Article

Roman Aqueduct Flow Estimation Using Geomatic Measurement

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Abstract: The aqueducts built by the ancient Romans are among the most impressive evidence of their engineering skills. The water inside the aqueducts was transported for kilometers, exploiting only the slight but constant differences in altitude throughout the route. To keep the differences in height constant, the aqueducts could proceed underground or aboveground on well-known arched structures that supported lead, ceramic or stone pipes. In order to reconstruct the characteristics of these structures, it is necessary to carry out an accurate survey of the orthometric heights, and therefore the most suitable technology is geometric levelling. In this case, however, it is not applicable, and therefore here we propose an alternative methodology. The final goal of this work was to estimate the flow of some sectors of these aqueducts preserved in the area south of the city of Rome. This has two main purposes: The first is to reconstruct the flow rate of these aqueducts for historical studies; the second is to check how much the orthometric heights have changed over the centuries, in order to reconstruct the movements from a geophysical and geodynamic point of view. The latter analysis will be developed in a following phase of this research. For this purpose, a high-precision geomatic survey was carried out in the area under study, partly retracing a survey already carried out in 1917 whose purpose and methodologies are not known. The area has been affected by a gradual subsidence over centuries, including since 1917. The observed sections of the aqueducts showed average inclinations, slightly lower than the 2 per thousand that is reported in the literature for similar aqueducts. The measurements carried out allowed the flow rate of the two specific aqueducts to be estimated more accurately, both as they were originally and in the presence of deposits that have accumulated during the years of use of the aqueducts. The reconstruction of the initial geometry will later be used as a reference to estimate how much the geodynamic deformations of the area have deformed the aqueducts themselves.

Keywords: Roman aqueducts; ancient Rome; levelling; water flow; GNSS; Colli Albani

1. Introduction

The aqueducts built by the ancient Romans are generally considered their most original and impressive engineering achievements. Rome was founded at its location due to the presence of a river island (Tiber island), which made it possible to easily cross the Tiber river, which the Romans had a special emotional bond with. The ancient Romans managed to satisfy their water needs for centuries thanks to the river, and the springs and wells around it. In 312 BC, however, with the rapid development of the city, more water became necessary. This was due both to the increase in the number of inhabitants, but also to the increasing spread of the thermal baths that characterized the entire history of the Roman Empire [1]. For this reason, they started building aqueducts to bring to Rome water from
all the neighboring regions (Figure 1), which was then used not only for the thermal baths but also for fountains and private houses. The water in the aqueducts was moved only by a slight but constant difference in altitude, which created pressure differences which were preserved through the use of ceramic, stone, and lead pipes [2].

![Figure 1. Main Roman aqueducts, and the two rivers (light blue), in the bold black circle are the stretches studied here.](image)

The use of the latter for drinking water was widespread, even though the Romans were already aware of the health risk associated with the use of lead pipes for drinking water, and the possible occurrence of related diseases [3]. This is why, in reality, most of the aqueducts had stone and ceramic pipes, despite the greater technical difficulty of keeping the pipes watertight, and the pipes we examined were also all made of stone. To maintain a slight but constant decrease in the hilly morphology around Rome, the aqueducts proceeded underground (Figure 2) and along elevated stretches (Figure 3) on well-known arched structures. In some cases, the aqueducts were supplemented by sedimentation and distribution tanks, as well as sluice gates, to correctly regulate the flow according to specific local requirements.

![Figure 2. General view (a) and interior (b) of the *Anio Novus* specus (water channel) Reprinted with permission from ref. [4]. Copyright 2016, D. Blanco.](image)
In the present work, the path of some existing stretches of Roman aqueducts south of the city was studied. For this purpose, high precision levelling was carried out in the south-east area of the present city, repeating, in part, a survey carried out for the same purpose almost exactly one century ago (1917) by the Faculty of Engineering of the University of Rome [5].

The study of the characteristics of these engineering works requires the use of the techniques and methods of the historical metric survey, which has been described in detail in the literature [6,7].

The study for the survey and conservative documentation of monuments, buildings, and historical-archaeological remains is the subject of research and investigation by researchers all over the world. Generally, the aim is to document the remains in detail in order to be able to correctly plan their restoration or make it possible to visit them virtually.

