Transversly Polarized Proton-Proton Collisions and the Collins Fragmentation Function

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Abstract

The Sivers Function is briefly reviewed and the Collins Fragmentation Function to produce a hadron from a polarized quark is defined. We estimate $\pi^+, \pi^- + X$ production, where $X$ represents particles which are not detected, via proton collisions with a polarized proton target using the Collins Fragmentation Function derived using the gluon distribution functions for polarized p-p collisions at $E=\sqrt{s}=200$ GeV.

PACS Indices:12.38.Aw,13.60.Le,14.40.Lb,14.40Nd

1 Introduction

We estimate inclusive $\pi^+, \pi^- + X$ production, where $X$ represents particles not detected, via transversely polarized p-p collisions at 200 GeV, an extension of our recent work[1] on $D$ production via unpolarized p-p collisions at $E=\sqrt{s}=200$ GeV. The E1039 Collaboration, see Ref.[2] for the Letter of Intent, plans to carry out a Drell-Yan experiment with a polarized proton target, with the main objective to measure the Sivers function[3]. A number of Deep Inelastic Scattering experiments[4, 5, 6] have measured non-zero values for the Sivers Function. See these references for references to earlier experiments.

Another important function is the Collins fragmentation function[7], which describes the fragmentation of a transversly polarized quark into an unpolarized hadron, such as a pion. For many years there have been theoretical studies of the Collins fragmentation functions, such as Refs.[8, 9, 10]. Recently a study of extracting the Collins fragmentation function from experiment has been carried out[11, 12].

Ref.[1] used the method of Braaten et. al.[13, 14, 15] to estimate the fragmentation of a charm quark to a D, a meson consisting of a charm and anti-light quark. Ref[16] used Ref.[13] and Ref.[17] used Ref.[14] for estimates of heavy quark state production via RHIC (Relativistic Heavy Ion Collisions). In the present work we calculate fragmentation of a
polarized light quark to a pion. Although both estimates of fragmentation use the method of Ref.[13] the magnitudes of the resulting cross section for $p^+ + p \rightarrow \pi + X$ is much larger than $p + p \rightarrow D + X$, due to the large difference in the charm and light quark masses, as will be shown.

2 Sivers and Collins Functions

The Sivers and Collins functions are defined by the target assymmetry, $A(\phi_h, \phi_S)$, in the scattering of an unpolarized lepton beam by a transversely polarized target[6]:

$$A(\phi_h, \phi_S) \simeq A_C \sin(\phi_h + \phi_S) + A_S \sin(\phi_h - \phi_S), \quad (1)$$

where $A_C, A_S$ are the Collins, Silvers moments with $\phi_h$ the hadron momentum azimuthal angle and $\phi_S$ the target spin azimuthal angle with respect to the lepton scattering plane.

In this section we briefly review the Sivers Function and the Collins Fragmentation Function.

2.1 Sivers Function

See, e.g., Ref[4] for a discussion of the Sivers Function in terms of experimental cross sections.

The Sivers term of the cross section for the production of hadrons using an unpolarized lepton beam on a transversely polarized target is[4]

$$\sigma(\phi_h, \phi_S) = \sigma_{UU} |S_T| [2 \sin(\phi_h - \phi_S) >_{UT} \times \sin(\phi_h - \phi_S) + ...], \quad (2)$$

where $\phi_h$ and $\phi_S$ were defined above. $\sigma_{UU}$ is the $\phi_h$-independent part of the polarization-independent cross section; and $U_T$ denotes the unpolarized beam and transverse target polarization w.r.t. the virtual photon direction. The Sivers function is obtained by an expansion of $2 < \sin(\phi_h - \phi_S) >_{UT}$ in quarks using a quark-parton model[4, 18]. As mentioned in the Introduction, a number of Deep Inelastic Scattering experiments have measured the Sivers function and obtained non-zero values.

