Symmetry of the Optical Phonons in LuVO$_4$: A Raman Study

Peter Rafailov $^1$, Dimitre Dimitrov $^{1,2,3,*}$, Yen-Fu Chen $^3$, Chi-Shen Lee $^4$ and Jenh-Yih Juang $^3$$^\dagger$

$^1$ Institute of Solid State Physics, Bulgarian Academy of Sciences, 72 Tzarigradsko Chaussee Blvd., 1784 Sofia, Bulgaria; rafailov@issp.bas.bg
$^2$ Institute of Optical Materials and Technologies, Bulgarian Academy of Sciences, Acad. G. Bonchev Str., 109, 1113 Sofia, Bulgaria
$^3$ Department of Electrophysics, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 30010, Taiwan; al0271227@hotmail.com (Y.-F.C.); jyjuang@g2.nctu.edu.tw (J.-Y.J.)
$^4$ Department of Applied Chemistry, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 30010, Taiwan; chishen@mail.nctu.edu.tw
* Correspondence: dzdimitrov@issp.bas.bg

Received: 21 March 2020; Accepted: 21 April 2020; Published: 26 April 2020

Abstract: A thorough analysis of the first-order vibrational spectrum of LuVO$_4$ is presented by using polarized micro-Raman spectroscopy with special focus on the phonon modes with the weakest intensity and occasional controversial assignment. Group-theory analysis is carried out to demonstrate the determination of numbers and symmetries of the Raman active modes. Crystal- and correlation-field splitting effects in the vibrational spectrum of LuVO$_4$ are discussed. Under conditions adjusted to minimize the birefringence effects we recorded, in each main scattering configuration, a series of Raman spectra in different sample orientations achieved by rotating the sample around the incident laser beam. The dependence of the Raman intensity on the rotational angle allowed us to identify the correct symmetry of the phonons with exceptionally weak scattering cross-section. A complete assignment of all twelve first-order Raman active phonons of LuVO$_4$ is thus obtained.

Keywords: rare-earth orthovanadates; single crystal; LuVO$_4$; Raman spectroscopy; optical phonons

1. Introduction

Rare-earth orthovanadates (RVO$_4$, with R being a lanthanide element) are intensively studied because of their interesting magnetic, optical, and electronic properties. These compounds show prospects for applications in a wide range of research fields, such as laser host crystals [1,2], scintillators [3], sensors [4], phosphor materials [5], and catalysts [6]. Due to their exceptional optical properties, such as wide optical transparency and large birefringence [7], the RVO$_4$ materials are suitable for various components for polarizing optics. Besides the technological importance, these compounds attract considerable theoretical interest due to the presence of 4f electrons.

Among them, LuVO$_4$ single crystals are of particular interest as laser host material due to the broad bandwidth, larger absorption cross-section near 800 nm, and larger emission cross section at 1.064 µm in comparison to those of other vanadate crystals [2]. These features are highly desirable for diode pumped solid-state lasers, as they make it feasible to achieve highly efficient pumping and to realize low threshold laser operation with high optical-to-optical efficiency. Recently, the vanadates GdVO$_4$ and LuVO$_4$ doped with several trivalent lanthanide ions showed promising results in diode pumped lasers [8]. Subsequent works demonstrated different $\chi$(3)-nonlinear generation effects in tetragonal LuVO$_4$ vanadate under picosecond pumping [9]. LuVO$_4$ thus turned out to be a promising material for utilization of the high-gain stimulated Raman scattering (SRS), especially in the development of...
novel Raman converters such as self-Raman lasers with lanthanide dopants. Four of the fundamental vibrational modes of LuVO₄ were identified as \( \chi(3) \)-promoting modes (113, 261, 826, and 900 cm\(^{-1} \)) [10]. Furthermore, cross-cascaded \((\chi(3) \leftrightarrow \chi(3))\) nonlinear lasing effects were observed with the participation of two pairs of SRS-active LuVO₄ vibrations (900 cm\(^{-1} \) together with 826 cm\(^{-1} \) and 900 cm\(^{-1} \) together with 261 cm\(^{-1} \)) in LuVO₄ [10]. These properties were detected in almost all known SRS-active tetragonal RVO₄ vanadates based on the same phonons and with similar Raman gain coefficients. The octave-spanning Stokes and anti-Stokes lasing comb generation was also demonstrated [10].

Given the large number of LuVO₄ normal modes participating in the SRS-effect, precise knowledge of the LuVO₄ vibrational spectrum is of vital importance for better understanding of its lasing properties. An impressive deal of Raman work has been performed in recent decades to explore the vibrational spectra of LuVO₄ and other orthovanadates. Nevertheless, some contradictions in the assignment of the Raman active modes still persist, most probably due to the large birefringence of the material and the vast difference in the Raman intensity of the distinct normal modes. In this paper, we present a Raman investigation of LuVO₄ with full utilization of the symmetry properties of the different modes. This allows us to detect modes with exceptionally low Raman intensity and to assign all 12 Raman active vibrations.

2. Materials and Methods

LuVO₄ single crystals were grown by the high-temperature solution growth method. As a first step, polycrystalline LuVO₄ seed was synthesized by solid state reaction and subsequently enriched with V₂O₅. The mixture was further melted and annealed at 1100 °C in a platinum crucible. Single crystals were obtained by slow cooling of the solution from 1100 to 700 °C. Details of the growth procedure are given in [7]. The obtained crystals with tetragonal rectangular shape and typical size of \( 8 \times 10 \times 1 \) mm\(^3 \) were collected from the bottom and the walls of the crucible.

A single crystal specimen of LuVO₄ was used for the X-ray crystallographic analysis. X-ray intensity data were obtained at room temperature on Bruker AXS Smart Apex three-circle diffractometer equipped with a CCD detector. Data collection was carried out with operating parameters of 50 kV and 30 mA and graphite monochromatic Mo-Kα radiation (\( \lambda = 0.71073 \) Å). Details of the X-ray crystallographic analysis are given in [7]. Additionally, X-ray Powder Diffraction for phase analysis was performed on LuVO₄ powder prepared from the single crystal sample at a D8 Bruker powder diffractometer [11].

For the spectroscopic measurements, single crystals with elongated shapes along the c-axis with naturally grown \( \{100 \} \) and \( \{001 \} \) surfaces were selected. The Raman spectra were measured in backscattering geometry in the range of 80–1000 cm\(^{-1} \) on a HORIBA Jobin Yvon Labram HR visible spectrometer equipped with a Peltier-cooled CCD detector. The 632.8 nm line of a He-Ne laser (power 5 mW) as well as the 514.5 nm line of an Ar⁺ laser (power 8 mW) was used for excitation. Both diffraction gratings of the instrument were used: with 600 grooves/mm and 1800 grooves/mm, the latter giving an absolute accuracy of 0.3 cm\(^{-1} \) for \( \lambda = 632.8 \) nm and 0.55 cm\(^{-1} \) for \( \lambda = 514.5 \) nm.

