Abstract

We use the method of QCD Sum Rules to estimate the masses of the charm baryon, $\Lambda_c^+$, bottom baryon $\Lambda_b^0$, strange baryon $\Lambda_s^0$ and compare them to their experimental values.

Keywords: Charmed baryon, Bottom baryon, Strange baryon, QCD Sum Rules

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1 Introduction

The method of QCD sum rules were introduced by Shifman, Vainshtein, and Zakharov \cite{1} to estimate properties of hadrons using 2-point correlators. See L.J. Reinders, et.al.\cite{2} for a detailed description of this method for estimating various properties of hadrons. We use this method for estimating the masses of the charm baryon $\Lambda_c^+$, bottom baryon $\Lambda_b^0$, and strange baryon $\Lambda_s^0$. Since the contribution from the quark condensates\cite{3} and the gluon condensate\cite{4} is negligible for the 2-point correlator needed to estimate the mass of the $\Lambda_c^+$, $\Lambda_b^0$, $\Lambda_s^0$ we only use the main diagram, shown in Figure 1 for $\Lambda_c^+$ in the following section. The experimental values of the masses are found in Ref\cite{5}. After the figures we review previous publications using QCD sum rules to estimate baryon and heavy quark masses.

2 QCD Sum Rules for the mass of the $\Lambda_c^+$

The two-point correlator in momentum space is used to estimate the mass of the $\Lambda_c^+$, a charmed baryon. The two-point correlator is

$$\Pi_2(p) = i \int d^4x \, e^{ipx} < 0 \vert T[\eta_{\Lambda_c^+}(x) \bar{\eta}_{\Lambda_c^+}(0)] \vert 0 >,$$

where $\eta_{\Lambda_c^+}$ is the current for $\Lambda_c^+$:

$$\eta_{\Lambda_c^+} = \epsilon_{efg}[u^T C \gamma_\mu d^f] \gamma^5 \gamma^\mu e^g,$$

with $e, f, g$ color indices and $\epsilon_{efg}$ resulting in $\eta_{\Lambda_c^+}$ having color=0, $u, d$ are up and down quarks and $c$ a charm quark.
The two-point correlator for $\Lambda_c^+$ is illustrated in Figure 1

![Diagram of $\Lambda_c^+$ correlator](image)

Figure 1: The $\Lambda_c^+$ (udc)

Using the methods of QCD Sum Rules[2] the $T[\eta_{\Lambda_c^+}(x) \bar{\eta}_{\Lambda_c^+}(0)]$ is replaced by the Trace

$$
\Pi_2(p) = i \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} Tr[S_c(k_1)\gamma^5\gamma^\mu S_u(k_2)\gamma^5\gamma^\mu S_d(p-k_1-k_2)]
$$

(3)

where $S_c, S_u, S_d$ are the charm, up, and down quark propagators. With $\not{p} = p_\mu \gamma^\mu$

$$
S_c(p) = i \frac{\not{p} + M_c}{p^2 - M_c^2}
$$

(4)

where $M_c \approx 1.27$ GeV is the mass of the charm quark. $S_u, S_d$ have similar expressions with $m_u \simeq m_d \equiv m \ll M_c$. From Eqs(3, 4)

$$
\Pi_2(p) = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{2M_cm^2 + M_ck_2 \cdot (p-k_1-k_2) + 2mk_1 \cdot (p-k_1-k_2) + mk_1 \cdot k_2}{[(k_1^2 - M_c^2)(k_2^2 - m^2)((p-k_1-k_2)^2 - m^2)]}
$$

(5)

Carrying out the trace one obtains

$$
\Pi_2(p) = 8 \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{2M_cm^2 + M_ck_2 \cdot (p-k) + 2mk_1 \cdot (p-k) + mk_1 \cdot k}{[(k^2 - m^2)((p-k)^2 - m^2)]}
$$

(6)

