Observational challenges in Ly\textalpha{} intensity mapping

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ABSTRACT
Intensity mapping (IM) is sensitive to the cumulative line emission of galaxies. As such it represents a promising technique for statistical studies of galaxies fainter than the limiting magnitude of traditional galaxy surveys. The strong hydrogen Ly\textalpha{} line is the primary target for such an experiment, as its intensity is linked to star formation activity and the physical state of the interstellar (ISM) and intergalactic (IGM) medium. However, to extract the meaningful information one has to solve the confusion problems caused by interloping lines from foreground galaxies. We discuss here the challenges for a Ly\textalpha{} IM experiment targeting $z > 4$ sources. We find that the Ly\textalpha{} power spectrum can be in principle easily (marginally) obtained with a 40 cm space telescope in a few days of observing time up to $z \sim 8$ ($z \sim 10$) assuming that the interloping lines (e.g. H\textalpha{}, [O ii], [O iii] lines) can be efficiently removed. We show that interlopers can be removed by using an ancillary photometric galaxy survey with limiting AB mag $\sim 26$ in the NIR bands (Y, J, H, or K). This would enable detection of the Ly\textalpha{} signal from $5 < z < 9$ faint sources. However, if a [C ii] IM experiment is feasible, by cross-correlating the Ly\textalpha{} with the [C ii] signal the required depth of the galaxy survey can be decreased to AB mag $\sim 24$. This would bring the detection at reach of future facilities working in close synergy.

Key words: cosmology: observations - intergalactic and interstellar medium - intensity mapping - large-scale structure of universe

1 INTRODUCTION
One of the key open problems in cosmology is the origin and evolution of galaxies and their stars. In the last decade astonishing technological progresses have allowed to probe galaxies located within less than one billion year from the Big Bang (Bouwens et al. 2014a; Oesch et al. 2014, 2015; Ouchi et al. 2010, 2008; Matthee et al. 2015). These searches reveal an early Universe in which complex phenomena were simultaneously taking place, ranging from the formation of supermassive black holes (Volonteri & Bellovary 2012) to the reionization process (Barkana & Loeb 2001), along with the metal enrichment by the first stars (Ferrara 2016).

High redshift sources are very faint and their detection is remarkably challenging: up to now, less than 1000 galaxies have been detected at $z \sim 8$, and among them only a handful are at $z \sim 10$ (e.g. Bouwens et al. 2014b; McLeod et al. 2016; Calvi et al. 2016). Moreover, it is believed that low-mass galaxies have a dominant role in driving reionization, while the most-luminous ones appear to be only rare outliers. Such ultra-faint galaxies are likely to remain undetected even by the next generation observatories, such as JWST\textsuperscript{1}, TMT\textsuperscript{2} or E-ELT\textsuperscript{3}.

A novel approach has been proposed to overcome the problem and study, at least statistically, the early faint galaxy population. Basically theidea is to trade the ability to resolve individual sources, with a statistical analysis of the cumulative signal produced by the entire population (Kashlinsky 2005; Cooray 2016). Intensity mapping (IM, see e.g. Visbal & Loeb 2010; Visbal et al. 2015) is one implementation of such concept and aims at detecting 3D large scale emission line fluctuations. In the last years this concept has become very popular and several lines have been proposed as candidates. Among these are the HI 21cm (Furlanetto et al. 2006), CO (Lidz et al. 2011; Rigby et al. 2008; Breysse et al. 2014), C ii (Gong et al. 2012; Silva et al. 2014; Yue et al. 2015), H$_2$ (Gong et al. 2013), HeII (Visbal et al. 2015) and Ly$\alpha$ (Pullen et al. 2014; Silva et al. 2013; Comaschi & Ferrara 2016) emission lines.

Although IM experiments seem indeed promising, their

\textsuperscript{1}http://www.jwst.nasa.gov
\textsuperscript{2}http://www.tmt.org
\textsuperscript{3}https://www.eso.org/sci/facilities/eelt/
reliability has not yet been convincingly demonstrated. In particular, continuum foregrounds dominate over line intensity by several orders of magnitude: cleaning algorithms have been developed for 21cm radiation \cite{Wang2006,Chapman2015,Wolz2015}, but not comparably well understood for other lines (Yue2013). Moreover, some lines (such as Ly\(\alpha\)) lines suffer from line confusion: for example the H\(\alpha\) line (\(\lambda_H\alpha = 6563\ \mu m\)) if emitted at \(z = 0.48\) can be misclassified as a Ly\(\alpha\) line emitted at \(z = 7\) (Gong et al. 2014). We will refer to such interfering sources as interlopers.

Considering that the first generation of instruments devoted to IM are starting to be proposed or funded \cite{Dore2014,Cooray2016,Crites2014}, it is essential to gain a deeper understanding of the difficulties implied by an IM experiment. This forms the motivation of this work and we will pay particular attention to the Ly\(\alpha\) emission line which is the most luminous UV line and one of the most promising candidates for an IM survey in the near infrared (NIR) spectral region.

Ly\(\alpha\) emission is associated with UV and ionizing radiation and therefore is strongly correlated with the star formation rate (SFR) in galaxies. Moreover, the reprocessing of UV photons by neutral hydrogen in the IGM also produces Ly\(\alpha\) photons. Some recent works have predicted the power spectrum (PS) of the target line and assessed its observability. \cite{Pullen2014} and \cite{Silva2013} developed analytical models for the Ly\(\alpha\) PS and showed that it is at reach of a small space instrument. \cite{Gong2014} used the model developed by \cite{Silva2013} to study the problem of line confusion, finding that masking bright voxels can represent a viable strategy. In a similar attempt, \cite{Breysse2015} pointed out that masking bright voxels is an effective strategy for the removal of the interlopers, but it might jeopardize the recovered line PS, causing loss of astrophysical information.

