Real-time propagators at finite temperature and chemical potential

S. Mallik
Theory Division, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India

Sourav Sarkar
Variable Energy Cyclotron Centre, 1/AF, Bidhannagar, Kolkata, 700064, India
(Dated: January 9, 2009)

We derive a form of spectral representations for all bosonic and fermionic propagators in the real-time formulation of field theory at finite temperature and chemical potential. Besides being simple and symmetrical between the bosonic and the fermionic types, these representations depend explicitly on analytic functions only. This last property allows a simple evaluation of loop integrals in the energy variables over propagators in this form, even in presence of chemical potentials, which is not possible over their conventional form.

PACS numbers: 11.10.Wx

I. INTRODUCTION

An advantage of the real-time formulation of field theory in medium over its imaginary-time counterpart is its close resemblance with the field theory in vacuum. A matrix propagator in this formulation is given, roughly speaking, by the vacuum propagator and another piece representing the medium. Also one deals here with an energy integral instead of a frequency sum. Thus, even though its matrix structure is looked upon as a source of complication, the real-time formulation has found many applications to study the properties of hadrons propagating through a medium.

However, a difficulty arises with the conventional form of propagators in presence of one or more chemical potentials. Such propagators have theta-functions in the energy variable separating the contributions of particles and antiparticles in the medium. Then, unlike the case in vacuum, the energy integral in a loop contribution cannot be carried out by the simple contour method.

In this work we rewrite all the real-time propagators in terms of their Källen-Lehmann spectral representations in a suitable form. Besides the spectral densities, these representations depend only on analytic functions, even in presence of chemical potentials, allowing contour evaluation of integrals in loop energies. The integrations over their known spectral densities are carried out.

In Sec 2 we consider the spectral representations for the spatial Fourier transform of the free propagators. In Sec 3 we show that the same form is also valid for the complete propagators. Then in Sec 4 we obtain the four dimensional transforms. In Sec 5 we work out an example of a loop integral with bosonic and fermionic propagators. Our discussion is contained in Sec 6.

II. FREE THEORY

Let us denote the ensemble average of an operator \( O \) by \( \langle O \rangle \),

\[
\langle O \rangle = \frac{\text{Tr}[e^{-\beta(H-\mu N)}O]}{Z}, \quad Z = \text{Tr}[e^{-\beta(H-\mu N)}],
\]

where \( H \) and \( N \) denote the Hamiltonian and the number operator of the system under consideration, maintained at temperature \( 1/\beta \) and chemical potential \( \mu \).

We begin with the free propagators for a scalar field \( \phi(x) \) (with no chemical potential) and a (Dirac) spinor field \( \psi(x) \) (with chemical potential \( \mu \)),

\[
D^{(0)}(x-x') = i\langle T_\tau \phi(x)\phi(x') \rangle, \quad S^{(0)}(x-x') = i\langle T_\tau \psi(x)\bar{\psi}(x') \rangle
\]

where \( x^\mu = (\tau, \vec{x}) \). Here \( \tau \) and \( \tau' \) are any two 'times' on a contour in the complex time plane. The ensemble average with the Boltzmann weight and the consequent analyticity domain, \( -\beta \leq \text{Im}(\tau - \tau') \leq 0 \), constrains the contour to begin at \( -T \), say, on the real axis and end

\[\text{at} X.\text{0901.1045v1 [hep-ph]} 8\text{Jan}2009\]
at $-T - i\beta$, nowhere moving upwards [10]. Apart from these requirements, we let the contour be arbitrary at this stage. Here $T_c$ (and $\theta_c$ below) denote time ordering (and $\theta$-function) with respect to such a contour.

