Fault Detection of Discrete-time LPV System Based on Event Triggering Mechanism

Shuai Mao

School of Electrical Engineering and Information, Northeast Petroleum University, Heilongjiang Province, Daqing 163318, China

Abstract: For discrete-time linear parameter varying systems with distributed time delays, the problem of fault detection under event triggering mechanism is studied. Based on the parameter dependent Lyapunov function method, the sufficient conditions for the system to be asymptotically stable and meet the $H_{\infty}$ performance criterion are given. The parameter matrix of fault detection filter is obtained by solving LMI technology. In addition, the event triggering strategy is introduced to save network resources.

Keywords: Fault detection, Discrete-time LPV system, $H_{\infty}$ performance, LMI.

1. Introduction

As a kind of time-varying system with uncertain parameters, linear parameter variation (LPV) system has been widely used in aerospace, industrial manufacturing, wind power generation and other fields [1-3]. No matter in which field, the operation of the control system is inseparable from the network environment. On the one hand, due to some disadvantages of the network control system itself, on the other hand, due to the influence of external complex working conditions, various faults will appear in the operation process of the control system [4-5]. If these faults can not be found and solved in time, it will either lead to the temporary collapse of the control system [4-5]. If these faults can not be found and solved in time, it will either lead to the temporary collapse of the control system, or lead to the empty state of human and financial resources. Therefore, it is very meaningful for the system to detect the fault information as soon as possible and take necessary measures before the fault occurs. However, the network control system is limited by the network bandwidth, and a large number of data will be congested in the transmission process [6-8]. In addition, the previous control strategy based on time trigger requires the system to sample in each cycle. If the sampling cycle interval is too small, the system will sample and send data frequently, which can not effectively filter useful information. In this regard, a new control strategy based on event triggering is introduced to make the system execute tasks under specific triggering conditions, the required information is transmitted and the useless information is filtered [9]. Based on the mode dependent switching strategy and the Lyapunov stability theory of dual dependence of parameters and modes, reference [10] studies the design of asynchronous $H_{\infty}$ fixed order filters for LPV time-delay switched systems under the mode dependent mean residence time method. Literature [11] gives sufficient conditions for the existence of filters for discrete LPV systems by using linear matrix inequality technology and parameter dependent quadratic Lyapunov function method. On this basis, literature [12] and literature [13] give $H_{\infty}$ filter design methods for parameter bounded discrete LPV systems and observer design methods for discrete LPV systems with unknown inputs respectively. The former also verifies the effectiveness of the proposed filter design scheme through a numerical simulation.

2. Problem Statement

Consider the following form of discrete-time switched LPV system

$$
\begin{align*}
\dot{x}(k+1) &= A_{\lambda}(\rho(k))x(k) + B_{\lambda}(\rho(k))y(k) \\
r(k) &= C_{\lambda}(\rho(k))x(k) + D_{\lambda}(\rho(k))y(k)
\end{align*}
$$

(1)

where $x(k) \in \mathbb{R}^n$ is the system state vector, $y(k) \in \mathbb{R}^n$ is the measurement output vector of controlled object, $f(k) \in \mathbb{R}^p$ is the fault signal of system, $\omega(k) \in \mathbb{R}^m$ is disturbance input variable in $L_2[0, \infty]$. For the convenience of description, we use $\rho_i$ for $\rho(k)$ and $A(\rho_i), A_i(\rho_i), B_i(\rho_i), B(\rho_i), C(\rho_i), D(\rho_i)$ are the functions of time-varying parameter $\rho_i$ with its parameter variation rate $\dot{\rho}_i(k)$ bounded in $[\rho_i, \rho_i]$, $l = 1, 2, \ldots, s$. The value of parameter $\rho_i$ can be measured online. $\tau(k)$ is the discrete time-varying delay, which meets $0 < \tau_i \leq \tau(k) \leq \tau^*$. The parameter $\lambda_i \geq 0$ and $\bar{X} = \sum_{i \in \mathcal{I}_c} \lambda_i \leq \sum_{i \in \mathcal{I}} \lambda_i < +\infty$.

