Detecting Neuronal Communities from Beginning of Activation Patterns

Luciano da Fontoura Costa
Institute of Physics at São Carlos, University of São Paulo,
PO Box 369, São Carlos, São Paulo, 13560-970 Brazil
(Dated: 27th Jan 2008)

The detection of neuronal communities is addressed based on two important concepts from neuroscience: facilitation of neuronal firing and nearly simultaneous beginning of activation of sets of neurons. More specifically, integrate-and-fire complex neuronal networks are activated at each of their nodes, and the dissemination of activation is monitored. As the activation received by each neuron accumulates, its firing gets facilitated. The time it takes for each neuron, other than the source, to receive the first non-zero input (beginning activation time) and the time for it to produce the first spike (beginning spiking time) are identified through simulations. It is shown, with respect to two synthetic and a real-world (C. elegans) neuronal complex networks, that the patterns of beginning activation times (and to a lesser extent also of the spiking times) tend to cluster into groups corresponding to communities of neurons in the original complex neuronal network. Such an effect is identified to be a direct consequence of the almost simultaneous activation between the nodes inside the same community in which the source of activation is placed, as well as of the respective trapping of activation implied by the integration of activation prior to firing. Interestingly, the accumulation of activity and thresholds inside each neuron were found to be essential for constraining the initial activations within each respective community during the transient activation (no clear clusters were observed when using overall activation or spiking rates). In addition to its intrinsic value for neuroscience and structure-dynamics studies, these results confirm the importance of the consideration of transient dynamics in complex systems investigations.

PACS numbers: 87.18.Sn, 05.40Fb, 89.70.Hj, 89.75.Hc, 89.75.Kd

I. INTRODUCTION

Much has been investigated about neuronal systems from both the biological and exact sciences points of view (e.g. [1]). More recently, neuronal networks met complex networks (e.g. [2, 3, 4, 5, 6, 7, 8, 9, 10]). Such an interface is particularly promising, as it allows the emphasis of dynamical systems characterizing research in neuronal networks to be integrated with the structural approaches of complex networks research (e.g. [11, 12, 13, 14]). The intersection of these two major areas, which lies at the heart of the structure and dynamics relationship (e.g. [12, 15]), is henceforth called Complex Neuronal Networks research. However, despite the growing attention to this area, few works have considered simple neuronal models such as the integrate-and-fire. In addition, rather few studies have addressed transient dynamics (e.g. [16]) or the accumulation of stimuli responsible for the facilitation of firing [1].

In a recent study [17], the transient dynamics of integrate-and-fire networks underlain by several types of connectivity was characterized with respect to a series of dynamical properties, including the activation of nodes, spiking, and onset times for activation and spiking. As the activation received by each neuron was stored inside it as its state (facilitation [1]), it became possible to investigate the activation and spiking separately. The neuronal dynamics was found to vary markedly with respect to the connectivity, with abrupt transitions of initiation of generalized spiking being observed for some complex networks models, as well as for the C. elegans network. The current work continues such a investigation in order to investigate for possible simultaneous neuronal activation as a consequence of concentrated interconnectivity between groups of neurons. The basic idea is that more intensely interconnected groups of neurons, i.e. communities of nodes (e.g. [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]), would imply concentration of the accumulated activation because of the thresholds, delaying the dissemination of the activation to other parts of the network. More specifically, external activation is fed into each of the N neurons, and the unfolding activation is monitored. The beginning activation time of each neuron \( i \) (except that used as source) is experimentally determined, corresponding to the time from the beginning of the external activation to the first arrival of non-zero stimuli at neuron \( i \). Therefore, a pattern of \( N - 1 \) beginning activation times are obtained for each place of stimulation. Because of the high dimensionality of these patterns, the statistical method known as Principal Component Analysis — PCA (e.g. [14, 30]) is applied for dimensionality reduction (through optimal decorrelation), allowing the visualization of the distribution of patterns of beginning activation times. Interestingly, these patterns were found to be organized in well-defined clusters for two synthetic networks, allowing the immediate identification of the neuronal communities. Structure distributions, also containing clusters, was also observed for the C. elegans net-
work. Similar, though a little less effective, results were obtained by considering the beginning spiking times. No clusterings are observed in case the activation of spiking patterns at specific times (or even averages) are taken into account. Such results corroborates the importance of transient dynamics in complex systems investigations.

