Weighing neutrinos in the presence of a running primordial spectral index

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Abstract. The three-year Wilkinson Microwave Anisotropy Probe observations, combined with other cosmological observations from galaxy clustering and type Ia supernovae, favour a non-vanishing running of the primordial spectral index independent of the low cosmic microwave background multipoles. Motivated by this feature we study a cosmological constraint on the neutrino mass, which is strongly dependent on what prior we adopt for the spectral shape of primordial fluctuations, taking possible running into account. As a result we find a more stringent constraint on the sum of the three neutrino masses, $m_\nu < 0.76 \text{ eV (2}\sigma)$, compared with $m_\nu < 0.90 \text{ eV (2}\sigma)$ for the case where a power law prior is adopted for the primordial spectral shape.

Keywords: cosmological neutrinos, inflation

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The three-year Wilkinson Microwave Anisotropy Probe observations (WMAP3) [1]–[5] have marked another milestone in the precision cosmology of the cosmic microwave background (CMB) radiation. The simplest six-parameter power law $\Lambda$CDM cosmology is in remarkable agreement with WMAP3 together with the large scale structure (LSS) of galaxy clustering as measured by 2dF [6] and SDSS [7] and with the type Ia supernovae (SNIa) as measured from the Riess ‘gold’ sample [8] and the first-year SNLS [9]. This agreement between the above ‘canonical’ cosmological model and observations can be used to test a number of possible new physics features, such as the equation of state of dark energy, neutrino masses, time variation of fundamental constants, etc.

Among them, the constraint on the neutrino or hot dark matter mass can be obtained from the free-streaming modification of the transfer function of the matter power spectrum. We should note, however, that if one allowed any shape of primordial spectrum, the free-streaming effect could easily be compensated by some non-trivial shape of the primordial spectrum, so that one cannot obtain a sensible limit on the neutrino mass. That is, we can obtain a non-trivial bound on neutrino mass if and only if we adopt some prior on the shape of the primordial power spectrum such as a simple power law. From the above argument, we expect that as we allow more degrees of freedom on the primordial spectrum beyond a power law, the constraint on the neutrino mass will be less stringent in general.

As for the shape of the primordial power spectrum, it is noteworthy that a significant deviation has been observed in WMAP3 from the simplest Harrison–Zel’dovich spectrum, and that this feature is more prominent with the combination of all the currently available CMB, LSS and SNIa data (dubbed the ‘All’ case in [5]). Moreover, a non-trivial negative running of the scalar spectral index $\alpha_s$, whose existence was studied even before the WMAP epoch [10,11], was favoured by the first-year WMAP papers [12]–[14]. But its favouring was somewhat diminished as corrections to the likelihood functions were made [15]. However, the new WMAP3 data favour again a negative running in the ‘All’ combination [5].

If confirmed, a non-vanishing running of $\alpha_s$ would not only constrain inflationary cosmology significantly [16]–[21], but also affect the cosmological constraint on the neutrino mass. In the LSS power spectrum, the effect of massive neutrinos may be compensated by a non-vanishing running of the primordial spectrum. On the other hand, if it is established that the running is negative, this will lead to even more stringent constraints on the neutrino mass compared with fittings in the constant scalar spectral index ($n_s$) cosmology, because they both lead to a damped power on small scales.

The actual problem, however, cannot be solved by the above simple one-to-one correspondence, because the effects of a non-vanishing neutrino mass on the CMB are much less dramatic than those of a non-vanishing running $\alpha_s$. Alternatively, on the scales probed via the CMB, especially near the third peak, there is a large degeneracy between $\alpha_s$ and the matter density $\Omega_m$ [22]–[26].

Hence in the concordance analysis the correlation between the running and neutrino mass needs to be addressed in a combined study of CMB, LSS and SNIa data, where SNIa helps significantly to determine the matter density. We report the results of such a combined analysis in this paper using the Markov chain Monte Carlo method in constraining the total neutrino mass, $m_\nu = \sum_{i=1}^{3} m_{\nu i} = 94.4\Omega_\nu h^2$ eV. Here $\Omega_\nu$ is the density parameter of the neutrino and $h$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$. 
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Table 1. Mean with 1σ (2σ) constraints on the spectral index, the running, and the neutrino mass based on LSS and SNIa with WMAP3 (with/without \( l < 24 \) CMB contributions) and with/without introducing a running of the primordial spectral index \( \alpha_s \) and with/without massive neutrinos. The rows starting with \( \Delta \chi^2 \) show the corresponding reduction of \( \chi^2 \) values compared with the power law ΛCDM cosmology.

