Viscosity in the excluded volume hadron gas model

M.I. Gorenstein,1,2 M. Hauer,3 and O.N. Moroz4,1

1Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine
2Frankfurt Institute for Advanced Studies, Frankfurt, Germany
3Helmholtz Research School, University of Frankfurt, Frankfurt, Germany
4National Technical University, Kiev, Ukraine

Abstract

The shear viscosity $\eta$ in the van der Waals excluded volume hadron-resonance gas model is considered. For the shear viscosity the result of the non-relativistic gas of hard-core particles is extended to the mixture of particles with different masses, but equal values of hard-core radius $r$. The relativistic corrections to hadron average momenta in thermal equilibrium are also taken into account. The ratio of the viscosity $\eta$ to the entropy density $s$ is studied. It monotonously decreases along the chemical freeze-out line in nucleus-nucleus collisions with increasing collision energy. As a function of hard-core radius $r$, a broad minimum of the ratio $\eta/s \approx 0.3$ near $r \approx 0.5$ fm is found at high collision energies. For the charge-neutral system at $T = T_c = 180$ MeV, a minimum of the ratio $\eta/s \approx 0.24$ is reached for $r \approx 0.53$ fm. To justify a hydrodynamic approach to nucleus-nucleus collisions within the hadron phase the restriction from below, $r \geq 0.2$ fm, on the hard-core hadron radius should be fulfilled in the excluded volume hadron-resonance gas.

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I. INTRODUCTION

One of the important discoveries in Au+Au collisions at BNL RHIC is evidence for strong collective effects in the form of transverse flow and asymmetric azimuthal (elliptic) flow. To account for this, one needs almost perfect fluid hydrodynamics (i.e. a small ratio, $\eta/s \leq 0.2$, of shear viscosity $\eta$ to entropy density $s$) starting at the very early stage of nucleus-nucleus (A+A) collision (see, e.g., [1, 2, 3, 4] and reference therein). Perturbative QCD calculations [5] have led to a much higher value of the viscosity to entropy density ratio, $\eta/s \geq 1$ (see Ref. [6]). Based on these results, the ‘almost perfect fluid’ in the deconfined phase is assumed to be the strongly interacting quark-gluon plasma (sQGP) [7]. In Ref. [6], different possibilities for a structure of a minimum of the ratio $\eta/s$ at the transition between hadron and quark-gluon phases are discussed. An absolute minimum of this ratio was identified with the critical point of QCD matter [3, 6]. This conclusion has been based in Ref. [6] on an empirical observation in liquid-gas transitions for other known substances. In Ref. [8] it was shown that certain field theories have the ratio $\eta/s = 1/4\pi$, and it was conjectured that $\eta/s \geq 1/4\pi$ for all other substances. This restriction appears to be close to the quantum (due to Heisenberg uncertainty principle) kinetic lower bound of Ref. [9], $\eta/s \geq 1/15$, for the gas of quarks and gluons (see Ref. [2]).

It is of both practical and fundamental importance to know the ratio $\eta/s$ in the hadron phase, in particular, near the transition to the sQGP. This will be a subject of the present paper. We will use the statistical model with full hadron-resonance spectrum and take into account the short-range repulsion effects within the hard-core approximation. Earlier calculations of the ratio $\eta/s$ were done in Ref. [10] for the pion gas and pion-kaon mixture (see recent pion gas results in Refs. [11, 12, 13]). There were also the calculations of the hadron system viscosity within the microscopic transport models: UrQMD in Ref. [14] and URASiMA event generator in Ref. [15].

