Galaxy cluster mass estimation from stacked spectroscopic analysis

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ABSTRACT

We use simulated galaxy surveys to study: (i) how galaxy membership in redMaPPer clusters maps to the underlying halo population, and (ii) the accuracy of a mean dynamical cluster mass, $M_\sigma(\lambda)$, derived from stacked pairwise spectroscopy of clusters with richness $\lambda$. Using ∼130 000 galaxy pairs patterned after the Sloan Digital Sky Survey (SDSS) redMaPPer cluster sample study of Rozo et al., we show that the pairwise velocity probability density function of central–satellite pairs with $m_i < 19$ in the simulation matches the form seen in Rozo et al. Through joint membership matching, we deconstruct the main Gaussian velocity component into its halo contributions, finding that the top-ranked halo contributes ∼60 per cent of the stacked signal. The halo mass scale inferred by applying the virial scaling of Evrard et al. to the velocity normalization matches, to within a few per cent, the log-mean halo mass derived through galaxy membership matching. We apply this approach, along with miscentring and galaxy velocity bias corrections, to estimate the log-mean matched halo mass at $z = 0.2$ of SDSS redMaPPer clusters. Employing the velocity bias constraints of Guo et al., we find $\langle \ln (M_{200c}(\lambda)) \rangle = \ln (M_{30}) + \alpha_m \ln (\lambda/30)$ with $M_{30} = 1.56 \pm 0.35 \times 10^{14} M_\odot$ and $\alpha_m = 1.31 \pm 0.06_{\text{stat}} \pm 0.13_{\text{sys}}$. Systematic uncertainty in the velocity bias of satellite galaxies overwhelmingly dominates the error budget.

Key words: methods: statistical – galaxies: clusters: general – galaxies: haloes.

1 INTRODUCTION

The most massive dark matter haloes to emerge in the universe host clusters of galaxies. Ongoing and near-future cosmological surveys are dedicated to identifying clusters for the purpose of studying cosmology and fundamental physics through spatiotemporal counts and other statistical properties of the cluster population (Allen, Evrard & Mantz 2011). The largest cluster samples are identified using photometric data, through colour-based (Gladders & Yee 2005; Koester et al. 2007; Dong et al. 2008; Murphy, Geach & Bower 2012; Oguri 2014; Stanford et al. 2014; Bleem et al. 2015; Licitra et al. 2016) or photometric redshift-based (Milkeraitis et al. 2010; Durret et al. 2011; Soares-Santos et al. 2011) algorithms.

Predicting such cluster counts for a given cosmology requires convolving the halo mass function (spatial number density as a function of mass and redshift) with a likelihood function linking observable cluster properties to total halo mass. As a result, the true halo mass of clusters is a crucial element in the methodology of cluster count cosmology.

Because photometric data provide only coarse resolution in redshift, projection of galaxies along the line of sight (LOS) to a massive halo limits the ability of cluster-finding algorithms to uniquely identify the galaxies that are members of a particular massive halo. Spectroscopic data provide improved distance and mass estimators for group and cluster selection (e.g. Robotham et al. 2011), but projection and miscentring still pose challenges for these methods (see e.g. Duarte & Mamon 2014, and references therein).

These sources of confusion are fundamentally rooted in the fact that clusters and haloes are identified in different spaces: sky-redshift or sky-colour space for clusters and 3D real space or 6D phase space for haloes. Peculiar velocities can blend distinct haloes in real space into a single structure in redshift space (e.g. van den Bosch et al. 2004; Biviano et al. 2006; Wojtak et al. 2007; Saro et al. 2013; Duarte & Mamon 2014). In addition, the fact that high-mass haloes in cold dark matter (CDM) cosmologies are dynamically evolving at late times means that substructure and mergers can create complex, transient phase-space structure. In simulations, this

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complexity can confuse assignment of subhaloes hosting galaxies to their parent haloes (Knebe et al. 2011).

In practice, assigning galaxies as members of either clusters or haloes is a matter of convention, defined by application of specific, algorithm-dependent rules to galaxy samples. Regardless of the particulars, the joint likelihood, \( P_\alpha(k) \) that a galaxy, \( k \), is a member of both cluster \( \alpha \) and halo \( i \) offers a means to map from one space to the other (Gerke et al. 2005).

The total galaxy content, or richness, of a cluster can then be considered as a sum of partial contributions from haloes closely aligned along a common sightline. In this work, we apply such a membership-matching approach in simulations to build a network linking clusters to haloes, with network edges weighted by fractional cluster membership.

We investigate the membership properties of the redMaPPer cluster finding algorithm (Rykoff et al. 2014). The method, which identifies clusters through their red sequence galaxy population, outputs background-corrected membership probabilities (Rozo et al. 2009; Rykoff et al. 2012) to each galaxy in a cluster as well as central galaxy probability for up to four cluster members. The method is designed to make optimal use of data from large, multicolour photometric surveys such as the Sloan Digital Sky Survey (SDSS; York et al. 2000) and the Dark Energy Survey (DES; Flaugher 2005). The SDSS redMaPPer cluster catalogue (Rykoff et al. 2014) has been extensively studied with multiwavelength data, including comparisons to existing X-ray and Planck satellite Sunyaev–Zel’dovich catalogues (Rozo & Rykoff 2014; Sadibekova et al. 2014; Planck Collaboration XXVII 2015; Rozo et al. 2015a).

The latest study in the redMaPPer series uses stacked spectroscopic analysis of cluster member pairwise velocities to investigate photometrically assigned membership probabilities (Rozo et al. 2015b, hereafter RMIV). In that work, very good agreement was found between spectroscopic and photometric definitions of cluster membership after a small number of modest corrections for blue cluster members, correlated LOS structure, and photometric noise.

Using only SDSS data, the RMIV study could not study membership from the perspective of the underlying halo population. Instead, spectroscopic members are defined in velocity space using an assumed Gaussian form for the pairwise velocity probability density function (PDF) of central and satellite cluster members. In this work, we use simulations to link spectroscopic cluster members to the underlying halo population, leading to an estimate of the log-mean matched halo mass.

In Section 2, we apply the redMaPPer algorithm to a 10,000 deg\(^2\) synthetic photometric galaxy catalogue derived from light cone outputs of N-body simulations. We then employ a membership-based matching algorithm, described in Section 3, to build bipartite graphs\(^1\) in which each cluster links to a set of haloes ranked by their fractional member contribution to that cluster, a measure we term membership strength. This method is used to deconstruct the stacked pairwise velocity distribution of central–satellite galaxies in Section 4.

In Section 5, we apply the N-body simulation-based virial scaling of Evrard et al. (2008) to estimate the total mass at fixed cluster richness from the velocity dispersion model of Section 4. We show that this dynamical mass recovers the log-mean mass of haloes matched by cluster membership to better than one per cent. Confounding effects of miscentring and velocity bias are then discussed. Using current estimates for the magnitudes of these sources of systematic error, in Section 6 we estimate the halo mass scale of the RMIV sample using their stacked velocity dispersion measurements. Our results are summarized in Section 7.