Our form for the propagators in momentum space follow directly from the following spectral representations of their spatial Fourier transforms [10],

$$D^{(0)}(\tau - \tau', k) = i \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \rho^{(0)}(k_0', \vec{k}) e^{-ik_0'(\tau - \tau')} \{\theta_c(\tau - \tau') + f(k_0')\}$$

(3)

$$S^{(0)}(\tau - \tau', p) = i \int_{-\infty}^{+\infty} \frac{dp}{2\pi} \sigma^{(0)}(p_\mu', \vec{p}) e^{-ip_\mu'(\tau - \tau')} \{\theta_c(\tau - \tau') - \tilde{f}(p_\mu')\}$$

(4)

where $\rho^{(0)}$ and $\sigma^{(0)}$ are the bosonic and the fermionic spectral functions for the free theory,

$$\rho^{(0)}(k_0, \vec{k}) = 2\pi \epsilon(k_0) \delta(k^2 - m_B^2)$$

(5)

$$\sigma^{(0)}(p_\mu, \vec{p}) = 2\pi \epsilon(p_\mu) (\not{p} + m_F) \delta(p^2 - m_F^2)$$

(6)

and the functions $f$ and $\tilde{f}$ are given by

$$f(k_0) = \frac{1}{e^{\beta k_0} - 1}, \quad \tilde{f}(p_\mu) = \frac{1}{e^{\beta(p_\mu - \mu)} + 1}$$

(7)

As $k_0$ and $p_\mu$ run over the entire real axis, $f$ and $\tilde{f}$ cannot be interpreted as particle distribution functions, which are

$$n(k_0) = \frac{1}{e^{\beta k_0} - 1} \quad \text{(boson)}$$

$$n_+(p_\mu) = \frac{1}{e^{\beta(p_\mu - \mu)} + 1} \quad \text{(fermion)}$$

$$n_-(p_\mu) = \frac{1}{e^{\beta(p_\mu + \mu)} + 1} \quad \text{(antifermion)}$$

(8)

But they may be readily expressed in terms of the latter ones as

$$f(k_0) = n\epsilon(k_0) - \theta(-k_0), \quad \tilde{f}(p_\mu) = N_1 \epsilon(p_\mu) + \theta(-p_\mu)$$

(9)

where

$$N_1(p_\mu) = n_+(p_\mu) + n_-(p_\mu)$$

(10)

A simple way to derive the results (3) and (4) is to note that each of the propagators in medium satisfy the same differential equation as the corresponding one in vacuum. Taking their spatial Fourier transforms we get one dimension equations in time $\tau$. A particular solution to such an inhomogeneous equation is given by the one with Feynman boundary condition. To get the most general solution, we add to it the two independent (plane wave) solutions of the homogeneous equation with arbitrary coefficients. When these coefficients are determined by applying the thermal, so-called Kubo-Martin-Schwinger (KMS) boundary condition [12,13], we get the above solutions. One can, of course, readily verify these solutions by showing that they satisfy the respective differential equation as well the boundary condition.

III. INTERACTING THEORY

Before we proceed, let us consider complete propagators for interacting theories in the Heisenberg picture and obtain the Källen-Lehmann representation for their spatial Fourier transforms. Consider first the full propagator for the interacting scalar field $\Phi(x)$,

$$D(x - x') = i \langle T_c \Phi(x) \Phi(x') \rangle$$

(11)

Now it does not satisfy a simple differential equation any more. But we may analyze the field product by introducing a complete set of eigenstates $|m\rangle$ of the four-momentum operator $P^\mu$ with eigenvalues $(E_m, \vec{k}_m)$. We insert this set twice in Eq. (11), once to evaluate the ensemble trace and then to separate the operators in the product. Using translational invariance to extract the $x$-dependence of the operators and introducing a delta-function in the energy variables, the spatial Fourier transform of $\langle \Phi(x) \Phi(x') \rangle$ in the $T_c$-product becomes

$$\int d^4 x e^{-ikx} \langle \Phi(x) \Phi(x') \rangle = \int \frac{dk_0}{2\pi} \frac{e^{-ik_0(x' - x)}}{M^+(k_0, \vec{k})}$$

(12)

where the spectral density $M^+$ is given by

$$M^+(k_0, \vec{k}) = (2\pi)^4 \sum_{m,n} e^{-\beta E_m} \delta^{(4)}(k + k_m - k_n) \times \langle m|\Phi(0)|n\rangle \langle n|\Phi(0)|m\rangle / Z$$

(13)

In just the same way we work out the Fourier transform $\langle \Phi(x)\Phi(x) \rangle$ with a spectral density $M^-$, that may be obtained from $M^+$ by changing the signs of $k_m$ and $k_n$ in the delta-function. The two spectral densities are related by the KMS condition

$$M^+(k) = e^{\beta k_0} M^-(k)$$

(14)

which may be obtained simply by interchanging the dummy indices $m,n$ in one of $M^\pm(k)$ and using the energy conserving $\delta$-function [19].

