Towards Asymptotic Sum Capacity of MIMO Cellular Two-Way Relay Channel

Zhaoxi Fang, Xiaojun Yuan, Member, IEEE and Xin Wang, Senior Member, IEEE

Abstract—In this paper, we consider the transceiver and relay design for multiple-input multiple-output (MIMO) cellular two-way relay channel (cTWRC), where a multi-antenna base station (BS) exchanges information with multiple multi-antenna mobile stations via a multi-antenna relay station (RS). We propose a novel two-way relaying scheme to approach the sum capacity of the MIMO cTWRC. A key contribution of this work is a new non-linear lattice-based precoding technique to pre-compensate the inter-stream interference at the relay, so as to allow efficient interference-free lattice decoding at the relay. We derive sufficient conditions for the proposed scheme to asymptotically achieve the cut-set outer bound of the single-input single-output (SISO) Gaussian TWRC in the high signal-to-noise ratio (SNR) regime. To fully exploit the potential of the proposed scheme, we also investigate the optimal power allocation at the BS and RS to maximize the weighted sum-rate of the MIMO cTWRC in the general SNR regime. It is shown that the problem can be formulated as a monotonic program, and a polyblock outer approximation algorithm is developed to find the global optimal solution with guaranteed convergence. We demonstrate by numerical results that the proposed scheme significantly outperforms the existing schemes and closely approaches the sum capacity of the MIMO cTWRC in the high SNR regime.

Index Terms—Two-way relay channel, lattice coding, dirty paper coding, monotonic optimization.

I. INTRODUCTION

Two-way communications can be dated back to Shannon [1] and have been rediscovered as an efficient method to mitigate the loss of spectral efficiency in conventional half-duplex one-way relaying [2]–[9]. Tremendous progresses have been made for efficient communications over the two-way relay channel (TWRC), in which two users want to exchange information via the help of a single relay. The main idea, termed physical-layer network coding (PNC), is to allow the two users to communicate with the relay simultaneously, and to allow each user to decode the message from the other user by exploiting the knowledge of the self-message. It was shown in [6] that PNC with nested lattice coding can achieve the cut-set outer bound of the single-input single-output (SISO) Gaussian TWRC within 1/2 bit. Later, the authors in [7]–[9] studied the multiple-input multiple-output (MIMO) TWRC, where both the users and the relay are equipped with multiple antennas. It was shown that near-capacity performance can be achieved in MIMO TWRCs by using nested lattice coding aided PNC.

PNC design for more sophisticated relay networks has recently attracted much research interest. In this regard, the authors in [10] generalized the TWRC model to the multiway relay channel (mRC), in which a relay simultaneously serves multiple clusters of users, and each user in a cluster wants to multicast its message to all the other users in the same cluster. Several special cases of the mRC have been studied in the literature. For example, the authors in [11]–[14] considered the multi-pair MIMO TWRC which is a special case of the mRC with each cluster consisting of two users; also, the Y channel proposed in [15] is a special case of the mRC with only one cluster. Various relaying protocols, including amplify-and-forward (AF), decode-and-forward (DF), compress-and-forward, and their mixtures, were investigated for these relay networks.

In this paper, we investigate the efficient PNC design for another important two-way relaying model, termed the cellular TWRC (cTWRC), where multiple users in a cellular network want to exchange information with a multiple-antenna base station (BS) via the help of a multiple-antenna relay. This MIMO cTWRC model was previously studied in [16]–[21]. In [16], linear precoding was applied at the BS to align the signals impinging upon the relay in such a way that each signal stream of the BS is aligned to the direction of the user’s signal stream to be exchanged with. In [17] and [18], linear precoding was applied at both the BS and the relay, and iterative algorithms were proposed to optimize the corresponding precoders based on various design criteria, such as to maximize the sum-rate or the minimum weighted signal-to-interference plus noise ratio (SINR). All these approaches are based on AF relaying which generally suffers from the noise propagation, as well as from the power inefficiency caused by transmitting analogue (instead of algebraic) superposition of the BS and user signals in the second phase.

To avoid the aforementioned disadvantages, the authors in [20] proposed a DF-based relaying scheme for the MIMO cTWRC involving linear precoding and nested lattice coding at the BS and users, and dirty-paper coding at the relay. It was shown that the achievable sum-rate of this scheme is much higher than the AF based schemes in [16]–[18], and this scheme can achieve the cut-set outer bound of the MIMO cTWRC if only the second phase is concerned. However, in general, the scheme in [20] performs far away from the capacity of the MIMO cTWRC, especially for a relatively large MIMO setup. This performance gap is largely attributed to the following fact: In the first phase, linear precoding can
applied only at the BS (as the users cannot cooperate), whereas the BS precoder alone fails to provide enough freedom to align the signals efficiently for interference-free PNC decoding at the relay.

In this paper, we propose a novel nested-lattice-coding aided PNC scheme to approach the sum capacity of the MIMO cTWRC. Compared with [20], one major difference and contribution of this work is a new non-linear precoding technique, called lattice precoding, employed in the first-phase system design. We show that, combined with linear precoding, nested lattice coding, and successive interference cancellation (SIC), the proposed lattice precoder at the BS efficiently pre-compensates for the inter-stream interference seen at the relay, so as to enable interference-free lattice decoding at the relay. We derive the achievable rates of the proposed scheme, and establish sufficient conditions that the proposed scheme asymptotically achieves the sum capacity of the MIMO cTWRC in the high SNR regime. Furthermore, we formulate a weighted sum-rate maximization problem for the proposed scheme to optimize the power allocation of the nodes in the network and show that this non-convex problem is solvable using monotonic programming (MP) [22]. An efficient polyblock outer approximation algorithm is developed to find the optimal power allocation. Numerical results demonstrate that the proposed scheme significantly outperforms its existing counterparts and closely approaches the capacity of MIMO cTWRC in the high SNR regime.

The rest of this paper is organized as follows. Section II describes the MIMO cellular TWRC model. Section III presents the proposed encoding and decoding scheme while Section IV analyzes the achievable sum-rate of the proposed scheme. Section V investigates the optimal power allocation problem. Section VI discusses the extension of the proposed scheme to MIMO cTWRCs with general antenna setups. The proposed scheme is tested and compared with the existing schemes in Section VII, followed by the conclusions in Section VIII.

Notations: The following notations are used throughout this paper. Boldface letters denote vectors or matrices. $(\cdot)^T$ denotes transpose. The $i$-th row of a matrix $A$ is denoted by $a^{(i)}$, and the $(i, j)$-th element of a matrix $A$ is denoted by $a(i, j)$. $\mathbb{R}^{K \times M}$ and $\mathbb{C}^{K \times M}$ denote the $K$-by-$M$ dimensional real and complex space, respectively. $\| \cdot \|_F$ denotes the Frobenius norm. $\text{tr}(\cdot)$ denotes the trace operation; $\text{diag}(a_1, \ldots, a_N)$ denotes a diagonal matrix with diagonal elements $(a_1, \ldots, a_N)$, and $(A)_{\text{diag}}$ denotes the diagonal matrix specified by the diagonal of matrix $A$; $I_K$ denotes a $K \times K$ identity matrix; $e_i$ denotes a unit vector with the only non-zero element in the $i$-th entry; $[x]^+$ denotes $\max(x, 0)$.

II. SYSTEM MODEL

We consider a MIMO cTWRC where a BS communicates with $K$ mobile stations (MSs) via a single relay station (RS), as shown in Fig. 1. There is no direct link between the BS and the MSs. The BS, the RS, and the $k$-th MS are equipped with $N_B$, $N_R$, and $N_{M,k}$ antennas, respectively. We consider quasi-static flat-fading channels and the channel coefficients keep unchanged in the duration of a transmission frame, denoted by $T$. All the nodes are half-duplex and the bidirectional transmission takes place in two phases. For presentation clarity, we consider single-antenna MSs, i.e., $N_{M,k} = 1$, for $k = 1, \ldots, K$, and assume $N_B = N_R = K$ in the following. The extension to a general antenna setup will be discussed in Section VI. Each MS exchanges one data stream in total in the considered MIMO cTWRC. The channel matrix from the BS to the RS is denoted by $H_{BR} \in \mathbb{C}^{K \times K}$, and the channel vector from the $k$-th MS to the RS by $h_{k,R} \in \mathbb{C}^{K \times 1}$. $H_{RB} \in \mathbb{C}^{K \times K}$ and $h_{R,k} \in \mathbb{C}^{1 \times K}$ are the corresponding channel matrices/vectors for the receive links. Following the convention (e.g., in [17] and [20]), we assume that all the nodes have global channel state information of all links.

The transmission protocol is described as follows. In the first phase, the BS and all the $K$ MSs transmit to the RS simultaneously. Let $X_B \in \mathbb{C}^{K \times T}$ and $x_{M,k} \in \mathbb{C}^{1 \times T}$ denote the transmit signal at the BS and the $k$-th MS, respectively. The received signal at the RS is given by

$$Y_R = H_{BR}X_B + H_{MR}X_M + \Psi_R,$$  (1)

where $H_{MR} := [h_{1,R}, \ldots, h_{K,R}] \in \mathbb{C}^{K \times K}$, $X_M := [x_{M,1}^T, \ldots, x_{M,K}^T]^T \in \mathbb{C}^{K \times T}$, and $\Psi_R \in \mathbb{C}^{K \times T}$ denotes the additive white Gaussian noise (AWGN) at the RS. It is assumed that each element in $\Psi_R$ is independent and identically distributed (i.i.d.) with zero mean and a variance of $\sigma^2$. The maximum transmit powers at the BS and the $k$-th MS are denoted by $P_B$ and $P_{M,k}$ respectively, i.e., $\frac{1}{T} \text{tr}(X_BX_B^H) \leq P_B$, and $\frac{1}{T} \|x_{M,k}\|^2 \leq P_{M,k}$.