Unless otherwise noted, our convention for the mass of a halo is \( M_{200c} \), the mass contained within a spherical region encompassing a mean density equal to 200 times the critical density of the universe, \( \rho_c(z) \). We also test the robustness of our results to the details of the synthetic galaxy population by implementing our analysis on an independent, higher resolution simulation, populated with a different galaxy prescription. Appendix A summarizes these results.

2 SIMULATION SAMPLES AND SYNTHETIC CLUSTER CATALOGUE

We employ N-body simulations produced with a lightweight version of the GADGET code developed for the Millennium Simulation (Springel et al. 2005). Three simulations, of 1.05, 2.6, and 4.0 h\(^{-1}\) Gpc volumes, are used to produce a sky survey realization covering 10,000 deg\(^2\) that resolves all haloes above 10\(^{13}\) M\(_\odot\) within \( z \leq 2 \). We refer to this suite of runs as the Aardvark simulation.

The resultant sky catalogue is built by concatenating continuous light cone output segments from the three different N-body volumes using the method described in Evrard et al. (2002). The smallest volume maps \( z < 0.35 \), the intermediate maps \( 0.35 \leq z \leq 1.1 \), and the largest volume covers \( 1.1 \leq z < 2 \). The simulations employ 2048\(^3\) particles, except for the 1.0 h\(^{-1}\) Gpc volume which uses 1400\(^3\), and corresponding particle masses are 0.27, 1.3, and 4.8 \times 10\(^{11}\) h\(^{-1}\) M\(_\odot\). The Aardvark suite assumes a ΛCDM cosmology with cosmological parameters: \( \Omega_m = 0.23, \Omega_\Lambda = 0.77, \Omega_b = 0.047, \sigma_8 = 0.83, h = 0.73, \) and \( n_s = 1.0 \). The Rockstar algorithm is used for halo finding (Behroozi, Wechsler & Wu 2013).

2.1 Galaxy population and halo membership

Galaxy properties are assigned to particles using the ADDGALS algorithm (Wechsler 2004; Busha et al. 2013; Chang et al. 2015; Wechsler et al. 2016). The algorithm is empirical, using the observed r-band luminosity function and trend of galaxy colour with local environment as input. The method assigns central galaxies to resolved haloes, but satellites as well as centrals in unresolved haloes are assigned to dark matter particles in a probabilistic manner weighted by a local dark matter density estimate. This density assignment scheme is tuned to match the clustering properties of a subhalo assignment matching (SHAM) approach applied to a 400 h\(^{-1}\) Mpc simulation using 2048\(^3\) particles.

Central galaxies are placed at the centre of resolved haloes and assigned a velocity at rest relative to the halo’s mean dark matter velocity within \( R_{200c} \). We explore the issue of non-zero central galaxy velocities in the analysis below. All other galaxies are assigned the positions and velocities of the corresponding particles to which they are assigned. Note that no particle can host more than one galaxy. The velocity assignment implies that the velocity dispersion of central–satellite pairs is expected to follow the same scaling with halo mass as that identified in the simulation ensemble of Evrard et al. (2008).

Regarding halo membership, our convention is that a galaxy, \( n \), is assigned to one and only one halo. Thus, if galaxy \( n \) is assigned to halo \( j \), then the probability that galaxy \( n \) belongs to halo \( i \) is \( P_{n,j}(i) = \delta_{ij} \). A spherical region of radius \( R_{200c} \) is used when defining halo membership. This region approximately defines the

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\(^{1}\) A bipartite graph links two disjoint sets of nodes, \( U \) and \( V \), with edges, each of which connects a node in \( U \) with one in \( V \). In our case \( U \) is the set of clusters and \( V \) the set of haloes.
hydrostatic region of massive haloes but it does not extend to the outer caustic, or back splash, edge which contains a mix of infalling and outgoing material (Busha et al. 2005; Cuesta et al. 2008; More, Diemer & Kravtsov 2015). We note that $R_{200c}$ is similar in scale to the search radius used by the redMaPPer cluster finding algorithm.

In regions where two or more haloes spatially overlap, the galaxy is assigned to the nearest halo. In the ADDGALS algorithm, galaxies can reside outside of a resolved $N$-body halo; 13 per cent of $m_l < 19$ galaxies reside beyond $R_{200c}$ of a resolved halo.

While not strictly a halo occupation distribution (HOD) method, ADDGALS produces an effective HOD for which intrinsic richness scales as a power law with halo mass. At low redshift, $\lambda_{int}$ defined as the number of galaxies with $M_r - 5 \log h \leq -19$ within $R_{200c}$, scales with halo mass in a sublinear fashion, $\lambda_{int} \propto M^\alpha$ with $\alpha \sim 0.8$. To test the robustness of our conclusions to the intrinsic HOD structure of massive haloes, we repeat the analysis on the galaxy catalogues of Hearin & Watson (2013) extracted from the Bolshoi simulation, which have a slightly steeper slope, $\alpha \sim 1.0$, and smaller intrinsic scatter in $\lambda_{int}$ compared to the Aardvark galaxy catalogue. Further details are provided in Appendix A.

The redMaPPer algorithm assumes that red galaxies are the prominent population occupying high-mass haloes. In Fig. 1, we show the distribution of $g - r$ colour as a function of $r$-band magnitude, $m_r$, for Aardvark galaxies in haloes of mass $M_{200c} > 10^{14} h^{-1} M_\odot$, and in the narrow redshift interval, $0.19 < z < 0.21$. A red sequence is evident, containing 78 per cent of galaxies brighter than 19th mag. The line shows the ridge-line approximate red sequence population. The slope and intercept are consistent with those than 19th mag. The line shows the ridge-line approximate red sequence as a function of redshift (Rykoff et al. 2014). We note that redMaPPer is continuously updated, so there is no unique redMaPPer catalogue. Here, we rely on the redMaPPer v5.10 SDSS catalogue, as this constitutes the most recently publicly available version.

The redMaPPer cluster finder is a matched filter algorithm with components that characterize the luminosity function, red-sequence colour, and projected number density of cluster galaxies. Writing the projected galaxy distribution in sky-magnitude space as a sum of cluster members and a locally uniform background component, the algorithm works iteratively to eventually tag each galaxy in the vicinity of a cluster, $\alpha$, with a probability, $P_{mem, \alpha}$, of being a member of that cluster. The richness, $\lambda$, is defined as the sum of the membership probabilities over the set, $G_\alpha$, of all member galaxies:

$$\lambda_\alpha = \sum_{n \in G_\alpha} P_{mem, \alpha}(n).$$

The redMaPPer algorithm applied to the Aardvark galaxy sample yields 3927 clusters with $\lambda > 20$ and redshift of $[0.1-0.3]$ over 10 400 deg$^2$. By comparison, there are 4522 clusters in the redMaPPer v5.10 DR8 cluster sample. Fig. 2 shows differential sky number counts, $dn/dz$, in units of number per 10 000 deg$^2$, for clusters with $\lambda > 20$ (upper lines) and 80 (lower lines) in the Aardvark and SDSS DR8 samples.

