RESEARCH PAPER

Determination of the Astrophysical S-factor and Thermonuclear Reaction Rates of the (α,n) Medium Elements Reactions

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ABSTRACT:

Cross-sections of the (α,n) medium elements reactions as a function of energies of alpha (α)-particle such as $^{45}$Sc(α,n)$^{48}$V, $^{48}$Ti(α,n)$^{51}$Cr, $^{51}$V(α,n)$^{54}$Mn, $^{50}$Cr(α,n)$^{53}$Fe, $^{53}$V(α,n)$^{56}$Co, $^{53}$Ni(α,n)$^{56}$Cu, $^{54}$Mn(α,n)$^{58}$Co, $^{55}$Mn(α,n)$^{58}$Co, and $^{66}$Zn(α,n)$^{69}$Ge have been interpolated from threshold to 10 MeV in step of 0.05 MeV by using the Program of MATLAB. Weighted averages of the cross-sections in (mb) have been utilized to calculate the astrophysical S-factor and thermonuclear reaction rates as a function of the energy of the center of mass, $E_{c.m.}$ and $T_9$ Which is the temperature in units of $10^9$K ($T_9 = 10^{-9^\circ}T$) respectively. Polynomial relationships have been utilized to fit the computed astrophysical S-factor and thermonuclear reaction rates to determine the astrophysical S-factor at various $E_{c.m.}$ and thermonuclear reaction rates at various $T_9$ from best fitting equations with the minimum Chi-Square. Empirical formulae of set of reactions $^{45}$Sc(α,n)$^{48}$V, $^{48}$Ti(α,n)$^{51}$Cr, $^{51}$V(α,n)$^{54}$Mn, $^{50}$Cr(α,n)$^{53}$Fe, $^{53}$Co(α,n)$^{56}$Cu, $^{54}$Ni(α,n)$^{56}$Zn, $^{55}$Co(α,n)$^{58}$Co, and $^{66}$Zn(α,n)$^{69}$Ge have been utilized to compute astrophysical S-factor as a function of $E_{c.m.}$ and $Z$ and thermonuclear reaction rates as a function of $T_9$ and the target nucleus atomic number $Z$. The results have been compared with the embraced astrophysical S-factor and thermonuclear reaction rates that have been calculated from the fitting equations which have a good agreement.

KEY WORDS: Cross-sections; astrophysical S-factor; thermonuclear reaction rates; Gamow factor; Gamow energy; Sommerfeld parameter.

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INTRODUCTION:

The astrophysical S-factor, $S(E)$, has covered a large area which used in the field to remove the energy dependence of the Coulomb barrier penetration from the cross-section, $\sigma(E)$ (Jose, 2016). As stellar energies are much lower than the Coulomb barrier, the cross sections hardly depend on energy (Descouvemont, 2011).

Thermonuclear reactions play an important role in supplying the major source of energy in stars in particular during hydrogen burning. This burning process in the stellar interiors consists of the proton-proton (pp) chain and the carbon-nitrogen-oxygen (CNO) cycle (Abdul Aziz, 2008). The quantity of interest in computing thermonuclear reaction rates for astrophysical aims is $N_A<\sigma>\nu$, which is the product of Avogadro’s number with the average value of the cross section times velocity, averaged over a Maxwell-Boltzmann distribution of temperature (Roughton et al., 1983). Total Cross-sections of
the \((\alpha,n)\) medium element reactions, that is a function of center of mass energy, have been calculated by a few authors, which are reminded by various references such as \(^{45}\)Sc\((\alpha,n)\)^{48}V (Vlieks, Morgan and Blatt, 1974; Hansper et al., 1989; Haider, 2012), \(^{48}\)Ti\((\alpha,n)\)^{51}Cr (Chang et al., 1973; Vonach, Haight and Winkler, 1983; Levkovski, 1991; Morton et al., 1992; Baglin, Coral et al., 2004), \(^{51}\)V\((\alpha,n)\)^{54}Mn (Levkovski, 1991; Hansper et al., 1993; Sonzogni et al., 1993; Peng, He and Long, 1999; Noori, 2008; Haider, 2012), \(^{50}\)Cr\((\alpha,n)\)^{53}Fe (Vlieks, Morgan and Blatt, 1974; Morton et al., 1994; Haider, 2012), \(^{55}\)Mn\((\alpha,n)\)^{58}Co (Rizvi et al., 1989; Levkovski, 1991; Tims et al., 1993; Haider, 2012), \(^{54}\)Fe\((\alpha,n)\)^{57}Ni (Houck and Miller, 1961; Vlieks, Morgan and Blatt, 1974; Tims et al., 1991; Haider, 2012), \(^{58}\)Co\((\alpha,n)\)^{62}Cu (Stelson and McGowan, 1964; D’auria et al., 1968; Zhukova et al., 1972; Tims et al., 1988; Noori, 2008), \(^{62}\)Ni\((\alpha,n)\)^{65}Zn (Stelson and McGowan, 1964; Levkovski, 1991; Haider, 2012), \(^{62}\)Cu\((\alpha,n)\)^{66}Ga (Stelson and McGowan, 1964; Zhukova et al., 1970; Haider, 2012), and \(^{66}\)Zn\((\alpha,n)\)^{69}Ge (Stelson and McGowan, 1964; Levkovski, 1991) respectively. The goal of this work is to determine the empirical formulae to compute the astrophysical S-factor, \(S(E)\), and thermonuclear reaction rates, \(N_{\Lambda} < \sigma v >\), utilizing the altered cross-sections of the reaction of the medium elements. The outcomes were compared with those published in the previous work.

2. Theory

Atomic masses of each medium element and isotopes related to this present work have been taken from the nuclear wallet cards published by the National Nuclear Data Center (NNDC) (Tuli, 2011). The \(Q\)-Value of the reaction \(X(\alpha, n)Y\), is defined as the difference between the initial and the final rest mass energies (Meyerhof, 1967):

\[
Q = [M_\alpha + M_X - (M_Y + M_n)c^2]
\]

(1)

Where \((M_\alpha, M_X, M_Y, \text{and } M_n)\) are the atomic masses of the incident, target particles, product nucleus and neutron (outgoing particle), respectively and \((c^2 = 931.494013 \text{ MeV/u})\); where u=atomic mass unit (amu) =1.66x10\(^{-27}\) kg). This equation is called the \(Q\)-value equation. If \(Q\) is positive, the reaction called exoergic; if \(Q\) is negative, it is endoergic.

The amount of energy needed for an endoergic reaction is called the threshold energy and can be calculated easily (Kaplan, 1962).

\[
E_{\text{th}} = -Q(1 + \frac{M_\alpha}{M_X})
\]

(2)

Fusion requires two (or more) interacting particles to approach closely enough, within the short range of the (attractive) strong nuclear force, \(\lesssim 10^{-15}\) m, to construct a new nucleus with \(A = A_1 + A_2\). The so-called height \(V_c\) of the barrier is its maximum value, which occurs at the nuclear radius, and is (Evans, 1955).

\[
V_c = \frac{Z_1Z_2e^2}{R}
\]

(3)

Where \(Z_1\) and \(Z_2\) are the charges of the projectile and target nuclei, and \(R\) and \((R = R_1 + R_2)\) is their separation, \(e\) is the charge of electron \((e^2 = 1.44 \text{ MeV fm})\), and the radius of the nucleus is given by \(R = 1.3 \times 10^{-13}\) \(A^{1/3}\) \(\text{cm}\), where \(A\) is the mass number (atomic weight) (Shaviv, 2012). Then Eq. (3) leads to

\[
V_c = E_c = \frac{1.44}{13} \left( \frac{Z_1Z_2}{A_1^{1/3} + A_2^{1/3}} \right)
\]

(4)

Where \(E_c\) is the coulomb barrier (Coulomb energy) in MeV, \(A_1^{1/3}\) and \(A_2^{1/3}\) are the mass numbers of the charges of bombarding and targeting nuclei respectively.

