MHD Boundary Layer Flow in Double Stratification Medium

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Abstract. Magnetohydrodynamic (MHD) fluid flow over an exponentially permeable stretching sheet in a thermally and chemically stratified porous medium with heat source is investigated. The partial differential equations are reduced to ordinary differential equations using suitable similarity transformation, which are then solved numerically with MAPLE software. The effect of the pertinent parameters on the flow, heat and mass transfer characteristics are examined graphically. The results indicated that the velocity profile decreases with increasing value of porosity, magnetic, and suction parameters. The temperature increases with porosity, magnetic and heat source parameters. The concentration profile increases with the increase in porosity and magnetic parameters. Thermal stratification reduces the temperature while chemical stratification reduces the concentration of the fluid.

1. Introduction

Boundary layer fluid flow on stretching surface have many applications in engineering processes. The quality of the products usually depends on the heat transfer rate at the stretching surface during the manufacturing processes. These applications include the glass fiber production, extraction of polymer sheet, petroleum production, exotic lubricants and suspension solutions, drawing of plastic films, hot rolling, wire drawing, solidation of liquid crystals, continuous cooling and fiber spinning [1].

MHD is the interaction of moving conducting fluids with electric and magnetic fields. MHD flows are of great significant in industrial processes. Magnetic field is involved in the process to purify molten metal’s from non-metallic inclusions [2]. MHD flow over stretching surface have practical applications in chemical engineering, electrochemistry and polymer processing [3]. Some of the works in MHD flow has been reported in the literature [4,5,6]. Yusof et al. [7] and Ibrahim [8] reported that when magnetic field parameter increases the fluid velocity decreases. This is due to the existence of the Lorentz force which opposes the motion of fluid and therefore the velocity profile decreases.

Suction and blowing processes are important in engineering fields, that include the design of bearings and radial diffuser clogging and thermal oil recovery [9]. Removing and adding of reactants in chemical processes involved suction and blowing respectively. Blowing is used to reduce the the surfacetemperature, surface drag and also to prevent corrosion or scaling.

Stratification appear in flow field due to the variation in temperature, difference in concentration or fluids with different densities. Stratification (thermal and solutal) effects are important in studying boundary layer flow due to their importance in convective transport phenomena. MHD boundary layer
flow in a thermally stratified medium was studied by [10]. Sekhar [11] investigated the boundary layer phenomena and heat transfer in a thermally stratified medium and found that thermal stratification increase the heat transfer rate at the surface. Singh and Kumar [12] analysed the MHD free convection flow in a micropolar fluid with the presence of double stratification, chemical reaction, heat generation and ohmic heating.

Porous media is the medium of permeable for any fluids where fluids can pass through the material and emerge on the other side. To maintain a heated body, porous media are widely used as an insulator. Porous media are also useful in removing the natural free convection that appeared on the vertical surface [13]. Several authors [14-16] have studied flow in porous medium under different physical situations.

The heat generation and absorption are of great importance in the industrial process. Flow stability is influenced by their presence. Ibrahim and Suneetha [17] studied the heat source, Soret, viscous, chemical and Joules dissipation effect on the MHD convection fluid flow embedded in a porous medium. Devi et al. [18] studied the heat source, radiation and mass transfer effect on MHD flow and found that the temperature increases when the heat source parameter increases.

Chemical reaction in fluid flow also occurs in many industrial applications [19]. The flow field with chemical reaction was discussed by [20-21].

In view of the technical applications discussed above, the aim of the present study is to analyse MHD boundary layer flow over an exponentially permeable stretching sheet in a thermally and chemically stratified porous medium with heat source.

2. Formulation of the problem

Consider a steady two dimensional MHD flow of an incompressible, viscous, and electrically conducting fluid over an exponentially stretching surface. It is assumed that the surface is stretched with velocity $U$ along the $x$-axis, and the $y$ axis normal to the $x$-axis. A variable magnetic field $B = B_0 e^{\frac{x}{m}}$ is applied normal to the surface where $B_0$ is constant. The surface is of temperature $T_w(x)$ and concentration $C_w(x)$ which is embedded in a thermally stratified medium of variable ambient temperature $T_\infty(x)$ and variable ambient concentration $C_\infty(x)$ where $T_w(x) > T_\infty(x)$ and $C_w(x) > C_\infty(x)$ respectively. It is assumed that $T_w(x) = T_0 + be^{\frac{x}{m}}$, $T_\infty(x) = T_0 + ce^{\frac{x}{m}}$, $C_w(x) = C_0 + me^{\frac{x}{m}}$ and $C_\infty(x) = C_0 + ne^{\frac{x}{m}}$ where $T_0$ is the reference temperature and $C_0$ is the reference concentration where $b, c, m$ and $n$ are positive constants.

