Cosmological Constraint on the Scale of the Supersymmetric Singlet Majoron †

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Abstract

In a supersymmetric theory with a spontaneously broken global symmetry, $G$, if the scale of supersymmetry breaking, $M_s$ is smaller than the scale $M_G$ of the global symmetry, the Nambu-Goldstone boson, $\chi$ is accompanied by two massive superpartners ( a fermionic, $\Psi_{\chi}$ and a scalar boson, $\sigma_{\chi}$ ) with mass of order $M_s$. Cosmological considerations imply stringent constraints on the couplings of $\Psi_{\chi}$ and $\sigma_{\chi}$. Application of these considerations to the supersymmetric singlet Majoron (SUSYSM) model leads to an upper limit on the scale $V_{BL}$ of global $U(1)_{B-L}$ symmetry to be $\leq 10^4$GeV, for reasonable values of parameters in the theory.

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It is widely believed that the fundamental particle interactions may be supersymmetric beyond the TeV scale in order to solve the problem of mass hierarchy between the Fermi and Planck scales. There are also various reasons for considering the existence of global symmetries (perhaps only $U(1)$ symmetries) of nature which are spontaneously broken. Examples are that of $U(1)_{PQ}$ symmetry needed to solve the strong CP problem\cite{1} or the $U(1)_{B-L}$ symmetry, widely discussed in understanding the nature of massive neutrinos\cite{2}. It is conceivable that a complete theory of nature is one that encompasses a high energy supersymmetry as well as a spontaneously broken global symmetry. It is, then, of interest to see if cosmological considerations imply any new constraints on such theories.

In a supersymmetric theory with a spontaneously broken global symmetry, $G$, if the scale, $M_G$ of the global symmetry breaking is much larger than the scale $M_s$ of supersymmetry breaking, then the effective theory for $\mu \ll M_G$ contains a (or a set of) massless Nambu-Goldstone boson(s) corresponding to the broken generator(s) of $G$ and its (their) superpartners, which have masses of order, $M_s$. Specializing to the case where $G$ is a $U(1)$ symmetry, the Nambu-Goldstone boson ($\chi$) will be accompanied by a 2-component neutral fermion ($\Psi_\chi$) and a scalar boson $\sigma_\chi$. Cosmological constraints on these particles for the case of SUSY $U(1)_{PQ}$ models have been widely discussed in the literature\cite{3}. In this note, we focus our attention on a supersymmetric theory where global $U(1)_{B-L}$ symmetry is spontaneously broken\cite{4} by a $SU(2)_L \times U(1)_Y$ singlet field. In such theories, the scale, $V_{BL}$ of the B-L symmetry breaking is connected to the neutrino mass via the see-
saw mechanism. Any information on $V_{BL}$ will therefore provide information on the nature of light neutrino masses. So far, only very weak constraints ($i.e.$ $V_{BL} \geq O(100 \text{ GeV})$) can be deduced from the astrophysics of red giant stars[4]. Recently, some more constraints on $V_{BL}$ have been deduced if one assumes that the Planck scale effects break the global B-L symmetry by dimension 5 operators[5-8]. In this letter, we discuss constraints on $V_{BL}$ that arise if the singlet Majoron model is made supersymmetric. There are two possible points of view: one is to consider a minimal extension of the SUSY standard model (MSSM) by including the right-handed neutrino superfield $\nu^c$ and a singlet field $S$ with $L = +2[9,10]$. In this model, scale $V_{BL}$ and $M_s$ are necessarily of the same order in order for spontaneous breaking of $U(1)_{B-L}$ to occur. There is then spontaneous breaking of R-parity[11] in this model. The second possibility, not discussed in the literature to date, is to consider $V_{BL} \gg M_s$, which, as we discuss below, necessarily requires two more $SU(2)_{L} \times U(1)_{Y}$ singlet superfield in addition to $\nu^c$ and $S$. We will show that in this class of theories, cosmological constraints imply that $V_{BL} \leq 10^{4}\text{GeV}$.

**The SUSYSM Model:** In order to derive the above mentioned constraint on $V_{BL}$, we will work with a generic supersymmetric theory where $V_{BL} \gg M_s$. The superfield content of the model along with their transformations under $SU(2)_{L} \times U(1)_{Y}$ as well as B-L global symmetry is shown in table I.

