Towards a model-independent constraint of the high-density dependence of the symmetry energy

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Abstract

Neutron-proton elliptic flow difference and ratio have been shown to be promising observables in the attempt to constrain the density dependence of the symmetry energy above the saturation point from heavy-ion collision data. Their dependence on model parameters like microscopic nucleon-nucleon cross-sections, compressibility of nuclear matter, optical potential, and symmetry energy parametrization is thoroughly studied. By using a parametrization of the symmetry energy derived from the momentum dependent Gogny force in conjunction with the Tübingen QMD model and comparing with the experimental FOPI/LAND data for $^{197}$Au+$^{197}$Au collisions at 400 MeV/nucleon, a moderately stiff, $x=-1.35\pm1.25$, symmetry energy is extracted, a result that agrees with that of a similar study that employed the UrQMD transport model and a momentum independent power-law parametrization of the symmetry energy. This contrasts with diverging results extracted from the FOPI $\pi^-/\pi^+$ ratio available in the literature.

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I. INTRODUCTION

The isovector part of the equation of state (asy-EoS) of asymmetric nuclear matter, known as symmetry energy (SE), represents one of the remaining open questions of nuclear physics. It comprises the lesser known part of the nuclear matter equation of state (EoS) that can be approximately described by the expansion

\[ E(\rho, \beta) = E_0(\rho, \beta = 0) + E_{\text{sym}}(\rho) \beta^2, \]

where \( \beta = (\rho_n - \rho_p) / (\rho_n + \rho_p) \) with \( \rho_n, \rho_p \) and \( \rho \) denoting the neutron, proton and total nucleon densities, respectively. Knowledge of its precise density dependence is mandatory for the proper understanding of the structure of rare isotopes, dynamics and spectra of heavy-ion collisions and most importantly for certain astrophysical processes such as neutron star cooling and supernovae explosions [1, 2]. Intermediate energy nuclear reactions involving stable and radioactive beams have allowed, by studying the thickness of neutron skins, deformation, binding energies and isospin diffusion, to constrain the density dependence of SE at densities below saturation (\( \rho_0 \)) [3–6]. Existing theoretical models describing its density dependence generally agree with each other in this density regime, but their predictions start to diverge well before regions with densities \( \rho \geq 2\rho_0 \) are reached [2].

Nuclear matter at suprasaturation densities is created in the laboratory in the processes of collisions of heavy nuclei. Several observables that can be measured in such reactions have been determined to bear information on the behavior of the SE above \( \rho_0 \): the ratio of neutron/proton yields of squeezed out nucleons [7], light cluster emission [8], \( \pi^-/\pi^+ \) multiplicity ratio in central collisions [9–11], double neutron to proton ratios of nucleon emission from isospin-asymmetric but mass-symmetric reactions [12] and others.

The FOPI experimental data for the \( \pi^-/\pi^+ \) ratio [13] have been used to set constraints on the suprasaturation density behavior of SE by various authors with contradicting results: Xiao et al. [9] made use of the IBUU transport model supplemented by the isovector momentum dependent Gogny inspired parametrization of SE [14] to point toward a soft asy-EoS, the study of Feng and Jin [10], which employed IQMD and a power-law parametrization of the potential part of the symmetry energy,

\[ S(\rho) = S_0 (\rho/\rho_0)^\gamma, \]

favors a stiff SE. Most recently Xie et al. [11] addressed the same issue within the Boltzmann-
Langevin approach and a power-law parametrization of asy-EoS presenting support for a super-soft scenario for the symmetry energy.

Constraints on the high density dependence of $S(\rho)$ extracted from elliptic flow ratios of neutrons and protons (npEFR) $v_2^{n/p} = v_2^n/v_2^p$ and of neutrons and hydrogen $v_2^{n/H}$ have been presented by Russotto et al. [15]. The experimental data taken by the FOPI-LAND Collaboration for $^{197}$Au+$^{197}$Au collisions at 400 MeV/nucleon incident energy [16, 17] have been reanalyzed allowing for a reduction of systematical and statistical uncertainties. To model heavy-ion collisions a version of the UrQMD model [18, 19] and the momentum independent power-law parametrization of SE mentioned above have been employed. A comparison of the theoretical and experimental elliptic flow ratios of neutrons vs. protons ($v_2^n/v_2^p$) and neutrons vs. hydrogen ($v_2^n/v_2^H$) has led to a constraint compatible with a linear density dependence for the potential part $S(\rho)$: $\gamma=0.9\pm0.4$ [15].

