The Parameterized Post-Friedmannian Framework for Nonminimal Derivative Coupling with General Cosmological Perturbation Metric

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Abstract. We study the parameterization of cosmological model where derivative of an additional scalar field coupled to curvature tensor. We extend the Parameterized Post-Friedmannian framework for interacting dark energy theories to the model. Starting from general cosmological perturbation metric, we derive perturbed energy-momentum tensor for scalar field and parameterized the tensor. Based on the value of the parameters, we compare the model with relevant models in current literature. As the results, we find that parameterization for type 1 theories of dark energy which is explicitly coupled to dark matter, gives only 4 non-zero Parameterized Post-Friedmannian coefficients.

1. Introduction

The development of observational evidence shows that the universe is filled with dark energy and dark matter. Both of these dark sectors are the main component of the current universe that is in the phase of accelerated expansion. Based on cosmological data from Cosmic Microwave Background (CMB) measurement, Supernova Ia, and Baryon Acoustic Oscillation (BAO) Surveys, the component that fill the universe in the form of weak interaction are Cold Dark Matter (CDM) around 25% and approximately 70% in the form of dark energy [1, 2]. Interacting dark energy has been reviewed in the framework of the theory of gravitational braneworld with Lorentz violation [3, 4, 5].

The dark energy modelling theoretically can be done by modified the left hand side of the Einstein’s equation (or so called modified gravity approach) and modified the right hand side of the Einstein’s equation (modified matter) [6]. Modified gravity can be done by adding the new fields to Einstein’s equation (fields of scalar, vector, tensor, or the combinations) or by adding the extra dimensions. The example of modified gravity with the addition of fields are \( f(R) \) gravity, the scalar-tensor theory, and the Gauss-Bonnet model. For the modification by adding the extra dimensions, we have braneworld models of Horava-Witten, Arkani-Hamed-Dvali-Dimopoulos (ADD), Randall-Sundrum (RS), Dvali-Gabadadze-Porrati (DGB), and universal extra dimension (UED). For modified matter, we have models of quintessence, k-essence, phantom, coupling of dark energy, and the chameleon scalar field. Modeling dark energy by nonminimal derivative coupling (NMDC) between
scalar fields and curvature will also give us an accelerated expansion. This coupling has been studied in the context of cosmological inflation and late time acceleration.

Currently, a wide variety of candidate theories of modified gravity may still be proposed. If these theories are able to be understood and can solve the problem of dark energy, these theories must be confronted with cosmological data. This situation is inline with the experimental study of general relativity in the early 1970s, known as the “Golden Age” of general relativity. At that time there were a number of alternative gravitational theories that needed to be confronted with the constraints of Solar System measurement such as The Parameterized Post-Newtonian (PPN). In PPN formalism, the theory of gravity is mapped with integrated parameters and is limited simultaneously using data from laboratory test, around lunar lasers, and initial satellite experiment [7]. However, PPN framework cannot be applied in the cosmological scale so we need another approach to make a parameterization framework that can be used for the concept of modified gravity of cosmology in general [8].

The Parameterized Post-Friedmannian (PPF) approach is a new framework used to test the concept of modified gravity of cosmology in general. The construction of the PPF framework does not depend on modified gravity but considers the number of ways to limit the Einstein’s linear equation so that it can be extended while maintaining the properties of the theory of gravity that are relevant physically. This PPF for malism tests an independent model of modified gravity so that it can pave the way for an independent model to classify and test the theory of dark interactions. The PPF framework has been applied to the theory of modified gravity including the interaction of dark energy [9].

In this paper, we will study cosmological perturbation from general perturbation metrics with nonminimal derivative coupling (NMDC) of scalar field. Then, we apply the PPF formalism in the cosmological model of coupled dark matter to dark energy type 1. From article [10], the type 1 models are classified via $F(n,Y,Z) = F(Y,\phi) + be^{\alpha(\phi)}$ where $\alpha(\phi)$ is free function of the field $\phi$. Thus, the characteristic and physical meaning of the model can be revealed by looking at the PPF coefficients resulting from linear perturbation of scalar mode in variables of metric and fluid.

