Computationally-Efficient Algorithms for Multiuser Detection in Short Code Wideband CDMA TDD Systems

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Abstract: This paper derives and analyzes a novel block fast Fourier transform (FFT) based joint detection algorithm. The paper compares the performance and complexity of the novel block-FFT based joint detector to that of the Cholesky based joint detector and single user detection algorithms. The novel algorithm can operate at chip rate sampling, as well as higher sampling rates. For the performance/complexity analysis, the time division duplex (TDD) mode of a wideband code division multiplex access (WCDMA) is considered. The results indicate that the performance of the fast FFT based joint detector is comparable to that of the Cholesky based joint detector, and much superior to that of single user detection algorithms. On the other hand, the complexity of the fast FFT based joint detector is significantly lower than that of the Cholesky based joint detector and less than that of the single user detection algorithms. For the Cholesky based joint detector, the approximate Cholesky decomposition is applied. Moreover, the novel method can also be applied to any generic multiple-input-multiples-output (MIMO) system.

Index Terms: Code division multiple access (CDMA), inter-symbol interference (ISI), minimum mean squared error (MMSE) equalizers, multiuser detection, zero-forcing.

I. INTRODUCTION

In some communication systems, such as frequency division duplex code division multiple access (FDD-CDMA) and time division duplex CDMA (TDD-CDMA), multiple communications are sent over the same frequency spectrum. These communications are differentiated by their channelization (spreading) codes. TDD-CDMA communication systems use repeating frames divided into timeslots for communication. A single communication sent in such a system will have one or multiple associated codes and timeslots assigned to it. However for wide-band code division multiple access (WCDMA) systems [1], [2], conventional RAKE receivers suffer from severe degradation in frequency selective fading channels because of significant multi-access interference and inter-symbol interference. For such systems, minimum mean squared error (MMSE) based multiuser detectors [3]–[10] and joint detectors [11]–[19], which jointly removes multi-access interference (MAI) and inter-symbol interference (ISI), have attracted attention. Joint detector algorithms are characterized by good performance with high complexity. However, for short-code TDD-CDMA systems, codes have a length equal to the symbol period, which allows the development of joint detectors. However, spreading codes, called long spreading or scrambling codes, used in FDD-CDMA systems have a period much longer than the symbol period, for which it is difficult to design multiuser detectors, as stated in [19], [20].

Another approach to removing multi-access and inter symbol interference is single user detection [11], [12], [16], [21]–[25]. This is an approach based on channel equalization and is applicable to the downlink of a CDMA system, without any transmit diversity. In single user detection, the received signal is passed through an equalization stage, followed by de-spreading for recovering the data of a single mobile. The equalization stage can be implemented using an approximate Cholesky decomposition described in this paper. Single user detection also has recent practical significance, as in recent references [21] for impulse radio ultra wideband (IR-UWB) systems, and [22] for a multiuser interference channel with multi-antenna transmitters and single antenna receivers, restricting each transmitter to a Gaussian input and each receiver to a single-user detector, all boundary points of the achievable rate region can be achieved.

An earlier fast Fourier transform (FFT) based implementation of the joint detector was indicated in [4], [5]. However the prior approach is for chip rate sampling and can be extended to oversampling or multiple chip rate sampling, although needing some adjustments. Also, it is not clear as to how the channel matrix based methods in [4], [5], with addition of block columns and block rows, would perform, in single chip rate or oversampled case. No analysis has been done on the degradation in [4], [5], whereas an analysis of the degradation in the novel algorithm is undertaken in our paper. This paper provides a novel fast joint detector algorithm that is applicable at any sampling rate. Practical receivers typically operate at twice the chip rate or higher rates. The fast implementation is achieved through a block circulant approximation of the correlation matrix, unlike prior approaches. Research in joint detectors still has recent relevance as witnessed in [11]–[17], particularly [12], page 57, where advanced, computationally efficient joint detectors, like the one proposed in this paper, will play very crucial role in future TDD systems, like evolution of time division synchronous code division multiple access (TD-SCDMA) to TD-SCDMA to future terrestrial universal radio environment (FuTURE) TDD, including long-term evolution TDD and FuTURE Beyond 3G TDD.
This paper also shows that single user detection based algorithms exhibit some degradation, as compared to the joint detector algorithm of [3]–[18], [20] at the downlink of a TDD-CDMA system. One motivation to developing the fast joint detector algorithm in this paper was to develop an algorithm, low in complexity compared to the existing joint detectors, but superior in performance to that of the single user detection algorithms, at the downlink of a TDD-CDMA system. Simulations are presented to illustrate the relative performance of the new algorithm for different multipath channels specified in [1], [2]. The performance is analytically explained for different channel conditions.

Single user detection algorithms are also used in the case of multi-code transmission systems, where the data of a particular user may have been sent using multiple codes. WCDMA TDD mode has the option to support higher data rates, like the two Mbps data service, for which twelve codes, out of sixteen codes in a timeslot, of spreading factor sixteen each, are allocated to a single user. In this paper, we extend the new joint detector algorithm to the multi-code scenario. In particular, we study the performance of the new fast joint detector algorithm with that of the single user detector and other joint detector algorithms for this application. Computational complexities of the different algorithms are analyzed and it is seen that the novel algorithm in this paper provides far superior performance, at less computational complexity, compared to single user detection algorithms.

This paper also provides an analysis of the performance degradation of the single user detector algorithms in different multipath channels, and compares it to that of the fast joint detector algorithm in this paper. The paper also provides results at higher than chip rate sampling and illustrates the advantages of the new algorithm, as compared to the existing fast joint detectors and single user detectors. The paper also considers some issues in the design of the fast joint detector algorithm, like the implementation of the inverse required in the method. The novel method in this paper can also be applied to any generic multiple-input multiple-output (MIMO), MIMO-CDMA systems like [26], [27], as briefly explained in subsection IVF.

The paper is organized as follows. Section II gives the signal model, while the algorithm derivation is given in Section III. Section IV provides details regarding implementation of the novel detector; as well as comparison of computational complexities among the detectors, along with comparison with single-user detectors. Simulation results are in Section V, while conclusions are included in Section VI.

Notations: Bold upper-case symbols \(A\) are used to denote matrices. Bold lower-case symbol \(h\) is used to denote vectors. \(I_k\) is an identity matrix of size \((k \times k)\), \(O_{i,j}\) is a zero matrix of size \((i \times j)\).

II. SIGNAL MODEL

A typical universal terrestrial radio access (UTRA) WCDMA TDD communication burst is shown in Figure 1, in which the uplink and downlink transmissions are confined to different time-slots. Within each time-slot, multiple signals are multiplexed using CDMA. A typical communication burst has a mini-damble, a guard period and two data fields \(D_1\) and \(D_2\). The mini-damble separates the two data fields and contains a training sequence. The guard period separates the communication bursts to allow for the difference in arrival times of bursts transmitted from different transmitters. The two data fields contain the communication burst's data.

The fast joint detector in this paper is developed for this communication system. The receiver receives a combination of the data of \(K\) users (bursts) arriving simultaneously. If certain users are using multiple codes in a particular time slot, the \(K\) bursts may be for less than \(K\) users. Each data field of a timeslot has a predefined number of transmitted symbols, such as \(N_s\). For the \(k\)th user, each of its \(N_s\) symbols is spread by its code \(c^{(k)}\) with spreading factor \(SF\); accordingly, each data field has \((N_s \times SF)\) chips. After passing through a channel having an impulse response of \(W\) chips, each received data field has a length of \((SF \cdot N_s + W - 1)\) chips, which is denoted by \(N_c\). A typical value of \(W\) is 57 chips. Each \(k\)th user is received at the receiver and can be written, in the noiseless case, as

\[
r^{(k)} = A^{(k)} d^{(k)}, \quad k = 1, 2, \ldots K
\]

where \(r^{(k)}\) is the received contribution of the \(k\)th burst; \(A^{(k)}\) is the combined channel and spreading response, being an \((N_c \times N_s)\) matrix. Each \(j\)th column in \(A^{(k)}\) is a zero-padded and shifted version of the symbol response for the \(j\)th element of \(d^{(k)}\), see below in (4). The combined channel response is the convolution of the multipath response \(h^{(k)}\) and spreading code \(c^{(k)}\) for the \(k\)th burst. \(d^{(k)}\), of size \((N_s \times 1)\), is the unknown data symbols transmitted in the burst. The multipath channel response for each \(k\)th burst, \(h^{(k)}\), has length \(W\). In the downlink, without any transmit diversity, all the bursts, from the base-station intended for different users, pass through the same channel \(h^{(k)} = h\) to a particular user. In uplink, the multipath responses \(h^{(k)}\), from the different users to the base-station are different. If transmit diversity is employed in downlink, then also \(h^{(k)}\) is different for each \(k\). The overall received vector from all \(K\) bursts sent over the wireless channel is

\[
r = \sum_{k=1}^{K} r^{(k)} + n.
\]