The number of clusters with $\lambda > 20$ in our simulation is lower by $\sim 20$ per cent relative to the SDSS DR8 catalogues. While this suppression may partly reflect the underpopulation of the inner $\sim 150$ kpc regions of the most massive simulated haloes, which suppresses the membership probability PDF at high $P_{mem}$ values for cluster members, this effect is not the only potential cause. The

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**Figure 1.** Colour–magnitude diagram for Aardvark simulation galaxies occupying haloes of mass $M_{200c} > 10^{14} h^{-1} M_\odot$ in the redshift interval $0.19 < z < 0.21$. The line indicates the red sequence ridge-line, $g - r = 1.65 - 0.32 m_r$, 78 per cent of galaxies brighter than $m_r = 19$ lie within 0.2 mag of this ridge-line.

**Figure 2.** Differential sky number counts per 10 000 deg$^2$ of clusters with richness, $\lambda > 20$ (thin lines) and 80 (bold lines) are shown for the Aardvark simulated galaxy catalogue run with RMv6.3.3 (solid) and SDSS DR8 run with RMv5.10 (dashed; RMIV) samples.
lower central galaxy density of massive Aardvark haloes also makes it more difficult for redMaPPer to centre clusters correctly. We note that the simulation matches well the observed trend of increasing counts with redshift. Finally, the difference may reflect differences in the underlying cosmological parameters. The Aardvark simulation has a smaller dark matter density \((\Omega_m = 0.23)\) than most current observational constraints, which implies a lower space density at fixed halo mass. The small difference in overall counts does not influence the spectroscopic analysis below.

### 2.3 Cluster and spectroscopic samples

The full redMaPPer cluster catalogues for both observations and simulated galaxy catalogues consist of clusters with \(\lambda > 20\) in the redshift range \(z = 0.1–0.3\).

To evaluate the sensitivity of our analysis to miscentring, we identify a correctly centred subsample of simulated clusters, those for which the central cluster galaxy is also the central galaxy of the top-ranked, matched halo. Throughout this work, we refer to this correctly centred subsample as CEN, and denote the full simulated cluster sample as ALL.

Our spectroscopic membership study is limited to cluster member galaxies with \(m_i < 19\). The limit of \(m_i < 19\) is a compromise value lying between the SDSS and Galaxy And Mass Assembly (GAMA) limits used by RMIV. Because satellite galaxies in haloes trace the dark matter kinematics by construction, our results are not strongly sensitive to the choice of magnitude limit.

Table 1 summarizes the number of clusters, number of galaxies, and number of central–satellite galaxy pairs in the simulation samples used below.

### 3 CLUSTER–HALO MEMBERSHIP MATCHING

To match redMaPPer clusters to haloes, we build a bipartite network between clusters and haloes with edges weighted by joint cluster–halo membership. The network is built using all photometric redMaPPer members of the cluster. Edges are weighted by the membership strength between cluster \(\alpha\) and halo \(i\), defined as

\[
S_{\alpha,i} = \frac{1}{N_{\alpha}} \sum_{n\in G_{\alpha}} P_{\text{mem},\alpha}(n)P_{\text{halo},i}(n),
\]

where \(G_{\alpha} = \{\text{ID}\}_{\alpha}\) is the list of galaxy IDs associated with cluster \(\alpha\), \(P_{\text{halo},i}(n)\) is a Boolean set to 1 if galaxy \(n\) is a member of halo \(i\), as described in Section 2. The strength, normalized to lie between 0 and 1, gives the fraction of the total membership of cluster \(\alpha\) contributed by halo \(i\).

Recall that \(\lambda_{\alpha}\) is the cluster richness defined in equation (1). In essence, the measured optical richness of a cluster can be expressed as a series of decreasing halo contributions:

\[
\lambda_{\alpha} = \sum_{r=1}^{N} S_{\alpha,i(r)},
\]

where the halo list, \(i(r)\), is rank ordered such that \(S_{\alpha,i(1)} \geq S_{\alpha,i(2)} \geq \cdots \geq S_{\alpha,i(N)}\). The matched halo of a cluster is defined as the halo with the highest strength; we use the terms ‘matched halo’ and ‘top-ranked halo’ interchangeably throughout this work. The mapping is not exclusive; two clusters can be mapped to one halo. In practice this happens infrequently. Out of 3927 redMaPPer clusters of redshift 0.1–0.3 only 38 clusters shared top rank halo. These 38 clusters mapped to 19 haloes.

Our approach is similar to that of Gerke et al. (2005), who introduced the concept of the largest joint member fraction to map clusters to haloes. However, that work uses a boolean measure of cluster membership. The probabilistic approach of redMaPPer makes the strength definition equivalent to the largest group fraction used in Gerke et al. (2005). Note that Rozo & Rykoff (2014) use a similar approach to match pairs of clusters derived from different search algorithms applied to the same SDSS data.

### 4 PAIRWISE VELOCITY PDF: HALO CONTRIBUTIONS TO SPECTROSCOPIC MEMBERSHIP

The study of RMIV assessed the validity of redMaPPer photometric membership probabilities by using spectroscopic redshifts. That work models the LOS velocity distribution of central–satellite pairs as a Gaussian distribution with zero mean and a dispersion that scales with cluster richness and, implicitly, with halo mass. After removing projected pairs having larger than escape velocities, the PDF of the remaining normalized pairwise velocities is modelled as a Gaussian plus a uniform background.

We begin by demonstrating that the simulated galaxy sample displays similar characteristics to the observations. Unlike the observations, our knowledge of the halo membership of each galaxy allows us to deconstruct the spectroscopic likelihood into distinct halo contributions.

#### 4.1 Constructing the velocity PDF of cluster central–satellite pairs

Using redshifts of cluster members in the spectroscopic samples described in Section 2.3, we determine pairwise velocities of each cluster’s satellite galaxies relative to its central galaxy:

\[
v = c \left( \frac{z_{\text{gal}} - z_{\text{cen}}}{1 + z_{\text{cen}}} \right),
\]

where \(c\) is the speed of light, and \(z_{\text{gal}}\) and \(z_{\text{cen}}\) are redshifts of satellite and central galaxies, respectively. The galaxy redshifts in the simulation are used with zero measurement error. Recall that central galaxies of resolved haloes are at rest with respect to their host halo.

In equation (4), the central galaxy is defined by the redMaPPer cluster-finding algorithm. In the CEN subsample this is also the central galaxy of the matched halo. For clusters in the CEN sample with high strength, we expect the root mean square velocity to be an unbiased estimate of the dark matter velocity dispersion of the matched halo.

