We present the first QCD spectral sum rules analysis of the $SU(3)$ breaking parameter $\xi$ and an improved estimate of the renormalization group invariant (RGI) bag constant $\hat{B}_{B_d}$ both entering into the $B^0_{d,s}$-$\bar{B}^0_{d,s}$ mass-differences. The averages of the results from the Laplace and moment sum rules to order $\alpha_s$ are $f_B \sqrt{\hat{B}_B} \simeq (247 \pm 59)$ MeV and $\xi \equiv f_B \sqrt{\hat{B}_B} / f_B \sqrt{\hat{B}_B} \simeq (1.18 \pm 0.03)$, in units where $f_B = 130.7$ MeV. Combined with the experimental data on the mass-differences $\Delta M_{d,s}$, one obtains the constraint on the CKM weak mixing angle $|V_{td}/V_{ts}|^2 \geq 0.9(1.1)$. Alternatively, using the weak mixing angle from the analysis of the unitarity triangle and the data on $\Delta M_d$, one predicts $\Delta M_u = 18.0(2.2)$ ps$^{-1}$ in agreement with the present experimental lower bound and within the reach of Tevatron 2.

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1. Introduction

$B^0_{d,s}$ and $\bar{B}^0_{d,s}$ are not eigenstates of the weak Hamiltonian, such that their oscillation frequency is governed by their mass-difference $\Delta M_q$. The measurement by the UA1 collaboration [1] of a large value of $\Delta M_d$ was the first indication of the heavy top quark mass. In the SM, the mass-difference is approximately given by

$$\Delta M_q \simeq \frac{C^2}{4\pi^2} M_W^2 |V_{tq}|V_{tb}|^2 S_0 \left( \frac{m_t^2}{M_W^2} \right) \eta_B C_B(\nu) \times \frac{1}{2 M_{B_q}} \langle \bar{B}^0_q | O_q(x) | B^0_q \rangle,$$

where the $\Delta B = 2$ local operator $O_q(x)$ is defined as

$$O_q(x) \equiv (\bar{b}_{\gamma\mu} L_q) (\bar{b}_{\gamma\mu} L_q),$$

with $L \equiv (1 - \gamma_5)/2$ and $q \equiv d, s$. $S_0$, $\eta_B$ and $C_B(\nu)$ are short distance quantities calculable perturbatively. (Here we are following the notation of Ref. [2].) On the other hand, the matrix element $\langle \bar{B}^0_q | O_q | B^0_q \rangle$ requires non-perturbative QCD calculations, and is usually parametrized for as

$$\langle \bar{B}^0_q | O_q | B^0_q \rangle \simeq \frac{4 f_{B_q}^2}{3} M_{B_q}^2 B_{B_q}. \quad(3)$$

$f_{B_q}$ is the $B_q$ decay constant normalized as $f_\pi = 130.7$ MeV, and $B_{B_q}$ is the so-called bag parameter which is $B_{B_q} = 1$ if one uses a vacuum saturation of the matrix element and equal to $3/4$ in the large $N_c$ limit. From Eq. (1), it is clear that the measurement of $\Delta M_d$ provides one of the CKM mixing angles $|V_{td}|$ if one uses $|V_{tb}| \simeq 1$. One can also extract this quantity from the ratio

$$\frac{\Delta M_s}{\Delta M_d} = \frac{|V_{ts}|^2 M_{B_d} \langle \bar{B}^0_q | O_q | B^0_q \rangle}{|V_{td}| \langle \bar{B}^0_q | O_q | B^0_q \rangle} \simeq \frac{|V_{ts}|^2 M_{B_d}}{|V_{td}| M_{B_d} \xi^2}, \quad(4)$$

since in the SM with three generations and unitarity constraints, $|V_{ts}| \simeq |V_{cb}|$. Here

$$\xi \equiv \sqrt{g_s/g_d} \equiv \frac{f_{B_d} \sqrt{\hat{B}_{B_d}}}{f_B \sqrt{\hat{B}_B}}.$$  

The great advantage of Eq. (4) compared with the former relation in Eq. (1) is that in the ra-
tio, different systematics in the evaluation of the matrix element tends to cancel out, thus providing a more accurate prediction. However, unlike \( \Delta M_d = 0.479(12) \, \text{ps}^{-1} \), which is measured with a good precision \( \text{[3]} \), the determination of \( \Delta M_s \) is an experimental challenge due to the rapid oscillation of the \( B^0_s-B^0_s \) system. At present, only a lower bound of 13.1 \( \text{ps}^{-1} \) is available at the 95\% CL from experiments \( \text{[4]} \), but this bound already provides a strong constraint on \( |V_{td}| \).

