Relativistic Charge Form Factor of the Deuteron
from (np)–Scattering Phase Shifts

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Relativistic integral representation in terms of experimental neutron–
proton scattering phase shifts is used to compute the charge form fac-
tor of the deuteron $G_{Cd}(Q^2)$. The results of numerical calculations of
$|G_{Cd}(Q^2)|$ are presented in the interval of the four–momentum transfers
squared $0 \leq Q^2 \leq 35 \text{ fm}^{-2}$. Zero and the prominent secondary maximum in
$|G_{Cd}(Q^2)|$ are the direct consequences of the change of sign in the experi-
mental $^3S_1$– phase shifts. Till the point $Q^2 \simeq 20 \text{ fm}^{-2}$ the calculated total
relativistic correction to $|G_{Cd}(Q^2)|$ is positive and reaches the maximal value
of 25% at $Q^2 \simeq 14 \text{ fm}^{-2}$.

Deuteron is the brightest example of intersection of nuclear and particle physics. During
more than sixty years it serves as a source of important information about the nuclear forces,
mesonic and baryonic degrees of freedoms in nuclei, relativistic effects and a possible role of
quarks in nuclear structure. Therefore it is not surprising that currently the electromagnetic
(EM) structure of the deuteron is a subject of intensive theoretical (the list of publication
is immense) and experimental investigations.

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With new experimental data from Jefferson Lab on elastic electron-deuteron scattering \cite{1,2} at momentum transfers in the GeV-range, one needs to develop relativistic approaches to the (np)-bound state problem. Recent experimental results from MIT-Bates \cite{3} provided the first experimental evidence for a zero in the deuteron charge form factor $G_{Cd}$ at about $Q^2 = 20 \text{ fm}^{-2}$ predicted in a number of theoretical models (or not predicted, as in some kinds of quark models).

Here we report the new results of numerical calculations of relativistic deuteron charge form factor (RDCFF). These calculations are based on the approach to the relativistic impulse approximation, which was briefly discussed in ref. \cite{4} (see also the review \cite{5} and, especially, the references therein). The more detailed formulae are contained in ref. \cite{6}. In this approach the deuteron electromagnetic form factors (EMFF) are expressed in terms of experimental neutron–proton $(n-p)$ phase shifts in the triplet channel of $n-p$ scattering and nucleon EMFFs. The approach was developed during a number of years and is still being developed. The key components are as follows.

1. General relativistic parametrization of the matrix elements of the EM current between the n-p scattering states $\langle n'p'|j_\mu|np \rangle$ in terms of the invariant EMFFs of the n-p system. This parametrization is carried out in the canonical basis $|P_\mu JlSm,J \rangle$ of the two-particle state vectors. Here $P_\mu = (p_\mu + n_\mu)$ is the total 4-momentum of n-p system; $J, m_J$ are the total angular moment and its projection; $l, S$ are the orbital moment and total spin of n-p system. As the result for the n-p scattering state in the elastic channel with deuteron’s quantum numbers ($J = S = J' = S' = 1; l, l' = 0, 2$) we obtain sixteen invariant FFs $G_{li}^{ll'}(s, s', t, ...)$, where $s \equiv (p_\mu + n_\mu)^2$, $t \equiv q_\mu^2 \equiv -Q^2 < 0$ and the index $i$ numerates the charge(C), magnetic(M), quadrupole(Q) and the quadrupole of the second generation, or toroidal(T) FFs.

\footnote{1 For the choice of kinematical variables see App.A.}
2. The decomposition of the total $G^-$ into the sum

$$G = g + G^{int},$$

(1)

where $g^-$ is the FFs of unconnected part of the current (see Fig.1). This part is defined as usual:

$$\langle p'|j_\mu|p\rangle\langle n'|n\rangle + \langle n'|j_\mu|n\rangle\langle p'|p\rangle.$$ 

The second term in eq.(1) describes the "real" n-p interaction. The decomposition in eq.(1) is relativistic invariant due to the existence of the one-particle singular invariants of the type $2p_0\delta(\vec{p} - \vec{p}')$.

