Two-particle contributions and nonlocal effects in the QCD sum rules for the axialvector tetraquark candidate \(Z_c(3900)\)

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Abstract

In this article, we study the \(Z_c(3900)\) with the QCD sum rules in details by including the two-particle scattering state contributions and nonlocal effects between the diquark and antidiquark constituents. The two-particle scattering state contributions cannot saturate the QCD sum rules at the hadron side, the contribution of the \(Z_c(3900)\) plays an un-substitutable role, we can saturate the QCD sum rules with or without the two-particle scattering state contributions. If there exists a barrier or spatial separation between the diquark and antidiquark constituents, the Feynman diagrams can be divided into the factorizable and nonfactorizable diagrams. The factorizable diagrams consist of two colored clusters and lead to a stable tetraquark state. The nonfactorizable Feynman diagrams correspond to the tunnelling effects, which play a minor important role in the QCD sum rules, and are consistent with the small width of the \(Z_c(3900)\).

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1 Introduction

In 2013, the BESIII collaboration observed the \(Z_c(3900)\) in the \(\pi^\pm J/\psi\) mass spectrum with a mass of \((3899.0 \pm 3.6 \pm 4.9)\) MeV and a width of \((46 \pm 10 \pm 20)\) MeV, respectively \cite{1}. The \(Z_c(3900)\) was also observed by the Belle collaboration \cite{2} and was confirmed by the CLEO collaboration \cite{3}. The average values of the mass and width of the \(Z_c(3900)\) listed in the Review of Particle Physics are \(M = (3887.2 \pm 2.3)\) MeV and \(\Gamma = (28.2 \pm 2.6)\) MeV, respectively \cite{4}. There have been several possible assignments for the \(Z_c(3900)\), such as tetraquark state \cite{5 6 7 8}, molecular state \cite{9 10}, hadro-charmonium \cite{11}, rescattering effect \cite{12}.

In 2014, the LHCb collaboration confirmed the \(Z_c(4430)\) state in the \(B^0 \rightarrow J/\psi \pi^- K^+\) decays and established its spin-parity \(J^P = 1^+\), the measured mass and width are \(M = (4475 \pm 7^{+15}_{-25})\) MeV and \(\Gamma = (172 \pm 13^{+37}_{-34})\) MeV, respectively \cite{13}. In 2017, the BESIII collaboration determined the spin and parity of the \(Z_c(3900)\) state \(J^P = 1^+\) \cite{14}.

The \(Z_c(3900)\) and \(Z_c(4430)\) can be assigned to be the ground state and the first radial excitation of the axialvector tetraquark states respectively according to the analogous decays, \(Z_c(3900)\rightarrow J/\psi\pi^\pm, Z_c(4430)^\pm \rightarrow J/\psi\pi^\pm,\) and the mass gaps \(M_{Z_c(4430)} - M_{Z_c(3900)} = 588\) MeV = \(m_{\psi'} - m_{J/\psi} = 589\) MeV \cite{15 16 17 18}.

In Ref.\cite{7}, we study the axialvector hidden-charm tetraquark states with the QCD sum rules, and explore the energy scale dependence of the QCD sum rules for the tetraquark states in details for the first time. The calculations support assigning the \(X(3872)\) and \(Z_c(3900)\) to be the \(J^{PC} = 1^{++}\) and \(1^{+-}\) diquark-antidiquark type tetraquark states, respectively. In Refs.\cite{8 19 20}, the two-body strong decays of the \(Z_c(3900)\) are studied with the QCD sum rules, which also support assigning the \(Z_c(3900)\) to be the \(J^{PC} = 1^{+-}\) diquark-antidiquark type tetraquark state. In Ref.\cite{21}, we take the diquark and antidiquark operators as the basic constituents to construct the scalar, axialvector and tensor currents to study the mass spectrum of the ground state hidden-charm tetraquark states with the QCD sum rules in a comprehensive way, and revisit the assignments of the \(X(3860), X(3872), X(3915), X(3940), X(4160), Z_c(3900), Z_c(4020), Z_c(4050), Z_c(4055), Z_c(4100), Z_c(4200), Z_c(4250), Z_c(4430), Z_c(4600)\), etc.
In those studies [7, 8, 19, 20, 21], the axialvector current,

\[ J_\mu(x) = \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}} \left\{ u_j^T(x)C\gamma_5 c_k(x)d_m(x)\gamma_\mu C\epsilon_n^T(x) - u_j^T(x)C\gamma_\mu c_k(x)d_m(x)\gamma_5 C\epsilon_n^T(x) \right\}, \quad (1) \]

is chosen to interpolate the \( Z_c(3900) \), where the \( i, j, k, m, n \) are color indices. The current \( J_\mu(x) \) has the quantum numbers \( J^{PC} = 1^{+-} \), the quantum field theory does not forbid the couplings to the two-particle scattering states \( J/\psi\pi^+ \), \( \eta, p^+ \), \( (D^*D^*)^+ \), \( \cdots \) with the \( J^{PC} = 1^{+-} \). Up to now, the contributions of the two-particle scattering states in the QCD sum rules for the hidden-charm tetraquark states have not been studied quantitatively. On the other hand, there maybe exist a barrier or spatial separation between the diquark and antidiquark constituents [22, 23, 24, 25], the nonlocal effects have not been taken into account in the QCD sum rules for the tetraquark states yet. In this article, we study the \( Z_c(3900) \) with the QCD sum rules in details by including the contributions of the two-particle scattering states and nonlocal effects between the diquark and antidiquark constituents, the conclusion is expected to survive for other diquark-antidiquark type tetraquark states.

The article is arranged as follows: in Sect.2, we derive the QCD sum rules by including the contributions of the two-particle scattering states and nonlocal effects between the diquark and antidiquark constituents; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusion.

2 Two-particle contributions and nonlocal effects in the QCD sum rules

Firstly, we write down the two-point correlation function \( \Pi_{\mu\nu}(p) \),

\[ \Pi_{\mu\nu}(p) = i \int d^4xe^{ipx} \langle 0|T \{ J_\mu(x, \epsilon)J_\nu^\dagger(0, \epsilon) \} |0 \rangle, \quad (2) \]

where

\[ J_\mu(x, \epsilon) = \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}} \left\{ u_j^T(x)C\gamma_5 c_k(x)d_m(x + \epsilon)\gamma_\mu C\epsilon_n^T(x + \epsilon) - u_j^T(x)C\gamma_\mu c_k(x)d_m(x + \epsilon)\gamma_5 C\epsilon_n^T(x + \epsilon) \right\}, \quad (3) \]

the \( i, j, k, m, n \) are color indices. We choose the current \( J_\mu(x, \epsilon) \) to interpolate the \( Z_c(3900) \), and take into account the nonlocal effects between the diquark and antidiquark constituents in the current \( J_\mu(x, \epsilon) \) by adding a finite four vector \( \epsilon \). The diquark-antidiquark type tetraquark state can be plausibly described by two diquarks in a double well potential separated by a barrier [22, 23]. At long distances, the diquark and antidiquark serve as two point color charges, and attract each other strongly. At shorter distances, those effects beyond the naive one-gluon exchange force increase at decreasing distances and produce a repulsion between diquark and antidiquark constituents thus form a barrier to avoid destroying the diquark and antidiquark, if large enough [24]. The diquark-antidiquark type tetraquark states emerge as QCD molecular objects made of spatially separated colored two-quark lumps in the Born-Oppenheimer approximation [24]. In the dynamical picture of the tetraquark states, the large spatial separation between the diquark and antidiquark leads to small wave-function overlap between the quark and antiquark constituents [25], which suppresses the Fierz rearrangements.

