On the measurement and generalisation of urban form

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Abstract. Developments in the provision and quality of digital data are opening up possibilities for more detailed measures of the form of urban areas. This paper begins with a review of some of the new data sources that are available in the United Kingdom, specifically the Ordnance Survey’s ADDRESS-POINT product. The authors go on to develop a fine-scale data model of population densities, and fractal measures of the way in which urban development fills space. The research findings are compared with those of previous research that used less detailed data models.

1 Introduction
Judgment about what constitutes ‘good’ urban theory is relative to what is known already. The objective of this paper is to contribute towards well-found theory about the form and functioning of urban settlements. If improved generalised description of urban form is central to the cumulative development of such theory, it follows that ‘better’ data and appropriate measurement schemes are precursors to progress. Here we will suggest some ways in which better measures of urban phenomena based upon a better digital data infrastructure can lead to better description and thence to better theory. The innovation of geographic information systems (GIS), allied to the proliferation of new, detailed, and disaggregate data sources, is ushering in a new era of data-led generalisation about the empirical characteristics of urban systems at a variety of scales. This in turn is creating a resurgence of interest in system-wide models of urban form and function (Clarke et al, 1997; White, 1999) which may contribute to a reinvigorated approach to rational planning (Batty, 1995).

The new urban modelling is very different from the old, however. 1970s-style urban models sought to draw ambitious system-wide generalisations, yet in practice the data that provided the foundations to analysis were outdated, zonally coarse, surrogate measures (Batty, 1981). The more recent history of urban modelling has been of more successful prediction and forecasting, achieved through richer specification of much more selective aspects of urban systems. That is, the domain of urban models has been restricted in spatial and/or substantive terms (Heggie and Jones, 1978), with applied retail forecasting of store catchments providing the most fertile and predictively successful applications domain (Birkin et al, 1996). Yet more broadly, our ability to develop understanding of physical and socioeconomic distributions through urban modelling remains limited by the quality and scope of data available.

At least two classes of problems arise in practice. First, data models use what is available, and this frequently entails the use of GIS to carry out operations such as spatial averaging across apparently uniform areas, or spatial object class transformations between point, line, and area data (see Martin, 1996, pages 50 – 70). Thus the assumptions invoked in class and object transformations may of necessity be heroic, particularly when between geographical objects of high order (for example, choropleth
map tiles) and low order (for example, points in space). These can generate fundamental and pervasive problems for spatial analysis for reasons that are well known—there are rarely any ‘natural’ units of aggregation in geography and confidentiality constraints restrict the availability of socioeconomic data in particular to (sometimes arbitrary) aggregations (Openshaw, 1984). A second class of generic problems in urban modelling is that attribute data are often very imperfect surrogate indicators of the attributes of socioeconomic systems—with census data having achieved particular notoriety for the absence of critical income and socioeconomic information, for example.

Data quality and quantity clearly prescribe the scope of spatially extensive urban analysis and thence the development of urban theory. Our understanding of the morphology of urban land use is one area in which this is clearly the case, and previous research has begun to devise a number of measures of urban structure and space filling (Batty and Longley, 1988; Frankhauser, 1994). We have both contributed to a paper previously published in this journal (Mesev et al, 1995) which began to develop a hybrid data model using satellite measurements and the UK Census of Population. This paper also applied a number of morphological measures to the analysis of form, an approach which was developed using small area TIGER file data from the US Census by Batty and Xie (1996). Here we will describe the ways in which five years of digital data creation in the United Kingdom have set new benchmarks for urban morphological measures and, in our conclusions, will assess the impacts of these and other changes for the development of new theories of urban form and evolution.

