Modeling an integrated hospital management planning problem using integer optimization approach

Suryati Sitepu¹, Herman Mawengkang² and Irvan³
¹University HKBP Nomensen Medan
²Mathematics Department, University of Sumatera Utara, Indonesia
³Universitas Muhammadiyah Sumatera Utara
E-mail: hmaawengkang@yahoo.com

Abstract. Hospital is a very important institution to provide health care for people. It is not surprising that nowadays the people’s demands for hospital is increasing. However, due to the rising cost of healthcare services, hospitals need to consider efficiencies in order to overcome these two problems. This paper deals with an integrated strategy of staff capacity management and bed allocation planning to tackle these problems. Mathematically, the strategy can be modeled as an integer linear programming problem. We solve the model using a direct neighborhood search approach, based on the notion of superbasic variables.

1. Introduction
Globally, the needs to ask for health service are increased from time to time. This event happens due to the exponential growth of population in the world, and the fast connection between one country to other countries. This condition also happens in Medan city, Indonesia.

Hospitals as a health service institution are urged to provide health care for people. It is not surprising that nowadays the people’s demand for hospitals is increasing. However, hospitals in Medan face serious problems related to financial pressure and capacity management. The increasing cost for operations and maintenance, and healthcare cost to the patients have added the dimension of problems [1, 2]. This situation brings serious consequence, i.e., there are more and more people from Medan who seek health care from neighbour countries, such as, Malaysia and Singapore. Undoubtedly, the urgent need to tackle this situation is to improve health service performance of hospitals.

A hospital can be thought as a production system, which has limited capacities to support the patient flow. The demand of the system consists of patients with different pathologies that need to be served in the hospital according to their own time. The supply side consists of the available personnel to provide health service (e.g., doctors, nursing staff) and the material resources (e.g., rooms, beds). The objective of a hospital is to match its supply and demand side in the best possible way, resulting in a reliable, quick and efficient patient service [3].

Capacity planning decisions are important to health care industry because not only it relates to the management of highly specialized and costly resources (i.e., nurses, doctors, and advanced medical equipment), but also it makes a difference between life and death in critical conditions [4]. Public hospitals have, in general, more demand for health services than available capacity. Therefore it is...
important to forecast and manage demand with good precision, in order to adjust capacity or take alternative courses of action for example transfer demand to other facilities. Demand forecasting and management is part of a larger design that intends to provide a systemic solution to global hospital management. Such solution is commonly based on the design of a general process structure for hospitals and which defines the management processes that are needed to optimize the use of resources in doing so and to ensure a predefined service level for patients. The general process structure allowed us to determine the key processes where implementation of new practices would generate most value [5, 6].

Most approaches for bed resources capacity planning reported in the literature fall into the use of mathematical programming and simulation. [7] develop a model to simultaneously determine the timing and the number of changes in bed capacity such that minimize capacity cost considering the desired level of facility performance over a finite planning horizon. They model the problem as a large scale nonlinear integer program. However, the model only cover bad capacity problem. They do not include other factors, such as, doctors and nurses.

[8] present a mathematical programming model approach. They propose a multi-objective decision aiding model for solving allocation of beds problem in a hospital. Their model is based on queuing theory and goal programming (GP). In order to obtain some essential characteristics of access to various departments within a hospital, a waiting line model is explored. The results are used to construct a goal programming frame work, taking account of targets and objectives related to customer service and profits from the hospital manager and all department heads. [9] create a mathematical model based on a dynamic dispatching approach for bed resources allocation considering hospitalization demands, bed capacity and income. The objectives of their model are to maximize income and to minimize the costs to use supplementary beds. Another goal programming model approach for solving re-allocation bed in a hospitals was proposed by [10]. The constraints considered in their model are total number of beds, nursing work hours, waiting time, the definite bed allocation to patient, and the definite bed allocation to ward. An integer linear programming model approach is featured by [11] to solve a hospital bed management problem constrained with budgetary cuts. They implemented their result in French hospitals.