A realistic Ly\(\alpha\) model has to deal with all the astrophysics processes (e.g. star formation, radiative transfer) self-consistently. This is rather challenging even for high resolution hydrodynamic simulations. Alternatively, a viable strategy for studying such complex processes is to develop an analytical model that includes all the theoretical uncertainties represented by a few parameters: in this way it is possible to understand easily how the results depend on the unknowns and what is the available parameter space of the problem yielding solution compatible with existing observations. \cite{Comaschi2016} (hereafter CF16) developed an analytical model for diffuse Ly\(\alpha\) intensity and its PS, with a focus on IM at the epoch of reionization (EoR). The model is observation-driven and it includes the most recent determinations both for galaxies and IGM. They associated dust-corrected UV luminosity to dark matter halos by the abundance matching technique \cite{Conroy2009,Behroozi2010,Yale2004}, using the LF from the Hubble legacy fields \cite{Bouwens2013}, and the UV luminosity spectral slope in \cite{Bouwens2014}. Then using a template spectral energy distribution (SED) from \textsc{starburst99} \cite{Leitherer1999,Vazquez2005}, they were able to model self-consistently the interaction of ionizing photons with the interstellar medium (ISM) and the IGM, calibrating the poorly constrained parameters in order to have a realistic reionization history \cite{PlanckCollaboration2015}.\cite{Fan2009}

CF16 found that for Ly\(\alpha\) absolute intensity is dominated by recombinations in ISM, and Lyman continuum absorption and relaxation in the IGM, with the latter being about a factor 2 stronger. However, intensity fluctuations are mostly contributed by the ISM emission on all scales \(< 100\ h^{-1}\text{Mpc}\). Such scale essentially corresponds to the distance at which UV photons emitted by galaxies are redshifted into Ly\(\alpha\) resonance.

We present in the following a feasibility study of a Ly\(\alpha\) IM survey based on CF16 results. In particular, we tackle the problem of (i) required sensitivity; (ii) suppression of line confusion through interlopers removal; (iii) detectability of the cross-correlation with the C \(\text{II}\) line. The paper is organized as follows: in Sec. 2 we compute in a general way the signal-to-noise ratio (S/N) of an IM observation; in Sec. 3 we model the sensitivity of an intensity mapper and compute the S/N of an observation; in Sec. 4 we analyse the problem of line confusion. Sec. 5 contains a study of the cross-correlation between Ly\(\alpha\) and C \(\text{II}\) emission and of the S/N of a realistic observation.

2 SIGNAL POWER SPECTRUM

In this Section we derive the PS (auto-correlation PS and cross-correlation PS) of the measured intensity fluctuations and its variance, with an approach similar to \cite{Visbal2010}. For simplicity we assume that the detected intensity includes three components: (i) the signal; (ii) the instrumental white noise; (iii) the interloping lines which are redshifted to the same frequency as the signal line, namely:

\[ I(\Omega, \nu) = I_s(\Omega, \nu) + I_N + \sum I_f(\Omega, \nu). \] (1)

Throughout work we will neglect the possible presence of continuum foregrounds, assuming that they can be easily removed thanks to the smoothness of the frequency spectrum \cite{Wang2006,Chapman2015,Wolz2015}.

Note that comoving coordinates are related to angle and frequency displacement from an arbitrary origin, \(x^0\), as follows:

\[ x_1, x_2 = \chi(z_o) \Delta \theta + x_1^0, x_2^0 \] (2)

\[ x_3 = \frac{\Delta \nu}{d\nu} \] (3)

where \(\chi(z_o)\) is the comoving distance from the observer to the signal, \(d\chi/d\nu = c(1+z_o)|H(z_o)|^{-1}\), \(\Delta \theta, \Delta \nu\) are the displacements in angle and frequency from the origin \(x^0\) (center of the survey). In this process a subtlety arises \cite{Visbal2010} because \(I_f\) is not emitted

\footnote{We assume a flat $\Lambda$CDM cosmology compatible with the latest Planck results: $h = 0.677$, $\Omega_m = 0.31$, $\Omega_b = 0.049$, $\Omega_\Lambda = 1 - \Omega_m$, $n = 0.97$, $s_8 = 0.82$ (Planck Collaboration et al. 2015).}
at \( z_0 \). Therefore, in that term we should consider deviations that are the projection at \( z_0 \) of the real coordinates at \( z_i \):

\[
I(x) = I_o(x, z_0) + I_N + \sum_i I_f \left( \frac{\chi(z_i)}{\chi(z_0)} \right)^2 \frac{(1 + z_0)H(z_0)}{(1 + z_i)H(z_i) - z_i} \tag{4}
\]

where \( 1 + z_i = (1 + z_0)\lambda_0/\lambda_i \).

