APPLICATION OF SUPERSTATISTICS TO ATMOSPHERIC TURBULENCE

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We successfully apply the recent developed superstatistics theory to a temporal series of turbulent wind measurements recorded by the anemometers of Florence airport. Within this approach we can reproduce very well the fluctuations and the pdfs of wind velocity returns and differences.

1. Introduction

During the last decades an enormous effort has been devoted to understand the physical origin of turbulence, both at the theoretical level and at the experimental one. Although many steps forward have been done, a well established theory of turbulence does not yet exist and many fundamental aspects of this phenomenon remain still unclear.

Atmospheric turbulence is a challenge per se being characterized by very high Reynolds numbers ($Re \sim 10^8$) and very intermittent distributions. On the other hand beyond basic research its interest lies also on the several engineering and meteorological applications. The Kolmogorov hypotheses were verified in the sixties doing measurements in the atmospheric boundary layer considering the flow velocity for a relative short time, with a high sampling rate and for relatively constant mean flow, in order to control and maintain constant the Reynolds number. In general this is not always possible.

Although atmospheric turbulence can have peculiar features regarding the non-stationarity character of the wind data and the high turbulence intensity $^a$

\[ I_v = \frac{\sigma_v}{\langle V \rangle} \]

Typical values for complex terrain are $I_v \geq 0.2$ while, on the other hand for microscale turbulence one usually has $I_v \sim O(10^{-2})$.

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many similarities with microscopic turbulence exists. For a detailed comparison between the two, one can see the recent paper by Böttcher et al.\[9\] In this short paper we discuss a study of a turbulent wind data series recently measured at Florence airport for a period of six months. We show by means of a statistical analysis that we can describe this example of atmospheric turbulence by means of the nonextensive approach adopted in refs. \[8\], \[10\], \[11\], \[12\], \[14\], \[15\], \[16\] within the more general superstatistics formalism introduced in ref. \[13\]. The latter justifies the successful application of Tsallis statistics in different fields, and more specifically in turbulence experiments \[11\], \[12\], \[13\], \[14\], \[15\], \[16\]. We will show that such an approach is meaningful and can reveal very interesting features which could have also a very practical utility for safety reasons when applied to air traffic control services. Part of this study has just been published \[17\] and a longer paper with a complete and exhaustive analysis is in preparation \[18\].

2. Statistical analysis of wind measurements

The wind velocity measurements were taken at Florence airport and were done for a time interval of six months, from October 2002 to March 2003. Data were recorded by using two 3-cups runway heads anemometers, each one mounted on a 10 m high pole, located at a distance of 900 m and with a sampling frequency of one measure every 5 minutes. Although in our experiment we actually could not control the Reynolds number, as usually done in microscopic turbulence, and despite our low sampling frequency (3.3 · 10^{-3} Hz) and the high intermittency of our wind data, we found several features of canonical turbulence as we will discuss in the following. We performed, on our time series, a statistical analysis using conventional mathematical tools which are normally adopted in small scale physical turbulence studied in laboratory. In particular we investigated correlations, spectral distributions as well as probability density functions of velocity components of returns and differences see refs. \[14\], \[15\] for more details. In this short contribution for simplicity and lack of space we discuss only returns of the longitudinal velocity components measured by one of the two anemometers (in the present case the one closest to the runway head 05 and labeled RWY05) defined by the following expression

\[
x(t+\tau) = V_x^{RWY05}(t+\tau) - V_x^{RWY05}(t) ,
\]

\(V_x(t)\) being the longitudinal velocity component at time \(t\) and \(\tau\) being a fixed time interval \(^b\). The same analysis was done also for the transversal components and for velocity difference between the two anemometers with similar results \([14,15]\).

\(^b\)Returns are here defined in a slight different way from those used in econophysics, i.e.: \(\frac{x(t+\tau) - x(t)}{x(t)}\).
2.1. Correlations and power spectra

Our data show very strong correlations and power spectra with the characteristic $-5/3$ law in the high-mid portion of the entire spectrum, see fig. 1 in ref. [17]. However the dissipation branch in the high-range frequency, well known in micro-scale (or high-frequency) turbulence analysis [16], is here missing due to the low-frequency sampling used [17,18]. Correlation functions also show an initial exponential decay, followed by a power law-decay modulated by the day-night wind periodicity which is a well known phenomenon. No significant difference was found for day and night periods, when air traffic is almost absent. For more details please see refs. [17,18].

2.2. The superstatistics approach for wind velocity pdfs

The superstatistics formalism proposed recently by C. Beck and E.G.D. Cohen is a general and effective description for nonequilibrium systems. For more details see the original article and the paper by Beck in this volume [13]. In the superstatistics approach one considers fluctuations of an intensive quantity, for example the temperature, by introducing an effective Boltzmann factor

$$B(E) = \int_{0}^{\infty} f(\beta) e^{-\beta E} d\beta \quad (2)$$

where $f(\beta)$ is the probability distribution of the fluctuating variable $\beta$, so that we have for the probability distribution

$$P(E) = \frac{1}{Z} B(E) \quad , \quad (3)$$

with the normalization given by

$$Z = \int_{0}^{\infty} B(E) dE \quad . \quad (4)$$

One can imagine a collection of many cells, each of one with a defined intensive quantity, in which a test particle is moving. In our atmospheric turbulence studies, the time series of the wind velocity recordings, are characterized by a fluctuating variance, so the returns (1), cannot be assumed to be by a ”simple” Gaussian process. They show a very high intermittent behavior stronger than that one usually found in small-scale fluid turbulence experiments.

