Improved storage of coherent and squeezed states in imperfect ring cavity

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We propose a method of improving quality of a ring cavity which is imperfect due to non-unit mirror reflectivity. The method is based on using squeezed states of light pulses illuminating the mirror and gradual homodyne detection of a radiation escaping from the cavity followed by single displacement and single squeezing operation performed on the released state. We discuss contribution of this method in process of storing unknown coherent and known squeezed state and generation of squeezing in the optical ring cavities.

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I. INTRODUCTION

Recently, quantum information processing and quantum communication experiments utilizing photons as information carrier (for a review [1]) are performed without an utilizing advantage of quantum memories. The quantum memories can store a quantum state carrying information for a further processing as well as they can store an entanglement resource for an actual time quantum communication. However, the photons are relatively difficult to store and an implementation of a practical quantum memory for photons remains a challenging problem. Earlier proposals for a quantum memory were mainly dedicated to a storing quantum states of individual photons in a high-Q cavity [2] in collective atomic excitations [3] or in a fiber loop [4]. To enhance storing individual photon in the fiber loop, a linear-optical quantum computing circuit that runs an error-correction code was proposed [5]. To protect a qubit against decoherence, the schemes based on the decoherence-free subspace were also proposed and implemented for trapped ions [6].

Quantum information processing with continuous variables (CVs) based on a manipulation with Gaussian states of many-photon systems [7] represents an interesting alternative to the quantum information and communication protocols exploiting individual photons. In the CV quantum information protocols, the coherent states are mainly utilized as the information carrier and the squeezed states are basic resource for a production of the CV entangled states. To perform more complex and collective CV quantum information protocols we would like to implement a quantum memory which is able to store unknown coherent state or known squeezed state for a long time. To store a continuous-variable information encoded as simultaneous amplitude and phase modulation of coherent state for a long time we can utilize a simple quantum memory device based on optical-fiber loop or a ring cavity. We lock the continuous-wave field into the high-fidelity fiber loop or ring cavity and release it when we need. Recently, a similar method has been proposed to store the polarization qubit in a long length fiber [8]. However, in the ring cavity an unavoidable losses at mirrors deteriorate information encoded in the coherent state and typically restricts the storage time. Further, to produce a pronouncedly squeezed state for efficient CV quantum information processing, a ring cavity containing a nonlinear crystal is frequently used in a continuous-wave optical parametric oscillator/amplifier (OPO/OPA) [9]. Here an unavoidable losses of the imperfect mirrors restricts a maximal value of the produced squeezing. A method of the squeezing generation in the ring cavity was used, for example, to produce entangled state for a teleportation of coherent states.

In this paper we propose a method how to increase the quality of the ring cavity if we are able to inject an auxiliary squeezed light to the cavity through the imperfect mirror and detect a field leaving outside from the mirror by a homodyne detection in every cycle of the field in the ring cavity. According to the measured data, a
simply joint feed-forward correction by a modulation of the final state can be performed after many cycles in the ring cavity and we can preserve an unknown coherent or known squeezed state for a long time as will be shown in Sec. II. For the storing of the coherent state it is even necessary only to perform an additional squeezing operation to obtain a fidelity of the storing which is independent on unknown input amplitude. The presented method is based on CV quantum erasing procedure [12] which allows us to restore at least partially the input state after an interaction. Thus a reflectivity of the mirror is in fact enhanced using the erasing procedure. It can also used to enhance maximal amount of the squeezing generated in OPO/OPA in which the ring cavity is filled by a nonlinear crystal as will be demonstrated in Sec. III. Thus we could consequently stimulate an increase of fidelity of CV quantum information protocols. This method is different to the error correcting codes since it works only with information which leaves from the memory unit.

II. PROTECTION OF CV STATE

In this Section, we demonstrate usefulness of quantum erasing to achieve an enhancement of a time of the storage of unknown coherent state or known squeezed state. We consider an empty imperfect ring cavity (without the crystal C) depicted in Fig. 1 which consists of two mirrors $M1, M2$ with almost unit reflectivity at a frequency of stored field. The third imperfect mirror $M(R)$ has a less reflectivity $R$ than $M1, M2$ however still $R > 0.99$ as is typical in this kind of experiments experiments. The rest of the losses and imperfections are assumed to be negligible in our analysis.

