Design and locomotion analysis of a novel cube mechanism with probabilistic rolling

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Abstract. This paper proposes a novel cube mechanism that is capable of probabilistic rolling locomotion. The mechanism consists of 8 vertex modules and 12 links. A kinematic model is presented, then the deformation and locomotion characteristics of the mechanism are demonstrated by kinematics analysis. So, we know that there are two probable orientations in the rolling locomotion. This stochastic process accords with Bernoulli experiment in ideal condition. The stability and the tumbling condition are discussed by the rolling locomotion analysis. The dynamics model is built and the locomotion simulation is done. The results show that the rolling locomotion can be realized with alternate two types of gaits which are advancing gait and turning gait. In both of advancing gait and turning gait, the orientations are probabilistic. A prototype is manufactured, the advancing and turning function is verified through the tests.

1. Introduction

Among the various locomotion modes of mobile robot, rolling is the one with higher efficiency. Lee et al. [1-3] proposed an icosahedral rolling mechanism, which uses modular vertices to form multiple closed rings in space, and achieves overall deformation by changing the length of the connecting rod, so that the robot rolls. Tadakuma et al. [4] proposed a projectile deformable tetrahedral robot, which can move in a narrow space through deformation, and it has the ability to switch motion modes and omnidirectional motion functions which is used for search and rescue and detection. Zagal et al. [5] proposed a deformable octahedral pipeline robot, which achieves deformation through the control of 12 edges by binary expansion, and realized continuous movement in the pipeline. Nozaki H et al. [6] proposed a moving spherical robot similar to the movement mode of amoeba, which can achieve continuous deformation and rolling with 32 telescopic legs and complete linear and curve movement.

At present, the polyhedral rolling robot can achieve rolling by changing its shape to change its center of gravity. Because the mechanism itself has many degrees of freedom, the control is complicated and the cost is high.

In this paper, a new novel cube mechanism is proposed. Due to the mechanism proposed is a kind of multi-loop overconstrained mechanism, so we can know that it is one degree-of-freedom [7-9]. The mechanism can realize deformation by scaling, and it can realize rolling locomotion by kinematics and rolling locomotion analysis. By the locomotion simulation, we know that the mechanism has two types of rolling gait: advancing gait and turning gait, and in both of them, the orientations are probabilistic. A prototype is manufactured, and the advancing and turning function is verified through the tests.
2. Mechanism description
A novel cube mechanism with probable rolling, as shown in figure 1, composes of 8 vertex modules (denoted as A, B, ..., and H, respectively) and 12 links (denoted as 1, 2, ..., and 12, respectively). Figure 1(a) shows the schematic diagram of the mechanism. figure 1(b) shows the 3D model.

As shown in figure 2, the revolute joints \(R_1, R_2, R_3\) in each vertex module are coplanar and 60° to each other, and the revolute joints \(R_2, R_4\) at the ends of each link are parallel to each other.

If each vertex module is simplified into one point respectively, as shown in figure 3, we get a geometric model denoted as \(ABCD-EFGH\). Take vertex module \(E\) for example, the three axes of joints \(R_{E1}, R_{E2}, \) and \(R_{E3}\) are parallel to line \(AH, AF,\) and \(FH\), respectively. Then the angle between vertex module \(E\) and the plane \(EFGH\) is \(\alpha\). According to the geometry of a cube, we know that \(\cos\alpha = \sqrt{6}/3\). For the mechanism, the angles between each vertex module and the plane that formed by any two links connected to it are equal to \(\alpha\).

3. Kinematics analysis
As shown in figure 4, the origin \(O\) of the coordinate system is set at the center of squerae \(EFGH\), x-axis passes through point \(F\), y-axis passes through point \(G\), and z-axis is determined by the right-handed helix rule. As the mechanism is symmetrical, it is projected to the xOz plane. The projection of point \(A\)
and $C$ is point $P$, which is the center of square $ABCD$, and the projection of point $E$ and $G$ is point $O$, so the mechanism is equivalent to a planar 6-bar linkage for analysis.

![Figure 4](image4.png)

**Figure 4.** Projection diagram.

It is equivalent to place the drive motor on any one of the revolute joints. Here, the drive motor is arranged on $R_{B1}$, as shown in figure 5. So, $\theta$ is the driving angle. In order to ensure that the distribution of mass is symmetrical, the counterweights of the same mass as the driving motor are placed on the other seven vertex modules.

![Figure 5](image5.png)

**Figure 5.** The arrangement of the driving angle.

Define that, at the initial state of locomotion, the three links connected in each vertex modules are perpendicular to each other. When the drive motor is controlled to run, the planar 6-bar linkage is deformed into the shape shown in figure 6. Then the drive motor is stopped. Now, the planar 6-bar linkage lands on only point $O$. So it is going to rotate around point $O$. At this moment, there two probable locomotion orientations (clockwise and counterclockwise). This stochastic process accords with Bernoulli experiment in ideal condition. Suppose, the planar 6-bar linkage rotates counterclockwise, so link $FO$ lands.

