Spontaneous fission half-lives of nuclei in a phenomenological model

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A simple phenomenological model, based on the Świątecki idea for evaluation of the spontaneous fission half-lives is proposed. The model contains only one adjustable parameter fixed to the data for even-even nuclei and two additional hindrance factors to the life-times, which give the effect of odd particles. A good agreement with the experimental data for all fissioning nuclei is achieved.

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I. INTRODUCTION

Close correlations between spontaneous fission half-lives and ground state masses of nuclei were noticed in 1955 by W. J. Świątecki [1]. He has proposed a simple formula, which has joined the observed fission lifetimes with the difference between experimental and liquid drop masses ($\delta M$). A regular dependence of $\log_{10} T_{sf}^{1/2}$ obtained by him after adding an empirical correction proportional to $\delta M$ is presented in Fig. 1 taken from Ref. [1] as function of the fissility parameter $Z^2/A$.

The aim of the present paper is to check if this 58 years old Świątecki brilliant idea still works. A modern version of the liquid drop model derived in Ref. [2] and all up to now measured spontaneous fission half-lives [3] are taken in our analysis analysis.

II. THE LIQUID DROP MODEL

The experimental ground-state masses of all nuclei with the proton number $Z \geq 8$ and neutron number $N \geq 8$ were calculated in Ref. [2] using the macroscopic-microscopic model. The macroscopic part of this Lublin-Strasbourg Drop (LSD) mass formula (in MeV units) was following:

$$M_{LSD}(Z, N, \text{def}) = 7.289034 \cdot Z + 8.071431 \cdot N - 0.0001433 \cdot Z^{2.39}$$

$$- 15.4920(1 - 1.8601I^2)A$$

$$+ 16.9707(1 - 2.2938I^2)A^{2/3} B_{\text{surf}}(\text{def})$$

$$+ 3.8602(1 + 2.3764I^2)A^{1/3} B_{\text{cur}}(\text{def})$$

$$+ 0.70978 Z^2 / A^{1/3} B_{\text{Coul}}(\text{def}) - 0.9181 Z^2 / A$$

$$- 10 \exp(-4.2|I|) B_{\text{cong}}(\text{def}) \ .$$

while the ground state microscopic corrections $E_{\text{micro}}$ were taken from the mass tables of Möller at al. [4]. In Eq. (1) $A = Z + N$ denotes the mass number, $I = (N - Z)/A$ reduced isospin and $B_{\text{surf}}$, $B_{\text{cur}}$, $B_{\text{Coul}}$ and $B_{\text{cong}}$ are relative to the sphere: surface, curvature, Coulomb and congruence (see Ref. [5]) energies. The parameters in the first and the last row in Eq. (1) are taken from Ref. [4], while the rest 8 parameter were fitted to the data.

It was shown in Refs. [2, 6, 7] that the LSD model [1], which parameters were fitted to the experimental ground-state masses only, is able to reproduce well the fission barrier heights of light, medium and heavy nuclei when the microscopic part of the ground-state binding energy is taken into account according to the topograph-

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The liquid drop barrier height of actinides decreases almost linearly in function of \( Z \) from 4.3 MeV for \( Z = 90 \) to 0 for \( Z \geq 103 \). The fission barrier of finite heights appears in the super-heavy nuclei mostly due to the shell effects in the ground state.

### III. CALCULATION DETAILS

Following Ref. [1], we have subtracted from the logarithm of the spontaneous fission half-lives in years:

\[
f(Z, N) = \log_{10} \left( \frac{T_{1/2}^{sf}(Z, N)}{y} \right) + k\delta M(Z, N) .
\]  

(2)

the experimental value of ground-state microscopic energy \( \delta M(Z, N) \) can be evaluated using the experimental and the liquid drop \([1]\) mass difference:

\[
\delta M_{\text{mexp}}(Z, N) = M_{\text{exp}}(Z, N) - M_{\text{LSD}}(Z, N, 0)
\]

(3)

multiplied by an arbitrary factor \( k \).

