Non-Gaussian fixed point candidates in the 4D compact U(1) gauge theories

W. Franzki, J. Jersák*
Institut für Theoretische Physik E, RWTH Aachen, Germany

C. B. Lang
Institut für Theoretische Physik, Karl-Franzens-Universität Graz, Austria

T. Neuhaus
FB8 Physik, BUGH Wuppertal, Germany

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Some interesting nonperturbative properties of the strongly coupled 4D compact U(1) lattice gauge theories, both without and with matter fields, are pointed out. We demonstrate that the pure gauge theory has a non-Gaussian fixed point with $\nu = 0.365(8)$ at the second order confinement-Coulomb phase transition. Thus a non-asymptotic free and nontrivial continuum limit of this theory, and of its various dual equivalents, in particular of a special case of the effective string theory, can be constructed. Including a scalar matter field (compact scalar QED), we confirm the Gaussian behavior at the endpoint of the Higgs phase transition line. In the theory with both scalar and fermion matter fields, we demonstrate the existence of a tricritical point. Here, the chiral symmetry is broken, and the mass of unconfined composite fermions is generated dynamically. Apart from the Goldstone bosons, the spectrum contains also a massive scalar. This resembles the Higgs-Yukawa sector of the SM, albeit of dynamical origin, like the Nambu–Jona-Lasinio model. However, the scaling behavior is different from that in the NJL model and the nonperturbative renormalizability might thus be possible.

I. INTRODUCTION

The only firmly established quantum field theories in four dimensions (4D) are either asymptotically free or so-called trivial theories. Both are defined in the vicinity of Gaussian fixed points. But in abelian gauge theories at strong coupling several candidates for a non-Gaussian fixed point exist. The best known example is the non-compact QED with a chiral phase transition. In this contribution we describe the recent progress in a systematic study of critical behavior in several compact U(1) gauge models on the lattice, both pure and with matter fields. It provides us with an information about the possible fixed point structure of compact QED. (Various results in this report have been obtained by various subsets of the present authors.)

II. PURE GAUGE THEORY

A. Problem of the two-state signal

First we reconsider the oldest candidate for a non-Gaussian fixed point in the 4D lattice field theory, the phase transition between the confinement and the Coulomb phases in the pure compact U(1) gauge theory with Wilson action and extended Wilson action,

$$S = - \sum_P w_P \left[ \beta \cos(\Theta_P) + \gamma \cos(2\Theta_P) \right].$$

(2.1)

Here $w_P = 1$ and $\Theta_P \in [0, 2\pi)$ is the plaquette angle, i.e. the argument of the product of U(1) link variables along a plaquette $P$. Taking $\Theta_P = a^2 g F_{\mu\nu}$, where $a$ is the lattice spacing, and $\beta + 4\gamma = 1/g^2$, one obtains for weak coupling $g$ the usual continuum action $S = \frac{1}{4} \int d^4 x F_{\mu\nu}^2$.

The detailed investigations performed as usual on toroidal lattices were hindered mainly by a weak two-state signal at $\gamma = 0$. This could either be a finite size effect, or it could imply that the phase transition at $\gamma = 0$ is actually of 1st order, preventing a continuum limit there.

In the model with the extended Wilson action, it was found that the confinement-Coulomb phase transition is clearly of 1st order for $\gamma > 0.2$, and weakens with decreasing $\gamma$. Various studies trying to take into account finite size effects suggested that the transition becomes 2nd order at slightly negative $\gamma$, or around $\gamma = 0$. Nevertheless, the two-state signal persisted on finite size lattices even at $\gamma = -0.5$.

We demonstrate that the problems encountered, when considering the continuum limit at this phase transition, can be surmounted. The clues are the observation that the two-state signal disappears on lattices with sphere-like topology, the construction of homogeneous spherical lattices, the use of modern finite size scaling (FSS) analysis techniques, and larger computer resources. A more detailed account of our work, as well as relevant references, can be found in Refs. [5,6].

