Analysis of the Two Dimensional Datta-Das Spin Field Effect Transistor

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Abstract

An analytical expression is derived for the conductance modulation of a ballistic two-dimensional Datta-Das Spin Field Effect Transistor (SPINFET) as a function of gate voltage. Using this expression, we show that the recently observed conductance modulation in a two-dimensional SPINFET structure does not match the theoretically expected result very well. This calls into question the claimed demonstration of the SPINFET and underscores the need for further careful investigation.

Key words: spintronics, spin field effect transistor, Ramsauer resonances

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Even two decades after the original proposal of the Datta-Das Spin Field Effect Transistor (SPINFET) [1], the exact analytical expression for the channel conductance of a two-dimensional device structure has remained somewhat obscure [see the Note at the end]. Although Datta and Das proposed a two-dimensional transistor structure in their original work [1], the expression they derived for the channel conductance as a function of gate voltage was based on the assumption that the carrier’s wavevector component transverse to the direction of current flow is zero, which effectively corresponds to a one-dimensional structure. No expression was derived for the conductance modulation in a two-dimensional structure, possibly because Datta and Das realized that the conductance modulation will be severely suppressed in a two-dimensional system.

Recently, a report has appeared in the literature claiming demonstration of the Datta-Das SPINFET for the first time. The claim is predicated on the fact that a conductance modulation was observed in a two-dimensional SPINFET structure as a function of gate voltage, which could be fitted exactly with the equation

\[ \Delta G = A \cos \left( 2m^* \alpha [V_G] \frac{L}{\hbar^2} + \phi \right) \, , \]  

where \( \alpha [V_G] \) is the gate-controlled Rashba spin-orbit interaction strength in the two-dimensional channel, \( V_G \) is the gate voltage, \( m^* \) is the charge carrier’s effective mass, \( L \) is the source-to-drain separation (channel length) and \( \phi \) is an arbitrary phase shift. The authors of [2] measured the expected amplitude \( A \) and the quantity \( \alpha [V_G] \) in their structure independently, and then using \( \phi \) as the only fitting parameter, they could fit the experimentally observed conductance modulation \( \Delta G \) in their structure with Equation (1). This “fit”
(among others) was offered as proof that the Datta-Das transistor has been demonstrated.

Ref. [2] took Equation (1) from ref. [1], not realizing that it applies only to a strictly one-dimensional channel since ref. [1] had derived it assuming that the wavevector component transverse to the direction of current flow is exactly zero. Equation (1) does not hold for a two-dimensional channel since there the transverse wavevector component will not be zero. Recently, one of us pointed this out [3] and derived the correct equation for a two-dimensional channel (of finite width) assuming that only the electron energy is conserved in ballistic transport. Subsequently, it was pointed out [4] that if the width of the channel is semi-infinite so that periodic boundary conditions can be imposed along the width, then the transverse wavevector (perpendicular to the direction of current flow) is also a good quantum number and will be conserved in ballistic transport. This is reminiscent of two-dimensional coherent resonant tunneling devices of semi-infinite width, where the transverse wavevector is conserved during tunneling [5].

Conservation of the transverse wavevector greatly simplifies the equation derived in [3]. Additionally, low temperature and low bias conditions cause further simplification, resulting in the following simple equation for the conductance modulation in a two-dimensional SPINFET:

$$\Delta G = B \int_0^{k_F} dk_z \left[ 1 - \frac{k_z^2}{k_F^2} \right] \cos \left[ \Theta \left( k_F, k_z, \alpha [V_G] \right) L \right],$$

(2)

1 The expression derived in ref. [1] did not contain the phase shift $\phi$, but it could arise if $\alpha [V_G] \neq 0$ when $V_G = 0$. 
where \( B \) is a constant and

\[
\Theta (k_F, k_z, \alpha [V_G]) = -\frac{(2m^*\alpha [V_G]/\hbar^2) k_F + (m^*)^2 \alpha^2 [V_G]/\hbar^4}{\sqrt{k_F^2 - k_z^2}}.
\]  (3)

Here, \( k_z \) is the transverse wavevector component (along the width) and \( k_F \) is the Fermi wavevector. Derivation of Equation (2) is given in Appendix I.