The astrophysical S-factor, \(S(E)\), in the unit \((\text{MeV-b})\) is related to the cross-section by (Li, J. et al., 2012):

\[
S(E) = E\sigma(E) \exp(2\pi\eta)
\]

(5)

Where \(E\) is the center-of-mass energy \((E_{\text{c.m.}})\) in MeV, \(\sigma(E)\) is the cross-section of the reaction in \((\text{mb})\), \(2\pi\eta\) is the Gamow factor, and \(\eta\) is Sommerfeld parameter (Angulo et al., 1999):

\[
\eta = \frac{Z_1Z_2e^2}{h\nu} = 0.1575Z_1Z_2\sqrt{\frac{\mu(\mu)}{E(\text{MeV})}}
\]

(6)

\(\hbar\) is Planck’s constant over 2\(\pi (1.0546 \times 10^{-27}\) ergs), \(\nu\) is the relative velocity, \(\mu\) is the reduced mass. The Gamow factor \(G\) (E) or \(2\pi\eta\) can be written as in (Jose, 2016):

\[
2\pi\eta = 0.98951Z_1Z_2\sqrt{\frac{\mu(\mu)}{E(\text{MeV})}}
\]

(7)

The reduced mass \(\mu\) in u (amu) is determined by the relationship (Clayton, 1968):

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\[ \mu = \frac{m_1m_2}{m_1+m_2} \]  \hskip1cm (8)

Where \( m_1 \) and \( m_2 \) represent the masses of the bombarding and target nucleus in units of (amu), respectively. The energy of the center of mass of pair of particles \( E_{\text{c.m.}} \) is related to the laboratory energy, \( E_{\text{lab.}} \) of the projectile particle by the equation (Meyerhof, 1967):

\[ E_{\text{c.m.}} = \frac{m_2}{m_1+m_2} E_{\text{lab.}} \]  \hskip1cm (9)

The Gamow energy \( E_G \), in MeV (Brown, 2015):

\[ E_G = 2\pi^2\mu C^2\alpha^2(Z_1Z_2)^2 = 0.979\mu(Z_1Z_2)^2 \]  \hskip1cm (10)

Where \( \alpha = \frac{1}{137} = \frac{e^2}{\hbar c} \) is the fine-structure constant.

The thermonuclear reaction rates, \( N_A(\sigma\nu) \) in unit \((cm^3 mol^{-1} s^{-1})\) (Angulo et al., 1999):

\[ N_A(\sigma\nu) = \left(\frac{8}{\mu}\right)^{1/2} \frac{1}{(k_B T)^{3/2}} N_A \int_{0}^\infty E\sigma(\nu) \exp(-E/k_B T) dE \]  \hskip1cm (11)

Where \( N_A \) is the Avogadro’s number \((6.022 \times 10^{23} mol^{-1})\), \( k_B \) is the Boltzmann’s constant \((1.38 \times 10^{-16} \text{erg}/K)\), and \( T \) is the temperature respectively. Eq. (11) leads to (Angulo et al., 1999):

\[ N_A(\sigma\nu) = 3.7313 \times 10^{16} u^{-1/2} T_9^{-3/2} \int_{0}^{\infty} E\sigma(\nu) \exp(-11.605E/T_9) dE \]  \hskip1cm (12)

Where \( T_9 \) is the temperature in units of \(10^9K\) \((T_9 = 10^{-9}T)\)

The weighted averages of the Cross-sections of medium elements \( \sigma_0(mb) \) and the uncertainty (errors) \( \Delta\sigma_0(mb) \) are expressed by the following Eqs. (Bevington and Robinson, 2003):

\[ \sigma_0(mb) = \frac{\sum i(\sigma_i/\delta_i^2)}{\sum i(1/\delta_i^2)} \]  \hskip1cm (13)

Where \( \sigma_i \) and \( \delta_i(\Delta\sigma_i) \) are the cross-section and the uncertainties of \( i^{\text{th}} \) reference, relating to each value of \( \sigma_i \),

\[ \Delta\sigma_0(mb) = \pm \sqrt{\sum_i(1/\delta_i^2)} \]  \hskip1cm (14)

The considered formalism type is the polynomial fit expression of the shape:

\[ Y = C_0 + C_1X + C_2X^2 + C_3X^3 + \ldots + C_NX^N = \sum_{i=0}^{N} C_iX^i \]  \hskip1cm (15)

This polynomial is obtained by the Excel computer program (Format Trendline). Where \((C_0, C_1, C_2, C_3, \ldots)\) are free parameters (coefficients of polynomial), and \((i = 0, 1, 2, 3, \ldots, M)\), and

\[ C_i = \sum_{j=0}^{N} C_{ij}K^j \]  \hskip1cm (16)

Are considered in this work, then by combining the Eqs. (15) & (16), the following relation has been acquired:

\[ Y = \sum_{i=0}^{M} \left( \sum_{j=0}^{N} C_{ij}K^j \right)X^i \]  \hskip1cm (17)

Where \( Y = \ln[S(E)] \) or \( \ln[N_A(\sigma\nu)] \), \((i=0,1,2,\ldots M), \) \((j=0,1,2,\ldots N), (C_{00}, C_{01}, C_{02}, \ldots)\) are coefficients of polynomials, \( K \) is the energy of the center of mass or \( T_9 \) according to the S(E) or \( N_A(\sigma\nu) \), and \( X \) is atomic number \( Z \). The Excel computer program has been utilized to acquire the best fit relationship corresponding to various energies ranges near threshold up to \(10 MeV\) in the center of mass system or \( T_9 \) ranges from \((1 \to 10) \times 10^9K\).

The data of these extents were avoided in each step, till a possible value of the determination coefficient \( R^2 \approx 1 \) was come to. The best fit adopted data was acquired with increasing order to supply the minimum value of Chi-Square (\( \chi^2 \)) by using the Eq. (Belgaid et al., 2005):

\[ \chi^2 = \frac{1}{(N-M)} \sum_{i}^{N} \left( \frac{Y_{\text{exp}}^i - Y_{\text{cal}}^i}{\Delta Y_{\text{exp}}^i} \right)^2 \]  \hskip1cm (18)

Where \( N \) is the data points’ number, \( M \) is the fitting coefficients number, \( Y_{\text{exp}}^i \) and \( \Delta Y_{\text{exp}}^i \) are the experimental (adopted value) of \( \ln[S(E)] \) or \( \ln[N_A(\sigma\nu)] \) and its error (uncertainty) respectively, \( Y_{\text{cal}}^i \) is the calculated \( \ln[S(E)] \) or \( \ln[N_A(\sigma\nu)] \).

