Laser wakefield acceleration of polarized electron beams

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Abstract. The acceleration of highly polarized electron beams are widely used in state-of-the-art high-energy physics experiments. In this work, a model for investigation of polarization dynamics of electron beams in the laser-plasma accelerator depending on the initial energy of electrons was developed and tested. To obtain the evolution of the trajectory and momentum of the electron for modeling its acceleration the wakefield structure was determined. The spin precession of the beam electron was described by Thomas–Bargman–Michel–Telegdi equations. The evolution of the electron beam polarization was investigated for zero-emittance beams with zero-energy spread.

1. Introduction

Polarization is an essential feature of particle beams in the most of modern high-energy physics experiments [1]. It significantly affects the interaction cross section of polarized beams and allows, for example, to perform precision tests of the standard model [2] or to probe directly the weak interaction [3]. Most accelerators have been modified or originally designed with the possibility of the use of polarized particles sources. The spin dynamics in traditional accelerators was investigated [1] and methods of controlling the direction and degree of polarization were developed [4]. However, maximal acceleration gradients that can be achieved in conventional accelerators are limited by the breakdown threshold of the waveguide wall material. The laser-plasma acceleration (LPA) can be an alternative to traditional accelerators due to high acceleration gradients of about 100 GeV per meter [5] (it is several orders of magnitude higher compared to field gradients in traditional accelerators) [6]. Therefore, it is necessary to investigate depolarization of the beam accelerated in LPA.

In this work evolution of the beam polarization was studied for the laser wake field accelerator (LWFA) scheme. In this scheme a laser pulse passes through a preformed parabolic plasma channel, which allows to extent the propagation of the laser radiation over the distance of many Rayleigh lengths without diffraction spreading [7], and exits plasma wake fields suitable for the electron acceleration.
2. Dynamics of the electron beam polarization in LWFA

To study the spin precession of the beam single electron Thomas–Bargman–Michel–Telegdi (T-BMT) equations were used [1]

\[
\frac{d\vec{s}}{dt} = -\frac{e}{mc} \left( a_m + \frac{1}{\gamma} \right) \vec{B} - \frac{a_m \gamma}{\gamma + 1} (\vec{B} \vec{B} - \left( a_m + \frac{1}{\gamma + 1} \right) \vec{\beta} \times \vec{E}) \times \vec{s},
\]

(1)

where \(\vec{s}\) is a spin normalized to the absolute value of the electron spin, \(c\) is the speed of light, \(a_m \approx 0.001\) is the anomalous magnetic moment of the electron, \(\vec{B}\) and \(\vec{E}\) are external (focusing and accelerating) magnetic and electric fields normalized to \(mc\omega_{p0}/e\), \(\omega_{p0} = \sqrt{4\pi e^2 N_0/m}\) is the plasma frequency, with \(e\) and \(m\) being the electron charge and mass, \(N_0\) is the initial electron plasma density at the channel axis, \(\gamma\) is a relativistic gamma factor of an electron and \(\vec{\beta} = \vec{v}_e/c\) corresponds to a dimensionless electron velocity.

Polarization \(\vec{P}\) of the beam is considered as an average spin of beam electrons [8]:

\[
\vec{P} = \sum_{n=1}^{N_b} \frac{\vec{s}(n)}{N_b},
\]

(2)

where \(N_b\) is the total number of electrons in the beam and \(\vec{s}(n)\) is the spin of the n-th electron. Connected with polarization the depolarization is defined as \(|\Delta \vec{P}| = |\vec{P} - \vec{P}_0|\), where \(\vec{P}_0 = \{P_{x0}, P_{y0}, P_{z0}\}\) is the initial beam polarization.

