Design of a robust LMI-based model predictive control method for surge instability in interconnected compressor systems in the presence of uncertainty and disturbance

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ABSTRACT
Surge is the most significant instability observed in the compressors, and its control requires the exact dynamics of the compressor systems. Uncertainty in compressor characteristics and unknown opening percentage of the throttle and spillback valves, as well as disturbances in compressor’s flow and pressures, are among the major issues to be addressed in controller design for surge instability. Furthermore, in compressor systems that consist of several individual compressors, their reciprocal effects should also be taken into account. This paper presents an LMI-based decentralized robust model predictive control to ensure the stability of the compressor system against surge instability, uncertainty, and disturbance. The proposed scheme benefits from the optimized control signal with minimum computational complexity to overcome the destabilizing effects in a complex compressor system. The considered working class for this compressor system is a continuous-time nonlinear system. Through this method, the optimization problem is designed for a worst-case scenario. The implementation results of the presented robust controller for a compressor system, consisting of three parallel and series compressors, suggest the effectiveness of the presented method.

1. Introduction
A compressor is a mechanical device that increases the pressure of a gas by reducing its volume. Centrifugal compressors, sometimes called radial compressors, are a subclass of dynamic turbo-machinery and achieve a pressure rise by adding kinetic energy/velocity to a continuous flow of fluid through the rotor or impeller (Brun & Kurz, 2018; Tibrewala et al., 2014). The most important applications of centrifugal compressors are in oil refineries, natural-gas processing, petrochemical, chemical plants, air-conditioning and refrigeration and HVAC, in industry and manufacturing to supply compressed air for all types of pneumatic tools and in air separation plants to manufacture purified and product gases (Bloch & Soares, 1998).

Surge is flow phenomenon instability at low mass flow rate operation for which the impeller cannot add enough energy to overcome the system resistance or backpressure (Semlitsch & Mihăescu, 2016). At low mass flow rate operation, the pressure ratio over the impeller is high. The high back pressure, downstream of the impeller, pushes flow back over the tips of the rotor blades towards the impeller eye (inlet) (Sundström et al., 2018). The rapid reversal flow exhibits a strong rotational component, which affects the flow angles at the leading edge of the blades. The deterioration of the flow angles causes the impeller to be inefficient and less flow is delivered downstream. Thereby, the plenum downstream of the impeller is emptied and the (back) pressure drops. As a result, less flow reverses over the rotor tips and the impeller gain becomes again efficient. These cyclic events cause large vibrations, increase temperature and change rapidly the axial thrust. These occurrences can damage the rotor seals, rotor bearings, the compressor driver and cycle operation. Most turbo machines are designed to easily withstand occasional surging. However, if the machine is forced to surge repeatedly for a long period of time, or if it is poorly designed, repeated surges can result in a catastrophic failure.

Surge control is of greater importance in complex compressor systems consisting of several parallel and series compressors for which a higher fluid pressure increase is expected. A complex system consists of many components that interact with each other, and due to dependencies, competitions, relationships, or other types of interactions between their parts or between a given
system and its environment; its modelling and controlling are inherently difficult (Ladyman et al., 2013). Systems that are ‘complex’ have certain characteristics that result from these relationships, such as nonlinearity, spontaneous emergence and order, compatibility, and feedback loops among others. The type of surge control system specified for a given system is largely determined by the type of compressor being used and the compressor system dynamics. If a system has a single compressor with a very steady behaviour, a simple control system may be appropriate. On the other hand, a complex system with multiple compressors, varying demand, and many types of end uses will require a more sophisticated surge control strategy. Due to the reciprocal effects of the compressors, their behaviour is more sensitive adjacent to the surge line, and thus a more careful control design is imperative. In any case, careful consideration should be given to compressor control system selection because it can be the most important single factor affecting system performance, efficiency and safety.

With increasing interest to the robust control approaches in various applications (Chen et al., 2020; Shi et al., 2017a; Shi et al., 2017b; Wang et al., 2021; Xiong et al., 2016; Yu et al., 2020; Zhang et al., 2018; Zhu et al., 2020), several methods have been proposed for active surge control in constant speed compressors. In Dominic et al. (2016), the feed-forward control method has been used to control the compressor pressure, which is robust to uncertainty and disturbance. Due to the high performance of adaptive methods (Huang et al., 2021; Ma & Xu, 2020; Zhang et al., 2020), references (Ghanaati et al., 2017a; Imani, Malekizade, et al., 2018b; Ghanavati, Salahshoor, Motlagh, et al., 2018a; Zibari, et al., 2017) have used adaptive and back-stepping approaches to overcome the effects of uncertainty and disturbance. In recent years, model predictive control methods have also been proposed to control surge instability in fixed speed compressors (Imani et al., 2017a; Imani et al., 2017b; Imani, Malekizade, et al., 2018; Marrani et al., 2019). Nonlinear model predictive control method is designed for compressors Geritzer model considering the CCV actuator (Imani et al., 2017a; Imani, Malekizade, et al., 2018). In Marrani et al. (2019), Tube-MPC is designed for discrete time systems where $H_{\infty}$ method has been used as an auxiliary controller. By taking into account the effects of pipe, robust model predictive control is provided in Imani et al. (2017b).

It is well established that in interconnected systems, the centralized control framework for each subsystem must use the data of the whole system (D’Andrea & Dullerud, 2003). In decentralized control schemes, the controller designed for each subsystem can only use its own local data. Due to the high independence of decentralized controllers, some people have tried to create design approaches that can ensure the stability and performance of the system.