3. Data Reduction and Analysis

The Atomic masses have been taken into consideration to determine the Q-Value, threshold energy, Coulomb barrier, reduced mass, and the ratio between \((E_{\text{c.m.}}/E_{\text{lab.}})\) of \((\alpha, n)\) medium elements reactions using the Eqs. (1, 2, 4, 8, and 9); the results have been shown in the table (1). Eqs. (6,7,10, and 5) taken into consideration to
determine the Sommerfeld parameter($\eta$), Gamow factor G(E), Gamow energy ($E_G$), and the S-factor of astrophysical, S(E) of the ($\alpha$,n) medium element reactions. The results are shown in table (2). The cross-sections of ($\alpha$,n) reactions of medium elements in present work such as ($^{45}$Sc, $^{48}$Ti, $^{51}$V, $^{50}$Cr, $^{55}$Mn, $^{54}$Fe, $^{59}$Co, $^{62}$Ni, $^{63}$Cu, and $^{66}$Zn), which are available in the literature review has been taken and plotted again, and using the MATLAB software to interpolate to acquire the cross-sections in fine step of 0.05 MeV. The weighted average of the altered Cross-sections of $i^{th}$ references for the medium elements which cross-section ($\sigma_0$) and uncertainty ($\Delta\sigma_0$) have been computed by using Eqs. (13) and (14) respectively. The acquired results have been utilized to calculate the astrophysical S-factor and thermonuclear reaction rates of ($\alpha$,n) reactions as a function of the center of mass energies $E_{c.m}$ by using eq. (5) and (12). The acquired equations to compute the S-factor of the reminded reactions are shown in Table 2.

The final formula for each astrophysical S-factor, $S(E)$ and thermonuclear reaction rate $N_A(\sigma v)$ is shown in Eq. (17) where $Y = \ln[S(E)]$ or $Y = \ln[N_A(\sigma v)]$.

**Table 1.** Q-Value, threshold energy ($E_{threshold}$), Coulomb barrier $E_c$, reduced mass ($\mu$), and the ratio between ($E_{c.m.}/Elab.$) of ($\alpha$, n) medium elements reactions.

| ($\alpha$,n) | Medium Element Reaction | Q-value (MeV) | $E_{threshold}$ (MeV) | Coulomb Barrier $E_c$ (MeV) | Reduced Mass ($\mu$) (amu) | $E_{c.m.}/Elab.$ |
|--------------|-------------------------|---------------|-----------------------|---------------------------|---------------------------|-----------------|
| $^{4}$He($\alpha$,n)$^{7}$Li | -2.241E+00 | 2.440E+00 | 2.241E+00 | 9.044E+00 | 3.675E+00 | 9.182E-01 |
| $^{4}$He($\alpha$,n)$^{7}$Be | -2.687E+00 | 2.911E+00 | 2.687E+00 | 9.334E+00 | 3.694E+00 | 9.230E-01 |
| $^{4}$He($\alpha$,n)$^{7}$Be | -2.294E+00 | 2.474E+00 | 2.294E+00 | 9.622E+00 | 3.711E+00 | 9.272E-01 |
| $^{4}$He($\alpha$,n)$^{7}$Be | -4.961E+00 | 5.359E+00 | 4.961E+00 | 1.009E+01 | 3.706E+00 | 9.258E-01 |
| $^{4}$He($\alpha$,n)$^{7}$Be | -3.512E+00 | 3.767E+00 | 3.512E+00 | 1.027E+01 | 3.731E+00 | 9.321E-01 |
| $^{4}$He($\alpha$,n)$^{7}$Be | -5.817E+00 | 6.249E+00 | 5.817E+00 | 1.073E+01 | 3.726E+00 | 9.309E-01 |
| $^{4}$He($\alpha$,n)$^{7}$Be | -5.089E+00 | 5.434E+00 | 5.089E+00 | 1.091E+01 | 3.748E+00 | 9.364E-01 |
| $^{4}$He($\alpha$,n)$^{7}$Be | -6.480E+00 | 6.899E+00 | 6.480E+00 | 1.119E+01 | 3.760E+00 | 9.393E-01 |
| $^{4}$He($\alpha$,n)$^{7}$Be | -7.502E+00 | 7.979E+00 | 7.502E+00 | 1.154E+01 | 3.763E+00 | 9.402E-01 |
| $^{4}$He($\alpha$,n)$^{7}$Ge | -7.445E+00 | 7.897E+00 | 7.445E+00 | 1.181E+01 | 3.774E+00 | 9.428E-01 |

**Table 2.** The Sommerfeld parameter($\eta$), Gamow factor G(E), Gamow energy ($E_G$), and the astrophysical S-factor, S(E) of the ($\alpha$,n) medium elements reactions.

| ($\alpha$,n) | Medium Element Reaction | Sommerfeld Parameter $\eta$ | Gamow factor G(E) | Gamow Energy $E_G$(MeV) | Astrophysical S-factor S(E) |
|--------------|-------------------------|-----------------------------|---------------------|----------------------------|----------------------------|
| $^{4}$He($\alpha$,n)$^{7}$Li | 1.268E+01/$E_{c.m.}$ | 7.967E+01/$E_{c.m.}$ | 6.348E+03 | $E_{c.m.}/(\eta E_{c.m.})$ |
| $^{4}$He($\alpha$,n)$^{7}$Be | 1.331E+01/$E_{c.m.}$ | 8.368E+01/$E_{c.m.}$ | 7.003E+03 | $E_{c.m.}/(\eta E_{c.m.})$ |
| $^{4}$He($\alpha$,n)$^{7}$Be | 1.395E+01/$E_{c.m.}$ | 8.769E+01/$E_{c.m.}$ | 7.689E+03 | $E_{c.m.}/(\eta E_{c.m.})$ |
| $^{4}$He($\alpha$,n)$^{7}$Be | 1.455E+01/$E_{c.m.}$ | 9.143E+01/$E_{c.m.}$ | 8.360E+03 | $E_{c.m.}/(\eta E_{c.m.})$ |
| $^{4}$He($\alpha$,n)$^{7}$Be | 1.502E+01/$E_{c.m.}$ | 9.556E+01/$E_{c.m.}$ | 9.132E+03 | $E_{c.m.}/(\eta E_{c.m.})$ |
| $^{4}$He($\alpha$,n)$^{7}$Be | 1.558E+01/$E_{c.m.}$ | 9.932E+01/$E_{c.m.}$ | 9.865E+03 | $E_{c.m.}/(\eta E_{c.m.})$ |
| $^{4}$He($\alpha$,n)$^{7}$Be | 1.646E+01/$E_{c.m.}$ | 1.034E+02/$E_{c.m.}$ | 1.070E+04 | $E_{c.m.}/(\eta E_{c.m.})$ |
| $^{4}$He($\alpha$,n)$^{7}$Be | 1.707E+01/$E_{c.m.}$ | 1.074E+02/$E_{c.m.}$ | 1.154E+04 | $E_{c.m.}/(\eta E_{c.m.})$ |
| $^{4}$He($\alpha$,n)$^{7}$Be | 1.771E+01/$E_{c.m.}$ | 1.113E+02/$E_{c.m.}$ | 1.240E+04 | $E_{c.m.}/(\eta E_{c.m.})$ |
| $^{4}$He($\alpha$,n)$^{7}$Be | 1.835E+01/$E_{c.m.}$ | 1.153E+02/$E_{c.m.}$ | 1.330E+04 | $E_{c.m.}/(\eta E_{c.m.})$ |

4. Results and Discussion

In general, we can write Eq. (15), and instead of $X$ insert center of mass energies $E_{c.m.}$. Then the Eq. (15) becomes
\[
Y = C_0 + C_1 K + C_2 K^2 + C_3 K^3 + \cdots + C_N K^N \\
= \sum_{i=0}^{N} C_i K^i
\]  
(19)

Where \(C_0, C_1, C_2, \ldots\) are free parameters, \(K\) are parameters that represent the C.M energy or \(T_9\), \((i=0, 1, 2, 3 \ldots M)\), and \(Y=\ln[S\text{-factor (MeV\text{-}b)}]\) or \(Y=\ln[N_A<n\nu>\text{ (cm}^3\text{mol}^{-1}\text{sec}^{-1})]\).

