A recent analysis of $B \to \pi K$ decays concludes that present data do not clearly indicate whether (i) the standard model (or $\Delta I = 0$ new physics) is sufficient, or (ii) $\Delta I = 1$ new physics is needed. We show that these two possibilities can be distinguished by whether a sum rule relating the CP asymmetries of the four $B \to \pi K$ decays is valid. If case (i) is favored, the sum rule holds, and one predicts $A_{CP}(\pi^0 K^0) = -0.15$, while in case (ii) fits to new physics involving large values of a color-suppressed tree amplitude entail $A_{CP}(\pi^0 K^0) = -0.03$. The current experimental average $A_{CP}(\pi^0 K^0) = -0.01 \pm 0.10$ must be measured a factor of at least three times more precisely in order to distinguish between the two cases.

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Several suggestions have been made in the past few years that data for $B \to \pi K$ decays cannot be simply explained within the Cabibbo-Kobayashi-Maskawa (CKM) framework. In most cases these claims suffered from uncertainties in QCD calculations of hadronic matrix elements using a heavy-quark expansion. The purpose of this note is to outline some of these claims, proposing a way for using improved data in order to make a more robust statement about new physics.

The four $B \to \pi K$ decay amplitudes are related by isospin [1],

$$A(B^0 \to \pi^- K^+) - \sqrt{2}A(B^+ \to \pi^0 K^+) + \sqrt{2}A(B^0 \to \pi^0 K^0) - A(B^+ \to \pi^+ K^0) = 0 \ .$$

(1)

The four amplitudes may be written in terms of penguin ($P_{tc}$ and $P_{uc}$), tree ($T$), color-suppressed ($C$), annihilation ($A$), color-favored electroweak penguin ($P_{EW}$), and color-suppressed electroweak penguin ($P_{EW}^C$) contributions [2]:

$$-A(B^0 \to \pi^- K^+) = P_{tc} + P_{uc} + T + \frac{2}{3}P_{EW}^C \ ,$$

(2)

$$-\sqrt{2}A(B^+ \to \pi^0 K^+) = P_{tc} + P_{uc} + T + C + A + P_{EW} + \frac{2}{3}P_{EW}^C \ ,$$

(3)

$$\sqrt{2}A(B^0 \to \pi^0 K^0) = P_{tc} + P_{uc} - C - P_{EW} - \frac{1}{3}P_{EW}^C \ ,$$

(4)

$$A(B^+ \to \pi^+ K^0) = P_{tc} + P_{uc} + A - \frac{1}{3}P_{EW}^C \ .$$

(5)

The two amplitudes $P_{tc}$ and $P_{uc}$ behave like isoscalars ($\Delta I = 0$) while the remaining terms are mixtures of isoscalar and isovector ($\Delta I = 1$). The terms $P_{tc}$, $P_{EW}$ and $P_{EW}^C$ involve a CKM factor $V_{tb}^*V_{ts}$ with weak phase $\pi$, while $P_{uc}, T, C$ and $A$ contain $V_{ub}^*V_{us}$ with phase $\gamma$. Each of these amplitudes has its own unknown strong phase, but some strong phases can be related to each other approximately. Using flavor SU(3) symmetry, the amplitudes $P_{EW}$ and $P_{EW}^C$ are given to a good approximation in terms of $T$ and $C$ [3]:

$$P_{EW} = \frac{3c_9 + c_{10}}{4c_1 + c_2}R(T + C) + \frac{3c_9 - c_{10}}{4c_1 - c_2}R(T - C) \approx \frac{3c_9 + c_{10}}{2c_1 + c_2}RT \ ,$$

(6)

$$P_{EW}^C = \frac{3c_9 + c_{10}}{4c_1 + c_2}R(T + C) - \frac{3c_9 - c_{10}}{4c_1 - c_2}R(T - C) \approx \frac{3c_9 + c_{10}}{2c_1 + c_2}RC \ .$$

(7)

Here, $c_i$ are Wilson coefficients [4] obeying $(c_9 + c_{10})/(c_1 + c_2) \approx (c_9 - c_{10})/(c_1 - c_2)$, while $R \equiv |(V_{tb}^*V_{ts})/(V_{ub}^*V_{us})| = 48.9 \pm 1.6$ [5]. The proportionality coefficient relating $P_{EW}$ to $T$ and $P_{EW}^C$ to $C$ is numerically of order one and negative: $\delta_{EW} \equiv (3/2)[(c_9 + c_{10})/(c_1 + c_2)]R = -0.60 \pm 0.02$. SU(3) breaking introduces a theoretical error of about 10% in the magnitude of this coefficient and an error of 5° in its strong phase [6].

