Analysis of Dynamic Response of Permittivity in a Liquid Crystal Cell with Flow

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Abstract. Microwave and millimeter-wave devices including a liquid crystal (LC) are increasingly attractive for the use in adaptive and controllable devices. Various types of microwave phase shifters having an LC transmission line have been studied (e.g. microstripline, coplanar waveguide, and rectangular waveguide). In conventional microwave devices, the response time after removal of voltage is slow because the LC layer in the devices is usually thick. In this study, the time response of the LC permittivity is studied experimentally and theoretically. Experimentally measured response curves having two time constants for decay are examined by using Frank’s continuum theory with and without the backflow effect.

1. Introduction

With the increasing use of mobile communications, satellite broadcasting, and intelligent transport systems, liquid crystal (LC) materials have become increasingly attractive for use in adaptive and controllable devices in the microwave and millimeter-wave fields. LCs have various anisotropies, such as permittivity, refractive index, and viscosity, owing to their molecular shape [1]. The electrically control of the LC permittivity is a basic concept of the active devices for microwave and millimeter-wave. Various types of microwave phase shifters having an LC transmission line have been studied, for example, microstripline, coplanar waveguide, and rectangular waveguide [2-5]. We have also proposed the LC microstripline and coplanar waveguide phase shifters [6,7]. In the former studies, the effective dielectric birefringence of an LC layer, the variation of the phase shift, and the response time of microwave phase shifters have been examined experimentally and theoretically.

LC configuration and its dynamics are key roles in the study of the time response. In particular, the dynamics of LC molecular motions is very important because a slow response time of microwave devices is a major barrier for practical use. The typical cell thickness of the LC microwave phase shifters is several ten microns that is a cause of the slow response. Relaxation process governs the molecular reorientation after removal of voltage. In general, the response of the relaxation process can be fitted with an exponential curve. However, we observed a response curve having two time constants for decay in experiments of a microwave phase shifter. In this paper, we have experimentally and theoretically studied the dynamic response of permittivity in typical LC cells to investigate the response curve with two time constants.
2. Experiments of time response of LC permittivity

LC cells with different thicknesses were used to examine time responses of permittivity caused by molecular reorientation. The LC cells with thicknesses of 12.7, 19.0, and 44.8 μm were prepared with polyethylene terephthalate films. The thicknesses were estimated by measuring the capacitances of the cells with an impedance analyzer (4294A, Agilent) before infiltrating LC into the cells. In the cells, indium-tin oxide (ITO) glass substrates were spin-coated with polyimide (AL-1254, JSR) and the polyimide layer was rubbed unidirectionally to obtain a planar cell. The LC cells were filled with a nematic LC 4-cyano-4A-pentylbiphenyl (5CB) (K-15, Merck).

A rectangular voltage at a frequency of 1 kHz supplied from a function generator (AFG3022, Tektronix) was applied to the cells via a power amplifier (4010, NF). The amplitudes of the voltages for all cells were set to be an electric field of 2.0 V/μm. That is, the amplitudes of the voltages for the 12.7-, 19.0-, and 44.8-μm cells were 2.54, 3.80, and 8.96 V, respectively. The permittivity of the LC cell at 100 kHz was measured with the impedance analyzer and a lock-in amplifier (SR844, Standford research systems).

Figure 1 shows the measured time-response waveforms of the LC permittivity after removing the applied voltage. Before 0 second, the electric field of 2.0 V/μm had been applied to each cell, and then the electric field was immediately decreased to zero at 0 second. In figure 1(a), the permittivity of the 12.7-μm cell shows an exponential decay curve with one time constant. On the other hand, in the 19.0- and 44.8-μm cells, the response curves are slightly different from the decay curve of the 12.7-μm cell. In the 19.0- and 44.8-μm cells, the permittivity quickly decreases at the moment that the voltage is zero, and then they exponentially decrease with a large time constant. In other word, the decay curve of the 12.7-μm cell has one time constant, while the decay curves of the 19.0- and 44.8-μm cells have two time constants.

Figure 1. Measured response waveforms of permittivity for the 12.7-, 19.0-, and 44.8-μm LC cells.

3. Calculations of time response of LC permittivity

3.1 Continuum theory

LC director configurations in the cells were calculated using Frank’s continuum theory [1,8] to investigate the characteristics of the permittivity response. Using one-constant approximation, the free energy density $f$ for nematic LC is given by

$$f = \frac{1}{2} K \left( \frac{d\theta}{dz} \right)^2 - \frac{1}{2} \varepsilon_0 \Delta \varepsilon E^2 \sin^2 \theta,$$

(1)

where $K$ is a Frank elastic constant, $\theta$ is the director angle, $\varepsilon_0$ is the vacuum permittivity, $\Delta \varepsilon$ is the anisotropy of LC permittivity, and $E$ is the electric field. The director configuration can be obtained by substituting $f$ into the Euler-Lagrange equation. The torque balance equation for LC is written as

$$\gamma \frac{\partial \theta}{\partial t} = K \frac{\partial^2 \theta}{\partial z^2} + \varepsilon_0 \Delta \varepsilon E^2 \sin \theta \cos \theta,$$

(2)
where $\gamma$ is the rotational viscosity of LC.

