Arbitrary qubit rotation through nonadiabatic evolution

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We show how one can perform arbitrary rotation of any qubit, using delayed laser pulses through nonadiabatic evolution, i.e., via transitions among the adiabatic states. We use a double-Λ scheme and use a set of control parameters such as detuning, ratio of pulse amplitudes, time separation of the two pulses for realizing different rotations of the qubit. We also investigate the effect of different kinds of chirping, namely linear chirping and hyperbolic tangent chirping. Our work using nonadiabatic evolution adds to the flexibility in the implementation of logic gate operations and shows how to achieve control of quantum systems by using different types of pulses.

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I. INTRODUCTION

The growing field of quantum computation and quantum information involves logical operations with quantum states. The single qubit operation is its inversion and rotation whereas the controlled-NOT and phase-gates require two qubits. Any specific computation consists of proper manipulation using these operations [1].

All these can be implemented on the basis of one’s ability to prepare the atomic system in desired states. Such a preparation has been studied using a variety of techniques using external fields. For example, by Rabi flopping [2] one can transfer the population completely to another orthogonal state. However this requires perfect control on pulse area. A more efficient way to transfer populations has been proposed by Oreg et al. [3]. They have shown how to transfer the population between the dipole forbidden levels in multilevel systems using the idea of adiabatic following [4]. They have predicted the use of two delayed pulses in a three level Λ system in the large detuning domain for this purpose. Their idea of counter-intuitively ordered pulses has been implemented in a technique of population transfer called Stimulated Raman Adiabatic Passage (STIRAP) [5]. Under conditions of adiabaticity, the atom follows the evolution of one of the eigenvectors of the time-dependent Hamiltonian. Grobe et al., under the same condition of adiabaticity, used the idea of delayed pulses in counterintuitive sequence in the context of formation of shape-preserving pulses (adiabatons) [6]. Note that, in STIRAP, both the pulses can in principle be kept resonant with the corresponding atomic transitions. Vitanov and Stenholm [7] investigated the effect of Raman detuning on the final state population in STIRAP starting from one of the two lower states. They used two delayed Gaussian pulses for this purpose. They found that, as the detuning increases, the final state population decreases. Sola et al. [8] used two contemporary chirped pulses and investigated how the final state population varies with the chirp rate of the pulses. They demonstrated the role of detuning in minimizing the intermediate level population.

All the above references discuss the population transfer between the unperturbed states of the atoms. Chang et al. [9] found a way to produce a final state which was a superposition of the bare atomic states. They used two contemporary chirped pulses. They have shown how the selectivity of the final states depends upon the relative sign of chirping rate and detuning. Further, Unanyan et al. in the tripod-STIRAP scheme [10] have shown how one can create an arbitrary superposition of two atomic states using a sequence of three pulses in a four level system and how the superposition can be controlled by changing the relative delay between the pulses. Marte et al. [11] and more recently Vitanov et al. [12] suggested another method, called fractional STIRAP, in which, the two counterintuitive pulses terminate at the same time to produce appropriate superposition states. In a recent development, Král and Shapiro [13] has found another unique way of creating a superposition of many energy-eigenstates by the method of shaped adiabatic passage (SAP).

So far we have discussed different methods of either the population transfer or the preparation of superpositions, all starting from a given unperturbed state. Now the question arises whether one can create a superposition, starting from a superposition state, i.e., produce essentially the rotation of a qubit. Reuzoni and Stenholm [14] used four laser pulses which coupled four ground levels to a common excited level and created a superposition of two ground states, starting from a superposition of two other ground states. Most recently Kis and Renzoni [15] have discovered how to create the superposition in the same basis as of the initial superposition state. They have used the tripod-STIRAP scheme to show how the relative phase of the control pulse controls the superposition of the atomic states in the final superposition. This is quite relevant in the context of qubit rotation. If one thinks of the initial superposition state as one qubit state, then creating another superposition in the same basis refers to the rotation of the qubit in the Hilbert space of those two basis states.

All the above methods are based on the adiabatic evolution of atomic system. The question arises, whether
one can go beyond the adiabatic limit. In the present paper, we explore this possibility in the context of a three level Λ-system interacting with two laser pulses. In nonadiabatic evolution, transition occurs between the different adiabatic states of the system [16]. During this rotation the state no longer remains confined in the two-dimensional Hilbert space of the basis states comprising the superposition. Thus a different superposition of the basis states is prepared in the long time limit.

