Incentive Mechanism Design for Federated Learning: Hedonic Game Approach

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ABSTRACT

Incentive mechanism design is crucial for enabling federated learning. We deal with clustering problem of agents contributing to federated learning setting. Assuming agents behave selfishly, we model their interaction as a stable coalition partition problem using hedonic games where agents and clusters are the players and coalitions, respectively. We address the following question: is there a family of hedonic games ensuring a Nash-stable coalition partition? We propose the Nash-stable set which determines the family of hedonic games possessing at least one Nash-stable partition, and analyze the conditions of non-emptiness of the Nash-stable set. Besides, we deal with the decentralized clustering. We formulate the problem as a non-cooperative game and prove the existence of a potential game.

KEYWORDS

Federated Learning, Hedonic Games, Optimal Clustering

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1 INTRODUCTION

Data protection is a major concern. If we do not trust someone withholding our data, we may opt for federated learning by privately developing intelligent systems to create privacy-preserving AI. Federated learning enables privacy-preserving machine learning in a decentralized way [13]. It is used in situations where data is distributed among different agents and training is impossible due to the difficulty to collect data centrally. All data is kept on device while a shared (global) learning model is trained in each device and aggregated (combined) centrally. Formally, we consider the following setting: i) data owner agents which locally trains the shared learning model, and ii) model aggregating entity (MAE) which combines learning model of its own with the agents. MAE and agents contribute to the same shared learning model. Federated learning has been identified as a distributed machine learning framework which sees rapid advances and broad adoption in next generation networking and edge systems [4, 7, 9, 10, 13, 15, 17, 20]. Obviously, the motivation to implement federated learning is to reduce the variance in a learned model by accessing more data.

A very crucial question is how would MAE motivate the agents to participate in federated learning. Designing the mechanism of agents’ incentives can be performed by utilizing various frameworks such game theory, auction theory, etc [15]. Any clustering among agents (players) being able to make strategic decisions becomes a coalition formation game when the players – for various individual reasons – may wish to belong to a relative small coalition rather than the grand coalition – the set of all players. Players’ moves from one to another coalition are governed by a set of rules. Basically, an agent (player) will move to a new coalition when it may obtain a better gain from this coalition. We shall not consider any permission requirements, which means that a player is always accepted by a coalition to which the player is willing to join. Based on those rules, the crucial question in the game context is how a stable partition exists. This is essential to enable federated learning.

We study the hedonic coalition formation game model of the agents and analyze the Nash stability [8]. A coalition formation game is called hedonic if each player’s preferences over partitions of players depend only on the members of his/her coalition. Finding a stable coalition partition is the main question in a coalition formation game. We refer to [1] discussing the stability concepts associated to hedonic conditions. In the sequel, we concentrate on the Nash stability. The definition of the Nash stability is quite simple: a partition of players is Nash stable whenever no player deviates from its coalition to another coalition in the partition.

In this work, we deal with the following problem: having coalitions associated with their gain, we seek the answer of how much the coalition gain be allocated to the players in order to obtain a stable coalition partition. Clearly, the fundamental question is to determine which gain allocation methods may ensure a Nash-stable partition. Note that the answer of this enables to find the family of hedonic games that possess at least one Nash-stable partition of players. We first propose the definition of the Nash-stable set which is the set of all possible allocation methods resulting in Nash-stable partitions. We show that additively separable and symmetric gain allocation always ensures Nash-stable partitions. Moreover, our work aims also at finding the partitions in a decentralized setting which basically corresponds to finding stable decentralized clustering. We model this problem as a non-cooperative game and show that such a game is a potential game.

A recent work that considers the clustering of agents in the form of hedonic games can be found in [6] where the authors study the agents decisions to participate in federated learning setting in case of a biased global model. In [14] a federated learning based privacy-preserving approach is proposed to facilitate collaborative machine learning among multiple model owners in mobile crowdsensing. Another work in [11] implements mechanism design and differential
We assume that agents may not agree to be in the same cluster. We consider a set of agents $i$, where multiple disjoint clusters may occur. In Figure 1, we illustrate with a deep reinforcement learning based approach to design the incentive mechanism and find the optimal trade-off between model training cost and communication delay. The work in [18] deals with a real reinforcement learning based approach to design the learning model. The parameters of learning model of MAE and agents contribute in the same global entity part. MAE and agents participate in the federated learning setting, and a incentive mechanism and find the optimal trade-off between model training cost and communication delay. The work in [18] deals with a deep reinforcement learning based approach to design the incentive mechanism and find the optimal trade-off between model training cost and communication delay. 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Coalition formation game

We can define the problem as a game where the agents strategically decide to which cluster to join; thus, we can change the language of problem formulation using game theoretic terms, i.e.

\[ \phi \in \mathbb{R} \text{ is the clustering gain of agent } i \text{ by joining cluster } S, \]  

and we set \( \phi^S_i = 0 \) for all \( i \in N \). Such a setting enables to deal with the clustering gains. Thus, the fundamental problem becomes to find clustering gains \( \phi \) so that the agents agree not to change their cluster.

