Dynamics and control of space debris during its contactless ion beam assisted removal

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Abstract. The space debris removal is one of the most important challenges of astronautics. The creation of contactless transportation systems based on the ion beam is a promising direction for solving this problem. The orientation of the debris object in the ion beam has a great influence on the magnitude and direction of the force transmitted by this beam. The aims of the work are to study the attitude dynamics of space debris during its transportation by ion beam, to search angular motion modes, which are most favourable from the point of view of the force transmission efficiency by the ion beam, and to develop the engine thrust control law that ensures the transfer of the object to these motion modes. A mathematical model describing the space debris motion under the influence of gravitational, and ion beam forces and torques is developed. An analysis of the cylindrical space debris object’s undisturbed attitude motion is carried out, and the most favourable orientation of space debris is determined. Control law for transferring the space debris in the required angular motion mode by a change in the thrust of the ion engine are considered. Numerical simulation confirmed the effectiveness of the proposed control law.

1. Introduction
The space debris removal is one of the most important challenges of modern astronautics. Non-functioning satellites and rocket stages pose a great potential danger, since in a mutual collision they can cause the formation of a large number of small space debris fragments. Various ways to large space debris removal have been actively discussed in recent years [1-3]. The creation of contactless transportation systems based on the ion beam is a promising direction for solving this problem. Such systems are safe because they do not require capturing or docking with space debris, besides, they can be built on the basis of existing technologies. The idea of using an additional electrodynamic engine to generate drug force on a space debris object by blowing it with a ions flow was expressed by C. Bombardelli, J. Pelaez and S. Kitamura in the works [4, 5]. To compensate the thrust of the electrodynamic engine, it is proposed to use a second oppositely directed engine. An alternative approach involves the use of a more economical double-sided ion thruster [6]. To date, quite a detailed study of a space debris object’s center of mass motion during its ion beam assisted removal [5, 7]. The angular motion of a space debris object relative to its center of mass under the action of an ion beam was studied in [8, 9]. A methodology for synthesizing the parameters of an active spacecraft for ion beam removal mission was proposed in [10]. Of great interest is a monograph [11] containing important and interesting scientific results on the topic of ion beam transportation. Studies [8, 11-13] show that the orientation of the debris object in the ion beam has a great influence on the magnitude and direction of the force transmitted by this beam. Obviously for space debris of given form, its angular position at which the force generated by the ion beam will be maximum can be found. The aims of the work are to study the attitude dynamics of space debris during its transportation by ion beam, to search angular motion modes, which are most favourable from the point of view of the force...
transmission efficiency by the ion beam, and to develop the engine thrust control law that ensures the transfer of the space debris object to these motion modes from an arbitrary initial state.

2. Mathematical model

A considered mechanical system consists of an active spacecraft and a space debris object. The spacecraft is the material point A, and the space debris is a rigid body with the center of mass at point B (figure 1). The system moves in the orbit plane under the action of gravitational, and ion beam forces and torques. The position of the space debris object can be determined using the angle $\nu$, distance $r = OB$ and the angle of the object’s axis $Bx_o$ deviation $\theta$. The position of the point can be determined in the orbital coordinate system associated with space debris $Bx_o,y_o$ by the Cartesian coordinates $x_d, y_d$. The axis $Bx_o$ is directed from the center of the Earth to point B, and the axis $By_o$ is directed towards the orbital flight of the object. The total thrust of the active spacecraft is presented in the form of two projections $P_x$ and $P_y$ on the axis of the orbital coordinate system $Bx_o,y_o$. The ion beam force generated at the space debris object by the flow of ions created by the active spacecraft’s engine is presented in the form of projections $F_x$ and $F_y$. Since the ion beam force application point does not coincide with the center of mass of the object, in addition to the force, the ion beam torque $L_z$ acts on the object. The magnitude of the ion beam force and torque depends on the parameters of the spacecraft ‘s engine, the distance $AB$ and orientation of the object with respect to the ion beam axis.

