Kinematic modelling and analysis of single leg in hybrid wheel-legged mobile robot

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Abstract. Mobile manipulators are extensively used in various applications. Most of the mobile manipulators are built on a wheeled mobile platform. A wheeled mobile platform performs well on flat terrains but is not suitable for navigation in rough terrains. The legged robots are commonly used for rough terrain applications. However, the navigation speed of legged robots is lesser than wheeled mobile robots. This paper focuses on the development of hybrid wheel-legged robots to overcome the above difficulties. The proposed model is quadruped with five degrees of freedom in each leg. The development of the kinematic model, its dimensional synthesis, and motion study for a single leg is presented in detail. DH-conventions are used for kinematic modelling, manipulability index is used for dimension synthesis, and motion study is carried out in ADAMS.

Keywords. Hybrid wheel-legged mechanism, rough terrains, kinematic modelling, Dimensional synthesis

1. Introduction

A mobile manipulator refers to the mounting of the manipulator over a mobile robot platform. The mobile manipulator's main advantage is its increased workspace compared to the industrial manipulators. This improvement in the workspace makes them more viable and gives greater flexibility for manipulation. A mobile manipulator also can be used as a service robot and has domestic employment potential. They can also be used in a military operation like surveillance, rescue, and combat. Legged robotic systems mounted with manipulators have been drawing much attention from both the industrial and academic community for their advantages of combining manipulation and locomotion in uneven terrains. The remaining of the paper is structured as follows. Section 2 describes the proposed mechanism and its mobility, whereas Section 3 deals with forwarding kinematic modelling in a single leg. Section 4 gives the inverse kinematic analysis of the proposed mechanism in a single leg, Section 5 discusses the dimension synthesis and then in Section 6, a motion study was carried out. Results were discussed in Section 7 and the paper was concluded in Section 8.

Very few works are carried out in the past based on Mobile manipulators. The major complexity in the development of the mobile manipulators is the design of controllers to simultaneously track the mobile base as well as the end effector’s trajectory. It is to be noted that most of these works involve the use of the wheeled mobile robot for mobile manipulators [1-6]. These wheeled mobile platforms are suitable mainly for flat terrains. In order to overcome the complexity in control, various techniques were used, such as impedance with sliding mode control [1], task space formulation [2], extended Cartesian
coordinates [3]. Few of the works [4-7] are focused on unknown terrains, which makes control more challenging. In spite of the above challenges, the wheeled robots are not suitable for navigation in rough terrains. Therefore, legged robots were developed to overcome the above disadvantages. Compared to legged robots, the wheeled legged robots combine the advantage of both wheel and legged robots. Obtaining the mathematical model of the wheeled legged robot has various difficulties. An external camera based approach was used for development of inverse kinematics by Teka et al. [8]. Grand et al. [9] assumed frames arbitrarily for the formulation of kinematic models. Du et al. [10] proposed a centroidal momentum model for the analyses and control of the wheel-legged mobile robot. Niu et al. [11] developed a wheel-legged robot that mimics the human leg. However, the developed model was very heavy and resulted in bad dynamic behaviour. A cost-efficient design was proposed by Nakajima [12], yet the robot lacks the control along its yaw axis in rough terrains. Ding et al. [13] studied the effects of slip ratio of the wheels with the change in the vertical height of the wheel-legged robotic platform.

It is clear from the above literature that most of the works are carried out for flat terrains and are focused on using a wheeled mobile platform. This restricts the use of mobile manipulators only for indoor environments. Very few works are carried out on wheel-legged type mobile robots and are currently being explored. This work discusses a novel design of a hybrid wheel-legged mechanism to overcome the above difficulties.

2. Model description
Figure 1 shows the conceptual model of the proposed wheel-legged hybrid robot. This model consists of five revolute joints, four links and a wheel on each leg and a base platform. This arrangement allows the robot to navigate in a flat as well in rough terrain. The focus of this study is to achieve the walking motion without undergoing any change in the angle of the platform. The motion of this robot is divided into two cases, namely 1. The motion of the leg 2. The motion of platform/base

![Figure 1. Proposed hybrid wheel legged mechanism](image)

2.1. Mobility Analysis
Mobility analysis is done to obtain the number of independent parameters required to control the mechanism. Kutzbach-Grubler criterion is used to determine these independent parameters (M) and is shown in equations (1) and (2).

\[
M = 6 (N - J - 1) + \sum f_i \quad \text{For mechanism} \tag{1}
\]

\[
M = 6 (N - J) + \sum f_i \quad \text{For chain} \tag{2}
\]

where, \(N\) denotes the number of links, \(J\) denotes the number of joints, and \(f_i\) denotes Degrees of freedom (DOF) of the \(i^{th}\) joint.

