The MHV lagrangian vertices and the Parke-Taylor formula

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Abstract: We explicitly calculate the vertices of the MHV-rules lagrangian in 4-dimensions. This proves that the vertices in the lagrangian obtained by a canonical transformation from light-cone Yang-Mills theory coincide to all order with the Parke-Taylor formula, filling the gap originally left in the lagrangian derivation of the CSW rules.

Keywords: Gauge symmetry, QCD
1. Introduction

The standard perturbative calculation for pure Yang-Mills theory is known to be challenging because the number of Feynman diagrams contributing to a scattering amplitude increases rapidly with the number of legs even at tree-level. A number of techniques has been devised to simplify the task. In [1] Parke and Taylor conjectured the general formula for colour-ordered MHV n-gluon scattering amplitudes. (Amplitudes with the most plus helicity gluons possible.) The formula was later proved by Berends and Giele [2] using recursion methods. In [3] Cachazo, Sr\v{r}ek and Witten discovered a set of remarkably simple rules inspired by twistor string theory to calculate scattering amplitudes with generic helicity configurations. The CSW rules take off-shell continued MHV amplitudes and scalar propagators as their building blocks and have been successfully applied at tree level [4, 5, 6] and several loop-level amplitudes [7, 8, 4, 10]. The proof of the CSW rules however, was provided using another approach. Britto, Cachazo, Feng and Witten [12, 13] derived a new recursion relation by analysing singularities of scattering amplitudes. Using Cauchy’s theorem, it was shown that the analytically continued amplitude can be equally obtained from a sum over residues [14, 17, 16, 7, 18]. The BCFW recursion method has been generalised to include massive particles [19, 20, 21, 22], superpartners [23] and to gravity [24, 25].

In [26] a lagrangian derivation of CSW rules was found. Starting from the Yang-Mills lagrangian in light-cone gauge, a canonical transformation was applied to transverse components of the gauge field to rearrange the self-dual part of the lagrangian into a free field theory

\[ \mathcal{L}^{++}[A] + \mathcal{L}^{-+-}[A] = \mathcal{L}^{++}[B] \]

After the transformation the equivalent lagrangian theory contains only vertices that have maximum helicity contents, in agreement with the CSW prescription. The method
was extended to QCD \cite{28} and supersymmetric theories \cite{30, 29}. In particular, a corresponding D-dimensional MHV-rules lagrangian has been developed to incorporate dimensional regularisation and to explain the non-vanishing all-plus amplitudes at one-loop level which do not appear in the CSW construction \cite{31, 32}. Alternatively, one can choose to work in 4-dimension provided a suitable regulator is imposed. In \cite{10} Brandhuber, Spence, Travaglini and Zoubos used the light-cone friendly regularisation scheme of Thorn \cite{11}. In this approach the “missing” all-plus amplitudes were explained by the extra contribution from counterterms.

In \cite{26} an indirect argument was given to show that in 4-dimension, the vertices of the MHV-rules lagrangian were argued to have the same form as the Parke-Taylor formula. At tree level, scattering amplitudes and the vertices can only differ by factors that contain squares of momenta, which vanish on-shell. Since the vertices in 4-dimension were known to be holomorphic such factors must be absent. Explicit calculations verified that the vertices agree with the Parke-Taylor formula up to 5-points \cite{27}, however a general verification to all vertices has been missing. In this short paper we present the proof to show that n-point vertices match with the Parke-Taylor formula. The notation used throughout the paper follow the definitions in \cite{27} and is summarised in the appendix.

