An Improved Method to Measure the Cosmic Curvature

Jun-Jie Wei\textsuperscript{1,2} and Xue-Feng Wu\textsuperscript{1,3,4}

\textsuperscript{1} Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China; jjwei@pmo.ac.cn
\textsuperscript{2} Guangxi Key Laboratory for Relativistic Astrophysics, Nanning 530004, China
\textsuperscript{3} School of Astronomy and Space Science, University of Science and Technology of China, Hefei, Anhui 230026, China
\textsuperscript{4} Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing University-Purple Mountain Observatory, Nanjing 210008, China

Received 2016 July 12; revised 2017 February 15; accepted 2017 March 14; published 2017 April 5

Abstract

In this paper, we propose an improved model-independent method to constrain the cosmic curvature by combining the most recent Hubble parameter $H(z)$ and supernovae Ia (SNe Ia) data. Based on the $H(z)$ data, we first use the model-independent smoothing technique, Gaussian processes, to construct a distance modulus $\mu_H(z)$, which is susceptible to the cosmic curvature parameter $\Omega_k$. In contrary to previous studies, the light-curve-fitting parameters, which account for the distance estimation of SN ($\mu_{SN}(z)$), are set free to investigate whether $\Omega_k$ has a dependence on them. By comparing $\mu_H(z)$ to $\mu_{SN}(z)$, we put limits on $\Omega_k$. Our results confirm that $\Omega_k$ is independent of the SN light-curve parameters. Moreover, we show that the measured $\Omega_k$ is in good agreement with zero cosmic curvature, implying that there is no significant deviation from a flat universe at the current observational data level. We also test the influence of different $H(z)$ samples and different Hubble constant $H_0$ values, finding that different $H(z)$ samples do not have a significant impact on the constraints. However, different $H_0$ priors can affect the constraints of $\Omega_k$ to some degree. The prior of $H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ gives a value of $\Omega_k$ a little bit above the $1\sigma$ confidence level away from 0, but $H_0 = 69.6 \pm 0.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$ gives it below $1\sigma$.

Key words: cosmological parameters – cosmology: observations – galaxies: general – supernovae: general

1. Introduction

Cosmic curvature is one of the fundamental parameters in modern cosmology. The intriguing question of whether cosmic space is open, flat, or closed is closely related to many important problems such as the evolution of our universe, the nature of dark energy, etc. A significant detection of a nonzero curvature will have far-reaching consequences for mankind’s views of fundamental physics and inflation theory, since a flat universe is supported by most of the observational data, including the latest Planck results (Planck Collaboration et al. 2016).

However, as a result of the strong degeneracy between the spatial curvature and the dark energy equation of state, it is fairly difficult to constrain these two parameters simultaneously. The curvature parameter is generally treated as zero in a dark energy analysis, or conversely, some specific models of dark energy (e.g., the cosmological constant) are assumed when constraining the curvature. It should be underlined that a simple flatness assumption may lead to incorrect reconstruction in the equation of state of dark energy even if the real curvature is tiny (Clarkson et al. 2007), and some confusion between the flat $\Lambda$CDM model and a dynamical dark energy non-flat model may be caused by a cosmological constant assumption (Virey et al. 2008). In order to overcome the defects of a zero curvature assumption, a direct model-independent method for determining the curvature by combining measurements of the angular diameter distance $D_A(z)$ (or the luminosity distance $D_L(z)$) and the Hubble parameter $H(z)$ has been proposed (Clarkson et al. 2007, 2008):

$$\Omega_k = \frac{[H(z)D'(z)]^2 - c^2}{H_0D(z)^2},$$

where $c$ is the speed of light, $H_0$ is the Hubble constant, $D(z) = (1+z)D_A(z) = D_L(z)/(1+z)$ is the comoving angular diameter distance, and $D'(z) = dD(z)/dz$ represents the derivative with respect to the redshift $z$.

Since this method was proposed, it has been used to determine the curvature parameter in several instances, including the following representative cases. Shaﬁeloo & Clarkson (2010) used the luminosity distances derived from Type Ia supernovae (SNe Ia) observations, Hubble rate measurements inferred from passively evolving galaxies and from baryon acoustic oscillation (BAO) data, and found no evidence for deviation from flatness (see also Mortess & Jonsson 2011). Saponi et al. (2014) compared four different measurement techniques to test cosmic curvature from the most recent Hubble rate and SNe Ia data. Li et al. (2014) determined the curvature parameter using $H(z)$ and $D_A(z)$ pairs from BAO measurements. Cai et al. (2016) used the model-independent smoothing technique (i.e., the Gaussian process) to reconstruct $H(z)$ from differential ages of galaxies and from radial BAO data and $D_L(z)$ from the SNe Ia Union2.1 data sets and then measure the curvature. L’Huillier and Shaﬁeloo (2017) tested the flatness of the universe at redshifts 0.32 and 0.57 using the most recent BAO and SNe Ia data, and they found that the current observations are compatible with a flat universe. Yu & Wang (2016) constrained the curvature to be $\Omega_k = -0.09 \pm 0.19$, combining the measurements of $H(z)$ derived from differential ages of galaxies and from radial BAO data with $D_A(z)$ estimated from BAO data.