Rarer is the survey for reverse engineering, i.e., to reconstruct the functioning of an instrument [8] or an infrastructure as in the case of this study.

Surveys for documentation purposes have the main objective of detecting in an efficient manner a large number of three-dimensional points that characterize the shape of the object. For this type of application, the most widespread methodology for many years has been terrestrial laser scanning, which allows reconstructing accurate point clouds in a relatively short time with high efficiency [9]. The only disadvantage of this technique is that it is usually not possible to acquire the whole object from a single position and the alignment of the various acquisitions can be time consuming and complex, which has stimulated the search for smart techniques to automate these processes [10]. Another technique often used for this type of survey is terrestrial photogrammetry [11], which has recently benefited from the increasing application of algorithms already known in other application fields such as structure from motion (SFM) [12]. SFMs, which often exploit acquisition redundancy to perform camera self-calibration [13], have made it possible to develop new techniques for indoor surveys [14] and, above all, to make the most of drone surveys [15–17].
On the other hand, in the case in which the survey of archaeological sites needs reverse engineering, it is necessary to reconstruct the three-dimensional coordinates of a number of points generally much lower than in the previous case, but with even greater three-dimensional accuracy [8].

In the specific case of the survey of aqueducts, the coordinate of greatest interest is obviously the height or its differences along the route. In particular, the height of some notable points that can characterize the height variations of the lowest part of the pipeline is of interest. This survey, in the design phase, can currently be carried out with great accuracy by means of geometric levelling. Similar methods were probably also used in ancient times, but unfortunately, they require access to the lower part of the conduit and this, once the aqueduct was completed, is generally no longer possible for obvious reasons of accessibility. Therefore, it is possible to use the already illustrated techniques of laser scanning or drone photogrammetry, but the accuracy is not entirely adequate for the specific investigation. In the case under study, advantage was taken of the numerous interruptions in the aqueduct itself, which made the lowest part of the conduits observable (although not stationable), allowing them to be surveyed using a total station.

The collapses that caused the interruptions to the aqueduct could be due to crustal movements affecting the area [18].

This paper aimed to evaluate the flow rate and to compare it with historical values, as well as to some results in the literature, including interpretations and experimental conclusions that can be drawn mainly with two main objectives:

- To reconstruct the differences in historical elevation (comparing the recent survey with the original construction and the survey of 100 years ago), considering that the area is affected by crustal movements;
- to investigate the geometry and the flow rates (also to understand if the aqueducts worked under pressure) of the original aqueducts.

This reconstruction work is part of a wider scientific project which is still in development, and whose main aim is to verify if and how much the area has been deformed in the 2000 years since the construction of the aqueducts, and since the survey was carried out a century ago. To be able to hypothesize what the original geometry of the aqueducts was, and therefore estimate the deformations, the flow rate data is obviously of fundamental importance. The most accurate possible knowledge of the flow rates is also of historical importance, as a number of scholars are debating whether the currently known aqueducts were sufficient for the considerable water needs of Imperial Rome or how they intersected and combined with each other [4,19,20].

Where possible, the same points as used in the 1917 campaign were remeasured, considering the difference in height altimetric datum [21–24].

The two aqueducts we studied, built in the same period, followed their path one above the other taking the supply of their waters in the distant Aniene valley so their complete original length is about 70 km, while their cross-sectional area is approximately 2 square meters. However, their geometry will be described in more detail in the paragraph on hydraulic modelling.

Here, we follow a path that, from the most extreme part of the current city (Roma Capannelle), arrives in the most urbanized area called ‘Porta Maggiore’. Specifically, the measurable points of the ‘Aqua Claudia’ end in the ‘Mura Latine’ area, while for the ‘Anio Novus’ aqueduct they end in the so-called ‘Porta Furba’ area. To correctly estimate the gradients, the aqueduct was divided into sections that were representative of the original trend, since the subsidence that has already been preliminarily studied in a first phase of this research project [2] reversed the gradient in other parts of the aqueduct, making them unreliable for the reconstruction of the flows.

