Dephasing of interacting tunneling systems
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Abstract

We investigate the phase coherence time of weakly interacting tunneling systems (TSs). We show that all neighbors of a given TS form together with the TS of interest an entangled cluster as long as the linewidth of the excitation of the neighbor is smaller than its interaction energy with the TS of interest. Thus, the relaxation of all neighbors in the cluster contributes to the dephasing of the TS of interest. This mechanism dominates the transversal decay of the TSs and it explains recent two-pulse echo experiments in which the exponential decay rate could not be explained within spectral diffusion consistent with internal friction data. However, since the proposed mechanism predicts only an exponential decay, the Gaussian like decay at short times remains unexplained.

Keywords: interacting tunneling systems, dephasing rate, spin-echo

1 Introduction

The low temperature properties of glasses are governed by tunneling systems (TSs) and their behavior is well described by the phenomenological Tunneling Model (TM) down to temperatures of about 100 mK. Therein, the TSs are considered as independent two-level systems (TLSs) with energy splitting $\epsilon = \sqrt{\Delta^2 + \Delta^2}$. The tunneling splitting $\Delta_0$ and the asymmetry $\Delta$ are assumed to be broadly distributed. The TSs interchange energy with phonons via their elastic moment leading to a relaxation towards thermal equilibrium. At low temperatures the one-phonon process dominates yielding the relaxation rate $\gamma_l = \gamma_0 \Delta^2 \coth (\beta \epsilon / 2)$ and the phase coherence time $\tau_p = 1 / \gamma_l = 2 / \gamma_l$.

For temperatures below about 100 mK the tunneling model fails in various aspects which is commonly attributed to small interactions between the tunneling systems. Although the assumption that with decreasing temperature the importance of interactions increases seems natural, the theoretical understanding of the influence of interactions between tunneling systems to their dynamics is still unsatisfactory. The importance of including interactions was firstly realized in polarization echo experiments which are analogous to magnetic spin echos. For example in so-called two pulse echos one is able to investigate directly the phase coherence time of a subset of TSs with fixed energy splitting. Thereby the decay of the phase coherence is partly due to the one-phonon process but the dominating process is due to the interactions between the TSs. All TSs with energy splittings smaller than the temperature undergo continuously thermal transitions which are accompanied by fluctuating strain fields in their environment. Since every TSs under investigation within an echo experiment will have many such neighbors, this leads to a fluctuating energy splitting for these TSs. This process know as spectral diffusion destroys the phase coherence of the echo amplitude.
Within spectral diffusion one distinguish between two limits. At short times $t_{12} \ll \tau_{\text{min}}(T) := 1/\gamma(\epsilon=\Delta_0=T)$ the decay of the two-pulse echo amplitude $A(t_{12})$ is expected to be non-exponential, varying with the time $t_{12}$ between the two pulses like $\propto \exp(-2t_{12}/\tau_{\text{p}1})$ with $\tau_{\text{p}1} = \sqrt{h\tau_{\text{min}}/(2\delta\epsilon)}$. Thereby $\tau_{\text{min}}$ is the minimal relaxation time (governed by the one-phonon process) of thermal TSs, which contribute the most to spectral diffusion, and $\delta\epsilon = (\Delta/\epsilon)\tau_{\text{min}}P_0k_BT$ is the spectral width of the energy splitting $\epsilon$ of a TS with asymmetry $\Delta$ due to the interaction with its neighbors. $J_{\text{min}}$ is the minimal coupling between the TSs and $P_0$ is the prefactor of the TM-distribution $P(\Delta_0, \Delta) = P_0/\Delta_0$. The expression $J_{\text{min}}P_0k_BT$ is approximately the mean coupling between thermal TSs. In the long time regime $t_{12} \gg \tau_{\text{min}}$ one finds an exponential decay: $\propto \exp(-2t_{12}/\tau_{\text{p}2})$ with $\tau_{\text{p}2} = h/(4\delta\epsilon)$. From the experimental point of view both time regimes where found but the extracted minimal relaxation time $\tau_{\text{min}}$ is by several orders of magnitude shorter than the values extracted form internal friction experiments.

