Heat and mass transfer analysis of MHD stagnation point flow of carbon nanotubes with convective stretching disk and viscous dissipation

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Abstract
The research of single and multi-wall carbon nanotubes (SWCNTs/MWCNTs) mixed in sodium alginate-based nanofluid with MHD stagnation-point flow on a convective heated stretching disk with viscous dissipation and suction effects is being done with the intention of decoding the heat and mass transmission mechanism. It is possible to transform PDEs that govern the boundary layer into ODEs. MATLAB’s Bvp5c is used to numerically solve the revised equations. The Yamada-Ota model and the Buongiorno model are used in this work to scrutinize the flow, heat, and mass transfer parameters. The following parameters were brought up for discussion: volume fraction nanoparticle, magnetic parameter, suction, Brownian motion, thermophoresis, Lewis number, Eckert number, Biot number, stretching, and thermophoresis. This study found that nanofluid (SWCNT/sodium alginate) has a superior flow, heat, and mass transfer rate than nanofluid (MWCNT/sodium alginate). Graphical representations of the effects of various factors are shown, and a comparison of current and prior findings is given in a table. A comparison of current and previous findings reveals a 0% relative inaccuracy. The velocity ratio parameter has solutions that look close to the separation value. The performance of heat and mass transfer operations may be improved by increasing suction parameters. Increases in Brownian motion Nb and suction decrease the temperature profile, whereas increases in velocity ratio and magnetic parameters increase velocity. This research is critical for estimating flow, temperature, and concentration behavior for CNTs with incorporated physical properties.

Keywords
Suction, stagnation point flow, convective heated disk, heat and mass transfer, CNTS (SWCNTS and MWCNTS), magnetohydrodynamics

Date received: 30 July 2022; accepted: 6 September 2022
Handling Editor: Chenhui Liang

Introduction
The concept of stagnation point flow may be found almost everywhere in the disciplines of technology and science. A flow may be obstructed in one of three different ways: by a solid wall; by a free stagnation point; or by a line that is located inside the core of the fluid domain. By using a similarity transformation, Hiemenz¹ in 1911 became the first person to pioneer

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and successfully solve the study of two-dimensional stagnation-point flow. Physically, the highest pressure, heat transfer, and mass deposition rates are found in the stagnation zone of a system. Transverse magnetic fields in boundary layer flow present an important and fundamental challenge for magnetohydrodynamics (MHD). In a broad range of technological fields, the MHD flow and heat transfer of viscous fluids due to a stretching/shrinking sheet may be used effectively, including plasma research, the petroleum industry, and the extraction of geothermal energy by Shateyi and Makinde. In addition, there are practical uses such as the thermal processing of sheet-like resources that transpire in the creation of tissue, plastic layer, sequestering resources, fine-grit flatness, and the boundary layer beside a molten film in the condensation progression. It is necessary to maintain control of the drag and heat flux in order to assure the greatest possible product quality. This is because the expanding and contracting sheet creates motion in the surrounding fluid that is parallel to itself. Problems involving sheets that may stretch and shrink become of major relevance in an industry in which the fluid flow is either redirected toward the origin of the surface or stretched away from the surface. On the other hand, the flow over a sheet that is being stretched out looks quite different from the flow above a sheet that is being contracted. According to Goldstein, this flow is really a reverse flow, and it shows physical features that are entirely distinct from those observed in the stretched flow. These characteristics may be shown to be wholly different from one another.

The requirement for emission and cost-effective heat transfer systems led to the conception of anchoring solid particles with high thermal conductivity in inherently low thermally conductive traditional fluids, such as in the use of heat pumps. This idea has led to the development of a number of innovative heat transfer systems. This is the potential technique to raise the thermally conductive traditional fluids, such as in the use of heat pumps. This idea has led to the development of a number of innovative heat transfer systems. Since it has major applications in diverse of engineering challenges including groundwater level infrastructures, bioengineering, blood research, thermal tenacity storing gadgets, MHD generating electricity, air circulation of nuclear reactors, quartz development, heat transfer, steam engines, energy loss from pipes, industrial applications, and boundary layer m, the heat and mass transport attributes of nanoliquids across curved geometries have become an important topic in

