On the impact of Helium abundance on the Cepheid Period-Luminosity and Wesenheit relations and the Distance Ladder

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ABSTRACT

This work analyses the effect of the Helium content on synthetic Period-Luminosity Relations (PLRs) and Period-Wesenheit Relations (PWRs) of Cepheids and the systematic uncertainties on the derived distances that a hidden population of He-enhanced Cepheids may generate. We use new stellar and pulsation models to build a homogeneous and consistent framework to derive the Cepheid features. The Cepheid populations expected in synthetic colour-magnitude diagrams of young stellar systems (from 20 Myr to 250 Myr) are computed in several photometric bands for $Y = 0.25$ and $Y = 0.35$, at a fixed metallicity ($Z = 0.008$). The PLRs appear to be very similar in the two cases, with negligible effects (few %) on distances, while PWRs differ somewhat, with systematic uncertainties in deriving distances as high as $\sim 7\%$ at $\log P < 1.5$. Statistical effects due to the number of variables used to determine the relations contribute to a distance systematic error of the order of few percent, with values decreasing from optical to near-infrared bands. The empirical PWRs derived from multi-wavelength datasets for the Large Magellanic Cloud (LMC) is in a very good agreement with our theoretical PWRs obtained with a standard He content, supporting the evidence that LMC Cepheids do not show any He effect.

Key words: stars: abundances - stars: variables: Cepheids - cosmology: distance scale

INTRODUCTION

Through the period-luminosity relations (PLRs), classical Cepheids are fundamental standard candles for the calibration of secondary distance indicators (as Supernovae Ia, surface brightness fluctuations, etc...), for the determination of the cosmological distances and the Hubble constant (Freedman et al. 2001; Saha et al. 2001). In spite of this, the general question of the dependence of the Cepheid properties and consequently their PLRs on the chemical composition, discussed by many authors over the years, still lacks of firm conclusions, and the size and even the sign of the effects are still disputed (see e.g. Sasselov et al. 1997; Kennicutt et al. 1998; Bono et al. 1999a; Fiorentino et al. 2002; Storm et al. 2004; Marconi et al. 2005; Romaniello et al. 2008; Freedman & Madore 2010; Efstathiou 2014). In particular, the effect of He abundance on Cepheid pulsation properties has been theoretically investigated by Bono et al. (2000), Fiorentino et al. (2002) and Marconi et al. (2005) for $Z \gtrsim 0.02$. At fixed metallicity, an increase in the He content produces a shift towards higher effective temperature. If both the metallicity and the He abundances increase, as expected, the two effects on the instability-strip topology tend to compensate each other. Moreover, at fixed He to metal enrichment ratio ($\Delta Y/\Delta Z$), as the metallicity increases from $Z = 0.03$ to $Z = 0.04$ the instability strip tends to narrow. As a result of the complex interplay between He and metal variations, in the case of metal-rich ($Z \gtrsim 0.01$) Cepheid samples, the predicted correction for metallicity to the LMC-based PLR was found to be dependent on the assumed $\Delta Y/\Delta Z$ (see Fiorentino et al. 2002 and Marconi et al. 2005 for details) and to show an opposite sign when comparing MW and MC Cepheids, in agreement with Romaniello et al. (2008). To reduce the influence of the instability-strip topology and of the contribution of metals, the period-Wesenheit relations (PWRs, Madore 1982) are often adopted. These relations have the additional advantage of widely removing the reddening effect, and of significantly reducing the dispersion of visual magnitudes at a given period. In fact, the dispersion of the PWRs for the LMC Cepheids is $\sim 2 \div 3$ times...
smaller compared to the optical PLRs (Tanvir 1999; Udalski et al. 1999; Fouquè et al. 2007; Soszynski et al. 2008; Ngeow et al. 2009). This reduces the random errors in the evaluation of distances. It is worth noting that the PWRs rely on the assumed extinction law, and any deviation of the extinction coefficients could introduce systematic errors on the inferred distances, with major impact in star forming regions, like 30 Doradus (see e.g. De Marchi et al. 2016). The effect of variations in the He content on the PWRs has been predicted by Carini et al. (2014) at the LMC typical metal content ($[\text{Fe}/\text{H}] = -1.1$). At fixed mass, Cepheids with higher He abundance pulsate with longer periods and, consequently, the PWRs have different slopes with respect to those of standard relations, as shown in Fig. 10 and Table 5 in Carini et al. (2014). In this paper, we analyse the effect of He content on synthetic PLRs and PWRs and on the inferred Cepheid-based distance determinations, by relying on stellar pulsation, stellar evolution and population synthesis models. We explore the role of He-enhanced Cepheids might have if present in a sizable fraction. In particular, we will verify whether and how the PLRs and PWRs may be affected by the presence of Cepheids with non-standard He content, through comparisons with samples of Cepheids observed in the LMC. This has relevant implications in view of the present efforts to reach the 1% level to properly confront the local value of $H_0$ with the microwave background measurements.

Moreover, it is now well known that multiple stellar populations (MSPs) are present in the globular clusters (GCs) of the Milky Way (MW, see the review by Piotto 2009), where the chemistry of stars belonging to the second generation(s) is altered with respect to the abundances of the original gas from which the cluster has been formed (Gratton et al. 2004; Carretta et al. 2009a; Gratton et al. 2012). Some studies suggested that the newly formed stars should be He-enhanced (up to $Y \sim 0.40$, e.g. Piotto et al. 2007; Gratton et al. 2010; Marino et al. 2014), while their metallicity appears nearly constant (the differences are less than 0.05 dex, Carretta et al. 2009b). Such an evidence seems to favour the hypothesis that the progenitors of the GCs with MPs should be very massive ($M > 10^7 M_\odot$), with second generation (SG) stars formed during the first $\sim 150$ million years of the cluster’s life (D’Ercole et al. 2008; Degl’Innocenti et al. 2007; de Mink et al. 2009; Bastian et al. 2013). If a similar formation history is assumed, one expects signatures of second generation stars in (relatively) young massive clusters. For instance, Cepheids with different chemical content (with respect to the bulk of stars) might exist as well, even if to date they have not been detected. He-enhanced variables in observed samples of Cepheids could, in principle, alter the slope and zero point of PLRs and PWRs, with respect to those derived from He-standard variables. Unfortunately, observations available up to now do not support or exclude in a conclusive way the presence of SG He-enhanced stars in young stellar systems. Some intermediate-age ($\sim 1-2$ Gyr) clusters of the Large Magellanic Cloud (LMC) show distinct main-sequence turn-offs, as well as few younger clusters (Milone et al. 2016). This occurrence has been interpreted as a sign of (at least) two main episodes of star formation, although alternative explanations have been invoked (different rotation rates, binaries, etc ..., e.g. Milone et al. 2009, 2017). The picture is still uncompleted and deserve more investigations using different approaches. In this paper we make a theoretical investigation to explore the impact of multiple populations with different He content on the observed Cepheids properties of a stellar system.

