On the consistency of warm inflation

Ian G Moss and Chun Xiong
School of Mathematics and Statistics, Newcastle University, Newcastle Upon Tyne NE1 7RU, UK
E-mail: ian.moss@ncl.ac.uk

Received 7 October 2008
Accepted 6 November 2008
Published 25 November 2008

Online at stacks.iop.org/JCAP/2008/i=11/a=023
doi:10.1088/1475-7516/2008/11/023

Abstract. Conditions are obtained for the existence of a warm inflationary attractor in the system of equations describing an inflaton coupled to radiation. These conditions restrict the temperature dependence of the dissipative terms and the size of thermal corrections to the inflaton potential, as well as the gradient of the inflaton potential. When these conditions are met, the evolution approaches a slow-roll limit and only curvature fluctuations survive on super-horizon scales. Formulae are given for the spectral indices of the density perturbations and the tensor/scalar density perturbation amplitude ratio in warm inflation.

Keywords: cosmological perturbation theory, inflation

ArXiv ePrint: 0808.0261
On the consistency of warm inflation

Contents

1. Introduction 2
2. Basic equations 3
3. Stability analysis 5
4. Density fluctuations 7
5. Conclusion 9
Acknowledgments 10
References 10

1. Introduction

Inflationary models [1]–[3] have proved very successful in explaining many of the large scale features of the universe (see e.g. [4]). An essential feature of these inflationary models is their stability, meaning in particular that inflation solutions are attractors in the solution space of the relevant cosmological equations (see e.g. [5, 6]), at least up until inflation ends and the universe becomes radiation dominated. Without this feature, inflation might never have begun, and certainly would not have lasted long enough to affect the large scale structure of the universe.

Warm inflation is an alternative inflationary scenario in which a small but significant amount of radiation survives during the inflationary era due to continuous particle production [7]–[9]. The coupling between radiation and the inflaton field leads to thermal dissipation and fluctuations in the time evolution of the inflaton field. The stability of the inflationary solutions in warm inflationary models has only been addressed in a limited form previously [10], and we will present the full stability analysis here. We shall examine conditions under which warm inflation is an attractor and give conditions for a prolonged period of warm inflation.

We shall show that the stability of warm inflation can be related to conditions on two parameters describing the temperature dependence of terms in the inflaton equation of motion which were not taken into account in the earlier stability analysis [10]. The first condition says that if the dissipation term in the equation of motion falls off too rapidly at low temperature then the temperature is driven to zero, and we fall into the conventional inflationary scenario. The second condition limits the temperature dependence of the inflaton potential. In many models large dissipation implies large thermal corrections to the potential which prevent warm inflation from taking place. This argument is essentially the one first presented in [11]. Nowadays, we know that there are models where thermal corrections to the inflaton potential are suppressed by supersymmetry, and these models may allow warm inflation as an attractor [12]–[14].

The duration of the period of inflation is related to a set of slow-roll parameters which were introduced in [15]. We shall re-derive the slow-roll conditions for warm inflation as part of the stability analysis. A well understood feature of the slow-roll conditions
for warm inflation is that they can be less restrictive than the slow-roll conditions for conventional inflation.

We stress that we are concerned here with the self-consistency of the warm inflationary scenario for given equations of motion. We shall not address how the equation of motion for the inflaton field is obtained from non-equilibrium thermal field theory. A discussion of the derivation of the equations of motion can be found in a recent review [12]. However, we would like to point out that some of the criticisms of warm inflation have been based on models which do not satisfy the fundamental stability conditions derived here, and are therefore not inconsistent with the validity of warm inflation in general [16].

The stability of warm inflation has consequences for the origin and evolution of cosmological density fluctuations [15]. In warm inflation, density fluctuations originate from thermal fluctuations [8,17]. In particular, the fact that the inflationary solution depends on only one parameter means that only one perturbation mode, the curvature perturbation, survives on super-horizon scales despite the fact that there are entropy perturbations present on sub-horizon scales. We shall give formulae for the spectral indices of the scalar and tensor modes and say a little about the tensor/scalar ratio.