The governing partials differentials involved in such flow are :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (1)

$$\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} u - \frac{\nu}{K} \frac{\partial u}{\partial y}$$  \hspace{1cm} (2)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p} (T - T_\infty)$$  \hspace{1cm} (3)

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 (C - C_\infty)$$  \hspace{1cm} (4)

where $u$ and $v$ are the components of velocity in the $x$ and $y$ directions respectively, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, $\mu$ is the coefficient of fluid viscosity, $\rho$ is the fluid density, $\sigma$ is the electrical conductivity of the fluid, $B$ is the magnetic field variable. The permeability is in the form of
\( K' = k^* e^{-\frac{x}{L}} \) where \( k^* \) is a constant. \( \alpha \) is the thermal diffusivity, \( T \) is the temperature of fluid, \( Q = Q_0 e^{-\frac{x}{L}} \) is the dimensional heat generation where \( Q_0 \) is a constant. \( c_p \) is the specific heat at constant pressure, \( C \) is the fluid concentration, \( T_w \) is the temperature of the surface and \( D \) is the solute diffusion coefficient. \( k_1 \) is the variable rate of chemical conversion of the first-order irreversible reaction where \( k_1 = \frac{1}{2} k_0 e^{L} \), \( k_0 \) is a constant.

The boundary conditions are:

\[
\begin{align*}
u &= -V(x) = -V_0 e^{\frac{x}{L}}, \quad T = T_w(x) = T_0 + b e^{\frac{x}{L}}, \\
C &= C_w(x) = C_0 + m e^{\frac{x}{L}}, \quad \text{at} \quad y = 0, \\
\nu \to 0, \quad T = T_w(x) = T_0 + c e^{\frac{x}{L}}, \quad C = C_w(x) = C_0 + m e^{\frac{x}{L}}, \quad \text{as} \quad y \to \infty,
\end{align*}
\]

where \( U_0 \) is the reference velocity. \( V(x) > 0 \) is the suction velocity while \( V(x) < 0 \) is the blowing velocity. The initial strength of suction is denoted as \( V_0 > 0 \) and \( V_0 < 0 \) is the initial strength of blowing.

The mathematical analysis is done with the following the dimensionless similarity variables:

\[
\begin{align*}
\eta &= \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{L}} y, \quad u = U_0 e^{\frac{x}{L}} f'(
eta), \\
v &= -\sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{L}} \{ f(\eta) + \eta f'(\eta) \}, \quad \theta(\eta) = \frac{T - T_w}{T_w - T_0}, \quad \text{and} \quad \phi(\eta) = \frac{C - C_w}{C_w - C_0}.
\end{align*}
\]

Substituting (7) into the Eqs. (2), (3) and (4), the ordinary differential equations obtained are:

\[
\begin{align*}
f'''' + f'' - 2 f' - K f' - M f' &= 0, \\
\theta'' + Pr \left( \theta f' - \theta f' - (St)f' + f\theta' \right) &= 0, \\
\phi'' + Sc (f\phi' - \phi f' - (St_1)f' + \beta \phi) &= 0.
\end{align*}
\]

The transformed boundary conditions are:

\[
\begin{align*}
f' &= 1, \quad f = S, \quad \theta = 1 - St, \quad \phi = 1 - St_1, \quad \text{at} \quad \eta = 0, \\
f' &\to 0, \quad \theta \to 0, \quad \phi \to 0, \quad \text{as} \quad \eta \to \infty.
\end{align*}
\]

where the prime denotes differential with respect to \( \eta \), \( K = \frac{2L
u}{k*U_0} \) is porosity parameter, \( M = \frac{2\sigma_b^2 \nu L}{\rho U_0} \) is the magnetic parameter, \( Pr = \frac{\nu}{\alpha} \) is the Prandtl number, \( Q_H = \frac{2LQ_0}{U_0 \alpha \nu} \) is the heat source parameter,

\[
\begin{align*}
St &= \frac{c}{b} \quad \text{is the thermal stratified parameter}, \quad Sc = \frac{D}{\nu} \quad \text{is the Schmidt number}, \quad St_1 = \frac{n}{m} \quad \text{is the chemically}
\end{align*}
\]
stratified parameter and $\beta = \frac{k_\ell L}{U_0}$ is rate of reaction parameter. $S = \frac{V_0}{\sqrt{\nu U_0}}$ is the suction or blowing parameter. $S > 0$ is the suction parameter while $S < 0$ is the blowing parameter. The physical significances of the flow are the skin friction coefficient $C_f$, local Nusselt number $Nu$ and local Sherwood number $Sh$ defined as:

$$C_f = \frac{\tau_w}{\rho U^2/2}, \quad Nu = \frac{xq_w}{k(T_w - T_0)}, \quad Sh = \frac{xJ_w}{D(C_w - C_0)}$$ \hspace{1cm} (13)

where the wall shear stress $\tau_w$, the surface heat flux $q_w$ and the surface mass flux are given by:

$$\tau_w = \mu(\partial u/\partial r)_{r=0}, \quad q_w = -k(\partial T/\partial r)_{r=0}, \quad J_w = -D(\partial C/\partial r)_{r=0}$$ \hspace{1cm} (14)

