Quest for the Dynamical Origin of Mass *)

An LHC perspective from Sakata, Nambu and Maskawa

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I review the dynamical symmetry breaking (DSB) approach to the Origin of Mass, which is traced back to the original (2008 Nobel prize) work of Nambu based on the BCS analogue of superconductor where mass of nucleon (then elementary particle) arises due to Cooper pairing and pions are provided as massless Nambu-Goldstone (NG) bosons, being composite as in Fermi-Yang/Sakata model. In this talk I will focus on the modern version of DSB or composite Higgs models: Walking/Conformal Technicolor, Hidden Local Symmetry (HLS) or Moose, and Top Quark Condensate, with the their extra dimension versions closely related with HLS. Particular emphasis will be placed on the large anomalous dimension and conformal symmetry at the conformal fixed points, developed along the line of the pioneering work of Maskawa and Nakajima. Due to (approximate) conformal symmetry these models do have composite Higgs particle (“Techni-dilaton”, “Top-sigma” etc.). Weakly coupled composite gauge boson is realized at “Vector Manifestation” formulated at conformal fixed point, which may be applied to the composite W/Z boson models. They will be tested in the upcoming LHC experiments.

§1. Introduction

The most urgent problem of the modern particle theory is to reveal the Origin of Mass. In the Standard Model (SM) the spontaneous symmetry breaking (SSB) of the electroweak symmetry is attributed to a single parameter, $v = 246$ GeV, the vacuum expectation value (VEV) of the field of a hypothetical elementary particle, the Higgs boson, which then is distributed via gauge and Yukawa couplings $g_i$ to all the particles having mass $m_i$: $m_i \sim g_i v$. Yet the nature of Higgs boson remains mysterious. In order to develop the VEV the mass squared of the Higgs in the Lagrangian should be tuned negative (tachyon!) on the weak scale in an ad hoc manner. Particle theorists looking desperately beyond the SM have been fighting on this central problem over 30 years without decisive experimental information. Now we are facing a new era that LHC experiments will tell us which theory is right while others are not.

It should be recalled that the very concept of SSB was created by the 2008 Nobel prize work of Nambu[1,2] in the concrete form of DSB where the nucleon mass was dynamically generated via Cooper pairing of (then elementary) nucleon and anti-nucleon, “nucleon condensate”, based on the Bardeen-Cooper-Schrieffer (BCS) analogue of superconductor: Accordingly, there appeared pions as massless Nambu-Goldstone (NG) bosons which were dynamically generated to be nucleon composites in the same sense as in the Fermi-Yang/Sakata model[3]. **Thus the SSB was born**
as DSB! Before advent of the concept of SSB, low energy hadron physics was well described by the effective theory of Gell-Mann-Levy (GL) linear sigma model with an elusive scalar boson, the sigma meson, which was unjustifiably assumed to have negative mass squared. Actually, the GL linear sigma model Lagrangian is a model equivalent to the SM Higgs Lagrangian.

The real physical meaning of this mysterious tachyonic mode was actually revealed by Nambu as the BCS instability where attractive forces between nucleon and anti-nucleon give rise to the nucleon Cooper paring (tachyonic bound state) which changes the vacuum from the original (free) one into the true one having no manifest symmetry. The Nambu’s theory for the nucleon mass was later developed into DSB in the underlying microscopic theory QCD where the gluonic attractive forces again generate the Cooper paring of quark and antiquark (instead of nucleon and anti-nucleon), quark condensate \( \langle \bar{q}q \rangle \), which then gives rise to the BCS instability and the dynamical mass of quarks: Pions are now quark composites. Hence Nambu’s idea was established in a deeper level of matter.

This DSB in QCD is the prototype of the Technicolor (TC)\(^4\). Just in the same way as the GL sigma model was for QCD, the SM Higgs Lagrangian may be regarded as an effective theory for the hypothetical underlying gauge theory like QCD, “Technicolor”: Higgs boson may be regarded as the composite particle like the sigma meson (“techni-sigma”). In much the same way as the sigma meson condensate \( \langle \sigma \rangle = f_\pi (= 93\text{MeV}) \sim A_{\text{QCD}} \) was an effective description of the quark condensate \( \langle q\bar{q} \rangle \sim A_{\text{QCD}}^3 \) in QCD, the Higgs condensate in SM \( \langle H \rangle = F_\pi (= 246\text{GeV}) \sim A_{\text{TC}} \) would be replaced by the “techni-fermion” condensate \( \langle \bar{F}F \rangle \sim A_{\text{TC}}^3 \) in TC, where \( A_{\text{QCD}} \) and \( A_{\text{TC}} \) are intrinsic scales of the respective theories, with roughly a scale up of \( A_{\text{TC}} \sim (F_\pi / f_\pi) A_{\text{QCD}} \sim 2600 \cdot (250\text{MeV}) \sim 700\text{GeV} \).

In order to accommodate mass of quarks/leptons \( m_q/l \), we should further introduce interactions between the technifermion and the quarks/leptons. This is most typically done by Extended Technicolor (ETC)\(^5\), which yields mass of quarks/leptons \( m_q/l \sim \frac{1}{A_{\text{ETC}}} \langle \bar{F}F \rangle_{A_{\text{ETC}}} \), where \( \langle \bar{F}F \rangle_{A_{\text{ETC}}} \) is the condensate evaluated at the scale of ETC \( A_{\text{ETC}} \gg A_{\text{TC}} \), which would be \( \langle \bar{F}F \rangle_{A_{\text{ETC}}} \sim \langle \bar{F}F \rangle_{A_{\text{TC}}} \sim A_{\text{TC}}^3 \) if the TC is a simple scale-up of QCD. Then we would have \( m_q/l \sim A_{\text{TC}}^3 / A_{\text{ETC}}^2 \ll (700)^3 / (10^6)^2 \text{MeV} \sim 0.3\text{MeV} \), if we impose a constraint \( A_{\text{ETC}} > 10^6\text{GeV} \) in order to avoid the excessive Flavor-Changing-Neutral-Currents (FCNC). Then the typical mass (s-quark mass) would be roughly \( 10^{-3} \) smaller than the reality. To avoid this problem, Holdom\(^7\) simply assumed that the TC has an ultraviolet fixed point and the anomalous dimension becomes larger than unity \( \gamma > 1 \) in the ultraviolet region so that the technifermion condensate at ETC scale is enhanced \( \langle \bar{F}F \rangle_{A_{\text{ETC}}} = Z_{m}^{-1} \langle \bar{F}F \rangle_{A_{\text{TC}}} \), where \( Z_{m}^{-1} = (A_{\text{ETC}} / A_{\text{TC}})^{\gamma m} \), more than \( 10^3 \) times the simple scale-up of QCD which has a vanishingly small anomalous dimension \( \gamma_m \approx 0 \).

The Holdom’ mechanism unfortunately had no support by concrete dynamical arguments and no prediction for the value of the anomalous dimension. It is my paper with M. Bando and K. Matumoto\(^8\) (receipt date on December 24, 1985) that

\footnote{\textsuperscript{4}) The same can be done in a composite model where quarks/leptons and technifermions are composites on the same footing.\textsuperscript{5)}
did demonstrate existence of such a theory having a concrete value of the large anomalous dimension \( \gamma_m = 1 \) as desired, based on the Spontaneous Chiral Symmetry Breaking (S\( \chi \)SB) solution of the ladder Schwinger-Dyson (SD) equation for fermion full propagator \( S_F(p) \) parameterized as \( iS_F^{-1} = A(p^2)\hat{p} - B(p^2) \) with non-running (ideal limit of the “walking”) gauge coupling, \( \alpha(Q) \simeq \alpha = \text{constant} \). (See Fig. 1) In 1986 similar enhancement effects of the condensate were also studied within the same ladder SD equation, without use of the renormalization-group equation (RGE) concepts of anomalous dimension and fixed point, rather emphasizing the asymptotic freedom of the TC theories with walking coupling.