### 4.1. Astrophysical S-factor Empirical Formulae

The adopted astrophysical S-factor has been used to acquire the fitting parameters by using the expressions of the polynomial (18), (20) and (19) as shown in the steps:

1. The polynomial relations which are utilized in eq. (19) to fit the computed astrophysical S-factor, \(S(E)\) in the natural logarithm of the calculated elements to compute the adopted (taken on) natural logarithm of astrophysical S-factor from the best fitting with a minimum \(\chi^2\) using Eq. (20). The acquired best fitting relations of the reminded reactions were presented in Eqs. (20, 21, 22, 23, 24, 25, 26, 27, 28, and 29) for the reactions \(^{45}\text{Sc}(\alpha,\text{n})^{48}\text{V}\), \(^{48}\text{Ti}(\alpha,\text{n})^{51}\text{Cr}\), \(^{51}\text{V}(\alpha,\text{n})^{54}\text{Mn}\), \(^{50}\text{Cr}(\alpha,\text{n})^{53}\text{Fe}\), \(^{55}\text{Mn}(\alpha,\text{n})^{58}\text{Co}\), \(^{54}\text{Fe}(\alpha,\text{n})^{57}\text{Ni}\), \(^{59}\text{Co}(\alpha,\text{n})^{62}\text{Cu}\), \(^{62}\text{Ni}(\alpha,\text{n})^{65}\text{Zn}\), \(^{63}\text{Cu}(\alpha,\text{n})^{66}\text{Ga}\), and \(^{66}\text{Zn}(\alpha,\text{n})^{69}\text{Ge}\) respectively.

\begin{align}
\text{ln}[S\text{-factor (MeV\text{-}b)}] &= 0.0062E^3 - 0.2075E^2 + 1.0183E + 30.7 \\
&= C_0 + C_1 E + C_2 E^2 + C_3 E^3
\end{align}
(20)

\begin{align}
\text{ln}[S\text{-factor (MeV\text{-}b)}] &= -0.0867E^4 + 2.5599E^3 - 28.125E^2 + 135.15E - 206.65 \\
&= C_0 + C_1 E + C_2 E^2 + C_3 E^3 + C_4 E^4
\end{align}
(21)

\begin{align}
\text{ln}[S\text{-factor (MeV\text{-}b)}] &= -0.0261E^3 + 0.4478E^2 - 3.1956E + 41.906 \\
&= C_0 + C_1 E + C_2 E^2 + C_3 E^3
\end{align}
(22)

\begin{align}
\text{ln}[S\text{-factor (MeV\text{-}b)}] &= 0.0763E^3 - 1.9295E^2 + 15.185E - 5.9137 \\
&= C_0 + C_1 E + C_2 E^2 + C_3 E^3
\end{align}
(23)

\begin{align}
\text{ln}[S\text{-factor (MeV\text{-}b)}] &= 0.0153E^3 - 0.4515E^2 + 3.3053E + 28.436 \\
&= C_0 + C_1 E + C_2 E^2 + C_3 E^3
\end{align}
(24)

\begin{align}
\text{ln}[S\text{-factor (MeV\text{-}b)}] &= 0.2583E^3 - 6.0359E^2 + 45.678E - 78.034 \\
&= C_0 + C_1 E + C_2 E^2 + C_3 E^3
\end{align}
(25)

\begin{align}
\text{ln}[S\text{-factor (MeV\text{-}b)}] &= 0.0872E^3 - 2.2052E^2 + 17.564E - 8.1943 \\
&= C_0 + C_1 E + C_2 E^2 + C_3 E^3
\end{align}
(26)

\begin{align}
\text{ln}[S\text{-factor (MeV\text{-}b)}] &= 0.1946E3E^3 - 4.9013E^2 + 40.02E - 69.093 \\
&= C_0 + C_1 E + C_2 E^2 + C_3 E^3
\end{align}
(27)

\begin{align}
\text{ln}[S\text{-factor (MeV\text{-}b)}] &= 2.2587E^3 - 57.223E^2 + 480.3E - 1299.5 \\
&= C_0 + C_1 E + C_2 E^2 + C_3 E^3
\end{align}
(28)

\begin{align}
\text{ln}[S\text{-factor (MeV\text{-}b)}] &= 0.7816E^3 - 20.777E^2 + 183.26E - 498.11 \\
&= C_0 + C_1 E + C_2 E^2 + C_3 E^3
\end{align}
(29)

2. At fixed values of energy in center-of-mass, the change of the S-factor in natural logarithm with the Z has been fitted to the polynomial relation utilizing Eq. (19). The acquired results were used to determine the free parameters (coefficients of polynomial) \((C_i)\).

3. The free parameters \((C_i)\) were plotted against each value of the center of mass energies and fitted to sufficient the polynomial relation were shown in Eq. (16).

4. The last formula of a set of reactions has been calculated by utilizing the combination of the two polynomials to show the systematic manner of the reactions which are shown in Eq. (17). The \(Y\) Variable is the astrophysical S-factor.

### 4.1.1 The Empirical Formulae Relating the Astrophysical S-factor to Center of Mass Energy and the Atomic Number Z of the Target Nucleus

The empirical formulae related to the astrophysical S-factor (MeV-b) with both of center of mass energy \(E_{\text{cm}}\) and the atomic number \(Z\) were performed as the steps below:

1. At fixed values of the center of mass energies from 5.5 to 10 MeV in steps of 0.25 MeV for the \(^{45}\text{Sc}(\alpha,\text{n})^{48}\text{V}\), \(^{48}\text{Ti}(\alpha,\text{n})^{51}\text{Cr}\), \(^{51}\text{V}(\alpha,\text{n})^{54}\text{Mn}\), \(^{50}\text{Cr}(\alpha,\text{n})^{53}\text{Fe}\), \(^{55}\text{Mn}(\alpha,\text{n})^{58}\text{Co}\), \(^{54}\text{Fe}(\alpha,\text{n})^{57}\text{Ni}\), \(^{59}\text{Co}(\alpha,\text{n})^{62}\text{Cu}\), \(^{62}\text{Ni}(\alpha,\text{n})^{65}\text{Zn}\), \(^{63}\text{Cu}(\alpha,\text{n})^{66}\text{Ga}\), and \(^{66}\text{Zn}(\alpha,\text{n})^{69}\text{Ge}\) reactions, the astrophysical S-factor in natural logarithm will vary with the atomic number(Z), as shown in Fig. (1). The data was fitted into the accompanying polynomial expression:

\[
Y = \sum_{i=0}^{2} C_i X^i
\]
(30)

Where \(Y = \ln[S(E)]\), and \(X=Z\), with free parameters \((C_0, C_1, \text{and } C_2)\).

2. The S-factor, \(S(E)\), which is was adopted, has been utilized as a function of atomic number \(Z\) of target nucleus at the fixed center of mass energies using the computer program Excel to acquire the fitting relations and then it was utilized to
compute the fitting parameters. The acquired results are shown in Table 3.