1The speaker at the Warsaw conference
B. Spherical lattice

Two of the present authors performed simulations at $\gamma = 0$, using the 4D surface of a 5D cubic lattice instead of the torus. They observed that on such lattices with sphere-like topology the two-state signal vanishes \[7,8\]. This suggests that the two-state signal at $\gamma \leq 0$ is related to the nontrivial topology of the toroidal lattice. It has been checked in spin models, that weak two-state signals are not washed out on lattices with sphere-like topology, if they are due to a genuine 1st order transition \[9\]. However, the lattice on the surface of a cube is rather inhomogeneous and causes complex finite size effects, preventing a reliable FSS analysis.

For our present study at $\gamma \leq 0$, we have chosen lattices with sphere-like topology, but make them approximately homogeneous. We use lattices obtained by projecting the 4D surface $SH[N]$ of a 5D cubic lattice $N^5$ onto a concentric 4D sphere. On such a spherical lattice $S[N]$, the curvatures concentrated on the corners, edges, etc., of the original lattice $SH[N]$ are approximately homogenized over the whole sphere by the weight factors

$$w_P = A'_P/A_P$$

in the action (2.1). $A_P$ and $A'_P$ are the areas of the plaquette $P$ on $S[N]$, and of its dual, as on any irregular, e.g. random lattice \[10\].

It has been checked in some spin and gauge models with 2nd order transitions that universality for spherical lattices holds, and that the FSS analysis works very well if $V^{1/D}$ is used as a linear size parameter, $V \equiv \frac{1}{6} \sum P w_P$ being the volume of the sphere $S[N]$ \[9,11\].

C. 2nd order scaling behavior

The measurements have been performed at $\gamma = 0, -0.2, -0.5$ on lattices of the sizes $N = 4, ..., 12$. We find that at the confinement-Coulomb phase transition at strong bare gauge coupling, $g = O(1)$, the model exhibits a 2nd order scaling behavior well described by the values of the correlation length critical exponent $\nu$ in the range $\nu = 0.35 - 0.40$. The FSS behavior of the Fisher zero, specific heat, some cumulants and pseudocritical temperatures has been studied.

The most reliable measurement is provided by the FSS analysis of the first zero $z_0$ of the partition function in the complex plane of the coupling $\beta$ (Fisher zero). Applying the multihistogram reweighting method for its determination, we have studied the approach of $z_0$ to the real axis, shown in Fig. 1. The expected FSS behavior

$$\text{Im } z_0 \propto V^{-1/D\nu}$$

has been used for measuring $\nu$. This has turned out to be superior to – though consistent with – the more common FSS analysis of specific heat and cumulant extrema.

\[\nu = 0.365(8)\].

In Fig. 2 we show that the scaling behavior of the pseudocritical temperatures, which have been determined from several different observables, is consistent with the value (2.4). Further data and technical details are presented in Ref. \[6\].
FIG. 3. Consistency of the FSS of various pseudocritical temperatures with the value $\nu = 0.365$.

D. Physical implications

The results for $\nu$ are quite different from $\nu = 0.25$, expected at a 1st order transition, as well as from $\nu = 0.5$, obtained in a Gaussian theory or in the mean field approximation. This strongly suggests the existence of a continuum pure U(1) gauge theory with properties different from theories governed by Gaussian fixed points with or without logarithmic corrections. It can be obtained from the lattice theory by the RG techniques. To our knowledge, the existence of such a continuum quantum field theory in 4D is in no way indicated by the perturbation expansion.

The physical content of the continuum limit of the pure compact U(1) gauge theory at the confinement-Coulomb phase transition depends on the phase from which the critical line is approached. In the confinement phase, a confining theory with monopole condensate is expected, as the string tension scales with a critical exponent consistent with the value (2.4) [12]. The physical spectrum consists of various gauge balls, whose spectrum is currently under investigation [13]. In the Coulomb phase, massless photon and massive magnetic monopoles should be present. The renormalized electric charge $e_r$ is large but finite [12], and has presumably a universal value. The numerical result $e_r^2/4\pi = 0.20(2)$ [12] agrees with the Lüscher bound.