In an independent study [20] the neutron-proton elliptic flow difference (npEFD) $v_2^{n-p} = v_2^n - v_2^p$ has been proposed as a viable observable for constraining the supersaturation density dependence of SE. Its dependence on model parameters like in-medium microscopic nucleon-nucleon cross-sections, compressibility of symmetric nuclear matter and width of the gaussian wave packet of nucleons is a rather small fraction of the sensitivity to the changes between a stiff and a soft asy-EoS for kinematical acceptances close to those of the FOPI experiment. A comparison with published FOPI-LAND impact parameter dependent data [16, 17] for $v_2^{n-p}$ was found problematic due to a highly non-monotonous dependence of the experimental data on that variable. Still, the experimental $v_2^{n-H}$, viewed as an upper bound of $v_2^{n-p}$, allowed the exclusion of the super-soft asy-EoS from the list of possible scenarios.

The present Article aims at an update of the results of References [15, 20] by extending the analysis of the former to both neutron-proton elliptic flow differences $v_2^{n-p}$ and ratios $v_2^{n/p}$ and by addressing the model dependence due to the momentum dependent part of the EoS and of the momentum dependence of the SE. A recent overview addressing the relevance of elliptic flow in the study of the SE at supra-saturation density can be found in Ref. [21].
II. THE FRAMEWORK

A. The model

In the present study, heavy-ion collisions have been simulated by using the QMD transport model developed in Tübingen [22, 23] and expanded to accommodate density-dependent nucleon-nucleon cross-sections and an isospin dependent EoS. The same model has been previously used to study dilepton emission in heavy-ion collisions [24–26], stiffness of the equation of state of symmetric nuclear matter [27] and various in-medium effects relevant for the dynamics of heavy-ion collisions [23, 28]. Most of the results of the following Section have been obtained by making use of the Gogny inspired momentum dependent parametrization of the isovector part of the equation of state [14]. It contains a parameter denoted $x$ which has been introduced to allow adjustments in the stiffness of asy-EoS, negative and positive values corresponding to a stiff and a soft density dependence of the symmetry energy, respectively. To assess the importance of the momentum dependent part of asy-EoS, the momentum-independent power-law parametrization (Eq. (2)) is used where indicated. Further details of the model, relevant for the current study, can be found in [20].

B. Experimental data

In the original release of the FOPI-LAND data [16, 17] the extraction of proton spectra required, due to insufficient calorimeter resolution, conservative cuts to be applied in order to eliminate deuterium and tritium events. As a result proton elliptic flow values show a non-monotonous dependence on the impact parameter, in contrast to neutrons, making a comparison with predicted values troublesome. The data have been recently reanalyzed [15] in order to determine the optimum conditions for the new ASY-EOS experiment [29] and to extract constraints on the density dependence of the symmetry energy. This effort has also resulted in a drastic improvement of the impact parameter dependence of the elliptic flow of protons. Several data sets, corresponding to different cuts in rapidity $0.25 < y/y_P < 0.75$ (B) and $0.45 < y/y_P < 0.55$ (C) and transverse momentum $0.3 < p_T < 1.0$ and $0.3 < p_T < 1.3$) are available. The experimental values for the elliptic flow of neutrons and protons, as well as hydrogen, used to obtain the results in this work are displayed in Tab. (I). For the original FOPI-LAND data only the rapidity cut $0.40 < y/y_P < 0.60$
| Data set | Centrality | b (fm) | \(v_2^n\) | \(v_2^p\) | \(v_2^H\) |
|----------|------------|--------|-----------|-----------|-----------|
| B        | E2         | 7.2    | -0.0939±0.0059 | -0.0966±0.0052 | -0.1045±0.0040 |
|          | E3         | 4.7    | -0.0711±0.0057 | -0.0705±0.0054 | -0.0758±0.0040 |
|          | E4         | 3.4    | -0.0615±0.0066 | -0.0324±0.0066 | -0.0501±0.0047 |
|          | E5         | 1.9    | -0.0245±0.0068 | -0.0201±0.0072 | -0.0222±0.0049 |
|          | E2-E5      | <7.5   | -0.0655±0.0031 | -0.0627±0.0030 | -0.0681±0.0022 |
| C        | E2         | 7.2    | -0.1065±0.0111 | -0.1008±0.0101 | -0.1249±0.0079 |
|          | E3         | 4.7    | -0.0681±0.0108 | -0.0583±0.0110 | -0.0841±0.0080 |
|          | E4         | 3.4    | -0.0552±0.0125 | -0.0356±0.0129 | -0.0593±0.0090 |
|          | E5         | 1.9    | -0.0259±0.0126 | 0.0007±0.0148  | -0.0178±0.0095 |
|          | E2-E5      | <7.5   | -0.0668±0.0058 | -0.0586±0.0059 | -0.0771±0.0043 |