2. Two-point function sum rule

Ref. \( \text{[3]} \) has extended the analysis of the \( K^0-\bar{K}^0 \) systems of Ref. \( \text{[1]} \), using two-point correlator of the four-quark operators into the analysis of the quantity \( f_{B \overline{B}} \) which governs the \( B^0 \rightarrow \overline{B}^0 \) mass difference. The two-point correlator defined as

\[
\psi_H(q^2) \equiv i \int d^4x \, e^{iqx} \left( 0 \mid T \mathcal{O}_q(x) \mathcal{O}_q(0) \right) \left( 0 \right),
\]  

(6)

is built from the \( \Delta B = 2 \) weak operator given in Eq. \( \text{(2)} \). The two-point function approach is very convenient due to its simple analytic properties which are not the case of approach based on three-point functions \( \text{[3]} \). However, it involves non-trivial QCD calculations which become technically complicated when one includes the contributions of radiative corrections due to non-factorizable diagrams. These perturbative (PT) radiative corrections due to factorizable and non-factorizable diagrams have been already computed in Ref. \( \text{[6]} \) (referred as NP), where it has been found that the factorizable corrections are large while the non-factorizable ones are negligibly small. NP analysis has confirmed the estimate in Ref. \( \text{[3]} \) from lowest order calculations, where under some assumptions on the contributions of higher mass resonances to the spectral function, the value of the bag parameter \( B_B \) has been found to be

\[
B_B (4M_B^2) \simeq (1 \pm 0.15).
\]

(7)

This value is comparable with the one \( B_{B_d} = 1 \) from the vacuum saturation estimate, which is expected to be a quite good approximation due to the relative high-scale of the \( B \)-meson mass. Equivalently, the corresponding RGI quantity is

\[
\hat{B}_{B_d} \simeq (1.5 \pm 0.2),
\]

(8)

where we have used the relation

\[
\hat{B}_{B_d} = B_{B_d}(\nu)\alpha_s \left \{ 1 + \left[ \frac{5165}{12696} \left( \frac{\alpha_s}{\pi} \right) \right] \right \},
\]

(9)

1 For detailed criticisms, see \( \text{[6]} \).

with \( \gamma_0 = 1 \) being the anomalous dimension of the operator \( \mathcal{O}_q \) and \( \beta_1 = -23/6 \) for 5 flavours. \( \nu \) is the subtraction point. The NLO corrections have been obtained in the \( \overline{\text{MS}} \) scheme \( \text{[3]} \). We have also used the value of the bottom quark pole mass \( \text{[3]} \) \( M_b = (4.66 \pm 0.06) \, \text{GeV} \).

In the following, we study \( \text{(for the first time)} \), from the QCD spectral sum rules (QSSR) method \( \text{[3]} \), the \( SU(3) \) breaking effects on the ratio \( \xi \) defined previously in Eq. \( \text{(3)} \), where a similar analysis of the ratios of the decay constants has given the values \( \text{[4]} \)

\[
\frac{f_{D_s}}{f_D} \simeq 1.15 \pm 0.04, \quad \frac{f_{B_s}}{f_B} \simeq 1.16 \pm 0.04.
\]

(11)

We shall also improve the previous result of Ref. \( \text{[4]} \) on \( B_{B_d} \) by the inclusion of the \( B_qB^*_q \) and \( B^*_qB^*_q \) resonances into the spectral function.

3. Inputs for the sum rule analysis

We shall be concerned here with the two-point correlator defined in Eq. \( \text{(3)} \). The hadronic part of the spectral function can be conveniently parametrized using the effective realization \( \text{[3]} \)

\[
\mathcal{O}_q^{\text{eff}} = \frac{1}{3} g_q \overline{q}_B B^0_q \overline{n} \gamma_5 B_0^q + ..., \]

