A Simple Comptonization Model

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Received 2009 March 1; accepted 2009 September 23; published 2009 October 16

ABSTRACT. We present an empirical model of Comptonization for fitting the spectra of X-ray binaries. This model, named SIMPL, has been developed as a package implemented in XSPEC. With only two free parameters, SIMPL is competitive as the simplest model of Compton scattering. Unlike the pervasive standard power-law model, SIMPL incorporates the basic features of Compton scattering of soft photons by energetic coronal electrons. Using a simulated spectrum, we demonstrate that SIMPL closely matches the behavior of physical Comptonization models that consider the effects of optical depth, coronal electron temperature, and geometry. We present fits to RXTE spectra of the black hole transient H1743–322 and a BeppoSAX spectrum of LMC X–3 using both SIMPL and the standard power-law model. A comparison of the results shows that SIMPL gives equally good fits, while eliminating the troublesome divergence of the standard power-law model at low energies. SIMPL is completely flexible and can be used self-consistently with any seed spectrum of photons. We show an example of how SIMPL—unlike the standard power law—teamed up with DISKBB (the standard model of disk accretion) provides a uniform disk normalization that is unaffected by moderate Comptonization.

1. INTRODUCTION

Spectra of X-ray binaries typically consist of a soft (often blackbody or bremsstrahlung) component and a higher-energy tail component of emission, which we refer to generically as a “power law” throughout this work. The origin of the power-law component in both neutron star and black hole systems is widely attributed to Compton upscattering of soft photons by coronal electrons (White et al.1995; Remillard & McClintock 2006, hereafter RM06). While this interpretation is not unique, in this work, we adopt the prevailing view that Compton scattering is the mechanism that generates the observed power law. This component is present in the spectra of essentially all X-ray binaries, and it occurs for a wide range of physical conditions.

The tail emission is generally modeled by adding a simple power-law component to the spectrum, e.g., via the model POWERLAW in the widely used fitting package XSPEC (Arnaud 1996). A few of the many applications where power-law models are employed include: modeling the thermal continuum (Shafee et al. 2006) or the relativistically-broadened Fe K line (Miller et al. 2008) in order to obtain estimates of black hole spin; modeling the surrounding environment of compact X-ray sources, such as a tenuous accretion-disk corona (White & Holt 1982) or a substantial corona that scatters photons up to MeV energies (Gierliński et al. 1999); and classifying patterns of distinct X-ray states, e.g., in black hole binaries (RM06).

Because of the importance of the power-law component, several physical models have been developed to infer the conditions of the hot plasma that causes the Comptonization. Models of this variety that are available in XSPEC are COMPXT (Titarchuk 1994), EQPAIR (Coppi 1999), COMPBB (Farinelli et al. 2008), BMC (Titarchuk et al. 1997), COMPBB (Nishimura et al. 1986), THCOMP (Życki et al. 1999), COMPLS (Lamb & Sanford 1979), and COMPS (Poutanen & Svensson 1996). It is essential to use such physical models when one is focused on understanding the physical conditions and structure of a scattering corona or other Comptonizing plasma.

Often, however, the physical conditions of the Comptonizing medium are poorly understood or are not of interest, and one is satisfied with an empirical model that seeks to match the data with no pretense that the model can sufficiently discern the underlying physics. The model POWERLAW is one such empirical model which has been extraordinarily widely used in modeling black hole and neutron star binaries (see text and references in White et al. 1995; Tanaka & Lewin 1995; RM06) and AGN (e.g., Zdziarski et al. 2002; Brenneman & Reynolds 2006). However, POWERLAW introduces a serious flaw: at low energies it rises without limit. The divergence at low energies, which is not expected for Comptonization, significantly corrupts the parameters returned by the model component with which it is teamed (e.g., the widely used disk-blackbody component DISKBB; § 3).
An excellent alternative to the standard power-law model for describing Compton scattering is given by convolution using a scattering Green’s function, formulated decades ago (Shapiro, Lightman & Eardley 1976; Rybicki & Lightman 1979; Sunyaev & Titarchuk 1980; Titarchuk 1994). In this approach, the power law is generated self consistently via Compton upscattering of a seed photon distribution; consequently, the power law naturally truncates itself as the seed distribution falls off at low energies.

In this article, we present our implementation of a flexible convolution model named SIMPL that can be used with any spectrum of seed photons. For a Planck distribution we show that SIMPL gives identical results to BMC, as expected since the two models are functionally equivalent (§ 2.3). Although SIMPL has only two free parameters, the same number as the standard POWERLAW, this empirical model is nevertheless able to very successfully fit data simulated using COMPTT, a prevalent physical model of Comptonization (§ 2.2). We analyze data for two black hole binaries and illustrate the flexibility of SIMPL by convolving SIMPL with DISKBB, the workhorse accretion disk model which has been used for decades (Mitsuda et al. 1984). The principal result is that SIMPL in tandem with DISKBB enables one to obtain values for the disk-normalization parameter for more heavily Comptonized data that are consistent with those found for weakly-Comptonized data (see § 3.2). The standard power law, on the other hand, delivers very inconsistent normalization values. This is shown in greater detail in Steiner et al. (2009).

In § 2 we outline the model and in § 3 we present a case study with several examples. We discuss a prospective application of the model in § 4 and conclude with a summary in § 5.

