Scalar field emulator via anisotropically deformed vacuum energy: Application to dark energy

Özgür Akarsu,1 Nihan Katricı,1 Anjan A. Sen,2 and J. Alberto Vázquez3

1Department of Physics, Istanbul Technical University, Maslak 34469 Istanbul, Turkey
2Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi-110025, India.
3Instituto de Ciencias Físicas, Universidad Nacional Autónoma de México, Cuernavaca, Morelos, 62210, México

We introduce a generalization of the usual vacuum energy, called ‘deformed vacuum energy’, which yields anisotropic pressure whilst preserving zero inertial mass density. It couples to the shear scalar in a unique way, such that they together emulate the canonical scalar field with an arbitrary potential. This opens up a new avenue by reconsidering cosmologies based on canonical scalar fields, along with a bonus that the kinetic term of the scalar field is replaced by an observable, the shear scalar. We further elaborate the aspects of this approach in the context of dark energy.

Introduction – The ΛCDM (Lambda cold dark matter) model is established on the spatially maximally symmetric and flat Robertson-Walker (RW) spacetime, and general relativity with a positive cosmological constant. It is in good agreement with most of the currently available data [1–5], but suffers from theoretical issues related to Λ [6,7]. This led to a more general ‘dark energy’ (DE) concept, for which the quintessence—described by a canonical scalar field (SF)—has been the most natural candidate [11–13]. It has been customary to justify the spatially flat RW background via the standard inflationary scenarios employing canonical SF [10–13], wherein the space dynamically flattens and very efficiently isotropizes (cosmic no-hair theorem [16–19]). Allowing anisotropic expansion factors—while retaining isotropically deformed vacuum energy (see, e.g., [55–59]). Search for anisotropic expansion occupies an important place in the upcoming missions such as the Euclid satellite [55], as it could reveal more on the nature of DE, viz., generically, modified gravity theories induce non-zero anisotropic stresses that lead to corresponding shear scalar evolutions, see, e.g., [56–59].

We introduce a generalization of the usual vacuum energy, called deformed vacuum energy, which yields anisotropic pressure whilst preserving zero inertial mass density. It couples to the shear scalar in a unique way, such that they together emulate the canonical SFs with an arbitrary potential. This leads to the opportunity of reconsidering the cosmologies employing canonical SF along with a bonus that the kinetic term of the SF is replaced by a new independent observable, the shear scalar.

Deformed vacuum energy – We begin with the locally rotationally symmetric (LRS) Bianchi I metric given by

\[ ds^2 = -dt^2 + a^2 [e^{\frac{\sigma}{2}}d\tau^2 + e^{-\frac{\sigma}{2}}(d\rho^2 + dz^2)], \]

which simply allows a different scale factor along one of the principal axes of the spatially flat RW metric, while preserving the isotropic spatial curvature [20–22].

\[ s \equiv v(t)^{1/3} \] is the mean scale factor with comoving volume scale factor \( v(t) \), from which the average Hubble parameter is defined as \( H \equiv \frac{1}{3} (H_{x} + 2H_{y}) \), where \( H_{i} \) (\( i = x, y, z \)) are the directional Hubble parameters along the \( x \)-, \( y \)-, and \( z \)-axes, and dot denotes the comoving time derivative. The term \( \varphi \) is related to the shear scalar \( \sigma^2 = \frac{1}{3} (H_{x} - H)^2 \), quantifying the anisotropic expansion, as \( \sigma^2 = \frac{\varphi}{3} \). We use the geometrised units \( c = 1 = 8\pi G \).

In a generic inertial frame, the most general matter-energy-momentum tensor accommodated by the metric [1–5] can be decomposed relative to a unique four-velocity \( u^\mu \) \((u_\mu u^\mu = -1 \) and \( \nabla_\mu u^\mu = 0 \)) in the form

\[ T_{\mu\nu} = \rho u_\mu u_\nu + p_{\text{iso}} h_{\mu\nu} + \pi_{\mu\nu}. \]

Here \( \rho = \rho(u^\mu) = T_{\mu\nu} u^\mu u^\nu \) is the relativistic energy density relative to \( u^\mu \), \( p_{\text{iso}} = \frac{1}{3} T_{\mu\nu} h_{\mu\nu} \) is the isotropic pressure and \( \pi_{\mu\nu} \) is the trace-free anisotropic pressure, where \( h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu \) (\( g_{\mu\nu} \) being the metric tensor) is the projection tensor into the instantaneous
rest frame of comoving observers. The set of equations arises from the twice-contracted Bianchi identities, which by means of Einstein field equations $G_{\mu\nu} = -T_{\mu\nu}$, implies the conservation equations. Projecting parallel and orthogonal to $w^\mu$, we obtain the energy and momentum conservation equations, correspondingly,

$$\dot{\rho} + \Theta (\rho + p_{\text{iso}}) + \sigma_{\mu\nu} \pi^{\mu\nu} = 0, \quad (3)$$

$$D^\mu p_{\text{iso}} + (\rho + p_{\text{iso}} + \pi_{\mu\nu}^\rho) u^\mu + (\text{div} \pi)^\rho = 0, \quad (4)$$

