Analytical Performance Assessment of Beamforming Efficiency in Reconfigurable Intelligent Surface-Aided Links

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Abstract
Future wireless networks designed to operate in the millimeter-wave and terahertz bands are expected to be highly assisted by Reconfigurable Intelligent Surfaces (RIS). The role of the RIS will be to mediate possible non-line-of-sight link by redirecting the incident beam from the transmitter to the receiver and possibly modifying its characteristics, in order to optimize the beamforming efficiency and to maximize the signal power at the receiver. Therefore, it is crucial to understand what are the bounds imposed on the received power and how these bounds depend on the system design parameters. In this paper we show that, contrary to typical line-of-sight links, the increase of the transmitter gain does not always guarantee power increase at the receiver, even for an infinitely large RIS, and we explain how the RIS size can further affect the received power. We present an analytical model that captures the performance of the RIS-aided link in the limit of very large RIS, and we demonstrate numerical examples that provide insight on the interplay between the RIS size and the properties of the transmitter beam.

Index Terms
Beamforming efficiency, beam steering, reconfigurable intelligent surface, THz communications.

I. INTRODUCTION
In recent years, the dramatic increase in demand for high data rates has lead the research towards unallocated high frequency bands such as the millimeter-wave (mmWave) (30-100 GHz) and terahertz (THz) bands (0.1-10 THz) [1]-[5]. However, because with increasing frequency pathloss increases and diffraction becomes stronger [5]-[7], the transmitted signals are expected to be more vulnerable to the presence of obstacles (blockage), as compared to operation in the low GHz, for example. In particular, in [5], the authors show that the losses from the human body (e.g. the user) can reach values of 40 dB or greater at 73.5 GHz, while in [8] they show that the user’s hand may induce a mean loss of about 15 dB. Therefore, at such high frequencies it is possible that the line-of-sight (LoS) link is blocked even at relatively short distances (in the order of a few meters), thus limiting the practicality of the mmWave and THz bands to mainly short range and LoS scenarios. In order to expand their use to non-line-of-sight (nLoS) scenarios, the use of RISs has been proposed for both in-door ([9]-[12]) and more general scenarios ([13]-[28]).

In [9] a practical RIS phase shift model is presented, along with the joint optimization of the AP transmit beamforming and the RIS beamforming, in order to minimize the transmit power. In [10] the authors investigate the performance of both indoors and outdoors RIS-aided systems and show that a RIS-aided link can even outperform the equivalent LoS link. The authors in [11] propose two schemes to optimize the channel capacity in indoor mmWave environments. The authors in [12], propose a method for improving the focusing of the transmitted signal, using deep neural networks. In [13], a tutorial on the RIS technology is provided to address challenges, such as the channel estimation, the optimization of the reflection and the deployment of the RISs. The authors of [14] derive the far-field pathloss...
In [27], the improvement of coverage with the help of RIS in laws as a function of the number of scattering elements. For the characterization of the SNR distribution and its scaling algorithm. In [26] the authors present an analytical approach for the received SNR with the help of a coverage maximization distance and the orientation of the RIS in order to maximize in [25] the authors explore the optimization of the RIS-AP signalling to optimize the data rate at the receiver. Moreover, transmit beamforming under proper and improper Gaussian joint design of the reflection coefficients of the RIS and the their trade-off. The authors in [24] address the problem of multiple UEs. The authors in [23] explore the optimization the location of the RIS, in order to maximize the sum rate of the transmit power allocation of the access point (AP). In [19], the coverage probability in the form of a closed-form expression for a RIS-aided scenario with the use of physical optics. In [15], the authors develop a general formula to describe the free-space pathloss in RIS-aided systems. Furthermore, they propose three free-space pathloss models for individual scenarios and validate them with experiments. In [16], the authors explore the similarities and differences between relays and RISs that function as anomalous reflectors. Their study shows that RISs of sufficiently large size reduce the complexity of the implementation, while surpassing the relays in terms of data rate. In [17] the authors propose a phase shift design for large intelligent surfaces and evaluate their performance in terms of ergodic spectral efficiency. Furthermore, in [18], the optimization of the data rate and energy efficiency of a RIS-aided multi-user downlink multiple input single output (MIMO) system is performed with two algorithms for the values of the RIS reflectors and the transmit power allocation of the access point (AP). In [19], the coverage probability in the form of a closed-form expression is derived for RIS-aided THz wireless systems. In [20] and [21], the optimal position of the RIS is determined with the prospect of maximizing the signal-to-noise-ratio (SNR) at the receiver. The results show that the RIS should ideally be placed closer to the receiver. In [22], the authors optimize the location of the RIS, in order to maximize the sum rate of multiple UEs. The authors in [23] explore the optimization of the number of elements and the phase shifts of the RIS, in order to maximize the data rate, the energy efficiency and their trade-off. The authors in [24] address the problem of joint design of the reflection coefficients of the RIS and the transmit beamforming under proper and improper Gaussian signalling to optimize the data rate at the receiver. Moreover, in [25] the authors explore the optimization of the RIS-AP distance and the orientation of the RIS in order to maximize the received SNR with the help of a coverage maximization algorithm. In [26] the authors present an analytical approach for the characterization of the SNR distribution and its scaling laws as a function of the number of scattering elements. In [27], the improvement of coverage with the help of RIS in the form of an intelligent wall is explored. Furthermore, in [28] the optimization of beamforming on the RIS is studied in order to minimize the transmit power of the AP, while keeping the received power at the user equipment (UE) constant.