3- The obtained free parameters $C_i$ ($C_0$, $C_1$, and $C_2$), presented in Table (3) are plotted against with the fixed values of center of mass energies from 5.5 to 10 MeV in step of 0.25 MeV as shown in Fig.(2), and then the acquired coefficients of polynomials $C_i$ have been fitted to the polynomial expression below:

$$C_i = \sum_{j=0}^{2} C_{ij}E^j \quad (31)$$

The combination of the two polynomials Eq. (30) and Eq. (31) takes the form of the formula below of energy ranged from 5.5 to 10 MeV in the step of 0.25 MeV:

$$Y = \sum_{i=0}^{2} \left( \sum_{j=0}^{2} C_{ij}E^j \right)X^i \quad (32)$$

Where $Y=\ln[S(E)]$, $X=$atomic number $Z$

$$Y = \sum_{i=0}^{2} \left( C_{i0}E^0 + C_{i1}E^1 + C_{i2}E^2 \right)X^i$$

Where $(C_{00}, C_{01}, C_{02}, C_{10}, C_{11}, C_{12}, \ldots, C_{22})$ are free parameters and their values are shown in the matrix below:

$$\begin{bmatrix}
C_{00} & C_{01} & C_{02} \\
C_{10} & C_{11} & C_{12} \\
C_{20} & C_{21} & C_{22}
\end{bmatrix} = \begin{bmatrix}
-2.0079 & -14.163 & 1.4295 \\
2.5083 & 1.1719 & -0.1305 \\
-0.0466 & -0.0217 & 0.0027
\end{bmatrix}, \quad R^2 = 0.6757$$

$$\begin{bmatrix}
R^2 = 0.7426 \\
R^2 = 0.7585
\end{bmatrix}$$

The acquired formula of a set of reactions such as $^{45}$Sc($\alpha$,n)$^{48}$V, $^{48}$Ti($\alpha$,n)$^{51}$Cr, $^{51}$V($\alpha$,n)$^{54}$Mn, $^{55}$Mn($\alpha$,n)$^{58}$Co, and $^{59}$Co($\alpha$,n)$^{62}$Cu has been used to calculate the astrophysical S-factor S(E) for each of the above reactions and compared with the adopted astrophysical S-factor calculated from the fitting expressions and shown to be in a good agreement and the comparison of the two results are shown in Table (4).

Table 3. Free parameters $C_i$ ($C_0$, $C_1$, and $C_2$) as a function of the energy of the center of mass.

| Ec.m. (MeV) | $C_0$ | $C_1$ | $C_2$ |
|-------------|-------|-------|-------|
| 5.5         | -49.997 | 6.1814 | -0.1106 |
| 5.75        | -41.303 | 5.3931 | -0.093 |
| 6           | -34.449 | 4.7574 | -0.0787 |
| 6.25        | -29.331 | 4.2685 | -0.0675 |
| 6.5         | -25.812 | 3.9172 | -0.0593 |
| 6.75        | -23.719 | 3.6906 | -0.0538 |
| 7           | -22.843 | 3.5724 | -0.0508 |
| 7.25        | -22.944 | 3.5423 | -0.0497 |
| 7           | -23.743 | 3.5767 | -0.0501 |
| 7.75        | -24.928 | 3.6484 | -0.0515 |
| 8           | -26.154 | 3.7264 | -0.053 |
| 8.25        | -27.038 | 3.7763 | -0.054 |
| 8.5         | -27.164 | 3.7599 | -0.0535 |
| 8.75        | -26.081 | 3.6355 | -0.0508 |
| 9           | -23.304 | 3.3579 | -0.0446 |
| 9.25        | -18.311 | 2.8781 | -0.0339 |
| 9.5         | -10.548 | 2.1436 | -0.0176 |
| 9.75        | 0.5769  | 1.0983 | 0.0057 |
| 10          | 15.687  | -0.3175 | 0.0374 |

Table 4. Comparison between polynomial fitting expression (Best Fitting) of the adopted astrophysical S-Factor of ($\alpha$,n) medium element reactions with those computed from Eq. (33).
| . (MeV) | ln[S-factor(MeV-b)] (Best Fitting) 4.04% | ln[S-factor(MeV-b)] (Formula) | ln[S-factor(MeV-b)] (Best Fitting) 4.215% | ln[S-factor(MeV-b)] (Formula) | ln[S-factor(MeV-b)] (Best Fitting) 3.801% | ln[S-factor(MeV-b)] (Formula) | ln[S-factor(MeV-b)] (Best Fitting) 3.028% | ln[S-factor(MeV-b)] (Formula) | ln[S-factor(MeV-b)] (Best Fitting) 1.974% | ln[S-factor(MeV-b)] (Formula) |
|--------|----------------------------------|-----------------------------|----------------------------------|-----------------------------|----------------------------------|-----------------------------|----------------------------------|-----------------------------|----------------------------------|-----------------------------|
| 5.5    | 31.055±1.255 31.301 32.461±1.368 32.684 33.534±1.275 33.897 35.503±1.075 35.819 36.214±0.715 37.067 |
| 5.75   | 30.873±1.247 31.182 32.466±1.368 32.584 33.375±1.269 33.821 35.422±1.073 35.803 36.473±0.720 37.128 |
| 6      | 30.679±1.239 31.048 32.325±1.363 32.467 33.216±1.263 33.727 35.319±1.069 35.769 36.645±0.723 37.174 |
| 6.25   | 30.473±1.231 30.899 32.087±1.352 32.334 33.054±1.256 33.615 35.193±1.066 35.716 36.737±0.725 37.203 |
| 6.5    | 30.255±1.222 30.735 31.791±1.340 32.183 32.886±1.250 33.484 35.046±1.061 35.645 36.758±0.726 37.217 |
| 6.75   | 30.026±1.213 30.556 31.472±1.327 32.016 32.712±1.243 33.336 34.881±1.056 35.556 36.715±0.725 37.216 |
| 7      | 29.787±1.203 30.362 31.154±1.313 31.832 32.527±1.236 33.170 34.698±1.051 35.449 36.618±0.723 37.198 |
| 7.25   | 29.539±1.193 30.152 30.853±1.300 31.631 32.329±1.229 32.986 34.498±0.454 35.324 36.474±0.720 37.165 |
| 7.5    | 29.281±1.183 29.928 30.577±1.289 31.413 32.117±1.221 32.784 34.284±0.383 35.180 36.292±0.716 37.117 |
| 7.75   | 29.015±1.172 29.689 30.328±1.278 31.179 31.887±1.212 32.564 34.056±0.313 35.019 36.079±0.712 37.052 |
| 8      | 28.741±1.161 29.434 30.096±1.269 30.928 31.637±1.203 32.326 33.816±1.024 34.839 35.844±0.708 36.972 |
| 8.25   | 28.459±1.150 29.165 29.865±1.259 30.659 31.365±1.192 32.070 33.566±1.016 34.641 35.595±0.703 36.876 |
| 8.5    | 28.171±1.138 28.880 29.613±1.248 30.374 31.068±1.181 31.796 33.306±1.009 34.424 35.340±0.698 36.764 |
| 8.75   | 27.877±1.126 28.581 29.306±1.235 30.072 30.744±1.169 31.504 33.039±1.000 34.190 35.088±0.693 36.637 |
| 9      | 27.577±1.114 28.266 28.903±1.218 29.754 30.391±0.155 31.195 32.766±0.992 33.937 34.846±0.688 36.494 |
| 9.25   | 27.272±1.102 27.937 28.357±1.195 29.418 30.005±0.140 30.867 32.488±0.984 33.666 34.622±0.683 36.335 |
| 9.5    | 26.963±1.089 27.592 27.611±1.164 29.066 29.584±0.124 30.521 32.206±0.975 33.377 34.426±0.680 36.161 |
| 9.75   | 26.649±1.077 27.232 26.600±1.121 28.696 29.127±0.107 30.157 31.923±0.967 33.070 34.264±0.676 35.971 |
| 10     | 26.333±1.064 26.858 25.250±1.064 28.310 28.630±0.088 29.776 31.639±0.958 32.745 34.146±0.674 35.765 |
Fig. 1. The variation of the natural logarithm of the astrophysical S-factor $S(E)$ with the atomic number ($Z$) for the $^{45}$Sc($\alpha$,n)$^{48}$V, $^{48}$Ti($\alpha$,n)$^{51}$Cr, $^{51}$V($\alpha$,n)$^{54}$Mn, $^{55}$Mn($\alpha$,n)$^{58}$Co, and $^{59}$Co($\alpha$,n)$^{62}$Cu reactions at fixed values of center of mass energies.
4.2. Thermonuclear Reaction Rates Empirical Formulae

