New observational constraints on $f(R)$ gravity from cosmic chronometers

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ABSTRACT: We use the recently released cosmic chronometer data and the latest measured value of the local Hubble parameter, combined with the latest joint light curves of Supernovae Type Ia, and Baryon Acoustic Oscillation distance measurements, in order to impose constraints on the viable and most used $f(R)$ gravity models. We consider four $f(R)$ models, namely the Hu-Sawicki, the Starobinsky, the Tsujikawa, and the exponential one, and we parametrize them introducing a distortion parameter $b$ that quantifies the deviation from $\Lambda$CDM cosmology. Our analysis reveals that a small but non-zero deviation from $\Lambda$CDM cosmology is slightly favored, with the corresponding fittings exhibiting very efficient $AIC$ and $BIC$ Information Criteria values. Clearly, $f(R)$ gravity is consistent with observations, and it can serve as a candidate for modified gravity.

KEYWORDS: Modified gravity, $f(R)$ gravity, Dark energy, Observational constraints, Cosmic chronometers, Information Criteria
1 Introduction

According to the concordance model of cosmology the universe must have experienced two accelerated expansion phases, at early and late times respectively. This behavior cannot be reproduced within the standard framework of general relativity and Standard Model of particles, and thus extra degrees of freedom should be introduced. Firstly, one can attribute these extra degrees of freedom to new, exotic forms of matter, such as the inflaton field at early times (for reviews see [1, 2]) and/or the dark energy concept at late times (for reviews see [3, 4]). Alternatively, one can consider the extra degrees of freedom to have gravitational origin, i.e. to arise from a gravitational modification that possesses general relativity as a particular limit (see [5, 6] and references therein). Note that the latter approach has the additional advantage that it might improve renormalizability and thus alleviate the difficulties towards quantization [7, 8].

In the usual approach to gravitational modification one adds higher-order corrections to the Einstein-Hilbert action. The simplest such modification arises from extending the Ricci scalar $R$ to an arbitrary function $f(R)$, which can lead to interesting behavior at early times [9], as well as explain the late-time acceleration [10-21], or describe both phases in a unified...
way [22–24], and can explain the large scale structure distribution in the universe [25, 26]. However, one can construct many other classes of curvature-based modified gravities, such as \( f(G) \) gravity [27, 28], Lovelock gravity [29, 30], Weyl gravity [31, 32], Galileon theory [33–36], or even extend to torsion-based modifications, such as \( f(T) \) gravity [37–42], \( f(T, T_G) \) gravity [43–45], etc.

An important and probably the most justified question in all gravitational modifications is what is the choice of the involved arbitrary function. A first constraining of the possible forms comes from theoretical arguments, such that the requirement for a ghost-free theory that possesses stable perturbations [10], or the desire for the theory to possess Noether symmetries [46, 47]. However, in order to further constrain the remaining huge class of theories the main tool is the use of observational data and the requirement for a successful reproduction of the universe history, as well as of the local/solar system behavior. In the case of \( f(R) \) gravity such a confrontation with cosmological data was performed in [48] (using data from cosmic microwave background (CMB) probes), in [49, 50] (using Large Scale Structure (LSS) data), in [51] (using neutron stars mass-radius data), in [52] (using Supernovae type Ia (SNe Ia) and Hubble Parameter data), in [53] (using SNe Ia and CMB data), in [54, 55] (using SNe Ia, CMB and data from baryonic acoustic oscillations (BAO) probes), in [56] (using Hubble Parameter and BAO data), in [57] (using CMB, BAO, Hubble Parameter data), in [58–61] (using SNe Ia, CMB, and growth rate data), in [62] (using SNe Ia, CMB, BAO, Hubble Parameter and cluster abundance constraints), and in [63] (using SNe Ia, CMB, BAO, Hubble Parameter, gravitational lenses and growth rate data). Additionally, the comparison with solar system data was performed in [64–70]. Hence, constructions that pass all the above constraints are called viable models.

In the present work we intend to provide updated observational constraints on \( f(R) \) gravity models, using the latest released cosmic chronometer data set and the latest released local value of the Hubble parameter with 2.4% precision, along with the standard cosmological probes for dark energy analysis, such as Supernovae type Ia and baryonic acoustic oscillations data. In particular, we will consider four viable \( f(R) \) models, namely (i) Hu-Sawicki model, (ii) Starobinsky model, (iii) Tsujikawa model and finally (iv) exponential \( f(R) \) model, and we will provide the updated constraints and contour plots for the involved parameters. The plan of the manuscript is the following: In section 2 we briefly review \( f(R) \) gravity and its cosmological application, focusing on four viable \( f(R) \) models. In section 3 we describe the data sets used for the observational confrontation, while in section 4 we provide the results of our analysis, namely the updated observational constraints on the various model parameters and observational quantities. Finally, we close our work in section 5, with a summary and discussion.