We use a double-Λ configuration for the demonstration of the rotation of a qubit. We note that the light propagation through an atomic medium in such a configuration has been studied [17,18].

The structure of the paper is as follows. In Sec. II, we describe the model configuration and relevant equations for the process of qubit rotation. In Sec. III, we present detailed numerical results on the rotation of a qubit. We discuss the role of detuning and other pulse parameters in this context. In Sec. IV, we present a comparison of our results with those obtained using two-level approximation. In Sec. V, we discuss the nonadiabaticity in the evolution. In Sec. VI, we consider the effect of chirped pulses on the rotation of a qubit. It may be noted that pulses with appropriate time-dependent phases have been used to achieve coherent control of quantum systems [19]. In the Appendix, we show how the traditional STIRAP can be used to prepare the atomic system in a state which is orthogonal to the initial superposition.

II. MODEL FOR QUBIT ROTATION

Consider the four level configuration as shown in Fig. 1. The levels |g⟩ and |f⟩ are coupled to the upper excited levels |e⟩ and |1⟩ by input pulses. We will show how specific choices of these pulses help in preparing some initial superposition of |g⟩ and |f⟩ and how this superposition can be rotated with the help of two delayed pulses. Note that the basis states of the qubit, under consideration, are |g⟩ and |f⟩. The states |g⟩ and |f⟩ are long-lived states and that the initial and final states should not have any population in the excited states |e⟩ and |1⟩.

FIG. 1. Level configuration for a double-Λ system. Here $G_1$ and $G_2$ are one-half of the Rabi frequencies of the applied classical fields which couple the level |1⟩ to |g⟩ and |f⟩, respectively. The classical fields with Rabi frequencies $2\Omega_1(t)$ and $2\Omega_2(t)$ couple the level |e⟩ to |g⟩ and |f⟩. $\Delta_1$ and $\Delta_2$ are their respective one-photon detunings.

A. Preparation of the initial superposition state

We start with the atom initially in the level |g⟩. We apply two cw fields with constant Rabi frequencies $2G_1$ and $2G_2$ which couple the state |1⟩ with |g⟩ and |f⟩, respectively. We assume these fields to be resonant with the corresponding transitions. When the cw fields are switched off, the system is prepared in the state

$$|i\rangle = \alpha|g\rangle + \beta e^{i\phi}|f\rangle,$$

(1)

where, $\alpha = G_1/G$, $\beta = G_2/G$, $G = \sqrt{G_1^2 + G_2^2}$ and $\phi$ is the relative phase between |g⟩ and |f⟩. This is the desired state, which will be considered as the initial state of the qubit in the next phase of evolution.

B. Basic equations for qubit rotation

We will now discuss how one can rotate the qubit in state (1), i.e., how one can create another superposition of |g⟩ and |f⟩. When the cw fields are switched off, we apply two chirped laser pulses with electromagnetic fields given by

$$E_1(t) = \varepsilon_1(t) e^{-i\omega_1 t} e^{-i\delta_1(t)} + c.c.,$$

(2a)

$$E_2(t) = \varepsilon_2(t) e^{-i\omega_2 t} e^{-i\delta_2(t)} + c.c.,$$

(2b)

where, $\varepsilon_{1,2}(t)$ are the envelopes of the pulses, $\omega_{1,2}$ are the carrier frequencies, $\delta_{i}(t) = \chi_i F_i(t)$ ($i = 1, 2$) corresponds to the applied chirping with chirping parameter $\chi_{1,2}$ and time-dependence of $F_i(t)$ represents the chirping profile. These pulses couple the level |e⟩ with |g⟩ and |f⟩, respectively.