(12)

where \( ... \) indicates higher resonances and \( g_q \equiv f_{B_{B_q}}^2 B_{B_q} \). Retaining the \( BB, BB^* \) and \( B^* B^* \) resonances and parametrizing the higher resonances with the QCD continuum contribution, it gives

\[
\frac{1}{\pi} \text{Im} \psi_0^q(t) = \frac{2}{9} \left( \frac{g_q}{8\pi} \right)^2 t^2 \left[ \left( 1 - \frac{2M_{B_q}^2}{t} \right)^2 \sqrt{1 - \frac{4M_{B_q}^2}{t} \theta(t - 4M_{B_q}^2)} + \left( 1 - \frac{4M_{B_q}^2}{t} + \frac{12M_{B_q}^2}{t^2} \right) \times \right.
\]

\[
\left. \sqrt{1 - \frac{4M_{B_q}^2}{t} \theta(t - 4M_{B_q}^2)} + 2\lambda^{3/2} \left( 1, \frac{M_{B_q}^2}{t}, \frac{M_{B_q}^2}{t} \right) \times \theta(t - (M_{B_q} + M_{B_q}^2)^2) \right]
\]

(13)

below the QCD continuum threshold \( t_c \). The function \( \lambda(x, y, z) \) is a phase space factor,

\[
\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2yx - 2yz - 2zx.
\]

(14)

Our preliminary results have been presented in \( \text{[3]} \).
We have used, to a first approximation, the large $M_b$ and vacuum saturation relations:

$$g_q \equiv f_{B_q}^0 B_{B_q} \simeq g_{q^*} \equiv f_{B_q^*}^0 B_{B_q^*}$$

among the couplings. The results $f_{B} \approx f_{B^*}$ have been also obtained from QCD spectral sum rules \cite{4,7}, while the vacuum saturation relation $B_{B_q^*} \simeq B_{B_q}$ is a posteriori expected to be a good approximation as indicated by the result obtained later on in this paper. The short distance expression of the spectral function is obtained using the Operator Product Expansion (OPE) including non-perturbative condensates \cite{11}. The massless ($m_q = 0$) expression for the lowest perturbative and gluon condensate contributions has been obtained in Ref. \cite{4}. Radiative factorizable and non-factorizable corrections to the perturbative graphs in the massless light quark case have been obtained in NP \cite{4}.

4. \textbf{SU(3) breaking contributions}

The lowest order perturbative contribution for $m_s \neq 0$ to the two-point correlator is

$$\frac{1}{\pi} \text{Im} \psi_s^{pert}(t) = \theta(t - 4(M_b + m_s)^2)$$

$$\times \frac{t^4}{1536 \pi^6} \times \int_{(1-\sqrt{3}-\sqrt{5})^2}^{(1+\sqrt{3})^2} dz \int_{(\sqrt{3}+\sqrt{5})^2}^{((1+\sqrt{3})^2} dz \int_{\sqrt{u} \chi(1, z, u \times \lambda^{1/2}(1, z, u) \lambda^{1/2}(1, \delta, \delta') \lambda^{1/2}(1, \delta, \delta') \lambda^{1/2}(1, \delta, \delta')$$

$$\times \left[ 4 f \left( \frac{\delta}{z} \frac{\delta'}{z} \right) f \left( \frac{\delta}{u} \frac{\delta'}{u} \right) - 2 f \left( \frac{\delta}{z} \frac{\delta'}{z} \right) g \left( \frac{\delta}{u} \frac{\delta'}{u} \right) - 2 g \left( \frac{\delta}{z} \frac{\delta'}{z} \right) f \left( \frac{\delta}{u} \frac{\delta'}{u} \right) + \frac{(1 - z - u)^2}{zu} g \left( \frac{\delta}{z} \frac{\delta'}{z} \right) g \left( \frac{\delta}{u} \frac{\delta'}{u} \right) \right].$$

(16)

Here $\delta \equiv M_b^2/t$ and $\delta' \equiv m_s^2/t$, respectively. The functions $f(x, y)$ and $g(x, y)$ are defined by

$$f(x, y) \equiv 2 - x - y - (x - y)^2,$$

$$g(x, y) \equiv 1 + x + y - 2(x - y)^2.$$

We include the $O(\alpha_s)$ correction from factorizable diagrams by using the results in the $\overline{MS}$ scheme

$$\text{Im} \psi_s^{pert}(t) = \theta(t - 4(M_b + m_s)^2)$$

$$\times \frac{t^4}{6 \pi^4} \int_{(1 - \sqrt{3} - \sqrt{5})^2}^{(1 + \sqrt{3})^2} dz \int_{(\sqrt{3} + \sqrt{5})^2}^{((1 + \sqrt{3})^2} dz \int_{\sqrt{u} \chi(1, z, u \times \lambda^{1/2}(1, z, u) \lambda^{1/2}(1, \delta, \delta') \lambda^{1/2}(1, \delta, \delta') \lambda^{1/2}(1, \delta, \delta')$$

$$\times \left[ \text{Im} \Pi_{\mu \nu}^{pert}(zt) \text{Im} \Pi^{pert}(ut) + \text{Im} \Pi_{\mu \nu}^{pert}(zt) \text{Im} \Pi^{pert}(ut) \right].$$

