A Theoretical Model of Dual Tunable Optoelectronic Oscillator

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Abstract. A theoretical model of an electronically tunable multi-loop optoelectronic oscillator (OEO) based on photonic and microwave multiferroic delay lines has been developed. The theory takes into account a dispersion of electromagnetic waves in optical fibers and a dispersion of spin-electromagnetic waves in ferrite-ferroelectric structures. An expression for transfer function of the OEO circuitry was derived. The theory enables one to calculate transmission characteristics and suppression of spurious harmonics of the multi-loop OEOs. The features of both magnetic and electric tunabilities are investigated.

Electronically tunable oscillators are one of the most important components of various radioelectronic systems. Optoelectronic oscillators (OEOs) based on high-Q whispering gallery mode optical resonators or long optical fibers offer the best spectral purity and the short-term frequency stability as well as the low phase noise. Due to these unique features OEOs have attracted a great attention of researchers in the last two decades [1].

Oscillation frequency in a classical configuration of the oscillator based on the single fiber loop is determined by the phase shift in the fiber and microwave (MW) power regeneration in a feedback containing an amplifier and a bandpass filter [2], [3]. An increase in the optical fiber length provides increasing the delay time and rising the Q-factor, hence decreasing the phase noise level. Nevertheless, it causes simultaneously a decrease in a free spectral range (FSR) that leads to appearance of unwanted spurious modes. Earlier, in the work [4]–[6] the multi-loop configuration was suggested to solve this problem.

The tunable OEOs are based on tunable filters in a microwave frequency domain [7] or an optical domain [8]. Spin-wave delay lines based on epitaxial yttrium iron garnet films were recently suggested for the magnetic tuning of an oscillation frequency of the OEO [9]. In order to increase functionality and realize a dual electric and magnetic tunable OEO one is required to use a multiferroic delay lines, which are usually fabricated with a combination of ferrite and ferroelectric layers [10].

The purpose of this work is to develop a theoretical model of the multi-loop optoelectronic oscillators based on photonic and microwave multiferroic delay lines. In contrast to earlier works, the developed theory takes into account the specific features of wave processes in both delay lines.

A theoretical model was developed for a common multi-loop OEO circuitry shown in Fig. 1. The OEO has a ring circuitry consisting of the microwave and optical paths. The optical path includes a monochromatic optical signal source (LD), an electro-optical modulator (EOM) of Mach–Zehnder type, an optical fiber splitter with N channels and splitting coefficients \( \eta_j \) for each channel, \( N \) single-
mode optical fibers of lengths \( l_j \), an optical fiber combiner, and a photodetector (PD). The main elements of the MW path are a MW amplifier with gain coefficient \( G \) and a multiferroic delay line.

Let us consider a circulation of microwave signal in the oscillator ring circuitry. The MW signal applied to the driving port of the electro-optical modulator modulates amplitude of the monochromatic optic radiation. The modulated radiation is divided by the splitter between the single-mode fibers. The optical signals propagating in different fibers accumulate different phase shifts and then interfere in the combiner. The optical radiation reaches the photodetector. After detection the electric signal of MW frequency propagates in the microwave path reaching the multiferroic delay line. The MW signal excites hybrid spin-electromagnetic wave that propagates in multiferroic structure and accumulates a delay time in accordance with its dispersion law. After the multiferroic delay line, the MW signal is introduced to an amplifier. The MW amplifier together with a variable attenuator controls amplification in the ring. A small part of the circulating MW signal is taken out of the ring by the output directional coupler. The other part of the MW signal is supplied to the driving input of the electro-optical modulator. As a result, the ring becomes closed. When the amplification compensates the total loss of the signal in the ring and an open-loop gain reaches unity, the microwave generation begins.