To demonstrate a protection by the erasing effect, the standard setup of imperfect cavity (in the box) is completed by the generator $S$ of squeezed states, balanced homodyne detection $HD$, displacement correction $D_T$ and squeezing correction $S_T$. The erasing procedure can be performed as follows. In every round-trip of the quantum field in cavity, a state squeezed in the quadrature variable $X_M$ is mixed with a state of an internal cavity mode and the output state leaving from the mirror $BS$ is detected by homodyne measurement producing the current $i_P$ proportional to a measured quadrature of the field. During many cycles in the cavity, the currents $i_{P1}, i_{P2}, \ldots, i_{PN}$ corresponding to every cycle are registered in a computer memory. After $N$ cycles corresponding to storage time, a quantum state leaving the cavity by a reducing the reflectivity of the mirror $M2$ can be corrected by a total displacement $D_T$ calculated from the measured values $i_{P1}, i_{P2}, \ldots, i_{PN}$, known reflectivity $R$ and total number $N$ of the round trips. In addition, we can convert the corrected state to another having the same mean values as the input state by total squeezing operation $S_T$, depending on the reflectivity $R$ and number $N$ of the cycles.

Now, let us look at this procedure in detail. Using Heisenberg picture representing each mode of light by a pair of the conjugate quadrature operators $X$ and $P$ satisfying the commutation relations $[X, P] = i$, the $k$-th pass through the cavity mirror $M(R)$ can be represented by transformation relations

$$X_k = RX_{k-1} + TX_{Mk}, \quad P_k = RP_{k-1} + TP_{Mk}$$

$$X'_{Mk} = TX_{k-1} - RX_{Mk}, \quad P'_{Mk} = TP_{k-1} - RP_{Mk}, \quad \text{(1)}$$

where $X_k, P_k$ denote the quadrature variables of the signal mode after $k$-th round and $X_{Mk}, P_{Mk}$ stand for the quadrature variable of the meter mode used to inject a squeezed state into the mirror. Detecting a field from the mirror by the homodyne detection, the operator $P'_{Mk}$ collapses on a real number $i_{Pk}$. Using relation (1) we can straightforwardly derive an evolution of the quadrature operators

$$X_N = R^N X_m + T \sum_{k=1}^{N} R^{k-1} X_{Mk},$$

$$P_N = R^{-N} P_m - T \sum_{k=1}^{N} R^{-k} i_{Pk} \quad \text{(2)}$$

after $N$ round trips in the cavity. Here $X_m$ and $P_m$ describe the initial quadrature operators of the signal mode which we are trying to protect. Now to suppress the decoherence effect in the signal mode after the series of the $N$ passages through the mirror, we can use all measured values $i_{P1}, \ldots, i_{PN}$ and implement on signal mode the total displacement operation

$$X'_N = X_N, \quad P'_N = P_N + T \sum_{k=1}^{N} R^{-k} i_{Pk} \quad \text{(3)}$$

followed by additional squeezing operation

$$X_{out} = \frac{X'_N}{R^N}, \quad P_{out} = R^N P'_N \quad \text{(4)}$$

to achieve an universal character of the protection. Universality means that the mean values of both complementary variables are preserved. Thus the resulting universal transformation

$$X_{out} = X_{in} + \frac{T}{R^N} \sum_{k=1}^{N} R^{k-1} X_{Mk}, \quad P_{out} = P_{in} \quad \text{(5)}$$

of the quadrature operators is obtained. Through our activities any unknown input state was fully restored in the momentum. Further, the mean value of both the coordinate and momentum are unchanged if the injected states have vanishing mean values of the quadratures. However the variance of the quadrature $X_{out}$ will differ from the variance $X_{in}$. If we consider the injected states to be independent and having the same variance $\sigma^2_{X_M}$, we can write the variance of $X_{out}$ in the following form