3. Results and Discussion

3.1. Characterization of the Vibrational Properties and Vibrational Splitting in LuVO₄ Crystals

According to X-ray powder diffraction analysis [11], the as-grown crystals had a single-phase zircon structure. All of the diffraction peaks were identified to result from LuVO₄ and no peaks of a second phase were found. The lattice structure of this crystal is depicted in Figure 1. It can be considered as made up of chains formed by alternating LuO₈ polyhedra (bisdisphenoid) and VO₄ tetrahedra along the c-axis. These chains are joined together by sharing other edges of the LuO₈ polyhedra along the a- and b-directions to form a three-dimensional lattice with centrosymmetric tetragonal structure with the point group symmetry \( D_{4h} \) and space group \( I4/amd \) (\( D_{4h} \)). Despite the different coordination environment, both Lu³⁺ and V⁵⁺ ions occupy sites of \( D_{2d} \) symmetry [12].
The unit cell contains 4 formula units (2 formula units per primitive cell) with lattice parameters of \( a = b = 7.0236\AA \) and \( c = 6.2293\AA \) [11]. Consequently, 12 atoms per primitive cell with 36 degrees of freedom should be taken into account.

![Figure 1. Structure of the zircon-type crystal lattice of LuVO₄. The two Lu atoms and the two VO₄ tetrahedra in the primitive cell are marked by vertical black arrows [13].](image)

According to the determined crystal structure [7,11,12] and the data available in the International Tables for Crystallography for the space group D₄h\(^{19}\), the occupied Wyckoff positions are as follows: 4a (Lu\(^{3+}\) ions), 4b (vanadium) and 16h (oxygen). As the mirror plane \( \sigma_v \) is the common symmetry element of the point group of the crystal D₄h and the site symmetry group C₄, C₄(\( \sigma_v \)) is identified as the correct site symmetry for the oxygen atoms, thus removing the ambiguity in the 16h positions for this structure. By carrying out the nuclear site group analysis [14], shown in detail in Table 1, one ultimately obtains the representation of the Raman active modes as follows:

\[
\Gamma_{\text{Raman}} = 2A_{1g} + 4B_{1g} + B_{2g} + 5E_g,
\]

(1)

| Atomic Species | Wyckoff Position and Site Symmetry | Irreducible Representations (Symmetries of Phonon Modes) Resulting for These Species’ Vibrations |
|----------------|-----------------------------------|-----------------------------------------------------------------------------------------------|
| Lu\(^{3+}\)    | 4a (D₂d)                          | \( A_{2u} + B_{1g} + E_g + E_u \)                                                             |
| V\(^{5+}\)     | 4b (D₂d)                          | \( A_{2u} + B_{1g} + E_g + E_u \)                                                             |
| O\(^2-\)       | 16h (C₄(\( \sigma_v \)))         | \( 2A_{1g} + A_{1u} + A_{2g} + 2A_{2u} + 2B_{1g} + B_{1u} + B_{2g} + 2B_{2u} + 3E_g + 3E_u \) |

It is also instructive to classify the vibrational modes with the molecular site group analysis, which is suitable for treating isolated ionic or molecular species within a crystal matrix [14,15]. The orthovanadate structure consists of heavy rare-earth ions and VO\(_4\(^{3-}\)\) tetrahedra that can be approximately regarded as separate units due to the strong internal bonds within each tetrahedron. Therefore, the lowest-frequency vibrations of LuVO₄ should comprise of translations and librations of the VO\(_4\(^{3-}\)\) tetrahedra as rigid units against the rare-earth ions, while the higher-frequency phonons should be almost entirely due to internal vibrations of the tetrahedra [16]. The isolated VO\(_4\(^{3-}\)\) complex
has the relatively high symmetry T_d and its degrees of freedom can be classified in terms of irreducible representations of the T_d group as follows:

\[ \Gamma_{XY4} = A_1(\nu_1) + E(\nu_2) + F_1(\text{pure rotation}) + F_2(\text{pure translation}) + F_2(\nu_3) + F_2(\nu_4). \]  

(2)

The pure rotation and translation comprise zero-frequency motions of the complex as rigid structure that transform into librations and external translational modes, respectively, in the LuVO_4 crystal structure [16]. The remaining four terms represent the four fundamental internal vibrations of the VO_4^{3−} tetrahedron. The first publications analyzing the Raman phonons in zircon-type structure and dividing them into external and internal modes according to the vibrating species were made by Miller et al. in 1968 [16] and Elliot et al. in 1972 [17]. Since then, these modes have been extensively studied by many authors for a number of orthovanadates. On lattice sites with D_{2d} symmetry (see Table 1), the vibrational states undergo partial (crystal-field-induced) splitting according to the T_d/D_{2d} group-subgroup correlation rules. Then one should take into account the interaction between the two formula units in the primitive cell, which is equivalent to induction from the representations of the D_{2d} group to those of the D_{4h} group. In this way, a doubling of the vibrational states to the real number of degrees of freedom per primitive cell takes place and this doubling indicates the Davidov or correlation-field splitting, which is a measure of the interaction between non-equivalent tetrahedra in the primitive cell and also of the magnitude of the phonon dispersion in the Brillouin zone [15].

From this analysis, the same total representation of the Raman active modes (Equation (1)) can be constructed with the valuable additional information that there will be 2A_{1g} + 2B_{1g} + 2B_{2g} + 2E_g internal (higher-frequency) VO_4^{3−} modes; 2B_{1g} + 2E_g external (translational) VO_4^{3−} modes and a librational E_g mode of the VO_4^{3−} tetrahedron [16]. The infrared active vibrations are: A_{2u} + E_u translational, an E_u librational and 2A_{2u} + 2E_u internal VO_4^{3−} modes. The distribution of the optically active internal modes shall be as follows: A_1 (\nu_1) \rightarrow A_{1g}; E (\nu_2) \rightarrow A_{1g} + B_{2g} and for both \nu_3 and \nu_4 from Equation (2): F_2 (\nu_i) \rightarrow B_{1g} + E_g + A_{2u} + E_u. This picture is of course an approximation of the real situation, where there are no purely internal or external modes, but still very helpful for the Raman mode assignment [16,17]. It also becomes clear that no Davidov splitting can be established from Raman measurements because the Davidov counterpart of each Raman mode is an odd-parity one. Combined Raman and IR measurements can detect the Davidov doublet E_g + E_u of the librational mode and the doublets A_{2u} + B_{1g} and E_g + E_u resulting from the F_2(\nu_3) and F_2(\nu_4) internal vibrations of the VO_4^{3−} tetrahedron.

