Diagnostics of Accelerated Electrons Anisotropy from Solar Hard X-rays

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Abstract. In most acceleration models only the energy distribution of the particles can be obtained, while the angular part of the distribution remains still unclear. The article represents the possibility of detecting the electron distribution anisotropy. The Hard X-ray spectrums, polarization degree and directivity for different models of anisotropy are calculated. The calculations claimed the possibility of determining the longitudinal-transverse anisotropy of electrons.

1. Introduction

Modern theories of solar flares suggest a different spatial location of the particle acceleration region and injection region into magnetic loop plasma. If take into account that distribution function is being transformed when electrons pass from acceleration to emission region, the task of energy and angular-dependencies segregation in situ is complicating. The inability of direct measurements of accelerated electrons requires a thorough analysis of the electromagnetic radiation from these electrons. The electron spectrum can be inferred from observable photon flux, solving a remote-sensing problem of flaring plasma. Because hard X-rays (HXR) are slightly affected by the propagation effects, and sources could be considered optically thin, it is possible to study the properties of their origin. Algorithms, including Tikhonov regularizations, and forward fitting using a parametric form consisting of a double power law with low/high cutoffs plus an isothermal component allow us to determine the energy dependence of the electron distribution function, while the pitch-angular dependence is still not clear [1, 2, 3, 4]. Nevertheless X-ray and microwave observations confirm the presence of accelerated electrons anisotropic distributions [5, 6]. The observation of bright coronal sources at the top of magnetic loops [7] assumes a quasi-transverse electron distribution, while particles with a quasi-longitudinal distribution pass through a loss cone into dense plasma of the footpoints of flaring loops, emitting X-rays. For HXR sources located in loop footpoints the non-thermal electron distribution function is considered isotropic due to intense Coulomb collisions in a dense plasma, or quasi-longitudinal in a rarefied plasma. It is well known that anisotropic distributions of non-thermal electrons produce anisotropic and polarized bremsstrahlung [8, 9]. The degree of X-ray polarization is usually a complex function of the strength and topology of the magnetic field. In addition, for such cases, the observed polarization degree is related to the photon directivity, which depends both on the electron beaming features and on the viewing angle. The highest polarization could
be produced at the loop top (up to 30-50%), while photons observed from the footpoints (in
the region of the dense chromosphere) would be polarized to the level of less than 10% [10].
The most favorable for the analysis of the X-ray polarization degree are powerful GOES X-class
flares. Currently, several X-ray polarization measurements are known with an accuracy of about
10% for high-power X-class flares [11]. In general, polarization can only indicate the presence
or absence of anisotropy in the distribution of electrons. The distribution of the flash along
the solar disk [12] is generally ambiguous, though is able to provide qualitative information
in relation to the angular distribution of HXR which is associated with electron distribution.
The purpose of the present study is to evaluate the influence of the various model pitch-angle
distributions on the slope of integrated HXR spectrum, HXR polarization and directivity.

2. Bremsstrahlung and hard X-ray energy spectra

The hard X-rays of solar flares are formed mainly by the bremsstrahlung of high-energy electrons
scattered on protons in a dense hydrogen plasma. HXR flux on the line of sight from source
with the mean plasma density \( \bar{n} \) and volume \( V \) at the distance \( R \) has the following form:

\[
I(\epsilon, \Omega) = \frac{\bar{n}V}{4\pi R^2} \int_\epsilon^\infty \int_{\Omega} F(E, \Omega) \frac{d^2\sigma}{d\epsilon d\Omega}(\epsilon, E, \theta') dE d\Omega \quad \text{[phot s}^{-1}\text{cm}^{-2}\text{keV}^{-1}] \tag{1}
\]