2. Data Collection and Analysis

The two investigated aqueducts, built in the same historic period, followed the same path with one above the other (Figure 4), taking the supply of their water from the upper
part of the Aniene river valley. As already mentioned, the water was only moved by gravity, for this reason, the water had to be taken from sources at a higher altitude than the city, in particular from the area east of the city. From there, the entire route of the aqueduct had to be designed with extreme care to make it compatible with the morphology of the terrain and, where possible, exploit the natural slopes. For these estimations, we used the ellipsoidal height differences measured with GNSS differential receivers and total stations, the area of transit sections of flow rates, reconstructed in previous studies in the literature, and the equations of flow in conditions of uniform motion in channels. Obviously, the ellipsoidal elevations were converted into orthometric elevations according to the procedure described in more detail in the section on surveying operations.

Figure 4. Anio Novus on top of the Aqua Claudia.

The reconstruction of the gradient profiles, through the measurement of the heights of several points on the bottom of the ‘specus’ along the two aqueducts of interest, allowed us to better study the aqueducts and the vertical movement of the surrounding area. We hypothesized, through the knowledge of the flow rates of the historical runoff, the placement of sections of the ancient channels, making possible a more accurate reconstruction of the ancient structures. It is well known that in order to reconstruct height differences for hydraulic calculations, it is necessary to use a technique capable of measuring orthometric height differences. The most accurate technique for this type of measurement is geometric levelling, with accuracies of up to a tenth of a millimeter on a single levelling section.

Unfortunately, this technique requires accessibility to the points to be measured, which in a historical aqueduct that has many collapsed sections, is not realistically feasible. This consideration required us to look into an alternative technique and raised the question as to which technique was used in 1917 by Reina et al. [5]. From the documentation available to us, it would appear that long geometric levelling was carried out at the ground level, from which a series of points were then ‘launched’ onto the internal structure of the pipelines where the various collapses made them visible. However, this second part described here is
not extensively documented in the reports we have, and therefore the methods and relative accuracy are not known for sure.

To compare these surveys, a specific study was also necessary to reconstruct the original height datum, which is different from that presently used and that can be estimated by measuring unchanged benchmarks as they are described in contemporary maps [24], as we will explain in more detail below.

2.1. Geomatic Surveys

All the 1917 recognizable surveyed points were measured again, referring to modern height datum realized by the GNSS network managed by the Lazio regional administration. As it is well known, the heights and elevations calculated using a network based on GNSS permanent stations are ellipsoidal and not orthometric with the well-known physical and numerical differences. The levelling performed in the first part of this research allowed the reconstruction of the gradients of the aqueducts ‘Aqua Claudia’ and ‘Anio Novus’. For both aqueducts, the surveys made it possible to estimate, with sufficient approximation, the flow within the channels. These flows were subsequently compared with those used by the Romans.

To estimate the gradients, the elevations of the lower part of the channels (specus) and the distances along the route obtained from the plan-altimetric survey performed were taken into account.

To compare the elevations measured in 1917 and those of today on the same points, one must consider both the probable variation of the altimetric reference system (datum), and any altimetric modifications due to geodynamic deformations. In this regard, it must be remembered that the altimetric datum used in the course of a century can be profoundly varied. In particular today, in peninsular Italy, an altimetric datum is used which conventionally refers to 1 January 1942 [24,25], which is certainly not the one used in 1917. Unfortunately, from the official documentation of the time [5,26,27], it is not possible to reconstruct with certainty the datum used at that time, nor were useful parameters provided for the conversion between different datums. It was deduced that the only way to compare the two surveys carried out with different reference systems was to try to reconstruct the heights of some benchmarks already in existence at that time, and still preserved today, in order to estimate the relative variation of height. Unfortunately, only two points levelled in 1917 are still recognizable, but their undisturbed maintenance is not certain. In fact, the pillar at ‘Piazza San Giovanni’ is visibly tilted (Figure 5), and it is not certain whether it was also tilted in 1917.

Additionally, the last step at the church of Santa Croce in Gerusalemme is on a staircase that may have been subject to wear over the years (Figure 6).

The differences between the heights in 1917 and those of today, however, gave decidedly discordant values of $-38$ cm for the first benchmark and $-59$ cm for the second.