2 Data infrastructure and urban analysis
Insofar as the fundamental problem of representing spatial distributions lies in the aggregation of geographical individuals, a partial solution is now available through ‘framework’ data sources (Rhind, 1997) and data sets created using global positioning systems (Lange and Gilbert, 1999). In the United Kingdom, Ordnance Survey ADDRESS-POINT® contains digital records of every UK (domestic and nondomestic) mail delivery point correct to 0.1 m resolution. A ‘mail delivery point’ is, in practice, deemed to be the seed point of the polygon that represents the built structure of the delivery address in the Ordnance Survey Land-Line product. This product was completed in the mid-1990s under a rolling programme and is subject to very frequent updating through changes to the UK Postcode Address File and ground survey. It provides a considerably enhanced and disaggregate means of representing microscale spatial distributions. Yet used alone, it does not provide a panacea for analysis for a number of reasons. First, ADDRESS-POINT does not discriminate between domestic and nondomestic records (this information is available only for unit postcode aggregations using the Yellow Point® and Ordnance Survey Code-Point® products). Related to this, there is a very imperfect, and geographically variable, correspondence between addresses and households, and households are notoriously difficult to identify (Martin and Higgs, 1997). Second, although the points are precise representations of a mail delivery ‘point’ they are nevertheless abstractions of the areal extent of the built structures or activity spaces of resident households. Thus pursuit of what is in many senses the ultimate abstraction and disaggregation nevertheless begs further questions about the shape and extent of built structures. Moreover, within the socioeconomic realm, there are very few spatially extensive public domain data sets that can be referenced at the scale of individual addresses. Thus, although ADDRESS-POINT provides a valuable new framework for analysis, it is still necessary to model attributes of the built environment and characteristics of human populations. We will return to this point again in our discussion, when we will anticipate that ADDRESS-POINT forces us to clarify our thinking about what we actually mean by terms such as ‘density’ and ‘space filling’.
Mesev et al (1995) sought to detect and model the urban form of Bristol, by using data that were available in the mid-1990s. In their paper, they reported the successful creation of a hybridised socioeconomic and physical representation of Bristol, by using remotely sensed (RS) LANDSAT 5 Thematic Mapper (TM) satellite imagery and a population-surface transformation (Bracken and Martin, 1989) of small area 1991 Census data. The LANDSAT TM image had a notional spatial resolution of 30 m, and the notional resolution of the population surface was 200 m. Even though the resolution of the census data was rather coarse, they nevertheless provided an important source of ancillary information that could be used in image training and postclassification sorting, and produced better classification of built-up and urban land uses than maximum likelihood classification of the image alone. Moreover, the use of ancillary census information also made it possible to produce sharper estimates of the spatial distribution of property types (terraced, detached, semidetached, flatted) than could be obtained from the census alone. Such disaggregation provided a firmer base to analysis of the density of settlement in the study area, and also established a firmer base to temporal analysis of urban land-use change.

The innovation of ADDRESS-POINT and associated data products amounts to a considerable upgrading of the available digital data infrastructure, with favourable consequences for the range and depth of urban analysis that can be developed. In this paper, we will use the empirical study of Mesev et al (1995) as a baseline against which new morphological measures for the City of Bristol may be compared. Almost every record in ADDRESS-POINT is known by the Post Office to be in current use, although the file also contains other minor categories, such as properties under construction and other delivery points not in current use. Frequent maintenance updates by the Ordnance Survey ensure that the product presents a much more contemporary picture than sources such as the census, for example. The points are typically the locations of households or commercial businesses, although multiple properties (apartments) may be referenced at the same location, and some addresses bear scant relation to physical locations (for example, Post Office boxes). The individual records allow discrimination of at least three kinds of microscale land use—residential, nonresidential, and built-up (the third being the sum of the first two). As noted above, these databases focus analysis at very fine scales, yet additional information is required in order to establish a more direct correspondence with real-world activity spaces and built forms. In this paper we will confine most of our attention to modelling of the microscale distribution of residential population, although we will also make some suggestions about the spatial distribution of built structures and suggest directions for future research.

A central weakness of conventional choropleth map representations of census data is that they imply uniformity of within-zone population distributions, and thus constrain analysis of the variability of socioeconomic characteristics to a mosaic geography (Martin, 1996, pages 31 – 49). Using algorithms developed by Martin and Bracken (1991), Mesev et al (1995) used population-surface transformations of small area (enumeration district, ED) census data that were based upon distance decay from subjectively positioned digital population centroids to develop continuous population-density measures. These allow the data model to move beyond the uniform within-area density assumption implicit in choropleth representations and to reduce dependence on the geographical tessellation of census zones. However, such representations are dependent upon the subjective placing of the population centroid (one per ED) and the density-attenuation function used—and so, as a rule of thumb, the resulting surfaces are usually deemed accurate only to about 200 m resolution. This is rather coarser than is desirable for heterogeneous urban areas. In this analysis, the ADDRESS-POINT data set was
used to distribute known 1991 Census populations within each ED—effectively by establishing one population centroid per residential address.

