Trapping $^{171}$Yb Atoms into a One-Dimensional Optical Lattice with Small Waist

Akio Kawasaki,† Boris Braverman,† Edwin Pedrozo-Peñafiel,† Chi Shu,1,2 Simone Colombo,1 Zeyang Li,1 and Vladan Vuletić1

1Department of Physics, MIT-Harvard Center for Ultracold Atoms and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
2Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

In most experiments with atoms trapped in optical lattices, the transverse size of the optical lattice beams is on the order of tens of micrometers, and loading many atoms into smaller optical lattices has not been carefully investigated. We report trapping 1500 $^{171}$Yb atoms in a one-dimensional optical lattice generated by a narrow cavity mode at a distance of 0.14 mm from a mirror surface. The simplest approach of loading atoms from a mirror magneto-optical trap (MOT) overlapped with the cavity mode allows the adjustment of the loading position by tuning a uniform bias magnetic field. The number of atoms trapped in the optical lattice exhibits two local maxima for different lattice depths, with a global maximum in the deeper lattice. These results open a way to quantum mechanical manipulation of atoms based on strong interaction with a tightly focused light field.

I. INTRODUCTION

Optical trapping is widely used in various fields ranging from physics to biology as a way to spatially confine dielectric objects. Optical tweezers were the first realization of this idea, where a 10 µm-sized particles were levitated in low vacuum by the radiation pressure of a single vertically-propagating laser beam. For dielectric particles smaller than a trapping light wavelength, the origin of the potential is the electric polarization induced by the light field, which makes the particle’s energy smaller at positions with more intense light, when the trapping light is red-detuned from electromagnetic resonant frequency. In the context of atomic physics, this effect is equivalent to the AC Stark shift. Optical trapping is applied both as a dipole trap originating from a single beam and as an optical lattice generated by counter-propagating laser beams. These traps are employed in a variety of experiments, such as optical tweezers (dipole traps) for a single atom, and atom arrays, and optical lattices for Bose-Einstein condensates, degenerate Fermi gases, optical lattice clocks, and cavity quantum electrodynamics (cQED).

Atoms are loaded into these optical traps directly from a MOT, leading to the transfer of a large number of atoms into the optical lattice. In most experiments with atoms trapped in optical lattices, the transverse size of the optical lattice beams is on the order of tens of micrometers, and loading many atoms into smaller optical lattices has not been carefully investigated. We report trapping 1500 $^{171}$Yb atoms in a one-dimensional optical lattice generated by a narrow cavity mode at a distance of 0.14 mm from a mirror surface. The simplest approach of loading atoms from a mirror magneto-optical trap (MOT) overlapped with the cavity mode allows the adjustment of the loading position by tuning a uniform bias magnetic field. The number of atoms trapped in the optical lattice exhibits two local maxima for different lattice depths, with a global maximum in the deeper lattice. These results open a way to quantum mechanical manipulation of atoms based on strong interaction with a tightly focused light field.

Several different methods have been demonstrated to trap a large number of atoms within a cavity mode. One method is to first load atoms into an auxiliary trap adjacent to the cavity mode, such as a magnetic trap or another optical lattice, and to move the atoms to overlap spatially with the cavity mode. This guarantees a good initial overlap between the MOT and the auxiliary trap, leading to the transfer of a large number of atoms, as the atom cloud can be compressed after loading into the second trap. However, this approach adds technical complexity in the system for moving atoms into the cavity mode. Furthermore, not all atomic species are magnetically trappable. When an optical cavity is in near-confocal configuration, a MOT can be generated in the center of the cavity, and direct loading to a lattice formed by the narrow cavity mode is possible. In this configuration, loading of $\sim 10^5$ atoms into the cavity mode with 16 µm waist has been reported. However, in that experiment, the broad distribution of the atoms along the axial direction of the cavity over as much as 500 µm lowers the average single-atom cooperativity down to 10% of the maximum single-atom cooperativity expected for atoms at the cavity waist, reducing the utility of this approach if the goal is maximizing the atom-light interaction.