Carbon nanotubes are often regarded as the most effective nanoparticles. Carbon nanotubes, abbreviated as CNT, are tubular structures made of carbon atoms and are sometimes referred to as graphene sheets. Single Wall Carbon Nanotubes (SWCNTs) have a cylindrical structure with a diameter of 0.5–1.5 nm and are formed of a single layer of graphene elements, keeping all the atoms in one location. SWCNTs are also known as carbon nanotubes with a single wall. The multi-walled carbon nanotube, often known as MWCNT, is a cluster of graphene tubes with diameters that increase exponentially. When compared to other nanoparticles and nanotubes, carbon nanotubes (CNT) exhibit mechanical, physicochemical, and thermal characteristics that are six times superior to those of other nanoparticles and nanotubes. The fields of ophthalmology, metallurgy, petrochemical plants, bioengineering, and the conditioning of microelectronics as well as biomedical applications and wastewater treatment, all benefit significantly from the use of CNTs. Xue looked at both experimental and theoretical explanations of CNTs in their research. The “Darcy-Forchheimer flow of water-based SWCNT and MWCNT” was described by Hayat et al. across a revolving disk that was heated convectively. According to what they found, the velocity constraint of the fluid increases as the amount of SWCNT and MWCNT that make up the fluid’s volume percentage does as well. In their study, Imtiaz et al. conducted research on the convective flow of CNTs between two revolving disks. In his research, looked at the short refining and dissolving of carbon nanotube fibers spume from the floating catalyst technique. Sreevani et al. investigated the impact that MHD, suction/injection, and chemical reaction have on the flow of mass and heat transfer of water-based MWCNT and SWCNT along a porous vertical conic section. They demonstrated their understanding that...
recent years. This is due to the fact that it has become a key subject in recent years. Curved bodies may have many different forms, including wavy channels, cylinders, wedges, ellipses, conical disks, toroidal geometries, and spherical geometries. “The influence of thermophoresis and the Brownian motion number on Buongiorno’s model of nanofluid flow close to a non-isothermal wedge has been investigated” by Chamkha and Rashad. In addition to this, they came to the conclusion that a rise in the values of the magnetic parameters caused a spike in the levels of the non-dimensional rates of mass, heat, and velocity transfer. This was shown to be the case. Researchers of Kandasamy et al. investigated the properties of unsteady CNT nanofluid flow across a porous wedge using seawater and water as base fluids.” They found that the magnitude and appearance of SWCNTs may play a crucial starring role in the increase in heat transfer. By taking thermophoresis and the Brownian motion number into consideration, authors of Khan et al. were able to determine the effect that the generation/absorption of heat and the condition of zero mass flux had on the unsteady flow of Falkner-Skan-Carreau nanofluid over a stretched sheet. Khan et al. conducted research on the thermal performance of the Falkner–Skan nanofluid flow that was suggested by Buongiorno across a moving–static wedge. They found that there was a decline in the concentration profiles as the estimates of the chemical reaction constraints intensified. The researchers of Mahdy and Chamkha recognized “MHD unstable Buongiorno’s model nanofluid flow across a tangentially extending porous hyperbolic wedge.” They also discovered an amplification in the rates of skin friction coefficient with growing estimates of the Weissenberg number. Authors of Berrehal and Maougal investigated the properties of MWCNT across a wedge by using water as the base fluid with entropy production. They discovered that the rates of entropy generation decreased when the radiation parameter was increased. Several authors recently provided heat and mass transfer analyses of nanofluid flow among a variety of geometries. Since that time, a significant amount of research that takes MHD into consideration has apparently been carried out, as shown by Al-Kouz et al., Bin Mizan et al., Salah et al., Shamshuddin et al.

However, the references that were discussed before only addressed the flat plate surface, there are still voids that may be filled in order to meet the criterion for further investigation. When it comes to operations that take place in the real world, fluid motion is not restricted to a flat surface; rather, it may be exposed to a variety of surfaces, such as a disk, cylinder, or curve shape. As a result, the primary persistence of this research is to use numerical computing to investigate the flow, temperature, and concentration characteristics of carbon nanotubes (CNTs) that are suspended in nanofluids of varying lengths and diameters and that are caused by a disk surface that is heated convectively. The MHD stagnation point nanofluid flow, heat, and mass transfer of carbon nanotube across a stretched heated disk with viscous dissipation is studied, along with the Yamada-Ota model operating in the presence of the suction effect. Few studies have been done on the molecular interaction of CNTs (SWCNT and MWCNT), although many researchers are discussing it. This interaction is important when reflecting the stagnation point and viscosity dissipation in base fluid. This research also takes into consideration sodium alginate as a base fluid and carbon nanotubes (CNTS), which includes single- and multi-wall carbon nanotubes (SWCNTs and MWCNTs) as a nanoparticle. This is due to the fact that CNT have a high thermal conductivity and have the potential to increase heat and mass transmission. The findings, which were collected in numerical form using bvp5c in MATLAB and are shown in the manner of graphs to depict the comportment of this research and associated with earlier published findings to ensure that there is a high level of agreement between the two sets of findings. We have no doubt that this ground-breaking discovery is important to the study of carbon nanotubes (CNTS) nanofluids and has the potential to capture academics, those working in heat and mass transfer. We are under the impression that no other work that is comparable is being examined, which lends credibility to the originality and importance of this work.

Description of the problem

We investigate a stable stagnation-point flow across a convectively heated stretched disk with sodium alginate as the basis fluid, incorporating single-and multi-wall CNTs, as illustrated in Figure 1, where the cylindrical coordinate (r, z) is employed with the surface at z = 0 while the flow is axisymmetric at z = 0. It is presumed that the flow is of a laminar and incompressible nature. It is anticipated that the base fluid and the CNTs will be in an updraft equilibrium state. The disk is linearly stretched with a velocity of $u_r(r) = ar$, and the free stream velocity (inviscid flow) is given by $u_r(r) = b r$, where $a$ and $b$ are constants, $a$ being greater than 0 for a stretching disk, and $b$ being a positive constant. It is also believed that the velocity of the mass flow is $w_0(r)$, where $w_0(r) > 0$ for injection and $w_0(r) < 0$ for suction, respectively. Nanofluid containing homogeneous nanoparticles is suspended around a CNTS. The thermophysical characteristics of the nanofluids are not considered since the base fluid and the nanoparticles are manufactured to generate a stable combination in the process of making the nanofluids. It is also thought that nanoparticles aren’t involved in the chemical reaction which we excluded here. The magnetic field is
applied in a direction that is normal to the surface of the disk. This causes an effect known as magnetohydrodynamics (MHD), which has a constant intensity of $B_0$. There is not going to be much of induced magnetic field created because of the mobility of the electrically conducting fluid. While the temperature $T_f$ is supposed for the hot CNTs, the bottom of the disk is heated convectively with a heat transfer coefficient of $h_f$. The ambient temperature of CNTs-SA is constant and kept as $T_\infty$. Following the completion of the study referred to earlier, the boundary layer equations for the model may be stated as follows\textsuperscript{34,35}:

Equation of conservation of mass:

$$u = u_w(r) = ar, w = w_w(r), -k_{cnt} \frac{\partial T}{\partial z} = h_f(T_f - T), C = C_f \text{ at } z = 0,$$

$$u \to u_c(r) \to hr, T \to T_\infty, C \to C_\infty \text{ as } z \to \infty.$$

$u$, and $w$ symbolizes the velocity component along the $r$- and $z$-axis correspondingly. The temperature of the CNTS is $T$. $C$ is nanoparticle concentration of the CNTS suspended nanofluid, $C_\infty$ is nanoparticle concentration far from disk, and $C_f$ is nanoparticle concentration at the heated disk. Thermophoretic diffusion coefficient $D_r$, Brownian diffusion coefficient $D_B$, heat capacity ratio $\tau = \left(\rho C_P\right)_c / \left(\rho C_P\right)_f$. The dynamic viscosity is $\mu_{cnt}$ of CNTS, the density is $\rho_{cnt}$ of CNTS, the thermal conductivity of CNTS is $k_{cnt}$ and the heat capacity $\left(\rho C_P\right)_c$ of CNTS. For the solution of the similarity equations (1) to (3), we will take $T_f = T_0 \left(\frac{r}{L}\right)^2$, where $T_0$ is a distinctive temperature of the disk and $L$ is a distinguishing length of the disk. Because of following, a similarity transformation is now suggested to arrive at similarity solutions\textsuperscript{34,35}:

$$u = b r \phi'(\eta), w = -2 \sqrt{b} \phi'(\eta), \theta(\eta)$$

$$= \frac{T - T_\infty}{T_f - T_\infty}, \eta = \sqrt{\frac{b}{2} r^2}, \Phi(\eta) = \frac{C - C_\infty}{C_f - C_\infty}. \tag{6}$$

The following ordinary differential equations are drawn by including (2), (3), and (4) in the steady-state equations.

$$f'' + \frac{\rho_{cnt}/\mu_f}{\mu_{cnt}/\mu_f} \left[2f'' - f'^2 + 1 - M(f' - 1)\right] = 0, \tag{7}$$
Along transformation of boundary conditions (4) obsessed by

\[
\begin{align*}
 f(0) &= S, \quad f'(0) = \lambda, \quad \frac{k_{\text{cm}}}{k_f} \theta'(\eta) \\
 &= Bi[1 - \theta(0)], \quad \Phi'(\eta) = 1, \quad \text{as} \ \eta \to 0, \\
 f'(\eta) &\to 1, \quad \theta(\eta) \to 0, \quad \Phi(\eta) \to 0 \quad \text{as} \ \eta \to \infty. \\
\end{align*}
\]

(10)

It has come to our attention that when \( Bi \to \infty \), the outcome is \( \theta(0) = 1 \), which indicates that the disk surface is isothermal. In the preceding equations \( Pr = \frac{(\mu_f C_p)}{k_f} \) denotes the Prandtl number. \( \lambda = \frac{\theta}{x} \) symbolizes as the stretching/shrinking parameter, mass transpiration parameter \( S \) where \( S > 0 \) denotes the suction parameter. \( Ec = \frac{k_f^2}{(C_f)(T_f - T_s)} \) denotes Eckert number,

\[
Le = \frac{\nu}{k_f} \quad \text{is Lewis number}, \quad Nb = \frac{(\rho C_p D_h (C_w - C_e))}{(\rho_f C_p) \nu} \quad \text{is Brownian motion}, \quad \text{and} \quad M = \frac{\sigma B_1^2}{B_0} \quad \text{is magnetic parameter}, \quad Bi = \frac{h_1}{k_f} \sqrt{\frac{\nu}{k_f}} \quad \text{is Biot number.}
\]

It is possible to express the formulas for the physical quantities of interest in this way.

\[
C_f = \frac{2 \tau_w}{\rho \mu_c}, \quad N_{\text{tu}} = \frac{rq_w}{k_f (T_f - T_s)}, \quad Sh_r = \frac{rq_m}{D_h (C_w - C_e)},
\]

\[
\text{(11)}
\]

where,

surface shear stress, \( \tau_w = \mu_{\text{cm}} \frac{\partial u}{\partial z} \mid_{z=0} \),

surface heat flux, \( q_w = -k_{\text{cm}} \frac{\partial T}{\partial z} \mid_{z=0} \),

mass heat flux, \( q_m = -D_h \frac{\partial C}{\partial z} \mid_{z=0} \).

\[
\text{(12)}
\]