The interaction Hamiltonian in the interaction picture and in the rotating wave approximation is

$$H = \hbar \left[ \Omega_1(t) e^{-i\Delta_1 t} e^{-i\phi_1(t)} |e\rangle \langle g| + \Omega_2(t) e^{-i\Delta_2 t} e^{-i\phi_2(t)} |e\rangle \langle f| + c.c. \right]$$

(3)

where, $2\Omega_1(t) = 2d_{eg} \varepsilon_1(t)/\hbar$ and $2\Omega_2(t) = 2d_{ef} \varepsilon_2(t)/\hbar$ are the time-dependent Rabi frequencies, $d_{ej}$ ($j = g, f$) are the dipole matrix elements of transitions |e⟩ ↔ |j⟩, $\Delta_1 = \omega_1 - \omega_{eg}$, $\Delta_2 = \omega_2 - \omega_{ef}$ are the respective one-photon detunings of the fields, and $\omega_{ej}$ are the frequencies of the atomic transitions |e⟩ ↔ |j⟩. Here we choose the envelopes $\Omega_1(t)$ and $\Omega_2(t)$ as

$$\Omega_1(t) = \Omega_{01} \exp \left[ -t(T - T)^2/\tau^2 \right] e^{-i\delta},$$

(4a)

$$\Omega_2(t) = \Omega_{02} \exp \left[ -t^2/\tau^2 \right],$$

(4b)

where, $\Omega_{0j}$ ($j = 1, 2$) is one-half of the peak Rabi frequency, $\tau$ is the half-width of both the pulses, $T$ is their
relative time separation, and $\delta$ is the relative phase difference of the pulses. The wave function of the atomic system can be expanded in the basis states $|e\rangle, |g\rangle, |f\rangle$ as

$$|\Psi(t)\rangle = c_e(t)|e\rangle + c_g(t)|g\rangle + c_f(t)|f\rangle,$$  

(5)

where, $c_j(t)$ ($j = e, g, f$) are the probability amplitudes for the corresponding states. Using Eq. (3) and the following transformations of the amplitudes

$$d_e = c_e e^{i\Delta_1 t} e^{i\phi_1(t)},$$  

$$d_g = c_g,$$  

$$d_f = c_f e^{i(\Delta_1 - \Delta_2) t} e^{i[\phi_1(t) - \phi_2(t)]},$$  

we arrive at the following time dependent equations for the $d_j(t)$'s:

$$\dot{d}_e = i(\Delta_1 + \dot{\phi}_1) d_e - i[\Omega_1(t) d_g + \Omega_2(t) d_f],$$  

(7a)

$$\dot{d}_g = -i\Omega_1^* (t) d_e,$$  

(7b)

$$\dot{d}_f = i[\Delta_1 - \Delta_2 + (\phi_1 - \phi_2)] d_f - i\Omega_2^* (t) d_e.$$  

(7c)

These equations can be solved numerically under the initial conditions [Eq. (1)]

$$d_g(-\infty) = \alpha, \quad d_f(-\infty) = \beta e^{i\phi}.$$  

(8)

FIG. 2. Effect of the detuning parameter $\Delta$ on the time dependence of the populations of the levels $|e\rangle$, $|g\rangle$, and $|f\rangle$ for different values of $\Delta \tau = 45$ (solid curve), $\Delta \tau = 60$ (dot-dashed curve), and $\Delta \tau = 120$ (dashed curve). The other parameters are $\alpha = 0.3, \phi = \pi/2, \Omega_0 = \Omega_2 = 15, \delta = 0, \chi_1 = \chi_2 = 0$, and $T = 4\tau/3$. The variation of populations of the levels $|g\rangle$ (dashed curve) and $|f\rangle$ (dot-dashed curve) with $\Delta$ at long time ($t = 15\tau$) is shown in the inset with the same numerical parameters as above.

III. ARBITRARY QUBIT ROTATION
CONTROLLED BY DIFFERENT PULSE PARAMETERS

A. Effect of one-photon detuning

We assume that the two delayed pulses (4) are equally detuned from the corresponding transitions, i.e., $\Delta_1 = \Delta_2 = \Delta$, say. This means that both the pulses are one-photon detuned, but satisfy the condition of two-photon resonance. We also assume that they are unchirped, i.e., $\chi_1 = \chi_2 = 0$. We further assume that, the pulses do not have any relative phase difference, i.e., $\delta = 0$ and have equal magnitudes, i.e., $\Omega_0 = \Omega_2$. We start with the initial atomic superposition (1) choosing $\alpha = 0.3$ and $\phi = \pi/2$. We integrate Eqs. (7) to obtain the time-dependence of the various amplitudes.