In the case of an asymmetric micromirror cavity, a MOT can also be created directly between the two cavity mirrors. To put atoms in the region of narrowest cavity mode and thus the strongest atom-light interaction, atoms in the MOT have to be very close to the mirror sur-
face, which potentially shortens the lifetime of atoms in the MOT. This hinders efficient loading, and risks of contaminating the mirror surface with atoms, decreasing the finesse of the cavity and thus the strength of atom-light interaction. Loading of atoms to a MOT near a mirror surface itself has been previously demonstrated using a mirror MOT [30], but overlapping the MOT with a small cavity mode further increases the complexity of the experiment. Also, the smallest distance between the atoms and the mirror reported in Ref. [30] is not small enough for atoms to reach the narrowest part of the cavity mode described in Ref. [29].

In this paper, we demonstrate the loading of atoms into a narrow-waist optical lattice near the surface of a mirror comprising one of the two mirrors in a high-finesse cavity. The loading efficiency and the properties of the trap are discussed. It is also shown that the increase in the optical loss of the cavity is slow enough to maintain a finesse above $10^4$ for wavelength $\lambda = 556$ nm for several years. The advantage of this setup is that we directly load a mirror MOT into the cavity volume, and easily manipulate the atoms’ loading position into the intra-cavity optical lattice using a bias magnetic field to move the zero location of the quadrupole magnetic field. This method is applicable to atomic species that have narrow transitions suitable for generating a compact MOT, including alkaline earth and alkaline earth-like atoms.

II. MIRROR MOT FOR YTTERBIUM

The experimental setup is shown in Fig. 1. The MOT is in a mirror MOT configuration [30], where a rectangular flat substrate of dimensions $12 \text{ mm} \times 25 \text{ mm}$ serves as the mirror. Laser beams for the MOT are sent from two directions: horizontally and diagonally. The horizontal beam propagates in $y$ direction, and the diagonal beam has an angle of incidence (AOI) to the mirror of 46 degrees in the $x-z$ plane, both being retroreflected at the output side. The MOT beams initially have 1 cm-radius circular shape, and appropriately shaped apertures minimize random scattering on the structure surrounding the mirrors by blocking parts of the MOT beams. MOT coils are on the same axis as the incoming diagonal beam. The atomic beam is produced by an oven $\sim 5$ cm away from the MOT region, providing a total atom flux of $\sim 10^{10} \text{s}^{-1}$ at oven temperature of $\sim 700 \text{ K}$. From the MOT, we load the atoms into a 759 nm optical lattice formed within a high-finesse cavity. The cavity is formed by a micromirror of $\sim 150 \mu\text{m}$ diameter, and $\sim 344 \mu\text{m}$ radius of curvature (ROC) fabricated into the flat substrate [31] on one side, and a 25 mm ROC mirror located 25.0467(10) mm below the flat mirror substrate on the other side (see Ref. [29] for additional details of the cavity structure). For 759 nm light used for optical trapping, the $1/e^2$ beam radius at the waist of the cavity mode is $w_0 = 5.4 \mu\text{m}$, and the Rayleigh range is 0.12 mm. This gives beam waist of 6.6 $\mu\text{m}$ at $Z = 0.14 \text{ mm}$, where $Z$ is the distance of the atoms from the micromirror substrate.

Because the mirror MOT geometry and the structure of the vacuum chamber limit the maximum quadrupole magnetic field gradient, we use two-color MOT (TC-MOT) [18] to trap $^{177}\text{Yb}$, where 399 nm and 556 nm lasers near resonant to the $^1S_0 \rightarrow ^1P_1$ transition and the $^1S_0 \rightarrow ^3P_1$ transition, respectively, are sent simultaneously (see Ref. [18] for the energy level diagram). Properties of the transitions and parameters of the MOT beams for the TCMOT are summarized in Table I. In addition to the MOT beams, a longitudinal cooling beam counterpropagating to the atomic beam is sent with a power of 1.8 mW and a detuning of $\sim 4.6\Gamma_s$ from the $^1S_0 \rightarrow ^1P_1$ transition, which has a radius of 1 mm at the MOT region. With a magnetic field gradient of $14.4 \text{ G/cm}$, typically $10^4$ atoms are trapped in the TCMOT.

