Note on the $8_{18}$-Knot

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Abstract

We define some physical variables associated with the traversing sequences of electrons along the orbit which is a 2D projection of $8_{18}$-knot. The configuration is regular but the resulting contributions, which are related to the physical variable, of those combinations from all the possible states to the fixed spatial sites show certain irregular behavior near the over- or under-crossing points of this knot. The possible explanation for this kind of direct geometric consequences is made to linked to the physical insight.

Keywords: Writhing number, entropy, energy, electron orbit, phase, system.

1 Introduction

Knots are classified according to the minimum number of intersections on their projections [1]. Their applications to statistical mechanics or topology of polymers and DNA dated back to early 70s [2]. Knots are usually categorized in terms of topological properties that are invariant under changes in a knot’s spatial configuration [3,4]. For the special interests of our study here, we will investigate certain property of $8_{18}$-knot [5] which has configurationally similar but topologically distinct relationship with some orbitals of electrons [6]. This knot has the polynomial invariant \[ \Delta(x) = 1 - 5x + 10x^2 - 13x^3 + 10x^4 - 5x^5 + x^6 \] [3] and zero writhing number.

Recently studies of random knots and the knot probability in lattice polygons have aroused many problems in the discrete form of knots or lattice knots [7]. We shall give some preliminary results about the discrete energy/entropy of $8_{18}$-knot in this paper. Our main focus will lie in the principles of indistinguishability and the micro-reversibility related to the states of electrons with certain spin after the electrons had traversed along this knot starting from any preassigned original sites in its $xy$-projection within one cycle.

2 General Approach

Our idea is to consider the $8_{18}$-knot as the traversing orbitals of electrons which have certain spin. In the modeling of superconducting, e.g. [8], the two pairing electrons avoid the Coulomb interaction by arranging themselves in space in a $d_{x^2-y^2}$ orbit which is similar to the 2D projection of $8_{18}$-knot. We start from the finite sequences of temporary electron-site which here are the crossings and the vertices of its projection or 2D $xy$-configuration for simplicity. These locations or moves of electron-site could be well defined in certain lattice. There are actually 12 characteristic points as shown in Fig. (i). This regular configuration is isotropic to any rotations in the $xy$-plane for the consideration of 4 branches of the $8_{18}$-knot. Starting from any one of these 12 points and then traversing along this knot for one cycle, the phase changes in 2D $xy$-projection are $6 \pi$ with respect to the origin o. The center of an atom or a nucleus is located
at the origin \( o \).

For simplicity, we treat those electrons moving within one cycle as a system. The energy and entropy are assumed to be dependent on the size of this system (as well as each element of this system, e.g., electron with certain spin because of the spin-spin coupling, or when the electrons are replaced by, vortices in the xy model [9]), such as the traversed area of this system in 2D case here. We can also imagine the energy/entropy increment as the phase difference between the temporary and original starting sites (we arbitrarily set) of the electrons.

Fig. (i) Schematic diagram of a loosely hard \( 8_{18} \)-knot. Those points of solid circle could be prescribed as vertices or lattice sites.

There are 10 different states or cases for the 2D configuration (xy-projection of the \( 8_{18} \)-knot here) in consideration because it is so regular and isotropic as we have assumed. These states come from the starting sites being at the center of each branch (e.g. I,J,K,L in Fig. 1), the outer shoulder (e.g. E,F,G,H in Fig. 1), and the inner shoulder (e.g. A,B,C,D in Fig. 1), respectively. As the object (i.e. electron here) traverses, the area (or entropy) increases. We thus associate the moving object (in a sequence) with the increasing area (or entropy) corresponding to their orbital position (the same as the traversing time-sequence) relative to the original starting position in our 2D configuration. For convenience, we set the increment between any two of the 12 characteristic points (in our observation) to be 1 (unit). The original starting value is 1. But, those shoulder points, being the overcrossing or undercrossing with respect to the same position in the xy-projection, must be identified and given the corresponding traversed values.

Now, once the object is starting from the centers of the branch, it has two choices (either clockwise or counterclockwise) to start traversing because of the absent crossings. As the initial site moves to the outer and inner shoulders, there are 4 choices for different traversing.

3 Results and Discussions

The results of the main different combinations are arranged and shown in Fig. 2 (a,b,c,d,e). Those values shown at the left-hand-side of "/" or upper side of "−" come from the overcrossing situations whereas those shown at the right-hand-side of "/" or lower side of "−" come from the undercrossing. We neglect those mirrored cases of the 2D xy-projection of the \( 8_{18} \)-knot in this presentation. We only plot those clockwise traversing cases which are half of the 10 states within one revolution cycle in our consideration. Thus there are missing 5 variants related to
Fig. 2 (a,b,c,d,e) corresponding to the counterclockwise traversing cases starting from locations K, F, A, respectively. We can easily figure out these states which are summarized in Table 1.

