Component Importance and Interdependence Analysis for Transmission, Distribution and Communication Systems

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Abstract—For critical infrastructure restoration planning, the real-time scheduling and coordination of system restoration efforts, the key in decision-making is to prioritize those critical components that are out of service during the restoration. For this purpose, there is a need for component importance analysis. While it has been investigated extensively for individual systems, component importance considering interdependence among transmission, distribution and communication (T&D&C) systems has not been systematically analyzed and widely adopted. In this study, we propose a component importance assessment method in the context of interdependence between T&D&C networks. Analytic methods for multilayer networks and a set of metrics have been applied for assessing the component importance and interdependence between T&D&C networks based on their physical characteristics. The proposed methodology is further validated with integrated synthetic Illinois regional transmission, distribution, and communication (T&D&C) systems, the results reveal the unique characteristics of component/node importance, which may be strongly affected by the network topology and cross-domain node mapping.

Index Terms—Multilayer network, interdependence, computer networks.

NOMENCLATURE

| Symbol | Description |
|--------|-------------|
| $T$    | Transmission network |
| $D$    | Distribution network |
| $C$    | Communication network |
| DER    | Distributed energy resource |
| CNA    | Complex network analysis |
| $G$    | Graph of a network |
| $N$    | Number of nodes in a graph |
| $L$    | Number of edges in a graph |
| $V$    | Node/vertex set in a graph |
| $v$    | A single node/vertex in a graph |
| $E$    | Edges in a graph |
| $E_{ij}$ | Edge between nodes $i$ and $j$ |
| $A$    | Adjacency matrix of a graph |
| $i,j$  | Node/vertex index |
| $a_{ij}$ | Element in an adjacency matrix for nodes $i$ and $j$ |
| $Z_{ij}$ | Impedance between nodes $i$ and $j$ |
| $R_{ij}$ | Resistance between nodes $i$ and $j$ |
| $X_{ij}$ | Reactance between nodes $i$ and $j$ |
| $I_{ij}$ | Magnitude of impedance between nodes $i$ and $j$ |
| $w_{ij}$ | Normalized edge weight between nodes $i$ and $j$ |
| $c_i$ | Cross-closeness centrality |
| $d_{pq}$ | Shortest path between vertices $p$ and $q$ |
| $G_i, G_j$ | Graph of two different networks |
| $N_i$ | Number of nodes/vertices in graph $G_i$ |
| $N_j$ | Number of nodes/vertices in graph $G_j$ |
| $V_i$ | Node/vertex set in graph $G_i$ |
| $V_j$ | Node/vertex set in graph $G_j$ |
| $p$ | Node/vertex in graph $G_i$ |
| $q$ | Node/vertex in graph $G_j$ |
| $b_p$ | Cross-betweenness centrality |
| $\sigma_{pq}$ | Total number of shortest paths between $p$ and $q$ |
| $\sigma_{pq}(v)$ | Number of shortest paths between $p$ and $q$ that include $v$ |
| $E_G$ | Cross-efficiency between graph $G_i$ and graph $G_j$ |
| $V_p$ | Voltage of node $p$ |
| $k_{pq}$ | Coefficient between nodes $p$ and $q$ for cross-efficiency calculation |
| $\Delta E_p(v)$ | Node efficiency drop |
| $E_G(V)$ | Cross-efficiency after the removal of the vertex |
| $E(G)$ | Network efficiency of graph $G$ |
| $<k>$ | Average degree of a node in a graph |
| $k_{max}$ | Max degree of a node in a graph |

I. INTRODUCTION

Identifying critical components in a system could help plan or prioritize the restoration procedure for distributed infrastructure systems, including transportation networks [1] and power systems [2], as well as other applications like structural vulnerability assessment and cascading blackouts prediction [3]. However, previous efforts and analyses were limited to one stand-alone system. There is a lack of systematic
analysis and understanding of component importance and interdependency for power transmission and distribution systems, and their interdependent infrastructures, particularly in the context of system restoration. This motivates us to perform this research to support coordinated T&D&C system restoration decision-making.