![Figure 1. Considered mechanical system.](image)

Lagrange equations of the second kind can be used to compose equations of motion

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = -\frac{\partial U}{\partial q_i} + \dot{Q}_i,$$

(1)
where $T$ is the total kinetic energy of the system, $U$ is the potential energy of the system, $q_i$ is the component of generalized coordinates vector $\mathbf{q} = [r, v, \theta, x_A, y_A]$, $Q_i$ is the generalized force. The kinetic energy of the system is

$$T = \frac{m_A V_A^2}{2} + \frac{m_B V_B^2}{2} + \frac{J_z \omega_B^2}{2},$$

where $m_A$ is the mass of the spacecraft, $m_B$ is the mass of the space debris object, $V_j$ is the velocity of the $j$-th point, $J_z$ is the moment of the inertia around object’s body frame axis $Bz_b$, $\omega_B = \theta + \nu$ is the object’s angular velocity. Finding coordinates of points A and B in inertial reference frame $Ox_p, y_p$ and differentiating allows to determine projections of the points velocities on $Ox_p, y_p$ axes. Summing the squares of these projections gives

$$\sum_{i} (v_i^2 + \dot{v}_i^2) = \sum_{i} (r^2 \ddot{v}_i^2 + \dot{r}^2 \dot{v}_i^2),$$

where $r$ is the distance between points A and B.

The potential energy of the system is the sum of the spacecraft’s and the space debris object’s potential energies in the central gravitational field of the Earth

$$U = \frac{-\mu m_A}{r} - \frac{\mu m_B}{r} + \mu \left( \frac{J_z}{r^2} + \frac{3}{2} \frac{\mu J_z \cos^2 \phi + \mu J_z \sin^2 \phi + J_z}{2 r^3} \right),$$

where $\mu$ is the gravitational constant of the Earth, $J_x$ and $J_y$ are the moments of the inertia around object’s body frame axis $Bx_b$ and $By_b$ respectively. Generalized forces of non-potential forces and torques are

$$Q_A = F_x + F_y, \quad Q_B = F_x - r y_A + P_y (r + x_A) + L_z, \quad Q_{\phi} = L_z, \quad Q_{x_A} = P_x, \quad Q_{y_A} = P_y.$$

Substitution of equations (2)-(5) in (1) and the expression of the second derivatives give

$$\ddot{v} = \dddot{v} r - \dddot{\theta} \frac{r^2}{r^3} + \frac{F_x}{m_A} + \frac{3 \mu (J_x \cos^2 \theta + J_x \sin^2 \theta - J_x) + 2 J_z}{2 m_A r^4},$$

$$\ddot{\theta} = \frac{2 \dot{v} \dot{\theta}}{r} + \frac{F_y}{m_B r} + \frac{L_z}{J_z} + \frac{3 \mu (J_x - J_y) \sin \theta \cos \theta}{r^3} \left( \frac{1}{m_A r^2} + \frac{1}{m_B r^2} + \frac{1}{J_z} \right),$$

$$\dddot{x}_A = \dot{v}^2 x_A + 2 \dot{v} \dot{x}_A - \frac{2 r \dot{v} y_A}{r} + \frac{\mu (r + x_A)}{r^3} - \frac{\mu (r + x_A)}{r^3} \left( \frac{1}{m_A r^2} \right),$$

$$\dddot{y}_A = \dot{v}^2 y_A - 2 \dot{v} \dot{y}_A + \frac{2 r \dot{v} x_A}{r} - \frac{\mu y_A}{r^3} + \frac{P_y}{m_A} - \frac{F_y}{m_B} + \frac{x_A F_y}{m_B r} - \frac{3 \mu (J_x \cos^2 \theta + J_x \sin^2 \theta - J_x) + 2 J_z}{2 m_A r^4} - \frac{3 \mu J_x \sin 2 \theta}{2 m_A r^4}.$$