Clearly, Equation (2) has no similarity with Equation (1). Therefore, the conductance modulations in the one- and the two-dimensional cases are very different. Particularly, Equation (1) predicts that the conductance modulation could reach 100\%, whereas Equation (2) shows unambiguously that it will never reach 100\% because ensemble averaging represented by the integration over the transverse wavevector component \( k_z \) will dilute the modulation considerably. Only in a strictly one-dimensional channel where Equation (1) holds, the conductance modulation can be 100\%, while in a two-dimensional channel, it will never be 100\%.

Ref. [6] has independently derived Equation (2) for a two-dimensional SPIN-FET and found that it can be approximated as

\[
\Delta G \approx \frac{hB}{2\sqrt{\pi m^* \alpha [V_G] L}} \cos \left[ 2m^* \alpha [V_G] L/\hbar^2 + \pi/4 \right].
\]  (4)

Equation (4) does not quite match Equation (1) either since the amplitude of the cosine function in Equation (4) is not constant, but gate-voltage dependent. Therefore, Equation (2) or Equation (4) cannot be reconciled with Equation (1). However, ref. [6] also found that for the particular experimental parameters of ref. [2], Equation (1) and Equation (2) yield similar curves for \( \Delta G \) versus \( V_G \) over the range of \( V_G \) used in the experiment. This similarity is coincidental and will not be sustained over extended ranges of \( V_G \). More
importantly, we have found that if we use the values of $m^*, \alpha [V_G], k_F$ and $L$ reported in [2], then the $\Delta G$ versus $V_G$ curve computed from the correct Equation (2) does not match the experimental $\Delta G$ versus $V_G$ curve reported in [2] very well. We show these two curves in Fig. 1. This disagreement between the correct theoretical result and the experimental observation casts doubt on the claimed demonstration of the Datta-Das SPINFET.

We emphasize that the above disagreement however does not establish conclusively that the Datta-Das SPINFET was not demonstrated in [2]. Instead, it casts doubt on the claimed demonstration and highlights the need for further investigation. Finally, the important question is if the observed voltage modulation was not due to the Datta-Das effect, what could it have been due to? Ref. [2] showed that the conductance modulation $\Delta G$ versus $V_G$ disappeared if the source and drain contacts were magnetized in a direction such that they injected and detected spins parallel to the effective magnetic field caused by the Rashba interaction. The modulation reappeared when the direction of magnetization was rotated by $90^\circ$ so that the injected spins became perpendicular to the effective magnetic field. This is consistent with the Datta-Das effect which relies on precession of the injected spins around the effective magnetic field caused by Rashba interaction. Since precession cannot occur if the spins are parallel to the effective magnetic field, the Datta-Das modulation will disappear in that case. The precession will occur if the injected spins are perpendicular to the effective magnetic field, so that the Datta-Das effect is recovered when the contacts’ magnetizations are rotated by $90^\circ$. This observation is certainly supportive of the Datta-Das effect, but it could also be caused by other phenomena. One likely phenomenon is Ramsauer resonances in the channel [7] which can also give rise to a voltage modulation $\Delta G$. 

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versus $V_G$. Ramsauer resonances are exacerbated by a magnetic field in the direction of current flow. In the experiment, when the contacts were magnetized in the direction perpendicular to the effective magnetic field caused by the Rashba interaction, they caused a real magnetic field to appear in the channel in the direction of current flow. This could have induced Ramsauer resonances. When the direction of magnetization of the contacts was rotated by 90°, the real magnetic field in the channel disappeared, which could have quenched or abated the Ramsauer resonances. Thus, the observed effect is also consistent with Ramsauer resonances. Consequently, further tests are required to identify the origin of the observed conductance modulation unambiguously. The expected oscillation periods for Ramsauer resonances and the Datta-Das effect are of course very different, but since barely one oscillation period was observed in the experiment of ref. [2], it is difficult to discriminate between these two effects from the observed modulation.

