Response Analysis of Free-Membrane Transition-Edge Detectors with Thin Substrates

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Abstract. In this work, we have investigated and presented the bolometric response of free-standing membrane type of superconductive transition-edge detectors at micro-scale and submicron-scale substrate thicknesses. The technique applied here is based on the finite-size scaling method which is now extensively used for the calculation of thermal parameters of materials at low dimensions when the significance of surface energy contribution is not negligible compared to that of the total bulk energy. This method is used to evaluate the variation of specific heat per unit volume and heat conductivity and hence its effect on the total heat capacitance and conductance of the substrate versus its thickness. By using the thermal parameters obtained from the applied method we calculated the IR response in these types of the devices. This response when inspected at low thicknesses of substrates showed anomalous behavior with respect to previous response analyses. Discussed in this paper, we also obtained the optimized substrate thickness for maximum responsivity of the device. The obtained results are compared to previous response calculations and experimental measurements.

1. Introduction

Superconductive bolometers have attracted much attention because of their high sensitivity at long wavelengths. Three basic types of these detectors have been introduced: the bolometer on a solid substrate, the bolometer on a thermal filament and the bolometer on a membrane. Measurements indicated that the free-membrane type exhibits more sensitivity than the other two basic types of bolometric detectors [1]. The empirical result compares very well with previous theoretical analyses, which have been derived in [2], where the IR response of bolometric detectors is presented as follow,

\[ \frac{r_v}{G + j \omega C} = \frac{\eta I}{G + j \omega C} \]  \hspace{1cm} (1)

where \( G \) is the total thermal conductance (steady-state and oscillating conductivity), \( I \) is DC bias current, \( \eta \) is the fraction of the incident power absorbed by the bolometer and \( C \) is the total heat capacitance observed by the film. For the free-membrane type, these two parameters are considered to be less than that of the other types. This leads to a higher value of response for the free-membrane type. Therefore, the theoretically predicted sensitivity for the free-membrane type is more than the thermal filament and solid substrate types as confirmed by experiments.
It can be shown that the application of a thinner substrate would result in a lowering of $C$ and $G$ and therefore more sensitivity. It seems that we can increase the responsivity to any desired value by thinning the substrate. This might be true if the specific heat per volume and thermal conductivity of the substrate remains constant, while its thickness is decreased. But empirical work shows that this assumption is not right and indicates the variation of thermal parameters of substrates with respect to their thickness especially for very thin substrates. It has been shown that by decreasing the substrate thickness the heat conductivity increases in the range of micrometer [3] and decreases in the range of hundreds of nanometers [4] through two different mechanisms. Theoretically this dependency of specific heat and thermal conductivity on the dimensions of a material is described by finite size effects [5]. In the adopted method, we consider the total energy of a substance as a sum of its surface and bulk energies. When the dimensions are large, the surface energy is usually much less than the bulk one and so its effect on the total energy is negligible. However, when the substance is miniaturized, this effect becomes more pronounced, and the term of surface energy becomes dominant in comparison to the bulk energy. Thus, for small size bodies we cannot neglect the surface energy in the calculation of the total energy. It is known that the specific heat is the rate of variation of total energy with respect to temperature. Also, the thermal conductivity per volume in electrically isolated solids is linearly related to specific heat through the sound velocity and phonon mean free path in the substrate [6]. Knowing these relations of thermal parameters to total energy, we conclude that for small dimensions, surface effects are required in the calculation of specific heat per unit volume and thermal conductivity as it is done for total energy. As we will illustrate in the next section, the inclusion of surface effects in the calculation of thermal parameters for a thin substrate caused it to become dependent on substrate thickness which is confirmed by the measurements as we mentioned before. In this work, we obtained the dependencies of specific heat per unit volume and thermal conductivity on substrate thickness and used them to calculate the total heat capacitance and thermal conductance versus substrate thickness. Then these parameters are substituted in equation (1) to obtain the IR response for membrane type of superconductive bolometers. The results are also applied for the optimization of the detectivity of this type of detectors.