III. ALGORITHM DEVELOPMENT

The algorithm is derived for the downlink situation without transmit diversity, in which case, \(h^{(k)} = h\). The novel method is for multi-user data estimation, assuming that the channel \(h\) is exactly known i.e., no channel estimation is involved. The spreading codes of all the users, as well as the scrambling code, for a particular base-station, is assumed to be known perfectly. The algorithm can be extended to the uplink situation, by having the multipath responses \(h^{(k)}\), from the different users to the base-station, different from each other. However, since in this paper, comparison with “single user detection” algorithms, designed for specifically for the downlink situation, without transmit diversity, will be made, the proposed joint detector will be derived only for the downlink. By combining the \(A^{(k)}\)'s for all
the data bursts into matrix $A$ and the unknown data for user $k$ as $d^{(k)}$ into the vector $d$, we have

\[
r = \sum_{k=1}^{K} c^{(k)} + n = \sum_{k=1}^{K} A^{(k)} d^{(k)} + n = [A^{(1)}, A^{(2)}, \ldots, A^{(K)}] \begin{bmatrix} d^{(1)} \\ d^{(2)} \\ \vdots \\ d^{(K)} \end{bmatrix} + n = Ad + n \tag{3}
\]

where $A = [A^{(1)}, A^{(2)}, \ldots, A^{(K)}]$ is the concatenated channel response and $d = [d^{(1)^T}, d^{(2)^T}, \ldots, d^{(K)^T}]^T$ is the concatenated data vector for the $K$ users. By the assumption of perfect channel and code knowledge, each $A^{(k)}$, and hence $A$, is known exactly. Since each $A^{(k)}$ is of size $(N_c \times N_s)$ and $d^{(k)}$ is of size $(N_c \times 1)$, the concatenated channel response $A$ is of size $(N_c \times K N_s)$ and the concatenated data vector $d$ is of size $(K N_s \times 1)$. The above model includes both MAI and ISI in the received signal. In (3), $r$ is the chip rate sampled received vector of length $N_c$ chips, and $n$ is the zero-mean noise vector. The maximum number of users is $K$, with $b^{(k)}(i)$ being the convolution of the channel response $h$ and the spreading code $e^{(k)}$, for the $k$th user at the $j$th chip interval of the $i$th symbol interval. Then $A^{(k)}$, for $k$th user, can be written as

\[
A^{(k)} = \begin{bmatrix} b^{(1)}_1(0) & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \cdots & b^{(k)}_1(1) \\
0 & 0 & \cdots & \cdots & b^{(k)}_{SF}(1) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\end{bmatrix}_{k = 1, 2, \ldots, K}. \tag{4}
\]

Now defining the block as

\[
B(i) = \begin{bmatrix} b^{(1)}_1(i) & \cdots & b^{(K)}_1(i) \\
\vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots \\
0 & \cdots & b^{(1)}_{SF}(i) \\
0 & \cdots & b^{(K)}_{SF}(i) \\
\end{bmatrix}
\]

Then we can write the transfer matrix $A$, in (3), in terms of $B(i)$, by re-arranging columns of $A$, in (3), so that the first columns of each $A^{(k)}$, for $k = 1, 2, \ldots, K$, are put side by side to first form the first $K$ columns of $A$, then the second columns of each $A^{(k)}$ are put side by side to first form the $(K + 1)$th to $2K$th columns of $A$, and continuing in this fashion.

\[
A = \begin{bmatrix} B(0) & 0 & 0 & \cdots & 0 \\
B(1) & B(0) & 0 & \cdots & 0 \\
B(2) & B(1) & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & B(2) & B(1) & \cdots \\
\end{bmatrix}
\]

Note that the right-most matrix on right hand side of (6) is a column re-arranged version of the actual $A$, which we still denote by $A$. Then the normal equations for the optimal least squares solution to the joint detection problem is given by

\[
d = R^{-1}(A^H r) = (A^H A + \sigma^2 I)^{-1}(A^H r), \tag{7}
\]

where the new data vector $d$, in (7) above, is of size $(K N_s + L - 1) \times 1$. $R = (A^H A + \sigma^2 I)$ is called the channel correlation matrix. The matrix $R$ is block Toeplitz as well as banded and algorithms for banded block Toeplitz system can be applied to it, e.g. approximate Cholesky joint detection [4]. For illustration purposes, a simplified example of $R$ with $N_s = 10$, is considered. Let $L$ be the combined multipath and spreading delay (in symbols), where $L = \lceil (SF + W - 1)/SF \rceil$. In the example below, let $L = 2$. Then

\[
R = \begin{bmatrix} R_0 & R_1^H & R_2^H & 0 & 0 & 0 & 0 & 0 & 0 \\
R_1 & R_0 & R_2^H & R_0^H & 0 & 0 & 0 & 0 & 0 \\
R_2 & R_1 & R_0 & R_2^H & R_1^H & 0 & 0 & 0 & 0 \\
0 & R_2 & R_1 & R_0 & R_2^H & R_2^H & 0 & 0 & 0 \\
0 & 0 & R_2 & R_1 & R_0 & R_1^H & R_1^H & 0 & 0 \\
0 & 0 & 0 & R_2 & R_1 & R_0 & R_0^H & R_0^H & 0 \\
0 & 0 & 0 & 0 & R_2 & R_1 & R_0 & R_2^H & R_2^H \\
0 & 0 & 0 & 0 & 0 & R_2 & R_1 & R_0 & R_1^H \\
0 & 0 & 0 & 0 & 0 & 0 & R_2 & R_1 & R_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & R_2 & R_1 \\
\end{bmatrix}
\]

This example is easily extendable for any value of $N_s$ and $L$. In general, the matrix $R$ is of size $(K N_s + K N_s) \times (K N_s + K N_s)$, each element $R_{ij}$ being a $(K \times K)$ block. The matrix $R$ has $N_s$ block rows and $N_s$ block columns, with $(L + 1)$ non-zero blocks in the first and last block-rows, and $(2L + 1)$ non-zero blocks in each block-row, in the middle portion of the matrix, i.e., the sub-matrix within the dotted lines of the matrix. $R$ also has an additional structure — the banded and Toeplitz structure of $R$ makes the sub-matrix, within the dotted lines of $R$, block-circulant. The portion of $R$, which is not block-circulant, depends on the value of $L$. Practical receivers for TDD system operate at twice the chip rate or a multiple of the chip rate to provide robustness to timing errors. For multiple chip rate sampling, the received signal model is
where sampling is at \( N \) times the chip rate. The sequence \( r_n \)
corresponds to the \( n \)th chip rate sub-vector, out of \( N \) such
sequences corresponding to oversampling by a factor of \( N \). In
this case, the correlation matrix \( \mathbf{R} \) can be written as

\[
\mathbf{R} = \mathbf{A}^H \mathbf{A} + \sigma^2 \mathbf{I} = \sum_{i=1}^{N} \mathbf{A}_i^H \mathbf{A}_i + \sigma^2 \mathbf{I}.
\]  

(10)

The structure of the \( \mathbf{R} \) matrix, for multiple chip rate sampling,
is still the same as in (8). The number of row-blocks of \( \mathbf{R} \) that is
outside the block-circulant structure, i.e., outside the dotted lines
of \( \mathbf{R} \), is equal to \( 2L \), actually \( L \) block-rows are outside the
dotted lines of \( \mathbf{R} \), at the top and \( L \) block-rows at the bottom of \( \mathbf{R} \).
The algorithm, derived in this paper, is based on block-circulant
extension of the matrix \( \mathbf{R} \). Since a block circulant approxima-
tion of the correlation matrix \( \mathbf{R} \), in (10), is used, the fast joint
detector can be easily developed for higher rate sampling. This is
in contrast with the approaches in [4], [5], where a circulant
approximation to the data matrix, not correlation matrix, is con-
sidered and requires addition of block columns and block rows
to make it block square. In the case of oversampling given by
(9), this will be cumbersome, needing some adjustments. Also,
it is not clear as to how the channel matrix based methods in [4],
[5], with addition of block columns and block rows, would per-
form in single chip rate or oversampled case. No analysis has
been done on the degradation in [4], [5], whereas an analysis of
the degradation in the novel algorithm is undertaken in our pa-
per. The novel approach in this paper has no such problems and
has been extended to the oversampling case in a straightforward
manner.

