The Viterbi Algorithm: A Personal History

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Abstract
The story of the Viterbi algorithm (VA) is told from a personal perspective. Applications both within and beyond communications are discussed. In brief summary, the VA has proved to be an extremely important algorithm in a surprising variety of fields.

1 Introduction
Andrew J. Viterbi is rightly celebrated as one of the leading communications engineers and theorists of the twentieth century. He has received almost every professional award possible, including election not only to the National Academy of Engineering (USA) but also to the National Academy of Sciences (USA), where he chairs the Computer Science section. His award citations usually cite “invention of the Viterbi algorithm” as his most notable accomplishment.

On the other hand, Andy would be the first to tell you that other people deserve much of the credit for recognizing its theoretical properties and its practical attractiveness, and for extending its domain of application. He has often told this story himself (see, e.g., [33]).

Nevertheless, no one doubts that Andy’s awards are entirely deserved, and that their focus on the Viterbi algorithm (VA) is appropriate. This article will attempt to explain why, by briefly recounting the history of the VA. It is a “personal history,” because the story of the VA is so intertwined with my own history that I can recount much of it from a personal perspective.

2 Invention of the Viterbi algorithm
The Viterbi algorithm was first presented in Andy’s famous 1967 paper [30] to help prove an asymptotically optimum upper bound on the error probability of convolutional codes, which had previously been derived by Yudkin in the context of sequential decoding [37]. In this paper, the VA is presented just as we understand it today. This paper introduces the important concept of survivors (a term possibly borrowed from tennis elimination tournaments), and shows that only $q^K$ survivors need be retained to decode a convolutional code with constraint length $K$ over the $q$-ary field $GF(q)$. Compared to a block code with $q^K$ codewords, such a convolutional code is shown to have a much better error exponent, particularly near capacity.
Andy recalls in a 1999 interview [22] that

“the Viterbi algorithm for convolutional codes . . . came out of my teaching . . . . I found information theory difficult to teach, so I started developing some tools . . . I wrote the first paper in March ’66, but it wasn’t published until April ’67. . . . At one point I was actually discouraged from publishing the algorithm details. Fortunately, one of the reviewers, Jim Massey, encouraged me to include the algorithm. . . . Nobody thought that it had any potential for practical value . . . ”

It is clear from the paper that at this point Andy had no idea that the VA was actually an optimum (maximum likelihood) decoder, nor that it was potentially practical. Indeed, the paper states that “this decoding algorithm is clearly suboptimal,” and concludes: “Although this algorithm is rendered impractical by the excessive storage requirements, it contributes to a general understanding of convolutional codes and sequential decoding through its simplicity of mechanization and analysis” [30].

3 Discovery that the VA is optimum

I believe that I received a copy of Andy’s paper prior to publication, probably via Jim Massey. At that time I was working at Codex Corp., a small start-up company aiming at practical applications of convolutional codes. Our primary focus was initially on threshold decoding, which was the subject of Jim’s doctoral thesis [21]; Jim was a consultant. Subsequently, we developed a sequential decoding system [36] for the Pioneer deep-space satellite program, which became the first code in space [5].

I had been trying to understand why in practice convolutional codes were generally superior to block codes, so I studied Andy’s paper with great interest. I realized that the path-merging property of convolutional codes could be depicted in what I called a trellis diagram, to contrast with the then-conventional tree diagram used in the analysis of sequential decoding. It was then only a small step to see that the Viterbi algorithm was an exact recursive algorithm for finding the shortest path through a trellis, and thus was actually an optimum trellis decoder. I believe that at that point I called Andy, and told him that he had been too modest when he asserted that the VA was “asymptotically optimum.”

These results were written up in a 1967 technical report [6] for NASA Ames Research Center. They were not published in journal form until many years later, in [9] and [10].

Shortly afterward, in a paper submitted in May 1968 [23], Jim Omura observed that the VA was simply the standard forward dynamic programming solution to maximum-likelihood decoding of a discrete-time, finite-state dynamical system observed in memoryless noise. Beyond proving optimality in a different way, he thus made the first connection between the VA and system and control theory. It is interesting to speculate whether the history of the VA would have been different if it had simply been called “dynamic programming” from the beginning.

At this point, none of us had recognized that the VA might be practical. Jim’s paper concludes: “. . . the decoding algorithm discussed here grows exponentially in complexity with constraint length $\nu$ and is therefore impractical for large $\nu$ . . . .” More embarrassingly, in a 1970 IEEE Spectrum paper [7] describing practical coding schemes for the space channel, I wrote:
Sequential decoding is the best-performing practical technique known for memoryless channels like the space channel, and will probably be the general-purpose workhorse for these channels in the future.

[The Viterbi algorithm] is competitive in performance with sequential decoding for moderate error rates, but cannot achieve very low error rates efficiently. On the other hand, it is capable of extremely high speeds (tens of megabits), where sequential decoders become uneconomic. It therefore may find application in high-data-rate systems with modest error requirements, such as digitized television.