2. THE MODEL: SIMPL

The model SIMPL (SIMple Power Law) functions as a convolution that converts a fraction of input seed photons into a power law (see eq. [1]). The model is currently available in XSPEC1. In addition to SIMPL-2, which is our implementation of the classical model described by Shapiro et al. (1976) and Sunyaev & Titarchuk (1980), which corresponds to both up- and downscattering of photons, we offer an alternative “bare-bones” implementation in which photons are only upscattered in energy. The physical motivations behind the two versions of the model are described in § 2.1, and the corresponding scattering kernels—the Green’s functions—are given in equation (4) and equation (5), respectively.

The parameters of SIMPL and the standard POWERLAW model are similar. Their principal parameter, the photon index Γ, is identical. However, in the case of SIMPL, the normalization factor is the scattered fraction SC, rather than the photon flux. The goal of SIMPL is to characterize the effects of Comptonization as simply and generally as possible. In this spirit, all details of the Comptonizing medium, such as its geometry (slab vs. sphere) or physical characteristics (optical depth, temperature, thermal vs. nonthermal electrons), which would require additional parameters for their description, are omitted.

It is appropriate to employ SIMPL when the physical conditions of the Comptonizing medium are poorly understood or are not of interest. When the details of the Comptonizing medium are known, or are the main object of study, one should obviously use other models (e.g., COMPTT, COMPSS, THCOMP, etc.), which are designed specifically for such work. SIMPL, on the other hand, is meant for those situations in which a Compton power-law component is present in the spectral data and needs to be included in the model but is not the primary focus of interest. SIMPL should thus be viewed as a broad-brush model with the same utility as POWERLAW but designed specifically for situations involving Comptonization.

By virtue of being a convolution model, SIMPL mimics physical reprocessing by tying the power-law component directly to the energy distribution of the input photons. The most important feature of the model is that it produces a power-law tail at energies larger than the characteristic energy of the input photons, and that the power law does not extend to lower energies. This is precisely what one expects any Compton-scattering model to do and is a general feature of all the physical Comptonization models mentioned. In contrast, the model POWERLAW simply adds to the spectrum a pure power-law component that reaches all the way downward to arbitrarily low energies. The difference between SIMPL and POWERLAW is thus most obvious at soft X-ray bands where SIMPL cuts off in a physically natural way, as appropriate for Comptonization, whereas POWERLAW continues to rise without limit (e.g., see Yao et al. 2005).

Two assumptions underlie SIMPL. The first is that all soft photons have the same probability of being scattered (e.g., the Comptonizing electrons are distributed spatially uniformly). This is a reasonable assumption when one considers that, even in the best of circumstances, almost nothing is known about the basic geometry of the corona. For example, usually the corona is variously and crudely depicted as a sphere, a slab, or a lamp post. The second assumption is that the scattering itself is energy independent. This is again reasonable given the soft thermal spectra of the seed photons that are observed for black hole and neutron star accretion disks, with typical temperatures of ~1 keV and a few keV, respectively. For example, in the extreme case of a 180° backscatter off a stationary electron, a 3 keV seed photon suffers only a 1% loss of energy, and even a 10 keV photon loses only 4% of its initial energy.

Figure 1 shows sample outputs from SIMPL when the input soft photons are modeled by the multitemperature disk black-body model DISKBB (Mitsuda et al. 1984). Results are shown for both SIMPL-2 and SIMPL-1, our alternative version of SIMPL that includes only upscattering of photons; the spectra are shown for Γ = 2.5 and a range of values of SC. Note the power-law tails in the model spectra at energies above the peak.

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1 See http://heasarc.nasa.gov/xanadu/xspec/manual/XSmodelSimpl.html.
of the soft thermal input and the absence of an equivalent power-law component at lower energies. This is the primary distinction between SIMPL and POWERLAW. SIMPL-2 and SIMPL-1 give similar spectra, but the spectrum from SIMPL-1 has a somewhat stronger power-law tail for the same value of $f_{SC}$. This is because SIMPL-1 transfers all the scattered photons to the high-energy tail, whereas SIMPL-2 has double-sided scattering. Therefore, for the same value of $f_{SC}$, fewer photons are scattered into the high-energy tail with SIMPL-2. Correspondingly, when fitting the same data, SIMPL-2 returns a larger value of $f_{SC}$ compared to SIMPL-1 (for examples, see § 3 and Table 2).

### 2.1. Green’s Functions

Given an input distribution of photons $n_{\text{in}}(E_0)dE_0$ as a function of photon energy $E_0$, SIMPL computes the output distribution $n_{\text{out}}(E)dE$ via the integral transform:

$$n_{\text{out}}(E)dE = (1 - f_{SC})n_{\text{in}}(E)dE + f_{SC} \left[ \int_{E_{\text{min}}}^{E_{\text{max}}} n_{\text{in}}(E_0) G(E; E_0) dE_0 \right] dE.$$  

(1)

A fraction $(1 - f_{SC})$ of the input photons remains unscattered (the first term on the right), and a fraction $f_{SC}$ is scattered (the second term). Here, $E_{\text{min}}$ and $E_{\text{max}}$ are the minimum and maximum photon energies present in the input distribution, and $G(E; E_0)$ is the energy distribution of scattered photons for a δ-function input at energy $E_0$, i.e., $G(E; E_0)$ is the Green’s function describing the scattering.