The resulting system of equations describes the motion of the considered mechanical system. Consider the case when the control system of the active spacecraft holds its constant relative position $(x_A = const, \ y_A = const)$. Since the forces created by the ion beam are small, it is assumed that the center of mass of the space debris object moves in a Kepler orbit with radius
Using equations (11), we pass in equation (8) to a new independent variable
\[ \tau = \Omega \nu, \]  
where \( \Omega = \sqrt{\frac{3(J_z - J_x)}{2J_x}} \). It is assumed that \( J_z > J_x \). Time derivatives can be defined as follows
\[ \dot{\theta} = \theta' \dot{\nu} = \theta' \Omega \sqrt{\frac{\mu}{p^3}} (1 + e \cos(\Omega^{-1}r))^2, \]
\[ \dot{r} = r' \dot{e} = e \sqrt{\frac{\mu}{p}} \sin(\Omega^{-1}r), \]
\[ \ddot{r} = \frac{d}{d\tau} \left( \theta' \Omega \sqrt{\frac{\mu}{p^3}} (1 + e \cos(\Omega^{-1}r))^2 \right) \dot{\nu} = \left( \theta'' (1 + e \cos \nu) - \frac{2\theta' \theta \sin(\Omega^{-1}r)}{\Omega} \right) \frac{\mu \Omega^2 (1 + e \cos(\Omega^{-1}r))^3}{p^3}, \]  
Substituting (11), (13), and (14) into (8) gives
\[ \theta'' = \frac{L_p^3}{J_x \Omega^2 \mu (1 + e \cos(\Omega^{-1}r))^4} \frac{\sin 2\theta}{1 + e \cos(\Omega^{-1}r)} + \frac{2(1 + \theta \Omega) \sin(\Omega^{-1}r)}{\Omega^2 (1 + e \cos(\Omega^{-1}r))}. \]  
Assuming that the eccentricity of the orbit is small, the expansion of (15) in the Maclaurin series and discarding nonlinear terms gives
\[ \theta'' - \frac{L_p^3}{J_x \Omega^2 \mu} \sin 2\theta = e \left( \frac{\sin 2\theta \cos(\Omega^{-1}r)}{\Omega^2} + \frac{2(1 + \theta \Omega) \sin(\Omega^{-1}r)}{\Omega^2} - \frac{4L_p^3 \cos(\Omega^{-1}r)}{J_x \Omega^2 \mu} \right). \]  
Equation (16) approximately describes the motion of a space debris object in an elliptical orbit. Since the form of the dependence \( L_p(\theta) \) is determined by the geometrical parameters of the space debris object and the parameters of the ion beam, the analysis of equation (16) in the general case is a very difficult task. Further analysis is performed for a specific object similar in parameters to Cosmos 3M upper stage [8].

### 3. Results of numerical simulation

#### 3.1. Unperturbed motion analysis

The parameters of the considered object and the ion beam are given in Table 1. Using the calculation procedure described in [8, 11], the dependences for the projections of the ion beam forces and the torque were found (Figure 2-4). The dependences shown in the figures are not \( \pi \)-periodic, since the object’s center of mass is shifted from the geometric center to the bottom end.

