NEW RESULTS ON INCLUSIVE QUARKONIUM DECAYS

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I review some recent progress, leading to a substantial reduction in the number of non-perturbative parameters, in the calculation of inclusive quarkonium decay widths in the framework of non-relativistic effective field theories.

1 Introduction

Considered as bound states, heavy quarkonia (e.g. $\Upsilon$, $\chi_b$, $\psi$, $\chi_c$, ...) have two main characteristics: 1) a mass scale, which is in the perturbative regime: $m_b \simeq 5 \text{ GeV}$, $m_c \simeq 1.5 \text{ GeV}$; 2) level splittings of a relative size that is typical of non-relativistic systems. Being non-relativistic bound states, heavy quarkonia are characterized by at least three hierarchically ordered energy scales: the hard scale $m$, the momentum scale $mv$ and the bound-state energy scale $mv^2$, where $v \ll 1$ is the heavy-quark velocity inside the bound state.

The different scales entering the quarkonium dynamics may be systematically integrated out, leading from QCD to simpler but equivalent effective field theories (EFTs). NRQCD is the EFT obtained by integrating out the hard scale $m$. This $m$ being larger than the scale of non-perturbative physics, $\Lambda_{\text{QCD}}$, the matching to NRQCD can be done order by order in $\alpha_s$. Hence, the NRQCD Lagrangian can be written as a sum of terms like $f_n O_n^{(d_n)} / m^{d_n-4}$, ordered in powers of $\alpha_s$ and $v$. More specifically, the Wilson coefficients $f_n$ are series in $\alpha_n(m)$ and encode the ultraviolet physics that has been integrated out from QCD. The operators $O_n^{(d_n)}$ of dimension $d_n$ describe the low-energy dynamics and are counted in powers of $v$. Heavy quarkonium decays are processes that take place at an energy-transfer scale of the order of the heavy-quark mass. Their signature is encoded in the imaginary part of the Wilson coefficients of the 4-fermion operators $(O_n^{(d_n)}) = \psi \dagger K_n \chi \chi \dagger K_n \psi$ in the NRQCD Lagrangian. The NRQCD factorization formula for quarkonium inclusive decay widths into light hadrons (LH) reads

$$\Gamma(H \to LH) = \sum_n \frac{2 \text{Im} f_n}{m^{d_n-4}} \langle H | \psi \dagger K_n \chi \chi \dagger K_n \psi | H \rangle.$$  (1)

The 4-fermion operators are classified with respect to their rotational and spin symmetry (e.g. $O(2S+1 S_J)$, $O(2S+1 P_J)$, ...) and of their colour content (octet, $O_8$, and singlet, $O_1$, operators). Singlet operator expectation values may be easily related to the square of the quarkonium wave functions (or derivatives of it) at the origin. These are unknown non-perturbative parameters and may be fitted from experimental data, calculated on the lattice or taken from phenomenological potential models. Octet operator expectation values are also unknown non-perturbative parameters to be fitted from the experimental data, or to be calculated on the lattice.

NRQCD still contains dynamical degrees of freedom associated with energy scales larger than the ultrasoft scale $mv^2$. Hence, pushing further the EFT programme for non-relativistic bound states, further simplifications occur if we integrate out those degrees of freedom. We call pNRQCD the resulting EFT. We will consider pNRQCD under the condition $\Lambda_{\text{QCD}} \gg mv^2$. Then, two situations are possible. First, the situation when $mv \gg \Lambda_{\text{QCD}} \gg mv^2$. In this case

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* As a consequence the power counting of the NRQCD operators is not unique, since they depend, in general, on the scales $mv$, $mv^2$ and $\Lambda_{\text{QCD}}$. 

