Magnetic field barriers in graphene: an analytically solvable model

Enrique Milpas¹, Manuel Torres¹ and Gabriela Murguía²

¹ Instituto de Física, Universidad Nacional Autónoma de México, Apartado Postal 20-364, México Distrito Federal 01000, Mexico
² Facultad de Ciencias, Universidad Nacional Autónoma de México, Apartado Postal 21-092, México Distrito Federal 04021, Mexico

E-mail: torres@fisica.unam.mx

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Abstract
We study the dynamics of carriers in graphene subjected to an inhomogeneous magnetic field. For a magnetic field with a hyperbolic profile the corresponding Dirac equation can be analyzed within the formalism of supersymmetric quantum mechanics, and leads to an exactly solvable model. We study in detail the bound-state spectrum. For a narrow barrier the spectrum is characterized by a few bands, except for the zero energy level that remains degenerated. As the width of the barrier increases we can track the band’s evolution into the degenerated Landau levels. In the scattering regime a simple analytical formula is obtained for the transmission coefficient, this result allows us to identify the resonant conditions at which the barrier becomes transparent.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The discovery of graphene [1–3], a single layer of carbon in a honeycomb lattice has generated a lot of excitement, due to its unique electronic properties [4–6] and its potential application in electronic devices. Electrons in graphene are described by a massless two-dimensional relativistic Dirac equation [7–9], that yields a gapless linear spectrum close to the \( K \) and \( K' \) points of the first Brillouin zone. Graphene exhibits a variety of pseudo-relativistic phenomena, providing an unexpected connection between condensed matter physics and quantum-relativistic phenomena. Among others we can cite the Zitterbewegung and its relation with the minimal electrical conductivity at vanishing carrier concentration [2, 9, 10], the unconventional quantum Hall effect [2, 3, 11], and Klein tunneling [12–15]. The Klein tunneling effect has important implications for the future design of graphene based electronic devices, because massless Dirac fermions cannot be effectively confined by electrostatic barriers; in particular for normal incidence the barrier becomes completely transparent [13].

Some schemes have been proposed to avoid the obstacle that represents Klein tunneling, and produce confined graphene based structures. An interesting proposal refers to the use of inhomogeneous magnetic fields [16]. Previous studies have considered the cases of single [16], double [17], and multiple square-well magnetic barriers [18–20]; and also of delta function magnetic barriers [21] and electron waveguides [22].

The previous studies of magnetic barriers in general consider profiles with sharp edges. In this paper we consider a magnetic barrier in which the edges are smoothed out. We select a magnetic field with a hyperbolic profile. We show that the corresponding Dirac equation can be analyzed within the formalism of supersymmetric quantum mechanics, and leads to an exactly solvable model. We study in detail the bound-state spectrum. For a narrow barrier the spectrum displays a series of bands separated by gaps; as the width of the barrier increases the bands evolve into the degenerated Landau levels. In the scattering regime a simple analytical formula is obtained for the transmission coefficient, this result allows us to identify the resonant conditions at which the barrier becomes transparent.

The use of inhomogeneous magnetic fields has received considerable attention both in the experimental [23–27] and theoretical [23, 28–30] studies of two-dimensional electron gases (2DEGs) in semiconductor heterostructures. Various configurations of local inhomogeneous magnetic fields have been created using microfabricated ferromagnetic and superconducting structures deposited on top of a 2DEG. Interesting transport phenomena have been observed, among
and the edge smearing length of the graphene lattice spacing, along the x direction we seek solution of the form $\psi_A(B, \kappa) \propto \text{sech}(\kappa x)$ and $\psi_B(B, \kappa) \propto \text{sech}(\kappa x)$ where the Fermi velocity is given by $v_F \approx c/300$. The magnetic field profile for the hyperbolic expression in equation (1) provides a good approximation to the shape of the magnetic barrier produced by a ferromagnetic film deposited in a 2DEG [24]. Although there exist a number of experimental realizations of similar configurations in graphene, their production in the near future appears to pose no fundamental problem. Hence we expect that apart from their intrinsic theoretical interest, the results obtained in this work will be useful in order to analyze the confinement by magnetic barriers in graphene samples.

