Vacua and Gaugings of Maximal $D = 4$ Supergravity

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Abstract. I review an effective method to find vacua of gauged supergravity and apply it to the maximal theory in four dimensions. This leads in particular to new Anti de Sitter and Minkowski vacua with broken supersymmetry, as well as new gauge theories. Interesting links between different gaugings, their vacuum solutions and the stability properties as well as the allowed values of the cosmological constant are outlined.

1. Introduction
Maximal supergravity in 4 dimensions has received a lot of attention because of its unique matter content and its special properties. Despite the fact that the absence of chiral gauge interactions makes the theory less interesting for phenomenology, it is still at the centre of current investigations because of its possible perturbative finiteness and because its gaugings may be related via the AdS/CFT correspondence to the effective theories of stacks of M2-branes and hence find applications on strongly coupled gauge theories and condensed matter systems. Moreover, a systematic study of gauge maximal supergravity and its vacuum solutions deserves attention as it can help to understand why it is so difficult to build supersymmetric models giving rise to a vanishing and especially positive cosmological constant, and what ingredients in the compactification scheme may or may not help in such task.

Based on the results of the main paper [1], we discuss here how by using the embedding tensor formalism [2] for gauged supergravity and by taking advantage of the fact that the scalar manifold is a homogeneous space, a classification of the possible gauge models and their vacuum solutions in maximal supergravity can be simplified. Indeed, so far only a handful of analytical solutions has been found [3] and only recently numerical techniques have allowed for a more systematic scan of the vacua of the SO(8) theory [4]. Moreover, until now the only known way to break $\mathcal{N} = 8$ supergravity on a Minkowski vacuum has been the Scherk–Schwarz model or equivalently the reduction on twisted tori [5, 6], which lead to flat gauge groups of the type $U(1) \ltimes T^r$. We will provide here some new examples.

Although a full classification of both gaugings and vacua of $\mathcal{N} = 8$ supergravity in four dimensions might still be a too ambitious goal and would probably require numerical computations, we will show how the methods applied here allow us to easily identify both new gaugings and interesting vacuum solutions.

2. Gauged maximal supergravity
The bosonic content of maximal $D = 4$ supergravity is given by the graviton, 28 vector fields $A_\mu^A$ and 70 scalars $\phi$ parametrizing the coset space $E_{7(7)}/SU(8)$. The ungauged theory can
be seen as arising from the compactification of Type II or 11 dimensional supergravity on a torus. In that case the vectors are abelian gauge fields, but no matter is charged under the corresponding U(1)$^{28}$. \text{E}_{7(7)}(\mathbb{R})$ acts on scalar fields as isometry transformations and on vectors as electromagnetic duality, which we recall is only a symmetry at the level of the equations of motion. In order to make this explicit, one can introduce “magnetic dual” vector fields $A_{\mu\Lambda}$ to form the $56$ of \text{E}_{7(7)}$: $A_{\mu} = (A_{\mu}^{A}, A_{\mu\Lambda})$. Of course only half of these vectors are physical fields. Gauged deformations of this theory are obtained when a subgroup of its global symmetries is promoted to local. This is most effectively achieved by means of the embedding tensor formalism \cite{2}, which allows to completely encode the choice of gauging in a single object and to treat all gaugings at the same footing in a formally \text{E}_{7(7)}$-covariant fashion. We will denote the generators of $\text{E}_{7(7)}$ by $t_{\alpha}$, $\alpha = 1, \ldots 133$. A subgroup $G_{\text{gauge}}$ of $\text{E}_{7(7)}$ can then be gauged by introducing the embedding tensor $\Theta^{\alpha}_{M}$, so that for instance the gauge covariantization of derivatives is given by

$$D_{\mu} \equiv \partial_{\mu} - g^{\alpha}_{\mu} \Theta^\alpha_{M} t_{\alpha}, \quad t_{\alpha} \in \mathfrak{e}_{7(7)}. \quad (1)$$

The matrix $\Theta^\alpha_{M}$ encodes how the algebra $\mathfrak{g}_{\text{gauge}}$ is embedded in $\mathfrak{e}_{7(7)}$, as well as what linear combinations of the vector fields $A_{\mu\Lambda}^{\mu}$ are used to build the gauge connection. Note that also ‘magnetic’ vector fields $A_{\mu\Lambda}$ can take part in the gauge connection, and one can even gauge a group which is a symmetry only at the level of the equations of motion\footnote{however there always exist symplectic frames in which $G_{\text{gauge}}$ is a symmetry of the action and magnetic fields do not appear in the gauge connection, namely $\Theta^{\alpha\Lambda} = 0$.}. In any case, as only 28 vectors are physical, the rank of $\Theta^\alpha_{M}$ and hence the dimension of $G_{\text{gauge}}$ can be at most 28.

