Anomalous positron excess from Lorentz-violating QED

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ABSTRACT: We entertain the idea that a suitable background of cold (very low momentum) pseudoscalar particles or condensate, may trigger a background that effectively generates Lorentz-invariance violation. This æther-like background induces a Chern-Simons modification of QED. Physics is different in different frames and, in the rest frame of the pseudoscalar background, high momentum photons can decay into pairs. The threshold for such decay depends quadratically on the rest mass of the particles. This mechanism could explain in a natural way why antiprotons are absent in recent cosmic ray measurements. A similar signal could be used as a probe of pseudoscalar condensation in heavy ion collisions.

KEYWORDS: Space-Time Symmetries, Gamma and Cosmic Rays, QCD Phenomenology.
1. Introduction

Recent results from the PAMELA collaboration [1] reveal an apparent excess of high-energy positrons in cosmic rays, which is not accompanied by a corresponding excess in antiprotons [2]. This excess had been found previously by ATIC [3] and PPB-BETS [4]. This enhancement has been confirmed recently by the FERMI collaboration (Gamma Ray Space Telescope) [5] (in accordance with previous indications from HEAT [6] and AMS-01 [7]). The latter missions cannot separate positrons from electrons, however.

It has been pointed out that the positron excess can have a purely astrophysical interpretation being possibly due to nearby pulsars in our galaxy or other astrophysical phenomena [8], but the possibility of the excess being due to dark matter annihilation or decay is of course very interesting and has been recently studied in some detail [9].

None of the dark matter interpretations however addresses satisfactorily the basic puzzling question; namely why antiprotons are so conspicuously absent from the PAMELA measurements. One would be forced to conclude that either the relevant dark matter component is abnormally leptophilic (hadrophobic) [10] or the thresholds for production of highly energetic $e^+e^-$ vs. $\bar{p}p$ pairs are very different due to some unexplained effect.

In this work we address the second possibility and propose a simple mechanism by virtue of which high-energy real photons could decay into particle-antiparticle pairs with a momentum threshold that depends on the rest mass of the particle. We hurry to say that this is impossible in a Lorentz-invariant theory. Indeed the mechanism is based on the condensation of either axions [11], vector fields [12] or simply neutral pions [13, 14], forming a Lorentz violating background uniform in space (or just slowly varying in comparison to the hard photon/high-energy positron Compton wave lengths). This condensate should however be varying in time (in contrast to [13]) for the proposed mechanism to be possible. We shall discuss later tentative possibilities for the origin of such effect and the orders or magnitude involved.
It turns out that the Lorentz-violating effects \cite{15, 16, 17, 18, 19} induced by this \textit{æther-like} pseudoscalar density generate successive thresholds for pair productions. Obviously the first threshold corresponds to $e^+e^-$ pairs. Photons with increasing larger values of the momenta could eventually generate $\mu^+\mu^-$ pairs (eventually decaying to electrons and positrons too) and for even higher energy photons $\bar{p}p$ pairs. A unambiguous prediction of the model is that the successive thresholds appear for photons whose momenta are in a relation roughly identical to the the ratio of the mass squared of muons or protons to that one of electrons.

We shall also see that this phenomenon could also be present in heavy ion collisions if the conditions are such that a pseudoscalar condensate may form. In fact the emergence of electron pairs with the very specific characteristics that we will find out later would be a novel and interesting signal of the presence of parity violation in dense baryon matter as it has recently been suggested \cite{14}. Another type of P-violation in hot metastable nuclear bubbles has been proposed in \cite{20} and induced photon instability could be also relevant to detect it.

\section{Lorentz violation in a Chern-Simons background}

Suppose that an spatially homogeneous and isotropic background may be induced by condensation of pseudoscalars and examine how photons may split $\gamma \rightarrow e^- + e^+$ via a mechanism that has a fair analogy with the Cherenkov radiation.