The availability of ADDRESS-POINT for analysis forces some early and crucial decisions about the practicalities of analysis. The data file comprises the precise geographical locations of mail delivery points, which are dimensionless. In our analysis they are taken to represent the living spaces of households, which clearly fill more space. But what are the appropriate dimensions to ascribe to the seed address-points? In terms of built structures, a modern detached house might typically be about $10 \times 10$ m, with older detached and semidetached properties being larger but most other types being smaller. Yet such a measure does not take into account other immediate aspects of life space—not just residential gardens, access conduits, and parking spaces, but also the road infrastructure (and not just that immediately fronting the property) which is necessary to support the residential function. Such considerations might realistically be expected to extend the area reasonably associated with an address-point. The average values of built structures and immediate support infrastructure will vary considerably across any large urban area, and it is possible to generate individual property estimates of each through painstaking analysis of the Land-Line.Plus product (Lake et al., 1998). However, such analysis lies beyond the scope of this research, and thus two sets of analysis were carried out, assigning plausible but unverified mean property parcel sizes of $10 \times 10$ m and $20 \times 20$ m to each address-point. In many senses, this is a crucial and early choice, because it directly determines the amount of space that is filled and thence the measures of ‘density’ that are obtained in our analysis. The scenarios invoked here clearly should be subjected to verification and geographical sensitivity analysis in further work.

Our empirical procedure began with the extraction of ED population counts from the 1991 UK Census of Population [held on-line at Manchester Information and Associated Services (MIMAS, University of Manchester)], and the location of postal address from ADDRESS-POINT tiles. Each ADDRESS-POINT tile covers 1 km$^2$, and thus an initial stage was to stitch the 81 tiles that make up the (approximately 9 km $\times$ 9 km) study area together in ArcView. The precision of the ADDRESS-POINT records (0.1 m, relative to the Great Britain National Grid) and the nature of the geographical entities that they represent (mail delivery points) renders them, almost, dimensionless. An objective of the analysis was to compute various measures of the ways and extent to which these points fill space, and thus each point had to be assigned a space-filling measure. As outlined above, we chose $10 \ m$ and $20 \ m$ as appropriate unit dimensions and used the GRID routine in ArcView to create $10 \ m$ and $20 \ m$ coverages. Each individual postal address in ADDRESS-POINT has a number of fields. These include unique individual property record numbers, unit postcodes, accuracy measures, a tag identifying whether a record has been updated, and, more importantly for work in this paper, a field for names of the postal address. This was used in (very time-consuming) manual sifting of the data to infer function, that is, whether the address was a school, hospital, retirement home, or whether the address field suggested that it was currently used in any of a range of commercial capacities. This stage was carried out using Microsoft Excel and led to the creation of separate built, residential, and nonresidential files. The combined residential data set is shown in figure 1(a), and that for nonresidential addresses is shown in figure 1(b). This figure presents a vivid and detailed picture of the form of Bristol, and a starting point for a range of possible indicators of development—such as the microscale geography of employment and service-delivery points. Raster-to-raster and pixel-to-raster conversion routines in ERDAS Imagine (version 8.3: Erdas Inc., Atlanta, GA) were used to create a series of rectangular ASCII arrays of spatially referenced cells—one for each of the three land uses at cell resolution $10 \ m \times 10 \ m$, and one for each land use at cell resolution $20 \ m \times 20 \ m$. The six resulting files each had three columns—$x$ and $y$ coordinates of each
Figure 1. The configuration of the study area: (a) residential addresses, and (b) nonresidential addresses.
cell and a tag identifying whether or not the cell was occupied. These were then fed into purpose-written (C++) programs to obtain space filling dimensions and density profiles.

