Large Phase of $B_s$-$\bar{B}_s$ Mixing in Supersymmetric Grand Unified Theories

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(Dated: May 20, 2008)

We consider the possibility of a large phase of $B_s$-$\bar{B}_s$ mixing in supersymmetric SU(5) and SO(10) models. We find that in the SU(5) model, the magnitude of this phase is correlated with the branching ratio of $\tau \rightarrow \mu \gamma$ and the phase can be within $2\sigma$ of the recent UTfit analysis. In the case of SO(10) models, this correlation can be relaxed and a large phase can be obtained. We predict the sparticle mass ranges for the LHC for these models once the UTfit result is accommodated and discuss the dark matter and the anomalous magnetic moment constraints on this analysis.

PACS numbers: 12.10.Dm, 12.60.Jv, 12.15.Ff

Recently, CDF and DØ collaborations have announced the analysis of the flavor-tagged $B_s \rightarrow J/\psi \phi$ decay. The decay width difference and the mixing induced CP violating phase, $\phi_s$, were extracted from their analysis [1]. In the Standard Model (SM), the CP violating phase is predicted to be small, $\phi_s = 2\beta_s \equiv 2 \arctan (-V_{ts} V_{tb}^* / V_{ts} V_{tb}^*) \simeq 0.04$. However, the measurements of the phase are large:

\[
\phi_s (CDF) \in [0.32, 2.82] \quad (68\% \text{ CL}) \quad (1)
\]

\[
\phi_s (DØ) = 0.57^{+0.30}_{-0.23} \text{ (stat)} +0.02_{-0.07} \text{ (syst)} \quad (2)
\]

The UTfit group made a combined data analysis including the semileptonic asymmetry in the $B_s$ decay, and find that the CP violating phase deviates more than 3$\sigma$ from the SM prediction [2]. This implies the existence of new physics (NP) and that the NP model requires a flavor violation in $b$-$s$ transition.

Supersymmetry (SUSY) is the most attractive candidate to construct the NP models. In SUSY models, the flavor universality is often assumed in squark and slepton mass matrices to avoid large flavor changing neutral currents (FCNC) in the meson mixings and the lepton flavor violations (LFV) [3]. Even if we assume the flavor universality, the non-universality is generated from the evolution of renormalization group equations (RGE). In the minimal extension of SUSY standard model (MSSM) with right-handed neutrinos, the induced FCNCs from RGE effects are not large in the quark sector, while sizable effects can be generated in the lepton sector due to the large neutrino mixing angles [4]. In the grand unified theories (GUT), the loop effects due to the large neutrino mixings can induce sizable effects in the quark sector since GUT scale particles can propagate in the loops. The patterns of the induced FCNCs highly depend on the unification scenario, and therefore, it is important to investigate the FCNC effects to obtain a footprint of the GUT models.

If the quark-lepton unification is manifested in GUT models, the flavor violation in $b$-$s$ transition can be responsible for the large atmospheric neutrino mixing [5], and thus, it has to be related to the $\tau \rightarrow \mu \gamma$ decay [6]. The branching ratio of the $\tau \rightarrow \mu \gamma$ is being measured at the $B$-factory, and thus, the future results of LFV and the phase of $B_s$-$\bar{B}_s$ mixing will provide an important information to probe the GUT scale physics. In this Letter, we study the correlation between $\text{Br}(\tau \rightarrow \mu \gamma)$ and $\phi_s$ in SU(5) and SO(10) GUT models, and investigate the constraints in these models from the observations in order to decipher GUT models.