where $\Theta = D^\mu u_\mu$ is the volume expansion rate, $\sigma_{\mu\nu} = D_{[\mu} u_{\nu]}$ is the shear tensor, and we used $\nabla_\nu u_\mu = D_\nu u_\mu - u_\nu u_\mu$ with $D_{\nu} u_{\mu} = \frac{1}{2} \Theta h_\mu^\nu + \sigma_{\mu\nu}$. We note that $u^\mu$ is the four acceleration, and thereby the multipliers $\rho + p_{\text{iso}} + \pi_{\mu\nu}^\rho$ for the spatial components in $u^\mu$ define the inertial mass densities along the principal axes as $\rho_{\text{inert},x} \equiv \rho + p_{\text{iso}} + \pi_{11}^\rho$ and $\rho_{\text{inert},y} = \rho_{\text{inert},z} \equiv \rho + p_{\text{iso}} + \pi_{22}^\rho$. Furthermore, we can define an average inertial mass density as $\bar{\rho}_{\text{inert}} \equiv \frac{1}{3} \left( \rho_{\text{inert},x} + 2 \rho_{\text{inert},y} \right)$ leading to

$$\bar{\rho}_{\text{inert}} = \rho + p_{\text{iso}} + \frac{1}{3} \pi_{11}^\rho + \frac{2}{3} \pi_{22}^\rho. \quad (5)$$

As the pressure along $x$-axis is $p_x = p_{\text{iso}} + \pi_{11}^\rho$ and the ones along $y$- and $z$-axes are $p_y = p_z = p_{\text{iso}} + \pi_{22}^\rho$, we can write $p_y = p_{\text{iso}} + \gamma \rho$ with $\gamma \equiv \sqrt{\frac{2}{3}} - 1$ measuring the deviation of $p_y$ from $p_x$, so that $\bar{\rho}_{\text{inert}}$ can be written as

$$\bar{\rho}_{\text{inert}} = \rho + p_{\text{iso}} + \frac{2}{3} \gamma \rho. \quad (6)$$

Staying loyal to the zero inertial mass density of the usual vacuum energy ($\rho_v + p_v = 0$), we thus assume

$$\bar{\rho}_{\text{inert}} = 0, \quad (7)$$

leading to $p_x = -\rho - \frac{2}{3} \gamma \rho$, from which, we reach a particular kind of anisotropic stress;

$$T_{\mu\nu} = \text{diag} \left[ -1, -1 - \frac{2}{3} \gamma, -1 + \frac{1}{3} \gamma, -1 + \frac{1}{3} \gamma \right] \rho. \quad (8)$$

which, henceforth, we call deformed vacuum energy (dv). Here, for convenience, we use the notation $T_{\mu\nu} = \text{diag} \left[ -1, w_x, w_y, w_z \right] \rho = \text{diag} \left[ -1, w_x, w_y + \gamma, w_z + \gamma \right] \rho$ with $\gamma = w_y - w_z$ being the skewness parameter providing a measure for the anisotropy of the fluid.

This is a well behaved anisotropic generalization of the usual vacuum energy of quantum field theory, which is isotropic ($\gamma = 0$). Such that, if we set a cosmic triad $u^\mu$, that is a set of three identical of them pointing mutually in orthogonal spatial directions, then these three resemble exactly the usual vacuum energy. Similarly, arbitrary number of them oriented in arbitrary directions would on average lead, stochastically, to the usual vacuum energy, cf. [67]. Besides, it does not represent any of the well known anisotropic sources such as vector fields, topological defects, etc. [25]. For instance, the EoS of a vector field $A_\mu$ with a mass $m$, $-w_x = w_y = w_z = \frac{A^2 - m^2 A^2}{A^2 + m^2 A^2}$ [67], implies $w_x = -\gamma/2$ with $-2 \leq \gamma \leq 2$ for $m^2 \geq 0$, which do not satisfy [8]. Similarly, the topological defects such as cosmic strings $\{w_x, \gamma\} = \{-1, 1\}$, or domain walls $\{w_x, \gamma\} = \{0, -1\}$, do not satisfy [8].

**Emulator for canonical scalar field** – The Einstein field equations in the presence of the deformed vacuum energy [8] for the simplest anisotropic background [1] can be given by the following set of equations:

$$3H^2 = \frac{1}{2} \sigma^2 + \rho_{\text{dv}}, \quad (9)$$

$$-2\dot{H} - 3H^2 = \frac{1}{2} \sigma^2 - \rho_{\text{dv}}, \quad (10)$$

$$\ddot{\sigma} + 3H \sigma = -\sqrt{\frac{2}{3}} \gamma \rho_{\text{dv}}, \quad (11)$$

