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ABSTRACT
Due to the complex phase change and heat transfer processes, the mechanisms of cavitation bubble collapse near a rigid boundary are well recognized to be complicated. Based on a modified large-density ratio multi-relaxation-time pseudo-potential lattice Boltzmann model, a single and a dual cavitation bubble collapse process near a rigid boundary with large-density and various viscosity ratios are simulated in the present study. Effects of density ratio, viscosity ratio, initial pressure difference, and distance between the cavitation bubble and wall on the cavitation process are studied. Furthermore, the evolution of maximum pressure, micro-jet velocity, lifetime, deformation index, and the first introduced total kinetic energy of cavitation bubbles are analyzed in the development of cavitation. Simulations show that the interaction mode of the bubbles and the distance between the rigid boundary and the lower bubble are key factors in determining the effect of aeration reduction. The study also shows that the proposed lattice Boltzmann pseudo-potential model is a robust and effective tool for studying the collapse of near-wall cavitation bubbles and has potential to predict the interaction of cavitation bubbles in the presence of complex boundaries.

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I. INTRODUCTION
Near-wall cavitation is a widespread phenomenon in both natural and engineering processes, such as hydraulic engineering, navigation, and humoral circulation. In growth and collapse process, there is a rapid phase transformation between gas and liquid phases, accompanied by a large increase in temperature and the generation of micro-jet and shock wave, which results in noise, vibration, and cavitation erosion. Although a substantial quantity of research has been conducted in the last few decades, some mechanisms are still not clear, including the evolution of the temperature field, deformation of cavitation bubbles, and threshold parameters of bubble formation and collapse near the complex boundary. However, cavitation in the near-wall region is more complicated and the cavitation bubble interface changes very sharply. It is difficult to theoretically study cavitation bubble collapse in the near-wall region.

To study the evolution behavior of bubbles while interacting with other bubbles or complex boundaries, a series of experiments were conducted. How the cavitation bubble deforms and the causes of formation of micro-jet and shock wave were studied. Blake studied the interaction mode of two cavitation bubbles near a rigid boundary. Later, bubbles collapsing and interacting with each other under complex boundary conditions were also studied experimentally. Through extensive observations, Philipp concluded that the mechanism of cavitation damage is a hydro-combined action of micro-jet, shock wave, and dramatic temperature change. All the experimental studies mainly focused on how the micro-jet, shock wave, and high temperature form, and how the bubbles deform and interact with each other.

With the development of computer technology, computational fluid dynamics (CFD) has become an important tool to study the evolution process of cavitation bubbles. When compared to
experiments, the numerical simulation could test the evolution of cavitation bubbles under various condition combinations or even extreme ones, which are difficult to be achieved in experiments with little expense. In addition, the development of all physical fields (e.g., pressure, velocity, and density fields) could be checked in higher temporal and spatial resolution. Traditional CFD methods to study the evolution of cavitation bubbles mainly depend on the Navier–Stokes equation and Rayleigh–Plesset equation. Depending on different assumptions regarding the compressibility of cavitation bubbles and different interface capture methods, numerical simulation methods of macroscopic cavitation bubbles can be divided into three categories to study the causes of micro-jet, shock wave, high temperature, and cavitation noise.19–21 The first method assumes that the gas in the bubble and the surrounding liquid has the same fluid component. The gas–liquid phase change during bubble growth and collapse is then based on the equation of state (EOS) relating the density, pressure, and temperature of this fluid, and the compressibility of the fluid is also taken into account.21–27 According to the research of Ghahramani,21 a small time step and fine grids are needed to capture the liquid–gas interface. The second method is based on the Navier–Stokes equations together with a mass transfer equation to obtain the mass exchange between the gas and liquid phases. To improve its accuracy, some empirical constants are usually introduced into the Rayleigh–Plesset equation of this kind of model to obtain the gas–liquid mass transfer rate.22,28 The third method, also called the discrete bubble model (DBM), uses the Euler equation to simulate the continuum flow and a Lagrange method to capture the changes in shape and the motion of cavitation bubbles.19,24 This method also needs a fine grid, and can be used to simulate complex gas–liquid interface changes, such as the movement and collapse of a group of cavitation bubbles.