Estimating the datum shift on these two points alone is therefore not statistically significant, and an independent way to verify the estimated datum differences was needed. In order to do this, it was first assumed that since the survey was carried out by the Italian Military Geographical Institute (IGMI), they had used the same altimetric datum that they were using at the time for detailed maps of some urban centers.

A map of the IGMI, at a scale of $1:5000$ from 1924, was available from previous research [27–29], on which the benchmarks used for its realization were reported. The cartography of 1924 contained both the heights of many benchmarks surveyed with geometric levelling, and therefore with millimetric/centimetric precision, and also a significantly greater number of points surveyed with lower precision. Therefore, it was decided to carry out a comparison between the elevations of the points on the 1924 map and a DEM obtained from the spot points of a modern (2005) map at the same scale, of which the altimetric datum was therefore certain (Genova, 1942).
Additionally, the last step at the church of Santa Croce in Gerusalemme is on a stair-case that may have been subject to wear over the years (Figure 6).

**Figure 5.** San Giovanni in Laterano square 1917 benchmark.

**Figure 6.** Santa Croce in Gerusalemme church 1917 (a) benchmark detail (b).
The estimation was carried out on 139 points extracted from historical cartography which were compared with the DEM obtained from modern vector cartography. A 4-metre filter was imposed to eliminate any outliers due to errors in the interpolation of the DEM from vector mapping and 26 points were thus eliminated. The remaining 113 points (Figure 7) had a minimum difference value of $-2.93$ and a maximum value of $2.17$ m. We were aware that the accuracy of the heights shown on the two maps was certainly not of the order of that obtainable with a geometric levelling, but we assumed that the large number of points compared could provide a statistically more reliable and therefore significant sample.

![Figure 7. The 1924 map benchmarks on a 2005 map DEM.](image)

The difference value estimated in this way was $-42$ cm. On the one hand, this partly confirms the measurements made on the two benchmarks (although it does not coincide with either of them, but only with their average). On the other hand, it indicates that the altimetric variation between the two altimetric datums (1917 and 1941) is of the decimetric order, and cannot be ignored.

Considering that the uncertainty of the 1917 elevation datum could create the exact same shift at all points measured in 1917, it is clear that comparisons between differences in elevation measured in 1917 and those measured today are more meaningful than the differences between absolute elevations measured in 1917 and those measured today. This has been taken into account in this work, and therefore the trend differences between the two sets of measurements were considered more significant.

In order to acquire the heights of the remaining points, it was not possible (as already mentioned) to use geometric levelling, and therefore it was decided to employ GNSS levelling using dual-frequency geodetic receivers, and to ‘launch’ from the same stationary points the total station measurements of the visible points, effectively measuring trigonometric levelling.

The points were measured using a total station ‘Leica TCR703’ (Figure 8a), and the total station’s positions were surveyed using a Topcon Legacy-E double frequency GNSS receiver (Figure 8b).
The GNSS data were processed in a post-processing mode using RINEX data provided by the network of permanent stations managed by the Lazio region [10] (Figure 9).

The heights obtained from trigonometric levelling calculated from the GNSS heights of the same station points must obviously be considered as calculated with reference to the WGS84 ellipsoid in the same realization of the reference network (RDN2008) [21,23].

An average accuracy of less than 2 cm can be attributed to these heights, mainly due to the difficulty of collimating with certainty the main points of the specus from the ground.

These elevations were converted to orthometric elevations in the national elevation datum using data from the ITALGEO2005 geoid model, which has an estimated vertical accuracy of about 2.5 cm [30] when using the specific interpolation algorithms as implemented in the GeoTrasformer software [31]. We could identify and measure six levelled points from the 1917 measurement campaign: Points belonging to the ‘Marcio’, ‘Tepula’, and ‘Julia’ aqueducts at Porta Maggiore (Figure 10a), a point belonging to the ‘Aqua Claudia’ aqueduct at ‘Mura Latine Street’, and points belonging to the ‘Aqua Claudia’ and ‘Anio Novus’ aqueducts (Figure 10b) located in the Aqueduct Park (Figure 10c). These points are the only ones that were identified and measured with certainty.

