A control of hexapod based on the neural networks solution for forward kinematics

Y A Zhukov¹, E B Korotkov¹, A V Moroz¹, V V Zhukova and A M Abramov³

¹ Baltic State Technical University «VOENMEH»
² Peter the Great St.Petersburg Polytechnic University
³ Novgorod State University, ul. B. St. Petersburgskaya, 41, 173003 Velikiy Novgorod, Russia

Abstract. This research is a part of the work implemented by BSTU "VOENMEH" under the financial support of the Ministry of Education and Science of the Russian Federation for design and development of a precision mechanism with parallel kinematics called "Hexapod". In the paper the inverse kinematics problem of Hexapod is described. The solution of the forward kinematics problem is solved by the iterative Newton-Raphson method. A solution of the forward kinematics problem and approximation of the Jacobian matrix of the hexapod based on the apparatus of artificial neural networks is proposed. The results of synthesis of neural networks are presented. An algorithm for the application of neural network kinematics in the hexapode control system is described. A model of a digital control system based on the neural network algorithm under investigation is implemented. The estimation of control accuracy in the positional mode is obtained.

Introduction

BSTU "VOENMEH" and JSC ISS-Reshetnev Company are working together on creating a number of multi-step mechanisms of parallel kinematics [1, 2] (MPK) to ensure the precise positioning and orientation of spaceborne instruments and devices [3]. The object of our research is the MPK ("Hexapod") based on six linear drives with stepper motors, depicted in Figure 1. The hexapod, constructed according to the Stewart platform scheme [4], consists of a fixed base and a movable platform controlled by six identical linear actuators - legs (bars, posts). Each leg consists of two half-stems connected with hinges to the base and platform.

The task of the hexapod control system is to develop the positions of the mobile platform given in the Cartesian coordinates with the respect to the base with an accuracy of ± 10 μm and orientation with an accuracy of ± 30 arc sec.

The main difficulty of controlling the MPK is that when regulating in Cartesian coordinates, it is required to generate forces in linear drives-the legs of the hexapod. The most simple and popular approach for solving this problem is to implement a separate control of the lengths of the hexapod's legs [5], in which the control system is divided into six regulators by each leg. The input of each regulator is fed with a signal of the required length of the leg, calculated on the basis of the solution of the inverse kinematics problem [6], control is formed on the basis of the signal from the linear actuator...
feedback sensor. Disadvantages of separate control are especially acute when positioning and orienting a large object with large moments of inertia and a remote center of mass.

Figure 1. Sketch of hexapod.

In our work, we study the approach to controlling based on the solution of the forward kinematics problem [6], in which regulation with the legs of the hexapod is based on a control error in Cartesian coordinates. The main difficulties of this approach are connected with obtaining a fast solution of the forward kinematics problem, as well as with the transformation of regulation signals from the space of Cartesian coordinates to the space of legs lengths. In [6-8], a forward kinematics problem was solved by iterative numerical methods, but an approach based on artificial neural networks (ANN) is of particular interest.

At present, many researchers are considering with great interest the possibilities of applying ANN theory to solving the problems of controlling mechanisms of parallel kinematics. In known works on the application of ANN for the forward kinematics problem of the Stuart platform, only a part of the problems are solved. In [9-12] the possibility of successful application of ANN direct propagation is shown. In [13-14], the possibilities of recurrent networks (RNN), and in work [15] - nonlinear autoregressive exogenous neural network (NLARX). Essential shortcomings of the research are their poor orientation to the hardware implementation and the insufficient validity of the ANN device application in comparison with the traditional approaches to solving of the forward problem of kinematics.

Thus, the main goal of the present paper is to obtain a neural network solution of the forward kinematics problem and to study its application in the control problem in Cartesian coordinates with a hexapod of space purpose.

**Kinematics of hexapod**

When managing the hexapod, two kinematics problems are solved [2, 6]:

1) inverse problem of kinematics (IPK) - determination of the length of the legs (half-stems) according to the given position and orientation of the mobile platform;

2) forward problem of kinematics (FPK) - determination of the position and orientation of the platform for the given length of the legs.

To solve kinematics problems, we introduce coordinate systems, as shown in Figure 2. The fixed coordinate system OXYZ is connected to the base, the mobile coordinate system O’X’Y’Z’ is connected to the platform. We define the initial “zero” position of the symmetrical hexapod, in which the legs have the same elongation. Thus, in this position, the coordinate system O’X’Y’Z’ relative to the OXYZ coordinate system is shifted along the axis OZ by the parameter h0.

