Di-neutron dynamics in medium-mass neutron-rich nuclei

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Abstract. Investigating the pair correlation in medium-mass nuclei near the neutron drip-line, we find that the pairing involving several weakly bound neutrons exhibits a significant spatial correlation of the di-neutron type. We suggest that the strong spatial correlation is an inherent property of the pair correlation in nuclear matters at low densities. The di-neutron correlation influences strongly properties of the soft dipole and octupole excitations of the neutron-rich nuclei.

1. Introduction

The di-neutron correlation has been discussed extensively in connection with the two-neutron halo nucleus \(^{11}\text{Li}\), for which the correlation among the neutrons is essential for the halo structure and the binding mechanism of the last two neutrons. It is predicted that the halo neutrons form a pair correlated spatially at short relative distances\([1, 2, 3, 4, 5]\). It is also argued, though not confirmed yet experimentally, that the pair correlation of this di-neutron type may be linked to the characteristic soft dipole excitation in the two-neutron halo nuclei\([1, 2, 6, 7]\).

We would like to generalize the concept of di-neutron correlation to a wider class of neutron-rich nuclei, including those in medium or heavy mass region which may have several neutrons in the low density skin region but do not necessarily exhibit a halo. We wish to see whether the di-neutron correlation is an inherent property of the nuclear pair correlation of nucleon many-body systems characterized by the weak binding and/or the low density. For this purpose, we first discuss the spatial structure of the Cooper pair in the superfluid low density nuclear matter. We shall show that the spatial correlation of the di-neutron type indeed manifests itself in this situation. We then turn to medium-mass neutron-rich nuclei, for which we demonstrate the presence of the di-neutron correlation and its influence on the soft modes of excitation.

2. Coordinate-space HFB and continuum QRPA

In order to describe the pair correlation in both uniform matters and nuclei near the drip-line, we utilize the generalized mean-field method, i.e. the Hartree-Fock-Bogoliubov (HFB) scheme. The HFB treatment is necessary for near-drip-line nuclei as the pair correlation may vary in different regions (inside, surface, skin and/or halo parts of the system). The HFB, if formulated in the coordinate-space representation, can describe also correct behaviors at large distances of weakly bound and unbound nucleon orbits which are important near the drip-line\([8, 9]\). We also emphasize that the HFB mean-field method is suitable for the description of the di-neutron correlation. It is known that the Bogoliubov’s mean-field method can be applied not only to the
The size of the Cooper pair (the coherence length $\xi$) plotted as a function of the density for the symmetric nuclear matter calculated with the Gogny D1 force. The dotted line is the average inter-neutron distance $d$. (b) The relative weight $P(r_d)$ concentrated at short relative distances $r < r_d$ of the probability distribution of the pair wave function, plotted as a function of the density. The inset shows the probability distribution $|\Psi_{\text{pair}}(r)|^2$ of the pair wave function at $\rho/\rho_0 = 1/2$.

By extending the same mean-field scheme to time-dependent problems, i.e. by means of the quasiparticle random phase approximation (QRPA) one can describe the dynamics associated with the pair or di-neutron correlations. It is again very important to treat the correct asymptotics of the nucleon wave function as the excitation causes actual emission of the weakly bound nucleons. This is achieved in the continuum QRPA method[14] which utilizes the Green function technique[15]. By means of the continuum QRPA, we can study the di-neutron dynamics which may take place far outside the nucleus and may lead to a pair emission.

