Moduli Space and Scattering of D0-Branes in Noncommutative Super Yang-Mills Theory

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Abstract

We study the moduli space of the D0-brane system on Dp-branes realized in the noncommutative super Yang-Mills theory. By examining the fluctuations around the solitonic solutions generated by solution generating technique, we confirm the interpretation of the moduli as the positions of D0-branes on Dp-branes. Low-energy scattering process is also examined for two D0-branes. We find that the D0-branes scatter at right angle for head-on collision in the D0-D4 system. For D0-D6 and D0-D8 systems we find special solutions which reduce to the D0-D4 case, giving the same behavior. This suggests that the scattering at right angle for head-on collision is a universal behavior of this kind of soliton scatterings.

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1 Introduction

D-branes are important solitons in string theory and have revealed not only various dualities in string theory but also nonperturbative aspects of field theories \[1\]. Especially D-brane effective theories with background NS-NS $B$-field have proved to be noncommutative gauge theories \[2, 3, 4\], and this realization has been used to study the nonperturbative dynamics of noncommutative field theories \[4\].

Conversely D\(_p\)-branes on D\(_{p'}\)-branes ($p < p'$) can be described as solitons in noncommutative theories \[3, 4, 8, 10, 12\]. This allows investigation of the D-brane dynamics, e.g. tachyon condensation, in terms of noncommutative gauge theories. By T-duality, these D-brane systems can be mapped to D0-D\(_p\) ($p = 0, 2, 4, 6, 8$) systems in type IIA theory, on which we will focus in this paper. It is interesting that some non-BPS D-brane systems can be BPS in appropriate background $B$-field \[13, 15, 17, 18\].

Exact noncommutative solitons are very useful to study the dynamics of D0-branes. There are mainly two powerful methods to construct exact (BPS) solitons in noncommutative gauge theories; “solution generating technique” \[19\] and ADHM construction \[20\]. Solution generating technique is a transformation which keeps field equations satisfied and generates nontrivial solutions from trivial ones. ADHM construction is a method based on the one-to-one correspondence between instanton moduli space and the solution space of ADHM equation. We can get all instanton solutions by solving ADHM equation.

Noncommutative gauge theories are non-local and have no local observables. However noncommutative solitons have the moduli parameters which represent the positions of the solitons. In previous work, evidence is given for the interpretation of the moduli parameters as the positions of the $k$ solitons in matrix theory \[14\], by use of the Wilson lines \[21\], by exact Seiberg-Witten map \[22\] and by ADHM construction \[23, 24\]. In this letter, we provide another evidence for this interpretation by examining the fluctuations around the soliton solutions. The fluctuations correspond to the open strings between D0-branes. What we find is that the mass eigenvalues are proportional to the length of the stretched string, confirming the above interpretation.

The moduli parameters can also be used to study the low-energy D0-brane scattering on D\(_p\)-branes. This has been discussed for the so-called GMS solitons \[6\] in noncommutative scalar field theories \[25, 26, 27, 28\]. However, they are approximate solutions in the leading order in the noncommutativity parameters, and the result is valid only in the leading approximation in the large noncommutativity parameters. It is then natural to ask what is the exact result for the soliton solutions. Here we examine this problem in noncommutative super Yang-Mills theory, which admits exact BPS soliton solutions.

The scattering is described by geodesic motion, and we obtain the result without approximation in the noncommutativity parameters. In particular, we find that the low-energy scattering occurs at right angle for zero impact parameter, a typical result for soliton scattering including monopoles \[29, 30, 31\], though the solitons obtained by solution gen-
erating technique scatters trivially. Our results indicate that this feature is a universal behavior of this kind of soliton scatterings.