2 \( f(R) \) gravity and cosmology

In this section we briefly review \( f(R) \) gravity and we proceed to its cosmological application. Then we examine four specific \( f(R) \) models, which, amongst the variety of \( f(R) \) scenarios, pass the basic theoretical and observational tests and thus they are considered as viable ones.
2.1 $f(R)$ gravity

In $f(R)$ gravitational theories one extends the Einstein-Hilbert action to

$$S = \int d^4x \sqrt{-g} \frac{f(R)}{16\pi G} + S_m + S_r,$$  

(2.1)

with $R$ the Ricci scalar and $G$ the gravitational constant, and where we have also considered the actions for the matter and radiation sectors, $S_m$ and $S_r$ respectively. Following the metric formulation, variation of the action (2.1) with respect to the metric $g_{\mu\nu}$ leads to the field equations

$$FG_{\mu\nu} = -\frac{1}{2}g_{\mu\nu}(FR - f) + \nabla_{\mu}\nabla_{\nu}F - g_{\mu\nu}\Box F + 8\pi G \left[T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(r)}\right],$$  

(2.2)

with $G_{\mu\nu} = R_{\mu\nu} - (1/2)g_{\mu\nu}R$ the Einstein tensor, $\nabla_{\mu}$ the covariant derivative, $\Box \equiv g^{\mu\nu}\nabla_{\mu}\nabla_{\nu}$, and where we have defined $F(R) \equiv f_{,R} = df(R)/dR$. Additionally, $T_{\mu\nu}^{(m)}$ and $T_{\mu\nu}^{(r)}$ are respectively the energy-momentum tensors for the matter and radiation sectors, corresponding to $S_m$ and $S_r$.

Before proceeding, and for completeness, we mention that apart from the above metric (or second order) formulation of $f(R)$ gravity, in which the field equations are derived through variation of the action with respect to the metric tensor, and where the affine connection depends only on the metric, one could have the Palatini (or first order) formulation, where the metric and the connection are treated as independent variables in the action variation, under the assumption that the matter part of the action does not depend on the connection [10]. For a general $f(R)$ form these two approaches lead to different field equations, and only in the General Relativity case, i.e for $f(R) = R$, the two formulations coincide. Finally, one could also have the metric-affine formulation, in which the Palatini variation is used but without the additional assumption that the matter action is connection-independent (the metric-affine formulation reduces to metric or Palatini formulations if extra considerations are made). In the present work we focus on the standard metric formulation, since Palatini formalism might exhibit difficulties in being compatible with observations and experiments, as well as it faces problems with the formulation of the Cauchy problem due to the presence of matter fields higher-derivatives in the field equations (see [10] and the references therein).

2.2 $f(R)$ cosmology

We now proceed to the cosmological application of $f(R)$ gravity. Hence, we consider the usual homogeneous and isotropic geometry, characterized by the Friedmann-Robertson-Walker (FRW) background metric

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{(1 - kr^2)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)\right],$$  

(2.3)

with $a(t)$ the scale factor and $k$ the spatial curvature (with $k = 0, -1, +1$ for flat, open and closed universe respectively). Focusing for simplicity to the flat case, and inserting the
FRW metric into the field equations (2.2), we obtain the modified Friedmann equations

\[ 3FH^2 = 8\pi G (\rho_m + \rho_r) + \frac{1}{2} (FR - f) - 3H \dot{F}, \tag{2.4} \]

\[ -2F \ddot{H} = 8\pi G (\rho_m + P_m + \rho_r + P_r) + \ddot{F} - H \dot{F}, \tag{2.5} \]

where \( H \equiv \dot{a}/a \) is the Hubble parameter, with dot denoting derivatives with respect to the cosmic time \( t \). Furthermore, we have considered that the matter and radiation sectors correspond to perfect fluids with energy densities \( \rho_m, \rho_r \) and pressures \( P_m, P_r \) respectively. Finally, note that in flat FRW geometry one obtains the useful relation

\[ R = 6 \left( 2H^2 + \dot{H} \right). \tag{2.6} \]

Observing the form of the Friedmann equations (2.4), (2.5), and comparing to the usual ones, namely

\[ 3H^2 = 8\pi G (\rho_m + \rho_r + \rho_{DE}) \]

as well as

\[ -2\dot{H} = 8\pi G (\rho_m + P_m + \rho_r + P_r + \rho_{DE} + P_{DE}), \]

we deduce that in the scenario at hand we obtain an effective dark energy sector, with dark energy density and pressure defined as

\[ \rho_{DE} \equiv \frac{1}{8\pi G} \left[ \frac{1}{2} (FR - f) - 3H \dot{F} + 3(1 - F) H^2 \right], \tag{2.7} \]

\[ P_{DE} \equiv \frac{1}{8\pi G} \left[ -\frac{1}{2} (FR - f) + \ddot{F} + 2H \dot{F} - (1 - F) \left( 2\dot{H} + 3H^2 \right) \right], \tag{2.8} \]

while its effective equation-of-state parameter reads:

\[ w \equiv P_{DE}/\rho_{DE}. \tag{2.9} \]

