Enhanced photon production near the light–cone by a hot plasma * †

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Strong collinear divergences, although regularized by a thermal mass, result in a breakdown of the standard hard thermal loop expansion in the calculation of the production rate of photons by a plasma of quarks and gluons using thermal field theory techniques.

1. INFRARED AND COLLINEAR PROBLEMS AT FINITE T

1.1. Hard thermal loop resummation

In this talk, we study the production of soft photons by a hot quark gluon plasma. Indeed, this is a good way of testing our understanding of the infrared and collinear behavior of thermal gauge theories. Intuitively, these difficulties are increased with respect to the \( T = 0 \) case, because of the presence in the Feynman rules of Bose-Einstein factors \( n_B(k_0) = 1/(\exp(k_0/T) - 1) \) which can be large if \( k_0 \) is much smaller than the temperature \( T \). Concerning the collinear singularities, they can no longer be factorized in the hadronic structure functions since we are now in a deconfined phase.

To improve the infrared behavior of thermal field theories, a resummation, known as the hard thermal loop (HTL) resummation, has been proposed by [1]. Basically, this resummation lies in the fact that certain thermal radiative corrections can be as large as the bare corresponding quantity. In particular, this is the case when the external legs of a propagator or vertex carry only soft momenta (i.e. of order \( gT \)).

1.2. Effective propagators and vertices

In QCD, we need effective gluon and quark propagators, as well as effective vertices. The resulting effective theory turns out to be gauge invariant. For later reference, let us quote the effective gluon propagator:

\[
D^{\mu\nu}(L) = \sum_{\alpha = T, L} \frac{-iP^{\mu\nu}_\alpha(L)}{L^2 - \Pi_\alpha(L) + i\epsilon} + \frac{i\xi P^{\mu\nu}_\alpha(L)}{L^2 + i\epsilon},
\]

(1)

in a covariant gauge with gauge parameter \( \xi \), where the sum runs over the transverse and longitudinal modes. The \( P^{\mu\nu}_\alpha \) are the adequate projectors. The corresponding self–energies are (see [1]):

\[
\Pi_T(L) = 3m^2_{\text{g}} \left[ \frac{l_0^2 - l^2}{2l^2} + \frac{l_0(l^2 - l_0^2)}{4l^3} \ln \left( \frac{l_0 + l}{l_0 - l} \right) \right],
\]

(2)

\[
\Pi_L(L) = 3m^2_{\text{g}} \left[ \frac{l^2 - l_0^2}{l^2} \right] \left[ 1 - \frac{l_0}{2l} \ln \left( \frac{l_0 + l}{l_0 - l} \right) \right],
\]

(3)

where \( m^2_{\text{g}} = g^2T^2[N + N_f/2]/9 \). This resummed gluon propagator possesses two mass shells above the light cone: a transverse and a longitudinal one. Therefore, everything happens as if the gluon had acquired a kind of mass through thermal corrections, this mass being of order \( gT \).

Concerning the effective quark propagator, we will need only its hard momentum limit:

\[
S(P) = \frac{iP}{P^2 - M_f^2 + i\epsilon},
\]

(4)

where \( M_f^2 = g^2C_f T^2/4 \) is a thermal mass of order \( gT \) (we started from massless bare quarks).

To achieve gauge invariance in the effective theory obtained by this resummation, we need also

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some effective vertices. In actual calculations, they are simply replaced by their diagrammatic expansion (another way to deal with these vertices is to use the Ward identities verified by the HTLs). For example, the effective $qqg$ vertex reads (where a solid dot denotes an effective vertex or propagator):

\[
\begin{array}{c}
\includegraphics{vertex1.png}\\
= \includegraphics{vertex2.png} + \includegraphics{vertex3.png}
\end{array}
\]

It is important to note that may exist some effective vertices without bare equivalent, like the $\gamma\gamma qq$ vertex:

\[
\begin{array}{c}
\includegraphics{vertex4.png}\\
= \includegraphics{vertex5.png}
\end{array}
\]

1.3. Physical interpretation

Physically, this resummation is closely related to the Debye screening in a plasma. Indeed, if we look at the static limit of the self-energies $\Pi_{T,L}$ for space-like gauge bosons, we may obtain a mass:

\[
m_{T,L}^2 \equiv \lim_{l_0 \to 0} \Pi_{T,L}(l_0 = 0, l).
\]

The interaction range is the inverse of this mass, and we have an exponential screening of the field of a test charge put in the plasma. In QED, where exists the classical model of Debye and Huckel, we can verify that the mass $m_L$ is precisely the classical Debye mass for electric fields, whereas the nullity of $m_T$ corresponds to the fact that static magnetic fields are not screened.