The ladder SD equation with non-running coupling was first consistently analyzed, with an explicit cutoff, by Maskawa and Nakajima: They discovered that the SSB can only take place for strong coupling \( \alpha > \alpha_{cr} = \mathcal{O}(1) \), non-zero critical coupling. From the explicit form of the Maskawa-Nakajima S\( \chi \)SB solution of the fermion mass function \( \Sigma(Q) = B(p^2)/A(p^2) \) in Landau gauge (\( A(p^2) \equiv 1 \)), we found:

\[
\Sigma(Q) \sim 1/Q \quad (Q \equiv \sqrt{-p^2} \gg \Lambda_{TC}) \quad \text{at } \alpha \to \alpha_{cr}, \tag{1.1}
\]

and

\[
\gamma_m = 1, \quad m_{q/t} \sim A_{TC}^2/A_{ETC}, \tag{1.2}
\]

in comparison with the operator product expansion \( \Sigma(Q) \sim 1/Q^2 \cdot (Q/A_{TC})^{\gamma_m} \) and \( \langle \bar{F}F \rangle_{A_{ETC}} = -\text{Tr}S_F(p) \sim A_{ETC}A_{TC}^2 \) or \( Z_{m}^{-1} = (A_{ETC}/A_{TC})^1 \), where the critical coupling \( \alpha_{cr} \) was identified with a nontrivial UV stable fixed point of the RGE a la Miransky \( \alpha = \alpha(A) \to \alpha_{cr} \) as \( A \to \infty \), to keep finite the solution \( \Sigma(0) \)

\[
\Sigma(0) \sim A \exp (-\pi/\sqrt{\alpha/\alpha_{cr} - 1}), \tag{1.3}
\]

which is often called “Miransky scaling” with an essential singularity at \( \alpha = \alpha_{cr} \).

Since the ladder SD equation is scale-invariant (except for the explicit cutoff), with the critical coupling identified as the conformal fixed point, we called the theory “Scale-invariant Technicolor” (would be “Conformal TC” in a currently fashionable language) and predicted a “Techni-dilaton”, a relatively light Higgs-like composite object due to approximate conformal symmetry.

Today the “Walking/Conformal TC” is simply characterized by near conformal property with \( \gamma_m \simeq 1 \) (For a review see Ref.14). Such a theory should have an almost non-running and strong gauge coupling (larger than a certain non-zero critical

\( ^{\ast} \) Earlier work in the ladder SD equation with non-running coupling all confused explicit breaking solution with the SSB solution and thus implied \( \alpha_{cr} = 0 \).
coupling for $S\chi$SB) to be realized either at UV fixed point or IR fixed point, or both ("fusion" of the IR and UV fixed points), as was characterized by "Conformal Phase Transition (CPT)". In contrast to the simple QCD scale-up which is widely believed to have no composite Higgs particle ("higgsless"), a salient feature of the walking/conformal TC is the prediction of the composite Higgs as a techni-dilaton.

In this talk I shall describe the DSB with Large Anomalous Dimension (see Ref. for basis and classics before 1996), namely a class of composite models based on the walking/conformal gauge theories having large anomalous dimension characteristic to the conformal UV/IR fixed point.

- Walking/Conformal TC
  Modern version realized through the Banks-Zaks (BZ) IR fixed point of the "Large $N_f^\ast$ QCD". Several issues:
  - Phase Structure, or CPT
  - Top Quark Mass in the Walking/Conformal TC with $\gamma_\ast > 1$
  - Techni-dilaton and Light Composite Spectra
  - $S$ Parameter in the Walking/Conformal TC

- Hidden Local Symmetry (HLS) and Holography in the Walking/Conformal Theories
  - "Vector Manifestation" at CPT
  - Realization of a Weakly Coupled Composite Gauge Boson
  - Holographic Walking/Conformal TC as an Extension of HLS

- Top Quark Condensate (Top Mode Standard Model, TMSM)
  - TMSM in Higher Dimensions at UV fixed point
  - Top Mode Walking/Conformal TC

All these models are based on the chiral phase transition (dynamical symmetry breaking) near the conformal region ("conformal window") associated with the UV/IR fixed point, which inevitably develops large anomalous dimension due to strong coupling. I expect that they will be tested in the upcoming LHC experiments.

§2. Walking/Conformal Technicolor

2.1. Large $N_f$ QCD as a walking/conformal TC

Modern version of the walking/conformal TC is based on the BZ IR fixed point in the "large $N_f$ QCD" which is the QCD with many flavors $N_f (\gg 3)$ of massless "quarks" (fundamental color representation): The two-loop beta function is $\frac{d}{d\mu}\alpha(\mu) = -\frac{b_0}{\alpha} \alpha^2(\mu) - c \alpha^3(\mu)$, where $b = (11N_c - 2N_f)/(6\pi)$, $c = [34N_c^2 - 10N_f N_c - 3N_f(N_c^2 - 1)/N_c] / (24\pi^2)$. When $b > 0$ and $c < 0$, i.e., $N_f^\ast <$
Note that $\alpha_s = \alpha_s(N_c, N_f) \to 0$ as $N_f \to 11N_c/2$ and hence there exists a certain range $N_f^\text{cr} < N_f < 11N_c/2$ (“Conformal Window”) satisfying $\alpha_s < \alpha_{\text{cr}}$, where the gauge coupling $\alpha(Q) (< \alpha_s)$ gets so weak that attractive forces are no longer strong enough to trigger the $S\chi$SB, namely the chiral symmetry gets restored and deconfinement takes place (non-Abelian Coulomb phase). Here $\alpha_{\text{cr}}$ may be evaluated by the ladder SD equation\textsuperscript{13} \[ \alpha_{\text{cr}} = \pi/3C_2 = (\pi/3)(2N_c/(N_c^2 - 1)), \] in which case $N_f^\text{cr}$ is evaluated by the condition $\alpha_s(N_c, N_f) = \alpha_{\text{cr}}$, yielding $N_f^\text{cr} \simeq 4N_c (= 12$ for $N_c = 3)$.\textsuperscript{12, 13} Thanks to the IR fixed point the gauge coupling is actually walking $\alpha(Q) \simeq \alpha_s$ over the range $0 < Q < \Lambda$ for the conformal window, where $\Lambda$ is a two-loop RG invariant analogue of the $\Lambda_{\text{QCD}}$.

Here we are interested in the SSB phase slightly off the conformal window, $0 < \alpha_s - \alpha_{\text{cr}} \ll 1$ ($N_f \simeq N_f^\text{cr}$) where the fermion gets a tiny mass $\Sigma(0)$ (or more properly, $m$) such that $\Sigma(m) = m$ from the SSB in the form\textsuperscript{17}

\[ m \sim \Lambda \exp \left( -\pi/\sqrt{\alpha_s/\alpha_{\text{cr}} - 1} \right) \ll \Lambda \quad (\alpha_s \simeq \alpha_{\text{cr}}), \] (2.2)

which is based on the same equation as the ladder SD equation with $\alpha(Q) \simeq \alpha_s$ and hence the same form as the Miransky scaling\textsuperscript{13} Eq. (1.3), of the Maskawa-Nakajima solution\textsuperscript{12, 13}. We also have the same result as Eqs. (1.1, 1.2):

\[ \Sigma(Q) \sim 1/Q, \quad \gamma_m = 1 \quad \text{at} \quad \alpha_s = \alpha_{\text{cr}}. \] (2.3)

Note that the exact BZ IR fixed point no longer exists in the SSB phase where the fermions acquire mass and hence get decoupled from the beta function, namely the gauge coupling quickly runs/grows up (strong asymptotically-free) for $Q < m (\ll \Lambda)$. Nevertheless, remnant of the IR fixed point, $\alpha(Q) \simeq \alpha_s$, dictates the coupling to walk for a wide region $m < Q < \Lambda$ which is the region most relevant to the physics of TC. See Fig.\textsuperscript{2}

Such a large separation between the $S\chi$SB scale and the intrinsic scale of the theory, $m \ll \Lambda$, is a salient feature of the walking/conformal gauge theory in contrast with the situation in the ordinary QCD.

