Design of backstepping controller for T-type network inverter

Guanghui Gao¹, Jiaqi Zeng, Jianding Han and Xiaoben Lei
Air Force Engineering University, Xi’an, China

¹Email: 1521211541@qq.com

Abstract. In the process of power conversion of new energy generation, complex two-stage converter is needed. T-type network inverter is a kind of transformer that can realize three-phase power output from two single-phase output inverters. Functional Topology, the purpose of this paper is to design the circuit topology and controller so that the inverter output voltage meets the power supply requirements. First, the DC power input circuit and transformer structure are designed according to the principle of voltage vector synthesis. Then the controller is designed by state space average modeling method and backstep method. Finally, the system model is built by Matlab/Simulink simulation software. The conversion of 27V DC power to 220V / 50Hz three-phase power for market power supply verified The effectiveness of The design content of this paper.

1. Introduction
In photovoltaic, wind, fuel cell and other renewable energy systems, two-stage integrated topologies (DC-DC and DC-AC) are usually used to connect to the power grid or supply loads. The common two-stage circuit topology needs to boost the output voltage of pv and then invert it. This topology makes the circuit complex, difficult to control, reducing the power supply quality and reliability of the circuit.[1] Traditional inverters include full-bridge inverters, neutral-point-clamped inverters, z-source inverters, etc. However, none of them has the function of boosting voltage to make the low DC voltage invert to 220V market power, so it is necessary to go through the step-up link first. This has undoubtedly done a great disservice.

The T-type network inverter is composed of two full-bridge modules and a group of vector synthesis transformers. Because of the transformer, the DC-DC boost module can be directly omitted, at the same time, the vector transformer can combine two single phase AC voltages into three phase AC voltages.

The T-type network inverter achieves the output of three-phase power from two single-phase full-bridge inverters through a tapped transformer. Since the two single-phase inverters have basically the same control method, the control of T-type network inverter is actually the control of single-phase full-bridge inverter, so the modeling methods of t-type network inverter are various and the control strategies are abundant. The T-type network inverter topology is in line with the Modular design concept and is conducive to improving the design of new energy efficiency and fault tolerant structures.

The inverter is a kind of switching converter, and its main component, the switch tube, has very strong nonlinear characteristics. Traditional control methods, such as PID control, predictive control, can not describe the nonlinear characteristics of the inverter effectively. In recent years, the rapid development of nonlinear control theory methods mainly includes neural network method, Lyapunov method, precise linearization method and so on. [2] The neural network method needs a lot of data
training, and the Lyapunov method is more complex to solve the lie derivative, whose stability criterion needs more computation and requires higher controller.[3] The exact linearization method is based on the differential geometry theory, which requires the control object to have an exact mathematical model. There are many uncertain factors in the inverter system, which will lead to the low robustness of the system and more susceptible to load when the method is applied to the inverter system. [4] In 1991, Kokotovic, a professor of Control Science in the United States, proposed a nonlinear control theory method "backstepping" based on Lyapunov equation, which can effectively deal with the uncertain parameters in the inverter system. It can be effectively realized in engineering practice. [5]

2. Working principle of T-Type network inverter

Through two single-phase full-bridge inverters, the T-type network inverter transforms the DC input into the single-phase AC output, and then achieves the effect of the three-phase AC output finally based on the vector synthesis principle and the transformer group transformation. How it works is shown in Figure 1, and the topology is shown in Figure 2. [6]

The T-type network inverter topology makes the direction of the output voltage vector of two singlephase full-bridge inverters present certain angle relation, and the amplitude value assumes certain proportion relation. Based on vector synthesis principle, finally realized two-phase inverter circuit output three-phase voltage. The principle of voltage vector synthesis is shown in Figure 3. According to Figure 2, $U_2$ is the voltage obtained by transformer transformation for the output voltage of inverter 2, and $U_1$ is the voltage obtained by transformer transformation for the output voltage of inverter 1.

In order to make them perpendicular to each other, a phase-shifting device is designed to make the output voltage of Inverter 1 lag the voltage $\frac{\pi}{2}$ angle of inverter 2.

