Cosmological Particle Creation in the Presence of Lorentz Violation

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In recent years, the effects of Lorentz symmetry breaking in cosmology has attracted considerable amount of attention. In cosmological context several topics can be affected by Lorentz violation, e.g., inflationary scenario, CMB, dark energy problem and baryogenesis. In this paper we consider the cosmological particle creation due to Lorentz violation (LV). We consider an exactly solvable model for finding the spectral properties of particle creation in an expanding space-time exhibiting Lorentz violation. In this model we calculate the spectrum and its variations with respect to the rate and the amount of space-time expansion.

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I. INTRODUCTION

Particle creation in the cosmological context is one of the most interesting feature of QFT in curved space time [1]. In this process, a quantum field propagates on an expanding space-time and the field quanta (particles) are generated through an impulse of the space-time expansion. The density and the rate of particle production depend on the vigor of the expansion.

Furthermore, it is well known that if a conformally invariant field propagates in a space-time which is conformally equivalent to Minkowski (conformal triviality) no particle production occurs. Thus, we expect that massless quanta of Maxwell and Dirac fields do not arise from the expansion of Universe. But if a term is added to the field equation such that conformal invariance is broken then these particles are created [1, 2].

QFT in curved background is based on the hypothesis that the field equations are locally Lorentz invariant. But in recent years there have been many attentions to the possibility of Lorentz symmetry breaking at high energies. The initial motivation came from string theory [3], and more recently this symmetry breaking has been discussed in the context of noncommutative geometry [4, 5]. Also, there has been evidence that this symmetry may be broken in at least three different phenomena:

i) Observation of ultra-high energy cosmic rays with energies beyond the so-called GZK cutoff, \( E_{GZK} \approx 10^{19} \text{eV} \) [6, 7].

ii) Events involving gamma radiation with energies beyond 20 TeV from distant sources such as Markarian 421 and Markarian 501 blazars [8].

iii) Studies of the evolution of air showers produced by ultra high-energy hadronic particles suggest that pions live longer than expected [9]. These observations can be explained via the breaking of Lorentz symmetry [10, 11]. Now, if we want to consider the effect of LV in QFT in curved space-time, an important question for investigation is: How dose Lorentz violation affect the particle creation in an expanding space-time?

The possible effects of LV physics in inflationary cosmology has been studied in [12, 13, 14, 15]. The authors found that the spectrum of fluctuations in inflationary cosmology and so the spectrum of temperature anisotropies in the Cosmic Microwave Background can be affected by LV. Also it is shown that Lorentz violation is relevant to the dark energy problem and baryogenesis [16, 17, 18, 19]. In the present work we look for the effects of LV in particle creation process in an expanding space-time. Our interest is to study the form and properties of particle creation spectrum of a scalar field. For this purpose we choose an exactly solvable model and we find the characteristics of the spectrum.

We benefit from the model that has been introduced in [20, 21] for LV to study this subject. In this model the usual dispersion relation is modified by adding a term \( \alpha^2 k^4 \). The modified dispersion relation preserves rotational invariance but violates the boost invariance of Lorentz symmetry. It has been shown that adding a term \( \alpha^2 k^4 \) to the dispersion relation is equivalent to putting a vector field which is coupled with matter field in the Lagrangian of the model. This vector field can physically be assumed as the four velocity of a preferred inertial observer. The additional term in the Lagrangian which enforces the LV, breaks the conformal invariance. Therefore we expect that the massless field quanta are created during the space-time expansion.

The organization is as follows: in section II we quantize the Lorentz violation model introduced in [20] on Minkowski space-time. In section III we define an expanding cosmological model and consider the quantization of the LV model on this space-time. Finally, we obtain and discuss the spectrum characteristics of created massless particles in details.

II. LORENTZ VIOLATION MODEL AND QUANTIZATION IN MINKOWSKI SPACE-TIME

As was mentioned before, in the model introduced for LV in [20] the dispersion relation is modified by an additional term \( \alpha^2 k^4 \) and the modified dispersion relation
reads as follows
\[ \omega^2(\vec{k}) = |\vec{k}|^2 - \alpha^2 |\vec{\varphi}|^4. \] (1)

