Measurement of mirror birefringence with laser heterodyne polarimetry

Harold Hollis,1 a) Gabriel Alberts,1 b) D. B. Tanner,1 c) and Guido Mueller1 d)
Department of Physics, University of Florida, PO Box 118440, Gainesville, Florida 32611, USA
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A laser heterodyne polarimeter (LHP) designed for the measurement of the birefringence of dielectric super-mirrors is described and initial results are reported. The LHP does not require an optical resonator and so promises unprecedented accuracy in the measurement of the birefringence of individual mirrors. The working principle of the LHP can be applied to the measurement of vacuum birefringence and potentially ALPS (Any Light Particle Search).

I. INTRODUCTION

Precision measurement of dielectric mirror birefringence (BF) is important1 for the many applications of high-finesse optical resonators; the effects of the BF of the mirrors forming the resonator is amplified by the resonator finesse which poses problems for high-precision applications of high-finesse resonators2. Due to this amplification, the BF of the optical cavity as a whole is often investigated rather than an individual mirror’s BF1–4. For example, Fleisher et al., report a precision measurement of mirror BF using a high-finesse optical resonator and cavity ring-down spectroscopy. The net (due to both mirrors) BF was measured to be $40.1 \mu\text{rad}$ with a precision of $0.1 \mu\text{rad}$. This sensitivity and precision comes with the price of the extra complexity of the alignment and locking of the optical resonator5–7.

In the following, we describe a laser heterodyne polarimeter we developed for the precision measurement of dielectric mirror BF, without the complications of an optical resonator, by probing an individual mirror with two orthogonally polarized laser fields for which the polarization axes are rotated with constant angular speed. The reflected fields then contain an oscillating phase relationship with amplitude determined by the mirror BF. The noise floor of our current LHP implementation is approximately $2 \mu\text{rad}/\sqrt{\text{Hz}}$, allowing us to reach sub $0.1 \mu\text{rad}$ precision for averaging times greater than 400 seconds.

II. OPERATIONAL OVERVIEW

A schematic of the experimental apparatus is shown in Fig.1. The LHP uses two single frequency lasers8. The laser frequencies are phase locked with a phase-locked loop (PLL) to a constant angular frequency offset $\Omega$ using standard methods9; the offset frequency is chosen to be within the bandwidth of the photodetectors. A Faraday isolator at the output of each laser prevents backscattered light from entering the laser while a HWP behind the Faraday isolator sets the initial field polarization. The HWPs are adjusted so that one laser’s field is p-polarized (blue line) while the other is s-polarized (red line). The two orthogonally polarized laser fields are then superimposed (purple line) with a polarizing beam splitter (PBS) and split into two paths.

One path leads to the phase-locked loop photodetector (PLL PD) which produces a beat signal (at the offset frequency) for use by the PLL. Since the two laser fields are orthogonally

a)Electronic mail: halhollis@ufl.edu.
b)Electronic mail: galberts@ufl.edu.
c)http://www.phys.ufl.edu/faculty/tanner.shtml
d)http://www.phys.ufl.edu/faculty/mueller.shtml
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FIG. 1. Schematic diagram of laser heterodyne polarimeter. Each laser beam passes through a Faraday isolator (which prevents backscattered light from entering the laser) and a half-wave plate (HWP) to set the polarization. Each beam is then reflected from a glass plate (reflectance of a few percent) to reduce the optical power within the polarimeter to a value that does not saturate the phase-lock loop (PLL), reference, and signal photodetectors (PLL PD, RPD, and SPD respectively). The initial polarizing beam splitter (PBS) combines the two orthogonally polarized laser beams and a power beam splitter (BS) splits the combined beams into a reference beam and probe beam.

Polarized, a beat signal will not be produced by the PLL PD unless the polarizations are first rotated into alignment. This is accomplished by a HWP, set to 22.5°, which rotates both fields by 45°, and a PBS which selects the s-polarized component from each laser field to be sent to the PLL PD.

The other path leads to the birefringence detector which begins with the power beam splitter (BS). The BS splits the beam into a reference beam and a probe beam. The reference path leads to the reference photodetector (RPD) which produces a reference beat signal. As with the PLL path, a HWP and PBS are used to rotate the polarizations into alignment.