2. MHV vertices in 4 dimensions and the Parke-Taylor formula

In the MHV lagrangian theory \cite{26} components of the gauge field $A$ and $\bar{A}$ in the light-cone coordinates are expanded into a new set of field variables $B$ and $\bar{B}$ using a canonical transformation. The new vertices are then derived by translating the $A$ and $\bar{A}$ fields attached to the vertex terms in the original lagrangian into $B$ and $\bar{B}$. For the 4-dimensional theory, Ettle and Morris \cite{27} have shown that the n-th order term in the $A$ field expansion can be summarised by a simple formula.

\[ B_2 \]
\[ \cdots \]
\[ B_n \]
\[ \mathcal{A}_1 \]

\begin{figure}[h]
  \centering
  \includegraphics[width=0.5\textwidth]{mhzv.png}
  \caption{MHV vertices in 4 dimensions and the Parke-Taylor formula.}
\end{figure}

\begin{equation}
  f(p)\hat{A}^\nu(p) = \int d^4q \frac{\delta B(q)}{\delta \hat{A}^\nu(p)} f(q)\bar{B}(q)
\end{equation}

\footnote{Note that the holomorphic behaviour of the MHV vertices is exclusive to the choice implicitly taken by the canonical transformation, where the canonical conjugate momentum $\hat{p}\hat{A}(p)$ is assumed to have the inverse transformation relation from the field variable $A(p)$. One can show that for a generic function of momentum $f(p)$, the transformation

\[ f(p)\hat{A}^\nu(p) = \int d^4q \frac{\delta B(q)}{\delta \hat{A}^\nu(p)} f(q)\bar{B}(q) \]

also preserves the integration measure. Following the method originally used in \cite{26} one can obtain vertices that give the same helicity configurations as the vertices used in the CSW rules. The generic translation kernel derived from the condition \cite{11} is not holomorphic, and cannot be expressed as products of round brackets. The MHV vertices derived from general measure-preserving transformations are not the same as the Parke-Taylor formula.}
Similarly, the $\tilde{A}$ expansion was shown to have the form

$$\tilde{A}_1 \rightarrow \frac{\hat{1} \hat{3} \hat{4} \cdots \hat{n}-1}{(23) \cdots (n-1,n)} B_2 B_3 \cdots B_n$$

(2.1)

In the following we shall prove that the new vertices have the same form as the Parke-Taylor formula by first proving that the MHV vertices described above and the Parke-Taylor formula can both be spanned by terms of the form

$$\frac{1}{(23) (34) \cdots (k-1,k)} \times \frac{1}{(k+1,k+2) \cdots (m-1,m)}$$

$$\times \frac{1}{(m+1,m+2) \cdots (l-1,l)} \times \frac{1}{(l+1,l+2) \cdots (n,1)}$$

(2.3)

together with terms of the form

$$\frac{1}{(23) (34) \cdots (k-1,k)} \times \frac{1}{(k+1,k+2) \cdots (l-1,l)} \times \frac{1}{(l+1,l+2) \cdots (n,1)}$$

(2.4)

and then we shall check that the coefficients of the expansion agree with each other. The denominators of (2.3) and (2.4) contain only round brackets of adjacent legs and are split into three and four groups of sequential products respectively. (Note however, the product is taken as 1 if it starts and ends at the same leg.) The vertices and the Parke-Taylor formula are regarded as functions of tilde component variables $\tilde{p}$ contained in the round brackets while expansion coefficients depend only on hat components $\hat{p}$. For example, the 5-point Parke-Taylor formula can be rewritten as

$$\frac{(12)^3}{(23) (34) (45) (51)} = A \frac{(12)}{(23) (34)} + B \frac{(12)}{(23) (45)} + C \frac{(12)}{(23) (51)}$$

$$+ D \frac{(12)}{(34) (45)} + E \frac{(12)}{(34) (51)} + F \frac{(12)}{(45) (51)}$$

$$+ G \frac{1}{(23)} + H \frac{1}{(34)} + I \frac{1}{(45)} + J \frac{1}{(51)}$$

(2.5)

The coefficients can be easily determined by the method of partial fractions. To calculate $A$ we set (23) and (34) to be zero. These conditions allow us to solve $3$ and $4$ in terms of $2$. 

