Model dependence of the bremsstrahlung effects from the superluminal neutrino at OPERA

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Abstract

We revisit the bremsstrahlung process of a superluminal neutrino motivated by OPERA results. From a careful analysis of the plane wave solutions of the superluminal neutrino, we find that the squared matrix elements contain additional terms from Lorentz violation due to the modified spin sum for the neutrino. We point out that the coefficients of the decay rate and the energy loss rate significantly depend on the details of the model, although the results are parametrically similar to the ones obtained by Cohen and Glashow \cite{1}. We illustrate this from the modified neutral current interaction of neutrino with Lorentz violation of the same order as in the modified dispersion relation.

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1 Introduction

Lorentz invariance is one of the cornerstones of the modern quantum field theory, and it was completely compatible with all previous experiments and observations. Recently, an intriguing result has been presented by the OPERA collaboration [2], claiming that the muon neutrino speed exceeds the speed of light by \((v - c)/c \simeq 2.37 \pm 0.32^{+0.34}_{-0.24} \times 10^{-5}\). This result has been recently confirmed by a test performed using short-bunch wide-spacing beam in the revised version of Ref. [2]. There are similar measurements from other oscillation experiments, for example, MINOS, resulting in \((v - c)/c = 5.1 \pm 2.9 \times 10^{-5}\) [3], but the OPERA is the first experiment which observed a positive \((v - c)\) for neutrinos with high statistical significance of about 6.2\(\sigma\). There is no statistically significant energy dependence of superluminality at OPERA where the results for the low- and high-energy samples with averaged neutrino energies, 13.8 GeV and 40.7 GeV, respectively, are in agreement [2].

Such a drastic result is seemingly in contradiction with a set of other neutrino observations\(^1\). The detection of neutrinos emitted from the SN1987A supernova [5, 6, 7] puts a stringent bound on the electron anti-neutrino speed, \(|v - c|/c < 2 \times 10^{-9}\). Furthermore, the observation of neutrino oscillations demands that neutrino velocities for different neutrino flavors should be equal up to \(|v_i - v_j| \lesssim 10^{-19}\) for \(i \neq j\), otherwise the coherence is lost and the oscillation pattern is smeared [8, 9]. This contradiction could be explained in two ways. One is to make the neutrino velocity energy dependent [10, 11], because the typical neutrino energy is 10 MeV for the supernova neutrinos, and is about 28 GeV for the CNGS neutrino beam, used by OPERA. Another is to make the neutrino superluminal only within the Earth radius [12, 11, 13, 14, 15], or only inside matter [16]. A different route to the solution could be taken by considering models with energy non-conservation or deformed Lorentz invariance [17] in the neutrino interactions. But this would be hard to formulate in the language of ordinary quantum field theory so we do not pursue this option in this article.

In any model explaining the OPERA results it is important to check, whether the creation, propagation, and detection in the OPERA setup can be explained. There are very strong statements that invalidate most of the proposed models for the OPERA results. First, the superluminal neutrino can radiate electron-positron pairs (in a way analogous to the Cherenkov radiation) [1], thus losing energy before reaching the detector. Second, the decay of a fast moving pion is modified, and even the initial neutrino spectrum should have a strong cutoff at energy, which is below the average energy detected by OPERA [18, 19, 20]. Both of these results rely on the following assumption—the only thing modified in the theory is the dispersion relation of the neutrino. As the neutrino speed is given by the derivative of the dispersion relation \(v = dE/dp\), a constant neutrino speed at OPERA means that the neutrino dispersion relation has the form \(E = vp = (1 + \delta/2)p\) with \(\delta \simeq 5 \times 10^{-5}\). A stronger claim, based on the result of Ref. [1] was made by the analysis

\(^1\)A review on the bounds on Lorentz violation in the neutrino sector before OPERA can be found in Ref. [4].
of the ICARUS results [21]. The ICARUS detector, while not being able to measure the arrival time of the neutrinos, can carefully measure their energy spectrum. The comparison of the expected and measured spectra provides strict bounds on the neutrino speed, because the energy loss by electron-positron radiation [1] would significantly change the spectrum.

