Mechanical Model for Vegetal Fibers-Reinforced Composite Materials

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Abstract

Composite materials reinforced with vegetal fibers appear to be a good alternative for engineering applications. The objective of this work was to develop a 2D mechanical model for the vegetal fibers-reinforced composite materials. This approach gives more possibilities for combining the two different materials without any more experimental tests for each kind of composition and position of the fibers. It’s a two-dimensional theory taking into account the one-dimensional behavior of the fibers, but considered, geometrically, as a two-dimensional system. The vegetal fibers are considered as simple cords and they are represented by an equivalent geometrical surface having the mechanical characteristics, including bending property and the coupling between twist-extension. The analytical solutions can be used to determine the optimization parameters leading to maximize the stress in the vegetal fibers and minimize it in the matrix.

1. Introduction

Materials reinforced by vegetal fibers constitute a current area of interest in composites research. These composites have many advantages over traditional glass fibers or inorganic mineral-filled materials, including lower cost, lower density, environmental friendliness, acceptable specific mechanical properties and biodegradability. The utilization of vegetal fibers is driven by growing market trends in terms of energy economy through weight reduction and recyclability after the end of the life cycle of the component. Moreover, vegetal fibers do not cause abrasion of processing tools and although time and
temperature have a significant influence on their properties [1]. Vegetal fiber-reinforced composite structures are being used in automotive, cosmetics, furniture, tools, etc.

Several authors treated the fiber-reinforced composites from simple homogeneous models and a limited sophisticated representation. Tsai and Pagano [2] presented a micromechanical model, which has been utilized to predict modulus of the composites with randomly-oriented fibers, the modulus of a ply reinforced by randomly dispersed fibers is given by the following expression [3]:

$$E = \frac{3}{8}E_L + \frac{5}{8}E_T$$

where $E$ is Young’s modulus of random fiber-reinforced composites, $E_L$ is the longitudinal modulus and $E_T$ the transverse modulus of the unidirectional ply. Recently, Fornes and Paul [4] showed that, in the case of completely random fiber orientation in all three orthogonal directions, fiber modulus reinforcement followed the relation [5]:

$$E = 0.184E_L + 0.816E_T$$

The longitudinal modulus and the transverse modulus for a ply reinforced by short unidirectional fibers is given by the following equations proposed by Halpin–Tsai [6]:

$$E_L = E_m \left[ \frac{(1+2l/d)\eta_f V_f}{1-\eta_f V_f} \right]$$
$$E_T = E_m \left[ \frac{1+2\eta_f V_f}{1-\eta_f V_f} \right]$$

where the constant $\eta_L$ and $\eta_T$ are defined as:

$$\eta_L = \frac{E_f / E_m - 1}{E_f / E_m + 2l/d}$$
$$\eta_T = \frac{E_f / E_m - 1}{E_f / E_m + 2}$$

in which $E_f$ and $E_m$ are respectively Young’s modulus of fibers and matrix, $l/d$ is the aspect ratio of fibers and $V_f$ is the volume fraction of fibers in the composites.

Akasaka and Hirano [7] presented a simplified homogeneous orthotropic model for the cord-composite structures, in which the system of the two materials is considered as an orthotropic composite material with the following mechanical properties

$$E_1 = c_1 E_c \eta_c \quad \text{and} \quad E_2 = \frac{4E_m}{3}$$

where $E_1$ and $E_2$ are respectively the Young modulus in the $x1$ and $x2$ directions, the constants $E_c$ and $E_m$ are the Young modulus of the cord and the matrix, respectively. $\eta_c$ is the cord volume fraction.

Miraoui and Hassis [8] developed a two-dimensional model for the cord-composite laminated cylindrical shells in which the concept of an equivalent 2D model is adopted.

In the homogeneous models, a part of the mechanical response of the vegetal fiber (or of the cord) is neglected which includes the twist-extension coupling behavior and the exact distribution of the fibers in the fiber-composite layer.