3. The calculation of $g^-$ as a function of invariant variables and nucleon EMFFs. These calculations were done by the methods of relativistic kinematics (without any $v/c$-expansion).

It's also evident from the two previous points that the nucleon EMFFs, entering to $g^-$, describe the nucleons on their mass shell.

4. The local analyticity of the FFs

$$G^{int} = \lim_{\epsilon,\eta \rightarrow 0} G^{int}(s \pm i\epsilon, s' \mp i\eta, ...),$$

(2)

in the vicinity of the physical region of the variables $s, s' \geq 4M^2$. As we defined above, $s$ and $s'$ are the squared invariant masses of n-p system in initial and final states.

5. The representation of the n-p scattering matrix $S(s)$ in the physical region in terms of the Jost matrix $B(s)$:

$$S(s) = B(s - i\epsilon)B^{-1}(s + i\epsilon), \quad s \geq 4M^2,$$

(3)

i.e. the solution of the boundary problem

$$S(s)B_+(s) = B_-(s), \quad s \geq 4M^2$$

(4)

in the scattering theory. As it well known, in the model one-channel case the the solution of eqs.(3,4) has the simple form
\[ B(s) = \left( 1 - \frac{M_d^2 - 4M^2}{s - 4M^2} \right) \cdot \exp \left\{ \frac{1}{\pi} \int_4^\infty \frac{\delta(u)du}{s - u} \right\} \]

where \( \delta \) is the phase-shift for one channel \( S \)-matrix \((S = \exp(2i\delta))\). But in the channel of \( n-p \) scattering with the deuteron’s quantum numbers the scattering matrix is \( 2 \times 2 \) matrix:

\[ S(s) \equiv S[\delta, \eta, \varepsilon] = \begin{pmatrix} \cos 2\varepsilon \cdot e^{2i\delta} & i \sin 2\varepsilon \cdot e^{i(\delta+\eta)} \\ i \sin 2\varepsilon \cdot e^{i(\delta+\eta)} & \cos 2\varepsilon \cdot e^{2i\eta} \end{pmatrix}. \tag{5} \]

In this case the solution of the boundary problem (eq.(4)) is much more complicated, but it appears to be possible to find \( B \) in the form of series with fast convergence (see below). It is essential that every element of the inverse matrix \( B^{-1} \) has a simple pole at the point of bound state \( s = M_d^2 \).

6. Solution of the linear conjugation problem for \( G_{-,-} \). Relative to variable \( s \), for example, the initial equation is

\[ [g(s,..) + G(s + i\varepsilon,..)] = [g(s,..) + G(s - i\varepsilon,..)] \cdot S(s). \tag{6} \]

Following by this way and combining the solution of eqs.(3, 6) for \( s \)-variable and the analogous one for \( s' \)-variable, we can obtain the integral representation for the relativistic EMFFs of \( n-p \) system in terms of \( g \) and \( B \). The resulting formulae are cumbersome enough and we omit them here.

7. Taking the residues at the deuteron poles \( s = s' = M_d^2 \) we obtain the final formulae for the DEMFFs (see,for example, eqs.(7,8).

After this sketch of the method we return to the analysis of RDCFF. Among the three DEMFFs the \( DCFF \) is the most interesting for us when investigating the deuteron structure. The point is that the most informative features of the deuteron structure — zero (or "diffractional minimum") and prominent secondary maximum in the theoretical predictions for DEMFFs — appear already in the DCFF at not so large values of \( Q^2 (\sim 20 - 30 \text{ fm}^{-2}) \). The measurements of \( |G_{cd}(Q^2)| \) for such \( Q^2 \) are real [1,2]. On the contrary, the theoretical picture of DM(Q)FFs is so that indicated above the fine structure in this FFs appears
for relatively high $Q^2$, which hardly can be reached in current and planned polarization experiments.

