Proton polarizability effect in the hyperfine splitting of the hydrogen atom

R.N.Faustov

Dorodnicyn Computing Centre RAS, Vavilov Street 40, Moscow, 119991, Russia

I.V.Gorbacheva, A. P. Martynenko

Samara State University, Pavlov Street 1, Samara, 443011, Russia.

The contribution of the proton polarizability to the ground state hyperfine splitting in the hydrogen atom is evaluated on the basis of isobar model and evolution equations for the parton distributions. The contributions of the Born terms, vector meson exchanges and nucleon resonances are taken into account in the construction of the proton polarized structure functions $g_{1,2}(W,Q^2)$. Numerical values of this effect are equal $(2.2 \pm 0.8) \times 10^{-6}$ times the Fermi splitting in electronic hydrogen and $(4.70 \pm 1.04) \times 10^{-4}$ times the Fermi splitting in muonic hydrogen.

PACS numbers: 36.10.Dr, 12.20.Ds, 31.30.Jv

Keywords: Proton polarizability, hyperfine structure, nucleon polarized structure functions

I. INTRODUCTION

The precise investigation of the energy levels of hydrogenic atoms (muonium, positronium, hydrogen atom, deuterium, helium ions et al.) allows to obtain more exact values for many fundamental physical constants such as the lepton masses, the ratio of the lepton and proton masses, the fine structure constant, the Rydberg constant which are used for creating standards of units [1]. The insertion of new simple atomic systems in the range of experimental investigation can lead to significant progress in solving of these problems. The measurement of muonic hydrogen Lamb shift at PSI (Paul Sherrer Institute) with a precision of 30 ppm will allow to improve our knowledge of the proton charge radius by an order of the magnitude [2]. Another important problem is connected with the measurement of the ground state hyperfine splitting (HFS) in muonic hydrogen [3, 4]. In the case of electronic hydrogen HFS was measured with extremely high accuracy many years ago [5]:

$$
\Delta E^{\text{exp}}_{\text{HFS}} = 1420 \, 405 \, 751.7667(9) \, kHz.
$$

The corresponding theoretical expression of the hydrogen hyperfine splitting can be written in the form $(\Delta E^\text{th}_{\text{HFS}} = 2\pi \hbar \Delta \nu^\text{th}_{\text{HFS}})$ [6]:

$$
\Delta E^\text{th}_{\text{HFS}} = E_F(1 + \delta^{QED} + \delta^S + \delta^P), \quad E_F = \frac{8}{3} \alpha^4 \frac{\mu_p m_p^2 m_e^2}{(m_p + m_e)^3},
$$

\(*\) faustov@theory.sinp.msu.ru

\(†\) mart@ssu.samara.ru
where $\mu_p$ is the proton magnetic moment, $m_e, m_p$ are the masses of the electron and proton. The calculation of different corrections to the Fermi energy $E_F$ has a long history. Modern status in the theory of hydrogenic atoms was presented in details in [6]. $\delta^{QED}$ denotes the contribution of higher-order quantumelectrodynamical effects. Corrections $\delta^S$ and $\delta^P$ take into account the influence of strong interaction. $\delta^S$ describes the effects of proton finite-size and recoil contribution. $\delta^P$ is the correction due to the proton polarizability. Basic uncertainties of theoretical result (2) are related with $\delta^S$ and $\delta^P$.

\[
\Delta E_Z = F_F \frac{2 \mu\alpha}{\pi^2} \int \frac{dp}{p^4} \left[ \frac{G_E(p^2)G_M(p^2)}{1 + \kappa} - 1 \right] = E_F(-2\mu\alpha)R_p. \tag{3}
\]

The Zemach radius $R_p$ is determined by the densities of electric charge and magnetic moment. It is considered as a fundamental parameter of the proton structure along with the proton charge radius. There exist three possibilities to determine the numerical value $R_p$ [8, 9, 10, 11]. One approach based on the analysis of the world data on $e - p$ scattering gives $R_p = 1.086 \pm 0.012$ fm [8]. Another method uses the comparison of theoretical and experimental results (1)-(2) for the hydrogen atom. In this case the value $R_p = 1.043(16)$ fm is obtained in [8]. The third method can be based on the comparison of the future experimental data and theoretical result for muonic hydrogen [11, 12]. The proton polarizability correction is important among other contributions of order $10^{-6}$. Numerical estimation of $\delta^P$ obtained in [13] serves at present as a reliable guide for defining the total value of the HFS in the hydrogen [8, 9, 10, 11].

