Design of Deterministic Sparse Fractal Array with Haferman Counter-Diagonal Fractal Tapering Technique for Modern Wireless Systems

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Abstract. Deterministic fractal structure is a repetitive geometry based structure having applications in various regions of science and engineering. Fractal antennas and array antennas are artistic structures and they are finding much attention in recent years owing to their feasible properties. A few deterministic fractal arrays owing to the fractal dimensional design methodology are thinned arrays, but the great concern for these types of array antennas is the great number of antennas present at higher level iteration and as well as expansion levels. The present research work discusses the feasibility of reducing antenna elements at higher expansion and iteration levels of fractal arrays with a new type of tapering technique named as Haferman counter-diagonal fractal tapering. Different types of fractal arrays created using a sub array of concentric elliptical ring design methodology have been considered for the application proposed tapering technique. Owing to the proposed technique, nearly 40% to 50% of the antenna elements thinned while maintaining fully populated array factor properties. The proposed technique can simplify the design complexity of fractal arrays and reduce the designing cost.

1. Introduction
When a complex wireless based communication system like a celestial system demands better gain, a more directive pattern with low side lobe (SdLb) ratio, multibeam characteristics, or any other attribute that a solitary antenna element cannot perform, an array antenna made up of group of distinct antenna elements may proffer an answer to this type of problems [1-6]. Ultra wide band and multibeam performance of an antenna and the antenna arrays are instrumental in celestial and other modern wireless based communication system [7, 8]. Fractal array antennas (FAA’s) are the better solution for this type of antenna performance. FAA’s are repetitive geometry based thinned and multibeam arrays having applications in satellite and celestial communications [9-12]. Geometrical construction of these arrays is basically three types like conventional array antennas. Linear FAA’s are one dimensional arrays having fractal geometric nature. Even and odd numbered Cantor linear array antennas are the best example for this type of arrays [13, 14]. Generally, deterministic FAA’s are generated by geometric design procedures. Concentric circular and a sub-array of elliptical ring designing technique...
and concentric sphere generators are ideal for 2D and 3D fractal geometric methodologies. Triangular, square, pentagonal, hexagonal, heptagonal, octagonal, and Sierpinski carpet arrays are ideal for planar FAA’s [15-18]. The array factor performance of these arrays is dependent on the number of iteration levels (p) and the scaling or expansion component (S). These two components are dependent on the fractal geometric design procedures of FAA. Fractal organization pattern of array elements generates a thinned FAA in turn accomplish desired radiation performance. Generally tapering techniques can be helpful in the process of thinning antenna elements with reduction in side lobe (SdLb) ratio. Array-tapering means eliminating a few antenna elements over an array surface according to array factor requirements [19, 20].

This report investigated deterministic rhombic FAA with Haferman anti-diagonal tapering technique (HADTT). Fractal numerical sequences are obeying repetitive nature as similar to fractal geometric shapes. Technical findings reported on fractal tapering techniques applied to FAA’s are quite less. A few findings reported various sequences such as Morse-Thue, atomic functional, and Fibonacci as the current -amplitudes in order to decrease SdLb ratio in conventional and also fractal arrays [20-25]. Haferman counter-diagonal fractal tapering, related formula, and expressions have discussed in the section 2 of this paper. Design equations of the considered fractal array presented in the section 3, corresponding array factor details and graphs presented in the results and discussion session of this paper. Finally need of the proposed paper and future ways of this research work presented in the conclusions session.

2. Haferman’s Counter0diagonal fractal Tapering Technique

Hagerman’s counter-diagonal sequence pattern is ideal for partly fractal sequence. This type of sequences never follows the fractal nature up to n\textsuperscript{th} iteration level. Haferman’s matrix is the modified variant of Sierpinski matrix. Actually in the Sierpinski matrix, the value of ‘M\textsubscript{22}’ is zero and remaining all values is one. But in this case both diagonal and counter-diagonal matrices are zero. This tapering technique for the design of sparse deterministic rhombic FAA is obtained from the Haferman’s matrix. In the implementation procedure of tapering technique, the count of antenna elements which are in ‘ON’ condition for every single iteration can be determined by equation (1)

\[
\frac{1}{14}\left(-1\right)^p5^{p+1} + 9^{p+1}
\]

Generally, this type of sequence can have infinite iterations. In the present work, successive five iterations are taken to achieve thinning of array antenna. Due to the expanding nature of the taken sequence, all the five iterations have reported nearly fifty percentage of thinning. The thinning percentage is maintained constantly throughout all the iterations. The complexity involved in designing of deterministic rhombic FAA can be lowered with the simple tapering behavior. The conventional Haferman matrix with corresponding counter-diagonal bit pattern till n\textsuperscript{th} iteration is given below;

Iteration -I (p=1) of Hagerman’s matrix,

\[
\begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 1 \\
0 & 1 & 0 \\
\end{bmatrix}
\]

Counter-Diagonal Sequence of above matrix is,

\[
[011000110]
\]
Iteration two (p=2) of Haferman’s matrix,

\[
\begin{bmatrix}
111010111 \\
11101111 \\
111010111 \\
010111010 \\
101111101 \\
010111010 \\
111010111 \\
111101111 \\
111010111
\end{bmatrix}
\]

Counter-Diagonal Sequence of above matrix is,

\[11111110110111000000111111111111111110111111111111101110111111111]