A Lagrangian that can lead the above dispersion relation for a scalar field \( \phi \) is
\[ \mathcal{L} = \frac{1}{2} (\partial^\mu \phi \partial_\mu \phi + \alpha^2 (D^2 \phi)^2), \] (2)
where \( \alpha \) is a constant (of order the Planck energy) that sets the scale of the Lorentz violation and \( D^2 \) is the spatial Laplacian, that is
\[ D^2 \phi = -D^\mu D_\mu \phi = -q^{\mu \nu} \partial_\nu (q_\gamma^{\alpha \beta} \partial_\alpha \phi), \] (3)
where \( q_{\mu \nu} \) is the (positive definite) spatial metric orthogonal to the unit timelike vector \( u^\mu \)
\[ q_{\mu \nu} = -\eta_{\mu \nu} + u_\mu u_\nu, \quad \eta^\mu_{\nu} u_\mu u_\nu = 1. \] (4)
The vector field \( u^\mu \) can physically be interpreted as the four velocity of a preferred inertial observer. The rest frame of this preferred observer may be called aether. In this rest frame we have \( u^\mu = (1, 0, 0, 0) \).

For the subsequent consideration in this chapter we shall take a coordinate system in which \( u^\mu \) is constant (The aether is a particular example for such a coordinate system). With this assumption the equation of motion for the Lagrangian (2) is
\[ [\Box - \alpha^2 q^{\mu \nu} q^{\gamma \delta} \partial_\mu \partial_\gamma \partial_\nu \partial_\delta] \phi = 0. \] (5)
The complex mode solution of (5) are taken as
\[ u_\phi \propto e^{-i k_\mu x^\mu}, \] (6)
where
\[ \omega^2(\vec{k}) = |\vec{k}|^2 - \alpha^2 q^{\mu \nu} q^{\gamma \delta} k_\mu k_\nu k_\gamma k_\delta. \] (7)

In general the relation (7) admits imaginary frequencies. These frequencies lead to instability and unboundedness [2]. It is necessary in this context to restrict ourselves to ordinary solutions with real frequencies. In this case the mode solutions (6) become positive frequency mode solutions.

We define the scalar product for two solutions \( \varphi_1 \) and \( \varphi_2 \) of Eq. (6) as follows
\[ (\varphi_1, \varphi_2) = -i \int_\Sigma \varphi_1(x) \overline{\partial^\mu} - \alpha^2 q^{\mu \nu} q^{\gamma \delta} \partial_\mu \partial_\gamma \partial_\nu \partial_\delta \varphi_2^*(x) d\Sigma_\mu, \] (8)
where \( d\Sigma_\mu = n_\mu d\Sigma \), with \( n_\mu \) a future-directed unit vector orthogonal to the space like hypersurface \( \Sigma \) and \( d\Sigma \) is the volume element in \( \Sigma \). The hypersurface \( \Sigma \) is taken to be a Cauchy surface in the space-time and we can show, using Gauss’ theorem, that the value of \( (\varphi_1, \varphi_2) \) is independent of \( \Sigma \). The notation \( \overline{\partial_\mu \partial_\nu \partial_\delta} \) is defined by
\[ \varphi_1 \overline{\partial_\mu \partial_\nu \partial_\delta} \varphi_2 = +\varphi_1 \partial_\mu \partial_\nu \partial_\delta \varphi_2 - \varphi_2 \partial_\mu \partial_\nu \partial_\delta \varphi_1
- \partial_\mu \varphi_1 \partial_\nu \partial_\delta \varphi_2 + \partial_\mu \varphi_2 \partial_\nu \partial_\delta \varphi_1
+ \partial_\nu \varphi_1 \partial_\mu \partial_\delta \varphi_2 - \partial_\nu \varphi_2 \partial_\mu \partial_\delta \varphi_1
- \partial_\delta \varphi_1 \partial_\mu \partial_\nu \varphi_2 + \partial_\delta \varphi_2 \partial_\mu \partial_\nu \varphi_1. \] (9)