A significant portion of critical component assessments have been focused on analyzing the component structural importance using network science or complex network theory [2]. In these efforts, infrastructure networks are modeled as graphs or networks, with components being modeled as either nodes or edges. Important components are identified based on their topological features [1]. For the power grid systems, complex network analysis has revealed that most of them are scale-free networks, in which the majority of nodes only have a few links and only few nodes have higher degrees. The most widely used metrics include the degree centrality and betweenness centrality [4]. Following the blackout event in 2003, Albert et al. [5] studied the North American power grid from a network perspective. Using the connectivity loss as the measurement, they showed that failure of only 4% of the high degree or high load nodes could result in up to 60% of connectivity loss, and the loss would be even higher with the consideration of cascading failures. Kinney et al. [6] used network global efficiency between generators and distribution substations as the metric. They demonstrated that the loss of one critical substation with high betweenness and high degree in the North American power grid could lead to 25% loss of transmission efficiency with the triggering of the cascading effects. Crucitti et al. [7] came to the similar conclusion studying the Italian electric power grid. They found that the system can be very vulnerable to the failures that occur on the nodes with highest betweenness centrality. It should be noted that these works focus on the transmission system only. With increasing penetration of distributed energy resources (DERs) and blurred boundary between T&D systems, it is imperative to include distribution systems and DERs in critical component analysis.

One of the main shortcomings of pure complex network analysis (CNA) approach is that only topology or connection information is considered, while the physical or functional characteristics of components and systems are ignored. Recent studies have been focusing on extended approaches that incorporate physical properties of power grids to pure complex network analysis, such as impedance, max power, power flow directions, and simulating a power grid as a directed and weighted graph ([8], [9]). For example, Wei and Liu [10] proposed the combination of impedance and line voltage level for line weights. They showed that their model was more effective in identifying vulnerable part of a network.

Many of pure CNA and extended CNA studies are mainly focused on a single, non-interacting infrastructure system. Yet, modern systems and critical infrastructures are highly interconnected and mutually dependent in complex ways [11 - 14]. The power incidents also disrupted several key industries, and stressed the entire U.S. Western Interconnection, causing far-reaching security and reliability concerns [11]. On the other hand, electric power infrastructure also depends significantly on other critical infrastructures. Since early 2000, researchers from different domains and areas have devoted significant efforts to identifying, understanding and leveraging interdependency between different networks and infrastructures. Donges et al. [15] investigated the topology of interacting networks, especially the importance and role of single vertex for the interaction or communication between different subnetworks, in a systematic and comprehensive way. A graph-theoretical framework for investigating the topology of interacting networks was proposed, and selected local and global network measures by extending the classical centrality measures such as degree, closeness, betweenness, and cluster coefficient were defined to quantify and investigate the interaction topology of networks on different topological scales. The proposed metrics were used to quantify the structural role of single vertices with respect to the interactions between a pair of subnetworks on both local and global scales. From a risk and vulnerability analysis perspective, Wang et al. [16] pointed out that features of the infrastructure systems must be considered from the perspective of global analysis. They proposed a method for ranking critical components in interdependent infrastructures for protection purposes. In their approach, the importance of an element was evaluated by considering the performance degradation of the network due to the disconnection of the component. The network performance was measured by global safety efficiency of the network $S(G)$, component outage results in a new system denoted by $S(G^*)$. The importance of a component can be assessed by the global efficiency drop after the removing of this component: $\Delta S(C) = (S(G) - S(G^*)/S(G))$. Using the similar approach, Milanovic and Zhu [17] defined the cross-efficiency drop for the coupled electric power system (EPS) network and information and communication technology (ICT) network. They proposed a three-dimensional model, which uses unidirectional edges for power flows and bidirectional edges for information exchanges and demonstrated that efficiency related metrics were most effective in identifying critical interconnected components. However, they only focused on T&C systems, and did not consider the physical properties and importance differences of EPS nodes of different nominal voltage levels. Furthermore, they calculated the metrics for each individual network and summed them up as a global metric, which means they did not explicitly consider cross-network interdependence.

This study aims to fill the gap by leveraging the recent development in the field of multilayer networks and several emerging analysis methods, and extend our previous work studying the interdependency between the transmission and communication systems (T&C) [18]. Our contributions are:

1. We for the first time included D network in interdependency analysis and proposed a multilayer network representation for T&D&C system and utilized it for component importance and interdependent analysis.
2. We developed a new approach for the edge weight calculation to reflect individual network physical properties, incorporating both line impedances and connected node voltages for both the power transmission
and distribution networks to better distinguish them. In addition, we extended several single-network-oriented metrics [15] and [17], to multilayer networks to assess the node importance for interdependent systems.

3) We compared these network theoretic analysis results with physical-model-based simulation results, including communication network performance analysis using ns-3 simulation [19] and maximum load delivery analysis using PowerModelsRestoration.jl [20, 21] to help understand the effectiveness and limitations of network theoretic analysis and how they could be used in conjunction with physical-based approaches.

