Kirson-Babu-Brown core polarization diagrams and low-momentum shell model effective interactions

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Abstract. The Kirson-Babu-Brown (KBB) induced interaction approach is applied to the derivation of the sd-shell nucleon effective interactions. We start from a low-momentum NN interaction $V_{\text{low-}}$ obtained by integrating out the high-momentum components of modern realistic NN potentials beyond a decimation momentum $\Lambda$. Then a large class of core polarization diagrams are summed up to all orders by way of the KBB self-consistent equations. It is found that the solution of these equations is simplified by the use of $V_{\text{low-}}$, which is energy independent, and by treating them in terms of the Green’s functions in the particle-particle and the particle-hole channels. The effective interactions calculated with the all-order KBB core polarizations and those with the polarization in second-order perturbation theory are found to be rather similar to each other, typical difference between them being less than 10%.

1. Introduction
First let me (TTSK) thank the organizers, particularly Prof. Otsuka, for inviting me to this conference. It is my great pleasure to come to this gathering honoring Prof. Akito Arima. I have known Prof. Arima for a long time, since about forty years ago when we were both at Princeton and Stony Brook. Today I would like to report on a recent work of Holt, Holt, Kuo, Brown and Bogner [1]; they have carried out an all order summation of the core polarization diagrams for the sd-shell effective interaction using the induced interaction approach of Kirson [2] and Babu and Brown [3].

A very nice feature of the nuclear shell model is its simplification of nuclear structure. To illustrate, this model allows us to reduce the complicated $^{18}\text{O}$ many-body problem to an effective two-body problem with two “dressed” valence particles outside a closed $^{16}\text{O}$ core. These dressed particles are of course different from the bare particles. The core is not closed in reality and consequently the renormalization effects due to core excitations have to be included. The bare nucleon-nucleon (NN) interaction is short ranged. But the effective interaction for the dressed valence nucleons is different; it has in addition important long-range components mediated by core excitations (phonon exchanges) [4]. Renormalizations of this type are generally referred to as the core polarization (CP) effects.

The importance of CP effects to nuclear structure was already pointed out about 50 years ago by the pioneering work of Arima and Horie [5]. Later, Bertsch [6] and Kuo and Brown [7] calculated the CP effects for the shell-model effective interactions ($V_{\text{eff}}$). As is well known, they found that the CP effects were very important and their inclusion is very desirable in improving
the agreement between theory and experiment. However, it should be pointed out that in these works the CP effects were calculated to second-order (in the G-matrix) in perturbation theory, namely the familiar second-order CP diagram of Fig. 1(a).

There are many many more higher-order CP diagrams. How important are the CP diagrams beyond second order? How to calculate them? These have been long standing questions. In fact the effective interactions including only the second order CP diagrams have been very successful [8] for a wide range of nuclei. To understand this success, one has to study the effect of the higher order CP diagrams.

We use the folded-diagram method [9] to derive the shell-model effective interaction \( V_{\text{eff}} \). In this method, the full space nuclear many-body problem \( H \Psi_n = E_n \Psi_n \) is reduced to a model space problem \( H_{\text{eff}} \chi_m = E_m \chi_m \), with \( H = H_0 + V \) and \( H_{\text{eff}} = H_0 + V_{\text{eff}} \). Here \( V \) denotes the bare NN interaction. The effective interaction \( V_{\text{eff}} \) is given by the folded-diagram expansion

\[
V_{\text{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \cdots, \tag{1}
\]

where \( \hat{Q} \) is the irreducible vertex function. (\( \hat{Q}' \) is the same as \( \hat{Q} \) except with its first order diagrams removed.) Once \( \hat{Q} \) is obtained, the above folded diagram series can be summed up to all orders using the Lee-Suzuki iteration method [10]. Thus the central task in the folded-diagram method is the calculation of the \( \hat{Q} \)-box vertex function.