Next, the dimensionless extents of pragmatic significance are the friction factor \( C_f \), local Nusselt number \( N_{\text{tu}} \), and the local Sherwood number \( Sh_r \), the following are referred to as:

\[
\frac{1}{Pr} \left( \frac{k_{\text{cm}}}{k_f} \right) \theta'' + 2f \theta' - 2f' \theta + Nb \Phi' \theta' + Nt \theta'^2 \\
+ \frac{\mu_{\text{cm}}}{\mu_f} \frac{(\rho C_p)_f}{(\rho C_p)_f} Ec \left( f'' \right)^2 = 0,
\]

\[
\Phi'' + Le \theta' + \frac{Nt}{Nb} \theta'' = 0.
\]

(8)

(9)

where \( Re_r = \frac{r m}{\nu} \) denote the local Reynolds number.

### Numerical scheme and validation

Several distinct methods may be used to solve the non-linear differential equations, including the issue with the boundary layer flow. This is something that is achievable. Analytical methods, semi-analytical methods, and numerical methods such as the shooting method, Runge–Kutta–Fehlberg method, Keller Box techniques, Homotopy perturbation method (HPM), Finite difference method, Variational iteration method, and Akbari–Ganji (AGM) method are all examples of the different types of methods that can be used. In the MATLAB software, the bvp5c solver is one of the powerful utensils that can be put to use to compute nonlinear ordinary differential equations. These equations include the reduced equations that come from the boundary layer flow problem. Equations (7) to (9) were numerically solved with the help of the built-in function bvp5c in MATLAB, and the results were subjected to the boundary conditions described in equation (10). In this context, the four-stage Lobatto IIIa formula is implemented using a finite difference code called bvp5c. This is referred to as a collocation formula, and the collocation polynomial is responsible for providing a \( C^1 \) continuous solution that is accurate to the fifth order. The only differential equations that can be used with bvp5c are those of the first order. It does not support the use of second order differential equations. As a result, equations (7) to (9) are altered into a system of differential equations of the first order. As a result, new variables will be defined as follows:

\[
\begin{align*}
 f'(\eta) &= X_1, \quad \theta'(\eta) = X_5, \quad \Phi'(\eta) = X_8, \\
 f''(\eta) &= X_2, \quad \theta''(\eta) = X_6, \quad \Phi''(\eta) = X_9, \\
 f''''(\eta) &= X_3, f''''(\eta) = XX_1, \theta''''(\eta) = XX_2, \Phi''''(\eta) = XX_3.
\end{align*}
\]

(14)

After entering the variables into the MATLAB bvp5c, the equations (7) to (9) are transformed into the following form of first order equations:

\[
XX_1 = -\frac{\rho_{\text{cm}}}{\mu_f} \left[ 2X_1 X_3 - X_2^2 + 1 - M(X_2 - 1) \right],
\]

(15)
\[ XX_2 = - \left( Pr \frac{(\rho C_P)_{\text{cnt}}/(\rho C_P)_f}{k_{\text{cnt}}/k_f} \right) \left( 2X_1X_6 + NbX_2X_6 + NrX_5^2 + \frac{\mu_{\text{cnt}}/\mu_f}{(\rho C_P)_{\text{cnt}}/(\rho C_P)_f} Ec(X_3)^2 \right), \]  

(16)

\[ XX_3 = - \left[ LeX_1X_6 + \frac{Nr}{Nb} XX_2 \right]. \]  

(17)

Similarly, BCs in equation (10) takes on the form:

\[ X_{a1}(0) = S, \ X_{a2}(0) = \lambda, -\frac{k_{\text{cnt}}}{k_f} X_{a6}(0) = B(1 - X_{a5}(0)), \]

\[ X_{a6}(0) = 1, \ X_{b2}(\infty) = 1, \ X_{b3}(\infty) = 0, \ X_{b5}(\infty) = 0. \]  

(18)

Where \( X_a \) and \( X_b \) are the respective codes for the initial boundary condition and the infinite boundary condition. In practice, the outer border of the solution, denoted by \( \eta \rightarrow \infty \), even though this barrier is infinite in theory. For the sake of all the calculations, the variable \( \eta_{\text{max}} \) is assumed to be 7. Because \( \eta_{\text{max}} = 7 \) is sufficiently high, the solution does not dramatically shift even if \( \eta_{\text{max}} \) is increased to a value greater than 7.

The fundamental method for determining whether the model formulation is accurate is to (a) compare the numerical values of recent and previously published studies, and (b) ensure that the far-field boundary condition (10) in the equations is satisfied. As a result, the validity of this model can be shown by linking the values of \( f''(0) \) in Tables 3 and 4 with those found in Khashi‘ie et al.34 for a specific instance of a viscous fluid. To evaluate the precision of the current result, we additionally compute the estimated percent relative inaccuracy by using the formula \( \epsilon_A = \left( \frac{\text{Present solution} - \text{Previous solution}}{\text{Present solution}} \right) \times 100\% \). Tables 3 and 4 show that the difference, shown by the symbol \( \epsilon_A \), between the most recent numeric data and the previous inputs is suitably minimal. This demonstrates that the current bvp5c code and model are accurate to a high degree. These tables illustrate that the precision of the existing code and model is justifiable.

