When teaching:

Out with magnitudes,
in with monochromatic luminosities!

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Abstract

The goal of this document is to illustrate that teaching the concepts of magnitudes is a needless complication in introductory astronomy courses, and that use of monochromatic luminosities, rather than arbitrarily defined magnitudes, leads to a large gain in transparency. This illustration is done through three examples: the Hertzsprung-Russell diagram, the cosmic distance ladder, and interstellar reddening. I provide conversion equations from the magnitude-based to the luminosity-based system; a brief discussion; and a reference to sample lecture notes.

I suggest that we, astronomers in the 21st century, **abolish magnitudes and instead use (apparent) monochromatic luminosities** in non-specialist teaching. Given the large gain in transparency I further propose that we seriously consider using (apparent) monochromatic luminosities also in research papers, bringing optical astronomy in line with astronomy at other wavelengths.

I would appreciate your opinion, comments and suggestions (see Sect. 5).

1 The Hertzsprung-Russell diagram

In analogy with the total or bolometric luminosity, we define the **monochromatic luminosity** $L_\lambda$ as

$$L_\lambda = 4\pi R^2 F_\lambda = 4\pi d^2 f_\lambda$$  \hspace{1cm} (1)$$

Here $F_\lambda$ and $f_\lambda$ are fluxes per unit wavelength, i.e. monochromatic fluxes, at the stellar surface and at Earth, respectively, $R$ and $d$ are the radius of and distance to
Figure 1: Monochromatic luminosity $L_{550}$ as a function of colour $F_{550}/F_{440}$ of 20305 stars with an Hipparcos distance more accurate than 10%. The corresponding $M_V$ and $B - V$ are shown at the right hand side and top.

the star. For the Sun, the flux at 550 nm in a band of 1 nm is $f_{550} \approx 2 \text{ W m}^{-2} \text{ nm}^{-1}$, and thus with $d = 1 \text{ AU}$, $L_{550} \approx 5.6 \times 10^{23} \text{ W nm}^{-1}$. (In fact, this flux is averaged over the Johnson V filter, and one may prefer to denote it $f_V$.)

The Hipparcos catalogue provides distances $d$ and monochromatic fluxes $f_{550}, f_{440}$ which with Eq. 1 give the monochromatic luminosity $L_{550}$ and colour $F_{550}/F_{440} = f_{550}/f_{440}$. (For the moment we assume that interstellar absorption is negligible.) We combine these in a colour - luminosity diagram Figure 1. This Figure tells us immediately that a B main-sequence star emits about 100 times more photons at 550 nm than the Sun, and a K dwarf about 10 times fewer. It also tells us directly that an early G star emits about as much flux at 550 nm as at 440 nm.

Such information is also present in the diagram labeled with $M_V$ and $B - V$, but very much hidden.

2 The cosmic distance ladder

For distances beyond the reach of parallax measurements, several methods are available that are based on monochromatic luminosities. For these we define the apparent monochromatic luminosity $L_{\lambda a}$, based on an assumed distance $d_a$, as:

$$L_{\lambda a} = 4\pi d_a^2 f_{\lambda}$$

(2)

We allow any choice for $d_a$, which means that the apparent luminosity must be given in tandem with an assumed distance. The logarithms of the real and apparent
monochromatic luminosities differ, with Eqs. 1 and 2, by:

\[ \Delta L \equiv \log L_\lambda - \log L_{\lambda a} = 2 \log \frac{d}{d_a} \]

\[ = \log \frac{L_\lambda}{L_\odot} - \log \frac{L_{\lambda a}}{L_\odot} \quad (3) \]

When we can determine this shift, the distance follows as

\[ d = 10^{0.5 \Delta L d_a} \quad (4) \]

2.1 Main-sequence fitting

As an example I show in Figure 2 the apparent luminosity at 550 nm for stars in the Pleiades for an assumed distance \( d_a = 1 \) pc, together with the lower envelope of the main-sequence for nearby stars, from Figure 1. A vertical shift of \( \Delta L \simeq 4.4 \) will put the main sequence of the Pleiades just above the lower envelope for nearby stars. The distance of the Pleiades follows with Eq. 4 as about 160 pc.

2.2 The Cepheid period-luminosity relation

Figure 3 shows the period - luminosity relation for Cepheids in the Small Magellanic Cloud (left frame; the luminosity shown for each Cepheid is the average of the maximum and minimum value of \( \log L_{550} \)) and lightcurves of three Cepheids in the galaxy IC 4182 (right frame; apparent luminosities for an assumed distance \( d_a = 1 \) Mpc). The brightest Cepheid in IC 4182 shown has a period of 42 d and an average apparent luminosity \( \log L_{550 a}/L_{550 \odot} \simeq 3.15 \). From the left frame of Figure 3 we see that the correct luminosity at 42 d is \( \log L_{550}/L_{550 \odot} \simeq 4.2 \), i.e. \( \Delta L \simeq 1.05 \). With Eq. 4 a distance of about 3.3 Mpc follows for IC 4182.

\[ ^1 \text{This requires a distance for the SMC; in my lecture notes I determine the distance to h and \( \chi \) Persei by main-sequence fitting, and the distance to the SMC by demanding that the period-luminosity relation passes through the values for the Cepheids in h and \( \chi \) Persei} \]
Figure 3: The period-luminosity relation for Cepheids in the Small Magellanic Cloud (left frame), and apparent monochromatic luminosities $L_{550\alpha}$ for assumed distance $d_a = 1$ Mpc for three Cepheids in IC 4182. The periods of the Cepheids in IC 4182 are for decreasing luminosity: 42 d, 9.24 d and 4.26 d.