The derivation of a block FFT based joint detector is outlined
below. This will enable an appreciation of the final form of the
algorithm. The block circulant extension of the correlation ma-
x

\[
\mathbf{R}_c = \begin{bmatrix}
\mathbf{R}_0 & \mathbf{R}_1^H & 0 & 0 & 0 & 0 & 0 & \mathbf{R}_2 & \mathbf{R}_1 \\
\mathbf{R}_1 & \mathbf{R}_0 & \mathbf{R}_1^H \mathbf{R}_2 & 0 & 0 & 0 & 0 & \mathbf{R}_2 & \mathbf{R}_1 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
\mathbf{R}_2 & \mathbf{R}_1 & \mathbf{R}_0 & \mathbf{R}_0^H & \mathbf{R}_1 & 0 & 0 & 0 & 0 \\
0 & \mathbf{R}_2 & \mathbf{R}_3 & \mathbf{R}_0 & \mathbf{R}_0^H & 0 & 0 & 0 & 0 \\
0 & 0 & \mathbf{R}_2 & \mathbf{R}_1 & \mathbf{R}_0 & \mathbf{R}_1^H & \mathbf{R}_2 & 0 & 0 \\
0 & 0 & 0 & \mathbf{R}_2 & \mathbf{R}_1 & \mathbf{R}_0 & \mathbf{R}_0^H & \mathbf{R}_1 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
\mathbf{R}_2^H & 0 & 0 & 0 & 0 & \mathbf{R}_2 & \mathbf{R}_1 & \mathbf{R}_0 & \mathbf{R}_1^H \\
\mathbf{R}_1^H & \mathbf{R}_2^H & 0 & 0 & 0 & 0 & \mathbf{R}_2 & \mathbf{R}_1 & \mathbf{R}_0 \\
\end{bmatrix}
\]  

(11)

Objective: It is required to compute \( \mathbf{R}^{-1} \), in order to es-
timate the transmitted data, by the least squares criterion in (7).
In order to reduce the computational complexity of computing
the inverse of \( \mathbf{R} \), we approximate \( \mathbf{R} \) by the block circulant ma-
x

\[\mathbf{R}_c = \mathbf{DA} \mathbf{D}^H,\]  

where \( \mathbf{D} \) is a block unitary matrix and \( \mathbf{A} \) is a block diagonal
diagonal matrix. Hence we need to determine \( \mathbf{D} \) and \( \mathbf{A} \).
Theorem 1 Let the eigen-decomposition of \( \mathbf{R}_c \) be given by

\[\mathbf{R}_c = e^{\mathbf{KH}} \mathbf{I}_K \mathbf{D} e^{\mathbf{K}^H} = \mathbf{D} \mathbf{A} \mathbf{D}^H,\]

where \( \mathbf{I}_K \) is a \((K \times K)\) identity matrix. \( \mathbf{D} \) is a "block
discrete Fourier transform (DFT)"- like matrix, i.e., each of its entry is a
\((K \times K)\) block. Furthermore, it can be shown that \( \mathbf{D}^H \mathbf{D} =
\mathbf{N}_s \mathbf{I}_{K\mathbf{N}_s} \), where \( \mathbf{I}_{K\mathbf{N}_s} \) is the \((K\mathbf{N}_s \times K\mathbf{N}_s)\) identity
matrix. Also, the block-diagonal matrix \( \mathbf{A} \), of size \((K\mathbf{N}_s \times K\mathbf{N}_s)\), is

\[
\mathbf{A} = \begin{bmatrix}
\mathbf{A}^{(1)} \\
\mathbf{A}^{(2)} \\
\vdots \\
\mathbf{A}^{(N_s)}
\end{bmatrix},
\]  

(13)

with each entry

\[
\mathbf{A}^{(i)} = \begin{bmatrix}
\lambda_1^{(i)} & \cdots & \lambda_i^{(i)} \\
\lambda_1^{(i)} & \cdots & \lambda_i^{(i)} \\
\vdots & \ddots & \vdots \\
\lambda_1^{(i)} & \cdots & \lambda_i^{(i)}
\end{bmatrix},
\]  

(14)

of size \((K \times K)\). Then it can be shown that

\[
\mathbf{A}^{(k)} = \mathbf{R}_L e^{-j2\pi(k-1)s/N_s} + \cdots + \mathbf{R}_1 e^{-j2\pi(k-1)s/N_s} \]

\[
+ \mathbf{R}_0 \mathbf{R}_L e^{-j2\pi(k-1)s/N_s} + \cdots + \mathbf{R}_0 \mathbf{R}_1 e^{-j2\pi(k-1)s/N_s},
\]  

(15)

Proof: See Appendix A.
computing the block-FFT of \( \{0,0,\ldots,R_2,R_1,R_0,R_H,R_H,\ldots,0,0\} \), or as the FFT of a two-sided sequence of \((K \times K)\) blocks \(\{R_0,R_1,\ldots,R_L\}\). As shown in the above equation (15), all the \((\Lambda(k))'s\) can be calculated using one block-row of \(R_i\); thus \(R_i\) does not need to be explicitly computed. Any block-row of \(R\), which is at least \(L\) block-rows from both the top and the bottom of the \(R\) matrix, can be used to calculate \(\Lambda^{(k)}\)'s, since these block-rows have the full set of \(R_i\)'s. In general form, \(\Lambda^{(k)}\) is given by (15), which requires calculation of \(K^2\) FFTs, each of length \(N_s\). Using the \((\Lambda(k))\) and \(D\) matrices, the block-circulant matrix \(R_c\) can be written as

\[
R_c = \frac{1}{N_s} D \Lambda D^H
\]  
(16)

where \(D\) and \(\Lambda\) are each of size \((KN_s \times KN_s)\). Since \(D^H D = N_s I_{KN_s}\), \(D^{-1} = (1/N_s) D^H\). Thus we have

\[
R_c^{-1} = N_s [(D^H)^{-1} \Lambda^{-1} D^{-1}] = N_s [D/(N_s) \Lambda^{-1} (D^H/N_s)] = (D/N_s) \Lambda^{-1} D^H.
\]  
(17)

The detected data vector \(\hat{d}\), of size \((KN_s \times 1)\), can be estimated using the least squares criterion as

\[
\hat{d} = R_c^{-1} (A^H r) = (D/N_s) \Lambda^{-1} D^H (A^H r),
\]  
(18)

which can be written as

\[
D^H \hat{d} = \Lambda^{-1} [D^H (A^H r)].
\]  
(19)

Now

\[
\Lambda = \begin{bmatrix}
\Lambda^{(1)} \\
\Lambda^{(2)} \\
\vdots \\
\Lambda^{(N_s)} 
\end{bmatrix}
\]  

\[
\Rightarrow \Lambda^{-1} = \begin{bmatrix}
\Lambda^{(1)}^{-1} \\
\Lambda^{(2)}^{-1} \\
\vdots \\
\Lambda^{(N_s)}^{-1}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
[A^{(1)}]^{-1} \\
[A^{(2)}]^{-1} \\
\vdots \\
[A^{(N_s)}]^{-1}
\end{bmatrix}.
\]  
(20)

The inversion requires \(N_s\) inversions of \((K \times K)\) matrices \(\Lambda^{(k)}\). As a result, the data estimation equation (19) can be re-written as

\[
[F(\hat{d})]_k = [\Lambda^{(k)}]^{-1} [F(A^H r)]_k
\]  
(21)

for each frequency point \(k\). \(F(.)\) refers to the FFT operation, since \((D^H \hat{d})_k = [F(\hat{d})]_k\). Equation (21) is applicable to both receivers, that sample the received signal at the chip rate and those that oversample the received signal at a multiple of the chip rate, such as twice the chip rate. For multiple chip rate receivers, the matrix \(R\), corresponding both to multiple chip rate sampling or just chip rate signaling, is of the form as (8), being approximately block-circulant, so all the above analysis holds.

