Modeling and Application of Passenger Transport Network Capacity in New Town Based on Control Indicators

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Abstract

In order to reflect the uncertainty of traffic demand and the orienting of planning schemes in new town, a bi-level programming model of passenger transport network capacity is proposed here based on travel choice behavior model. In the model, the objective function is to maximize the road network capacity. The saturation of road sections, the community development intensity and the public transit trip contribution rate are defined as control indicators, among which the interactive control relationships are revealed. Furthermore, a corresponding solution algorithm is also presented. At last, a numeric example is studied to verify the effectiveness of the model and the algorithm. The results show that the model and the algorithm are of highly effective and available in practice.

Keywords: New town; Passenger transport network; Capacity of the road network; Control indicator; Bi-level programming;

1. Introduction

The capacity of urban passenger transport network reveals the ability of road network in processing traffic demand. Measuring the capacity of planning network and making the oriented control process of transportation planning are very important for the development of a new town. What kinds of scale and layout in urban road network can meet the traffic demand? Which part of the network should be expanded or reformed in the case of insufficient capacity? How to balance the development intensity and the traffic facility capacity in the process of developing large urban functional area? All these problems are related to a basic problem, namely how to quantify the capacity of urban road network. Therefore, modeling of the road network capacity plays a key role in traffic planning and management(Cheng Lin et al, 2007).

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The study of road network capacity attached much attention in traffic engineering. Saito and Masuva(1989) presented an application of T-region in a linear programming problem to the calculation of zonal trip generation and attraction based on the max-flow and min-cut theorem; Zhou et al. (1996) showed time-space sources of the network of urban road and space-capacity of traffic; Yang et al. (2000) modeled the capacity and level of service of urban transportation networks; Xie et al. (2012) proposed an evaluation model of urban road network system capacity. Furthermore, Bhat (1998) optimized the allocations of urban road resources so as to maximize the capacity; Talen (2003) suggested that high density of road network can improve the average lane capacity, which would make the road network capacity increase; Huang et al. (2007) studied the network capacity and the minimum time flow at the aspect of the blocking flow by introducing the network-blocking-minimum-flow indicator; Leng et al. (2007) proposed the network capacity calculation model on the capacity limitations of parking facilities; Zhan et al. (2008) established a macroscopic model to measure the support capacity of a metropolitan transportation system based on the concept of "vehicle on road". However, all these models are based on the determine traffic demand, they can not reveal the dynamic change of traffic demand and travel structure, moreover, they lack of the feedback mechanisms between traffic demand forecasting and planning scheme and did not involve in the traffic assignment problem of multi-mode passenger transport network. Therefore, these models can’t be applied to new town directly.

The focus of our work is attempts to establish a bi-level programming model to analyze the interactive control relationships among the three indicators: saturation of road sections, intensity of regional development and traffic mode structure. Briefly then, the outline of this paper is as follows. Section 2 describes in detail the bi-level programming model of passenger transport network capacity. A solution algorithm to solve the model is presented in section 3. Section 4 introduces a numerical example and presents numerical results for testing the model and algorithm. Section 5 concludes the paper with a summary of findings and observations.

2. The bi-level programming model of passenger transport network capacity

2.1. Model hypothesis

In order to facilitate the analysis, some conditions should be restricted according to the actual situation.
(1) Road network should maintain high efficiency in a certain service level.
(2) The community development intensity is limited. In the model, the time scale is the peak hour.
(3) Considers car, bus and rail transit these three travel modes.
(4) The impedance of road section is a strictly monotone increasing function of road traffic flow. In this paper, BPR function is adopted.
(5) Passengers select their travel route in stochastic user equilibrium mode.