The aim of this work consists in the investigation of the proton polarizability correction in the hydrogen HFS. We performed new calculation of $\delta^P$ using the isobar model describing the processes of photo- and electroproduction of $\pi, \eta$ mesons, nucleon resonances on the nucleon in the resonance region, and on evolution equations for the parton distributions in deep inelastic region.
II. GENERAL FORMALISM

The main contribution to $\delta^P$ is determined by two-photon diagrams, shown in Fig. 1. The corresponding amplitudes of virtual Compton scattering on the proton can be expressed through nucleon polarized structure functions $G_1(\nu, Q^2)$ and $G_2(\nu, Q^2)$. Inelastic contribution of the diagrams (a), (b) Fig. 1 can be presented in the form [13, 14, 15, 16, 17]:

$$\Delta E_{HFS}^P = \frac{Ze m_e}{2\pi m_p (1 + \kappa)} E_F (\Delta_1 + \Delta_2) = (\delta_1^P + \delta_2^P) E_F = \delta^P E_F,$$

where $\kappa$ is the Bjorken variable, $\nu$ is the invariant mass of the pion-nucleon system:

$$\nu_{th} = m_\pi + \frac{m_\pi^2 + Q^2}{2m_p},$$

and the functions $\beta_{1,2}$ have the form:

$$\beta_1(\theta) = 3\theta - 2\theta^2 - 2(2 - \theta) \sqrt{\theta(\theta + 1)},$$

$$\beta_2(\theta) = 1 + 2\theta - 2\sqrt{\theta(\theta + 1)}, \quad \theta = \nu^2/Q^2.$$  

$F_2(Q^2)$ is the Pauli form factor of the proton, $\kappa$ is the proton anomalous magnetic moment: $\kappa = 1.792847351(28)$ [1].

The polarized structure functions $g_1(\nu, Q^2)$ and $g_2(\nu, Q^2)$ enter in the antisymmetric part of the hadronic tensor $W_{\mu\nu}$, describing lepton-nucleon deep inelastic scattering [18]:

$$W_{\mu\nu} = W_{\mu\nu}^{[S]} + W_{\mu\nu}^{[A]},$$

$$W_{\mu\nu}^{[S]} = \left(-g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^2}\right) W_1(\nu, Q^2) + \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu\right) \left(P_\nu - \frac{P \cdot q}{q^2} q_\nu\right) \frac{W_2(\nu, Q^2)}{m_p^2},$$

$$W_{\mu\nu}^{[A]} = \epsilon_{\mu\nu\alpha\beta} q^\alpha \left\{ S^\beta g_1(\nu, Q^2) + [(P \cdot q) S^\beta - (S \cdot q) P^\beta] \frac{g_2(\nu, Q^2)}{(P \cdot q)^2} \right\},$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor in four dimensions, $g_1(\nu, Q^2) = m_p^2 \nu G_1(\nu, Q^2)$, $g_2(\nu, Q^2) = m_p^2 \nu^2 G_2(\nu, Q^2)$, $P$ is the four-momentum of the nucleon, $x = Q^2/2m_p \nu$ is the Bjorken variable, $S$ is the proton spin four-vector, normalized to $S^2 = -1$, $q^2 = -Q^2$ is the square of the four-momentum transfer. The invariant quantity $P \cdot q$ is related to the energy transfer $\nu$ in the proton rest frame: $P \cdot q = m_p \nu$. The invariant mass of the electroproduced hadronic system, $W$, is then $W^2 = m_p^2 + 2m_p \nu - Q^2 = m_p^2 + Q^2 (1/x - 1)$. Here $W_1$ and $W_2$ are the structure functions for unpolarized scattering. In the DIS regime the invariant mass $W$ must be greater than any resonance in the nucleon. The threshold between the resonance region and the deep-inelastic region is not well defined, but it is usually taken to be at about $W^2 = 4 GeV^2$. 

