Physical modeling of reinforced concrete structures exposed to emergency loads

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Abstract. The article examines the role of dynamic collapse modelling for reinforced concrete structures exposed to emergency dynamic loads. The authors reviewed methodological approaches to test simulation of emergency loads applied to reinforced concrete structures with the aim to consider the scale effects revealed in manufacturing of test models of reinforced concrete structures. The comparative analysis is based on the examples of the authors’ previous computational and experimental investigations of the reinforced concrete building frames made of cast-in-situ reinforced concrete. The article determines regularities in the impact of physical and geometric similarities applicable to models and full-scale specimens. The obtained results can become a base for a mathematical description of the impact produced by scale effects that shall be considered in simulation of emergency dynamic loads for the given structures.

1. Introduction
The experience of theoretical and experimental investigations of reinforced concrete structures reveals a number of peculiarities that do not allow to fully rely on purely theoretical justification of the complex designed structure configuration which is based on the number of assumptions and hypotheses. Sometimes mechanical study cannot be made by means of mathematical reasoning and calculations. Very often there is not any mathematical formulation of the problem at all, as the mechanical subject under study is so complex that it still does not have a satisfactory mathematical interpretation. In such cases, the key role is played by experimental research methods based on which one can shape laws governing the considered phenomenon and write them down mathematically.

In construction, the importance of experimental investigation rises drastically when the additional dynamic loads and their consequences are analysed. Physical modelling in structural dynamics is an efficient way of testing the objects under study using geometrically similar models. Similarity theory is to be used to justify modelling results reliability, i.e. to prove that, after being converted, the information obtained in experiments with a model is credible enough to be applied to a full-scale specimen. As sometimes full compliance with the similarity requirements cannot be achieved, it is necessary to evaluate practical relevance and to try to compensate for the variance caused by the scale effects; that is why similarity in structural deformation processes and in the way of the dynamic load transmission should be provided.

Studies [1-7] considered the use of quasi-static and dynamic calculation models rendering the structure behaviour for the progressive collapse hazard analysis. Among the experimental
investigations aimed to study structure response to emergency dynamic load, one can note studies [8-12].

Recent publications describe tests involving models along with full-scale specimen research (for example, studies by Yu J., K.H. Tan [13], R. Ahmadi [14], A.T. Fam [15]). The main drawback of these studies is that the emergency load is transferred with the help of hydraulic test systems that do not allow studying load redistribution after one of the load-bearing elements has failed; this fact is considered in the studies by G.A. Geniev [16], V.I. Travush [17], V.I. Kolchunov [18], N.V. Fedorova [19], that are devoted to defining deformation and collapse parameters of the reinforced concrete structures exposed to dynamic emergency loads by means of experiment where the dynamic load is modelled by immediate break-up of linear or moment connection and the static operation load is applied directly to the studied elements [20-21], this way fully reflects the specific features of the stress strain state of the analysed structures.

2. Research methods

As an example, let us consider the bearing structure of cast-in-situ reinforced concrete described in the study [22] and the model the experimental investigation of which is represented in the studies [19-21], Figure 1a and b.

Figure 1. General View of the Full-Scale (a) and Modelled (b) Reinforced Concrete Building Part

Although tests with the use of models are attractive and cheap, experience shows that, sometimes, there is certain – in some cases insignificant – deviation from the classical similarity theory. This phenomenon is called a large-scale effect, which reveals itself in case similarity of the reinforced concrete material structure is violated. It is characterized by various size defects decreasing its resistance.

The bigger is the sample size, the higher is the probability of defects and as a result the lower is the strength. However, it is rather difficult to determine a quantitative dependence of the material strength upon the inhomogeneity distribution. The scale effect \( \varphi \) can be identified in testing materials of a full-scale specimen and a model. In this case, the results of the model tests are converted to be used for a full-scale specimen with allowance for the scale effect.

There is a number of complex empirical dependences [23] to define the scale effect, but they are not accurate enough.
The study [23] experimentally determined the dependence of the scale effect \( \varphi \) on the coefficient of variation of the material prism strength \( \nu \), that is \( \varphi = f(\nu) \). Here, the coefficient of variation \( \nu \) is a ratio of the prism strength standard deviation \( \sigma_{st} \) to its mathematical expectation \( R_{st} \), i.e.

\[
\nu = \frac{\sigma_{st}}{R_{st}}
\]

(1)

Naturally, the larger is the coefficient of variation, the stronger is the impact produced by the scale effect as it decreases from unity (ideal conditions and characteristics of the material) to a certain value (\( \approx 0.85 \)).