In summary, we have shown that the conductance modulation observed in ref. [2] cannot be fitted very well by the correct equation governing such a device, contrary to the claim of ref. [2]. Moreover, there can be alternate explanations for the origin of the observed conductance modulation of the device. Therefore, further careful study is required to resolve these controversies definitively.

*Note:* After the submission and acceptance of this work, we became aware of a paper [M. G. Pala, M. Governale, J. König and U. Zülicke, Europhys. Lett., 65, 850 (2004)] which has derived an analytical expression for the channel conductance of a two-dimensional SPINFET as a function of different orientations of the contacts’ magnetization. That expression reduces to Equation (2) when the contacts are magnetized in the +x-direction. We thank Prof. Ulrich Zülicke for bringing this to our attention.
Appendix I

In this Appendix, we derive the expression for the channel conductance of a two-dimensional Datta-Das SPINFET as a function of gate voltage.

Consider the two-dimensional channel of a Spin Field Effect Transistor (SPINFET) in the x-z plane (shown in Fig. 2(a)), with current flowing in the x-direction. An electron’s wavevector components in the channel are designated as \( k_x \) and \( k_z \), while the total wavevector is designated as \( k_t \). Note that \( k_t^2 = k_x^2 + k_z^2 \) as shown in Fig. 2(b).

The gate terminal induces an electric field in the y-direction which causes Rashba interaction. The Hamiltonian operator describing an electron in the channel is

\[
H = \frac{p_x^2 + p_z^2}{2m^*} [I] + \alpha [V_G] (\sigma_z p_x - \sigma_x p_z), \tag{5}
\]

where the \( p \)-s are the momentum operators, the \( \sigma \)-s are the Pauli spin matrices and \([I]\) is the 2×2 identity matrix. Since this Hamiltonian is invariant in both x- and z-coordinates, the wavefunctions in the channel are plane wave states \( e^{i(k_x x + k_z z)} \). Consequently, in the basis of these states, the Hamiltonian is

\[
H = \begin{pmatrix}
\frac{\hbar^2 k_t^2}{2m^*} + \alpha [V_G] k_x & -\alpha [V_G] k_z \\
-\alpha [V_G] k_z & \frac{\hbar^2 k_t^2}{2m^*} - \alpha [V_G] k_x
\end{pmatrix}. \tag{6}
\]

Diagonalization of this Hamiltonian yields the eigenenergies and the eigen-spinors in the two spin-split bands in the two-dimensional channel:
\[
E_l = \frac{\hbar^2 k_t^2}{2m^*} - \alpha [V_G] k_t \text{ (lower band)}; \quad E_u = \frac{\hbar^2 k_t^2}{2m^*} + \alpha [V_G] k_t \text{ (upper band)}.
\]

(7)

\[
[\Psi]_l = \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix} \text{ (lower band)}; \quad [\Psi]_u = \begin{bmatrix} -\cos \theta \\ \sin \theta \end{bmatrix} \text{ (upper band)}.
\]

(8)

where \( \theta = (1/2) \arctan (k_z/k_x) \). The energy dispersion relations in the two bands (one broken and the other solid) are plotted in Fig. 3. Note that an electron of energy \( E \) has two different wavevectors in the two bands given by \( k_t^{(1)} \) and \( k_t^{(2)} \).

We will assume that the source contact of the SPINFET is polarized in the \(+x\)-direction and injects \(+x\)-polarized spins into the channel under a source-to-drain bias. We also assume that the spin injection efficiency at the source is 100\%, so that only \(+x\)-polarized spins are injected at the complete exclusion of \(-x\)-polarized spins. An injected spin will couple into the two spin eigenstates in the channel. It is as if the \( x \)-polarized beam splits into two beams, each corresponding to one of the channel eigenspinors. This will yield:

\[
\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = C_1 \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix} + C_2 \begin{bmatrix} -\cos \theta \\ \sin \theta \end{bmatrix},
\]

\(+x\) - polarized

(9)

where the coupling coefficients \( C_1 \) and \( C_2 \) are found by solving Equation (9).

