Identification of radiant source in an enclosure by reduced model

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Abstract. We propose an original method to recover from a few measurement points the integrity of the temperature field of a furnace heated by a radiant thermal source. The radiant thermal source is first identified via a low order reduced model based on a low order reduced model based on AROMM (Amalgam Reduced Order Modal Model) method which preserves the integrity of the geometry. The minimization is performed via a trust-region reflective least squares algorithm implemented in MATLAB “lsqcurvefit” function. From that identified heat flux, the integrity of the thermal field is then recovered by direct simulation thanks to a reduced model of higher rank to have a better precision. The treated application is a complex titanium piece heated by two radiant panels placed in a furnace. With four measuring points, the temperature of the whole thermal scene is retrieved at all times with an average error around 1 K on the studied object.

1. Introduction
The direct numerical resolution of industrial applications is today mastered and generally carried out by finite elements. However for complex geometries, the consideration of thermal radiation substantially increases the computation time, which excludes iterative procedures as parameters identification in a reasonable time. An alternative approach is to use reduced modal models [1, 2]. Modal methods do not degrade the geometry, and give access to the entire temperature field, which is mandatory for a correct modeling of radiation. A recent work adapted the Amalgam Reduced Order Modal Model (AROMM) in order to incorporate thermal radiation by radiosity method [3]. This article presents an extension of this previous work: the identification of a radiant heating source to reconstruct a thermal scene in a short time.

2. Physical problem
A titanium object with a complex shape is placed on a stand in an industrial furnace (see Fig. 1 and Table 1). This object is heated by two radiant tubes surrounded by parabolic reflectors located at the top of the furnace. The heat exchange between the furnace and the outside is modeled by an equivalent global exchange coefficient \( h_{ext} = 1 \text{W.m}^{-2}\cdot\text{K}^{-1} \) (this low value models the furnace insulation). The outside temperature is supposed to be constant at \( T_{ext} = 293.15\text{K} \). The convective exchanges between the interior surfaces of the furnace are represented by a constant coefficient \( h_{int} = 5 \text{W.m}^{-2}\cdot\text{K}^{-1} \) and a fluid temperature \( T_{int} \) that depends on the temperature of all internal surfaces. Finally, the initial condition is the ambient temperature \( T(t = 0) = T_{ext} \), and the simulation duration is \( 5 \times 10^4 \text{s} \).
Reflector
Titanium
object
Stand
Tube
Wall
Measurement
points
Point A

Figure 1: The considered geometry.

Figure 2: The radiant panel temperature.

Table 1: Thermophysical characteristics of the different components

|                | Heat capacity $c$ [J.m$^{-3}$.K$^{-1}$] | Thermal conductivity $k$ [W.m$^{-1}$.K$^{-1}$] | Emissivity $\varepsilon$ | Thickness $e$ [m] |
|----------------|------------------------------------------|-----------------------------------------------|--------------------------|------------------|
| Piece          | $2.35 \times 10^6$                      | 21.9                                          | 0.8                      | 0.001            |
| Stand          | $3.95 \times 10^6$                      | 16.3                                          | 0.95                     | 0.005            |
| Wall           | $0.18 \times 10^6$                      | 45                                            | 0.95                     | 0.01             |
| Tube           | $3.4 \times 10^6$                       | 45                                            | 0.95                     | 0.01             |
| Reflector      | $3.95 \times 10^6$                      | 16.3                                          | 0.3                      | 0.001            |

3. Reduced modal model

In modal methods, temperature is searched as a sum of known spatial functions $V_i(M)$ weighted by unknown temporal amplitudes $x_i(t)$:

$$ T(M,t) \approx \sum_{i=1}^{\tilde{N}} x_i(t) V_i(M) $$  \hspace{1cm} (1)

Temporal amplitudes are numerically obtained by solving the following equation:

$$ V^t C V \frac{dX}{dt} = V^t [K + H] VX + V^t U_0 + V^t R_{rad} T^t $$  \hspace{1cm} (2)

with

$$ T = T^{UR} V X $$  \hspace{1cm} (3)

$V$ $[N \times \tilde{N}]$ is the matrix gathering the $\tilde{N}$ discrete reduced modes $V_i$, and $X$ is the vector of amplitudes. $C$, $K$, $H$ are $[N \times N]$ matrices issuing from finite elements discretization: $C$ is the thermal inertia matrix, $K$ the conductivity matrix and $H$ is the matrix describing convective exchanges. $U_0$ is a vector characterizing the external known solicitations. Vector $T$ of dimension $[N_p]$ contains mean temperatures of every radiant patch and is easily calculated with matrix $U_R$ of dimension $[N_p \times N]$. It is the average operator which allows to compute the mean temperature on the $N_p$ patches from the temperature at the $N$ nodes of the mesh.
Matrix problem defined by Eqs. (2)-(3) is built in two steps: creation of radiation matrix (view factors and finite element matrices) in $t_{\text{Rad CPU}} \approx 9h$ and the computation of reduced modal basis in $t_{\text{RM CPU}} \approx 7\ min$. The computation of modal basis is described in [3]. To validate the AROMM method, the problem is solved with a modal model of order 50, and compared with a finite elements simulation. The whole thermal field of the titanium object is recovered with a maximum error of 38$K$ and an average error of 1.5$K$, compared to the magnitude of the thermal evolution of the object (646$K$). These good results are obtained in a short computation time (2 min) in relation to the CPU time of the finite elements model (8 h).