As a mesoscopic method, the lattice Boltzmann method (LBM), which is based on Boltzmann kinetic molecular dynamics, has become a robust and efficient tool to simulate the multiphase phenomenon. All the established LBM multiphase models can be categorized into color gradient model,26–27 pseudo-potential model,28,29 free energy model,28,30 interface tracking model,30 and entropy model.31 The pseudo-potential model, proposed by Shan and Chen,26,27 uses the interaction force between particles to form the interface automatically, and this special treatment makes interface tracking easier. With a non-ideal gas EOS introduced in this model, the pseudo-potential model is capable of simulating the phase change processes, such as liquid boiling and crystallization.3 After the first application of the pseudo-potential model in studying the growth of cavitation bubbles by Sukop,32 LBM gradually occupies a competitive place in the field of the numerical study of cavitation. When the Bhatnagar–Gross–Krook (BGK) collision operator33–36 is used to simulate the growth and collapse of cavitation bubbles, the calculated density ratio is 30–70, while an multi-relaxation-time (MRT) collision operator37–40 is used, the calculated density ratio can be increased to 720, range of density ratio values found in actual cavitation phenomena.38–44 The particle distribution functions with external force terms can be expressed as

$$f_i(x + e_i \Delta t, t + \Delta t) = f_i(x, t) - \sum_j \Lambda_{ij} (f_j - f_j^{eq})|e_i| \Delta t$$

where \(f_i\) is the particle distribution function, \(f_j^{eq}\) is the equilibrium distribution, \(x\) is the spatial position, \(e_i\) is the discrete velocity of ith direction, \(\Delta t\) is the time step, \(I_{ij}\) is the unit matrix, \(\Lambda = M^{-1} A\) is the collision matrix, and the diagonal matrix \(A\) can be described as

$$A = \text{diag}(\tau_p^{r_1}, \tau_p^{r_1}, \tau_p^{r_1}, \tau_q^{r_1}, \tau_q^{r_1}, \tau_q^{r_1}, \tau_v^{r_1}, \tau_v^{r_1}, \tau_v^{r_1})$$

### II. MODEL DESCRIPTION

Compared to the BGK collision operator, the pseudo-potential model with an MRT collision operator has greater numerical stability and lower spurious currents when simulating some high-density ratio multiphase phenomenon.26–31 The particle distribution functions with external force terms can be expressed as

$$f_i(x + e_i \Delta t, t + \Delta t) = f_i(x, t) - \sum_j \Lambda_{ij} (f_j - f_j^{eq})|e_i| \Delta t$$

where \(f_i\) is the particle distribution function, \(f_j^{eq}\) is the equilibrium distribution, \(x\) is the spatial position, \(e_i\) is the discrete velocity of ith direction, \(\Delta t\) is the time step, \(I_{ij}\) is the unit matrix, \(\Lambda = M^{-1} A\) is the collision matrix, and the diagonal matrix \(A\) can be described as

$$A = \text{diag}(\tau_p^{r_1}, \tau_p^{r_1}, \tau_p^{r_1}, \tau_q^{r_1}, \tau_q^{r_1}, \tau_q^{r_1}, \tau_v^{r_1}, \tau_v^{r_1}, \tau_v^{r_1})$$

### Table I. Parameters of the relevant research on cavitation bubble collapse.

| Researchers | Collision operator | Force scheme | \(T/T_c\) | \(\rho/\rho_g\) | \(v_p/v_l\) |
|-------------|--------------------|--------------|----------|----------------|---------------|
| Shan35      | BGK                | EDM          | 0.689    | 40             | 1             |
| Mao36       | BGK                | EDM          | 0.68     | 40             | 1             |
| Shan47      | MRT                | Li19        | 0.5      | 720            | 1             |
| Xue39       | MRT                | Li15        | 0.8      | 12.8           | 1             |
where the relaxation time $\tau_5$ is correlated with the kinematic viscosity $\nu = 1/c_s^2(\tau_5 - 0.5)\Delta t$, $c_s$ is the lattice sound speed, $M$ is the transformation matrix, and $M^{-1}$ is its inverse matrix. Using $M$, the equilibrium moments $m^{eq}$ can be obtained by projecting the equilibrium distribution $f^{eq}$ onto the moment space, and for the D2Q9 LBM model, $m^{eq}$ is expressed as
\[
 m^{eq} = Mf^{eq} = \rho\left(1 - 2 + 3(u_i^2 + u_j^2), 1\right)
 \]
\[
 - 3\left(u_i^2 + u_j^2\right)u_i - u_j, u_i - u_j, u_i - u_j, u_i - u_j, u_i - u_j\right). \tag{3}
\]

Li et al.\cite{55,56} proposed a modified external forcing scheme $S$, which can achieve thermodynamic consistency, given by the following expression:
\[
 S = \rho\left[0.6(u_iF_x + u_jF_y) + \frac{12c|F_m|^2}{\psi^2\Delta t(\tau_5 - 0.5)}(u_xF_x + u_yF_y)\right]
 \]
\[
 - \frac{12c|F_m|^2}{\psi^2\Delta t(\tau_5 - 0.5)}(F_x, F_y, F_z, 2(2u_xF_x - u_iF_y), 2u_xF_x + u_yF_y), \tag{4}
\]

where the value of the parameter $\epsilon$ can be adjusted to achieve thermodynamic consistency. This modified forcing term enlarges the range of density ratio and increases the numerical stability of the model. $\rho$ is the macroscopic density and $u$ is the macroscopic velocity, which can be obtained as
\[
 \rho = \sum_i f_i \rho u = \sum_i f_i \rho \Delta t, \tag{5}
\]