\textsuperscript{1)} The value should not be taken seriously, since $\alpha_s = \alpha_{\text{cr}}$ is of $O(1)$ and the perturbative estimate of $\alpha_s$ is not so reliable there, although the chiral symmetry restoration in large $N_f$ QCD has been supported by many other arguments, most notably the lattice QCD simulations, which however suggest diverse results as to $N_f^\text{cr}$. Some recent results do $8 < N_f^\text{cr} < 12$ (Kogut-Susskind fermion)\textsuperscript{11} while other does $6 < N_f^\text{cr} < 7$ (Wilson fermion)\textsuperscript{12}.
where \( m \sim \Lambda_{QCD} \). Then the scale \( \Lambda \) (although an analogue of \( \Lambda_{QCD} \)) plays a role of cutoff in the SD equation and hence we may set the situation \( \Lambda \sim \Lambda_{ETC} \), while \( m \) is directly tied to the weak scale \( F_\pi = 246 \text{ GeV} \) and hence plays a role of \( \Lambda_{TC}(\ll \Lambda_{ETC}) \) in the previous discussions: \( m \rightarrow \Lambda_{TC} \).

2.2. Conformal Phase Transition

Such an essential singularity scaling law like Eq. (2.2) characterizes an unusual phase transition, what we called “Conformal Phase Transition (CPT)”, where the Ginzburg-Landau effective theory breaks down. Although it is a second order (continuous) phase transition where the order parameter \( m(\alpha^* > \alpha_{cr}) \) is continuously changed to \( m = 0 \) in the symmetric phase (conformal window, \( \alpha^* < \alpha_{cr} \)), the spectra do not, i.e., while there exist light composite particles whose mass vanishes at the critical point when approached from the side of the SSB phase, no isolated light particles do not exist in the conformal window, recently dubbed as “unparticle”.

The essence of CPT may be illustrated by a simpler model, 2-dimensional Gross-Neveu Model. This is the \( D \rightarrow 2 \) limit of the \( D \)-dimensional Gross-Neveu model (\( 2 < D < 4 \)) which has the beta function and the anomalous dimension:

\[
\beta(g) = -2g^2, \quad \gamma_m|_{g=0} = 0 \quad (D = 2),
\]

where \( g = g_*(\equiv D/2 - 1) = g_{cr} \) and \( g = 0 \) are respectively the UV and IR fixed points of the dimensionless four-fermion coupling, \( g \), properly normalized (as \( g_* = 1 \) for the \( D = 4 \) NJL model). There exist light composites \( \pi, \sigma \) near the UV fixed point \( g \simeq g_* \) in both sides of symmetric (\( 0 < g < g_* \)) and SSB (\( g > g_* \)) phases as in the NJL model.

Now we consider \( D \rightarrow 2 (g_* \rightarrow 0) \) where we have a well-known effective potential: \( V(\sigma, \pi) \sim (1/g - 1)\rho^2 + \rho^2 \ln(\rho^2/\Lambda^2) \), or \( \partial^2 V/\partial \rho^2|_{\rho=0} = -\infty \), where \( \rho^2 = \pi^2 + \sigma^2 \). This implies breakdown of the Ginzburg-Landau theory which distinguishes the SSB (\( \rho < 0 \)) and symmetric (\( \rho > 0 \)) phases by the signature of the finite \( \partial^2 V/\partial \rho^2 \) at the critical point \( g = 0 \). Eq. (2.4) now reads:

\[
\beta(g) = -2g^2, \quad \gamma_m|_{g=0} = 0 \quad (D = 2),
\]

namely a fusion of the UV and IR fixed points at \( g = 0 \). Now the symmetric phase is squeezed out to the region \( g < 0 \) (conformal phase) which corresponds to a repulsive four-fermion interaction and no composite states exist, while in the SSB phase (\( g > 0 \)) there exists a composite state \( \sigma \) of mass \( M = 2m \) where \( m \) is the dynamical mass of the fermion \( m^2 \sim \Lambda^2 \exp(-1/g) \rightarrow 0 \) (\( g \rightarrow +0 \)), which shows an essential singularity scaling, in accord with the beta function \( \beta(g) = M \partial g/\partial \Lambda = -2g^2 \). Note the would-be composite mass in the symmetric phase \( |M|^2 \sim \Lambda^2 \exp(-1/g) \rightarrow \infty (g \rightarrow -0) \).

Now look at the walking/conformal TC as modeled by the large \( N_f \) QCD: When the walking coupling is close to the critical coupling, \( \alpha(Q) \simeq \alpha_* = \alpha_{cr} \), we should
include the *induced* four-fermion interaction which becomes relevant operator due to the anomalous dimension $\gamma_m = 1$, and the system becomes “gauged Nambu-Jona-Lasinio” model\textsuperscript{[23]} whose solution in the full parameter space was obtained in Ref.\textsuperscript{[37]}

Thus we may regard the *Walking/Conformal TC* as the gauged Nambu-Jona-Lasinio model. Based on the solution\textsuperscript{[23]} the RGE flow in $(\alpha, g)$ space was found to be along the line of $\alpha = \alpha_s$ ($\alpha$ does not run), on which the (properly normalized dimensionless) four-fermion coupling $g$ runs, with the beta function and anomalous dimension given by \textsuperscript{[38][35][39]}

$$\beta(g) = -2(g - g_+)(g - g_-), \quad \gamma_m = 2g + \alpha_s/(2\alpha_{cr}), \quad (2.6)$$

where $g = g_\pm \equiv (1 \pm \sqrt{1 - \alpha_s/\alpha_{cr}})^2/4$ are the UV/IR fixed points (fixed lines) for $\alpha_s \leq \alpha_{cr}$. The anomalous dimension takes the value $\gamma_m = 1 \pm \sqrt{1 - \alpha_s/\alpha_{cr}}$ at the UV/IR fixed lines. Light composite spectra only exist near the UV fixed line $g \approx g_+$ in both SSB ($g > g_+$) and symmetric ($g > g_+$) phases as in NJL model.

Thus it follows that as $\alpha_s \rightarrow \alpha_{cr}$. Eq. (2.6) takes the form

$$\beta(g) = -2(g - g_+)^2, \quad \gamma_m |_{g = g_+} = 1, \quad (\alpha_s = \alpha_{cr}), \quad (2.7)$$

with $g_\pm \rightarrow 1/4 \equiv g_*$, and hence we again got a fusion of UV and IR fixed lines with the essential-singularity scaling of $m^2 \sim \Lambda^2 \exp(-1/(g - g_*))$\textsuperscript{[37]}. Again there is a composite state with $M^2 \sim n^2 \rightarrow 0$ as $g \rightarrow g_* + 0$, while there are no composites $|M|^2 \sim \Lambda^2 \exp(-1/(g - g_*)) \rightarrow \infty$ for $g \rightarrow g_* - 0$.

