Research on Unsteady Characteristics of Pump Turbine in Pumping Mode

Yong Liu, Hongjuan Ran and Dezhong Wang
School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China
E-mail: lytbbf@126.com

Abstract. A study on the unsteady characteristics of pump turbine in pumping mode was made. Basing on the existing research, this paper is aimed at providing valuable reference for hydraulic design, structure optimization, fault diagnosis of pump turbine. CFX simulation verified by experiment was made to analyze the distribution law and propagation characteristic of pressure and its pulsation in the whole fluid field. Research shows that the pressure pulsation owns different propagation characteristic from the flow velocity pulsation. The pressure pulsation component with guide blade frequency has very special properties in stationary flow passages, while the pressure pulsation component with runner blade passing frequency is absolutely different. Blade constraints have significant effect on the unsteady properties of pressure and flow velocity. Besides, some other analysis was talked about, such as the external characteristics, pressure pulsation near the volute tongue, and so on.

1. Pump Turbine Specification and Numerical Simulation Methods

Pumped Storage is a special power source safeguarding the stability of electric system. In the foreseeable future, Pumped Storage is the most environmentally friendly and efficient technique to be spread in large, of which pump turbine is the key equipment. As the core hydraulic component of the pumping station, the pump turbine plays a vital role in peak-cutting, frequency-modulation and phase-modulation. Promoting research on pump turbine would help improve the safety, reliability and economy of pumping stations. Since 1931 when the pump turbine came into service, scholars at home and abroad have done a lot of research to improve its performance. Nevertheless, there are still some matters restricting the further improvement of the performance of the Pumped Storage, so the related research has a long way to go.

In most of the time, the pump turbine runs at the design operating point. This paper researches on the unsteady characteristics of pump design working condition of pump turbine, which is aimed at applying some valuable reference for hydraulic design, structure optimization, fault diagnosis and research on the non-designed operating point. When studying non-design conditions such as small flow working condition, it is necessary to compare the design conditions to accurately clarify the main features caused by non-design conditions. In addition, the rotor-stator interaction and unstable flow studied in this paper significantly affect the modal and vibration of the pump turbine. Therefore, this paper will provide a valuable reference for the vibration and modal research as well. [11-13]

Ran [1], Nakamura T et al. [5] studied on the flow field and pressure pulsation of runner and draft tube. Wang et al. [2][3] analysed the effect of guide vane opening on the pump working condition of
pump turbine by SIMPLEC arithmetic and SST k-ω turbulence model, and the pressure pulsation in vaneless area as well as other key location are still researched. Barrio et al. [6] analysed the axial-radial force of pump working condition of pump turbine and compared it with other working conditions. Sun et al. [7] studied the influence of the distribution circle diameter on the pressure fluctuation of the vaneless zone under pump working condition. Basing on the dynamic grid technology, Wang et al. [4] simulated the pressure pulsation located at key points in guide vane and volute under pump working condition. The above research analyses the flow field and pressure pulsation under pumping conditions, but talks little about differences in unsteady characteristics of different parts, and there are few studies on the unsteady characteristics of flow velocity pulsation and external characteristics. Based on the above research, this paper further analyses the unsteady characteristics of pump working condition, and try to provide valuable reference for other scholars' follow-up research.

2. Pump Turbine Specification and Numerical Simulation Methods

2.1. Geometric Model and Mesh Generation

The research object of this paper is the linearly reduced model of the pump turbine of Xiang Shuijian Pumped Storage Power Station in Anhui Province, China. The key geometric parameters are showed in table 1 and the model for calculating is shown as in Figure 1(a).

| Parameters     | Value |
|----------------|-------|
| $D_{in}$       | 80    |
| $D_{out}$      | 80    |
| $Z_r$          | 9     |
| $Z_g$          | 20    |
| $Z_s$          | 20    |

$D_{in}$——diameter at pump inlet under pump working condition, mm;

$D_{out}$——diameter at pump outlet under pump working condition, mm;

$Z_r$——number of runner blades, piece;

$Z_g$——number of guide vane blades, piece;

$Z_s$——number of stay vane blades, piece.