Equation (1) assumes that every photon ultimately escapes to infinity, either unscattered (the first term on the right-hand side) or after Compton scattering (the second term). In the context of a disk-corona model we note that as much as half the scattered photons (the exact fraction depends on geometry) are redirected toward the disk. Computing the fate of these photons is the goal of sophisticated reflection models, e.g., REFLION (Ross & Fabian 2005) and PEXRAV (Magdziarz & Zdziarski 1995). Equation (1) ignores all these details and simply assumes that all the photons that return to the disk are effectively scattered from the surface with no change in energy. In the opposite extreme, we may wish to assume that all the returning photons are fully absorbed and thermalized. In this limit, we would replace equation (1) with

$$n_{\text{out}}(E)dE = (1 - f_{SC})n_{\text{in}}(E)dE + (f_{SC}/2) \left[ \int_{E_{\text{min}}}^{E_{\text{max}}} n_{\text{in}}(E_0) G(E; E_0) dE_0 \right] dE.$$  

(2)

Clearly, the real situation is somewhere in between (1) and (2). The version of SIMPL currently implemented in XSPEC makes use of equation (1), though it would be straightforward to change it to equation (2).

We now describe the specific prescriptions we use for SIMPL-2 and SIMPL-1. We also discuss the physical motivations behind these prescriptions, drawing heavily on the theory of Comptonization as described by Rybicki & Lightman (1979) (hereafter RL79).

#### 2.1.1. SIMPL-2

In § 7.7, RL79 discuss the case of unsaturated repeated scattering by nonrelativistic thermal electrons. Following Shapiro et al. (1976), they solve the Kompaneets equation and show that Comptonization produces a power-law distribution of photon energies (eq. 7.76d in RL79). There are two solutions for the photon index $\Gamma$:

$$\Gamma_1 = -\frac{1}{2} + \sqrt{\frac{9}{4} + \frac{4}{y}}$$

$$\Gamma_2 = -\frac{1}{2} - \sqrt{\frac{9}{4} + \frac{4}{y}}$$

where the Compton $y$ parameter is given by $y = (4kT_e/m_e c^2) \text{Max}(\tau_{es}, \tau_{es}^2)$. Here, $kT_e$ is the electron temperature and $\tau_{es}$ is the optical depth to electron scattering. Upscattered photons have a power-law energy distribution with photon index $\Gamma_1$ and downscattered photons have a different power-law distribution with photon index $\Gamma_2$.

We model this case of nonrelativistic electrons with the following Green’s function (Sunyaev & Titarchuk 1980; Titarchuk 1994; Ebisawa 1999), which corresponds to the model SIMPL-2:
\[ G(E; E_0)dE = \frac{(\Gamma - 1)(\Gamma + 2)}{(1 + 2\Gamma)} \times \left\{ \begin{array}{ll}
(\frac{E}{E_0})^{-\Gamma}dE/E_0, & E \geq E_0 \\
(\frac{E}{E_0})^{\Gamma+1}dE/E_0, & E < E_0.
\end{array} \right. \] (3)

The function is continuous at \( E = E_0 \), is normalized such that it conserves photons, and holds for all \( \Gamma > 1 \). Substituting (3) in (1), we see that \textsc{simpl}-2 has two parameters: \( f_{\text{SC}} \) and \( \Gamma \). Note that although the model makes use of two power laws, their slopes are not independent.

As in the case of the standard power law, \textsc{simpl} includes no high-energy cutoff. Technically, for any complete model of Comptonization, the upscattered power-law distribution is cut off for photon energies larger than \( kT_e \). To avoid increasing the complexity of our model, we have ignored this detail; extra parameters could easily be added to account for high-energy attenuation if desired. By keeping the model very basic, \textsc{simpl} is a direct two-parameter replacement for the standard power law while bridging the divide between the latter model and physical Comptonization models.

### 2.1.2. \textsc{simpl}-1

The Green’s function (3) is obtained by solving the Kompaneets equation, which assumes that the change in energy of a photon in a single scattering is small. This assumption is not valid when the Comptonizing electrons are relativistic.

In § 7.3 of their text, RL79 discuss Compton scattering by relativistic electrons with a power-law distribution of energy:

\[ n_e(E_e)dE_e \propto E_e^{-\Gamma}dE_e. \]

In the limit when the optical depth is low enough that we only need to consider single scattering, they show that the Comptonized spectral energy distribution (SED) is a power law of the form \( P(E)dE \propto E^{-\Gamma}. \) Equivalently, the photon energy distribution takes the form \( n(E)dE \propto E^{-\Gamma} \), with a photon index \( \Gamma = (p + 1)/2 \). Hardly any photons are downscattered in energy.

In their § 7.5, RL79 show that repeated scatterings produce a power-law SED even when the relativistic electrons have a non-power-law distribution (see also Titarchuk & Lyubarskij 1995). In terms of \( \tau_{\text{ee}} \) and the mean amplification of photon energy per scattering \( A \), the Comptonized photon energy distribution takes the form \( n(E)dE \propto E^{-\Gamma} \) with a photon index \( \Gamma = 1 - \ln \tau_{\text{ee}}/\ln A \). For the specific case of a thermal distribution of electrons with a relativistic temperature \( kT_e > m_ec^2 \), the amplification factor is given by \( A = 16(kT_e/m_ec^2)^3 \). Once again, hardly any photons are downscattered.

For both cases discussed above, Comptonization is dominated by upscattering and produces a nearly one-sided power-law distribution of photon energies. This motivates the following Green’s function, valid for \( \Gamma > 1 \), which we refer to as the model \textsc{simpl}-1:

\[ G(E; E_0)dE = \begin{cases} 
(\Gamma - 1)(E/E_0)^{-\Gamma}dE/E_0, & E \geq E_0 \\
0, & E < E_0.
\end{cases} \] (4)

The normalization factor \((\Gamma - 1)\) ensures that we conserve photons.