The appropriate Lagrangian in the presence this background has three pieces

$$\mathcal{L} = \mathcal{L}_{\text{INV}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{LIV}}; \quad (2.1)$$

\begin{align*}
\mathcal{L}_{\text{GF}} &= A^\lambda(x) \partial_\lambda B(x) + \frac{i}{2} \varpi B^2(x), \quad (2.2) \\
\mathcal{L}_{\text{INV}} &= -\frac{1}{4} F^{\alpha\beta}(x) F_{\alpha\beta}(x) + \bar{\psi}(x) \{ \gamma^\mu [i \partial_\mu - e A_\mu(x)] - m_e \} \psi(x), \quad (2.3) \\
\mathcal{L}_{\text{LIV}} &= \frac{1}{2} \eta_\alpha A_\beta(x) \bar{F}^{\alpha\beta}(x), \quad (2.4)
\end{align*}

where $A_\mu$ and $\psi(x)$ stand for the photon and matter field, respectively, $\bar{F}^{\alpha\beta}(x) = \frac{1}{2} \varepsilon^{\alpha\beta\rho\sigma} F_{\rho\sigma}(x)$ is the dual field tensor, while $B$ is the gauge-fixing auxiliary scalar field with $\varpi \in \mathbb{R}$. LIV stands for Lorentz invariance violating.

The vector $\eta_\alpha \simeq \langle \partial_\alpha \theta \rangle \simeq \delta_{\alpha0} \langle \dot{\theta}(t) \rangle$, coupled via the anomaly term to photons, is supposedly induced by a pseudoscalar density which form the background in which high momentum photons, of some unspecified origin, propagate. Similar formulae can be derived for propagating electrons and positrons at high energies in a Lorentz violating background \cite{17}, but let us follow the argumentation of KSVZ \cite{21} and neglect the kinematical distortion induced on them.

Since this is an unfamiliar setting it is advisable to examine this theory carefully. In momentum space we obtain the free field equations for the LIV massive vector field and the auxiliary scalar field

\begin{align*}
\left\{ g^{\lambda\nu} k^2 - k^\lambda k^\nu + i \varepsilon^{\lambda\alpha\beta} \eta_\alpha k_\beta \right\} \tilde{A}_\lambda(k) + i k^\nu \tilde{B}(k) &= 0, \quad (2.5) \\
i k^\lambda \tilde{A}_\lambda(k) + \varpi \tilde{B}(k) &= 0. \quad (2.6)
\end{align*}
In the Feynman gauge \( \kappa = 1 \) we get
\[
\tilde{B}(k) + i k \cdot \tilde{A}(k) = 0, \quad (2.7)
\]
\[
k^2 \tilde{B}(k) = k^2 \eta \cdot \tilde{A}(k) = 0, \quad (2.8)
\]
\[
\left\{ g^{\nu \lambda} k^2 + i \varepsilon^{\lambda \mu \rho \sigma} \eta_\alpha k_\beta \right\} \tilde{A}_\lambda(k) \equiv K^\nu_\lambda \tilde{A}_\lambda(k) = 0. \quad (2.9)
\]

The Levi-Civita symbol in four dimensional Minkowski space-time is
\[
\varepsilon_{0123} = -\varepsilon_{0123} \equiv 1.
\]

We now define
\[
S^\nu_\lambda \equiv \varepsilon^{\mu \nu \alpha \beta} \eta_\alpha k_\beta \equiv K^\nu_\lambda \tilde{A}_\lambda(k) = 0.
\]

which satisfies the following properties
\[
S^\nu_\lambda \eta^\lambda = S^\nu_\lambda k^\lambda = 0, \quad S^\mu_\nu S^\nu_\lambda = \frac{S}{2} S^\mu_\lambda, \quad S = S^\nu_\nu = 2[(\eta \cdot k)^2 - \eta^2 k^2]. \quad (2.11)
\]

Notice that for a time-like and spatial isotropic pseudoscalar æther\( ^1 \) \( \eta^\mu = (\eta, 0, 0, 0) \) we find \( S = 2\eta^2 k^2 > 0 \). Next, it is convenient to introduce the two orthonormal hermitian “projectors”
\[
P^\pm_\mu = \frac{S^\mu_\nu}{S} \pm \frac{i}{\sqrt{S/2}} \varepsilon^{\mu \nu \alpha \beta} \eta_\alpha k_\beta, \quad (2.12)
\]

which enjoy the properties \( \forall k^\mu = (k_0, k) \)
\[
P^\pm_\mu \eta_\nu = P^\pm_\mu k_\nu = 0, \quad g^\mu_\nu P^\pm_\mu = 1, \quad (2.13)
\]
\[
P^\pm_\mu P^\mp_\nu = P^\pm_\nu, \quad P^\pm_\mu P^\pm_\nu = 0, \quad P^\pm_\mu + P^\mp_\mu = \frac{2}{S} S^\mu_\nu. \quad (2.14)
\]