The paper is arranged as follow: Section 1 describes the stellar evolution and pulsation models, and introduces the adopted stellar population synthesis models. Our theoretical PL and PW relations for the two values of initial He are compared with the most updated empirical ones derived from multi-wavelength datasets for the LMC in Section 2. In Section 3 our theoretical results are compared to Cepheids in LMC adopting OGLE III (Udalski et al. 2008; Soszynski et al. 2008) and VMC (Cioni et al. 2011; Ripepi et al. 2012) releases. In Section 4 we evaluate the stochastic effects on deriving distances when only a small sample ($\sim 50$) of Cepheids is available. A brief discussion closes the paper.

1 MODEL COMPUTATIONS
1.1 Stellar evolution and pulsation models
Stellar evolution models have been computed using the ATON evolutionary code (Ventura et al. 1998) and published in Carini et al. (2014). Here, we briefly recall the main assumptions of the code. The convective instability is described by means of the Full Spectrum of Turbulence (FST) model developed by Camuto & Mazzitelli (1991); the mass-loss rate in the ATON case is determined via the Blöcker’s formulation (Blöcker 1995):

$$M = 4.83 \times 10^{-22} \eta_R M^{-3.1} L^{3.7} R,$$

where $M$, $L$, and $R$ are denoted in solar units, $\eta_R$ is a free parameter. We used $\eta_R = 0.02$.

The mixing of chemicals and nuclear burning is coupled with a diffusive approach, according to the scheme suggested by Cloutman & Eoll (1976). The overshoot of convective eddies into radiatively stable regions is described by an exponential decay of the velocity beyond the formal border found via the Schwarzschild criterion. The extent of the overshoot is given by the free parameter $\xi$, in this study we put $\xi = 0.02$, in agreement with the calibration given in Ventura et al. (1998). No overshooting is used for the evolutionary phases following the core He-burning phase.

The evolutionary models used in the present work have metallicity $Z = 0.008$, initial He $Y = 0.25$ and $Y = 0.35$; the mixture is $\alpha$-enhanced, with $[\alpha/\text{Fe}] = +0.2$, with the reference solar mixture taken from Grevesse & Sauval (1998). The star evolution with both He contents is followed from the main sequence (MS) to the beginning of the phase of thermal pulses; the range in mass is from $0.4 M_\odot$ to $12 M_\odot$.

The sets of pulsation models computed for the above chemical compositions in Carini et al. (2014) have been extended with the computation of a finer grid of stellar masses with the same nonlinear pulsation code (see Marconi et al. 2005, 2010 and references therein). The adopted physical and numerical assumptions are the same as in Marconi et al. (2005) (see also Boni et al. 1998; Boni et al. 1999a for details). Here, we only remind that the adopted
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1.2 Stellar population models

To simulate the properties of stellar populations and of synthetic Cepheids, we used the stellar population synthesis code SPoT, described in Brocato et al. (1999, 2000); Raimondo et al. (2005); Raimondo (2009). Here, we recall that the code starts directly from stellar evolutionary tracks and relies on the Monte Carlo technique for populating the initial mass function (IMF). Since our purpose is to have a reasonable number of variables in the instability strip, we simulated only stars with mass greater than 1 M⊙ and assumed a Salpeter IMF (Salpeter 1955) with exponent α = 2.35. The cut at low mass stars does not affect the conclusions of this paper because we are interested to bright high mass stars evolving off the main sequence. All the evolutionary phases, from the main sequence (MS) up to the asymptotic giant branch, are covered by models. For each set of simple stellar population (SSP) parameters (i.e., age, t; metallicity, Z; and He abundance, Y), 100 independent simulations each containing 5000 stars (in the mass range 1-12 M⊙) are computed. Each single simulation corresponds to a stellar population having a total mass of the order of ∼ 10^4 M⊙, including low mass stars down to 0.1 M⊙.

We computed synthetic magnitudes and colours (in the U, B, V, R, I, J, H and K bandpasses) by adopting Castelli & Kurucz (2003) stellar atmospheres library. The code has been implemented by including specific routines to compute the number of predicted Cepheids and, for each variable, the pulsation mode and period (Carini et al. 2014). For populations with Y = 0.25, we adopted the instability strip calculated by Bono et al. (1999a) and Bono et al. (2001), for Y = 0.35 we used the values published in Carini et al. (2014). We consider ages from 20 Myr up to about 250 Myr (with an age step of ∼ 2 Myr up to 50 Myr, and ∼ 10 Myr up to 250 Myr) for the first stellar generation (Y = 0.25), and from 20 Myr up to 150 Myr (with the same age steps) for Cepheids belonging to the second generation of stars (Y = 0.35). For older ages no Cepheids are found in the populations (see Fig. 1). The age steps are chosen to reproduce in detail the main features of the Cepheid populations as a function of the age. The mean mass, luminosity and effective temperature of the stars at the termination of the Main Sequence luminosity function, for each population, are listed in Table 1.

In Fig. 1, we present the brightest part of the Hertzsprung-Russel (HR) diagrams expected from a sample of stellar populations having He abundances Y = 0.25 (left panel) and Y = 0.35 (right panel) and selected ages. The number of stars in each synthetic model shown in Fig. 1 is quite high to ensure that the He-burning phase is well populated also when computing very young stellar populations. This is obtained by collecting together all the 100 independent simulations, in this way each plotted model represents
a single burst stellar population with a mass of $\sim 10^6 M_\odot$. The instability strips derived from the pulsation models are also plotted: the blue short-dashed line is the first overtone blue edge (FOBE), the blue solid line is the fundamental blue edge (FBE), the red dotted line is the first overtone red edge (FORE) and the red solid line is the fundamental red edge (FRE). Green circles are the stars predicted to be variables.

Within the theoretical framework described in the previous sections, we are able to put constraints on the oldest stellar populations where Cepheids can be observed. In the case of $Y = 0.25$ (left panel) the oldest synthetic population expected to host Cepheids is found to be 254 Myr old, while for $Y = 0.35$ (right panel) the Cepheids should not be observed for ages older than 156 Myr. The exact age values are obtained by computing synthetic models with a very narrow step in age. The difference in the age limits is due to the fact that stars with a high He abundance, due to the higher molecular weight, evolve in a shorter time and are more luminous and hotter than stars with a lower He abundance at fixed mass. Therefore, the population with $Y = 0.25$ and age $t \sim 150$ Myr contains stars with mass lower than $\sim 3 M_\odot$, as well as a population with age $\sim 250$ Myr but with $Y = 0.25$ (Table 2). The reason for the age limits is well known. In old population, the hotter extremity of the blue loop of the evolving stars becomes cold enough to lie outside of the instability strip, so that no star falls inside the instability strip and no Cepheid is foreseen, as clearly shown in Fig. 1. This implies that there are no Cepheids with $Y = 0.35$ pulsating in the fundamental mode with log $P \lesssim 0.50$ and no Cepheids with $Y = 0.25$ at log $P \lesssim 0.2$. In Table 2 we report the maximum age of the stellar system hosting Cepheids, and the physical properties of pulsating stars, namely mass, luminosity, temperature and period for the two analysed He contents.