The ordinary modes \( u_\phi \) are orthogonal
\[ (u_\phi, u_{\phi'}) = 0, \quad \vec{k} \neq \vec{k'}, \] (10)
and if we choose
\[ u_\phi = [2(2\pi)^3 (\omega - 2\alpha^2 q^{\mu \nu} q^{\gamma \delta} k_\mu k_\gamma k_\delta)]^{-\frac{1}{2}} e^{-ik_\mu x^\mu}, \] (11)
these ordinary modes are normalized in the sense of the scalar product (8). An ordinary solution of Eq. (5) may be expanded in term of the ordinary modes (11) and their complex conjugates
\[ \varphi(t, \vec{x}) = \sum_k a_k \phi_k(t, \vec{x}) + a_k^* \phi_k^*(t, \vec{x}) \quad \omega \in \mathbb{R}, \] (12)
and the system is quantized in the canonical quantization scheme by imposing the following commutation relations
\[ [a_\phi, a_{\phi'}^*] = 0, \quad [a_{\phi'}^*, a_{\phi'}] = 0, \quad [a_\phi, a_{\phi'}^*] = \delta_{kk'}. \] (13)
With these commutation relations, \( a_{\phi} \)'s and \( a_{\phi'}^* \)'s are annihilation and creation operators and the vacuum of the Lorentz violation model in Minkowski space-time is defined by
\[ a_\phi \mid 0_{LV} \rangle = 0, \quad \omega \in \mathbb{R}. \] (14)

The above vacuum was defined with respect to a coordinate system in which the components of \( u^\mu \) were constant. We shall work in a coordinate system which corresponds to the rest frame of a preferred inertial observer (aether). In this case \( u_\mu \) takes the form \( (1, 0, 0, 0) \). With respect to the aether the dispersion relation (7) and the normalization constant in (11) are transformed to Eq. (11) and \( \omega^{-1/2} \) , respectively.

It is important to note that in what sense is the Lorentz invariance violated. The Lagrangian (2) does not exhibit the invariance under particle boost transformations [22, 23]. These are transformations of a physical system (particles or localized fields) within a fixed coordinate system. It is in this sense that Lorentz invariance is violated. Therefore the above vacuum may not be considered as an invariant state under a particle Lorentz transformation transforming a physical system within the rest frame of the preferred observer (aether).

III. COSMOLOGICAL MODEL AND PARTICLE CREATION

To show how particle creation occurs in the cosmological context, we use a two-dimensional Robertson-Walker space-time with the line element
\[ ds^2 = dt^2 - a^2(t) dx^2, \] (15)
and consider the generalization of the Lagrangian (2) to this cosmological space-time, namely
\[ \mathcal{L}(\varphi, u_\mu, \lambda) = \frac{1}{2} \sqrt{-g} g^{\mu \nu} \nabla_\mu \varphi \nabla_\nu \varphi - \xi R \varphi^2 + \alpha^2 (D^2 \varphi)^2 + \lambda (1 - u^\mu u_\mu)), \] (16)
where $D^2 \varphi$ is the covariant spatial Laplacian (the covariant analogue of (19))
\[
D^2 \varphi = -D^\mu D_\mu \varphi = -q^{\mu \nu} \nabla_\mu (q_\mu \nabla_\nu \varphi),
\]
(17)
and $g_{\mu \nu} = -g_{\mu \nu} + u_\mu u_\nu$ with $g_{\mu \nu}$ corresponding to (15). $\xi$ is a coupling constant between the scalar field and scalar curvature. The vector field $u_\mu$ is as a non-dynamical vector field to be specified by the conditions of the theory. By introducing the conformal time $\eta$, defined by $d\eta = dt/a(t)$, the metric (15) takes the form
\[
ds^2 = c(\eta)(d\eta^2 - dx^2), \quad c(\eta) = a^2(t).
\]
(18)

The Lagrange multiplier $\lambda$ in (16) imposes the following constraint on $u^\mu$
\[
g_{\mu \nu} u^\mu u^\nu = 1.
\]
(19)

In the context of the homogeneous and isotropic cosmological metric (18) the vector field $u_\mu$ is taken as to satisfy the isotropic property of cosmological metric and equation (16). This leads to
\[
u_\mu \equiv \left(\sqrt{c(\eta)}, 0\right).
\]
(20)

Thus $q_{\mu \nu}$ is
\[
q_{00} = q_{01} = q_{10} = 0 \text{ and } q_{11} = c(\eta).
\]
(21)

Now setting the variation of the action $S = \int \mathcal{L}(\varphi) d^4x$ with respect to $\varphi$ equals to zero yields the equations of motion for $\varphi$ in the metric (18) as follows
\[
\Box \varphi + \xi R \varphi - \frac{\alpha^2}{c^2(\eta)} \partial^4 \varphi = 0.
\]
(22)

We want to quantize $\varphi$ in equation (22) when $\xi = 0$ (it is the conformal coupling case in two dimension in absence of the Lorentz violation term). For this purpose we will obtain the mode solutions $u_k(x)$ of equation (22).