2. Basic equations

We start with a flat, homogeneous universe with expansion rate $H$. The matter content consists of a homogeneous inflaton field $\phi$ and thermal radiation of temperature $T$. We restrict attention to the warm inflationary regime where $T > H$, with the radiation close to thermal equilibrium. The evolution of the inflaton is governed by a potential $V(\phi, T)$ and a damping coefficient $\Gamma(\phi, T)$, such that the inflaton field satisfies the basic equation

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V_{,\phi} = 0,$$

where a comma before a function denotes a derivative. The expansion rate is related to the energy density $\rho$ by the Friedmann equation

$$3H^2 = 8\pi G \rho,$$

where $G$ is Newton’s constant.

It is important to realize that the potential appearing in the inflaton equation is the free energy density, rather than the potential energy density. The potential energy density is given by the thermodynamic relation $V + Ts$, where $s$ is the entropy density

$$s = -V_{,T}.$$

The total energy density, including the inflaton’s kinetic energy density, is therefore

$$\rho = \frac{1}{2}\dot{\phi}^2 + V + Ts.$$

This includes the contributions of the scalar field and the radiation. It is not always possible to separate the scalar and radiation components of the energy density in an unambiguous way.

The final equation is the one which governs the transfer of energy from the inflaton to the radiation field. This can easily be derived from the stress–energy tensor $T_{\mu\nu}$ [15],

$$T_{\mu\nu} = Ts u_{\mu} u_{\nu} - V g_{\mu\nu} + \nabla_{\mu} \phi \nabla_{\nu} \phi - \frac{1}{2} (\nabla \phi)^2 g_{\mu\nu},$$

Journal of Cosmology and Astroparticle Physics 11 (2008) 023 (stacks.iop.org/JCAP/2008/i=11/a=023)
where $u_\mu$ is the radiation fluid 4-velocity vector, $g_{\mu\nu}$ is the metric and $\nabla_\nu$ is the spacetime derivative operator. Conservation of the stress–energy gives
\[ T(s + 3Hs) = \Gamma \dot{\phi}^2. \] (6)

This version of the second law of thermodynamics shows clearly how the friction converts the inflaton’s energy into heat. We now have a complete set of evolution equations which can be solved for $\phi$ and $s$ given a set of initial conditions.

Inflation is associated with a slow-roll approximation which consists of dropping the leading derivative term in each equation. The slow-roll equations are therefore
\[ \dot{\phi} = \frac{-V_{,\phi}}{3H(1 + Q)}, \] (7)
\[ Ts = Q\dot{\phi}^2, \] (8)
\[ H^2 = \frac{8\pi GV}{3}, \] (9)

where the strength of the dissipation is quantified by the parameter $Q$,
\[ Q = \frac{\Gamma}{3H}. \] (10)

Note that, since equation (7) is first order in time derivatives, any solution to the slow-roll equations has just one constant of integration.

The validity of the slow-roll approximation will depend on the size of a set of slow-roll parameters [15]. We use the following set of ‘small’ parameters:
\[ \epsilon = \frac{1}{16\pi G} \left( \frac{V_{,\phi}}{V} \right)^2, \quad \eta = \frac{1}{8\pi G} \frac{V_{,\phi\phi}}{V}, \quad \beta = \frac{1}{8\pi G} \frac{V_{,\phi}\Gamma_{,\phi}}{V \Gamma}. \] (11)

An additional pair of parameters describe the temperature dependence,
\[ b = \frac{TV_{,\phi T}}{V_{,\phi}}, \quad c = \frac{TT_{,T}}{\Gamma}. \] (12)

We use $b$ in place of the parameter $\delta$ defined in [15].

The parameter $b$ is very important in the theory of warm inflation. It measures the size of contributions to the potential from thermal quantum fields. In ordinary circumstances, we would expect $b$ to be order 1. Consider, for example, an inflaton of mass $m_\phi$ coupled to a set of $g_\ast$ light scalar fields and temperature $T > H > m_\phi$. The effective potential is
\[ V(\phi, T) = -\frac{\pi^2}{90}g_\ast T^4 - \frac{1}{12}m_\phi^2 T^2 + v(\phi), \] (13)

where $v(\phi)$ is the effective potential at $T = 0$. The parameter $b$ for the potential (13) is approximately 2. Values of $b$ can be much smaller in supersymmetric theories, where there is a cancellation of leading order thermal corrections when the temperature $T < \Lambda_\text{S}$, where $\Lambda_\text{S}$ is the supersymmetry breaking scale [18]. We shall find limits on $b$, and conclude that warm inflation only takes place when there is a mechanism, like supersymmetry, which reduces the size of the thermal corrections to the potential [19].
3. Stability analysis

