Numerical Analysis of Inertance Pulse Tube Refrigerator

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Abstract. The inertance type pulse tube refrigerator (IPTR) has been extensively used in industries due to its small size and efficiency. The physical phenomena of oscillating flow in IPTR is complex and it can be examined by using a commercial computational fluid dynamics (CFD) software. Researchers are not so much interested in the 3D analysis of IPTR due to high computational time but for better visualization, it is very beneficial. A two-dimensional axis-symmetric model and 3D model is considered for simulation. The temperature at the cold heat exchanger (CHX) and phase difference between pressure and mass flow rate at CHX is demonstrated. In this paper, CFD analysis of IPTR has been done and various issues involved in the simulation are presented for the betterment in the desired solution.

1. Introduction

Pulse tube refrigerator (PTR) is one of the promising contenders to the conventional Gifford-McMahon and Stirling cryocoolers, due to absence of moving parts in the cold region. PTR is a highly reliable device with low vibration and high life. A PTR is an oscillating flow loop type device consists of various components in which thermal interactions take place between solid and fluid. Continuous improvements in the performance of PTR is going on since basic PTR invented [1], which lowers the achievable refrigeration temperature. In all PTRs, the correct nature of physical operation occurring in the system is not well understood and the reason behind this is the poor understanding of phase difference and thermal relaxation. Another reason is the processes of pressurization and depressurization not well defined. Most important among all is the phase lag between the mass flow rate and pressure. This phase lag is adjusted by the inertance tube in various IPTC designs.

Figure 1. Different analysis techniques of the IPTR system
The different modelling techniques for the analysis of Inertance pulse tube refrigerator (IPTR) system is shown in Figure 1. Theoretical modelling is done by using various approaches like cyclic analysis, nodal analysis, or phasor analysis for the analysis of PTRs [2-4]. These models are convenient and simple but do not have sufficient considerations for the study of fluid flow and heat transfer.

Computational models have the capacity and it considers heat transport and fluid flow details for the analysis of PTR. The tool for the numerical solution of Navier-stokes equations is CFD commercial code, such as Fluent. Using CFD simulation, various difficulties are overcome like heat transfer phenomenon in regenerators and heat exchangers, modelling of oscillating flow in the system.

CFD analysis of simple orifice type of pulse tube cryocooler operating at high frequency is done by Barrett and Arsalan [5] using the axis-symmetric model. The solid material and gas physical properties are taken as constant while doing analysis. They have shown the swirling flow pattern presented in the velocity vectors. Numerical investigation of IPTR is done to observe the significance and extent of multi-dimensional flow effects in the IPTR system operating at high-frequency with two-dimensional axisymmetric model [6]. Chen [7] studied the thermodynamic cycles in an IPTR system by means of CFD simulation method. Numerical study of a 3D IPTR system from a design perspective has been done by Kumar [8]. Various issues like the simulation of the compressor and negative volume error occurred in it, are investigated which are involved in CFD simulation of an IPTR in detail. In this paper, two-dimensional axisymmetric and 3D model CFD simulations are conducted to analyze the IPTR. The IPTR model considered for the analysis is taken from the work reported by Gawali [9].

2. Simulated System

2.1. Model building and meshing

The pulse tube refrigerator contains all cylindrical components. The model is generated using ANSYS workbench DesignModeler.

2.1.1. 2D-Axisymmetric model. The schematic diagram of IPTR is shown in Figure 2. The dimensions of IPTR is given in Table 1. The different components are generated using the function of ‘Add frozen’ instead of ‘Add material’. This is done to avoid a single solid as a body because boundary conditions should be given to specific components of IPTR. So after forming all solids then do ‘Form new part’ which act as one body of different solids components. This geometry is drawn symmetric along the x axis to take advantage of the axisymmetric model. IPTR 2D axisymmetric model is as shown in Figure 3.

ANSYS meshing module is used for the meshing of the object using elements of triangular and rectangular shape. However, in the current case, piston oscillates and generates pressure waves of the high amplitude of approximately ±5 bar. Operating pressure in this case in the system is 16 bar. The compressor is the most critical and essential component from a meshing point of view.
Table 1. Dimensions of IPTR [2]

| Component          | Radius (m) | Length (m) |
|--------------------|------------|------------|
| A(Compressor)      | 0.03090    | 0.01300    |
| B(Transfer line)   | 0.00300    | 0.03796    |
| C(Aftercooler)     | 0.01400    | 0.0107     |
| D(Regenerator)     | 0.01400    | 0.05400    |
| E(CHX)             | 0.01400    | 0.00879    |
| F(Pulse tube)      | 0.00480    | 0.12850    |
| G(HHX)             | 0.00500    | 0.009      |
| H(Inertance tube)  | 0.00125    | 2.00000    |
| I (Reservoir)      | 0.03889    | 0.10000    |

It is obvious that to obtain pressure waves dynamic meshing should be done [10]. Compressor meshing is done by rectangular elements only to avoid negative cell volume detected error and the aspect ratio has to be maintained nearly equal to one to achieve better numerical stability.