Following [6] the formulae for RDCFF $G_{cd}(Q^2)$ appear as

$$G_{cd}(Q^2) = (\rho \tilde{B}^{20} + \tilde{B}^{22})^2 G_{cd}^{00}(Q^2) - (\rho \tilde{B}^{20} + \tilde{B}^{22})(\rho \tilde{B}^{00} + \tilde{B}^{02})[G_{cd}^{02}(Q^2) + G_{cd}^{20}(Q^2)] + (\rho \tilde{B}^{00} + \tilde{B}^{02})^2 G_{cd}^{22}(Q^2),$$

(7)

$$G_{cd}^{ll'} = \Gamma^2 \int_{4M^2}^{s_2} ds \Delta B_{ll'}(s) \int_{s_1}^{s_2} \frac{ds'g_c(s, s't)\Delta B(s')}{s' - M_d^2},$$

$$s_{2,1} = 2M^2 + \frac{1}{2M^2}(2M^2 - t) \cdot (s - 2M^2) \pm \frac{1}{2M^2} \sqrt{(-t)(4M^2 - t)(s - 4M^2)}. \tag{8}$$

In eq. (7) $\rho$ is the constant, which describes the mixing of two $n - p$ states with different orbital moments ($l = 0$ and $l = 2$) in the point of the bound state, i.e., the deuteron. This constant is defined by the correspondence principle. Analyzing the nonrelativistic limits of eqs. (7, 8), we can prove that $\rho$ appears to be the standard asymptotic $D/S$–ratio of the radial deuteron wave functions, so $\rho = 0.0277$ (numerical calculations show that the dependence of DCFF on the variation of $\rho$ is very weak). All four elements of the matrix $\tilde{B}_{ll'}(s) (l, l' = 0, 2)$ are taken at the bound state point $s = M_d^2$ ($M_d = 2M - \varepsilon$, where $M_d, M$ being deuteron and nucleon masses, and $\varepsilon$– being the deuteron binding energy).

In eq. (8) $\Gamma^2$ is the normalization constant, which is calculated from the condition $G_{cd}(0) = 1$. Matrix functions $\Delta B_{ll'}(s) = B_{ll'}(s + i\varepsilon) - B_{ll'}(s - i\varepsilon)$ are the discontinuities of the Jost matrix $B(s)$. The reduced Jost matrix $\tilde{B}$ in eq. (7) is the solution of the same as for $B$, eq. (8) with the scattering matrix $\tilde{S} \equiv S[\tilde{\delta}, \tilde{\varepsilon}, \tilde{\eta}].$ The reduced phase shifts $\tilde{\delta}, \tilde{\varepsilon}, \tilde{\eta}$ have the auxiliary nature and describe the $n$-$p$ scattering without bound state ($\tilde{\delta}(E \to 0) \to 0$). Roughly speaking, matrix $\tilde{B}$ is the rest of matrix $B$ after taking the residue at the point of the deuteron’s pole. The formulae for $\tilde{B}$ and $B$ in terms of $n - p$ phase shifts are cumbersome
and are summarized in Appendix B. All relativistic aspects of the two–nucleon problem are contained in $G^{ll'}$–matrix.

As was mentioned above, the matrix functions $g^{ll'}_c(s, s', t)$ of three variables are the relativistic CFF of the unconnected part of the matrix element of EM current $\langle n'p'|j_\mu|np \rangle$. The results of the calculations of $g^{ll'}_c$ are given in Appendix C. It is interesting to note that in general case in the relativistic region $g^{ll'}_c$–functions are not factorized in $s, s'$ variables, whereas in the nonrelativistic limit such factorization takes place. It means that in the framework of the used relativistic approach [4]–[6] it is impossible to introduce some kind of concept of relativistic deuteron wave function.

The $n − p$ phase shifts were taken from the recent analysis of Virginia Tech group [7] and is shown in Fig. 2. This analysis was made in the energy range $0 < E_{lab} \leq 1300$ MeV. Extrapolation to higher energies is not so important for the calculations of $G_{Cd}$ for the small and intermediate values of $Q^2$ (see below). The only essential circumstance is that $^3S_1$–phase shifts change sign from positive to negative and have the minimum near the energy $E_{lab} \sim 1$GeV, then go to zero in accordance with the Levinson’s theorem. Any realistic $n − p$ $^3S_1$–phase shift analysis has such a behavior. Two other states ($^3D_1$ and $^3\varepsilon_1$) give a relatively small contribution to $G_{Cd}$.