The absence of the composites in the symmetric phase $g < g_*$ may be understood as in the 2-dimensional Gross-Neveu model for $g < 0$, namely the *repulsive* four-fermion interactions: From the analysis of the RG flow, it was argued\textsuperscript{[33]} that the IR fixed line $g = g_{(-)}$ is due to the *induced* four-fermion interaction by the walking TC dynamics itself, while deviation from that line, $g - g_{(-)}$, is due to the *additional* four-fermion interactions, repulsive ($g < g_{(-)}$) and attractive ($g > g_{(-)}$), from UV dynamics other than the TC (i.e., ETC). It is clear that no light composites exist for repulsive four-fermion interaction $g < g_{(-)}$, which becomes $g < g_*$ at $\alpha_s = \alpha_{cr}$.

### 2.3. Top Quark Mass in the Walking/Conformal TC

Note that the anomalous dimension at the UV fixed line $g = g_+$ is even larger than unity $\gamma_m > 1$\textsuperscript{[23]} due to the additional four-fermion coupling from other than the walking/conformal gauge dynamics, i.e., ETC. When $g(g_{ETC} + g_{(-)}) > g_+$, or $g_{ETC} > g_{ETC}^g \equiv g_{(+) - g_{(-)} = \sqrt{1 - \alpha_s/\alpha_{cr}}}$, SSB takes place with large anomalous dimension $\gamma_m = 1 + \sqrt{1 - \alpha_s/\alpha_{cr}} > 1$, which would enhance the condensate more dramatically and accommodate the large top quark mass within the TC framework.

If, on the other hand, the top quark instead of techni-fermion has a strong four-fermion interaction with scale $\Lambda_t$ close to the critical line but subcritical, $g = g_{(+)} - \epsilon$, then the ETC-induced top mass $m_t^{ETC}(\ll m_t)$ will be enhanced by the anomalous dimension $\gamma_m \simeq 2$, or $Z_m^{-1} = (\Lambda/m_t)^2$ (See Eq. (2.6)). Thus we have the realistic top mass $m_t = Z_m^{-1} \cdot m_t^{ETC}$ without producing the light top-pion (See Subsection 4.2).
2.4. Confronting $S,T,U$ Parameters

Now the next problem is the so-called $S,T,U$ parameters\textsuperscript{[11]} measuring possible new physics in terms of the deviation of the LEP precision experiments from the SM. In particular, $S$ parameter excludes the TC as a simple scale-up of QCD which yields $S = (N_f/2) \cdot \hat{S}$ with $\hat{S}_{\text{QCD}} = 0.33 \pm 0.04$. For a typical ETC model with one-family TC, $N_f = 8$, we would get $S = \mathcal{O}(1)$ which is much larger than the experiments $S < 0.1$. This is the reason why many people believe that the TC is dead. However, since the simple scale-up of QCD was already ruled out by the FCNC as was discussed before, the real problem is whether or not the walking/conformal TC which solved the FCNC problem is also consistent with the $S$ parameter constraint above.

There have been many arguments\textsuperscript{[42]} that the $S$ parameter value could be reduced in the walking/conformal TC than in the simple scale-up of QCD. Here we present the most straightforward computation of the $S$ parameter for the large $N_f$ QCD (for $N_c = 3$), based on the SD equation and (inhomogeneous) BS equation in the ladder approximation.\textsuperscript{[21]} (see Fig. 3). The $S$ parameter is given by $\hat{S} = S/(2N_f) = -4\pi \left[ \frac{d}{dQ^2} (\Pi_{VV} - \Pi_{AA}) \right]_{Q^2=0}$, where $\Pi_{VV}(\Pi_{AA})$ is the vector (axialvector) current correlator, which is obtained by closing the fermion legs (solution of SD equation) of the BS amplitudes (solution of BS equation). Although the region studied was only $0.89 < \alpha_s < 1$, still somewhat away from the critical point $\alpha_s = \alpha_{cr} = \pi/4 = 0.79$, the result shows that $\hat{S}$ gradually decreases $\hat{S}/N_c \simeq 0.30/N_c$ ($\alpha_s = 1$) to $\hat{S}/N_c \simeq 0.25/N_c$ ($\alpha_s = 0.89$) as we approach the conformal window $\alpha_s \rightarrow \alpha_{cr}$ ($N_f \rightarrow N_{cr}^f$) and is definitely smaller values than that in the ordinary QCD. The reduction does not seem to be so dramatic so far, due to technical limitation for the present computation to get further close to the conformal window. It is highly desirable to extend the computation further close to the conformal window.

However, the results may imply nontrivial: The ladder SD and BS method tends to overestimate $\hat{S}$ in QCD, which could be understood as scale ambiguity of $\Lambda_{\text{QCD}}$, $\Lambda_{\text{QCD}} \approx 725\text{MeV}$ to reproduce the realistic value of QCD $\hat{S} \approx 0.33$, while $\Lambda_{\text{QCD}} \approx 500\text{MeV}$ reproducing other quantities yields $\hat{S} \approx 0.47$. Thus the reduction can read $\hat{S}_{\text{QCD}}^{\text{ladder}} = 0.47 \rightarrow \hat{S}_{\text{large}N_f}^{\text{ladder}} = 0.25$ more than $40\%$ reduction! Then the actual value near the conformal phase transition point should be properly re-scaled by a factor roughly $2/3$ to compare the QCD value within the ladder SD/BS equation: If this is done, then the value could be

$$\hat{S}^{(\text{re-scaled})}/N_{\text{TC}} \simeq 0.067(\alpha_s = 1.0) \rightarrow 0.056(\alpha_s = 0.89), \quad (2.8)$$

which could be barely consistent with the experiments if $N_{\text{TC}} = 2$. Since the walk-
ing/conformal theories are strong coupling theories and the ladder approximation would be no more than a qualitative hint, more reliable calculations are certainly needed, including the lattice simulations, before drawing a definite conclusion about the physics predictions. We will see.

2.5. Walking/Conformal Signatures in LHC?

Walking/conformal TC would predict several characteristic phenomena in TeV region to be tested in the ongoing Tevatron and the upcoming LHC. There are huge varieties of the TC models on the market, which predict rather diverse phenomena. To be definite, however, here we shall take a concrete model of ETC\textsuperscript{14} one family model of Farhi and Susskind embedded into a typical $SU(N_{TC} + 3)$ ETC which consists of $SU(N_{TC})$ TC of the $N_f = 8$ (one-family) techni-quarks/leptons and the $SU(3)$ horizontal gauge group for 3 families of quarks and leptons.

Since the weak scale is $(246\text{ GeV})^2 = (N_f/2) \times F_\pi^2$ we have $F_\pi = 123\text{ GeV}$, half of the naive scale-up of the $N_f = 2$ QCD. As already noted in the footnote, the estimate of $N_f$ in the large $N_f$ QCD has some ambiguity, $N_f^{CT} \sim [6 - 12] (N_c/3)$ in the literature, we may assume $N_f^{CT} = 2N_c - 4N_c$, i.e., the walking/conformal TC, $N_f \simeq N_f^{CT}$, for the one-family model with $N_f = 8$ would be realized for $2 \leq N_{TC} \leq 4$, or $N_{TC} = 3 \pm 1$.

Then the light spectra of the one-family TC would be as follows:

- **Pseudo NG Bosons (Techni-pions)**
  This is heavily model-dependent (Some models have no such objects). In the one-family model the SSB of the global chiral symmetry $SU(8)_L \times SU(8)_R$ for $N_f = 8$ flavors produces 63 NG bosons: 3 would-be NG bosons absorbed into W/Z bosons and 60 pseudo NG bosons acquiring mass from explicit breaking due to various gauge interactions other than the TC. Most problematic pseudo NG bosons would be colorless ones, “techni-axions”, which may have mass mostly from ETC or Pati-Salam type interaction (if any). It takes the form $m_{pNG}^2 \sim (FF)^2_{ETC}/(F_\pi^2 A_{ETC}^2) \sim (A_{ETC}/A_{TC})^{2\gamma_m} A_{TC}^2/A_{ETC}^2$ which reads $m_{pNG}^2 \sim A_{TC}^2 \sim (350\text{ GeV})^2$ for $\gamma_m = 1$\textsuperscript{5}.