When considering the Fourier transform of the fluctuations, this projection introduces (i) a global extra factor that multiplies the PS; (ii) anisotropies due to the different projection of modes along and across the line of sight; (iii) a loss of correspondence between comoving and observed k-modes:

\[
\delta I(k) = 7_o(b) \sigma_{\alpha}^2 + \delta_k^N + \sum_i C(z_i) T_f(b) \delta_{k_i}(k) \tag{5}
\]

where \( T \) and \( \langle b \rangle \) with each subscript are the mean intensity and halo luminosity weighted mean bias of each line; \( \delta_k^N \) is the instrumental noise (see Sec. [3]). The global extra factor is

\[
C(z_i) = \left( \frac{\chi(z_0)}{\chi(z_i)} \right)^2 \frac{(1 + z_0)H(z_0)}{(1 + z_i)H(z_i)} \tag{6}
\]

\[
\mathbf{k}'(k) = \left( k_1 \frac{\chi(z_0)}{\chi(z_i)} , k_2 \frac{\chi(z_0)}{\chi(z_i)}, k_3 \frac{(1 + z_0)H(z_0)}{(1 + z_i)H(z_i)} \right) \tag{7}
\]

From the above equations, the PS of the measured intensity fluctuations becomes

\[
P(k) = \langle \delta I(k) \delta I^*(k) \rangle = P_o(k) + P_N + \sum_i P_f \tag{8}
\]

where

\[
P_o(k) = \mathcal{T}_o(b) \sigma_{\alpha}^2 \rho_{\text{dm}}(k, z_0),
\]

\[
P_N = \langle \delta \mathcal{N} \delta \mathcal{N} \rangle,
\]

\[
P_f(k) = C(z_i) \mathcal{T}_f(b) \sigma_{\alpha}^2 \rho_{\text{dm}}(k', z_i),
\]

in deriving the last line the relation \( \langle \delta k | \delta k' \rangle = C^{-1} \rho_{\text{dm}}(k') \delta(k - k') \) is used and \( \rho_{\text{dm}} \) is the dark matter PS. The noise component \( P_N \) is well known and easily subtracted; the interlopers power spectrum, \( P_f(k) \), is however unknown and yet must be removed in order to extract the astrophysical PS signal.

The variance of \( P(k) \) is

\[
\sigma^2_P(k) = \delta P^2(k) = \langle (\delta I(k) \delta I^*(k))^2 \rangle - \langle \delta I(k) \delta I^*(k) \rangle^2 \tag{9}
\]

Using that fact that noise and interloping lines only correlate with themselves, and that \( \langle \delta k_i \rangle^2 = 2 \sigma_{\alpha}^2 \) and \( \langle \delta \mathcal{N} \rangle^2 = 2 P_N \), it is easy to prove (see Appendix [A] for the full calculation)

\[
\sigma^2_P(k) = \left[ P_o(k) + P_N + \sum_i P_f(k) \right]^2. \tag{10}
\]

From this equation we can see that the variance depends strongly on the detector noise and on the PS of the interlapping lines.

In case the PS is isotropic, \( P(k) = P(k) \), several independent modes can be combined to reduce the PS variance at given k:

\[
P(k) = \left( \sum_k \frac{1}{\sigma^2_P(k)} \right)^{-1}\sum_k P(k) \sigma^2_P(k). \tag{11}
\]

Lyor intensity mapping

In order to estimate the S/N we have to consider the PS variance due to the finite survey volume and resolution. In this case the power k-modes are discrete and multiples of \( (2\pi/L_1, 2\pi/L_2, 2\pi/L_3) \), where \( L_1, L_2, L_3 \) are the dimensions of the survey volume. Suppose the survey has a resolution \( l_\parallel \) and \( l_\perp \) along and perpendicular to the line-of-sight (generally \( l_\parallel \gg l_\perp \), respectively). Then only modes satisfying \( 2\pi/L_1, 2\pi/L_2 < k_1, k_2 < 2\pi/L_1 \) and \( 2\pi/L_3 < k_3 < 2\pi/l_\perp \) are accounted.

Sometimes it is useful to estimate the total PS variance and S/N for all modes with \( k_{\min} < k < k_{\max} \) (Pullen et al. 2014):

\[
\langle \sigma^2_P \rangle = \left( \int \frac{dk}{\Delta k^2} \frac{1}{\sigma^2_P(k)} \right)^{-1}; \tag{12}
\]

\[
\langle (S/N)^2 \rangle = \left( \int \frac{dk}{\Delta k^3} \frac{P(k)}{\sigma^2_P(k)} \right)^2, \tag{13}
\]

\[
\delta X^2 = (2\pi)^3/V_S \text{ is the k-space volume occupied by each discrete mode and the integral is over all wavenumbers with } k_{\min} < k < k_{\max}, k_1, k_2 < 2\pi/L_1 \text{ and } k_3 < 2\pi/l_\perp.
\]

The contamination in the auto-correlation PS (Eq. [5]) could be suppressed by cross-correlating different measurements targeting two different signals, \( \alpha \) and \( \beta \), that are contaminated by uncorrelated interloping lines (Visbal & Loeb 2010). The cross-correlation PS is (Visbal & Loeb 2010):

\[
P_{\alpha, \beta}(k) = \langle I_o(b) \rangle \langle I_o(b) \rangle \delta_l \rho_{\text{dm}}(k), \tag{14}
\]

where only the signal term is left as noise and interloping terms are uncorrelated for \( \alpha \) and \( \beta \). Nevertheless, noise and interloping lines increase the variance:

\[
\sigma^2_{P_{\alpha, \beta}} \equiv \frac{1}{2} \left[ P_{\alpha, \beta}^2 + \left( P_{\alpha} + P_{N,1} + \sum_i P_{f,1} \right) \left( P_{\beta} + P_{N,2} + \sum_i P_{f,2} \right) \right], \tag{15}
\]

where the subscripts 1, 2 represent the qualities in the two measurements respectively. We will apply this suppression method to our model and discuss more specific details in Sec. [I].