Results and discussion

Analysis of results

The similarity solutions may be obtained by first translating the equations into bvp5c code and then utilizing the MATLAB program’s bvp5c application to solve equations (7) to (9). This allows the application to be accessed from inside the MATLAB software (detailed are in previous section). It is briefly discussed how the velocity, temperature, concentration, skin friction, local heat, and mass transfer curves are affected by respective parameters such as nanoparticle volume fraction \( \phi \), suction \( S \), stretching \( \lambda \), Biot number \( Bi \), Eckert number \( Ec \), magnetic parameter \( M \), Brownian motion \( Nb \), thermophoresis \( Nr \), and Lewis number \( Le \).

To avoid confusion with the prior argument, the Prandtl number is set to \( Pr = 6.2 \) for the current study’s investigation of the sodium alginate-based carbon nanotubes suspended by nanofluid. The volume percentage of single wall and multi wall nanoparticles in the modern effort is selected between 0.0 \( \leq \phi \leq 0.2 \). In addition, a number of values for regulatory parameters have been determined to fall between the following ranges: 0.0 \( \leq M \leq 1.0 \), 0 \( \leq \lambda \leq 2 \), 0.0 \( \leq S \leq 1.0 \), 3.0 \( \leq Le \leq 6.0 \), 0.05 \( \leq Nr \leq 1.0 \), 0.05 \( \leq Nb \leq 1.0 \), 0.0 \( \leq Ec \leq 1.0 \), and 0.1 \( \leq Bi \leq 1.0 \) for the purpose of ensuring the accuracy of the findings.

A physical model of the flow of carbon nanotubes (CNTs) that are suspended in nanofluid is shown in Figure 1. Flow chart of current problem is represented in Figure 2. Schematic diagram for bvp5c is depicted in Figure 3. The effects of significant constraints on the fields of velocity, temperature, concentration, skin friction, Nusselt and Sherwood numbers are established in Figures 4 to 32 respectively. Yamada-Ota thermal conductivity model was used for CNTS (single wall and multi wall) sodium alginate based suspended nanofluid in this study.
Discussion of results

In the coming sections, graphical comparison of SWCNTs/SA and MWCNTs/SA nanofluids are discussed. Also included in the form of a table for discussion is a comparison of the current findings with those that have previously been published.

Velocity curve \( f(\eta) \). The distribution of the velocity profile along with a few other values of \( \phi \) is shown in Figure 4 for the CNTs (SWCNTs and MWCNTs) that was suspended using the nanofluid model. Figure 4 shows that a growth in the volume percentage of the fluid outcomes in a reduction in the fluid’s velocity. It
Figure 8. Impression of $\phi$ over $\theta(\eta)$.

Figure 9. Impression of $S$ over $\theta(\eta)$.

Figure 10. Impression of $Nt$ over $\theta(\eta)$.

Figure 11. Impression of $Nb$ over $\theta(\eta)$.

Figure 12. Impression of $\lambda$ over $\theta(\eta)$.

Figure 13. Impression of $Ec$ over $\theta(\eta)$.
Figure 14. Impression of \(Bi\) over \(\theta(\eta)\).

Figure 15. Impression of \(\lambda\) on \(\Phi(\eta)\).

Figure 16. Impression of \(Le\) on \(\Phi(\eta)\).

Figure 17. Impression of \(Nb\) on \(\Phi(\eta)\).

Figure 18. Impression of \(Nt\) on \(\Phi(\eta)\).

Figure 19. Impression of \(S\) on \(\Phi(\eta)\).
has been discovered that when nanoparticles are present, there is a tendency for the flow of the fluid to become more sluggish. The speeds of nanofluid in the presence of SWCNTs are found to be much quicker than the velocities of nanofluid in the presence of MWCNTs. The dispersion of the velocity profile $f'(\eta)$ heading toward $S$ is shown in Figure 5. When the value of $\eta_w = 5$, the boundary condition is said to be asymptotically satisfied. As can be seen in Figure 5, it has been shown that an increase in $f'(\eta)$ may be seen in the
solution’s velocity profile in response to an increase in $S$. It has been demonstrated that there is a correlation between increases in $S$ and increases in the flow resistance of the solution measured in $f'(\eta)$.

The variation of $f'(\eta)$ relative to the velocity ratio parameter $\lambda$ can be observed in Figure 6, which is presented in this article. When the values of $\lambda$ are increased, the speed at which the liquid is moving
increases proportionally. When the $\lambda = 1$, the flow condition is referred to as axisymmetric flow. However, when the $\lambda = 0$, the flow condition is referred to as two-dimensional flow. Figure 5 illustrates this point well. The velocity of SWCNTS is much slower in comparison to that of MWCNTS. A graphical illustration of the velocity profile with respect to the magnetic $M$ effect is shown in Figure 7. The Lorentz force that is spawned by the magnetic parameter causes the velocity of the fluid to slow down and the velocity profile of the fluid to lower as the magnitude of the magnetic constraint increase. When compared with MWCNT-sodium alginate, it has also been observed that the velocity distribution is dominated by SWCNT in Figures 4, 5 and 7.