One can easily see that \( \rho_{DE} \) and \( P_{DE} \) defined in (2.7), (2.8) satisfy the usual evolution equation

\[ \dot{\rho}_{DE} + 3H(\rho_{DE} + P_{DE}) = 0. \tag{2.10} \]

Finally, the equations close considering the standard matter and radiation evolution equations, namely

\[ \dot{\rho}_m + 3H(\rho_m + P_m) = 0, \tag{2.11} \]

\[ \dot{\rho}_r + 3H(\rho_r + P_r) = 0, \tag{2.12} \]

respectively.

### 2.3 Specific \( f(R) \) models

In this subsection we review the most used and viable \( f(R) \) models. First of all, a given \( f(R) \) model must satisfy some basic theoretical constraints, namely to possess a positive effective gravitational constant, as well as to exhibit stable cosmological perturbations \cite{10}. In particular, one should have

\[ f_R > 0 \text{ for } R \geq R_0, \tag{2.13} \]
with $R_0$ the present value of the Ricci scalar, in order to avoid a ghost state, and
\[ f_{,RR} > 0 \quad \text{for} \quad R \geq R_0, \quad (2.14) \]
in order to avoid the scalar-field degree of freedom to become tachyonic. Additionally, a given $f(R)$ model must satisfy some basic observational requirements. Specifically, one should have
\[ f(R) \to R - 2\Lambda \quad \text{for} \quad R \geq R_0, \quad (2.15) \]
in order to be able to reproduce the matter era and to obtain consistency with equivalence principle and local gravity constraints, and
\[ 0 < \frac{Rf_{,RR}}{f_{,R}}(r) < 1 \quad \text{at} \quad r = -\frac{Rf_{,R}}{f} = -2, \quad (2.16) \]
in order to have the presence and stability of a late-time de Sitter solution [10]. Hence, considering viable models that have up to two parameters, one can write them as
\[ f(R) = R - 2\Lambda y(R, b), \quad (2.17) \]
where the function $y(R, b)$ quantifies the deviation from Einstein gravity, i.e. the effect of the $f(R)$ modification, through the distortion parameter $b$.

Having these in mind, one can construct four viable $f(R)$ models, that have been investigated in detail in the literature, which are given below.

1. The Hu-Sawicki model [70].

This model corresponds to
\[ f(R) = R - \frac{c_1 R_{\text{HS}} (R/R_{\text{HS}})^p}{c_2 (R/R_{\text{HS}})^p + 1}, \quad (2.18) \]
where $c_1$, $c_2$ and $R_{\text{HS}}$ are parameters and $p > 0$ a positive constant. Note that not all of these parameters are independent since, using the first Friedmann equation (2.4) at present, one of them can be eliminated in favor of the present values of the density parameters $\Omega_i_0 = \frac{8\pi G \rho_i_0}{3H_0^2}$ as well as the present value of the Hubble function $H_0$ (the subscript “0” denotes the current value of a quantity). One can easily rewrite (2.18) to the form (2.17), with
\[ y(R, b) = 1 - \frac{1}{1 + \left(\frac{R}{M_b}\right)^p}, \quad (2.19) \]
where the two free model parameters read as $\Lambda = \frac{c_1 R_{\text{HS}}}{2c_2}$ and $b = 2c_2^1 - 1/p / c_1$. Hence, one can see that for $b \to 0$ (i.e. for $c_1 \to \infty$, $R_{\text{HS}} \to 0$, with $c_1 R_{\text{HS}} \to 2c_2\Lambda$) the Hu-Sawicki model reduces to $\Lambda$CDM cosmology since $f(R) \to R - 2\Lambda$. We mention here that the above reduction/mapping to two parameters (plus $p$) offers an effective way in order to be able to investigate the fittings on all parameters through a reconstruction method via error propagation. In principle one could try to fit all parameters independently, however the existing data (in terms of quantity and precision) cannot lead to a good precision fits.
2. The Starobinsky model [71].

This model corresponds to

$$f(R) = R - \lambda R_S \left[ 1 - \left( 1 + \frac{R^2}{R_S^2} \right)^{-n} \right],$$  \hspace{1cm} (2.20)

with $\lambda(> 0)$ and $R_S$ the free parameters and $n > 0$ a positive constant. One can rewrite (2.20) to the form (2.17), with

$$y(R, b) = 1 - \frac{1}{1 + \left( \frac{R}{\Lambda b} \right)^2},$$  \hspace{1cm} (2.21)

where $\Lambda = \lambda R_S/2$ and $b = 2/\lambda$. Thus, for $b \to 0$ (i.e. for $\lambda \to \infty$, $R_S \to 0$, with $\lambda R_S \to 2\Lambda$) the Starobinsky model reduces to $\Lambda$CDM cosmology, namely $f(R) \to R - 2\Lambda$. Note that the mapping to two parameters (plus $n$) is performed similarly to the previous Hu-Sawicki model.