(19)

Here $\Pi_{\mu \nu}^{pert}(q^2)$ and $\Pi_{\mu \nu}^{pert}(q^2)$ are respectively the lowest and the next-to-leading order QCD contribution to the two point correlator $\Pi_{\mu \nu}(q^2)$ defined by

$$\Pi_{\mu \nu}(q^2) \equiv i \int d^4 x \ e^{iqx} \langle 0 | T(\bar{b}(x) \gamma_\mu Ls(x))(\bar{s}(0) \gamma_\nu Lb(0)) | 0 \rangle.$$ 

(20)

The quark condensate contribution reads

$$\frac{1}{\pi} \text{Im} \psi_s^{pert}(t) = \theta(t - 4(M_b + m_s)^2)$$

$$\times \frac{1}{384 \pi^6} m_s \langle \bar{s}s \rangle$$

$$\times \int_{(M_b + m_s)^2}^{(4q^2 - \frac{\partial}{\partial q^2})} dq^2 \sqrt{\lambda_0} \left[ 4 q^2 - M_b^2 \right]$$

$$\times f_1 \left( 1 + \frac{M_b^2}{q^2} \right) \left( q^2 - M_b^2 \right) \right].$$

(21)

Here $\lambda_0$, $\lambda_1$, and $f_1$ are defined by

$$\lambda_0 \equiv \lambda \left( 1, \frac{q^2}{q^2}, \frac{M_b^2}{q^2} \right),$$

$$\lambda_1 \equiv \lambda \left( 1, \frac{M_b^2}{q^2}, \frac{m_s^2}{q^2} \right),$$

$$f_1 \equiv 1 + \frac{M_b^2}{q^2} + \frac{m_s^2}{q^2} - 2 \left( \frac{M_b^2 - m_s^2}{q^2} \right)^2.$$ 

(22)

5. \textbf{The sum rule analysis}

For the sum rule analysis, we shall work like \cite{4,7} with the moments

$$M_q^{(n)} = \int_{4(M_b + m_s)^2}^{t_n} dt \ t^n \frac{1}{\pi} \text{Im} \psi_q(t),$$

(25)

We shall neglect the nonfactorizable corrections in our analysis, according to the results in NP \cite{4} obtained in a slight variant of the $\overline{MS}$ scheme.
In so doing, in addition to the pQCD input parameters given previously, we shall need the values of the QCD condensates and SU(3) breaking parameters, which we give in Table 1. We show in Fig. 1 the moment sum rules analysis of $f_{B_{(s)}} \sqrt{B_{B_{(s)}}}$ for different values of $r_q$ and $n$: $f_{B_d} \sqrt{B_B}$ versus: a) $r_d$ at $n = -30$, b) $n$ at $r_d = 1.13$; $f_{B_s} \sqrt{B_B}$ versus: c) $r_s$ at $n = -26$, d) versus $n$ at $r_s = 1.17$. Dotted curve: lowest order perturbative contribution; dashed curve: lowest order perturbative + $m_s$ (only for c) and d) + $(\alpha_s G^2)$ condensates. Solid curves: total contribution to order $\alpha_s$.

and with the Laplace sum rule

$$L(\tau)_q = \int_{4(M_b + m_q)^2} \tau \left( t - \tau \right) \frac{1}{\pi} \text{Im} \psi_q(t).$$