**Temperature curve.** The fluctuations in temperature $\theta(\eta)$ that occur with an increase in $\phi$ are seen in Figure 8. As a direct result to the temperature of the fluid increases, the width of the thermal boundary layer also increases. Also rises. The addition of appropriate nanoparticles produces a significant rise in the temperature of the base fluid. This is because carbon nanotubes have a high thermal conductivity and a low temperature gradient in compared to the base fluid. MWCNT-nanoparticles also have lower temperatures than SWCNT-nanoparticles, as shown by the comparison of their temperatures. See Table 1 for further information. It can be observed that the density of MWCNT is lower than the density of SWCNT. The temperature distribution profile of CNTS (both SWCNT and MWCNT) is shown in Figure 9 against a range of different suction $S$ values. According to the findings of this graph, the temperature profile exhibits a decreasing trend as the suction parameter increases.

Figure 10 shows a breakdown and analysis of the impact that the parameters $Nt$ have on the temperature distribution $\theta(\eta)$. The form that the graph takes demonstrates that an upsurge in the value of the thermophoresis constraint $Nb$ initiates a more rapid increase in the temperature of the CNTS (both the SWCNT and the MWCNT) of the Yamada-Ota model. A temperature gradient is employed to produce the thermophoretic force, which is then used to form a rapid stream across the surface of the water. The presence of a thermal boundary layer that is much thicker is indicative of the fact that this influence is a contributing component to the current problem condition.

The fundamental attribute that performs a part in boosting the capacity of heat transfer is the Brownian motion $Nb$, which takes place when CNTS nanoparticles (both SWCNT and MWCNT) are immersed in base fluids. This motion takes place when the nanoparticles undergo Brownian motion. The goal of Figure 11 is to conduct an investigation of the effect that $Nb$ has on temperature profiles. When the value of the parameter $Nb$ is increased, the corresponding change in the distribution $\theta(\eta)$ is a narrowing for the CNTS (SWCNTs and MWCNTs), as is seen by this graphic representation, which illustrates this correlation. In addition to this, when taking into consideration the CNTS that are suspended by nanofluid, temperature profiles start to asymptotically disappear further away from the surface. It is expected that the movement of nanoparticles in an arbitrary zigzag pattern will result in a decrease in the system’s temperature. This is due to the fact that the nanoparticles collide with one another, transferring heat energy from one particle to the next as they do so. As a result, the Brownian movement is the primary feature that is responsible for the extremely effective regulation of the temperature.

The impact that growing the velocity constraint $\lambda$ has on the temperature of the fluid is seen in Figure 12. As $\lambda$ rises, there is a corresponding decrease in temperature. The goal of examining the changes in the parameter $Ec$ and determining how those changes affect the deviation in the thermal distribution $\theta(\eta)$ is what Figure 13 is all about. It has been noted that there is an increase in temperature profile, which is evidence that temperature sketches about the stretching surface are elevated than the temperature of the free stream. The fact that there is an increase in temperature profile was the observation that led to the discovery of this. The Yamada-Ota model of SWCNT is shown to be more accurate than the MWCNT model when $Ec$ is in the range of $0.0 \leq \eta \leq 1.0$ as shown in the figure. Due to the immediate repercussions of this, the temperature inside the CNTS rises as a direct result of the significant heat release. The CNTS particles (both SWCNT and MWCNT) encounter one another because of the dissipation parameter reaching its maximum value. When the particles collide, they generate more heat, which is then distributed throughout the system.
The influence that the Biot number $Bi$ has on the temperature profiles is exemplified in Figure 14. There is an evident rising tendency in the temperature distributions that can be detected inside the fluid flow when the Biot number increases. This pattern cannot be denied. Biot numbers are defined mathematically, and a rise in their values signifies an increase in convective heat transfer coefficient, which in turn allows an increase in the quantity of heat transfer from the surface. Because of this, the fluid will begin to heat up, which will ultimately result in an increase in the fluid temperature distributions of CNTS (SWCNT and MWCNT).

Concentration profile. Figure 15 illustrates how the value of $\alpha$ affects the concentration profile $\Phi(\eta)$ even when all other parameters remain the same. It has been shown that an intensification in the amount of nonlinear stretching outcomes in a decline in the concentration profile. The solution concentration curve is shown in Figure 16 for a wide range of Lewis number ($Le$) values. An increase in intensity of the Lewis number results in a reduction in the concentration profile. Higher values of the Lewis number are correlated with lower values of the molecular diffusivity and reduced thicknesses of the boundary layers. Figures 17 and 18 demonstrate, respectively, the influence that the parameters $Nb$ and $Nt$ have on the concentration profile of CNTS (both SWCNT and MWCNT) for the surface that is being stretched. These charts make it clear that higher values have a significant influence on the outcome of $Nb$ and $Nt$ has the effect of decreasing the concentration profile. The impact that the suction $S$ had on the dimensionless concentration profile is seen in Figure 19. According to this diagram, suction results in a reduction of the dimensionless concentration. The physical meaning of this is that the fluid is moved closer to the surface, which in turn lowers the width of the solute boundary layer. This takes place when the circumstances are ambient.

Skin friction with heat and mass transfer. This subsection provides the outcomes for essential engineering concerns, such as the skin fraction, heat, and mass transfer rates across numerous dimensionless constraints, for CNTS (SWCNT and MWCNT) suspended by nanofluid. Figures 20 to 32 show the variation of the $\phi$, $S$, $A$, $Nt$, $Nb$, $Le$, $M$, $Bi$, and $Ec$ on the skin friction coefficient, heat, and mass transfer rates of CNTS.

