Four-fermion production near the W-pair production threshold

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I report on recent results for the total production cross section of the process $e^- e^+ \rightarrow \mu^- \bar{\nu}_\mu u \bar{d} X$ near the W-pair production threshold up to next-to-leading order in $\Gamma_W/M_W \sim \alpha \sim v^2$ obtained in the framework of unstable-particle effective field theory. Remaining theoretical uncertainties and their impact on the experimental determination of the W mass are discussed.
1. Introduction

An accurate measurement of the W mass is of primary interest for precision tests of the Standard Model and for search of New-Physics effects through virtual-particle exchange. The total error on $M_W$ could be lowered to 6 MeV by measuring the four-fermion production cross section near the $W$-pair production threshold [1] at a future International Linear Collider (ILC), provided that the theoretical uncertainties are well below 1%. This is a difficult task, requiring gauge-invariant inclusion of finite-width effects and calculation of QCD and electroweak radiative corrections to the full $2 \rightarrow 4$ process. Previous NLO calculations in the double-pole approximation [2] were supposed to break down near threshold for kinematical reasons. The recent computation of the complete NLO corrections to $e^- e^+ \rightarrow 4f$ in the complex-mass scheme [3] is valid both near threshold and in the continuum, but is technically difficult, requiring the computation of one-loop six-point functions.

Here I present NLO results for the total cross section of the process

$$e^- e^+ \rightarrow \mu^- \bar{\nu}_\mu u d X$$

(1.1)

near the $W$-pair production threshold [4] computed with effective field theory (EFT) techniques [5, 6, 7]. Section 2 reviews briefly the formalism, while the calculation of the Born cross section and of radiative corrections is outlined in Sections 3 and 4. Section 5 presents numerical results together with an estimate of the remaining theoretical uncertainties and a comparison with [3].

2. Unstable-particle effective field theory

The EFT approach [7] exploits the hierarchy of scales $M \Gamma \ll M^2$ which characterizes processes involving unstable particles, $M$ and $\Gamma$ being the mass and width of the intermediate resonance. The degrees of freedom of the full theory are classified according to their scaling into short-distance ($k^2 \sim M^2$) and long-distance ($k^2 \lesssim M \Gamma$) modes. The fluctuations at the small scale (resonant particles, soft and Coulomb photons,...) represent the field content of the effective Lagrangian $\mathcal{L}_{\text{eff}}$. “Hard” fluctuations with $k^2 \sim M^2$ are not part of the effective theory and are integrated out. Their effect is included in $\mathcal{L}_{\text{eff}}$ through short-distance matching coefficients, computed in standard fixed-order perturbation theory. The systematic inclusion of finite-width effects is relevant for modes with virtuality $k^2 \lesssim M \Gamma$ and is obtained through complex short-distance coefficients in $\mathcal{L}_{\text{eff}}$ [7].

The specific process (1.1) is primarily mediated by production of a pair of resonant $W$s. The total cross section is extracted from appropriate cuts of the forward-scattering amplitude [8], which after integrating out the hard modes with $k^2 \sim M^2$ reads [7]

$$i \mathcal{A} = \sum_{k,l} \int d^4 x \langle e^- e^+ | T[i \Theta^{(k)}_p (0) i \Theta^{(l)}_p (x)] | e^- e^+ \rangle + \sum_k \langle e^- e^+ | i \Theta^{(k)}_{4e} (0) | e^- e^+ \rangle.$$

(2.1)

The operators $\Theta^{(l)}_p$ ($\Theta^{(k)}_p$) in the first term on the right-hand side of (2.1) produce (destroy) a pair of non-relativistic resonant $W$ bosons. The second term accounts for the remaining non-resonant contributions. The computation of $i \mathcal{A}$ is split into the determination of the matching coefficients of the operators $\Theta^{(l)}_p$, $\Theta^{(k)}_p$ and the calculation of the matrix elements in (2.1). Both quantities are computed as power series in the couplings $\alpha, \alpha_s$, the ratio $\Gamma_W / M_W$ and the non-relativistic velocity of the intermediate resonant $W$ pair $v^2 \equiv (\sqrt{s} - 2M_W) / (2M_W)$, collectively referred to as $\delta / \alpha_s^2 \sim \alpha \sim \Gamma_W / M_W \sim v^2$. 

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The effective Lagrangian describing the non-relativistic W bosons up to NLO in $\delta$ is \[ \mathcal{L}_{\text{NRQED}} = \sum_{a=1}^{4} \left[ \Omega_{ia}^{\perp} \left( iD^0 + 2 \frac{D^2}{2M^2_W} - \frac{\Delta}{2} \right) \Omega_{ia}^{\perp} + \Omega_{ia}^\perp \left( \frac{\Delta}{8M^2_W} \right)^2 \Omega_{ia}^{\perp} \right]. \] (2.2)

$\Delta$ is the matching coefficient $\Delta \equiv (\bar{s} - M_W^2)/M_W$, where $\bar{s}$ is the complex pole of the W propagator. The field $\Omega_{ia}^\perp = \sqrt{2M_W}\Omega_{ia}^\perp$ describes the three physical polarizations of non-relativistic Ws, and the covariant derivative $D_\mu \Omega_{\pm} = (\partial_\mu \mp i e A_\mu) \Omega_{\pm}$ contains the interaction of the resonant fields $\Omega_{\pm}$ with soft and potential photons (see Section 4). To complete $\mathcal{L}_\text{eff}$ one has to add to (2.2) the effective production vertices $\mathcal{O}_p^{(i)}$ and the four-fermion operators $\mathcal{O}_{4f}^{(i)}$ with the corresponding matching coefficients computed to the desired order in $\delta$. These are presented in Sections 3 and 4.

