The Photon Sector in the Quantum Myers-Pospelov Model: an improved description

C. M. Reyes, L. F. Urrutia and J. D. Vergara

Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, A. Postal 70-543, 04510 México D.F., México

Abstract

The quantization of the electromagnetic sector of the Myers-Pospelov model coupled to standard fermions is studied. Our main objective is to construct an effective quantum theory that results in a genuine perturbation of QED, such that setting zero the Lorentz invariance violation (LIV) parameters will reproduce it. This is achieved by introducing an additional low energy scale $M$, together with a physically motivated prescription to take the QED limit. The prescription is successfully tested in the calculation of the electron self-energy in the one loop approximation. The LIV radiative corrections turn out to be properly scaled by very small factors for any reasonable values of the parameters, no fine-tuning problems are found at this stage and the choice for $M$ to be of the order of the electroweak symmetry breaking scale is consistent with the stringent bounds for the LIV parameters, in particular with those arising from induced dimension three operators.

Key words: Quantum effective models, Lorentz violations, QED extension.

PACS: 12.20.-m, 11.30.Cp, 04.60.Cf, 11.30.Qc

1. Introduction

Recently there has been a great deal of interest in the study of effective field theory models that describe violations of Lorentz and CPT invariance [1] in order to correlate the numerous and diverse experimental and observational test carried to probe those symmetries [2]. Such interest has been enhanced from the theoretical perspective since the proposal of Ref. [3] suggesting that Lorentz invariance violation (LIV) could arise due to a foamy...
or granular structure of space-time. Observation of high energy photons arriving from astrophysical sources was also proposed as a method to test such possibility [4]. The above suggestion sparked immediate interest in identifying fundamental theories that could generate these effects. The most natural choice to look for is a dynamical theory of space-time at the quantum level, that is to say quantum gravity, where most of the developing theories share the belief that the description of space-time will suffer important deviations from its standard view as a continuum, when we are in the Planck scale regime. Preliminary estimations of the induced corrections in particle propagation at standard model energies appeared in Refs. [5, 6, 7, 8]. Nevertheless, up to now there is no systematic derivation of any semiclassical approximation starting from a fundamental quantum gravity theory, for example, that could determine the exact nature of the possible corrections, if any, arising from such modifications of space-time. This situation has prompted the construction and analysis of effective field theory models which capture the basic ingredients that we expect to survive at standard model energies. At present, all observational test of LIV lead to negligible violation, codified in the very stringent bounds set upon the LIV parameters. In this way, the proposed effective models have to provide highly suppressed radiative corrections to comply with observation [9]. Radiative corrections to LIV theories have been also considered in Refs. [10]. Fine tuning problems arising from LIV theories have been found in Refs. [11] and [12]. This last reference deals with the Myers-Pospelov model (MPM), to be discussed in this Letter from a different perspective which eliminates those fine tuning problems. The MPM [13] incorporates particle (active) LIV parameterized by dimension five operators together with a non-dynamical timelike four-vector \( n_\mu \) that can be interpreted as the four velocity of a preferred frame. It respects observer (passive) Lorentz covariance among concordant frames. Due to the presence of \( n^\mu \) the full MPM exhibits higher order derivative (HOD) corrections in the kinetic terms of the Lagrangian entering as dimension five operators. As it is well known such theories have additional degrees of freedom with respect to the standard ones, present unitarity and causality violations together with unbounded Hamiltonians which are non-analytical in the coefficients \( \xi_A \) that control the higher dimensional operators. Many different approaches to cope with such problems in the Lorentz covariant case have appeared in Refs. [14] and a final answer has not been provided yet. Of particular interest to the case of the full MPM considered as a perturbation of QED is the work in Ref. [15], where a consistent perturbation procedure in terms of the HOD
operators is developed, that allows to calculate corrections to the physics of the original low energy degrees of freedom (i.e. those corresponding to standard QED in our case). When dealing with the full MPM as a perturbation of QED, such drawbacks must be taken carefully into account with the additional requirement of extending the analysis to the LIV case.