How to calculate the \( \hat{Q} \)-box? In the past, it was mostly calculated from some low-order diagrams, among them being the second-order CP diagram of Fig. 1(a). There has been much interest in studying higher-order CP diagrams [2, 3, 11, 12, 13, 14, 15]. Third-order core polarization diagrams, including folded diagrams, were studied in detail by Barrett and Kirson [11] for the \( sd \)-shell effective interactions. Horth-Jensen et al. [10] have carried out extensive investigations of the third order CP diagrams for the tin region. A main result of these studies is that the effect of the third-order diagrams is generally comparable to that of the second order; the former cannot be ignored in comparison with the latter. High-order CP diagrams are difficult to calculate, largely because the number of such CP diagrams grows rapidly as one goes to higher orders in perturbation theory. The number of diagrams at third order is already quite large, though still manageable. But at fourth order there are just ”too many” diagrams and a complete fourth order calculation has never been carried out. It was soon realized that an order by order calculation of CP diagrams beyond third order is not practicable. To fully assess the effects of core polarization to high order, a non-perturbative method is called for.

In general higher-order CP diagrams are difficult to calculate. But fortunately an elegant and rigourous method for summing up CP diagrams to all orders has been provided by the Kirson-Babu-Brown (KBB) induced interaction approach [2, 3]. It is a nonperturbative approach. In the past, one generally used the G-matrix interaction for microscopic CP calculations. Since such interaction depends on both starting energy and the Pauli exclusion operator, it adds complications to the KBB calculation. In the present work, we shall employ the low-momentum interaction \( V_{\text{low-k}} \) [16, 17, 18, 19, 20, 21, 22] to the derivation of the \( sd \)-shell effective interaction using the KBB approach. \( V_{\text{low-k}} \) does not have the above difficulties, and thus considerably simplifies the calculation. In the following, we first briefly how we apply the KBB method to the all-order summation for the CP diagrams in section 2. Some initial results will be presented and discussed in section 3.

2. Kirson-Babu-Brown vertex function

A basic idea of the KBB approach is that the irreducible vertex function should be calculated self-consistently, in the sense that the core polarization term contained in the vertex function be generated self-consistently from the same vertex function.
To illustrate this approach, let us first consider the particle-hole (ph) vertex function $f$. As displayed in Fig. 2, $f$ is given by the sum of $V$, the driving term, and core polarization diagrams which themselves are dependent on $f$. $f$ obeys the self-consistent equation

$$ f = V + \Sigma g_{ph} \Sigma + \Sigma g_{ph} f g_{ph} \Sigma + \Sigma g_{ph} f g_{ph} f g_{ph} \Sigma + \cdots. \tag{2} $$

In the above $g_{ph}$ is the free ph Green’s function. The $\Sigma$’s denote the vertex functions for particle-core and hole-core couplings. For simplicity, the bra and ket indices in the above equation have been supressed. For example, in Eq.(2) the $f$ on the LHS represents $\langle 12^{-1} | f | 34^{-1} \rangle$, while the second and third $\Sigma$’s on the RHS represent $\langle 1 ph^{-1} | \Sigma | 3 \rangle$ and $\langle 2^{-1} | \Sigma | p/h^{-1} 1^{-1} \rangle$, respectively.

The vertex function $f$ as defined above contains CP diagrams to all orders. To see this, let us consider $\Sigma = V$. In this case, Eq. (2) becomes

$$ f = V + V g_{ph} V + V g_{ph} f g_{ph} V + V g_{ph} f g_{ph} f g_{ph} V + \cdots. \tag{3} $$

Since $f$ appears on both sides of this equation, it is clear by iteration that $f$ contains core polarization diagrams to all orders, including those with “bubbles inside bubbles” as displayed in Fig. 1(b).