In Ref. [7], we observe that the axialvector tetraquark current \( J_\mu(x, 0) \) couples potentially to the \( Z_c(3900) \),

\[ \langle 0|J_\mu(0)|Z_c(p) \rangle = \lambda_Z \varepsilon_\mu, \quad (4) \]
where the $\lambda_Z$ is the current-hadron coupling constant or pole residue, the $\varepsilon_\mu$ is the polarization vector.

If we perform Fierz rearrangement both in the color and Dirac-spinor spaces, we can change the diquark-antidiquark type current $J_\mu(x, \varepsilon)$ into a special superposition of the color singlet-singlet type currents [26, 27],

$$J_\mu = \frac{1}{2\sqrt{2}} \left\{ i\vec{c}i\gamma_5 c \bar{d} \gamma_\mu u - i\vec{c}i\gamma_\mu c \bar{d} \gamma_5 u + \bar{c}a \bar{d} \gamma_\mu \gamma_5 c - \bar{c} \gamma_\mu \gamma_5 u \bar{d} c 
- i\vec{c}i\gamma_\mu c \bar{d} \sigma_\mu u + i\vec{c}i\sigma_\mu c \bar{d} \gamma_\mu \gamma_5 u - i\vec{c}i\sigma_\mu \gamma_5 u \bar{d} \gamma_\mu c + i\vec{c}i\gamma_\mu u \bar{d} \sigma_\mu \gamma_5 c \right\} ,$$

$$= \frac{1}{2\sqrt{2}} \left\{ iJ^1_\mu - iJ^2_\mu - iJ^3_\mu + iJ^4_\mu + J^5_\mu - J^6_\mu + J^7_\mu + iJ^8_\mu \right\} , \quad (5)$$

where

$$J^1_\mu(x, \varepsilon) = \bar{c}(x + \varepsilon)i\gamma_5 c(x)\bar{d}(x + \varepsilon)\gamma_\mu u(x) , 
J^2_\mu(x, \varepsilon) = \bar{c}(x + \varepsilon)i\gamma_\mu c(x)\bar{d}(x + \varepsilon)i\gamma_5 u(x) , 
J^3_\mu(x, \varepsilon) = \bar{c}(x + \varepsilon)\gamma^\alpha \gamma_5 c(x)\bar{d}(x + \varepsilon)\sigma_\mu u(x) , 
J^4_\mu(x, \varepsilon) = \bar{c}(x + \varepsilon)\sigma_\mu c(x)\bar{d}(x + \varepsilon)\gamma^\alpha \gamma_5 u(x) , 
J^5_\mu(x, \varepsilon) = \bar{c}(x + \varepsilon)u(x)\bar{d}(x + \varepsilon)\gamma^\alpha \gamma_5 c(x) , 
J^6_\mu(x, \varepsilon) = \bar{c}(x + \varepsilon)\gamma_\mu \gamma_5 u(x)\bar{d}(x + \varepsilon)c(x) , 
J^7_\mu(x, \varepsilon) = \bar{c}(x + \varepsilon)\sigma_\mu c(x)\bar{d}(x + \varepsilon)\gamma^\alpha c(x) , 
J^8_\mu(x, \varepsilon) = \bar{c}(x + \varepsilon)\gamma^\alpha u(x)\bar{d}(x + \varepsilon)\sigma_\mu c(x) . \quad (6)$$

In fact, the barrier or spatial separation between the diquark and antidiquark pair frustrates the Fierz rearrangements [22, 23, 24, 25].

The color singlet-singlet type currents couple potentially to the meson-meson pairs or molecular states. In the following, we write down the couplings to the meson-meson pairs explicitly,

$$\langle 0 | J^1_\mu(0) | \eta_c(q) \rho(p - q) \rangle = \frac{f_{\eta_c} m^2_{\eta_c}}{2m_c} f_{\rho} m^2_{\rho} \varepsilon^\rho_\mu ,$$

$$\langle 0 | J^2_\mu(0) | \pi(q) J/\psi(p - q) \rangle = \frac{f_{\pi} m^2_{\pi}}{m_\pi + m_d} f_{J/\psi} m_{J/\psi} \varepsilon^\pi_\mu \varepsilon^{J/\psi}_\mu ,$$

$$\langle 0 | J^3_\mu(0) | \pi(q) \psi'(p - q) \rangle = \frac{f_{\pi} m^2_{\pi}}{m_\pi + m_d} f_{\psi'} m_{\psi'} \varepsilon^\pi_\mu \varepsilon^{\psi'}_\mu , \quad (7)$$

$$\langle 0 | J^4_\mu(0) | \eta_c(q) \rho(p - q) \rangle = -if_{\eta_c} q^\alpha i f^T_{\rho} \left[ \varepsilon^\rho_\mu (p - q)_\alpha - \varepsilon^\rho_\mu (p - q)_\mu \right] ,$$

$$\langle 0 | J^5_\mu(0) | \eta_c(q) b_1(p - q) \rangle = -if_{\eta_c} q^\alpha i f_{b_1} \varepsilon_{\mu \sigma \tau} \varepsilon^{b_1}_{\mu \sigma \tau},$$

$$\langle 0 | J^6_\mu(0) | \chi_{c1}(q) \rho(p - q) \rangle = f_{\chi_{c1}} m_{\chi_{c1}} \varepsilon^{\alpha}_{\chi_{c1}} i f^T_{\rho} \left[ \varepsilon^\rho_\mu (p - q)_\alpha - \varepsilon^\rho_\mu (p - q)_\mu \right] , \quad (8)$$

$$\langle 0 | J^7_\mu(0) | \pi(q) J/\psi(p - q) \rangle = -if_{\pi} q^\alpha i f^T_{J/\psi} \left[ \varepsilon^J/\psi_\mu (p - q)_\alpha - \varepsilon^J/\psi_\mu (p - q)_\mu \right] ,$$