- **Vector/Axialvector Mesons (Techni-\(\rho\)/Techni-\(a_1\))**
  Since in the vicinity of the conformal window in the large $N_f$ QCD, only the low energy scale is the tiny dynamical mass of fermion $m(\ll \Lambda)$, one would expect that mass of techni-\(\rho\)/techni-\(a_1\) would also be vanishingly small $m_{\text{techni-}\rho/a_1}/\Lambda \sim m/\Lambda \ll 0$ as $N_f \gg N_f^{CT}$\textsuperscript{15}. A straightforward calculation based on the SD and (homogeneous) BS equation (without the first diagram in the right-hand side of Fig. \textsuperscript{3}) in fact yields $m_{\text{techni-}\rho}/F_\pi \sim 11$, $m_{\text{techni-}a_1}/F_\pi \sim 12$, with $F_\pi/\Lambda \sim m/\Lambda \rightarrow 0$ as $N_f \gg N_f^{CT}$\textsuperscript{20}. The ratio for techni-\(\rho\) appears somewhat larger than that in the case of the \(\rho\) meson in QCD $m_\rho/f_\pi \simeq 8.5$. Hence in the one-family model with $N_{TC} = 3 \pm 1$, $F_\pi \simeq 125\text{ GeV}$, this would imply that $m_{\text{techni-}\rho/a_1} \sim (11 - 12) F_\pi \simeq 1.3 - 1.5\text{ TeV}$.

- **Techni-dilaton (Techni-sigma)**
  The original walking/conformal TC predicted a (massive) dilaton, “techni-dilaton”, as a pseudo NG boson of the spontaneous breakdown of the approximate scale invariance\textsuperscript{9} This looks like a Higgs boson in the SM. In the
vicinity of the conformal window of the large $N_f$ QCD, we also expect a massive dilaton\[^{15}\] whose mass may be estimated in the gauged NJL model as $m_{\text{technidilaton}} \simeq \sqrt{2} m_{\pi}\[^{14}\]$ which is consistent with the straightforward ladder SD/BS computation\[^{20}\] in the large $N_f$ QCD: The scalar mass sharply drops when approaching the conformal window, $m_{\text{technidilaton}} \sim 4F_{\pi} \simeq 1.5m \simeq \sqrt{2} m$ as $N_f \nearrow N_f^c$. This would imply that $m_{\text{technidilaton}} \simeq 500$ GeV in the case of the one-family TC model.

It should be emphasized that the dynamics near conformal window of large $N_f$ QCD is strong coupling and hence the value of the BZ IR fixed point and the critical spectra such as in the lattice calculations.

Before drawing a definite phenomenological conclusion, we would need more reliable coupling in the ladder approximation would be no more than a qualitative hint. Here we take the Hidden Local Symmetry (HLS) model\[^{22}\] which extends the Callan-Coleman-Wess-Zumino (CCWZ) construction of the nonlinear sigma model consisting of quantum fields for the light composite spectra as an effective field theory. In contrast to the underlying microscopic theory, we may take a different approach, namely, an effective field theory. To incorporate $\rho, a_1, \ldots$ as composite gauge bosons. Note that the HLS is the induced gauge symmetry at the composite level which does not exist in the underlying theory. There is nothing wrong with this, since the gauge symmetry is not a symmetry. The concept of HLS is often described by a later notion, Moose\[^{46}\] (actually the condensed Moose\[^{40}\]).

\section{Hidden Local Symmetry (HLS) and Holography in the Walking/Conformal Theories}

Since at this moment there are some limitations on directly solving the strong coupling gauge theories near conformal window, we may take a different approach, namely, an effective field theory. In contrast to the underlying microscopic theory, the effective field theory consists of quantum fields for the light composite spectra as the dynamical variables. Here we take the Hidden Local Symmetry (HLS) model\[^{22}\] which extends the Callan-Coleman-Wess-Zumino (CCWZ) construction of the nonlinear sigma model consisting of $\pi$ so as to incorporate $\rho, a_1, \ldots$ as composite gauge bosons. Note that the HLS is the induced gauge symmetry at the composite level which does not exist in the underlying theory. There is nothing wrong with this, since the gauge symmetry is not a symmetry. The concept of HLS is often described by a later notion, Moose\[^{46}\] (actually the condensed Moose\[^{40}\]).

\subsection{Hidden Local Symmetry}

It is generally shown\[^{22}\] that the nonlinear sigma model based on the coset space $G/H$ is gauge equivalent to another model having a symmetry $G_{\text{global}} \times H_{\text{local}}$, where the gauge symmetry $H_{\text{local}}$ ("Hidden Local Symmetry") as well as the global symmetry $G_{\text{global}}$ is spontaneously broken to give rise to a mass $m_{\rho}$ of the gauge boson $\rho$ ("$\rho$ meson" and the flavor partners), and the left-over (global) symmetry $H$ is a diagonal sum of the $H_{\text{global}}(\in G_{\text{global}})$ and $H_{\text{local}}$, which is identified with the original symmetry $H$ of the $G/H$. The latter model consists of massless $\pi$ and massive $\rho$.

In the case of QCD with $N_f$ massless flavors, the CCWZ Lagrangian takes the form $\mathcal{L}_{\text{CCWZ}} = (F_{\pi}^2/4) \text{tr}(\partial_\mu U^\dagger \partial_\mu U)$, where $U(\pi(x)) = e^{i2\pi(x)/F_{\pi}} = \xi \xi^\dagger$, with $\xi/\xi^\dagger \equiv e^{\pm i\pi(x)/F_{\pi}}$, which transform as $U(\pi) \rightarrow U(\pi') = g_L U(\pi) g_R^\dagger$, $(\xi, \xi^\dagger) \rightarrow (\xi^\dagger, \xi') = h(g, \pi(x)) (\xi, \xi^\dagger) g_{L,R}^\dagger$, with $g = (g_L, g_R) \in SU(N_f)_{L,R}$ and $h \in SU(N_f)_{V}$.

Now, we may rewrite $U = \xi \xi^\dagger$ into the form $U = \xi_{L,R}^L(x) \xi_{R,L}^R(x)$, where $\xi_{L,R}(x) \rightarrow \xi_{L,R}^L(x) = h(x) \xi_{L,R}(x) \xi_{L,R}^R$, with $h(x) \in H_{\text{local}} = [SU(N_f)_{L,R}]_{\text{local}}$, $g_{L,R} \in G_{\text{global}} = [SU(N_f)_{L} \times SU(N_f)_{R}]_{\text{global}}$. Here we have introduced a gauge symmetry $H_{\text{local}}$ as an
ambiguity of dividing $U$ into two parts, which is independent of the global symmetry $G_{\text{global}}$. Thus a theory of $\xi_{L,R}(x)$ has a symmetry $G_{\text{global}} \times H_{\text{local}}$ larger than that of the original model, whose lowest order Lagrangian takes the form

$$\mathcal{L}_{\text{HLS}} = \mathcal{L}_A + a \mathcal{L}_V - (1/2g^2) \text{tr} F^2_{\mu\nu},$$

$$\mathcal{L}_{A/V} = -(F^2_{\mu\nu}/4) \text{tr}(D_\mu \xi_L \cdot \xi^\dagger_L + D_\mu \xi_R \cdot \xi^\dagger_R),$$

where $D_\mu \xi_{L,R} = (\partial_\mu - iV_\mu)\xi_{L,R}$, $a$ is a parameter and $F_{\mu\nu}$ is the field strength of the HLS gauge field $V_\mu$ of $H_{\text{local}}$ ($\rho$ meson) which transforms as $V_\mu \rightarrow V'_\mu = h(x)V_\mu h^\dagger(x) - i\partial_\mu h(x) \cdot h^\dagger(x)$, and $g$ is the HLS coupling constant. We can further gauge (a part of) the $G_{\text{global}}$ as $D_\mu \xi_{L,R} = (\partial_\mu - iV_\mu)\xi_{L,R} + i\xi_{L,R}(\mathcal{L}_\mu, R_\mu)$, where $\mathcal{L}_\mu, R_\mu$ contain $\gamma$ and $W, Z$.