4 Recognition that the VA is practical

Andy has always said that Jerry Heller was the first person to realize that the VA might be practical. Jerry simulated the performance of short-constraint-length codes at the Jet Propulsion Laboratory (JPL) in 1968-69, and found that with only a 64-state code he could obtain a sizable coding gain, of the order of 6 dB.

In 1968, Andy, Irwin Jacobs, and Len Kleinrock incorporated Linkabit Corp. in San Diego as a vehicle to pool their consulting efforts and to obtain small government study contracts. All kept their jobs as professors. In 1969, Jerry Heller was hired as Linkabit’s first full-time employee. Linkabit obtained some small Navy and NASA contracts, which enabled the construction of a VA prototype in 1969-70. “It was a big monster filling a rack.”

The first IEEE Communication Theory Workshop in 1970 in St. Petersburg became famous as the “coding is dead” workshop, after Ned Weldon and other speakers worried publicly that coding theory had come to a dead end. But what I remember best from that session is Irwin Jacobs standing up in the back row, flourishing an integrated circuit (a 4-bit shift register, I believe), and asserting that this represented the future of coding. He was quite right. (Unfortunately, by this time Codex had made a business decision to get out of coding.)

By 1971, Linkabit had implemented a 2 Mb/s, 64-state Viterbi decoder. In a special issue on coding of the IEEE TRANSACTIONS ON COMMUNICATION TECHNOLOGY in October 1971, Heller and Jacobs discuss this decoder and many practical issues in careful detail. They compare the VA with sequential decoding, and conclude that the VA will often be preferable because it can use quantized soft decisions easily, and is less sensitive to channel and equipment variations. In the same issue, Cohen, Heller and Viterbi describe a system using orthogonal convolutional codes and the VA for asynchronous multiple-access communications, and Viterbi introduces generating-function analysis techniques for the VA.

During the 1970s, through the leadership of Linkabit and JPL, the VA became part of the coding standard for deep-space communication, ultimately in a concatenated coding system with a Reed-Solomon (RS) outer code. Linkabit developed a relatively inexpensive and flexible VA chip, and the VA became a nice little business for Linkabit. It didn’t hurt that the inventor of the Viterbi algorithm was a Linkabit founder. The VA also began to be incorporated in many other communications applications.

In the early 1990s, JPL built a 2^14-state “Big Viterbi Decoder” (BVD) with 8192 parallel add-compare-select (ACS) units, which operated at a rate of the order of 1 Mb/s. As far as I know, the BVD remains the biggest Viterbi decoder ever built.
When the primary antenna failed to deploy during the Galileo mission in 1992, JPL devised an elaborate concatenated coding scheme involving a $2^{14}$-state rate-1/4 inner convolutional code and a set of variable-strength RS outer codes, and reprogrammed it into the spacecraft computers. This scheme was able to operate within about 2 dB of the Shannon limit at a bit error probability of less than $10^{-6}$, which was the world record prior to the advent of turbo codes [5].

5 The VA and intersymbol interference channels

In the late 1960s, Codex turned its attention to the voiceband modem business. Our first-generation product was a single-sideband (SSB) 9600 b/s modem with a so-called Class IV or $1 - D^2$ “partial response.” About 1969, I recognized that the symbol correlation that was thus introduced could be exploited by an ad hoc error correction algorithm, which was able to improve the noise margin by about 2–3 dB. This little decoder extended the commercial life of this marginal-performance modem by perhaps a year or two.

It took me a while to understand that I had in fact invented a maximum-likelihood sequence detector for this modem. Over time, I realized that this was nothing more than the Viterbi algorithm again, streamlined for the $1 - D^2$ response. This led to a 1972 paper that showed that the VA could be used as a maximum-likelihood sequence detector for digital sequences in the presence of intersymbol interference (ISI) and AWGN noise.

Meanwhile, Jim Omura had recognized independently at UCLA that the VA could be used on intersymbol interference channels, because of their convolutional character. Indeed, a tantalizing hint in this direction appears in a book review by Andy Viterbi in 1970. After visiting UCLA, Hisashi Kobayashi further developed this idea, particularly for practical applications in partial response modems and magnetic recording.

The VA proved to be too complicated for general use as an equalizer on ISI channels. However, it stimulated many suboptimal approximations, and analysis of its performance gave bounds on the best possible performance of any sequence detector.

However, the VA did become standard in the related application of high-density magnetic recording. In so-called PRML systems (“partial-response equalization with maximum-likelihood sequence detection”), the magnetic recording channel is first equalized to a simple “partial response” such as $1 - D^2$, and the resulting sequence is then detected by the VA, or by a simplified version thereof, as Kobayashi had envisioned. In retrospect, it seems possible that my little SSB modem decoder was the first implementation of such a PRML scheme.

6 Trellis-coded modulation

After Gottfried Ungerboeck published his invention of trellis-coded modulation in 1982, the VA became the workhorse decoder for the next several generations of voiceband modems. Ungerboeck extended trellis coding to multilevel constellations by constructing trellis codes in which each branch of the trellis represents a subset of constellation symbols, rather than a single symbol. By clever constellation partitioning and attention to distances between subsets, he was able to obtain coding gains in the bandwidth-limited regime comparable to those that can be obtained in the power-limited regime.
For example, the V.32 modem (1986) used an 8-state trellis code to obtain a coding gain of about 3.5 dB, while the later V.34 modem (1994) used 16 to 64-state trellis codes to obtain coding gains of 4.0 to 4.5 dB.

