SUPERNOVA NEUTRINO OPACITIES

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In a supernova (SN) core during infall and after collapse, neutrinos are trapped by their interactions with nucleons. For the axial-vector current interactions it is not enough to include static particle-particle correlations. Even the single-nucleon spin autocorrelation reduces the scattering cross section dramatically. Therefore, the dynamical structure of the spin-density structure function has an important impact on neutrino transport in supernovae. At the present time, SN neutrino opacities cannot be calculated to a controlled degree of precision.

1 Introduction

The collapsed cores of supernovae provide the only known environments which are so dense and hot that neutrinos are trapped. The transport of energy and lepton number is governed by the neutrino transport coefficients, i.e. by their effective scattering and absorption cross sections with the medium constituents. While electrons cannot be entirely ignored, it is the interaction with nucleons which is thought to dominate the neutrino opacities.

The scattering cross sections can be dramatically modified by correlation effects. In the context of SN physics, the best-known example is the coherent enhancement of the neutral-current scattering cross section off large nuclei which is instrumental for the lepton-number trapping in a SN core during the infall phase. Recently it has been recognized that this enhancement is partly undone by the nucleus-nucleus anticorrelations caused by their Coulomb repulsion.

After collapse when the shock wave has dissociated the remaining layer of large nuclei, the neutrino opacities are governed by single protons and neutrons. In the nonrelativistic limit the neutral-current scattering cross section is $(G_F^2/4\pi) (C_V^2 + 3C_A^2) E_\nu^2$ where $C_V$ and $C_A$ are the axial-vector current couplings; $C_V^2 = 1$ for neutrons and $C_V^2 \approx 0$ for protons. Further, $C_A^2 \approx 1$ for both protons and neutrons, but the exact values in a nuclear medium are not known. What is relevant for neutrino transport is actually an angular average weighted with $(1 - \cos \theta)$, leading to the so-called transport cross section $(G_F^2/3\pi) (C_V^2 + 5C_A^2) E_\nu^2$. Either way, neutrino scattering is now dominated by the axial-vector current interactions because in this medium of single nucleons the coherent enhancement of the vector-current cross section no longer operates.
Nucleons interact by a spin-dependent force so that one expects spin-spin correlations. In nuclei, the nucleon spins tend to pair off, leading to a coherent reduction of the axial-vector current cross section, in contrast with the coherent enhancement of the vector current part. For the hot nuclear matter of a nascent neutron star, the static spin-spin correlations have been studied once, indicating a significant pairing effect and thus a suppression of the effective scattering cross section.

More recently, it has been recognized that there is also a very significant suppression effect from the dynamical structure of the spin correlation function. The basic idea is that the target spin evolves “during” the collision with a neutrino. As a result, the neutrino “sees” a reduced average spin and thus scatters less efficiently. This reduction effect has been the subject of a series of papers involving the present author and has also been studied by other groups. In the nonrelativistic limit, there is no corresponding vector-current effect because the zeroth component of the vector current (the charge density) is conserved. A charge which is being kicked around cannot “cancel itself”, in contrast with a spin when it flips back and forth due to collisions.

2 The Spin-Density Structure Function

Sawyer has shown that the cross-section reduction by nucleon spin fluctuations can be derived perturbatively by calculating the $\nu N$ scattering cross section in the presence of a spin-dependent external potential for the nucleons $N$. However, a deeper understanding is achieved in the language of linear-response theory.

I assume that only a single species of nucleons is involved. The neutral axial-vector current neutrino interaction is based on the Hamiltonian density $(C_{AV}G_F/2\sqrt{2})\bar{\psi}_N\gamma_\mu\gamma_5\psi_N\bar{\psi}_\nu\gamma^\mu(1-\gamma_5)\psi_\nu$. If I further focus on an isotropic, nonrelativistic, nondegenerate medium of baryon density $n_B$ and temperature $T$, the axial-current transition rate for a neutrino of four momentum $(E_1, k_1)$ to $(E_2, k_2)$ is found to be $(C_{AV}^2G_F^2/4) n_B (3-\cos \theta) S_\sigma(\omega, k)$ with $\theta$ the scattering angle and $(\omega, k) = (E_1 - E_2, k_1 - k_2)$ the energy-momentum transfer. Here, the dynamical spin-density structure function is defined by

$$S_\sigma(\omega, k) = \frac{4}{3n_B} \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \sigma(t, k) \cdot \sigma(0, -k) \rangle,$$

where $\sigma(t, k)$ is the spatial Fourier transform at time $t$ of the nucleon spin-density operator. The expectation value $\langle \ldots \rangle$ is taken over a thermal ensemble so that detailed balance $S_\sigma(\omega, k) = S_\sigma(-\omega, -k) e^{\omega/T}$ is satisfied.

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If one neglects the momentum transfer from neutrinos to nonrelativistic nucleons (long-wavelength approximation), only $S_\sigma(\omega) = \lim_{k \to 0} S_\sigma(\omega,k)$ is used. After an angular integration the axial-current scattering cross section is

$$
\frac{d\sigma_A}{dE_2} = \frac{3C_A^2G_F^2}{4\pi} \frac{E_2^2 S_\sigma(E_1 - E_2)}{2\pi},
$$

(2)

where $S_\sigma(\omega) = \frac{4}{3n_B} \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \sigma(t) \cdot \sigma(0) \rangle$ with $\sigma(t) \equiv \sigma(t,0)$ is the total spin operator for the complete ensemble of nucleons. In a noninteracting medium the nucleon spins do not evolve, and so $\sigma(t) = \sigma(0)$. Then the time integration yields $S_\sigma(\omega) = 2\pi \delta(\omega)$. In Eq. (2) one thus recovers the standard expression for the scattering cross section where recoil effects have been ignored.

