Experiment and Research on Calculation Methods on Flexural Mechanical Behaviors of Non-Metallic Reinforced ECC Beams

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Abstract. Engineered cementitious composites (ECC) has high tenacity and the characteristics of strain hardening. In the bending test, ECC beams equipped with carbon fiber reinforced polymer (CFRP) in the tensile region show good deformation ability and bending ability. In this paper, the constitutive models of ECC and CFRP materials are established by uniaxial tensile compression test. Based on the plane section supposition, the analysis theory of bending capacity of cross section for ECC beams with non-metallic reinforcement is proposed. The calculation results are compared with the four-point bending test results. The results show that: (1) ECC beams with non-metallic reinforcement reflect the advantages of the two materials, and have strong deformation capacity before bending failure. (2) The calculation results based on the theory of flexural capacity of normal section are in good agreement with the experimental results, and the maximum error is less than 4%. The research results can provide the basis and reference for the calculation analysis and practical application of non-metallic reinforced high toughness material structure.

Keywords. Non-Metallic reinforced beam, Engineered cementitious composites (ECC), Carbon fiber reinforced polymer (CFRP), Flexural capacity of normal section, Constitutive relationship.

1. Introduction

Reinforced concrete makes full use of the advantages of materials and is widely used in engineering. However, the deformation capacity of traditional reinforced concrete members is poor, and the steel reinforcement inside the structure is corrosion susceptible to environmental impact after cracking [1], which reduces the load-bearing capacity of the structure and leads to poor durability. In order to solve some problems [2], experts and scholars try to use new materials instead of steel reinforcement to improve the durability of the structure. Carbon fiber reinforced polymer CFRP bar is a new type of material with ordinary steel reinforcement shape made of carbon fiber tow and matrix, which has the advantages of light weight, high strength, corrosion resistance and fatigue resistance [3]. There are many theoretical research and practical applications of CFRP reinforced concrete, such as the mechanical behaviors test and analysis of CFRP reinforced concrete beams, and the fatigue performance and shear performance of concrete beams are researched by applying a small amount of prestressed to CFRP bars [4].
ECC and CFRP bars are high-performance engineering materials emerging in recent years, which have been widely concerned and researched by engineering and technical personnel. However, there are relatively few reports on the composition of CFRP-ECC composite structure by combining the two [5]. The main research content of this paper is to configure CFRP bars in the tensile area of ECC beam, make them into CFRP-EC beam for four-point bending test, and conduct theoretical research and verification of flexural bearing capacity analysis. Due to the material constitutive relation of CFRP bars and ECC is significantly different from that of steel bars and concrete in ordinary structures, the normal section bending analysis theory of ordinary reinforced concrete is not applicable [6]. Based on the uniaxial tensile and compressive constitutive relation of ECC, combined with the tensile stress-strain relationship of CFRP bars, the calculation theory of normal section bending capacity for CFRP-ECC structure is proposed. The correctness of the calculation theory is verified by comparing and analyzing the test results [7].

2. Theory of Flexural Capacity of Cross Section

2.1. The Constitutive Relation of ECC

The tensile and compressive specimens of 1×5×15 cm and 4×4×16 cm were made by ordinary ECC, and the uniaxial tensile and compressive tests were carried out. The stress-strain curve was fitted according to the experimental data, as shown in figure 1.

![ECC uniaxial tensile and compressive test data and fitting curve](image)

Figure 1. ECC uniaxial tensile and compressive test data and fitting curve.

The fitting curve is described by equation as follows:

Tensile:

\[
\sigma = \begin{cases} 
\sigma_{t0} \left( \frac{\varepsilon}{\varepsilon_{t0}} \right)^{\alpha_\varepsilon} & 0 \leq \varepsilon < \varepsilon_{t0} \\
\sigma_{t0} + \frac{\varepsilon - \varepsilon_{t0}}{\varepsilon_{tp} - \varepsilon_{t0}} \left( \sigma_{tp} - \sigma_{t0} \right) & \varepsilon_{t0} \leq \varepsilon < \varepsilon_{tp} \\
0 & \varepsilon_{tp} \leq \varepsilon < \varepsilon_{tu}
\end{cases}
\]

(1)

