**CMBE constraints on inflation models with a massive non-minimal scalar field**

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We derive power spectra of the scalar- and tensor-type structures generated in an inflation model based on a massive non-minimally coupled scalar field with the strong coupling assumption. We make analyses in both the original-frame and the conformally transformed Einstein-frame. We derive contributions of both structures to the anisotropy of the cosmic microwave background radiation, and compare the contributions with the four-year COBE-DMR data. Previous study showed that sufficient amount of inflation requires a small coupling parameter. In such a case the spectra become near Zeldovich spectra, and the gravitational wave contribution becomes negligible compared with the scalar-type contribution which is testable in future CMBR experiments.

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1. Introduction: Inflation scenario, although still in hypothetical stage, now seems to have its firm presence in the theoretical study of the early universe preceding the radiation dominated big bang era. The main necessity of the inflation stems from the natural mechanism it provides concerning the origin of the large scale structures in the paradigm of spatially homogeneous and isotropic world model. If one accepts the presence of inflation and its ability to generate seeds for the large scale structures, one can use the data of the observed structures to constrain the model parameters for the inflation models. These days, the trouble is not the lack of plausible inflation models but the presence of too many of them mostly based on toy models which are specially designed for the successful inflation and some based on hopeful theories for the physics in high-energy regime, i.e., the lack of standard model \([1]\). However, often the observational constraints are strong enough to exclude models based on some promising high-energy physics; like the classical evolution processes of the scalar- and tensor-type structures applicable in a wide class of generalized gravity theories \([2,4]\), and made applications to string gravity \([4]\) and nonminimally coupled scalar field with a self coupling \([5]\). In this paper we will make another application to a chaotic type inflation model based on a nonminimally coupled massive scalar field proposed by Futamase and Maeda \([6]\) and will derive constraints on the model using the recent COBE-DMR data. The main results are in Eqs. (8,15) and (10,16), and discussions below Eq. (16).

2. Gravity theories: We consider the following form of generalized gravity theory

\[
\mathcal{L} = \sqrt{-g} \left[ \frac{1}{2} f(\phi, R) - \frac{1}{2} \omega(\phi) \phi^2 \phi^2 - V(\phi) \right].
\]

(1)

Unified analyses of the quantum generation and the classical evolution processes of the scalar- and tensor-type structures based on this gravity theory were presented in \([13,14]\). A non-minimally coupled massive scalar field is a case with \(f = (\kappa^2 - \xi \phi^2)R, \ \omega = 1\), and \(V = \frac{1}{2} m^2 \phi^2\) where \(\kappa^2 \equiv 8\pi m_{pl}^{-2}\); we call it the original-frame. Through a conformal transformation \([17,18]\) this gravity theory can be transformed to Einstein gravity \((f = \kappa^2 R, \ \omega = 1)\) with a special potential, which is also a special case of Eq. (1); we call it the Einstein-frame. Since the general results concerning cosmic structures based on Eq. (1) were presented in \([13,14]\), in the following we will use those results freely.
3. The original-frame: Equations for the background are in Eq. (5) of [13]. Assuming the strong coupling condition $\kappa^2 |\xi|^2 > 1$ we have the following solutions for the background (we consider $\xi < 0$ case) [13,13]:

$$a \propto e^{Ht}, \quad H_i \equiv \sqrt{\frac{1 - 4\xi}{2\kappa(1 - 6\xi)(3 - 16\xi)}},$$

$$\phi \propto e^{\alpha mt}, \quad \alpha \equiv \sqrt{\frac{2\kappa}{(1 - 6\xi)(3 - 16\xi)}}. \quad (2)$$

Thus, we have a near exponentially expanding period which can provide a plausible inflationary era.

The on-shell Lagrangians for the scalar- and tensor-type structures are presented in Eq. (7) of [13] and Eq. (3.24) of [14], respectively. The background evolution in Eq. (3) leads to

$$\nu = \nu_g = \frac{3 - 16\xi}{2(1 - 4\xi)}, \quad (3)$$

which are defined in Eqs. (14,21) of [13] and Eqs. (8,11) of [14]. Thus, the general mode function solutions in Eq. (14) of [13] and Eq. (29) of [14] include our case as a special subset. In the large-scale limit, the general power spectra based on vacuum expectation values in Eq. (16) of [13] and Eq. (32) of [14] lead to the following:

$$P_{\phi^2}^{1/2} \propto \frac{H^2}{\phi^2} \frac{\Gamma(\nu)}{\Gamma(\nu + 1/2)} \sqrt{1 - 16\xi} \left(\frac{|k|}{a}\right)^{3/2 - 2\nu} \times |c_2(k) - c_1(k)|, \quad (4)$$

$$P_{C_{\alpha\beta}}^{1/2} = \frac{\kappa H}{\sqrt{2\pi}} \frac{1}{\Gamma(3/2)} \left(\frac{1}{2}\right)^{3/2 - \nu_g} \times \left(\frac{2}{2}\right) \sum_{|c_2(k) - c_1(k)|^2}, \quad (5)$$

where $\ell$ indicates the two polarization states of the gravitational wave. In these forms, $\xi = 0$ reproduces correctly the minimally coupled limit; see Eqs. (56,57,6) of [13] and Eqs. (34,37) of [14]. $c_i(k)$ and $c_{\ell i}(k)$ are constrained by the quantization conditions $|c_2|^2 - |c_1|^2 = 1$, and $|c_2|^2 - |c_{\ell i}|^2 = 1$. Notice the general dependences of the power spectra on the scale $k$ through the vacuum choices which fix $c_1$ and $c_{\ell i}$. Only if we choose the simplest vacuum states $c_2 = 1$ and $c_{\ell 2} = 1$ the power spectra show power-law dependence on $k$.

As an ansatz we identify the power spectra based on the vacuum expectation value during the inflation era $[P_{\phi^2}^\ast$ and $P_{C_{\alpha\beta}}^\ast$] with the classical power spectra based on the spatial average $[P_{\phi^2}^\ast$ and $P_{C_{\alpha\beta}}^\ast$]; by this identification we have ignored the possible contributions from classicalization processes. Then, we have the same results in Eqs. (13) now valid for $P_{\phi^2}^\ast$ and $P_{C_{\alpha\beta}}^\ast$. In [13,14] we have shown that, ignoring the transient solutions, $\phi^\ast_{\phi^2}$ and $C_{\alpha\beta}^\ast$ are generally conserved independently of changing gravity [from non-minimally coupled to minimally coupled], changing potential, and changing equation of state, as long as the scale remains in the large-scale; this is the case for the observationally relevant scales before the second (inward) horizon crossing in the matter dominated era. Consequently, even in the matter dominated era Eqs. (13) remain valid as the power spectra in the large-scale limits [the super-sound-horizon for the scalar-type structure and the super-horizon for the gravitational wave]. The spectral indices of the scalar- and tensor-type structures are given as $n - 1 = 3 - 2\nu$ and $n_g = 3 - 2\nu_g$; see below Eq. (60) of [13] and below Eq. (52) of [14]. Thus, ignoring the vacuum dependences in the inflation era (i.e., taking the simplest vacuum state), we have

$$n - 1 = n_g = \frac{4\xi}{1 - 4\xi}, \quad (6)$$

which is less than zero, thus showing redder spectra compared with the scale independent ones ($n - 1 = 0 = n_g$); results up to this point are valid for general negative value of $\xi$. In order to be consistent with the COBE data with $n - 1 \simeq 0 \simeq n_g$ we need $|\xi| \ll 1$. In this limit we have:

$$P_{\phi^2}^{1/2} = \sqrt{\frac{3}{-2\xi}} H^2, \quad P_{C_{\alpha\beta}}^{1/2} = \sqrt{\frac{2}{-\xi}} \frac{H_i}{2\pi m}, \quad \sqrt{\frac{3}{-2\xi}} H^2. \quad (7)$$

In order to confront the theory with the observational data we derive the contribution to angular power-spectrum of the cosmic microwave background radiation. The multipole $a^2 \equiv \langle a_m^2 \rangle$ is related to $P_{\phi^2}^\ast$ and $P_{C_{\alpha\beta}}^\ast$ through formulae in Eq. (61) of [13] and Eq. (56) of [14]. Thus, the observed values (or limits) of $a^2$ constrain directly $P_{\phi^2}^\ast$ and $P_{C_{\alpha\beta}}^\ast$, thus constrain free parameters in the model.