We now define a third spectral density related to the commutator of the field operators,

$$\rho(k) = M^+(k) - M^-(k) \equiv \int d^4 x e^{ikx} \langle [\Phi(x), \Phi(x')] \rangle$$

(15)

in terms of which $M^\pm(k)$ can be expressed as

$$M^+(k) = \{1 + f(k_0)\} \rho(k), \quad M^-(k) = f(k_0) \rho(k)$$

(16)

Using these relations, we see that the three-dimensional Fourier transform of the complete propagator is given exactly by Eq.(3) with the free density $\rho^{(0)}$ replaced by the interacting one, $\rho$.

The complete propagator for the interacting spinor field $\Psi(x)$,

$$S(x - x') = i \langle T_c \Psi(x) \Psi(x') \rangle$$

(17)
can be treated in the same way. The three dimensional Fourier transform of $\langle \Psi(x)\overline{\Psi}(x') \rangle$ is given by an equation like (12) with the spectral density

$$\tilde{M}^+(p_0, \vec{p}) = (2\pi)^4 \sum_{m,n} e^{-\beta(E_m - \mu N_m)} \delta^{(4)}(p + p_m - p_n) \times \langle m|\overline{\Psi}(0)|n\rangle \langle n|\overline{\Psi}(0)|m\rangle / Z, \quad (18)$$

while that of $\langle \overline{\Psi}(x')\Psi(x) \rangle$ is given by a second spectral density $\tilde{M}^-(p_0, \vec{p})$, obtained from the first one by interchanging $\overline{\Psi}(0)$ and $\Psi(0)$ and changing the signs of $p_m$ and $p_n$. Here the integers $N_m$ are the eigenvalues of the operator $N$, the fermion number conservation restricting them to $N_m = N_n - 1$. The two spectral densities are again related by the KMS condition

$$\tilde{M}^+(p) = e^{\beta(p_0 - \mu)} \tilde{M}^-(p), \quad (19)$$

Defining a third spectral density in terms of the anticommutator of the field operators,

$$\sigma(p) = \tilde{M}^+(p) - \tilde{M}^-(p) = \int d^4x e^{ip \cdot (x - x')} \{\Psi(x), \overline{\Psi}(x')\}$$

we can express the first two as

$$\tilde{M}^+(p) = \{1 - \tilde{f}(p_0)\} \sigma(p), \quad \tilde{M}^-(p) = \tilde{f}(p_0) \sigma(p) \quad (20)$$

We thus again get the complete spinor propagator as given by Eq.(4) with $\sigma^{(0)}$ replaced by $\sigma$.

It should be noted that in deriving these representations for the complete scalar and spinor propagators, we have not used any property specific to these field operators, except to define their spectral densities by the commutator and the anticommutator of the respective fields. We thus conclude that these are the most general representations for the spatial Fourier transforms of two-point functions of any bosonic and fermionic local operators. Below we construct their real time representations, again without requiring any property of the spectral densities at all, so that although we refer to them as propagators, they actually remain valid for any two-point function.

In passing, let us compare the derivation of the Källen-Lehmann representation for the ensemble average of two time-ordered fields with that for its vacuum expectation value. In the vacuum case, the intermediate energies are all positive and Lorentz invariance makes the spectral densities to depend on $\theta(k_0)$ and $k^2 = (k_0^2 - \vec{k}^2)$ ($\theta(p_0)$ and $p^2 = (p_0^2 - \vec{p}^2)$). There is, of course, no KMS condition here, but since the commutator (anticommutator) vanishes outside the light-cone, the two parts of spectral densities must be equal \[20\]. In the medium, there is no Lorentz invariance (or we may have it at the cost of introducing a four-velocity vector for the medium). As a result, the vanishing of the commutator (anticommutator) does not lead to any simple condition on the spectral densities.

\[\text{FIG. 1: Contour in time plane for real time formalism}\]