The decays $B \to \pi K$ provide nine measurements: four branching fractions $B$, four direct CP asymmetries $A_{CP}$, and the mixing-induced CP asymmetry $S_{CP}$ in $B^0 \to \pi^0 K^0$. Assuming values for the weak phases $\beta$ and $\gamma$ as determined in a global CKM fit [5], these observables can be expressed in terms of nine parameters: the magnitude of the five independent amplitudes, $P_{tc}$, $P_{uc}$, $T$, $C$, $A$, and their four
relative strong phases. Neglecting the amplitude $A$, which vanishes to leading order in $1/m_b$ and $\alpha_s$ \cite{7}, one may perform a best fit with two degrees of freedom. Such a fit was made about two years ago \cite{8} with data available in early 2007. The fit neglected $P_{uc}$ but kept $\gamma$ as a free parameter which was extracted successfully in agreement with its determination in the CKM fit. A good fit, corresponding to $\chi^2_{min}/d.o.f. = 1/3 \ (80\% \ C.L.)$, required $|C/T| = 1.6 \pm 0.3$. In contrast, values of $|C/T|$ calculated in QCD, using a heavy-quark expansion and various assumptions about $A_{QCD}/m_b$ corrections, do not exceed 0.6 \cite{9} \cite{10}. The larger value of $|C/T|$ obtained in the fit \cite{8} was thus considered a possible indication for New Physics (NP) \cite{11}. (Another good fit to the data, involving a sizable contribution from $P_{uc}$, gave a lower value $|C/T| = 0.8 \pm 0.1$. However, its value of $\gamma$ was in disagreement with the global CKM fit.)

Recently, an update of the fit to $B \to \pi K$ data was performed using the latest information as of early 2009 \cite{12}. The current data are summarized in Table 1 \cite{13}, compared with the data of two years ago in parentheses. The only significant change occurred in the central values of the two CP asymmetries $A_{CP}$ and $S_{CP}$ in $B^0 \to \pi^0 K^0$, which involve the largest experimental errors. The current experimental situation with respect to the need for NP has now become less clear. Although the best fit \cite{12}, including $P_{uc}$ as a parameter and using $\gamma$ as an input, is relatively poor [corresponding to $\chi^2_{min}/d.o.f. = 3.2/2 \ (20\%)$], it now requires $|C/T| = 0.58 \pm 0.24$. This value is consistent with QCD calculations (within their uncertainties) and with flavor SU(3) fits combining $B \to \pi K$ and $B \to \pi\pi$ data \cite{14}.

A test for the CKM framework based on the asymmetry $S_{CP}(\pi^0 K^0)$ was suggested in Ref. \cite{15}. Assuming flavor SU(3) in order to obtain the quantity $|T+C|$ from the decay rate for $B^+ \to \pi^+\pi^0$ \cite{16}, the asymmetry $S_{CP}(\pi^0 K^0)$ can be determined using as inputs this rate measurement and all the $B \to \pi K$ measurements except $S_{CP}(\pi^0 K^0)$. It was noted, however, that this test is sensitive to input values of $B(B^0 \to \pi^0 K^0)$, $B(B^0 \to \pi^- K^+)$ and to SU(3) breaking in $\delta_{EW}$ \cite{17}.

Table 1: Branching fractions and CP asymmetries for $B \to \pi K$ decays, as of today and for early 2007 (in parentheses) \cite{13}.

| Mode | $B \ (10^{-6})$ | $A_{CP}$ | $S_{CP}$ |
|------|-----------------|-----------|-----------|
| $B^0 \to \pi^-K^+$ | $19.4 \pm 0.6$ | $-0.098^{+0.012}_{-0.011}$ | |
| | $(19.7 \pm 0.6)$ | $(-0.093 \pm 0.015)$ | |
| $B^+ \to \pi^0K^+$ | $12.9 \pm 0.6$ | $0.050 \pm 0.025$ | |
| | $(12.8 \pm 0.6)$ | $(0.047 \pm 0.026)$ | |
| $B^0 \to \pi^0K^0$ | $9.8 \pm 0.6$ | $-0.01 \pm 0.10$ | $0.57 \pm 0.17$ |
| | $(10.0 \pm 0.6)$ | $(-0.12 \pm 0.11)$ | $(0.33 \pm 0.21)$ |
| $B^+ \to \pi^+K^0$ | $23.1 \pm 1.0$ | $0.009 \pm 0.025$ | |
| | $(23.1 \pm 1.0)$ | $(0.009 \pm 0.025)$ | |
The branching fractions in Table 1 and the lifetime ratio \( \tau_+/\tau_0 \equiv \tau(B^+)/\tau(B^0) = 1.073 \pm 0.008 \) imply that the corresponding decay widths are approximately in the ratio 2:1:1:2. This indicates the dominance of the isospin-preserving (\( \Delta I = 0 \)) amplitude \( P_{tc} \). Indeed, in the best fit of Ref. [12], \(|T|\) is about 12% of \(|P_{tc}|\) while other contributions are smaller. Assuming that \( C \) is suppressed relative to \( T \) and can be neglected, it was shown [18] that the rate differences