This article starts by presenting the basic concepts in complex networks, integrate-and-fire complex neuronal networks and the statistical method of Principal Component Analysis. The results, discussion and perspectives for future investigations are presented subsequently.

II. BASIC CONCEPTS

A directed, unweighted network $\Gamma$ can be fully represented in terms of its adjacency matrix $K$. Each edge extending from node $i$ to node $j$ implies $K(j,i) = 1$. The absence of connection between nodes $i$ and $j$ is represented as $K(j,i) = 0$. The out-degree of a node $i$, henceforth expressed as $k_i$, corresponds to the number of outgoing edges of that node. An analogue definition holds for the in-degree. The immediate neighbors of a node $i$ are those nodes which can be reached from $i$ through an outgoing edge. Two nodes are adjacent if they share an edge; two edges are adjacent if they share a node. A walk is a sequence of adjacent edges, with possible repetitions of nodes and edges. A path is a walk without repetition of nodes or edges. The length of a walk (or path) is equal to the number of the edges it contains. The shortest path length between two nodes is the length of the shortest path between them.

An integrate-and-fire neuron involves two successive stages: (i) summation of the inputs $x_i$, i.e. $S = \sum x_i$; and (ii) thresholding, i.e. a spike is produced whenever $S$ is larger than a threshold $T$. In this work, each node represents an integrate-and-fire neuron. The incoming activation (from dendrites) is accumulated as its state until a spike occurs, in which case the accumulated activation is completely flushed out through the outgoing connections (axons). In order to ensure conservation of the total activation, each outgoing axon conveys a fraction $S/k(i)$ of the respective previously accumulated activation, where $k(i)$ is the out-degree of neuron $i$. The activation and spiking of all neurons in the network can be represented through the respective activogram and spikogram, namely matrices storing the activation or occurrence of spikes for every node along all considered times. In this article, all neurons have the same threshold $T = 1$, and the external activation of the network is always performed by injecting activation of intensity 1 at each of the neurons. For each of these activations (with the source of activation placed at node $v$), the time one neuron $i$ takes, from the beginning of the external initiation, to receive the first non-zero input is henceforth called its respective beginning activation time $T_s(i,v)$. The time it takes for that neuron to produce the first spike is the beginning spiking time $T_s(i,v)$. The dimensionality of the measurement space defined by the onset activation times for each node in a network involves $N$ measurements (the beginning times). So, we have a total of $N$ measurements for each of the $N$ nodes, which is a high dimensional space involving several correlations between the times. The dimensionality of such measurements spaces can be convenient and optimally reduced (decorrelation) by using the Principal Component Analysis (PCA) methodology (e.g. [14, 30]). Let each of the $N$ observations $v = \{1, 2, \ldots, N\}$, characterized by the set of beginning times at each neuron $i$ as a consequence of activation placed at node $v$, be organized as respective feature vectors $f_v$, with respective elements $f_v(i) = T_s(i,v)$, $i \in \{1, 2, \ldots, N\}$. Let the covariance matrix between each pair of measurements $i$ and $j$ be defined in terms of its elements $C(i,j) = \frac{1}{N-1} \sum_{v=1}^{N} (f_v(i) - \mu_i)(f_v(j) - \mu_j)$

where $\mu_i$ is the average of $f_v(i)$ over the $N$ observations. The eigenvalues of $C$, sorted in decreasing order, are represented as $\lambda_i$, $i = 1, 2, \ldots, M$, with respective eigenvectors $\vec{v}_i$. The following matrix, obtained from the eigenvectors of the covariance matrix, defines the stochastic linear transformation known as the Karhunen-Lo` eve Transform [14, 30].