Defining $\bar{p} = p - k_1, k = k_2$,

$$
\Pi_{k_1}(p) = \int \frac{d^4k}{(2\pi)^4} \frac{2M_cm^2 + M_ck \cdot (\bar{p} - k) + 2mk_1 \cdot (\bar{p} - k) + mk_1 \cdot k}{[(k^2 - m^2)((\bar{p} - k)^2 - m^2)]}
$$

(7)

Defining

$$
\Pi_1(p) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{[(k^2 - m^2)((\bar{p} - k)^2 - m^2)]}
$$

(8)

and using $1/(k^2-m^2) = \int_0^\infty d\alpha e^{-\alpha(k^2-m^2)}, D = 4 - \epsilon, \bar{k} = k - \beta \bar{p}/(\alpha + \beta), \int \frac{d^Dk}{(2\pi)^D} e^{-(\alpha+\beta)k^2} = 1/(4\pi(\alpha+\beta))^{D/2}$, and $\alpha \rightarrow \rho\alpha, \beta \rightarrow \rho\beta, \delta(\rho - \alpha - \beta) = \delta(1 - \alpha - \beta)/\rho$, one obtains for $\Pi_1(p)$
\[ \Pi_1(p) = \int_0^\infty d\rho \int_0^1 d\alpha \int_0^1 d\beta \frac{1}{(4\pi)^{D/2}} \rho^{1-D/2} e^{-\rho(-m^2 + \alpha(1-\alpha)p^2)} \delta(1 - \alpha - \beta). \] (9)

Integrating Eq(9) by parts one obtains

\[ \Pi_1(p) = \frac{1}{(4\pi)^2}(2m^2 - \bar{p}^2/2)I_0(\bar{p}) \text{ with } \]
\[ I_0(p) = \int_0^1 d\alpha \frac{1}{-m^2 + \alpha(1-\alpha)p^2}. \] (10)

From Eq(7) we also need

\[ \Pi_{\mu_1}(p) = \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu}{((k^2 - m^2)((\bar{p} - k)^2 - m^2))} = \frac{\bar{p}^\mu}{(4\pi)^2}[(m^2 - \bar{p}^2/4)I_0(\bar{p}) - \frac{7}{4}], \] (11)

Therefore from Eqs(7,10,11),

\[ \Pi_{k_1}(p) = \frac{1}{(4\pi)^2}(2mk_1 \cdot \bar{p} + m^2M_c)(2m^2 - \bar{p}^2/2)I_0(\bar{p}) \]
\[ + (M_c\bar{p}^2 - mk_1 \cdot \bar{p})(m^2 - \bar{p}^2/4)I_0(\bar{p}) - \frac{7}{4}]. \] (12)

From Eqs(6,12), dropping the 7/4 term which vanishes with a Borel transform

\[ \Pi_2(p) = \frac{8}{(4\pi)^4} \int_0^1 d\alpha (2m^4M_cI_1(p) + 3m^3I_2(p) + \frac{3m^2M_c}{2}I_3(p) \]
\[ - \frac{3m}{4}I_4(p) - \frac{M_c}{4}I_5(p)). \] (13)

With \( k = k_1 \)

\[ I_1(p) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M_c^2)[(\alpha(1-\alpha)(p-k)^2 - m^2]} \] (14)
\[ I_2(p) = \int \frac{d^4k}{(2\pi)^4} \frac{k \cdot (p-k)}{(k^2 - M_c^2)[(\alpha(1-\alpha)(p-k)^2 - m^2]} \] (15)
\[ I_3(p) = \int \frac{d^4k}{(2\pi)^4} \frac{(p-k)^2}{(k^2 - M_c^2)[(\alpha(1-\alpha)(p-k)^2 - m^2]} \] (16)
\[ I_4(p) = \int \frac{d^4k}{(2\pi)^4} \frac{k \cdot (p-k)(p-k)^2}{(k^2 - M_c^2)[(\alpha(1-\alpha)((p-k)^2 - m^2] \] (17)
\[ I_5(p) = \int \frac{d^4k}{(2\pi)^4} \frac{(p-k)^2(p-k)^2}{(k^2 - M_c^2)[(\alpha(1-\alpha)((p-k)^2 - m^2]} \] (18)
Dropping terms that vanish with a Borel transform one can show that

\[
I_3(p) = \frac{m^2}{\alpha(1 - \alpha)} I_1(p)
\]

\[
I_4(p) = \frac{m^2}{\alpha(1 - \alpha)} I_2(p)
\]

\[
I_5(p) = \frac{m^4}{\alpha(1 - \alpha)^2} I_1(p).
\]