Obviously, this is the stable clustering problem under selfishness where the agents strategically decide to which cluster to join; thus, we can define the problem as a coalition formation game. We then change the language of problem formulation using game theoretic terms, i.e.

\[ \text{agents } \rightarrow \text{ players} \]  
\[ \text{cluster } \rightarrow \text{ coalition} \]  
\[ \text{clustering } \rightarrow \text{ partition} \]

In the sequel, we deal with figuring out family of coalition formation games that ensure stable clusterings.

3.2 Clustering under Selfishness

As agents can behave selfishly, the fundamental question is to find clusters which are stable under selfishness. Let us consider that agent \( i \) shall get some monetary gain by joining cluster \( S \) as following:

\[ \phi^S_i = \pi_i + \phi_i^S \quad (10) \]

where \( \phi^S_i \in \mathbb{R} \) is the clustering gain of agent \( i \) by joining cluster \( S \), and we set \( \phi^S_i = 0 \) for all \( i \in N \). Such a setting enables to deal with the clustering gains. Thus, the fundamental problem becomes to find clustering gains \( \phi \) so that the agents agree not to change their cluster.

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3. HEDONIC GAME

A hedonic coalition formation game (in short, hedonic game) is given by a pair \( \langle N, \succ \rangle \), where \( \succ = (\succ_1, \succ_2, \ldots, \succ_n) \) denotes the preference profile, specifying for each player \( i \in N \) his preference relation \( \succ_i \), i.e. a reflexive, complete and transitive binary relation.

Given \( N \), called as coalition partition, and \( i, S, \pi(i) \) denotes the set \( S \in \Pi \) such that \( i \in S \). Moreover, \( \Pi \) is the set of all possible coalition partitions over \( N \). In its partition form, a coalition formation game is defined on the set \( N \) by associating a gain \( u(S) \) to each subset of any partition \( \Pi \) of \( N \). The gain of a set is independent of the other coalitions, and therefore, \( u(S) = u(S_T) \). The games of this form are more restrictive but present interesting properties to reach a stability. Practically speaking, this assumption means that the gain of a group is independent of the other players outside the group. Hedonic games fall into this category with an additional assumption:

Definition 3.1. A coalition formation game is hedonic if

- the gain of any player depends solely on the members of the coalition to which the player belongs, and
- the coalitions arise as a result of the preferences of the players over their possible coalitions’ set.

3.1 Preference Relation

The preference relation of a player can be defined over a preference function. We consider the case where the preference relation is chosen to be the gain allocated to the player in a coalition. Thus, player \( i \) prefers the coalition \( S \) to \( T \) iff,

\[ \phi^S_i \geq \phi^T_i \Leftrightarrow S \succ_i T. \quad (11) \]

3.2 The Nash Stability

The stability concepts for a hedonic game are various. In the literature, a hedonic game could be individually stable, Nash stable, core stable, strict core stable, Pareto optimal, strong Nash stable or, strict Nash stable. We refer to [1] for a thorough definition of these different stability concepts. In this paper, we are only interested in the Nash stability because the players do not cooperate to take their decisions jointly.

Definition 3.2 (Nash Stability). A partition of players is Nash-stable whenever no player has incentive to unilaterally change...
its coalition to another coalition in the partition which can be mathematically formulated as follows: partition $\Pi^\text{NS}$ is said to be Nash-stable if no player can benefit from moving from his coalition $S_{\Pi^\text{NS}}(i)$ to another existing coalition $T \in \Pi^\text{NS}$, i.e.:

$$S_{\Pi^\text{NS}}(i) \succeq_T T \cup i, \quad \forall T \in \Pi^\text{NS} \cup \emptyset; \forall i \in N. \quad (12)$$

which can be similarly defined over preference function as follows:

$$\phi^S_i(T) = \phi^S_i(T \cup i), \quad \forall T \in \Pi^\text{NS} \cup \emptyset; \forall i \in N. \quad (13)$$

Nash-stable partitions are immune to individual movements even when a player who wants to change does not need permission to join or leave an existing coalition [5].