The DOF analysis was done for three different scenarios:

1. Platform / base
2. Link 1
3. Link 2
4. Link 3
5. Link 4 with two revolute joints
6. Wheels
1. Considering single leg of the mechanism with $N = 6; J = 5; \sum f_i = 5$ yields $M = 5$. This shows all the five revolute joints are used in control of a single leg.

2. Considering the whole mechanism including the wheel yields $M = 24$, where $N = 25; J = 24; \sum f_i = 24$. This shows twenty revolute joints are used for the control of all four legs and remaining four for the wheels.

3. For the Legged model without wheels with $N = 13; J = 12; \sum f_i = 12$ yields $M = 12$ that denotes three joints each leg are used in control.

3. **Forward kinematic modelling**

This section deals with kinematic modelling of the mechanism. Compared to the conventional use of forward kinematics in Serial manipulators, two different forward kinematic equations are developed for the proposed robot. In both the cases, Denavit and Hartenberg modelling principles were used. The kinematic model was carried out on one leg and the same can be extended to all the four legs due to their identical structure. Figure 2 shows the frame assignment to the joints on the proposed model while Figure 3 shows the axis assignment to these frame as per the DH convention. In Figure three, numbers within the {} refers to the frame number.

![Figure 2. Frame assignment for proposed model in single leg](image)

![Figure 3. DH frame assignment for proposed model in single leg](image)
Here $L_1$, $L_2$, $L_3$, $L_4$ denotes the link lengths between the joints $\theta_1$, $\theta_2$, $\theta_3$, $\theta_4$, $\theta_5$ are the joint variables representing the five revolute joints. $2P$ and $2Q$ are the length and width of the platform respectively. The determination of these link lengths are discussed in section 5.

Two types of forward kinematic analysis were performed based on the above frame assignments. In case 1, frame \{1\} remains fixed and wheel frame \{wheel\} is the end effector. Thus, case 1 is used for motion of leg in which the wheel acts as end effector and \{1\} acts as fixed frame. This equation is used to obtain the desired trajectory of the wheel. Table 1 shows the DH parameters for case 1 and the final transformation matrix can be obtained by post multiplying the transformation matrix of each joint.

| Parameter | $\theta$ (deg) | d (mm) | a (mm) | $\alpha$ (deg) |
|-----------|----------------|--------|--------|---------------|
| 1         | $\theta_1$     | $-L_1$ | 0      | 90            |
| 2         | $\theta_2$     | 0      | $L_2$  | 0             |
| 3         | $\theta_3$     | 0      | $L_3$  | 0             |
| 4         | $\theta_4$     | 0      | 0      | -90           |
| 5         | $\theta_5$     | 0      | $L_4$  | 0             |

The transformation matrices for various joints were obtained by substituting the DH parameters in the equation (3).

$$i^{-1}T = \begin{bmatrix}
\cos \theta_i & -\sin \theta_i \cos \alpha_i & \sin \theta_i \sin \alpha_i & a_i \cos \theta_i \\
\sin \theta_i & \cos \theta_i \cos \alpha_i & -\cos \theta_i \sin \alpha_i & a_i \sin \theta_i \\
0 & \sin \alpha_i & \cos \alpha_i & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}$$

(3)

Then resulting final transformation matrix are shown in equations (4) and (5).

$$\text{Wheel}^1T = \frac{1}{2}T * \frac{3}{2}T * \frac{4}{3}T * \frac{5}{4}T * \text{Wheel}^5T$$

(4)

$$\text{Wheel}^1T = \begin{bmatrix}
c_1 & -c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 \\
c_1 & -c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 \\
c_1 & -c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 \\
c_1 & -c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 - c_5 & c_4 & c_3 & c_2 & c_1 \\
\end{bmatrix}$$

(5)

For case 2, wheel frame \{Wheel\} remains fixed and frame \{1\} is the end effector. Table 2 shows the DH parameters for case 2 and the corresponding final transformation matrices are shown in equation (6) and (7). This equation is used to obtain the desired pose of the platform.

