A Constituent Codes Oriented Code Construction Scheme for Polar Code-Aim to Reduce the Decoding Latency

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Abstract—This paper proposes a polar code construction scheme that reduces constituent-code related decoding latency. Constituent codes are the sub-codewords with specific patterns. They are used to accelerate the successive cancellation decoding process of polar code with negligible performance degradation. We modify the traditional construction approach to yield increased number of desirable constituent codes that speeds the decoding process. For \((n, k)\) polar code, instead of directly setting the \(k\) best and \((n-k)\) worst bits to the information bits and frozen bits, respectively, we swap the locations of some information and frozen bits carefully according to the qualities of their equivalent channels. We conducted the simulation of 1024 and 2048 bits length polar codes with multiple rates and analyzed the decoding latency for various length codes. The numerical results show that the proposed construction scheme generally is able to achieve at least around 20% latency deduction with an negligible loss in gain with carefully selected optimization threshold.

I. INTRODUCTION

Recently, polar code [1] attract more and more research interests due to that it is the first code which provably achieves the channel capacity and its low coding complexity. Such property makes it very promising for real scenario such as wireless communication and storage. Successive cancellation (SC) [1], list successive cancellation (LSC) [2] and belief propagation (BP) [3] are the three most widely known decoding algorithms. Among those, SC and LSC receive more attention due to their simpler hardware complexity compared with that of BP. Due to its serial property, SC decoder suffers from the high decoding latency. LSC considered as an extension of SC, similarly, has the same problem. The latency reduction of SC decoder is able to benefit the LSC decoder as well. Thus, a lot of efforts have been done on the SC decoder to reduce the latency from both hardware and algorithm aspects.

C. Leroux [4] proposed both tree and line architecture of SC decoder. For length \(n\) polar code, it takes \((2n-2)\) clock cycles to decode. Later on, he [5] proposed a semi-parallel architecture for both tree and line SC decoder, which makes a trade-off between hardware complexity and latency. C. Zhang [6] proposed a low latency SC decoder with pre-computing and overlapped architecture. Pre-computing technology reduces the latency to \((n-1)\) clock cycles, and the overlap scheme significantly the throughput with multiple frame situation. B. Yuan [7] proposed an architecture with applying the 2-bit decoding at the last stage and some gate level optimizations, this further reduce the latency to \((3/4n-1)\) clock cycles. Alamdar-Yazdi [8] proposed the simplified SC (SSC) which can significantly reduce the latency via some certain pattern sub-codewords. These kinds of sub-codewords are also called constituent codes. This is the first time the concept of constituent codes has been mentioned. Inspired by this, Sarkis [9] proposed the fast-SSC which can further reduce the latency by exploring more kinds of constituent codes. T. Che [10] proposed the hardware architecture of constituent codes based polar codes decoder. It allows the decoding processes are compatible with both conventional and constituent codes based polar codes. P. Giard [11] proposed an unrolled architecture with fast-SSC is able to achieve 237 Gbps throughput. P. Giard [12] also proposed a low-complexity decoder for low rate polar code. In that work, he further utilized the potential of constituent codes by changing the frozen and information sets. Additionally, the idea of constituent codes also benefits other decoding algorithm. J. Xu [13] applied the constituent concept to the BP decoding which significantly reduces the computing complexity. G. Sarkis [14] proposed an constituent codes based LSC decoding. T. Che [15] also pointed that the constituent codes can benefit the overlapped LSC architecture in term of hardware efficiency.

Introducing constituent code is an approach to reduce decoding latency. Most of the aforementioned works stress on the decoding sides. In this paper, we explore the potential of constituent codes from an opposite angle. We stress on the construction scheme to make the codeword more constituent-code-friendly. By adjusting the traditional construction approach, more expected types of constituent codes are manually produced for decoding. For \((n, k)\) polar code, instead of directly setting the \(k\) best and \((n-k)\) worst bits to the information bits and frozen bits, respectively, we thoughtfully swap the locations of some information and frozen bits according to the qualities of their equivalent channels. The constituent codes oriented polar code construction algorithm is described. We conducted the simulation of 1024 and 2048 length polar codes with multiple rates and analyzed the decoding latency for various length codes. The numerical result shows that the proposed construction scheme typically achieves 4-20% latency reduction with negligible loss in decoding performance with carefully selected optimization threshold. Some relevant discussions are also presented.