In so doing, in addition to the pQCD input parameters given previously, we shall need the values of the QCD condensates and SU(3) breaking parameters, which we give in Table 1. We show in Fig. 1 the moment sum rules analysis of $f_{B_{(s)}} \sqrt{B_{B_{(s)}}}$ for different values of $r_q$ and $n$. As one can see from Fig. 1, the stability regions of the quantity, $\sqrt{g_d} = f_{B_d} \sqrt{B_B}$ versus the number $n$ of moments and the continuum threshold

$$r_d \equiv \frac{t_c^{(d)}}{4M_b},$$

are obtained for large ranges ending with an extremum for

$$n \simeq -30, \quad r_d \simeq 1.13.$$

These range of values of the sum rule parameters are in good agreement with previous results in Ref. [5] and NP [6]. Analogous values of $n$ and $r_s$ stabilities are also obtained in the analysis of

$$\sqrt{g_s} = f_{B_s} \sqrt{B_B},$$

(see Fig. 1b, d), with

$$n \simeq -26, \quad r_s \simeq 1.17.$$

One can notice that the stabilities in the continuum for $\hat{g}_d$ and $\hat{g}_s$ differ slightly as a reflection of the SU(3) breakings, which one can parametrize numerically as

$$r_s \simeq \left( \frac{\sqrt{r_d + m_s}}{M_b} \right)^2 \simeq r_d + 0.05.$$

Similar analysis is done with the Laplace sum rules. We show in Fig. 2 the predictions of $f_{B_{(s)}} \sqrt{B_{B_{(s)}}}$ for different values of $r_q$ and $\tau$, where an extremum is obtained for

$$\tau \simeq 0.3 \text{ GeV}^{-2}.$$

6. Results and implications on $|V_{ts}/V_{td}|^2$ and $\Delta M_s$

We take as a conservative result for $\hat{g}_d$, from the moments sum rule analysis, the one from a large range of $n = -10$ to $-30$ and for $r_d = 1.06$ to 1.17. Adding quadratically the different sources of errors in Table 1, we obtain

$$f_B \sqrt{B_B} \simeq (245 \pm 57) \text{ MeV},$$

(33)
from the moments and Laplace sum rules results. As a final result, we take the arithmetic average
\[ \langle \alpha \rangle_{\nu} \approx \tau \approx \Lambda_5 \]
in units where \( r = 130.7 \) MeV. The box marked with – means that the error is zero or negligible.

**Sources**

| Sources | \( \Delta \left( f_B \sqrt{B_B} \right) \) [MeV] | \( \Delta \xi \times 10^2 \) |
|-----------------|----------------------------------|-------------------------------|
| \( n \approx (30 \sim 10) \) | 8.3 | 1.5 |
| \( \tau \approx (0.1 \sim 0.31) \) GeV\(^{-2} \) | – | – |
| \( r_d \approx 1.06 \sim 1.17 \) | 7.9 | 1.0 |
| \( \Lambda_b = (216^{+53}_{-40}) \) MeV | 0.4 | 0.1 |
| \( \nu = M_b \sim 2M_b \) | 8.7 | 0.2 |
| \( \alpha^2 \): geometric PT series | 43.0 | 0.6 |
| \( M_b = (4.66 \pm 0.06) \) GeV | 34.6 | 1.2 |
| \( \langle \alpha_s G^2 \rangle = (0.07 \pm 0.01) \) GeV\(^4 \) & 1.3 | – |
| \( \langle \bar{u}u \rangle (2) = (254 \pm 15)^3 \) MeV & – | – |
| \( \langle \bar{s}s \rangle / \langle \bar{u}u \rangle \) & – | – |
| \( m_s (2) = (117 \pm 23) \) MeV & – | – |

Total | 57.1 | 2.6 |

in units where \( r = 130.7 \) MeV. The most relevant errors given in Eq. (33) come from \( M_b \) and the truncation of the PT series. We have estimated the latter by assuming that the coefficient of the \( \alpha_s^2 \) contribution comes from a geometric growth of the PT coefficients. The other parameters \( n, r_d, \Lambda, \langle \bar{q}q \rangle, \langle \alpha_s G^2 \rangle \) and \( \nu \) (subtraction point) induce smaller errors as given in Table 1. We proceed in a similar way for \( \tilde{g}_s \). Then, we take the range \( n = -10 \) to \(-26 \) and \( r_s = 1.10 \) to \( 1.21 \), and deduce the ratio

\[ \xi = \frac{f_B \sqrt{B_B}}{f_B \sqrt{B_B}} \approx 1.174 \pm 0.026 \]  

