Analysis of cascading failures of interdependent networks based on time-delay coupled map lattices model

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Abstract-The robustness of coupled network under cascading failure has attracted a lot of attention. In real word, the networks don’t exist in isolation, and the interdependent network is an important network model. In the process of fault propagation, there is usually delay, and a node may have multiple interdependent links. A cascading failure model based on coupled map lattices for small word interdependent network is built in this paper. The research shows that when the external perturbation 3.7<R<4.0, the cascading failure ranges of high-degree and random strategy in interdependent network are close, and the different time-delay can induce different range of cascading failure in interdependent network. The thresholds of the external perturbation are common under different time-delay when the whole interdependent network is failed. The time-delay also can prolong the failure spreading time during which measures can be taken to suppress cascading failures. The research can provide a reference for building high-robust transport interdependent network.

1. INTRODUCTION
Cascading failures are widespread in real life. Cascading failures describe that when some nodes or edges are failed, the failure will spread over a large area of the network [1]. The cascading failure can happen in the power grids, Internet and hydraulic system because some nodes are failed. For example, one power station in the power grid is in overload will induce many power stations overload and cause a massive shutdown of the power grid. The blackout in North America is a typical cascading failure, and the accident caused huge economic losses.

Form the above analysis, the study of cascading failure in complex networks has great practical demands, and study is mainly concerned with cascading failed modeling [2-5], the influence of different
attack strategies [6,7] and the precaution strategies [8-11]. In cascading failure modeling, it can be divided into the cascading failure of single network [2,3] and multilayer network [4,5]. Attack methods can be divided into random attacks and deliberate attacks [6]. The deliberate attacks have high-degree strategy and high-clustering coefficient strategy and so on [6]. The precaution strategies involve precaution strategies and optimization strategies. In real life, the networks do not exist in isolation, and they are coupled to each other. Interdependent is an important coupling way of networks. It is of practical importance to study cascading failure of interdependent network. In the study of cascading failure in interdependent network [12-14], some are topologies researches [12], some are analyzing the cascading failure with the node load [13,14]. The load flows through the network according to certain rules, and the node failure occurs when the load exceeds capacity.

Coupled map lattice (CML) is a dynamical system with discrete time, discrete space and continuous state variables. Wang and Xu [15] study the cascading failure in globally coupled mapping network, small-world network and scale-free network. Most of the researches based on the cascading failure of coupled map lattices model are single network [7,16]. For multilayer networks, Shao et al. [17] investigated the cascading failure of coupled networks, but each node has at most one coupling-link. Peng et al. [18] investigated cascading invulnerability of multi-coupling-links coupled networks based on time-delay coupled map lattices model. Time-delay exists widely in reality, it is important to investigate the network with time-delay. This paper will investigate the cascading failures of interdependent networks based on time-delay coupled map lattices model, and the effects of attack method and time-delay.

2. THE DESCRIPTION OF MODEL

2.1 the interdependent networks model

The two layers of interdependent networks which contain sub-network A and sub-network B are introduced in this paper. The links between network A and network B are called interdependent links. The links in network A or network B are called intra-links. The intra-degree distribution of network A and network B is \( p(A,k) \) and \( p(B,k) \), and the inter-degree distributions of network A and network B is \( p(A,d) \) and \( p(B,d) \). In this paper, we set \( \langle k^A \rangle = \langle k^B \rangle \) and \( \langle d^A \rangle = \langle d^B \rangle \). And the sizes of the two sub-network are same, \( N_A = N_B = N/2 \). The two sun-networks A and B are produced by using Watts and Strogatz (WS) network model. The construction algorithm of the WS network model are as follows: 1) start from a nearest-neighbor coupled network: consider a nearest-neighbor coupled network with N nodes arranged on a ring, and every node connected to its K/2 nodes on either side. 2) random rewriting: with a probability p, each link is rewired to a node over all the node in the network except self itself. If the p=0, the network will not change. And the network will become random network. If the node i in network A relies on the node j in network B but the node j dose not rely on node i, the failure of node j will cause the failure of node i. The structure of the interdependent networks under attacking is shown in Fig.1.