In the present work we introduce some simplifications that make the quantization of the electromagnetic sector of the MPM much simpler, without loosing some of the general features of the complete model. In the first place we retain only the LIV parameter associated to the photon field. In this way our starting point is the Lagrangian

\[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2M} (n^\mu F_{\mu\nu}) (n^\alpha \partial_\alpha) (n_\rho \epsilon^{\rho\mu\nu\lambda} F_{\nu\lambda}) + \bar{\Psi} (i\gamma^\mu (\partial_\mu + ieA_\mu) - m) \Psi, \]

where LIV is codified by the parameter \( \xi \) and \( m \) is the electron mass. The quantity \( \bar{M} \) is assumed to arise from a fundamental theory and determines the scale where quantum gravity effects dominate. As a second simplification, we work in the rest frame of the preferred system, where \( n^\mu = (1, 0) \). In this frame the HOD term leads to a contribution quadratic in the field velocities, thus representing a correction over the standard photon propagation modes with no additional degrees of freedom involved, which can be dealt with in the standard way. The theory described by the Lagrangian in Eq. (1) is an effective one which is valid only up to distances of the order of \( 1/\bar{M} \). Guided by our goal to recover QED in the limit \( \xi \to 0 \) we introduce an additional scale \( M \). Intuitively such scale corresponds to that entering in the regularization procedure, via Pauli-Villars factors for example, required in standard QED, so that we expect \( M \ll \bar{M} \). In this way we introduce it through the same choice of regulating factors as required in the Lorentz invariant situation. In the present case this prescription amounts to the following modification of the photon propagator

\[ \Delta_{\mu\nu}(k) \longrightarrow \Delta_{\mu\nu}(k) \mathcal{I}(k), \quad \mathcal{I}(k) = \frac{M^2}{M^2 - k^2}, \quad M \gg m. \]

Notice that the scale \( M \) has been introduced in a fully Lorentz covariant way such that all LIV is still codified by the parameter \( \xi \). Consistency with the choice of \( M \) provides the prescription to recover QED from the quantum modified model: (i) first take \( \xi \to 0 \), for fixed \( M \) and (ii) subsequently
take $M \to \infty$. In this work we focus on the calculation of the electron self-energy, with special emphasis upon all LIV terms that are good candidates to induce fine-tuning problems by generating lower dimensional operators with unsuppressed corrections.

2. The quantization of the photon sector

To be on the safe side and motivated by the difficulties inherent to HOD theories, some of which are still present in spite of our simplifications, we proceed to quantize the system using a standard canonical approach, which allows a good control over the conflicting issues. Moreover, taking advantage of the selected reference frame we choose to incorporate the corrections as part of an exact free propagator which induces modified dispersion relations. The canonical approach applied to the $3 + 1$ description of the Lagrangian in Eq. (1) is analogous to the standard case in the Coulomb gauge, except for modifications of order $g$ in the momenta canonically conjugated to the fields $A_i$

$$\Pi_i = \frac{\partial L}{\partial \dot{A}_i} = \dot{A}_i + \partial_i A^0 + 2g\epsilon^{ijk}\partial_j \dot{A}_k, \quad g = \frac{\xi}{M}. \quad (3)$$

The elimination of the velocities in terms of the momenta can in fact be performed starting from Eq. (3), but requires the introduction of the non-local inverse of the operator $(\delta^{ik} + 2g\epsilon^{ijk}\partial_j)$, which can be exactly calculated. The canonical transformation $A_T \to \bar{A}_T$, $\Pi_T \to \bar{\Pi}_T$

$$A_T^i = \sqrt{1 + \frac{W}{\sqrt{2}}} \left[ \delta^{iq} - \frac{2g}{(1 + W)} \epsilon^{imq} \partial_m \right] \bar{A}_q^i,$$

$$\Pi_T^r = \sqrt{1 + \frac{W}{\sqrt{2}}} \left[ \delta^{rq} + \frac{2g}{(1 + W)} \epsilon^{rmq} \partial_m \right] \bar{\Pi}_q^r, \quad (4)$$

with the notation $W = \sqrt{1 + 4g^2\nabla^2}$ leads to the Hamiltonian

$$H = \int d^3x \left( \frac{1}{2} \bar{\Pi}_p^T \bar{\Pi}_p^T - \frac{1}{2} \dot{A}_T^i \left( \frac{\nabla^2}{W^2} \right) [\delta^{rp} - 2g\epsilon^{rmp}\partial_n] \bar{A}_T^p + \frac{1}{2} J^0 \left( -\frac{1}{\nabla^2} \right) J^0 - J^i A_T^i (\bar{A}_T^k) \right). \quad (5)$$