where $F = F_m + G + \cdots$ is the total force acting on the fluid particles, $F_m$ is the interaction force between particles, which can be obtained as
\[
 F_m = -G\psi(x) \sum_i \omega_i \psi(x + \xi_i)\Delta t, \tag{6}
\]

where $G$ is the interaction strength between two particles, $\psi$ is the inter-particle potential, and $\omega_i$ are the weights. For the D2Q9 LBM model, $\omega_{1-4} = 1/3$ and $\omega_{5-8} = 1/12$. $\psi$ can be calculated with a non-ideal gas EOS, which is introduced as
\[
 \psi(\rho) = \sqrt{\frac{2(p_{SOS} - \rho c_s^2)}{Gc^2}}, \tag{7}
\]

where $p_{SOS}$ is the pressure calculated by the non-ideal gas EOS.

Finally, the collision process can be described as
\[
 m(RHS) = \rho - \overline{X}(m - m^{eq}) + \Delta t\left(I - \frac{\overline{X}}{2}\right)S, \tag{8}
\]

and the stream process can be expressed as
\[
 f_i(x + \xi_i\Delta t, t + \Delta t) = f_i(x, t) = M^{-1}m(RHS). \tag{9}
\]

III. NUMERICAL RESULTS AND DISCUSSION
A. Model validation

The computational domain is a $401 \times 401$ $lu^2$ square domain as shown in Fig. 1. A Zou–He pressure inlet condition\cite{48} is applied at the upper boundary, non-equilibrium extrapolation conditions are applied at both the left and right boundaries, and the no-slip boundary conditions\cite{48} in Eq. (11) are applied at the bottom boundary. A spherical bubble with radius $R_{ini} = 50lu$ is initialized in the computational domain, $d$ is the distance between the bubble center and rigid boundary, $p_i$ is pressure inside the bubble, and $p_{\infty}$ is the ambient and the inlet boundary pressures,
\[
 f_2 = f_4, \quad f_5 = f_7 - 0.5(f_1 - f_3) - 0.25\Delta t(F_x + F_y), \tag{10}
\]
\[
 f_6 = f_8 + 0.5(f_1 - f_3) + 0.25\Delta t(F_x - F_y).
\]

The Carnahan–Starling (C–S) EOS is applied in the present study.\cite{58}

![FIG. 1. Schematic of the computational domain for the evolution of a single cavitation bubble.](image-url)
The density field is initialized as follows:

$$\rho(x, y) = \frac{(\rho_l + \rho_g)}{2} + \frac{(\rho_l - \rho_g)}{2} \times \tanh \left[ \frac{2\sqrt{(x - x_0)^2 + (y - y_0)^2}}{w} - R_{ini} \right],$$

(12)

where \((x_0, y_0)\) is the center of the bubble and the initial interface width is set as 5 \(lu\). In the present study, the interaction strength is set as \(G = -1\), and the relaxation parameters are chosen as follows: \(\tau_p = \tau_g = 1.0\), \(\tau_l = 1.1\), and \(\tau_e = 1.1\). Considering the density ratio of the actual cavitation phenomena in water as about \(\rho_l/\rho_g = 750\), we choose the temperature \(T = 0.5T_c\) with the corresponding liquid–gas density as \(\rho_l/\rho_g \approx 720\). The viscosity relaxation time \(\tau_e\) can be obtained by the following linear interpolation:

$$\tau_v = \tau_g + \left( \frac{\rho - \rho_g}{\rho_l - \rho_g} \right) (\tau_l - \tau_g),$$

(13)

where \(\tau_g\) is the relaxation time of gas, \(\tau_l\) is the relaxation time of liquid, \(\rho_l\) and \(\rho_g\) are the gas density and liquid density, respectively. Equation (13) provides a tunable viscosity ratio method for the present study. \(\varepsilon\) is chosen as 0.112 to maintain the numerical stability and thermodynamic consistency.

Some normalized parameters are introduced in this paper to describe the effect of the rigid boundary, including the deformation parameter \(Ra = h/l\) and the distance ratio of radius to distance between the center of the bubble and wall \(y = d/R_{ini}\). In addition, the dimensionless time step \(t' = tl/t_{max}\) and dimensionless maximum pressure \(p' = p_{max}/\Delta \rho\) are also introduced in our study. The units of measurement discussed in this paper include mass unit (\(mu\)), time step (\(tu\)), velocity unit (\(lu \cdot tu^{-1}\)), and pressure unit (\(mu \cdot lu^{-1} tu^{-2}\)).

