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Peristaltic mechanism of hydromagnetic Jeffrey fluid having variable thermal conductivity and slip conditions

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Abstract: In this study, we focus on the slip effects on the peristaltic unsteady flow of magnatoahydromagnetic Jeffrey fluid in a flow passage with non-conducting and flexible boundary walls. The effect of the magnetic field with varying thermal conductivity is taken under the influence of heat transfer analysis. The dimensionless system of PDEs is solved analytically, and the obtained results are computed for the temperature, pressure drop, the axial pressure gradient, axial velocity, and then these results are discussed for different values of the physical parameters of our interest. For the stream functions, the contour plots are also obtained which indicates the exact flow behavior within the flow channel, and the effects of the physical parameters on Jeffery fluid within the flow channel are discussed briefly. Our results indicate that the heat transfer coefficient decreases with an increase in thermal slip and velocity slip parameters. Furthermore, it shows that the size of the trapped bolus is greater for the inclined magnetic field as compared to the transverse magnetic field.

Keywords: Jeffrey fluid; Peristaltic flow; Magnetohydrodynamic; Thermal conductivity; Heat transfer; Velocity slip condition; Temperature slip condition.

MSC: 76A05; 35B35; 35D35; 35B40.

1. Introduction

In magnetohydrodynamics some recent studies [1–11] have considered the peristaltic flow of Newtonian and non-Newtonian fluid and investigated the result of magnetic field on them. Stud et al., [12] investigated the interaction of Poiseuille with peristalsis flow. In an asymmetric tube, the effects of peristalsis of viscous fluid are studied by Mekheimer [13].

In the diagnoses of blood circulation and measurements of blood glucose, the study of heat transfer in blood flow plays a vital role. Our normal blood temperature is about 37°C, but our proteins start damaging when the temperature is increased from 41°C. In peristalsis flow, the effect of heat transfer has been investigated by many scholars, but a review of heat transfer by Nadeem and Akbar [14] is quite famous in the era of 2003.

Most of the natural physiological fluids like polymer melts, bubbly fluids, hydrocarbons, and many nuclear aggressive materials are based on complicated stress-strain relations as compared to Newtonian fluids. The governing equations of such fluids give challenges to researchers and scholars. The Jeffrey version of the Oldroyed [15–22] is an important example of non-Newtonian fluid models in which time derivative of strain tensor. Peristaltic action of non-Newtonian fluid for small wave amplitude has been described by Raju and Devanathan [23], the long-wavelength peristalsis analysis is performed by Radhakrishnamacharya [24].

In all the studies mentioned above slip effects in presence of variable thermal conductivity have been ignored. Therefore it is of great interest to study the peristaltic mechanism of hydromagnetic Jeffrey fluid having variable thermal conductivity with slip conditions and we did this analysis because of its practical applications in the field of biomedical engineering and food industries in fluid delivery.
2. Mathematical formulation

The unsteady flow of an incompressible hydromagnetic fluid in an asymmetric two-dimensional tube having two walls $h_1$ and $h_2$ is considered in this study.

The wall geometries as shown in Figure 1 can be expressed as follows:

$$h_1 (\bar{X}, \bar{t}) = \ell_1 + b_1 \cos \left[ \frac{2\pi}{\lambda} (\bar{X} - c\bar{t}) \right],$$  \hspace{1cm} (1)

$$h_2 (\bar{X}, \bar{t}) = -\ell_2 - b_2 \cos \left[ \frac{2\pi}{\lambda} (\bar{X} - c\bar{t}) + \theta \right],$$  \hspace{1cm} (2)

where $\ell_1$ and $\ell_2$ are the channel half widths, $\lambda$ is the wavelength, $c$ is the wave speed, $b_1$ and $b_2$ are the amplitudes, $\bar{t}$ is the time, and $\theta$ is the phase difference. Furthermore, $\theta$ fulfill the condition

$$b_1^2 + b_2^2 + 2b_1b_2 \cos \theta \leq (\ell_1 + \ell_2)^2. \hspace{1cm} (3)$$

![Figure 1. Geometry of the physical model.](image)

It is assumed that $T_1$ is upper wall temperature and $T_2$ is the lower wall temperature and $T_1$ must be less than $T_2.$

In MHD fluid the governing equations are expressed as

$$\nabla \cdot \bar{V} = 0,$$  \hspace{1cm} (4)

$$\rho \left[ \frac{\partial \bar{V}}{\partial \bar{t}} + (\bar{V} \cdot \nabla) \bar{V} \right] = \nabla \cdot \bar{T} + \rho \bar{f},$$  \hspace{1cm} (5)

$$\rho c_p \left[ \frac{\partial \bar{T}}{\partial \bar{t}} + (\bar{V} \cdot \nabla) \bar{T} \right] = -\nabla \cdot \bar{q} + S_t. \hspace{1cm} (6)$$

In above equation, $\nabla$ is the gradient operator, and $\bar{V} = [U (\bar{X}, \bar{Y}, \bar{t}), V (\bar{X}, \bar{Y}, \bar{t}), 0]$ is the two-dimensional velocity field vector. $\bar{S}, \bar{T},$ and $\bar{I}$ are respectively the extra stress tensor for Jeffrey fluid, Cauchy stress tensor and the Identity tensor, $S_t$ represent the source term, $c_p$ being the specific heat, $\bar{T}$ is the temperature and $\bar{q}$ is the heat flux vector.

The constitution Jeffrey fluid model equations are

$$\bar{T} = -\rho \bar{I} + \bar{S},$$  \hspace{1cm} (7)

$$\bar{S} = \frac{\mu}{1 + \lambda_1} [\bar{\tau} + \lambda_2 \bar{\tau}],$$  \hspace{1cm} (8)

$$\bar{\tau} = \bar{K} + \bar{K}^T,$$  \hspace{1cm} (9)
where \( \bar{S} \) is extra stress tensor, relaxation time is denoted by \( \lambda_1 \), fluid viscosity by \( \mu \), and retardation time by \( \lambda_2 \). \( \bar{p} \) is pressure, and \( \bar{I} \) is the identity tensor.

