A note on Boltzmann brains

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ABSTRACT

Understanding the observed arrow of time is equivalent, under general assumptions, to explaining why Boltzmann brains do not overwhelm ordinary observers. It is usually thought that this provides a condition on the decay rate of every cosmologically accessible de Sitter vacuum, and that this condition is determined by the production rate of Boltzmann brains calculated using semiclassical theory built on each such vacuum. We argue, based on a recently developed picture of microscopic quantum gravitational degrees of freedom, that this thinking needs to be modified. In particular, depending on the structure of the fundamental theory, the decay rate of a de Sitter vacuum may not have to satisfy any condition except for the one imposed by the Poincaré recurrence. The framework discussed here also addresses the question of whether a Minkowski vacuum may produce Boltzmann brains.

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1. Introduction

At first sight, the fact that we observe that time flows only in one direction may seem mysterious, given that the fundamental laws of physics are invariant under reversing the orientation of time. 

Upon careful consideration, however, one notices that the problem is not the unidirectional nature per se. As discussed in Refs. [1,2], given any final state $|f⟩$ whose coarse-grained entropy is lower than the initial state $|i⟩$, the evolution history is overwhelmingly dominated by the $\text{CPT}$ conjugate of the standard (entropy increasing) process $|f⟩ → |i⟩$. This implies that a physical observer, who is necessarily a part of the whole system, sees virtually always, i.e. with an overwhelmingly high probability, that time flows from the “past” (in which correlations of the observer with the rest of the system are smaller) to the “future” (in which the correlations are larger).

The problem of the arrow of time, therefore, is not to understand its unidirectional nature, but to explain why physical predictions are (probabilistically) dominated by what we observe in our universe, i.e. a flow from a very low coarse-grained entropy state to a slightly higher entropy state. In particular, it requires the understanding of the following facts:

- At least one set of states representing our observations, which are mutually related by time evolution spanning the observation time, are realized in the quantum state representing the whole universe/multiverse. (Here and below we adopt the Schrödinger picture.) This is the case despite the fact that these states have very low coarse-grained entropies.
- The answer to a physical question, which may always be asked in the form of a conditional probability [6], must be determined by the class of low coarse-grained entropy states described above. In particular, the probability should not (always) be dominated by the states in which the unconditioned part of the system has the highest coarse-grained entropies.

Because of the Hamiltonian constraint, the full universe/multiverse state is expected to be static, i.e. not to depend on any time parameter [3,4]. We may, however, talk about effective time evolution if we focus on branches of the whole universe/multiverse state, since they are not (necessarily) invariant under the action of the time evolution operator $e^{-iHt}$. This is the picture we adopt in this paper. Note that this time evolution still does not have to be the same as "physical time evolution" defined through correlations among physical subsystems, e.g., as in Ref. [5]. In the static-state picture, the statement here is phrased such that the state of the universe/multiverse contains components representing our observations despite the fact that they are not generic in the relevant Hilbert space.

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These elements comprise (essentially) the well-known Boltzmann brain problem [78]. The problem of the arrow of time is thus equivalent to the Boltzmann brain problem [4] under (rather general) assumptions that went into the line of argument given above.

Any realistic cosmology must accommodate the two facts listed above. Is it trivial to do so? In a seminal paper [7], Dyson, Kleban, and Susskind pointed out that it is not. In particular, they considered a de Sitter vacuum representing our own universe and argued that if it lives long enough, thermal fluctuations in de Sitter space lead to Boltzmann brains observing chaotic worlds, who overwhelm ordinary, ordered observers like us. If true, this would give an upper bound on the lifetime of our universe which is much stronger than that needed to avoid the Poincaré recurrence (barring the possibility that the observed vacuum energy relaxes into a zero or negative value in the future). In this paper we argue that this consideration needs to be modified, based on the picture of the microscopic structure of quantum gravity advanced recently [9] to address the black hole information problem [10,11]. We discuss implications of this modification for our own universe and the eternally inflating multiverse. We also discuss implications of the framework for the possibility [12] of Boltzmann brain production in a Minkowski vacuum.

2. de Sitter space in quantum gravity

We first extend the discussion of Ref. [9], which mainly focused on a system with a black hole, to de Sitter space. In cosmology, de Sitter space appears as a meta-stable state in the middle of the evolution of the universe/multiverse, and in this sense it is similar to a spacetime with a dynamically formed black hole. Indeed, string theory suggests that there is no absolutely stable de Sitter vacuum in full quantum gravity; it must decay, at least, before the Poincaré recurrence time [13]. This implies that what we call de Sitter space cannot be an eigenstate of energy (at least in this context).