In view of the constantly increasing volume of related works, it becomes clear that there are several parameters that need to be taken into account in order to assess the performance of a RIS-aided link. For example, the received power will strongly depend on the positioning of the RIS with respect to the transmitter and receiver, and will be affected by the RIS size and the properties of the transmitter beam. Therefore, there is a need for a simple analytical model that can clarify the crucial bounds imposed by the system design parameters on the link performance. In this work, in an attempt to provide a generalized approach, the efficiency of the RIS beamforming is studied in terms of the received power. The main contributions are summarized as follows.

- The system performance is qualitatively divided into two regimes of operation, with respect to the RIS size relative to the footprint of the transmitter beam on the RIS.
- For relatively small footprint, a simple analytical model is derived, providing insight on the interplay between crucial parameters, such as the positioning of the RIS with respect to the transmitter and receiver, and the properties of the transmitter beam. The model clarifies why the received power does not increase monotonically with increasing gain, but rather acquires a maximum.
- For relatively large footprint, the impact of the RIS size on the maximum received power is demonstrated through simulations.
- The approach followed in this work provides simple guidelines for the algorithmic design of rules for optimized performance.

II. SYSTEM MODEL
A RIS-aided system model is shown in Fig. 1, where the RIS is centered at the origin of the coordinate system and

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**FIGURE 1.** System model. (a) Schematic of a RIS-aided link, illustrating the positions and relative angles between the AP, RIS and UE. The RIS consists of $M \times N$ elements (here shown $4 \times 5 = 20$ elements). (b) Details on the orientation of the RIS elements and single unit cell with schematic representation of its radiation pattern. The angle $\phi_{m,n}'(\theta_{m,n}', \phi_{m,n}')$ denotes the elevation angle from the transmitting (receiving) antenna to the unit cell. The angles $\phi_{m,n}'(\theta_{m,n}', \phi_{m,n}')$ denote the elevation and azimuth angles from the unit cell to the transmitting (receiving) antenna. Along the designated direction, the normalized power radiation pattern of the unit cell, $U_R$, acquires the value $U_R(\phi_{m,n}', \theta_{m,n}')$. The RIS consists of $M \times N$ elements (here shown $4 \times 5 = 20$ elements). (b) Details on the orientation of the RIS elements and single unit cell with schematic representation of its radiation pattern. The angle $\phi_{m,n}'(\theta_{m,n}', \phi_{m,n}')$ denotes the elevation angle from the transmitting (receiving) antenna to the unit cell. The angles $\phi_{m,n}'(\theta_{m,n}', \phi_{m,n}')$ denote the elevation and azimuth angles from the unit cell to the transmitting (receiving) antenna. Along the designated direction, the normalized power radiation pattern of the unit cell, $U_R$, acquires the value $U_R(\phi_{m,n}', \theta_{m,n}')$. The RIS consists of $M \times N$ elements (here shown $4 \times 5 = 20$ elements). (b) Details on the orientation of the RIS elements and single unit cell with schematic representation of its radiation pattern. The angle $\phi_{m,n}'(\theta_{m,n}', \phi_{m,n}')$ denotes the elevation angle from the transmitting (receiving) antenna to the unit cell. 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consists of $M \times N$ elements (unit cells), periodically arranged along the $x$– and $y$– axes, with periodicity $dx$ and $dy$ respectively. The AP is located at $(x_{AP}, y_{AP}, z_{AP})$ and is equipped with a highly directional antenna with tunable beam-width. The AP illuminates the RIS, which subsequently redirects the incident beam to the UE, located at $(x_{UE}, y_{UE}, z_{UE})$. In this work, misalignment between the AP and RIS or the RIS and UE is not considered, however the results can be directly extended to account for this possibility. The power received by the UE can be calculated using the power density, $S_r$, at the UE position and the effective aperture of the UE, $A_r$, as:

$$P_r = S_r A_r \equiv \frac{|E_r|^2}{2Z_o},$$  \hspace{1cm} (1)$$

where $E_r$ is the electric field at the position of the UE, $Z_o$ is the characteristic impedance of air and $A_r = \frac{\Omega A_r^2}{4\pi}$, with $G_r$ being the antenna gain of the UE and $\lambda$ the free-space wavelength. The total electric field at the UE position is:

$$E_r = \sum_{m=1}^{M} \sum_{n=1}^{N} E_{m,n},$$  \hspace{1cm} (2)$$

where the contribution of the $(m, n)$ RIS element to the total electric field is given by [15], [19]:

$$E_{m,n} = \sqrt{2Z_o} P_t G_t A_{RIS} G_{RIS} \frac{\sqrt{U^t(t_{RIS}, r_{RIS}) U^r}}{4\pi l_{m,n}^2 \gamma_{m,n}} \times |R_{m,n}| \exp(-j\phi_{m,n}) \exp \left(-\frac{2\pi}{\lambda} (l_{m,n}^t + l_{m,n}^r) \right).$$  \hspace{1cm} (3)$$

In Eq. (3) $P_t, G_t$ are the transmitted power and antenna gain of the AP, respectively, $A_{RIS}, G_{RIS}$ are the effective aperture and antenna pattern gain of each RIS element, respectively, $l_{m,n}^t, l_{m,n}^r$ are the distances between the $(m, n)$ RIS element and the AP and UE respectively; $R_{m,n} = |R_{m,n}| \exp(-j\phi_{m,n})$ is the complex reflection coefficient introduced by the $(m, n)$ RIS element. The normalized power radiation pattern of the AP, UE and the RIS element is denoted by $U^t, U^r$ and $U^{RIS}$, respectively. In particular, $U^t \equiv U^t(\theta_{m,n}^t, \Phi_{m,n}^t)$ is the value of $U^t$ along the direction defined by the AP and the $(m, n)$ element, as shown in Fig. (1b), with the AP targeting the center of the RIS (origin of coordinate system). Similarly, $U^r \equiv U^r(\Theta_{m,n}^r, \Phi_{m,n}^r)$ is the value of $U^r$ along the direction defined by the UE and the $(m, n)$ element, with the UE also targeting the center of the RIS. Last, $U^{RIS, t/r}$ is the value of $U^{RIS}$ along the direction defined by the positions of the $(m, n)$ element and the AP/UE, as denoted with the superscript $t/r$, i.e. $U^{RIS, t/r} \equiv U^{RIS}(\theta_{m,n}^{t/r}, \phi_{m,n}^{t/r})$. Because the RIS elements are identical, they all share a common radiation pattern $U^{RIS} = \cos^2\theta_{m,n}^{t/r}$, with $\theta_{m,n}^{t/r} \in [0, \pi/2]$ and $\phi_{m,n}^{t/r} \in [0, 2\pi]$ defined with respect to the coordinate system $(x', y', z')$ [see Fig. (1b), right panel]. Note that, although $U^{RIS}$ is common to all RIS elements, the angles $\theta_{m,n}^{t/r}, \phi_{m,n}^{t/r}$ change with $(m, n)$ and, therefore, $U^{RIS, t/r}$ depends on the coordinates of each $(m, n)$ element. Last, regarding the phase introduced by the RIS, it should be noted that a RIS design that aims to redirect a beam from the AP to the UE should provide the linear phase gradient given by [15], [19]:

$$\phi_{m,n} = \frac{2\pi}{\lambda} \left[ (\sin \theta_{UE} \cos \phi_{UE} + \sin \theta_{AP} \cos \phi_{AP}) x_m + (\sin \theta_{UE} \sin \phi_{UE} + \sin \theta_{AP} \sin \phi_{AP}) y_m \right].$$  \hspace{1cm} (4)$$

where $x_m = m \times d_x, m = 1, \ldots, M$ and $y_n = n \times d_y, n = 1, \ldots, N$ are the coordinates of the $(m, n)$ RIS element.