| \( \nu \) | \( \alpha_s \) | \( \nu + \alpha_s \) |
|---|---|---|
|Normal WMAP3|
| \( n_s \) | \( 0.947^{+0.016}_{-0.016} \) | \( 0.880^{+0.038}_{-0.037} \) | \( 0.899^{+0.040}_{-0.041} \) |
| \( \alpha_s \) | Set to 0 | \( -0.051^{+0.025}_{-0.025} \) | \( -0.037^{+0.028}_{-0.028} \) |
| \( m_\nu \) | \( 0.460^{+0.113}_{-0.100} \) | Set to 0 | \( 0.314^{+0.079}_{-0.079} \) |
| \( \Delta \chi^2 \) | 2 | -3.4 | 4.1 |

\( l < 24 \) dropped

| \( n_s \) | \( 0.942^{+0.021}_{-0.022} \) | \( 0.862^{+0.049}_{-0.049} \) | \( 0.899^{+0.058}_{-0.059} \) |
| \( \alpha_s \) | Set to 0 | \( -0.072^{+0.043}_{-0.042} \) | \( -0.045^{+0.054}_{-0.050} \) |
| \( m_\nu \) | \( 0.470^{+0.131}_{-0.130} \) | Set to 0 | \( 0.378^{+0.099}_{-0.099} \) |
| \( \Delta \chi^2 \) | 1 | -2.1 | -2.2 |

To break possible degeneracy among the cosmological parameters, we make a global fit to the aforementioned current data with the publicly available Markov chain Monte Carlo package cosmomc [10, 27]. Our most general parameter space is

\[
p \equiv (\omega_b, \omega_c, \Theta_S, \tau, m_\nu, n_s, \alpha_s, \log[10^{10} A_s])
\]

where \( \omega_b = \Omega_b h^2 \) and \( \omega_c = \Omega_c h^2 \) are the physical baryon and cold dark matter densities relative to the critical density, \( \Theta_S \) characterizes the ratio of the sound horizon and angular diameter distance, \( \tau \) is the optical depth and \( A_s \) is defined as the amplitude of the initial power spectrum. The pivot scale for \( n_s \) and \( \alpha_s \) is chosen at \( k = 0.05 \) Mpc\(^{-1}\).

Assuming a flat Universe and in terms of the Bayesian analysis, we vary the above eight parameters and fit the theory to the observational data with the MCMC method. For CMB we have only adopted WMAP3. The bias factors of LSS have been used as nuisance parameters and hence essentially we have used only the shapes of 2dF and SDSS power spectra. As for the SNIa data, while the WMAP team uses both SNLS and the Riess sample simultaneously in their ‘All’ data set, here we adopt only one of them, namely, the Riess ‘gold’ sample, rather than combining with SNLS. This is because these two groups use somewhat different methods in their analysis and it would not be appropriate to put them together simply\(^3\).

Regarding the first-year WMAP data Bridle et al [28] found that the claimed favouring of a negative \( \alpha_s \) was merely due to the lowest WMAP multipoles. In order to probe the sensitivity of the running to the lower multipoles we analyse the running and neutrino properties using the CMB data with and without the contributions of lower multipoles

\(^3\) Currently the SNLS and Riess ‘gold’ sample are comparable in the determination of cosmological parameters, as shown in [5], and each data set has its own nice features. In the present work we use only the Riess sample.
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Figure 1. Two-dimensional posterior constraints on the primordial spectral index versus its running at $k = 0.05 \text{ Mpc}^{-1}$, using 2dF, SDSS, SNIa and WMAP3 (with/without $l < 24$ CMB contributions) and with/without the presence of massive neutrinos.

which suffer from large cosmic variance. Specifically, we truncate naturally at $l = 24$ given the current likelihood of WMAP3 [1]–[3], [5].

As a result we find a more stringent constraint on the neutrino mass in the presence of running compared with the analysis with constant $n_s$. We also find that currently the favouring of a negative running is fairly independent of the WMAP3 low CMB quadrupoles and hence relatively robust.

In table 1 we show the mean $1\sigma$ ($2\sigma$) constrains on the relevant cosmological parameters combining 2dF, SDSS and the Riess ‘gold sample with WMAP3. We have addressed the cases with/without $l < 24$ CMB contributions, with/without introducing $\alpha_s$ and with/without massive neutrinos. The last row shows the corresponding reduction of $\chi^2$ values compared with the power law $\Lambda$CDM cosmology. For normal LSS + SNIa + WMAP in the seven-parameter fittings we get $\alpha_s = -0.0512^{+0.0506}_{-0.0480}$ at $2\sigma$. Our results are considerably less stringent than the ‘All’ combination of the WMAP team [5], as we have not included SNLS and other CMB observations. On the other hand the favouring of non-vanishing $\alpha_s$ is larger than $2\sigma$ in both cases. The favouring of negative running still remains even if we drop the WMAP3 $l < 24$ contributions, when we get $\alpha_s = -0.0717^{+0.0833}_{-0.0886}$ at $2\sigma$. This may imply that a negative running is indeed preferred non-trivially in the combination of WMAP + LSS + SNIa, even without the presence of WMAP3 small $l$ contributions. This can also be understood through the Akaike information criterion (AIC [29]), which is defined by

