A Confinement Potential for Leptons and Their Tunneling Effects into Extra Dimensions

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Considering the five-dimensional warped spacetime $AdS_5$ with the $D3$-brane, we derive a potential in the fifth dimension, according to which ordinary particles are initially confined on the $D3$-brane. It is estimated, however, that the lightest neutrino with mass $m_1$ is tunneling away into the extra dimension. Hence there is a possibility that no neutrinos with mass $m_1$ exist in cosmic background neutrinos, but surviving neutrinos are those with heavier masses $m_2$ and $m_3$. The other possibilities are also discussed.

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I. INTRODUCTION

In the present paper we would like to discuss some behaviors of particles in extra dimensions. We take the so-called braneworld picture. In this scenario, ordinary matter is trapped in a three-dimensional space, called the $D3$-brane, embedded in a higher dimensional space. This idea must be contrasted with the traditional view of extra dimensions, the Kaluza-Klein picture, where matter fields live everywhere in compact extra dimensions. Any such higher dimensional field can be described as an infinite collection of four-dimensional fields, the so-called KK modes, with masses depending on the size of the extra dimensions. Non-observation of KK modes in the present collider experiments suggests the size to be very small. Hence we do not take such a picture here.

We focus our attention on the problem about what kinds of particles can move to the extra dimension. We confine ourselves to consider only leptons, because this possibility may be large for particles with small masses. We consider the five-dimensional warped spacetime $AdS_5$ with the $D3$-brane [1]-[3]. We derive a potential in the fifth dimension, according to which ordinary leptons are initially confined on the $D3$-brane. Then we would like to discuss whether these particles on the brane can move to the fifth dimension by something like a tunneling effect [4].

Main results are the following: Initial numbers of neutrinos will be found to decrease because of escapes into the fifth dimension by tunneling. From the data of solar neutrinos [6] we estimate the half-life times of neutrinos with masses $m_1, m_2$, and $m_3$, respectively. The lightest neutrino with $m_1$ is only tunneling away into the extra dimension. Hence there is a possibility that no neutrinos with mass $m_1$ exist in cosmic background neutrinos, but surviving neutrinos are those with heavier masses $m_2$ and $m_3$. The other possibilities are also discussed.

In Sec. II we derive such a potential in the extra dimension. In Sec. III the tunneling effect is discussed. In Sec. IV we derive the dispersion relation in the bulk. The final section is devoted to concluding remarks.

II. A POTENTIAL IN THE EXTRA DIMENSION

Let us consider a five-dimensional spacetime with three-dimensional isotropy and homogeneity metric

$$ds^2 = e^{-2\eta(y)}(dt^2 - dr^2) - dy^2,$$  

(2.1)
where the brane-universe is located at \( y = 0 \) and is spatially flat \( \mathbb{R}^{1,3} \).

This metric has been obtained from the five-dimensional Einstein equations
\[
G_{AB} + \Lambda g_{AB} = \kappa^2 T_{AB} ,
\]
where \( \Lambda \) is a cosmological constant in the bulk. Here the energy-momentum tensor \( T_{AB} \) is decomposed into a bulk and a brane. The former function in the exponential factor comes from this delta potential should be positive at \( \epsilon < y < \epsilon \). In order that the particle with \( y = \dot{y} = 0 \) is fixed to \( \epsilon_0 - (\epsilon^2 - \epsilon^2)/m^2 \) \( . \)

Now we would like to discuss whether the ordinary leptons on the brane can move along the geodesic line of Eq. (2.11) by something like a tunneling effect \( 4 \). In order to see this we consider the action
\[
I = \int d\tau \mathcal{L} ,
\]
which reduces to, by choosing \( \tau = t \),
\[
I = \int dt L ,
\]
where \( m \) is a parameter with the mass-dimension. We fix its value to be the four-dimensional particle mass, when \( y = \dot{y} = 0 \).