TABLE I: Experimental FOPI-LAND values for elliptic flow of neutrons, protons and hydrogen for two choices of the kinematical cuts and referred to in the text as data sets B and C. The applied kinematical cuts are shown in the first column, the values of the transverse momentum \(p_T\) are in units of GeV/c. Changes in the elliptic flow values are minute (fourth digit) when the transverse momentum cut is relaxed to \(0.3 \leq p_T \leq 1.3\).

(A) has been applied, the kinematical acceptance of the FOPI-LAND detector in the chosen rapidity window setting constraints for the transverse momentum as well: \(0.27 < p_T < 1.06\). The values of the elliptic flow of neutrons and protons for this data set have been derived using the experimental values for the squeeze-out factor \(R_N\) presented in Ref. [17].

Constraints on the stiffness of asy-EoS extracted by a comparison of the data sets corresponding to different rapidity windows, integrated over impact parameter, with model predictions of EFD \(v_2^{n-p}\) agree with each other. One obtains for the \(x\) parameter the following values: \(x = -2.5 \pm 1.5\), \(x = -1.5 \pm 0.75\), and \(x = -2.0 \pm 0.75\) for the data sets A, B and C, respectively. Differences between the choices \(p_T < 1.0\) and \(p_T < 1.3\) are negligible. The theoretical estimates were obtained using the set of model parameters employed in Section III to generate the central solution, the uncertainty in the values of the \(x\) parameter originating solely from the error bars of experimental elliptic flow values.
III.  MODEL DEPENDENCE

A.  Parametrizations of the potential

The nucleon optical potential is an important ingredient of transport models, the sensitivity of heavy-ion observables in general \[30-32\] and collective flows \[33\] in particular to its momentum dependence being well documented. In a recent study \[34\] the effects of the momentum dependence of the symmetry energy on transverse and elliptic flows have been investigated with the conclusion that the neutron-proton elliptic flow difference presents a small sensitivity to the momentum dependent part of the isovector nucleon potential within the constraint of an asy-soft EoS. This is an important finding since the momentum dependence of the isovector part of the nucleon potential is still an open question. Parametrizations of it with various momentum dependences, or none at all, are commonly employed.

On the theoretical side, the optical potential has been extracted from first principles \[35\], and similar approaches have later been extended to also extract the symmetry potential \[36-38\]. Alternatively, it has been possible to extract the momentum dependence of the bare nucleon interaction within an effective model \[39\] starting from the optical potential of Refs. \[40, 41\] obtained within a relativistic Dirac-equation description of experimental data of proton scattering on Ca and heavier nuclei. The results of the two approaches are somewhat different, the Brueckner-Hartree-Fock approach and its relativistic counterpart favor a potential that is attractive at all values of the momentum, while the relativistic Dirac approach delivers a potential that becomes repulsive above a certain momentum threshold depending on which experimental data sets are considered. Additionally, the Brueckner-Hartree-Fock approach predicts an optical potential that is almost momentum independent at moderate values of the momentum.

To account for this model dependence we have simulated heavy-ion collisions by considering three different parametrizations of the optical potential. The first one stems from the isoscalar part of the Gogny interaction \[14\] while the last two mimic the parametrizations presented in Ref. \[39\]

\[
V_{\text{opt}}^{(MDI)}(\vec{p}_i, \vec{p}_j) = (C_l + C_u) \frac{1}{1 + (\vec{p}_i - \vec{p}_j)^2/\Lambda^2} \frac{\rho_{ij}}{\rho_0}
\]

\[
V_{\text{opt}}^{(HA)}(\vec{p}_i, \vec{p}_j) = \{V_0 + v \ln^2[\alpha (\vec{p}_i - \vec{p}_j)^2 + 1]\} \frac{\rho_{ij}}{\rho_0}
\]  