The pertinent Raman tensors for the D_{4h} point group are the following [15]:

\[
A_{1g}: \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix} ; B_{1g}: \begin{bmatrix} c & 0 & 0 \\ 0 & -c & 0 \\ 0 & 0 & 0 \end{bmatrix} ; B_{2g}: \begin{bmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \text{ and } E_g,1: \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & e \\ 0 & e & 0 \end{bmatrix} ; E_g,2: \begin{bmatrix} 0 & 0 & -e \\ 0 & 0 & 0 \\ -e & 0 & 0 \end{bmatrix}.
\]

The scattering intensity of a particular mode with Raman tensor \( \alpha \) is given by:

\[ I \sim |e_1 \cdot \alpha \cdot e_s|^2 \]

(3)

where the unit vectors \( e_1 \) and \( e_s \) denote the polarization of the incident and scattered light, respectively. For the polarized Raman measurements, we use the notations X (100), Y (010) and Z (001) for the main crystal axes, and X’(110) and Y’(−110) for diagonal directions in the XY plane. The applied scattering configurations are labelled by Porto notations. Raman selection rules may suffer from depolarisation effects caused by the extremely strong birefringence of the LuVO_4 crystal. These effects lead to partial mixing of allowed and forbidden intensity between different Raman modes, which, nevertheless, can be minimized by using focusing/collection optics with small numerical aperture (N.A.), while keeping the beam path within the sample short [18]. The above two requirements were satisfactorily met by...
using a 20× microscope objective with N.A. = 0.4. Special care was also taken to achieve as perfect as possible back-scattering conditions on samples with smooth and optically defect-free surfaces. The main crystallographic planes of our samples were identified by X-ray diffractometry [11].

The spectra also contain features that do not obey the Raman selection rules and obviously do not pertain to the first-order vibrational spectrum of the LuVO₄ crystal. Their intensity turns out to be dependent on the investigated sample spot and the excitation wavelength λ with strongest intensity appearing upon red (λ = 632.8 nm) excitation. Our earlier experience with similar polarized Raman measurements on LuVO₄ at 632.8 nm excitation [11] shows that these features also display various polarization behaviour. As no significant impurities or second-phase inclusions were detected by XRD, we attribute these features to photoluminescence lines from electronic transitions of other rare-earth ions that are unavoidably present as substitutional defects in LuVO₄. In the spectra excited at 514.5 nm photoluminescence features are largely suppressed and upon blue excitation (488 nm) they are virtually missing. Besides, the birefringence of LuVO₄ is minimal in the blue spectral region [7]. The 488 nm laser thus appears to be the obvious choice of excitation wavelength. We, nevertheless, used the 514.5 nm laser line in the present study because of indications that the generally missing Raman active Eₘ(4) mode might be resonantly enhanced in the green spectral region, and carefully selected sample regions with minimum and non-variable intensity of impurity features. For some spectra when the luminescence lines lie far from the studied Raman modes, the He-Ne laser excitation (632.8 nm) was used.

The measured Raman spectra are presented in Figure 2. Although Raman selection rules are clearly discernible, in view of the known birefringence, the question may arise if the mode intensities in different scattering configurations are directly comparable. To address this question, we recall that besides the scattering cross section (given by the magnitude of the tensor elements), the Raman signal is influenced by the reflectivity $R$ at the sample surface, the possible absorption within the sample, and the size of the volume, in which the detected Raman scattering effectively occurs (scattering volume). The latter is determined by the focus spot depending entirely on the focusing optics and the depth of focus. The correction factor resulting from reflection losses has the form $(1 - R)^2$ because both the incident and the scattered radiation undergo reflection. By means of the Fresnel equation for normal-incidence reflectivity, we obtain $(1 - R)^2 = 16n^2/(n + 1)^2$. We shall inspect that factor for the spectra labelled X(YY)X and X(ZZ)X representing the two limiting cases: ordinary wave ($n = n_o = 2.1$) and extraordinary wave ($n = n_e = 2.3$), respectively. Using the results for the refraction index $n$ from [7], we obtain a value of 0.764 for spectrum X(YY)X and for spectrum X(ZZ)X $(1 - R)^2 = 0.714$. According to Gaussian beam theory [19], the depth of focus is of the order $\lambda/(N.A.)^2$ but it also linearly depends on $n$. Thus, the larger value of $n_e$ effectively increases the scattering volume by about 10%, entirely compensating the higher reflection losses. Even if we neglect one of these counter-acting effects and consider only the other, the expected change in Raman signal does not exceed 10%. The depth of focus is of the order 10 µm and the crystal exhibits high transparency above 500 nm [11]; therefore, possible absorption losses should be negligible. Consequently, the different spectra in Figure 2 are perfectly comparable for the purpose of Raman mode assignment and even for establishing relations between the Raman tensor elements $a$ and $b$ for the $A_{1g}$ modes. The relative peak intensity of the $A_{1g}(2)$ line in spectra X(YY)X and X(ZZ)X is $\approx 17000$ and $\approx 19900$ (arb. u.), respectively. This implies $|a| \approx |b|$ for the $A_{1g}(2)$ mode. However, the intensity of the $A_{1g}(1)$ line in spectrum X(YY)X is about four times smaller than in spectrum X(ZZ)X. The $A_{1g}(1)$ mode thus has a strongly anisotropic Raman tensor ($|b| \approx 2|a|$).
The measured Raman spectra are presented in Figure 2. Although Raman selection rules are tetrahedron \([20,21]\). We also included in Table 2 the peak-intensities and the linewidths in order to highlight the SRS-promoting phonons by means of their two most important properties: high Raman efficiency and long lifetime. As noted in \([22]\), the steady-state Raman gain coefficient of a certain phonon mode in SRS depends linearly on the ratio of its scattering cross-section (integral intensity) and its linewidth in spontaneous Raman scattering. This ratio is proportional to the peak intensity \([22]\). Indeed, for phonons that were identified as SRS-promoters \([10]\), the peak intensity (given in bold font in Table 2) is many times higher than for most of the other normal vibrations in LuVO\(_4\), and the linewidth is small.
Table 2. Frequencies and assignment of the Raman active phonons of LuVO$_4$. Additionally, for the librational $E_g$ and the internal $B_{1g}$ and $E_g$ modes, in the first column, the symmetries and frequencies of their IR active Davidov counterparts are given in brackets, as taken from [23]. In those doublets, both VO$_4^{3-}$ complexes in the primitive cell execute the same vibration: $u$-modes in phase and $g$-modes – out of phase. The data for the problematic modes discussed in Section 3.2 are given in red bold font.