$$\langle 0 | J^8_\mu(0) | \pi(q) \psi'(p - q) \rangle = -if_{\pi} q^\alpha i f_{\psi'} \left[ \varepsilon^\psi'(p - q)_\alpha - \varepsilon^\psi'(p - q)_\mu \right] ,$$

$$\langle 0 | J^9_\mu(0) | \pi(q) h_c(p - q) \rangle = -if_{\pi} q^\alpha i f_{h_c} \varepsilon_{\mu \sigma \tau} \varepsilon^{h_c}_{\mu \sigma \tau},$$

$$\langle 0 | J^{10}_\mu(0) | \pi(q) h_c'(p - q) \rangle = -if_{\pi} q^\alpha i f_{h_c'} \varepsilon_{\mu \sigma \tau} \varepsilon^{h_c'}_{\mu \sigma \tau},$$

$$\langle 0 | J^{11}_\mu(0) | a_1(q) J/\psi(p - q) \rangle = f_{a_1} m_{a_1} \varepsilon^{\alpha}_{a_1} i f^T_{J/\psi} \left[ \varepsilon^J/\psi_\mu (p - q)_\alpha - \varepsilon^J/\psi_\mu (p - q)_\mu \right] . \quad (9)$$
the \( \varepsilon_\mu \) are the polarization vectors of the vector and axialvector mesons, the \( f_\eta, f_\rho, f_\tau, f_{J/\psi}, f_{\psi'}, f_{J/\psi}', f_{\psi}', f_{\psi'}, f_{\psi'} \), \( f_{J/\psi}', f_{\psi}', f_{\psi}' \), \( f_{1} \), \( f_{0} \), \( f_{1} \), \( f_{0} \), \( f_{2} \), \( f_{2} \) and \( f_{2} \) are the decay constants.

We write the correlation function \( \Pi_{\mu\nu}(p) \) as

\[
\Pi_{\mu\nu}(p) = \Pi(p^2) \left( -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \Pi_0(p^2) \frac{p_\mu p_\nu}{p^2},
\]

according to Lorentz covariance, where the \( \Pi(p^2) \) and \( \Pi_0(p^2) \) denote the spin 1 and 0 components respectively. In this article, we study the component \( \Pi(p^2) \).

At the hadron side, we take into account the contributions of the tetraquark candidate \( Z_c(3900) \) and the two-particle scattering state contributions from the \( \pi J/\psi, \pi h_c, \eta_c \rho_1, D^* D^*, D_0 D, \eta b_1, \eta b_1 \), \( \eta b_1 \), \( \eta b_1 \), \( \eta b_1 \), \( \eta b_1 \) below the threshold of the \( Z_c(4430) \), which can be assigned to the first radial excitation of the \( Z_c(3900) \) \[15, 16, 17\].

\[
\Pi(p^2) = \frac{\lambda_Z^2}{M_Z^2 - p^2} + \lambda_{\eta_c \rho_1}^2 \int_{m_{\eta_c \rho_1}^2}^{s_0} ds \frac{1}{s - p^2} \frac{\sqrt{\lambda(s, m_{\eta_c}^2, m_{\rho_1}^2)}}{s} \left[ 1 + \frac{\lambda(s, m_{\eta_c}^2, m_{\rho_1}^2)}{12s m_{\rho_1}^2} \right]
\]

\[
+ \lambda_{\pi J/\psi; 22}^2 \int_{m_{\pi J/\psi}^2}^{s_0} ds \frac{1}{s - p^2} \frac{\sqrt{\lambda(s, m_{\pi}^2, m_{J/\psi}^2)}}{s} \left[ 1 + \frac{\lambda(s, m_{\pi}^2, m_{J/\psi}^2)}{12s m_{J/\psi}^2} \right]
\]

\[
+ \lambda_{\pi J/\psi; 22}^2 \int_{m_{\pi J/\psi}^2}^{s_0} ds \frac{1}{s - p^2} \frac{\sqrt{\lambda(s, m_{\pi}^2, m_{J/\psi}^2)}}{s} \left[ (s - m_{\pi}^2 - m_{J/\psi}^2)^2 - \frac{\lambda(s, m_{\pi}^2, m_{J/\psi}^2)(s - m_{J/\psi}^2)}{3s} \right]
\]

\[
+ \lambda_{\pi J/\psi; 22}^2 \int_{m_{\pi J/\psi}^2}^{s_0} ds \frac{1}{s - p^2} \frac{\sqrt{\lambda(s, m_{\pi}^2, m_{J/\psi}^2)}}{s} \left[ \frac{\lambda(s, m_{\pi}^2, m_{J/\psi}^2)}{6} - (s - m_{\pi}^2 - m_{J/\psi}^2) \right]
\]

\[
+ \lambda_{\pi J/\psi; 22}^2 \int_{m_{\pi J/\psi}^2}^{s_0} ds \frac{1}{s - p^2} \frac{\sqrt{\lambda(s, m_{\pi}^2, m_{J/\psi}^2)}}{s} \left[ \frac{\lambda(s, m_{\pi}^2, m_{J/\psi}^2)}{6} - (s - m_{\pi}^2 - m_{J/\psi}^2) \right]
\]

\[
+ \lambda_{\eta b_1}^2 \int_{m_{\eta c}^2}^{s_0} ds \frac{1}{s - p^2} \frac{\sqrt{\lambda(s, m_{\eta c}^2, m_{b_1}^2)}}{s} \frac{\lambda(s, m_{\eta c}^2, m_{b_1}^2)}{6}
\]

\[
+ \lambda_{\chi c \rho}^2 \int_{m_{\chi c}^2}^{s_0} ds \frac{1}{s - p^2} \frac{\sqrt{\lambda(s, m_{\chi c}^2, m_{\rho}^2)}}{s} \frac{\lambda(s, m_{\chi c}^2, m_{\rho}^2)}{12s m_{\chi c}^2}
\]
\begin{align}
&\lambda_{\pi_{\psi,11}}^2 = \frac{f_{h_1}^2 m_{h_1}^2 f_{\rho}^2 m_{\rho}^2}{128\pi^2 4m_{\pi}^2}, \\
&\lambda_{\pi_{J/\psi,22}}^2 = \frac{f_{h_1}^2 m_{h_1}^2 f_{\rho}^2 m_{\rho}^2}{128\pi^2 (m_u + m_d)^2}, \\
&\lambda_{\eta_{c,33}}^2 = \frac{f_{h_1}^2 f_{h_1}^2 f_{T}^2}{128\pi^2}, \\
&\lambda_{h_1,33}^2 = \frac{f_{h_1}^2 f_{h_1}^2}{128\pi^2}, \\
&\lambda_{X_{c,33}}^2 = \frac{f_{X_{c}}^2 m_{X_{c}}^2 f_{T}^2}{128\pi^2}, \\
&\lambda_{\pi_{\psi,44}}^2 = \frac{f_{h_1}^2 f_{h_1}^2 f_{T}^2}{128\pi^2}, \\
&\lambda_{\pi_{h_1,44}}^2 = \frac{f_{h_1}^2 f_{h_1}^2}{128\pi^2}, \\
&\lambda_{a_1,44}^2 = \frac{f_{a_1}^2 m_{a_1}^2 f_{T}^2}{128\pi^2}. 
\end{align}