It follows from this that we can build up a pair of complex and space-like chiral polarization vectors by means of a constant and space-like four vector: for example \( \varepsilon^\nu_\nu = (0, 1, 1, 1)/\sqrt{3} \), in such a manner that we can set
\[
\varepsilon^\mu_\pm(k) \equiv \left[ \frac{k^2 - (\varepsilon \cdot k)^2}{2k^2} \right]^{-1/2} P^\mu_\pm \varepsilon^\nu, \quad (2.15)
\]

which satisfy the orthogonality relations
\[
- g^\mu_\nu \varepsilon^\mu_\pm(k) \varepsilon^\nu_\pm(k) = 1; \quad g^\mu_\nu \varepsilon^\mu_+ (k) \varepsilon^\nu_+ (k) = 0, \quad (2.16)
\]
as well as the closure relation
\[
\varepsilon^\mu_+ (k) \varepsilon^\nu_+ (k) + \varepsilon^\mu_- (k) \varepsilon^\nu_- (k) + \text{c.c.} = -\frac{4}{S} S^\mu_\nu. \quad (2.17)
\]

Now we are ready to find the general solution of the free field equations (2.9) in the Feynman gauge. Taking into account the relations (2.13) and (2.14) we readily obtain
\[
K^\mu_\nu = \delta^\mu_\nu k^2 + \sqrt{\frac{S}{2}} (P^\mu_\nu - P^\mu_-). \quad (2.18)
\]

\( ^1 \)This type of æther was considered for the first time in [15] but in the context of the large-scale structure of the universe whereas here we assume its existence near some astrophysical objects – or in heavy ion collisions.
Therefore
\[ K^\mu \varepsilon_\pm^\nu (k) = \left( k^2 \pm \sqrt{S^2 / 2} \right) \varepsilon_\pm^\nu (k), \] (2.19)
which shows that they are solutions of the equations of motion iff
\[ k^\mu_\pm = (\omega_\pm(k), k) \quad \omega_\pm(k) = \sqrt{k^2 \pm \eta |k|}. \] (2.20)
Evidently for “+” polarization the photon energy at sufficiently low momenta, $|k| < \eta$, becomes imaginary signifying the instability of photon vacuum \[11, 15, 17, 18\].

For strictly vanishing photon mass the instability affecting the “−” polarization sets up already for zero-momentum photons. However in-medium photons do acquire a mass without breaking any fundamental gauge principle, so we will take $m_\gamma \neq 0$. The instability then sets up for $\eta > 2m_\gamma$ (see \[22\]) and whether the vacuum instability it is relevant or not depends on the relative values of $m_\gamma$ and $\eta$. However in this work we are interested in high-energy phenomena when photon wave vectors are much larger than the scale set up by $\eta$ (in fact assuming that the very concept of a spatially constant pseudoscalar background is valid at microscopic scales and makes sense for sufficiently high photon frequencies). Accordingly we neglect this subtlety and refer the reader for its treatment to \[22\].

It is also important to realize that for “−” photons, even after having introduced a mass $m_\gamma > \eta/2$ the condition $k^2 \geq 0$, which is necessary for genuine causal propagation, holds only and only if the spatial momentum $k$ stands below the momentum cutoff $\Lambda_\gamma$, i.e., it belongs to the causality/stability region
\[ |k| < \frac{m_\gamma^2}{\eta} \equiv \Lambda_\gamma, \] (2.21)
Above this bound, photons of both chiralities would obviously decay (via the box diagram) to three photons of negative chiralities which in turn could also decay and so on until they are completely red-shifted. The decay process is a slow one, nevertheless being of order $\alpha^4$. It happens because photons of negative chiralities obtain an effective mass of tachyonic type having nevertheless group velocities less than the conventional speed of light \[17\].