We point out that our results depend on the evolutionary scenario we have adopted. Different assumptions concerning the surface boundary condition, solar calibrated mixing length, model of convection, chemical composition, equation of state (EOS), gravitational settling, mass loss and other physics, may result in differences in the evolutionary path in the HR diagram. To throw some light on this point, in Fig. 2 we compare our tracks (solid black lines) with the ones from other popular databases: the Padova database\(^1\) (short dashed red lines, $Z = 0.008, Y = 0.26, 0.34$), and the Basti database\(^2\) (long dashed blue lines, $Z = 0.008, Y = 0.256$; see Bertelli et al. 2009; Pietrinferni et al. 2004, 2006, for details). We outline that the $Z$ and $Y$ values are not exactly the same, but are chosen as similar as possible, when available. In Fig. 2 we plot the evolutionary tracks for models of $4 M_\odot$ and $8 M_\odot$, for the labelled He abundance values. The black vertical lines represent the instability strip boundaries.

First, we can see that the effect of He abundance is fairly similar in our and Padova tracks. The ATON (black) He-rich

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**Table 1.** Mean value of the mass, luminosity and temperature of the stars at the termination of the Main Sequence luminosity function for each age.

| $t$ (Myr) | $M/M_\odot$ | $\log L/L_\odot$ | $\log T_{\text{eff}}$ |
|----------|-------------|------------------|------------------|
| 22       | 10.90       | 4.23             | 4.35             |
| 24       | 10.34       | 4.15             | 4.34             |
| 26.5     | 9.73        | 4.07             | 4.33             |
| 28       | 9.40        | 4.02             | 4.33             |
| 30       | 9.04        | 3.97             | 4.31             |
| 35       | 8.30        | 3.84             | 4.29             |
| 40       | 7.74        | 3.74             | 4.28             |
| 45       | 7.26        | 3.64             | 4.26             |
| 50       | 6.88        | 3.56             | 4.25             |
| 60       | 6.28        | 3.42             | 4.23             |
| 70       | 5.84        | 3.34             | 4.19             |
| 80       | 5.49        | 3.21             | 4.20             |
| 90       | 5.20        | 3.13             | 4.18             |
| 100      | 4.95        | 3.05             | 4.17             |
| 110      | 4.75        | 2.99             | 4.16             |
| 120      | 4.56        | 2.92             | 4.15             |
| 130      | 4.41        | 2.87             | 4.14             |
| 140      | 4.27        | 2.81             | 4.13             |
| 150      | 4.15        | 2.77             | 4.12             |
| 160      | 4.02        | 2.71             | 4.11             |
| 170      | 3.93        | 2.68             | 4.10             |
| 180      | 3.84        | 2.64             | 4.10             |
| 190      | 3.75        | 2.60             | 4.09             |
| 200      | 3.66        | 2.56             | 4.09             |
| 210      | 3.60        | 2.53             | 4.08             |
| 220      | 3.52        | 2.50             | 4.07             |
| 230      | 3.46        | 2.46             | 4.07             |
| 240      | 3.39        | 2.43             | 4.06             |
| 250      | 3.34        | 2.41             | 4.06             |

| $Z = 0.008 Y = 0.35$ |
|----------------------|
| $t$ (Myr) | $M/M_\odot$ | $\log L/L_\odot$ | $\log T_{\text{eff}}$ |
| 20       | 8.97        | 4.11             | 4.36             |
| 22       | 8.46        | 4.02             | 4.35             |
| 24       | 8.03        | 3.95             | 4.34             |
| 28       | 7.38        | 3.83             | 4.32             |
| 30       | 7.10        | 3.77             | 4.32             |
| 35       | 6.53        | 3.65             | 4.30             |
| 40       | 6.09        | 3.54             | 4.28             |
| 45       | 5.75        | 3.46             | 4.26             |
| 50       | 5.46        | 3.38             | 4.25             |
| 60       | 5.09        | 3.24             | 4.23             |
| 70       | 4.65        | 3.13             | 4.21             |
| 80       | 4.37        | 3.03             | 4.20             |
| 90       | 4.15        | 2.95             | 4.18             |
| 100      | 3.96        | 2.88             | 4.17             |
| 110      | 3.80        | 2.81             | 4.17             |
| 120      | 3.66        | 2.75             | 4.15             |
| 130      | 3.54        | 2.69             | 4.14             |
| 140      | 3.43        | 2.65             | 4.13             |
| 150      | 3.34        | 2.60             | 4.12             |

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**Table 2.** Mean values for key quantities of variables in the oldest stellar population where Cepheids are expected, for the two He contents.

| $Y$ | $t$ (Myr) | $M/M_\odot$ | $\log L/L_\odot$ | $\log T_{\text{eff}}$ | $\log P$ |
|-----|----------|-------------|------------------|------------------|--------|
| 0.25| 254      | 3.60        | 2.67             | 3.76             | 0.28   |
| 0.35| 156      | 3.56        | 2.97             | 3.75             | 0.57   |

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1. [http://pleiadi.pd.astro.it/](http://pleiadi.pd.astro.it/)
2. [http://basti.oa-teramo.inaf.it/](http://basti.oa-teramo.inaf.it/)

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Figure 2. The evolutionary path in the HR diagram of a 4 $M_\odot$ and a 8 $M_\odot$ model calculated with the ATON code (black solid line) is compared to that from other databases: Basti (long dashed blue line) and Padova (short dashed red lines) for $Y = 0.25$ (left panel) and $Y = 0.35$ (right panel, see text). The FRE (dot dashed black line), FORE (dotted black line), FBE (solid black line) and FOBE (short dashed black line) derived in Carini et al. (2014) are also shown.

4 $M_\odot$ model is more luminous of $\sim 0.2$ dex at the TO and of $\sim 0.3$ dex at the He-burning loop than the standard one; for the 8 $M_\odot$ model the overluminosity values are 0.15 dex and 0.25 dex, respectively. The extension of the loop for the 4 $M_\odot$ models is similar for the two He abundances, while for the 8 $M_\odot$ models the He-enhanced path is more extended in temperature of about 0.15 dex. The Padova tracks show a similar behaviour in luminosity while the extension of the loop remains comparable for the two He values. In the Basti database there are not He-enhanced tracks for intermediate-mass stars in the He burning evolutionary phase.