The superpotential for the model can be written as a sum of two terms:

$$W = W_0 + W_1 ,$$
Table I. Superfields and their transformations under $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$

| Superfield | $SU(2)_L \times U(1)_Y$ | $U(1)_{B-L}$ |
|------------|-----------------------|-------------|
| Q          | (2, 1/3)              | +1/3        |
| $u^c$      | (1, -4/3)             | -1/3        |
| $d^c$      | (1, +2/3)             | -1/3        |
| L          | (2, -1)               | -1          |
| $e^c$      | (1, +2)               | +1          |
| $\nu^c$   | (1, 0)                | -1          |
| $H_u$      | (2, +1)               | 0           |
| $H_d$      | (2, -1)               | 0           |
| S          | (1, 0)                | +2          |
| $S'$       | (1, 0)                | -2          |
| Z          | (1, 0)                | 0           |

where

$$W_0 = h_u Q H_u u^c + h_d Q H_d d^c + h_e L H_d e^c + h_{\nu} L H_u \nu^c$$

$$+ f \nu^c \nu^c S + \mu H_u H_d \ ,$$

and

$$W_1 = \lambda (S S' - M_1^2) Z \ .$$

We choose $M_1 \gg V_{WK}$, the electroweak scale. The vanishing of F-terms at the scale $M_1$ (which is of order $V_{BL}$) in order to maintain supersymmetry is satisfied by

$$< S >= V_S; \quad < S' >= V_{S'}; \quad < Z >= 0; \quad < \tilde{\nu}^c >= 0,$$

with $V_S V_{S'} = M_1^2$. $V_S$ and $V_{S'}$ will be assumed to be of the same order $(i.e. \ V_{BL} \simeq M_1 \gg V_{WK})$. It is easy to work out the particle spectrum for $\mu \ll V_{BL}$. Apart from the quark, lepton and Higgs fields $H_{u,d}$, there are three massless fields:
i) the Majoron $\chi$:

$$\chi = \frac{V_S \chi S - V_S' \chi S'}{\sqrt{V_S^2 + V_S'^2}} ;$$  \hspace{1cm} (3)

ii) Majorino $\Psi_\chi$:

$$\Psi_\chi = \frac{V_S \Psi S - V_S' \Psi S'}{\sqrt{V_S^2 + V_S'^2}} ;$$  \hspace{1cm} (4)

iii) Smajoron $\sigma_\chi$:

$$\sigma_\chi = \frac{V_S \sigma S - V_S' \sigma S'}{\sqrt{V_S^2 + V_S'^2}} ,$$  \hspace{1cm} (5)

where we have written the superfield as:

$$S = \frac{1}{\sqrt{2}}(\sigma S + i\chi S) + \sqrt{2}\theta \Psi S + \theta^2 F_S ,$$  \hspace{1cm} (6)

and similarly for $S'$. In the absence of explicit supersymmetry breaking terms, all of the light fields ($Q$, $L$, $H_{u,d}$, $\chi$, $\sigma_\chi$, $\Psi_\chi$) are massless and the electroweak symmetry is unbroken. In order to make the theory realistic, we will add to the lagrangian the soft SUSY breaking (but B-L conserving) terms which have the form:

$$V_s = \sum_a \mu_a^2 \phi_a^\dagger \phi_a + \sum_a A_a \int d^2 \theta^2 W_a$$

$$+ \sum_a m_{\lambda_a} \lambda_a \lambda_a + h.c ,$$  \hspace{1cm} (7)

$\phi_a$ goes over all scalar superpartner of light fields and $\lambda_a$ are gaugino fields. $W_a$ denotes each term in the superpotential. The origin of the soft SUSY breaking potential, $V_s$ is irrelevant to our subsequent discussion. We will assume the parameters of $V_s$ to be such that they induce electroweak symmetry breaking \textit{i.e.} $< H_u >= V_u$ and $< H_d >= V_d$ as usual. Two immediate
consequences follow from eq.7. Firstly, that since the terms in $V_s$ respect B-L symmetry, the Majoron, $\chi$ remains massless. On the other hand, the SUSY breaking terms impart a mass to $\sigma_\chi$ of order $M_s$. We will assume $M_s \simeq 1\text{TeV}$ in the subsequent discussion.

In order to discuss the Majorino mass, we first note that once supersymmetry is broken, the scalar component of the singlet field $Z$ acquires a vev:

$$<z> \simeq M_s V S V S' / M_s^2 \sim M_s.$$  

This then gives a tree level mass to the Majorino of order $M_s$, i.e., $m_{\Psi \chi} \simeq M_s$, which can be of the order of a TeV.