During last decades statistical models of hadron-resonance gas (HG) have served as an important tool to investigate high energy nuclear collisions. The main object of the study have been mean multiplicities of produced hadrons (see, e.g., recent papers [16, 17, 18] and references therein). An extension to the ideal gas picture based on the van der Waals (VDW) excluded volume procedure was suggested in Refs. [19, 20, 21] to phenomenologically take into account repulsive interactions between hadrons. This leads to the well known VDW suppression of hadron number densities. However, the particle number ratios are (in Boltzmann approxima-
tion) independent of the proper volume parameter \( \nu \), if it is the same for all hadron species. Thus, a rescaling of the total volume \( V \) leads to exactly the same hadron yields as those in the ideal hadron gas. Recently the VDW HG model has been used in Ref. [22] to calculate particle number fluctuations. It was demonstrated [22] that multiplicity fluctuations are suppressed in the VDW HG gas. This suppression is qualitatively different from that of particle yields. In contrast to average multiplicities, the suppression of multiplicity fluctuations cannot be removed by rescaling of the total volume of the system. Thus, the hard-core radius of the hadrons can be, in principle, straightforwardly connected with data on multiplicity fluctuations.

The aim of the present paper is to make calculations of the ratio \( \eta/s \) in the VDW HG model. We present our results for different values of temperature \( T \) and baryon chemical potential \( \mu_B \) along the chemical freeze-out line. In this \( T - \mu_B \) region hydrodynamic expansion within the hadron phase is expected. The paper is organized as follows. In Section II we present the results for the thermodynamic functions in the multi-component VDW gas. In Section III the expression for the viscosity in the VDW HG is obtained. In Section IV the viscosity to entropy density ratio is calculated along the chemical freeze-out line. This gives the VDW HG predictions for this ratio in central nucleus-nucleus collisions in the hadron phase at different collision energies. A summary, presented in Section V, closes the paper.

II. VDW HADRON GAS

The VDW excluded volume procedure gives the following transcendental equation for the system pressure \( p \) [19]:

\[
p = \exp \left( - \frac{\nu p}{T} \right) T \phi ,
\]

where \( \nu \) is the hadron excluded volume parameter,

\[
\nu = 4 \cdot \frac{4}{3} \pi r^3 ,
\]

with \( r \) being the particle hard-core radius. In what follows it is assumed to be the same for all hadron species. In Eq. (1), \( \phi \) is the total particle number density in the ideal HG:

\[
\phi(T, \mu_B) = \sum_i \phi_i = \sum_i \frac{g_i}{2\pi^2} m_i^3 T K_2 \left( \frac{m_i}{T} \right) \exp \left( \frac{\mu_i}{T} \right) ,
\]
with \( \phi_i \) in Eq. (3) being the ideal gas particle number density of \( i \)th particle species. As in Ref. [22], we use the Boltzmann approximation, neglecting small effects of Bose and/or Fermi statistics. In Eqs. (1,3), \( T \) is the system temperature, \( g_i \) the degeneracy factor of \( i \)th particle species, \( m_i \) the particle mass, and

\[
\mu_i = b_i \mu_B + s_i \mu_S + q_i \mu_Q
\]

the chemical potential due to the \( i \)th particle charges \( (b_i, s_i, q_i) \) — baryon number, strangeness, electric charge. The summation in Eq. (3) is taken over all hadron species. Finally, \( K_2 \) in Eq. (3) is the modified Hankel function.

The \( i \)th particle number density in the VDW HG equals to:

\[
n_i = \frac{\exp(-vp/T) \phi_i}{1 + v \exp(-vp/T)} \phi \equiv \frac{x_i}{1 + vx}.
\]

In Eq. (4) the notations, \( x_i \equiv \exp(-vp/T) \phi \), and \( x \equiv \sum_i x_i \), have been introduced. The factor \( R = \exp(-vp/T) (1 + vx)^{-1} \) presents the VDW suppression factor. Since the same excluded volume parameter for all particle species is assumes, it is the same for all particle densities, and the VDW HG energy density.

