Neural network approach to solving the inverse problem of surface-waves generation

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Abstract. The paper proposes an approach to the inverse tasks of the surface-waves generation, based on information technologies of machine learning. On the basis of this approach, a method has been developed for determining the parameters of a cylinder and a wing profile moving in fluid according to a laboratory experiment about a free surface liquid disturbed by them.

1. Introduction
Radar and optical means of remote sensing of the marine environment do not allow one to directly observe the processes occurring at a considerable depth. But, with their help, parameters of the sea surface can be measured. The need to study the processes occurring in the marine deepness according to parameters of its surface led to the creation of computer radio tomography of the marine environment [1].

The theoretical basis of this section of science and technology consists of various approaches to solving the task of restoring the internal structure of a heavy fluid flow according to its free surface. In particular, the inverse task of the generation of surface waves, related to determining the parameters of sources of disturbances from the features of the waves generated by them on the liquid surface, is of considerable interest.

This task of continuing the fields in the direction of their sources belongs to the class of incorrectly stated tasks [2]. Its solution is unstable in relation to small changes in the original data, which makes it difficult to obtain information about the sources of disturbances.

For the first time, some approaches to solving the inverse task of generating surface waves when a plane flow goes around a point-like hydrodynamic feature were proposed in [3, 5]. However, during test application of the proposed algorithms to processing the data of laboratory experiments [6, 7], in which measurements were made of the angles of inclination of the water surface perturbed by the moving cylinder and wing profile aroused some difficulties appropriate to incorrect tasks of determining the parameters of sources of disturbances.

In this paper, a fundamentally new approach to solving the inverse task of generating surface waves is proposed, based on the methods of machine learning, namely, the theory of neural networks, which is now widely used in data processing and analysis [8, 10].
This is explained by the fact that modern learning algorithms for neural networks such as a stochastic gradient descent, an impulse or a packet method are based on the principles of overcoming mathematical incorrectness.

2. Solution

To use machine learning technology, you must have a training set. It should be noted here that such a set can be obtained in various ways. For example, in the current task, it can be obtained by multiple numerical determinations of waves on the surface of a stream flowing around point-like hydrodynamic features that simulate an obstacle in the stream, using the formulas [7].

To simulate real conditions, some noise can be applied to the solutions obtained in this way. In any case, this will be a certain model solution, since neither the imposed noise, nor the system of hydrodynamic features, which simulates the obstacle streamlined by the flow, can not exactly correspond to reality.

In the present work, the training samples were obtained by measuring the slopes of the surface of the water, perturbed by the wing profiles and cylinders of various diameters moving in its thickness. The experiments were carried out on an experimental facility [3, 5], which is a 300 mm x thickness. The experiments were carried out on an experimental facility [3, 5], which is a 300 cm x 50 cm channel filled with water. In the coordinate system moving with the model, \( y = S(x) \) is the deviation of the free surface of the liquid from the equilibrium position of \( y = 0 \). At various points \( x_n \), the tangent of the angle \( \alpha_n(n = 1, N) \) of the inclination of the free water surface to the horizon, that is, the derivative \( S'(x_n) \), was measured. In [6], these results are presented in tabular form. One series of experiments was carried out with a symmetrical wing profile moving at a depth of \( H = 175 mm \) with different velocities \( V = 482 - 938 mm/s \). The other series - with a circular cylinder with a diameter of \( D = 6 mm \), moving with velocities \( V = 280 - 1056 mm/s \) at depths of \( H = 66 - 126 mm \). The third one - with a circular cylinder with a diameter of \( D = 20 mm \), moving with velocities \( V = 324 - 565 mm/s \) at depths \( H = 103 - 133 mm \).