The radius of evolution of cavitation bubble in the near-wall region can be predicted by the Rayleigh–Plesset equation, \(^{46}\)

$$\ln \left( \frac{d}{R_b} \right) \left( \frac{R_b^2}{R_g^2} + R_g \frac{\dot{R}_b}{R_b} \right) - \frac{R_b^2}{2} - 2\nu_l \frac{\dot{R}_b}{R_b} + \frac{\sigma}{\rho_l R_b^2} \frac{\dot{p}_l - p_{\infty}}{\rho_l} = \frac{\Delta \rho'}{\rho_l}.$$

(14)

where \(R_b\) is the bubble radius, \(\nu_l\) is the liquid viscosity, and \(\sigma\) is the surface tension. The evolution of a bubble with \(d = 1.6R_{ini}\) is simulated, at an initial pressure difference of 0.00253 \(mu \cdot lu^{-1} tu^{-2}\), with the corresponding liquid viscosity \(\nu_l = 0.0125\) and surface \(\sigma = 0.0105\). The comparison between the normalized result of LBM and Rayleigh–Plesset prediction is shown in Fig. 2. It is found that the simulation result of LBM agrees quite well with the Rayleigh–Plesset prediction.

$$p_{\text{ROS}} = \rho RT \frac{1 + b\rho/4 + (b\rho/4)^2 - (b\rho/4)^3}{(1 - b\rho/4)^2} - a\rho^2,$$

(11)

where \(a = 0.4693R^2 T_c^2 / \rho_c\) and \(b = 0.1873RT_c / \rho_c\) are the two parameters of the C-S EOS. The parameter \(a\) can affect the interface thickness \(w\); considering both numerical stability and validity of the results, the initial interface thickness is suggested to be chosen as \(w = 4 \sim 5\) lattice units.\(^{50}\) Therefore, the parameters in C-S EOS can be chosen as follows: \(a = 0.25\), \(b = 4\), and \(R = 1\).

The units of measurement discussed in this paper include mass unit (\(mu\)), time step (\(tu\)), velocity unit (\(lu \cdot tu^{-1}\)), and pressure unit (\(mu \cdot lu^{-1} tu^{-2}\)).

B. The evolution process of a single bubble near a rigid boundary

Philipp \(^{\text{12}}\) referred that when \(d > 2.2R_{ini}\), the effect of the wall on the evolution of the cavitation bubble can be neglected. The influence of different parameters on the evolution characteristics of the cavitation bubble is studied in detail in this part.

For the LBM pseudo-potential model with non-ideal EOS, the pressure and surface tension are related to the temperature and the liquid/gas density ratio, which makes it difficult to compare the maximum pressure, maximum micro-jet velocity, and lifetime of the cavitation bubble directly under different density ratios. Therefore,
a dimensionless initial pressure \( \Delta p' = \Delta p_{\text{max}}/\Delta p_e \) is introduced to study the effect of the density ratio on cavitation bubble collapse, where \( \Delta p_{\text{max}} \) is the maximum pressure difference that can maintain the numerical stability during the cavitation bubble collapse process under different density ratios, and \( \Delta p_e \) is the pressure difference of the cavitation bubble in the equilibrium state. Four different density ratios \( \rho_l/\rho_g = 12.8, 40, 135.6, \) and 720 are chosen to study the effect of the liquid/gas density ratio, and the change in \( \Delta p' \) with density ratio is shown in Fig. 3. It is found that \( \Delta p' \) decreases with the increase in the density ratio. The surface tension of the cavitation bubble increases with the increase in the density ratio of the LBM pseudo-potential model, leading to the increase in the equilibrium pressure difference \( \Delta p_e \). Due to the density gradient on the interface, the numerical stability decreases with the increase in the density ratio, which also makes the dimensionless initial pressure \( \Delta p' \) decrease with the increase in the density ratio.

The fluid viscosity can be changed with the relaxation time in LBM. In order to study the influence of fluid viscosity on the evolution process, different relaxation times are chosen for gas and liquid. The initial pressure difference \( \Delta p = p_{\infty} - p_v \) is set as 0.00253 \( \text{mu} \cdot \text{lu}^{-1} \cdot \text{tu}^{-2} \) and dimensionless distance \( \gamma \) is 2.0. The lifetime \( t_{\max} \), maximum micro-jet velocity \( u_{\max} \), and maximum pressure \( p_{\max} \) during the evolution process vary with gas relaxation times \( \tau_g \) and liquid relaxation times \( \tau_l \), which are shown in Fig. 4.