We now fix the gauge of $H_{\text{local}}$ as $\xi^\dagger_L = \xi_R = \xi$ (unitary gauge), so that $(\xi_L, \xi_R)$ coincide with the CCWZ bases $(\xi^\dagger, \xi)$, and $g_{L,R}$ and $h(x)(=h(\gamma, \pi(x)))$ are no longer independent of each other, leaving the symmetry $G$ spontaneously broken to $H$ which is a diagonal subgroup of $H_{\text{global}}(\subset G_{\text{global}})$ and $H_{\text{local}}$. It is easy to see that $\mathcal{L}_A$ is the same as the original nonlinear sigma model $\mathcal{L}_{\text{CCWZ}}$ with $G$ gauged by $\mathcal{L}_\mu, R_\mu$, while $a \mathcal{L}_V$ contains mass term of $V_\mu$ with a mass $m_\rho^2 = ag^2F^2_\pi = g^2F^2_\rho$, “photon mass” $m_\gamma^2 = ae^2F^2_\pi$, $\rho - \gamma$ mass mixing $m_{\rho\gamma}^2 = eg_{\rho\pi}$, $g_{\rho} = agF^2_\pi$ and $\rho - \pi - \pi$ coupling $g_{\rho\pi\pi} = (a/2)g$. The direct $\gamma - \pi - \pi$ coupling comes from both $\mathcal{L}_A$ and $a \mathcal{L}_V$, yielding $g_{\gamma\pi\pi} = (1 - a/2)e$. Then we have a successful KSRF(1) relation as an a-independent result: $g_{\rho} = 2g_{\rho\pi\pi}F^2_\pi$ (KSRF (II)), $g_{\rho\pi\pi} = g$ (universality), $g_{\gamma\pi\pi} = 0$ (vector meson dominance).

In the case of $N_f = 2$ the mass term indicates $SU(2)_{\text{local}} \times [U(1)_{\tau_3}]_{\text{global}}$, with $[U(1)_{\tau_3}]_{\text{global}}(\subset G_{\text{global}})$ now gauged by a photon coupling, which is spontaneously broken down to the $U(1)_{\tau_3}$, with the true (diagonalized) photon mass being precisely 0. This is precisely the same Higgs mechanism as in the Standard Model, $\rho^\pm$ corresponding to $W^\pm$ and $\rho^0$ to $Z^0$: $m_{\rho^\pm}/m_{\rho^0} = 1 + e^2/g^2$.

3.2. Vector Manifestation: Weakly Coupled Composite Gauge Boson Realized near the Conformal Window

Composite gauge bosons are usually regarded as strongly coupled, as we know about the $\rho$ meson, which is actually a conceptual barrier against model building for the composite $W$ and $Z$ bosons. Here I will discuss such a possibility as “Vector Manifestation (VM)” realized at the CPT. Such a dynamical possibility may be applied to the composite $W/Z$ boson model. The VM of chiral symmetry was also vigorously advanced in the chiral phase transition of the hot and dense QCD.

Let us discuss again the large $N_f$ QCD, where we have seen that there exists the Banks-Zaks IR fixed point $\alpha_s$ and a conformal window $N^c_f < N_f < 11N_c/2$ ($0 < \alpha_s < \alpha_{crit}$). If we regard the HLS model as an effective field theory for the underlying large $N_f$ QCD, then we may expect that the chiral phase transition also takes place in the HLS model for a certain large $N_f$ corresponding to the conformal window.

It was found that this is indeed the case, once we match the HLS model (the
simplest model $G_{\text{global}} \times H_{\text{local}}$ for $\pi$ and $\rho$ mesons) with the underlying large $N_f$ QCD through OPE for current correlators at some scale $\Lambda$ where each theory gives a reasonable description. In the HLS Lagrangian matched in this way (the bare HLS theory defined at $\Lambda$) the bare $\pi$ “decay constant” $F_\pi(\Lambda)$ is given by $F_\pi^2(\Lambda) \simeq N_c \times (\Lambda/4\pi)^2 \neq 0$ even when the chiral restoration takes place $\langle \bar{q}q \rangle \rightarrow 0$ in the QCD side and hence looks as if in the broken phase at the scale $\Lambda$. Nevertheless, it receives quantum corrections of $\pi$ and $\rho$ loops including quadratic divergence, $-(N_f/2)(\Lambda/4\pi)^2$, which may change the phase of the full quantum theory into the symmetric phase when we increase $N_f$: The true decay constant $F_\pi(0)$ as an order parameter (π pole residue) is given as

$$F_\pi^2(0) = F_\pi^2(\Lambda) - (N_f/2)(\Lambda/4\pi)^2 \rightarrow 0$$  \hfill (3.2)\

for $N_f \not> N_f^c \simeq 2N_c = 6$ (more detailed analysis yields $N_f^c \simeq 5$), which is compared with a lattice value$^{32}$ $6 < N_f^c < 7$ and the value $N_f^c \sim 4N_c$ given by the ladder SD equation and the two-loop BZ IR fixed point. Thus the HLS theory also has a chiral phase transition at large $N_f$.

In the limit of $\langle \bar{q}q \rangle \rightarrow 0$ in the QCD side, the above matching is possible only when $a(\Lambda) = 1$ and $g(\Lambda) \rightarrow 0$ ("Vector Limit")$^{23}$, so that $m_\rho^2/F_\rho^2 = g^2 \rightarrow 0$ with $F_\pi^2 = F_\pi^2(0) \rightarrow 0$ ($N_f \not> N_f^c$). Thus we encounter a new situation of the chiral symmetry restoration (Wigner phase): The $\rho$ meson becomes massless, with the longitudinal $\rho$ (NG boson field) degenerate with $\pi$ as a chiral partner. We called it “Vector Manifestation (VM)” of Wigner phase of chiral symmetry.$^{23}$ The HLS coupling now vanishes at the conformal window.

3.3. **Summing up HLS Towers, or Holography**

If we apply the VM to the TC instead of the composite $W/Z$ model, the HLS gauge boson is the techni-$\rho$. The current correlator dominated by $\pi, \rho$ takes the form $(\Pi_{VVV} - \Pi_{AA})/Q^2 = F_\pi^2/(Q^2 + M_\rho^2) - F_\rho^2/Q^2$, which yields the $S$ parameter: $\hat{S} = 4\pi(F_\rho/M_\rho)^2 = 4\pi/g^2$, with $g$ being the HLS gauge coupling for techni-$\rho$. Thus we would have $\hat{S} \rightarrow \infty$ as $N_f \not> N_f^c$, which is in opposite direction to the straightforward ladder/BS equation calculation mentioned in Sec 2.3. This is due to the infrared divergence of $\rho$ which at VM would become massless as a chiral partner of massless $\pi$, while the $\pi$ contribution accidentally drops out in our definition of $\hat{S}$ which is not identical to the quantity measured by the LEP precision experiments. In the case of QCD infrared divergence due to massless $\pi$ also takes place in $\hat{S}$ (or $L_{10}$) which is regularized by the pion mass in reality. It would be nice to find a proper definition of $\hat{S}$ to keep track of the chiral partner between $\pi$ and $\rho$ and to be compared with the LEP experiments.