$$\sigma^2_{X_{out}} = \sigma^2_{X_{in}} + (R^{-2N} - 1)\sigma^2_{X_M} \quad \text{(6)}$$
From this follows that we can reduce the noise in the quadrature $X_{\text{out}}$ as well if we use a state squeezed in the coordinates $X_{M_k}$ and thus achieve almost perfect protection of an unknown coherent or known squeezed state. The reason is that the mixing the signal with appropriately squeezed states on the mirror $M(R)$ approaches the ideal quantum non-demolition measurement of the certain field quadrature, which can be reversed perfectly by the erasing procedure [12]. Note that an amount of fluctuations in the complementary variables $P_{M_k}$ does not influence the proposed protection procedure and only the squeezing of single quadrature is relevant.

For a comparison, without any measurements performed, the resulting variances would look as

$$
\sigma^2_{X_{\text{out}}} = R^{2N} \sigma^2_{X_{\text{in}}} + (1 - R^{2N}) \sigma^2_{X_{M}},
\sigma^2_{P_{\text{out}}} = R^{2N} \sigma^2_{P_{\text{in}}} + (1 - R^{2N}) \sigma^2_{P_{M}},
$$

(7)

if the protected state has mean values equal to zero. However, if this was not true, we would have to perform phase-insensitive amplification to achieve universality and the variances would be

$$
\sigma^2_{X_{\text{out}}} = \sigma^2_{X_{\text{in}}} + (R^{-2N} - 1)\sigma^2_{X_{M}} + (R^{-2N} - 1)\sigma^2_{X_{Z}},
\sigma^2_{P_{\text{out}}} = \sigma^2_{P_{\text{in}}} + (R^{-2N} - 1)\sigma^2_{P_{M}} + (R^{-2N} - 1)\sigma^2_{P_{Z}},
$$

(8)

with $X_{Z}, P_{Z}$ being the operators of ancillary mode of the amplifier. Qualitatively, the protection method reduces noise completely in the quadrature $P_{\text{out}}$ and also partially in the quadrature $X_{\text{out}}$ as can be seen from Eq. (7).

To demonstrate an usefulness of the protection quantitatively we fix the reflexivity $R$ of the mirror $M(R)$ and the squeezing in the auxiliary modes $M_k$ since we will compare different strategies with the same resources. In CV quantum computing there are important two basic types of quantum states, coherent and squeezed vacuum state, and our aim was to show how the protection we proposed increases efficiency of storage of these states. We consider and compare four different strategies: (A) protection with vacuum squeezed in $Xk$ injected into the modes $M_k$ and homodyne detections, (B) protection with unsqueezed vacuum injected in the modes $M_k$ and homodyne detections, (C) unprotected storing and (D) only injecting vacuum squeezed in $Pk$ in into the modes $M_k$ without the homodyne detections.

To compare the different strategies we use the fidelity of protection

$$
F = \left[ (\sigma^2_{X_{\text{in}}} + \sigma^2_{X_{\text{out}}} + \sigma^2_{P_{\text{in}}} + \sigma^2_{P_{\text{out}}}) \right]^{-1/2},
$$

(9)

which is the overlap between initial and resulting Gaussian state. Due to universality of storage only the variances of the initial and resulting state must be calculate to infer the fidelity. For unknown initial coherent state, the fidelity of protection strongly depends on squeezing of meter mode. In Fig. 3 we use for illustration a feasible squeezed state generated with the variance $\sigma^2_{X_{M}} = 0.5 \exp(-2)$ corresponding to 3dB squeezing injected to the imperfect mirror with $R = 0.99$. It is evident that using erasing with the squeezed state is the best strategy for a long time preservation of unknown coherent state. We can see a qualitative change from an exponential decoherence in the case $C$ to the non-exponential one in the case $A$. The strategy of squeezed vacuum injection (D) is counterproductive since any unbalancing noise in the complementary quadratures at output results in formidable decrease of fidelity.