exhibiting the proper normalization of the $\bar{\Pi}^2$ term. Next we determine the corresponding normal modes of the free ($J^\mu = 0$) Hamiltonian in Eq. (5).
starting from the expansion

$$\vec{A}_T(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3}} \sum_{\lambda = \pm 1} \sqrt{\frac{1}{2\omega_\lambda(k)}} \left[ a_\lambda(k) \varepsilon^i(\lambda, k)e^{-ik(\lambda) \cdot x} + a_\lambda^\dagger(k) \varepsilon^{i*}(\lambda, k)e^{+ik(\lambda) \cdot x} \right], \quad (6)$$

in terms of creation-annihilation operators $a_\lambda^\dagger(k)$, $a_\lambda(k)$, respectively. The notation is $|k(\lambda)\rangle_\mu = (\omega_\lambda(k), -k)$, together with $k(\lambda) \cdot x = \omega_\lambda(k)x^0 - k \cdot x$, where the modified normal frequencies will be consistently determined and the polarization vectors $\varepsilon^i(\lambda, k)$, are chosen in the helicity basis. Assuming the standard creation-annihilation commutation rules $[a_\lambda(k), a_{\lambda'}^{\dagger}(k')] = \delta_{\lambda\lambda'}\delta^3(k - k')$ and starting from Eq. (6) we recover the basic field commutator corresponding to the quantum mechanical extension of the standard transverse Dirac brackets. The modified dispersion relations are

$$\omega^2_\lambda(k) = \frac{|k|^2}{1 + 2\lambda g|k|} \quad (7)$$

which is exact in $g$. With no loss of generality we assume from now on that $g > 0$. One can further verify that the resulting free Hamiltonian is in fact positive definite and that has the expected expression in terms of the previously introduced creation-annihilation operators and the frequencies given in Eq. (7). Also, the Hamiltonian is Hermitian as far as the frequencies remain real, which is the case in the region $|k| < 1/(2g)$.

Let us notice that in Eq. (6) the four-vector $[k(\lambda = +1)]_\mu$ is space-like, while $[k(\lambda = -1)]_\mu$ is timelike. At this stage we are confronted with two problems that usually arise in LIV theories: (i) on one hand, the frequency $\omega_-(k)$ will become imaginary when $|k| > 1/(2g)$ and diverges when $|k| = |k|_{\text{max}} = 1/(2g)$. From an intuitive point of view we consider $1/(2g)$ as the analogous of the value $|k|_{\text{max}} = \infty$ in the standard case and we will cut all momentum integrals at this value. (ii) on the other hand, since $[k(\lambda = +1)]_\mu$ is space-like, we can always perform an observer Lorentz transformation such that $\omega_+(k)$ becomes negative thus introducing stability problems in the model. For a given momentum $k$ this occurs when $1/\sqrt{1 + 2g|k|} < |v| < 1$. Then, the maximum allowed momentum $|k| = 1/(2g)$ leads to the requirement that the allowed concordant frames in which the quantization will remain consistent are such that $\beta < 1/\sqrt{2}$, with respect to the rest frame.
Next we calculate the modified free photon propagator, given by the standard expression
\[ i\bar{\Delta}_{ij}(x - y) = \langle 0 | T \left( \bar{A}^T_i(x) \bar{A}^T_j(y) \right) | 0 \rangle. \]
Nevertheless some care is required in implementing a perturbation theory based on the Hamiltonian \( \mathcal{H} \) because the interaction is described by \( \bar{A}^T_i \), propagating with \( \bar{\Delta}_{ij} \), instead of \( \bar{A}^T_i \). The propagator \( \bar{\Delta}_{ij} \) is directly obtained from \( \bar{\Delta}_{ij} \) via the canonical transformation in Eq.(4). The further inclusion of the instantaneous Coulomb term appearing in Eq. (5), following the steps of Ref. [16], leads to the four dimensional propagator
\[ \Delta_{\mu\nu}(k) = \frac{1}{((k^2)^2 - 4g^2 |k|^2 k_0^4)} \left[ -k^2 \eta_{\mu\nu} + 2igk_0^2 \epsilon^{lmr} k_m \eta_{\mu r} \eta_{\nu l} \right. \]
\[ \left. -\frac{4g^2k_0^4}{k^2} k_l k_r \delta^l_\mu \delta^r_\nu + \frac{4g^2k_0^4 |k|^2}{k^2} \eta_{\mu l} \eta_{\nu r} \right]. \quad (8) \]