The VDW HG entropy density \( s \) is equal to:

\[
s = \frac{\partial p}{\partial T} = \frac{x}{1 + vx} \left(1 + v x + \frac{T}{\phi} \frac{\partial \phi}{\partial T}\right).
\]

### III. SHEAR VISCOSITY

The shear viscosity in a non-relativistic gas of hard-core spheres can be calculated analytically (see e.g., Ref. [23]):

\[
\eta = \frac{5}{64} \frac{\sqrt{mT}}{\sqrt{\pi r^2}}.
\]

The Eq. (6) demonstrates a simple dependence of the viscosity on the system temperature \( T \), particle mass \( m \), and hard-core particle radius \( r \). The derivation of Eq. (6) from the molecular-kinetic theory demonstrates the dependence of the viscosity,

\[
\eta \propto n l \langle |p| \rangle,
\]

on the particle density \( n \), the mean free path \( l \propto 1/(nr^2) \), and the particle average thermal momentum:

\[
\langle |p| \rangle = \frac{\int_0^\infty p^2 dp \exp[-p^2/(2mT)]}{\int_0^\infty p^2 dp \exp[-p^2/(2mT)]} = \sqrt{\frac{8mT}{\pi}}.
\]
The Eq. (8) can be easily extended to the relativistic thermal motion:

\[ \langle |p| \rangle = \int_0^\infty \frac{p^2 dp}{\sqrt{p^2 + m^2/T}} \exp[-\sqrt{p^2 + m^2/T}] = \int_0^\infty \frac{p^2 dp}{\sqrt{p^2 + m^2/T}} \exp[-\sqrt{p^2 + m^2/T}] = \sqrt{\frac{8mT}{\pi}} \frac{K_{5/2}(m/T)}{K_2(m/T)}. \] (9)

At \( m/T \gg 1 \), the factor \( K_{5/2}(m/T)/K_2(m/T) \) goes to 1, and Eq. (9) is reduced to Eq. (8). In the opposite limit, \( m/T \ll 1 \), Eq. (9) goes to the well known result for massless particles, \( \langle |p| \rangle = 3T \).

For the the mixture of particle species with different masses, but with the same hard-core radius \( r \), Eq. (6) is transformed into:

\[ \eta = \frac{5}{64 \sqrt{\pi}} \frac{\sqrt{T}}{r^2} \sum_{i=1}^{n} \sqrt{m_i} \frac{K_{5/2}(m_i/T)}{K_2(m_i/T)} \frac{n_i}{n}, \] (10)

where \( n_i \) is given by Eq. (4), and \( n = \sum_i n_i \) is the total VDW gas particle number density. In Eq. (10), the relativistic correction factors, \( K_i \equiv K_{5/2}(m_i/T)/K_2(m_i/T) \) from Eq. (9), are taken into account. To obtain Eq. (10) the same mean free path has been assumed for different \( i \)th hadron species. This is approximately valid because of the same hard-core radius \( r \). The corrections because of different particle masses \( m_i \) are neglected. Eq. (10), with the summation over all particles and resonance species listed in THERMUS [24], will be used to calculate the hadron viscosity. The ratios \( n_i/n \) in Eq. (10) can be substituted by the ideal gas values, \( n_i/n = \phi_i/\phi \), as we have assumed the same hard-core radius \( r \) for all hadrons. The VDW suppression factor \( R \) is then the same for all hadron species, and it is canceled out in the ratios of particle densities.

Note that the viscosity \( \eta \) is not proportional to the number of hadron species. If one assumes the same hadron masses \( m_i = m \) and \( m \gg T \), Eq. (10) is reduced back to the case of one particle species (6). On the other hand, the entropy density \( s \) increases with the number of species. This fact has been used in recent paper [25] to construct a counterexample to the lower bound of \( \eta/s = 1/4\pi \) assumed in Ref. [8].