Upon receiving $Y_R$, the transmit signal at the RS is regenerated as $X_R = g_R(Y_R) \in \mathbb{C}^{K \times T}$, where $g_R(\cdot)$ denotes the RS decoding and re-encoding function. The RS' transmit power is constrained as $\frac{1}{T} \text{tr}(X_RX_R^H) \leq P_R$, where $P_R$ is the power budget at the relay.

In the second phase, the RS broadcasts $X_R$ to the BS and the $K$ MSs. The received signals at the BS and the $k$-th MS are respectively given by

$$Y_B = H_{RB}X_R + \Psi_B,$$  (2)

$$y_{M,k} = h_{R,k}X_R + \psi_{M,k},$$  (3)

Fig. 1. A MIMO cTWRC with $K$ mobile stations.
where $\Psi_B$ and $\psi_{M,k}$ are the AWGN at the BS and the $k$-th MS, respectively. With $Y_M = [y_{M,1}^T, \ldots, y_{M,K}^T]^T$, $H_{RM} = [h_{R,1}^T, \ldots, h_{R,K}^T]^T$, and $\Psi_M = [\psi_{M,1}^T, \ldots, \psi_{M,K}^T]^T$, we have
\[ Y_M = H_{RM}X_R + \Psi_M. \quad (4) \]

With the knowledge of the self message $X_B$, the BS estimates the designated message from the received signal $Y_B$. Meanwhile, for $k = 1, \ldots, K$, the $k$-th MS decodes the BS’s message from $y_{M,k}$ with the knowledge of $x_{M,k}$.

For the MIMO cTWRC, let $R_{B,k}$ be the transmission rate from the BS to the $k$-th MS, and $R_{M,k}$ be the transmission rate from the $k$-th MS to the BS. A rate tuple $(R_{B,1}, \ldots, R_{B,K}, R_{M,1}, \ldots, R_{M,K})$ is said to be achievable if there exist transmit encoding functions, MIMO processing functions, and receive decoding functions at the BS and MSs such that the decoding error probability tends to zero as the codeword length $T \to \infty$. From the cut-set theorem, two rate outer bounds of the MIMO cTWRC are given by [20]
\[ \sum_{k=1}^K R_{B,k} \leq \min \left\{ \frac{1}{2} \log |I + H_{BR}Q_BR^H|, \frac{1}{2} \log |I + H_{RM}Q_R^H| \right\}, \quad (5a) \]
\[ \sum_{k=1}^K R_{M,k} \leq \min \left\{ \frac{1}{2} \log |I + H_{RB}Q_R^H|, \frac{1}{2} \log |I + H_{MR}Q_M^H| \right\}, \quad (5b) \]
where $Q_S = \frac{1}{T}E(X_SX_S^H), S \in \{B, R, M\}$, are the corresponding signaling covariance matrices. These bounds will be used as a benchmark of the system design for the MIMO cTWRC.

**III. PROPOSED TWO-WAY RELAYING SCHEME**

In this section, we propose a novel two-phase two-way relaying scheme to approach the sum-rate capacity of the MIMO cTWRC. The key novelty of our scheme, compared to [20], is that lattice precoding and random dithering are employed in the first phase to pre-compensate for the inter-stream interference seen at the relay. Building on this theme, encoding and decoding operations at the BS, MSs and the RS are carefully developed to enable efficient interference-free PNC at the relay, even with the restriction of non-cooperation among MSs.

**A. Channel Trianglization**

To start with, we describe a linear precoding technique to triangularize the channel matrices involved in the two transmission phases, following the approach in [20].

We consider the first phase. Let the QR decomposition of $H_{MR}$ be
\[ H_{MR} = Q_{MR}R_{MR}, \quad (6) \]
where $Q_{MR}$ is a unitary matrix and $R_{MR}$ is an upper-triangular matrix. Further let the RQ decomposition of $Q_{MR}^H H_{BR}$ be
\[ Q_{MR}^H H_{BR} = R_{BR}Q_{BR}, \quad (7) \]
where $Q_{BR}$ is a unitary matrix and $R_{BR}$ is an upper-triangular matrix. By multiplying $Q_{MR}^H$ to the RS received signal $Y_R$, we obtain
\[ \hat{Y}_R = Q_{MR}^H Y_R = R_{BR}X_B + R_{MR}X_M + \hat{\Psi}_R, \quad (8) \]
where $\hat{X}_B = Q_{BR}X_B$, and $\hat{\Psi}_R = Q_{MR}^H \Psi_R$.

Let $s_{B,k} \in \mathbb{C}^{1 \times T}$ be the coded vector of the BS transmitted to the $k$-th MS. The transmit signal of the BS is linearly precoded as $X_B = Q_{BR}^H S_B$, where $S_B = [s_{B,1}^T, \ldots, s_{B,K}^T]^T \in \mathbb{C}^{K \times T}$ is the codeword matrix. The transmit signal of the $k$-th MS is directly generated as $x_{M,k} = s_{M,k}$, where $s_{M,k} \in \mathbb{C}^{1 \times T}$ denotes the coded vector of the $k$-th MS. With such linear precoding, the RS obtains
\[ \hat{Y}_R = R_{BR}S_B + R_{MR}S_M + \hat{\Psi}_R, \quad (9) \]
where $S_M = [s_{M,1}^T, \ldots, s_{M,K}^T] \in \mathbb{C}^{K \times T}$. Correspondingly, the power constraints at the BS and the $k$-th MS can be equivalently written as $\frac{1}{T} \text{tr}(S_B S_B^H) \leq P_B$, and $\frac{1}{T} \|s_{M,k}\|_F^2 \leq P_{M,k}, k = 1, \ldots, K$.

The channels from the RS to the BS and MSs can be triangularized in a similar way. Let $\Phi$ be an arbitrary permutation matrix. We will see in Section III-E that $\Phi$ specifies the DPC re-encoding order at the relay. Let the LQ decomposition of $\Phi H_{RM}$ be
\[ \Phi H_{RM} = L_{RM}Q_{RM}, \quad (10) \]
where $Q_{RM}$ is unitary and $L_{RM}$ is lower-triangular. The transmit signal of the RS is given by
\[ X_R = Q_{RM}^H X_{R,DPC}, \quad (11) \]
where $X_{R,DPC} \in \mathbb{C}^{K \times T}$ is the DPC codeword matrix to be elaborated in Subsection E. As $Q_{RM}$ is unitary, the power constraint of the relay can be equivalently written as $\frac{1}{T} \text{tr}(X_{R,DPC} X_{R,DPC}^H) \leq P_R$.

The permuted received signal $\hat{Y}_M = \Phi Y_M$ at all $K$ MSs can be expressed as
\[ \hat{Y}_M = \Phi H_{RM} X_R + \Phi \Psi_M = L_{RM}X_{R,DPC} + \hat{\Psi}_M, \quad (12) \]
where $\hat{\Psi}_M = \Phi \Psi_M$ is still an AWGN matrix.

Now consider the received signal at the BS. Let the QR decomposition of $H_{RB}Q_{RM}$ be
\[ H_{RB}Q_{RM} = Q_{RB}^H L_{RB}. \quad (13) \]

By multiplying $Q_{RB}^H$ to the received signal, the BS obtains
\[ \hat{Y}_B = Q_{RB}^H Y_B = L_{RB}X_{R,DPC} + \hat{\Psi}_B, \quad (14) \]
where $\hat{\Psi}_B = Q_{RB}^H \Psi_B$. In the above channel triangularization, only unitary transforms are involved, implying that the new signal model in (9), (12) and (14) has the same capacity as the original MIMO cTWRC. Therefore, we henceforth focus on the signaling design for the equivalent MIMO cTWRC given by (9), (12) and (14).
B. Partitions of Upper-Triangular Matrices $\mathbf{R}_{BR}$ and $\mathbf{R}_{MR}$

In the following, we describe the main ideas behind our novel lattice precoding and decoding scheme in detail, based on channel models given in (9), (12) and (14). We first consider the phase-1 channel model in (9). As will be detailed later, successive interference cancellation is employed at the relay for efficient PNC decoding. To this end, we represent successive interference cancellation is employed at the relay

\[ \mathbf{Y}_R = \left( \mathbf{R}_{BR} \mathbf{S}_B + (\mathbf{R}_{MR})_{\text{diag}} \mathbf{S}_M \right) + \left( \mathbf{R}_{BR} - \mathbf{R}'_{BR} \right) \mathbf{S}_B + (\mathbf{R}_{MR} - (\mathbf{R}_{MR})_{\text{diag}}) \mathbf{S}_M + \mathbf{\Psi}_R, \]

where $\mathbf{R}'_{BR}$ is an upper-triangular matrix to be determined momentarily. The first under-braced term in the right hand side (RHS) of (15) represents the signal to be decoded at the RS. Here, $\mathbf{R}'_{BR}$ is allowed to be upper-triangular, as the proposed lattice precoding at the BS, as will be elaborated in the sequel. The second under-braced term in the RHS of (15) denotes the residue inter-stream interference. We need to properly chose $\mathbf{R}_{BR}$ such that this interference term can be successively cancelled at the RS with decoding ordered from the $K$-th spatial stream to the first stream. First, to ensure that the second term in (15) only contains inter-stream interference, $(\mathbf{R}_{BR} - \mathbf{R}'_{BR})$ is required to be strictly upper-triangular, i.e., the diagonal of $\mathbf{R}'_{BR}$ is chosen the same as $\mathbf{R}_{BR}$. For convenience, we refer to the $k$-th row of $(\mathbf{R}'_{BR} \mathbf{S}_B + (\mathbf{R}_{MR})_{\text{diag}} \mathbf{S}_M)$ as the $k$-th network-coded message. In decoding the $k$-th network-coded message, the relay only knows the $(k+1)$-th to $K$-th network-coded messages. This implies that the relay is only able to cancel linear combinations of the last $(K-k)$ entries of $(\mathbf{R}'_{BR} \mathbf{S}_B + (\mathbf{R}_{MR})_{\text{diag}} \mathbf{S}_M)$ in decoding the $k$-th network-coded message. To completely cancel the inter-stream interference at the relay, we require that there exists a strictly upper-triangular matrix $\mathbf{U}_R$ satisfying