The result is

\[
C_1 = C_1 (k_x, k_z) = \sin (\theta + \pi/4)
\]

\[
C_2 = C_2 (k_x, k_z) = -\cos (\theta + \pi/4)
\]

(10)
Note that the coupling coefficients depend on $k_x$ and $k_z$.

At the drain end, the two beams recombine and interfere to yield the spinor of the electron impinging on the drain. Here, we are neglecting multiple reflection effects between the source and drain contacts in the spirit of ref. [1]. Since the two beams have the same energy $E$ and transverse wavevector $k_z$ (these are good quantum numbers in ballistic transport), they must have different longitudinal wavevectors $k_x^{(1)}$ and $k_x^{(2)}$ since $k_t^{(1)} \neq k_t^{(2)}$. Therefore, these two beams have slightly different directions of propagation in the channel. In other words, the channel behaves like a “birefringent” medium where waves with antiparallel spin polarizations travel in slightly different directions.

Hence, the spinor at the drain end will be:

$$[\Psi]_{\text{drain}} = C_1 \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix} e^{i(k_x^{(1)} L + k_z W)} + C_2 \begin{bmatrix} -\cos \theta \\ \sin \theta \end{bmatrix} e^{i(k_x^{(2)} L + k_z W)}$$

$$= e^{i k_z W} \begin{bmatrix} \sin(\theta + \pi/4) \sin \theta e^{i k_x^{(1)} L} + \cos(\theta + \pi/4) \cos \theta e^{i k_x^{(2)} L} \\ \sin(\theta + \pi/4) \cos \theta e^{i k_x^{(1)} L} - \cos(\theta + \pi/4) \sin \theta e^{i k_x^{(2)} L} \end{bmatrix}$$

(11)

where $L$ is the channel length (distance between source and drain contacts) and $W$ is the transverse displacement of the electron as it traverses the channel.

Since the drain is polarized in the same orientation as the source, it transmits only $+x$-polarized spins, so that spin filtering at the drain will yield a transmission probability $|T|^2$ where $T$ is the projection of the impinging spinor on the eigenspinor of the drain. It is given by
\[ T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \sin(\theta + \pi/4) \sin\theta e^{ik_x L} + \cos(\theta + \pi/4) \cos\theta e^{ik_x L} \\ \sin(\theta + \pi/4) \cos\theta e^{ik_x L} - \cos(\theta + \pi/4) \sin\theta e^{ik_x L} \end{bmatrix} e^{ik_x W} \]

\[ = e^{ik_x W} \left[ \sin^2(\theta + \pi/4) e^{ik_x L} + \cos^2(\theta + \pi/4) e^{ik_x L} \right]. \quad (12) \]

Here, we have assumed 100% spin filtering efficiency.

Therefore,

\[ |T|^2 = \cos^4(\theta + \pi/4) \left| 1 + \tan^2(\theta + \pi/4) e^{i [k_x^{(1)} - k_x^{(2)}] L} \right|^2 \]

\[ = \cos^4(\theta + \pi/4) + \sin^4(\theta + \pi/4) + \frac{1}{2} \cos^2(2\theta) \cos(\Theta L), \quad (13) \]

where \( \Theta = k_x^{(1)} - k_x^{(2)}. \)

From Equation (8), we get that \( k_t^{(1)} - k_t^{(2)} = -2m^*\alpha [V_G] / \hbar^2. \) Expressing the wavevectors in terms of their x- and z-components, we get:

\[ \sqrt{[k_x^{(1)}]^2 + k_z^2} - \sqrt{[k_x^{(2)}]^2 + k_z^2} = -2m^*\alpha [V_G] / \hbar^2, \quad (14) \]

which yields

\[ \Theta = k_x^{(1)} - k_x^{(2)} = \frac{-2m^*\alpha [V_G] / \hbar^2 k_t^{(2)} + 2 (m^*)^2 \alpha^2 [V_G] / \hbar^4}{[k_x^{(1)} + k_x^{(2)}] / 2}. \quad (15) \]