In addition to the particle-hole vertex $f$, our calculation requires also the particle-particle (pp) vertex function $\Gamma$. Like $f$, $\Gamma$ is given by a driving term plus core polarization terms, where the latter now depend on both $\Gamma$ and $f$. Furthermore, the diagramatic representation of $\Gamma$ is identical to Fig. 2 except with the hole lines 2 and 4 replaced by the corresponding particle lines. This gives the self-consistent equation for $\Gamma$

$$ \Gamma = V + \Sigma g_{ph} \Sigma + \Sigma g_{ph} f g_{ph} \Sigma + \Sigma g_{ph} f g_{ph} f g_{ph} \Sigma + \cdots. \tag{4} $$
Figure 3. Valence-core coupling vertices $\Sigma_{pp}$ and $\Sigma_{ph}$.

We note that the external lines of the $\Sigma$ vertices in $\Gamma$ are different from those shown in Fig. 2. The third $\Sigma$ vertex, for example, now represents $\langle 2|\Sigma|ph^{-1}\rangle$. These different $\Sigma$ vertices can be related to each other, however, via appropriate particle-hole transformations.

The final step in this procedure is to couple the vertex functions $f$ and $\Gamma$ via the coupling vertex $\Sigma$. In the present work we choose

$$\Sigma = V + \Sigma_{pp} + \Sigma_{ph},$$

$$\Sigma_{pp} = Vg_{pp}V + Vg_{pp}f_{pp}V + Vg_{ph}f_{ph}f_{ph}V + \cdots,$$

$$\Sigma_{ph} = Vg_{pp}V + Vg_{pp}\Gamma g_{pp}V + Vg_{pp}\Gamma g_{pp}\Gamma g_{pp}V + \cdots,$$

where $g_{pp}$ is the free pp Green’s function. Clearly, $\Sigma$ depends on both $f$ and $\Gamma$.

The self-consistent vertex functions $f$ and $\Gamma$ are determined from Eqs. (2), (4) and (5). They are very similar to the equations used by Kirson [2], except a difference about the coupling vertex $\Sigma$. Our $\Sigma$ includes both $\Sigma_{ph}$ and $\Sigma_{pp}$, while in Kirson’s calculations only $\Sigma_{ph}$ is included [2, 23]. As illustrated in Fig. 3, $\Sigma_{pp}$ and $\Sigma_{ph}$ are both coupling vertices between valence lines and core excitations, their difference being that the former has particle-particle ladder interactions while the latter has particle-hole ones. To see the role of these vertices, let us consider the CP diagram of Fig. 1(c). Here the lower particle-core vertex belongs to $\Sigma_{pp}$, while the upper one belongs to $\Sigma_{ph}$. If we set $\Sigma_{pp}=0$, CP diagrams of this type would be suppressed. In our present work, we include this type of CP diagrams to all orders and hence it is necessary to include $\Sigma_{pp}$. Our equations are equivalent to those of Kirson when $\Sigma_{pp}$ is set to zero, and in this case $\Gamma$ does not enter the calculation of $f$.

We now discuss the solution of Eqs. (2) and (4-5) for the vertex functions $f$ and $\Gamma$. As described in ref.[1], these equations can be solved by iteration after regrouping them in terms of the true particle-particle and particle-hole Green’s functions. The solution is also facilitated by the use of the low-momentum NN interaction $V_{low-k}$ which is energy independent.

3. Results and discussion

We have just discussed how to sum up the CP diagrams using the KBB induced interaction approach. To actually calculate these diagrams, we must have a NN interaction. There are a number of realistic models for $V_{NN}$, such as those of [24, 25, 26, 27]. Although they all reproduce the empirical two-nucleon data very well, these potentials are in fact quite different from each other[20]. Thus there is the uncertainty about which of these models should one use. These models all contain strong short range repulsions, and to use them in nuclear structure calculations one usually first calculate the G-matrix, which is dependent on the starting energy as well as the Pauli exclusion operator. Since the vertex functions $f$ and $\Gamma$ both depend on the starting energy, there would be off-energy-shell effects present in many CP diagrams if the G-matrix interaction were chosen. This would make the calculation very complicated.