The ICEM is used to divide the unstructured grid. To guarantee the $y^+$, grids near the blade surface are refined. According to the mesh independence verification, the final mesh scheme shown in Figure 2 is set as 12 million elements and 2.07 million nodes.
2.2. Numerical Simulation Settings

The simulation in the paper is completed by the ANSYS CFX 19.0. The wall surface is defined as smooth and non-slip, the boundary condition is set to the static pressure inlet and the mass flow rate outlet. The SST k-ω turbulence model with better treatment near wall is adopted. The solution format and the non-stationary solution format are respectively High Resolution and Second Order Backward Euler. The residual RMS convergence criterion is set to 10^-4. Total time is set as 1.7s during which the runner rotates 37 turns, and the timestep is set as the time during which the runner rotates by 2°. After removing the unstable results in the initial stage based on the fluctuation of pressure at monitoring points, the remaining valid data covers the frequency domain of 1~3900Hz, which satisfies the needs of this paper.

2.3. Monitoring Points

5 streamlines were set in flow passage of runner and every streamline contained 3 monitoring points. Respectively, Figure 3(a), (b) and (c) showed streamlines near pressure side and shroud, streamlines near suction side and hub, streamline in the middle of the flow passage. The streamlines LA and LB in the stationary flow unit consisted of monitoring points showed in Figure 3(d), and LA was near the tongue of volute. As shown in Figure 3(e), monitoring W1-W8 were placed in volute clockwise starting from tongue. Monitoring points at outlet of runner were set circumferentially and uniformly, the same as inlet of guide vane and inlet of stay vane.

3. Results and Discussions

3.1. Geometric Model and Mesh Generation

The parameters about the key characteristics of the designed point under pump working condition of the model machine studied in this paper are shown as in Table 2. To verify the accuracy of calculation, an experiment was conducted. The testing model and other equipment are shown as in Figure 1(b). As shown in Table 3, the comparison between calculation data and experiment data proves that calculation in this paper is feasible. [8]

![Figure 2. Mesh Generation](image)

| \( n \) (rev/min) | \( Q_n \) (kg/s) | \( H_n \) (m) | \( n_s \) | \( \gamma \) (°) |
|------------------|-----------------|---------------|---------|---------------|
| 1300             | 7.8             | 3.07          | 181     | 26            |

| \( H \)          | \( \Delta H/H \) |
|------------------|------------------|
| Experiment       | 2.973m           | 3.221%         |
| Calculation      | 3.126m           | 2.904%         |
| Error            | 5.146%           | -9.8%          |

Table 2. Parameters about Key Characteristics of Pump Working Conditions of the Model Machine

Table 3. Verification of Feasibility of Calculation
3.2. Analysis of the Unsteady Characteristic along the Flow Direction
The unsteady characteristic of flow is reflected by almost all the parameters relating to flow. This paper studies two main parameters, namely the static pressure (hereinafter is collectively called as pressure) and the flow velocity. This paper analyzes the full flow field containing all the flow passage components along the flow direction and the circumferential direction respectively to obtain the spread and distribution characteristics of pressure, flow velocity, pressure fluctuation and flow velocity fluctuation.

In this paper, dimensionless processing is carried out for both the pressure and flow velocity:

\[ C_p = \frac{(p-p_1)}{0.5 \rho U^2} \]  \hspace{1cm} (1)

\[ V_p = \frac{v}{U} \]  \hspace{1cm} (2)

In the formula, \( C_p \) is the pressure coefficient, \( V_p \) is the velocity coefficient, \( p \) is the static pressure, \( v \) is the flow velocity, \( p_1 \) is the benchmark static pressure which adopts the runner inlet pressure, and \( U \) is the benchmark velocity which adopts the circumferential velocity on the intersection point of the runner vane inlet and the shroud.
Spread and transmission of the unsteady characteristic along the flow direction is analyzed firstly below.

3.2.1. Analysis of the Unsteady Characteristic in the Runner. Monitoring for flow line in the flow field is shown as in Figure 3 - Figure 6 are the time domain figure of pressure and flow velocity on five flow lines in the flow passage of runner, the fft frequency domain figure and the figure about key information of time - frequency domain respectively. Figure 5 indicates that rotor-stator interaction not only can result in pressure fluctuation but also can result in flow velocity fluctuation, and the dominant frequency of both pressure fluctuation and flow velocity fluctuation is the blade frequency of the guide vane. Different from the pressure fluctuation, flow velocity fluctuation attenuates quickly in the opposite direction of the flow direction, so there is no obvious fluctuation in the runner other than near the outlet.