\[ \text{---} - 3 - \text{---} \]
\[ \tilde{3} = \frac{3}{2}, \quad \tilde{4} = \frac{4}{2} \quad (2.6) \]

Brackets formed by momenta 3 and 4 with other legs \( p \) \( q \) can therefore be replaced by brackets of 2 with \( p \) \( q \).

\[
(3, p) = \frac{3}{2} (2, p), \quad (4, q) = \frac{4}{2} (2, q) \quad (2.7)
\]

Together with momentum conservation the remaining brackets (45) and (51) can be expressed in terms of (12). Matching both sides of the equation gives us the coefficient \( A \). For terms like \( G \) that do not have (12) in the numerator we set (12) and (23) zero. The other coefficients are then determined through the same procedure.

\[
A = \frac{235}{1 (2 + 3 + 4)}, \quad B = \frac{2 \left(1 + \hat{2} + \hat{3}\right)^2}{1 (2 + 3)}, \quad C = \frac{124}{(1 + 5) (2 + 3)}, \quad (2.8)
\]

\[
D = \frac{4 (3 + \hat{4} + \hat{5})^2}{12}, \quad E = \frac{-14 (1 + \hat{2} + \hat{5})^2}{23 (1 + 5)}, \quad F = \frac{135}{2 (1 + 4 + 5)} \quad (2.9)
\]

\[ G = H = I = J = 0 \quad (2.10) \]

From the method described above, it is clear that the Parke-Taylor formula does not contribute to terms independent of (12). At the end of the argument we shall show this is generally also true for the n-point MHV vertices, but for convenience for the moment we will retain such terms in the expansion.

2.1 Partial fraction expansion

To justify the expansion we need to show (2.3) and (2.4) are sufficient to describe MHV vertices and the Parke-Taylor formula. An n-point vertex in the MHV lagrangian theory consists of terms splitted from the 3-point and 4-point LCYM vertices. The contributions from the 4-point vertex (Fig.1) naturally are of the form (2.3). For vertices that originate from the 3-point vertex (Fig.2), translating \( A \) and \( \bar{A} \) fields associated with each leg into \( B \) and \( \bar{B} \) produces a series of products of brackets. Using the bilinear property the factor \((1 + \cdots (l + 1), 2 + \cdots k)\) in the numerator can be expanded into brackets of single leg momenta \((p, q)\) with \( p \) and \( q \) running through 1 to \( l + 1 \) and 2 to \( k \) respectively. Each term \((p, q)\) can then be rewritten as a linear combination of brackets of adjacent momenta by noticing that

\[
\frac{(p, q)}{pq} = \tilde{q} \tilde{p} - \tilde{p} \tilde{q} = \tilde{q} \tilde{p} - \tilde{p} \tilde{q} + \tilde{p} \tilde{q} - \tilde{p} \tilde{q} + \tilde{p} \tilde{q} - \tilde{p} \tilde{q} = \frac{(p - 1, q)}{p - 1 q} + \frac{(p, p - 1)}{p - 1 p - 1} \quad (2.11)
\]
Applying (2.11) repeatedly $p$ and $q$ can be moved toward 1 and 2, resulting in a term of the form (2.4) while brackets of adjacent momenta produced in the process cancel brackets in the denominator, resulting terms of the form (2.3).

![Figure 1: Translated 4-point vertex](image)

![Figure 2: Translated 3-point vertex](image)

The Parke-Taylor formula can also be spanned by (2.3) and (2.4). To show this is true we need to express two of the $(12)$ factors in the numerator as a linear combination of products of two different brackets $(ab)(cd)$. To replace a first $(12)$, notice that the momentum labels are defined cyclically

$$\sum_{k=1}^{n} (k, k+1) = \sum_{k=1}^{n} \left( \frac{k+1}{k+1} - \frac{k}{k} \right) = 0 \quad (2.12)$$