| Symmetry Species ($D_{4h}$), Frequency and Origin in Isolated VO$_4^{3-}$ ($T_d$) | Vibrational Pattern | Peak Intensity (arb. u.) | Linewidth (cm$^{-1}$) | Frequencies (cm$^{-1}$) from Available References |
|-------------------------------------------------|---------------------|-------------------------|----------------------|-----------------------------------------------|
| $E_g$(1): 103 cm$^{-1}$ pure translation | Lu$^{3+}$/VO$_4^{3-}$ translation | 220; 8 | — | — |
| $B_{1g}$(1): 113 cm$^{-1}$ pure translation | Lu$^{3+}$/VO$_4^{3-}$ in-phase translation | 3120; 6 | 114 | 112 113 |
| $E_g$(2): 157.5 cm$^{-1}$ pure translation | Lu$^{3+}$/VO$_4^{3-}$ translation | 800; 9 | 158 | 157 250 |
| $E_g$(3): 252 cm$^{-1}$ (E$_u$ at 248 cm$^{-1}$) pure rotation | VO$_4^{3-}$ libration | 580; 12 | 251 | 259 158 |
| $B_{2g}$(2): 261.5 cm$^{-1}$ $E(v_2)$ | $v_2$, O–V–O asymmetric bending | 19800; 4.5 | — | 259 826 |
| $B_{1g}$ (2): 269 cm$^{-1}$ pure translation | Lu$^{3+}$/VO$_4^{3-}$ out-of-phase translation | 200; 12 | 261 | — 262 |
| $A_{1g}$(1): 380 cm$^{-1}$ $E(v_2)$ | $v_2$, O–V–O symmetric bending | 4900; 27 | 382 | 381 379 |
| $E_g$(4): 393 cm$^{-1}$ (E$_u$ still missing) $F_2(v_4)$ | $v_4$, VO$_4^{3-}$ asymmetric bending/stretching | 120; 11 | — | — — |
| $B_{2g}$(3): 493 cm$^{-1}$ (A$_{2u}$ at 451 cm$^{-1}$) $F_2(v_4)$ | $v_4$, VO$_4^{3-}$ asymmetric bending/stretching | 1460; 10 | 493 | 492 495 |
| $B_{1g}$(4): 826 cm$^{-1}$ (A$_{2u}$ at 819 cm$^{-1}$) $F_2(v_4)$ | $v_3$, VO$_4^{3-}$ asymmetric stretching (synchronic parallel Z-motion of all 4 oxygens against the V$^{5+}$ ion) | 9300; 6 | 826 | 824 826 |
| $E_g$(5): 846 cm$^{-1}$ (E$_u$ at 790 cm$^{-1}$) $F_2(v_3)$ | $v_3$, VO$_4^{3-}$ asymmetric stretching (synchronic parallel X- and Y-motion of all 4 oxygens against the V$^{5+}$ ion) | 7000; 6.5 | 847 | 846 847 |
| $A_{1g}$(2): 900 cm$^{-1}$ $A_1(v_1)$ | $v_1$, VO$_4^{3-}$ symmetric stretching (breathing of the tetrahedron) | 19900; 7.5 | 901 | 900 901 |
Table 2 contains some new results on the LuVO₄ Raman mode assignment, which will be discussed in detail in the next subsection. Here, we will briefly review the crystal-field splitting $\Delta \omega_{\text{CF}}$ and Davidov splitting $\Delta \omega_D$ within the manifolds arising from the fundamentals $\nu_4$ and $\nu_4$, all components of which are optically active. Within those manifolds, the frequency difference between the $E_g(E_u)$ and the $B_{1g}(A_{2u})$ mode provides an estimate for the crystal-field splitting while the separation in the $E_g/E_u$ or $B_{1g}/A_{2u}$ doublet serves as a measure for the Davidov splitting. Although IR measurements on LuVO₄ [6,23] are scarce, its IR active modes are long known [23], except for the $E_u$ in the $\nu_4$ manifold. Nevertheless, this mode can be credibly assumed to lie between 300 and 350 cm$^{-1}$ by analogy to the well-studied internal vibrations of YVO₄ [16,27,28]. Additionally, only $\Delta \omega_{\text{CF}}$ can be estimated for $\nu_2$ by the separation $A_{1g}(1)-B_{2g}$ for their odd-parity counterparts $A_{1u}$ and $B_{2u}$ are not optically active.

Table 3 represents the estimated splitting using the first column of Table 2. Three observations can be made from Table 3, which should be more or less valid for all members of the orthovanadate family: (i) $\Delta \omega_{\text{CF}}$ decreases monotonically in going from bond-bending to bond-stretching vibrations. (ii) For the mixed bending/stretching vibrations of the $\nu_4$ manifold, $\Delta \omega_{\text{CF}}$ and $\Delta \omega_D$ are comparable with slight prevalence of $\Delta \omega_{\text{CF}}$. (iii) In the $\nu_3$ manifold, $\Delta \omega_D$ for the E-modes dominates over $\Delta \omega_{\text{CF}}$ and is eight times stronger than $\Delta \omega_D$ for the $B_{1g}(4)/A_{2u}$ doublet. The fundamental frequency $\nu_3 = 825$ cm$^{-1}$ of isolated VO$_4^{3-}$ [21] appears like a “common center of gravity” of both doublets. For this effect, we propose a simple qualitative explanation. In crystals of complex ions as VO$_4^{3-}$, the interaction causing Davidov splitting is mainly governed by short-range repulsion forces [15]. Therefore, $\Delta \omega_D$ should primarily depend on the relative motion of the vibrating ions for the modes of a particular doublet. In the rectangular unit cell, the difference in the Z-coordinates of the two non-equivalent tetrahedra of the primitive cell is more than twice smaller than their horizontal distance [17] (see also Figure 1). The $E_g(5)$ mode has equally charged ions of the two tetrahedra vibrating horizontally against each other, while the corresponding $E_u$ mode has these ions executing the same vibration in phase. Due to the proximity of their Z-coordinates, a considerable additional repulsive force should arise between the two tetrahedra during the $E_g(5)$ vibration, unlike the corresponding $E_u$ motion, which completely lacks such a force. In the analogous case with the $B_{1g}(4)$ mode and its Davidov counterpart $A_{2u}$, the vibrational motion takes place along the vertical Z-axis. Due to the larger horizontal distance between the two non-equivalent tetrahedral, the mutual influence between motions of equally charged ions during the $B_{1g}(4)$ vibration should be much weaker, which might explain the small $\Delta \omega_D$ of only 7 cm$^{-1}$ in the $B_{1g}(4)/A_{2u}$ doublet. This splitting resembles only a slight perturbation of the fundamental $F_2(\nu_3)$.

