Quantum antiferromagnet at finite temperature: a gauge field approach

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Starting from the $CP^{N-1}$ model description of the thermally disordered phase of the $D = 2$ quantum antiferromagnet, we examine the interaction of the Schwinger-boson spin-1/2 mean-field excitations with the generated gauge (chirality) fluctuations in the framework of the $1/N$ expansion. This interaction dramatically suppresses the one-particle motion, but enhances the staggered static susceptibility. This means that actual excitations in the system are represented by the collective spin-1 excitations, whereas one-particle excitations disappear from the problem. We also show that massive fluctuations of the constraint field are significant for the susceptibility calculations. A connection with the problem of a particle in random magnetic field is discussed.

PACS numbers: 75.10; 75.30D

Some time ago, two self-consistent approaches to the finite temperature properties of the low-dimensional magnets were developed: the modified spin wave theory \cite{1} and the Schwinger-boson theory \cite{2}. Both of them correctly describe the generation of the correlaton length by the thermal fluctuations, by complying with the Mermin-Wagner theorem \cite{3}. Due to its conservation of the spin rotational invariance in the thermally (or quantum) disordered (paramagnetic) phase the Schwinger-boson theory is a suitable starting point. Moreover, it allows the generalization of the Heisenberg Hamiltonian to that of an $SU(N)$-invariant model \cite{2}, for which a powerful $1/N$ expansion can be developed \cite{2}. Perturbation expansion in a parameter $1/N$ around the mean-field ($N = \infty$) saddle point provides a convenient way of treating the fluctuations in the isotropic system.

The applicability of the Schwinger-boson transformation of the spin operator relies completely on the necessary constraint at every site: the number of boson degrees of freedom has to be equal to $NS$, where $S$ is the the value and $N$ is the number of boson "colors" \cite{2}. Due to the constraint only $N - 1$ out of $N$ bosons per site are really independent. Usually, this constraint is introduced through the Lagrange field $\lambda$, defined on every lattice site. In the mean-field approximation on-site constraint is approximated by the averaged one \cite{2}, and it makes all $N$ bosons free.

As was shown in \cite{2}, the fluctuations of the constraint field $\lambda$ above the mean-field value $m^2$ and the phases of the Hubbard-Stratonovich decoupling field form three components of the $U(1)$ gauge field in the continuum limit. Also it was proved that the continuum limit of the Schwinger-boson large-$N$ theory coincides with the same limit of the $CP^{N-1}$ model at distances much larger than the lattice spacing \cite{4}.

Despite the fact that $CP^{N-1}$ model at $N = 2$ is just another representation of the $O(3)$ nonlinear $\sigma$ model, there is no direct correspondence between these models at arbitrary $N$. In principle, identical results should be obtained from the exact solutions of both models for the case of physical spins. It is the perturbative expansion around different $N = \infty$ saddle points of the models that makes the predictions of the models very different. Namely, mean-field analysis of the $CP^{N-1}$ model in the disordered phase produces two branches of elementary excitations with spin-1/2, whereas the $O(N)$ nonlinear $\sigma$ model gives triply degenerate spin-1 excitations in the disordered phase at $N = \infty$ \cite{5}. At the same time expressions for the spin-correlation length obtained from these models in the renormalized classical region are similar at the mean-field level.

In this communication we investigate the corrections beyond the mean-field level in the $CP^{N-1}$ model for the special case of the thermally disordered phase (or renormalized classical phase at $T \neq 0$, which corresponds to the Neel-type state at zero temperature) of the $CP^{N-1}$ model, by exploiting the abovementioned equivalence of it to the continuum limit of the Schwinger-boson theory. It is nessecary to stress here that in the analysis to follow we consider the lowest-energy excitations with momentum (and energy) much smaller than the inverse correlation length. In that regime thermal fluctuations of locally Neel ordered regions destroy completely long range Neel order and antiferromagnet is disordered, while inside the regions of the size of correlation length the system behaves as in the broken symmetry phase \cite{2}. The model contains two collective modes: the massless $U(1)$ gauge field (due to the famous mechanism of the dynamical generation of the kinetic energy of the gauge field \cite{2}) and the massive fluctuations of the constraint field $\lambda$. $\lambda$-fluctuations produce only the short-range density-density interaction term, whereas the coupling to the gauge field produces the long-range current-current interaction. Due to their masslessness, the gauge fluctuations change significantly the mean-field picture. As we will show here, it is the interaction with the generated gauge field that supresses spin-1/2 excitations, while only slightly affecting collective excitations with spin 1. At the same time, interaction of the massive $\lambda$-fluctuations with the mean-field excitations is much weaker, nevertheless it gives a susceptibility correction of the same order of magnitude as the interaction with gauge fluctuations.