Figure 8. Survey phases using forced centerings with the total station (a) and GNSS antenna (b).

Figure 9. CORS network managed by Lazio regional administration.

Figure 10. Map of the Lazio region showing the locations of the Marcio, Tepula, Julia, Aqua Claudia, Mura Latine Street, and Anio Novus aqueducts.
In the work by Reina et al. [5], levelling campaigns that were performed on the aqueducts ‘Anio Novus’, ‘Anio Vetus’, ‘Marcia’, and ‘Claudia’ were described. They surveyed three different lines of levelling: From Rome to the ‘Capannelle’ area, from Anagnina street to the area ‘Pallavicina’, and from the ‘Pallavicina’ area to Cineto Romano.

The section of our interest, however, was only the first part of Rome-Capannelle, along which the aqueducts ‘Anio Novus’ and ‘Claudia’ are present (Figures 11 and 12).

2.2. Hydraulics of the Roman Aqueducts

The Romans developed very high-level hydraulic engineering to support the development of the water supply to their main cities, especially Rome. Several aqueducts were realized, and their design was undertaken under consideration of the absence of pumping systems. The flow through them is entirely driven by gravity, flowing slowly from higher elevations. Thus, the aqueduct path was designed to respect the continuous decrease in the topographic level, considering a given gradient to avoid a high flowrate and its possible consequences. The Romans aimed for a slope of 0.02%, as reported in the Natural History by Plinius, who indicated “channel must be as solid as possible, and their slope should not be less than twenty centimetres per kilometre of length”. This value of 0.02% ensured the proper flow velocity without requiring a pumping system. The requirement of a solid channel was to guarantee the solidity of the structure, and its reliability during the water flow. The larger part of the Roman structures work at atmospheric pressure due to the difficulty, at that time, of creating piping systems that were solid enough. Roman aqueducts were usually built as buried masonry channels, with a rectangular internal profile and a semi-circular vault. The floor and the sidewalls were covered with a characteristic red waterproof cement, named opus signinum, with a special quarter round structure to strengthen the joints of the walls and floor. The channel was usually only half [33] to two-thirds full of water (Figure 13).
Figure 11. Points used to calculate the height difference along the entire route.

Figure 12. Points used to calculate the height difference in the area known as the "Parco degli Acquedotti".
This assumption seems to be confirmed by Blackman, who cites ‘the language used by Frontinus and others implies a free surface’. Following these assumptions, the flow rate was calculated with equations of flow in a uniform steady-state open channel flow.

The most common are Chézy’s equation and Manning’s equations [34,35]. To calculate the aqueduct discharge, the Manning equation and the cross-sectional area are used. Thus, the flow rate is calculated by the following formula:

\[
Q = A \frac{1}{n} R_h^{\frac{2}{3}} \sqrt{S}
\]  

(1)

where \(R_h\) is the hydraulic radius (the ratio between the area and the wetted perimeter), \(A\) is the flow area, \(S\) is the hydraulic gradient or the bottom slope of the channel, and \(n\) is the Manning coefficient. To evaluate the capacity of the Aqua Claudia and the Aqua Anio Novus, the cross-sectional area characteristics, the slope of the channel, and the roughness of the walls are needed.

2.2.2. Reconstruction of the Outflow Sections and Flow Characteristics

The reconstruction of the duct section is not an easy task for two reasons:

- The collapse and deformation of the masonry parts;
- the formation of deposits of sediment inside the ducts themselves. The first indication of the cross section characteristics were reported by Ashby [36]. He undertook some surveys following the ones done by Reina et al. [5]. In the investigated stretch, four sections were identified on the Aqua Claudia and only one on the Aqua Anio Novus (Table 1).
Blackman [37] analyzed only the width of the *specus*, giving a distribution of all measurements. The mean values were 1.1 m for *Aqua Claudia* and 1.15 m for the *Aqua Anio Novus*. The approach followed by Blackman was to search for the depth of water having assumed a rectangular cross section.