3. The Tsujikawa model [72].

This model corresponds to

$$f(R) = R - \mu R_T \tanh \left( \frac{R}{R_T} \right),$$  \hspace{1cm} (2.22)

where $\mu(> 0)$ and $R_T(> 0)$ are two positive constants. One can rewrite (2.22) as (2.17), defining

$$y(R, b) = \tanh \left( \frac{R}{b\Lambda} \right),$$  \hspace{1cm} (2.23)

where $\Lambda = \mu R_T/2$, and $b = 2/\mu$. The Tsujikawa model reduces to $\Lambda$CDM cosmology for $b \to 0$ (i.e. for $\mu \to \infty$, $R_T \to 0$, with $\mu R_T \to 2\Lambda$).

4. The exponential gravity model [73–75].

This case corresponds to

$$f(R) = R - \beta R_E \left( 1 - e^{-R/R_E} \right),$$  \hspace{1cm} (2.24)

with $\beta$, $R_E$ the model parameters. One can rewrite (2.24) to the form (2.17), with

$$y(R, b) = 1 - e^{-R/(\Lambda b)},$$  \hspace{1cm} (2.25)

where $\Lambda = \beta R_E/2$, and $b = 2/\beta$. This model reduces to $\Lambda$CDM cosmology for $b \to 0$ (i.e. for $\beta \to \infty$, $R_E \to 0$, with $\beta R_E \to 2\Lambda$).
3 Current Observational Data

In this work we are interested in constraining $f(R)$ gravity using observational data acquired from probes that map the expansion history of the late-time universe, namely lying in the redshift region $z < 2.36$. The main ingredient of our analysis is the Hubble parameter measurements obtained with the cosmic chronometers (CC) technique, which are the latest and model-independent measurements of the Hubble parameter, and thus provide better constraints on a cosmological model. In addition, we consider standard probes such as Supernovae Type Ia (SNe Ia), local Hubble parameter value $H_0$ ones, and Baryon Acoustic Oscillation (BAO) distance measurements, in order to reduce the degeneracy between the free parameters of the models. We mention here that it would be both interesting and necessary to try to constrain $f(R)$ gravity on smaller scales, too. Although at galaxies and smaller scales the effect of modified gravity is expected to be very small and hardly detectable, indeed at galaxy clusters it might lead to observational constraints. This interesting subject lies beyond the scope of the present work, and it is left for a future project. The following subsections describe the employed data sets for our analysis.

3.1 Cosmic chronometer dataset and local value of the Hubble constant

The Cosmic Chronometer (CC) approach is a very powerful implementation in understanding the universe evolution. It was first introduced in [76], and the method determines the Hubble parameter data through the differential age evolution of the passively evolving early-type galaxies. Since the Hubble parameter for FRW universe can be expressed as $H = -(1 + z)^{-1}dz/dt$, by measuring the quantity $dz/dt$, one can directly measure the Hubble parameter data. Hence, the CC data are very powerful in order to provide better constraints on cosmological models. For a detailed description on the implementation of CC data, all possible kind of uncertainties, as well as some related issues, we refer the reader to [77]. Here we consider the compilation of Hubble parameter measurements as provided in [77, 78]. The data set contains 30 $H(z)$ measurements [77–81] obtained through the CC approach in the redshift range $0 < z < 2$, and it roughly covers about 10 Gyr of cosmic time. Moreover, in addition to the CC data, in our investigation we include the new local value of $H_0$ as measured by [82] with a 2.4 % determination, which yields $H_0 = 73.02 \pm 1.79$ km s$^{-1}$ Mpc$^{-1}$.

3.2 Type Ia Supernovae

SNe Ia provided the first signal for a universe acceleration [83, 84], and they serve as the main observational data set to probe the late-time, dark-energy epoch. In this work we consider the latest “joint light curves” (JLA) sample [85] containing 740 SNe Ia in the redshift range $z \in [0.01, 1.30]$. From the observational point of view, the distance modulus of a SNe Ia can be abstracted from its light curve, assuming that supernovae with identical color, shape and galactic environment, have on average the same intrinsic luminosity for all redshifts. This hypothesis is quantified by an empirical linear relation, yielding a standardized distance modulus $\mu = 5 \log_{10}(d_L/10pc)$ of the form

$$\mu = m_B^* - (M_B - \alpha \times X_1 + \beta \times C),$$  

(3.1)
where \( m^*_B \) corresponds to the observed peak magnitude in rest frame B band and \( \alpha, \beta, \) and \( M_B \) are nuisance parameters in the distance estimate. The absolute magnitude is related to the host stellar mass \( (M_{\text{stellar}}) \) by a simple step function: \( M_B = M_B \) if \( M_{\text{stellar}} < 10^{10} M_\odot \), otherwise \( M_B = M_B + \Delta M \). The light-curve parameters \( (m^*_B, X_1 \) and \( C) \) result from the fit of a model of the SNe Ia spectral sequence to the photometric data. In our analysis we assume \( M_B, \Delta M, \alpha \) and \( \beta \) as nuisance parameters.