Figures 20 and 23 depict, respectively, how skin friction and the local Nusselt number are impacted by the nanoparticle volume fraction $\phi$ toward $\lambda$. It has been discovered that the magnitude of the Nusselt number and the skin friction coefficient both rise with an increase in the value of $\phi$. The presence of nanoparticles in the base fluid triggers an increase in the velocity of the fluid due to collisions that occur between the nanoparticles and the particles of the base fluid. This phenomenon is brought about by the presence of nanoparticles. As a result, the thickness of the momentum boundary layer will decrease, which will ultimately result in an increase in skin friction and heat transfer at the surface.

The effect that the magnetic constraint $M$ has on the skin friction coefficients, which are represented by (SWCNT/SA and MWCNT/SA), is shown in Figure 21. The graph demonstrates an increase in $\lambda$ conjunction with the rise in $M$. The magnetic field is thought to theoretically generate a resistance Lorentz force, which works against the speed at which the fluid is moving and, as a consequence, slows down the process of the boundary layer separating from the surrounding fluid. On the other hand, the fact that the skin friction coefficient is increasing in Figure 21 shows that the free stream flow and suction parameter are the ones that facilitate the movement of the fluid the most.

Not only does the suction parameter $S$, which is one of the associated components in this study, influence the pace at which heat, and mass are moved from one location to another, but it also drives the rate at which these transfers occur. Figures 22, 24 and 29 illustrate, in the appropriate order, the impact that a change in the suction parameter $S$ has on the skin friction, local heat, and mass transfer rates. When the suction component is included, the values in Figure 22 perceive the skin section to be bigger than it really is. This is an essential point to keep in mind. The suction effect at the boundary slows down the CNTS motion, which also increases the velocity difference along the side of the permeable stretching disk. An unanticipated velocity gradient is produced as a result of the advent of suction. This causes the heated fluid motions that are close to the wall to begin, and as a consequence, the buoyant strengths that are induced by the impact of the robust viscosity are slowed down. Suction causes the creation of an unanticipated velocity gradient. Next, the estimations of the local heat and mass transfer rates are revealed in Figures 24 and 29, correspondingly. The information in these graphs shows that as $S$ was exposed along the expanding surface, the rate of heat and mass transfer increased. Most of the time, an growth in the magnitude of $S$ will lead to an increase in both the rate of heat transfer and the rate of mass transfer. When the value of the suction parameter was increased, the thermal boundary layer became thinner. This, in turn, caused the temperature disparity at the surface to become greater. The findings indicate that there is an increase in the amount of frictional drag that is exerted, which, as a possible consequence, may slow down the development of boundary layer separation on the decreasing surface as the skin fraction rises.
In Figures 25 and 30, one can see the changes in the local heat and mass transfer rates for a number of distinct thermophoresis constraints $Nt$ that are directed toward both the rate of heat transfer and the rate at which mass is transferred are slowed down as a result of the fluctuations in $Nt$. The variations of the local Nusselt number and the local Sherwood number, separately, for varied evaluations of the Brownian motion constraint $Nb$, are revealed in Figures 26 and 31, respectively. These figures demonstrate the results of the calculations. An increase in $Nb$ has the opposite effect in Figure 31, where it brings about an expansion in the rate of mass transfer. This can be seen by looking at the graphs, which show that an increase in $Nb$ tends to bring about a drop in the rate of heat transfer in Figure 26. However, an increase in $Nb$ has the opposite effect in Figure 31.

In the meantime, the expansion in the Eckert number $Ec$ intensity advances to an improvement in the thermal state of the fluid, which in change causes an elevation in the thermal boundary layer, as illustrated in Figure 27. This cascade of events results in a rise in the temperature of the fluid as a whole. The Nusselt numbers have been shown to be lower when there is an increase in the amount of dissipation that is taking place, and they have been revealed to be higher when there is no dissipation taking place at all.

Figure 28 illustrates the variations in the rate of heat transfer that occur depending on the value of the Biot number $Bi$ in relation to the convectively heated stretched disk. This graph demonstrates that there is a general upward trend in the solutions’ heat transfer rate, which coincides with the boost in the $Bi$ values. The ratio of the conduction resistance found inside the disk to the convection resistance found on the disk is denoted by the Biot number. As shown in Figure 28, a rise in the Biot number, which is linked to an increase in the efficiency of convective heating, is shown to effectively bring down the fluid temperature in the solutions. This observation was made. However, a high Biot number does not always imply an increase in the disk internal temperature compared to that in its border layer. This is the case due to the fact that the temperature distribution profile spikes as the value of $Bi$ rises, which suggests that the internal thermal resistance of the disk is less.

Lewis number leads in an increase in the mass transfer rate. This is because, from a purely physical standpoint, the Lewis number causes an improve in the mass transfer rate of CNTS (both SWCNT and MWCNT). In addition to this, it demonstrates that there is a rise in the concentration gradient near the surface of the stretched disk. In addition to this, when the Lewis number goes up, the concentration near the surface of the stretched disk goes down. From these figures, it is seen that SWCNTS has higher profiles as compared to MWCNTS.

