The 1995 Revision of the Joint US/UK Geomagnetic Field Models—I. Secular Variation

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We present the methods used to derive mathematical models of global secular variation of the main geomagnetic field for the period 1985 to 2000. These secular-variation models are used in the construction of the candidate US/UK models for the Definitive Geomagnetic Reference Field at 1990, the International Geomagnetic Reference Field for 1995 to 2000, and the World Magnetic Model for 1995 to 2000 (see paper II, Quinn et al., 1997). The main sources of data for the secular-variation models are geomagnetic observatories and repeat stations. Over the areas devoid of these data secular-variation information is extracted from aeromagnetic and satellite data. We describe how secular variation is predicted up to the year 2000 at the observatories and repeat stations, how the aeromagnetic and satellite data are used, and how all the data are combined to produce the required models.

1. Introduction

The main requirements of data for secular-variation prediction and modelling are that (1) they have been measured at exactly the same location through time so that they are uncontaminated by crustal fields when differences are taken between data at different times, (2) they have been measured over a long period of time (i.e., greater than one year) so that future predictions can be made more confidently based on the historical records, (3) they have been measured over continuous periods of time, or near other data measured over continuous periods of time, relative to the periods of the external field variations so that these effects can be evaluated and eliminated, (4) they are evenly distributed over the globe, and (5) the most recent data are available to minimise the errors caused by prediction. Data from magnetic observatories satisfy requirements (1), (2) and (3) and to a lesser extent, especially with regard to requirements (2) and (3), so do repeat-station data. The observatories and repeat stations, however, do not cover the ocean areas of the Earth and even their distribution on land is far from ideal. To satisfy requirement (4), secular-variation information has been extracted from Project MAGNET 3-component aeromagnetic data and the Polar Orbiting Geomagnetic Survey (POGS) satellite data. Finally, particular emphasis is placed on requirement (5) in all data types. Advantage was taken of the World Data Centre status of the Geomagnetism Group at BGS for the timely collection of observatory annual means and repeat-station data. For the Project MAGNET and POGS data the existing collaboration between BGS and the US Naval Oceanographic Office (for the production of the World Magnetic Models) ensured that the latest data were incorporated into the secular-variation predictions.

To predict secular variation from the different data types various methods were used. For the observatory data, linear prediction filters were constructed and the only assumption made about the secular variation is that the spectral characteristics remain constant through time. For the repeat-station data, linear regression was used for prediction and it was therefore assumed that the secular variation varied linearly with time. For the other data types (Project MAGNET and POGS), it was assumed that secular variation was constant throughout the period.

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Sixteen annual secular-variation models are derived, each valid from the mid-point of one year to the mid-point of the next year, for the period 1985 to 2000. By calculating annual models, continuous and discontinuous changes in the secular variation can be modelled more accurately. However, as the requirement of both the IGRF and the World Magnetic Model is for five-year average models of secular variation, means were taken of the coefficients of the annual secular-variation models. For example, for the secular-variation model for 1995.0 to 2000.0, a mean of the coefficients of the 1995.0, 1996.0, 1997.0, 1998.0, 1999.0 and 2000.0 models was taken, giving half weight to the 1995.0 and 2000.0 models and full weight to the other models. The secular-variation models for 1985.0 to 1990.0 and 1990.0 to 1995.0 were produced in an analogous way.

2. Observatories

Data from a total of 182 observatories, with annual means for 1982 and later, were extracted from the computer file of observatory annual means maintained by BGS. External fields, such as transient disturbances, diurnal variations and seasonal effects, are largely removed by taking mean values over a year. As these fields are modulated by the solar cycle, small residual effects with an 11-year fundamental period may remain in the time series of annual means (McLeod, 1992). Figure 1 shows the positions of the observatories and Fig. 2 shows the number of annual means from these observatories available for each year from 1980 to 1993. If an observatory was moved for any reason, the data recorded at the original site are adjusted to the new site using measured site-differences. By taking differences between consecutive annual means an estimate of the secular variation is obtained.