Formally, the task of machine learning is reduced to the construction of a neural network and is formulated as a mathematical problem of approximation. Let a region \( \Omega \subset R^3 \) be given and two sets of points be given: the input data set \( Q_N = \{r_n\} \in \Omega(n = 1, N) \) and the target output data set \( Q_M = \{r_m\} \in \Omega(m = 1, M) \). Let \( f(Q_N) \) be the function values at the points of the training set, and then the approximating function \( F(r) \) is realized with the help of a neural network as a continuous mapping \( F : Q_n \rightarrow Q_m \). The coordinates of the point \( r \in \Omega \) arrive at the input. After the signal is converted in the neural network, the output is a signal \( F(r) \).

In such tasks, training with a teacher is always used, that is, for each training input vector there is a training output vector. For this, a network of direct signal propagation of the multilayer perceptron type was used, which was taught by algorithms based on the method of back propagation of error [8].

An important problem, along with the acquisition of data for training and their processing, is a search of the characteristic features of the training set. In our case, the training set was a sequence of tangent values of the angles of inclination of the free surface of the liquid, measured at different points \( x_n \). To select the main features of such a set that characterize the properties and behavior of the process or object under study, recently, deep learning networks [12, 15] are used, but for this, besides a large amount of initial information, considerable computational resources or a long time of the learning process are required, which unattainable in real conditions and does not meet the requirements of operational information processing when determining the parameters of the source of the perturbation of the liquid. Therefore, to identify the characteristic features of the observed process and reduce the training set to the data obtained during the experiment, namely, to a sequence of tangents of the angles of inclination of the liquid surface, measured at different points, spectral analysis was applied. It is known that the
spectrum of the observed signal is invariant with respect to the displacement along the spatial coordinate.

This feature made it possible to move from the whole sequence of observed coordinates of the training vector to several lines of the spectrum, which significantly reduced the size of the neural network and the duration of its learning process. As a result of spectral analysis of experimentally obtained sequences of values of tangents of angles of inclination of the water surface, perturbed by different models towed in the fluid, three main frequencies were identified and found to have similar values for all series of laboratory experiments.

Statistical analysis of a sample of the amplitudes of the spectral lines obtained as a result of processing the experimental data revealed a strong correlation between them and the characteristics of the sources of disturbances. Therefore, for training the neural network, the calculated parameters of the training vector with three coordinates that have the values of the main amplitudes of the spectrum were used.

After conducting a sufficient number of numerical experiments, the neural network architecture was chosen. From the initial data, a larger sample was formed by 5% noise with a uniform density. The choice of the noise interval was identified by computational experiment.

For learning on the variability of factors, data were normalized, that is, reduced to relative units in the interval \((0, 1)\) by dividing each parameter by the maximum value. Neural network training is reduced to minimizing the error function, by adjusting the weights of synaptic connections between neurons. The expression for the root-mean-square error is represented as follows.

\[
E = \frac{1}{2p} \sum_{j=1}^{p} (d_j - y_j)^2,
\]

where \(y_j\) is the actual result, \(d_j\) is the desired network response.

The classical gradient descent method is based on choosing the value of the weights of connections between neurons in order to achieve the minimum approximation error of the objective function in the minimum number of learning steps. The disadvantage of this method is that in some cases the minimization process does not converge to a global value. This task can be solved by the method of stochastic gradient descent (SGD) [9], [12, 13, 14, 15, 16, 17].

Features of this approach are that the method of stochastic gradient descent helps to avoid "sticking" the model at a local minimum point, is not sensitive to small disturbances, is simpler to calculate, is not sensitive to small disturbances, and is not picky about computational resources. Numerical experiments with a neural network confirmed their coincidence with the results of laboratory experiments with an accuracy of 5.

### 3. Conclusion

Methods of machine learning and preparation of training samples require significant computational resources and can be successfully implemented by means of parallel programming on computing clusters or graphic processors like Tesla.

Since they have shown their effectiveness in solving the inverse task of determining the parameters of a moving body according to the disturbances of the free surface of a fluid created by this moving body, it is necessary to extend the described approaches to other classes of incorrectly posed tasks of fluid mechanics and geophysics.

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