Effect of the interaction with generated gauge field on the properties of free spin-1/2 spinons was investigated in ref. \cite{6} for the case of quantum disordered phase of the $SU(N)$ quantum antiferromagnet, and it was found that due to the "hedgehog"-like instanton tunneling events spinons are confined into the spin-1 pairs. Contrast to these we are considering the thermally disordered phase without topological defects.
Our starting point is the nonlinear $\sigma$ model Euclidian action in the continuum limit

$$S = \frac{c}{2g} \int_0^{\beta} d\tau \int d^2r \left\{ \frac{1}{e^2}(\partial_i n)^2 + (\partial_i n)^2 \right\}, i = x, y$$

(1)

with the constraint $n^2 = 1$. Vector $n$ describes local staggered magnetization. Putting $c = 1$, using the $CP^1$ representation $n = z^+ \sigma z$ ($z$ is a complex two-component field), and introducing $A_\mu = -\frac{i}{2}(z^+ \partial_\mu z - (\partial_\mu z^+)z)$ the action becomes [5]

$$S = \frac{1}{g} \int_0^{\beta} d\tau \int d^2r \left\{ |(\partial_\mu - i A_\mu)|^2 \right\}, \mu = \tau, x, y$$

(2)

where Lagrange multiplier $\lambda$ enforces the constraint $z^+ z = 1$.

Generalizing the doublet $z$ to the $N$-component vector we arrive at the $CP^{N-1}$ model action. The saddle point equation for the $\lambda$-field produces the gap $m$ in the spectrum of $z$-quanta, and finally we get (we have rescaled $z$ by $\sqrt{g}$)

$$Z = \int Dz \int DA \int D\lambda e^{\beta S} - \int_0^{\beta} d\tau \int d^2r \left\{ |(\partial_\mu - i A_\mu)|^2 + \frac{m^2}{\beta^2} |z|^2 + i\lambda' |z|^2 \right\},$$

(3)

where we have used parametrization [7,8]:

$$i\lambda(x, \tau) = m^2 + i\lambda'(x, \tau),$$

with the condition

$$\int_0^{\beta} d\tau \int d^2r \lambda'(x, \tau) = 0.$$

It is important for us that in the thermally disordered phase the gap $m$ is exponentially small as a function of temperature [9]

$$m \simeq T \exp - \frac{\text{const}}{T}.$$  

(4)

We have briefly described the derivation of the $CP^{N-1}$ model from the nonlinear $\sigma$ model for the sake of completeness of our discussion. Microscopical derivation of the expression (3) was done in [4], where the coupling of $A$ field to the spin-Peierls order parameter was also found, but we will not consider this coupling here.

Integration over $z$ fields produces the effective action for $A$ and $\lambda'$ fields. In the large-$N$ limit, the gaussian (quadratic) fluctuations around the saddle point $<A> = 0$, $<\lambda'> = 0$ are leading, because the higher orders of $A$ ($\lambda'$) will have additional $\frac{1}{\beta^2}$ factors [5]. This quadratic approximation is equivalent to the calculation of the polarization operator of $z$ fields. The corresponding diagrams, describing generation of the gauge field energy [8], are shown on Fig.1 and in the Coulomb gauge ($A_0 = 0$) an explicit expression for the polarization is

$$\Pi_{\mu\nu}(q, \omega_n) = -\frac{1}{\beta} \sum_l \int \frac{d^2p}{(2\pi)^2} G(p, \epsilon_l) G(p + q, \epsilon_l + \omega_n) (2p + q)_{\mu}(2p + q)_{\nu} - 2\delta_{\mu\nu} \frac{1}{\beta} \sum_l \int \frac{d^2p}{(2\pi)^2} G(p, \epsilon_l),$$

(5)

where $G(p, \epsilon)$ is the Green’s function of $z$-bosons in the absence of gauge fields

$$G(p, \epsilon_l) = -\frac{1}{\epsilon_l^2 + p^2 + m^2}$$

and $\epsilon_l = 2\pi l T, \omega_n = 2\pi n T$ are bosonic Matsubara frequencies.