EDs are the finest spatial scale for which population counts are available, and in the study area the average population size of an ED is 290 persons. In order to present greater detail in the population density profiles, each address-point was assigned to a census ED in a further stage of the data preparation. Using each point's geometric coordinates, a basic point-in-polygon search was carried out in ArcView. This was essentially a joint operation between two databases (ADDRESS-POINT and the census) where the matched fields were coordinates from ADDRESS-POINT and ED areas from the census. The process was particularly time consuming with regard to EDs that straddled the boundary of the study area, because of the need to identify which address-points lay either side of the boundary. The outcome of this lengthy computational process was a table of the numbers of address-points in each ED, and thence measures of the average number of persons per residential address in each ED (that is, ED population divided by total number of residential addresses in that ED).

Table 1 and figure 2 show that 1991 average ‘pseudo household size’ shows considerable and spatially systematic variation across the study area. The address-points can be aggregated into any zonal scheme (here we will reaggregate into radial distance bands) and thus combination of ED averages with ADDRESS-POINT allows much finer average estimates to be created than hitherto.\(^1\) Additionally, a nonresidential data set was created from the residual nondomestic addresses, and a built-up data set comprised the sum of residential and nonresidential addresses. Finally, as a precursor to the distance-band analysis, the straight-line distance of each address-point from the centre of the city (deemed to be Bristol Bridge) was determined. All points were allocated to 10 m and 20 m bands for two sets of analysis. The stages of the data-processing procedure are set out in figure 3 (see over).

New theories are emerging concerning the ways in which urban land use is structured, focusing upon the underlying order and structure that characterise the apparently irregular geometry of urban land use (Batty and Longley, 1994). Urban land-use policies are tightly prescribed by the geometrical configuration of the cities themselves, their densities, scale, and dimension, yet more generic changes are taking place in the location and constellation of retail, residential, industrial, commercial, and recreational spaces.

\begin{table}
\caption{Census and ADDRESS-POINT statistics of the study area.}
\begin{tabular}{ll}
Mean: & 2.12 persons per address-point \\
Range in ED (enumeration district) average household size: & 0.17–3.93\(^a\) \\
Standard deviation of ED mean household size: & 0.66 \\
Total study area: & 82.08 km\(^2\) \\
Total study area population: & 258,412 \\
Number of residential address-points: & 122,059 \\
Number of nonresidential address-points: & 8426 \\
Total number of address-points: & 130,485 \\
Number of EDs (some partly outside study area): & 679 \\
\end{tabular}
\end{table}

\(^a\) The lower bound is set by a very small number of city centre EDs, in which small numbers of nondomestic properties were probably not identified.

\(^1\) Multiple population weighting is given where multiple addresses occur at a single location. All estimates assume that within-ED household size is uniform, which is unlikely to be the case in EDs with heterogeneous housing stock. Some further analytical refinement, not attempted here, is possible—for example, by varying household size within EDs according to density of address-points, and constrained by known ED totals of different house construction types as recorded in the 1991 Census, or by use of the ED/postcode directory as described by Martin and Higgs (1997).
In addition to producing a range of urban density profiles, our own 1995 analysis used new techniques informed by fractal geometry that have been developed to measure the density, size, shape, scale, and dimension of urban land use. The broader remit of such analysis was to begin to develop wide and small area morphological comparisons which allow the effects of historical contingency and physical features to be isolated from other forces of change. In this way, the argument went, urban planning and regeneration policy (including that based on a ‘predict and provide’ basis) might be put on a surer footing. The data sets created for this study represent a further advance towards these goals.