We introduce two further orthonormal polarization four-vectors, i.e., the temporal and longitudinal polarization vectors, respectively
\[ \varepsilon_\mu^T(k) \equiv \frac{ik^\mu}{\sqrt{k^2}} \quad (k^2 > 0), \] (2.22)
\[ \varepsilon_\mu^L(k) \equiv (k^2 D)^{-1/2} (k^2 \eta^\mu - k^\mu \eta \cdot k) \quad (k^2 > 0), \] (2.23)
which fulfill by construction
\[ g_{\mu\nu} \varepsilon_\mu^T(k) \varepsilon_\nu^T(k) = - g_{\mu\nu} \varepsilon_\mu^L(k) \varepsilon_\nu^L(k) = 1; \quad g_{\mu\nu} \varepsilon_\mu^T(k) \varepsilon_\nu^L(k) = 0. \] (2.24)
Thus we have at our disposal $\forall k^\mu$ with $k^2 > 0$ a complete orthonormal set of four polarization four vectors: namely $\varepsilon_\mu^A(k)$ with $A = T, L, +, −$. They satisfy
\[ g_{\mu\nu} \varepsilon_\mu^A(k) \varepsilon_\nu^B(k) = g_{AB}; \quad g_{AB} \varepsilon_\mu^A(k) \varepsilon_\nu^B(k) = g^{\mu\nu}, \] (2.25)
where $g^{AB} = \text{diag}(1, -1, -1, -1)$.

Then for massive photons the physical subspace consists of the three polarizations $A = \pm, L$, but one of the two chiral transverse states with complex polarization vectors $\varepsilon_\nu^\pm (k_\pm)$, namely $\varepsilon^\nu_-(k_-)$ exists only iff $|k| < \Lambda_\gamma \Leftrightarrow k^2 > 0$. If we take $m_\gamma = 0$ this helicity state becomes superluminal for all values of the momenta and produces a sort of Cherenkov radiation, gradually splitting into three photons with negative polarizations (see \cite{24} for similar arguments but for space-like background vectors). We have to stress that, kinematically, the high-energy photon with positive polarization can also undergo splitting into the negative polarization photons. Both splittings are kinematically allowed as it can be easily read out from the inequality for the forward decay (we neglect here the photon mass),

$$\omega_\pm(k) = \sqrt{k^2 + \eta |k|} > 3\omega_-(\frac{k}{3}). \quad (2.26)$$

Thus if the phenomenon of positron excess is accounted for by the instability of photons in a pseudoscalar background an accompanying effect might be the suppression of high-energy $\gamma$ rays from the same region, depending on the value of the effective photon mass, bearing in mind that this process is anyway a one-loop effect and the threshold is $\sim m_\gamma^2/\eta$ (perhaps the results reported in \cite{23} might be a hint of this phenomenon). In addition, there is the possibility of “radiative” LIV decays $e^- \to e^-\gamma$; the momentum threshold being $|k| > m_\gamma m_\epsilon/\eta$. This effect will change the energy spectrum of the $e^+e^-$ pair produced in LIV $\gamma \to e^+e^-$ decays, but it is suppressed by a power of $\alpha$ and the cross-section must be proportional to $\eta$ too.

3. Decay amplitudes

We are now ready to derive the lowest order decay amplitude for the process $\gamma \to e^+e^-$. The Feynman rules are formally the usual ones except for the addition of thresholds such as the one implied by (2.21).

$$|\mathcal{M}(k, p, q)|^2 = 2\pi \alpha g_{\rho\sigma} g_{\mu\nu} \varepsilon_A^\mu(k) \varepsilon_B^{\mu\nu}(k) \times \bar{u}_s(p) \gamma_\nu v_s(q) \bar{v}_s(q) \gamma_\sigma u_r(p). \quad (3.1)$$

Let us now determine the thresholds for the different polarizations involved in the decay process. For the longitudinal polarization $A = L$ the dispersion relation is $k_0^2 = k^2 + m_\gamma^2$ so that the energy-momentum conservation forbids the decay process for $m_\gamma \ll m_\epsilon$. Conversely, for the transverse chiral polarizations $A = \pm$ we find

$$\omega_\pm(k) = \sqrt{k^2 + m_\gamma^2 \pm \eta |k|} = \sqrt{p^2 - m_\epsilon^2 + \sqrt{(k - p)^2 + m_\epsilon^2}} \equiv E(p) + E(q), \quad (3.2)$$

with $q = k - p$. From the energy-momentum conservation we get

$$k_\pm^2 = m_\gamma^2 \pm \eta |k| \simeq \pm \eta |k| = 2m_\epsilon^2 + 2E(p)E(q) - 2p \cdot q, \quad (3.3)$$

since $m_\gamma \ll m_\epsilon$. It is evident that photons with a negative chiral polarization $A = (-)$ cannot decay as they are tachyonic-like in such a background, while photons of positive
chiral polarization $A = (+)$ do undergo the decay iff the photon momentum is above the threshold
\[ |k| \geq \frac{4m_e^2}{\eta} \equiv k_{\text{th}}. \] (3.4)