This paper is organized as follows. The relative background is reviewed in section II. Then, the proposed construction scheme are described in section III. After that, the numerical results and relevant discussions are presented in section IV. Finally, this paper is concluded in section V.
II. BACKGROUND

A. Polar code

As described by E. Arikan [1], a polar code is constructed by successively performing channel polarization. Polar codes are linear block codes of length $n = 2^m$. The coded codeword $\mathbf{x} \equiv (x_1, x_2, \ldots, x_n)$ is computed by $\mathbf{x} = \mathbf{uG}$ where $G = F^\otimes m$, and $F^\otimes m$ is the $m$-th Kronecker power of $F = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. Each row of $G$ corresponds to an equivalent polarizing channel. For an $(n, k)$ polar code, $k$ bits that carry source information are called information bits. They are transmitted via the $k$ best channels. While the rest $n-k$ bits, called frozen bits, are set to zeros and are placed at the $n-k$ worst channels. Fig. 1(a) shows an example of the construction of an 8-bit polar code, where information bits and frozen bits are denoted by black nodes and white nodes on the most left side, respectively.

Polar codes can be decoded by recursively applying successive cancellation to estimate $\hat{u}_i$ using the channel output $y_0^{n-1}$ and the previously estimated bits $\hat{u}_0^{i-1}$. This method is referred to as successive cancellation (SC) decoding. Actually, SC decoding can be regarded as a binary tree traversal as described in Fig. 1(b). The number of bits of one node in stage $m (m = 0, 1, 2, \ldots)$ is equal to $2^m$. The soft reliability value, typically is log-likelihood ratio (LLR). Each left and right child nodes can calculate the LLR for current node via $f$ and $g$ functions, respectively [9]. However, in order to compute $g$ function, a feedback $\beta_t$ from left child of the same parent node is needed. This kind of feedback is called partial sum. Actually, this serial property of feedback operation limits the throughput of SC decoding.

B. Constituent codes based SC decoding

The recursive processing of getting the partial sum from each node significantly constrains the decoding speed. Thus, in order to obtain partial sum directly without performing tree traversal, constituent codes based SC decoding has been proposed [8], [9]. Certain patterns in the codewords allows us to decode the sub-codewords and get their corresponding partial sums immediately, which significantly reduces the partial-sum-constrained latency.

$\mathcal{N}^0$, $\mathcal{N}^1$, $\mathcal{N}^{SPC}$ and $\mathcal{N}^{REP}$ are the four most common constituent codes. $\mathcal{N}^0$ and $\mathcal{N}^1$ only contain either frozen bits or information bits, respectively. $\mathcal{N}^{SPC}$ and $\mathcal{N}^{REP}$ contain both frozen bits and information bits. In the $\mathcal{N}^{SPC}$ codes, only the first bit is frozen. It makes the length $n$ constituent codes as a rate $(n-1)/n$ single parity check (SPC) code. In the $\mathcal{N}^{REP}$ codes, only the last bit is information bit. In this case, all the corresponding partial sums should be the same since they all are the reflections of the last information bit. All the above four constituent codes can be decoded very quickly. According to T. Che’s implementation [10], the latency of length $n$ constituent code can be reduced from $2n - 2$ to $1, log_2(n + 1) + 1$ and $log_2 n$ for $\mathcal{N}^0$, $\mathcal{N}^1$, $\mathcal{N}^{SPC}$ and $\mathcal{N}^{REP}$ codes, respectively. Fig. 2 shows an example of how constituent code can simplify the SC decoding tree.

