Hybrid Method for Power Control Simulation of a Single Fluid Plasma Thruster

S. Jaisankar¹ and T. S. Sheshadri²

Department of Aerospace Engineering, Indian Institute of Science, Bangalore, India

Corresponding e-mail: ¹jshankar.s@gmail.com, ¹jshankar@aero.iisc.ernet.in, ²tss@aero.iisc.ernet.in

Abstract. Propulsive plasma flow through a cylindrical-conical diverging thruster is simulated by a power controlled hybrid method to obtain the basic flow, thermodynamic and electromagnetic variables. Simulation is based on a single fluid model with electromagnetics being described by the equations of potential Poisson, Maxwell and the Ohm’s law while the compressible fluid dynamics by the Navier Stokes in cylindrical form. The proposed method solved the electromagnetics and fluid dynamics separately, both to segregate the two prominent scales for an efficient computation and for the delivery of voltage controlled rated power. The magnetic transport is solved for steady state while fluid dynamics is allowed to evolve in time along with an electromagnetic source using schemes based on generalized finite difference discretization. The multistep methodology with power control is employed for simulating fully ionized propulsive flow of argon plasma through the thruster. Numerical solution shows convergence of every part of the solver including grid stability causing the multistep hybrid method to converge for a rated power delivery. Simulation results are reasonably in agreement with the reported physics of plasma flow in the thruster thus indicating the potential utility of this hybrid computational framework, especially when single fluid approximation of plasma is relevant.

1. Introduction

Magneto-plasma dynamic (MPD) thruster is an electric propulsion device having applications ranging from near-Earth orbit raising maneuvers to long duration interplanetary high impulse space missions [1]. Such a thruster in its simple form consists of a cylindrical cathode surrounded by a concentric anode and the arc between the electrodes ionizes a gaseous propellant. The interaction of the current with the self-induced magnetic field accelerates the plasma to produce thrust. Numerous experimental investigations leading to plasma thruster development have been reported and so also on the simulation of single or two fluid plasma using cartesian or cylindrical co-ordinate based models. A single fluid simulation with a detailed model including viscosity, Hall effect, electrostatic forces etc., along with rated power control is seldom, which is being attempted here.

Here, a hybrid solver is proposed in which the electromagnetic equations in steady form and fluid equation evolving in time are solved separately, so as to have a reasonable combination of robustness and speed of convergence. The hyperbolic treatment in the magnetic transport is employed following...
Heiermann et al.[2]. The overall method uses generalized finite differences [3] which is particularly useful in simplifying the computation when cross derivatives are involved with arbitrary shaped mesh. The separation of fluid solver simplified the coding efforts as up-gradation for plasma propulsion is done just by the addition of source terms.

2. Governing Model
The basic electrodynamics equations include (i) the Poisson equation (1) for potential ($\phi$) written as

$$\nabla^2 \phi = -\frac{q}{\varepsilon_0}$$

(1)

with $q$ and $\varepsilon_0$ being the space charge distribution and free space permittivity, respectively (ii) the electric field distribution given by

$$\vec{E} = -\Delta \phi$$

(2)

and the magnetic transport equation (5) which could be derived by combining Maxwell's equations (3)

$$\Delta \vec{B} = 0; \quad \Delta \times \vec{B} = \mu_0 \vec{j}; \quad \Delta \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}; \quad \Delta \times \vec{E} = -\frac{q}{\varepsilon_0}$$

(3)

along with (iv) the generalized Ohm's law (4) which relates current ($\vec{j}$) to the quantities of plasma velocity ($\vec{v}$), electric field ($\vec{E}$) and magnetic field ($\vec{B}$) as

$$\vec{j} = \sigma \left[\vec{E} + (\vec{v} \times \vec{B})\right] - \frac{H_p}{B} (\vec{j} \times \vec{B})$$

(4)

Following (3) and (4), divergence free magnetic flux is expressed as

$$\frac{\partial \vec{B}}{\partial t} = rot(\vec{v} \times \vec{B}) - rot \left( \frac{rot \vec{B}}{\mu_0 \sigma} + \frac{H_p}{\sigma B} rot \vec{B} \times \vec{B} \right)$$