3 LINE DETECTABILITY

We start by assessing first the detectability of the Ly\( \alpha \) PS without considering the interloping contamination. Our discussions are based on different setup parameters of a small space telescope that can map efficiently a large sky area in the visible (corresponding to \( 2.2 < z_\alpha < 4.8 \)) and NIR (\( z > 4.8 \)) spectral bands. We do not aim at proposing an optimal setup of such instrument, but rather at understanding to what extent the Ly\( \alpha \) IM is a viable tool for studying high-z galaxies.

The size of the voxel is one of the most relevant factors for detectability. The voxel size along the line-of-sight is given by \( l_\parallel = \frac{dV}{dE} \Delta z = \frac{dV}{dE} \frac{z}{(1+z)^2} \); in the perpendicular direction it is instead \( l_\perp = \frac{\chi(z)}{\theta_{\min}} \). As such, it depends on the spectral resolution, \( R \), and angular resolution, \( \theta_{\min} \), of the telescope. The choice of an optimal \( l_\parallel \) and \( l_\perp \) is crucial:
a small voxel results in a smaller volume loss following interlopers removal but requires a longer time to complete a survey for a given area; large voxels suffer from the opposite problem. Moreover, as we will discuss in Sec. 3.2 there are additional limitations imposed by the ancillary imaging-survey used to identify the interlopers. The latter sets the minimum voxel size to the precision of the redshift measurement (i.e. typically \( 0.05(1 + z) \) for photometric surveys) along the line-of-sight. It is necessary to find a balanced choice that is specific to the IM experiment configuration and goals.

Our fiducial instrument has a \( \theta_{\text{min}} = 6 \) arcsec beam FWHM (full width at half maximum), a spectrometer with resolution \( R = \lambda / \Delta \lambda = 100 \) and a survey area of 250 deg\(^2\) (Pullen et al. 2014) Silva et al. 2013 Doré et al. 2014 Cooray et al. 2016). Therefore the sample space has voxels with \( \Delta l _ { l } = 35.3 \) and 28.1 Mpc, and \( \Delta l _ { \perp } = 214 \) and 257 kpc, for \( z = 4 \) and 7 respectively.

In our setup, the voxel size is always larger than the galaxy correlation length (typically \( \approx 1 \) Mpc\(^3\)); therefore we expect that each voxel contains several galaxies. Also, as \( \Delta l _ { l } \gg \Delta l _ { \perp } \) (typically \( \approx 20 - 30 \) Mpc vs. \( \approx 200 - 300 \) kpc), only transversal modes contribute to the PS measurement at \( k > 0.1 \) hMpc\(^{-1}\).

Another relevant crucial point is the instrumental noise. A space telescope is usually background limited, i.e. the noise level is set by Poisson fluctuations of the background light. For Ly\( \alpha \) observations the most important background is the Zodiacal Light (ZL). In this case, \( \sigma _ { \text{N}} \approx 1.37 \epsilon ^{-1/2} \mu \text{Wm}^{-2} \text{sr}^{-1} \times \left[ \frac{\mu \text{m}}{\lambda} \right] \frac{ R }{ 10^3 \text{nWm}^{-2} \text{sr}^{-1}} \frac{ I _ { \text{ZL}} }{ \pi D _ { \text{obs}}^2 } \frac{ 0.126 \text{m}^2 }{ 8.5 \times 10^{10} \text{sr}^{-1} } \frac{ 10 \text{s}}{ \Omega _ { \text{pix}} } \frac{ h^5 }{ \text{obs} } \right]^{1/2}, \) \( \text{(16)} \)

where \( I _ { \text{ZL}} = \nu I _ { \nu } \approx 2 \times 10^{-2} \text{nWm}^{-2} \text{sr}^{-1} \) is the typical ZL flux in the relevant frequency range (Kashlinsky 2005 Doré et al. 2014). An efficiency \( \epsilon \) is added to account for the photons loss by mirrors and integral field unit (IFU), here assumed conservatively to be \( \epsilon = 0.25 \).

### 3.1 Power spectrum observations

The first generation of intensity mappers are likely to have a limited S/N that will allow only to probe EoR Ly\( \alpha \) fluctuations power-spectrum. In this Section we discuss an instrument designed for this aim. Nevertheless in the future more powerful instruments could undertake tomographic observations and we will discuss such possibility in Sec. 3.2.

The PS of the instrumental noise is \( P _ { \text{N}}(z) = \sigma _ { \text{N}}^2 V _ { \text{pix}}, \) \( \text{(17)} \)

where \( \sigma _ { \text{N}} \) is from Eq. (16) and \( V _ { \text{pix}} \) is the comoving voxel volume. We then compute the S/N using \( \sigma _ { k}^2 = (P _ { o} + P _ { N})^2 \) and Eq. (13) (we use \( \Delta z = 1 \) for the survey volume \( V _ s \) and divide \( k \)-space in \( k \)-bins with \( \Delta k = 1.2 k \)).

6 We ignore dark current and readout noise as they depend strongly on survey implementation; this approximation is safe at least for instruments similar to SPHEREx (M. Zemcov, private communication).

7 This survey set-up is rather conservative; a deeper survey should be possible.
us to find the best observational strategy for a PS observation: given a fixed total observing time we want to find the optimal exposure time per pointing. Considering only the instrumental noise, from Eq. (13), (16) and (17) we have

\[
(S/N)^2 \propto \frac{A_{\text{surv}}}{\sigma_N^2}, \tag{18}
\]

Since \(A_{\text{surv}} \propto t_{\text{obs}}^{-1}\) and \(\sigma_N^2 \propto t_{\text{obs}}^{-1}\), the S/N does not depend on the depth of the survey as long as the cosmic variance term negligibly appears in Eq. (10). In other words the best strategy for an IM experiment is to carry out a shallow, however large area survey.