### 3.3 Baryon Acoustic oscillation

Another potential cosmological test comes from the baryon acoustic oscillations (BAO) data. In our analysis, we adopt the following BAO data to constrain the expansion history of the universe: the measurement from the Six Degree Field Galaxy Survey (6dF) \[86\], the Main Galaxy Sample of Data Release 7 of Sloan Digital Sky Survey (SDSS-MGS) \[87\], the LOWZ and CMASS galaxy samples of the Baryon Oscillation Spectroscopic Survey (BOSS-LOWZ and BOSS-CMASS, respectively) \[88\], and the distribution of the LymanForest in BOSS (BOSS-Ly) \[89\]. These measurements and their corresponding effective redshift \( z \) are summarized in Table 1.

| Survey          | \( z \) | Parameter | Measurement    | Reference |
|-----------------|--------|-----------|---------------|-----------|
| 6dF             | 0.106  | \( r_s/D_V \) | 0.327 ± 0.015 | \[86\]    |
| SDSS-MGS        | 0.10   | \( D_V/r_s \) | 4.47 ± 0.16   | \[87\]    |
| BOSS-LOWZ       | 0.32   | \( D_V/r_s \) | 8.47 ± 0.17   | \[88\]    |
| BOSS-CMASS      | 0.57   | \( D_V/r_s \) | 13.77 ± 0.13  | \[88\]    |
| BOSS-Lyα        | 2.36   | \( c/(Hr_s) \) | 9.0 ± 0.3     | \[89\]    |
| BOSS-Lyα        | 2.36   | \( D_A/r_s \) | 10.08 ± 0.4   | \[89\]    |

**Table 1.** Baryon acoustic oscillation (BAO) data measurements included in our analysis.

### 4 Observational Constraints

In this section we shall present the main observational constraints, extracted for the four viable \( f(R) \) models reviewed in subsection 2.3. We use the data described in the previous section, and we first perform fittings using Cosmic Chronometer (CC) + \( H_0 \) observations. Then, we proceed to the combination of all data sets, namely of SNe Ia “joint light curves” (JLA) + BAO + CC + \( H_0 \). To fit the free parameters in these \( f(R) \) scenarios we use the publicly available code CLASS \[90\] in the interface with the public Monte Carlo code Monte Python \[91\]. Moreover, in our analysis we use the Metropolis Hastings algorithm as our sampling method. In the following subsections we shall separately discuss the observational results on the various \( f(R) \) models.

#### 4.1 Constraints on Hu-Sawicki model

We fit the Hu-Sawicki model of (2.18), following the above procedure, and in Fig. 1 we present the contour plots of various quantities and model parameters, for both used data
sets. We mention that since there is a known degeneracy between \( p \) and \( \Omega_{m0} \) that requires to fix \( p \) a priori, we choose the case \( p = 1 \) since it is the most used case in the literature [60] (in principle one could perform the fittings for higher \( p \) too, nevertheless higher values have difficulties in fitting the data). Additionally, in Table 2 we summarize the best fit values of the data analysis for the two data sets respectively. As we observe, and interestingly enough, the parameter \( b \) which quantifies the deviation from ΛCDM cosmology is favored to have nonzero values for both data sets (for the combined analysis, i.e. for JLA + BAO + CC + \( H_0 \), the contours come closer to zero comparing to the CC + \( H_0 \) case, but the zero value is only marginally allowed), although the zero value is still inside the allowed region.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{contour_plots}
\caption{Contour plots for the free parameter \( b \), as well as for the present value of the matter density parameter \( \Omega_{m0} \) and for the dimensionless Hubble parameter \( h \), for the Hu-Sawicki model of (2.18). The red and pink regions correspond to 1σ and 2σ confidence level in the case of CC + \( H_0 \) data sets, while the blue and light blue regions correspond to 1σ and 2σ confidence level for the combined analysis of JLA + BAO + CC + \( H_0 \) data sets. Additionally, we present the corresponding marginalized one-dimensional posterior distributions. The parameter \( \Omega_m \) includes both baryons and cold dark matter, i.e. \( \Omega_m = \Omega_{cdm} + \Omega_b \), and \( h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1} \).}
\end{figure}
at both 1σ and 2σ confidence level. Hence, observations seem to slightly favor a small but not non-zero deviation from ΛCDM cosmology. This is one of the main results of the present work. Although some indications towards this direction were previously obtained in [60], in the present work, with the addition of CC data, this behavior is enhanced.