A beam of polarized electrons is accelerated in a plasma channel by wake field self-consistently generated by a laser pulse. The laser pulse propagation in the channel can be described by the Maxwell’s wave equation for a dimensionless envelope of the pulse \(a = eE_L/(mc\omega)\) [9]:

\[
\left\{ 2i k \left( \frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) + \Delta_\perp + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right\} a = \frac{\omega_{p0}^2}{c^2} \frac{n}{N_0 \gamma_p} a.
\]

(3)

Here \(\Delta_\perp\) is a transverse part of the Laplace operator, \(n\) is the density of free plasma electrons, \(k = \omega/c\) is a wave number of laser radiation. The complex amplitude of the laser field \(\vec{E}_L\), slowly varying on the time \(\omega^{-1}\) and spatial \(k^{-1}\) scales, is connected with the high frequency electric field of the laser pulse \(\vec{E}\) by the expression

\[
\vec{E} = \vec{e}_L \Re(E_L e^{-i(\omega t - kz)}),
\]

(4)

where \(\vec{e}_L\) is the unit vector of the polarization. To describe the slowly varying motions and fields in a plasma Maxwell’s equations with relativistic hydrodynamic equations for cold plasma electrons were used and gamma factor of plasma electrons was given by [10]

\[
\gamma_p = \sqrt{1 + \frac{p^2}{m^2 c^2} + \frac{|a|^2}{2}}.
\]

(5)

Here \(p\) is the electron momentum. In the case of the axial symmetry the equation (3) can be rewritten in the form

\[
\left\{ 2i k \frac{\partial}{\partial \zeta} + \frac{k_p0}{k} \left( \Delta_\perp + 2 \frac{\partial^2}{\partial \zeta \partial \bar{\zeta}} \right) \right\} a = \frac{k_p0\nu}{k\gamma_p} a
\]

(6)

by using dimensionless comoving with the laser pulse variables

\[
\xi = k_p0(z - ct), \quad \zeta = k_{\rho0} z, \quad \bar{\rho} = k_{p0} \bar{r}_\perp, \quad \bar{p}_{\rho0} = \frac{\omega_{p0}}{c},
\]

(7)

Here \(k_{\rho0}\) is the dimensionless laser pulse width in the transverse direction.
where $\nu = n/N_0$ and $\Delta_{\perp\rho} = (1/\rho) \partial/\partial \rho (\rho \partial/\partial \rho)$. The nonlinear relativistic plasma response $\nu/\gamma_p$ can be expressed through the single scalar function $\Phi = \gamma_p - p_z/(mc)$ (which is called a wake field potential and normalized to $mc^2/e$) [9]

$$\frac{\nu}{\gamma_p} = \nu_0 + \Delta_{\perp\rho} \Phi$$

where $\nu_0 = n_0/N_0$ and $n_0 = n_0(\rho, \zeta)$ is an initial distribution of the plasma density in the channel. The quasistatic equation (the temporal variations of slow quantities are negligible on the time equal to the laser pulse duration [11]) for the wake field potential is derived in a hydrodynamic approach in case of a wide pulse (in comparison with $1/k_{p\rho}$) [9]:

$$\left\{ (\Delta_{\perp\rho} - \nu_0) \frac{\partial^2}{\partial \zeta^2} - \frac{\partial \ln \nu_0}{\partial \rho} \frac{\partial^3}{\partial \rho \partial \zeta^2} + \nu_0 \Delta_{\perp\rho} \right\} \Phi - \nu_0^2 \frac{1}{2} \left\{ 1 - \frac{1 + |a|^2/2}{(1 + \Phi)^2} \right\} = \nu_0 \Delta_{\perp\rho} |a|^2/4 \quad (9)$$