This research aims to develop mechanical model for the vegetal fibers-reinforced composite materials based on the different mechanical properties of each constituent material in fiber-matrix composites, and in which the one-dimensional properties of the fibers are considered in an equivalent 2D model.
mechanical model takes into account the geometrical position of the vegetal fibers and the twist-extension coupling behavior.

2. Fibers and matrix behavior

A fiber consists of micro fibrils made of cellulose chains in an amorphous matrix of lignin and hemicelluloses. The micro fibrils are twisted in a helical form defined by the helix angle $\alpha$. We consider the fiber as a simple cord, consisting of many strands, which represented by a central strand and $n$ helical twists defined by the helix angle $\alpha$.

Fiber (see fig. 1) is assumed to be able to carry an axial force $N$, a twisting moment $C$ and a bending moment $M$. The fiber behavior is defined by the following relationship:

$$N = A E f \epsilon_{xx} \quad ; \quad C = E f r^2 \tau \quad ; \quad M = E f r^4 \kappa$$

where $A$ is cross sectional area of the fiber, $r$ is the radius of the fiber, $\epsilon_{xx}$ is the axial strain (where $x$ is the fiber's direction), $\kappa$ is curvature and $\tau$ is the twist strain.

Fig. 1. SEM micrograph of a vegetal fiber [9]

The matrix is considered as a homogeneous isotropic material (following the Mindlin assumption) characterized by the membrane tensor, $N_m$, the moment tensor, $M_m$, and the shear stress vector, $T_m$.

2. Presentation of the mechanical model

We consider a thin pipe (or plate) which is reinforced by vegetal fibers, oriented in one or two directions (longitudinal direction $\bar{x}^1$ and circumferential direction $\bar{x}^2$). The geometrical position of the fibers is characterized by the in-plane distance between fibers ($d^l$ and $d^c$, respectively in the $\bar{x}^1$ and the $\bar{x}^2$ directions) and the thickness position (heights) of the fibers, $e^l$ and $e^c$ (see fig. 2).

By considering a multiphase (fiber and matrix) elementary volume (fig. 2.a), and for one fiber belonging to the layer number $i$, the internal forces are represented by:

$$\begin{pmatrix} \bar{X}^i_l \\ \bar{M}^i_l \end{pmatrix}_{\text{long. cord}} : \bar{X}^i_l = \begin{pmatrix} N^i_l \\ 0 \end{pmatrix} ; \bar{M}^i_l = \begin{pmatrix} C^i_l \\ M^i_l \end{pmatrix}_{\text{circ. cond}} : \bar{X}^i_c = \begin{pmatrix} 0 \\ N^i_c \end{pmatrix} ; \bar{M}^i_c = \begin{pmatrix} M^i_c \\ 0 \end{pmatrix}_{\text{circ. cond}}$$
The internal force (resultant) takes the following expression when it is written in the mid-plane of the elementary volume (fig. 2.b)

\[
\sum_i \left( \frac{\ddot{X}_i^t}{M_i^t} + \ddot{X}_i^c e_i^t \ddot{x}_i \right) = \sum_i \left( \frac{\ddot{X}_i^c}{M_i^c} + \ddot{X}_i^e e_i^t \ddot{x}_i \right)
\]

In order to construct a 2D mechanical model, the fibers are geometrically distributed in the elementary surface defined by \( dx_1 dx_2 \) (fig. 2.c) with the following equivalent wrench (distributed)

\[
\sum_i \left( \frac{\ddot{X}_i^t}{M_i^t} + \ddot{X}_i^c e_i^t \ddot{x}_i \right) + \sum_i \left( \frac{\ddot{X}_i^c}{M_i^c} + \ddot{X}_i^e e_i^t \ddot{x}_i \right) = \sum_i \left( \frac{\ddot{X}_i^c}{M_i^c} + \ddot{X}_i^e e_i^t \ddot{x}_i \right) + \sum_i \left( \ddot{X}_i^e e_i^t \ddot{x}_i \right)
\]

where \( \ddot{W} \) is the distributed quantity of \( \ddot{F} \).

