Anomalous Couplings in the Two Higgs Doublet Model

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(Dated: July 6, 2021)

Abstract

We make a complete one loop calculation of the $tbW$ couplings in the Two Higgs Doublet Model. We evaluate both the anomalous couplings $g_L$ and $g_R$ as well as left handed and right handed component of $tbW$. The computation is done in the Feynman gauge using the on-shell scheme renormalization for the Standard Model wave functions and parameters. We first show that the relative corrections to these anomalous couplings are rather small in most regions of the parameter space. We then analyze the effects of these anomalous couplings on certain observables such as top quark polarization in single top production through $t$–channel as well as $W^\pm$ boson helicity fractions in top decay.
I. INTRODUCTION

Top quark is the heaviest particle discovered by D0 \[1\] and CDF \[2\] collaborations at the Tevatron-Fermilab with mass \(m_t = 173.21 \pm 0.51(\text{stat.}) \pm 0.71(\text{syst.})\) GeV. Some of its properties have been studied by the first run of LHC and will get improved by the new LHC run. It is well known that LHC machine with 13-14 TeV center of mass energy will act as a top factory since the total cross section for top quark pair production will reach one nanobarn. At the LHC, the top quark production will be two orders of magnitude larger than in the Tevatron. At low luminosity phase of LHC, one expects about ten millions top pairs per year and this number will increase during the high luminosity phase. Therefore, with such extremely large number of top anti-top, it is expected that top quark properties (top mass, top spin and decay rates...) can be examined with very good precision.

The main decay of the top quark is into W boson and bottom quark. At the tree level, this decay proceeds through the left handed V-A charged weak interaction which is directly proportional to Cabbibo-Kobayachi-Maskawa \(V_{tb}\) which can be measured in single top production. Both in the Standard Model (SM) and Beyond SM, loop effects can modify the structure of \(tbW\) vertex. Such modifications are typically described by anomalous couplings \(g_L\) and \(g_R\) as well as modifications to left handed \((V_L)\) and right handed \((V_R)\) components of \(tbW\). The QCD corrections to the anomalous coupling \(g_R\) have been evaluated a while ago in \[3\], while the SM electroweak and QCD corrections to \(g_{L,R}\) and \(V_R\) have been studied in \[4\] and \[5\]. It turns out that the anomalous couplings \(g_L, g_R\) as well as top quark right coupling \(g_R\) in the \(tbW\) are dominated by the QCD corrections.

It is well known that, the anomalous \(tbW\) couplings could be probed by measuring the W boson helicity fractions in the top quark decay \[6, 7\]. These polarization states are proven to be sensitive to new physics effects \[8\]. Moreover, top quark due to its short lifetime, decays before it hadronizes. Therefore, the information about its polarization may be preserved in its decay products which can be viewed as top spin analyzers.

In this study, we are interested in computing the complete one loop contribution to the anomalous \(tbW\) couplings in the framework of Two Higgs Doublet Model (2HDM). We evaluate both the anomalous couplings \(g_L\) and \(g_R\) as well as left handed \(V_L\) and right handed \(V_R\) component of \(tbW\). We stress that, evaluation of the top anomalous couplings in the framework of the two Higgs Doublet Model (2HDM) has been studied some times ago in
and recently in [10]. In [10], only the computation of the tensorial anomalous couplings $g_R$ and $g_L$ has been considered. We will perform, in addition to tensorial couplings $g_{L,R}$, a complete one-loop computation of the left and right chiral couplings $V_L$ and $V_R$ and quantify their effects on top quark polarisation in single production through $t$-channel and $W^\pm$ helicity fractions.

The 2HDM effects are found to be below percent level. In the present computation, we perform a comparative study and will include all the virtual effect of the 2HDM as well as the real emission of photon and gluon in the final state that are necessary for the computation of the one loop contribution to $V_L$ in order to have infra-red finite result.

There have been several experimental searches for anomalous coupling of the top quark. One of the most strongest constraints comes from measurement of $Br(\bar{B} \to X_s\gamma)$ [11]. Tevatron also has reported limits on the anomalous couplings in the search of new physics in top quark decays [13]. We note also that there are limits from ATLAS and CMS collaborations on anomalous couplings from the measurements of the $W^\pm$ helicity fractions in top quark decay [6, 7]. In this regard, the first measurement was reported by the CMS collaboration [14] assuming $V_L = 1, g_L = V_R = 0$, they have found the following value $g_R = 0.070 \pm 0.053(stat.)^{+0.081}_{-0.073}(syst.)$. But this measurement suffers from large statistical and systematic uncertainties. The sensitivity of the ATLAS experiment to the anomalous $tbW$ couplings has been studied in [15]. Finally, we stress here that the anomalous couplings might be measured from the measurement of single top production cross section at the LHC [16], from the measurements of Laboratory frame observables constructed in [17] through single top production at the LHC [18], and from the observables that were considered for the case of a future $e^-p$ collider [19].

In fact, all measurements of top quark properties performed so far are in perfect agreement with the SM theoretical predictions. We would like to investigate the top quark $tbW$ anomalous couplings as well as left and right handed $tbW$ couplings in the 2HDM, and quantize their effects on some top quark observables such as top polarization in single top production through $t$–channel as well as $W^\pm$ helicity fractions in top decay.

The outline of this paper is the following: In section (II), we introduce the two Higgs Doublet Model, its parameters and the constraints that we will use during the numerical analysis. In section (III), we describe the experimental status of the anomalous couplings
and the theoretical set-up used in our calculation while in section (V), we present and discuss our numerical results. Our conclusions are drawn in section (V). The appendix is devoted to analytical expression for the one-loop anomalous couplings given for the first time in terms of Passarino-Veltman functions and comparison with some results from literature.

II. THE TWO-HIGGS-DOUBLET-MODEL

In the Two-Higgs-Doublet Model, two scalar doublets under $SU(2)_L$ with Hypercharge $Y_{H_1,2} = 1/2$ are used to generate fermion and gauge boson masses. The inclusion of the two doublets may give rise to sizeable flavor changing neutral current processes (FCNC) at tree level. In order to avoid such tree level FCNC, a discrete symmetry $Z_2$ (where for example $H_1 \rightarrow H_1$ and $H_2 \rightarrow -H_2$) is imposed [20]. Hence, there are 4 different combinations of the Yukawa Lagrangian depending on the $Z_2$ charge assignment to the leptons and quarks fields [21, 22]. There are four different models of Yukawa interactions. In type-I model, only the second doublet $H_2$ interacts with all the fermions while in type-II model where the doublet $H_2$ interacts with up-type quarks and $H_1$ interacts with the charged leptons and down-type quarks. In type-X model, charged leptons couple to $H_1$ while all the quarks couple to $H_2$. Finally, in type-Y model, charged leptons and up-type quarks couple to $H_2$ while down-type quarks acquire masses from their couplings to $H_1$. Given that the Higgs couplings to quarks are the same in type-I (resp type-II) and in type-X (resp type-Y), in what follow we will discuss only 2HDM type-I and II.

The Lagrangian representing the Yukawa interactions is given by:

$$-\mathcal{L}_{Yuk} = \bar{q}_L \mathcal{Y}_u \tilde{H}_2 u_R + \bar{q}_L \mathcal{Y}_d H_d d_R + \bar{l}_L \mathcal{Y}_l H_l l_R + \text{H. c.}$$  

(1)

Where $H_i, i = l, d$ is either $H_1$ or $H_2$ and $\mathcal{Y}_i$ is a set of Yukawa matrices.