3 Morphological measurement and profiling

Increases in personal mobility, manufacturing and service decentralisation, and the fission of contemporary lifestyles all provide powerful shifts in the locus of urban development. Urban density profiles, focused upon historic central business district (CBD) areas, nevertheless remain centrally important to urban analysis for a number of reasons. First, cities historically grow outwards from their central seed sites, and built form freezes the social processes that led to the production of the built environment.
Second, the quest for more sustainable urban futures implies a refocusing of functions back towards city centres. And, third, an urban density profile provides a transect across the maximum spanning distance of most urban settlements which is likely to be representative of all stages of urban growth. In some of our previous work we have focused upon settlements which are not strongly constrained by their physical setting (for example, Longley and Mesev, 1997a). Here, however, our priority is to compare the new digital data infrastructure with what was previously available (Longley and Mesev, 1997a; Mesev et al, 1995) and thus our focus here will be upon Bristol. This is not ideal, as far as any quest for establishing space-filling ‘norms’ is concerned, because Bristol’s physical setting is characterised by varied topography and the physical constraint of the Avon Gorge, which abruptly truncates development to the west of the city.

The population profiles created with the new data set are more detailed and accurate than those previously available. Figure 4(a) shows how the numbers of people resident in successive (larger area) distance bands increases with distance from the historic centre of the city, but that the general rate of increase peaks at about 2 km from the centre. Thereafter most, if not all, of the increased area available for development appears to be assimilated through decreased densities of settlement. The logged incremental population density profile shown in figure 4(b) bears some similarities to the classic escarpment profile of spatial structures described by Fotheringham et al (1989), in which the highest y value marks the edge of the ‘complete’ urban structure, and all development beyond this point occurs in the ‘growth zone’ of the structure. Longley and Mesev (1997b, figure 5) have previously provided some comparison of this normative profile with respect to

Figure 3. The data preparation and processing procedures.
RS - GIS’ measures of ‘urban’, ‘built-up’, ‘residential’, and ‘property’ geographies. Of the
four different geographies, the profile shown in figure 4(b) bears the closest resemblance
to the ‘property’ geography measured by Longley and Mesev (1997b), which was obtained
by using census information to augment further an RS - GIS data model. The trend that
seems to emerge from this comparison is that increasingly disaggregate measurements
provide increasing support for the emergence of space-filling norms in the established
parts of city structures. This has implications for our understanding of the rate and
mechanisms of growth of ‘edge cities’ and the extent to which such structures might be
deemed characterised by space-filling norms. Figure 4(b) suggests that ‘complete’ built
structures may evolve in a more gradual way than previously thought—if indeed current
planning policies lead to the development of settlements that are ‘complete’ in population
terms at all. The primary comparative remit of this paper with respect to previous
research means that the chosen study area does not extend far enough to provide
definitive evidence about the edge of Bristol. This will be the focus of future research.

In physical development terms, the residential density, \( \rho(R) \) at any radial distance,
\( R \), from the city centre is given by the number of occupied cells in that distance band
(that is, 10 m \times 10 m or 20 m \times 20 m cells based around an address-point), divided by
the area of the distance band. The logged residential density, \( \ln[\rho(R)] \), profile for

![Figure 4. Population profiles of Bristol.](image)
Bristol is shown, at the 20 m and 10 m pixel scales, in figure 5(a). This illustrates the tailing off in residential densities that takes place with distance from the historic centre of the city area, until a near-constant proportion of cells is occupied in each successive distance band. The selection of either the 20 m or 10 m levels of resolution has an evident shift effect upon the absolute measure of density (with 20 m measures filling more space), although figure 5(a) shows that the absolute magnitude of the difference is more or less constant at all radial distances. If each address-point were to be assigned a succession of larger areas, occurrences of more than one point per cell would become increasingly common and this would offset the increased magnitude of recorded density that is attributable to coarser pixel size. In further work we hope to conduct sensitivity analysis of such ‘overlap effects’, because they are likely to provide an insight into the (spatially variable) way in which human activity spaces are configured in residential areas. The built-density profile is similar to that for residential land use, because of the dominance of residential land use across the city as a whole. The profile for non-residential land use [figure 5(b)] shows a rapid attenuation of density with distance from the centre. It also exhibits rather more pronounced scale effects—in particular with respect to the recorded densities of nonresidential land use towards the outer zones, where the 10 m pixel mesh leaves more vacant interstices between developed cells.

