Quantum Phase Slip as a Dual Process to Josephson Tunneling

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Abstract. Superconducting properties of metallic nanowires can be entirely different from those of bulk superconductors because of the dominating role played by thermal and quantum fluctuations of the order parameter. Fundamental attributes of superconductivity such as zero resistivity, persistent currents in closed loops, energy gap in excitation spectra can be drastically violated by fluctuations. Here we report the experimental study of I-V characteristics of thin superconducting titanium nanowires governed by quantum phase slips. The thinnest samples imbedded in high-ohmic environment demonstrated counterintuitive behavior for a superconductor: Coulomb blockade. The magnitude of the Coulomb gap correlates with the rate of quantum phase slips. The observation confirms the similarity of quantum charge dynamics in a Josephson junction and in a quasi-one-dimensional superconducting channel governed by quantum fluctuations of the order parameter.

1. Introduction
The subject of quasi-one-dimensional (1D) superconductivity has attracted a significant interest [1]. It has been demonstrated that in sufficiently narrow channels quantum fluctuations of the complex order parameter $\Delta = |\Delta| e^{i\varphi}$ may significantly alter the text-book attributes of superconductivity such as zero resistivity [2], persistent currents [3,4] and energy gap in excitation spectra [5,6]. The particular manifestation of quantum fluctuations corresponding to momentary nulling of the order parameter modulus $|\Delta|$ and ‘slippage’ of the phase $\varphi$ by $2\pi$ is called quantum phase slip (QPS). It has been pointed out that the QPS process, being formally equal to tunnelling of magnetic flux through a superconductor, is dual to tunnelling of a Cooper pair through an insulating layer of a Josephson junction (JJ) [7]. The observation leads to a counterintuitive effect: current-biased narrow superconducting channel governed by quantum fluctuations (QPS junction – QPSJ) demonstrates insulating behaviour – the Coulomb blockade. The objective of this paper is to experimentally study the phenomenon.

2. Theory background
It has been demonstrated [8] that physical properties of a QPSJ are described by the Hamiltonian:

$$\hat{H}_{\text{QPS}} = \frac{E_L}{(2\pi)^2} \hat{\varphi}^2 - E_{\text{QPS}} \cos(2\pi \hat{q}) + \hat{H}_{\text{coup}} + \hat{H}_{\text{env}}$$

(1)
which is dual to the Hamiltonian of a JJ

\[ \hat{H}_J = E_C \hat{q}^2 - E_J \cos \hat{\phi} + \hat{H}_{\text{coup}} + \hat{H}_{\text{env}} \]  

(2)

if to swap the conjugated operators of charge \( \hat{q} \) and phase \( \hat{\phi} \), which are not commuting \([\hat{q}, \hat{\phi}] = -i\).

The term \( \hat{H}_{\text{env}} \) stands for external electromagnetic environment, and \( \hat{H}_{\text{coup}} \) describes the interaction with this environment. Parameters \( E_{QPS}, E_C, E_L \) and \( E_J \) are the characteristic energies related to QPS process, capacitance, inductance and Josephson coupling, correspondingly. Simple comparison of Hamiltonians (1) and (2) allows one to establish their equivalence after substitution

\[ E_C \leftrightarrow \pi^2 E_L, E_J \leftrightarrow 2E_{QPS}, \phi \leftrightarrow \pi q / 2e \]  

(3)

For particular case of a current-biased superconducting nanowire connected in series with high-ohmic environment \( R_S > R_0 = h/(2e) = 6.47 \text{ k}\Omega \) when condition \( E_{QPS} > E_L \) is satisfied, the voltage drop across the QPSJ is given by:

\[ V(t) = V_C \sin(2\pi q) + 2e \left( L \frac{d^2 q}{dt^2} + R_S \frac{dq}{dt} \right) \]  

(4)

Expression (4) is identical to the well-known in Josephson physics motion of a quantum particle in ‘tilted wash-board’ potential, given that current \( I \) is substituted with voltage \( V \) and the ‘coordinate’ is not the phase \( \phi \), but charge \( q \). The critical current \( I_C \) of a JJ is substituted by critical voltage \( V_C = (2\pi/2e)E_{QPS} \) of the QPSJ. The described similarity established the fundamental quantum duality between these two systems. A JJ with transparent barrier is equivalent to a superconductor with critical current \( I_C \), and a thin superconducting nanowire, governed by quantum fluctuations, is equivalent to a chain of JJs, demonstrating a Coulomb blockade with gap \( \delta V_{CB} = V_C \). Expression (4) leads to a counterintuitive conclusion: at small bias a QPSJ should demonstrate a dielectric behavior: the Coulomb blockade. In this regime the quantum dynamics of charge is described by expressions formally equivalent to a Bloch electron in a periodic potential of a crystal lattice [9]. The objective of the paper is to experimentally study the Coulomb blockade effect in a QPSJ.