Hadronic tensor $W_{\mu \nu}$ is proportional to the imaginary part of the off-shell Compton amplitude for the forward scattering of virtual photons on nucleons: $\gamma^* N \rightarrow \gamma^* N$. The photon-nucleon interaction depends on the photon polarization as well as on the nucleon one. This gives four independent helicity amplitudes of the form $M_{ab,cd}$, with a, b, c, d values for the helicities of the photon and nucleon initial and final states:

$$M_{1,1/2,1/2}, M_{1,-1/2,1/2}, M_{0,1/2,0,1/2}, M_{1,1/2,0,-1/2}.$$  

These components correspond to the four structure functions $W_1, W_2, g_1, g_2$. All other possible combinations of initial and final photon and nucleon helicities are related to the above by time reversal and parity transformation.

The proton spin structure functions can be measured in the inelastic scattering of polarized electrons on polarized protons. Recent improvements in polarized lepton beams and nucleon targets have made it possible to make accurate measurements of nucleon polarized structure functions $g_1, g_2$ in experiments at SLAC, CERN and DESY [19, 20, 21, 22, 23, 24]. The spin dependent structure functions can be expressed in terms of virtual photon-absorption cross sections [18]:

$$g_1(\nu, Q^2) = \frac{m_2 \cdot K}{8\pi^2 \alpha(1 + Q^2/\nu^2)} \left[ \sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2) + \frac{2\sqrt{Q^2}}{\nu} \sigma_{TL}(\nu, Q^2) \right], \quad (13)$$

$$g_2(\nu, Q^2) = \frac{m_2 \cdot K}{8\pi^2 \alpha(1 + Q^2/\nu^2)} \left[ -\sigma_{1/2}(\nu, Q^2) + \sigma_{3/2}(\nu, Q^2) + \frac{2\nu}{\sqrt{Q^2}} \sigma_{TL}(\nu, Q^2) \right], \quad (14)$$

where $K$ is the kinematical flux factor for virtual photons, $\sigma_{1/2}, \sigma_{3/2}$ are the virtual photoabsorption transverse cross sections for the total photon-nucleon helicity of $1/2$ and $3/2$ respectively, $\sigma_{TL}$ is the interference term between the transverse and longitudinal photon-nucleon amplitudes. In this work we calculate contribution $\Delta E_{HFS}^P$ on the basis of the latest experimental data on the structure functions $g_{1,2}(\nu, Q^2)$ and theoretical predictions for the cross sections $\sigma_{1/2,3/2,TL}$.

The proton polarizability contribution to HFS in the resonance region is determined by the processes of photo- and electroproduction on nucleons of the pions and some prominent baryon resonances. The amplitudes of such reactions are shown in Fig.2.

To obtain correction (4) in the resonance region ($W^2 \leq 4 GeV^2$) we use the Breit-Wigner parameterization for the photoabsorption cross sections in Eqs.(13)-(14), suggested in [25, 26, 27, 28, 29, 30, 31, 32]. In the considered region of the variables $k^2, W$ the most contribution is given by five resonances: $P_{13}(1232), S_{11}(1535), D_{13}(1520), P_{11}(1440), F_{15}(1680)$. Accounting the resonance decays to the $N\pi^-$ and $N\eta^-$ states we can express the absorption cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ as follows:

$$\sigma_{1/2,3/2} = \left( \frac{k_R}{k} \right)^2 \frac{W^2 \Gamma_{\gamma} \Gamma_{R\rightarrow N\pi}}{(W^2 - M^2_R)^2 + W^2 \Gamma^2_{tot}/M^2_R} \frac{4m_p}{M_R \Gamma_R} |A_{1/2,3/2}|^2, \quad (15)$$

where $A_{1/2,3/2}$ are transverse electromagnetic helicity amplitudes,

$$\Gamma_{\gamma} = \Gamma_R \left( \frac{k}{k_R} \right)^{j_1} \left( \frac{k_R^2 + X^2}{k^2 + X^2} \right)^{j_2}, \quad X = 0.3 GeV. \quad (16)$$
The resonance parameters $\Gamma_R, M_R, j_1, j_2, \Gamma_{tot}$ are taken from [27, 28, 33, 34]. In accordance with [27, 29, 34] the parameterization of one-pion decay width is