(5)

with the Hall parameter $H_p$ for a degree of ionization $D$ and propellant molecular weight $W$ being defined as

$$H_p = 10^{-11} \frac{W T^{3/2}}{D \rho} B$$

(6)

The fluid model being employed here is that of two dimensional cylindrical Navier-Stokes (7).

$$\frac{\partial U}{\partial t} + \frac{\partial A}{\partial z} + \frac{\partial B}{\partial r} + C + D = 0$$

(7)

where

$$U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{pmatrix}, \quad D = \begin{pmatrix} 0 \\ (\vec{j} \times \vec{B} + q \vec{E})_z \\ (\vec{j} \times \vec{B} + q \vec{E})_r \\ \vec{j} \cdot \vec{E} \end{pmatrix}, \quad A = \begin{pmatrix} \rho u \\ p + \rho u^2 - \tau_{zz} \\ \rho uv - \tau_{rz} \\ pu + \rho u E + q_z - u \tau_{zz} - v \tau_{rz} \end{pmatrix}$$

(8)

$$B = \begin{pmatrix} \rho v \\ \rho uv - \tau_{rz} \\ \rho vv + p - \tau_{rr} \\ pv + pv E + q_r - u \tau_{rz} - v \tau_{rr} \end{pmatrix}, \quad C = \begin{pmatrix} \rho v \\ \rho uv - \tau_{rz} \\ \rho vv - \tau_{rr} \end{pmatrix}$$

(9)
The molecular stress tensor components are

\[
\tau_{zz} = \frac{2\mu}{3} \left( \frac{2}{\partial z} \frac{\partial u}{\partial r} - \frac{\partial v}{\partial r} \right); \quad \tau_{rr} = \frac{2\mu}{3} \left( \frac{2}{\partial r} \frac{\partial u}{\partial z} - \frac{\partial v}{\partial z} \right); \quad \tau_{rz} = \mu \left( \frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} \right)
\]  

(10)

with the diffusive heat-flux vector described as

\[
q_z = -k \frac{\partial T}{\partial z} \quad \text{and} \quad q_r = -k \frac{\partial T}{\partial r}
\]

(11)

The plasma viscosity \( \mu \) and thermal conductivity \( k \) are computed using logarithm law with temperature while electrical conductivity \( \sigma \) is calculated using Spitzer-Harm formulation [4]. Charge density \( q \) is computed using the divergence of electric field (3).

2.1 Numerical Solution

Solution of the governing model consists of obtaining the voltage potential across the electrodes using the Poisson equation, from the divergence of which the electric field distribution could be obtained. Electric field is then used in magnetic transport equation and general Ohm's law (4) which yields a combined elliptic-hyperbolic equation. Each of these elliptic potential Poisson (1) and steady magnetic transport (5) equations can be cast in a general form as

\[
\gamma_1 \frac{\partial^2 \psi}{\partial r^2} + \gamma_2 \frac{\partial \psi}{\partial r} + \gamma_3 \frac{\partial \psi}{\partial z} \frac{\partial^2 \psi}{\partial z^2} + \gamma_4 \psi + \gamma_6 = 0
\]

(12)

Finite difference discretization of (12) is used to solve for \( \psi \). Central differences for second order derivatives and sign of co-efficients(of \( \gamma_2, \gamma_3 \)) based backward or forward differences for the first derivatives are applied. \( \psi \) either as a potential or magnetic field is obtained iteratively using successive relaxation till their residual reached just below \( 10^{-3} \). Current, magnetic and electric fields thus obtained define the source term of the fluid solver. As the source could cause stiffness in the flow solution, very low CFL number is used for time marching in addition to source terms being incrementally added with iterations to avoid blow up of the code. The fluid solver adopted is a mesh-free method [5] in which the flux derivatives are discretized as (13) and (14).

\[
\left( \frac{\partial \Psi}{\partial z} \right)_p = \frac{\sum (\Delta r_m)^2 \sum \Delta z_m \Delta \Psi_m - \sum \Delta z_m \Delta r_m \sum \Delta r_m (\sum \Delta \Psi_m)}{\sum (\Delta z_m)^2 \sum (\Delta r_m)^2 - (\sum \Delta z_m \Delta r_m)^2}
\]