The most general scalar potential which is gauge-invariant, re-normalizable and CP-invariant is:

$$V(H_1, H_2) = \mu_{11}^2 |H_1|^2 + \mu_{22}^2 |H_2|^2 - \mu_{12}^2 (H_1^\dagger H_2 + H_2^\dagger H_1) + \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1^\dagger H_2|^2 + \frac{\lambda_5}{2} [ (H_1^\dagger H_2)^2 + \text{H.c}]$$  

(2)

where $\mu_{11,22}^2, \lambda_{i, i=1...4}$ are real parameters while $\mu_{12}^2$ and $\lambda_5$ could be complex for CP violating case. Note that in the above potential the $Z_2$ symmetry is only broken softly by dimension
2 term $\mu_{12}^2(H_1^H H_2)$ while dimension four terms are not introduced in our potential. The two Higgs doublets $H_1$ and $H_2$ are given by:

$$H_i = \left( \frac{\phi_i^+}{v_i + \frac{1}{\sqrt{2}}(h_i + i\omega_i)} \right), \quad i = 1, 2$$

where $v_1$ and $v_2$ are the vacuum expectation values of the two doublets. After electroweak symmetry breaking, one has five additional degrees of freedom; a pair of charged scalar bosons $H^\pm$, one CP-odd $A^0$ and two CP-even $h^0, H^0$ where the lightest CP-even scalar boson is identified as the SM Higgs boson. These eigenstates are defined as follow:

\[
\begin{pmatrix}
    h_1 \\
    h_2
\end{pmatrix}
= O(\alpha) \begin{pmatrix}
    H^0 \\
    h^0
\end{pmatrix}, \quad
\begin{pmatrix}
    \phi_1^+ \\
    \phi_2^+
\end{pmatrix}
= O(\beta) \begin{pmatrix}
    G^\pm \\
    H^\pm
\end{pmatrix}, \quad
\begin{pmatrix}
    \omega_1 \\
    \omega_2
\end{pmatrix}
= O(\beta) \begin{pmatrix}
    G^0 \\
    A^0
\end{pmatrix}
\]

where:

$$O(\theta) = \begin{pmatrix}
    \cos \theta & -\sin \theta \\
    \sin \theta & \cos \theta
\end{pmatrix}.$$ 

The Yukawa Lagrangian in eq. (1) becomes:

\[
-\mathcal{L}_{Yuk} = \sum_{\psi=u,d,l} \left( \frac{m_{\psi}}{v} \kappa_{\psi}^h \bar{\psi} \psi H^0 + \frac{m_{\psi}}{v} \kappa_{\psi}^H \bar{\psi} \psi H^0 - i \frac{m_{\psi}}{v} \kappa_{\psi}^A \bar{\psi} \gamma_5 \psi A^0 + \frac{m_{\psi}}{\sqrt{2}v} \bar{\psi} m_{\psi} \kappa_{\psi}^A P_L + m_{\psi} \kappa_{\psi}^A P_R \right) dH^+ + \frac{m_{\psi} \kappa_{\psi}^A}{\sqrt{2}v} \bar{l}_{R} l_{R} H^+ + H.c.
\]

where $\kappa^S_i$ are the Yukawa couplings in the 2HDM. We give, in table I, the values of the couplings in the four types of Yukawa interactions of the 2HDM with softly broken $Z^2$ symmetry. We will identify the light CP-even Higgs $h^0$ as the 125 GeV SM Higgs, the other parameters of the 2HDM are not yet measured by any experiment, hence we will apply the following theoretical and experimental constraints on the parameter space of the model:

- Vacuum stability of the scalar potential \[23\].

|        | $\kappa^h_u$ | $\kappa^h_d$ | $\kappa^h_l$ | $\kappa^H_u$ | $\kappa^H_d$ | $\kappa^H_l$ | $\kappa^A_u$ | $\kappa^A_d$ | $\kappa^A_l$ |
|--------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Type-I | $c_\alpha/s_\beta$ | $c_\alpha/s_\beta$ | $c_\alpha/s_\beta$ | $s_\alpha/s_\beta$ | $s_\alpha/s_\beta$ | $c_\beta/s_\beta$ | $-c_\beta/s_\beta$ | $-c_\beta/s_\beta$ | $s_\beta/c_\beta$ |
| Type-II| $c_\alpha/s_\beta$ | $-s_\alpha/c_\beta$ | $-s_\alpha/c_\beta$ | $s_\alpha/s_\beta$ | $c_\alpha/c_\beta$ | $c_\beta/s_\beta$ | $s_\beta/c_\beta$ | $s_\beta/c_\beta$ | $c_\beta/s_\beta$ |
| Type-X | $c_\alpha/s_\beta$ | $c_\alpha/s_\beta$ | $-s_\alpha/c_\beta$ | $s_\alpha/s_\beta$ | $c_\alpha/c_\beta$ | $c_\beta/s_\beta$ | $s_\beta/c_\beta$ | $c_\beta/s_\beta$ | $-c_\beta/s_\beta$ |
| Type-Y | $c_\alpha/s_\beta$ | $-s_\alpha/c_\beta$ | $c_\alpha/s_\beta$ | $s_\alpha/s_\beta$ | $c_\alpha/c_\beta$ | $c_\beta/s_\beta$ | $s_\beta/c_\beta$ | $c_\beta/s_\beta$ | $-c_\beta/s_\beta$ |

TABLE I. Yukawa couplings in terms of mixing angles in the 2HDM Type I, II, X and Y.
- Tree-level perturbative unitarity [24][26].
- We will impose constraints on the $\rho$ parameter using the PDG update on electroweak fits [27].
- We impose constraints from the ATLAS measurement [28] of the signal strength $\mu_{XX}$ defined by:

$$
\mu_{XX} = \frac{\sigma(pp \rightarrow h^0)^{2HDM} \Gamma(h^0 \rightarrow XX)^{2HDM}}{\sigma(pp \rightarrow h^0)^{SM} \Gamma(h^0 \rightarrow XX)^{SM}}
$$

where $XX$ represents the following channels: $W^\pm W^\mp, ZZ^*, \gamma\gamma$, and $\tau^+\tau^-$. While $\sigma(pp \rightarrow h^0)$ includes the following Higgs production mechanisms at the LHC: $ggF$, Vector Boson fusion VBF, Higgs-strahlung $W^\pm h^0$, $Zh^0$ and $t\bar{t}h^0$.
- We will use the results of indirect constraints on the charged Higgs boson mass from processes at the one-loop order, e.g $b \rightarrow s\gamma$ and $R_b$ [29][36]. In our analysis, we assume that $m_{H^\pm} \geq 480$ GeV in 2HDM type-II.
- Constraints from direct searches of charged Higgs bosons at LEP [37] and the LHC [38][40] will be used.

### III. ANOMALOUS $tbW$ COUPLINGS

Owing to Lorentz invariance, the amplitude of top quark decay $t(p_t) \rightarrow b(p_b)W^+(q)$ can be written as:

$$
\mathcal{M}(t \rightarrow bW^+) = \frac{-e}{\sqrt{2}\sin \theta_W} \bar{u}_b(p_b) \left[ (V_LP_L + V_RP_R)\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{M_W} (g_LP_L + g_RP_R) \right] u_t(p_t)\epsilon^*_\mu(q) \nonumber
$$

where $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$ are the projection operators, $p_t, p_b$ and $q = p_t - p_b$ are respectively the four-momenta of the top, bottom and $W^+$ boson. The three particles are assumed to be on-shell. For the case of $W^+$ being off shell, there are two-additional terms which should be added to the matrix element\footnote{In other words, two form factors $f_L$ and $f_R$ have to be added as follows $\bar{u}_b(p_b)\frac{i\sigma^{\mu\nu}(p_t + p_b)_\nu}{M_W}(f_LP_L + f_RP_R)u_t(p_t)\epsilon^*_\mu(q)$} in eq. (7). At tree level, in the SM, $V_L = V_{tb}$ and $V_R = g_R = g_L = 0$, while radiative corrections in the SM induce non-zero values for $V_{R,L}$, $g_R$ and $g_L$. Note that renormalizable theories beyond the SM might induce non-zero
values for the right chiral coupling $V_R$ even at tree level, but $g_R$ and $g_L$ have to be induced only at one-loop order. Before discussing the details of our calculations, we recapitulate the experimental status of the direct searches for the anomalous couplings as well as the indirect constraints coming from one-loop induced processes.