The paper is organized as follows. In section 2 we study the model for Dirac fermion dynamics in graphene when the system is subjected to an inhomogeneous magnetic field. We show that the system can be analyzed within the formalism of supersymmetric quantum mechanics. The effective potentials and the explicit analytical solution for the wavefunction are discussed. In section 3 we study in detail the bound-state spectrum and analyze its corresponding degeneracy, both for a narrow barrier, and also in the limit in which the barrier width becomes comparable to the size of the system. In section 4 the dispersion regime is analyzed in detail. In section 5 the conclusions are presented.

2. Graphene in an inhomogeneous magnetic field

We focus on an electron in a single graphene layer subject to an inhomogeneous perpendicular magnetic field, which varies along the x direction. In order to study a smooth magnetic barrier we select a profile given by

$$B = B_0 \text{sech}^2 \left( \frac{x}{2d} \right) e_z,$$  \hspace{1cm} (1)

where $e_z$ is the unit vector normal to the graphene plane. This expression for the magnetic field presents several advantages. (i) We obtain an analytically solvable model, that allows us to analyze in detail the bound-state spectrum, as well as the transmission through the magnetic barrier. (ii) In order to have conditions that are physically relevant to the study of graphene, we need a magnetic field $B(x)$ that varies slowly on the scale of the graphene lattice spacing, $a = 0.246$ nm. Selecting $a \ll d$, we observe that both the half-width ($\Delta \approx 3.25d$) and the edge smearing length $[(1/B)(dB/dx)]^{-1} \geq d$ of the magnetic barrier satisfy the required conditions. (iii) As shown in figure (1), the magnetic field in equation (1) provides a good approximation to the shape of the magnetic field barrier that is produced by a ferromagnetic film deposited on the top of a two-dimensional system [24]. Thus the present formalism could be useful to analyze a similar arrangement of inhomogeneous magnetic barriers in graphene samples.

The gauge can be selected in such a way that the vector potential is written as

$$A(x) = 2 B_0 d \tan \left( \frac{x}{2d} \right).$$  \hspace{1cm} (2)

Notice that if we consider that the system is confined within a square box of area $L \times L$, then in the $d \gg L$ limit, the magnetic field can be considered homogeneous, and the vector potential reduces to the Landau gauge expression $A = B_0(0, x, 0)$.

Figure 1. Magnetic field profile for the hyperbolic expression in equation (1) (solid line) as compared to the profile $B(x) = -\mu_0 M/(4\pi) \ln[(x^2 + k^2)/(x^2 + (l + h)^2)]$ [24], produced by a ferromagnetic strip with a magnetization $\mu_0 M$, thickness $l$ and separated at a distance $h$ from the two-dimensional system (dashed line). The parameters are selected in such a way that the peak and half-width of both profiles have the same value.

On low energy scales the dynamics of quasiparticles in graphene is described by two independent 2+1 dimensional Dirac equations; the equations remain decoupled as long as the magnetic field is not too strong. The scale of the energy separation between the K and the K' points is of the order $\hbar v_F / a$, which is much larger than the scale of the energy eigenvalues (see equation (17)) if $B_0 \ll \hbar/e a^2 \sim 10^4$ T, a condition that is widely satisfied in any realistic situation. The time independent Dirac equation describing low energy excitations around the K point in the Brillouin zone is written as

$$H \psi(x, y) = v_F \delta \cdot [p + eA(x)] \psi(x, y) = E \psi(x, y).$$  \hspace{1cm} (3)