More recent studies give some evidence of the real cross-section of the aqueducts. Keenan-Jones [38] and Motta et al. [39] highlighted the presence of travertine concretions on the bottom and walls of the *Aqua Novus* aqueduct. They studied sketches near Tivoli and in Rome, near the “Casale Roma Vecchia” area, and focused on the travertine concretions to undertake a hydraulic evaluation of the discharge through the *Anio Novus* aqueduct.

For the present study, the reconstruction of the *Anio Novus* section near point I.4 from Reina et al. [5] was considered. The reconstruction of the section based on the characteristics reported by Keenan-Jones et al. [38] is reported in Figure 14.

Table 1. *Aqua Claudia* (II) and *Aqua Anio Novus* (I) dimensions and levels (Ashby, 1935).

| Site | Type  | Width (m) | Height (m) | Specus Level (Orthometric Height, m) |
|------|-------|-----------|------------|--------------------------------------|
| I.4  | Vault | 1.14      | 1.25       | 77.66                                |
| II.3 | Flat  | 1.14      | 1.75       | 65.76                                |
| II.4 | Flat  | 1.14      | 1.75       | 66.23                                |
| II.5 | Flat  | 1.10      | 1.79       | 69.78                                |
| II.6 | Flat  | 1.10      | 1.79       | 75.00                                |

For the *Aqua Claudia*, the cross-section was derived from the available documentation, and its shape was determined to be rectangular due to the superposition of the *Anio Novus* (the roof of *Claudia* is the floor of the *Anio Novus*). In Figure 15 the geometry is shown. No information has been found regarding a travertine presence in the *specus* of *Aqua Claudia*.

The flow characteristic to be defined is the Manning coefficient. It depends on the roughness of the surfaces in contact with water. In case the water flowed in contact with the mortar lining, the estimate of the roughness coefficient was 0.014 s/m$^{1/3}$, as suggested by Fahlbusch [40]. In other studies, lower values have been proposed.

The presence of travertine at the bottom of the *specus* was also well documented in the past. It was possible to survey the surface in contact with the flowing water, and its roughness was determined. Figure 16 shows the thickness of the travertine deposits and the surface created on the *specus*, such as a riverbed with cobbles of nearly 15 cm in diameter. The appearance of the surface suggests a Manning roughness coefficient around 0.035 s/m$^{1/3}$. The actual surface did not respond to the status during ancient Rome, and was probably smoother due to the presence of a muddy layer, a roughness coefficient in agreement with Fahlbusch’s estimation (1982) of 0.019 s/m$^{1/3}$ was used.
Figure 15. Cross-section geometry for the *Aqua Claudia*: Point II.3. (a) Full section (b) with travertine deposits (From: Ashby, 1935).

Figure 16. (a) A view of the *Anio Novus* aqueduct’s travertine deposits on the specus. (b) View from the top of the travertine deposits and their roughness.

3. Results and Discussion

The first result of the study is the slope analysis of the aqueducts. In fact, to find the hydraulic gradient under the hypothesis of uniform open-channel flow, the slope of the aqueduct floor is required. To reconstruct the bottom profile of the two aqueducts, rather than using the well-known levels from Reina et al. [5], the data collected during the geomatics survey were used. The main objective was to verify the acquired level in 1917, and use them to evaluate the slope of the bottom of the channels of the ancient aqueducts.

The two aqueducts, as presented in Figure 17, follow the same path with one above the other, maintaining a distance between the channels that is almost constant with an average of 3 m and a standard deviation of 0.26 m. Ashby [36] reports that the distance between the bottom of the *Anio Novus* and the intrados of the *Claudia* is about 1.00 m.
Considering the height of the Claudia of 1.75 m and the thickness of the slab of 1 m, the distance between the two specus is 2.75 m. In our data, the values are always greater. In the three points taken as reference from the Reina et al. (1915) levelling survey (II.3); the distance between the specus is 2.85 m for the points I.1 and I.2 and 1.7 m for point I.3. It is clear that there is a mistake in the reported data. Comparing the level measurements of the bottom of the Claudia with the present survey, the difference of 1 m confirms the potential mistake.

In Figure 17, the reconstructed data, indicated with x, are reported as well as the connecting lines with dashed ones, having assumed a distance between the specus of 3 m. The point highlighted with a circle corresponds to an outlier of this distance. In the stretch, a high variability of the level and also inverse slopes appear. Those behaviors are probably due to ground or structural movements.