2. Theoretical Model
The calculations were performed on a thin layer crystal substrate with a square surface. Thus, the surface energy was as important as the volume energy when the thickness of substrate was brought down. The total energy of the substrate could be considered as the addition of the energies of the volume and the surface,

$$E_{\text{tot}} = E_V + E_S$$ \hspace{1cm} (2)

Here, $E_V$ and $E_S$ are the total volume and surface energies. $E_V$ is a function of the substrate volume and $E_S$ is a function of its surface area. Dividing both sides of above equation by the volume of the substrate leads to the finite size scaling method where the energies per volume are described as below [5],

$$f_{\text{tot}} = f_V + \frac{2}{d} f_S + ...$$ \hspace{1cm} (3)

where $d$ is the substrate thickness and $f_V, f_S, f_{\text{tot}}$ are the energies per unit volume as discussed above and the factor of two comes from taking into account the two possible polarizabilities for the two-dimensional method discussed in the following to calculate the $U_{2D}$. These energies are calculated by using phonon heat transfer equations and Debye model at low temperatures [6, 7]. For calculating the two dimensional density of states, periodic boundary conditions are assumed over $N^2$ primitive samples within a square plane of side $L$, which calls for the quantization of the momentum vector elements on the accompanying plane coordinates, $x$ and $y$, i.e. they can only take the following values,

$$0, \pm \frac{2\pi}{L}, ... (N \times \frac{2\pi}{L})$$ \hspace{1cm} (4)
Consequently, there is only one value of $K$ per unit area $(2\pi/L)^2$ on $k$-plane and $(2\pi/L)^2$ allowed values of $K$ per unit area of $k$-plane for each polarization and each branch. The area of the specimen, $S$ is $L^2$. Hence the total number of modes with a wavevector with a magnitude less than $K$ for each polarization in a circular area of radius $K$ on this plane is $N=SK^2/4\pi$.

By direct differentiation with respect to frequency, $D(\omega)$ the density of states for each polarization is:

$$D(\omega) = \frac{S}{2\pi} K \frac{dK}{d\omega}$$

which, by using Debye approximation, is reduced to be $S\omega/2\pi\nu$, where $\nu$ is the speed of sound in the material and is assumed constant.

By neglecting the contributions of phonon modes with frequencies higher than the Debye frequency, $\omega_D$ which is the maximum frequency that makes up the crystal, the two-dimensional internal energy, $U_{2D}$ is found by integration of $D(\omega)$ over $\omega$-space to the boundary value $\omega_D$ by considering the fact that a set of identical harmonic oscillators in thermal equilibrium take a Plank distribution. The result divided by the area of the substrate turns out to be,

$$U_{2D} = \frac{\hbar}{d\pi\nu^2} \int_0^{\omega_D} \frac{\omega^2 d\omega}{\exp\left(\frac{\hbar\omega}{k_BT}\right)}$$

where the two-dimensional specific heat per unit area is found by differentiation of the $U_{2D}$ with respect to temperature, at constant surface area, $S$.

To evaluate $U_{3D}$, a three-dimensional cubic space with edges of length $L$ is assumed, the same strategy approached above leads to the quantization of $K_x$, $K_y$ and $K_z$ to only the following values,

$$0, \pm \left(\frac{2\pi}{L}\right), ..., \left(\frac{N \times 2\pi}{L}\right)$$

Hence there is only one value of $K$ per unit volume $(2\pi/L)^3$ in $k$-space and $(2\pi/L)^3$ allowed values of $K$ per unit volume of $k$-space for each polarization and branch. The volume of the specimen, $V$ is $L^3$. Hence the total number of modes corresponding to a wavevector with magnitude less than $K$ for each polarization in a spherical volume of radius $K$ is,

$$N = \left(\frac{L}{2\pi}\right)^3 \times \left(\frac{4\pi K^3}{3}\right)$$

Therefore, the three-dimensional density of states after utilizing Debye’s approximation turns out to be,

$$D(\omega) = \frac{V \omega^3}{2\pi^2 V^3}$$

Neglecting the contributions of the modes with frequencies higher than Debye frequency $\omega_D$ and using the Plank distribution function, and for the sake of brevity assuming the phonon velocity independent of the polarization, the result is multiplied by a factor of three to yield the three-dimensional internal energy as,

$$U_{3D} = \frac{3\hbar}{4\pi^2 V^3} \int_0^{\omega_D} \frac{\omega^2 d\omega}{\exp\left(\frac{\hbar\omega}{k_BT}\right)}$$

Now the specific heat per unit volume and area can be found in two and three dimensions as follow,

$$c_{V2D} = \left(\frac{\partial U_{2D}}{\partial T}\right)_S$$

$$c_{V3D} = \left(\frac{\partial U_{3D}}{\partial T}\right)_S$$
The total heat capacitance could be assumed as the sum of the two latter heat capacitances,

\[ c_v = c_{V3D} + \frac{2}{d} c_{V2D} \]  

(13)

By using 1D model, diffusion length is calculated as below,

\[ L_f = \left( \frac{D}{\pi f} \right) \]  