From Equation (8), we also get that

\[ k_t^{(2)} = \frac{m^*\alpha [V_G]}{\hbar^2} \pm \sqrt{\left( \frac{m^*\alpha [V_G]}{\hbar^2} \right)^2 + k_0^2} \approx k_0 + \frac{3 m^*\alpha [V_G]}{2 \hbar^2}, \quad (16) \]

where \( k_0 = \sqrt{2m^*E/\hbar}. \)

Now, if \( \alpha [V_G] \) is small, then \([k_x^{(1)} + k_x^{(2)}] / 2 \approx \sqrt{k_0^2 - k_z^2}. \) Substituting these results in Equation (15), we get
\[
\Theta = -\frac{(2m^*\alpha [V_G]/\hbar^2)k_0 - (m^*)^2\alpha^2[V_G]/\hbar^4}{\sqrt{k_0^2 - k_z^2}}
\]

\[
= -\frac{(2m^*\alpha [V_G]/\hbar^2)\sqrt{2m^*E/\hbar} - (m^*)^2\alpha^2[V_G]/\hbar^4}{\sqrt{2m^*E/\hbar^2 - k_z^2}}.
\]  
(17)

The current density in the channel of the SPINFET (assuming ballistic transport) is given by the Tsu-Esaki formula:

\[
J = \frac{q}{W_y} \int_0^\infty \frac{1}{\hbar} dE \int \frac{dk_z}{\pi} |T|^2 \left[ f(E) - f(E + qV_{SD}) \right],
\]

where \( q \) is the electronic charge, \( W_y \) is the thickness of the channel (in the \( y \)-direction), \( V_{SD} \) is the source-to-drain bias voltage and \( f(\eta) \) is the electron occupation probability at energy \( \eta \) in the contacts. Since the contacts are at local thermodynamic equilibrium, these probabilities are given by the Fermi-Dirac factor.

In the linear response regime when \( V_{SD} \to 0 \), the above expression reduces to

\[
J = \frac{q^2V_{SD}}{W_y} \int_0^\infty \frac{1}{\hbar} dE \int \frac{dk_z}{\pi} |T|^2 \left[ -\frac{\partial f(E)}{\partial E} \right].
\]

(19)

This yields that the channel conductance \( G \) is

\[
G = \frac{I_{SD}}{V_{SD}} = \frac{JW_yW_z}{V_{SD}} = \frac{q^2W_z}{\pi\hbar} \int_0^\infty dE \int dk_z|T|^2 \left[ -\frac{\partial f(E)}{\partial E} \right],
\]

(20)

where \( I_{SD} \) is the source-to-drain current and \( W_z \) is the channel width.

Using Equations (13) and (17), we finally get that the channel conductance is

\[
G = G_0 + \frac{q^2W_z}{2\pi\hbar} \int_0^\infty dE \int dk_z \cos^2(2\theta)\cos(\Theta L) \left[ -\frac{\partial f(E)}{\partial E} \right]
\]

\[
= G_0 + \frac{q^2W_z}{2\pi\hbar} \int_0^\infty dE \int dk_z \frac{k_x^2}{k_x^2 + k_z^2} \cos(\Theta L) \left[ -\frac{\partial f(E)}{\partial E} \right]
\]
\[ G_0 + \frac{q^2 W_z}{2\pi \hbar} \int_0^\infty dE \int dk_z \left[ 1 - \frac{\hbar^2 k_z^2}{2m^* E} \right] \cos (\Theta L) \left[ -\frac{\partial f(E)}{\partial E} \right], \]  
\( 21 \)

where \( G_0 \) is a constant independent of \( \Theta \) and hence the gate voltage. It is easy to show that \( G_0 = \frac{q^2 W_z}{2\pi \hbar} \int_0^\infty dE \int dk_z \left[ 1 + \frac{\hbar^2 k_z^2}{2m^* E} \right] \left[ -\frac{\partial f(E)}{\partial E} \right]. \)

