The stable topology of the planetary systems of two 2:1 resonant companions: application to HD 82943

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Abstract

We have numerically explored the stable planetary geometry for the multiple systems involved in a 2:1 mean motion resonance, and herein we mainly concentrate on the HD 82943 system by employing two sets of the orbital parameters (Mayor et al. 2004; Ji et al. 2004). In the simulations, we find that all stable orbits are related to the 2:1 resonance that can help to remain the semi-major axes for two companions almost unaltered over the secular evolution for $10^8$ yr. In addition, we also show that there exist three possible stable configurations: (1) Type I, only $\theta_1 \approx 0^\circ$, (2) Type II, $\theta_1 \approx \theta_2 \approx \theta_3 \approx 0^\circ$ (aligned case), and (3) Type III, $\theta_1 \approx 180^\circ$, $\theta_2 \approx 0^\circ$, $\theta_3 \approx 180^\circ$ (antialigned case), here the lowest eccentricity-type mean motion resonant arguments are $\theta_1 = \lambda_1 - 2\lambda_2 + \varpi_1$ and $\theta_2 = \lambda_1 - 2\lambda_2 + \varpi_2$, the relative apsidal longitudes $\theta_3 = \varpi_1 - \varpi_2 = \Delta \varpi$. And we find that other 2:1 resonant systems (e.g., GJ 876) may possess one of three stable orbits in their realistic motions. Furthermore, we show that the assumed terrestrial bodies cannot survive near the habitable zones for HD 82943 due to strong perturbations induced by two resonant companions, but such low-mass planets can be dynamically habitable in the GJ 876 system at $\sim 1$ AU in the numerical surveys.

Key words: methods: N-body simulations — celestial mechanics — planetary systems — stars: individual (HD 82943, GJ 876)

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1 Introduction

The discovery of the extrasolar planets should show the diversity of the planetary systems in the universe and also sketch out an innovative world outside our solar system. Mayor & Queloz (1995) began such great revolution in our minds when they found the first extrasolar giant Jupiter-51 Peg, and to date there are more than 100 planetary systems (see also a daily updated website\(^1\); Butler et al. 2003), which most of them were detected with the radial velocity technique in the surveys of nearby young stars. At present, a dozen of multiple planetary systems were discovered in recent years and this number is undoubtedly cumulative as time elapses. Hence, it is necessary to categorize discovered multiple systems according to their statistical characteristics (such as the distribution of the planetary masses, semi-major axes, eccentricities and metallicity), then to study the correlation between mass ratio and period ratio (Mazeh & Zucker 2003) and to improve the understanding of the correlation between the planet occurrence rate and stellar metallicity (Santos et al. 2003; Fischer, Valenti, & Marcy 2004). The other key point is to investigate possible stable configurations for the multiple systems, in which the observations reveal that most of them are typically characterized by mean motion resonance (MMR) and (or) apsidal phase-locking between their orbiting companions (Fischer et al. 2003; Ji et al. 2003a), so that one can better understand the full dynamics of these systems.

2 Stable Planetary Geometry in 2:1 Mean Motion Resonance

Amongst the discovered extrasolar planetary systems, the resonant pairs are quite common for the multiple systems, e.g., HD 82943 (Gozdziewski & Maciejewski 2001) and GJ 876 (Lee & Peale 2002; Ji, Li & Liu 2002) in a 2:1 resonance, and 55 Cancri in a 3:1 MMR (Marcy et al. 2002; Ji et al. 2003b). In addition, as the observation results show that most of the systems can potentially harbor more than one planet, thus we turn to the fascinating topic of studying the stable planetary geometry locking into a resonance, which such investigations are expected to be helpful in an understanding of the orbital evolution of the planets and the dynamical mechanisms of sustaining the stability of such systems. In the present study, we mainly focus our attention on the HD 82943 system. In Table 1 are listed the orbital parameters for this system, where two sets of the best-fit solutions are respectively given according to Mayor et al. (2004) (Fit 2) and their original orbital data (Fit 1; as of July 31, 2002), which are adopted in our simulations.

\(^1\) [http://cfa-www.harvard.edu/planets/bibli.html](http://cfa-www.harvard.edu/planets/bibli.html)
Table 1. Orbital parameters of HD 82943 planetary system. The stellar mass $M_c$, for Fit 1, $M_c = 1.05M_\odot$, while for Fit 2, $M_c = 1.15M_\odot$.