$$\Gamma_{R \rightarrow N\pi}(q) = \frac{M_R}{M} \left( \frac{q}{q_R} \right)^3 \left( \frac{q_R^2 + C^2}{q^2 + C^2} \right)^2, \quad C = 0.3 \text{ GeV} \tag{17}$$

for the $P_{33}(1232)$ and

$$\Gamma_{R \rightarrow N\pi}(q) = \Gamma_R \left( \frac{q}{q_R} \right)^{2l+1} \left( \frac{q_R^2 + \delta^2}{q^2 + \delta^2} \right)^{l+1}, \tag{18}$$

for $D_{13}(1520), P_{11}(1440), F_{15}(1680)$. $l$ is the pion angular momentum and $\delta^2 = (M_R - m_p - m_\pi)^2 + \Gamma_R^2/4$. Here $q (k)$ and $q_R (k_R)$ denote the c.m.s. pion (photon) momenta of resonances with the mass $M$ and $M_R$ respectively. In the case of $S_{11}(1535)$ we take into account $\pi N$ and $\eta N$ decay modes [29, 34]:

$$\Gamma_{R \rightarrow \pi,\eta} = \frac{q_{\pi,\eta}}{q} \Gamma_R \frac{q_R^2 + C_{\pi,\eta}^2}{q^2 + C_{\pi,\eta}^2}, \tag{19}$$

where $b_{\pi,\eta}$ is the $\pi$ ($\eta$) branching ratio.

The cross section $\sigma_{TL}$ is determined by an expression similar to Eq.(15), containing the product $(S_{1/2}^* \cdot A_{1/2} + A_{1/2}^* S_{1/2})$ [19]. The calculation of helicity amplitudes $A_{1/2}, A_{3/2}$ and longitudinal amplitude $S_{1/2}$, as functions of $Q^2$, was done on the basis of constituent quark model (CQM) in [35, 36, 37, 38, 39, 40]. In the real photon limit $Q^2 = 0$ we take corresponding resonance amplitudes from [33].

The two-pion decay modes of the higher nucleon resonances ($S_{11}(1535), D_{13}(1520), P_1(1440)$ and $F_{15}(1680)$ were described phenomenologically using two-step process as in [27]. The high-lying nucleon resonance $R$ can decay first into $N^*$ ($P_{33}(1232)$ or $P_{11}(1440)$) and a pion or into a nucleon and $\rho$- or $\sigma$-meson. Then the new resonances decay into a nucleon and a pion or two pions:

$$R \rightarrow r + a = \begin{cases} N^* + \pi \rightarrow N + \pi + \pi, \\ \rho(\sigma) + N \rightarrow N + \pi + \pi. \end{cases}$$
The total decay width of such processes can be presented as a phase space weight integral over the mass distribution of the intermediate resonance $r = N^* , \rho, \sigma \ (a = \pi, N)$:

$$
\Gamma_{R \rightarrow r+a}(W) = \frac{P_{2\pi}}{W} \int_{0}^{W-m_a} d\mu \cdot p_f \frac{2}{\pi} \frac{\mu^2 \Gamma_{r, tot}(\mu)}{(\mu^2 - m_r^2)^2 + \mu^2 \Gamma_{r, tot}(\mu)} \frac{(M_R - m_2 - 2m_\pi)^2 + C^2}{(W - m_2 - 2m_\pi)^2 + C^2},
$$

where $C = 0.3 \ GeV$, the factor $P_{2\pi}$ must be taken from the constraint condition: $\Gamma_{R \rightarrow r+a}(W_R)$ coincides with the experimental data, $p_f$ is the three momentum of the resonance $r$ in the rest frame of $R$. $\Gamma_{r, tot}$ is the total width of the resonance $r$. The decay width of the meson resonance is parameterized similarly to that of the $P_{33}(1232)$:

$$
\Gamma(\mu) = \Gamma_r \frac{m_r}{\mu} \left( \frac{q}{q_r} \right)^{2J_r+1} \frac{q_r^2 + \delta^2}{q^2 + \delta^2}, \quad \delta = 0.3 \ GeV,
$$

where $m_r$ and $\mu$ are the mean mass and the actual mass of the meson resonance, $q$ and $q_r$ are the pion three momenta in the rest frame of the resonance with masses $\mu$ and $m_r$, $J_r$ and $\Gamma_r$ are the spin and decay width of the resonance with the mass $m_r$.