**IV. FORM OF PROPAGATORS**

Having established the results for the spatial Fourier transforms of the propagators, where we kept their time coordinates on an arbitrary contour in the complex time plane, we now choose an appropriate contour to get the real time field theory. Our choice is that of Fig. 1, which for $T \to \infty$, reduces to two parallel lines, the real line and the one shifted by $-i\beta/2$, to be denoted by subscripts 1 and 2 respectively \[1\]. Thus $\tau_1 = t$, $\tau_2 = t - i\beta/2$, $\theta_1(\tau_1 - \tau_1') = \theta(t - t')$, $\theta_2(\tau_1 - \tau_2') = 0$, etc. We can now define the Fourier transform of the four components of a propagator with respect to real time. We thus get from Eqs. (3, 4) the four dimensional Fourier transform of the components respectively of the boson and the fermion propagator as

$$D_{ab}(k_0, \vec{k}) = \int_{-\infty}^{+\infty} dt e^{i\kappa_a(t - t')} D(\tau_a - \tau_1', \vec{k}), \quad a, b = 1, 2 \quad (22)$$

$$S_{ab}(p_0, \vec{p}) = \int_{-\infty}^{+\infty} dt e^{i\kappa_a(t - t')} S(\tau_a - \tau_2', \vec{p}), \quad a, b = 1, 2 \quad (23)$$

It is now simple to work out the matrix propagators. For the scalar field, it is

$$D(k_0, \vec{k}) = \int_{-\infty}^{+\infty} \frac{dk_0'}{2\pi} \rho(k_0', \vec{k}) \Lambda(k_0', k_0) \quad (24)$$

with the elements of the matrix $\Lambda$ given by

$$\Lambda_{11} = -\Lambda_{22}^* = \frac{1}{k_0' - k_0 - i\eta} + 2\pi i f(k_0')\delta(k_0' - k_0) \quad (25)$$

The corresponding results for the spinor propagator is

$$S(p_0, \vec{p}) = \int_{-\infty}^{+\infty} \frac{dp_0'}{2\pi} \sigma(p_0', \vec{p}) \Omega(p_0', p_0) \quad (26)$$

where the matrix $\Omega$ has the elements,

$$\Omega_{11} = -\Omega_{21}^* = \frac{1}{p_0' - p_0 - i\eta} - 2\pi i \tilde{f}(p_0')\delta(p_0' - p_0) \quad (27)$$

Eqs. (24,25) and (26,27) constitute our form for the spectral representation of any bosonic and any fermionic two-point function respectively. The two matrices $\Lambda$ and
$\Omega$ are symmetrical, being related by the replacement $f \leftrightarrow -f$. These describe the common propagation properties of all elementary (particle) and composite fields, the dependence on spin, etc being contained in the respective spectral function.

We also note that the distribution-like functions $f$ and $\bar{f}$ given by Eq. (7) which occur in Eqs. (25) and (27) are analytic functions, in contrast to the distribution functions themselves given by Eq. (8). In particular, the matrices $\Lambda$ and $\Omega$ do not contain any $\theta$-function in the energy variables $k_0(p_0)$, which are present in the conventional form of the propagators in presence of chemical potential. Thus, as we show in Sec. 5, the use of the above spectral representations avoids the difficulty encountered in integration of loop energies over the conventional form of propagators.

Recalling Eq. (9), we can, of course, readily restate the matrix elements given by Eqs. (25, 27) in terms of the particle distribution functions themselves as

$$
\Lambda_{11} = -\Lambda_{22} = \frac{1}{k_0' - k_0 - \text{i}\epsilon(k_0)} + 2\pi i e(k_0)\delta(k_0' - k_0)
$$

$$
\Lambda_{12} = \Lambda_{21} = 2\pi i \sqrt{n(1 + n)} e(k_0)\delta(k_0' - k_0)
$$

$$
\Omega_{11} = -\Omega_{22} = \frac{1}{p_0' - p_0 - \text{i}\epsilon(p_0)} - 2\pi i N_1 \epsilon(p_0)\delta(p_0' - p_0)
$$

$$
\Omega_{12} = -e^\beta\Omega_{21} = -2\pi i e^{\beta\mu/2} N_2 \epsilon(p_0)\delta(p_0' - p_0)
$$