We notice that Eq. (8) is just the propagator in the Lorentz gauge obtained from the equations of motion arising from the Lagrangian in Eq. (1). As our exact calculation shows, there is no high-momentum pole arising from the denominator in the above equation. This justifies a posteriori an expansion in powers of \( g \) which should amount to treat the LIV corrections as insertions in the original QED action.

3. The electron self-energy

As a first step in testing the proposed construction we consider the electron self-energy with the dynamical modifications introduced only via the LIV photon propagator. The starting point is
\[ \Sigma^g(p) = -ie^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \left[ \frac{\gamma (p - k) + m}{((p - k)^2 - m^2 + i\epsilon)} \right] \gamma^\nu \Delta_{\mu\nu}(k) \, \mathcal{I}(k) \, \theta \left( \frac{1}{2g} - |k| \right). \quad (9) \]

Next we expand the self energy in powers of the external momentum obtaining, up to first order,
\[ \Sigma^g(p) = AI + \bar{A} \gamma^0 \gamma^5 + (B - C) p^0 \gamma_0 + C p^\mu \gamma_\mu + ip^i \bar{C} \gamma^j \gamma^k \epsilon^{ijk} + O(p^2). \quad (10) \]

The general strategy to evaluate the required integrals is the following. Within the region of integration (\( |k| < 1/(2g) \)), the poles in the complex \( k_0 \) plane of the denominators in (9) have the form \( k_{01} = \mathcal{E}(|k|) - i\epsilon, \quad k_{02} = -\mathcal{E}(|k|) + i\epsilon \), with \( \mathcal{E}(|k|) > 0 \). Here \( \mathcal{E}(|k|) \) stands for any of the involved energies \( \omega_{\pm}(k) \)
and $E(k) = \sqrt{k^2 + m^2}$. In this way, it is always possible to perform a Wick rotation to the Euclidean signature such that $k_0 = i k_4$. Due to the remaining rotational symmetry, together with the symmetrical integration over $k$, one is finally left with only two integration variables which are $k_4$ and $|k|$ that can be conveniently rewritten in polar form $k_4 = r \cos \alpha$, $|k| = r \sin \alpha$. The details of the exact calculation are given in Refs. \[17, 18\]. The potentially dangerous contributions arise from the following terms

$$B - C = \frac{e^2}{\pi^2} (gM)^2 \left( -0.070 + 0.010 \ln(gM) \right) + \ldots , \quad (11)$$

$$\tilde{A} = \frac{e^2}{6\pi^2} gM^2 \left( 0.018 + 0.063 \ln(gM) \right) + \ldots , \quad (12)$$

$$\tilde{C} = \frac{e^2}{48\pi^2} (gm) \ln \left( \frac{m}{M} \right) + \ldots , \quad (13)$$

$$A = \frac{e^2 m}{\pi^2} \left( \frac{M^2}{2(m^2 - M^2)} \ln \left( \frac{M}{m} \right) + (gM)^2 \left( 0.75 + 0.047 \ln(gM) \right) \right) + \ldots , \quad (14)$$

where we have written only the dominant parts.