The adopted thermonuclear reaction rates $N_A \langle \sigma v \rangle$ have been utilized to acquire the fitting parameter by utilizing the polynomial expressions (16), (18) and (19) by the steps below:

1. Polynomial expressions were utilized in eq. (19) to fit the computed thermonuclear reaction rates natural logarithm $N_A \langle \sigma v \rangle$ of the thoughtful medium elements to set the embraced natural logarithm of thermonuclear reaction rates $N_A \langle \sigma v \rangle$ from the best fitting with a minimum ($\chi^2$) utilizing Eq. (18). The acquired best fitting relationships of the remembered reactions are shown in Eqs. (34, 35, 36, 37, 38, 39, 40, 41, 42, and 43) for the reactions $^{45}$Sc$(\alpha,n)^{48}$V, $^{48}$Ti$(\alpha,n)^{51}$Cr, $^{51}$V$(\alpha,n)^{54}$Mn, $^{50}$Cr$(\alpha,n)^{53}$Fe, $^{55}$Mn$(\alpha,n)^{58}$Co, $^{54}$Fe$(\alpha,n)^{57}$Ni, $^{59}$Co$(\alpha,n)^{62}$Cu, $^{62}$Ni$(\alpha,n)^{65}$Zn, $^{63}$Cu$(\alpha,n)^{66}$Ga, and $^{66}$Zn$(\alpha,n)^{69}$Ge respectively.

$$
\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = 0.001875 - 0.063674 + 0.909473 - 6.713472 + 27.4637 - 42.483
$$

$$
\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = -0.15674 + 0.458487 - 5.1037^2 + 26.927 - 48.328
$$

$$
\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = -0.0157^4 + 0.458487 - 5.1037^2 + 26.927 - 48.328
$$

$$
\ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = 0.02817^3 - 0.82537^2 + 8.95137 - 24.344
$$

$$
55\text{Mn}(\alpha,n)^{58}\text{Co} \quad x^2=0.318
$$

$$
ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = -0.00537^4 + 0.17987^3 - 2.4157^2 + 16.1227 - 34.73
$$

$$
54\text{Fe}(\alpha,n)^{57}\text{Ni} \quad x^2=0.57
$$

$$
ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = -0.00937^4 + 0.29847^3 - 3.70557^2 + 22.317 - 47.654
$$

$$
59\text{Co}(\alpha,n)^{62}\text{Cu} \quad x^2=0.849
$$

$$
ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = 0.00137^5 - 0.05047^4 + 0.7943^3 - 6.55017^2 + 30.0537 - 55.065
$$

$$
62\text{Ni}(\alpha,n)^{65}\text{Zn} \quad x^2=0.597
$$

$$
ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = -0.01037^4 + 0.33097^3 - 4.11587^2 + 24.7497 - 53.036
$$

$$
63\text{Cu}(\alpha,n)^{66}\text{Ga} \quad x^2=1.475
$$

$$
ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = 0.00487^5 - 0.16847^4 + 2.36057^3 - 16.767^2 + 63.1157 - 100.95
$$

$$
66\text{Zn}(\alpha,n)^{69}\text{Ge} \quad x^2=0.422
$$

$$
ln[N_A(\sigma v)(cm^3 s^{-1} mol^{-1})] = 0.00487^5 - 0.16897^4 + 2.37157^3 - 16.8887^2 + 63.9737 - 102.72
$$

2. At fixed values of $T_9$, the variation of the natural logarithm of the thermonuclear reaction rates with the physical parameter atomic number $Z$ has been fitted to the polynomial expression utilizing Eq. (19). The acquired outcomes are contemplated to set the coefficients of polynomials ($C_i$).

3. The coefficients of polynomials $C_i$ are plotted versus each value of $T_9$ and fitted to satisfactory
the polynomial expression were shown in Eq. (16).
4. The last formula of a set of reactions has been determined by utilizing the combination of the two polynomials to show the systematic manner of the reactions, which is shown in Eq. (17). The Y Variable is the thermonuclear reaction rates.

4.2.1. The Empirical Formulae Relating the Thermonuclear Reaction Rates to \( T_9 \) and the Atomic Number \( Z \) of the Target Nucleus

The empirical formulae relating to the thermonuclear reaction rates \( N_A<\sigma v> \) (cm\(^3\) s\(^{-1}\) mol\(^{-1}\)) with both \( T_9 \) and \( Z \) were performed as the steps below:

1. At fixed values of the \( T_9 \) from 6 to 10 \( 10^9 \) K in steps of 0.25 \( 10^9 \) K for the \( ^{45}\text{Sc}(\alpha,n)^{48}\text{V} \), \( ^{48}\text{Ti}(\alpha,n)^{51}\text{Cr} \), \( ^{51}\text{V}(\alpha,n)^{54}\text{Mn} \), \( ^{55}\text{Mn}(\alpha,n)^{58}\text{Co} \), \( ^{62}\text{Ni}(\alpha,n)^{65}\text{Zn} \), and \( ^{66}\text{Zn}(\alpha,n)^{69}\text{Ge} \) reactions, the natural logarithm of the thermonuclear reaction rates will vary with the atomic number \( Z \) this shown in Fig. (3). The data fitted to the polynomial expression as the same as Eq. (30), Where \( Y=\ln[N_A<\sigma v>] \), \( X=Z \), with free parameters \( C_i \) (\( C_0, C_1, \) and \( C_2 \)).

2. The adopted thermonuclear reaction rates have been used as a function of \( Z \) at fixed \( T_9 \) utilizing the computer program Excel to acquiring the fitting expressions and then used to calculate the fitting parameters. The obtained results are presented in Table (5).