Therefore we have

$$\frac{(12)}{12} = -\frac{(23)}{23} - \frac{(34)}{34} - \cdots - \frac{(n1)}{n1} \quad (2.13)$$

Using (2.13) to expand one of the $(12)$ factors gives us a sum over terms of the form $(12)^2 (ab)$. Another $(12)$ can be replaced by first applying conservation of momentum to substitute one of the legs

$$\frac{(12)}{12} = - (13) - (14) - \cdots - (1n) \quad (2.14)$$

Applying (2.11) again, all of the round brackets on the right hand side can be expressed in terms of brackets of adjacent momenta. We then have two equations (2.13) and (2.14) for $(12)$ and $(ab)$. Solving the equations will give us an expression for $(12)$ in terms of brackets other than $(ab)$, which can be used to replace the second $(12)$ in the numerator of the Parke-Taylor formula.
2.2 Matching expansion coefficients

Since the MHV vertices and the Parke-Taylor formula are spanned by functions of round brackets with coefficients depending on hat components only, as in the 5-point case shown at the beginning of this section we are free to adjust all of the tilde component variables on both sides of the expansion equation to solve for the coefficients. First let us check the coefficients of (2.4). For the n-point MHV vertex, the contributions to (2.4) come solely from terms translated from the 3-point LCYM vertex (Fig. 2). Following the convention introduced in [32] for graphical notation this gives

\[ \hat{2} \hat{3} \cdots \hat{k} - 1 \]
\[ \hat{l} + 1 + \cdots \hat{1} \]
\[ \hat{k} + 2 \cdots \hat{l} - 1 \]
\[ \hat{1} \hat{l} + 2 \cdots \hat{n} \]
\[ \cdots \]
\[ \hat{k} + 1 + \cdots \hat{l} \]
\[ \hat{l} + 1 + \cdots \hat{1} \]
\[ \hat{k} + 1 + \cdots \hat{l} \]
\[ \hat{l} + 1 + \cdots \hat{1} \]
\[ \hat{k} + 1 + \cdots \hat{l} \]
\[ \hat{l} + 1 + \cdots \hat{l} \]
\[ \hat{l} - 1 = \frac{\hat{l} - 1}{\hat{l}} \]

Using conditions

\[ \hat{k} = \frac{\hat{k}}{\hat{k} - 1}, \cdots \hat{3} = \frac{\hat{3}}{2} \]
\[ \hat{l} + 1 = \frac{\hat{l} + 1}{\hat{l} + 2}, \cdots \hat{n} = \frac{\hat{n}}{1} \]
\[ \hat{k} + 1 = \frac{\hat{k} + 1}{\hat{k} + 2}, \cdots \hat{l} - 1 = \frac{\hat{l} - 1}{\hat{l}} \]

and conservation of momentum

\[ \hat{l} = -\hat{l} - 2 - \cdots - \hat{l} - 1 - \hat{l} + 1 \cdots - \hat{n} \]

the numerator simplifies to

\[ ((l + 1) + \cdots 1 + 2 + \cdots k) = \left( \frac{\hat{l} + 1 + \cdots \hat{1}}{\hat{k} + 1 + \cdots \hat{l}} \left( \frac{\hat{2} + \cdots \hat{k}}{12} \right) \right) \]
\[ \left( \frac{\hat{l} + 1 + \cdots \hat{1}}{\hat{k} + 1 + \cdots \hat{l}} \left( \frac{\hat{2} + \cdots \hat{k}}{12} \right) \right) \]

Similarly, for the expanded Parke-Taylor formula we have

\[ (k, k + 1) = \frac{\hat{k}}{2} \left( \frac{\hat{l} + 1 + \cdots \hat{1}}{\hat{k} + 1 + \cdots \hat{l}} \right) \]
\[ \hat{i} \left( \frac{\hat{k} + 1 + \cdots \hat{l}}{12} \right) \]
\[ (l, l + 1) = \frac{\hat{l} + 1}{1} \left( \frac{\hat{k} + \cdots \hat{2}}{2} \right) \]
\[ \hat{i} \left( \frac{\hat{k} + 1 + \cdots \hat{l}}{12} \right) \]
Collecting terms, both (2.15) and the Parke-Taylor formula give the same coefficient for (2.4).