3. Low-density nuclear matter

The nuclear matter becomes a superfluid at low densities due to the pair correlation in the $1S$ channel. Although there is a large uncertainty concerning the higher order medium effects, many calculations for the neutron matter performed at the mean-field level predict essentially the same results[16] pointing to a large pair gap $\Delta = 2 - 3$ MeV at the densities $\rho/\rho_0 = 5 \times 10^{-1} - 5 \times 10^{-2}$ where $\rho$ (and $\rho_0$) is the neutron density (and that of the normal nuclear matter). This enough suggests importance of the pair correlation in the low-density many-nucleon systems. An insight to the spatial behavior of the paired neutrons or the size of the Cooper pair may be obtained by evaluating the Pippard’s coherence length $\xi = h^2k_F/m^*\pi\Delta[10]$. Fig.1(a) shows the coherence length $\xi$ for the symmetric matter calculated with the effective Gogny D1 force. The coherence length exhibits dramatic decrease from a large value of the order of $\sim 40$ fm at the normal density $\rho/\rho_0 \sim 1$ to a considerably small value of $\sim 4$ fm at the low densities $\rho/\rho_0 \sim 2 \times 10^{-1} - 1 \times 10^{-2}$. If we compare with the average inter-neutron distance $d = \rho^{-1/3}$, the size of the Cooper pair is comparable to or a little smaller than $d$ in a very wide interval extended to much lower densities $\rho/\rho_0 \sim 2 \times 10^{-1} - 1 \times 10^{-4}$. The relation $\xi < d$ may be regarded as a condition for strong di-neutron correlation. If we make a nominal correspondence in terms of the local densities, the above analysis suggests that the strong di-neutron correlation may be realized both in the region...
of the neutron skin $\rho/\rho_0 = 2 \times 10^{-1} - 1 \times 10^{-2}$ and in that of the halo $\rho/\rho_0 = 10^{-2} - 10^{-4}$ (we here regard $\rho/\rho_0 = 10^{-2}$ as a boundary between the skin and the halo[17, 18]).

We can see the spatial structure of di-neutron correlation more explicitly by looking into the wave function of the Cooper pair. The inset of Fig.1(b) shows an example of the Cooper pair wave function

$$\Psi_{\text{pair}}(\mathbf{r} \uparrow, \mathbf{r}' \downarrow) \equiv \langle \Phi_0 | \psi_{\uparrow}^1(\mathbf{r} \uparrow)\psi_{\downarrow}^1(\mathbf{r}' \downarrow) | \Phi_0 \rangle = \sum_{\mathbf{k}} u_k v_k e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')} \quad \text{(1)}$$

plotted as a function of the relative distance $r = |\mathbf{r} - \mathbf{r}'|$ of the correlated $^1S_0$ pair of neutrons. In the example shown ($\rho/\rho_0 = 1/2$), the calculated pair wave function exhibits a significant amplitude at the short relative distances $r < 2 - 3$ fm whereas the coherence length $\xi = 9.5$ fm at this density is much larger than this distance scale. If we evaluate the relative weight $r < r^*$ for pairs of spin anti-parallel neutrons at positions $\mathbf{r}$ and $\mathbf{r}'$ as a function of the relative distance $\mathbf{r}'$, we obtain $P(r_d) \sim 0.4 - 0.5$ for $r_d = 2 - 3$ fm (see Fig.1(b)), implying that two neutrons of a Cooper pair often come at short relative distances $r < r_d = 2$ fm with large probability around 40% even at the modest density $\rho/\rho_0 = 1/2$. At $\rho/\rho_0 \sim 10^{-1}$ corresponding to the skin region, the size of the Cooper pair is the smallest $\xi \sim 4$ fm, and the probability $P(r_d)$ for the range $r < r_d = 3$ fm reaches up to 70%.

4. Di-neutron correlation in the ground state of medium-mass neutron-rich nuclei

The di-neutron correlation is seen also in the ground state of medium-mass neutron-rich nuclei. To show this, we analyze the Cooper pair in the ground state $\Phi_0$ obtained in the coordinate-space HFB calculation, where a density-dependent delta interaction is used as the effective interaction for the neutron pairing, and a Woods-Saxon potential is adopted as a substitute for the Hartree-Fock potential[19]. The use of the delta interaction requires an energy cut-off, for which we adopt $E_{\text{cut}} = 50$ MeV to define the maximal quasiparticle energy.