In the equation, \(\sigma_{t0}\) is the cracking stress, \(\varepsilon_{t0}\) corresponds to the cracking strain, \(\sigma_{tp}\) is the ultimate stress, and \(\varepsilon_{tp}\) is the strain of the ultimate stress. When the strain exceeds \(\varepsilon_{tp}\), the material enters the softening stage. Due to the small contribution to the structure, the corresponding stress is considered to be 0. From the experimental data, it can be seen that the ultimate strain of the material exceeds 5 %, and has obvious strain hardening characteristics. The first curve is not straight, and the equation is described in exponential form. When \(\alpha_\varepsilon = 1\), ECC becomes ideal elasticity in the first stage. In the equation derivation of flexural capacity, the main parameters of the constitutive relationship are: \(\sigma_{t0} = 2.5\, MPa, \varepsilon_{t0} = 0.005, \sigma_{tp} = 3\, MPa, \varepsilon_{tp} = 0.04, \alpha_\varepsilon = 1\).
Compression curve fitting reference method, using an equation for fitting:

Compression:

\[ \sigma = \sigma_{cp} \cdot \frac{n(\varepsilon/\varepsilon_{cp})}{1+(n-2)(\varepsilon/\varepsilon_{cp})+\varepsilon/\varepsilon_{cp}^2} \quad \varepsilon < 0, \ n > 0 \]  

(2)

In the equation, \( \sigma_{cp} \) is the ultimate compressive stress and \( \varepsilon_{cp} \) is the strain corresponding to the ultimate compressive stress. From the experimental data, it can be seen that the mechanical behavior of ECC during compression is basically similar to that of conventional concrete. The short cut PVA added to ECC has little contribution to the compressive bearing capacity, and the main role is to improve the tensile properties of ECC. In the derivation of the following bearing capacity equation, the parameters based on the test results are: \( \sigma_{cp} = 40 \text{MPa}, \ \varepsilon_{cp} = 0.05, \ n = 2 \).

2.2. The Structural Relationship of CFRP Bars

According to literature and the experimental results of CFRP bars in the laboratory, the tensile failure of CFRP bars is brittle failure, and there is no plastic property (yield) [8]. The stress-strain relationship of failure is linear elastic, and the stress-strain relationship of CFRP bars can be determined by figure 2.

![Figure 2. Stress-strain curve of CFRP bars under tension.](image)

In the figure, \( \varepsilon_{fu} \) is the ultimate tensile strain, corresponding to \( f_{fu} \) is the ultimate tensile stress of FRP bars, \( \varepsilon_{fd} \) is the ultimate tensile strain, corresponding to \( f_{fd} \) is the ultimate tensile stress. The ultimate tensile strain and tensile stress of CFRP bars are determined by experiments. The design value can be determined by reference to article 3.2.12 of National Standard Specification for Application of Fiber Reinforced Composites in Construction Engineering (GB 50608 - 2010):

\[ f_{da} = \frac{f_{fu}}{\gamma_f \gamma_e} \]  

(3)

In the equation, \( \gamma_f \) is the partial coefficient of FRP, and the fiber sheet and FRP bars are 1.4. \( \gamma_e \) is the environmental impact coefficient of FRP materials, usually outdoor environment, for CFRP bars can be taken 1.1.

According to the experimental results, the parameters are \( f_{fu} = 1870 \text{MPa}, \ \varepsilon_{fu} = 0.011, \ f_{fd} = 1200 \text{MPa}, \ E_s = 170 \text{GPa}. \)

2.3. Theoretical Research of Cross Section Bending

The deformation of CFRP-ECC beam is large when the test is loaded to failure, when the concrete in the compression area is crushed, the bearing capacity reaches the maximum value. Because there is no yield stage of CFRP bars, the reasonable failure of the beam should be judged by ECC crushing in the
compression area [9]. At this time, the CFRP bars do not reach the ultimate tensile strength. Therefore, the components should be designed and calculated according to the super-reinforced beam, and the equation should also be deduced according to the ECC crushing stage in the compression area [10]. During the experiment, with the increase of the tensile stress at the lower edge of the section, the CFRP bars are in the elastic stage throughout the bending process, while the toughness of ECC material is high. The tensile strength of ECC contributes to the tensile force by CFRP bars and ECC [12]. According to the above analysis, referring to the analysis idea of ordinary reinforced concrete, the bending of CFRP-ECC cross section is simplified into two key stages, namely, ECC exactly cracking and beam exactly compression failure:

(1) When ECC cracks in the tensile area, the corresponding crack bending moment is the cracking moment $M_{cr}$, as shown in figure 3.

