INTRODUCTION

Roof-collapse accidents are one of the most common accidents that occur during underground coal mining. The unpredictability of such accidents increases the difficulties in roof control, thereby posing a serious threat to mining safety. According to statistics provided by the Mine Safety and Health Administration (MSHA), USA, regarding production safety accidents in mines, 7738 casualties were recorded between 1966 and 2005 in the USA. In other words, 1.75 roof...
accidents occurred every 200 000 employee work hours. In 2015, 1 fatality, 117 nonfatal days-lost injuries, and 48 no-days-lost injuries occurred due to roof collapses in underground coal mines in the US, as reported by the MSHA. In recent years, China has ranked first in total global coal production. However, there has also been a significant increase in the number of safety accidents each year, resulting in massive casualties, as shown in Figure 1.

The cumulative excavation length of roadways in China can reach up to 10 000 km each year. However, approximately two to three roof failure accidents occur during roadway excavations every 10 000 m, resulting in 0.2-0.3 casualties. According to statistics, in 2015, approximately 171 safety accidents occurred during coal mine production in China, with roof-collapse accidents being the most common and accounting for 32.42% of the total number of accidents, as shown in Figure 2.

Roof-fall accidents are problematic because the exact mechanism governing their occurrence has not been determined thus far, and there exists a growing concern regarding safe coal production. Therefore, formulating reasonable support plans and preventative countermeasures is essential for improving and ensuring safety in mining operations. The main types and causes of roof-fall accidents that have been identified thus far are listed in Table 1.

Thoroughly studying and determining both the physical and mechanical properties of roof strata for roadways, the stress distribution regularity in surrounding rocks, and the influencing factors (such as buried depth and crossing section) are essential aspects for accurately determining the mechanism of roadway roof collapses. Numerous researchers have conducted studies in this regard. Wang Chen applied the complex function theory to analyze the stress distribution law of rectangular roadway roofs. A 3D elasto-plastic model, developed by Pandit B, was used in FLAC3D modeling to simulate the stress distribution in overlying strata.

Lu Chaiping explained that the stress distribution and roof fracturing trends could be determined by using the mining-induced microseisms method. Considering that clamped supported beams and simply supported beams are widely applied to support plans and preventative countermeasures is essential for improving and ensuring safety in mining operations. The main types and causes of roof-fall accidents that have been identified thus far are listed in Table 1.

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Lu Chaiping explained that the stress distribution and roof fracturing trends could be determined by using the mining-induced microseisms method. Considering that clamped supported beams and simply supported beams are widely applied to analyze the damage mechanism of roadway roofs, Yiouta-Mitra P and Sofianos AI proposed an iterative algorithm for evaluating the stability of the voussoir beam structure. Li Zhu investigated the stability of the voussoir beam structure considering the loading distribution characteristics of hinged broken blocks. Liu Hongtao reported that the caved height can be determined using clamped beam modeling.

A method for calculating the critical height for roof caving by applying Poisson’s ratio and Young’s modulus was proposed by Yan Hong. He Fulian analyzed the tensile and compressive stresses acting on a cross-section and studied the safety mechanism for anticaving considering a thick and fractural coal roof. Hu Jianhua established a geo-mechanical model with an induction caving roof simplified as a triple twist arch model. Furthermore, a rectangular elastic thin plate (RETP) model was also used to evaluate the stress distribution, caving, and subsidence of roofs. Zhang Bei studied the stress distribution during the initial breakage of an overlying roof based on the elastic theory and offered analytical solutions to calculate the deflection of roof rock. A mechanical model of a plate on an elastic foundation was established by Wang Jinan to analyze the failure processes in roof strata. Bekir Akgoz established a model analyzing micro-sized plates resting on an elastic medium by using the modified couple stress theory. Based on the elastic thin plate theory, the stress distribution in and roof control methods for soft rock roadways were studied by Jia Housheng. However, the stratification characteristics of coal measure strata and the extent of damage or destruction in different rock layers have not been considered in previous studies. In addition, although high regional stresses and strains can be determined through stress analyses, regional rock damage does not necessarily lead to a structural collapse. Therefore, studying the law of energy change during the process of rock failure and its relationship with structural strength would be useful in thoroughly understanding the internal relationship between changes in rock strength and overall failure under external loads. Such
research can be used to evaluate the completeness of a roof rock and guide the development of appropriate roadway support methods, thereby preventing roof-fall accidents.