![Figure 1. The structure interdependent network](image-url)
2.2 The cascading failure model of interdependent networks based on time-delay CML

The cascading failure model of interdependent networks based on time-delay CML can be described as follows, for the sub-network A:

\[
x_i(t+1) = \left[ (1 - \alpha_i - \alpha_{ab}) f(x_i(t)) + \alpha_i \sum_{j=1}^{N/2} a_{ij} f(x_j(t - \tau_{ij})) / k_i^a + \alpha_{ab} \sum_{m=1}^{N/2} c_{im} f(y_m(t - \tau_{im})) / d_i^a \right], \quad i = 1, 2, \ldots, N/2
\]

For the sub-network B:

\[
y_n(t+1) = \left[ (1 - \alpha_n - \alpha_{ba}) f(y_n(t)) + \alpha_n \sum_{m=1}^{N/2} b_{nm} f(y_m(t - \tau_{mn})) / k_n^b + \alpha_{ba} \sum_{j=1}^{N/2} c_{nj} f(x_j(t - \tau_{jn})) / d_j^b \right], \quad n = 1, 2, \ldots, N/2
\]

The \( x_i(t) \) and \( y_n(t) \) are the state of \( i \)th node in network A and \( n \)th node in network B at \( t \)th time step. \( A = (a_{ij})_{N/2 \times N/2} \) and \( B = (b_{nm})_{N/2 \times N/2} \) are the adjacent matrix of network A and network B. If node \( i \) is connected to node \( j \), the \( a_{ij} = 1 (b_{ij} = 1) \), otherwise, \( a_{ij} = 0 (b_{ij} = 0) \). \( \alpha_i \) and \( \alpha_n \) are the coupling strength in network A and network B. \( \alpha_{ab} \) and \( \alpha_{ba} \) are the coupling strength between network A and network B. \( \tau_{ij}, \tau_{mn} \) and \( \tau_{im}, \tau_{jn} \) are the time delay. In this paper we assume \( \tau_{ij} = \tau_{mn} = \tau_1 \), \( \tau_{im} = \tau_{jn} = \tau_2 \), \( \alpha_i = \alpha_n = \alpha_1 \) and \( \alpha_{ab} = \alpha_{ba} = \alpha_2 \). The formulas (1) and (2) can be rewritten as follows:

\[
x_i(t+1) = \left[ (1 - \alpha_i - \alpha_2) f(x_i(t)) + \alpha_i \sum_{j=1}^{N/2} a_{ij} f(x_j(t - \tau_{ij})) / k_i^a + \alpha_2 \sum_{m=1}^{N/2} c_{im} f(y_m(t - \tau_{im})) / d_i^a \right], \quad i = 1, 2, \ldots, N/2
\]

\[
y_n(t+1) = \left[ (1 - \alpha_n - \alpha_2) f(y_n(t)) + \alpha_n \sum_{m=1}^{N/2} b_{nm} f(y_m(t - \tau_{mn})) / k_n^b + \alpha_2 \sum_{j=1}^{N/2} c_{nj} f(x_j(t - \tau_{jn})) / d_j^b \right], \quad n = 1, 2, \ldots, N/2
\]

Here, mapping function \( f(x) \) denotes the local dynamics which is chosen as the logistic map \( f(x) = \lambda x (1 - x) \), \( \lambda \leq 4 \). The absolute value sign is guaranteed that the state of each node is non-negative. In this paper, we chose the \( \lambda \) as 4, and the state of node is chaotic.