In terms of simulation technique for solving bed allocation problem, the works in the literatures belong to the use of simulation only and a combined simulation-optimization. Due to the inherent uncertainties in the problem, it is reasonable to use simulation technique only. In particular, the use of simulation we should be able to get useful insights on bed allocation. These results can be found in [12], and [13]. [14] use discrete event simulation for solving bed allocation problem in the emergency department of a hospital. An interesting review of using simulation in healthcare is addressed in [15]. However the use of only simulation may not reach the best solution, particularly when the problems involve combinatorial nature. It is not surprising that some literatures propose a combine strategy simulation and optimization. [16] propose a simulation optimization approach for solving resource allocation in an emergency department. The use of multi-objective optimization combined with simulation to tackle the bed allocation problem can be found in Wang [17].

2. Problem Description
Increasing demand for healthcare through hospitals created heavy challenges to their managers and decision makers. The challenges involve high costs, limited budget, and limited resources. Most of hospitals in Medan are encountered with some pressures, such as, shortages of qualified healthcare professionals, limited hospital equipments and facilities, increasing operational costs.

Capacity planning, for hospitals in particular, is concerned with making sure of balancing the quality of health care delivered with the cost of providing that care. Such planning involves predicting the quantity and particular attributes of resources required to deliver health care service at specified levels of cost and quality. The most fundamental measure of
hospital capacity planning is the number of inpatient beds accordingly the number of doctors and the number of nurses. Hospital bed capacity decisions have traditionally been made based on target occupancy levels. Certain units in the hospital, such as, intensive care units (ICUs) are often run at much higher utilization levels because of their high costs.

The other important thing that should be considered in order to enhance the service performance of hospitals is waiting time, due to the bed capacity allocation system [10, 11]. Alternative strategy to overcome this situation is to have a well coordinated hospital capacity management along with bed allocation system.

3. Mathematical Model
The decision problem, for our problem, is to maximally coordinate the utilization of multifold resources within the hospital. In our case the multifold resources are doctors, nurses, beds and rooms. In this case the most appropriate model to be created is a linear integer programming problem.

Some notations are necessary to be defined to be used for the mathematical model.

\[ \text{I} \] : Set of departments with index \( i \)
\[ \text{J} \] : Set of doctors type with index \( j \)
\[ \text{K} \] : Set of nurses type with index \( k \)
\[ \text{L} \] : Set of available beds with index \( l \)
\[ \text{R} \] : Set of rooms with index \( r \)

The decision variables are defined as follows.

\[ DA_{ij} \] : Initial number of type \( j \) doctors in department \( i \)
\[ SA_{ij} \] : Initial number of type \( k \) nurses in department \( i \)
\[ SBA_{ij} \] : Initial number of type \( k \) nurse-aids in department \( i \)
\[ D_{ij} \] : Type \( j \) doctors added in department \( i \)
\[ S_{ij} \] : Number of type \( k \) nurses added in department \( i \)
\[ SB_{ij} \] : Number of type \( k \) nurse-aids added in department \( i \)
\[ TPA_{il} \] : Initial number of beds \( l \) in department \( i \)
\[ TP_{il} \] : Number of beds \( l \) added in department \( i \)

There are binary variables involved

\[ x_{ilr}^i \] \( \text{Equals to 1, if bed } l \text{ are allocated for room } r \in R \text{ in department } i \), \( \text{Equals to 0 otherwise.} \)
\[ y_{ir}^i \] \( \text{Equals to 1 if room } r \text{ is used in department } i \), \( \text{Equals to 0 otherwise} \)

Parameters

\[ bd_{ij} \] : Cost of \( j \) type doctors in department \( i \)
\[ bs_{ik} \] : Cost of type \( k \) nurses in department \( i \)
\[ bsa_{ik} \] : Cost of type \( k \) nurse-aids in department \( i \)
\[ bt_{il} \] : Cost of operating beds \( l \) in department \( i \)
\[ bw_{il} \] : Waiting cost for beds \( l \) in department \( i \)
\[ ba_{lr} \] : Cost for allocating bed \( l \) for room \( r \) in department \( i \)
\[ cr_r \] : Cost to operate room \( r \) in department \( i \)
\[ md_{ij} \] : Maximum number of doctor for each type can be allocated to each department
\[ mn_{ik} \] : Maximum number of nurse each type can be allocated to each department
\(mna_i\) : Maximum number of nurse-aids each type can be allocated to each department

\(mb_i\) : Maximum number of beds \(l\) can be allocated to each department

\(\rho_i\) : Maximum fund available for department \(i\)

The capacity management problem for a hospital should decide the number of doctors, nurses and bed to be provided such as to minimize the overall operating costs.