In principle, the nuisance parameters characterizing SN light curves should be optimized simultaneously with the cosmological parameters when using SNe Ia as standard candles. But it is shown that the nuisance parameters have extremely little covariance with the cosmological parameters (see Marriner et al. 2011). In previous works, the luminosity distances of SNe Ia were obtained directly from Hubble diagrams where the light-curve-fitting (nuisance) parameters were inferred from global fitting within the context of a cosmological model. To
confirm if the cosmic curvature parameter has a dependence on the nuisance parameters or not, we keep them free in our analysis. On the other hand, following the method of Clarkson et al. (2007, 2008), one needs to estimate the derivative function of \( D(z) \) from a fitting function (see Equation (1)), which will introduce a large uncertainty (Yu & Wang 2016). In order to avoid the shortcoming of this method, we perform an improved model-independent method to achieve a reasonable and compelling test of the cosmic curvature. Moreover, we also investigate the impact of the Hubble constant \( H_0 \) on this test.

The rest of this paper is organized as follows. In Section 2, we briefly describe the data used in our work, including the most recent SNe Ia and \( H(z) \) data. In Section 3, we introduce our improved method for testing the curvature. The constraints on the curvature are shown in Section 4. Finally, we summarize our conclusions in Section 5.

### 2. Observational Data

In the following, we describe the data sets that we will use in the present analysis.

#### 2.1. SNe Ia Sample

We use a joint light-curve analysis (JLA) sample of 740 SNe Ia processed by Betoule et al. (2014). The observed distance modulus of each SN is given by

\[
\mu_{\text{SN}} = m_B^* + \alpha \cdot X_1 - \beta \cdot C - M_B,
\]

where \( m_B^* \) is the observed peak magnitude in the rest-frame \( B \)-band, \( X_1 \) describes the time stretching of the light curve, and \( C \) corresponds to the supernova color at maximum brightness. The absolute \( B \)-band magnitude \( M_B \) is assumed to be related to the host stellar mass \( (M_{\text{stellar}}) \) by a simple step function (Betoule et al. 2014):

\[
M_B = \begin{cases} 
M_B^0 & \text{for } M_{\text{stellar}} < 10^{10}M_\odot, \\
M_B^0 + \Delta M & \text{otherwise}.
\end{cases}
\]

Note that \( \alpha, \beta, M_B^0, \) and \( \Delta M \) are nuisance parameters in the distance estimate, which should be fitted simultaneously with the cosmological parameters. Meanwhile, \( m_B^*, X_1, \) and \( C \) are obtained from the observed SN light curve.

For each SN, the theoretical distance modulus \( \mu_{\text{th}} \) can be calculated from the measured redshift \( z \) by the definition:

\[
\mu_{\text{th}} = 5 \log \left( \frac{D_L(z)}{\text{Mpc}} \right) + 25,
\]

where \( D_L(z) \) is the cosmology-dependent luminosity distance. Betoule et al. (2014) fit a }\text{CDM cosmology to the JLA sample by minimizing the \( \chi^2 \) statistic:

\[
\chi^2 = \Delta \hat{\mu}^T \cdot \text{Cov}^{-1} \cdot \Delta \hat{\mu},
\]

where \( \Delta \hat{\mu} = \hat{\mu}_{\text{SN}}(\alpha, \beta, M_B^0, \Delta M; z) - \hat{\mu}_{\text{th}}^{\text{LCDM}}(\Omega_m, H_0; z) \) is the data vector and \( \text{Cov} \) is the full covariance matrix, defined by

\[
\text{Cov} = D_{\text{stat}} + C_{\text{stat}} + C_{\text{sys}}.
\]

Here \( D_{\text{stat}} \) is the diagonal part of the statistical uncertainty, given by

\[
(D_{\text{stat}})_{ii} = \sigma_{m_B}^2 + \alpha^2 \sigma_{X_1}^2 + \beta^2 \sigma_{C}^2 + 2 \alpha \beta \sigma_{M_{\text{stellar}}} \cdot \sigma_{C} - 2 \alpha \sigma_{M_{\text{stellar}}} \cdot \sigma_{X_1} - 2 \beta \sigma_{M_{\text{stellar}}} \cdot \sigma_{C} + \sigma_{\text{sys}}^2 + \frac{5 \sigma_{\text{sys}}^2}{z_i \ln 10} + \sigma_{\text{int}}^2.
\]

where the last three terms stand for the variation of magnitudes arisen from gravitational lensing, the uncertainty in cosmological redshift caused by peculiar velocities, and the intrinsic variation in SN magnitude, respectively. \( \sigma_{m_B}, \sigma_{X_1}, \text{ and } \sigma_{C} \) represent the standard errors of the peak magnitude and light-curve parameters of the \( i \)-th SN. The terms \( C_{m_B}, C_{X_1}, \text{ and } C_{C} \) denote the covariances among \( m_B, X_1, \text{ and } C \) for the \( i \)-th SN.

