Analysis of the NMSSM Higgs Boson Masses at One-Loop Level

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Abstract

For a reliable prediction of the NMSSM Higgs boson signatures at present and future high-energy colliders and a proper distinction of the NMSSM and MSSM Higgs sector the precise knowledge of the Higgs boson masses including higher-order corrections is indispensable. In this paper, the one-loop corrections to the neutral NMSSM Higgs boson masses and mixings are calculated in three different renormalisation schemes. In addition to the DR renormalisation scheme, existing in the literature, two other schemes are adopted. Furthermore, the dependence on the value of the top quark mass is investigated. The resulting Higgs mass corrections have been compared and the residual theory error due to missing higher-order corrections can be estimated to be of the order of 10%. 

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1 Introduction

Supersymmetric theories \[1,2\] provide a natural solution to the hierarchy problem \[2,3\]. The latter is related to the fact that the Standard Model (SM) Higgs boson mass receives radiative corrections which are quadratic in the cut-off scale, usually chosen to be the GUT scale \(M_{\text{GUT}} = 10^{16}\) GeV. To keep the Higgs mass of the order of the electroweak (EW) scale an extreme fine-tuning of the model parameters is necessary. Supersymmetry (SUSY) introduces a new symmetry between fermionic and bosonic degrees of freedom, leading to extra contributions to the radiative corrections of the Higgs boson mass due to the new SUSY particles which cancel the dangerous quadratic divergence of their corresponding SM counterpart. In this way the Higgs mass is naturally kept at a phenomenologically valid level even in the presence of high mass scales.

As the superpotential must be analytic in the chiral superfields, two complex Higgs doublets have to be introduced, \(H_u\) to provide masses to the up-type fermions and \(H_d\) to ensure non-zero down-type fermion masses. In this way the theory is also kept anomaly-free. In the superpotential of the Minimal Supersymmetric Extension of the Standard Model (MSSM) \[4\] the two Higgs doublet fields mix through the term \(\mu H_u \epsilon H_d\) which involves the higgsino mass parameter \(\mu\). Phenomenology requires \(\mu\) to be of the order of the electroweak scale \[5\]. On the other hand, \(\mu\) is present in the superpotential before the symmetry breaking and hence not a priori of the order of the EW scale. In the Next-to-Minimal Supersymmetric Extension of the Standard Model (NMSSM) \[6–9\] this \(\mu\) problem is solved by dynamically relating the value of \(\mu\) to the EW scale. The parameter \(\mu\) arises here as the vacuum expectation value of the neutral component of an additional Higgs field \(S\) which is a singlet field with respect to the SM gauge groups. It couples to the MSSM Higgs fields via the interaction term \(\lambda S(H_u \epsilon H_d)\). With the scalar field \(S\) acquiring a non-zero vacuum expectation value (VEV) \(v_s/\sqrt{2}\) an effective \(\mu\) term is generated, naturally expected to be of the order of the EW scale, \(\mu = \lambda \langle S \rangle \equiv \lambda v_s/\sqrt{2}\). Furthermore, new contributions to the quartic coupling increase the tree-level mass value of the lightest Higgs boson. In the MSSM the tree-level mass of the lightest Higgs boson is predicted to be below the mass of the \(Z\) boson. Large radiative corrections involving top and stop loops are necessary to lift the Higgs mass value beyond the lower bound from direct searches at LEP \[10\]. The additional NMSSM contributions lift the mass of the SM-like Higgs boson more easily beyond the LEP bound.

The extension of the Higgs sector by two more degrees of freedom through the introduction of an additional singlet superfield \(\hat{S}\) leads to a total of 7 Higgs bosons after electroweak symmetry breaking, three neutral CP-even, two neutral CP-odd and two charged Higgs bosons. The fermionic component of \(\hat{S}\) mixes with the MSSM higgsinos and neutral gauginos to yield five neutralinos. Among the considerable phenomenological modifications in the Higgs and neutralino sector compared to the MSSM are possible new Higgs-to-Higgs decays, as \(e.g.\) the decay of a SM-like scalar Higgs boson into a pair of lighter pseudoscalar Higgs states, so that the present Tevatron and LHC search studies for supersymmetric Higgs particles have to be revisited \[11\]. Such a Higgs boson could have escaped the LEP bounds \[12\].

From the above discussions it is clear that the precise knowledge of the Higgs boson masses is indispensable to distinguish between MSSM and NMSSM predictions, to properly define scenarios

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\footnote{1We denote superfields by a hat over the field. Fields without hat are the corresponding component fields.}
with new Higgs-to-Higgs decays within the NMSSM and to correctly interpret the experimental data. In the MSSM, where radiative corrections are crucial to accommodate the light Higgs boson mass with the LEP limits, the Higgs boson masses have been calculated with impressive accuracy. After the calculation of the dominant one-loop contributions due to top/stop loops \cite{13} the full one-loop corrections have been provided in \cite{14}. The leading logarithmic two-loop effects obtained through renormalisation group equations \cite{15} have been completed by the genuine $\mathcal{O}(\alpha_t \alpha_s)$ \cite{16, 20}, $\mathcal{O}(\alpha_t^2)$ \cite{16, 19, 21}, $\mathcal{O}(\alpha_t \alpha_s)$ \cite{22} and $\mathcal{O}(\alpha_t \alpha_b + \alpha_b^2)$ \cite{23} two-loop corrections in the limit of zero external momentum. The electroweak two-loop effects including part of the external momentum dependence have been calculated \cite{24} and the leading three-loop contributions have been evaluated in \cite{25}. Also in the case of a CP-violating MSSM a great effort has been undertaken to calculate the higher-order corrections. After first investigations \cite{26}, they have been evaluated at one-loop order in the effective potential approach \cite{27} and with the renormalisation group improved effective potential method through next-to-leading order \cite{28}. In the Feynman diagrammatic approach the one-loop leading $m_t^2$ corrections have been provided in \cite{29}, a full one-loop calculation in \cite{30}, and more recently the leading two-loop contributions of $\mathcal{O}(\alpha_t \alpha_s)$ have been evaluated in \cite{31}. The corrections have been implemented in the public computer code \texttt{FeynHiggs} \cite{18, 30, 32}. \texttt{CPsuperH} \cite{33}, another public code, is based on the renormalisation group improved effective potential approach \cite{28}. The MSSM Higgs mass spectrum for real parameters can also be obtained from the spectrum calculators \cite{34–36}.

In the NMSSM, however, the higher-order calculations to the Higgs boson masses have not yet reached the same level of accuracy as in the MSSM. The leading one-loop contributions due to top/stop and bottom/sbottom loops have been calculated in the effective potential approach \cite{37}, the one-loop contributions due to chargino, neutralino and scalar loops have been evaluated in leading logarithmic order in Ref. \cite{38}. These corrections and the leading logarithmic two-loop terms of $\mathcal{O}(\alpha_t \alpha_s)$ and $\mathcal{O}(\alpha_t^2)$, taken over from the MSSM results, have been implemented in the public computer code \texttt{NMHDECAY} \cite{39}. Furthermore, the full one-loop contributions have been computed in the $\overline{\text{DR}}$ renormalisation scheme and the $\mathcal{O}(\alpha_t \alpha_s + \alpha_b \alpha_s)$ corrections have been provided in the approximation of zero external momentum \cite{40}. They have been implemented in \texttt{NMHDECAY} as well as in the spectrum calculator \texttt{SPheno} \cite{36}.

In this paper we complement the effort to reach a higher level of accuracy in the computation of the NMSSM Higgs boson masses by providing the full one-loop corrections in a mixed $\overline{\text{DR}}$–on-shell and in an on-shell (OS) renormalisation scheme. By comparison with the results of Ref. \cite{40} in the $\overline{\text{DR}}$ scheme, the dependence on the renormalisation schemes can be studied. In this way an estimate of the theoretical error due to missing higher-order corrections can be derived. Having available the corrections in the mixed and in the OS renormalisation scheme in addition enables a comparison of the NMSSM results in the MSSM limit with the corresponding MSSM corrections given in the OS scheme.

The paper is organised as follows. In section \ref{sec:2} we present the details of our calculation starting by introducing the NMSSM Higgs sector and setting up our notation in \ref{sec:2.1}. After presenting the chargino and neutralino sector in \ref{sec:2.2} we list the parameters, which we employ, in section \ref{sec:2.3}. The renormalisation is explained in detail in \ref{sec:2.4}. The explicit computation of the one-loop corrected Higgs boson masses and mixing matrix elements is described in \ref{sec:2.5}. Section \ref{sec:3} finally is devoted to the numerical analysis. Our results are summarised in \ref{sec:4}.
2 Calculation

2.1 The NMSSM Higgs Boson Sector

The Higgs mass matrix is obtained from the NMSSM Higgs potential, which is derived from the NMSSM superpotential \( W_{NMSSM} \), the corresponding soft SUSY breaking terms and the \( D \) term contributions. The NMSSM superpotential for the Higgs superfields \( \tilde{H}_u, \tilde{H}_d, \tilde{S} \) in our conventions reads

\[
W_{NMSSM} = W_{MSSM} - \epsilon_{ab} \lambda S \tilde{H}_d^a \tilde{H}_u^b + \frac{1}{3} \kappa S^3 ,
\]

where \( a, b = 1, 2 \) are the \( SU(2)_L \) fundamental representation indices and \( \epsilon_{ab} \) denotes the totally antisymmetric tensor with \( \epsilon_{12} = \epsilon^{12} = 1 \). The Higgs superfield which couples to up-type (down-type) fermion superfields is given by \( \tilde{H}_{u(d)} \). The parameters \( \lambda \) and \( \kappa \) are dimensionless and, working in the CP-invariant NMSSM, are chosen to be real. The first term in Eq. (1) is the MSSM superpotential, which in terms of the quark and lepton superfields \( \hat{Q}, \hat{U}^c, \hat{D}^c, \hat{L}, \hat{E}^c \) reads

\[
W_{MSSM} = \epsilon_{ab} [y_d \tilde{H}_d^a \tilde{L}^c + y_d \tilde{H}_d^a \tilde{Q}^b \tilde{D}^c - y_u \tilde{H}_u^a \tilde{Q}^b \tilde{U}^c] ,
\]

where we have omitted colour and generation indices. Note that the MSSM \( \mu \) term in NMSSM constructions is commonly assumed to be zero as well as terms linear and quadratic in \( \tilde{S} \). The soft SUSY breaking terms in the NMSSM in terms of the component fields \( H_u, H_d, S \) are given by

\[
\mathcal{L}_{soft} = \mathcal{L}_{soft, MSSM} - m_S^2 |S|^2 + (\epsilon_{ab} \lambda A_{\lambda} S \tilde{H}_d^a H_u^b - \frac{1}{3} \kappa A_{\kappa} S^3 + h.c.) ,
\]

with the MSSM soft SUSY breaking Lagrangian

\[
\mathcal{L}_{soft, MSSM} = -m_{H_d}^2 H_d^\dagger H_d - m_{H_u}^2 H_u^\dagger H_u - m_Q^2 \tilde{Q}^\dagger \tilde{Q} - m_{\tilde{L}}^2 \tilde{L}^\dagger \tilde{L} - m_{\tilde{U}}^2 \tilde{U}^\dagger \tilde{U} - m_{\tilde{D}}^2 \tilde{D}^\dagger \tilde{D} - m_{\tilde{E}}^2 \tilde{E}^\dagger \tilde{E} - (\epsilon_{ab} [y_d A_{\lambda} \tilde{H}_d^a \tilde{L}^c + y_d A_{\lambda} \tilde{H}_d^a \tilde{Q}^b \tilde{D}^c - y_u A_{\kappa} \tilde{H}_u^a \tilde{Q}^b \tilde{U}^c] + h.c.) - \frac{1}{2} (M_{\tilde{B}} \tilde{B} \tilde{B} + M_{\tilde{W}} \tilde{W} \tilde{W} + M_{\tilde{G}} \tilde{G} \tilde{G} + h.c. ,
\]

where in the first two lines tilde denotes the scalar component of the corresponding quark and lepton superfield, and \( \tilde{Q} = (\tilde{u}_L, \tilde{d}_L)^T, \tilde{L} = (\tilde{e}_L, \tilde{\nu}_L)^T \). Note that e.g. \( \tilde{u}_R^* \) is the scalar component of \( \tilde{U}^c \). In the last line the soft SUSY breaking gaugino mass terms for the gaugino fields \( \tilde{B}, \tilde{W}_i, \tilde{G} \) are given. All soft SUSY breaking trilinear couplings \( A_k \) \( (k = \lambda, \kappa, d, u, e) \) and gaugino mass parameters \( M_i \) \( (i = 1, 2, 3) \) are assumed to be real, and squark and slepton mixing between the generations is neglected. Furthermore, possible soft SUSY breaking terms linear and quadratic in the singlet field \( S \) are set to zero in accordance with the majority of phenomenological NMSSM constructions.\(^3\)

The neutral components of the Higgs fields can be parametrised in terms of CP-even and CP-odd fluctuations around their vacuum expectation values,

\[
H_u^1 = \frac{1}{\sqrt{2}} (v_u + h_u + i a_u), \quad H_u^2 = \frac{1}{\sqrt{2}} (v_u + h_u + i a_u), \quad S = \frac{1}{\sqrt{2}} (v_s + h_s + i a_s),
\]

\(^{2}\)The superscript c denotes charge conjugation.