Therefore, a new method for determining the height of the caving zone and the damaged area on the roof of a roadway is proposed herein. In the proposed method, first, considering the layer characteristics of the roof rock, different rocks are regarded as RETPs. Second, the stress distribution in the RETPs is calculated, and the stability of each RETP is evaluated based on a damage criterion. Third, a bottom-up statistical analysis is performed to obtain the damaged range profile of each stratum. Finally, the cumulative collapse height of the roof strata is obtained. Based on the results, this method is deemed promising for predicting the roof-collapse height of roadways and determining reasonable support parameters for roadways.

2 | THEORETICAL MODELING

To analyze the stress distribution and deflection characteristics of a roadway roof, the theoretical RETP model was used;

| Researchers                  | Type and cause                                      |
|------------------------------|-----------------------------------------------------|
| Jia Mingkui (2005)           | Deterioration of strata combination, strata fissures, and stress concentration |
| Li Ji (2014)                 | Buried depth, fissure development in surrounding rock, water situation, roof rock properties, and bolt support quality |
| Ma Nianjie (2015)            | Fracture scale, low bonding strength, and rock mass leakage |
| Bai Qingsheng (2020)         | Jointed roof and different support schemes          |
| O’Mahoney, Laurence (1978)   | Joints, geological structures, roof rock composition, clay veins and slips, and topography |
| Van der Merwe J N, et al (2001) | Insufficient joint support, excessive bolt spacing, weathering, horizontal stress/weak roof rock, and inappropriate mining |
| Singh K B, et al (1994)      | Geological discontinuities such as faults, joints, cleats, and bedding planes |

The roof stratum of a roadway can be regarded as an elastic thin plate whose four sides are in a fixed state. Its boundary conditions can be expressed as in Equation (1), while its equilibrium differential equation of deflection can be expressed as in Equation (2):

\[ \omega|_{x=0,a} = \frac{\partial \omega}{\partial x}|_{x=0,a} = 0, \quad \omega|_{y=0,b} = \frac{\partial \omega}{\partial y}|_{y=0,b} = 0 \]  

\[ \frac{\partial^4 \omega}{\partial x^4} + 2 \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} = \frac{Q}{D} \]  

Here, \( D \) represents the bending stiffness of the RETP; \( D = Eh^3/12(1 - \mu^2) \). \( E \) represents the elastic modulus of the RETP; \( \mu \) represents Poisson’s ratio, which is the ratio of the transverse deformation of a material to its longitudinal deformation; \( Q \) represents the load borne by the plate, and \( \omega \) represents the deflection volume of the plate.

Solutions to the abovementioned equilibrium differential equations for deflection volume can be determined in the form of trigonometric functions. Therefore, the deflection volume of a four-sided thin plate with fixed supports can be obtained using Equation (3); thereafter, the deflection volume of each point on the plate can be calculated accordingly:
\( \omega(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \left( 1 - \cos \frac{2\pi mx}{a} \right) \left( 1 - \cos \frac{2\pi ny}{b} \right) (3) \) 

In Equation (3), \( A_{mn} \) is the undetermined coefficient. In terms of the principle of minimum potential energy, \( A_{mn} \) can be described as

\[
A_{mn} = \frac{Q}{4\pi^2 D \left( \frac{3m^4}{a^4} + \frac{2m^2n^2}{a^2b^2} + \frac{3n^4}{b^4} \right)}
\]

Similarly, the deflection volume can be expressed as.