2. Formalism

The equation of gravitational field for theory of dark interaction can be written as follows:

$$G_{\mu\nu} = 8\pi G \left( T^{(SM)}_{\mu\nu} + T^{(GDM)}_{\mu\nu} + T^{(DE)}_{\mu\nu} \right)$$

where $G_{\mu\nu}$ is Einstein’s tensor, $T^{(SM)}_{\mu\nu}$ is energy-momentum tensor of matter form of standard model (baryon, foton, neutrino, etc), $T^{(GDM)}_{\mu\nu}$ is energy-momentum tensor of generalized dark matter (GDM), and $T^{(DE)}_{\mu\nu}$ is energy-momentum tensor which produces the effect of dark energy. The equation of Einstein’s tensor (1) must obey Bianchi’s identity $\nabla_{\mu} G_{\mu}^{\nu} = 0$, so that

$$\nabla_{\mu} \left( T^{(SM)}_{\mu\nu} + T^{(GDM)}_{\mu\nu} + T^{(DE)}_{\mu\nu} \right) = 0.$$ 

The particle of standard model are considered to be not coupled to the dark sector explicitly so $\nabla_{\mu} T^{(SM)}_{\mu\nu} = 0$ and $\nabla_{\mu} \left( T^{(GDM)}_{\mu\nu} + T^{(DE)}_{\mu\nu} \right) = 0$. This condition led to the coupling current that represents the exchange of energy and momentum between the component of dark sector in the form of,

$$\nabla_{\mu} T^{(GDM)}_{\mu\nu} = J_{\nu} = -\nabla_{\nu} T^{(DE)}_{\mu\nu}.$$ 

The cosmological model that will be associated with PPF is a model that has a Friedmann-Robertson-Walker (FRW) solution as a background cosmology. This FRW model is a model of an
expanding universe with metric that contain time-dependent functions. The spacetime of FRW background is described in the form,

\[ ds^2 = a^2 \left( -d\tau^2 + \delta_{ij}dx^i dx^j \right) \]  

(4)

where \( i, j \) is index of three-dimensional space coordinates, \( d\tau \) is the conformal time, and \( \delta_{ij} \) is the spacial metric. The function \( a \) is the scale factor and depends on the time coordinate \( t \). In this work, we choose a flat spacetime in regard to the observation from Wilkinson Microwave Anisotropy Probe (WMAP) [11]. Due to symmetry of the spacetime of FRW background, the non zero component of \( \bar{T}_{\mu\nu} \) is energy density \( \bar{\rho} = -\bar{T}^0_0 \) and pressure \( \bar{T}^i_j = \bar{P}\delta^i_j \). The components Einstein’s tensor in equation (1) for the equation metric (4) are

\[ 3H^2 = 8\pi G \left( \bar{\rho}_{SM} + \bar{\rho}_{GDM} + \bar{\rho}_{DE} \right) \]  

(5)

and

\[ H^2 - 2\frac{\ddot{a}}{a} = 8\pi G \left( \bar{P}_{SM} + \bar{P}_{GDM} + \bar{P}_{DE} \right), \]  

(6)

where \( H = \frac{\dot{a}}{a} \) is the conformal Hubble parameter and dot shows derivative with respect to conformal time \( \tau \).

The coupling current with non zero component is

\[ \bar{J}_0 = Q \]  

(7)

where \( Q(\tau) \) is the background coupling function. The continuity equation that obtained from the equation metric (4) is

\[ \dot{\bar{\rho}}_i + 3H\bar{\rho}(1 + w_i) = s_i Q, \]  

(8)

where index \( i \) represents the combination of components (matter of standard model, generalized dark matter, and dark energy), \( w_i = P_i / \rho_i \) shows the equation of state for each component-1 , and \( s_i \) shows a constant that is 1 for dark energy, 0 for standard model, and -1 for generalized dark matter. The coupling current in PPF formalism is parameterized to produce the field equation that containing at most two time derivatives or equivalent to the first order linear field equation in the FRW background. Then we consider a linear perturbation from the spacetime of FRW background to see its effect on the parameterization.