Through analysis of Figure 4(a), 5(a) and 6(a), it can be found that the pressure and the pressure fluctuation amplitude inside the runner increase gradually along the flow direction. The former is affected by rotation of the runner, and the latter is resulted from rotor-stator interaction. The position where is closer to the outlet of runner will be liable to be affected by rotor-stator interaction. From Figure 6(a), it can be found that along the flow direction, under the constraint of the runner, pressures of different positions on the cross section tend to be uniform, and at a cost, nonuniformity of pressure fluctuation amplitude of different positions on the cross section becomes increasingly larger.

Similarly, through analysis of Figure 6 (b), it can be found that, under the constraint of the runner, the flow velocity of different positions on the cross section along the flow direction tends to be uniform, and, at a cost, the nonuniformity of flow velocity fluctuation amplitude becomes increasingly larger.

The total energy of the fluid during the process of flowing through the runner increases. Since the area of the cross section of the runner flow passage changes increase by a small margin, the flow velocity will not change greatly and the function of the runner on the fluid is mainly embodied in the increase of the static pressure, which can be clearly known by comparing Figure 6(a) and Figure 6(b).

(a) Time Domain about Pressure in the Runner  (b) Time Domain about Flow Velocity in the Runner

Figure 4. Time Domain about the Pressure and Flow Velocity in the Runner
3.2.2. **Analysis of the Unsteady Flow Characteristic in Guide Vane and Stay Vane.** Figure 7 - Figure 9 are the time domain figure of pressure and flow velocity when the fluid flows through the guide vane and stay vane. The fft frequency domain figure and the figure about key information of time - frequency domain respectively. From Figure 7, it can be easily found that the pressure of the fluid flowing through the guide vane increases while the flow velocity decreases, which is resulted from that the runner can decrease the flow velocity and increase the pressure. The pressure fluctuation and the flow velocity fluctuation are mainly analysed below.

The shaft frequency is recorded as fn, thus, the runner blade frequency is recorded as 9fn and the vane frequency is recorded as 20fn. Figure 8(a) and Figure 9(a) indicate that under the action of rotor-stator interaction, 9fn and 2*9fn and 20fn occur to the flow field of guide vane and stay vane.

During the spread process of the pressure wave along the flow direction, the pressure fluctuation of vane frequency does not weaken almost while the pressure fluctuation of the runner blade frequency weakens quickly, wherein, the one-time runner blade frequency 9fn has the fastest weakening speed. Since different spread characteristics corresponding to different frequency components of pressure waves, the dominant frequency of the pressure fluctuation inside the vanes along the flow direction changes, and the vane frequency gradually substitutes the runner blade frequency to be the first dominant frequency inside the guide vane and stay vane.

The spread characteristics of flow velocity fluctuation inside the vanes are basically the same as those inside the runner. As shown in Figure 8(b) and Figure 9(b), the flow velocity fluctuation inside the runner of the guide vane mainly includes two frequency components, namely the one-time runner frequency 9fn and the two-time runner blade frequency 2*9fn. During the spread process along the flow direction and away from the rotor-stator interaction area, flow velocity fluctuation inside the vanes weakens quickly, so the flow velocity fluctuation inside the stay vane has become quite weak.

Comparison of similarities and differences of the unsteady characteristics between the flow line LB and the flow line LA as shown in Figure 9(a)(b) and Figure 9(c)(d) targets at analyzing the effects of the casing tongue on flow inside the vanes. It can be found out easily that the pressure, flow velocity, pressure fluctuation intensity and flow velocity fluctuation closer to the casing tongue are stronger.
Figure 7. Time Domain about the Pressure and Flow Velocity in the Vanes

(a) Time Domain about Pressure in the Vanes
(b) Time Domain about Flow Velocity in the Vanes

Figure 8. Frequency Spectrum of Pressure and Flow Velocity in the Vanes

(a) Frequency Spectrum of Pressure in the Vanes
(b) Frequency Spectrum of Flow Velocity in the Vanes

Figure 9. Time - Frequency Domain of Pressure and Flow Velocity in the Vanes

(a) Time - Frequency Domain of Pressure on LB Flow Line in the Vanes
(b) Time - Frequency Domain of Flow Velocity on LB Flow Line in the Vanes
(c) Time - Frequency Domain of Pressure on LA Flow Line in the Vanes
(d) Time - Frequency Domain of Flow Velocity on LA Flow Line in the Vanes
3.2.3. **Analysis of the Unsteady Flow Characteristic in the Spiral Case.** Figure 10 - Figure 12 are figures about key information about the time domain of pressure and flow velocity, frequency domain and time - frequency domain during the process when the fluid flows through the spiral case. As shown in Figure 10 and Figure 12(a)(c), along the flow direction, the pressure inside the spiral case becomes large gradually while the flow velocity becomes smaller, which is resulted from the gradually increasing area of the cross section of the spiral case along the flow direction.