**Table 2.** Summary of the best fit values and main results for the free parameter $b$, as well as for the present value of the matter density parameter $\Omega_{m0}$ and for the dimensionless Hubble parameter $h$, for the Hu-Sawicki model of (2.18), using CC+ $H_0$ and JLA+BAO+CC + $H_0$ observational data. The parameter $\Omega_{m}$ includes both baryons and cold dark matter, i.e. $\Omega_{m} = \Omega_{cdm} + \Omega_{b}$, and $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$.

| Parameters | CC+ $H_0$ | JLA + BAO + CC + $H_0$ |
|------------|-----------|-------------------------|
| $b$        | $0.107^{+0.316}_{-0.158}$ | $0.045^{+0.062}_{-0.077}$ |
| $h$        | $0.729^{+0.034}_{-0.034}$  | $0.722^{+0.042}_{-0.043}$ |
| $\Omega_{m}$ | $0.264^{+0.069}_{-0.058}$ | $0.264^{+0.050}_{-0.052}$ |

4.2 Constraints on Starobinsky model

For the case of Starobinsky model of (2.20) with $n = 1$ (similarly to the Hu-Sawicki model there is a degeneracy between $n$ and $\Omega_{n0}$ that requires to fix $n$ a priori, and we choose the value $n = 1$ since it is the most used case in the literature), and similarly to the previous model, we perform the fittings using two different data sets, namely CC + $H_0$ data, and JLA + BAO + CC + $H_0$. In Fig. 2 we depict the contour plots of various quantities, while in Table 3 we provide the corresponding best fit values. In this model, the distortion parameter $b$ which quantifies the deviation of the model from ΛCDM cosmology has a slight preference to be non zero (as can be especially seen by the marginalized one-dimensional posterior distribution), however the zero value is clearly allowed, at both 1σ and 2σ confidence level, and hence this model can observationally coincide with ΛCDM scenario.

**Table 3.** Summary of the best fit values and main results for the free parameter $b$, as well as for the present value of the matter density parameter $\Omega_{m0}$ and for the dimensionless Hubble parameter $h$, for the Starobinsky model of (2.20), using CC+ $H_0$ and JLA+BAO+CC+ $H_0$ observational data. The parameter $\Omega_{m}$ includes both baryons and cold dark matter, i.e. $\Omega_{m} = \Omega_{cdm} + \Omega_{b}$, and $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$.

| Parameters | CC+ $H_0$ | JLA + BAO + CC + $H_0$ |
|------------|-----------|-------------------------|
| $b$        | $0.229^{+0.254}_{-0.720}$ | $0.111^{+0.070}_{-0.286}$ |
| $h$        | $0.72^{+0.031}_{-0.031}$  | $0.71^{+0.030}_{-0.028}$ |
| $\Omega_{m}$ | $0.26^{+0.062}_{-0.050}$ | $0.26^{+0.050}_{-0.042}$ |
Figure 2. Contour plots for the free parameter $b$, as well as for the present value of the matter density parameter $\Omega_{m0}$ and for the dimensionless Hubble parameter $h$, for the Starobinsky model of (2.20). The red and pink regions correspond to 1σ and 2σ confidence level in the case of $CC + H_0$ data sets, while the blue and light blue regions correspond to 1σ and 2σ confidence level for the combined analysis of $JLA + BAO + CC + H_0$ data sets. Additionally, we present the corresponding marginalized one-dimensional posterior distributions. The parameter $\Omega_m$ includes both baryons and cold dark matter, i.e., $\Omega_m = \Omega_{cdm} + \Omega_b$, and $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$.

4.3 Constraints on Tsujikawa model

For the case of Tsujikawa model of (2.22), in Fig. 3 we depict the contour plots arisen from the fitting analysis, while in Table 4 we present the corresponding best fit values for both used data sets, namely for $CC + H_0$ and for $JLA + BAO + CC + H_0$ observational data. As we observe, in this case the distortion parameter $b$ is clearly non-zero, with the zero value only very marginally allowed. Thus, Tsujikawa model exhibits an observable deviation from $\Lambda$CDM cosmology. This is one of the main results of the present work.
4.4 Constraints on exponential model