which are the energy density [9], average pressure [10] and shear propagation [11] equations, respectively. Comparing the density and average pressure equations, we see that the shear scalar term $\sigma^2/2$ and the density of the deformed vacuum $\rho_{\text{dv}}$ together resemble the canonical SF, namely, they appear as the kinetic term $\dot{\phi}^2/2$ and potential $V$ of a canonical SF, correspondingly. Further the shear propagation equation (11) resembles the scalar field (Klein-Gordon) equation. Thus, this system (9)-(11) has the same mathematical form of the standard isotropic Friedmann equations in the presence of a canonical SF

$$3H^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad (12)$$

$$-2\dot{H} - 3H^2 = \frac{1}{2} \dot{\phi}^2 - V(\phi), \quad (13)$$

$$\ddot{\phi} + 3H \dot{\phi} = -\frac{dV}{d\phi}, \quad (14)$$

under the following transformations:

$$\mathcal{H} \rightarrow H, \quad \sigma \rightarrow \dot{\phi}, \quad \rho_{\text{dv}} \rightarrow V(\phi), \quad \gamma \rightarrow \sqrt{\frac{3}{2}} \frac{1}{V} \frac{dV}{d\phi}. \quad (15)$$

Accordingly, given that the energy density and pressure of a canonical SF are $\rho_{\phi} = \frac{1}{2} \dot{\phi}^2 + V(\phi)$ and $p_{\phi} = \frac{1}{2} \dot{\phi}^2 - V(\phi)$, if we define effective energy density and pressure as $\rho_{\text{eff}} = \frac{1}{2} \sigma^2 + \rho_{\text{dv}}$ and $p_{\text{eff}} = \frac{1}{2} \sigma^2 - \rho_{\text{dv}}$, correspondingly, we further have the transformations:

$$\rho_{\text{eff}} \rightarrow \rho_{\phi}, \quad p_{\text{eff}} \rightarrow p_{\phi} \text{ and } w_{\text{eff}} \equiv \frac{p_{\text{eff}}}{\rho_{\text{eff}}} \rightarrow w_{\phi} \equiv \frac{p_{\phi}}{\rho_{\phi}}. \quad (16)$$

It is straightforward to see that as the Klein-Gordon equation (14) leads to the continuity equation, $\dot{\rho}_{\phi} + 3H (\rho_{\phi} + p_{\phi}) = 0$, for the SF, the shear propagation equation (11) leads to the continuity equation

$$\dot{\rho}_{\text{eff}} + 3H \rho_{\text{eff}} (1 + w_{\text{eff}}) = 0, \quad (17)$$

for the effective source defined from the cooperation of the deformed vacuum energy with the shear scalar—as
long as the shear propagation equation is not altered by any other anisotropic contribution.

While the condition for a canonical SF to be able to drive accelerated expansion, $w_\phi < -1/3$, implies $\dot{\phi}^2 < V$ (or $\dot{\phi}^2/2 < \rho_\phi/3$), in our model, correspondingly, $w_{\text{eff}} < -1/3$ implies $\sigma^2 < \rho_{\text{eff}}$ (or $\sigma^2/2 < \rho_{\text{eff}}/3$). On the other hand, to give rise to an accelerated expansion using SF, it is often required a flat potential satisfying $\dot{\phi}^2 \ll V$, which leads to $w_\phi \simeq -1 + \frac{3}{2} \epsilon$, where $\epsilon < 1$ is the so-called slow roll parameter defined as $\epsilon = \frac{\dot{V}}{V} \frac{\dot{\phi}^2}{V}$. Considering the relations given in (14) and (15), it turns out that the role of the slow-roll parameter is taken over by the skewness of the deformed vacuum energy as $\gamma^2/3 \to \epsilon$ and hence one should require small anisotropy $\sigma^2 \ll \rho_{\text{eff}}$, which leads to $w_{\text{eff}} \simeq -1 + \frac{3}{2} \gamma^2$ with $|\gamma| \ll \sqrt{3}$. And, the role of the flatness of the potential (quantified by $\epsilon$) is taken over by the ratio-squared of the rate of change of the energy density of the deformed vacuum to the shear scalar, namely, $\epsilon \to \frac{\sigma^2}{\gamma^2} \simeq \frac{1}{2} \frac{\rho_{\text{eff}}}{\rho_{\phi}}$.

There is no-go theorem which forbids a single canonical SF (real SF $\dot{\phi}^2 \geq 0$ with a non-negative potential $V(\phi) \geq 0$) to cross below the $w = -1$ boundary of the usual vacuum energy, viz., its EoS parameter is confined to the range $-1 \leq w_\phi \leq 1$. In line with that, in our model, the non-negativity condition on the density of the deformed vacuum energy—as an actual physical source with negative density would be physically ill—($\rho_{\text{eff}} > 0$) guarantee that $-1 \leq w_{\text{eff}} \leq 1$.

**Cosmology with deformed vacuum energy** — We proceed with an investigation of the cosmologies in the presence of the deformed vacuum. We consider the isotropic perfect fluids—representing usual cosmological fluids such as dust and radiation—described by $p_i / \rho_i = w_i = \text{const}$ ($i$ stands for $i$th fluid) for the other sources present along with the deformed vacuum energy in the Universe. As these are isotropic, these alter neither the form of the shear propagation equation (11) nor the features that arise from the deformed vacuum energy, see (15)-(17).

