Gate-controlled mid-infrared light bending with aperiodic graphene nanoribbons array

Eduardo Carrasco1,5, Michele Tamagnone1,2,5, Juan R Mosig2, Tony Low3,4 and Julien Perriussel-Carrier1

1 Adaptive MicroNano Wave Systems, Ecole Polytechnique Federale de Lausanne (EPFL), 1015 Lausanne, Switzerland
2 Laboratory of Electromagnetics and Acoustics (LEMA), Ecole Polytechnique Federale de Lausanne (EPFL), 1015 Lausanne, Switzerland
3 Department of Physics & Electrical Engineering, Columbia University, NY 10027, USA
4 Department of Electrical & Computer Engineering, University of Minnesota, MN 55455, USA

E-mail: tlow@umn.edu

Received 15 December 2014, revised 21 January 2015
Accepted for publication 22 January 2015
Published 11 March 2015

Abstract
Graphene plasmonic nanostructures enable subwavelength confinement of electromagnetic energy from the mid-infrared down to the terahertz frequencies. By exploiting the spectrally varying light scattering phase at the vicinity of the resonant frequency of the plasmonic nanostructure, it is possible to control the angle of reflection of an incoming light beam. We demonstrate, through full-wave electromagnetic simulations based on Maxwell equations, the electrical control of the angle of reflection of a mid-infrared light beam by using an aperiodic array of graphene nanoribbons, whose widths are engineered to produce a spatially varying reflection phase profile that allows for the construction of a far-field collimated beam towards a predefined direction.

Keywords: graphene, plasmon, metamaterials

Introduction
Graphene plasmonics has emerged as a very promising candidate for terahertz to mid-infrared applications [1], namely the frequency range where it supports plasmonic propagation. [1, 3–10]. Today the terahertz to mid-infrared spectrum, ranging from 10 to 4000 cm−1, is finding a wide variety of applications in information and communication, homeland security, military, medical sciences, chemical and biological sensing and spectroscopy, among many others [11–13]. The bi-dimensional and semi-metallic nature of graphene allows for electrical tunability not possible with conventional metals by simply biasing electrostatically the graphene device. In addition, graphene is also an excellent conductor of electricity, with highest attained carrier mobility reaching 1 000 000 cm2 Vs−1 in suspended samples [14] and 100 000 cm2 Vs−1 for ultra-flat graphene on boron nitride [15]. These attributes, in addition to its stability and compatibility with standard silicon processing technologies, have also raised interest in a myriad of graphene-based passive and active photonic [2, 22, 23] and terahertz devices [3, 7, 17–21].

Electromagnetic surfaces able to dynamically control the reflection angle of an incident beam have been studied for decades in the microwave community [16, 24], with applications in satellite communications, terrestrial and deep-space radio links. Non-conventional reflecting surfaces for optical frequencies have also been proposed recently, using metal elements having certain fixed configurations and operating in the plasmonic regime [25, 26]. However, the high carrier concentration in metals prohibits dynamic control of the reflected beams via these surface elements. In this letter, we show how the reflection angle of a mid-infrared incident beam can be electrically controlled with an aperiodic array of graphene nanoribbons, by utilizing its intrinsic plasmonic resonance behavior. We demonstrate that the proposed effect is...
experimentally realizable by performing numerical Maxwell simulations of the device with experimentally feasible parameters.

**Basic concepts**

Consider an aperiodic array of graphene nanoribbons of varying widths lying at the interface of two half spaces as illustrated in figure 1(a), with $p$-polarized light at incidence angle $\theta_i$, which is reflected and transmitted at angle $\theta_r$ and $\theta_t$ respectively. Each nanoribbon constitutes a plasmonic resonator, which can effectively produce a scattering phase $\phi$ between 0 and $\pi$ depending on the frequency of free-space light $\omega$ with respect to the plasmon resonance frequency $\omega_0$. In plasmonic metasurfaces consisting of nanoribbons of varying widths, the scattering phase in general can also vary across the interface, namely as $\phi(x_j)$, where $x_j = j\Delta x$ and $\Delta x$ being the distance between the centers of adjacent ribbons, which is considered constant in this contribution. In the limit where ray optics is applicable, i.e. $\lambda \gg \Delta x$, where $\lambda$ is the free-space wavelength, the generalized Snell’s law dictates that [25]

$$
\sin(\theta_t) \sqrt{\epsilon_2} - \sin(\theta_i) \sqrt{\epsilon_1} = \frac{\lambda}{2\pi} \int \frac{d\phi}{dx},
$$

$$
\sin(\theta_r) - \sin(\theta_t) = \frac{\lambda}{2\pi\sqrt{\epsilon_1}} \frac{d\phi}{dx}.
$$