In fact, in some parts, due to the subsidence of the structures, the trend of the profile does not respect the hydraulic requirements for the outflow of waters. To estimate the gradients, it was decided to divide the route into sections, for which a positive slope of the aqueducts is guaranteed. In Table 2, the calculated slopes are given, along with the average between the positive values according to our calculations, and the average gradient between the extreme points.

Table 2. Aqua Claudia and Aqua Anio Novus slopes.

| Slopes            | Aqua Claudia (%) | Aqua Anio Novus (%) |
|-------------------|------------------|---------------------|
| Average value from data | 1.66             | 2.60                |
| Over the entire stretch | 1.63             | 1.31                |
| From Reina data    | 1.57             | 1.54                |

Figure 17. Specus level for the two aqueducts: On the x-axis the progressive distance along the aqueduct; on the y-axis the orthometric heights from levelling.
Following the suggestion from Keenan-Jones on the incorrect positioning of benchmark 37, corresponding to point I.4, a verification of the more probable position was done. From the representation of Figures 18 and 19 in Reina et al., is clear that point I.4 is about 350 m downstream from the indicated place. In this case, the slope of the two aqueducts over the entire stretch is 1.68‰ for Aqua Claudia and 1.64‰ for Aqua Anio Novus.

Figure 18. Slope profile of Aqua Anio Novus.

From historical documents and technical studies, is assumed that the ancient Romans made an effort to achieve an almost constant slope of 2‰. This slope could vary slightly concerning the routes that the aqueducts were to follow, the obstacles to be overcome, and the flow velocity desired.

The calculated average slope for the Anio Novus aqueduct in the analyzed section is 1.31‰, and for the Aqua Claudia aqueduct is 1.63‰. Those values agree with values previously cited in historical documents.

Evaluation of the Flow Capacity of Roman Aqueducts

To evaluate the volume capacity of the Claudia and Anio Novus aqueducts, two different calculations have been adopted. First, using the historical flow rate and the characterization of the aqueduct, the depth of water in the specific section is evaluated. The reference section lies in the stretch of the ‘Parco degli Acquedotti’ as previously reported.

The second calculation is the direct evaluation of the discharge flow rate based on the measured slopes and the sections in different flowing conditions.

The first stage is similar to the one followed by Blackman [37]. The basic assumption was the uniform channel flow, and the section was previously presented. Two different conditions were considered: A fully available section, and a restricted section due to travertine deposits.

From the calculation reported in Table 3, the water depth in Anio Novus is always greater than the maximum height of the specus (1.79 m) in the case of presence of travertine deposits. Without travertine deposits the conditions are in an open channel flow. On the contrary, Aqua Claudia always flows in an open channel condition. Of note, the water depth for the full section was in line with two-thirds of the height of the specus, as reported in the literature.
Figure 19. Slope profile of *Aqua Claudia*.

Table 3. *Aqua Claudia* and *Aqua Anio Novus* water depths for historical flow rates.

| Reference          | Flow Rate (m³/d) | Full Section (m) | With Travertine (m) |
|--------------------|------------------|------------------|---------------------|
| *Aqua Claudia*     |                  |                  |                     |
| Di Fenizio [41] (1916) | 191,190          | 1.26             | 1.70                |
| De Feo [33] et al. (2013) | 184,280          | 1.22             | 1.65                |
| *Aqua Anio Novus*   |                  |                  |                     |
| Di Fenizio (1916)  | 196,627          | 1.52             | 2.04                |
| De Feo et al. (2013) | 189,520          | 1.46             | 2.05                |

The second approach is based on direct calculation of the flow rate under the assumption of uniform steady-state open channel flow. For the Manning coefficient, different values have been assumed to have also a sensitivity analysis of discharge flow. The Mannings coefficients used were: 0.14 for the full section while for the case with travertine, 0.19 for low and 0.035 for high. The extreme value of 0.035 has been adopted only as reference for high roughness surfaces considering the limitation in the flow rate. The calculated discharge flow rates are reported in Table 4 for the three different conditions just reported. The assumption in the calculation of flow rates are: Maximum flow rate corresponding to the full section and the standard flow rate with the maximum water depth of 2/3 of the height section.