The components of the IPTR system have meshed with finer elements for better numerical stability as shown in Figure 4. Rectangular elements have been used to mesh the inertance tube, pulse tube and regenerator for suitable wave propagation. In the current case, total nodes are created 3916.
2.1.2. 3D model. 3D IPTR model is drawn using geometry design modeller by taking same dimensions as given in Table 1 and it is shown in Figure 5.

![3D IPTR model](image)

Figure 5. 3D IPTR model

Meshing of 3D IPTR model has been done and it is shown in Figure 6. For optimal meshing 3,96,568 nodes are created.

![Meshing of 3D model](image)

Figure 6. Meshing of 3D model

2.2. Governing equations
The mass, momentum and energy equations are the governing equations for the current 2-D axisymmetric model [11]. The governing equations for the gas domain are given by

**Mass:**

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0
\]  

(1)

**Momentum:**

\[
\frac{\partial (\rho \vec{v})}{\partial t} + \nabla \cdot (\rho \vec{v} \vec{v}) = -\nabla p + \nabla \cdot (\bar{T})
\]  

(2)
Energy:
\[
\frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\bar{v}(\rho E + p)) = \nabla \cdot \left( k_{\text{eff}} \nabla T + (\tau_{\text{eff}} \cdot \bar{v}) \right)
\]  
\[(3)\]

Where \(\rho\), \(\bar{v}\), \(p\), \(T\), \(k_{\text{eff}}\) and \(E\) are the fluid density, velocity, static pressure, temperature, thermal conductivity and total energy of the fluid, respectively. Stress tensor is denoted by \(\tau\).

Similarly, the governing equation for the porous zones includes mass, momentum and energy equations are given as However, for porous zones, the properties of the solid matrix should be taken into account.

Mass:
\[
\frac{\partial(\varepsilon \rho)}{\partial t} + \nabla \cdot (\varepsilon \rho \bar{v}) = 0
\]  
\[(4)\]

Momentum:
\[
\frac{\partial}{\partial t}(\varepsilon \rho \bar{v}) + \nabla \cdot (\varepsilon \rho \bar{v} \bar{v}) = -\varepsilon \nabla p + \nabla \cdot (\varepsilon \bar{v}) + S_i
\]  
\[(5)\]

Where \(\varepsilon\) is the void number, \(S_i\) is the source term added in the momentum equation. Source term made of two parts: the Darcy term and the Forchheimer term [11]:
\[
S_i = -\left( \frac{\mu}{\alpha} v_j + C_2 \frac{1}{2} \rho_{\text{mag}} v_j \right)
\]  
\[(6)\]

Where \(\alpha\) is the permeability and \(C_2\) is the inertial resistance factor.
\[
\alpha = \frac{D_p^2}{150} \frac{\varepsilon^3}{(1 - \varepsilon)^2}
\]  
\[(7)\]

\[
C_2 = \frac{3.5}{D_p} \frac{(1 - \varepsilon)}{\varepsilon^3}
\]  
\[(8)\]

Energy:
\[
\frac{\partial}{\partial t} \left( \varepsilon \rho_f E_f + (1 - \varepsilon) \rho_s E_s \right) + \nabla \cdot \left( \bar{v} \left( \varepsilon \rho_f E_f + p \right) \right) = \nabla \cdot \left( k_{\text{eff}} \nabla T + (\tau_{\text{eff}} \cdot \bar{v}) \right)
\]  
\[(9)\]

Where, \(E_f\) = Fluid energy
\(E_s\) = Solid medium energy
\(\varepsilon\) = void number
\(k_{\text{eff}}\) = effective thermal conductivity

2.3. Fluent

The fluent configuration for the current case and boundary conditions applied to different components are tabulated in Table 2 and Table 3 respectively.