(28)

where $N_1$ is defined by Eq. (10) above and $N_2$ by

$$
N_2(p_0) = \sqrt{n_+ (1 - n_+) \theta(p_0) - \sqrt{n_- (1 - n_-) \theta(-p_0)}
$$

(29)

Note that we choose primed variables $(k_0', p_0')$ for the functions multiplying the delta-functions in Eqs. (25, 27), while we choose unprimed ones $(k_0, p_0)$ for such functions in Eq. (28). The first choice allows us to integrate over loop energies in a simple way (see Eq. (46) below). The second choice makes the matrices diagonalising the propagators independent of the integration variable in the spectral representations (see Eqs. (32, 33) below).

The matrices $\Lambda$ and $\Omega$ as defined by Eq. (28) can now be diagonalised respectively by $U$ and $V$,

$$
U(k_0) = \left( \frac{\sqrt{1 + n}}{\sqrt{n}} \right),
$$

$$
V(p_0) = \left( \frac{N_2/\sqrt{N_1}}{\sqrt{N_1} e^{-\beta\mu/2}} \right),
$$

which in turn diagonalises the two-point correlation functions themselves,

$$
D(k_0, \vec{k}) = U(k_0) \begin{pmatrix} \bar{D} & 0 \\ 0 & -D^* \end{pmatrix} U(k_0)
$$

$$
\bar{S}(p_0, \vec{p}) = V(p_0) \begin{pmatrix} \bar{S} & 0 \\ 0 & -S \end{pmatrix} V(p_0),
$$

(32)

where the analytic amplitudes $\bar{D}$ and $\bar{S}$ are given by

$$
\bar{D}(k_0, \vec{k}) = \int_{-\infty}^{\infty} \frac{dk_0'}{2\pi} \frac{\rho(k_0', \vec{k})}{k_0' - k_0 - \text{i}\epsilon(k_0)}
$$

$$
\bar{S}(p_0, \vec{p}) = \int_{-\infty}^{\infty} \frac{dp_0'}{2\pi} \frac{\sigma(p_0', \vec{p})}{p_0' - p_0 - \text{i}\epsilon(p_0)}
$$

(35)

To find the spectral densities $\rho$ and $\sigma$, we need not refer to their defining equations (15) and (20) any more. It suffices to calculate the imaginary component of any one, say the 11-component of the two point functions to get them. Thus from Eqs. (25) and (27), we get

$$
\rho(k_0, \vec{k}) = \frac{2\text{tanh}(\beta k_0/2)}{\text{Im}D_{11}(k_0, \vec{k})}
$$

$$
\sigma(p_0, \vec{p}) = \frac{2\text{coth}(\beta(p_0 - \mu)/2)}{\text{Im}S_{11}(p_0, \vec{p})}
$$

(36)

(37)

We close our discussion of the form of two-point functions by obtaining explicitly the conventional form of the free propagators for the scalar and the spinor fields $\Phi, \bar{\Phi}$.

With the free field spectral densities given by Eqs. (5, 6), Eqs. (34, 35) give immediately

$$
\bar{D}^{(0)}(k) = \frac{-1}{k^2 - m_B^2 + \text{i}\eta} \equiv \Delta(k, m_B)
$$

$$
\bar{S}^{(0)}(p) = (\bar{p} + m_F) \Delta(p, m_F)
$$

(38)

which are just the Feynman propagators in vacuum. Then multiplying out the matrices in Eqs. (32, 33) we get the scalar propagator as

$$
D^{(0)}(k_0, \vec{k}) = \left( \begin{array}{cc} d_{11} & d_{12} \\ d_{21} & d_{22} \end{array} \right)
$$

(39)

where the matrix elements are

$$
d_{11} = -d_{22} = \Delta(k, m_B) + 2\pi i n k^2 - m_B^2
$$

$$
d_{12} = d_{21} = 2\pi i \sqrt{n(1 + n)} \delta(k^2 - m_B^2)
$$

(40)

and the spinor propagator as

$$
S^{(0)}(p_0, \vec{p}) = (\bar{p} + m_F) \left( \begin{array}{cc} s_{11} & s_{12} \\ s_{21} & s_{22} \end{array} \right)
$$