The components of extra stress tensor \( \bar{S} \) are

\[
\begin{align*}
\bar{S}_{XX} &= \frac{2\mu}{1 + \lambda_1} \left[ 1 + \lambda_2 \left( \frac{\partial}{\partial t} + \bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right) \right] \frac{\partial \bar{U}}{\partial \bar{X}}, \\
\bar{S}_{XY} &= \frac{\mu}{1 + \lambda_1} \left[ 1 + \lambda_2 \left( \frac{\partial}{\partial t} + \bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right) \right] \left( \frac{\partial \bar{V}}{\partial \bar{X}} + \frac{\partial \bar{U}}{\partial \bar{Y}} \right), \\
\bar{S}_{YY} &= \frac{2\mu}{1 + \lambda_1} \left[ 1 + \lambda_2 \left( \frac{\partial}{\partial t} + \bar{U} \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right) \right] \frac{\partial \bar{V}}{\partial \bar{Y}}, \\
\end{align*}
\]

and

\[
\bar{S}_{XY} = \bar{S}_{X\bar{Y}}, \quad \bar{S}_{XZ} = \bar{S}_{Z\bar{X}} = \bar{S}_{2\bar{X}} = \bar{S}_{2\bar{Z}} = 0.
\]

The current density \((\bar{J} = \bar{S}_t | \bar{V} \times \bar{B}_0|)\) where magnetic field under the neglignce of impose electric field and induce electric field represent by \( \bar{B}_0 (= \{ \bar{B}_0 \cos \phi, \bar{B}_0 \sin \phi, \bar{0} \}) \). Thus magnetic body force take the form

\[
\bar{J} \times \bar{B}_0 = \left[ -\bar{S}_0 \bar{B}_0^2 \left( \bar{U} \sin^2 \phi - \bar{V} \cos \phi \sin \phi \right), \bar{S}_0 \bar{B}_0^2 \left( \bar{U} \cos \phi \sin \phi - \bar{V} \cos^2 \phi \right), 0 \right].
\]

Using above results, Eqs (4)-(6) are modified as

\[
\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0,
\]

\[
\rho \left( \frac{\partial \bar{U}}{\partial t} + \bar{U} \frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V} \frac{\partial \bar{U}}{\partial \bar{Y}} \right) = -\frac{\partial \bar{p}}{\partial \bar{X}} + \frac{\partial \bar{S}_{XX}}{\partial \bar{X}} + \frac{\partial \bar{S}_{XY}}{\partial \bar{Y}} - \bar{S}_0 \bar{B}_0^2 \left( \bar{U} \sin^2 \phi - \bar{V} \cos \phi \sin \phi \right),
\]

\[
\rho \left( \frac{\partial \bar{V}}{\partial t} + \bar{U} \frac{\partial \bar{V}}{\partial \bar{X}} + \bar{V} \frac{\partial \bar{V}}{\partial \bar{Y}} \right) = -\frac{\partial \bar{p}}{\partial \bar{Y}} + \frac{\partial \bar{S}_{XY}}{\partial \bar{X}} + \frac{\partial \bar{S}_{YY}}{\partial \bar{Y}} - \bar{S}_0 \bar{B}_0^2 \left( \bar{U} \sin \phi \cos \phi - \bar{V} \cos^2 \phi \right),
\]

\[
\rho \epsilon_0 \left( \frac{\partial \bar{T}}{\partial t} + \bar{U} \frac{\partial \bar{T}}{\partial \bar{X}} + \bar{V} \frac{\partial \bar{T}}{\partial \bar{Y}} \right) = \frac{\partial}{\partial \bar{X}} \left[ \kappa(\bar{T}) \left( \frac{\partial \bar{T}}{\partial \bar{X}} + \frac{\partial \bar{T}}{\partial \bar{Y}} \right) \right] + \frac{\partial}{\partial \bar{Y}} \left[ \kappa(\bar{T}) \left( \frac{\partial \bar{T}}{\partial \bar{X}} + \frac{\partial \bar{T}}{\partial \bar{Y}} \right) \right] + \Phi_a + \Phi_b.
\]

In above Eqs (15)-(21), we define the specific heat as \( c_p \), density of the fluid as \( \rho \), and the fluid temperature as \( \bar{T} \); whereas, temperature dependent thermal conductivity \( \kappa(\bar{T}) \), the viscous dissipation \( \Phi_a \), and Joule dissipation \( \Phi_b \) are defined as follows:

\[
\kappa(\bar{T}) = \kappa_0 \left[ 1 + \epsilon_1 \left( \bar{T} - \bar{T}_0 \right) \right],
\]

\[
\Phi_a = \frac{\mu}{1 + \lambda_1} \left[ 1 + \lambda_2 \left( \frac{\partial}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial \bar{Y}} \right) \right] \times \left[ \frac{2}{\left( \frac{\partial \bar{U}}{\partial \bar{X}} \right)^2} + \frac{2}{\left( \frac{\partial \bar{V}}{\partial \bar{Y}} \right)^2} + \frac{\left( \frac{\partial \bar{U}}{\partial \bar{X}} \right)^2}{\left( \frac{\partial \bar{V}}{\partial \bar{Y}} \right)^2} + \frac{\left( \frac{\partial \bar{V}}{\partial \bar{Y}} \right)^2}{\left( \frac{\partial \bar{U}}{\partial \bar{X}} \right)^2} + \frac{\left( \frac{\partial V}{\partial \bar{Y}} \right)^2}{\left( \frac{\partial \bar{U}}{\partial \bar{X}} \right)^2} + \frac{\left( \frac{\partial \bar{U}}{\partial \bar{X}} \right)^2}{\left( \frac{\partial V}{\partial \bar{Y}} \right)^2} \right],
\]

and

\[
\Phi_b = \bar{S}_0 \bar{B}_0^2 \left( \bar{U} \sin \phi - \bar{V} \cos \phi \right),
\]

where \( \epsilon_1 \) is a constant and \( \kappa_0 \) is the dynamic thermal conductivity.