In this article we re-analyze the bremsstrahlung process of a superluminal neutrino that has been done in Ref. [1]. The main point of the previous analysis follows from the kinematical possibility for the neutrino with non-standard dispersion relation to “decay” into other particles. For example, for the process $\nu \rightarrow \nu + e^+ + e^-$, the “masses” (or squares of the four momenta) of the initial and final neutrinos are different, enabling the process to take place. However, the exact calculation for the process rate is more involved. Specifically, when the dispersion relation for neutrino is modified, one has to use the modified plane wave solutions for the neutrino. In turn, the spin sum for the final neutrino gets modified, leading to the additional terms of the order of $\delta$ in the expression for the squared matrix elements, as compared to the previous calculations [1]. Moreover, when there are modifications of the same order in the electroweak interaction vertex of neutrino as in the neutrino dispersion relation, there appear more terms of the similar order in the squared matrix elements too. All these effects are added up to give a nontrivial result, which depends on the additional modifications of the same order. This is due to the fact that the squared matrix elements (obtained by the standard rules for the spin sum for Lorentz invariant fermions) is only of the order of $\delta^2$ in the kinematically allowed region. As a result, due to various cancellations in the matrix element, the final probability of the bremsstrahlung process depends on the details of the Lorentz violation in the model.

We explicitly construct (at the level of Fermi four fermion interactions) two models with broken Lorentz symmetry. Both models have a common property that the neutrino kinetic term contains a Lorentz violating term as inspired by the modification of the metric for the neutrino [12]. The difference is that one of the models keeps the interaction terms Lorentz invariant while the other model introduces a similar Lorentz violation in the electroweak neutral current of neutrino too.

In section 2 we introduce the Lagrangians for the models. Then, in section 3 we obtain the free solutions for the neutrinos and present the rules for “summation” over the spin states. Consequently, in section 4 we provide the detailed calculation of the neutrino decay width and the rate of energy loss. Finally, conclusions are drawn.

## 2 Models

We first define the framework for the calculation of the decay (bremsstrahlung) of a superluminal neutrino into an electron-positron pair. Following the ideas in [8, 22] the lowest order Lorentz violating operator in the Lagrangian for the massless (Weyl) fermion looks like

$$
\mathcal{L} = i\bar{\nu} \gamma^{\mu} \tilde{g}_{\mu\nu} \partial^\nu (1 - \gamma^5) \nu, \tag{1}
$$
where the Lorentz violating “metric” can be chosen as
\[ \tilde{g}_{\mu\nu} = \text{diag}(1, -v, -v, -v), \tag{2} \]
with the neutrino speed \( v \equiv 1 + \delta/2 \). Quite obviously, this action gives rise to the superluminal neutrino dispersion relation, \( E = v |\mathbf{p}| \). In the limit of Lorentz invariance, we get 
\[ \tilde{g}_{\mu\nu} = \text{diag}(1, -1, -1, -1) \equiv g_{\mu\nu}. \]

The next component of the model is the interaction term. We will not go in the details of how the model emerges from the underlying electroweak gauge theory, because this would lead to complications due to the different velocities for the left-handed electron and neutrino of the same multiplet. We will take the purely phenomenological approach and analyze two types of the four fermion neutral current interaction. The first one will be the usual Lorentz invariant one (model I)
\[ L_{\text{int1}} = \frac{G_F}{\sqrt{2}} \left[ \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu \right] \left[ \bar{e} \gamma^\mu (v_e - a_e \gamma^5) e \right]. \tag{3} \]

The second one is inspired by a “gauge invariant” Lagrangian, where the covariant derivative enters in the same way as in (1) (model II)
\[ L_{\text{int2}} = \frac{G_F}{\sqrt{2}} \left[ \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu \right] \tilde{g}_{\mu\nu} \left[ \bar{e} \gamma^\nu (v_e - a_e \gamma^5) e \right]. \tag{4} \]

In fact, this interaction term does not follow from the gauge invariant SM, as far as electroweak gauge symmetry is unbroken. In this work, we assume this possibility but do not consider a microscopic model with Lorentz symmetry/electroweak symmetry breaking for that.