| Parameter                        | Value  |
|----------------------------------|--------|
| Total mass \( m_B \)             | 1400 kg|
| Length                           | 6.5 m  |
| Radius                           | 1.2 m  |
| Distance from the bottom to the center of mass | 1.5 m |
| Principal moment on inertia \( J_z \) | 1300 kg·m\(^2\) |
| Principal moments on inertia \( J_y, J_z \) | 6800 kg·m\(^2\) |
| Plasma density                   | \(2.6 \times 10^{16}\) m\(^3\) |
| Mass of particle                 | \(2.18 \times 10^{-25}\) kg |
| Radius of the beam at the beginning of the far region | 0.1 m |
| Ion beam axial velocity          | 38 000 m/s |
| Ion beam divergence angle        | 15º    |
| Distance \( AB \)                | 15 m   |
The unperturbed motion of the space debris object in a circular orbit \( (e = 0) \) is described by the equation
\[
\theta^\prime - \frac{L_z(\theta) p^3}{J_z \Omega^2 \mu} + \sin 2\theta = 0. \tag{17}
\]
This equation admits the energy integral...
The type and position of the singular points of equation (17) depends on the distance \( p = R_E + h \), where \( R_E \) is the Earth radius, \( h \) is the altitude. Figure 5 shows a bifurcation diagram where the solid lines indicate the stable equilibrium positions of the center type, and the dashed lines shows the unstable equilibrium states of the saddle type. It can be seen that with increasing orbit altitude \( h \), the stable equilibrium positions shift closer to the unstable position \( \theta = \frac{3\pi}{2} \). The deformation of the phase portrait with increasing altitude is associated with a decrease in the influence of the gravity gradient torque in comparison with the ion beam torque. In addition to the equilibrium positions, Figure 5 also shows the points of separatrix intersection with the abscissa axis (dash-dotted lines), and the intersection points of the phase trajectory, at which the maximum ion beam force \( F_y \) averaged over the period of \( \theta \) oscillation is observed (thin solid lines).

Figure 5. Bifurcation diagram.

Figure 6 shows a phase portrait of equation (17) corresponding to the altitude of 5000 km. Oscillations are \( 2\pi \)-periodic. Three areas of oscillation are observed. The external oscillation region is bounded by separatrices connecting points \( s_1 \) and \( s_3 \). Two inner regions are bounded by separatrices that begin and end at saddle point \( s_2 \). Centers \( c_1 \) and \( c_2 \) are located inside the inner oscillation regions. Comparing Figures 4 and 6, it can be noted that the centers do not correspond to the points of the maximum of the ion beam force \( F_y \) modulus (\( \theta_{\text{min}} = 2.4784 \text{ rad} \) \( F_{y\text{min}} = -8.2652 \times 10^{-3} \)). On the other hand, at the minimum point there is no equilibrium point in the phase portrait (Figure 6). Thus, to transport the object of space debris in a position corresponding to the minimum effective force \( F_{y\text{min}} \), additional efforts are required to stabilize the object in this position. In the absence of control, the space debris object will leave position \( \theta_{\text{min}} \). The corresponding phase trajectory is shown in Figure 6 by a dotted line.
3.2. Search of favorable angular position

To determine the most favorable angular position of the space debris object for its contactless transportation, a series of numerical integration of (17) with various initial conditions was performed, after which the average forces over the object’s oscillation period $T_{osc}$ were calculated for each initial position by formulas

$$F_x = \frac{1}{T_{osc}} \int_0^{T_{osc}} F_x(\theta(t)) dt,$$
$$F_y = \frac{1}{T_{osc}} \int_0^{T_{osc}} F_y(\theta(t)) dt. \tag{20}$$

Figures 7 and 8 show the dependences of the averaged forces $F_x$ and $F_y$ on various initial values $\theta_0$. From figure 8 it follows that the most favorable trajectory from the point of view of transmission of ion beam force is the trajectory passing through points $a_1$ and $a_2$. The corresponding phase trajectory is shown in Figure 6 by a solid line. It is noteworthy that the force $F_x$ for these trajectories is close to zero. This means that there is no force that tends to shift the center of mass of the space debris object from the axis of the ion flux. The average force $F_x$ is close to zero in the entire external oscillation area. The minimum points $F_y$ of the internal oscillation regions are close to the centers $c_1$ and $c_2$. However, their modules are smaller than for trajectories passing through points $a_1$ and $a_2$. Another drawback is the presence of a large modulus of force $F_x$, to parry which will have to make additional efforts.

![Figure 6. Phase portrait for $h = 5000km$.](image)

Figures 7 and 8 show the dependences of the averaged forces $F_x$ and $F_y$ on various initial values $\theta_0$.