Three kinds of frequency, namely 9fn, 2*9fn and 20fn, occur to the pressure fluctuation inside the spiral case, as shown in Figure 11. Through analysis of Figure 12(b), during the process of pressure spreading along the flow direction, 9fn frequency fluctuation weakens fast, 2*9fn frequency fluctuation becomes stronger and then weak, while the 20fn frequency fluctuation intensity keeps unchanged almost. During spread of the pressure waves in the spiral case, the dominant frequency transits from the runner blade frequency 9fn to the guide blade frequency 20fn gradually.

There is a tongue structure in the spiral case, which is one important reason of circumferential asymmetry of the spiral case. As shown in Figure 10 and Figure 12, the pressure, flow velocity, pressure fluctuation intensity and flow velocity fluctuation intensity nearby the tongue is obviously larger than that of other positions in the spiral case.

What needs to be pointed out particularly is that the main frequency components (9fn, 2*9fn, and 20fn) of pressure fluctuation at the tongue have the same intensity, which is an important characteristic different from all other positions of the flow passage component.

Compared with the pressure fluctuation, the flow velocity fluctuation inside the spiral case is weaker. Only 9fn frequency obviously occurs to flow velocity fluctuation nearby the tongue.
3.3. Analysis of the Unsteady Characteristic along the Circumferential Direction

Distribution characteristics of the unsteady characteristic in the circumferential direction are analyzed below.

3.3.1. Analysis of the Unsteady Characteristic along the Circumferential Direction at the Outlet of Runner. Figure 13 and Figure 14 are the time domain figure and frequency domain figure of pressure and flow velocity on the outlet of the runner respectively. From the figures, it can be found out that the dominant frequency of pressure and flow velocity fluctuation at the outlet of runner is the 20fn. As shown in Figure 15, the distribution pattern of the pressure, flow velocity and pressure fluctuation and flow velocity fluctuation at the outlet of runner in the circumferential direction is similar, and they are all multipolar and asymmetric.
3.3.2. Analysis of the Unsteady Characteristic in the Vanes in the Circumferential Direction. As shown in Figure 18 and Figure 21, the distribution patterns of the pressure, flow velocity, pressure fluctuation and flow velocity fluctuation in the vanes in the circumferential direction are similar. The distribution pattern of pressure and velocity in Guide Vane in the circumferential direction is complementary, which is resulted from the Bernoulli effect.

Compared with Figure 15(a), Figure 18(a) and Figure 21(a), it can be found that the guide vane and stay vane make pressure distribution in the circumferential direction tend to be uniform. Compared with Figure 15(c), Figure 18(c) and Figure 21(c), it can be found that the guide vane and the stay vane make distribution of flow velocity in the circumferential direction tend to be uniform.
However, compared with Figure 15(b), Figure 18(b), Figure 21(b), Figure 15(d), Figure 18(d), Figure 21(d), it can be easily found that the guide vane and stay vane intensify non-uniformity of pressure fluctuation and velocity fluctuation in the circumferential direction. It can be found that, vanes can promote uniformity of distribution of pressure and flow velocity in the circumferential direction, while this function is at the cost of increasing of non-uniformity of distribution of pressure fluctuation and flow velocity fluctuation in the circumferential direction.