1.4. Subsidiary problems

Even if this HTL resummation is carried on, there remain some residual divergences.

The first kind of problems is related to the fact that static “magnetic” (transverse) fields are not screened in a plasma. This fact is well known in a QED plasma, for which it has been proven that the “magnetic mass” $m_T$ does vanish at all orders of perturbation theory. For QCD and other non abelian gauge theories, the status of this magnetic mass is not so clear; nevertheless it is strongly expected that it is beyond the abilities of perturbative methods and of order $g^2T$ or smaller. A long standing problem where this kind of transverse infrared divergence does appear is the calculation of the fermion damping rate.

Other divergences which are not cured by the HTL resummation, at least in its minimal version, are collinear ones. Indeed, collinear divergences may occur whenever we have massless particles in a diagram. Since in the standard HTL framework only soft lines are resummed and can therefore acquire a thermal mass, we can still have problems with hard massless particles. If we come back to the circumstances which necessitate a resummation, we see that we should compare the quantities $P^2$ and the self-energy $\Pi(P) = O(g^2T^2)$. Of course, as said before, we need a resummation when the components of $P$ are soft; but we need also effective propagators when the components of $P$ are hard, but $P$ is close to the light-cone. It has been shown recently in [3] that this supplementary resummation still preserves gauge invariance. We will see in the next section on the example of photon production that it can cure the collinear divergences, and, unexpectedly, some transverse infrared divergences.

2. SOFT PHOTON PRODUCTION

2.1. Various approaches

2.1.1. Semi-classical methods

Recently, in order to compute the $\gamma$ emission rate by an ultrarelativistic parton of the plasma, semi-classical methods have witnessed a renewed interest. Basically, the idea of these approaches is to look at the emission of a single photon by a quark undergoing multiple scatterings while crossing the plasma. An essential hypothesis of this method is to assume that the scattering centers are static, so that only longitudinal bosons are exchanged in the scattering process. Also, the interaction is assumed to be Debye screened.

The essential physical result of this approach is a suppression of the emission rate at small $\gamma$ energies, known as the Landau-Pomeranchuk-Migdal (LPM) effect [4]. Indeed, the lifetime of a quark emitting a $\gamma$ of energy $\omega$ at an angle $\theta$ after a scattering is $\tau = 1/\omega \theta^2$. If this lifetime is greater than the mean free path $\lambda$ of the quark in the medium,
then the $\gamma$ emission is suppressed because a new scattering occurs before the $\gamma$ emission: this is the case when $\omega < 1/\lambda$.

2.1.2. Thermal field theory

In thermal field theory, the emission rate per unit volume of the plasma is related to the imaginary part of the $\gamma$ self-energy by:

$$ q_0 \frac{dN}{d^3q^4x} = -\frac{1}{(2\pi)^3}n_\rho(q_0)\text{Im} \Pi^{\mu\nu}(Q), \quad (5) $$

the imaginary part being itself obtained by a generalization at finite $T$ of the standard cutting rules. In the standard HTL framework at 1 loop [4], we have three diagrams:

\begin{align*}
\text{Im} & \left\{ \qquad \mu \qquad \frac{q_0}{q_0} \mu \qquad \right\} \sim e^2g^4 \frac{T}{q_0} \ln g^4 \quad (a) \\
\text{Im} & = O_{HTL} \quad (b) \\
\text{Im} & = O_{HTL} \quad (c)
\end{align*}

where the symbol $0_{HTL}$ means that we obtain a null result within the standard rules for the HTL expansion. Some of the relevant corresponding processes are:

\begin{align*}
\left[ \qquad \frac{r}{r} \qquad \frac{r}{r} \qquad \frac{r}{r} \qquad \right] \times \left[ \qquad \frac{r}{r} \qquad \frac{r}{r} \qquad \frac{r}{r} \qquad \right] \ast \quad (a') \\
\left[ \qquad \frac{r}{r} \qquad \frac{r}{r} \qquad \frac{r}{r} \qquad \right] \times \left[ \qquad \frac{r}{r} \qquad \frac{r}{r} \qquad \frac{r}{r} \qquad \right] \ast \quad (b') \\
\left[ \qquad \frac{r}{r} \qquad \frac{r}{r} \qquad \frac{r}{r} \qquad \right] \times \left[ \qquad \frac{r}{r} \qquad \frac{r}{r} \qquad \frac{r}{r} \qquad \right] \ast \quad (c')
\end{align*}

We can see that the diagram $(a)$ involves the interference of a “bremsstrahlung-like” process with a very complicated one (see $(a')$), whereas the diagrams $(b)$ and $(c)$ involve only bremsstrahlung processes (see $(b')$ and $(c')$). From a physical point of view, it seems very surprising that bremsstrahlung is subdominant in front of the complicated process of diagram $(a)$. This was our motivation to study the last two diagrams beyond the standard HTL scheme. Moreover, since the parton exchanged in the scattering is soft and since in that kinematical regime a bosonic statistical weight dominates over a fermionic one, the diagram $(c)$ is dominant with respect to $(b)$.