![Fig. 2. Distributed geometrical model](image)

Then when the behavior or the internal generalized forces are evoked, each fiber is considered as a one-dimensional element, but it is distributed on a two geometrical surface defined by the plane \( \omega \) (middle surface of the matrix).

3. The equilibrium equations

By applying the virtual works principle \[10\] on the fibers-matrix system, the equilibrium equations are

\[
\begin{align*}
\text{div} N_a - \text{div}(M_a C) - C \ddot{T} + \text{div} N_e + \ddot{F}_a &= 0 \\
\text{div} M_a - \ddot{T}_a + \text{div} M_e + \ddot{m}_a &= 0 \\
N_a : \epsilon + \text{div} \ddot{T}_a - (M_a C) : C + N_e : \epsilon + F_e &= 0
\end{align*}
\]
and the boundary conditions are:

\[
\begin{align*}
- \left[ N_m - M_m C + N_x \right] \vec{v} + \left( \vec{F}_x \right)_n &= \vec{0} \\
- \left[ M_m + M_x \right] \vec{v} + \left( \vec{m}_x \right)_n &= \vec{0} \\
- \vec{t}_n \cdot \vec{v} + F^3 &= 0
\end{align*}
\]

where \( \vec{v} \) is a vector normal to the boundary and belonging to the middle surface and \( C \) is the curvature tensor. \( N_x \) and \( M_x \) are the fiber's internal tensors, introducing the effect of the fibers on the matrix:

\[
N_x = \hat{x}^i \otimes \hat{X}^j + \hat{x}^j \otimes \hat{X}^i + \frac{N_x^i}{d_i} \hat{x}^i \otimes \hat{x}^j \quad \text{and} \quad M_x = \left[ \hat{x}^i \otimes \left( \hat{M}^j + \hat{X}^j \wedge \hat{e}^j \hat{x}^i \right) + \hat{x}^j \otimes \left( \hat{M}^i + \hat{X}^i \wedge \hat{e}^i \hat{x}^j \right) \right]
\]

\( F^i \) and \( F^j \) are respectively the \( i^{th} \) component of the surface and the linear density forces, obtained, by integration, from volumetric density forces. \( m_i \) and \( m_j \) are respectively the \( j^{th} \) components of the surface and the linear density moments (\( o \) is the middle surface of the matrix).

4. Application

To analyze the reinforcement effects on the displacements and internal efforts, we consider a fiber-composite pipe consisting of one longitudinal central layer. The cylindrical shell is simply supported (on its base) and submitted to a pressure, \( p \), and axial tension, \( q \), on its top boundary. In order to simplify the analysis, the membrane theory is adopted. In this case, the problem has an analytical solution. The solution with and without reinforcement is computed and compared (see table 1), and the optimizing reinforcement parameter is detected (see fig. 3).

Table 1. Displacements and internal efforts without and with reinforcement

| Solution without reinforcement | Solution with longitudinal reinforcement |
|-------------------------------|----------------------------------------|
| \( u_0(x^i) = \frac{(-v_{m_{pR}} + q)x^i}{D_2(1-v_{m_{pR}}^2)} \) | \( u_0(x^i) = \frac{(-v_{m_{pR}} + q)x^i}{D_2[1-v_{m_{pR}}^2] + A'E_1^C_1} \) |
| \( u_0(x^i) = \frac{R(-v_{m_{pR}}pR + v_{m_{pR}}q)}{D_2(1-v_{m_{pR}}^2)} \) | \( u_0(x^i) = \frac{R(-pR + v_{m_{pR}}q - pR_A^C_1)}{D_2[1-v_{m_{pR}}^2] + A'E_1^C_1} \) |
| \( N_0^m(x^i) = 0 \) | \( N_0^m(x^i) = 0 \) |
| \( N_0^m(x^i) = q \) | \( N_0^m(x^i) = q(1-v_{m_{pR}}^2) + pR_A^C_1 \) |
| \( N_{11}^m(x^i) = pR \) | \( N_{22}^m(x^i) = pR ; \quad N_{12}^m(x^i) = 0 \) |
where \( N^m(x^1) \) the normal stress of the matrix, \( N^r(x^1) \) the normal stress of the reinforcement and \( R \) the radius of the middle surface of the cylindrical shell.