Once the MOT loading stage of a few seconds is completed, atoms are transferred from the TCMOT into a MOT with only the 556 nm light (triplet MOT). The sequence to generate the triplet MOT, which is optimized to maximize the transfer efficiency from the TCMOT to the triplet MOT, is summarized in Fig. 2. The gradual changes in parameters maintain larger number of atoms in the MOT, as compared to sudden step-function-like quenches. Note that tuning the $x$ direction bias magnetic field actually moves the atom position along the $z$ axis, due to the tilted quadrupole field from the MOT.
TABLE I. Properties of the two transitions used for the MOT and parameters of the MOT beams for the TCMOT.

| Transition      | $^1S_0 \rightarrow ^1P_1$ | $^1S_0 \rightarrow ^3P_1$ |
|----------------|--------------------------|--------------------------|
| Wavelength (nm) | 399.911                  | 556.799                  |
| Linewidth/2π (MHz) | $\Gamma_s = 29.1$  | $\Gamma_t = 0.184$  |
| Saturation intensity (mW/cm$^2$) | $I_{sat,s} = 57$  | $I_{sat,t} = 0.14$  |
| Laser intensity | $0.10I_{sat,s}$         | $50I_{sat,t}$         |
| Laser detuning  | $-0.71\Gamma_s$         | $-38\Gamma_t$         |

The temperature of atoms in the triplet MOT is measured by the time-of-flight method with absorption imaging by a CCD camera using a 399 nm laser resonant with the $^1S_0 \rightarrow ^1P_1$ transition, sent horizontally with a $\sim 15$ degree angle relative to the $y$ axis. The temperature of the atoms in the triplet MOT is $\sim 15 \mu K$.

The atom position $Z$ relative to the flat mirror is measured by a CCD camera imaging along an angled direction, with a tilt of $14.2 \pm 0.2$ degrees to the $xy$ plane. Both a direct image and a reflected image in the mirror of the MOT are visible in the camera when the MOT is sufficiently close to the mirror (Fig. 3). $Z$ is calculated from the separation between the two images of the MOT. To remove image artifacts due to significant amount of light scattered from the mirror substrate by surface defects, background is subtracted by acquiring a reference image at the end of each experimental sequence, after removing all remaining atoms with a pulse of 399 nm light. $Z$ is affected by the detuning of the 556 nm laser from the atomic resonance due to the influence of gravity and imbalance between the incident and reflected MOT beam intensities. With a fixed 556 nm laser frequency, the triplet MOT position is stable within $10 \mu m$ in experimental runs spanning an hour.

The imaging of the triplet MOT also gives the size of the MOT, as shown in Fig. 4. A detuning of $\Delta = -\Gamma_t$ at the cooling stage compresses the MOT down to root-mean-square (RMS) radius of $55 \mu m$ and optimizes the lattice loading efficiency. The lifetime of atoms in the triplet MOT at different positions $Z$ is measured by continuously monitoring the amount of fluorescence from the triplet MOT by an avalanche photodiode, and extracting the exponential decay rate from the total fluorescence of the MOT. Fig. 5 shows the change in the triplet MOT lifetime according to the distance from the mirror. The decay rate increases at $Z \lesssim 0.7 \, \text{mm}$, and decay becomes too fast to observe at $Z = 0.6 \, \text{mm}$. This is reasonably consistent with Ref. [32]. Note that this measurement is performed at the
magnetic field gradient of 9 G/cm. With a larger magnetic field gradient of 14.4 G/cm and repeated optimization over time of the beam alignment and polarizations, atoms are trapped by the triplet MOT for a few hundreds of milliseconds at $Z = 0.14 \text{ mm}$.

III. LOADING TO THE OPTICAL LATTICE

The loading sequence into an optical lattice made of 759 nm light is summarized in Fig. 2. As the 759 nm trap light is always circulating in the cavity mode (see Section IV), simply turning off the 556 nm laser with atoms at the desired location transfers atoms to the optical lattice. In the last 20 ms of the triplet MOT, the intensity of the 556 nm laser is reduced to $2I_{\text{sat},t}$, in addition to an increase in the detuning to $-2.7\Gamma_t$. This larger red-detuning is necessary to compensate for the AC Stark shift induced by the trapping light, which is 20% larger for the $^3\text{P}_1$ excited state than the $^1\text{S}_0$ ground state. (It is assumed that information for $^{174}\text{Yb}$ in Ref. [33] is the same for $^{171}\text{Yb}$ when averaged over the hyperfine structure.) The quadrupole magnetic field is kept constant over the entire sequence up to this point.