It is worth noting that distribution of intensity of 20fn pressure fluctuation of guide vane and stay vane in the circumferential direction is very uniform. Compared with other frequency pressure fluctuation, it can be approximately considered that distribution of intensity of vane frequency 20fn pressure fluctuation in guide vane is equal in the circumferential direction, and the stay vane has the same characteristic.
(b) Amplitude of Pressure Fluctuation Dominant Frequency in Guide Vane in the Circumferential Direction

(c) Time Domain of Flow Velocity in Guide Vane in the Circumferential Direction
(d) Amplitude of Flow Velocity Fluctuation Dominant Frequency 9fn in Guide Vane in the Circumferential Direction

**Figure 18.** Distribution of Time - Frequency Domain of Pressure and Flow Velocity in Guide Vane in the Circumferential Direction

(a) Time Domain of Pressure in Stay Vane in the Circumferential Direction
(b) Time Domain of Flow Velocity in Stay Vane in the Circumferential Direction

**Figure 19.** Time Domain of Pressure and Flow Velocity in Stay Vane in the Circumferential Direction

(a) Frequency Spectrum of Pressure in Stay Vane in the Circumferential Direction
(b) Frequency Spectrum of Flow Velocity in Stay Vane in the Circumferential Direction

**Figure 20.** Frequency Spectrum of Pressure and Flow Velocity in Stay Vane in the Circumferential Direction
3.4. Analysis of the Fluctuation Characteristic of External Characteristics

The unsteady characteristic of the flow field in the pump turbine finally will result in the unsteadiness of external characteristics such as the head, shaft torque, axial force and radial force, which will affect performance of the pump turbine and other components of the pumping and storage system such as the penstock and the tunnel.

Figure 22(a) is the time domain figure of the total head and the separate head. It can be found that the head formed through transition of the mechanical energy of the runner to the pressure energy of the fluid is the most important constituent of the total head. Figure 23(a) is the head frequency domain figure. The total head has two kinds of frequency fluctuation, namely 2*9fn and 20fn, among which 2*9fn fluctuation is mainly produced in the spiral case, while 20fn fluctuation is mainly produced in the runner. Similarly, from Figure 22(b) and 23(b), it can be found that the shaft torque also has fluctuation of 9fn, 2*9fn and 20fn frequency.

Time domain of the axial force and radial force is shown in Figure 24(a). Fluctuation of the radial force Fxy is obvious while that of the axial force is weak. Decomposition of Fxy in vertical directions can form two component forces (i.e. Fx and Fy) with consistent fluctuation laws while phase position of 180° difference. Frequency domain of axial force and radial force is shown in Figure 24(b). It can be easily found that Fxy mainly has fluctuation of 9fn, 2*9fn and 20fn frequency, among which the 9fn is the first dominant frequency. In Figure 26, the component of pressure fluctuation with the largest amplitude is 9fn frequency. The component force of radial force Fx and Fy have obvious shaft frequency fluctuation and linear combination frequency fluctuation of blade frequency and shaft frequency, which is resulted from rotation of radial force Fxy at the rotating speed of the runner as shown in Figure 25. Thus, Fx and Fy is closely related to the shaft frequency, and there is a 180° phase difference between the two.
Figure 22. Time Domain of the Head and Shaft Torque

Figure 23. Frequency Spectrum of the Head and the Shaft Torque

Figure 24. Time-Frequency Domain of Axial Force and Radial Force
4. Conclusion

After the study in this paper, the unsteady characteristic of the pump turbine at the designed pump operating condition can be concluded that:

1) The spread characteristic of pressure fluctuation and flow velocity fluctuation is different. The flow velocity fluctuation weakens fast during the spreading, so there is obvious flow velocity fluctuation only at the outlet of runner and in guide vane; Pressure fluctuation weakens slowly during spreading, so there is obvious pressure fluctuation in almost all the flow passage components;

2) The intensity of the vane frequency component of pressure fluctuation is approximately equal in static flow passage components everywhere;

3) In static flow passage components, the dominant frequency of pressure fluctuation is the blade frequency and its harmonic frequency and vane frequency. During spreading in the flow direction, the intensity of vane frequency keeps nearly unchanged, while the component of blade frequency and its harmonic frequency weakens continuously. Thus, the first dominant frequency of pressure fluctuation in the static flow passage component transits from the blade frequency to the vane frequency;

4) Constraints of the vanes and blades have obvious effects on distribution of the unsteady characteristic. The blades make the pressure in different positions of the cross section tend to be uniform, which is at the cost of intensification of non-uniformity of distribution of pressure fluctuation on the cross section. The vanes make distribution of pressure, flow velocity in the circumferential direction uniform, which results in intensification of non-uniformity of the distribution of pressure fluctuation and flow velocity fluctuation in the circumferential direction;

5) The intensity of the fluctuation of the three dominant frequencies (one-time / two-time blade frequency and the vane frequency) of pressure fluctuation in the tongue is approximately equal, which is an important characteristic distinguishing the tongue from other positions. In addition, pressure...
fluctuation and flow velocity fluctuation nearby the tongue is more violent than that of other positions in the circumferential direction.