The embedding tensor must satisfy a set of algebraic constraints in order to define a consistent gauging. We define gauge generators $X_{M} \equiv \Theta^\alpha_{M} t_{\alpha}$ and will use in particular those in the $56$ representation of $\text{E}_{7(7)}$: $X_{MN} \equiv \Theta^\alpha_{M}[t_{\alpha}]N^{P}$. The constraints are:

$$\Theta^\alpha_{M} \Theta^\beta_{N} \Omega^{MN} = 0, \quad [X_{M}, X_{N}] = -X_{MN}^{P}X_{P}, \quad X_{MN} = 0, \quad (t^{\alpha}t_{\alpha})_{M}^{N} \Theta^{\alpha}_{N} = -\frac{1}{2} \Theta^{\beta}_{M}. \quad (2)$$

The quadratic constraints guarantee locality of the theory and closure of the gauge algebra, while the linear ones are imposed by supersymmetry and force the embedding tensor to belong to the $912 \subset 56 \times 133$ representation of $\text{E}_{7(7)}$.

### 3. Finding vacua of gauged maximal supergravity

The scalar potential that arises in gauged maximal supergravity is of course also parametrized by the embedding tensor. We denote the representatives of the coset space $\text{E}_{7(7)}/\text{SU}(8)$ by $L(\phi)_{MN}$, which depend on the 70 scalar fields $\phi$. We can write the scalar potential as:

$$V(\phi) = \frac{g^{2}}{672} \left( X_{MP} X_{NQ} \mathcal{M}(\phi)^{MN} \mathcal{M}(\phi)^{PQ} \mathcal{M}(\phi)_{RS} + 7X_{MP} X_{NR} \mathcal{M}(\phi)_{MN} \right), \quad (3)$$

where $\mathcal{M}_{MN} \equiv (LL^{T})_{MN}$ and $\mathcal{M}^{MN}$ is its inverse. Note in particular that $V(\phi)$ and $\mathcal{M}(\phi)$ are invariant under the local SU(8) transformations that define the coset. Studying this form of the scalar potential is very difficult, given that the dependence on scalar fields is at best given by a combination of exponentials and polynomials in 70 variables. Moreover one would have to study $V(\phi)$ separately for each possible gauging. We observe instead that the scalar potential only depends on the contraction of the embedding tensor with the coset representatives: $V(\phi, \Theta) = V(L(\phi)^{-1}\Theta)^{\phi}_{\Theta}$. As the scalar manifold is a homogeneous space, we can map any point (and hence any extremum of the potential) to the origin $\phi = 0$ by some $\text{E}_{7(7)}$ element $U$, provided

\footnote{we write $L(\phi)^{-1}\Theta$ as a shorthand notation for $L(\phi)^{-1}_{M} \Theta_{N}^{\alpha} L(\phi)^{-1}_{N} \beta^{\alpha}$}
that we redefine $\Theta$ by the action of the same transformation. Therefore, given an appropriate $U$ we can always write

$$V(\phi, \Theta) = V(0, U^{-1} \Theta), \quad U \in E_7(7).$$

(4)

At such point, the scalar potential is a simple quadratic function of the embedding tensor and the problem of finding extrema is reduced to quadratic conditions on $\Theta$. We have therefore translated the search for extrema of $V(\Theta)$ into the study of the dependence of $V(\Theta) \equiv V(\phi = 0, \Theta)$ on the $E_7(7)$ orbits of the embedding tensor. Moreover, it is also possible to study the vacua of whole classes of gaugings together, by constructing embedding tensors that satisfy the constraints (2) as well as the stationary point conditions of $V(\Theta)$.