| Parameter | $\theta$ (deg) | d (mm) | a (mm) | $\alpha$ (deg) |
|-----------|----------------|--------|--------|---------------|
| 1         | $\theta_3$     | 0      | $-L_4$ | 90            |
| 2         | $\theta_4$     | 0      | $-L_3$ | 0             |
| 3         | $\theta_3$     | 0      | $-L_2$ | 0             |
| 4         | $\theta_2$     | 0      | 0      | -90           |
| 5         | $\theta_1$     | $L_1$  | 0      | 0             |
Similarly, from equation (3), the final transformation matrix is given by

\[
\begin{align*}
\text{Wheel}_1^T &= \text{Wheel}_5^T \cdot \frac{5}{4}T \cdot \frac{3}{2}T \cdot \frac{2}{1}T \\
\text{Wheel}_5^T &= \begin{bmatrix}
C_1C_5C_{234} - S_1S_{234} & -C_5S_{234} - C_1S_{5} & -C_5S_{234} - C_1S_{5} & (L_4 + L_3C_4 + L_2S_{234} + L_2C_{34}) \\
S_5C_1C_{234} - c_{1}S_{5} & S_1S_{234} - C_1C_{5} & -S_5C_{234} - S_1S_{5} & (L_4 + L_3S_4 + L_1C_{234} + L_2S_{34}) \\
C_1S_{234} & -S_1S_{234} & C_{234} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\]

(6)

(7)

4. Inverse kinematic analysis

This section deals with formulation of inverse kinematic equations which are needed for the control of individual joints. In this work, an alternative approach is used to find the joint angles compared to the conventional approach followed for serial manipulator. In order to ensure the undisturbed wheel position for the robot to execute wheeled motion at any instant of time, wheel frame needs to be parallel to frame \{1\}. This constraint reduces the complexity in deriving the inverse kinematic equation. Based on this constraint, the obtained transformation matrix between the frame \{1\} and wheel frame \{wheel\} is:

\[
\begin{bmatrix}
1 & 0 & 0 & x \\
0 & 1 & 0 & y \\
0 & 0 & 1 & z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(8)

Here \((x, y, z)\) represents the wheel position coordinates. For case 1, equation (4) gives

\[
\text{Wheel}_1^T \cdot [\text{Wheel}_5^T]^{-1} = \frac{5}{2}T \cdot \frac{3}{2}T \cdot \frac{2}{1}T
\]

(9)

From (9), the inverse kinematic equations are obtained and are shown in equations (10-14).

\[
\begin{align*}
\theta_1 &= \tan^{-1}\left(\frac{y}{x-L_4}\right) \\
\theta_2 &= \tan^{-1}\left(\frac{(L_1+x)+(L_3C_3+L_2)-(L_3S_3)+(x+c_1+yS_1-L_4C_3)}{(L_3C_3+L_2)+(x+c_1+yS_1-L_4C_3)+(L_1+x)+(L_3S_3)}\right) \\
\theta_3 &= -\cos^{-1}\left(\frac{(x+c_1+yS_1-L_4C_3)^2+(x+L_4)^2-L_3^2}{2L_3L_4}\right) \\
\theta_4 &= -(\theta_2 + \theta_3) \\
\theta_5 &= -\theta_4
\end{align*}
\]

(10)

(11)

(12)

(13)

(14)

In case 2, \((x, y, z)\) represents the platform coordinates. By using similar decoupling method the equation (6) yields

\[
\begin{bmatrix}
\text{Wheel}_1^T \cdot [\text{Wheel}_5^T]^{-1} = \frac{5}{2}T \cdot \frac{3}{2}T \cdot \frac{2}{1}T
\end{bmatrix}
\]

(15)

From equation (15) solved inverse kinematic are shown in equations (16-20).

\[
\begin{align*}
\theta_1 &= -\theta_5 \\
\theta_2 &= -(\theta_3 + \theta_4)
\end{align*}
\]

(16)

(17)
\[ \theta_3 = -\cos^{-1}\left(\frac{(x+c_5+y+s_5+L_4*c_1)^2+(z-L_1)^2-L_2^2-L_3^2}{2*L_2*L_3}\right) \]  
(18)

\[ \theta_4 = \tan^{-1}\left(\frac{(L_2^2-L_3^2-L_3^2)*(x+c_5+y+s_5+L_4*c_1)}{(L_2^2+L_3^2-L_3^2)*(x+c_5+y+s_5+L_4*c_1)}\right) \]  
(19)

\[ \theta_5 = \tan^{-1}\left(\frac{y}{x}\right) \]  
(20)

5. Dimensional synthesis
The determination of the link lengths \(L_1, L_2, L_3, L_4\) and the dimension of the platform are discussed in this section. The desired step length of the proposed mechanism was considered to be 600mm [19]. Maximum step height was limited to 280 mm [21].