We probe here the spatial structure of the correlated pairs by evaluating the two-body correlation density

$$\rho_2(\mathbf{r} \uparrow, \mathbf{r}' \downarrow) = \langle \Phi_0 | \sum_{i \neq j \in n} \delta(\mathbf{r} - \mathbf{r}_i)\delta(\mathbf{r}' - \mathbf{r}_j)\delta_{\sigma_i \uparrow}\delta_{\sigma_j \downarrow} | \Phi_0 \rangle - \rho_{n}(\mathbf{r} \uparrow)\rho_{n}(\mathbf{r}' \downarrow) \approx |\Psi_{\text{pair}}(\mathbf{r} \uparrow, \mathbf{r}' \downarrow)|^2 \quad \text{(2)}$$

for pairs of spin anti-parallel neutrons at positions $\mathbf{r}$ and $\mathbf{r}'$. This quantity is essentially the same as the probability distribution of the the Cooper pair wave function $\Psi_{\text{pair}}(\mathbf{r} \uparrow, \mathbf{r}' \downarrow)$ defined by Eq.(1). A plot of this quantity for $^{64}$Ni is drawn in Fig.2 by fixing the position $\mathbf{r}'$ of one neutron at the surface position $z' = R_{\text{surf}} = 4.8$ fm along the $z$-axis. It is seen that the correlation is significantly enhanced at short relative distances $|\mathbf{r} - \mathbf{r}'| < 2 - 3$ fm. This aspect is qualitatively in accordance with the spatial correlation seen in the Cooper pair wave function in the uniform matter shown in the inset of Fig.1(b). If we evaluate the relative weight $P(r_d)$ at short relative distances $|\mathbf{r} - \mathbf{r}'| < r_d$ of the probability distribution $|\Psi_{\text{pair}}(\mathbf{r} \uparrow, \mathbf{r}' \downarrow)|^2$, we obtain $P(r_d) = 0.61(0.49)$ for $r_d = 3(2)$ fm in the case of Fig.2. If we evaluate the same quantity at the internal and external positions $z' = R_{\text{surf}} \pm 2$ fm, they read $P(r_d) = 0.38(0.32)$ and $0.47 (0.25)$, respectively. These numbers of $P(r_d)$ roughly correspond to the curves in Fig.1(b), if we make a local density assignment $\rho/\rho_0 \approx 1, 0.5$ and 0.05 for $z' = R_{\text{surf}} - 2$ fm, $R_{\text{surf}}$ and $R_{\text{surf}} + 2$ fm, respectively. This correspondence however should be seen only qualitatively as the local density approximation is not fully justified.

We can clarify the nature of the spatial correlation from the viewpoint of the configuration mixing of participating quasiparticle orbits. In Fig.2(a), we plot results obtained by reduced cut-off energies $E_{\text{cut}} = 5, 10, 15, \cdots 30$ MeV chosen smaller than the the reference value $E_{\text{cut}} = 50$ MeV. It is seen that the smaller cut-off energies produce the weaker di-neutron correlation.
Namely configuration mixing of the neutron orbits far above the Fermi energy (i.e. having large quasiparticle energies) plays a crucial role. Fig.2(b) displays different decomposition of the correlation density in terms of the orbital angular momentum $l$ of the neutron orbits. In $^{84}$Ni, only the orbits with $l \leq 4$ exist around the neutron Fermi energy. Nevertheless, it is found in Fig.2(b) that high-$l$ orbits with $l = 5 - 7$ give a large contribution to produce the prominent correlation at short relative distances. Keeping in mind the uncertainty relation between the relative angle and the orbital angular momentum, and the relative distance and the linear momentum, the configuration mixing of the high-$l$ and the high energy orbits simply reflects the strong correlation at short relative distances.