Fig. 3 shows the distribution of pairwise velocity magnitudes against cluster richness for the ALL sample. The structure is very similar to that found by RMIV for the SDSS+GAMA spectroscopic data (see their fig. 2), with a main component at low velocities, referred to as the signal, and a cloud of projected pairs lying at high velocities.
The left-hand panel of Fig. 4 shows the PDF of the pair velocities normalized by the expected velocity dispersion for the CEN cluster sample. The structure of the full sample is similar. We bootstrap the cluster sample to compute means and standard deviations of the PDF in 50 bins between $-5$ and 5 in $v/\sigma_v$, shown as the points with error bars. The black line is a Gaussian of zero mean and unit variance plus the constant distribution, with amplitude given by the best-fitting model. The model is not a good fit to the data ($\chi^2$/dof = 82/16 over the signal region, $v/\sigma_v \in [-2.5, 2.5]$).

We find parameters that are similar to the RMIV fit to the SDSS redMaPPer sample. The CEN sample’s Gaussian magnitude, $p = 0.919 \pm 0.002$, and velocity–richness slope, $\alpha = 0.405 \pm 0.008$, are very similar to the SDSS values of 0.916 $\pm$ 0.004 and 0.44 $\pm$ 0.02, respectively. The ALL sample has reduced magnitude, $p = 0.885 \pm 0.002$, and a slightly shallower slope, $\alpha = 0.387 \pm 0.007$, differences that we discuss further in Section 5.2.2 below.

The velocity normalization, $\sigma_p$, is generally $\sim 10$ per cent lower than the RMIV value. As we discuss in Section 5, non-zero central galaxy velocities, satellite galaxy velocity bias, cosmology, and miscentring frequency all play a role in setting the normalization.

As an independent check that explores the sensitivity of these parameters to the galaxy assignment scheme, we repeat the analysis using measurements at known halo locations of the Bolshoi simulation catalogues of Hearin & Watson (2013). That work uses age distribution matching, a method for predicting how galaxies of magnitude $r$ and colour $g - r$ occupy haloes, to populate haloes with galaxies at redshift $z = 0$. When using the catalogue from the Bolshoi simulation, we rely on a $z = 0$ snapshot rather than a properly constructed light cone. We note the Hearin & Watson (2013) catalogue has only $g$ and $r$ data available, rather than the full five-band photometry available in the SDSS and Aardvark.

Results of this exercise, details of which are given in Appendix A, produce a velocity PDF of similar shape to the Aardvark CEN subsample. The best-fitting parameters show a similar Gaussian magnitude, $p = 0.89$, but a shallower slope, $\alpha = 0.30$, that reflects the steeper HOD slope in the Bolshoi galaxy catalogue compared to the Aardvark galaxy catalogue.

As found by RMIV, the best-fitting model does not have an acceptable $\chi^2$, as reflected by the deviations seen in the left-hand panel of Fig. 4 ($\chi^2$/dof = 82/16). We show below that the deviations from the simple flat-plus-Gaussian model arise from galaxies lying along the LOS in haloes outside the matched halo.

4.3 Halo-ranked contributions to the velocity PDF

The cluster–halo membership network allows us to determine what fraction of pairs in the main Gaussian PDF component arise from the matched halo. For the CEN sample, we find that, on average, 62 per cent of galaxy pairs arise from within the matched halo. For the full sample, the mean value decreases somewhat to 58 per cent. For the Bolshoi catalogue, in which all clusters are correctly centred on the full sample, the mean value decreases somewhat to 58 per cent. For the Bolshoi simulation, we rely on a snapshot rather than a properly constructed light cone. We note the Hearin & Watson (2013) catalogue has only $g$ and $r$ data available, rather than the full five-band photometry available in the SDSS and Aardvark.

The middle panel of Fig. 4 shows only the matched halo’s contribution to the pairwise velocity PDF of the CEN sample. As before, error bars are produced via bootstrap resampling of the cluster sample using 50 bins between $-5$ and 5 in $v/\sigma_v$. The black line shows a Gaussian with dispersion given by the best fit to the entire spectroscopic sample (left-hand panel), listed in Table 2. The principal difference with the left-hand panel is that we force $p = 1$, meaning no background component. While there exists moderate kurtosis in this distribution, the high-velocity wings of the PDF are not well populated. Relative to the full CEN sample, the goodness

2 In this work, the RMIV normalization is calculated using pivot richness, $\sigma_p = 30$, and redshift, $z_p = 0.2$, slightly different from the published RMIV pivot values.
of fit is improved by nearly a factor of 2 ($\chi^2$/dof = 47/16 over $v/\sigma_v \in [-2.5, 2.5]$).

The good match seen in the middle panel is important in that it indicates that the best-fitting velocity derived from the spectroscopic data set accurately recovers the velocity dispersion of the top-ranked halo. This finding offers leverage for a mean dynamical mass estimate as a function of cluster richness that we explore in the next section.

The right-hand panel of Fig. 4 shows the contribution from satellite galaxies outside of the matched halo. Clearly, a constant background does not adequately capture this component, which is a sum over second and higher ranked haloes. For the CEN sample, an average of 38 per cent of spectroscopic pairs are not contributed by the top-ranked halo. This finding offers leverage for a mean dynamical mass estimate as a function of cluster richness that we explore in the next section.

The classical virial theorem balances the kinetic energy with (modulo surface terms) half the gravitational potential energy of a halo, $\frac{1}{2}M_\text{halo} \sigma_v^2 = \frac{1}{2} \mu \lambda_1 (1 + \beta)$. Here, $\sigma_v$ is the dispersion, $M_\text{halo}$ is the mass, $\mu$ is the mean baryon fraction, and $\lambda_1$ is the ratio of the mean dark matter and baryon velocity dispersions within that radius. The dispersion is measured with respect to the mean dark matter velocity with within that radius.

### 5 MASS ESTIMATION

In this section we derive a scaling relation between total mass and optical richness by applying the virial velocity scaling of massive haloes to the pairwise velocity dispersion model described above.

We compare this stacked dynamical mass to that derived from membership matching to haloes, and find excellent agreement with the log-mean matched mass at fixed richness.

We begin by using the CEN sample to avoid uncertainties caused by miscentring, then investigate miscentring in Section 5.2.2.

### 5.1 Cluster mass from dark matter virial scaling

The classical virial theorem balances the kinetic energy with (modulo surface terms) half the gravitational potential energy of a halo, thereby offering a scaling law between velocity dispersion and mass within an enclosed radius. In a study of multiple, independent N-body and adiabatic hydrodynamic simulations, Evrard et al. (2008, hereafter E08) calibrated the dark matter virial relation.