The numerical solution of the kinematics problems will depend on the design parameters of the hexapod shown in Figure 2:
\( R_b, R_p \) are the radii of the pitch circles of the base and platform, respectively, on which the hinges are placed;

\( C_b, C_p \) is the distance between adjacent pairs (1-2, 3-4, 5-6) of the base joints and pairs (2-3, 4-5, 6-1) of the platform hinges, respectively. The length of the legs in the "zero" position is defined as \( L_0 \) - "zero leg lengths". Let's enter the leg numbers, as shown in Figure 2.

During the design of the hexapod construction, the following parameters are selected: \( h_0 = 0.4 \) m; \( R_b = 0.175 \) m; \( R_p = 0.15 \) m; \( C_b = 0.045 \) m; \( C_p = 0.045 \) m.

The position of the center \( O' \) platform with respect to the fixed coordinate system is specified with the Cartesian coordinates \( X, Y, Z \). The Euler angles are used to determine the orientation of the platform [16] \( \varphi, \theta, \psi \). Thus, the linear position and the angular orientation of the platform are given with the vector \( \mathbf{q} = [X, Y, Z, \varphi, \theta, \psi]^T \).

The set working range of the hexapod is: with respect to the X coordinate \( \pm 100 \) mm, the Y coordinate \( \pm 100 \) mm, the Z coordinate \( \pm 25 \) mm, the angular coordinates \( \pm 7 \) deg.

![Figure 2. Kinematic scheme and design parameters of the hexapod.](image)

For the connection between a fixed and a moving coordinate system, we use the matrix transformation in homogeneous coordinates [16]:

\[
\mathbf{r} = \mathbf{T} \times \mathbf{r'},
\]

(1)

where \( \mathbf{r} = [X, Y, Z, 1]^T \) is the vector of homogeneous coordinates of some point in the OXYZ coordinate system, \( \mathbf{T} \) is the index denoting the transpose of the vector, \( \mathbf{r'} \) is the vector of homogeneous coordinates of the same point relative to the mobile coordinate system.

In transformation (1), the expanded matrix \( \mathbf{T} \) is defined through \( \mathbf{p} = [X', Y', Z']^T \) - the coordinates of the origin of the mobile system with respect to the fixed - and the rotation matrix \( \mathbf{R} \)

\[
\mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{p} \\ 0 & 0 & 0 & 1 \end{bmatrix},
\]

(2)

Rotation matrix for choosing Euler angles - expression

\[
\mathbf{R} = \begin{bmatrix}
\cos \varphi \cos \psi + \sin \varphi \sin \theta \sin \psi & \sin \varphi \sin \theta \cos \psi - \cos \varphi \sin \psi & \sin \varphi \cos \theta \\
\cos \theta \sin \psi & \cos \theta \cos \psi & -\sin \theta \\
\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi & \sin \varphi \sin \psi + \cos \varphi \sin \theta \cos \psi & \cos \varphi \cos \theta
\end{bmatrix},
\]

(3)
Calculating from (1-3) the transformation matrix $T$ for a given vector $q$ and knowing the coordinates of the attachment points of the hinges of the legs to the base ($r_{Ai}$ - the vector of homogeneous coordinates of the point $A_i$ in the OXYZ system) and to the platform ($r_{Bi}$ - the vector of homogeneous coordinates of the point $B_i$ with respect to $O'X'Y'Z'$), it is easy to solve the inverse kinematics problem-the length of the $i$-th leg of the hexapod

$$L_i = \| r_{Ai} - T \times r_{Bi} \|, \quad (4)$$

Essentially, the solution of the inverse kinematics problem is represented by a set of nonlinear algebraic expressions

$$L = H(q), \quad (5)$$

The solution of the forward kinematics problem can be represented by finding the inverse function

$$q = H^{-1}(L). \quad (6)$$

Since an analytical solution of problem (6) is difficult, in practice, numerical iterative Newton-Raphson methods [6] are used to obtain vector estimates $q_{k+1}$ from a given vector $L_k$,

$$q_{k+1} = q_k + J^{-1}(L_k - H(q_k)), \quad (7)$$

where $J$ is the Jacobi matrix of the Stewart platform, an algorithm for computing which is given in [17].

**Neural network solution of kinematics problems**

In the present paper, the apparatus of artificial neural networks is used for the approximation solution (6). The investigations have shown that the separate neural network approximation of the components of the vector $q$, the scheme of which is shown in Figure 3, is the best way to use ANN.

![Figure 3](image_url)

**Figure 3.** The scheme of the neural network solution of the forward kinematics problem.

The application of this approach makes it possible to accelerate and simplify the process of learning and synthesizing ANN for the solution of a FPK with a specified accuracy, during which the following are determined: algorithm for forming a training sample, the network architecture (network type, number of hidden layers, number of neurons in each layer), learning algorithm, matrix network implementation.