5. Di-neutron dynamics in the soft modes
The di-neutron correlation is expected to bring about strong influence on the soft excitation modes which emerge in medium-mass nuclei near the drip-line. Using the same model adopted for the description of the ground state, we perform the continuum QRPA calculation[19] by including the particle-hole residual interaction, for which a density dependent delta force of the Skyrme-type is assumed. We here discuss the soft dipole excitation, taking $^{84}$Ni as an example. It is seen in the calculated E1 strength distribution (Fig.3(a)) that a large amount of the strength is distributed around $E_x = 2 - 7$ MeV, indicating the soft dipole mode. As the excitation energy of this mode is larger than the threshold energy ($\sim 2$ MeV, the arrow in the figure) for one- and two-neutron separation, this mode can decay by emitting two neutrons.

The neutron transition densities provide a useful tool to infer the nature of this mode. In addition to the standard particle-hole transition density $\rho_{i}^{ph}(r) = \langle \Phi_i | \sum_{\sigma} \psi_n^\dagger(\sigma r) \psi_n(\sigma r) | \Phi_0 \rangle$, we evaluate also the particle-particle transition density $P_{i}^{pp}(r) = \langle \Phi_i | \psi_n^\dagger(\uparrow r) \psi_n^\dagger(\downarrow r) | \Phi_0 \rangle$, which probes the motion of the spin anti-parallel neutron pairs. In addition to the usual features in the particle-hole amplitude $\rho_{i}^{ph}(r)$, the particle-particle amplitude $P_{i}^{pp}(r)$ points to a remarkable aspect: $P_{i}^{pp}(r)$ is greater than the $\rho_{i}^{ph}(r)$ in the external region $r > 6$ fm. This indicates a dominant particle-particle character of this mode rather than a particle-hole nature.

In Fig.3(b), we plot also results of calculations where the contribution of two quasiparticle configurations is limited by a cut-off $l_{cut}$ with respect to the orbital angular momentum $l$ of the
Figure 3. (a) The calculated E1 strength function in $^{84}\text{Ni}$. The arrow indicates the threshold energy for one and two neutron separation. (b) The particle-hole transition density $r^2 \rho_{\text{ph}}^2 (r)$ (in the upper panel), and the particle-particle transition density $r^2 P_{\text{pp}}^2 (r)$ (in the lower panel) for neutrons in $^{84}\text{Ni}$, evaluated at $E_x = 4.0$ MeV, plotted with the solid line as a function of the radial coordinate $r$ from the nuclear center. The thick dashed line represents the result with the RPA correlation neglected, while the other lines are the results obtained with various cut-off $l_{\text{cut}} = 5, 7, 9, \cdots$ on the orbital angular momentum of the quasiparticle orbits[19].

It is seen that two-quasiparticle configurations $[l \times (l + 1)]_{L=1}$ consisting of high-$l$ orbits build up to produce the large amplitude of the particle-particle transition density. In the far external region $r > 10$ fm, even the orbits with $l > 10$ give a sizable contribution. This readily indicates that this mode having the dominant particle-particle character cannot be described as a simple two-quasiparticle excitation nor an independent particle excitation: A RPA correlation plays a central role (the thick dashed line in Fig.3(b) is the results without the RPA correlation). If we recall the configuration mixing analysis (Fig.2(b)) of the di-neutron correlation in the ground state, we find a similarity with respect to the role of high-$l$ orbits. This suggests that the enhancement of the particle-particle amplitude is a manifestation of the di-neutron correlation in the soft dipole excitation. It is further noticed in Fig.3(b) that the particle-particle amplitude $P_{\text{pp}}^2 (r)$ exhibits an oscillatory behavior as a function of $r$ in the nuclear exterior. Such an oscillation is possible only if two neutrons are emitted simultaneously from the excited state. Combining these observations, we can deduce that a motion of a neutron pair having the spatial di-neutron correlation dominates in the soft dipole excitation while the correlated neutron pair is eventually emitted out of the nucleus.