A fault detection filter in the form of formula (2) is designed to obtain the residual signal of the system and detect the fault of the system

$$
\begin{align*}
x_f(k+1) &= A_f(\rho_f) x_f(k) + B_f(\rho_f) y(k) \\
r_f(k) &= C_f(\rho_f) x_f(k) + D_f(\rho_f) y(k)
\end{align*}
$$

(2)

where $x_f(k) \in \mathbb{R}^n$ is the state of the designed filter, $r_f(k) \in \mathbb{R}^n$ is the residual signal. The parameter-varying matrices $A_f(\rho_f), B_f(\rho_f), C_f(\rho_f), D_f(\rho_f)$ are represent the parameter matrix of fault detection filter to be solved.

Setting the event-triggered condition

$$k_{rv} = \min \{k > k_{rv-1} | \varepsilon(k) < \varepsilon y_r(k) y(k)\}
$$

(3)

where $\varepsilon$ is the event-triggered threshold,
\[ e_i(k) = y(k_i) - y(k), \quad k \in [k_i, k_{i+1}] \]

Combining (1)-(3), we can get

\[ \sum_{k_{i-1}}^{k_{i+1}} \left[ \xi^T(k) x_i^T(k) \right] = e(k) = r(k) - f(k), \]

where \( H = [0, I] \), \( \mathcal{A}(\rho) = \begin{bmatrix} A(\rho) & 0 \\ B_1(\rho) C(\rho) & A_r(\rho) \end{bmatrix} \),

\[ \mathcal{A}(\rho) = \begin{bmatrix} A(\rho) & 0 \\ B_1(\rho) D_1(\rho) & A_r(\rho) \end{bmatrix}, \]

\[ \mathcal{C}(\rho) = \begin{bmatrix} D_1(\rho) C(\rho) \\ C_1(\rho) \end{bmatrix}, \]

\[ \mathcal{D}(\rho) = \begin{bmatrix} D_1(\rho) D_2(\rho) \\ D_1(\rho) D_2(\rho) - I \end{bmatrix} \]

Definition 1. The switched LPV system (1) has \( H_{\infty} \) performance, if there exist event-triggered condition (3), filter (2) such that the following conditions are satisfied

1. When \( \omega(k) = 0 \), the system (4) is asymptotically stable.
2. Under the zero initial condition, for all nonzero \( \omega(k) \in L_2[0, \infty) \), the filtering error system and the control system satisfies

\[ \sum_{i=0}^{k} e_i^T(k) e(k) < \gamma^2 \sum_{i=0}^{k} \psi^T(k) \psi(k). \]

### 3. Main Results

For the fault detection system (4), by selecting the appropriate Lyapunov Krasinski functional, a sufficient condition is proposed to make the fault detection system (4) asymptotically stable and meet the \( H_{\infty} \) performance criterion under the event trigger mechanism, which provides a theoretical basis for the later design of fault detection filter.

**Theorem 1.** If there exist positive definite matrix functions \( P(\rho) > 0 \), \( Q > 0 \) and \( M > 0 \), such that the following matrix inequalities hold

\[ \begin{bmatrix} \Pi & 0 & 0 & 0 & 0 \\ * & -Q & 0 & 0 & 0 \\ * & * & -1 & M & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} \begin{bmatrix} \mathcal{C}(\rho) P(\rho) \mathcal{C}(\rho) & \mathcal{C}(\rho) & \sqrt{2} \mathcal{C}(\rho) \\ * & -Q & 0 & 0 & 0 \\ * & * & -1 & M & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \] (5)

where \( \Pi = -P(\rho) + \mu H^T QH + \Xi H^T M H, \quad \mu = \tau_z - \tau_i + 1 \),

\[ \Xi = \begin{bmatrix} C(\rho) & 0 \\ 0 & D(\rho_1) D_1(\rho) \end{bmatrix}, \]

Then the fault detection system (4) is asymptotically stable, and its \( H_{\infty} \) performance is guaranteed.

**Proof.** Choosing the multiple parameter-dependent Lyapunov functions as

\[ V_i(k, \xi(k)) = V_1(k, \xi(k)) + V_2(k, \xi(k)) + V_3(k, \xi(k)) + V_4(k, \xi(k)) \]

\[ V_4(k, \xi(k)) = \sum_{j=0}^{k} \zeta^T(j) H^T M H \xi(j), \]

\[ V_1(k, \xi(k)) = \xi^T(k) P(\rho_i) \xi(k), \]

\[ V_2(k, \xi(k)) = \sum_{j=i}^{k} \zeta^T(i) H^T QH \zeta(i), \]

\[ V_3(k, \xi(k)) = \sum_{j=i}^{k} \zeta^T(i) H^T QH \zeta(i). \]