Using \(1/[\alpha(1 - \alpha)(p - k)^2 - m^2] = \int_0^\infty d\lambda e^{-\lambda[(1 - \alpha)(p - k)^2 - m^2]}, \)

\[
I_1(p) = \int \frac{d^4k}{(2\pi)^4} \int_0^\infty dk d\lambda \ e^{-\lambda(k^2 - M_c^2)} e^{-\lambda[(1 - \alpha)(p - k)^2 - m^2]} e^{-\kappa(k^2 - M^2) - \lambda[-m^2 + \alpha(1 - \alpha)(p - k)^2]}
\]

Carrying out the momentum integral with \(d^4k \to d^Dk, D = 4 - \epsilon, \) using \(\kappa \to \rho \kappa, \lambda \to \rho \lambda, \delta(\rho - \kappa - \lambda) = \delta(1 - \kappa - \lambda)/\rho; \) and carrying out the \(\lambda\) integral, and using \(\kappa \to \rho \kappa, \lambda \to \rho \lambda, \delta(\rho - \kappa - \lambda) = \delta(1 - \kappa - \lambda)/\rho\) one obtains

\[
I_1(p) = \frac{1}{(4\pi)^2} \int_0^\infty dp \int_0^1 d\kappa \rho(1 - D/2) \frac{1}{[\kappa + \alpha(1 - \alpha)(1 - \kappa)]^{D/2}} e^{-\rho a}
\]

with

\[
a = \frac{\kappa(1 - \kappa)\alpha(1 - \alpha)p^2}{\kappa + \alpha(1 - \alpha)(1 - \kappa)} - \kappa M_c^2 - (1 - \kappa)m^2
\]

Carrying out \(\int_0^\infty dp\) and using \(a^{-\frac{\epsilon}{2}} = e^{-(\epsilon/2) \ln a} = 1 - (\epsilon/2) \ln a\)

\[
I_1(p) = \frac{1}{(4\pi)^2} \Gamma \left(\frac{\epsilon}{2}\right) \int_0^1 d\kappa \frac{a^{-\frac{\epsilon}{2}}}{[\kappa + \alpha(1 - \alpha)(1 - \kappa)]^{D/2}} \times (-\ln a).
\]

Evaluating \(I_2(p)\) in a similar way one finds

\[
I_2(p) = \int \frac{d^4k}{(2\pi)^4} \frac{k.(p - k)}{(k^2 - M_c^2)[-m^2 + \alpha(1 - \alpha)(p - k)^2]}
\]

\[
= p_\mu \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu}{(k^2 - M_c^2)[-m^2 + \alpha(1 - \alpha)(p - k)^2]}
\]

\[
- \int \frac{d^4k}{(2\pi)^4} \frac{M_c^2}{(k^2 - M_c^2)[-m^2 + \alpha(1 - \alpha)(p - k)^2]}
\]

\[
= p_\mu \Pi^{\mu}_1(p) - M_c^2 I_1(p),
\]

with

\[
\Pi^{\mu}_1(p) = \int \frac{d^4k}{(2\pi)^4} k^\mu \int_0^\infty d\kappa d\lambda \ e^{-\kappa(k^2 - M_c^2) - \lambda[-m^2 + \alpha(1 - \alpha)(p - k)^2]}
\]

\[
= \int_0^\infty d\kappa d\lambda \int \frac{d^4k}{(2\pi)^4} k^\mu \exp \left[-(\kappa + \lambda \alpha(1 - \alpha)) \left(k - \frac{\lambda \alpha(1 - \alpha)p}{\kappa + \lambda \alpha(1 - \alpha)}\right)^2\right]
\]