Due to the unique capabilities of the MPC controller in the centralized regulation of large systems with numerous input and output variables, so far little attention has been paid to decentralized MPC algorithms with guaranteed stability. This is due to the following reasons: (1) the multivariate nature of MPC, which makes it easy to form a centralized regulator. (2) Problems of ensuring the stability of decentralized algorithms due to the implicit control law obtained from the MPC optimization process (Mayne et al., 2000).

Constructing coordinated decentralized control systems needs dynamic interaction among different units in the design process of control systems. Also, the disturbance, uncertainty and nonlinear properties are reflected in the process of optimization and stability assurance for the designing of decentralized MPC controllers (Alessio et al., 2011; Alessio & Bemporad, 2008, June; Gudi & Rawlings, 2006; Magni & Scattolini, 2006). In Tuan et al. (2015), a new decentralized predictive control scheme has been purposed for a plant made of interconnected systems that provide a method to stabilize a nominal large-scale plant using limited and decentralized controllers.

Despite these efforts, the decentralized controller design faces several major challenges. The first difficulty lies in the stability proof of the general closed-loop system in the presence of the controllers, which only use the local information of their respective subsystems. In other words, it is evident that the stability of each subsystem might not always ensure the stability of the whole system and plant. Another challenge is due to the limitation of the available data and lack of connection between various controllers, which can lead to reduced closed-loop performance under a decentralized control platform (Cui & Jacobsen, 2002). The necessity of addressing the nonlinear dynamics of the subsystems, the disturbances present in the system dynamics, and the parameter uncertainties of the system are other concerns regarding the decentralized robust predictive controller design. To the best knowledge of the authors, there has been no effective control method presented for cooperated decentralized control of a complex compressor system consisting of several parallel and series compressors.

This paper presents a novel LMI-based decentralized robust predictive control scheme for complex compressor systems which covers the followings:

1. The proposed decentralized method is able to address the mutual effects of compressors, and the most important property of this model is its independent communication with various local controllers.
The presented method is based on a predictive model in which the state and control signal constraints are taken into account, and the optimization of the objective function is also performed.

In order to reduce the computational time and complexity, the LMI method is employed to solve the optimization problem in each time step.

The effects of disturbance on compressor flow and pressure are considered.

The effects of uncertainty in the throttle valve, Spill-back valve, and also compressor characteristics are covered.

The stability is ensured by the Lyapunov method using the proposed controller, and the mutual effects of the compressors, as well as the effects of nonlinearity, uncertainty, and disturbances with unspecified upper bound, are addressed at the same time.

The organization of this paper is as follows. In Section 2, the preliminaries are presented. Section 3 describes the implemented method for complex compressor system surge control in detail. The implementation of the proposed method for complex compressor system surge control is provided in Section 4. Sections 5 and 6 present the simulation results and conclusions, respectively.

2. Preliminaries

Consider the following continuous-time nonlinear system
\[ \dot{x}(t) = Ax(t) + Bu(t) + w(t,x), \] (1)
where \( x(t) \in R^{nx} \) shows the system states, \( u(t) \in R^{nu} \) the control input, \( w(t,x) : R^{nx} \rightarrow R^{nw} \) continuous nonlinear uncertainty function. \( w(t,x) \) is considered in the following set
\[ W = \{ w(t,x) \in R^{nw} | w \leq w_{\text{max}} \}. \] (2)

The system has the following limitations \( x(t) \in \tilde{X}, u(t) \in \tilde{U}, \forall t > 0 \). Where \( \tilde{X} \subset R^{nx} \) is bounded and \( \tilde{U} \subset R^{nu} \) is the compact.

\textbf{Lemma 2.1 (Yu et al., 2010, June):} Let \( S : R^{nx} \rightarrow [0, \infty) \) be a continuously differentiable function and \( \alpha_1(x) < S(x) < \alpha_2(x) \), where \( \alpha_1, \alpha_2 \) are \( K_{\infty} \) class functions. Suppose \( u : R \rightarrow R^{nu} \) is chosen, and there exists \( \lambda > 0 \) and \( \mu > 0 \) such that
\[ \dot{S}(x) + \lambda S(x) - \mu w^T(t,x)w(t,x) \leq 0, \] (3)
With \( x \in X, w \in W \). Then, the system trajectory starting from \( x(t_0) \in \Omega \subseteq X \) will remain in the set \( \Omega \), where
\[ \Omega = \left\{ x \in R^{nx} | S(x) \leq \frac{\mu w_{\text{max}}^2}{\lambda} \right\}. \] (4)

\textbf{Lemma 2.2 (Poursafar et al., 2010):} Let \( M, N \) be real constant matrices and \( P \) be a positive matrix of compatible dimensions. Then
\[ M^T P N + N^T P M \leq \varepsilon M^T P M + \varepsilon^{-1} N^T P N \] (5)
holds for any \( \varepsilon > 0 \).