| Planet | Mass(Mjup) | $P$(day) | $a$(AU) | $e$  | $\omega$(deg) | $\tau$(JD) |
|--------|------------|----------|---------|------|--------------|------------|
| b (Fit 1) | 1.63 | 444.6(8.8) | 1.16 | 0.41(0.08) | 96(7) | 2451620.3(12) |
| c (Fit 1) | 0.88 | 221.6(2.7) | 0.73 | 0.54(0.05) | 138(13) | 2451630.9(5.9) |
| b (Fit 2) | 1.84 | 435.1(1.4) | 1.18 | 0.18(0.04) | 237(13) | 2451758(13) |
| c (Fit 2) | 1.85 | 219.4(0.2) | 0.75 | 0.38(0.01) | 124(3) | 2452284(1) |

In this paper, we employ N-body codes (Ji et al. 2002) to perform direct numerical simulations by using RKF7(8) and symplectic integrators (Wisdom & Holman 1991) for this system. In the simulations, we always take the stellar mass and the minimum planetary masses from Table 1. The time step size is usually adopted to be $\sim$ 1%-2.5% of the orbital period of the innermost planet, which is sufficiently smaller for the integration. Additionally, the numerical errors were effectively controlled with the accuracy of $10^{-14}$ over the integration timescale.

In our simulations for HD 82943 system, we found three possible stable configurations for a system in a 2:1 MMR: (I) only $\theta_1$ librates about 0$^\circ$, (II) $\theta_1 \approx \theta_2 \approx \theta_3 \approx 0^\circ$ (aligned case), (III) $\theta_1 \approx 180^\circ$, $\theta_2 \approx 0^\circ$, $\theta_3 \approx 180^\circ$ (antialigned case). The definition of the lowest eccentricity-type 2:1 resonant arguments are $\theta_1 = \lambda_1 - 2\lambda_2 + \varpi_1$ and $\theta_2 = \lambda_1 - 2\lambda_2 + \varpi_2$. And the relative apsidal longitude is $\theta_3 = \Delta\varpi = \varpi_1 - \varpi_2$, where $\lambda_{1,2}$ are, respectively, the mean longitudes of the inner and outer planets, and $\varpi_{1,2}$, respectively, their periastron longitudes. Obviously, for above three angles, no more than two are linearly independent. Besides, these stable planetary geometry is related to the existence of symmetric stable equilibrium solutions, where we find that other 2:1 resonant systems (e.g., GJ 876) may possess one of the above three stable orbits in their realistic motions.

As a paradigm, we simply show the dynamical evolution for one of the types of the stable orbits for HD 82943 and the reader may refer to Ji et al. (2004) for a detailed study. Figures 1 and 2 exhibit that the long-term orbital evolution for one set of stable solutions derived from Fit 2, where $\theta_1$ and $\theta_3$ both librate about 0$^\circ$ (Type II) for $10^8$ yr, meanwhile the semi-major axes and eccentricities for two massive planets perform slight vibrations. Still, one may understand that the 2:1 resonance can help remain the semi-major axes for two companions almost unaltered over the secular evolution (see Figs. 1 and 2), and the apsidal phase-locking between two orbits can further enhance the stability for this system, because the eccentricities are simultaneously preserved to restrain the planets from frequent close encounters. Our results are well consistent with previous works, e.g., Hadjidemetriou (2002) numerically studied the families of periodic orbits for the HD 82943 system under the ro-
tating framework, and indicated that two kinds of families orbits can survive for the 2:1 resonant planets. Moreover, Beauge et al. (2003) further showed the asymmetric stable solutions for the 2:1 resonant system using both analytical and numerical means. In summary, all the above outcomes imply the potential planetary configurations that the extrasolar systems involved in 2:1 resonance may hold.

In order to understand the dynamics of this system, in a companying paper (see Ji et al. 2004), we analytically plot the evolution of the eccentricities and the relative apsidal longitudes. The semi-analytical contour chart exhibits a good agreement with the numerical investigations of maintaining the eccentricities with larger $\theta_3$-libration amplitudes (ref. to Fig.4b of Ji et al. 2004; Type II), and the figure also implies that the $\theta_3$-libration will be broken up if the libration amplitudes exceed or approach the critical value of $90^\circ$ near the separatrix. Bois (2003, private communication) also confirmed that the stable strip of the alignment for the initial $\varpi_1 - \varpi_2$ is much narrower than that of the antialignment in the calculations for HD 82943.

3 Habitable Zones

The Habitable zones (HZ) are generally convinced to be suitable places for terrestrial planets that can provide the liquid-water, subtle temperature and atmosphere environment, and other proper conditions (Kasting et al. 1993), supporting the development and biological evolution of life on their surfaces. And herein, we performed numerical surveys to examine the potential habitable zones for the systems (e.g. HD 82943 and GJ 876) with a stable geometry in 2:1 resonance. We generated 100 seed planets that all bear the same masses as Earth in each run. The distribution of the initial orbital elements for these postulated planets are as follows: they move on the much less inclined belt with the relative inclination with regard to the orbital plane of the resonant companions less than $5^\circ$, with $0.96 \ AU \leq a \leq 1.05 \ AU$ and $0 \leq e \leq 0.1$.