FIG. 3. The variation of final state population at long time with $\Delta$ for $\alpha = 1$. Other parameters are the same as in Fig. 2.

FIG. 4. Temporal variation of the relative phase between the levels $|g\rangle$ and $|f\rangle$ for different values of $\Delta$ is shown. The other parameters are the same as in Fig. 2. Note that the phase becomes constant in the long time limit.

We plot the variation of populations in the levels $|e\rangle$, $|g\rangle$, and $|f\rangle$ with time for different values of $\Delta$, in Fig. 2. We note that in the long time limit the population gets redistributed in the levels $|g\rangle$ and $|f\rangle$ with almost no population in $|e\rangle$. As the value of $\Delta$ varies, the pop-
ulations in $|g\rangle$ and $|f\rangle$ also vary. We show in the inset of Fig. 2 how the populations in $|g\rangle$ and $|f\rangle$ in the long time limit vary with $\Delta$. In this limit, the intermediate level population becomes almost zero. This is the case even when $\alpha = 1$, i.e., if the initial state is $|g\rangle$. Fig. 3 clearly shows remarkably large change in the final state composition, which is controlled by one-photon detuning $\Delta$.

The relative phase $\phi$ of the levels $|g\rangle$ and $|f\rangle$ is given by

$$\cos \phi(t) = \text{Re}[c_g^* c_f / |c_g|^2 |c_f|].$$

(9)

In Fig. 4, we have shown the time-dependence of $\cos \phi(t)$. Clearly, the phase at the long time limit becomes constant. This means, a different coherent superposition of $|g\rangle$ and $|f\rangle$ has been created. This implies to a rotation of the concerned qubit, which is controlled by one-photon detuning $\Delta$.

Note that for intermediate times, the level $|e\rangle$ no longer remains confined in the two-dimensional Hilbert space of $|g\rangle$ and $|f\rangle$. Rather, it travels through the three-dimensional Hilbert space of the levels $|e\rangle$, $|g\rangle$, and $|f\rangle$ and finally comes back to the initial space of states $|g\rangle$ and $|f\rangle$, but with a different composition of them.

From the above discussions, one can infer that the amount of rotation can be suitably controlled by the one-photon detuning parameter $\Delta$. For example, for $\Delta \tau \sim 180$, one can prepare an equal superposition state of the levels $|g\rangle$ and $|f\rangle$ with phase $\phi = 2\pi/3$ (see inset, Fig. 2). We emphasize that for $\Delta = 0$, the level $|e\rangle$ remains populated at large times. So large values of the detuning $\Delta$ are needed for proper qubit rotation.

It is interesting to note that the time separation $T$ between the pulses affects the qubit rotation as shown in Fig. 5 which depicts the variation of populations in $|g\rangle$ and $|f\rangle$ with $T$. It should be noted that at long time, the level $|e\rangle$ does not get populated.

It should be mentioned here that one can get approximate analytical solutions of Eqs. (7), putting $\chi_i = 0$ ($i = 1, 2$) [7,20,21]. However, all these solutions are for $\alpha = 1$. For $\alpha \neq 1$, as discussed in the present section, it is very difficult to obtain the solution analytically.

![FIG. 6. Variation of populations in the levels $|e\rangle$, $|g\rangle$ and $|f\rangle$ with the ratio of the amplitudes of the Rabi frequencies $\Omega_{01}/\Omega_{02}$ at long time ($t = 15\tau$). The numerical parameters used are $\Delta = \pi/2$, $\Omega_{02}\tau = 15$, $\delta = 0$, $\chi_1 = \chi_2 = \chi = 0$, and $\Delta \tau = 75$.](image)

![FIG. 7. Variation of populations in long time limit ($t = 15\tau$) with the ratio $\Omega_{01}/\Omega_{02}$ for $\alpha = 1$ and $\Delta \tau = 0$ is shown. All the other parameters are the same as in Fig. 6.](image)