4. Identification procedure

The principle of the identification procedure is to find $T_{\text{tube}}$ which minimizes a quadratic criterion between a measured temperature (noted $Y$) at $N_{\text{mes}}$ points and $N_t$ times, and a corresponding simulated temperature noted $\hat{Y}(T_{\text{tube}})$:

$$J = \frac{1}{2} \sum_{i=1}^{N_t} \sum_{j=1}^{N_{\text{mes}}} (Y_{ij} - \hat{Y}_{ij}(T_{\text{tube}},i))^2 \quad (4)$$

The minimization is performed thanks to MATLAB optimization function “lsqcurvefit” with algorithm “trust-region-reflective” and default parameters. The minimization is an iterative procedure which requires at each step the resolution of Eqs. (2)-(3).

4.1. Results

There are 5000 parameters to identify (one per time), which is way too important to be performed as a whole. Thus, a strategy with a sliding time window of constant size is chosen. Its size is a question of compromise: if it is too small, the data is insufficient, but if it is too large, the increase of parameters to identify multiply the computation effort [1]. After numerical tests, a size of $\Delta t = 50\ s$ has been chosen, and four measurement points are taken to add information (see Fig. 1). Data $Y$ for the four measurement points at every 10$s$ is obtained from a finite elements simulation. A white noise with a standard deviation $\sigma = 0.2\ K$ has been added to mimic experimental data. The identification procedure is conducted with a reduced model. Its order is also a question of compromise: a low order reduced model naturally regularizes the solution and quickens computations, but if the order is too small, the reduced model will not be representative of the physics. In this study, an order of 50 has been chosen. With these specifications, the procedure is performed in 970$s$ and gives the temperature of radiant panel represented in figure 3. The order of magnitude of the temperature is properly recover along time. We notice a very good identification of the temperature during strong variations as between 20000 and 35000$s$, as the error between the exact and identified temperature is of the order of 5%. The noise that appears during step phase is due to the small size of the temporal window. Considering the computation time and the “experimental” duration, a more elaborate post-treatment (as a low-frequency filter) could be applied to reduce the noise.

We used the identified radiant temperature of the tube as an input for Eqs. (2)-(3), in order to recover the whole temperature field. The order of the reduced model used to compute the thermal field is increased to 300 to improve the quality of the reconstruction, which is done in a CPU time of 530$s$. Figure 4 illustrates the performance of identification in term of precision: it shows the evolution of temperature at point A (see Fig. 1), where the error on the object is maximal. Dashed line depicts the temperature computed with the exact source and a finite elements code, while the plain line represents the reconstructed temperature. The agreement is very good: the error is below 20$K$ and most of time is below 5$K$. We notice that despite a noisy source, the time evolution of the temperature is rather smooth. This is in part due the inertia of the material, and in part to the natural filtering of reduced model: modes with short time
constant are discarded during the reduction step. Figure 5 depicts the reconstructed thermal field (the scale is adapted to the temperature range of the object) at $t = 6000\, s$, i.e. when the error is maximal. Contours produced by the complex radiative heat transfer and the mask effects are remarkably well recovered. The difference between the finite element model computed with the exact source, and the reduced model with the identified source is represented in Fig. 6. This field is erratic, which is characteristic of modal reduction. The difference barely exceeds $20\, K$, which has to be related to the temporal variation of $1000\, K$ of radiant tubes. The average error (in space and time) on the object is less than $1\, K$.

### 4.2. Conclusion

In [3], the AROMM and radiosity methods were combined to build a reduced model able to deal with heat transfer problem dominated by radiative effect. In this communication, we use that model to identify a time dependent radiative heat source in a reasonable time from few measurement points, and then reconstruct the whole thermal scene. In the presented example the total procedure (identification and reconstruction) do not exceed $1500\, s$, for an average error below $1\, K$ on the studied object. These very satisfying first results demonstrates the interest of the AROMM method for radiation dominated heat transfer problem.

![Figure 3: The identified tube temperature.](image1)

![Figure 4: Temperature on the object](image2)

![Figure 5: The reconstructed thermal field](image3)

![Figure 6: The error field](image4)

### References

[1] Carmona S, Rouizi Y, Quéméner O, Joly F and Neveu A 2019 *Int. J. Therm. Sci.* 131 94-104

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[3] Gaume B, Joly F and Quéméner O 2019 *Int. J. Heat Mass Transf.* 141 779-788