Main nonresonant contribution to the cross sections $\sigma_{T,L}$ in the resonance region is determined by the Born terms constructed on the basis of Lagrangians of $\gamma NN, \gamma \pi \pi, \pi NN$.
FIG. 4: The proton polarized structure function $g_2(W,Q^2)$ as the function of variables $Q^2$ (0 ÷ 1) Gev$^2$ and $W$ (1.1 ÷ 2.0) GeV.

 interactions. Another part of nonresonant background comprises the $t$- channel contributions of $\rho$, $\omega$ mesons obtained by means of effective Lagrangians $\gamma\pi V$, $VNN$ interactions ($V = \rho, \omega$) [31]. In the unitary isobar model accounting the Born terms, the vector meson, nucleon resonance contributions and the interference terms we calculated the cross sections $\sigma_{T,L}$ by means of numerical program MAID (http://www.kph-uni-mainz.de/MAID) in the resonance region as the functions of two variables $W$ and $Q^2$. The obtained nucleon polarized structure functions $g_{1,2}(W,Q^2)$ are presented in Figs.3-4. These results for the structure function $g_{1}(W,Q^2)$ are in qualitative agreement with experimental data. The particular significance in the study of the spin-dependent properties of baryon resonances belongs to Gerasimov-Drell-Hearn (GDH) sum rule [41]

$$-\frac{k^2}{4m^2} = \frac{1}{8\pi^2\alpha} \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu} [\sigma_{1/2}(\nu,0) - \sigma_{3/2}(\nu,0)].$$  \hspace{1cm} (22)

The GDH sum rule rests on the basic physical principles and an unsubtracted dispersion relation applied to the forward Compton amplitude.

To construct the spin-dependent structure functions of the proton in deep inelastic region we can use the $Q^2$ evolution equations for the quark and gluon distributions [42]:

$$\frac{dq_i(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \int_{x}^{1} \frac{dy}{y} \left[ q_i(y,Q^2) P_{qq} \left(\frac{x}{y}\right) + g(y,Q^2) P_{qg} \left(\frac{x}{y}\right) \right],$$  \hspace{1cm} (23)
\[
\frac{dg(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[ \sum_i q_i(y, Q^2) P_{qq}(x/y) + g(y, Q^2) P_{gg}(x/y) \right],
\]

where the sum is considered over all quarks and antiquarks. \( P_{qq}, P_{qg}, P_{gg}, P_{gg} \) are the quark-gluon splitting functions \[43\]. Numerical solution of the integrodifferential evolution equations (23), (24) by means of the method suggested in \[44\] allows to obtain the parton distributions and the structure functions \( g_{1,2}(x, Q^2) \) for different values of a photon momentum squared \( Q^2 \). Corresponding numerical results are in good agreement with the world experimental data \[13, 21, 22, 23, 24\].

\textbf{III. NUMERICAL RESULTS}

In this paper we calculate the proton polarizability correction to the hyperfine splitting of the ground state in the hydrogen atom on the basis of the isobar model describing the processes of low-energy scattering of virtual photons on nucleons and the evolution equations for the parton distributions. These two significant ingredients of the calculation allow to construct the absorption cross sections for transversely and longitudinally polarized photons by nucleons and to express the structure functions \( g_{1,2}(W, Q^2) \) (13), (14), which determine required contribution (4).

The values of contributions \( \delta_1^P, \delta_2^P \) and the total contribution \( \delta^P \), obtained after the numerical integration in the resonance and nonresonant regions are as follows in electronic and muonic hydrogen correspondingly:

\[
\delta_1^P = 2.6 \text{ ppm}, \quad \delta_2^P = -0.4 \text{ ppm}, \quad \delta^P = (2.2 \pm 0.8) \text{ ppm},
\]

\[
\delta_1^P = 5.18 \times 10^{-4}, \quad \delta_2^P = -0.48 \times 10^{-4}, \quad \delta^P = (4.70 \pm 1.04) \times 10^{-4}.
\]