In that work, the one-dimensional velocity dispersion of a halo, $\sigma_v$, is defined in an orientation-averaged fashion using particles within $R_{200c}$. The dispersion is measured with respect to the mean dark matter velocity within that radius.
E08 showed that the halo velocity dispersion of the population follows a power law form with approximately log-normal scatter, meaning the conditional probability, $P(\ln(\sigma_\lambda)|M, z) = N(\ln(\sigma_{DM}(M, z)), 0.046)$, where $N$ denotes a normal distribution, $\sigma_{DM}(M, z)$ is the log-mean velocity dispersion at fixed mass and redshift, and 0.046 is the scatter in $\ln(\sigma_\lambda)$ at fixed mass.

The log-mean velocity dispersion follows the scaling:

$$\ln(\sigma_{DM}(M_{200c}, z)) = \pi_\sigma + \alpha_\sigma \ln(h(z)M_{200c}/10^{15}M_\odot),$$  

with amplitude $\pi_\sigma = \ln(1082.9 \pm 4.0)$ and slope $\alpha_\sigma = 0.3361 \pm 0.0026$. Here, $h(z) = H(z)/100$ km s$^{-1}$ Mpc is the dimensionless Hubble parameter. The ellipsoidal collapse model of Okoli & Afshordi (2015) offers a first-principles explanation of the form and parameter values of this calibration.

At fixed mass, the distribution of velocity dispersion seen in the E08 simulation ensemble is very close to log-normal, with a modest tail to higher values driven by actively merging systems. Saro et al. (2013) show that the 1D LOS velocity dispersion has higher scatter compared to angle-averaged velocity dispersion. The normalization and slope of their scaling relation, found using subhaloes as galaxy tracers, are within $\pm 3$ per cent of the E08 values.

For a halo ensemble uniformly sampled in mass, the inverse of the above scaling relation provides an unbiased estimate of the log-mean halo mass at fixed velocity dispersion, $P(\ln(M)|\ln(\sigma_\lambda), z)$. For samples drawn from the expected cosmic mass function, the log-mean mass will be biased low by approximately 5 per cent, as detailed in Evrard et al. (2014). The magnitude of this correction is subdominant to systematic errors discussed below, so we choose to ignore it in this work.

To estimate halo mass as a function of richness in the redMaPPer cluster population, we apply the inverse to the log-mean halo virial scaling relation found in E08:

$$\ln(h(z)M_\lambda(\lambda, z)/10^{15}M_\odot) = 3 \ln\left(\frac{\sigma_\lambda(\lambda, z)}{1083\text{ km s}^{-1}}\right),$$  

where $\sigma_\lambda(\lambda, z)$ is the velocity dispersion scaling of central–satellite pairs analysed in Section 4 and the simple cubic power is consistent with the slope found in the E08 simulation ensemble.

If intrinsic galaxy richness, $\lambda$, were a nearly perfect tracer of halo mass, and if cluster finders cleanly identified halo members, then the log-normal form of the PDF relating velocity to mass (or vice versa) implies that the virial-scaled mass, $\ln(M_\lambda(\lambda, z))$, should accurately measure the log-mean mass, $\langle \ln(M(\lambda, z)) \rangle$, at fixed richness and redshift. Introducing (log-normal) scatter in richness at fixed mass can produce shifts that depend on the local slope and curvature of the mass function as well as the covariance of $\lambda$ and $\sigma_\lambda$ at fixed $M$ (Evrard et al. 2014). We defer a more detailed examination of these issues to future work.

Galaxy joint member matching provides an independent mass estimate for each cluster – the matched halo mass – that can used to assess the meaning of the stacked dynamical mass estimate, equation (8).

Fig. 5, a key result of this work, compares the mass scale inferred from the scaled velocity dispersion with membership matched masses for the CEN sample. The thick black line shows the mass–richness scaling at redshift 0.2 inferred from virial scaling, equation (8), while the points show individual $M_{200c}$ values of matched haloes for individual correctly centred clusters of redMaPPer richness, $\lambda$, within redshift range of [0.1, 0.3]. The red dots with error bars show the median and 68 per cent inclusion region of matched halo mass in different richness bins.

The blue line and shaded blue region are the mean and 95 per cent inclusion region of membership-matched masses in this redshift range, with shaded region showing 95 per cent confidence uncertainties in this mean relation at redshift 0.2.

![Figure 5](http://mnras.oxfordjournals.org/Downloaded from)
samples indicate that miscentring plays an additional role. In addition, variance in the velocity dispersion of clusters of fixed richness, reflective of the variance in matched halo mass, can introduce bias.

The following sections address these issues in turn, finding that the first two are more important than the third. How satellite galaxies trace dark matter kinematics is the key source of systematic error.

5.2.1 Central galaxy velocities and satellite galaxy velocity bias

The degree to which galaxy velocities trace the kinematics of dark matter particles in haloes is a central issue for virial mass calibration. By construction, the central galaxy is at rest with respect to its host halo in our simulations. In reality, central galaxies are measured to have a non-zero velocity dispersion with respect to their host clusters.

In cases of actively merging systems the rest frame of a cluster is often difficult to define. In the post-merger phase, the central galaxy will settle to the centre of cluster due to dynamical friction on a timescale on the order of 1 Gyr (White 1976; Bird 1994), during which time the central galaxy will have a net velocity with respect to the full halo. Based on a sample of nearly 500 Abell clusters with 10 or more redshifts, Coziol et al. (2009) find that brightest cluster galaxies have velocities with root mean square magnitude $\sim 0.5\sigma_{\text{cl}}$, with $\sigma_{\text{cl}}$ the LOS velocity dispersion of the host cluster. A similar ratio of 0.25 is found by Lauer et al. (2014) using 178 clusters with 50 or more member spectra. Martel, Robichaud & Barai (2014) find a similar thermal motion for central galaxies in a sample of 18 massive haloes extracted from a large cosmological, hydrodynamic simulation.

Redshift-space distortion studies also support non-zero values for central galaxy velocities (Skibba et al. 2011; Guo et al. 2015a, b). If the central galaxy population has velocity dispersion as some fraction, $\sigma_{\text{c}}$, of the host halo dispersion, $\sigma_{\text{halo}} = \alpha_{\text{c}} \sigma_{\text{halo}}$, then the central–satellite pairwise velocity normalization, $\sigma_{p}$, will be enhanced by a factor $(1 + \alpha_{\text{c}}^2)^{1/2} \simeq 1 + \alpha_{\text{c}}^2/2$, the latter if $\alpha_{\text{c}}$ is small compared to unity. Mass estimates derived from virial scaling will be boosted by a factor $(1 + \sigma_{\text{c}}^2)^{1/2} \simeq 1 + 3 \sigma_{\text{c}}^2/2$ relative to the case of cold centrals ($\alpha_{\text{c}} = 0$). These factors assume that the satellite galaxy velocities are unbiased with respect to the dark matter.

The velocity dispersion of satellite galaxies relative to the halo rest frame may also biased (Carlberg 1994), so that $\sigma_{\text{sat}} = \alpha_{\text{sat}} \sigma_{\text{halo}}$, where $\alpha_{\text{sat}}$ is the satellite galaxy velocity bias. The simulation study of Wu et al. (2013) that combines $N$-body and hydrodynamic models indicates that $\alpha_{\text{sat}}$ lies near unity, with brighter galaxies tending to have values less than 1 and fainter galaxies slightly above unity, asymptotically reaching a value of 1.05. This pattern is not seen in the redshift-space distortion work of Guo et al. (2015b), discussed below.