For the calculations of $G_{Cd}$ we used (as a first step) the simplest choice of the nucleon form factors: $G_{Ep} = (1 + Q^2/18.23 \text{ fm}^{-2})^{-2}$, $G_{Mp}/\mu_p = G_{Mn}/\mu_n = G_{Ep}$, $G_{En} \equiv 0$ for all $Q^2$.

The result of the calculations are presented in Fig. 3. Our brief conclusions are as follows.

1. The appearance of zero and secondary maximum in $|G_{Cd}(Q^2)|$ at intermediate values of $Q^2$ is the direct consequence of the change of sign of the experimental $^3S_1$–phase shifts at intermediate energies. It is easy to calculate that the model’s $\delta(E)$, which decreases monotonically with $E$ and is always positive ($\delta(E) > 0$ for all $E$), immediately leads to monotonically decreasing with $Q^2$ values of $|G_{Cd}(Q^2)|$ without any fine structure.
2. Almost up to the point of zero \(Q^2 \simeq 20 \text{ fm}^{-2}\) of \(|G_{Cd}(Q^2)|\) the total relativistic correction (TRC), \textit{i.e.}, the difference between \(G_{Cd}\) calculated relativistically \((1,2)\) and its nonrelativistic limit, is positive and appears to be not small. For example, for \(Q^2 \simeq 14 \text{ fm}^{-2}\) it reaches the value of 25%.

3. TRC becomes large in the region of the secondary maximum of \(|G_{Cd}(Q^2)|\), increasing the magnitude of the form factor.

4. The obtained results are consistent with the available data on \(G_{Cd}\) from MIT-Bates \([8]\). New data from Jefferson Lab E–94-018 \([1]\) are extremely important to test the proposed relativistic approach in the region of higher transferred momenta, where relativistic corrections appear to be significant.

We would like to make the following comments to the obtained results. Firstly, as it was mentioned above, for the fixed till \(E_{lab} = 1300 \text{ MeV}\) \(^3S_1\)–phase shifts set of ref. \([7]\) the character of extrapolation to high energies is not essential when calculating the RDCFF. Two other, than in Fig.\([2]\) possible extrapolations of \(\delta(E \to \infty)\) are shown in Fig.\([4]\). Their foundations are the following. If we include the so-called forbidden bound states in NN system (possible in some kinds of theories) in the formulation of Levinson’s theorem, then the immediate consequence for \(\delta(E)\) would be \(\delta(E \to \infty) \to -\pi\) \((\text{see, for example, ref. } [8])\). The second extrapolation is connected with the phase shift analysis of proton–proton scattering for the several GeV region \((\text{ref. } [7])\). It is natural to think that for high energies the contribution of Coulomb effects are small, so n–p and p–p phase shifts in the equal spin states are very close to each other. The results of the calculations of \(|G_{Cd}(Q^2)|\) with the phase shifts from Fig.\([4]\) are shown in Fig.\([5]\). It is evident that the difference between these three curves is negligible. But when we turn to other phase shifts, which are differing from the results of ref. \([7]\) in the intermediate energy region, \(E_{lab} < 1 \text{ GeV}\), the situation change essentially. The dependence of \(|G_{Cd}(Q^2)|\) structure on the choice of different sets of experimental \(n–p\) phase shifts available from the literature \((\text{ref. } [10])\) is strong enough. Possible variation of
δ, ε, η may shift the position of zero in |G_{Cd}(Q^2)| from the indicated point Q^2 = 20 fm\(^{-2}\) to the point Q^2 = 16 fm\(^{-2}\) or to the point Q^2 = 23 fm\(^{-2}\). At the same time the secondary maximum is located in the interval 26 \(\leq Q^2 \leq 32\) fm\(^{-2}\), and its height may change by a factor of seven. We can see that for improving our understanding of |G_{Cd}(Q^2)| it would be desirable to have a more detailed phase shifts analysis of n – p scattering in triplet channel in intermediate energy region E_{lab} \(\leq 1\) GeV. Secondly, let us indicate the dependence of |G_{Cd}(Q^2)| on the possible choice of nucleon EM form factors. Since the uncertainties of G_{Ep}(Q^2) in the considered range of Q^2 are very small, the main effect in |G_{Cd}(Q^2)| may be caused only by variation of G_{En}(Q^2). It seems to be generally accepted that the maximal deviation of G_{En}(Q^2) from the zero-value approximation G_{En} \equiv 0 is given by known formula G_{En}(Q^2) = -\mu_n \tau G_{Ep}(Q^2), where \(\mu_n = -1.91\) is the neutron anomalous magnetic moment and \(\tau = Q^2/4M^2\). The results of the calculations of |G_{Cd}(Q^2)| with this nonzero values of G_{En}(Q^2) are shown in fig.2. One can see that the effect is sizable and the contributions of relativistic effects and nonzero G_{En} have a similar behavior.