The unsteady flow in fixed frame \((\bar{X}, \bar{Y}, \bar{T})\) can be treated as steady in the wave frame \((\bar{x}, \bar{y})\) if we define

\[
\begin{align*}
\bar{u}(\bar{x}, \bar{y}) &= \bar{U}(\bar{X}, \bar{Y}, \bar{T}) - c, \\
\bar{v}(\bar{x}, \bar{y}) &= \bar{V}(\bar{X}, \bar{Y}, \bar{T}), \\
\bar{x} &= \bar{X} - ct, \\
\bar{y} &= \bar{Y},
\end{align*}
\]

where \( \bar{u} \) and \( \bar{v} \) are the respective components of velocity in \( \bar{x} \)-direction and \( \bar{y} \)-directions. Equations (15)-(18) after using equations (20)-(22) give

\[
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0,
\]
Introducing the dimensionless variables as follows

\[ \rho \left( \frac{\partial u}{\partial x} + \sigma \frac{\partial u}{\partial y} \right) = \rho \left( \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{u}}{\partial y} \right) = \frac{\partial \bar{p}}{\partial x} + \left( \frac{\partial \bar{S}_{xx}}{\partial x} + \frac{\partial \bar{S}_{xy}}{\partial y} \right) - S_c B_0^2 \left( \sin^2 \phi (\bar{u} + c) - \frac{\sigma}{2} \sin 2\phi \right), \] \hspace{1cm}(24)\\
\[ \left( \frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{v}}{\partial y} \right) = -\frac{\partial \bar{p}}{\partial y} + \left( \frac{\partial \bar{S}_{xy}}{\partial y} + \frac{\partial \bar{S}_{yy}}{\partial y} \right) + S_c B_0^2 \left( -\sigma \cos^2 \phi + \frac{\bar{u} + c}{2} \sin 2\phi \right), \] \hspace{1cm}(25)\\
and,

\[ c_p \left( \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{T}}{\partial y} \right) = \frac{\partial}{\partial x} \left[ \kappa (T) \left( \frac{\partial \bar{T}}{\partial x} + \frac{\partial \bar{T}}{\partial y} \right) \right] + \frac{\partial}{\partial y} \left[ \kappa (T) \left( \frac{\partial \bar{T}}{\partial x} + \frac{\partial \bar{T}}{\partial y} \right) \right] + \frac{\mu}{1 + \lambda_1} \left[ 1 + \lambda_2 \left( \frac{\partial \bar{u} + c}{\partial x} + \frac{\partial \bar{v}}{\partial y} \right) \right] \] 
\times \left[ 2 \left\{ \left( \frac{\partial (\bar{u} + c)}{\partial x} \right)^2 + \left( \frac{\partial \bar{v}}{\partial y} \right)^2 \right\} + \left( \frac{\partial (\bar{u} + c)}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right)^2 \right] + S_c B_0^2 \left[ (\bar{u} + c) \sin \phi - \sigma \cos \phi \right]. \] \hspace{1cm}(26)\\

Introducing the dimensionless variables as follows

\[
\begin{align*}
    x &= \frac{\bar{x}}{\bar{L}_x}, \quad y = \frac{\bar{y}}{\bar{L}_y}, \quad S = \frac{\bar{S}_1}{\bar{S}_0}, \quad u = \frac{\bar{u}}{\bar{U}_1}, \quad v = \frac{\bar{v}}{\bar{U}_1}, \quad \nu = -\delta \left( \frac{\partial \bar{T}}{\partial y} \right), \\
    T &= \frac{T - T_0}{T_1 - T_0}, \quad \ell = \frac{\bar{L}}{\bar{L}_x}, \quad \kappa = \frac{\bar{\kappa}}{\bar{\kappa}_0}, \quad \epsilon = \epsilon (T_1 - T_0), \quad u = \frac{\bar{u}}{\bar{U}_1}. \hspace{1cm}(27)
\end{align*}
\]

We define dimensionless pressure \( p \), wave number \( \delta \), Hartman number \( M \), Brinkman number \( Br = \frac{Pr \times Ec}{2} \), Eckert number \( Ec \), Prandtl number \( Pr \), and Reynolds number \( Re \) as follows:

\[
\begin{align*}
    p &= \frac{\bar{p}_1}{\bar{S}_1}, \quad Br = \frac{\mu \bar{c}_1}{\bar{S}_0}, \quad M = \sqrt{\frac{\bar{B}_0}{\bar{S}_0}} \bar{c}_1, \\
    Re &= \frac{\rho \bar{c}_1}{\mu}, \quad \delta = \frac{\bar{L}}{\bar{L}_x}, \quad Pr = \frac{\mu \bar{c}_1}{\bar{S}_0}, \quad Ec = \frac{c^2}{\bar{c}_1} \bar{S}_0. 
\end{align*}
\]

Using Eqs (27) and (28) and adopting the long wavelength procedure, the Eqs (23)-(26) reduce to

\[ \frac{\partial \bar{p}}{\partial x} = \frac{\partial}{\partial y} \left( \frac{1}{1 + \lambda_1} \frac{\partial^2 \phi}{\partial y^2} \right) - M^2 \sin^2 \phi \left( \frac{\partial \phi}{\partial y} + 1 \right), \] \hspace{1cm}(29)\\
\[ \frac{\partial \bar{p}}{\partial y} = 0, \] \hspace{1cm}(30)\\
and

\[ \frac{\partial}{\partial y} \left( \kappa (T) \frac{\partial \bar{T}}{\partial y} \right) + Br \left[ \frac{1}{1 + \lambda_1} \left( \frac{\partial^2 \phi}{\partial y^2} \right)^2 + M^2 \sin^2 \phi \left( \frac{\partial \phi}{\partial y} + 1 \right)^2 \right] = 0, \] \hspace{1cm}(31)\\
where

\[ \kappa(T) = 1 + \epsilon T. \] \hspace{1cm}(32)