Using $dt = -\frac{dz}{H(1+z)}$, where $z$ is the average redshift, defined from the mean scale factor as $z = \frac{a(z=0)}{a(z)} - 1$, we reach the following anisotropic Friedmann equation

$$3H^2 = \sum_i \rho_i(1 + z)^3(1 + w_i) + \rho_{\text{eff}},$$

where

$$\rho_{\text{eff}} = \rho_{\text{eff}} 0 \frac{3}{(1 + w_{\text{eff}})} \frac{d \ln (1+z)}{dz}.$$

This is mathematically exactly the same with the usual Friedmann equation, but physically different. Note that here $\rho_{\text{eff}} = \rho_{\sigma^2} + \rho_{\text{deformed}}$ consists of the energy density corresponding to the shear scalar, i.e., expansion anisotropy,

$$\rho_{\sigma^2} \equiv \frac{\sigma^2}{2} = \frac{1 + w_{\text{eff}}}{2} \rho_{\text{eff}},$$

and the energy density of the deformed vacuum,

$$\rho_{\text{deformed}} = \frac{1 - w_{\text{eff}}}{2} \rho_{\text{eff}},$$

whose EoS parameter is skewed as

$$\gamma = \frac{w_{\text{eff}}(1 + z) - 3(1 - \frac{w_{\text{eff}}}{2})}{\sqrt{2} + 2(1 - w_{\text{eff}}) \sqrt{1 + \sum_i \rho_i \rho_{\phi}}},$$

where $\gamma$ denotes $d\sigma/dz$.

We see that the ratio of the energy density corresponding to the expansion anisotropy to that of the deformed vacuum energy is determined solely by $w_{\text{eff}}$ as $\rho_{\text{eff}} / \rho_{\phi} = \frac{\Omega_{\text{deformed}}}{\Omega_{\phi}} = \frac{1 + w_{\text{eff}}}{1 - w_{\text{eff}}}$, $\Omega = \rho / \rho_{\text{cr}}$ (with $\rho_{\text{cr}} = 3H^2$ being the critical energy density) is the density parameter of the component denoted by its subscript. Accordingly, if the energy density/EoS of a SF is given in terms of redshift, then one can straightforwardly study its correspondence via the transformations $\rho_\phi(z) \to \rho_{\text{eff}}(z)$ or $w_\phi(z) \to w_{\text{eff}}(z)$ [see (16)]—, upon first replacing the RW background by the Bianchi I (viz., $H \to H$). If the potential, $V(\phi)$, described the SF is given, the same procedure can be utilized upon obtaining $\rho_\phi(z)$ from the exact solutions of the considered cosmological model. The emulator of the given SF will give exactly the same expansion history for the comoving volume element. Yet, each of the SF will correspond to a specific evolution of the shear scalar, which would allow, in principle, to observationally distinguish between these two corresponding models.

We continue the exploration of the model by focusing on DE. Dark energies described by a canonical SF with a non-trivial potential in which the field slowly rolls down is the epitome of quintessence models [15–62], and thereby the shear scalar-deformed vacuum energy alliance would also be. To begin with, in the case of emulating a SF with $w_\phi = \text{const}$ leading to $\rho_\phi \propto (1 + z)^{3(1 + w_\phi)}$ (the so called $w$CDM model but with a lower bound $w = -1$), the ratio $\frac{\Omega_{\text{deformed}}}{\Omega_{\phi}} = \frac{1 + w_{\text{eff}}}{1 - w_{\text{eff}}} = \frac{1 + w_{\text{eff}}}{1 - w_{\text{eff}}}$ is a fixed value, so that, as $w_\phi \to w_{\text{eff}}$, for the shear scalar we have $\sigma^2 \propto (1 + z)^{3(1 + w_{\text{eff}})}$, i.e., it tracks $\rho_\phi \propto (1 + z)^{3(1 + w_{\text{eff}})}$, which, in principle, is an observable and can be utilized to distinguish between our model and the usual $w$CDM based on RW background. As for the $w$CDM observations suggest $w \sim -1$, this implies $\sigma^2 \sim \text{const}$ along with $\Omega_{\text{deformed}}$ increasing with the expansion of the Universe (anisotropization of the Universe as it expands) in contrast to the simple Bianchi I generalization of the standard $\Lambda$CDM (or of any isotropic DE) for which $\sigma^2 \propto (1 + z)^6$. The situation changes in the case of early DE which was suggested for addressing the so called Hubble tension as it increases the early expansion rate while leaving the later evolution of the Universe unaltered [63]. It behaves like a cosmological constant, before some critical redshift, $w \sim -1$ for $z < z_c$, but its energy density then increases like that of radiation with the increasing redshift, $w \sim \frac{1}{3}$ for $z > z_c$. Accordingly, if we emulate early DE, then $\sigma^2 \sim \text{const}$ and
\( \Omega_{x,0} \sim 0 \) (almost isotropic Universe) before some critical redshift, as \( w_{\text{eff}} \sim -1 \) for \( z < z_c \), but afterwards shear scalar increases like the energy density of radiation and \( \Omega_{x,0} \sim \frac{1}{z_c} \), as \( w_{\text{eff}} \sim \frac{1}{z} \) for \( z > z_c \). Thus, the emulator of a quintessence can be distinguished via the modified redshift dependence of the shear scalar.