4. Final comments

Contrary to the case of the second Ref. \[13\] we admit here the appearance of induced lower dimensional operators. In order to make some numerical estimations we take $\bar{M} = M_P = 10^{19}$ GeV. Let us begin with the discussion of the term $\tilde{A}$, that will provide an improved interpretation of $M$ as a low energy scale, which we take to be the electroweak symmetry breaking scale, i.e. $M \approx 250$ GeV. This point was not fully addressed in Ref. \[18\], that was mainly concerned with the $\xi \rightarrow 0$ limit of the MPM. In order to take properly into account the radiative correction induced by $\tilde{A}$ we have to carry on part of the renormalization process related to the bare coupling $\xi$. First we rewrite the photon modification term in Eq. (1) in terms of the renormalized coupling $\xi_R$ by introducing $\xi = \xi_R + \xi_R (\xi/\xi_R - 1)$ and subsequently we treat the second contribution arising from the previous splitting as a counterterm. In this way we have to change $\xi \rightarrow \xi_R$ in all results of the previous section. We take the upper bound $\xi_R = 10^{-10}$, given in Ref. \[19\]. Also we denote the tuning coefficient $(\xi/\xi_R - 1) = \mu$. From Eq. (10) we realize that the radiative correction proportional to $\tilde{A}$ gives rise to the dimension three operator $(\Delta L)_{RC} = b_0(\xi_R) \left[ \bar{\psi} \gamma^\mu \gamma^5 \psi \right]$, dominated by
$b_0(\xi_R) = 0.063 \left(\frac{e^2}{6\pi^2}\right) \left(\frac{\xi_R M^2}{\bar{M}}\right) \ln \left(\frac{\xi_R M}{\bar{M}}\right)$. This means that we had better started with the corresponding bare term in the original Lagrangian (1), which we write in the analogous form $(\Delta L)_{BARE} = -b_0(\xi) \left[\bar{\psi}\gamma^0\gamma^5\psi\right]$. In this way we obtain $(b_0)_{EXP} = b_0(\xi_R) - b_0(\xi)$ for the observable prediction of such coefficient. Under the approximation $(1 + \mu) \ln(1 + \mu) \simeq \mu$, we have $|b_0|_{EXP} = \mu \times 0.063 \left(\frac{e^2}{6\pi^2}\right) \left(\frac{\xi_R M^2}{\bar{M}}\right) \left[1 + \ln(\xi_R + \ln(M/\bar{M}))\right]$. The bound $|b_0|_{EXP} < 10^{-29} \text{ GeV}$ [20] leads to $\mu < 3.4 \times 10^{-2}$, which we consider acceptable. The absence of the additional scale $M$ would lead to a tuning coefficient regulated only by $\bar{M}$, with a value of $\mu \approx 10^{-34}$. Regarding the $(B - C)$ contribution, we observe that the use of any covariant regulator $F(k^2/M^2)$ to introduce the scale $M$ leads to a zero value for the finite $g^2$ independent piece, thus eliminating the large unsuppressed corrections reported in Ref.[12]. We verify that the remaining contribution is consistent with recent observations. In our specific case, this LIV contribution produces an additional dimension four term in the Lagrangian, given by $(\Delta L)_2 = \left(\frac{e^2}{\pi^2}\right) \delta \bar{\psi}\gamma^0 i\partial_0\psi$, where our calculation leads to a prediction dominated by $|\delta| \sim 10^{-2} \times (\xi_R M^2/\bar{M}) \mid \left[1 + \ln(\xi_R + \ln(M/\bar{M}))\right] = 3.8 \times 10^{-54}$, which falls comfortably within the observational range $|\delta| < 10^{-21}$ established in Ref.[12]. The term proportional to $\tilde{C}$ provides a contribution $(\Delta L)_3 \sim 2\tilde{C} \bar{\psi}\gamma^0\gamma^5(\gamma^k i\partial_k)\psi = 2\tilde{C}m \bar{\psi}\gamma^0\gamma^5\psi$, where, for the sake of an estimation, we have used the zeroth-order equation of motion for $\psi$, together with dropping a remaining total time derivative term. Again, this corresponds to a dimension three operator with $|b'_0| = 2\tilde{C}m = 10^{-39} \text{ GeV}$. Finally, the term $A$ is unsuppressed but induces a contribution to the Lorentz covariant fermion mass term, which should be dealt with via the fermion mass renormalization procedure. The remaining LIV contributions to the electron self-energy given in Eq.(10), including corrections up to second order in the external momentum, are calculated in analogous way and produce highly suppressed corrections of similar type, as shown in Ref.[18]. These results, with the exception of $A$, have precisely the expected property that reduce to zero when we turn off the LIV correction parameterized by $\xi_R$, keeping $M$ fixed. In this letter we have presented the construction of a sector of the quantum MP effective model emphasizing the recovering of the correct QED limit in relation with the absence of fine-tuning problems. A low energy scale $M \approx 250 \text{ GeV}$ has been introduced which is consistent with an ultraviolet cutoff of the order of the Planck mass, together with the very stringent bounds upon the LIV parameters, in particular with those associated to dimension three operators. It is very remarkable that our procedure allows to
relate such very different UV and IR scales in a way consistent with observations, including the absence of fine tuning. C. M. R. acknowledges support from DGAPA-UNAM through a postdoctoral fellowship. L.F.U is partially supported by projects CONACYT # 55310 and DGAPA-UNAM-IN109107. J. D. V acknowledges support from the projects CONACYT # 47211-F and DGAPA-UNAM-IN109107.