One of the most strongest constraints comes from $Br(\bar{B} \to X_s\gamma)$ [11]. The enhancement factor $m_t/m_b$ implies that these constraints are stronger for $V_R$ and $g_L$ and rather weaker for $g_R$:

$$-0.15 \leq \text{Re}(g_R) \leq 0.57$$
$$-7 \times 10^{-4} \leq V_R \leq 2.5 \times 10^{-3}$$
$$-1.3 \times 10^{-3} \leq g_L \leq 4 \times 10^{-4}$$

The above limit on $V_R$ and $g_L$ would be improved if one can measure more accurately $Br(\bar{B} \to X_s\gamma)$ at the Super B factory [12] and/or LHCb.

There are also $2\sigma$ limits available for $g_{L,R}$ using LHC simulations [11];

$$-0.026 < g_R < 0.024 \quad \text{and} \quad -0.058 < g_L < 0.026$$

In the search of new physics in top quark decays [13], Tevatron has reported 95% CL limit on anomalous couplings as follow:

$$|V_R|^2 < 0.30 \quad , \quad |g_L|^2 < 0.05 \quad \text{and} \quad |g_R|^2 < 0.12$$

It was assumed $V_L = V_{tb}$.

There are also 95% limits [12] on all the anomalous couplings including the left chiral coupling $V_L$ from a global fit to the experimental data which corresponds to single top production cross section (all the channels were included) and $W^\pm$ helicity fraction at Tevatron and the LHC. These limits are:

$$-0.142 \leq g_R \leq 0.023 \quad , \quad -0.081 \leq g_L \leq 0.049,$$
$$0.902 \leq V_L \leq 1.081 \quad \text{and} \quad -0.112 \leq V_R \leq 0.162$$

Moreover, global fit of the anomalous $Wtb$ couplings has been performed in [13] where correlations among the different effective operators have been investigated.
On the other hand, limits on tensorial anomalous couplings $g_L$ and $g_R$ have been studied in [45, 46] by combining several constraints from $b \to s\gamma$, helicity fractions, single top production, electroweak precision test (mainly from the S-parameter) and the electric dipole moments. It was found that the real part of $g_R$ is strongly constrained by the helicity fractions while the strongest constraint on Re$[g_L]$ comes from $b \to s\gamma$ branching ratio. On the other hand, the imaginary part of tensorial couplings is severely constrained by the electric dipole moments (EDM); e.g the strongest constraint on Im$[g_L]$ comes from neutron EDM while on Im$[g_R]$ comes from electron EDM.

The ATLAS collaboration [44] has reported 95% CL limits on the ratios of the anomalous couplings $g_R$ and $V_L$ from the measurement of the double differential decay rate of the top quark in single top production through $t$-channel process at $\sqrt{s} = 7$ TeV taking $V_R = g_L = 0$. The limits are:

$$\text{Re}\left[\frac{g_R}{V_L}\right] \in [-0.36, 0.10] \text{ and } \text{Im}\left[\frac{g_R}{V_L}\right] \in [-0.17, 0.23]$$ \hspace{1cm} \text{(11)}$

Recently, Ref. [47] puts 95% CL limits on the real and imaginary part of the anomalous couplings which were obtained from a global fit to data using the following observables:

- $t$-channel single top production cross section at the LHC at $\sqrt{s} = 7, 8$ and 13 TeV and at Tevatron $\sqrt{s} = 1.96$ TeV,

- $s$-channel $tW$ associated production at both the LHC 7 + 8 TeV and Tevatron

- Results from $W^\pm$ helicity fractions in $t\bar{t}$ production at $\sqrt{s} = 8$ TeV

- Expected results corresponding the $t$-channel production cross section at $\sqrt{s} = 14$ and 33 TeV assuming that $V_L = V_{tb} \approx 1$.

We stress that these limits are rather weak for $V_R$, and $g_L$ and slightly stronger for the case of $g_R$.

We will do a complete analysis of all the anomalous couplings present in eq. (7). The corresponding Feynman diagrams are depicted in figure 1. For the calculation of $g_L$, $g_R$ and $V_R$, there is no need to renormalize the theory since these couplings are absent at tree level. In fact, infrared divergences are also absent in the case of these couplings. For instance, in the case of $g_L$ and $g_R$, contributions from diagrams with exchange $tW^\pm\gamma$ and $tG^\pm\gamma$ (from $bW^\pm\gamma$ and $bG^\pm\gamma$) are individually infrared divergent but their sum is infrared finite. On the other
hand, in the case of $V_R$, all the diagrams involving the photon/ gluon are IR finite.

Before computing the anomalous couplings in the framework of the 2HDM, we have calculated them in the SM and compared with the results of [4] for the case of $g_L$ and $g_R$ and with [5] for the case of the right chiral coupling $V_R$. The numerical values of $g_L$, $g_R$ and $V_R$ in the SM are tabulated in appendix (A) while their analytical expressions are given for the first time in terms of Passarino-Veltmann functions in appendix (B).

The one-loop corrections to $V_L$ involves divergent integrals. In order to get meaningful results we should add appropriate counter-term to the bare coupling. In order to achieve that, we will be working in the on-shell renormalization scheme [48, 49] where necessary redefinition of the fields and parameters is performed such that the total amplitude (un-renormalized and counter term) is UV-finite. The counter-term of $tbW$ coupling is given

FIG. 1. Feynman diagrams that contribute to one loop $tbW$ coupling in the 2HDM
by:

\[ \delta M_{tbW} = \bar{u}_t(p_t)ie\gamma^\mu P_L\delta C_\gamma u_b(p_b)\epsilon_\mu^*(q) \]  

(12)

where \( \delta C_\gamma \) is:

\[ \delta C_\gamma = \frac{1}{\sqrt{2s_W}} \left( \delta Z_e - \frac{\delta s_W}{s_W} + \frac{1}{2} \delta Z_W + \frac{1}{2}(\delta Z_t,L+ + \delta Z_b,L) \right) \]  

(13)

where we have assumed that \( V_{tb} = 1 \) and \( \delta V_{tb} = 0 \). The renormalization constants \( \delta Z_e, \delta s_W, \delta Z_W, \delta Z_{t,L}^\dagger \) and \( \delta Z_{b,L}^\dagger \) are determined as usual by suitable mass and field renormalization conditions.

The Feynman diagrams and the corresponding amplitudes have been generated with FeynArts and FormCalc packages [51]. The output was passed to LoopTools [52] for numerical integration of the one-loop functions. UV divergences and renormalization scale independence have been checked analytically with FormCalc and numerically with LoopTools. However, due to the contribution of virtual photons and gluons, the corrections to \( V_L \) are infrared divergent. These IR divergences are canceled after introducing real photons and gluons emissions in the final state. We have checked that indeed, the total amplitude consisting of virtual, soft and hard photons/gluons emissions are independent of the effective cutoff \( \lambda_{IR}^2 \). This cancellation has been checked analytically by computing the IR divergent part of the three-points Passarino Veltman function \( C_0 \) in the soft limit using analytical expressions from [50] and the real (soft and hard) emission factors extracted from [49]. With LoopTools, We checked numerically that the total contribution:

\[ 2\text{Re}(M^*_{\text{tree}}M_{\text{virtual}}) + |M_{\text{real}}|^2 \]  

(14)

is independent of \( \lambda_{IR}^2 \) by computing the sum [14] for different values of \( \lambda_{IR}^2 \in [10^{-10} : 10^6] \) and have found that the sum is \( \lambda_{IR}^2 \) independent.