\[ \mathbf{U}_R \left( \mathbf{R}'_{BR} \mathbf{S}_B + (\mathbf{R}_{MR})_{\text{diag}} \mathbf{S}_M \right) = (\mathbf{R}_{BR} - \mathbf{R}'_{BR}) \mathbf{S}_B + (\mathbf{R}_{MR} - (\mathbf{R}_{MR})_{\text{diag}}) \mathbf{S}_M \]

for arbitrary $\mathbf{S}_B$ and $\mathbf{S}_M$. We can equivalently write the above condition as

\[ \mathbf{U}_R \mathbf{R}'_{BR} = \mathbf{R}_{BR} - \mathbf{R}'_{BR} \]

\[ \mathbf{U}_R (\mathbf{R}_{MR})_{\text{diag}} = \mathbf{R}_{MR} - (\mathbf{R}_{MR})_{\text{diag}}. \]

From (18), we have

\[ \mathbf{U}_R = (\mathbf{R}_{MR} - (\mathbf{R}_{MR})_{\text{diag}})(\mathbf{R}_{MR})^{-1}_{\text{diag}}. \]

Substituting (19) into (17), we further obtain

\[ \mathbf{R}'_{BR} = \left( \mathbf{I}_K + \mathbf{U}_R \right)^{-1} \mathbf{R}_{BR} = (\mathbf{R}_{MR})_{\text{diag}} \mathbf{R}_{MR}^{-1} \mathbf{R}_{BR}. \]

Let $\mathbf{y}^{(k)}_R$ and $\mathbf{\tilde{y}}^{(k)}_R$ be the $k$-th row of $\mathbf{Y}_R$ and $\mathbf{\tilde{Y}}_R$, respectively. Then, from (15), the $k$-th subchannel, scaled by the factor $\frac{1}{r_{BR}(k,k)}$, can be expressed as

\[ \mathbf{y}_{R,k} = \frac{\mathbf{y}^{(k)}_R}{r_{BR}(k,k)} = \mathbf{s}_{B,k} + \alpha_k \mathbf{s}_{M,k} + \mathbf{u}_k + \mathbf{v}_k + \mathbf{\psi}_{R,k}, \quad (21) \]

where

\[ \mathbf{u}_k = \sum_{j=k+1}^{K} \left( \frac{r_{BR}(k,j)}{r_{BR}(k,k)} \right) \mathbf{s}_{B,j} \right) + \left( \frac{r_{MR}(k,j)}{r_{BR}(k,k)} \right) \mathbf{s}_{M,j}; \]

\[ \mathbf{v}_k = \sum_{j=k+1}^{K} \left( \frac{r_{MR}(k,j)}{r_{BR}(k,k)} \right) \mathbf{s}_{B,j}; \]

\[ \mathbf{\psi}_{R,k} = \frac{\mathbf{\tilde{y}}^{(k)}_R}{r_{BR}(k,k)}. \]

Note that the decoded signal of the $k$-th stream is $\mathbf{s}_{B,k} + \alpha_k \mathbf{s}_{M,k} + \mathbf{u}_k$. With $\mathbf{U}_R$ and $\mathbf{R}_{BR}$ respectively given in (19) and (20), then $\mathbf{u}_k$ in (21) can be expressed as $\mathbf{u}_k = \sum_{j=k+1}^{K} \mathbf{u}_{R}(k,j)(\mathbf{s}_{B,j} + \alpha_j \mathbf{s}_{M,j} + \mathbf{v}_j)$ (see (16)), i.e., $\mathbf{u}_k$ is in fact a weighted sum of the signals already decoded at the relay, with the decoding ordered from the $K$-th spatial stream to the first stream. Hence, it can be cancelled from $\mathbf{y}_{R,k}$. Moreover, $\mathbf{v}_k$ in (21) is a weighted sum of the last $K-k$ rows of $\mathbf{s}_B$. The encoding order of the BS is ordered from the $K$-th spatial stream to the first stream. Then, when encoding the $k$-th spatial stream, $\mathbf{v}_k$ is known to BS; thus it can be pre-cancelled using lattice precoding at the BS, as will be elaborated in the sequel.

C. Phase 1: Encoding at the BS and MSs

In the following, we describe the encoding and decoding operations involved in the proposed scheme. We start with the encoding operations at the BS and MSs.

Let $\mathcal{W}_{M,k} = \{1, 2, ..., 2^{TR_{B,k}}\}$ be the message set for the spatial data stream at the $k$-th MS, and $w_{M,k} \in \mathcal{W}_{M,k}$ be the corresponding message. The $(1 \times T)$-dimensional coded vector for the spatial data stream at the $k$-th MS is denoted as $\mathbf{s}_{M,k} = \mathbf{f}_{M,k}(w_{M,k})$, where $\mathbf{f}_{M,k}(\cdot)$ is the encoding function to be specified in the following. Similarly, the message of the $k$-th spatial stream to be transmitted to the $k$-th MS at the BS is denoted by $w_{B,k} \in \mathcal{W}_{B,k}$, where $\mathcal{W}_{B,k} = \{1, 2, ..., 2^{TR_{B,k}}\}$ is the message set. The corresponding $(1 \times T)$-dimensional encoded vector is denoted by $\mathbf{s}_{B,k} = \mathbf{f}_{B,k}(w_{B,k})$.

Nested lattice coding is applied to each message pair $(w_{B,k}, w_{M,k})$. An $n$-dimensional lattice $\Lambda$ is a subgroup of $\mathbb{R}^n$ under normal vector addition. A lattice $\Lambda$ is nested in the lattice $\Lambda_1$ if $\Lambda \subseteq \Lambda_1$. The main idea of nested lattice codes is to use the coarse lattice $\Lambda$ as a shaping region and the lattice points from the fine lattice $\Lambda_1$ contained within the Voronoi region of the coarse lattice $\Lambda$ as the codewords. The details of nested lattice codes can be found, e.g., in [6], [23]–[25] and the references therein.

The encoding functions $\{f_{B,k}(\cdot)\}_{k=1}^{K}$ and $\{f_{M,k}(\cdot)\}_{k=1}^{K}$ are described as follows. The encoding of the BS follows the order from the $K$-th stream to the first stream sequentially. Without

...
loss of generality, we assume $R_{B,k} \leq R_{M,k}$. For the $k$-th subchannel in (21), we construct a nested lattice chain $\Lambda_{B,k}$, $\Lambda_{M,k}$, and $\Lambda_{C,k}$ satisfying $\Lambda_{M,k} \subseteq \Lambda_{B,k} \subseteq \Lambda_{C,k}$. Here, $\Lambda_{B,k}$ and $\Lambda_{M,k}$ are simultaneously Rogers-good and Poltyrev-good while $\Lambda_{C,k}$ is Poltyrev-good [6]. Let $c_{X,k} \in C_{X,k}$ denote the codeword mapped from the message $w_{X,k}$, $X \in \{B, M\}$, where $C_{X,k}$ is the nested lattice code defined by $\Lambda_{B,k}$ and $\Lambda_{C,k}$. The coding rate of the nested lattice code $C_{X,k}$ can approach any $\xi_X > 0$ as $T \to \infty$ [6], i.e.,
\[
R_{X,k} = \frac{1}{T} \log \left( \frac{\text{Vol}(\Lambda_{X,k})}{\text{Vol}(\Lambda_{C,k})} \right) = \xi_X + o_T(1),
\]
where $\text{Vol}(\Lambda)$ is the the volume of the Voronoi region of a lattice $\Lambda$, and $o_T(1) \to 0$ as $T \to \infty$.

For the $k$-th subchannel in (21), $\Lambda_{B,k}$ and $\Lambda_{M,k}$ are chosen to meet
\[
\frac{\text{Vol}(\Lambda_{M,k})}{\text{Vol}(\Lambda_{B,k})} = \left( \frac{R_{BR}(k,k)}{|P_{BR}(k,k)|} \right)^2 P_{B,k} + o_T(1),
\]
where $P_{B,k}$ and $P_{M,k}$ denote the average power of the $k$-th spatial stream at the BS and the $k$-th MS, respectively. Then, the relation between $R_{B,k}$ and $R_{M,k}$ can be written as [6]
\[
R_{B,k} = R_{M,k} + \frac{1}{2} \log \left( \frac{|P_{BR}(k,k)|^2}{|P_{MR}(k,k)|^2} \right) + o_T(1).
\]

The encoding at the BS is as follows. Let $d_{X,k}$ be a random dithering vector that is uniformly distributed over the Voronoi region of $\Lambda_{X,k}$, $X \in \{B, M\}$. With random dithering, the $k$-th transmit signal $s_{B,k}$ at the BS is constructed as
\[
s_{B,k} = (c_{B,k} - v_k - d_{B,k}) \mod \Lambda_{B,k},
\]
where the inter-stream interference $v_k$ is a priori known when encoding the $k$-th stream at the BS.