Figure 2. Dependence of the Scale Effect \( \varphi \) on the Coefficient of Variation \( \nu \) of the Concrete Prism Strength \( R_{st} \).

It is expedient to describe the obtained graph by means of a dependence relation. For this purpose, using the least squares method, we can define a range of mathematical functions enlisted in Table 1.

| №  | Regression equation | Least squares system of equations |
|----|---------------------|----------------------------------|
| 1  | \( \varphi = a + bV \) | \( a \cdot n + b \sum V = \sum \varphi \)
|    |                     | \( a \sum V + b \sum V^2 = \sum (V \cdot \varphi) \) |
| 2  | \( \varphi = a + \lg V \) | \( a \cdot n + b \sum \lg V = \sum \varphi \)
|    |                     | \( a \sum \lg x + b \sum (\lg x)^2 = \sum (\lg V \varphi) \) |
| 3  | \( \varphi = ab^V \) | \( n \cdot \lg a + \lg b \sum V = \sum \lg \varphi \)
|    |                     | \( \lg a \sum \lg V + \lg b \sum V^2 = \sum (V \cdot \lg \varphi) \) |
| 4  | \( \varphi = a + bV + cV^2 \) | \( a \cdot n + b \sum V + c \sum V^2 = \sum \varphi \)
|    |                     | \( a \sum V + b \sum V^2 + c \sum V^3 = \sum (V \cdot \varphi) \)
|    |                     | \( a \sum V^2 + b \sum V^3 + c \sum V^4 = \sum (V^2 \cdot \varphi) \) |

Table 1. The effect of concrete strength on the scale factor.

Here, we introduce the variable \( V \) to denote the function argument - concrete strength \( R_{st} \).

The proposed mathematical expressions (Table 1) present the functional dependence of the scale effect on the prism strength of concrete in an analytical way to automate calculations.

Analysis of the dependence presented in Fig. 1 allows for the conclusion that the scale effect has insignificant impact on the accuracy of calculations when converting the results obtained for a model to be used for a full-scale specimen. Moreover, researchers’ disregard for it does not imply any hazards, as actually, the scale effect reduces the full-scale load and increases the structural element deflection.

The dependence shown in Fig. 1 gives the general idea about the impact of the concrete prism strength on the scale effect. That is why every individual research involving the use of model concrete grade requires its own dependence relation that is to be defined in the course of standard tests. The obtained results are approximated by means of a formula chosen from Table 1. The best equation is characterized by minimal variance.
2.1. Scale Factor for Geometric Similarity

To ensure accurate behaviour of the studied reinforced concrete structure in the stress strain state throughout all the stages of its operation, model material features shall be identical to the features of the full-scale specimen, i.e.

$$\begin{cases}
\sigma_c^m = \sigma_c^n = \sigma_r^c, E_c^m = E_c^n = E_r^c \\
\sigma_m^m = \sigma_m^n = \sigma_m^m, E_m^m = E_m^n = E_m^m
\end{cases}$$

(2)

here \( \sigma_c \) – concrete stress; \( \sigma_m \) – reinforcement strength; \( E_c \) – concrete modulus of elasticity; \( E_m \) – reinforcement modulus of elasticity.

In case of beam extension and deflection, the following conditions are to be ensured:
- with monoaxial extension, the following rigidity ratio is to be observed
  $$\frac{A_n E_n}{A_m E_m} = \text{const},$$

(3)

- in case of deflection, the following conditions are to be complied with
  $$\frac{E_n I_n}{E_m I_m} = \text{const}.$$

(4)

Let us choose the linear scale factor for the model. It must be only one to provide for compliance with the geometric similarity in general. However, sometimes a number of model creation conditions in the strict geometric similarity cannot be observed. That is why, in particular cases, we can deviate from the purely geometric observation to evaluate the stress strain state of the structure.