![Figure 3. Diagram of Cracking Moment Calculation.](https://example.com/figure3)

Figure 3 shows that the strain at the ECC edge of the tensile area reaches $\varepsilon_{t0}$, and the strain of CFRP is less than the ultimate strain $\varepsilon_{fu}$, and the ECC edge of the compressive area is less than the ultimate strain $\varepsilon_{cp}$. According to the principle of mechanical balance, the equation is obtained:

$$
\begin{aligned}
F &= C - T - \sigma_s \cdot A_s = 0 \\
M_{cr} &= C \cdot (h - x_c - a) + T \cdot (x_t - a) \\
M_{cp} &= T \cdot (h - x_c - x_t) + \sigma_s \cdot A_s \cdot (h - x_c - a)
\end{aligned}
$$

In the equation, $C$ is the ECC resultant force of the compression area, $x$ is the height of the compression area, $x_c$ is the distance between the resultant force $C$ and the upper edge, $T$ is the ECC resultant force of the tension area, $x_t$ is the distance between the resultant force $T$ and the lower edge. According to equation (1) and equation (2) ECC constitutive relation, when $\alpha_e = 1$, $n = 2$, $x_c = x/3$, $x_t = (h - x)/3$, $C$ and $T$ calculation equation is:

$$
\begin{aligned}
F &= \int_0^x b \cdot \sigma_{cp} \cdot \frac{2(\varepsilon/\varepsilon_{cp})}{1 + (\varepsilon/\varepsilon_{cp})} \cdot d\varepsilon \\
T &= \frac{1}{2} E_t \cdot \varepsilon_{t0} \cdot (h - x) \cdot b = \frac{1}{2} \sigma_{t0} \cdot (h - x) \cdot b
\end{aligned}
$$

where $E_t$ is the tensile elastic modulus of ECC. According to the plane section assumption, $\varepsilon_c = \varepsilon_{t0} \cdot x/(h - x)$, $\varepsilon_s = \varepsilon_{t0} \cdot (h - x - a)/(h - x)$ can be obtained. Combining the horizontal equilibrium equation $F = 0$ and Equation (5), after integration and sorting, we can obtain:

$$
F = \sigma_{cp} \cdot b \cdot \ln \left[ 1 + \left( \frac{\varepsilon_{t0} \cdot x}{\varepsilon_{cp} \cdot (h - x)} \right) \right]^2 - \frac{1}{2} \sigma_{t0} \cdot (h - x) \cdot b - E_s \cdot \varepsilon_{t0} \cdot b \cdot \frac{(h - x - a)}{(h - x)} = 0
$$

In the equation, $E_s$ is the elastic modulus of CFRP bars. At this point, equation (6) is only
unknown \( x \), and the rest are known. The cracking moment \( M_{cr} \) can be obtained by substituting the solution \( x \) into equation (4).

(2) When ECC in the compression area reaches the ultimate compressive strain and destroys, the CFRP-ECC beam reaches the ultimate state of bearing capacity, and the corresponding bending moment is the ultimate bending moment \( M_u \). The calculation diagram is shown in figure 4.

![Figure 4](image)

**Figure 4.** schematic diagram of ultimate bending moment calculation.

In figure 4, the strain at the edge of ECC in the corresponding compression area reaches \( \varepsilon_{cp} \), and the strain of CFRP is still less than the ultimate strain \( \varepsilon_{fu} \). The ECC at the edge of the tensile area has entered the strengthening stage, and its stress is less than \( \varepsilon_{tp} \) and greater than \( \varepsilon_{t0} \). Similarly, according to the principle of mechanical equilibrium, the calculation equation is the same as that in equation (4), but the cracking moment \( M_{cr} \) becomes the ultimate moment \( M_u \).

Similarly, the equation (5) can be used to calculate the ECC resultant force \( C \) in the compression area, but the ECC resultant force \( T \) in the tension area is composed of the elastic stage and the hardening stage. According to the equation (1), the ECC tensile constitutive relation can be obtained:

\[
T = \frac{1}{2} \sigma_{t0} \cdot x_{t0} \cdot b + \frac{1}{2} \left( \sigma_{t0} + \sigma_{t0} \cdot \frac{\varepsilon_{t0}}{\varepsilon_{tp} - \varepsilon_{t0}} \left( \sigma_{tp} - \sigma_{t0} \right) \right) \cdot \left( h - x - x_{t0} \right) \cdot b \tag{7}
\]

In order to simplify the calculation, it is assumed that the ECC cracking stress \( \sigma_{t0} \) is the same as the ultimate stress \( \sigma_{tp} \), and equation (7) is simplified as:

\[
T = \sigma_{t0} \cdot \left( h - x - \frac{1}{2} x_{t0} \right) \cdot b \tag{8}
\]

According to the plane section assumption, we can get \( x_{t0} = x \cdot \frac{\varepsilon_{t0}}{\varepsilon_{cp}} \cdot \left( h - x - a \right) / x \). Combined with the horizontal force balance equation \( F = 0 \) and equation (8), after integration and sorting, we can obtain:

\[
F = \sigma_{cp} \cdot b \cdot \ln \left[ 1 + \left( \frac{x}{\varepsilon_{cp}} \right)^2 \right] - \sigma_{t0} \cdot \left( h - x - \frac{1}{2} x_{t0} \right) b - E_s \cdot \varepsilon_{cp} \cdot b \cdot \frac{\left( h - x - a \right)}{x} = 0 \tag{9}
\]

At this point, the equation (9) is only an unknown number \( x \), and the rest are known quantities. The ultimate moment \( M_u \) can be obtained by substituting \( x \) into equation (4).