For what concerns differences in the stellar evolution codes and in the adopted input physics we focus on He-standard models (left panel), because they are available in each database (even with slightly different $Y$ values), making possible a complete comparison. The left panel of Fig. 2 shows that the tracks slightly differ in luminosity ($\Delta \log L/L_\odot \approx 0.1$ dex) and in the extension of the core-He burning loop ($\Delta \log T_{\text{eff}} \lesssim 0.1$ dex). For example, the 4 $M_\odot$ ATON model (black line) is less luminous but it has a blue loop more extended than the others. The behaviour in luminosity is the same for the model with 8 $M_\odot$, but in this case the Padova track extends to hotter temperatures. A deep analysis of the impact of different evolutionary codes and input physics on the evolutionary behaviour of stellar models is beyond the aim of the present work, and we refer the interested reader to Chiosi & Matteucci (e.g. 1982); Castellani et al. (e.g. 1990); Ventura et al. (e.g. 1998, 2005). The comparison illustrated above suggests that the number and thermodynamic properties of the synthetic Cepheids may slightly change according to the used set of stellar models. However, the differences are expected to affect only marginally the main results obtained in this work. Consequently, the synthetic PLRs and PWRs based on evolutionary tracks by other authors show slopes and zero points similar to ours, within the uncertainties. This is the case of results by Bono et al. (2010), whose relations are in very good agreement with ours, even taking into account that their Wesenheit relations are calculated by adopting $R_V = 3.23$ instead of $R_V = 3.1$. The PLRs and PWRs by Caputo et al. (2000) and Fiorentino et al. (2007) are also in agreement with our results, when statistical effects in the number of Cepheids are taken into account (see below Tab. 7 in Sec. 4).

2 HE-ENHANCED CEPHEIDS AND COMPARISON WITH EMPIRICAL RELATIONS.

Our models make possible to simulate the expected number and properties of Cepheids in stellar systems. In particular, we are interested to investigate the observational features of Cepheids in stellar populations having a metallicity similar to the typical value of the LMC ($Z \sim 0.008$) and different He abundances: $Y = 0.25$, which represents the classical case, the higher value $Y = 0.35$, and a simple mixture of the two stellar populations. As a first step, we assembled together all the simulations computed assuming $Z = 0.008$ and $Y = 0.25$, considering all the ages. The resulting large sample of Cepheids corresponds to that of a stellar population generated from a series of close star-formation bursts with short age steps (2-10 Myr). It is worth noting that, this can be considered as a stellar population whose stars are gen-
stars of ended up with two stellar systems having a total mass in $Y = 0$ ple stellar populations (Renzini & Buzzoni 1986). Theoretical PLRs and PWRs for fundamental classical Cepheids derived from a linear fit: $M_\lambda$ (or $W_{\text{band},\lambda}$) = $\beta \times \log P + \alpha$. The standard deviation of the slopes $\sigma_\beta$ and the intercepts $\sigma_\alpha$ are also reported.

| Band | $\beta$  | $\sigma_\beta$ | $\alpha$  | $\sigma_\alpha$ |
|------|---------|---------------|---------|---------------|
| $Z = 0.008$ and $Y = 0.25$ |
| B    | -2.598  | 0.006         | -0.638  | 0.003         |
| V    | -2.836  | 0.005         | -1.132  | 0.002         |
| I    | -3.003  | 0.003         | -1.735  | 0.001         |
| J    | -3.137  | 0.003         | -2.138  | 0.001         |
| K    | -3.252  | 0.002         | -2.471  | 0.001         |
| W(B,V)| -3.574 | 0.002         | -2.664  | 0.001         |
| W(V,I)| -3.262 | 0.002         | -2.663  | 0.001         |
| W(V,K)| -3.306 | 0.001         | -2.645  | 0.001         |
| W(J,K)| -3.318 | 0.001         | -2.663  | 0.001         |
| $Z = 0.008$ and $Y = 0.35$ |
| B    | -2.591  | 0.015         | -0.798  | 0.011         |
| V    | -2.859  | 0.011         | -1.172  | 0.008         |
| I    | -3.053  | 0.008         | -1.680  | 0.006         |
| J    | -3.195  | 0.006         | -2.022  | 0.004         |
| K    | -3.319  | 0.004         | -2.295  | 0.003         |
| W(B,V)| -3.691 | 0.003         | -2.332  | 0.002         |
| W(V,I)| -3.350 | 0.003         | -2.463  | 0.002         |
| W(V,K)| -3.378 | 0.003         | -2.441  | 0.002         |
| W(J,K)| -3.390 | 0.003         | -2.452  | 0.002         |

erated according to a constant Star Formation Rate (SFR) from 20Myr to 250Myr. Myr. We tested this assumption by computing a stellar population model in which a continue star formation has been reproduced by the series of SSPs with the adopted age steps. This is a well known result in stellar population synthesis, as a complex stellar population can always be expanded in a series of simple stellar populations (Renzini & Buzzoni 1986).

The same assembling procedure has been performed for $Y = 0.35$ models with age ranging from 20 to 150 Myr. We ended up with two stellar systems having a total mass in stars of $\sim 10^7 M_\odot$, and a total number of predicted Cepheids of $\sim 10000$ for $Y = 0.25$ and $\sim 7700$ for $Y = 0.35$.

From the analysis of the two samples, we derived the PLRs and the PWRs in different photometric bands, namely in $B$, $V$, $I$, $J$, and $K$, and $W(B,V)$, $W(V,I)$, $W(V,K)$, $W(J,K)$. The mean slopes and intercepts are reported in Table 3.

These theoretical relationships will be compared with the empirical ones, derived by observed samples of LMC Cepheids, in the following subsections.

### 2.1 PL relations

Our PLRs in the $V$, $I$, $J$ and $K$ bands are compared to those derived by Storm et al. (2011), who analysed 22 Cepheids pulsating in the fundamental mode in the LMC. The authors selected stars which simultaneously have high-quality near-infrared light-curves from Persson et al. (2004) and very accurate optical photometry from OGLE-III (Udalski et al. 2008; Soszynski et al. 2008). Storm et al. determined the following relations from a linear regression to the absolute magnitudes and $\log P$, from the infrared surface brightness analysis:

$$M_V = -2.78 (\log P - 1) - 4.00$$
$$M_I = -3.02 (\log P - 1) - 4.74$$
$$M_J = -3.22 (\log P - 1) - 5.17$$
$$M_K = -3.28 (\log P - 1) - 5.64$$

The dispersion around the fits is 0.26 mag in the $V$ band, and 0.21 mag in the others. Note that their relations are in very good agreement within the uncertainties with our determinations (Table 3), as shown in Fig. 3, where the difference between the empirical and theoretical PLRs is reported as function of the photometric band.

The first thing that stands out is that the relations derived from models with the two considered pure He abundances are very similar. In each band both the theoretical relations are in agreement with the empirical ones within the uncertainty, but a better agreement is found when assuming the standard He abundance. In the $K$ band the relation obtained from models with $Y = 0.35$ diverges from the observed ones at $\log P < 1.5$, with a maximum difference in magnitude of $\sim 0.05$ mag at $\log P = 0.6$. The opposite trend
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This implies a $\Delta BC_V = -0.07$ mag, $\Delta BC_I = 0.05$ mag, and $\Delta BC_K = 0.2$ mag. In the second case ($t = 30$ Myr), with a $\Delta T_{eff} \sim 230$ K, the differences in the bolometric corrections are $\Delta BC_V = -0.09$ mag, $\Delta BC_I = 0.01$ mag, and $\Delta BC_K = 0.12$ mag. Therefore, by increasing $\Delta T_{eff}$, $\Delta BC_K$ increases, but $\Delta BC_V$ decreases. For this reason, the two relations log $P$-$K$ are inverted with respect to the log $P$-$V$ relations, and their magnitude difference is higher at shorter periods than at longer ones.