3. Boundary conditions

The appropriate dimensionless velocity slip conditions can be expressed as

\[
\begin{align*}
    \phi &= \frac{\epsilon}{2}, \quad \frac{\partial \phi}{\partial y} + \beta_1 \frac{\partial^2 \phi}{\partial y^2} = -1, \quad \text{at} \quad y = h_1 \left( x \right), \\
    \phi &= -\frac{\epsilon}{2}, \quad \frac{\partial \phi}{\partial y} - \beta_1 \frac{\partial^2 \phi}{\partial y^2} = -1, \quad \text{at} \quad y = h_2 \left( x \right). \hspace{1cm}(33)
\end{align*}
\]

The temperature jump conditions are defined as

\[
\begin{align*}
    T + \beta_2 \frac{\partial T}{\partial y} &= 0, \quad \text{at} \quad y = h_1 \left( x \right), \\
    T - \beta_2 \frac{\partial T}{\partial y} &= 1, \quad \text{at} \quad y = h_2 \left( x \right). \hspace{1cm}(34)
\end{align*}
\]

In above equation, \( \beta_1 \) is the velocity slip parameter, whereas, \( \beta_2 \) is the thermal slip parameter.
4. Method

Taking cross differentiation of Eqs (29) and (30), we arrive at

\[ 0 = \frac{\partial^2}{\partial y^2} \left( \frac{1}{1 + \lambda_2} \frac{\partial^2 \psi}{\partial y^2} \right) - M^2 \sin^2 \phi \left( \frac{\partial^2 \psi}{\partial y^2} \right). \]  

(35)

The general solution of Eq. (35) is

\[ \psi(x, y) = D_1 + D_2 y + D_3 \cosh \left[ M \sin \phi \sqrt{1 + \lambda_2 y} \right] + D_4 \sinh \left[ M \sin \phi \sqrt{1 + \lambda_2 y} \right], \]  

(36)

where \(D_1, D_2, D_3\) and \(D_4\) are constants. The constants are obtained using boundary condition (33). The axial pressure gradient has been obtained as

\[ \frac{dp}{dx} = -\frac{D_5 D_6 + D_7}{R_{11} + R_{12} + R_{13} R_{14}}. \]  

(37)

Since Eq. (31) is highly non linear, so it is difficult to find the analytical solution. In this study, a perturbation technique has been used to calculate the temperature distribution of Jeffery fluid, so we construct perturbation solution for a small \(\varepsilon\) as

\[ T(x, y) = T^{(0)}(x, y) + \varepsilon T^{(1)}(x, y), \]  

(38)

where \((0 < \varepsilon < 1)\).

Using Eq. (38) into Eqs (31) and (34) give yield zeroth order and 1st order systems of the temperature distribution, correspondingly. These are given by

Zeroth order system

\[ \frac{\partial^2 T^{(0)}}{\partial y^2} + Br \left[ \frac{1}{1 + \lambda_1} \frac{\partial^2 \psi}{\partial y^2} + M^2 \sin^2 \phi \left( \frac{\partial \psi}{\partial y} + 1 \right)^2 \right] = 0, \]  

(39)

\[ \begin{cases} 
T^{(0)} + \beta_2 \frac{\partial T^{(0)}}{\partial y} = 0, \text{ at } y = h_1(x), \\
T^{(0)} - \beta_2 \frac{\partial T^{(0)}}{\partial y} = 1, \text{ at } y = h_2(x). 
\end{cases} \]  

(40)

First order system

\[ \frac{\partial^2 T^{(1)}}{\partial y^2} + \frac{\partial}{\partial y} \left( T^{(0)} \frac{\partial T^{(0)}}{\partial y} \right) = 0, \]  

(41)

\[ \begin{cases} 
T^{(1)} + \beta_2 \frac{\partial T^{(1)}}{\partial y} = 0, \text{ at } y = h_1(x), \\
T^{(1)} - \beta_2 \frac{\partial T^{(1)}}{\partial y} = 1, \text{ at } y = h_2(x). 
\end{cases} \]  

(42)

Solving zeroth and first order systems, we find that

\[ T^{(0)} = -f_0(x, y) + D_8 y + D_9, \]  

(43)

\[ T^{(1)} = -f_1(x, y) + D_{10} y + D_{11}. \]  

(44)

The heat transfer coefficient defined as

\[ Z = \frac{\partial h_2}{\partial x} \times \frac{\partial T}{\partial y}. \]  

(45)

5. Graphical illustrations

This section is aimed to study the slip effects and their importance on a peristaltic mechanism under the influence of different emerging parameters. The graphs have been drawn for various concerned parameters. Note that the values of the parameters are fixed as \(s_1 = 0.75, s_2 = 0.75, \eta = 1, \theta = \pi/4\) for all graphs and Tables.
Figure 2. Pressure drop $\Delta P_\lambda$ vs $Q$ for (a): $M = 4, \lambda_1 = 1, \beta_1 = 0.2$, (b): $\phi = \pi/4, \beta_1 = 0.2, \lambda_1 = 1$, (c): $\phi = \pi/4, \beta_1 = 0.2, M = 1$, (d): $M = 4, \lambda_1 = 1, \phi = \pi/4$.