If the temperature is low so that \(-\frac{\partial f(E)}{\partial E} \approx \delta(E - E_F)\), then the last equation reduces to

\[ G = G_0 + \frac{q^2 W_z}{2\pi \hbar} \int_0^\infty dE \int dk_z \left[ 1 - \frac{\hbar^2 k_z^2}{2m^* E} \right] \cos (\Theta L) \delta(E - E_F) \]

\[ = G_0 + \frac{q^2 W_z}{2\pi \hbar} \int_0^{k_F} dk_z \left[ 1 - \frac{k_z^2}{k_F^2} \right] \cos \left( \Theta(k_F, k_z, \alpha[V_G]) L \right) \]

\[ = \frac{q^2 W_z}{\pi \hbar} \int_0^{k_F} dk_z \left[ 1 - \frac{k_z^2}{k_F^2} \right] F(\alpha[V_G], L, k_F, k_z), \]  
\( 22 \)

where

\[ F(\alpha[V_G], L, k_F, k_z) = \left\{ \cos^2 \left[ \Theta(k_F, k_z, \alpha[V_G]) L \right]/2 + \frac{k_z^2}{k_F^2} \sin^2 \left[ \Theta(k_F, k_z, \alpha[V_G]) L \right]/2 \right\}, \]  
\( 23 \)

and \( k_F \) is the Fermi wavevector. Therefore,

\[ \Delta G = G - G_0 = \frac{q^2 W_z}{2\pi \hbar} \int_0^{k_F} dk_z \left[ 1 - \frac{k_z^2}{k_F^2} \right] \cos \left( \Theta(k_F, k_z, \alpha[V_G]) L \right). \]  
\( 24 \)

The last equation is identical with Equation (2).
Appendix II

In this appendix, we will derive an expression for $\Delta G$ assuming non-ideal spin injection and detection. Let us call the spin injection efficiency at the source contact $\eta_S$ and the spin filtering efficiency in the drain contact $\eta_D$. If these efficiencies are less than 100\%, then the probability of a $+x$-polarized spin being injected by the source is $\frac{1 + \eta_S}{2}$ and the probability of it being filtered at the drain is $\frac{1 + \eta_D}{2}$ when both contacts are magnetized in the $+x$-direction. Therefore the contribution to $\Delta G$ arising from $+x$-polarized injection and $+x$-polarized detection is

$$[\Delta G]_{+x,+x} = \frac{(1 + \eta_S)(1 + \eta_D)}{4} \frac{q^2 W_z}{2\pi h} \int_0^{k_F} dk_z \left[ 1 - \frac{k_z^2}{k_F^2} \right] \cos \left[ \Theta \left( k_F, k_z, \alpha [V_G] \right) \right] L$$

(25)

Next consider the situation when the source injects a $+x$-polarized spin, but it transmits into the $-x$-polarized band in the drain because spin filtering is imperfect.

In this case, since the spin injected from the source is $+x$-polarized, Equation (11) is still valid for the spinor of the electron impinging on the drain. However, we now have to re-calculate the projection of the impinging spinor on the $-x$-polarized state in the drain, which will give

$$T_{+x,-x} = \frac{1}{\sqrt{2}} \left[ \begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \right] \sin \left( \theta + \pi/4 \right) \sin \theta e^{ik_{x}^{(1)} L} + \cos \left( \theta + \pi/4 \right) \cos \theta e^{ik_{x}^{(2)} L}$$

$$= \frac{1}{2} e^{ik_{x} W} \cos \left( 2\theta \right) \left[ e^{i k_{x}^{(2)} L} - e^{i k_{x}^{(1)} L} \right].$$