Consider a semiclassical de Sitter space with Hubble radius \( \alpha \). (We focus on 4-dimensional spacetime for simplicity, but the extension to other dimensions is straightforward.) Following the complementarity hypothesis [14], and in particular its implementation in Refs. [2,6], we adopt a “local description,” in which quantum states represent physical configurations on equal-time hypersurfaces foliating the causal patch associated with a freely falling frame. We assume that the timescale for the evolution of microstates representing the de Sitter space is of order \( \Delta t \approx \alpha \), where \( t \) is the proper time measured at the spatial origin, \( \rho_0 \), of the reference frame. The uncertainty principle then implies that a state representing this space must involve a superposition of energy eigenstates with a spread of order \( \Delta E \approx 1/\alpha \). Associating this energy with the vacuum energy density \( \rho_{\text{vac}} \), integrated over the Hubble volume, \( E \approx O(\rho_{\text{vac}} \Delta t^2) \approx O(\alpha/\ell_P^2) \), this spread is translated into \( \Delta \alpha \approx O(\ell_P^2/\alpha) \), where \( \ell_P \) is the Planck length.

How many different independent ways are there to superpose the energy eigenstates to arrive at the semiclassical de Sitter space described above? As in the black hole case, we assume that the Gibbons–Hawking entropy [15]

\[
S_{\text{GH}} = \frac{A}{4\ell_P^2} = \frac{\pi \alpha^2}{\ell_P^2},
\]

gives the logarithm of this number (at the leading order in expansion in inverse powers of \( A/\ell_P^2 \), where \( A = 4\pi \alpha^2 \) is the area of the de Sitter horizon [16]. In particular, there are exponentially many independent de Sitter vacuum states—the states that do not have a field or string theoretic excitation in the semiclassical background—which all represent the same de Sitter vacuum at the semiclassical level.

The analysis of physics in this de Sitter vacuum is parallel to that on a black hole background in Ref. [9]. Denoting the index representing the exponentially many de Sitter vacuum states by \( k = 1, \ldots, e^{\tilde{h}} \),

\[
|S_0 - S_{\text{GH}}| \approx O(\mathcal{A}^q/\ell_P^q, q < 1),
\]

states at late times on this vacuum can be expanded in terms of the microstates of the form \( |\Psi_{\text{vac}}(\alpha)\rangle \).

Here, \( \alpha \) and \( a \) label excitations of the stretched horizon, located at \( r = \alpha - O(\ell_P^2/\alpha) = \alpha_s \), and the interior region, \( r < \alpha_s \), respectively, where \( r \) is the static radial coordinate with \( r = 0 \) taken at \( \rho_0 \). Note that excitations here are defined as fluctuations with respect to a fixed background, so their energies as well as entropies can be either positive or negative, although their signs must be the same. The contribution of the excitations to the entropy is subdominant in the \( \ell_P^2/A \) expansion, so that the total entropy of this de Sitter system (not necessarily of the vacuum states) is still given by \( S = A = 4\ell_P^2 \) at the leading order.

The indices for the excitations, \( \tilde{a} \) and \( a \), and the vacuum, \( k \), do not fully “decouple”. In particular, operators in the semiclassical theory representing modes whose energies defined at \( r = 0 \) are \( \omega \lesssim T_{\text{GH}} \),

\[
\omega \lesssim T_{\text{GH}},
\]

act nontrivially on both \( a \) and \( k \) indices, where \( T_{\text{GH}} = 1/2\pi\alpha \) is the Gibbons–Hawking temperature. This allows us to understand the thermal nature of the semiclassical de Sitter space in the following manner. The fact that all the independent microstates with different \( k \) lead to the same geometry (within the quantum mechanical uncertainty) suggests that the semiclassical picture is obtained after coarse-graining the degrees of freedom represented by this index, which we call the vacuum degrees of freedom. In this picture, the de Sitter vacuum in the semiclassical description is represented by the density matrix

\[
\rho_0(\alpha) = \frac{1}{e^{\tilde{h}}} \sum_{k=1}^{e^{\tilde{h}}} |\Psi_{\text{vac}}(a_k, \alpha); \Psi_{\text{vac}}(\alpha)\rangle.
\]

To obtain the response of this state to the operators in the semiclassical theory, we may trace out the subsystem \( \hat{C} \) on which they do not act. Consistently with our identification of the origin of the Gibbons–Hawking entropy, we identify the resulting reduced density matrix as the thermal density matrix

\[
\tilde{\rho}_0(\alpha) = \text{Tr}_{\hat{C}} \rho(\alpha) \approx \frac{1}{Z} e^{-\beta H_{\text{vac}}/\hbar},
\]

where \( Z = \text{Tr} e^{-\beta H_{\text{vac}}/\hbar} \), and \( H_{\text{vac}}(\alpha) \) is the Hamiltonian of the semiclassical theory.

