Reduction of Dynamic Loads in Mine Lifting Installations

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Abstract. Article is devoted to a problem of decrease in the dynamic loadings arising in transitional operating modes of the mine lifting installations leading to heavy oscillating motions of lifting vessels and decrease in efficiency and reliability of work. The known methods and means of decrease in dynamic loadings and oscillating motions of the similar equipment are analysed. It is shown that an approach based on the concept of the inverse problems of dynamics can be effective method of the solution of this problem. The article describes the design model of a one-ended lifting installation in the form of a two-mass oscillation system, in which the inertial elements are the mass of the lifting vessel and the reduced mass of the engine, reducer, drum and pulley. The simplified mathematical model of this system and results of an efficiency research of an active way of reduction of dynamic loadings of lifting installation on the basis of the concept of the inverse problems of dynamics are given.

1. Introduction
The high speeds of movement of mine lifting installations lead to the emergence of large dynamic loads, elastic deformations of structural elements and oscillating movements, causing excessive stress, fatigue, increased wear and, as a result, deterioration of durability and safety in operation. The tighten of characteristics of safety and reliability of modern lifting installations places high demands on the level of their dynamic calculations and necessitates taking into account the elastic properties of the design and development of methods and tools for reducing dynamic loads and vibrations [1–3].

To solve this problem, in [4–7] proposed a special mathematical model that takes into account all important features of the mechanics of mine lifting installation under braking, and in [8] considered the impact of different parameters on the dynamic behavior of mine lifts during extreme braking. The various circuit designs that can be configured to implement dynamic braking on a mine hoisting system are offered and selection of the load resistance for the desired dynamic braking system performance is addressed [9]. The present state of knowledge of dynamics of hoisting systems and available modern computer hardware enable to develop and analyze simulation models used for optimizing of dynamic characteristics of the system of mine lifting installations [10–12].

In reference [13], computer monitoring systems and registers of the parameters of the lifting installations are proposed, which allow to determine the occurrence causes of additional loads and to develop measures for their elimination. The system of automatically controlled safety braking is
proposed in [14] for minimize dynamic overloads in the mechanical part and increase the safety of the mine lifting installations. This system is designed for use on all types of inclined drum and vertical multi-channel lifts with radial and disc brakes. In reference [15–17] the use the smooth laws of motion of the lifting vessels are encouraged for reduce the dynamic loads in mine hoisting installations. The possibilities of application of the retarding force on the vessel during the working of mine lifts in mode of lifting are investigated in [18]. The main disadvantages of these methods reduce the dynamic loads are either low efficiency under varying operating conditions or the performance of the mining installations.

In this regard, particular interest is the use of the method of reduction of dynamic loads and elastic vibrations of the links of managed machines based on the use of the concept of inverse problems of dynamics, proposed in [19–23]. The results of research, related to the implementation of this method to reduce of dynamic loads and damping of elastic vibrations on the example of one-ended lifting installation, are given in this article.

2. Problem statement

Currently the developed methods of dynamic synthesis, which would allow to realize a controlled movement with the desired dynamic properties, are absent in the dynamics of controlled machines with elastic links. Most of the known solutions in this field based on engineering experience and intuition than on scientific – methodological basis. Analysis of known work in this area has shown that an effective method of problems solving of control synthesis, providing the mechanical movement with the desired properties, is the use of methods of solution of inverse problems of dynamics for a given type of vibrational motion. A principal scheme of one-ended lifting installation is shown in figure 1, for which considered the implementation of this approach for problem solving of reducing of dynamic loads.