![Figure 1. Working principle diagram of T-type network inverter.](image1)

![Figure 2. T-type network inverter topology diagram.](image2)

![Figure 3. Voltage Vector Synthesis schematic diagram.](image3)

![Figure 4. Single phase voltage inverter topology.](image4)
In order to obtain three-phase symmetrical AC output voltage, the amplitude of them should be controlled. From the geometric relations of congruent triangles

$$\frac{l_{ab}}{l_{ad}} = 2\sqrt{3}$$

Therefore, the amplitude ratio of them should be controlled as $2\sqrt{3}$. In order to achieve this goal, it can be adjusted from two links. In the first scheme, the original side windings of the two transformers are the same, so that the output voltage amplitude of the two inverters is proportional to $2\sqrt{3}$, and the output voltage amplitude of the inverters is determined by the DC input voltage, so it is necessary to design the DC input circuit of the inverter. The ratio of DC input voltage of two inverters can be controlled as $2\sqrt{3}$ by using capacitor voltage divider. In the second scheme, keep the output of the two inverters the same and the ratio of the transformer $TP_1$ and $TP_2$ is presented as $2\sqrt{3}$ by designing the primary side winding of the transformer.

Two schemes are compared. The first scheme is easy to be realized in engineering practice, and the design proportion can be approximated accurately by using adjustable capacitance. The second scheme requires higher winding process of transformer, higher cost and larger error. Therefore, we choose the first scheme, namely through the inverter front-stage circuit adjustment, in order to obtain the ideal amplitude.

In order to achieve the vector synthesis effect shown in Figure 2.4, the secondary side windings of the transformer need to be designed. According to the mathematical geometry analysis, the $D$ point is the midpoint of the $BC$ side in $\Delta ABC$, and the point $O$ is the lower equilateral point of the $AD$ side. A tap is drawn from the midpoint $O$ of the transformer $TP_1$ to divide $U_i$ into $U_i'$ and $U_i''$, and they have the same direction and amplitude; A tap is drawn from the upper third of the transformer $TP_2$ to divide $U_z$ into $U_z'$ and $U_z''$, and they are in the same direction, with a magnitude ratio of 2:1.

3. State average modeling

The state space description is an internal description of the system. Compared with the "black box" external description, the internal description can reflect the dynamic characteristics of the system more completely. The mathematical description of a system state space generally includes two equations, the input equation describes the relationship between the output and the state variables, and the output equation describes the relationship between the output variables and the state variables. [7]

The inverter circuit is a kind of nonlinear time-varying system, and different switch combination corresponds to different commutation circuit, so the state space description of one working cycle of the inverter system often needs multi-group state equations. If the system is described in this way, the process of analysis and solution will be very complicated. In order to simplify the analysis process, we use the state space average method to unify the state equations of the inverter circuit into the same expression.

Since the control methods of the two single-phase inverter circuits are the same, the inverter circuit 1 and its filter circuit are analyzed and modeled. The topology is as shown in Figure 4. $V_{dc}$ is the input DC voltage of the inverter, and $s_{ij}$ ($j \in \{1,2,3,4\}$) is the full-bridge circuit power switch. $R_i$, $L_i$, $C_i$ are respectively filter circuit resistance, inductance and capacitance. $R_T$, $L_T$ are the transformer $TP_1$ equivalent resistance, inductance, and $U_0$ is the full-bridge circuit output voltage.

The switching state of inverter is analyzed, and the state function $s_{ij}$ ($j \in \{1,2,3,4\}$) of the switching tube is defined. The switching tube is off, $s_{ij} = 0$, while the switching tube is turned on, $s_{ij} = 1$. According to the working principle of the inverter, only one switch tube is turned on in each leg.
during a working cycle. The corresponding relationship between the switching function $s_{ij}$ and the output voltage $u_O$ of the full-bridge circuit is shown in Table 1.

**Table 1.** Relationship between switch function and output voltage of full bridge circuit.

|      | s11 | s12 | s13 | s14 | uO    |
|------|-----|-----|-----|-----|-------|
| State 0 | 0   | 0   | 1   | 1   | 0     |
| State 1 | 0   | 1   | 1   | 0   | Vdc   |
| State 2 | 1   | 0   | 0   | 1   | -Vdc  |

When single-phase full-bridge inverter is working, state 1 and State 2 work alternately, each half cycle, that is, each bridge arm’s upper and lower switch-on time is complementary, so the state function of upper switch-on tube is only taken. The following relationships can be observed in the table

$$u_o = (s_{11} - s_{12})V_{dc}$$  \(2\)