Another manifestation of the non-decoupling nature of the \( a \) and \( k \) indices is that for states having a negative energy excitation, the range over which \( k \) runs is smaller than that in Eq. (2)—this is the meaning that a negative energy excitation carries a negative entropy. As discussed in the next section, this fact is important in ensuring unitarity in the process in which a physical detector held at constant \( r \) is excited due to interactions with the de Sitter spacetime.

3. Vacuum degrees of freedom

The expression in Eq. (6) implies that the spatial distribution of the information in \( k \) follows the thermal entropy calculated using the local temperature
Namely, the vacuum degrees of freedom can be viewed (in a given quasi-static reference frame) as being distributed according to the thermal entropy calculated using \( T(r) \) in the semiclassical theory. Since the thermal nature is the crucial element in the argument of Boltzmann brains, we must ask: what is the dynamics of the vacuum degrees of freedom?

In Ref. \[9\], I have argued, together with Sanches and Weinberg, that the thermal nature of semiclassical theory should not be taken “too literally.” Specifically, it does not mean that the degrees of freedom described within the semiclassical theory are actually in thermal equilibrium with local temperature \( T(r) \). Rather, the thermal nature implies that the vacuum degrees of freedom—which are already coarse-grained to obtain the semiclassical theory—interact with the excitations in the semiclassical theory—e.g. a detector located in de Sitter space—as if these excitations are immersed in the thermal bath of temperature \( T(r) \). In particular, the dynamics of the vacuum degrees of freedom themselves cannot be described by the semiclassical Hamiltonian \( H_{\text{sc}}(\alpha) \). These degrees of freedom are neither matter nor spacetime; they are “some stuff” that reveal either feature of matter or spacetime depending on the question one asks—the phenomenon we referred to as \textit{spacetime-matter duality}.^3

To elucidate this point further, let us consider a physical detector held at some constant \( r = r_d \). (For \( r_d \neq 0 \), this is an accelerating detector.) The detector then responds as if it is immersed in the thermal bath of temperature \( T(r_d) \), and correspondingly extracts information from the vacuum degrees of freedom \( k \). It need not, and in fact does not, imply that the semiclassical degrees of freedom are actually in thermal equilibrium with the temperature in Eq. (7). Due to energy conservation, this response is accompanied by a creation of a negative energy excitation, which propagates (or free-falls) toward larger \( r \) and collides with the stretched horizon. The background spacetime will then eventually relax into the one whose horizon area is (slightly) smaller, reflecting the existence of the detector with a higher energy. This whole process is depicted schematically in Fig. 1. Note that because the negative energy excitation has a negative entropy, each step in the process can be separately unitary. For more details, see the discussion on the analogous process of black hole mining in Ref. \[9\].

The second process we consider is the Hawking–Moss transition \[17\] from a de Sitter vacuum to another vacuum. As discussed, e.g., in Ref. \[18\], this transition can be viewed as a thermal process occurring through a field climbing up the potential barrier separating the two vacua. In our picture, this interpretation becomes valid only in the context of a theory built on the daughter vacuum to which the original vacuum decays. In particular, the existence of the transition does not imply that the semiclassical degrees of freedom built on the original de Sitter vacuum were actually in thermal equilibrium with Eq. (7) before the transition.

The two examples above illustrate that the thermal nature of spacetime acquires a clear semiclassical interpretation only in the context of the vacuum degrees of freedom interacting with other degrees of freedom, either semiclassical excitations (e.g. when a physical detector exists) or another system beyond the one built on the vacuum degrees of freedom themselves (e.g. when the vacuum decays). In the case of a black hole, the former corresponds to the situation in black hole mining, while the latter to spontaneous

\[
T(r) = \frac{1}{2\pi a^2} \frac{1}{\sqrt{\frac{1}{2} - \left(\frac{r}{r_d}\right)^2}}.
\]

Fig. 1. A schematic depiction of the process in which a detector located in de Sitter space interacts with the spacetime: time flows from the top to the bottom. The detector initially in some state (unfilled dot in the top panel) will react to the local Gibbons–Hawking temperature (filled dot in the middle panel). This is accompanied by the creation of a negative energy excitation, which has a negative entropy and propagates to the stretched horizon (dashed arrow in the middle panel). The background system then eventually relaxes into a new space whose horizon area is smaller than the original one (the bottom panel).