**Remark 3.1.** Stability concepts being immune to individual deviation are Nash stability, individual stability, contractual individual stability. Nash stability is the strongest one. The notion of core stability has been used already in some models where immunity to coalition deviation is required [8].

**Remark 3.2.** In [2], the authors propose some set of axioms which are non-emptiness, symmetry pareto optimality, self-consistency, and they analyze the existence of any stability concept that can satisfy these axioms. It is proven that for any game $|N| > 2$, there does not exist any solution which satisfies these axioms.

### 3.3 Aggregated Learning Model Parameters

When a stable partition exists, then this means that all the players (agents) are agreed to participate to federation. As a result of this, MAE utilizes the following aggregation of learning model parameters:

$$\theta^F = w\theta_{\text{MAE}} + (1 - w) \sum_{i \in N} x_i m_i \theta_i$$

where $0 \leq w \leq 1$ is a weighting parameter showing how much MAE favors the aggregated learning model parameters of agents (players), $\theta_{\text{MAE}}$ shows the learning parameters of MAE’s local model. In summary, we have the following procedure:

```python
while Nash-stable partition $\Pi^\text{NS}$ exists
1. Player (agent) $i$ sends information of $\theta_i$, for all $i \in N$
2. MAE calculates aggregated learning model parameters $\theta^F$
end
```

Given $\theta^F$, the expected value of loss function in federation can be calculated as following:

$$\mathbb{E}_x [L(\theta^F)] = \sum_{x \in X} \mathbb{E}_x [L(\theta^F; x)] = (\text{Jensen’s inequality})$$

Note that calculating $\mathbb{E}_x [L(\theta^F)]$ may be more difficult than $\mathbb{E}_x [L(\theta^F; x)]$. Therefore, it can be also an option to define the gain of a cluster using $\mathbb{E}_x [\theta^F; x]$ in eq. (7).

### 4 THE NASH-STABLE SET

As the gain $u$ associated with all possible coalitions are known, we are interested in finding a gain distribution to ensure Nash stability. We thus define an allocation method $\phi \in \mathbb{R}^N$ where $k = n^2 - 1$ as following:

$$\phi = \{\phi^S_i : \forall i \in S, \forall S \in 2^N\} \quad (17)$$

which directly sets up a preference profile. The set of all possible allocation methods is denoted by $F \subset \mathbb{R}^N$. We define the mapping $M$, which for any allocation method $\phi$, it finds corresponding all possible Nash-stable partitions, i.e. $M(\phi) \subset F$.

We define the Nash-stable set which includes all those allocation methods that build the following set:

$$M_{\text{stable}} = \{\phi \in \mathbb{R}^N : \exists \Pi \in M(\phi) | \phi^S_i(T) \geq T \cup i, \quad \forall T \in \Pi \cup \emptyset; \forall i \in N\}. \quad (18)$$

Essentially, the Nash-stable set includes the family of hedonic games, each one having a different preference profile that derives from a different allocation method. Thus, before finding a Nash-stable partition, we need to find the hedonic game (i.e., an allocation method) for which a Nash-stable partition exists.

Let us define the set of constraints stemming from the preference function in order to check if the Nash-stable set is non-empty. Due to the gain bound, for any allocation method $\phi$, we have

$$\sum_{i \in S} (\pi_i + \phi_i^S) \leq u(S), \quad \forall S \in 2^N$$

called as *budget balanced* gain allocation which further can be given by

$$\sum_{i \in S} f \left( \frac{p_i}{\sum_{j \in S} f(\theta_j)} \right) + \sum_{i \in S} \phi_i^S \leq f \left( \frac{1}{\sum_{j \in S} f(\theta_j)} \right) - c(S), \quad \forall S \in 2^N. \quad (19)$$

