Application of Fuzzy Model Analysis of Default and Prepayment Behaviours

Lai-Wang Wang¹, Chiu-Hsiung Chen²*, and Cheng-Chung Wu³

¹ Department of Industrial Engineering Management, National Kaohsiung University of Science and Technology, Kaohsiung, Taiwan
² Department of Industrial Engineering Management, National Kaohsiung University of Science and Technology, Kaohsiung, Taiwan
³ Department of Finance, Suqian College, Jiangsu, China

*Corresponding author e-mail: sunecohansa@yahoo.com.tw

Abstract. This paper has debated Fuzzy Model in the context of default and prepayment behaviour, through theoretical and empirical analysis, this paper shows the law of default and repayment and the main factors affecting the risk of default and repayment. On this basis, it chooses appropriate measurement tools to control the risk, and provides some basic information for the default risk of housing mortgage loan. At the same time, it also provides commercial banks with evasion of personal housing mortgage. The default and repayment risks faced by loans provide further quantitative information.

1. Introduction

Because the micro-variables related to borrowers and the macro-variables that borrowers cannot resist affect the default risk and prepayment risk of mortgage loans, this paper explores the actual risks faced by individual housing mortgage loans of commercial banks through a reasonable risk control model, uses the model to carry out empirical analysis, supplemented by qualitative research, and at the same time is the same. At the same time, relevant theoretical research will be carried out. In addition, through theoretical research and empirical analysis, this paper explores the causes and main risk factors leading to the risk of personal housing loans, and then chooses appropriate risk prediction and control measurement tools to improve the level of risk management model and risk prevention ability of commercial banks, improve the profitability of banks, and solve the problem of individual house purchase. Problems are of great significance in social practice.

2. Literature Review

Empirical studies on default and prepayment risk focus on the correlation between borrower characteristics, loan characteristics, real estate characteristics, regional characteristics and default and prepayment risk.

2.1. Literature Regarding Defaults and Prepayments

Merton (1974) is based on the analysis that the default process is endogenous and related to the capital structure. Foster and Van Order (1984) discussed this issue in terms of the theory of expectancy right, regarding default as put option and early repayment as call option, and concluded that housing equity is an important variable to reveal borrowers ‘default.
Figure 1. Concept of Option in Asset Mortgages

Kau et al. (1995) used contingency requirement theory to treat default as a reasonable decision. When the value of housing is reduced to less than the amount of mortgage loan, default will occur. This method regards strategic mortgage default as a selling option for the mortgage itself.

2.2. Application of Research Models

After considering all the above factors and the method of past research, this paper concludes that the most frequently used statistical methods in the estimation of defaults and prepayments are discriminant analysis and logistic regressions. Although Tam and Kiang (1992) suggest that logistic methods are not necessarily superior to discriminant analysis, as sometimes discriminant analysis yields a more accurate rate than logistic regressions. Therefore, there is no concrete evidence concerning the pros and cons of discriminant analysis and logistic analysis. That would be the reason; it also depends on the data attributes and the assumptions of analysis methods in varying situations. Despite these arguments of past researches, there is no consistent thought between these methods. We can think of three criteria which might influence the method of our choice among the approaches as follows:

1. It must to be theoretical what it is exactly we are trying to model.
2. Practical which is there valid exclusion restrictions, without the sample selection model may under-perform.
3. We are also often interested in the effect of variables such as what are the influential factors of default and prepayment behaviours with mortgage. We could observe the behaviours of people who do something default and prepayment on their loans in all probability will be those people only able to achieve a relatively behaviours. That is, we are interested in modeling the potential factors of behaviours with people who loaning from business bank, which default and prepayment occurring. We could then estimate the effect of a covariate such as influential factors of these behaviours.

3. Methodology

3.1. Classic S-Curve Theory

Miskawi (1989) development of S-curve equation, the envelope of S-curve is generated. The equation can be used in various applications of project control. The research includes establishing equations and then conducting various experiments. The results show that the theoretical data are closely related to the actual data of various projects implemented around the world. The S-curve model has the following form:

\[
P = \frac{3^T}{2} \sin \left( \frac{\pi(1-T)}{2} \right) \sin(\pi T) \log \left( \frac{T + (1.5 - T_p)}{T_p + T} \right) - 2T^3 + 3T^2
\]

The S-curve can also be used to indicate an S shaped chart resulting from a cumulative likelihood distribution. In this function, an S-curve is a tool of quantitative risk analysis which project management would use to determine the possible dangers of any given course of action.