Here the Fermi velocity is $v_F \approx c/300$, $p = -i\hbar \nabla$ is the momentum operator and the pseudospin Pauli matrices $\sigma_i$ operate in the spinor $\psi(x, y) = (\psi_A, \psi_B)^T$, that represent the electron amplitude on two sites (A and B) in the unit cell of the graphene lattice. Taking into account the translational invariance along the y direction we seek solution of the form $\psi_A = \exp(ik_y y) \psi_+$ and $\psi_B = \exp(ik_y y) \psi_-$. The Dirac equation yields the coupled equations

$$\Delta \psi_+(x) = \left( -\frac{\partial}{\partial x} - i W(x) \right) \psi_-(x),$$

$$\Delta \psi_-(x) = \left( -\frac{\partial}{\partial x} + i W(x) \right) \psi_+(x).$$  \hspace{1cm} (4)

With the magnetic length given by $l_B = \sqrt{\hbar/eB_0}$, it is convenient to define the following dimensionless quantities: $\tilde{x} = x/l_B$, $\tilde{y} = y/l_B$, $\kappa_y = l_B k_y$, $\delta = d/l_B$, and $\Delta = E l_B/\hbar v_F$. The function $W(x)$ is given by

$$W(x) = \kappa_y + \frac{el_B A(x)}{\hbar} = \kappa_y + 2\delta \tanh \left( \frac{x}{2\delta} \right).$$  \hspace{1cm} (5)
Combining the two equations in (4) we obtain the decoupled equations

$$H_\pm \psi_\pm(x) = \left( \frac{d^2}{dx^2} + V_\pm \right) \psi_\pm(x) = \Delta^2 \psi_\pm(x),$$

(6)

where the effective potentials $V_\pm$ are given by

$$V_\pm(x) = W^2 \pm \frac{dW}{dx} = \left[ \left( \kappa_y + 2\delta \tanh \left( \frac{x}{2\delta} \right) \right)^2 \pm \text{sech}^2 \left( \frac{x}{2\delta} \right) \right].$$

(7)

As seen in figure (2), depending on the values of $\kappa_y$ and $\delta$, the effective potentials can assume the form of potential wells or steps. In the following sections the bound-state spectrum and scattering properties will be analyzed in detail.

It is interesting to point out that the Dirac equation in the presence of an external magnetic fields possesses a structure that can be analyzed within the formalism of supersymmetric quantum mechanics (SUSY-QM) \[31–34\]. The potentials $V_x$ and $V_y$ in (7) are known as the super-partner potentials, they are obtained from the superpotential function $W(x)$ by the relation in (7). The explicit expressions for $V_\pm$ for the gauge potential in (2) are identified as the Rosen–Morse II potentials \[31, 32\] and the corresponding Schrödinger equations are exactly solvable. There are important properties of SUSY-QM that relate the spectrum and eigenfunctions of the effective Hamiltonians of $H_+$ and $H_-$ in equation (6). In particular, except from the ground state, $H_+$ and $H_-$ share the same energy eigenvalues.

It is convenient to define the operators

$$L^\pm = -\frac{d}{dx} \pm iW(\tilde{x}).$$

(8)

In terms of these operators the relation between the upper and lower components in (4) simply read

$$\psi_+(x) = \frac{1}{\Delta} L^- \psi_-(x), \quad \psi_-(x) = \frac{1}{\Delta} L^+ \psi_+(x).$$

(9)

Let us introduce the auxiliary variable

$$\xi = \frac{1}{1 + \exp(\tilde{x}/\delta)},$$

(10)

that varies from 0 to 1, as $\tilde{x}$ goes from $\infty$ to $-\infty$. In the new variable equation (6) becomes

$$\frac{d^2}{d\xi^2} + \frac{1 - 2\xi}{\xi(1 - \xi)} \frac{d}{d\xi} + \delta^2 \left[ \Delta - \left[ \kappa_y + 2\delta (1 - 2\xi) \right]^2 \pm 4\xi(1 - \xi) \right] \psi_\pm = 0.$$

(11)

These equations have the asymptotic solutions $\psi_\pm \sim \xi^\rho$ for $\xi \to 0$ ($x \to \infty$) and $\psi_\pm \sim (1 - \xi)^\sigma$ for $\xi \to 1$ ($x \to -\infty$), where the asymptotic behavior is determined by

$$\rho = \delta \sqrt{(\kappa_y + 2\delta)^2 - \Delta^2}, \quad \sigma = \delta \sqrt{(\kappa_y - 2\delta)^2 - \Delta^2}. $$