It is interesting to see whether the di-neutron correlation emerges also in other modes of excitation. We have investigated octupole excitations by means of the same model as above. The octupole strength calculated for $^{84}\text{Ni}$ is shown in Fig.4(a), where a large strength at low energies $E_x = 2–5$ MeV is noticeable. Inspecting in more detail the shape of the strength and the isoscalar and isovector contributions, we see that there are two modes of excitation overlapping in this energy region: one is a sharp peak at $E_x \approx 4.4$ MeV which is mainly an isoscalar mode and corresponds to the low-lying collective vibration widely present in stable nuclei, and the second is a broad bump around $E_x \approx 2–5$ MeV just above the neutron threshold. The transition densities of the latter mode exhibits a remarkable similarity to the soft dipole excitation: The particle-particle amplitude $P_{\text{pp}}^2 (r)$ is dominating over the particle-hole amplitude $\rho_{\text{ph}}^2 (r)$ in the external region $r > 6$ fm, where contributions of the high-$l$ orbits are also significant. It indicates that
Figure 4. (a) The calculated octupole strength function in $^{84}$Ni. The isoscalar (isovector) strength is plotted with the solid (dashed) lines. (b) The particle-hole transition density $r^2\rho_{ph}(r)$ (in the upper panel), and the particle-particle transition density $r^2\rho_{pp}(r)$ (in the lower panel) for neutrons in $^{84}$Ni, evaluated at $E_x = 3.0$ MeV and plotted with the solid line. The thick dashed line represents the result with the RPA correlation neglected, while the other lines are the results obtained with the angular momentum cut-off $l_{cut} = 7, 9, 11, \cdots$

the second mode having smooth strength distribution just above the threshold energy exhibits a character of the di-neutron motion. The soft excitations reflecting the di-neutron correlation emerge both in the dipole and in the octupole multipolarity.

References

[1] Bertsch G F and Esbensen H 1991 *Ann. Phys.* 209 327; Esbensen H and Bertsch G F *Nucl. Phys.* A 542 310
[2] Ikeda K 1992 *Nucl. Phys.* A 538 355c
[3] Zhukov M V et al 1993 *Phys. Rep.* 231 151
[4] Barranco F, Bortignon P F, Broglia R A, Coló G and Vigezzi E 2001 *Eur. Phys. J.* A 11 385
[5] Aoyama S, Katō K and Ikeda K 2001 *Prog. Theor. Phys. Suppl.* 142 35
[6] Danilin B V, Thompson I J, Vaagen J S, Zhukov M V 1998 *Nucl. Phys.* A 632 383
[7] Myo T, Aoyama S, Katō K and Ikeda K 2003 *Phys. Lett.* B 576 281
[8] Dobaczewski J, Flocard H and Treiner J 1984 *Nucl. Phys.* A 422 103
[9] Bulgac A 1980 Hartree-Fock-Bogoliubov Approximation for Finite Systems *Preprint* nucl-th/9907088.
[10] de Gennes P G 1966 *Superconductivity of Metals and Alloys* (Benjamin)
    Tinkham M 1975 *Introduction to Superconductivity* (McGraw-Hill)
[11] Leggett A J 1980 *Modern Trends in the Theory of Condensed Matter* (Springer-Verlag) p 13
[12] Nozières P and Schmitt-Rink S 1985 *J. Low Temp. Phys.* 59 195
[13] Randeria M 1995 *Bose-Einstein Condensation* ed A Griffin, D W Snoke *et al* (Cambridge Univ. Press) p 355
[14] Matsuo M 2001 *Nucl. Phys.* A 696 371
[15] Belyaev S T, Smirnov A V, Tolokonnikov S V and Fayans S A 1987 *Sov. J. Nucl. Phys.* 45 783
[16] Dean D J and Hjorth-Jensen M 2003 *Rev. Mod. Phys.* 75 607
[17] Ozawa A *et al* 2001 *Nucl. Phys.* A 691 599
[18] Fukunishi N, Otsuka T and Tanihata I 1993 *Phys. Rev.* C 48 1648
[19] Matsuo M, Mizuyama K and Serizawa Y 2005 *Phys. Rev.* C in press (*Preprint* nucl-th/0408052)