### III. RIS ILLUMINATION REGIMES: RIS SIZE VS AP BEAM FOOTPRINT

In a conventional LoS link, the beam travels from the AP directly to the UE with increasing width –due to free-space propagation– and therefore with decreasing power density, which can be further reduced by possible atmospheric absorption and scattering. As a result, for a certain emitted beam power, the narrower the beam-width at the UE, the higher the power density and the more the power that can be received by the UE. However, when an intermediate re-radiating element, such as a RIS, comes into play, this intuitive picture can change entirely. The reason is that the RIS is essentially a secondary antenna; it is driven by the incident field from the AP and is re-emitting a secondary beam (towards the UE), the propagation characteristics of which depend on both the incident power density on the RIS and the illuminated area.

For highly directional antennas, as considered here, the received power will be largely defined by the main lobe of the AP antenna, regardless of the exact radiation pattern, which will depend on the particular design specifications. Therefore, in order to be able to examine a simple analytical model that can capture the overall performance, beyond the antenna’s particular characteristics, a convenient way to approximate the main lobe of a generalized AP antenna and to introduce tunability is to consider a Gaussian radiation pattern (see e.g. [29]) of the form $G_t U^t \equiv G_t \exp \left(-\frac{\pi}{4} \sin^2 \Theta_{m,n}^t \right)$, with $\Theta_{m,n}^t \in [0, \pi/2]$ and $\Phi_{m,n}^t \in [0, 2\pi]$ defined with respect to the coordinate system located at $(x_{AP}, y_{AP}, z_{AP})$ as shown in Fig. (1b), with $m = 1 \ldots M, n = 1 \ldots N$ (see Appendix A for details). With the AP targeting the center of the RIS, the AP gain becomes maximum (equal to $G_t$) for the $(m, n)$ element located at the origin (where $U_t = 1$).
For highly directional beams, this form for the radiation pattern ensures that the total power radiated from the AP remains practically constant upon beam-width change, i.e. $\int (P_f / 4\pi d^2) dS \approx P_f$, where $dS$ is the elementary surface element of a sphere of radius $d$ centered at the AP.

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In this notation, low $G_t$ means a relatively wide AP beam that can illuminate the entire RIS; therefore, the incident power residing beyond the RIS surface cannot be captured by the RIS, leading effectively to power loss. Intuitively, to compensate for the lost power, $G_t$ must be increased, in order to reduce the area of the AP beam-footprint on the RIS and achieve the utilization of the entire AP beam. Simultaneously, the power density of the beam will increase, as now the same incident power will be concentrated in a smaller region on the RIS, and more power is expected to be received by the UE, similarly to what would happen in a LoS link with increasing AP gain. However, a smaller AP footprint will lead the secondary beam (from the RIS to the UE) to undergo stronger spreading in air, subsequently reducing its power density and possibly leading to the opposite result.

From this discussion it becomes apparent that, besides the incident power density, the size of the RIS illuminated area plays a decisive role on the overall performance of the RIS-aided link and, therefore, the trade-off between the RIS size, AP beam-width and AP power density, can lead to unexpected results with respect to the power received by the UE. To study this trade-off, two regimes of operation can be identified with respect to the RIS size. On the one hand, for relatively low $G_t$, the AP beam illuminates the entire RIS and, therefore, the footprint of the secondary beam does not depend on the AP beam width, rather is determined by the actual area of the RIS (the effective aperture of the RIS is determined by the finite area of the RIS); we refer to this case as finite RIS regime. On the other hand, for relatively high $G_t$, the AP beam illuminates only a portion of the RIS and, therefore, the width of the secondary beam on the RIS is defined entirely by the AP beam footprint, i.e. the actual RIS size does not matter (the effective aperture of the RIS is determined by the AP beam footprint); in this case the AP beam sees an effectively infinite RIS and we refer to this case as infinite RIS regime.

IV. INFINITE RIS REGIME

When the entire AP beam is captured by the RIS we can follow an alternative approach to model the RIS-aided link; we can leave aside the discrete nature of the RIS and treat it as a continuous surface, i.e. we can treat the beam as a continuous distribution. The advantage of this approach over the summation given by Eq.(2) is that the electric field at the position of the UE can be expressed in a simple closed form, enabling the derivation of analytical expressions that provide insight to the RIS operation.