In the limit \( r \rightarrow 0 \) the excluded volume parameter \( v \) (2) goes to zero. In this limit, the thermodynamical functions of the VDW HG, \( p \) (11), \( n_i \) (4), and \( s \) (5) converge to the corresponding ideal gas expressions, \( p^{id} = T\phi \), \( n_i^{id} = \phi_i \), and \( s^{id} = \phi + T\partial\phi/\partial T \). On the other hand, according to Eq. (10) the viscosity \( \eta \) goes to infinity as \( \eta \propto 1/r^2 \). Thus, the ratio \( \eta/s \) diverges as \( r^{-2} \rightarrow \infty \) in the ideal gas limit of \( r \rightarrow 0 \).
IV. VDW HG MODEL RESULTS

In this section we present the results of the VDW HG for the ratio $\eta/s$ along the chemical freeze-out line in central A+A collisions for the whole energy range from SIS to LHC. The values of $T$, $\mu_B$, and $\gamma_s$ at the chemical freeze-out at different collision energies are presented in Table I. They are identical to those values in Table I of Ref. [22].

![Table I](image)

| $\sqrt{s_{NN}}$ | $T$ (GeV) | $\mu_B$ (MeV) | $\gamma_s$ | $K_\pi(r = 0.1 \text{ fm})$ | $K_\pi(r = 0.3 \text{ fm})$ | $K_\pi(r = 0.5 \text{ fm})$ |
|---------------|---------|---------------|-----------|----------------|----------------|----------------|
| [ GeV ] | [ MeV ] | [ MeV ] | $R$ | $\eta/s$ | $R$ | $\eta/s$ | $R$ | $\eta/s$ |
| 2.32 | 64.3 | 800.8 | 0.64 | 1.50 | 0.998 | 16.6 | 0.944 | 1.94 | 0.788 | 0.82 |
| 4.86 | 116.5 | 562.2 | 0.69 | 1.85 | 0.994 | 6.92 | 0.870 | 0.87 | 0.603 | 0.44 |
| 6.27 | 128.5 | 482.4 | 0.72 | 1.92 | 0.993 | 5.68 | 0.844 | 0.73 | 0.552 | 0.39 |
| 7.62 | 136.1 | 424.6 | 0.73 | 1.97 | 0.992 | 5.01 | 0.825 | 0.66 | 0.519 | 0.37 |
| 8.77 | 140.6 | 385.4 | 0.75 | 2.00 | 0.991 | 4.64 | 0.812 | 0.62 | 0.498 | 0.35 |
| 12.3 | 149.0 | 300.1 | 0.79 | 2.04 | 0.990 | 4.03 | 0.786 | 0.55 | 0.459 | 0.33 |
| 17.3 | 154.4 | 228.6 | 0.83 | 2.07 | 0.989 | 3.66 | 0.766 | 0.51 | 0.432 | 0.31 |
| 62.4 | 160.6 | 72.5 | 0.98 | 2.11 | 0.987 | 3.18 | 0.738 | 0.46 | 0.397 | 0.30 |
| 130 | 161.0 | 35.8 | 1.0 | 2.11 | 0.986 | 3.14 | 0.735 | 0.46 | 0.393 | 0.29 |
| 200 | 161.1 | 23.5 | 1.0 | 2.11 | 0.986 | 3.13 | 0.735 | 0.46 | 0.393 | 0.29 |
| 5500 | 161.2 | 0.9 | 1.0 | 2.11 | 0.986 | 3.13 | 0.735 | 0.46 | 0.393 | 0.29 |