The probe beam passes through a HWP rotating with angular speed $\omega_p$ and is then reflected (cyan line) from the birefringent mirror, which can be rotated with angular speed $\omega_m$, to pass back through the rotating HWP and into the BS. The reflected probe beam from the BS, after passing through a HWP and PBS to bring the polarizations into alignment, produces a phase modulated beat signal at the signal photodetector (SPD).

Both the reference beat signal and probe beat signal are used as inputs to a 2-channel phasemeter (PM). The PM measures the relative phase of the reference and probe inputs producing an output proportional to the phase modulation of the probe beat signal.

As is shown in the appendix, mirror BF will result in phase modulation of the probe beat signal.
signal at angular frequency $4\omega_\pi - 2\omega_m$ while any departure from ideal half-wavelength retardation in the rotating HWP will produce a component at $2\omega_\pi$.

III. EXPERIMENTAL SETUP AND INSTRUMENTATION

For our initial LHP implementation, we chose 1064 nm lasers. An analog PLL locks the frequency offset to 5 MHz. A Thorlabs DDR05 motorized rotation stage is used to rotate a HWP at a rate of 1800° per second (5 Hz). For this rotation rate, we expect to see the 1st order HWP imperfection signal at 10 Hz and the mirror BF signal at 20 Hz (when the mirror is not rotating). A motorized mirror mount can rotate the BF mirror at rates between 3 Hz and 4 Hz.

For our measurements, we have access to two different phasemeter instruments: (1) a Liquid Instruments Moku:Lab running the phasemeter instrument and (2) an extended range phasemeter developed in-house for another experiment. In either case, the phasemeter samples the phase of the two input signals and the resulting phase estimate time series is recorded to disk for post processing.

To extract the phase modulation signals due to the mirror BF and HWP imperfection, and to reduce common mode phase noise, the difference of the the recorded time series for each channel is used for the data analysis. Data are typically recorded for 5 minutes though we have taken data for as long 12 hours. The amplitude of the HWP imperfection and mirror BF signals is then determined by demodulating the time series at the appropriate frequency.

![Image](image.png)

FIG. 2. Spectrum of difference of phasemeter channels with HWP rotating at 5 Hz. The HWP imperfection signal at 10 Hz and the mirror BF signal at 20 Hz are apparent. There are, in addition, several spurious signal at multiples of 5 Hz of unknown origin which are likely due to mechanical resonance, misalignment, or electrical pickup.

In Fig. the spectrum of the difference of the phasemeter channels is plotted for a 5 minute data run with an off-the-shelf broadband dielectric mirror as the BF source (the mirror was not rotating for this data run). As expected, there are components at 10 Hz, due to the HWP, and at 20 Hz due to the BF mirror. (Also evident are the fundamental at 5 Hz as well as other higher order harmonics; the source of these components is currently under investigation.) The amplitude of the 20 Hz component is $1.202 \text{ mrad}$ which is the measured BF for this mirror.

The large 10 Hz component due to the imperfect HWP allows precise pinpointing of the
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frequency to demodulate the phasemeter time series and extract the amplitude of the BF signal component. Our approach is to demodulate at precisely twice the imperfect HWP signal frequency. This will be important when mirrors with much lower BF are measured such that the mirror BF signal is near the noise floor.

![Graph of spectrum difference between phasemeter channels with HWP only rotating at 5 Hz (no peak) and with both HWP and mirror rotating at 5 Hz and 3.5 Hz respectively. The peak at 27 Hz is the mirror BF signal with an amplitude of 1.256 mrad.]

To gain some confidence that the 20 Hz signal is entirely due to mirror BF, we exploited the fact that the phase of the mirror BF signal is $4\theta - 2\theta_m$. If the mirror is rotated with constant angular speed $\omega_m$, the frequency of the mirror BF signal becomes

$$\omega_{BF} = 4\omega_\pi \pm 2\omega_m \quad (1)$$

while the frequency of the HWP imperfection signal remains $2\omega_\pi$. We mounted the BF mirror in a rotation mount which was then driven by a motor at about 3.5 Hz in the opposite sense of the HWP rotation giving a mirror BF signal frequency of about 27 Hz. In Fig. 3 the spectrum in the vicinity of 27 Hz is shown with and without the mirror rotating. The amplitude of the mirror BF signal at about 27 Hz is 1.256 mrad which is about 4.5% higher than the amplitude measured with just the HWP rotation.

