Giant monopole resonance in even-\(A\) Cd isotopes, the asymmetry term in nuclear incompressibility, and the “softness” of Sn and Cd nuclei

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**Abstract**

The isoscalar giant monopole resonance (ISGMR) in even-\(A\) Cd isotopes has been studied by inelastic \(\alpha\)-scattering at 100 MeV/u and at extremely forward angles, including 0°. The asymmetry term in the nuclear incompressibility extracted from the ISGMR in Cd isotopes is found to be \(\tau = -555 \pm 75\) MeV, confirming the value previously obtained from the Sn isotopes. ISGMR strength has been computed in relativistic RPA using NL3 and FSUGold effective interactions. Both models significantly overestimate the centroids of the ISGMR strength in the Cd isotopes. Combined with other recent theoretical effort, the question of the “softness” of the open-shell nuclei in the tin region remains open still.

The equation of state (EOS) of nuclear matter plays an important role in our understanding of a number of interesting phenomena such as the collective behavior of nucleons in the nuclei, the massive stellar collapse leading to a supernova explosion, nuclear properties including the neutron-skin thickness of heavy nuclei, and the radii of neutron stars \(1,2\). The nuclear incompressibility, \(K_\infty\), is the curvature of EOS of nuclear matter at saturation density \(3\). \(K_\infty\) is, thus, a measure of nuclear stiffness and thereby imposes significant constraints on theoretical descriptions of the effective nuclear interactions. However, even more stringent constraints emerge as one studies the evolution of the incompressibility coefficient as the system becomes neutron rich. Neutron-rich systems are sensitive to the poorly-known density dependence of the symmetry energy and the experiment reported here is of vital importance in this regard.

The study of the isoscalar giant monopole resonance (ISGMR) provides a direct experimental tool to study nuclear incompressibility in finite nuclear systems. The centroid energy of ISGMR, \(E_{\text{ISGMR}}\), can be directly related to the nuclear incompressibility of finite nuclear matter, \(K_A\), as:

\[
E_{\text{ISGMR}} = \hbar \sqrt{\frac{K_A}{m\langle r^2 \rangle}}
\]

where, \(m\) is the nucleon mass and \(\langle r^2 \rangle\) is the mean square radius of the nucleus \(4,5\). \(K_A\) may be further parameterized as \(5,6\):

\[
K_A \approx K_{\text{vol}}(1 + cA^{-1/3}) + K_T ((N - Z)/A)^2 + K_{\text{Coul}}A^{-4/3}.
\]

Here, \(K_{\text{vol}}\) is the volume term, directly related to \(K_\infty\) \(c \sim -1\) \(7\). \(K_{\text{Coul}}\) is essentially a model-independent term \(8\), and \(K_T\) is the asymmetry term. Although closely related, the finite-nucleus asymmetry term \(K_T\) should not be confused with the corresponding term in infinite nuclear matter—a quantity also denoted by \(K_T\) at times, but written here as \(K_T^\infty\). Indeed, \(K_T^\infty\) should never be regarded as the \(A \to \infty\) limit of the finite-nucleus asymmetry \(K_T\) \(6\). Yet, the fact that \(K_T^\infty\) is both experimentally accessible and strongly correlated with \(K_T^\infty\) is vital in placing stringent constraints on the density dependence of the symmetry energy. Recall that \(K_T^\infty\) is simply related to a few fundamental parameters of the equation of state \(9\):

\[
K_T^\infty \sim 155 - 420\alpha - 130\beta.
\]

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where $Q_0$ is the “skewness” parameter of symmetric nuclear matter and $L$ and $K_{sym}$, respectively, are the slope and curvature of the symmetry energy. It is the strong sensitivity of $K_{sym}^{\infty}$ to the density dependence of the symmetry energy that makes the present study of critical importance in constraining the EOS of neutron-rich matter.

