Search for Spatial Structures at Scales $Z \sim 1$.
III. The Effect of Lensing on QSO ?

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Abstract. We carried out a search for peak inhomogeneities in the distribution of matter - namely clumps and voids, within the range $Z \sim 1 - 3$. We used a new method, based on the lensing of quasars by a combination of lenses, belonging to the above sought inhomogeneities in the matter distribution. This work confirms the evidence of the existence of inhomogeneities found by us earlier - of a clump (superattractor N.1), and of a void (supervoid). Besides, the existence of a new gigantic clump (superattractor N.2) was also discovered at $Z \sim 3$. These clumps could well serve as centers of the Bose-condensation in the early Universe; in particular - as Anselm’s arion condensate, which leads to the formation of quasiperiodic structures with a period $p \sim 100 - 200$ Mpc.

Keywords: Grav.lensing,quasars,matter clumps

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1. Introduction

The structure of the observed part of our Universe—the Metagalaxy—has been studied so far along the lines of spatial distribution of galactic clusters and superclusters. These clusters have dimensions ranging from about a few Mpc for the clusters to about tens of Mpc for the superclusters. It has been assumed that these structures were formed about 10-15 billion years ago. The more earlier structures could define cosmic objects that were formed before the usual galaxies; quasars, for example. Work that has been carried out on the statistical analyses of quasar catalogs over the last decade, clearly outlines the quasiperiodic structures with a characteristic scale around 100 Mpc (Kunth, 1989; Litvin et al., 1993; Ryabinnikov et al., 1998). It is possible, according to Anselm (1990), that the factor responsible for the evolution of such anomalies in the matter at the above mentioned scales, could be a scalar field, which, at a temperature less than the temperature of separation of radiation from a substance, turns into a condensate. Anomalies in the matter distribution of such a model arise due to a gravitational link between the matter and the scalar field. However, a strongly unambiguous quasiperiodic structure requires that the condensate layers be spherically symmetric. This fact was pointed out in the works by Litvin et al. (1994), and it was also shown that a symmetry like this could be obtained with the presence of a center (or centers) being condensed. The role of such a center could be taken by the so-called Superattractor (SA) discovered by Litvin et al. (1994).

The hypothesis about the existence of the SA in a given direction on the celestial sphere agrees with the existence of a maximum density of quasars with absorption lines in that direction (Litvin et al., 1994). Pertaining to this, a case of special interest is one introduced by Dravskikh & Dravskikh (1994), on the amplification effect of the observed brightness of quasars having absorption lines, as opposed to those without absorption lines. This result was confirmed by Van Den Berk et al. (1996). A possible explanation of this effect in the above works could possibly be the lensing of quasar radiation by massive objects which make up the absorbing medium. Thus, the search for directions of maximum lensing of quasars over the entire celestial sphere could become an approach to solving the problem of bringing out the structures in the distribution of gravitating matter at scales $Z \sim 1$. In the present paper, we propose a method to carry out such a search.

We continue with an analysis, started in the works by Litvin et al. (1997), of Hewitt & Burbidge’s quasar catalog. The analysis method is based on the search for assumed directions of maximum lensing, and the results obtained are presented in section 2, with a discussion in...
section 3. What is important in the present paper, as compared to the
one by Litvin et al. (1997), is the transformation from equatorial to
galactic coordinates. This allows us to most effectively follow random
structures that could arise around the region of the galactic equator, as
a result of strong random fluctuations. Details on the probability of the
formation of such structures and statistical analyses of their results are
discussed in sections 3 and 4. A more general discussion and conclusion
is then given in section 5.

2. Method used to search for the lensed matter

As a parameter characterizing the lensing of quasars, we chose $K_i$, which is the ratio of: the mean apparent magnitude of the quasar
population, falling within the $i$-th standard cell (in a circle of radius
$R = 40^\circ$ on the celestial sphere), $V_i$, to the mean apparent magnitude
over the entire celestial sphere, $V_{4\pi}$.

$$K_i = \frac{V_i}{V_{4\pi}}$$

The standard cells are located over the celestial sphere at 150 nodes of
a mesh. The positions of these nodes are chosen by means of a principle,
where the celestial sphere quasiuniformly covers these nodes.