### 3 Free solutions for the neutrinos and spin sums

The action (1) leads to the Dirac equation of the form (in momentum representation and two component form for simplicity)
\[ (E \sigma^0 - v p^i \sigma^i) \chi = 0, \tag{5} \]
where \( \sigma^0, \sigma^i \) are the unit \( 2 \times 2 \) matrix and Pauli matrices, respectively. Let us also (using invariance under \( O(3) \) spatial rotations) align the momentum along the 3rd spatial axis. Then we immediately get two solutions
\[ E = vp^3 \text{ with } \chi = \sqrt{2E} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{6} \]
\[ E = -vp^3 \text{ with } \chi = \sqrt{2E} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{7} \]
where \( \sqrt{2E} \) is the standard overall normalization. The first solution corresponds to the neutrino and the second solution to the antineutrino, both with velocity \( v \). Notice that
the dispersion relation is modified, while the spinors have the usual form. The spin “sum” for the neutrino (which is in this case trivial, as far as there is only one spin state for the neutrino) is
\[ \sum_{s=1/2} \chi_s \chi_s^\dagger = 2E \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = p^\mu \tilde{g}_{\mu\nu} \tilde{\sigma}^\nu \neq p^\mu g_{\mu\nu} \tilde{\sigma}^\nu. \] (8)

The important observation here is that the spin sum is not given by the standard expression, but with the momentum contracted with the sigma matrices using the superluminal metric. The difference is nontrivial in the first order in \( \delta \), which is essential for the calculation of the decay width.

The generalization to the usual four component spinors is obvious,
\[ \sum_s \nu_s \bar{\nu}_s = p^\mu \tilde{g}_{\mu\nu} \gamma^\nu \equiv \tilde{p}, \] (9)

where the momentum with the tilde is a shorthand for \( \tilde{p}_\mu \equiv \tilde{g}_{\mu\nu} p^\nu \). Note, that raising and lowering the indices is always done with the normal metric, while the tilde means the additional factor of \( v \) for the spatial part in the scalar product.

Now we are ready to evaluate the matrix elements and the squared of them.

4 Decay width calculation

Following the standard rules for the decay process, we get for the bremsstrahlung process \( \nu_\mu \rightarrow \nu_\mu + e^+ + e^- \) with four momenta \( p, p', q_1, \) and \( q_2, \)
\[ \Gamma = \frac{(2\pi)^4}{2E_\nu} \int \prod_f \frac{d^3 \tilde{p}_f}{(2\pi)^3 2E_f} \delta^4(p - p' - q_1 - q_2) \sum_{\text{spin}} |\mathcal{M}|^2. \] (10)
Here, note that we do not average over the spin states of the neutrino, which was done in the calculation of [1]. In the current setup the neutrino has explicitly only one spin state, instead of two spin states for a massive neutrino. The square of the matrix elements is the following: in the model I (3),
\[ \sum |\mathcal{M}_I|^2 = \frac{G_F^2}{2} M^{\alpha\beta} E_{\alpha\beta}, \] (11)
and in the model II (4),
\[ \sum |\mathcal{M}_{II}|^2 = \frac{G_F^2}{2} M^{\alpha\beta} \bar{g}_{\alpha\gamma} \tilde{g}_{\beta\delta} E^{\gamma\delta}. \] (12)
Here the individual traces are
\[ M^{\alpha\beta} = \text{tr} \left[ \tilde{p}' \gamma^\alpha (1 - \gamma_5) \tilde{p}(1 + \gamma_5) \gamma^\beta \right], \] (13)
\[ E_{\alpha\beta} = \text{tr} \left[ \bar{q}_2 \gamma_\alpha (v_e - a_e \gamma_5) \bar{q}_1 (v_e + a_e \gamma_5) \gamma_\beta \right]. \] (14)
There are two differences from the standard calculation with Lorentz invariance: the spatial parts of $p$ and $p'$ momenta in $M^{\alpha\beta}$ are multiplied with $v$, and the indices between $M$ and $E$ are contracted with superluminal metric for the model II.