For the case of exponential $f(R)$ gravity model of (2.24), in Fig. 4 we present the likelihood contours arisen from the fitting analysis, while in Table 5 we provide the corresponding best fit values for both used data sets, namely for CC + $H_0$ and JLA + BAO + CC + $H_0$. Similarly to the previous model, the parameter $b$ that quantifies the deviation from ΛCDM cosmology is clearly non-zero, with the zero value only very marginally allowed. Hence, exponential $f(R)$ gravity could be observationally distinguished from ΛCDM paradigm. Furthermore, note that this scenario exhibits a very similar behavior with Tsujikawa model,
Figure 4. Contour plots for the free parameter $b$, as well as for the present value of the matter density parameter $\Omega_m$ and for the dimensionless Hubble parameter $h$, for the exponential $f(R)$ gravity model of (2.24. The red and pink regions correspond to 1σ and 2σ confidence level in the case of CC $+ H_0$ data sets, while the blue and light blue regions correspond to 1σ and 2σ confidence level for the combined analysis of JLA $+$ BAO $+$ CC $+$ $H_0$ data sets. Additionally, we present the corresponding marginalized one-dimensional posterior distributions. The parameter $\Omega_m$ includes both baryons and cold dark matter, i.e. $\Omega_m = \Omega_{cdm} + \Omega_b$, and $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$.

which was expected due to the relation of the hyperbolic tangent with the exponentials.

4.5 Model comparison

We close the observational analysis with the present subsection, in which we compare the fittings of the various models, using the standard information criteria. There are two main such criteria, namely the Akaike Information Criterion ($AIC$) [92] and the Bayesian or Schwarz Information Criterion ($BIC$) [93]. These are respectively defined as

$$AIC = -2 \ln \mathcal{L} + 2d = \chi^2_{\text{min}} + 2d,$$

(4.1)
Table 4. Summary of the best fit values and main results for the free parameter $b$, as well as for the present value of the matter density parameter $\Omega_{m0}$ and for the dimensionless Hubble parameter $h$, for the Tsujikawa model of (2.22), using CC+ $H_0$ and JLA+BAO+CC+ $H_0$ observational data. The parameter $\Omega_m$ includes both baryons and cold dark matter, i.e. $\Omega_m = \Omega_{cdm} + \Omega_b$, and $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$.

| Parameters | CC+ $H_0$ | JLA + BAO + CC + $H_0$ |
|------------|-----------|-------------------------|
| $b$        | $0.429^{+0.400}_{-0.424}$, $-0.0.424$ | $0.196^{+0.124}_{-0.195}$, $-0.0.195$ |
| $h$        | $0.726^{+0.031}_{-0.031}$, $-0.0.031$ | $0.709^{+0.031}_{-0.035}$, $-0.0.035$ |
| $\Omega_m$ | $0.261^{+0.063+0.059}_{-0.056-0.081}$ | $0.284^{+0.041+0.048}_{-0.044-0.052}$ |

Table 5. Summary of the best fit values and main results for the free parameter $b$, as well as for the present value of the matter density parameter $\Omega_{m0}$ and for the dimensionless Hubble parameter $h$, for the exponential $f(R)$ gravity model of (2.24), using CC+ $H_0$ and JLA+BAO+CC+ $H_0$ observational data. The parameter $\Omega_m$ includes both baryons and cold dark matter, i.e. $\Omega_m = \Omega_{cdm} + \Omega_b$, and $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$.

| Parameters | CC+ $H_0$ | JLA + BAO + CC + $H_0$ |
|------------|-----------|-------------------------|
| $b$        | $0.289^{+0.341+0.635}_{-0.289-0.289}$ | $0.139^{+0.089+0.118}_{-0.130-0.130}$ |
| $h$        | $0.724^{+0.031+0.046}_{-0.032-0.047}$ | $0.711^{+0.030+0.039}_{-0.026-0.033}$ |
| $\Omega_m$ | $0.261^{+0.064+0.100}_{-0.055-0.080}$ | $0.284^{+0.040+0.043}_{-0.049-0.062}$ |

and

$$BIC = -2 \ln \mathcal{L} + d \ln N = \chi^2_{min} + d \ln N,$$

(4.2)

where $\mathcal{L} = \exp(-\chi^2_{min}/2)$ is the maximum likelihood function, $d$ is the number of model parameters and $N$ denotes the total number of data points used in the statistical analysis. Definitely, one must also introduce a reference scenario, with respect of which the comparisons will be performed, and obviously this is $ΛCDM$ cosmology. Hence, for any given model denoted by $M$, and calculating the difference $\Delta X = X_M - X_{ΛCDM}$ (where $X = AIC$ or $BIC$), one may result to the following conclusions [94]; (i) If $\Delta X \leq 2$, then the concerned model has substantial support with respect to the reference model (i.e. it has evidence to be a good cosmological model), (ii) if $4 \leq \Delta X \leq 7$ it is an indication for less support with respect to the reference model, and finally, (iii) if $\Delta X \geq 10$ then the model has no observational support. Note that including the nuisance parameters arising from Supernova Type Ia, we have 6 model parameters in $ΛCDM$ paradigm, while in all $f(R)$ models we have 7 free parameters.