where

\begin{align}
&+\lambda_{\pi_{\psi,11}}^2 \int_{m_{h_1}^2}^{s_0} ds \frac{1}{s - p^2} \sqrt{\lambda(s, m_{\pi}^2, m_{\rho}^2)} \frac{\lambda(s, m_{\pi}^2, m_{\rho}^2)}{6} \\
&+\lambda_{a_1,11}^2 \int_{m_{a_1}^2}^{s_0} ds \frac{1}{s - p^2} \sqrt{\lambda(s, m_{a_1}^2, m_{1/\psi}^2)} \frac{\lambda(s, m_{a_1}^2, m_{1/\psi}^2)}{12sm_{a_1}^2} \\
&+\lambda_{D_0^0 D,55/66}^2 \int_{m_{D_0^0 D}^2}^{s_0} ds \frac{1}{s - p^2} \sqrt{\lambda(s, m_{D_0^0}^2, m_{D}^2)} \frac{\lambda(s, m_{D_0^0}^2, m_{D}^2)}{12s} \\
&+\lambda_{D^{*+},77/88}^2 \int_{m_{D^{*+}}^2}^{s_0} ds \frac{1}{s - p^2} \sqrt{\lambda(s, m_{D^{*+}}^2, m_{D^*}^2)} \left[ \frac{m_{D^*}^2 + \lambda(s, m_{D^*}^2, m_{D}^2)(s + m_{D}^2)}{6sm_{D^*}^2} \right] \\
&+\lambda_{D_0^0 D^{*},77/88}^2 \int_{m_{D_0^0 D^{*}}^2}^{s_0} ds \frac{1}{s - p^2} \sqrt{\lambda(s, m_{D_0^0}^2, m_{D^{*+}}^2)} \frac{\lambda(s, m_{D_0^0}^2, m_{D^{*+}}^2)}{6} \\
&+\lambda_{\pi_{\psi,22}}^2 \int_{m_{\pi_{\psi}}^2}^{s_0} ds \frac{1}{s - p^2} \sqrt{\lambda(s, m_{\pi_{\psi}}^2, m_{\rho}^2)} \left[ 1 + \frac{\lambda(s, m_{\pi_{\psi}}^2, m_{\rho}^2)}{12sm_{\rho}^2} \right] \\
&+\lambda_{\pi_{\psi,44}}^2 \int_{m_{\pi_{\psi}}^2}^{s_0} ds \frac{1}{s - p^2} \sqrt{\lambda(s, m_{\pi_{\psi}}^2, m_{\rho}^2)} \left[ \frac{(s - m_{\pi}^2 - m_{\rho}^2)^2 - \lambda(s, m_{\pi_{\psi}}^2, m_{\rho}^2)(s - m_{\rho}^2)}{3s} \right] \\
&+\lambda_{\pi_{\psi,44}}^2 \int_{m_{\pi_{\psi}}^2}^{s_0} ds \frac{1}{s - p^2} \sqrt{\lambda(s, m_{\pi_{\psi}}^2, m_{\rho}^2)} \left[ \frac{\lambda(s, m_{\pi_{\psi}}^2, m_{\rho}^2)}{6} - (s - m_{\pi}^2 - m_{\rho}^2) \right] \\
&+\lambda_{\pi_{\psi,44}}^2 \int_{m_{\pi_{\psi}}^2}^{s_0} ds \frac{1}{s - p^2} \sqrt{\lambda(s, m_{\pi_{\psi}}^2, m_{\rho}^2)} \left[ \frac{\lambda(s, m_{\pi_{\psi}}^2, m_{\rho}^2)}{6} \right] \\
&+\lambda_{\pi_{\psi,44}}^2 \left[ \pi_{\psi}^2 \rightarrow \pi_{\psi''}^2 \right] + \cdots ,
\end{align}
\[\lambda_{\eta_c;\rho;13}^2 = \frac{1}{128\pi^2} \frac{f_{\eta_c}^2 m_{\rho}^2 f_{\rho}^T}{2m_c},\]
\[\lambda_{\pi J/\psi;24}^2 = \frac{1}{128\pi^2} \frac{1}{m_u + m_d},\]
\[\lambda_{D_0 D;55/66}^2 = \frac{1}{128\pi^2} \frac{f_{D_0}^2 m_{D_0}^2 f_{D}^T}{m_u + m_d},\]
\[\lambda_{D^* D;77/88}^2 = \frac{1}{128\pi^2} \frac{1}{m_u + m_d},\]
\[\lambda_{\psi';22}^2 = \frac{1}{128\pi^2} \frac{f_{\psi'}^2 m_{\psi'}^2}{(m_\mu + m_\rho)^2},\]
\[\lambda_{\psi';44}^2 = \frac{1}{128\pi^2} \frac{f_{\psi'}^2 m_{\psi'}^2}{m_u + m_d},\]
\[\lambda_{h_c;44}^2 = \frac{1}{128\pi^2} \frac{f_{h_c}^2 m_{h_c}^2}{m_u + m_d},\]
\[\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc,\]
\[m_{\eta_c;\rho}^2 = (m_{\eta_c} + m_{\rho})^2,\]
\[m_{\pi J/\psi}^2 = (m_{\pi} + m_{J/\psi})^2,\]
\[m_{\eta_c;\rho;13}^2 = (m_{\eta_c} + m_{\rho})^2,\]
\[m_{\pi J/\psi;24}^2 = (m_{\pi} + m_{J/\psi})^2,\]
\[m_{D_0 D;55/66}^2 = (m_{D_0} + m_{D})^2,\]
\[m_{D^* D;77/88}^2 = (m_{D^*} + m_{D})^2,\]
\[m_{\psi';22}^2 = (m_{\psi'}^2)^2,\]
\[m_{\psi';44}^2 = (m_{\psi'}^2)^2,\]
\[m_{h_c;44}^2 = (m_{h_c}^2)^2,\]
\[\sqrt{s_0} \leq M_{Z_{(4430)}}.\]
Figure 1: The factorizable Feynman diagrams contribute to the perturbative term, $\langle \bar{q}q \rangle$, $\langle \bar{q}q, \sigma G q \rangle$, $\langle \alpha_s GG \pi \rangle$, and $\langle \bar{q}q \rangle \langle \alpha_s GG \pi \rangle$, where the solid lines and dashed lines denote the light quarks and heavy quarks, respectively. Other diagrams obtained by interchanging of the light quark lines and heavy quark lines are implied.
Figure 2: The factorizable Feynman diagrams contribute to the $\langle \bar{q}q \rangle \langle \frac{\alpha_s G G}{\pi} \rangle$, $\langle \bar{q}q \rangle^2$, $\langle \bar{q}q \rangle \langle \bar{q}g_s G q \rangle$, $\langle \bar{q}q \rangle^2 \langle \frac{\alpha_s G G}{\pi} \rangle$ and $\langle \bar{q}g_s G q \rangle^2$. Other diagrams obtained by interchanging of the light quark lines and heavy quark lines are implied.
Figure 3: The factorizable Feynman diagrams contribute to the $\langle \bar{q}g_\sigma q G \rangle^2$. Other diagrams obtained by interchanging of the light quark lines and heavy quark lines are implied.