3. EFT approximation to the Born cross section

The lowest-order production operator of two non-relativistic resonant Ws is \[ \mathcal{O}_p^{(0)} = \frac{\pi e^2}{M_W^2} (\bar{e}_{c_{L}} (\gamma^\mu n^\nu + \gamma^\nu n^\mu) e_{c_{L}}) \left( \Omega_{ia}^{\perp} \Omega_{ia}^{\perp} \right). \] (3.1)

Its matching coefficient is extracted from the on-shell process $e^- e^+ \to W^- W^+$, where “on-shell” means $k^2 = \bar{s}$. The four-fermion operators $\mathcal{O}_{4f}^{(i)}$ do not contribute to $\mathcal{A}$ at this order, and the forward-scattering amplitude is simply

$$
\mathcal{A}^{(0)} = \int d^4x \langle e^- e^+ | T [i \mathcal{O}_p^{(0)^+}(0) i \mathcal{O}_p^{(0)}(x)] | e^- e^+ \rangle = \frac{i\alpha^2}{s_w^2} \frac{\sqrt{E + i\Gamma_W^{(0)}}}{M_W},
$$

with $E = \sqrt{s} - 2M_W$ and $s_w = \sin \theta_W$. The total cross section for (1.1) is extracted from appropriate cuts of (3.2). At lowest order this is correctly done by multiplying the imaginary part of $\mathcal{A}^{(0)}$ with the LO branching ratios of the decays $W^- \to \mu^- \bar{\nu}_\mu, W^+ \to ud$, so that $\sigma^{(0)} = \frac{1}{27\alpha} \operatorname{Im} \mathcal{A}^{(0)}$.

Beyond the leading term $\sigma^{(0)}$ there are contributions which can be identified with terms of the expansion in $\delta$ of a full-theory Born result computed with a fixed-width prescription. The first class of corrections arises from four-electron operators in (2.1). The imaginary part of their matching coefficients are extracted from suitable cuts of hard two-loop SM diagrams [6]:

\[ \Omega_{ia}^{\perp} = \sum_{\alpha=1}^{4} \left[ \Omega_{ia}^{\perp} \left( iD^0 + 2 \frac{D^2}{2M^2_W} - \frac{\Delta}{2} \right) \Omega_{ia}^{\perp} + \Omega_{ia}^\perp \left( \frac{\Delta}{8M^2_W} \right)^2 \Omega_{ia}^{\perp} \right]. \] (3.3)

Compared to the LO cross section $\sigma^{(0)} \sim \alpha^2 \sqrt{\delta}$ the new term is suppressed by $\alpha/\sqrt{\delta} \sim \sqrt{\delta}$ and is denoted as “$\sqrt{\text{NLO}}$”. True NLO contributions to $\mathcal{A}^{(0)}$ arise from higher-dimension production operators and propagator corrections. The former come from the matching of the effective theory on the on-shell process $e^- e^+ \to W^- W^+$ at order $\nu (\mathcal{O}_p^{(1/2)^+})$ and $\nu^2 (\mathcal{O}_p^{(1)^+})$ [6]. The latter correspond to the term $\left( \frac{\Delta}{8M^2_W} \right)^2 (\sqrt{s} - 2M_W)$ in (2.2). A comparison of the EFT Born approximations with the full result computed with Whizard [8] shows a good convergence of the series [4]. However partial inclusion of $\text{N}^{1/2}\text{LO}$ corrections is necessary to obtain an agreement of $\sim 0.1\%$ at 170 GeV and $\sim 10\%$ at 155 GeV [4].
4. Radiative corrections

A complete NLO prediction must include radiative corrections to the Born result. These are electroweak and QCD corrections to the matching coefficient of $O_p^{(0)}$ and loop contributions to the EFT matrix elements. At NLO the flavor-specific final state is selected by multiplying the total cross section with NLO branching ratios. The $O(\alpha)$ correction to the matching coefficient of \cite{5} is obtained from the one-loop amplitude of $e^-e^+ \to W^-W^+$. Many of the 180 one-loop diagrams do not contribute due to threshold kinematics and the result reads \cite{5}:

$$C_p^{(1)} = \frac{\alpha}{2\pi} \left[ \left( -\frac{1}{\varepsilon^2} + \frac{3}{2\varepsilon} \right) \left( \frac{4M_W^2}{\mu^2} \right)^{-\varepsilon} + c_p^{(1,\text{mix})} \right] \quad (4.1)$$

The one-loop corrections to the matrix elements arise from exchange of potential ($\sim M_W(\delta, \sqrt{\delta})$) and soft ($\sim M_W(\delta, \delta)$) photons. Loops containing $n$ potential photons are enhanced by inverse powers of $\alpha$, $\Delta \alpha \sim \alpha^{n+1} \sim \alpha^{n/2}$, so that the first and second Coulomb corrections must be included in a NLO calculation. Near threshold they amount respectively to $\sim 5\%$ and $\sim 0.2\%$ of $\sigma(\alpha)$ \cite{4}.