$$
G = \begin{bmatrix}
\vec{v}_1 & \rightarrow \\
\vec{v}_2 & \rightarrow \\
\vdots & \vdots \\
\vec{v}_m & \rightarrow 
\end{bmatrix}
$$

where $m = N$. Because such a transformation concentrates the variance of the observations along the first axes (the so-called principal axes), it is frequently possible to reduce the dimensionality of the measurements without losing much information (the variances along the other axes tend to be small as a consequence of correlations between the original measurements) by considering in the above matrix only the $m < N$ eigenvectors associated to the largest eigenvalues. The new measurements $\vec{g}$, with dimension $m$, can now be straightforwardly obtained as

$$
\vec{g} = G\vec{f}.
$$

III. RESULTS AND DISCUSSION

The potential of the neuronal community detection approach reported in this article is illustrated with respect to the three following directed networks: (a) a synthetic network (Net1) containing 3 small communities (Figure 1); (b) a medium-sized synthetic network (Net2) containing 4 communities (Figure 1); and (c) the network...
of *C. elegans* (*NetCe*)\(^{22}\). The two synthetic communities were obtained by randomly assigning direct edges among each of the communities, extracting the connected component, and interconnecting the communities according to a fixed probability. Each of the three communities in *Net1* contains 5, 7 and 7 nodes, respectively. Each of the four communities in *Net2* contains 20, 37, 22 and 24 nodes. The largest strongly connected component in the *C. elegans* network, used in this work, contained 239 nodes.

**FIG. 1:** A simple network (*Net1*) involving 3 communities with 5, 7 and 7 nodes considered in this work for illustrative purposes.

Figure 3 shows the activograms and the spikegrams, as well as the respective beginning activation and beginning spiking times diagrams for activation placed at node 3 (a), 9 (b) an 16 (c). It is clear from the respective beginning activation time diagram in Figure 3(a) that the activation being received by node 3 implied in early and simultaneous conveyance of non-zero activation to the other nodes in the community to which node 3 belongs (i.e. the community including nodes 1 to 5). Observe that the beginning spiking times, shown in the respective diagram, are larger than the activation time, because the respective neuronal firing requires accumulation of the activation received by the dendrites of the neurons in that community. Similar nearly simultaneous activations of the other communities can be identified in Figure 3(b) and (c). However, a less uniform initiation of activation in the this community is observed in Figure 3(c).

Figure 4 shows the distribution of the beginning activation times after PCA projection onto a two-dimensional space defined by the principal variables *pca*\(_1\) and *pca*\(_2\). The three communities yielded well-defined respective clusters, which can be immediately identified by using traditional clustering methods (e.g. \(^{30}\)). Observe that the nodes appearing at the borders of each of the three clusters in Figure 4 correspond precisely to those nodes implementing the intercommunity connections (see also \(^{23}\)).

Figure 5 depicts the clustering structure obtained for network *Net2*. Again, each of the communities was clearly mapped into respective clusters in the two-dimensional PCA projected space. Again, the bordering nodes in each cluster can be found to correspond to the interface nodes between communities in the original network. It is interesting to observe that, compared to the previous example, the larger number of communities and nodes in this network tended to imply a more cluttered distribution, especially at the interface between the red and magenta communities. Substantially more separated clusters have been observed in three-dimensional PCA projected spaces.