\textbf{Lemma 2.3 (Schur complements (Boyd et al., 1994):} The LMI
\[ \begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0 \] (6)
In which \( Q(x) = Q^T(x) \), \( R(x) = R^T(x) \) and \( S(x) \) are affine functions of \( x \), and are equivalent to
\[ R(x) > 0 \quad Q(x) - S(x)R^{-1}(x)S^T(x) > 0 \] \[ Q(x) > 0 \quad R(x) - S(x)Q^{-1}(x)S^T(x) > 0 \]. (7)

3. Robust LMI-based decentralized MPC

Consider a process plant \( \Sigma \) consisting of \( h \) interconnected nonlinear systems, each denoted as \( s_i, i = 1, \ldots, h \) in the continuous time state space
\[ s_i : \dot{x}_i(t) = A_ix_i(t) + B_iu_i(t) + f_i(x_i, \theta_i) + d_i(t) + g_i(x), \] (8)
where \( x_i(t) \in R^{nx_i} \) shows the system states, \( u_i(t) \in R^{nu_i} \) the control input, \( f_i(x_i, \theta_i) : R^{nx_i} \rightarrow R^{nx_i} \) specifies continuous nonlinear function, \( \theta_i(t) \in R^{nu_i} \) indicates uncertainty in the system, \( d_i(t) \in R^{nx_i} \) specifies bounded and unknown system disturbances, \( g_i(x) : R^{nx_i} \rightarrow R^{nx_i} \) is the interactive (or coupling) continuous nonlinear function, and \( x(t) \in R^{nx} \) shows the process plant \( \Sigma \) states. The disturbances are considered in the following set
\[ D_i = \{ d_i(t) \in R^{nx_i} | d_i \leq d_{\text{max}} \}. \] (9)

The system has the following limitations
\[ x_i(t) \in X_i, u_i(t) \in U_i, \forall t > 0, \] (10)
where \( X_i \subset R^{nx_i} \) is bounded and \( U_i \subset R^{nu_i} \) is the compact. with
\[ w_i(t,x) = f_i(x_i, \theta_i) + d_i(t) + g_i(x), \] (11)
will have
\[ \dot{x}_i(t) = A_ix_i(t) + B_iu_i(t) + w_i(t,x). \] (12)

The state-feedback control law for system (12) in \( kT \) time is chosen as
\[ u_i(kT + \tau, kT) = K_ix_i(kT + \tau, kT) (\tau \geq 0). \] (13)
These control signals are true for the following constraint
\[ u_i(kT + \tau, kT)_2 \leq u_{i\text{max}}. \] (14)

Finally, the chosen infinite horizon quadratic cost function is specified as:
\[
J_i = \int_{0}^{\infty} (x_i(kT + \tau, kT)^T Q_i x_i(kT + \tau, kT)
+ u_i(kT + \tau, kT)^T R_i u_i(kT + \tau, kT)
- \mu_i w_i(kT + \tau, kT) \lambda_i w_i(kT + \tau, kT)) \, d\tau, \mu_i > 0, \quad (15)
\]
where \( Q_i \) and \( R_i \) are positive definite weight matrices. In the objective function (15), the uncertain but negative effect term is introduced with weight \( \mu_i \), where \( \mu_i \) is a positive constant (Tahir & Jaimoukha, 2011).

**Theorem 3.1:** Consider system (12), \( x_i(kT) \) is the measure value in sampling time of \( kT \). There is a state-feedback control law (13) that is true in stability condition and in input constraint (14) in every moment. If the optimization problem with LMI constraints can be feasible.

\[
\begin{align*}
\min_{\gamma_i} \gamma_i & \quad \mathbf{X}_i \\
\text{subject to} & \quad \gamma_i, \mathbf{X}_i \\
& \quad \mathbf{X}_i > 0, \quad \mathbf{Y}_i \text{ are matrixes obtained from the above-mentioned optimization problem. By this way, the state-feedback matrix in every moment is obtained as } K_i = Y_i X_i^{-1}.
\end{align*}
\] (16)

**Proof:** Considering a quadratic Lyapunov function, we have
\[ V_i(x_i(t)) = x_i(t)^T P_i x_i(t), \quad P_i > 0. \] (17)
In the sampling time of \( kT \) assume that \( V_i(x_i(t)) \) is true in the following condition
\[ x_i(t)^T P_i x_i(t) < \gamma_i \] (18)
\[
\frac{dV_i(x_i(kT + \tau, kT))}{dt} \leq -(x_i(kT + \tau, kT)^T Q_i x_i(kT + \tau, kT)
+ u_i(kT + \tau, kT)^T R_i u_i(kT + \tau, kT)
- \mu_i w_i(kT + \tau, kT)^T w_i(kT + \tau, kT)). \quad (19)
\]
In order to obtain the robust efficiency, we should have \( x_i(\infty, kT) = 0 \) which results \( V_i(x_i(\infty, kT)) = 0 \). By integrating both sides of the equation (19), we have
\[ J_i \leq V_i(x_i(kT)) \] (20)
where \( \gamma_i \) is a positive scalar (the upper bound of the objective (15)).

In order to obtain an MPC robust algorithm, the Lyapunov function should be minimized considering the upper bound (Ghaffari et al., 2013, June). So
\[
\begin{align*}
\min_{\gamma_i} \gamma_i \quad \mathbf{X}_i \\
\text{subject to} & \quad \gamma_i, \mathbf{X}_i \\
& \quad \mathbf{X}_i > 0, \quad \mathbf{Y}_i \text{ are matrixes obtained from the above-mentioned optimization problem. By this way, the state-feedback matrix in every moment is obtained as } K_i = Y_i X_i^{-1}.
\end{align*}
\] (21)