**B. Effect of relative amplitudes of the pulses**

The relative amplitude of the two pulses is expected to determine the final state of the atom. We continue to keep both the pulses equally detuned by an amount of $\Delta$. In Fig. 6, we show the long time behavior of populations in the levels $|e\rangle$, $|g\rangle$, and $|f\rangle$ as functions of the parameter $\Omega_{01}/\Omega_{02}$. We present results for both the cases when the atom starts in the state $|g\rangle$ and the state $|i\rangle$ with $\alpha = 0.3$ and $\phi = \pi/2$. Note that the population in the level $|e\rangle$ is negligible for large times. The population is redistributed in the levels $|g\rangle$ and $|f\rangle$. Thus, a different superposition is created at the end of evolution. The composition varies if one changes the ratio $\Omega_{01}/\Omega_{02}$. For
example, for $\Omega_{01}/\Omega_{02} = 1$, one gets an equal superposition of $|g\rangle$ and $|f\rangle$, for $\alpha = 0.3$. It is thus clear that the ratio of the amplitudes of the two pulses can serve as a control parameter for producing rotation of a qubit. Note that for $\alpha = 1$ and $\Delta = 0$ only orthogonal rotation of the qubit is possible for all values of $\Omega_0$ [see Fig. 7].

When the detuning $\Delta$ of the pulses becomes much larger than the pulse amplitudes, one can ignore the time variation of $\alpha [3]$. The Eqs. (7) in the $(|i\rangle, |k\rangle)$ basis can be written as

$$\begin{pmatrix}
\dot{d}_i \\
\dot{d}_k
\end{pmatrix} = -i \begin{pmatrix}
\Delta_e & \Omega_e \\
\Omega_e^* & -\Delta_e
\end{pmatrix}
\begin{pmatrix}
d_i \\
d_k
\end{pmatrix},$$

(11)

where,

$$f_1(t) = \alpha \Omega_1^*(t) + \beta e^{-i\phi} \Omega_2^*(t),$$

$$f_2(t) = \beta e^{i\phi} \Omega_1^*(t) - \alpha \Omega_2^*(t),$$

(12)

and $\Omega_e = f_1 f_2^* / \Delta$, $\Delta_e = (|f_1|^2 - |f_2|^2) / 2\Delta$. Thus the three-level atomic system can be approximated as a two-level system with effective Rabi-frequency $\Omega_e$ and effective detuning $\Delta_e$ [Eq. (11)]. It is well-known that in a two-level atomic system, if detuning of the pulse is much larger than the pulse amplitude, then the induced polarization of the atom follows the time-evolution of the population term [4]. This is referred to as adiabatic following. Similarly, if $|\Delta_e| \gg |\Omega_e|$, then, adiabatic following occurs in a three-level system also.

C. Effect of relative phase of the two pulses

We will now discuss how the amount of qubit rotation can be controlled by the relative phase $\delta$ between the two pulses. We work under the two-photon resonance condition and take the pulses to have equal amplitudes. The periodic dependence of the populations of $|g\rangle$ and $|f\rangle$ on $\delta$ in the long time limit for the initial condition (8) is shown in Fig. 8. Thus the relative phase of the input pulses can be used as an useful control parameter for the qubit rotation.

It should be mentioned here that, if $\alpha = 1$, i.e., if the atom is initially prepared in the state $|g\rangle$, then there is no $\delta$-dependence of the populations in the states $|g\rangle$ and $|f\rangle$. This is because the Eqs. (7) can be redefined with an extra complex phase term in the probability amplitudes. For example, the replacements $d_e = d_e e^{i\phi}$, $d_f = d_f e^{i\phi}$, and $d_g = d_g$ keep the form of the equations unchanged. Thus $\delta$ does not show up in the square moduli of these amplitudes. However, the wave functions would be different for different values of $\delta$ and the probability amplitudes become different. Thus, if the atom is initially in the state $|g\rangle$, then a change in $\delta$ cannot produce arbitrary rotation of $|g\rangle$.

IV. COMPARISON WITH THE RESULTS IN TWO-LEVEL APPROXIMATION

We write the state orthogonal to $|i\rangle$ [Eq. (1)] as

$$|k\rangle = \beta e^{-i\phi} |g\rangle - \alpha |f\rangle.$$ (10)

FIG. 8. Variation of the population of the levels $|g\rangle$ and $|f\rangle$ with the relative phase $\delta$ of the two applied electric fields at long time ($t = 15\tau$) for the parameters $\alpha = 0.3$, $\phi = \pi/2$, $\Omega_{01}/\Omega_{02} = 1.0$, $\Delta \tau = 75$, and $\chi = 0$.