The presence of the travertine deposits can highly reduce the capacity of discharge of the aqueducts in the range of 30% to 68%. This was well known by Romans that were doing maintenance work by removing the scaling from the wall and from the basement.

The discharge capacity of standard flow rate highlights the requirements of higher water depth in the analyzed sections to accomplish the historical flow rate. This is not mandatory, but it gives an idea of the possible working range of the aqueducts.

The *Anio Novus* was designed without clearance for historical flow rates. The section restricted by travertine deposits has a reduced capacity of 1.27 m³/s (i.e., 109,454 m³/d). Thus, it will flow in pressure and to avoid pressure inside the channel, the flow should be less.
Table 4. *Aqua Claudia* and *Aqua Anio Novus* estimated discharge capacity and historical flow rates.

| Maximum Flow Rate               | Aqua Claudia (m³/d) | Aqua Anio Novus (m³/d) |
|--------------------------------|---------------------|------------------------|
| Full section                   | 244,940             | 198,156                |
| With travertine–low            | 166,714             | 109,454                |
| With travertine–high           | 90,502              | 59,418                 |
| **Standard flow rate**         |                     |                        |
| Full section                   | 172,864             | 157,709                |
| With travertine–low            | 126,448             | 101,782                |
| With travertine–high           | 68,643              | 55,253                 |
| Di Fenizio (1916)              | 191,190             | 196,627                |

4. Conclusions

The techniques used here made it possible to reconstruct an orthometric height difference with an accuracy prudentially limited to more or less 4 cm, allowing the geometric characteristics of two Roman aqueducts to be reproduced for the observable sections. Since it was not possible to reconstruct the difference in the altimetric datum between the 1917 elevations and the current reference system with absolute certainty, comparisons of height differences measured in the two surveys must be considered more significant than the absolute differences in elevation. The comparison between the current survey and that of 1917 showed a difference of the orthometric heights datum of about 42 cm due to the different altimetric datum used in 1917 and presently (Genova, 1942).

In particular, the ‘Anio Novus’ was surveyed and reconstructed from the area of ‘Mandrione’ street to the area of the ‘Aqueduct Park’, where the aqueduct remains almost intact. The ‘Aqua Claudia’ aqueduct was taken over and rebuilt from the area of Via Latina up to the same area of the ‘Aqueduct Park’, where the survey of the other aqueduct was completed. In both cases, the average gradient was close to 2%, which is the value reported in the historical reference texts for the design of Roman hydraulic works. Specifically, ‘Anio Novus’ currently has a slope of 1.31‰, while ‘Aqua Claudia’ has a slope of 1.63‰. These slope values made it possible to estimate the flow of water, and consequently to verify the conditions of the channels in which the water flowed. To reconstruct the cross-section of both aqueducts, the study by Keenan-Jones et al. [39] and the study by Ashby [36] were used. Based on the comparison between the calculated flow rate and water heights and the data reported in the literature, two different conclusions can be reached for the two aqueducts. The *Aqua Claudia* has a calculated discharge flow rate higher than expected, and therefore the channel worked with a 70–95% filling of the section, as reported in the literature. The *Anio Novus* aqueduct, on the other hand, currently has a discharge flow rate lower than historically, suggesting that in the study area it was working under pressure at the time.

This work has highlighted the possibility to investigate the hydraulic of Roman aqueducts starting from a geodetic survey. The importance of the survey is clear when comparing the historical documents and data. The more difficult aspect is to understand after centuries which is the original path and relative slopes measured on ruins that can be exposed to ground movements.

A better knowledge of the hydraulic characteristics of the aqueduct will help in the subsequent phases of this research project where the current strain of the structure will allow confirming or refuting some theories on the deformations taking place in this area.

Author Contributions: The authors contributed equally to this work. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data presented in this study are available on request from the corresponding author. The data are not publicly available for privacy reasons.
Acknowledgments: The authors would like to thank all the people who helped carry out the surveys by allowing access to numerous private areas, and the facilities that enabled the research work to be carried out.

Conflicts of Interest: The authors declare no conflict of interest.

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