6) The distribution pattern of pressure, flow velocity, pressure fluctuation and flow velocity fluctuation at the outlet of runner in the circumferential direction is similar (the distribution is asymmetric), while the distribution pattern of pressure and velocity in Guide Vane and stay vane is complementary.

7) The total head is mainly transited from the mechanical energy of the runner. The total head has obvious fluctuation of two-time blade frequency and vane frequency. Respectively, these two kinds of fluctuation are resulted from the runner head component and the spiral case head component. The dominant frequency of fluctuation of the shaft torque and the radial force is consistent with that of the total head. Through decomposition of the radial force in the horizontal direction and the vertical direction, two component forces with the same dominant frequency and the phase difference of 180° can be obtained. The component force of the radial force has obvious shaft frequency fluctuation and the linear combination frequency of the blade frequency and the shaft frequency. Compared with other forces, the axial force is large while the fluctuation is rather weak.

References

[1] Ran H J, Xu H Y, Luo X W, Liu S H. 2008 The Numeric Simulation and Performance Analysis of the Pump Turbine[J]. Large Motor Technology, 2008(04):45-49.
[2] Wang L Q, Liu J T, Zhang L F, Qin D Q, Jiao L. 2012 Study of pump-turbine’s pumping mode at different openings of guide vane[J]. JOURNAL OF HYDROELECTRIC ENGINEERING, 31(02):222-227.
[3] Wang L Q, Liu Y Y, Liu W J, Qin D Q, Jiao L. 2013 Pressure fluctuation characteristics of pump-turbine at pump mode[J]. Journal of Drainage and Irrigation Machinery Engineering, 31(01):7-10+35.
[4] Wang H J, Gao Y H, Li D Y, Gong R Z, Wei X Z, Qian D Q, Ming L, Nie W Z. 2015 Analysis of Pressure Fluctuation of Pump-Turbine in Pump Mode Based on Dynamic Mesh[J]. Large Motor Technology, 2015(05):46-49+56.
[5] Nakamura T, Nishizawa H, Yasuda M, et al. 1996 Study on High Speed and High Head Reversible Pump -Turbine[M]// Hydraulic Machinery and Cavitation. Springer Netherlands, 210-219.
[6] Barrio R, Fernández J, Blanco E, et al. 2012 Performance characteristics and internal flow patterns in a reverse-running pump–turbine[J]. ARCHIVE Proceedings of the Institution of Mechanical Engineers Part C Journal of Mechanical Engineering Science 1989-1996 (vols 203-210), 226(3):695-708.
[7] Sun Y K, Zuo Z G, Liu S H, et al. 2012 Numerical simulation of the influence of distributor pitch diameter on performance and pressure fluctuations in a pump-turbine[C]// 2037.
[8] Guo, L., et al. 2014 "Pressure fluctuation propagation of a pump turbine at pump mode under low head condition." Science China Technological Sciences 57(4): 811-818.
[9] Li, D., et al. 2018 "Analysis of Pressure Fluctuations in a Prototype Pump-Turbine with Different Numbers of Runner Blades in Turbine Mode." Energies 11(6): 1474.
[10] Sun, Y., et al. 2014 "Distribution of Pressure Fluctuations in a Prototype Pump Turbine at Pump Mode." Advances in Mechanical Engineering 6: 923937.
[11] Jia W, Liu J S, Pang L J, Lv G P, Wang H J. 2014 Dynamic and Static Interference and Vibration Analysis of Pump Turbine in Pumped Storage Power Station[J]. Journal of Vibration Engineering, 27(04):565-571.
[12] Pang L J, Lv G P, Liu J S, Jia W. 2013 Anti-vibration and Crack Control Design of High-head Pump-turbine Runner[J]. JOURNAL OF MECHANICAL ENGINEERING, 49(04):140-147.
[13] Rodriguez C G. 2007 Frequencies in the Vibration Induced by the Rotor Stator Interaction in a Centrifugal Pump Turbine[J]. Journal of Fluids Engineering, 129(11):1428-1435.