In practice, though, the optimal \(t_{\text{obs}}\) is set by the technical implementation of the survey, which should take into account the following limitations: (i) \(t_{\text{obs}}\) cannot be shorter than, or even comparable to, the instrumental pointing time; (ii) with a large survey area it is impossible to avoid sky regions with higher foregrounds; (iii) as we will discuss in Sec. 4 the IM survey might need deep ancillary galaxy surveys for interloper removal, and therefore the data available for final analysis is limited to the overlapping sky regions.

### 3.2 Tomography

Alternatively, an IM experiment allows us to make tomographic maps of the Ly\(\alpha\) intensity, although only the low-z part of the signal is accessible to fiducial space telescope design introduced above.

In CF16 we found that the mean Ly\(\alpha\) intensity at \(z = 4\) is \(I_{\alpha} \approx 0.1\) nW m\(^{-2}\) sr\(^{-1}\). At the same redshift the dark matter field has a fluctuations level of \(\sigma_N \approx 0.23\) on 10 Mpc scales, and the mean Ly\(\alpha\) bias \(b_{\alpha} \approx 3\). Therefore if the survey has voxels of volume \((10\, \text{Mpc})^3\), corresponding to \(R \approx 350\) and \(\Delta \theta_{\text{pix}} = 4.7\text{arcmin at } z = 4\), the 1\(\sigma\) Ly\(\alpha\) fluctuations level is

\[
\sigma_N = I_{\alpha} b_{\alpha} \sigma_{\text{dm}} \approx 0.07 \text{ nW/m}^2/\text{sr}, \tag{19}
\]

which is larger than the noise level \(\sigma_N \approx 0.04\) nW m\(^{-2}\) sr\(^{-1}\) in Eq. (16) for \(t_{\text{obs}} = 10^6\) s. Therefore even this small intensity mapper can observe directly the spatial fluctuations of Ly\(\alpha\) emission from low redshift galaxies, although with a modest S/N.

The tomographic observation of the Ly\(\alpha\) signal from the EoR is more challenging, as the Ly\(\alpha\) intensity drops by one order of magnitude. Thus a tomographic map of the EoR signal requires a more powerful instrument. Fig. 4 shows the S/N = \(\sigma_N/\sigma_N\) as a function of \(z\) and \(t_{\text{obs}}\) for a 2 m space telescope and same voxels of \((10\, \text{Mpc})^3\) volume. The observation requires an integration time of at least few months and even so it will be only feasible for the late stages of the EoR. The experiment can be even more challenging once the confusion by interloping lines such as H\(\alpha\) and [O \(\text{iii}\)] that dominate over Ly\(\alpha\) emission are accounted for.

### 4 INTERLOPING LINES

Low redshift emission lines could significantly contribute to the observed intensity fluctuations (see Eq. (8)). Particularly important for Ly\(\alpha\) experiments are the H\(\alpha\) (0.6563 \(\mu\)m), [O \(\text{iii}\)] (0.5007 \(\mu\)m) and [O \(\text{ii}\)] (0.3727 \(\mu\)m) (Gong et al. 2014, Pullen et al. 2014) lines. Their power spectra may dominate the Ly\(\alpha\) signal, and are distorted and
amplified due to coordinate projection effects. Their contribution must therefore be accurately removed from the received flux. In what follows we investigate the power spectra of these interloping lines and suggest a technique to remove them.

### 4.1 Power spectra of interloping lines

The abundance matching technique required to compute the power-spectrum of interloping lines involves the knowledge of the line LF, which is not as easy as the continuum LF to measure. Fortunately our Lyα signal is only contaminated by interlopers at low redshift ($z < 2$), where observations are more easily available. We use the Schechter LF parameterization (Schechter 1976) in Ly et al. (2007) and Drake et al. (2013) (D13); for simplicity, we consider only works that fitted their LF with a Schechter function, neglecting, for example, Gunawardhana et al. (2013). We use the raw LF, without dust correction.

| Hα | $z$ | $\phi$ | $L_*$ | $\alpha$ |
|----|----|------|------|-------|
| L07 | 0.24 | $-2.98 \pm 0.40$ | $41.25 \pm 0.34$ | $-1.70 \pm 0.10$ |
| 0.4 | $-2.40 \pm 0.14$ | $41.29 \pm 0.13$ | $-1.28 \pm 0.07$ |
| D13 | 0.25 | $-2.43 \pm 0.19$ | $40.83 \pm 0.18$ | $-1.03 \pm 0.16$ |
| 0.4 | $-2.44 \pm 0.16$ | $41.16 \pm 0.12$ | $-1.14 \pm 0.14$ |
| 0.5 | $-2.23 \pm 0.11$ | $41.24 \pm 0.08$ | $-1.23 \pm 0.13$ |