(26)
This yields that

\[
|T_{x,-x}|^2 = \frac{\cos^2(2\theta)}{2} [1 - \cos(\Theta L)] = \frac{1}{2} \left[ 1 - \frac{k_z^2}{k_0^2} \right] \{1 - \cos [\Theta (k_0, k_z, \alpha [V_G]) L] \}.
\]  \hspace{1cm} (27)

Since the probability of injecting a +x-polarized spin at the source contact is \((1 + \eta_S)/2\) and the probability of its transmitting into the -x-polarized band at the drain contact is \((1 - \eta_D)/2\), the corresponding contribution to \(\Delta G\) will be

\[
[\Delta G]_{x,-x} = \frac{(1 + \eta_S) (1 - \eta_D)}{4} \frac{q^2 W_z}{2\pi h} \int_0^{k_F} dk_z \left[ 1 - \frac{k_z^2}{k_F^2} \right] \{1 - \cos [\Theta (k_F, k_z, \alpha [V_G]) L] \}.
\]  \hspace{1cm} (28)

Now, consider the situation when the source injects a -x-polarized spin, but it transmits into the +x-polarized band in the drain.

In this case, the spin injected from the source contact is -x-polarized and we will have to recalculate the spinor of the electron impinging on the drain.

Equation (9) will now be replaced by

\[
\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = C'_1 \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix} + C'_2 \begin{bmatrix} -\cos \theta \\ \sin \theta \end{bmatrix},
\]  \hspace{1cm} (29)

which yields

\[
C'_1 = \sin (\theta - \pi/4), \\
C'_2 = -\cos (\theta - \pi/4).
\]  \hspace{1cm} (30)

Therefore, the spinor of the electron impinging on the drain is
\[ \ket{\Psi}_{\text{drain}} = C_1' \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix} e^{i(k_y^{(1)} L + k_z W)} + C_2' \begin{bmatrix} -\cos \theta \\ \sin \theta \end{bmatrix} e^{i(k_y^{(2)} L + k_z W)} \]

\[ = e^{ik_z W} \begin{bmatrix} \sin (\theta - \pi/4) \sin \theta e^{ik_z^{(1)} L} + \cos (\theta - \pi/4) \cos \theta e^{ik_z^{(2)} L} \\ \sin (\theta - \pi/4) \cos \theta e^{ik_z^{(1)} L} - \cos (\theta - \pi/4) \sin \theta e^{ik_z^{(2)} L} \end{bmatrix}, \]

(31)

The projection of this spinor on the +x-polarized state in the drain gives

\[ T_{-x,+x} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \sin (\theta - \pi/4) \sin \theta e^{ik_z^{(1)} L} + \cos (\theta - \pi/4) \cos \theta e^{ik_z^{(2)} L} \\ \sin (\theta - \pi/4) \cos \theta e^{ik_z^{(1)} L} - \cos (\theta - \pi/4) \sin \theta e^{ik_z^{(2)} L} \end{bmatrix} e^{ik_z W} \]

\[ = \frac{1}{2} e^{ik_z W} \cos(2\theta) \left[ e^{ik_z^{(2)} L} - e^{ik_z^{(1)} L} \right]. \]

(32)

Therefore, once again,

\[ |T_{+x,-x}|^2 = \frac{\cos^2(2\theta)}{2} [1 - \cos(\Theta L)] \]

\[ = \frac{1}{2} \left[ 1 - \frac{k_z^2}{k_0^2} \right] \{1 - \cos [\Theta (k_0, k_z, \alpha [V_G]) L] \}. \]

(33)

Since the probability of injecting a -x-polarized spin at the source is \((1 - \eta_S)/2\) and the probability of its transmitting into the +x-polarized band at the drain is \((1 + \eta_D)/2\), the corresponding contribution to \(\Delta G\) is

\[ [\Delta G]_{-x,+x} = \frac{(1 - \eta_S)(1 + \eta_D)}{4} \frac{q^2 W_z}{2\pi \hbar} \int_{k_F} k_z^2 d(k_z) \left[ 1 - \frac{k_z^2}{k_0^2} \right] \{1 - \cos [\Theta (k_F, k_z, \alpha [V_G]) L] \}. \]

(34)
Finally, consider the situation when a -x-polarized electron is injected at the source and transmits into the -x-polarized band of the drain contact.

