The upper kHz QPO: a gravitationally lensed vertical oscillation

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ABSTRACT

We show that a luminous torus in the Schwarzschild metric oscillating along its own axis gives rise to a periodically varying flux of radiation, even though the source of radiation is steady and perfectly axisymmetric. This implies that the simplest oscillation mode in an accretion flow, axisymmetric up-and-down motion at the meridional epicyclic frequency, may be directly observable when it occurs in the inner parts of accretion flow around neutron stars and black holes. The high-frequency modulations of the X-ray flux observed in low-mass X-ray binaries at two frequencies (twin kHz QPOs) could then be a signature of strong gravity both because radial and meridional oscillations have different frequencies in non-Newtonian gravity, and because strong gravitational deflection of light rays causes the flux of radiation to be modulated at the higher frequency.

Subject headings: X-rays: general

1. Highest frequency in accreting black holes and neutron stars

The highest frequencies modulating the X-ray flux observed from accreting neutron stars and black holes continue to attract attention because their values are as high as those of orbital frequencies close to the neutron star surface or to the circular photon orbit around a black hole. The origin of the modulations, known as quasi-periodic oscillations (QPOs) because they are not quite coherent, still remains a major puzzle (see van der Klis 2000 for a review).

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Typically these QPOs come in pairs, with the higher frequency (as high as 1.2 kHz for neutron stars and 0.5 kHz for black holes) larger by about 50% than the lower frequency of the pair. It has been recognized for some time that the HF QPOs may correspond to accretion disk oscillations not present in Newtonian 1/r gravity, e.g., modes trapped close to the maximum of the epicyclic frequency\(^1\) (Wagoner 1999, Kato 2001). Non-axisymmetric modes have been preferred, as it was thought that a considerable degree of non-axisymmetry is a necessary condition for modulating the X-rays. It has also been suggested that the HF QPO phenomenon is caused by a non-linear resonance between two modes of oscillation of the accretion disk or torus (Kluźniak & Abramowicz 2001, Abramowicz & Kluźniak 2001). In a resonance, there should be a rational ratio of frequencies, and indeed the observed pairs of high-frequency QPOs in black holes are in a 3:2 ratio (McClintock and Remillard 2003). More recently, it has been specifically suggested that the higher frequency corresponds to vertical oscillations of the accretion disk/torus occurring at the meridional epicyclic frequency (Kluźniak & Abramowicz 2002, 2003; Kluźniak et al. 2004; Lee, Abramowicz & Kluźniak 2004). However, the mechanism of X-ray modulation remained a puzzle.

Here, we show that gravitational lensing of the photon trajectories in Schwarzschild metric suffices to appreciably modulate the flux observed at infinity even if the source is symmetric about the axis of a black hole, provided that it moves parallel to the symmetry axis. Specifically, we show that a toroidal source oscillating about the equatorial plane of the black hole, but otherwise steady, gives rise to a periodically modulated flux. If, in addition, the source is strongly variable at another frequency, the flux will strongly vary at two frequencies, with the power ratio depending on the inclination angle of the observer.

2. Calculation of trajectories and observer flux

In order to compute the amount of radiation coming from the source we have developed a new three-dimensional ray-tracing code. Following the method used by Rauch & Blandford (1994) we integrate geodesic and geodesic deviation equations in the Schwarzschild space-time. Photon trajectories are integrated backward in time from the observer positioned at infinity at some inclination angle \(i\) with respect to the \(z\) axis. At certain points along the trajectory the current position, momentum, time delay and magnification are recorded. This information is then used to reconstruct each photon’s path and calculate the total amount

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\(^1\)**The radial epicyclic frequency may also have a maximum for rapidly rotating Newtonian stars, and it is even possible that an innermost (marginally) stable circular orbit may then exist outside the stellar surface (Amsterdamski et al. 2002, Zdunik & Gourgoulhon 2001).**
of incoming radiation.