This procedure allows us to construct, with the help of a software
called Winsurfer, a topography $K_i (l,b)$ (in the galactic coordinate
system) as a combination of lines of equal height for the given red-
shift interval $Z_e$, which is defined over QSO emission lines. Similar
to the works by Litvin et al. (1997) we used four red-shift intervals,$Z_e[0.5; 2.5]; Z_e[1; 3]; Z_e[1.5; 3.5]; Z_e[2; 4.5]$. Fig.1 shows the corre-
sponding topographies for four populations of quasars from the He-
witt & Burbidge catalog (1993). The main characteristic on all four
topographies is the presence of well-defined maxima and minima.

3. Discussion of results obtained by the above method

Let us try to discuss a possible interpretation of the presence of these
“spots” (i.e. regions, where the parameter, $K_i$ is extremal).
3.1. “Spots” with a minimum value for $K_i$

One observes two such spots on the topographies for the four QSO populations, at $l_1 = (250^\circ \pm 20^\circ); b_1 = (-5^\circ \pm 20^\circ)$ and at $l_2 = (120^\circ \pm 20^\circ); b_2 = (5^\circ \pm 20^\circ)$ respectively. Let us consider each “spot” separately.

3.1.1. “Spot” $l_1 = (250^\circ \pm 20^\circ); b_1 = (-5^\circ \pm 20^\circ)$

The existence of the region where $K_i$ is minimum in the given direction may be related to the lensing of quasars, since it is in this direction that Litvin et al.(1994) had earlier discovered the maximum density of quasars with absorption lines, and this fact supports the amplification effect of lensing according to Dravskikh & Dravskikh. Besides, in this direction, an anomalous behaviour of the slope of the straight line on the Hubble diagram also favors the lensing hypothesis. This was explained by Litvin et al.(1994) with the Superattractor (SA) hypothesis according to which, an increase in the average brightness of quasars in this direction may be due to an increased probability of quasar lensing by matter from the SA. On the other hand, according to Baryshev and Yezova (1997), in the case of strong lensing, under a fractal distribution of matter along the line of sight, the most probable position of the lens would be, either closer to the observer or to the source. If the above mentioned fact materializes in our case, then a maximum lensing effect will be seen for an interval $Z_\epsilon$ closer to the approximated position of the SA ($Z_{SA} = 2.13 \pm 0.60$ according to Litvin et al.1997), i.e. for the interval $Z_\epsilon[1;3]$. And indeed, in Fig.1, we notice that the region of minimum $K_i$ at $l_1 = (250^\circ \pm 20^\circ); b_1 = (-5^\circ \pm 20^\circ)$ is most brightly expressed within the interval $Z_\epsilon[1;3]$.

3.1.2. “Spot” $l_2 = (120^\circ \pm 20^\circ); b_2 = (5^\circ \pm 20^\circ)$

This spot also appears on the topographies for all four sets of quasars, but the effect is maximum for the interval $Z_\epsilon[1.5;3.5]$ and it reduces with a decrease in $Z_\epsilon$. Thanks to an increased probability of strong lensing of quasars in a given direction, we shall follow the above suggested interpretation for the region of minimum $K_i$ as being the direction of maximum amplification of mean quasar brightness. Let us therefore consider the behaviour of the topography of the relative density of absorbers in this region, for the interval $Z_\epsilon[1.5;3.5]$. The topography corresponding to this region, which was constructed by us according to the method proposed by Litvin et al.(1997), in galactic coordinates, is presented in Fig.2A (whereas for comparison, Fig.2B has the real topography for the lensing parameter $K_i$ within the interval $Z_\epsilon[1.5;3.5]$). The presence of a spot where the value of the relative density of absorbers is maximum $l = (135^\circ \pm 20^\circ); b = (10^\circ \pm 20^\circ)$ in Fig.2A confirms
the above hypothesis of the connection between the discussed spot $l_2 = (120^\circ \pm 20^\circ)$; $b_2 = (5^\circ \pm 20^\circ)$ in the topography in Fig.1 and the effect of lensing.

For the given spot, we shall not be discussing results on how the topography of the slope of the straight line on Hubble’s diagram behaves in galactic coordinates. This is because the method used in defining the slope is very sensitive to random fluctuations which arise due to minor changes in the number of quasars falling within a particular cell. The effect becomes even stronger with an increase in $Z_e$, since an increase in $Z_e$ means a decrease in the total amount of data in Hewitt & Burbidge’s catalog (1993). This way, wherever there is a discussion on the topography of the slope of the straight line on Hubble’s diagram we use only a qualitative comparison with results from the works by Litvin et al. (1997), since in those works we had used equatorial coordinates but had not considered the region around the North Pole due to strong distortions in the Mercatorial projections.