The starting sites K-J-L-I/F-E-H-G/A-D-C-B, however, are indistinguishable respectively because any rotations without deformations cannot change the regularity of 2D $xy$-configuration we impose for this knot and besides, there are no fixed labels for the sites. We only temporarily mark those electron-sites with the 2D spatial projection-positions in the $xy$-plane in order to observe the electron orbit conveniently and instantly. The time sequences are artifically created by us and might not be the real situations which, however, could be approximately emulated by random processing subjected to certain physical rules if the sampling numbers are large enough. That is to say, to start from position K makes no differences with starting from position J or I or L. Similar conclusions can also be made for those startings from the inner and outer shoulders. After long-time averaging for all the possible states or traversing sequences, the equilibrium contributions (entropy or interaction energy) from the electron-motion to those 2D spatial sites with over- or under-crossing should be almost the same if we accept that for the same group (center of branch/outer shoulder/inner shoulder) since they are equally distant from the origin o (there is the nucleus) the interactions should be nearly of the same values. But, from our results, as shown in Fig. 2, we can observe some mismatches by simply checking the addition of those values at the shoulders. We have no idea about these kinds of defects related to the regular 8$18$-knot. These could be compensated only when the irregular hopping of the electron happens around those shoulders of 8$18$-knot during the traversing. We make this statement by the direct geometric combination-results and would like to wait for the explanations from the rigorous theories of physics or well-measured experiments presently or in the future.

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Fig. 1  Schematic diagram of a *loosely* hard $8_{18}$-knot.

(a)  
(b)

(c)  
(d)  
(e)

Fig. 2 (a,b,c,d,e) Sequence of different traversing along a $8_{18}$-knot.
Table 1. Possible traversing sequences and energy/entropy allocations. case (b),(d),(h),(f),(j) correspond to Fig. 2 (a),(b),(c),(d),(e) respectively. case (k) is the mirrored case of (a). The values shown in the upper rows of each case are at the overcrossing whereas others in the lower rows are for the undercrossing of the 2D projection of the $8_{18}$-knot.

| case | A | B | C | D | E | F | G | H | I | J | K | L |
|------|---|---|---|---|---|---|---|---|---|---|---|---|
| a    | 13 | 8 | 3 | 18 | 5 | 20 | 15 | 10 | 11 | 6 | 1 | 16 |
|      | 19 | 14 | 9 | 4 | 12 | 7 | 2 | 17 |   |   |   |   |
| b    | 9  | 14 | 19 | 4 | 17 | 2 | 7 | 12 | 11 | 16 | 1 | 6 |
|      | 3  | 8 | 13 | 18 | 10 | 15 | 20 | 5 |   |   |   |   |
| c    | 14 | 9 | 4 | 19 | 6 | 1 | 16 | 11 | 12 | 7 | 2 | 17 |
|      | 20 | 15 | 10 | 5 | 13 | 8 | 3 | 18 |   |   |   |   |
| d    | 8  | 13 | 18 | 3 | 16 | 1 | 6 | 11 | 10 | 15 | 20 | 5 |
|      | 2  | 7 | 12 | 17 | 9 | 14 | 19 | 4 |   |   |   |   |
| e    | 7  | 2 | 17 | 12 | 19 | 14 | 9 | 4 | 5 | 20 | 15 | 10 |
|      | 13 | 8 | 3 | 18 | 6 | 1 | 16 | 11 |   |   |   |   |
| f    | 15 | 20 | 5 | 10 | 3 | 8 | 13 | 18 | 17 | 2 | 7 | 12 |
|      | 9  | 14 | 19 | 4 | 16 | 1 | 6 | 11 |   |   |   |   |
| g    | 1  | 16 | 11 | 6 | 14 | 9 | 4 | 19 | 13 | 8 | 3 | 18 |
|      | 15 | 10 | 6 | 20 | 7 | 2 | 17 | 12 |   |   |   |   |
| h    | 7  | 12 | 17 | 12 | 15 | 20 | 5 | 10 | 9  | 14 | 19 | 4 |
|      | 1  | 6 | 11 | 16 | 8 | 13 | 18 | 3 |   |   |   |   |
| i    | 1  | 16 | 11 | 6 | 13 | 8 | 3 | 18 | 19 | 14 | 9 | 4 |
|      | 7  | 2 | 17 | 12 | 20 | 15 | 10 | 5 |   |   |   |   |
| j    | 1  | 6 | 11 | 16 | 9 | 14 | 19 | 4 | 3  | 8 | 13 | 18 |
|      | 15 | 20 | 5 | 10 | 2 | 7 | 12 | 17 |   |   |   |   |
| k    | 19 | 14 | 9 | 4 | 12 | 7 | 2 | 17 | 11 | 6 | 1 | 16 |
|      | 13 | 8 | 3 | 18 | 5 | 20 | 15 | 10 |   |   |   |   |