In the following, according to Lemma 2.1, for system (12) we have
\[ S_i(x_i(t)) + \lambda_i S_i(x_i(t)) - \mu_i w_i(t, x)^T w_i(t, x) \leq 0 \] (23)
Then, according to (17), we have
\[ x_i(t)^T P_i x_i(t) + x_i(t)^T P_i x_i(t) + \lambda_i x_i(t)^T P_i x_i(t) \\
- \mu_i w_i(t, x)^T w_i(t, x) \leq 0. \] (24)
According to Lemma 2, we have
\[ w_i(t, x)^T P_i w_i(t, x) \leq \alpha_i x_i(t)^T P_i x_i(t) \\
+ \alpha_i^{-1} w_i(t, x)^T P_i w_i(t, x) \] (25)
By substituting (26) in (25), we have
\[ x_i(t)^T ((A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i) + \lambda_i P_i) x_i(t) \\
+ \alpha_i^{-1} w_i(t, x)^T P_i w_i(t, x) - \mu_i w_i(t, x)^T w_i(t, x) \leq 0. \] (27)
Consider
\[ P_i \leq \lambda_{i\text{max}} I \leq \varepsilon I, \] (28)
Finally, the input constraint is investigated in (Poursafar et al., 2010), then
\[ x_i(t)^T ((A_i + B_iK_i)^T P_i + P_i (A_i + B_iK_i) + (\alpha_i + \lambda_i) P_i) x_i(t) + (\alpha_i^{-1} \epsilon_i - \mu_i) w_i(t, x)^T w_i(t, x) \leq 0. \]  
(29)

By choosing
\[ \mu_i = \frac{\epsilon_i}{\alpha_i} \]  
(30)

Equation (29) is reduced to
\[ x_i(t)^T ((A_i + B_iK_i)^T P_i + P_i (A_i + B_iK_i) + (\alpha_i + \lambda_i) X_i\gamma_i I) x_i(t) \leq 0. \]  
(31)

Substituting \( P_i = \gamma_i X_i^{-1} \) and \( \gamma_i = Y_i X_i^{-1} \),
\[ ((A_i + B_iY_i X_i^{-1})^T X_i^{-1} + X_i^{-1} (A_i + B_iY_i X_i^{-1}) + (\alpha_i + \lambda_i) X_i^{-1}) \gamma_i \leq 0. \]  
(32)

Pre- and post-multiplying by \( X_i \),
\[ (A_i X_i + B_i Y_i)^T + A_i X_i + B_i Y_i + (\alpha_i + \lambda_i) X_i \leq 0. \]  
(33)

Given (28), we have
\[ P_i \leq \epsilon_i \]  
(34)

Substituting \( P_i = \gamma_i X_i^{-1} \) and pre-multiplying by \( X_i \), we have
\[ -X_i + \gamma_i \epsilon_i^{-1} I \leq 0 \]  
(35)

Finally, the input constraint is investigated in (Poursafar et al., 2010). According to (14), we have
\[ u_i(kT + \tau, kT) \leq u_{i \max} \]  
(36)

Consistent with (18) and (20), it is known that the states of \( x_i(kT + \tau, kT) \) are determine and ellipsoid invariant set
\[ S_i = \{ x_i \mid x_i^T X_i x_i \leq 1 \} \]  
(37)

Therefore,
\[ u_i(kT + \tau, kT)^2 = K_i X_i(kT + \tau, kT)^2 \]
\[ = Y_i X_i^{-1} \left( X_i^{-T} X_i(kT + \tau, kT) \right)^2 \]
\[ \leq Y_i X_i^{-2}. \]  
(38)

From (36) and (37), the input two-norm constraint in (36) can be rewritten as
\[ Y_i X_i^{-1} Y_i - u_{i \max}^2 \leq 0 \]  
(39)

It is the same as following inequality by applying Schur complemen.(39)
\[ \left[ -u_{i \max}^2 \ Y_i^T \right] \leq 0 \]  
(40)

So, the proof is completed. ■

### 4. Robust model predictive control on surge

Surge is a condition that occurs on compressors when the amount of gas is insufficient to compress and the turbine blades lose their forward thrust, causing a reverse movement in the shaft. It can cause extensive structural damage in the machine because of the violent vibration and high thermal loads that generally accompany the instability. In recent years, several methods have been proposed to control the instability (Fazeli et al., 2019; Imani, Jahed-Motlagh, et al., 2018; Taleb Ziaabari et al., 2012; Taleb Ziaabari et al., 2017). For this reason, compressor system control as one of the most practical systems is considered in this section.

#### 4.1. Compressor model

In this section, given the proposed decentralized predictive controller, a surge controller is designed for serial and parallel compressors. First, a compressor model is investigated and then a decentralized controller is designed for these combinations: two parallel compressors and one serial compressor. Moore and Greitzer’s surge model of the centrifugal compressor is as follows
\[ \dot{\psi} = \frac{1}{4B^2 l_c} (\phi - \phi_T (\psi) - d_\phi (t)) \]
\[ \dot{\phi} = \frac{1}{l_c} (\psi - \phi + d_\phi (t)) \]  
(41)

where \( \psi \) is the coefficient of compressor pressure, \( \phi \) is the coefficient of compressor’s mass flow, \( d_\phi (t) \) and \( d_\psi (t) \) are the disturbances of flow and pressure, respectively. Also, \( \phi_T (\psi) \) is the characteristic of throttle valve and \( \psi_C (\phi) \) is the characteristic of the compressor. \( B \) is the Greitzer’s parameter and \( l_c \) shows the length of ducts. Moore and Greitzer’s (1986) compressor characteristic is defined as
\[ \psi_C (\phi) = \psi_{c0} + H \left( 1 + \frac{3}{2} \left( \frac{\phi}{W} - 1 \right) - \frac{1}{2} \left( \frac{\phi}{W} - 1 \right)^2 \right) \]  
(42)

where \( \psi_{c0} \) is the value of characteristic curve in zero dB, \( H \) is the half of the height of the characteristic curve, and \( W \) is the half of the width of the characteristic curve. The equation for throttle valve characteristic is also derived from (Ziaabari, et al., 2017) and is as follows
\[ \phi_T (\psi) = \gamma_T \sqrt{\psi} \]  
(43)

where \( \gamma_T \) is also the valve’s yield. Values of compressor parameters are used in simulation according to (Greitzer, 1976).

