The 1986 CODATA Recommended Values Of the Fundamental Physical Constants

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This paper gives the values of the basic constants and conversion factors of physics and chemistry resulting from the 1986 least-squares adjustment of the fundamental physical constants as recently published by the CODATA Task Group on Fundamental Constants and as recommended for international use by CODATA. The new, 1986 CODATA set of recommended values replaces its predecessor published by the Task Group and recommended for international use by CODATA in 1973.

Key words: CODATA; conversion factors; fundamental physical constants; least-squares adjustments; recommended values; Task Group on Fundamental Constants.

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CODATA (Committee on Data for Science and Technology) has recently published a report of the CODATA Task Group on Fundamental Constants prepared by the authors [1] under the auspices and guidance of the Task Group. The report summarizes the 1986 least-squares adjustment of the fundamental physical constants and gives a set of self-consistent values for the basic constants and conversion factors of physics and chemistry derived from that adjustment. Recommended for international use by CODATA, this 1986 set of values is reprinted here for the convenience of the many readers of the Journal of Research of the National Bureau of Standards and to assist in its dissemination throughout the scientific and technological communities. The 1986 CODATA set entirely replaces its immediate predecessor, that recommended for international use by CODATA in 1973. This set was based on the 1973 least-squares adjustment of the fundamental physical constants which was also carried out by the authors under the auspices and guidance of the Task Group [2,3].

As in previous least-squares adjustments of the constants [3,4,5], the data for the 1986 adjustment were divided into two groups: auxiliary constants and stochastic input data. Examples of the 1986 auxiliary constants are the speed of light in vacuum $c=299792458$ m/s; the permittivity of vacuum $\varepsilon_0=4\pi \times 10^{-7}$ N/A$^2$; the Rydberg constant for infinite mass $R_\infty$; and the quantity $E=483594.0 \times 10^9$ Hz/V which is equal numerically to the value of the Josephson frequency-voltage ratio $2e/h$ (e is the elementary charge and $h$ is the Planck constant) adopted in 1972 by the Consultative Committee on Electricity of the International Committee of

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1CODATA was established in 1966 as an interdisciplinary committee of the International Council of Scientific Unions. It seeks to improve the compilation, critical evaluation, storage, and retrieval of data of importance to science and technology. Dr. David R. Lide, chief of the NBS Office of Standard Reference Data, is the current President of CODATA.

2Figures in brackets indicate literature references.
Weights and Measures for defining laboratory representations of the volt [6, 7]. Quantities in this category are either defined constants such as $c$, $\mu_0$, and $E$ with no uncertainty, or constants such as $R_\infty$ with assigned uncertainties sufficiently small in comparison with the uncertainties assigned the stochastic input data with which they are associated in the adjustment that they can be taken as exact (i.e., their values are not subject to adjustment in contrast to the stochastic data). In the 1986 adjustment the uncertainty of each auxiliary constant was no greater than 0.02 parts-per-million or ppm. In contrast, the uncertainties assigned the 38 items of stochastic input data considered in the 1986 adjustment were in the range 0.065 to 9.7 ppm. (The 38 items were of 12 distinct types with the number of items of each type ranging from one to six.) Examples of such data are measurements of the proton gyromagnetic ratio $\gamma_p$ (uncertainty in the range 0.24 to 5.4 ppm), the molar volume of silicon $M\,(\text{Si})/\rho\,(\text{Si})$ (1.15 ppm), and the quantized Hall resistance $R_H = h/e^2$ (0.12 to 0.22 ppm).

Because new results which can influence a least-squares adjustment of the constants are reported continually, it is always difficult to choose an optimal time at which to carry out a new adjustment and to revise the recommended values of the constants. In the present case, all data available up to 1 January 1986 were considered for inclusion, with the recognition that any additional changes to the 1973 recommended values that might result by taking into account more recent data would be much less than the changes resulting from the data available prior to that date.

Each of the 38 items of stochastic data are expressed (using the auxiliary constants as necessary) in terms of five quantities that serve as the "unknowns" or variables of the 1986 adjustment. These are $\alpha^{-1}$, the inverse fine-structure constant; $K_V$, a dimensionless quantity relating the SI (International System of Units) volt $V$ to the unit of voltage $V_{76\text{-B1}}$ maintained at the International Bureau of Weights and Measures (BIPM) using a value of the Josephson frequency-voltage ratio using $e$, $\hbar$, $m_e$, $N_A$, and $F$, and the Josephson frequency-voltage ratio $2e/h$:

$$e = \frac{E}{K_V} \quad \hbar = \frac{h}{e^2} \quad m_e = \frac{m}{e^2} \quad N_A = \frac{N}{e^2} \quad F = \frac{F}{e^2} \quad \frac{2e}{h} = \frac{2e}{h}$$

of the proton. "Best" values in the least-squares sense for these five quantities, with their variances and covariances, are thus the immediate output of the adjustment.