Hawking emission in which the far exterior region (outside the “zone”) serves as the other system \[9\]. We note that the picture described here does not affect the standard calculation of density fluctuations in an inflationary universe \[19\], since these fluctuations are interpreted in a late-time universe with much smaller vacuum energy (analogous to the case of a vacuum decay).

4. Boltzmann brains

Consider a de Sitter vacuum \( J \) with Hubble radius \( a_J \). What are the conditions that its decay rate \( \Gamma_J \) must satisfy? If the fundamental theory does not have a stable de Sitter vacuum, as suggested by string theory, then the vacuum must decay before the Poincaré recurrence time \( t_{\text{rec.} J} \):

\[
\Gamma_J \gtrsim \frac{1}{t_{\text{rec.} J}}.
\]

Since the Gibbons–Hawking entropy is obtained under the (implicit) assumption that the dynamics of the degrees of freedom it represents takes a local form, it is appropriate to use for \( t_{\text{rec.} J} \) the expression for the classical Poincaré recurrence time

\[
t_{\text{rec.} J} \sim a_J \epsilon_{\text{GH}} J,
\]

where \( S_{\text{GH}} J = \pi a_J^2 / l_p^2 \). Note that this does not necessarily mean that the dynamics of the vacuum degrees of freedom is local in the original de Sitter space. It only implies that, assuming the vacuum degrees of freedom indeed comprise \( O(S_{\text{GH}} J) \) quantum degrees of freedom, their dynamics can be organized to take a local form in

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^3 This situation reminds us of wave–particle duality, which played an important role in early days in the development of quantum mechanics—a quantum object exhibited dual properties of waves and particles, while the “true” (quantum) description did not fundamentally rely on either of these classical concepts.
some (“holographic”) space, i.e., the interaction Hamiltonian takes a special nearest-neighbor form in this space.\footnote{If the dynamics of \(O(\Sigma_{2d})\) degrees of freedom took a “fully ergodic” form in the sense that a generic initial state probes, as time passes, the entire Hilbert space without any “classicalization,” then \(t_{\text{rec,J}}\) would be given by the quantum Poincaré recurrence time, \(\alpha_J e^{\gamma J} t_{\text{rec,J}}\).}

Another requirement for \(J\) is that Boltzmann brains do not overwhelm ordinary observers in this vacuum, \(J\). This leads to the condition \([4,20]\)

\[
\Gamma_J \gtrsim \frac{n_J}{\Gamma_{BB,J}},
\]

(10)

where \(n_J\) and \(\Gamma_{BB,J}\) are, respectively, the number of ordinary observers and the rate of producing Boltzmann brains, both counted with a common rule in the spacetime region causally accessible from \(p_0\). Given how the universe enters into \(J\), \(n_J\) can be computed (in principle) using semiclassical theory built on \(J\). The question is: how to calculate, or estimate, \(\Gamma_{BB,J}\)?

Traditionally, \(\Gamma_{BB,J}\) has been estimated using the semiclassical theory built on \(J\) with the assumption that the degrees of freedom in the theory are in thermal equilibrium with the Gibbons–Hawking temperature \([7,20–22]\). In our picture, however, it is the internal dynamics of the vacuum degrees of freedom that is relevant for the production of Boltzmann brains, which—as we have argued in Section 3—cannot be captured by the semiclassical Hamiltonian \(H_{\text{sc,J}}\).\footnote{We assume that the process of “consciousness” needed to characterize Boltzmann brains, as well as ordinary observers, is defined by a set of states spanning the time for the process (not just by an instantaneous state), and thus depends on the Hamiltonian generating the time evolution.} In fact, we may expect that this dynamics is very different from that given by \(H_{\text{sc,J}}\). (Note that if the two were identical, it would reintroduce the firewall problem of Ref. [11].) In particular, we know that \(n_J\) is nonvanishing in the vacuum representing our universe, but this does not mean that the internal dynamics of the corresponding vacuum degrees of freedom (which we do not know yet) must produce intellectual observers. If this dynamics does not support any intellectual observer, then the timescale for Boltzmann brain production need not be much shorter than the Poincaré recurrence time, i.e., the timescale in which the vacuum spontaneously creates semiclassical excitations with significant probabilities. The decay rate of our universe then need not be much larger than \(1/t_{\text{rec,J}}\), where \(t_{\text{rec,J}}\) is given by Eq. (9).