The starting point in this approach is to identify the properties of the AP footprint on the RIS. First, a beam of total power $P_f$ illuminates the RIS, as illustrated schematically in Fig. (2a). For a Gaussian beam, as considered here, the AP footprint on the RIS is also Gaussian with radius $w_{RIS}$ and power density:

$$S_{RIS}^{inc} = \frac{2P_f}{\pi w_{RIS}^2} \exp \left( -2\frac{x^2 + y^2}{w_{RIS}^2} \right). \quad (6)$$

Next, the AP footprint is reflected by the RIS and propagates as a tilted Gaussian beam along the direction defined by the angles $\theta_B, \phi_B$. The non-orthogonal local beam coordinate system follows the beam as it propagates; $z_B$ is the origin location along the paraxial propagation direction and the transverse coordinates, $x_B$ and $y_B$, lie on a plane parallel to the $x, y$ plane.

$$x_B = x - z_B \sin \theta_B \cos \phi_B$$
$$y_B = y - z_B \sin \theta_B \sin \phi_B$$
$$z_B = z / \cos \theta_B.$$
The term $\Phi$ expresses the phase advance of the redirected beam and is given by: $\Phi = x_B \sin \theta_B \cos \phi_B + y_B \sin \phi_B + z_B$. For $\theta_B = \theta_{UE}$, $\phi_B = \phi_{UE}$ the RIS steers the beam towards the UE and, therefore, exactly at the UE position $(x_{UE}, y_{UE}, z_{UE})$ where $x_B = 0$, $y_B = 0$ and $z_B = d_{UE}$. For the power density at the UE is simply given with the aid of Eq. (5) by:

$$S_r^{inf} = \frac{2P_t}{\pi w_{RIS}^2} \frac{|R|^2}{\left(1 + \frac{d_{UE}^2}{z_R^2}\right) \left(1 + \frac{d_{UE}^2}{z_R^2 \cos^4 \theta_{UE}}\right)}$$

(8)

where the superscript $'inf'$ emphasizes the fact that the result of Eq. (8) derives from the continuous approach in the infinite RIS regime. Then, the power collected by the UE is $P_r^{inf} = S_r^{inf} A_r$, where $A_r$ is the UE effective aperture introduced in Eq. (1).

The denominator of Eq. (8) implies that what is of importance is the RIS-UE distance ($d_{UE}$) relative to the Rayleigh length $z_R$, which is a function of $w_{RIS}$ and the wavelength (see Appendix A for details):

$$z_R = \frac{k_0 w_{RIS}^2}{2} = 4k_0 d_{AP}^2 G_t$$

(9)

In the limit $d_{UE}/z_R \ll 1$, Eq. (8) acquires a simple form:

$$d_{UE} \ll z_R : P_r^{inf} = A_r|R|^2 \frac{2P_t}{\lambda} \frac{1}{z_R}$$

(10)

i.e., $P_r^{inf} \sim G_t$ [see Eq. (9)], as in typical LoS links. However, in the other extreme:

$$d_{UE} \gg z_R : P_r^{inf} = A_r|R|^2 \frac{2P_t}{\lambda} \frac{\cos^2 \theta_{UE}}{d_{UE}^2} z_R$$

(11)

i.e., $P_r^{inf} \sim 1/G_t$, which means that $P_r^{inf}$ decreases with increasing $G_t$, contrary to what might be intuitively expected, as in the previous case. The two limits imply that there should be a maximum $P_r^{inf}$ for some value of $G_t$, which can be found by differentiating Eq. (8) with respect to $z_R$. The maximum power received by the UE is:

$$\max(P_r^{inf}) = A_r|R|^2 \frac{2P_t}{\lambda d_{UE}} \frac{\cos^2 \theta_{UE}}{1 + \cos^2 \theta_{UE}}$$

(12)

and occurs when:

$$z_R \cos \theta_{UE} = d_{UE}$$

(13)

Interestingly, the maximum $P_r^{inf}$ does not depend on the position of the AP. This is a direct consequence of the fact that the RIS size is much larger than the AP beam footprint and, hence, the latter can be tuned without limitation. Furthermore, because the Rayleigh length $z_R$ is the distance denoting the transition from the near-field to the far-field of the beam, the result of Eq. (13) dictates that the maximum $P_r^{inf}$ occurs when the UE resides in the vicinity of this transition. In essence, a change in $G_t$, the gain of the AP, modifies the relative distance $z_R$ with respect to the position of the UE [see Eq. (9)]. By increasing $G_t$, $w_{RIS}$ decreases, in turn reducing the distance $z_R$ and effectively transferring the UE from the near- to the far-field of the beam. Therefore, relatively low $G_t$ essentially means that $d_{UE} < z_R$ and increasing $G_t$ will increase $P_r^{inf}$ [see Eq. (10)] until the maximum $P_r^{inf}$ is reached [Eq. (12)]. Further increase in $G_t$ will lead to $d_{UE} > z_R$ and to gradual reduction of $P_r^{inf}$ [see Eq. (11)]. The necessary AP gain to reach the maximum $P_r^{inf}$ can be derived from Eq. (13) with the aid of Eq. (9) as:

$$G_t^{inf} = \frac{4k_0 \cos \theta_{UE} d_{AP}^2}{P_t}$$

(14)