According to Figure 4, the discrete state equation of single-phase inverter is obtained

$$\begin{cases} C \frac{du_c}{dt} = -\frac{u_c}{R_c} + i_i \\ L \frac{di_i}{dt} = -u_c - R_c i_i + (s_{11} - s_{12})V_{dc} \end{cases}$$  \(3\)

In order to transform the discrete equation in equation (3) into a continuous model, the average operator [8] is introduced

$$\langle x(t) \rangle_c = \frac{1}{T_c} \int_{0}^{T_c} x(t)dt$$  \(4\)

The average operator is introduced into (3) to transform the discrete equation into a continuous model

$$\begin{cases} C \frac{d\langle u_c \rangle_c}{dt} = -\langle u_c \rangle_c + \langle i_i \rangle_c \\ L \frac{d\langle i_i \rangle_c}{dt} = -\langle u_c \rangle_c - R_c \langle i_i \rangle_c + (s_{11} - s_{12})V_{dc} \end{cases}$$  \(5\)

Because there are capacitor inductors and other energy storage elements in the inverter, the changing frequency of variable $u_c, i_i$ is much lower than the changing frequency of the switch

$$\langle u_c(t) \rangle_c \approx u_c$$

$$\langle i_i(t) \rangle_c \approx i_i$$

Defines a turn-on time of switch $S_{11}, S_{12}$ as $T_{s1}, T_{s2}$, and a duty cycle of $d_1, d_2$ have the following relationship

$$T_s = T_{s1} + T_{s2}$$  \(7\)

$$d_k = \frac{T_{s_k}}{T_s} (k \in \{1, 2\})$$  \(8\)

The average operator of the switching function is obtained by the additivity of the definite integral

$$\langle s_{11} - s_{12} \rangle_c = \frac{1}{T_s} \int_{0}^{T_s} (s_{11} - s_{12})dt$$  \(9\)

Simultaneous (7) and (8) are available
\[ d_1 + d_2 = 1 \quad (10) \]

Replace (9) with (10)

\[ \{s_1 - s_2\}_i = 2d_1 - 1 = 2d - 1 \quad (11) \]

By substituting Formula (6)(11) into formula (5), the state average model of single-phase full-bridge inverter is obtained

\[
\begin{align*}
C_i \frac{d e_i}{dt} &= \frac{V_c}{R_L} + i_L \\
L \frac{di_L}{dt} &= -u_c - R_L i_L + (2d - 1) V_c
\end{align*}
\quad (12)
\]

4. Backstepping controller design

Define the output quantity \( x_1 = u_c \), the state variable \( x_2 = i_L \), and the input control quantity \( u = 2d - 1 \), and the expression (12) becomes

\[
\begin{align*}
\dot{x}_i &= -\frac{1}{C_i R_L} x_i + \frac{1}{C_i} x_1 \\
\dot{e}_i &= -\frac{1}{L} x_2 - \frac{R_L}{L} x_i + \frac{V_c}{L} u
\end{align*}
\quad (13)
\]

In order to obtain the exact parameter feedback form required by the backstepping method, the substitution (14) is used

\[
\begin{align*}
A &= \frac{1}{C_i} \\
B &= \frac{V_c}{L} \\
f_1 &= \frac{1}{C_i R_L} x_1 \\
f_2 &= -\frac{1}{L} x_2 - \frac{R_L}{L} x_i
\end{align*}
\quad (14)
\]

The state equation of single-phase full-bridge inverter is obtained in the form of strict true parameter feedback

\[
\begin{align*}
\dot{x}_i &= Ax_i + f_1 \\
\dot{x}_2 &= Bu + f_2
\end{align*}
\quad (15)
\]

Then the backstepping method is used to design the control variables \( u \). Firstly, the error variables are defined as follows [9]

\[
\begin{align*}
e_1 &= x_1 - \alpha_1 \\
e_2 &= x_2 - \alpha_2
\end{align*}
\quad (16)
\]

Among them, \( \alpha_1 \), \( \alpha_2 \) are virtual control volume. Define \( \alpha = u_{\text{ref}} \) (\( u_{\text{ref}} \) is defined as the target voltage). The Lyapunov function of the single-phase inverter system

\[ V_1 = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 \quad (17) \]

Derivation of Lyapunov function

\[ \dot{V}_1 = e_1 \dot{e}_1 + e_2 \dot{e}_2 \quad (18) \]