| VO$_4^{3-}$ Fundamental | $\Delta \omega_{\text{CF}}$ (cm$^{-1}$) | $\Delta \omega_D$ (cm$^{-1}$) |
|-------------------------|---------------------------------|------------------|
| $\nu_2$                | $A_{1g}-B_{2g}$                 | 120              | —    |
| $\nu_4$                | $E_g-B_{1g}$                    | 100              | $E_g-E_u$ 50–100 |
|                        | $E_u-A_{2u}$                    | $\approx 100$   | $B_{1g}-A_{2u}$ $\approx 50$ |
| $\nu_3$                | $E_g-B_{1g}$                    | 20               | $E_g-E_u$ $\approx 55$ |
|                        | $E_u-A_{2u}$                    | $\approx 30$    | $B_{1g}-A_{2u}$ $\approx 7$ |

3.2. Assignment of Problematic Raman Active Modes Using Their Rotational Transformation Properties

Thanks to several detailed group-theoretical and spectroscopic investigations of orthovanadate crystals, the symmetry properties of the stronger lines in their Raman spectra are well-established and their assignment has been known for decades [16–18,24,29]. However, the modes $E_g(1)$, $B_{1g}(2)$ and $E_g(4)$ are known to have exceptionally weak scattering intensity in almost all orthovanadates of the zircon-family [28]. Due to their low intensity and complications from birefringence effects, these
modes are frequently missing [24] or wrongly assigned [10,25,26] in some recent Raman studies of orthovanadates. Therefore, to complete the mode assignment for the LuVO₄ crystal, i.e., to obtain the results shown in Table 2, we measured Raman spectra as a function of the rotation angle φ by rotating the sample around the exciting beam in its polarization plane. This approach utilizing the differences in the transformation properties of phonons of different symmetry has proved to be helpful in many instances. Beside its conventional use for checking phonon symmetry, it has also been applied for determination of crystal structure [30] and detection of lattice defects [31].

For the present experiments, φ is defined as the angle between the X-axis and the actual polarization direction of the exciting laser beam. For rotating scattering configurations in perpendicular polarization xₓφ, yφ, and zφ are orthogonal φ-dependent directions in the XY and the XZ plane, respectively. For rotation in parallel polarization, the transition from the initial to the final polarization is denoted. For instance, Y(XX → ZZ) is gradual transition from Y(XX) to Y(ZZ) through rotation about the Y axis. Vertically stacked rotation-angle dependent spectra are shown in Figures 3–6 for various frequency regions and scattering configurations. The theoretically predicted dependence of the Raman intensity (Equation (3)) on the angle φ for the examined configurations is given in Table 4. The angular dependences of the measured mode intensities normalized to the maximum intensity for each line of interest are plotted in Figure 7. Theoretical curves of the corresponding functions in Table 4 are also plotted in Figure 7 for comparison.

![Figure 3. Rotation-angle dependent Raman spectra of the Eg(1) mode of the LuVO₄ crystal in parallel (a) and perpendicular (b) polarization. The spectra with the highest Eg(1) intensity are plotted in red. The laser excitation wavelength is 632.8 nm.](image-url)
The rotation-angle dependent Raman spectra of the B$_{1g}$(2) mode of the LuVO$_4$ crystal in parallel (a) and perpendicular (b) polarization in the XZ plane. The spectra with the highest B$_{1g}$(2) intensity are plotted in red. The laser excitation wavelength is 632.8 nm. Due to interference-fringe effects, the spectra are smoothed out by means of FFT filtering. The asterisks mark luminescence features. Inset: Rotation-angle dependent spectra of the luminescence lines found in the spectra of panel (b) around 450 cm$^{-1}$.

![Figure 4](image_url)

Figure 4. Rotation-angle dependent Raman spectra of the B$_{1g}$(2) mode of the LuVO$_4$ crystal in parallel (a) and perpendicular (b) polarization in the XZ plane. The spectra with the highest B$_{1g}$(2) intensity are plotted in red. The laser excitation wavelength is 632.8 nm. Due to interference-fringe effects, the spectra are smoothed out by means of FFT filtering. The asterisks mark luminescence features. Inset: Rotation-angle dependent spectra of the luminescence lines found in the spectra of panel (b) around 450 cm$^{-1}$.

The rotation-angle dependent Raman spectra of the B$_{1g}$(2) mode of the LuVO$_4$ crystal in parallel (a) and perpendicular (b) polarization in the XY plane. The spectra with the highest B$_{1g}$(2) intensity are plotted in red. The laser excitation wavelength is 632.8 nm.

![Figure 5](image_url)

Figure 5. Rotation-angle dependent Raman spectra of the B$_{1g}$(2) mode of the LuVO$_4$ crystal in parallel (a) and perpendicular (b) polarization in the XY plane. The spectra with the highest B$_{1g}$(2) intensity are plotted in red. The laser excitation wavelength is 632.8 nm.
The rotation-angle dependent spectra of the lowest-energy translational modes of the LuVO₄ crystal are displayed in Figure 3. From Figure 7c and the observed behavior of the line at 103 cm⁻¹, it is evident that its ϕ-dependent intensity oscillation is in excellent agreement with the formulae listed in the last row of Table 4. Consequently, we can unambiguously assign the E₉(1) mode to the line at 103 cm⁻¹.

To confirm the assignment of the B₁₈(2) mode, which has extremely weak intensity, we performed thorough investigations by rotating in two perpendicular planes: XZ (excitation along Y) and XY (excitation along Z) using the high-resolution diffraction grating (1800 gr/mm). The corresponding sets of Raman spectra are shown in Figures 4 and 5, respectively. Because of interference fringes, the spectra in Figure 4 were smoothed out by means of fast Fourier Transform (FFT) filtering. The first impression from Figure 4 is that both 261.5 cm⁻¹ and 269 cm⁻¹ lines satisfy the theoretical ϕ-dependence of the intensity of the B₁₈ mode. Thus, if only the polarizations used in Figure 4 are considered, one can easily declare that the stronger 261.5 cm⁻¹ line is associated with the B₁₈(2) mode. However, due to its high intensity in XY polarization (Figure 2) and its ϕ-dependent behavior (Figure 5), the 261.5 cm⁻¹ line is undoubtedly recognized as the only B₂₈ mode of the LuVO₄ crystal. In Figure 7a,b, the ϕ-dependence

| Mode | Symmetry | Z(xᵦxᵦ)Z(XX→YY) | Z(xᵦyᵦ)Z(XY→YY) | Y(xᵦxᵦ)y(XX→ZZ) | Y(xᵦzᵦ)y(XZ→ZX) |
|------|----------|-----------------|-----------------|-----------------|-----------------|
| A₁₈ | | | | | |
| B₁₈ | | | | | |
| B₂₈ | | | | | |
| E₉ | | | | | |
of the 269 cm\(^{-1}\) line intensity is presented together with theoretical curves according to the formulae listed in the second row of Table 4.