After a thorough analysis using a number of least-squares algorithms, the initial group of 38 items of stochastic input data was reduced to 22 items by deleting those that were either highly inconsistent with the remaining data or had assigned uncertainties so large that they carried negligible weight. The adjusted values of the five unknowns, and hence all the other 1986 recommended values that were subsequently derived from them (with the aid of the auxiliary constants), are therefore based on a least-squares adjustment with 17 degrees of freedom.

The 1986 adjustment represents a major advance over its 1973 counterpart; the uncertainties of the recommended values have been reduced by roughly an order of magnitude due to the enormous advances made throughout the precision measurement-fundamental constants field in the last dozen years. This can be seen from the following comparison of the 1973 and 1986 recommended values for the inverse fine-structure constant $\alpha^{-1}$, the elementary charge $e$, the Planck constant $\hbar$, the electron mass $m_e$, the Avogadro constant $N_A$, the proton electron mass ratio $m_p/m_e$, the Faraday constant $F$, and the Josephson frequency-voltage ratio $2e/h$:

| Quantity | 1973 | 1986 | Change in 1973 recommended value in ppm resulting from 1986 adjustment |
|----------|------|------|-------------------------------------------------------------------------|
| $\alpha^{-1}$ | 0.82 | 0.045 | -0.37                                                                   |
| $e$       | 2.90 | 0.30 | -7.4                                                                    |
| $\hbar$   | 5.40 | 0.60 | -15.2                                                                   |
| $m_e$     | 5.10 | 0.59 | -15.8                                                                   |
| $N_A$     | 5.10 | 0.59 | +15.2                                                                   |
| $m_p/m_e$ | 0.38 | 0.020 | +0.64                                                                   |
| $F$       | 2.80 | 0.30 | +7.8                                                                    |
| $2e/h$    | 2.60 | 0.30 | +7.8                                                                    |

It is also clear from this comparison that unexpectedly large changes have occurred in the 1973 recommended values of a number of these constants (i.e., a change which is large relative to the uncertainty assigned the 1973 value). These changes are a direct consequence of the 7.8 ppm decrease from 1973 to 1986 in the quantity $K_V$, and the high correlation between $K_V$ and the calculated values of $e$, $\hbar$, $m_e$, $N_A$, and $F$. Since $2e/h = E/K_V$, the 1986 value of $K_V$ also implies that the value of the Josephson frequency-voltage ratio adopted by the Consultative Committee on Electricity in 1972, which was believed to be consistent with the SI value and which most national standards laborato-

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3Throughout, all uncertainties are one standard deviation estimates.
ries adopted to define and maintain their laboratory unit of voltage, is actually 7.8 ppm smaller than the SI value. This unsatisfactory situation should be rectified in the near future [8,9].

The large change in $K_v$ and hence in many other quantities between 1973 and 1986 would have been avoided if two determinations of $F$ which seemed to be discrepant with the remaining data had not been deleted in the 1973 adjustment. In retrospect, the disagreement was comparatively mild. In view of this experience it is important to recognize that there are no similar disagreements in the 1986 adjustment; the measurements which were deleted were so discrepant that they obviously could not be correct, or of such low weight that if retained the adjusted values of the five unknowns would change negligibly. Thus, it is unlikely that any alternate evaluation of the data considered in the 1986 least-squares adjustment could lead to significant changes in the 1986 recommended values. Moreover, the quality of the 1986 data and its redundancy would seem to preclude future changes in the 1986 recommended values relative to their uncertainties comparable to the changes which occurred in the 1973 values.

The 1986 recommended values of the fundamental physical constants are given in five tables. Table 1 is an abbreviated list containing the quantities which should be of greatest interest to most users. Table 2 is a much more complete compilation.

### Table 1. Summary of the 1986 recommended values of the fundamental physical constants.

An abbreviated list of the fundamental constants of physics and chemistry based on a least-squares adjustment with 17 degrees of freedom. The digits in parentheses are the one-standard-deviation uncertainty in the last digits of the given value. Since the uncertainties of many of these entries are correlated, the full covariance matrix must be used in evaluating the uncertainties of quantities computed from them.