\(^{3}\)In the MSSM there is an additional soft SUSY breaking term \( m_{\tilde{H}_d H_u}^2 \epsilon_{ab} \tilde{H}_d^a \tilde{H}_u^b \) with \( m_{\tilde{H}_d H_u}^2 \) usually expressed in terms of the soft SUSY breaking parameter \( B \) and the higgsino parameter \( \mu \) as \( m_{\tilde{H}_d H_u}^2 = B \mu \). This term is not explicitly added to the NMSSM soft SUSY breaking Lagrangian but is generated dynamically via the term \( \epsilon_{ab} A_{\lambda} S \tilde{H}_d^a \tilde{H}_u^b \).
with the VEVs $v_d, v_u, v_s$ chosen to be real and positive. The parameter $\mu$ then arises dynamically from the singlet field expanded about its VEV, cf. Eq. (10),

$$\mu = \frac{\lambda v_s}{\sqrt{2}}. \quad (6)$$

The minimisation conditions of the tree-level scalar potential can be applied to replace the soft SUSY breaking Higgs mass parameters $m_{H_u}^2, m_{H_d}^2, m_S^2$ by combinations of $\lambda, \kappa$, the electroweak gauge couplings $g$ and $g'$, the VEVs and the trilinear couplings $A_\lambda, A_\kappa$, so that the tree-level mass matrix $M_S^2$ of the neutral CP-even Higgs bosons obtained from the second derivative of the Higgs potential with respect to the fields in the vacuum, in the basis $h^S = (h_d, h_u, h_s)^T$ can be cast into the form

$$M_S^2 = \begin{pmatrix}
\lambda^2 - \bar{g}^2 & \lambda^2 & \lambda^2 \\
\lambda^2 & \lambda^2 & \lambda^2 \\
\lambda^2 & \lambda^2 & \lambda^2
\end{pmatrix} + \begin{pmatrix}
\frac{(R_\lambda + R v_s/2)v_s v_d}{v_d} & (R_\lambda + R v_s/2)v_s v_s - (R_\lambda + R v_s) v_d \\
(R_\lambda + R v_s/2)v_s v_s - (R_\lambda + R v_s)v_d & \frac{(R_\lambda + R v_s/2)v_s v_u}{v_u} \\
(R_\lambda + R v_s/2)v_s v_u & \frac{(R_\lambda + R v_s/2)v_s v_s}{v_s}
\end{pmatrix} \quad (7)$$

where we have defined

$$g^2 = \frac{1}{4} (g^2 + g'^2), \quad R = \lambda \kappa, \quad R_\lambda = \frac{1}{\sqrt{2}} \lambda A_\lambda, \quad R_\kappa = \frac{1}{\sqrt{2}} \kappa A_\kappa. \quad (8)$$

Note, that the MSSM limit can be recovered by $\lambda, \kappa \to 0$ (with the ratio $\kappa/\lambda$ kept constant for a smooth approach) and keeping the parameter $\mu = \lambda v_s/\sqrt{2}$ as well as the parameters $A_\lambda$ and $A_\kappa$ fixed. In this limit we hence have $v_s \to \infty$.

The CP-even mass eigenstates $H_i$ ($i = 1, 2, 3$) are obtained by an orthogonal transformation $R^S$ (the summation over paired indices is implicit),

$$H_i = R^S_{ij} h^S_j, \quad j = d, u, s. \quad (9)$$

They are ordered by ascending mass with $M_{H_1} \leq M_{H_2} \leq M_{H_3}$. The CP-odd fields $(a_d, a_u, a_s)$ can be rotated first to separate a massless Goldstone boson $G$

$$\begin{pmatrix}
a \\
da_u \\
a_s
\end{pmatrix} = \begin{pmatrix}
s_{\beta_n} & c_{\beta_n} & 0 \\
0 & 0 & 1 \\
c_{\beta_n} & -s_{\beta_n} & 0
\end{pmatrix} \begin{pmatrix}
a_d \\
a_u \\
a_s
\end{pmatrix}, \quad (10)$$

with the abbreviations $c_x \equiv \cos x$ and $s_x \equiv \sin x$, in the following also $t_x \equiv \tan x$. Starting from the tree-level CP-odd mass matrix squared in the basis $(a_d, a_u, a_s)^T$ and using Eq. (10) results in the mass matrix squared $M_P^2$ in the basis $h^P = (a, a_s, G)^T$, which reads

$$M_P^2 = \begin{pmatrix}
(2R_\lambda + R v_s) v_s^{\frac{\Delta \beta}{\Delta \beta}} & (R_\lambda - R v_s)v_s c_{\Delta \beta} & (R_\lambda + R v_s/2)v_s^{\frac{\Delta \beta}{\Delta \beta}} \\
(R_\lambda - R v_s)v_s c_{\Delta \beta} & \left(2R + R_\lambda/v_s\right) v_s^{\frac{\Delta \beta}{\Delta \beta}} - 3R_\kappa v_s & (R_\lambda - R v_s) v_s s_{\Delta \beta} \\
(R_\lambda + R v_s/2)v_s^{\frac{\Delta \beta}{\Delta \beta}} & (R_\lambda - R v_s) v_s s_{\Delta \beta} & (2R + R v_s) v_s^{\frac{\Delta \beta}{\Delta \beta}}
\end{pmatrix} \quad (11)$$
with $\Delta \beta = \beta - \beta_n$ and $v^2 \equiv v_u^2 + v_d^2$. The angle $\beta_n$ coincides at tree-level with the angle $\beta$ defined through the ratio of the VEVs $v_u, v_d$, $\tan \beta = v_u / v_d$. Hence $\Delta \beta = 0$ at tree-level, leading to a massless Goldstone boson, which decouples as it should. The first entry $(M_P^2)_{11}$ at tree-level,

$$(M_P^2)_{11} = \frac{(2R\lambda + Rv_s)v_s}{s_2\beta},$$

(12)

becomes the mass of the pseudoscalar Higgs boson in the MSSM limit\(^4\). Applying an orthogonal rotation $R^P$ to $h^P = (a, a_s, G)^T$, the CP-odd mass eigenstates $A_i \equiv A_1, A_2, G$ ($i = 1, 2, 3$) are obtained,

$$A_i = R^P_{ij} h^P_j,$$

(13)

where at tree-level $R^P_{33} = 1$ and $R^P_{3i} = R^P_{i3} = 0$ for $i \neq 3$. The pseudoscalar masses are ordered by ascending mass, $M_{A_1} \leq M_{A_2}$. The CP-even and CP-odd Higgs mass values squared are given by the eigenvalues of the respective mass matrices $M_S^2$ and $M_P^2$. Analytic expressions would be rather complicated if not expanded in special parameter regions as e.g. in Ref. [41], where a comprehensive investigation of the NMSSM Higgs boson sector has been performed. In our analysis the mass eigenvalues are derived numerically. The charged Higgs boson mass on the other hand takes a simple form after the massless charged Goldstone boson has been separated by an orthogonal rotation with a mixing angle $\beta_c$. At Born level we have $\beta_c = \beta$ and get

$$M_{H^\pm}^2 = M_W^2 + \sqrt{2} \lambda \frac{v_s}{s_2\beta} (A_\lambda + \kappa \frac{v_s}{\sqrt{2}}) - \frac{\lambda^2 v^2}{2} = M_W^2 + (M_P^2)_{11} - \frac{\lambda^2 v^2}{2},$$

(14)

with $M_W$ being the $W$ boson mass and where we have applied the definition given in Eq. (12).

### 2.2 The Neutralino and Chargino Sector

The chargino sector remains unchanged with respect to the MSSM. For completeness and to set up our notation we briefly repeat the chargino system. The chargino mass matrix depends on the wino mass parameter $M_2$, the effective Higgsino parameter $\mu = \lambda v_s / \sqrt{2}$ and the ratio of the vacuum expectation values $\tan \beta$. In the interaction eigenbasis it is given by [42]

$$M_C = \begin{pmatrix} M_2 & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{pmatrix}.$$

(15)

It is diagonalised by two real matrices $U$ and $V$,

$$U^* M_C V^\dagger \rightarrow U = O_+ \quad \text{and} \quad V = \begin{cases} O_+ & \text{if } \det M_C > 0 \\ \sigma_3 O_+ & \text{if } \det M_C < 0 \end{cases},$$

(16)

with the Pauli matrix $\sigma_3$ to render the chargino masses positive. The rotation matrices $O_{\pm}$ are given in terms of the mixing angles

$$\tan 2\theta_- = \frac{2\sqrt{2}M_W (M_2 c_\beta + \mu s_\beta)}{M_2^2 - \mu^2 - 2M_W^2 c_\beta}, \quad \tan 2\theta_+ = \frac{2\sqrt{2}M_W (M_2 s_\beta + \mu c_\beta)}{M_2^2 - \mu^2 + 2M_W^2 c_\beta},$$

(17)

\(^4\)A_\lambda takes the role of the soft SUSY-breaking parameter $B$ in the MSSM.
and the two chargino masses read

\[ m_{\tilde{\chi}^{\pm}}^2 = \frac{1}{2} \left\{ M_2^2 + \mu^2 + 2M_W^2 + [(M_2^2 - \mu^2)^2 + 4M_W^2(M_2^2 - c_{2\beta} + M_2^2 + \mu^2 + 2M_2\mu s_{2\beta})]^{\frac{1}{2}} \right\}. \] (18)

In the neutralino sector, the mixing of the fermionic component of the singlet superfields \( \tilde{S} \) with the neutral gauginos \( \tilde{B}, \tilde{W}_3 \) and higgsinos \( \tilde{H}_d^0, \tilde{H}_u^0 \) yields in the Weyl spinor basis \( \psi^0 = (\tilde{B}, \tilde{W}_3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S})^T \) the \( 5 \times 5 \) neutralino mass matrix

\[
M_N = \begin{pmatrix}
M_1 & 0 & -c_\beta s_W M_Z & s_\beta s_W M_Z & 0 \\
0 & M_2 & c_\beta c_W M_Z & -s_\beta c_W M_Z & 0 \\
-c_\beta s_W M_Z & c_\beta c_W M_Z & 0 & -\lambda v_s/\sqrt{2} & -\lambda v_u/\sqrt{2} \\
s_\beta s_W M_Z & -s_\beta c_W M_Z & -\lambda v_s/\sqrt{2} & 0 & -\lambda v_d/\sqrt{2} \\
0 & 0 & -\lambda v_u/\sqrt{2} & -\lambda v_d/\sqrt{2} & \sqrt{2}\kappa v_s
\end{pmatrix}. \] (19)

We have introduced the short-hand notation \( s_W \equiv \sin \theta_W \), \( c_W \equiv \cos \theta_W \) for the Weinberg angle \( \theta_W \), and \( M_Z \) denotes the Z boson mass. Diagonalisation with a unitary matrix \( \mathcal{N} \) yields the five neutralino mass eigenstates \( \chi_i^0 \) \( (i = 1, \ldots, 5) \),

\[
\chi_i^0 = \mathcal{N}_{ij} \psi_j^0, \quad j = 1, \ldots, 5.
\] (20)

They are ordered by ascending mass, \( m_{\chi_1^0} \leq \ldots \leq m_{\chi_5^0} \). With our assumption of CP invariance and allowing for negative neutralino mass eigenvalues, the mass matrix \( \mathcal{N} \) is real.