![Image](image.png)

**Figure 6.** Rotation-angle dependent Raman spectra of the Eg(4) mode of the LuVO\(_4\) crystal in parallel (a) and perpendicular (b) polarization in the XZ plane. The laser excitation wavelength is 514.5 nm. The lowest two traces are model Lorentzians for the modes A\(_{1g}(1)\) and Eg(4) used to fit the spectrum at 45° (a) and at 0° (b). The asterisks mark luminescence features.

![Image](image.png)

**Figure 7.** Angular dependence of the relative intensity of Raman lines (see the text and Table 2). The polarization configurations are indicated in the plots. Panels (a) and (b): B\(_{1g}(2)\) mode at 269 cm\(^{-1}\). Panel (c): Eg(1) mode at 103 cm\(^{-1}\). Panel (d): The Eg(4) mode at 393 cm\(^{-1}\) and the A\(_{1g}(1)\) mode at 381 cm\(^{-1}\) in parallel polarization. Panel (e): The Eg(4) mode at 393 cm\(^{-1}\) and the A\(_{1g}(1)\) mode at 381 cm\(^{-1}\) in perpendicular polarization. The solid lines are graphs of the theoretically predicted functional dependencies.

The excellent agreement between theory and experiment manifested in Figure 7a,b confirms the assignment of the 269 cm\(^{-1}\) line to the B\(_{1g}(2)\) mode. This assignment is further corroborated by Figure B1 of [32], where the lines Eg(3), B\(_{2g}\) and B\(_{1g}(2)\) of LuVO\(_4\) are detected in the same sequence and at nearly the same frequencies. From that figure it is seen that the B\(_{1g}(2)\) mode crosses over the B\(_{2g}\) mode in going from NdVO\(_4\) to LuVO\(_4\), while the B\(_{2g}\) frequency remains nearly constant at 260–261 cm\(^{-1}\). The same behaviour may be deduced from Table 1 of [24], if the labels B\(_{2g}(\nu_2)\) and B\(_{1g}(2)\) are interchanged. We therefore argue that it is the B\(_{2g}\) mode that remains relatively unchanged for all rare-earth vanadates while the B\(_{1g}(2)\) mode undergoes a slight hardening over the rare-earth vanadate series from NdVO\(_4\) to LuVO\(_4\). This is, however, contradictory to the mass-dependent frequency behavior \(\omega \sim (\mu)^{-1/2}\) (here \(\mu\) is the reduced mass of the rare-earth and the VO\(_4^{3-}\) ions) for its vibrational pattern (out-of-phase translation) expected from the simple model adopted in [24]. This seemingly contradictory behavior is supposed to be due to the mixing of the three close-by lying modes Eg(3), B\(_{2g}\) and B\(_{1g}(2)\) of totally different character. B\(_{2g}\) as internal tetrahedral mode should be least susceptible to such mixing which supports the assumption for its nearly constant frequency. Additionally, the B\(_{2g}\) vibrational pattern having all 4 oxygens move perpendicular to the V-O bonds should also keep this phonon insensitive to a change in the rare earth ion radius. On the other hand, the mass-dependent frequency behavior of the in-phase translational B\(_{1g}(2)\) mode (113 cm\(^{-1}\) for LuVO\(_4\)) agrees qualitatively with the analogue model expected for its vibrational pattern, namely, \(\omega \sim (m)^{-1/2}\) with \(m\) being the total mass of the rare-earth and the VO\(_4^{3-}\) ions [24].
The $E_g(4)$ mode is expected to lie in the vicinity of the $A_{1g}(1)$ mode at 381 cm$^{-1}$ for all members of the orthovanadate family with a frequency nearly independent of the mass of rare-earth ion [28]. It is the second lowest in energy of the internal Raman modes of the VO$_4^{3−}$ complex and seems to have exceptionally low scattering intensity. The experimental detection of the $E_g(4)$ mode in orthovanadates has almost never been reported, except for the study of Sanson et al. [28] on YVO$_4$, where the $E_g(4)$ mode was found at 387 cm$^{-1}$. Interestingly, in the present study, we find in the spectra excited at 514.5 nm a line at 393 cm$^{-1}$ with promising polarization behaviour (see Figures 2 and 6a). In parallel polarization, the relative intensity of this line resembles more or less the $\phi$-dependence for an $E_g$ mode, as depicted in Figure 7d. However, its intensity drops significantly when going from $Y(XZ)\bar{Y}$ to $Y(ZX)\bar{Y}$ scattering configuration (see Figure 2), which is extremely unusual for a phonon mode. We therefore scanned its angular dependence in perpendicular polarisation in the full range from $0^\circ$ to $180^\circ$ using the high-resolution diﬀraction grating (1800 gr./mm). The obtained spectra are shown in Figure 6b and the resulting $\phi$-dependences are plotted in Figure 7e. It is seen that, despite the intensity drop towards $90^\circ$, the relative intensity of this line tends to form a local minimum at about $45^\circ$ and has a weak local maximum at $90^\circ$. It thus appears that there is an accidental coincidence of two lines: a weak one with a period of $90^\circ$ and a stronger one with a period of $180^\circ$ in the angular dependence of their intensities. The former one has the right angular dependence for an $E_g$ mode, while the latter one has the right angular dependence for a polarized luminescence line. Although luminescence features normally appear in optical spectra in form of wider bands, rare-earth compounds are an exception. Sharp phonon-like luminescence lines are typical for rare-earth ions due to the radial confinement of their $f$-electrons [33]. Radiative discharge of some excited states in RE ions via dipole-allowed transitions was shown to produce polarized spectra [34] having $cos^2\phi$-like angular dependence (hence period of $180^\circ$), which is typical for radiation of an oscillating dipole with fixed orientation. We also established a $cos^2\phi$-like angular dependence for the intensity of some luminescence features around 400 cm$^{-1}$ in the spectra from Figure 4b, which we have depicted in the inset of Figure 4b. Therefore, to provide theoretical curves for panels (d) and (e) of Figure 7, we modelled the intensity behaviour of the 393 cm$^{-1}$ line with functions proportional to $sin^2\phi + A sin^22\phi$ for parallel and $(cos^2\phi + A cos^22\phi)$ for perpendicular polarization, respectively. The constant $A$ was estimated to be $\approx 1/3$ from comparison of the line’s intensity in crossed polarization at $0^\circ$ where both components should be at maximum and $90^\circ$ where the Raman line is again at maximum while the luminescence vanishes. The model shows satisfactory agreement with the measured data in both polarizations, especially in view of the fact that the region 300–450 cm$^{-1}$ is rich in luminescence lines and there might be more than one of them interfering with the Raman modes $A_{1g}(1)$ and $E_g(4)$. For instance, in [35] luminescence spectra of rare-earth-doped LuVO$_4$ nanoleaves were reported containing lines around 526 nm, which corresponds to a Raman shift of $\approx 400$ cm$^{-1}$ in a spectrum excited at 514.5 nm. We consider the presented arguments sufficient to assign the 393 cm$^{-1}$ line to the so far missing $E_g(4)$ mode. This assignment is corroborated by the presence of a weak line in an $E_g$ Raman spectrum of LuVO$_4$ at the same frequency in Figure B3 of [32] for 514.5 nm excitation and at 386 cm$^{-1}$ in Figure 5 of [25] for 532 nm excitation. In both publications, these faint features were not discussed probably due to their weak intensity. We should also mention the work of Voron’ko et al. [36] who reported to have found the $E_g(4)$ mode around 440 cm$^{-1}$ in $X(ZY)\bar{X}$ Raman spectra of YVO$_4$ and GdVO$_4$. In their temperature-dependent Raman study, these authors observed a double-peak structure for the $A_{1g}(1)$ line, which they explained by thermally-activated population of higher local minima in the lattice potential through rotation of some VO$_4^{3−}$ tetrahedra. The higher-energy component, whose $A_{1g}$ character was reportedly confirmed by polarization studies [36], nearly coincides in frequency with the line identified by us as the $E_g(4)$ mode. Although we do not rule out a possible inhomogeneous broadening of the $A_{1g}$ line in our LuVO$_4$ crystal due to VO$_4^{3−}$ to rotation at higher temperatures, we emphasize that we do not detect any splitting in its lineshape despite the large linewidth. We also have confirmed the symmetry of the $E_g(4)$ mode at 393 cm$^{-1}$ in the relevant polarization configurations, and additionally by demonstrating its rotational transformation properties. Furthermore, the $E_g(4)$ mode is expected at a nearly constant
position between 370 and 410 cm$^{-1}$ for all orthovanadates and has already been detected at 387 cm$^{-1}$ for YVO$_4$ in full agreement with theoretical model calculations [28].