The spacetime metric of general linear perturbation are

\[ ds^2 = -a^2 (1 + 2\Psi) d\tau^2 - 2a^2 \nabla_\xi d\tau dx^i + a^2 (1 + \frac{1}{3}h) \gamma_{ij}y^i y^j + a^2 D_jy^i] dx^i dx^j \]  

(9)

where \( \Psi \), \( \zeta \), \( h \), \( \nu \) are four function of time and space (four scalar mode) and \( D_j = \nabla_j - \frac{1}{3}\gamma^j_{ij} \nabla^2 \) is derivative operators that project of longitudinal, traceless, and spatial parts of perturbation. From that metric, geometric quantities such as the perturbation of Ricci tensor is

\[ \delta R_{00} = \partial_i \Psi' - \frac{1}{3} \bar{h} - \partial_i \zeta_i + H \left( 3\Psi' - \frac{1}{3} \bar{h} \right) - \partial_i \zeta_i, \]  

(10)

\[ \delta R_{0i} = -H \partial_i \zeta_i - \frac{1}{3} \bar{h} + \frac{1}{2} \delta_i \bar{\zeta}_i + \frac{1}{2} \left( \partial_i \partial_j \zeta_j - \partial_j \partial_i \zeta_j \right) - 2H^2 \zeta_i + 2H\Psi_i, \]  

(11)

\[ \delta R_{00} = -H \partial_i \zeta_i - \frac{1}{3} \bar{h} + \frac{1}{2} \delta_i \zeta_i + \frac{1}{2} \left( \partial_i \partial_j \zeta_j - \partial_j \partial_i \zeta_j \right) - 2H \zeta_i + 2H \Psi_i, \]  

(12)

\[ \delta R_{ij} = H \left[ (-2\Psi + \frac{1}{3} \bar{h}) \partial_j \bar{\nu}_i + H \bar{\nu}_i \right] + H \left[ (-\Psi + \frac{2}{3} \bar{h}) \bar{\partial}_j \bar{\nu}_i + \bar{\partial}_j \bar{\nu}_i \right] + \frac{1}{2} \bar{\partial}_i \bar{\partial}_j \zeta_i + \frac{1}{2} \bar{\partial}_i \bar{\partial}_j \zeta_j \]  

(13)

\[ -\frac{1}{6} \bar{\partial}_i \bar{\partial}_j \bar{\partial}_k \bar{\partial}_l \bar{\nu}_a + \frac{1}{6} \bar{\partial}_i \bar{\partial}_j \bar{\partial}_k \bar{\partial}_l \bar{\nu}_a + H \bar{\partial}_i \bar{\partial}_k \bar{\partial}_l \bar{\partial}_j \zeta_i + \frac{1}{2} \bar{\partial}_i \bar{\partial}_j \bar{\partial}_k \bar{\partial}_l \bar{\nu}_a - \bar{\partial}_j \Psi_i \]  

\[ -\frac{1}{2} \bar{\partial}_i \bar{\partial}_j \bar{\partial}_k \bar{\partial}_l \bar{\nu}_a + 2H^2 \left[ (-2\Psi + \frac{1}{3} \bar{h}) \bar{\nu}_i + \bar{\partial}_i \bar{\nu}_a \right] + H \left( \bar{\partial}_i \zeta_i + \bar{\partial}_j \zeta_j \right) + \frac{1}{2} \bar{\nu}_a \bar{H} \bar{\partial}_i \zeta_i, \]  

the perturbation of Ricci scalar,
\[ \delta R = a^2 \left[ -12 \Psi \dot{H} - 2 \dot{\Psi} \dot{\Psi}' + \dot{h} + 2 \dot{\zeta} \dot{\zeta}' + 3 \H (-2 \dot{\Psi} + \dot{h}) - 12 \Psi \dot{H}^2 \right] + 6 \dot{\zeta} \dot{H} - \frac{3}{2} \partial_i \partial^i h + \frac{1}{2} \partial_j \partial^i v_j \],

(14)

and the perturbation of Einstein’s tensor,

\[ \delta G_{00} = \H \left( \dot{h} + \psi' \right) + 2 \H \partial^i \zeta_i - \frac{1}{2} \partial_i \dot{h} + \frac{1}{2} \partial_i \partial^i v_i \]

(15)

\[ \delta G_{0i} = 2 \H \zeta_i - \frac{1}{2} \partial_i \dot{h} + \frac{1}{2} \partial_i \partial^i v_i + \frac{1}{2} \left( \partial_j \partial^i \zeta_i - \partial_i \partial^i \zeta_j \right) + \H^2 \zeta_i + 2 \H \Psi_i \]

(16)

\[ \delta G_{i0} = 2 \H \zeta_i - \frac{1}{2} \partial_i \dot{h} + \frac{1}{2} \partial_i \partial^i v_i + \frac{1}{2} \left( \partial_j \partial^i \zeta_i - \partial_i \partial^i \zeta_j \right) + \H^2 \zeta_i + 2 \H \Psi_i \]