For the present calculation we have chosen to use the low-momentum NN interaction $V_{low-k}$ [16, 17, 18, 19, 20, 21, 22]. A central theme of the renormalization group and effective field theory (RG-EFT) approach [28, 29] is that physics in the infrared region is insensitive to the
details of the short-distance dynamics. One can therefore have infinitely many theories that differ substantially at small distances, but still give the same low-energy physics if they possess the same symmetries and the “correct” long-wavelength structure [28, 29]. The fact that the various meson models for $V_{NN}$ share the same one pion tail, but differ significantly in how they treat the shorter distance pieces illustrates this explicitly as they give the same phase shifts and deuteron binding energy. In RG-EFT language, the short-distance (high momentum) pieces of $V_{NN}$ are like irrelevant operators since their detailed form cannot be resolved from low-energy data.

Motivated by these observations, we derive a low-momentum $NN$ potential $V_{\text{low}−k}$ by integrating out the high-momentum components of $V_{NN}$ using the following T-matrix equivalence approach. Since its details have been given elsewhere [16, 17, 18, 19, 20, 21, 22], here we only provide a brief description. We define $V_{\text{low}−k}$ through the T-matrix equivalence equations

$$T(k', k, k^2) = V_{NN}(k', k) + P \int_{0}^{\Lambda} q^2 dq V_{NN}(k', q) \frac{1}{k^2 - q^2} T(q, k, k^2), \quad (6)$$

$$T_{\text{low}−k}(p', p, p^2) = V_{\text{low}−k}(p', p) + P \int_{0}^{\Lambda} q^2 dq V_{\text{low}−k}(p', q) \frac{1}{p^2 - q^2} T_{\text{low}−k}(q, p, p^2), \quad (7)$$

$$T(p', p, p^2) = T_{\text{low}−k}(p', p, p^2); \quad (p', p) \leq \Lambda, \quad (8)$$

where $V_{NN}$ represents some realistic NN potential and $\Lambda$ is the decimation momentum beyond which the high-momentum components of $V_{NN}$ are integrated out. $V_{\text{low}−k}$ preserves both the deuteron binding energy and the low-energy scattering phase shifts of $V_{NN}$. Since empirical nucleon scattering phase shifts are available only up to the pion production threshold ($E_{\text{lab}} \sim 350 \text{ MeV}$), beyond this momentum the realistic NN potentials cannot be uniquely determined. Accordingly, we choose $\Lambda \approx 2.0 \text{ fm}^{-1}$. In fact for this $\Lambda$, the $V_{\text{low}−k}$ derived from various NN potentials [24, 25, 26, 27] are all nearly identical [20]. We note that $\Lambda$ is not a parameter; its appropriate value is around $2 \text{ fm}^{-1}$.

We have carried out some initial calculations and would like to present our results. Our aim is to derive the folded diagram effective interaction $V_{\text{eff}}$ with the $\hat{Q}$-box calculated with the

![Figure 4. $^{18}O$ spectra calculated with the Argonne-V18 potential.](image-url)
KBB self-consistent equations. A model space of four oscillator shells (namely 0s, 0p, 0d1s and 1p0f) and oscillator constant \( h\omega = 14 \text{ MeV} \) are employed, which are the same as those used in the second-order core polarization calculations [16, 18].

We have found that the sd-shell effective interactions calculated with the all order CP diagrams are remarkably close to those given by the second-order ones, on average the former being about 10% weaker than the latter. As illustrated in Fig. 4, the energy spectra of \(^{18}\text{O}\) calculated with second- and all-order CP diagrams are quite similar. Note that for \(^{18}\text{F}\) the deviation is slightly larger [1].

In summary, we would like to mention the following: Core polarization is well known to be a physically very important process. In the past, most such calculations were carried out using second-order perturbation theory. We have demonstrated that core polarization diagrams can be summed up to all orders in a rather convenient way, using the Kirson-Babu-Brown induced interaction approach together with the energy independent low-momentum interaction \( V_{\text{low}-k} \). This energy independence has significantly simplified the solution of the KBB self-consistent equations. The effective interactions given by the all-order core polarizations are remarkably similar to those given by the second order ones. Core polarizations are also important for effective operators such as magnetic moments. It would be of much interest to extend our present study to effective operators.

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