The rest of the paper is organized as follows: Section II introduces the multilayer network representation of T&D&C networks and multilayer network metrics; Section III describes the details of the T&D&C system; Section IV presents the network properties for each network from the complex network perspective; Section V compares the results between complex network analysis, the ns-3 simulation, and maximum loading capability analysis; the interdependency between network are elaborated in Section VI; Section VII presents the comparison between the proposed edge weight approach and the traditional approach using T-only network; and the conclusions are presented in Section VII.

II. MULTILAYER NETWORKS AND METRICS FOR ANALYSIS

The emerging and unique features of T&D&C systems can be better described and highlighted by directly comparing with previously studied T-only and T&C systems, as follows:

• Distribution systems are not modeled as passive loads connected to transmission buses but detailed subsystems. Their components have their unique functional and physical properties such as radial structure, low voltage and power, sparingly connected with communication systems. Correspondingly, edge and node weights should be properly designed to distinguish components in T&D systems.

• Communication was not considered at the edge of the grid in previous studies. We can capture it with T&D&C modeling. Note that compared with T-network, there is yet much communication coverage in distribution systems except a few nodes connected with C-network. These nodes essentially become communication hubs in distribution systems, and such hubs are usually not observed in T-network.

We have fully considered these features in network representations (mainly edge and node weights), and proposed several important metrics for analyzing the component importance and interdependence in this paper.

Researches in CNA and extended CNA have been focusing on graph topology related centralities: degree, betweenness, closeness, and efficiency [3]. Researches from Kinney et al. [6] showed that the power grid system can be very vulnerable to the failures that occur on the nodes with the highest betweenness centrality or the highest closeness centrality. Jovica et al. [17] showed efficiency metrics, which use system efficiency and system efficiency drop after removing individual, is the one of the most effective in identifying critical interconnected components. We extended basic centralities and used the following metrics in multilayer interdependent networks for the study of interdependency of T&D&C networks:

• Cross-closeness centrality: it measures the closeness of the node p in G_i to G_j along shortest paths.

• Cross-betweenness centrality: it measures the importance of node v in G_i that is responsible for passing the information from G_i to G_j.

• Node efficiency drop: it measures the cross-efficiency drop passing information from G_i to G_j after removing the node v in G_i.

A. Multilayer Network Representation of T&D&C Networks

Using complex network theory, T, D, or C networks can be represented using a graph G = (V, E), where V = {V_t, V_d, ..., V_N} is the set of N nodes/vertices in G and E = {E_t, E_d, ..., E_N,N} is the set of edges/links. Adjacency matrix A = (a_ij)_{N x N} is used to represent the connectivity between nodes i and j: a_ij = 1 (or weight for a weighted graph) if there is an edge between node i and node j and a_ij = 0 otherwise.

To simulate the interdependency between T, D, and C networks, we also need to represent the inter-layers edges/links between networks. Since a node i in layer α can be connected to any node j in any layer β in a multilayer framework, we adopted the following adjacency matrix to add layer information: a(i, α, j, β) = 1 (or weight for a weighted graph) if node i in layer α is connected to node j in layer β, otherwise 0.

B. Edge weights

Edge weights are assigned to take into account of physical properties of T, D, and C networks. For intralayer edges that are within the T and D networks, their weights are calculated using the line impedance: let Z_o = R_o + jX_o be the impedance of the edge between nodes i and j, then I_{i,j} = \sqrt{R_{i,j}^2 + X_{i,j}^2}; the weight for edge E_{i,j} is the normalized I_{i,j}, i.e., w_{i,j} = I_{i,j} / mean(I_{i,j}). For edges in the C network, their edges are all set to 1. For all interlayer edges between two networks, their weights are set to 0 because the two nodes for interlayer connections are usually co-located (or at least very close) and thus their distance is zero.

C. Cross-closeness centrality

Cross-closeness centrality measures the topological closeness of a vertex in one network to the other network along the shortest path. It quantifies the efficiency of interaction between the vertex and the other network. A vertex with high cross-closeness is likely to be important for the fast exchange of energy and/or information with a network [15]. For a vertex p \in V_i in G_i the cross-closeness centrality \( c_{iq} \) between p and G_j is:

\[
c_{iq} = \frac{N_j}{\sum_{q \in V_j} d_{pq}} \tag{1}
\]

where \( d_{pq} \) is the shortest path between vertices p and q \in G_j. If no path exists between p and q, \( d_{pq} = N_i + N_j - 1 \) is set as
the upper bound, where \( N_i \) and \( N_j \) are the number of vertices in \( G_i \) and \( G_j \), respectively. For generality, \( d_{pq} \) is not restricted to any order of vertices and may contain any vertices \( v \in V_i \cup V_j \) in any order depending on the topology of the network.