Although \textsc{simpl}-1 is most relevant for relativistic Comptonization, it can also be used as a stripped-down version of \textsc{simpl}-2 for nonrelativistic coronae. The reason is that the low-energy power law \((E/E_0)^{\Gamma+1}\) in equation (3) almost never has an important role. There is not much power in this component, and what little contribution it makes is indistinguishable from the input soft spectrum. Therefore, even for the case of nonrelativistic thermal Comptonization, for which the Green’s function (3) is designed, there would be little difference if one were to use \textsc{simpl}-1 instead of \textsc{simpl}-2.

### 2.2. Comparison to \textsc{comptt}

To illustrate the performance of \textsc{simpl} relative to other Comptonization models, we have simulated a 2 \times 10^6-count \textit{BeppoSAX} (Boella et al. 1997) observation using the \textsc{comptt} model in XSPEC, version 12.4.0x.

For our source spectrum, we adopt disk geometry, a Wien distribution of seed photons at \( kT_0 = 1 \) keV, and a hydrogen column density of \( N_H = 10^{21} \) cm\(^{-2}\). We set the optical depth and temperature of the Comptonizing medium to \( \tau_c = 2 \) and \( kT_c = 40 \) keV. Our simulation uses the LECS, MECS, and PDS detectors on \textit{BeppoSAX}, which span a wide energy range ~0.1–200 keV (for details on the instruments, see § 3). The total number of counts in the simulated spectra (\sim 2 \times 10^9) corresponds to a 3 ks observation of a 1 Crab source.

We analyze the simulated data with a model consisting of a blackbody (BB) coupled with \textsc{simpl}. We refer to this model as \textsc{simpl@bb} (the \( \oplus \) is to emphasize that \textsc{simpl} represents a convolution). The best fits achieved have reduced chi-squared values of \( \chi^2_r = 1.00 \) (\textsc{simpl}-1) and \( \chi^2_r = 1.06 \) (\textsc{simpl}-2). The fitted BB temperatures are respectively 1.14 \pm 0.02 keV and 1.29 \pm 0.01 keV compared to 1 keV in the original \textsc{comptt} model. Figure 2 shows the fit using \textsc{simpl}-1 and Table 1 lists the best-fit parameters for both models.

In comparison, \textsc{comppbb}, an alternative model of Compton scattering that assumes slab geometry, fits our simulated spectrum comparably well as \textsc{simpl} with \( \chi^2_r = 1.05 \) (Table 1). \textsc{comppbb} returns the same temperature as \textsc{simpl}-2, \( kT_{bb} = 1.29 \pm 0.01 \) keV. Compared to the \textsc{comptt} progenitor, \textsc{comppbb} gives similar estimates of the coronal temperature \( kT_c \) and optical depth \( \tau_c \) (Table 1). Even though \textsc{comppbb} is a physically more realistic model of coronal scattering than \textsc{simpl}, it does not outperform \textsc{simpl} in terms of fitting the \textsc{comptt}-generated data. Meanwhile, the model BB + \textsc{powerlaw} performs quite poorly, yielding \( \chi^2_r > 2 \). Parameters for this fit are given in Table 1. Note that the derived \( N_H \) using \textsc{powerlaw} is much higher than either the original value or those from fits with \textsc{simpl}.
Fig. 2.—The data correspond to a simulated BeppoSAX observation with a total of \(2.1 \times 10^6\) counts; the spectrum was generated using \textsc{compTT}. The histogram shows the fit achieved using \textsc{simpl}.1. This fit is performed over the recommended energy ranges of the narrow-field instruments (NFI), as given by the Cookbook for BeppoSAX NFI Spectral Analysis, yielding \(\chi^2 = 1.00\). For details, see Table 1. This example demonstrates the ability of \textsc{simpl} to match a representative spectrum generated by a physical model of Comptonization.

Though \textsc{simpl} is a purely empirical model, we see that it can deliver a remarkably successful fit to data simulated using the physical model \textsc{compTT}. Even for a very cool corona with electron temperatures as low as \(kT_e = 20\) keV, which causes \textsc{compTT} to produce noticeable curvature in the high-energy spectrum, we find that \textsc{simpl} and \textsc{simpl} achieve reasonable fits with \(\chi^2 < 1.2\).

A significant virtue of \textsc{simpl} relative to the physical Comptonization models in XSPEC is that \textsc{simpl} can be employed in conjunction with any source of seed photons. The physical models, on the other hand, are typically restricted to treating only one or two predefined photon distributions. One standard choice of continuum model that is widely used in fitting Comptonized accretion disks is \textsc{diskbb} + \textsc{comptt}. With \textsc{simpl}, one would instead employ the model \textsc{simpl}×\textsc{diskbb}. The latter not only generates the power law self-consistently via upscattering of the seed photons, but it also has two fewer control parameters.

### 2.3. Bulk Motion Comptonization

The model \textsc{bmc} describes the Comptonization of black-body seed photons by a converging flow of isothermal gas that is freely falling toward a compact object, i.e., bulk motion Comptonization (see, e.g., Shrader & Titarchuk 1998; Titarchuk et al. 1997). \textsc{bmc} is an alternative to coronal Comptonization models and is structured \textit{identically} to \textsc{simpl} × \textsc{bb}; both models are specified with just four parameters. As a direct demonstration in XSPEC that \textsc{simpl} × \textsc{bb} and \textsc{bmc} are identical, we analyzed our simulated BeppoSAX spectrum described in § 2.2 using both models. We found that the returned values of the column density \(N_H\), the blackbody temperature \(kT\), and the photon index \(\Gamma\) agreed in each case to four or more significant figures.