(34)

where the errors come almost equally from \( n, r_q, m_s, M_b \) and the \( \alpha_s^2 \) term. As expected, we have smaller errors for the ratio \( \xi \) due to the cancellation of the systematics. We proceed in the same way with the Laplace sum rules where we take the range of \( \tau \) values from 0.1 to 0.37 GeV\(^{-2} \) (see Fig. 2 a, b) in order to have a conservative result. Then, we deduce

\[ f_B \sqrt{B_B} \approx (249 \pm 61) \) MeV, \]
\[ \xi \approx 1.187 \pm 0.038 \]  

(35)

As a final result, we take the arithmetic average from the moments and Laplace sum rules results. Then, we deduce

\[ f_B \sqrt{B_B} \approx (247 \pm 59) \) MeV, \]
\[ \xi \approx 1.18 \pm 0.03 \]  

(36)

in the unit where \( r = 130.7 \) MeV. These results can be compared with different lattice \( f_B \sqrt{B_B} \approx (230 \pm 32) \) MeV, \( \xi \approx 1.14 \pm 0.06 \), and global-fit of the CKM mixing angles giving \( f_B \sqrt{B_B} \approx (231 \pm 15) \) MeV quoted in [17,20]. By comparing our results Eq. (36) with the one of \( f_B \sqrt{B_B} \) in Eq. (11), one can conclude (to a good approximation) that

\[ \hat{B}_B \approx \hat{B}_{B} \approx (1.65 \pm 0.38) \] \( \Rightarrow \) \[ B_{B_s \bar{u}} (4M_L^2) \approx (1.1 \pm 0.25) \]  

(37)

indicating a negligible SU(3) breaking for the bag parameter. For a consistency, we have used the estimate to order \( \alpha_s \) [22]

\[ f_B \approx (1.47 \pm 0.10) f_B \approx (192 \pm 19) \) MeV \]

(38)

and we have assumed that the error from \( f_B \) compensates the one in Eq. (36). The result is in excellent agreement with the previous result of Ref. [3].

\footnote{One can notice that similar strengths of the SU(3) breakings have been obtained for the \( B \to K^* \gamma \) and \( B \to Klq \) form factors [4].}
in Eqs. (7) and (8), and agrees within the errors with the lattice estimates [19,21].

Using the experimental values

\[ \Delta M_d = 0.479(12) \text{ ps}^{-1}, \]
\[ \Delta M_s \geq 13.1 \text{ ps}^{-1} \quad (95\% \text{ CL}), \quad (39) \]

one can deduce from Eq. (4)

\[ \rho_{sd} \equiv \left| \frac{V_{ts}}{V_{td}} \right|^2 \geq 20.0(1.1). \quad (40) \]

Alternatively, using

\[ \rho_{sd} \simeq \frac{1}{\lambda^2 \left[ (1 - \tilde{\rho})^2 + \eta^2 \right]} \simeq 28.4(2.9) \quad (41) \]

with [19]

\[ \lambda \simeq 0.2237(33), \]
\[ \tilde{\rho} \equiv \rho \left( 1 - \frac{\lambda^2}{2} \right) \simeq 0.223(38), \]
\[ \tilde{\eta} \equiv \eta \left( 1 - \frac{\lambda^2}{2} \right) \simeq 0.316(40), \quad (42) \]

\( \lambda, \rho \) and \( \eta \) being the Wolfenstein parameters, we deduce

\[ \Delta M_s \simeq 18.6(2.2) \text{ ps}^{-1}, \quad (43) \]

in agreement with the present experimental lower bound and within the reach of Tevatron run 2 experiments.

7. Conclusions

We have applied QCD spectral sum rules for extracting (for the first time) the \( SU(3) \) breaking parameter \( \xi \), and for improving the estimate of the quantity \( f_B \sqrt{\bar{B}_B} \). Our predictions are given in Eq. (30). The phenomenological consequences of our results for the CKM mixing angle and\( B_{d,s}^0 - \bar{B}_{d,s}^0 \) mass-differences are given in Eqs. (40) and (43).

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20. C. Bernard, *Nucl. Phys. (Proc. Suppl.)* B94 (2001) 159 and references therein.

21. S. Narison, *Phys. Lett.* B327 (1994) 354; *ibid* B337 (1994) 163; *ibid* B283 (1992) 384.

22. S. Narison, *Nucl. Phys. (Proc. Suppl.*) B74 (1999) 304 and [hep-ph/9712386](http://arxiv.org/abs/hep-ph/9712386).