A canonical SF with an exponential potential, \( V(\phi) = V_0 e^{\lambda \phi} \), with \( \lambda = \text{const.} \) corresponds to the case of the deformed vacuum energy with a constant skewness parameter, \( \gamma = \text{const.} \). The corresponding transformation which reads \( \gamma \rightarrow \sqrt{\frac{2}{3}} \lambda \) from (15). One of the two Swampland criteria on an effective field theory consistent with string theory is that given a point in field space, the derivative of the SF potential has to satisfy the lower bound \( \frac{dV/da}{V} > c \sim O(1) \). For exponential potential of the form \( V(\phi) = V_0 e^{\lambda \phi}, \lambda = c \). The dark energies that barely distinguished from the \( \Lambda \) re-

ternal potential of the form 

| \( w_{\text{eff}} \) | \( \frac{\lambda}{w_{\text{eff}}} \) |
|---|---|
| \( \rho_{\text{sf}} \) | \( \rho_{\text{sf}}(1 + z)^6 \) |
| \( \gamma \) | \( -3(1 + m_0) \) |

\( \Omega_{x,0} \lesssim 10^{-15} \), for which anisotropy becomes irrelevant to the matter-radiation equality redshift and the peak of the matter perturbations, but the CMB quadrupole temperature changes up to values beyond its actual value, viz., \( \sim 11 \text{mK} \). Besides, it was suggested that the anisotropy has no significant effect on the standard Big Bang Nucleosynthesis (BBN) provided that \( \Omega_{x,0} \lesssim 10^{-23} \), for which anisotropy remains irrelevant to the CMB quadrupole temperature. On the other hand, in the d\( \nu \)CDM model, the shear scalar tracks the deformed vacuum energy—both of which evolve as \( (1 + z)^{3(1 + w_{\text{eff}})} \)—and hence, as like the DE in the usual \( w \)CDM, it would reach considerable values at late times only, and consequently the constraints on the anisotropy can be relaxed. Namely, in this case, the Universe anistropizes as it expands, which implies that the expansion anisotropy would be irrelevant to the dynamics of the early Universe and the tight constraints on its present-day density parameter from its effect on the expansion rate on the comoving volume of the early Universe (e.g., from BBN) would be evaded. This relaxed amount of anisotropic expansion would allow us to manipulate the CMB quadrupole temperature on top of its statistical value. This is the observationally distinguishing feature of the d\( \nu \)CDM model from the usual \( w \)CDM model.

**Manipulating CMB quadrupole temperature**—The observed quadrupole power spectrum of temperature fluctuations in the CMB (multipole \( l = 2 \), the corresponding angular scale \( \theta = \pi/2 \)) is \( \Delta T_{\text{PLK}} \approx 14 \mu K \) [27] lower than the CMB predicted value, \( \Delta T_{\text{st}} \approx 34 \mu K \) (or \( \Delta T_{\text{st+variance}} \approx 28 \mu K \) when the cosmic variance is included) [22] [73]. It is suggested in [22] [41] that this discrepancy can be addressed by an ellipsoidal expansion (within LRS Bianchi I spacetime) driven by an anisotropic DE. The evolution of the free streaming photon temperature in the \( i^{th} \) direction can be given as \( T_i = T_0 \frac{\Delta m_i}{\Delta m_0} = T_0 e^{-\int H_i dt} \approx T_0 - T_0 \int H_i dt \) (i.e., \( x = y, z \)) where \( T_0 = 2.7255 \pm 0.0006 \text{ K} \) [74] is the present-day CMB monopole temperature [23] [73]. Thus, as \( H_x = \frac{\dot{H}}{\sqrt{2}} \gamma \), \( H_y = \gamma - \frac{\dot{\gamma}}{\sqrt{2}} \), and \( \Omega_{x,0} = \sigma_{x,0}^2 \frac{\dot{\gamma}}{H^2} \), the difference between photon temperatures along the \( x \)- and \( y \)-axes since the recombination (\( z_{\text{rec}} = 1100 \)) to the present time (\( z = 0 \))

**Table I. Equations for ACMDM\(_{x,2}\) and d\( \nu \)wCDM\(_{x,2}\) models.**

| ACMDM\(_{x,2}\) | d\( \nu \)wCDM\(_{x,2}\) |
|---|---|
| \( \rho_{\text{eff}} \) | \( \rho_{\text{dot}}(1 + z)^2 \) |
| \( \rho_{\text{dot}} \) | \( \rho_{\text{dot}}(1 + z)^2 \) |
| \( \gamma \) | \( 0 \) |