### A. Further Reductions in Computational Complexity

To estimate the transmitted data, of \(K\) users by (21), one needs to compute \([F(A^H r)]\). Further reductions in complexity in computing the matched filter \((A^H r)\) can be achieved by exploiting the special structure of the overall channel response matrix \(A\), which is again accomplished by a block FFT method. A glance at the matrices \(A\) and \(B(i)\), in (5) and (6), shows that the matrix \(A\), of size \(N_s SF \times KN_s\), is a non-square matrix; however a portion of \(A\) is also block-circulant. \(L\) block-rows at the top and \(L\) block-rows at the bottom prevent \(A\) from being circulant. A block-circulant extension of the matrix \(A\) is denoted by \(A_c\).

Theorem 2 \(A_c\) can be written as

\[
A_c = D_1 A_1 D_2^H,
\]  
(22)

where \(D_1\) is a \((N_s SF \times N_s SF)\) matrix, \(D_2\) is a \((N_s K \times N_s K)\) matrix and \(A_1\) is a \((N_s SF \times N_s K)\) block-diagonal matrix. The matrix \(A_1\) is of the same form as \(A\) in (20), except that each block \(A^{(k)}\) is non-square, being of size \((SF \times K)\), and

\[
D_1 = \begin{bmatrix}
I & e^{j(\pi/4) SF} & \cdots & e^{j(\pi/4)(N_s-1) SF} \\
I & e^{j(\pi/4) SF} & \cdots & e^{j(\pi/4)(N_s-1) SF} \\
\vdots & \vdots & \ddots & \vdots \\
I & e^{j(\pi/4) SF} & \cdots & e^{j(\pi/4)(N_s-1) SF}
\end{bmatrix}.
\]  
(23)

\(D_2\) is of the same form as the \(D\) in (12). Then it can be shown that

\[
A_1^{(k)} = [B(0) + B(1)e^{-j(\pi/4) SF} + \cdots + B(L)e^{-j(\pi/4)(L-1) SF}],
\]  
(24)

\(k = 0,1,\cdots, N_s\).

Proof: See Appendix B.

As a result, each \(A^{(k)}\) can be determined as the one-sided FFT of \((SF \times K)\) blocks \(B(i)\)'s. From (22) and using the fact that \(D_1^H D_2 = N_s I_{KN_s}\)

\[
A_c^H r = D_2 A_1^H (D_1^H r),
\]  
(25)

\[
D_2^H (A_c^H r) = N_s [A_1^H (D_1^H r)].
\]  
(26)

Accordingly, \(F(A_c^H r)\) can be determined using FFTs per

\[
[F(A_c^H r)]_k = N_s [A_1^{(k)}]_H [F(r)]_k.
\]  
(27)

Similarly, since the matrix \(A\) is approximately block-circulant, \(R = (A^H A + \sigma^2 I)\) can be implemented using block FFTs using \(A_1\).

### B. Implementation of the Inverse

The optimal least squares estimate of the transmitted data is obtained by using (21). It requires computation of \([F(A_c^H r)]_k\).
efficient methods for calculating which are outlined in sub-
section III.A, and $|\Lambda^{(k)}|^{-1}$. An efficient method for the imple-
mation of $|\Lambda^{(k)}|^{-1}$ is detailed in this subsection. The matrix
$|\Lambda^{(k)}|$ is Hermitian, but has no special structure. To reduce the
computational complexity of calculating $|\Lambda^{(k)}|^{-1}$, the solu-
tion of (21) can be performed using a LU (lower-upper) decom-
position. Each $|\Lambda^{(k)}|$ is a $(K \times K)$ matrix, whose LU de-
composition is given by

$$
\Lambda^{(k)} = LU,
$$

(28)

where $L$ is a lower triangular matrix and $U$ is an upper triangular
matrix. Equation (21) can then be solved by using forward and
backward substitution. The forward substitution uses the lower
triangular matrix $L$, starting from (21),

$$
[A^{(k)}][F(\hat{d})]_k = [F(A^H r)]_k = LU[F(\hat{d})]_k = [F(A^H r)]_k \\
\Rightarrow Ly = [F(A^H r)]_k,
$$

(29)

where $y = U[F(\hat{d})]_k$, and finds an estimate of $y$. The backward
substitution then uses the upper triangular matrix $U$ to solve for

$$
F(\hat{d})_k = y.
$$

(30)

To improve the bit error rate (BER) for the data symbols at the
ends of each data field, samples from the midamble portion and the
guard period are used in the data estimation algorithm as shown in
Fig. 1. To collect all the samples of the last symbols in the
data fields, the samples $r$ are extended by $W - 1$ chips into the
midamble and guard period. This permits utilization of all the
multipath components of the last data symbols for data esti-
mation purposes. The known midamble sequences are canceled
from the received samples, corresponding to the midamble, prior
to data estimation. Similarly, the guard period is used for the
second data field.

IV. ANALYSIS OF NOVEL ALGORITHM

A. Analysis of Approximations Involved in Novel Algorithm

The proposed novel block FFT algorithm is an approximate
algorithm, where the banded and block-Toeplitz autocorrelation
matrix $R$ in (8) is approximated by a block-circulant matrix
$R_c$ in (11), resulting in the degradation in the performance of
the proposed detector. From (8) and (11), it is seen that the
block rows, within the dotted lines of both $R$ and $R_c$, are the
same. Thus, $(N_s - 2L)$ block rows of both $R$ and $R_c$ are ex-
actly the same. The approximation lies in the $L$ block-rows at the
top of $R$, where some zero blocks in top-right of $R$ are re-
placed by non-zero blocks in $R_c$, as well as in $L$ block-rows at the
bottom of $R$, where some zero blocks in bottom-left of $R$ are re-
placed by non-zero blocks in $R_c$, resulting in $2L$ block-
rows of $R$ being outside the block-circulant structure, and thus
$2L$ block-rows of $R$ and $R_c$ are different. Since the least squares
estimate of the transmitted data in (7) requires $R$, but to obtain
reduced computational complexity, we are actually estimating the
transmitted data by

$$
\hat{d} = R_c^{-1}(A^H r),
$$

(31)

with more terms appearing in the corners of $R_c$. Since $2L$ block-
rows of $R$ and $R_c$, outside the dotted lines in (8) and (11), differ,
obviously the estimate of transmitted data using (31) is de-
graded, with respect to the optimal least squares estimate of trans-
mitted data obtained using (7). Thus, a measure of the de-
gradation of the proposed block-FFT detector is the fraction of
non-circulant block-rows in $R$, which is $(2L/N_s)$, since both
$R$ and $R_c$ have $(N_s \times N_s)$ blocks. For channels with small
multipath spread and thus small $L$, $R$ is close to being a circulant
matrix, i.e. close to $R_c$, and hence (7) and (31) are almost equi-
alent and the novel detector performs well as the optimal least squares
detector. However for channels with larger multipath spread, i.e.,
with larger $L$, $R$ deviates from the circulant structure, i.e., $R_c$,
leading to divergence of (31) from (7), leading to larger degrada-
tion of the proposed block-FFT detector relative to the optimal least squares joint detector. In the above analy-
sis, it is assumed that the powers of the multipath profile are the
same in both the channels, one with small $L$ and the other with
large $L$. As an illustration, the multipath profiles specified in 3rd
Generation Partnership Project for UTRA TDD radio transmis-
sion and reception document [1] and denoted by tddWq4Case1,
tddWq4Case2, and tddWq4Case3 are considered. Case 1 and
Case 3 channels have multipath spread within 5 or 6 chip in-
tervals. Case 2 channel however has equal power multipaths at
delays of 1, 5, and 47 chips. For multipath spread $W = 6$, as
in Case1 and Case3 channels, and spreading factor $SF = 16$,
$L = [(SF + W - 1)/SF] = 2$, which means only two block
rows at the top, and two block rows at the bottom, of $R$ and
$R_c$ are different. The degradation measure, introduced above,
is $(2L/N_s) = 4/61 = 0.065$, where $N_s = 61$ in TDD Burst
Type I. Simulation results show that the degradation of the novel
joint detector, in Case 1 and 3 channels, is small compared to the
optimal joint detector. However, for Case 2 channel with multi-
path spread $W = 47$, $L = [(SF + W - 1)/SF] = 4$, which
means that four block rows at the top, and four block rows at the
bottom, of $R$ and $R_c$ are different, which results in a degrada-
tion measure of $(2L/N_s) = 8/61 = 0.1311$, resulting in large
degradation of the novel detector in Case 2 channel compared to
optimal joint detector. Simulations indicate that even for equal
power multipaths, if the maximum delay spread $L$ is reduced, in
say, a modified channel, the performance of the proposed detector
improves.