III. PROPOSED CONSTRUCTION SCHEME

As described before, constituent codes are decoded faster than conventional polar codes. Thus, in order to reduce the decoding latency, we expect more constituent codes, especially constituent codes with large length. According to the definition of constituent codes, the initial distribution of constituent codes is determined by the location of information and frozen bits. If we can change the locations, we are able to manually produce expected constituent codes. However, the location of frozen and information bits are very sensitive, and random changes may cause negative influence on the coding performance. Thus, a thoughtful construction scheme to produce more expected constituent code is on demand. The division of information and frozen bits is decided by the qualities of the equivalent channels which are corresponding to each bit. Based on the channel model, the equivalent channel qualities can be calculated accordingly [11] [16] [17] [18]. This gives us a hint that if we swap some information and frozen bits those with similar channel qualities, this might only incur a very slight performance loss. The numerical simulation results in the following section prove this idea. In this work, binary erasure channel (BEC) is used as our channel model, and thus Bhattacharyya parameter is used as the metric for equivalent channel quality. This method can be extended to any other kind of channel model.

Now, we have the idea about how to change the division of constituent codes. Next, we need to consider what kind of changes are desired. For any length $n$ polar code, it can be regarded as a combination of the following four types of sub-codewords.

- **Type-I:** All the bits are frozen bits. This is also $\mathcal{N}^0$ constituent code.
- **Type-II:** All the bits are information bits. This is also $\mathcal{N}^1$ constituent code.

![Fig. 2. SC decoding tree simplified by constituent codes](image-url)
Type-III: Only one bit is information bit, the rest are frozen bits. This can be regarded as the combination of one $N^\text{REP}$ and multiple $N^0$ constituent codes.

Type-IV: Only one bit is frozen bit, the rest are information bits. This can be regarded as the combination of one $N^\text{SPC}$ and multiple $N^1$ constituent codes.

According to T. Che’s results [10], there is only one clock cycle needed for decoding $N^0$ and $N^1$ node, thus, it is unnecessary to optimize type-I and type-II codes since they are already fully optimized. Our target should be focused on the type-III and type-IV. These two types are similar, they all only have one node with different type with others. There are two situations we need to deal with. The first situation is the optimization for single type-III or type-IV sub-tree. We can swap the different node between the two types to make it a $N^\text{SPC}$ or $N^\text{REP}$ nodes for type-IV or type-III sub-tree, respectively. The second situation is that the optimization for a combination of type-III and type-IV sub-trees. For this case, we can swap the different node between the two types to make them became one type-I and one type-II sub-trees. For the first situation, suppose we have one Type-III node at stage $m + 1$. It consist of one $N^\text{REP}$ and one $N^0$ node at stage $m$. This is shown in Fig. 3(a). Totally, it needs $1 + \log_2 2^m = m + 1$ to finish decoding. If we move the place of the information bit to make it a $N^\text{REP}$ constituent code, the new latency for decoding should be $\log_2 2^{m+1} = m + 1$. There is no change if we do this modification. This is also similar to type-IV situation. For the second situation, suppose we have one type-III and one type-IV nodes at stage $m$ as shown in Fig. 3(b) the totally latency for decoding should be $2m + 1$. If we swap the information bit in type-III and frozen bit in type-IV, we get one $N^0$ and one $N^1$ nodes. The total should be reduced to 2. This makes a huge difference. Thus, our target should be the swap operation between type-III and type-IV nodes.

Based on above discussion, the constituent code oriented polar code construction algorithm is proposed in algorithm (1).

Algorithm 1 constituent code oriented polar code construction

1. get the sub-codeword-type look up table $T$ from $L$; this table gives the sub-codeword type information and its index
2. if $T[index]$ is type-III sub-codeword then
3. get the index $i$ of the information bit in $T[index]$, search all the next type-IV sub-codewords and find the one whose frozen bit’s Bhattacharyya parameter has the minimum difference with $\epsilon_n[f]$. Recode the index $f$ of that.
4. if $|\epsilon_n[i] - \epsilon_n[f]| < T_h$ then
5. swap the property of $L[i]$ and $L[f]$, update $T$.
6. end if
7. end if
8. if $T[index]$ is type-IV sub-codeword then
9. get the index $f$ of the frozen bit in $T[index]$, search all the next type-III sub-codewords and find the one whose information bit’s Bhattacharyya parameter has the minimum difference with $\epsilon_n[f]$. Recode the index $i$ of that.
10. if $|\epsilon_n[i] - \epsilon_n[f]| < T_h$ then
11. swap the property of $L[i]$ and $L[f]$, update $T$.
12. end if
13. end if
14. increase index by 1, repeat to 2, until to the end of $T$.