### D. Cross-betweenness centrality

For a vertex \( v \in V_i \cap G_i \), cross-betweenness centrality \( b^f_v \) indicates the role of \( v \) for mediating or controlling interactions/communication between two networks \( G_i \) and \( G_j \). It is defined as

\[
b^f_v = \sum_{p \in V_i} \sum_{q \in V_j} \sigma_{pq}(v) / \sigma_{pq}
\]

, where \( \sigma_{pq} \) is the total number of shortest paths between \( p \) and \( q \) and \( \sigma_{pq}(v) \) is the number of shortest paths between \( p \) and \( q \) that include \( v \).

Cross-betweenness centrality describes the importance of vertex \( v \in V_i \) as a relational hub between two networks \( G_i \) and \( G_j \). A vertex with high cross-betweenness may serves as a robust transmitter between two networks.

### E. Cross-efficiency and node efficiency drop

Considering two independent networks \( G_i \) and \( G_j \), cross-efficiency measures how efficiently the information is exchanged between nodes in these two networks. Assuming the information is transmitted along the shortest paths, information exchange between two nodes in two different networks is disproportional to the length of the shortest path between two nodes, i.e., more efficient if the shortest paths are shorter. We adopted the single-network efficiency metric from Milanović and Zhu [17], and further proposed the cross-efficiency between two networks by adding a coefficient \( k_{pq} \) to represent the weight for the starting node. The cross-efficiency of the interconnected network \( G = (G_i \cup G_j) \) denoted by \( E_G \) can be calculated as follows:

\[
E_G = \frac{1}{N_i N_j} \sum_{p \in V_i} \sum_{q \in V_j} k_{pq} / \sigma_{pq}, \quad k_{pq} = V_p
\]

where \( d_{pq} \) is the shortest path between vertices \( p \in V_i \) and \( q \in V_j \), and \( N_i \) and \( N_j \) are the number of vertices in \( G_i \) and \( G_j \), respectively. \( k_{pq} \) is set to the nominal voltage (e.g., 500 kV) of node \( p \); \( V_p \), which gives higher weights to nodes with higher voltages. \( N_e \) is the number of shortest paths from \( p \in V_i \) to \( q \in V_j \).

For a vertex \( v \in V_i \), the cross-efficiency drop \( \Delta E_G(v) \) is defined as the drop in the cross-efficiency after removing \( v \) from the \( V_i \).

\[
\Delta E_G(v) = \frac{E_G - E_G(V - v)}{E_G}
\]

where \( E_G(V - v) \) is the cross-efficiency of \( G \) after the removal of the vertex \( v \).

Cross-efficiency drop for the vertex \( v \in V_i \) describes the importance of the \( v \) in the interaction between two networks \( G_i \) and \( G_j \), with higher cross-efficiency drop indicating higher node importance. For example, a vertex with \( \Delta E_G(v) = 1 \) indicates that all the interactions between two vertices from \( G_i \) and \( G_j \) need to go through the vertex \( v \). The removing of \( v \) would lead to the total disconnection between \( G_i \) and \( G_j \).

**III. SYSTEM CONFIGURATION**

The simulation system consists of three networks: transmission network (\( T \)), communication network (\( C \)), and distribution network (\( D \)). \( T \) network is a synthetic Illinois regional 200-bus transmission system [22], which consists of 111 substations (nodes) that are named from node 1 to node 111. Twenty substations with the highest load are connected to distribution feeders. Two simplified feeder models from the 24 prototypical radial distribution feeder models are considered in this work, namely R5-12.47-1 and R5-12.47-2 [23]. The numbers of nodes are 142 and 67 for these two feeders, respectively. Each of the twenty substations is connected to an appropriate feeder model according to the size of its load. Among the 20 substations that have feeders attached, 17 of them are connected to R5-12.47-1 feeder, and 3 of them are connected to R5-12.47-2 feeder (Table I). Feeder nodes are named using two numbers separated by a dot as “XYZABC”. The number before the dot (“XYZ”) represents the substation name, while the number after the dot (“ABC”) represents the node index within the feeder. For example, node 65.046 shows that this feeder node is within a feeder attached to the substation 65 and the node index inside the feeder is 046. For each feeder, we assume 3 nodes are equipped with DERs, they are randomly selected among nodes other than the substation. Substations and DERs are assumed to have communication capabilities. Therefore, each of them has a corresponding node in the \( C \) network. Substation nodes in the \( C \) network share the same name with their corresponding names in the \( T \) network, and DER nodes in the \( C \) network share the same name with their corresponding names in the \( D \) network. In the \( C \) network, DERs are directly connected to their respective substations (Fig. 2 – 3), while the connections among substations are synthesized by randomly rewiring 10% of the \( T \) network edges. The resultant topology of \( T \) and \( C \) networks are shown in Fig. 1 with distribution-level nodes hidden for clarity.
IV. GENERAL PROPERTIES OF T&D&C SYSTEM