\(n^{th}\) iteration (p=n) of Haferman’s carpet matrix,

\[
\begin{bmatrix}
111010----------n \\
111011----------n \\
111010----------n \\
----------
----------
----------
----------
----------
----------
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----------
----------
\end{bmatrix}
\]

Counter-Diagonal Sequence of above matrix is,

\[11111111101110111100000111111111111111110111111111111101110111111111]

3. Design Equations of Deterministic Rhombic FAA

The minimum number of elements required to generate a rhombic array antenna is four. And this can be reiterated up till infinite times. This works reports till fourth iteration. The antenna elements in the array maintained uniform distance for expansion behavior (S) of one and two. Owing to recursive character of the rhombic FAA near to ternary of the total populated antenna elements set to OFF condition in every single iteration and thus led to multibeam characteristics. The expansion behavior of the FAA determines the actual number of elements required and their respective positions. This section handout the two different expansion factors which are taken into consideration for rhombic fractal array. The Fig. 1 depicts the antenna array pattern when expansion factor is one. The corresponding array factor is determined by equation-(2).

\[
AF_p(\theta, \phi) = |\prod_{p=1}^{p} \sum_{m=1}^{M} \sum_{n=1}^{N} I_{mn} e^{jk(S)(m-n)}|^{1/2}
\]

Where, ‘p’ represents iteration value or the iteration level, ‘m’ gives count of concentric rings, ‘n’ is gives the number of antenna elements, \(I_{mn}\) represents the current-amplitudes, k gives wave count, ‘S’
is the representation of expansion factor, where $S = 1$. Equation “(3)” determines the antenna Array factor when expansion factor is two. The behavior of array is changed based on its expansion factor. Dimensional structure of rhombic fractal array when expanding factor is two is show in Fig 2. Because of the expansion nature, no elements in the array are thinned for the given configuration.

\[
A.F_p(\theta, \phi) = \prod_{p=1}^{P} \left[ \sum_{m=1}^{M} \sum_{n=1}^{N} I_{mn} e^{jk(2)^{p-1} \phi_{mn}} \right]
\]

(3)

Figure 1. Successive four iterations levels of rhombus fractal array antenna where expansion factor is taken as one.
4. Results and Discussion

A wide amount of thinning is achieved in the Haferman counter-diagonal tapered rhombic array antennas for both expansion levels (S= 1 and 2). Nearly 50% of thinning has obtained in all iterations of deterministic rhombic FAA. Array factor properties are shown in Fig 3 (a) and 3 (b). Percentage of thinning and the parameters are given in table 1. The proposed tapered arrays are providing superior performance in terms of thinning percentage and the array factor behavior than an array which consists of all the actual number of elements. Haferman’s counter-diagonal tapering method has achieved high and uniform thinning percentage in comparison with Sierpinski counter-diagonal tapering for all levels of iterations. First iteration of FAA has achieved -4.6 SdLb level, 42° beamwidth, and 87° side lobe level angle with 50% of thinning. Second iteration of FAA has achieved -13 SdLb level, 25° beamwidth, and 89° side lobe level angle with 55% of thinning. Third and forth iterations achieved -22 and -31.3 SdLb levels respectively with good thinning. It is clear from the considered sequence that if the
iterations are increasing, the thinning percentage is also increasing. It is a good sign for the fractal array antenna thinning especially at higher orders.

The tapering techniques are feasible for practical design considerations due to its low profile, higher tapering efficiency, and less amplitude weights. These tapering methods can be suggested for satellite, celestial, and other modern wireless systems, where antenna aperture plays a vital role.

**Table 1.** Array properties of rhombic fractal antenna array with Haferman counter-diagonal binary semi-fractal distribution function of four successive level of iterations (P=1, 2, 3, 4) where expansion factors is taken as S=1 and 2.

| S | Expansion Factor (S) = 1 |  |  |
|---|--------------------------|---|---|
| Ite. (P) | Total No. of elements | SdLb level (dB) | HPBW (Degree) | SdLb Level Angle (Degree) | No. of off elements |
| 1 | 04 | -4.6 | 42 | 87 | 02 |
| 2 | 09 | -13 | 25 | 89 | 05 |
| 3 | 16 | -22 | 19.4 | 89 | 06 |
| 4 | 25 | -31.3 | 16 | 89 | 08 |

| S | Expansion Factor (S) = 2 |  |  |
|---|--------------------------|---|---|
| Ite. (P) | Total No. of elements | SdLb level (dB) | HPBW (Degree) | SdLb Level Angle (Degree) | No. of off elements |
| 1 | 04 | -4.6 | 42 | 87 | 02 |
| 2 | 16 | -15.4 | 15.4 | 34.8 | 08 |
| 3 | 64 | -22.3 | 7.4 | 40 | 32 |
| 4 | 256 | -22.4 | 3.6 | 21 | 128 |
Figure 3. Array factor (P=1, 2, 3, 4) of Haferman counter-diagonal tapered rhombic fractal antenna array where (a) S=1, (b) S=2.

5. Conclusion

Haferman counter-diagonal fractal tapering technique to achieve thinning of antenna elements in deterministic rhombic FAA for two different expansion factors has been presented. In both cases, nearly 50% of elements become OFF and SLL lowered in most of the iterations compare to fully-populated FAA. The comparative analysis gives remarkable improvement of SLL with notable decrement of number total elements required by the proposed technique, which led to reduction of complexity and cost-effective.

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