In order to avoid the well known ambiguities in the particle concept in curved space-time [3], we suppose that the space-time can be treated as asymptotically Minkowskian in the remote past and future. We refer to the remote past and future as the in and out regions, respectively. We take $c(\eta)$ as
\[
c(\eta) = A + B \tanh(\rho \eta),
\]
(23)
where $A$, $B$ and $\rho$ are some constants such that $2B$ and $\rho$ represent the amount and the rate of expansion in conformal time $\eta$, respectively [see fig. (1)]. Then in the remote past and future the space-time becomes Minkowskian since
\[
c(\eta) \to A \pm B, \quad \eta \to \pm \infty.
\]
(24)

This kind of conformal scale factor first introduced by Bernard and Duncan in [24] and then has been used in many works [25, 26, 27, 28] for considering the cosmological particle creation.

We can solve Eq. (22) and obtain $u_k(x)$ by the method of separation of variables. The mode solution of this equation is
\[
u_k(\eta, x) = (2\pi)^{-\frac{1}{2}} e^{ikx} \chi_k(\eta),
\]
(25)

such that $\chi_k(\eta)$ satisfies the following equation
\[
\frac{d^2}{d\eta^2} \chi_k(\eta) + (k^2 - \frac{\alpha^2}{c(\eta)} k^4) \chi_k(\eta) = 0.
\]
(26)

This equation can be solved in terms of hypergeometric functions. The normalized modes which behave like the positive frequency Minkowski space modes in the remote past ($\eta \to -\infty$) are
\[
u_k^{in}(\eta, x) = (4\pi \omega_{in})^{-\frac{1}{2}} \exp(ikx - i\omega_+ \eta)
\]
\[
- \frac{i\omega}{\rho} \ln[(A + B) e^{\rho \eta} + (A - B) e^{-\rho \eta}] \times F(1 + i\omega - \rho, i\omega - \rho; 1 - i\omega_{in}/\rho; z),
\]
and the modes which behave like positive frequency Minkowski modes in the out region as $\eta \to +\infty$ are found to be
\[
u_k^{out}(\eta, x) = (4\pi \omega_{in})^{-\frac{1}{2}} \exp(ikx - i\omega_+ \eta)
\]
\[
- \frac{i\omega}{\rho} \ln[(A + B) e^{\rho \eta} + (A - B) e^{-\rho \eta}] \times F(1 + i\omega - \rho, i\omega - \rho; 1 - i\omega_{out}/\rho; 1 - z),
\]
where
\[
z = \frac{1}{2} (A + B) \tanh(\rho \eta) + \frac{1}{A + B \tanh(\rho \eta)}.
\]
(29)
The energies \(\omega_{\text{in}}\) and \(\omega_{\text{out}}\) can be imaginary. But we limit ourselves to the modes with real frequencies (the ordinary modes). The ordinary modes \(u_k^{\text{in}}(\eta, x)\) are orthonormal in the following conserved inner product

\[
(\varphi_1, \varphi_2) = -i \int_{\Sigma} \sqrt{-g} \varphi_1(x) \left( \overleftrightarrow{\partial}^\mu - \alpha^2 q^{\mu \nu} q^{\gamma \delta} \overleftrightarrow{\partial}_\nu \partial_\gamma \partial_\delta \right) \varphi_2(x) d\Sigma^\mu,
\]

where \(\varphi_1(x)\) and \(\varphi_2(x)\) are regarded by the aether in the curved space-time and \(\overleftrightarrow{\partial}_\nu \partial_\gamma \partial_\delta\) is defined in \(\text{fig.}\). Thus we may expand one ordinary solution of Eq. (22) with respect to \(u_k^{\text{in}}(\eta, x)\) as

\[
\varphi(t, x) = \sum_k a_k^{\text{in}}(t, x) + a_k^{\text{in}}(t, x) + a_k^{\text{out}}(t, x) + \omega_{\text{in}} \in \mathbb{R}
\]

and the second quantization is implemented in the same way as the Minkowski space-time by the following commutation relations

\[
[a_k^{\text{in}}, a_k^{\text{out}}] = \delta_{kk'}, [a_k^{\text{in}}, a_k^{\text{out}}] = \delta_{kk'}.
\]