The degree of a node, which is defined as the number of edges that are connected to the node, has been shown to be a good indicator of the network topological importance, especially its probability distribution ([5], [18]). For T, D, and C networks, majority of the nodes only have 1 or 2 edges (57.66%, 69.95%, and 65.51%, respectively), and only few nodes have higher number of degrees (> 5): 2.7%, 11.5%, and 2.1% for T, C, and D networks, respectively (Fig. 4), indicating that all 3 networks are vulnerable to targeted attacks on nodes with high degrees (hubs) and robust to random failures of nodes with low degrees. Among all three networks, T network has the highest average degree of 2.81 and D network has the lowest average degree of 1.98 (Table II). The highest degree is 8 for both T and D networks, and 9 for the C network.

The network efficiency, which is inversely proportional to the shortest distance in a network, indicates how efficiently energy and/or information can be exchanged within the network. The efficiency of a network, \( E(G) \), is defined as

\[
E(G) = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}},
\]

where \( d_{ij} \) is the shortest path between node \( i \) and \( j \). The efficiency for the T network is slightly higher than the C network, 0.319 and 0.257, respectively, suggesting that the energy flow is more efficient within the T network than the information exchange within the C network. Because those 20 individual feeder networks in D network are not connected with each other, each node in D network only has shortest paths with nodes in its feeder network, while not with nodes in other feeder networks. As a result, the overall efficiency of the D network becomes very small and is only 0.012.

| Network | \( N \) | \( L \) | \( k_{\text{max}} \) | \( E(G) \) |
|---------|------|------|---------|--------|
| T       | 111  | 156  | 8       | 0.319  |
| C       | 111(T)+72(DERs) | 226  | 9       | 0.257  |
| D       | 2615 | 2595 | 8       | 0.012  |

\( N \) is the number of nodes; \( L \) is the number of edges, \( k_{\text{max}} \) is the average degree of a node; \( E(G) \) is the network’s efficiency.

V. COMPARISON Of NETWORK ANALYSIS AND PHYSICS-BASED SIMULATION RESULTS

We first analyzed the C network and identified the top 10 critical nodes using the proposed network metrics, which are compared with the results from the ns-3 simulator [18] (Table III). For the ns-3 simulation, we sent packages between every two nodes in the C network and recorded package delays (\( \Delta T \)) for all packages. The simulation was repeated 100 times, and the median package delays of the 100 simulations was calculated and recorded. Then we did the same simulation 100 times while removing each individual node in the network to assess how each node affected the package delays in the network.
Nodes were ranked based on how they impact the package delays using the median package delays from high to low.

The difference of how the shortest path distance was used led to the difference between the results from the network analysis and the ns-3 simulations. For example, although both the node efficiency drop in the network analysis and the package delays in ns-3 simulations used the shortest path distance between nodes as the input, the node efficiency drop in (3) was calculated using the reciprocal of the shortest path distance, while the package delays in ns-3 simulations were proportional to the length of the shortest path distance. To be more specific, in the network analysis results (Table III), node 69 is the top node using all three network metrics, which was expected by observing Fig. 1 where node 69 is in upper left region. The highest closeness centrality of node 69 indicates that it has the shortest path length on average to other nodes in the network, and the highest betweenness centrality of node 69 suggests that it falls on the shortest path between other pairs of nodes in the network. As a result, the removal of node 69 led to the highest efficiency drop of the network.

On the other hand, package delay is a unique characteristic between two communication nodes, which may separate it from other graph metrics; in comparison, electric potential is a unique characteristic of nodes in grid network. It should be noted that the highest ranked node was node 66 from the ns-3 simulations: the removal of node 66 caused the highest package delays than any other nodes; potential reasons include its location in this network, as well as the simulation parameter settings/ configurations in ns-3 simulations. For top ranked node from the ns-3 simulations, both nodes 65 and 66 were also top ranked nodes using three network metrics, but not for other top ranked nodes from the ns-3 simulations.

The maximum loading capability analysis for the T network was done using PowerModelsRestoration.jl. For each node in the T network, we simulated the power flow after single node removal, as well as removing the generators/loads attached to the node, if applicable. Then we calculated the ratio of system generation vs system load, which indicated the extra cost of the power system re-balancing. The top 10 ranked nodes from the PowerModelsRestoration.jl simulation were compared with the network analysis results for the T network (Table IV). Their results agree quite well: 6 of the top ranked nodes from the PowerModelsRestoration.jl simulation are also among the top ranked nodes from the network node efficiency drop analysis.