In our analysis we considered the following quantities: (a) the wind velocity returns $x$ defined by eq. (1), (b) the corresponding variance of the returns $x$ which we indicate with $\sigma$, (c) the fluctuations of $\sigma$, whose variance we indicate with the symbol $\Sigma$.

We extracted from the experimental data, using an fixed time interval $\tau$, the distribution for the fluctuations of the longitudinal wind component variance. The aim is to slice the time series in ”small” pieces in which the signal is almost Gaussian and apply superstatistics theory. This fluctuating behavior of $\sigma$ is plotted in Fig. 1 for a time interval $\tau = 1$ hour. In Figs. 2 and 3 we then plot the probability
Figure 1. Variance fluctuations of the longitudinal wind velocity component for the anemometer RWY05 obtained with a moving time window $\tau$ of one hour.

Figure 2. Standardized pdf of the fluctuating variance corresponding to the previous figure (open points) are compared with a Gamma distribution (full line) and with a Log-normal distribution (dashed line) sharing the same mean ($\sigma_0$) and variance ($\Sigma$) extracted from experimental data. The Log-normal is not able to reproduce the experimental data.

distribution of the variance $\sigma$ for $\tau = 1$ and 3 hours respectively. In the same figures we plot for comparison a Gamma (full curve) and a Log-normal (dashed curve) distribution characterized by the same average and variance extracted from the experimental data. In this sense, the curves are not fitted to the data. The comparison clearly shows that the Gamma distribution is able to reproduce very nicely the experimental distribution of the $\sigma$ fluctuations and that this type of distribution show robustness for different time windows choices. This is at variance with the Log-normal distribution which is usually adopted in microscopic turbulence.
and which in this case is not able to reproduce the experimental data.

In general using Beck and Cohen notation \[13\] we have for the Gamma distribution

\[
f(\beta) = \frac{1}{b\Gamma(c)} \left(\frac{\beta}{b}\right)^{c-1} e^{-\beta/b}
\]

with

\[
c = \left(\frac{\sigma(\beta)}{b}\right)^2 = \frac{1}{q - 1}, \quad bc = <\beta> = \beta_0
\]

where \(2c\) is the actual number of effective degrees of freedom and \(b\) is a related parameter. Inserting this distribution into the generalized Boltzmann factor (2)

\[
P(x) = (1 - (1 - q)\beta_0 E(x)) \frac{1}{1 - q x^{1-q}}
\]

In our analysis we have \(E = \frac{1}{2} x^2\) with \(x\) defined by eq.(1). \[16\] Considering the fluctuations of the variance \(\sigma\) of the returns \(x\), we get the following correspondence with the original superstatistics formalism

\[
\beta = \sigma_\tau, \quad \sigma(\beta) = \Sigma(\sigma_\tau), \quad \beta_0 = <\sigma_\tau> = \sigma_0
\]

In the present case, we get for the Gamma distributions which describe the experimental variance fluctuations reported in Figs.2 and 3 the characteristic values \(c = 2.70\) and \(c = 3.22\), for a time interval of 1 and 3 hours, from which, using eq. (6), we get the corresponding q-values \(q = 1.37\) and \(q = 1.31\). In Fig.4 we plot the probability density function \(P(x)\) of the experimental longitudinal returns for different
time intervals, i.e. 1 hour (full circles), 3 hours (open diamonds) and 24 hours (open squares). For comparison we plot a Gaussian distribution (dashed curve) and the q-exponential curves (7) characterized by the q-values extracted from the Gamma distributions of Figs.2 and 3 for a time interval $\tau$ corresponding to 1 and 3 hours. The q-exponential curves reproduce very well the experimental data which, on the other hand, are very different from the Gaussian pdf. However one can notice that for a very long time interval, i.e. $\tau =$ 24 hours, the data are not so far from being completely decorrelated and therefore the corresponding experimental pdf is closer to the Gaussian curve. Notice that the theoretical curves are not fitted, and that the superstatistic approach, in a self-consistent and elegant way, is able to explain and characterize in a quantitative way the wind data. In a similar way one can extract theoretical curves which reproduce the wind velocity differences pdfs with similar entropic q-values, although in that case an asymmetry correction has to be considered to better reproduce the tails of the pdfs.\[17,18\]

![Figure 4](image.png)

Figure 4. Comparison between standardized longitudinal velocity returns pdfs for three different time intervals ($\tau =$ 1, 3, 24 hours) and the q-exponential curves with the q-value extracted from the $c$ parameter of the Gamma distribution shown in the previous figures. A Gaussian pdf is also shown as dashed curve. See text.

In our analysis the large and intermittent wind velocity variance fluctuations are reproduced very well by a Gamma type superstatistics excluding the Log-normal one and this gives exactly the Tsallis q-exponential for the velocity returns pdfs. However one has to say that the situation is much more difficult for the less fluctuating velocity flow of the microscale fluid turbulence.

We add as a final remark that, very recently a similar method has been adopted by a research group at the NASA Goddard Space Flight Center to analyze the solar wind speed fluctuations.\[16\]
3. Conclusions

We have studied a temporal series of wind velocity measurements recorded at Florence airport for a period of six months. The statistical analysis for the velocity components shows intermittent fluctuations which exhibit power-law pdfs. Applying the superstatistics formalism, it is possible to extract a Gamma distribution from the probability distributions of the variance fluctuations of wind data. The characteristic parameter $c$ of this Gamma distribution gives the entropic index $q$ of the Tsallis $q$-exponential which is then able to reproduce very well the velocity returns and differences pdfs. Beyond the successful application of superstatistics and Tsallis thermostatistics for turbulent phenomena and the corresponding theoretical implications, we think that this work shows a useful and interesting method to characterize and study in a rigorous and quantitative way atmospheric wind data for safety flight conditions in civil and military aviation.

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