Both one-loop integrals in (5) are ultraviolet divergent but the Pauli-Villars regularization makes them finite. Because the gap $m$ is much smaller than the temperature $T$, the lowest excitations (i.e. those inside the gap) correspond to $\omega_n = 0$ in (5), which is equivalent to a static approximation. Making an expansion over $\frac{a^2}{m^2}$ in the integrand we get

$$\Pi_{\mu\nu}(q, \omega_n = 0) = \frac{1}{12\pi} \left( \frac{1}{\beta} \sum_l \frac{1}{\epsilon_l^2 + m^2} \right) (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) q^2.$$  

(6)
Using
\[
\frac{1}{\beta} \sum_l \frac{1}{\epsilon_l^2 + m^2} = \frac{1}{m^2} \frac{1}{2} + \frac{1}{e^{\beta m} - 1} \approx \frac{1}{\beta m^2},
\]
we obtain the following expression for the Green’s function of the gauge field
\[
D_{\mu\nu}(q) = - \langle TA_\mu(q)A_\nu(-q) \rangle = - \frac{1}{N} \Pi_{\mu\nu}^{-1}(q) = - \frac{12\pi\beta m^2}{Nq^2} (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}),
\]
which describes static random magnetic fields with the correlator
\[
\chi(q, 0, \omega = 0) = \frac{12\pi\beta m^2}{Nq^2}.
\]

Physically, the magnetic field \( h(r) \) describes the chirality through
\[
\frac{1}{2} \mathbf{n} \cdot (\partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n}) = \partial_\mu A_\nu - \partial_\nu A_\mu = h.
\]

The nature of static approximation is easy to understand: the minimal energy of the \( z \)-boson is \( m \), whereas \( A \) fields are lying inside the gap \( m \) and, hence, they can be considered as the static ones in comparison with fast \( z \)-quanta.

Knowing the Green’s function, it is easy to calculate the contribution of the gauge fluctuations to the specific heat
\[
\delta C_v \simeq m^2 \log T.
\]
Because of the \( m^2 \) prefactor, this contribution is small.

The problem of a particle subject to random magnetic field has arisen in the gauge theory of the \( t - J \) model \[8,9\], in the vortex lines dynamics of high-\( T_c \) materials, and in the study of the transport properties of the normal film -superconducting film sandwiches in a magnetic field \[10\]. Significant progress has been made recently in understanding of the problem, mainly due to the development of nonperturbative approaches \[11\]. Due to the long range nature of the vector potential \( A \) and two-dimensionality of the problem, the interaction between the \( z \)-particle and the gauge field is strong enough to produce divergences in the leading corrections to the particle’s self-energy and vertex \[8,9\].

As we will show later, \( z \)-bosons also have similar divergences. The divergence of the self-energy implies that the averaged over the gauge fluctuations Green’s function is zero, or the \( z \)-particle does not propagate. But it is necessary to keep in mind that \( z \)-boson describes an \( S = \frac{1}{2} \) excitation at \( N = 2 \), whereas physical excitations (i.e. those corresponding to poles in the susceptibility) are described by the pair of the \( z \)-particles , which form an \( S = 1 \) excitation \[12\].

At the mean field level the susceptibility is given by the bubble diagram (Fig.2), for an \( N \)-component boson system
\[
\chi_{mf}(q = 0, \omega = 0) = N \frac{1}{4\pi\beta} \sum_l \frac{1}{\epsilon_l^2 + m^2} = \frac{N}{4\pi\beta m^2}.
\]
\( \chi_{mf}(q = 0, \omega = 0) \) diverges at \( T = 0 \) \( (m = 0) \) indicating the transition into ordered Neel-type state (since \( \mathbf{n} \) is local staggered magnetization, \( q = 0 \) actually corresponds to antiferromagnetic vector \( (\pi, \pi) \) in the original problem). The first \( 1/N \) correction to \( \chi_{mf} \) is generated by the self-energy insertions to the boson lines and by the vertex correction (Fig.3, solid line denotes \( z \) propagator, wiggly - gauge-field propagator). The corresponding analytical expressions are:
\[
\chi^1_{mf}(q = 0, \omega = 0) = \frac{1}{\beta^2} \sum_l \int \frac{d^2k}{(2\pi)^2} \int \frac{d^2q}{(2\pi)^2} G^3(k, \epsilon_l)G(k - q, \epsilon_l)D_{\mu\nu}(q)(2k - q)_{\mu}(2k - q)_{\nu},
\]
\[
\chi^1_{mf}(q = 0, \omega = 0) = \frac{1}{\beta^2} \sum_l \int \frac{d^2k}{(2\pi)^2} \int \frac{d^2q}{(2\pi)^2} G^3(k, \epsilon_l)D_{\mu\nu}(q),
\]
\[
\chi^1_{mf}(q = 0, \omega = 0) = \frac{1}{\beta^2} \sum_l \int \frac{d^2k}{(2\pi)^2} \int \frac{d^2q}{(2\pi)^2} G^3(k, \epsilon_l)G^2(k - q, \epsilon_l)D_{\mu\nu}(q)(2k - q)_{\mu}(2k - q)_{\nu}
\]
Diagram 3b can be calculated exactly, while expansion over \( q^2 \) is necessary in other expressions. Straightforward calculations give the following answer