4. Gauge groups in $SL(8, \mathbb{R})$

We choose to consider the class of gaugings contained in $SL(8, \mathbb{R})$, which can be taken as the global symmetry of the action by an appropriate choice of symplectic frame. Under $SL(8)$, the embedding tensor (which we recall sits in the $912$ of $E_7(7)$) decomposes in the irreducible representations: $912 \to 36 + 36' + 420 + 420'$. The $36$ and $36'$ in particular can be represented by symmetric matrices $\xi_{AB}$ and $\theta_{AB}$ respectively, with $A, B, \ldots = 1, \ldots 8$. The corresponding form of the embedding tensor is:

$$\Theta_{AB, CD} \propto \delta_{[A} \theta_{B]} D, \quad \Theta^{AB, CD} \propto [A \xi_{B]} C.$$  

(5)

The quadratic constraints when only these terms of the embedding tensor are switched on reduce to the conditions:

$$\xi = c\theta^{-1}, \quad c \in \mathbb{R}, \quad \text{or} \quad \theta \xi = 0.$$  

(6)

When $\xi = 0$ one recovers the well-know class of $SO(p, q)$ and $CSO(p, q, r)$ gaugings of maximal $D = 4$ supergravity $[2, 8]$, with $p + q + r = 8$. They are classified by the positive, negative and vanishing eigenvalues of $\theta_{AB}$, which plays the role of the quadratic invariant of the gauge algebra in the vector representation. In particular, $\theta = 1_8$ gives rise to the $SO(8)$ gauged supergravity. The $CSO(p, q, r)$ groups are contractions of the semi-simple ones and arise when $\theta$ is singular. These gaugings are electric in the symplectic frame of $SL(8)$ and also only involve electric vectors $A^\Lambda_{\mu} \equiv A^0_{[\Lambda \mu]}$. Turning on $\xi$, we can construct a different form of the $SO(p, q)$ gaugings and also obtain new non semisimple contractions when $\theta \xi = 0$:

$$G_{\text{gauge}} = (SO(p, q) \times SO(p', q')) \ltimes T^{(8-r)+(8-r')(r'+r-8)},$$  

(7)

the gauge algebra generated by $U^{-1} \Theta$ is generally only isomorphic to the original one and the vectors involved in the gauge connection may change as a consequence of the duality transformation.
where \( p + q + r = p' + q' + r' = 8 \). These are indeed new gaugings of maximal supergravity. They are still contained in SL(8), however now the gauge connection must involve magnetic vector potentials \( A_{\mu \lambda} = A_{\mu} [ A_{\lambda} ] \). A more detailed discussion of the semi-simple case \( \xi \propto \theta^{-1} \) is given in [9].

As explained in the previous section, we can look for vacua of these classes of gauge theories by keeping the scalar fields to zero and studying the orbits of the embedding tensor under \( E_{7(7)} \). However, the full orbits will necessarily involve the introduction of the \( 420 \) and \( 420' \) components of the embedding tensor. We then restrict to studying the submanifold given by the coset space \( SL(8)/SO(8) \), generated by the real parts of the scalar fields. Even if this analysis is not going to be exhaustive, it still allows us to find new interesting results. Taking \( \phi = 0 \), a direct calculation of the potential gives

\[
V(\theta', \xi') = \frac{1}{8} \text{Tr}(\theta'^2) - \frac{1}{16}(\text{Tr} \theta')^2 + \frac{1}{8} \text{Tr}(\xi'^2) - \frac{1}{16}(\text{Tr} \xi')^2,
\]

and the stationary point condition can be cast in the form:

\[
2(\theta'^2 - \xi'^2) - (\theta \text{Tr} \theta - \xi \text{Tr} \xi) \propto 1_8.
\]

In other words, the traceless part of \( 2(\theta'^2 - \xi'^2) - (\theta \text{Tr} \theta - \xi \text{Tr} \xi) \) must vanish. The problem of finding vacua is now formulated in terms of the eigenvalues of \( \theta \) and \( \xi \). Although a full scan of the resulting vacuum solutions will likely still require numerics, we find a set of new analytic solutions, summarized in Table 1, where the spectra for all scalar fields are also included. All anti de Sitter vacua turn out to be unstable as they do not satisfy the Breitenlohner–Freedman bound. Minkowski vacua break supersymmetry completely. We refer to the main paper for a more thorough discussion of the results and presentation of all the explicit solutions. By performing a suitable symplectic transformation, one can show that the \( SO(8) \) vacua included here were known [1] and belong to the standard \( SO(8) \) theory, where the fields taking a vev are pseudoscalars, which also parametrize a (different) \( SL(8)/SO(8) \) coset space.