2.2. Mathematical model

According to the above hypothesis, the bi-level programming model of passenger transport network capacity is as follows:

The upper model: the maximization model of passenger transport network capacity.

\[ \max Z(q) = \sum_r \sum_s (q_{rs}^c + q_{rs}^b + q_{rs}^m) \]  

s.t.

\[ v_a \leq \lambda_a \times c_a \]
\[ q^*_r = \sum_s (q^{c^*_s} + q^{b^*_s} + q^{m^*_s}) \leq q^{\text{max}}_r - q^0_r \]  
(3)

\[ q^*_s = \sum_r (q^{c^*_r} + q^{b^*_r} + q^{m^*_r}) \leq q^{\text{max}}_s - q^0_s \]  
(4)

\[
\frac{q^b + q^c}{q^c + q^b + q^m} = l - \omega_1
\]  
(5)

\[
\frac{q^b}{q^c + q^b} = \omega_2
\]  
(6)

\[ q^{c^*_r}, q^{b^*_r}, q^{m^*_r} \geq 0 \]  
(7)

Where \( r, s \) means trip origin and trip destination, respectively; \( a \) means road section number. \( v_a \) and \( c_a \) are the flow and capacity of road section \( a \), units are pcu/h. \( \lambda_a \) means saturation of road \( a \), and it is dimensionless. \( q^0_r, q^*_r, q^{\text{max}}_r \) respectively mean the existing trip production, increasing trip production and maximum trip production of the origin \( r \). \( q^0_s, q^*_s, q^{\text{max}}_s \) are the existing trip attraction, increasing trip attraction and maximum trip attraction of the destination \( s \). \( q^{c^*_r}, q^{b^*_r}, q^{m^*_r} \) are the increased number of people travel from \( r \) to \( s \) by car, bus and rail transit. The units of above \( q \) are number of people per hour. \( \omega_1, \omega_2 \) respectively mean the proportion of rail transit in all traffic and the proportion of bus in the road traffic. In addition, the objective function type (1) is maximization reserve capacity of the network. Type (2) is road section capacity constraint while saturation at \( \lambda_a \). Type (3) and (4) are development intensity constraints of points O and points D. Type (5) is the rail transit trip proportion of all traffic, and type (6) is the bus trip proportion constraint of the road traffic. Type (7) is nonnegative constraint. The control indicators are saturation \( \lambda_a \), maximum trip production \( q^{\text{max}}_r \), maximum trip attraction \( q^{\text{max}}_s \) and the traffic structure \( \omega_1, \omega_2 \). However, \( v_a, q^*_r \) come from the lower model.

The lower model: multi-mode passenger traffic network equilibrium assignment model

\[
\min Z(\nu, q) = \sum_a \int_{q^0_a}^{q^*_a} t_a(v_a) \, dv + \frac{1}{\theta_1} \sum_{rsk} f^{c}_{rsk} \ln f^{c}_{rsk} + \frac{1}{\theta_2} \sum_{rsk} f^{b}_{rsk} \ln f^{b}_{rsk}
\]  
(8)

s.t.

\[
\sum_k f^{c}_{rsk} = q^{c^*_r} + q^{c^*_s} \quad \forall r, s
\]  
(9)

\[
\sum_k f^{b}_{rsk} = q^{b^*_r} + q^{b^*_s} \quad \forall r, s
\]  
(10)

\[
x^c_a = \sum_r \sum_s \sum_k f^{c}_{rsk} \delta^{c^*_r}_a \quad \forall a
\]  
(11)

\[
x^b_a = \sum_r \sum_s \sum_k f^{b}_{rsk} \delta^{b^*_r}_a \quad \forall a
\]  
(12)

\[
v_a = v^c_a + v^b_a = x_a \frac{U}{A} + x_b \frac{U}{A} \quad \forall a
\]  
(13)
$$t_a(v_a) = t_a^0 \left[ 1 + 0.15 \left( \frac{v_a}{c_a} \right)^q \right]$$  \hspace{1cm} (14)$$