\( \omega(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{Q}{4\pi^2 D \left( \frac{3m^4}{a^4} + \frac{2m^2n^2}{a^2b^2} + \frac{3n^4}{b^4} \right)} \left( 1 - \cos \frac{2\pi mx}{a} \right) \left( 1 - \cos \frac{2\pi ny}{b} \right) (5) \)

The internal stresses, \( \sigma_x \) and \( \sigma_y \), of the differential element in Figure 4 can be determined using Equation (7) based on the relationship between the stress component and the deflection function in the elasticity theory, which is expressed in Equation (6). Moreover, the internal stress \( \sigma_z \) generated due to the load acting on the RETP, that is, \( Q \), can be calculated according to the key strata theory. Note that the calculation method is not repeated in this paper for brevity.

\[
\begin{align*}
\sigma_x &= -\frac{E_z}{1 - \mu^2} \left( \frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial y^2} \right) \\
\sigma_y &= -\frac{E_z}{1 - \mu^2} \left( \frac{\partial^2 \omega}{\partial y^2} + \mu \frac{\partial^2 \omega}{\partial x^2} \right)
\end{align*}
(6)
\]

\[
\sigma_x = \frac{4\pi^2 E_z Q}{\mu^2 - 1} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{m^2 \cos \frac{2\pi mx}{a}}{a^2} \left( 1 - \cos \frac{2\pi ny}{b} \right) + \mu \frac{n^2 \cos \frac{2\pi nx}{b}}{b^2} \left( 1 - \cos \frac{2\pi my}{a} \right) \right\} \\
\sigma_y = \frac{4\pi^2 E_z Q \cos \alpha}{\mu^2 - 1} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{n^2 \cos \frac{2\pi mx}{a}}{b^2} \left( 1 - \cos \frac{2\pi ny}{b} \right) + \mu \frac{m^2 \cos \frac{2\pi nx}{b}}{a^2} \left( 1 - \cos \frac{2\pi my}{a} \right) \right\}
(7)
\]

\[ U^e = \frac{1}{2} \sigma_1 \varepsilon_1^e + \frac{1}{2} \sigma_2 \varepsilon_2^e + \frac{1}{2} \sigma_3 \varepsilon_3^e = \frac{1}{2E} \left[ \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu \left( \sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1 \right) \right]
(10)\]

\[
\varepsilon_i^e = \frac{1}{E_i} \left[ \sigma_i - \mu_i \left( \sigma_j + \sigma_k \right) \right]
(11)\]

In the above equations, \( \sigma_1, \sigma_2, \) and \( \sigma_3 \) represent the three principal stresses; \( \varepsilon_1, \varepsilon_2, \) and \( \varepsilon_3 \) and \( \varepsilon_1^e, \varepsilon_2^e, \) and \( \varepsilon_3^e \) represent the three principal total strains and elastic strains, respectively.

During actual mining activities, the rock mass surrounding the roadway is deformed and destroyed under external forces. However, the work done by these external forces on the rock mass is transformed into dissipated energy inside the rock. This dissipated energy mainly manifests as fractures in the rock. Over time, these fractures gradually expand and reduce the integrity and strength of the rock mass. The remaining energy is stored within the rock mass and converted into elastic strain energy. This energy is released rapidly once it exceeds the maximum value of the surface energy of the rock mass, causing macroscopic fragmentation of rock masses in the form of collapses of the roof rock and coal slabs.