Equation (1) implies that the reflected or transmitted beam can be effectively bent such that $\theta_r \neq \theta_{t_0}$ if a constant spatial gradient in the scattering phase is imposed (i.e. $d\phi/dx = \text{constant}$). Simple estimates from equation (1) suggest that it is possible to bend normal incident ($\theta_i = 0$) mid-infrared light far from broadside (i.e. $\theta_t \neq 0$). Assuming $\lambda = 10 \mu m$ (e.g from a CO2 laser), $\Delta x = 100 \text{ nm}$ (typical ribbon width where plasmon resonance resides in the mid-
infrared), one obtains $\Delta \phi \approx \pi$, suggesting that it is indeed possible to induce a gradual spatial variation in $\phi$ across the interface.

The scattering phase due to a graphene plasmonic resonator, and its design space, can be examined more quantitatively as follows. We consider graphene on a SiO$_2$ substrate and with an electrolyte superstrate, which can also serve as a top gate for inducing high doping in graphene [28]. We solve the Maxwell equations for the reflection coefficient of a $p$-polarized light, $r_p(q, \omega)$, where graphene is modeled by its dynamic local conductivity obtained from the random phase approximation [29, 30]. The scattering phase $\phi$ then follows from $\phi(\omega, \omega) = \text{arg}[r_p(q, \omega)]$. The nanoribbon resonator frequency $\omega_0$ can be estimated from the scattering coefficients for a continuous monolayer, where the in-plane wave-vector $q$ is related to the width $W$ of the nanoribbon via $q = \pi/W$, after accounting for the anomalous reflection phase off the edges [27]:

$$\omega_0 = \frac{3e^2\mu}{8W\hbar^2\epsilon_{\text{env}}},$$

where $\epsilon_{\text{env}} = \frac{1}{2}(\epsilon_1 + \epsilon_2)$, $\epsilon_1$ and $\epsilon_2$ are the relative permittivities of the superstrate and substrate respectively. The Lorentz oscillator model provides then a simple expression for the scattering phase:

$$\phi(W, \omega) \approx \tan^{-1}\left(\frac{\omega}{\tau_0} \left(\frac{\omega^2 - \frac{3e^2\mu}{8W\hbar^2\epsilon_{\text{env}}}}{\pi W}\right)^{-1}\right).$$

In this simple model calculation, we simply take $\epsilon_2 = 3.9$ and $\epsilon_1 = 6$. Assuming $\lambda = 10$ $\mu$m, figures 1(b)–(c) depicts $\phi(W, \omega)$ for varying $W$, with graphene at different chemical potentials $\mu$, and electronic lifetimes $\tau_0$. The phase $\phi$ changes most rapidly with $W$ when the ribbon is at resonance. For a given ribbon plasmon resonance frequency, increasing the doping would allow for the same resonance frequency at a larger $W$ as depicted in figure 1(b). On the other hand, decreasing $\tau_0$ dampens the plasmon resonance, causing a smoother variation in $\phi$ with $W$ as shown in figure 1(c). This dampening can be understood, considering the quality factor of the plasmonic resonance, which is proportional to $\tau_0\omega$, i.e. a lower quality factor implies a smoother phase step. Equation (3) therefore provides a simple intuitive understanding of the scattering phase of a graphene plasmonic resonator.

**Device simulation**

In principle, the tunable scattering phase of a graphene plasmonic resonator allows the design of surface elements which controls the angle of reflection or transmission of an incoming beam. In this work, we consider the former i.e. a reflectarray [16]. Figures 2(a)–(b) shows a detailed schematic of the proposed device, designed for a working frequency of 27 THz (900 cm$^{-1}$). The graphene nanoribbons array is between an electrolyte gating superstrate of 200 nm and a 1.2 $\mu$m SiO$_2$ dielectric substrate with a metal layer underneath, which serves as a reflector, reflecting most of the incoming light. The full dielectric function of SiO$_2$ is used in the simulation [31]. For the electrolyte superstrate,
we assumed a dielectric constant of 6. Hence, the effective
dielectric constant of the graphene’s environment, which
ultimately determines its plasmonic response, is approxi-
mately five, similar to [3]. The aperiodic array of nanor-
ibbons has a designed inter-ribbon separation \( p \) taken to be
140 nm, and the width of each nanoribbon is to be chosen
so that the spatial variation in \( \phi_r \) satisfies \( d\phi_r/dx = \text{constant} \),
as discussed previously. As explained later, the final
reflection phase \( \phi_r \) of the reflecting cell is not given simply
by the response \( \phi \) of the ribbon, since the contribution of
the ground plane must also be taken into account. We
consider a Gaussian beam illuminating the array at an
incident angle 45° with respect to the normal of the \( xz \)
plane as illustrated. Figure 2(c) depicts the top view of the
nanoribbons array, including the elliptical projection of the
impinging beam on the \( xz \) plane, where \( b_1 = 31.36 \mu\text{m} \) and
\( b_2 = 22.18 \mu\text{m} \).