2.1 Prediction of secular variation by linear prediction filters

The historical time series of secular-variation values at each observatory is characterised in terms of
frequencies present by using maximum entropy spectral analysis (Press et al., 1986; Barraclough et al., 1992). From this characterisation the coefficients of a linear prediction filter are determined. This filter can be used to extrapolate the time series beyond the last secular-variation value and any dominant periodicities in the original time series are preserved in the forecast.

As this method requires the secular-variation values to be evenly spaced in time, any missing values in the secular-variation records were replaced with estimates obtained by linear interpolation if the gap was no more than five years long; otherwise, the earlier part of the record was discarded. In addition, annual means that were not computed from a full twelve months of data were adjusted using the neighbouring annual means so that the resulting secular-variation values were at exactly one-year intervals. Some early parts of records that were very noisy, especially in the vertical component, were also discarded.

As the predicted values of secular variation depend, to some extent, on the number of poles used in the maximum entropy spectral analysis, the final predictions were taken as the weighted mean of the predicted values for all possible numbers of poles up to an arbitrarily imposed maximum of 25. This limit was set in order to damp out the higher frequency components of the spectrum. The weights were the inverse squares of the errors in estimating the last five years of the observed time series from the preceding observed values using different numbers of poles. The observed and predicted secular-variation values were smoothed with a 3-point filter with coefficients (0.25, 0.50, 0.25) before being used for the spherical harmonic analyses.

The performance of this method has been compared with linear regression at 160 observatories where there were long enough records (Macmillan and Barraclough, 1993). Up to 10 secular-variation values were used in the regression to estimate secular variation for the next five years. The results were compared with the last five years of observed secular variation, which were not used in the predictions. When predicting more than three years ahead, linear prediction filters were always more accurate than linear regression. For up to three years ahead, linear prediction filters give similar results to linear regression.

![Fig. 2. The number of observatory annual means available for each year from 1980 to 1993.](image-url)
2.2 Weighting scheme for spherical harmonic analyses

For each observatory, estimates of the annual secular change in the northerly (X), easterly (Y) and vertical (Z) components were obtained for each of the sixteen years from 1985 to 2000 using the observed values if available, or the predicted values otherwise. The weights for the subsequent spherical harmonic analyses assigned for each observatory, each year, and each component are based on (a) the noisiness, $E_n$, expressed as the root mean square deviation of the observed or predicted values, for a five-year period centred on the year of interest, from a quadratic function of time fitted to them using the method of least squares, and (b) an empirical estimate of the error in prediction, $E_p$. The prediction error was obtained by (i) determining, for those observatories having complete 1985–90 data, the errors in prediction of the 1985–90 mean value as data before 1990 were successively eliminated, back to a maximum of 10 years before; and (ii) computing, by least-squares fit of these errors, a general expression where the error increases with an increase in both the time lag from the end of the data record to the date of prediction and the standard deviation per pole, and decreases with an increase in the length of the record. A general expression fulfilling these criteria is of the form

$$E_p = a \tau^b (s+1)^c t^{-d}. \tag{1}$$

Here, $\tau$ is the time lag between the end of the data used for prediction and the final year predicted, in years; $s$ is the standard deviation per pole of the predicted values for all the poles used, in nT/year/pole; $t$ is the length of record used, in years; and $a$, $b$, $c$ and $d$ are unknown parameters determined in the least-squares fit. For $\tau = 0$ (and $b > 0$) $E_p$ is zero. The use of $s$ reflects the assumption that the prediction should be more accurate if it varies less with the different number of poles used; the standard deviation per pole is used because there is less source of variation with fewer poles. The length of record ($t$) is included because longer records should predict the lower frequency part of the secular variation better than short records. Regression analysis gave values for $a$, $b$, $c$ and $d$ equal to 1.456, 0.712, 3.057 and 0.149 respectively.