(41)

with the matrix elements

$$
s_{11} = -s_{22} = \Delta(p, m_F) - 2\pi i N_1 \delta(p^2 - m_F^2)
$$

$$
s_{12} = -e^{\beta\mu} s_{21} = -2\pi i e^{\beta\mu/2} N_2 \delta(p^2 - m_F^2)
$$

(42)

V. AN EXAMPLE

Finally we evaluate in some detail the energy integral in a loop involving a scalar (with $\mu = 0$) and a spinor (with $\mu \neq 0$) propagator, both exact, using their spectral representations given by Eqs. (24-27). The matrix amplitude of such a loop, call it $\Sigma$, belongs to the complete spinor two-point function and by Eqs. (35, 37) its analytic amplitude $\Sigma$ must have the form

$$
\Sigma(p_0, \vec{p}) = \int_{-\infty}^{\infty} \frac{dp_0'}{2\pi} \frac{\sigma_1(p_0', \vec{p})}{p_0' - p_0 - \text{i}\epsilon(p_0)}
$$

(43)
where the one-loop spectral density is given by Eq. (37) as
\[ \sigma_1(p_0, \vec{p}) = 2 \coth(\beta(p_0 - \mu)/2) \text{Im} \Sigma_{11}(p_0, \vec{p}). \] (44)

Ignoring the spin and isospin structures of the interaction vertices, the 11-component of the matrix amplitude is given by
\[ \Sigma_{11}(p_0, \vec{p}) = i \int \frac{d^4q}{(2\pi)^4} S_1(q) D_{11}(p - q) = \int \frac{d^3q}{(2\pi)^3} \times \int_{-\infty}^{\infty} \frac{dq_0}{2\pi} \sigma(q_0', \vec{q}; \vec{p}) \rho(q_0', \vec{p} - \vec{q}) I(q_0', q_0', p_0) \] (45)
where
\[ I(q_0', q_0', p_0) = i \int_{-\infty}^{+\infty} \frac{dq_0}{2\pi} \Omega_{11}(q_0', q_0) \Lambda_{11}(q_0', q_0, p_0 - q_0) \] (46)
Rewriting the propagators in \( I \) as
\[ \Omega_{11}(q_0', q_0) = \frac{1 - \bar{f}(q_0')}{q_0' - q_0 - i\eta} + \frac{\bar{f}(q_0')}{q_0' - q_0 + i\eta} \]
\[ \Lambda_{11}(q_0', q_0) = \frac{1 + \bar{f}(q_0')}{q_0' - q_0 - i\eta} - \frac{\bar{f}(q_0')}{q_0' - q_0 + i\eta} \] (47)
we can integrate the resulting four terms over \( q_0 \) by just closing the contour in the \( q_0 \) plane to get
\[ I(q_0', q_0', p_0) = \frac{1 - \bar{f}(q_0')}{p_0 - q_0 - q_0' + i\eta} + \frac{\bar{f}(q_0')}{p_0 - q_0 - q_0' - i\eta} \] (48)
Had we taken the spinor propagator from Eq.(28), the \( \theta \)-functions in it would not have allowed such a simple evaluation of \( I \) [12].

The imaginary part of \( \Sigma_{11} \) is given by that of \( I \)
\[ \text{Im} I(q_0', q_0, p_0) = -\pi \{ (1 - \bar{f})(1 + f) - \bar{f}f \} \delta(p_0 - q_0 - q_0'), \]
which has the 'wrong' relative sign between the two terms in the bracket. It can be reversed by taking out a factor of \( \tanh[\beta(p_0 - \mu)/2] \). Then \( F \) reduces to
\[ \sigma_1(p_0, \vec{p}) = -2\pi \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dq_0}{2\pi} \sigma(q_0', \vec{q}) \int_{-\infty}^{\infty} \frac{dq_0''}{2\pi} \rho(q_0'' \vec{p} - \vec{q}) \{ 1 - \bar{f}(q_0') + f(q_0'') \} \delta(p_0 - q_0 - q_0'') \] (49)