| Quantity | Symbol | Value | Units | Relative Uncertainty (ppm) |
|----------|--------|-------|-------|---------------------------|
| speed of light in vacuum | $c$ | 299792458 | m s$^{-1}$ | (exact) |
| permeability of vacuum | $\mu_o$ | $4\pi \times 10^{-7}$ | N A$^{-2}$ | (exact) |
| permittivity of vacuum | $\varepsilon_0$ | $1/\mu_0 c^2$ | $8.854 \, 187 \, 817 \ldots \times 10^{-12}$ F m$^{-1}$ | (exact) |
| Newtonian constant of gravitation | $G$ | 6.67259(85) | $10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$ | 128 |
| Planck constant | $h$ | 6.6260755(40) | $10^{-34}$ Js | 0.60 |
| $h/2\pi$ | | | | |
| Planck constant | $\hbar$ | 1.05457266(63) | $10^{-34}$ Js | 0.60 |
| elementary charge | $e$ | 1.60217733(49) | $10^{-19}$ C | 0.30 |
| magnetic flux quantum, $h/2e$ | $\Phi_0$ | 2.06783461(61) | $10^{-15}$ Wb | 0.30 |
| electron mass | $m_e$ | 9.1093897(54) | $10^{-31}$ kg | 0.59 |
| proton mass | $m_p$ | 1.6726231(10) | $10^{-27}$ kg | 0.59 |
| proton-electron mass ratio | $m_p/m_e$ | 1836.152701(37) | | 0.020 |
| fine-structure constant, $\mu_{e} c e^2/2h$ | $\alpha$ | 7.2973539(33) | $10^{-3}$ | 0.045 |
| inverse fine-structure constant | $\alpha^{-1}$ | 137.0359895(61) | | 0.045 |
| Rydberg constant, $m_e c \alpha^2/2h$ | $R_{\infty}$ | 10.973731.534(13) | m$^{-1}$ | 0.0012 |
| Avogadro constant | $N_A, L$ | 6.0221367(36) | $10^{23}$ mol$^{-1}$ | 0.59 |
| Faraday constant, $N_A e$ | $F$ | 96485.309(29) | C mol$^{-1}$ | 0.30 |
| molar gas constant | $R$ | 8.314510(70) | J mol$^{-1}$ K$^{-1}$ | 8.4 |
| Boltzmann constant, $R/N_A$ | $k$ | 1.380658(12) | $10^{-23}$ J K$^{-1}$ | 8.5 |
| Stefan–Boltzmann constant, $(\pi^2/60)k^4/h^3c^2$ | $\sigma$ | 5.67051(19) | $10^{-8}$ W m$^{-2}$ K$^{-4}$ | 34 |

Non-SI units used with SI

| electron volt, $(e/C)J = \{e\} J$ | eV | 1.60217733(49) | $10^{-19}$ J | 0.30 |
| (unified) atomic mass unit, $1 \, u = m_u = \frac{1}{12} m(12C)$ | u | 1.6605402(10) | $10^{-27}$ kg | 0.59 |
Table 2. The 1986 recommended values of the fundamental physical constants.

This list of the fundamental constants of physics and chemistry is based on a least-squares adjustment with 17 degrees of freedom. The digits in parentheses are the one-standard-deviation uncertainty in the last digits of the given value. Since the uncertainties of many of these entries are correlated, the full covariance matrix must be used in evaluating the uncertainties of quantities computed from them.

| Quantity | Symbol | Value | Units | Relative Uncertainty (ppm) |
|----------|--------|-------|-------|-----------------------------|
| speed of light in vacuum | $c$ | 299792458 | m s$^{-1}$ | (exact) |
| permeability of vacuum | $\mu_0$ | $4\pi \times 10^{-7}$ | N A$^{-2}$ | (exact) |
| permittivity of vacuum | $\varepsilon_0$ | $1/\mu_0 c^2$ | | |
| Newtonian constant of gravitation | $G$ | 6.67259(85) | $10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$ | 128 |
| Planck constant | $h$ | 6.6260775(40) | 10$^{-34}$ J s | 0.60 |
| in electron volts, $h/\{e\}$ | | 4.1356692(12) | 10$^{-15}$ eV s | 0.30 |
| $h/2\pi$ | | 1.05457266(63) | 10$^{-34}$ J s | 0.60 |
| Planck mass, $(hc/G)^{1/2}$ | $m_p$ | 2.17671(14) | 10$^{-8}$ kg | 64 |
| Planck length, $h/m_pc = (hc/c^2)^{1/2}$ | $l_p$ | 1.61605(10) | 10$^{-35}$ m | 64 |
| Planck time, $l_p/c = (hc/c^5)^{1/2}$ | $t_p$ | 5.39056(34) | 10$^{-44}$ s | 64 |

### GENERAL CONSTANTS

#### Universal Constants

| Quantity | Symbol | Value | Units | Relative Uncertainty (ppm) |
|----------|--------|-------|-------|-----------------------------|
| elementary charge | $e$ | 1.60217733(49) | $10^{-19}$ C | 0.30 |
| magnetic flux quantum, $h/2e$ | $e/h$ | 2.41798836(72) | $10^{14}$ A J$^{-1}$ | 0.30 |
| Josephson frequency–voltage ratio | $2e/h$ | 4.8359767(14) | $10^{14}$ Hz V$^{-1}$ | 0.30 |
| quantized Hall conductance | $e^2/h$ | 3.87404614(17) | $10^{-5}$ S | 0.045 |
| quantized Hall resistance, $h/e^2 = \mu_o c/2\alpha$ | $R_H$ | 25812.8056(12) | $\Omega$ | 0.045 |
| Bohr magneton, $e\hbar/2m_e$ | $\mu_B$ | 9.2740154(31) | $10^{-24}$ JT$^{-1}$ | 0.34 |
| in electron volts, $\mu_B/\{e\}$ | | 5.78838263(52) | $10^{-5}$ eV T$^{-1}$ | 0.089 |
| in hertz, $\mu_B/h$ | | 1.39962148(42) | $10^{10}$ Hz T$^{-1}$ | 0.30 |
| in wavenumbers, $\mu_B/hc$ | | 46.686437(14) | m$^{-1}$ T$^{-1}$ | 0.30 |
| in kelvins, $\mu_B/k$ | | 0.6717099(57) | KT$^{-1}$ | 8.5 |
| nuclear magneton, $e\hbar/2m_p$ | $\mu_N$ | 5.0507866(17) | $10^{-27}$ J T$^{-1}$ | 0.34 |
| in electron volts, $\mu_N/\{e\}$ | | 3.15251662(28) | $10^{-6}$ eV T$^{-1}$ | 0.089 |
| in hertz, $\mu_N/h$ | | 7.62259142(23) | MHz T$^{-1}$ | 0.30 |
| in wavenumbers, $\mu_N/hc$ | | 2.54262281(77) | m$^{-1}$ T$^{-1}$ | 0.30 |
| in kelvins, $\mu_N/k$ | | 3.658246(31) | $10^{-4}$ KT$^{-1}$ | 8.5 |
Table 2. The 1986 recommended values of the fundamental physical constants (continued).