In Table 6 we present the values of $\Delta X$ for the four analyzed models, for both used data sets, namely for CC+ $H_0$ and JLA+BAO+CC+ $H_0$ ones. As we can see, for both data sets $\Delta AIC \leq 2$, and hence these models are very efficient and in very good agreement with observations, and they fit the data slightly better than $ΛCDM$ paradigm. Concerning
we observe that it acquires slightly larger values, and therefore according to this criterion ΛCDM scenario is slightly favored, although all $f(R)$ models are still very efficient. In summary, we deduce that all models behave very efficiently, and especially the Hu-Sawicki and Starobinsky ones seem to have a better fitting behavior comparing to ΛCDM paradigm.

Table 6. Summary of the AIC and BIC values, as well as of their difference from the reference model of ΛCDM cosmology, for the CC+ $H_0$ and JLA+BAO+CC+ $H_0$ data sets, for all four analyzed $f(R)$ models.

| Models          | CC+ $H_0$ | JLA + BAO + CC + $H_0$ |
|-----------------|-----------|------------------------|
|                 | AIC | $\Delta$AIC | BIC | $\Delta$BIC | AIC | $\Delta$AIC | BIC | $\Delta$BIC |
| ΛCDM Model      | 28.205 | 0 | 36.809 | 0 | 721.084 | 0 | 749.017 | 0 |
| Hu-Sawicki Model| 28.744 | 0.539 | 38.782 | 1.973 | 720.840 | −0.244 | 753.428 | 4.411 |
| Starobinsky Model| 29.096 | 0.891 | 39.134 | 2.325 | 721.726 | 0.642 | 754.314 | 5.297 |
| Tsujikawa Model | 29.407 | 1.202 | 39.445 | 2.636 | 722.966 | 1.882 | 755.554 | 6.537 |
| Exponential Model| 29.310 | 1.105 | 39.347 | 2.538 | 722.548 | 1.464 | 755.136 | 6.119 |

5 Conclusions

In this manuscript we have implemented the recently released cosmic chronometer data in order to impose constraints on the viable and most used $f(R)$ gravity models. In particular, we used the recent cosmic chronometer data set, along with the latest measured value of the local Hubble parameter, $H_0 = 73.02 \pm 1.79$ km s$^{-1}$ Mpc$^{-1}$ [82], while we additionally performed a combined analysis using the latest “joint light curves” (JLA) SNe Ia sample [85] in the redshift range $z \in [0.01, 1.30]$, as well as baryon acoustic oscillation (BAO) data points from various probes.

We examined four specific $f(R)$ models, namely the Hu-Sawicki, the Starobinsky, the Tsujikawa, and the exponential one, and we parametrized them introducing a distortion parameter $b$ that quantifies the deviation from ΛCDM cosmology. Thus, we used the above observational data in order to fit this parameter, along with various other cosmological quantities.

For the Hu-Sawicki scenario the parameter $b$ is favored to have nonzero values for both data sets, although the zero value is still inside the allowed region at both 1σ and 2σ confidence level, and thus a small but not non-zero deviation from ΛCDM cosmology is slightly favored. For the Starobinsky scenario $b$ has a slight preference to be non zero, however the zero value is clearly allowed, at both 1σ and 2σ confidence levels, and hence this model can observationally coincide with ΛCDM scenario. However, for the Tsujikawa and exponential models the distortion parameter $b$ is clearly non-zero, with the zero value only very marginally allowed. Hence, both these models exhibit an observable deviation from ΛCDM cosmology. This is one of the main results of the present work. Note that although some indications towards this direction had been previously obtained in the literature, in the present work, with the addition of CC data, this behavior is much more clear.
Finally, we performed a comparison of the fitting procedure with ΛCDM paradigm, using the AIC and BIC Information Criteria. According to AIC, for both data sets all four \( f(R) \) models are very efficient and sightly better than ΛCDM one, while according to BIC the ΛCDM scenario is slightly better, nevertheless with all \( f(R) \) models quite efficient.

In summary, using for the first time the recently released cosmic chronometer data, combined with data from other probes, we fitted the viable and most used \( f(R) \) gravity models. As we saw, clearly \( f(R) \) gravity is consistent with observations. Additionally, a small but non-zero deviation from ΛCDM cosmology is slightly favored, with the corresponding fittings exhibiting very efficient information criteria values. These features indicate that \( f(R) \) gravity may serve as a good candidate for gravitational modifications.

Acknowledgments

S.P. acknowledges Science and Engineering Research Board (SERB), Govt. of India, for awarding National Post-Doctoral Fellowship (File No: PDF/2015/000640). E.M.C.A. thanks CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico), and the Brazilian scientific support federal agency, for partial financial support, under Grants numbers 302155/2015-5, 302156/2015-1 and 442369/2014-0 and the hospitality of Theoretical Physics Department at Federal University of Rio de Janeiro (UFRJ), where part of this work was carried out. This article is based upon work from COST Action “Cosmology and Astrophysics Network for Theoretical Advances and Training Actions”, supported by COST (European Cooperation in Science and Technology).

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