Antinomies of Symmetry and Information ‡

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Abstract: There are several apparent antinomies within the studies of information, symmetry, and the mutual relationship between symmetry and information. These antinomies are only apparent as the antinomial opposition of statements about information, symmetry, and their mutual relationship is a result of differences between the use of these fundamental concepts in the narrow contexts of their early applications or the common-sense discourse and more rigorous and general study. When information and symmetry are conceptualized at a sufficiently high level of generality and with sufficient precision, the apparent antinomies are eliminated together with the causes of misconceptions within them. In this paper, a selection of examples of such antinomies is followed by a nutshell overview of their solutions, achieved mainly through the clarification of conceptual confusion without the need for revision of the already existing methodology.

Keywords: symmetry; information; complexity; reduction of complexity

1. Introduction

This paper proposes a resolution of several apparent antinomies within the studies of information, symmetry, and the mutual relationship between symmetry and information. These antinomies are only apparent as the antinomial opposition of statements about information, symmetry, and their mutual relationship is a result of differences between the use of these fundamental concepts in the narrow contexts of their early applications or the common-sense discourse and more rigorous and general study. When information and symmetry are conceptualized at a sufficiently high level of generality and with sufficient precision the apparent antinomies are eliminated, together with the causes of misconceptions within them. In the following, a selection of examples of such antinomies is followed by a nutshell overview of their solutions.

The solutions proposed in this paper are based on particular conceptual frameworks for information and symmetry. The former, which makes the distinction between selective and structural manifestations of information, may not necessarily satisfy all who pursue the most adequate definition [1–3]. However, the sole existence of the approach in which the antinomies are eliminated shows that they are not inherent in the study of information. The latter is a standard way symmetry is understood in mathematics, physical sciences, and all rigorous studies of this concept, and as such it is very unlikely to be contested. The issue is that even this standard approach is frequently misunderstood. This does not mean that the study of symmetry has reached its final form and is an already closed field of research [4].

2. Examples of Apparent Antinomies

The earliest example of opposition in the views on information can be found in the critical reaction to the claim of Shannon’s foundational work denying importance to the semantic aspects of communication [5]. This denial exposed his work to the objection that it is not about information, right from the beginning of its triumphal declaration as...
“the theory of information” [6,7]. The issue was never completely resolved, although it faded with the increased popularity of naive claims that the problem disappears when we demand in the definition that whatever information is (for instance data, as if data could serve as a genus for information, not the other way around), it has to be true.

The relationship between the measure of information given by Shannon in the form of entropy and the measure called negentropy introduced by Schrödinger as a magnitude which, although being non-negative, has its value opposite to the non-negative entropy, is antinomial [5,8]. This curious pairing, although considered sufficiently harmless not to attract much attention, is the tip of an iceberg of much deeper internal opposition in the view of information. Shannon’s view of information is tied to the uncertainty of the recipient of a message, i.e., it is a change of the state of an observer as a result of the actualization of the selection from a set of possible choices with different probabilities. Schrödinger’s negentropy is a numerical characteristic of the acquired freedom in forming organized structure within the system. The two views of information are not explicitly opposed as entropy and negentropy, but they require opposition in the identification of the locus of information. One of them identifies information with the selection performed by an external entity, and the other with the structure within the medium of information.

An example representing antinomies of symmetry has the form of opposition of two oppositions. One of them is between the artificial, intentional character of symmetry associated with human aesthetical preference, and the natural character of asymmetry associated in a common-sense view with spontaneous, unconstrained generation of forms. The other reversed opposition is provided by the biological evolution in which the steps in the transition to a higher form of life are marked by diverse forms of breaking symmetry, leading from the highly symmetric proto-organismic simple systems to the complex human organism with its asymmetric functional specialization. The presence of highly symmetric organic or mineral forms and their beauty serves as an argument for the necessity of divine intervention in their creation.

Finally, there is an example of the opposition in views on the relationship between information and symmetry with its main axis between the claim that information has its foundation in symmetry and the view that it is asymmetry which is its source. The former view is quite intuitive as symmetry, especially in its traditional understanding as harmony, guides us towards truth and beauty [9]. In physics, the celebrated theorem of Emmy Noether demonstrated that the conserved physical magnitudes such as momentum, energy, etc., which we distinguish in our description of physical reality are not distinguished because they are conserved, but they are those magnitudes that are conserved when we want to have the description of physical reality invariant with respect to the transformations related to the change of reference frames (observers) [10]. Noether’s Copernican revolution associated objective description of reality, i.e., objective information about reality with symmetry understood as invariance with respect to the groups of transformations that constitute objectivity rather than mysterious distinguished entities conserved in time.

This position originates in Pierre Curie’s Principle of Dissymmetry, which states that symmetric causes cannot have asymmetric effects justifying the focus on asymmetry [11], as it can guide us to the actual causes of phenomena. The early formulation of Gregory Bateson’s metaphor of “information as a difference which makes difference in some later event” still paired with the definition of information as “that which excludes certain alternatives” was in his explanation of the rules of biological asymmetry postulated by his father in a book published the same year that Curie’s paper appeared [12,13]. The younger Bateson linked the biological process of breaking symmetry with the requisite information.