| Quantity | Symbol  | Value                  | Units       | Relative Uncertainty (ppm) |
|----------|---------|------------------------|-------------|-----------------------------|
| **ATOMIC CONSTANTS** |         |                        |             |                             |
| fine-structure constant, \(\mu e^2/2h\) | \(\alpha\) | 7.297 353 08(33)     | \(10^{-3}\) | 0.045                       |
| inverse fine-structure constant | \(\alpha^{-1}\) | 137.035 9895(61)      | \(10^{-3}\) | 0.045                       |
| Rydberg constant, \(m_e c a^2/2h\) | \(R_\infty\) | 10 973 731.534(13)   | \(m^{-1}\) | 0.0012                      |
| in hertz, \(R_\infty c\) |            | 3.289 841 9499(39)   | \(10^{15}\) Hz | 0.0012                      |
| in joules, \(R_\infty hc\) |            | 2.179 8741(13)      | \(10^{-18}\) J | 0.60                        |
| in eV, \(R_\infty hc/\{e\}\) |            | 13.605 6981(40)     | eV           | 0.30                        |
| Bohr radius, \(\alpha/4\pi R_\infty\) | \(a_0\) | 0.529 177 249(24)    | \(10^{-10}\) m | 0.045                       |
| Hartree energy, \(e^2/4\pi e_0 a_0 = 2 R_\infty hc\) | \(E_h\) | 4.359 7482(26)     | \(10^{-18}\) J | 0.60                        |
| in eV, \(E_h/\{e\}\) |            | 27.211 3961(81)      | eV           | 0.30                        |
| quantum of circulation | \(h/2m_e\) | 3.636 94807(33)     | \(10^{-4}\) m^2 s^{-1} | 0.089                       |
| | \(h/m_e\) | 7.273 89614(65)      | \(10^{-4}\) m^2 s^{-1} | 0.089                       |
| **Electron** |         |                        |             |                             |
| electron mass | \(m_e\) | 9.109 3897(54)      | \(10^{-31}\) kg | 0.59                        |
| in electron volts, \(m_e c^2/\{e\}\) |            | 5.485 799 03(13)    | \(10^{-4}\) u | 0.023                       |
| electron–muon mass ratio | \(m_e/m_\mu\) | 4.836 332 18(71)    | \(10^{-3}\) | 0.15                        |
| electron–proton mass ratio | \(m_e/m_p\) | 5.446 170 13(11)    | \(10^{-4}\) | 0.020                       |
| electron–deuteron mass ratio | \(m_e/m_d\) | 2.724 437 07(6)     | \(10^{-4}\) | 0.020                       |
| electron–\(\alpha\)-particle mass ratio | \(m_e/m_\alpha\) | 1.370 933 54(3)     | \(10^{-4}\) | 0.021                       |
| electron specific charge | \(-e/m_e\) | -1.758 819 62(53)   | \(10^{11}\) C kg^{-1} | 0.30                        |
| electron molar mass | \(M(e), M_e\) | 5.485 799 03(13)    | \(10^{-7}\) kg/mol | 0.023                       |
| Compton wavelength, \(h/m_e c\) | \(\lambda_c\) | 2.426 310 58(22)    | \(10^{-12}\) m | 0.089                       |
| \(\lambda_c/2\pi = a_0 a_e = \alpha^2/4\pi R_\infty\) | \(\lambda_c\) | 3.86 159 323(35)    | \(10^{-13}\) m | 0.089                       |
| classical electron radius, \(\alpha^2 a_0\) | \(r_e\) | 2.817 940 92(38)    | \(10^{-15}\) m | 0.13                        |
| Thomson cross section, \((8\pi/3)r_e^2\) | \(\sigma_e\) | 0.665 246 16(18)    | \(10^{-28}\) m^2 | 0.27                        |
| electron magnetic moment | \(\mu_e\) | 928.47701(31)       | \(10^{-6}\) J T^{-1} | 0.34                        |
| in Bohr magnetons | \(\mu_e/\mu_B\) | 1.001 159 652 193(10) | \(1x10^{-5}\) |                             |
| in nuclear magnetons | \(\mu_e/\mu_N\) | 1838.282 000(37) |                             | 0.020                       |
| electron magnetic moment anomaly, \(\mu_e/\mu_B - 1\) | \(a_e\) | 1.159 652 193(10)   | \(10^{-3}\) | 0.0086                       |
| electron g-factor, \(2(1 + a_e)\) | \(\gamma_e\) | 2.002 319 304 386(20) | \(1x10^{-5}\) |                             |
| electron–muon magnetic moment ratio | \(\mu_e/\mu_\mu\) | 206.766 967(30) |                             | 0.15                        |
| electron–proton magnetic moment ratio | \(\mu_e/\mu_p\) | 658.210 6881(66) |                             | 0.010                       |
| **Muon** |         |                        |             |                             |
| muon mass | \(m_\mu\) | 1.883 5327(11)       | \(10^{-28}\) kg | 0.61                        |
| in electron volts, \(m_\mu c^2/\{e\}\) |            | 0.113 428 913(17)   | u                        | 0.15                        |
| muon–electron mass ratio | \(m_\mu/m_e\) | 206.768 262(30)     | \(10^{-4}\) kg/mol | 0.15                        |
| muon molar mass | \(M(\mu), M_\mu\) | 1.134 289 13(17) | \(10^{-4}\) kg/mol | 0.15                        |
| muon magnetic moment | \(\mu_\mu\) | 4.490 4514(15)     | \(10^{-26}\) J T^{-1} | 0.33                        |
| in Bohr magnetons, | \(\mu_\mu/\mu_B\) | 4.841 970 97(71) | \(10^{-3}\) | 0.15                        |
| in nuclear magnetons, | \(\mu_\mu/\mu_N\) | 8.890 5981(13) |                             | 0.15                        |
| Quantity | Symbol | Value | Units | Relative Uncertainty (ppm) |
|----------|--------|-------|-------|---------------------------|
| muon magnetic moment anomaly, \([\mu_\mu/(e\hbar/2m_\mu)] - 1\) | \(a_\mu\) | 1.1659230(84) | 10^{-3} | 7.2 |
| muon g-factor, 2(1 + \(a_\mu\)) | \(g_\mu\) | 2.002331846(17) | | 0.0084 |
| muon–proton magnetic moment ratio | \(\mu_\mu/\mu_p\) | 3.18334547(47) | | 0.15 |