After the 556 nm laser is turned off instantaneously, the quadrupole magnetic field and radial bias fields $B_x, B_y$ are ramped down to zero, while the axial magnetic field $B_z$ is ramped to a specific value, typically 13.6 G. The turning off of the quadrupole magnetic field is performed gradually over 40 ms, to avoid a mechanical kick to the cavity structure leading to oscillations that prevent reliable probing of the cavity resonance.

IV. OPTICAL LATTICE PROPERTIES

The one-dimensional optical lattice consists of 759 nm light circulating in the cavity mode, with a finesse of $3.14(6) \times 10^3$ at 759 nm. Because of imperfect mode matching between the input light and the cavity mode and losses at the mirror surfaces, coupling efficiency of the input light to the cavity mode is limited to 19%. Input power of $P_{759} = 6.7 \text{ mW}$ of the 759 nm laser to the cavity therefore generates an optical lattice in the cavity equivalent to a 1.3 W retroreflected beam. The cavity is locked to the 759 nm laser by Pound-Drever-Hall (PDH) technique [34] to ensure that the power enhancement is always present, as well as to maintain the cavity resonant frequency at a specific value. The 759 nm light is generated by a distributed Bragg reflector laser that is PDH locked to a separate stable reference cavity. To reduce the heating of atoms by intensity fluctuations of the optical lattice converted by the cavity from laser frequency fluctuations, the laser has an electro-optic modulator (EOM) feedback system that reduces the linewidth down to $\sim 1 \text{ kHz}$ [35]. Furthermore, intensity feedback to reduce the power fluctuation inside the cavity, particularly in case the laser is intrinsically noisy, is implemented. The power of the transmitted light is monitored by a photodiode, and the input power is actively stabilized by a power modulating EOM in the path of the input light. This reduces the intensity fluctuation of the intracavity power by a factor of $\sim 10$ at $\sim 100 \text{ kHz}$ [36, 37].

The trapping frequency of the optical lattice is measured by modulating the trap depth. Typical timescale $T$ for heating of atoms in the lattice obeys the following formula [38]:

$$\frac{1}{T} = \pi^2 \nu^2 S(2\nu),$$

where $\nu$ is the trapping frequency, and $S$ is the noise spectral density of the trap depth. According to this formula, $T$ decreases significantly compared to other modulation frequencies, when the modulation frequency is twice as large as the trapping frequency. The population of atoms in the trap is measured at $Z = 1.99 \text{ mm}$ by the dispersive
shift of the cavity resonant frequency \[29\,30\,37\], which is fitted by an exponential function to extract \(T\). Measured radial and axial trapping frequencies are \(\nu_r = 125 \pm 5 \text{ Hz} \) (radial) and \(\nu_{ax} = 67 \pm 2 \text{ kHz} \) (axial) for 1.92 W intracavity power, corresponding to input power of 9.9 mW. The value is reasonably consistent with expected values from Ref. \[33\] and Eqs. (2) and (3).

Figure 6 shows the comparison of the measured lifetime and the expected lifetime derived from the noise spectral density of the transmission light and Eq. (1). Different trapping frequencies are obtained by changing the input power \(P_{759}\). At \(\nu_{ax} \geq 100 \text{ kHz} \) or more, measured lifetime has a reasonable agreement with the expected lifetime. Deviation at lower frequencies is likely due to the lattice being too shallow to trap atoms. This measurement is performed without the intensity feedback, and the lifetime observed in the lattice with intensity feedback enabled is more than one second. With the latest version of the EOM feedback to the trapping laser \[35\], the frequency noise is so low that the lifetime of the atoms in the lattice is 2 s or more even without the intensity feedback. The temperature of the atoms in the lattice is \(\sim 30 \mu\text{K}\), measured by the Doppler profile of the \(^1S_0 \rightarrow ^3P_1\) clock transition. Compared with the temperature in the triplet MOT (\(\sim 15 \mu\text{K}\)), the atom temperature in the optical lattice is hotter, which is consistent with the previous observations in strontium system \[40\,41\].