This agreement verifies the applicability of network analysis results for planning or prioritizing the restoration procedure: giving higher priorities to top ranked nodes identified from the network efficiency can not only increase the overall network efficiency, but also improve the cost of re-balancing the system.

VI. INTERDEPENDENCY BETWEEN T& D&C NETWORKS

A. Interdependency Between T and C Networks

Fig. 5 shows histogram of the cross-efficiency drop when passing information from the T network to the C network, and from C network to T network, when removing each individual node. Clearly, removing most of nodes in both networks only cause less than 1% cross efficiency drop. Moreover, the removing of majority of nodes in both networks: 96.4% in T network and 96.7% in C network, only cause less than 3% efficiency drop. In C network, the single node removal of 83% of nodes only cause efficiency drop of 1%. Table V lists the top 10 nodes in the T network that can cause the highest cross-efficiency drop when being removed. The most critical nodes are nodes 69, 65, and 3, which would cause 6.21%, 5.97%, and 4.92% efficiency drop if being removed, respectively. In comparison, the removing of any other nodes would cause 3.61% or less efficiency drops. Among top 10 nodes, both nodes 69 and 3 are top ranked nodes for cross-closeness centrality and cross-betweenness centrality in the T network, indicating that they are both the closest nodes and the most important information hub when passing information from T network to C network.

Because of the different topologies and edge weights between C network and T network, the identified critical nodes using cross-efficiency drop for C network are different than the T network: the top 5 nodes are 14, 27, 65, 69, and 75 (Table VI). Among all nodes in the C network, removing each of the top 4 nodes on the list could cause over 4% cross-efficiency drop, which is higher than any other lower ranked nodes. There is also a strong overlapping of critical nodes identified using cross betweenness centrality and cross-efficiency drop. These two lists share 7 nodes between each other. In comparison, only 3 nodes identified using cross closeness centrality are on the top 10 node list identified using the cross-efficiency drop, which
suggests that node with higher cross betweenness centrality (the information hub) is more critical than nodes with higher cross closeness centrality (closer) when passing information from C network to T network. However, using cross betweenness centrality alone may not find all critical nodes. For example, although node 69 is not on the top 10 nodes list using the cross betweenness centrality, it is still one of the critical nodes that can cause higher cross-efficiency drop when being removed.

### Critical Nodes in the T Network for the Interdependency between T & C Networks

#### Table V

| Rank | \( c_v^l \) | \( b_v^l \) | \( \Delta E_c(v) \) |
|------|---------------|---------------|------------------|
| 1    | 69            | 27            | 14               |
| 2    | 3             | 24            | 12               |
| 3    | 59            | 42            | 23               |
| 4    | 72            | 65            | 16               |
| 5    | 72            | 69            | 16               |
| 6    | 72            | 72            | 16               |
| 7    | 72            | 72            | 16               |
| 8    | 72            | 72            | 16               |
| 9    | 72            | 72            | 16               |
| 10   | 72            | 72            | 16               |

### Critical Nodes in the C Network for the Interdependency between T & C Networks

#### Table VI

| Rank | \( c_v^l \) | \( b_v^l \) | \( \Delta E_c(v) \) |
|------|---------------|---------------|------------------|
| 1    | 69            | 27            | 14               |
| 2    | 3             | 24            | 12               |
| 3    | 59            | 42            | 23               |
| 4    | 72            | 65            | 16               |
| 5    | 72            | 69            | 16               |
| 6    | 72            | 72            | 16               |
| 7    | 72            | 72            | 16               |
| 8    | 72            | 72            | 16               |
| 9    | 72            | 72            | 16               |
| 10   | 72            | 72            | 16               |

### Critical Nodes in the D Network for the Interdependency between T & D Networks

#### Table VII

| Rank | \( c_v^l \) | \( b_v^l \) | \( \Delta E_c(v) \) |
|------|---------------|---------------|------------------|
| 1    | 69            | 27            | 14               |
| 2    | 3             | 24            | 12               |
| 3    | 59            | 42            | 23               |
| 4    | 72            | 65            | 16               |
| 5    | 72            | 69            | 16               |
| 6    | 72            | 72            | 16               |
| 7    | 72            | 72            | 16               |
| 8    | 72            | 72            | 16               |
| 9    | 72            | 72            | 16               |
| 10   | 72            | 72            | 16               |