In this case, Equation (31) will describe the spinor of the electron impinging on the drain and the projection of this spinor on the -x-polarized state in the drain will give

\[
T_{-x,-x} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \sin(\theta - \pi/4) \sin \theta e^{ik_F^1 L} + \cos(\theta - \pi/4) \cos \theta e^{ik_F^2 L} \\ \sin(\theta - \pi/4) \cos \theta e^{ik_F^1 L} - \cos(\theta - \pi/4) \sin \theta e^{ik_F^2 L} \end{bmatrix} e^{ik_z W} 
= e^{ik_z W} \left\{ \sin^2(\theta - \pi/4) e^{ik_F^1 L} + \cos^2(\theta - \pi/4) e^{ik_F^2 L} \right\}. \tag{35}
\]

Consequently, the contribution to \( \Delta G \) will be

\[
[\Delta G]_{-x,-x} = \frac{(1 - \eta_S)(1 - \eta_D)}{4} \frac{q^2 W_z}{2\pi \hbar} \int_0^{k_F} dk_z \left[ 1 - \frac{k_z^2}{k_F^2} \right] \cos \left[ \Theta \left( k_F, k_z, \alpha [V_G] \right) \right] L \]
\[= \eta_S \eta_D \frac{q^2 W_z}{2\pi \hbar} \int_0^{k_F} dk_z \left[ 1 - \frac{k_z^2}{k_F^2} \right] \cos \left[ \Theta \left( k_F, k_z, \alpha [V_G] \right) \right] L. \tag{36}
\]

Using the Principle of Superposition, the total gate voltage dependent conductance modulation will be

\[
\Delta G = \left[ \frac{(1 + \eta_S)(1 + \eta_D)}{4} - \frac{(1 + \eta_S)(1 - \eta_D)}{4} - \frac{(1 - \eta_S)(1 + \eta_D)}{4} + \frac{(1 - \eta_S)(1 - \eta_D)}{4} \right] 
\times \frac{q^2 W_z}{2\pi \hbar} \int_0^{k_F} dk_z \left[ 1 - \frac{k_z^2}{k_F^2} \right] \cos \left[ \Theta \left( k_F, k_z, \alpha [V_G] \right) \right] L 
\]
\[= \eta_S \eta_D \frac{q^2 W_z}{2\pi \hbar} \int_0^{k_F} dk_z \left[ 1 - \frac{k_z^2}{k_F^2} \right] \cos \left[ \Theta \left( k_F, k_z, \alpha [V_G] \right) \right] L. \tag{37}
\]

Therefore, non-ideal spin injection and filtering reduces the amplitude of any non-local voltage modulation by the factor \( \eta_S \eta_D \).
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Fig. 1. Plots of channel conductance modulation $\Delta G$ of a two-dimensional SPINFET versus gate voltage $V_G$. The solid lines are theoretical results calculated from Equation (2) where we have used the values of $m^*$, $k_F$ and $\alpha [V_G]$ reported in reference [2] and the points are (approximated) experimental results reported in [2]. The amplitudes of the theoretical plots are adjusted to match the experimental results as closely as possible. Note that neither the periods, nor the phases of the experimental plots agree very well with the theoretical plots. The plots are for two different channel lengths of $L = 1.65 \mu m$ and $1.25 \mu m$ used in the experiments of ref. [2]. The experiments were carried out at low temperatures and biases. The spin splitting energy in the dot specified in Fig. 2 as a function of Rashba interaction strength.
Fig. 2. (a) A two-dimensional SPINFET channel, and (b) the wavevector components in the plane of the channel.
Fig. 3. Schematic representation of the dispersion relations in the two spin split bands, under the influence of the gate voltage inducing Rashba interaction in the channel.