3.2. “Spot” with a maximum value for $K_i$

The position of this spot on the celestial sphere $l_3 = (60^\circ \pm 20^\circ)$; $b_3 = (0^\circ \pm 20^\circ)$ coincides, within a margin of error, with an earlier discovered, but uninterpreted region (Litvin et al., 1994; Litvin et al., 1997) where the slope of the straight line on Hubble’s diagram is maximum. In both cases, the spots were observed only within the interval $Z_e \in [2; 4.5]$ unlike the spots of minimum $K_i$ considered earlier.

As a cause for the appearance of this spot one could assume the existence of a gigantic Supervoid in the region $Z_e \sim 3$.

Let us consider, for simplicity, a spherical void. Quasars, located at the farther (relative to the observer) boundary of the void, are drawn into motion by an unbalanced gravitational attraction of mass outside the void region. As a result, the observed red-shift of such quasars increases. On the other hand, this effect leads to a decrease in the red-shift of quasars located at the nearer boundary of the sphere. Because of this we observe an increase in the slope of the line on Hubble’s diagram, which leads to the formation of a spot of maximum slope in the direction of the void.

In order to explain, within the bounds of the present hypothesis, the formation or occurrence of a spot with a maximum mean stellar value of quasars on the topography in Fig.1D, it is necessary to take into account the distribution of quasars as a function of $Z_e$ in Hewitt & Burbidge’s catalog (1993). The number of quasars in a unit interval around $Z_e \sim 4.5$ is negligibly small when compared to their number around $Z_e \sim 2$. Thus the effect by which the red-shift of quasars around
$Z_e \sim 2$ decreases, can be considered a dominant factor. At the same time, quasars having a $Z_e > 2$, but lying around the boundary $Z_e = 2$, could "fall out" from the interval $Z_e = [2; 4.5]$, these being the brightest for the given interval. As a result, we observe an overall increase in the mean apparent stellar magnitude within the interval $Z_e = [2; 4.5]$ in the direction of the void (spot of maximum $K_i$ in Fig.1D). It should be noted that in the absence of the void within other intervals (Figs.1:A,B,C) the above effect is not observed.
4. A statistical analysis of the data obtained

A detailed analysis of the topograms, about which we had spoken in the previous section, shows, that all extremal spots lie close to the plane of the galactic equator. This carries a definitive scepticism in relation to the proposed interpretation of the results obtained by us. Poor statistics in the region near the galactic equator leads to an increased influence of random fluctuations in the data analysis. In this way, the probability of occurrence of random structures, analogous to the spots in Figs.1 and 2, increases. Moreover, a non-uniform choice of objects from the catalog for different areas of the celestial sphere can lead to a selection. In order to evaluate the influence of these factors, we carried out an analysis of the Hewitt & Burbidge (1993) catalog using two different methods:

1.) An estimate was carried out of the probability of random occurrence of “extremal” spots on the topograms in Figs.1 and 2.

Similar to section 2, we used the method of constructing topographies for \( K_i \) by taking data from the same catalog. The role of standard cells was taken over by \( N \) identical domains, e.g. circles of radius \( R \), whose centers form a mesh on the celestial sphere. (For the sake of convenience, we used a mesh that was rectangular in the Mercatorial projection of the celestial sphere). The size and the corresponding number of domains were defined by the necessary condition that \( N_{\text{obj}} \geq 5 \) objects per standard cell. That way, after creating the topography of the celestial sphere, we obtained a set of domains, wherein each of which, the lensing parameter \( K_i \) was defined.

Now, let the number of such domains be equal to \( n \); ( where, for a spot of minimum \( K_i \), the relation \( K_i \leq K_{cr} \) is true; or where, for a spot of maximum \( K_i \), the relation \( K_i \geq K_{cr} \) is true; \( K_{cr} \) being some critical cut-off parameter on the topogram ).

Also, let the lensing topogram have a spot made up of \( m \) domains, which satisfy \( K_i \leq K_{cr} \) ( or \( K_i \geq K_{cr} \), in the case of a maximum ). By “spot” we imply a set of domains in a rectangular mesh, where the center of each new element differs from the previous element by exactly one step of the mesh.