References

[1] D. Colladay and V. A. Kostelecký, Phys. Rev. D55, 6760 (1997) [arXiv:hep-ph/9703464]; Phys. Rev. D58, 116002 (1998) [arXiv:hep-ph/9809521]. For a review see for example: D. Mattingly, Living Rev. Rel. 8, 5 (2005) [arXiv:gr-qc/0502097]; T. Jacobson, S. Liberati and D. Mattingly, Ann. Phys.[N.Y.] 321, 150 (2006) [arXiv:astro-ph/0505267]; G. Amelino-Camelia, Quantum Gravity Phenomenology, arXiv:gr-qc/0806.0339.

[2] For a review see for example Proceedings of the Meeting on CPT and Lorentz Symmetry, ed. V. A. Kostelecký (World Scientific, 1999); C. M. Will, Living Reviews in Relativity 4, 4 (2001) [arXiv:gr-qc/0103036]; Proceedings of the Second, Third and Fourth Meeting on CPT and Lorentz Symmetry, ed. V. A. Kostelecký (World Scientific, 2002, 2004 and 2007).

[3] G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and S. Sarkar, Nature 393, 763 (1998) [arXiv:astro-ph/9712103].

[4] J. Ellis, N. E. Mavromatos, D. V. Nanopoulos and A. S. Sakharov, Astron. Astrophys. 402, 409 (2003) [arXiv:astro-ph/0210124]; J. Ellis, N. E. Mavromatos, D. V. Nanopoulos, A. S. Sakharov and E. K. G. Sarkisyan, Astropart. Phys. 25, 402 (2006) [arXiv:astro-ph/0510172], erratum Astropart. Phys. 29, 158 (2008) [arXiv:astro-ph/0712.2781]; MAGIC Collaboration and Other Contributors (J. Albert et al.), Phys. Lett. B668, 253 (2008) [arXiv:astro-ph/0708.2889].

[5] R. Gambini and J. Pullin, Phys. Rev. D59, 124021 (1999) [arXiv:gr-qc/9809038]; R. Gambini and J. Pullin, in Proceedings of the Second Meeting on CPT and Lorentz Symmetry, ed. V.A. Kostelecky, World Scientific, 2002 [arXiv:gr-qc/0110054].
[6] J. Alfaro, H.A. Morales-Técostl, L.F. Urrutia, Phys. Rev. Lett. 84, 2318 (2000)[arXiv: gr-qc/990979]; J. Alfaro, H.A. Morales-Técostl and L.F. Urrutia, Phys. Rev. D65, 103509 (2002)[arXiv: hep-th/0108061].

[7] J. Alfaro, H.A. Morales-Técostl and L.F. Urrutia, Phys. Rev. D66, 124006 (2002)[arXiv:hep-th/0208192].

[8] J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Rev. D61, 027503 (1999) [arXiv:gr-qc/9906029]; J. R. Ellis, K. Farakos, N. E. Mavromatos, V. A. Mitsou and D. V. Nanopoulos, Astrophys. J. 535, 139 (2000) [arXiv:astro-ph/9907340]; J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Rev. D62, 084019 (2000) [arXiv:gr-qc/0006004]; J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and G. Volkov, Gen. Rel. Grav. 32, 1777 (2000) [arXiv:gr-qc/9911055]; J. R. Ellis, E. Gravanis, N. E. Mavromatos and D. V. Nanopoulos, *Impact of low-energy constraints on Lorentz violation*, arXiv:gr-qc/0209108; J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. B66, 412 (2008)[arXiv:hep-th/0804.3566]

[9] J. Alfaro, Phys. Rev. Lett. 94, 221302 (2005) [arXiv:hep-th/0412295]; J. Alfaro, Phys. Rev. D 72, 024027 (2005) [arXiv:hep-th/0505228].