| O II | $z$ | $\phi$ | $L_*$ | $\alpha$ |
|----|----|------|------|-------|
| L07 | 0.89 | $-2.25 \pm 0.13$ | $43.33 \pm 0.09$ | $-1.27 \pm 0.14$ |
| 0.91 | $-1.97 \pm 0.09$ | $41.40 \pm 0.07$ | $-1.20 \pm 0.10$ |
| 1.18 | $-2.20 \pm 0.10$ | $41.74 \pm 0.07$ | $-1.15 \pm 0.11$ |
| 1.47 | $-1.97 \pm 0.06$ | $41.60 \pm 0.05$ | $-0.78 \pm 0.13$ |
| D13 | 0.35 | $-2.31 \pm 0.24$ | $40.90 \pm 0.18$ | $-1.06 \pm 0.36$ |
| 0.53 | $-2.85 \pm 0.35$ | $41.13 \pm 0.20$ | $-1.68 \pm 0.36$ |
| 1.19 | $-2.41 \pm 0.08$ | $41.61 \pm 0.07$ | $-0.95 \pm 0.14$ |
| 1.46 | $-2.03 \pm 0.05$ | $41.76 \pm 0.05$ | $-0.91 \pm 0.11$ |
| 1.64 | $-1.68 \pm 0.47$ | $41.73 \pm 0.11$ | $-0.91 \pm 0.11$ |

| O III | $z$ | $\phi$ | $L_*$ | $\alpha$ |
|----|----|------|------|-------|
| L07 | 0.48 | $-2.55 \pm 0.25$ | $41.17 \pm 0.22$ | $-1.49 \pm 0.11$ |
| 0.42 | $-2.38 \pm 0.22$ | $41.11 \pm 0.21$ | $-1.25 \pm 0.13$ |
| 0.62 | $-2.58 \pm 0.17$ | $41.51 \pm 0.15$ | $-1.22 \pm 0.13$ |
| 0.83 | $-2.54 \pm 0.50$ | $41.53 \pm 0.11$ | $-1.44 \pm 0.09$ |
| D13 | 0.14 | $-3.67 \pm 0.28$ | $41.6 \pm 0.28$ | $-1.63 \pm 0.42$ |
| 0.63 | $-2.57 \pm 0.12$ | $41.44 \pm 0.09$ | $-1.27 \pm 0.11$ |
| 0.83 | $-2.25 \pm 0.80$ | $41.28 \pm 0.09$ | $-0.76 \pm 0.21$ |
| 0.99 | $-3.00 \pm 0.23$ | $41.70 \pm 0.13$ | $-0.78 \pm 0.20$ |

### 4.2 Interlopers removal

Removing the interloping lines requires a strategy that is different from that used to deal with continuum foregrounds. A possible strategy is to mask the contaminated pixels (Gong et al. 2014; Pullen et al. 2014; Breysse et al. 2015). This is feasible because the galaxy population emitting the interloping lines is very different from the signal sources at EoR: bright galaxies are very rare at high redshift because they are exponentially suppressed in the LF. Hence, if we remove the most luminous pixels from the survey, most of them would be occupied by low-$z$ galaxies and the intensity of interloping lines could be reduced significantly.

However, although straightforward this approach has two drawbacks: (i) if the S/N of the observation is not high, bright voxels can result from noise or foreground fluctuations; (ii) it removes also Lyα flux (Breysse et al. 2015). For this reason in this work we will use a different approach relying on ancillary galaxy surveys for the identification of the interlopers (Pullen et al. 2014; Silva et al. 2015; Yue et al. 2015). This strategy would affect only weakly the Lyα PS; however, ancillary surveys have to be sufficiently deep, wide and galaxy redshifts have to be estimated precisely.

To demonstrate the feasibility of such approach, we first perform a calculation similar to that shown in the left panel of Fig. 5 but imposing an upper limit to the mass of the interloping galaxies. We assume that the pixels containing galaxies larger than this upper limit are removed from the survey. Fig. 5 (right) shows the PS of Lyα signal at $z = 7$ and interlopers, normalizing all power spectra at $k = 0.1 h\text{Mpc}^{-1}$.

---

6 The emission redshift is $1 + z_{em} = (1 + z)(\nu_{em}/\nu_{o})$, therefore at $z_{o} = 6, 7.8$ the corresponding emission redshifts for the interlopers are $z_{H_{\alpha}} = 0.30, 0.48, 0.67$, $z_{OII} = 1.28, 1.61, 1.94$ and $z_{OIII} = 0.70, 0.94, 1.19$. 

---
In the left panel of Fig. 5 we show the minimum mass of halos that have to be removed from the survey to reach a interloper-to-signal ratio \( r \) (defined as the PS ratio at scale \( k = 0.1 \, h\text{Mpc}^{-1} \)) for PS of Ly\( \alpha \) from redshift \( z \). We find that an effective interloper removal requires to resolve galaxies hosted by halos with \( M \gtrsim 10^{11} M_\odot \) and line flux \( f \gtrsim 10^{-46} \text{erg cm}^{-2} \). This can be challenging for a large area survey. The fraction of the volume loss can be substantial, as shown by the right panel of Fig. 5 when considering a 5% (\( R = 20 \)) redshift uncertainty in the ancillary galaxy survey, resulting in more than one voxels discarded per galaxy. We find that if more than \( \sim 30\% \) of the survey volume is masked, the PS reconstruction can be unfeasible (Kashlinsky et al. 2009). From the right panel of Fig. 5 we conclude that cleaning a Ly\( \alpha \) IM survey can be intrinsically difficult at \( z > 12 \), while the volume loss is not problematic for observations at later epochs.