**Cosmological Constraints:** In generic SUSY models of the type we are considering, the dominant interactions of $(\chi, ~ \Psi \chi, ~ \sigma \chi)$ are with the super-heavy particles, such as $\nu_{c}^c$, $(V_{S'}S + V_{S}S')$ superfields. Any interaction with light particles arises via the coupling $h_{\nu} L \nu_{c}^c H_d$ and is therefore suppressed by the inverse powers of $V_{BL}$. This observation has important implications for cosmology, since for the epochs of the universe below the temperature $T < V_{BL}$, all the heavy particles annihilate and disappear. As a result, the interactions (scatterings as well as decays) of the particles $\Psi \chi$ and $\sigma \chi$ become very weak. In order to study their impact on the evolution of the universe, we have to find the temperature, at which $\Psi \chi$ and $\sigma \chi$ decouple from the rest of the particles since this determines their abundance at subsequent epoch until nucleosynthesis temperature. If this abundance is significant, we have to find their life time to study their impact on nucleosynthesis.

a) **Determination of decoupling temperature:**

We are interested in the case where $V_{BL} \gg V_{WK}$. For $T \geq V_{BL}$, all
particles are massless and are in equilibrium. For \( T < V_{BL} \), the dominant effective interactions that can keep the \((\chi, \sigma, \Psi)\) in equilibrium with leptons and quarks is of the form:

\[
\mathcal{L}_{eff} \simeq \frac{\epsilon_1}{V_{BL}^2} (\partial_\mu \chi)^2 \tilde{l}_a \tilde{b} + \frac{\epsilon_2}{V_{BL}^2} (\partial_\mu \sigma \chi)^2 \tilde{l}_a \tilde{b} \\
+ \frac{\epsilon_3}{V_{BL}^2} \bar{\Psi} \gamma_\mu \partial_\mu \Psi \chi \tilde{l}_a \tilde{b} + \ldots
\]  

(8)

The \( \ldots \) stands for Higgs fields replacing \( \tilde{l} \). These interactions arise from the D-type terms induced at the one loop, therefore \( \epsilon_i \) are expected to be small, typically\([F.1]\)

\[
\epsilon_i \simeq \frac{h_\nu^2}{16\pi^2},
\]  

(9)

where \( h_\nu \) is the coupling of \( \nu^c \) to the heaviest light neutrino. The order of magnitude of the decoupling temperature for \( \Psi_\chi \) and \( \sigma_\chi \) is then determined by the condition:

\[
\frac{\epsilon_i^2 T^5}{V_{BL}^4} \leq \left[ g_\ast(T) \right]^{1/2} \frac{T^2}{M_{pl}}.
\]  

(10)

This leads to

\[
T_D \leq 10^{4.3} V_{BL} \left( \frac{V_{BL}}{M_{pl}} \right)^{1/3} \left( \frac{10^{-6}}{\epsilon_i} \right)^{2/3} \text{GeV}.
\]  

(11)

In the specific model under consideration, since the neutrinos cannot decay fast enough\([12]\), we would expect their masses to satisfy the cosmological

\[\text{[F.1]}\] Here as well as in the rest of the paper, we will only carry out the order of magnitude estimates for parameters. More precise statements than this would require detailed structure of the model, which is not important at the present stage.
constraint, $m_\nu \leq 40 \text{ eV}[13]$, which would imply that,

$$h_\nu^2 \leq \frac{40 \text{ (eV)} f V_{BL}}{V_{WK}^2}. \quad (12)$$

Using this equation and eq.11 we find (all parameters in GeV units)

$$T_D \leq 10^{9.6} \left( \frac{V_{BL}^2}{M_{pl} f^2} \right)^{1/3} \left( \frac{40 \text{eV}}{m_\nu} \right)^{2/3} \text{GeV} ; \quad (13.a)$$

or

$$T_D \sim V_{BL} , \quad (13.b)$$

whichever is lower.

For $T < T_D$ until the particles $\sigma_\chi$ and $\Psi_\chi$ decay, their number density decreases only due to expansion of the universe ($n \sim T^3$) except at various annihilation thresholds for massive particles. Therefore, for temperature $T(\tau) < T < T_D$ (where $T(\tau)$ is the temperature at the decay epoch of these particles),

$$\left. \frac{n(\sigma_\chi)}{n_\gamma} \right|_T \simeq \frac{g_*(T)}{g_*(T_D)} . \quad (14)$$

Since $M_{\sigma_\chi} \simeq M_s \simeq 1 \text{TeV}$, the Smajoron will dominate the mass density of the universe below approximately $T \simeq 10 \text{GeV}$ and will completely upset the discussions of nucleosynthesis, if it is stable. In order to maintain our present excellent understanding of the nucleosynthesis[14], we demand that the heavy Majorino and Smajoron decay before $t \leq 10^{-2} \text{ sec}$.