FIG. 9. The variation of populations in the levels $|g\rangle$ and $|f\rangle$ as obtained from exact numerical results and from two-level approximation for $\Delta \tau = 30$. The other parameters are $\alpha = 0.3$, $\phi = \pi/2$, $\Omega_{01}\tau = \Omega_{02}\tau = 15$, $\delta = 0$, $\chi_1 = \chi_2 = 0$, and $T = 4\tau/3$.

FIG. 10. The variation of populations in the levels $|g\rangle$ and $|f\rangle$ as obtained from exact numerical results and from two-level approximation for $\Delta \tau = 45$. The other parameters are the same as in Fig. 9.
We present the exact and two-level approximated results for the variation of the populations in $|g\rangle$ and $|f\rangle$ in Figs. 9 and 10 for different values of $\Delta$. Clearly, for smaller values of $\Delta$, the two-level approximation is not valid. But still rotation of the qubit occurs. Only when $\Delta \gg \Omega_0$, exact numerical result and two-level approximated result match, as we stated earlier. This is because, in this case, the level $|e\rangle$ is hardly populated during the evolution and the system thereby keeps itself confined in the Hilbert plane of $|g\rangle$ and $|f\rangle$. Thus the rotation of a qubit can be obtained even for values of $\Delta$ for which two-level approximation is not valid.

![Variation of populations in the adiabatic states](image)

FIG. 11. Variation of populations in the adiabatic states $|a_0\rangle$ and $|a_+\rangle$ [Eqs. (14)] with time for $\Delta \tau = 60$. The other parameters are same as in Fig. 9.

V. TRANSITIONS AMONG ADIABATIC STATES - NONADIABATICITY

We further address the question of nonadiabaticity. The evolution of a quantum system is called adiabatic, if the populations of the adiabatic states (which are the eigenstates of the time-dependent Hamiltonian) remain constant during the evolution. This condition requires that the nonadiabatic coupling between the adiabatic states is much smaller than their energy difference [22,23]. To attain this, the Hamiltonian should be smoothly varying with time. In a three level $\Lambda$ system, if one uses two smoothly varying pulses with large amplitudes in counterintuitive sequence, then the complete transfer of population occurs between the two lower lying levels of the system and also the evolution of the system becomes adiabatic [5]. One has to use the pulses in Raman resonance. Even when the pulses are one-photon resonant, then for large enough values of the pulse amplitudes the evolution becomes adiabatic. In this light we examine the nonadiabaticity of the process of rotation described here. The Hamiltonian (under the conditions $\Delta_1 = \Delta_2 = \Delta$ and $\chi_1 = \chi_2 = 0$) can be written in the $(|e\rangle, |g\rangle, |f\rangle)$ basis as

$$H = \begin{pmatrix} -\Delta & \Omega_1 & \Omega_2 \\ \Omega_1^* & 0 & 0 \\ \Omega_2^* & 0 & 0 \end{pmatrix}. \quad (13)$$

The eigenstates (or the adiabatic states) of $H$ are given by,

$$|a_0\rangle = \cos \Theta |g\rangle - \sin \Theta |f\rangle, \quad (14a)$$

$$|a_+\rangle = \sin \Theta \cos \Phi |g\rangle - \cos \Theta \cos \Phi |f\rangle, \quad (14b)$$

$$|a_-\rangle = \sin \Theta \sin \Phi |g\rangle + \cos \Phi |f\rangle, \quad (14c)$$

where

$$\tan \Theta = \frac{\Omega_1(t)}{\Omega_2(t)}, \quad (15)$$

and

$$\tan \Phi = \frac{2\sqrt{|\Omega_1|^2 + |\Omega_2|^2}}{\Delta + \sqrt{\Delta^2 + 4(|\Omega_1|^2 + |\Omega_2|^2)}} \quad (16)$$

The state (14a) is the well-known coherent population trapping state. It is independent of the two-photon detuning.

We have shown the temporal variation of the populations in these adiabatic states in Fig. 11. It is clear that during the course of evolution, transfer of population occurs between the states, namely $|a_0\rangle$ and $|a_+\rangle$. Thus the nonadiabatic coupling is no more small enough to keep the process adiabatic. Clearly the system evolves nonadiabatically in the chosen parameter domain.