The results of the multiplication are (in the approximation of a purely axial electron neutral current, $a_e = -1/2$ and $v_e = 0$)

$$
\sum |M_I|^2 = 8G_F^2 \left[ (\tilde{p}q_1)(\tilde{p}'q_2) + (\tilde{p}'q_1)(\tilde{p}q_2) \right],
$$

and

$$
\sum |M_{\Pi}|^2 = 8G_F^2 \left[ (\tilde{p}q_1)(\tilde{p}'q_2) + (\tilde{p}'q_1)(\tilde{p}q_2) \right. - (\tilde{p}p')(\tilde{q}_1\tilde{q}_2) - (\tilde{p}'p')(q_1q_2) + \frac{1 + 3v^2}{2}(\tilde{p}p')(q_1q_2)].
$$

Here the tilde always means one factor of $v$ in front of the spatial product, i.e. $(\tilde{p}q) \equiv p^0q^0 - v \cdot q$, $(\tilde{p}q) \equiv p^0q^0 - v^2 \cdot q$, $(\tilde{p}p') \equiv p^0p'^0 - v^2 \cdot p'$, etc. Note that in model II the second line in (16) does not vanish. For comparison to our model I, in Ref. [23], the second term in the squared amplitude (15) was missing, the spin average for the initial neutrino was taken, and the modified plane-wave solutions for neutrino were not taken into account for the spin sum of the neutrino.

The rest of the calculation is rather straightforward, and consists just of careful integration over the final momenta. The safest way is to perform the calculation in the lab frame directly. We will only sketch the derivation here.

First, we perform integration over the momenta of the electron and positron, in the limit of zero electron mass (this is fine as far as we are interested in the decay of high energy neutrinos)

$$
\int q_{1\mu}q_{2\nu} \frac{d^3q_1}{E_1} \frac{d^3q_2}{E_2} \delta^4(k - q_1 - q_2) = \frac{\pi}{6}(k^2g_{\mu\nu} + 2k_{\mu}k_{\nu}),
$$

where $k \equiv q_1 + q_2$ is the momentum of the electron-positron pair. It is convenient to rewrite the remaining integration as the integration over the modulus of the final neutrino momentum $|p'|$ and the cosine of the angle $\theta$ between $p$ and $p'$ (in the lab frame). Calculating all the scalar products together with the dispersion relations, $p^0 = v|p|$ and $p'^0 = v|p'|$, we obtain the decay rate as follows,

$$
\Gamma = \frac{G_F}{96\pi^3v^2|p|} \int |p'|d|p'|d\cos \theta I
$$

where $I = I_{I,\Pi}$ for models I and II are given by

$$
I_I = \left( v^2 - 1 \right) (|p|^2 + |p'|^2) - 2|p||p'|(v^2 - \cos \theta) v^2|p||p'|(1 - \cos \theta)
$$
$$
- 2v^2 \left( (v - 1)|p|^2 - |p||p'|(v - \cos \theta) \right) \left( (v - 1)|p'|^2 - |p||p'|(v - \cos \theta) \right)
$$

(19)
and
\[ I_2 = \left[ (1+3v^2)\left( (v^2-1)(|p|^2+|p'|^2)-2|p||p'|(v^2-\cos \theta) \right) + 2v^2|p||p'| (1-\cos \theta) \right] v^2|p||p'| (1-\cos \theta) \\
-2 \left( (v^2-1)(|p|^2+|p'|^2)-2|p||p'|(v^2-\cos \theta) \right) v^2|p||p'| (1-v^2 \cos \theta) - 2v^4|p|^2|p'|^2 (1-\cos \theta)^2. \]

(20)

The integration over momenta is governed by the positivity of the electron-positron pair invariant mass, \( k^2 > 0 \). For \( v = 1 + \delta/2 \) this means that we should integrate over the whole region \(-1 < \cos \theta < 1 \) for \( 0 < |p'| < p_c \equiv |p| \delta/(4+\delta) \), but only over \( 1 > \cos \theta > \cos \theta_{\text{min}} \approx (-\delta|p|^2 - \delta|p'|^2 + 2(1+\delta)|p||p'|)/(2|p||p'|) \) for \( p_c < |p'| < |p| \). The latter region gives in fact the major contribution to the integral. The integral for the rate of the energy loss is similar but with the integrand multiplied by \(- (E - E')\).