(12)

In order to obtain consistent solutions, we recall that besides solving the effective Schrödinger equation in (11), the wavefunction components are interrelated by the Dirac equation via (9). Then, one can consider the following options. (a) Equation (11) is solved for the lower component $\psi_-$, the corresponding upper component $\psi_+$ is obtained from the first relation in (9). (b) Equation (11) is solved for the upper component $\psi_+$, and the lower component is obtained from the second relation in (9). We consider the first option, the second option gives equivalent solutions, except for the $n = 0$ state. Taking into account the asymptotic behavior, we propose an ansatz of the form $\psi_-(\xi) = \xi^\rho (1 - \xi)^\sigma f(\xi)$, substituting in equation (11) we find that $f(\xi)$ satisfies the hypergeometric equation. Two linear independent solutions can be chosen as \[35\] $f(\xi) = F(\alpha, \beta, \gamma; \xi)$ and $f(\xi) = \xi^{-2\rho} F(\alpha - \gamma + 1, \beta - \gamma + 1, 2 - \gamma; \xi)$, where $F(\alpha, \beta, \gamma; \xi)$ is the hypergeometric function. However, the second solution does not have the correct asymptotic behavior and has to be discarded. The corresponding upper component is obtained from the first equation in (9). The complete solution is then given as

$$\psi = Ce^{\alpha\xi^\rho} \xi^\rho (1 - \xi)^\sigma \left[ \frac{1}{\Delta} G(\xi) F(\alpha, \beta, \gamma; \xi) + \xi (1 - \xi)^{\frac{\beta}{\gamma}} F(\alpha + 1, \beta + 1, \gamma + 1; \xi) \right].$$

(13)

where $G(\xi) = [(\rho - 2\beta^2)(1 - \xi) - \xi(\sigma - 2\delta^2) - \delta\kappa_y]$, C is the normalization constant and

$$\alpha = \rho + \sigma - 4\delta^2, \quad \beta = \rho + \sigma + 4\delta^2 + 1, \quad \gamma = 2\rho + 1.$$  

(14)
3. Bound-state spectrum

Our aim is now to discuss the bound-state spectrum. First we notice that the electron–hole symmetry is preserved by the inhomogeneous magnetic field. This follows from the fact that the Hamiltonian in (3) anticommutes with the $\sigma_3$ matrix: $[H, \sigma_3] = 0$. Then, if $\psi$ is an eigenvector of $H$ with eigenvalues $E, \sigma_3 \psi$ is also an eigenvector with eigenvalue $-E$.

The fact that the Hamiltonians $H_+ \text{ and } H_-$ share the same eigenvalues implies that the existence of bound states require that both $V_+$ and $V_-$ have the form of potential wells. This condition is obtained if the minima of $V_\pm$ are attained for a finite value of $x$; it is given by

$$|\kappa_y| < 2 \delta. \quad (15)$$

As seen in figure 2(a), when the previous condition holds both $V_+$ and $V_-$ have the form of asymmetric potential wells around the guiding center position

$$\tilde{x}_c = 2 \delta \arctanh \left(\frac{\kappa_y}{2 \delta}\right). \quad (16)$$

Bounds states are found if the wells are sufficiently deep. In the limit in which the transverse momentum vanishes ($\kappa_y = 0$), the effective potentials reduce to symmetric wells centered at the maximum of $B(x)$. For values of $\kappa_y$ for which the condition (15) is not obeyed, at least one of $V_\pm$ takes the form of a potential step figure (2(b)), and bound states are not supported.