In order to estimate the probability of the random formation of the given spot in any part of the topogram, we carried out the following procedure:

\( n \) domains with \( K_i \leq K_{cr} \) ( or \( K_i \geq K_{cr} \), in the case of a maximum) were randomly thrown over a sphere and the resulting topogram was then scanned to check for at least a single case in which the spot would occur, within \( m \) domains satisfying \( K_i \leq K_{cr} \) ( or \( K_i \geq K_{cr} \), in the case of a maximum).
The next step was to calculate \( q \) such occurrences for \( Q \) throws or iterations. Finally, the ratio \( p = q/Q \) gives us the required probability value.

Results of our calculations for spots with minimum and maximum values of the parameter \( K_i \) (Figs.1:A,B,C,D) are presented in Tables I,a,b,c,d; II,a,b,c,d and III.

Similarly, we can estimate the probability of a random occurrence of extremal spots on the topograms of the relative density of absorbers (see Fig.2A and also Litvin et al.,1997). These estimations are presented in Table IV.

Calculations (Table I.) show that the occurrence of two of the spots of minimum \( K_i \) on the lensing topography (the first one in the direction \( l_1 = (250^\circ \pm 20^\circ) \); \( b_1 = (-5^\circ \pm 20^\circ) \) within the intervals \( Z_{e\epsilon}[0.5;2.5]; Z_{e\epsilon}[1;3] \); and \( Z_{e\epsilon}[2;4.5] \); and the second one in the direction \( l_2 = (120^\circ \pm 20^\circ) \); \( b_2 = (5^\circ \pm 20^\circ) \) within the interval \( Z_{e\epsilon}[1.5;3.5] \)) can be considered non-random. However, while doing so, it is understood that effects of a selection cannot be omitted (see the corresponding estimate below). The same can be said about spots of maximum \( K_i \) on the lensing topogram within the interval \( Z_{e\epsilon}[2;4.5] \) (Fig.1D and Table III) and on the topogram of the relative density of absorbers within the interval \( Z_{e\epsilon}[1.5;3.5] \) (Fig.2A and Table IV).

An estimate was also carried out of the probability of occurrence of extremal spots on the topogram of Fig.1 as a result of possible quasar selection effects from the catalog by Hewitt & Burbidge (1993).

In this method, we carried out a reapproximation ("mixing") of quasar parameters (namely apparent stellar magnitude and redshift), but their coordinates remained fixed. At the same time, using the standard procedure (see section 2.), we defined a set of \( N_{knots} \) values for the lensing parameter \( K_i \) (where, \( N_{knots} \) is the number of standard cells, chosen by us, and defined by the radius of the cells, in particular, by the condition that a sphere entirely covers the cells with a minimum excess covering).

For a given \( p \)-th set of cells we defined a maximum \( K_{p,\text{max}} \) and a minimum \( K_{p,\text{min}} \) for the value of the lensing parameter \( K_i \).

We then calculated \( n_1 \) sets for which the following condition was satisfied:

\[
(K_{p,\text{max}} - K_{p,\text{min}}) \geq (K^{\text{max}} - K^{\text{min}}),
\]

where \( K^{\text{max}} \) and \( K^{\text{min}} \) - corresponding maximum and minimum values for the lensing parameter, defined for the catalog by Hewitt & Burbidge (1993) without a "mixing".
The ratio, \( p_1 = n_1/Q_1 \) then gives us the estimate of the probability of the formation of spots on the topograms (Figs.1 & 2) taking into account the effects of a selection. The results of these calculations within four \( Z_e \) intervals and for three values of the radius \( R \) of a standard cell, are presented in Table V.a,b,c,d.

An analogous estimate of the probability for the topography of absorbers in Fig.2A is presented in Table VI.

Results from Tables V & VI support the fact that the probability of occurrence of extremal spots due to closed selection effects are very small. This, in conjunction with the results obtained above of an estimate of the probability of random occurrence of these spots (see above for discussion of Tables I-IV), leads us to conclude that the role of random fluctuation effects is very small, and the formation of anomalies on the topographs (Figs.1 & 2) can be interpreted as a physical effect, according to section 2.