(13)

\[
\left( \frac{\partial \Psi}{\partial r} \right)_p = \frac{\sum (\Delta z_m)^2 \sum \Delta r_m \Delta \Psi_m - \sum \Delta r_m \Delta z_m \sum \Delta z_m (\sum \Delta \Psi_m)}{\sum (\Delta z_m)^2 \sum (\Delta z_m)^2 - (\sum \Delta z_m \Delta r_m)^2}
\]

(14)

where \( \Delta z_m = z_m - z_p; \quad \Delta r_m - r_p \quad \text{and} \quad \Delta \Psi_m = (\Psi_m - \Psi_p) \)

Flux derivative at a point ‘\( p \)’ is obtained using the neighborhood points, say = 1.8 . \( \Psi_m \) is the mid-point state between points ‘\( p \)’ and ‘\( t \)’ and defined as per the physics of the term. For convective terms, \( \Psi_m \) represents the flux and being hyperbolic is defined here as \( \Psi_m = \left[ \frac{1}{2} (\psi_p + \psi_t - |\alpha| (U_p - U_t)) \right], \forall i \), following the diffusion regulated flux [6, 7]. \( \alpha \) is the numerical viscosity co-efficient and \( U_t, U_p \) are conserved variables in (8). For diffusive terms (thermal and viscous) \( \Psi \) (which could be
velocity or temperature) $\Psi_m = \left[ \frac{1}{2} (\Psi_p + \Psi_l) \right] \Psi_i$ is the mean of two states in the stencil. A schematic of the steps involved in computation is given in fig. 1.

![Flow Chart of Steps in the Hybrid Computational Method](image)

**Figure 1.** Flow Chart of Steps in the Hybrid Computational Method

The thruster is evaluated for thrust generation, specific impulse and efficiency. Thrust is evaluated by integrating as $F = \int_e 2\pi r dr \dot{m} V_e$. Specific impulse ($I_{sp}$) and efficiency ($\eta$) are calculated as

$$I_{sp} = \left. \frac{1}{g} \int_e 2\pi r \dot{m} V_e dr \right|_{r}^{\frac{1}{2}}$$

$$\eta = \frac{\int_e \left( p_e + \frac{1}{2} p_e V_e^2 \right) V_e dA_e}{(V_a - V_c) \int_a J_a dA_a}$$

with subscripts ‘e’, ‘c’ and ‘a’ denoting exit, cathode and anode, respectively. $V_e$ is the exit velocity, $V_a$ and $V_c$ are voltage at the electrodes.

### 2.2 Power Control

In this simulation with constant voltage supply, magnetic field is obtained by solving each of Poisson equation and magnetic transport flux equation separately. Current is then computed using the Ohm's law to obtain the integrated power over the volume of the thruster. The relevant source terms are included in the fluid solver and the whole computational sequence is repeated till convergence as depicted in fig. 1. When a constant magnetic field through constant total discharge current is not applied, there is a possibility of nonphysically more current getting drawn into the system as current and electric field are functions of fluid variables which are computed later in the multistep computational method used here. This can cause a runaway in the simulation and hence power supply is to be controlled. Power drawn by thruster can be controlled by any of the three ways (i) direct control of external energy in fluid solver (ii) by regulating the current through conductivity or temperature or (iii) by voltage control. Direct energy control in conservation law made a disproportionate source or widely varying scales between energy and momentum term which imposed strong restrictions on the marching time of the fluid solver. Power control by regulating current drawn at every discretized volume by modifying electrical conductivity and hence the magnetic field is found extremely slow in convergence and required multiple control points. This is due to space and time varying current being highly non-linear function of magnetic field and electrical conductivity. But here, power control by regulating the voltage across electrode is attempted using (16)

$$\Delta \phi_{new} = w' \Delta \phi' + w \Delta \phi$$

and

$$\Delta \phi' = \left( \frac{\text{Rated Power}}{\text{Drawn Power}} \right) \Delta \phi$$

(16)
where \( w \) and \( w' \) are the weights of \( \Delta \phi \) and \( \Delta \phi_{\text{new}} \), respectively. Control strategy (16) is simple as electric field in energy term is linear to potential difference which is a constant quantity. Control of supply voltage is depicted in Fig. 1. Voltage control of power is found to be repetitively converging and faster than the other methods. Constant power thruster in the entire of literature is achieved by a constant total discharge current, which is quite different from rated power operation attempted here.