Nucleon-nucleon collisions with a spin-dependent force cause a nontrivial evolution of $\sigma(t)$. Still, in $\int_{-\infty}^{+\infty} d\omega S_\sigma(\omega)$ the $e^{i\omega t}$ factor gives one $2\pi \delta(t)$. Therefore, one finds that the “sum” $\int_{-\infty}^{+\infty} d\omega S_\sigma(\omega)/2\pi = (4/3n_B) \langle \sigma(0) \cdot \sigma(0) \rangle$ is independent of the time evolution of $\sigma(t)$. In the present discussion I ignore NN correlations so that one finds the simple sum rule $\int_{-\infty}^{+\infty} d\omega S_\sigma(\omega) = 2\pi$.

In an interacting medium the nucleon spins “forget” their initial orientation after a timescale $\Gamma^{-1}_\sigma$ where $\Gamma_\sigma$ is the spin-fluctuation rate due to collisions. It roughly represents the width of the Fourier-transformed nucleon spin autocorrelation function $S_\sigma(\omega)$. For $\omega \gtrsim \Gamma_\sigma$ one can calculate $S_\sigma(\omega)$ perturbatively. It is easiest to compute $d\sigma_A/dE_2$ directly for the $\nu NN \to N\nu N$ process according to the usual Feynman rules, including a $NN$ interaction potential. One can then extract $S_\sigma(\omega)$ by removing coupling constants and phase-space factors according to Eq. (2). This calculation is closely related to that of the bremsstrahlung emission of $\nu\nu$ pairs in $NN$ collisions. Generically, the result is of the form $S_\sigma^{\text{brems}}(\omega) = (\Gamma_\sigma/\omega^2) s(\omega/T) b(\omega/T)$ where $b(x) = 1$ for $x > 0$ and $e^x$ for $x < 0$ ensures the detailed-balance condition. The function $s(x)$ is even, slowly varying, and normalized according to $s(0) = 1$. For large energy transfers it represents details about the assumed $NN$ interaction potential.

A perturbative calculation cannot reveal $S_\sigma(\omega)$ in the $\omega \lesssim \Gamma_\sigma$ regime where multiple-scattering effects dominate. Motivated by the classical analogy of a spin vector kicked around by a random force one may use the Lorentzian ansatz

$$
S_\sigma(\omega) = \frac{\Gamma_\sigma}{\omega^2 + \Gamma^2/4} s(\omega/T) b(\omega/T),
$$

(3)

where $\Gamma$ is chosen such that the sum-rule is fulfilled.
3 Cross-Section Reduction

One may then proceed to calculate the total neutrino scattering cross section, averaged over a thermal distribution of neutrino energies. For noninteracting nucleons with $S_{\sigma}(\omega) = 2\pi \delta(\omega)$ it is found to be $\sigma_T = 9G_A^2 G_F^2 T^2/\pi$. With the ansatz Eq. (3) one finds that the cross section varies with $\Gamma_\sigma$ as shown in Fig. 1 where $s(x) = 1$ has been chosen.

The dashed line is the tangent to the solid line at the point $\Gamma_\sigma = 0$. It actually does not depend on the Lorentzian ansatz for the low-$\omega$ behavior. Any modification of $S_{\sigma}^{\text{brems}}(\omega)$ which is chosen such that the sum rule is obeyed yields the same result for the dashed line. It can also be derived directly with perturbative methods.

Except in the dilute-medium limit ($\Gamma_\sigma \ll T$) it is not obvious how to compute $\Gamma_\sigma$ as a function of the medium’s temperature and density. A naive extension of a perturbative calculation based on a one-pion exchange potential between the nucleons indicates that in a SN core $\Gamma_\sigma \gg T$ which would lead to a vast suppression of the neutrino opacities. Clearly, neither the naive nor the perturbatively suppressed cross sections shown in Fig. 1 are adequate proxies for the true neutrino scattering cross section in a SN core.

One would expect that the true nucleon spin-fluctuation rate is smaller than indicated by a perturbative calculation. Sigl3 has derived an $f$-sum rule for $S_{\sigma}(\omega)$ which indicates that $\Gamma_\sigma$ never exceeds a few $T$. The same conclusion is empirically reached on the basis of the SN 1987A neutrino signal which should have been much shorter than observed if the neutrino opacities had been far below their “standard” (i.e. naive) values.
4 Conclusion

In a SN core, nucleon spin autocorrelations lead to a significant reduction of the axial-vector neutrino scattering cross section. However, the exact magnitude of this effect cannot be calculated, at the present time, on the basis of first principles. In addition, spin-spin correlations must be included which complicate the problem even further. Of course, in the spirit of a dimensional analysis the overall magnitude of the neutrino opacities in a SN core are correctly estimated by the naive values which ignore all correlation effects. However, the true opacities appear to be reduced by a factor of order unity which is not calculable at the present time. The duration of the SN 1987A signal seems to indicate that the opacity suppression cannot have been too severe, but this conclusion rests on the very small number of late events in the SN signal. Significant theoretical efforts are needed to perform a meaningful calculation of the neutrino opacities in a nuclear medium. Of course, a statistically more significant neutrino signal from a future galactic SN would go a long way at resolving these problems empirically!

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