A key assumption in the theory of spectral diffusion is that the interaction $J$ with a neighboring TS is smaller than the linewidth of the excitation energy of that neighbor. Accordingly a flip of that neighbor is possible without energy transfer between the two TSs and they can still be treated as isolated. Due to the broad distribution of interaction energies we expect that many neighbors will not fulfill that condition. In that case, where the linewidth is smaller than the coupling, the TSs are entangled and we have to treat them as a coupled cluster.

In the following we investigate the phase coherence time of a TS within such a cluster.

## 2 Interacting tunneling systems

Since the couplings are considered as weak, it is convenient to switch directly to the diagonal basis of the uncoupled TLSs leading to the Hamiltonian

$$
\hat{H} = \sum_i \frac{\epsilon_i}{2} \sigma^{(i)}_x - \sum_{i\neq j} \bar{u}_i \bar{u}_j J_{ij} \sigma^{(i)}_x \sigma^{(j)}_x
$$

with $u_i = \Delta_{0i}/\epsilon_i$, $\bar{u}_i = \Delta_i/\epsilon_i$ and $\epsilon_i = \sqrt{\Delta_{0i}^2 + \Delta_i^2}$. The main approximation of the present paper is to discuss the four coupling terms separately; thus, we are left with three cases: the $\sigma^{(i)}_x \sigma^{(j)}_x$-, the $\sigma^{(i)}_z \sigma^{(j)}_z$- and the $\sigma^{(i)}_z \sigma^{(j)}_x$-coupling. This separation neglects correlation effects between the different coupling terms but it reveals first order effects like a perturbative approach.

We find that the term $\propto \sigma^{(i)}_x \sigma^{(j)}_x$ only yield corrections to the dynamics of the order $O((J/\epsilon)^2)$, thus they are negligible for weak couplings. The same holds true for the transversal coupling term $\propto \sigma^{(i)}_z \sigma^{(j)}_z$ unless the TSs are resonant, meaning that they have comparable energy splitting $\epsilon_i \approx \epsilon_j$; to be precise, the difference should

1The $\sigma^{(i)}_x \sigma^{(j)}_x$-coupling term is equivalent to the $\sigma^{(i)}_z \sigma^{(j)}_x$-term.
be smaller than the coupling between them: $|\epsilon_i - \epsilon_j| \leq J_{ij}$. Since the probability of such pairs is $\leq 10^{-3}$, this term is also neglectable. Burin et al. [7] found that resonant pairs lead to a new relaxation mechanism at lowest temperature in glasses. But this effect bases inherently on the combination of the $\sigma_x^{(i)\sigma_x^{(j)}}$ and the $\sigma_x^{(i)\sigma_z^{(j)}}$-term and, thus, it is beyond the scope of the present paper. We emphasize that for weak coupling the most important coupling term is the $\propto \sigma_x^{(i)\sigma_x^{(j)}}$-term leading to the Hamiltonian: $\hat{H} = -\sum_i \frac{\epsilon_i}{2} \sigma_x^{(i)} - \sum_{i\neq j} \tilde{u}_{i\sigma_x^{(i)}\sigma_x^{(j)}} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)}$.

In order to describe dynamics exhibiting relaxation and phase decoherence, we couple additionally each TSs to phonons. This problem is investigated in detail in ref. [8]. The Hamiltonian is diagonal and accordingly the eigenstates of the many-body problem are the product states of the isolated TSs. Nevertheless, the interaction leads to entanglement between the TSs.

For example, consider a TS with energy splittings $\epsilon_1$ coupled just to one neighbor with splitting $\epsilon_2$ and an interaction energy $J_{12}$. If we couple with an field to the TS 1, the excitation energy $\omega = \epsilon_1 \pm 2J_{12}$ depends on the present state of the second TS. The dephasing or transversal rate of the TS 1 depends also on the state of the second TS [8]. As long as the coupling energy $J_{12}$ is bigger than the linewidth $\gamma_2$ of the second TS (given by the transversal rate of the isolated second TS), the dephasing rate for the first TS is the sum $\Gamma_1 = \gamma_1 + 2\gamma_{2\uparrow}/4$ with the one-phonon rate $\gamma_1$ of the isolated TS 1 and the decay rate of the second TS due to phonon emission $\gamma_{2\uparrow} = \gamma_0 u_2^3 \epsilon_2 (1 + n(\epsilon_2))$ or phonon absorption $\Gamma_{2\downarrow} = \gamma_0 u_2^3 \epsilon_2^3 n(\epsilon_2)$ with the Bose factor $n(\epsilon_2)$. Here and in the following we approximate $\epsilon_1 \pm J_{ij} \approx \epsilon_i$. If $\gamma_2 > J_{12}$ the coupling is negligible and the dynamics of the first TS is independent of the second one.