### Critical Nodes in the D Network for the Interdependency between T & D Networks

#### Table VIII

| Rank | \( c_v^l \) | \( b_v^l \) | \( \Delta E_c(v) \) |
|------|---------------|---------------|------------------|
| 1    | 69            | 27            | 14               |
| 2    | 3             | 24            | 12               |
| 3    | 59            | 42            | 23               |
| 4    | 72            | 65            | 16               |
| 5    | 72            | 69            | 16               |
| 6    | 72            | 72            | 16               |
| 7    | 72            | 72            | 16               |
| 8    | 72            | 72            | 16               |
| 9    | 72            | 72            | 16               |
| 10   | 72            | 72            | 16               |

### Interdependency Between T and D Networks

Fig. 6 shows the histogram of cross-efficiency drop when passing information from T network to D network, and from D network to T network, when removing each individual node. Because of the tree topology of the D network, the removing of a single node in the D network would only cause less than 1% cross-efficiency drop for 86.73% of nodes. The removing of any of the top 10 nodes in the T network could result in the cross-efficiency drop ranging from 6.94% to 14.08% (Table
In comparison, the max cross-efficiency drop is less than 6.5% in the D network and all identified top nodes are either the feeder head (65, 24, 69) or the node directly connected with the feeder head (65, 27, 69) in the D network (Table VIII). The removing of those top three nodes in the T network: 65, 24, and 69, could lead to over 10% cross-efficiency drop from the T network to the D network. The median efficiency drops for removing nodes with feeders are much higher than removing nodes without feeders in T network: 6.71% and 0.95%, respectively (Fig. 7). However, two of top 10 nodes: node 69 and 17 have no loads/feeders and removing either of them could cause higher cross-efficiency drop than the remaining 18 nodes that have feeders.

C. Interdependency Between C and D Networks

Fig. 8 shows the histogram of cross-efficiency drop of passing informing from C network to D network, and from D network to C network, when removing each individual node. The removing of any of the top 10 nodes in C network could cause efficiency drop ranging from 8.05% to 10.83% (Table IX). The comparison of nodes with/without feeders in the C network shows that the median cross-efficiency drops for removing nodes with feeders are much higher than removing nodes without feeders in C network: 7.86% and 0.82%, respectively (Fig. 9). The removing of any nodes with feeders would cause the cross-efficiency drop > 4%, except for the node 48. For nodes without feeders, although the removing of most of them would only cause the cross-efficiency drop < 4%, the removing of node 69 would cause the cross-efficiency drop 7.5%.

Like the interdependency between T and D, the removing 97.40% of D nodes only cause less than 1% cross-efficiency drop, and the max cross-efficiency drop is 5.6% (Table X), indicating that the removing of an individual node in D network doesn’t affect much of the overall cross-efficiency, which is expected because each feeder network has 3 DERs. When one DER node is removed, information can still be passed from D network to C network through the other two DER nodes. Unlike identified top nodes from D network to T network, which are
TABLE XI
CRITICAL NODES IN THE T NETWORK IDENTIFIED USING BOTH TRADITIONAL APPROACH AND THE PROPOSED EDGE WEIGHT APPROACH

| Rank | Edge Weight = 1 | Physical-based Edge Weight |
|------|-----------------|-----------------------------|
|      | \( \Delta E_p(v) \) (Node) | \( \Delta E_p(v) \) (%) | Voltage | \( \Delta E_p(v) \) (Node) | \( \Delta E_p(v) \) (%) | Voltage |
| 1    | 69              | 13.62%                      | 230     | 69              | 15.37%                      | 230     |
| 2    | 65              | 5.86%                       | 230     | 65              | 10.56%                      | 230     |
| 3    | 24              | 5.19%                       | 230     | 17              | 9.00%                       | 230     |
| 4    | 17              | 4.41%                       | 230     | 24              | 7.98%                       | 230     |
| 5    | 41              | 4.22%                       | 115     | 47              | 5.67%                       | 230     |
| 6    | 34              | 4.00%                       | 230     | 95              | 5.49%                       | 230     |
| 7    | 95              | 3.57%                       | 230     | 34              | 5.22%                       | 230     |
| 8    | 33              | 3.07%                       | 115     | 16              | 4.97%                       | 230     |
| 9    | 29              | 2.99%                       | 230     | 3               | 4.28%                       | 230     |
| 10   | 3               | 2.93%                       | 230     | 29              |                             | 230     |

Fig. 10. Locations of Identified Top 10 Nodes using Both the Proposed Physical-based Edge Weight approach and the Traditional Approach.

all root nodes or nodes next to root nodes (Table VIII), identified top nodes in D network using the cross efficiency drop include both DERs (6.023, 6.03, …) and non-DERs (105.047, …). Another notable observation is that because of the tree-like nature of the feeder graph, removing some critical non-DER nodes could cut the graph into disconnected pieces and significantly increase the length of shortest paths from D to C for some nodes, which in turn increases the overall cross-efficiency drop.