(3)
The parameters present in $V_{opt}^{(MDI)}(\vec{p}_i, \vec{p}_j)$ can be found in Ref. [14], while for $V_{opt}^{(HA)}(\vec{p}_i, \vec{p}_j)$ they read: $V_0=-0.054$ GeV, $v=0.00158$ GeV and $a=500$ GeV$^{-2}$, $V_0=-0.0753$ GeV, $v=0.002526$ GeV and $a=500$ GeV$^{-2}$ for the old and new parametrization in Ref. [39], respectively; $\rho_{ij}$ is the contribution to the density at the location of nucleon $j$ due to nucleon $i$, recovering in the infinite nuclear matter limit the parametrization of Ref. [14] and the right EoS. The $V_0$ parameter is absorbed in the linearly density dependent term of the single nucleon potential $V = \alpha \frac{\rho}{\rho_0} + \beta (\frac{\rho}{\rho_0})^\gamma + V_{opt}$. For completeness, the values of the remaining parameters, producing a soft (K=210 MeV) isoscalar EoS, read: $\alpha=-0.3901$ GeV, $\beta=0.3203$ GeV, $\gamma=1.14$ and $\alpha=-0.2017$ GeV, $\beta=0.1861$ GeV, $\gamma=1.2104$ for the two HA parametrizations.

The momentum dependence of the symmetry energy is currently an unsettled issue and consequently various parametrizations have been employed in the literature. To estimate the impact of this unknown on elliptic flow observables, we have selected two of the most widely employed parametrizations for the current study: the momentum dependent parametrization that follows from the Gogny interaction and the power-law momentum independent parametrization $S(\rho) = S_0 (\frac{\rho}{\rho_0})^\gamma$ ($S_0=18.5$ MeV). For values of the parameter $\gamma=2.0, 1.5, 0.5$ density dependencies close to those provided by the Gogny interaction for values $x=-2.0, -1.0, 0.0$, respectively, are obtained. To mimic the soft and super-soft symmetry energy provided by the Gogny interaction with $x=1$ and respectively $x=2$ the following modified power-law parametrizations are employed above the saturation point: $S(\rho) = a + (18.5 - a) (\frac{\rho}{\rho_0})^\gamma$ [MeV]; the sets of parameters $a = 23.0$ MeV, $\gamma=1.0$ and $a = 31.0$ MeV, $\gamma=2.0$ reproduce a soft and super-soft density dependence respectively. Below saturation, the standard power-law parametrization with $\gamma=1.0$ is employed in these cases. The soft power-law parametrizations, while producing discontinuous force terms at saturation density, have the advantage of having an identical functional density dependence of the force term as its stiff counterpart and generating a SE stiffness below saturation point compatible with the experimental result for that density region.

B. Model dependence of observables

Elliptic flows of protons and neutrons cannot be used separately to constrain the isovector part of the equation of state above the saturation point due to their sizable dependence on particular values of transport model parameters, that are either inaccurately determined or
FIG. 1: (Color online) Variations in the values of the impact parameter integrated \((b \leq 7.5 \text{ fm})\) npEFD (top) and npEFR (bottom) due to different choices for the optical potential \((V_{\text{opt}})\), parametrization of symmetry-energy \((S)\) as well as the combined, quadratically added, uncertainty. We do not represent measurable quantities, like in-medium nucleon-nucleon (NN) cross-sections, compressibility modulus of nuclear matter, width of the nucleon wave function and strength of the optical potential. A precise knowledge of the dependence on these parameters would allow the elimination of the most uncertain ones leaving one with a set of observables that bear no or almost no model dependence. In practice, one is forced to make assumptions that are either verified or disproved by employing a definite transport model. Neutron-proton elliptic flow ratios require a scenario in which elliptic flows scale linearly with model parameters, while for the differences the variation of elliptic flows of neutrons and protons with respect to model parameters should be equal in order to be able to extract a model independent constraint on the density dependence of SE.

To extract constraints on the stiffness of the SE the following procedure is employed. We require that the experimental elliptic flow values are reproduced as closely as possible for a particular asy-EoS scenario, while keeping model parameters within limits commonly found in the literature. As a consequence, npEFD and npEDR cannot be treated as independent
observables anymore. Differences in the constraints extracted by using each of them independently are related to how close the experimental elliptic flow values are reproduced for the favored asy-EoS scenario.