To demonstrate the above, next, a D-band in-door scenario is considered. The operating frequency is 150 GHz and the AP is placed at distance $d_{AP} = 1$ m from the RIS with $\theta_{AP} = 0^\circ$, while the UE is placed at $d_{UE} = 2$ m with $\theta_{UE} = 20^\circ$ [see Fig. 1]. The AP, RIS and UE are all on the plane $y = 0$, i.e., $\phi_{AP} = 180^\circ$ and $\phi_{UE} = 0^\circ$. The AP transmits a beam of tunable gain, $G_t$, and constant power $P_t = 1$ W, while the UE gain, $G_t$, is 20 dB. For a theoretically infinite RIS, the power received by the UE is found using Eq. (8) with the aid of Eq. (9) and is shown in Fig. (3a), as a function of the AP gain, $G_t$. The equivalent numerical calculation using the discrete model of Eq. (2) is also shown, verifying the theoretically expected power levels. To account for the theoretically infinite RIS in the numerical model, a panel with 1200 x 1200 elements of size $dx = dy = \lambda/5$ has been considered. For the AP-RIS distance and AP gain levels used in this example, this size is adequate for the RIS to essentially capture the entire AP beam and operate as an effectively infinite panel. In the same Figure, Eq. (10) and Eq. (11) are shown as dashed lines, marking the asymptotic behavior of the communication link in the limit of relatively low and high AP gain. Essentially, the asymptotes serve as an upper threshold for the received power when the link resides in the infinite RIS regime. The vertical dashed line marks the AP gain that provides the maximum power at the UE and is extended to Fig. (3b), to emphasize that this is in agreement with $G_t^{inf}$ as predicted by Eq. (14), which results from the crossing of $z_R \cos \theta_{UE}$ with $d_{UE}$ [graphic solution of Eq. (13)] and marks the transition from the near- to the far-field of the beam.

In Fig. (4) further examples for variable AP and UE positions are presented. In Fig. (4a) the AP is at the position previously examined in Fig. (3) and the UE position is scanned along the direction $\theta_{UE} = 20^\circ$. For high AP gain ($d_{UE} \gg z_R$) the pathloss increases with increasing $d_{UE}$ and the received power $P_r^{inf}$ decreases, as intuitively expected and also predicted by Eq. (11). For low AP gain ($d_{UE} \ll z_R$), however, the received power is independent of the UE position, as also captured by Eq. (10); this is a direct consequence of the fact that, because the beam propagation is deeply in the near-field, the peak power of the secondary beam is practically constant and the pathloss is therefore similar for all examined cases. Furthermore, with increasing
distance, \(d_{\text{UE}}\), the maximum received power drops and moves to lower AP gain, as predicted by Eqs. (12),(14). Physically, this means larger AP footprint, i.e. weaker beam spreading and higher power density at the UE, thus compensating for the increased pathloss at the UE that is placed at larger distances from the RIS. In Fig. (4b) the UE is at the position previously examined in Fig. (3) and the AP position is scanned along the direction \(\theta_{\text{AP}} = 0^\circ\). In this case, the maximum received power is independent of the position of the AP [see Eq. (12)] and undergoes a shift towards higher AP gain with increasing \(d_{\text{AP}}\) [see Eq. (14)]. The reason is that, while an increasing AP-RIS distance leads to an increasing AP footprint and lower power density, with higher AP gain, the same power levels can be restored at a fixed UE position, as demonstrated here. In essence, in the infinite RIS regime, there is always a combination of AP-RIS distance and AP gain that can lead to practically the same power at the UE. This does not hold always, as we will show next, when the finite RIS size comes into play.

V. IMPACT OF FINITE RIS SIZE: FROM PENCIL BEAMS TO WIDE BEAMS AND FULL RIS COVERAGE

Infinitely large RIS does not necessarily imply an actually infinite surface. In practice, it suffices to have a finite-size RIS that is large enough, so that the incident beam is captured entirely by the RIS surface and further increase in the RIS size does not impose any change on the propagation properties of the re-directed beam. Therefore, a direct way to quantify the RIS size is via the percentage of the incident power that is collected by the RIS, as shown in Fig. (5). Clearly, by increasing either the RIS size with fixed AP gain [panel (a)] or the AP gain with fixed RIS size [panel (b)], the power received by the RIS increases until the entire beam is captured. At this limit, the total power captured by the RIS reaches 100% of the incident power and the communication link enters the regime of infinitely large RIS.