**Table 2. Setup**

| Setup                              | Selection                                      |
|------------------------------------|------------------------------------------------|
| Solver                             | Pressure based solver                          |
| Computation                        | Double precision mode                          |
| Material                           | Ideal Gas (Helium)                             |
| Energy and Turbulent Model         | Enabled and Standard \(\kappa - \varepsilon\) model (default mode) |
| Solution Methods                   | Pressure velocity coupling, SIMPLE, second-order transient |
Table 3. Cell zone and Boundary conditions

| Component | Cell zone and Boundary conditions |
|-----------|-----------------------------------|
| Compressor, Transfer line, Pulse tube, Reservoir | Cell zone conditions: Fluid: Helium, Solid: Steel  
Boundary conditions: Walls: Adiabatic |
| Aftercooler, Hot end heat exchanger | Cell zone conditions: Porous medium (Copper)  
Viscous-resistance (D): $2.47 \times 10^9 \text{ m}^{-2}$  
Inertial resistance (C): $26254 \text{ m}^{-1}$, Porosity ($\phi$): 0.664  
Boundary conditions: Wall Temperature: 300K |
| Regenerator | Cell zone conditions: Porous medium (Steel)  
Viscous-resistance (D): $4.09 \times 10^{10} \text{ m}^{-2}$  
Inertial resistance (C): $107306 \text{ m}^{-1}$, Porosity ($\phi$): 0.662, Boundary conditions: Walls: Adiabatic |
| Cold end heat exchanger (CHX) | Cell zone conditions: Porous medium (Copper)  
Viscous-resistance (D): $2.10 \times 10^9 \text{ m}^{-2}$, Inertial resistance (C): $9234 \text{ m}^{-1}$, Porosity ($\phi$): 0.69, Boundary conditions: Walls: Adiabatic |
| Inertance tube | Cell zone conditions: Fluid: Helium, Solid: Copper  
Boundary conditions: Walls: Adiabatic |

In the dynamic mesh section, layering mesh method is used. Three different sections considered in the compressor. First one is piston, second one is axis and last one is compressor wall. In fluent language, compressor axis and wall is defined as deforming and piston is defined as rigid body. In current case, piston motion is horizontal (x direction). Oscillating motion of piston is obtained by using a user defined function (UDF). This UDF is shown below, which produce desired pressure waves. The velocity is given to piston at 50 Hz frequency.

```c
#include "udf.h"
DEFINE_CG_MOTION (velocitycomp, dt, vel, omega, time, dtime) {
  real frequency = 50.0;
  real dw = 2.0 * 3.14 * frequency;
  real disp = 0.005;
  NV_S(vel, = ,0.0);
  NV_S(omega, = ,0.0);
  vel[0] = dw * disp * cos (dw * time);
}
```

The convergence criteria for the simulation and operating system parameters are considered as per tabulated in Table 4 and Table 5 respectively. Standard initialization technique is used for the initialization. Time step size taken as $3e^{-4}$ sec.

Table 4. Convergence criteria

| Convergence criteria     | Value   |
|--------------------------|---------|
| Continuity, Velocity, $\kappa - \varepsilon$ | $1e-03$ |
| Energy                   | $1e-06$ |
Table 5. Operating parameters

| Working fluid | Helium |
|---------------|--------|
| Frequency     | 50 Hz  |
| Operating pressure | 16 Bar |

3. Results and Discussions

3.1. 2D-axisymmetric

The simulation of PTR is done using FLUENT software for mainly studying the temperature drop at the cold end of the pulse tube i.e. CHX. The simulation should be carried out until the cyclic steady state is achieved.

![Figure 7. Cool down curve at no load condition](image)

Experimental temperature achieved at CHX by Gawali [2] is 39 K at no load condition. Numerical temperature value achieved at CHX is 63.8 K as shown in Figure 7. Main reason for the difference between numerical and experimental result is due to aftercooler modelling. Physically aftercooler is slotted heat exchanger but in the numerical simulation it is taken as porous section due to difficulty in the 2D axisymmetric modelling. Temperature contour of the PTR is shown in Figure 8.

![Figure 8. Temperature contour](image)

3.2. 3D-PTR Model:

Temperature achieved at CHX is 26.5 K as shown in Figure 9. Gawali [2] achieved 39K experimentally. Difference between numerical and experimental result is due to heat transfer losses occurred in the different part of the system.
Pressure and mass flow rate at cold end of pulse tube are plotted in Figure 10. This plot is used to find out phase angle between pressure and mass flow rate which is crucial for refrigerating effect.

4. Conclusion
Summarizing, the methodology for analysing an IPTR in CFD, using 2D-axisymmetric and 3D model is proposed. Gawali’s IPTR model [2] is validated and results of the presented model are adequately agree with the Gawali model. This simulation demonstrates the time varying temperature at cold heat exchanger. Also the phase difference in pressure and mass flow rate at cold end of pulse tube is also predicated. This methodology can be used for design of IPTR and same methodology can be extended to do the modifications in the system and analyze it.

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