![Figure 1. A principal scheme of lifting installation:](image)

- the rigidity of the gear reducer 3, the couplings 2, 6 and shafts is much higher than the stiffness of the rope 8;
- the mass of the rope 8 is concentrated at the drum 7 and lifting vessel 10, the rope is a weightless and non-viscous elastic system with a constant stiffness coefficient;
- the aerodynamic air resistance and friction force in the conductors of the shaft are absent;
- the movement is considered from the position of static equilibrium of the lifting vessel;

The following assumptions are adopted to obtain the dynamic model of the mine lifting installation:

- the rigidity of the gear reducer 3, the couplings 2, 6 and shafts is much higher than the stiffness of the rope 8;
- the mass of the rope 8 is concentrated at the drum 7 and lifting vessel 10, the rope is a weightless and non-viscous elastic system with a constant stiffness coefficient;
- the aerodynamic air resistance and friction force in the conductors of the shaft are absent;
- the movement is considered from the position of static equilibrium of the lifting vessel;
the influence of dynamic processes in the drive motor or braking device is not taken into account.

A design scheme of the mine lifting installation can be represented as a two-mass mechanical system, shown in figure 2.

![Figure 2. Two-mass design scheme of the lifting installation.](image)

An inertial elements of the lift installation are the mass of the lifting vessel with part of the rope mass and the resulting mass of the engine, reducer, drum, parts of a rope and pulley, connected by a weightless elastic rope. The differential equations of motion of two-mass system, obtained on the basis of the D’alembert principle, are as

\[ m_n \ddot{q}_1 + c(q_1 - q_2) = Q_n - F_m; \quad (1) \]
\[ m \ddot{q}_2 + c(q_2 - q_1) = 0, \quad (2) \]

where \( m_n \) – the reduced mass of the engine, reducer, drum, parts of a rope and pulley; \( m \) – the reduced mass of the lifting vessel with part of the rope mass; \( c \) – the reduced coefficient of rope stiffness; \( Q_n, F_m \) – the reduced values of the driving force of the actuator and the braking force; \( q_1 \) – the generalized coordinate of the mass movement \( m_n \); \( q_2 \) – the generalized coordinate of the lifting vessel movement.

Summing to equations (1) and (2) and denoting the elastic deformation of the rope \( \Delta q = q_2 - q_1 \), obtain the equation of motion of the first mass

\[ (m + m_n) \ddot{q}_1 + m \Delta \ddot{q} = Q_n - F_m. \quad (3) \]

The expression (2) takes the form

\[ m \Delta \ddot{q} + m \ddot{q}_1 + c \Delta \ddot{q} = 0. \quad (4) \]

It is required to determine the law of change of motive (\( Q_n \)) or braking (\( F_m \)) forces in expression (1), providing a minimum dynamic load and compensation of elastic vibrations of the lifting vessel, when it goes up, by solving the inverse dynamic problem for a given law of change of elastic deformation of the rope and limiting the movement acceleration of the first mass value \( |\ddot{q}_1| \leq a \).

3. Theory

Find the control action \( Q_n \), by which the elastic vibrations of the lifting vessel is fully extinguished for any valid values of acceleration time (deceleration) and the motion acceleration of the first mass does not exceed a predetermined value \( |\ddot{q}_1| \leq a \). Consider the mode of the vessel lifting, assuming that the elastic vibrations are absent in the initial time, i.e. \( \Delta q(0) = \Delta \dot{q}(0) = 0 \), in this case the elastic deformation of the rope is changed under the harmonic law

\[ \Delta q = \gamma_p \left( 1 - \cos \frac{2\pi}{T_p} t \right), \quad (5) \]

where \( \gamma_p \) – a constant value; \( T_p \) – acceleration time (deceleration). Then the conditions \( \Delta q = \Delta \dot{q} = 0 \) will be implemented when \( t = 0 \) and \( t = T_p \) and on the basis expression (5). After double
differentiating in time (5), the law of acceleration change of the elastic deformation of the rope is obtained

$$\Delta \ddot{q} = \frac{4\pi^2 \gamma_p}{T_p} \cos \frac{2\pi}{T_p} t.$$  \hspace{1cm} (6)

Express from expression (1) when $F_m = 0$ the acceleration

$$\ddot{q}_1 = \frac{Q_n + c \Delta q}{m_n}.$$  \hspace{1cm} (7)

Combining expressions (7) with (4), is obtained

$$\Delta \ddot{q} + \omega^2 \Delta q = - \frac{Q_n}{m_n},$$  \hspace{1cm} (8)

where $\omega = \sqrt{\frac{c(m + m_n)}{mm_n}}$ – the frequency of free oscillations of the system.