\( \sigma_{x,0} \lesssim 10^{-15} \), for which anisotropy becomes irrelevant to the matter-radiation equality redshift and the peak of the matter perturbations, but the CMB quadrupole temperature changes up to values beyond its actual value, viz., \( \sim 11 \text{mK} \). Besides, it was suggested that the anisotropy has no significant effect on the standard Big Bang Nucleosynthesis (BBN) provided that \( \Omega_{x,0} \lesssim 10^{-23} \), for which anisotropy remains irrelevant to the CMB quadrupole temperature. On the other hand, in the d\( \nu \)CDM model, the shear scalar tracks the deformed vacuum energy—both of which evolve as \( (1 + z)^{3(1 + w_{\text{eff}})} \)—and hence, as like the DE in the usual wCDM, it would reach considerable values at late times only, and consequently the constraints on the anisotropy can be relaxed. Namely, in this case, the Universe anistropizes as it expands, which implies that the expansion anisotropy would be irrelevant to the dynamics of the early Universe and the tight constraints on its present-day density parameter from its effect on the expansion rate on the comoving volume of the early Universe (e.g., from BBN) would be evaded. This relaxed amount of anisotropic expansion would allow us to manipulate the CMB quadrupole temperature on top of its statistical value. This is the observationally distinguishing feature of the d\( \nu \)CDM model from the usual wCDM model.

Manipulating CMB quadrupole temperature—The observed quadrupole power spectrum of temperature fluctuations in the CMB (multipole \( l = 2 \), the corresponding angular scale \( \theta = \pi/2 \)) is \( \Delta T_{\text{PLK}} \approx 14 \mu K \) [27] lower than the CMB predicted value, \( \Delta T_{\text{st}} \approx 34 \mu K \) (or \( \Delta T_{\text{st+variance}} \approx 28 \mu K \) when the cosmic variance is included) [22] [73]. It is suggested in [22] [41] that this discrepancy can be addressed by an ellipsoidal expansion (within LRS Bianchi I spacetime) driven by an anisotropic DE. The evolution of the free streaming photon temperature in the \( i^{th} \) direction can be given as \( T_i = T_0 \frac{\Delta m_i}{\Delta m_0} = T_0 e^{-\int H_i dt} \approx T_0 - T_0 \int H_i dt \) (i.e., \( x = y, z \)) where \( T_0 = 2.7255 \pm 0.0006 \text{ K} \) [74] is the present-day CMB monopole temperature [23] [73]. Thus, as \( H_x = \frac{\dot{H}}{\sqrt{2}} \gamma \), \( H_y = \gamma - \frac{\dot{\gamma}}{\sqrt{2}} \), and \( \Omega_{x,0} = \sigma_{x,0}^2 \frac{\dot{\gamma}}{H^2} \), the difference between photon temperatures along the \( x \)- and \( y \)-axes since the recombination (\( z_{\text{rec}} = 1100 \)) to the present time (\( z = 0 \))

\( \sigma_{x,0} \lesssim 10^{-15} \), for which anisotropy becomes irrelevant to the matter-radiation equality redshift and the peak of the matter perturbations, but the CMB quadrupole temperature changes up to values beyond its actual value, viz., \( \sim 11 \text{mK} \). Besides, it was suggested that the anisotropy has no significant effect on the standard Big Bang Nucleosynthesis (BBN) provided that \( \Omega_{x,0} \lesssim 10^{-23} \), for which anisotropy remains irrelevant to the CMB quadrupole temperature. On the other hand, in the d\( \nu \)CDM model, the shear scalar tracks the deformed vacuum energy—both of which evolve as \( (1 + z)^{3(1 + w_{\text{eff}})} \)—and hence, as like the DE in the usual wCDM, it would reach considerable values at late times only, and consequently the constraints on the anisotropy can be relaxed. Namely, in this case, the Universe anistropizes as it expands, which implies that the expansion anisotropy would be irrelevant to the dynamics of the early Universe and the tight constraints on its present-day density parameter from its effect on the expansion rate on the comoving volume of the early Universe (e.g., from BBN) would be evaded. This relaxed amount of anisotropic expansion would allow us to manipulate the CMB quadrupole temperature on top of its statistical value. This is the observationally distinguishing feature of the d\( \nu \)CDM model from the usual wCDM model.
due to the anisotropic expansion, $\Delta T_{\sigma z} = T_x - T_y$, reads

$$\Delta T_{\sigma z} = T_0 \int_{t_{rec}}^{t_0} (H_x - H_y) dt = 3T_0 \int_{t_{rec}}^{t_0} \sqrt{\Omega_{\sigma z}} d\ln(1 + z).$$