Proton

| Quantity | Symbol | Value | Units | Relative Uncertainty (ppm) |
|----------|--------|-------|-------|---------------------------|
| proton mass | \(m_p\) | 1.6726231(10) | 10^{-27} kg | 0.59 |
| in electron volts, \(m_p e^2/\{e\}\) | | 1.007276470(12) | u | 0.012 |
| proton–electron mass ratio | \(m_p/m_e\) | 1836.152701(37) | | 0.020 |
| proton–muon mass ratio | \(m_p/m_\mu\) | 8.8802444(13) | | 0.15 |
| proton specific charge | \(e/m_p\) | 9.5788309(29) | 10^7 C kg^{-1} | 0.30 |
| proton molar mass | \(M(p), M_p\) | 1.007276470(12) | 10^{-3} kg/mol | 0.012 |
| proton Compton wavelength, \(h/m_p c\) | \(\lambda_{C,p}/2\pi\) | 2.1038937(19) | 10^{-16} m | 0.089 |
| proton magnetic moment | \(\mu_p\) | 1.41060761(47) | 10^{-26} J T^{-1} | 0.34 |
| in Bohr magnetons | \(\mu_p/\mu_B\) | 1.521032202(15) | 10^{-3} | 0.010 |
| in nuclear magnetons | \(\mu_p/\mu_N\) | 2.792847386(63) | | 0.023 |
| diamagnetic shielding correction for protons in pure water, spherical sample, 25 °C, 1 - \(\mu_p/\mu_p\) | \(\sigma_{H_2O}\) | 25.689(15) | 10^{-6} | - |
| shielded proton moment | \(\mu_p^I\) | 1.41057138(47) | 10^{-26} J T^{-1} | 0.34 |
| (H₂O, sph., 25 °C) in Bohr magnetons | \(\mu_p^I/\mu_B\) | 1.520993129(17) | 10^{-3} | 0.011 |
| in nuclear magnetons | \(\mu_p^I/\mu_N\) | 2.792775642(64) | | 0.023 |
| proton gyromagnetic ratio | \(\gamma_p\) | 26752.2128(81) | 10^{4} s^{-1} T^{-1} | 0.30 |
| | \(\gamma_p/2\pi\) | 42.577469(13) | MHz T^{-1} | 0.30 |
| uncorrected (H₂O, sph., 25 °C) | \(\gamma_p^I\) | 26751.5255(81) | 10^{4} s^{-1} T^{-1} | 0.30 |
| | \(\gamma_p^I/2\pi\) | 42.576375(13) | MHz T^{-1} | 0.30 |