$$q_n^0, q_n^+, q_n^{bo}, q_n^{bh}, f_n^c, f_n^b \geq 0$$  \hspace{1cm} (15)$$

Where $k$ means travel route. $f_n^c, f_n^b$ are the number of people travel from $r$ to $s$ through route $k$ by car and by bus, the units are number of people per hour. $\delta_{rk}^a$ is a binary variable, while the route $k$ include the road $a$, $\delta_{rk}^a = 1$, otherwise, $\delta_{rk}^a = 0$. $t_a(v_a)$ is the road impedance function, here use BPR function. $t_a^0$ is the free flow time of road $a$. $\theta_1$ and $\theta_2$ are the estimation error parameters of route impedance. $x_a^c$ and $x_a^b$ mean the traffic demand of car and the traffic demand of bus on the road $a$, units are number of people per hour; $U_c, U_b$ are standard car conversion coefficient of car and bus; $A_c, A_b$ mean average passenger number of car and bus. As the objective function, type (8) doesn't have any intuitive economic significance. In the type, the first is Beckmann transformation, the last two are the entropy function of travel route choice, which can be derived to Logit polynomial. The above form of the objective function is compatible with SUE traffic assignment model; Type (9) and (10) are travel demand conservation conditions; Type (11), (12) are road traffic demand and route traffic demand; Type (13) is relationship between road traffic and road travel demand; Type (14) is BPR function, and type (15) is nonnegative constraint.

In the planning stage, according to the sustainable development of the city requirements, the government management department control the service level, development intensity, environmental benefits and efficiency. On the other hand, users make the personal travel route choice in the network facilities. One significant characteristic of this model is taking network service level, community development intensity and traffic structure as control indicators, based on travel choice behavior, control passenger’s travel demand and traffic structure through the transportation planning scheme.

3. Algorithms

From the description of the model, we can know that the proposed model is a nonlinear bi-level programming model. The objective function of upper model is a maximum flow problem, and the lower model is a stochastic-user-equilibrium assignment problem. This kind of nonlinear bi-level programming model need to design corresponding algorithm. The upper model calculate how many flow each OD pair can accommodate according to the result of the lower model, that is so called "reserve capacity"; And the lower model assign the flow to each route in SUE mode. So repeatedly iterate until the result become convergent, the maximum capacity of the road network will be got.

F-W algorithm is effective for calculating the lower model. The detail progresses are as follow steps:

Step 1: Initialize the network, compute the impedance of each section with the initial flow, and set up a steps $i = 1$.

Step 2: Find the drop direction of objective function, and then calculate a optimal step length, intercept optimal step length at the steepest descent direction, get the next starting point, repeated iterate until the optimal solution come out.

The steepest descent direction is determined by solving the first order Taylor expansion of the objective function, just like the type below:

$$\min Z(f_{n,k}^{hi}, f_{n,k}^{ei}) = \sum_{rk} [c_{rk}^{hi} + \frac{1}{2} \ln f_{n,k}^{hi}] j \times j_{n,k}^{hi} + \sum_{rk} [c_{rk}^{ei} + \frac{1}{2} \ln f_{n,k}^{ei}] j \times j_{n,k}^{ei}$$ \hspace{1cm} (16)$$

s.t.

$$\sum_k j_{n,k}^{hi} + \sum_k j_{n,k}^{ei} = d_{n}^{i+}$$ \hspace{1cm} (17)$$
\[
\sum_k j_{r,s,k}^{b(i)} + \sum_k j_{r,s,k}^{c(i)} = q_{r,s}^{(i)}
\]  
(18)

\[
 j_{r,s,k}^{b(i)}, j_{r,s,k}^{c(i)} \geq 0
\]  
(19)

Where \( c_{r,s,k}^{b(i)} \) and \( c_{r,s,k}^{c(i)} \) are the travel cost of taking bus and taking car from \( r \) to \( s \) through route \( k \). Assume the impedances are fixed, using an all or nothing assignment can make the objective function minimization. The descent direction of objective function is determined as \( f_{r,s,k} - f_{r,s,k} \).

Step 3: According to the descent direction above, use the dimensional search method to determine the optimal step length \( \lambda \).