Conjugate momenta are given as
\[
\bar{p} = \frac{\partial L}{\partial \dot{r}} = \frac{m \dot{r} e^{-2\eta |y|}}{\sqrt{e^{-2\eta |y|} (1 - \dot{r}^2) - \dot{y}^2}} ,
\]
\[
p_y = \frac{\partial L}{\partial \dot{y}} = \frac{m \dot{y}}{\sqrt{e^{-2\eta |y|} (1 - \dot{r}^2) - \dot{y}^2}} .
\]
The Hamiltonian is
\[
H = \bar{p} \cdot \dot{r} + p_y \dot{y} - L
\]
\[
= \frac{m e^{-2\eta |y|}}{\sqrt{e^{-2\eta |y|} (1 - \dot{r}^2) - \dot{y}^2}} .
\]
Eliminating \( \dot{r} \) and \( \dot{y} \) from Eqs. (2.5), (2.7), we have
\[
H = \sqrt{\bar{p}^2 + e^{-2\eta |y|} (p_y^2 + m^2)} .
\]
Hamilton’s equations of motion are
\[
\dot{\bar{p}} = \frac{\partial H}{\partial \dot{r}} = \bar{p} / H ,
\]
\[
\dot{p}_y = - \frac{\partial H}{\partial p_y} = 0 ,
\]
\[
\dot{y} = \frac{\partial H}{\partial p_y} = \frac{p_y}{H} e^{-2\eta |y|} ,
\]
\[
\dot{p}_y = - \frac{\partial H}{\partial y} = \pm \frac{ke^{-2\eta |y|}}{H} (p_y^2 + m^2) ,
\]
\[
(+ \text{ for } y > 0 , \ - \text{ for } y < 0)
\]
From Eqs. (2.8)-(2.9) we get
\[
\bar{p} = c_1 = \text{const} . ,
\]
\[
H = \sqrt{c_1^2 + e^{-2\eta |y|} (p_y^2 + m^2)} = c_0 = \text{const} . ,
\]
\[
\dot{r} = \text{const} . .
\]
An equation for the extra dimension is obtained from Eq. (2.11) as
\[
p_y^2 / 2m + U(y) = E_y = 0 ,
\]
where
\[
U(y) = \frac{m}{2} \left[ 1 - q e^{2\eta |y|} \right] ,
\]
\[
q = (c_0^2 - c_1^2) / m^2 .
\]
Here \( p_y^2 / 2m \) and \( U(y) \) can be regarded as a kinetic energy and a potential of the particle in the extra dimension, respectively, with the total energy \( E_y = 0 \).

Let us suppose that the brane at \( y = 0 \) is initially an extended object with a thin width, \(-\epsilon < y < \epsilon \). In order that the particle with zero-energy is confined in this region, the potential should be positive at \( y = \pm \epsilon \). So, we require the condition
\[
U(\pm \epsilon) \equiv \frac{m}{2} (1 - q) \equiv Q > 0 ,
\]
where \( 1 - q = -p_y^2 (\pm \epsilon) / m^2 > 0 \) is fixed to be a positive constant. The other case, \( i.e. \),
FIG. 1: A plot of the potential $U(y)$ as a function of $y$. The $D3$-brane at $y = 0$ is initially an extended object with a thin width, $-\epsilon < y < \epsilon$.

$p_y^2(\pm \epsilon) \geq 0$, the particle with zero-energy can not be confined in this region, $-\epsilon < y < +\epsilon$. So, we do not consider such a case. In the region, $-\epsilon < y < \epsilon$, the potential $U(y)$ is assumed to be zero.

The potential is symmetrical as depicted in Fig.1. The curve crosses the $y$-axis at $y = \pm y_0$, satisfying $\exp(-2\eta|y_0|) = q$. Particles are initially confined in the $D$-brane. Hence each of them is supposed to be in a virtually bound state $-\epsilon < y < \epsilon$. Some kinds of particles could escape from the inside to the outside of the potential by the tunneling effect. In the next section we calculate this tunneling probability $P$.

The reflection probability $R$ is, of course, given by $R = 1 - P$.