Figure 2 shows the director configurations in the LC cells. In these calculations, we assumed that the pretilt angle of the LC is 1.0° and that the boundary condition is weak anchoring. We set to be 0 second when the applied voltage was removed after the application of voltage with a sufficient time. Almost all of the directors direct to 90° at 0 second. After removing the applied voltage, the director angles gradually decrease with time. The spatial distributions of the LC permittivity were calculated using the calculated director angles. The time responses of permittivity were obtained by averaging the spatially distributed LC permittivity. Figure 3 shows the response waveforms of the LC permittivity calculated using Frank’s continuum theory. We obtained three decay curves which show a good agreement with the experimental data in terms of time scale. However, the decay curves for the 19.0- and 44.8-μm cells do not have two time constants.

![Figure 2](image1.png)

**Figure 2.** LC director configurations calculated using Frank’s continuum theory.

![Figure 3](image2.png)

**Figure 3.** Calculated response waveforms of LC permittivity using Frank’s continuum theory.

### 3.2 Dynamics with backflow

In section 3.1, we calculated the LC director configurations and the time-response curves of the LC permittivity using Frank’s continuum theory. The calculated decay curves had the same time scale in comparison with the experimental results. However, the calculated curves did not have two time constants which were the characteristic features of the experimental results. To investigate more details, we calculated the LC director dynamics taking into account flow of LC molecules. Backflow of LCs is a well-known phenomenon in display applications [9]. The torque balance equation including the backflow effect is written as

$$
\gamma \frac{\partial \theta}{\partial t} = K \frac{\partial^2 \theta}{\partial z^2} + \varepsilon_0 \Delta \varepsilon E^2 \sin \theta \cos \theta - (\alpha_3 \cos^2 \theta - \alpha_2 \sin^2 \theta) \frac{\partial v_x}{\partial z},
$$

(3)

where $v_x$ is a flow velocity, $\alpha_2$ and $\alpha_3$ are the Leslie coefficients. In addition to equation (3), the Navier-Stokes equations must be solved to obtain $v_x$. In this study, equation (3) was numerically solved using the Crank-Nicolson method.

Figure 4 shows the flow velocities in the cells obtained by solving the Navier-Stokes equations. Flow occurs in the LC cells at the moment that the voltage is removed. Therefore, the flow velocities at 0.01 seconds are the largest in each graph. Since the boundary effect from the glass substrate is
relatively weak in a thick cell, the flow velocity in the 44.8-μm cell is larger than those in the 12.7- and 19.0-μm cells. The flow is symmetry in the LC cell, and the flow velocity at the center of the cell is always zero.

The LC director configurations calculated using Frank’s continuum theory with backflow are shown in figure 5. After removing the applied voltage, the director angle decreases to 0°. Note that the director angle in the center area temporarily increased from 90°. This is caused by the backflow effect and this is the difference with the results shown in figure 2. The 44.8-μm cell shows the largest increase in director angle because the flow velocity is also the largest. The spatial distributions of the LC permittivity were calculated as the same manner in section 3.1, and the time responses of the permittivity were obtained by averaging the spatially distributed LC permittivity. Figure 6 shows the response waveforms of the LC permittivity calculated using Frank’s continuum theory including the backflow effect. It is noteworthy that the time-response curves in figures 6(b) and 6(c) have two time constants. In contrast, the time-response curve in figure 6(a) appears to be an exponential curve with one time constant. The reason can be explained as follows: The LC permittivity at 90° is the permittivity along the long molecular axis that is the largest permittivity for the LC. When the backflow occurs in the cell, the director angle increases beyond 90°, and then the angle decreases to 0° via 90°. Therefore, in the molecular reorientation, the LC permittivity at the center area decreases, increases, and decreases again. This is the reason that the time-responses in the 19.0-μm and 44.8-μm
cells do not show an exponential decay curve with one time constant.

4. Conclusions
The time response of LC permittivity was experimentally and theoretically investigated. The measured responses of the 19.0- and 44.8-μm cells after removal of voltage showed an exponentially decay curve with two time constants. The responses calculated by Frank’s continuum theory did not show a decay curve with two time constants but showed an exponential decay curve with one time constant. We also calculated the LC director dynamics with taking into account of the backflow effect. The response curve including the backflow consisted of an exponential decay with two time constants. The calculations showed a qualitative agreement with the experimental results. The two time constants of a decay curve were explained by the backflow effect.

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References
[1] de Gennes P G and Prost J 1993 The Physics of Liquid Crystals, 2nd ed. (Oxford Science, Oxford)
[2] Lim K C, Margerum J D and Lackner A M 1993 Appl. Phys. Lett. 62 1065
[3] Utsumi Y, Kamei T, Saito K and Moritake H 2005 IEEE Transactions on Microwave Theory and Techniques 53 3345
[4] Moritake H, Morita S, Ozaki R, Kamei T and Utsumi Y 2007 Jpn. J. Appl. Phys. 46 L519
[5] Nose T, Yanagihara S, Sato Y, Ito R and Honma M 2008 Jpn. J. Appl. Phys. 47 8483
[6] Utsumi Y, Bach N T, Kamei T, Ozaki R and Moritake H 2009 IEICE Trans. Electron. J92-C 778 (In Japanese)
[7] Nguyen T, Nguyen B T, Ozaki R and Moritake H 2013 Jpn. J. Appl. Phys. 52 121701
[8] Frank F C 1958 Discuss. Faraday Soc. 25 19
[9] Berreman D W 1975 J. Appl. Phys. 46 3746