On the other hand, squeezed states are asymmetrical in the variances so that we can greatly benefit from asymmetry of our method, if we know the orientation of the squeezed state which could be protected. In our method, a noise in a quadrature is suppressed at the cost of blurring the other one. Since our correction completely
preserves the squeezed quadrature and the noise added to the conjugate one is diminutive compared to noise already present, the fidelity is almost unity. Also the squeezing of the mode $M_k$ is not of great importance for almost perfect reflectivity $R$, as can be seen in Fig. 1. A larger distinction between the strategies (A) and (B) arises for a large number $N$ of the rounds in the cavity. Both other strategies (C,D) are worse than (A,B). Here the fidelity strongly depends on the proportion of variances of squeezed quadratures of the signal and meter modes. If we consider the input state is squeezed substantially more then the meter modes, what is reasonable demand, we never find the fidelity for strategy (D) comparable with the protected one. It turned out that for both unknown coherent and known squeezed state protection is always the best strategy.

III. PROTECTION OF SQUEEZING GENERATION

Because of a strong squeezing producing a large entanglement is required for the CV quantum information protocols, the optical cavity often used to form OPO/OPA so that the down-converted fields pass the nonlinear medium numerous times. For the single pass case, the interaction in continuous-wave regime is weak and requested gain is obtained after sufficient number of rounds in the cavity, thus equivalently lengthening the interaction distance. As an example we assume the cavity filled by a nonlinear crystal exhibiting (degenerate or non-degenerate) down-conversion process. It consists of two mirrors $M_1, M_2$ with almost unit reflectivity at a frequency of down-converted beams and almost unit transitivity for the pump beam. This setup operating as a continuous-wave frequency degenerate but polarization non-degenerate subthreshold OPO with collinear phase-matched type-II down-conversion in KTP was frequently used in the many experiments. It produces a squeezed state in two linear polarization modes which can be simply converted to single mode squeezing by $\lambda/2$-wave plate along a direction $\pi/4$ relative to these polarizations. The nonlinear crystal is pumped by a pulse from the ring laser with intra-cavity frequency doubling. For simplicity, a pumping part of the experimental setup from the ring laser with intra-cavity frequency doubling.

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To protect the operation we apply a slight modification of above described procedure. The ring cavity containing nonlinear medium performing a weak squeezing of the signal field during each trip in the cavity can be described by the following transformation relations

$$X_k = RG X_{k-1} + TX_{M_k}, \quad P_k = \frac{R}{G} P_{k-1} + TP_{M_k}$$

$$X'_k = GT X_{k-1} - RX_{M_k}, \quad P'_k = \frac{R}{G} P_{k-1} - RP_{M_k}$$

for the quadrature operators after the $k$-th cycle. Here $G$ is a gain of squeezing per single cycle in the cavity which is typically small. In an analogy with the previous case, after the displacement operation

$$X'_N = X_N, \quad P'_N = P_N + \frac{T}{R} \sum_{k=1}^{N} (RG)^{1-k} i p_k, \quad (11)$$

we find the resulting state have the variances

$$\sigma^2_{X_{out}} = \frac{1}{2} (RG)^{2N} + T^2 \frac{1 - (RG)^{2N}}{1 - (RG)^2} \sigma^2_{XM}, \quad (12)$$

$$\sigma^2_{P_{out}} = \frac{1}{2} (RG)^{-2N}.$$ 

if we consider unsqueezed vacuum to be the initial state. Note, in contrast to protection of the state, we have not performed any additional squeezing $S_T$ of the signal mode after it lefted the cavity.

Again we will compare few different strategies, so for evaluation is needed to find the variances of signal field quadrature operators

$$\sigma^2_{X_{out}} = \frac{1}{2} (RG)^{2N} + T^2 \frac{1 - (RG)^{2N}}{1 - (RG)^2} \sigma^2_{XM}, \quad (13)$$

$$\sigma^2_{P_{out}} = \frac{1}{2} (R/G)^2 + T^2 \frac{1 - (R/G)^{2N}}{1 - (R/G)^2} \sigma^2_{PM} \quad (14)$$

if no active protection was performed. Recall now, we are trying to protect the squeezing operation. That is, if our initial state was vacuum then our target state should be a minimal uncertainty squeezed state. There are two criteria of a quality that we have taken into our account. First one is overall fidelity between the target and actually produced state, the second one is an amount of achievable single quadrature squeezing. Again, four different strategies (A,B,C,D) were considered as in the previous Section.