### 2.3 Parameter Basis

The NMSSM Higgs potential depends on 12 independent parameters in the CP-conserving case. They are given by the soft SUSY breaking mass parameters \( m_{H_u}^2, m_{H_d}^2, m_S^2 \), the gauge couplings \( g, g' \), the vacuum expectation values \( v_u, v_d, v_s \), the dimensionless parameters \( \lambda, \kappa \) and the soft SUSY breaking couplings \( A_\lambda, A_s \). For the physical interpretation it is convenient to replace some of these parameters: The minimisation of the Higgs potential \( V \) requires the terms linear in the Higgs fields to vanish in the vacuum. Hence for the scalar fields,

\[
\left\langle \frac{\partial V}{\partial h_u} \right\rangle = \left\langle \frac{\partial V}{\partial h_d} \right\rangle = \left\langle \frac{\partial V}{\partial h_s} \right\rangle = 0,
\] (21)

where the brackets denote the vacuum. The corresponding coefficients, which are called tadpoles, therefore have to be zero. At lowest order the tadpole conditions for the CP-even fields read\(^5\)

\[
\left\langle \frac{\partial V}{\partial h_d} \right\rangle \equiv t_{h_d} = v_dm_{H_d}^2 - R_\lambda v_u v_s + \frac{g^2 + g'^2}{8} v_d(v_d^2 - v_u^2) + \frac{\lambda^2}{2} v_d(v_u^2 + v_s^2) - \frac{R}{2} v_d v_s^2 = 0 \quad (22)
\]

\[
\left\langle \frac{\partial V}{\partial h_u} \right\rangle \equiv t_{h_u} = v_u m_{H_u}^2 - R_\lambda v_d v_s + \frac{g^2 + g'^2}{8} v_u(v_u^2 - v_d^2) + \frac{\lambda^2}{2} v_u(v_d^2 + v_s^2) - \frac{R}{2} v_d v_s^2 = 0 \quad (23)
\]

\[
\left\langle \frac{\partial V}{\partial h_s} \right\rangle \equiv t_{h_s} = v_s m_S^2 - R_\lambda v_d v_u + R_\kappa v_s^2 + \frac{\lambda^2}{2} v_s(v_d^2 + v_u^2) + \kappa^2 v_s^3 - R v_d v_u v_s = 0. \quad (24)
\]

\(^5\)As we work in the real NMSSM the derivatives of the Higgs potential with respect to the CP-odd fields are zero and no additional conditions have to be required.
Equations (22, 23, 24) can be exploited to replace $m_{H_u}^2, m_{H_d}^2, m_S^2$ by the tadpole parameters $t_h, t_{h_s}$ and $t_{h_b}$. The parameters $g, g', v_u, v_d$ are replaced by the electric charge $e$, the gauge boson masses $M_W, M_Z$ and the ratio of the vacuum expectation values $\tan \beta \equiv v_u/v_d$ through the relations

$$
g = \frac{eM_Z}{\sqrt{M_Z^2 - M_W^2}} \quad g' = \frac{eM_Z}{M_W},$$

$$
v_u = \sqrt{\frac{M_Z^2 - M_W^2}{1 + t^2_\beta} M_W t_\beta \ v_s} \quad v_d = \sqrt{\frac{M_Z^2 - M_W^2}{1 + t^2_\beta} M_W e M_Z}.
$$

Finally, $A_\lambda$ is replaced by the charged Higgs boson mass $M_{H\pm}^2$,

$$
A_\lambda = \frac{s^2 \beta}{\sqrt{2}} \lambda v_s \left( \frac{v^2 \lambda^2}{2} - M_W^2 + \frac{M_{H\pm}^2}{c^2_\Delta \beta} \right) \ - \frac{\kappa v_s}{\sqrt{2} \lambda v_s} \left( \frac{s^2 \beta}{c^2_\Delta \beta} t_{h_d} + \frac{c^2 \beta}{c^2_\Delta \beta} t_{h_u} \right)
$$

with

$$
v = \frac{2M_W}{e} \sqrt{1 - \frac{M_W^2}{M_Z^2}},
$$

where we have kept the dependence on the tadpole parameters and the mixing angles $\beta_c = \beta_n \equiv \beta_B$. At tree-level they coincide with $\beta$. It is the angle $\beta$ which will be needed for the renormalisation: According to our renormalisation schemes, presented in section 2.4, the renormalisation procedure can be performed before the transformation into mass eigenstates. This means that the mixing matrices, also those separating the neutral and charged Goldstone bosons from the Higgs bosons and hence the angles $\beta_n$ and $\beta_c$, which appear in these matrices, do not receive counterterms. Therefore, special care has to be taken to determine the elements of the mass matrices without inserting the tree-level relation $\beta_B = \beta$ and to apply the renormalisation procedure uniquely for $\tan \beta$ given by the ratio of $v_u$ and $v_d$.

To summarise, we work with the following parameter set

$$
t_{h_u}, t_{h_d}, t_{h_s}, e, \ M_W^2, M_Z^2, \tan \beta, M_{H\pm}^2, \lambda, \kappa, v_s, A_\kappa.
$$

2.4 Renormalisation Schemes

For the determination of the loop-corrected Higgs boson masses the Higgs self-energies have to be calculated. They develop ultraviolet (UV) divergences. Evaluating the self-energies in $D = 4 - 2\epsilon$ dimensions, the divergences can be parametrised by $1/\epsilon$ leading to poles in $D = 4$ dimensions. To get a finite result the parameters entering the loop calculation have to be renormalised by the introduction of appropriate counterterms absorbing the UV divergences. Note, that at higher orders also the terms linear in the Higgs fields get loop contributions. Therefore, also the tadpole parameters $t_{h_i} \ (i = u, d, s)$ have to be renormalised, in order to fulfill the tadpole conditions Eq. (21). Before we describe the determination of the one-loop corrected Higgs boson masses in section 2.5 the renormalisation schemes which have been adopted, shall be presented in detail in the following.

We start with a renormalisation scheme which is a mixture between on-shell and DR renormalisation conditions. In order to make contact and to compare to earlier results presented in Ref. [40]..
our result will also be converted to a pure DR scheme. To get an estimate of the uncertainties due to missing higher-order corrections we study the influence of the renormalisation scheme on the one-loop Higgs mass corrections. We therefore also compare to a pure OS scheme. In all three renormalisation schemes, mixed, DR and OS, we start out from the parameter set Eq. (29) and replace the parameters by the renormalised ones and their corresponding counterterms,

\[

d_{H} \rightarrow d_{H} + \delta d_{H},
\]

\[

M_{Z}^{2} \rightarrow M_{Z}^{2} + \delta M_{Z}^{2},
\]

\[

M_{W}^{2} \rightarrow M_{W}^{2} + \delta M_{W}^{2},
\]

\[

M_{H}^{2} \rightarrow M_{H}^{2} + \delta M_{H}^{2},
\]

\[

e_{e} \rightarrow (1 + \delta Z_{e}) e,\quad \tan \beta \rightarrow \tan \beta + \delta \tan \beta,
\]

\[

\lambda \rightarrow \lambda + \delta \lambda,\quad \kappa \rightarrow \kappa + \delta \kappa,
\]

\[

v_{s} \rightarrow v_{s} + \delta v_{s},\quad A_{\kappa} \rightarrow A_{\kappa} + \delta A_{\kappa} .
\]

For the field renormalisation, the Higgs boson doublet and singlet fields are replaced by the renormalised ones and a corresponding single field renormalisation constant for each doublet and the singlet, respectively,

\[

H_{u} \rightarrow \sqrt{Z_{H_{u}}} H_{u} = \left(1 + \frac{1}{2} \delta Z_{H_{u}}\right) H_{u},
\]

\[

H_{d} \rightarrow \sqrt{Z_{H_{d}}} H_{d} = \left(1 + \frac{1}{2} \delta Z_{H_{d}}\right) H_{d},
\]

\[

S \rightarrow \sqrt{Z_{S}} S = \left(1 + \frac{1}{2} \delta Z_{S}\right) S .
\]

Applying this renormalisation procedure, the renormalised self-energies in the basis \( \{ h_{i}^{S} | i = 1, 2, 3 \} = \{ d_{u}, h_{u}, h_{s} \} \) for the CP-even Higgs bosons and in the basis \( \{ h_{i}^{P} | i = 1, 2, 3 \} = \{ a, a_{s}, G \} \) for the CP-odd ones can be derived as \( (X = S, P) \)

\[

\tilde{\Sigma}_{h_{i}^{S} h_{j}^{S}}(k^{2}) = \Sigma_{h_{i}^{S} h_{j}^{S}}(k^{2}) + \frac{1}{2} k^{2} [\delta Z_{h_{i}^{S} h_{j}^{S}} + \delta Z_{h_{j}^{S} h_{i}^{S}}] - \frac{1}{2} [\delta Z_{h_{i}^{S} h_{j}^{S}} (M_{X}^{2})_{ij} + (M_{X}^{2})_{ik} \delta Z_{h_{k}^{S} h_{j}^{S}}] - \delta (M_{X}^{2})_{ij} \quad \text{with} \quad i, j = 1, 2, 3 ,
\]

where

\[

\delta Z_{h_{1}^{S} h_{1}^{S}} = \delta Z_{H_{u}}, \quad \delta Z_{h_{2}^{S} h_{2}^{S}} = \delta Z_{H_{d}}, \quad \delta Z_{h_{3}^{S} h_{3}^{S}} = \delta Z_{S}, \quad \delta Z_{h_{i}^{S} h_{j}^{S}} = 0 \quad \text{for} \quad i \neq j ,
\]

and

\[

\delta Z_{h_{1}^{P} h_{1}^{P}} = s_{\beta}\delta Z_{H_{u}} + c_{\beta}\delta Z_{H_{d}}, \quad \delta Z_{h_{2}^{P} h_{2}^{P}} = \delta Z_{S}, \quad \delta Z_{h_{3}^{P} h_{3}^{P}} = c_{\beta}\delta Z_{H_{d}} + s_{\beta}\delta Z_{H_{u}} ,
\]

\[

\delta Z_{h_{1}^{P} h_{2}^{P}} = s_{\beta}c_{\beta}(\delta Z_{H_{u}} - \delta Z_{H_{d}}) ,
\]

\[

\delta Z_{h_{2}^{P} h_{3}^{P}} = s_{\beta}c_{\beta}(\delta Z_{H_{u}} - \delta Z_{H_{d}}) .
\]

Here, \( M_{S}^{2} \) and \( M_{P}^{2} \) are the tree-level CP-even and CP-odd Higgs boson mass matrices squared, respectively. For the derivation of the counterterm matrices \( \delta (M_{S}^{2}) \) and \( \delta (M_{P}^{2}) \), the CP-even and CP-odd Higgs boson mass matrices squared are expressed in terms of the parameter set Eq. (29) including also their dependence on the tadpole parameters as well as on the mixing angle \( \beta_{B} \). As the expressions are quite lengthy they are given in Appendix A. Then, the parameters entering
these matrices are replaced according to Eqs. (30) and an expansion about the counterterms is performed. The part of $M^2_S$ and $M^2_p$, respectively, which is linear in the counterterms, corresponds to $\delta(M^2_S)$ and $\delta(M^2_p)$.

The renormalised self energies in Eq. (32) are related to the ones in the basis of the mass eigenstates through, cf. Eqs. (31,33),

$$
\hat{\Sigma}_{H,H_i}(k^2) = \mathcal{R}^S_i \mathcal{R}^S_j \hat{\Sigma}_{h_i^R h_j^R}(k^2) \quad (37)
$$

$$
\hat{\Sigma}_{A,H_i}(k^2) = \mathcal{R}^P_i \mathcal{R}^P_j \hat{\Sigma}_{h_i^P h_j^P}(k^2) \quad \text{with} \quad i, j, k, l = 1, 2, 3 \quad (38)
$$

The field renormalisation constants $\delta Z_{H_d}, \delta Z_{H_u}, \delta Z_S$ are obtained from

$$
\delta Z_{H,H_i} = |R^S_{i1}|^2 \delta Z_{H_d} + |R^S_{i2}|^2 \delta Z_{H_u} + |R^S_{i3}|^2 \delta Z_S, \quad i = 1, 2, 3 \quad (39)
$$

with

$$
\delta Z_{H,H_i} = \frac{\partial \Sigma_{H,H_i}(k^2)}{\partial k^2} \Big|_{k^2=(M^0_H_i)^2}, \quad i = 1, 2, 3 \quad (40)
$$

where $(M^0_H_i)^2$ denotes the corresponding tree-level mass squared. The field renormalisation constants are defined in all three renormalisation schemes via DR conditions. This is indicated by the superscript 'div' and means that in the field renormalisation only the divergent part $\Delta = 2/(4-D) - \gamma_E + \ln(4\pi)$ is kept with $\gamma_E$ being the Euler constant. Solving Eq. (39) for $\delta Z_{H_d}, \delta Z_{H_u}, \delta Z_S$ results in

$$
\delta Z_{H_d} = \left[ (|R^S_{23}|^2 |R^S_{32}|^2 - |R^S_{22}|^2 |R^S_{33}|^2) \delta Z_{H_d,H_1} + \left( |R^S_{12}|^2 |R^S_{13}|^2 - |R^S_{13}|^2 |R^S_{33}|^2 \right) \delta Z_{H_d,H_2} \right. \\
\left. + \left( |R^S_{12}|^2 |R^S_{23}|^2 - |R^S_{12}|^2 |R^S_{23}|^2 \right) \delta Z_{H_d,H_3} \right] / R^S_r \\
\delta Z_{H_u} = \left[ (|R^S_{23}|^2 |R^S_{31}|^2 - |R^S_{23}|^2 |R^S_{33}|^2) \delta Z_{H_u,H_1} + \left( |R^S_{11}|^2 |R^S_{12}|^2 - |R^S_{12}|^2 |R^S_{32}|^2 \right) \delta Z_{H_u,H_2} \right. \\
\left. + \left( |R^S_{11}|^2 |R^S_{23}|^2 - |R^S_{11}|^2 |R^S_{23}|^2 \right) \delta Z_{H_u,H_3} \right] / R^S_r \\
\delta Z_S = \left[ (|R^S_{22}|^2 |R^S_{31}|^2 - |R^S_{22}|^2 |R^S_{32}|^2) \delta Z_{H_d,H_1} + \left( |R^S_{11}|^2 |R^S_{12}|^2 - |R^S_{12}|^2 |R^S_{32}|^2 \right) \delta Z_{H_u,H_2} \right. \\
\left. + \left( |R^S_{11}|^2 |R^S_{21}|^2 - |R^S_{11}|^2 |R^S_{21}|^2 \right) \delta Z_{H_u,H_3} \right] / R^S_r \\
$$

where

$$
R^S_r = -|R^S_{11}|^2 |R^S_{22}|^2 |R^S_{33}|^2 + |R^S_{11}|^2 |R^S_{23}|^2 |R^S_{32}|^2 + |R^S_{12}|^2 |R^S_{21}|^2 |R^S_{33}|^2 \\
- |R^S_{11}|^2 |R^S_{23}|^2 |R^S_{31}|^2 - |R^S_{12}|^2 |R^S_{21}|^2 |R^S_{32}|^2 + |R^S_{12}|^2 |R^S_{22}|^2 |R^S_{33}|^2 \
$$

Contrary to the field renormalisation constants the renormalisation conditions for the remaining parameters are different in the three chosen renormalisation schemes, as will be described in the following.