The distribution of nodes obtained for the *C. elegans* network by two-dimensional PCA projection is shown in Figure 6. A concentration of nodes can be observed at the left-hand side of the space, containing the nodes with higher numbers (the numbers follow the original assignment as in \(^{22}\)). A community can also be discerned at the lower right-hand side of the transformed measurement space. Because of the large number of nodes in this network, it is interesting to consider additional dimensions in the PCA projection. The measurement space defined by the principal variables *pca*\(_1\) and *pca*\(_2\) is shown in Figure 7. This additional projection shows that the denser cluster at the left-hand side of Figure 6 is actually scattered along the third variable *pca*\(_3\), with a more compact cluster of nodes appearing at the upper left-hand side of Figure 7. In addition, a small cluster involving nodes 136, 146, 148, 149, 234, 235 and 236 is now identifiable at the lower left-hand side of Figure 7.

Less definite results were obtained in all cases by considering the beginning spiking times, and no clear cluster structure was observed when other activation of spiking measurements (at given instants or averaged) were used.

**IV. CONCLUDING REMARKS**

This work has addressed an important issue related to the structure-dynamics paradigm in neuronal and complex networks research. More specifically, we have investigated how communities of neurons in directed complex neuronal networks can be identified by considering the transient dynamics of beginning activation of nodes. As a consequence of the integration period required for reaching the firing threshold in each integrate-and-fire neuron, the activation incoming from the source node tends to be trapped inside the respective community, unfolding to other portions of the network only after most of the
neurons in that community have started spiking. The distribution of the activation flushed outside each neuron at the spikings, required for the conservation of the activation, was also critical for the compartmentalization of the activation inside communities. The distinct patterns of beginning activation times obtained by placing the activation source at each of the neurons of each community were clearly revealed by the optimal statistical method of Principal Component Analysis. More specifically, the nodes tended to cluster into respective groups, with the nodes at the borders of such groups corresponding to those nodes implementing the intercommunity connection in the original network. In addition to its intrinsic value for biological neuroscience, these results also provide effective and simple practical means for obtaining neuronal communities.

Several interesting future works are possible. First, it would be important to perform a more systematic and comprehensive study of the separability of the communities by considering other types of networks, with distinct interconnectivity between communities, among other possibilities. Also interesting is to use hierarchical clustering methods (e.g. [14, 19, 30]) in order to obtain a hierarchical organization of the neuronal communities, as well as investigating how such hierarchies (e.g. [29]) are organized with respect to time. As the suggested method can be immediately extended to identification in other types of networks, including non-directed structures, it would be interesting to compare this method with other more traditional approaches not involving
FIG. 3: The activograms and spikegrams, as well as the respective beginning activation and beginning spiking times, are shown with respect to the situations where the activation source corresponded to nodes 3 (a), 9 (b) and 16 (c). The beginning time diagrams show in black the time instants preceding the first activation or spiking of each neuron. For instance, in the beginning spiking times diagram in (a), neuron 5 started spiking at the 8th time step from the initiation of the external activation arriving at node 3.

thresholds. Because abrupt beginning of spiking has been observed [17] for several types of complex networks, it would be also interesting to search for possible phase transitions of activation inside each community, which could be ultimately responsible for the activation trapping inside each neuronal community during the transient activation period.

All in all, the findings and perspectives reported in this article have supported the fact that investigations of transient non-linear dynamics are specially promising and useful in the study of complex systems (see also [17, 31–34]).

Acknowledgments

Luciano da F. Costa thanks CNPq (308231/03-1) and FAPESP (05/00587-5) for sponsorship.

[1] L. R. Squire, F. E. Bloom, S. K. McConnell, J. L. Roberts, N. S. Spitzer, and M. J. Zigmond, Fundamental Neuroscience (Academic Press, 2003).
[2] D. Stauffer, L. Aharony, L. da F. Costa, and J. Adler, Eur. Phys. J. B 32, 395 (2003).
[3] L. da F. Costa and D. Stauffer, Physica A 330, 37 (2003).
[4] L. da F. Costa (2005), arXiv:q-bio/0503041.
[5] B. J. Kim, Phys. Rev. E 69, 045101 (2004).
FIG. 4: The PCA projection of the patterns of beginning activation times obtained for Net1. Each of the three original communities can be clearly identified from the three respective clusters in this scatterplot.