IV. CONCLUSION AND OUTLOOK

We have measured the birefringence of an off-the-shelf broadband dielectric mirror using a laser heterodyne polarimeter. While the measurement results confirm the ideal theoretical calculations for the mirror BF signal at $4\omega_\pi - 2\omega_m$ and the imperfect HWP signal at $2\omega_\pi$, we continue to investigate the source of the signals at $\omega_\pi$ and other harmonics not predicted by the ideal model.

At present, the maximum rotation rate of the HWP is limited by the Thorlabs DDR05 to 5 Hz. We can improve our LHP sensitivity by increasing the rotation rate in order to move the BF signal into a higher frequency region of the spectrum where the noise floor is lower.
V. ACKNOWLEDGEMENTS

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Appendix - Theoretical Calculations

As will now be shown, mirror BF will result in phase modulation of the probe beat signal at angular frequency $4\omega_m - 2\omega_m$ while any departure from ideal half-wavelength retardation in the rotating HWP will produce a component at $2\omega_m$. In the following, we assume ideal polarizing and power beam splitters. The Jones vector representation of the combined, orthogonally polarized, phase-locked laser fields is given by:

$$\hat{E} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} A_p e^{i\omega t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} A_s e^{i(\omega + \Omega)t}$$

(2)

where the amplitudes $A_p$ and $A_s$ are taken to be real. The reflected field from the (ideal) BS is given by

$$\hat{E}_R = \frac{1}{\sqrt{2}} \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} A_p + \begin{pmatrix} 0 \\ 1 \end{pmatrix} A_s e^{i\Omega t} \right) e^{i\omega t}$$

(3)

The field incident on the RPD, after passing the HWP-PBS combination, is then

$$\hat{E}_{RPD} = \frac{1}{\sqrt{2}} \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \hat{E}_R$$

$$= \frac{1}{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} (A_p - A_s e^{i\Omega t}) e^{i\omega t}$$

(4)

with intensity of

$$I_{rpd} = |\hat{E}_{RPD}|^2 = \frac{1}{4} \left( A_p^2 + A_s^2 \right) - \frac{1}{2} A_p A_s \cos \Omega t$$

(5)

Following the Siegman convention\textsuperscript{12} (p. 406), the transmitted field from the BS is given by

$$\hat{E}_T = \frac{i}{\sqrt{2}} \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} A_p + \begin{pmatrix} 0 \\ 1 \end{pmatrix} A_s e^{i\Omega t} \right) e^{i\omega t}$$

(6)

This field propagates through an ideal HWP rotated at angle $\theta_\pi$, reflects from the mirror with reflection phases $\phi_x, \phi_y$ rotated at angle $\theta_M$, and propagates back through the HWP. The Jones matrix representation for this combination is

$$\begin{pmatrix} e^{i\phi_x} \cos^2 \gamma + e^{i\phi_y} \sin^2 \gamma & \frac{1}{2} \left( e^{i\phi_x} - e^{i\phi_y} \right) \sin 2\gamma \\ \frac{1}{2} \left( e^{i\phi_x} - e^{i\phi_y} \right) \sin 2\gamma \sin 2\gamma & e^{i\phi_y} \cos^2 \gamma + e^{i\phi_x} \sin^2 \gamma \end{pmatrix}$$

(7)

where $\gamma = 2\theta_\pi - \theta_M$. After reflection at the BS and propagating through the HWP-PBS combination, the field incident on the SPD is then

$$\hat{E}_{SPD} = \frac{i}{4\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \left\{ \begin{pmatrix} e^{i\phi_x} - e^{i\phi_y} \\ e^{i\phi_x} \end{pmatrix} \left( A_p + A_s e^{i\Omega t} \right) \cos 2\gamma \\
- \left( e^{i\phi_x} - e^{i\phi_y} \right) \left( A_p - A_s e^{i\Omega t} \right) \sin 2\gamma \\
+ \left( e^{i\phi_x} + e^{i\phi_y} \right) \left( A_p - A_s e^{i\Omega t} \right) \right\} e^{i\omega t}$$