b) Decay of Majorino ($\Psi_\chi$) and Smajoron ($\sigma_\chi$) : Let us first consider the Smajoron decay. Above the electroweak phase transition temperature, the
$\sigma_\chi$ is absolutely stable. For $T < V_{WK}$, however, the decay $\sigma_\chi \rightarrow \nu \nu$ can occur with a coupling strength of order due to non-zero Dirac mass of the neutrino:

$$g(\sigma_\chi \rightarrow \nu \nu) \simeq \frac{h_\nu^2}{f} \left( \frac{V_{WK}}{V_{BL}} \right)^2 .$$

(15)

In eq.15, we have kept only the heaviest of the light left-handed neutrinos. Requiring $\tau_{\sigma_\chi} < 10^{-2} \text{s}$[15], we then find,

$$V_{BL} \leq 10^8 h_\nu \left( \frac{m_{\sigma_\chi}}{1 \text{TeV}} \right)^{1/4} \text{GeV} .$$

(16)

Again using the see-saw formula for neutrino masses $m_\nu \simeq h_\nu^2 V_{WK}^2 / f V_{BL}$, we have that

$$V_{BL} \leq 10^4 \left( \frac{m_\nu}{40 \text{eV}} \right) \left( \frac{m_{\sigma_\chi}}{1 \text{TeV}} \right)^{1/2} \text{GeV} .$$

(17)

As pointed out above that in the minimal singlet Majoron model the neutrinos are likely to have a long life time[12], we use the cosmological upper limit of 40 eV on the mass $m_\nu$ of stable neutrino[13] and get $V_{BL} \leq 10^4$ GeV. In non-minimal Majoron models, neutrinos may be unstable and therefore may be heavier than 40 eV. The upper limit on $V_{BL}$ is then less stringent.

Turning now to the decay of the Majorino, let us assume that, $m_{\Psi_\chi} \geq m_{\tilde{H}}$, where $\tilde{H}$ is the lightest neutralino. Present data therefore implies that $m_{\Psi_\chi}$ should be in the 100 GeV range (or higher). The case $m_{\Psi_\chi} \leq m_{\tilde{H}}$ is discussed later on. The dominant decay of $\Psi_\chi$ is non-vanishing only for $T < V_{WK}$, and is a tree-level process mediated by virtual $\tilde{\nu}_e$ exchange, leading to $\nu \nu \tilde{H}$ as a final state. The strength of the $\Psi_\chi \rightarrow \nu \nu \tilde{H}$ coupling is

$$g_{\Psi_\chi \rightarrow \nu \nu \tilde{H}} \simeq \frac{h_\nu^2 f V_{WK}}{2\sqrt{2} V_{BL}} .$$

(18)
Again requiring $\tau_{\Psi \chi} \leq 10^{-2} \text{ sec.}$, we get, using the see-saw formula

$$V_{BL} \leq 4 \times 10^3 \left( \frac{m_\nu}{40 \text{eV}} \right)^{1/2} f \left( \frac{m_{\Psi \chi}}{1 \text{ TeV}} \right)^{5/4} \text{ GeV} \ .$$  \hspace{1cm} (19)

This bound is of the same order as in eq.17.

The case where $m_{\Psi \chi} < m_{\tilde{H}}$ is interesting because, in this case, either the abundance $\Psi \chi$ must be reduced by annihilation process or the SUSYSM model is ruled out. Note that, since in this case, R-parity is an exact symmetry, the decay of $\Psi \chi$ is forbidden, if it is lighter than the lightest neutralino. And its annihilation channels are also inefficient if $V_{BL} > \text{TeV}$ (see eqs.13). Therefore, if $m_{\Psi \chi} < m_{\tilde{H}}$, our conclusion is that $V_{BL}$ must be less than a TeV.

A general concern in the case of late decaying of heavy particles is the possible dilution of baryon to entropy ratio below the observed value. This question has been analyzed in detail by Scherrer and Turner in ref.15. For $m_{\sigma, \Psi \chi} \leq 1 \text{ TeV}$, and lifetime $\tau \leq 10^{-2} \text{ sec.}$ considered here, they have shown that neither nucleosynthesis nor baryon to photon ratio is effected by their late decay.

In summary, we have found that if spontaneous breaking of global B-L symmetry occurs in a supersymmetric model, the scale $V_{BL}$ is likely to be in the TeV range. It is therefore quite likely to manifest itself in rare decay processes. It is also worth emphasizing that, while we have carried out our discussion using the minimal singlet Majoron model, all our considerations hold for more elaborate versions of it.
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