3. Simulation
Simulation of plasma flow through the classical cylindrical-conical thruster geometry [8, 9] shown in Fig. 2 is studied. A hybrid method for the simulation of fully-ionized argon plasma thruster with a single fluid model is used. The scheme features constant controlled power input and is general for any cylindrical-conical geometries and physical input parameters. Each of the solvers for Poisson, magnetic transport and cylindrical compressible flow are allowed to converge for a fall in numerical residue up to \( 10^{-3} \) or lesser. The computational domain is divided into (axial radial) nodes and the solution sequence comprises of potential Poisson computation, followed by magnetic transport equations to obtain current and magnetic field, and hence source terms for the fluid solver. Fluid equations with added source are then solved to reach steady state. The whole procedure is iterated till the residual due to potential distribution and exhaust velocity reached a very low value (say \( 10^{-5} \)). The solving of steady magnetic transport equation requires an initial value, more so it should be a careful guess as the electromagnetic solver is called several times in the power control loop. After some initial experiments, \( B_p(z,r,0) = -\mu_0 \sigma E_c \Delta Z \) is taken, where \( \Delta Z = Z - Z_l \), with \( Z_l \) being length of cathode. In every subsequent call of electromagnetic solver only \( E_c \) is to be changed.

![Figure 2. Geometry of Cylindrical Conical Diverging Plasma Thruster: \( R_l = 7\, mm, R_o = 12.2\, mm \), and \( R_c = 2\, mm \) – radius of inley, outlet and cathode, (ii)](image)

3.1 Boundary Conditions
Flow and solid electrodic boundaries are marked in fig. 2. Propellant mass flow rate and density are specified at inlet while the static pressure is evaluated by the first-order extrapolation at entrance due to subsonic flow. Velocity at inlet follows a developed annular flow [10] while temperature is set by ideal gas law. At the exit of the thruster, the boundary condition is governed by whether the outflow is supersonic or subsonic. If supersonic, all the independent variables are evaluated by Neumann condition with zero gradient. The flow in the boundary layer is possibly subsonic, however, this region has little influence on the entire flow characteristics and that too when the objective is steady-state solution.

On the electrode boundaries, no slip condition for velocity and zero normal gradient for pressure are imposed. The electrode surface is treated as a adiabatic thermal boundary with temperature being set to that of contacting plasma. The discharge voltage is imposed across the electrode surfaces and controlled iteratively so that the calculated total power amounts to the rated value. At the inlet, center axis, and exit boundary, the electric field perpendicular to the boundary is set zero. Upstream and
downstream boundaries for all electromagnetic variables are set for no current entering or leaving the thruster along with the fluid flow, which translates to \( E_x - H_p E_r = -H_p v_x B_\theta \).

### 3.2 Results and Discussion

Simulation results for argon flow in MPD thruster presented in figs. 3-10 exhibit the following on relevant physical and computation related parameters.

**Electromagnetics:** Electric field in fig. 3b is established normal to the equipotential lines of fig. 3a. The magnetic field as seen in fig. (4)a is negative maxima at the entrance which gradually reached minimum magnitude at the thruster exit. Cathodic current as seen in fig. 4b is concentrated close to the entrance and exit, similar to the trend reported [11].

