Content-sensitive warped filter equalization for loudspeaker systems

Peng Wang
School of Electronic and Information Engineering, Changshu Institute of Technology, Changshu, Jiangsu Province, China
one_pen@126.com

Abstract. The technique of frequency-warped filtering has been recommended as a promising solution to the problem of loudspeaker equalization. According to the characteristics of audio signal, this paper puts forward a method to improve the warping effect and thus the overall equalization effect, and the effectiveness of the proposed method is validated via computer simulations.

1. Introduction
The technique of frequency-warped filtering has received much attention in literature as a promising candidate to the problem of loudspeaker equalization [1]-[7]. A warped filter is known to be able to produce flexible frequency resolutions. The warped filter equalization can, accordingly, be designed to provide a variable degree of compensation for distortions at different frequencies, which makes it highly attractive for the equalization of audio systems, e.g., loudspeakers, where the distortions at different frequencies are generally regarded to make a different degree of degradation to the quality of the audio signal.

The frequency warping in a warped filter results from a nonlinear mapping mechanism between the conventional linear frequency domain and the so-called warped frequency domain. The warping effect, which prescribes the means of redistribution of the frequency resolving capability and hence the equalization effect over frequency, is mainly determined by the value of the warping parameter. In [3], an optimum warping parameter in the sense of the mean-square error is derived, and in [5], the equalization is optimized with respect to the characteristics of the human auditory system. However, neither of the two works pays attention to the difference of the audio signals. This paper introduces a method to optimize the warping process so as to achieve better equalization effect according to the frequency characteristics of the audio signal.

The paper is organized as follows. Section 2 reviews the principle of warped filter equalization. Section 3 introduces how the equalization process can benefit from the knowledge of the audio signal. Section 4 gives the simulation results that support the effectiveness of the proposed scheme. The paper ends with the concluding remarks in Section 5.

2. Principle of warped filter equalization
The basic principle of warped filtering is to process the signal in the warped frequency domain instead of in the linear frequency domain. Firstly, the target signal is warped to obtain its representation in the warped frequency domain. The frequency band where an enhanced resolution is needed is expanded at the cost of the others being suppressed. The conventional signal processing techniques, e.g., loud-
speaker equalization, can then be applied to the warped signal. As the final step, the output signal needs to be unwarped implicitly to restore its representation in the linear frequency domain.

The frequency-warping effect results from the linear equalization of the warped signal, which implies that, the redistribution of the equalization potential over different frequencies is dominantly determined by the warping and the counterpart unwarping processes.

The coupled processes, warping and unwarping, can be achieved by the use of a special filter structure, warped filter. To construct a warped filter, the unit delays in a normal filter are replaced with first-order all-pass function, \( D_1(z) \), and

\[
D_1(z) = z^{-1} = \frac{z^{-1} - \lambda}{1 - \lambda z^{-1}}
\]

wherein, \( z' = e^{j\omega} \), \( z = e^{j\omega} \), \( \omega' \) and \( \omega \) are respectively the warped and the linear circular frequencies, and \( \lambda \) is the warping parameter ranging between -1 and 1. The warping effect can be quantified by the partial derivative of \( \omega' \) with respect to \( \omega \), which is clearly a function with a unique parameter \( \lambda \). As a result, the value of \( \omega' \) solely determines the means of redistribution of the frequency resolving potential of the system, and thereby, the equalization effect over frequency in the case of loudspeaker equalization. In general, positive values of \( \lambda \) enhance the low-frequency resolution while negative values enhance the high-frequency resolution. Since the human auditory system is more sensitive to low-frequency distortions [4], the positive warping parameters, i.e., the low-frequency enhancement, have been the focus of attention for audio engineers.

3. Optimization based on signal characteristics

The previous works dedicated to the optimization of the warping parameter, e.g., [3] and [5], attempt to quantify and minimize the differences between the ideal and the equalized responses of the target loudspeaker, without considering the characteristics of the target signals. However, as the warped filter equalization involves a nonlinear redistribution of the equalization effect over frequency, the signal characteristics have a direct impact on the quality of the output signal and therefore should not be neglected in the optimization of the loudspeaker equalizer.

Imagine a quite common scenario where the warped filter has been designed to provide an optimum equalization on the full frequency band while the spectrum of the target signal is mostly centered at a narrow band. Clearly, in this case, the strategy of full-frequency optimization would not be able to work well, and a better solution is to focus specifically on the distortions falling into the signal bandwidth.