(8)
For $\delta \equiv \phi_x - \phi_y$ small enough, the intensity is approximately

$$I_{SPD} \approx \frac{1}{8} (A_p^2 + A_s^2) - \frac{1}{4} A_p A_s (\delta \cos 2\gamma \sin \Omega t + \cos \Omega t)$$

\[
\approx \frac{1}{8} (A_p^2 + A_s^2) - \frac{1}{4} A_p A_s \cos (\Omega t - \phi) \tag{9}
\]

where $\phi \equiv \delta \cos 2\gamma$. By rotating the HWP in front of the mirror with constant angular speed $\omega$ such that $\theta_r = \omega t$, the phase of the SPD beat signal is modulated at an angular frequency of $4\omega$ with an amplitude of $\delta$

$$\phi = \delta \cos 2\gamma = \delta \cos (4\omega t - 2\theta_m) \tag{10}$$

We model an imperfect rotating HWP by setting the retardation to $\pi + \epsilon$. Working to first order in $\epsilon$ and $\delta$, an additional term of order $\epsilon$ is added to the small signal approximation above:

$$\epsilon \frac{A_p A_s}{2} \cos 2\theta_r \sin \Omega t \tag{11}$$

and then the phase becomes

$$\phi = \delta \cos 2\gamma - 2\epsilon \cos 2\theta_r \tag{12}$$

As before, for $\theta_r = \omega t$, the time dependent phase term becomes

$$\phi = \delta \cos 2\gamma = \delta \cos (4\omega t - 2\theta_m) - 2\epsilon \cos 2\omega t \tag{13}$$

The rotating imperfect HWP modulates the phase at an angular frequency of $2\omega$ with an amplitude of $2\epsilon$. It’s worth going to second order in $\epsilon$ and find the following additional term to the small signal approximation

$$- \frac{\epsilon^2}{4} A_p A_s (1 + \cos 4\theta_r) \cos \Omega t \tag{14}$$

so that the phase becomes

$$\phi \approx \delta \cos 2\gamma - 2\epsilon \cos 2\theta_r - \frac{\epsilon^3}{3} \cos 6\theta_r \tag{15}$$

which shows that, ideally, there is no imperfect HWP signal at the frequency of the mirror BF signal.

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1. A. J. Fleisher, D. A. Long, Q. Liu, and J. T. Hodges, “Precision interferometric measurements of mirror birefringence in high-finesse optical resonators,” Physical Review A - Atomic, Molecular, and Optical Physics 93, 1–7 (2016).
2. J. L. Hall, J. Ye, and L.-s. Ma, “Measurement of mirror birefringence at the sub-ppm level : Proposed application to a test of QED,” 62, 1–8 (2000).
3. M. Uphoff, M. Brekenfeld, G. Rempe, and S. Ritter, “Frequency splitting of polarization eigenmodes in microscopic Fabry-Perot cavities,” New Journal of Physics 17, 013053 (2015).
4. J. B. Camp, W. Kells, M. M. Fejer, and E. Gustafson, “Measurement of birefringence of low-loss, high-reflectance coating of m-axis sapphire,” Applied Optics 40, 3753 (2001).
5. R. W. P. Drever, J. L. Hall, F. V. Kowalski, J. Hough, G. M. Ford, A. J. Munley, and H. Ward, “Laser phase and frequency stabilization using an optical resonator,” Applied Physics B 31, 97–105 (1983).
6. D. a. Shaddock, M. B. Gray, and D. E. McClelland, “Frequency locking a laser to an optical cavity by use of spatial mode interference,” Optics letters 24, 1499–1501 (1999) arXiv:0809.0545.
7. M. D. Harvey and A. G. White, “Frequency locking by analysis of orthogonal modes,” Optics Communications 221, 163–171 (2003).
8. M. J. Kane and R. L. Byer, “Monolithic, unidirectional single-mode nd:yag ring laser,” Opt. Lett. 10, 65-67 (1985).
9. M. Prevedelli, T. Freegarde, and T. Haensch, “Phase locking of grating tuned diode lasers,” apb, 61, S241–S248 (1995).
10. Moku:Lab, “Phasemeter,” http://www.liquidinstruments.com/phasemeter, accessed: 2017-07-28.
11. J. Eichholz, Digital Heterodyne Laser Frequency Stabilization for Space-Based Gravitational Wave Detectors and Measuring Coating Brownian Noise at Cryogenic Temperatures, Ph.D. thesis, University of Florida (2015).
12. A. E. Siegman, LASERS (University Science Books, 1986).