Since the pure U(1) lattice gauge theory with the Villain (periodic Gaussian) action presumably belongs to the same universality class, rigorous dual relationships imply that also the following 4D models possess a continuum limit described by the same fixed point: the Coulomb gas of monopole loops [14], the noncompact U(1) Higgs model at large negative squared bare mass (frozen 4D superconductor) [15][16], and an effective string theory equivalent to this Higgs model [17][18].

These findings raise once again the question, whether in strongly interacting 4D gauge field theories further non-Gaussian fixed points exist, that might possibly be of interest for theories beyond the standard model.

The pursuit of this question requires an introduction of matter fields. Therefore we investigate, what extensions of the pure compact U(1) gauge theory might hide interesting fixed points. We introduce fermion and scalar matter fields of unit charge, either each separately or both simultaneously.

III. STATUS OF COMPACT QED WITH STAGGERED FERMIONS

Similar to the pure compact U(1) gauge theory, also the compact QED with staggered fermions on the lattice is known to have both the confinement and the Coulomb phases. In the strongly coupled confinement phase the chiral symmetry is broken in a similar way as in QCD. The possibility to construct the continuum limit in this phase would thus imply the existence of an abelian continuum theory with many phenomena analogous to QCD. However, it would be strongly interacting at short distances.

The existence of the continuum limit when approaching the phase transition between both phases from the Coulomb phase would mean that the usual QED can be extended to strong coupling in such a way that a new, non-Gaussian fixed point is encountered. This might solve the old problem of the Landau pole and of the triviality of QED, suggested by the perturbation theory.

These considerations represent a strong impetus to investigate the phase transition in the compact QED with staggered fermions. Earlier investigations using the Wilson action for the gauge field found a strong 1st order transition, however [19]. This does not allow to perform the continuum limit, and therefore the compact theory has been abandoned and the noncompact one preferred. Later it has been pointed out [20] that, similar to the pure gauge theory, the strength of the 1st order transition decreases with decreasing $\gamma$ if the extended Wilson action is used, and that a 2nd order transition can be found at sufficiently large negative $\gamma$.

We have tried to check this conjecture [13]. With better statistics than in [21] the appearance of the 2nd order could not be confirmed. However, the weakening of the two-state signal is a fact. Of course, in all these investigations the toroidal lattices were used. The experience with the pure gauge theory suggests that the use of the spherical lattice might reveal a 2nd order. Unfortunately,
because of the irregularities on the lattice constant scale, the introduction of the staggered fermions on the spherical lattice is as yet an open problem.

Nevertheless, assuming that at $\gamma < 0$ a 2nd order part of the confinement-Coulomb phase transition can be found, one can investigate its properties in the quenched approximation. For this purpose it is sufficient to use toroidal lattices and stay at some distance from the phase transition line at $\gamma = -0.2$, in order to avoid the spurious two-state signal. First results [3] suggest that also the meson masses in the confinement phase scale in a similar way as the pure gauge observables. They might thus survive in the continuum limit, which might be nontrivial.

IV. ENDPOINT IN THE COMPACT SCALAR QED

A natural step in the search for critical points in the strongly coupled compact QED is to introduce a scalar field $\phi$ of unit charge:

$$S_{U\phi} = S_U + S_{\phi}$$
$$S_U = \beta \sum_P (1 - \text{Re} U_P)$$
$$S_{\phi} = -\kappa \sum_x \sum_{\mu=1}^2 (\phi^\dagger_U U_{x,\mu} \phi_{x+\mu} + h.c.).$$

The $|\phi| = 1$ constraint does not restrict the model qualitatively.

As is well known, when the hopping parameter $\kappa$ increases, the scalar field induces a phase transition. At weak gauge coupling, $\beta > 1$, this is the Higgs phase transition between the Coulomb phase and the Higgs region of the confinement-Higgs phase. Its numerical investigation in the eighties [21,22] strongly suggested that it is of weak first order, and thus does not influence the triviality of the Higgs sector of the standard model.

A possible candidate for the continuum limit remained to be the critical endpoint $E_\infty$ of the Higgs phase transition line at low $\beta$. Early analytic estimates and a numerical investigation of the scaling along the first order phase transition line separating the confinement and the Higgs regions [23] suggested the mean field value $\nu = 1/2$.