The forward difference along the system (4) can be obtained

\[ \Delta V_1(k, \xi(k)) = \xi^T(k+1) P(\rho_{i+1}) \xi(k+1) - \xi^T(k) P(\rho_i) \xi(k) \]

\[ \Delta V_2(k, \xi(k)) = \sum_{j=i}^{k} \zeta^T(i) H^T QH \zeta(i) \]

\[ \Delta V_3(k, \xi(k)) = \sum_{j=i}^{k} \zeta^T(i) H^T QH \zeta(i). \]

Defining

\[ \eta(k) = \left[ \begin{array}{c} \xi^T(k) \\ - \tau_z \xi^T(k) \end{array} \right] \]

we can get

\[ \Delta V(k, \xi(k)) = \sum_{j=i}^{k} \zeta^T(i) H^T M H \xi(j) \]

The performance index in the form of formula (6) is introduced

\[ J = \sum_{i=0}^{k} \left[ e_i(k) e(k) - \gamma^2 \psi^T(k) \psi(k) \right] \] (6)

Under zero initial conditions, \( V(k, \xi(k)) \big|_{k=0} = 0 \), \( V(k, \xi(k)) \big|_{k=0} > 0 \), we can get

\[ J \leq \sum_{i=0}^{k} e_i(k) e(k) - \gamma^2 \psi^T(k) \psi(k) + \Delta V(k, \xi(k)) \big|_{k=0} = \sum_{i=0}^{k} \psi^T(k) \Phi \psi(k) \] (7)

According to theorem 1, for any non-zero vector \( \psi(k) \), \( J < 0 \), then

\[ \sum_{i=0}^{k} e_i(k) e(k) - \gamma^2 \psi^T(k) \psi(k) < 0, \]

so we can get

\[ \sum_{i=0}^{k} \psi^T(k) \psi(k) < 0 \]

**Theorem 2.** If there exist positive definite matrix functions \( P(\rho) \), \( Q \), \( M \) and \( U \), such that the following matrix inequalities hold

\[ \begin{bmatrix} \Pi & 0 & 0 & 0 \\ * & -Q & 0 & 0 \\ * & * & -1 & M \\ * & * & * & -I \end{bmatrix} \begin{bmatrix} \mathcal{C}(\rho) & \sqrt{2} \mathcal{C}(\rho) \\ * & 0 \\ * & * \\ * & * \end{bmatrix} < 0 \] (8)

then the fault detection system (4) is asymptotically stable, and its \( H_{\infty} \) performance is guaranteed.

**Proof.** It is necessary to prove that inequality (5) is
equivalent to inequality (8). If inequality (5) holds, defining $V = V^T = P(\rho_{1zi})$, substitute in (5), so we can get (8). Therefore, if (8) holds, according to $-V^T - P(\rho_{1zi}) < 0$ and $P(\rho_{1zi}) > 0$, we have
\[
\begin{align*}
(V - P(\rho_{1zi}))^T P^{-1}(\rho_{1zi})(V - P(\rho_{1zi})) &\geq 0, \\
-V^T P^{-1}(\rho_{1zi})W &\leq -V^T + P(\rho_{1zi}).
\end{align*}
\]

So
\[
\begin{bmatrix}
-P(\rho_{1zi}) + h^T Q h \\
* \\
* \\
* \\
* \\
* \\
* \\
* \\
* \\
* \\
* \\
\end{bmatrix}
\begin{bmatrix}
X(\rho_{1zi})W \\
L(\rho_{1zi}) \\
Q \\
I \\
Q \\
\eta I \\
-I \\
I \\
-I \\
-I \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
< 0.
\]

Assuming $V$ reversibility, using $\text{diag} \{ I, I, I, V^T P(\rho_{1zi}), I \}$ congruent transformation of inequality, we can get (8), theorem proving.

