Students’ critical thinking skills in making mathematical problems

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Abstract. Critical thinking is not only required in solving mathematical problems, but also in making problems. This qualitative descriptive study was aimed at tracing critical thinking skills of 25 students of elementary school teacher education program in the process of making mathematical problems. Students were given the task of making problems on the topic of the triangle. Furthermore, an interview was conducted to explore the process or reason for making the problems. The results of this study indicated that students had not been skilled at critical thinking in making problems. The activity of critical thinking in the process of making problems only included determining the focus of the problem and decision making. Nevertheless, the decision which was taken was not based on logical reasoning. The activity of examining the accuracy of supporting data and reflecting on problems had not been accomplished, so as to produce incorrect problems. Thus, critical thinking is of urgency to produce good and right problems. The findings of this study can be utilized to develop critical thinking skills in making problems and solving mathematical problems.

1. Introduction

Critical thinking is one of the goals of learning mathematics [1]. It implies that mathematics learning must develop critical thinking skills. In mathematics learning, critical thinking is developed by means of mathematical problem solving activities [2]. Therefore, mathematical problems must encourage students to think critically. Mathematical problems must be prepared carefully and critically by the teacher. This shows that the mathematics teacher must be proficient in critical thinking when it comes to making mathematical problems.

Critical thinking skills have been studied from various perspectives. Critical thinking has a relationship with problem solving abilities [3] and is such an effective way to improve the understanding of mathematical concepts [4]. Critical thinking skills can be improved through a cooperative learning [5]; a mixture of six thinking hats techniques, brainstorming, role playing, Socrates seminars, and anyone here an expert [6]. The CORT program will be able to improve critical thinking skills of students who have experienced learning difficulties [7]. However, low critical thinking skills of students are still being found in the Middle East [8] and Malaysia [9]. Critical thinking skills of pre-service teachers of elementary school in Turkey are still at an intermediate level.
[10]. Even the majority of the pre-service teachers of mathematics in Malang are considered non-critical thinkers [11].

In general, the measurement of critical thinking skills is just carried out in problem solving activities [3,11]. So far, investigation has not been found on critical thinking skills in the process of making problems. Mathematical problems should be utilized to develop critical thinking skills. Therefore, this study aimed at describing the critical thinking skills of elementary school teacher education program students in the process of making mathematical problems. The results of this study can be used as a basis for developing skills to make good and right problems, so that these problems can be utilized to develop critical thinking skills in solving mathematical problems.

2. Method

This research was a qualitative descriptive study. The research subjects were represented by 25 students of elementary school teacher education program at one of the private universities in Malang. Subjects had studied the triangle topic and its problems. Subjects also had learned how to solve problems and to teach the topic of triangle. Essay test and interview guidelines were employed in this research as the research instruments. Essay test was provided in the form of assignments to making problems, related to triangle topic. Subjects were free to choose the form of problems, such as objective questions, subjective questions, word problems, and so on. The interview was aimed to get data about the process or the basic thinking of the subjects related to the process of making the problems.

Data analysis began by examining the accuracy of the problem content, then grouping the data. After that, transcribing the interview results of 6 S1 group subjects and 4 S2 group subjects. Furthermore, an analysis of the making problem process was carried out departing from the critical thinking elements, comprised: (a) focusing on problems and facts, (b) giving the reasons for decision support, (c) making reasonable conclusions, (d) understanding the situation, (e) understanding the meaning of the term, and (f) reflecting on the decisions taken [12]. Understanding of the situation and the meaning of the term appears in all elements of critical thinking, while determining focus is divided into two, namely: clarifying the problem and examining the data/facts on the problem. Therefore, indicators of critical thinking skills in making problems include (a) determining the subject matter, (2) examining supporting facts/data, (3) giving logical reasons, (4) drawing conclusions, and (5) reflecting.

3. Results and discussion

Data analysis based on the content of the problem produced two groups of data, namely the correct problem (S1 data) and the wrong problem (S2 data). The data grouping are shown in table 1.

| Table 1. Grouping problems which were made by students on the topic of the triangle. |
|---------------------------------|-------------------------------|
| Group  | The process of making problems | Numbers of problems |
| S1     | Adoption                       | 15                          |
|        | Modification                   | 6                           |
| S2     | Adoption                       | 0                           |
|        | Modification                   | 4                           |

Table 1 shows that S1 data consist of two groups, i.e 15 adoption problems (S1.1 data) and 6 modification problems (S1.2 data). S2 data comprise 4 modification problems. The problem of adoption is a problem that is made by directly quoting a problem, both of which have been studied in learning or they are contained on the internet. Modification problems are problems that are made by changing some or several parts of the problems, or changing the form of the problems.
3.1. S1 data
S1 data have the same pattern or form of problems. Therefore, this section presents data that represents S1 data. S1.1 data and the problems that were referred to are presented in figure 1.

Figure 1. (a) Example of S1.1 data and (b) Referred problem.