Two-loop diagrams with soft photons connecting different hard subprocesses of \cite{5} give the so-called non-factorizable corrections. As a consequence of the residual gauge-invariance of $\mathcal{L}_{\text{eff}}$, and in agreement with previous results \cite{9}, only the initial-initial state interferences survive:

$$\begin{align*}
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\text{\includegraphics[width=0.2\textwidth]{diagram.png}}
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\end{align*}
= \frac{4\pi^2 \alpha^2}{s_W^4 M_W^4} \frac{\alpha}{\pi^2} \int \frac{d^4 r}{(2\pi)^4} \frac{1}{\eta_- \eta_+} \left[ \left( \frac{1}{\varepsilon^2} + \frac{5}{12} \varepsilon^2 \right) \left( -\frac{2\eta_-}{\mu} \right)^{-2\varepsilon} \right] \quad (4.2)
$$

with $\eta_- = r_0 - \frac{\vec{p}^2}{2M_W} + i\frac{\Gamma^{(0)}_W}{2}$ and $\eta_+ = E - r_0 - \frac{\vec{p}^2}{2M_W} + i\frac{\Gamma^{(0)}_W}{2}$.

5. Results and remaining theoretical uncertainties

Because of the approximation $m_e = 0$, the sum of the corrections calculated in Section \cite{4} is not infrared safe, containing uncanceled $\varepsilon$-poles. The result should be convoluted with $\overline{\text{MS}}$ electron distribution functions after minimal subtraction of the pole. Since the distributions available in the literature are computed in a different scheme, which assumes $m_e$ as infrared regulator, it is more convenient to convert our result from $\overline{\text{MS}}$ to this scheme. This is done by adding contributions from the hard-collinear ($k^2 \sim m_e^2$) and soft-collinear ($k^2 \sim m_e^2 \Gamma_{\text{soft}}$) regions. These cancel the $\varepsilon$-poles, but introduce large logs of $2M_W/m_e$ \cite{8}. The large logs are resummed by convoluting the NLO cross section with the structure functions $\Gamma^{\text{eff}}_{\text{ee}}$ used in \cite{8} after subtracting the double counting terms \cite{9}. Since only leading logs are resummed in $\Gamma^{\text{eff}}_{\text{ee}}$, one can equivalently choose to convolute only the Born cross section with the structure functions, as done for example in \cite{3}, the difference being formally NLL. Fig. \cite{8} shows the percentual correction to the Born result due to initial-state radiation alone (solid black), full NLO corrections with ISR improvement of the Born cross-section only (dot-dashed red), and complete NLO corrections with full ISR improvement (dashed blue). The contribution of genuine electroweak and QCD corrections amounts to $\sim 8\%$ at threshold. It must also be noted that the difference between the two implementations of ISR is numerically important,
reaching \( \sim 2\% \) at threshold. A comparison of the EFT approximation with \([3]\) reveals a discrepancy which is never larger than \( \sim 0.6\% \) in the range \( 161\, \text{GeV} < \sqrt{s} < 170\, \text{GeV} \). More precisely we have for the full calculation \( \sigma_{4f}(161\, \text{GeV}) = 118.12(8)\, \text{fb} \), \( \sigma_{4f}(170\, \text{GeV}) = 401.8(2)\, \text{fb} \) \([3]\), while in the EFT one obtains \( \sigma_{\text{EFT}}(161\, \text{GeV}) = 117.38(4)\, \text{fb} \), \( \sigma_{\text{EFT}}(170\, \text{GeV}) = 399.9(2)\, \text{fb} \) \([4]\).

The dominant remaining theoretical uncertainty comes from an incomplete NLL treatment of ISR. This translates into an uncertainty on the \( W \) mass of \( \sim 31\, \text{MeV} \) \([4]\). Further uncertainties come from \( N^{3/2}\)LO corrections in the EFT. The missing \( O(\alpha) \) corrections to the four-electron operator \([\Sigma]\), which are included in \([3]\), contributes an estimated uncertainty of \( \sim 8\, \text{MeV} \) \([4]\), while interference of potential and soft photon exchange accounts for additional \( \sim 5\, \text{MeV} \) \([4]\). This means that with a NLL treatment of initial-state radiation, which seems realistically achievable in the near future, and further inputs from \([3]\) the total theoretical error on \( M_W \) could be reduced to the level required for phenomenological applications at linear colliders.

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