Observation of quantum Zeno effect in a superconducting flux qubit

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Abstract
When a quantum state is subjected to frequent measurements, the time evolution of the quantum state is frozen. This is called the quantum Zeno effect. Here, we observe such an effect by performing frequent discrete measurements in a macroscopic quantum system, a superconducting quantum bit. The quantum Zeno effect induced by discrete measurements is similar to the original idea of the quantum Zeno effect. By using a Josephson bifurcation amplifier pulse readout, we have experimentally suppressed the time evolution of Rabi oscillation using projective measurements, and also observed the enhancement of the quantum state holding time by shortening the measurement period time. This is a crucial step to realize quantum information processing using the quantum Zeno effect.

1. Introduction
Quantum measurements project qubit states into an eigenstate of the measured observable [1]. Quantum mechanics includes two types of evolutions, a unitary operation and a non-unitary operation, and the quantum measurements belong to the latter one. While the unitary evolution is continuous, the quantum projective measurement occurs in a discrete way. Also, while the unitary evolution is deterministic, the quantum measurements contain a probabilistic nature, because the measurement result occurs in a stochastic way. The properties of quantum measurements are quite unique, and these are related to the fundamental understanding of quantum mechanics, and so the quantum measurements have been paid attention to by many researchers [1–3].

Quantum measurements are useful for many fields such as quantum state stabilization [4], quantum control [5, 6], initialization [7], entanglement generation [8], one-way quantum computation [9], and quantum error corrections [10]. The quantum Zeno effect (QZE) is one of the applications of the quantum measurements [11, 12]. Frequent discrete measurements suppress time evolution of the qubit state, and this remarkable phenomenon in quantum mechanics is known as the quantum Zeno effect. Depending on the measurement interval and the properties of the environment, frequent general measurements can also enhance the decay [3, 13, 14]. This is called the anti-Zeno effect. These phenomena were first observed in a radio frequency (RF) transition between two \(^9\)Be\(^+\) ground state hyperfine levels [15], then nuclear magnetic resonance (NMR) [16] and atomic systems [17–25].

QZE can freeze the evolution of the system, but also can confine the dynamics in a specific subspace [26]. The frequent measurements on a subspace allows the state to evolve from the initial state, but prohibits the state away from the target subspace due to the frequent measurements on the other subspace. This is called the quantum Zeno subspace, and is experimentally demonstrated [22, 23].

A superconducting qubit is also a promising system with which to demonstrate the QZE. We can construct quantum discrete energy levels in a superconducting energy gap by making superconducting circuits [27–31]. Such superconducting circuits work as artificial two-level atoms, and so we can manipulate this system as a qubit. By using a linear detector and induced continuous dephasing, the time evolution suppression of Rabi oscillation has been observed on a superconducting qubit [32].
The QZE has many potential applications for quantum information processing. By using the QZE, it is possible to suppress the undesirable time evolution of a qubit state such as decoherence, which provides us with efficient control of quantum systems [33]. In particular, in a superconducting flux qubit, 1/f noise is a dominant source of decoherence [34, 35]. A quantum Zeno scheme has been suggested for suppressing dephasing caused by 1/f noise in a superconducting qubit [36]. Also, if we can prepare three qubits coupled in a row, it has been theoretically proposed that we can generate an entanglement between qubits at both ends by performing frequent discrete measurements on the middle qubit [37]. Since coupling between three superconducting qubits has already been demonstrated [38], a superconducting qubit is an attractive candidate for the realization of such a Zeno-based entanglement operation. Another application of the QZE is quantum metrology. It has been shown that the QZE is useful to enhance the sensitivity of the quantum magnetic field sensor under the effect of low-frequency noise [39, 40]. It is worth mentioning that strong discrete projective measurements are typically assumed for these applications. So it is preferable to demonstrate the QZE using such strong discrete projective measurements rather than using weak continuous measurements.

In this paper, as a proof of a principle experiment for the QZE in a superconducting flux qubit, we demonstrate the suppression of the time evolution of Rabi oscillation by using strong periodic projective measurements. Also, we propose a way to estimate measurement errors via QZE. Actually, by using this scheme, we succeeded in estimating a projection error rate from the analysis of our quantum Zeno experiment.