![Figure 7. Averaged ion beam force $F_x$ for $h = 5000km$.](image)
3.3. Space debris attitude control by ion beam thrust

In the previous subsection, the phase trajectory, for which the absolute value of the average ion beam force, was determined numerically. The control law of the active spacecraft ion engine thrust, which provides the transfer of the phase space point from a certain initial position to the indicated phase trajectory, will be proposed here. In the case of unperturbed motion, the phase trajectory of equation (17) is determined by the value of the total energy (18). The idea of the control is to track the energy and its derivative, and if the operation of the ion engine helps bring the current value of energy closer to the energy \( E^* \) corresponding to the required phase trajectory, then the engine turns on. Otherwise it turns off. To implement the described idea, we multiply the ion beam torque and forces in the equations of motion by the control coefficient \( u(\theta, \theta') \). For the case of unperturbed motion the equation (17) takes form

\[
\theta^* = \frac{uL_1(\theta) p^3}{J_s \Omega^2 \mu} \sin 2\theta, \tag{21}
\]

The control coefficient can be determined as follows

\[
u = \begin{cases} 
0, & \text{when } E'(E_* - E) > 0, \\
1, & \text{otherwise}; \end{cases} \tag{22}
\]

where \( E' \) is derivative of energy (18) calculated taking into account (21) for \( u = 0 \).

\[
E' = \theta \dot{\theta}^* - \frac{L_1(\theta) p^3 \dot{\theta}'}{J_s \Omega^2 \mu} - \dot{\theta} \sin 2\theta = \theta \left( \frac{uL_1(\theta) p^3}{J_s \Omega^2 \mu} - \sin 2\theta \right) - \frac{L_1(\theta) p^3 \dot{\theta}'}{J_s \Omega^2 \mu} + \theta' \sin 2\theta = -\frac{L_1(\theta) p^3 \dot{\theta}'}{J_s \Omega^2 \mu}. \tag{23}
\]

As an example, consider the angular motion of the space debris object in a circular orbit of 5000km high, the phase portrait of which is shown in Figure 6. It is assumed that engine thrust control (22) is used to transfer the imaging point to the phase trajectory shown in Figure 6 by a solid line and passing through the points \( a_1 \) and \( a_2 \). This trajectory goes through the point \( \theta_0 = 2.0958 \text{rad}, \dot{\theta}_0 = 0 \), and corresponding energy is \( E_* = 5.6321 \). Figure 9 shows the result of numerical simulation of the controlled motion of a space debris object. Figure 10 shows the change in the energy of a space debris object during the controlled motion.

It should be noted that the proposed control law does not allow translating the phase space point into the internal oscillation area, since one energy value corresponds to two trajectories in different oscillation regions. Using the control law (22), it is impossible to tell in advance in which of the inner areas the phase space point will fall.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{Averaged ion beam force \( F_y \) for \( h = 5000 \text{km} \).}
\end{figure}
4. Conclusion
The attitude motion of a space debris object under the influence of gravitational, and ion beam forces and torques was considered in this study. The mathematical model was developed using the Lagrange formalism. A simplified equation describing the angular oscillations of the object in a Kepler orbit with low eccentricity was obtained. The undisturbed oscillations of the cylindrical space debris with the given parameters were investigated. A bifurcation diagram describing the location and type of equilibrium positions depending on the height of the orbit was constructed. It was shown that the orientation of the cylindrical object, corresponding to the maximum value of the force generated by the ion beam, does not coincide with stable equilibrium positions. To keep the object in this position, additional efforts are required. A phase trajectory on which the average ion beam force is maximum in absolute value was determined as a result of a series of numerical calculations with different initial conditions. The law of the active spacecraft engine thrust control, which ensures the transfer of the space debris object into a motion mode corresponding to movement along the found phase trajectory, was proposed. Numerical simulation confirmed the effectiveness of the proposed control law.

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