Towards the continuum limit of the lattice Landau gauge gluon propagator

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Abstract. The infrared behaviour of the lattice Landau gauge gluon propagator is discussed, combining results from simulations with different volumes and lattice spacings. In particular, the Cucchieri-Mendes bounds are computed and their implications for D(0) discussed.

Keywords: confinement, Landau gauge, lattice QCD, gluon propagator

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INTRODUCTION AND MOTIVATION

The link between the deep infrared behaviour of the gluon and ghost propagators and confinement, has motivated a great effort on computing these quantities on the lattice. Besides checking gluon confinement criteria, another important goal is to compare recent solutions of the Dyson-Schwinger equations with lattice results. In particular, the scaling solution [1] predicts a vanishing gluon propagator and a divergent ghost propagator at zero momentum. This solution complies with Gribov-Zwanziger [2] and Kugo-Ojima [3] confinement criteria. On the other hand, the decoupling solution [4] claims that the zero momentum gluon propagator is connected with a dynamical generated gluon mass.

In this paper we report on our current results for the Cucchieri-Mendes bounds in SU(3) lattice gauge theory.

CUCCHIERI-MENDES BOUNDS

The Cucchieri-Mendes bounds [5] provide upper and lower bounds for the zero momentum gluon propagator of lattice Yang-Mills theories in terms of the average value of the gluon field. In particular, they relate the gluon propagator at zero momentum \( D(0) \) with

\[
M(0) = \frac{1}{d(N_c^2 - 1)} \sum_{\mu, a} |A_{\mu}^a(0)|,
\]

where \( d \) is the number of space-time dimensions, and \( N_c \) the number of colors. In the above equation, \( A_{\mu}^a(0) \) is the \( a \) color component of the gluon field at zero momentum, defined by

\[
A_{\mu}^a(x) = \frac{1}{V} \sum_x A_{\mu}^a(x)
\]

where \( A_{\mu}^a(x) \) is the \( a \) color component of the gluon field in the real space. \( D(0) \) is related with \( M(0) \) by

\[
\langle M(0) \rangle^2 \leq \frac{D(0)}{V} \leq N_d(N_c^2 - 1) \langle M(0) \rangle^2. \tag{3}
\]

In the last equation \( \langle \cdot \rangle \) means Monte Carlo average over gauge configurations. For convenience we will use the definition \( N_d = N_d(N_c^2 - 1) \). The bounds in equation (3) are a direct result of the Monte Carlo approach. The interest on these bounds comes from allowing a scaling analysis which can help understanding the finite volume behaviour of \( D(0) \): assuming that each of the terms in inequality (3) scales with the volume according to \( A/V^\alpha \), the simplest possibility and the one considered in [5], an \( \alpha > 1 \) for \( \langle M(0) \rangle^2 \) clearly indicates that \( D(0) \rightarrow 0 \) as the infinite volume is approached. In this sense, this scaling analysis allows to investigate the behaviour of \( D(0) \) in the infinite volume limit.

For the SU(2) Yang-Mills theory [5], the results show a \( D(0) = 0 \) for the two dimensional theory, but a \( D(0) \neq 0 \) for three and four dimensional formulations.

RESULTS FOR SU(3) GAUGE THEORY

We have studied the Cucchieri-Mendes bounds within SU(3) lattice gauge theory for three values of the gauge coupling: \( \beta = 6.0 \) [6, 7], \( \beta = 5.7 \) [7], and \( \beta = 6.2 \).

Scaling analysis for \( \beta = 6.0 \)

In table 1 we present the lattice setup for \( \beta = 6.0 \), pointing out the differences to [6, 7].

Figure 1 shows the results for the bounds, together with the fits to \( \omega/V^\alpha \). Assuming this simple scaling
behaviour, our results for the exponent α support $D(0) = 0$ – see table 2. However, when one assumes a scaling behaviour like $C/V + \omega V^{-\alpha}$, the results support $D(0) \neq 0$ – see table 3. In this sense, a finite and non-vanishing value for $D(0)$ in the infinite volume is not excluded.