Note that the conditions for average energy per particle, $\langle E \rangle/\langle N \rangle = 1 \text{ GeV}$, zero value of the net total strangeness, $S = 0$ (this defines $\mu_S = \mu_S(T, \mu_B)$), and the charge to baryon ratio, $Q/B = 0.4$ (this defines $\mu_Q = \mu_Q(T, \mu_B)$), remain the same as in Refs. [27, 28]. The only
tiny difference of $T$ and $\mu_B$ values from those in Refs. [27, 28] comes because of the Boltzmann statistics approximation used in the present paper and in Ref. [22]. The dependence of $\mu_B$ on the collision energy is parameterized as [16], $\mu_B \left( \sqrt{s_{NN}} \right) = 1.308 \text{ GeV} \left( 1 + 0.273 \sqrt{s_{NN}} \right)^{-1}$, where the c.m. nucleon-nucleon collision energy, $\sqrt{s_{NN}}$, is taken in GeV units. The strangeness saturation factor, $\gamma_S$, is parameterized as [17], $\gamma_S = 1 - 0.396 \exp (-1.23 T/\mu_B)$. Both these relations are the same as in Refs. [22, 27, 28]. The resulting ratio $\eta/s$ in the VDW HG is presented in Fig. 1 as a function of hard-core radius $r$ for several chemical freeze-out points from Table I.

FIG. 1: The ratio $\eta/s$ in the VDW HG as the function of hard-core radius $r$. The lines correspond to different $T - \mu_B$ freeze-out points from Table I: SIS ($\sqrt{s_{NN}} = 2.32 \text{ GeV}$), AGS ($\sqrt{s_{NN}} = 4.86 \text{ GeV}$), SPS ($\sqrt{s_{NN}} = 17.3 \text{ GeV}$), and RHIC ($\sqrt{s_{NN}} = 200 \text{ GeV}$).

The values of $\sqrt{s_{NN}}$ quoted in Table I correspond to the beam energies at SIS ($2A \text{ GeV}$),
AGS (11.6\,A GeV), SPS (20A, 30A, 40A, 80A, and 158A GeV), colliding energies at RHIC ($\sqrt{s_{NN}} = 62.4$ GeV, 130 GeV, and 200 GeV), and LHC ($\sqrt{s_{NN}} = 5500$ GeV). The excluded volume correction factor, $R = \exp\left(-v_p/T\right)/(1 + vx)^{-1}$, and the entropy density, Eq. (5), are calculated using the THERMUS package [24]. The relativistic enhancement factor, $K_\pi = K_{5/2}(m_\pi/T)/K_2(m_\pi/T)$, for average thermal momentum is also presented in Table I at different freeze-out points for pions, which are the lightest hadrons. The viscosity is calculated according to Eq. (10). The Fig. 1 demonstrates a presence of a broad minimum of the ratio $\eta/s$ near $r \approx 0.5$ fm at high collision energies. For fixed hard-core radius $r$, the ratio $\eta/s$ monotonously decreases along the freeze-out line with increasing of collision energy. This is shown in Fig. 2.

FIG. 2: The ratio $\eta/s$ in the VDW HG along the chemical freeze-out line as the function of $\sqrt{s_{NN}}$ at different hard-core radiiuses. The solid line presents the results for $r=0.5$ fm and dashed line for $r=0.3$ fm.
The presence of strong collective flow in A+A collisions indicates a rather small viscosity. According to Ref. [29], the following inequality, \( \eta/s \leq 5 \), should be satisfied. As seen from Fig. 1, this leads to the restriction, \( r \geq 0.2 \text{ fm} \), on the hard-core radius from below in the VDW HG.

The Fig. 3 shows the dependence of the ratio \( \eta/s \) on the temperature \( T \) in the VDW HG for charge-neutral system \((Q = B = S = 0\) and \( \gamma_S = 1 \)).