Accordingly, provided that the orientation of the expansion anisotropy is set properly, by means of $\Delta T_{\sigma z} \approx 20 \mu K$ it is possible to reduce $\Delta T_{\sigma z} \approx 34 \mu K$ in $\Lambda$CDM to the observed value $\Delta T_{PLK} \approx 14 \mu K$ [27]. However, within $\Lambda$CDM$_{\sigma z}$, it is not possible to have this reduction, since the upper limit $\Omega_{\sigma z0} \approx 10^{-23}$ from BBN allows only up to $\Delta T_{\sigma z} \sim 1 \mu K$ reduction [74]. In our model (even in the simplest case, $dvw$CDM$_{\sigma z}$), we are able to manipulate the evolution of $\Omega_{\sigma z}$ so as to evade this limit on $\Omega_{\sigma z0}$ from BBN and manipulate $\Delta T_{\sigma z}$ at the required amount. This can be done by demanding, for instance, from the simplest case $dvw$CDM$_{\sigma z}$, to lead to $\Delta T_{\sigma z} \sim 20 \mu K$ change on top of $\Delta T_{\sigma z} \approx 34 \mu K$ via a suitably choosing, e.g., present-day value of the expansion anisotropy, or, in a robust way, e.g., by including $\Delta T_{PLK} \approx 14 \mu K$ as a prior while modelling anisotropic distribution of the data in the sky.

**Observational constraints** — We perform a parameter estimation and provide observational constraints of the model-free parameters given in Table I. In order to explore the parameter space, we make use of a modified version of a simple and fast Markov Chain Monte Carlo (MCMC) code, named SimpleMC [76, 77], that computes expansion rates and distances using the Friedmann equation. For the model $dvw$CDM$_{\sigma z}$, the Friedmann equation [18] in the presence of radiation ($w_r = \frac{1}{3}$) and dust ($\text{CDM}+\text{baryons}$) ($w_m = 0$) reads:

$$\frac{H^2}{H_0^2} = \Omega_{\text{eff}0}(1 + z)^3(1+w_{\text{eff}}) + \Omega_{m0}(1+z)^3 + \Omega_{\text{v}0}(1+z)^4,$$

where $\Omega_{\text{eff}0} = \Omega_{\sigma z0} + \Omega_{\text{v}0}$. The code uses a compressed version of the recent Planck data (PLK), a recent reanalysis of Type Ia supernovae (SN) data, and high-precision Baryon Acoustic Oscillation measurements (BAO) at different redshifts up to $z = 2.36$ [77]. For a detailed description about the data sets used see [77]. We also include a collection of currently available measurements on $H(z)$ from cosmic chronometers ($H$) (see [78] and refs. therein). See [79] for an extended review of cosmological parameter inference procedure. Throughout the analysis we assume flat priors over our sampling parameters: $\Omega_{m0} = [0.05, 0.5]$ for the dust density parameter today, $\Omega_{k0}h_0^2 = [0.02, 0.025]$ for the physical baryon density parameter today and $h_0 = [0.4, 1.0]$ for the reduced Hubble constant, $h_0 = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. As our main purpose here is to demonstrate how $dvw$CDM$_{\sigma z}$ works in comparison with the $\Lambda$CDM ($w_{\text{eff}} = -1$) and $\Lambda$CDM$_{\sigma z}$ models, rather than providing robust observational analyses, for the sake of obtaining tight constraints consistent with $\Delta T_{\sigma z} \sim 20 \mu K$, we take samples from the posterior distribution of the parameter-space by imposing the condition $\Omega_{\sigma z0} = 4 	imes 10^{-21}$ for $\Lambda$CDM$_{\sigma z}$ and

| Parameter | $\Lambda$CDM | $\Lambda$CDM$_{\sigma z}$ | $dvw$CDM$_{\sigma z}$ |
|----------|-------------|----------------|----------------|
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | 68.20(49) | 68.84(49) | 67.65(85) |
| $\Omega_m$ | 0.302(6) | 0.298(6) | 0.307(8) |
| $\Omega_{\sigma z0}$ | 0 (4 × 10$^{-21}$) | 6.58(8) [10$^{-12}$] |
| $\Omega_{\text{v}0}$ | 0.698(6) | 0.701(6) | 0.693(8) |
| $1 + w_{\text{eff}}$ | 0 (0) | 0 (1.900 × 10$^{-11}$) |
| $\gamma_0$ | 0 | 0 | -11.108(49) [10$^{-6}$] |
| $\Delta T_{\sigma z} [\mu K]$ | 0 | 21.12(21) | 20.08(27) |
| $k_{\text{eq}} [\text{Mpc}^{-1}]$ | 0.01024(7) | 0.01032(7) | 0.01026(8) |
| $\Omega_{\sigma z}(z = z_{\text{BBN}})$* | 0 | 0.803(2) | 8.91(32) [10$^{-42}$] |