In conclusion, it is interesting to note a certain characteristic in the behaviour of the spot at minimum \( K_i \) as a function of \( Z_e \), in the direction \( l_1 = (250^\circ \pm 20^\circ); b_1 = (-5^\circ \pm 20^\circ) \). For this direction, the lensing effect is maximum within the interval \( Z_e \in [1; 3] \). On increasing or decreasing \( Z_e \) within the intervals \( Z_e \in [0.5; 2.5] \) and \( Z_e \in [1.5; 3.5] \) the effect weakens (within \( Z_e \in [1.5; 3.5] \) the effect is almost statistically negligible—see Table Ic). However, within \( Z_e \in [2.5; 4.5] \) the effect is stronger and an occurrence by chance of the corresponding spot is highly improbable (Table Id.). At first sight, such a “strange” behaviour of the anomaly in the direction \( l_1 = (250^\circ \pm 20^\circ); b_1 = (-5^\circ \pm 20^\circ) \) can be connected to different “regimes” of lensing of radiation from near and far (relative to the SA) quasars by components of matter from the SA. For quasars located nearer to the SA, in \( Z_e \in [1; 3] \) (i.e. in the region of maximum concentration of lenses) there exists an increased probability of strong lensing effects as a result of a strong probability that the observer could fall in the region of a conical caustic (Baryshev & Yezova, 1997). This was already mentioned in section 2. On increasing the relative lens-quasar distance, the probability of strong lensing drops, but the probability of multiple weak-lensing increases (if the distance to the lens is greater than half the distance to the quasar). In this case, the maximum probability of lensing occurs, if the lens is exactly half-way between the observer and source. Hence, when the lenses are concentrated in the region around \( Z \sim 2 \), one should expect an amplified lensing effect of quasars at \( Z_e \sim 4 \), i.e. in our case, within the interval \( Z_e \in [2; 4.5] \).
5. Discussion

An analysis of the results obtained with the help of the method proposed in this paper, of finding the direction of maximum lensing of quasars, is indeed a continuation for the search of large-scale inhomogeneities in the distribution of matter at scales $Z \sim 1$. The existence of one such inhomogeneity around $Z \sim 2$ - the SA, which was found by us earlier while analysing the spatial distribution of absorbers, confirms the presence of a maximum in the mean observed brightness of quasars in the direction of the SA. Yet another maximum in the mean observed quasar brightness was discovered in the region $Z \sim 3$ in a direction (equatorial coordinates) that we had not searched. This anomaly, as in the previous case, corresponds to a maximum in the density of quasars with absorption lines. In this way, we can talk about the existence of yet another large-scale inhomogeneity in the distribution of matter at scales $Z \sim 3$, which analogous to the first, was named SA2.

To observable parts in the Universe the inhomogeneity picture in the distribution of matter also outlines the existence of a gigantic void (SV) at distances, estimated by us to be within $Z \sim 2 - 3$. In a very definitive sense, the SA and SV form a gravitational dipole. The direction, defined by the axis of this dipole, is quite close to the “Axis of the Universe”, which in turn is defined by the orientation of radiogalaxies (Amirkhanjan, 1994).

We assume that the main reason why we observe structures on the topographies is because of effects related to the “anomalous” amplification or absorption of quasar radiation during their path through large scale matter non-uniformities; and not because of effects that arise due to random fluctuations or selections. The basis for such an assumption are the statistical calculations described in section 4. Besides, we carried out a series of tests to estimate the possible extra effects, connected with very poor statistics in the galactic equator region. In particular, with the help of a standard algorithm (section 2) we had calculated the behaviour of the median of the apparent stellar magnitude of quasars in different directions on the celestial sphere. The median, unlike the mean stellar magnitude, is less sensitive to random fluctuations arising due to poor statistics; and the fact that the structures observed on it’s topography agree with the behaviour of the extremums on the lensing topography (Fig.1) speak in favor of a statistical significance of these extremums. However, we cannot totally exclude selection effects. Results of yet another test, related to the analysis of the depth of selection of catalog objects, show that there exists some correlation between that region on the celestial sphere where the limiting apparent magnitude is minimum (i.e. possibly only bright objects were accounted for) and
the direction towards SA2. With the presence of such an observable selection it could be possible to also explain the “effective” amplification in the mean brightness of quasars along a given direction and the presence of a maximum in the absorption bands of their spectra.