In conclusion, we confirm that the He content has an effect not detectable on the PLRs with the sensitivity of the current instruments, so that we cannot discriminate whether the population is He-enhanced or not. The log $P$-$V$ and log $P$-$K$ relations show an opposite behaviour which depends on the difference in the effective temperature between the variables having same period but different He. This effect will be accentuate in the Wesenheit relations, which depends on the colour of the stars (see next section).

2.2 Wesenheit relations

We compare our theoretical Wesenheit relations with the ones derived by Ripepi et al. (2012) and Inno et al. (2013). The former authors analysed $K_s$-band light curves of the Cepheids in the LMC observed by the VISTA Magellanic Cloud survey\(^3\) (VMC, Cioni et al. 2011; Ripepi et al. 2012). The stars are detected in the $Y$, $J$ and $K_s$ filters; the Cepheid $K_s$ light curves are very well sampled, with at least 12 epochs, and with typical errors of 0.01 mag. They mapped two fields centred on the south ecliptic pole and the 30 Doradus star-forming regions, respectively. We used only their results from the second field, because the sample is the richest one, being composed by 172 classical Cepheids pulsating in the fundamental mode and 150 in the first-overtone. Since the longest period in the data-set is 23 days, Ripepi and collaborators complemented the sample with literature data, including other 80 Cepheids pulsating in the fundamental mode from Persson et al. (2004). From all data they derived:

$$W(V, K) = -3.325 \log P + 15.870$$

with a dispersion of 0.078. No correction for the inclination of the LMC disc was applied.

We plot the comparison of the empirical relation with the present results in the top panel of Fig. 5. The theoretical relation obtained with the standard He abundance (red short-dashed line) overlaps the observed relation (black solid line) over the full range of periods, as clearly seen in the lower panel of the figure, where the difference between the empirical and theoretical PWRs is shown. The red line is nearly constant because the two relations are similar (throughout the paper, we assume the LMC distance modulus of $\mu=18.50$), the small difference between the slopes of the two relations (0.019) leads to a maximum difference $\Delta W(V, K)$

\(^3\) Based on observations made with ESO Telescopes at the La Silla or Paranal Observatories under programme ID(s) 179.B-2003(D), 179.B-2003(C), 179.B-2003(B)
Table 4. Thermodynamic values, magnitudes in the $V$, $I$ and $K$ bands and the bolometric corrections of two couple of stars at log $P=0.6$ and 1.59, and age $t = 130$ Myr and 30 Myr, respectively.

| $Y$ (Myr) | $t$ (Myr) | $M/M_\odot$ | $T_{eff}$ (K) | log $g$ | log $P$ | $V - K$ (mag) | $V$ (mag) | $I$ (mag) | $K$ (mag) | $BC_V$ (mag) | $BC_I$ (mag) | $BC_K$ (mag) |
|-----------|-----------|-------------|---------------|--------|---------|---------------|-----------|-----------|-----------|-------------|------------|-------------|
| 0.25      | 130       | 4.67        | 5495          | 3.06   | 1.97    | 0.60          | -2.77     | -3.50     | -4.42     | -0.13       | 0.60       | 1.51        |
| 0.35      | 130       | 3.84        | 5888          | 3.10   | 1.95    | 0.60          | -2.94     | -3.55     | -4.31     | -0.06       | 0.55       | 1.31        |
| 0.25      | 30        | 9.78        | 4897          | 4.19   | 0.93    | 1.59          | -5.41     | -6.37     | -7.63     | -0.32       | 0.64       | 1.90        |
| 0.35      | 30        | 7.82        | 5128          | 4.19   | 0.91    | 1.59          | -5.50     | -6.38     | -7.50     | -0.23       | 0.65       | 1.77        |

Figure 5. Upper panel: Comparison between the $W(V,K)$ relation (black solid line) of Ripepi et al. (2012) and the theoretical ones: the red short-dashed line corresponds to $Y = 0.25$ and the blue dot-long dashed line to $Y = 0.35$ (see Table 3). Lower panel: Difference between theoretical and empirical Wesenheit relations plotted in the upper panel. The used distance modulus of the LMC is 18.50 mag (Ripepi et al. 2012). Symbols and colours are as in the previous figure.

Figure 6. Difference between theoretical and empirical PWRs by Inno et al. (2013) in the $(V,I)$, $(J,K)$ and $(V,K)$ bands. Symbols and colours are as in the previous figure.
particularly evident in the case of mixed population, where the uncertainties are 12%, 5%, 5%, and 10% for the pure He abundance population is similar (within \( \sim \) 3%) of the mixed population is similar (within \( \sim \) 3-7%) in evaluating the distance uncertainties. For the pure He abundance, the uncertainty in the evaluation of the distance modulus, with a \( Y \) enhancement of the order of 40%. We test this population split, because it can be considered one of the worst case scenarios, useful to derive an upper limit of the uncertainty on the galaxy distances caused by different helium abundances in the galaxy field populations. In addition, it may be interesting for young cluster populations.

In this case, the relations are tilted with respect to those with a pure abundance of He, as already shown in Carini et al. (2014). Here, we extended the computation and the results on PLRs and PWRs as reported in Table 5. The systematic uncertainty in the evaluation of distances can be considered negligible only in the I band, because the PLR of the mixed population is similar (within \( \sim \) 1%) to the one derived for a pure \( Y = 0.25 \) PLR population. The other uncertainties are 12%, 5%, 5%, and 10% for the \( B, V, J, K \) bands and decrease when the period decreases. Since the PLR in the K band is less affected by systematic errors due, for example, to the finite width of the instability strip, non linearity and chemical composition (e.g. Bono et al. 1999b; Caputo et al. 2000) or reddening and intrinsic dispersion (e.g. Madore & Freedman 1991) to stochastic effects (see Table 7 in sec. 4), the effect of helium seems to be the potentially the most important one. This becomes particularly evident in the case of mixed population, where the uncertainty due to the helium effect on the distance determination can be as high as 10%. This can be seen in the upper panel of Fig. 7, that shows the PLRs in the K band for three different cases: pure \( Y = 0.25 \) (black solid line), pure \( Y = 0.35 \) (red dotted line) and the mixed population \( Y_{\text{MIXED}} \) (blue dot-long dashed line). The black solid line and the red dotted line are nearly superimposed, while the third line is tilted with respect to the other ones, particularly for \( \log P > 1.0 \). This is more evident in the bottom panel of the same figure, where the difference between the empirical \( M_\odot \) relation by Storm et al. (2011) and our theoretical relation obtained from a pure \( Y = 0.25 \) (red short-dashed line) and from the mixed population (blue dot-long dashed line) are shown.