[10] V. A. Kostelecký and R. Pottting, Phys. Rev. D51,3923 (1995); A. A. Andrianov and R. Soldati: Phys. Letts. B435, 449 (1998); R. Jackiw and V. A. Kostelecký, Phys. Rev. Lett. 82, 3572 (1999); M. Pérez-Victoria, Phys. Rev. Lett. 83, 2518 (1999); J.M. Chung, Phys. Lett. B461, 138 (1999); W.F. Chen and G. Kunstatter, Phys. Rev. D62, 105029 (2000); V. A. Kostelecký, C. D. Lane and A. G. M. Pickering, Phys. Rev. D65, 056006 (2002); A. A. Andrianov, P. Giacconi and R. Soldati: JHEP 0202, 030 (2002); B. Altschul, Phys. Rev. D70, 101701 (2004); J. Alfaro, A. A. Andrianov, M. Cambiaso, P. Giacconi and R. Soldati, Phys. Lett. B639, 586 (2006).

[11] J. Collins, A. Pérez, D. Sudarsky, L. F. Urrutia and H. Vucetich, Phys. Rev. Lett. 93, 191301 (2004) [arXiv:gr-qc/0403053].

[12] P. M. Crichigno and H. Vucetich, Phys. Lett. B 651, 313 (2007) [arXiv:hep-th/0607214].
[13] R. C. Myers and M. Pospelov, Phys. Rev. Lett. 90, 211601 (2003) [arXiv:hep-ph/0301124]; P. A. Bolokhov and M. Pospelov, Phys. Rev. D77, 025022 (2008) [arXiv:hep-ph/0703291].

[14] See for example: P. T. Mattews, Phys. Rev. 76, 684 (1949); T. D. Lee and G. C. Wick, Nucl. Phys. B9, 209 (1969); R. E. Cutkowsky, P. V. Landshoff, D. I. Olive and J. C. Polkinghorne, Nucl. Phys. B12, 281 (1969); C. W. Bernard and A. Duncan, Phys.Rev. D11, 848 (1975); C. Grosse-Knetter, Phys.Rev. D49, 6709 (1994) [arXiv:hep-ph/9306321]; C. Grosse-Knetter, Dissertation, Equivalence of Hamiltonian and Lagrangian Path Integral Quantization, [arXiv:hep-ph/9311259]; B. Grinstein, D. O'Connell and M. B. Wise, Phys. Rev. D77, 025012 (2008) [arXiv:0704.1845 [hep-ph]]; ibid. Causality as an emergent macroscopic phenomena: The Lee-Wick O(N) model, [arXiv:0805.2156 [hep-th]].

[15] T.-C. Cheng, P.-M. Ho and M.-C. Yeh, Nucl. Phys. B 625, 151 (2002); Phys. Rev. D 66, 085015 (2002).

[16] See for example, S. Weinberg, The Quantum Theory of Fields I, Cambridge University Press, Cambridge, 1995, p. 350.

[17] C. M. Reyes, L. F. Urrutia and J. D. Vergara, Quantization of the Myers-Pospelov model: A Progress report, Proceedings of From Quantum to Emergent Gravity: Theory and Phenomenology, PoS QG-PH:040.2007 [arXiv:0712.3489 [hep-ph]]; ibid. AIP Conf. Proc. 977, 214 (2008) [arXiv:0806.2166 [hep-ph]].

[18] C. M. Reyes, L. F. Urrutia and J. D. Vergara, Phys. Rev. D78, 125011 (2008) [arXiv:0810.5379 [hep-ph]].

[19] L. Maccione, S. Liberati, A. Celotti and J. G. Kirk, JCAP 0710, 013 (2007) [arXiv:0707.2673 [astro-ph]].

[20] V.A. Kostelecky and N. Russell, Data Tables for Lorentz and CPT Violation, [arXiv:0801.0287 [hep-ph]].