$$U_{ij}(x),\ D_{ij}(x),$$

$$S_{ij}(x) = \frac{i\delta_{ij} \not{x}}{2\pi^2 x^4} - \frac{\delta_{ij} \langle \bar{q}q \rangle}{12} - \frac{\delta_{ij} x^2 \langle \bar{q}g_\sigma q G \rangle}{192} - \frac{i\bar{g}_s G_{\alpha\beta} t_{ij}^\alpha (f^\alpha + s^\alpha \not{x})}{32\pi^2 x^2} - \frac{1}{8} \langle \bar{q}_i \sigma^{\mu\nu} q_j \rangle \sigma_{\mu\nu} + \cdots, \quad (20)$$

$$C_{ij}(x) = \frac{i}{(2\pi)^4} \int d^4 k e^{-i k \cdot x} \left\{ \frac{\delta_{ij}}{\vec{k} - m_c} - \frac{g_s G_{\alpha\beta} t_{ij}^\alpha}{4} \frac{\sigma^{\alpha\beta}(\vec{k} + m_c) + (\vec{k} + m_c)\sigma^{\alpha\beta}}{(k^2 - m_c^2)^2} \right\} = \frac{g_s^2(t^n t^b)_{ij} G_{\alpha\beta} G^{\mu\nu}}{4(k^2 - m_c^2)^b} \left( f^{\alpha\mu\nu} \sigma^{\alpha\beta} f^{\alpha\mu\nu} + f^{\alpha\mu\nu} + f^{\alpha\mu\nu} \right) + \cdots, \quad (21)$$

and $t^n = \frac{\lambda^n}{3}$, the $\lambda^n$ is the Gell-Mann matrix [7, 28, 29].

As there exists a spatial separation $\epsilon$ between the diquark and antidiquark constituents, we split the point 0 (and $x$) into two points,

$$0 \to 0, \ 0 + \epsilon,$$

$$x \to x, \ x + \epsilon, \quad (22)$$

to distinguish the diquark and antidiquark contributions in drawing the Feynman diagrams. We classify the Feynman diagrams as factorizable diagrams and nonfactorizable diagrams respectively. In Figs.13 we draw the factorizable Feynman diagrams, in which the contributions come from the diquark loops and antidiquark loops are factorizable due to the nonzero spatial separation $\epsilon$. They contribute to the perturbative terms, $\langle \bar{q}q \rangle$, $\langle \bar{q}g_\sigma q G \rangle$, $\langle \bar{q}q \rangle^2$, $\langle \bar{q}q \rangle^2$, $\langle \bar{q}g_\sigma q G \rangle$, $\langle \bar{q}g_\sigma q G \rangle^2$, $\langle \bar{q}g_\sigma q G \rangle^2$ and $\langle \bar{q}g_\sigma q G \rangle^2$. In Figs.13 we draw the nonfactorizable Feynman diagrams, in which the contributions come from the diquark loops and antidiquark loops are nonfactorizable even if we take into account the nonzero spatial separation $\epsilon$. They contribute to the $\langle \bar{q}g_\sigma q G \rangle$, $\langle \bar{q}g_\sigma q G \rangle$, $\langle \bar{q}g_\sigma q G \rangle$, $\langle \bar{q}g_\sigma q G \rangle^2$, $\langle \bar{q}g_\sigma q G \rangle^2$, and $\langle \bar{q}g_\sigma q G \rangle^2$.

We compute both the factorizable and nonfactorizable Feynman diagrams, and obtain the correlation function $\Pi(p^2)$ at the quark level, then obtain the QCD spectral density through dispersion relation. We match the hadron side with the QCD side of the correlation function $\Pi(p^2)$ below the continuum threshold parameter $s_0$, then perform the Borel transformation with respect to the variable $P^2 = -p^2$ to obtain the QCD sum rules,

$$\lambda_2^2 \exp\left(-\frac{M^2}{T^2}\right) + \Pi_{RC}(T^2) = \int_{4m_c^2}^{s_0} ds \left[ \rho_f(s) + \rho_n(s) \right] \exp\left(-\frac{s}{T^2}\right), \quad (23)$$
Figure 4: The nonfactorizable Feynman diagrams contribute to the $\langle \bar{q}g_\pi Gq \rangle$, $\langle \alpha_s GG \rangle$ and $\langle \bar{q}q \rangle \langle \alpha_s GG \rangle$. Other diagrams obtained by interchanging of the light quark lines and heavy quark lines are implied.
Figure 5: The nonfactorizable Feynman diagrams contribute to the \(\langle \bar{q}q, \sigma Gq \rangle\), \(\langle \bar{q}g, \sigma Gq \rangle^2\) and \(\langle \bar{q}q \rangle^2\). Other diagrams obtained by interchanging of the light quark lines and heavy quark lines are implied.