3. Experimental

The QPS, responsible for phenomena studied in the work, is an essentially size-dependent effect observed exclusively in quasi-one-dimensional nanoscale superconductors. In order to enable the high rate of quantum fluctuations of the phase \( \phi \) one should stabilize the conjugated parameter – the charge \( q \). As electric current is just the time derivative of charge \( I = dq/dt \), the problem is reduced to stabilization of current. However utilization of high-quality laboratory current source is mandatory, but not sufficient. The QPSJ should be connected in series with high-ohmic (or high-impedance) environment to enable Copper pair transport with precise rate. Conventional lift-off e-beam lithography followed by ultra-high vacuum deposition of materials was used for fabrication of the nanostructures (Fig. 1, inset). Fine adjustment of the nanowire diameter, enabling the desired rate of quantum phase slips, was obtained by low-energy \( Ar^+ \) ion milling [10]. The samples were analysed by scanning electron (SEM) and atomic force (AFM) microscopes. Error in characterization of the nanowire effective diameter ~3 nm comes from the AFM tip deconvolution effect. As it has been already demonstrated in our earlier reports [10, 4], the low-energy ion milling does not damage the metal matrix, except the very top layer of thickness ~2 nm, which is comparable to the thickness of naturally grown oxide. At low acceleration voltages < 1 kV the milling process is so gentle, that it does not destroy extremely delicate tunnel junctions [4,11]. The resulting roughness of about ± 1 nm [16] is comparable to the surface of perfect quasi-one-dimensional single crystals (whiskers) [12,13,14], where the effect of classical (thermal) fluctuations has been studied. The ‘polishing’ effect of the low-energy ion milling simplifies the interpretation of results in terms of quantum fluctuations, as the corresponding model [1] deals with objects of uniform cross section.
The length of the 'body' of the sample (the QPSJ itself) $L$ was equal to 10 $\mu$m. The $T_C$ and $\rho_N$ of e-beam evaporated titanium thin films depend on the residual pressure of oxygen and the deposition rate. The observation enables fabrication of structures with the thinnest part (the QPS nanowire) made of relatively clean titanium, while the contact probes – from highly resistive bismuth and/or ‘dirty’ high-ohmic titanium, showing no traces of superconductivity down to the lowest achievable temperatures. In all our structures the resistivity per square area of both the nanowire $\rho_{NQPS} \leq 300 \Omega/\square$ and the electrodes $\rho_{Nprobe} \leq 1 \mathrm{k}\Omega/\square$ were on the metallic side of metal-to-insulator transition.

Experiments were made using $^3\mathrm{He}^4\mathrm{He}$ dilution refrigerator located in electromagnetically shielded room. All input/output lines between the analogue front-end electronics located inside the shielded room and the rest of the measuring set-up (PC, DMMs, lock-in) were carefully filtered passing through the walls of the shielded room. The multi stage RLC filtering system inside the cryostat [15] was used to suppress the undesired influence of external noisy electromagnetic environment minimizing the overheating of electrons above the base (phonon) temperature.

4. Results and discussion
Four-probe configuration was used to study $R(T)$ and $V-I$ characteristics of titanium nanowires of identical geometry. In a full accordance with our earlier studies [16], all structures demonstrated very broad $R(T)$ dependencies, which can be nicely described by the QPS model [1].

![Figure 1. I-V characteristic of 39 nm titanium nanowire at T=20 mK. The arrow indicates the direction of the I-V dependence recording. Inset: SEM image of a typical nanostructure.](image)

The I-V dependencies of the thinnest nanowires, connected in series with current-biasing 10 M$\Omega$ ballast resistors, demonstrate the clear Coulomb blockade (Fig. 1). The value of the Coulomb gap $\delta V_{CB}$ nicely correlates with estimations of the QPS energy $\delta V_{QPS}=E_{QPS}/e$, obtained from fitting the $R(T)$ dependencies. The Coulomb gap can be modulated by external gate capacitively coupled with the QPSJ [17]. The period of the modulation is in a reasonable agreement with sample geometry. At small bias currents $\lesssim 50$ pA the system jumps from current to voltage biased mode demonstrating S-shape I-V dependence. The corresponding the back-bending (the ‘Bloch nose’) is the consequence of non-trivial electrodynamics, described by Eq. (4).

It should be noted that the essentially non-linear I-V dependencies disappear at temperatures above $\sim 400$ mK, which corresponds to the critical temperature $T_C$ of the titanium nanowires. Let us assume that our samples with characteristic diameter $\sim 30$ nm might contain unintentionally formed tunnel junction(s). If the case, an elementary calculation gives that one should observe the single electron
Coulomb blockade up to temperatures ~ 7 K. The observation confirms that in our nanowires there are no ‘conventional’ weak links and the observed Coulomb blockade is the dynamic process originating from the essentially superconducting phenomenon – the QPS events.

To summarize, our experiments demonstrate quite unusual effect – an insulating state of a superconducting nanowire. The phenomenon originates from coherent QPS events. The finding proves the quantum duality between QPS process and tunneling in a JJ.

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