We shall consider the consistency of the slow-roll approximation by performing a linear stability analysis to determine the conditions which are sufficient for the system to remain close to the slow-roll solution for many Hubble times. It may be worthwhile considering first what happens in the alternative cold inflationary scenario (see e.g. [4]). The slow-roll equation in this case is first order in time derivatives, and the general solution has the form \( \phi = f(t - t_0) \), where \( t_0 \) is an arbitrary constant. There always exists a homogeneous perturbation which is equivalent to changing the value of \( t_0 \), and is hugely important for the existence of density perturbations. Another mode decays on the Hubble timescale. It proves convenient to exclude the time-translation mode by using the value of the field as the time coordinate, which is possible if the inflaton field has non-vanishing time derivative. We shall follow the same procedure for the analysis of warm inflation.

With the inflaton as independent variable we can rewrite the system of equations in first-order form,

\[
x' = F(x). \tag{14}
\]

Primes denote derivatives with respect to \( \phi \) and

\[
x = \begin{pmatrix} u \\ s \end{pmatrix}, \tag{15}
\]

where \( u = \dot{\phi} \). Equations (1) and (6) become

\[
u' = -3H - \Gamma - V_{,\phi}u^{-1}, \tag{16}
\]

\[
s' = -3Hsu^{-1} + T^{-1}\Gamma u, \tag{17}
\]

with the temperature determined implicitly by equation (3) and \( H \) given by equation (2). We take a background \( \bar{x} \) which satisfies the slow-roll equations (7)–(9) and then the linearized perturbations satisfy

\[
\delta x' = M(\bar{x})\delta x - \bar{x}', \tag{18}
\]

where \( M \) is the matrix of first derivatives of \( F \) evaluated at the slow-roll solution.

Consider stability first of all. We can express all the components of the matrix \( M \) in terms of the slow-roll parameters (12). For example,

\[
\Gamma_{,s} = \Gamma_{,T} T_{,s} = \frac{c\Gamma}{T}T_{,s} = c\frac{\Gamma}{3s} = c\frac{QH}{s}, \tag{19}
\]

where we have used \( T_{,s} = (sT)^{-1} = T/3s \). Following a similar procedure for all of the first derivatives gives a final expression for \( M \),

\[
M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \tag{20}
\]
On the consistency of warm inflation

where

\[ A = \frac{H}{u} \left\{ -3(1 + Q) - \frac{\epsilon}{(1 + Q)^2} \right\}, \]  
\[ B = \frac{H}{s} \left\{ -cQ - \frac{Q}{(1 + Q)^2}\epsilon + (1 + Q)b \right\}, \]  
\[ C = \frac{H s}{u^2} \left\{ 6 - \frac{\epsilon}{(1 + Q)^2} \right\}, \]  
\[ D = \frac{H}{u} \left\{ c - 4 - \frac{Q\epsilon}{(1 + Q)^2} \right\}. \]  

We require the determinant,

\[ \det M = \frac{H^2}{u^2} \left\{ 12(1 + Q) + 3(Q - 1)c - 6(1 + Q)b 
+ \left( \frac{3Q^2 + 9Q + 4}{(1 + Q)^2} - \frac{c}{1 + Q} \right) \epsilon + \frac{bc}{1 + Q} \right\}, \]  

and the trace,

\[ \text{tr} M = \frac{H}{u} \left\{ c - 4 - 3(1 + Q) - \frac{\epsilon}{1 + Q} \right\}. \]  

In the cold inflationary case, where \( Q = b = c = s = 0 \), the decaying modes have the approximate form \( \delta u \propto \exp(-3N) \) and \( \delta s \propto \exp(-4N) \), where \( N \) is the number of e-folds of inflation [5].

Sufficient conditions for stability of the warm inflationary solution are that the matrix \( M \) varies slowly and

\[ |c| \leq 4 - 2b, \quad b \geq 0. \]  

Evidently, the trace is negative and the determinant is positive if these conditions hold. The linear equation therefore has two negative eigenvalues and both eigenmodes decay. The slow variation of \( M \), which allows us to diagonalize the linear system, follows if the forcing term in equation (18) is small, and so we turn to this term next.