Figure 3 summarizes the main results of the simulations for the HD 82943 system - no Earth-mass planet can survive about 1 AU in the final system, where 95% of the orbits are usually ejected at $t \leq 10^5 \ yr$ due to the gravitational scattering arising from two close massive planets. The scenario of a typical ejected orbit may replay that the semi-major axis $a$ grows from $\sim 1 \ AU$ to tens of AU, meanwhile the eccentricity $e$ undergoes a rapid increase from $\sim 0.1$ to 1. Hence, the assumed Earth-like planet is thrown out from the two-planet system in less than $10^5 \ yr$. As for GJ 876, all tests about 1 AU are stable at least for $10^6 \ yr$. Figure 4 exhibits the typical orbital evolution for the Earth-like planet, both the semi-major axis and the eccentricity execute small fluctuations about 1 AU and 0.06, respectively, and the inclination also
remains less than 2 degrees in the same time span. And there is no sign to indicate that such regular orbits at $\sim 1$ AU with low eccentricities will become chaotic for much longer time even for the age of the star.

In addition, we also carried out extended integrations for the HD 82943 planetary system to investigate whether there may exist the analogous structure of asteroidal belts that the residual planetesimals are left in the system in later planetary formation. We let several hundreds of test particles initially located at the regions of the 2:1, 3:1 and 5:2 MMRs with the inner massive planet and then performed the simulations for 1 Myr. The results show that most of the small bodies can be removed at the dynamical time $\sim 10^4$ yr, which suggests that no survivals can finally stay at above three resonant regions. In this sense, it is quite similar to Kirkwood gaps for the main belt asteroids in our solar system. However, there is much prospect of the space observations with high resolutions (e.g., Spitzer) revealing the evidence of the disk debris structure in other extrasolar planetary systems in future.

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References

Beauge, C., Ferraz-Mello, S., & Michtchenko, T.A. 2003, ApJ, 593, 1124
Butler, R. P., et al. 2003, ApJ, 582, 455
Fischer, D., et al. 2003, ApJ, 586, 1394
Fischer, D. A., Valenti, J., & Marcy, G. 2004, in ASP Conf. Ser., S219, Stars as Suns: Activity, Evolution and Planets, ed. A. Dupree and A. Benz, (San Francisco: ASP), 29
Gozdziewski, K., & Maciejewski, A.J. 2001, ApJ, 563, L81
Hadjidemetriou, J. 2002, Cel. Mech. & Dyn. Astron.,83,141
Ji, J., Li, G., & Liu, L. 2002, ApJ, 572, 1041
Ji, J., et al. 2003a, ApJ, 591, L57
Ji, J., et al. 2003b, ApJ, 585, L139
Ji, J., et al. 2004, [astro-ph/0403386]
Kasting, J. F., et al. 1993, Icarus, 101, 108
Lee, M.H., & Peale,S.J. 2002, ApJ, 567, 596
Marcy, G. W., et al. 2002, ApJ, 581, 1375
Mayor, M., & Queloz D. 1995, Nature, 378, 355
Mayor, M., et al. 2004, A&A, 415, 391
Mazeh,T., & Zucker,S. 2003, ApJ, 590, L115
Santos, N., et al. 2003, A&A , 398, 363
Wisdom, J., & Holman, M. 1991, AJ, 102, 1528
Fig. 1. One set of the stable solutions derived from Fit 2 (aligned orbits). Long-term orbital evolution for $\theta_1$ and $\theta_3$, where $\theta_1$ librates (by yellow color) about 0° with a moderate amplitude of $\sim 60^\circ$, $\theta_3$ (by green color) about 0° with a larger amplitude of $\sim 70^\circ$ for $10^8$ yr, in association with Type II orbit. The 2:1 resonance for this system is confirmed by the modulations of the resonant angles, which reveals the regular motion for two massive planets (see also Figure 2).
Fig. 2. Long-term orbital evolution for $a$ and $e$ for two planets (the same as Figure 1, yellow and green colors, respectively, for the inner and outer planets). The semi-major axes $a_1$ and $a_2$ are almost unchanged and they modulate about 0.75 and 1.18 AU with relatively smaller amplitude for $10^8$ yr. In the same time, the amplitude of the oscillations for $e_1$ and $e_2$ are not so large and they are just wandering in the span (0.35, 0.45) and (0, 0.20), respectively. No signs show that this system will be chaotic for even longer evolution.
Fig. 3. The numbers of the ejected terrestrial planet vs ejecting timescale. Notice that 95% of the orbits are ejected at $t \leq 1.0 \times 10^5$ yr.
Fig. 4. Orbital evolution of an Earth-mass planet placed in GJ 876 at 1 AU. The results show that such orbits may exist and be stable in the planetary system.