Step 4: Determine a new iterative starting point: \( f_{r,s,k}^{i+1} = f_{r,s,k}^{i} + \lambda (f_{r,s,k} - f_{r,s,k}) \).

Step 5: Convergence test, if \( f_{r,s,k}^{i+1} - f_{r,s,k}^{i} \leq \xi_i \), end the algorithm. Otherwise, continue and turn back to step 1, calculate the road impedance.

To sum up, the general iterative algorithm process of the whole model is shown below:

Step I: Initialize network, find out a set of feasible route flows as the initial solution;

Step II: Transformed the route flow to OD traffic flows, and bring it into the lower model to do a SUE assignment, then update the route flows of OD pairs;

Step III: Bring the route into the upper model. Using the simplex method to find each route reserve capacity under the current traffic loading;

Step IV: Judge the each route reserve capacity is less than the default convergence parameters or not. if not, take the route reserve capacity as a new flow loading, then return to the step 2; If so, then stop the iteration, output the route flows data.

4. Numerical example

Construct a simple traffic network diagram like Figure 1, in which there are 9 nodes and 12 sections. This road network includes four OD pairs (1-3, 1-9, 7-3, 7-9). There are three traffic modes: car, bus and rail transit. The saturation of section and initial travel time are shown in Tab 1 Parameters of road section. The limit of each node's development intensity is pre-established by planning department, take the number of people who live in the area as the development intensity, as it shown in Figure 1 (the numbers below the nodes), and the peak hour travel rate is 0.5. There are 8 valid routes by calculating the route impedance, 1→2→3, 1→4→5→2→3, 1→4→5→6→3, 1→4→5→6→9, 7→4→5→2→3, 7→4→5→6→3, 7→4→5→6→9, 7→4→5→6→9, 7→8→9.

![Fig. 1 Network Diagram](image-url)
Table 1 Parameters of road section

| Section Number | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
|----------------|----|----|----|----|----|----|----|----|----|----|----|----|
| t₀ (min)       | 15 | 10 | 5  | 10 | 10 | 17 | 8  | 5  | 6  | 10 | 10 | 5  |
| C (pcu/h)      | 1600 | 3200 | 4800 | 3200 | 1920 | 3200 | 1280 | 3200 | 3200 | 960 | 1920 |

Assume that the constraints of section saturation $\lambda_s = 0.7$, route selection random factors $\theta_1 = \theta_2 = 1$, standard car conversion coefficient $U_c = 1, U_b = 3$, the average passengers $A_c = 1.8, A_b = 30$, the proportion of rail transit $\omega_r = 0.4$, bus proportion of road traffic $\omega_2 = 0.5$ or $\omega_2 = 0.3$ or $\omega_2 = 0.2$, convergence criteria $\xi_1 = \xi_2 = 0.01$. According to the algorithm proposed above, we can get the maximum capacity of network and each route flow and impedance. As shown in Tab 2.

Table 2 Route flow and route impedance

| OD Pair | Route | $w_2 = 0.2$ | $w_2 = 0.3$ | $w_2 = 0.5$ |
|---------|-------|-------------|-------------|-------------|
|         |       | Route flow  | Impedance   | Route flow  | Impedance   | Route flow  | Impedance   |
| (1,3)   | 1→2→3 | 1122.24     | 25.75       | 1117.76     | 25.74       | 1116.16     | 25.72       |
|         | 4→5→2→3 | 1117.76     | 25.74       | 1116.16     | 25.72       | 1115.60     | 25.75       |
| (1,7)   | 7→4→5→2→3 | 1115.60     | 25.75       | 1114.98     | 25.74       | 1114.46     | 25.72       |
|         | 4→5→6→3 | 1114.98     | 25.74       | 1114.46     | 25.72       | 1114.04     | 25.71       |
|         | 4→5→6→9 | 1114.04     | 25.71       | 1113.60     | 25.70       | 1113.16     | 25.68       |
| (7,3)   | 7→4→5→2→3 | 1113.16     | 25.68       | 1112.72     | 25.67       | 1112.32     | 25.65       |
|         | 4→5→6→3 | 1112.72     | 25.67       | 1112.32     | 25.65       | 1111.92     | 25.63       |
|         | 4→5→6→9 | 1111.92     | 25.63       | 1111.52     | 25.61       | 1111.12     | 25.59       |