Finally, we show for comparison in Fig.3 the results of calculation of G_{Cd} in a relativistic approach developed in ref. [8]. It may be seen that zero of |G_{Cd}(Q^2)| predicted in ref. [8] is shifted to the lower values of Q^2 and the height of the secondary maximum is approximately the same as in our calculations.

Here we restricted ourselves only to the discussion of the deuteron charge form factor G_{Cd}. Even in this case we omitted such interesting questions as an analytical representation of relativistic corrections in different orders in (v/c)^2, the new representation for realistic deuteron wave functions, the role of relativistic rotation of nucleon spins and orbital momentum \(l = 2\) in the deuteron, the problem of extraction, using the present approach, of G_{En}(Q^2) for ultralow values of Q^2 from experimental data on elastic ed–scattering, and contributions from meson–exchange currents. It would also be interesting to perform a detailed comparison of the present approach with other relativistic approaches to the description of
deuteron structure.

All these questions, as well as the calculations of the deuteron magnetic and quadrupole form factors will be discussed in forthcoming publications.

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APPENDIX A: KINEMATIC VARIABLES.

By definition \( s \) is the invariant mass of \( n – p \) system squared:

\[
    s = (p_n + p_p)_\mu^2.
\]

In laboratory (LS) and center-of-mass (CMS) systems we have

\[
    s = 4M^2 + 2E = 4M^2 + 4p^2,
\]

where \( E \) is the nucleon energy in LS and \( p \) is the magnitude of the nucleon 3–momentum in CMS.

\( Q^2 \) is the magnitude of the 4–momentum transfer squared:

\[
    Q^2 \equiv -q^2_\mu \equiv -t > 0.
\]

APPENDIX B: JOST MATRICES \( B, \tilde{B} \).

The formulae for pairs \((S, B)\) and \((\tilde{S}, \tilde{B})\) have the most convenient form in the \( p \)–plane:

\[
    S(p)B_+(p) = B_-(p), \quad -\infty < p < \infty,
\]
where \( S \equiv S[\delta(p), \eta(p), \varepsilon(p)] \), see eq.(5). Let us introduce two new matrices \( \tilde{S} \) and \( \tilde{B} \):

\[
\tilde{B}_\pm(p) = R(\mp p)B_\pm(p),
\]

\[
R(p) = I - \frac{2i\alpha}{(p+i\alpha)(1+p^2)} \cdot \begin{pmatrix} 1 & -\rho \\ -\rho & \rho^2 \end{pmatrix}, \ (\alpha^2 = M\varepsilon).
\]

Now the equation for \( \tilde{B} \) has the form

\[
\begin{cases}
\tilde{S}(\tilde{p})\tilde{B}_+(\tilde{p}) = \tilde{B}_-(\tilde{p}), & -\infty < \tilde{p} < \infty, \\
\tilde{S}(\tilde{p}) = R(p)S(p)R^{-1}(-p) \equiv \tilde{S}[\tilde{\delta}, \tilde{\eta}, \tilde{\varepsilon}].
\end{cases}
\tag{B1}
\]

The last equation defines the reduced phase shifts \( \tilde{\delta}, \tilde{\varepsilon}, \tilde{\eta} \) as functions of input experimental phase shifts \( \delta, \varepsilon, \eta \).