We use CST microwave studio to numerically compute
the reflection coefficient and \( \phi_r \) for a nanoribbon of particular
width. Using the Floquet’s theory for periodic arrays, mutual
coupling between neighboring nanoribbons is accounted for.
Graphene is modeled (using a 2D boundary condition) by its
local dynamic conductivity \( \sigma(\omega) \), assuming typical electronic
lifetime of \( \tau_e = 0.1 \text{ ps} \). Figure 3(a) shows the calculated \( \phi_r \)
of the reflection coefficient, as a function of the chemical
potential \( \mu \) and width \( W \) of the nanoribbon. For a chemical
potential \( \mu = 1.0 \text{ eV} \), \( \phi_r \) varies between 0° and −340° by
adjusting the width of the ribbons between 40 and 140 nm. As \( \mu \)
decreases, the range of \( \phi_r \) decreases, which approaches a
constant \( \phi_r = −340° \) at \( \mu = 0.3 \text{ eV} \).

We design the reflectarray device to perform a binary
operation: consisting of producing a reflected far-field beam
in the broadside direction (0°) and a reflected far-field beam in
the specular direction (45°). The spatial phase profile required

Figure 3. (a) Phase \( \phi \) of the reflection coefficient as a function of chemical potential \( \mu \) for different widths \( W \) of a nanoribbon, ranging from 40 to 140 nm. Mutual coupling between neighboring ribbons is taken into account. (b) Spatial phase profile required at each nanoribbon along the \( x \)-axis in order to produce a far-field beam in the broadside direction (0°) and a reflected far-field beam in the specular direction (45°). (c) Width of the nanoribbons across the array (\( x \)-axis) of the reflectarray device. (d) Phase \( \phi \) of the reflection coefficient as a function of chemical potential \( \mu \) for different widths \( W \) of a nanoribbon using the equivalent circuit model described in figure 2(d).
to achieve these reflection angles can be determined based on equation (1), and are shown in figure 3(b) for these two cases. For specular beam, a zero phase difference between the nanoribbons would suffice, regardless of the absolute value of that phase. Otherwise, the sequence of ribbons’ widths has to be chosen such that it provides the necessary spatial phase profile to produce a reflected far-field beam at \( \mu = 1.0 \text{ eV} \). As mentioned above, with ribbons between 40 and 140 nm, \( \phi_r \) can be made to vary between 0° and −340°. The missing phase values (from −340° to −360°) were found not to have noticeable impact in the far-field generation. The widths of the nanoribbons along the array are displayed in figure 3(c).

In order to produce a reflected far-field beam in the specular direction, a constant \( \phi_r \) is required along all the elements of the array. This can be achieved by decreasing \( \mu \) to 0.3 eV as shown. In other words, the binary operation of our reflectarray device can be achieved by electrically controlling the doping \( \mu \).

In figures 4(a) and (b), we compute the far-field produced by our reflectarray device, for broadside \( (\mu = 1.0 \text{ eV}) \) and specular radiation \( (\mu = 0.3 \text{ eV}) \) respectively. The possibility of bending the light beam in an aperiodic array of graphene nanoribbons is clearly demonstrated by these simulations.

**Discussion**

A constant phase gradient is needed for producing a collimated beam. At intermediate dopings, where the ribbon array phases are not designed with constant phase gradient, the produced far field beam can be highly distorted. The design of smooth beam steering is possible, but would require a more complicated gating scheme that addresses the doping of individual ribbons separately as typically done in microwave reflectarrays [33] and more recently also for terahertz graphene-based reflectarrays [32]. The simplicity in the reflectarray array scheme proposed in this work has the obvious appeal of providing a design much easier to be implemented experimentally.