\textsc{bmc} has been variously used to support claims that Compton scattering off infalling gas within several gravitational radii gives rise to the observed high-energy power law in several black hole binaries (e.g., Shrader & Titarchuk 1998, 1999; Borozdin et al. 1999). However, this is only one interpretation of the model; \textsc{simpl} × \textsc{bb}, and therefore \textsc{bmc}, can equally be used to support a more standard model of coronal scattering (operating with uniform efficiency at all energies, see § 2 and § 2.1.2). Thus, although \textsc{bmc} is designed specifically to model relativistic accretion inflows, its function is actually quite general.

A virtue of \textsc{simpl} is that it fully incorporates the utility of \textsc{bmc} while allowing complete flexibility in the choice of the spectrum of seed photons; e.g., \textsc{simpl}×\textsc{diskbb} is more appropriate for modeling Comptonization in accretion disks than \textsc{bmc}, which is hardwired to a Planck function.

The theory of bulk motion Comptonization is developed further and rigorously in Titarchuk et al. (1997). This paper

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**Table 1: Results of Fitting a Simulated \textsc{compTT} Spectrum**

| MODEL           | \(\chi^2/\nu\) | \(N_H\) (\(10^{22}\) cm\(^{-2}\)) | \(\Gamma\) | \(f_{SC}\) | Norm(PL)\(^a\) | \(kT_0\) (keV) | Norm\(^b\) | \(kT_e\) (keV) | \(\tau_e\) |
|----------------|----------------|----------------------------------|------------|----------|----------------|----------------|------------|----------------|----------|
| \textsc{compTT} | \ldots        | 0.1                              | \ldots     | \ldots   | \ldots         | 1.0            | 0.001      | 40.0           | 2.0       |
| \textsc{simpl} × \textsc{bb} | \ldots      | 1.00/731                         | 0.28 ± 0.01| 1.41 ± 0.02| 0.84 ± 0.01    | 1.142 ± 0.015 | 10.9 ± 0.4 | \ldots          | \ldots    |
| \textsc{simpl} × \textsc{bb} | \ldots      | 1.06/731                         | 0.31 ± 0.01| 1.37 ± 0.02| 0.87 ± 0.01    | 1.292 ± 0.01    | 7.8 ± 0.3  | \ldots          | \ldots    |
| \textsc{compph} | \ldots      | 1.05/731                         | 0.31 ± 0.01| \ldots    | \ldots         | 1.292 ± 0.01    | 19.7 ± 0.7  | 43.6 ± 2.2     | 2.21 ± 0.03|
| \textsc{bb} + \textsc{powerlaw} | \ldots     | 2.02/731                         | 0.68 ± 0.01| 1.00 ± 0.01| \ldots         | (5.0 ± 0.2) × 10\(^{-3}\) | 1.700 ± 0.008| 0.89 ± 0.02   | \ldots    |

\(^a\) \text{POWERLAW} normalization given at 1 keV in photons s\(^{-1}\) cm\(^{-2}\) keV\(^{-1}\).

\(^b\) BB and \textsc{compph} normalization = \(R/(D/\text{km})\)^\(^2\) for a blackbody of radius \(R\) at a distance \(D\); \textsc{compTT} normalization is undefined.

\(^c\) \textsc{compTT} model set to disk geometry (geometry switch = 1).
describes a Green’s function that is more appropriate than the one used in BMC. A complete version of this Green’s function is incorporated into the more sophisticated model COMP3B. However, this model is again limited to treating scattering from a predefined set of (blackbody-like) seed photon distributions and includes additional free parameters. We find that the fitting results obtained using this Green’s function are intermediate between those given by SIMPL-1 and SIMPL-2 so long as the temperature of the inflowing electrons, \( kT_e \), is above the observed energy range.

3. DATA ANALYSIS

In this section, we apply SIMPL to a sample of observations to illustrate how SIMPL compares with POWERLAW. To this end, we have selected two black hole binaries, H1743–322 and LMC X–3. H1743–322 (hereafter H1743) is an ideal black hole transient for this exercise (see Remillard et al. 2006) since, for much of its 2003 outburst, its spectrum can be satisfactorily modeled with just absorbed (\( N_H \approx 2.2 \times 10^{22} \) cm\(^{-2}\)) thermal-disk and power-law components (McClintock et al. 2009, hereafter M09). In particular, the 122 days of contiguous spectral data on which we focus do not require any additional components to accommodate the reflection or absorption features that are often present in the spectra of black hole binaries.

The spectra of H1743 were acquired by the Rossi X-Ray Timing Explorer (RXTE) PCU-2 module (Swank 1999), RXTE’s best-calibrated PCU detector, and were taken in “standard 2” format. All spectra have been background subtracted and have typical exposure times \( \sim 3000 \) s. A systematic error of 0.6% has been added to all energy channels. The resultant pulse-height spectra are analyzed from 2.8–25 keV using XSPEC (see M09 for further details).

While RXTE provides good spectral coverage in hard X-rays (\( \gtrsim 10 \) keV), which is most important for constraining the power-law component, it is not sensitive at low energies (\( \lesssim 2.5 \) keV). Therefore, RXTE data are generally insensitive to \( N_H \). To complement the RXTE observations presented here, we have selected a BeppoSAX observation of LMC X–3, a persistent and predominantly thermal black hole source with a very low hydrogen column (\( N_H \approx 4 \times 10^{20} \) cm\(^{-2}\); Page et al. 2003; Yao et al. 2005). In analyzing these spectra, we have left \( N_H \) free in order to best illustrate the systematic differences between fits achieved using SIMPL and POWERLAW.