\[
\Delta(\pi^-K^+) \equiv \Gamma(B^0 \to \pi^+K^-) - \Gamma(B^0 \to \pi^-K^+)
\]

and

\[
2\Delta(\pi^0K^+) \equiv 2[\Gamma(B^- \to \pi^0K^-) - \Gamma(B^+ \to \pi^0K^+)]
\]

should be approximately equal. As the CP-averaged rate for \( B^0 \to \pi^-K^+ \) is about double that for \( B^+ \to \pi^0K^+ \), this translates to a prediction of approximately equal CP asymmetries,

\[
A_{CP}(\pi^-K^+) \approx A_{CP}(\pi^0K^+).
\]

The current measured asymmetries given in Table 1 differ by more than 5\( \sigma \). This has occasionally been taken to indicate the presence of NP in \( B \to \pi K \) [19]. It was noted, however [20], that a nonzero contribution of \( C \) at a level suggested in SU(3) fits [14] (and recently in the \( B \to \pi K \) fit [12]) could account for this difference. A necessary requirement is that the strong phase difference \( \arg(C/T) \) is large and negative [21]. A large negative phase has been obtained in flavor SU(3) fits to \( B \to \pi K \) and \( B \to \pi\pi \) data [14]. A phase of about \(-130^\circ\), obtained in the best fit in Ref. [12], and the central value \(|C/T| = 0.58\) account well for the difference between the two asymmetries. While this magnitude of \( C/T \) can be accounted for in QCD calculations, its large negative phase seems a problem for certain QCD calculations [7, 9] but not for others [10].

A very robust sum rule was suggested a few years ago [22] combining all four \( CP \) rate asymmetries in \( B \to \pi K \):

\[
\Delta(\pi^-K^+) + \Delta(\pi^+K^0) \approx 2\Delta(\pi^0K^+) + 2\Delta(\pi^0K^0),
\]

where \( \Delta(\pi^+K^0) \) and \( \Delta(\pi^0K^0) \) are defined similarly to \( \Delta(\pi^-K^+) \) and \( \Delta(\pi^0K^+) \) in Eqs. (8) and (9). This approximate sum rule is based on the amplitude isospin relation [Eq. (1)] and its CP conjugate, in which the isoscalar (\( \Delta I = 0 \)) and isovector (\( \Delta I = 1 \)) parts vanish separately. Consider the difference between the left-hand side of Eq. (11) and its right-hand side. The leading terms in this difference involve interference between the dominant isoscalar amplitude \( P_{tc} \) and a linear combination of smaller isoscalar and isovector amplitudes, each of which vanishes because of Eq. (1). The remaining terms arise from the interference of smaller \( \Delta I = 1 \) amplitudes with one another. These terms were found to vanish in the flavor SU(3) and heavy-quark limits [22]. A simplified and slightly less precise version of the sum rule [Eq. (11)], which assumes that the rates in Table 1 are in the ratio 2:1:1:2, relates the CP asymmetries directly:

\[
A_{CP}(\pi^-K^+) + A_{CP}(\pi^+K^0) \approx A_{CP}(\pi^0K^+) + A_{CP}(\pi^0K^0).
\]
The sum rule in Eq. (11) holds also in the presence of a $\Delta I = 0$ NP amplitude which can be absorbed in $P_c$ in the above argument. Furthermore, since the sum rule can only be violated by terms which are quadratic in $\Delta I = 1$ amplitudes, a substantial violation of the sum rule requires isovector NP amplitudes which are not much smaller than $P_c$. This will be the basis of our following diagnostic for NP in $B \to \pi K$ decays.

While any evidence for NP in $B \to \pi K$ is currently weak, suppose that NP is present in these decays. It was argued in Ref. [23] that all NP strong phases are negligible. Given that all strong phases are equal, there are two classes of NP amplitudes contributing to $B \to \pi K$, differing only in their color structure [24]:

$$A^q e^{i\Phi_q} \equiv \sum \langle \pi K | \bar{s}_\alpha \Gamma_i b_\alpha \bar{q}_\beta \Gamma_j q_\beta | B \rangle ,$$

$$A^{C,q} e^{i\Phi_q} \equiv \sum \langle \pi K | \bar{s}_\alpha \Gamma_i b_\alpha \bar{q}_\beta \Gamma_j q_\alpha | B \rangle ,$$

where $\Gamma_{i,j}$ represent Lorentz structures and $q = u, d$. (Despite the index C standing for color-suppression, the matrix elements $A^{C,q} e^{i\Phi_q}$ are not necessarily smaller than $A^q e^{i\Phi_q}$.) Here, $\Phi_q$ and $\Phi_q^C$ are the NP weak phases; the strong phases are taken to be zero. The NP amplitudes $A^q e^{i\Phi_q}$ and $A^{C,q} e^{i\Phi_q^C}$ are equivalent to the amplitudes $\Delta P_q$ and $\Delta P_q^c$ defined in Ref. [25].