It is convenient to create and train a neural network suitable for realizing the problems of approximation of multidimensional nonlinear functions using modern mathematical modeling
environments, which include the Matlab Neural Network Toolbox expansion package [18]. The toolkit of this package was used in the design of neural networks for solving the FPK.

The research has shown that the architecture of the three-layer cascade ANN of direct propagation with two hidden layers, the architecture of which is shown in Figure 4, satisfies the given working range and the required accuracy of the FPK.

![Figure 4. Structure of a cascade neural network of direct propagation.](image)

The synthesis procedure is performed for six ANNs approximating the components of the vector \( q \), in this paper the results for the ANN1 are demonstrated. On the basis of the solution of the inverse problem of kinematics (5), a training sample is formed as a linearly distributed array for the values of the working range of the hexapod. The dimension of the array for the training sample was \( 6 \times P_6 \) (\( P = 5 \)). A three-layer network with two hidden layers was selected, 24 neurons were specified in the hidden layers, and a hyperbolic tangent was used as an activation function. In the output layer there were linear neurons. In the course of ANN training the Levenberg-Marquardt method was used. Learning outcomes for 1000 epochs are shown in the figure 5.

![Figure 5. Learning outcomes of the neural network](image)

To test the accuracy of the solution, a test sample of dimension \( 6 \times P_6 \) (\( P = 10 \)) was formed, the maximum positional error of the solution of FPK for the coordinate \( X - \Delta_{\text{max}} = 1.4 \mu m \) was calculated, which satisfies the requirements of the permissible error of hexapod control.

For each synthesized ANN, a matrix realization, defined with expression
where the matrices \( W_{11}, W_{21}, W_{31}, W_{22}, W_{32}, W_{33} \) set the synaptic weights in the layers of the neural network, and \( b_1, b_2, b_3 \) - the displacements in each layer, their values were obtained during the training of the ANN; \( K_{inp}, x_{0}, x_{min} \) - matrices of coefficients of valuation of input values (values of the length vector of legs \( L \)); \( K_{out}, y_0, y_{min} \) - matrices of recovery coefficients of the output of the network \( y \) are determined in the normalization of training samples; \( F_{th} \) - the activation function of the hyperbolic tangent, is found from expression

\[
F_{th}(z) = \frac{2}{1 + e^{-2z}} - 1.
\]

A similar approach was used to synthesize the neural network of approximation of the inverse Jacobi matrix in expression (7). The three-layer network was trained; its structure is shown in Figure 6. Learning outcomes are shown in Figure 7.

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**Figure 6.** The structure of the cascade network of approximating the inverse Jacobi matrix.

**Figure 7.** The results of the approximation of the inverse Jacobi matrix.
The maximum absolute error of approximation of the inverse Jacobi matrix on the test sample for the synthesized network was \( \Delta_{\text{max}} = 0.062 \text{ rad/m} \).

**Hexapod Management System Model**

An estimate of the quality of hexapod control is realized for the algorithm of the stabilizing proportional-differential control in Cartesian coordinates with the calculation of the control forces based on the solution of the inverse dynamic problem of Lagrange’s equation of the second kind for the mobile platform [19]

\[
M(q)\ddot{q} + N(q, \dot{q}) = J^T \cdot F,
\]

where \( M(q) \) is the matrix of inertia of the platform, \( N(q, \dot{q}) \) is the vector of centrifugal, gravitational and Coriolis forces. The algorithm for calculating the components of expression (10) is described in detail in [17, 19].

When implementing the control method on the basis of the estimated moment (called "computed-torque control") of the hexapod control forces \( F \) are derived from expressions

\[
\begin{align*}
F & = (J^T)^{-1} \cdot (M(q)\ddot{q} + N(q, \dot{q}) + K_p e + K_d \dot{e}) \\
\dot{e} & = q_{dd} - H^{-1}(L) \\
\dot{e} & = \dot{q}_{dd} - J^{-1} \cdot \dot{L}
\end{align*}
\]

where \( K_p, K_d \) - the diagonal matrices of the adjustment factors of the PD controller, the \( e \)-vector of the control errors \( \ddot{q}, \dot{q}, q \), - the accelerating signals, velocities and positions in Cartesian coordinates obtained from the trajectory planner, \( L, \dot{L} \) - measured with the speed sensors and the length of the hexapod’s legs, \( H^{-1}(L) \) - the iterative solution of the forward problem kinematics (7).