It is assumed that all observed data can be obtained in general. Therefore, in this study, we use T-S fuzzy model to develop a basic S-curve model, which can be used to analyze default and prepayment risk by constructing the behavior evaluation of borrowers. The proposed T-S fuzzy model will be used to smoothly connect the fuzzy regression curves of large and small engineering projects.

3.2. T-S Fuzzy Model via Fuzzy S-curve

There are two methods to identify T-S fuzzy model: establishing T-S fuzzy model by motion
equation and identifying model parameters by using fuzzy C-means clustering algorithm, least squares method, genetic algorithm and other fitting algorithms based on input and output data. In this method, firstly, the motion of the system is analyzed, then the state space form (non-linearity) of the motion state is obtained, and then the T-S fuzzy model of the system is obtained by piecewise approximation of the T-S fuzzy model.

\[ y = a_k x^k \]

T-S fuzzy model can generate more complex non-linear functions with a small number of fuzzy rules, which can effectively reduce the number of fuzzy rules when dealing with multivariable systems, and thus has great advantages. However, because the conclusion parameter is a linear function rather than a fuzzy number, the conclusion part of the actual system can not be obtained directly from expert experience and operational data, and must be trained through certain algorithms. Therefore, the identification of model parameters becomes the main problem in the establishment of T-S fuzzy system.

In this study, the data clusters of default and prepayment behaviours is distributed into overlapping regions representing the outlay for the borrowers’ behaviours of five commercial bank in Taiwan. These clusters are shown by the membership functions of fuzzy sets \( C_1, C_2, \ldots, C_i \). Therefore, according to the first rule of fuzzy reasoning, a set of fuzzy IF-THEN rules will be used to describe it. The formulas are as follows. (Hsiao et al., 2005; Pereira et al., 2008):

Rule 1: IF \( x \) is \( C_1 \) THEN \( y_1 = a_{1k} x^k \)

Rule 2: IF \( x \) is \( C_2 \) THEN \( y_2 = a_{2k} x^k \)

\[ \vdots \]

Rule 2: IF \( x \) is \( C_i \), THEN \( y_i = a_{ik} x^k \)

In this case, \( x \) is the input, representing the variable; \( y_i \) (\( i = 1,2 \)) is the output, representing the actual result. Where \( r \) is the number of IF-THE rules, and it is a prerequisite variable. The final output can be inferred by using the fuzzy method, product reasoning and a single fuzzy controller.

\[ y = \frac{\sum_{i=1}^{r} w_i y_i}{\sum_{i=1}^{r} w_i} = \sum_{i=1}^{r} h_i y_i. \]

It is assumed that \( w_i \geq 0, \ i = 1, 2, \ldots, r; \sum_{i=1}^{r} w_i > 0 \). Therefore, \( h_i \geq 0 \) and \( \sum_{i=1}^{r} h_i = 1 \). If the rule=2 can be simplified as made clear by the following Remark.

**Remark 1:** \( w_i \) is the degree of membership of either the low (\( i = 1 \)) or high (\( i = 2 \)) fuzzy set. When \( x < C^L \), the regression model of rule 1 will be used. Conversely, when \( x > C^H \), the regression model of rule 2 is used. When \( C^L \leq x \leq C^H \), the two equations are simultaneously utilized with continuously varying weights. For example, when the value of \( x \) rises in the interval \([C^L, C^H]\), more weight is given. Regression model of rule 1 has less weight to the regression model of rule 2.

4. **Results**

That is, as the borrowers’ behaviour, there are one or more times to be occurred for the default and prepayment. Besides, it is hard to distinguish how many times the behaviours of default and prepayment there is high proportion to appear, i.e. how old of age for borrowers or how much of monthly income they earn would be high rate to default or prepayment, have the absolutely or relative difference. That is why we need to induce the modifying fuzzy two parts of regression and fuzzy regression model with the fuzzy variable.
Based on the previous analysis with this study by different regression model, it develops a set of predictive models for the application with default and prepayment behaviours to the construction the worth references to bank industry, which emphasized the importance of pre-understanding the behaviours and the influence of various default and prepayment factors. These behaviours forecasting models were the kind of traditional regression model that could only analyze the influence of each factor upon the behaviours of default and prepayment, taking into account variation in different possible influential assessing factors of measurement variation, etc. It is particularly important to study the modeling of nonlinear systems. For the non-linear system, it is difficult to obtain accurate mathematical model. Even if the mathematical model can be established, it is often too complex, which makes the traditional control difficult to achieve the desired control effect.