Let $\sigma_{p,0}$ be the normalization of the central–satellite pairwise velocity dispersion determined through the simulation analysis presented in Section 4.2. Recall that the simulations are constructed to have $\alpha_{\text{c}} = 0$ and $\alpha_{\text{sat}} = 1$. Introducing uncorrelated central and satellite galaxy velocity biases modifies the pairwise velocity PDF normalization to

$$\sigma_{p} = (\alpha_{\text{c}}^2 + \alpha_{\text{sat}}^2)^{1/2} \sigma_{p,0}. \quad (9)$$

If these effects alone are responsible for the normalization difference between the SDSS and Aardvark CEN samples (see Table 2), then we would require $(\alpha_{\text{c}}^2 + \alpha_{\text{sat}}^2)^{1/2} = 1.13$. 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{Figure6.png}
\caption{The normalization and slope of mass–richness scaling at redshift 0.2 inferred from stacked dynamical masses (black contours) and membership matching in the redshift range [0.1, 0.3] (blue) for correctly centred redMaPPer clusters. Contours show 68 and 95 per cent statistical uncertainties.}
\end{figure}
5.2.2 Cluster miscentring

While the analysis of Section 4.2 focused on the well-centred sub-sample of clusters, the pairwise velocity PDF of the full sample has a similar form. However, the fit parameters in Table 2 indicate that the normalization of the full sample is enhanced, 585 km s$^{-1}$ (ALL) versus 547 km s$^{-1}$ (CEN), and the slope $\alpha$ is slightly decreased. Because of the simulation limitations discussed in Section 2, the miscentred fraction of simulated redMaPPer clusters in the ALL sample is larger than that of the SDSS sample. Comparing to X-ray observations of a joint sample of more than 100 clusters, Rozo & Rykoff (2014) find that 86 ± 4 per cent of high-mass clusters are correctly centred on the X-ray counterpart. This statistic is weighted towards higher richness values, $\lambda \sim 100$, but preliminary results of ongoing redMaPPer sample analysis indicate that the full sample of $\lambda > 20$ redMaPPer clusters has a similar fraction of well-centred clusters.

We exploit the differences in the CEN and ALL samples to estimate, using a weighted sampling approach, how velocity PDF parameters shift as the fraction of miscentred clusters is varied.

The ALL cluster sample contains both miscentred and correctly centred clusters. Let $f_{\text{cen}}$ be the fraction of ALL galaxy pairs lying in the latter (CEN) sample. Our approach is to simply create simulated central–satellite pairs drawn in proportion from the CEN and (ALL–CEN) cluster samples in order to achieve a desired $f_{\text{cen}}$ fraction.

Specifically, for a given $f_{\text{cen}}$ value, we randomly draw without replacement a total of 10 000 galaxy pairs from these two cluster subpopulations in a way that satisfies the $f_{\text{cen}}$ fraction. We run the Markov chain Monte Carlo (MCMC) chains for these samples to find the best-fitting velocity PDF parameters for a total of 2000 realizations uniformly spanning $0.5 \leq f_{\text{cen}} \leq 1$.

Fig. 7 shows how the velocity PDF parameters change with correctly centred fraction, $f_{\text{cen}}$. The black lines are the best linear fits as a function of $f_{\text{cen}}$, with fit parameters and their root mean square deviations, $\sigma$, listed in the legend of each panel.

As the fraction of miscentred clusters increases (lower $f_{\text{cen}}$ values), the velocity dispersion normalization, $\sigma_p$, increases while the slope, $\alpha$, and Gaussian amplitude, $p$, both decrease. As expected, the limit of $f_{\text{cen}} = 1$ recovers parameters of the CEN catalogue (see Table 2).

We use this behaviour to correct for the effect of miscentring on the RMIV pairwise velocity normalization. Assuming the fraction of correctly centred SDSS redMaPPer clusters with $\lambda > 20$ to be $f_{\text{cen}} = 0.85 \pm 0.05$ leads to a ~3 per cent normalization correction for correctly centred systems,

$$\sigma_p,\text{RMIV},\text{CEN} = 582 \pm 8 \text{ km s}^{-1}. \quad (10)$$

We use this value to evaluate the mass scale of SDSS redMaPPer clusters in Section 6 below. The miscentring correction to the slope, $\alpha$, is smaller than 0.01 and is not applied below.

5.2.3 Velocity dispersion variance at fixed richness

The satellite–central velocity likelihood model employs a single Gaussian of width $\sigma_p(\lambda, z)$ at fixed richness, $\lambda$, but there is non-zero variance in velocity dispersion values of a fixed $\lambda$ population that reflects the variance in matched halo mass. Scatter in halo mass at fixed $\lambda$ is already incorporated into the simulations; the scatter in matched halo masses shown in Fig. 5 is 0.85 in $\ln M$. We perform here an explicit test, independent of the simulated samples, to confirm that this scatter does not strongly affect the recovered velocity PDF parameters.

We create ensembles of 10 000 galaxy pairs drawn from Gaussian distributions with dispersion values log-normally distributed about a scaling mean relation, $\sigma_p(\lambda, z)$, equation (5) with variance $\sigma_{\ln \sigma}^2$. Sampling in $\lambda$ and redshift uniformly covers the observed ranges of $[20, 200]$ and $[0.1, 0.3]$, respectively. We then perform the stacked velocity PDF analysis on each simulated pair ensemble.

We find that the model parameters remain unbiased until $\sigma_{\ln \sigma} > 0.2$, after which the tails of the velocity distribution begin to affect the normalization $p$ at the 1 per cent or greater level. The recovered values of $\sigma_p$ and $\alpha$, the key parameters involved in mass estimation, are unaffected up to values of $\sigma_{\ln \sigma} = 0.5$, or 1.5 scatter in $\ln M$. This degree of mass scatter is larger than either the simulated or observed (Rozo & Rykoff 2014) values. Variance in host halo velocity dispersion at fixed richness is therefore a negligible source of systematic error in the velocity PDF modelling and resultant mass estimates.

5.2.4 Orientation and shape selection bias

Because dark matter haloes are aspherical, optical cluster selection and richness estimation on the sky are sensitive to halo orientation, with preferential selection of structures elongated along the LOS (e.g. Dietrich et al. 2014). The Hubble Volume simulation analysis of Kasun & Evrard (2005) finds alignment of position and velocity ellipsoids in massive haloes, with median alignment angle of $22^\circ$. Orientation biases in an optically selected cluster sample such as redMaPPer could produce shifts in the mean stacked velocities. Along these lines, Skielboe et al. (2012) show that the LOS velocity dispersion of galaxies lying along the major axis of SDSS clusters is

\footnote{The value of 0.85 ± 0.05 is slightly more conservative than that published for higher richness clusters in RMIV.}
slightly larger than that of galaxies lying along the minor axis. Simet et al. (2016) use analytic arguments to estimate a 4 ± 2 per cent orientation bias (overestimate) in stacked weak lensing masses for redMaPPer selected clusters.