In the following the sensitivity of npEFD and npEFR with respect to model parameters will be presented. For the central estimates, a reproduction of the experimental elliptic flow data for a value of the asy-EoS stiffness parameter \( x = -1 \) within 10-15\% was possible with the following set of parameter values: stiffness of the isoscalar EoS set to \( K = 210 \) MeV, width of the nucleon wave function \( L = 4.33 \) fm\(^2\), the new version of the \( V_{\text{opt}}^{(HA)} \) optical potential parametrization and Cugnon nucleon-nucleon cross-sections. We would like to note that very few of the possible combinations of model parameters produce values for the elliptic flow compatible with the experimental values, many combinations underestimate its strength severely, sometimes up to a factor of 2. Once the stiffness of the scalar part of the EoS is set to a soft one (\( K = 210 \) MeV) the choices to enhance elliptic flow towards realistic values are to decrease the nucleon wave function width and/or choose an optical potential that is as repulsive as possible in its higher energy region. The Cugnon nucleon-nucleon cross-section parametrization has been used in the collision term. It should be noted that Cugnon neutron-proton cross-sections are lower than the experimental vacuum ones below an incident kinetic energy of \( T = 100 \) MeV, but significantly higher than the in-medium theoretical predictions at saturation density. They can, therefore, be thought of as effectively simulating some in-medium effects.

Results for the sensitivity of npEFD and npEFR to both the momentum dependent part of the isospin symmetric EoS and various parametrizations of the symmetry energy are displayed in Fig. (1) as a function of the stiffness of the asy-EoS. Collisions of Au+Au at 400 MeV/nucleon have been simulated and the kinematical cuts labeled ‘B’ above have been applied. The widths of the bands represent the variations of npEFD or npEFR when switching between parametrizations of the optical potential while keeping the SE parametrization fixed and vice-versa. The results presented correspond to averages over different choices of the quantity that was kept fixed. Conclusions as e.g. the sensitivity/insensitivity of the studied observables to the momentum dependence of symmetry energy for an asy-soft scenario (as was presented in Ref. [34]) can thus not be drawn from this figure. Each of the possible combinations of optical potential parametrization and symmetry energy parametrization (6 in total) usually yields a different outcome in this respect. The result of Fig. (1) should there-
FIG. 2: (Color online) Model dependence of npEFD and npEFR and comparison with FOPI-LAND experimental data, integrated over impact parameter $b \leq 7.5$ fm. Sensitivity to the different model parameters, compressibility modulus ($K$), width of nucleon wave function ($L$), optical potential ($V_{opt}$) and parametrization of the symmetry energy ($S$) are displayed. The total model dependence is obtained by adding, in quadrature, individual sensitivities.

Therefore be considered as an estimate. Nevertheless, it can be concluded that the uncertainties in the optical potential and the momentum dependence/independence of SE have an important impact on elliptic flow observables like npEFD and npEFR. For precisely constraining the symmetry energy at high density from elliptic flow data, an accurate knowledge of the optical potential will, therefore, be required and the problem of the momentum dependence or independence of the asy-EoS will have to be resolved.

In Fig. (2) the model dependence of impact parameter integrated npEFR and npEFD to variations of the compressibility modulus ($K$), width of the nucleon wave function ($L$), optical potential ($V_{opt}$) and SE parametrization ($S$) are presented. The central curve (full line) was obtained employing the parameter values mentioned above. By using the vacuum Li-Machleidt nucleon-nucleon cross-sections [42, 43] instead of the Cugnon ones, while keeping the other model parameters unchanged, both npEFD and npEFR remain practically the same irrespective of the value of $x$. As such the sensitivity to cross-section parametrizations has not been included in the total sensitivity bands. The commonly employed value for the compressibility modulus, a soft $K=210$ MeV, has been extracted from the multiplicity
ratio of $K^+$ production in heavy (Au+Au) over light (C+C) nuclei at incident energies close to 1 AGeV by the KaoS Collaboration \cite{27,44,45} but at lower incident energies the situation is not as clear: the KaoS result points to an even softer EoS while the study of sidewards flow or stopping by the FOPI collaboration \cite{46,47} does not exclude a somewhat stiffer isoscalar EoS. For the case of the nucleon wave function width, the value $L=4.33 \text{ fm}^2$ is commonly employed in transport models simulations of collisions of lighter nuclei while for the simulation of heavier nuclei an increase to the value $L=8.66 \text{ fm}^2$ is found necessary to prevent nucleon evaporation. The optical model dependence and SE parametrization dependence are the same as presented in Fig. (1).