\[ B = 1.8, \ l_c = 3, \ H = 0.18, \ W = 0.25, \ \psi_{c0} = 0.3 \]  
(44)

Figure 1 is the diagram of compression system with Close Couple Valve (CCV).
The system model equations, considering a CCV, are

\[ \dot{\psi} = \frac{1}{4B^2l_c}(\psi - \phi \dot{T}(\psi) - d_\psi(t)) \]
\[ \dot{\phi} = \frac{1}{l_c}(\psi_c(\psi) - \psi - \psi(\phi) + d_\phi(t)) \tag{45} \]

Consider \( \psi(\phi) \) as the input for system control.

\[ \dot{\psi} = \frac{1}{4B^2l_c}(\psi - \phi \dot{T}(\psi) - d_\psi(t)) \]
\[ \dot{\phi} = \frac{1}{l_c}(\psi_c(\psi) - \psi - u + d_\phi(t)) \tag{46} \]

According to Figure 2, the equations for the two parallel compressors and one serial compressor are

\[ \dot{\psi}_1 = \frac{1}{4B^2l_c}(\psi_1 - \phi \dot{T}_1(\psi_1) - d_{\psi_1}(t) - \vartheta_1\phi_2) \]
\[ \dot{\phi}_1 = \frac{1}{l_c}(\psi_{c1}(\psi_1) - \psi_1 - u_1 + d_{\psi_1}(t) + \vartheta_2\psi_2 - \psi_{SB}(\phi_3)) \]
\[ \dot{\psi}_2 = \frac{1}{4B^2l_c}(\psi_2 - \phi \dot{T}_2(\psi_2) - d_{\psi_2}(t) - \vartheta_3\phi_1) \]
\[ \dot{\phi}_2 = \frac{1}{l_c}(\psi_{c2}(\psi_2) - \psi_2 - u_2 + d_{\psi_2}(t) + \vartheta_4\psi_1 - \psi_{SB}(\phi_3)) \]
\[ \dot{\psi}_3 = \frac{1}{4B^2l_c}(\psi_3 - \phi \dot{T}_3(\psi_3) - d_{\psi_3}(t) - \vartheta_5(\phi_1 + \phi_2)) \]
\[ \dot{\phi}_3 = \frac{1}{l_c}(\psi_{c3}(\phi_3) - \psi_3 - u_3 + d_{\psi_3}(t) + \vartheta_6\psi_1) \tag{47} \]

where \( \vartheta_j, j = 1, \ldots, 6 \) are uncertain parameters and

\[
\begin{align*}
    d_{\psi_1}(t) &= 0.02 \sin(0.1t) + 0.02 \cos(0.4t) \\
    d_{\phi_1}(t) &= 0.02 \sin(0.1t) + 0.02 \cos(0.4t) \\
    d_{\psi_2}(t) &= 0.02 \sin(0.1t) + 0.02 \cos(0.4t) \\
    d_{\phi_2}(t) &= 0.02 \sin(0.1t) + 0.02 \cos(0.4t) \\
    d_{\psi_3}(t) &= 0.15e^{-0.01\xi} \cos(0.2t) \\
    d_{\phi_3}(t) &= 0.1e^{-0.005\xi} \sin(0.3t) \tag{48}
\end{align*}
\]

The characteristic of the spill back valve is

\[ \psi_{SB}(\phi_3) = \gamma_{SB}\phi_3^2 \tag{49} \]

where \( \gamma_{SB} \) is also the valve’s yield. In designing a surge controller in the compressor systems (49), it is assumed that the value of throttle valve, as well as the compressor characteristic, is not known.

### 4.2. Controller design

According to equations (8) and (47), each subsystem is rewritten as

\[ A_i = \begin{bmatrix} 0 & \frac{1}{4B^2l_c} \\ -\frac{1}{l_c} & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ -\frac{1}{l_c} \end{bmatrix} \tag{50} \]

\[ f_i(\phi_1, \psi_1) = \begin{bmatrix} -\frac{1}{4B^2l_c}\gamma_{T_i}\dot{\phi}_i \\ 0 \end{bmatrix} \sqrt{\psi_{c_i}(\phi_i)} \tag{51} \]

\[ d_i(t) = \begin{bmatrix} \frac{1}{4B^2l_c}d_{\psi_i}(t) \\ \frac{1}{l_c}d_{\phi_i}(t) \end{bmatrix} \tag{52} \]

\[ g_1(\phi_1, \psi_1, \phi_2, \psi_2, \phi_3, \psi_3) = \begin{bmatrix} -\vartheta_2\psi_2 - \psi_{SB}(\phi_3) \\ \vartheta_4\psi_1 - \psi_{SB}(\phi_3) \end{bmatrix} \tag{53} \]

\[ g_2(\phi_1, \psi_1, \phi_2, \psi_2, \phi_3, \psi_3) = \begin{bmatrix} -\vartheta_3\phi_1 \\ -\vartheta_5(\phi_1 + \phi_2) \end{bmatrix} \tag{54} \]

\[ g_3(\phi_1, \psi_1, \phi_2, \psi_2, \phi_3, \psi_3) = \begin{bmatrix} -\vartheta_6\phi_1 + \phi_2 \\ \vartheta_6\psi_1 \end{bmatrix} \tag{55} \]