## 2 Moduli as positions of D0-branes

We begin by describing the D0-Dp \((p = 2, 4, 6, 8)\) systems in type IIA theory with background constant \(B\)-field. The Dp-brane fills the directions \(x_0, \cdots, x_p\) and the \(B\)-field is block-diagonal and is taken to lie in the directions \((x_1, x_2, \cdots, x_{p-1}, x_p)\):

\[
B = \text{diag}([B_1], \cdots, [B_{p/2}]) = \frac{\epsilon}{2\pi\alpha'} \text{diag}([b_1], \cdots, [b_{p/2}]),
\]

where \([B_i]\) and \([b_i]\) \((i = 1, \cdots, p/2)\) are \(2 \times 2\) matrices

\[
[B_i] = \begin{pmatrix} 0 & -B_i \\ B_i & 0 \end{pmatrix}, \quad [b_i] = \frac{\epsilon}{2\pi\alpha'} \begin{pmatrix} 0 & -b_i \\ b_i & 0 \end{pmatrix}.
\]

The metric on the string worldsheet is written as \(g_{ab} = \epsilon\delta_{ab} (a, b = 1, \cdots, p)\), \(g_{00} = -1\). Here \(\epsilon\) is a parameter to define the zero slope limit in order to give noncommutative theories \([4]\):

\[
\alpha' \sim \epsilon^{1/2} \rightarrow 0, \quad B: \text{finite}, \quad b_i \sim \epsilon^{-1/2} \rightarrow \infty.
\]

In the present letter, we concentrate on the zero slope limit \((3)\), and consider the corresponding \((p + 1)\)-dimensional noncommutative \(U(1)\) gauge theory \([2, 3, 4]\)

\[
S = -\frac{1}{4g_s^2G_s/g_s} \int dt d^p x \sqrt{-G} G^\mu\lambda G^{\nu\sigma} F_{\mu\nu} \ast F_{\lambda\sigma},
\]

where \(g_s\) is the string coupling and satisfies \(g_s^2 = (2\pi)^{p-2}(\alpha')(p-3)/2 g_s\) and

\[
G_{ab} = g_{ab} - (2\pi\alpha')^2 (Bg^{-1}B)_{ab} \rightarrow \epsilon b^2 \delta_{ab},
\]

\[
G_s = g_s \left( \frac{\text{yet}(g + 2\pi\alpha'B)}{\text{yet} g} \right)^{1/2} \rightarrow g_s \prod_{i=1}^{p/2} b_i,
\]

in the zero slope limit. Though we should supplement \((4)\) with fermionic terms when some supersymmetry is preserved, it is enough to consider only the bosonic terms for our purpose.

The above representation is in terms of star-product. There is another formulation of noncommutative theories, known as operator formalism which is equivalent to the above via Weyl transformation. Let us now switch to the operator formalism. The noncommutativity of the space coordinates implies

\[
[x_0^{2i-1}, x_0^{2i}] = i\theta_i, \quad \theta_i = \frac{2\pi\alpha'}{\epsilon b_i} = \frac{1}{B_i}, \quad (i = 1, \cdots, p/2),
\]
where we assume \( b_i, \theta_i \geq 0 \). We define complex coordinates

\[
z_j = \frac{1}{\sqrt{2}}(x^{2j-1} + ix^{2j}), \quad \bar{z}_j = \frac{1}{\sqrt{2}}(x^{2j-1} - ix^{2j}),
\]

and creation/annihilation operators \( a_i^\dagger = \bar{z}_i / \sqrt{\theta_i} \) and \( a_i = z_i / \sqrt{\theta_i} \). In the temporal \( A_0 = 0 \) gauge, we can rewrite (4) as

\[
S = -\frac{\prod_{i=1}^{p/2}(2\pi b_i)}{g_{\text{NYM}}^2} \int dt \mathcal{L},
\]

\[
\mathcal{L} = \text{Tr} \left[ \frac{1}{2} \sum_{i=1}^{p/2} \left( -\partial_t C_i \partial_t \bar{C}_i + \frac{1}{2} \left( [C_i, \bar{C}_i] + 1/b_i \right)^2 \right) \right.
\]

\[
+ \sum_{i<j} \left( [C_i, C_j] [C_j, C_i] + [C_i, C_j] [C_j, C_i] \right) \right]
\]