Performing the momentum integrals we obtain the decay rate and the rate of the energy loss as follows,
\[ \Gamma = a G_F^2 \frac{c^5}{192\pi^3} E^5, \]
\[ \frac{dE}{dx} = -a' G_F^2 \frac{c^6}{192\pi^3} E^6 \]

(21)
(22)

Although the results are parametrically similar to the ones in Ref. \([1]\), the numerical coefficients turn out to be model dependent. We have that for the model I,
\[ a_1 = \frac{1}{420}(v-1)^3 (v+1)(53 + 20v - 5v^2) \approx \frac{17}{420} \delta^3, \]
\[ a'_1 = \frac{1}{672}(v-1)^3 (v+1) v(67 + 28v - 7v^2) \approx \frac{11}{336} \delta^3, \]

(23)
(24)

and for the model II,
\[ a_2 = \frac{1}{420}(v^2-1)^3 (5v^2 + 19) \approx \frac{2}{35} \delta^3, \]
\[ a'_2 = \frac{1}{672}(v^2-1)^3 v(7v^2 + 23) \approx \frac{5}{112} \delta^3. \]

(25)
(26)

For comparison, the results in Ref. \([1]\), which are obtained with the standard “Lorentz invariant” expression for the squared matrix element, are
\[ a_{CG} = \frac{(v^2-1)^3}{14v^2} \approx \frac{1}{14} \delta^3, \]
\[ a'_{CG} = \frac{25}{448} \frac{(v^2-1)^3}{v} \approx \frac{25}{448} \delta^3. \]

(27)
(28)

We find that in all the cases, the decay rate and energy loss are proportional to \( \delta^3 \). This \( \delta^3 \) dependence in our models reminds us of the argument in Ref. \([1]\) based on the kinematics.
that a superluminal neutrino gets an “effective” mass such that the decay rate is proportional to $\delta^3 E^5$. However, since the numerical coefficients are model dependent, it might be possible to construct models of Lorentz violation that allow for a reduction or cancellation of $\delta^3$ terms.

We obtain the lifetime of a superluminal neutrino as compared to the results in [1]:

$$\tau = \Gamma^{-1}$$

is 1.76(1.25) $\tau_{\text{CG}}$ for the model I(II). The mean fractional energy loss due to a single pair emission is $E^{-1}(dE/dx)/\Gamma \simeq 0.81(0.78)$ for model I(II), which is very similar to 0.78, the value given in [1]. The terminal energy of the superluminal neutrino is given by [1]

$$E_T = \left( \frac{5G_F^2}{192\pi^3} a'L \right)^{-1/5}. \quad (29)$$

For the OPERA baseline of 730 km, we have 13.9 GeV and 13.1 GeV for the models I and II, which is numerically very close to the value 12.5 GeV in [1].

5 Conclusions

We calculated the decay rate and the energy loss of a superluminal neutrino in two models where the Lorenz violation is introduced in the kinetic term of the neutrino in the action. We found that due to the change in the form of the solutions of the free field (Weyl) equation for neutrinos, the modified spin sum rules must be used for the calculation of the matrix element. We also found that the final result depends explicitly on the form of the Lorentz violation in the action. In the analyzed models, the energy loss by electron-positron bremsstrahlung still makes the models incompatible with the observation of high energy neutrinos at OPERA (and ICARUS as well as IceCube [1, 24]), but it advises us for a very careful calculation of the neutrino decay rate in more complicated models (for example, models with energy dependent velocity or modified velocities of electrons). A model with cancellation of $\delta^3$ contribution in the decay rate (if it exists) may evade the neutrino decay constraint.

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