$$\text{AIC} = -2 \ln \mathcal{L} + 2k,$$

(2)
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Figure 2. One-dimensional posterior constraints on the sum of neutrino masses, using 2dF, SDSS, SNIa and WMAP3 (with/without \( l<24 \) CMB contributions) and with/without introducing a running of the primordial spectral index \( \alpha_s \). Note that due to the non-Gaussian distribution of \( m_\nu \), LSS + SNIa + WMAP3 (black line) seems to indicate a non-zero neutrino mass, but this is not the case, as can be seen in table 1.

where \( \mathcal{L} \) is the maximum likelihood, \( k \) is the number of fitting parameters. This quantity gives a criterion for how many parameters we should use to fit a data set, and it implies that it is worth introducing a new parameter if \( \Delta \chi^2 \equiv -2\Delta \ln \mathcal{L} \) improves by more than 2. From table 1 we can find the reduction of \( \chi^2 \) is larger than 2 in the case of running compared with the simple power law \( \Lambda \)CDM cosmology; hence according to AIC, we should introduce the running.

The favouring of a negative running can also be obtained in the two-dimensional posterior contours of \( n_s - \alpha_s \), as depicted in figure 1. For the seven-parameter case with one additional parameter of \( \alpha_s \), although the contour without WMAP3 low \( l \) contributions is larger than that in the left panel, a constant \( n_s \) lies close to the \( 2\sigma \) lines in both cases. The enlarged contour without \( l<24 \) is easily understood due to the \( n_s - \tau \) degeneracy.

The correlation between \( m_\nu \) and the shape of the primordial spectrum is obvious, as can be seen from table 1 and figure 1. In the presence of massive neutrinos the error bars on \( n_s \) and \( \alpha_s \) get increased with or without small \( l \) CMB contributions. And it is noteworthy that, in the presence of massive neutrinos, a scale-invariant primordial spectrum is consistent with the observations at \( \sim 2\sigma \) if we drop the small \( l \) WMAP3 contributions.

We find that, while almost all of the remaining parameters get less stringently constrained, the neutrino mass is an exception: a more tightened bound on \( m_\nu \) is achieved in the presence of a non-zero \( \alpha_s \) than for the case with constant \( n_s \), as shown in table 1, and for the one-dimensional constraints, as in figure 2. This has shown that running is indeed strongly correlated with neutrino mass, which is mainly due to the physics of LSS.

In figure 3 we display the two-dimensional posterior constraints on the sum of neutrino masses versus matter density in the same case as for figure 2. While the allowed
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Figure 3. Two-dimensional posterior constraints on the sum of neutrino masses versus matter density, using 2dF, SDSS, SNIa and WMAP3 (with/without \( l < 24 \) CMB contributions) and with/without introducing a running of the primordial spectral index \( \alpha_s \).

Parameter space for \( \Omega_m \) is significantly enlarged in the presence of running, neutrino mass is constrained strongly in cases with non-zero \( \alpha_s \). The correlation between \( \Omega_m \) and \( m_\nu \) is rather strong and the accumulation of the observational data, such as SNAP, PLANCK and SDSS, will help significantly to break the degeneracy, for detecting the features of the primordial spectrum as well as the nature of neutrinos.

It has been claimed that a running of the spectral index will be excluded in the presence of SDSS Lyman \( \alpha \) observations [30]–[33]; the systematics of Lyman \( \alpha \) data are less constrained [1, 23] and we leave this to a separate investigation to be reported in [34], and detailed analysis with other additional possible degeneracies to [35].

For many years neutrinos have played a fundamental role in both physics and astrophysics, and provided areas of new physics such as parity violation and oscillations with tiny masses. Surely neutrinos will continue to play a crucial role in our understanding of the Universe. While the resulting reduction in neutrino mass in this paper is less than 0.2 eV, such an effect will one hopes help to change our understanding of the ultimate detection of neutrino mass with future cosmological surveys and the difference is already larger than the low limit on neutrino mass from oscillation experiments. On the other hand there are also some mild tensions in the determinations of the background cosmological parameters with current CMB, LSS and SNIa data. We may still need some better understanding of each data set before entering the precision cosmology arena [36], and in cases where all of the observations have similar tendencies in the favouring of a negative running we can, we hope, get more prominent effects in the probing of neutrino mass.
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The distinctive feature probed in the current paper will also open new windows to relevant studies, such as probing neutrino mass with a non-zero running in gravitational lensing surveys, N-body simulations in the presence of running and massive neutrinos.

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