where we have set \( g_{\text{NYM}}^2 = g_{\text{YM}}^2 \prod_{i=1}^{p/2} b_i, \bar{b}_i = \epsilon b_i^2 \theta_i = 2\pi \alpha' b_i \) and

\[
C_j = C_{z_j} = \frac{1}{\sqrt{eb_j}} \left( -iA_{z_j} + \frac{1}{\sqrt{\theta_j}} a_j \right), \quad \bar{C}_j = C_{\bar{z}_j} = \frac{1}{\sqrt{eb_j}} \left( iA_{\bar{z}_j} + \frac{1}{\sqrt{\theta_j}} a_j \right).
\]

In addition to the equations of motion, the gauge condition \( A_0 = 0 \) induces the Gauss law constraint

\[
\sum_{i=1}^{p/2} \left( [C_i, \partial_t \bar{C}_i] + \left[ \bar{C}_i, \partial_t C_i \right] \right) = 0.
\]

On D0-Dp, we can construct exact solitonic solutions by applying “solution generating technique” [19]. This is defined by the following “almost gauge transformation”:

\[
C_i \rightarrow S_k^i C_i S_k + \sum_{l=1}^{k} \xi_i^l |p_l\rangle \langle p_l|,
\]

where \( |p_l\rangle \) is orthogonal and normalized states of the oscillators, and \( S_k \) is an almost unitary operator, which is usually called a partial isometry and satisfies

\[
S_k S_k^\dagger = 1, \quad S_k^\dagger S_k = 1 - P_k,
\]

where \( P_k = \sum_{l=1}^{k} |p_l\rangle \langle p_l| \) is a projection operator whose rank is \( k \). A typical example of the partial isometry is a shift operator, given, for example, in [19]. It has been argued that the complex parameters \( \xi_i^l \) represent the positions of the \( k \) solitons [10, 21, 22, 23, 24]. We are now going to present another evidence for this interpretation by showing that the fluctuation spectra of two D0-branes on Dp-branes are proportional to \( |\xi_1^1 - \xi_2^1|^2 \) which is the distance between two solitons.
The transformation (12) leaves the equation of motion and the Gauss law constraint satisfied [13], and generates the following nontrivial solution from the vacuum solution $A_i = 0$:

$$C_i^{(0)} = \frac{1}{\sqrt{b_i}}S_k^i a_i^+ S_k + \sum_{l=1}^k \xi_l^i |p_l\rangle \langle p_l|,$$

(14)

which corresponds to the D-brane system of $k$ D0-branes on a D$p$-brane. The parameters $\xi_i$ are arbitrary and are called the moduli of the solitons.

Let us investigate small fluctuations around the exact solution (14) represented by

$$C_i = C_i^{(0)} + \delta C_i$$

$$= C_i^{(0)} + P_k A_i P_k + P_k W_i (1 - P_k) + (1 - P_k) \tilde{T}_i P_k + S_{i}^k D_i S_k.$$  \hspace{1cm} (15)

The mass matrix of the fluctuations is obtained by substituting (15) into the action (9). Just for simplicity we study $k = 2$ case and focus on the fluctuations $A_i$ which correspond to 0-0 strings. The classical solution is

$$C_i^{(0)} = \frac{1}{\sqrt{b_i}}S_k^i a_i^+ S_k + \xi_i^1 |p_1\rangle \langle p_1| + \xi_i^2 |p_2\rangle \langle p_2|,$$