2.2 Collins Fragmentation Function

The definition of the Collins Function[11, 12]is similar to that of the Sivers Function in Eq(2):

$$\sigma_C(\phi_h, \phi_S) \propto F_{UU} (1 + A_{UT} \times \sin(\phi_h + \phi_S)),$$

with $\phi_h$ and $\phi_S$ defined above, $F_{UU}$ the spin-averaged structure function, and $A_{UT}$ the asymmetry that can be calculated from quark distribution and fragmentation, discussed in detail in the next section. See Refs.[11, 12] for definitions of $\sigma_C, F_{UU}$ and derivation of $F_{UU}$.
From Ref.[19] (also see Ref.[20] and earlier references for the derivation of $H^q \equiv H_1^{q\perp}$) the fragmentation probability to produce a hadron, $h$, from a transversely polarized quark in $e^+e^-$ annihilation is given by the function

$$D_{hq\uparrow}(z, k_T^2) = D^q(z, K_T^2) + H^q(z, K_T^2) \frac{(\vec{k} \times \vec{K}_T/k) \cdot s_q}{z M_h},$$

where $M_h$ is the hadron mass, $\vec{k}$ the quark momentum, $s_q$ the quark spin vector, $\vec{K}_T$ the hadron momentum transverse to $\vec{k}$, $z$ the light-cone momentum fraction of $h$ wrt the fragmenting quark, and $D^q$ the unpolarized fragmentation function. The Collins fragmentation function is $H^q \equiv H_1^{q\perp}$.

3 Differential $p\uparrow + p \to \pi + X$ and $p + p \to \pi + X$ cross sections

This work on estimating the cross section $\sigma_{p\uparrow p \to \pi X}$ is an extension of our previous work on $J/\Psi, \Psi'(2S), \Upsilon(nS)$ production[21] for $E=\sqrt{s}=200$ GeV using the color octet model[22, 14, 15], but for the production of pions rather than heavy quark mesons. The differential cross section for a proton collision with a proton polarized orthogonal to the scattering plane is

$$\frac{d\sigma_{p\uparrow p \to \pi X}}{dy} \simeq \frac{1}{x(y)} \Delta f_g(x(y), 2m) \Delta f_g(a/x(y), 2m) \sigma_{q\uparrow + g \to \pi X},$$

where rapidity $y$ is defined in Eq(12), $\Delta f_g(x)$ is the gluon distribution function for polarized $p$-$p$ collisions (see Ref.[21]) and $a = 4m^2/s$, $m \simeq 3.5$ MeV for light quarks; and with[13]

$$\sigma_{q\uparrow + g \to \pi X} = 2\sigma_{q+g \to q\bar{q}} D_{q\rightarrow q\pi},$$

where $\sigma_{q+g \to q\bar{q}}$ is similar to the charmonium production cross section in Ref.[21], but with light quarks rather than charm quarks. $D_{q\rightarrow q\pi}$ is the total fragmentation probability for a polarized light quark, $q$, to fragment to a $\pi(q\bar{q})$ meson plus a $q$ quark. Theoretical equations similar to Eq(5) were also used in Refs.[16],[17].

Note there are also contributions to $d\sigma_{p\uparrow p \to \pi X}/dy$ from $\Delta f_d(x(y), 2m)\Delta f_d(a/x(y), 2m)$ and $\Delta f_u(x(y), 2m)\Delta f_u(a/x(y), 2m)$ quarks and anti-quarks. However, as shown in Ref.[21] these contributions are more than an order of magnitude smaller that those from gluons. Also, since the target proton is polarized, resulting in a polarized quark $q\uparrow$ the cross section depends on the gluon distribution functions $\Delta f_g(x)$ rather than $f_g(x)$ used in Ref.[1]

The Collins fragmentation function for a polarized light quark fragmenting to a $q\bar{q}(\pi)$ meson plus $q$, a light quark, is illustrated in Figure 1 (see Ref.[13], Sec.II, Fig. 1).
Ref.[13] explains above Eq(31) that a $1/m_Q$ expansion was used so Eq(31) also gives light quark fragmentation, $D_{q^\uparrow \rightarrow q\bar{q}}(z)$, with a light quark polarized $q^\uparrow$ replacing a heavy unpolarized quark $Q$. Note also that the $\pi$ production is non-perturbative as $g \rightarrow q\bar{q} \rightarrow \pi$ involves a nonperturbative q gluon-quark interaction. With $r \equiv (m_\pi - m_q)/m_\pi \simeq 1.0$, $m_q \simeq 3.5$ MeV, $\alpha_s(m_q) \simeq .26$, $|R(0)|^2 \simeq 4m_q^2$, $N \simeq 2.125 \times 10^{-3}$. From Ref.[13] Eq(31) one finds

$$D_{q^\uparrow \rightarrow q\bar{q}}(z) \simeq 2.125 \times 10^{-3}(6z + 6z^2 - 15z^3 - 12z^4 + 15z^5),$$

with $0 \leq z \leq 1$.