Through analyzing the result, each route flow between OD pair basically match the result assigned by Logit mode. Take the 3 routes of OD pair between (1, 3) as example, when $\omega_2$ proportion are 0.900, 0.065 and 0.035, these 3 routes’ Logit selection proportion are 0.900, 0.065 and 0.035. And their proportions of total traffic flow are 0.906, 0.062 and 0.032. These two data prove that tow proportions are totally coincide with each other, which means the travelers’ route selection is based on the SUE principle.

By further analyzing the saturation of road section and the development intensity, we can find out limit section and limit node, shown in Tab 3 and Tab 4. It is obviously that the limit section and the limit node are distinct in the different traffic structures. For example, when the bus proportion is 0.2, the limit sections will be 1, 5, 8, 12 and no limit node. However, when the proportion of bus is 0.5, limit sections will be 1, 5 and limit node are 1, 2.

Table 3 Road section saturation

| Section | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
|---------|----|----|----|----|----|----|----|----|----|----|----|----|
| $V_s/C_s$ | $w_2 = 0.2$ | 0.700 | 0.610 | 0.530 | 0.528 | 0.700 | 0.420 | 0.415 | 0.698 | 0.112 | 0.387 | 0.000 | 0.693 |
| $w_2 = 0.3$ | 0.698 | 0.607 | 0.528 | 0.520 | 0.700 | 0.420 | 0.406 | 0.657 | 0.112 | 0.387 | 0.000 | 0.680 |
| $w_2 = 0.5$ | 0.698 | 0.598 | 0.404 | 0.346 | 0.698 | 0.419 | 0.235 | 0.622 | 0.102 | 0.372 | 0.000 | 0.406 |

Table 4 The ratio of developed intensity to maximum development intensity

| Node number | 1  | 7  | 3  | 9  |
|-------------|----|----|----|----|
| $q_r$ (or $q_l)/q_r^{max}$ | $w_2 = 0.2$ | 0.910 | 0.722 | 0.659 | 0.599 |
| $w_2 = 0.3$ | 0.990 | 0.800 | 0.728 | 0.657 |
| $w_2 = 0.5$ | 1.000 | 1.000 | 0.902 | 0.867 |
Through analysis of the example result, some feedback suggestions of this planning program can be drawn as follows.

I. When the bus-contribution rate is 0.2 of road traffic, all the 4-node maximum development intensity cannot be satisfied and the road network is completely unable to support the development of the region. Therefore, the road network has become a bottleneck and limits the development of regional economy. To deal with this big problem, capacity of the limit section should be expanded.

II. When the bus-contribution rate is increased to 0.3, the road network can meet the requirement of the node 1 maximum development intensity. In this case, we can adjust road section expanding and increase development intensity to get balance.

III. When the bus-contribution rate is 0.5, the maximum flow of the road network has far exceeded the maximum development intensity of nodes 1, 2. Necessary of raising the maximum node development intensity should be considered at this time.

IV. Sections 11 have no flow in these three contribution rate modes. So, as an invalid section, the construction necessity should be re-analyzed again.

5. Conclusions

Basing on the traffic planning characteristics in a new town, this paper attempts to establish a bi-level programming model to analyze the interactive control relationships among the three indicators: saturation of road sections, intensity of regional development and traffic mode structure, and design a heuristic algorithm to solve the model. Also, this paper uses a numerical example to illustrate the effect of the method. According to the research method, the model can be extended to solve multi-mode new town transportation planning problem, in which the impact of the relationship between the different kinds of transport modes and the effects of intersections impedance should be considered more carefully. More effective algorithms should be applied in real road network in future research works.

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