For the scale independent spectra in Eq. (13) $\langle a^2 \rangle$ can be integrated [13]. The quadrupole anisotropy is the following

$$\langle a_2 \rangle = \langle a_2^2 \rangle_{S} + \langle a_2^2 \rangle_{T} = \frac{\pi}{15} P_{\phi^2} + \frac{7.74}{480\pi^2} \frac{1}{5} \frac{3}{52} P_{C_{\alpha\beta}} = \frac{1}{480\pi^2} \left(\frac{m}{\phi}\right)^2 \left(\frac{1}{15\phi^2} + \frac{3 \times 7.74}{4\pi}\right). \quad (8)$$

Thus, if $|\xi| < 1/27.7$ the gravitational wave contribution is smaller compared with the scalar-type one. The four-year COBE-DMR data shows [20]:

$$Q_{\text{rms-Ps}} = 18 \pm 1.6 \mu K, \quad T_0 = 2.725 \pm 0.020 K,$$

$$\langle a_2 \rangle = \frac{4\pi}{5} \left(\frac{Q_{\text{rms-Ps}}}{T_0}\right)^2 \simeq 1.1 \times 10^{-10}. \quad (9)$$

This leads to a constraint

$$m \phi \simeq 1.6 \times 10^{-3} |\xi| \sqrt{\frac{|\xi|}{1 + 27.7 |\xi|}}, \quad (10)$$

4. The Einstein-frame: Exactly similar analyses can be made in this gravity theory. Assuming the strong
coupling limit \((\kappa^2 |\xi|^2 \gg 1)\) in the original-frame, the potential in the Einstein-frame shows an asymptotic form \(V = \frac{m^2}{2\kappa^2 |\phi|^2} e^{-\frac{2\kappa \phi}{m}}\). Due to the exponential potential, we have a power-law type accelerated expansion \([4]\):

\[
a \propto t^p, \quad p = 3 - \frac{1}{2\kappa},
\]

\[
\kappa \phi = \sqrt{2p} \ln \left( \frac{2}{(1 - 6\xi)(3 - 16\xi)} \right). \tag{11}
\]

This leads to the same result in Eq. \((3)\) for \(\nu\) and \(\nu_g\). The large-scale power spectra become:

\[
P^{1/2}_{\phi} = \frac{H^2}{2\pi |\phi| \Gamma(3/2)} \Gamma(\nu) \left( \frac{k |\phi|}{2} \right)^{3/2 - \nu} \times |c_2(k) - c_1(k)|, \tag{12}
\]

\[
P^{1/2}_{C_{\alpha \beta}} = \frac{\kappa H}{2\pi \sqrt{1 - 6\xi}} \left( \frac{k |\eta|}{2} \right)^{3/2 - \nu_g} \times \sqrt{\sum_{\ell} c_2(k) - c_1(k)}^2, \tag{13}
\]

with similar constraints on \(c_1(k)\) and \(c_\ell(k)\) as below Eq. \((1)\). These results, of course, are consistent with Eq. \((57)\) in \([4]\) and Eq. \((37)\) in \([4]\).

After identifying the power spectra based on the vacuum expectation values with the classical power spectra based on the spatial average, and using the conservation properties in the large-scale limit, we have the same results in Eqs. \((11,12)\), now valid for \(P_{\phi \phi}\) and \(P_{C_{\alpha \beta}}\). Ignoring the vacuum dependences in the inflation era we have the same result in Eq. \((1)\) now valid for the spectral indices \(n\) and \(n_g\). Thus, to be consistent with the COBE data, again we need \(|\xi| < 1. In this limit we have:

\[
P^{1/2}_{\phi} = \frac{1}{\sqrt{-2\pi} \kappa m_{pl}}, \quad P^{1/2}_{C_{\alpha \beta}} = \frac{2}{\sqrt{\pi}} m_{pl}. \tag{14}
\]

For the scale independent spectra in Eq. \((14)\) the quadrupole anisotropy becomes

\[
\langle a_2^2 \rangle = \frac{1}{10} \left( \frac{H}{m_{pl}} \right)^2 \left( \frac{1}{-15\xi} + \frac{3 \times 7.74}{4\pi} \right). \tag{15}
\]

Thus, for \(|\xi| < 1/27.7\) the gravitational wave contribution is smaller compared with the scalar-type one. Using the four-year COBE-DMR data in Eq. \((4)\) we have a constraint

\[
\frac{H}{m_{pl}} \simeq 1.3 \times 10^{-4} \sqrt{\frac{|\xi|}{1 + 27.7|\xi|}}. \tag{16}
\]

Based on analyses in the Einstein frame, in order to have successful amount of inflation, authors of \([4]\) showed \(|\xi| < 10^{-3}\). In such a case the spectra of both structures in Eq. \((4)\) show near Zeldovich spectra which is consistent with the COBE-DMR data. According to our above analyses such a small coupling parameter leads to two important consequences. Firstly, based on Eq. \((5)\) it predicts that the contribution of the gravitational wave should be negligible compared with the scalar-type perturbation. This is a testable result in future CMBR experiments like MAP and Planck Surveyor missions with high accuracy temperature and polarization anisotropies. Secondly, based on Eq. \((14)\) we have a limit on the expansion rate during inflation like \(H < 4 \times 10^{-5} m_{pl}\).