where the two particle scattering state contributions (RC),

\[
\Pi_{RC}(T^2) = \lambda^2_{\eta_c;11} \int_{m_{\eta_c}^2}^{s_0} ds \frac{\sqrt{\lambda(s, m_{\eta_c}^2, m_{\rho}^2)}}{s} \left[ 1 + \frac{\lambda(s, m_{\eta_c}^2, m_{\rho}^2)}{12sm_{\rho}^2} \right] \exp\left(-\frac{s}{T^2}\right) \\
+ \lambda^2_{\pi, J/\psi;22} \int_{m_{\pi, J/\psi}^2}^{s_0} ds \frac{\sqrt{\lambda(s, m_{\pi, J/\psi}^2, m_{\rho}^2)}}{s} \left[ 1 + \frac{\lambda(s, m_{\pi, J/\psi}^2, m_{\rho}^2)}{12sm_{\rho}^2} \right] \exp\left(-\frac{s}{T^2}\right) \\
+ \lambda^2_{\eta_c;33} \frac{1}{4} \int_{m_{\eta_c}^2}^{s_0} ds \frac{\sqrt{\lambda(s, m_{\eta_c}^2, m_{\pi, J/\psi}^2)}}{s} \left[ (s - m_{\pi, J/\psi}^2 - m_{\rho}^2)^2 - \frac{\lambda(s, m_{\eta_c}^2, m_{\rho}^2)(s - m_{\rho}^2)}{3s} \right] \exp\left(-\frac{s}{T^2}\right) \\
+ \lambda^2_{\pi, J/\psi;44} \frac{1}{4} \int_{m_{\pi, J/\psi}^2}^{s_0} ds \frac{\sqrt{\lambda(s, m_{\pi, J/\psi}^2, m_{\pi, J/\psi}^2)}}{s} \left[ (s - m_{\pi, J/\psi}^2 - m_{\rho}^2)^2 - \frac{\lambda(s, m_{\pi, J/\psi}^2, m_{\pi, J/\psi}^2)(s - m_{\rho}^2)}{3s} \right] \exp\left(-\frac{s}{T^2}\right) \\
+ \lambda^2_{\eta_c;13} \int_{m_{\eta_c}^2}^{s_0} ds \frac{\sqrt{\lambda(s, m_{\eta_c}^2, m_{\rho}^2)}}{s} \left[ \frac{\lambda(s, m_{\eta_c}^2, m_{\rho}^2)}{6} - (s - m_{\eta_c}^2 - m_{\rho}^2) \right] \exp\left(-\frac{s}{T^2}\right) \\
+ \lambda^2_{\pi, J/\psi;24} \int_{m_{\pi, J/\psi}^2}^{s_0} ds \frac{\sqrt{\lambda(s, m_{\pi, J/\psi}^2, m_{\rho}^2)}}{s} \left[ \frac{\lambda(s, m_{\pi, J/\psi}^2, m_{\rho}^2)}{6} - (s - m_{\pi, J/\psi}^2 - m_{\rho}^2) \right] \exp\left(-\frac{s}{T^2}\right) \\
+ \lambda^2_{\eta_c b_1;33} \int_{m_{\eta_c b_1}^2}^{s_0} ds \frac{\sqrt{\lambda(s, m_{\eta_c b_1}^2, m_{b_1}^2)}}{s} \frac{\lambda(s, m_{\eta_c b_1}^2, m_{b_1}^2)}{6} \exp\left(-\frac{s}{T^2}\right) \\
+ \lambda^2_{\chi_c;33} \int_{m_{\chi_c}^2}^{s_0} ds \frac{\sqrt{\lambda(s, m_{\chi_c}^2, m_{\rho}^2)}}{s} \frac{\lambda(s, m_{\chi_c}^2, m_{\rho}^2)(2s + m_{\rho}^2 + 2m_{\chi_c}^2)}{12sm_{\chi_c}^2} \exp\left(-\frac{s}{T^2}\right)
\]
The QCD spectral densities $\rho_f(s)$ and $\rho_{nf}(s)$ receive contributions from the factorizable and nonfactorizable Feynman diagrams, respectively. The explicit expressions of the QCD spectral densities $\rho_f(s)$ and $\rho_{nf}(s)$ are available upon request by contacting me via E-mail.

If there exists a barrier or spatial separation between the diquark and antidiquark constituents, the Feynman diagrams can be divided into factorizable and nonfactorizable diagrams. The factorizable Feynman diagrams correspond to the stable diquark-antidiquark type contributions. The barrier or spatial separation frustrates dissociation of the diquark and antidiquark states to form the color singlet quark-antiquark pairs $q\bar{q}$, $Q\bar{Q}$ or $Q\bar{Q}$, $q\bar{Q}$, the colored diquark and antidiquark constituents are confined objects, which lead to a stable tetraquark state. The nonfactorizable Feynman diagrams correspond to the tunnelling effects between diquark and antidiquark constituents, and facilitate dissociation of the diquark and antidiquark states to form the color singlet quark-antiquark pairs $q\bar{q}$, $Q\bar{Q}$ or $Q\bar{Q}$, $q\bar{Q}$.

In this case, the QCD sum rules in Eq. (23) can be replaced with two QCD sum rules,

$$\lambda_2^2 \exp \left(-\frac{M_2^2}{T^2}\right) = \int_{4m_2^2}^{s_0} ds \rho_f(s) \exp \left(-\frac{s}{T^2}\right),$$  \hspace{1cm} (25)

$$\Pi_{RC}(T^2) = \int_{4m_2^2}^{s_0} ds \rho_{nf}(s) \exp \left(-\frac{s}{T^2}\right).$$  \hspace{1cm} (26)

On the other hand, if the effects of the barrier or spatial separation between the diquark and antidiquark constituents are neglectful, we can perform Fierz rearrangement for the diquark-antidiquark type current in the color and Dirac-spinor spaces freely to obtain a special superposition...
of the color singlet-singlet type currents, which couple potentially to the meson-meson pairs, it is not necessary to take into account the pole term of the \( Z_c(3900) \), in other words, the axialvector tetraquark state does not exist as a real resonance, it is a virtual state and embodies the net effects. And it is not necessary to divide the Feynman diagrams into factorizable and nonfactorizable parts. We saturate the hadron side of the QCD sum rules with the two-particle scattering state contributions,

\[
\Pi_{RC}(T^2) = \kappa \int_{4\pi}^{s_0} ds \left[ \rho_f(s) + \rho_{nf}(s) \right] \exp \left( -\frac{s}{T^2} \right),
\]

where we introduce a parameter \( \kappa \) to measure the deviation from the ideal value 1.