(17)

\[ \delta G_{ij} = \H \left[ (4 \Psi - \frac{2}{3} h) \delta_{ij} - 2 \nu_{ij} \right] + \H \left[ (2 \Psi - \frac{2}{3} h) \delta_{ij} + \nu_{ij} \right] + \frac{1}{2} \left( \partial_j \partial^i \zeta_i - \partial_i \partial^i \zeta_j \right) + \left( -\frac{1}{2} \partial_i \delta_{ij} + \frac{1}{2} v_{ij} \right) \]

(18)

where \( \zeta_i = \nabla \zeta \) and \( \nu_{ij} = D_j \nu \). The component of energy-momentum tensor for fluid are

\[ T^0_0 = -\bar{\rho} (1 + \delta) \]

(19)

\[ T^0_i = - (\bar{\rho} + \bar{P}) \tilde{\nabla}^i \theta \]

(20)

\[ T^i_0 = - (\bar{\rho} + \bar{P}) \tilde{\nabla}_i \theta \]

(21)

\[ T^i_j = \bar{\rho} (w + \Pi) \delta^i_j + (\bar{\rho} + \bar{P}) D_j \Sigma \]

(22)

where \( \delta \) is density \( (\delta = \delta \rho / \bar{\rho}) \), \( \theta \) is the scalar mode of momentum \( u_i = a \nabla \theta \), \( \Pi \) is the dimensionless pressure perturbation \( (\Pi = \delta P / \bar{\rho}) \), and \( \Sigma \) is scalar mode from shear \( (\Sigma_{ij} = D_j \Sigma) \). The perturbation of Einstein’s equation are

\[ \H \left( \dot{h} + 2 \dot{\zeta} \right) - 6 H^2 \Psi - \frac{1}{2} \partial_i \partial^i h + \frac{1}{2} \partial_i \partial^i \zeta_i = 8 \pi G a^2 \sum_i \bar{\rho}_i \delta_i \]

(23)

\[ -2 \H \Psi' + \frac{1}{2} \partial_i \dot{h} - \frac{1}{2} \nu_i - \left( \partial_j \partial^i \zeta_i - \partial_i \partial^i \zeta_j \right) - 2 \dot{H}^2 \zeta_i + 2 \H \dot{H} \zeta_i = 8 \pi G a^2 \sum_i (\bar{\rho}_i + \bar{P}_i) \tilde{\nabla}_i \theta_i \]

(24)

\[ -2 \H \Psi' + \frac{1}{2} \partial_i \dot{h} - \frac{1}{2} \partial_i \nu_i - \left( \partial_j \partial^i \zeta_i - \partial_i \partial^i \zeta_j \right) = 8 \pi G a^2 \sum_i (\bar{\rho}_i + \bar{P}_i) \tilde{\nabla}_i (\theta_i - \zeta_i) \]

(25)

\[ \left\{ -\frac{1}{3} \delta_i \partial^i \left( \frac{1}{3} h + \Psi \right) + \H \left( 2 \Psi - \frac{2}{3} h \right) + (4 \H + 2 \H^2) \Psi - \partial^i \zeta_i - 2 \dot{H} \zeta_i - \frac{1}{2} \partial_i \partial^i \zeta_i \right\} \delta_i 

+ \partial^i \partial_j \left( -\frac{1}{3} h - \Psi \right) + \frac{1}{2} \partial^i \partial^j \zeta_i + \partial^i \partial^j \zeta_j \right) + \H \left( \partial_j \zeta_i + \partial^i \zeta_j \right) + \frac{1}{2} \partial^i \nu_i \]

(26)

\[ + \frac{1}{2} \partial_i \partial^i \nu_i + \H \nu = 8 \pi G a^2 \sum_i \left[ \bar{\rho}_i \Pi_i \delta_i + (\bar{\rho}_i + \bar{P}_i) D_j \Sigma \right] \]

Let us define two scalar modes of perturbation \( q \) and \( S \),

\[ q = \dot{J}_0 \text{ and } \quad \tilde{\nabla} S = \delta J_i \]

(27)