(16)

and the fluctuations around it are written as

$$C_i = \left( B_i, \frac{W_i}{\tilde{T}_i}, \frac{S_k^i (a_i^+/\sqrt{b_i} + D_i) S_k}{2} \right), \quad B_i = \left( A_{i1}^1 + \xi_i^1, A_{i2}^1, A_{i1}^2 + \xi_i^2 \right).$$  \hspace{1cm} (17)

where $A_{jk}^i$ are the fluctuations that we are interested in. The mass terms for the fluctuations in the Lagrangian are found to be

$$\mathcal{L} = \sum_{i,j=1}^{p/2} \left\{ 2 |\xi_i^j - \xi_j^i|^2 |A_{i1}^j|^2 + 2 |\xi_i^j - \xi_j^i|^2 |A_{i2}^j|^2 - (\xi_i^i - \xi_j^i)(\xi_j^i - \xi_i^j) A_{i1}^i A_{i2}^j (\xi_i^i - \xi_j^i) A_{i1}^i A_{i2}^j - (\xi_i^i - \xi_j^i)(\xi_j^i - \xi_i^j) A_{i1}^i A_{i2}^j (\xi_i^i - \xi_j^i) A_{i1}^i A_{i2}^j \right\}. $$  \hspace{1cm} (18)

Diagonalizing this mass matrix, we get the mass spectra in terms of the properly normalized coordinates $x_i^j \equiv \sqrt{2b_\theta \xi_i^j}$:

$$0, \quad \frac{\epsilon}{(2\pi \alpha')^2} \sum_{i=1}^{p/2} |x_i^1 - x_i^2|^2. $$  \hspace{1cm} (19)

It can be shown that the zero eigenvalue corresponds to the unphysical mode specified by the Gauss law (11). The other eigenvalues show that the open string stretched between two D0-branes has the mass proportional to $|x_1 - x_2|$. This is consistent with the picture that the parameters $\xi_1^i$ and $\xi_2^i$ correspond to the positions of the two D0-branes with strings stretched between them, and the string tension is given by $\sqrt{\epsilon/2\pi \alpha'}$, as we expected.

Though we have explicitly checked this interpretation for $k = 2$, there should be no difficulty in extending our method to arbitrary $k$. 

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3 Low-energy scattering of D0-D0 on Dp-branes

As an interesting application of our results, let us discuss low-energy scattering of two BPS D0-branes on Dp-branes. Since the solutions we are considering are BPS without static force, the scattering can be described by the geodesic in the moduli space [29]. To be more explicit, let us consider two D0-branes on D4-branes whose moduli are those of $U(1)$ two instantons. To examine the scattering of D0-branes, it is necessary to know the metric of the moduli for the relative positions of D0-branes. This can be read off from the kinetic terms in the action of the Dp-branes when the soliton solutions with time-dependent positions are substituted [30, 31]. It turns out that the metric for the D0-branes generated by solution generating technique is flat, so that the scattering is trivial. However, it is possible to construct more general BPS solitons by using ADHM construction. The metric of the BPS instanton moduli is then equivalent to the solution space of ADHM equation. We can determine the metric of the moduli space from the general solutions of ADHM equation. This enables us to derive the geodesic on it and discuss the classical scattering of two D0-branes. We discuss this problem for each D0-Dp system separately.

3.1 D0-D4 System

First let us consider the system of $k$ D0-branes on $N$ D4-branes with background $B$-field. This system corresponds to self-dual $G = U(N)$ $k$-instanton on noncommutative $\mathbb{R}^4$. The moduli space of the system is described by the deformed ADHM equation:

$$
[\Phi_1, \Phi_1^\dagger] + [\Phi_2, \Phi_2^\dagger] + II^\dagger - J^\dagger J = \zeta,
$$

$$
[\Phi_1, \Phi_2] + IJ = 0.
$$

(20)

where $\Phi_i$ ($i = 1, 2$) and $I, J^\dagger$ are $k \times k$ and $k \times N$ matrices and correspond to 0-0 strings and 0-4 strings, respectively. The real parameter $\zeta$ is given in terms of the noncommutativity parameters as $\zeta = \theta_1 - \theta_2$. Note that the self-dual case corresponds to $\zeta = 0$.