The calculation of the total decay width is quite standard:
\[
\Gamma_+(k) = \frac{1}{2\omega_+(k)} \int \frac{d\mathbf{p}}{(2\pi)^3 2E(p)} \int \frac{d\mathbf{q}}{(2\pi)^3 2E(q)} (2\pi)^4 \delta^{(4)}(k - p - q)
\times \sum_{r,s=1,2} |\mathcal{M}_{rs}(k,p,q)|^2
\]
\[ = \frac{1}{32\pi^2 \omega_+(k)} \int \frac{d\mathbf{p}}{E(p)E(q)} \delta\left(\omega+(k) - E(p) - E(k - p)\right)
\times \sum_{r,s=1,2} |\mathcal{M}_{rs}(k,p,k - p)|^2. \] (3.5)

From the equality (3.3) in the form
\[ k_+^2 = \eta |k| = 2E(p) \left[ E(p) + E(k - p) \right] - 2 |k| |p| \cos \theta, \] (3.6)
we readily get with $k = |k|$ and $p = |p|$ the following condition for the energy delta function to be satisfied
\[ \frac{1}{2} \eta k = \sqrt{(p^2 + m_e^2)(k^2 + k\eta)} - kp \cos \theta = p_\mu k_\mu^\mu, \] (3.7)
which gives the physical values of electron/positron momenta,
\[ p_\pm = \frac{\eta \cos \theta \pm 2m_e \sqrt{\left(1 + \frac{\eta}{2}\right)\left(\frac{\eta^2}{4m_e^2} - \sin^2 \theta\right)}}{2\left(\sin^2 \theta + \frac{\eta}{k}\right)}, \] (3.8)
and requires the following inequality to hold
\[ \sin^2 \theta \leq \frac{\eta^2}{4m_e^2} \left(1 - \frac{4m_e^2}{\eta k}\right). \] (3.9)

As we see, the emitted pair is produced inside a narrow forward cone with an angle that, typically, should be small as we expect $\eta << m_e$; that is $\theta_{\text{max}} < \eta/2m_e$.

For the adjacent momentum $\mathbf{q} = \mathbf{k} - \mathbf{p}$ we define $q = |\mathbf{k} - \mathbf{p}|$, $\mathbf{q} \cdot \mathbf{k} = qk \cos \bar{\theta}$ and find the relation
\[ q \sin \bar{\theta} = -p \sin \theta, \] (3.10)
wherefrom and from eq.(3.7) it can be obtained that the two angles are similarly small but the two momenta are complementary. For instance, if $k \gg k_{\text{th}}$ for the lowest limit $p_- \approx m_e^2/\eta$ one finds $q_+ \approx k - m_e^2/\eta$ and vice versa.

The angular integration in the phase space integral [3.7] resolves the delta-function in a nontrivial way when
\[ \cos \bar{\theta} = -\frac{\eta}{2p} + \sqrt{\left(1 + \frac{\eta}{k}\right)\left(1 + \frac{m_e^2}{p^2}\right)} \leq 1, \] (3.11)
that bounds the momentum
\[
\min p_\pm \simeq \frac{m^2}{\eta} \leq p \leq \max p_+ \simeq k - \frac{m^2}{\eta},
\]  
(3.12)
for \(k \gg k_{th}\). Accordingly the phase space integral can be evaluated
\[
\frac{1}{32\pi^2\omega_+(k)} \int \frac{dp}{E(p)E(k-p)} \delta\left(\omega_+(k) + E(p) + E(k-p)\right) \simeq \frac{1}{16\pi^2} \int \frac{dp}{\frac{m^2}{\eta}},
\]  
(3.13)
for \(\cos \theta\) obeying eq.(3.11).

Finally, setting \(m_\gamma = 0\), the squared decay amplitude for a photon of positive chiral polarization is
\[
\sum_{r,s=1,2} |M_{rs}(k,p,q)|^2 = 16\pi \alpha \theta (|k| - k_{th}) \times \left[2p_\mu p_{\rho} P_+^{\mu\rho}(k_+) + p \cdot k_+\right]
\]
\[
= 16\pi \alpha \left\{m^2_e - \frac{1}{4}\eta^2 + \frac{\eta}{2k} \left(E^2(p) + E^2(k-p)\right)\right\}
\]
\[
\simeq \frac{8\pi \alpha \eta}{k} \left(2p^2 + k^2 - 2kp\right)
\]  
(3.14)
for sufficiently large \(k \gg k_{th}\). Eventually the integration over phase space entails
\[
\Gamma_+ = \tau_+^{-1} \simeq \frac{\alpha \eta}{3},
\]  
(3.15)
which is essentially constant for high-energy photons.