5. Discussions: In Sec 2. we mentioned about the relation between the two frames through the conformal transformation \([22]\). Using the conformal transformation properties in Eqs. \((11-14)\) of \([13]\) we can show that through the conformal transformation the original-frame results in Eqs. \((11,12,13)\) produce correctly the Einstein-frame results in Eqs. \((14,15)\). In Eq. \((25)\) of \([18]\) we showed that \(\varphi_{\phi \phi}\) and \(C_{\alpha \beta}\) are invariant under the conformal transformation. As far as we can tell, however, the two theories related by the conformal transformation are not necessarily equivalent physically: for interesting discussions, see \([23]\). Still, as long as the structure generation and evolution processes based on the linear theory are concerned, as noted above, the conformal transformation provides a simple and useful mathematical trick for practical calculations.

In this paper we have derived the the scalar- and tensor-type structures generated from quantum fluctuations in a chaotic inflation model based on a massive non-minimally coupled scalar field. We derived contributions of both structures to the anisotropy of the cosmic microwave background radiation and compared them with the four-year COBE-DMR data on quadrupole anisotropy. The power spectra in Eqs. \((11,12,13)\) and their corresponding classical ones after classicalization and subsequent conserved evolution, and the spectral indices in Eq. \((4)\), are valid for general negative value of \(\xi\). In order to investigate the constraints from the COBE data in such a general \(\xi\) situation we can numerically integrate the general multipole formulae in Eq. \((61)\) of \([13]\) and Eq. \((56)\) of \([4]\). For a small \(|\xi|\) the spectra are nearly scale-independent and the results in the original frame and the Einstein frame are presented in Eqs. \((4,4)\) respectively. Also, for very small \(|\xi|\) the gravitational wave contribution becomes negligible compared with the scalar-type contribution. In fact, for a successful amount of inflation in the Einstein frame the authors of \([4]\) showed that \(|\xi| < 10^{-3}\) and in such a case the gravitational wave should give negligible contribution to the CMBR temperature anisotropy which is a testable result in future CMBR experiments. That is, an excessive amount of gravitational wave signature detected in the future CMBR experiments can rule out the inflation scenario proposed by Futamase and Maeda in \([4]\). A way of detecting gravitational wave signature using a particular
combination of polarization parameters was discussed in \cite{24}. Expected accuracy of temperature and polarization anisotropies in future CMBR experiments using MAP and Planck satellite missions are investigated in \cite{25}.

Recently, the Sakai and Yokoyama \cite{19} proposed a scenario based on a topological inflation \cite{26} realized in the domain wall formed at the maximum of the effective potential in the Einstein frame. Analyzing the scalar and tensor-type contributions to CMBR in this scenario would be an interesting subject which will be addressed in a future occasion.