\[
- \lambda \alpha(1 - \alpha)p^2 + \frac{(\lambda \alpha(1 - \alpha)p)^2}{\kappa + \lambda \alpha(1 - \alpha)} + (\kappa M_c^2 + \lambda m^2).
\]
Using $k^\mu \rightarrow k^\mu + \frac{\lambda \alpha (1-\alpha) p^\mu}{\kappa + \lambda \alpha (1-\alpha)}$, one finds

$$
\Pi_1^\mu(p) = \int_0^\infty d\kappa d\lambda \int \frac{d^4k}{(2\pi)^4} \left[ k + \frac{\lambda \alpha (1-\alpha) p}{\kappa + \lambda \alpha (1-\alpha)} \right]^\mu \times \exp \left[ - (\kappa + \lambda \alpha (1-\alpha)) k^2 - \lambda \alpha (1-\alpha) p^2 + \frac{\lambda \alpha (1-\alpha) p^2}{\kappa + \lambda \alpha (1-\alpha)} + \left( \kappa M^2 + \lambda m^2 \right) \right].
$$

Integrating over the four-momenta

$$
\Pi^\mu(p) = \frac{p^\mu}{(4\pi)^2} \int_0^1 d\kappa \frac{\alpha (1-\alpha) (1-\kappa)}{[\kappa + \alpha (1-\alpha) (1-\kappa)]^3} \times (- \ln a);
$$

and therefore

$$
I_2(p) = p^2 \Pi(p) - M^2 c I_1(p),
$$

where

$$
\Pi(p) = \frac{1}{(4\pi)^2} \int_0^1 d\kappa \frac{\alpha (1-\alpha) (1-\kappa)}{[\kappa + \alpha (1-\alpha) (1-\kappa)]^3} \times (- \ln a).
$$

Therefore from Eqs(13,19)

$$
\Pi_2(p) = \frac{8}{(4\pi)^4} \int_0^1 d\alpha \left[ 2m^4 M c I_1(p) + 3m^3 \left\{ p^2 \Pi(p) - M^2 c I_1(p) \right\} + \frac{3m^4 M c}{2\alpha (1-\alpha)} I_1(p) \right]
$$

$$
\quad - \frac{3m^3}{4\alpha (1-\alpha)} \left\{ p^2 \Pi(p) - M^2 c I_1(p) \right\} - \frac{M c m^4}{4\alpha^2 (1-\alpha)^2} I_1(p)
$$

$$
= \frac{8}{(4\pi)^4} \int_0^1 d\alpha \left[ \left\{ 2m^4 M c - 3m^3 M^2 c + \frac{3M c m^4}{2\alpha (1-\alpha)} + \frac{3m^3 M^2 c}{4\alpha (1-\alpha)} - \frac{M c m^4}{4\alpha^2 (1-\alpha)^2} \right\} I_1(p)
$$

$$
\quad + \left\{ 3m^3 - \frac{3m^3}{4\alpha (1-\alpha)} \right\} p^2 \Pi(p) \right] .
$$

A Borel transform $\mathcal{B}$ of $\Pi_2(p)$ gives

$$
\Pi_2(M_B) = \frac{8}{(4\pi)^4} \int_0^1 d\alpha \left[ \left\{ 2m^4 M c - 3m^3 M^2 c + \frac{3M c m^4}{2\alpha (1-\alpha)} + \frac{3m^3 M^2 c}{4\alpha (1-\alpha)} - \frac{M c m^4}{4\alpha^2 (1-\alpha)^2} \right\} \mathcal{B}[I_1(p)]
$$

$$
\quad + \left\{ 3m^3 - \frac{3m^3}{4\alpha (1-\alpha)} \right\} \mathcal{B}[p^2 \Pi(p)] \right] .
$$