The initial state of node in the network A and network B is in \((0, 1)\). If there is no external perturbations, and then the interdependent network will be in normal state. To study the cascading failure
in the interdependent network, we add the external perturbation $R \geq 1$ to one node in network A at sth time step. The model is as follows:

$$x_i(s) = \left(1 - \alpha_1 - \alpha_2\right)f(x_i(s-1)) + \alpha_1 \sum_{j=1}^{N_A} a_{ij} f(x_j(s-1)) / k_i^a + R, \quad (5)$$

$$+ \alpha_2 \sum_{m=1}^{N_B} c_{im} f(y_m(s-1)) / d_i^b$$

$$i = 1, 2, \ldots, N / 2$$

The state of $c$th node in network $x_i(s) > 1$, and the node is failed at sth time step. The state of the node will be 0 at all $t > s$. And other nodes in the interdependent network will be affected by the failed node. And the other node states evolve according to the formula (3) and (4). The cascading failure will spread to the whole interdependent network. The total failed nodes in network A and network B is $I_A$ and $I_B$, and the cascading failure size of the interdependent network is

$$S = \frac{I_A + I_B}{N} \quad (6)$$

3. RESULTS & DISCUSSION

3.1 The effects of different attack strategies on the robustness of interdependent network

To trigger the cascading failure of the interdependent network, we attack one node in the network A. When the node is attacked, the external perturbation $R$ is added to the state of the node. $R$ is the strength of the external perturbation. In this paper, three attack strategies are used to investigate the cascading failure. The three strategies are as follows:

Random strategy (RS):

For random strategy (RS), one node is selected randomly from the initial network.

High-degree strategy (HDS):

The degree, an important local property of complex networks, is the number of node’s direct connections. Attack high-degree node is a widely used intentional attacking strategy. Under HDS, we sort node in descending order of node’s degree and chose top node from the sorting list to form the perturbed node.

Low-degree strategy (LDS):

LDS is the reverse of LDS.

We fix the $\alpha_1 = \alpha_2 = 0.5, \quad \tau_1 = \tau_2 = 5, \quad N = 1000$, the rewriting probability $P=0.3$. We can obtain the $S \sim R$ curves for three attack strategies as Figure 2.
Figure 2. The $S - R$ curves for three attack strategies

$S$ is the size of cascading failure in interdependent network. In Figure 2, the $S$ corresponding to each $R$ is the average value of the results obtained after 100 simulations. The stronger the external perturbation is, the larger the range of cascading failure is. The high-degree strategy will induce the largest range cascading failure. And the interdependent network is most robust when the attack strategy is low-degree strategy. When $3.7 < R < 4.0$, the cascading failure ranges of high-degree and random strategy are close. In the later simulation, the attack strategy adopts high-degree attack.

3.2 The effects of time-delay on the robustness of interdependent network

The effects of time-delay can be divided into two aspects. One aspect is that different time-delay can induce different cascading failure range. The other is that time-delay make the failure time longer.

For the aspect one, we fix the $\alpha_1 = \alpha_2 = 0.5$, $N = 1000$, the rewriting probability $P=0.3$. We can obtain the $S - R$ curves for different time-delay as Figure 3.

As show in Figure 3, different time-delay can induce different cascading failure range in interdependent network. But we can’t obtain how time-delay affect the range of cascading failure. We can conclude that the thresholds of the external perturbation $R$ are common under different time-delay when the whole interdependent network is failed. And when the external perturbation $R$ is small, the cascading failure ranges are close under different time-delay.

Figure 3. The $S - R$ curves for different time-delay

For the aspect one, we fix the $\alpha_1 = \alpha_2 = 0.5$, $N = 1000$, $R = 5.5$, and the rewriting probability $P=0.3$. We can obtain the $S - t$ curves for different time-delay as Figure 4.
Figure 4. the $s-t$ curves for different time-delay

From Figure 4, we can conclude that the larger time-delay is, the longer cascading failure duration is. But the time-delay can’t affect the range of cascading failure. Time-delay prolongs the failure propagation time and buys time for taking protective measures.

4. CONCLUSIONS

In this paper, the cascading failure model of interdependent network based on time-delay coupled map lattices is proposed. We mainly investigate the effects of different attack strategies and time-delay on cascading failures. The stronger the external perturbation is, the larger the range of cascading failure is, and the interdependent network is most robust when the attack strategy is low-degree strategy. When $3.7 < R < 4.0$, the cascading failure ranges of high-degree and random strategy are close. For the effect of time-delay, different time-delay can induce different range of cascading failure, and the time-delay can prolong the failure spreading time during which measures can be taken to suppress cascading failures. When the external perturbation $R$ is small, the cascading failure ranges are close under different time-delay. When improving the robust of interdependent network, the parameters can be selected referring to the relevant research in this paper.