On the other hand, the signal $s_{M,k}$ at the $k$-th MS is encoded as
\[
s_{M,k} = \frac{1}{\alpha_k} ((c_{M,k} - d_{M,k}) \mod \Lambda_{M,k}),
\]
where $d_{M,k}$ is a random dithering vector uniformly distributed over the Voronoi region of $\Lambda_{M,k}$.

It is worth mentioning that the authors in [20] presumed that random dithering cannot be used in the first phase of the MIMO eTWRC due to the non-cooperation among MSs. Interestingly, we will show that through a careful precoder design, random dithering can be in fact employed to improve performance.

D. Relay’s Operation: Lattice Decoding

We now consider the operations at the relay. The relay’s decoding order is from the $K$-th stream to the first stream. For the $k$-th subchannel in (21), the relay intends to decode the combinations $s_{B,k} + \alpha_k s_{M,k} + v_k$. Recall that $u_k$ in (21) is a weighted sum of the network-coded signals already decoded; hence it can be cancelled from the received signal

\[y_{R,k}.\] Together with the knowledge of the dithering vectors $d_{B,k}$ and $d_{M,k}$, the relay constructs
\[
w_{R,k} = y_{R,k} - u_k + d_{B,k} + d_{M,k}
\]
\[
= s_{B,k} + d_{B,k} + v_k + \alpha_k s_{M,k} + d_{M,k} + \psi_{R,k}
\]
\[
= \hat{c}_{B,k} + \hat{c}_{M,k} + \psi_{R,k},
\]
where $\hat{c}_{B,k} = s_{B,k} + d_{B,k} + v_k$, and $\hat{c}_{M,k} = \alpha_k s_{M,k} + d_{M,k}$. From (20), we see that $\hat{c}_{M,k} = \hat{c}_{B,k} - v_k - d_{B,k}$ mod $\Lambda_{B,k} + d_{B,k} + v_k$, which is a lattice point in $\Lambda_{C,k}$. Similarly, $\hat{c}_{M,k} = (\Lambda_{M,k} - d_{M,k})$ mod $\Lambda_{M,k} + d_{M,k}$ is a lattice point in $\Lambda_{C,k}$. Hence, $(\hat{c}_{B,k} + \hat{c}_{M,k}) \in \Lambda_{C,k}$ is decodable using lattice decoding, provided that [6]
\[
R_{B,k} \leq R_{B \to R,k}(P_{B,k}) \triangleq \left[ \frac{1}{2} \log \left( \frac{|R_{BR}(k,k)|^2 P_{B,k}}{\sigma^2} \right) \right]^+. \tag{29}
\]
From (25), we also obtain
\[
R_{M,k} \leq R_{M \to R,k}(P_{M,k}) \triangleq \left[ \frac{1}{2} \log \left( \frac{|R_{MR}(k,k)|^2 P_{M,k}}{\sigma^2} \right) \right]^+. \tag{30}
\]

Once the lattice point $\hat{c}_{B,k} + \hat{c}_{M,k}$ is decoded (with a vanishing error probability as $T \to \infty$), the relay can reconstruct $s_{B,k} + \alpha_k s_{M,k} + v_k = \hat{c}_{B,k} + \hat{c}_{M,k} - d_{B,k} - d_{M,k}$, which is used as the known signal to cancel the residue inter-stream interference in subsequent decoding; see (21) and the discussions therein.

E. Relay’s Operation: Re-encoding

After lattice decoding, the relay calculates
\[
s_{R,k} = (\hat{c}_{B,k} + \hat{c}_{M,k}) \mod \Lambda_{B,k}, \tag{31}
\]
for $k = 1, \ldots, K$. Then, one-side DPC encoding is applied to $s_{R,1}, \ldots, s_{R,K}$ with an order $\mu_1, \ldots, \mu_K$, so that each MS receives an interference-free signal. Note that the decoding order $[\mu_1, \ldots, \mu_K]^T$ is specified by $\Phi$ as $\phi(k, \mu_k) = 1$, and $\phi(k, j) = 0, \forall j \neq \mu_k, k = 1, \ldots, K$. From (21), the received signal at the $k$-th MS can be expressed as
\[
y_{M,k} = (R_{MR}(q_k, k) \alpha_{R,k} \psi_{R,k}) + t_{M,k} + \gamma_{M,k}, \tag{32}
\]
where $\mu_{q_k} = k$, $\gamma_{M,k}$ denotes the $q_k$-th row of $\hat{\Psi}_{M}$, $t_{M,k} = \sum_{n=1}^{N_k-1} l_{RM}(q_k, n) x_{R,DPC,n}$ and $x_{R,DPC,n}$ denotes the $n$-th DPC encoded signal. The interference $t_{M,k}$ can be pre-cancelled at the RS using dirty paper precoding as
\[
x_{R,DPC,q_k} = \left( s_{R,k} - \beta_{M,k} \frac{t_{M,k}}{l_{RB}(q_k, k)} - d_{R,k} \right) \mod \Lambda_{B,k}, \tag{33}
\]
where $d_{R,k}$ is a random dither vector that is known by the BS and the MSs, and $\beta_{M,k} = \frac{l_{RM}(q_k, k)^2 P_{R,k}}{(|l_{RM}(q_k, k)|^2 P_{R,k} + \sigma^2)}$ is the minimum mean square error coefficient for decoding at the MS [24], with $P_{R,k}$ being the transmit power of the $k$-th stream of the RS satisfying $P_{R,k} \leq P_R$. The DPC encoded signal $x_{R,DPC} = [x_{R,DPC,1}, \ldots, x_{R,DPC,K}]^T$ is then linearly precoded as (11) and broadcast to the BS and MSs in the second phase.
where the last step holds provided this interference can be successively cancelled at the BS since \( \tilde{x} \) recovers \( F \).

### Phase 2: MS Decoding

The decoding operations at each MS is as follows: For \( k = 1, \ldots, K \), the \( k \)-th MS first decode \( s_{R,k} \) from the received signal \( y_{M,k} \); then it recovers \( c_{B,k} \) from \( \tilde{s}_{R,k} \) (i.e., the decoded \( s_{R,k} \)) with the help of the knowledge of the self-message \( c_{M,k} \) and the dither signal \( d_{M,k} \). For the first step, it has been shown in [20] that the probability \( \tilde{s}_{R,k} \neq s_{R,k} \) vanishes as \( T \to \infty \) provided that

\[
R_{B,k} \leq R_{R \to M,k} (P_{R,k}) \leq \frac{1}{2} \log \left( 1 + \frac{|l_{RM}(q_k,q_k)|^2 P_{R,k}}{\sigma^2} \right). \tag{34}
\]

Here we focus on the second step, i.e., recovering \( c_{B,k} \) from \( \tilde{s}_{R,k} \). Note that \( s_{R,k} \) in (31) can be written as

\[
s_{R,k} = (\tilde{c}_{B,k} + \tilde{c}_{M,k}) \mod \Lambda_{B,k} \tag{35a}
\]
\[
= (s_{B,k} + d_{B,k} + v_k + \omega_k s_{M,k} + d_{M,k}) \mod \Lambda_{B,k} \tag{35b}
\]
\[
= (c_{B,k} + (c_{M,k} - d_{M,k}) \mod \Lambda_{M,k} + d_{M,k}) \mod \Lambda_{B,k}. \tag{35c}
\]

As \( c_{M,k} \) and \( d_{M,k} \) are known, the \( k \)-th MS obtains \( c_{B,k} \) as

\[
\tilde{c}_{B,k} = (\tilde{s}_{R,k} - (c_{M,k} - d_{M,k}) \mod \Lambda_{M,k} - d_{M,k}) \mod \Lambda_{B,k}, \tag{36a}
\]
\[
= (\tilde{s}_{R,k} - s_{R,k} + c_{B,k}) \mod \Lambda_{B,k} \tag{36b}
\]
\[
= c_{B,k}, \tag{36c}
\]

where (36b) follows from the equality of \( x \mod \Lambda_{M,k} \) mod \( \Lambda_{B,k} = x \mod \Lambda_{B,k} \) as \( \Lambda_{M,k} \subseteq \Lambda_{B,k} \), and the last equality holds provided \( \tilde{s}_{R,k} = s_{R,k} \).

### Phase 2: BS Decoding

The BS first decodes \( s_{R,k}, k = 1, \ldots, K \), from the received signal \( \bar{Y}_B \) in (14). Inter-stream interference still exists at the BS since DPC encoding is only applied to the RS-MS link. However, if the decoding order at the BS is the same as the encoding order at the RS, it was shown in [20] that this interference can be successively cancelled at the BS since \( L_{RB} \) is lower-triangual. The corresponding error probability \( \tilde{s}_{R,k} \neq s_{R,k} \) vanishes as \( T \to \infty \) provided that

\[
R_{M,k} \leq R_{R \to B,k} (P_{R,k}) \leq \frac{1}{2} \log \left( 1 + \frac{|l_{RB}(q_k,q_k)|^2 P_{R,k}}{\sigma^2} \right). \tag{37}
\]

With the knowledge of the self-message \( c_{B,k} \), the BS then recovers \( c_{M,k} \) from \( \tilde{s}_{R,k} \) by calculating

\[
\tilde{c}_{M,k} = (\tilde{s}_{R,k} - c_{B,k}) \mod \Lambda_{M,k}. \tag{38}
\]

To see this, we obtain from (31) that

\[
\tilde{c}_{M,k} = (\tilde{s}_{R,k} - s_{R,k} + (c_{M,k} - d_{M,k}) \mod \Lambda_{M,k} + d_{M,k}) \mod \Lambda_{M,k}, \tag{39a}
\]
\[
= (\tilde{s}_{R,k} - s_{R,k} + c_{M,k}) \mod \Lambda_{M,k} = c_{M,k}, \tag{39b}
\]

where the last step holds provided \( \tilde{s}_{R,k} = s_{R,k} \).