In the principal stress space, the probability of the release of this elastic strain energy along the direction of the maximum compressive stress, \( \sigma_1 \), is relatively low. However,
this energy is more likely to be released along the direction of the minimum compressive stress or the tensile stress, \( \sigma_3 \). Accordingly, the failure criteria for rock mass elements subjected to compression and tension are as follows:

1. Differential element compressed along with three directions (\( \sigma_1 > \sigma_2 > \sigma_3 \geq 0 \)).

The strain energy release rate is calculated based on the difference between the maximum compressive stress and the minimum compressive stress, which represents the energy consumed during crack propagation in the roof stratum.\(^{32,33}\) The energy release rate can be expressed as “\( F_i \),” and \( K_i \) represents the material coefficient:

\[
F_i = K_i (\sigma_1 - \sigma_i) U^e \quad (i = 1, 2, 3)
\]

Under this condition, the maximum energy release is likely to occur along the direction of the minimum compressive stress, as expressed in Equation (13):

\[
F_3 = K_3 (\sigma_1 - \sigma_3) U^e
\]

When the maximum energy release rate, \( F_3 \), reaches the critical value, \( F_c \), the strain energy stored in the roof rock layer is released preferentially in the direction. If the strain energy, \( U^e \), becomes equal to the surface energy required for unit failure, \( U^0 \), then all the elastic strain energy is released, triggering the failure of the entire rock mass. The energy difference between \( U^e \) and \( U^0 \) is the kinetic energy of the broken rock mass in the surrounding rock; this kinetic energy is responsible for the ejection of broken rock masses.

Based on the aforementioned analyses, when a rock element fails during a uniaxial compression experiment (i.e., \( \sigma_1 = \sigma_c \) and \( \sigma_2 = \sigma_3 = 0 \)), Equations (14) and (15) can be adapted to determine the elastic strain energy:

\[
F_3 = K_3 (\sigma_1 - \sigma_3) U^e = F_c
\]

\[
U^e = \frac{\sigma_c^2}{2E}
\]

where \( F_c \) is a material constant, namely the maximum value of the energy release rate; it can be determined via uniaxial compression tests.

The overall failure criterion for a rock mass subjected to triaxial compression can be deduced from Equation (16) by incorporating Equation (15) into Equation (14); furthermore, it can be expressed as in Equation (17):

\[
(\sigma_1 - \sigma_3) U^e = \frac{\sigma_c^2}{2E}
\]

\[
(\sigma_1 - \sigma_3) [\sigma_1^2 + \sigma_3^2 - 2\mu (\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1)] = \sigma_c^2
\]

1. Tensile stress applied to an elementary volume (\( \sigma_3 < 0 \)).

The tensile strengths of underground engineering rock materials are considerably lower than their compressive strengths. This implies that tensile stresses are more likely to cause damage and that the energy release will occur along the direction of the principal tensile stress. Therefore, when at least one tensile stress among the three principal stresses exists, it is expected to contribute significantly to the energy release. The elastic strain energy will be released depending on the values of the three principal stresses; moreover, maximum strain energy release will occur along the direction of the maximum principal tensile stress, as expressed in Equation (18):

\[
F_i = K_i \sigma_i U^e
\]

\[
F_3 = K_3 \sigma_3 U^e = F_i
\]

where \( K_i \) is a material constant; its maximum value is observed for an element subjected to various tensile stresses, and it can be determined using the Brazilian split test.

Based on the abovementioned analysis, elastic strain energy can be calculated using Equation (20) under the conditions of uniaxial tension (\( \sigma_3 = \sigma_1 = \sigma_2 = 0 \)):

\[
U^e = \frac{\sigma_i^2}{2E}
\]
where \( \sigma_t \) represents the uniaxial tensile strength.

Therefore, the overall failure criterion for a rock mass subjected to triaxial compression can be obtained as shown in Equation (21), which can further be rewritten as Equation (22):

\[
\sigma_3 U^c = \frac{\sigma_1^2}{2E} (21)
\]

\[
\sigma_3 \left[ \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2 \mu \left( \sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1 \right) \right] = \sigma_1^2 (22)
\]

### 2.3 Damage evolution rule for RETPs

Based on the abovementioned method for calculating energy, the strength criterion for an elementary volume can be determined; the compressive and tensile states will occur in the upper and lower portions of the neutral layer of the RETP, respectively. Thus, a roadway roof can be considered as a combination of multilayered RETPs. This classification and the number of layers are illustrated in Figure 5.