where \( F(E, \Omega) \) is the electron distribution function, \( \frac{d^2\sigma}{d\epsilon d\Omega}(\epsilon, E, \theta') \) is the relativistic
bremsstrahlung cross-section, given by [9], and \( \theta' \) is the angle between the photon momentum
\( \mathbf{k} \) (emitted along the line of sight) and the momentum vector of the scattered electron. In order
to take into account angular and energy dependence separately, we consider that the electron
distribution function could be factorized: \( F(E, \Omega) = N(E)g(\theta, \phi) \). Here \( \theta \) is the pitch angle of
the electron, and \( \phi \) is the azimuthal angle in the system where magnetic line is taken as Z-axis
(generally as the normal to the photosphere). Owing to azimuthal symmetry along the magnetic
lines it is assumed that \( g(\theta, \phi) = g(\theta) \). We represent the electron energy spectrum in the form of
the power-law: \( N(E) = kE^{-\delta} \). Thus bremsstrahlung photon flux (in phot s}^{-1}\text{keV}^{-1}\text{ster}^{-1}\text{cm}^{-3}\) is given by [8]:

\[
Q(\alpha, \epsilon) = \pi k \int_\epsilon^\infty dE \int_{-1}^{+1} d(\cos \theta) \int_0^{2\pi} d\phi g(\theta) E^{-\delta} \frac{d^2\sigma}{d\epsilon d\Omega}(\epsilon, E, \theta') v(E) \tag{2}
\]

Here \( v(E) \) is the relativistic electron velocity. Angle \( \theta' \) between electron and photon momentum
can be expressed from the observation angle \( \alpha \) (between line of sight and magnetic field line),
and polar coordinates \( \theta, \phi \):

\[
\cos \theta' = \cos \alpha \cos \theta + \sin \alpha \sin \theta \cos \phi. \tag{3}
\]

Electron-nucleus bremsstrahlung, given by [6] is significantly anisotropic. The angle
corresponding to the maximum cross-section is changing with increasing photon energy (\( \theta' = 0^\circ \)
for \( \epsilon < 50 \text{ keV} \), and \( \theta' \approx 40^\circ \) for \( \epsilon > 50 \text{ keV} \)), and the degree of anisotropy is growing with
increasing electron energy. Hence energetic electron populations are more sensitive to their
angular distributions, it will affect the parameters of HXR radiation. In current study we
examine various electron angular distributions:

\[
g(\theta) = \begin{cases} 
1, & \text{isotropic case} \\
\sin^8(\theta), & \text{quasi-transversal case} \\
\cos^8(\theta), & \text{quasi-longitudinal case} \\
\exp\left(-\frac{\theta^2}{\Delta \theta^2}\right), & \text{normal distribution} 
\end{cases} \tag{4}
\]
Figure 1. Hard X-ray flux \( Q \) logarithm (measured in photons \( s^{-1} \) keV\(^{-1} \) ster\(^{-1} \) cm\(^{-3} \)) for different initial electron spectra with respect to different observation angles.

HXR spectrum with respect to slope \( \delta \) of initial electron spectrum and observation angles \( \alpha \) is shown in figure 1. Different observation angles correspond to flare position on solar disc: \( \alpha = 0 \ldots 90 \), varied from the center to limb for the footpoint sources and vice versa for the coronal ones. It is also necessary to take into account the spatial orientation and tilt angle of the flaring magnetic loop. HXR power-law slopes \( \gamma \) for each \( g(\theta) \) were acquired by fitting the spectrum in \( 20 - 100 \) keV energy range. Due to power-law electron energy spectrum, the contribution of high energy electrons (\( E \geq 20 \) MeV) to HXR spectrum in energy range up to 150 keV can be neglected. From figure 1 it follows that the most distinct slopes \( \gamma \) with a difference \( \Delta \gamma \approx 1 \) take place for the hard electron spectrum \( \delta = 2, 3 \). In this case variations from quasi-transverse to quasi-longitudinal angular distributions become observable, though in
Figure 2. Hard X-ray polarization degree via observation angles for angular distributions (4), different quantum energy and spectrum exponents.

Flare events steeper ones are more common. Analysis of observational spectroscopic data from RHESSI using OSPEX subroutines [13, 14] usually determines the slope of HXR spectrum with accuracy $\Delta \gamma = 0.2 \ldots 0.5$. Therefore, the variations of photon spectrum power-law exponents obtained for different angular distributions for $\gamma > 5$ are in fact indistinguishable. Also figure 1 shows the steepening of the photon spectrum with the growth of the observational angle for quasi-longitudinal anisotropy and flattening for quasi-transverse case.