The statistical and systematic covariance matrices, \( C_{\text{stat}} \) and \( C_{\text{sys}} \), are given by

\[
C_{\text{stat}} + C_{\text{sys}} = V_0 + \alpha^2 V_0 + \beta^2 V_0 + 2 \alpha V_0 V_0 - 3 V_{\text{int}} - 2 \alpha V_{\text{int}} - 2 \beta V_{\text{int}} - 2 \alpha \beta V_{\text{int}},
\]

where \( V_0, V_0, V_0, V_0, \text{ and } V_{\text{int}} \) are matrices available in Betoule et al. (2014). Since the Hubble constant \( H_0 \) is degenerate with \( M_B \) when constructing an SN Hubble diagram, it is not free if \( M_B \) is considered as one of the optimized variables. Betoule et al. (2014) fixed the value of \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \), and they obtained \( (\alpha, \beta, M_B^0, \Delta M) = (0.141 \pm 0.006, 3.101 \pm 0.075, -19.05 \pm 0.02, -0.070 \pm 0.023) \) including both statistical and systematic errors.

In this work, we directly adopt the observational quantities \( (m_B^*, X_1, C) \) from the JLA sample to constrain the curvature. By marginalizing the nuisance parameters \( (\alpha, \beta, M_B^0, \Delta M) \), one can obtain a cosmology-independent constraint on the curvature and justify whether the curvature has a dependence on the nuisance parameters.

#### 2.2. Hubble Parameter Data

The \( H(z) \) measurement can be obtained via two methods. One is calculating the differential ages of passively evolving galaxies (e.g., Jimenez & Loeb 2002; Simon et al. 2005; Stern et al. 2010), usually called a cosmic chronometer (hereafter CC \( H(z) \)). The other is based on the detection of radial BAO features (e.g., Gaztañaga et al. 2009; Blake et al. 2012; Samushia et al. 2013). For convenience, we refer to this kind of \( H(z) \) as BAO \( H(z) \). We compile the latest 41 \( H(z) \) data points in Table 1, including 31 CC \( H(z) \) data and 10 BAO \( H(z) \) data. These are all independent data sets and analyses.

#### 3. New Model-independent Method

Within the framework of the Friedmann–Robertson–Walker metric, the proper distance can be written as (Hogg 1999)

\[
d_p(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')},
\]

where \( E(z) = H(z)/H_0 \). For the base \( \Lambda \text{CDM } \) model, \( E(z) \) has the form of \( E(z) = \sqrt{\Omega_m (1 + z)^3 + \Omega_k + \Omega_{\Lambda} (1 + z)^2} \).

Inspired by the work of Yu & Wang (2016), we employ an improved approach to acquire proper distances that are
The Astrophysical Journal, 838:160 (9pp), 2017 April 1

Wei & Wu

Table 1
The Latest $H(z)$ Measurements from the Differential Age Method (I) and the Radial BAO Method (II)

| $z$  | $H(z)$ (km s$^{-1}$ Mpc$^{-1}$) | Method | References |
|------|--------------------------------|--------|------------|
| 0.09 | 60 ± 12                        | I      | Jimenez et al. (2003) |
| 0.17 | 83 ± 8                         | I      |                     |
| 0.27 | 77 ± 14                        | I      |                     |
| 0.4  | 95 ± 17                        | I      |                     |
| 0.9  | 117 ± 23                       | I      | Simon et al. (2005) |
| 1.3  | 168 ± 17                       | I      |                     |
| 1.43 | 177 ± 18                       | I      |                     |
| 1.53 | 140 ± 14                       | I      |                     |
| 1.75 | 202 ± 40                       | I      |                     |
| 0.48 | 97 ± 62                        | I      | Stern et al. (2010) |
| 0.88 | 90 ± 40                        | I      |                     |
| 0.35 | 82.1 ± 4.9                     | I      | Chuang & Wang (2012) |
| 0.179| 75 ± 4                         | I      |                     |
| 0.199| 75 ± 5                         | I      |                     |
| 0.352| 83 ± 14                        | I      |                     |
| 0.593| 104 ± 13                       | I      | Moreasco et al. (2012) |
| 0.68 | 92 ± 8                         | I      |                     |
| 0.781| 105 ± 12                       | I      |                     |
| 0.875| 125 ± 17                       | I      |                     |
| 1.037| 154 ± 20                       | I      |                     |
| 0.07 | 69 ± 19.6                      | I      |                     |
| 0.12 | 68.6 ± 26.2                    | I      | Zhang et al. (2014) |
| 0.2  | 72.9 ± 29.6                    | I      |                     |
| 0.28 | 88.8 ± 36.6                    | I      |                     |
| 1.363| 160 ± 33.6                     | I      | Moreasco (2015)     |
| 1.965| 186.5 ± 50.4                   | I      |                     |
| 0.3802| 83 ± 13.5                      | I      |                     |
| 0.4004| 77 ± 10.2                      | I      |                     |
| 0.4247| 87.1 ± 11.2                    | I      | Moreasco et al. (2016) |
| 0.4497| 92.8 ± 12.9                    | I      |                     |
| 0.4783| 80.9 ± 9                      | I      |                     |
| 0.24 | 79.69 ± 2.65                   | II     | Gaztañaga et al. (2009) |
| 0.43 | 86.45 ± 3.68                   | II     |                     |
| 0.44 | 82.6 ± 7.8                     | II     | Blake et al. (2012) |
| 0.6  | 87.9 ± 6.1                     | II     |                     |
| 0.73 | 97.3 ± 7                       | II     |                     |
| 0.35 | 84.4 ± 7                       | II     | Xu et al. (2013)    |
| 0.57 | 92.4 ± 4.5                     | II     | Samushia et al. (2013) |
| 2.3  | 224 ± 8                        | II     | Busca et al. (2013) |
| 2.36 | 226 ± 8                        | II     | Font-Ribera et al. (2014) |
| 2.34 | 222 ± 7                        | II     | Delubac et al. (2015) |