The forcing term in equation (18) depends on \( \vec{x}' \). This term is present because the background chosen is not an exact solution to the full set of equations. The slow-roll approximation can only be valid when \( \vec{x}' \) is small. If we work with time derivatives, the magnitude of \( \vec{x}' \) depends on the dimensionless quantities \( \dot{u}/(Hu) \) and \( \dot{s}/(Hs) \), and small values represent slow variation on the timescale of the Hubble time.

We only quote the leading terms in \( \epsilon \). Starting from the slow-roll equation (9) and taking the time derivative gives

\[ \frac{\dot{H}}{H^2} = -\frac{1}{1 + Q}\epsilon. \]  


By combining the other slow-roll equations (7) and (8) we eventually arrive at
\[
\frac{\dot{u}}{Hu} = \frac{1}{\Delta} \left\{ -3c(1 + Q)b - \frac{c(1 + Q) - 4}{1 + Q} \epsilon + (c - 4)\eta + \frac{4Q}{1 + Q} \beta \right\},
\]
(29)
\[
\frac{\dot{s}}{Hs} = \frac{1}{\Delta} \left\{ +3(cQ + Q + 1 - c)(1 + Q)b + \frac{3Q + 9}{1 + Q} \epsilon - 6\eta + \frac{3(Q - 1)}{1 + Q} \beta \right\},
\]
(30)
where \(\Delta = 4(1 + Q) + (Q - 1)c\). The slow-roll approximation requires \(\dot{u} \ll Hu\) and \(\dot{s} \ll Hs\), and sufficient conditions for this are
\[
\epsilon, |\beta|, |\eta| \ll 1 + Q; \quad 0 < b \ll \frac{Q}{1 + Q}; \quad |c| < 4.
\]
(31)
The conditions on \(\epsilon\) and \(\eta\) agree with a previous stability analysis [10]. These are weaker than the corresponding conditions for cold inflation, and this fact is a well known feature of warm inflation.

The physical interpretation of the condition \(c < 4\) is evident from equation (6): that radiation must be produced at a rate (\(\Gamma \propto T^c\)) exceeding the rate at which radiation is removed by the expansion of the universe (\(Ts \propto T^4\)). The condition on \(b\) can only be met if there is a mechanism for suppressing thermal corrections to the potential because, as we mentioned at the end of section 2, high temperature thermal corrections would otherwise lead to \(b \approx 2\). Models which include a mechanism for suppressing thermal corrections can be found, for example, in [13,14,19].

4. Density fluctuations

The results which have been obtained as part of the stability analysis are also helpful for analysing various features of the density fluctuation spectrum. We therefore take the opportunity, whilst the results are to hand, of giving some formulae which might be useful for observational tests of warm inflation.

The origin of density fluctuations in warm inflationary scenarios is due to thermal fluctuations in the radiation. These are coupled to the inflaton as a consequence of the friction term in the inflaton equation of motion, and their amplitude is fixed by a fluctuation-dissipation theorem. This means that both entropy and curvature perturbations must be present. However, on length scales larger than the horizon, we know from the stability argument that the coupled inflaton plus radiation system approaches the slow-roll solution which has only one free parameter. Consequently, on large scales only the pure curvature perturbation survives. This has been confirmed in particular models by solving the full set of density fluctuation equations numerically [15].

Even though the entropy perturbations decay on large scales, they can sometimes leave behind an impression on the curvature fluctuations. If the friction term depends on temperature, then the entropy and curvature fluctuations on sub-horizon scales become coupled. The situation is similar to the sympathetic oscillations of a double pendulum [20]. When the curvature fluctuations ‘freeze out’, the amplitude of the sympathetic oscillation may be anywhere between zero and its maximum value, leading to oscillations in the wavenumber dependence of the curvature fluctuation spectrum. The amplitude given below therefore has only limited use when \(b\) and \(c\) are non-zero and refers only to the envelope of these oscillations.
On the consistency of warm inflation