**Mixed renormalisation scheme**

In the mixed renormalisation scheme we divide the parameters into parameters defined through on-shell conditions and into parameters defined via DR conditions:

$$
\begin{align*}
&M_Z, M_W, M_{H^\pm}, t_{h_u}, t_{h_d}, t_{h_s}, e, \tan \beta, \lambda, v_s, \kappa, A_{\kappa} \\
on-shell scheme \quad &\text{DR scheme}
\end{align*}
\quad (45)
$$

\(^8\)In slight abuse of the language we also call the renormalisation conditions for the tadpole parameters on-shell.
In the following, the various counterterms shall be specified in more detail.

(i,ii) Gauge boson masses

The gauge boson masses are defined through on-shell conditions,

$$\text{Re} \Sigma^T_{ZZ}(M_Z^2) = 0, \quad \text{Re} \Sigma^T_{WW}(M_W^2) = 0,$$

where $T$ denotes the transverse part of the respective self-energy. For the mass counterterms this yields

$$\delta M_Z^2 = \text{Re} \Sigma^T_{ZZ}(M_Z^2), \quad \delta M_W^2 = \text{Re} \Sigma^T_{WW}(M_W^2).$$

Note, that the NMSSM gauge boson self-energies differ from the MSSM case due to the introduction of an additional superfield $\hat{S}$.

(iii) Mass of the charged Higgs boson

The mass of the charged Higgs boson is determined through the on-shell condition,

$$\text{Re} \Sigma_{H^+H^-}(M_{H^\pm}^2) = 0,$$

resulting in the corresponding counterterm

$$\delta M_{H^\pm}^2 = \text{Re} \Sigma_{H^+H^-}(M_{H^\pm}^2).$$

(iv-vi) Tadpole parameters

The tadpole coefficients are required to vanish also at one-loop order, yielding

$$t_{h_i}^{(1)} - \delta t_{h_i} = 0, \quad i = d, u, s,$$

where $t_{h_i}^{(1)}$ stands for the contributions coming from the corresponding genuine Higgs boson tadpole graphs. As the tadpole graphs are calculated in the mass eigenstate basis, they have to be transformed to the interaction basis. Applying Eq. (9) we have

$$\delta t_{h_i} = R_{jH_i}^S t_{H_j}^{(1)}, \quad i = d, u, s, \quad j = 1, 2, 3.$$

(vii) Electric charge

The electric charge is defined to be the full electron-positron photon coupling for on-shell external particles in the Thomson limit, so that all corrections to this vertex vanish on-shell and for zero momentum transfer. The counterterm for the electric charge is then given in terms of the transverse part of the photon-photon and photon-Z self-energies

$$\delta Z_e = \frac{1}{2} \frac{\partial \Sigma^T_{\gamma\gamma}(k^2)}{\partial k^2} \bigg|_{k^2=0} + \frac{s_W \Sigma^T_{\gamma Z}(0)}{c_W M_Z^2}.$$

Note that the sign of the second term in Eq. (52) differs from the one in [43] due to our conventions in the Feynman rules.
For the renormalisation of \( \tan \beta \) we adopt the \( \overline{\text{DR}} \) scheme. Applying Eq. (31) and
\[
v_i \rightarrow v_i + \delta v_i \quad i = u, d
\]
results in
\[
\delta \tan \beta = \tan \beta \left[ \frac{1}{2} (\delta Z_{H_u} - \delta Z_{H_d}) + \left( \frac{\delta v_u}{v_u} - \frac{\delta v_d}{v_d} \right) \right]_{\text{div}} = \left[ \frac{\tan \beta}{2} (\delta Z_{H_u} - \delta Z_{H_d}) \right]_{\text{div}},
\]
where we have used in the last step \( \delta v_u/v_u \big|_{\text{div}} = \delta v_d/v_d \big|_{\text{div}} \). The field renormalisation constants \( \delta Z_{H_d} \) and \( \delta Z_{H_u} \) are given in Eqs. (41, 42).

In the mixed renormalisation scheme \( \lambda \) is defined as a \( \overline{\text{DR}} \) parameter. The counterterm is determined via the renormalised self-energy \( \hat{\Sigma}_{h^+_i h^-_i} \), see Eq. (32), using that
\[
\left. \hat{\Sigma}_{h^+_i h^-_i} \right|_{\text{div}} = 0 \iff \delta (M_{h^+_i}^2)_{11} = \Sigma_{h^+_i h^-_i} \left( (M_{h^+_i}^2)_{11} \right)_{\text{div}}.
\]
As \( \delta (M_{h^+_i}^2)_{11} \) contains the counterterm \( \delta \lambda \), Eq. (55) can be solved for \( \delta \lambda \) resulting in,
\[
\delta \lambda = \frac{e^2}{4 \lambda M_W^2 s_W^2} \left[ \Sigma_{h^+_i h^-_i} \left( (M_{h^+_i}^2)_{11} \right) - \delta M_{h^+_i}^2 + \delta M_{h^-_i}^2 \left( 1 + \frac{2 \lambda^2 (c_w^2 - s_w^2)}{e^2} \right) - \frac{2 \lambda^2 c_w^4}{e^2} \delta M_Z^2 \right]_{\text{div}}.
\]
The self-energy \( \Sigma_{h^+_i h^-_i} \) is obtained from the self-energies in the mass eigenstate basis \( \Sigma_{A_i A_j} \) (\( i, j = 1, 2, 3 \)) through
\[
\Sigma_{h^+_i h^-_i} = \mathcal{R}_{i1} \Sigma_{A_i A_j} \mathcal{R}_{j1}^T.
\]

The vacuum expectation value \( v_s \) of the singlet field is renormalised in the \( \overline{\text{DR}} \) scheme. The counterterm is derived by exploiting the chargino sector. In fact, the lower right entry of the chargino mass matrix \( M_C \) in the interaction basis reads, \( \text{cf.} \) Eq. (15),
\[
(M_C)_{22} = \frac{\lambda v_s}{\sqrt{2}}.
\]
Applying Eq. (58), expanding around the counterterms and extracting the terms linear in the counterterms yields
\[
\delta v_s = \left[ \frac{\sqrt{2}}{\lambda} \delta (M_C)_{22} - v_s \frac{\delta \lambda}{\lambda} \right]_{\text{div}},
\]
\[^a\text{Compare with Eq. (12).}\]
with $\delta \lambda$ given by Eq. (56). The counterterm $\delta (M_C)_{22}$ is obtained from the renormalised chargino self-energies in the following way. Defining the general structure of a fermionic self-energy as,

$$\Sigma_{ij}(k^2) = \delta \Sigma_{ij}^L(k^2) \mathcal{P}_L + \delta \Sigma_{ij}^R(k^2) \mathcal{P}_R + \Sigma_{ij}^{LS}(k^2) \mathcal{P}_L + \Sigma_{ij}^{RS}(k^2) \mathcal{P}_R,$$

(60)

where $\mathcal{P}_{LR} = (\mathds{1} \mp \gamma_5)/2$ are the left- and right-handed projectors, we use the condition

$$\left[(M_C)_{22} [V^T \Sigma^L_{\chi^\pm}(k^2) V + U^T \Sigma^R_{\chi^\pm}(k^2) U^*]_{22} + [U^T \Sigma^L_{\chi^\pm}(k^2) V + V^T \Sigma^R_{\chi^\pm}(k^2) U^*]_{22}\right]_{\text{div}} = 0.$$  

(61)

Note that $\hat{\Sigma}_{\chi^\pm}$ is a $2 \times 2$ matrix with the entries given by the renormalised self-energies of the charginos in the mass eigenbasis. The renormalised self-energies in terms of the unrenormalised ones, the mass counterterms and field renormalisation constants are given in Appendix B, Eqs. (122)-(125). The structure of the condition Eq. (61) has been chosen such that the (divergent) contributions of the chargino field renormalisation constants drop out. Replacing the renormalised self-energy by the relations (122)-(125) leads to the counterterm $\delta (M_C)_{22}$,

$$\delta (M_C)_{22} = \frac{1}{2} \left[(M_C)_{22} [V^T \Sigma^L_{\chi^\pm}(k^2) V + U^T \Sigma^R_{\chi^\pm}(k^2) U^*]_{22} + [U^T \Sigma^L_{\chi^\pm}(k^2) V + V^T \Sigma^R_{\chi^\pm}(k^2) U^*]_{22}\right]_{\text{div}}.$$  

(62)

Note that the $k^2$ dependence in Eq. (62) drops out as only the divergent part is taken.

\[\text{Coupling } \kappa\]

The counterterm for the DR renormalised parameter $\kappa$ is derived from the neutralino sector in an analogous procedure as the chargino sector was exploited to determine $\delta v_s$. The lower right entry $(M_N)_{55}$ of the neutralino mass matrix Eq. (19) reads

$$(M_N)_{55} = \sqrt{2} \kappa v_s,$$

(63)

leading to the counterterm $\delta \kappa$,

$$\delta \kappa = \frac{1}{\sqrt{2} v_s} \delta (M_N)_{55} = \kappa \frac{\delta v_s}{v_s}.$$  

(64)

The determination of $\delta v_s$ has been described in the previous paragraph. For the determination of $\delta (M_N)_{55}$ we use the condition

$$\left[(M_N)_{55} [\mathcal{N}^T (\Sigma^L_{\chi^0}(k^2) + \hat{\Sigma}^R_{\chi^0}(k^2)) \mathcal{N}]_{55} + [\mathcal{N}^T (\Sigma^L_{\chi^0}(k^2) + \hat{\Sigma}^R_{\chi^0}(k^2)) \mathcal{N}]_{55}\right]_{\text{div}} = 0,$$

(65)

where the fermionic self-energy structure Eq. (61) has been applied for the decomposition of the renormalised neutralino $5 \times 5$ self-energy matrix $\hat{\Sigma}_{\chi^0}$. Once again the condition has been chosen such that the divergent parts of the field renormalisation constants cancel in Eq. (65). Rewriting the equation in terms of the unrenormalised self-energies, cf. Eqs. (126)-(129), yields

$$\delta (M_N)_{55} = \frac{1}{2} \left[(M_N)_{55} [\mathcal{N}^T (\Sigma^L_{\chi^0}(k^2) + \Sigma^R_{\chi^0}(k^2)) \mathcal{N}]_{55} + [\mathcal{N}^T (\Sigma^L_{\chi^0}(k^2) + \Sigma^R_{\chi^0}(k^2)) \mathcal{N}]_{55}\right]_{\text{div}},$$

(66)

which is inserted in Eq. (64) to determine $\delta \kappa$. 

\footnote{The decomposition can be applied both for unrenormalised self-energies $\Sigma$ and renormalised self-energies $\hat{\Sigma}$.}
(xii) Trilinear coupling $A_\kappa$

The trilinear coupling $A_\kappa$ is also defined as a DR parameter. For the derivation of the counterterm we use that
\[
\hat{\Sigma}_{h_2^P h_2^P} \left( (M_P^2)_{22} \right) \bigg|_{\text{div}} = 0 .
\] (67)

Equation (67) depends on $\delta A_\kappa$ via the mass matrix squared counterterm $\delta (M_P^2)_{22}$. Solving for $\delta A_\kappa$ we have
\[
\delta A_\kappa = \left[ -\frac{\sqrt{2}}{3\kappa v_s} \Sigma_{h_2^P h_2^P} \left( (M_P^2)_{22} \right) - \delta f \right] - A_\kappa \left[ \frac{\delta \kappa}{\kappa} + \frac{\delta v_s}{v_s} \right] \bigg|_{\text{div}} .
\] (68)

The counterterm $\delta f$ is derived from
\[
f = \frac{t_{h_u}}{v_s} - \frac{2M_W s_W s_\beta c_\beta^2}{e v_s^2 \Delta_\beta} [t_{h_u} + t_{h_d} t_{\beta_B}^2] + \frac{M_W^2 s_W^2 s_\beta^2}{e^2 v_s^2 \Delta_\beta} [M_{H^\pm}^2 - M_W^2 c_\beta^2]
+ \frac{\lambda M_W^2 s_W^2 s_\beta^2}{e^4 v_s^2} [2\lambda M_W^2 s_W^2 s_\beta^2 + 3\kappa e^2 v_s^2] ,
\] (69)

with $\Delta_\beta = \beta - \beta_B$. This is done by replacing the parameters $t_{h_u}, t_{h_d}, t_{h_s}, e, M_Z, M_W, M_{H^\pm}, \tan \beta, \lambda, \kappa,$ and $v_s$ according to Eq. (30), performing an expansion about the counterterms, extracting the part linear in the counterterms and finally applying the tree-level relations for the tadpole parameters $t_{h_u} = t_{h_d} = t_{h_s} = 0$ and for the mixing angle, $\beta_B = \beta$.