![Figure 1. Block-diagram of the multi-loop ferrite-ferroelectric OEO](image)

Let us describe the proposed theoretical model of a multi-loop resonator. The complex transmission coefficient \( H(\omega) \) is calculated as the ratio of the complex amplitudes of the output and input signals:

\[
H(\omega) = \frac{A_{\text{out}}(\omega)}{A_{\text{in}}(\omega)}.
\]

We assume that the input signal is monochromatic. For definiteness, let the phase of the input signal at the initial instant of time be equal to zero:

\[
A_{\text{in}}(\omega) = A_0 \exp(i\omega t).
\]

As was discussed above modulated optical radiation is divided by the splitter with splitting coefficient \( \eta_j \) between the single-mode fibers of length \( l_j \). Note, that \( \sum \eta_j = 1 \). During the signal transmission in the optical waveguide it accumulates a phase shift proportional to the fiber length and the wavenumber of the carrier optical signal. The optical signals are combined and interfere before the photodetector. Then the signal propagates through the remaining elements of the ring. The transfer coefficient \( T(\omega) \) of \( M \) optical fibers connected in parallel is described by the following expression:

\[
T(\omega) = \sum_{j=1}^{M} \eta_j \exp\left( -ik_{\text{opt}}(\omega)l_j - \alpha_{\text{opt}}(\omega)l_j \right).
\]
where $k_j(\omega)$ is the wavenumber of the carrier optical signal propagating in the optical fiber having a number $j$, $\alpha_j(\omega)$ is a spatial damping decrement in the fiber.

Dispersion equation for optical waves in the fiber [11]:

$$
\left[ \frac{\varepsilon_1 \alpha_j^2 J_j^{(1)}(\kappa_j a) + i \gamma_j a}{\varepsilon_2 \kappa_j J_j^{(1)}(\kappa_j a)} + i \gamma_j a \right] + \left[ \frac{\alpha_j^2 J_j^{(1)}(\kappa_j a) + i \gamma_j a}{\kappa_j J_j^{(1)}(\kappa_j a)} + i \gamma_j a \right] = \left[ \frac{\varepsilon_1}{\varepsilon_2 - 1} \frac{1}{\kappa_j^2} \right]^2 ,
$$

(4)

where $\varepsilon_1$ and $\varepsilon_2$ are dielectric permittivities of the fiber core and cladding, respectively; $a$ is a core radius; $J_j^{(1)}(\kappa_j a)$ is the Bessel function of the 1-st kind in the core; $H_j^{(1)}(\kappa_j a)$ is the derivative of the Bessel function of the 1-st kind; $H_j^{(1)}(i \gamma_j a)$ is the Hankel function of the 1-st kind in the cladding;

$H_j^{(1)}(i \gamma_j a)$ is the derivative of the Hankel function of the 1-st kind; $\nu$ is the order of Bessel and Hankel functions; $k_\perp^2 = \omega^2 \mu_0 \varepsilon_0 \varepsilon_2$; $\gamma_j^2 = \omega^2 \mu_0 \varepsilon_0 \varepsilon_1 - k_\perp^2$; $\gamma_j^2 = k_\perp^2 - \omega^2 \mu_0 \varepsilon_0 \varepsilon_2$; $\mu_0 = 4\pi \cdot 10^{-7} \text{ H} \cdot \text{m}^{-1}$ is a magnetic permeability and $\varepsilon_0 = 8.85 \cdot 10^{-12} \text{ F} \cdot \text{m}^{-1}$ is an electric constant.

Propagation of the hybrid spin-electromagnetic wave through the multiferroic delay line utilizing ferrite-ferroelectric layered structures is described by the term $\exp[-i(k_{SEW}(\omega) - i\alpha_{SEW}(\omega))l_{SEW}]$, where $k_{SEW}(\omega)$ is a wavenumber of spin-electromagnetic wave, $\alpha_{SEW}(\omega)$ is a damping decrement, $l_{SEW}$ is a length of a multiferroic delay line. For the quasi-surface spin-electromagnetic wave in a two-layered ferrite-ferroelectric structure the wavenumber $k_{SEW}(\omega)$ is calculated from an appropriate dispersion equation [12]

$$
\frac{\tan(\kappa_d a_d)}{\kappa_d} \left( (\kappa_0^2 - \kappa_\perp^2) \mu_\perp - P(\omega, k) \right) + \left( 2 \frac{\mu_\perp^2 - \mu_\perp^2}{\mu^2} \right) \tan(\kappa_f a_f) + 2 \kappa_0 \mu_\perp = 0 ,
$$