B. Higher Data Rate Services

For higher data rate services, WCDMA TDD uses a multi-
code, multi-slot strategy. For 2 Mbps data service, twelve codes
of spreading factor 16 each and twelve out of the 15 timeslots
in a radio frame are allocated to one user. Refer to (5) for the
entries of matrix $B(i)$, which are used to form the system matrix
$A$. If $M$ codes are allocated to one user, then $M$ columns of
the matrix $B(i)$ are allocated to that user, while the remaining
$(K - M)$ columns of the matrix $B(i)$ are for the remaining users.
Simulation results are shown in this paper for the 2 Mbps data
service.

C. Review of Existing Algorithms

In this section, we review some existing data estimation algo-
rithms, both of joint detection and single detection categories,
for comparison purposes. First, within the “joint detector” category, an approximate Cholesky factor of the correlation matrix $R$-based computationally efficient block algorithm [3], [6], widely used in a TDD system and denoted by JDChol, computes only the first few lower triangular submatrices of the Cholesky factor, and making the rest of lower triangular matrix i.e., the Cholesky factor, block Toeplitz, then estimates the transmitted data by (7), using forward and backward substitutions, with some loss in accuracy. The novel block FFT based joint detector introduced in this paper is denoted by JDFFT, while the matched filter based joint detector, denoted by MF, computes $(A^H r)$.

Next, we review some existing data estimation algorithms within the “single user detection” category [12], [21]–[24], which designed for the downlink without any transmit diversity, for which the received signal, for an oversampling factor of $N$, can be expressed as (starting from (9)),

$$
\begin{bmatrix}
    r_1 \\
    r_2 \\
    \vdots \\
    r_N
\end{bmatrix} =
\begin{bmatrix}
    A_1 \\
    A_2 \\
    \vdots \\
    A_N
\end{bmatrix} d + n =
\begin{bmatrix}
    H_1 \\
    H_2 \\
    \vdots \\
    H_N
\end{bmatrix}
\begin{bmatrix}
    C_1 & C_2 & \cdots & C_K
\end{bmatrix} d + n. \quad (32)
$$

This follows from $A_\nu$, for the $i$th oversampling branch, containing the convolution of the channel response $h^{(k)}$ and spreading code $c^{(k)}$, for the $i$th user; $d^{(k)}$ is the unknown data symbols transmitted to the $i$th user. In the downlink, without any transmit diversity, all the bursts, intended for the different users, pass from the base-station through the same channel $h^{(k)} = h$ to a particular user. In (32), $H_i$ is the associated channel matrix for the $i$th oversampling branch, and is the same for all the users.

Let $s = \begin{bmatrix} C_1 & C_2 & \cdots & C_K \end{bmatrix} d$. Then $s$ can be estimated by the least squares criterion,

$$
\hat{s} = (H^H H + \sigma^2 I)^{-1} (H^H r) \quad (33)
$$

where

$$
H = [H_1^H H_2^H \cdots H_N^H]^H,
$$

and then de-spreading $\hat{s}$ by the code-matrix $[C_1 C_2 \cdots C_K]$, to estimate the data symbols of all $K$ users $\hat{d}$. If the approximate Cholesky algorithm is applied to the matrix $(H^H H + \sigma^2 I)$ to estimate $\hat{s}$ by (33) (referred to as chip-level equalization), then the method is referred to as SDChol [23]. Next, a FFT algorithm, within the “single user detection” category, introduced in this paper and denoted by SDFFT, is obtained by observing that

$$
R \sim H^H H + \sigma^2 I = \sum_{i=1}^{N} H_i^H H_i + \sigma^2 I \quad (35)
$$

is also approximately circulant, not block-circulant, and the portion, that is not circulant, is equal to $2W$ rows. A derivation, similar to above, gives us

$$
F(\hat{s}) = \frac{F(H^H r)}{N_F F(R_i)}, \quad (36)
$$

where $F(R_i)$ is the scalar Fourier transform of a suitable column of $(R_i)_i$; then $\hat{d}$ is obtained by de-spreading $\hat{s}$.

\section{D. Computational Complexity}

An analysis of the computational complexity of the novel joint detector and its comparison with other data detectors are now undertaken. The complexity of calculating $A$ is K.S.F.W. The computational complexity of calculating $A^H A$ is $((K^2 + K)[2(SF + W - 1) - (n_{max} - 1)]/2 : n_{max}/2 - ((K - K)(SF + W - 1))/2$, where $n_{max} = \min(N_s, (SF + W - 1)/SF + 1)$. Calculating $A^H r$, a matrix vector multiplication, requires $K N_s (SF + W - 1)$ calculations. Calculating the FFT of the $k$th block-column of $R$ requires $K^2 (N_s \log_2 N_s)$ computations. Calculating $F(F(\hat{d}))$, without LU decomposition, requires $K^3$ calculations, which for $N_s$ frequency points, requires $N_s K^3$ number of calculations. Calculating $[F(F(\hat{d}))]_k = [A^{(k)}]^{-1} [F(A^H r)]_k$ requires $K^2$ calculations, each for $N_s$ frequency points, resulting in $N_s K^2$ total number of calculations. The inverse FFT of $[F(F(\hat{d}))]$ requires $K(N_s \log_2 N_s)$ computations.

To illustrate the complexity for fast joint (multiuser) detection, the number of million real operations per second (MOPS) for processing a TDD Burst Type I [1], with $N_c = 976$, $N_s = 61$, spreading factor $SF = 16$, number of codes (users) $K = 8$, $W = 57$ chips, is determined. The calculations of $A$, $A^H A$, a block-column of $R$, $A(k), [A^{(k)}]^{-1}$ are performed once per TDD burst, i.e., 100 times per second, for which MOPS is shown in Table 1. The calculations $A^H r$, $F(A^H r)$, computing and inverse FFT of $[F(F(\hat{d}))]$ are performed twice per burst, i.e., 200 times per second, for which MOPS is shown in Table 2. Four calculations are required to convert a complex operation into a real operation.

Thus, total number of MOPS for calculation efficient multiuser (joint) detection is $65.02$ MOPS, where $(A^H r)$ is calculated directly as a matrix-vector multiplication. If FFTs are used to calculate $(A^H r)$, using (27), the computational complexity then reduces from 65.02 to 63.99 MOPS. Though this reduction is not significant in the case of this novel fast joint detector, using (27) gives significant computational advantages over direct matrix-vector multiplication in the case of FFT based single-user detection algorithm SDFFT. Also, if a LU decomposition is used to determine $[A^{(k)}]^{-1}$, using (28)–(30), the calculation load of the novel FFT-based joint detector reduces to 54.87 MOPS. Comparison of the computational complexities of the fast joint detection and other detection techniques, for a TDD Burst Type I with $SF = 16$ and $K = 8$, is given below in Table 3.

The complexity of SDChol is much higher than JDChol, because it involves chip level equalization. If for higher data rates, there are 12 codes of $SF = 16$ each, the complexity of JDChol
is 177 MROPS while the complexity of JDFFT is 90 MROPS, which is about 50% reduction in complexity.