The PWRs show the same behaviour (Fig. 7 right panel); the empirical \( W(V,K) \) relation is by Ripepi et al. (2012). The error in the distance determinations from the PWRs could be as high as 10% (or more) for \( \log P > 1.5 \). At shorter periods the error decreases, for example at \( \log P = 1 \) it is about 7%. These percentages are similar in the different bands.

The unexpected behaviour in the case of the mixed population is due to the different distribution of Cepheids. In fact, by modelling the distributions one finds:

- at \( \log P < 0.5 \) Cepheids with \( Y = 0.25 \) are brighter and more numerous than ones with \( Y = 0.35 \):
  
  \[ N_{Y=0.25}(\log P < 0.5) = 8503, \]
  
  \[ N_{Y=0.35}(\log P < 0.5) = 0; \]

- for \( 0.5 \leq \log P \leq 1 \) the Cepheids with \( Y = 0.25 \) are brighter and less numerous than the ones with \( Y = 0.35 \) (\( \Delta W(V,K) \) \( \sim \) 0.15 mag):
  
  \[ N_{Y=0.25}(0.5 \leq \log P \leq 1.0) = 1193, \]
  
  \[ N_{Y=0.35}(0.5 \leq \log P \leq 1.0) = 7171; \]

- for \( 1.0 \leq \log P \leq 1.5 \) the number of stars for both \( Y = 0.25 \) and \( Y = 0.35 \) are much less numerous than that found at shorter periods:
  
  \[ N_{Y=0.25}(1.0 \leq \log P \leq 1.5) = 144, \]
  
  \[ N_{Y=0.35}(1.0 \leq \log P \leq 1.5) = 409; \]

- for \( 1.5 \leq \log P \leq 2.0 \) the number of Cepheids are much less numerous than ones found at shorter periods:
  
  \[ N_{Y=0.25}(1.5 \leq \log P \leq 2.0) = 76, \]
  
  \[ N_{Y=0.35}(1.5 \leq \log P \leq 2.0) = 59. \]

The distributions described above show that the mixed population considered here is composed of two major bulks of Cepheids, one at \( \log P < 0.5 \) (\( Y = 0.25 \)) and the second one at \( 0.5 < \log P < 1.0 \) (\( Y = 0.35 \)), with the latter fainter than typical Cepheids with \( Y = 0.25 \). This forces a simple linear fitting to provide a relation more tilted than that from a pure He abundance population.

In other words, at \( \log P < 1 \) a sample fully composed by Cepheids with a standard and a sample with enhanced He content produce very similar PWRs. Therefore, if the samples used to calibrate the relations contain, in addition to Cepheids with standard He abundances (\( Y = 0.24 – 0.25 \)), variables with a He abundance up to 0.35, these latter stars do not affect the calibration relations significantly. A differ-

| Band    | \( \beta \)   | \( \sigma_\beta \) | \( \alpha \)   | \( \sigma_\alpha \) |
|---------|---------------|-------------------|---------------|-------------------|
| \( Z = 0.008 Y_{\text{MIXED}} \) |               |                   |               |                   |
| B       | -2.784        | 0.005             | -0.612        | 0.003             |
| V       | -2.909        | 0.004             | -1.120        | 0.002             |
| I       | -2.989        | 0.003             | -1.733        | 0.002             |
| J       | -3.057        | 0.002             | -2.146        | 0.001             |
| K       | -3.109        | 0.002             | -2.487        | 0.001             |
| W(B,V)  | -3.296        | 0.003             | -2.695        | 0.002             |
| W(V,I)  | -3.113        | 0.002             | -2.678        | 0.001             |
| W(V,K)  | -3.135        | 0.002             | -2.664        | 0.001             |
| W(J,K)  | -3.139        | 0.002             | -2.683        | 0.001             |
ent behaviour is expected at longer periods, where a high contamination of He-enhanced Cepheids (>40%) may affect the derived relationships. Of course, this results require further work with different mixture of stellar populations. Nevertheless, it provides interesting warning against simple interpolation between pure populations to infer indications on properties of mixed population.

Figure 7. Upper panels: Comparison between the $P - M_k$ (left panel) and $W(V, K)$ (right panel) relations derived from a pure $Y = 0.25$ population (black solid line), pure $Y = 0.35$ population (red dotted line), and a mixed population $Y_{\text{MIXED}}[60\%(0.25); 40\%(0.35)]$ (blue dot-long dashed line). Lower panel: The differences between theoretical and empirical relations, i.e $M_k[Y = 0.25] - M_k[\text{empirical}]$ (right panel), $W(V, K)[Y = 0.25] - W(V, K)[\text{empirical}]$ (left panel) represented by a red short-dashed line, and $M_k[Y_{\text{MIXED}}[60\%(0.25); 40\%(0.35)]] - M_k[\text{empirical}]$ (left panel) represented by a blue dot-long-dashed line. The empirical $P - M_k$ relation is from Storm et al. 2011 ($P - M_k$), while the $W(V, K)$ relation is from Ripepi et al. 2012. The 1σ uncertainties are reported as filled yellow ($Y = 0.25$) and shaded green ($Y_{\text{MIXED}}[60\%(0.25); 40\%(0.35)]$).

3 COMPARISON WITH CEPHEIDS IN LMC

We used our models to search for systematic effects possibly associated to the presence of Cepheids with higher He abundances with respect to the primordial value in an observed sample. As a true observational sample, we adopt the Cepheids of the OGLE III catalogue (Soszynski et al. 2008) for the data in the V and I band, and the VMC (Cioni et al. 2011; Ripepi et al. 2012) for the Ks magnitudes. The first catalogue includes 3361 classical Cepheids, 1848 of them pulse in the fundamental mode. The stars are detected in the V and I bands, the accuracy of the photometric calibrations is better than 0.02 mag. Since the Cepheids are distributed along all the LMC, a differential reddening is expected (e.g. Soszynski et al. 2008 Zaritsky et al. 2004). The presence of a differential reddening affecting the LMC Cepheids in the OGLE sample broadens the observed sequences (see below) and the uncertainty on the apparent magnitude increases. For example we estimated from Fig. 8 a spread of the order of 0.2 mag at log $P = 0.6$. The second catalogue is described in Sec 2.2.

In Fig. 8 observational data (red circles) are compared with Cepheids predicted by models described in Sect. 1.2 with both He abundances ($Y = 0.25$, blue triangles; $Y = 0.35$, green squares), in the V (left panel), I (central panel) and Ks (right panel) bands.