7 Applications in mobile and broadcast communications

The mobile communications channel is subject to fading, bursts, and multiuser interference, and is a much more difficult medium than the AWGN and linear Gaussian channels discussed above. The designers of second-generation (2G) cellular systems used every tool available at the time (early 1990s) to provide reliable communication on this difficult channel.

The CDMA system developed by Qualcomm uses a $2^8$-state, rate-1/3 convolutional code with interleaved 64-orthogonal modulation, and of course a Viterbi decoder. The TDMA system developed for GSM uses the VA not only to decode a 16-state, rate-1/2 convolutional code, but also for equalization. A soft-output Viterbi algorithm (SOVA) is often used in the latter application.

VA decoders are currently used in about one billion cellphones, which is probably the largest number in any application. However, the largest current consumer of VA processor cycles is probably digital video broadcasting. A recent estimate at Qualcomm is that approximately $10^{15}$ bits per second are now being decoded by the VA in digital TV sets around the world, every second of every day.

8 General application to hidden Markov models

In 1973, I wrote a tutorial paper on the Viterbi algorithm for the PROCEEDINGS OF THE IEEE that has turned out to be my most cited paper by far. A recent search using Google Scholar shows 734 citations, far more than the 181 for my next-most-cited reference.

One of the main points of that paper was that the VA can be applied to any problem that involves detecting the output sequence of a discrete-time, finite-state machine in memoryless noise—i.e., to detection and pattern recognition problems involving hidden Markov models (HMMs). Of course, decoding of convolutional codes and sequence detection on ISI channels were the main applications discussed in that paper.

During the 70s and 80s, the VA became widely used in a variety of pattern recognition problems that could be described by HMMs, particularly for speech recognition; see [26]. Here the VA is often used as the M-step of an EM algorithm, which also adjusts HMM parameters.

Indeed, a recent search of IEEE Xplore shows that most current IEEE references to the VA occur in such Transactions as PATTERN ANALYSIS AND MACHINE INTELLIGENCE or SYSTEMS, MAN AND CYBERNETICS, rather than in COMMUNICATIONS or INFORMATION THEORY. It seems that everyone in these fields knows how to “Viterbi the data.”

Finally, in the past decade, the VA has become widely used in much more distant fields such as computational biology, e.g., to locate genes in DNA sequences. See for example [16], with its “Viterbi Exon-Intron Locator” (VEIL).
9 Related algorithms

In the past decade, the development of the field of “codes on graphs” and their related decoding algorithms has led to a remarkable conceptual unification of a variety of detection and estimation algorithms which have been introduced under various names for various applications.

In his 1996 dissertation, generalizing the earlier work of Gallager [12] and Tanner [27], Niclas Wiberg [34, 35] developed the generic “sum-product” and “min-sum” decoding algorithms for cycle-free graphs which may include both symbol (observable) and state (hidden) variables. For trellis graphs, he showed that these reduce to the BCJR algorithm [2] and an algorithm equivalent to the Viterbi algorithm, respectively. For capacity-approaching codes such as turbo codes and low-density parity-check (LDPC) codes, the sum-product algorithm with an appropriate schedule becomes the standard iterative decoding algorithm that is normally used with such codes.

Later authors (e.g., [1, 20]) have shown that the sum-product algorithm is equivalent to Pearl’s “belief propagation” algorithm for statistical inference on Bayesian networks; the Baum-Welch or “forward-backward” algorithm for inference with hidden Markov models; and the Kalman smoother for linear Gaussian state-space models.

However, it is important to note that the min-sum algorithm is a two-way “backward-forward” algorithm. The VA obtains the same result with a “forward-only” algorithm by storing a path history with each survivor. Of course, “forward-only” is a key simplification, particularly for real-time communications; the min-sum algorithm would never have been adopted in practice as widely as the VA has been.1

10 Conclusion

The Viterbi algorithm has been tremendously important in communications. For moderately complex (not capacity-approaching) codes, it has proved to yield the best tradeoff between performance and complexity both on power-limited channels, such as space channels, and on bandwidth-limited channels, such as voiceband telephone lines. In practice, in these regimes it has clearly outstripped its earlier rivals, such as sequential decoding and algebraic decoding. (However, it seems likely that it will be superseded in many of its principal communications applications by capacity-approaching codes with iterative decoding.)

Moreover, the VA has become a general-purpose algorithm for decoding hidden Markov models in a huge variety of applications, from speech recognition to computational biology.

Andy Viterbi clearly did not envision the full import of the VA when he first introduced it. However, he and his colleagues at Linkabit and Qualcomm were largely responsible for making it practical, and for driving its widespread adoption in communications. The history might have been otherwise, but it wasn’t. In actual fact, no one deserves more credit for this tremendously important invention than its actual inventor.

1Interestingly, Ungerboeck discovered both the sum-product and the min-sum algorithms for equalization applications in his thesis [28]; however, he missed the forward-only version.
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