**III. TUNNELING EFFECTS**

Initially let a particle be confined inside the $D$-brane with the binding energy $E_0$. This means that the total energy should be replaced by

$$p_y^2/(2m) + U(y) = E_0$$

The tunneling probability is then given by

$$P \equiv \exp\left[-\frac{2}{\hbar} \int_{-\epsilon}^{\epsilon} dy \sqrt{2m(U(y) - E_0)}\right].$$

(3.2)

in the WKB approximation. Here $y_1$ is given by $U(y_1) = E_0$. The integral is carried out exactly as follows:

$$I = \frac{2}{\hbar} \int_{-y_0}^{y_0} dy \sqrt{2m(U(y) - E_0)}$$

$$= \frac{2}{\hbar} \int_{-y_0}^{y_0} dy \sqrt{2mU'(y)} ,$$

(3.3)

where

$$U'(y) = \frac{m'}{2}(1 - q \exp(2\eta y)) ,$$

(3.4)

$$m' = m \sqrt{1 - \frac{2E_0}{m}} ,$$

(3.5)

$$q' = q \frac{2}{1 - \frac{2E_0}{m}} ,$$

(3.6)

and $\epsilon \to 0$. Then it follows that

$$I = \frac{2m'}{\hbar\eta} f(q')$$

(3.7)

with

$$f(q') = -\sqrt{1 - q'} + \frac{1}{2} \ln \frac{2\sqrt{1 - q'} + 2 - q'}{q'} .$$

(3.8)

Here $q'$ does not depend on particle masses. This property is nothing but the equivalence principle of General Relativity. Substituting the result (3.7) into Eq. (3.2) we get

$$P \equiv \exp\left[-\frac{2m'}{\hbar\eta} f(q')\right] .$$

(3.9)

From Eq. (3.9) we see that the tunneling probability depends sharply on the mass parameter $m'$. For $\eta^{-1} = 10^{-2}$ cm, we have tunneling probabilities for leptons other than neutrinos listed in Table I. Here we have assumed to be $2E_0/m < 1$, hence $m' \cong m$ and $q' \cong q$. The tunneling probabilities seem to be very small, actually regarded so as to be zeros because of large masses of leptons, if $f(q)$ is not so small. In fact we will see later a fact that $f(q)$ should take values larger than 10.8.

As for masses of three mass-eigenstates of neutrinos $\nu_1$, $\nu_2$, and $\nu_3$, we take $m_1 = 0.001$ eV/$c^2$, $m_2 = 0.01$ eV/$c^2$, and $m_3 = 0.05$ eV/$c^2$. Three kinds of flavor neutrinos $\nu_e$, $\nu_\mu$, and $\nu_\tau$ are composed of $\nu_1$, $\nu_2$, and $\nu_3$. The tunneling probabilities for $\nu_1$, $\nu_2$, and $\nu_3$ are
Let the initial number of solar neutrinos decrease in this flight by $500P_1 = 1/100$, hence $P_1 = 1/50000 \text{ s}^{-1}$. This means that the half-life time is given by $T_1 = 9.6 \text{ h}$ and $f = 10.8$, as shown in Table III. The decreasing rate $1/100$ of solar neutrinos may be likely in the error of the observable values $\bar{\nu}_e$.

As a result, there is a possibility that no neutrinos with $m_1$ exist in cosmic background neutrinos, but surviving neutrinos are those with heavier masses $m_2$ and $m_3$. All cases of $f = 10.8-41$ may be also likely for solar neutrinos and for SN1987A-neutrinos. For the special case of $f = 41$ all of $\nu_1$, $\nu_2$, and $\nu_3$ appear to be stable in our universe, though we are not interested in such a case.

### IV. Dispersion Relations

Let us consider two points A and B with a distance $L_{AB}$ on the D3-brane. A neutrino can move directly on the D3-brane from A to B without tunneling. The propagation time $T$ of such a neutrino is, of course, given by $T = L_{AB}/|\vec{v}|$, where the neutrino velocity is given by, in a conventional notation

$$|\vec{v}| = \frac{|\vec{p}|}{E} = \frac{\sqrt{E^2 - m^2}}{E}. \quad (4.1)$$

On the other hand, the neutrino can move through the bulk from A to B according to reflections at the potential walls. This means as follows: One neutrino has a tunneling probability $P$ through the potential from 0 to $y_1$, then it escapes into the bulk and will simply continue its propagation in the bulk, never coming back on the brane. The initial numbers of neutrinos will decrease because of escapes by tunneling.