We derive Eq.(23), Eq.(25) and Eq.(27) with respect to \( \tau = \frac{s}{\mu} \), and obtain the QCD sum rules for the tetraquark mass and other QCD sum rules,

\[
M^2_Z = -\frac{d}{d\tau} \left\{ \int_{4\pi}^{s_0} ds \left[ \rho_f(s) + \rho_{nf}(s) \right] \exp \left( -\frac{s}{\mu^2} \right) - \Pi_{RC}(T^2) \right\} \bigg|_{\tau = \frac{s}{\mu}},
\]

\[
M^2_Z = -\frac{d}{d\tau} \int_{4\pi}^{s_0} ds \rho_f(s) \exp \left( -\frac{s}{\mu^2} \right) \bigg|_{\tau = \frac{s}{\mu}},
\]

\[
\frac{d}{d\tau} \Pi_{RC}(T^2) = \kappa \frac{d}{d\tau} \int_{4\pi}^{s_0} ds \left[ \rho_f(s) + \rho_{nf}(s) \right] \exp \left( -\frac{s}{2T^2} \right) \bigg|_{\tau = \frac{s}{\mu}}.
\]

3 Numerical results and discussions

At the QCD side, we take the standard values of the vacuum condensates \( \langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{GeV})^3 \), \( \langle \bar{q}g_\sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle \), \( m_0^2 = (0.8 \pm 0.1) \text{GeV}^2 \), \( \langle \bar{q}g_\sigma Gq \rangle = (0.33 \text{GeV})^4 \) at the energy scale \( \mu = 1 \text{GeV} \), and choose the \( \overline{MS} \) mass \( m_c(m_c) = (1.275 \pm 0.025) \text{GeV} \) from the Particle Data Group [4]. Moreover, we take into account the energy-scale dependence of the parameters,

\[
\langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{m_0^2}{\mu^2}},
\]

\[
\langle \bar{q}g_\sigma Gq \rangle(\mu) = \langle \bar{q}g_\sigma Gq \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{m_0^2}{\mu^2}},
\]

\[
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{m_0^2}{\Lambda^2}},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0} + \frac{b_2^2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^2 t^2} \right],
\]

where \( t = \log \frac{\mu^2}{\Lambda^2} \), \( b_0 = \frac{33-2\alpha_s}{12\pi} \), \( b_1 = \frac{153-19\alpha_s}{24\pi^2} \), \( b_2 = \frac{2857-663\alpha_s+\frac{33}{2}\alpha_s^2}{128\pi^4} \), \( \Lambda = 210 \text{MeV}, 292 \text{MeV} \) and \( 332 \text{MeV} \) for the flavors \( n_f = 5, 4 \) and 3, respectively [1 32], and evolve all the parameters to the ideal energy scale \( \mu \) with \( n_f = 4 \) to extract the axialvector tetraquark mass as the c-quark is concerned.

At the hadron side, we take the hadronic parameters as \( m_{\pi} = 2.9839 \text{GeV}, m_{J/\psi} = 3.0969 \text{GeV}, m_{h_c} = 3.5254 \text{GeV}, m_{X_{c1}} = 3.5107 \text{GeV}, m_{p} = 0.7753 \text{GeV}, m_{\pi} = 1.2300 \text{GeV}, m_{b_1} = 1.2295 \text{GeV}, m_{\pi} = 0.1396 \text{GeV}, m_{D} = 1.8672 \text{GeV}, m_{D^*} = 2.0086 \text{GeV}, m_{D_{s0}} = 2.3245 \text{GeV}, m_{\psi'} = 3.6861 \text{GeV}, \)
\[ m_{\psi'} = 4.0396 \text{ GeV} \text{ from the Particle Data Group} \]  
\[ m_{h'} = 3.9560 \text{ GeV} \text{ for the Godfrey-Isgur model} \]  
\[ f_{\psi'} = 0.418 \text{ GeV}, \quad f_{h'} = 0.387 \text{ GeV}, \quad f_{T_{\psi'}} = 0.410 \text{ GeV}, \quad f_{h_T} = 0.235 \text{ GeV} \text{ from Lattice QCD} \]  
\[ f_{\chi_{c0}} = 0.338 \text{ GeV} \text{ from Lattice QCD} \]  
\[ f_{\rho} = 0.205 \text{ GeV}, \quad f_{\rho_T} = 0.160 \text{ GeV} \text{ from Lattice QCD} \]  
\[ f_{h_1} = 0.180 \text{ GeV} \text{ from Lattice QCD} \]  
\[ f_{a_1} = 0.238 \text{ GeV} \]  
\[ f_D = 0.208 \text{ GeV}, \quad f_{D^+} = 0.263 \text{ GeV}, \quad f_{D_0} = 0.373 \text{ GeV} \text{ from the QCD sum rules} \]  
\[ f_{\pi} = 0.130 \text{ GeV}, \quad f_{\psi} = 0.295 \text{ GeV}, \quad f_{\psi'} = 0.187 \text{ GeV} \text{ extracted from the experimental data} \]  
\[ f_{\psi''} = 0.166 \text{ GeV} \text{ estimated in the present work} \]  
\[ f_{M_\pi^2}/(m_a + m_d) = -2\langle\bar{q}q\rangle/f_{\pi} \text{ from the Gell-Mann-Oakes-Renner relation} \]

From Table 1, we can see that the largest thresholds are \( m_{a_{1,\psi}} \) and \( m_{D_{0,D}} \). We take into account the uncertainties of the input parameters, and obtain the values of the mass and pole residue, which are shown explicitly in Figs 6–7.

\[
M_Z = 3.85 \pm 0.09 \text{ GeV},
\]
\[
\lambda_Z = (1.35 \pm 0.24) \times 10^{-2} \text{ GeV}^5 \quad \text{with} \quad \Pi_{RC}(T^2),
\]
\[
M_Z = 3.89 \pm 0.08 \text{ GeV},
\]
\[
\lambda_Z = (1.66 \pm 0.25) \times 10^{-2} \text{ GeV}^5 \quad \text{with} \quad \Pi_{RC}(T^2).
\]

Although the values of the mass are both in consistent with the experimental data \( M_{Z,(3900)} = (3887.2 \pm 2.3) \text{ MeV} \), the value \( M_Z = 3.89 \pm 0.08 \text{ GeV} \) with the \( \Pi_{RC}(T^2) \) is better. In Figs 6–7, we plot the mass and pole residue with variations of the Borel parameter \( T^2 \) at much larger range than the Borel window.

If we switch off the two-particle scattering state contributions \( \Pi_{RC}(T^2) \), we can obtain the values:

\[
M_Z = 3.90 \pm 0.08 \text{ GeV},
\]
\[
\lambda_Z = (2.09 \pm 0.33) \times 10^{-2} \text{ GeV}^5.
\]
Figure 6: The mass of the $Z_c(3900)$ with variations of the Borel parameter $T^2$, where the $A$ and $B$ correspond to the $\Pi_{RC}(T^2)$ and $\Pi_{RC}(T^2)$, respectively.