For simplicity, let us define marginal gain as following:

$$\Lambda_\theta(S) = \left\{ f \left( \frac{1}{\sum_{j \in S} f(\theta_j)} \right) - c(S) - \sum_{i \in S} f \left( \frac{p_i}{\sum_{j \in S} f(\theta_j)} \right), \quad \forall S \in 2^N \setminus \emptyset \right\}, \quad (20)$$

which results in the following constraints:

$$\mathcal{C}_\theta^1(\phi) := \left\{ \left. \sum_{i \in S} \phi_i^S \right| \Lambda_\theta(S) \subseteq \mathbb{R}^N \right\}. \quad (21)$$

which are the constraints that stem from budgeted balancedness. On the other hand, for any $\phi$, the constraints that ensure the Nash stability are given by

$$\mathcal{C}_\theta^2(\phi) := \left\{ \exists \Pi \in M(\phi) | \phi^S_i(T) \geq \phi^S_{T \cup i}, \forall T \in \Pi \cup \emptyset; \forall i \in N \right\}. \quad (22)$$

Based on these two constraints represented by $\mathcal{C}_\theta^1(\phi)$ and $\mathcal{C}_\theta^2(\phi)$, we can define the Nash-stable set.

$$M_{\text{stable}}(\theta) = \{\phi \in \mathbb{R}^N : \mathcal{C}_\theta^1(\phi) \text{ and } \mathcal{C}_\theta^2(\phi)\}. \quad (23)$$

Then, the non-emptiness of the Nash-stable set is crucial. The theorem below states the necessary conditions about the non-emptiness of the Nash-stable set:
We then define a method to generate additively separable and symmetric preferences: $S_{ij}$ and $S_{ji}$ for all possible $i,j$. The meaning of $S_{ij}$ is the mutual gain of player $i$ and $j$ whenever the preference set $S$ contains both $i$ and $j$. We, in fact, model the problem of finding a Nash-stable partition as a non-cooperative game. We propose to formulate an optimization problem for finding the values of $v(i)$, for all $i$. The constraints that define the Nash-stable set are then given by

$$\mathcal{N} = \left\{ v \in \mathbb{R}^{\mathcal{V}(N)} : \sum_{(i,j) \in \mathcal{V}(S)} v(i,j) \leq \frac{\Delta_{\theta}(S)}{2}, \forall S \in 2^N \right\}$$

(26)

Finding the values of $v(i,j)$ in eq. (26) satisfying $v(i,j)$ conditions can be done straightforward. However, we propose to formulate as an optimization problem for finding the values of $v(i,j)$. A feasible solution of the following linear program guarantees the non-emptiness of $\mathcal{A}_{\text{stable}}(\theta)$:

$$\max \sum_{(i,j) \in \mathcal{V}(S)} v(i,j) \text{ subject to } \sum_{(i,j) \in \mathcal{V}(S)} v(i,j) \leq \frac{\Delta_{\theta}(S)}{2}, \forall S \in 2^N,$$

(27)

where note that any feasible solution $v^*$ is upper bounded by $\sum_{(i,j) \in \mathcal{V}(S)} v^*(i,j) \leq \Delta_{\theta}(S)/2$. Furthermore, the coalition partition that stems from $v^*$ is given by $\Pi^{NS} \in M(v^*)$ which is Nash-stable.

5 DECENTRALIZED CLUSTERING

In this section, we study finding a Nash-stable partition in a decentralized setting which corresponds to finding stable decentralized clustering. We, in fact, model the problem of finding a Nash-stable partition as a non-cooperative game.
A hedonic coalition formation game is equivalent to a non-cooperative game. Denote as $\Sigma$ the set of strategies. We assume that the number of strategies is equal to the number of players, i.e., $|\Sigma| = n$. This is sufficient to represent all possible choices. Indeed, the players that select the same strategy are interpreted as a coalition. For example, if every player chooses different strategies, then this corresponds to the coalition partition comprised of singletons.

Consider the best-reply dynamics where in a particular step, only one player chooses its best strategy. A strategy tuple is represented as $\sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_n\}$, where $\sigma_i \in \Sigma$ is the strategy of player $i$. In every step, only one dimension is changed in $\sigma$. We further define

$$S_\sigma(i) = \{j \in N : \sigma_i = \sigma_j\}$$

and

$$\Pi(\sigma) = \{S_\sigma(i), \forall i \in N\}$$

the set of players that share the same strategy with player $i$ and partition of players with respect to strategy tuple $\sigma$. Thus, note that $\cup_{i \in N} S_\sigma(i) = N$ for each step. The gain of player $i$ in case of strategy tuple $\sigma$ is represented by $\phi_i(\sigma)$ which verifies the following relation:

$$\phi_i(\sigma) \geq \phi_i(\sigma') \iff S_\sigma(i) \supseteq_i S_{\sigma'}(i).$$

Any sequence of strategy tuple in which each strategy tuple differs from the preceding one in only one coordinate is called a path, and a unique deviator in each step strictly increases the gain he receives is an improvement path. Obviously, any maximal improvement path which is an improvement path that can not be extended is terminated by stability.