As we mentioned before we have chosen a conformally flat expanding space-time which is in the in and out regions Minkowskian. With using the Eqs. (20) and (23) we define the aether in the in and out regions as a frame in which \(u_\mu\) is \((\sqrt{A - B}, 0)\) and \((\sqrt{A + B}, 0)\), respectively. The vacuum states with respect to the aether in the in and out regions are defined by

\[
\begin{align*}
a_k^{\text{in}} &\mid 0^{M}_{L^4} \rangle^{\text{in}} = 0 \quad \omega_{\text{in}} \in \mathbb{R} \\
d_k^{\text{out}} &\mid 0^{M}_{L^4} \rangle^{\text{out}} = 0 \quad \omega_{\text{out}} \in \mathbb{R}.
\end{align*}
\]

Suppose the quantum field in the remote past resides in the state \(\mid 0^{M}_{L^4} \rangle^{\text{in}}\) and suppose we are working in the Heisenberg picture. Thus in the out region, the quantum field is also in the state \(\mid 0^{M}_{L^4} \rangle^{\text{in}}\). But this state is not regarded by the aether in the out region as the physical vacuum, this role being reserved for the state \(\mid 0^{M}_{L^4} \rangle^{\text{out}}\).

We want to calculate the number of particles detected in the out region in the state \(\mid 0^{M}_{L^4} \rangle^{\text{in}}\). If we use the linear transformation properties of hypergeometric functions, we expand \(u_k^{\text{in}}\) with respect to \(u_k^{\text{out}}\) and obtain the following relation

\[
u_k^{\text{in}}(\eta, x) = \alpha_k u_k^{\text{out}}(\eta, x) + \beta_k u^{-}_{k}^{\text{out}}(\eta, x),
\]

where

\[
\alpha_k = \frac{\omega_{\text{out}}}{\omega_{\text{in}}} \frac{\Gamma(1 - (i\omega_{\text{in}}/\rho)) \Gamma(-i\omega_{\text{out}}/\rho)}{\Gamma(-i\omega_{\text{in}}/\rho) \Gamma(1 - i\omega_{\text{out}}/\rho)}
\]

and

\[
\beta_k = \frac{\omega_{\text{out}}}{\omega_{\text{in}}} \frac{\Gamma(1 - (i\omega_{\text{in}}/\rho)) \Gamma(i\omega_{\text{out}}/\rho)}{\Gamma(-i\omega_{\text{in}}/\rho) \Gamma(1 - i\omega_{\text{out}}/\rho)}.
\]

In the above relation \(\beta_k^2\) is dependent on the parameters \(\alpha, A, B, \text{ and } \rho\). We set the first parameter \(\alpha\) in natural units \((\hbar = c = 1)\) as \(\alpha = 1^{-}\) because we assume that \(LV\) occurs in the scale of Planck energy. The three last parameters are related to the conformal scalar factor \(c(\eta)\) by (23). These parameters determine the spectrum of created particles with respect to the coordinates \((\eta, x)\) and metric \((15)\) through \(\beta_k^2\) in \(30\).

It is instructive to relate \(\beta_k^2\) to the physical parameters in the free-fall frame which is determined by the coordinates \((t, x)\) and the metric \((\text{fig.})\). To do this we first find the analogue of the parameters \(A, B, \text{ and } \rho\) in the free-fall frame. The parameters \(A, B, \text{ and } \rho\) have a good meaning in the free-fall frame because as we have shown in fig. \((11)\), \(\sqrt{A - B}\) and \(\sqrt{A + B}\) are initial and final scale factors in the free fall frame. For some advantages we put \(A - B = 1\), it implies that the conformal time is the same as the proper time in the remote past also the initial scale factor is 1 and the final scale factor is \(\sqrt{2B + 1}\). Now let us consider the analogue of \(\rho\). This parameter is proportional to the inverse of time interval of expansion with respect to the conformal time. We show this by the following argument. We define an effective time interval of expansion, \(\Delta\eta\). As we have shown...
FIG. 3: The above figures show the properties of the spectrum of created particles. (a) and (b) show $\beta^2(k_{\text{max}})$ and $k_{\text{max}}$ versus the final amount of the scale factor, $\rho$, rate of expansion in free fall frame, namely $f = 1$. (c) and (d) show $\beta^2(k_{\text{max}})$ and $k_{\text{max}}$ versus the rate of expansion in free fall frame, $\rho_f$, where $\sqrt{2B + 1} = 10^5$. (Note $\alpha = 1^\circ$, $\epsilon = 0.99$ and $A - B = 1$)