(5)

where $a_d$ and $a_f$ are thicknesses of ferroelectric and ferrite layers, respectively;

$\kappa_d = \sqrt{\omega^2 \mu_0 \varepsilon_0 a_d - k_{SEW}^2}$, $\kappa_0 = \sqrt{k_{SEW}^2 - \omega^2 \mu_0 \varepsilon_0}$, $\kappa_f = \sqrt{\omega^2 \mu_0 \varepsilon_0 f \mu_\perp^2 - k_{SEW}^2}$, $\varepsilon_d$ and $\varepsilon_f$ are dielectric permittivities of ferroelectric and ferrite layers, respectively;

$$
P(\omega, k) = \tan(\kappa_f a_f) \left[ \left( \frac{k_{SEW}^2 \mu_f}{\mu_f} + \kappa_0 \mu_\perp \left( \frac{\kappa_0 k_{SEW}^2 \mu_a}{\kappa_f \mu_f} + \kappa_0 \mu_\perp \right) \right) \right] + \kappa_0 \mu_f ,
$$

$$
\mu_a = \frac{\omega_0 \omega_M}{\omega_H^2 - \omega^2}, \quad \mu_\perp = \frac{\mu^2 - \mu_\perp^2}{\mu}, \quad \omega_H = |\gamma| H, \quad \omega_M = 4\pi |\gamma| M_0, \quad |\gamma| = 2.8 \text{ MHz/Oe} \text{ is the gyromagnetic ratio for electrons}, \ H \text{ is an external magnetic field}, \ M_0 \text{ is a saturation magnetization}. \text{ The spatial damping decrement for the quasi-surface spin-electromagnetic wave is calculated as [13]}

$$
\alpha_{SEW}(\omega) = \frac{\partial k_{SEW}(\omega)}{\partial H} \Delta H + \frac{\partial k_{SEW}(\omega)}{\partial \varepsilon} \frac{\varepsilon_d}{\varepsilon} \tan \delta,
$$

(6)

where $\Delta H$ is a half of the ferromagnetic resonance line-width and $\tan \delta$ is a dielectric loss tangent.

The microwave signal passed the multiferroic delay line is fed to the microwave amplifier used to compensate the propagation loss in the elements of the scheme. The signal transmission through the microwave amplifier is described by a factor $\exp(g(\omega))$ assuming that the amplifier does not introduce
an additional phase shift. Note, that transfer functions of electro-optical modulator and photodetector could be taken into account with using of the small signal approximation as described in [2].

\[ A_{out}(\omega) = A_0 \left[ \sum_{n=1}^{\infty} \left( T(\omega) \exp \left( g(\omega) - \alpha_{SEW}(\omega) l_{SEW} - ik_{SEW}(\omega) l_{SEW} \right) \right)^n \right], \]  

(7)

where \( T(\omega) \) is the transmission coefficient of parallel connection described by expression (3).

Using the mathematical approach described above, the complex transmission coefficient (1) can be represented as:

\[ H(\omega) = \sqrt{H_P(\omega)} \exp(i\varphi(\omega)), \]  

(8)

where \( H_P(\omega) \) is the power transfer function and \( \varphi(\omega) \) is the phase shift between the input and output signals.

The numerical analysis of the single-, dual-, and triple-loop OEO resonant spectra were carried out. In order to do it, we calculate the transmission characteristics in the cases of equal optical power division, i.e. \( \eta = 0.5 \) for the dual-loop, and \( \eta = 1/3 \) for the triple-loop OEO. For the calculations, we utilized the typical parameters for the single-mode optical fiber: \( \varepsilon_1 = 2.1543, \varepsilon_2 = 2.0875, a = 4.1 \) μm, and \( \alpha_f(\omega) = 0.2 \) dB/km; and for the ferrite-ferroelectric structure composed of yttrium iron garnet film and barium strontium titanate (BST) ceramic layer: \( M_0 = 1750 \) G, \( H = 1500 \) Oe, \( \Delta H = 0.5 \) Oe, \( \varepsilon_f = 14, a_f = 5.2 \) μm, \( a_d = 500 \) μm, \( \varepsilon_d = 1500, \tan \delta = 0.01, \) and \( l_{SEW} = 3 \) mm.