The difference of obtained number for electronic hydrogen from the result of our previous work \(1.4 \div 0.6\) ppm \[13\] is connected with new contributions (nonresonant background in the resonance region and two pion decays of the resonances) which we considered in this study. There exists a number of theoretical uncertainties associated with quantities entering in the correction (4). In the improved isobar model \[31, 45, 46\] containing 14 resonances, we can omit theoretical error which arises due to the insertion of other high-lying nucleon resonances. On our sight the main theoretical error is closely related with the calculation of the helicity amplitudes \( A_{1/2}(Q^2), A_{3/2}(Q^2), S_{1/2}(Q^2) \) in the quark model based on the oscillator potential \[18\]. Only systematical experimental data for the helicity amplitudes of the photoproduction on the nucleons \( A_{1/2}(0), A_{3/2}(0) \) are known with sufficiently high accuracy to the present \[33\]. In the case of amplitudes for the electroproduction of the nucleon resonances experimental data contain only their values at several points \( Q^2 \). So, we have no consistent check for the predictions of the oscillator model. Possible theoretical uncertainty connected with the calculation of amplitudes \( A_{1/2}(Q^2), A_{3/2}(Q^2), S_{1/2}(Q^2) \) with the account of relativistic corrections can attain the value of order 10%. Then the theoretical error for the correction (4) in the resonance region comprises 20% from the obtained value. We solved DGLAP equations in the NLO approximation, so possible uncertainty in \( \delta^P \) can comprise near 10% of obtained result in the nonresonant region. The other source of the theoretical uncertainty arises from the experimental data errors in the \( Q^2 \leq 1 \text{ GeV}^2 \) region. We estimated it at a level of about 20% of the contribution \( \delta^P \) at \( Q^2 \leq 1 \text{ GeV}^2 \) in the nonresonant region.
FIG. 5: Plots of $\pi^+$ virtual photoproduction cross sections at $Q^2 = 0.4$ $GeV$ as the functions of the $W (1100 \div 1600)$ $MeV$, obtained on CLAS detector. The solid curves correspond to the results from MAID.
New experimental data for the electroproduction cross sections in the reaction $ep \rightarrow e'\pi^+n$ in the resonance region were obtained recently on CLAS detector [47]. Their comparison with the calculations carried out on the basis of the unitary isobar model (MAID) is presented in Fig.5. It evidently shows that MAID, which we used in our calculation, gives the numerical values for the electroproduction cross sections in the regions of the $\Delta$ isobar and resonances $D_{11}(1520), S_{11}(1535)$, which are slightly higher (approximately by 5%) than the experimental data.

The work was supported by the Russian Foundation for Basic Research (grant No. 06-02-16821).