Scalar modes \( q \) and \( S \) are parameterized as a form of linear combination from variables of metric, fluid, and gauge. There are 12 variables for each \( q \) and \( S \). Then, the gauge transformation is done with two constraint equations for each \( q \) and \( S \) so that it can reduce the number of perturbation variables to 10 for \( q \) and \( S \) respectively. The results are

\[ q = - \frac{1}{2} \left( \dot{Q} - H Q \right) V + Q \Psi - 6 A_1 \hat{\Phi} - 6 A_2 \hat{\Phi} + A_3 \delta_{DE} + A_4 \delta_{GDM} + A_5 \hat{\Phi}_{DE} + A_6 \hat{\Phi}_{GDM} \]

(28)

\[ + A_7 \hat{\Pi}_{DE} + A_8 \hat{\Pi}_{GDM} + A_9 \Sigma_{DE} + A_{10} \Sigma_{GDM} \]

and
\[ S = \frac{1}{2} Q V - 6 B_{i} \dot{\Phi} - 6 B_{2} \dot{\Gamma} + B_{4} \dot{\delta}_{DE} + B_{5} \dot{\theta}_{GDM} + B_{6} \dot{\theta}_{GDM} \]
\[ + B_{7} \dot{\Pi}_{DE} + B_{8} \dot{\Pi}_{GDM} + B_{9} \Sigma_{DE} + B_{10} \Sigma_{GDM}. \]  

(29)

In the special case of CDM, CDM fluid is a perfect fluid so that \( w_{GDM} = \Pi_{GDM} = \Sigma_{GDM} = 0 \). Furthermore, we assume dark energy does not have a shear \( \Sigma_{DE} = 0 \) and write the equation of state \( w_{DE} = w \). The form of pressure perturbation in this case will be expressed in terms of \( \delta_{DE} \) and \( \theta_{DE} \) through the following equation of state

\[ \Pi_{DE} = c_{a}^{2} \delta_{DE} + \left( c_{a}^{2} - c_{a}^{2} \right) \left[ 3(1 + w)H - \frac{Q}{\rho_{DE}} \right] \theta_{DE} + \mu \left( \theta_{C} - \theta_{DE} \right) \]  

(30)

where \( c_{a}^{2} \) and \( c_{a}^{2} \) are effective and adiabatic sound speed. The adiabatic sound speed can be expressed in the equation of state \( w \) as

\[ c_{a}^{2} = w + \frac{Q}{\rho_{DE}} - 3H(1 + w). \]  

(31)

From the description above, it can be determined that \( A_{i} \) and \( B_{i} \) are zero.

Furthermore, we choose condition of conformal Newtonian gauge with \( \zeta = \nu = 0 \) and \( V = 0 \). The parameterization equation \( q \) and \( S \) in the form of the invariant gauge variable for case CDM becomes

\[ q = Q \dot{\Psi} - 6 A_{1} \Phi - 6 A_{2} \left( \dot{\Phi} + H \Psi \right) + A_{4} \delta_{DE} + A_{4} \delta_{C} + A_{5} \theta_{DE} + A_{6} \theta_{C} \]

(32)

and

\[ S = -6 B_{1} \Phi - 6 B_{2} \left( \dot{\Phi} + H \Psi \right) + B_{4} \delta_{DE} + B_{4} \delta_{C} + B_{5} \theta_{DE} + B_{6} \theta_{C} \]

(33)

where \( A_{i} \) and \( B_{i} \) are the unknown function with \( i \in 1 \ldots 6 \).

3. The PPF Framework for NMDC

Scalar fields generally appear in the Lagrangian of cosmological model in three forms: minimal coupling, nonminimal coupling, and nonminimal derivative coupling (NMDC). The third coupling form of scalar fields was introduced by Amendola (1993) [12]. Amendola showed that the presence of NMDC allows attractor inflation, if none, so inflation will be absence. This model has been studied in the context of inflation and late time acceleration. The behaviour of de Sitter universe with coupling constants of NMDC can be used to restore the cosmological constant [13].

The application of NMDC model has been extended to the five dimensional Universal Extra Dimension (UED) model and produces solutions of an accelerated expansion of the universe [14, 15]. In addition, the cosmological constant can be expressed in combination of the coupling constant of this model. The expansion of NMDC model has also been carried out on five dimensional Randall-Sundrum braneworld model [16]. A study of the effect of NMDC on changes in the gravitational constant \( G \) value has also been carried out in [15].