5.1. Pumping features

The variation of pressure drop per wavelength $\Delta P_\lambda$ verses flow rate $Q$ is shown in Figure 2. Influence of inclination angle $\phi$ on pressure drop $\Delta P_\lambda$ is depicted in Figure 2(a). It is observed that in the peristaltic pumping region, the pumping rate starts increasing when $\phi$ is increased from 0 to $\pi/2$, whereas it decreases in the augmented pumping region. It shows opposite behavior for $\phi \in [0, \pi/2]$. Figure 2(b) gives us an opportunity to study the connection between Hartman number $M$ and $\Delta P_\lambda$. In the positive feasible solution area which is the peristaltic pumping area, a rise in pressure drop is observed as we increase Hartman number $M$. In the augmented pumping region action of pumping performance remains increasing. Figure 2(c) is plotted to investigate the behavior on $\Delta P_\lambda$ by Jeffrey fluid parameter $\lambda_1$. It is seen that the pumping rate under the effects of $\lambda_1$ shows increasing behavior. Figure 2(d) illustrates the effects of thermal slip parameter $\beta_1$ on $\Delta P_\lambda > 0$. It is seen that the pumping rate under the effects of $\lambda_1$ shows increasing behavior.

Figure 3 shows the pressure gradient profile for one wavelength. It illustrates that in the wider part of channel $x \in [0, 0.2]$ and $x \in [0.65, 1]$ the flow can pass easily without implementation of a large pressure.

| $\epsilon$ | $\pi/4$ | $\pi/2$ | $2\pi/3$ | $5\pi/6$ | $7\pi/8$ |
|----------|--------|--------|----------|----------|--------|
| 0.0      | 1.6713 | 2.41269| 2.04214  | 1.29947  | 1.14463|
| 0.1      | 1.6961 | 2.41624| 2.05731  | 1.33195  | 1.17971|
| 0.2      | 1.7209 | 2.41979| 2.07247  | 1.36443  | 1.21479|
0.2 0.3 0.4 0.5 0.6  

\[ \phi = 0 \]
\[ \phi = \pi/6 \]
\[ \phi = \pi/2 \]
\[ \phi = 2\pi/3 \]

\( dp/dx \)

\[ \lambda_1 = 0 \]
\[ \lambda_1 = 1 \]
\[ \lambda_1 = 2 \]
\[ \lambda_1 = 6 \]

\( M = 0 \)
\( M = 2 \)
\( M = 3 \)
\( M = 4 \)

\[ \beta_1 = 0 \]
\[ \beta_1 = 0.05 \]
\[ \beta_1 = 0.2 \]
\[ \beta_1 = 0.5 \]

Figure 3. \( dp/dx \) (pressure gradient) vs \( x \in [-0.5, 0.5] \) for \( Q = 0.1 \), (a): \( M = 4, \beta_1 = 0.2, \lambda_1 = 1 \), (b): \( \phi = \pi/4, \beta_1 = 0.2, \lambda_1 = 1 \), (c): \( \phi = \pi/4, \lambda_1 = 1, \beta_1 = 0.2, M = 4 \), (d): \( M = 4, \lambda_1 = 1, \phi = \pi/4 \).

gradient, that’s why pressure gradient is small in that region, but in the narrow part of the channel \( x \in [0.3, 0.6] \), flow can pass with same flux under the large pressure gradient, especially the narrowest position \( x = 0.45 \). This phenomenon is well in accordance with the physical situation. We also observe the effect of \( \phi, M, \lambda_1 \) and \( \beta_1 \) on \( dp/dx \) by using fixed values of other parameters. The amplitude of \( dp/dx \) increases with increase in values of \( \phi \) and \( M \) while it decrease as \( \lambda_1 \) and \( \beta_1 \) increases.

The velocity profile for different parameters is shown in Figure 4. We observe that the velocity profile shows decreasing behavior for a gradually increase in \( \phi, M, \lambda_1 \) and \( \beta_1 \).

5.2. Heat transfer characteristics

Figure 5 shows the variation of fluid temperature for various parameters. After examining Figure 5 we concluded that, if \( \phi \) is raised from 0 to \( \pi/2 \) then temperature increases. As \( \phi \) is raised from \( \pi/2 \) to \( \pi \) then temperature decreases. When Heartman number \( M \) is increased then the temperature is also increased, while an increase in \( \lambda_1 \) the temperature shows decreasing behavior. Under the effects of viscous and Joule

| \( \varepsilon \) | 1      | 2      | 3      | 4      | 5      |
|-----------------|--------|--------|--------|--------|--------|
| 0.0             | 0.97064| 1.11242| 1.34606| 1.67129| 2.08847|
| 0.1             | 1.00824| 1.14800| 1.37690| 1.69610| 2.10229|
| 0.2             | 1.04583| 1.18358| 1.40931| 1.72090| 2.11611|

Table 2. \( Z \) (Heat transfer coefficient) as function of \( M \) and \( \varepsilon \). Here \( x = 0.1, Q = -1.2, \beta_1 = 0.2, \beta_2 = 0.2, \phi = \pi/4, \lambda_1 = 1, \text{Br} = 0.5 \).
dissipation, the temperature got increases with an increase in $Br$. An increase in thermal slip parameters and velocity slip parameter temperature shows decreasing behavior.

Tables 1-6 give us the numerical results of heat transfer coefficient $Z$ various parameters. From Table 1, we observe that the heat transfer coefficient is less for hydrodynamic fluid but large for the hydromagnetic fluid. The value of the heat transfer coefficient $Z$ increases under the effect of \( \phi \in [0, \pi/2] \); however, opposite behavior is observed for \( \phi \in [\pi/2, \pi] \). Table 2 tells us that the value of $Z$ increases as we move from Jeffrey fluid to Newtonian fluid. Table 4 witness that the heat transfer coefficient $Z$ increases under the consideration of the viscous and Joule dissipation. Table 5 and Table 6 tell us that the heat transfer coefficient $Z$ decreases as we increase thermal slip parameters and velocity slip parameters.