Next, the existing constraints in the compressor systems should be incorporated in the optimization problem (15). The first established constraint is on the control input. Since the control signal is a CCV output, so we have

\[ \dot{u}_i(t) > 0, \quad i = 1, 2, 3 \tag{56} \]

The next constraint and limitation is that the flow has some maximum and minimum values. This constraint should also be considered.

\[ -\phi_{mi} \leq \phi_i(t) \leq \phi_{Chokei}, \quad i = 1, 2, 3 \tag{57} \]

The LMI parameters are selected

\[ \alpha_i = 10^{-3}, \lambda_i = 10^{-3}, \mu_i = 10, \varepsilon_i = 10^{-2}, \quad i = 1, 2, 3 \tag{58} \]

### 5. Simulation

In this section, simulation is performed in Matlab to demonstrate the performance of the proposed control method. The applied scenario is similar to the scenario presented in the article (Fazeli et al., 2019), and the obtained results for the proposed controller are compared to the robust decentralized tube MPC adaptive control method. The considered scenario is given as follows:
In $t = 40s$ the first compressor throttle valve value reduces from $\gamma_T_1 = 0.65$ to $\gamma_T_1 = 0.6$, which leads to surge in the first compressor. As such, in $t = 50s$, the second compressor throttle valve value reduces from $\gamma_T_2 = 0.7$ to $\gamma_T_2 = 0.6$, and the second compressor also experiences surge. Also, in $t = 60s$, the third compressor throttle valve value reduces from $\gamma_T_3 = 0.75$ to $\gamma_T_3 = 0.6$, and the third compressor also leads to surge. Finally, in $t = 20s$ the spill back valve value changes from $\gamma_{SB} = 0.2$ to $\gamma_{SB} = 0$. After applying the presented predictive controller and simulating the compressors behaviour, the following results are obtained.

Figures 3–6 display the pressure, flow, control signal, and trajectory of the first compressor system, respectively. Figure 3 shows a higher pressure increase using the proposed method compared to the reference (Fazeli et al., 2019). Figure 5 also exhibits the lower fluctuations of the compressor flow with the proposed controller. The obtained control signal also has smaller amplitude in the range of 0 and 1, indicative of its practicality. The traversed path on the compressor’s characteristic curve shown in Figure 6 ensures compressor operation near the surge line without entering the surge region.

The pressure, flow, the control signal, and trajectory associated with the second compressor are shown in Figures 7–10, respectively. Similar to the first case, the proposed method achieved a higher pressure increase, smaller fluctuations in flow, smaller amplitude for the control signal of the compressor two, and ensured working close to the surge line without entering it.

Figures 10–14 also show the obtained results for compressor 3 in the compressor system. By employing the presented controller, the obtained results for compressor 3 are in agreement with two first compressors.

The obtained results from these figures indicate the effectiveness of the presented method for the compressor system stabilization, surge avoidance, control signal optimization, higher pressure increase, and reduced flow fluctuations.
Figure 4. Flow of compressor 1.

Figure 5. Control signal of compressor 1.

Figure 6. Trajectories of compression system 1.

Figure 7. Pressure of compressor 2.

Figure 8. Flow of compressor 2.

Figure 9. Control signal of compressor 2.
6. Conclusions

This paper presents an LMI-based decentralized robust predictive control scheme for a particular class of complex compressor systems, which also includes the multi-stage compressor system model. The presented control method was implemented to avoid surge instability as the most significant compressor instability. The effects of the nonlinear factors, disturbance, and uncertainty on the compressor characteristic curve and throttle valve and spillback valve are also addressed. Furthermore, the objective function is optimized in each step using the LMI method to obtain the control signal. The simulation results suggest that our proposed approach, compared to the robust adaptive tube MPC control method, was superior in terms of the pressure increase, flow fluctuation, and control signal. Optimal compressor operation
near the surge line with the ability to increase the pressure more and the ability to work with less flow are among the results obtained from the simulation section which ensures minimal fluctuations on flow and pressure during disturbance and uncertainty in the complex compressor system, despite the control signal with the lowest amplitude.

**Disclosure statement**

No potential conflict of interest was reported by the author(s).

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**References**

Alessio, A., Barcelli, D., & Bemporad, A. (2011). Decentralized model predictive control of dynamically coupled linear systems. *Journal of Process Control*, 21(5), 705–714. https://doi.org/10.1016/j.jprocont.2011.03.003

Alessio, A., & Bemporad, A. (2008, June). Stability conditions for decentralized model predictive control under packet drop communication. In 2008 American Control Conference, Seattle, WA, USA (pp. 3577–3582). IEEE. https://doi.org/10.1109/ACC.2008.4587048

Bloch, H. P., & Soares, C. (1998). *Process plant machinery*. Elsevier.

Boyd, S., El Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). *Linear matrix inequalities in system and control theory*. Society for industrial and applied mathematics.

Brun, K., & Kurz, R. (Eds.). (2018). *Compression machinery for oil and gas*. Gulf Professional Publishing.