The self-energy $\Sigma_{h_2^P h_2^P}$ in terms of the corresponding self-energies in the mass eigenbasis is given by
\[
\Sigma_{h_2^P h_2^P} = \mathcal{R}_{i2}^P \Sigma_{A_i A_j} \mathcal{R}_{j2}^P , \quad i, j = 1, 2, 3 .
\] (70)

Alternatively, we could have derived the counterterms $\delta v_s$ and $\delta \kappa$ from the Higgs sector instead of resorting to the chargino and neutralino sector. We have explicitly verified that this leads to the same results for the one-loop corrected Higgs boson masses. Our choice of renormalisation allows a non-trivial cross-check of the renormalisation procedure. Moreover, it paves the way for an extension of the one-loop corrections to the Higgs decays into charginos and neutralinos.

**On-shell renormalisation scheme**

In this renormalisation scheme we keep the conditions (i)-(viii) ($\tan \beta$ is still renormalised in the DR scheme) but the parameters $v_s, \lambda, \kappa$ and $A_\kappa$ are determined via on-shell renormalisation conditions. This is done by applying on-shell renormalisation conditions on the two CP-odd Higgs boson mass eigenstates $A_1, A_2$, on the mass eigenstates of the two charginos and on the mass eigenstates of the two lightest neutralinos. As OS conditions are imposed on the mass eigenstates not only single elements of the counterterm mass matrices occur in the conditions, in contrast to the DR conditions exploited within the mixed scheme. Thus, the equations get more involved and include also the parameters $M_1, M_2$. This is why six renormalisation conditions are required, although in the end the counterterms for $M_1, M_2$ are not needed.

Requiring on-shell masses for the CP-odd Higgs bosons as well as for the two charginos and for
the lightest and next-to-lightest neutralinos leads to the following relations,

\[
\left( \mathcal{R}^P \delta(M_2^2) (\mathcal{R}^P)^T \right)_{11} = \Re \Sigma_{A_1} \delta_M^{(0)}(M_{A_1}^{(0)})^2, \quad \left( \mathcal{R}^P \delta(M_2^2) (\mathcal{R}^P)^T \right)_{22} = \Re \Sigma_{A_2} \delta_M^{(0)}(M_{A_2}^{(0)})^2, \\
\left( U^* \delta(M_C) V^* \right)_{11} = \frac{1}{2} \Re \left[ m_{\chi_1^+} (\Sigma_{\chi_1^+}^L (m_{\chi_1^+}^2) + \Sigma_{\chi_1^+}^R (m_{\chi_1^+}^2)) + \Sigma_{\chi_1^0}^L (m_{\chi_1^0}^2) + \Sigma_{\chi_1^0}^R (m_{\chi_1^0}^2) \right], \\
\left( U^* \delta(M_C) V^* \right)_{22} = \frac{1}{2} \Re \left[ m_{\chi_2^+} (\Sigma_{\chi_2^+}^L (m_{\chi_2^+}^2) + \Sigma_{\chi_2^+}^R (m_{\chi_2^+}^2)) + \Sigma_{\chi_2^0}^L (m_{\chi_2^0}^2) + \Sigma_{\chi_2^0}^R (m_{\chi_2^0}^2) \right], \\
\left( N^* \delta(M_N) N^T \right)_{11} = \frac{1}{2} \Re \left[ m_{\chi_1^0} (\Sigma_{\chi_1^0}^L (m_{\chi_1^0}^2) + \Sigma_{\chi_1^0}^R (m_{\chi_1^0}^2)) + \Sigma_{\chi_1^0}^L (m_{\chi_1^0}^2) + \Sigma_{\chi_1^0}^R (m_{\chi_1^0}^2) \right], \\
\left( N^* \delta(M_N) N^T \right)_{22} = \frac{1}{2} \Re \left[ m_{\chi_2^0} (\Sigma_{\chi_2^0}^L (m_{\chi_2^0}^2) + \Sigma_{\chi_2^0}^R (m_{\chi_2^0}^2)) + \Sigma_{\chi_2^0}^L (m_{\chi_2^0}^2) + \Sigma_{\chi_2^0}^R (m_{\chi_2^0}^2) \right],
\]

where \((M_{A_{1,2}}^{(0)})^2\) are the tree-level masses squared of the pseudoscalar mass eigenstates \(A_1, A_2\). The counterterm matrix \(\delta(M_C^2)\) is derived as described above. The entries \(\delta(M_C)_{ij} (i, j = 1, 2)\) of the counterterm matrix \(\delta(M_C)\) for the charginos read

\[
\delta M_{C11} = \delta M_2 \\
\delta M_{C12} = \delta M_{W}^2 \frac{s_{\beta}}{\sqrt{2} M_W} + \tan \beta \sqrt{2} c_{\beta} M_W \\
\delta M_{C21} = \delta M_{W}^2 \frac{c_{\beta}}{\sqrt{2} M_W} - \tan \beta \sqrt{2} c_{\beta} M_W s_{\beta} \\
\delta M_{C22} = \delta \lambda \frac{v_3}{\sqrt{2}} + \delta v_3 \frac{\lambda}{\sqrt{2}}.
\]

And finally, the entries \(\delta(M_N)_{ij} = \delta(M_N)_{ji} (i, j = 1...5)\) of the counterterm matrix \(\delta(M_N)\) for the neutralinos can be cast into the form

\[
\delta M_{N11} = \delta M_1 \\
\delta M_{N13} = (\delta M_{W}^2 - \delta M_{Z}^2) \frac{c_{\beta}}{2 M_Z s_{\beta}} + \tan \beta \frac{c_{\beta} M_Z s_{\beta} s_{2\beta}}{2} \\
\delta M_{N14} = (\delta M_{W}^2 - \delta M_{W}^2) \frac{s_{\beta}}{2 M_Z s_{\beta}} + \frac{c_{\beta} M_Z s_{W} s_{2\beta}}{c_W} \\
\delta M_{N22} = \delta M_2 \\
\delta M_{N23} = \delta M_{W}^2 \frac{c_{\beta}}{2 M_W} - \tan \beta \frac{c_{\beta}^2 M_W s_{\beta}}{e} \\
\delta M_{N24} = -\delta M_{W}^2 \frac{s_{\beta}}{2 M_W} - \tan \beta \frac{c_{\beta} M_W s_{\beta}}{e} \\
\delta M_{N34} = -\delta \lambda \frac{v_3}{\sqrt{2}} - \delta v_3 \frac{\lambda}{\sqrt{2}} \\
\delta M_{N35} = -c_W^2 \delta M_{Z}^2 - 2c_W^2 \delta M_{W}^2 + \delta M_{W}^2 \frac{\lambda s_{\beta}}{\sqrt{2} e M_W s_{W}} - \frac{\sqrt{2} \lambda c_{\beta} M_W s_{\beta}}{e} \\
+ (\lambda \delta Z_e - \delta \lambda) \frac{\sqrt{2} M_W s_{W} s_{\beta}}{e} \\
\delta M_{N45} = -c_W^2 \delta M_{Z}^2 - 2c_W^2 \delta M_{W}^2 + \delta M_{W}^2 \frac{\lambda c_{\beta}}{\sqrt{2} e M_W s_{W}} + \frac{\lambda c_{\beta} M_W s_{W} s_{2\beta}}{\sqrt{2} e} \\
+ (\lambda \delta Z_e - \delta \lambda) \frac{\sqrt{2} c_{\beta} M_W s_{W}}{e}.
\]
\[
\delta M_{N55} = \sqrt{2}\kappa v_s + \sqrt{2}\kappa \delta v_s \\
\delta M_{N12} = \delta M_{N53} = \delta M_{N43} = \delta M_{N44} = 0.
\]

With these ingredients the explicit expressions for the equations (71) can be derived. In our calculation, the system of equations (71) is solved numerically, keeping, however, the dependence on the divergent part \(\Delta\) explicitly.

**DR renormalisation scheme**

The DR renormalisation scheme differs in the conditions (i)-(vii) from the mixed one, which means in the conditions for \(M_Z, M_W, M_{H^\pm}, t_h, t_{h_d}, t_{h_s} , \epsilon\). Note that a change of the renormalisation condition for \(M_{H^\pm}\) can be interpreted as a change of the condition for \(A_\lambda\), cf. Eq. (14). For these parameters instead of OS conditions DR renormalisation conditions are adopted now. The other renormalisation conditions do not change.

### 2.5 Loop Corrected Higgs Boson Masses and Mixing Matrix Elements

The one-loop corrected scalar Higgs boson masses squared are extracted numerically as the zeroes of the determinant of the two-point vertex functions \(\hat{\Gamma}^S\),

\[
\hat{\Gamma}^S(k^2) = 
i \begin{pmatrix}
k^2 - (M_{H_1}^{(0)})^2 + \hat{\Sigma}_{H_1H_1}(k^2) & \hat{\Sigma}_{H_1H_2}(k^2) & \hat{\Sigma}_{H_1H_3}(k^2) \\
\hat{\Sigma}_{H_2H_1}(k^2) & k^2 - (M_{H_2}^{(0)})^2 + \hat{\Sigma}_{H_2H_2}(k^2) & \hat{\Sigma}_{H_2H_3}(k^2) \\
\hat{\Sigma}_{H_3H_1}(k^2) & \hat{\Sigma}_{H_3H_2}(k^2) & k^2 - (M_{H_3}^{(0)})^2 + \hat{\Sigma}_{H_3H_3}(k^2)
\end{pmatrix}
\]

In the same way the pseudoscalar masses squared are obtained from \(\hat{\Gamma}^P\),

\[
\hat{\Gamma}^P(k^2) = \begin{pmatrix}
k^2 - (M_{A_1}^{(0)})^2 + \hat{\Sigma}_{A_1A_1}(k^2) & \hat{\Sigma}_{A_1A_2}(k^2) \\
\hat{\Sigma}_{A_2A_1}(k^2) & k^2 - (M_{A_2}^{(0)})^2 + \hat{\Sigma}_{A_2A_2}(k^2)
\end{pmatrix}.
\]

The superscript (0) denotes the tree-level values of the masses squared. It should be noted that in Eq. (88) the mixing with the Goldstone bosons is not taken into account. We have checked explicitly that the numerical effect is negligible. The unrenormalised self-energy and tadpole contributions that occur implicitly in Eqs. (87) and (88) are evaluated at one-loop order. They contain fermion, Goldstone and Higgs boson, gauge boson and ghost loops as well as loops from the corresponding superpartners i.e. sfermions, charginos and neutralinos.

The mass eigenvalues are obtained iteratively. In order to obtain the lightest scalar Higgs boson mass e.g., in the first iteration the external momentum squared \(k^2\) in the renormalised self-energies \(\hat{\Sigma}_{H_jH_j}\) is set equal to the lightest scalar tree-level mass squared. Then, the mass matrix part of \(\hat{\Gamma}^S\), meaning \((\hat{\Gamma}^S + k^2\mathbb{I})\), is diagonalised and the resulting mass eigenvalues squared are used in the next iteration where \(k^2\) is set equal to the lightest of the obtained mass eigenvalues. Once again the mass eigenvalues are obtained. The procedure is repeated until the deviation between the lightest eigenvalue and the one of the previous iteration is less than \(10^{-9}\). The other Higgs mass eigenvalues are derived accordingly.

Due to the radiative corrections, not only the masses of the particles receive contributions but also the fields are affected. To take these effects into account, new matrices, \(\mathcal{R}^{S,H}, \mathcal{R}^{P,H}\), are
introduced which transform the fields \( h_u, h_d, h_s \) and \( a_a, a_s \) into the corresponding one-loop mass eigenstates, respectively. These matrices are no physical observables and beyond lowest order they depend on the external momentum in the self-energies. For the derivation of the radiatively corrected matrices, \( \mathcal{R}^{S,s.l}, \mathcal{R}^{P,s.l} \), we follow the procedure applied in Ref. [30]. It ensures the correct on-shell properties for the external particle in processes with external on-shell Higgs bosons at higher orders and thus accounts also for the mixing between the Higgs bosons. This leads to finite wave function correction factors. In the scalar case e.g. we have to apply the additional factor \( Z_f^S \) to the tree-level matrix \( \mathcal{R}^S \), which rotates the interaction states \( (h_d, h_u, h_s)^T \) to the mass eigenstates \( (H_1, H_2, H_3)^T \), to get the one-loop matrix elements,

\[
\mathcal{R}^{S,s.l}_{il} = (Z_f^S)_{ij} \mathcal{R}^S_{jl}, \quad i, j = H_1, H_2, H_3, \quad l = h_d, h_u, h_s.
\]

The correction factor is given by

\[
(Z_f^S)_{ij} = \sqrt{Z_i^S Z_{ij}^S},
\]

with

\[
Z_i^S = \frac{1}{1 + (\text{Re} \Sigma_{ii}^{\text{eff}})'(M_{H_i})^2}.
\]

The prime denotes the derivative with respect to \( k^2 \), and \( M^2_{H_i} \) is the one-loop corrected Higgs boson mass squared. The effective self-energy \( \Sigma_{ii}^{\text{eff}} \) appears in the diagonal Higgs boson propagators

\[
\Delta_{ii}(k^2) = -\left[(\Gamma^S(k^2))^{-1}\right]_{ii} = \frac{i}{k^2 - (M^0_{H_i})^2 + \Sigma_{ii}^{\text{eff}}(k^2)}.
\]