2. Quantum Zeno effect

Let us begin by summarizing the QZE. For the time evolution of a two-level system, the initial state \(|\psi_0\rangle\) gradually evolves into an orthogonal state \(|\psi_f\rangle\). Here, we can define the fidelity of the state as \(F = \langle \psi_f | \rho (\tau) | \psi_0 \rangle\) where \(\rho (\tau)\) denotes the density matrix of the system after a certain time \(\tau\). Interestingly, if the initial decay of the fidelity is approximated as a quadratic decay as \(F \approx 1 - \Omega^2 \tau^2\) where \(\Omega\) denotes the transition frequency of this system from the initial state, we can observe the QZE [11]. We perform \(N\) projective measurements on this system during a time \(\tau\) to distinguish whether or not the system is still in its initial state. By postselecting the case where the state is projected into the initial state for all measurements in the previous sequence, we can define a success probability \(P_s\). This probability is given by \(P_s \approx (1 - \Omega^2 (\frac{\tau}{\tau_m})^2)^N \approx 1 - \Omega^2 \tau_m \tau\) for \(\Omega^2 \tau_m \tau \ll 1\) where \(\tau_m = \frac{\tau}{\pi}\) denotes the time interval of the measurements. This probability is near unity in the limit of short \(\tau_m\). This is called the QZE. Although this process requires a postselection regarding the measurement results, there is another class of QZE without any postselections [41]. If we perform projective measurements on this system where we do not know the measurement results, this process is equivalent to applying a superoperator as \(\hat{E} (\rho) = |\psi_0\rangle \langle \psi_0 | \rho | \psi_0 \rangle \langle \psi_0 | + |\psi_f\rangle \langle \psi_f | \rho | \psi_f \rangle \langle \psi_f |\). By performing \(N\) non-selective projective measurements on this system during time \(\tau\), we obtain \(\rho \approx (1 - \Omega^2 \tau_m \tau |\psi_0\rangle \langle \psi_0 | + \Omega^2 \tau_m \tau |\psi_f\rangle \langle \psi_f |\) again for \(\Omega^2 \tau_m \tau \ll 1\) [36]. So we can freeze the system in the initial state for the limit of a short \(\tau_m\) by performing non-selective measurements. In this paper, we demonstrate the latter type of QZE.

3. Experimental setup and qubit property

To observe the QZE, it is necessary to utilize quantum non-demolition measurements [42–44]. Here, the interaction Hamiltonian between the system and the measurement apparatus commutes with the system Hamiltonian. This guarantees both an accurate measurement result and minimum disturbance to the relevant system. With a superconducting flux qubit, the supercurrent direction is different between the qubit ground and excited state. By detecting the small magnetic field from the supercurrent, we can distinguish the qubit state. For this purpose, we use the bifurcation phenomenon of a non-linear resonator with the Josephson bifurcation amplifier (JBA) [45]. In this scheme, the Josephson junction is not turned into the voltage state and the transition frequency of the system is near unity. This scheme guarantees both an accurate measurement result and minimum disturbance to the relevant system.

We use a long coplanar-type superconducting transmission line fabricated on a substrate by the angled aluminum evaporation method. Here, we terminate both ends of the line with capacitors. We set the length between the terminals as half of the microwave wavelength along the line. The transmission line has a role of a resonator. Our superconducting flux qubit was constructed with a superconducting loop, which had Josephson junctions (figure 1(c inset)). It was designed to couple magnetically to the superconducting quantum interference device (SQUID) structure. We embedded the SQUID structure at the center of the transmission line resonator. The SQUID provides us with a non-linearity. We call this non-linear resonator the JBA resonator. Due to this non-linearity, the JBA resonator shows a bistable behavior, and the JBA resonator becomes sensitive against small changes under some conditions, which we can use for the readout of the flux qubit. To control the qubit state, we prepared a microwave line near the qubit (figure 1(a)). We can measure properties of our
superconducting flux qubit and JBA resonator sample. The resonance frequency of the JBA resonator was 6.5 GHz and the qubit gap frequency was $\Delta = 3.3$ GHz at the degeneracy point (figure 1(b)).