The solution in equation (13) leads to a divergence in $\xi \to 1(x \to -\infty)$ except for $\alpha$ or $\beta$ being a negative integer. Letting $\alpha = -n$, and utilizing equation (14) we obtain the energy spectrum, that can be conveniently written as

$$\Delta \equiv \Delta_{n,k_\gamma} = \pm \sqrt{2 \left[n - \frac{n^2}{8 \delta^2}\right] \left[1 - \left(\frac{\kappa_y/2 \delta}{n/4 \delta^2}\right)^2\right]} \quad (17)$$

The index $n$ takes the values $n = 0, 1, 2, 3, \ldots, n_{\text{max}}$. Both the allowed values of $n$ and $\kappa_y$ are restricted in order to satisfy the square integrability condition:

$$n_{\text{max}} \leq 4 \delta^2, \quad \kappa_{y,\text{max}} \leq \frac{[4 \delta^2 - n^2]}{8 \delta^3}. \quad (18)$$

The first condition determines the highest bound state supported for a given width of the barrier and eliminates the possible singularity in (17), whereas the second condition determines the allowed values of $\kappa_y$ for a given $n$ and is related to the fact that the electron group velocity is limited by the free velocity in graphene, i.e. $|\partial E_{n,k_\gamma}/\partial \kappa_y| \leq v_F$.

The two conditions guarantee that $\Delta_{n,k_\gamma} < \min[V_\pm(\pm \infty)]$, so the electron does not escape toward $\pm \infty$; equivalently the coefficients in (12) that determine the wavefunction asymptotic behavior satisfy $\rho, \sigma > 0$.

The wavefunction for the zero energy level takes a simple form that can be obtained by solving the first equation in (4) when $\Delta = 0$, it reads

$$\psi = C e^{i\kappa_y \tilde{x}} e^{-\kappa_y \tilde{x}} \left[\text{sech} \left(\frac{\tilde{x}}{2 \delta}\right)\right]^{4 \beta} \left[\begin{array}{c} 0 \\ 1 \end{array}\right]. \quad (19)$$

![Figure 3. Bound-state spectrum $\Delta_{n,k_\gamma} = l_B E_{n,k_\gamma}/\hbar v_F$ as a function of $\kappa_y = l_B k_\gamma$ for a magnetic barrier with $\delta = 1.5$. According to (18) $n_{\text{max}} = 9$, and for every $n$-level the allowed values of $\kappa_y$ are delimited by the free-electron spectrum $\Delta = \pm \kappa_y$ (dashed lines). In the figure we observe seven of the nine predicted bound states: positive energy not counting the $n = 0$ energy state. The reason is that according to equations (18) the $n = 9$ state is represented by a single point at $(\kappa_y = 0, \Delta = 3)$ and the energy values of the $n = 7$ and $8$ states are too close to be distinguishable in this figure.](image)
that varies between $\kappa$, allowed. For a small value of $\Delta^1$, dashed line, barrier when the system is confined by a square box of area $L \times L$ with $L/l_0 = 50$. The dark zones are the allowed energy values. The dashed line $\Delta = 2\delta$ delimits the minimum barrier width that supports bound states. For every level $n$ the transverse momentum varies between $\kappa_y = 0$ and $K_y$.

Figure 4. Energy spectrum as a function of $\delta = d/l_0$ for a magnetic barrier when the system is confined by a square box of area $L \times L$ with $L/l_0 = 50$. The dark zones are the allowed energy values. The dashed line $\Delta = 2\delta$ delimits the minimum barrier width that supports bound states. For every level $n$ the transverse momentum varies between $\kappa_y = 0$ and $K_y$.

integer part $[(2\delta \bar{L}/\pi) \tanh(\bar{L}/\delta)]$, whereas the momentum limit imposed by the size of the system reads $\kappa^L_{y, \text{max}} = 2\delta \tanh(\bar{L}/\delta)$. It is interesting to notice that the degeneracy coincides with the number of flux quanta piercing the system, i.e. $N = \Phi/\Phi_0$, since the total flux $\Phi$ for $B(x)$ in (1) is calculated as $\Phi = (4\hbar/e) \delta \bar{L} \tanh(\bar{L}/\delta)$ and $\Phi_0 = h/e$ is the elementary fluxon. In order to track the degeneracy evolution as the barrier is modified from narrow to broad as compared to $L$, we define

$$K_y = \frac{1}{\kappa^L_{y, \text{max}}} + \frac{1}{\kappa^L_{y, \text{max}}}.$$  