Consider a pure squeezed state as a target state and ask how is it close to our prepared state. To quantify it we can count the fidelity

$$F = \frac{1}{2} (\sigma_{X_{out}} + \sigma_{X_{target}})(\sigma_{P_{out}} + \sigma_{P_{target}})^{-1/2}. \quad (15)$$

between target and obtained state and compare it using the strategies (A,B,C,D). The result is depicted in Fig. III. For a small number of cycles the strategy of squeezed vacuum injection (D) looks better but only because meter mode squeezing is still comparable or even better than the squeezing of signal field, so interaction on the mirror actually improves the state in the cavity. However, at some point the relevant quadrature cannot
FIG. 4: Fidelity of squeezed state produced in the protected ring cavity: $R = 0.99$, the gain of squeezing in a single cycle $G = \exp(0.02)$, the variance of the squeezed quadrature of meter modes $\sigma^2_M = 0.5 \exp(-2)$. A target mode is squeezed in quadrature $P$ with variance $\sigma^2_{P\text{target}} = 0.5 \exp(-5)$.

FIG. 5: Logarithm of squeezed variance of state produced in the protected ring cavity: $R = 0.99$, the gain of squeezing in a single cycle $G = \exp(0.02)$, the variance of squeezed quadrature of meter modes $\sigma^2_M = 0.5 \exp(-2)$. A target mode is squeezed in quadrature $P$ with variance $\sigma^2_{P\text{target}} = 0.5 \exp(-5)$.

be squeezed further due to the cavity losses and each cycle in cavity only adds additional noise to the conjugate quadrature what results in a fidelity decrease. If we apply the correction procedure the squeezing buildup is slower, but inevitable. We can represent squeezing protected by our method as nearly ideal (squeezed quadrature ideal, antisqueezed quadrature slightly disturbed), but with the gain $RG$. From this interpretation is apparent we need reflexivity and gain choose so that $RG > 1$ to obtain high fidelity results.

Next we can study how the evolution of noise in the quadrature which we are trying to squeeze depend on the number $N$ of cycles. The results can be understand from Fig. [Fig. III]. The logarithmic scale was chosen to clearly show what is happening. First, for protected case (A,B), the variance of momentum is not dependent on squeezing in modes $Mk$ and it has obviously no theoretical lower bound. Here also is not necessary to use a squeezed meter modes for a generation of the squeezing in a single quadrature with no respect to the noise in the complementary one. On the other hand, for the strategies (C,D) when the protection is not performed the squeezed variances saturate at a point

\[ \lim_{N \to \infty} \sigma_{P_{\text{out}}} = G^2 \frac{1 - R^2}{G^2 - R^2} \sigma_{PM} \]

determined by the proprieties of the cavity and the used squeezing in the modes $P_k$. In Fig. [Fig. III] we use in the case (D) the same amount of squeezing $\sigma_M P = 0.5 \exp(-2)$ as in the cases (A,B) however in the complementary quadrature.

From this follows, if we are interested only in achieving squeezing in one quadrature, what is often sufficient in the CV quantum information protocols, such as optimal CV teleportation, or protection of state depicted in previous section, it is not necessary use a squeezing in the modes $Mk$ to protect the squeezing. We need only perform an effective homodyne measurement followed by a single displacement operation at the end.

IV. CONCLUSION

We have shown how collective quantum erasure performed on single mode in different times can be used to increase the quality of the cavity and the efficiency of storing and squeezing generation processes within. Our method formidably increases the fidelity of storing of unknown coherent state, but due to its asymmetrical nature allows almost perfect storage of known squeezed state. If our protocol is applied to process of squeezing generation it removes the lower bound for one quadrature squeezing attainable and also allows production of pure squeezed states with high fidelity.

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