2.2. Bremsstrahlung diagrams

The remaining of this talk will be devoted to the study of the diagram $(c)$. In order to do this calculation, we replace the $\gamma\gamma gg$ effective vertex by the corresponding loops, which gives:

\begin{align*}
\text{Im} & \Pi^{\mu\nu}(Q) \approx 8(-1)_{L}e^2g^2 \int d^4R d^4L \left( \frac{2\pi}{T} \right)^6 \times \\
q_0 & T n_\rho(\rho_0)(1 - n_\rho(\rho_0))n_\rho(\rho_0)\rho_{T,R}(\rho_0, l) \times
\end{align*}
\[
\frac{r^2}{l^2} \left( L^2 \right)^2 \delta(P^2 - M_f^2) \delta((R + L)^2 - M_f^2) \left( R^2 - M_f^2 \right) \left( (P + L)^2 - M_f^2 \right)
\]
\[
, \quad (6)
\]

where \( n_{B,P}(\omega) = 1/(\exp(\omega/T) + 1) \) and \( \rho_{T,L} \)
is the imaginary part of the longitudinal or transverse part of the effective gluon propagator: \( \rho_{T,L}(l_0, l) = 2\text{Im}(L^2 - \Pi_{T,L}(L) + i\epsilon)^{-1} \). The symbol \((-1)_L\) denotes an extra minus sign in the longitudinal gluon exchange contribution. It is straightforward to verify that the gauge dependent part of this propagator disappears when the self-energy and vertex corrections are combined.

Two kinds of contributions arise in \( \rho_{T,L} \), corresponding to very different features of the analytic structure of the effective propagator. The first contribution, arising at \( L^2 > 0 \), is made of Dirac delta functions corresponding to the thermal mass shells of the resummed propagator (i.e. from poles in the \( l_0 \) plane). The second contribution to this imaginary part comes from the Landau damping (damping of a wave by absorption of virtual photons by quarks of the plasma).

In fact, in our rate, the term containing the pole part of the functions \( \rho_{T,L} \) corresponds to a production by Compton effect, where a thermalized on-shell gluon is absorbed by a quark which emits a photon, rather than bremsstrahlung. In \( (a'), (b') \) and \( (c') \) we represented only the amplitudes corresponding to bremsstrahlung. Nevertheless, a careful analysis of the Dirac delta constraints in (6) shows that the region \( L^2 > 0 \) is forbidden as far as \( Q^2/q_0^2 \ll 1 \), which is the situation we are interested in. Therefore, the bremsstrahlung production dominates over the Compton one.

2.2.2. Enhancement mechanism

Taking into account the constraints provided by the two Dirac delta functions, we obtain for the denominators (in the region where \( P, R \sim T \) and \( Q, L \ll T \)):

\[
R^2 - M_f^2 \approx 2q_0 r \left[ 1 - \cos \theta + \frac{M_{\text{eff}}^2}{2r^2} \right]
\]
\[
2\pi \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{1}{(P + L)^2 - M_f^2} \approx (2q_0 r)^{-1} \times
\]
\[
\left[ \left( 1 - \cos \theta + \frac{M_{\text{eff}}^2}{2r^2} + \frac{L^2}{2r^2} \right)^2 - \frac{L^2 M_{\text{eff}}^2}{r^4} \right]^{-1/2}
\]
\[
, \quad (8)
\]

where \( \theta \) is the angle between the spatial components of \( R \) and \( Q \), \( \phi \) is the azimuthal angle between the spatial parts of \( Q \) and \( L \), and \( M_{\text{eff}}^2 \equiv M_f^2 + Q^2 r^2/q_0^2 \) (we integrated over \( \phi \) here since \( (P + L)^2 - M_f^2 \) is the only place where \( \phi \) appears). At that point, it is clear that the potential collinear divergence at \( \theta = 0 \) is regularized by a combination of the fermion thermal mass and of the photon virtuality. At \( Q^2 = 0 \), it appears to be essential to keep this fermion thermal mass, despite the fact that the fermion has a hard momentum.