It is given by (no summation over \( i, j, l \))

\[
\Sigma_{ii}^{\text{eff}}(k^2) = \Sigma_{ii} - i \frac{2\Gamma_{ij}^S(k^2)\Gamma_{jl}^S(k^2)\Gamma_{lj}^S(k^2) - \Gamma_{ij}^S(k^2)\Gamma_{jl}^S(k^2) - \Gamma_{ij}^S(k^2)\Gamma_{lj}^S(k^2)}{\Gamma_{ij}^S(k^2)\Gamma_{lj}^S(k^2) - \Gamma_{jl}^S(k^2)}.
\]

The off-diagonal Higgs boson propagator \( \Delta_{ij} \) (\( i \neq j \), no summation over \( i, j, l \)) reads

\[
\Delta_{ij}(k^2) = \frac{\Gamma_{ij}^S\Gamma_{il}^S - \Gamma_{ij}^S\Gamma_{il}^S}{\Gamma_{ii}^S\Gamma_{jj}^S\Gamma_{ll}^S + 2\Gamma_{ij}^S\Gamma_{jl}^S\Gamma_{lj}^S - \Gamma_{ij}^S\Gamma_{ll}^S - \Gamma_{ij}^S\Gamma_{lj}^S - \Gamma_{jj}^S\Gamma_{ll}^S - \Gamma_{lj}^S\Gamma_{ij}^S}.
\]

The argument \( k^2 \) in \( \Gamma_{ij}^S \) has been dropped for better readability. For \( Z_{ij}^S \) we have in terms of the propagators (again no summation over the indices)

\[
\hat{Z}_{ij}^S \quad = \quad \left. \frac{\Delta_{ij}(k^2)}{\Delta_{ii}(k^2)} \right|_{k^2=M_{H_i}^2}.
\]

\[
\hat{Z}_{ii}^S \quad = \quad 1.
\]
In case of the pseudoscalar $2 \times 2$ mixing matrix $R_{il}^{P,il}$ the effective self-energy reduces to

$$\hat{\Sigma}_{ii}^{\text{eff}}(k^2) = \hat{\Sigma}_{ii} + i \frac{\hat{\Gamma}_{jj}^{P}(k^2)}{\hat{\Gamma}_{jj}^{P}(k^2)}, \quad i, j = A_1, A_2$$

(97)

and the off-diagonal propagator reads

$$\Delta_{ij}(k^2) = \frac{\hat{\Gamma}_{ij}^{P}}{\hat{\Gamma}_{ii}^{P} \hat{\Gamma}_{jj}^{P} - \hat{\Gamma}_{ij}^{P2}}.$$  

(98)

The thus derived mixing matrix elements include the full momentum dependence and imaginary parts of the Higgs boson self-energies. Alternatively we could have set $k^2 = 0$ which corresponds to the result in the effective potential approximation and yields a unitary mixing matrix. For our parameter sets used in the numerical analysis we found that the differences in the two approaches are negligible. Furthermore, the imaginary parts of the mixing matrix elements are small compared to the real parts.

3 Numerical Analysis

The calculation of the Higgs and gauge boson self-energies, of the tadpoles and the counterterms has been performed numerically in two different calculations. In the first calculation all necessary Feynman rules have been derived from the NMSSM Lagrangian and implemented in a FeynArts model file [45]. In the second calculation the Feynman rules have been obtained with the Mathematica package SARAH [46]. The Feynman rules in the two approaches have been cross-checked against each other and also against the rules given in Ref. [8]. Subsequently, in both calculations FormCalc [47] was used to evaluate the self-energy and tadpole diagrams in the 't Hooft-Feynman gauge, in which the Goldstone bosons and the ghost fields have the same masses as the corresponding gauge bosons. The divergent integrals are regularised applying the constrained differential renormalisation scheme [48] which has been shown to be equivalent to the SUSY conserving dimensional reduction scheme [50]. The numerical computation of the integrals has been performed with LoopTools [47]. Two Mathematica programs have been written to evaluate the counterterms, diagonalise numerically the one-loop corrected Higgs boson mass matrices and extract the mass eigenvalues.

For our numerical analysis we follow the SUSY Les Houches Accord (SLHA) [51] and use as input values the Fermi constant $G_F = 1.16637 \cdot 10^{-5} \text{ GeV}^{-2}$ and the $Z$ boson mass $M_Z = 91.187 \text{ GeV}$. For the electroweak coupling we set $\alpha = 1/137$. From these input values we derive the parameters of our input set defined in Eq. (29). The top quark pole mass is given by $M_t = 173.3 \text{ GeV}$. As we cross-check our results against the ones of Ref. [40] which uses the running $\overline{\text{DR}}$ quark masses, we need to calculate these as well. In order to obtain the $\overline{\text{DR}}$ top quark mass we convert $M_t$ at the scale $Q_t = M_t$ in the corresponding running mass. The SM renormalisation group equations are then used to evolve the top mass up to a common scale $Q$ chosen to be of the order of the SUSY breaking scale, where the gluino corrections are added. We denote the running top mass by $m_t$ in the following. The same procedure is applied to the bottom mass starting from the SLHA input value $m_b(m_b)^{\overline{\text{MS}}}$ set equal to 4.19 GeV. The masses of the light quarks are chosen as $m_u = 2.5 \text{ MeV},$
$m_c = 1.27\text{ GeV}, m_d = 4.95\text{ MeV}, m_s = 101\text{ MeV}$ [52]. The $\tau$ mass has been set to $m_\tau = 1.777\text{ GeV}$.

In the following we will discuss the results for various scenarios which exemplify different effects of the higher-order corrections. For our scenarios, we took care not to violate unitarity bounds by choosing $\lambda, \kappa$ such that $\sqrt{\lambda^2 + \kappa^2} \lesssim 0.7$. Furthermore, $v_s$ has been chosen to be of the order of the vacuum expectation value $v$. As the one-loop corrections to the pseudoscalar masses are small, we mostly show plots for the scalar masses and comment briefly on the pseudoscalar masses. Furthermore, the corrections for the heaviest scalar Higgs boson are not shown, as in all cases they are negligible.

**Variation of $\lambda$** We first verify that we reproduce the results of Ref. [40] by adopting a DR renormalisation scheme and choosing the same parameter set

$$\kappa = \lambda/5, \quad \tan \beta = 2, \quad A_\lambda = 500\text{ GeV}, \quad A_\kappa = -10\text{ GeV}, \quad \mu = 250\text{ GeV} \quad (99)$$

with $\lambda$ being a free parameter and $v_s$ given by $v_s = \sqrt{2}\mu/\lambda$. For the squarks and sleptons a common soft SUSY breaking mass $M_S = 300\text{ GeV}$ has been adopted, and the remaining soft SUSY breaking parameters have been chosen as

$$A_t = A_b = A_\tau = -1.5M_S, \quad M_1 = M_S/3, \quad M_2 = 2/3M_S, \quad M_3 = 2M_S. \quad (100)$$

The renormalisation scale has been set equal to $Q_0 = M_S$ and for the top and bottom quark mass the running DR mass at the scale $Q_0$ has been used. Furthermore, the light quark masses have been set to zero as in Ref. [40]. We find agreement for the one-loop corrected Higgs boson masses. Starting from this scenario in the DR scheme, in order to investigate the effect of different renormalisation schemes, the DR input values have been converted to the mixed renormalisation scheme as defined above, as well as to a pure on-shell scheme, and the Higgs boson masses and the matrix $\mathcal{R}^S$ are evaluated accordingly. For the definitions of the schemes, see section 2.4. In Fig. 1 we show the matrix elements squared $(\mathcal{R}_{13}^S)^2$ and $(\mathcal{R}_{23}^S)^2$, respectively, of the mixing matrix $\mathcal{R}^S$. These matrix

![Figure 1: The matrix element squared $(\mathcal{R}_{13}^S)^2$ (left), $(\mathcal{R}_{23}^S)^2$ (right) for the two lightest CP-even Higgs bosons as function of $\lambda$ at tree-level (yellow/full) and at one-loop level adopting a DR (blue/dotted), a mixed (red/dashed) and an on-shell (green/small dotted) renormalisation scheme.](image-url)
elements are a measure of the strength of the singlet component of the two lightest Higgs bosons. They are shown for the three different renormalisation schemes compared to the tree-level result as a function of \( \lambda \). In order to match the result of Ref. [40] they have been evaluated at vanishing external momentum \( k^2 = 0 \). At tree-level, for small values of \( \lambda \), the lightest CP-even Higgs boson is dominantly MSSM-like and the next-to-lightest is dominantly singlet-like. With increasing \( \lambda \) the mixing increases and at tree-level there is a cross-over at \( \lambda \approx 0.94 \): the next-to-lightest Higgs boson is now more MSSM-like than the lightest Higgs boson. The higher-order corrections change the amount of the singlet component and for this parameter set there is no cross-over below \( \lambda = 1 \). Furthermore, we see that for \( \lambda > \approx 0.2 \) the amount of the singlet component is hardly affected by the renormalisation scheme. The curves for the three renormalisation schemes lie on top of each other for \( \lambda \gtrsim 0.65 \). For smaller values of \( \lambda \), however, after having crossed \( (R_{13}^S)^2 = 0 \) and \( (R_{23}^S)^2 = 1 \), respectively, \( (R_{13}^S)^2, (R_{23}^S)^2 \) in the OS-scheme start to differ largely from the corresponding values in the two other schemes. The reason is that the finite parts of the counterterms involved in the conversion from the \( \overline{\text{DR}} \) parameters to on-shell parameters contain a division by \( \lambda \), so that these finite counterterm contributions blow up in the limit \( \lambda \to 0 \). The \( H_1 \) mass squared even turns negative for \( \lambda \lesssim 0.1 \). The matrix elements and masses in the OS scheme are therefore not plotted any more for \( \lambda \) values below this value.

Figure 2 shows the one-loop corrected mass \( M_{H_1} \) of the lightest (left) and \( M_{H_2} \) of the next-to-lightest (right) CP-even Higgs boson as a function of \( \lambda \) in the three different renormalisation schemes compared to the tree-level result. The tree-level masses increase with rising \( \lambda \) due to the NMSSM contribution \( \sim \lambda^2 \sin^2 2\beta \) from the Higgs quartic coupling. As \( H_1 \) is dominantly MSSM-like its one-loop corrections are much more important than for the singlet dominated \( H_2 \). For the latter they are negligible at small \( \lambda \) and more important for large \( \lambda \) where \( H_2 \) is more MSSM-like.

As can be inferred from the figures the deviations between the different schemes is negligible. The curves for all three schemes lie on top of each other. Apart from small values of \( \lambda \) where the one-loop corrected \( H_1 \) and \( H_2 \) mass in the on-shell scheme start to differ from the \( \overline{\text{DR}} \) and the mixed scheme.

Figure 2: The mass \( M_{H_1} \) of the lightest (left) and \( M_{H_2} \) of the next-to-lightest (right) CP-even Higgs boson as function of \( \lambda \) at tree-level (yellow/full) and at one-loop level adopting a \( \overline{\text{DR}} \) (blue/dotted), a mixed (red/dashed) and an on-shell (green/small dotted) renormalisation scheme.
The mass corrections and hence also the strengths of the singlet and MSSM component, respectively, strongly depend on the value of the top quark mass, which reflects the fact that the main part of the higher-order corrections stems from the top sector. Figure 3 shows the mixing matrix elements squared \((R_{13}^S)^2\), \((R_{23}^S)^2\) as functions of \(\lambda\) at one-loop level, calculated in the \(\overline{\text{DR}}\) scheme, with the top quark mass taken as the running \(\overline{\text{DR}}\) mass \(m_t = 150.6\) GeV in one case and as the pole mass \(M_t = 173.3\) GeV in the other case. For comparison the tree-level values are shown as well. Figure 4 displays the corresponding Higgs masses. In contrast to the case where the top quark mass has been set to the \(\overline{\text{DR}}\) value, using the top pole mass leads to a one-loop corrected lightest Higgs boson which is singlet-like and to a next-to-lightest Higgs boson being MSSM-like.
Furthermore, the Higgs mass corrections are more important for a higher top quark mass value. Defining the relative correction $\Delta M_H / M_H$ as

$$\Delta M_H = \frac{|M_H - M_H^{(0)}|}{M_H^{(0)}},$$

where $M_H (M_H^{(0)})$ denotes the 1-loop corrected (tree-level) Higgs boson mass, the relative correction for $H_1$ amounts to maximally 55% for the running mass and 79% for the pole mass. The large corrections also explain the change of the one-loop corrected $H_1$ from a MSSM-like to singlet-like Higgs boson. In fact, due to the large corrections the one-loop corrected $H_1$ mass gets shifted above the one-loop corrected mass of $H_2$. Due to our convention to label by ascending indices the Higgs bosons with increasing mass, the $H_1$ one-loop corrected mass is assigned to $H_2$ and vice versa, so that $H_1$ and $H_2$ interchange their roles and $H_1$ becomes singlet-like at 1-loop whereas $H_2$ becomes MSSM-like.