We first describe the SU(5) and SO(10) GUT models which we investigate in this Letter. In the SU(5) model, the right-handed down-type quarks ($D^c$) and left-handed lepton doublets ($L$) are unified in 5 representation. The quark doublets ($Q$), right-handed up-type quarks ($U^c$), and right-handed charged-leptons ($E^c$) are unified in 10, and the right-handed neutrino ($N^c$) is a singlet under SU(5). The superpotential which involves the Yukawa interaction is: $W_Y = Y_d^{\nu} \bar{10}^{5} \bar{10}^{5} H_5 + Y_q^{ij} \bar{10}^{5} L_5 H_5 + Y_{e}^{ij} 5_i N_7^c H_5$, where $i,j$ denote the generation indices, and $H_5$ and $H_5$ are the Higgs fields in which colored Higgs fields are unified with the Higgs doublets. The charged-lepton Yukawa coupling $Y_d$ is unified to the down-type quark Yukawa coupling $Y_d$, $Y_d = Y_d^T$, in the minimal SU(5) setup. In the basis where $Y_e$ and Majorana right-handed neutrino mass matrix $M_N$ are diagonal, the Dirac neutrino Yukawa coupling is denoted as $Y_e = U_L Y_d^{\nu} U_R^T$. When $U_R = 1$, the unitary matrix $U_L$ is same as a mixing matrix of neutrino oscillation, and it contains large mixing angles. The Dirac neutrino Yukawa interaction can generate off-diagonal elements of the (squared) scalar mass matrix for 5, $M_5^2$, due to a loop diagram in which $N^c$ and $H_5$ propagate. The induced flavor violating term in the scalar mass matrix is proportional to $Y_d^{\nu} U_L^T U_R^T$. When $U_R = 1$, the unitary matrix $U_L$ is same as a mixing matrix of neutrino oscillation, and it contains large mixing angles. The Dirac neutrino Yukawa interaction can generate off-diagonal elements of the (squared) scalar mass matrix for 5, $M_5^2$, due to a loop diagram in which $N^c$ and $H_5$ propagate. 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as $-1/2 m_\nu^2 \kappa \sin 2\theta_{23} e^{i\alpha}$, where $\theta_{23}$ is a 2-3 mixing in $U_{13}$, which is large and responsible for the large atmospheric neutrino mixing unless there exists a fine-tuned relation among $Y_{\nu}$ and $M_N$. The phase $e^{i\alpha}$ generates a phase of SUSY contribution for $B_s$-$\bar{B}_s$ mixing amplitude, $M_{12}^{\text{SUSY}}$, and the absolute value of $M_{12}^{\text{SUSY}}$ is controlled by $\kappa \sin 2\theta_{23}$. The off-diagonal elements of the scalar mass matrix for 10 are also generated by colored Higgs loop, in addition to the MSSM contribution, but they are small since they get generated from quark mixings.

In the SO(10) GUT model, which we use in this Letter, all matter species are unified in the spinor representation 16. The matter representation can couple to the 10, $\mathbf{126}$ and 120 Higgs representations, $W_Y = h_{ij}16, 16, 10 + f_{ij}16, 16, 10 + h'_{ij}16, 16, 120$. The Majorana neutrino mass is generated from the $f$ coupling term when the $B$ - $\bar{B}$ direction of $\mathbf{126}$ gets a vacuum expectation value (VEV). The fermion Yukawa couplings, $Y_{u,d,e,\nu}$, are given by the linear combinations of $h$, $f$, $h'$ multiplied by Higgs mixings. If there is no cancellation among the $h$, $f$, $h'$ couplings, the Dirac neutrino Yukawa coupling does not have large mixing in the basis where the charged lepton mass matrix is diagonal, due to the presence of right-handed neutrino in 16. Thus, the large neutrino mixings may not originate from $Y_{\nu}$. However, they can originate from the relative mixing angle of $h$ and $f$ couplings in the model. Such a structure can be constructed even in the SU(5) models with additional Higgs fields. The SO(10) model, however, is more predictive since the Majorana coupling and the contributions to Dirac fermion masses are unified to the $f$ coupling. In the basis where $Y_{\nu}$ is diagonal, the $f$ coupling is denoted as $U_f^{-\dagger} i \, \mathbf{T}$, and the unitary matrix $U$ is the neutrino mixing matrix when the triplet term of the type II seesaw is dominant. Through the loop diagram in which the heavy particles from $\mathbf{126}$ (or 120) propagate, flavor violations term can be generated for all fermion mass matrices and it is proportional to $f f^\dagger$ (or $h h^\dagger$). The off-diagonal elements are similarly denoted in the SU(5) case. The phase of SUSY contribution for the $B_s$-$\bar{B}_s$ mixing amplitude $M_{12}^{\text{SUSY}}$ can be generated from the phases of 23 elements of $M_Q$ and $M_D^{\nu}$. The two phases are independent in the SO(10) model in the basis where $Y_d$ is real diagonal, and the freedom of choosing one of these phases governs the phase of $M_{12}^{\text{SUSY}}$. It is important that the absolute value of $M_{12}^{\text{SUSY}}$ is enhanced if both left- and right-handed squark mass matrices have off-diagonal elements. Therefore, a large phase of $B_s$-$\bar{B}_s$ mixing can be expected in SO(10) model which is much larger compared to the SU(5) model.