(21)

$K_y$ is the effective cut for the transverse momentum. Notice that $K_y$ interpolates between $\kappa^L_{y, \text{max}}$, valid for a narrow barrier, and $\kappa^L_{y, \text{max}}$ valid when $\delta > \bar{L}$. Figure (4) shows the resulting energy spectrum as a function of $\delta$, the dark zones are the allowed energy values. For every level $n$ the transverse momentum varies between $\kappa_y = 0$ and $K_y$. The restriction on the level index $n$ given by the first equation in (18), translates into the following equation for the separatrix $\Delta = 2\delta$ (dashed line); energies to the left of this line are not valid for a narrow barrier, whereas energy conservation allow us to relate the transmitted and incident longitudinal momenta as

$$K_y' = \sqrt{\kappa^L_{y} - 16\delta^2 - 8\delta\kappa_y}.$$  

(24)

Equation (23) implies that for a critical angle $\phi_c = \arcsin(1 - 4\delta/\Delta)$ no transmission is possible. Furthermore, when the following condition applies:

$$\Delta \leq 2\delta.$$  

(25)

the transmission vanishes regardless of the incident angle $\phi$. The condition in (25) establishes that states with an average cyclotron radius (in the barrier region) smaller than $2\delta$ will bend by the magnetic field and are completely reflected; a similar condition was obtained in the case of a square-well magnetic barrier [16]. Comparing equations (22) with (12), we observe that the longitudinal momenta $\kappa_x$ and $\kappa_y'$ are related to the asymptotic coefficients $\rho$ and $\sigma$ as follows: $\sigma = -i\kappa_x'\delta$ and $\rho = \delta \sqrt{-\kappa^L_y}$. It is verified that if condition (25) holds $\rho$ is real, instead when (25) is not valid we replace $\rho = -i\kappa_x'\delta$. Utilizing the properties of the hypergeometric functions it is verified that in the limit $x \rightarrow \infty (\xi \rightarrow 0)$ the wavefunction in (13) yields the correct asymptotic expression

$$\psi \sim e^{i(\kappa_x'x + \kappa_y'y)} \left[\frac{xe^{-i\phi}}{1}\right].$$  

(26)

where $s = \text{sgn} E$. The asymptotic wavefunction for $x \rightarrow -\infty (\xi \rightarrow 1)$, is obtained using the linear transformation formulas [35] that relate $F(\alpha, \beta, \gamma, z)$ with hypergeometric functions evaluated at $1 - \xi$, to obtain

$$\psi \sim e^{i(\kappa_x'x + \kappa_y'y)} \left[\frac{\Gamma(\gamma - \alpha - \beta)}{\Gamma(\gamma - \alpha)\Gamma(\gamma - \beta)\Gamma(\alpha)\Gamma(\beta)}\right] \left[\frac{ze^{-ib}}{1}\right].$$  

(27)

From this equation the reflection coefficient $R$ is obtained as

$$R = \left|\frac{\Gamma(\rho - i\kappa_x\delta + 4\delta^2 + 1)\Gamma(\rho - i\kappa_x\delta - 4\delta^2)}{\Gamma(\rho + i\kappa_x\delta + 4\delta^2 + 1)\Gamma(\rho + i\kappa_x\delta - 4\delta^2)}\right|^2.$$  

(28)

Under condition (25) $\rho$ is real, thus $R = 1$ and, as expected, the transmission coefficient vanishes. But if the condition (25) is not obeyed, $\rho$ is substituted by $\rho = -i\rho x'\delta$, and the transmission probability $T = 1 - R$ can be written in a simple closed form as

$$T = \frac{\sinh[2\pi \kappa_x]}{\sin^2[\pi \kappa_x]} + \sinh[2\pi \kappa_x'] + \sinh[(\kappa_x + \kappa_x')]^2.$$  

(29)
A plot for the transmission coefficient as a function of the incidence angle is shown in figure (5) for a fixed energy and several values of $\delta$. The qualitative behavior in this plot is similar to that obtained for the case of a square-well magnetic barrier [16]. However an advantage of the result in (29) is that similar to that obtained for the case of a square-well magnetic barrier. In the first case figure 7(a), coefficient are shown in figure 6(b). Contour plots for the transmission to the resonant condition in equation (30) with $j = 1, 2, 3, \ldots$.

