Hyperon vector coupling $f_1(0)$ from 2+1 flavor lattice QCD

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Abstract. We present results for the hyperon vector form factor $f_1$ for $\Xi^0 \rightarrow \Sigma^+ l \bar{\nu}$ and $\Sigma^- \rightarrow n l \bar{\nu}$ semileptonic decays from dynamical lattice QCD with domain-wall quarks. Simulations are performed on the 2+1 flavor gauge configurations generated by the RBC and UKQCD Collaborations with a lattice cutoff of $a^{-1} = 1.7$ GeV. Our preliminary results, which are calculated at the lightest sea quark mass (pion mass down to approximately 330 MeV), show that a sign of the second-order correction of SU(3) breaking on hyperon vector coupling $f_1(0)$ is likely negative.

Keywords: CKM unitarily, lattice QCD, SU(3) breaking

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INTRODUCTION

The matrix element for hyperon beta decays $B \rightarrow b l \bar{\nu}$ is composed of the vector and axial-vector transitions, $\langle b(p')|V_\alpha(x)+A_\alpha(x)|B(p)\rangle$, which is described by six form factors: the vector ($f_1$), weak-magnetism ($f_2$), and induced scalar ($f_3$) form factors for the vector current, and the axial-vector ($g_1$), weak electricity ($g_2$), and induced pseudo-scalar ($g_3$) form factors for the axial current. The experimental rate of the hyperon beta decay, $B \rightarrow b l \bar{\nu}$, is given by

$$\Gamma = \frac{G_F^2}{60\pi^5}(M_B-M_b)^5(1-3\delta)|V_{us}|^2[f_1^{B \rightarrow b}(0)]^2 \left[ 1 + 3 \frac{|g_1^{B \rightarrow b}(0)|^2}{|f_1^{B \rightarrow b}(0)|^2} + \cdots \right], \quad (1)$$

where $G_F$ and $V_{us}$ denote the Fermi constant and an element of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix respectively [1]. Here, $M_B$ ($M_b$) denotes the rest mass of the initial (final) state. The ellipsis can be expressed in terms of a power series in the small parameter $\delta = (M_B-M_b)/(M_B+M_b)$, which is regarded as a size of flavor SU(3) breaking [2]. The first linear term in $\delta$, which should be given by $-4\delta [g_2(0)/f_1(0)]^2_{B \rightarrow b}$, is safely ignored as small as $\mathcal{O}(\delta^2)$ since the nonzero value of the second-class form factor $g_2$ [3] should be induced at first order of the $\delta$ expansion [2]. The absolute value of $g_1(0)/f_1(0)$ can be determined by measured asymmetries such as electron-neutrino correlation [1, 2]. Therefore a theoretical estimate of vector coupling $f_1(0)$ is primarily required for the precise determination of $|V_{us}|$.

The value of $f_1(0)$ should be equal to the SU(3) Clebsch-Gordan coefficients up to the second order in SU(3) breaking, thanks to the Ademollo-Gatto theorem [4]. As the mass splittings among octet baryons are typically of the order of 10-15%, an expected size of the second-order corrections is a few percent level. However, either the size, or the sign of their corrections are somewhat controversial among various theoretical
TABLE 1. Theoretical uncertainties of \( \tilde{f}_1 = |f_1/f_{1}^{SU(3)}| \) for various hyperon beta-decays. HBChPT and IRChPT stand for heavy baryon chiral perturbation theory and the infrared version of baryon chiral perturbation theory.

| Type of result                                      | \( \Lambda \to p \)  | \( \Sigma^- \to n \) | \( \Sigma^- \to \Lambda \) | \( \Xi^0 \to \Sigma^+ \) | Reference |
|-----------------------------------------------------|-----------------------|-----------------------|-----------------------------|-----------------------------|-----------|
| Bag model                                           | 0.97                  | 0.97                  | 0.97                        | 0.97                        | [5]       |
| Quark model                                         | 0.987                 | 0.987                 | 0.987                       | 0.987                       | [6]       |
| Quark model                                         | 0.976                 | 0.975                 | 0.976                       | 0.976                       | [7]       |
| 1/Nc expansion                                       | 1.02(2)               | 1.04(2)               | 1.10(4)                     | 1.12(5)                     | [8]       |
| Full \( O(p^4) \) HBChPT                           | 1.027                 | 1.041                 | 1.043                       | 1.009                       | [9]       |
| Full \( O(p^4) \) + partial \( O(p^5) \) HBChPT    | 1.066(32)             | 1.064(6)              | 1.053(22)                   | 1.044(26)                   | [10]      |
| Full \( O(p^4) \) IRChPT                           | 0.943(21)             | 1.028(02)             | 0.989(17)                   | 0.944(16)                   | [11]      |
| Full \( O(p^4) \) IRChPT + Decuplet                 | 1.001(13)             | 1.087(42)             | 1.040(28)                   | 1.017(22)                   | [11]      |
| Quenched lattice QCD                                | N/A                   | 0.988(29)             | N/A                         | 0.987(19)                   | [12, 13]  |

Studies at present as summarized in Table 1. A model independent evaluation of SU(3)-breaking corrections is highly demanded. Although recent quenched lattice studies suggest that the second-order correction on \( f_1(0) \) is likely negative [12, 13], we need further confirmation from (2+1)-flavor dynamical lattice QCD near the physical point.