A simple circuit model for each nanoribbon cell, as shown in figure 2(d), is useful for better understanding of the proposed device. The substrate and superstrate can be modelled with two transmission line segments having propagation constants and characteristic impedance equal to the two media which they model. The equivalent of the ground plane is simply a short circuit, while the graphene ribbons can be represented by an equivalent \( Z_g \) impedance in parallel, that models the surface currents induced in the ribbons by the tangential electric field. An approximate closed form expression for \( Z_g \) can be found by modeling resonant ribbons as Lorentz oscillators as explained previously. This can be represented by a simple RLC series circuit, where \( R \) and \( L \) are simply found from the real and imaginary parts of the surface conductivity of graphene \( \sigma \), while \( C \) represents the quasi-static electric fields associated with the plasmons, and can be obtained by enforcing the resonant frequency of the RLC circuit \( (1/\sqrt{LC}) \) to be equal to the ribsoms resonant frequency \( \omega_0 \):

\[
L = \frac{\pi h^2}{e^2 \mu} \quad R = \tau^{-1} L, \quad C = \frac{1}{\omega_0^2 L}, \quad (4)
\]

where

\[
\omega_0 \approx \frac{3e^2 \mu}{8W^2 \varepsilon_{\text{env}}}. \quad (5)
\]

The final equivalent impedance of graphene nanoribbons is then

\[
Z_g = R + j \omega L + \frac{1}{j \omega C} = \frac{\pi h^2}{e^2 \mu} \left( \omega_0^2 + j \omega \tau / \omega_0^2 \right). \quad (6)
\]

This circuit model provides for an estimated value for the reflection phase \( \phi_r \) as function of \( W \) and \( \mu \). Figure 3(d) shows the calculated reflection phase \( \phi_r \) which agrees qualitatively...
with the numerical simulations presented in figure 3(a). The full wave simulations account for higher order phenomena not captured by this simple model. Importantly, while in the Lorentz oscillator the phase range is always less than π for the nanoribbons alone (i.e. equation (3)), the presence of the substrate and of the ground plane allows here a much wider phase range, which is a key point for the device presented in our work.

Our device has an average element loss of 1.32 dB for the $\mu = 0.3$ eV configuration, and 1.77 dB for $\mu = 1$ eV. Hence, most of the incoming light is reflected. The loss of the device is directly related to the assumed electronic lifetime $\tau$, as evident from equation (4), and can be improved with better graphene quality or by using a better substrate such as boron nitride [34].

Conclusion

In this work, we show how the reflection angle of a mid-infrared beam can be dynamically controlled by employing tunable graphene plasmons in an aperiodic array of graphene nanoribbons according to the generalized laws of reflection. The use of a simple gating scheme makes the proposed design particularly attractive because of its easy experimental fabrication. Numerical Maxwell device simulations convincingly demonstrate the dynamic bending of mid-infrared beams, assuming experimentally accessible physical parameters.

Acknowledgments

This work was supported by the European Union (Marie Curie IEF 300934 RASTREO project), the Hasler Foundation (Project 11149) and the Swiss National Science Foundation (Project 133583), and the University of Minnesota. E Carrasco, M Tamagnone, T Low and JR Mosig would like to dedicate this paper to the memory of J Perruisseau-Carrier, who passed away during the preparation of this work.