Theorem 3. If there exist positive definite matrix functions $P(\rho_{1zi})$, $Q$, $M$ and filter parameters $\mathcal{A}(\rho_{1zi})$, $\mathcal{B}(\rho_{1zi})$, $\mathcal{C}(\rho_{1zi})$, $D(\rho_{1zi})$, such that the following matrix inequalities hold
\[
\begin{bmatrix}
\Lambda_{11} - \mathcal{P}(\rho_{1zi}) \\
\Sigma \\
\Sigma \\
\Sigma \\
\Sigma \\
\Sigma \\
\Sigma \\
\Sigma \\
\Sigma \\
\Gamma \\
\end{bmatrix}
\begin{bmatrix}
\mathcal{T}(\rho_{1zi}) \\
\mathcal{P}(\rho_{1zi}) \\
\mathcal{P}(\rho_{1zi}) \\
\mathcal{P}(\rho_{1zi}) \\
\mathcal{P}(\rho_{1zi}) \\
\mathcal{P}(\rho_{1zi}) \\
\mathcal{P}(\rho_{1zi}) \\
\mathcal{P}(\rho_{1zi}) \\
\mathcal{P}(\rho_{1zi}) \\
\mathcal{P}(\rho_{1zi}) \\
\end{bmatrix}
< 0.
\]

where
\[
\begin{align*}
\Lambda_{11} &= \mathcal{P}(\rho_{1zi}) + h^T Q h + \Sigma M, \\
\Lambda_{11} &= [\mathcal{A}(\rho_{1zi})], \\
\Lambda_{11} &= [\mathcal{A}(\rho_{1zi}) X \mathcal{A}(\rho_{1zi})], \\
\Lambda_{11} &= [\mathcal{A}(\rho_{1zi}) X + \mathcal{C}(\rho_{1zi}) \mathcal{B}(\rho_{1zi})], \\
\Lambda_{11} &= [\mathcal{A}(\rho_{1zi}) X \mathcal{A}(\rho_{1zi}) Y], \\
\Lambda_{11} &= [\mathcal{B}(\rho_{1zi}) \mathcal{B}(\rho_{1zi})], \\
\Lambda_{11} &= [\mathcal{B}(\rho_{1zi}) X + \mathcal{D}(\rho_{1zi}) \mathcal{B}(\rho_{1zi}) \mathcal{B}(\rho_{1zi})], \\
\Lambda_{11} &= [\mathcal{B}(\rho_{1zi}) Y + \mathcal{D}(\rho_{1zi}) \mathcal{B}(\rho_{1zi}) \mathcal{B}(\rho_{1zi})], \\
\Lambda_{11} &= [\mathcal{D}(\rho_{1zi}) \mathcal{D}(\rho_{1zi}) + I], \\
\Lambda_{11} &= [\mathcal{D}(\rho_{1zi}) \mathcal{D}(\rho_{1zi}) - I], \\
\Lambda_{11} &= [\mathcal{D}(\rho_{1zi}) \mathcal{D}(\rho_{1zi})].
\end{align*}
\]

Then the fault detection system (4) is asymptotically stable, and its $H_\infty$ performance is guaranteed. The parameters of fault detection filter are
\[
\begin{bmatrix}
A(\rho_{1zi}) \\
B(\rho_{1zi}) \\
C(\rho_{1zi}) \\
D(\rho_{1zi}) \\
\end{bmatrix}
= \begin{bmatrix}
Z^{-1} \\
0 \\
1 \\
0 \\
\end{bmatrix}
\begin{bmatrix}
\mathcal{A}(\rho_{1zi}) \\
\mathcal{B}(\rho_{1zi}) \\
\mathcal{C}(\rho_{1zi}) \\
D(\rho_{1zi}) \\
\end{bmatrix}
\]

\[
\text{Proof. Defining } P(\rho_{1zi}) = \begin{bmatrix}
P_{1zi}(\rho_{1zi}) & P_{2zi}(\rho_{1zi}) \\
P_{2zi}(\rho_{1zi}) * & P_{3zi}(\rho_{1zi}) \\
V = & \begin{bmatrix} V_t & V_z \end{bmatrix}, \\
\Omega = & \begin{bmatrix} I & 0 \\
0 & V_t \end{bmatrix}, \text{ using } \text{diag} \{ I, I, I, I, I, I \} \text{ to make congruent transformation of inequality (8), and defining}
\begin{align*}
\mathcal{P}(\rho_{1zi}) &= \Omega^T \mathcal{P}(\rho_{1zi}) \Omega, \\
\mathcal{T}(\rho_{1zi}) &= \mathcal{T}(\rho_{1zi}) \mathcal{V}(\rho_{1zi}) \mathcal{V}(\rho_{1zi})^T, \\
\mathcal{F}(\rho_{1zi}) &= \mathcal{F}(\rho_{1zi}) \mathcal{V}(\rho_{1zi}) \mathcal{V}(\rho_{1zi}), \\
\mathcal{F}(\rho_{1zi}) &= \mathcal{F}(\rho_{1zi}) \mathcal{V}(\rho_{1zi}) \mathcal{V}(\rho_{1zi}) + \mathcal{D}(\rho_{1zi}), \\
\mathcal{C}(\rho_{1zi}) &= \mathcal{C}(\rho_{1zi}) \mathcal{V}(\rho_{1zi}) \mathcal{V}(\rho_{1zi}) + \mathcal{D}(\rho_{1zi}).
\end{align*}
\]