the soft scale $mv$ can be integrated out perturbatively. This leads to an intermediate EFT that contains singlet and octet quarkonium fields and ultrasoft gluons as dynamical degrees of freedom. The octet quarkonium field and the ultrasoft gluons are eventually integrated out by the (non-perturbative) matching to pNRQCD. Second, the situation when $\Lambda_{\text{QCD}} \sim mv$. In this case the (non-perturbative) matching to pNRQCD has to be done in one single step. Under the circumstances that other degrees of freedom develop a mass gap of order $\Lambda_{\text{QCD}}$ or that they play a negligible role, the quarkonium singlet field $S$ remains as the only dynamical degree of freedom in the pNRQCD Lagrangian, which reads $\mathcal{L}_{\text{pNRQCD}} = \text{Tr} \left\{ S^\dagger (i \partial_0 - \mathcal{H}) S \right\}$, $\mathcal{H}$ being the pNRQCD Hamiltonian, to be determined by matching pNRQCD to NRQCD. The inclusive quarkonium decay width into light hadrons is given by

$$\Gamma(H \to LH) = -2 \text{Im} \langle n, L, S, J | H | n, L, S, J \rangle,$$

where $|n, L, S, J\rangle$ is an eigenstate of $\mathcal{H}$ with the quantum numbers of the quarkonium state $H$. From the matching we obtain schematically:

$$\text{Im} \mathcal{H} = \delta^2(r) \sum_n \frac{\text{Im} f_n}{m_{d_n-4}} A_n + \{\delta^2(r), \Delta\} \sum_n \frac{\text{Im} f_n}{m_{d_n-4}} B_n + \nabla^i \delta^2(r) \nabla^j \sum_n \frac{\text{Im} f_n}{m_{d_n-4}} C_{ij} + \ldots,$$

where $f_n$ are the matching coefficients inherited from NRQCD, and $A_n$, $B_n$, ... are non-perturbative operators, which are universal in the sense that they do not depend either on the heavy-quark flavour or on the specific quantum numbers of the considered heavy-quarkonium state. Inserting Eq. (3) into (2) and comparing with Eq. (1), we see that all NRQCD matrix elements, including the octet ones, can be expressed through pNRQCD as products of the universal non-perturbative factors by the squares of the quarkonium wave functions (or derivatives of it) at the origin. This drastically reduces the number of unknown non-perturbative factors to be introduced in order to describe the whole set of charmonium and bottomonium inclusive decays and makes new theoretical predictions possible. As an example, in the following we will discuss the case of $P$-wave inclusive quarkonium decays into light hadrons.

## 2 P-wave decays in NRQCD and pNRQCD

In NRQCD the $P$-wave inclusive decay width for the $S = 0$ ($h$) and $S = 1$ ($\chi$) quarkonium states is given at leading (non-vanishing) order in $v$ by:

$$\Gamma(h \to LH) = \frac{9 \text{Im} f_1(1P)}{\pi m^4} \left| R'_P \right|^2 + \frac{2 \text{Im} f_8(1S_0)}{m^2} \langle h | O_8(1S_0) | h \rangle,$$

$$\Gamma(\chi_J \to LH) = \frac{9 \text{Im} f_1(3P_J)}{\pi m^4} \left| R'_P \right|^2 + \frac{2 \text{Im} f_8(3S_1)}{m^2} \langle h | O_8(1S_0) | h \rangle, \text{ for } J = 0, 1, 2$$

where $R'_P$ is the derivative of the $P$-wave function at the origin. We stress that, according to the power counting of NRQCD, the octet contribution $\langle h | O_8(1S_0) | h \rangle$ is as relevant as the singlet contribution; the above formula may by no means be used without considering it (for instance in extracting $\alpha_s(m)$, as can still be found in some recent literature). We also recall that the octet contribution reabsorbs the dependence on the infrared cut-off $\mu$ of the Wilson coefficients $\text{Im} f_1(P)$. Finally, we see that in NRQCD the 8 $P$-wave bottomonium states ($1P$, $2P$) and the 4 $P$-wave charmonium states ($1P$), which lie under threshold, depend at leading order in the velocity expansion on 6 non-perturbative parameters ($3$ wave functions + $3$ octet matrix elements).