The reconstruction function $f(z)$ at different points $z$ and $\bar{z}$ are correlated by a covariance function $k(z, \bar{z})$, which only depends on two hyperparameters $l$ and $\sigma_f$. Both $l$ and $\sigma_f$ would be determined by GP with the observational data. Therefore, the GP method does not specify any form of $f(z)$ and is model-independent. There is a python package of GP developed by Seikel et al. (2012a), which has been widely used in various studies (e.g., Bilicki & Seikel 2012; Seikel et al. 2012b; Shafieloo et al. 2012; Seikel & Clarkson 2013; Busti et al. 2014; Yahya et al. 2014; Yang et al. 2015; Cai et al. 2016; Yu & Wang 2016; Zhang & Xia 2016). We refer the reader to Seikel et al. (2012a) for more details on the GP method and the GP code.

3. We normalize the $H(z)$ data using an independent measurement of the local Hubble parameter $H_0$, thus we get the dimensionless Hubble parameter $E(z) = H(z)/H_0$. Note that the initial condition $E(z=0)=1$ should be taken into account in our calculation. Considering the uncertainty of the Hubble constant, the propagated error of $E(z)$ can be calculated by $\sigma_E^2 = (\sigma_H^2/H_0^2) + (H^2/H_0^4)\sigma_H^2$. To explore the influence of the Hubble constant on the reconstruction and then on the test of the curvature parameter (more on this below), we follow the treatment of Zhang & Xia (2016) and adopt two recent measurements, $H_0 = 69.6 \pm 0.7$ km s$^{-1}$ Mpc$^{-1}$ with 1σ uncertainty (Bennett et al. 2014), and $H_0 = 73.24 \pm 1.74$ km s$^{-1}$ Mpc$^{-1}$ with 2.4σ uncertainty (Riess et al. 2016), respectively. Moreover, we study the potential impact on the results from different $H(z)$ samples (i.e., the only CC $H(z)$ data and the total $H(z)$ data). We show the results in Figures 1 and 2.

Using the GP method, the reconstructions of $E(z)$ for the CC $H(z)$ data with $H_0 = 69.6 \pm 0.7$ km s$^{-1}$ Mpc$^{-1}$ and with $H_0 = 73.24 \pm 1.74$ km s$^{-1}$ Mpc$^{-1}$ are shown in Figures 1(a) and (c), respectively. The solid lines are the means of the reconstructions and the shaded regions are the 1σ and 2σ confidence regions of the reconstructions. Because of the poor quality of data at higher redshifts, the errors become larger. For a comparison, we also fit the observational data points of $E(z)$ using the flat $\Lambda$CDM model (dashed lines). One can see from these plots that the reconstructions of $E(z)$ are consistent with the best-fit flat $\Lambda$CDM model within their 1σ confidence regions, indicating that the GP method can give a reliable reconstructed function from the observational data. With the observations and reconstructions of $E(z)$, we can use Equation (9) to derive the observed $d_P(z)$ together with their 1σ errors and the reconstructed $d_P(z)$ together with the 1σ and 2σ confidence levels at a certain $z$, respectively. As shown in Figures 1(b) and (d), both the observed (red points) and reconstructed (solid lines) $d_P(z)$ are also consistent with those determined from the best-fit flat $\Lambda$CDM model (dashed lines). Not surprisingly, the comparison between the top and bottom panels in Figure 1 shows that the best-fit values of $\Omega_m$ for the flat $\Lambda$CDM model are different, since different $H_0$ priors are adopted.

We follow the same procedure for the total $H(z)$ data, first considering a prior of $H_0 = 69.6 \pm 0.7$ km s$^{-1}$ Mpc$^{-1}$ (the top row of Figure 2), followed by the other one of $H_0 = 73.24 \pm 1.74$ km s$^{-1}$ Mpc$^{-1}$ (the bottom row of Figure 2). The comparison between these two $H(z)$ samples may be summarized as follows: the reconstructions of $E(z)$ and $d_P(z)$ are consistent with
the flat ΛCDM model for both the CC \( H(z) \) and total \( H(z) \) data, suggesting that the GP method can reconstruct the \( E(z) \) and \( d_P(z) \) functions well; the errors of the reconstructions for the total \( H(z) \) data become smaller owing to the added BAO \( H(z) \) data.