We note that a detailed simulation of LMC SFR in beyond the aim of this paper. Differently from what assumed in our models, where a constant SFR in the age interval 20-250 Myr is adopted, recent LMC SFR estimations are found to vary greatly from subregion to subregion (e.g. Rubele et al. 2012, and reference therein), hence, we prefer to not introduce a further not well known parameter in the present discussion. The predicted variables occupy the same region of the diagram independently of the He content, and all of them overlay the observed ones, therefore, the presence of He-enhanced stars in the observed sample is not to be excluded. From the comparison, it is clear that in the period range typical of LMC Cepheids, it is difficult to disentangle the two He abundances. We stress that the comparison is only a by-eye analysis (including projection effect and differential reddening effects), while the coefficients of the PLRs and of the PWRs offer a more robust test for disentangling the two populations. From them, it appears that the use of a He-enhanced population gives discrepancies with what is observed, supporting the lack of a He-rich population in the LMC, together with different initial condition in the forma-
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Figure 8. Period-Luminosity diagram in the $V$ and $I$ and $K_s$ bands for the classical fundamental-mode Cepheids in the LMC, taken from the catalogue OGLEIII ($V$ and $I$) and VMC ($K_s$) (red circles). The blue triangles and green squares are the simulation done with $Y = 0.25$ and $Y = 0.35$, respectively.

Figure 9. Period-W($V$,I) diagram for the classical fundamental-mode Cepheids in the LMC, taken from the catalogue OGLEIII (red circles). Symbols are the same as in the previous figures.

To investigate in more details the possible presence of He-enhanced Cepheids, we need to study simultaneously the period, the age and the magnitude of variables. This is because He-enhanced stars evolve more rapidly, they are brighter and pulsate with a longer period with respect to Cepheids of same mass but with $Y = 0.25$. Note that, as we have already shown, Cepheids with log $P \lesssim 0.5$ cannot be He-enhanced stars.

Our models also predict a non negligible number of Cepheids ($\sim 200$ for $Y = 0.25$ and 60 for $Y = 0.35$) at log $P > 1.5$ and many (hundreds) with log $P < 0.4$ not present in the OGLE III catalogue. The long-period Cepheids are very bright ($M_I \lesssim -6.0$ mag) and have an age $t \lesssim 40$ Myr, the short periods ones have $M_I < -2.0$ mag and $t > 150$ Myr. The lack of Cepheids observed at log $P > 1.5$, could be due to several reasons including a bias effect in the observations. In the OGLE III catalogue the most bright stars ($I > 13$ mag) are saturated in the $I$ band and they are not measured in the $V$ filter, for the latter filter the saturation starts to be relevant at periods longer than $\sim 50$ days (Fig. 8, right panel). Moreover, in the OGLE III catalogue the Cepheids with log $P < 0.4$ suffer of incompleteness because in this domain the PL sequences overlap with various types of pulsating variables, so it is difficult to distinguish the characteristic shapes of the light curves of Cepheids. In the VMC catalogue the magnitude limits in the $K_s$ band of the Cepheids sample are $11.4 \lesssim K_s \lesssim 20.7$ mag.

From the figure, we also note a large dispersion in magnitude in the $V$ and $I$ bands, which is due to differential reddening as we have already mentioned. This is also supported by the small dispersion found in the $K_s$ band (see left panel of Fig. 8). Further, if we use the Wesenheit relations this spread disappears, as shown in Fig. 9 (the symbols are the same of Fig. 8). Again, simulated Cepheids with both the He abundances overlap very well the observational data.

4 STOCHASTIC EFFECTS DUE TO THE NUMBER OF CEPHEIDS

In this section we analyse how the number of Cepheids included in the sample affects the determination of the slope and zero point of PL and PW relationships, that is the distance evaluation. To this purpose, by taking advantage of the Monte Carlo procedures used to simulate stellar populations, we used the results obtained from our sets of models. We assembled together the Cepheids belonging to each simulation at different ages and fixed chemical composition, ending up with 100 independent samples of Cepheids. Each
Table 6. Mean standard deviations of the differences between the simulated magnitude of stars in each mini-sample and that calculated from the reference relations, representing the distance modulus $dM_X$, where $X$ refers to the generic band.

| $Y$  | $\sigma(dM_B)$ (mag) | $\sigma(dM_V)$ (mag) | $\sigma(dM_J)$ (mag) | $\sigma(dM_K)$ (mag) | $\sigma(dM_{W(V,J)})$ (mag) | $\sigma(dM_{W(V,K)})$ (mag) | $\sigma(dM_{W(J,K)})$ (mag) |
|------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 0.25 | 0.14                | 0.10                | 0.07                | 0.05                | 0.04                | 0.04                | 0.03                |
| 0.35 | 0.22                | 0.17                | 0.12                | 0.09                | 0.06                | 0.04                | 0.05                |

Figure 10. Standard deviations of the differences between the magnitude calculated with the reference relation and that derived from evolution models. This procedure is done for each mini-sample in the $V$ band (top panels) and $K$ band (middle panels), and for $W(V,K)$ in the lower panels. The red horizontal lines are the mean values of standard deviations. Results for $Y = 0.25$ ($Y = 0.35$) are reported in the left (right) panels.

sample contains nearly 50-100 Cepheids. Hereinafter, we refer to these small samples of Cepheids as mini-samples.

The standard procedure to derive distance moduli is to determine the magnitude difference between the individual star and the adopted reference relation and then average these values. Following this procedure for each mini-sample, we quantify the stochastic uncertainties due to the small number of Cepheids of a given stellar population. To this aim, we computed the difference between the magnitude of the Cepheids of each mini-sample with the one obtained using the reference PL relations for each photometric band (Table 3). For each mini-sample, the average of these differences is obviously equal to zero ($\sim 10^{-4}$) because we are dealing with absolute magnitudes. What is interesting is the standard deviation ($\sigma$) which provides the theoretical evaluation of the stochastic uncertainty. This is shown in Fig. 10, where the $\sigma$ values in the $V$ and $K$ bands are plotted for all our mini-samples and for the two helium abundances. The average values are marked as red lines, and reported in Table 6. The uncertainties decrease from optical to near-infrared bands for both He abundances: for Cepheids with $Y = 0.25$ in the $B$ and $V$ bands the $\sigma$ is as high as 0.14 mag and 0.10 mag, respectively, while in the $K$ band it reaches 0.04 mag. This implies that, infrared measurements remain quite reliable for deriving the galaxy distances even taking into account the stochasticity. As far as the He abundance is concerned, the stochastic uncertainties for $Y = 0.35$ appear slightly more relevant than for $Y = 0.25$.

The procedure described above has been adopted to evaluate the stochastic uncertainties to derive distances with the Wesenheit relations and, the results are presented in Table 6. Once again, the powerfulness of the Wesenheit relations is confirmed, in fact the values of the distance uncertainties are nearly one order of magnitude smaller than what found for PLRs. In particular, the $1 - \sigma$ is of the order of 0.03 for $Y = 0.25$ and $\sim 0.04$ for $Y = 0.35$. This can be seen in the lower panels of Fig. 10.