Neutron

| Quantity | Symbol | Value | Units | Relative Uncertainty (ppm) |
|----------|--------|-------|-------|---------------------------|
| neutron mass | \(m_n\) | 1.6749286(10) | 10^{-27} kg | 0.59 |
| in electron volts, \(m_n e^2/\{e\}\) | | 1.008664904(14) | u | 0.014 |
| neutron–electron mass ratio | \(m_n/m_e\) | 1838.683662(40) | | 0.022 |
| neutron–proton mass ratio | \(m_n/m_p\) | 1.001378404(9) | | 0.009 |
| neutron molar mass | \(M(n), M_n\) | 1.008664904(14) | 10^{-3} kg/mol | 0.014 |
| neutron Compton wavelength, \(h/m_n c\) | \(\lambda_{C,n}/2\pi\) | 1.31959110(12) | 10^{-15} m | 0.089 |
| neutron magnetic moment * | \(\mu_n\) | 0.96623707(40) | 10^{-26} J T^{-1} | 0.41 |
| in Bohr magnetons | \(\mu_n/\mu_B\) | 1.04187563(25) | 10^{-3} | 0.24 |
| in nuclear magnetons | \(\mu_n/\mu_N\) | 1.91304275(45) | | 0.24 |
| neutron–electron magnetic moment ratio | \(\mu_n/\mu_e\) | 1.04066882(25) | 10^{-3} | 0.24 |
| neutron–proton magnetic moment ratio | \(\mu_n/\mu_p\) | 0.68497934(16) | | 0.24 |
Table 2. The 1986 recommended values of the fundamental physical constants (continued).

| Quantity                                      | Symbol | Value                     | Units          | Relative Uncertainty (ppm) |
|-----------------------------------------------|--------|---------------------------|----------------|---------------------------|
| Deuteron                                      |        |                           |                |                           |
| Deuteron mass                                 | $m_d$  | $3.3435860(20)$           | $10^{-27}$ kg  | 0.59                      |
| in electron volts, $m_d e^2/|e|$             |        | $2.013553214(24)$         | u              | 0.012                     |
| Deuteron-electron mass ratio                  | $m_d/m_e$ | $3670.483014(75)$      | MeV            | 0.020                     |
| Deuteron-proton mass ratio                    | $m_d/m_p$ | $1.999007496(6)$        |                | 0.003                     |
| Deuteron molar mass                           | $M(d), M_d$ | $2.013553214(24)$          | $10^{-3}$ kg/mol | 0.012                     |
| Deuteron magnetic moment *                    | $\mu_d$ | $0.43307375(15)$          | $10^{-26}$ J T$^{-1}$ | 0.34                      |
| in Bohr magnetons,                           | $\mu_d/\mu_B$ | $0.4669754479(01)$         | $10^{-3}$     | 0.019                     |
| in nuclear magnetons,                        | $\mu_d/\mu_N$ | $0.857438230(24)$         |                | 0.028                     |
| Deuteron-electron magnetic moment ratio      | $\mu_d/\mu_e$ | $0.4664345460(91)$         | $10^{-3}$     | 0.019                     |
| Deuteron-proton magnetic moment ratio         | $\mu_d/\mu_p$ | $0.3070122035(51)$        |                | 0.017                     |

PHYSICO-CHEMICAL CONSTANTS

- Avogadro constant
  $$N_A, L = 6.0221367(36) \times 10^{23} \text{ mol}^{-1}$$
- Atomic mass constant, $\frac{1}{12} m^{(12)}(C)$
  $$m_u = 1.6605402(10) \times 10^{-27} \text{ kg}$$
- Faraday constant
  $$F = 96485.309(29) \text{ C mol}^{-1}$$
- Molar Planck constant
  $$N_A h = 3.99031323(36) \times 10^{-10} \text{ J s}$$
- Molar gas constant
  $$R = 8.314510(70) \text{ J mol}^{-1} \text{ K}^{-1}$$
- Boltzmann constant, $R/N_A$
  $$k = 1.380658(12) \times 10^{-23} \text{ J K}^{-1}$$
- Molar volume (ideal gas), $RT/p$
  $$V_m = 22.41410(19) \text{ L mol}^{-1}$$
- Loschmidt constant, $N_A/V_m$
  $$n_0 = 2.686763(23) \times 10^{25} \text{ m}^{-3}$$
- Wien displacement law constant
  $$b = 2.897756(24) \times 10^{-3} \text{ m K}$$

**The scalar magnitude of the neutron moment is listed here. The neutron magnetic dipole is directed oppositely to that of the proton, and corresponds to the dipole associated with a spinning negative charge distribution. The vector sum, $\mu_0 = \mu_+ + \mu_-$, is approximately satisfied.**

**The entropy of an ideal monatomic gas of relative atomic weight $A$, is given by $S = S_n + \frac{3}{2} R \ln A - R \ln \left( \frac{p}{p_A} \right) + \frac{1}{2} R \ln (T/K)$.
Table 3 is a list of related "maintained units and standard values," while Table 4 contains a number of scientifically, technologically, and metrologically useful energy conversion factors. Finally, Table 5 is an extended covariance matrix containing the variances, covariances, and correlation coefficients of the unknowns and a number of different constants (included for convenience) from which the like quantities for other constants may be readily calculated. Such a matrix is necessary, of course, because the variables in a least-squares adjustment are statistically correlated. Thus, with the exception of quantities which depend only on aux-

The variable $d_{220}$ is omitted from Table 5 because there is little need for its correlations with other quantities. Moreover, since the more significant and related quantity $N_A$ is included (note that $N_A = d_{220}$), there is no loss of information by omitting $d_{220}$.