\[ \nu_r = \frac{1}{2\pi} \sqrt{\frac{4U_0}{w^2m}} \]  

The 556 nm laser is detuned by \(\Delta_1 = -2.7T_1\) from the \(^1S_0 \rightarrow ^3P_1\) transition during the final cooling stage for all experiments described in this section.

For the shallow-lattice regime, \(U_0 \approx 3 \text{ MHz} \) and \(\nu_{ax} \approx 50 \text{ kHz} \). This roughly means \(U_0 \approx -\Delta_1\), within a linewidth, where an atom can dissipate all its excess kinetic energy acquired when moving into a lattice potential minimum by scattering a single photon at the atom’s resonance frequency. As the lattice becomes deeper, this cooling mechanism no longer works and the loading efficiency decreases.

In the deep-lattice region, \(U_0 \approx 3 \text{ MHz} \) and \(\nu_{ax} \approx 200 \text{ kHz} \). The trap depth \(U_{3P_1}\) for the \(^3P_1\) excited state is higher than the depth \(U_{1S_0}\) for the \(^1S_0\) ground state \[39\], primarily due to coupling to the 6s7s\(^3\)S\(_1\) state. This means that as the lattice becomes deeper, the effective detuning of the cooling laser near the bottom of the potential becomes smaller in magnitude, and eventually sideband cooling becomes possible when \(\Delta_{eff} = \Delta_{556} + (U_{3P_1} - U_{1S_0}) \approx -\nu_{ax}\). This leads to an increased trap loading efficiency as atoms are cooled into the minimum of the trapping potential.

\[ \nu_{ax} = \frac{1}{2\pi} \sqrt{\frac{2U_{ax}k^2}{m}} \]  

\[ \nu_r = \frac{1}{2\pi} \sqrt{\frac{4U_0}{w^2m}} \]  

VI. LATTICE LOADING NEAR THE MICROMIRROR SURFACE

Previous study of the mirror MOT \[32\] reports that the atom lifetime decreases rapidly as the atoms are brought closer than 0.2 mm from a surface. However, our system has two advantages compared to \[32\]: atoms in the MOT are kept near the mirror surface only for a short time before they are loaded into the lattice, and the triplet MOT is very compact with an RMS radius of 55 \(\mu\text{m}\), due to the narrow linewidth of the cooling transition. To load the atoms very near the micromirror surface, MOT location is moved upward by simply ramping the bias magnetic field and then MOT beams are suddenly turned off.

As shown in Figs. 7 and 8, the smallest distance between the atom loading position in the cavity and the micromirror surface is 0.14 mm. At this position, \(N\eta \approx 10^4\) and \(\eta = 10\) are observed \[29\], which implies a total atom number \(N_{tot} = (3/2)N \approx 1500\) (see Ref. 44 or Ref. 44 for the derivation). The lifetime of atoms in the lattice is shorter when \(Z\) is small, estimated to be around 0.5 s for \(Z < 0.25 \text{ mm}\), whereas the typical lifetime is more than one second for larger \(Z\). We believe that the atom lifetime in the trap is shorter for smaller \(Z\) primarily due to the stronger probe-induced atom loss during the measurement of the atom number at smaller \(Z\), as the single atom cooperativity \(\eta\) is larger for smaller \(Z\). Indeed, we observe a linear rather than exponential decay of the atom number with stronger probing, which is consistent with probing-induced loss rather than one-body loss.
FIG. 7. (a) MOT brightness (blue squares) and atom number loaded into the optical lattice (red circles) at a fixed atom position $Z = 0.54$ mm, as a function of the input 759 nm laser power. Stars around 1 mW and 8 mW correspond to the shallow lattice and deep lattice data points in (b), respectively. (b) Values of trap laser power that locally maximize atom loading efficiency. The two points at $Z = 0.54$ mm correspond to the maxima shown in (a). The dashed lines correspond to ones in Fig. 8 converted into the incident trap beam power necessary to generate the trapping frequencies.