By using the reconstructed \( d_P(z) \) function together with its 1σ uncertainty \( \sigma_{d_P} \), the luminosity distance \( D_L^H(z) \) from the \( H(z) \) data can be expressed as

\[
D_L^H(z) = \frac{c}{H_0} \sqrt{\Omega_k} \sin \left( \sqrt{\Omega_k} d_P(z) \frac{H_0}{c} \right) \quad \text{for } \Omega_k > 0
\]

\[
= \frac{c}{H_0} \sqrt{\Omega_k} \sin \left( \sqrt{\Omega_k} d_P(z) \frac{H_0}{c} \right) \quad \text{for } \Omega_k = 0,
\]

\[
= \frac{c}{H_0} \sqrt{\Omega_k} \sin \left( \sqrt{\Omega_k} d_P(z) \frac{H_0}{c} \right) \quad \text{for } \Omega_k < 0
\]

with its corresponding uncertainty

\[
\sigma_{D_L^H} = \begin{cases} 
(1 + z) \cosh \left( \sqrt{\Omega_k} d_P(z) \frac{H_0}{c} \right) \sigma_{d_P} & \text{for } \Omega_k > 0 \\
(1 + z) \sigma_{d_P} & \text{for } \Omega_k = 0, \\
(1 + z) \cos \left( \sqrt{\Omega_k} d_P(z) \frac{H_0}{c} \right) \sigma_{d_P} & \text{for } \Omega_k < 0
\end{cases}
\]

where we emphasize that the spatial curvature \( \Omega_k \) is the only one free parameter. Then, we can further obtain the reconstructed distance modulus \( \mu_H(\Omega_k; z) \) from the \( H(z) \) data by

\[
\mu_H(\Omega_k; z) = 5 \log \left( \frac{D_L^H(\Omega_k; z)}{\text{Mpc}} \right) + 25.
\]
The propagated uncertainty of $\mu_H(\Omega_k; z)$ is given by

$$\sigma_{\mu_H} = \frac{5}{\ln 10} \sigma_{D^{H}}$$

Now, we use a $\chi^2$ minimization to constrain $\Omega_k$.

$$\chi^2(\alpha, \beta, M^J, \Delta_M, \Omega_k) = \Delta \mu^T \cdot \text{Cov}^{-1} \cdot \Delta \mu,$$

where $\Delta \mu = \tilde{\mu}^{\text{SN}}(\alpha, \beta, M^J, \Delta_M; z) - \tilde{\mu}_H(\Omega_k; z)$ is the difference between the distance moduli $\mu^{\text{SN}}$ of SNe Ia derived from Equation (2) and the constructed distance moduli $\mu_H$ from the $H(z)$ data, and $\text{Cov} = \text{D}_{\text{stat}} + \text{C}_{\text{stat}} \pm \text{C}_{\text{sys}}$ is the full covariance matrix. Here $\text{D}_{\text{stat}}$ is the diagonal part of the statistical uncertainty, given by

$$(\text{D}_{\text{stat}})_{ii} = (\text{D}_{\text{stat}}^{\text{SN}})_{ii} + \sigma_{\mu_{ii}}^2,$$

where $\text{D}_{\text{stat}}^{\text{SN}}$ of SNe Ia comes from Equation (7). The statistical and systematic covariance matrices, $\text{C}_{\text{stat}}$ and $\text{C}_{\text{sys}}$, are given by Equation (8).

The likelihood distributions of free parameters can be obtained by $L(\alpha, \beta, M^J, \Delta_M, \Omega_k) \propto \exp(-\chi^2/2)$. We use the Markov Chain Monte Carlo technique to generate sample points distributed in parameter space according to the posterior probability, using the Metropolis–Hastings algorithm with uniform prior distributions. Then we apply a public python package “triangle.py” from Foreman-Mackey et al. (2013) to plot our constraint contours.

4. Constraints on Cosmic Curvature

To investigate the influence of Hubble constant $H_0$ on the reconstruction of $E(z)$ and then on the test of the curvature parameter, we take into account two priors of $H_0$. We also
Figure 3. (a): 1D marginalized distributions and 2D joint distributions with the 1σ and 2σ contours corresponding to the cosmic curvature $\Omega_k$ and the SNe Ia nuisance parameters ($\alpha$, $\beta$, $M_B^{\text{CC}}$, $\Delta_{\text{MO}}$), using the CC $H(z)$ + SNe Ia data with the prior of $H_0 = 69.6 \pm 0.7$ km s$^{-1}$ Mpc$^{-1}$. The vertical solid lines denote the best-fits, and the vertical dashed lines enclose the 1σ confidence region. (b): Same as panel (a), but now with the prior of $H_0 = 73.24 \pm 1.74$ km s$^{-1}$ Mpc$^{-1}$. 

The Astrophysical Journal, 838:160 (9pp), 2017 April 1 Wei & Wu
compare the tests from different \( H(z) \) samples (i.e., the only \( C C \) \( H(z) \) data and the total \( H(z) \) data).