### Table 3. Maintained units and standard values.

A summary of "maintained" units and "standard" values and their relationship to SI units, based on a least-squares adjustment with 17 degrees of freedom. The digits in parentheses are the one-standard-deviation uncertainty in the last digits of the given value. Since the uncertainties of many of these entries are correlated, the full covariance matrix must be used in evaluating the uncertainties of quantities computed from them.

| Quantity | Symbol | Value | Units | Relative Uncertainty (ppm) |
|----------|--------|-------|-------|-----------------------------|
| electron volt, $(e/C) J = \{e\} J$ (unified) atomic mass unit, $1 u = m_u = \frac{1}{12} m(^{12}C)$ standard atmosphere | eV $u$ atm $g_n$ | 1.60217733(49) 1.6605402(10) 101325 | $10^{-19}$ J $10^{-27}$ kg Pa | 0.30 0.59 (exact) |
| standard acceleration of gravity | | 9.80665 | m s$^{-2}$ | (exact) |
| BIPM maintained ohm, $\Omega_{69-BI}$ $\Omega_{Bl85} \equiv \Omega_{69-BI} (1$ Jan 1985) | $\Omega_{Bl85}$ | $1 - 1.563(50) \times 10^{-6}$ | $\Omega$ | 0.050 |
| Drift rate of $\Omega_{69-BI}$ | $d\Omega_{69-BI}/dt$ | $-0.0566(15)$ | $\mu\Omega/a$ | — |
| BIPM maintained volt, $V_{76-BI} \equiv 483.594$ GHz $(h/2e)$ | $V_{76-BI}$ | $1 - 7.59(30) \times 10^{-6}$ | V | 0.30 |
| BIPM maintained ampere, $A_{Bl85} = V_{76-BI}/\Omega_{69-BI}$ | $A_{Bl85}$ | $1 - 6.03(30) \times 10^{-6}$ | A | 0.30 |
| Cu x-unit: $\lambda(CuK\alpha) \equiv 1537.400$ xu | xu$(CuK\alpha)$ | 1.00207789(70) | $10^{-13}$ m | 0.70 |
| Mo x-unit: $\lambda(MoK\alpha) \equiv 707.831$ xu | xu$(MoK\alpha)$ | 1.00209938(45) | $10^{-13}$ m | 0.45 |
| $\Lambda^*$: $\lambda(WK\alpha) \equiv 0.209100 \Lambda^*$ | $\Lambda^*$ | 1.00001481(92) | $10^{-10}$ m | 0.92 |
| lattice spacing of Si (in vacuum, 22.5 °C), $^+ d_{220} = a/\sqrt{8}$ | $d_{220}$ | 0.192015540(40) | nm | 0.21 |
| molar volume of Si, $M(Si)/\rho(Si) = N_A a^3/8$ | $V_m(Si)$ | 12.0588179(89) | cm$^3$/mol | 0.74 |

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*The lattice spacing of single-crystal Si can vary by parts in 10$^7$ depending on the preparation process. Measurements at PTB indicate also the possibility of distortions from exact cubic symmetry of the order of 0.2 ppm.*
iliary constants, the uncertainty associated with a quantity calculated from other constants in general can be found only with the use of the full covariance matrix.

To use table 5, note that the covariance between two quantities $Q_k$ and $Q_s$, which are functions of a common set of variables $x_i (i = 1, \ldots, N)$ is given by

$$v_{ks} = \sum_{i,j=1}^{N} \frac{\partial Q_k}{\partial x_i} \frac{\partial Q_s}{\partial x_j} v_{ij}$$

where $v_{ij}$ is the covariance of $x_i$ and $x_j$. In this general form, the units of $v_{ij}$ are the product of the units of $Q_k$ and $Q_s$.

For most cases of interest

Table 4. Energy conversion factors.

To use this table note that all entries on the same line are equal; the unit at the top of a column applies to all of the values beneath it.

| J   | kg           | m$^{-1}$          | Hz   |
|-----|--------------|-------------------|------|
| 1   | $1/\{c^2\}$ | $1.11265006 \times 10^{-17}$ | $1/\{h\}$ |
| 1kg | $8.987551787 \times 10^{16}$ | $1$ | $1.50918897(90) \times 10^{33}$ |
| 1m$^{-1}$ | $1.9864475(12) \times 10^{-25}$ | $2.2102209(13) \times 10^{-42}$ | $1/\{c\}$ |
| 1Hz | $6.6260755(40) \times 10^{-34}$ | $1/\{c^2\}$ | $4.5243437(27) \times 10^{41}$ |
| 1K  | $1.380658(12) \times 10^{-23}$ | $3.35640952 \times 10^{-9}$ | $1/\{c\}$ |
| 1eV | $1.60217733(49) \times 10^{-19}$ | $1/\{c\}$ | $299792458$ |
| 1u  | $1.49241909(88) \times 10^{-10}$ | $1/\{c\}$ | $2.083674(18) \times 10^{10}$ |
| 1hartree | $4.3597482(26) \times 10^{-16}$ | $4.8058741(29) \times 10^{-35}$ | $21947463.067(26)$ |