It can be seen from the results given in the table 1, that the main parameters of the reinforcement are not the volumetric proportion of the fibers and the Young modulus ratio between fibers and matrix (as it considered in other papers), but there is only one which is:

\[
x = \frac{A'E'}{Dd} = \left( \frac{A'}{d} \right) \left( \frac{1 - \nu_m^2}{\nu_m} \right) = \frac{E'}{E_m} \left( 1 - \nu_m^2 \right)
\]

where \( \nu_m \) the Poisson coefficient of the matrix and \( h \) is the thickness of the matrix.

It can be used as optimizing parameter. The evolution, as a function of the parameter \( x \), of the relative normal stress of the matrix and the fibers are represented in figure 3. To optimize the structure is equivalent to maximize the normal stress in the fibers and to minimize it in the matrix.

![Fig. 3. Evolution of the normal stress for the matrix (o o o o) and for the reinforcement (____) as a function of the parameter x](image)

5. Conclusion

A mechanical model for the vegetal fibers-reinforced composite materials is presented. It is a two-dimensional theory taking into account the one-dimensional behavior of the fibers but considered, geometrically, as a two dimensional system. The analytical solutions can be used to optimize the disposition, the stiffness ratio and the volumetric proportion between fibers and matrix. This model is different from the homogenous theory, which does not give any importance to the position of the fibers. In a forthcoming work, it would be interesting to study the effects of the extension-twist coupling.

References

[1] Baley C, Pillin I, Grohens Y. Etat de l’art sur les matériaux composites biodégradables. Revue des Composites et des Matériaux Avancés, Vol.2, pp. 135–66, 2004.
[2] Tsai SW, Pagano NJ. Invariant properties of composite materials. Composite Materials Workshop, Stamford, pp. 233–253, 1968.
[3] Gibson R. Principles of composite material mechanics. New York: McGraw-Hill, 1994.
[4] Fornes TD, Paul DR. Crystallization behavior of nylon 6 nanocomposites. Polymer, Vol. 44 (14), pp. 3945–3961, 2003.
[5] Liu H, Wu Q, Zhang Q. Preparation and properties of banana fiber-reinforced composites based on high density polyethylene (HDPE)/Nylon-6 blends. Bioresource Technology, Vol. 100, pp. 6088-6097, 2009.
[6] Halpin J, Kardos J. The Halpin-Tsai equations: a review. Polymer Engineering and Science, Vol. 16 (5), pp 344–52, 1976.
[7] Akasaka T, Hirano M. Approximate elastic constants of fiber reinforced rubber sheet and its composite laminate. Composite Materials and Structures, vol. 1, pp. 70-76, 1972.
[8] Miraoui I, Hassis H. Modeling of cord composite laminated cylindrical shells using a 2D general formulation. International Journal of Mathematical, Physical and Engineering Sciences, Vol. 3 (1), pp. 23-30, 2009.

[9] Bourmaud A, Baley C. Rigidity analysis of polypropylene/vegetal fiber composites after recycling. Polymer Degradation and Stability, Vol. 94, pp. 297–305, 2009.

[10] Miraoui I. Sur la modélisation des structures minces renforcées par des cordes avec prise en compte de l’interaction fluide structure et la cinématique d’ordre supérieur. Thèse de doctorat de l’ENIT, 2008.