Figure 4: The lightcurve at 550 nm for supernova of type Ia 1937C in IC 4182, and lightcurves of apparent monochromatic luminosities for supernovae of type Ia in the Virgo and Coma clusters (computed for $d_a = 1$ Mpc, the lightcurve for Coma is a composite of several supernovae).

2.3 Maximum luminosity of supernova Ia

In 1937 a supernova of type Ia was observed in IC 4182. With the distance of this galaxy derived from the Cepheids in it, we can compute the monochromatic luminosities of this supernova. The resulting lightcurve is shown in Figure 4. In Figure 4 I also show the lightcurve for a supernova Ia in the Virgo cluster, and a composite lightcurve of several supernovae Ia in the Coma cluster, computing apparent luminosities for an assumed distance $d_a = 1$ Mpc. We see from the maxima
(at $t = 0 \text{ d}$) that $\Delta L \simeq 2.5$ for Virgo and $\Delta L \simeq 4$ for Coma, giving distances with Eq. 4 of about 18 Mpc and 100 Mpc, respectively.

### 3 Interstellar absorption and reddening

Absorption and scattering of radiation by interstellar matter is characterized by the optical depth $\tau_\lambda$, where the subscript indicates the wavelength dependence. The flux $f_\lambda^c$ corrected for the absorption is given by

$$ f_\lambda^c = f_\lambda e^{\tau_\lambda} \tag{5} $$

*It is this corrected flux which should be entered in Eqs. [1][2] when $\tau_\lambda > 0$.*

Because of the wavelength dependence of the absorption, the colours of the star change. With Eq. 5 we can relate the change in colour to the optical depths at the different wavelengths. For the visual-to-blue flux ratio, we find

$$ \log \frac{f_{550}^c}{f_{440}^c} = \log \frac{f_{550}}{f_{440}} + (\tau_{550} - \tau_{440}) \log e \tag{6} $$

Generalizing this equation to arbitrary $\lambda$, we see that by taking the ratio of the intrinsic colour $f_\lambda^c/f_{550}^c$ and the observed colour $f_\lambda/f_{550}$ we can derive the ratio $\tau_\lambda/\tau_{550}$. Figure 5 shows how this works. From the line strengths we can derive the spectral types of the stars as O5 and A0 V, even if the spectrum is affected by interstellar absorption. The dependence of $\tau_\lambda$ on wavelength is found by dividing the observed, absorbed spectrum by the correct, standard spectrum. The result is shown in Figure 5 (right).
If we know the intrinsic colour of the star, we can derive the interstellar absorption even if we do not know the distance. For example, from the data used to make Figure 5 we have \( \tau_{440} \approx 1.32 \tau_{550} \). Hence

\[
\tau_{550} \approx 3.1 (\tau_{440} - \tau_{550})
\]  

(7)

where the right hand side is found from Eq. 6.

If we have measurements at three wavelengths, i.e. two independent colours, we can also determine the reddening, as illustrated in Figure 6. The left frame shows the main sequence colours in the absence of reddening, the right frame shows a reddened star that due to reddening no longer lies on this sequence. Corrections to the colours have been applied for assumed values of \( \tau_{550} \) in steps of 0.15. It is seen that a correction for \( \tau_{550} = 0.3 \) puts the star on the main-sequence colour-colour relation, close to a G5 V star. With the intrinsic luminosity \( L_{550} = L_{550\odot} \), the observed flux \( f_{550} = 5.65 \times 10^{-16} \text{ W s}^{-1} \text{ nm}^{-1} \), and the determined reddening \( \tau_{550} = 0.3 \) the distance of the star can be determined. The Figure also shows that the solution is not unique: an early F star reddened by \( \tau_{550} \approx 1.2 \), or an even more reddened B star, is also possible. Often the use of other colours allow selection of a unique solution.