The intensity observed at infinity is an integration of the emissivity $f$ over the path length along geodesics and it can be written down as

$$I_{\text{obs}}(t) = \int f(r, \theta, \phi, t - \Delta t) \sqrt{-g_{tt}} k^t g^4 d\lambda.$$  \hspace{1cm} (1)

The integration is along the light ray parametrized by an affine parameter $\lambda$. Here, $k^t$ is the time component of photon’s 4-momentum, $g$ is the red-shift factor and $\Delta t$ is the photon time delay.

3. A luminous, vertically oscillating torus

We consider an isolated, luminous, optically thin and geometrically slender torus around a non-rotating black hole of mass $M$. In this problem, all radii scale with $M$ (we use $G/c^2 = 1.5 \text{ km}/M_\odot$), and all frequencies scale as $1/M$. For a convenient comparison with the observed frequencies we chose $M = 1.4M_\odot$.

The torus is assumed to be circular in cross-section and to oscillate harmonically parallel to its axis. In Sections 4 and 5, we allow the same torus to execute radial oscillations as well. The circle of its maximum pressure is at $\tilde{r}_0 = 10.8 M(G/c^2)$ and $\tilde{z}(t) = \delta \tilde{z}_0 \sin(\omega_\theta t)$, in cylindrical co-ordinates related to the Schwarzschild co-ordinates through $\tilde{r} = r \sin \theta$, $\tilde{z} = r \cos \theta$. Here, $\omega_\theta = \Omega_K = \sqrt{GM/r^3}$ is the vertical epicyclic frequency, in Schwarzschild geometry equal to the Keplerian orbital frequency. The amplitude of vertical motion is $\delta \tilde{z}_0 = 0.1 M(G/c^2)$. The intrinsic emissivity per unit co-moving volume of the torus is held constant in time, as is its cross-sectional radius $R_0 = 1.5 M(G/c^2)$.

In this Section, the only time variation of the torus is in its vertical position. We find that in spite of this, the flux observed at infinity clearly varies at the oscillation frequency (Fig. 1). This is caused by relativistic effects at the source (lensing and beaming), and no other cause need be invoked to explain in principle the highest-frequency modulation of X-rays in luminous black-hole binary sources.

4. Two oscillations of a torus

Now consider radial oscillations of the torus under discussion. Keeping the cross-sectional radius $R_0$ fixed we allow the central circle of the torus to vary as

$$\tilde{r}(t) = \tilde{r}_0 + \delta \tilde{r}_0 \sin(\omega_r t),$$  \hspace{1cm} (2)
Fig. 1.— The power spectrum of the radiation flux of an axi-symmetric torus oscillating vertically in Schwarzschild geometry. The observer is at infinity, the inclination angle is $i = 45^\circ$, the power scale is arbitrary. To the level of parts per thousand, all the harmonic content is in the frequency of vertical oscillation. If, in addition, the torus is oscillating radially at a different frequency, two strong Fourier components are seen (compare Fig. 3).

Simultaneously with

$$\ddot{z}(t) = \delta \dot{z}_0 \sin(\omega_\theta t).$$

(3)

This results in a periodic change of volume of the torus. Because the optically thin torus is assumed to be filled with a polytropic gas radiating by bremsstrahlung cooling, there is a corresponding change of luminosity, with a clear periodicity at $2\pi/\omega_r$. With our choice of $\tilde{r}_0$, we have $\omega_r = \frac{2}{3} \omega_\theta$.