[6] R. M. Memmesheimer and M. Timme, Physica D 224, 182 (2006).
[7] G. V. Osipov, J. Kurths, and C. Zhou, Synchronization in Oscillatory Networks (Springer, 2007).
[8] H. Hasegawa, Phys. Rev. E 70, 066107 (2004).
[9] H. Hasegawa, Phys. Rev. E 72, 056139 (2005).
[10] S. M. Park and B. J. Kim, Phys. Rev. E 74, 026114 (2006).
[11] R. Albert and A. L. Barabási, Rev. Mod. Phys. 74, 47 (2002).
[12] M. E. J. Newman, SIAM Rev. 45, 167 (2003).
[13] S. N. Dorogovtsev and J. F. F. Mendes, Adv. in Phys. 51, 1079 (2002).
[14] L. da F. Costa, F. A. Rodrigues, G. Travieso, and P. R. V. Boas, Adv. in Phys. 56, 167 (2007).
[15] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D. Hwang, Phys. Rep. 424, 175 (2006).
[16] L. da F. Costa and O. Sporns, Intl. J. Bif. Chaos 17, 2387 (2007).
[17] L. da F. Costa (2008), arXiv:0801.3056.
[18] M. Girvan and M. E. J. Newman, Proc. Natl. Acad. Sci. USA 99, 7821 (2002).
[19] H. Zhou, Phys. Rev. E 67, 061901 (2003).
[20] M. E. J. Newman, Eur. Phys. J. B 38, 321 (2004).
[21] F. Radicchi, C. Castellano, F. Cecconi, V. Loreto, and D. Parisi, Proc. Natl. Acad. Sci. USA 101, 2658 (2004).
[22] D. J. Watts and S. H. Strogatz, Nature 393, 409 (1998).
[23] R. Guimerà and L. A. N. Amaral, Nature 433, 895 (2005).
[24] J. Bagrow and E. M. Bollt, Phys. Rev. E 72, 046108 (2005).
[25] A. Capocci, V. D. P. Servedio, G. Caldarelli, and F. Colaiori, Phys. A 352, 669 (2005).
[26] F. A. Rodrigues, G. Travieso, and L. da F. Costa, Intl. J. Mod. Phys. C 18, 937 (2006).
[27] J. Hopcroft, O. Khan, B. Kulis, and B. Selman, Proc. Natl. Acad. Sci. USA 101, 5249 (2004).
[28] M. Latapy and P. Pons, Proc. 20th Intl. Sympo. Comp. and Inf. Sci pp. 284–293 (2005), arXiv:physics/0512106.
[29] A. Arenas, A. Fernandez, and S. Gomez (2008), arXiv:physics/0703218.
[30] L. da F. Costa and R. M. Cesar, Shape Analysis and Classification: Theory and Practice (CRC Press, 2001).
[31] V. Latora and M. Baranger, Physical Review Letters 82, 520 (1999).
[32] J. G. Gardener and V. Latora (2007), arXiv:0712.0278.
[33] L. da F. Costa (2008), arXiv:0801.0380.
[34] L. da F. Costa (2008), arXiv:0801.2520.
[35] Though a total of $N-1$ non-zero beginning times is obtained for each activation, we consider a vector of $N$ measurements by incorporating the zero time respective to the activation source.
FIG. 5: The PCA projection of the patterns of beginning activation times obtained for Net2. Each of the four original communities can be clearly identified from the respective clusters in this scatterplot, with the nodes at the borders of the clusters corresponding to the nodes at the borders of the original communities.
FIG. 6: The distribution of nodes obtained for the *C. elegans* network considering the two principal PCA variables *pca1* and *pca2*. 
FIG. 7: The distribution of nodes obtained for the *C. elegans* network considering the first and third principal PCA variables pca1 and pca3.