This asymmetry term, $K_\tau$, can be studied over a series of isotopes for which the neutron–proton asymmetry, $(N - Z)/A$, changes by an appreciable amount. The first such investigation was carried out by Sharma et al. [10] and later much improved by Li et al. over the even–even $^{112-126}$Sn isotopes [11,12]. Li et al. obtained a value $K_\tau = -550 \pm 100$ MeV, a number consistent with values indirectly obtained from several other measurements [8,13,14]. Most intriguingly, the ISGMR centroid energies of the Sn isotopes were found to be consistently lower than those predicted by relativistic and non-relativistic calculations by as much as 1 MeV [11,12]. It bears noting that the interactions used in these calculations were the very same that reproduced the ISGMR centroid energies in “standard” nuclei, $^{208}$Pb and $^{90}$Zr, very well, leading to the question: Why are the Sn isotopes so fluffy? [15,16].

To confirm the value of $K_\tau$ obtained from the Sn isotopes, and to further explore the lack of success of the relativistic as well as non-relativistic calculations in correctly obtaining the centroids of the ISGMR strength in the open-shell nuclei, we have measured the ISGMR strength distributions in the even–even $^{106,110-116}$Cd isotopes. The asymmetry parameter, $(N - Z)/A$, changes by as much as 83% along this chain, making these nuclei very attractive, like the stable Sn isotopes, from the point of view of extracting the asymmetry term in the nuclear incompressibility. Preliminary results from this investigation have been presented previously [17,18].

The experiment was performed at the Ring Cyclotron facility of the Research Center for Nuclear Physics (RCNP), Osaka University, Japan. $^4$He particles at an incident energy of 100 MeV/u were scattered off self-supporting even–A $^{106,110-116}$Cd targets; highly-enriched Cd targets (>93%) with thicknesses ranging from 5 to 6.5 mg/cm$^2$ were used. Elastic as well as inelastic scattering measurements were performed over a wide range of angular settings—elastic scattering over $3^\circ$–$19^\circ$ and inelastic scattering at extremely forward angles ($0^\circ$–$9.8^\circ$). The primary justification for the difficult-to-do extremely forward angle measurements lies in the angular distribution patterns, which exhibit clear distinction between various multipoles at these angles and, with the ISGMR cross sections peaking at $0^\circ$, it is extremely important to make a measurement as close to $0^\circ$ as possible.

The scattered $\alpha$ particles were momentum analyzed by the high-resolution magnetic spectrometer, Grand Raiden [19], and focused onto the focal-plane detector system comprised of two MWDCs [20] and two plastic scintillator counters. The MWDCs allow measurement of the horizontal and vertical coordinates of the impact position of the $\alpha$ particle on the focal plane. This, in turn, allows the determination of the angle of incidence on the focal plane and the momentum of the scattered $\alpha$ particles. Using the ray-tracing technique for trajectory determination of scattered particles, energy spectra were obtained for specific scattering angles by subdividing the full angular opening. The focal-plane detector covered the excitation–energy range of $E_x \sim 8$ MeV to 31 MeV. Energy calibration runs were carried out with a $^{12}$C target at every angle for each target. The Grand Raiden spectrometer was used in the double-focusing mode in order to eliminate the instrumental background [12,21]. The background-subtracted spectra at an “average” angle of 0.7° for various Cd isotopes are shown in Fig. 1. Elastic-scattering cross sections were used to extract the optical model parameters (OMP) for these beam-target combinations. The parameters were determined using the “hybrid” potential proposed by Satchler and Khoa [22] and found to work very well for $\alpha$-scattering at medium energies (see, for example, Refs. [12,23,24]). In this procedure, the density-dependent single-folding model with a Gaussian $\alpha$-nucleon potential was used to determine the real part of the optical potential. The computer codes SDOLFIN and DOLFIN [25] were used to calculate the shape of the real part of the potential and the form factors, respectively. For the imaginary term, a Woods–Saxon potential was used and its parameters, together with the depth of the real part, $V$, were obtained by fitting the elastic-scattering cross sections using the minimization of chi-square technique, with the help of the computer code PTOLEMY [26]. Using the known $B(E2)$ values from the literature [27], the OMP thus obtained, the angular distributions of differential cross sections were calculated for the $2^+_1$ states. An excellent agreement between the calculated and experimental angular distributions of differential cross sections for the $2^+_1$ states established the appropriateness of the OMP.