We have confirmed recently this result by means of a study of the Fisher zero in the vicinity of $E_\infty$ [24,25]. Instead on the 1st order phase transition line we have approached this point in the $\kappa$ direction at fixed $\beta = 0.848$. One must take into account that the divergence of the correlation length $\xi$ is then determined by the critical exponent $\tilde{\nu}$,

$$\xi \propto (\kappa - \kappa_c)^{-\tilde{\nu}},$$

which is different from the standard $\nu$ defined along the 1st order line or its continuation. This is analogous to the situation e.g. in the Ising model, when the critical point is approached so that both the temperature and the external magnetic field vary simultaneously. The equation of state predicts $\tilde{\nu} = \nu/(\beta + \gamma)$. The mean-field values $\beta = 1/2, \gamma = 1$ and $\nu = 1/2$ lead then to the prediction for a Gaussian fixed point value $\tilde{\nu} = 1/3$.

The Fisher zero $z_0$ in the complex $\kappa$ plane scales as

$$\text{Im}(z_0) = A \cdot L^{-1/\tilde{\nu}}.$$ (4.2)

The data for $\text{Im}(z_0)$ are shown in Fig. 4. The fit gives $\tilde{\nu} = 0.324(2)$ and is thus consistent with the mean-field value. An analogous study in the $\beta$ direction results in $\tilde{\nu} = 0.322(2)$. Similar results have been obtained some years ago when the endpoint was approached in the SU(2) Higgs model [26,27].

Thus we have confirmed by means of the FSS of the Fisher zero the Gaussian character of the endpoint in the compact scalar QED. Simultaneously, we have checked the applicability of the method in the situation when the critical point is approached on some general path in a multiparameter space. This we expect to be of much use in the study of the tricritical point described in the next section.

![FIG. 4. FSS of $\text{Im}(z_0)$ at the endpoint in the scalar QED.](image)

V. COMPACT QED WITH FERMIONS AND SCALARS OF UNIT CHARGE

A. Action and phase diagram

The models discussed until now are presumably only of academic interest for the 4D QFT. In order to create a situation more realistic for particle physics we now introduce into the compact U(1) gauge theory simultaneously the fermion $\chi$, and the scalar $\phi$ of unit charges. The main motivation is the possibility to break spontaneously a chiral symmetry and to obtain a massive and unconfined fermion $F = \phi^\dagger \chi$. This resembles the Higgs-Yukawa sector of the SM, but the symmetry breaking is of dynamical origin. The idea is described in Ref. [28]
and the earlier results obtained during its pursuit are presented in Refs. [29–34].

The action of this “χUφ4 model” is

\[ S_{\chi U \phi} = S_{\chi} + S_{U} + S_{\phi} \]

\[ S_{\chi} = \frac{1}{2} \sum_{x} \sum_{\mu=1}^{2} \eta_{x\mu} (U_{x,\mu} \chi_{x+\mu} - U_{x-\mu,\mu} \chi_{x-\mu}) + a m_0 \sum_{x} \chi_{x} \]

\[ S_{U} = \beta \sum_{P} (1 - \text{Re} U_{P}) \]

\[ S_{\phi} = -\kappa \sum_{x} \sum_{\mu=1}^{2} \phi_{x}^{\dagger} U_{x,\mu} \phi_{x+\mu} + h.c. \].

The model has a global U(1) chiral symmetry in the limit case \( m_0 = 0 \).

The schematic phase diagram is shown in Fig. 5. We recognize the models previously discussed as special cases of the \( \chi U \phi_4 \) model:

- At \( \kappa = 0 \) and \( a m_0 = \infty \), the pure gauge theory with the Wilson action (\( \gamma = 0 \)).
- At \( \kappa = 0 \) and \( a m_0 \) finite, the gauge theory with fermions.
- At \( a m_0 = \infty \) and \( \kappa \) arbitrary, the scalar QED.

The \( \beta = 0 \) case corresponds to the Nambu–Jona-Lasinio (NJL) model, as the bosonic fields can be integrated out [35].