A potentially counteracting effect, found by Ragone-Figueroa et al. (2010) in the MARENOSTRUM UNIVERSEx simulation, is that, at fixed mass, more elongated haloes have smaller 3D-averaged velocity dispersion than less elongated systems. They link this effect to formation epoch, hence it is a form of assembly bias.

We perform two tests on the Aardvark simulation to estimate orientation biases on stacked dynamical.

First we ask whether the redMaPPer finder preferentially selects elongated haloes. To measure halo shape, we assume an ellipsoidal model and determine the three eigenvalues, $\lambda_i$, of the shape tensor in position space for galaxy members. The largest eigenvector gives the orientation. We define the elongation as $c/a$, where $c$ is the minor axis and $a$ the major axis of the shape tensor (see section 2.4 of Kasun & Evrard 2005 and section 2 of Zemp et al. 2011 for more detail).

We find that the distribution of shapes for matched haloes selected by redMaPPer matches well that of the overall halo population. Using bins of width 0.2 dex in mass, the median and quartile values of $c/a$ for the two populations match to within ~0.02 for haloes more massive than $10^{13.5} M_\bigodot$. Shape selection bias is not a large effect for this sample.

The second test concerns possible orientation bias of redMaPPer selection. The unbiased velocity dispersion is the 3D-averaged velocity dispersion of galaxies within the halo. We measure the LOS and 3D velocity dispersion for all galaxies inside matched haloes. Regressing both velocity dispersion values against mass, we find that the normalization of the LOS velocity dispersion is larger than the 3D value by ~1.1 per cent. This implies a 3.3 per cent overestimation of the stacked dynamical mass at fixed richness.

Because this level of bias is smaller than the other sources of uncertainty described in Section 6, we do not explicitly apply a correction. We note that the specific correction will depend on the algorithm employed for optical cluster selection.

### 6 STACKED DYNAMICAL MASS SCALING OF SDSS redMaPPer CLUSTERS

The above analysis indicates that the mass determined through virial scaling of the pairwise velocity PDF normalization offers an unbiased estimate of the log-mean mass of haloes matched via joint galaxy membership.

We now turn to estimate the characteristic $M_{\log_{10}}$ mass scale of correctly centred redMaPPer clusters as a function of richness $\lambda$, at the pivot redshift $z_p = 0.2$. Recall from Sections 5.2.1 and 5.2.2 that the pairwise velocity normalization depends on the miscentring frequency and the velocity bias of central and satellite galaxies. We need to estimate the magnitudes of these effects, and their uncertainties, into our mass estimate.

The normalization correction for miscentring, assuming $f_{\text{cen}} = 0.85 \pm 0.05$ for the SDSS redMaPPer sample, is already incorporated into the correctly centred estimate given in equation (10).

To estimate the velocity dispersion of the underlying dark matter from the pairwise satellite–central galaxy measurements, we need to divide the latter by the quadrature sum of the respective velocity bias factors,

$$\sigma_{v,\text{RMIV,DM}} = \frac{\sigma_{p,\text{RMIV,CEN}}}{\sqrt{\sigma^2 + \sigma^2}}.$$ (11)

The velocity bias of galaxies has been recently investigated by Guo et al. (2015a,b) using SDSS galaxy clustering measured both in projected separation and in redshift space. We employ the Guo et al. (2015b) estimates for the velocity bias factors of bright ($M_\ast \sim -21.5$, as appropriate for the bulk of the spectroscopic galaxies in this study) galaxies (see their fig. 8) of $\alpha_s = 0.30 \pm 0.05$ and $\alpha_c = 1.05 \pm 0.08$. Their central galaxy dispersion is in line with previous estimates based on explicit spectroscopy of cluster members (Coziol et al. 2009; Lauer et al. 2014) as well as with recent simulation expectations (Martel et al. 2014). There is more contention on the velocity bias of satellite galaxies. In recent simulations, values less than one have been measured for bright galaxies in massive haloes (Munari et al. 2013; Old, Gray & Pearce 2013; Wu et al. 2013). We note that the $2\sigma$ range of $\alpha_s \in [0.89, 1.21]$ admits values less than unity.

These velocity bias estimates imply a correction factor, $(\alpha_s^2 + \alpha_c^2)^{-1/2} = 0.92 \pm 0.07$, which leads to the dark matter velocity dispersion at the pivot richness and redshift of

$$\sigma_{p,\text{RMIV,DM}} = 535 \pm 41 \text{ km s}^{-1}.$$ (12)

Note that the uncertainty in this velocity is dominated by systematic error in the velocity bias estimate.

Finally, using this value in equation (8), we obtain an estimate of the log-mean mass of redMaPPer clusters at the pivot richness and redshift of

$$M_s(\lambda = 30, z_p = 0.2) = (1.56 \pm 0.35) \times 10^{14} M_\bigodot.$$ (13)

where to infer above mass scale we assume a LCDM cosmology with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, and $h(z) = 0 = 0.7$.

The scaling of the pairwise velocity normalization, $\sigma_p(\lambda, z)$, determines how the mean dynamical mass, $M_s(\lambda, z)$, scales with richness and redshift. Because of the relatively weak constraint on the redshift scaling behaviour of the SDSS cluster sample velocities, we defer analysis of redshift evolution to a later study and concentrate here on the behaviour with richness at the pivot redshift of 0.2. The simulations indicate the mean dynamical mass, $M_s(\lambda, z)$, matches the log-mean membership matched mass at the pivot richness, but as shown in Fig. 6, the best-fitting slope of log-mean mass with richness differs by 0.10 from the slope of $M_s(\lambda)$. We therefore include this difference as a systematic error term when quoting the slope.

The result is an estimate for the log-mean membership matched mass of the SDSS redMaPPer sample at redshift 0.2 of

$$\langle \ln (M_s/10^{14} M_\bigodot) | \lambda, z_p = 0.2 \rangle = \pi + \sigma_m \ln(\lambda/30),$$ (14)

with normalization $\pi = 0.44 \pm 0.22$ and slope $\sigma_m = 1.31 \pm 0.06_{\text{stat}} \pm 0.13_{\text{sys}}$.