Note that even when $\alpha = 1$, for larger $\Delta$ the evolution becomes nonadiabatic [Fig. 3] for a constant value of $\Omega_0$. If the system were to stay in the adiabatic state defined by $H(t)|a_0\rangle = 0$, then the final state population would be independent of the detuning. The $\Delta$-dependence of the population in Fig. 3 implies that the dynamics of the system does not remain confined to the adiabatic state $|a_0\rangle$ only. Further note that for a given value of $\Delta$, large values of $\Omega_0$ are required to make the evolution adiabatic. Fig. 7 supports the adiabatic result at large $\Omega_0$ ($\Omega_{01}/\Omega_{02} \gtrsim 0.3$) for $\Delta = 0$, i.e., the system remains essentially in the state (14a).

VI. QUBIT ROTATION CONTROLLED BY CHIRPING

So far we have discussed how the one-photon detuning of the pulses can be used to produce rotation of the qubit. We now consider use of the chirped pulses for obtaining arbitrary qubit rotation. We also examine the simultaneous effects of chirping and one-photon detuning.
FIG. 12. The variation of populations of $|g\rangle$ and $|f\rangle$ in the long time limit with $\chi$ (in case of linear chirping) for $\Delta\tau = 75$ and $\Delta\tau = -75$ is shown. The other parameters are $\alpha = 0.3$, $\phi = \pi/2$, $\Omega_{01}/\Omega_{02} = 1$, and $\delta = 0$.

A. Effect of linear chirping

We assume that the pulses are linearly chirped, i.e.,

$$
\frac{d}{d\left(\frac{t}{\tau}\right)} \phi_1 = \chi_1 \left(\frac{t - T}{\tau}\right),
$$

$$
\frac{d}{d\left(\frac{t}{\tau}\right)} \phi_2 = \chi_2 \left(\frac{t}{\tau}\right).
$$

We further assume that $\chi_1 = \chi_2 = \chi$. We put $\delta = 0$, $\alpha = 0.3$, and $\Omega_{01}/\Omega_{02} = 1$. We have seen from the temporal variation of populations in the levels $|e\rangle$, $|g\rangle$, and $|f\rangle$ (results not shown) that populations in $|g\rangle$ and $|f\rangle$ in the long time limit becomes constant at a value depending on $\chi$ whereas the level $|e\rangle$ remains unpopulated. Therefore by changing the chirping rate $\chi$, we produce arbitrary rotation to the state $|i\rangle$. In Fig. 12 we show the variations of populations of $|g\rangle$ and $|f\rangle$ in the long time limit with the chirping parameter $\chi$ for different values of $\Delta$. Clearly, we can have a rotation irrespective of the relative sign of $\chi$ and $\Delta$, but the amount of rotation is quite dependent on it. From Fig. 12, it is also clear that for a constant value of $\chi$, one can have different rotation for different values of detuning parameter $\Delta$. Thus by varying either $\Delta$ [or $\chi$] while keeping $\chi$ [or $\Delta$] constant, one has a way to achieve arbitrary rotation to the qubit state $|i\rangle$.

We have further found that making $\chi_1 \neq \chi_2$ can also control the rotation. In Fig. 13, we have shown the variation of populations in the levels $|g\rangle$ and $|f\rangle$ at long time limit with the ratio $\chi_2/\chi_1$ for two different values of $\Delta$. Note that the level $|e\rangle$ remains unpopulated. Thus either by varying the chirping parameter $\chi$ ($\chi_1 = \chi_2 = \chi$) or by changing the ratio $\chi_2/\chi_1$ one can achieve arbitrary rotation of a qubit.

We have checked that the populations in the adiabatic states does not remain constant during the evolution and hence the process becomes nonadiabatic in this case also.

FIG. 13. Variation of populations of $|g\rangle$ and $|f\rangle$ in long time limit with $\chi_2/\chi_1$ (taking $\chi_1 = 1$) is shown for $\Delta\tau = 75$ and $\Delta\tau = -75$. The other parameters are the same as in Fig. 12.