The variation of the K and L model parameters has been performed within ranges that take into account the facts mentioned in the previous paragraph together with the requirement that the simulated values for the elliptic flow of neutrons and protons for $x=-1$ be within 25% from the experimental ones. This limit was chosen because it represents about 3 standard deviations of the combined experimental and numerical uncertainties. A search for the allowed values for K and L that obey this constraint has been performed with the following outcome. Increasing the compressibility modulus results in higher elliptic flow absolute values, reaching the upper 25% off the experimental value boundary in the region $K=270\div280 \text{ MeV}$. By decreasing the compressibility modulus below $K=210 \text{ MeV}$ a saturation region is reached with values well within the 25% off region just below $K=190 \text{ MeV}$. The dependence of the elliptic flow values on the nucleon wave function width $L$ proves to be approximately parabolic with a maximum absolute value for the elliptic flows, compatible with the imposed constraint, reached close to $L=3.5 \text{ fm}^2$ and crossing the lower 25% boundary for the values $L=2.5 \text{ fm}^2$ and $L=7.0 \text{ fm}^2$. Consequently, to produce the results presented in Fig. (2) the following variation ranges for the K and L model parameters have been adopted: $K=190\div280 \text{ MeV}$ and $L=2.5\div7.0 \text{ fm}^2$. Increasing the compressibility modulus to $K=300 \text{ MeV}$ or the nucleon wave function width to $L=8.66 \text{ fm}^2$ produces elliptic flow values that can deviate from the experimental ones with up to 40-50% but with marginal impact on the allowed values for the asy-EoS stiffness parameter $x$.

The sensitivities of npEFD and npEFR to model parameter values are similar; both show a model dependence on the optical potential that is almost independent of the stiffness of asy-EoS while the averaged dependence on the parametrization of SE is more pronounced for asy-soft scenarios. The L dependence is slightly more important for an asy-stiff than for an
as-y-soft EoS for both npEFD and npEFR. The K dependence is important for both npEFD and npEFR irrespective of the value chosen for the as-y-EoS stiffness with the exception of the as-y-stiff region for npEFD where it represents the most important source of uncertainty. The model dependence due to these last two parameters makes up most of the sensitivity of npEFD, especially for the case of an as-y-stiff scenario. The same holds true for npEFR in the as-y-stiff region while for the case of an as-y-soft scenario the importance of the optical potential and SE parametrization is equally or slightly more important.

The total model dependence K+L+V_{opt}+S of both npEFD and npEFR is almost insensitive to the stiffness of as-y-EoS and is comparable in absolute magnitude with the experimental uncertainty of the respective quantity. For each observable the experimental value and its uncertainty are depicted in Fig. (2) by the horizontal line and hatched band, respectively. The separation of the theoretical and experimental bands is clear and it amounts to about one standard deviation effect in the super-soft scenario region.

C. Constraints on as-y-EoS

The plots of Fig. (2) allow the extraction of estimates for the stiffness of as-y-EoS: x=-1.50^{+1.75}_{-1.00} from npEFD and x=-1.25^{+1.25}_{-1.00} from npEFR. These constraints translate, using the parabolic expansion of the as-y-EoS around the saturation point

\[ E_{sym}(\rho) = E_{sym}(\rho_0) + \frac{L_{sym}}{3} \frac{\rho - \rho_0}{\rho_0} + \frac{K_{sym}}{18} \frac{(\rho - \rho_0)^2}{\rho_0^2}, \]

into the following estimates for the slope and curvature parameters of the symmetry energy: 
L_{sym}=129^{+46}_{-80} \text{ MeV}, K_{sym}=272^{+291}_{-508} \text{ MeV (npEFD)} and L_{sym}=118^{+45}_{-57} \text{ MeV}, K_{sym}=199^{+291}_{-362} \text{ MeV (npEFR)}. The obtained values for L_{sym} are larger by a factor of 2 and 50% than the ones extracted from an analysis of neutron skin thickness and isospin diffusion at lower energies [6], respectively. The central values of the npEFD and npEFR based constraints favor therefore a density dependence of the symmetry energy above the saturation point close to mildly stiff or linear. They are consistent with each other, the difference between the central values is a consequence of the imperfect theoretical description of the experimental elliptic flow data at the favored value for the x parameter. This difference can in principle be eliminated by a finer than here attempted tuning of model parameters.