Note that

$$
\mathcal{B}[\ln(p^2 - b^2)] = -M_B^2 e^{-b^2/M_B^2},
$$

$$
\mathcal{B}[p^2 \ln(p^2 - b^2)] = -M_B^2 (b^2 - M^2_B) e^{-b^2/M_B^2}.
$$
From Eqs(33,34)

\[
\Pi_2(M_B) = -\frac{8}{(4\pi)^6} \int_0^1 d\alpha \int_0^1 d\kappa \left( 2m^4 M_c - 3m^3 M_c^2 + \frac{3M_c m^4}{2\alpha(1-\alpha)} + \frac{3m^3 M_c^2}{4\alpha(1-\alpha)} - \frac{M_c m^4}{4\alpha^2(1-\alpha)^2} \right) \\
\times \left( -\frac{M_B^2 e^{-b^2/M_B^2}}{[\kappa + \alpha(1-\alpha)(1-\kappa)]^2} \right) \\
- \frac{8}{(4\pi)^6} \int_0^1 d\alpha \int_0^1 d\kappa \left( 3m^3 - \frac{3m^3}{4\alpha(1-\alpha)} \right) \left( -\frac{M_B^2 b^2 - M_B^2}{[\kappa + \alpha(1-\alpha)(1-\kappa)]^3} \right). 
\]

(35)

It is convenient for \(\Pi_2(M_B)\) to use

\[
\Pi_2(M_B) = \frac{8M_B^2}{(4\pi)^6} \int_0^1 d\alpha \int_0^1 d\kappa \frac{g(\alpha, \kappa) M_5(\alpha) + 3m^3 M_2(\alpha, \kappa)}{g(\alpha, \kappa)^3} \frac{e^{-b(\alpha, \kappa)^2/M_B^2}}{M_5(\alpha)}, 
\]

(36)

where

\[
g(\alpha, \kappa) = \kappa + \alpha(1-\alpha)(1-\kappa), 
\]

(37)

\[
b(\alpha, \kappa)^2 = g(\alpha, \kappa) \left[ \frac{M_c^2}{\alpha(1-\alpha)(1-\kappa)} + \frac{m^2}{\kappa\alpha(1-\alpha)} \right], 
\]

\[
M_2(\alpha, \kappa) = \left( 1 - \frac{1}{4\alpha(1-\alpha)} \right) [b(\alpha, \kappa)^2 - M_B^2], 
\]

\[
M_5(\alpha) = 2m^4 M_c - 3m^3 M_c^2 + \frac{3M_c m^4}{2\alpha(1-\alpha)} + \frac{3m^3 M_c^2}{4\alpha(1-\alpha)} - \frac{M_c m^4}{4\alpha^2(1-\alpha)^2}. 
\]

From Eqs(36,37), dropping the factor of \(\frac{8}{(4\pi)^6}\) as only the shape of \(\Pi_2(M_B)\) is needed to estimate the mass, as in Ref[6], obtains the result shown in Figure 2.

![Figure 2: Two-point correlator \(\Pi_2\) for \(\Lambda_c^+\) as a function of the Borel mass \(M_B\)](image Link)
3 QCD Sum Rules for the masses of the $\Lambda^0_b$ and $\Lambda^0_s$

The two point correlator illustrated in Figure 1 and derived in Eqs(1-6) for $\Lambda^+_c$ are the same for $\Lambda^0_b$, $\Lambda^0_s$ with quarks $c \to b$, $c \to s$ in Eqs(1,2) and the figure; and $M_c \to M_b$, $M_c \to M_s$ in Eqs(3-6).

The calculation of the masses of $\Lambda^0_b$ and $\Lambda^0_s$ use the equations Eqs(36,37) with $M_c \simeq 1.27$ GeV replaced by $M_b \simeq 4.18$ GeV for the mass of $\Lambda^0_b$ and $M_s \simeq 96$ MeV for the mass of $\Lambda^0_s$.

The results for $M_{\Lambda^0_b}$ and $M_{\Lambda^0_s}$ are shown in Figures 3 and 4.