i) costs for doctors

\[
\text{Cost}(1) = \sum_{i \in I} \sum_{j \in J} bd_{ij} (DA_j + D_y)
\]

ii) costs for nurses

\[
\text{Cost}(2) = \sum_{i \in I} \sum_{k \in K} bs_{ik} (SA_k + S_y)
\]

iii) costs for nurse–aids

\[
\text{Cost}(3) = \sum_{i \in I} \sum_{k \in K} bsa_{ik} (SBA_k + SB_k)
\]

iv) costs for beds

\[
\text{Cost}(4) = \sum_{i \in I} \sum_{l \in L} bt_{il} (TPA_i + TP_y)
\]

v) costs regarded to patience has to wait for beds in department \(i\)

\[
\text{Cost}(5) = \sum_{i \in I} \sum_{l \in L} bw_{il} (TPA_i + TP_y)
\]

vi) costs to allocate bed \(l \in L\) if it is allocated to room \(r \in R\)

\[
\text{Cost}(6) = \sum_{i \in I} \sum_{r \in R} \sum_{x \in X} ba_{ir} x^i_r
\]

vii) costs to utilize a room \(r\) if it is chosen

\[
\text{Cost}(7) = \sum_{i \in I} \sum_{r \in R} cr_{ir} y^i_r, \quad \forall i \in I, \forall r \in R
\]

It can be seen that there are seven components for costs.

The objective function can be written as to

\[
\text{Minimize} \sum_{j=1}^{7} \text{Cost}(j)
\]

The constraints of the problem are expressed as follows.

\[
DA_j + D_y \leq md_j, \quad \forall i \in I, \forall j \in J
\]

\[
SA_k + S_y \leq mn_k, \quad \forall i \in I, \forall k \in K
\]

\[
SBA_k + SB_k \leq mna_k, \quad \forall i \in I, \forall k \in K
\]

\[
TPA_i + TP_y \leq mb_i, \quad \forall i \in I, \forall l \in L
\]
\[
\sum_{i \in I} x_{ij}^r \leq y_i^r, \quad \forall i \in I, \forall r \in R
\] (13)
\[
\sum_{i \in I} \sum_{j \in J} b_{ij} x_{ij} + \sum_{i \in I} \sum_{k \in K} b_{ik} S_{ik} + \sum_{i \in I} \sum_{k \in K} b_{iak} s_{ik} + \sum_{i \in I} \sum_{k \in K} b_{ik} T_{ik} + \sum_{i \in I} \sum_{k \in K} b_{ik} T_{ik}^r + \sum_{i \in I} \sum_{k \in K} b_{ik} T_{ik}^r \geq 0 \quad \forall i \in I, \forall r \in L
\] (14)
\[
DA_{ij}, D_y, S_{ik}, S_{iak}, SB_{ik}, SB_{iak}, TP_{ij}, TP_i^r \geq 0 \quad \text{and integer}
\]
\[
\forall i \in I, \forall j \in J, \forall k \in K, \forall l \in L
\] (15)
\[
x_{ij}^r, y_i^r \in \{0, 1\} \quad \forall i \in I, \forall l \in L, \forall r \in R
\] (16)

Constraints (9) state that the number of doctor should not be greater the maximum number of doctors can be allocated to each department. Constraints (10) – (12) are just an analogy to Constraints (9), respectively, for the number of nurse, the number of nurse-aids and the number of beds. Constraints (13) are to make sure that beds are allocated to the available room. Constraints (14) state that the budget spent should not be greater than the fund provided. Expressions (15) and (16) are integrity constraints. The model is in the form of a large scale integer programming problem.

4. Feasible Neighborhood Heuristic Search

Branch-and-bound approach is a general method for solving linear integer programming problem. However, for large-scale problems such a procedure would be prohibitively expensive in terms of total computing time, and frequently the algorithm terminates without solving the problem. We have adopted the approach of examining a reduced problem in which most of the integer variables are held constant and only a small subset allowed varying in discrete steps.

In linear programming terms, there are three kind of variables involve. The first one is basic variables, which have nonnegative values. Secondly is nonbasic variables, which values are at their bounds (in this case the bounds are zero). The third variables are superbasic, which have positive values but do not belong to basic variables [18].