An estimate of the total error, $E$, is given by

$$E^2 = E_n^2 + E_p^2. \tag{2}$$

In general the records of secular variation in $Y$ are much smoother and therefore more predictable than those in $X$ or $Z$. This is because oscillations in the annual means due to external field effects related to the solar cycle are smaller on the east-west axis than on the north-south or vertical axes as a consequence of external current geometry. As a result the errors are lower for $Y$ (average of 7.6 nT/year for all years) than for $X$ (average of 9.3 nT/year for all years) or $Z$ (average of 10.6 nT/year for all years). To ensure that the final model is not biased in its fit to the $Y$ data, the preliminary errors for $X$ and $Z$ were adjusted so that the resulting mean errors for each year and for each of the three components were the same. The weights for the subsequent spherical harmonic analyses are given by

$$w_o = \exp(-0.5 E'). \tag{3}$$

where $E'$ is the adjusted error. This results in an approximately even spread of weights in the range $0 < w_o < 1$.

2.3 Selected results

The observatories selected as examples are two from areas with a relatively dense coverage of observatories (Europe and North America) and two from areas where there are few other observatories (Africa and Pacific Ocean). Figure 3 is for Eskdalemuir in the UK and Fig. 4 is for Victoria in Canada. Figure 5 is for Addis Ababa in Ethiopia and Fig. 6 is for Pamatat in Tahiti. They show observed, predicted
The observed secular-variation values are shown by the solid lines, the predicted values by the dashed lines, the final five-year model values for 1985 to 2000 by the dotted lines, the adjusted weights for the annual spherical harmonic analyses for 1985 to 2000 by the circles and the mean adjusted weights for all three elements at all observatories by the crosses. The code, name, latitude, longitude and starting date of the observatory are shown at the top of the plot.
Fig. 4. As Fig. 3 but for Victoria.
Fig. 5. As Fig. 3 but for Addis Ababa.
Fig. 6. As Fig. 3 but for Pamatai.
(not smoothed) and modelled secular variation in $X$, $Y$ and $Z$ for the period 1955–2000 and their corresponding weights. The observed secular-variation values are shown by the solid lines, the predicted values by the dashed lines and the final five-year model values for 1985 to 2000 by the dotted lines. The weights for the annual spherical harmonic analyses from 1985 to 2000 for each observatory are shown by the circles and the mean weights for all three elements at all observatories are shown by the crosses. Details of the how the final five-year models are calculated from the observed and predicted secular-variation values are given later. The code, name, latitude, longitude and starting date of the observatory are shown at the top of each plot. These plots allow for visual checks, and adjustments if necessary, on the behaviour of the linear predictor filter and estimated weights for each element at each observatory, and also on the fit of the final models to the observed and predicted data. For example, for thirty observatories where it was felt that the linear predictor filter was over-sensitive to past periodicities, particularly in the vertical component, and was giving unsatisfactory predictions, linear regression on the five most recent secular-variation values was used instead. For any changes made, the models were recalculated and the plots redrawn. By this iterative process the best set of models was achieved.

Both Eskdalemuir and Victoria show the observatory weights above the mean weights and a good agreement between the observed and predicted values and the modelled values. For Addis Ababa the secular variation in $Z$ is poorly fitted by the final model and the local prediction is dubious. It can be seen that the predicted secular-variation values have generally been allocated lower than average weights reflecting the fact that the latest data are for 1989 and that they are noisy. For Pamatai the observed values are less noisy, the predicted values are more realistic and the final model fits better.

3. Repeat Stations

Repeat stations with at least two occupations a minimum of six months apart and with a mid-date for the two most recent occupations after 1983.0 were selected from the BGS files. Observations that were anomalous, empirically defined as being more than 1000 nT different from the 6th generation IGRF, were not used. Figure 7 shows the locations of the 890 repeat stations that contributed useful secular-variation information to the models (after eliminating outliers), and also six recently-opened observatories (or reopened in the case of San Fernando in southern Spain) that have less than three years of data. These observatories have been treated in a similar fashion as the repeat stations but have been given higher weight in the subsequent spherical harmonic analyses. Figure 8 shows the number of occupations each year, including a count for the years between two occupations when an estimate of the secular variation at the site is possible.