Applying the above \( \chi^2 \)-minimization procedure, we find that the best-fit curvature parameter using the \( C C \) \( H(z) + JLA \) SNe Ia data with the prior of \( H_0 = 69.6 \pm 0.7 \text{ km s}^{-1} \text{ Mpc}^{-1} \) is \( \Omega_k = 0.09 \pm 0.25(1\sigma) \pm 0.49(2\sigma) \). For the case of \( H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1} \), we obtain \( \Omega_k = -0.28 \pm 0.22 (1\sigma) \pm 0.43(2\sigma) \). Our constraint results with these two \( H_0 \) priors are presented in Figures 3(a) and (b), respectively. We give the 1D distributions for each parameter \( (\Omega_k, \alpha, \beta, M_B^i, \Delta M) \), and 1\sigma, 2\sigma contours for the joint distributions of any two parameters. The corresponding best-fit parameters are summarized in Table 2, along with the 1\sigma and 2\sigma standard deviations for each. From Figures 3(a) and (b), one can easily see that the measured \( \Omega_k \) is consistent with zero cosmic curvature within the 1\sigma confidence level for both of the two \( H_0 \) priors, implying that there is no significant deviation from a flat universe at the current observational data [\( H(z) \) data and SNe Ia] level.\(^5\) However, a careful comparison of Figures 3(a) and (b) shows that different \( H_0 \) priors can affect the constraints on \( \Omega_k \) to some degree. The prior of \( H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1} \) gives a value of \( \Omega_k \) a little bit above 1\sigma away from 0, but \( H_0 = 69.6 \pm 0.7 \text{ km s}^{-1} \text{ Mpc}^{-1} \) gives it below 1\sigma. Note that SNe Ia data do not constrain \( H_0 \), so these different pulls on \( H_0 \) are coming from the \( H(z) \) constraints.

We show the constraints for the total \( H(z) + JLA \) SNe Ia data in Figure 4. The best-fit values corresponding to the priors of \( H_0 = 69.6 \pm 0.7 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and \( H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1} \) are \( \Omega_k = -0.02 \pm 0.24(1\sigma) \pm 0.47(2\sigma) \) and \( \Omega_k = -0.35 \pm 0.22(1\sigma) \pm 0.43(2\sigma) \), respectively (see Table 2). Evidence also shows that no significant deviation from flatness is found. The best-fit \( \Omega_k \) is in full agreement with zero spatial curvature at the 1\sigma confidence level, regardless of which prior of \( H_0 \) is adopted. However, the influence of \( H_0 \) in this type of data is still exist. That is, the prior of \( H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1} \) leads to a slightly bigger deviation from the flat universe. Comparison between Figures 3 and 4 (see also Table 2) shows that the best-fit results are more or less the same for both the \( CC \) \( H(z) + SNe Ia \) and total \( H(z) + SNe Ia \) data, for the same prior of \( H_0 \).

5. Summary and Discussion

Clarkson et al. (2007, 2008) have proposed a model-independent method for measuring the cosmic curvature. Using this method, several studies have been done. However, we find that the luminosity distances of SNe Ia used in past works were obtained directly from Hubble diagrams where the SN light-curve-fitting parameters were inferred from global fitting in the context of a cosmological model. In contrary to previous studies, we keep the light-curve-fitting parameters free to investigate whether the curvature parameter has a dependence on them. On the other hand, the estimation of the derivative function of comoving distance \( D(z) \) following the method of Clarkson et al. (2007, 2008) will introduce a large uncertainty (Yu & Wang 2016).

In this work, we propose an improved model-independent method to test cosmic curvature. The main idea of our method is to compare two kinds of distance moduli. One distance modulus \( \mu_B(\Omega_k) \) is constructed from the \( H(z) \) data, which is susceptible to the curvature parameter \( \Omega_k \). Based on the measurements of \( H(z) \), we use the GP method to reconstruct the \( E(z) \) function and use Equation (9) to derive the proper distance function \( d_p(z) \). Using the reconstructed \( d_p(z) \) function, the luminosity distance \( D_L^H(\Omega_k) \) and the corresponding distance modulus \( \mu_B(\Omega_k) \) from the \( H(z) \) data can be further calculated at a certain \( z \). The other distance modulus \( \mu_B(\alpha, \beta, M_B^i, \Delta M) \) is from the SNe Ia data, which is inferred directly from the observed SN light curve (i.e., the original data \( m_B^i, X_i, C_i \), but with some nuisance parameters \( (\alpha, \beta, M_B^i, \Delta M) \).

Our model-independent analysis suggests that the best-fit curvature parameter is constrained to be \( \Omega_k = -0.02 \pm 0.24 \), which is in good agreement with a flat universe. We also considered the impact of Hubble constant \( H_0 \) on the constraints, finding that different \( H_0 \) priors can affect the measurements of \( \Omega_k \) to some degree; the prior of \( H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1} \) leads to a slightly bigger deviation from the zero cosmic curvature than that of \( H_0 = 69.6 \pm 0.7 \text{ km s}^{-1} \text{ Mpc}^{-1} \). In addition, we also compared the constraints from different \( H(z) \) samples: (i) the only \( CC \) \( H(z) \) data; and (ii) the total \( H(z) \) data. We found that the optimized curvature parameters change quantitatively, though the qualitative results and conclusions remain the same, independent of which kind of the sample is used.