We can then translate the above constraints on a limiting apparent magnitude at which interloper galaxies must be removed. To do this we use the optical and NIR rest frame LFs in Helgason et al. (2012) and assign luminosities to DM halos using the abundance matching technique. The apparent AB magnitude at a specified wavelength is obtained from linear interpolation between two neighboring bands in Helgason et al. (2012). Fig. 6 shows the maximum depth needed by a survey to remove interlopers as a function of \( r \) and signal redshift in the Y, J, H and K bands. To access the signal from late EoR the ancillary survey must reach an AB mag \( \gtrsim 26 \). Compared with the designed sensitivity of future photometric surveys this is rather challenging. For example the EUCLID wide survey will reach a limiting magnitude of 24 in bands Y, J, and H: this can be enough only to clean the Ly\( \alpha \) PS at \( z < 4.4 \) (without the H\( \alpha \) line). Observing the EoR signal and reaching AB mag \( m = 27 - 28 \) is extremely challenging and is at the edge of the capabilities of future instruments, such as WFIRST[9] or FLARE.

5 CROSS-POWER SPECTRA

In Sec. 4.2 we have discussed an interloper removal method based on ancillary surveys. In spite of the optimistic assumptions (for example, we have neglected the scatter in the line luminosity, SFR and halo mass relations) the required masking depth is relatively demanding.

An alternative strategy would be to use the cross-correlation between two different intensity mapping experiments contaminated by different interloping lines. The 157.7 \( \mu \text{m} \) [C II] fine structure line is the brightest of all the metal lines, contributing generally up to \( \sim 1\% \) of the total galaxy IR luminosity. Its line luminosity scales tightly with the SFR, but is affected also by the ISM metallicity (Vallini et al. 2013, 2015). The removal of continuum foreground and interloping lines for [C II] auto-correlation PS measurements was investigated in Yue et al. (2015). In this section we investigate its cross-correlation with the Ly\( \alpha \) line. The interloping lines for these two signals are not correlated with each other because they are produced in non-overlapping redshift intervals.

5.1 [C II] line intensity and cross power

The mean [C II] intensity can be directly obtained from the galaxy line luminosity (Comaschi & Ferrara 2016): 

\[
I_{\text{CII}}(z) = \frac{c}{4\pi v_{\text{CII}} H(z)} \int \frac{dM}{dM} L_{\text{CII}}(M, z).
\]

It spatially fluctuates following the large scale DM density field multiplied by a line luminosity-weighted mean bias, \( (b)_{\text{CII}} \):

\[
\delta I_{\text{CII}} = I_{\text{CII}}(b)_{\text{CII}} \delta;
\]

References:

[9] http://sci.esa.int/euclid/

[10] http://wfirst.gsfc.nasa.gov
Figure 6. **Left**: Maximum mass of the galaxies contributing to the interlopers PS. The sharp discontinuity at $z = 4.4$ is due to the H$_\alpha$ line entering the survey. **Right**: Fraction of voxels that has to be removed to obtain a ratio $r$ between the interlopers and Ly$\alpha$ PS on scale $k = 0.1 \, h\text{Mpc}^{-1}$ at redshift $z$. A redshift uncertainty in the ancillary galaxy survey of 5% ($R = 20$) has been assumed, thus multiple voxels are discarded for each interloper.

Figure 7. Limiting magnitude in different spectral bands (Y, J, H, K) required for a survey to clean the Ly$\alpha$ PS signal from interlopers; $r$ is the ratio between the interlopers and Ly$\alpha$ PS on scale $k = 0.1 \, h\text{Mpc}^{-1}$ at redshift $z$. The sharp horizontal feature at $z = 4.4$ is due to the H$_\alpha$ PS from $z = 0$; it formally diverges (see Eq. (6)).
where $\delta$ is the DM density contrast,

$$\langle b \rangle_{\text{ClH}} = \frac{1}{\rho_{\text{ClH}}} \int dM \frac{dn}{dM} b(M, z) \rho_{\text{ClH}}(M, z). \quad (22)$$

The cross-correlation PS includes three main terms:

- **Large scale DM fluctuations** originating from the Ly$\alpha$ and [C ii] lines, both emitted by the ISM. This component dominates the PS on scales $\gtrsim 1$ Mpc. It can be written as

$$P_{\text{ClH},a}(k, z) = I_{\text{ClH}}(z) I_a(z) \langle b \rangle_{\text{ClH}} \rho_{\text{ClH}}(k, z), \quad (23)$$

where $I_a(z)$ is the Ly$\alpha$ emission from the ISM;

- **Fluctuations from UV continuum emission** resulting from the correlation between Ly$\alpha$ emission in the IGM and [C ii] emission in the ISM. Ly$\alpha$ fluctuations are produced by (i) UV emission from the galaxies, and (ii) Lyman absorption followed by relaxation in the IGM. We can express the spatial intensity fluctuations as

$$\delta I_{\text{ClH},a}(z) = \frac{c \rho_{\text{a}}}{4 \pi (1 + z)} \sum_{n=2}^{\infty} P_{ab}(n, z) f(n) \times$$

$$\times \int d\nu' \frac{\dot{n}_a(\nu', \nu')}{H(z')} \sum_{n'=n+1}^{\infty} T(n', z') \langle b(\nu') \rangle_{\nu'} \delta =$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta = \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta = \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta = \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

where $P_{ab}(n, z)$ is the IGM absorption probability of a Lyman-$\alpha$ photon at redshift $z$, $f(n)$ is the fraction of Ly$\alpha$ photons emitted by an HI atom during the decay from the $n$-th energy level, $\dot{n}_a$ is the number of UV photons emitted per unit time, volume, and frequency. $T(n, z) = 1 - P_{ab}(n, z)$ is the transmission probability; we refer to CF16 for details. The associated cross-correlation PS is

$$P_{\text{ClH},a}(k, z) =$$

$$\int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

where $\nu' = \nu'_{\text{Ly} \alpha}$ and $\nu'_{\text{Ly} \alpha} = c \int_{\infty}^{\infty} dx H(\nu'_{\text{Ly} \alpha})^{-1}$. It becomes important only on scales $\gtrsim 100$ Mpc as fluctuations on scales smaller than the typical mean free path of a photon with energy between the Ly$\alpha$ and the Lyman-limit are washed out.