### 4. Experiment and results

First, we measured the Rabi oscillation to determine the $\pi$ pulse length. The Rabi oscillation frequency $\Omega_{\text{Rabi}}$ was proportional to the amplitude of a qubit resonant microwave pulse. To suppress the Rabi oscillation with the QZE, the time period of the measurements $T_{\text{period}}$ should be much shorter than the Rabi period time $T_2 \propto \Omega_{\text{Rabi}}$. With our measurements, $T_{\text{period}}$ was limited by the JBA bifurcation time $\tau_{\text{JBA}} = Q_{\text{JBA}} / f_{\text{JBA}}$. Figure 1(c) shows the observed Rabi oscillation. All the measurements were performed in a dilution refrigerator at a temperature below 50 mK. We optimized the microwave power to set the Rabi period time at 88 ns. This value is longer than $T_{\text{period}}$ and $\tau_{\text{JBA}}$.

Next, we attempted to observe the QZE. To suppress the time evolution of the Rabi oscillations with the QZE, we used the pulse sequence shown in figure 2(a). The initial state of the qubit is the ground state because we wait longer than $T_1$ time. We applied a Rabi pulse tuned to the resonance frequency of the qubit. When the qubit state became an excited state, we applied several short readout pulses (which we call projection pulses) to project the qubit state. After that, we turned off the Rabi pulse and read the qubit state. The $\pi$ pulse length of the Rabi oscillation was $\pi / \Omega_{\text{Rabi}} = 44$ ns. The rising time of the projection pulse was 5 ns and it maintained its peak amplitude for 4 ns, which means that the length of one projection pulse was 9 ns. This is the shortest pulse length for which we succeeded in observing projection phenomenon. Before bifurcation of the JBA resonator increases, we turn off projection pulses to avoid pulse-induced relaxation. The roughly estimated value of $\tau_{\text{JBA}}$ was $\sim 7$ ns. We set the projection pulse period at $T_{\text{period}} = 11$ ns and measured the qubit state by sweeping the projection pulse height $h$ and number of projection pulses. This is the same as a measurement for observing Rabi
oscillations when \( h = 0 \) (no projection pulse). To observe the quantum Zeno effect with a periodic pulse measurement, we need a strong projective measurement. To confirm the projective measurement region, we swept \( h \). Here, we expect the projection to occur when \( h \) passes beyond a certain threshold as described in [48].

The projection pulse height \( h = 1.0 \) corresponded to the pulse height of the final readout pulse. We expect to observe Rabi oscillation without any projections. On the other hand, strong projection pulses whose height is above the threshold actually induce measurements to project the qubit state into the ground state or the excited state, and these projections are expected to suppress Rabi oscillation to keep the qubit state in the initial state.

We performed this sequence without and with the application of the Rabi pulse. Without the Rabi pulse, the pulse sequence simply induces projections of the qubit state into the ground state. By contrast, by applying the Rabi pulse, we can observe the time evolution of the system during our pulse sequence. Measured probability sometimes fluctuates by time due to an instability of measurement equipment. To subtract this uncertainty, we repeated a sequence to measure switching probabilities with a Rabi pulse and without a Rabi pulse. The measurement time of a single sequence is much shorter than the time scale of the fluctuation induced by the instability, and therefore we can obtain the correct switching probability by plotting the difference between the switching probabilities without and with a Rabi pulse. The measurement time of a single sequence is much shorter than the time scale of the fluctuation induced by the instability, and therefore we can obtain the correct switching probability by plotting the difference between the switching probabilities without and with a Rabi pulse. The relationship between this probability difference \( P_{\text{diff}} \) and the expected value of \( \sigma_i \) is described as \( \langle \sigma_i \rangle = \frac{2}{V} \cdot P_{\text{diff}} - 1 \). Here \( V \) is the visibility of the measurement system. Figure 2(b) shows the result of this measurement sequence. We set the qubit operating field at \( \Delta \Phi_{\text{ext}} = -0.4 \, \text{m} \Phi_0 \) (figure 1(b)). The operating point is near the degeneracy point (\( \Delta \Phi_{\text{ext}} = 0 \)) where the coherence time is relatively long but the visibility is low. When the amplitude of the projection pulses \( h \) was below 0.8, we clearly observed the Rabi oscillations after a \( \pi \) pulse. In this region, \( h \) is below the threshold \( h_c \), and

![Figure 2](image_url)
there is no projection of the qubit state. Above $h = 0.9$ up to 1.0, the Rabi oscillations disappear and we observed monotonic decay curves. The projection pulse height $h = 1.0$ is larger than threshold $h_c$, so we use $h = 1.0$ to observe QZE.