5.3. Trapping phenomena

The plots for streamlines for different values of $M$, $\phi$, $\beta_1$, $\beta_2$ and $\lambda_1$ are given in Figures 6-8. Figure 6 depicts that size of trapped bolus reduces as a result of increasing $\phi \in [0, \pi/2]$; however, the situation is reversed for $\phi \in [\pi/2, \pi]$, it displays an inverse behavior. Figure 7 concludes that the trapped bolus is smaller in size for hydromagnetic fluids than hydrodynamic fluids. It can be seen from Figure 8 that the size of the bolus is a decreasing function of Jeffrey fluid parameter $\lambda_1$. Figure 9 tells us that the size of the trapped bolus falls down with an increment in the value of velocity slip parameter $\beta_1$. 

**Figure 4.** Distribution of velocity ($u$) vs $y$ for $x = 0$, $Q = 1.2$, (a): $M = 4, \beta_1 = 0.2, \lambda_1 = 1$, (b): $\phi = \pi/4, \beta_1 = 0.2, \lambda_1 = 1$, (c): $\phi = \pi/4, \beta_1 = 0.2, M = 4$, (d): $M = 4, \lambda_1 = 1, \phi = \pi/4$. 

...
Figure 5. Distribution of temperature ($T$) vs $y$ for $x = 0$, $Q = -1.2$, (a): $\epsilon = 0.1$, $M = 4$, $\lambda_1 = 1$, $\beta_1 = 0.2$, $\beta_2 = 0.2$ $Br = 1$, (b): $\epsilon = 0.1$, $\phi = \pi/4$, $\beta_1 = 0.2$, $\beta_2 = 0.2$, $\lambda_1 = Br = 1$, (c): $\phi = \pi/4$, $M = 4$, $Br = 1.5$, $\beta_1 = 0.2$, (d): $\phi = \pi/4$, $M = 4$, $Br = 1$, $\beta_1 = 0.2$, $\beta_2 = 0.2$, (e): $\epsilon = 0.1$, $\phi = \pi/3$, $\beta_1 = 0.2$, $\beta_2 = 0.2$, $M = 4$, $\lambda_1 = 1$, (f): $\epsilon = 0.1$, $\phi = \pi/4$, $\lambda_1 = Br = 1$. 

(a) $\phi = 0$

(b) $\phi = \pi/4$

(c) $\phi = \pi/2$

(d) $\phi = 2\pi/3$

(e) $\phi = 0$

(f) $\phi = \pi/4$

(g) $\phi = \pi/2$
Table 3. \( Z \) (Heat transfer coefficient) as a function of \( \lambda_1 \) and \( \epsilon \). Here \( x = 0.1, Q = -1.2, \beta_1 = 0.2, \beta_2 = 0.2, \phi = \pi/4, M = 4, Br = 0.5 \).

| \( \epsilon \) | 1   | 2   | 3   | 4   | 5   |
|-----------------|-----|-----|-----|-----|-----|
| 0.0             | 1.67129 | 1.65697 | 1.64965 | 1.64522 | 1.64226 |
| 0.1             | 1.69610 | 1.68210 | 1.67496 | 1.67063 | 1.66774 |
| 0.2             | 1.72090 | 1.70724 | 1.70026 | 1.69604 | 1.69321 |

Table 4. \( Z \) (Heat transfer coefficient) as a function of \( Br \) and \( \epsilon \). Here \( x = 0.1, Q = -1.2, \beta_1 = 0.2, \beta_2 = 0.2, \phi = \pi/4, M = 4, \lambda_1 = 1 \).

| \( Br \) | 0.5 | 1   | 1.5  | 2    | 2.5  |
|---------|-----|-----|------|------|------|
| 0.0     | 1.67129 | 2.45597 | 3.24064 | 4.02532 | 4.80999 |
| 0.1     | 1.69610 | 2.45803 | 3.21107 | 3.99555 | 4.69048 |
| 0.2     | 1.72090 | 2.46010 | 3.18151 | 3.88513 | 4.57098 |

Table 5. \( Z \) (Heat transfer coefficient) as a function of \( \beta_1 \) and \( \epsilon \). Here \( x = 0.1, Q = -1.2, \beta_2 = 0.2, Br = 0.5, \phi = \pi/4, M = 4, \lambda_1 = 1 \).

| \( \epsilon \) | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|-----------------|-----|-----|-----|-----|-----|
| 0.0             | 1.70382 | 1.67129 | 1.65603 | 1.64778 | 1.64275 |
| 0.1             | 1.72792 | 1.69610 | 1.68124 | 1.67313 | 1.66822 |
| 0.2             | 1.75196 | 1.72090 | 1.70640 | 1.69848 | 1.69368 |

Table 6. \( Z \) (Heat transfer coefficient) as a function of \( \beta_2 \) and \( \epsilon \). Here \( x = 0.1, Q = -1.2, \beta_1 = 0.2, Br = 0.5, \phi = \pi/4, M = 4, \lambda_1 = 1 \).

| \( \epsilon \) | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|-----------------|-----|-----|-----|-----|-----|
| 0.0             | 1.73194 | 1.67129 | 1.61795 | 1.57066 | 1.52845 |
| 0.1             | 1.76865 | 1.69610 | 1.63254 | 1.57635 | 1.52629 |
| 0.2             | 1.80537 | 1.72090 | 1.64713 | 1.58205 | 1.52414 |
Figure 6. Effect on trapping by $\phi$ with $Q = 1.48$, $M = 1.2$, $\lambda_1 = 1$, (a); $\phi = 0$, (b); $\phi = \pi/6$, (c); $\phi = \pi/3$, (d); $\phi = \pi/2$.

Figure 7. Effect on trapping by $M$ with $Q = 1.48$, $\phi = \pi/4$, $\lambda_1 = 1$, (a); $M = 0$, (b); $M = 1$, (c); $M = 1.3$, (d); $M = 1.5$. 
Figure 8. Influence of $\lambda_1$ on trapping with $Q = 1.48$, $\phi = \pi/4$, $M = 1$ (a); $\lambda_1=0$, (b); $\lambda_1=2$, (c); $\lambda_1=3$, (d); $\lambda_1=4$.