Alternatively, we may consider a generalized HLS model$^{23}$ including the (techni-) $a_1$ as well as (techni-) $\rho$: $\hat{S} = 4\pi((F_\rho/M_\rho)^2 - (F_{a_1}/M_{a_1})^2) = (4\pi/g^2)(1 - (b/(b+c))^2)$, where $b,c$ are the parameters of this generalized HLS model to be running at loop level. The one loop contribution of this model is more involved,$^{30}$ which may suggest $^\dagger \pi$ loop alone gives a factor $N_f$ instead of $N_f/2$. Thus the HLS is essential to having a sensible result.
a possibility of a fixed point of the HLS parameters for giving a vanishing $\hat{S} \sim c/g^2 \rightarrow 0$ due to cancellation among $\rho$ and $a_1$ contributions at the chiral restoration point.

Now, we are interested in summing up higher resonances in HLS model. We can easily extend the HLS to incorporate higher vector resonances.\(^{22}\) In the low energy region with the momentum $p \ll m_\rho$ where the $\rho$ kinetic term may be ignored, we can integrate out the gauge boson $\rho$, or by use of the equation of motion for $\rho$, we get $a L_V = 0$, and hence we are left with $\hat{L}_A$ which is nothing but the original nonlinear sigma model on the coset space $G/H$. Similarly, the $G_{\text{global}} \times H_{\text{local}}$ HLS model is gauge equivalent to another model having a symmetry $G_{\text{global}} \times G_{\text{local}}$, with the gauge symmetry $G_{\text{local}}$ spontaneously broken down to $H_{\text{local}}$ giving mass to the axialvector meson $a_1$, where we have introduced another nonlinear sigma model (Higgs field) to be absorbed into $a_1$. In the energy region $m_\rho < p \ll m_{a_1}$ we may integrate out $a_1$ and get back to the $G_{\text{global}} \times H_{\text{local}}$ model. We can go further to $G_{\text{global}} \times H_{\text{local}} \times G_{\text{local}}$.\(^{22}\) In this way we can proceed indefinitely to incorporate higher vector/axialvector mesons by introducing infinite set of nonlinear sigma models (Higgs fields): $G_{\text{global}} \times G_{\text{local}} \times G_{\text{local}} \times G_{\text{local}} \times \cdots$. In the case of QCD, this chain of the larger HLS models may be summarized as $SU(N_f)_L \times SU(N_f)_R/SU(N_f)_V \Rightarrow [SU(N_f)_L \times SU(N_f)_R]_{\text{global}} \times [SU(N_f)_V]_{\text{local}} \Rightarrow [SU(N_f)_L \times SU(N_f)_R]_{\text{global}} \times [SU(N_f)_L \times SU(N_f)_R]_{\text{local}} \Rightarrow [SU(N_f)_L \times SU(N_f)_R]_{\text{local}} \times [SU(N_f)_L \times SU(N_f)_R]_{\text{local}} \times \cdots$.

Now we consider a special parameter choice: $a = 1$ and $g = 0$ (vector limit)\(^{23}\) in which case $\xi_L$ and $\xi_R$ get decoupled and hence we get two independent nonlinear sigma models $[SU(N_f)_L \times SU(N_f)_L/SU(N_f)_L \times SU(N_f)_R/SU(N_f)_R]$, where $\xi_L \rightarrow \xi'_L = g_L \xi_L g_R^\dagger$, and similarly for $\xi_R$. Conversely, switching on the HLS coupling get the global symmetry $G' = SU(N_f)_L \times SU(N_f)_R$ down to $H_{\text{local}} = [SU(N_f)_L \times SU(N_f)_R]_{\text{local}}$ and the model is reduced to the HLS model having $H_{\text{local}} \times G_{\text{global}}$ with $G_{\text{global}} = [SU(N_f)_L \times SU(N_f)_R]_{\text{global}}$. In terms of the (condensed) Moose language,\(^{24}\) this situation is identical to the “3-site linear moose”: $[SU(N_f)_L]_{\text{global}} - SU(N_f)_L - SU(N_f)_R]_{\text{global}}$, with the site of HLS $SU(N_f)_L$ in the middle being circled and with the $G_{\text{global}}$ split to both ends, $[SU(N_f)_L]_{\text{global}}$ and $[SU(N_f)_R]_{\text{global}}$, being circled (when gauged with external gauge fields $\mathcal{L}_\mu, \mathcal{R}_\mu$) or opened (when ungauged) and the link denoted by “−” being each nonlinear sigma model base. In this sense Moose is nothing but a reformation of HLS.

In a particular parameter choice (generalization of the choice of $a = 1$ in the vector limit) it is sometimes convenient to put it into the linear moose style (Fig.4): $[SU(N_f)_L]_{\text{global}} - [SU(N_f)_L]_{\text{local}} - [SU(N_f)_L]_{\text{local}} - \cdots - [SU(N_f)_R]_{\text{local}} - [SU(N_f)_R]_{\text{local}} - [SU(N_f)_R]_{\text{global}}$. By incorporating infinite tower of massive HLS gauge bosons in this way, one actually arrives at the gauge theory in 5 dimensions,\(^{25}\) the massive tower of HLS gauge bosons being nothing but the Kaluza-Klein (KK) tower of the massless 5-dimensional gauge boson $A^M(x^M)$ ($M = \mu, 5$). When the 5-th dimension is deconstructed/latticized\(^{26}\) the Wilson line $e^{if \int dz^5 A^5}$ (link variable in the lattice gauge theory) acts like a nonlinear sigma model (Higgs) corresponding to each “−” in the above moose and is adsorbed into the gauge bosons (KK modes).
of the HLS at the nearest sites (circles). Actually, the holographic approach to QCD gives the 5-dimensional gauge theory which is nothing but an infinite set of HLS gauge bosons.\textsuperscript{50, 51} Converse is true. We can always integrate out\textsuperscript{52} the KK tower of the holographic model to get back to the simplest HLS model with the lowest resonance, the \( \rho \) meson, plus \( \mathcal{O}(p^4) \) terms in the HLS chiral perturbation (See\textsuperscript{23}) with definite coefficients.

3.4. \textbf{Holographic Walking/Conformal TC}

The reduction of \( S \) parameter in the walking/conformal TC has been argued in a version of the holographic QCD\textsuperscript{53} deformed to the walking/conformal TC by tuning a parameter to simulate the large anomalous dimension \( \gamma_m \simeq 1 \).\textsuperscript{53} We recently examined\textsuperscript{23} such a possibility paying attention to the renormalization point dependence of the condensate. We explicitly calculated the \( S \) parameter in entire parameter space of the holographic walking/conformal technicolor (See Fig. 5). The \( S \) parameter was given as a positive monotonic function of \( \xi \) which is fairly insensitive to \( \gamma_m \) and continuously vanishes as \( \hat{S} \sim \xi^2 \to 0 \) when \( \xi \to 0 \), where \( \xi \) is the (dimensionless) vacuum expectation value of the bulk scalar field at the infrared boundary of the 5th dimension \( z = z_m \) and is related to the mass of (techni-) \( \rho \) meson \( (M_\rho) \) and the decay constant \( (f_\pi) \) as \( \xi \sim f_\pi z_m \sim f_\pi / M_\rho \) for \( \xi \ll 1 \). However, although \( \xi \) is related to the techni-fermion condensate \( \langle \bar{F}F \rangle \), we find no particular suppression of \( \xi \) and hence of \( S \) due to large \( \gamma_m \), based on the correct identification of the renormalization-point dependence of \( \langle \bar{F}F \rangle \) in contrast to the literature.\textsuperscript{54}

Curiously enough, a set of the values of \( \xi^2 \) (read from \( F_\pi / M_\rho \)) and \( \hat{S}/N_{TC} \) in Eq.\textsuperscript{23} in the straightforward calculation of ladder SD/BS equations fit in the line of the holographic result in Fig. 5.