The solution of eq.(B1) was found in ref. [11] in the form of series

\[
\tilde{B}_\pm(p) = \tilde{B}_{\pm,0}(p) \cdot \left[ I + \sum_{m=1}^{\infty} \tilde{B}_{\pm,m}(p) \right],
\]

where

\[
\tilde{B}_{\pm,0}(p) = \begin{pmatrix} \varphi_1(p)e^{\mp \delta(p)} & 0 \\ 0 & \varphi_2(p)e^{\pm \eta(p)} \end{pmatrix},
\]

\[
\varphi_1(p) = \exp\left[-\frac{1}{\pi}V.P. \int_{-\infty}^{\infty} \tilde{\delta}(p')dp' \right],
\]

\[
\varphi_2(p) = \exp\left[-\frac{1}{\pi}V.P. \int_{-\infty}^{\infty} \tilde{\eta}(p')dp' \right],
\]

\[
\tilde{B}_{\pm,m}(p) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{dp'}{p - p' \pm i0} \cdot \sum_{n=1}^{m} G_n(p') \tilde{B}_{+,0}(p') [\tilde{B}_{+,m-n}(p')]^{1-\delta_{mn}}.
\tag{B2}
\]

In eq.(B2) for odd \( n \)

\[
G_n(p) = i(-1)^{\frac{n-1}{2}} \cdot \frac{1}{n!} \cdot [2\tilde{\varepsilon}(p)]^n \cdot \begin{pmatrix} 0 & e^{i(\delta+\eta)} \\ e^{i(\delta+\eta)} & 0 \end{pmatrix}
\]

and for even \( n \)

\[
G_n(p) = i(-1)^{\frac{n}{2}} \cdot \frac{1}{n!} \cdot [2\tilde{\varepsilon}(p)]^n \cdot \begin{pmatrix} e^{2i\delta} & 0 \\ 0 & e^{2i\eta} \end{pmatrix},
\]

\( \delta_{mn} \) is the Kroneker delta.
APPENDIX C: $G_C^{LL'}$–MATRIX.

In terms of invariant variables $s, s', t$ and the nucleon EM form factors the matrix elements have the form:

$$g_{c}^{00}(s, s', t) =$$
$$g(s, s', t)[g_1(s, s', t)(\cos \alpha_1 \cos \alpha_2 - \frac{1}{3} \sin \alpha_1 \sin \alpha_2) \cdot G_{EN}^s(Q^2) +$$
$$+ \frac{1}{2M} g_2(s, s', t) \cdot (\frac{1}{3} \sin \alpha_1 \cos \alpha_2 - \cos \alpha_1 \sin \alpha_2) \cdot G_{MN}^s(Q^2)] ,$$

$$g_{c}^{02}(s, s', t) =$$
$$g(s, s', t)\{g_1(s, s', t)(-\sqrt{2}P_{20} \cos \alpha_1 \sin \alpha_2 + \frac{1}{\sqrt{2}} P_{21} \sin \alpha_1 \cos \alpha_2) \cdot G_{EN}^s$$
$$- \frac{1}{2M} g_2(s, s', t)(\sqrt{2}P_{20} \sin \alpha_1 \cos \alpha_2 + \frac{1}{\sqrt{2}} P_{21} \cos \alpha_1 \sin \alpha_2)G_{MN}^s\},$$