\[
\frac{\hat{i}\hat{2}\cdots\hat{n}}{kk + 1\hat{l}\hat{l} + 1} \cdot \frac{\left(\hat{k} + 1 + \cdots \hat{l}\right)^2}{\left(\hat{k} + \cdots \hat{2}\right)\left(\hat{l} + 1 + \cdots \hat{1}\right)}
\] (2.23)

As for the coefficient of terms (2.3), we receive contributions from graphs translated from the 4-point vertex (Fig.1), for which the translation kernels from the legs yield factors of the form (2.3), and contributions from graphs using the 3-point vertex as backbone (Fig.2 (a) to (c)), in which case the bracket in the numerator cancels another bracket coming from the kernel and splits the denominator into two sets of sequential products of brackets.

\[
\begin{align*}
&\hat{1} \hat{2} \cdots \hat{n} \\
&\hat{k} + \cdots \hat{2} \\
&\hat{m} + \cdots \hat{1} \\
&\hat{l} + \cdots \hat{1} \\
&\hat{1} \hat{2} \\
&\hat{k} + \cdots \hat{2} \\
&\hat{m} + \cdots \hat{1} \\
&\hat{l} + \cdots \hat{1} \\
&\hat{n}
\end{align*}
\] (2.24)

For simplicity we extract the common factors from each graph.

\[
\frac{\hat{2}\hat{3}\cdots\hat{k}-1}{(23)\cdots(k-1,k)} \times \frac{\hat{k} + 2\cdots\hat{m}-1}{(k+1,k+2)\cdots(m-1,m)}
\times \frac{\hat{m} + 2\cdots\hat{l}-1}{(m+1,m+2)\cdots(l-1,l)} \times \frac{\hat{l} + 2\cdots\hat{n}}{(l+1,l+2)\cdots(n,1)}
\] (2.24)

The remaining factors are then simplified by partial fractions. For graph (a), this is

\[
\frac{\hat{l}\hat{l} + 1}{(l,l+1)} \cdot \frac{(m+1)\cdots\hat{1} + 2\cdots\hat{k}}{(m+1\cdots\hat{1})(2\cdots\hat{k})} \cdot \frac{\hat{i}\hat{2} \left(\hat{k} + 1 + \cdots \hat{m}\right)}{\left(m+1\cdots\hat{1}\right)(2\cdots\hat{k})} = \frac{\hat{i}\hat{2} \cdot c^2 d}{(a + d)^2 b}
\] (2.25)

where \(a, b, c \) and \(d \) denote the momenta of the four lines stretching out of the 4-point vertex in (Fig.1).
Similarly for graph (b) we have

\[
\hat{l} \hat{k} + 1 \left( \frac{(l + 1) + \cdots + 2 + \cdots + m}{(l + 1 + \cdots + 1) (2 + \cdots + \hat{m})} \right) \frac{\hat{l} \hat{2} \left( m + 1 + \cdots + \hat{l} \right)}{\hat{l} + 1 + \cdots + 1} = \frac{\hat{l} \hat{2} \cdots}{(a + d)^2} \frac{cd^2}{a} \tag{2.30}
\]

After simplification graph (c) is proportional to (12), and therefore vanishes

\[
((l + 1) + \cdots + 1, 2 + \cdots + k) = \frac{\hat{l} + 1 + \cdots + \hat{l}}{2} \frac{\hat{2} + \cdots + \hat{k}}{2} (12) = 0 \tag{2.31}
\]

Putting (2.25) and (2.30) together cancels the contribution from (Fig.1)

\[
-\frac{\hat{l} \hat{2} \cdots}{(a + d)^2} \frac{cd}{ab} (ac + bd) \tag{2.32}
\]

we thus verified that all of the expansion coefficients for terms of the form (2.3) are zero, as claimed at the beginning of the section. Since the argument presented here does not depend on whether the negative helicity legs are adjacent to each other, the result generalises to all n-point vertices.