\[
\chi_{i}^{1/N}(q = 0, \omega = 0) = \frac{-1}{(2\pi\beta)^2} \sum_{l} \frac{1}{(\epsilon_{l} + m^2)^2} C_{l} \int \frac{dq}{q} \sum_{l} \frac{1}{(\epsilon_{l}^2 + m^2)^3} S_{l} \int dq
\]  

(12)

Numerical coefficients \( C_{l} \) and \( S_{l} \) are collected in Table I, as well as symmetry factors \((F)\) of diagrams.

As one should expect from the gauge invariance, the logarithmically divergent term disappears from the final expression for the first \( 1/N \) correction to susceptibility and, using \( m \) as an upper cut-off in the integration over \( q \), we get (subscript \( A \) denotes the gauge field origin of this correction)

\[
\chi_{A}^{1/N}(q = 0, \omega = 0) = \frac{0.7}{4\pi\beta m^2}
\]  

(13)

Unexpectedly, the susceptibility correction \( \chi_{A}^{1/N} \) is positive \((\chi = \chi_{mf} + \chi_{A}^{1/N})\), which implies that the gauge fluctuations enhance the short range antiferromagnetic order. This result is not as strange as it at first appears. Let us recall the phenomenon of the quantum stabilization of the classical order in the \( J_1 - J_2 \) model \([12]\): quantum fluctuations act to reinforce the classical order beyond the classical domain of stability.

The finite result for \( \chi_{A}^{1/N} \) means that despite the dramatic suppression of the one-particle motion (i.e. the divergency of the single-particle self-energy — diagrams 3a and 3b), the coherent propagation of a pair of \( z \)-particles is not affected significantly by the gauge fluctuations. This, in turn, means that the final physical picture is similar to that of the \( O(N) \) nonlinear \( \sigma \) model at \( N = \infty \): the elementary excitations in the disordered phase of the antiferromagnet are spin — 1 excitations.

It is possible to show the origin of the obtained result using an eikonal expansion \([14]\). Analogous to \([8]\), the self-energy divergence corresponds to the vanishing of the \( z \)-particle Green's function. In the eikonal approximation, the Green’s function depends upon the field \( A \) through the phase \( exp(i\Gamma) = \exp(i\int_{0}^{t} dr A_{\mu}(r) \frac{\partial A_{\nu}(r)}{\partial \tau} \)  , where \( r(\tau) \) is a straight line path connecting the initial \((r(0) = 0)\) and final \((r(t) = r')\) points. In the simplest approximation \( r(\tau) = r' \tau \), hence \( \Gamma = \int \frac{dq}{(2\pi)^2} \int_{0}^{t} dr A_{\mu}(q)e^{iqr'} \tau \approx r_{\mu} \int \frac{dq}{(2\pi)^2} A_{\mu}(q) \) for small enough \( q \). After averaging over gauge field fluctuations we have

\[
< e^{i\Gamma} > = \exp - \frac{1}{2} < (\Gamma)^2 > \approx \exp - \frac{1}{2} r_{\mu} r_{\nu} \int \frac{dq}{(2\pi)^2} < A_{\mu}(q)A_{\nu}(-q) >
\]

Using eq.(7) we arrive to the logarithmically divergent expression

\[
< e^{i\Gamma} > = \exp - \frac{1}{2} r_{\mu} r_{\nu} \frac{12\pi\beta m^2}{N} \int_{0}^{2\pi} d\theta sin \theta \int_{0}^{m} dq \frac{1}{(2\pi)^2 \frac{q^2}{q^2}}
\]

Thus, \( < e^{i\Gamma} > = 0 \) for any nonzero \( r' \). At the same time, in the susceptibility calculations we are dealing with closed trajectories \([8]\), where by the Stokes theorem \( \int \frac{dq}{(2\pi)^2} \int_{0}^{t} dr A(r) \frac{\partial A(r)}{\partial \tau} \) is the magnetic flux through the loop formed by trajectory \( r(\tau) \). According to (8) the magnetic field correlator is constant. Therefore the momentum integration will not produce infrared divergency.