The luminosity variations will depend on the properties of the torus. We take the emissivity in the local frame to be $f \propto \rho^2 T^{3/2}$, with $T = K \rho^{\gamma-1} \mu m_u/k_B$, where $\gamma = \frac{5}{3}$, $\mu = \frac{7}{4}$, $m_u$ and $k_B$ are polytropic index, molecular weight, atomic mass unit and the Boltzmann constant, respectively. To approximate the properties of an oscillating torus, we actually took the equipotential structure obtained by Taylor expanding in the $\tilde{z}$ direction equilibrium solutions of the relativistic Euler equation (Abramowicz et al. 1978) of a torus with uniform
angular momentum $\ell(\tilde{r}) = \ell_K(\tilde{r}_0) = \sqrt{M\tilde{r}_0^3/(\tilde{r}_0 - 2M)}$, so that $\rho = \left[\frac{\gamma - 1}{\gamma} K^{\gamma - 1} (e^{\Delta W} - 1)\right]^{1/\gamma - 1}$, and $\Delta W = (R_0^2 - R^2)/[2\tilde{r}_0^2(\tilde{r}_0 - 3M)]$. The net effect is best displayed as the power spectrum at infinity of a flux of photons propagating in Minkowski space, i.e., one obtained neglecting the relativistic effects responsible for modulating the flux at the vertical epicyclic frequency. The resulting power spectrum of a torus oscillating with amplitudes $\delta\tilde{z}_0 = \delta\tilde{r}_0 = 0.1M(G/c^2)$ about a Newtonian point mass $M$ is shown in Fig. 2.

Fig. 2.— The power spectrum of an axi-symmetric torus oscillating radially and vertically about a Newtonian point mass (the power scale is arbitrary). The calculation is performed exactly as for the other figures, except that the light trajectories are computed in Minkowski space, and Doppler beaming has been switched off. The radial oscillations result in a harmonic change in volume, and hence also in the luminosity, of the torus. Note that all the power is at the frequency of radial oscillation. Without lensing (or Doppler beaming) there is no modulation at the frequency of vertical motion (compare Figs. 1, 3, 4).

5. Modulation of the light curve

We have computed the light trajectories for several inclination angles in Schwarzschild geometry for the oscillating torus described in Section 4. In all cases, two strong periodic components are clearly seen in the light curves and in the power spectra, at the two oscillation
frequencies $\omega_r$ and $\omega_\theta$. The relative power in the two components depends on the inclination angle and the amplitude of oscillation (Figs. 3, 4).

Fig. 3.— Results of numerical simulations of the oscillating torus in Schwarzschild geometry. The equilibrium distance of the torus $\tilde{r}_0 = 10.8M$, its cross-section radius is $R_0 = 1.5M$, the oscillation amplitudes are $\delta \tilde{z}_0 = \delta \tilde{r}_0 = 0.1M$, and frequencies $\omega_\theta = \Omega_K$, $\omega_r = \frac{2}{3}\Omega_K$. (Top):— snapshots of an instant image, as viewed by a distant observer, (middle):—the computed light curve, and (bottom):—the corresponding power spectrum, for three different viewing angles, $i = 45^\circ$ (left), 60$^\circ$ (middle) and 80$^\circ$ (right).

6. Discussion

We have shown that gravitational lensing at the source will modulate the flux received at infinity from an axially symmetric emitter oscillating about the equatorial plane of a black hole. We also find, that in a torus executing simultaneous oscillations in the radial and vertical directions at frequencies $\omega_r$, and $\omega_\theta = (3/2)\omega_r$, as expected in the parametric resonance model (Kluźniak and Abramowicz 2002), both frequencies will show up in the power spectrum, with no other (e.g., harmonic) strong components. The lower of the frequencies
Fig. 4.— Same as Fig. 3, but for $R_0 = 2M$, $\delta\tilde{z}_0 = \delta\tilde{r}_0 = 1M$ and inclinations $i = 45^\circ$, $65^\circ$, and $85^\circ$.

may reflect changes in the emissivity of the torus, but the presence of the upper frequency is explained by effects of relativity alone. Strong-field gravity may thus have two signatures in the observed fast variability of black hole emission. First, it is responsible for the presence of two frequencies $\omega_\theta \neq \omega_r$, where in Newtonian gravity there is only one ($\omega_\theta = \omega_r = \Omega_K$). Second, it is responsible for modulation of the light curve (Figs. 3,4), where in Newtonian gravity there was none at the frequency of the vertical oscillations (Fig. 2).

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