3.1 Prediction of secular variation

Secular-variation estimates were calculated from regression lines fitted to the observed average annual secular-variation values at the mid-dates between occupations. Predictions of secular variation up to the year 2000 were made using these regression lines. Values for $X$, $Y$ and $Z$ were used if possible; otherwise the other elements were used, up to a maximum of three elements per station. In the cases where there are only two occupations at a repeat station the secular variation is assumed to be constant.

The secular-variation estimates from the regression lines were compared with those calculated from the 6th generation IGRF to identify outliers. The cut-off levels for eliminating outliers for each component were determined from histograms of the residuals. For declination and inclination they were set at ±0.25°/year and for all the other elements they were set at ±25 nT/year. Approximately 5% of the secular-variation estimates were rejected as outliers.

3.2 Weighting scheme

The weighting scheme for the subsequent annual spherical harmonic analyses takes into account whether the secular-variation estimate is effectively observed or predicted. If it is effectively observed (i.e., interpolated), the weighting scheme takes into account the time interval between occupations and
Fig. 7. The locations of the 890 repeat stations (circles) and the six observatories with short records (triangles, only 5 are visible, the 6th is San Fernando in Southern Spain) used for modelling secular variation.

Fig. 8. The number of repeat stations providing secular-variation estimates for each year from 1980 to 1993.
whether the data from each occupation have been reduced to a magnetically quiet level. Only 2% of observed secular-variation estimates were derived from observations that had not been reduced in any way to a magnetically quiet level. If the estimate is effectively predicted (i.e., involves extrapolation), the weighting scheme also takes into account the total number of occupations at a station and the time interval after the last occupation. Thus, for an estimate that is effectively observed the weight takes the form

\[ w_r = \frac{1}{4T} \]  

(4)

if data from both occupations are reduced to a magnetically quiet level, or

\[ w_r = \frac{1}{8T} \]  

(5)

if data from one or both occupations are not reduced to a magnetically quiet level. \( T \) is the number of years between the two occupations. For an estimate that is effectively predicted the weight takes the form

\[ w_r = \left( \frac{n}{n + 1} \right) \frac{1}{\tau} w_{\text{ave}}, \]  

(6)

where \( w_{\text{ave}} \) is the average of the weights determined for estimates that are effectively observed (using Eqs. (4) and (5)), \( n \) is the number of observed secular-variation estimates after 1983.0, and \( \tau \) is the number of years between the year of interest and the date of the first occupation if extrapolating back in time or the last occupation if extrapolating forward in time.

4. Extracting Secular-Variation Information from Aeromagnetic and Satellite Data

Aeromagnetic and satellite data are generally not observed at the same location through time. Thus, to extract information about secular variation, main-field models for a number of dates are derived and differences between these models calculated.

4.1 Aeromagnetic data

From 1953 to 1994 the US Navy, under its Project MAGNET programme, collected three-component aeromagnetic data, primarily over ocean areas, for the World Magnetic Model production. We were able to incorporate data from October 1988 to March 1994, released by the US Naval Oceanographic Office in mid-1994 into the secular-variation models. One-minute means were computed from the two-second data to reduce the volume of data. The method used to extract secular-variation information from these data follows that of Peddie (1992).