| K   | eV            | u                | hartree |
|-----|---------------|------------------|---------|
| 1   | $1/\{c\}$    | $6.2415064(19) \times 10^{18}$ | $1/\{2Roo \}$ |
| 1kg | $6.509616(55) \times 10^{39}$ | $1/\{m_u\}$ | $2.2937104(14) \times 10^{17}$ |
| 1m$^{-1}$ | $0.00438769(12)$ | $1.23984244(37) \times 10^{-6}$ | $1/\{2Roo \}$ |
| 1Hz | $4.799216(41) \times 10^{-11}$ | $4.4382224(40) \times 10^{-24}$ | $2.061484(12) \times 10^{34}$ |
| 1K  | $1/\{c\}$    | $9.617385(73) \times 10^{-5}$ | $1/\{2Roo \}$ |
| 1eV | $11604.45(10)$ | $1/\{c\}$ | $4.5563352672(54) \times 10^{-8}$ |
| 1u  | $1.0809478(91) \times 10^{13}$ | $1/\{c\}$ | $3.166829(27) \times 10^{-6}$ |
| 1hartree | $3.157733(27) \times 10^{5}$ | $1/\{c\}$ | $0.036749309(11)$ |

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The elements of the covariance matrix appear on and above the major diagonal in (parts in 10^9)^2; correlation coefficients appear in italics below the diagonal. The values are given to as many as six digits only as a matter of consistency.

The correlation coefficient between m_e and N_A appears as -1.000 in this table because the auxiliary constants were considered to be exact in carrying out the least-squares adjustment. When the uncertainties of m_p/m_e and M_p are properly taken into account, the correlation coefficient is -0.999 and the variances of m_e and N_A are slightly increased.

| α^-1 | K_V | K_N | μ_μ/μ_ρ | e | h | m_e | N_A | F |
|------|-----|-----|----------|---|---|-----|-----|---|
| 1997 | -1062 | 925 | 3267 | -3059 | -4121 | -127 | 127 | -2932 |
| K_V  | 0.080 | 87988 | 90 | -1737 | 89050 | 177038 | 174914 | -174914 | -85864 |
| K_N  | 0.416 | 0.006 | 2477 | 1513 | -835 | -744 | 1105 | -1105 | -1939 |
| μ_μ/μ_ρ | 0.489 | -0.040 | 0.207 | 21523 | -5004 | -6742 | -208 | 208 | -4796 |
| e    | -0.226 | 0.989 | -0.055 | -0.112 | 92109 | 181159 | 175042 | -175042 | -82933 |
| h    | -0.154 | 0.997 | -0.025 | -0.077 | 0.997 | 358197 | 349956 | -349956 | -168797 |
| m_e  | 0.005 | 0.997 | 0.038 | 0.002 | 0.975 | 0.989 | 349702 | 349702 | -174660 |
| N_A  | 0.005 | -0.997 | 0.038 | 0.002 | -0.975 | -0.989 | -1.000 | 349702 | 174660 |
| F    | -0.217 | -0.956 | -0.129 | -0.108 | 0.902 | -0.931 | -0.975 | 0.975 | 91727 |

where the quantities in brackets are auxiliary constants taken to be exact. Using eq (3) and letting α^-1 correspond to i = 1 and K_V to i = 2 gives

$$\varepsilon^2_{μ_μ} = Y_{i1}v_{11} + 2Y_iY_jv_{12} + Y_j^2v_{22}. \quad (6)$$

Comparing eq (5) with eq (2) yields Y_1 = -3 and Y_2 = 1. Thus eq (6) and table 5 lead to

$$\varepsilon^2_{μ_μ} = [9(1997) - 6(-1062) + 1(87988)] \times (10^{-9})^2 \quad (7)$$

or ε_μ_μ = 0.335 ppm. An alternate approach is to evaluate e/h/2m_e directly from table 5; then e corresponds to i = 5, h to i = 6, and m_e to i = 7 with Y_5 = Y_6 = 1 and Y_7 = -1. Then

$$\varepsilon^2_{μ_μ} = Y_{i5}v_{55} + 2Y_iY_6v_{56} + Y_6^2v_{66}$$

$$+ 2Y_6Y_{77} + 2Y_iY_{77} + Y_i^2v_{77} \quad (8a)$$

$$= [1(92109) + 2(181159) + 1(358197)]$$

$$- 2(175042) - 2(349956)$$

$$+ 1(349702)] \times (10^{-9})^2 \quad (8b)$$

which also yields ε_μ_μ = 0.335 ppm.

Note that in using eq (3), we set s = k, \( \varepsilon^i_k \rightarrow v_{kk} \), suppress k as a subscript on Y, and replace k with \( \mu_μ \).
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