| # | \( G_{\text{gauge}} \) | \( G_{\text{res}} \) | \( \Lambda_{\text{cosm}} \) | \( m^2 \) (multiplied) |
|---|---|---|---|---|
| vi | SO(2, 6) | SO(2) \( \times \) SO(6) | Mink | \( 2^{(2)}, \frac{1}{2} \left( 20 \right), 0 \left( 48 \right) \) |
| xii | SO(4) \( \times \) SO(2, 2) \( \times \) \( T^{16} \) | SO(2) \( \times \) SO(4) | Mink | \( 4^{(4)}, 2^{(12)}, 1^{(16)}, 0^{(28)} \) |
| xi | SO(7, 1) | SO(7) | AdS | \( 2^{(1)}, -\frac{4}{5} \left( 27 \right), -\frac{2}{5} \left( 35 \right), 0 \left( 7 \right) \) |
| vii | SO(3, 5) | SO(3) \( \times \) SO(5) | dS | \( 2^{(3)}, -\frac{4}{3} \left( 14 \right), -\frac{2}{3} \left( 5 \right), 0 \left( 15 \right) \) |
| viii | SO(7) \( \times \) \( T^{7} \) | SO(6) | AdS | \( 2^{(2)}, -1 \left( 20 \right), -\frac{1}{4} \left( 20 \right), 0 \left( 28 \right) \) |
| vi | SO(7, 1) | SO(7) \( \times \) \( T^{7} \) | SO(6) \( \times \) SO(1, 1) \( \times \) \( T^{12} \) | SO(5) | AdS | \( 2^{(3)}, -\frac{4}{3} \left( 14 \right), -\frac{2}{3} \left( 5 \right), 0 \left( 48 \right) \) |
| vii | SO(3, 5) | SO(3) \( \times \) SO(5) | dS | \( 2^{(1)}, 4^{(5)}, 2^{(10)}, 4^{(14)}, -\frac{2}{3} \left( 5 \right), 0 \left( 15 \right) \) |

Table 1. Mass spectra and residual symmetries for the new vacua. Known solutions of \( \text{CSO}(2, 0, 6) \) and \( \text{SO}(8) \) theories are given for reference. When \( \Lambda \neq 0 \), masses are normalized with respect to it, while we set \( g = 1 \). Supersymmetry is completely broken in all vacua.

One important thing we notice from Table 1 is that vacua belonging to different theories seem to share the same properties, at least up to the quadratic order in the expansion around
the vacuum solution. Indeed, one should expect that multiplicities in the mass spectrum fall into representations of the residual gauge symmetry group. However, even the sign of the cosmological constant and the values of the masses turn out to coincide (for AdS vacua we actually refer to the ratio of the masses over the cosmological constant). For instance the SO(6, 2) shares the same spectrum of the CSO(2, 0, 6) ≃ U(1) × T^{12} theory, which is the Scherk–Schwarz model with 4 identical parameters \[5\]. This is also true for vector and fermion spectra. However beyond the quadratic level the two models are not identical, as moving around the moduli space of the SO(6, 2) solution one finds instabilities, which are absent in the SS models. Moreover, moving along some flat directions in the SO(6, 2) theory till the boundary of the moduli space allows us to recover vacua which reflect the gauge structure of another model. To give an example and in order to show the explicit form of at least some of the new vacua, we can parametrize two flat directions of the SO(6, 2) vacuum as:

\[
\theta = \xi^{-1} = \text{diag}(-s, -s, s, s, u/s, u/s, 1/us, 1/us), \quad s, u \geq 0.
\]

By taking the singular limits \( g \sim s \rightarrow 0 \), \( g \sim u \rightarrow 0 \) and \( g \sim s, s \sim u \rightarrow 0 \) one recovers the other three Minkowski vacua in the table. More properties of the mass spectra of these vacua are discussed in \[10\].

As a last, more general, observation, we note that the scalar potential \[3\] taken at \( \phi = 0 \), in a symplectic frame where \( \mathcal{M}(0) = 1_{56} \) and choosing orthonormal generators \( t_\alpha \), can be written as

\[
V(\Theta) = \frac{g^2}{8g_\mathfrak{g}} \Theta_M^\alpha \Theta_M^\beta (\delta_\alpha^\beta + 7\eta_\alpha^\beta),
\]

where \( \eta_{\alpha\beta} \) is the Killing–Cartan metric on \( \mathfrak{e}_{7(7)} \), with negative eigenvalues for compact generators. \( V(\Theta) \) is clearly not \( \mathfrak{e}_{7(7)} \) invariant. One sees that whenever the vectors of the gauge connection all have the same normalization, the counting of compact and non-compact generators in \( g_{\text{gauge}} \) suffices to identify the sign of the cosmological constant. Although this argument is spoiled when acting with a general \( \mathfrak{e}_{7(7)} \) transformation on \( \Theta \) (and indeed, for example, the SO(8) potential is not negative definite), we observe that all the vacua known so far respect the pattern predicted by this simple observation.

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