The BeppoSAX narrow-field instruments provide sensitive measurements spanning a wide range in energy, from tenths to hundreds of keV. The low-energy concentrator system (LECS) and the medium-energy concentrator system (MECS) probe soft fluxes, from \( \sim 0.1–4 \) keV and \( \sim 1.5–10 \) keV, respectively. The phoswich detector system (PDS) is sensitive to hard X-rays from \( \sim 15–200 \) keV, and the high-pressure gas scintillation counter (HPGSPC) covers \( \sim 4–100 \) keV. In this analysis, we consider only the LECS, MECS, and PDS because the statistical quality of the HPGSPC data is relatively poor.

In reducing BeppoSAX data, we have followed the protocols given in the Cookbook for BeppoSAX NFI Spectral Analysis (Fiore et al. 1999). We use pipeline products and extract spectra from 8’ apertures centered on LMC X–3 for both the LECS and (combined) MECS detectors. For the PDS, which is a simple collimated phoswich detector, we selected the fixed rise-time spectrum. In our analysis, we have used standard response matrices and included blank-field background spectra with the appropriate scalings. No pile-up correction is necessary.

3.1. Steep Power-Law State

About a third of the way through its nine-month outburst cycle, H1743 repeatedly displayed spectra in the steep power-law (SPL) state that were devoid of absorption features. A salient feature of the SPL state is the presence of a strong power-law component of emission. (For a review of black hole spectral states and a precise definition of the SPL state, see Table 2 and text in RM06.)

Twenty-eight such featureless spectra were consecutively observed over a period of about three weeks (spectra 58–85; M09). We focus here on one representative spectrum, No. 77, whose spectral parameters (\( \Gamma, kT_e \), and both POWERLAW and DISKBB normalizations) are quite representative of the values observed for the other 27 spectra (see M09). In Figure 3 we show our fits and the associated unabsorbed models obtained using DISKBB + POWERLAW and SIMPL@DISKBB. Fitted spectral parameters are presented in Table 2.

The quality of fit (as measured by \( \chi^2 \)) using either model is comparable. Nevertheless, there are distinct differences between the models. The fits with SIMPL have a \( \sim 50\% \) larger disk normalization compared to POWERLAW and a \( \sim 40\% \) lower \( N_H \) (Table 2). The fit using POWERLAW diverges at low energies, as revealed by removing photoabsorption from the fitted models (panels on the right in Fig. 3). The effect is quite severe and has no obvious physical explanation. In contrast, the fit using SIMPL is well behaved and the unabsorbed model is not divergent.

3.2. Thermal Dominant State

The key feature of the thermal dominant (TD) state is the presence of a totally dominant and soft (\( kT_e \sim 1 \) keV) blackbody-like component of emission that arises in the innermost region of the accretion disk. The TD state is defined by three criteria, the most relevant of which here is that the fraction of the total 2–20 keV unabsorbed flux in the thermal component is \( \gtrsim 75\% \). For the full definition of this state, see Table 2 in RM06.

Here we have chosen H1743 spectrum 91, which belongs to a sequence of \( \sim 50 \) featureless spectra (spectra 86–136; M09) in the TD state. This spectrum has \( \Gamma \sim 2 \), which is somewhat harder than usual, but is otherwise typical of H1743’s TD state.
| Source State | Mission | Detector | MJD     | $\chi^2/\nu$ | $\frac{R_{\text{in}}}{D}$ (10$^{22}$ cm$^{-2}$) | $N_{\text{H}}$ | $\nu_{\text{max}}$ (keV) | $\nu L_{\text{disk}}/L_{\text{Edd}}$ | $a$ | $f_{\text{sc}}$ | $kT_\gamma$ (keV) | Norm | $\Gamma$ | Norm(PL)$^d$ |
|--------------|---------|----------|---------|--------------|---------------------------------|-----------|-------------------|--------------------------------|-----|-------------|-----------------|------|---------|----------------|
| H1743        | RXTE    |          | 52797.6 | 1.11/44      | 0.21                            | 1.89      | 0.11              | ...                           | ... | ...         | 1.189 ± 0.010  | 564 ± 19 | 2.64    | 9.77 ± 0.42  |
| SPL          | PCA     |          |         | 1.11/44      | 0.29                            | 1.12      | 0.10              | S1                           | ... | 0.169 ± 0.003 | 1.159 ± 0.008  | 867 ± 32 | ...      | ...            |
| H1743        | RXTE    |          | 52811.5 | 0.93/44      | 0.24                            | 1.50      | 0.10              | ...                           | ... | ...         | 1.107 ± 0.004  | 864 ± 23 | 0.51    | ...            |
| TD           | PCA     |          |         | 0.93/44      | 0.25                            | 1.44      | 0.10              | S1                           | 1.98 | 0.030 ± 0.001 | 1.105 ± 0.004  | 904 ± 24 | ...      | ...            |
| LMC X–3      | BeppoSAX |         | 50415.5 | 1.05/729     | 0.58                            | 0.073     | 0.008             | ...                           | ... | ...         | 1.279 ± 0.011  | 24.5 ± 0.8 | ...     | 0.055 ± 0.010 |
| TD           | LECS, MECS |        | 1.08/729 | 0.60         | 0.044 ± 0.003                   | S1        | 2.41 ± 0.45       | 0.062 ± 0.021                | ... | ...         | 1.238 ± 0.013  | 30.4 ± 1.2 | ...     | ...            |
| PDS          |         |          | 1.08/729 | 0.59         | 0.044 ± 0.003                   | S2        | 2.46 ± 0.48       | 0.085 ± 0.033                | ... | ...         | 1.239 ± 0.012  | 30.3 ± 1.1 | ...     | ...            |

**Note.**—All errors are presumed Gaussian and quoted at 1$\sigma$.

$^a$ Bolometric (0.1–20 keV) luminosity of the disk component in Eddington units. For H1743, we adopt nominal values: $M = 10 M_\odot$, $D = 9.5$ kpc, and $i = 60^\circ$. The fiducial values used for LMC X–3 are $M = 7.5 M_\odot$ and $i = 67^\circ$ (Cowley et al. 1983; Orosz 2003). For fits using SIMPL, this quantity describes the seed spectral luminosity.