Now we discuss the phase of $B_s$-$\bar{B}_s$ mixing in the GUT models. We use the model-independent parameterization of the NP contribution: $C_B e^{2i \theta_{23}} = M_{12}^{\text{full}}/M_{12}^{\text{SM}}$, where ‘full’ means the SM plus NP contribution, $M_{12}^{\text{full}} = M_{12}^{\text{SM}} + M_{12}^{\text{NP}}$. The NP contribution can be parameterized by two real parameters $C_B$ and $\phi_{BS}$. The time dependent CP asymmetry ($S = \sin \phi_s$ in $B_s \to J/\psi \phi$) is dictated by the argument of $M_{12}^{\text{full}}$: $\phi_s = -\arg M_{12}^{\text{full}}$, and thus $\phi_{BS}$ is the deviation from the SM prediction: $\phi_s = 2(\beta_s - \phi_{BS})$. It is important to note that the large SUSY contribution is still allowed even though the mass difference of $B_s$-$\bar{B}_s$ [11] is fairly consistent with the SM prediction. This is because the mass difference can be just twice the absolute value of $M_{12}^{\text{full}}$. The consistency of the mass difference just means $C_B \simeq 1$, and a large $\phi_{BS}$ is still allowed. Actually, when $C_B \simeq 1$, the phase $\phi_{BS}$ is related as $2 \sin \phi_{BS} \simeq A_s^{\text{NP}}/A_s^{\text{SM}}$, where $A_s^{\text{NP}} = |M_{12}^{\text{NP}}/M_{12}^{\text{SM}}|$. In the model-independent global analysis by the UTfit group, the fit result is

$$A_s^{\text{NP}}/A_s^{\text{SM}} \in [0.24, 1.38] \cup [1.50, 2.47]$$

at 95% probability. The argument of $M_{12}^{\text{NP}}$ being free in GUT models is due to the phase in off-diagonal elements in SUSY breaking mass matrix (in the basis where $Y_d$ is a real diagonal matrix), and one can choose an appropriate value for the new phase in the NP contribution. Therefore, the experimental data constrain $A_s^{\text{NP}}/A_s^{\text{SM}}$, and that means $\kappa \sin 2\theta_{23}$ is constrained for a given SUSY particle spectrum. Among the flavor violating decay modes, the current bound for $Br(\tau \to \mu \gamma)$ is less than $4.5 \times 10^{-8}$ [12] and $Br(b \to s \gamma) = (3.55 \pm 0.26) \times 10^{-4}$ [13]. In fact, in the models where gaugino masses are unified at the GUT scale (neglecting anomaly mediated SUSY breaking contribution), the $\tau \to \mu \gamma$ constraint is stronger than the $b \to s \gamma$ constraint in most of the parameter space. This is because the squark masses are raised much more compared to the slepton masses due to the gaugino loop contribution since the gluino is heavier than the Bino and the Wino at low energy. This gaugino effect is also important to allow a large phase in the $B_s$-$\bar{B}_s$ mixing. The gaugino loop effects are flavor invisible and they enhance the diagonal elements of the scalar mass matrices while keeping the off-diagonal elements unchanged. If the flavor universal scalar masses at the cutoff scale ($m_5, m_{10}$ in our notation) become larger, both $Br(\tau \to \mu \gamma)$ and $\phi_s$ are suppressed. However, $Br(\tau \to \mu \gamma)$ is much more suppressed compared to $\phi_{BS}$ for a given $\kappa \sin 2\theta_{23}$ because the low energy slepton masses are sensitive to $m_5$ and $m_{10}$ while squark masses are not so sensitive due to the gluino loop contribution to their masses.