- **Shot noise** due to the discrete nature of the sources dominates on small scales:

$$P_{\text{ClH},a}(k, z) =$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

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$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta,$$

$$= \frac{1}{4\pi} \int d\Omega \int_{\infty}^{\infty} d\nu' A(z, \nu') \dot{n}_a(\nu', \nu') \delta.$$
6 CONCLUSIONS

We have investigated the feasibility of a Lyα intensity mapping experiment targeting the collective signal from galaxies located at $z > 4$. We have used a recently developed analytical model to predict the Lyα power spectrum, and carefully studied the main observational challenges. These are ultimately quantified by the expected S/N for various observational strategies.

We found that in principle the Lyα PS for $z < 8$ is well at reach of a small space telescope (40 cm in diameter); detections with low S/N are possible only in some optimistic cases up to $z \sim 10$. However, the foreground from interloping lines represent a serious source of confusion and must be removed. The host galaxies of these interloping lines can be resolved via an ancillary photometric galaxy survey in the NIR bands (Y, J, H, K). If the hosts are removed down to AB mag $\sim 26$, then the Lyα PS for $5 < z < 9$ can be recovered with good S/N. We further found that, by cross-correlating the Lyα emission with [C ii] emission from the same redshift, the required depth of the ancillary galaxy survey could be within reach of Euclid (AB mag $\sim 24$).

The results of this work show the yet unexplored, remarkable potential of Lyα IM experiments. By using a small space telescope and a few days observing time it is possible to probe galaxies hosted by DM halos with $M \approx 10^{10} M_\odot$ well into the EoR. Such galaxies emit the bulk of the collective Lyα radiation. However, the technical difficulty is represented by the interloping lines removal, which sets demanding requirements to the ancillary survey: the combination of very large survey areas ($\sim 250$ deg$^2$) and significant depth (AB mag $\sim 26$) appear to be challenging also for the next generation telescopes. We have suggested however, that such problem can be overcome by cross-correlating the Lyα IM with other lines (as the 157.7 μm [C ii] fine structure line), thus making a strong synergy between programs targeting different bands almost mandatory.

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In this Appendix we discuss the derivation of the deviation of Eq. (10). For simplicity we consider only two components: the line intensity $I_\alpha$ and the detector noise $I_N$; we will work in $k$-space:

$$\delta I = \delta I_\alpha + \delta I_N. \quad (A1)$$

Since other components do not correlate with $I_\alpha$ and with $I_N$, adding them to the results is trivial.

In this paper we use the Fourier convention from Padmanabhan (1993):

$$\delta k = \int d^3 \mathbf{x} e^{-i k \cdot \mathbf{x}} d^3 \mathbf{x}; \quad (A2)$$

$\delta k$ has a Gaussian Probability Distribution function (PDF). If $\delta k$ is written in polar coordinates, $\delta k = r_k \exp[i \phi_k]$, the PDF assumes the form

$$g(\delta k, \phi_k)dr_k d\phi_k = \frac{2r_k dr_k}{\sigma_k} \left( \frac{d\phi_k}{2\pi} \right) e^{-\frac{r_k^2}{2\sigma_k^2}}. \quad (A3)$$

With Eq. (A3) it is easy to prove that $\langle \delta k \delta_k^* \rangle = \eta_{k\alpha} \sigma_k^2$ (where $\eta_{k\alpha}$ is the Kronokecker delta function). The cases for $\delta I_\alpha$ and for $\delta I_N$ are similar with the only exception that the variance of $\sigma_N$ does not depend on $k$.

Expanding the first term in Eq. (10), we get

$$\langle \delta I \delta I^* \rangle = \langle \delta I_\alpha \delta I_\alpha^* \rangle + \langle \delta I_\alpha \delta I_N^* \rangle + \langle \delta I_N \delta I_\alpha^* \rangle + \langle \delta I_N \delta I_N^* \rangle\quad (A4)$$
where terms like $\langle (\delta I_\alpha^*)^2 \delta I_\alpha^2 \rangle$ are null because of the averaging over the phase $\phi$ in Eq. (A3):

$$\langle (\delta I_\alpha^*)^2 \delta I_\alpha^2 \rangle \propto \int d\phi_\alpha d\phi_N e^{-2i\phi_\alpha} e^{2i\phi_N} = 0. \quad (A5)$$

The second term in Eq. (9) is

$$\langle (\delta I \delta I^*)^2 \rangle = \langle \langle |\delta I_\alpha|^2 \rangle + \langle |\delta N|^2 \rangle \rangle^2. \quad (A6)$$

Using the fact that both $\delta I_\alpha$ and $\delta I_N$ are Gaussian we have

$$\langle |\delta_k|^4 \rangle = \int r_k^4 \frac{dr_k^2}{\sigma_k^2} e^{-\frac{r_k^2}{2\sigma_k^2}} = 2\sigma_k^4, \quad (A7)$$

and finally

$$\langle (\delta I \delta I^*)^2 \rangle - \langle (\delta I \delta I^*)^2 \rangle = \langle \langle |\delta I_\alpha|^2 \rangle + \langle |\delta N|^2 \rangle \rangle^2. \quad (A8)$$