We will use two linear scale factors:
- geometric scale factor of the model in the longitudinal direction of the beam (spans, spacings, floor height)
  $$m_{l1} = \frac{l_n}{l_m};$$

(5)

- for evaluation of the structural element cross sections (collar beams, columns)
  $$m_{l2} = \frac{l_n}{l_m}. $$

(6)

In this case, if the material properties are kept identical, based on the mathematical expressions (3) and (4), one can define the ratio for cross section characteristics.

$$\begin{cases}
A_n = m_{l2}^2 A_m \\
I_n = m_{l2}^3 I_m
\end{cases}$$

(7)

This approach is, in particular, used in the study [24].

Although, in this case, the Poisson’s ratios for the model and for the full-scale specimen are not equal, it does not change the overall picture of the modelled structure element collapse [24].

Let us again refer to the monoaxial extension. We find the internal force \( F_{n(m)} \) given stresses \( \sigma_{n(m)} \) and the cross-sectional area of the structure \( A_{n(m)} \),

$$F_n = \sigma_n A_n, \quad F_m = \sigma_m A_m.$$

(8)

Their ratio can be presented as follows:
\[
\frac{F_n}{F_m} = \frac{\sigma_n A_n}{\sigma_m A_m} = \frac{A_n}{A_m} = m_{12}^2
\]

Hence
\[
F_n = m_{12}^2 F_m
\]

Now let us consider the modelling condition that requires relevant elongations to be equal, i.e.
\[
\varepsilon_n = \varepsilon_m
\]  
(10)

\[
\frac{F_n}{A_n E_n} = \frac{F_m}{A_m E_m}
\]

or
\[
\frac{F_n}{A_n} = \frac{F_m}{A_m}
\]  
(11)

It follows that
\[
F_n = m_{12}^2 F_m
\]  
(12)

We see that dependences (9) and (12) are completely identical.

Now we refer to the internal bending moment which is the product of the resultant force in the compressed or extended area by the arm of the internal couple of the reinforced concrete cross section.

In case one of the supports collapsed, the collapse area is affected by the weight of the above structures. In this case, the bending moment occurs in the adjacent supports.

\[
M = \frac{Gl}{4},
\]

(13)

where \( G \) is the weight affecting the beam in the collapsed support area

To model the bending moment properly, the following similarity criterion is to be complied with:

\[
\frac{M_n}{\rho_n l_{n}^5} = \frac{M_m}{\rho_m l_{m}^5}
\]  
(14)

When identical materials are used in a model and a full-scale specimen, the material density is reduced when finding the scale factor. Analyzing criterion (14), we can also define the time scale factor of the structure collapse.

\[
\frac{t_n^2}{t_m^2} = \frac{M_n l_{m}^5}{M_m l_{n}^5} = m_{11}^{5} \frac{1}{m_m}
\]  
(15)

Let us substitute the scale factor of weight and linear dimension for the bending moment scale factor according to the equation (13). Weight is described as the product of material density by the structure volume, i.e.

\[
G = \rho V g
\]

Then, the scale factor of the bending moment can be represented as follows

\[
m_m = \frac{M_n}{M_m} = \frac{\rho_n V_n l_n}{\rho_m V_m l_m} = m_{11}^{5} m_{11} = m_{11}^6
\]  
(16)

Considering (16), we can simplify the equation (15)
\[ m_i^2 = \frac{m_{1i}^8}{m_{1i}^4} = m_{1i} \]

or

\[ m_i = \sqrt{m_{1i}} \quad (17) \]

Thus, this is the way to convert the results of model studies for further use in a full-scale specimen. Time dependence must be observed with structures exposed to dynamic loads.

3. Conclusions

The specific feature of the reinforced concrete structure modelling is the difference of concrete mixtures used for a model and for a full-scale specimen which results in scale effect dependent on the main linear scale factor of the model. Decreasing size of the model as compared with the full-scale specimen reduces accuracy in defining concrete prism strength up to 15% in some cases. The impact of the scale effect can be determined with reliable accuracy in the course of additional experimental investigations that are described by analytical dependences shown in Table 1.

Squared longitudinal force depends on the cross-section scale factor \( m_{ls} \); for the bending moment, this relation will be as follows \( m_m = m_{1m} \), i.e. it will quartically depend on the geometric scale factor of the model. Exposure time scale factor is dependent on the geometric scale factor of the model and is defined as its square root.

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