Concerning the fits to $\omega/V^\alpha$, the reasons for the differences in the values of $\alpha$ reported here and in [5] – and therefore on the behaviour of $D(0)$ in the infinite volume limit – are not clear. The simulations use different gauge groups. Although there it is generally believed that the SU(2) and SU(3) propagators are equivalent for momenta above 1 GeV [8, 9], a recent direct comparison for smaller momenta has shown a measurable difference in the infrared region [10].

Moreover, the physical volumes used in [5] are much larger – up to $(27\text{fm})^4$ – than the ones used here – up to $(8\text{fm})^4$. However, the reader should be aware that in the SU(2) case the lattice spacing used is about twice the lattice spacing considered here.

TABLE 2. Fits to $\omega/V^\alpha$ using lattice data at $\beta = 6.0$.
\begin{tabular}{c|cc|c}
\hline
 & $\omega$ & $\alpha$ & $\chi^2$ \\
\hline
$\langle M(0) \rangle$ & 9.53(36) & 0.5255(26) & 0.80 \\
$D(0)/V$ & 149 ± 10 & 1.0542(49) & 0.63 \\
$N_{cd} \langle M(0)^2 \rangle$ & 2927 ± 221 & 1.0504(54) & 0.83 \\
\hline
\end{tabular}

TABLE 3. Fits to $C/V + \omega V^{-\alpha}$ using lattice data at $\beta = 6.0$.
\begin{tabular}{cccc}
\hline
 & $\omega/1000$ & $\alpha$ & $C/100$ & $\chi^2$ \\
\hline
$\langle M(0) \rangle^2$ & 0.23(24) & 1.22(11) & 0.337(50) & 0.47 \\
$D(0)/V$ & 0.27(23) & 1.19(10) & 0.49(11) & 0.42 \\
$N_{cd} \langle M(0)^2 \rangle$ & 7.1 ± 7.3 & 1.22(11) & 11.0 ± 1.7 & 0.55 \\
\hline
\end{tabular}

LATTICE SPACING EFFECTS IN THE GLUON PROPAGATOR

In order to disentangle possible lattice effects due to the use of a different lattice spacing, we carried out simulations at $\beta = 5.7$ and $\beta = 6.2$. The lattice setup is shown in tables 4 and 5 respectively.

TABLE 4. Lattice setup for $\beta = 5.7$. The lattice spacing is $a = 0.1838(11)\text{fm}$.
\begin{tabular}{cccccccc}
\hline
$L^4$ & $L/(\text{fm})$ & # conf. & 51 & 149 & 149 & 149 & 132 & 100 & 55 \\
\hline
$4^4$ & 1.47 & 1.84 & 2.57 & 3.31 & 4.78 & 6.62 & 8.09 \\
$5^4$ & 1.74 & 2.32 & 3.49 & 4.65 & 5.81 & 7.1 & 8.6 & 10.0 & 5.5 \\
$6^4$ & 2.03 & 2.44 & 3.25 & 4.88 & 6.50 & 8.13 \\
$7^4$ & 2.57 & 3.31 & 4.78 & 6.62 & 8.09 & 10.0 & 12.0 \\
$8^4$ & 3.25 & 4.88 & 6.50 & 8.13 & 10.0 & 12.0 & 14.0 & 16.0 & 18.0 \\
$9^4$ & 4.88 & 6.50 & 8.13 & 10.0 & 12.0 & 14.0 & 16.0 & 18.0 & 20.0 \\
\hline
\end{tabular}