The results of the numerical calculations of the power transmission coefficient \( H_P \) for the single-, dual- and triple-loop OEOs are shown in Fig. 2. The simulations were carried out for the ring gain just below the self-generation threshold. The characteristics of the single-loop OEO with fiber length of 1000 m, 500 m, and 250 m are shown in Fig. 2 (a), (b), and (c), respectively. The characteristic of the dual-loop OEO based on the two fibers with lengths of 500 m and 1 km is shown in Fig. 2 (d). The characteristic of the triple-loop OEO contained the three fibers with lengths of 250 m, 500 m, and 1000 m and the same OEO with multiferroic delay line is shown in Fig. 2 (e).

![Figure 2. Transmission characteristics: of the single-loop OEO with fiber length of 1000 m (a), 500 m (b), and 250 m (c); of the dual-loop OEO with fiber lengths of 500 m and 1 km (d); the triple-loop OEO with fiber lengths of 250 m, 500 m, and 1 km by solid curves and the same triple-loop OEO with multiferroic delay line by dashed curves (e)
One can see that the transmission characteristic of single-loop OEO consists of series of resonant peaks. The increase of fiber length from 250 m to 1000 m provides rising the Q-factor from $9 \cdot 10^5$ up to $7.9 \cdot 10^5$, but simultaneous decrease in the FSR from 800 kHz down to 200 kHz (see Fig. 2 (a), (b), and (c)). Transmission characteristic of dual-loop configuration consists of the series of stopbands and passbands (see Fig. 2 (d)). The passbands correspond to the resonant frequencies that are resulted by constructive interference of the optical signals in the combiner. Positions of the stopbands are defined by destructive interference of the optical signals. It gives possibility to suppress odd spurious harmonics in the resonant spectrum, therefore the FSR of dual-loop configuration is equal to the FSR of single-loop with fiber length of 500 m (400 kHz). The triple-loop circuity provides increase of the FSR up to 800 kHz (see solid line in Fig. 2 (e)). Therefore, increasing of short loop number allows to suppress the spurious at higher offsets from the oscillation frequency, otherwise, to achieve higher FSR. Number of short loops is limited by decreasing of the total Q-factor. The Q-factor of multi-loop configuration is described by following expression: $Q_\Sigma = \sum_{j=1}^{M} \eta_j Q_j$. It is seen form the Fig. 2 (e) that Q-factor of the triple-loop configuration (solid line) $Q = 5.2 \cdot 10^5$ less than Q-factor of single-loop $Q = 7.9 \cdot 10^5$ (see Fig. 2 (a)). Multiferroic delay line implements frequency selection as well as introduces an additional time delay (see dash line in Fig. 2 (e)).

The magnetic and electric tuning of oscillation frequency of the triple-loop OEO with fiber lengths of 250 m, 500 m, and 1000 m connected with the multiferroic delay line are shown in Fig. 3 (a) and (b), respectively.

The electric tunability of the multiferroic delay line is realized due to the dependence of dielectric permittivity of the ferroelectric layer on external electric field. In particular, for the BST layer in paraelectric state the dependence of the dielectric permittivity on the electric field can be approximated from the experimental characteristic as

$$
\varepsilon_d(E) = \frac{\varepsilon_d(0) - \varepsilon_d(E_{\text{max}})}{1 + E^2 / E_0^2} + \varepsilon_d(E_{\text{max}}),
$$