In JLA (Betoule et al. 2014), the SN nuisance parameters \( (\alpha, \beta, M_B^i, \Delta M) \) are derived from a fit to the flat \( \Lambda \)CDM model. In other words, Betoule et al. (2014) compared \( \mu_{\Lambda CDM}(\Omega_m) \) to find the best-fit cosmological parameters and nuisance parameters, which were \( \Omega_m = 0.30 \pm 0.03, \ \alpha = 0.14 \pm 0.01, \ \beta = 3.10 \pm 0.08, \ M_B^i = -19.05 \pm 0.02, \) and \( \Delta M = -0.07 \pm 0.02 \). In our analysis, we adopt the constructed \( \mu_B(\Omega_k) \) from the \( H(z) \) data, instead of the cosmology-dependent \( \mu_{\Lambda CDM}(\Omega_m) \), and then derive the best-fit curvature parameter and nuisance parameters by comparing \( \mu_B(\Omega_k) \) with \( \mu_{\Lambda CDM}(\Omega_m) \). We find that our constraints on the nuisance parameters (see Table 2) are very similar to those results of Betoule et al. (2014),\(^6\) not only attesting to the reliability of our calculation, but also confirming

\(^{5}\) A similar estimation of \( \Omega_k \) from the \( H(z) + SNe Ia \) data was given in Li et al. (2016), which we received while working on this paper.

\(^{6}\) Note that the only parameter that is different is \( M_B^i \). Since it is degenerate with \( H_0 \), it has to change if \( H_0 \) changes.
Figure 4. Same as Figure 3, except now using the total $H(z)$ + SNe Ia data.

(a) $H_0 = 69.6 \pm 0.7$ km s$^{-1}$ Mpc$^{-1}$

(b) $H_0 = 73.24 \pm 1.74$ km s$^{-1}$ Mpc$^{-1}$
that the curvature parameter is independent of the nuisance parameters. To check the validity and efficiency of our new method, we also run the more conventional (non-Gaussian processes) method and just leave the curvature parameter free to see what value we get and if it is different. Following the conventional method, we allow \( \Omega_k \) to be free along with the matter energy density \( \Omega_m \) in the \( \Lambda \)CDM model, and compare \( H_{\text{obs}}(\alpha, \beta, M_0, \Delta_M) \) with \( H_{\text{ADCM}}(\Omega_m, \Omega_k) \) (or compare \( H_{\text{obs}}(z) \) with \( H_{\text{ADCM}}(z; \Omega_m, \Omega_k) \)). In Figure 5, we display the confidence regions of \( (\Omega_k, \Omega_m) \) in the \( \Lambda \)CDM model determined with the conventional method for \( CC \) \( H(z) \) (dark cyan dashed–dotted lines) and SNe Ia (blue dashed lines), respectively. The contours show that at the 1\( \sigma \) confidence level, the best fits are \( (\Omega_k = 0.30 \pm 0.39, \Omega_m = 0.20 \pm 0.17) \) for SNe Ia and \( (\Omega_k = 0.03^{+0.64}_{-0.55}, \Omega_m = 0.33^{+0.19}_{-0.21}) \) for \( CC \) \( H(z) \). The corresponding contours of \( (\Omega_k, \alpha) \) from our GP method for the \( CC \) \( H(z) \) + SNe Ia data (red solid lines) are also shown in Figure 5 for comparison. One can see that the determined \( \Omega_k \) from the conventional method are also consistent with a flat universe within error limits. But the errors on these measured \( \Omega_k \) are at the levels of \( \sigma_{\Omega_k} \approx 0.39 \) and \( \sigma_{\Omega_k} \approx 0.64 \), which are not as good as those of our GP method (\( \sigma_{\Omega_k} \approx 0.22 \)). What’s more, our constraint on \( \Omega_k \) with the GP method is more robust and more widely applicable, as it does not depend on the cosmological model. If in the future the quality of observational data are much improved, the prospects for constraining the cosmic curvature with this method will be very promising.

We are very grateful to the anonymous referee for providing a thoughtful review and making several important suggestions that have improved the manuscript significantly. We also acknowledge Gabriel R. Bengochea for useful communications. This work is partially supported by the National Basic Research Program (“973” Program) of China (Grant No. 2014CB845800), the National Natural Science Foundation of China (Grant Nos. 11673068 and 11603076), the Youth Innovation Promotion Association (2011231 and 2017366), the Key Research Program of Frontier Sciences (QYZDB-SSW-SYS005), the Strategic Priority Research Program “Multi-waveband gravitational wave universe” (Grant No. XDB23040000) of the Chinese Academy of Sciences, the Natural Science Foundation of Jiangsu Province (Grant No. BK20161096), and the Guangxi Key Laboratory for Relativistic Astrophysics.