Since focusing ($F_r = \partial \Phi / \partial \rho$) and accelerating ($F_z = \partial \Phi / \partial \xi$) forces acting on beam electrons can be expressed through the potential $\Phi$, equations of motion for the $n$-th electron can also be expressed through derivatives of $\Phi$:

$$\begin{align*}
dq_{z(n)} / d\zeta &= \frac{1}{\beta_{z(n)}^2} \frac{\partial \Phi}{\partial \xi} , \quad dq_x(n) / d\zeta = \frac{1}{\beta_{z(n)}^2} \frac{\partial \Phi}{\partial \rho} \cos \phi(n) , \quad dq_y(n) / d\zeta = \frac{1}{\beta_{z(n)}^2} \frac{\partial \Phi}{\partial \rho} \sin \phi(n) , \\
d\xi(n)/d\zeta &= 1 - \frac{1}{\beta_{z(n)}^2} , \quad dx(n)/d\zeta = \frac{q_x(n)}{q_z(n)} , \quad dy(n)/d\zeta = \frac{q_y(n)}{q_z(n)} .
\end{align*}$$

Spin precession equations can be written from (1) as follows [12]:

$$\begin{align*}
ds_x(n) / d\zeta &= s_z(n) \left( a_m + \frac{1}{\gamma(n)} \right) \frac{\partial \Phi}{\partial \rho} \cos \phi(n) , \\
ds_y(n) / d\zeta &= s_z(n) \left( a_m + \frac{1}{\gamma(n)} \right) \frac{\partial \Phi}{\partial \rho} \sin \phi(n) , \\
ds_z(n) / d\zeta &= \frac{1}{\beta_{z(n)}^2} \left( a_m + \frac{1}{\gamma(n)} \right) \frac{\partial \Phi}{\partial \rho} \left[ s_z(n) \cos \phi(n) + s_y(n) \sin \phi(n) \right] ,
\end{align*}$$

where $\phi(n) = \arctan(y(n)/x(n))$, $q(n) = p_z(n)/(mc)$ is a dimensionless momentum and $\gamma(n)$ is a gamma factor of the $n$-th electron. Thus, the self-consistent solution of equations (6), (9)–(14) allows describing the spin precession of electrons accelerated by generated wake fields in the plasma channel.

3. Numerical Simulations
Depolarization of the electron beam is studied during the acceleration in the parabolic channel [13,14] with the unperturbed plasma radial profile

$$n_0 = N_0 \left( 1 + \frac{\nu^2}{R_{ch}^2} \right) ,$$

with the plasma density at the channel axis $N_0 = 10^{17} \text{ cm}^{-3}$ and the corresponding plasma wave number $k_{p\rho} = 0.0595 \text{ mm}^{-1}$. The channel radius is chosen to be $R_{ch} = 305.1 \text{ mm}$, herewith
Figure 1. Comparison between the numerical solution of equations (6), (9)–(14) and theoretical results (18) for the beam with $\sigma_r/k_p0 = 4.2 \ \mu m$ and $E_{e,\text{inj}} = 67.5 \ \text{MeV}$, accelerating $E_{\text{acc}} \cong 0.47$ and focusing $F_r = 0.075 \rho$ forces are obtained by averaging of forces acting on electrons in LWFA. The solid line corresponds to the numerical result and the dashed line corresponds to the theoretical prediction.

$k_{p0}R_{ch} = 18.16$ (compare with the matched channel radius $R_{ch}^m = 0.5k_{p0}r_L^2 = 236.34 \ \mu m$, $k_{p0}R_{ch}^m = 14.06$ [7]).

The dimensionless envelope of the laser pulse is assumed to be the Gaussian at the entrance of the channel,

$$a(\xi, \rho, \zeta = 0) = a_0 \exp \left(\frac{-\rho^2}{\rho_L^2} - \left(\frac{\xi - \xi_{L0}}{\tau_L}\right)^2\right),$$

here the normalized amplitude $a_0 = 1.414$, the laser spot size $r_L = 89.13 \ \mu m$ ($\rho_L = k_{p0}r_L = 5.303$), the pulse duration $t_L = 56 \ \text{fs}$ ($\tau_L = t_L\omega_{p0} = 1$ and $\xi_{L0} = 7$). The laser intensity and laser pulse power are $I_L = 4.28 \times 10^{18} \ \text{W/cm}^2$ and $P_L = 534 \ \text{TW}$ respectively.