Figure 9. Effect on trapping by $\lambda_1$ with $Q = 1.48$, $\phi = \pi/4$, $M = 1$, $\lambda_1 = 1$ (a); $\beta_1=0$, (b); $\beta_1=0.1$, (c); $\beta_1=0.2$, (d); $\beta_1=0.3$.,
6. Concluding remarks

The peristaltic flow of a hydromagnetic Jeffrey fluid under the influence of an inclined magnetic field and variable thermal conductivity with slip conditions has been analyzed. The influence of various parameters is discussed graphically. The main finding can be summarized as listed below.

1. The magnetic field for $\phi \in [0, \pi/2]$ shows opposite behavior when compared with $\phi \in [\pi/2, \pi]$.
2. For magnetic fluid, the value of axial pressure gradient is higher rather than Newtonian fluid.
3. An increase in $\phi$ and $M$ leads to an increase in $dp/dx$, whilst it decreases when $\lambda_1$ is increased.
4. The magnitude of $dp/dx$ show decreasing behavior when $\beta_1$ is increased.
5. The amplitude of velocity $u$ is a decreasing function of $\phi \in [0, \pi/2]$, $\beta_1$, $M$ and $\lambda_1$.
6. The temperature of the fluid $T$ falls down under the effect of $\phi \in [0, \pi/2]$; Contrarily, it decreases with an increase in conductivity parameter $\epsilon$.
7. Ascending values of $\lambda_1$ decreases the value of heat transfer coefficient $Z$, whilst, an increment in the values of $M$, $\epsilon$ and $Br$ leads to an increase in heat transfer coefficient $Z$.
8. The heat transfer coefficient decreases as we increase thermal slip parameters $\beta_2$ and velocity slip parameter $\beta_1$.
9. Size of the trapped bolus is greater for the inclined magnetic field as compared with the transverse magnetic field. Moreover, increasing $M$, $\beta_1$, and $\lambda_1$ decreases the size of the trapped bolus.

Appendix

Here we include the values of $D_i(i = 1, 2, \ldots, 11)$ and $R_i(i = 1, 2, \ldots, 26)$.

\begin{align*}
R_1 &= 8(h_1 + h_2)\sqrt{1 + \lambda_1}, \\
R_2 &= \frac{R_1}{2}(-2 - FM^2\beta_1(1 + \lambda_1) + FM^2\beta_1(1 + \lambda_1) \cos 2\phi)\cosh((h_1 - h_2)M\sqrt{1 + \lambda_1}\sin \phi), \\
R_3a &= M(1 + \lambda_1)\beta_1/2(h_1 + h_2)(-4\beta_1 - F(2 + M^2\beta_1^2)(1 + \lambda_1)), \\
R_3b &= FM^2\beta_1^2(1 + \lambda_1)\cos 2\phi \sinh(M(\sqrt{1 + \lambda_1}(h_1 - h_2)\sin \phi)), \\
R_4 &= -16\sqrt{1 + \lambda_1}, \\
R_5 &= 8\sqrt{1 + \lambda_1}(2 - M^2\beta_1(1 + \lambda_1) + (h_1 - h_2)M^2\beta_1(1 + \lambda_1)), \\
R_3b &= \cos 2\phi \cosh(M\sqrt{1 + \lambda_1}\sin \phi(h_1 - h_2)), \\
R_6a &= 8M(1 + \lambda_1)\sin \phi(h_1 - h_2 - 2\beta_1), \\
R_6b &= (h_1 - h_2)M^2\beta_1^2(1 + \lambda_1)\sin \phi^2 \sinh(M(\sqrt{1 + \lambda_1}\sin \phi(h_1 - h_2))), \\
R_7 &= \cosh(h_1M\sqrt{1 + \lambda_1}\sin \phi)\sin \phi, \\
R_8 &= FM\sqrt{1 + \lambda_1}\cosh(h_2M\sqrt{1 + \lambda_1}\sin \phi)FM\sqrt{1 + \lambda_1}\sin \phi, \\
R_9 &= (2 + FM^2\beta_1(1 + \lambda_1)\sin \phi^2), \\
R_{10} &= \sinh(\sqrt{1 + \lambda_1}h_1M\sin \phi) - \sinh(\sqrt{1 + \lambda_1}\sin \phi h_2M), \\
R_{11} &= \cosh(h_1M\sqrt{1 + \lambda_1}\sin \phi)\sin \phi, \\
R_{12} &= (h_1 - h_2)M\sqrt{1 + \lambda_1}\cosh(h_2M\sqrt{1 + \lambda_1}\sin \phi)(h_1 - h_2)M\sqrt{1 + \lambda_1}\sin \phi, \\
R_{13} &= -2 + (h_1 - h_2)M^2\beta_1(1 + \lambda_1)\sin \phi^2, \\
R_{14} &= -\sinh(\sin \phi h_1M\sqrt{1 + \lambda_1}) - \sinh(\sqrt{1 + \lambda_1}h_2M\sin \phi), \\
R_{15} &= \cosh(h_1M\sqrt{1 + \lambda_1}\sin \phi)(F + h_1 - h_2)\sqrt{1 + \lambda_1}, \\
R_{16} &= \cosh(h_2M\sqrt{1 + \lambda_1}\sin \phi)(F + h_1 - h_2), \\
R_{17} &= (F + h_1 - h_2)M\beta_1(1 + \lambda_1)\sin \phi \sinh(h_2M\sqrt{1 + \lambda_1}\sin \phi), \\
R_{18} &= (F + h_1 - h_2)M\beta_1(1 + \lambda_1)\sin \phi \sinh(h_2M\sqrt{1 + \lambda_1}\sin \phi), \\
R_{19} &= (h_1 - h_2)M^2\beta_1(1 + \lambda_3^2/2 + (1 + \lambda_1)\cos 2\phi), \\
R_{20} &= \cosh(\sqrt{1 + \lambda_1}\sin \phi(h_1 - h_2)M), \\
R_{21} &= (h_1 - h_2)\sin \phi(2\beta_1 + M^2\beta_1^2(1 + \lambda_1)\sin \phi^2)M(1 + \lambda_1), \\
R_{22} &= \sinh(h_1 - h_2)M\sqrt{1 + \lambda_1}\sin \phi, \\
R_{23} &= \sin \phi \beta_1 + \lambda_1 \cosh(h_1M\sqrt{1 + \lambda_1}\sin \phi)(F + h_1 - h_2)M(F + h_1 - h_2)M,
\end{align*}
\[ R_{24} = \sin \phi \cosh(h_2 M \sqrt{1 + \lambda_1 \sin \phi})(F + h_1 - h_2), \]
\[ R_{25} = \sqrt{(1 + \lambda_1) \sinh(h_1 M \sqrt{1 + \lambda_1 \sin \phi})(F + h_1 - h_2)}, \]
\[ R_{26} = \sinh(h_2 M \sqrt{1 + \lambda_1 \sin \phi}) \sqrt{(1 + \lambda_1)(F + h_1 - h_2)}, \]
\[ D_1 = \frac{R_1 + R_2 + R_{3a} + R_{3b}}{R_4 + R_{5a} + R_{5b} - R_{6a} - R_{6b}}, \]
\[ D_2 = \frac{R_7 + R_8 + R_9 R_{10}}{R_{11} + R_{12} + R_{13} R_{14}}, \]
\[ D_3 = \frac{R_{15} - R_{16} + R_{17} + R_{18}}{- (R_{19} R_{20} - R_{23} R_{22})}, \]
\[ D_4 = \frac{R_{23} + R_{24} + R_{25} - R_{26}}{(R_{19} R_{20} - R_{21} R_{22})}, \]
\[ D_5 = 2(F + h_1 - h_2) M^3 \cosh((h_1 - h_2) M \sqrt{1 + \lambda_1 \sin \phi}) \sin \phi^3, \]
\[ D_6 = \sqrt{1 + \lambda_1 \cosh(\frac{1}{2}(h_1 - h_2) M \sqrt{1 + \lambda_1 \sin \phi}), \]
\[ D_7 = M \beta_1 + \lambda_1 \sin \phi \sinh(\frac{1}{2}(h_1 - h_2) M \sqrt{1 + \lambda_1 \sin \phi}), \]
\[ D_8 = \frac{-1 + f_0(x, h_2) + f_0(x, h_1) + \beta_2 (f_0(x, h_2) + f_0(x, h_1))}{2 \beta_2 + h_1 + h_2}, \]
\[ D_9 = \frac{\beta_2 + h_1 + f_0(x, h_1) (\beta_2 + h_2) + f_0(x, h_2) (\beta_2 + h_1) - \beta_2 (\beta_2 + h_1) (f_0(x, h_2))}{2 \beta_2 + h_1 + h_2}, \]
\[ + \frac{f_0(x, h_1) (2 \beta_2^2 + \beta_2 h_1 + \beta_2 h_2 - \beta_2 - h_1)}{2 \beta_2 + h_1 + h_2}, \]
\[ D_{10} = \frac{-f_1(x, h_2) + f_1(x, h_1) + \beta_2 (f_1(x, h_2) + f_1(x, h_1))}{2 \beta_2 + h_1 - h_2}, \]
\[ D_{11} = \frac{-f_1(x, h_1) (-1 + 2 \beta_2 + h_1 - h_2) + f_1(x, h_2) + (f_1(x, h_2))}{2 \beta_2 + h_1 - h_2}, \]
\[ - \frac{f_1(x, h_1) (\beta_2 h_1 - \beta_2 h_2 + 2 \beta_2^2 + \beta_2) + f_1(x, h_2)}{2 \beta_2 + h_1 - h_2}. \]