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\begin{thebibliography}{99}
\bibitem{1} M. S. Turner, in \textit{The cosmic microwave background}, edited by C. H. Lineweaver, \textit{et al.}, (Kluwer Academic Publishers, London, 1997) 309p.
\bibitem{2} J. M. Bardeen, P. S. Steinhardt and M. S. Turner Phys. Rev. D \textbf{28}, 679 (1983).
\bibitem{3} D. H. Lyth and A. Riotto, preprint \texttt{hep-ph/9807278}.
\bibitem{4} R. Brustein, M. Gasperini, M. Giovannini, V. Mukhanov and G. Veneziano, Phys. Rev. D \textbf{51}, 6744 (1995); J. Hwang, Astroparticle Phys. \textbf{8}, 201 (1998).
\bibitem{5} J. E. Lidsey, \textit{et al.}, Rev. Mod. Phys. \textbf{69}, 373 (1997).
\bibitem{6} B. S. DeWitt, \textit{Dynamical theory of groups and fields} (New York, Gordon and Breach, 1965); A. D. Sakharov, Sov. Phys. Dokl. \textbf{12}, 1040 (1968); N. D. Birrell and P. C. W. Davies, \textit{Quantum fields in curved space} (Cambridge, Cambridge University Press, 1982).
\bibitem{7} M. Green, J. Schwarz and E. Witten, \textit{Superstring Theory}, Vol. 1 and 2 (Cambridge Univ. Press: Cambridge 1987). E. Alvarez, Rev. Mod. Phys. \textbf{61}, 561 (1989); Y. M. Cho, Phys. Rev. Lett. \textbf{68}, 3133 (1992).
\bibitem{8} A. A. Starobinsky, Phys. Lett. B \textbf{91}, 99 (1980).
\bibitem{9} B. L. Spokoiny, Phys. Lett. B \textbf{147}, 39 (1984); D. La and P. J. Steinhardt, Phys. Rev. Lett. \textbf{62}, 376 (1989); M. D. Pollock and D. Sahdev, Phys. Lett. B \textbf{222}, 12 (1989); R. Fukir and W. G. Unruh, Phys. Rev. D \textbf{41}, 1783 (1990); G. Veneziano, Phys. Lett. B \textbf{265}, 287 (1991).
\bibitem{10} A. H. Guth, Phys. Rev. D \textbf{23}, 347 (1981).
\bibitem{11} V. F. Mukhanov, L. A. Kofman and D. Yu. Pogosyan, Phys. Lett. B \textbf{193}, 427 (1987); D. S. Salopek, J. R. Bond and J. M. Bardeen, Phys. Rev. D \textbf{40}, 1753 (1989); N. Makino and M. Sasaki, Prog. Theor. Phys. \textbf{86}, 103 (1991); V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rep. \textbf{215}, 203 (1992), and references thererin.
\bibitem{12} J. Hwang, Phys. Rev. D \textbf{53}, 762 (1996); Class. Quant. Grav. \textbf{14}, 3327 (1997).
\bibitem{13} J. Hwang and H. Noh, Class. Quant. Grav. \textbf{15}, 1387 (1998).
\bibitem{14} J. Hwang, Class. Quant. Grav. \textbf{15}, 1401 (1998).
\bibitem{15} J. Hwang and H. Noh, Phys. Rev. Lett. \textbf{81}, 5274 (1998).
\bibitem{16} T. Futamase and K. Maeda, Phys. Rev. D \textbf{39}, 399 (1989).
\bibitem{17} J. Hwang, Class. Quant. Grav. \textbf{7}, 1613 (1990).
\bibitem{18} J. Hwang, Class. Quant. Grav. \textbf{14}, 1981 (1997).
\bibitem{19} N. Sakai and J. Yokoyama, preprint \texttt{hep-ph/9901336}.
\bibitem{20} C. L. Bennett, Astrophys. J. \textbf{464}, L1 (1996); K. M. Górski, \textit{et al.}, \textit{ibid.} \textbf{464}, L11 (1996); K. M. Górski, \textit{et al.}, Astrophys. J. Suppl. \textbf{114}, 1 (1998).
\bibitem{21} F. Lucchin and S. Matarrese, Phys. Rev. D, \textbf{32}, 1316 (1985).
\bibitem{22} R. H. Dicke, Phys. Rev. \textbf{125}, 2163 (1962); B. Whitt, Phys. Lett. B \textbf{145}, 176 (1984).
\bibitem{23} G. Magnano and L. M. Sokolowski, Phys. Rev. D \textbf{50}, 5139 (1994); V. Faraoni, E. Gunzig, and P. Nardone, preprint \texttt{gr-qc/9811047}.
\bibitem{24} U. Seljak and M. Zaldarriaga, Phys. Rev. Lett. \textbf{78}, 2054 (1997).
\bibitem{25} G. Jungman, K. Kamionkowski, A. Kosowsky, and D. N. Spergel, Phys. Rev. D \textbf{54}, 1332 (1996); J. R. Bond, G. Efstathiou, and M. Tegmark, Mon. Not. R. Astron. Soc. \textbf{291}, L33 (1997); M. Zaldarriaga and U. Seljak, Phys. Rev. D \textbf{55}, 1830 (1997); M. Kamionkowski, A. Kosowsky, and A. Stebbins, \textit{ibid.} \textbf{55}, 7368 (1997).
\bibitem{26} A. Linde, Phys. Lett. B \textbf{327}, 208 (1994); A. Vilenkin, Phys. Rev. Lett. \textbf{72}, 3137 (1994).
\end{thebibliography}