3. The obtained free parameters \( C_i \) (\( C_0, C_1, \) and \( C_2 \)), as presented in Table (5) are plotted versus with the fixed values of \( T_9 \) from 6 to 10 \( 10^9 \) K in steps of 0.25 \( 10^9 \) K as presented in Fig. (4), and then the acquired coefficients of polynomials \( C_i \) have been fitted to the polynomial expression:

\[
C_i = \sum_{j=0}^{2} C_{ij} T_9^j \quad \text{(44)}
\]

The combination of the two polynomials Eq. (30) and Eq. (44) takes the shape of the following formula range \( T_9 \) from 6 to 10 \( 10^9 \) K in steps of 0.25 \( 10^9 \) K:

\[
Y = \sum_{i=0}^{2} \left( \sum_{j=0}^{2} C_{ij} T_9^j \right) X^i \quad \text{(45)}
\]

Where \( Y=\ln[N_A<\sigma v>] \), \( T_9 \) is the temperature in \( 10^9 \) K, and \( X=\text{atomic number} Z \).

\[
Y = C_{00} T_9^2 X^0 + C_{01} T_9 X^1 + C_{02} T_9^2 X^2 + C_{10} T_9 X^1 + C_{11} T_9 X^2 + C_{12} T_9^2 X^3 + C_{20} T_9^3 X^2 \quad \text{(46)}
\]

Where \( (C_{00}, C_{01}, C_{02}, \ldots, C_{22}) \) are free parameters and their values are shown in the matrix below:

\[
\begin{bmatrix}
C_{00} & C_{01} & C_{02} \\
C_{10} & C_{11} & C_{12} \\
C_{20} & C_{21} & C_{22}
\end{bmatrix}
\]

\[
\begin{bmatrix}
42.691 & -16.998 & 1.7602 \\
-3.0522 & 1.4998 & -0.145 \\
0.0375 & -0.0265 & 0.0027
\end{bmatrix}
\]

\[ R^2 = 0.9717 \]
\[ R^2 = 0.9645 \]
\[ R^2 = 0.9643 \]

The acquired formula of a set of reactions such as \( ^{45}\text{Sc}(\alpha,n)^{48}\text{V} \), \( ^{48}\text{Ti}(\alpha,n)^{51}\text{Cr} \), \( ^{51}\text{V}(\alpha,n)^{54}\text{Mn} \), \( ^{55}\text{Mn}(\alpha,n)^{58}\text{Co} \), \( ^{62}\text{Ni}(\alpha,n)^{65}\text{Zn} \), and \( ^{66}\text{Zn}(\alpha,n)^{69}\text{Ge} \) has been used to calculate the thermonuclear reaction rates \( N_A<\sigma v> \) for each of the above reactions and compared with the adopted thermonuclear reaction rates calculated from the fitting expressions and shown to be in a good agreement and the comparison of the two results are shown in Table (6).

Table 7 presents the comparison of thermonuclear reaction rates of some (\( \alpha,n \)) medium elements reactions with other works as Roughton et.al. (Roughton et al., 1983)
## Comparison between polynomial fitting expression (Best Fitting) of the adopted astrophysical S-Factor of $(\alpha,n)$ medium element reactions with those computed from Eq. (46).

| T9 (109 K) | $^{45}$Sc$(\alpha,n)^{48}$V | $^{48}$Ti$(\alpha,n)^{51}$Cr | $^{51}$V$(\alpha,n)^{54}$Mn | $^{55}$Mn$(\alpha,n)^{58}$Co | $^{61}$Ni$(\alpha,n)^{64}$Zn | $^{66}$Zn$(\alpha,n)^{69}$Ge |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|            | ln[Na<σv> (cm$^3$ s$^{-1}$ mol$^{-1}$)] (Best Fitting) | ln[Na<σv> (cm$^3$ s$^{-1}$ mol$^{-1}$)] (Formula) | ln[Na<σv> (cm$^3$ s$^{-1}$ mol$^{-1}$)] (Best Fitting) | ln[Na<σv> (cm$^3$ s$^{-1}$ mol$^{-1}$)] (Formula) | ln[Na<σv> (cm$^3$ s$^{-1}$ mol$^{-1}$)] (Best Fitting) | ln[Na<σv> (cm$^3$ s$^{-1}$ mol$^{-1}$)] (Formula) |
| 6          | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ |
| 6.25       | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ |
| 6.75       | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ |
| 7          | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ |
| 7.5        | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ |
| 8          | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ |
| 8.5        | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ |
| 9          | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ |
| 9.5        | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ |
| 9.75       | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ |
| 10         | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ | $^{668}$±$^{0.046}$ |

Table 7. Comparison between polynomial fitting expression (Best Fitting) of the adopted astrophysical S-Factor of $(\alpha,n)$ medium element reactions with those computed from Eq. (46).
Fig. 3. The variation of the natural logarithm of the thermonuclear reaction rates with the atomic number $Z$ for the $^{45}\text{Sc}(\alpha,n)^{48}\text{V}$, $^{48}\text{Ti}(\alpha,n)^{51}\text{Cr}$, $^{51}\text{V}(\alpha,n)^{54}\text{Mn}$, $^{55}\text{Mn}(\alpha,n)^{58}\text{Co}$, $^{62}\text{Ni}(\alpha,n)^{65}\text{Zn}$, and $^{66}\text{Zn}(\alpha,n)^{69}\text{Ge}$ reactions at fixed values of $T_9$. 

| $T_9$ (109 K) | $^{54}\text{Fe}(\alpha,n)^{57}\text{Ni}$ | $^{59}\text{Co}(\alpha,n)^{62}\text{Cu}$ | $^{63}\text{Cu}(\alpha,n)^{66}\text{Ga}$ | $^{66}\text{Zn}(\alpha,n)^{69}\text{Ge}$ |
|--------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| $T_9=2$      | Roughton et al. 1983 | Present Work | Roughton et al. 1983 | Present Work | Roughton et al. 1983 | Present Work | Roughton et al. 1983 | Present Work |
| $T_9=3$      | -18.526                      | -18.474                      | -16.811                      | -17.051                      | -25.945                      | -25.653                      | -25.759                      | -26.138                      |
| $T_9=4$      | -6.908                       | -6.867                       | -5.991                       | -6.195                       | -10.871                       | -11.147                       | -10.820                       | -11.238                      |
| $T_9=5$      | 2.708                        | 2.699                        | 3.526                        | 3.311                        | 1.335                        | 0.427                        | 1.386                        | 0.826                        |
| $T_9=6$      | 5.193                        | 5.179                        | 6.131                        | 5.865                        | 4.431                        | 3.291                        | 4.522                        | 3.841                        |
| $T_9=7$      | 7.003                        | 6.978                        | 8.039                        | 7.726                        | 6.659                        | 5.317                        | 6.791                        | 5.979                        |
| $T_9=8$      | 8.434                        | 8.339                        | 9.547                        | 9.134                        | 8.343                        | 6.821                        | 8.517                        | 7.568                        |
| $T_9=9$      | 9.547                        | 9.401                        | 10.714                       | 10.231                       | 9.680                        | 7.975                        | 9.852                        | 8.789                        |
| $T_9=10$     | 10.519                       | 10.248                       | 11.695                       | 11.104                       | 10.736                       | 8.887                        | 10.951                       | 9.753                        |
Fig. 4. $C_i$ coefficients against $T_9$, for $C_0$, $C_1$, and $C_2$ respectively. The solid line represents the fitted curve through the data.
5. Conclusions

1-The astrophysical S-factor, $S(E)$, was starting with an increase and then decreased irregularly by increasing the center of mass energy, this because of Coulomb barrier penetration $\exp(2\pi\eta)$.

2-The astrophysical S-factor increased with increasing atomic number $Z$ of target nuclei at a fixed center of mass energy.

3-The thermonuclear reaction rates, $N_A<\sigma_v>$, were increased with increasing $T_9$ because by increasing the $T_9$ the charged interacting particles need to overcome the existing Coulomb barrier.

4-The thermonuclear reaction rates decreased with increasing atomic number $Z$ of target nuclei at fixed $T_9$ because as $Z$ increased Coulomb barrier increased.

5-The astrophysical S-factor and Thermonuclear reaction rates calculated in the present work are in good agreement with those measured previously by other works.