To demonstrate the effect of the finite RIS size, the scenario of Fig. (3) is re-examined in Fig. (6) under variable RIS size. Fig. (6a) shows the received power versus AP gain, \(G_t\), for a RIS-aided link residing in the infinite RIS regime, with a RIS of \(M = N = 1200\) elements. (a) Variable RIS-UE distance, \(d_{\text{UE}}\), along the direction \(\theta_{\text{UE}} = 20^\circ\) with fixed AP position \((d_{\text{AP}} = 1 \text{ m}, \theta_{\text{AP}} = 0^\circ)\). (b) Variable RIS-AP distance, \(d_{\text{AP}}\), along the direction \(\theta_{\text{AP}} = 0^\circ\) with fixed UE position \((d_{\text{UE}} = 2 \text{ m}, \theta_{\text{UE}} = 20^\circ)\). The AP, RIS and UE lie on the same plane, the transmitted power is \(P_t = 1 \text{ W}\) and the UE gain is \(G_{\text{UE}} = 20 \text{ dB}\).
as can be identified in Fig. (6c). As a result, the numerically calculated power shown in Fig. (6a) agrees with the theoretically expected for the infinite RIS, and the maximum received power marked with the cyan dashed line in Fig. (6b) is located at the position predicted by Eqs. (12), (14). With decreasing RIS size the system enters the finite RIS regime as can be identified in Fig. (6c), with implications on the power received by the UE. For low AP gain, the received power decreases with decreasing RIS size [Fig. (6a)], as the amount of $P_t$ captured by the RIS is reduced. For high AP gain, on the other hand, the received power is independent of the RIS size as the footprint of the AP beam on the RIS is always smaller than the RIS size (for the examples considered here) and, therefore, the propagation properties of the secondary beam are not affected by the RIS size. Furthermore, with decreasing RIS size, higher AP gain is required to reach the maximum $P_r$. This is a direct consequence of the fact that the finite RIS size truncates the incident power and, to compensate for the lost power, the AP footprint must become smaller, so that the entire incident power can be captured by the RIS. Interestingly, the AP gain that restores the power lost due to the finite RIS size is the gain that transfers the system to the infinite RIS regime, i.e. where the power received by the RIS approaches 100% of $P_t$. Further increase in the AP gain will result in no further power compensation, but to a smaller AP footprint and, therefore, stronger beam spreading and lower power density at the UE.

Last, the example of a UE with variable position - previously studied in Fig. (4a) for an infinitely large RIS- is now examined in Fig. (7) for a small RIS consisting of 80 x 80 elements. Here, in constrast to the infinitely large RIS case, the received power drops with increasing AP-RIS distance, regardless of the AP gain and the maximum $P_r$ [panel (a)]. In accordance with the behavior observed in Fig. (6), this is a direct consequence of the fact that, in all cases examined, the theoretically expected AP gain that marks the maximum $P_r$ [panel (b)] is located at lower AP gain levels than the gain that marks the transition from the infinite to the finite RIS regime [panel (c)]. In other words, the power loss from the AP footprint on the RIS due to the finite (relatively small) RIS size dominates, leading to this more intuitive and typical performance.

VI. DISCUSSION

In a communication environment, where the AP beam properties and the RIS operation are tuned according to the UE requirements, there is a need for simple algorithmic
FIGURE 7. RIS-aided link residing in the finite RIS regime, with a RIS of $M = N = 80$ elements. (a) Received power, $P_r$, (b) graphic solution of Eq. (13), and (c) power received by the RIS, vs AP gain, $G_t$, for variable UE positions along the direction $\theta_{UE} = 20^\circ$. The transmitted power is $P_t = 1$ W, the UE gain is $G_{UE} = 20$ dB and the AP, RIS and UE lie on the same plane, with the AP position fixed ($d_{AP} = 1$ m, $\theta_{AP} = 0^\circ$).

description of the possible decisions that have to be made in order to maximize certain key performance indicators (KPIs). With the analysis presented in this work, the dimensionality of the multi-parameter communication space reduces to a few quantities, such as the size of the RIS and the gain of the AP. Then, depending of the KPIs of the particular link, decisions can be made based on these quantities.