**FIG. 3:** The ratio \( \eta/s \) for charge-neutral system \((Q = B = S = 0\) and \( \gamma_S = 1 \)) as a function of \( T \). We fix the crossover temperature between HG and sQGP as \( T_c = 180 \text{ MeV} \). At \( T \leq T_c \) the VDW HD results are shown at different hard-core radiiuses, the solid line corresponds to \( r = 0.5 \text{ fm} \) and dashed line to \( r = 0.3 \text{ fm} \). The dotted line corresponds to the lower bound, \( \eta/s = 1/4\pi [8] \). At \( T > T_c \), the triangle symbol corresponds to Ref. [31] of the pure gluon plasma in the lattice QCD, and the square is the estimate of Ref. [32] in the quasiparticle model of the sQGP.
The VDW HG for \( r = 0.3 \) fm and \( r = 0.5 \) fm are presented at \( T < T_c \), with \( T_c = 180 \) MeV assumed as the temperature of a crossover between the confined and deconfined phase. At \( T = T_c = 180 \) MeV a minimum of the ratio \( \eta/s \approx 0.24 \) is reached for \( r \approx 0.53 \) fm. Note that at high collision energies the baryon chemical potential is small, and a hydrodynamic expansion within the hadron phase at \( T < T_c \) is expected in A+A collisions.

In the deconfined phase there were several estimates of the ratio \( \eta/s \). The estimates of Ref. [6] based on the perturbative calculations [5] give the large value, \( \eta/s > 1 \). The non-perturbative results correspond to smaller values of the ratio \( \eta/s \). In the pure SU(3) gauge model on the lattice a shear viscosity to entropy density ratio of \( \eta/s < 1 \) in the temperature region \( 1.4 \leq T/T_c \leq 1.8 \) was found in Ref. [30]. The same upper bound, \( \eta/s < 1 \), and the estimate \( \eta/s = 0.134 \pm 0.033 \) at \( T = 1.65 T_c \) were obtained in Ref. [31]. In the quasiparticle description of the sQGP it was found [32] a shear viscosity to entropy density ratio \( \eta/s \approx 0.34 \) at \( T \approx 1.5 T_c \).