Define that, $\beta$ is $\angle POP$, $\phi$ is $\angle PFH$, $\delta$ is the angle between line $OD$ and $x$-axis, $a$ is the length of the sides of the cube.

![Figure 6](image6.png)

**Figure 6.** Kinematics model.

Then we can figure out the coordinates of $F$, $H$, $D$, $P$ and $B$ in the $xOz$ plane as follows:
According to the geometric characteristics of the planar 6-bar linkage, we can figure out the coordinates of F′, H′, D′, P′ and B′ as follows:

\[
[r_f] = \begin{bmatrix}
\frac{\sqrt{3}}{2}a\cos\theta \\
\frac{1}{2}a - \frac{\sqrt{6}}{4}a\cos\theta \\
\end{bmatrix}
\]

\[
[r_h] = \begin{bmatrix}
\frac{\sqrt{3}}{2}a\cos\theta \\
\frac{1}{2}a - \frac{\sqrt{6}}{4}a\cos\theta \\
\end{bmatrix}
\]

\[
[r_d] = \begin{bmatrix}
a\sqrt{\frac{3}{4} - \frac{9}{8}\cos^2\theta + \frac{\sqrt{6}}{4}\cos\theta} \\
\ centrifugal}
\]

\[
[r_p] = \begin{bmatrix}
0 \\
\frac{1}{2}a + \frac{\sqrt{6}}{4}a\cos\theta \\
\end{bmatrix}
\]

\[
[r_b] = \begin{bmatrix}
\frac{1}{2}a + \frac{\sqrt{6}}{4}a\cos\theta \\
\end{bmatrix}
\]

Among them, (x_F, z_F), (x_H, z_H), (x_P, z_P) and (x_D, z_D) refer to equation (1), besides:

\[
\sin\beta = \frac{z_F}{\sqrt{x_F^2 + z_F^2}}
\]

\[
\cos\beta = \frac{x_F}{\sqrt{x_F^2 + z_F^2}}
\]

\[
\cos\phi = \frac{\sqrt{3}}{3}
\]

\[
\sin\phi = \frac{\sqrt{6}}{3}
\]

\[
\cos\delta = \frac{x_D}{\sqrt{x_D^2 + z_D^2}}
\]
\[
\sin \delta = \frac{z_0}{\sqrt{x_0^2 + z_0^2}}
\]

4. Rolling locomotion analysis

The condition that the mechanism can realise rolling locomotion is the Zero moment point (ZMP) falls outside the support zone. Equation (4) is used to calculate the ZMP [10]. Since the projection of ZMP onto the \( y \)-axis (denoted as \( y_{ZMP} \)) is always square to 0, we only figure out the projection of ZMP onto the \( x \)-axis (denoted as \( x_{ZMP} \)).

\[
x_{ZMP} = \frac{\sum_{i=1}^{n} m_i (z_i + g_z) x_i - \sum_{i=1}^{n} m_i \ddot{z}_i}{\sum_{i=1}^{n} m_i (z_i + g_z)}
\]

(4)

Where \( n \) is the number of links, \( m_i \) is the mass of link \( i \), \( x_i \) and \( z_i \) are the coordinates of the center of mass of link \( i \), \( \ddot{x}_i \) and \( \ddot{z}_i \) are the accelerations along the \( x \)-axis and \( z \)-axis respectively, and \( g_z \) is the acceleration due to the gravity. As shown in figure 7, \( r_i \) \((i=1, 2, \ldots, 7)\) is the center of mass of link \( i \).

![Figure 7. Rolling locomotion analysis.](image)

Note that \( r_i \) can be determined by the coordinates shown in equation (2). Then, take the quadratic derivative of \( r_i \) with respect to time \( t \) and add it into equation (4) to obtain:

\[
x_{ZMP} = \frac{dp - 6d\dot{u} \sin \gamma}{8dv + 12g_z}
\]

(6)

Among them,

\[
d = a \cos(\arcsin(\cos(\theta/2)/\sqrt{2}))
\]

\[
y = a \cos((3\cos(\theta-1)/(3-\cos(\theta))
\]

\[
u = (8 \dot{\theta}^2 (3\cos(\theta-1) + (\dot{\theta}(3-\cos(\theta)-2\sqrt{2} \dot{\theta}^2 \sin(\theta) \times (3-\cos(\theta)) \times \sin(\gamma))/(3-\cos(\theta))^3
\]

\[
p = -5d \cos(\gamma-6g_z \cos(\gamma-d^2 \cos(\gamma-2dg_z \cos(\gamma+4dv+6g_z)
\]

(7)

According to the following calculation example, the condition that the mechanism can realise rolling locomotion is discussed. Substitute the structural parameters listed in table 1 into equation (6), and we can figure out \( x_{ZMP} \) shown in figure 8.

|   |   |   |   |
|---|---|---|---|
| parameter | \( m \) / kg | \( a \) / m | \( g_z \) / kg/s\(^2\) |
| value     | 1  | 0.1 | 9.8 |
Figure 8. The change in $x_{ZMP}$.