FIG. 8. Trap depth $U_0$ (red circles) and axial (blue squares, $\nu_{\text{ax}}$) and radial (green triangles, $\nu_r$) trapping frequencies of trap depths locally maximizing $N\eta$ in the lattice at different atom position $Z$; $\nu_r$ is the geometrical mean of two orthogonal radial directions. The deep and shallow lattice regimes correspond to axial vibrational frequencies $\nu_{\text{ax}}$ of approximately 53 kHz and 160 kHz, respectively.

Although the cloud of atoms is located relatively close to the mirror surface, the deterioration of the quality of the mirror is slow, as Fig. 9 shows. The rate 6.76(1) ppb/h of mirror loss increase obtained by the fit is similar to the room temperature case in Ref. [45] (see Table II for comparison). As for the same top layer material of SiO$_2$, 100 °C data in Ref. [45] is converted to 1 ppb/h at room temperature, following Fig. 3 of Ref. [45], which is of the same order of magnitude as the measured rate of the loss increase. Other systems we can compare with are Ref. [46] and Ref. [47]. Our system has significantly smaller rate of the loss increase compared to Ref. [46], and larger than Ref. [47]. In spite of this larger increase in the loss than Ref. [47], the rate of the loss increase is still small enough to maintain the high finesse over the time scale of many years. This atomic ensemble near a mirror surface without excessive contamination is suitable for performing a wide variety of cQED experiments in the strong coupling regime, where $\eta > 1$.

| Ref. | $dL/dt$ (ppb/h) | Temperature $\lambda$ (nm) | Top layer |
|------|-----------------|-----------------------------|-----------|
| This work | 6.76 ± 0.01 | 30 °C | 556 nm | SiO$_2$ |
| [45] | 12.3 ± 4.3 | 21 °C | 370 nm | Ta$_2$O$_5$ |
| [45] | 230 ± 30 | 100 °C | 370 nm | SiO$_2$ |
| [46] | 2300 ± 200 | R.T. | 369 nm | SiO$_2$ |
| [47] | 0.9 ± 3.5 | R.T. | 369 nm | SiO$_2$ |
FIG. 9. Change of the cavity loss at 556 nm over time: the red dots are the measured values, and the blue line is a linear fit. The fluctuation of the loss away from the trendline significantly exceeding the error bar results from the finesse drift induced by slight changes in cavity alignment. To avoid large effects from these outliers, Lorentzian weight is used for the fit, instead of the Gaussian weight in the standard \( \chi^2 \) fit. The slope obtained by the fit is \( 6.76(1) \) ppb/h. The black line is a fit by exponential function

\[
L = L_0 + A (1 - \exp\left(-\frac{t}{\tau}\right))
\]

described in [45]. The fitted parameters are \( L_0 = 88 \pm 1 \) ppm, \( A = 346 \pm 4 \) ppm, and \( \tau = 1490 \pm 75 \) days. As the modified \( \chi^2 / ndf = 1.6 \) for both linear and exponential fits, only the result of linear fit is discussed in the main text. Vertical grid lines correspond to the first day of the year shown in the tick label. Typical atom position is \( Z = 1.4 \) mm until July, 2017, and after that, various different distances are used ranging from \( Z = 0.14 \) mm to 1.4 mm, settling down to \( Z = 0.42 \) mm in February 2018.

VII. CONCLUSION

We have demonstrated the loading of \(^{171}\)Yb atoms into a tight one-dimensional optical lattice in an optical cavity close to the surface of a mirror. Two distinct regimes of efficient loading of atoms from a mirror MOT into the optical lattice are observed, due to two different loading mechanisms. The loading of atoms in the lattice is performed simply by putting a MOT at a desired location, and up to 1500 atoms are trapped in an optical lattice with 6.6 \( \mu \)m waist at a distance of 0.14 mm from the mirror surface. These results open a simple way to realize a system suitable for quantum mechanical manipulation of atoms in the strong coupling regime.

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