TABLE 5. Lattice setup for $\beta = 6.2$. The lattice spacing is $a = 0.07261(85)\text{fm}$.
\begin{tabular}{cccccccc}
\hline
$L^4$ & $L/(\text{fm})$ & # conf. & 51 & 56 & 87 & 99 & 150 & 150 & 85 \\
\hline
$4^4$ & 1.74 & 2.32 & 3.49 & 4.65 & 5.81 & 7.1 & 8.6 & 10.0 & 5.5 \\
$5^4$ & 2.03 & 2.44 & 3.25 & 4.88 & 6.50 & 8.13 & 10.0 & 12.0 \\
$6^4$ & 2.57 & 3.31 & 4.78 & 6.62 & 8.09 & 10.0 & 12.0 & 14.0 & 16.0 \\
$7^4$ & 3.25 & 4.88 & 6.50 & 8.13 & 10.0 & 12.0 & 14.0 & 16.0 & 18.0 \\
$8^4$ & 4.88 & 6.50 & 8.13 & 10.0 & 12.0 & 14.0 & 16.0 & 18.0 & 20.0 \\
$9^4$ & 6.50 & 8.13 & 10.0 & 12.0 & 14.0 & 16.0 & 18.0 & 20.0 & 22.0 \\
$10^4$ & 8.13 & 10.0 & 12.0 & 14.0 & 16.0 & 18.0 & 20.0 & 22.0 & 24.0 \\
$11^4$ & 10.0 & 12.0 & 14.0 & 16.0 & 18.0 & 20.0 & 22.0 & 24.0 & 26.0 \\
\hline
\end{tabular}

Some differences have been seen between the gluon propagator computed at different lattice spacings for similar physical volumes. An example can be seen in figures 2 and 3, where the infrared $\beta = 6.2$ data does not agree with data from $\beta = 5.7$ and 6.0 simulations. These differences deserve further investigations to clarify any possible effects due to finite lattice spacing.

Scaling analysis for $\beta = 5.7$ and $\beta = 6.2$

In what concerns the fits to $\omega/V^\alpha$, the analysis of the data coming from both sets still supports a vanishing $D(0)$ in the infinite volume limit – see tables 6 and 7.

Similarly to the case studied before, the lattice data is also well described by the functional form $C/V + \omega V^{-\alpha}$ – see tables 8 and 9. Although the $\beta = 5.7$ case supports $D(0) \neq 0$, for $\beta = 6.2$ the statistical errors do not allow to take any conclusion. In fact, although $C = 0$ within statistical errors, we also get $\alpha = 1$. For this case, it is worth an increase of statistics.

FIGURE 1. Cucchieri-Mendes bounds for $\beta = 6.0$. 

![Graph showing Cucchieri-Mendes bounds for $\beta = 6.0$.]
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Comparing the gluon propagator computed using different lattice spacings at the same physical volume $V \sim (4.8\, fm)^4$.

### TABLE 6. Fits to $\omega V^{-\alpha}$ using lattice data at $\beta = 5.7$. In order to keep $\chi^2_\nu < 2$, the $26^4$ lattice data has been excluded.

| $\omega$  | $\alpha$ | $\chi^2_\nu$ |
|-----------|-----------|--------------|
| $\langle M(0) \rangle / V$ | 4.63(12) | 0.524(23) | 1.92 |
| $D(0)/V$ | 32.8±1.6 | 1.046(42) | 1.14 |
| $N_{cd}(M(0))^2$ | 696±37 | 1.048(47) | 1.72 |

### CONCLUSIONS

We have studied the scaling behaviour of Cucchieri-Mendes bounds using ensembles generated at several lattice spacings. Fits of the data to a pure power law in the volume strongly support $D(0) = 0$, but the use of other ansatze do not allow to take definitive conclusions.

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Comparing the gluon propagator computed using different lattice spacings at the same physical volume $V \sim (6.5\, fm)^4$.

### TABLE 9. Fits to $C/V + \omega V^{-\alpha}$ using lattice data at $\beta = 6.2$. Data for $M(0)$ does not include $48^4$.

| $\omega/1000$ | $\alpha$ | $C/1000$ | $\chi^2_\nu$ |
|--------------|-----------|-----------|--------------|
| $\langle M(0) \rangle^2$ | 0.34(66) | 1.13(29) | 0.4±1.2 | 0.13 |
| $D(0)/V$ | 0.366(47) | 1.07(29) | 0.04±5.6 | 0.95 |
| $N_{cd}(M(0))^2$ | 8.6±6.7 | 1.08(28) | 4±85 | 0.25 |

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