When $\omega=0$, $x_{ZMP}$ is always within the critical value of support zone, and the mechanism cannot realize rolling. When $\omega=\pi \text{rad/s}$, the mechanism can realize rolling when $\theta=1.37 \text{rad/s}$; When $\omega=2\pi \text{rad/s}$, the mechanism can realize rolling when $\theta=1.06 \text{rad/s}$. As a result, the mechanism can realize rolling locomotion, and with the increase of $\omega$, it will be more prone to rolling.

5. Locomotion simulation

As shown in figure 9, ADAMS$^\text{TM}$ dynamic model is established and locomotion simulation is carried out. Figure 9(a) shows the initial state. The blue square and the yellow square indicates the starting position and the ending position of the motion, respectively. As shown in figure 9(b), the drive motor is controlled to rotate in positive direction and link $F$ and $H$ are off the land. As shown in figure 9(c), the mechanism rolls around the axis of link $E$ and $G$, and link $F$ lands. As shown in figure 9(d), the mechanism keeps rolling around link $F$ in the same direction as before until link $B$ land. In this moment, link $E$ and $G$ are off the land.

As shown in figure 9(e), link $G$ lands, causing the mechanism to turn. Then the drive motor is controlled to rotate in reverse and $\theta$ is decreased to $\theta_0$, as shown in figure 9(f), so the mechanism is deformed into the initial state. At this time, link $B$, $C$, $G$ and $F$ land. Figure 9(a)-(f) show a motion cycle. Similarly, figure 9(f)-(k) show the next motion cycle of the mechanism. After the third motion cycle, as shown in figure 9(k)-(p), the mechanism moves to the ending position.
By definition, the gait which the mechanism rolling along the normal direction of two vertex modules is probable advancing gait in a motion cycle, as shown in figure 9(b)-(d); the gait which the mechanism rolling along one link is probable turning gait, as shown in figure 9(e), figure 9(j) and figure 9(o). The drive motor is controlled to rotate in reverse and $\theta$ is decreased to $\theta_0$, as shown in figure 9(f), so the mechanism is deformed into the initial state.

6. Prototype and tests
Make a prototype, as shown in figure 10. The overall size of the prototype is $350\text{mm} \times 350\text{mm} \times 350\text{mm}$, the overall mass of the prototype is 2.2kg.
The deformation test of the prototype is carried out, shown in figure 11(a)-(d). As shown in figure 12, the rolling locomotion test of the prototype is carried out. The position of the blue ball and the yellow one marks the starting point and the ending point, respectively.

Figure 10. The prototype.

Figure 11. Deformation test.

Figure 12. The rolling locomotion test.

Figure 12(a) shows the initial state. Figure 12(b)-(d) shows the process of the mechanism's first probabilistic rolling locomotion, and figure 12(e)-(g) shows the second probabilistic rolling
locomotion; Figure 12(h)-(j) shows the third probabilistic rolling locomotion. The gaits of the rolling locomotion test are basically the same as those of the locomotion simulation.

7. Conclusions
This paper proposed a novel cube mechanism which could realize probabilistic rolling locomotion. By kinematics analysis and locomotion simulation, the mechanism could roll in two probable orientations in probable advancing gait and probable turning gait. A prototype was made and tests were carried out to verify the locomotion performance. This probabilistic locomotion mode has potential application value in demining, territorial exploration, military reconnaissance and other fields.

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References
[1] Lee W H and Sanderson A C 2000 Proc. Int. Conf. on Robotics and Automation (San Francisco CA: USA/American Elsevier) p 2840
[2] Lee W H and Sanderson A C 2000 Proc. Int. Conf. on Intelligent Robots and Systems (Takamatsu: Japan/American Elsevier) p 2178
[3] Lee W H and Sanderson A C 2002 Robotics and Automation 18 32
[4] Tadakuma K, Nagatani K, et al 2009 Proc. Int. Conf. on Intelligent Robots and Systems (St. Louis: USA/American Elsevier) p 2801
[5] Zagal J C, Armstrong C and Li S 2012 Artificial life 13 (Cambridge: MIT Press) p 109
[6] Nozaki H, Kujirai Y, Niiyama R, et al 2018 Proc. Int. Conf. on Intelligent Robots and Systems (San Francisco, CA: USA/American Elsevier) p 2721
[7] Li R M, Yao Y A and Kong X W 2016 Mech. Mach. Theory 105 09
[8] Wei G W and Dai J S 2014 J. Mech. Rob. 6 021010
[9] Wohlhart H C 1999 Proc. 10th. Int. Conf. on the theory of machines and mechanisms (Oulu: Finland/American Elsevier) p 683
[10] Takanishi A, Tochizawa M, Takeya T, Karaki H and Kato I 1989 Proc. 4th. Int. Conf. on Intelligent Robots and Systems (Columbus: USA/American Elsevier) p 299