E. Comparison with Single User Detection

Performance of the five algorithms enumerated in subsection IV.C, i.e., JDChol, JDFFT, MF, SDChol, and SDFFT, will be analyzed in this subsection. Simulation results, in Section V, indicate that single user detection algorithms, like SDChol and SDFFT, suffer from appreciable degredation in performance, compared to joint detector algorithms (JDChol, JDFFT) in some multipath channels. The degradation in performance of a FFT based single user detection algorithm (SDFFT) can be attributed to two factors:

1. Degradation due to single user detection framework: In single user detection in the downlink, the user only uses its own spreading code for decoding the data. This has an advantage over joint detectors, which requires the spreading codes of all users, in that there is no need to estimate the codes for the other users (referred to as blind code detection). However, this leads to a degradation in its performance, compared to joint detector algorithms, which uses information about the spreading codes of all the the users in its data estimation [3]–[10].

2. Degradation due to using a FFT approximation: The degradation in performance, due to the FFT approximation, has been analyzed in subsection IV.A, where it is seen that the FFT based algorithms degrade more in channels with large multipath spread. Since single user detection algorithms are computed at the chip level, it is seen that a total of 2W rows, not block-rows, of \((HH^H + \sigma^2I)\) in (35), are outside a circulant structure.

Thus a measure of degradation in SDFFT can be given by \((2W/(N_sSF)))\), similar to JDFFT case in subsection IV.A. For Case 1 and Case 3 channels \((W = 6)\), a degradation measure in SDFFT is \((12/(61 \times 16)) = 0.0123\), whereas for Case 2 channel, \((W = 47)\), a degradation measure in SDFFT is \((94/(61 \times 16)) = 0.0963\). As expected, simulations show that the degradation of SDFFT in Case 2 channel is much worse than Case 1 and Case 3 channels. To illustrate the degradation due to “single user detection” framework only (factor number one only), performance of SDChol is compared with joint detector algorithms (JDChol, JDFFT) and it exhibits appreciable degradation with respect to them. However, SDFFT, though of much reduced complexity, suffers more degradation, due to both “single user detection” framework as well as FFT approximation, and performs worse than JDChol, JDFFT as well as SDChol.

The development of JDFFT in this paper was motivated by the need to remove factor number one for degradation in performance, while achieving reduction in complexity as compared to JDChol.

F. Extension to Multi-Input Multi-Output (MIMO) Systems

In a general MIMO system with \(J\) transmit antennas and \(M\) receive antennas, the MIMO channel matrix at the \(l\)th tap delay is given by

\[
H(l) = \begin{bmatrix} h^{(1,1)}(l) & h^{(1,2)}(l) & \ldots & h^{(1,J)}(l) \\
\vdots & \vdots & \ddots & \vdots \\
\end{bmatrix},
\]

(37)
of size \((M \times J)\), where \(h^{(m,j)}(l)\) is the channel, at \(l\)th tap delay, between the \(j\)th transmit antenna and \(m\)th receive antenna. Then the overall channel matrix, of size \((MN_s \times J(N_s + L - 1))\), is

\[
\tilde{H} = \begin{bmatrix} H(0) & H(1) & \ldots & H(L) & 0 \\
0 & H(0) & H(1) & \ldots & H(L) \\
\end{bmatrix}.
\]

(38)

Then the received signal \(r\), of size \((MN_s \times 1)\), at \(M\) receive antennas, over \(N_s\) symbols, is

\[
r = \tilde{H}d + n
\]

(39)

where \(d\) is the \((J(N_s + L - 1) \times 1)\) data vector, from \(J\) transmit antennas. Then the normal equations for the optimal least squares solution is given by \(d = (\tilde{H}^H\tilde{H} + \sigma^2I)^{-1}\tilde{H}^Hr\).

Then the channel autocorrelation matrix \(R = (\tilde{H}^H\tilde{H} + \sigma^2I)\) has the same structure as the channel correlation matrix of a CDMA system in (8), and thus the matrix \(R\), of this MIMO system, can be approximated by the block-circulant matrix \(R_c\) in (11). Then a block-FFT algorithm can be developed can be developed for MIMO system, along the same lines as the CDMA system, which has the same analysis and advantages as above for the CDMA system above and can easily be extended even to a MIMO-CDMA system. Comparison of the novel algorithm with [4], [5] is provided in Appendix C.

V. SIMULATION RESULTS

The performance of JDFFT, JDChol, SDChol, and SDFFT along with matched filtering (MF), for the parameters for Burst Type I, is illustrated in the following plots. Channels specified by the WCDMA TDD specifications [1] were tested. For the approximate Cholesky based joint detector (JDChol) and the FFT based joint detector (JDFFT), it was assumed that the spreading codes of all the users are known. In general, the joint detector algorithms have to estimate the spreading codes of the other users, which adds to the complexity. All the simulations are for
the downlink situation, without any transmit diversity. Simulation results are provided for the following cases. The simulations were performed over 800 timeslots.

In the first case, with $SF = 16$ and $K = 8$, Figs. 2–4 show the un-coded bit error rate (BER) for Case 1, Case 3, and Case 2 channels respectively. For both Case 1 and Case 3 channels, the performance of $J_DFFT$ is very close to $J_DChol$, which, for example, is a standard algorithm used in WCDMA TDD standard. Under the "single user detection" (SUD) category, even the Cholesky based $SDChol$ exhibits a lot of degradation, compared to $J_DChol$ and $J_DFFT$ algorithms, for all channels, including Case 1 and Case 3 channels. FFT based single user detection $SDFFT$ performs even worse. However, there is some degradation in the performance of $J_DFFT$ with respect to $J_DChol$ in Case 2 channel, because, as analyzed earlier, of the degradation in FFT based algorithms in channels with large multipath spread.

Case 2 channel is characterized by equal power multipaths at delays of 1 and 5 chip intervals, and at a significant delay of 47 chip intervals. To study the effect of multipath spread on detector performance, a modification to the Case 2 channel has equal power multipaths at delays of 1, 5, and 9, instead of 47, chip intervals. The simulation results are shown in Fig. 5, where it is seen that $J_DFFT$ exhibits less degradation, with respect to $J_DChol$ than in Fig. 4 for Case 2 channel, thereby illustrating that FFT based algorithms are susceptible to channels with multipath spread at long delays. The fast joint detector in this paper is suitable for higher sampling rates. Simulation results are provided, for oversampling by a factor of 2 and for high data rate services (2 Mbps data service) in Figs. 6–8.

VI. CONCLUSIONS

The single user detection method has an advantage in that it requires only the spreading code of the particular mobile unit at the downlink. It therefore obviates the need for blind code detection at the downlink of a TDD system. In this paper, a novel fast joint detector, based on block-FFT, is developed, which requires spreading codes of all the users. The novel $J_DFFT$ algorithm performs very close to $J_DChol$, at much reduced complexity. Also $J_DFFT$ also shows a significant improvement in performance over single-user detection algorithms ($SDChol$ and $SDFFT$), at lower complexity. FFT based joint detector ($J_DFFT$) is thus a good choice in a TDD system and can be employed to deliver high data rate service in the downlink. The $J_DFFT$ algorithm can also be extended to the uplink case. However, since in this paper, comparison with "single user detec-
\(R_c \cdot D =
\begin{bmatrix}
(\mathbf{R}_0 + \mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_H) \\
(\mathbf{R}_0 + \mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_H^H) \\
(\mathbf{R}_1 d + \mathbf{R}_0 d^2 + \mathbf{R}_H d^3 + \mathbf{R}_2 d^4 + \mathbf{R}_H d^10) \\
\vdots \\
(\mathbf{R}_1 d + \mathbf{R}_0 d^2 + \mathbf{R}_H d^3 + \mathbf{R}_2 d^4 + \mathbf{R}_H d^10) \\
\end{bmatrix}
\times 
\begin{bmatrix}
(\mathbf{R}_0 d (N_s - 1) + \mathbf{R}_1 d (N_s - 1) + \mathbf{R}_2 d (N_s - 1) + \mathbf{R}_H d (N_s - 1) + \mathbf{R}_H d (N_s - 1) N_s) \\
(\mathbf{R}_1 d (N_s - 1) + \mathbf{R}_0 d (N_s - 1) + \mathbf{R}_H d (N_s - 1) + \mathbf{R}_2 d (N_s - 1) + \mathbf{R}_H d (N_s - 1) N_s) \\
\vdots \\
(\mathbf{R}_1 d (N_s - 1) + \mathbf{R}_0 d (N_s - 1) + \mathbf{R}_H d (N_s - 1) + \mathbf{R}_2 d (N_s - 1) + \mathbf{R}_H d (N_s - 1) N_s) \\
\end{bmatrix}
\]

\[ (A.1) \]