In Figure 5 (a), RETP “I” is divided into two parts by the neutral layer. The upper section “I_u” is in a compressive state, whereas “I_t” is in a tensile state. The tensile strength of a rock material is typically lower than its compressive strength. Thus, the roof stratum is more prone to tensile failure. Therefore, the boundary stress of the roof stratum can be expressed as in Equation (24):

\[
\sigma_1 = \min(\sigma_x, \sigma_y, \sigma_z); \sigma_2 = \max(\sigma_x, \sigma_y) (24)
\]

On substituting Equation (24) into Equation (22), Equation (25) is obtained, and the damaged elementary volume and boundary of the damaged area can be identified.

\[
\sigma_3 \left[ (\sigma_x)_3^2 + \sigma_2^2 + \sigma_3^2 - 2 \mu ((\sigma_x)_3 \sigma_1 + \sigma_2 \sigma_3 + \sigma_3 \sigma_2) \right] = \sigma_3^3 (25)
\]

1. The maximum tensile stress is \( \sigma_3 = \sigma_{x_t} \).

In this case, the boundary stress of the roof stratum can be expressed as in Equation (26):

\[
\sigma_1 = \min(\sigma_x, \sigma_y, \sigma_z); \sigma_2 = \sigma_{x_t}; \sigma_3 = \sigma_y (26)
\]

The boundary of damaged area of the RETP can be determined based on Equation (27), which is deduced by substituting Equation (26) into Equation (22):

\[
\sigma_x \left[ (\sigma_x)_x^2 + \sigma_2^2 + \sigma_3^2 - 2 \mu ((\sigma_x)_x \sigma_1 + \sigma_2 \sigma_3 + \sigma_3 \sigma_2) \right] = \sigma_x^3 (27)
\]

### 3 MECHANISM OF ROOF COLLAPSE

A schematic model of a roadway support is presented in Figure 6, with the roadway roof shown as a multilayered RETP. The stratification layer, namely the interface between adjacent RETPs, is considered the neutral layer. The roof shown in Figure 6 is schematically divided into four RETPs.

### 3.1 Identifying damage in I_t of the RETP

The elementary volume of I_t below the neutral layer underwent damage due to tensile stress, and the area of I_t subjected to tensile stress can be determined using Equation (22). When the minimum principal stress is \( \sigma_t \) or \( \sigma_y \) in the principal stress space, as shown in Equation (23), two situations need to be considered:

\[
\sigma_1 = (\sigma_z); \sigma_2 = \min(\sigma_x, \sigma_y); \sigma_3 = \max(\sigma_x, \sigma_y) (23)
\]

1. The maximum tensile stress is \( \sigma_3 = \sigma_{y_t} \).

In this case, the boundary stress of the roof stratum can be expressed as in Equation (24).

\[
\sigma_1 = (\sigma_z); \sigma_2 = \sigma_{x_t}; \sigma_3 = \sigma_y (24)
\]

### 3.2 Identifying damage in I_c of the RETP

According to the internal force distribution shown in Figure 4, Equation (17) can be used to identify the compressive area I_c of the RETP. As the uniaxial compressive strength of a rock material is 4-25 times greater than the uniaxial tensile strength, the roof of the roadway does not undergo compressive damage; however, tension and shear failures are more likely to occur. Thus, the compressive damage in the RETP is minimal, and even negligible, in most cases.

### 3.3 Determining damaged area and caving height

The horizontal stresses, \( \sigma_x \) and \( \sigma_y \), acting on the lower surface of the RETP can be calculated using Equation (8), and their contours are shown in Figure 7.