In the SU(5) model, we find that $m_5$ has to be larger than 1.2 TeV in order to obtain a large $\phi_{BS}$ at 95% probability of the UTfit result if $m_{10}, \mu < 1$ TeV ($\mu$ is the Higgsino mass) for a universal gaugino mass $m_{1/2} = 300$ GeV and $\tan \beta = 10$, which is a ratio of Higgs fields’ VEVs. On the other hand, in the SO(10) model, we find that $m_{16} (= m_5 = m_{10})$ needs to be larger than 500 GeV if $\mu < 1$ TeV and $m_{1/2} = 300$ GeV and $\tan \beta = 10$. The reason of a smaller scalar mass being allowed in the SO(10) model is due to a left-right enhancement effect.
in the box diagram. The constraint on masses based on UTfit result provides an important guidance to search for the SUSY particles at the LHC. Actually, in general, if quark-lepton unification is manifested in the GUT models, the slepton masses needs to be heavy (especially in the SU(5) model) in order to suppress $\tau \rightarrow \mu \gamma$ and to obtain a large phase of $B_s - \bar{B}_s$ mixing.

The diagrams for $\tau \rightarrow \mu \gamma$ which can provide important effects are the chargino loop diagrams. This contribution can be suppressed if $\mu$ and/or $m_\tau$ are large. If $m_{10}$ is small (which means that right-handed sleptons are light), the Bino diagram can contribute. In SU(5) model, the off-diagonal elements of the right-handed sleptons are small, but the Bino diagram can generate LFV through left-right mixing of sleptons. The amplitude of $\tau \rightarrow \mu \gamma$ is proportional to $\tan \beta$, while $\phi_{B_\mu}$ does not depend on $\tan \beta$ much. Therefore, in order to obtain a larger $\phi_{B_\mu}$, heavier SUSY particles and smaller $\tan \beta$ are favored. In Fig.1, we show the contour plot for $A_s^{NP}/A_s^{SM}$ when the $B_r(\tau \rightarrow \mu \gamma)$ bound is saturated. To draw the figure, we choose $m_{1/2} = 300$ GeV, $m_5 = 2$ TeV, and $\tan \beta = 10$. We assume that SUSY breaking Higgs masses, $m_{H_u}$ and $m_{H_d}$, are free to make $\mu$ to be a free parameter, but we assume to $m_{H_u} = m_{H_d}$ at the GUT scale just for simplicity. In the figure, the blue shaded region shows 1σ allowed region and the yellow shaded region is enclosed for 2σ allowed region. The right-bottom area in the figure is excluded since the stability condition of the Higgs potential is not satisfied. In the area, closer to the excluded region, the mass of the charged Higgs boson is light and therefore, $b \rightarrow s \gamma$ becomes large. If $\mu$ is large, the gluino contribution becomes large for right-handed operators of $b \rightarrow s \gamma$ (which is often called $C_{7R}, C_{8R}$) due to the large left-right mixing for sbottom, and in this region, $b \rightarrow s \gamma$ is more important rather than $\tau \rightarrow \mu \gamma$. One can see that large $A_s^{NP}/A_s^{SM} > 0.38$ (which is at 68% probability) is allowed for a large value of $\mu$. This is because the chargino contribution for $\tau \rightarrow \mu \gamma$ is suppressed in those area. The contours are curved down for small $m_{10}$ because the Bino diagram starts contributing. It is worth noting that the stau coannihilation region (where the lightest stau and the lightest neutralino mass difference is $\sim 5-15$ GeV) $m_{10} \sim 100$ GeV to satisfy the dark matter content is not allowed at 95% probability in this figure. To revive the stau coannihilation region, $\tan \beta \sim 5$ is needed and then scalar trilinear coupling has to be chosen appropriately to satisfy the lightest Higgs mass bound. The Higgsino dark matter is not very favored (though it is allowed for $\tan \beta = 10$ in 95% probability) since a small value of $\mu$ does not suppress the chargino contribution for $\tau \rightarrow \mu \gamma$. The stop coannihilation mechanism or $A$-funnel region would be more suitable to explain the dark matter content if the quark-lepton unification is manifested and the experimental constraint from flavor violation is considered. A detailed discussion on dark matter constraint will be presented elsewhere.