Substituting expressions (5) and (6) into equation (8), the required law of change of driving (braking) force is determined

$$Q_n = -m_n \gamma_p \left[ \omega^2 \left( 1 - \cos \frac{2\pi}{T_p} t \right) + \frac{4\pi^2}{T_p^2} \cos \frac{2\pi}{T_p} t \right].$$  \hspace{1cm} (9)

The definition of constant $\gamma_p$. Substituting expressions (5) and (9) into equation (7) is obtained

$$\dot{q}_1 = -\gamma_p \left[ \frac{4\pi^2}{T_p^2} \cos \frac{2\pi}{T_p} t + \omega_0^2 \left( 1 - \cos \frac{2\pi}{T_p} t \right) \right].$$  \hspace{1cm} (10)

where $\omega_0 = \sqrt{c/m}$ – the partial oscillation frequency of the mass of the lifting vessel.

After double integrating (10) the speed and coordinate the movements of the first mass are

$$\dot{q}_1 = -\gamma_p \left[ \frac{2\pi}{T_p} \sin \frac{2\pi}{T_p} t - \omega_0 \left( t - \frac{T_p}{2\pi} \sin \frac{2\pi}{T_p} t \right) \right] + C_1;$$  \hspace{1cm} (11)

$$q_1 = \gamma_p \cos \frac{2\pi}{T_p} t - \omega_0^2 \gamma_p \left( \frac{t^2}{2} + \frac{T_p^2}{4\pi^2} \cos \frac{2\pi}{T_p} t \right) + C_1 t + C_2.$$  \hspace{1cm} (12)

The constants of integration $C_1$ and $C_2$ are found using the initial conditions $q_1(0) = \dot{q}_1(0) = 0$

$$C_1 = 0; \quad C_2 = -\gamma_p \left( 1 - \frac{\omega_0^2 T_p^2}{4\pi^2} \right).$$  \hspace{1cm} (13)

Constant $\gamma_p$ is defined using the boundary conditions $\dot{q}_1(T_p) = V_{ycm}$, where $V_{ycm}$ is the steady-state value of the movement speed. Substituting the last condition in expression (11) is obtained

$$\gamma_p = \frac{V_{ycm}}{\omega_0 T_p}.$$  \hspace{1cm} (14)

Given expression (14) the laws of change of the driving (braking) forces and acceleration of the first mass are of the form

$$Q_n = \frac{(m + m_n) V_{ycm}}{T_p} \left( 1 - \cos \frac{2\pi}{T_p} t \right) + \frac{4\pi^2 m_n V_{ycm}}{\omega_0^2 T_p^3} \cos \frac{2\pi}{T_p} t;$$  \hspace{1cm} (15)

$$\dot{q}_1 = \frac{V_{ycm}}{T_p} \left( 1 - \cos \frac{2\pi}{T_p} t \right) + \frac{4\pi^2 V_{ycm}}{\omega_0^2 T_p^3} \cos \frac{2\pi}{T_p} t.$$  \hspace{1cm} (16)
Substituting in expression (16) instead $\ddot{q}_1$ of its limit value $\ddot{q}_1^{np} = a$, the expression for determining of the permissible value of acceleration time (deceleration) is

$$a = \frac{V_{ycm}}{T_p} \left[ 1 - \left(1 - \frac{4\pi^2}{\omega_0^2 T_p^2} \cos \frac{2\pi}{T_p} \right) \right]. \quad (17)$$