In our analysis of function \( f(Z, N) \) were evaluated for 62 even-even fissioning isotopes with \( Z \leq 114 \). Smooth dependence of \( f(Z, N) \) on \( Z \), in which shell effects were almost wash out, was achieved for \( k = 7.5 \) MeV\(^{-1}\) as one can see in Fig 2. For given element the dependence of the function \( f(Z, N) \) on neutron number is very weak, so we omit this dependence in further consideration. The function \( f(Z) \) which represents the global trend in the fission life-times can be approximated by two crossing straight lines:

\[
f(Z) = \begin{cases} 
-4 \cdot Z + 371.2 & \text{for } Z < 104 , \\
-42.8 & \text{for } Z \geq 104 .
\end{cases}
\]  

(4)

The coefficients of these straight lines are simply given by the systematics of points representing the corrected (according to Eq. (2)) fission life-times. They are not free adjustable parameters. The above dependence is very similar to the dependence of the liquid-drop barrier heights in function of \( Z \). It is also visible in Fig. 2 that the slope of similar data for odd-\( A \) and odd-odd nuclei is almost the same but the corresponding curves are shifted by a constant. This shift, which can be called global hindrance factor is equal to \( h = 10.3 \) for odd-\( A \) nuclei and 17.8 for odd-odd systems. It represents the fission life-time increase given both by the odd-even effects in the ground state energy and the fission barrier enhancement due to the angular momentum and parity conservation of unpaired nucleons. Contrary to Świątecki analysis [1] we assume here the linear in \( Z \) form of the \( f(Z) \) and we have extrapolated it by a constant for the super-heavy nuclei.

Using the approximation (4) one can estimate the spontaneous fission half-lives by the following formula:

\[
\log_{10} \left( \frac{T_{1/2}^{sf}(Z, N)}{y} \right) = -4Z + 371.2 - 7.5\delta M(Z, N)
\]

\[
+ \begin{cases} 
0 & \text{for even – even ,} \\
10.3 & \text{for odd-\( A \),} \\
17.8 & \text{for odd – odd ,}
\end{cases}
\]

(5)

where the experimental microscopic correction \( \delta M \) is defined in Eq. (3). The estimates of \( T_{1/2}^{sf} \) obtained with formula (5) are compared in Fig. 3 with the experimental data taken from Ref. [3].

Surprisingly good agreement of the model estimates with the data is achieved for all actinide and even super-heavy nuclei. The root mean square deviation of \( \log T_{1/2}^{sf} \) for the even-even isotopes with \( Z < 104 \) is 1.52 and it grows to 2.02 when odd-\( A \) and odd-odd nuclei are taken into account. The rms deviation reaches 2.47 when one includes to the analysis the super-heavy isotopes (for which the liquid drop barrier vanishes). To foresee \( T_{1/2}^{sf} \) for unknown nuclei one can evaluated \( \delta M \) using mass estimates from one of existing on the market mass tables.

In order to understand this striking for the first sight result it is good to remind the topographical theorem, proposed by Myers and Świątecki [1], the mass of a nucleus in a saddle point is determined by the macroscopic part of the binding energy. The shell effects at the saddle are negligible, so the fission barrier heights is approximately determined by the difference of the macroscopic (here LSD) and the experimental masses:

\[
V_B(Z, N) = M_{\text{LSD}}(Z, N, \text{saddle}) - M_{\text{exp}}^{Z_{\text{odd}}}.
\]

(6)

The function \( F(Z) \) (5) represents the liquid-drop estimate of the fission life-time and the \( k \) parameter in Eq. (5) gives enhancement of the life-time due to increase of the fission barrier due to the ground state microscopic effects. This parameter \( k \) plays a similar role as the hindrance factor \( h \) which origins from the barrier augmentation due to spin and parity conservation of an odd-\( A \) nucleus along the path to fission.
IV. SUMMARY

The following conclusions can be drawn from our investigation:

- Simple phenomenological formula for the spontaneous fission half-lives depending on proton number and microscopic energy correction in the ground state was found.

- Model estimates of the spontaneous fission half-lives of even-even nuclei are in a surprisingly good agreement with experimental data.

- Quality of evaluation for odd nuclei is much worse, what is due to fact that the effect of an odd-particle on the barrier penetrability is described here by a single constant, independent on angular momentum or parity of the odd nucleon.

- Our simple formula with the microscopic energy corrections taken e.g. from the tables [4] can serve to rough estimates of the fission life-times of new undiscovered yet isotopes.

The present phenomenological formula for the spontaneous fission half-lives and similar formula for the alpha decay half-lives derived in Ref. [10] can serve a useful tool to estimate stability of non discovered yet isotopes. The only input which one needs in the both formulas are the binding energy estimates which can be taken from one of the mass model existing on the market.

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