### Achievable Rates of the Overall Scheme

Combining the discussions in Subsections C-G, we have the following theorem for the proposed two-way relaying scheme.

**Theorem 1:** As \( T \to +\infty \), a rate tuple of \( (R_{B,1}, \ldots, R_{B,K}, R_{M,1}, \ldots, R_{M,K}) \) of the MIMO cTWRC is achievable if

\[
R_{B,k} \leq \min (R_{B \to R,k} (P_{B,k}), R_{R \to M,k} (P_{R,k})), \tag{40}
\]
\[
R_{M,k} \leq \min (R_{R \to M,k} (P_{M,k}), R_{B \to R,k} (P_{B,k})), \tag{41}\]

\( k = 1, \ldots, K \), where \( R_{B \to R,k} (P_{B,k}), R_{R \to M,k} (P_{R,k}), R_{M \to R,k} (P_{M,k}) \) and \( R_{B \to R,k} (P_{B,k}) \) are respectively given in (29), (34), (30) and (37).

### Analysis of the Sum-Rate Performance

In this section, we analyze the sum-rate performance of the proposed two-way relaying scheme in the high SNR regime. We first consider the cut-set bound in [5]. It is known that, in the high SNR regime, equal power allocation at the BS and RS is optimal, i.e., \( Q_{B}^{opt} = \frac{P_B}{K} I_K \), and \( Q_{M}^{opt} = \frac{P_M}{K} I_K \). Also, as the MSs cannot cooperate, the optimal \( Q_M \) is given by \( Q_M^{opt} = \text{diag}(P_{M,1}, \ldots, P_{M,K}) \). Then the sum-rate of cut-set bound at high SNR can be written as

\[
R_{sum,cs} = \sum_{k=1}^{K} (R_{B,k} + R_{M,k}) \tag{41a}
\]
\[
\approx \min \left( \sum_{k=1}^{K} \frac{1}{2} \log \left( \frac{\lambda_{RB,k} P_B}{K \sigma^2} \right), \sum_{k=1}^{K} \frac{1}{2} \log \left( \frac{\lambda_{RM,k} P_R}{K \sigma^2} \right) \right) \tag{41b}
\]

where \( \lambda_{X,k} \) denotes the square of the \( k \)-th singular value of \( H_X \), \( X \in \{ B, R, M \} \), and \( x \sim y \to 0 \) as \( \sigma^2 \to 0 \).

We now consider the proposed two-way relaying scheme. From (40), the achievable sum-rate of the proposed scheme is given by

\[
R_{sum} = \sum_{k=1}^{K} \left( \min (R_{B \to R,k} (P_{B,k}), R_{R \to M,k} (P_{R,k}))) + \min (R_{M \to R,k} (P_{M,k}), R_{B \to R,k} (P_{B,k}))) \right). \tag{42}
\]

It is difficult to derive a closed-form expression for the optimal power allocation, even in the high SNR regime. In the following, we assume equal power allocation, i.e., \( P_{B,k} = P_B / K, P_{R,k} = P_R / K, \forall k \), which in general gives a lower-bound of achievable sum-rate. We next show that the proposed scheme with equal power allocation can asymptotically achieve the cut-set bound (41) under certain conditions.

To start with, we consider the situation that the transmission rate of the BS-MS link is bottle-necked by the BS-RS link. For the cut-set bound (41), it is not difficult to see that the achievable sum-rate of the BS-RS link is no greater than the relayed RS-MS link if the transmit power of the BS satisfies

\[
P_B \leq \rho_B P_R, \tag{43}
\]
where $\rho_B = \frac{K}{\prod_{k=1}^{K} (\lambda_{RM,k}/\lambda_{BR,k})}$. On the other hand, for the proposed scheme with equal power allocation, the transmission rate of the $k$-th spatial stream from the BS to the RS is less than or equal to that from the RS to the $k$-th MS if
\[
P_R \leq \rho_{B,k} P_R. \tag{44}
\]
where $\rho_{B,k} = |r_{RM}(q_k, q_k)|^2 / |r_{BR}(k, k)|^2$. We will show that if both (43) and (44) hold for all the $K$ spatial streams, i.e., $P_R \leq \min (\rho_B, \min_k \rho_{B,k} P_R)$, then the proposed scheme achieves the sum-rate cut-set bound of the BS-to-MS link in the high SNR regime. Similarly, the proposed scheme asymptotically achieves the sum-rate cut-set bound if the data transmission of the BS-to-MS link is bottlenecked by the RS-MS link. Furthermore, it can be shown that the cut-set bound of the MS-to-BS link can be achieved in the high SNR regime if the data transmission from the MSs to the BS is bottlenecked by either the MS-RS link or the RS-BS link. Following these lines, we define the conditions C1-C4 as follows:

- **C1:** $P_B \leq \min (\rho_B, \min_k \rho_{B,k} P_R)$;
- **C2:** $P_B \geq \max (\rho_B, \max_k \rho_{B,k} P_R)$;
- **C3:** $P_R \leq \min \left( \rho_M \prod_{k=1}^{K} P_{M,k}^{1/K} \right)$, $\min_k \rho_{M,k} P_{M,k}$;
- **C4:** $P_R \geq \max \left( \rho_M \prod_{k=1}^{K} P_{M,k}^{1/K} \right)$, $\max_k \rho_{M,k} P_{M,k}$,

where $\rho_M = \sqrt{\prod_{k=1}^{K} (\lambda_{RM,k}/\lambda_{RB,k})}$, and $\rho_{M,k} = |r_{MR}(k, k)|^2 / |r_{RB}(q_k, q_k)|^2$. Then we can establish that:

**Theorem 2:** The proposed scheme achieves the cut-set bound if one of the conditions C1-C2 holds and one of the conditions C3-C4 holds for given DPC order $\Phi$.

**Proof:** See Appendix A.

Note that the conditions C1-C4 depend on the DPC order (i.e. different encoing order $\Phi$ results in different conditions). Theorem 2 states that the proposed scheme is able to asymptotically achieve the sum capacity of the MIMO cTWRC for arbitrary DPC order, as long as the corresponding C1-C4 hold so that the data exchanges between the BS and the MSs are simultaneously constrained by either the BS-RS link or the MS-RS link. In contrast, for the preceding and decoding scheme proposed by Yang in [20], it is required that the SNR from the BS to RS should be large enough compared to the SNR from the RS to MSs such that the rate-loss due to precoding at the BS is negligible. Hence, the Yang’s scheme in [20] can only asymptotically achieve the sum capacity when the data exchanges are simultaneously bottlenecked by the MS-RS link. For the proposed novel precoding/decoding design, only unitary transforms are involved for the channels, and interference-free PNC can be performed with QR decomposition based SIC at the relay [26]. As a result, the symmetry between the BS-RS and MS-RS links is no longer a prerequisite to achieve near-capacity performance for the proposed scheme.

**V. Optimal Power Allocation**

We have shown that, under certain conditions, the proposed two-way relaying scheme achieves the sum-capacity of the MIMO cTWRC with equal power allocation in the high SNR regime. To fully exploit the potential of the proposed scheme, we next investigate the optimal power allocation at the BS and RS to maximize the weighted sum-rate of the MIMO cTWRC at general SNR (which includes the sum-rate in (42) as a special case where all weights are equal). We will show that this optimization problem can be formulated as a monotonic program and can be solved by a polyblock outer approximation algorithm.

It is clear that, the optimal power allocation for each MS is to transmit with maximum power $P_{M,k}$. Let $P_B := [P_{B,1}, \ldots, P_{B,K}]^T$ and $P_R := [P_{R,1}, \ldots, P_{R,K}]^T$ be the power allocation profiles at the BS and RS, respectively; and denote $P := [P_B^T, P_R^T]^T$. Consider the weighted sum-rate maximization problem:

\[
\max_P \sum_{k=1}^{K} (w_{B,k} R_{B,k} + w_{M,k} R_{M,k}) \tag{45a}
\]

s. t. \[\sum_{k=1}^{K} P_{B,k} \leq P_B, \quad \sum_{k=1}^{K} P_{R,k} \leq P_R. \tag{45b}\]

where $w_{B,k}$ and $w_{M,k}$ denote the weights assigned to the $k$-th data stream at the BS and the data stream of the $k$-th MS, respectively.