where $\varepsilon_d(0)$ and $\varepsilon_d(E_{\text{max}})$ are the dielectric permittivities in the absence of electric field and in case of maximum electric field, respectively, and $E_0$ is an electric field on the half of experimental characteristic of the dielectric permittivity versus electric field. The magnetic tuning is provided by a dependence of magnetic permeability of ferrites on the bias magnetic field. Therefore, the resulting spectrum of spin-electromagnetic waves is dually controllable by the both electric and magnetic fields.
As is seen in Fig. 3 (a), an increase of the magnetic field shifts the oscillation frequency towards higher frequencies in accordance with the shift of the wave spectrum in the multiferroic delay line. The range of magnetic tuning is typical for spin-wave device based on yttrium iron garnet film and belongs to the range from 3 GHz up to 9 GHz for the magnetic field variation in the range from 500 Oe up to 2500 Oe. One can see, that magnetic tuning has step-like behavior. The value of steps equal to the distance between the resonant frequencies. For the triple-loop configuration described above it is equal to 800 kHz. An increase of the electric field shifts the wave spectrum in the multiferroic delay line towards low wavenumbers that gives possibility to smooth tuning the oscillation frequency in the free spectral range (see Fig. 3 (b)).

In conclusion, the theory has been developed for electronically tunable multi-loop optoelectronic oscillators that are based on optical and microwave multiferroic delay lines. An expression for transfer function of the OEO circuitry consisting of an arbitrary number of optical delay lines in optical path and a dispersive multiferroic delay line in the microwave path was derived. Numerical simulations of the resonant frequency spectra showing its transformation with increasing a number of optical loops were carried out. The theory enables one to investigate transmission characteristics and suppression of spurious frequency harmonics of the multi-loop OEOs. It was shown that oscillation frequency of multi-loop OEO with multiferroic delay line are dual tunable by both electric and magnetic fields.

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References

[1] Maleki L. Optoelectronic oscillators for microwave and mm-wave generation 2017 18th International Radar Symposium (IRS) IEEE 1-5
[2] Yao X S and Maleki L Optoelectronic microwave oscillator 1996 J. Opt. Soc. Am. B 13 1725-1735
[3] Belkin M E, Loparev A V, Semenova Y et al Tunable RF-band optoelectronic oscillator and optoelectronic computer-added design model for its simulation 2011 Microwave and Optical Technology Letters 53 2474-2477
[4] Yao X S and Maleki L Multiloop optoelectronic oscillator 2000 IEEE Journal of Quantum Electronics 36 79-84
[5] Bánky T, Horváth B and Berceli T Optimum configuration of multiloop optoelectronic oscillators 2006 J. Opt. Soc. Am. B 23 1371-1380
[6] Hong J, Zhanga S, Yoa S et al Comparison of both type injection locked and parallel dual-loop OEO 2015 Opt.-Int. J. for Light and Elect. Opt. 126 4410-4413
[7] Eliyahu D and Maleki L Low phase noise and spurious level in multi-loop opto-electronic oscillators 2003 Proc. of the 2003 IEEE Int. Frequency Control Symp. and PDA Exhibition Jointly with the 17th European Frequency and Time Forum 405-410
[8] Cen Q, Dai Y, Yin F et al Rapidly and continuously frequency-scanning opto-electronic oscillator 2017 Optics Express 25 635-641
[9] Ustinov A B, Nikitin A A and Kalinikos B A Magnetically Tunable Microwave Spin-Wave Photonic Oscillator 2015 IEEE Magnetics Letters 6 1-4
[10] Ustinov A B, Srinivasan G and Kalinikos B A Ferrite-ferroelectric hybrid wave phase shifters Applied physics letters 2007 90 No 3 031913
[11] Agrawal G P Nonlinear fiber optics 2007 Academic press
[12] Nikitin A A, Ustinov A B, Vitko V V et al Spin-electromagnetic waves in planar multiferroic multilayers 2017 Journal of Applied Physics 122 014102
[13] Ustinov A B, Kalinikos B A, Tiberkevich V S et al Q factor of dual-tunable microwave resonators based on yttrium iron garnet and barium strontium titanate layered structures 2008 Journal of Applied Physics 103 No 6 063908