\( F_{\text{ann}}(L) \) - matrix function of the implementation of the neural network solution of the FPK, obtained on the basis of (8) and (9) for the ANN structure shown in Figure 3.

Applying the neural network approximation for the components of the kinematic problem in (11), we obtain a control algorithm based on the estimated moment with a neural network solution of the FPK and approximation of the inverse Jacobi matrices, which is determined by the relations

\[
\begin{align*}
F & = F_{\text{ann2}}(L) \cdot (M(q)\ddot{q} + N(q, \dot{q}) + K_p e + K_d \dot{e}) \\
\dot{e} & = q_{dd} - F_{\text{ann}}(L) \\
\dot{e} & = \dot{q}_{dd} - F_{\text{ann1}}(L) \cdot \dot{L}
\end{align*}
\]

where \( F_{\text{ann}}(L) \) is the matrix function of the implementation of the neural network solution of the FPK, obtained on the basis of (8) and (9), \( F_{\text{ann2}}(L) \) - neural network approximation of the reciprocal transposed Jacobi matrix, \( F_{\text{ann1}}(L) \) - neural network approximation of the inverse Jacobi matrix.

Based on the structure of the regulator (12) and the results of the ANN synthesis in the Matlab Simulink mathematical modeling environment, a digital hexapod control system model was created, shown in Figure 8.
The hexapod control is simulated under conditions of zero gravity. The hexapod control system generates foot forces on the basis of the regulator (12) realized in the S-Function "computed torque control xyz". The input of the S-Function block includes: the signals of position, velocity, acceleration in Cartesian coordinates ("xyz_ref", "Dxyz_ref", "D2xyz_ref") and feedback signals - position and speed of leg lengths. Commanders in Cartesian coordinates are calculated according to the desired position of the platform on the basis of solving the problem of planning trajectories.

The hexapod dynamics model consists of the following elements: "Base" - base, "Plate" - platform, ("Leg1" – “Leg6”) - models of the hexapod's legs dynamics, "Machine Environment"- general environment parameters of the simulation system (zero acceleration of free fall, the accuracy of assembly of the model of mechanics), "Ground" - the basis of the absolute reference frame, "Weld" - the base soldering with the absolute coordinate system. The model allows you to calculate the positioning and orientation errors of the platform by the signal from the absolute position sensor "Body Sensor". On the input "Outer Force" signals of external forces are applied to the center of mass of the platform. The same elements "Go to" and "From" determine the connections in the model. The model of the hexapod leg, shown in Figure 9, consists of two bodies of semi-stems "Base Cylinder" and "Plate Cylinder", connected by a cylindrical hinge "Leg Cylindrical", two-axis hinges of the base and platform represented by the objects "2d joint base" and "2d joint plate" respectively. The "Joint Sensor" block - an element of the information system - generates signals in the model for the linear position and speed of the drive, "Joint Actuator" transmits to the mechanics model the resulting force that determines the motion of bodies in the linear drive. The model takes into account the viscous friction force, linearly dependent on the rate of change in leg length.
Modeling of the control system

During the simulation of the hexapod control system, the parameters of the regulator PD - the matrix values $K_p, K_d$ - were synthesized. Synthesis was carried out in the mode of position control that is transfer of the system from the initial position $q_i = [0, 0, 0.35, 0, 0, 0]^T$ to the final position $q_f = [0.1, -0.1, 0.375, -0.122, 0.122, -0.122]^T$. The parameters of the hexapod mechanics are specified: the mass of the platform with the inertial load is 100 kg, the main moments of inertia of the platform $J_{xx} = 4900 \text{ kg} \cdot \text{m}^2, J_{yy} = 4900 \text{ kg} \cdot \text{m}^2, J_{zz} = 6300 \text{ kg} \cdot \text{m}^2$. For the digital realization of the algorithm (12), the time-quantization period $T = 0.001 \text{ sec}$ is determined. An estimate of the hexapod tracking errors, which are shown in Figure 10, is obtained.

As can be seen from the figures, the use of the neural network approximation of the hexapod kinematics provides a stable control; the steady-state control error is within the required range.

Conclusion

In the course of this research, algorithms for managing a hexapod based on the neural network solution of the forward kinematics problem and approximation of the Jacobi matrices are developed. A model of a digital hexapod control system has been implemented. Modeling of the hexapod control system with a workload in the position control mode was carried out. The performed studies showed the effectiveness of the application of neural network approximation of kinematic problems, as well as their successful implementation in the hexapod control system. When implementing neural network kinematics problems in parallel computing systems, the algorithms under study will get a higher speed than traditional ones based on iterative solutions.

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