In pNRQCD the $P$-wave inclusive decay widths are given at leading order in $v$ by:

$$\Gamma(h \to LH) = \left| R'_P \right|^2 \left[ \frac{9 \text{Im} f_1(1P)}{\pi m^4} + \frac{\text{Im} f_8(1S_0)}{9} \mathcal{E} \right],$$

(6)
The quarkonium-state dependence factorizes in the pNRQCD formulas. This allows some new statements. Using the values of $\mathcal{E}$ from Fig. 1, we get at leading order in $v$ the relation between the octet matrix element of NRQCD and $\mathcal{E}$: $\langle h|O_{S}(^{1}S_{0})|h\rangle = |R'_{P}|^{2}\mathcal{E}/(18\pi m^{2})$. The quarkonium-state dependence factorizes in the pNRQCD formulas. This allows some new predictions with respect to NRQCD, which are synthetized by the formula (valid at leading order in $v$):

$$
\frac{\Gamma(H(2^{S}+1nP_{J}) \to LH)}{\Gamma(H(2^{S'}+1nP'_{J'}) \to LH)} = \frac{81 \Im f_{1}(2^{S}+1P_{J}) + \Im f_{8}(2^{S}+1S_{J})\mathcal{E}}{81 \Im f_{1}(2^{S'}+1P'_{J'}) + \Im f_{8}(2^{S'}+1S'_{J'})\mathcal{E}}, \tag{9}
$$

where the left-hand side is a ratio between inclusive decay widths of $P$-wave quarkonia with the same principal quantum number $n$ and the right-hand side no longer depends on $n$ and has the whole flavour dependence encoded in the Wilson coefficients, which are known quantities.

In practice, the 12 $P$-wave quarkonium states, which lie under threshold, depend only, in pNRQCD at leading (non-vanishing) order in the velocity expansion, on 4 non-perturbative parameters (3 wave functions + 1 chromoelectric correlator $\mathcal{E}$). The reduction by 2 in the number of unknown non-perturbative parameters with respect to NRQCD, allows us to formulate two new statements. Using the values of $\mathcal{E}$ from Fig. 1, we get (at NLO):

$$
\frac{\Gamma(\chi_{b0}(1P) \to LH)}{\Gamma(\chi_{b0}(1P) \to LH)} = \frac{\Gamma(\chi_{b0}(2P) \to LH)}{\Gamma(\chi_{b0}(2P) \to LH)} = 8.0 \pm 1.3, \tag{10}
$$
or alternatively

$$\frac{\Gamma(\chi_b(1P) \rightarrow \text{LH})}{\Gamma(\chi_b(2P) \rightarrow \text{LH})} = \frac{\Gamma(\chi_b(2P) \rightarrow \text{LH})}{\Gamma(\chi_b(2P) \rightarrow \text{LH})} = 0.50^{+0.06}_{-0.04}. \tag{11}$$

The errors refer only to the uncertainties displayed in Fig. 1. In Fig. 2 we plot the above ratios of decay widths as functions of the factorization scale $\mu$. The figures show a stable result.

3 Conclusions

By integrating out degrees of freedom associated with energy scales larger than $m v^2$, octet matrix elements in NRQCD can be written as products of wave functions at the origin by universal non-perturbative factors. This reduces the number of unknown matrix elements of NRQCD and enables definite predictions for inclusive decay ratios of states with different flavour or principal quantum number. As an explicit example we have shown the case of $P$-wave quarkonium inclusive decay into light hadrons. The same program may be carried out for $S$-wave decays. At $O(m v^5)$ the hadronic inclusive decay widths of the 10 $S$-wave quarkonium states, which lie under threshold, are described in NRQCD by about 30 non-perturbative matrix elements: 10 wave functions and 20 further matrix elements. These 20 matrix elements reduce dramatically in pNRQCD to few universal correlators, which may be fixed phenomenologically or by lattice calculations. Noting that in the hadronic/electromagnetic ratios the wave-function dependence drops out, we expect that in the near future pNRQCD will enable QCD to access a large number of new results concerning heavy quarkonium inclusive and electromagnetic decays.

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