Chen, Z., Wang, J., Ma, K., Huang, X., & Wang, T. (2020). Fuzzy adaptive two-bits-triggered control for nonlinear uncertain system with input saturation and output constraint. *International Journal of Adaptive Control and Signal Processing*, 34(4), 543–559. https://doi.org/10.1002/acs.3098

Cui, H., & Jacobsen, E. W. (2002). Performance limitations in decentralized control. *Journal of Process Control*, 12(4), 485–494. https://doi.org/10.1016/S0959-1524(01)00015-4

D’Andrea, R., & Dullerud, G. E. (2003). Distributed control design for spatially interconnected systems. *IEEE Transactions on Automatic Control*, 48(9), 1478–1495. https://doi.org/10.1109/TAC.2003.816954

Dominic, S., Löh, Y., Schwung, L., & Ding, S. X. (2016). PLC-based real-time realization of flatness-based feedforward control for industrial compression systems. *IEEE Transactions on Industrial Electronics*, 64(2), 1323–1331. https://doi.org/10.1109/TIE.2016.2612160

Fazeli, S., Abdollahi, N., Imani Marrani, H., Malekizadeh, H., & Hosseinzadeh, H. (2019). A new robust adaptive decentralized tube model predictive control of continuous time uncertain nonlinear large-scale systems. *Cogent Engineering*, 6(1), 1680093. https://doi.org/10.1080/23319196.2019.1680093

Ghaffari, V., Naghavi, S. V., Safavi, A. A., & Shafiee, M. (2013, June). An LMI framework to design robust MPC for a class of nonlinear uncertain systems. In 2013 9th Asian Control Conference (ASCC), Istanbul, Turkey (pp. 1–5). IEEE. https://doi.org/10.1109/ASCC.2013.6606169

Ghanavati, M., Salahshoor, K., Jahed-Motlagh, M. R., Ramezani, A., & Moarefianpur, A. (2018). A novel combined control based on an adaptive generalized back-stepping method for a class of nonlinear systems. *Cogent Engineering*, 5(1), 1471787. https://doi.org/10.1080/23319196.2018.1471787

Ghanavati, M., Salahshoor, K., Motlagh, M. R. J., Ramezani, A., & Moarefianpur, A. (2018). A novel combined approach for gas compressors surge suppression based on robust adaptive control and backstepping. *Journal of Mechanical Science and Technology*, 32(2), 823–833. https://doi.org/10.1007/s12206-018-0133-1

Greitzer, E. M. (1976). Surge and rotating stall in axial flow compressors—Part I: Theoretical compression system model.

Gudi, R. D., & Rawlings, J. B. (2006). Identification for decentralized model predictive control. *AIChE Journal*, 52(6), 2198–2210. https://doi.org/10.1002/aic.10781

Huang, Y., Wang, J., Wang, F., & He, B. (2021). Event-triggered adaptive finite-time tracking control for full state constraints nonlinear systems with parameter uncertainties and given transient performance. *ISA Transactions*, 108, 131–143. https://doi.org/10.1016/j.isatra.2020.08.022

Imani, H., Jahed-Motlagh, M. R., Salahshoor, K., Ramezani, A., & Moarefianpur, A. (2017a). A novel tube model predictive control for surge instability in compressor system including piping acoustic. *Cogent Engineering*, 4(1), 1409373. https://doi.org/10.1080/23319196.2017.1409373

Imani, H., Jahed-Motlagh, M. R., Salahshoor, K., Ramezani, A., & Moarefianpur, A. (2017b). Constrained nonlinear model predictive control for centrifugal compressor system surge including piping acoustic using closed coupled valve. *Systems Science & Control Engineering*, 5(1), 342–349. https://doi.org/10.1080/20445597.2017.1367732

Imani, H., Jahed-Motlagh, M. R., Salahshoor, K., Ramezani, A., & Moarefianpur, A. (2018). Robust decentralized model predictive control approach for a multi-compressor system surge instability including piping acoustic. *Cogent Engineering*, 5(1), 1483811. https://doi.org/10.1080/23319196.2018.1483811

Imani, H., Malekizadeh, H., Asadi Bagal, H., & Hosseinzadeh, H. (2018). Surge explicit nonlinear model predictive control using extended Greitzer model for a CCV supported compressor. *Automatika: časopis za automatiku, mjerjenje, elektroniku, računarstvo i komunikacije*, 59(1), 43–50. https://doi.org/10.1002/acs.3098

Ladyman, J., Lambert, J., & Wiesner, K. (2013). What is a complex system? *European Journal for Philosophy of Science*, 3(1), 33–67. https://doi.org/10.1007/s13194-012-0056-8

Ma, H. J., & Xu, L. X. (2020). Decentralized adaptive fault-tolerant control for a class of strong interconnected nonlinear systems via graph theory. *IEEE Transactions on Automatic Control*. https://doi.org/10.1109/TAC.2020.3014292

Magni, L., & Scattolini, R. (2006). Stabilizing decentralized model predictive control of nonlinear systems. *Automatica*, 42(7), 1231–1236. https://doi.org/10.1016/j.automatica.2006.02.010

Marrani, H. I., Fazeli, S., Malekizadeh, H., & Hosseinzadeh, H. (2019). Tube model predictive control for a class of nonlinear discrete-time systems. *Cogent Engineering*, 6(1), 1629055. https://doi.org/10.1080/23319196.2019.1629055

Mayne, D. Q., Rawlings, J. B., Rao, C. V., & Scokaert, P. O. (2000). Constrained model predictive control: Stability and optimality. *Automatica*, 36(6), 789–814. https://doi.org/10.1016/S0005-1098(99)00214-9
Moore, F. K., & Greitzer, E. M. (1986). A theory of post-stall transients in axial compression systems: Part I – Development of equations.