It should be pointed out, however, that no Raman feature at 393 cm$^{-1}$ is detectable upon blue excitation at 488 nm where the spectrum is almost free of luminescence lines. This implies a possible resonance character of the E$_g$(4) mode. A strong dependence of another E$_g$ mode intensity on the excitation energy has already been established for YVO$_4$ [28]. Unfortunately, no complete resonance excitation profile can be constructed for this mode because upon red excitation (633 nm) the region 350–500 cm$^{-1}$ is overshadowed by intense luminescence features stronger than the phonon lines. Still, measurements at other excitation wavelengths in the green region and, possibly at low temperatures, could provide ultimate confirmation of our E$_g$(4) mode assignment.

4. Conclusions

Large single crystals of lutetium vanadate (LuVO$_4$) have been successfully grown by the high-temperature solution method. Group-theoretical analysis is performed and the determination of the various types of Raman active modes is demonstrated. An assignment of three controversial Raman lines of LuVO$_4$ with exceptionally low scattering intensity was achieved by investigation of polarized rotational-angle dependent Raman spectra. Thus, the correct assignment of all twelve first-order Raman active phonons of LuVO$_4$ is completed. We also show how details of the crystal structure of LuVO$_4$ are related to crystal- and correlation-field splitting effects in its vibrational spectrum.

Author Contributions: This article was coordinated by J.-Y.J., most of the text was written by P.R., who also performed the Raman measurements, with all authors giving input throughout. In particular, D.D. wrote most of the Introduction and Materials and Method, and provided the crystal samples, Y.-F.C. and C.-S.L. performed XRD measurements and their data interpretation. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Ministry of Science and Technology, Taiwan, grant numbers MOST 106-2112-M-009-013-MY3 and MOST 108-2811-M-009-538” and by Bulgarian National Fund (BNF) project DCOST 01/2.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Maunier, C.; Douglas, J.L.; Moncorge, R.; Speghini, A.; Bettinelli, M.; Cavalli, E. Growth, spectroscopic characterization, and laser performance of Nd: LuVO$_4$ a new infrared laser material that is suitable for diode pumping. J. Opt. Soc. Am. B 2002, 19, 1794–1800. [CrossRef]

2. Zhang, H.; Liu, J.; Wang, J.; Xu, X.; Jiang, M. Continuous-wave laser performance of Nd:LuVO$_4$ crystal operating at 1.34 μm. Appl. Opt. 2005, 44, 7439–7441. [CrossRef] [PubMed]

3. Liu, J.F.; Li, Y.D. General synthesis of colloidal rare earth orthovanadate nanocrystals. J. Mater. Chem. 2007, 17, 1797–1803. [CrossRef]

4. Tsipis, E.V.; Patrakeev, M.V.; Kharton, V.V.; Vyshatko, N.P.; Frade, J.R. Ionic and p-type electronic transport in zircon-type Ce$_{1-x}$A$_x$VO$_4$±δ (A = Ca, Sr). J. Mater. Chem. 2002, 12, 3738–3745. [CrossRef]

5. Yu, M.; Lin, J.; Wang, S.B. Effects of x and R$^{3+}$ on the luminescent properties of Eu$^{3+}$ in nanocrystalline Y$_2$P$_2$O$_7$:Eu$^{3+}$ and RVO$_4$:Eu$^{3+}$ thin-film phosphors. Appl. Phys. A 2005, 80, 353–360. [CrossRef]

6. Au, C.T.; Zhang, W.D. Oxidative dehydrogenation of propane over rare-earth orthovanadates. J. Chem. Soc. Faraday Trans. 1997, 93, 1195–1204. [CrossRef]

7. Dimitrov, D.Z.; Rafailov, P.M.; Chen, Y.F.; Lee, C.S.; Todorov, R.; Jiang, J.Y. Growth and characterization of LuVO$_4$ single crystals. J. Cryst. Growth. 2017, 473, 34–38. [CrossRef]

8. Liu, J.; Mateos, X.; Zhang, H.; Wang, J.; Jiang, M.; Griebner, U.; Petrov, V. Continuous-wave laser operation of Yb:LuVO$_4$. Opt. Lett. 2005, 30, 3162–3164. [CrossRef]

