The economic success of any enterprise is determined by the balance between its production and sales. One of the tasks of an enterprise is to optimally expand its market niche and bring the production capacity in line with the current demand for its products [1; 2]. In theoretical studies, an important role is played by the development of models which connect all the basic parameters of a logistics system in a single system of equations. Such models should be the basis for co-optimization of an enterprise’s production capacity and retail sales.

Many aspects of planning the current activities of enterprises are investigated in [3; 4]. However, the mentioned works do not sufficiently cover the quantitative relationship between the parameters of an enterprise’s logistics system and the current characteristics of the consumer market: potential demand for products, purchasing rate of products. This shortcoming complicates studying the modern theory of the joint planning of an enterprise’s production capacity and retail sales. Work [4] proposes a model which fundamentally meets the formulated requirements. This model makes it possible to take into account the detailed characteristics of the market. In [5], the further development and generalization of this model is carried out. To date, there are no effective
methodologies to plan production capacity of an enterprise in real-time with consideration for the enterprise’s logistics and, in particular, its retail network (RN).

The aim of the article is to develop an economic and mathematical model of an enterprise’s production activity, with consideration for its retail sales; application of the developed model for the co-optimization of the production capacity and retail sales of consumer goods, taking into account the peculiarities of their realization.

The work studies an enterprise whose logistics corresponds to the scheme presented in Fig. 1.

![Diagram of the enterprise's logistics](image)

Working in a competitive market requires the director of an enterprise to pay attention to increasing the sales of the enterprise’s products or at least maintaining the sales at some acceptable level. One of the main ways to achieve this goal is to align the enterprise’s retail sales with its production capacity.

Building the model begins with the description of the influence of advertising on the potential demand \( Q \). Time will be treated as a discrete variable \( i \) \((i = 0, 1, 2, ..., T)\). We will study the project with the \( T \) length (the planning horizon). Let us assume that the influence of advertising on the potential current demand \( Q_{i+1} \) in the \((i + 1)\text{th}\) period is described by the formula:

\[
Q_{i+1} = Qr(Z) \cdot \left(1 - \exp \left(-\frac{i}{t}\right)\right) + Qn,
\]

where \( Qn \) represents the value of potential demand in the absence of advertising; \( Qr(Z) = Qm \cdot \left(1 - \exp \left(-\frac{Z}{tz}\right)\right) \) is the maximum value of additional potential demand due to advertising; \( t \) and \( tz \) are the parameters of lagging.

Formula (1) means that the impact of the \( Qr \) advertising campaign on the potential demand is described by the first-order lag model [7; 8]:

Let us formulate a system of equations which describe the logistics system of the enterprise depicted in Fig. 1. We will assume that the company is fully provided with working capital.

1. The change in the demand \( Q_i \) for products on the market is an input impact for the enterprise whose task is to bring its output in line with the demand:

\[
r_{i+1} = rR_i \cdot (Q_i - V_i),
\]

2. The inventory level at the retail is determined by the recurrence formula:

\[
R_{i+1} = R_i + Td \cdot (so_i - r_i),
\]

where \( so \) is the rate of goods flow which goes into the warehouse to retailers; \( Td \) is the period of discretization of the model, the time interval between decisions.

3. The level \( R_i \) should be within the limits \( 0 \leq R_i \leq R_m \), where \( R_m \) is the maximum possible inventory level at the retail. The requirement is described by the following formula for the rate of delivery from the warehouse to retailers:

\[
so_{i+1} = \min \left[ r_i \cdot \left(1 + \frac{R_m - R_i}{R_m}\right), \frac{R_m - R_i}{Td}, S_i/Td \right],
\]

where \( S_i \) represents the inventory level at the wholesale warehouse.

Publication [5] substantiates the need for averaging when performing the calculations with the proposed model:

\[
\text{ps} = \left(\text{so}_{i=1-ps}\right),
\]

where \( ps \) is the averaging time interval.

4. The production rate \( y_i \) is determined by the following formulas:

\[
y_{i+1} = \left(y_i + \frac{ym - y_i}{ty}\right) \cdot A(S_i),
\]

\[
A(S_i) = \begin{cases} 1, & \text{if } S_i < S_m - 2, \\ 0.5, & \text{otherwise}, \end{cases}
\]

where \( y_i \) is the production capacity in the \( i\text{th}\) period; \( ym \) is the planned value of production capacity; \( S_m \) is the maximum inventory level at the wholesale warehouse. These formulas allow to avoid the overflow of the wholesale warehouse.

5. The inventory level at the wholesale warehouse \( S_i \) is calculated using the following formula:

\[
S_{i+1} = S_i + Td \cdot (y_i - so_i),
\]

where \( y_i \) is the rate of goods flow which goes into the wholesale warehouse from the production.
6. To determine the net profit of the enterprise, the following formulas are applied:

\[
M_i = (1 - kp) \cdot [(1 - kad) \cdot p \cdot r_i - p \cdot c \cdot yp_i - \\
- k_2 \cdot S_i - z - R_m - z_1 \cdot (R_m)^2 - gz \cdot Z] - B(y_i),
\]

(8)

\[
B(y_i) = \begin{cases} 
0, & \text{if } i < 1, \\
qy|y_i - y_{i-1}|, & \text{otherwise}, 
\end{cases}
\]

(9)

where \(c\) is the share of the prime cost in the price for products; \(p\) is the price for a production unit; \(zR, zS\) are the costs for the storage of a production unit during one period at retail and wholesale warehouse, respectively; \(kp\) is the income tax rate; \(kad\) is the value-added tax rate; \(qy\) is the cost of ‘including,’ ‘excluding’ a unit of production capacity.