![Figure 4](image-url)
First, we set the liquid relaxation time \( t_l \) as 0.5375, and the gas relaxation time \( t_u \) is set from 0.5375 to 1.0625, with the corresponding gas viscosity \( \nu_l \) ranging from 0.0125 to 0.1875, to discuss the effect of gas viscosity. As shown in Fig. 4(a), the numerical results indicate that the lifetime \( t_{\text{max}} \), maximum velocity \( u_{\text{max}} \), and maximum pressure \( p_{\text{max}} \) of different gas relaxation times \( t_u \) are consistent with each other. Then, we set the gas relaxation time \( t_u \) as 1.0625, and the liquid relaxation times \( t_l \) ranging from 0.5375 to 0.7, with the corresponding liquid viscosities \( \nu_l \) ranging from 0.0125 to 0.0667. The results show that the lifetime of the bubble decreases from 769 to 714 with \( \nu_l \) decreasing, while \( u_{\text{max}} \) and \( p_{\text{max}} \) decrease with \( \nu_l \) increasing. Maximum pressure \( p_{\text{max}} \) increases about 11 times with \( t_u \) change from 0.021 \( \text{mu} \cdot \text{lu}^{-1} \text{tu}^{-2} \) to 0.266 \( \text{mu} \cdot \text{lu}^{-1} \text{tu}^{-2} \), while \( \nu_l \) decreases from 0.0667 to 0.0125. Meanwhile, the maximum micro-jet velocity increases 110% from 0.448 \( \text{lu} \cdot \text{tu}^{-1} \) to 0.943 \( \text{lu} \cdot \text{tu}^{-1} \) with the reduction in \( \nu_l \) from 0.0667 to 0.0125. The numerical results are consistent with the conclusion of Popinet and Minster, and lower liquid viscosity is preferred to obtain a higher cavitation pressure and larger micro-jet velocity.

To investigate the influence of initial pressure difference between the bubble and ambient liquid on the evolution process, five different initial pressure differences are chosen from 0.0013 \( \mu \text{m} \cdot \text{lu}^{-1} \text{tu}^{-2} \) to 0.0068 \( \mu \text{m} \cdot \text{lu}^{-1} \text{tu}^{-2} \) in our study. Figure 5 shows that the deformation index changes with dimensionless time under different \( \Delta p \), the maximum \( Ra \) stays in the range between 1.1 and 1.2, and the deformation index \( Ra \) decreases with the increase in the initial pressure difference \( \Delta p \). Figure 6 shows the lifetime \( t_{\text{max}} \), maximum micro-jet \( u_{\text{max}} \), and maximum pressure \( p_{\text{max}} \) change with an initial pressure difference \( \Delta p \). The maximum pressure \( p_{\text{max}} \) and maximum micro-jet \( u_{\text{max}} \) velocity increase with the increase in \( \Delta p \). When \( \Delta p \) increases from 0.0013 \( \mu \text{m} \cdot \text{lu}^{-1} \text{tu}^{-2} \) to 0.0068 \( \mu \text{m} \cdot \text{lu}^{-1} \text{tu}^{-2} \), \( u_{\text{max}} \) increases 5.19 times from 0.047 \( \text{lu} \cdot \text{tu}^{-1} \) to 2.44 \( \text{lu} \cdot \text{tu}^{-1} \), and \( p_{\text{max}} \) increases 88.57 times from 0.014 \( \mu \text{m} \cdot \text{lu}^{-1} \text{tu}^{-2} \) to 1.24 \( \mu \text{m} \cdot \text{lu}^{-1} \text{tu}^{-2} \). On the contrary, it is found that the collapse time of the cavitation bubble decreases with the increase in the initial pressure difference, when \( \Delta p \) increases from 0.0013 \( \mu \text{m} \cdot \text{lu}^{-1} \text{tu}^{-2} \) to 0.0068 \( \mu \text{m} \cdot \text{lu}^{-1} \text{tu}^{-2} \) and \( t_{\text{max}} \) decreases from 925 \( \text{tu} \) to 487 \( \text{tu} \).

Five dimensionless distances \( y \) are chosen to study their relationship with the lifetime \( t_{\text{max}} \). Figure 7(a) shows that when the bubble stays in the near-wall region, where \( d < 2.2 Ra_{\text{ini}} t_{\text{max}} \), \( t_{\text{max}} \) increases linearly when \( y \) decreases. The rigid wall affects the evolution of the pressure field between the wall and cavitation bubble, making the deformation index \( Ra \) larger, as shown in Fig. 7(b).

The evolution process of the cavitation bubble is presented in this section where the initial pressure difference \( \Delta p = 0.0038 \mu \text{m} \cdot \text{lu}^{-1} \text{tu}^{-2} \), the initial radius \( R_{\text{ini}} = 50 \text{lu} \), and the corresponding gas–liquid viscosity ratio is 15. The evolution process of the
FIG. 7. The effect of the dimensionless distance $\gamma$ between the bubble center and rigid boundary on (a) the lifetime of bubble and (b) deformation parameters change.

cavitation bubble obtained by our simulation agrees well with the experimental result,\textsuperscript{12} as shown in Fig. 8.

