system travel with road profile 1

Figure 14. Active versus passive system travel with road profile 1

Figure 15. Active vs Passive System travel with Road profile 1

Figure 16. Active versus passive system travel with road profile 2
**Figure 17.** Active versus passive system travel with road profile 2

**Figure 18.** Active versus passive system travel with road profile 2

**Figure 19.** Active versus passive system travel with road profile 2

**Figure 20.** Active versus passive system travel with road profile 2
Evaluation between Active and Passive (P / PD / PI / PID controller) model: software modeling study was carried out on Eqs. (1)- (8). There is a comparison for the quarter-vehicle model between passive and active suspension. Suspension performance effects on different road profiles. The effectiveness of the active suspension system is tested under different road profiles between the passive controller and the PID controller. Because of the time-increasing output parameters, the settling period, the percentage of overruns and the steady-state error are all lower for the PID controller than for the passive, active with the P, PD, PI controller system and also for the PID control system that is shown in Figure 10 to Figure 21. A proportional controller (Kp) will decrease, but never eliminate, the steady-state mistake, the time of the rise and decrease. The steady-state error will be removed by an important control (Ki), but it may aggravate the transient response. A derivative control (Kd) increases the strength of the scheme, reduces overflow and improves the transient response.

CONCLUSION

Active and passive systems were explored and the two undertook a complete relative study. The methodology for designing an active suspension for a passenger vehicle has been established by creating a controller that increases system efficiency. It has been discovered that the PID controller development strategy is the most effective approach for the autonomous scheme. In a passive system, suspension travel was discovered to be decreased to over half its importance. By counting an active element in the suspension, instead of using solely passive components, it is feasible to enhance its efficiency. Using the PID controller layout, the potential for better ride comfort and better road handling is studied. This study’s objective was accomplished and the design for separate road profiles was checked. Thus, the rider can attain excellent handling and feel greater comfort in bad highway circumstances by using Active suspension with the PID controller. The test findings conducted using SimMechanics and Simulink indicate that the PID module application reduces the active suspension body speed to nearly half the passive suspension. Thus, the passenger's ride comfort can be improved by applying a PID controller. As the spring stiffness rises and reduces as the damping coefficient rises, the findings also indicate an improvement in body acceleration. Comparison of both outcomes indicates that amplitudes of the suspension body's active and passive acceleration are comparable, but it requires longer in Simulink than in SimMechanics to dissipate passive suspension body acceleration.

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