In order to get an estimate of the missing higher-order corrections we investigate the influence of the renormalisation scale $Q_0$. The results are shown (in the DR scheme) in Fig. 5 for the mixing matrix elements squared. They are plotted as a function of $\lambda$ at tree-level and at one-loop level for three different values $Q_0 = 150, 300$ and 600 GeV. Note that the scale $Q_0$ also changes the value of the running $b$ and $t$ quark masses. The corresponding plots for the masses of the two lightest Higgs bosons are depicted in Fig. 6. Whereas the change of the renormalisation scale alters the mixing matrix elements considerably, the effect on the Higgs boson masses is less pronounced. With rising $Q_0$ the running top quark mass decreases, leading to smaller one-loop Higgs boson mass corrections. The relative correction for the lightest Higgs boson mass is 63% for $Q_0 = 150$ GeV and 50% for $Q_0 = 600$ GeV. The relative correction of the next-to-lightest Higgs boson mass is changed less, with $\Delta M_{H_2} / M_{H_2} = 15\%$ and 8% for $Q_0 = 150$ and 600 GeV. This is due to the higher tree-level mass value. The residual theoretical uncertainties due to missing higher-order corrections can thus be estimated to $O(10\%)$. 

\[ \text{Figure 5: The matrix element squared } (R_{13})^2 \text{ (left), } (R_{23})^2 \text{ (right) for the two lightest CP-even Higgs bosons as function of } \lambda \text{ at tree-level (yellow/full), at one-loop level at the renormalisation scale } Q_0 = 150 \text{ GeV (blue/dotted), 300 GeV (red/dashed) and 600 GeV (green/small dotted).} \]
The amount of singlet component of the Higgs bosons strongly affects their couplings to fermions and gauge bosons and hence their phenomenology. In particular, with the small $H_1$ and $H_2$ mass values between $\sim 90$ and $\sim 180$ GeV, the question arises if the scenario has been excluded by LEP, Tevatron or LHC. We have explicitly verified that there are still regions in $\lambda$ which have not been excluded. The allowed and excluded regions are shown in Fig. 7. The exclusion limit for $\lambda \leq 0.6$ where $M_{H_1} \leq 113$ GeV is due to the LEP exclusion of $M_{H_1}$ in the channel $e^+e^- \rightarrow ZH \rightarrow Zb\bar{b}$ [5]. The Tevatron [54] and the present LHC results [55,56] do not constrain the scenario. Note, that our

The exclusion regions always apply to the one-loop corrected Higgs boson masses.
limits represent a rough estimate and cannot replace a sophisticated study of exclusion limits set by a combination of the experimental results. We have cross-checked though our exclusion limits against those obtained with HiggsBounds \cite{HiggsBounds} and have found agreement.

Concerning the pseudoscalar Higgs bosons, the lighter state $A_1$ is singlet-like, whereas the heavier one is MSSM-like, and the corrections are small. They are negative and in this scenario the relative corrections are below 2\%. The masses increase with rising $\lambda$. The one-loop corrected mass $M_{A_1}$ of the lighter pseudoscalar Higgs boson ranges between 40 and 140 GeV, and $M_{A_2}$ takes values of 586 to 598 GeV.

**Variation of $A_\kappa$** In the following the variation of the soft SUSY breaking coupling parameter $A_\kappa$ is investigated. We have chosen a common soft SUSY breaking squark mass $m_0 = 1.1$ TeV and $M_S = 600$ GeV, large enough to fulfill the present exclusion limits on the squark masses of the first two generations and the gluino mass set by the LHC experiments \cite{LHClimits}. The full parameter set is given by

$$\begin{align*}
\lambda &= 0.6, \quad \kappa = \lambda/3, \quad \tan \beta = 2, \quad A_\lambda = 500 \text{ GeV}, \quad \mu = 275 \text{ GeV} \\
A_t &= A_b = A_\tau = -1.5 M_S, \quad M_1 = M_S/3, \quad M_2 = 2/3 M_S, \quad M_3 = 2 M_S. \quad (102)
\end{align*}$$

The top and bottom quark masses have been chosen to be the pole masses, with $m_{tb}^{\text{pole}} = 4.88$ GeV. The renormalisation scheme is the mixed scheme as defined in section 2.4. The mixing matrix elements squared quantifying the amount of the singlet component of $H_1$ and $H_2$ are presented in Fig. 8 (left). They have been obtained by the procedure described in section 2.5. We have explicitly verified that the difference to the values obtained by setting $k^2 = 0$ is negligible. The tree-level and one-loop corrected $H_1$ and $H_2$ masses are shown in Fig. 8 (right). As can be inferred from the figure, for large negative values of $A_\kappa$ the lightest Higgs boson is mostly singlet-like and the heavier one MSSM-like. With increasing $A_\kappa$ the mixing increases developing a rapid cross-over at $A_\kappa \approx -294$ GeV. The one-loop corrections shift the cross-over to a larger value $A_\kappa \approx -240$ GeV. This behaviour is also reflected in the Higgs boson masses. Below (above) the cross-over the
lightest (next-to-lightest) Higgs boson is mostly singlet-like and exhibits a strong dependence on $A_{\kappa}$, whereas in the MSSM-like case the Higgs bosons hardly depend on $A_{\kappa}$. This behaviour results from the fact that the soft SUSY breaking term which contains $A_{\kappa}$ is a cubic coupling in the singlet field $S$. A variation of $A_{\kappa}$ can hence only be communicated through a singlet contribution of the Higgs boson fields. Since the mixing is small, only fields which are mainly singlet-like are affected. Despite the singlet character of $H_1$ for small $A_{\kappa}$ the one-loop corrections can be important and more than triple the mass. This is mainly due to the small tree-level value. In case of the next-to-lightest Higgs boson $H_2$ the one-loop corrections in the singlet-like case are less pronounced and of $\mathcal{O}(10 \text{ GeV})$. The corrections to the MSSM-like Higgs bosons increase the masses by about 37 GeV. No Higgs boson mass values $M_{H_{1,2}}$ have been excluded so far by the experiments. With a slightly higher value of $\lambda$, however, the MSSM-like Higgs boson mass gets shifted above 141 GeV, so that the scenario would be excluded by the new LHC exclusion limits [56].

For the pseudoscalar masses the relative corrections are very small, at the 6% level for the singlet-like $A_1$ and below 0.2% for the heavier $A_2$. The one-loop masses decrease with decreasing absolute value of $A_{\kappa}$ and are 110 GeV $\lesssim M_{A_1} \lesssim 345$ GeV and 639 GeV $\lesssim M_{A_2} \lesssim 640$ GeV for 0 GeV $\geq A_{\kappa} \geq -400$ GeV.

Variation of $M_{H^\pm}$ The variation of $A_3$ corresponds to a variation of the charged Higgs boson mass, cf. Eq. (12). The other parameters are kept fixed and chosen as

$$\lambda = 0.65 \ , \ \ \kappa = \lambda/3 \ , \ \ \tan \beta = 2 \ , \ \ A_{\kappa} = -10 \text{ GeV} \ , \ \ \mu = 225 \text{ GeV}$$

$$A_t = A_b = A_{\tau} = -1.5 M_S \ , \ \ M_1 = M_S/3 \ , \ \ M_2 = 2/3 M_S \ , \ \ M_3 = 2 M_S \ . \ \ (103)$$

The common soft SUSY breaking squark mass is once again taken to be $m_0 = 1.1 \text{ TeV}$ and $M_S = 600$ GeV. The parameter $A_\lambda$ is varied between 250 and 600 GeV, which corresponds to $M_{H^\pm} \approx 420...610$ GeV. Outside this parameter range the $H_1$ mass squared becomes negative.\(^\text{11}\)

The results for the singlet components and Higgs boson masses at tree-level and one-loop are shown in Fig. 9. The lightest Higgs boson $H_1$ is dominantly MSSM-like with a 100% MSSM component at $M_{H^\pm} \approx 520$ GeV. Here $H_2$ is maximally singlet-like, though not completely. The heaviest scalar Higgs boson $H_3$ takes over the remaining singlet component (not shown here), so that $\sum_{i=1...3}(R_{i3}^S)^2 = 1$ as demanded by unitarity of the mixing matrix.\(^\text{12}\) The one-loop corrections increase the mixing between $H_1$ and $H_2$ away from the maximum and minimum singlet values. They lead to a more pronounced maximum and minimum in the singlet component and slightly shift their positions to smaller $H^\pm$ masses, $M_{H^\pm} \approx 518$ GeV. The $H_1$ and $H_2$ masses are maximal and minimal, respectively, at the position of minimal mixing. Due to its MSSM nature the lightest Higgs boson receives large one-loop corrections. The correction is $\sim 35$ GeV for $M_{H_1}$ at its maximum value, which is taken at $M_{H^\pm} \approx 515$ GeV, and can even triple the tree-level mass at the borders of the $M_{H^\pm}$ range. The latter is mostly the effect of an already very small tree-level mass of $\sim 30$ GeV. The singlet-like $H_2$ on the other hand receives smaller corrections which can nevertheless reach 10 GeV. The investigated parameter range has been partially excluded as indicated in Fig. 9 (right). For $M_{H^\pm} \leq 452$ GeV light Higgs boson masses $M_{H_1} \lesssim 112$ GeV have been excluded by the LEP

\(^\text{11}\)See also Ref. [11] for a discussion of the Higgs boson masses and their dependence on the NMSSM parameters.

\(^\text{12}\)It should be noted that this is only approximately true for the matrix obtained via the procedure described in Sect. 2.5. In general this matrix, in contrast to the one obtained using $k^2 = 0$, is not unitary. As already mentioned the difference between the two approaches is small for our scenarios.
searches in $ZH \to Zb\bar{b}$ [53]. The right exclusion limit $M_{H^\pm} \geq 585$ GeV is also due to the LEP exclusion of $M_{H_1} \lesssim 113.5$ GeV in the $ZH \to Zb\bar{b}$ channel. The Tevatron results do not exclude Higgs mass values. The exclusion limits obtained with HiggsBounds agree with ours. The maximum mass value taken by the MSSM-like lightest scalar Higgs boson is 140.5 GeV. It is just below the value excluded by the newest LHC limits [56]. In view of the uncertainties associated with the 1-loop Higgs mass corrections and also taking into account the fact that our exclusion limits are only a rough estimate, the scenario might be excluded for charged Higgs mass values around 515 GeV.

The influence of the renormalisation scheme on the one-loop corrected $H_{1,2}$ masses is shown in

**Figure 9:** Left: The matrix element squared $(R_{33}^e)^2$ ($i = 1, 2$) as function of $M_{H^\pm}$ at tree-level (dashed) and one-loop (full) for $H_1$ (blue/dark grey) and $H_2$ (red/light grey). Right: The corresponding tree-level and one-loop corrected masses with the exclusion limits.

**Figure 10:** The mass $M_{H_1}$ of the lightest (left) and $M_{H_2}$ of the next-to-lightest (right) CP-even Higgs boson as function of $M_{H^\pm}$ at tree-level (yellow/full) and at one-loop level adopting a DR (blue/dotted), a mixed (red/dashed) and an on-shell (green/small dotted) renormalisation scheme.
They are shown as functions of $M_{H^\pm}$ in the OS, mixed and pure DR scheme and compared to the tree-level result. For the chosen parameter set the differences in the mixed and OS scheme are negligible in contrast to the DR scheme. This is to be expected as both in the mixed and the OS scheme $A_\lambda$, respectively $M_{H^\pm}$, are renormalised on-shell, whereas in the DR scheme only the divergent part is included in the counterterm. The differences in the renormalisation schemes are, however, of maximally $\mathcal{O}(10\%)$ for the masses of the light Higgs boson. For the heavier scalar particles they are only $\sim 1\%$ of the one-loop corrected mass.

In Figure 11 we show the tree-level and one-loop corrected masses of the pseudoscalar Higgs bosons as functions of $M_{H^\pm}$. Whereas for the heavier CP-odd boson the corrections are negligible they can be of $\mathcal{O}(5\%)$ for the lighter one at large values of $M_{H^\pm}$. Nevertheless they are much smaller than the corrections for the lightest scalar Higgs boson.

Figure 11: The mass $M_{A_1}$ of the lighter (left) and $M_{A_2}$ of the heavier (right) pseudoscalar Higgs boson as a function of $M_{H^\pm}$ at tree-level (dashed/blue) and at one-loop (full/red).

4 Summary and Conclusions

In summary, we have calculated the one-loop corrections to the NMSSM Higgs masses in a renormalisation scheme which mixes on-shell and DR conditions, by applying the latter solely to the parameters $\tan \beta, v_s, \lambda, \kappa, A_\kappa$. We have compared our result to a pure DR and a pure OS-scheme. Apart from special parameter regions which unphysically blow up the counterterms, the results differ by at most 10% for the masses of the lightest scalar Higgs boson. Another estimate of the effect of the missing higher-order corrections is given by the variation of the renormalisation scale. We found the effect on the Higgs mass corrections to be $\mathcal{O}(10\%)$ or less. Altogether, the residual theoretical uncertainty at one-loop level can be estimated to be of the order of 10%.

The bulk of the one-loop corrections stems from the top quark sector as it is known from the MSSM. This is reflected in the difference of the relative corrections for the lightest Higgs boson mass. The relative correction in the scenario, which we investigated, is equal to 55% when adopting the running DR mass and 79% in case of the top quark pole mass. Furthermore, the higher-order
corrections can shift the point of cross-over between singlet-like and MSSM-like behaviour of the Higgs bosons. This dictates also the amount of coupling to the gauge bosons and fermions and hence influences the Higgs boson phenomenology. The precise knowledge of the higher-order corrections is therefore indispensable for a proper investigation of specific production and decay scenarios and a proper interpretation of the experimental results.