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## Nomenclature

| Symbol | Description |
|--------|-------------|
| $\bar{X}, \bar{Y}$ | Dimensional form of $X$ coordinate and $Y$ coordinate in laboratory frame |
| $\bar{u}, \bar{v}$ | Dimensional form of velocity components in wave frame |
| $\bar{x}, \bar{y}$ | Dimensional form of $x$ coordinate and $y$ coordinate in wave frame |
| $u, v$ | Dimensionless form of velocity components in wave frame |
| $\bar{H}_1, \bar{H}_2$ | Wall structures dimensional form in laboratory frame |
| $h_1, h_2$ | Wall structures dimensionless form in wave frame |
| $\theta_1, \theta_2$ | Wave amplitudes |
| $s_1, s_2$ | Ratios of Amplitude |
| $c$ | Wave speed |
| $\ell_1, \ell_2$ | Channel widths |
| $\gamma$ | Phase difference |
| $\eta$ | Ratios of channel widths |
| $\lambda$ | Wavelength |
| $\phi$ | Inclination angle of magnetic field |
| $\bar{t}$ | Dimensional form of time |
| $t$ | Dimensionless form of time |
| $\bar{P}$ | Dimensional form of Pressure |
| $p$ | Dimensionless pressure |
| $\Delta P_\lambda$ | Pressure drop per wavelength |
| $\psi$ | Stream function |
| $Q$ | Rate of flow of volume |
| $F$ | Instantaneous rate of flow of volume |
| $\bar{\sigma}$ | Extra stress tensor of Jeffrey fluid |
| $\mathbf{I}$ | Identity tensor |
| $\mathbf{J}$ | Current density |
| $\mathbf{B}_0$ | Magnetic field |
| $M$ | Hartman number |
| $\text{Br}$ | Brinkman number |
| $\text{Re}$ | Reynolds number |
| $\rho$ | Fluid density |
| $\mu$ | Fluid viscosity |
| $\lambda_1$ | Jeffrey fluid parameter |
| $\lambda_2$ | Retardation time |
| $\beta_1$ | Velocity slip parameter |
| $\beta_2$ | Thermal slip parameter |

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