IV. NUMERICAL RESULTS

The input parameters of the SM are taken from the Particle Data Group [27]:

| Parameter | Value       |
|-----------|-------------|
| \( m_t \) | 173.21 GeV  |
| \( m_b \) | 4.66 GeV    |
| \( M_W \) | 80.385 GeV  |
| \( M_Z \) | 91.1876 GeV |
| \( \alpha_S \) | 0.118        |
| \( m_H \) | 125 GeV     |
TABLE II. Parameter space of the Two-Higgs-Doublet model Type-I and -II over which the scan has been performed

| Type-I                                      | Type-II                                      |
|---------------------------------------------|----------------------------------------------|
| $100 \text{ GeV} \leq m_{H^\mp} \leq 900 \text{ GeV}$ | $480 \text{ GeV} \leq m_{H^\mp} \leq 900 \text{ GeV}$ |
| $90 \text{ GeV} \leq m_{A^0} \leq 900 \text{ GeV}$   | $90 \text{ GeV} \leq m_{A^0} \leq 800 \text{ GeV}$ |
| $125 \text{ GeV} \leq m_{H^0} \leq 900 \text{ GeV}$ | $125 \text{ GeV} \leq m_{H^0} \leq 900 \text{ GeV}$ |
| $0 \leq \sin(\beta - \alpha) \leq 1$        | $0 \leq \sin(\beta - \alpha) \leq 1$        |
| $1 \leq \tan \beta \leq 30$              | $1 \leq \tan \beta \leq 30$                |
| $-25 \leq \lambda_5 \leq 25$             | $-25 \leq \lambda_5 \leq 25$               |

FIG. 2. Relative contribution $\Delta g_L$ in 2HDM type-I (upper panels) and type-II (lower panels) shown as a scatter plot in the $(t_\beta, \kappa^h_d)$ plan (left), $(m_{H^0}, m_{A^0})$ plan (middle) and $(s_{\beta-\alpha}, c_\alpha)$ plan (right)

while the parameter space of the 2HDM is scanned over the range specified in table [II].

For our numerical analysis, we define the following ratios $\Delta O_i$:

\[
\Delta O_i = \frac{O_{i}^{2HDM} - O_{i}^{SM}}{O_{i}^{SM}}
\]  

(15)

where $O_i = \text{Re}(g_L), \text{Re}(g_R), \text{Re}(V_R)$ and $\text{Re}(V_L) + V_{tb}$. 

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A. Anomalous couplings

The relative correction $\Delta g_L$ is shown in figure 2 in different plan $(\tan \beta, \kappa^h_d)$ (left panels), $(m_H, m_A)$ (middle panels) and $(\sin(\beta - \alpha), \cos \alpha)$. Upper panels are for 2HDM-I and lower panels for 2HDM-II. One can see that $g_L$ gets enhancement for type-I (in most regions of the parameter space) while it is always suppressed with respect to the SM for 2HDM type-II. Most of the regions in the parameter space correspond to $\Delta g_L < 2\%$. In type-I, $\Delta g_L$ reaches 3.5\% for $\tan \beta \sim 1$ (left panel) and for all values of $\kappa^h_d$ between 1 and 1.3 and also for $m_{H^0, A^0} \in [100 : 300]$ GeV (middle panel). A decoupling behavior is easily observed for large values of $\tan \beta \geq 25$, $\kappa^h_d \sim 1$ and also for heavy scalars $m_{A^0, H^0} > 700$ GeV.

In 2HDM type-II, We see that $\Delta g_L$ is always negative while it approaches 0\%, (SM regime), for $\kappa^h_d \sim 1$ and $\tan \beta \sim 1$.

In figure 3, we plot the correction to $g_R$ in 2HDM type-I (upper panels) and 2HDM type-II (lower panels). In 2HDM type-I, one can see that the corrections can reach $-5\%$ for $\tan \beta \sim 1$ and the enhancement attains 1\%. However, in type-II 2HDM, the suppression of the correction to $g_R$ with respect to the SM result is smaller and the enhancement is quite bigger than type-I 2HDM, i.e. $\max (\Delta g_R) \simeq 1.5\%$ and $\min (\Delta g_R) \simeq -2\%$. 
FIG. 4. Relative contribution $\Delta V_R$ in 2HDM type-I (top) and 2HDM type-II (bottom) shown as a scatter plot in ($t$, $\kappa^h_\alpha$) plan (left), ($m_{H^0}, m_{A^0}$) plan (middle) and ($s_{\beta-\alpha}, c_\alpha$) plan (right).

In figure 4 (upper panels), we illustrate the correction to $V_R$ in 2HDM type-I. It is clear from these plots that the corrections hardly reach 2% while the maximum of suppression is about $-3.5\%$. The decoupling limit where $V_R^{2HDM} = V_R^{SM}$ is attained for large $\tan \beta$, large masses $m_{H^0, A^0} > 700$ GeV and for $s_{\beta-\alpha} \simeq 1$.

In the lower panels of figure 4, we can see that contrarily to 2HDM type-I, the enhancement of $V_R$ in 2HDM type-II is larger and reaches 6% for $m_{H^0}$ and $m_{A^0} \in [300 : 400]$ GeV while the suppression is hardly fulfilled and reaches only $-1\%$. In figure 5 we have shown corrections to the left chiral coupling $V_L$. We see that the corrections are very small (not exceeding 0.4%) in most regions of the parameter space. The suppression of $V_L$ with respect to its SM value reaches $-0.5\%$ in the 2HDM type-I and II.

B. Top Polarization

We also studied numerically the top polarization in the channel $qg \rightarrow q't\bar{b}$ at the LHC for $\sqrt{s} = 14$ TeV. The relative correction is defined as:

$$\Delta P_i = \frac{P_i^{2HDM} - P_i^{SM}}{P_i^{SM}}, \quad i = x, y, z$$

Following Ref. [55], the axes were defined as follow; $z$ axis is the direction of the spectator quark $q'$, the $y$-axis is orthogonal to the direction of the momentum of the initial quark $q$.
FIG. 5. Relative contribution $\Delta V_L$ in 2HDM type-I (top) and 2HDM type-II (bottom) shown as a scatter plot in $(t_\beta, \kappa_d^h)$ plan (left), $(m_{H^0}, m_{A^0})$ plan (middle) and $(s_{\beta-\alpha}, c_\alpha)$ plan (right).

FIG. 6. Scatter plots in $(m_{H^0}, m_{A^0})$ plan where the palette shows the values of $\Delta P_x$ (left panels), $\Delta P_y$ (middle panels) and $\Delta P_z$ (right panels) in the 2HDM type-I (upper panels) and 2HDM type-II (lower panels) and the momentum of the spectator quark $q'$ and the $x$-axis is chosen such that the system is right handed. We have taken the expressions of the components of the polarization vector from [55] for both the top and anti-top quarks at 14 TeV. We quote the results for the
case of the SM in table. (III). In figure 6 (upper panels), we plot the relative correction

|         | Top quark       | Anti-top quark          |
|---------|-----------------|-------------------------|
| $P_x$   | $-0.0179074$    | $-0.107771$             |
| $P_y$   | $0.0040848$     | $9.66629 \times 10^{-6}$|
| $P_z$   | $0.880908$      | $-0.850601$             |

TABLE III. Values of the polarization of $t/\bar{t}$ in the SM at $\sqrt{s} = 14$ TeV. Formula are taken from Ref. [55]

$\Delta P_i, i = x, y, z$ in the 2HDM type-I for the top quark in the $(m_{H^0}, m_{A^0})$ plan. We see that $\Delta P_x$ reaches 2% as a maximum of enhancement. The suppression of $P_x$ with respect to the SM value reaches $-5\%$. Corrections to $P_y$ are shown in the middle panel of figure 6, the corrections are rather smaller than those corresponding to $P_x$: max $\Delta P_y \sim 0.5\%$ and the suppression is of order of $-0.5\%$ which implies that non significant deviation from the SM is attained. We note also that the corrections to $P_z$ (right panel of figure 6) are even more smaller ($0.001\%$ as a maximum).