As the distance between the RETP and the coal seam increases, the damaged size of the RETP decreases till intactness can be obtained in certain RETPs. The width and length
of the damaged area and the size of each RETP can be determined using Equation (28):

\[
a_{i+1} = \frac{b_i}{\pi} \arcsin \left( a_i \sqrt{\frac{\mu}{b_i^2 + 2\mu a_i^2}} \right)
\]

\[
b_{i+1} = \frac{a_i}{\pi} \arcsin \left( b_i \sqrt{\frac{\mu}{a_i^2 + 2\mu b_i^2}} \right)
\]  

(28)

By summing the thicknesses of the damaged RETPs, the collapse height can be obtained as follows:

\[
h_d = \sum_{i=1}^{n} h_{\text{RETP}_i}
\]  

(29)

According to the equations above and the mechanics of the surrounding rock of roadways, the failure height of a roadway roof stratum and the extent of horizontal damage in the different layers of the strata can be determined. These calculations are significantly important for determining the installation depth and inclination of bolts and cables, and they also serve as a valuable reference for formulating appropriate roadway support measures.

4 | CASE STUDY

The No. 30102 working face of the Nanliang Coal Mine, located in the northern part of Shaanxi Province, is deployed in the No. 3 coal seam, where the dip angle is 1-3° and the average mining height is 2.02 m. The immediate roof of the No. 3 coal seam is composed of argillaceous siltstone and mudstone, with a thickness of 1-3 m. The main roof is also composed of argillaceous siltstones. Owing to well-developed fissures and a low overall rock strength, the immediate roof

![Diagram of RETP evolution rule](image)

FIGURE 5 Sectional diagram for the evolution rule of damage in RETPs
of the No. 30102 working face is weak; consequently, roof-fall accidents are likely to occur.

Roof-fall accidents have occurred in the open-off cut of the No. 30102 working face over a range of up to 30 m. The layout of the working face and images illustrating the roof-fall accident at the mining site are shown in Figures 8 and 9, respectively.

Although this roof failure did not cause casualties, it damaged the drift machines, and some equipment was buried. At the accident site, as shown in Figure 9, the bolt mesh support was damaged, and strata had fallen off in layers. The roof-collapse height was approximately 2.2 m. The block of siltstone was large; above the caved block, a smaller loose mudstone block with a thickness of 0.4 m was noted, as shown in Figure 9.

4.1 Collapse height and area of roof layers

After the roof-collapse accident, the collapse height and caved range of the roof strata were measured. The stratigraphic columnar section of the roof strata and the outline of the caved region, indicated in red lines, are shown in Figure 10.

The supports used at the time of the roof collapse and the lithology and thicknesses of each roof layer are presented in Figure 10. The initial dimensions of the excavated section were 4500 × 2200 mm, and bolts (Ø 16 mm × (l) 2200 mm) with wire mesh, featuring an inter-row spacing of 900 × 900 mm, were used for support.

According to the key strata theory, the fine sandstone layer with a thickness of 4.8 m is regarded as the key stratum.
Therefore, the rock layers with thicknesses of 1.85 and 0.33 m need to be supported by the artificial support structure. Based on the analysis presented in Section 2.1, the vertical load acting on the lowermost layer of the roadway roof can be calculated using Equation (29):

$$\sigma_z = \frac{E_1h_1^3(\gamma_1h_1 + \gamma_2h_2 + \cdots + \gamma_nh_n)}{\sum_{i=1}^{n} E_i h_i^3} = \frac{2840 \times 1.85^3 (2429 \times 1.85 + 2411 \times 0.33)}{2840 \times 1.85^3 + 2440 \times 0.33^3} \times 10^{-5} = 0.053 \text{MPa}$$  (29)

To identify the damaged area of the roof, the roof is divided into several layers with thicknesses of 0.5 m for ease of calculation, as shown in Figure 5. Parameters, $a_i$ and $b_i$, representing the width and length of RETP I, are set to 10 m and 20 m, respectively, in order to accurately reflect the area and boundary of the damaged region. The results showed that nine RETPs suffered damage according to the criteria based on the energy method, expressed in Equations (25) and (27). The physical and mechanical parameters of each strata are listed in Table 2.