The action for NMDC model is given as follows:

\[ S = \int d^{5}x \sqrt{-g} \left( R + \frac{1}{2} g_{\mu \nu} \partial^{\mu} \phi \partial^{\nu} \phi + \frac{\varepsilon}{2} g_{\mu \nu} R \partial^{\mu} \phi \partial^{\nu} \phi + \frac{\eta}{2} g_{\mu \nu} \partial^{\mu} \phi \partial^{\nu} \phi + V(\phi) \right). \]

(34)

The action is varied with respect to the metric tensor to get the Einstein’s equation,

\[ G_{\mu \nu} = 8\pi G T_{\mu \nu}(\phi), \]

(35)

where
\( T_{\mu\nu}(\phi) = \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla_\alpha \phi \nabla^\alpha \phi \) \\
\( \quad + \xi \left[ R \nabla_\mu \phi \nabla_\nu \phi + G_{\mu_1 \nu_1 \phi} \nabla_{\alpha_1} \phi - \nabla_\mu \nabla_\nu \left( \nabla^\alpha \phi \nabla_\alpha \phi \right) + g_{\mu\nu} \nabla_\alpha \phi \nabla^\alpha \left( \nabla_\mu \phi \nabla_\nu \phi \right) \right] \)

\( \frac{-\eta}{2} \left[ g_{\mu\nu} R_{\alpha\beta} \nabla^\alpha \phi \nabla^\beta \phi - 2 R_{\mu_1 \nu_1 \phi} \nabla_{\alpha_1} \phi \nabla^2 \phi - 2 R_{\alpha_1 \phi} \nabla_{\mu_1} \phi \nabla^2 \phi \nabla_{\nu_1} \phi \nabla_{\alpha_1} \phi \nabla_{\mu_1} \phi \nabla_{\nu_1} \phi \right] + \nabla_\mu \nabla_\nu \left( \nabla^\alpha \phi \nabla_\alpha \phi \right) - g_{\mu\nu} \nabla_\alpha \phi \nabla_\beta \left( \nabla^\alpha \phi \nabla^\beta \phi \right) \right]. \quad (36) \)

We choose condition \(2\xi + \eta = 0\) for the energy-momentum equation with NMDC so that the third derivative terms of scalar fields can be eliminated. The equations that resulting from this NMDC action still have a complicated form, so it is very difficult to find a solution directly. Therefore, it is necessary to review certain cases in order to obtain constraints that make the equation become much simpler. The solution reviewed in this work is De Sitter solution that provides a constraint in the form of constant Hubble parameter \(H\) value and scalar field is described in a linear function with respect to time \((\dot{\phi} \text{ constant})\) as in [6]. By the selection of these conditions, the density values and background pressure that obtained from the NMDC action are

\[ \rho = \frac{\dot{\phi}^2}{2a^2} + V(\phi) + a^4\xi(2\dot{H} + 15H^2)\dot{\phi}^2, \quad (37) \]

\[ P = \frac{\dot{\phi}^2}{2a^2} - V(\phi) + a^4\xi(-2\dot{H} + 5H^2)\dot{\phi}^2, \quad (38) \]

and the perturbation of density values and background pressure that obtained from the NMDC action are

\[ \delta \rho_{DE} = a^2(\dot{\phi} + \dot{\psi}) + V_{\phi}\phi + a^4\xi\left[ \left( 2\dot{H} - 48H^2 \right)\psi + 3H(\dot{\psi} + \dot{\psi}) \right] \dot{\phi}^2 + 6H^2\dot{\phi}\phi + \left( -6hH^2 + \frac{1}{2}hH - \frac{1}{3}\partial' h \right) \dot{\phi}^2 + \left( -\partial' \xi, \dot{\phi}^2 + 8H - 17H^2 \right) \dot{\phi}\phi \]

\[ (39) \]

\[ \delta P = a^2(\dot{\phi}\dot{\psi} - \dot{\phi}\dot{\psi}) - \delta V(\phi) + a^4\xi\left[ \left( 16\psi H^2 + 3\dot{\psi} + 3\dot{\psi} \right) \dot{\phi}^2 + (8H - 17H^2) \dot{\phi}\phi \right] + \left( 1 + 3(10h^2 + 16) \dot{H} + \frac{1}{6}hH - \frac{1}{6}h \right) \dot{\phi}^2 + \left( -H\partial'_i \xi - 3\partial' \xi - \frac{1}{2}\partial' \dot{\psi} \right) \dot{\phi}^2 \right]. \]

\[ (40) \]