**Figure 5.** 10 m and 20 m density profiles: (a) residential, and (b) nonresidential land uses.
These profiles are generally comparable with those constructed for 1991 by Mesev et al (1995). However, particularly with regard to the residential density profile, the 1995 analysis suggested an increase in residential densities towards the outermost zones—which, at the time, was interpreted as the effect of the profile moving beyond the physical constraint of the Avon Gorge. However, the profile based on address-points shows a continuing downward drift at these distances. The 1995 classification was based upon inference of land use from land cover, using physical measures of surface reflectance (albeit supplemented with small area population data used in the classification process). As such it was an areal classification, and the density profile was based upon the incidence of elements in a mosaic of areal land-use classes. This indicator of space filling is different in conception to the density of address-points which are seeded inside individual built structures. The implication of the comparison is that proportionally more land is covered by built structures towards the suburbs, yet the density of addresses continues to decline. The difference most likely arises because more land cover is attributed to residential land use towards the outer suburbs, yet the number of mail delivery points continues to decrease.

At one level it seems trivial to labour the point that suburbs and more central areas are characterised by different built structures, yet the differences between the measurements reported here and those recorded in 1995 provide a clear indication of the ways in which conception and measurement prescribe analysis. The comparison brings together two different conceptions of density—a measurement of land cover (hybridised by the use of population land use to modify classification) and a more direct indicator of land use and human activity. The measurement thus sharpens our thinking about ‘density’ and the need for a ‘horses for courses’ approach to data provision. Within the realm of urban sustainability studies, for example, urban ecological analysis might be justified in asserting the primacy of the extent of different built structures, while prescriptive planning applications might be more concerned with the geography of population needs as indicated by address-points.

Such thinking has implications for the way we might devise and interpret space-filling norms as well. The profiles in figures 4 and 5 suggest that, consistent with conventional space–distance trade-off models (Alonso, 1964), the density gradient of residential addresses decreases steadily with distance from the centre of the city. This apparent finding stands in contrast to the result of the 1995 analysis, which suggested no monotonic decrease in the densities of built structures. To these observations we should add our own impression that the analysis of address-points represents a considerable improvement on the 1995 data model. The population-surface model was used in 1995 to reduce land-cover misclassifications, yet its coarse resolution rendered it incapable of attributing households to precise geographical areas, even within the constrained physical carcass provided by the LANDSAT image. Visual inspection of the original LANDSAT image also suggests that the apparent increase in density in the inner suburbs may in part reflect the misclassification of nonresidential built structures as residential, and the failure of the coarse-scale population-surface model to detect this.

The 1995 analysis also presented count profiles of the cumulative number of pixels occupied with increasing distance from the ‘seed’ site of the city (Bristol Bridge). The corresponding profiles for the address-point data are shown in figures 6(a) and 6(b) (see over). The residential [figure 6(a)] and built (not shown) profiles are very similar, yet show a smoother build up in the cumulative structure than was suggested by the 1995 analysis. The nonresidential profile [figure 6(b)] exhibits clear sensitivity to scale effects—with the 10 m profile better bringing out the concentration of nonresidential mail delivery points in the downtown area. This profile is rather different from profiles for nonresidential land use created for 1981 and 1991 by Mesev et al (1995). The profiles
created as the basis to temporal comparison in the 1995 paper are consistent with one another. However, this only serves to underline the extent to which conception of the phenomenon conditions the outcome of the measurement exercise.

Fractal geometry can be used to provide numerical summaries of the kinds of spatial distributions portrayed in figures 5 and 6. Indeed, in a general descriptive sense, Goodchild and Mark (1987, page 267) have suggested that such indexes “may be the most important parameter of an irregular cartographic feature, just as the arithmetic mean and other measures of central tendency are often used as the most characteristic parameters of a sample”. With regard to urban density functions, measurement of fractal dimensions provides a means of linking the form of development to its spread and extent (Batty and Longley, 1994; Makse et al, 1999). Here we will assess the values of parameters which measure how space is filled, and measure the rate at which the nature of space filling changes with respect to distance from the CBD. We will also draw some comparisons between the results of analysis of our ADDRESS-POINT data set and measurements made for 1981 and 1991 using the LANDSAT–census data sets created by Mesev et al (1995). Fractal dimensions can be inferred from the slope coefficients of best-fit regression lines fitted through density profiles to measure the attenuation effects of distance and as measures of space filling: here we will concentrate on the latter.