The surface of the Earth is divided into 412 approximately equal-area tesserae. The tesserae are 10° of latitude by 10° of longitude at the equator and approximately 1113 km by 1113 km in area. For each tessera in which there were at least two flights at least six months apart, spherical harmonic models with maximum degree and order \((n^*)\) equal to one were fitted to a minimum of three selected one-minute means from each flight. There were 75 such tesserae. The number of flights in these tesserae with valid data ranged from 2 to 8. The mean time span of selected data from any one flight crossing a tessera is 81 minutes, so there will be only small changes caused by the diurnal variation and, in any case, most of the flights are done at night-time. The one-minute means were only selected if they occurred during three-hour periods when the global disturbance index \(Kp\) was less than or equal to 3. and if the flights were above 15000 ft (4.572 km) to reduce the effect of crustal field contamination. An estimate of secular variation
was then made for each tessera, at the mean position of all the selected data, from a model derived from the slopes of the regression lines to each set of spherical harmonic coefficients when plotted against time. The differences between these secular-variation estimates and those synthesised from the 6th generation IGRF model are much larger than expected. On further investigation it was realised that models with $n^* = 1$ were too simple to synthesize main-field variations in the selected data along each flight segment. For example with such a model $Y$ does not vary with latitude, only with longitude. Further work is required to determine the best $n^*$ to use. Only 40 Project MAGNET secular-variation estimates in either $X$, $Y$ or $Z$ at the 32 locations shown in Fig. 9 were used in the final models and these had fairly low weighting ($\bar{w}_o/2\sqrt{2}$, where $\bar{w}_o$ is the mean of all the weights for estimates from the observatories for the year in question).

4.2 Satellite data

Quiet-time data from the POGS satellite corrected for ionospheric, magnetometer bias and drift, and clock-drift effects, were grouped into fourteen time groups over the interval 1991.0 to 1993.7. This work was undertaken by the US Naval Oceanographic Office (paper II, Quinn et al., 1995, 1997). To counteract the Backus effect which would result from modelling with only scalar data, selected Project MAGNET data from October 1988 to December 1993 were reduced to each of the fourteen model epochs using secular-variation models derived from observatory and repeat-station data, supplemented with estimates from the 6th generation IGRF. Fourteen spherical harmonic main-field models, each with $n^* = 12$, were calculated. A secular-variation model based largely on POGS data for 1990.0 to 1995.0, with $n^* = 12$, was then derived from the slopes of the regression lines through each set of spherical harmonic coefficients plotted against time.
5. Derivation of Initial Secular-Variation Models

For the interval 1985.0 to 1990.0 an initial secular-variation model (with $n^* = 8$) was computed from the relevant annual models (see Section 1). These were derived from observatory and repeat-station data, supplemented with estimates from the 6th generation IGRF for 14 locations in ocean areas in the southern hemisphere.

For the interval 1990.0 to 1995.0 the model of secular variation derived from POGS data truncated to $n^* = 8$ was combined (by simple averaging of coefficients) with a model computed from the relevant annual models. These were derived from observatory and repeat-station data, again supplemented with estimates at 14 locations in ocean areas in the southern hemisphere from the 6th generation IGRF. This resulted in an initial model (with $n^* = 8$) for 1990.0 to 1995.0 that includes POGS data.

The difference between two secular-variation models (both derived from observatory and repeat-station data and values from the 6th generation IGRF) for the intervals 1985.0 to 1990.0 and 1990.0 to 1995.0 effectively represents the secular acceleration for the interval 1985.0 to 1995.0. This model of secular acceleration was added to the initial secular-variation model for 1990.0 to 1995.0 to give an initial secular-variation model (with $n^* = 8$) for 1995.0 to 2000.0 that includes POGS data.

6. Derivation of Final Secular-Variation Models

The time interval between the derivations of the initial models and the final models was about six months, during which more recent observatory and repeat-station data were collected and the secular-variation information from Project MAGNET data was incorporated. New annual means at 74 observa-

Fig. 10. The 240 locations where estimates from an initial model, including secular-variation information from POGS data, were used in the final models.
Stories and values at 133 repeat stations for 1993 were included in the prediction algorithms, giving new secular-variation estimates up to the year 2000. Estimates of secular variation in $X$, $Y$ and $Z$ at 240 locations (shown in Fig. 10) in areas where there were no observatories or repeat stations were computed from the initial models. For 1990.0 and onwards these estimates include POGS data. This new data set was then used to produce the annual models from which the final models for 1985.0 to 1990.0, 1990.0 to 1995.0 and 1995.0 to 2000.0 were derived. The weighting scheme between the different types of data was as follows:

![RMS residuals to input data](plot.png)

**Fig. 11.** The increase in the misfit of the annual secular-variation models to the input data with time.

**Table 1.** US/UK candidate secular-variation model for 1995.0 to 2000.0.

| $n$ | $m$ | $g_n^m$ (nT/year) | $h_n^m$ (nT/year) |
|-----|-----|------------------|------------------|
| 1   | 0   | 17.6             |                  |
| 1   | 1   | 13.2             | -18.0            |
| 2   | 0   | -13.7            |                  |
| 2   | 1   | 4.0              | -14.6            |
| 2   | 2   | -0.3             | -7.2             |
| 3   | 0   | 0.8              |                  |
| 3   | 1   | -6.6             | 4.0              |
| 3   | 2   | -0.5             | 2.2              |
| 3   | 3   | -8.5             | -12.6            |
| 4   | 0   | 1.2              |                  |
| 4   | 1   | 1.1              | 1.3              |
| 4   | 2   | -0.8             | 1.0              |
| 4   | 3   | 0.3              | 2.5              |
| 4   | 4   | -4.5             | -1.2             |
| 5   | 0   | 0.9              |                  |
| 5   | 1   | 0.5              | 0.5              |
| 5   | 2   | -1.4             | 1.5              |
| 5   | 3   | -1.7             | 0.6              |
| 5   | 4   | 0.0              | 1.7              |
| 5   | 5   | 2.1              | 0.6              |
| 6   | 0   | 0.4              |                  |
| 6   | 1   | -0.3             | 0.7              |
| 6   | 2   | 0.3              |                  |
| 6   | 3   | 2.1              |                  |
| 6   | 4   | 0.0              |                  |
| 6   | 5   | -0.4             |                  |
| 6   | 6   | -0.4             |                  |
| 7   | 0   | -0.3             |                  |
| 7   | 1   | -1.1             |                  |
| 7   | 2   | -0.5             |                  |
| 7   | 3   | 0.5              |                  |
| 7   | 4   | 1.3              |                  |
| 7   | 5   | 0.1              |                  |
| 7   | 6   | 0.0              |                  |
| 7   | 7   | -0.9             |                  |
| 8   | 0   | 0.1              |                  |
| 8   | 1   | 0.0              |                  |
| 8   | 2   | 0.4              |                  |
| 8   | 3   | 0.3              |                  |
| 8   | 4   | -1.3             |                  |
| 8   | 5   | 0.5              |                  |
| 8   | 6   | 0.4              |                  |
| 8   | 7   | -0.9             |                  |
| 8   | 8   | 0.1              | -1.1             |
| Data type                              | Weight          | Number of component data/year |
|---------------------------------------|-----------------|-------------------------------|
| Observatories                         | $w_o$ (maximum 1) | 546                           |
| Observatories with 1–3 years’ data    | $\bar{w}_o$     | 18                            |
| Repeat stations                       | $w_i$ (maximum 1/4) | 2400–2505                    |
| Project MAGNET                        | $\bar{w}_o / 2\sqrt{2}$ | 40                             |
| Initial models including POGS data    | $\bar{w}_o / 2$ | 720                           |

Expressions for the weights $w_o$ and $w_i$ are given in Eqs. (3)–(6), and $\bar{w}_o$ is the mean weight of all secular-variation estimates from the observatories for the year in question.

Figure 11 shows the weighted root mean square residuals between each of the annual models and all the observed or predicted secular-variation values used in deriving them. As expected the ability of the models to fit the input data gets worse with time because the data are increasingly derived from predictions.

The 80 coefficients for the 1995.0 to 2000.0 secular-variation model are listed in Table 1. This is the US/UK candidate secular-variation model for this interval and also forms the secular-variation part of the World Magnetic Model. The models for 1985.0 to 1990.0, 1990.0 to 1995.0 were used to reduce data to the base epochs of the US/UK candidate main-field models at 1990.0 and 1995.0 and also of the World Magnetic Model at 1995.0.

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