In Table II we observe that there exists no significant difference between the constraints on the parameters $H_0$, $\Omega_m0$ and $\Omega_{\text{v}0}$ of the models, and that the present-day density parameter corresponding to the anisotropic expansion, $\Omega_{\sigma z0}$, is of the order $O(10^{-20})$ for $\Lambda$CDM$_{\sigma z}$, and $O(10^{-11})$ for $dvw$CDM$_{\sigma z}$, which cannot be detected locally today—as they are much below the model independent upper bounds of order $O(10^{-4})$. Further, we notice no significant difference between the constraints on $k_{\text{eq}} = H_{\text{eq}}^{-1} = \frac{H_{\text{eq}}}{1 + z_{\text{eq}}}$ (the wavenumber of a mode of density perturbations that enter the horizon at the radiation-matter transition, which is highly sensitive to the modifications to $\Lambda$CDM, and related to the dynamics of the Universe.
at matter-radiation equality redshift $z_{eq} \sim 3400$ larger than the recombination redshift $z_{rec} \sim 1100$ related to the CMB. All these imply that, when the evolution of the comoving volume element [viz., $H(z)$] is considered, the $\Lambda$CDM$_{a,z}$ and $dvw$CDM$_{a,z}$ models are observationally indistinguishable from $\Lambda$CDM all the way to the matter-radiation transition epoch. Yet, both these can be distinguished from $\Lambda$CDM as they predict $\Delta T_{a,z} \approx 20\mu K$, i.e., reduction of $\Delta T_{a} \approx 34\mu K$ in the $\Lambda$CDM to the observed value $\Delta T_{PLK} \approx 14\mu K$ [27]. However, the anisotropic expansion by this modification in the CMB quadrupole temperature does not spoil the successful description of the radiation dominated Universe (including standard BBN) only for the $dvw$CDM$_{a,z}$. We see in Table [1] and in Figure [1] that the expansion anisotropy dominates 80% of the Universe at BBN epoch for $\Lambda$CDM$_{a,z}$, while it is irrelevant to make any change on the standard BBN model for $dvw$CDM$_{a,z}$. This implies in $dvw$CDM$_{a,z}$ that it is not the BBN, but the quadrupole temperature putting the tightest constraints on the expansion anisotropy. Accordingly, while $\Lambda$CDM$_{a,z}$ prohibits a significant modification in the CMB quadrupole temperature due to the small BBN upper bound on the present-day expansion anisotropy, $dvw$CDM$_{a,z}$ is able to manipulate it. Figure [1] is very demonstrative for the difference between these two anisotropic models. In $\Lambda$CDM$_{a,z}$, as $\rho_{a,z} \propto (1+z)^{3}$, the Universe isotropizes as it expands: the density parameter corresponding to expansion anisotropy $\Omega_{a,z}$ rapidly increases—thereby the model deviates from $\Lambda$CDM—with increasing redshift, and eventually the expansion anisotropy dominates over the radiation and spoils the Universe leaving this epoch, $\Omega_{a,z} \approx \frac{2}{3}$0 during the quasi-de Sitter epoch (when $w_{eff} \approx -1$) and then, while the Universe leaving this epoch, $w_{eff}$ increases, so does $\Omega_{a,z}$. This implies that emulators of the standard inflationary scenarios will generally predict an anisotropization process of the Universe by the end of inflation. This anisotropization (anisotropic hair) can occur in non-trivial ways, whence the reheating mechanisms and/or an actual scalar field is also included into the model.

Throughout the paper, we have considered the LRS Bianchi I spacetime (the simplest spatially flat anisotropic metric). Extending this work to Bianchi I or V (spatially open) spacetimes, in principle, would not change our results as these two are atypical in that they bring no restoring ‘force’ term in the shear propagation equation [22], whereas one set of such terms come, in more complicated anisotropic spacetimes, anisotropic spatial curvature [25]. For instance, the most general spatially flat (or open) anisotropic spacetimes, Bianchi VII$\text{h}$ (or VII$\text{b}$), yield anisotropic spatial curvature that mimics traceless anisotropic fluid. Thus, consideration of the deformed vacuum energy in more general anisotropic spacetimes would extend our approach presented here to a family of non-canonical scalar fields.

**Acknowledgements** – The authors thank to Shahin Sheikh-Jabbari for valuable discussions. O.A. acknowledges the support by the Turkish Academy of Sciences in scheme of the Outstanding Young Scientist Award (TÜBA-GEBİP). N.K. acknowledges the post-doctoral research support from the Istanbul Technical University (ITU). A.A.S. acknowledges funding from DST-SERB, Govt of India, under the project NO. MTR/2019/000599. J.A.V. acknowledges the support provided by FOSEC SEP-CONACYT Investigación Básica A1-S-21925, and UNAM-DGAPA-PAPIIT IA102219.

---

* akarsuoa@itu.edu.tr
† nihan.katirci@itu.edu.tr
‡ aasen@jmi.ac.in
§ javazquez@icf.unam.mx

[1] A. G. Riess et al. [Supernova Search Team], Astron. J. 116, 1009 (1998), astro-ph/9805201
[2] P. A. R. Ade et al. (Planck Collaboration), Astron. Astrophys. 594, A13 (2016), 1502.01589
[3] S. Alam et al. (BOSS Collaboration), Mon. Not. Roy. Astron. Soc. 470, 2617 (2017), 1607.03155
[4] T. M. C. Abbott et al. (DES Collaboration), Phys. Rev. D 98, 043526 (2018), 1708.01530
[5] N. Aghanim et al. (Planck Collaboration), 1807.06209
[6] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989)
[7] V. Sahni, A. A. Starobinsky, Int. J. Mod. Phys. D 9, 373 (2000), astro-ph/9904398
[8] P. J. E. Peebles, B. Ratra, Rev. Mod. Phys. 75, 559