**APPENDIX A: Proof of Theorem 1.**

The block-circulant matrix \(R_c\) is multiplied by the matrix \(D\) to form the matrix \(R_c \cdot D\), with \((K \times K)\) blocks as entries; this is shown below, in (A.1) (top of next page), with \(d = e^{j2\pi/N_s}\)

Similarly, the \(D\) matrix is multiplied by the \(\Lambda\) in (16). The system of equations obtained by equating each block-row of \(R_c \cdot D\) and \(D\Lambda\) is consistent. Then

\[
D\Lambda =
\begin{bmatrix}
\Lambda^{(1)} & \Lambda^{(2)} & \Lambda^{(3)} & \cdots & \Lambda^{(N_s)} & \Lambda^{(N_s)} (N_s - 1) \\
\Lambda^{(1)} & \Lambda^{(2)} & \Lambda^{(3)} & \cdots & \Lambda^{(N_s)} & \Lambda^{(N_s)} (N_s - 1) \\
\vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
\Lambda^{(1)} & \Lambda^{(2)} & \Lambda^{(3)} & \cdots & \Lambda^{(N_s)} & \Lambda^{(N_s)} (N_s - 1) \\
\end{bmatrix}
\]

\[ (A.2) \]

Accordingly, the same set of equations are obtained by equating any block-row of \(R_c \cdot D\) with \(D\Lambda\).
The first block-row of \( \mathbf{DA} \) is given by
\[
\left[ \mathbf{A}^{(1)} \, \mathbf{A}^{(2)} \, \mathbf{A}^{(3)} \, \mathbf{A}^{(4)} \right]. \tag{A.3}
\]
Equating (A.2) with (A.3), the solution of \( \mathbf{A}^{(i)} \)'s is given by
\[
\mathbf{A}^{(1)} = \left( \mathbf{R}_0 + \mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_3^T + \mathbf{R}_4^T \right)
\]
\[
\mathbf{A}^{(2)} = \left( \mathbf{R}_0 + \mathbf{R}_1^T \mathbf{d}^T + \mathbf{R}_2^T \mathbf{d}^T + \mathbf{R}_3^T \mathbf{d}^{-2} + \mathbf{R}_4^T \mathbf{d}^{-1} \right)
\]
\[
\vdots
\]
\[
\mathbf{A}^{(N_K)} = \left( \mathbf{R}_0 + \mathbf{R}_1^T \mathbf{d}^{(N_K-1)} + \mathbf{R}_2^T \mathbf{d}^{(N_K-1)}
+ \mathbf{R}_3^T \mathbf{d}^{-2} \mathbf{d}^{(N_K-1)} + \mathbf{R}_4^T \mathbf{d}^{-1} \mathbf{d}^{(N_K-1)} \right). \tag{A.4}
\]

Although (A.2) to (A.4) are derived using the first block-row of \( \mathbf{R}_c \mathbf{D} \) and \( \mathbf{DA} \), same results are obtained using any block-row of \( \mathbf{R}_c \mathbf{D} \) and \( \mathbf{DA} \). In general form,
\[
\mathbf{A}^{(k)} = \begin{bmatrix}
\mathbf{R}_0 e^{-j \pi N_s (k-1) / N_s} \\
\vdots \\
\mathbf{R}_c e^{-j \pi N_s (N_K - 1) / N_s}
\end{bmatrix} + \begin{bmatrix}
\mathbf{R}_0 e^{-j \pi N_s (k-1) / N_s} \\
\vdots \\
\mathbf{R}_c e^{-j \pi N_s (N_K - 1) / N_s}
\end{bmatrix}
\]
\[
\mathbf{R}_n + \mathbf{R}_1^T \mathbf{d}^{(k-1)} \mathbf{d}^{(N_K-1)} + \mathbf{R}_2^T \mathbf{d}^{(k-1)} \mathbf{d}^{(N_K-1)} + \cdots + \mathbf{R}_n^T \mathbf{d}^{(k-1)} \mathbf{d}^{(N_K-1)},
\]
\[
k = 0, 1, \ldots, N_s. \tag{A.5}
\]

which is the same as (15).

APPENDIX B: Proof of Theorem 2.

Pre-multiplying \( \mathbf{A}_c \) by \( \mathbf{D}_2 \), it is seen, in (B.1) (next page), that blocks of size \( SF \times K \) are formed.

Similarly, in multiplying \( \mathbf{D}_1 \) by \( \mathbf{A}_1 \), blocks of size \( SF \times K \) are formed. Comparing any row of \( \mathbf{A}_c \mathbf{D}_2 \) with any row of \( \mathbf{D}_1 \mathbf{A}_1 \), it is seen that
\[
\mathbf{A}^{(1)} = \begin{bmatrix}
\mathbf{B}(0) + \mathbf{B}(1) + \mathbf{B}(2)
\end{bmatrix}
\]
\[
\mathbf{A}^{(2)} = \begin{bmatrix}
\mathbf{B}(0) + \mathbf{B}(1) e^{-j \pi N_s / N_s} + \mathbf{B}(2)
\end{bmatrix} e^{-j \pi N_s / N_s}
\]
\[
\vdots
\]
\[
\mathbf{A}^{(N_s)} = \begin{bmatrix}
\mathbf{B}(0) + \mathbf{B}(1) e^{-j \pi N_s (N_s - 1) / N_s} + \mathbf{B}(2)
\end{bmatrix} e^{-j \pi N_s (N_s - 1) / N_s}. \tag{B.2}
\]

In general form, we have
\[
\mathbf{A}^{(k)} = \begin{bmatrix}
\mathbf{B}(0) + \mathbf{B}(1) e^{-j \pi N_s (k-1) / N_s} + \cdots + \mathbf{B}(L) e^{-j \pi N_s (k-1) / N_s}
\end{bmatrix},
\]
\[
k = 0, 1, \ldots, N_s, \tag{B.3}
\]
which is the same as (24).

APPENDIX C: Comparison of Novel Detector with [4], [5]

In this paper, the block-circulant approximation to the correlation matrix is used, as opposed to the channel matrix \( \mathbf{A} \) in (3) in [4], [5]. In this paper, the correlation matrix is already block-square of size \( (N \times K) \), while the channel matrix in [4], [5] is of size \( N \times (N, K) \), where \( N = (SF \cdot N_s + W - 1) \).

For circulant approximation to be applied to a matrix, the matrix must be block-square. In this paper, the correlation matrix is approximated by \( \mathbf{R}_c \). However the matrix \( \mathbf{R} \) is already block-square, and does not addition of block rows and block columns to make it block square. Both the matrices \( \mathbf{R} \) and \( \mathbf{R}_c \) are of the same size \( (N \times K) \). The only difference is in the entries in top right and bottom left block rows of the two matrices for \( L \) block rows at each end. This degradation, due to circulant approximation, is analyzed in subsections IV.A and IV.E. But the overall structure of \( \mathbf{R} \) and \( \mathbf{R}_c \) are the same.

In contrast, in [4], [5], the channel matrix is non-block square. With blocks of size \( SF \times K \), the channel matrix has \( N \times [(W - 1) / SF] \) block rows and \( N \times SF \) block columns, making the channel matrix non-block square. As indicated in [5], to make the channel matrix block square, the number of columns to be added is \( [(SF \cdot W - 1) / SF] - 1 \), which can be significant if \( W \) is large, as in Case 2 channel, with \( W = 57 \) chips and \( SF = 16 \). Moreover, block rows also have to be added, such that the channel matrix has \( D \times D \) blocks, where \( D = N_s + [(SF \cdot W - 1) / SF] - 1 \).

This addition of block columns and rows creates a substantial change in the size and structure of the channel matrix and may lead to substantial degradation. On the other hand, in this paper, the correlation matrix is already block-square, with individual blocks also square, of size \( K \times K \), and the sizes of \( \mathbf{R} \) and its circulant extension, \( \mathbf{R}_c \), are the same. Thus, there is no significant change in size and structure when \( \mathbf{R} \) is extended to \( \mathbf{R}_c \). Thus, circulant extension of correlation matrix, rather than the channel matrix, seems to be more general.