To determine the solution space of ADHM equation (20), we have to find its general solutions. Those for $G = U(1)$ and $k = 2$ are found in [32] to be

$$
\Phi_i = w_i^c + \frac{w_i^r}{2} \begin{pmatrix} 1 & \sqrt{2b}a \\ 0 & -1 \end{pmatrix}, \quad I = \sqrt{\zeta} \begin{pmatrix} \sqrt{1 - b} & \sqrt{1 + b} \\ \sqrt{1 + b} & \sqrt{1 - b} \end{pmatrix}, \quad J = 0,
$$

(21)

where

$$
a = \frac{|w_1^c|^2 + |w_2^c|^2}{2\zeta}, \quad b = \frac{1}{a + \sqrt{1 + a^2}}.
$$

(22)

The complex parameters $w_i^c \sim (\xi_i^1 + \xi_i^2)/2$ and $w_i^r \sim \xi_i^1 - \xi_i^2$ correspond to the center of mass and relative positions, respectively.
The metric of the moduli space is also derived in \cite{32} by considering infinitesimal gauge transformation $\delta$ and linearized Gauss law:

$$ds^2 = 2\text{tr}(\delta\Phi_1\delta\Phi_1^\dagger + \delta\Phi_2\delta\Phi_2^\dagger).$$

(23)

The metric naturally decomposes into the parts of the center of mass and the relative motions. The latter part turns out to be

$$ds^2_{\text{rel}} = f(r)\left(dr^2 + \frac{1}{4}r^2\sigma_z^2\right) + \frac{1}{4}f(r)^{-1}r^2(\sigma_x^2 + \sigma_y^2),$$

(24)

where

$$f(r) = \sqrt{1 + \frac{4\zeta^2}{r^4}},$$

(25)

and

$$\sigma_x = -\sin\psi d\theta + \cos\psi \sin\theta d\varphi,$$
$$\sigma_y = \cos\psi d\theta + \sin\psi \sin\theta d\varphi,$$
$$\sigma_z = d\psi + \cos\theta d\varphi,$$

(26)

are the SU(2) invariant one-forms. We note that the metric (24) becomes flat in the case $\zeta = 0$, that is, if noncommutativity parameter $\theta_i$ is self-dual. This is the case for the BPS solitons generated by the solution generating technique, and hence we again find here that the scattering is trivial in that case.

Now let us find the geodesic on the moduli space. The geodesic equation is given as the equation of motion following from the action:

$$I = m \int d\tau \ g^\text{rel}_{\mu\nu} \frac{du^\mu}{d\tau} \frac{du^\nu}{d\tau},$$

(27)

where the metric $g^\text{rel}_{\mu\nu}$ is read from (24) and $u^\mu = (r, \theta, \psi, \varphi)$ are the coordinates of the moduli space. The variational equation $\delta I = 0$ yields

$$\frac{1}{f(r)} r^2 + \frac{r^2}{4f(r)} (\dot{\psi} + \cos\theta\dot{\varphi})^2 + \frac{1}{4}f(r)r^2(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2) = E,$$
$$\frac{r^2}{f(r)} (\dot{\psi} + \cos\theta\dot{\varphi}) = L,$$
$$L \cos\theta + f(r)r^2 \sin^2\theta\dot{\varphi}^2 = C,$$
$$\frac{d}{d\tau} (f(r)r^2\dot{\theta}) + L \sin\theta\dot{\varphi} - f(r)r^2 \sin\theta \cos\theta\dot{\varphi}^2 = 0,$$

(28)

where the dot stands for the differentiation with respect to the parameter $\tau$ describing scattering process (which can be regarded as time), and $E, L$ and $C$ are the integration constants. The solution of our interest for these equations is

$$\dot{\varphi} = 0, \quad \dot{\theta} = 0, \quad \dot{\psi} = L\frac{f(r)}{r^2}, \quad \dot{r}^2 + V(r) = 0,$$

(29)
where
\[ V(r) = f(r) \left( \frac{L^2 f(r)}{4r^2} - E \right). \] (30)

Our problem thus reduces to the classical dynamics for the scattering of zero-energy particles with potential \( V(r) \). Introducing the impact parameter \( \rho = L/2\sqrt{E} \) and the turning point \( r = r_0 \) defined by \( V(r_0) = 0 \), we get
\[ \frac{dr}{d\psi} = r \sqrt{\frac{r^2}{\rho^2 f(r)} - 1}. \] (31)