The problem (45) is not a convex problem. Yet, it can be reformulated as a monotonic program, which can be efficiently solved by a polyblock outer approximation method [22]. Using $\log(x)^+ = \log(1 + (x - 1)^+)$, the achievable rate $R_{B,k}$ in (40) can be expressed as
\[
R_{B,k} = \frac{1}{2} \log(1 + SNR_{BM,k}(P)), \tag{46}
\]
where
\[
SNR_{BM,k}(P) = \min \left( \frac{|r_{BR}(k, k)|^2 P_{B,k}}{\sigma^2} - 1 \right)^+, \tag{47}
\]

Similarly, $R_{M,k}$ in (40) can be expressed as
\[
R_{M,k} = \frac{1}{2} \log(1 + SNR_{MB,k}(P)), \tag{48}
\]
where
\[
SNR_{MB,k}(P) = \min \left( \frac{|r_{MR}(k, k)|^2 P_{M,k}}{\sigma^2} - 1 \right)^+, \tag{49}
\]

Then, the weighted sum-rate $R_{ws}$ in (45) can be expressed as
\[
R_{ws} = \sum_{i=1}^{2K} w_i \frac{1}{2} \log(1 + SNR_i(P)). \tag{50}
\]
where $w_i$ is the $i$-th element of the weight vector $w = [w_{B,1}, \ldots, w_{B,K}, w_{M,1}, \ldots, w_{M,K}]^T$, $SNR_i(P) = SNR_{BM,i}(P)$, and $SNR_{K+i}(P) = SNR_{MB,i}(P)$, for $i = 1, \ldots, K$. 

\[\]
Define the set $S := \{ P \mid \sum_{k=1}^{K} P_{R,k} \leq P_B, \sum_{k=1}^{K} P_{R,k} \leq P_R, k = 1, \ldots, K \}$. Introducing an auxiliary vector $z = [z_1, \ldots, z_{2K}]^T$, we can rewrite (53) into

$$
\max_{z \in S} \Gamma(z) := \sum_{i=1}^{2K} w_i \log_2(z_i).
$$

where the feasible set $S := \{ z \mid z_1 \leq z_i \leq z_{n_i} + \text{SNR}_i(P), i = 1, \ldots, 2K, \forall P \in S \}$. Let

$$
G := \{ z \mid 0 \leq z_i \leq z_{n_i} + \text{SNR}_i(P), i = 1, \ldots, 2K, \forall P \in S \}.
$$

It can be shown that $G$ is a compact normal set with nonempty interior [22]. Further $H := \{ z \mid z_i \geq 1, \forall i \}$ is a reverse normal set. Then, (51) becomes a standard MP [22] as

$$
\max \Gamma(z) \quad \text{s.t.} \quad z \in G \cap H.
$$

For the MP in (53), we can use a polyblock outer approximation method to find its global optimal solution [22]. This method has been used for power control [27], multi-cell coordinated beamforming [26], etc. The main idea of the iterative polyblock outer approximation method is to construct a series of outer polyblocks $P_n$ to approximate $G \cap H$. Given any finite set $\mathcal{T}_n = \{ v_i \mid i = 1, \ldots, l \}$, the union of all the sets $\{ z \mid 0 \leq z \leq v_i \}$ is a polyblock with vertex set $\mathcal{T}_n$. A polyblock $P_n$ is an outer polyblock of $S$ if $S \subseteq P_n$ [22]. Usually, the algorithm starts from a one-vertex outer polyblock $P_0$ of $G \cap H$, and a smaller new outer polyblock is constructed in each iteration. A key step in the construction of the new outer polyblock $P_{n+1}$ from $P_n$ is to find the following projection by solving (52):

$$
\theta^n = \max \{ \alpha \mid \alpha z^n \in G \},
$$

where $z^n = \arg \max_{z \in \mathcal{T}_n} \Gamma(z)$ denotes the maximizer among the vertices in $\mathcal{T}_n$. With $\theta^n$, the projection $y^n = \theta^n z^n$ is then the unique point where the halfline from 0 through $z^n$ meets the upperboundary of $G$. From (52), $\theta^n$ can be determined as

$$
\theta^n = \max \{ \alpha \mid \alpha z^n \in G \} = \max \{ \alpha \mid \alpha \leq \min_{i=1,\ldots,2K} \frac{1 + \text{SNR}_i(P)}{z^n_i}, \forall P \in S \}
$$

$$
= \max_{P \in S} \min_{i=1,\ldots,2K} \frac{1 + \text{SNR}_i(P)}{z^n_i}.
$$

With the definition of $\text{SNR}_{BM,k}(P)$ in (47) and $\text{SNR}_{MB,k}(P)$ in (49), the above max-min problem (55) can be decoupled into the following two sub-problems:

**P1:** $\theta_1^n = \max_{P_B} \min_k \frac{1}{z^n_k} \left( 1 + \left( \frac{r_{RM}(k,k)^2 P_{MB,k}}{\sigma^2} - 1 \right) \right)$

s.t. $\sum_{k=1}^{K} P_{R,k} \leq P_B$.  

**P2:** $\theta_2^n = \max_{P_B} \min_k \left( \frac{1}{z^n_k} \left( 1 + \frac{[r_{RB}(k,k)^2 P_{RB,k}]^2}{\sigma^2} \right) \right)$

s.t. $\sum_{k=1}^{K} P_{R,k} \leq P_R$.  

The solutions for two sub-problems P1 and P2 are given in the following lemma.

**Lemma 1:** For P1, let $\theta^n_i = \frac{\sigma^2}{[r_{RB}(i,i)^2], i = 1, \ldots, K$, sort $z^n_i, i = 1, \ldots, K$, as $z^n_0 \leq z^n_2 \leq \ldots \leq z^n_K$, and define $z^n_0 = 0$. Then the optimal $\theta^n_1$ for problem P1 is given by

$$
\theta^n_1 = \max \left( \frac{1}{z^n_k} \left( \frac{1}{\sum_{k=1}^{K} z^n_k} \frac{P_B}{\sigma^2} \right) \right),
$$

where $\ell$ is an integer satisfying $1 \leq \ell \leq K$, and

$$
\frac{1}{z^n_k} \sum_{k=1}^{K} z^n_k \sigma^2_{\ell} \leq P_B < \frac{1}{z^n_k} \sum_{k=1}^{K} z^n_k \sigma^2_{\ell-1}.
$$

Proof: see Appendix B.

With $\theta^n_1$ and $\theta^n_2$ provided by Lemma 1, the optimal $\theta^n$ for (53) is then given by $\theta^n = \min(\theta^n_1, \theta^n_2, \theta^n_3)$, where

$$
\theta^n_3 = \min_{k=1,\ldots,K} \frac{1}{z^n_k} \left( 1 + \left( \frac{r_{RM}(k,k)^2 P_{MB,k}}{\sigma^2} - 1 \right) \right).
$$

Note that $\theta^n_3$ simply follows from $\min_{k=1,\ldots,K} \frac{1}{z^n_k} \left( 1 + \frac{[r_{RB}(i,i)^2 P_{RB,k}]^2}{\sigma^2} \right)$.

Let $z^n$ denote the global optimal solution of (53). For a given accuracy tolerance $\epsilon > 0$, we say that a feasible $z$ is an $\epsilon$-optimal solution if $(1 + \epsilon) \Gamma(z) \geq \Gamma(z^{\text{opt}})$. Based on the max-min solution to (53), we propose the following algorithm to find an $\epsilon$-optimal solution to (53).

**Algorithm 1:** for weighted sum-rate maximization

**Initialize:** select an accuracy level $\epsilon > 0$, let $n = 0$, and CurrentBestValue(CBV) = $-\infty$. Initialize vertex set $\mathcal{T}_0$ with a selected outer vertex to construct the initial outer polyblock $P_0$.

**Repeat:** 1. Finds $z^n \in \mathcal{T}_n$ that maximizes $\Gamma(z)$ and solve (53) to obtain $\theta^n$ and $y^n = \theta^n z^n$.

2. If $y^n \in H$ and $\Gamma(y^n) > \text{CBV}$, then $\text{CBV} = \Gamma(y^n)$ and $z = y^n$.

3. Let $z^n(i) = z^n - (z^n - y^n) e_i, i = 1, \ldots, 2K$, where $z^n$ and $y^n$ are the $i$-th entry of $z^n$ and $y^n$, respectively; i.e., $z^n(i)$ is obtained by simply replacing the $i$-th entry of $z^n$ by $y^n(i)$. Clearly, $y^n \leq z^n \leq z^n(i) \leq z^n$.

4. Let $\mathcal{T}_{n+1} = \{ z^n \cup \{ z^n(i) \} \} \cap H$; i.e., obtain a new vertex set $\mathcal{T}_{n+1}$ by replacing the vertex $z^n$ in $\mathcal{T}_n$ with $2K$ new vertices $z^n(i), i = 1, \ldots, 2K$. Further remove from $\mathcal{T}_{n+1}$ any $v_j \in \mathcal{T}_{n+1}$ that satisfying $\Gamma(v_j) \leq \text{CBV}(1 + \epsilon)$. By construction
of \( z^n(i) \), the new outer polyblock \( \mathcal{P}_{n+1} \) with vertex set \( T_{n+1} \) satisfies \( \mathcal{P}_{n+1} \subseteq \mathcal{P}_n \).

5. Set \( n = n + 1 \) and goto Step 1 until \( T_n \) is empty.

\textbf{Output:} \( \bar{z} \) and CBV as the \( \epsilon \)-optimal solution for \( \mathcal{G} \).

Per iteration \( n \) of Algorithm 1, we have \( y^n = \theta^n z^n \in \mathcal{G} \). If \( y^n \in H \) too, we obtain a feasible point \( y^n \in \mathcal{G} \cap H \). In this case, we update CBV \( = \max \{ \text{CBV}, \Gamma(y^n) \} \). This implies CBV is the current best value so far, and the corresponding \( \bar{z} \) is the current best solution for \( \mathcal{G} \).

Observe that for any \( v_j \in T_{n+1} \) satisfying \( \Gamma(v_j) \leq \text{CBV}(1 + \epsilon) \), we have \( (1 + \epsilon) \text{CBV} \geq \Gamma(y), \forall y \in [0, v_j] \), due to monotonicity of \( \Gamma(\cdot) \). Hence, \( v_j \) can be removed from \( T_{n+1} \) for further consideration since \( \bar{z} \) will be the desired \( \epsilon \)-optimal solution if \( z^n_{\text{opt}} \in [0, v_j] \).

Algorithm 1 is essentially a “smarter” branch-and-bound method. A key requirement for its guaranteed convergence is that \( z \in \mathcal{G} \cap H \) is lower bounded by a strictly positive vector. Since \( z \geq 1 > 0 \) in \( \mathcal{G} \), it readily follows from [22] Theorem 1 that

**Proposition 1:** Algorithm 1 globally converges to an \( \epsilon \)-optimal solution for \( \mathcal{G} \).

Before leaving this section, we emphasize that the sum-rate optimization in this section works for a fixed DPC order at the relay, i.e., the permutation matrix \( \Phi \) in (10) is fixed. The global optimum will be obtained by enumerating all possible DPC orders.