The same holds true for many neighbors of a given TS $\alpha$ leading to a dephasing rate

$$\Gamma_t = \gamma_{\alpha} + 2 \sum_{i \in \langle J_{i\alpha} > \gamma_i \rangle} \left\{ \begin{array}{ll} \gamma_{i\uparrow} & \text{for TLS } i \text{ in the ground state} \\ \gamma_{i\downarrow} & \text{for TLS } i \text{ in the excited state} \end{array} \right.$$  \hspace{1cm} (1)

Thereby we distinguish between neighboring TSs fulfilling $J_{i\alpha} > \gamma_i$, which contributes to the rate of TS $\alpha$ and the TSs whose coupling to the TS $\alpha$ is to weak. This second group of neighbors causes spectral diffusion. Assuming homogeneously distributed TSs and a dipole like interaction between them, we expect a distribution of interaction energies like $P(J) \propto 1/J^2$ with equal probability for both signs. Since the above introduced conditions only involve the absolute values of the coupling energies we can neglect the sign of the coupling in the following. In order to gain simple analytical expressions we separate the neighbors sharply between thermally active and inactive ones with the further approximation $\exp(-\epsilon_i/(k_B T)) \approx \Theta(k_B T - \epsilon_i)$.

Using the tunneling model distribution we get for the dephasing

$$\langle e^{-\Gamma_t} \rangle \approx e^{-\gamma_{\alpha} t - \Gamma_{ww} t} \text{ with } \Gamma_{ww} = 2\tilde{u}_{\alpha}(P_0 J_{\min}) (k_B T) \ln \left( \frac{k_B T}{\Delta_{\min}} \right)$$  \hspace{1cm} (2)

with the minimal interaction energy $J_{\min}$ and the minimal tunneling element $\Delta_{\min}$. Thereby $\langle \cdot \rangle$ denotes the thermal averaging of the neighbors. $(P_0 k_B T)$ is the number of thermally active TSs and $J_{\min}$ is approximately the mean interaction between two TSs: $J \approx J_{\min} \ln(J_{\max}/J_{\min})$. 

3
Thus the entanglement between the TSs mediated by weak couplings results in a considerably fastened dephasing.

3 Discussion and Summary

In our approach we divide the neighbors of a given TS $\alpha$ into two groups. The TSs, fulfilling $J_{i\alpha} > \gamma_i$, form a cluster with the TS $\alpha$ leading to an enhanced decoherence rate for the TS $\alpha$. The dynamics of the other neighbors with $J_{i\alpha} < \gamma_i$ still cause spectral diffusion. The dephasing rate due to the entangled cluster is comparable with the exponential decay of spectral diffusion in the long time limit: $\Gamma_{ww} \simeq 1/\tau_{p2}$. In typical echo experiments the decay in the long time regime should also be faster than the one of the short time regime since $\tau_{\text{min}}(T) > \tau_{p2}$. Accordingly we expect the decay $\exp(-\Gamma_{ww}t)$ due to the entanglement to dominate the decay studied in two pulse echo experiments.

So far, the present theory describes fairly well the temperature dependence as well as the absolute value of the exponential decay measured in two-pulse polarization echoes in $\text{B}_2\text{O}_3$ [10] and in $\text{SiO}_2$ (Suprasil I) [9]. But within this theory we can not account for the initial non-exponential decay found at lowest temperatures. At least, the crossover depends no longer on the minimal relaxation time $\tau_{\text{min}}(T)$ of the thermal TSs. Thus the inconsistency with data of the internal friction is resolved.

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