$$g_{c}^{20}(s, s', t) = g_{c}^{02}(s', s, t) ,$$

$$g_{c}^{22}(s, s', t) =$$
$$g(s, s', t)\{g_1(s, s', t)\left[\frac{1}{3}P_{21}P_{21}' + \frac{2}{3}P_{20}P_{20}'\right] \cdot \cos(\alpha_1 - \alpha_2) +$$
$$+ \frac{1}{12} P_{22}P_{22}' + \frac{1}{3} P_{20}P_{20}'\right] \cdot \cos \alpha \cos \alpha_2 +$$
$$+ \left[\frac{1}{12}(P_{22}P_{21}' - P_{21}P_{22}') + \frac{1}{2}(P_{21}P_{20}' - P_{20}P_{21}')\right] \sin(\alpha_1 - \alpha_2) -$$
$$- \frac{1}{6}(P_{22}P_{20}' + P_{20}P_{22}') \sin \alpha_1 \sin \alpha_2\} \cdot G_{EN}^s - \frac{1}{2M} g_2(s, s', t) \cdot$$
$$\left[\frac{1}{12}(P_{21}P_{21}' - P_{22}P_{22}') + \frac{1}{2}(P_{20}P_{21}' - P_{21}P_{20}')\right] \cdot$$
$$\cos(\alpha_1 - \alpha_2) - \frac{1}{12} P_{22}P_{22}' + \frac{1}{3} P_{20}P_{20}'\right) \cdot \cos \alpha_1 \sin \alpha_2 -$$
$$- \frac{1}{6}(P_{22}P_{20}' - P_{20}P_{22}') \sin \alpha_1 \cos \alpha_2 +$$
$$+ \left[\frac{1}{3}P_{21}P_{21}' + \frac{2}{3}P_{20}P_{20}'\right] \sin(\alpha_1 - \alpha_2)\} \cdot G_{MN}^s\} ,$$
where

\[ g(s, s', t) = \frac{g_1(s, s', t)(-t)}{\sqrt{(s-4M^2)(s'-4M^2)}} \cdot \frac{1}{|\lambda(s, s', t)|^{1/2}} \cdot \frac{1}{\sqrt{1+\tau}}, \]

\[ g_1(s, s', t) = s + s' - t, \]

\[ g_2(s, s', t) = \left[ (-1)(M^2\lambda(s, s', t) + ss't) \right]^{1/2}, \]

\[ \lambda(s, s', t) = s^2 + s'^2 + t^2 - 2(ss' + st + s't). \]

\( P_{lm} \) are the Legendre polynomials, \( P_{lm} \equiv P_{lm}(x) \) and \( P'_{lm} \equiv P_{lm}(x') \), where

\[ x(s, s', t) = \frac{\sqrt{s'(s'-s-t)}}{\sqrt{(s'-4M^2)\lambda(s, s', t)}}, \]

\[ x'(s, s', t) = -x(s', s, t). \]

The angles \( \alpha_1, \alpha_2 \) of the relativistic rotation of nucleon spins in deuteron are

\[ \alpha_1 = \arctan \frac{g_2(s, s', t)}{M[(\sqrt{s}+\sqrt{s'})^2-t]+\sqrt{ss'}(\sqrt{s}+\sqrt{s'}+2M)}, \]

\[ \alpha_2 = \arctan \frac{g_2(s, s', t)(\sqrt{s}+\sqrt{s'}+2M)}{M(s+s'-0)(\sqrt{s}+\sqrt{s'}+2M)+\sqrt{ss'}(4M^2-t)}. \]

\( \tau = Q^2/4M^2; G^a_{E,MN} = \frac{1}{2}(G_{E,Mp} + G_{E,Mn}) \) are the nucleon isoscalar charge and magnetic form factors.

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FIG. 1. Separation of the unconnected part of the two-nucleon electromagnetic current.
FIG. 2. Neutron–proton phase shifts from ref. [7] and their extrapolation.
FIG. 3. Relativistic deuteron charge form factor and its nonrelativistic limit. RDCFF with nonzero values of $G_{En} = -\mu_n \tau G_{Ep}$ is also shown. For comparison DCFF in relativistic approach of [8] and experimental results from MIT–Bates [8] are given, too.
FIG. 4. The various extrapolations of \(^3S_1\)-phase shifts to high energies. Curve 1 is the same as in Fig.2. The curves 2,3 are described in the text.
FIG. 5. RDCFF for the phase shifts from Fig. 4.