Due to the influence of the rigid boundary, the vertical compression rate of the cavitation bubble is larger than the horizontal one with evolution time from $t = 0$ to $t = 470 \, tu$. At this collapse stage, the velocity on the interface is mainly the radial velocity along the radius of the centroid of the bubble, making the cavitation bubble change from spherical to ellipsoid, with the deformation parameter $Ra$ varying from 1 to 1.104. Then, a dent occurred at the top of the interface of the bubble, the shrinkage rate in the vertical direction is higher than that in the horizontal direction. The height of the cavitation bubble decreases rapidly, and $Ra$ changes from 1.104 to 0.

The flow field around the cavitation bubble is shown in Fig. 9. A high-pressure zone is formed, while a dent occurs at the top of the interface at $t = 500 \, tu$. In addition, the pressure of the area below the cavitation bubble is affected by the wall with a low-pressure zone formed here. With the dramatic deformation of the cavitation bubble in the collapse stage, the area of the high-pressure region and the maximum pressure $p_{\text{max}}$ increase rapidly, while the pressure difference between the high-pressure region and low-pressure region increases rapidly, which accelerates the deformation of the cavitation bubbles and eventually leads to the collapse of the cavitation bubbles. In addition, the pressure at the collapse point increases sharply to 0.529 $mu \cdot lu^{-1} \cdot tu^{-2}$, and the shock wave is formed. In addition, the maximum velocity and maximum pressure variation with time during the cavitation bubble collapse are shown in Fig. 10. The maximum pressure and the maximum velocity increase slowly in the early collapse process. However, with the violent deformation of the cavitation bubble during $t = 500–610 \, tu$, the maximum velocity increases rapidly to 1.36 $lu \cdot tu^{-1}$, and the maximum pressure increased to 0.529 $mu \cdot lu^{-1} \cdot tu^{-2}$.

The evolution of the cavitation bubble is a process with energy accumulation and release, and the TKE of the cavitation bubble is introduced to describe the collapse process from the perspective of energy. The surrounding liquid works on the cavitation bubble through pressure; as the volume of the cavitation bubble shrinks, the distance between gas molecules decreases, which makes the molecular potential energy and molecular total kinetic energy increase. In addition, the energy release to the surrounding liquid in a short time

FIG. 8. Collapse process of simulation results: (a) $t = 530 \, tu$, (b) $t = 550 \, tu$, (c) $t = 580 \, tu$, (d) $t = 590 \, tu$, (e) $t = 600 \, tu$, and (f) $t = 610 \, tu$, and the experimental results\textsuperscript{12} with $\Delta p = 0.0038 \, mu \cdot lu^{-1} \cdot tu^{-2}$.
FIG. 9. The flow field changes at the collapse stage: (a) $t = 100$ $tu$, (b) $t = 400$ $tu$, (c) $t = 500$ $tu$, (d) $t = 550$ $tu$, (e) $t = 580$ $tu$, and (f) $t = 610$ $tu$ at $\Delta p = 0.0038$ $mu \cdot lu^{-1} \cdot tu^{-2}$.

with the phase change happens and shock waves form on the macroscopic. The definition of TKE of the cavitation bubble is described as follows:

$$\text{TKE} = \sum \rho_i |u|^2.$$  \hfill (15)

The density cutoff value $\rho_c \leq (\rho_g + \rho_l)/2$ is used to identify the gas phase. The evolution of the cavitation bubble can be divided into two stages from the perspective of energy, energy accumulation, and release. The energy accumulation with the cavitation bubble shrinks in most of the collapse time. However, the TKE releases sharply in a short time with the gas phase transfer into liquid. As shown in Fig. 11, the time of accumulation stage of TKE is from $t = 0$ to $t = 585$ $tu$, reaching a maximum value of $1.041$ $mu \cdot lu^{-1} \cdot tu^{-2}$. Later, from $t = 585$ $tu$ to $t = 611$ $tu$, TKE releases radically in a short time, decreasing from $1.041$ $mu \cdot lu^{-1} \cdot tu^{-2}$ to $0.041$ $mu \cdot lu^{-1} \cdot tu^{-2}$.
C. The interaction of two bubbles near a rigid boundary

In most natural phenomena, cavitation occurs in large numbers; bubbles not only interact with the boundary, but also interact with each other. Previous studies indicate the interaction between multiple cavitation bubbles, and the wall may change the direction of the micro-jet, the propagation process of the shock wave, and the wall-effect range.