3. Polarization and Directivity of Hard X-Rays

X-ray linear polarization and directivity explicitly demonstrate the angular distribution of the initial electron beam. The polarization degree was calculated according to [8, 15], for cross-section components parallel and perpendicular to the plane spanned by magnetic field and photon momentum vectors (plane of emission):

$$P(\epsilon, \alpha) = \cos 2\psi \frac{Q_\perp - Q_\parallel}{Q} = \frac{1}{Q(\epsilon, \alpha)} \int_{\epsilon}^{\infty} dE \int_{\Omega} d\Omega \cos 2\psi \left( \frac{d^2 \sigma_\perp}{d\epsilon d\Omega} - \frac{d^2 \sigma_\parallel}{d\epsilon d\Omega} \right) g(\theta) KE^{-\delta} v(E)$$

(5)

where $\psi$ is the angle between plane of emission and plane spanned by electron and photon momentum vectors defined by: $\cos \psi = (\cos \alpha \sin \theta \cos \phi - \sin \alpha \cos \theta) / \sin \theta'$. Results are shown
in figure 2. For the isotropic electron distribution polarization degree does not exceed 1%. Let
us discuss the $\delta = 3$ case. For quasi-transverse angular distribution the polarization degree
is positive and reaches about 15% at 100 keV at $\alpha = 90^\circ$. For quasi-longitudinal angular
distribution the polarization degree is negative and reaches value less than -25% at 100 keV at
$\alpha = 90^\circ$. Figure 2 shows the higher photon energy, the less polarization degree for any angle
distributions. In the right column the hard X-ray polarization degree is drawn for $\delta = 5$ case.
As it was previously discussed, the difference in the spectral exponents for all models of 4 is
smaller for the $\delta = 5$, the opposite is true for the polarization degree. The absolute values of
polarization degree are rather higher: about +25% for quasi-transverse angular distributions
and about -50% for quasi-longitudinal angular distributions at 100 keV at $\alpha = 90^\circ$. There are
specific features of the values and sign for these models. In case the observation angle tends to
$\alpha = 0^\circ$ or $\alpha = 180^\circ$ the polarization degree tends to 0. Therefore the limb flares are favorable
to study X-ray polarization degree. Despite the coincidence in photon spectrum, symmetrical
g($\theta) = \cos^8(\theta)$ and asymmetrical \exp(-$\theta^2/\Delta\theta^2$) angular distributions can be distinguished using
polarization measurements.

Directivity is defined as photon flux at the direction $\alpha$, normalized on flux, averaged of all
observation angles: $D(\epsilon, \alpha) = Q(\epsilon, \alpha)/\langle Q(\epsilon) \rangle$. The directivity of HXR reflects the electron
angular distribution as shown in fig. 3. For quasi-transverse angular distribution the peak of

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Hard X-ray directivity versus observation angles for angular distributions (4),
different photon energies and electron spectrum exponents.}
\end{figure}
directivity corresponds to $\alpha = 90^\circ$, while for quasi-longitudinal case the peak of directivity corresponds to $\alpha = 0^\circ$ and $\alpha = 180^\circ$. The directivity in these cases is weakly dependent on the spectral exponent $\delta$ (see the columns in the middle and in the right). For isotropic distribution the HXR directivity is close to 1 practically, but cross-section anisotropy is responsible for weak variations.

4. Summary
There are several ways to determine the anisotropy of the accelerated electrons distribution. Firstly, the variations of photon spectrum power-law exponents obtained for different angular distributions (4) for exponent $\gamma < 5$ can be measured. Next, the experiments on measurements of HXR polarization degree are promising for these goals. HXR directivity also results in anisotropy of accelerated electrons. Besides the measurements, the theory and modeling of the particle acceleration and transport in a magnetically active plasma should be put forward. Thus, the combination of spectral and polarization observations with a numerical calculation of accelerated electrons transport for given magnetic field and plasma density distributions can provide an answer on the angular distribution of the electrons both at the moment of radiation and at the moment of injection into the magnetic loop during a flare.

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