VI. FURTHER DISCUSSIONS

The proposed two-way relaying scheme can be readily extended to the MIMO cTWRC with each MS equipped with multiple antennas. In this case, a straightforward approach is to assume that the antennas at each MS are not allowed to cooperate, i.e., each antenna at the MS can be treated as a virtual “user” (though the virtual users associated with a common MS share the power budget of this MS). Then, the proposed transceiver and relaying scheme developed in Section III direct applies. Furthermore, as antenna cooperation is physically allowed, we can introduce an extra linear precoder at each MS. With an appropriate precoding design, the system performance can be further enhanced.

The proposed scheme can be extended to a more general antenna setup in which the number of antennas at the BS and the RS may be unequal. In this case, the total number of data streams that can be supported by the two-way relaying network is given by \( N_s = 2 \min(N_B, N_R, \sum_{k=1}^{K} N_{M,k}) \) with \( N_s/2 \) streams in each way, where \( N_{M,k} \) represents the number of antennas at the \( k \)-th MS. When the number of antennas at the BS (or RS) is larger than \( N_s/2 \) (implying that this node has antenna redundancy in supporting \( N_s/2 \) independent data streams), an extra linear precoder can be applied at the BS (or RS) for beamforming. The detailed precoding design for a general antenna setup will be an interesting direction for future research.

VII. NUMERICAL RESULTS

In this section, numerical results are presented to demonstrate the performance of the proposed two-way relaying scheme. It is assumed that all elements of \( H_{BR} \) and \( h_{k,R}, k = 1, \ldots, K \), are independently drawn from a circularly symmetric complex Gaussian distribution with zero mean and unit variance. The channel matrices remain constant over the two phases and are reciprocal, i.e., \( H_{RB} = H_{BR}^T, h_{R,k} = h_{k,R}^T, \forall k \). All the MSs have the same power budget, i.e., \( P_{M,k} = P_M, \forall k \), and all the nodes have the same noise variance \( \sigma^2 \). The SNRs are defined as \( SNR_{X,R} = P_X/\sigma^2 \) and \( SNR_{R,X} = P_R/\sigma^2 \), where \( X \in \{B, M\} \). The SNR in the following figures is defined as \( SNR = SNR_{MB} = P_M/\sigma^2 \). Unless otherwise specified, it is also assumed that \( N_M = 1, N_B = N_R = K \), and each MS exchanges one data stream with the BS.

Fig. 2 shows the sum-rate of the proposed scheme for a MIMO cTWRC with \( K = 4 \) single-antenna MSs. Every node in the network has the same power budget, i.e., \( P_B = P_R = P_M \). Achieved sum-rate of the AF-based interference alignment (AF-IA) scheme in (16) and the scheme proposed by Yang in [20] are included in Fig. 2 for comparison. For the proposed scheme, we also consider a low-complexity sub-optimal alternative in which the DPC encoding order at the RS is fixed as \( \Phi = I_K \) and equal power allocations are used for all spatial streams. A similar sub-optimal alternative for Yang’s scheme is also shown, in which the BS uses ZF-based precoding, the RS employs fixed-order DPC and the transmit
powers are the same for all data streams. It is shown that AF-IA scheme in [16] suffers from the noise propagation problem and the achievable sum-rate is much lower than the other two schemes. The proposed scheme with optimal permutation \( \Phi \) and power allocation can approach the cut-set bound as the SNR increase, the gap is only 0.5 bps/Hz when \( SNR \geq 25 dB \). Compared with the sub-optimal scheme, there is about 1 bps/Hz gain for the optimal solution. It can be also seen that sum-rate gain is about 3 bps/Hz for the proposed scheme as compared with Yang’s scheme.

Fig. 3 compares the sum-rate performance of the proposed scheme and the Yang’s scheme [20] when there are \( K = 8 \) and \( K = 16 \) single-antenna MSs in the MIMO cTWRC. Note that there are \( K! \) possible DPC encoding orders in total at the RS. Even for a moderate \( K \), it is too time-consuming to find the optimal DPC encoding order and the corresponding optimal power allocations. Instead, we search over \( 5 \times 10^4 \) randomly chosen DPC encoding orders and use equal power allocation to determine the best DPC order. Even with this suboptimal approach, we see from Fig. 3 that the proposed scheme performs only 0.5 dB away from the cut-set bound. Compared with Yang’s scheme, we see that the performance gain increases as the number of MSs increases. Particularly, when there are 16 users in the network, more than 7 dB power gain is observed for the proposed scheme.

To further demonstrate the advantages of the proposed scheme, Fig. 4 illustrates the achievability of the cut-set bound under different \( SNR_{BR} \). The transmit power of the RS is 10 dB larger than the MSs, and the SNRs are fixed to be \( SNR_{MR} = 30 \) dB, \( SNR_{RM} = SNR_{RB} = 40 \) dB. It can be seen that the proposed scheme is able to achieve the cut-set bound when \( SNR_{BR} \geq 20 \) dB. However, for Yang’s scheme, it is required that \( SNR_{BR} \) is large enough compared to \( SNR_{RM} \) to meet the condition for the rate-loss due to precoding at the BS to be negligible.

The weighted sum-rate performance of the proposed scheme is shown in Fig. 5. We assume that the priority of the data transmission from the BS to the MSs are higher than that from the MSs to the BS. The weights are chosen as \( w_{B,k} = 0.4 \) and \( w_{M,k} = 0.1, \forall k \). The weighted sum-rate cut-set bound is derived using a similar monotonic optimization approach as in Section V. Again, the proposed scheme is able to approach the weighted sum-rate cut-set bound when the SNR is higher than 25 dB.

Finally, Fig. 6 shows the performance of the MIMO cTWRC with \( K = 2 \) multi-antenna MSs. The number of antennas at the BS and RS is set to \( N_B = N_R = 4 \) and each MS is equipped with \( N_M = 2 \) antennas. Each MS exchanges two data streams with the BS; at any of the two MSs, no extra precoding is performed. The transmit power of the RS is set bound when \( SNR \geq 30 \) dB. Compared with the sub-optimal scheme, there is about 1 bps/Hz gain for the optimal solution. It can be also seen that sum-rate gain is about 3 bps/Hz for the proposed scheme as compared with Yang’s scheme.

Fig. 4. Sum-rate vs. \( SNR_{BR} \) for MIMO cTWRC with \( K = 2 \) single-antenna MSs, \( SNR_{MR} = 30 \) dB, \( SNR_{RB} = SNR_{RM} = 40 \) dB.

Fig. 5. Weighted sum-rate performance of a MIMO cTWRC with single-antenna MSs, \( P_B = P_R = P_M, w_{B,k} = 0.4, w_{M,k} = 0.1, \forall k \).

Fig. 6. Performance of a MIMO cTWRC with \( K = 2 \) multi-antenna MSS. Each MS is equipped with \( N_M = 2 \) antennas, \( N_B = N_R = 4 \).

VIII. CONCLUSION

We proposed a novel two-way DF relaying scheme for the MIMO cTWRC. A non-linear lattice precoder was proposed at the BS to pre-compensate for the inter-stream interference, which enables efficient interference-free lattice decoding at the relay. The sufficient conditions for the achievability of the sum-rate cut-set bound were derived. The optimal power allocation for the proposed scheme was also obtained through monotonic programming. Numerical results demonstrated that the proposed scheme outperforms the existing alternatives and closely approaches the cut-set bound in the high SNR regime.
**APPENDIX A: PROOF OF THEOREM 2**

Consider the proposed scheme with equal power allocation, i.e., $P_{B,k} = P_B/K$ and $P_{R,k} = P_R/K, \forall k$. In the high SNR regime, the achievable rates $R_{B\rightarrow R,k}(P_{B,k})$ and $R_{R\rightarrow M,k}(P_{R,k})$ can be expressed as

$$R_{B\rightarrow R,k}(P_{B,k}) = \frac{1}{2} \log \left( \frac{[r_{BR}(k,k)]^2 P_{B,k}}{\sigma^2} \right)$$

$$\approx \frac{1}{2} \log \left( \frac{[r_{BR}(k,k)]^2 P_{B,k}}{\sigma^2} \right),$$

and

$$R_{R\rightarrow M,k}(P_{R,k}) = \frac{1}{2} \log \left( \frac{[l_{RM}(q_k,q_k)]^2 P_{R,k}}{\sigma^2} \right)$$

$$\approx \frac{1}{2} \log \left( \frac{[l_{RM}(q_k,q_k)]^2 P_{R,k}}{\sigma^2} \right).$$

We first consider Condition C1. If the inequalities in C1 hold, we obtain

$$P_B \leq \frac{[l_{RM}(q_k,q_k)]^2 P_{R}}{[r_{BR}(k,k)]^2}.$$

Then

$$R_{B\rightarrow R,k}(P_{B,k}) = \frac{1}{2} \log \left( \frac{[r_{BR}(k,k)]^2 P_{B}}{K \sigma^2} \right)$$

$$\leq \frac{1}{2} \log \left( \frac{[l_{RM}(q_k,q_k)]^2 P_{R}}{K \sigma^2} \right) = R_{R\rightarrow M,k}(P_{R,k}).$$

i.e., the achievable rate for the $k$-stream of the BS-RS link is lower than or equal to the link from the RS to the $k$-th MS. Consequently, the sum-rate of the proposed scheme with equal power allocation for the BS-to-MS links is given by

$$R_{\text{sum},B\rightarrow M} = \sum_{k=1}^{K} R_{B\rightarrow R,k}(P_{B,k})$$

$$= \sum_{k=1}^{K} \frac{1}{2} \log \left( \frac{[r_{BR}(k,k)]^2 P_{B}}{K \sigma^2} \right)$$

$$= \frac{1}{2} \log \det \left( \frac{P_B}{K \sigma^2} H_{BR} H_{BR}^H \right)$$

$$= \sum_{k=1}^{K} \frac{1}{2} \log \left( \frac{\lambda_{BR,k} P_{B}}{K \sigma^2} \right).$$