Effects of absorption and lensing of quasar radiation by massive objects that form the medium through which this radiation traverses, seem to be a real way to detect and analyze clustered structures of dark matter at scales $Z \sim 1$. In some cases, one can get more information about the presence of such clusters from the chemical analysis of quasar emission spectra. In the work by Gnedin & Ostriker (1997), for example, it says that the process of hierarchical clustering of neutral gas in the early Universe could have possibly led to the formation of clusters of massive stars. An intensive burning of light elements and a generation of heavier elements in these stars leads to a strong “enrichment” of nearer regions with heavy elements. So, the presence of an increased composition of heavy elements in the observable region may be proof of occurrences of strong matter-clustering processes there.

Using this method, we carried out a comparative analysis of spectral lines of quasars from the Hewitt & Burbidge catalog (1993) in different parts of the celestial sphere. The results obtained tell us about a slight excess over the background of the relative composition of heavy elements in regions adjoining the SA and SA2. Even though these are based on poor statistics, yet in conjunction with other results, they confirm our conclusions about the existence of gigantic clusters of matter at scales $Z \sim 1$. This is in agreement with the opinion of Sylos Labini (1996), that the known clusters and galactic superclusters are not the most large scaled structures.

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Captions to Figures

Figure 1. Topography of the lensing parameter $K_i$ in the Mercatorial projection of the celestial sphere, for four different intervals of $Z_e$. Radius of the standard cell, $R = 40^\circ$. Number of cells, $N_{knots} = 150$.
A) $Z_e\epsilon[0.5; 2.5]$
B) $Z_e\epsilon[1; 3]$
C) $Z_e\epsilon[1.5; 3.5]$
D) $Z_e\epsilon[2; 4.5]$

Figure 2. Topography of:
A) The relative density of absorbers, and
B) The lensing parameter $K_i(l, b)$
both, in the Mercatorial projection of the celestial sphere in the interval $Z_e\epsilon[1.5; 3.5]$. Radius of the standard cell, $R = 40^\circ$. Number of cells, $N_{knots} = 150$. 
Table I. Results of the probability estimate of random occurrence of a spot with a minimum $K_i$ on the lensing topography in the direction $l_1 = (250^\circ \pm 20^\circ)$; $b_1 = (-5^\circ \pm 20^\circ)$, for four intervals of $Z_e$. Radius of the standard cell, $R = 40^\circ$. Number of cells, $N_{knots} = 108$. Number of throws or iterations, $Q = 10^4$. Explanation is given in the text.

I.a) $Z_e \epsilon [0.5; 2.5]$

| $K_{cr}$ | 0.963 | 0.968 | 0.972 |
|---------|-------|-------|-------|
| m       | 5     | 6     | 8     |
| n       | 6     | 7     | 13    |
| p%      | 0.05  | 0.09  | 1.8   |

I.b) $Z_e \epsilon [1; 3]$

| $K_{cr}$ | 0.968 | 0.97  | 0.972 |
|---------|-------|-------|-------|
| m       | 5     | 6     | 7     |
| n       | 6     | 8     | 11    |
| p%      | 0.05  | 0.2   | 0.6   |

I.c) $Z_e \epsilon [1.5; 3.5]$

| $K_{cr}$ | 0.963 | 0.967 | 0.97  |
|---------|-------|-------|-------|
| m       | 2     | 2     | 2     |
| n       | 3     | 6     | 9     |
| p%      | 8     | 42    | 76    |

I.d) $Z_e \epsilon [2; 4.5]$

| $K_{cr}$ | 0.966 | 0.97  | 0.972 |
|---------|-------|-------|-------|
| m       | 5     | 6     | 9     |
| n       | 6     | 8     | 11    |
| p%      | 0.05  | 0.2   | 0.7   |
Table II. Results of the probability estimate of random occurrence of a spot with a minimum $K_i$ on the lensing topography in the direction $l_2 = (120^\circ \pm 20^\circ)$; $b_2 = (5^\circ \pm 20^\circ)$, for four intervals of $Z_e$. Radius of the standard cell, $R = 40^\circ$. Number of cells, $N_{knots} = 108$. Number of throws or iterations, $Q = 10^4$. Explanation is given in the text.