Further, if the propagators are the free ones, one can easily work out the \( q_0'' \) and \( q_0' \) integrals over the mass shell \( \delta \)-functions in the spectral densities to get \[ \omega_1 = \sqrt{q^2 + m^2}, \quad \omega_2 = \sqrt{\vec{p} - \vec{q})^2 + m_B^2} \]
\[ \sigma_1^{(0)}(p_0, \vec{p}) = -2\pi \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dq_0}{2\pi} \rho(q_0', \vec{p} - \vec{q}) \int_{-\infty}^{\infty} \frac{dq_0''}{2\pi} \{ (1 - n_+ + n) \delta(p_0 - \omega_1 - \omega_2) + (n_+ + n) \delta(p_0 - \omega_1 + \omega_2) \} \]
\[ -(\omega_1 \rightarrow -\omega_1, \omega_2 \rightarrow -\omega_2, n_+ \rightarrow n_-) \] (50)
in agreement with the computation in the imaginary time formulation [21].

VI. DISCUSSION

Despite many applications of the real-time propagators in medium, their conventional matrix form is not satisfactory in that the matrix components contain non-analytic functions, particularly when chemical potential is present. This causes difficulty in carrying out integrals in loop energies over such form of propagators.

In looking for improvement on this point, we take our cue from the Källén-Lehmann representation for the vacuum two-point functions, where the propagation properties common to all fields are displayed explicitly by analytic functions, keeping dependence on spin etc. confined to the spectral densities. We are thus led to study this representation at finite temperature and chemical potential for all two-point correlation functions.

Indeed, we find spectral representations here to depend explicitly only on analytic, distribution-like functions, along with propagation function expected in vacuum. They are only of two types, one for the bosonic and another for fermionic two-point functions. Also they are simple and symmetrical between the two types. We choose this representation even for free propagators. Then the loop integrals in the energy variables can be easily carried out, irrespective of the presence of chemical potentials in them. Integrals over the spectral densities can then be performed.

[1] G. W. Semenoff and H. Umezawa, Nucl. Phys. B 220, 196 (1983)
[2] A.J. Niemi and G.W. Semenoff, Ann. Phys. 152, 105 (1964).
[3] R.L. Kobes, G.W. Semenoff and N. Weiss, Z. Phys. C 29, 371 (1985).
[4] For a review, see N.P. Landsmann and Ch.G. van Weert, Phys. Rep. C 145, 141 (1987).
[5] H. Leutwyler and A.V. Smilga, Nucl. Phys. B 342, 302 (1990).
[6] A. Schenk, Phys. Rev. D 47, 5138 (1993).
[7] D. Toublan, Phys. Rev. D 56, 5629 (1997).
[8] J.F. Nieves, Phys. Rev. D42, 4123 (1990).
[9] R.D. Pisarski, M. Tytgat, Phys. Rev. D 54, 2989 (1996).
[10] S. Mallik and A. Nyffeler, Phys. Rev. C63, 065204 (2001).
[11] S. Mallik and S. Sarkar, Phys. Rev. D 65, 016002 (2001).
[12] R.L. Kobes and G.W. Semenoff, Nucl. Phys. B 260, 714 (1985).
[13] In simple cases, it is, of course, possible to bypass this difficulty with the conventional form of the propagators. One way is to use the Cutkosky rules to find the imaginary part. Another is to work in coordinate space and
write the propagators in terms of their three-dimensional Fourier transforms [10].

[14] G. Källen, Helv. Phys. Acta, 25 417 (1952)
[15] H. Lehmann, Nuovo Cim, 11, 342 (1954).
[16] R.L. Mills, Propagators for many-particle systems, (Gordon and Breach, New York, 1969).
[17] R. Kubo, J. Phys. Soc., Japan 12, 570 (1957).
[18] P.C. Martin and J. Schwinger, Phys. Rev. 115, 1342 (1959).
[19] There is also a symmetry relation, $M^+(k) = M^-(−k)$. But we shall not use it here.
[20] S. Weinberg, The quantum theory of fields, vol I, p.457 (Cambridge University press, 1995).
[21] H.A. Weldon, Phys. Rev. D 28, 2007 (1983).