$^b$ Version of SIMPL being used, i.e., S1 for SIMPL-1 and S2 for SIMPL-2.

$^c$ For an accretion disk inclined by $i$ to the line of sight, with inner radius $R_{\text{in}}$ at distance $D$, $N_{\text{H}} = \frac{(R_{\text{in}}/D)^2}{10^{22}} \cos i$.

$^d$ POWERLAW normalization given at 1 keV in photons s$^{-1}$ cm$^{-2}$ keV$^{-1}$.

$^e$ The cross-normalizations for $C_{\text{LM}}$ = LECS/MECS and $C_{\text{PM}}$ = PDS/MECS are fitted from 0.7–1 and 0.77–0.93 respectively. $C_{\text{LM}} = 0.802 \pm 0.283, 0.814 \pm 0.007, 0.813 \pm 0.008$ for the fits with POWERLAW, SIMPL-1, and SIMPL-2. $C_{\text{PM}}$ is pegged at 0.93 for the same fits.
Spectral fit results are shown in Figure 4. In addition, in order to further illustrate for the TD state the differences between SIMPL and POWERLAW at energies below the ≈2.5 keV response cutoff of RXTE, we use a BeppoSAX observation of LMC X–3; our results are illustrated in Figure 5. This observation was carried out on 1996 November 28 with exposure times of 1.8, 4.5, and 2 ks respectively for the LECS, MECS, and PDS.

As in § 3.1, we fit these data using DISKBB + POWERLAW and SIMPL⊗DISKBB. The best-fit spectral parameters are listed in Table 2. Due to a calibration offset between the various BeppoSAX instruments, we follow standard procedure and fit for the normalization of the LECS and PDS relative to the MECS, the best calibrated of the three. We adopt the canonical limits of 0.7–1 for LECS/MECS and 0.77–0.93 for PDS/MECS. These normalizations are included in the tabulated results.

A comparison of the results obtained with POWERLAW and SIMPL confirms the trends highlighted in § 3.1, namely the differences in normalization and $N_{\text{H}}$. However, they are more modest here because the Compton component is weaker in the TD state.

3.3. Comparison of SIMPL and POWERLAW

An examination of Table 2 reveals the following systematic differences in the derived spectral parameters returned when fitting with SIMPL versus POWERLAW: SIMPL yields (1) a stronger and softer thermal disk component, i.e., a larger normalization
and lower $kT_e$; (2) a generally steeper power-law component (larger $\Gamma$); and (3) a systematically lower $N_H$. As we now show, all of these effects can be simply understood.

Because POWERLAW produces higher fluxes than SIMPL at low energies, it tends to suppress the flux available to the (soft) thermal component, namely DISKBB in the examples given here. This explains why POWERLAW tends to harden the DISKBB component and to steal flux from it (i.e., reduce its normalization constant). Also, at low energies the POWERLAW component predicts high fluxes that, in order to conform to the observed spectrum, depress the value of $\Gamma$. These differences between SIMPL and POWERLAW are most pronounced when the power law is relatively steep, i.e., typically when $\Gamma \gtrsim 3$.

Modest and reasonable values of $N_H$ are returned in fits using SIMPL, as well as COMP TT and other Comptonization models, because the Compton tail is produced by the upscattering of seed photons and there is no power-law component at low energies. In contrast, POWERLAW continues to rise at low energies, which forces $N_H$ to increase in order to allow the model to fit the observed spectrum. This systematic difference is apparent in our fit results for the H1743 spectra and is especially prominent in the case of the LMC X-3 spectrum for which $N_H$ differs by a factor of 2. For H1743, the discrepancy in $N_H$ is much less for the TD spectrum than for the SPL spectrum because the SPL state has both a steeper and relatively stronger power-law component.

We turn now to consider the DISKBB normalization constant, which is proportional to $R_{in}^2$, the square of the inner disk radius (see footnotes to Table 2). For the pair of H1743 spectra, we see that the disk normalization obtained with POWERLAW is $\approx 33\%$ smaller in the SPL state than in the TD state (Table 2). With SIMPL, on the other hand, there is no significant change in the

Fig. 4.—Same as Fig. 3 except that the results shown here are for an RXTE observation of H1743 in the TD state. The systematic differences between the SIMPL and POWERLAW fits are greatly reduced compared to the differences shown for the SPL example in Fig. 3.
normalization, and hence both the SPL and TD states can be modeled with a disk that has the same radius. Although we present here only one example comparing TD and SPL states, a more detailed analysis may be found in Steiner et al. (2009). In that work, the full outburst of H1743 is analyzed with a relativistic disk model. Table 1 and Figure 2 there demonstrate that SIMPL is able to reconcile the derived values of the disk inner radius $R_{in}$ for TD and SPL state spectra, whereas POWERLAW performs quite poorly in this regard.

Kubota & Makishima (2004) similarly identified a constant normalization for the black hole binary XTE J1550–564 between the TD and SPL states in an analysis using the model DISKBB + THCOMP. Because THCOMP is implemented as an additive (i.e., nonconvolution) model, Kubota & Makishima had to employ a somewhat ad hoc procedure to obtain their result (see their Appendix). Their work improved upon a similar result obtained for black hole GRO J1655–40 (Kubota et al. 2001). With SIMPL, the modeling is significantly easier.