Appendix

A The CP-even and CP-odd Mass Matrices

In the following we display the mass matrices in terms of the parameters for which we apply our renormalisation conditions. We repeat them here for completeness, cf. also Eq. (29).

\[ t_h, t_d, t_s, e, M_W^2, M_Z^2, \tan \beta, M_{H^\pm}, \lambda, \kappa, v_s, A_\kappa. \] (104)

Note, that in the mass matrices we paid attention to keep the distinction in the angle \( \beta \) and the angle \( \beta_B \). The difference of these two angles is denoted as

\[ \Delta \beta = \beta - \beta_B. \] (105)

Here, \( \beta \) is defined by the ratio of the vacuum expectation values of the neutral components of the two Higgs doublets \( H_u \) and \( H_d \), \( \tan \beta = v_u/v_d \). The angle \( \beta_B \) on the other hand performs the rotation from the basis \((a_d, a_u, a_s)\) to the basis \((a, a_s, G)\) to separate a massless Goldstone boson. It is the angle \( \beta \) which receives a counterterm. The angle \( \beta_B \) also enters the scalar mass matrix. This is due to the replacement Eq. (27) of \( A_\lambda \) in terms of the parameter set Eq. (104).

The scalar \( 3 \times 3 \) mass matrix \( M_3^2 \) in the basis \( h^S = (h_d, h_u, h_s)^T \) is given by the entries \( M^2_{S\gamma} \) \( (i, j = 1, 2, 3) \), with

\[
M_{S_{11}}^2 = \frac{ec\beta c_\beta}{2MW_{SW}c_\Delta \beta} [t_h (2t_\beta t_{\beta_B} + 1) - t_h t_\beta] + \frac{s_\beta^2}{c_\Delta \beta} [M_{H^\pm}^2 + (M_Z^2 t_\beta - M_{W}^2) c_\Delta \beta]
+ \frac{2\lambda^2 M_{W}^2 s_{W}^2 s_{2\beta}}{e^2} \] (106)

\[
M_{S_{12}}^2 = \frac{ec\beta c_\beta}{2MW_{SW}c_\Delta \beta} [t_h t_\beta t_{\beta_B} + t_h] - \frac{s_\beta^2 c_\beta}{c_\Delta \beta} [M_{H^\pm}^2 + (M_Z^2 - M_{W}^2) c_\Delta \beta] + \frac{\lambda M_{W}^2 s_{W}^2 s_{2\beta}}{e^2} \] (107)

\[
M_{S_{13}}^2 = \frac{s_\beta^2 c_\beta^2}{v_s^2 c_\Delta \beta} [t_h t_\beta t_{\beta_B} + t_h] + \frac{2MW_{SW} s_{W}^2 s_{2\beta} c_\beta}{ev_s c_\Delta \beta} [M_{H^\pm}^2 c_\Delta \beta - M_{H^\pm}^2] + \frac{\lambda M_{W} s_{W} c_\beta v_s}{e} [2\lambda - \kappa t_{\beta}] + \frac{4\lambda^2 M_{W}^2 s_{W}^2 s_{3\beta} c_\beta}{e^2} v_s \] (108)

\[
M_{S_{22}}^2 = \frac{ec\beta c_\beta}{2MW_{SW}c_\Delta \beta} [-t_h s_\beta t_{\beta_B} + t_h s_\beta (t_{\beta_B} + 2)] + \frac{s_\beta^2}{c_\Delta \beta} [M_{H^\pm}^2 + (M_Z^2 t_\beta - M_{W}^2) c_\Delta \beta]
+ \frac{2\lambda^2 M_{W}^2 s_{W}^2 s_{2\beta}}{e^2} \] (109)
\[
M^2_{S_{23}} = \frac{c_{\beta}^2 t_{\beta}^2}{v_s^2 c_{\Delta}^2} [t_{h_d} t_{\beta} t_{\beta B} + t_{h_u}] + \frac{2M_W s_W s_{\beta} c_{\beta}^2}{e v_s c_{\Delta}^2} [M^2_W c_{\Delta}^2 - M^2_{H^\pm}] + \frac{\lambda M_W s_W c_{\beta} v_s}{e} [2\lambda t_{\beta} - \kappa] \\
+ \frac{-4\lambda^2 M^2_W s^2_W s_{\beta} c_{\beta}^2}{e^3 v_s} \\
M^2_{S_{33}} = \kappa A_* \frac{v_s}{\sqrt{2}} + 2\kappa^2 s^2_s + \frac{t_{h_u}}{v_s} + \frac{2M_W s_W s_{\beta} c_{\beta}^2}{e^2 v_s^2 c_{\Delta}^2} [2M^2_{H^\pm} M_W s_W s_{\beta} - e(t_{h_d} t_{\beta} t_{\beta B} + t_{h_u} c_{\beta B}^2)] \\
+ \frac{M^2_W s^2_W s_{\beta} c_{\beta}^2}{e^4 v_s^2} [2\lambda^2 M^2_W s^2_W s_{\beta} - \kappa e^2 v^2_s - M^2_W e^2 s_{\beta}^2] .
\]

The entries \(M^2_{P_{ij}} = M^2_{P_{ji}} (i,j = 1,2,3)\) of the pseudoscalar \(3 \times 3\) mass matrix \(M^2_P\) in the basis \(h^P = (a,s,G)^T\) read

\[
M^2_{P_{11}} = \frac{2\lambda^2 M^2_W s^2_W c_{\Delta}^2}{e^2} + M^2_{H^\pm} - M^2_W c_{\Delta}^2 \tag{112}
\]

\[
M^2_{P_{32}} = \frac{M^2_W s^2_W s_{\beta} c_{\beta}^2}{e v_s c_{\Delta}^2} [M^2_{H^\pm} - M^2_W c_{\Delta}^2] - \frac{c_{\beta}^2 c_{\beta B}^2}{v_s c_{\Delta}^2} [t_{h_u} + t_{h_d} t_{\beta} t_{\beta B}^2] \\
+ \frac{\lambda M_W s_W c_{\beta} c_{\beta}^2}{e^3 v_s} [2\lambda M^2_W s^2_W s_{\beta} - 3\kappa e^2 v^2_s] \tag{113}
\]

\[
M^2_{P_{33}} = M^2_{H^\pm} t_{\Delta} + \frac{M^2_W s^2_W c_{\beta} c_{\beta}^2}{e v_s c_{\Delta}^2} [t_{h_u} + t_{h_d} t_{\beta} t_{\beta B}^2] \\
+ \frac{\lambda M^2_W s^2_W s_{\beta} c_{\beta}^2}{e^3 v_s} [2\lambda^2 M^2_W s^2_W s_{\beta} + 3\kappa e^2 v^2_s] \tag{114}
\]

\[
M^2_{P_{22}} = -3A_* \frac{v_s}{\sqrt{2}} + \frac{t_{h_u}}{v_s} - \frac{2M_W s_W s_{\beta} c_{\beta}^2 c_{\beta B}^2}{e v_s c_{\Delta}^2} [t_{h_u} + t_{h_d} t_{\beta} t_{\beta B}^2] \\
+ \frac{\lambda M^2_W s^2_W s_{\beta} c_{\beta}^2}{e^3 v_s} [2\lambda M^2_W s^2_W s_{\beta} - 3\kappa e^2 v^2_s] \tag{115}
\]

\[
M^2_{P_{23}} = \frac{M^2_W s^2_W s_{\beta} c_{\beta}^2}{2e v_s c_{\Delta}^2} [2M^2_{H^\pm} t_{\Delta} - M^2_W s_{2\Delta}^2] - \frac{c_{\beta}^2 c_{\beta B}^2 t_{\Delta}^2}{v_s c_{\Delta}^2} [t_{h_u} + t_{h_d} t_{\beta} t_{\beta B}^2] \\
+ \frac{\lambda M_W s_W s_{\beta} c_{\beta}^2}{e^3 v_s} [2\lambda M^2_W s^2_W s_{\beta} - 3\kappa e^2 v^2_s] \tag{116}
\]

\[
M^2_{P_{33}} = M^2_{H^\pm} \tan^2 \Delta \beta + \frac{M^2_W \sin^2 \Delta \beta}{e^2} [2\lambda^2 s^2_W - e^2] \\
+ \frac{e}{2M_W s_W c_{\Delta}^2} [t_{h_d} c_{2\beta B} - t_{h_u} s_{\beta - 2\beta B}] \tag{117}
\]

**B Fermionic Self-Energies**

In this Appendix we give the renormalised fermionic self-energies which are needed in the determination of the counterterms \(\delta v_s\) and \(\delta \kappa\) from the chargino and the neutralino sector, respectively. We start from the general structure of a fermionic self-energy given by Eq. (60), which also applies to the corresponding renormalised self-energies. The left- and right-chiral chargino fields \(\psi^\pm_{L,R}\) in the interaction basis are given in terms of the Weyl spinors for the gaugino fields \(\tilde{W}_1, \tilde{W}_2\) and the charged components of the higgsino fields \(\tilde{H}_d^\pm, \tilde{H}_u^\pm\).

\[
\psi_R = \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_d^- \end{pmatrix}, \quad \psi_L = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix}, \tag{118}
\]
with $\tilde{W}^\pm = (\tilde{W}_1 \mp i\tilde{W}_2)/\sqrt{2}$. The chargino fields in the interaction basis are replaced by the renormalised fields and the corresponding field renormalisation constants,

$$\psi_{L,R}^\pm \rightarrow \left(1 + \frac{1}{2}\delta Z_{L,R}\right)\psi_{L,R}^\pm , \quad \text{with} \quad \delta Z_{L,R} = \begin{pmatrix} \delta Z_{L_1,R_1} & 0 \\ 0 & \delta Z_{L_2,R_2} \end{pmatrix} ,$$

and the $2 \times 2$ chargino mass matrix by

$$M_C \rightarrow M_C + \delta(M_C) .$$

Performing the rotation of $\psi_{L,R}^\pm$ to the fields $\chi_{L,R}^\pm$ in the mass eigenbasis with the unitary matrices $U$ and $V$, 

$$\chi_L^+ = V\psi_L^+ , \quad \chi_R^- = U\psi_R^-$$

yields for the renormalised self-energies $\hat{\Sigma}$ in the mass eigenbasis in terms of the unrenormalised self-energies, field renormalisation constants and mass matrix counterterm

$$\hat{\Sigma}_{\chi_L^\pm}(k^2) = \Sigma_{\chi_L^\pm}(k^2) + \frac{1}{2}U^*(\delta Z_R + \delta Z_R^*)U^T$$

$$\hat{\Sigma}_{\chi_R^\pm}(k^2) = \Sigma_{\chi_R^\pm}(k^2) + \frac{1}{2}V(\delta Z_L + \delta Z_L^*)V^\dagger$$

$$\hat{\Sigma}_{\chi_L^\mp}(k^2) = \Sigma_{\chi_L^\mp}(k^2) - \frac{1}{2}U^*(\delta Z_R M_C + M_C \delta Z_L)V^\dagger - U^*\delta(M_C)V^\dagger$$

$$\hat{\Sigma}_{\chi_R^\mp}(k^2) = \Sigma_{\chi_R^\mp}(k^2) - \frac{1}{2}V(\delta Z_L^* M_C^\dagger + M_C^\dagger \delta Z_R^*)U^T - V\delta(M_C^\dagger)U^T .$$

The self-energies are $2 \times 2$ matrices. The corresponding renormalised self-energies in the neutralino sector, which are $5 \times 5$ matrices, are obtained analogously and read

$$\hat{\Sigma}_{\chi_0^R}(k^2) = \Sigma_{\chi_0^R}(k^2) + \frac{1}{2}\mathcal{N}^*(\delta Z_R + \delta Z_R^*)\mathcal{N}^T$$

$$\hat{\Sigma}_{\chi_0^L}(k^2) = \Sigma_{\chi_0^L}(k^2) + \frac{1}{2}\mathcal{N}(\delta Z_L + \delta Z_L^*)\mathcal{N}^\dagger$$

$$\hat{\Sigma}_{\chi_0^L}(k^2) = \Sigma_{\chi_0^L}(k^2) - \frac{1}{2}\mathcal{N}^*(\delta Z_R M_N + M_N \delta Z_L)\mathcal{N}^\dagger - \mathcal{N}^*\delta(M_N)\mathcal{N}^\dagger$$

$$\hat{\Sigma}_{\chi_0^R}(k^2) = \Sigma_{\chi_0^R}(k^2) - \frac{1}{2}\mathcal{N}(\delta Z_L^* M_N^\dagger + M_N^\dagger \delta Z_R^*)\mathcal{N}^T - \mathcal{N}\delta(M_N^\dagger)\mathcal{N}^T ,$$

where $\mathcal{N}$ is the unitary $5 \times 5$ matrix which performs the rotation from the interaction basis to the neutralino mass eigenbasis. Allowing also for negative neutralino mass values, as we do, the matrix is real. The neutralino mass matrix $M_N$ is given in Eq. (13) and $\delta Z_{L,R}$ denote diagonal $5 \times 5$ matrices with the field renormalisation constants in the interaction eigenbasis $\delta Z_{L,R_i}$ ($i = 1, \ldots, 5$), as entries. Note that $\delta Z_L$ and $\delta Z_R$ are related due to the Majorana character of the neutralinos.

\textsuperscript{14}For better readability we did not choose a different notation for the field renormalisation constants in the chargino and the neutralino case. They are understood to be different, though.
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