5. Analysis and discussion

We analyzed these experimental results using the phenomenological model. When we do not apply projection pulses or apply projection pulses with the height below the threshold ($h < h_c$), we observed clear Rabi oscillations, shown in red curves in figure 2(b). These Rabi curves are well fitted by the following phenomenological formula:

$$\langle \sigma_z \rangle_{(h< h_c)} = f_{\text{Rabi}}(t) = A \exp\left(-\frac{t}{\tau_{\text{Rabi}}} \right) \cos\left(\Omega_{\text{Rabi}} t - \phi\right)$$  \hspace{1cm} (1)

Here $\tau_{\text{Rabi}}, \Omega_{\text{Rabi}}, \phi$ and $A$ denote Rabi decay time, Rabi frequency, phase shift, and proportional constant, respectively. On the other hand, when we apply strong projection pulses with the height above the threshold ($h > h_c$), the time evolution of $\langle \sigma_z \rangle$ shows a single exponential curve, where holding time denotes $T_{\text{hold}}$, as shown in the blue curves of figure 2(b). So the temporal behavior of $\langle \sigma_z \rangle$ satisfies the following formula.

$$\langle \sigma_z \rangle_{(h> h_c)} = f_{\text{decay}}(t) = \exp\left(-\frac{t}{T_{\text{hold}}} \right)$$  \hspace{1cm} (2)

When we apply periodic projection pulses shown as figure 2(a), the time evolution of the qubit does not depend on the height $h$ of the projection pulse between the projection pulses. This means that equation (1) and equation (2) match at $t = 0$ and $t = T_{\text{period}}$, so that we should obtain $f_{\text{Rabi}}(T_{\text{period}}) = f_{\text{decay}}(T_{\text{period}})$, as shown in figure 3(b). By using this condition, we can show the period time $T_{\text{period}}$ dependence of holding time $T_{\text{hold}}$.

$$T_{\text{hold}} = \frac{\tau_{\text{Rabi}} T_{\text{period}}}{\log \left[\cos\left(\Omega_{\text{Rabi}} T_{\text{period}} - \phi\right) / \cos \phi\right]}$$  \hspace{1cm} (3)

Figure 3(a) shows the results for a quantum Zeno sequence when we change the time period of the projection pulses ($T_{\text{period}}$). We fixed the height of the pulse $h = 1.0$ to realize periodic projections. After the projection about the qubit state occurs, the qubit state shows the time evolution by the Rabi pulse, and so the expected value of observed $\sigma_z$ decreases at a constant rate $\left(T_{\text{hold}}\right)$. The decay curves of the observed $\langle \sigma_z \rangle$ are well fitted by a single-exponential decay function of $\exp\left(-\frac{t}{T_{\text{hold}}} \right)$. As we increase $T_{\text{period}}$, the holding time $T_{\text{hold}}$ becomes shorter. In our JBA readout, the interaction between the qubit and the readout microwave pulse causes the qubit energy shift with the ac Stark effect. We observed a frequency shift about 100 MHz when the projection pulse height $h$ is larger than 0.9. On the other hand, we observed a much smaller qubit frequency shift when
h < 0.9. Since this frequency shift is much larger than the Rabi frequency \( \Omega_{\text{Rabi}} \), the qubit is off resonant when the projection pulse is applied to the JBA. This time \( T_p \) is related to the projection pulse width and \( \tau_{\text{JBA}} \). As long as the qubit is in an off-resonant condition, Rabi oscillation does not occur, and we obtain a condition \( \int_{\text{Rabi}} (T_{\text{period}} - T_w) = \int_{\text{decay}} (T_{\text{period}}) \) so that equation (3) should be modified as follows:

\[
T_{\text{hold}} = \frac{\tau_{\text{Rabi}} T_{\text{period}}}{T_{\text{period}} - T_w - \tau_{\text{Rabi}} \log \left( \cos (\Omega_{\text{Rabi}} (T_{\text{period}} - T_w - \phi) / \cos \phi \right)}
\]