In figure 6 (lower panels), we plot the relative corrections to the components of the polarization vector of the top in 2HDM type-II. We see that in this model, corrections are very small. max$\{$$\Delta P_x, \Delta P_y, \Delta P_z$$\} = \{2\%, 0.4\%, 0.0025\%\}$ and min$\{$$\Delta P_x, \Delta P_y, \Delta P_z$$\} = \{-2\%, -0.5\%, 0.0015\%\}$.

C. W helicity fractions

Anomalous $tbW$ couplings could be probed by measuring the $W$ boson helicity fractions in the top quark decay (unpolarized decay) [6, 7]. These polarization states are proven to be sensitive to new physics effects [8] where the $W$ boson could be produced with positive ($R$), negative ($L$) or zero ($0$) helicity states, $\Gamma(t \rightarrow bW^+) = \Gamma_L + \Gamma_R + \Gamma_0$. Expressions of the polarized widths in terms of the anomalous couplings are taken from [41]. The polarization of the $W$ boson could be measured by looking to the angular distributions of its decay products (especially into leptons). The differential decay rate of the unpolarized top quark
is given by:
\[
\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta^*_l} = \frac{3}{8} (1 + \cos \theta^*_l)^2 F_R + \frac{3}{8} (1 - \cos \theta^*_l)^2 F_L + \frac{3}{4} \sin^2 \theta^*_l F_0,
\]
(17)
with \( F_i = \Gamma_i / \Gamma \) are the helicity fractions and \( \theta^*_l \) is the angle between the lepton three-momentum in the rest frame of the parent W boson and the W boson three momentum in the top quark rest frame. The SM predictions are known up to NNLO in QCD [53]:

\[
F_0 = 0.687 \pm 0.005, \quad F_L = 0.311 \pm 0.005 \quad \text{and} \quad F_R = 0.0017 \pm 0.0001
\]
Calculations of the helicity fractions have been performed in the framework of the MSSM in [54].

For numerical analysis, we define the ratios \( \delta F_i \) by:
\[
\delta F_i = \frac{F_i^{2\text{HDM}} - F_i^{\text{SM}}}{F_i^{\text{SM}}}
\]
(18)
where \( F_i^{\text{SM}} \) includes complete one-loop corrections (in \( \alpha_s \) and \( \alpha \)) while \( F_i^{2\text{HDM}} \) contains additional the contribution from the extra particles of the 2HDM and its interference with the pure SM (EW and QCD) contribution.

In figure 7 (upper panels), we plot the contribution to the helicity fractions in 2HDM type-I. \( \delta F_0 \) is shown in the left-panel, where we observe that the corrections are quite small \( \max \delta F_0 = 2.2\% \) and always enhancing \( F_0 \) with respect to its SM value while the minimum of the correction is 1.5\%. Corrections to \( F_R \) is depicted in figure 7 (middle panel), we see that the corrections are always suppressing the SM value; \( \max \delta F_R \sim -3.4\% \) and \( \min \delta F_R \sim -5.4\% \). We notice that \( F_R \) is very small and vanishes in the limit \( m_b \to 0 \). We illustrate in the right panel of the same figure the correction to \( F_L \). As it can be seen, there is always suppression of \( F_L \) with respect to its SM value \(-4.8\% \leq \delta F_L \leq -3.4\% \).

In figure 7 (lower panels), corrections to \( F_i \) are shown in 2HDM type-II. In the left panel, we see that correction to \( F_0 \) is more or less of the same size as for the case of 2HDM type-I. The maximum of \( \delta F_0 \) is 2.1\% reached where the masses are quite small \( m_{H^0, A^0} \sim 300 - 400 \) GeV. In the middle panel of figure 7, we show the correction to \( F_R \). We see that \( F_R \) is always suppressed with respect to its SM value. \(-5.5\% \leq \delta F_R \leq -3\% \). Finally, the extra contribution to \( F_L \) is shown in the right panel of figure 7, one can see that the corrections are the same in 2HDM type-I and type-II, e.g \(-4.8\% \leq \delta F_R \leq -3.4\% \) while the maximum of suppression is reached for the region of low scalar masses.
FIG. 7. Scatter plot in the \((m_{H^0}, m_{A^0})\) plan where the palettes show the values of \(\delta F_0\) (left panel), \(\Delta F_R\) (middle panel) and \(\Delta F_L\) (right panel) in the 2HDM type-I (top) and 2HDM type-II (bottom)

V. CONCLUSION

We have computed the complete one loop contribution to the anomalous \(tbW\) couplings in the 2HDM. We give for the first time the analytical expressions of the anomalous couplings in terms of the Passarino Veltman functions. We have evaluated both the anomalous couplings \(g_L\) and \(g_R\) as well as left handed \(V_L\) and right handed \(V_R\) component of \(tbW\). The computation is done by diagrammatic method in the Feynman gauge using dimensional regularization in the On-shell renormalization scheme.

We show sensitivity of the 2HDM parameters to the various anomalous \(tbW\) couplings taking into account recent LHC constraints. We also illustrate the overall sensitivity to the 2HDM parameters to some LHC observables such as: top polarization in single top production through t-channel as well as \(W^\pm\) helicity fractions in top decay. We also project our numerical results on \(\kappa_D\), which is the Yukawa coupling of the Higgs to down quarks and also on \(\sin(\beta - \alpha)\) which measure departure from decoupling limit of 2HDM.

The effect on most of the observables we consider are rather small. It will be rather a difficult task to disentangle the 2HDM from SM even with the High luminosity LHC option. However, with the projected Super B Factory experiments with high luminosity, from the
precise measurement of $b \to s\gamma$ we would have a strong limit on $V_R$ and $g_L$.

**ACKNOWLEDGMENTS**

This work was supported by the Moroccan Ministry of Higher Education and Scientific Research MESRSFC and CNRST: ”Projet dans les domaines prioritaires de la recherche scientifique et du développement technologique”: PPR/2015/6. A.J would like to thank the STEP Programme (ICTP-Trieste) and GDRI P2IM Maroc-France (LAPTh, CNRS) for financial support during his stay where part of this work has been done. The authors would like to thank Fawzi Boudjema for careful reading of the manuscript.

**Appendix A: Anomalous Tensor couplings in the SM**

In this appendix, we give our numerical results for the anomalous tensor couplings in the SM and compare with results from [4] and [5].

In table (IV) we show the values of $g_L$. In most of the cases, there is an agreement between our results and those presented by the authors of [4] except for diagrams with $bW^+Z$ and $bG^+Z$ exchange where our results are two times larger. On the other hand, in the diagrams with $bW^+H$ and $bG^+G^0$ exchange, we have found that our imaginary part of $g_L$ has a different sign to that found in [4].

The table (V) shows the values of coupling $g_R$ for different diagrams and compares with the results of Vidal [4]. One can see that the same remarks apply here as for the case of tensor coupling $g_L$.