**Table 1.** Table of thermophysical property of different CNTS with base fluid.36,37

| Properties | SWCNT | MWCNT | Sodium alginate (SA) |
|------------|-------|-------|----------------------|
| $\rho$ (kg/m$^3$) | 2600 | 1600 | 989 |
| $C_p$ (J/kgK) | 425 | 796 | 4175 |
| $k$ (W/mK) | 6600 | 3000 | 0.6376 |
| $Pr$ | | | 6.2 |

**Table 2.** The effective thermo-physical property of CNTs suspended nanofluid.36

| Density $\rho_{tot} = (1 - \phi)\rho_f + \phi\rho_s,$ |
| Viscosity $\mu_{tot} = \frac{\mu_f}{(1 - \phi)^2},$ |
| Heat capacity $(\rho C_p)_{tot} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s,$ |
| Thermal conductivity (Yamada-Ota model) $\frac{k_{tot}}{r} = \frac{1 - \phi (L + 2 + \phi (\frac{L}{r})^{1/2} + 2\phi (\frac{L}{r})^{1/2} \ln(\frac{L}{r})^{1/2}) - \phi}{1 - \phi + 2\phi (\frac{L}{r})^{1/2} \ln(\frac{L}{r})^{1/2}}$ |

**Table 3.** $f''(0)$ for $\lambda$ and $M$ when $S = 0$ for the case of regular fluid is tabulated and compared.

| $\lambda$ | $M$ | Present | Khashi’ie et al.34 | $\epsilon_{\Delta}(\%)$ |
|----------|------|---------|-------------------|-----------------|
| 0.0      | 1.0  | 1.64532\text{167} | 1.64532\text{1652} | 0.0 |
| 0.2      | 1.38320\text{821} | 1.38320\text{819} | 0.0 |
| 0.5      | 0.9235\text{3421} | 0.9235\text{3419} | 0.0 |
|        | 0.0  | 0.78032\text{335} | 0.78032\text{3348} | 0.0 |
|        | 5.0  | 1.35768\text{817} | 1.35768\text{8160} | 0.0 |
|        | 10.0 | 1.75767\text{520} | 1.75767\text{5199} | 0.0 |

**Table discussion.** The Table 1 provides several thermo-physical properties of carbon nanotubes (CNTS), that is, single-wall and multi-wall carbon nanotubes (SWCNT and MWCNT) and sodium alginate (SA) as the base fluid. A mathematical depiction of the thermal possessions of nanofluid may be found in Tables 2 and 3. Tables 3 and 4 present the comparative analysis between present study with Khashi’ie et al.34 In present study we used bvp5c in MATLAB whereas Khashi’ie
et al. 34 used bvp4c in MATLAB and found excellent agreement between present method and previously used. In Table 1 φ is volume fraction of CNTs nanoparticles. ρf indicates the sodium alginate density, and k_f denotes the thermal conductivity of sodium alginate. In this work, the thermal property of CNTs was analyzed with the use of Yamada-Ota thermal conductivity model.

### Conclusion

This work explored the stable MHD stagnation points flow of CNTS (SWCNTS and MWCNT) toward convective heated stretching disk, considering the viscous dissipation with suction on heat and mass transmission, and found that it was effective. Utilizing the bvp5c in the programming environment provided by MATLAB allowed for the generation of the outputs. The correctness of the current model has been validated by the presence of a good agreement, in some instances, between the findings that are now being reported and those that have been previously published, with an error of roughly 0% relative. The material that was accessible was combed through to get the thermophysical properties that were then utilized in this experiment. Several factors that include the nanoparticle volume fraction φ, magnetic field M, suction S, thermophoresis Nt, Brownian motionNb, Lewis number Le as well as parameters like stretching λ, Biot number Bi, and Eckert number Ec. From the information shown up above, we may draw the following conclusions:

- Velocity profile of the CNTS (SWCNT and MWCNT) against various working constraints such magnetic, stretching, and suction increases whereas against volume fraction decreases.
- Enhancement of Nt, Ec, Bi and solid volume fraction escalates the fluid temperature and opposite behavior is shown by suction S, Nb, and λ.
- Concentration Profile decreases against Nb, Nt, S, Le, and λ.
- The skin friction coefficient profile against φ, S, and M toward λ boost up for CNTS.
- The rate of heat transmission dropped as Nt, Nb, and Ec increased, but when φ, S, and Bi boosted toward λ, the rate of heat transfer increased simultaneously.
- The rate of mass transfer boosted against S, Nb, and Le toward λ whereas mass transfer rate dropped down against Nt toward λ.
- Interestingly, single wall carbon nanotube (SWCNT) has superior velocity, thermal distribution, skin friction, heat, and mass transfer rate than the multi wall carbon nanotube (MWCNT) for Yamada-Ota model.
- The inquiry that is now being carried out has a significant impact on the ways in which science and technology are used in the modern world. Carbon nanotubes (CNTS), carbon-based nanotechnology has promptly developed as a platform technology for a wide range of applications, including biomedicals, wastewater treatments, and electronics.

### Acknowledgement

We would like to thank the reviewers for their thoughtful comments and efforts toward improving our paper.

### Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

### Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: The authors extend their appreciation to the Deanship for Research and Innovation, Ministry of Education in Saudi Arabia for funding this research work through the project number “IF_2020_NBU_453.”

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