According to Fig. 4 and Table I, we note that the proposed construction scheme generally is able to achieve at least around 20% latency reduction with an negligible gain loss. For a certain code length, we can find that the acceptable threshold is increasing along with the code rate. This is due to the nature of channel polarization. There are more frozen and information bits mixed in the front and middle part of codeword for lower rate codes, which gives more flexibility during the optimization. However, this does not indicate that this coding scheme is not working well on high rate. The interesting part is that the performance on high rate is as good as the low rate according to Fig. 4. It’s possibly attributable to the following two reasons. First is that the coding performance of high rate itself is much worse than that of low rate. This causes the difference after optimization is not so obvious. The second reason is that the simulated code length is still not long enough due to the limitation of simulation environment. This is per coding theory that suggests the longer polar code will generally yield higher polarization. For a certain code rate, we can find the acceptable threshold is decreasing along with the code length. Since the longer codes are more polarized, the difference of each equivalent threshold is decreasing along with the code length. This works fine with low and medium length but not so obvious with high length. This also can be explained by the two reason presented before.
TABLE I. LATENCY REDUCTION

| decoder | length | rate | threshold | latency | reduction(%) |
|---------|--------|------|-----------|---------|-------------|
| no optimization | 0.3 | 1e-13 | 303 | 288 | 4.9 |
| no optimization | 1e-12 | 260 | 14 |
| no optimization | 1e-11 | 234 | 22.7 |
| 1e-4 | 245 | - |
| 5e-4 | 218 | 18 |
| 1e-3 | 197 | 21.8 |
| no optimization | 0.7 | 172 | - |
| 0.1 | 165 | 4 |
| 0.2 | 137 | 20 |
| 0.4 | 126 | 23.6 |
| no optimization | 0.3 | 576 | - |
| 1e-18 | 549 | 4.6 |
| 1e-17 | 519 | 9.9 |
| 1e-16 | 493 | 14.4 |
| no optimization | 0.5 | 493 | - |
| 1e-6 | 487 | 1.2 |
| 1e-5 | 436 | 10.5 |
| 1e-4 | 323 | 33.6 |
| no optimization | 0.7 | 297 | - |
| 0.1 | 269 | 9.4 |
| 0.2 | 248 | 16.5 |
| 0.3 | 228 | 23.2 |
| no optimization | 0.3 | 3992 | - |
| 1e-50 | 3661 | 8.3 |
| 1e-45 | 3242 | 18.8 |
| 1e-40 | 2724 | 31.8 |
| no optimization | 0.5 | 2777 | - |
| 1e-13 | 3187 | 4.2 |
| 1e-12 | 2898 | 12.9 |
| 1e-11 | 2465 | 25.9 |
| no optimization | 0.7 | 1350 | - |
| 0.1 | 1260 | 6.6 |
| 0.2 | 1165 | 15.7 |
| 0.4 | 894 | 33.4 |

We compared the latency of proposed design with the decoder in [7]; this is the fastest non-constituent-codes-based polar code decoder to the best of our knowledge. We can see even constituent codes decoder without any optimization is much faster than that. Our proposed construction scheme is capable of achieving 20% or more latency reduction. This is very significant especially for very long code lengths.

Compared with [12] in which also changes the frozen and information sets to benefit the decoding, our work has two main differences. The first one is that we target a different decoder architecture, which results in different demanding of desirable constituent codes combination. The second difference is that the construction algorithm is different.

V. CONCLUSION

This paper presented a novel polar code construction scheme which reduces the decoding latency. The proposed constituent codes oriented polar code construction algorithm can automatically produce more types of constituent codes which are desirable. The simulation results show that the proposed construction scheme generally is able to achieve at least around 20% latency reduction with negligible decoding performance loss. Besides, compared with non-constituent-codes-based decoder, the constituent codes based decoder has a measurable advantage in term of latency. Our construction scheme is able to further enhance the timing performance.

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