**Figure 6.** Count profiles: (a) residential, and (b) nonresidential land uses.
Mesev et al (1995) demonstrate that the space-filling dimension can be obtained from the cumulative number of points \(N(R')\) occupied on a rectangular grid at radial distance \(R'\) by:

\[
D = \frac{1}{\ln R'} \ln \left( \frac{N(R')}{4} \right).
\]  

(1)

Experience has shown that, for a growing fractal structure, the mean of all the radial distance measures taken of a contiguous settlement, \(R'\), is often taken to be the most appropriate single value of \(R\).

A consistent reexpression of this in terms of density in continuous space rather than cumulative population is given by

\[
D \approx 2 + \frac{\ln \rho(R')}{\ln R'},
\]

(2)

where \(\rho(R')\) is the density of occupied pixels in distance band \(R'\). For both expressions, \(D\) should lie between 1 and 2, and intuitively can be thought of as the degree to which geographical objects fill more space than a line (dimension 1 having length but no breadth) but less than a plane (dimension 2 having length and breadth).

All previous results obtained using these equations have used a pixel mesh to identify and define geographical individuals—that is, the presence or absence of a land-use (or cover) category rather than the occurrence of a ‘true’ geographical individual observation. Thus the higher the number of pixels of finite area that are occupied by a land use or cover, the closer the fractal dimension of that land use or cover is to 2. Here we have applied equations (1) and (2) to the microscale distribution of three categorisations of address-points—which correspond to the ‘built-up’, ‘residential’, and ‘nonresidential’ categories of previous analyses using the sorting procedure described in section 2 above. The (essentially arbitrary) dimensions ascribed to the individual address-points here fundamentally condition the outcome of analysis—in that the raw data are for practical purposes dimensionless points having neither length nor breadth. Clear thinking is necessary to assign dimensions that are consistent with the aspect of form or function that the analysis purports to measure. We have again assigned the essentially arbitrary dimensions of 10 m × 10 m and 20 m × 20 m in our own two sets of measurements, and the results are presented in table 2.

Each set (10 m and 20 m) of results shown in table 2 shows the expected sequencing of

\[D(\text{built-up}) > D(\text{residential}) > D(\text{nonresidential}).\]

The measures obtained at the 10 m scale consistently fill less space than those made at 20 m for each of the land-use classes. Figure 7 (see over) shows how the dimensions are, in turn, consistently below those obtained from the satellite image-based model developed by Mesev et al in 1995. The 1991 figures are broadly consistent, and illustrate

**Table 2.** Space-filling and fractal dimensions of urban land use.

| Land use         | Spatial dimensions | Fractal dimensions |
|------------------|--------------------|--------------------|
|                  | resolution (m) | total raster size | occupied cells | density | count |
| Built-up         | 20                | 204 304            | 49 680         | 1.526   | 1.540  |
| Built-up         | 10                | 817 216            | 77 824         | 1.429   | 1.451  |
| Residential      | 20                | 204 304            | 46 928         | 1.515   | 1.530  |
| Residential      | 10                | 817 216            | 74 601         | 1.420   | 1.443  |
| Nonresidential   | 20                | 204 304            | 4 698          | 1.157   | 1.211  |
| Nonresidential   | 10                | 817 216            | 5 424          | 1.095   | 1.151  |
the basis to a scale-based analysis. After the absolute differences in measures that are obviously attributable to the scale measure of urban development have been taken into account, there is remarkable consistency between the 1998 results reported here and those reported in Mesev et al for 1991. Indeed, comparison of the 1991 and 1998 measures suggested greater similarity than existed between 1981 and 1991 data in Mesev et al’s temporal change analysis.