The formula for calculating the net income takes into account the quadratic dependence on the maximum volume of the retail sales, which is under the contract as signed to goods of a certain manufacturer. The dependence can occur for a number of reasons. For example, with an increase in the number of retail outlets, the transportation distance increases, etc.

Before carrying out the optimization, we need to check the adequacy of the model (1) – (9). For this purpose, we will perform calculations for the model (1) – (9) with the following values of its parameters:

\[
R_m = 80, qy = 100, Q = 1200, T = 1, n = 0.0001, \\
k_1 = 0.33, k_2 = 0.01, So = 100, Sm = 200, \\
Ro = 50, n_1 = 0.1, kp = 0.25, kad = 0.06, c = 0.6, \\
p = 10, z = 0.01, Se = 180, qy = 50.
\]

(10)

The enterprise decides to set the production capacity at the level of \(ym = 4.3\), with the maximum volume of retail sales of \(R_m = 80\). The results of the calculations for the model (1) – (9) with the values of the parameters (10) are shown in Fig. 2 – Fig. 5.

Fig. 2 demonstrates the behavior of the basic rates of the logistics system.

![Fig. 2. Behavior of basic rates of the logistics system](image)

It can be seen that in the initial period of work all rates, except for the rate of production, experience significant fluctuations, which negatively affects economic indicators (it requires labor supply, etc.). Fig. 3 shows the behavior of the inventory level at the retail. The figure shows that the inventory level at the retail \(R_i\) stabilizes (as the calculations show) at the level of 46 units. That is much less than the marginal (contractual) capacity (80 units).

![Fig. 3. Behavior of the inventory level at the retail \(R_i\)](image)

This means that in this case the RN is used ineffectively.

Fig. 4 shows the behavior of the inventory level at the wholesale warehouse \(S_i\). It is obvious that the warehouse is also used ineffectively.

![Fig. 4. The behavior of the inventory level at the wholesale warehouse \(S_i\)](image)

Fig. 5 demonstrates the behavior of the operating income of the enterprise.

![Fig. 5. Behavior of the operating income of the enterprise \(M_i\)](image)

In this case, the enterprise’s comprehensive income for the year will amount to

\[
\sum_{i=1}^{365} M_i = 436.5.
\]

(11)

Now the enterprise’s administration requires the managers to solve the following optimization problem:

\[
F_i(Rm) = \sum_{i=1}^{365} M_i \rightarrow \max,
\]

(12)

while keeping the production capacity at the same level of \(ym = 4.3\).
In other words, to determine the optimal value of the retail sales, which will provide for obtaining the maximum profit (with \( y_m = 4.3 \)).

The optimization task (12) should be solved at the limitations of the parameters set by the system of equations of the model (1) – (9), with the values of the parameters (10). Solving the task (12) results in: \( Rm_{opt} = 57.3 \), with the obtained profit \( F_1(Rm_{opt}) = 9558.2 \), which is twice as large as the value (11).

Fig. 6 illustrates the solution of the optimization task.

For an optimal solution, the time dependence of the basic characteristics of the LS will be similar to that in Fig. 7 – Fig. 10.

Fig. 7 demonstrates that in this case the fluctuations of the basic rates in the initial period are significantly smaller.

Fig. 8 illustrates that now the RN is used more effectively.

Fig. 9 allows to conclude that the warehouse is used ineffectively.

However, if, under the terms of the contract with the wholesale warehouse, the payment is made only based on the actual quantity of products \( S_i \) in the warehouse (8), it does not virtually affects the profit.

Fig. 11 demonstrate the behavior of the inventory level in the hands of the consumer for an optimal solution.

The optimization task can be solved by the enterprise's management in a more general setting: to find the maximum value of the target function (12) with the following variable parameters: \( y_m \) is the planned value of production capacity; \( Rm \) – the maximum volume of the retail sales.

Now, it will be an optimization task with two variable parameters:

\[
F_1(Rm, y_m) = \sum_{i=1}^{365} M_i \rightarrow \max.
\]
Solving the optimization task results in:

$$
\begin{align*}
R_{m\ _{\text{opt}}} &= 105.5 \\
y_{m\ _{\text{opt}}} &= 8.46
\end{align*}
$$

(13)

With such optimal values of the variable parameters, the value of the target function (the income obtained for the entire planning period $T$ – a year) will be

$$
F_1 = 12178.9
$$

which significantly surpasses the previous results. It should be noted that this result was obtained with the optimal values of the planned capacity and retail sales, which results in all links of the logistics system working with maximum efficiency (in fact, at the limit of their capabilities). The behavior of the basic rates of the logistics system for this case is shown in Fig. 12.

![Fig. 12. Behavior of the basic rates of the LS for optimal solution (13)](image)

**CONCLUSIONS**

Thus, in the article, a model of the enterprise logistics system is developed. Unlike the existing models, the proposed one allows to perform co-optimization of an enterprise’s production capacity and retail sales, i.e., those links that directly determine the production and sale of goods and determine the operation of all other links of the logistics system using a closed system of equations.

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