For example, in a link where the power received by the UE must be maximized, the entire problem can be expressed in terms of two parameters only, namely $G_t^{inf}$ [given by Eq. (14)] and the gain that marks the transition from the finite to the infinite RIS regime, which we denote here as $G_t^{M \times N}$ (the superscript emphasizes the dependence on the RIS size). From the examples of Fig. (6) and Fig. (7) it becomes apparent that if $G_t^{M \times N} > G_t^{inf}$, then $G_t = G_t^{M \times N}$, while if $G_t^{M \times N} < G_t^{inf}$, then $G_t = G_t^{inf}$. A simple algorithm is shown in the flow chart of Fig. (8a), where $G_t^{M \times N}$ is calculated only once in the beginning and $G_t^{inf}$ is evaluated in real-time, as the UE moves. Another example is when the RIS can be separated in independent blocks, to serve multiple users. In this case $G_t^{M \times N}$ can be calculated a priori for the entire RIS and each block individually, and be compared with $G_t^{inf}$ in a similar manner, as shown in the example of Fig. (8a).

As an implementation of the algorithm shown in Fig. (8a), a scenario of a moving UE is demonstrated in Fig. (8b). As the UE moves, the algorithm decides on the AP antenna gain that optimizes the received power at the UE, leading to an overall stronger reception as compared to the power received under constant AP antenna gain. In this example two cases of constant gain are shown (45 and 50 dB), and the optimized gain for each UE position is shown in the colorbar. The size of the RIS is $500 \times 500$ elements, the AP is located at $\theta_{AP} = 0^\circ$ and $d_{AP} = 2$ m, while $\theta_{UE} = 20^\circ$ and $d_{UE}$ ranges from 0.1 to 3 m.
VII. CONCLUSION

The successful adoption of RISs in future wireless systems is highly dependent on crucial choices for the optimal RIS placement and the efficient transceiver design. In turn, these choices are closely related to certain design parameters that need to be taken into account when designing a RIS-aided link. In this work, the interplay between the RIS size and the properties of the transmitter beam was investigated in terms of the RIS beamforming efficiency. The study revealed that, even for infinitely large RIS, the increase of the transmitter gain does not always guarantee increased power at the receiver, rather there is a maximum, which is further affected by the finite RIS size. The performance of the RIS-aided link was evaluated as a function of the AP gain, revealing two distinct operation regimes, with respect to whether the size of the AP beam footprint is larger or smaller than the RIS area, termed here as finite RIS regime and infinite RIS regime, respectively. In the infinite RIS regime an analytical model was presented, predicting the maximum received power and providing insight on the underlying mechanism. The theoretically expected performance was also verified by numerical examples of typical in-door scenarios, elucidating the role of variable RIS size, AP-RIS distance and RIS-UE distance.

APPENDIX A

The footprint of the AP beam on the RIS given by Eq. (6), can be written as:

\[ S_{\text{RIS}}^{\text{inc}} = \frac{2P_t \gamma}{\pi w_{\text{RIS}}^2} U_1, \]  

(15)

with

\[ U_1 = \exp\left(-\frac{x^2 + y^2}{w_{\text{RIS}}^2}\right) \]  

(16)

expressing the AP radiation pattern at distance \( d \sim d_{\text{AP}} \), where the RIS is located. The power density of Eq. (15) can be alternatively expressed in terms of \( G_t \), the AP gain, as:

\[ S_{\text{RIS}}^{\text{inc}} = G_t U_1 \frac{P_t}{4\pi d^2}, \]  

(17)

where \( d \) is the radius of the sphere centered at the AP, and for pencil beams \( d \sim d_{\text{AP}} \). From Eq. (15) and Eq. (17) we find:

\[ G_t = 8 \left( \frac{d_{\text{AP}}}{w_{\text{RIS}}} \right)^2. \]  

(18)

The result of Eq. (18) is used in Eq. (9) of the main text, to express \( z_R \) in terms of \( G_t \). Additionally, the radiation pattern given by Eq. (16) can be simplified using the result of Eq. (18) and the approximation \( \sqrt{x^2 + y^2} \sim d_{\text{AP}} \sin \theta \), leading to:

\[ U_1 = \exp\left(-\frac{d_{\text{AP}}^2 \sin^2 \theta}{w_{\text{RIS}}^2}\right) = \exp\left(-\frac{G_t}{4} \sin^2 \theta\right), \]  

(19)

where \( \theta \) corresponds to the angle \( \Theta_{m,n} \) shown in Fig.(1) and explained in the relevant text.
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