Of the 22 per cent error in the derived mass normalization, 21.5 per cent arises from systematic uncertainty in the velocity bias terms, particularly that of satellite galaxies. Miscentring contributes 2.6 per cent, and statistical uncertainties from the stacked pairwise velocity and virial calibration parameters are 3.2 per cent. The error in $\ln (M_s)$ is essentially triple the uncertainty in $\ln (\alpha_s)$. As a result, achieving 10 per cent error in mean mass would require knowing $\alpha_c$ to a fractional accuracy of ~0.03. It remains to be seen whether future spectroscopic campaigns, coupled with improved hydrodynamic simulations of galaxy formation in massive haloes to pin down systematic errors, can achieve this level of precision.
7 CONCLUSION

Using galaxy catalogues derived from large $N$-body simulations, we study the mapping of galaxy clusters identified in sky-photometry space to the underlying real-space population of haloes through membership matching. We measure membership strength, defined as the fraction of a cluster’s richness contributed by a given halo, and build bipartite graphs linking clusters to haloes with strength-weighted edges. The matched halo of a cluster maximizes this strength.

We then study pairwise velocities, and derived masses, from stacked spectroscopic analysis of clusters patterned after the spectroscopic analysis of SDSS redMaPPer clusters developed by RMIV. The structure in the simulated data is similar to that of the observations, with galaxy pairwise velocities having a main Gaussian provisionally identified as cluster members. We employ a subsample of correctly centred clusters – those for which the central cluster galaxy is also the central galaxy of the matched halo – as well as studying the full simulated cluster sample.

We then use our findings to estimate the log-mean, membership-matched mass of SDSS redMaPPer clusters at $z = 0.2$. Our detailed results are as follows.

(i) Although the pairwise velocity PDF model is not a good fit to data, the richness and redshift-dependent width of the PDF adequately reflects the log-mean velocity dispersion of matched haloes. Decomposing this main component into halo contributions, we find that the top-ranked, matched halo contributes an average of 62 per cent (58 per cent) of pairs in the correctly centred (full) cluster samples. The second-ranked halo contributes $\sim$10 per cent, the third $\sim$5 per cent, and the remainder contribute $\sim$20 per cent, in the mean. The projected component, consisting of all galaxy pairs not contributed by the top-ranked matched halo, has a pairwise velocity PDF described roughly by a Gaussian plus constant form.

(ii) Converting the velocity dispersion–richness relation to a mass–richness relation using the dark matter virial relation calibrated by independent simulations, we find this stacked dynamical mass recovers, to within a few per cent, the log-mean mass–richness relation using the dark matter virial relation calibrated roughly by a Gaussian plus constant form.

(iii) We model effects of cluster miscentring and galaxy velocity bias in order to correct the measured redMaPPer cluster velocity dispersion to reflect that of correctly centred, dark matter haloes. Using central and satellite velocity bias parameters $\alpha_c = 0.30 \pm 0.05$ and $1.05 \pm 0.08$, respectively (Guo et al. 2015b), we infer a log-mean matched halo mass of $M_{200,\rho} = (1.56 \pm 0.35) \times 10^{14} M_\odot$ at the pivot richness, $\lambda_p = 30$, and redshift $z_p = 0.2$, and a slope with richness of $1.31 \pm 0.06_{\text{stat}} \pm 0.13_{\text{sys}}$ for SDSS redMaPPer clusters.

Kinematic biases of central and, especially, satellite galaxies, are the dominant source of systematic error. Further work is needed, both empirically and through hydrodynamic simulations, to better constrain the relationship between galaxy velocities and dark matter. One possible approach is to invert the analysis presented here; comparing the stacked dynamical masses with stacked weak lensing masses of the same sample with the aim of constraining velocity bias.

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⁴ https://github.com/dfm/emcee
Appendix A: Spectroscopic Membership in the Bolshoi Simulation

We provide here a test of the sensitivity of the pairwise velocity PDF to the galaxy assignment method based on the Bolshoi simulation catalogue of Hearin & Watson (2013).

Hearin & Watson (2013) use age distribution matching, an empirical method for modelling how galaxies occupy haloes as a function of luminosity and colour. The method relates galaxy luminosity to subhalo size, and assigns colour to formation epoch at fixed size, with older systems being redder. Table A1 compares the HOD properties of the Aardvark and Bolshoi catalogues. The slope and intrinsic scatter for a luminosity cut, $M_r - 5 \log h = -19$, are given for halo samples limited in mass above $M_{200c} = 2 \times 10^{14} h^{-1} M_\odot$ (Aardvark) and $M_{vir} = 10^{14} h^{-1} M_\odot$ (Bolshoi). The Aardvark HOD is shallower and has larger variance than that of Bolshoi.

Despite these differences in intrinsic galaxy population, we find that Bolshoi and Aardvark simulation both produce similar pairwise velocity PDF structure. For the Bolshoi analysis, we place the $z = 0$ catalogue at an effective redshift 0.1, then use cylinders of length $120 h^{-1}$ Mpc (the size of the Bolshoi simulation) centred on known halo locations to extract projected galaxies around the position of the central galaxy. The $120 h^{-1}$ Mpc comoving length is equivalent to $\sim 0.03$ redshift shells at a central redshift of 0.1.

We measure redMaPPer richness and generate LOS velocity pairs based on redMaPPer cluster membership. There is no redshift evolution in colour because the Bolshoi redshift is fixed.

Fig. A1 shows the normalized, pairwise velocity PDFs for all members (left-hand panel) and using only member of the target halo (right-hand panel). These panels are the equivalent of the left-hand and middle panels of Fig. 4. The results are generally consistent with the Aardvark simulation and the observational data. Differences in the fit parameters listed in Table 2 reflect differences in the intrinsic HODs of the two simulations.

We find a Gaussian component amplitude of $p = 0.884$, only slightly lower than the values of the Aardvark CEN sample and the observational data. Using the membership definition of Hearin & Watson (2013) (used to calculate the intrinsic richness of the halo), we find that 70 per cent of spectroscopic member galaxies belong to the target halo. The shallower slope $\alpha$ reflects the steeper intrinsic HOD slope of Bolshoi. The larger strength, $S_{max}$ of the matched halo membership contribution to the main Gaussian component may be due to the smaller HOD scatter as well as the more limited treatment of projected contamination in that simulation.

### Table A1. Slope and log-normal scatter of intrinsic richness, $\lambda_{vir}$, versus mass, $M_{200c}$, for the Aardvark and Bolshoi (Hearin & Watson 2013) simulations applying a luminosity threshold of $M_r - 5 \log h = -19$. 

| Simulation | $z$ | Slope | Scatter |
|------------|----|-------|---------|
| Aardvark   | 0.15 | 0.75  | 0.33    |
| Aardvark   | 0.25 | 0.79  | 0.28    |
| Bolshoi    | 0    | 1.0   | 0.21    |
Figure A1. The left-hand panel shows the PDF of spectroscopic sample galaxy members’ LOS velocity in the Bolshoi simulation by normalizing the velocity according to equation (5) and setting the redshift evolution to zero. The black line shows the best fit for our likelihood model equation (6). The right-hand panel shows similar exercise for the target halo galaxy members in the Bolshoi simulation. The black line shows the best fit for our likelihood model equation (6) assuming $p = 1$. Error bars are $2\sigma$ using bootstrap method.

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