Following a different approach, the slopes of the PL and PW relations for each mini-sample (containing 50-100 Cepheids) are also computed. Clearly, they change respect to the reference ones reported in Tab 3, due to the smaller number of Cepheids. Table 7 lists the mean, minimum and maximum, values of the slopes and the intercepts for all the computed log $P$-$L$ and log $P$-$W$ relations and their standard deviation. As expected, the mean values converge to the ones derived in the previous section from a larger sam-
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Table 7. Mean, minimum and maximum values of the slopes and intercepts of the log $P$-mag and log $P$-W relations with the respective errors ($X= a \log P + b$).

| Band   | $a_{mean}$ | $a_{min}$ | $a_{max}$ | $b_{mean}$ | $b_{min}$ | $b_{max}$ |
|--------|------------|-----------|-----------|------------|-----------|-----------|
| Z = 0.008 Y = 0.25 |
| B      | -2.634     | -3.040    | -2.518    | -0.626     | -0.518    | -2.217    |
| V      | -2.859     | -3.141    | -2.570    | -1.125     | -1.050    | -1.217    |
| I      | -3.018     | -3.210    | -2.671    | -1.730     | -1.679    | -2.815    |
| J      | -3.144     | -3.267    | -2.104    | -1.937     | -1.900    | -2.176    |
| K      | -3.252     | -3.320    | -2.450    | -2.471     | -2.450    | -2.495    |
| W(B,V) | -3.556     | -3.662    | -2.635    | -0.058     | -0.018    | -2.705    |
| W(V,K) | -3.262     | -3.333    | -2.641    | 0.036      | 0.011     | -2.689    |
| W(V,I) | -3.303     | -3.351    | -2.632    | 0.025      | 0.008     | -2.663    |
| W(J,K) | -3.314     | -3.357    | -2.651    | 0.023      | 0.007     | -2.682    |

Z = 0.008 Y = 0.35

| B      | -2.591     | -3.149    | -2.225    | 0.183      | -0.441    | -1.016    |
| V      | -2.857     | -3.266    | -2.911    | 0.134      | -0.911    | -1.326    |
| I      | -3.049     | -3.347    | -2.854    | 0.098      | -1.490    | -1.796    |
| J      | -3.191     | -3.410    | -2.107    | 0.073      | -1.882    | -2.107    |
| K      | -3.314     | -3.458    | -2.361    | 0.051      | -2.203    | -2.361    |
| W(B,V) | -3.680     | -3.807    | -2.534    | 0.052      | -2.261    | -2.428    |
| W(V,K) | -3.346     | -3.472    | -2.531    | 0.049      | -2.382    | -2.531    |
| W(V,I) | -3.374     | -3.491    | -2.521    | 0.043      | -2.357    | -2.521    |
| W(J,K) | -3.386     | -3.507    | -2.532    | 0.041      | -2.366    | -2.532    |

5 SUMMARY AND CONCLUSIONS

We simulated the observational properties of Cepheids belonging to stellar populations with metallicity $Z = 0.008$ and two different He abundances, $Y = 0.25$ and $Y = 0.35$, in the age interval 20-250 Myr ($Y = 0.25$) and 20-150 Myr ($Y = 0.35$). With the help of population synthesis models we investigated and quantified if and how the presence of He-enhanced Cepheids in the observed samples could contribute to the PLRs and PWRs and to their uncertainty.

We find that Cepheids pulsating in the fundamental mode belonging to a stellar population with $Y = 0.35$ and $\log P \lesssim 0.5$ or with $Y = 0.25$ and $\log P \lesssim 0.2$ should not be observed, because these stars, during the He-burning phase, do not cross the instability strip. We analysed our whole sample of synthetic Cepheids and derived the PLRs and PWRs in different photometric bands from the optical to near-infrared wavelengths. The comparison between our theoretical and empirical relationships, obtained by Storm et al. (2011), Ripepi et al. (2012) and Imo et al. (2013) from multi-wavelength datasets of LMC, discloses a very good agreement in the case of synthetic Cepheids with a He abundance $Y = 0.25$, metallicity $Z = 0.008$ and Salpeter IMF.

We find that the differences in the PLRs obtained from stellar populations having different He abundance has a negligible impact (few percents) on the distance determination. Instead, if a mixed population of Cepheids (composed by $\sim 60\%$ of variables with $Y = 0.25$ and $\sim 40\%$ with $Y = 0.35$) is present in the observed sample and compared to empirical PLRs and PWRs to derive distances, the possible systematic uncertainties can be of the order of $5-10\%$ at $\log P > 1.5$, while at shorter periods the error is less than $7\%$, independently from the band taken into account. In the $I$ band the uncertainty is of the order of $1\%$ independently from the period. In addition, in the case of $Y = 0.35$, the Wesenheit relations are slightly tilted with respect to the empirical ones. Therefore, in principle, when evaluating the distance of a generic stellar population containing He-enhanced stars with the quoted metallicity, the uncertainty due to the Helium abundance could be negligible, being the systematic error of the order of $5-10\%$ in all bands for $\log P < 1$, with the exception of the $W(V, I)$ for which the error is as small as $\sim 3\%$.

With the aim of investigating the presence of He-enhanced Cepheids in an observed sample of variables, we compared our simulations with Cepheids in the LMC listed in the OGLE III and the VMC catalogues, finding a very...
good agreement. Unfortunately, there is not a clear separation between Cepheids having different He abundances, and some refinements in understanding Cepheids in LMC are still needed.

Finally, we studied the uncertainty introduced in the distance estimations when one deals with a small number of Cepheids (few tens) and derived the corresponding PL and PW relations.

Our analysis shows that the stochastic uncertainties due to the small number of Cepheids (∼50) used to derive distances with the Wesenheit procedure is nearly negligible (1 − σ ≲ 0.04 mag) in all bands and for both He values investigated here. Larger uncertainties are found for PL relations. Nevertheless, the 1 − σ uncertainties in the B band is ∼ 0.14 mag for Y = 0.25, while it becomes much smaller for the K band (∼ 0.04 mag). As far as it concerns Y = 0.35, the uncertainties are slightly larger, ranging from ∼ 0.22 mag for the B band up to 0.06 for the K band. From the present analyses we conclude that, in the optical bands, the population stochastic effect appears to be larger than the He effect on the estimation of the PLRs and PWRs, hence on the evaluation of the cosmological distance. We showed that the stochastic uncertainties are minimized in the NIR bands.

In conclusion, the present study suggests that when information on the He abundance of the Cepheids in a galaxy is not available and the variables are compared to PLRs and PWRs obtained from local samples, the derived distances might be affected by non negligible systematic uncertainties of the order of a few percent. These uncertainties are more effective if the number of Cepheids observed in the given galaxy is small. The effect appears more severe for optical than for near-infrared bands. Nevertheless, we found that PLR in the K band (log P ≲ 1.5) seems to be the relation most affected by variations in the Helium content. These results for the K band are particularly interesting in view of future Cepheid observations with the James Webb Space Telescope. Further fundamental step on this issue will be also provided by the large sample of Cepheids properties expected from the Gaia mission (Clementini et al. 2016) and in the faraway future from the LSST survey.

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