Figure 7: The pole residue of the $Z_c(3900)$ with variations of the Borel parameter $T^2$, where the $A$ and $B$ correspond to the $\Pi_{RC}(T^2)$ and $\Pi_{RC}(T^2)$, respectively.
Compared with the value $M_Z = 3.90 \pm 0.08 \text{GeV}$ from the single-pole approximation \cite{21}, the values $M_Z = 3.85 \pm 0.09 \text{GeV}$ and $3.89 \pm 0.08 \text{GeV}$ from the QCD sum rules where the two-particle scattering state contributions included are slightly smaller, see Fig.\ref{fig:fig3}. While the values of the pole residue $\lambda_Z = (1.35 \pm 0.24) \times 10^{-2} \text{GeV}^5$ and $(1.66 \pm 0.25) \times 10^{-2} \text{GeV}^5$ are much smaller than the value $(2.09 \pm 0.33) \times 10^{-2} \text{GeV}^5$ from the single-pole approximation, see Fig.\ref{fig:fig7}. In all the QCD sum rules, there appear Borel platforms in the Borel window $T^2 = (2.7 - 3.1) \text{GeV}^2$.

Moreover, the energy scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2M_c)^2}$ with the updated effective $c$-quark mass $M_c = 1.82 \text{GeV}$ \cite{26,41} is satisfied for the QCD sum rules with or without the two-particle scattering state contributions included are slightly smaller, see Fig.6. While the values of the pole contribution of the $Z_c(3900)$ is necessary, the current $J_\mu(x)$ couples potentially to the $Z_c(3900)$.

### 3.2 Two-particle contributions only

Again we choose the continuum threshold parameter $\sqrt{s_0} = 4.45 \pm 0.10 \text{GeV}$ and the optimal energy scale $\mu = 1.4 \text{GeV}$, the pole contribution is $PC = (43 - 66)\%$ at the Borel window $T^2 = (2.7 - 3.1) \text{GeV}$. As there are only the two-particle scattering state contributions, the QCD sum rules are reduced to two independent QCD sum rules in Eq.\ref{eq:eq27} and Eq.\ref{eq:eq30}.

We take into account the uncertainties of the input parameters, and obtain the values of the $\kappa$, which are shown explicitly in Fig.\ref{fig:fig8}

$$
\kappa = 0.55 \pm 0.11 \text{ from Eq.}\ref{eq:eq27},
\kappa = 0.57 \pm 0.11 \text{ from Eq.}\ref{eq:eq30},
$$

in the Borel window. If we take the replacement $\Pi_{RC}(T^2) \rightarrow \Pi_{RC}(T^2)$ to subtract the perturbative corrections, we obtain the corresponding values $\Sigma$,

$$
\Sigma = 0.39 \pm 0.08 \text{ from Eq.}\ref{eq:eq27},
\Sigma = 0.40 \pm 0.08 \text{ from Eq.}\ref{eq:eq30}.
$$

The values $\kappa, \Sigma \ll 1$, the two-particle scattering state contributions cannot saturate the QCD sum rules at the hadron side. If we insist on performing the Fierz rearrangement,

$$
J_\mu = \frac{1}{2\sqrt{2}} \left\{ iJ^1_\mu - iJ^2_\mu - iJ^3_\mu + iJ^4_\mu + J^5_\mu - J^6_\mu - iJ^7_\mu + iJ^8_\mu \right\},
$$

we should take into account the possible molecular states, such as $\eta_c \rho, \pi J/\psi, \cdots$,

$$
\langle 0|J^1_\mu(0)|\eta_c \rho(p) \rangle = \lambda_{\eta_c \rho,M} \epsilon_\mu,
\langle 0|J^2_\mu(0)|\pi J/\psi(p) \rangle = \lambda_{\pi J/\psi,M} \epsilon_\mu,
$$

$\cdots$, where the $\lambda_{\eta_c \rho,M}, \lambda_{\pi J/\psi,M}, \cdots$ are the pole residues, the $\epsilon_\mu$ are the polarization vectors. All in all, the two-particle scattering state contributions cannot saturate the QCD sum rules.

### 3.3 $Z_c(3900)$ only with the factorizable Feynman diagrams

We choose the continuum threshold parameter $\sqrt{s_0} = 4.40 \pm 0.10 \text{GeV}$ and the optimal energy scale $\mu = 1.4 \text{GeV}$, the pole contribution is $PC = (40 - 63)\%$ at the Borel window $T^2 = (2.7 - 3.1) \text{GeV}$ with both the factorizable and nonfactorizable Feynman diagrams.

In Fig.\ref{fig:fig9} we plot the contribution from nonfactorizable Feynman diagrams with variations of the Borel parameter $T^2$. From the figure, we can see that the nonfactorizable contribution is about 1% at the Borel window and play a minor important role in the QCD sum rules. No stable QCD sum rules can be obtained in Eq.\ref{eq:eq29}. The dominance contributions come from the factorizable
Figure 8: The \( \kappa \) with variations of the Borel parameter \( T^2 \), where the \( A \) and \( B \) correspond to the QCD sum rules in Eq. (27) and Eq. (30), respectively.

Figure 9: The contribution from nonfactorizable Feynman diagrams with variations of the Borel parameter \( T^2 \).
Feynman diagrams. The factorizable contributions consist of two colored clusters, a diquark cluster and an antidiquark cluster, the color confinement frustrates tunnelling effects and leads to stable tetraquark state, which is consistent with the small width of the $Z_c(3900)$.

From the QCD sum rules in Eq.(25) and Eq.(29), we can obtain the values of the mass and pole residue,

\begin{align}
M_{Z_c} &= 3.90 \pm 0.08 \text{ GeV}, \\
\lambda_{Z_c} &= (2.09 \pm 0.33) \times 10^{-2} \text{ GeV}^5,
\end{align}

the nonfactorizable contributions can be neglected safely in the Borel window.

4 Conclusion

In this article, we study the $Z_c(3900)$ with the QCD sum rules in details by including the contributions of the two-particle scattering states and nonlocal effects between the diquark and antidiquark constituents. The two-particle scattering state contributions cannot saturate the QCD sum rules at the hadron side, we have to take into account the pole contribution of the $Z_c(3900)$. If we approximate the hadron side of the QCD sum rules with the $Z_c(3900)$ plus two-particle scattering state contributions, we can obtain a mass which is consistent with the experimental data. In fact, we can saturate the QCD sum rules with or without the two-particle scattering state contributions, although different pole residues are needed. If there exists a barrier or spatial separation between the diquark and antidiquark constituents, the Feynman diagrams can be divided into the factorizable and nonfactorizable diagrams. The factorizable contributions consist of two colored clusters, a diquark cluster and an antidiquark cluster, the color confinement frustrates tunnelling effects and leads to a stable tetraquark state, which is consistent with the small width of the $Z_c(3900)$. The nonfactorizable Feynman diagrams correspond to the tunnelling effects, which play a minor important role in the QCD sum rules. If we equal the nonfactorizable contributions to the two-particle scattering state contributions, no stable QCD sum rules can be obtained. The factorizable contributions dominate the QCD sum rules at the QCD side. The present conclusion is expected to survive in other QCD sum rules for the diquark-antidiquark type tetraquark states.

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