Figure 3: Two-point correlator $\Pi_2$ for $\Lambda^0_b$ as a function of the Borel mass $M_B$

Figure 4: Two-point correlator $\Pi_2$ for $\Lambda^0_s$ as a function of the Borel mass $M_B$
More than three decades ago sum rules analogous to those of SVZ[1] were used[7] to estimate the mass difference between the $\Sigma(3/2^+)$ and the $\Delta(3/2^+)$. Using the strange quark mass $m_s = 150$ MeV, Ioffe estimated that $M_\Sigma - M_\Delta \simeq 125$ MeV, while current experimental results[5] are $M_\Sigma - M_\Delta \simeq -130$ MeV. Since the current value of $m_s \simeq 96$ MeV[5], this could be an explanation of Ioffe’s result. More recently Ioffe[8] gave evidence that the violation of chiral symmetry in the QCD vacuum is the origin of baryon masses, and estimated $M_\Delta = 1.30 \pm 0.18$ GeV while current experimental result[5] is $M_\Delta \simeq 1.51$ GeV.

Also, using QCD spectral sum rules[9] attempts to estimate the masses of the charm baryon $\Lambda_c^+$ and bottom baryon $\Lambda_b^0$ were made, but the authors were not able to make reliable predictions for the $\Lambda_c^+, \Lambda_b^0$ masses.

Sum rules have also been used to estimate the masses of quarks. Using a Pseudoscalar Sum Rule[10] the strange quark mass was estimated to be $m_s \simeq 105 \pm 6 \pm 7$ MeV while the current value[5] is $m_s \simeq 96$ MeV. Using QCD sum rules[11] the charm quark mass was estimated as $m_c = 1.46 \pm 0.07$ GeV while the current value[5] is $m_c = 1.27 \pm 0.03$ GeV. Using QCD spectral sum rules[12] Narison found the mass of the bottom quark $m_b = 4.23^{+0.03}_{-0.04}$ GeV while the current value[5] is $m_b = 4.18^{+0.04}_{-0.03}$ GeV.

4 Results and Conclusions

Using the method of QCD Sum Rules the mass of $\Lambda_c^+, \Lambda_b^0$, and $\Lambda_s^0$ were estimated. The masses and theoretical uncertainty are estimated using minimum and spread near the minimum in the plot of $\Pi_2(M_B)$ in Figures 2, 3, and 4.

From Figure 2 $M_{\Lambda_c^+} = 2.01 \pm 0.3$ GeV compared to the experimental value[5] 2.29 GeV. From Figure 3 $M_{\Lambda_b^0} = 5.34 \pm 0.25$ GeV compared to the experimental value[5] 5.62 GeV. From Figure 4 $M_{\Lambda_s^0} = 1.05 \pm 0.1$ GeV compared to the experimental value[5] 1.116 GeV.

We conclude from our results that the method of QCD Sum Rules[1, 2] can be used to estimate the masses of charm, bottom, and strange baryons.

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References

[1] M.A. Shifman, A.J. Vainshtein, and V.I. Zakharov, Nucl. Phys. B 147 (1979) 385.
[2] L.J. Reinders, H. Rubenstein, and S. Yazaki, Phys. Reports, 127, 1 (1985)
[3] Dmitri Antonov, Jose Emilio, and F.T. Ribeiro, Eur.Phys.J. C 72, 2179 (2012)
[4] N. Brambilla and A. Vairo, Phys. Lett. B 407, 157 (1997)
[5] Particle Data Group, Chin. Phys. C 40, 100001 (2016)
[6] Leonard S. Kisslinger and Steven Casper, Int. J. Theor. Phys. 54, 3825 (2015)
[7] B.L. Ioffe, Nucl. Phys. B 188, 317 (1981)
[8] B.L. Ioffe, Phys. Atom. Nucl. 72, 1214 (2009)
[9] E. Bagan, M. Chabab, H.G. Dosch and S.Narison, Phys. Lett. B 287, 176 (1992)
[10] K.G. Chetyrkin and A. Khodjamirian, Eur. Phys. J. C 46, 721 (2006)
[11] C.A. Dominguez, G.R. Gluckman and N. Paver, Phys. Lett. B 333, 184 (1994)
[12] S. Narison, Phys. Lett. B 34, 73 (1994)