The consistent differences between the sets of measurements highlights the importance of the way in which ‘urbanity’ is measured. A strength of fractal measures is that they can be used to develop comparisons across space and time (but see White, 1999). In an early review of fractal measures of urban form, Batty and Longley (1994, pages 234–244) suggested that measures for different cities were clustered around the ‘natural space filling’ value of 1.71, characteristic of space-filling structures produced by diffusion-limited aggregation. The results of this analysis demonstrate the degree to which this conclusion was predicated upon definition and measurement of urban structures, and the results presented here stand in quite stark contrast to the denser structures with higher dimensions detected by Frankhauser (1994). In retrospect it begins to appear that the consistency of the measures across space may in significant part have reflected the (consistent) spatial limitations of measurement methods of the time. There is a clear need to establish benchmark measurement standards in order to further the comparison of urban forms across space and through time. Although the population densities reported elsewhere in this paper constitute continuous measures, the fractal dimensions, like nearly all of those reported elsewhere in the urban literature, are essentially two-dimensional—that is, they are measures of whether a space is occupied or not. Cities are three-dimensional entities, and there is a need to develop the analysis presented here to encompass three-dimensional representations of city form.

### 4 Conclusions

Within the realm of urban morphology research ADDRESS-POINT and associated data products permit a leap forward in our abilities to measure the properties of socioeconomic systems. Much of urban theory to date has been based upon models which are overly simplistic in the spatial structures that they suggest. It is simply not good enough to seek to constrain the science or study of urban form to the idealised geometrical structures that are the hallmarks of classic urban models. Fractal measures

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**Figure 7.** Comparison of space-filling and count dimensions.
and density signatures provide measures of urban form which are much more suited to summarising the jaggedly irregular land-use patterns that characterise real-world cities. Although the notion of 'data-led' thinking or analysis has always had pejorative connotations in social science, the volume, detail, and quality of data that are now available are clearly conducive to richer inductive generalisation about urban systems. Furthermore, the properties of the data available to this study provide an example of the way in which such generalisation forces us to reconceptualise what we mean by terms such as 'density'. In a partial sense the new generation of digital framework data products also resolves ecological fallacy and modifiable areal unit problems in spatial analysis—although ethical constraints on socioeconomic data provision dictate that the assignment of socioeconomic attributes to address-point data will continue to require data modelling of spatial distributions. These assumptions will nevertheless be less heroic than hitherto.

The data used in this paper are a commercial product, the strategic and routine maintenance of which entail significant expenditure. There are some enduring doubts as to whether the information commerce through which these costs are recovered is consistent with the widest dissemination and use of data sets that are not in the public domain (Rhind, 1999). Certainly, the most rapid progress will be made if the best models are based upon the best data—indeed this is fundamental to effective monitoring of the unprecedented scale and rapidity of change in today's urban systems.

Further technical modifications might be made to the sorts of indices developed here—such as accommodating the scale, spacing, and configuration of development at different points in the history of urban systems, or the supplementation of straight-line distance metrics with generalised accessibility measures. The frequency and quality of data updating also make it possible for the first time to monitor incremental changes in urban form in something approaching real time. Moreover, there are also potential extensions beyond analysis of form along to microscale models of human activity patterns. Rich data are now becoming available through so-called 'lifestyles' sources (Longley and Harris, 1999), and these hold the prospect of better understanding how density distributions relate to human activity patterns and the provision of urban functions. All of this has important potential feeds into rational planning policy.

Finally, fundamental to the analysis presented in this paper is the premise that quantitative measurement of urban form can yield generalised insights about the form, and thence the functioning, of urban areas. Almost a quarter of a century has elapsed since the heyday of urban modelling, and the approach today accounts for a considerably reduced share of intellectual activity in urban geography and planning. For some (for example, Curry, 1995), the innovation of improved geographical information handling technologies and the huge upgrading of digital data infrastructure do not (and indeed, can never) result in any forward shift in our abilities to 're-present' social systems. The implications of this view for regional and local land-use planning policy are gloomy, for it implies a confinement of urban research to the idiographic, and a diminished interest in transparent generalisation across space and time. Adherence to this view also contributes to today's ironic situation in which a transformation in our abilities to depict the generalisable detail of urban systems coincides with new lows in the esteem in which 'predict and provide' planning is held. If urban geography and planning are to impact on anything but academic discourse, the goals of generalisation about spatial systems cannot be readily abandoned. The time is now ripe to renew the quest for generalisation across urban systems.

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