In table (VI), we show contribution to $V_R$ for different diagrams in the SM and compare with the recent results of Vidal et al reported in [5]. We have checked the correctness of our results, for certain diagrams where the results are not consistent with [4, 5], both by the Feynman parameterization and Passarino-Veltman reduction methods.
### Appendix B: Top Quark Anomalous Couplings $g_L, g_R$ and $V_R$ in the Two-Higgs-Doublet-Model

In this appendix, we present for the first time the analytical expressions of the anomalous couplings for different diagrams in the 2HDM in terms of Passarino-Veltman functions. Where $\kappa_d^h, \kappa_d^H$ and $\kappa_d^A$ are the Yukawa couplings defined in equation (5) and in table (I).
| Diagram          | Contribution to $g_R$ | J.Vidal et al |
|------------------|-----------------------|---------------|
| $tZW^-$          | -1.211                | -1.176        |
| $tHW^-$          | 0.26147               | 0.220         |
| $tG^0G^-$        | -0.3644               | -0.344        |
| $tG^-H$          | 0.56                  | 0.462         |
| $tZG^-$          | -0.02949              | -0.050        |
| $t\gamma W + t\gamma G^-$ | 0.5706               | 0.572         |
| $bW^+Z$          | $-1.33481 - 1.46899i$ | $-0.623 - 0.664i$ |
| $bW^+H$          | 0                     | 0             |
| $bG^+G^0$        | $0.0001675 - 0.0011i$ | $(1.5 + 11i) \times 10^{-4}$ |
| $bG^+H$          | $-0.000439 - 0.00117i$| $(4.3 + 8.6i) \times 10^{-4}$ |
| $bG^+Z$          | $-0.1820 - 0.132i$    | $-0.088 - 0.062i$ |
| $bW^+\gamma + bG^+\gamma$ | 0.118 - 0.503i       | 0.0114 - 0.509i |
| $Ztb$            | -0.4096               | -0.397        |
| $\gamma tb$     | 0.0669                | 0.068         |
| $G^0tb$          | -0.00069              | $-6.8 \times 10^{-4}$ |
| $Htb$            | -0.00077              | $-6.2 \times 10^{-4}$ |
| $\Sigma(EW)$    | $-1.95628 - 2.10655i$ | $-1.24 - 1.23i$ |
| $gtb$            | -6.60729              | -6.61         |

**TABLE V.** A comparison between our results and those of [4] corresponding to $10^3 g_R$
| Diagram          | Contribution to $V_R$ | Result of $[5]$ |
|------------------|-----------------------|-----------------|
| $tZW^\pm$        | $2.18162 \times 10^{-5}$ | $2.01 \times 10^{-5}$ |
| $t\gamma W^\pm$ | $-1.22114 \times 10^{-5}$ | $-1.10 \times 10^{-5}$ |
| $tHW^\pm$        | $0$                   | $0$             |
| $tG^\pm G^0 + tHG^\pm$ | $-1.67866 \times 10^{-5}$ | $-1.55 \times 10^{-5}$ |
| $tZG^\pm$        | $0.117165 \times 10^{-5}$ | $0.1 \times 10^{-5}$ |
| $t\gamma G^\pm$ | $0.76815 \times 10^{-5}$ | $0.69 \times 10^{-5}$ |
| $bW^\pm Z$       | $(1.19335 + 8.90489i) \times 10^{-4}$ | $(1.12 + 8.24i) \times 10^{-5}$ |
| $bW^\pm \gamma$ | $(8.97983 - 4.71769i) \times 10^{-5}$ | $(8.34 - 4.25i) \times 10^{-5}$ |
| $bW^\pm H$       | $0$                   | $0$             |
| $bG^\pm G^0 + bG^\pm H$ | $(1.05897 + 1.9014i) \times 10^{-5}$ | $(1.01 - 0.35i) \times 10^{-5}$ |
| $bG^\pm Z$       | $(0.00109755 + 0.360717i) \times 10^{-5}$ | $0.31i \times 10^{-5}$ |
| $bG^\pm \gamma$ | $(-4.82503 + 2.5363i) \times 10^{-5}$ | $(-4.47 + 2.29i) \times 10^{-5}$ |
| $Ztb$            | $-2.5271 \times 10^{-5}$ | $-2.30 \times 10^{-5}$ |
| $\gamma tb$     | $-2.98898 \times 10^{-5}$ | $-2.78 \times 10^{-5}$ |
| $G^0 tb + Htb$   | $-1.13206 \times 10^{-5}$ | $-1.03 \times 10^{-5}$ |
| $\Sigma (EW)$   | $(-0.0727959 + 8.98568i) \times 10^{-5}$ | $(0.06 + 6.23i) \times 10^{-5}$ |
| $gtb$            | $2.91224 \times 10^{-3}$ | $2.68 \times 10^{-3}$ |

**TABLE VI.** The right chiral coupling $V_R$ in the SM at the one-loop order
1. The Tensorial Coupling $g_L$

\[
\begin{align*}
\frac{g_{L}^{h0}}{} &= \frac{\alpha_c m_h m_t^{2} m_d^{2}}{16 M_W \pi s_\beta s_W^2} \{C_{12} - C_2\} \\
\frac{g_{L}^{\bar{H}0}}{} &= \frac{\alpha_s m_\bar{b} m_t^{2} m_d^{2} H}{16 M_W \pi s_\beta s_W^2} \{C_{12} - C_2\} \\
\frac{g_{L}^{A_{0}}}{-\alpha m_b m_t^{2} m_d^{2}} &= \frac{\alpha_s m_\bar{b} m_t^{2} m_d^{2} A}{16 M_W \pi s_\beta s_W^2} \{C_{12} + C_2\} \\
\frac{g_{L}^{G_0}}{-\alpha m_b m_t^{2} m_d^{2}} &= \frac{\alpha m_b m_t^{2} m_d^{2}}{16 M_W \pi s_\beta s_W^2} \{C_1 + C_12\} \\
\frac{g_{L}^{h0 H^+}}{} &= \frac{\alpha_c m_h m_t^{2} m_d^{2}}{16 \pi M_W s_\beta s_W^2} \{C_{12} + t_\beta \kappa_d^A (2 C_2 + C_{22})\} \\
\frac{g_{L}^{h0 H^+}}{} &= \frac{\alpha_c m_h m_t^{2} m_d^{2}}{16 \pi M_W s_\beta s_W^2} \{C_{12} + t_\beta \kappa_d^A (2 C_2 + C_{22})\} \\
\frac{g_{L}^{H^+ H^0}}{} &= \frac{-\alpha s m_\bar{b} m_t^{2} m_d^{2}}{16 M_W \pi s_\beta s_W^2} \{C_{12} + t_\beta \kappa_d^A (2 C_2 + C_{22})\} \\
\frac{g_{L}^{h0 H^+}}{} &= \frac{-\alpha m_b m_t^{2} m_d^{2}}{16 M_W \pi s_\beta s_W^2} \{C_{12} + t_\beta \kappa_d^A (2 C_2 + C_{22})\} \\
\frac{g_{L}^{A_{0} H^0}}{} &= \frac{-\alpha m_b m_t^{2} m_d^{2}}{16 M_W \pi s_\beta s_W^2} \{C_{12} + t_\beta \kappa_d^A (2 C_2 + C_{22})\} \\
\frac{g_{L}^{A_{0} H^0}}{} &= \frac{-\alpha m_b m_t^{2} m_d^{2}}{16 M_W \pi s_\beta s_W^2} \{C_{12} + t_\beta \kappa_d^A (2 C_2 + C_{22})\} \\
\end{align*}
\]
\[ g_L^{bG^+C^0} = \frac{\alpha m_b}{16 M_W \pi s_W^2} \{m_b^2(C_0 + C_1 + C_{12} + 2C_2 + C_{22}) + m_t^2(C_1 + C_{11} + C_{12})\} \]
\[ g_L^{\gamma b} = \frac{Q_t Q_b \alpha m_b M_W}{2\pi} \{C_1 + C_{11} + C_{12}\} \]
\[ g_L^{\gamma bZ} = \frac{\alpha m_b M_W (-3 + 4s_W^2)}{72c_W^2 \pi s_W^2} \{(-3 + 2s_W^2)(C_{12} + C_{22}) + 2s_W^2 C_2\} \]
\[ g_L^{\gamma b} = -\frac{C_F \alpha m_b M_W}{2\pi} \{C_1 + C_{12} + C_{11}\} \]
\[ g_L^{bG^\pm} = \frac{Q_b \alpha m_b M_W}{4\pi} \{C_0 + C_1 + C_2\} \]
\[ g_L^{bG^\pm Z} = \frac{-\alpha m_b M_W s_W^2}{12c_W^2 \pi} C_2 \]
\[ g_L^{hW} = g_L^{h^0W} = 0 \]
\[ g_L^{G^\pm} = \frac{-\alpha Q_t m_b M_W}{4\pi} \{C_0 + C_1 + C_2\} \]
\[ g_L^{G^\pm Z} = \frac{\alpha m_b M_W (-3 + 4s_W^2)}{24c_W^2 \pi} C_2 \]
\[ g_L^{hW^\pm} = \frac{\alpha m_b M_W s_{\beta-\alpha} k_d^h}{8\pi s_W^2} C_2 \]
\[ g_L^{H^0W^\pm} = \frac{-\alpha m_b M_W c_{\beta-\alpha} k_d^H}{8\pi s_W^2} C_2 \]
\[ g_L^{W^\pm} = \frac{Q_t \alpha m_b M_W}{4\pi} \{C_0 + C_1 - 2C_{12} - C_2\} \]
\[ g_L^{W^\pm} = \frac{Q_b \alpha m_b M_W}{4\pi} \{-C_0 + C_1 + C_2 + 2(C_{11} + C_{12})\} \]
\[ g_L^{W^\pm Z} = \frac{-\alpha m_b M_W (-3 + 4s_W^2)}{24\pi s_W^2} \{2C_{11} + 2C_{12} - C_2\} \]
\[ g_L^{ZW^\pm} = \frac{\alpha m_b M_W}{24\pi s_W^2} \{-(3 + 4s_W^2)C_1 + 2(3 - 2s_W^2)(C_{12} + C_{22}) - 6s_W^2 C_2\} \]