The electron beam of zero emittance is injected near the accelerating field maximum ($\xi_{\text{max}} = 3.0, \xi_{\text{inj}} = 3.2$) with the initial polarization $\vec{P}_0 = \{0.279, -0.334, 0.9\}$ ($|\vec{P}_0| = 1$) and energy $E_{e,\text{inj}} = 67.5 \ \text{MeV}$, which corresponds to the gamma factor $\gamma_0 = 132$ closed to $\gamma_{ph} = 1/\sqrt{1 - \beta_{ph}^2} \cong \omega/\omega_{p0}$, where $\beta_{ph}$ is a dimensionless phase velocity of the wake wave. The initial distribution of beam electrons is chosen to be the transversely cylindrically symmetric Gaussian distribution:

$$f_e = n_{b0} \exp \left(\frac{-\rho^2}{2\sigma_r^2}\right) \delta(z - z_0) \delta(\vec{v}_e) \delta(v_e - v_{e,0}) \delta(\vec{s} - \vec{F}_0),$$

with the electron beam transverse width $\sigma_r = 0.25$ ($\sigma_r/k_{p0} = 4.2 \ \mu m$), the peak density at the channel axis $n_{b0}$ and total number of electrons $N_b = 5 \times 10^5$. Depolarization $|\Delta \vec{P}|$ of this beam oscillates with the frequency determined by betatron oscillations of electrons. The depolarization amplitude for zero-emittance beams in the presence of the uniform accelerating field $F_z = E_{\text{acc}}$ (it should be noted that $\gamma \cong \gamma_0 + E_{\text{acc}}t$ in the assumption of a low transverse energy of electrons) and the linear focusing force $F_r = -\alpha \rho$ can be estimated by analytical expression [15]:

$$\frac{|\Delta \vec{P}|_{\text{env}}}{|\vec{P}_0|} = \frac{(1 + P_{z0}^2)\sigma_r^2(\alpha \gamma_0 \gamma)^{1/2}a_m^2}{8}.$$
Figure 2. Comparison between the numerical solution of equations (6), (9)–(14) and theoretical results (18) for the beam with $\sigma_r/k_0 = 4.2 \ \mu m$ and $E_{e\ inj} = 10.2 \ \text{GeV}$. Other parameters are the same as in figure 1. The solid line corresponds to the numerical result and the dashed line corresponds to the theoretical prediction.

The numerical solution of equations (6), (9)–(14) is compared with the formula (18) (it is assumed that the focusing force with $\alpha \approx 0.075$ and accelerating force $E_{\text{acc}} \approx 0.47$ corresponds to the average value of forces acting on beam electrons during acceleration) in figure 1.

At the end of acceleration (with acceleration length of $\sim 0.5 \ \text{m}$), when the average energy of beam electrons reaches about $7.5 \ \text{GeV}$, the numerical result for the final depolarization is in a good agreement with the analytical prediction, but the theory for constant fields (18) gives an underestimated value of the depolarization at low energies (see figure 1). Figure 2 shows that the theory is already consistent with the simulation throughout the acceleration region, if the initial beam energy is about 10 GeV.

4. Summary
In this work the dynamics of beam electrons spin precession with zero-emittance and zero-energy spread is investigated in the laser wake field acceleration. The wake field is generated by the laser pulse with the Gaussian envelop in the parabolic plasma channel. The spin precession of the beam electron is described by the self-consistent numerical solution of T-BMT equations and equations of the electron motion. It was found that obtained numerical results for the dynamics of beam polarization is in a good agreement with the analytical prediction in the area of high energy of beam particles and the theory underestimates the depolarization value for energies less than about $4 \ \text{GeV}$.

Acknowledgments
This work is supported by the project No. 3.522.2014/K of the Ministry of Education and Science of the Russian Federation.

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