3. Four-Point Bending Test

3.1. Experimental Loading Method

Four-point bending test was carried out by two-point symmetrical loading. The specimen size is 400 length × 50 width × 30 height, and the unit is mm. The center of CFRP bars tendon is 8 mm away from the bottom edge. The loading diagram is shown in figure 5.
3.2. Comparative Analysis of the Result

3.2.1. Theoretical Calculation. Based on equation (6), the basic parameters $\sigma_{cp} = 40\, MPa$, $b = 50\, mm$, $\varepsilon_{t0} = 0.005$, $\varepsilon_{cp} = 0.005$, $h = 30\, mm$, $\sigma_{t0} = 2.5\, MPa$, $E_s = 170\, GPa$, $a = 8\, mm$ are substituted into the equation to solve $x = 11.90\, mm$. Substituting it into equation (4), the cracking moment $M_{cr} = 26.23\, N.m$, the cracking load $T_{cr} = 6\, M/l = 524.60\, N$, where $l = 300\, mm$ is the calculated span.

Based on equation (9), the middle parameter $\varepsilon_{cp} = 0.005$, the rest is the same as above, the ECC compressive stress on the upper edge of the component reaches the ultimate compressive stress, and the height of the neutral axis $x = 13.57\, mm$ is obtained by numerical calculation. According to the equation (4), the ultimate moment $M_u = 145.7\, N.m$, corresponding to the ultimate load $T_u = 6\, M/l = 2914.00\, N$.

At this time, the stress of CFRP bar $\sigma_s = E_s \cdot \frac{x-a}{h-x} = 614\, MPa < f_{fd} = 1200\, MPa$. When the CFRP bar is crushed by ECC at the upper edge, the stress of the steel bar is less than the design strength, and it is still in the elastic stage, which conforms to the precondition of equation derivation.

3.2.2. Experimental Result. Through the load time curve obtained by the test load, the initial crack load and the polar load of the three specimens can be known. The theoretical calculation of the initial crack load and the ultimate load are drawn in figure 6, which is close to the experimental value.

3.2.3. Error Analysis. The maximum error between the theoretical calculation value of the initial crack load and the measured value of the test is 3.74\%, and the maximum error between the theoretical calculation value of the ultimate load and the measured value of the test is 3.77\%, not exceeding 4\%. The calculation results of the equation in this paper are close to the experimental measured values,
indicating the reliability of the calculation method of flexural capacity of CFRP-ECC beams. The experimental and theoretical values of the three specimens are shown in table 1.

**Table 1.** Theoretical and experimental results and error analysis table.

| Load type   | Experimental value (N) | Calculation(N) |
|-------------|------------------------|----------------|
|             | Specimen A | Specimen B | Specimen C |             |
| Cracking load | 539      | 528      | 505     | 524.60 |
| Ultimate load | 2804     | 2989     | 2817    | 2914.00 |

| Load Type   | Error (%) | Specimen A | Specimen B | Specimen C | Maximum error (%) |
|-------------|-----------|------------|------------|------------|--------------------|
| Cracking load | 2.74      | 0.84      | 3.74      | 3.74      |                   |
| Ultimate load | 3.77      | 2.57      | 3.33      | 3.77      |                   |

4. Conclusion

(1) CFRP has no yield stage and is fully elastic. If the CFRP-ECC beam is configured according to the appropriate reinforcement beam, brittle fracture will occur, which will affect the safety of the structure. It is suggested that the crushing of ECC in the compression area should be the failure criterion of the whole CFRP-ECC beam according to the super-reinforcement beam.

(2) ECC has the characteristics of high toughness and strain hardening, and participates in the work before failure. The tensile contribution of ECC in this experimental beam accounts for 16% -20% of the overall tensile force. Therefore, the tensile strength of ECC cannot be ignored when calculating the flexural capacity of CFRP-ECC members.

(3) The research results of this paper can provide reference for the calculation and analysis of non-metallic reinforced high-toughness material structure. When the constitutive relationship of the material changes, the research idea of this paper is still applicable.

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