Using Neutrino Oscillations to Measure $H_0$?

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Recently, the idea of using neutrino oscillations to measure the Hubble constant was introduced. We show that such a task is unfeasible because for typical energies of cosmic neutrinos, oscillations average out over cosmological distances and so the oscillation probability depends only on the mixing angles.

I. INTRODUCTION

In the course of the last decade, an intriguing inconsistency between measurements of the cosmic expansion rate based on early- and late-Universe probes has emerged. This inconsistency shows up as a discrepancy in the value of the Hubble constant $H_0$ [1], as inferred from measurements of the anisotropy of the cosmic microwave background, $H_0 = (67.27 \pm 0.60) \text{ km s}^{-1} \text{ Mpc}^{-1}$ at 68% CL [2], and as measured from a series of distance indicators in the local Universe. More precisely, the latest distance ladder measurement based on Type Ia supernovae (SNIa) calibrated by Cepheids gives $H_0 = (73.04 \pm 1.04) \text{ km s}^{-1} \text{ Mpc}^{-1}$ at 68% CL [3], whereas using the Tip of the Red Giant Branch to calibrate SNIa leads to $H_0 = (72.4 \pm 2.0) \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $H_0 = (69.6 \pm 0.8 \text{(stat)} \pm 1.7 \text{(sys)}) \text{ km s}^{-1} \text{ Mpc}^{-1}$ [4], both at 68% CL. Depending on which set of measurements one combines, the tension between the model-dependent and independent estimates of $H_0$ sits between 4.5σ to 6.3σ [5]. This so-called "$H_0$ tension" has become the new cornerstone of modern cosmology, and many new-physics setups are rising to the challenge [7].

In a recent publication, the possibility of using neutrino oscillations to measure $H_0$ was entertained. In this article, we show that such a task is unfeasible because for typical energies of neutrinos originating via cosmic-ray interactions in astrophysical sources, oscillations average out over cosmological distances and so the oscillation probability depends only on the angles of the leptonic mixing matrix.

The layout of the paper is as follows. In Section II, we go through the formalism of neutrino oscillations and show that oscillation averaged probabilities for transition between flavors have no dependence on the travel distance to the astrophysical sources of cosmic neutrinos. In Section III, we examine the conditions for the coherence loss of the neutrino wavepacket. Finally, in Section IV we present our conclusions.

II. OSCILLATIONS OF HIGH-ENERGY COSMIC NEUTRINOS

Neutrino oscillations are the outcome of nonzero neutrino masses and the certainty that virtually all useful neutrino sources are coherent. In other words, the neutrinos produced via charged-current weak interactions associated with $l_a$ charged leptons, $a = e, \mu, \tau$, can be described as coherent superpositions of neutrino states $\nu_j$ with different masses $m_{\nu_j}$, $j = 1, 2, 3$, weighted by the elements $U_{\alpha j}$ of the Pontecorvo–Maki–Nagakawa–Sakata (PMNS) matrix [9–11], i.e.,

$$|\nu_{\alpha}\rangle = \sum_j U_{\alpha j} |\nu_j\rangle. \quad (1)$$

The superposition of mass eigenstates is valid in no small part because neutrino masses are tiny when compared with their laboratory/cosmic energies. For a neutrino of energy $E_\nu$, traveling a distance $L$, we can conveniently parameterize the oscillation phase $\Delta \varphi_{ij}$ as

$$\Delta \varphi_{ij} = \frac{\Delta m_{ij} L}{4 E_\nu} \approx 1.27 \left( \frac{\Delta m_{ij}^2}{\text{eV}^2} \right) \left( \frac{L}{\text{km}} \right) \left( \frac{E_\nu}{\text{GeV}} \right)^{-1}, \quad (2)$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$ [12]. Now, for a relatively low redshift of $z = 0.05$, the distance traveled by the neutrinos is about 200 Mpc. Taking the highest energy measured at the IceCube facility $E_\nu \sim 10^7 \text{ GeV}$ [13] and the solar mass splitting, $\Delta m_{21}^2 = \Delta m_{10}^2 \approx 7.42 \times 10^{-5} \text{ eV}^2$ [14], Eq. (2) leads to $\Delta \varphi_{ij} \sim 6 \times 10^{10}$, which is the number of oscillation periods the neutrinos would experience. That is, the neutrinos would experience 60 billion oscillations to the Earth over a redshift $z = 0.05$. This implies that we would need to know $\Delta m_{ij}^2$ to one part in 60 billion and we would also have
to measure $E_\nu$ to one part in 60 billion. Both of these measurements are ridiculously unfeasible. After averaging over these uncertainties, we find the so-called oscillation averaged probability for transition of flavor $\alpha$ to $\beta$, which is given by

$$P(\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i}^2 U_{\beta i}^2,$$

(3)

and has no dependence on $L$ [12]. Note that for $z = 0.05$, an oscillation phase $\Delta_{ij} \sim 1$ would require an unrealistic neutrino energy of roughly $10^{16}$ GeV.