It is obvious that the maximum acceleration will occur in start and end moment of acceleration time ($t = 0$ and $t = T_p$). In this case expression (17) takes the form

$$a = \frac{4\pi^2 V_{ycm}}{\omega_0^2 T_p^3},$$

where permissible acceleration time

$$T_p \geq \frac{2\pi}{\omega_0}. \quad (18)$$

4. Discussion of results

A comparison of the effectiveness of the obtained compensating exposure with the optimal movement of two-mass system at a predetermined distance is held. We define the control providing a system movement from resting state

$$q_s(0) = \dot{q}_s(0) = \Delta q(0) = \Delta \dot{q}(0) \quad (19)$$

at a predetermined distance $q_0$, with vibration damping

$$q_s(T_p) = q_0; \quad \dot{q}_s(T_p) = \Delta q(T_p) = \Delta \dot{q}(T_p) = 0. \quad (20)$$

When $F_m = 0$ from expression (3) and (4) the acceleration is

$$\ddot{q} = -\omega^2 \Delta q + u(t), \quad (21)$$

where $u(t) = -Q/m$ – the driving force.

Assume that the driving force is limited in absolute value

$$|u(t)| \leq u_0. \quad (22)$$

Neglecting the influence of elastic oscillations on the actuator movement, the driving force according to expression (1) for the minimum time $T_p$ taking into account (22) is

$$u \approx \dot{q}_s. \quad (23)$$

Present expressions (21) and (23) in the form

$$\dot{z}_1 = z_2; \quad \dot{z}_2 = u; \quad \dot{z}_3 = z_4; \quad \dot{z}_4 = -\omega^2 z_3 + u, \quad (24)$$

where $z_1 = q_s; z_2 = \Delta q$.

The Hamilton function for the system (24) can be written

$$H = \psi_1 z_2 + \psi_2 u + \psi_3 z_4 + \psi_4 (u - \omega^2 z_3), \quad (25)$$

where $\psi_1, \psi_2, \psi_3, \psi_4$ are auxiliary variables that can be determined from the system of equations

$$\psi_1 = 0; \quad \psi_2 = -\psi_1; \quad \psi_3 = \psi_4; \quad \psi_4 = -\psi_3. \quad (26)$$

The optimum control, providing the maximum of the function (26), has the form

$$u = u_0 \text{ sgn}(\psi_2 - \psi_4). \quad (27)$$

According to expression (27) the optimal control $u(t)$ is a relay function taking values $\pm u_0$. The number of switching points and their positions are unknown. As an example we consider the control with one switching point $t = T_p/2$ of function $u(t)$.
\[
\begin{align*}
  u(t) = & u_0 \quad npu \quad 0 \leq t < T_p / 2; \\
  u(t) = & -u_0 \quad npu \quad T_p / 2 < t \leq T_p.
\end{align*}
\]  

Integrating the motion equation (24) with \( u = u_0 \) and initial conditions (19), we obtain
\[
q_1' = z_1 = \frac{u_0 t^2}{2}; \quad \dot{q}_1' = z_2 = u_0 t; \quad \Delta q = \frac{u_0}{\omega^2 (1 - \cos \omega \tau)}; \quad \Delta \dot{q} = \frac{u_0}{\omega \sin \omega \tau}.
\]

Similarly, we find the solution for \( u = -u_0 \) on the interval \( [T_p / 2, T_p] \) and the end conditions (20)
\[
q_1' = q_0 - \left( \frac{u_0 (T_p - t)^2}{2} \right); \quad \dot{q}_1' = u_0 (T_p - t); \quad \Delta q = \frac{u_0}{\omega^2 \cos \omega (T_p - t) - 1}; \quad \Delta \dot{q} = -\frac{u_0}{\omega \sin \omega (T_p - t)}.
\]

The found solutions expressions (29) and (30) must be continuous at \( t = T_p / 2 \). The continuity of the values of \( \dot{q}_s \) and \( \Delta \dot{q} \) takes place at any values of the coordinate \( q_0 \), the continuity condition for \( \dot{q}_s \) and \( \Delta \dot{q} \) is secured if
\[
q_0 = \frac{u_0 T_p^2}{4}; \quad \cos \left( \frac{\omega T_p}{2} \right) = 1.
\]

On the basis of expression (31) are
\[
T_p = \frac{4\pi}{\omega k}; \quad q_0 = \frac{4\pi^2 u_0 k}{\omega^2},
\]
where \( k = 1, 2, 3, \ldots \).