2. Tensorial Coupling \( g_R \)

\[ g_R^{h^0t} = \frac{\alpha c_b m_b^2 m_t k_d^h}{16 M_W \pi s_b s_W^2} \{C_0 - C_{11} - C_{12} + C_2\} \]
\[ g_R^{H^0t} = \frac{\alpha s_b m_b^2 m_t k_d^H}{16 M_W \pi s_b s_W^2} \{C_0 - C_{11} - C_{12} + C_2\} \]
\[ g_R^{A^0t} = \frac{-\alpha m_b^2 m_t k_d^A}{16 M_W \pi t_b s_W^2} \{C_{11} + C_{12}\} \]
\[ g_R^{b^t} = \frac{\alpha m_b^2 m_t}{16 M_W \pi s_W^2} \{C_0 + C_1 + C_{12} + 2C_2 + C_{22}\} \]
\[ g_{R}^{h0H^\pm} = \frac{\alpha_c c_{\beta - \alpha} m_t}{16\pi M_W s_{\beta} \bar{s}_{W}} \{ (m_t^2 - m_b^2 t_{\beta} \kappa_d^A) C_{12} + m_t^2 (2C_2 + C_{22}) \} \]
\[ g_{R}^{h0H^0} = -\frac{\alpha c_{\beta - \alpha} m_b^2}{16\pi M_W s_{\beta} \bar{s}_{W}} \{ -C_0 + C_{11} + C_{12} - C_2 + t_{\beta} \kappa_d^A (C_{12} + C_2 + C_{22}) \} \]
\[ g_{R}^{H^0H^\pm} = -\frac{\alpha s_{\beta - \alpha} m_t}{16\pi M_W s_{\beta} \bar{s}_{W}} \{ (m_t^2 - m_b^2 t_{\beta} \kappa_d^A) C_{12} + m_t^2 (2C_2 + C_{22}) \} \]
\[ g_{R}^{H^0H^0} = \frac{\alpha s_{\beta - \alpha} m_b^2 m_t \kappa_d^H}{16\pi M_W s_{\beta} \bar{s}_{W}} \{ -C_0 + C_{11} + C_{12} - C_2 + t_{\beta} \kappa_d^A (C_{12} + C_2 + C_{22}) \} \]
\[ g_{R}^{A^0H^\pm} = \frac{\alpha m_t}{16\pi M_W s_{\beta} \bar{s}_{W} t_{\beta}} \{ (m_t^2 + m_b^2 t_{\beta} \kappa_d^A) C_{12} + m_t^2 C_{22} \} \]
\[ g_{R}^{A^0bH^\pm} = -\frac{\alpha m_b^2 m_t \kappa_d^A}{16\pi M_W s_{\beta} \bar{s}_{W} t_{\beta}} \{ C_{11} + (1 + t_{\beta} \kappa_d^A) C_{12} \} \]
\[ g_{R}^{b^0G^\pm} = \frac{\alpha c_{\alpha} m_t s_{\beta - \alpha}}{16\pi M_W s_{\beta} \bar{s}_{W}} \{ m_t^2 (2C_1 + C_{11} + C_{12}) + m_b^2 C_{12} \} \]
\[ g_{R}^{h^0G^\pm} = \alpha m_b^2 m_t s_{\beta - \alpha} \kappa_d^h \{ C_{12} - C_0 + 2C_2 + C_{22} \} \]
\[ g_{R}^{H^0G^\pm} = -\frac{\alpha s_{\alpha} m_t c_{\beta - \alpha}}{16\pi M_W s_{\beta} \bar{s}_{W}} \{ m_t^2 (2C_1 + C_{11} + C_{12}) + m_b^2 C_{12} \} \]
\[ g_{R}^{b^0H^0G^\pm} = \frac{\alpha m_b^2 m_t c_{\beta - \alpha} \kappa_d^h}{16\pi M_W s_{\beta} \bar{s}_{W}} \{ C_{12} - C_0 + 2C_2 + C_{22} \} \]
\[ g_{R}^{b^0G^0G^\pm} = \frac{\alpha m_b^2 m_t}{16\pi M_W s_{\beta} \bar{s}_{W}} \{ C_{12} + C_2 + 2C_2 + C_{22} \} \]
\[ g_{R}^{\gamma t} = \frac{Q_t Q_b \alpha m_t M_W}{2\pi} \{ C_2 + C_{22} + C_{12} \} \]
\[ g_{R}^{b^0Z} = -\frac{\alpha m_t M_W (-3 + 2s_{W}^2)}{72\pi s_{\beta} \bar{s}_{W}} \{ (-3 + 4s_{W}^2) C_{12} - 3C_2 \} \]
\[ g_{R}^{b^0t} = -\frac{C_F \alpha_s m_t M_W}{2\pi} \{ C_2 + C_{12} + C_{22} \} \]
\[ g_{R}^{\gamma t} = -\frac{Q_b \alpha m_t M_W}{4\pi} \{ C_0 + C_1 + C_2 \} \]
\[ g_{R}^{b^0Z} = \frac{\alpha m_t M_W (-3 + 2s_{W}^2)}{24\pi s_{\beta} \bar{s}_{W}} \{ C_2 \} \]
\[ g_{R}^{b^0W} = \frac{\alpha c_{\alpha} m_t M_W s_{\beta - \alpha}}{8\pi s_{\beta} \bar{s}_{W}} \{ C_2 \} \]
\[ g_{R}^{H^0W} = \frac{\alpha s_{\alpha} m_t M_W c_{\beta - \alpha}}{8\pi s_{\beta} \bar{s}_{W}} \{ C_2 \} \]
\[ g_{R}^{t^0G^\pm} = \frac{\alpha Q_t m_t M_W}{4\pi} \{ C_0 + C_1 + C_2 \} \]
\[ g_{R}^{t^0Z} = -\frac{\alpha m_t M_W s_{W}^2}{6\pi \bar{s}_{W}} \{ C_2 \} \]
\[ g_{R}^{b^0h^0W^\pm} = 0 \]
\[ g_{R}^{\gamma t} = \frac{Q_t \alpha m_t M_W}{4\pi} \{ -C_0 + C_1 + C_2 + 2(C_{12} + C_{11}) \} \]
\[ g_R^{\beta W^\pm} = \frac{Q_{b\alpha m_t M_W}}{4\pi} \{C_0 + C_1 - 2C_{12} - C_2\} \]
\[ g_R^{\mu W^\pm z} = \frac{-\alpha m_t M_W}{24\pi s_W^2} \{(3 + 8s_W^2)C_1 - 2(3 - 4s_W^2)(C_{12} + C_{22}) + 12s_W^2C_2\} \]
\[ g_R^{\kappa Z W^\pm} = \frac{\alpha m_t M_W(-3 + 2s_W^2)}{24\pi s_W^2} \{-2(C_{11} + C_{12}) + C_2\} \]