REFERENCES

[1] Boccaletti S, Latora V, Moreno Y. (2006) Complex networks: structure and dynamics. Physics Reports, 424, 175-308
[2] Volker Turau, Christoph Weyer. (2019) Cascading failures in complex networks caused by overload attacks. Journal of Heuristics, 25(6).
[3] Xingzhao Peng, Hong Yao, Jun Du, Zhe Wang, Chao Ding. (2015) Invulnerability of scale-free network against critical node failures based on a renewed cascading failure model. Physica A: Statistical Mechanics and its Applications, 421.
[4] Chao Ding, Hong Yao, Jun Du, Xing Zhao Peng. (2018) Load-induced cascading failures in interconnected network systems. International Journal of Modern Physics C, 29(8).
[5] Yao Lu, Yanyan Chen, Jie Xiong, Ning Chen, Bin Zhou, Xuzhen Zhu. (2019) Effects of group size distribution on cascading failure in partially interdependent networks. Physica A: Statistical Mechanics and its Applications, 534.
[6] Guizhen Yang, Xiaogang Qi, Lifang Liu. (2020) Research on network robustness based on different deliberate attack methods. Physica A: Statistical Mechanics and its Applications, 545.
[7] Kai-Jun Xu, Chen Hong, Xu-Hong Zhang, (2020) Qing-Hua Sun, Ning He, Ming-Ming Xiao. Cascades in coupled map lattices with heterogeneous distribution of perturbations. Physica A: Statistical Mechanics and its Applications, 547.
[8] Tang Liang, Jiao Peng, Li Ji Kang. (2018) Study on cascading failure mechanism and robustness of complex networks with recovery strategy. Control and Decision-Making, 33(10):1841-1850
[9] Gao J, Buldyrev S V, Havlin S. Robustness of a network of network. Physical Review Letters, 107:195701
[10] Salo D R, Hu Y Q, (2014) Babino A. Avoiding catastrophic failure in correlated networks of networks, Nature Physics, 10(10):762-768
[11] Di Muro M A, La Rocca C E, Stanley H E. (2016) Recovery of interdependent networks, Scientific Reports, 6(1):22834
[12] Xin Su, Jinming Ma, Ning Chen, Xuzhen Zhu. (2019) Cascading failures on interdependent networks with multiple dependency links and cliques. Physica A: Statistical Mechanics and its Applications, 526.
[13] Meng Tian, Zhengcheng Dong, Mingjian Cui, Jianhui Wang, Xianpei Wang, Le Zhao. (2019) Energy-supported cascading failure model on interdependent networks considering control nodes. Physica A: Statistical Mechanics and its Applications, 522.
[14] Xingzhao Peng, Hong Yao, Jun Du, Zhe Wang, Chao Ding. (2015) Study on cascading invulnerability of multi-coupling-links coupled networks based on time-delay coupled map lattices model. Acta Phys. Sin., 64(04):355-362.
[15] Wang XF, and Xu J. (2004) Cascading failures in coupled map lattices. Phys. Rev. E, 70: 5.
[16] Shao J, Buldyrev SV, Havlin S, and Stanley HE. Cascade of failures in coupled network systems with multiple support-dependence relations. Phys. Rev. E, 83: 9.
[17] Wang E S, Hong C, Zhang X H, et al. (2019) Cascading failures with coupled map lattices on Watts–Strogatz networks. Physica A: Statistical Mechanics and its Applications, 525:1038-1045.
[18] Xingzhao Peng, Hong Yao, Jun Du, Chao Ding, Zhihao Zhang. (2014) Study on cascading invulnerability of multi-coupling-links coupled networks based on time-delay coupled map lattices model. Acta Phys. Sin., 63(07):440-446.