The control of the inverter can be abstracted as a precise tracking problem. The aim of the control is to make the output voltage \( u_c \) of the inverter converge to the target voltage \( u_{\text{ref}} \), that is to say, to control the exponent of error variable \( e_1, e_2 \) converge to zero. According to the second Lyapunov
Stability Theorem, the system is asymptotically stable when $V > 0$ and $\dot{V} < 0$. Make a construction of pair (18) as in pair (19)

$$\dot{V}_1 = e_1 (-k_1 e_1 + Ae_2) + e_2 (-k_2 e_2 - Ae_1)$$

$$\dot{V}_1 = -k_1 e_1^2 - k_2 e_2^2 < 0 \quad (19)$$

$\dot{V}_1$ is obviously less than zero in (19) and can be set

$$\begin{cases}
\dot{e}_1 = -k_1 e_1 + Ae_2 \\
\dot{e}_2 = -k_2 e_2 - Ae_1 
\end{cases} \quad (20)$$

The derivation of the pair (16) is obtained

$$\begin{cases}
\dot{e}_1 = Ax_1 + f_1 - \dot{\alpha}_1 \\
\dot{e}_2 = Bu + f_2 - \dot{\alpha}_2 
\end{cases} \quad (21)$$

Comparative Forms (20) and (21) are available

$$\begin{cases}
\alpha_1 = u_{ref} \\
\alpha_2 = \frac{1}{A} [\dot{\alpha}_1 - f_1 - k_1 e_1] \\
u = \frac{1}{B} [\dot{\alpha}_2 - f_2 - k_2 e_2 - Ae_1] 
\end{cases} \quad (22)$$

From equation (22), we can clearly see the idea of designing input control quantity by backstepping method, and deduce it one step forward from the system output. The reference voltage $u_{ref}$ is used as the first analog control quantity $\alpha_1$, the second analog control quantity $\alpha_2$ is calculated, and then the input control quantity $u$ is calculated by the first two analog control quantity.

5. Simulation validation

In order to verify the feasibility of the backstepping controller to control the single-phase full-bridge inverter and investigate its control effect, a full-bridge inverter model is built based on Matlab / Simulink simulation software, which is shown in Figure 5.

First, a backstepping controller is constructed according to formula (22), and then a signal modulation module is constructed according to formula (11) to convert the output control rate $u$ of the backstepping controller into the duty cycle $d$ of the inverter bridge switch $S_{11}$ shown in Figure 4. In the model, the Universal bridge module is used to simulate the single-phase full-bridge circuit. In the two-arm mode, the input amount required by the control input port $g$ is a 4-dimensional vector. According to formula (10), the other three elements in the control vector are obtained. After modulation by the PWM modulation module, input to port $g$ so realize the control of the inverter bridge.

Set simulation parameters as shown in Table 2, simulation step size set to 1e-5s.

6. Conclusions

The simulation results are shown in Figure 6. The sine waveform with amplitude $220\sqrt{2}$ V and frequency 50Hz is ideal, which proves that the state space average model derived in this paper is more accurate. The controller designed by backstepping method can effectively control the output ideal waveform of single-phase full-bridge inverter.

After the validity of the backstepping controller is verified, the T-type network inverter topology model is built based on the simulation structure of the single-phase full-bridge topology. The DC input of the two single-phase inverters is supplied by the DC power after two voltage capacitors, whose ratio is $2\sqrt{3}$. The voltage synthesis part is realized by connecting two side-tapped transformers. The model
is shown in Figure 7. The three-phase output voltage waveform with amplitude $220\sqrt{2}$ V, phase difference $\frac{2\pi}{3}$ is obtained by simulation, as is shown in Figure 8, which meets the requirement of power supply system.

**Figure 5.** Simulink simulation structure of singlephase voltage type full-bridge inverter.

| parameter | value   |
|-----------|---------|
| Vdc       | 27V     |
| R1        | 0.45Ω   |
| C1        | 3e-6F   |
| L1        | 5.6e-3H |
| RT        | 2Ω      |
| LT        | 6e-3H   |
| k1        | 8100    |
| k2        |         |

**Table 2.** The simulation parameters.

**Figure 6.** The simulation waveform of backstepping controller verifies.
Figure 7. T-type network inverse topology simulation structure.

Figure 8. T-type network inverter three-phase output waveform.

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