Thus, the control mode with one switching point satisfies the maximum principle only for magnitude values of the displacement \( q_0 \), defined by expression (31). For prove the optimality of the obtained control, it is sufficient to verify the existence of a nonzero vector of auxiliary variables, that providing \( H(T_p) \geq 0 \) and satisfy system (26). Solving this system, we define
\[
\psi_1 = -C; \quad \psi_2 = C \left( t - \frac{T_p}{2} \right); \quad \psi_3 = 0; \quad \psi_4 = 0,
\]
where \( C = const < 0 \).

Substituting expression (33) into (27), we have
\[
u = u_0 \operatorname{sgn} \left( \psi \left( t - \frac{T_p}{2} \right) \right).
\]

The calculation of the Hamilton function according to expression (25) leads to \( H(T_p) > 0 \).

We compare the acceleration time, expended for control with one switching point, with time expression (18), received for action, which provides the harmonic nature of the changes in the elastic coordinates. On the basis of expression (32) the minimum acceleration time can’t be less than the value
\[
T_{p_{\text{min}}} = 2\pi / \omega.
\]
Given \( \omega = \omega_0 \sqrt{\frac{m + m_n}{m_n}} < \omega_0 \), we have \( T_{p_{\text{min}}} < T_p \), defined from the expression (18).

In this case, the time difference grows with decreasing mass \( m_n \). Thus, management (34) provides
higher performance compared to the found control (15). However, management (34) requires instantaneous change in force, which makes difficult its practical feasibility using real drives.

The found the control law (15) is a software, independent on the regulator structure of the engine or the brakes. As a result, the possibility of problem solving, not only parametric but also structural synthesis of motion control system of the lifting installation appears, by using the expression of obtained time dependence on generalized coordinates. Using expressions (5) and (14), trigonometric function through elastic coordinate is expressed

$$\cos \frac{2\pi}{T} = 1 + \frac{\omega_0^2 T}{V_{ycm}} \Delta q. \quad (35)$$

Substituting expression (35) into (15), we obtain

$$Q_n(t) = Q_0 - c^* \Delta q, \quad (36)$$

where $c^* = \left(\frac{4\pi^2}{T_p^2} - \omega_0^2\right)m_n - c$; $Q_0 = \frac{4\pi^2 m_n V_{ycm}}{\omega_0^2 T_p^3}$ – the constant component of the force.

We find the control as a function of acceleration of elastic vibrations. After double differentiating expression (5) in time subject to (14), is obtained

$$\Delta \ddot{q} = -\frac{4\pi^2 V_{ycm}}{\omega_0^2 T_p^3} \cos \frac{2\pi}{T_p} t,$$

from which

$$\cos \frac{2\pi}{T_p} t = -\frac{\omega_0^2 T_p^3}{4\pi^2 V_{ycm}} \Delta \ddot{q}. \quad (37)$$

Substituting expression (37) into (15), we have

$$Q_n = Q_0' - m^* \Delta \ddot{q}, \quad (38)$$

where $Q_0' = \frac{(m + m_n)V_{ycm}}{T_p}$ – the constant component of the force;

$$m^* = \frac{\omega_0^2 T_p^2 + m_n (\omega_0^2 T_p^2 - 4\pi^2)}{4\pi^2}.$$

The coefficients $c^*$, $m^*$ in expressions (36) and (38) can be interpreted as an amplification coefficients of additional feedback on the elastic coordinate $\Delta q$ and its derivatives. Structural scheme of two-mass system with an additional feedback of the form (36) is shown in figure 3.

![Figure 3. Structural scheme of the system with additional feedback on the elastic coordinate](image-url)
In this diagram, the amplification coefficient of the additional feedback on the elastic coordinate are equal $k_p = -c^*$. Thus, the found control can be implemented using a closed-loop by the elastic deformation of the rope of control system providing for active damping of oscillations and the required level of dynamic load of mine lifting installation.

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