Poursafar, N., Taghirad, H. D., & Haeri, M. (2010). Model predictive control of non-linear discrete time systems: A linear matrix inequality approach. *IET Control Theory & Applications*, 4(10), 1922–1932. https://doi.org/10.1049/iet-cta.2009.0650

Semlitsch, B., & Mihăescu, M. (2016). Flow phenomena leading to surge in a centrifugal compressor. *Energy*, 103, 572–587. https://doi.org/10.1016/j.energy.2016.03.032

Sheng, H., Huang, W., Zhang, T., & Huang, X. (2014). Robust adaptive fuzzy control of compressor surge using backstepping. *Arabian Journal for Science and Engineering*, 39(12), 9301–9308. https://doi.org/10.1007/s13369-014-1448-1

Shi, K., Tang, Y., Liu, X., & Zhong, S. (2017a). Non-fragile sampled-data robust synchronization of uncertain delayed chaotic Lurie systems with randomly occurring controller gain fluctuations. *ISA Transactions*, 66, 185–199. https://doi.org/10.1016/j.isatra.2016.11.002

Shi, K., Tang, Y., Liu, X., & Zhong, S. (2017b). Secondary delay-partition approach on robust performance analysis for uncertain time-varying Lurie nonlinear control system. *Optimal Control Applications and Methods*, 38(6), 1208–1226. https://doi.org/10.1002/oca.2326

Sundström, E., Semlitsch, B., & Mihăescu, M. (2018). Generation mechanisms of rotating stall and surge in centrifugal compressors. *Flow, Turbulence and Combustion*, 100(3), 705–719. https://doi.org/10.1007/s10494-017-9877-z

Tahir, F., & Jaimoukha, I. M. (2011). Robust model predictive control through dynamic state-feedback: An LMI approach. *IFAC Proceedings Volumes*, 44(1), 3672–3677. https://doi.org/10.3182/20110828-6-IT-1002.03112

Taleb Ziabari, M., Jahed Motlagh, M. R., Salahshoor, K., & Ramazani, A. (2012). Surge control in constant speed centrifugal compressors using nonlinear model predictive control. *The Modares Journal of Electrical Engineering*, 12(1), 49–53.

Taleb Ziabari, M., Jahed-Motlagh, M. R., Salahshoor, K., Ramezani, A., & Moarefiapur, A. (2017). Tube-MPC for a class of uncertain continuous nonlinear systems with application to surge problem. *Kybernetika*, 53(4), 679–693. https://doi.org/10.14736/kyb-2017-4-0679

Tibrewala, A. P., Padave, T. J., Wagh, T. P., & Gajare, C. M. (2014). Flow analysis of upstream fluid flow using simulation for different positions of optimized inlet guide vane in centrifugal air compressor. *American Journal of Engineering Research*, 3(02), 148–156.

Tuan, H. D., Savkin, A., Nguyen, T. N., & Nguyen, H. T. (2015). Decentralised model predictive control with stability constraints and its application in process control. *Journal of Process Control*, 26, 73–89. https://doi.org/10.1016/j.jprocont.2015.01.002

Wang, J., Zhu, P., He, B., Deng, G., Zhang, C., & Huang, X. (2021). An adaptive neural sliding mode control with ESO for uncertain nonlinear systems. *International Journal of Control, Automation and Systems*, 19, 687–697. https://doi.org/10.1007/s12555-019-0972-x

Xiong, L., Zhang, H., Li, Y., & Liu, Z. (2016). Improved stability and $H_{\infty}$ performance for neutral systems with uncertain markovian jump. *Nonlinear Analysis: Hybrid Systems*, 19, 13–25. https://doi.org/10.1016/j.nahs.2015.07.005

Yu, D., Mao, Y., Gu, B., Nojavan, S., Jermitsiparsert, K., & Nasserli, M. (2020). A new LQG optimal control strategy applied on a hybrid wind turbine/solid oxide fuel cell/in the presence of the interval uncertainties. *Sustainable Energy, Grids and Networks*, 21, 100296. https://doi.org/10.1016/j.segan.2019.100296

Yu, S., Böhm, C., Chen, H., & Allgöwer, F. (2010, 30 June–2 July). Robust model predictive control with disturbance invariant sets. In Proceedings of the 2010 American Control Conference, Baltimore, Maryland, USA (pp. 6262–6267). IEEE.

Zhang, X., Jing, R., Li, Z., Li, Z., Chen, X., & Su, C. Y. (2020). Adaptive pseudo inverse control for a class of nonlinear asymmetric and saturated nonlinear hysteretic systems. *IEEE/CAA Journal of Automatica Sinica*, 8(4), 916–928. https://doi.org/10.1109/JAS.2020.1003435

Zhang, X., Wang, Y., Chen, X., Su, C. Y., Li, Z., Wang, C., & Peng, Y. (2018). Decentralized adaptive neural approximated inverse control for a class of large-scale nonlinear hysteretic systems with time delays. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 49(12), 2424–2437. https://doi.org/10.1109/TSMC.2018.2827101

Zhu, G., Wang, S., Sun, L., Ge, W., & Zhang, X. (2020). Output feedback adaptive dynamic surface sliding-mode control for quadrotor UAVs with tracking error constraints. *Complexity*, 2020, 5.

Ziabari, M. T., Jahed-Motlagh, M. R., Salahshoor, K., Ramezani, A., & Moarefiapur, A. (2017). Robust adaptive control of surge instability in constant speed centrifugal compressors using tube-MPC. *Cogent Engineering*, 4(1), 1339335. https://doi.org/10.1080/23311916.2017.1339335