NUMERICAL RESULTS

In this study, we use the RBC-UKQCD joint (2+1)-flavor dynamical DWF coarse ensembles on a \( 24^3 \times 64 \) lattice [14], which are generated with the Iwasaki gauge action at \( \beta = 2.13 \). For the domain wall fermions with the domain-wall height of \( M_5 = 1.8 \), the number of sites in the fifth dimension is 16, which gives a residual mass of \( am_{\text{res}} \approx 0.003 \). Each ensemble of configurations uses the same dynamical strange quark mass, \( am_s = 0.04 \). The inverse of lattice spacing is \( a^{-1} = 1.73(3) \) (\( a=0.114(2) \) fm), which is determined from the \( \Omega^- \) baryon mass [14]. We have already published our findings in nucleon structure from the same ensembles in three publications, Refs. [15, 16, 17].

In this study, we calculate the vector coupling \( f_1(0) \) for two different hyperon beta-decays, \( \Xi^0 \to \Sigma^+ l\bar{\nu} \) and \( \Sigma^- \to n l\bar{\nu} \), where \( f_1^{\Xi^0 \to \Sigma^+}(0) = +1 \) and \( f_1^{\Sigma^- \to n}(0) = -1 \) in the exact SU(3) limit. We will present our results for \( am_{ud} = 0.005 \), which corresponds to about 330 MeV pion mass. We use 4780 trajectories (the range from 940 to 5720 in molecular-dynamics time) separated by 20 trajectories. The total number of configurations is actually 240. We make two measurements on each configuration using two locations of the source time slice, \( t_{\text{src}} = 0 \) and 32. Details of our calculation of the quark propagators are described in Ref [16].

For convenience in numerical calculations, instead of the vector form factor \( f_1(q^2) \), we consider the so-called scalar form factor

\[
    f_S^{B \to b}(q^2) = f_1^{B \to b}(q^2) + \frac{q^2}{M_B^2 - M_b^2} f_3^{B \to b}(q^2),
\]

where \( f_3 \) represents the second-class form factor, which are identically zero in the exact SU(3) limit [3]. The value of \( f_S(q^2) \) at \( q^2_{\text{max}} = -(M_B - M_b)^2 \) can be precisely
evaluated by the double ratio method proposed in Ref. [12], where all relevant three-point functions are determined at zero three-momentum transfer $|\mathbf{q}| = 0$.

Here we note that the absolute value of the renormalized $f_S(q_{\text{max}}^2)$ is exactly unity in the flavor SU(3) symmetric limit, where $f_S(q_{\text{max}}^2)$ becomes $f_1(0)$, for the hyperon decays considered here. Thus, the deviation from unity in $f_S(q_{\text{max}}^2)$ is attributed to three types of the SU(3) breaking effect: (1) the recoil correction ($q_{\text{max}}^2 \neq 0$) stemming from the mass difference of $B$ and $b$ states, (2) the presence of the second-class form factor $f_3(q^2)$, and (3) the deviation from unity in the renormalized $f_1(0)$. Taking the limit of zero four-momentum transfer of $f_S(q^2)$ can separate the third effect from the others, since the scalar form factor at $q^2 = 0$, $f_S(0)$, is identical to $f_1(0)$. Indeed, our main target is to measure the third one.

The scalar form factor $f_S(q^2)$ at $q^2 > 0$ is calculable with non-zero three-momentum transfer ($|\mathbf{q}| \neq 0$) [13]. We can make the $q^2$ interpolation of $f_S(q^2)$ to $q^2 = 0$ together with the precisely measured value of $f_S(q^2)$ at $q^2 = q_{\text{max}}^2 < 0$. In Fig. 1, we plot the absolute value of the renormalized $f_S(q^2)$ as a function of $q^2$ for $\Xi^0 \to \Sigma^+$ (left) and $\Sigma^- \to n$ (right) at $am_{ud} = 0.005$. Open circles are $f_S(q^2)$ at the simulated $q^2$. The solid (dashed) curve is the fitting result by using the monopole (quadratic) interpolation form [13], while the open diamond (square) represents the interpolated value to $q^2 = 0$. As shown in Fig. 1, two determinations to evaluate $f_S(0) = f_1(0)$ from measured points are indeed consistent with each other.

We finally quote the values obtained from the monopole fit as our final values. The values of the renormalized $f_1(0)$ divided by the SU(3) symmetric value at $m_\pi = 330$ MeV are obtained as

$$[f_1(0)/f_1^{\text{SU(3)}}]_{\Xi \to \Sigma} = 0.981(8),$$

$$[f_1(0)/f_1^{\text{SU(3)}}]_{\Sigma \to n} = 0.962(14),$$

which are consistent with the sign of the second-order corrections on $f_1(0)$ reported in previous quenched lattice studies [12, 13], while our observed tendency of the SU(3) breaking correction disagrees predictions of both the latest baryon ChPT result [11] and large $N_c$ analysis [8].

**SUMMARY**

We have presented results of the flavor SU(3) breaking effects on hyperon vector coupling $f_1(0)$ for the $\Xi^0 \to \Sigma^+$ and $\Sigma^- \to n$ decays in (2+1)-flavor QCD using domain wall quarks. We have observed that the second-order correction on $f_1(0)$ is still negative for both decays at much smaller pion mass, $m_\pi = 330$ MeV, than in the previous quenched simulations. The size of the second-order corrections observed here is also comparable to what was observed in our DWF calculations of $K_{l3}$ decays [18]. To extrapolate the value of $f_1(0)$ to the physical point, our simulations at two different sea quark masses ($am_{ud} = 0.01$ and 0.02) are now in progress.

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1 We note that $q^2$ quoted here is defined in the Euclidean metric convention.
FIGURE 1. Interpolation of $|f_S(q^2)|$ to $q^2 = 0$ for $\Xi^0 \to \Sigma^+$ (left) and $\Sigma^- \to n$ (right) at $am_{ud} = 0.005$.

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