By fitting the experimental results of the Rabi oscillations, we estimated \( \tau_{\text{Rabi}} = 120 \) ns, \( \Omega_{\text{Rabi}} = 2 \pi \times 11 \) MHz, \( \phi = 0.12 \). In figure 3(c), we plot the observed \( T_{\text{hold}} \) and the curve given by equation (4). The behavior of \( T_{\text{hold}} \) clearly demonstrates the QZE. From equation (4), \( T_p \) is estimated to be around \( \sim 7 \) ns. In our measurement, the energy shift is around 100 MHz during the time of \( T_p \). From numerical calculations with this condition, asymmetric low amplitude oscillation should be observed, because the Rabi frequency increases and the qubit state does not reach the ground state. Since we cannot see such an oscillation in figure 2, this result suggests that projection only occurs in the region of the vanishing Rabi oscillation \( (h > 0.9) \). This is consistent with the fact that a projection occurs only when the height of the projection pulse is above the threshold for the JBA readout method [48].

Finally, we estimated the projection error rate. An imperfect projective measurement may induce a relaxation with a finite probability \( e_p \) from the excited state to the ground state during the measurement process (details are provided in the appendix). An expected value of the measured observable should be preserved before and after an ideal quantum non-demolition measurement. However, if such a projection error occurs, the expected value of the observable is decreased by measurements. So, \( e_p \) is an indicator of how non-destructively we can project the qubit state. This can be evaluated by the pulse sequence of the projection pulses and is not affected by the last readout pulse. Therefore, if we have sufficient visibility to observe the decay curves in figure 3(a), we can evaluate \( e_p \) precisely. Generally, due to projection error, the expected value after projection is \( 1 - 2e_p \). However, \( 1 - 2e_p \) is larger than the observed decay because the observed decay is described as a product of decay caused by time evolution and decay caused by projection error \( (1 - 2e_p) \).

\[
1 - 2e_p > \exp \left( -\frac{T_{\text{period}}}{T_{\text{hold}}} \right)
\]

From inequality (5), we can estimate the upper limit of the projection error rate by using \( T_{\text{hold}} \) data
\[
e_p < \frac{1}{2} - \frac{1}{2} \exp \left( -\frac{T_{\text{period}}}{T_{\text{hold}}} \right).
\]

When \( T_{\text{period}} = 11 \) ns, we obtain \( T_{\text{hold}} = 276 \) ns. The estimated projection error rate \( e_p \) is below 2.0 ± 0.4%. Therefore, this result demonstrates the projection part of our JBA readout method is suitable for a quantum non-demolition measurement.

6. Summary

We used JBA measurements to read out superconducting flux qubits and succeeded in suppressing the time evolution of Rabi oscillations by frequent measurements. To the best of our knowledge, this is the first demonstration of the QZE realized by periodic projective measurements in a superconducting qubit. We estimated the upper limit of the projection error for our projection pulse, and it was below 2%. This result suggests that the JBA readout method can provide an accurate projective measurement.

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Appendix

Quantum measurement theory

In a quantum measurement, there are three parameters such as a decoherence rate, detection error, and projection error, which quantify how accurate the measurement is. Here, we explain the strict mathematical definition of these according to quantum measurement theory [1, 3]. Also, we describe the difference between our results to estimate such measurement parameters and previous results in [46] where they also claim to estimate one of them.
Let us summarize the mechanism of quantum measurements [1, 3]. We can construct quantum measurements via an interaction between the system qubit and a measurement apparatus. Firstly, we prepare a superposition of the qubit and an initial state of the measurement apparatus such as \( \langle \alpha | g \rangle + \beta | e \rangle \otimes | M_{\text{ini}} \rangle \) where \( | M_{\text{ini}} \rangle \) denotes the initial state of the measurement apparatus. Secondly, by turning on the interaction between the qubit and measurement apparatus, we generate a correlation between them such as \( \alpha | g M_x \rangle + \beta | e M_x \rangle \) where \( | M_x \rangle \) and \( | M_y \rangle \) are orthogonal states of the measurement apparatus. Since this kind of entangled state usually decays quickly, we obtain a mixed state \( \rho = \langle \alpha | g M_x \rangle \langle g M_x | + \beta | e M_x \rangle \langle e M_x | \).