9. Kaminskii, A.A.; Rhee, H.; Eichler, H.J.; Ueda, K.; Oka, K.; Shibata, H. New χ(3)-nonlinear-laser manifestations in tetragonal LuVO$_4$ crystal: More than sesqui-octave Raman-induced Stokes and anti-Stokes comb generation and cascaded self-frequency “tripling”. Appl. Phys. B 2008, 93, 865–872. [CrossRef]
10. Kaminskii, A.A.; Lux, O.; Rhee, H.; Eichler, H.J.; Ueda, K.; Yoneda, H.; Shirakawa, A.; Zhao, B.; Chen, J.; Dong, J.; et al. New manifestations of $\chi^3$-nonlinear laser interactions in tetragonal LuVO$_4$ and YbVO$_4$ crystals attractive for SRS-converters and self-Raman lasers. Laser Phys. Lett. 2012, 9, 879–887. [CrossRef]
11. Dimitrov, D.; Rafaie, P.; Marinova, V.; Babeva, T.; Goovaerts, E.; Chen, Y.F.; Lee, C.S.; Juang, J.Y. Structural and optical properties of LuVO$_4$ single crystals. J. Phys. Conf. Ser. 2017, 794, 012029. [CrossRef]
12. Chakoumakos, B.C.; Abraham, M.M.; Boatner, L.A. Crystal Structure Refinements of Zircon-Type MVO$_4$ (M = Sc, Y, Ce, Pr, Nd, Tb, Ho, Er, Tm, Yb, Lu). J. Solid State Chem. 1994, 109, 197–202. [CrossRef]
13. Momma, K.; Iizumi, F. VESTA 3 for three-dimensional visualization of crystal, volumetric and morphology data. J. Appl. Cryst. 2011, 44, 1272–1276. [CrossRef]
14. Rousseau, D.L.; Bauman, R.P.; Porto, S.P.S. Normal mode determination in crystals. J. Raman Spectrosc. 1981, 10, 253–290. [CrossRef]
15. Zhizhin, G.N.; Mavrin, B.N.; Shabanov, V.F. Optical Vibrational Spectra of Crystals; Nauka: Moscow, Russia, 1984.
16. Miller, S.A.; Caspers, H.H.; Rast, H.E. Lattice Vibrations of Yttrium Vanadate. Phys. Rev. 1968, 168, 964–969. [CrossRef]
17. Elliott, R.J.; Harley, R.T.; Hayes, W.; Smith, S.R.P. Raman scattering and theoretical studies of Jahn-Teller induced phase transitions in some rare-earth compounds. Proc. R. Soc. Lond. Ser. A 1972, 328, 217–266.
18. Armbruster, A. Infrared reflection studies on the phosphates, arsenates and vanadates of lutetium and yttrium. J. Optoelectron. Adv. Mat. 2005, 7, 473–476.
19. Peppi, S.; Sapienza, P.; van der Beek, A.; Chelucci, A.; Giannini, F.; Mezzetti, M. Nonlinear optical properties of LuVO$_4$ single crystals. J. Raman Spectrosc. 2006, 37, 321–327. [CrossRef]
20. Santos, C.C.; Silva, E.N.; Ayala, A.P.; Guedes, I.; Pizani, P.S.; Loong, C.K.; Boatner, L.A. Raman investigations of rare earth orthovanadates. J. Appl. Phys. 2007, 101, 053511. [CrossRef]
21. Basiev, T.T.; Sobol, A.A.; Zverev, P.G.; Osiko, V.V.; Powell, R.C. Comparative spontaneous Raman spectroscopy of crystals for Raman lasers. Appl. Opt. 1999, 38, 594–598. [CrossRef]
22. Guedes, I.; Hirano, Y.; Grimsditch, M.; Wakabayashi, N.; Loong, C.K.; Boatner, L.A. Raman study of phonon modes in ErVO$_4$ single crystals. J. Appl. Phys. 2001, 90, 1843–1846. [CrossRef]
23. Milenov, T.I.; Rafaie, P.M.; Tomov, V.; Nikolova, R.P.; Skumryev, V.; Igartua, J.M.; Madariaga, G.; López, G.A.; Iturbe-Zabalo, E.; Gospodinov, M.M. Growth and characterization of Pb$_3$Ni$_1$Mn$_5$O$_{15}$ single crystal. J. Phys. Condens. Matter. 2011, 23, 156001. [CrossRef]
24. Rafaie, P.M.; Milenov, T.I.; Veleva, M.N.; Thomsen, C.; Gospodinov, M.M. A Raman spectroscopic study of defects in Bi$_2$Ge$_2$O$_{12}$ crystals. J. Optoelectron. Adv. Mat. 2005, 7, 473–476.
25. Levesque, Y. Étude Optique de Matériaux Laser:Yb$_3$(Lu, Nd)$_{1−x}$VO$_4$. Master Thesis, Sherbrooke University, Québec, QC, Canada, 2011.
26. Iturbe-Zabalo, E.; Gospodinov, M.M. Growth and characterization of Pb$_3$Ni$_1$Mn$_5$O$_{15}$ single crystal. J. Phys. Condens. Matter. 2011, 23, 156001. [CrossRef]
27. Rafaie, P.M.; Milenov, T.I.; Veleva, M.N.; Thomsen, C.; Gospodinov, M.M. A Raman spectroscopic study of defects in Bi$_2$Ge$_2$O$_{12}$ crystals. J. Optoelectron. Adv. Mat. 2005, 7, 473–476.
28. Levesque, Y. Étude Optique de Matériaux Laser:Yb$_3$(Lu, Nd)$_{1−x}$VO$_4$. Master Thesis, Sherbrooke University, Québec, QC, Canada, 2011.
29. Le Nguyen, A.D. Polarization Dependence of Two-Photon Transition Intensities in Rare-Earth Doped Crystals. Ph. D. Thesis, Department of Physics, University of California, Berkeley, CA, USA, 1996.
30. Eichhammer, E.; Utikal, T.; Götzinger, S.; Sandoghdar, V. Spectroscopic detection of single Pr$^{3+}$ ions on the $^3H_4-^1D_2$ transition. New J. Phys. 2015, 17, 083018. [CrossRef]
35. Xu, Z.; Bian, S.; Liu, T.; Wang, L.; Gao, Y.; Lian, H.; Lin, J. Self-assembled growth of LuVO$_4$ nanoleaves: Hydrothermal synthesis, morphology evolution, and luminescence properties. *RSC Adv.* **2012**, *2*, 11067–11077. [CrossRef]

36. Voron’ko, Y.K.; Sobol, A.A.; Shukshin, V.E.; Zagumennyi, A.I.; Zavartsev, Y.D.; Kutovoi, S.A. Raman Spectroscopic Study of Structural Disordering in YVO$_4$, GdVO$_4$, and CaWO$_4$ Crystals. *Phys. Solid State* **2009**, *51*, 1886–1893. [CrossRef]

© 2020 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).