The coupling current that will be used in parameterization with this NMDC is the coupling current from model of coupled dark matter to dark energy type 1 [10], namely

\[ J_\mu = -\beta_\phi \rho C \phi_\mu \phi \]

where \(\beta_\phi\) is free function to scalar field \(\phi\) (\(\beta_\phi \equiv \frac{\partial \phi}{\partial \phi}\)) and subscript C states CDM. The function of background coupling is

\[ Where Q dan S becomes \]

\[ q = -\beta_\mu \rho C \dot{\phi} \phi + Q \left( \frac{\delta C}{\phi} + \frac{\dot{\phi}}{\phi} \right) \quad \text{and} \quad S = Q \frac{\phi}{\dot{\phi}}. \]

\[ (43) \]

From the background equation of density and pressure with NMDC (37-38), we get

\[ \dot{\phi}^2 = \frac{a^4(\rho + P)}{a^2 + 20\xi H^2}. \]

And from the equation perturbation of density and pressure with NMDC (39-40), we get
\[
\frac{\dot{\phi}}{\phi} = \left( \frac{(3 + 16\xi \pi G \rho_{\text{DE}})}{(1 + 16\xi \pi G \rho_{\text{DE}})(1 + w)} - \frac{4\xi \pi G \rho_{\text{DE}}}{1 + 16\xi \pi G \rho_{\text{DE}}} \right) \delta_{\text{DE}} + \frac{1 + 8\xi \pi G (15\rho + P)}{1 + 16\xi \pi G \rho} \psi
\]

\[
- \frac{a^3 V_\phi}{(1 + 16\xi \pi G \rho)^2} \frac{\xi}{\phi} \theta_{\text{DE}} + \frac{\xi}{a^2 (1 + 16\xi \pi G \rho)^2} \left[ H^2 \psi - 3H \dot{\psi} - \partial_i \psi \partial^i + \frac{1}{2} \dot{h} + \frac{1}{6} \partial_i \partial^i h \right]
\]

Both of the above equations are substituted into the equation (43)

\[
q = \left( -\beta_\phi \overline{\rho}_c \dot{\phi}^2 - \frac{aV_\phi}{(1 + 16\xi \pi G \rho)^2} \theta_{\text{DE}} + Q \delta_{\text{DE}} + \frac{1 + 8\xi \pi G (15\rho + P)}{1 + 16\xi \pi G \rho} Q \psi \right)
\]

\[
+ \left( \frac{(3 + 16\xi \pi G \rho_{\text{DE}})}{(1 + 16\xi \pi G \rho_{\text{DE}})(1 + w)} - \frac{4\xi \pi G \rho_{\text{DE}}}{1 + 16\xi \pi G \rho_{\text{DE}}} \right) \delta_{\text{DE}}
\]

\[
+ \frac{\xi Q}{a^2 (1 + 16\xi \pi G \rho)^2} \left[ H^2 \Psi - 3H \dot{\Psi} - \partial_i \Psi \partial^i + \frac{1}{2} \dot{h} + \frac{1}{6} \partial_i \partial^i h \right]
\]

and

\[
S = Q \theta_{\text{DE}}
\]

where \( \theta_{\text{DE}} = \frac{\dot{\phi}}{\phi} \) is the momentum divergence for scalar field. If

\[
\frac{1 + 8\xi \pi G (15\rho + P)}{1 + 16\xi \pi G \rho} = 1,
\]

we will have the following PPF coefficients by comparing the equations (46) with (32) and (47) with (33),

\[
A_i = A_2 = A_5 = 0, \quad A_4 = \frac{Q}{(1 + 16\xi \pi G \rho) \dot{\phi}^2}, \quad A_6 = \frac{\beta_\phi \overline{\rho}_c \dot{\phi}^2 \dot{\phi}^2}{(1 + 16\xi \pi G \rho) \dot{\phi}^2}, \quad B_5 = \theta_{\text{DE}}.
\]

4. Conclusion

In this paper, parameterization of the model of coupled dark matter to dark energy type 1 has been done with NMDC in the framework of PPF. The parameterization for these theories depends on the chosen coupling function, the background coupling, the background field energy density, the potential, etc. For type 1 with NMDC, we get 3 non-zero coefficients for \( A \) and only 1 non-zero \( B \) coefficient. Article [17] shows that the acceleration of the universe occurs if the equation of state, \( w < -\frac{1}{3} \). So the equation of state that resulting from PPF with NMDC at the coupling current from model of coupled dark matter to dark energy type 1 fulfills that the universe is dominated by exotic energy called dark energy.

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