4 Conversion formulae

Absolute visual magnitudes to luminosity at 550 nm:

\[
\log \frac{L_{550}}{L_{550\odot}} = 0.4 [M_{V\odot} - M_V] = 0.4 [4.81 - M_V]
\]  

(8)
Colour to flux ratio (for $\tau_\lambda = 0$):

$$\log \frac{F_{550}}{F_{440}} = \log \frac{f_{550}}{f_{440}} = 0.4 [B - V - 0.66] \tag{9}$$

$$\log \frac{F_{806}}{F_{550}} = \log \frac{f_{806}}{f_{550}} = 0.4 [V - I - 1.35] \tag{10}$$

$$\log \frac{F_{440}}{F_{365}} = \log \frac{f_{440}}{f_{365}} = 0.4 [U - B - 0.54] \tag{11}$$

The constants follow from the definitions of zero magnitude for $U$, $B$, $V$, and $I$.

Apparent visual magnitudes to monochromatic luminosity:

$$\log \frac{L_{550}}{L_{550\odot}} = 2 \log \frac{d}{1 \text{ pc}} - 0.4 [V + 0.19] \tag{12}$$

Absorption in magnitudes to optical depth:

$$\tau_\lambda = \frac{0.4 A_\lambda}{\log e} \tag{13}$$

hence colour excess to difference in optical depth:

$$\tau_{440} - \tau_{550} = \frac{0.4 E(B - V)}{\log e} \tag{14}$$

5 Discussion

What is wrong with magnitudes? The definition of the absolute magnitude as used in the original form of the colour-magnitude diagram

$$M_\lambda = m_\lambda + 5 - 5 \log \frac{d}{1 \text{ pc}} = -2.5 \log f_\lambda + c_\lambda - 5 \log \frac{d}{10 \text{ pc}}$$

has no less than four arbitrary elements, which we have to explain to students:

1. the minus-sign, which causes high magnitudes to correspond to low fluxes
2. the factor 2.5
3. the flux at zero apparent magnitude $m_\lambda$, as expressed in $c_\lambda$ (which has to be defined for each filter – or $\lambda$ – in each filter system!)
4. the distance of 10 pc

Monochromatic luminosity has no arbitrariness. I have no quarrel with Claudius Ptolemaios for his assigning magnitudes between 1 and 7 to the stars that are visible to the naked eye, but I somehow think that he will forgive us if we switch to a more convenient notation.

Apparent luminosities. The advantage of leaving the assumed distance $d_a$ free, is that a suitable value can be chosen for each application. When determining the distance to the Coma cluster a reference distance of 10 pc is less suitable than a reference distance measured in Mpc. The disadvantage is that each apparent luminosity must be accompanied explicitly by a mention of the assumed distance.
This disadvantage can be removed if we agree to limit choices of $d_a$ to integer powers of 10. The power used can then be added to the index: $L_{550a6}$ would denote the apparent luminosity for $d_a = 1 \text{Mpc}$, and $L_{550a1}$ the apparent luminosity for $d_a = 10 \text{pc}$. The numbers 1-9 would suffice for any practical purpose.

In practice a flux often represents a weighted average over a filter. One can then replace the wavelength in the index with the filter name. Thus $L_{Ua4}$ is the apparent luminosity derived from the average flux in the $U$ filter, for $d_a = 10 \text{kpc}$.

**Sample lecture notes.** To test the viability of using monochromatic luminosities, I have re-written sections of my lecture notes for a course on *Life of the stars: from dust to black holes* that I teach to first year physics & astronomy students in Utrecht. The course is in Dutch and uses SI units. English versions of the re-written sections can be found on my website:

www.astro.uu.nl/~verbunt/onderwijs/intro/new

as sample lecture notes that use (apparent) monochromatic luminosities. More Tables and Figures can be found in these.

**Some final remarks.** If you have already found a similar, indeed better, way to replace magnitudes, please let me know.

It may be argued that an advantage of the magnitude system is that it makes explicit that any flux measurement is based on flux ratios with calibrated sources. It appears to me that this should be obvious anyway: remember that radio and X//$\gamma$ astronomers do very well without magnitudes.

If a miracle happens, i.e. if reason prevails, and enough astronomers agree that we should stop using magnitudes, we may wish to avoid a proliferation of different notations and choices by asking the appropriate committees of the *International Astronomical Union* to define the method and notation of preference.

If enough of you encourage me, I will ask the IAU to do so.

data used in the figures: (1) Perryman et al. 1995 A&A 304, 69. (2) Johnson et al. 1952 ApJ 117, 313. (3) Arp 1960 AJ 65, 404; Saha et al. 1994 ApJ 425, 14. (4) Baade & Zwicky 1938, ApJ 88, 411 (1937C); Kimeridze & Tsvetkov 1987 Soviet Astr. 25, 513; Barbon et al. 1989 A&A 220, 83 (1984A in NGC4419); Zwicky 1961 PASP 73, 185 (1961D), Zwicky & Barbon 1967 AJ 72, 1366 (1962A), Barbon 1978 AJ 83, 13 (1963F,M), Kohoutek & Kowal 1978 PASP 90, 565 (1975F) (5) Gunn & Stryker 1983 ApJS 52, 121, Seaton 1979 MNRAS 187, 75P (6) Bessell et al. 1998 A&A 333, 231