The inelastic-scattering cross sections were sorted into 1 MeV bins to reduce statistical fluctuations. The experimentally-obtained spectra consist of contributions from various multipoles. In order to extract the ISGMR contribution from these spectra at different scattering angles, multipole-decomposition analysis (MDA) was performed. The experimental double-differential cross sections are expressed as linear combinations of calculated distorted-wave Born approximation (DWBA) double-differential cross sections for different multipoles as follows:

$$\frac{d^2\sigma}{d\Omega \, dE} = \sum_{L=0}^{7} a_L(E_x) \frac{d^2\sigma^{\text{DWBA}}(E_m, E_x)}{d\Omega \, dE}$$

Fig. 1. Excitation-energy spectra at an “average” angle of 0.7° for the even–even Cd isotopes investigated in this work.

where $L$ is the order of the multipole and $a_L(E_x)$ is the percentage of the energy-weighted sum rule (EWSR) for multipolarity $L$. DWBA cross sections corresponding to 100% EWSR were calculated using transition densities and sum rules provided in Refs. [28,29]. DWBA calculations were performed for up to a maximum angular-momentum transfer of $L = 7$; addition of higher angular-momentum-transfer terms resulted in minimal or no change in the strength distributions. Further details on the MDA can be found in Refs. [21,30,31]. The isovector giant dipole resonance (IVGDR) contribution was subtracted out of the experimental spectra prior to the fitting procedure. Photonuclear data were used in conjunction with the fitting procedure to extract the ISGMR strength distributions. Further details on the MDA can be found in Refs. [21,30,31].
The parameters of the Lorentzian fit, and the customary moment ratios calculated over the excitation-energy range 10.5–20.5 MeV, as well as those from calculations for the FSUGold, NL3 and SLy5 (without pairing) interactions; \( m_0 \) is the 0th moment of the strength distribution; \( m_i = \int E S(E_x) dE_x \). For comparison, Lorentzian parameters from Liu et al. (Gaussian fits) are also provided, where available [24].

| Target  | \( E_{\text{ISGMR}} \) (MeV) | \( \Gamma \) (MeV) | \( \sqrt{\Gamma/\Gamma_{-1}} \) (MeV) | \( \sqrt{\Gamma/\Gamma_1} \) (MeV) | \( m_1/m_0 \) (MeV) |
|---------|----------------|----------------|----------------|----------------|----------------|
| \(^{106}\text{Cd}\) | 16.50 ± 0.19 | 6.14 ± 0.37 | – | 16.06 ± 0.05 | 16.83 ± 0.09 | 16.27 ± 0.09 | 16.73 | 17.25 | 16.92 |
| \(^{108}\text{Cd}\) | – | – | – | – | – | – | 16.05 | 17.17 | – |
| \(^{110}\text{Cd}\) | 16.09 ± 0.15 | 5.72 ± 0.45 | 15.71 ± 0.11 | 15.72 ± 0.05 | 16.53 ± 0.08 | 15.94 ± 0.07 | 16.59 | 17.09 | 16.65 |
| \(^{112}\text{Cd}\) | 15.72 ± 0.10 | 5.85 ± 0.38 | – | 15.59 ± 0.05 | 16.38 ± 0.06 | 15.80 ± 0.05 | 16.50 | 17.00 | 16.50 |
| \(^{114}\text{Cd}\) | 15.59 ± 0.20 | 6.41 ± 0.64 | – | 15.37 ± 0.08 | 16.27 ± 0.09 | 15.61 ± 0.08 | 16.38 | 16.90 | 16.47 |
| \(^{116}\text{Cd}\) | 15.43 ± 0.12 | 6.51 ± 0.40 | 15.17 ± 0.12 | 15.19 ± 0.06 | 16.14 ± 0.07 | 15.44 ± 0.06 | 16.27 | 16.77 | 16.36 |

\( m_0 \) = \int E S(E_x) dE_x.