3. Conclusion and discussions

We explicitly proved that a generic n-point vertex of the MHV lagrangian is described by the Parke-Taylor formula, which was originally argued by holomorphism and verified only up to 5-points. The derivation presented in this paper also directly showed that the Parke-Taylor formula defined by light-cone coordinate external leg momenta through (A.4) and (A.5) serves as the off-shell continued MHV amplitude used in the CSW rules derived from the light-cone Yang-Mills lagrangian.

The method described in this paper can also be extended to supersymmetric theories. Since the translation kernels in QCD and the $N = 4$ SYM theory [29, 30] were shown to contain the same translation kernel used in the pure YM multiplied by light-cone hat components of momenta only, it is straightforward to apply similar expansions and the method of partial fractions to verify that the vertices are given by the Parke-Taylor forumla multiplied by suitable ratios of spinor brackets restricted by SUSY Ward identity.
A. Notation

In this paper we adopted the shorthand notation \((\hat{p}, \hat{\mu}, p, \bar{p})\) to describe covariant vectors in light-cone coordinates, which are related to Minkowski coordinates by

\[
\hat{p} = (p_0 - p_3), \quad \hat{\mu} = (p_0 + p_3), \quad p = (p_1 - ip_2), \quad \bar{p} = (p_1 + ip_2).
\]  

(A.1)

In light-cone coordinates the metric becomes off-diagonal. The Lorentz invariant product of two vectors is given by

\[
p \cdot q = (\hat{p} \hat{q} + \hat{\mu} \bar{q} - \bar{p} \bar{q}) / 2.
\]  

(A.2)

To keep the derivation simple, the momentum components \(p_{n\mu}\) of the \(n^{th}\) external leg are simply denoted by number \(n\) with the appropriate decoration (\(\hat{n}, \hat{\mu}, n, \bar{n}\)). Note that a tilde is used for the \(p = p_1 - ip_2\) component to avoid possible confusion with numerical factors.

A 4-vector can be written in the form of a bispinor

\[
P_{\alpha\dot{\alpha}} = p^\mu \sigma_{\mu\alpha\dot{\alpha}} = \begin{pmatrix} \hat{p} & -p \\ -\bar{p} & \hat{\mu} \end{pmatrix}
\]  

by contracting with \(\sigma_\mu = (I_2, \sigma)\), where \(I_2\) is the \(2 \times 2\) identity matrix and \(\sigma\) stands for Pauli matrices.

If \(p_\mu\) is lightlike, \(\hat{p} = p\bar{p}/\hat{\mu}\) and the bispinor factorises \(P_{\alpha\dot{\alpha}} = \lambda^\alpha \bar{\lambda}_{\dot{\alpha}}\), where

\[
\lambda^\alpha = \begin{pmatrix} -p/\sqrt{\hat{\mu}} \\ \sqrt{\hat{\mu}} \end{pmatrix}, \quad \bar{\lambda}_{\dot{\alpha}} = \begin{pmatrix} -\bar{p}/\sqrt{\hat{\mu}} \\ \sqrt{\hat{\mu}} \end{pmatrix}.
\]  

(A.4)

Spinors \(\lambda^\alpha\) associated with different massless particles can be contracted to given a Lorentz invariant angle bracket

\[
\langle 12 \rangle = \epsilon^{\alpha\beta} \lambda^\alpha_{1\alpha} \lambda^\beta_{2\beta} = \frac{(12)}{\sqrt{12}}.
\]  

(A.5)

and we define a round bracket as

\[
(12) = \hat{1} \hat{2} - \hat{2} \hat{1}.
\]  

(A.6)

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