[1] P.J.Mohr, B.N.Taylor, Rev. Mod. Phys. 77, p.1, 2005.
[2] R.Pohl, A.Antognini, F.D.Amaro et al., Can. J. Phys. 83, p.339, 2005.
[3] K.Pachucki, Phys. Rev. A 53, p.2092, 1996.
[4] A.P.Martynenko, Phys. Rev. A 71, p.022506, 2005.
[5] H. Hellwig, R.F.C.Vessot, M.W.Levine, et al, IEEE Trans. IM-19, p.200, 1970.
[6] M.I.Eides, H.Grotch, V.A.Shelyuto, Phys. Rep.342, p. 2001.
[7] A.C.Zemach, Phys. Rev. 104, p.1771, 1956.
[8] J.L.Friar, J.Sick, Phys. Rev. Lett. 95, p.049101, 2005.
[9] S.J.Brodsky, C.E.Carlson, J.R.Hiller, D.S.Hwang, Phys. Rev. Lett. 94, p.022001, 2005.
[10] A.V.Volotka, V.M.Shabaev, G.Plunien, G.Soff, Eur. Phys. J. D 33, p.23, 2005.
[11] A.Dupays, A.Beswick, B.Lepetit et al. Phys. Rev. A 68, p.052503, 2003.
[12] E.V.Cherednikova, R.N.Faustov, A.P.Martynenko, Nucl. Phys. A 703, p.365 , 2002.
[13] R.N.Faustov, A.P.Martynenko, Eur. Phys. J. C 24, p.281, 2002.
[14] S.D.Drell, J.D.Sullivan, Phys. Rev. 154, p.1477, 1967.
[15] A.Vergenalakis, D.Zwanziger, Nuovo Cimento A 39, p.613, 1965.
[16] F.Guerin, Nuovo Cimento A 50, p.1, 1967.
[17] G.M.Zinov’ev, B.V.Struminsky, R.N.Faustov, V.L.Chernyak, Sov. J. Nucl. Phys. 11, p.715, 1970.
[18] R.P.Feynman, Photon-Hadron Interactions, W.A. Benjamin, Inc. Reading, Massachusets, 1972.
F.E.Close, An introduction to quarks and partons, Academic Press, N.Y., 1979.
[19] K.Abe, T.Akagi, P.L.Anthony et al., Phys. Rev. D 58, p.112003, 1998.
[20] K.Abe et al., Phys. Rev. Lett. 78, p.815, 1997.
[21] P.L.Anthony et al., Phys. Lett. B 458, p.529, 1999.
[22] G.S.Mitchell, Preprint SLAC-PUB-8104, 1999.
[23] D.Adams et al., Phys. Rev. D 56, p.5330, 1997.
[24] D.Adeva et al., Phys. Rev. D 60, p.072004, 1999.
[25] R.L.Walker, Phys. Rev. 182, p.1729, 1969.
[26] R.A.Arndt, R.L.Workman, Z.Li et al., Phys. Rev. C 42, 1864, 1990.
[27] S.Teis, W.Cassing, M.Effenberger et al., Z. Phys. A 356, p.421, 1997.
[28] M.Effenberger, A.Hombach, S.Teis et al., Nucl. Phys. A 613, 353, 1997.
[29] B.Krusche, J.Ahrens, G.Anton et al., Phys. Rev. Lett. 74, p.3736, 1995.
[30] N.Bianchi, V.Muccifora, E.Sanctis et al., Phys. Rev. C 54, p.1688, 1996.
[31] D.Drechsel, O.Hanstein, S.S.Kamalov et al., Nucl. Phys. A 645, p.145, 1999.
[32] Y.-B. Dong, Eur. Phys. Jour. A 1, p.347, 1998.
[33] Review of Particle Physics, Phys. Lett. B 592, p.1, 2004.
[34] M. Effenberger, A. Hombach, S. Teis et al., Nucl. Phys. A 614, p.501, 1997.
[35] Z. Li, Y.-B. Dong, Phys. Rev. D 54, p.4301, 1996.
[36] R. Koniuk, N. Isgur, Phys. Rev. D 21, p.1888, 1980.
[37] F. E. Close, Z. Li, Phys. Rev. D 42, p.2194, p.2207, 1990.
[38] S. Capstick, Phys. Rev. D 46, p.1965, p.2864, 1992.
[39] Zhenping Li, V. Burkert, Zhujun Li, Phys. Rev. D 46, p.70, 1992.
[40] M. Warns, W. Pfeil, H. Rollnik, Phys. Rev. D 42, p.2215, 1990.
[41] S. B. Gerasimov, Sov. J. Nucl. Phys. 2, p.430, 1966;
S. D. Drell, A. C. Hearn, Phys. Rev. Lett. 16, p.908, 1966.
[42] V. N. Gribov, L. N. Lipatov, Sov. J. Nucl. Phys. 15, p.438, 1972;
Yu. A. Dokshitzer, JETP 46, p.641, 1977;
G. Altarelli, G. Parisi, Nucl. Phys. B 126, p.298, 1977.
[43] E. Leader, E. Predazzi, An introduction to gauge theories and the ”New physics”, Cambridge University Press, NY, 1982.
[44] M. Hirai, S. Kumano, M. Miyama, Comp. Phys. Comm. 108, p.38, 1998.
[45] W.-T. Chiang, S. N. Yang, L. Tiator, D. Drechsel, Nucl. Phys. A 700, p.429, 2002.
[46] D. Drechsel, S. S. Kamalov, G. Krein, L. Tiator, Phys. Rev. D 59, p.094021, 1999.
[47] H. Egiyan, et al. Electroproduction on the proton in the first and second resonance regions at $0.25 \, GeV^2 \leq Q^2 \leq 0.65 \, GeV^2$ using CLAS, e-preprint nucl-ex/0601007.