Using the dimensionless variables in (7) we obtained :

$$f^*(0) = \frac{C_f}{2\sqrt{\Re \frac{L}{x}}}, \quad -\theta^*(0) = \frac{Nu(1-St)}{\sqrt{\Re \frac{L}{2}}}, \quad \text{and} \quad -\phi^*(0) = \frac{Sh(1-St)}{\sqrt{\Re \frac{L}{2}}}$$ \hspace{1cm} (15)

where $\Re = \frac{Ux}{\nu}$ is the local Reynolds number.

3. Results and discussion

To verify the validity of the result obtained, numerical results for the nusselt number $[-\theta'(0)]$ are compared to [6] for $K = 0, \quad M = 0, \quad Q_H = 0, \quad St = 0, \quad Sc = 0, \quad St_t = 0, \quad \beta = 0, \quad \text{and} \quad S = 0$. Table 1 shows the numerical values obtained in this study as compared to the values reported by [6], which are in good agreement.

| $Pr$ | [6] | Present Study |
|------|-----|---------------|
| 1    | 0.9547 | 0.9548 |
| 2    | 1.4714 | 1.4715 |
| 3    | 1.8691 | 1.8691 |

Numerical computation are performed for several values of the parameters and the obtained results are graphically plotted in Figures 1-15 so as to understand the physical aspect of the flow characteristics.

Figure 1 shows that an increase in the porosity parameter reduces the velocity of the fluid flow. The presence of porous medium causes more restriction to the fluid flow which decelerate the flow and thus decrease the velocity. Figure 2 and Figure 3 demonstrate that an increasing in porosity parameter, the temperature and concentration profile increase respectively. Figure 4 elucidates that velocity profile and it’s corresponding boundary layer thickness decreases due to an increase in the magnetic parameter. The presence of Lorentz force produces resistance to the transport, the rate of transport decreases as magnetic parameter increases. The flow of fluid becomes slow and fluid velocity decreases in boundary layer. Figure 5 and Figure 6 show both thermal and concentration boundary layer thickness increase as the magnetic parameter increases.
From Figure 7 it is observed that the thermal boundary layer thickness decreases with increasing Prandtl number. The ratio of momentum diffusivity to thermal diffusivity is represented by the Prandtl number. Fluids of higher Prandtl number have lower thermal conductivity and thinner thermal boundary layer structures. The temperature of the fluid decreases when Prandtl number increases number due to the increase in the rate of heat transfer. Figure 8 illustrates the temperature decreases when there is an increase in thermal stratification parameter. Figure 9 shows the increasing temperature profiles with increasing heat source parameter.
Figure 10 shows an increasing in Schmidt number leads to the thinning of the concentration boundary layer. Schmidt number signifies the ratio of momentum diffusivity to mass diffusivity. The mass diffusivity decreases due to increasing of Schmidt number which lead to the decreasing of the concentration boundary layer thickness. Figure 11 demonstrates the concentration boundary layer thickness decreases due to an increase in the rate of the reaction parameter. Influence of chemical stratification parameter on the concentration boundary layer is shown in Figure 12. It is noticed that the concentration boundary layer thickness decreases when the chemical stratification parameter increases.

Figure 7. Temperature Profiles with Prandtl Number, $Pr$

Figure 8. Temperature Profiles with Thermal Stratification Parameter, $St$

Figure 9. Temperature Profiles with Heat Source Parameter, $Q_{H}$

Figure 10. Concentration Profiles with Schmidt Number, $Sc$

Figure 11. Concentration Profiles with Rate of Reaction Parameter, $\beta$

Figure 12. Concentration Profiles with Chemical Stratification Parameter, $St_{i}$
Figures 13-15 demonstrate the influence of suction on the velocity, thermal and concentration in the boundary layer respectively. The effect of the suction parameter is to reduce the flow velocity, temperature and concentration distribution.

**Figure 13.** Velocity Profiles with Suction Parameter, $S$

**Figure 14.** Temperature Profiles with Suction Parameter, $S$

**Figure 15.** Concentration Profiles with Suction Parameter, $S$

4. Conclusions
This paper studied the MHD boundary layer flow over an exponentially permeable stretching sheet in thermally and chemically stratified porous medium with heat source. The transformed ordinary differential equations were solved numerically using MAPLE software. The results revealed that the velocity profiles decrease when the porosity, magnetic and suction parameters increase. The temperature profiles decrease as the Prandtl number, thermal stratification and suction parameters increase. The concentration profiles decrease when the Schmidt number, rate of reaction, chemical stratification and suction parameters increase.

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