Thus, we have yet another opposition to the views associating either asymmetry or symmetry with information. Now we have several antinomies to resolve, but it turns out that the task is getting easier when we consider them together; however, first we have to clarify misunderstandings obscuring the concepts of structure and symmetry.
3. Structure, Symmetry, and the Resolution of Antinomies

The apparent antinomy in the opposition between the views of information conceptualized in terms of selection (external view) or structure (internal view) can be resolved by a definition which involves both. An example of such conceptualization is in the definition introduced by the author of [1–3]. Information is understood as a resolution of the one–many opposition between unity and multiplicity which can be achieved by the selection of one out of many or by a structure of the many uniting them into a whole. Both the ideas of the one–many opposition and structure (at the earlier historical stages of philosophical reflection, usually called form) have very rich philosophical traditions. Moreover, both can easily be formalized in terms of set theory and algebra. It is important to recognize that these are not different forms of information but its different manifestations (selective or structural) that always coexist at different levels of analysis. We cannot consider a process of selection of “one” out of “many” without this “one” having a structure distinguishing it from the others in the “many”. On the other hand, the “many” to be considered “one”, i.e., a whole, has to have some structure. Someone who prefers a different definition of information may disagree with this one, but no matter what this preferred definition is, it has to incorporate the selective and structural aspects. Thus, the elimination of this apparent antinomy can be achieved by a sufficiently general definition incorporating both aspects, and the definition above shows that it is possible without leading to any contradiction. The next step is to develop tools for the study of structure.

It is an irony of modern intellectual history that the popularity of so-called Structuralism in the mid-20th century contributed to the more recent confusion and hostility towards the concept of a structure as a fundamental tool of inquiry outside of physical sciences. In mathematics and physical sciences, the study of structures acquired the form of the study of groups of transformations that preserve them, i.e., structures became invariants of groups of transformations (their automorphisms). The first step was in the revolutionary 1872 Erlangen Program of Felix Klein [14], in which a variety of different geometries with their diverse structures were incorporated into a unified study of groups of transformations. Noether’s theorem and a long sequence of developments in physics made symmetry and its breaking the central theme of the entire discipline. By 1972, when Philip W. Anderson, who soon later became a Nobel Prize laureate, wrote that “It is only slightly overstating the case that physics is the study of symmetry”, no physicist would object to it [15]. There is no direction of inquiry in mathematics or physics called “structuralism” as there is no alternative methodology.

In contrast to mathematics and physical sciences, the philosophical structuralism of humanities and cultural studies was openly or even demonstratively rejected after two decades of great popularity stimulated by the influential works of Hermann Weyl, Jean Piaget, and Claude Levi-Strauss promoting symmetry as the main tool for structural analysis [9,16,17]. The typical reason for this radical change was the objection to the ahistorical and static character of the central concept of structure. This objection is another example of antinomy when we contrast it with the use of symmetry in physics precisely for the inquiry of dynamical systems. When we associate structure with symmetry, i.e. with the invariance with respect to transformation, as it is done in physics, we do not have to make a choice between static and dynamic aspects of reality.

There is no mystery in the “post-structuralist” revolt, but simply an example of confusion. Structuralism in humanities had its source in the ideas of Ferdinand de Saussure related to linguistics [18]. He introduced the division of inquiries into idiographic and synchronic which in time became the keywords of reflection on the methodology of the studies outside of natural sciences. The idiographic methodology was associated with historicism, and synchronic methodology became associated with structuralism. The opposition of the two methodologies is meaningless for mathematics or natural sciences or in all domains influenced by Klein’s Erlangen Program. However, the very limited understanding of symmetry in other domains of inquiry made the space for the commonsense idea that any structure must be static because it is by (false) definition synchronic.
(devoid of a time aspect). In reality, symmetry determines the structure by the distinction of what does not change when the entire system transforms. There is no symmetry (and no structure) in the absence of transformations.

Another source of confusion is in considering the opposition symmetry–asymmetry. The word “symmetry” in its singular form represents the idea of the relationship between structures and transformations. There is usually no singular symmetry, but a hierarchy of symmetries corresponding to the hierarchy of subgroups of some group. Breaking symmetry means a transition to another symmetry or transition from one structure of higher symmetry to another structure of lower symmetry. This resolves the apparent antinomy of information associated with symmetry or asymmetry.

Information can be associated with the change of symmetry. Curie’s Principle means simply that the change of symmetry requires an instance of cause (using his terminology) or in other words, it requires the interaction that is not invariant with respect to the original group of transformations.

Finally, we can eliminate the apparent antinomy of the artificial vs. natural character of symmetry. Gregory Bateson was probably the first who recognized the relationship between symmetry and information. Symmetry in the organic world reduces the need for information in the reproduction of organisms. Instead of storing and transmitting separate information about several petals, the symmetry of the flower reduces the genetic information to that of one petal. In this case, it is a matter of quantitative reduction.

However, there is another aspect of the relation between symmetry and information that can explain why the presence of symmetry (especially its geometric forms) is usually associated with human action or preference and considered an indicator of the artificial. The explanation can be found in the proposed by the author main characteristic of intelligence as the ability to reduce the complexity of information [19]. This can explain our human (and not only human) preference for symmetry as a tool for the reduction of information. This does not mean that symmetry is not natural, but that it is sought in the environment. We notice the objects which are symmetric as the information about them is easier for cognitive processing and memory storage. This applies not only to spatial symmetry but also to its temporal forms (as for instance rhythm in music or rhyme in poetry). This is one more instance of the mysteries that can be explained when we combine the two conceptual tools of symmetry and information.

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