Instead of a global optimization effect, this paper aims at a local optimum equalization within the signal bandwidth. The power bandwidth of the signal is evaluated in real time and is adopted as the basis for the estimation of the warping parameter. The effective warping range (EWR), that is, the frequency band with enhanced resolution, always starts from 0 for positive parameter values and ends with \( \pi \) for negative values. Its width can thus be calculated as follows

\[
W(\lambda) = \begin{cases} 
\omega_0 & \text{if } \lambda > 0 \\
\pi - \omega_0 & \text{if } \lambda < 0,
\end{cases}
\]

wherein \( \omega_0 \) represents the linear circular frequency with zero warping effect, i.e.,

\[
\frac{\partial \omega'}{\partial \omega} \bigg|_{\omega'=\omega_0} = 0.
\]

The width of the EWR decreases as the warping parameter increases from 0 to 1. Therefore, in this study, the warping parameter is selected to just cover the signal bandwidth, i.e., the minimum warping parameter that satisfies the following criterion
where $B_s$ represents the real-time estimate of the signal bandwidth.

4. Computer simulations

A typical commercial loudspeaker system is employed as the target of equalization in the computer simulations. Its magnitude frequency response is shown in Figure 1. As can be seen, there are obvious distortions over the full frequency band, which substantiates the necessity of loudspeaker equalization.

The equalized frequency response is shown in Figure 2, where the conventional deconvolution method [8] is applied to the target loudspeaker and the filter length is set to 100. As can be seen, due to the uniform and thus limited frequency resolution, although the high-frequency distortions have received sufficient or even redundant mitigation, the low-frequency equalization effect is still unsatisfactory. By sacrificing a certain degree of high-frequency resolution for improving the low-frequency resolution, the warped filter equalization can partially overcome this problem. The advantage of the warped filter equalization is clearly demonstrated in Figure 3, where the equalized response of a warped filter equalizer with the same number of taps and warping parameter 0.6 is shown.

Figure 1. Magnitude frequency response of the target loudspeaker

Figure 2. Magnitude frequency response of the equalized loudspeaker: deconvolution method, 100 taps

Figure 3. Magnitude frequency response of the equalized loudspeaker: warped filter equalization, $\lambda = 0.6$, 100 taps

Figure 4. Mean square residual error after warped filter equalization with fixed and variable warping parameters, 100 taps
By taking into account the variation of the signal characteristics, as has been described in the previous section, the performance of warped filter equalization can be further improved. In Figure 4, two warped filter equalizers with fixed and variable warping parameters respectively are compared by the mean square residual error after equalization. As can be seen, the use of the proposed content-sensitive warping parameter can noticeably reduce the residual error.

5. Conclusions
The technique of warped filter equalization has attracted an increasing interest in literature. However, the previous works dedicated to this topic leave out the difference and the time variation of the signal characteristics, and accordingly, cannot provide generally optimum equalization effect.

In this paper, the problem is overcome by adjusting the warping parameter in real-time with respect to the power bandwidth of the processed signal. The effectiveness of the proposed scheme is supported by the simulation results.

References
[1] M. Karjalainen, E. Piirilä and A. Järvinene, "Loudspeaker response equalization using warped digital filters", in Proc. NorSig-96, (Espoo, Finland), pp.367-370, Sept. 1996.
[2] A. Hˇarmˇa, M. Karjalainen, L. Savioja, V. V’esta, U. K. Laine, and J. Huopaniemi, "Frequency-warped signal processing for audio applications," J. Audio Eng. Soc., vol. 48, no. 11, pp. 1011–1031, Nov. 2000.
[3] P. Wang, W. Ser and M. Zhang, "Multiband warped filter equalizer design for loudspeaker systems," in Proc. IEEE Int. Conf. Acoust. Speech, and Signal Process., pp. 913-916, Istanbul, Turkey, Jun. 2000.
[4] P. Wang, W. Ser and M. Zhang, "A dual-band equalizer for loudspeakers," J. Audio Eng. Soc. 2000, 48, 917–921.
[5] P. Wang, Wee Ser and M. Zhang, "Bark scale equalizer design using warped filter," in Proc. IEEE Int. Conf. Acoust., Speech, and Signal Process., pp. 3317-3320, Salt Lake City, USA, May 2001.
[6] Ramos, G.; López, J.J.; Pueo, B. Cascaded warped-FIR and FIR filter structure for loudspeaker equalization with low computational cost requirements. Digital Signal Process., 19, pp. 393–409, 2009.
[7] B. Bank, G. Ramos, "Improved Pole Positioning for Parallel Filters Based on Spectral Smoothing and Multiband Warping," IEEE Signal Process. Lett., IEEE, 18(5), pp. 299-302, 2011.
[8] J. Mournopoulos, “Digital equalization methods for audio systems,” presented at the 84th Convention of the Audio Engineering Society, J. Audio Eng. Soc. (Abstracts), vol. 36, p.384 (1988 May), preprint 2598.