VII. COMPARISON OF THE PROPOSED APPROACH AND EXISTING APPROACHES

To illustrate the difference between the proposed physical-based weight approach and the traditional approach that set all edge weights to 1 [17], we first compared the identified top 10 nodes from the node efficiency drop results using both approached for the single T network. As shown in Table XI, while results using both approaches were comparable: 8 identified nodes were shared by both approaches and only two nodes were different from between them (see Fig. 10). However, the proposed edge weight approach clearly gave more priorities to nodes with higher voltages: all identified 10 critical nodes have 230 kV nominal voltage, while the two nodes that were only identified using the traditional approach have lower nominal voltage (115 kV). The calculated efficiency drops were also higher using the proposed approach than the traditional approach, indicating that the identified nodes were more important than other lower ranked nodes when using the proposed edge weight approach. The difference was also observed in identified critical nodes for the interdependency from the T network to the C network (Table XII). Although nodes 69 and 65 were identified as the top two nodes using both approaches, 7 out of 10 critical nodes identified using the traditional approach have 115 kV nominal voltage. Using the proposed edge weight approach, only 1 critical node (78) had nominal voltage 115 kV and all other 9 nodes had higher nominal voltage 230 kV. Similarly, the calculated efficiency drops were also higher for critical nodes identified using the proposed edge weight approach than using the traditional approach.

Table XIII shows the critical nodes in the single T network and T&C network. Although both node sets share 7 same nodes and have the same top 2 nodes, ranks of the rest node are different, which suggests that the importance of each individual node may be different in multilayer networks than in a single layer network. For example, node 17 is ranked at the third place in the single T network, but it is outside the top 10 critical nodes list for the interdependency from the T network to the C

TABLE XII
CRITICAL NODES IN THE T NETWORK FOR THE INTERDEPENDENCY BETWEEN T&C NETWORKS IDENTIFIED USING BOTH TRADITIONAL APPROACH AND THE PROPOSED EDGE WEIGHT APPROACH

| Rank | Edge Weight = 1 | Physical-based Edge Weight |
|------|-----------------|-----------------------------|
|      | \( \Delta E_p(v) \) (Node) | \( \Delta E_p(v) \) (%) | Voltage | \( \Delta E_p(v) \) (Node) | \( \Delta E_p(v) \) (%) | Voltage |
| 1    | 69              | 2.74%                       | 230     | 69              | 6.21%                       | 230     |
| 2    | 65              | 2.04%                       | 230     | 65              | 5.97%                       | 230     |
| 3    | 27              | 1.95%                       | 230     | 3               | 4.92%                       | 230     |
| 4    | 41              | 1.74%                       | 115     | 47              | 3.61%                       | 230     |
| 5    | 78              | 1.73%                       | 115     | 95              | 3.61%                       | 230     |
| 6    | 105             | 1.60%                       | 115     | 78              | 3.44%                       | 115     |
| 7    | 75              | 1.59%                       | 115     | 16              | 3.30%                       | 230     |
| 8    | 7               | 1.58%                       | 115     | 27              | 3.28%                       | 230     |
| 9    | 14              | 1.57%                       | 115     | 29              | 3.27%                       | 230     |
| 10   | 60              | 1.51%                       | 115     | 66              | 3.21%                       | 230     |

TABLE XIII
CRITICAL NODES IN THE T NETWORK ONLY AND FOR THE INTERDEPENDENCY BETWEEN T&C NETWORKS IDENTIFIED USING THE PROPOSED EDGE WEIGHT APPROACH

| Rank | T Network Only | T&C |
|------|----------------|-----|
|      | \( \Delta E_p(v) \) (Node) | Voltage | \( \Delta E_p(v) \) (Node) | Voltage |
| 1    | 69              | 230     | 69              | 230     |
| 2    | 65              | 230     | 65              | 230     |
| 3    | 17              | 230     | 3               | 230     |
| 4    | 24              | 230     | 47              | 230     |
| 5    | 47              | 230     | 95              | 230     |
| 6    | 95              | 230     | 78              | 115     |
| 7    | 34              | 230     | 16              | 230     |
| 8    | 16              | 230     | 27              | 230     |
| 9    | 3               | 230     | 29              | 230     |
| 10   | 29              | 230     | 66              | 230     |
network. This indicates it is important to consider the network mutual interdependence and quantify the different node importance in interdependent T&D&C networks.

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