Based on the aforementioned analyses, the size of the RETP and the values of $a_i$ and $b_i$, for each layer can be calculated by substituting $a_i$ and $b_i$ into Eq. (30). In addition, the
damaged area and boundary of each layer can be determined by substituting $a_i$ and $b_i$ into Equation (27) and Equation (29) successively. To highlight the results of the calculation, diagrams of three RETPs are presented in Figure 11, where the rectangular and elliptical portions represent the boundary of the RETP and the damaged area, respectively.

The calculation results prove that nine RETPs were damaged. Therefore, the collapse height of the roof stratum can be calculated by summing the thicknesses of the damaged RETPs, as follows:

$$h_d = \sum_{i=1}^{n} h_{\text{ERTP}_i} = 9 \times 0.25 = 2.25 \text{ m}$$  \hspace{1cm} (30)

The profile and dimensions of the suspended roof are presented in Figure 12. The measurement results show that the falling height of the roadway roof was approximately 2.2 m, while the length and width of the suspended upper stratum above the caved region along the major and minor axes were 11.5 and 5.0 m, respectively. The caved height, obtained by using the abovementioned method, was 2.25 m, and the length and width of the suspended upper stratum were 10.56 and 4.81 m, respectively. Thus, the theoretical analysis results were in accordance with the on-site measurements, indicating that the proposed method is applicable for investigating the collapse heights of roadway roofs.

After determining the collapse height of the stratum of the roadway roof, the boundary of the caved range was further studied, and the law of the reducing range of a bottom-up collapse was determined. This is important for formulating roadway support schemes, especially when determining installation angles and the depth of anchor bolts for the roof and two sides of the roadway.

**TABLE 2** Mechanical parameters of rock

| Lithology | $\rho$ (kg/m$^3$) | $E$ (MPa) | $\mu$ | $\sigma_c$ (MPa) | $\sigma_t$ (MPa) | $c$ (MPa) |
|-----------|-----------------|---------|-------|----------------|----------------|--------|
| Coal      | 1299            | 2820    | 0.31  | 22.53          | 1.24           | 2.97    |
| Mudstone  | 2411            | 2440    | 0.36  | 20.35          | 0.78           | 1.85    |
| Siltstone | 2429            | 4190    | 0.24  | 37.24          | 1.73           | 3.37    |

**FIGURE 11** Boundary of the RETP and damaged area

**FIGURE 12** Dimensions and boundary of suspended roof in a cleaned-caved area
The fitting equation of the vertical boundary of the caved zone can be obtained by connecting and fitting the end vertices of the minor axis for the oval damaged area of the nine RETPs. The scattering plot and fitted curve can be obtained using Equation (31) and are shown in Figure 13:

\[ y = -0.31x^2 - 0.2x + 4.53 \]  

(Figure 13) Vertical boundary of the damaged RETPs in the caved zone

4.2 Optimization and improvement of support schemes

Based on the analyses discussed above, it was determined that the roadway roof collapsed because the length of the bolts was less than the total thickness of all the collapsed strata. The main function of the anchor bolts used for supporting roadways is to suspend and secure broken roof strata onto intact strata.

For the case study considered in this work, the collapse height of the roof was determined to be 2.25 m, via field measurements and calculations; however, the length of the bolts was only 2.2 m in the initial supporting scheme. Therefore, the bolts failed to suspend and secure the broken strata onto intact ones, a root cause of roof-collapse accidents.

Accordingly, by increasing the length of the bolts from 2.2 m to 2.8 m and placing their anchoring sections in the upper siltstone stratum, which has a thickness of 4.8 m as shown in Figure 10, an effective and secure suspension for broken surrounding rocks can be realized. Simultaneously, for ease of installation, the installation angle of the bolts at the two sides of the roadway was decreased from 75° to 70°. This renders the bolts perpendicular to the lateral boundary of the caving zone up to the maximum and gives full supporting effect of the bolts.