Thirdly, we amplify the signal of the experimental apparatus. Typically, just after the interaction with the qubit, the signal that the measurement apparatus has is too small to read under the effect of a noisy environment, and so it is necessary to amplify the signal so that one can obtain a better signal-to-noise ratio. So we have \( \rho = \langle \alpha | g M_x \rangle \langle g M_x | + \beta | e M_x \rangle \langle e M_x | \) via the amplification process where \( | M_x \rangle \) and \( | M_y \rangle \) are macroscopically distinguishable states of the measurement apparatus. Finally, we classically read out the state of the apparatus, namely a distinction whether the state is \( M_x \) or \( M_y \).

**Definition of measurement parameters**

By using the language of the quantum measurement theory, we show the definition of three parameters to quantify how accurate the measurement is.

The first parameter in the measurement process is a dephasing rate of the system qubit, which the measurement process itself generates. In the ideal case, we have a maximum correlation between the qubit and apparatus which completely removes off-diagonal terms of the reduced density matrix of the qubit such as \( \rho_{\text{qubit}} = \langle \alpha | g \rangle | g \rangle + \beta | e \rangle | e \rangle \). On the other hand, if there is no correlation between them, the initial state remains separable so that it is impossible to obtain the information of the system qubit from the apparatus. In this case, the measurement process itself does not affect the non-diagonal terms. So the dephasing rate to decrease the non-diagonal term of the qubit can be considered an indicator of such correlation as long as the dephasing of the qubit from other environments is smaller than that from the measurement during the measurement process. We define this parameter as \( \Gamma \).

The second parameter is a detection error, which is a failure rate for the readout of the measurement apparatus. If an unheralded transition between \( M_x \) and \( M_y \) occurs before the readout, the readout result does not always show the correct answer of the qubit state so that the experimentalists could get a wrong answer for the measurement result. The detection error is defined as \( P ( M_y | e ) \) and \( P ( M_x | g ) \), where \( P ( X | x ) \) denotes a probability to find the measurement apparatus in the state of \( X \) after the interaction with the qubit that was prepared in the state of \( x \).

The third parameter is a disturbance of the qubit, which we call a ‘projection error’. If the interaction Hamiltonian between the system qubit and apparatus is not ideal (or if there is unknown coupling with an environment), the measurement process may change the population of the ground state and the excited state of the qubit such as \( \langle g | \rho_{\text{qubit}} | g \rangle \neq \langle \alpha | \rho_{\text{qubit}} | \alpha \rangle \) and \( \langle e | \rho_{\text{qubit}} | e \rangle \neq \langle \beta | \rho_{\text{qubit}} | \beta \rangle \). As we discuss in the main paper, minimizing this error is relevant to observe QZE. It is worth mentioning that a quantum non-demolition (QND) measurement is defined as a measurement process that does not contain such a projection error, and therefore this projection error is considered an indicator of how far the measurement is from the QND measurement. We define this error rate as \( \epsilon_p \).

**The relationship between the visibility and other measurement parameters**

A visibility is commonly used in quantum information as a measure of how trustworthy the measurement result of the total process is. A visibility is defined as \( \nu = P ( M_x | e_{\text{initial}} ) - P ( M_y | e_{\text{initial}} ) \), where \( e_{\text{initial}} \) denotes the initial excited (ground) state of the qubit before starting the interaction between the measurement apparatus and qubit. This values take a unity for an ideal measurement while this takes zero for a completely failed measurement that does not transfer any information from the qubit to the apparatus. Typically, we can calculate the visibility from other parameters, such as the detection error and projection error, once we fix the measurement model with an imperfection of the measurement apparatus.

As a example, we use a simple model to describe the erroneous measurement process as shown in figure A1. Firstly, during the measurement process, the quantum state of the qubit is affected by the projection error, and we have a finite probability \( \epsilon_p \) to make the transition from the initial state into the other state. It is worth mentioning that this projection error typically increases as the necessary time of the measurement increases, because a longer measurement time induces an extra perturbation in the system due to the environment.