We now consider the cut-set bound in [1]. Note that C1 implies $P_B \leq \rho_B P_R$, the sum-rate of the BS-to-MS link is given by

$$R_{\text{sum},\text{cs},B\rightarrow M} = \sum_{k=1}^{K} R_{B,k} = \sum_{k=1}^{K} \frac{1}{2} \log \left( \frac{\lambda_{BR,k} P_{B}}{K \sigma^2} \right).$$

From (63) and (66), we see that the gap between the proposed scheme and the cut-set upper bound of the BS-to-MS link vanishes as $\frac{P_B}{\sigma^2} \rightarrow +\infty$. Therefore, the proposed scheme asymptotically achieves the cut-set upper bound when condition C1 holds. Similarly, it can be shown that when C2 holds, the proposed scheme asymptotically achieves the cut-set upper bound of the BS-to-MS link as $\frac{P_B}{\sigma^2} \rightarrow +\infty$.

Further, it can also be shown in a similar way that the sum-rate gap between the proposed scheme and the cut-set bound for the MS-to-BS link vanishes as $\frac{P_{B,k}}{\sigma_{\pi k}^2} \rightarrow +\infty$ and $\frac{P_{R,k}}{\sigma_{\pi k}^2} \rightarrow +\infty$ when either C3 or C4 holds. This concludes the proof of Theorem 2.

**APPENDIX B: PROOF OF LEMMA 1**

For the optimization problem P1, it is clear that $\sum_{k=1}^{K} P_{B,k} = P_B$ holds for the optimal $\theta_1$. Then P1 can be written as:

$$\theta_1^n = \max_{P_B} \min_{P_{\pi k}} \sum_{k=1}^{K} \frac{1}{2} \log \left( \frac{P_{B,k}}{\sigma_{\pi k}^2} \right) + \frac{1}{2} \log \left( \frac{P_{\pi k}}{\sigma_{\pi k}^2} \right) - \frac{1}{2} \log \left( \frac{P_{\pi k} - 1}{\sigma_{\pi k}^2} \right)$$

s.t. $\sum_{k=1}^{K} P_{B,k} = P_B,$

where $\sigma_{\pi k}^2 = \sigma_{\pi k}^2 / [r_{BR}(k,k)]^2$.

Sort $\left\{ \frac{z_{\pi k}}{P_{\pi k}} \right\}_{k=1}^{K}$ such that $z_{\pi 1}^n \leq z_{\pi 2}^n \leq \ldots \leq z_{\pi K}^n$ and define $z_{\pi 0}^n = 0$. Note that $\frac{1 + (\frac{P_{\pi k}}{\sigma_{\pi k}^2} - 1)}{z_{\pi k}^n}$ depends on the value of $P_B$, there are two possible cases:

Case A: $\theta_1^n = \frac{1}{z_{\pi 1}^n}$, where $1 \leq \ell \leq K$. The corresponding power allocation satisfies

$$P_{B,\pi_k} = 0, \quad \frac{1 + (\frac{P_{B,\pi_k}}{\sigma_{\pi_k}^2} - 1)}{z_{\pi_k}^n} = \frac{1}{z_{\pi_1}^n}, \quad k = 1, \ldots, \ell - 1,$$

$$0 \leq P_{B,\pi_k} \leq z_{\pi_k}^n, \quad 1 + \frac{P_{B,\pi_k}}{\sigma_{\pi_k}^2} - 1 = \frac{1}{z_{\pi_k}^n}, \quad k = \ell + 1, \ldots, K.$$
From (70b), we have $P_{B,\pi_k} = \frac{z_n^2}{\pi_k} \sigma^2 n \theta_k^0$, for $k = \ell, \ldots, K$. Then, $P_B = \sum_{k=1}^K P_{B,k} = \sum_{k=1}^K \frac{z_n^2}{\pi_k} \sigma^2 n \theta_k^0$, and $\theta_k^0 = \frac{\sigma^2}{\pi_k} n \theta_k^0$. From $\frac{1}{\pi_k} < \theta_k^0 < \frac{1}{\pi_{k-1}}$, we obtain the constraints on $P_B$ as

$$\frac{1}{\pi_k} \sum_{k=1}^K \frac{z_n^2}{\pi_k} \sigma^2 n < P_B < \frac{1}{\pi_{k-1}} \sum_{k=1}^K \frac{z_n^2}{\pi_k} \sigma^2 n.$$  

(71)

Combining the above two cases we arrive at (58).

For $P_2$, the optimal solution must satisfy

$$\frac{1}{\pi_k} \left( 1 + \left[ \frac{[R_{RM}(q_k,k)]^2}{\sigma^2} \right] P_{R,k} \right) \geq \theta_k^0.$$  

(72)

which implies that $P_{R,k} \geq \frac{1}{\pi_k} \left( \frac{(\theta_k^0)^2 - 1}{\pi_k} \right) \sigma^2 n$. Similarly, we have

$$P_{R,k} \geq \frac{(\theta_k^0)^2 - 1}{\pi_k} \sigma^2 n.$$

(73)

As a result, the maximum $\theta_k^0$ can be determined by solving the equation (59) via a simple bisection search.

REFERENCES

[1] C. E. Shannon, “Two-way communication channels,” in Proc. 4th Berkeley Symp. Math. Stat. Prob., Berkely, CA, vol. 1, pp. 611-644, 1961.

[2] S. Zhang, S.-C. Liew, and P. P. Lam, “Hot topic: Physical-layer network coding,” in Proc. ACM MobiCom, Los Angeles, CA, 2006.

[3] B. Nazer and M. Gastpar, “Efficient protocols for half-duplex fading relay channels,” IEEE J. Sel. Areas Commun., vol. 25, no. 2, pp. 379-389, Feb. 2007.

[4] X. Yuan, Y. Liao, and Y. Li, ““Two-way communication channel,”” ” IEEE Trans. WIRELESS COMMUN., vol. 1, no. 1, pp. 140-147, Jan. 2007.

[5] Z. Ding, I. Krikidis, J. Thompson, and K. K. Leung, “Physical layer network coding and precoding for the two-way relay channel in cellular systems,” IEEE Trans. Signal Process., vol. 59, no. 2, pp. 696-712, 2011.

[6] C. Sun, C. Yang, Y. Li, and B. Vucetic, “Transceiver design for multiuser multi-antenna two-way relay cellular systems,” IEEE Trans. Commun., vol. 60, no. 10, pp. 2893-2903, Oct. 2012.

[7] E. Chiu and V. K. N. Lau, “Cellular multiuser two-way MIMO AF relaying via signal space alignment: Minimum weighted SINR maximization,” IEEE Trans. Signal Process., vol. 60, no. 9, pp. 4864-4873, Sep. 2012.

[8] M. Gan, Z. Ding, and X. Dai, “Application of analog network coding to MIMO two-way relay channel in cellular systems,” IEEE Trans. Signal Process., vol. 60, no. 7, pp. 3793-3816, Sept. 2012.

[9] H. J. Yang, Y. Choi, N. Lee, and A. Paulraj, “Achieving sum-rate of MU-MIMO cellular two-way relay channels: Lattice code-aided linear precoding,” IEEE J. Sel. Areas Commun., vol. 3, no. 8, pp. 1304-1318, 2012.

[10] F. Wang, X. Yuan, S. C. Liew, and Y. Li, “Bidirectional cellular relay network with distributed relaying,” IEEE J. Sel. Areas Commun. Special Issue on Virtual MIMO, vol. 31, no. 10, pp. 2082-2098, Oct. 2013.

[11] H. Tuy, “Monotonic optimization: Problems and solution approaches,” SIAM J. Optim., vol. 11, no. 2, pp. 464-494, 2000.

[12] D. Tse and D. Yamamoto, “A nested linear/lattice codes for structured multiterminal binnning,” IEEE Trans. Inf. Theory, vol. 48, no. 6, pp. 2500-2524, June 2002.

[13] U. Erez and R. Zamir, “Achieving 1/2 log(1 + SNR) on the AWGN channel with lattice encoding and decoding,” IEEE Trans. Inf. Theory, vol. 50, no. 10, pp. 2293-2314, Oct. 2004.

[14] U. Erez, S. Litsyn, and R. Zamir, “Lattices which are good for (almost) everything,” IEEE Trans. Inf. Theory, vol. 51, no. 10, pp. 3401-3416, Oct. 2005.

[15] G. Caire and S. Shamai, “On the achievable throughput of a multiantenna Gaussian broadcast channel,” IEEE Trans. Inf. Theory, vol. 49, no. 7, pp. 1691-1706, July 2003.

[16] L. P. Qian, Y. J. Zhang, and J. Huang, “MAPEL: achieving global optimality for a non-convex wireless power control problem,” IEEE Trans. Wireless Commun., vol. 8, no. 3, pp. 1553-1563, Mar. 2009.

[17] W. Utschick and J. Brehmer, “Monotonic optimization framework for coordinated beamforming in multicell networks,” IEEE Trans. Signal Process., vol. 60, no. 4, pp. 1899-1909, 2012.