### 5.1 Equilibrium Analysis

The Nash equilibrium is defined as following:

$$\sigma^*_i \in \arg \max_{\sigma_i \in \Sigma} \phi_i(\sigma_i, \sigma_{-i}), \quad \forall i \in N.$$  

(31)

essentially corresponding to a Nash-stable partition in the original hedonic game, which is given by

$$S_{\sigma^*_i}(i) = \{j \in N : \sigma_i^* = \sigma_j\}, \quad \forall i \in N$$

$$\Pi^* = \{S_{\sigma^*_i}(i), \forall i \in N\}.$$  

(32)

In the sequel, we prove that the additively separable and symmetric gains result in a potential game where all the players have incentive to change their strategy according to a single global function called as potential function.

**Theorem 5.1.** Any additively separable and symmetric gain results in a potential game with potential function:

$$P_\nu(\sigma) = \sum_{S \in \Pi(\sigma)} \sum_{(i,j) \in V(S)} v(i, j).$$

(33)

**Proof.** A non-cooperative game is a potential game whenever there exists a function $P_\nu$ such that:

$$P_\nu(\sigma_i, \sigma_{-i}) - P_\nu(\sigma'_i, \sigma_{-i}) = \phi(\sigma_i, \sigma_{-i}) - \phi(\sigma'_i, \sigma_{-i})$$

where $\sigma_i, \sigma_{-i}$ is $\sigma$ and $\sigma_{-i}$ shows the strategies of the players other than $i$. This means that when player $i$ switches from strategy $\sigma_i$ to $\sigma'_i$ the difference of its gain can be given by the difference of a function $P$. We choose the following potential function:

$$P_\nu(\sigma) = \sum_{S \in \Pi(\sigma)} \sum_{(i,j) \in V(S)} v(i, j).$$

(34)

Let us denote as $i \in S$ and $i \notin S'$ the coalitions when player $i$ switches from strategy $\sigma_i$ to $\sigma'_i$, respectively. Potential function is given as following before and after switching

$$P_\nu(\sigma_i, \sigma_{-i}) = \sum_{(i,j) \in V(S)} v(i, j) + \sum_{(k,j) \in V(S')} v(k, j)$$

$$+ \sum_{T \in \Pi(\sigma) \setminus \{S, S'\}} \sum_{(k,j) \in V(T)} v(k, j)$$

where note that we have $S \rightarrow S \setminus i$ and $S' \rightarrow S' \cup i$ after switching. Thus, we have

$$\phi(\sigma_i, \sigma_{-i}) - \phi(\sigma'_i, \sigma_{-i}) = \sum_{(i,j) \in V(S)} v(i, j) + \sum_{(k,j) \in V(S')} v(k, j)$$

$$- \sum_{(k,j) \in V(S')} v(k, j) - \sum_{j \in S \setminus i} v(j, i)$$

$$\sum_{j \in S \cup i} v(j, i)$$

which concludes the proof that $P_\nu(\sigma_i, \sigma_{-i}) - P_\nu(\sigma'_i, \sigma_{-i}) = \phi(\sigma_i, \sigma_{-i}) - \phi(\sigma'_i, \sigma_{-i}).$ 

□

In a potential game, a Nash equilibrium shall result in an optimum in potential $P_\nu$. Therefore, $\sigma^* \in \arg \max_{\sigma} P_\nu(\sigma)$ corresponds to a coalition partition $\Pi(\sigma^*) \in M(\nu)$ which is Nash-stable.

### 6 CONCLUSIONS

We analyzed stable clustering problem in federated learning setting. Clusters are made up of the agents contributing to federated learning. We considered that every agent is better off when switching from one cluster to another one. We modeled the decisions of agents in the framework hedonic games which is a widely used cooperative game model for this type of problems. A fundamental question in hedonic games is to analyze the conditions how stable coalition partitions can occur. We studied the existence of stable coalition partitions by introducing the Nash-stable set, and analyzed the existence of decentralized coalition partitions.

As future work, it may be interesting to do stability analysis for different stability notions, to study other types of preference profiles ensuring Nash stability. On the other hand, it is essential
to do experiments with real data and specific learning models as well as realistic gain and communication cost functions.

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