Na\"ıvely, we obtain the following order for the integral over \( \cos \theta \):

\[
I \equiv \int_{-1}^{+1} \frac{d\cos \theta}{(R^2 - M_f^2)((P + L)^2 - M_f^2)} \sim \frac{1}{q_0^2 r^2}
\]
\[
, \quad (9)
\]

In order to explain the enhancement mechanism, we can roughly approximate:

\[
R^2 - M_f^2 \sim 2q_0 r (1 - \cos \theta + u^*)
\]
\[
(P + L)^2 - M_f^2 \sim 2q_0 r (1 - \cos \theta + \bar{u}^*),
\]
\[
, \quad (10)
\]

where \( u^*, \bar{u}^* \sim M_{\text{eff}}^2 / r^2 \) and \( u^* - \bar{u}^* \sim L^2 / r^2 \), so that we get:

\[
\text{If } u^* \leq L^2 / r^2 \quad I \sim \frac{1}{u^* - \bar{u}^*} \frac{1}{q_0^2 r^2} \gg \frac{1}{q_0^2 r^2}
\]
\[
\text{If } u^* \gg L^2 / r^2 \quad I \sim \frac{1}{u^* - \bar{u}^*} \frac{1}{q_0^2 r^2}.
\]
\[
, \quad (11)
\]

Therefore, we have an enhancement over the order one expects naïvely as far as \( u^* \ll 1 \), i.e. if \( Q^2 / q_0^2 \ll 1 \) (this is the meaning of “near the light-cone”). The origin of this enhancement lies in the residues of collinear singularities: potential collinear divergences, although regulated by a fermion thermal mass, are at the origin of the break-down of the power counting rules of the HTL expansion, which fails to handle properly collinear sensitive processes.

2.2.3. Final expression of \( \text{Im} \Pi_{\mu}^{\nu} \)

Finally, after taking into account the \( \delta \)-constraints and performing the angular integration,
we obtain:

\[
\text{Im } \Pi_{\mu \nu}(Q) \approx (-1) \frac{e^2 g^2 T^3}{\pi^2 q_0} \times \\
\int_0^{+\infty} dv \frac{v^2}{e^v + 1} \left[ 1 - \frac{1}{e^v + 1} \right] \times \\
\int_{0}^{1} \frac{dx}{x} \tilde{I}_{T,L}(x) \int_{0}^{+\infty} dw \sqrt{\frac{w}{w + 4}} \tanh^{-1} \sqrt{\frac{w}{w + 4}},
\]

where we introduced some useful dimensionless quantities:

\[
\tilde{R}_{T,L} \equiv \text{Re } \Pi_{T,L}/M_{\text{eff}}^2, \quad \tilde{I}_{T,L} \equiv \text{Im } \Pi_{T,L}/M_{\text{eff}}^2, \\
v \equiv r/T, \quad w \equiv -L^2/M_{\text{eff}}^2, \quad x \equiv l_0/l.
\]

In the expression above, we have factorized out the parameters fixing the dimension and the order of magnitude on the first line, so that the remaining two factors are only a numerical factor, which will be denoted by \(J_{T,L}\) in what follows.

3. CONCLUSIONS

Let us review some properties of the bremsstrahlung production rate quoted before in (12).

(i) The regions \(R \ll T\) and \(L \sim T\) are negligible in the integral, so that our approximations based on these assumptions are safe.

(ii) The result is infrared finite, without the need of a magnetic mass for the transverse gluon exchange. This feature is very important for QED, where this magnetic mass is known to be zero to all orders. At \(Q^2 = 0\), if we look at the formal limit \(M_f \ll m_g\), we obtain:

\[
J_L \sim \ln(m_g/M_f) \quad J_T \sim \ln^2(m_g/M_f).
\]

One power of that logarithm, common to both the transverse and longitudinal contributions, is due to the potential collinear singularity. The additional power of that logarithm in the transverse term is a remnant of an infrared divergence (related to the absence of magnetic mass at this order), unpredictably screened by the fermion thermal mass.

(iii) At \(Q^2 = 0\), the bremsstrahlung contribution is of order \(\text{Im } \Pi_{\mu \nu} \sim e^2 g^2 T^3/q_0\) and therefore dominates over the contribution of the soft fermion loop \((e^2 g^4 T^3/q_0)\).

(iv) Let us end with a plot of \(J_{T,L}\) as a function of \(\log(Q^2/q_0^2)\), with \(g = 0.1, N = 3\) and \(N_f = 3\):

![Plot of J_T and J_L]

We note a flatness at small \(Q^2\), reflecting the fact that the mass \(M_{\text{eff}}\) is dominated by the fermion thermal mass, and we see that the enhancement disappears if \(Q^2/q_0^2 \rightarrow 1\).

(v) The transverse and longitudinal contributions are of the same magnitude, which seems to indicate that the static scattering centers hypothesis of the semi-classical approach is wrong.

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