We also tried to calculate the \( 1/N \) correction to the staggered dynamical susceptibility. In that case, the analogous cancelation of the logarithmically divergent terms was found. As before, it is nothing but the consequence of the gauge invariance of \( \chi \). Unfortunately, no damping was found for the frequency less than the gap. Thus, interaction with gauge field can not eliminate (at least at \( 1/N \) order) another shortcoming of the mean-field Schwinger-boson theory - the absence of the relaxational modes in the system. Such hydrodynamical excitations were found by the summation of the ladder diagrams describing the boson-boson interaction beyond the mean-field approximation of the Schwinger-boson theory \([12]\). In addition, recent study of the \( O(N) \) quantum nonlinear \( \sigma \) model by an \( 1/N \) expansion \([8]\) shows that an interaction with massive fluctuations of the constraint field does produce damping in the susceptibility and nonzero density of states inside the gap region.

Because of all these facts, we understand the problem as following: the main role of the gauge fluctuations is not only to eliminate the mean-field excitations from the spectrum, but to conserve the coherent pair propagation. After that, the problem is equivalent to the initial nonlinear \( \sigma \) model, which naturally describes \( S = 1 \) excitations.

To confirm this point we will calculate the effect of the massive \( \lambda \)-fluctuations on \( \chi \). As in the case of gauge fluctuations, we need the polarization operator \( \Pi_{\lambda}(q, \omega) \) for \( \lambda \)-field, which is the dynamical susceptibility near \( \mathbf{q}_{AFM} = \)
(π, π), (Fig.2). To be consistent with previous consideration, only static part of Π_λ(q, ω) has to be kept. The same arguments about long-distance behaviour permit us to use an expansion over q^2/m^2, and we obtain

\[ Π_λ(q, 0) = -\frac{1}{β} \sum_l \int \frac{d^2p}{(2π)^2} G(p, ε_l) G(p + q, ε_l) (i)^2 = \frac{1}{πβ(q^2 + 4m^2)} \]

Correspondingly, the λ-field temperature Green’s function is

\[ D_λ(q) = -\frac{1}{N} Π_λ(q, 0) \].

Unlike the gauge fluctuations, the fluctuations of the constraint field do not produce infrared divergence in the z-particle self-energy. Nevertheless, they give correction to χ of the same order of magnitude as A-fluctuations. This can be shown by the calculation of the diagrams of Fig.3, using instead of D_µν the λ-field Green’s function D_λ and the corresponding vertex i. By this way we obtained (again using m as an upper cut-off in the q-integration)

\[ χ^{1/N}_λ = \frac{0.875}{4πβm^2} \]

Thus, being relatively unimportant for single-particle properties, fluctuations of the constraint field are as significant for the spin susceptibility as those of the gauge field. Summing both 1/N corrections together, finally we have

\[ χ^{mf}(q = 0, \omega = 0) + χ^{1/N}(q = 0, \omega = 0) = χ^{mf}(q = 0, \omega = 0)(1 + 1.575/N) \]

The resulting effect of the fluctuations is to enhance the short range antiferromagnetic order, in agreement with the results of [5]. Interestingly, this enhancement arises from the integration over spatial scales which are much longer than the correlation length. At the same time, the renormalization of the correlation length and the damping effects arise at smaller spatial scale, as was shown in [5].

Equation (16) shows that both types of fluctuations are important in the problem under consideration. To obtain the complete 1/N correction to the susceptibility it is necessary to carry out the calculations in the entire energy and momentum region, which is much more complicated problem [5] and is beyond the scope of our paper.

In conclusion, we have shown that the spin-1/2 mean-field excitations in the thermally disordered phase of the quantum antiferromagnet are suppressed by the interaction with generated gauge massless field. At the same time this interaction enhances the susceptibility, which describes the spin-1 collective excitations. Analogous enhancement is produced by the massive fluctuations of the constraint field.

ACKNOWLEDGMENTS

The authors would like to thank Z.Y.Weng for collaboration on the early stage of this work and helpful discussions, and D.N.Sheng and C.S.Ting for useful remarks. We’d like to acknowledge also useful comments of the referee. One of us (O.A.S.) is grateful to A.F.Barabanov, A.G.Abanov, V.Brazhnikov, and I.Gruzberg for stimulating discussions. The present work is supported by a grant from Robert A.Welch Foundation and the Texas Center for Superconductivity at the University of Houston.

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| Diagram | C   | S         | F   |
|---------|-----|-----------|-----|
| a       | 1/6 | -1/20     | 2   |
| b       | -1/4| 0         | 2   |
| c       | 1/6 | -1/60     | 1   |

TABLE I. Numerical values of $C_i$, $S_i$, and symmetry factors $F$ of diagrams from Fig.3