3. Right Chiral Coupling \( V_R \)

\[ V_R^{\beta h^0 t} = -\frac{\alpha c_{\beta} m_b m_t h^0}{16M_W^2\pi s_\beta s_W^2} \{-B_0(m_b^2, m_b^2, m_{h^0}^2) + 2C_{00} + m_b^2(C_1 + C_{11} + C_{12}) + M_W^2(C_1 + C_0 + C_2) - m_t^2(C_1 + C_0 + C_{11})\} \]
\[ V_R^{\beta h^0 t} = -\frac{\alpha m_b m_t h^0}{16M_W^2\pi s_\beta s_W^2} \{-B_0(m_b^2, m_b^2, M_Z^2) + 2C_{00} - M_W^2C_1 + m_b^2(C_1 - C_{11} - C_{12}) + m_t^2(C_1 - C_{12} - 2C_2 - C_{22})\} \]
\[ V_R^{\alpha h^0 t} = -\frac{\alpha m_b m_t h^0}{16M_W^2\pi s_\beta s_W^2} \{-B_0(m_b^2, m_b^2, M_Z^2) - 2C_{00} - m_b^2(C_{12} + C_2 + C_{22}) + m_t^2(C_0 + C_{12} + C_2) - M_W^2(C_0 + C_1 + C_2)\} \]
\[ V_R^{\alpha h^0 t} = -\frac{\alpha c_{\beta} - \alpha m_b m_t h^0}{16M_W^2\pi s_\beta s_W^2} \{t_\beta\kappa_d^A(2C_{00} + (m_t^2 - m_b^2)C_{12}) + m_t^2(1 + \kappa_d^A t_\beta)(2C_2 + C_{22})\} \]
\[ V_R^{\alpha h^0 t} = -\frac{\alpha c_{\beta} - \alpha m_b m_t h^0}{16M_W^2\pi s_\beta s_W^2} \{m_t^2((C_0 - C_{11} - C_{12} - C_2) + t_\beta\kappa_d^A(C_0 - C_{11} - 2C_{12} - C_{22})) - m_t^2(C_{12} + C_2 + C_{22})\} \]
\[ V_R^{\alpha h^0 t} = -\frac{\alpha s_\beta - \alpha s_\beta m_b m_t h^0}{16M_W^2\pi s_\beta s_W^2} \{2t_\beta\kappa_d^A C_{00} + (m_t^2 - m_b^2)t_\beta\kappa_d^A C_{12} + m_t^2(1 + t_\beta\kappa_d^A)(2C_2 + C_{22})\} \]
\[ V_R^{\alpha h^0 G^z} = \frac{\alpha c_{\beta} m_t h^0}{16M_W^2\pi s_\beta s_W^2} \{-2C_{00} + m_b^2(-1 + \kappa_d^A t_\beta) C_{11} + (m_t^2 - m_b^2) C_{12}\} \]
\[ V_R^{\alpha h^0 G^z} = \frac{\alpha c_{\beta} m_t h^0}{16M_W^2\pi s_\beta s_W^2} \{-2C_{00} + (m_b^2 - m_t^2)(C_{12} + C_2 + C_{22})\} \]
\[ V_R^{H_0 G^+} = \frac{\alpha c_{\beta \alpha} m_b m_t s_\alpha}{16 M_W^2 \pi s_W^2} \{ -2 C_{00} + (m_b^2 - m_t^2) C_{12} \} \]
\[ V_R^{bH_0 G^+} = \frac{\alpha c_{\beta \alpha} m_b m_t K_H^H}{16 M_W^2 \pi s_W^2} \{ -2 C_{00} + (m_b^2 - m_t^2)(C_{12} + C_2 + C_{22}) \} \]
\[ V_R^{G^+ G} = \frac{\alpha m_t m_t}{16 M_W^2 \pi s_W^2} \{ 2 C_{00} + m_b^2 (C_1 + C_{11} + C_{12}) + m_t^2 (2 C_0 + 3 C_1 + C_{11} + 3 C_{12} + 4 C_2 + 2 C_{22}) \} \]
\[ V_R^{bG^+ G} = \frac{\alpha m_t m_t}{16 M_W^2 \pi s_W^2} \{ 2 C_{00} + m_b^2 (2 C_0 + 3 C_1 + C_{11} + 3 C_{12} + 4 C_2 + 2 C_{22}) + m_t^2 (C_1 + C_{11} + C_{12}) \} \]
\[ V_R^{\gamma b} = \frac{Q_t Q_b a m_t m_t}{2 \pi} \{ C_{11} + 2 C_{12} + C_{22} \} \]
\[ V_R^{\gamma Z} = \frac{\alpha m_t m_t}{2 \pi c_W^2 s_W^2} \{ 8 s_W^4 C_0 + 2 s_W^2 (-9 + 8 s_W^2) C_2 + (9 - 18 s_W^2 + 8 s_W^4) C_{22} \} \]
\[ V_R^{\gamma Z} = \frac{C_F a s m_t m_t}{2 \pi} \{ C_{22} + 2 C_{12} + C_{11} \} \]
\[ V_R^{B G^+} = \frac{Q_b m_t m_t}{4 \pi} C_1 \]
\[ V_R^{B G^+ Z} = \frac{-a m_t m_t s_W^2}{12 c_W^2 \pi} \{ C_0 + C_1 + C_{2} \} \]
\[ V_R^{h_0 W^+} = V_R^{H_0 W^+} = 0 \]
\[ V_R^{\gamma W^+} = \frac{Q_t a m_t m_t}{4 \pi} C_1 \]
\[ V_R^{G^+ Z} = \frac{-a m_t m_t s_W^2}{6 c_W^2 \pi} \{ C_0 + C_1 + C_{2} \} \]
\[ V_R^{b_0 W^+} = V_R^{bH_0 W^+} = 0 \]
\[ V_R^{\gamma W^+} = \frac{-Q_t a m_t m_t}{4 \pi} (C_1 - 2 C_{11}) \]
\[ V_R^{bW^+} = \frac{-Q_b a m_t m_t}{4 \pi} \{ C_1 - 2 C_{11} \} \]
\[ V_R^{W^+ Z} = \frac{a m_t m_t}{12 \pi s_W^2} \{ -6 s_W^2 C_0 + (3 - 4 s_W^2) (C_{22} + C_{11} + 2 C_{12}) + (3 - 10 s_W^2) (C_1 + C_{22}) \} \]
\[ V_R^{bW^+ Z} = \frac{a m_t m_t}{12 \pi s_W^2} \{ -3 s_W^2 C_0 + (3 - 5 s_W^2) (C_1 + C_{22}) + (3 - 2 s_W^2) (C_{11} + 2 C_{12} + C_{22}) \} \]

Where \( C_{i,j} = C_{i,j}(m_b^2, m_t^2, M_W^2, m_A^2, m_B^2, m_C^2) \), \( A, B \) and \( C \) are the particles running in the loops. \( Q_b = 1/3, Q_t = 2/3 \) and \( C_F = 4/3, s_W = \sin \theta_W, c_W = \cos \theta_W, c_i = \cos i, s_i = \sin i \) and \( t_i = \tan i \) where \( i = \alpha, \beta \). Expressions in the case of the Standard Model are recovered by letting \( s_{\beta - \alpha} \rightarrow 1, c_{\beta - \alpha} \rightarrow 0 \) and \( s_{\beta} = c_{\alpha}, s_{\alpha} = -c_{\beta} \) in the previous formulae.

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