II.a) $Z_e[0.5; 2.5]$

| $K_{cr}$ | 0.971 | 0.973 | 0.975 |
|---|---|---|---|
| $m$ | 5 | 8 | 9 |
| $n$ | 12 | 17 | 19 |
| $p\%$ | 2.7 | 6.7 | 14 |

II.b) $Z_e[1; 3]$

| $K_{cr}$ | 0.972 | 0.974 | 0.976 |
|---|---|---|---|
| $m$ | 4 | 7 | 8 |
| $n$ | 11 | 15 | 18 |
| $p\%$ | 6.5 | 3.8 | 8.7 |

II.c) $Z_e[1.5; 3.5]$

| $K_{cr}$ | 0.965 | 0.97 | 0.974 |
|---|---|---|---|
| $m$ | 3 | 4 | 7 |
| $n$ | 5 | 9 | 14 |
| $p\%$ | 2.1 | 2.5 | 2.5 |

II.d) $Z_e[2; 4.5]$

| $K_{cr}$ | 0.974 | 0.976 | 0.978 |
|---|---|---|---|
| $m$ | 4 | 5 | 8 |
| $n$ | 15 | 16 | 20 |
| $p\%$ | 25 | 12 | 14 |
Table III. Results of the probability estimate of random occurrence of a spot with a maximum $K_i$ on the lensing topography in the direction $l_3 = (60^\circ \pm 20^\circ); b_3 = (0^\circ \pm 20^\circ)$, for the interval $Z_c \epsilon [2; 4.5]$. Radius of the standard cell, $R = 40^\circ$. Number of cells, $N_{knots} = 108$. Number of throws or iterations, $Q = 10^4$. Explanation is given in the text.

III. $Z_c \epsilon [2; 4.5]$

| $K_{cr}$ | 1.02 | 1.018 | 1.016 |
|---------|------|-------|-------|
| m       | 3    | 7     | 9     |
| n       | 5    | 7     | 9     |
| p%      | 2.1  | 0.08  | 0.2   |
Table IV. Results of the probability estimate of random occurrence of a spot with a maximum value for the relative density of absorbers in the direction $l = (135^\circ \pm 20^\circ)$; $b = (10^\circ \pm 20^\circ)$, for the interval $Z_e \in [1.5; 3.5]$. Radius of the standard cell, $R = 40^\circ$. Number of cells, $N_{knots} = 108$. Number of throws or iterations, $Q = 10^4$. Explanation is given in the text.

IV. $Z_e \in [1.5; 3.5]$

| $K_{cr}$ | 3.2 | 2.8 | 2.6 |
|---------|-----|-----|-----|
| m       | 3   | 4   | 6   |
| n       | 4   | 8   | 12  |
| p\%     | 0.69| 1.4 | 1.3 |
Table V. Results of the probability estimate of formation of extremal spots on the lensing topography, occurring due to selection effects. This was done for four intervals of $Z_e$. Number of throws or iterations is, $Q_1 = 10^3$. Explanation is given in the text.

V.a)

| $\Delta Z_e$ | $R^\circ$ | $p_1\%$ |
|--------------|-----------|---------|
| 0.5-2.5      | 30        | 12      |
| 0.5-2.5      | 35        | 0.0     |
| 0.5-2.5      | 40        | 0.4     |

V.b)

| $\Delta Z_e$ | $R^\circ$ | $p_1\%$ |
|--------------|-----------|---------|
| 1-3          | 30        | 24      |
| 1-3          | 35        | 0.0     |
| 1-3          | 40        | 0.9     |

V.c)

| $\Delta Z_e$ | $R^\circ$ | $p_1\%$ |
|--------------|-----------|---------|
| 1.5-3.5      | 30        | 19      |
| 1.5-3.5      | 35        | 0.2     |
| 1.5-3.5      | 40        | 0.9     |

V.d)

| $\Delta Z_e$ | $R^\circ$ | $p_1\%$ |
|--------------|-----------|---------|
| 2-4.5        | 30        | 0.9     |
| 2-4.5        | 35        | 0.1     |
| 2-4.5        | 40        | 1.3     |
Table VI. Results of the probability estimate of formation of extremal spots on the lensing topography of the relative density of absorbers, occurring due to selection effects. This was done for the interval $Z_\epsilon[1.5;3.5]$. The number of throws or iterations, $Q = 10^3$. Explanation is given in the text.

**VI. $Z_\epsilon[1.5;3.5]$**

| $\Delta Z_\epsilon$ | $R^\circ$ | $p_1\%$ |
|----------------------|-----------|---------|
| 1.5-3.5              | 30        | 1.7     |
| 1.5-3.5              | 35        | 0.2     |
| 1.5-3.5              | 40        | 1.2     |
Figure 0.
Fig. 2

Figure 0.