The thermal fluctuations produce a power spectrum of scalar density fluctuations of the form \[ P_S = \frac{\sqrt{\pi} H^3 T}{u^2} (1 + Q)^{1/2}. \] (32)
The spectral index \( n_S \) is defined by \[ n_S - 1 = \frac{\partial \ln P_S}{\partial \ln k}, \] (33)
evaluated when the amplitudes ‘freeze out’. To leading order in the slow-roll parameters we can take the freeze-out time to be the horizon crossing time when \( k = aH \), and then \[ n_S - 1 = \frac{\dot{P}_S}{H P_S}. \] (34)
We can use equations (28)–(30) to obtain
\[ n_S - 1 = \frac{1}{\Delta} \left\{ \frac{-3(2Q + 2 + 5Qc)(1 + Q)}{Q} b - \frac{9Q + 17 - 5c}{1 + Q} \epsilon - \frac{9Q + 1}{1 + Q} \beta \right. \]
\[ \left. - \frac{3Qc - 6 + 6Q + 2c}{1 + Q} \eta \right\}. \] (35)
If we consider \( b = c = 0 \), then important limits include the strong regime of warm inflation, \( Q \gg 1 \),
\[ n_S - 1 = -\frac{9}{4Q} \epsilon - \frac{9}{4Q} \beta + \frac{3}{2Q} \eta. \] (36)
This agrees with a partial result in [22] and the full result in [15]. In the weak regime of warm inflation, \( Q \ll 1 \), thermal fluctuations lead to the spectral index
\[ n_S - 1 = -\frac{17}{4} \epsilon - \frac{1}{4} \beta + \frac{3}{2} \eta. \] (37)
Previous results for the weak regime, though expressed in a less useful form, can be found in [23, 24]. Finally, the case \( Q \ll 1 \) and \( c = 3 \) is important because it corresponds to a class of warm inflationary models for which the friction coefficient \( \Gamma \) has been calculated [25],
\[ n_S - 1 = -2 \epsilon - \beta. \] (38)
The tensor modes have the same amplitude as they do in the cold inflationary models\(^1\),
\[ \mathcal{P}_T = 8\pi GH^2. \] (39)
The spectrum for the tensor modes is simply
\[ n_T - 1 = -\frac{2}{1 + Q} \epsilon. \] (40)
Unlike in the cold inflationary scenario, the tensor–scalar amplitude ratio cannot be expressed in terms of slow-roll parameters. Instead [22],
\[ \frac{\mathcal{P}_T}{\mathcal{P}_S} = \frac{2\epsilon H}{(1 + Q)^{3} T}. \] (41)
\(^1\) Our power spectrum convention for \( \delta_k \) is \( \langle \delta_k \delta_{k'} \rangle = (2\pi)^3 k^{-3} \mathcal{P}_S(k) \delta(k_1 - k_2). \)
Since $T > H$ for warm inflation, the tensor–scalar ratio is likely to be smaller than $1 - n_T$. Tensor modes are strongly suppressed relative to the scalar modes in the strong regime of warm inflation $Q \gg 1$, but they could be significant in the weak regime of warm inflation.

We conclude this section by finding the lower limit on the friction term which is required for warm inflation. We shall express this limit in terms of $Q = \Gamma / 3H$. Rewrite the scalar amplitude equation (32) as

$$P_S \approx \frac{T^4 H^3}{u^2 T^3} (1 + Q)^{1/2}.$$  \hspace{1cm} (42)

The first factor can be replaced using the slow-roll equation, equation (8), and the potential (13), and we obtain

$$\frac{T}{H} \approx \left(\frac{45}{2\pi^2 g_*}\right)^{1/3} (1 + Q)^{1/6} \left(\frac{Q}{P_S}\right)^{1/3}.$$  \hspace{1cm} (43)

The condition for warm inflation $T > H$ is therefore

$$Q > g_* P_S.$$  \hspace{1cm} (44)

Cosmic microwave background observations give a scalar power spectrum of $1 \times 10^{-10}$ on large scales; therefore very small amounts of dissipation can result in warm inflation.

5. Conclusion

We shall recapitulate the main points of this paper. There are conditions on six of the parameters defined in section 2 for the possibility of a stable period of warm inflation in the early universe:

- The parameter $Q$ which measures the strength of the friction term must satisfy

$$Q > g_* P_S,$$  \hspace{1cm} (45)

where $g_*$ is the effective particle number and $P_S$ is the scalar perturbation power spectrum on large scales.