Secondly, the measurement apparatus is affected by the detection error, and there is a probability that the meter of the apparatus shows a wrong measurement result after the interaction with the qubit due to an unwanted transition of the apparatus state such as \( M_x \rightarrow M_y \) or \( M_y \rightarrow M_x \).
In this model, we can calculate the visibility as follows. We have

\[
P_M(e) = \langle \tilde{e} | \tilde{e} \rangle = \langle e | g \rangle \langle g | e \rangle = e_p \langle e | g \rangle + (1 - e_p) \langle g | e \rangle.
\]

So we obtain

\[
\sigma (\tilde{e}^T) \sigma (\tilde{e}) = \langle e | g \rangle \langle g | e \rangle = e_p \langle e | g \rangle + (1 - e_p) \langle g | e \rangle.
\]

where we use

\[
P_M(e) = P (M_e | e) = e_p P (M_g | e) + (1 - e_p) P (M_e | e).
\]

Mechanism of the Josephson bifurcation amplifier readout method

Let us summarize the mechanism of a Josephson bifurcation amplifier (JBA) readout method [45–47, 49]. The pulse sequence for the JBA is described in figure A2. Firstly, we prepare a superposition of the flux qubit and the JBA is prepared in a vacuum state, and so we have

\[
\langle e | \tilde{e} \rangle = \langle e | g \rangle + \langle g | e \rangle.
\]

At this stage, the correlation between the flux qubit and JBA is generated. At a specific point of this plateau region, we have a state described as

\[
\langle e | \tilde{e} \rangle = \langle e | g \rangle + \langle g | e \rangle.
\]

Here, the modified high-amplitude state \(M_{\tilde{g}}\) and low-amplitude state \(M_{\tilde{e}}\) has a larger distance between them in the Q representation than that of \(M_g\) and \(M_e\). This distance decreases the detection error due to an

**Figure A1.** A model to describe an imperfect measurement process. The initial qubit state (\(|e_{\text{initial}}\rangle\) or \(|g_{\text{initial}}\rangle\)) is affected by a projection error, and the state may be changed into the other state with a finite probability \(e_p\). After this process, the measurement apparatus transfers the information of the qubit to the apparatus state with a finite error probability.

**Figure A2.** Schematic of the driving amplitude to perform the JBA readout. This pulse has two roles, namely the generation of the correlation with the measurement apparatus and amplification of the signal of the measurement apparatus. Since the signal amplification only decreases the detection errors, the signal amplification is irrelevant to observe the QZE with non-selective measurement, and so we use only the former part to construct a projection pulse in the QZE experiment described in the main text.
unwanted transition \(|M_{\text{g}}\rangle \leftrightarrow |M_{\text{e}}\rangle\) induced by environmental noise. On the other hand, in the latter plateau region, the voltage signal from the JBA to reflect the qubit state flows out of the refrigerator with further amplification. As a result, we obtain \(|\alpha|^2 \langle gM_{\text{g}} |gM_{\text{g}}\rangle + |\beta|^2 \langle eM_{\text{e}} |eM_{\text{e}}\rangle\), where \(|M_{\text{g}}\rangle\) and \(|M_{\text{e}}\rangle\) are macroscopically different so that we can read the state of the measurement apparatus.

It is worth mentioning that we experimentally determine when we obtain the correlated state between the qubit and measurement apparatus \(|\alpha|^2 \langle gM_{\text{g}} |gM_{\text{g}}\rangle + |\beta|^2 \langle eM_{\text{e}} |eM_{\text{e}}\rangle\) via the observation of the quantum Zeno effect (QZE). We swept the pulse length of the first plateau region without the second plateau region, and we checked whether the QZE is observed with that length of the pulse or not. This let us know a minimum length of the pulse to induce the projection, which we define as a projection pulse. If the length of the pulse is too short, the projection of the flux qubit does not occur so that the QZE cannot be observed. On the other hand, if we set a longer pulse length, then the projection error increases due to the relaxation of the flux qubit during the increasing process of the bifurcation shown in figure A2, which decreases the holding time of the qubit state by the QZE.

Importantly, the length of the projection pulse (9 ns) is much shorter than the rest of the pulse (50 ns) to increase the bifurcation in the first plateau, which makes our estimation of the projection error different from the other estimation by previous research \([46, 47]\). A projection error until point III (II) in figure A2 has been estimated in \([46]\) \((47)\). On the other hand, in our experiment, we perform several short projection pulses (until point I in figure A2) without inserting signal amplification pulses, and we have succeeded in estimating the upper bound of the genuine projection error purely induced by the interaction between the qubit and JBA, which is an order of 2% smaller than that estimated in \([46, 47]\).

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