S-curve mainly exists in Logit model and Logistic regression model. They can accurately reflect the accurate situation of bank mortgage loan by mathematics and predict the future development. They are useful tools for customer financial management behavior. At present, the banking industry will be more efficient in loan management. Mortgage loan management is the most important method in bank evaluation and analysis. It can clearly show the behaviours of customers to provide valuable references of potential customers in future. Therefore, if analysis and prediction of behaviors can be performed before construction on a mortgage business begins it helps to understand the loan of customers’ demands based on the predictive results during the construction stage in advance. Bankers could dispatch the relative practice to be caught aware with the possible loss that can be made earlier to avoid situations of insufficient loan turnover by their customers.

Using T-S Fuzzy theory is applied to solve the problem of huge reasoning rules in multidimensional fuzzy reasoning. The practical S-curve regression model is fitted to the sample data. Then, by using the S-curve regression model and the concepts of default and advance payment, the default behavior and prepayment behavior graph are obtained from the important factors of the sample tender.

Secondly, it establishes the basic mortgage management and control model of banks. This model is different from the previous analysis model. The concept of a prior analytical model with sample data being tested and used to derive corrections. Thereafter, the variance in behavior can be calculated, and the relative risk prediction can be established, and the probability of occurrence can be calculated. Finally, the model is evaluated and put into use (including the prediction of default and prepayment), and some conclusions are drawn.

5. Discussion and Suggestion

When the banking industry grants credit to the housing mortgage loan, what factors may influence the overdue loan and which ones are the most influential ones, and among many influencing factors of the overdue loan, several key factors are selected and simplified in order to study the different bank groups with different overdue ratios for housing mortgage loan. What are the significant differences in the perception of the possible influencing factors of overdue lending caused by credit granting? Therefore, this study considers that in terms of relative importance, there are many factors that vary with the credit route and the credit period. In practice, this model can provide borrowers as a measurement tool when applying for loans, and help banks as a reference for preliminary screening of borrowers, thus alleviating the serious information asymmetry between borrowers and lenders in the borrowing market.

References
[1] Foster, C., and Van Order, R. 1984. An Opiton-based Model of Mortgage Default, Housing Finance Review, 3(4): 351-372.
[2] Hsiao, S.F., M.C. Chen, M.Y. Tsai and C.C. Lin (2005) System on chip implementation of the whole advanced encryption standard processor using reduced XOR-based sum-of-product operations. IEEE Proc .Inf. Secur., 152(1), 21-30.
[3] Kau, J. B., and Keenan, D.C., and Muller, W.J., and Epperson, J.F. 1995. The Valuation at Origination of Fixed-Rate Mortgages with Default and Prepayment, Journal of Real Estate Finance and Economics, 11(1): 5-36.
[4] Merton, R. (1974) On the Pricing of Corporate Debt: The Risk Structure of Interest Rate. Journal of Finance, 29, 449-470.

[5] Miskawi, Z., (1989) An S-curve equation for project control, Construction Management and Economics, 7, 115-124.

[6] Pereira, E., Rodrigues, S. M., Otero M., Válega, M., Lopes, C. B., Pato, P., Coelho, J. P., Lillebø A.I., Pardal M.A., Rocha R., Duarte A.C. (2008) Evaluation of a proficiency inter laboratory exercise for total mercury in environmental samples: soils, sediments and fish tissue. TrAC, Trends in Analytical Chemistry, 27 (10), 959-970.

[7] Tam, K. Y. and Kiang, M. Y., (1992) Managerial applications of neutral networks: The case of bank failure predictions, Journal of Business Finance and Accounting, 12(1), 19-45.

[8] Takagi, T. and Sugeno, M. (1985) Fuzzy identification of systems and its application to modeling and control, IEEE Trans. on Syst., Man and Cybernetics, part B – Cybernetics, 15, 116-132.