The interaction of two cavitation bubbles near a rigid boundary is studied in the present study. The computational domain is a 401 × 601 lu² rectangular area, as shown in Fig. 12, and Zou–He pressure inlet boundary conditions are applied in the top of the domain; both the left and the right side are non-equilibrium extrapolation boundaries, and the no-slip boundary is used in the bottom. The density ratio is set as ρ_l/ρ_g ~ 720, and the relaxation time of the gas phase is τ_g = 1.0625 with the liquid phase set as τ_g = 0.5375, making the ratio of the viscosity ν_g/ν_l = 15. All these make the gas–liquid

![FIG. 10. Deformation parameter Ra, maximum pressure p_{max}, maximum velocity u_{max} changes with time at Δp = 0.0038 m_μ · lμ−1 tμ−2.](image)

![FIG. 11. TKE varies during the collapse process with Δp = 0.0038 m_μ · lμ−1 tμ−2.](image)

![FIG. 12. Schematic of the computational domain for the evolution of the interaction of two cavitation bubbles.](image)
TABLE II. Parameters for different interaction cavitation cases.

| Case  | $R_{ini1}$ (lu) | $R_{ini2}$ (lu) | $d$ (lu) | $d_2$ (lu) |
|-------|----------------|----------------|---------|-----------|
| Case 1| 50             | 50             | 311     | 159       |
| Case 2| 50             | 50             | 219     | 86        |
| Case 3| 50             | 50             | 180     | 50        |

density ratio and the viscosity ratio close to the actual cavitation phenomena. In our simulation, the initial pressure difference between the bubble and ambient liquid is $\Delta p = 0.0025 \text{mu} \cdot \text{lu}^{-1} \cdot \text{tu}^{-2}$. Three cases are simulated, and the corresponding parameters are shown in Table II. Two normalized parameters, $\gamma_1 = d/R_{ini2}$ and $\gamma_2 = d_2/R_{ini1}$, are introduced to describe the effect of the rigid boundary, where $R_{ini1}$ and $R_{ini2}$ are the radii of the lower bubble and upper bubble, respectively.

In case 1, $\gamma_2$ is set as 3.18, which means that the lower bubble stays out of the boundary-effect region, and the simulation...
results are shown in Fig. 12(a). The result shows that the interaction between cavitation bubbles makes the cavitation bubbles collapse toward each other; it seems there is a “wall” between the upper and lower cavitation bubbles. The upper cavitation bubble collapses a little faster than the lower one, and the differences of maximum deformation parameters $Ra$ and the lifetime of two bubbles are small. The evolution process means that the interaction between two bubbles is stronger than the interaction with

![Fig. 15. The evolution of the flow field in case 1 at (a) $t = 200$ $tu$, (b) $t = 500$ $tu$, (c) $t = 650$ $tu$, (d) $t = 700$ $tu$, (e) $t = 730$ $tu$, and (f) $t = 747$ $tu$.](image-url)
the rigid boundary in this case. Compared with the experimental result, the result of LBM seems more reasonable than that with the boundary integral method in the second column of Fig. 13(a).

When $\gamma_2$ is set as 1.72, the lower cavitation bubble is located in the wall-effect region. The simulation result shows that the deformation process of the upper cavitation bubble is unaffected by the wall, and its maximum deformation parameter $R_a$ is close to

![Image of flow field evolution](https://example.com/image.png)

**FIG. 16.** The evolution of the flow field in case 2 at (a) $t = 200$ tu, (b) $t = 500$ tu, (c) $t = 650$ tu, (d) $t = 700$ tu, (e) $t = 750$ tu, and (f) $t = 781$ tu.
1.1. On the contrary, the lower cavitation bubble is affected by both the wall and the upper cavitation bubbles, and it becomes an ellipsoidal bubble with maximum $Ra$ larger than 2.0. The lifetime of the lower bubble is longer than the upper one. The numerical result is shown in Fig. 13(b), and the result of LBM simulation agrees well with those from the boundary integral method and experiment.\(^7\)

When $y_2 = 1$, the lower bubble is in direct contact with the rigid boundary. The numerical result of case 3 is shown in Fig. 13(c). The deformation parameter $Ra$ of the upper cavitation bubble is close to 1.1. Due to the effect of the rigid boundary and upper bubble, the lower cavitation bubbles mainly shrink along the horizontal direction, and the vertical compression is very small.

The deformation parameters of two bubbles varying during the process are shown in Fig. 14. The time-deformation parameter curves of the upper bubble in three cases almost overlap with each other, and the maximum $Ra$ has only a mild difference. However, the time-deformation parameter curves of the lower bubble are different, when the lower cavitation bubble stays outside of the near-wall region; the time-deformation parameter curve is consistent with the upper bubble. When the lower bubble stays in the near-wall region, the shrinkage rate in the horizontal direction is much greater than that in the vertical direction. The smaller the $y_2$ is, the slower the vertical shrinkage rate is, thus gradually increasing the $Ra$ value.

In case 1, a dent occurs at the top of the interface of the upper bubble at $t = 600 \, tu$, and the high-pressure region is formed, while the pressure between the lower bubble and the rigid boundary is affected by the wall, and a low-pressure region is formed between two bubbles. At $t = 700 \, tu$, a dent also occurs at the bottom of the lower bubble, and a high-pressure region is formed between the lower bubble and the wall. It interacts with the high-pressure region of the upper cavitation bubble, making the upper bubble and lower bubble move toward each other. The influence of the wall makes the area of the high-pressure region of the lower bubble change slowly. However, the high-pressure region above the upper bubble increases rapidly, which accelerates the collapse of the upper bubble, as shown in Fig. 15. During the collapse process of two cavitation bubbles, the formation of the microjet can be observed, and the two cavitation bubbles collapse toward each other.