From Eq(7),

$$D_{q^\uparrow \rightarrow q\bar{q}} = \int_0^1 dzD_{q^\uparrow \rightarrow q\bar{q}}(z) = 2.87 \times 10^{-3}. \tag{8}$$

In order to extract $H^g$ from Eq(4) one also needs to evaluate the unpolarized fragmentation function, $D^g$. From From Ref.[13], Eq(33) with $r=m_q/(m_q + m_Q) \simeq 0$, 

$$D^g = \int_0^1 dzD^g(z) = 3N \frac{8}{15} \simeq 3.4 \times 10^{-3} \tag{9}$$

A fit to the parton distribution functions for polarized p-p collisions for $Q \simeq 1$ GeV obtained from CTEQ6[23] in the x range needed for the present work at $\sqrt{s}=200$ GeV is

$$\Delta f_g(x) \simeq 15.99 - 700.34x + 13885.4x^2 - 97888x^3. \tag{10}$$

From Eqs(5,6)

$$\frac{d\sigma_{p^\uparrow p \rightarrow \pi X}}{dy} = Aqq \Delta f_g(x(y), 2m) \Delta f_g(a/x(y), 2m) \frac{dx(y)}{dy} \frac{1}{x(y)} D_{q^\uparrow \rightarrow q\bar{q}}. \tag{11}$$
with rapidity $y$

$$
y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right)
$$

$$
x(y) = 0.5 \left[ \frac{m}{E} \left( \exp y - \exp (-y) \right) + \sqrt{\left( \frac{m}{E} \left( \exp y - \exp (-y) \right) \right)^2 + 4a} \right], \quad (12)
$$

where $Aqq$ is the matrix element for $q\bar{q}$ production modified by an effective mass $m_s$. For $E=200$ GeV $Aqq = 7.9 \times 10^{-4} (1.5 \text{GeV}/m_s)^3 \text{nb}$. With $m_s = 3.5 \text{MeV}$, $Aqq = 0.60 \times 10^{-5} \text{nb}$.

For unpolarized $pp \rightarrow \pi X$ the differential cross section is

$$
\frac{d\sigma_{pp \rightarrow \pi X}}{dy} \approx Aqq f_g(x(y), 2m)f_g(a/x(y), 2m) \frac{dx(y)}{dy} x(y) \frac{1}{D_q} \quad (13)
$$

$$
f_g(y) = 1334.21 - 67056.5x(y) + 887962.0x(y)^2.
$$

From Eqs(10,11,8, 12) $\frac{d\sigma_{pp \rightarrow \pi X}}{dy}$, and from Eqs(13,9) $\frac{d\sigma_{pp \rightarrow \pi X}}{dy}$ are found as shown in Fig. 2.
4 Conclusions

We have estimated the production of pions via polarized \( p^\uparrow - p \) collisions, deriving the Collins fragmentation function needed for the \( p^\uparrow - p \rightarrow \pi + X \) cross section, and for comparison the unpolarized \( p - p \rightarrow \pi + X \) cross section. Although the Drell-Yan unpolarized experiment described in Ref. [2] can determine the Sivers but not the Collins function, future experiments with \( p^\uparrow - p \) collisions might be able to measure the cross section for \( \pi + X \) production. From \( \frac{d\sigma}{dy} p^\uparrow p \rightarrow \pi X \), using Eqs(4,11,10,9), one would be able to measure \( H^q \equiv H^1_{1q} \), the Collins fragmentation function. The results of the present article could thereby be tested.

Acknowledgements

This work was supported in part by the DOE contracts W-7405-ENG-36 and DE-FG02-97ER41014. The author LSK was a visitor at Los Alamos National Laboratory, Group P25. The authors thank Dr. Xiaodong Jiang, LANL Group P25 and Dr. Zhong-Bo Kang, LANL Group T-2, for helpful discussions. The authors declare that there is no conflict of interest regarding the publication of this paper.

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