Unfortunately, in order to reduce the chargino contribution to $\tau \rightarrow \mu \gamma$, the sleptons must be heavy enough, and as a result we find that one cannot provide a solution for a muon $g - 2$ anomaly to obtain large $\phi_{B_\mu}$ if quark-lepton unification is manifested. Therefore, we should consider the possible breaking of the manifest quark-lepton unification. So, let us consider the possibility that $\kappa \sin 2\theta_{23}$ is different between the squark and slepton sectors. To begin with, the 23 mixing angle $\theta_{23}$ can be different between quarks and leptons because $Y_d$ and $Y_e$ may not be simultaneously diagonalized. In minimal SU(5) model, $Y_d$ and $Y_e$ are unified, but it gives a wrong prediction for the 1st and 2nd generation masses, and we need a correction from 45 Higgs field or non-renormalizable interaction. Actually, there is a freedom to choose $\theta_{23}^{\text{lepton}} < \theta_{23}^{\text{SM}}$ which is needed to relax the constraint arising from $\tau \rightarrow \mu \gamma$. However, the motivation to explain the large neutrino mixing is lost. This situation is same as in the SO(10) models. Thus, let us consider the possibility that $\kappa$ is different between quark and lepton sectors. Actually, it should be different since the flavor violation terms are generated from the loop diagram in which heavy particles propagate, and the heavy particles should split when GUT symmetry is broken. In SU(5) model, however, it always gives wrong direction, i.e., $\kappa^{\text{quark}} < \kappa^{\text{lepton}}$. The RGE effect to generate the flavor violation survives till the right-handed Majorana mass scale for left-handed slepton, but it ends at the colored Higgs mass scale for the right-handed down-type squarks. To satisfy the nucleon decay bounds, the colored Higgs need to be heavier rather than the right-handed Majorana mass scale when $Y_{\tau}$ is less than $O(1)$. Therefore, in a model, where the flavor violation originates from the Dirac neutrino Yukawa coupling, a large $\phi_{B_\mu}$ is disfavored. In this sense, the Fig.1 is drawn conservatively. When the Majorana mass is $10^{14}$ GeV, the colored Higgs mass is $10^{16}$ GeV, and the cutoff scale is
$10^{18}$ GeV, one then obtains $2\kappa_{\text{quark}} \simeq \kappa_{\text{lepton}}$. At that time, the contour values for $A^{\text{NP}}_{5}/A_{5}^{\text{SM}}$ should be reduced roughly by half to saturate the $\tau \rightarrow \mu \gamma$ bound and then $\mu$ has to be larger than 1 TeV.

In the SO(10) model, $\theta_{23}$’s for quarks and leptons are not very different if there is no huge cancellation in the Yukawa couplings, $h$, $f$, and $h'$. However, $\kappa$ can be different among the fermion species, and it depends on the SO(10) breaking vacua and the Higgs spectrum from $\mathbf{126}$. If the mass of one of the Higgs representation is light compared to the SO(10) breaking scale, the off-diagonal elements for some fermion species are generated. For example, when the SU(2)$_R$ symmetry remains unbroken and $(1, 1, \pm 1)$ fields from $\mathbf{126}$, which are SU(2)$_R$ Higgsinos, remain light, the off-diagonal elements for only right-handed slepton are generated. Obviously, these do not generate a large $\phi_{B_{X}}$. When $(8, 2, \pm 1/2)$ fields of $\mathbf{126}$ are light, only off-diagonal elements for squarks are generated while the same elements for sleptons are small. This is the proper way to generate large $\phi_{B_{X}}$ without suffering from the $\tau \rightarrow \mu \gamma$ constraint. It is interesting to note that the light $(8, 2, \pm 1/2)$ fields are good candidate to suppress the nucleon decay in these models. In this way, it is possible that the experimental measurements of large $\phi_{B_{X}}$ and Br($\tau \rightarrow \mu \gamma$) for a given SUSY particle spectrum can constrain the GUT scale particle spectrum. Thus, more experimental data is very important to probe the GUT scale physics.