in fig. (1), $\Delta \eta = \eta_2 - \eta_1$ is taken as a time interval such that the proportion $\varepsilon$ of the total expansion is done in this interval. We set $\varepsilon = 0.99$ that means 99% of total expansion occurs in $\Delta \eta$. By this definition one can find from (28), $\eta_2 = -\eta_1 = \tanh^{-1}(\varepsilon)/\rho$ and so

$$\rho = \frac{2 \tanh^{-1}(\varepsilon)}{\eta_2 - \eta_1},$$

(37)

where $\eta_1$ and $\eta_2$ are the initial and the final time of the effective expansion respectively, and $\tanh^{-1}(0.99) = 2.64$. The Eq. (37) shows $\rho \propto 1/\Delta \eta$ and the proportional constant is of order one. This relation makes it clear that $\rho$ is proportional to the inverse of the time interval of expansion so it can be interpreted as the rate of expansion. In analogy to $\rho$ we can define $\rho_f$ in free falling frame such that it is the inverse of expansion time interval in free falling frame, namely

$$\rho_f = \frac{1}{t(\eta_2) - t(\eta_1)}. \quad (38)$$

For evaluating $\rho_f$ in the above relation we find the relationship between $t$ and $\eta$ from (23) and $dt = \sqrt{c(\eta)}d\eta$. We get

$$t(\eta) = \frac{\sqrt{2B + 1}}{\rho} \tanh^{-1}\left(\frac{\sqrt{B[1 + \tanh(\rho\eta)]} + 1}{\sqrt{2B + 1}}\right) + \frac{1}{2\rho} \ln\left(\frac{\sqrt{B[1 + \tanh(\rho\eta)]} + 1}{\sqrt{B[1 + \tanh(\rho\eta)]} + 1}\right) + C, \quad (39)$$

where $C$ is a constant that sets the initial values of $\eta$ and $t$. From (39) together with (33) we find $\rho_f = \rho/\lambda$ where $\lambda$ is a function of $B$ and $\varepsilon$, namely

$$\lambda = \sqrt{2B + 1} \tanh^{-1}\left(\frac{\sqrt{B(1 + \varepsilon) + 1}}{\sqrt{2B + 1}}\right) - \sqrt{2B + 1} \tanh^{-1}\left(\frac{\sqrt{B(1 - \varepsilon) + 1}}{\sqrt{2B + 1}}\right) + \frac{1}{2} \ln\left(\frac{\sqrt{B(1 + \varepsilon) + 1} - 1}{\sqrt{B(1 + \varepsilon) + 1} + 1}\right) - \frac{1}{2} \ln\left(\frac{\sqrt{B(1 - \varepsilon) + 1} - 1}{\sqrt{B(1 - \varepsilon) + 1} + 1}\right). \quad (40)$$

From (40) we find that for sufficiently large amount of $B$, $\lambda$ can be approximated by $(2B)^{1/2}$. Substituting $\rho$ by $\lambda \rho_f$ in (30) we get $|\beta_k|^2$ in term of the physical parameter $\rho_f$ in the free falling frame. We can approximate (36) in the limit $\lambda \rho_f \gg 1$ by

$$|\beta_k|^2 = \frac{(\omega_{\text{out}} - \omega_{\text{in}})^2}{4\omega_{\text{in}}\omega_{\text{out}}}, \quad (41)$$

where in the region $B \gg 1$ and $k \neq 0$ the above relation takes the form

$$|\beta_k|^2 = \frac{(1 - \sqrt{1 - \alpha^2k^2})^2}{4\sqrt{1 - \alpha^2k^2}}, \quad (42)$$
which implies that $|\beta_k|^2$ is independent of $\lambda \rho_f$.

In the opposite limit where $\lambda \rho_f$ is small the spectrum shows an interesting behavior. The figure 2 shows the spectrum of created particles for $\sqrt{2B+1} = 10^3$ and $\rho_f = 10^{-5}$ so $\lambda \rho_f = 0.032$. It shows that particles are not created at low and high momenta. Most particles are produced with momenta around $k_{\text{max}}$. In figure (3) we show the properties of this spectrum. Figures (3,a) and (3,c) show the variations of the number of created particles at momentum $k_{\text{max}}$ with respect to the amount $\sqrt{2B+1}$, and the rate $\rho_f$ of the space-time expansion respectively. With increasing the amount and the rate of expansion the number of created particles increases. Figures (3,b) and (3,d) shows $k_{\text{max}}$ with respect to $\sqrt{2B+1}$ and $\rho_f$. We see that with increasing the amount and the rate of the expansion, the particles with higher momenta are created.

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