Now, suppose that the fundamental theory has a multitude of vacua, as suggested by string theory, and that it leads to the eternally inflating multiverse. Under rather general assumptions about the dynamics of the multiverse, the conditions for avoiding Boltzmann brain dominance can be written as

\[
\Gamma_J \gtrsim \Gamma_{BB,J},
\]

(11)

for all de Sitter vacua in the theory \([22]\). (The factors \(n_J\) do not play a significant role if the probability of producing ordinary observers in our universe is not double-exponentially suppressed, which seems to be the case.) In the traditional picture, this imposes a strong constraint on the decay rate of any de Sitter vacuum \(J\) supporting intelligent observers. In particular, it gives constraints on the decay rates of all the vacua that are similar to our own vacuum, which are much stronger than the ones needed to avoid the Poincaré recurrence. While there is a suggestion that these strong constraints may indeed be satisfied in (at least, a particular corner of) the string landscape \([21]\), one might feel disconcerting that such a fundamental property as the fact that we can comprehend the world relies on “accidental,” numerical features of the theory.

Our picture offers a much simpler possibility: the dynamics of the vacuum degrees of freedom may simply not support any intelligent observers. If this is the case, then

\[
\Gamma_{BB,J} \left\{ \begin{array}{ll}
\sim \frac{1}{\alpha_J} e^{-\gamma J} & \text{if the semiclassical theory in } J \text{ supports observers,} \\
= 0 & \text{otherwise,}
\end{array} \right.
\]

(12)

and the conditions in Eq. (11) can be easily satisfied under the assumption in Eqs. (8), (9). We may say that “spacetime cannot think.”

We finally note that while some features appearing in the present framework look similar to those discussed in Ref. [23], the underlying physical pictures are different, so that the physical implications of the two are also different. In the present picture, a semiclassical de Sitter vacuum is not an exact energy eigenstate and is subject to a nontrivial dynamics at the microscopic level (at least in cosmological contexts). The rate of Boltzmann brain production in such a vacuum then depends crucially on the (unknown) microscopic dynamics of quantum gravity. This issue was not discussed in Ref. [23].

5. Summary and discussion

Assuming that the concept of consciousness is defined physically (and that quantum mechanics provides a correct description of nature at the fundamental level), the fact that we observe an ordered, comprehensible world implies special structures of quantum operators characterizing our observations, which act on a Hilbert space in which the state representing the universe/multiverse lives. In particular, in the standard time-evolution picture, which arises from focusing on branches in the full universe/multiverse state, the decay rate of any de Sitter vacuum that is cosmologically populated must be larger than the production rate of Boltzmann brains, Eq. (11). Under certain weak assumptions on the dynamics of the multiverse, satisfying this condition is equivalent to explaining the origin of the arrow of time we observe in nature.

In this paper we have argued that, in contrast with the traditional view, the rate of Boltzmann brain production in a de Sitter vacuum cannot be calculated using the semiclassical theory built on this vacuum. It is determined, instead, by the (unknown) microscopic dynamics of the vacuum degrees of freedom, which is not the same as that of the usual semiclassical degrees of freedom despite the fact that they provide the origin of the Gibbons–Hawking entropy (the phenomenon referred to as spacetime-matter duality in Ref. [9]). In particular, the fact that a physical detector located in the de Sitter space sees a thermal bath of temperature \(T(r)\) in Eq. (7) does not imply that the semiclassical degrees of freedom, whose dynamics is determined by the semiclassical Hamiltonian, are actually in thermal equilibrium with temperature \(T(r)\). It only implies the existence of some degrees of freedom—the vacuum degrees of freedom—which interact with semiclassical degrees of freedom as if they are a thermal bath of temperature \(T(r)\).

The picture of the microscopic structure of quantum gravity described above offers a new possibility to avoid the dominance of Boltzmann brains—the dynamics of the vacuum degrees of freedom may simply not support any intelligent observers. The picture also addresses the question \([12]\) of whether a Minkowski vacuum produces Boltzmann brains—it depends on the microscopic dynamics of the vacuum degrees of freedom comprising that Minkowski vacuum. The fact that the Hawking temperature is zero in a Minkowski vacuum does not, by itself, guarantee the absence of Boltzmann brains; it simply means the absence of interactions between the semiclassical and vacuum degrees of freedom. Whether Boltzmann brains are produced in this vacuum then depends on
the internal dynamics of the vacuum degrees of freedom, which is not yet known.