4. DISCUSSION

A standard method of classifying X-ray states in black hole binaries involves spectral decomposition into two primary components—a multitemperature blackbody disk, DISKBB, and a Compton power law, POWERLAW (RM06). This method is compromised by the use of the standard power law when the photon index is large ($\Gamma \gtrsim 3$). In this case, at low energies the flux from the power law can rival or exceed the thermal component. As discussed in § 2, intrusion of the Compton component at low energies is fundamentally inconsistent with Compton scattering.
This difficulty in classifying states using POWERLAW is remedied by the use of SIMPL, because the latter model naturally truncates the power-law component at low energies. It is useful to consider the intrinsic differences between the two models and how they influence the classification of black hole X-ray states. Using POWERLAW, the thermal disk and tandem Compton emission are modeled independently. On the other hand, under SIMPL, all photons originate in the accretion disk. Some of these disk photons scatter into a power law en route from the disk to the observer. As described in § 3.3, fits employing SIMPL imply stronger disk emission and weaker Compton emission than those using POWERLAW. As a result, state selection criteria would need to be modified for classification using SIMPL. This application is beyond the scope of the present paper.

5. SUMMARY

We present a new prescription for treating Comptonization in X-ray binaries. While no new physics has been introduced by this model, its virtues lie in its simplicity and natural application to a wide range of neutron star and black hole X-ray spectra. SIMPL offers a generic and empirical approach to fitting Comptonized spectra using the minimum number of parameters possible (a normalization and a slope), and it is valid for a broad range of geometric configurations (e.g., uniform slab and spherical geometries). The scattering of a seed spectrum occurs via convolution, which self-consistently mimics physical reprocessing of photons from, e.g., an accretion disk. SIMPL has only two parameters, so it is straightforward to use it in place of POWERLAW whenever one is dealing with Comptonization.

Our model is valid for all $\Gamma > 1$. We have shown that SIMPL is able to provide a good fit to a demanding simulated data set, which was generated with the widely used Comptonization model COMPORD. Furthermore, we have demonstrated that SIMPL and POWERLAW give very comparable $\chi^2/\nu$ when fitting spectral data (see Table 2). This quality of performance holds true not only for spectra with weak Compton tails (TD state) but also for spectra requiring a large Compton component (SPL state). In the latter case, the model based on SIMPL gives physically more reasonable results for the soft end of the spectrum (e.g., see § 3.3).

Using SIMPL⊗DISKBB, it will be important to revisit the classification of black hole states (RM06) for two reasons. First, the selection of TD data will no longer be adversely affected by the presence of a steep power-law component. Secondly, this model will allow some degree of unification of the TD state and SPL state, the latter being a more strongly Comptonized version of the former.

The authors would like to thank George Rybicki for discussions on the physics of Comptonization as well as Jifeng Liu, Lijun Gou, Rebecca Shafee, and Ron Remillard for their input on SIMPL. J. F. S. thanks Joey Neilsen for enthusiastic discussions as well as comments on the manuscript, Ryan Hickox for ideas which improved this article, Irwin Shapiro for his suggestions on the manuscript, and Keith Arnaud for helping implement SIMPL in XSPEC. The authors thank Tim Oosterbroek for his indefatigable assistance with the BeppoSAX reduction software. J. F. S. was supported by the Smithsonian Institution Endowment Funds and R. N. acknowledges support from NASA grant NNX08AH32G and NSF grant AST-0805832. J. E. M. acknowledges support from NASA grant NNX08AJ55G.

APPENDIX

XSPEC IMPLEMENTATION

SIMPL is currently implemented in XSPEC. This version includes three parameters (two that can be fitted), the power-law photon index ($\Gamma$), the scattered fraction ($f_{sc}$), and a switch to set upscattering only (SIMPL-1: switch $> 0$) and double-sided scattering (SIMPL-2: switch $\leq 0$). Since SIMPL redistributes input photons to higher (and lower) energies, for detectors with limited response matrices (at high or low energies), or poor resolution, the sampled energies should be extended or resampled within XSPEC to adequately cover the relevant range. For example, when treating the RXTE data in § 3, which have no response defined below 1.5 keV, the command “energies 0.05 50 1000 log” was used to explicitly extend and compute the model over 1000 logarithmically spaced energy bins from 0.05–50 keV.

Using SIMPL can be problematic when $\Gamma$ is large, especially if the power-law component is faint or the detector response extends only to $\sim 10$ keV (e.g., Chandra, XMM, or ASCA). When the photon index becomes sufficiently large, a runaway process can occur in which $\Gamma$ steepens and the scattered fraction becomes abnormally high (typically $\geq 50\%$, inconsistent with a weak power law). This occurs because scattering redirects photons from essentially a $\delta$-function into a new function with characteristic width set by $\Gamma$. If $\Gamma$ reaches large values ($\gtrsim 5$), the scattering kernel will also act like a $\delta$-function, and the convolved spectrum will be nearly identical to the seed spectrum.

In such circumstances, we recommend bracketing $\Gamma$. In practice, the power-law spectral indices of black hole systems are found to lie in the range $1.4 \lesssim \Gamma \lesssim 4$ (Remillard & McClintock 2006). An upper limit of $\Gamma \sim 4$–4.5 is typically sufficient to prevent this runaway effect, and this constraint should be applied if it is deemed appropriate for the source in question.
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