On the other hand, under the condition

$$\delta = \frac{\sqrt{T}}{2}, \quad j = 1, 2, 3, \ldots$$  \hspace{1cm} (30)

In this case the barrier acts as an asymmetric filter, it behaves as perfectly transparent for angles in the region $-\pi/2 < \phi < \phi_0$, figure 6(a). In particular, for energy values slightly above the threshold condition in (25), $\Delta = 2\delta + \epsilon$, with $\epsilon \ll 1$, the width of the transparency region can be very narrow.

The value of $T$ is reduced, in particular for values of the energy slightly above the threshold condition in (25), $\Delta = 2\delta + \epsilon$, with $\epsilon \ll 1$, the transmission coefficient is strongly reduced, see figure 6(b). Contour plots for the transmission coefficient $T$ are shown in figure 7. Note that the transmission coefficient depends not only on the longitudinal momentum $k_x$, but also on the momentum $k_y$ perpendicular to the magnetic barrier. In the first case figure 7(a), $\delta = 1/2$ corresponds to the resonant condition in equation (30) with $j = 1$, we observe a wide region for the transmission as compared to the plot in figure (7(b)) corresponding to the minima condition in equation (31) with $j = 0$.

$$\delta = \frac{\sqrt{j + \frac{1}{2}}}{2}, \quad j = 0, 1, 2, 3, \ldots$$  \hspace{1cm} (31)

5. Conclusions

In this paper we have studied the dynamics of carriers in graphene subjected to an inhomogeneous hyperbolic magnetic field. The corresponding Dirac equations were analyzed within the formalism of supersymmetric quantum mechanics. We found compact analytical solutions for the energy eigenvalues and eigenfunctions for electrons and holes. The dispersion relation in (17) shows that the inhomogeneity of $B$ lifts the degeneracy for every quantum level $n$ and gives rise to a $k_x$-dependent dispersion relation, which leads to a drift velocity along the $y$ axis. This is valid, except for the $n = 0$ level that has zero energy, independent of the magnetic field for all values of $k_x$. For a narrow barrier the spectrum displays a series of bands separated by gaps; our results provide a detailed analysis of how the bound states inside the magnetic barrier evolve into the relativistic Landau levels when the barrier size exceeds the system size. In the scattering regime a simple analytical formula is obtained for the transmission coefficient.

This result allows us to identify the resonant conditions at which the barrier becomes transparent. It should be mentioned that recently an exact solution for an electrostatic potential with a trapezoid profile has been presented in order to assess the effects of a smooth finite slope potential step on Klein tunneling [36]. The solution allows the author to compute the conductance and Fano factor of graphene in the ballistic regime. More recently a combined model of electric and magnetic barriers [37] shows that the band structure is substantially modified near the Dirac point leading to strong modifications on the transmission coefficients. Based on the results of these papers and the present work, we plan to extend
our model in order to compute the conductance in the ballistic limit and to incorporate the effects of a scalar potential. In a future work we plan to address the problem of calculating the longitudinal conductivity as well as the Hall conductivity for the case of graphene under inhomogeneous magnetic fields. Finally, we expect that the results obtained in this work will be useful in order to analyze the confinement by magnetic barriers in graphene samples.

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Figure 7. (a) Contour plot of the transmission coefficient $T$ through a barrier with $\delta = 1/2$ (resonant condition in equation (30) with $j = 1$). (b) As in (a) for $\delta = 1/\sqrt{8}$ (equation (31) with $j = 0$).