If the state of the universe/multiverse in fact probes a Minkowski vacuum, or stays in a de Sitter vacuum supporting intelligent observers for a very long time, then the consideration in this paper becomes relevant. It is intriguing that such a basic fact that we observe an ordered, comprehensible world may deeply rely on the structure of quantum gravity at the fundamental level.

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References

[1] A. Aguirre, S.M. Carroll, M.C. Johnson, J. Cosmol. Astropart. Phys. 02 (2012) 024, arXiv:1108.0417 [hep-th].
[2] Y. Nomura, Found. Phys. 43 (2013) 978, arXiv:1110.4630 [hep-th].
[3] B.S. DeWitt, Phys. Rev. 160 (1967) 1113.
[4] Y. Nomura, Phys. Rev. D 86 (2012) 083505, arXiv:1205.5550 [hep-th].
[5] D.N. Page, W.K. Wootters, Phys. Rev. D 27 (1983) 2885.
[6] Y. Nomura, J. High Energy Phys. 11 (2011) 063, arXiv:1104.2324 [hep-th].
[7] L. Dyson, M. Kleban, L. Susskind, J. High Energy Phys. 10 (2002) 011, arXiv:hep-th/0208013.
[8] A. Albrecht, in: J.D. Barrow, P.C.W. Davies, C.L. Harper Jr. (Eds.), Science and Ultimate Reality: Quantum Theory, Cosmology, and Complexity, Cambridge University Press, Cambridge, 2004, p. 363, arXiv:astro-ph/0210527; D.N. Page, Phys. Rev. D 78 (2008) 065353, arXiv:hep-th/0610079.
[9] Y. Nomura, F. Sanches, S.J. Weinberg, J. High Energy Phys. 04 (2015) 158, arXiv:1412.7538 [hep-th]; Y. Nomura, F. Sanches, S.J. Weinberg, Phys. Rev. Lett. 114 (2015) 201301, arXiv:1412.7539 [hep-th].
[10] S.W. Hawking, Phys. Rev. D 14 (1976) 2460.
[11] A. Almheiri, D. Marolf, J. Polchinski, J. Sully, J. High Energy Phys. 02 (2013) 062, arXiv:1207.3123 [hep-th].
[12] D.N. Page, J. Korean Phys. Soc. 49 (2006) 711, arXiv:hep-th/0510003; M. Davenport, K.D. Olum, arXiv:1008.0808 [hep-th].
[13] S. Kachru, R. Kallosh, A. Linde, S.P. Trivedi, Phys. Rev. D 68 (2003) 046005, arXiv:hep-th/0301240.
[14] L. Susskind, L. Thorlacius, J. Uglum, Phys. Rev. D 48 (1993) 3743, arXiv:hep-th/9306009.
[15] C.W. Gibbons, S.W. Hawking, Phys. Rev. D 15 (1977) 2738.
[16] Y. Nomura, S.J. Weinberg, J. High Energy Phys. 10 (2014) 185, arXiv:1406.1505 [hep-th].
[17] S.W. Hawking, L.G. Moss, Phys. Lett. B 110 (1982) 35.
[18] E.J. Weinberg, Phys. Rev. Lett. 98 (2007) 251303, arXiv:hep-th/0612146.
[19] S.W. Hawking, Phys. Lett. B 115 (1982) 295; A.A. Starobinsky, Phys. Lett. B 117 (1982) 175; A.H. Guth, S.-Y. Pi, Phys. Rev. Lett. 49 (1982) 1110; See also, see also, V.F. Mukhanov, G.V. Chibisov, JETP Lett. 33 (1981) 532, Pis’ma Zh. Eksp. Teor. Fiz. 33 (1981) 549.
[20] R. Bouso, B. Freivogel, J. High Energy Phys. 06 (2007) 018, arXiv:hep-th/0610132.
[21] B. Freivogel, M. Lippert, J. High Energy Phys. 12 (2008) 096, arXiv:0807.1104 [hep-th].
[22] R. Bouso, B. Freivogel, I.-S. Yang, Phys. Rev. D 79 (2009) 063513, arXiv:0808.3770 [hep-th]; A. De Simone, A.H. Guth, A. Linde, M. Noorbala, M.P. Salem, A. Vilenkin, Phys. Rev. D 82 (2010) 063520, arXiv:0808.3778 [hep-th].
[23] K.K. Bodd, S.M. Carroll, J. Pollack, arXiv:1405.0298 [hep-th].