Therefore the exit angle is derived as
\[ \frac{\psi_{\text{exit}}}{2} = \int_{y_0}^{\infty} \frac{dy}{y \sqrt{\frac{2\zeta y^2}{\rho^2 \sqrt{y^2 + 1}} - 1}}, \] (32)

where \( y = r^2/2\zeta \) and \( y_0 = r_{0}^2/2\zeta \). The angle \( \psi \) covers the whole space twice for the range \( 0 \leq \psi \leq 2\pi \), so it is more convenient to call \( \psi_{\text{exit}}/2 \) the exit angle. It is plotted as a function of the logarithm of the impact parameter in Figure 1.

![Figure 1: exit angle versus log of impact parameter](image)

From this figure, we find that the exit angle is \( \pi \) for large impact parameter and gradually decreases if the impact parameter is decreased. In particular, the exit angle for the head-on collision is \( \pi/2 \), as is the case for monopoles and GMS solitons. We again note that for \( \zeta = 0 \), the scattering is trivial, which means that the D0-branes generated by solution generating technique scatter trivially. Our result indicates that the more general background \( B \)-field makes the scattering nontrivial.
3.2 D0-D6 System

Next we consider the system of \( k \) D0-branes on \( N \) D6-branes with background \( B \)-field. The moduli space of the system is determined in [15] by

\[
[\Phi_1, \Phi_1^\dagger] + [\Phi_2, \Phi_2^\dagger] + [\Phi_3, \Phi_3^\dagger] + II^\dagger = \zeta,
\]

\[
[\Phi_1, \Phi_2] = 0, \quad [\Phi_2, \Phi_3] = 0, \quad [\Phi_3, \Phi_1] = 0, \quad (33)
\]

where \( \Phi_i \) (\( i = 1, 2, 3 \)) and \( I \) are \( k \times k \) and \( k \times N \) matrices and correspond to 0-0 and 0-6 strings, respectively, as in D0-D4 system. The real parameter \( \zeta \) is a FI parameter and depends on the background \( B \)-field. Only when \( \zeta \geq 0 \), eq. (33) has solutions.

Since the general solution of (33) has not been found, let us investigate special solutions for \( G = U(1) \) case. If we restrict \( \Phi_3 = \omega_c^3 \), then eq. (33) reduces to ADHM equation (20), where \( \omega_c^3 \) represents the center of mass coordinate. Hence the moduli space of this simple solution is the same as that of D0-D4 system and the scattering process will be the same as D0-D4 case,[4] implying that the exit angle for the head-on collision is generally \( \pi/2 \) and only \( \zeta \neq 0 \) leads to nontrivial scattering. If \( \zeta = 0 \), in fact, the general solution is found as \( \Phi_i = \text{diag}(\xi_i^l), \quad I = 0 \) and the metric of the moduli becomes flat.

3.3 D0-D8 System

Finally consider \( k \) D0-branes on \( N \) D8-branes with background \( B \)-field. The equation for the moduli space is again not known explicitly. However there exists an eight-dimensional ADHM construction which gives rise to some class of eight-dimensional instantons [33]. We examine the eight-dimensional ADHM equations on noncommutative \( \mathbb{R}^8 \) [16] and focus on the subspace of the moduli space and the corresponding scattering process.

The eight-dimensional ADHM equations are given by [33, 16]

\[
[\Phi_1, \Phi_1^\dagger] + [\Phi_2, \Phi_2^\dagger] + II^\dagger - J J^\dagger = \zeta,
\]

\[
[\Phi_1, \Phi_2] + I J = 0,
\]

\[
[\Phi_1, \Phi_3^\dagger] + [\Phi_2, \Phi_4^\dagger] + IK^\dagger - L J^\dagger = 0,
\]

\[
[\Phi_1, \Phi_4] + [\Phi_3, \Phi_2] + IL + K J = 0,
\]