FIG. 14. Variation of populations in the levels $|g\rangle$ and $|f\rangle$ in long time limit with $\chi$ (in case of tanh chirping) is shown for positive and negative values of detunings. The other parameters are same as in Fig. 12.
the present context. It is clear that logic gate operations can be implemented as well by using nonadiabatic evolution. Finally note that one could preselect a definite time-dependent phase of the pulse to attain a desired final state.

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APPENDIX

We present here an alternative method of orthogonal rotation of a qubit initially in the superposition state \(|i\rangle\) [Eq. (1)]. The state orthogonal to \(|i\rangle\) is \(|k\rangle\) which is given by Eq. (10). The method described here for rotation of the qubit from the state \(|i\rangle\) to \(|k\rangle\) is a generalization of the STIRAP process. This is implemented through adiabatic evolution. The Hamiltonian of the system is the same as in (3), but with \(\Delta \tau = 0\) and \(\chi_1 = 0\) \((i = 1, 2)\). We rewrite this in terms of the basis states \((|i\rangle, |k\rangle, |e\rangle)\) in the following way:

\[
H = \hbar[\Omega_1 \alpha + \Omega_2 \beta e^{i\phi}]|e\rangle\langle i| + \Omega_1 \beta e^{-i\phi} - \Omega_2 \alpha)|e\rangle\langle k| + \text{h.c.},
\]

The relevant time-dependent equations for probability amplitudes in the basis \((|i\rangle, |k\rangle, |e\rangle)\) will be

\[
\begin{align*}
\dot{c}_i &= -i f_1(t) c_e, \\
\dot{c}_k &= -i f_2(t) c_e, \\
\dot{c}_e &= -i [f_1(t) c_i + f_2(t) c_k],
\end{align*}
\]

where

\[
\begin{align*}
f_1(t) &= \alpha \Omega_1(t) + \beta e^{-i\phi} \Omega_2(t), \\
f_2(t) &= \beta e^{i\phi} \Omega_1(t) - \alpha \Omega_2(t).
\end{align*}
\]

We solve these equations under the initial conditions

\[
\begin{align*}
c_i(-\infty) &= 1, \quad c_k(-\infty) = c_e(-\infty) = 0.
\end{align*}
\]

If one chooses the time-dependences of \(f_1\) and \(f_2\) in the following way

\[
\begin{align*}
f_1 &= \exp[-(t-T)^2/\tau^2], \\
f_2 &= \exp(-t^2/\tau^2),
\end{align*}
\]

then, one can transfer the population from \(|i\rangle\) to \(|k\rangle\) adiabatically. This is analogous to the well known STIRAP process in which population is transferred from the state \(|g\rangle\) to \(|f\rangle\). Using Eqs. (VII) and (VIII) we obtain the form of the required pulses \(\Omega_1(t)\) and \(\Omega_2(t)\)

\[
\begin{align*}
\Omega_1(t) &= \alpha \exp[-(t-T)^2/\tau^2] + \beta e^{-i\phi} \exp(-t^2/\tau^2), \\
\Omega_2(t) &= \beta e^{i\phi} \exp[-(t-T)^2/\tau^2] - \alpha \exp(-t^2/\tau^2)
\end{align*}
\]

VII. CONCLUSIONS

In conclusion, we have examined how one can produce a general rotation of the qubit in a superposition of the states \(|g\rangle\) and \(|f\rangle\) by using the nonadiabatic evolution. Two delayed pulses in two-photon resonance have been employed for this purpose. We have found that each of the parameters - one-photon detuning, amplitudes, phases, and chirpings of the two pulses can provide proper control of the qubit rotation. The effect of time separation of the pulses has also been investigated. We discussed the relevance of two-level approximation in
to transfer the population from the state $|i\rangle$ to its orthogonal state $|k\rangle$.

We thus find that, in the long time limit the population is transferred to the state $|k\rangle$ through adiabatic process while the level $|e\rangle$ remains almost unpopulated for all times. Thus the qubit is rotated from $|i\rangle$ to its orthogonal state $|k\rangle$. One may note that as the evolution described here is analogous to STIRAP, so the robustness of the process is similar to that of STIRAP. It is also worth mentioning that one could rotate $|i\rangle$ to any other arbitrary superposition of $|g\rangle$ and $|f\rangle$ by just chopping the pulses $\Omega_1(t)$ and $\Omega_2(t)$ at the desired moment. But this requires complete control on the pulse area.

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