If a suitable SO(10) breaking vacuum is selected, the $\tau \rightarrow \mu \gamma$ and $\phi_{B_{X}}$ relation can be completely broken. If both left- and right-handed squarks have off-diagonal elements in this vacuum, the large $\phi_{B_{X}}$ can be easily obtained. In Fig. 2, we demonstrate the allowed region for different $\kappa m_{5}^{2}$. We assume a universal scalar mass $m_{0} = m_{5} = m_{10} = m_{H_{u}} = m_{H_{d}}$. The figure is drawn in the case of $\tan \beta = 10$, though $\tan \beta$ is not an important parameter in the plot. When $m_{0}$ is small, the trilinear scalar coupling $A_{0}$ has to be large in order to make $\kappa$ large. If $\tau \rightarrow \mu \gamma$ is suppressed by the choice of a vacuum, the $b \rightarrow s \gamma$ constraint becomes more important especially for large $\tan \beta$. However, since the phase of the chargino contribution is free in the SO(10) boundary condition (actually it is independent of the phase of $M_{2}^{\text{SUSY}}$), experimentally allowed solutions can be found as long as the gluino contribution for $C_{BR}$, $C_{SR}$ is not very large. When the chargino contribution is large, the direct CP asymmetry for $B \rightarrow X_{s} \gamma$ may become large. This is similar to the case where $\phi_{B_{X}}$ is large while $C_{BR} \sim 1$. In Fig. 3, we plot the direct CP asymmetry and $A^{\text{NP}}_{5}/A_{5}^{\text{SM}}$ for allowed Br($B \rightarrow X_{s} \gamma$). Due to the freedom of the off-diagonal elements of up-type squark mass matrices, the $b \rightarrow s \gamma$ constraint can be satisfied even in the usually excluded region in the case of minimal supergravity model. Actually the parameter $m_{1/2} = 200$ GeV we choose to draw the Fig. 3 is not allowed in the minimal supergravity for $\tan \beta = 40$. Thus in this region, one needs off-diagonal elements of squark mass matrix, and thus, a non-zero value of $A^{\text{NP}}_{5}$ is predicted. If Br($B \rightarrow X_{s} \gamma$) is fixed to a particular value, this plot is just a circle. The dark matter and the anomalous magnetic moment constraints are easily satisfied in this scenario.

In this Letter, we have emphasized the importance of $\tau \rightarrow \mu \gamma$ and $\phi_{B_{X}}$ correlation in GUT models, since they can be correlated directly by 23 mixing. The constraint from $\mu \rightarrow e \gamma$ may be also important, but this Br calculation highly depends on the details of flavor structure which can have a freedom of cancellation. Therefore, we have not talked about the $\mu \rightarrow e \gamma$ constraint. We refer to the Ref. [10] for an analysis of flavor violation including the first generation.

In conclusion, we have investigated the large phase of $B_{s}-\bar{B}_{s}$ mixing by comparing SU(5) and SO(10) GUT models. The existence of $\phi_{B_{X}}$ in GUT models can tell us whether the flavor violation originates from Dirac neutrino Yukawa coupling or Majorana coupling such as the $\mathbf{126}$ Higgs coupling in the SO(10) model. At present, the SO(10) models are more preferred rather than the
Dirac neutrino induced flavor violation in SU(5) models. It is important to note that we can distinguish these two scenarios once more experimental data on $B_s - \bar{B_s}$ mixing phase and $\text{Br}(\tau \to \mu \gamma)$ decay are available along with the data from the LHC.

This work is supported in part by the DOE grant DE-FG02-95ER40917.

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