\[
[\Phi_3, \Phi_3^\dagger] + [\Phi_4, \Phi_4^\dagger] + KK^\dagger - L J^\dagger L = 0,
\]

\[
[\Phi_3, \Phi_4] + KL = 0, \quad (34)
\]

where \( \Phi_i \) (\( i = 1, 2, 3, 4 \)) and \( I, J, K, L \) are \( k \times k \) and \( k \times N \) matrices, respectively. The parameter \( \zeta \) depends not only on the background \( B \)-field but also on the matrices \( \Phi_3, \Phi_4, K, L \). These equations are the (restricted) D-flatness conditions in the worldvolume theory on the D0-branes, and then \( \Phi_i \) (\( i = 1, 2, 3, 4 \)) and \( I, J, K, L \) correspond to 0-0 and 0-8 strings, respectively.

[4] In D0-D0 scattering on a D4 system, we have restricted it to the \( r-\psi \) plane by taking \( \dot{\theta} = \dot{\phi} = 0 \), which might justify the discussion here.
As in the case of D0-D6, it is difficult to solve these equations fully. Hence we look for special solutions. A solution is obtained by putting $\Phi_3 = w_c^3$, $\Phi_4 = w_c^4$, $K = L = 0$. Then eq. (34) reduces to ADHM equation (24), and the problem is similar to the D0-D4 systems. By the same reasoning as D0-D6 system, we conclude that the scattering of D0-D0 on D8-branes would be the same as that of D0-D4 and the scattering would occur at right angle for the head-on collision.

4 Conclusions and Discussions

We have discussed moduli space of D0-Dp-brane systems. We have shown that the moduli parameters in solution generating technique represent the positions of the solitons by examining the fluctuation spectra corresponding to open strings between D0-branes. As an interesting application of our results, we have also examined the scattering process of D0-branes in the Dp effective theory for arbitrary noncommutativity parameters without approximation. The exit angle is determined as a function of the impact parameter, and in particular it turns out to be $\pi/2$ for the head-on collision, which is a universal result in low-energy soliton scattering. If $\zeta = 0$ which corresponds to self-dual solutions and those constructed by solution generating technique, the scattering becomes trivial. Hence the existence of the general background $B$-field is important to render the scattering nontrivial.

We have some comments on the universal results of such two soliton scatterings. In all cases, the two solitons are treated as bosons and the moduli spaces have $\mathbb{Z}_2$ symmetry. The metric (24) for the D0-D4 system is in fact equivalent to Eguchi-Hanson metric [36] which is a resolution of the orbifold $C^2/\mathbb{Z}_2$. Similarly the moduli spaces of D0-D6 and D0-D8 systems for $G = U(1)$ are considered to be resolutions of the orbifolds $C^3/\mathbb{Z}_2$ and $C^4/\mathbb{Z}_2$, respectively. The boundary of Eguchi-Hanson space at the infinite distance between two D0-branes is $S^3/\mathbb{Z}_2$. $S^3$ has a Hopf-fibration whose fiber is $S^1$ with the coordinate $\psi$. The $\mathbb{Z}_2$ symmetry would give rise to the right angle scattering for the head-on collision. Similarly the boundary of the moduli for D0-D8 system is $S^7/\mathbb{Z}_2$. $S^7$ also has a Hopf-fibration whose fiber is $S^3$ and this part corresponds to the boundary of Eguchi-Hanson space. This is why the moduli space of D0-D8 system contains that of D0-D4 system as is seen in subsection 3.3, and the universal scattering behavior is expected because of the $\mathbb{Z}_2$ symmetry.

There is another important BPS D-brane system corresponding to BPS monopoles: $k$ D1-branes ending on $N$ D3-branes [34]. For $N = 1$ and 2, the moduli space is unchanged by the presence of $B$-field on the D3-branes [35]. Hence the scattering process is all the same as commutative case; especially the noncommutative $U(2)$ monopoles scatter at right angle for the head-on collision, and get converted into noncommutative dyons.

Note added. It was pointed out to us that scattering of noncommutative solitons was also discussed in [37].
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