(14)

where \( f \) is the modulation frequency and \( D = k/cV \) is the thermal diffusivity of this substrate and \( k \) is thermal conductivity of material. The thermal conductance, \( G \) and heat capacitance, \( C \) of the substrate are:

\[ G_s = \frac{kS}{L_f} \]  

(15)

\[ C_s = c_{Vs}SL_f \]  

(16)

where \( S \) is the substrate area. Substrate thickness in membrane type bolometers is typically less than the thermal diffusion length and therefore for calculation of \( G \) and \( C \), thermal diffusion length is replaced by substrate thickness. We also know from the 1D model that \( G = G_t + G(0) \) where \( G_t \) is the thermal conductance at modulation frequency and \( G(0) \) is the steady-state thermal conductance due to lateral thermal diffusion. \( G_t \) is the series of two thermal conductances, first the substrate conductance and second the boundary conductance between the back of the active area of the substrate and the cold-head. So the total thermal conductance at modulation frequency is zero because in the membrane type the substrate is floating behind it there is no thermal contact. The only thermal conductance in this type of bolometer is the steady-state conductance, \( G(0) \) which corresponds to lateral heat flow through the substrate and it is also taken to be constant because it is mainly determined by the thermal boundary resistance of substrate to cold-head (we consider that the lateral length of the substrate is more than thermal diffusion length in a way that lateral heat flow at the modulation frequency can be neglected). By these assumptions responsivity is calculated using equation (1)

3. Results and Analyses

Here we present the results of our calculation for substrate with cubic active area. In figure 1 the specific heat per unit volume versus the thickness of the substrate is plotted. This figure shows that the specific heat per unit volume of the substrate at large thicknesses is approximately constant. This is in line with our theoretical analysis because as mentioned previously, when the thickness is large we expect that the surface has little effect and so the dependency on thickness is thought to be negligible. Below 100µm, specific heat varies with respect to the substrate thickness. This behaviour of specific heat is also expected because at this range of thickness, the surface energy is comparable to the bulk energy. Therefore, decreasing one dimension and so the total volume of the substrate makes the surface energy more significant with respect to bulk energy. In [8] similar results were obtained for thin substrates.

The heat capacitance versus substrate thickness is depicted in figure 3. As illustrated, heat capacitance decays until a specific thickness, called “new thickness” in this work. Below the new thickness, heat capacitance is moderately constant. The new thickness is strongly related to substrate material.

Variation of time constant with respect to the substrate thickness is presented in figure 4. Also in this figure, we observed two different behaviours below and above the new thickness. This behaviour is consistent with a previously reported measurement on the time constant of NTC thermal sensors,
which has analogous principle of operation and similar thermal model to that of transition-edge sensors [11].

In figure 2 the frequency-dependent response of the bolometer versus the substrate thickness is drawn at an excitation frequency of 4Hz. For this calculation, thermal boundary resistance at the substrate-cold-head of the samples, RSC is taken to be 6.82K\(^{-1}\)cm\(^2\) which is found experimentally for MgO crystal lattice connected to gold cold-head [2]. Using this value for RSC and neglecting thermal resistance of substrate, which is usually one order of magnitude below RSC, total thermal conductance, G calculated to be approximately 14.6Wmm\(^{-2}\)K\(^{-1}\).

It is noteworthy to see that as the thickness of the substrate diminishes the responsivity is enhanced until it reaches a fairly constant value at thicknesses of about 10µm. Thus thinning the substrate is desirable until an particular thickness of substrate and after that, the responsivity would not improve.
The three-dimensional plot of the frequency dependent response versus substrate thickness and excitation frequency of the bolometer is drawn in figure 3. Similar to figure 2, in this figure it is observed that responsivity saturates below a particular substrate thickness. As it is illustrated in this figure, this impact of the substrate thickness which was also predicted by the finite size scaling method is more tangible as the frequency is decreased. It should also be noted that in addition to saturating responsivity at small value of substrate thickness, it also become constant for thicker substrates at near-to-DC modulation frequencies. This is attributed to slight effect of the heat capacitance at low frequencies which is the only thickness dependent parameter in the calculation. Saturation of response at low frequencies for different substrate thicknesses is previously reported [8].
4. Conclusions
The magnitude of response in superconductive edge transition bolometers is contingent on thickness of the substrate material. As thickness is lowered the response shows an unexpected behaviour which relates to its surface energy. By using finite size scaling method and calculating surface energy the response behaviour can be explained as a function of substrate thickness.

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