V. SUMMARY

In summary, the shear viscosity \( \eta \) in the van der Waals excluded volume hadron-resonance gas model is calculated. We use the same hard-core radius \( r \) for all hadron species. The ideal gas limit appears to be a singular one: the ratio of the viscosity \( \eta \) to the entropy density \( s \) diverges as \( r^{-2} \) at \( r \to 0 \). This makes an ideal gas picture inappropriate for any kinetic or hydrodynamic calculations. Moreover, the ratio \( \eta/s \) should be small enough to justify the hydrodynamic approach to A+A collisions. Near the chemical freeze-out line this leads to the restriction from below, \( r \geq 0.2 \) fm, on the hard-core hadron radius. When the ratio \( \eta/s \) is considered as a function of \( r \), a broad minimum, \( \eta/s \approx 0.3 \) fm for \( r \approx 0.5 \) fm is found for the chemical freeze-out at high collision energies. For fixed hard-core radius \( r \), the ratio \( \eta/s \) monotonously decreases along the chemical freeze-out line with increasing collision energy. A similar behavior is found for the ratio \( \eta/s \) as a function of \( T \) for the charge-neutral system. The ratio \( \eta/s \) decreases with \( T \), and its minimal value is \( \eta/s \approx 0.24 \) at \( T = T_c = 180 \) MeV for \( r \approx 0.53 \) fm. Theoretical estimates of the ratio \( \eta/s \) in the deconfined phase require the non-perturbative calculations, and they are far from being complete. More efforts are needed to clarify the detailed behavior of the ratio \( \eta/s \) at the transition (crossover) between hadron and quark-gluon phases.
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[1] D. Teaney, Phys. Rev. C 68, 034913 (2003).
[2] T. Hirano and M. Gyulassy, Nucl. Phys. A 769, 71 (2006).
[3] R.A. Lacey, et al. Phys. Rev. Lett. 98, 092301 (2007).
[4] H. Drescher, A. Dumitru, C. Gombeaud, and J. Ollitrault, arXiv:nucl-th/0704.3553.
[5] P. Arnold, G.D. Moore, and L.G. Yaffe, J. High Energy Physics, 05 (2003) 051.
[6] L.P. Csernai, J.I. Kapusta, and L.D. McLerran, Phys. Rev. Lett. 97 152303 (2006).
[7] T.D. Lee, Nucl. Phys. A 750, 1 (2005); M. Gyulassy and L. McLerran, Nucl. Phys. A 750, 30 (2005); E.V. Shuryak, Nucl. Phys. A 750, 64 (2005).
[8] P.K. Kovtun, D.T. Son, and A.O. Starinets, Phys. Rev. Lett. 94 111601 (2005).
[9] P. Danielewicz and M. Gyulassy, Phys. Rev. D 31, 53 (1985).
[10] M. Prakash, M. Prakash, R. Venugopalan, and G. Welke, Phys. Rep. 227, 321 (1993).
[11] A. Dobado and F.J. Llanes-Estrada, Phys. Rev. D 69, 116004 (2004).
[12] J. Chen and E. Nakano, Phys. Lett. B 647, 371 (2007).
[13] E. Nakano, arXiv:hep-ph/0612255.
[14] A. Muronga, Phys. Rev. C 69, 044901 (2004).
[15] S. Muroya and N. Sasaki, Prog. Theor. Phys. 113, 457 (2005).
[16] J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys. Rev. C 73, 034905 (2006).
[17] F. Becattini, J. Manninen, and M. Gaździcki, Phys. Rev. C 73, 044905 (2006).
[18] A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A 772, 167 (2006).
[19] D.H. Rischke, M.I. Gorenstein, H. Stöcker, and W. Greiner, Z. Phys. C 51, 485 (1991); J. Cleymans, M.I. Gorenstein, J. Stahnacke, and E. Suhonen, Z. Phys. C 8, 347 (1993).
[20] Granddon D. Yen, M.I. Gorenstein, W. Greiner, and Shin Nan Yang, Phys. Rev. C 56, 2210 (1997); M.I. Gorenstein, H. Stöcker, Granddon D. Yen, Shin Nan Yang, and W. Greiner, J. Phys. G 24, 1777 (1998); Granddon D. Yen and M.I. Gorenstein, Phys. Rev. C 59, 2788 (1999).
[21] M.I. Gorenstein, V.K. Petrov, and G.M. Zinovjev, Phys. Lett. B 106, 327 (1981); M.I. Gorenstein, W. Greiner, and Shin Nan Yang, J.Phys. G 24, 725 (1998); M.I. Gorenstein, M. Gaździcki, and W. Greiner, Phys. Rev. C 72, 024909 (2005).

[22] M.I. Gorenstein, M. Hauer, and D.O. Nikolajenko, arXiv:nucl-th/0702081, Phys. Rev. C, in print.

[23] E. M. Lifschitz and L. P. Pitaevski, Physical kinetics, 2. ed. Pergamon Press, Oxford, 1981 (Landau-Lifschitz. Course of theoretical physics. V.10), Chap. 1, Par. 10.

[24] S. Wheaton and J. Cleymans, arXiv:hep-ph/0407174.

[25] T.C. Cohen, Phys. Rev. Lett. 99, 021602 (2007).

[26] J. Cleymans and K. Redlich, Phys. Rev. Lett. 81, 5284 (1998).

[27] V.V. Begun, M.I. Gorenstein, M. Hauer, V.P. Konchakovski, and O.S. Zozulya, Phys.Rev. C 74 044903 (2006).

[28] V.V. Begun, M. Gaździcki, M.I. Gorenstein, M. Hauer, V.P. Konchakovski, and B. Lungwitz arXiv:nucl-th/0611075, Phys. Rev. C, in print.

[29] L.P. Csernai, J.I. Kapusta, and L.D. McLerran, J. Phys. G 32 S115 (2006).

[30] A. Nakamura and S. Sakai, Phys. Rev. Lett. 94, 072305 (2005).

[31] H.B. Meyer, arXiv:hep-lat/0704.1801.

[32] B.A. Gelman, E.V. Shuryak, and I. Zahed, Phys. Rev. C 74, 044908 (2006).