**Hydrodynamics:** All the flow related variables experience a discontinuity close to the entrance which is the zone of concentrated current and magnetic field (fig. 4), hence that of maximum added electromagnetic energy (fig. 6). The maximum acceleration seen through velocity vector in fig. 5b and pressure maxima in fig. 6 are consistent with the understanding on maximum energy being added close to the cathodic entrance. This is also reflected in high gradient in density closer to the cathode entrance. Fig. 7 show the argon plasma being accelerated upto 10 km/s, as expected out of a high specific impulse operation in a MPD device.
Figure 5. Vector plot of (a) Current and (b) Velocity

Figure 6. (a) Density and (b) Pressure variation along the length of the thruster

Figure 7. Velocity variation along the length of the thruster

Computational: Current continuity observed in fig. 8 in 4-5 iterations of the full loop solver indicate the solver stability and efficiency. Thrust and efficiency in fig. 9 are quite comparable with the reported performance of such a MPD device [8, 12]. Figs. 8 to 10 show how the solution and performance variables are converging to their respective steady values in about 10 iterations, along with stability on varying mesh size.

In general, the physical understanding as indicated by the above simulation are reasonably consistent with the published literature on theory and experimental findings [11-13]. In the proposed hybrid multistep computation, with each part of the solver being allowed to converge up to a residue of $10^{-3}$, it takes about 15-20 iterations for the combined loop residue to reach the desire level of
Thus, the results show consistency with the known physics of the problem, the convergence of each part of the solver and grid stability of solution.

**Figure 8.** Convergence of Current for mesh sizes $80 \times 60$ (+), $100 \times 757$ (o) and $120 \times 90$ (*).

**Figure 9.** Convergence of Thrust (L) and Efficiency (R) for mesh sizes $80 \times 60$ (+), $100 \times 757$ (o) and $120 \times 90$ (*).

**Figure 10.** Convergence of fluid (L) and overall solver (R).

### 4. Summary

A hybrid multistep method for the simulation of plasma thruster with a single fluid model is presented. The procedure is developed to solve for electromagnetic aerothermodynamics of internal flows in MPD thrusters driven by an external power source at a constant voltage. The scheme is general for any cylindrical-conical geometries and physical input parameters. The computational method involved
solving for potential Poisson equation as a starter, followed by magnetic transport with Ohm's law and the cylindrical Navier Stokes, the above sequence to be repeated along with voltage control till convergence. Results of simulation for fully ionized argon plasma indicate residue of each computational module and that of the full solver converging to desired levels. Variables of electric field distribution, magnetic field intensity, current distribution, current at and near the electrodes and their effects on the fluid density, velocity and pressure are all observed to be in consistent with the known and reported physics of the problem. Performance variables of thrust, specific impulse and efficiency are quite comparable to those reported. In short, the method exhibited physical consistency, convergence as well as grid stability for the problem studied. The procedure accounts for only single fluid, its computational requirements are relatively low and hence, can be potentially explored as a reduced model for constant voltage driven plasma thruster simulation.

References
[1] LaPointe M R 1991 AIAA Paper 2341
[2] Heiermann J and Auweter-Kurtz M 2004 International Journal of Numerical Methods for Heat and Fluid Flow 14 559-572
[3] Jaisankar S, Shivashankar K and R Rao S V 2007 AIAA Paper 3946
[4] Spitzer Jr L and Ha’rm R 1953 Physical Review 89 977
[5] Jaisankar S and Sheshadri T S 2010 AIAA Paper 7022
[6] Jaisankar S and Rao S V R 2007 Journal of Computational Physics 221 577-599
[7] Jaisankar S and Sheshadri T S 2013 Journal of Computational Physics 233 83-99
[8] Yoshikawa T, Kagaya Y, Yokoi Y and Tahara H 17th International Electric Propulsion Conference, Tokyo, 1984 IEPC-84-58
[9] Jaisankar S, Sushma T V and Sheshadri T S 2012 AIAA Paper 0191
[10] Schlichting H 1968 Boundary-layer theory, Sixth Edition (McGraw-Hill, New York)
[11] Wolff M, Kelly A and Jahn R 17th International Electric Propulsion Conference, Tokyo, 1984 IEPC-84-32
[12] Jahn R G 1968 Physics of Electric Propulsion (McGraw-Hill, New York)
[13] Kubota K, Funaki I and Okuno Y 2009 IEEE Transactions on Plasma Science 37 2390-2398