However, there are neutrinos which are reflected from both points of the potential walls, 0 and $y_1$. The total reflection probability is, of course, given by $R = 1 - P$. Thus a neutrino has a probability such that it travels from A on the brane into the $y$-direction by tunneling and will come back to B on the brane after reflection at both points of the potential walls.

The neutrino energy inside the potential is given by Eq.(2.11), or in a conventional nota-
brane have the group velocity of neutrinos on the dispersion relation in the bulk. From Eq. (4.2), we have the group velocity of neutrinos on the D3-brane

\[ \tilde{\gamma}_{\text{bulk}} = \frac{\partial E}{\partial |\tilde{p}|} = \frac{\sqrt{E^2 - m^2 q}}{E}. \]  

(4.3)

where \( p_y^2 < 0 \). This can be regarded as a dispersion relation in the bulk. From Eq. (4.2), we have the group velocity of neutrinos on the D3-brane

\[ \tilde{\gamma}_{\text{bulk}} = \frac{\partial E}{\partial |\tilde{p}|} = \frac{\sqrt{E^2 - m^2 q}}{E}. \]  

(4.3)

Eqs. (4.1) and (4.3) tell us that there are two groups of neutrinos, one with the velocity (4.1) and the other with (4.3). However, the difference between both velocities is too small to distinguish, because of \( m^2, q \ll E \).

In our model we see trivially that there are no neutrinos with velocities faster than light because of the Poincare invariant metric of Eq. (2.1) or equivalently by Eq. (4.3), even if they take a shortcut from A to B through the bulk.

V. CONCLUDING REMARKS

We have derived the potential in the extra-dimension from the five-dimensional warped spacetime AdS_5. Initially the potential works well to confine ordinary leptons on the D3-brane. However, we have the tunneling effect through the potential. The tunneling probability (4.9) for heavy leptons depends sharply on \( m \). We have listed tunneling probabilities for heavy leptons in Table II. Their values seem to be very small, actually regarded so as to be almost zeros. They are hard to move into the extra dimension by the tunneling effect because of their large masses, but move only the D3-brane.

On the other hand neutrino masses may be too small, so that we have generally a non-zero tunneling probability. We have taken neutrino masses of three mass-eigenstates as \( m_1 = 0.001 \text{ eV}/c^2, m_2 = 0.01 \text{ eV}/c^2, \) and \( m_3 = 0.05 \text{ eV}/c^2 \). Initial numbers of neutrinos will be found to decrease because of escapes into the fifth dimension by tunneling. From the data of solar neutrinos [6] we estimate the half-life times of neutrinos with masses \( m_1, m_2, \) and \( m_3 \), respectively. The lightest neutrino with \( m_1 \) is only tunneling away into the extra dimension. Hence there is a possibility that no neutrinos with \( m_1 \) exist in cosmic background neutrinos, but surviving neutrinos are those with heavier masses \( m_2 \) and \( m_3 \).

All cases of \( f = 10.8-41 \) in Table II may be also likely for solar neutrinos and for SN1987A-neutrinos. For the special case of \( f = 41 \) all of \( \nu_1, \nu_2, \) and \( \nu_3 \) appear to be stable in our universe, though we are not interested in such a case.

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[4] As for the photon, we can take into account of a fact that it runs on geodesic line \( \tilde{r}^2 + \exp (2k|y|)\tilde{y}^2 = 1 \), which is derived from Eq. (2.1). From this we have \( \tilde{y} = 0 \) whenever \( |\tilde{r}| = 1 \), that is, the photon on the D3-brane never deviate into the extra dimension.
[5] The equation of motion for \( y \) is derived from Eq. (2.12) as \( \ddot{y} = m^2 \eta^2(2\exp(-4\eta y) - q \exp(-2\eta y))/c_0^2, \) for \( y \geq 0 \), where \( c_0^2/m^2 = \frac{c^2}{m^2} + \exp(-2\eta y)(1 + p_{y}^2/m^2) \). From Eqs. (2.5) and (2.6) it follows that both \( c_0^2/m^2 \) and \( p_y^2/m^2 \) are independent of \( m \). Hence we see that \( m^2/c_0^2 \) and \( q \) are independent of \( m \). This means that the acceleration \( \ddot{y} \) is independent of any mass, re-
flecting the equivalence principle of General Rel-

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