As shown in Figure 14, the support for the suspended roof in the collapsed area and the two rips in the caved zone were reinforced following an assessment of the root cause of the collapse. The length of the bolts was increased, and anchor cables were also employed for support. The row spacing between bolts was reduced from 900 mm to 600 mm. Moreover, the diameter of the steel wire used in the mesh was increased to 2 mm, and the width of the grid was reduced to 50 mm. Detailed support methods and the structural parameters of the anchor rods, anchor cables, steel belts, and steel mesh are listed below.

1. Anchor bolt specifications: diameter of 16 mm and length of 2800 mm, inter-row space of 600 × 600 mm, and steel pallet with dimensions of 120 × 120 × 8 mm;
2. Steel strap specifications: indented steel with a length of 3.6 m and width of 120 mm;
3. Wire mesh specifications: length of 3.8 m, width of 2.1 m, wire with diameter of 2 mm, and holes measuring 50 × 50 mm;
4. Cable specifications: diameter of 21.8 mm and length of 7500 mm, pallet with dimensions of 150 × 150 × 12 mm, space of 3000 mm, and pretightening force of 30 t.

In addition, steel band beams with a length of 3600 mm were added to the two sides of the caved zone. Wood cribs were built in the open-off cut with a spacing of 2000 mm. Two rows of hydraulic props were erected on both sides of the wood cribs, at a distance of 1000 mm. I-type steel beams were installed above the hydraulic props. The complete support scheme is depicted in Figure 14.

5 Conclusion

1. A mechanical model based on the theory of elastic thin plates was established to analyze the collapse height.
of the roadway roof. In the model, the roof stratum is divided into multiple layers, each with a thickness of just 0.3-0.5 m. The stress distribution law during the flexural subsidence of each layer was determined.

2. Based on the stress distribution characteristics and the law governing the evolution of elastic strain energy, the fracture boundary of each layer was analyzed and the thicknesses of all the fractured layers were summed to calculate the collapse height of the roadway roof strata. Concurrently, by fitting all the boundary points of the fracture layer, the falling boundary curve can be obtained. The theoretical analysis results were compared with actual measurements conducted at the No. 30102 working face of the Nanliang Coal Mine. The theoretically calculated caved height was 2.25 m, and the suspended length and width were 10.56 m and 4.81 m, respectively. The measured values were 2.2 m, 11.5 m, and 5.0 m, respectively. Thus, the theoretical analysis results were validated.

3. This study introduces a method for accurately calculating the roadway roof-fall height and the boundary shape of the roof-collapse area. This is significantly important for establishing roadway support methods, especially when determining the installation depths and inclination angles of bolts and anchor cables. With regard to the Nanliang Coal Mine roof-fall accident, the length of the bolts in the roof was increased from 2.2 m to 2.8 m and the installation angle of the bolts was adjusted from 75° to 70°, effectively ensuring the subsequent stability of surrounding rocks. Thus, these findings serve as a valuable reference for formulating reasonable roadway support plans.

ACKNOWLEDGMENTS

Financial support from the National Natural Science Foundation of China (No.51904201), Fundamental Research Program of Shanxi (201901D211035), and Scientific and Technological Innovation Programs of Higher Education Institutions in Shanxi (2019L0245) are greatly appreciated. The authors express sincerely their gratitude to associate professor Zhu Defu and his research team for his great help in theoretical derivation of the thesis and on-site measurement. And the authors also express their gratitude to the anonymous reviewers for their constructive comments and suggestions.

CONFLICT OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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**How to cite this article:** Li Z, Zhang H, Jiang Z, Feng G, Cui J, Ma J. Research on failure criteria and collapse height of roadway roof strata based on energy accumulation and dissipation characteristics. *Energy Sci Eng*. 2021;9:2461–2473. [https://doi.org/10.1002/ese.926](https://doi.org/10.1002/ese.926)