§4. \textbf{Top Quark Condensate in Walking/Conformal Theories}

Top Quark Condensate (Top Mode Standard Model, TMSM\textsuperscript{25, 26, 27}) is an idea alternative to the technicolor, and as such has potentiality to account for the origin

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Fig4.png}
\caption{Linear moose for arbitrary number of HLS's. Each circle stands for \( SU(N_f) \) HLS, and each bar connecting two circles stands for the nonlinear sigma model transforming under the connecting two HLS's. Two end points stand for \([SU(N_f)_{L,R}]_{\text{global}}\) which may be gauged by the external gauge fields \( L_\mu, R_\mu \). .}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Fig5.png}
\caption{Plot of \( \xi^2 \)-dependence of \( \hat{S}/N_{TC} \) with \( \gamma_m \simeq 1 \). The blob is the result of the ladder SD and BS equations, \( \xi \) from homogeneous BS equation\textsuperscript{23} and \( \hat{S} \) from inhomogeneous BS equation\textsuperscript{23}}}
\end{figure}
of mass of all the SM particles.\textsuperscript{25} (For a recent attempt see\textsuperscript{55}).

The original top quark condensate model was formulated in the gauged Nambu-Jona-Lasinio (NJL) model\textsuperscript{25} four-fermion theory plus Standard Model gauge couplings, whose phase structure (critical line) was revealed in the ladder SD equation.\textsuperscript{27} The gauged NJL model was shown\textsuperscript{19} to have a very large anomalous dimension $\gamma_m \simeq 2$ due to strong four-fermion interaction at the UV fixed line $g = g_{(\pm)}$ in Eq.\textsuperscript{24} (identified with the critical line) for small SM gauge coupling ($\alpha \sim 0$). The existence of critical line implies that a tiny difference among the four-fermion coupling (or the gauge coupling) of the top and bottom could result in $m_t \neq 0$ and $m_b = 0$, thus explaining a large hierarchy among top and bottom (and other quarks/leptons): $m_t \gg m_b, m_c, \ldots$. The model predicted (long before the discovery of the top quark) $m_t \approx 250$ GeV (large $N_c$ leading)\textsuperscript{25} and $m_t \approx 220$ GeV (including $N_c$ subleading effects)\textsuperscript{27} for the cutoff $\Lambda \sim 10^{16} - 10^{19}$ GeV. This reflects the reality at 0th order approximation that only the top mass (as well as $W, Z$ masses) is on the order of weak scale. However, if $\Lambda$ is a natural scale in TeV region, $m_t$ would be as large as 500 GeV. The model predicts a Higgs boson as a bound state of $\bar{t}t$ ("Top-sigma")\textsuperscript{25}, 26) whose mass is $m_H \approx 2m_t$ in the NJL model, but is changed to $m_H \approx 1.1m_t$\textsuperscript{27} in the gauged NJL model. Now the questions: What is the origin of the four-fermion interactions? How can we get a realistic top quark mass $m_t \approx 172$ GeV in a natural way?

4.1. TMSM with extra dimensions

Let us now come to an extension\textsuperscript{57}, 28) of the TMSM as simply the SM formulated in the higher dimensional bulk \textit{without ad hoc four-fermion interactions}, where the (dimensionless) color coupling in the bulk gets strong when the extra dimensions become operative in the high energies in TeV region. It was noted earlier\textsuperscript{53} that the bulk gluon exchanges (or the gluon KK mode exchanges) play the role of the four-fermion interactions triggering the top quark condensate in the original model and the bulk top quark give rise to KK modes of the top quark which can bring the top mass prediction of the original model down to the realistic value $m_t \approx 172$ GeV in a way similar to the top-seesaw.\textsuperscript{59}

Actually, the QCD with compactified ($D - 4$) extra dimensions becomes a walking/conformal gauge theory having a UV fixed point with large anomalous dimension $\gamma_m \simeq D/2 - 1$.\textsuperscript{28} Because of the UV fixed point, the dimensionless bulk QCD coupling does not grow indefinitely, while the $U(1)_Y$ bulk coupling has a Landau pole at high energy and hence dominates the QCD coupling at certain high energy scale to favor the $\tau$ condensate rather than the top quark condensate. Thus it is highly nontrivial whether or not there exists a region where the top quark condensate is a Most Attractive Channel (MAC) favored to others. We called such a region a “topped MAC (tMAC)” region which is identified with the cutoff where the composite Higgs of $tt$ is formed. Actually the tMAC region is so narrow, if existed at all, that we can predict the mass of top quark and also of Higgs boson as a $tt$ composite without much ambiguity: $m_t = 172 - 175$ GeV $m_H = 176 - 188$ GeV ($D = 8$ with $R^{-1} = 1 - 100$ TeV where $R (= 1/\Lambda)$ is the radius of compactified extra ($8 - 4 = 4$) dimensions.\textsuperscript{28} If one assumes that SM particles live in the 6-dimensional brane (5-
brane) in the $D = 8$ bulk where only the gluons live, one gets: $m_t = 177 - 178$ GeV $m_H = 183 - 207$ GeV. The Higgs in this characteristic mass range will be discovered immediately once the LHC started.

4.2. Top Mode Walking/Conformal TC

The top quark is very special in the ETC, since the top and bottom mass given by $m_{t/b} \sim \frac{1}{\Lambda_{\text{ETC}}} \langle \bar{F} F \rangle_{\Lambda_{\text{ETC}}}$ would require large isospin violation in the condensate $\langle \bar{F} F \rangle_{\Lambda_{\text{ETC}}}$ in order to produce large mass splitting $m_t \gg m_b$, which would conflict the $T$ parameter constraint. A possible way out, so-called “Topcolor-assisted TC (TC2)" is to introduce the (isospin violating) top quark condensate, in addition to the (isospin conserving) TC condensate which gives main contribution to the weak scale ($W/Z$ mass). Such a model generically predicts (besides the top-sigma) a salient light pseudo NG boson, top-pion, whose mass comes from the ETC-induced top/bottom mass. If we require the top mass mainly comes from the top condensate rather than the ETC-origin, then we found\(^{29}\) that the mass of the top-pion $m_{\pi t}$ is severely constrained as $m_{\pi t} < 70$ GeV due to the large anomalous dimension $\gamma_m \simeq 2$ of the dynamics for the top quark condensate.

§5. Conclusion

We have discussed composite models for the electroweak sector, based on walking/conformal gauge theories with large anomalous dimension near the IR/UV fixed point. Many such models predict Higgs-like composite spectra somewhat heavier than those anticipated in the typical SUSY theories and SM, and hence will be distinguished in the LHC experiments: Walking/Conformal TC will have techni-dilaton (techni-sigma) which is expected to have mass typically around $\sqrt{2} M_f (> 500 - 600 \text{ GeV})$, where $M_f$ is the techni-fermion mass. Top quark condensate will have a top-sigma whose mass slightly less than twice top quark mass $< 2 m_t \sim 350 \text{ GeV}$. If LHC did not find light Higgs with mass lighter than, say 180 GeV, there will be a good chance for the composite model and history will repeat itself on the old avenue that Sakata, Nambu, and Maskawa walked on. We will see.

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