\( \sqrt{m_1/m_0} \) = \frac{m_1}{m_0}.

\( \sqrt{m_1/m_{-1}} \) = \frac{m_1}{m_{-1}}.

\( \sqrt{m_1/m_1} \) = \frac{m_1}{m_1}.

\( \Gamma_{-1} \) = \frac{\Gamma}{\Gamma_{-1}}.

\( \Gamma_1 \) = \frac{\Gamma}{\Gamma_1}.

Table 1: Lorentzian-fit parameters for the ISGMR strength distributions in the Cd isotopes investigated in this work. Also presented are the various moment ratios calculated over the excitation-energy range 10.5–20.5 MeV, as well as those from calculations for the FSUGold, NL3 and SLy5 (without pairing) interactions; \( m_0 \) is the 0th moment of the strength distribution; \( m_i = \int E S(E_x) dE_x \). For comparison, Lorentzian parameters from Liu et al. (Gaussian fits) are also provided, where available [24].

The different centroids of ISGMR in \(^{208}\text{Pb}\) [40]. Yet, this same model significantly overestimates the ISGMR energies in the Sn isotopes [15]. Perhaps not surprisingly, our present theoretical results overestimate the centroid energies in the nearby Cd isotopes as well. The use of the NL3 effective interaction, with an incompressibility coefficient significantly larger than FSUGold, exacerbates the discrepancy between theory and experiment even further. Likewise, in a recently available calculation [36] within the Skyrme Hartree–Fock + BCS and quasiparticle RPA with the SLy5 parameter set (\( K_{\text{xc}} = 230 \text{ MeV} \)), which, incidentally, reproduces the ISGMR in \(^{208}\text{Pb} \) very well), the centroids of ISGMR strength distributions in the Cd isotopes (also shown in Fig. 3) are, again, significantly larger than the experimentally-obtained results. Thus, the question originally posed in Refs. [15,16,40] of “Why are the Sn isotopes so Fluffy” extends to the cadmium isotopes as well.

We conclude this theoretical discussion with a brief comment on the possible role of superfluid (or pairing) correlations on the softening of the isoscalar monopole response in the Sn and Cd isotopes. This was investigated previously by Civitarese et al. [41] and more recently by Li et al. [42] and Cao et al. [36]. Indeed, even with inclusion of the pairing effects, using a mixed pairing interaction [36], the centroids of the ISGMR remain well above the experiment.
natural values (see Fig. 3)—the net effect appears to be that of lowering the centroid by only ~100 keV in $^{106}$Cd to a maximum of ~240 keV in $^{116}$Cd. Thus, the impact of superfluid correlations on the isoscalar giant monopole resonance in the even-A $^{106,110–116}$Cd isotopes via inelastic scattering of $\alpha$ particles at extremely-forward angles, including $0^\circ$, with the aim to put on further experimental footing the results obtained earlier for the ISGMR in the Sn isotopes. The centroids of the ISGMR have been calculated in the relativistic RPA using the NL3 and FSUGold effective interactions. The calculated centroids for the Cd isotopes are significantly larger than the experimentally obtained values, similar to the results obtained for the Sn isotopes, leaving the question of the “softness” of the open-shell nuclei open still. The value of $\sim 555 \pm 75$ MeV for the asymmetry term in the nuclear-matter incompressibility, $K_\tau$, extracted from this measurement confirms the value obtained from the study of the Sn isotopes.

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