4. Physical scenarios

We have seen that the presence of an æther-like time-dependent pseudoscalar background leads to the possibility of rather exotic phenomena\(^2\) such as \(\gamma \to e^+ e^-\) (and \(e \to \gamma e\), \(\gamma \to \gamma \gamma \gamma\) controlled by an effective photon mass). For a LIV photon we have seen that this is a physical state only for a given polarization governed by the sign of the background vector \(\eta_\mu\) whereas photons of the opposite polarization are "tachyonic" or superluminal in the phase velocity. Furthermore the decay of physical photons can take place (due to energy-momentum conservation) for photons of sufficiently high 3-momentum. This leads to the possibility of successive thresholds for the production of progressively more massive \(\bar{f} f\) pairs. The key inequality is
\[
|k| > \frac{4m_f^2}{\eta}.
\]  
(4.1)

The accompanying processes \(e \to \gamma e\), \(\gamma \to \gamma \gamma \gamma\) are controlled by an effective photon mass and by an oscillation frequency of the axion background \(\eta\). They involve "tachyonic" photons as final states and, although they are rare, they potentially lead to a red shift in

\(^2\)For analogous processes triggered by a space-like CS vector, see the analysis in \[24\].
electron/positron and photon spectra as well as to the dominance of a particular photon polarization if \( \eta \) has a definite sign. But most plausibly \( \eta \) is slowly oscillating as being the derivative of an axion-like background which cannot grow up to extremely large values. Then if the phenomenon takes place in a sufficiently large volume one cannot register a definite polarization.

The decay width of the process \( e \to \gamma e \) is expected to be of the same order \( \sim \alpha \eta \) as the photon decay due to crossing symmetry. But the final photon obtains a tachyonic effective mass and is not involved further on into \( e^+ e^- \) pair creation (but rather decays in 3-photon states). Thus this decay enforces a red shift in the electron spectrum, in principle observable by Fermi-LAT \( ^3 \). In particular, the absence or softening of the electron-positron excess \( ^3 \) in the interval of \( 300 \div 800 \) GeV might be just accounted for by this red shift. Thus such a phenomenon may perhaps point out to a mechanism of anomalous \( e^+ e^- \) pair creation as a main source for explanation of the PAMELA data. In turn the decay width of photon splitting \( \gamma \to \gamma \gamma \gamma \) is estimated to be much smaller, proportional to \( \alpha^2 \eta^2 / m_f \) and is not expected to make an essential red shift of the photon spectrum. More accurate calculations have not been done yet for time-like \( \eta \) and this work is in progress.

Let us now discuss possible physical situations where this phenomenon might happen. Let us begin by discussion of heavy ion collisions and nuclear matter at high densities. It has been derived in \( ^{14} \) that a phase where parity is spontaneously broken may exist for baryonic densities corresponding to 3 to 8 times the usual nuclear density. Such a state could be produced in heavy ion collisions in conditions which are expected in the experiment CMB \( ^{25} \) at the FAIR facility or at the planned NICA accelerator \( ^{26} \). As the density in the center-of-mass frame of colliding ions is growing in time of collision one anticipates a time-dependent neutral pion condensate with a nearly constant derivative. The latter just produces a modification of QED and photon instability as described above. In this case the natural scale for \( \eta \) is

\[
\eta \sim \frac{\alpha \dot{\rho}}{2\pi f_\pi} \frac{\partial \langle \Pi \rangle}{\partial \rho},
\]

where \( f_\pi \) would be the in-medium pion decay constant (vacuum value: \( f_\pi \sim 100 \) MeV), \( \rho \) is the density and \( \langle \Pi \rangle \) is the value of the parity-breaking condensate. Although the actual magnitude depends on interaction details (such as \( \dot{\rho} \), which in turn depends on the hadronic matter compressibility) a natural expectation would be \( \eta \sim 1 \) keV implying that the threshold for anomalous LIV \( e^+ e^- \) pair production \( k_{th} \) is naturally in the GeV region and the photon decay width \( \Gamma_+ \) is of order 1 eV for \( k \gg k_{th} \) . As for hard photons, they are abundant due to interactions at the parton level with energies in the GeV region. So anomalous LIV \( e^+ e^- \) pair production is a realistic possibility and a potential way to probe hadronic matter in extreme conditions. It is remarkable that the generation of photon mass is also predicted in the presence of a neutral pion condensate \( ^{13} \).