References

Abdul Aziz, A. (2008) Charged-Particle Induced Thermonuclear Reaction Rates of Light Nuclei. (M.Sc. Thesis). University of Malaya, Malaysia.

Angulo, C. et al. (1999) ‘A compilation of charged-particle induced thermonuclear reaction rates’, Nuclear Physics A, 656(1), pp. 3–183. doi: 10.1016/S0375-9474(99)00030-5.

Baglin, Coral, M. et al. (2004) ‘Measurement of 107 Ag (α, γ ) 111 In Cross Sections’, Conf. on Nucl.Data for Sci.and Techn., Santa Fe 2004, 2, p. 1370.

Belgaid, M. et al. (2005) ‘Semi-empirical systematics of (n, p) reaction cross sections at 14.5 MeV neutron energy’, Nuclear Instruments and Methods in Physics Research, Section B: Beam Interactions with Materials and Atoms, 239(4), pp. 303–313. doi: 10.1016/j.nimb.2005.05.053.

Bevington, P. R. and Robinson, D. K. (2003) Data Reduction and Error Analysis for the Physical Sciences. McGraw-Hill Companies, Inc.

Brown, E. (2015) stellar astrophysics. Edward Brown.

Chang, C. N. et al. (1973) ‘Total cross section measurements by x-ray detection of electron-capture residual activity’, Nuclear Instrum.and Methods in Physics Res., 109(January), pp. 327–331.

Clayton, D. D. (1968) Principles of Stellar Evolution and Nucleosynthesis. McGraw-Hill Book Company.

D’auria, J. M. et al. (1968) ‘Reaction Cross Section for Low-Energy Alpha Particles on 59Co’, Physical Review, 168, p. 1224.

Descouvemont, P. (2011) ‘Theoretical Models in Nuclear Astrophysics PoS, p.008.’

Haider, R. D. (2012) The Empirical Formulae to Determine the Astrophysical S-Factor of (α,n) Reactions for some Medium Elements. (M.Sc. Thesis). University of Salahaddin, Kurdistan Region, Iraq.

Hanser, V. Y. et al. (1989) ‘The 45Sc(a, p)48Ti and 45Sc(a,γ)48N Cross Sections’, 504, pp. 605–620.

Hanser, V. Y. et al. (1993) ‘Cross sections and thermonuclear reaction rates for $51V(\alpha,p)54Mn$ and $51V(\alpha,\gamma)54Cr$’, Nuclear Physics, Section A, 551, p. 158.

Houck, F. S. and Miller, J. M. (1961) ‘Reactions of Alpha Particles with Iron-54 and Nickel-58’, Physical Review, 123(1), pp. 231–240.

Jose, J. (2016) Stellar Explosions Hydrodynamics and Nucleosynthesis. Taylor & Francis Group, LLC, Science. doi: 10.1126/science.218.4576.992.

Kaplan, I. (1962) Nuclear Physics. Addison-Wesley Publishing Company, Inc.

Levkovski, V. N. (1991) ‘Cross sections of medium mass nuclide activation (A=40-100) by medium energy protons and alpha-particles (E=10-50 MeV)’, Levkovskij,Act.Cs.By Protons and Alphas,Moscow.

Li, J., Y. et al. (2012) ‘New determination of the astrophysical 13C(p, γ )14N S(E) factors and reaction rates via the 13C(7Li, 6He) 14N reaction’, European Physical Journal A, 48(2), pp. 1–7. doi: 10.1140/epja/i2012-12013-x.

Meyerhof, W. E. (1967) Elements of Nuclear Physics. McGraw-Hill Book Company.

Morton, A. J. et al. (1992) ‘The 48Ti(a, n)51Cr and 48Ti(a,p )51V Cross Sections’, Nuclear Physics A, 537, pp. 167–182.

Morton, A. J. et al. (1994) ‘The 50Cr(a, n)53Fe and 50Cr(a, p)53 Mn cross sections’, Nuclear Physics, Section A, 573(2), pp. 276–290. doi: 10.1016/0167-5064(94)90171-6.

Noori, B. M. (2008) Empirical Formulae to calculate Neutron Yields for (α,n) Reactions Using Medium Elements and Various Alpha Emitters. (M.Sc. Thesis). University of Salahaddin, Kurdistan Region, Iraq.

Peng, X., He, F. and Long, X. (1999) ‘Excitation functions for α-induced reactions on vanadium’, Nuclear Instruments and Methods in Physics Research, Section B: Beam Interactions with Materials and Atoms, 152(4), pp. 432–436. doi: 10.1016/S0168-583X(99)00179-2.

Rizvi, I. A. et al. (1989) ‘ Pre-equilibrium emission of multiparticles in α-induced reactions with 55 Mn nucleus ’, Canadian Journal of Physics, 67(11), pp. 1091–1096. doi: 10.1139/p89-188.

Roughton, N. A. et al. (1983) ‘Thick-Target Measurements and Astrophysical Thermonuclear Reaction Rates: Alpha-Induced Reactions’, Atomic Data and Nuclear Data Tables, 28(November), pp. 341–353.
Shaviv, G. (2012) The Synthesis of the Elements, The Astrophysical Quest for Nucleosynthesis and What It Can Tell Us About the Universe. Springer-Verlag Berlin Heidelberg.

Sonzogni, A. A. et al. (1993) ‘Alpha and deuteron induced reactions on vanadium’, Journal of Radioanalytical and Nuclear Chemistry Articles, 170(1), pp. 143–156. doi: 10.1007/BF02134585.

Stelson, P. H. and McGowan, F. K. (1964) ‘Cross Sections for (a,n) Reactions for Medium-Weight Nuclei’, Physical Review, 133(February), pp. B911–B919.

Tims, S. G. et al. (1988) ‘The 59Co(a,n)62Ni and 59Co(a,n)62Cu Cross Sections’, Nuclear Physics A, 483(December), pp. 354–370.

Tims, S. G. et al. (1991) ‘The 54Fe(a, n)57Ni and 54Fe(a, P)57Co Cross Sections’, Nuclear Physics A524, 524, pp. 479–494.

Tims, S. G. et al. (1993) ‘Cross sections of the reactions 58Fe (p, y) 59Co, 58Fe(p,n)58Co, 55Mn(a,n)58Co, 55Mn(a,n)58Fe and 57Fe (p, n) 57Co’, Nuclear Physics A, 563, pp. 473–493.

Tuli, J. K. (2011) ‘Nuclear Wallet Cards’, National Nuclear Data Center. doi: 10.1080/03632415.2011.626737.

Vlieks, A. E., Morgan, J. F. and Blatt, S. L. (1974) ‘Total cross sections for some (a, n) and (a, p) reactions in medium-weight nuclei’, Nuclear Physics, Section A, 224(3), pp. 492–502. doi: 10.1016/0375-9474(74)90551-X.

Vonach, H., Haight, R. C. and Winkler, G. (1983) ‘(alpha,n) and total alpha-reaction cross sections for Ti-48 and V-51’, Physical Review, Part C, Nuclear Physics, 28, p. 2278.

Zhukova, O. A. et al. (1970) ‘Excitation functions of reactions induced by alpha particles with maximum energy of 38 MeV on copper isotopes.’, Izvestiya Akademii Nauk KazSSSR,Ser.Fiz.-Mat., 1970(4), p. 1.

Zhukova, O. A. et al. (1972) ‘Nuclear Reactions Produced Y Alpha Particles on Co-59’, Yadernaya Fizika, 16, p. 242.