inequality (9) can be obtained, that is, theorem 3 can be proved.

Since the matrix $P(\rho_{1zi})$ is positive definite, it can be obtained $\Omega^T \mathcal{P}(\rho_{1zi}) \Omega > 0$, that equation (10) holds. Substitute (12) into the following formula and simplify it appropriately
\[
\begin{align*}
\mathcal{T}(\rho_{1zi}) &= \mathcal{C}(\rho_{1zi}) \mathcal{V}(\rho_{1zi}) \mathcal{V}(\rho_{1zi}) \mathcal{V}(\rho_{1zi}) + \mathcal{D}(\rho_{1zi}), \\
\mathcal{T}(\rho_{1zi}) &= \mathcal{C}(\rho_{1zi}) \mathcal{V}(\rho_{1zi}) \mathcal{V}(\rho_{1zi}) \mathcal{V}(\rho_{1zi}) + \mathcal{D}(\rho_{1zi}).
\end{align*}
\]

thus, the filter parameter matrix with form (11) can be obtained. The certificate is completed.

In this section, numerical simulation will be used to verify the effectiveness of the fault detection scheme of discrete LPV system under the designed event triggering mechanism. Considering the LPV system with mixed delay (1), the system parameter matrix is given as follows
\[
\begin{align*}
A &= \begin{bmatrix}
-0.02 & 0.08 \\
-0.12 & 0.05 + 0.02 \\
\end{bmatrix}, \\
B &= \begin{bmatrix}
0.14 + 0.1\rho_{1zi} \\
0.16 \\
\end{bmatrix}, \\
C &= \begin{bmatrix}
0.03 \\
0.02 + 0.08 + 0.05\rho_{1zi} \\
\end{bmatrix}, \\
D &= \begin{bmatrix}
0.2 \\
0.4 \\
\end{bmatrix}, \\
\end{align*}
\]

The selection basis function $f(\rho) = 1$, $f(\rho) = \rho(k)$, and set the event trigger threshold $\varepsilon = 0.3$ at the same time. The $H_\infty$ performance index $\gamma_{\text{max}} = 0.9577$ is solved by convex optimization, and the parameters of fault detection filter are
\[
\begin{align*}
A_f &= \begin{bmatrix}
-0.0076 \\
-0.0221 \\
0.0001 \\
0.0654 \\
\end{bmatrix}, \\
B_f &= \begin{bmatrix}
0.0229 \\
-0.0043 \\
0.0770 \\
0.0668 \\
\end{bmatrix}, \\
C_f &= \begin{bmatrix}
0.4202 \\
-0.4720 \\
-0.1979 \\
-0.0025 \\
\end{bmatrix}, \\
D_f &= \begin{bmatrix}
0.7522 \\
0.4414 \\
\end{bmatrix}, \\
\end{align*}
\]
shows the transmission time and interval of event triggered data. The time length of analog signal is set to 120. After adopting the event trigger strategy, only 97 valid data are successfully sent, which effectively saves network resources.

4. Conclusion

In this paper, the problem of fault detection under event triggering mechanism is studied for discrete-time linear parameter varying systems with distributed delay. Based on the parameter dependent Lyapunov function method, the sufficient conditions for the system to be asymptotically stable and meet the $H_\infty$ performance criterion are given. The threshold logic method is used to detect the fault signal. The coupling between the system matrix and the Lyapunov function matrix is decoupled, and the parameter matrix of the fault detection filter is solved by LMI technology. Finally, the simulation results show that the fault detection filter designed in this chapter can accurately detect the fault signal of the system and achieve the expected goal. In addition, after introducing the event triggering strategy, it effectively avoids the transmission of some useless data, which not only saves network resources, but also improves the efficiency of network transmission.

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