III. COHERENCE LOSS OF COSMIC NEUTRINO OSCILLATIONS

There is also a loss of coherence during neutrino propagation [15, 16]. This is because different neutrino mass eigenstates of the same energy have different velocities, and so the wavepackets of the mass eigenstates composing a neutrino state will come apart as they propagate. For neutrinos traveling over cosmological distances, the flying path is so large that these components completely separate from each other. In the case of coherence loss, the oscillatory terms of the oscillation probabilities disappear.

Following [17], we define $\Delta E_{\nu, L}$ as the energy difference for which

$$\Delta_{ij}(E_\nu - \Delta E_{\nu, L}) - \Delta_{ij}(E_\nu, L) = \pi.$$  

(4)

A straightforward calculation leads to

$$\Delta E_{\nu, L} \simeq \frac{4\pi E_\nu^2}{\Delta m_{ij}^2} = \frac{\ell_{ij}}{L},$$

(5)

where $\ell_{ij} = 4\pi E_\nu / \Delta m_{ij}^2$ is the vacuum oscillation length. As neutrinos travel cosmological distances between their origins and us, they are essentially on their mass shells, satisfying $E_\nu^2 = p^2 + m_j^2$. The relativistic dispersion relation implies that $E_\nu \sigma_{E_\nu} = p \sigma_p$, where $\sigma_{E_\nu}$ and $\sigma_p$ are the uncertainties in the energy and the width of the wavepacket in the longitudinal direction. Because neutrinos are ultrarelativistic $E_\nu \approx p$, and hence $\sigma_{E_\nu} \approx \sigma_p$. It is easily seen that the interference between the effects of different mass eigenstates disappears if

$$\sigma_p > \Delta E_{\nu, L} = \frac{\ell_{ij}}{L};$$

(6)

an equivalent expression can be found in configuration space, where the size of the wavepacket at the production point is given by the inverse of the uncertainty $\sigma_p$, i.e., $\sigma_x \sim \sigma_p^{-1}$ [17].

If the parent pion does not undergo any interaction with matter or with the magnetic field before decay, the width of the wavepacket in the configuration space is estimated to be

$$\sigma_x \sim 2 \times 10^{-6} \left( \frac{E_\nu}{10^7 \text{ GeV}} \right)^{-1} \text{ cm}$$

(7)

whereas for a parent muon,

$$\sigma_x \sim 2 \times 10^{-4} \left( \frac{E_\nu}{10^7 \text{ GeV}} \right)^{-1} \text{ cm},$$

(8)

i.e., for the neutrinos produced by muons, $\sigma_x$ is larger by the ratio of the lifetime of the muon to that of the pion [17]. The dominant modification of $\sigma_x$ at the sources comes from the interaction of the parent charged particle with the magnetic field $B$, which can be parameterized as

$$\sigma_x \sim 6 \times 10^{-19} \left( \frac{\Gamma}{100} \right)^{1/2} \left( \frac{B}{10^7 \text{ G}} \right)^{-1/2} \left( \frac{E_\nu}{10^7 \text{ GeV}} \right)^{-3/2} \text{ cm},$$

(9)

for pions and

$$\sigma_x \sim 5 \times 10^{-17} \left( \frac{\Gamma}{100} \right) \left( \frac{B}{10^7 \text{ G}} \right)^{-1} \left( \frac{E_\nu}{10^7 \text{ GeV}} \right)^{-2} \text{ cm},$$

(10)
for muons, where $\Gamma$ is the Lorentz boost of the plasma [17].

A straightforward calculation shows that for typical $B$ fields of cosmic neutrino sources, the mass states will decohere after propagation over cosmological distances, i.e., the mass eigenstates will be separated enough that there is no overlap of the wavefunctions and we detect on Earth the three mass states each with their own probability given by Eq. [3]. Indeed, as shown in [18], oscillation averaging and full decoherence yield the same probability. This implies that it does not even matter if we measure $E_\nu$ and $\Delta m^2_{ij}$ to arbitrary precision.

**IV. CONCLUSIONS**

We have shown that for typical energies of cosmic neutrinos, oscillations average out over cosmological distances and so the oscillation probability depends only on the mixing angles of the PMNS matrix. As a consequence, neutrino oscillations cannot be used to estimate cosmological distances and therefore cannot be adopted as a probe to measure $H_0$.

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