At strong coupling, \( \beta < 1 \), the model has three sheets of 1st order phase transitions: The two “wings”, separating the confinement and Higgs regions also at finite \( a m_0 \), and the sheet at \( a m_0 = 0 \), separating the regions with nonzero chiral condensate of opposite sign. These three sheets have critical boundary lines \( E_{T} \) and \( E_{C} \), respectively. We have verified that within the numerical accuracy these 2nd order phase transition lines indeed intersect at one point, the tricritical point E. There is no known theoretical argument why this should be so.

To illustrate the existence of the tricritical point, we show in Fig. 6 the positions at small \( a m_0 \) of the chiral phase transition line (NETC), and of the Higgs phase transition line (ET and its continuation to larger \( \beta \)). Both transitions coincide on the ET line. There is also no cusp at E (a cusp would exclude E being a tricritical point).

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FIG. 6. Coincidence of the chiral and Higgs phase transitions on the line ET.

B. Spectrum in the Nambu phase

Of most interest is the Nambu phase at \( m_0 = 0 \), at small \( \beta \) and \( \kappa \). Because of confinement there is no \( \phi \)-boson, i.e. charged scalar, in the spectrum. There is also no fundamental \( \chi \)-fermion. The chiral symmetry is dynamically broken, which leads to the presence of the composite physical fermion \( F = \phi^{\dagger} \chi \) with the mass \( a m_F > 0 \). It scales, \( a m_F \downarrow 0 \), when the NE line is approached.

Further states include the “mesons”, i.e. the fermion-antifermion bound states: the Goldstone boson \( \pi \) with \( m_\pi \propto \sqrt{am_0} \), the scalar \( \sigma \), and the vector \( \rho \). Also “bosons”, the scalar-antiscalar bound states are present; the scalar \( S = \phi^{\dagger} \phi \) and the vector of a similar structure. Their mixing with mesons is expected. Finally there are some gauge-balls.

The mass \( a m_S \) of the \( S \)-boson vanishes on the lines \( E_{T} \) and \( E_{C} \), whereas \( a m_F \) vanishes at \( a m_0 = 0 \) on the line \( E_{T} \) and above it. This provides another check of the crossing of these three lines at the tricritical point. In Figs. 5 and 6 we show the minima of \( a m_S \) and the approximate vanishing of \( a m_F \), respectively, at three values of \( a m_0 \). Their positions shift in \( \beta \), as \( a m_0 \) decreases, but approach the same value at \( a m_0 = 0 \), the approximate \( \beta \) coordinate of the tricritical point.
C. Question of the continuum limit

In principle, the continuum limit can be considered at any point along the whole NE line. In the Nambu phase a massive fermion F could be expected. The question is whether the model is renormalizable. For this the existence of the tricritical point might be crucial.

We know that at $\beta = 0$ the model is equivalent to the nonrenormalizable NJL model [36]. This model, properly generalized, belongs to the universality class of the Gaussian fixed point of the 4D Yukawa theory. Strong coupling expansion in powers of $\beta$ suggests similar scaling properties for a finite $\beta$ interval. Indeed, it has been checked [32] that the scaling behavior nearly on the whole NE line is very similar to that at the point N at $\beta = 0$. However, it changes qualitatively in the vicinity of the point E.

This raises the hope that the renormalizability properties at the point E are different from the NJL model. This conjecture is supported by the experience that the tricritical points belong frequently to the universality classes different from those of the adjacent critical lines.

Furthermore, at the point E a new diverging correlation length $\xi_S = 1/am_S$ exists, which is finite along the rest of the NE line. In Fig. 7 we show that the ratio $am_S/am_F$ seems to approach a constant value when the fermion mass $am_F$ decreases in the vicinity of the point E. Thus, the continuum limit taken at the tricritical point contains presumably at least two massive particles, together with the massless Goldstone boson.

This, and some further results on the spectrum [24, 25], suggest that the tricritical point E does not belong to the universality class of the Gaussian fixed point of the Yukawa model with the same symmetries, and thus merits further investigation. For this purpose the determination of the tricritical exponent $\nu$ by means of the Fisher zero, as in the pure gauge theory and in the scalar QED, might be most suitable. However, with dynamical fermions this still represents a challenge.

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