In Fig. (3) the explicit constraints on the density dependence of SE obtained in this
study from the comparison of theoretical and experimental values of npEFD and npEFR are presented. As npEFD and npEFR are not independent observables, due to the constraint that experimental elliptical flow data be reproduced at a value of the stiffness parameter $x$ close to the extracted one, only one band, obtained from averaging the npEFD and npEFR constraints, is advanced for the allowed values for the asy-EoS stiffness. The result of Ref. [15] is added for comparison. The two studies employ independent flavors of the QMD transport model (Tübingen QMD vs. UrQMD) and parametrizations of isovector EoS that differ: Gogny inspired (momentum dependent) vs. power law (momentum independent).

Relinquishing the requirement of a close reproduction of the experimental elliptical flow values for the central estimates allows one to treat npEFD and npEFR as independent observables. The extracted constraints for the values of the $x$ parameter are wider if extracted from npEFD and npEFR independently, but the overlapping region of the two differs only slightly from the one presented in Fig. (3). This brings strong support to the conclusion that the obtained constraint for the symmetry energy stiffness is model independent by providing evidence that an asy-EoS stiffness close to $x=-1$ is favored.

The constraints on the density dependence of SE obtained with these different ingredients are in agreement with each other. This contrasts with the current status of the effort to constrain the SE from $\pi^-/\pi^+$ ratios: a study employing IBUU transport model and the Gogny inspired asy-EoS [9] favors a soft asy-EoS, a second study which uses IQMD and the power-law parametrization of SE [10] points towards a stiff asy-EoS while the work of [11] within the Boltzmann-Langevin approach supplemented by a power-law parametrization of SE concludes that a super-soft scenario is the realistic one. The reason for this strong disagreement has most likely nothing to do with the parametrization used for the EoS or its momentum (in)dependence but may originate in medium effects on both $\Delta$ resonance and pion production cross-sections (including their isospin dependent energy thresholds) or other related topics [48, 49].

The result presented in this Article is robust, the model dependence of the presented observables, while important, is well understood and constraints obtained by employing different parametrizations of the asy-EoS are compatible with each other. An improvement of the current theoretical model is mandatory to allow, together with more accurate experimental data of elliptic flow of neutrons and protons as expected to be delivered by the ASY-EOS Collaboration [29], a tighter constraint on the high density dependence of the
IV. CONCLUSIONS

Constraints on the high density dependence of the symmetry energy (SE) have been extracted by comparing theoretical predictions of neutron-proton elliptic flow differences (npEFD) and neutron-proton elliptic flow ratios (npEFR) with experimental results obtained from a recent analysis of the FOPI-LAND experimental data for Au+Au collisions at 400 MeV/nucleon. The Tübingen QMD model supplemented with the Hartnack-Aichelin and Gogny parametrizations of the isoscalar EoS and asy-EoS respectively, for the central values, has been employed. A thorough study of the model dependence of npEFD and npEFR has been performed concluding that, while the sensitivity to uncertainties in the model parameters is important, the two observables offer the opportunity to extract information about the SE above the saturation point. Furthermore, the results of the present study supplemented with those of Ref. [15] allow one to conclude that constraints on symmetry energy.
energy extracted from elliptic flow data are independent on its parametrization, suggesting that an almost model independent extraction can be achieved in this case. This contrasts with the case of $\pi^-/\pi^+$ ratios where the stiffnesses of asy-EoS extracted using different parametrizations for SE or transport models can be extremely different.

We have imposed that the experimental elliptic flow values of neutrons and protons are reproduced as closely as possible and accomplished that to within 10-15% by changing model parameters within limits commonly employed in the literature. Averaging the constraints extracted independently from npEFD and npEFR one obtains the following allowed values for the parameter describing the stiffness of the symmetry energy: $x=-1.35 \pm 1.25$. Together with the estimates obtained in Ref. [15] we advance the following constraint, obtained from averaging these results, on the stiffness of asy-EoS: $x=-1.0 \pm 1.0$. It corresponds to a moderately stiff to linear density dependence and excludes the super-soft and, with a lesser degree of confidence, the soft asy-EoS scenarios from the list of possibilities. An improvement of the current theoretical model, in the sense of reducing theoretical uncertainties, is mandatory to allow, together with more accurate experimental data of elliptic flow of neutrons and protons, a tighter constraint on the high density-dependence of the symmetry energy.

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[50] Units of momenta, GeV/c, are not displayed explicitly.