**Figure 4.** Shows the undefined dimensions of mechanism

Following are the constraints used in the dimensional synthesis of the proposed mechanism.

i. Initial clearance between the ground and platform is 450 mm.

ii. Initially assumed platform width = 300 mm and \(L_1 = 50\) mm, \(L_4 = 50\) mm. It is to be noted that \(L_1\) and \(L_4\) does not affect \(L_2\) and \(L_3\) dimensions.

iii. Step length is considered as 600 mm and its coordinates are shown Figure 5.

**Figure 5.** Step length of the proposed model

iv. In order to avoid collision between link 2 and obstacle, \(L_3\) should be greater than obstacle height. Hence, \(L_3 > 280\) mm.

v. Length of links \(L_2\) and \(L_4\) were limited within 300 mm in order to reduce the motor torque.

vi. Manipulability Index \((w)\) is a quality measure which describes the distance to singular configuration. It can be analyzed by measuring manipulability ellipsoid which is spanned by the singular vectors of the Jacobian. It should be maximum because it is proportional to the volume of ellipsoid in which no singularity is possible [18]

\[ w = \sqrt{|f(q) * f(q)^T|} \]  
(21)
where J is Jacobian matrix and q is joint variable

By using these constraints and inverse kinematic equations, a program was simulated in MATLAB software for some length combination. Then final possible dimensions are shown in Table 3.

| Parameter name          | Range (mm) |
|-------------------------|------------|
| Platform length         | 700        |
| Platform width          | 300        |
| Platform thickness      | 15         |
| L₁                      | 50         |
| L₂                      | 300        |
| L₃                      | 290        |
| L₄                      | 50         |

### 6. Motion study
The successful motion of the robot is divided into two stages, namely motion of the leg and motion of the platform. These motions are discussed in detail in the subsequent section. Motion study begins with obtaining the end effector's coordinates as per the desired motion and then solving for the inverse kinematic to obtain the necessary joint angles to trace the desired end-effector trajectory. The desired motion's joint trajectories were generated, and the torque required in each joint was determined.

#### 6.1. Motion of the legs
This section defines the joint torque for all the revolute joints when the platform is fixed. The maximum walking speed of the robot is considered to be 1km/hr and the robot can cover a distance of step length of 600mm in 2.16 seconds. The clearance between the platform and ground is maintained at 500 mm, the distance between wheel and platform is 200mm. In Figure 6, coordinates for the motion of the legs are shown. A parabolic path is considered for the motion of the legs and this also ensures that the wheel does not come in contact with the ground during the motion of the legs. Figure 7 shows the three-dimensional trajectory of the wheel frame.

**Figure 6.** Displays the coordinates for this motion  
**Figure 7.** Desired trajectory of wheel

#### 6.1.1. Position based Joint Trajectory
A suitable trajectory is designed for each joint to ensure the safe movement of each links without affecting the end effector trajectory. Based on the coordinates defined in Figure 6, the joint angles are obtained from inverse kinematic equations (10-14) and are shown in Table 3. Fifth order polynomial equation (22) is used to generate trajectory for joint control in every joint.
\[ \theta(t) = c_0 + c_1 \cdot t + c_2 \cdot t^2 + c_3 \cdot t^3 + c_4 \cdot t^4 + c_5 \cdot t^5 \]  \hspace{1cm} (22)

where \( C_0, C_1, C_2, C_3, C_4, C_5 \) are constants of the polynomial equation and \( t \) refers to time.

By using the values from the Table 3 and polynomial equation (22), the values of the constants are calculated and desired fifth order polynomial equation for each joint were obtained.

**Table 3. Data required for joint control**

| Time | x   | y   | z     | \( \theta_1 \) | \( \theta_2 \) | \( \theta_3 \) | \( \theta_4 \) | \( \theta_5 \) |
|------|-----|-----|-------|--------------|--------------|--------------|--------------|--------------|
| 0    | 200 | 300 | -500  | 63.4349      | -35.6524     | -35.9265     | 71.5789      | -63.4349     |
| 0.54 | 200 | 150 | -462.5| 45           | -25.3725     | -76.352      | 101.7246     | -45          |
| 1.08 | 200 | 0   | -450  | 0            | -26.7527     | -87.2329     | 113.9856     | 0            |
| 1.62 | 200 | -150| -462.5| -45          | -25.3725     | -76.352      | 101.7246     | 45           |
| 2.16 | 200 | -300| -500  | -63.4349     | -35.6524     | -35.9265     | 71.5789      | 63.4349      |

6.2. Motion of platform

This section analyses the ability of the single leg to displace the platform in vertical direction. As discussed earlier, wheel frame is fixed frame and frame \{1\} is the end effector frame. The displacement in the vertical direction (Z axis of wheel frame) is considered because the torque required for vertical motion comparatively higher than the torque required along other axis.