The approach may be implemented within the structure of a program by marking all integer variables at their bounds at the continuous solution as nonbasic and solving a reduced problem with these maintained as superbasic.

The procedure may be summarized as follows:

**Step 1**: Solve the problem ignoring integrality requirements. The problem equation (1) to equation (16) is solved by relaxing the integer restrictions equation (15) and equation (16).

**Step 2**: Obtain a (sub-optimal) integer feasible solution, using heuristic rounding of the continuous solution.

**Step 3**: Divide the set \( I \) of integer variables into the set \( I_1 \) at their bounds that were nonbasic at the continuous solution and the set \( I_2, I = I_1 + I_2 \).

**Step 4**: Perform a search on the objective function, maintaining the variables in \( I_1 \) nonbasic and allowing only discrete changes in the values of the variables in \( I_2 \).

**Step 5**: At the solution obtained in step 4, examine the reduced costs of the variables in \( I_1 \). If any should be released from their bounds, add them to set \( I_2 \) and repeat from step 4, otherwise terminate.

The above summary provides a framework for the development of specific strategies for particular classes of problems. For example, the heuristic rounding in step 2 can be adapted to suit the nature of the constraints, and step 5 may involve adding just one variable at a time to the set \( I_2 \).

At a practical level, implementation of the procedure requires the choice of some level of tolerance on the bounds on the variables and also their integer infeasibility. The search in step 4 is affected by such considerations, as a discrete step in a super basic integer variable may only occur if all of the basic integers remain within the specified tolerance of integer feasibility.
In general, unless the structure of the constraints maintains integer feasibility in the integer basic variables for discrete changes in the superbasic, the integers in the set \( I_2 \) must be made superbasic. This can always be achieved since it is assumed that a full set of slack variables is included in the problem.

5. Conclusion
This paper presents an integrated model to cover capacity management and bed allocation planning in a hospital. All variables in the model are defined as integer numbers and binary numbers. We propose a feasible neighbourhood integer search for solving the large scale integer programming problem.

References
[1] Brailsford S and Vissers J 2011 European Journal of Operational Research. 212(2) 223-234
[2] Rais A and Viana A 2010 International Transactions in Operational Research 18(1) 1-31
[3] Jack E P and Powers T L 2009 International Journal of Management Reviews 11(2) 149-174
[4] Hans E W, Van Houdenhoven M and Hulshof P J 2012 A framework for healthcare planning and control Handbook of Healthcare System Scheduling (Springer)
[5] Swayne L E, Duncan W J and Ginter P M 2012 Strategic Management of Health Care Organization John Wiley and Sons
[6] Ma G and Demeulemeester E 2013 Computers and Operations Research 40 2198-2207
[7] Ackali E, Cote M J and Lin C 2006 Healthcare Manag. Sci. 9 391-404
[8] Li X, Jones D and Tamiz M 2009 Journal of the Operational Research Society. 60 330-338
[9] Wang Y, Lee L H, Chew E P, Lam S S W, Low S K, Ong M E H, Li H. 2015 Multi-objective optimization for a hospital inpatient flow process via discrete event simulation In Proceedings of the 2015 Winter Simulation Conference, edited by Yilmaz L, Chan W K V, Moon I, Roeder T M K, Macal C, and Rosseti M D, 3622-3631 Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc
[10] Ataollahi F, Bahrami M A, Abesi M and Mobasheri F 2013 Middle East Journal of Scientific Research 18(11) 1537-1543
[11] Bachouch R B, Guinet A and Hajri-Gabouj S 2012 Int. J. Production Economics 140 833-843
[12] Harper P R and Shahani A K 2002 The Journal of the Operational Research Society 53(1) 11-18
[13] Cochrane J K and Bharti A 2006 Health Care Management Science 9(1) 31-45
[14] Laker L F, Froehle C M, Lindsell C J and Ward M J 2014 Annals of Emergency Medicine 64(6) 591-603
[15] Almagooshi S 2015 Prtocedia Manufacturing 3 301-307
[16] Keshtkar L, Salimifard K and Faghih N 2015 QScience Connect. 1 8
[17] Wang T, Guinet A and Besombes B 2009 Intelligent Patient Management. SCI 189 113-125
[18] Murtagh B A 1981 Advanced Linear Programming: Computation and Practice McGraw-Hill. ISBN 0-07-044095-6