References

[1] Low T and Avouris P 2014 Graphene plasmonics for terahertz to mid-infrared applications ACS Nano 8 1086–101
[2] Tamagnone M, Fallahi A, Mosig J R and Perruisseau-Carrier J 2014 Fundamental limits and near-optimal design of graphene modulators and non-reciprocal devices Nat. Photonics 8 556–63
[3] Ju L et al 2011 Graphene plasmonics for tunable terahertz metamaterials Nat. Nanotechnology 6 630–4
[4] Koppens F H, Chang D E and Garcia de Abajo F J 2011 Graphene plasmonics: a platform for strong lightmatter interactions Nano Lett. 11 3370–7
[5] Grigorenko A N, Polini M and Novoselov K S 2012 Graphene plasmonics Nat. Photonics 6 749–58
[6] Yan H, Low T, Zhu W, Wu Y, Freitag M, Li X, Guinea F, Avouris P and Xia F 2013 Damping pathways of mid-infrared plasmons in graphene nanostructures Nat. Photonics 7 394–9
[7] Yan H, Li X, Chandra B, Tulevski G, Wu Y, Freitag M, Zhu W, Avouris P and Xia F 2012 Tunable infrared plasmonic devices using graphene/insulator stacks Nat. Nanotechnology 7 330–4
[8] Chen J et al 2012 Optical nano-imaging of gate-tunable graphene plasmons Nature 487 77–81
[9] Fei Z et al 2012 Gate-tuning of graphene plasmons revealed by infrared nano-imaging Nature 487 82–5
[10] Jablani M, Buljan H and Soljani M 2009 Plasmonics in graphene at infrared frequencies Phys. Rev. B 80 245435
[11] Tonouchi M 2007 Cutting-edge terahertz technology Nat. Photonics 1 97–105
[12] Ferguson B and Zhang X-C 2002 Materials for terahertz science and technology Nat. Mater. 1 26–33
[13] Soref R 2010 Mid-infrared photons in silicon and germanium Nat. Photonics 4 495–7
[14] Du X, Skachko I, Barker A and Andrei E Y 2008 Approaching ballistic transport in suspended graphene Nat. Nanotechnology 3 491–5
[15] Dean C R et al 2010 Boron nitride substrates for high-quality graphene electronics Nat. Nanotechnology 5 722–6
[16] Huang J and Encinar J A 2008 Reflect array Antennas (USA: IEEE Press)
[17] Chen J et al 2012 2013 Terahertz metamaterial devices based on graphene nanoostructures IEEE Trans. Terahertz Sci. Tech. 3 748–56
[18] Padolou R Y, Yakolev A B, Kaipa C S R, Hanson G W, Medina F and Mesa F P 2013 Dual capacitive–inductive nature of periodic graphene patches: transmission characteristics at low terahertz frequencies Phys. Rev. B 87 115401
[19] Lee S H et al 2012 Switching terahertz waves with gate-controlled active graphene metamaterials Nat. Mater. 11 936–41
[20] Otsuji T, Tombet S B, Satou A, Fukidome H, Suemitsu M, Sano E, Popov V, Ryzhii M and Ryzhii V 2012 Graphene-based devices in terahertz science and technology J. Phys. D: Appl. Phys. 45 303001
[21] Vicarelli L, Vitiello M S, Coquillat D, Lombardo A, Ferrari A C, Knap W, Polini M, Pellegrini V and Tredicucci A 2012 Graphene field-effect transistors as room-temperature terahertz detectors Nat. Mater. 11 865–71
[22] Bao Q and Loh K P 2012 Graphene photonics, plasmonics, and broadband optoelectronic devices ACS Nano 6 5677–94
[23] Bonaccorso F, Sun Z, Hasan T and Ferrari A C 2010 Graphene photonics and optoelectronics Nat. Photonics 4 611–22
[24] Hum S and Perruisseau-Carrier J 2013 Reconfigurable reflectarrays and array lenses for dynamic antenna beam control: a review IEEE Trans. on Antennas and Propagation 62 183–198
[25] Yu N, Genevet P, Kats M A, Aieta F, Tetienne J-P, Capasso F and Gaburro Z 2011 Light propagation with phase discontinuities: generalized laws of reflection and refraction Science 334 333–7
[26] Ni X, Emani K N, Kildishev A V, Boltassee A and Shalaev V M 2012 Broadband light bending with plasmonic nanoantennas Science 335 427–427
[27] Nikitin A Y, Low T and Martin-Moreno L 2014 Anomalous reflection phase of graphene plasmons and its influence on resonators Phys. Rev. B 90 041407
[28] Efetov D K and Kim P 2010 Controlling electron–phonon interactions in graphene at ultrahigh carrier densities Phys. Rev. Lett. 105 256805
[29] Wunsch B, Stauber T, Sols F and Guinea F 2006 Dynamical polarization of graphene at finite doping New J. Phys. 8 318
[30] Hwang E H and Sarma S D 2007 Dielectric function, screening, and plasmons in two-dimensional graphene Phys. Rev. B 75 205418

[31] Palik E D (ed) 1998 Handbook of Optical Constants of Solids vol 3 (New York: Academic)

[32] Carrasco E, Tamagnone M and Perruisseau-Carrier J 2013 Tunable graphene reflective cells for THz reflectarrays and generalized law of reflection Appl. Phys. Lett. 102 104103

[33] Kamoda H, Iwasaki T, Tsumochi J, Kuki T and Hashimoto O 2011 60 GHz electronically reconfigurable large reflectarray using single-bit Phase shifters IEEE Trans. Antennas Propag. 59 2524–31

[34] Woessner A et al 2014 Highly confined low-loss plasmons in graphene–boron–nitride heterostructures Nat. Materials at press (doi:10.1038/nmat4169)