Figure 1a is the adoption of the problems in figure 1b. The process of making problems was conducted by determining the focus of the problem in the topic of the triangle. After that, the subjects looked up the examples of problems on the internet, then chose one of these problems. The subjects did not examine the accuracy of the data and reflect on the problems. An exploration through interview showed that the subjects never learned how to make problems, so they prefer the problems on the internet. This was such a contradiction to the professional standards of teaching mathematics, namely giving students the opportunity to propose or to create their own problems [13].

S1.2 data is a modification of the problems that have been studied by the subject or they-the problems- are contained on the internet. Modification problems and the problems that were referred to are presented in figure 2.

Figure 2. (a) Example of S1.2 data and (b) Referred Problem.

The problems in figure 2a are the modification of the word problem in figure 2b. The subjects employed the problem solving figure 2a to make a new problem, then asked the same thing. The
problems in the form of a word problem were converted into an image. The process of making the figure 2a was conducted by means of determining the focus of the problem. After that, looked up examples of problems that had been studied or found on the internet. Next, the subject changes the form of the problem. Through interviews, it was found that subjects were lack of confidence in the ability to understand and to solve mathematical problems, so they did not dare to make too many changes to the referred problems. According to Bandura, self-efficacy affects a person’s way of thinking and acting [14].

3.2. S2 Data
S2 data showed there were as many as 4 problems that had the same error. Figure 3 is a representation of the S2 data and the referred problems.

![Figure 3. (a) Example of S2 data, (b) Referred problem, (c) Problem solving.](image)

The problems in figure 3a are the modification of the problems that were found on the internet as shown in figure 3b. Modification was conducted by changing the length of one side of the triangle and adding the altitude of the triangle. The addition of the altitude of the triangle was aimed to facilitate the determination of the area of the triangle as shown in figure 3c. The results of the interview showed that the subjects were accustomed to obtaining the area triangle problem that could be solved by the formula: the area of the triangle = $\frac{1}{2} \times (\text{base}) \times (\text{altitude})$. This raised up an assumption that mathematical problems had only one way of solving [15]. As a result, the subject did not try to reflect in a way that they would be able to re-examine the problem in another way.

Errors that were caused by the addition of altitude size can be known if the reflection is done in a way:

$$s = \frac{1}{2} (a + b + c) = \frac{1}{2} (6 + 7 + 8) = 10.5$$
$$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{10.5(10.5-6)(10.5-7)(10.5-8)} = \sqrt{413.4375}$$

So, it is impossible for the same triangle to have a different area, namely 17.5 (figure 3c) and $\sqrt{413.4375}$.

The process of making S2 problems started with determining the focus of the problem, namely: the area and circumferences of the triangle. After that, the subject looked up examples of problems that had been studied or found on the internet. The next step was to make changes to the problems. Interview results indicated that the subject did not examine the accuracy of the data due to changes that had been made. Changes of problems were only to facilitate problem solving and not to take the theory or concept of triangles into account. This caused errors in the content of the problem. In this
case, critical thinking activities were not carried out, namely recognizing data inconsistencies, such as contradictory data [16].

Based on S1 and S2 data, students' critical thinking activities in making problems are presented in figure 4.

![Critical Thinking Activity](image)

**Figure 4.** Critical thinking activities in the process of making problems.

Figure 4 shows that the activity of critical thinking in making problems only includes determining the subject matter and determining the problem or drawing conclusions. Determination of the problem was not based on logical reasoning because it only quoted directly or made a slight change to a problem. This shows that making problems does not involve critical thinking activities such as interpreting, analyzing, and evaluating information as a basis for valid and reliable decision making [4].

After determining the focus of the problem, the subject looked up the examples of problems. Some problems were changed in certain parts. The subject did not examine the relationship of facts that were caused by the change, resulting in content errors in several questions. This shows that making problems does not involve critical thinking activities in the form of examining additional information on the problem and analyzing the situation [16]. If the activity was carried out properly, it would not produce the wrong problems. Therefore, critical thinking is of necessity in making mathematical problems.

Some subjects provided solutions to the problems that had been generated. However, the resolution of the problems was not utilized to reflect or to review the accuracy of the problem. This habit was caused by the previous learning process which did not direct and demand the subject to re-examine the accuracy of his work. In this case, the subject had problematic met-before, which is a learning experience that does not support or raise problems in current learning [17].

### 4. Conclusion

The findings of this study are some errors in student-made problems. These errors were discovered when the problems were being analyzed through critical thinking activities. This shows that making problems that are good and right is not a piece of cake and there is a relationship between the process of making a problem with the critical thinking. Therefore, the skill to make problems or problem posing needs to be the part of mathematics learning for all students and more importantly for pre-service teachers. The problems that are generated by students can be criticized, in order to provide additional benefits, namely developing critical thinking skills. In addition, errors in content that is made by students may not be limited to the topic of the triangle. There is a possibility of errors in the
content of problems on other mathematical topics, for instance on the topic of arithmetic, algebra, and so on. This can be followed up by conducting research on other mathematical problems.

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