References

Bennett, C. L., Larson, D., Weiland, J. L., & Hinshaw, G. 2014, \textit{ApJ}, 794, 135
Bouleux, M., Kessler, R., Guy, J., et al. 2014, \textit{A&A}, 568, A22
Bilicki, M., & Seikel, M. 2012, \textit{MNRAS}, 425, 1664
Blake, C., Brough, S., Colless, M., et al. 2012, \textit{MNRAS}, 425, 405
Busca, N. G., Delubac, T., Rich, J., et al. 2013, \textit{A&A}, 552, A96
Busti, V. C., Clarkson, C., & Seikel, M. 2014, \textit{MNRAS}, 441, L11
Cai, R.-G., Guo, Z.-K., & Yang, T. 2016, \textit{PhRvD}, 93, 043517
Chuang, C.-H., & Wang, Y. 2012, \textit{MNRAS}, 426, 226
Clarkson, C., Bassett, B., & Lu, H. T. H.-C. 2008, \textit{PhRvL}, 101, 011301
Clarkson, C., Cortés, M., & Bassett, B. 2007, \textit{JCAP}, 8, 011
Delubac, T., Bautista, J. E., Busca, N. G., et al. 2015, \textit{A&A}, 574, A59
Font-Ribera, A., Kirkby, D., Busca, N., et al. 2014, \textit{JCAP}, 5, 027
Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, \textit{PASP}, 125, 306
Gaztañaga, E., Cabré, A., & Hui, L. 2009, \textit{MNRAS}, 399, 1663
Hogg, D. W. 1999, arXiv:astro-ph/9905116
Jimenez, R., & Loeb, A. 2002, \textit{ApJ}, 573, 37
Jimenez, R., Verde, L., Treu, T., & Stern, D. 2003, \textit{ApJ}, 593, 622
L’Huillier, B., & Shafieloo, A. 2017, \textit{JCAP}, 1, 015
Li, Y.-L., Li, S.-Y., Zhang, T.-J., & Li, T.-P. 2014, \textit{ApJL}, 789, L15
Li, Z., Wang, G.-J., Liao, K., & Zhu, Z.-H. 2016, \textit{ApJ}, 833, 240
Marriner, J., Bernstein, J. P., Kessler, R., et al. 2011, \textit{ApJ}, 740, 72
Moresco, M. 2015, \textit{MNRAS}, 450, L16
Moresco, M., Pozzetti, L., Cimatti, A., et al. 2016, \textit{JCAP}, 5, 014
Moresco, M., Verde, L., Pozzetti, L., Jimenez, R., & Cimatti, A. 2012, \textit{JCAP}, 7, 053
Mortsell, E., & Jonsson, J. 2011, arXiv:1102.4485
Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2016, \textit{A&A}, 594, A13
Riess, A. G., Macri, L. M., Hoffmann, S. L., et al. 2016, \textit{ApJ}, 826, 56
Samushia, L., Reid, B. A., White, M., et al. 2013, \textit{MNRAS}, 429, 1514
Sapone, D., Majerotto, E., & Nesseris, S. 2014, \textit{PhRvD}, 90, 023012
Seikel, M., & Clarkson, C. 2013, arXiv:1311.6678
Seikel, M., Clarkson, C., & Smith, M. 2012a, \textit{JCAP}, 6, 036
Seikel, M., Yahya, S., Maartens, R., & Clarkson, C. 2012b, \textit{PhRvD}, 86, 083001
Shafieloo, A., & Clarkson, C. 2010, \textit{PhRvD}, 81, 083537
Shafieloo, A., Kim, A. G., & Linder, E. V. 2012, \textit{PhRvD}, 85, 123530
Simon, J., Verde, L., & Jimenez, R. 2005, \textit{PhRvD}, 71, 123001
Stern, D., Jimenez, R., Verde, L., Kamionkowski, M., & Stanford, S. A. 2010, \textit{JCAP}, 2, 008
Virey, J.-M., Talon-Esmieu, D., Ealet, A., Taxil, P., & Tilquin, A. 2008, \textit{JCAP}, 12, 008
Xu, X., Cuesta, A. J., Padmanabhan, N., Eisenstein, D. J., & McBride, C. K. 2013, \textit{MNRAS}, 431, 2834
Yahya, S., Seikel, M., Clarkson, C., Maartens, R., & Smith, M. 2014, \textit{PhRvD}, 89, 023503
Yang, T., Guo, Z.-K., & Cai, R.-G. 2015, \textit{PhRvD}, 91, 123533
Yu, H., & Wang, F. Y. 2016, \textit{ApJ}, 828, 85
Zhang, C., Zhang, H., Yuan, S., et al. 2014, \textit{RAA}, 14, 1221
Zhang, M.-J., & Xia, J.-Q. 2016, \textit{JCAP}, 12, 005

Figure 5. 1\( \sigma \) and 2\( \sigma \) constraint contours of \( (\Omega_k, \Omega_m) \) in the \( \Lambda \)CDM model determined with the conventional method for \( CC \) \( H(z) \) (dark cyan dashed–dotted lines) and SNe Ia (blue dashed lines), respectively. The red solid contours correspond to the confidence levels of \( (\Omega_k, \alpha) \) for the \( CC \) \( H(z) + \text{SNe Ia} \) data, obtained from the cosmological model-independent method (i.e., the GP method).