Compared with case 1, the lower bubble in case 2 is affected by the rigid boundary and the upper cavitation bubbles. The low-pressure regions generated at both the top and bottom of the lower bubble make the lower bubble compress in the horizontal direction, and the lower bubble becomes an ellipsoid type. The evolution of the flow field near the upper bubble is similar to case 1. When $t = 650 \, tu$, a dent occurs at the top of the upper bubble, and a local high-pressure region appears at the same time. The maximum pressure value of the high-pressure zone increases rapidly, making the upper cavitation bubble collapse in a short time. The micro-jet is formed and flows in the direction toward the lower bubble, as shown in Fig. 16.

The TKE of both upper and the lower bubbles in case 1 increases slowly in the early stage of the collapse process. Because the two bubbles stay outside of the wall-affect region, the curve of TKE in the early collapse stage overlaps with each other. However, with a dent occurring at the top of the upper bubble, the TKE of the upper bubble grows faster. The maximum value of TKE reached near $0.6 \, mu \cdot lu^{-1} \cdot tu^{-2}$ before the release process, and the energy releases rapidly with the bubble collapse later. The development of the high-pressure field near the lower bubble is affected by the wall; this decreases the TKE growth rate of the lower bubble. When the upper bubble collapses, the TKE of the lower bubble is $0.4 \, mu \cdot lu^{-1} \cdot tu^{-2}$, as shown in Fig. 17(a). In case 2, when the lower bubble is affected by the wall and the upper bubble, no high-pressure region is formed around it. When the upper bubble collapses, the lower bubble still stays in the collapse process. Before TKE of the upper bubble releases, it reaches the maximum value of $0.58 \, mu \cdot lu^{-1} \cdot tu^{-2}$, while the TKE of the lower bubble is only $0.29 \, mu \cdot lu^{-1} \cdot tu^{-2}$, as shown in Fig. 17(b).
IV. CONCLUSIONS

In this paper, an MRT pseudo-potential model with a high-density ratio and a high viscosity ratio is used to simulate the collapse process of a single bubble and a dual bubble collapse near a rigid boundary, with density ratio $\rho_l/\rho_g = 720$. The simulation agrees well with experiments and theoretical analysis, which means that the proposed LBM pseudo-potential model is a robust and effective tool for the study of the collapse of near-wall cavitation bubbles. Effects of the liquid/gas density ratio, viscosity, initial pressure difference, and bubble distance to the wall on the evolution of the cavitation bubble are studied. The deformation of cavitation bubbles, micro-jet velocity, and the maximum collapse pressure are analyzed. The TKE is first introduced to describe the formation of the shock wave from the perspective of energy. Simulation results indicate that

1. The gas viscosity $\nu_g$ does not affect the maximum pressure and maximum micro-jet velocity in the cavitation process, while the liquid viscosity $\nu_l$ does. The maximum velocity and pressure decrease with an increase in the liquid viscosity $\nu_l$, while the lifetime of the bubble increases. The initial pressure difference $\Delta p_{ini}$ mainly affects the time of the compression stage, but it has a little effect on the time of the collapse stage during cavitation bubble evolution. With an increase in $\Delta p_{ini}$, the maximum velocity and pressure increase, while the lifetime of the cavitation bubble decreases.

2. In the near-wall region, where $d < 2.2R_{ini}$, the lifetime of the cavitation bubble decreases with the increase in the distance ratio $\gamma$. The closer the cavitation bubble to the wall, the larger the deformation parameter $R_0$, and the longer the lifetime of the bubble.

3. The evolution of the cavitation bubble can be divided into two stages with TKE introduced in the present study; energy accumulation and energy release. The shock wave is formed with TKE release in a short time.

4. When two cavitation bubbles interact with each other in the near-wall region, the deformation curves and energy curves of the upper cavitation bubbles of different cases are similar to each other. The evolution of the lower cavitation bubble is related to the distance between the lower cavitation bubble and the wall. When $\gamma_l > 2.2$, the evolution of the lower cavitation bubbles is not affected by the wall, but the high-pressure of the lower cavitation bubble is affected by the rigid boundary, making the lower cavitation bubble collapse toward the upper one, and its lifetime is longer than that of the upper one. When $\gamma_l < 2.2$, the evolution of the lower cavitation bubble is affected by both the wall and the upper cavitation bubble, and low-pressure regions are formed at the top and bottom of the lower cavitation bubble. In addition, the lifetime of the lower cavitation bubble becomes much longer than that of the upper one.

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