- The parameters which describe the inflaton dependence of the effective potential and friction term satisfy

$$\epsilon \ll 1 + Q, \quad |\eta| \ll 1 + Q, \quad |\beta| \ll 1 + Q.$$  \hspace{1cm} (46)

- The temperature dependence of the potential and the friction term is restricted by

$$|b| \ll \frac{Q}{1 + Q}, \quad |c| < 4.$$  \hspace{1cm} (47)

The condition on $b$ implies that warm inflation is only possible when a mechanism, such as supersymmetry, reduces the size of the thermal corrections to the potential.

Models of elementary particles exist in which these conditions can be satisfied. The most convincing of these models use a combination of supersymmetry and a two-stage decay process, where there are no direct couplings between the inflaton and the radiation and all thermal effects are suppressed by factors of $T/\Lambda_S$, where $\Lambda_S$ is the supersymmetry breaking scale \cite{13, 14, 19}.
When the conditions listed above are satisfied, then the solutions to the equations of motion approach a slow-roll approximation during inflation. As a result, large scale density perturbations have only one degree of freedom, which we identify as the curvature perturbation. (Entropy perturbations can only be introduced by adding extra degrees of freedom to the system.)

Acknowledgments

Chun Xiong was supported by an ORS scholarship and by the School of Mathematics and Statistics, Newcastle University.

References

[1] Guth A H, 1981 Phys. Rev. D 23 347 [SPIRES]
[2] Linde A, 1982 Phys. Lett. B 108 389 [SPIRES]
[3] Albrecht A and Steinhardt P J, 1982 Phys. Rev. Lett. 48 1220 [SPIRES]
[4] Liddle A R and Lyth D H, 2000 Cosmological Inflation and Large-Scale Structure (Cambridge: Cambridge University Press) section 3.7
[5] Salopek D S and Bond J R, 1990 Phys. Rev. D 42 3936 [SPIRES]
[6] Liddle A R, Parsons P and Barrow J D, 1994 Phys. Rev. D 50 7222 [SPIRES] [arXiv:astro-ph/9408015]
[7] Moss I G, 1985 Phys. Lett. B 154 120 [SPIRES]
[8] Berera A and Fang L Z, 1995 Phys. Rev. Lett. 74 1912 [SPIRES]
[9] Berera A, 1995 Phys. Rev. Lett. 75 3218 [SPIRES]
[10] de Oliveira H P and Ramos R O, 1998 Phys. Rev. D 57 741 [SPIRES] [arXiv:gr-qc/9710093]
[11] Yokoyama J and Linde A, 1999 Phys. Rev. D 60 083509 [SPIRES]
[12] Berera A, Moss I G and Ramos R, 2008 Rep. Prog. Phys. submitted [arXiv:0808.1855]
[13] Bastero-Gil M and Berera A, 2007 Phys. Rev. D 76 043515 [SPIRES] [arXiv:hep-ph/0610343]
[14] Bueno Sanchez J C, Bastero-Gil M, Berera A and Dimopoulos K, 2008 Phys. Rev. D 77 123527 [SPIRES] [arXiv:0802.4354]
[15] Hall L M H, Moss I G and Berera A, 2004 Phys. Rev. D 69 083525 [SPIRES] [arXiv:astro-ph/0305015]
[16] Aarts G and Tranberg A, 2008 Phys. Rev. D 77 123521 [SPIRES] [arXiv:0712.1120]
[17] Berera A, 2000 Nucl. Phys. B 585 666 [SPIRES]
[18] Hall L M H and Moss I G, 2005 Phys. Rev. D 71 023514 [SPIRES] [arXiv:hep-ph/0408323]
[19] Berera A and Ramos R O, 2003 Phys. Lett. B 567 294 [SPIRES]
[20] Sommerfeld A, 1952 Mechanics (New York: Academic)
[21] Moss I G and Xiong C, 2007 J. Cosmol. Astropart. Phys. JCAP04(2007)007 [SPIRES] [arXiv:astro-ph/0701302]
[22] Taylor A N and Berera A, 2000 Phys. Rev. D 62 083517 [SPIRES]
[23] Hall L M H and Peiris H V, 2008 J. Cosmol. Astropart. Phys. JCAP01(2008)027 [SPIRES] [arXiv:0709.2912]
[24] Bastero-Gil M and Berera A, 2005 Phys. Rev. D 71 063515 [SPIRES] [arXiv:hep-ph/0411144]
[25] Moss I G and Xiong C, 2006 arXiv:hep-ph/0603266