In an astrophysical context, it has been seen by the PAMELA mission that there is an excess of positrons starting at around 10 GeV . So we assume \( k_{th} \) to be approximately 10 GeV. An obvious possibility would be to consider the relic cold axion \( ^{27} \) density. Then, for a light axion with \( f_a \sim 10^{11} \) GeV, a calculation analogous to the just presented would
lead to
\[ \eta \sim \frac{\alpha}{2\pi f_a} \sqrt{m_a n_a}, \]
(4.3)

where \( m_a \) is the axion mass < 1eV and \( n_a \) the number of cold axions per unit volume. Thus this possibility to explain the PAMELA excess is totally excluded as it would require \( n_a \) many orders of magnitude larger than the current cosmological bound. Somewhat lower bounds on \( f_a \leq 10^{10} \text{ GeV} \) have been obtained with different helioscopes \(^\text{[28]}\) to register solar/terrestrial axion-like particles \(^3\) (see the reviews in \(^\text{[27]}\)) but yet its value seems to be too low to trigger a visible photon instability within the solar system. Thus the electron-positron excess must have its origin far from the solar system.

Another possibility to consider is the apparent existing window for more massive axions \(^\text{[31]}\) which corresponds to \( f_a \sim 10^5 \text{ GeV} \). As we have said to generate anomalous LIV pair in the GeV region one needs \( \eta \) to be in the KeV region. This would require still very large axion densities, perhaps attainable only in axion stars. Finally, a very fast varying time dependent axion condensate could provide the seed for LIV anomalous pair production, but we do not have a concrete mechanism to propose.

If cold axions \(^\text{[27]}\) constitute the essential component of dark matter, we can estimate the value of \( \eta \) to be \( \eta \sim 10^{-29} \text{ eV} \). This may seem a tiny value, and indeed it is, but if we take our previous results at face value and use the upper limit for an effective photon mass in the range of \( 10^{-18} \text{ eV} \) \(^\text{[32]}\), we see that the threshold for the anomalous LIV ”radiative” process \( e^- \rightarrow e^- \gamma \) is \( 10^8 \text{ GeV} \) or lower, while the one for \( p \rightarrow p \gamma \) is \( 10^{11} \text{ GeV} \). Both should be relevant for high-energy cosmic rays, acting as an additional suppression on top of the usual GZK effect.

There may be more exotic causes to generate an axial-vector condensate. Condensation of vector “bumblebee“ fields in certain non-renormalizable models \(^\text{[12]}\) and of gradients of (nearly) massless axions due to Coleman-Weinberg mechanism \(^\text{[11]}\). Unfortunately in those approaches the identification of dark condensates cannot be easily done on terms of particle physics.

It is anyhow clear that to observe anomalous thresholds in the GeV region the natural scale is provided by strongly interacting pseudoscalars. The experimental signal would be rather unmistakable: the produced \( e^+e^- \) pair flies away in a very narrow cone and, of course, the process has a threshold that has nothing to do with the usual \( \gamma\gamma \rightarrow e^+e^- \) one.

5. Conclusions

In this paper we have investigated the phenomenology of Lorentz-violating QED, described by an additional Chern-Simons term including a time-like, but translational invariant, background. This æther-like background could be provided by any type of cold pseudoscalars. We have also explored the possibility that this LIV modification of QED may trigger

\(^3\)Some time ago, in the PVLAS experiments, much larger values for axion-like particle mass and much lower values for \( f_a \) have been announced. But later, after substantial improvement of the experimental technique the birefringence effect disappeared \(^\text{[29]}\) and the bounds are conservatively given by the CAST observations \(^\text{[30]}\).
anomalous $e^+e^-$ pair production. In fact it is totally natural in this type of models to have very different thresholds, thus explaining in a simple and natural way why $\bar{p}p$ pairs are conspicuously absent in some astrophysical phenomena.

Axions or axion-like particles are obvious candidates for this pseudoscalar background, but if the anomalous pair production is to take place around 10 GeV this requires an absurdly large density of relic cold axions. A more like scenario is that pseudoscalar condensation due to strong interactions may give some visible effects.

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