Figure 8 displays the planar motion coordinates in ZX plane such that the movement is along the Z direction. It is clear from Figure 8 that clearance of the platform from the ground is 500 mm and X direction it is maintained in 350 mm for maximum walkable position.

![Figure 8. Coordinates for the motion](image)

**Figure 8. Coordinates for the motion**

![Figure 9. Desired trajectory of platform](image)

**Figure 9. Desired trajectory of platform**

6.2.1. Position based trajectory for the joints

In order to achieve the linear motion of the platform, a sixth order polynomial equation (23) is considered as the joint trajectory. Based on the coordinates defined in Figure 8, the joint angles are obtained from inverse kinematic equations (16-20) and are shown in the Table 4

\[ \theta(t) = c_0 + c_1 \cdot t + c_2 \cdot t^2 + c_3 \cdot t^3 + c_4 \cdot t^4 + c_5 \cdot t^5 + c_6 \cdot t^6 \]  \hspace{1cm} (23)

where \( C_0, C_1, C_2, C_3, C_4, C_5, C_6 \) are constants of the polynomial equation and \( t \) refers to time.

These constant are calculated using the values from Table 4 and polynomial equation (23). This yields the desired sixth order polynomial equation for each joints.
Table 4. Joint value for this motion.

| Time | x    | y    | z    | θ1  | θ2   | θ3   | θ4   | θ5   |
|------|------|------|------|------|------|------|------|------|
| 0    | 350  | 0    | 0    | 0    | -60.2274 | 118.9033 | -58.6759 | 0    |
| 1.5  | 350  | 0    | 100  | 0    | -39.804 | 115.2052 | -75.4012 | 0    |
| 3    | 350  | 0    | 200  | 0    | -18.963 | 104.6817 | -85.7186 | 0    |
| 4.5  | 350  | 0    | 300  | 0    | 1      | 88.0568  | -89.0597 | 0    |
| 6    | 350  | 0    | 400  | 0    | 21.3241 | 64.1379  | -85.4619 | 0    |
| 7.5  | 350  | 0    | 500  | 0    | 50.4035 | 17.5511  | -67.9546 | 0    |

7. Results and discussion

In this work, MSC ADAMS VIEW software is used for the motion study of the single leg of the proposed hybrid wheel-legged mechanism. The obtained equations (22 and 23) for both cases were used to analyse the motion of joints and end effector.

Figures 10 - 14 shows the results of motion of legs (case 1). It is clear from Figure 7 and 10 the desired parabolic motion of wheel is achieved. Therefore, the obtained torque values of each joint are noted from the simulation study.

![Figure 10. Performance Analysis for case 1](image)

The wheel velocity in Figure 11 shows that the robot is able to achieve walking speed of 1.08km/hr which is close to the desired velocity of 1 km/hr. The negative sign in the wheel velocity shown in Figure 11 leg moving in negative direction of the assigned wheel frame.

![Figure 11. Achieved walking speed form simulation](image)

Figure 12 – 14 shows the torque values obtained for the motion of the leg. Joint 2 demands a heavy torque in the range of 1500 Nmm to 2500 Nmm while joint 5 demands a torque of 7.5 Nmm. Other joints 1, 3 and 4 demand torque less than 500 Nmm. The larger torque value of joint 2 compare to joint 1 is because joint 2 lifts subsequent joints against the gravity whereas joint 1 performs a yaw motion and demands less torque.
Figure 12. Torques curve at joint 1, 3 and 4

Figure 13. Torque curve at joint 2

Figure 14. Torque curve at joint 5

Figure 15–17 represents the results of platform motion (case 2). It is clear from Figure 8 and 15 that the desired linear motion for the platform is achieved. Therefore, obtained torque values are noted from simulated results.

Figure 15. Performance Analysis for case 2

Figure 16 and 17 displays the torque values obtained for the platform motion. Joint 2 needs 30 Nmm while joint 3 and 4 demands heavy torque of 5100 Nmm. This is because joint 3 and 4 lifts subsequent joints as well as platform against the gravity.
**Conclusion**

In this work, the hybrid wheel-legged robot was designed by considering the advantage of both wheel and legged locomotion. The motion study was successfully performed on a single leg, and the same can be extended to all four legs. The MSC ADAMS software is used for the simulation study. The resulting trajectories from the simulation study are in line with desired trajectories. Therefore the obtained torque values of each joint are considered for future work. Further, the torques are to be determined analytically and compared with the simulation study's torques. Also, the obtained dynamic model shall be used for the implementation of control strategies.

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