The model for oversampling, by a factor of \( N \), is given in (9) in our paper, where the received signal for 1st, 2nd, \ldots, \( N \)th oversampled branches are concatenated one below the other. However, even with model in (9), in this paper, the novel method deals with the correlation matrix, not channel matrix, and the correlation matrix, and its circulant extension are still the same as before, as shown in (10) and explained in the text below that equation.

However, for the channel matrix based detection method [4], [5], the extension to oversampled model is as follows. Let the received signal, on the \( n \)th oversampled branch, in (9), be given by \( r_n = [r_n(0), r_n(1), \ldots, r_n(N_K - 1)]^T \). Then defining \( \tilde{r}(k) = [r_1(k), r_2(k), \ldots, r_{N}(k)]^T \) and \( \hat{r} = [\hat{r}(0) \hat{r}(1) \ldots \hat{r}(N_K - 1)]^T \), the oversampled model of (9) can be rewritten as \( \tilde{r} = \tilde{A} \hat{d} + \tilde{\eta} \), where size of \( \tilde{A} \hat{d} \) is \( (N \cdot SF \cdot N_s + W - 1) \times (N, K) \). Adding block columns and rows, as in [4], [5], the \( \tilde{A} \hat{d} \) matrix can be made block-square, such that it has \( D \times D \) blocks, where
\[ D = N - \left\lceil \frac{(SF + W - 1)/SF} \right\rceil - 1, \text{ except that each block is now of size } N \cdot SF \times K. \]

The problem is that the blocks are non-square \((N \cdot SF \times K)\) for multiple chip rate sampling, compared to blocks of size \((SF \times K)\) for single chip rate sampling for methods in [4], [5]. In contrast, in this paper, blocks are of size \(K \times K\) (square blocks), for both single chip rate and multiple chip rate sampling.

This non-square blocks may create difficulties during the inversion process, (19) in [5] and (31) in [4], where block diagonal \(\Lambda^{(k)}\), for each frequency point \(k\), is of size \(N \cdot SF \times (\text{non-square})\) and has to be inverted. Instead of an exact inverse, one has to resort to a pseudo-inverse minimum norm solution method. In contrast, the inversion equation (21), in our paper, needs to invert a square matrix \(\Lambda^{(k)}\), of size \(K \times K\), and an exact inverse can be obtained.

From the above discussion, it is not clear how the channel matrix based methods in [4], [5], with addition of block columns and block rows, would perform, in single chip rate or oversampled case. No analysis has been done on the degradation in this case. On the other hand, analysis of the degradation of the novel algorithm has been done in this paper in subsections IVA and IVE.

REFERENCES

[1] 3rd Generation Partnership Project, “UTRA (UE) TDD: Radio transmission and reception” TSG RAN WG4.

[2] S. Wang, S. G. Kim, S. Kwon, and H. Kim, “Toward forward link interference cancellation,” CDMA Development Group Technology Forum, San Francisco, CA, 2006.

[3] A. Klein, G. K. Kaleh, and P. W. Baier, “Zero forcing and minimum mean-square error equalization for multiuser detection in code-division multiple-access channels,” IEEE Trans. Veh. Technol., vol. 45, no. 2, pp. 276–287, May 1996.

[4] M. Vollmer, M. Haardt, and J. Goette, “Comparative study of joint detection techniques for TD-CDMA based mobile radio systems,” IEEE J. Sel. Areas Commun., vol. 19, no. 8, pp. 1461–1475, Aug. 2002.

[5] M. Vollmer, J. Goette and M. Haardt, “Joint detection using fast Fourier transforms in TD-CDMA based mobile radio systems,” in Proc. IEEE ICT, 1999.

[6] H. Karimi and N. Anderson, “A novel and efficient solution to block-based joint detection using approximate Cholesky factorization,” in Proc. IEEE PIMRC, 1998.

[7] U. Madhow and M. Honig, “MMSE interference suppression for direct sequence CDMA,” IEEE Trans. Commun., vol. 42, no. 12, pp. 3178–3188, Dec. 1994.

[8] J. Zhang, E. Chong, and D. Tse, “Output MAI distributions of linear MMSE multiuser receivers in DS-CDMA systems,” IEEE Trans. Inf. Theory, vol. 47, no. 3, pp. 1128–1144, Mar. 2001.

[9] S. Buzzi, M. Lops, and M. Tulino, “A new family of MMSE multiuser receivers for interference suppression in DS/CDMA systems using BPSK modulation,” IEEE Trans. on Commun., vol. 49, no. 1, pp. 154–167, Jan. 2001.

[10] S. Miller, M. Honig, and L. Milstein, “Performance analysis of MMSE receivers for DS-CDMA in frequency-selective fading channels,” IEEE Trans. Commun., vol. 48, no. 11, pp. 1919–1929, Nov. 2000.

[11] E. Dahlman, S. Parkvall, Johan Skold, and P. Beming, “3G Evolution: HSDPA and LTE for mobile broadband,” Elsevier 2008.

[12] G. Liu et al., “Evolution map from TD-SCDMA to B3G TDD,”IEEE Commun. Mag., pp. 54–61, Mar. 2008.

[13] S. Buzzi et al., “Blind user detection in doubly dispersive DS/CDMA fading channels,” IEEE Trans. Signal Process., vol. 58, no. 3, pp. 1446–1453, Mar. 2010.

[14] P. Westermann et al., “Performance analysis of W-CDMA on a vector DSP,” in Proc. EUCSSC 2008.

[15] Y. Sung, L. Tong, and A.-J van der Veen, “Signal processing advances for 3G WCDMA: From rake receivers to blind techniques,” IEEE Commun. Mag., pp. 48–54, Jan. 2009.

[16] J. Zhang, T. Bhattacharya, and D. Mandyam, “Fourier transform based linear equalization for CDMA downlink,” US patent no. 7502312, granted Mar. 10, 2009.

[17] P. De et al., “Fast joint detection base station,” U.S. patent no. 7822103, granted Oct. 26, 2010.

[18] G. Boudreau et al., “Interference co-ordination and cancellation for 4G networks,” IEEE Commun. Mag., pp. 48–54, Apr. 2009.

[19] P. De, “A computationally efficient multiuser detection algorithm in long code CDMA system,” IEEE Trans. Veh. Technol., vol. 61, no. 5, pp. 2360–2369, June 2012.

[20] S. Wang, S. Chen, A. Wang, J. An, and L. Hanzo, “Joint timing and channel estimation for handily coded based MC-DS-CDMA: A low-complexity near-optimal algorithm and CRLB,” IEEE Trans. Commun., vol. 61, no. 5, pp. 1998–2011, May 2013.

[21] S. Nader-Esfahani et al., “On optimal front-end filter for single-user detection in IR-UWB systems,” IEEE Trans. Commun., vol. 60, no. 1, pp. 37–41, Jan. 2012.

[22] X. Shang, B. Chen, and H. V. Poor, “Multiserver MISO interference channels with single user detection: Optimality of beamforming and the achievable rate region,” IEEE Trans. Inf. Theory, vol. 57, no. 7, pp. 276–287, July 2011.

[23] P. De et al., “Low complexity data detection using the fast Fourier transform decomposition of the channel correlation matrix,” in Proc IEEE GLOBECOM, 2002.

[24] P. De, “Fast joint detector and comparison with single user detection (for WCDMA systems),” in Proc. IEEE VTC, Oct. 2003.

[25] P. De et al., “Method and apparatus for receiving plurality of data signals,” US patent no. 7768986, granted Aug. 3, 2010.

[26] P. De, “Linear Prediction Based Scheduling Channel Estimation for Multiuser OFDM with insufficient guard interval,” IEEE Trans. Wireless Commun., vol. 8, no. 12, pp. 5728–5737, Dec. 2009.

[27] N. Al-Dhahir and A. H. Sayed, “The finite-length multi-input multi-output MMSE-DFE,” IEEE Trans. Signal Process., vol. 48, no. 10, pp. 2921–2936, Oct. 2000.
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