There are three NP matrix elements which contribute to the $B \to \pi K$ amplitudes: $A^{comb} e^{i\Phi} \equiv -A^u e^{i\Phi_u} + A^d e^{i\Phi_d}, A^{C,u} e^{i\Phi_u^C},$ and $A^{C,d} e^{i\Phi_d^C}$ [24]. (These are equivalent to the three independent NP combinations, $-\Delta P_u + \Delta P_d, \Delta P_u^c,$ and $\Delta P_d^c$, contributing to $B \to \pi K$ amplitudes as discussed in Ref. [25].) The first operator corresponds to including NP only in the color-favored electroweak penguin amplitude: $A^{comb} e^{i\Phi} \equiv -P_{EW,NP} e^{i\Phi_{EW}}$. Nonzero values of $A^{C,u} e^{i\Phi_u^C}$ and/or $A^{C,d} e^{i\Phi_d^C}$ imply the inclusion of NP in both the gluonic and color-suppressed electroweak penguin amplitudes, $P_{NP} e^{i\Phi_P}$ and $P_{EW,NP} e^{i\Phi_{EW}}$, respectively [26]:

$$P_{NP} e^{i\Phi_P} \equiv \frac{1}{3} A^{C,u} e^{i\Phi_u^C} + \frac{2}{3} A^{C,d} e^{i\Phi_d^C} ,$$

$$P_{EW,NP} e^{i\Phi_{EW}} = A^{C,u} e^{i\Phi_u^C} - A^{C,d} e^{i\Phi_d^C} .$$

Thus, NP in $B \to \pi K$ is of one of the above three varieties. It can affect the gluonic penguin amplitude ($P_{NP} e^{i\Phi_P}$), the color-favored electroweak penguin amplitude ($P_{EW,NP} e^{i\Phi_{EW}}$), or the color-suppressed electroweak penguin amplitude ($P_{EW,NP}^c e^{i\Phi_{EW}}$). These three NP amplitudes are added to $P_{tc}$, $P_{EW}$ and $P_{EW}^c$, respectively. In the first case, the NP is $\Delta I = 0$ and, as mentioned, will not affect the sum rule of Eq. (11). In the other two cases, the NP breaks isospin by one unit, so that the sum rule will be violated. Ref. [12] performed fits with each of the three NP operators. In the SM-like fit it was found that it is not possible to constrain NP in the $\Delta I = 0$ gluonic penguin ($P_{NP} e^{i\Phi_P}$). This is not surprising – it is the same in $B \to \pi\pi$ decays [27]. However, the two non-SM-like fits did indeed involve substantial contributions from NP: $|P_{EW,NP}/P_{tc}| = 0.4, |P_{EW,NP}^c/P_{tc}| = 0.3$. They had lower values of $\chi^2/d.o.f.$ (0.4/2 and 2.5/2) than the SM-like fit (3.6/2). One potentially troublesome point is that they had very large values of $|C/T|$ (6 ± 11.
and 4.9 ± 3.8). However, the errors are also quite large, so this might not be a real problem. The key point here is that these two NP operators are of the type ΔI = 1, and so can violate the sum rules of Eqs. (11) and (12).

Using the CP asymmetries [all except for A_{CP}(B^0 \rightarrow \pi^0 K^0)] and branching ratios in Table 1 and translating ratios of branching ratios into ratios of rates using the lifetime ratio τ_+/τ_0 = 1.073±0.008 \cite{13}, the sum rule of Eq. (11) predicts

\[ A_{CP}(B^0 \rightarrow \pi^0 K^0) = -0.149 \pm 0.044 \]  

(17)

more than 3σ away from zero. The B^0 \rightarrow \pi^0 K^0 mode is thus predicted to exhibit the largest CP asymmetry of any of the four B \rightarrow \pi K modes.

If one uses the simplified version of the sum rule [Eq. (12)], which assumes the rates in Table 1 are in the ratio 2:1:1:2, the CP asymmetries are related directly:

\[ A_{CP}(\pi^0 K^0) = A_{CP}(\pi^- K^+) + A_{CP}(\pi^+ K^0) - A_{CP}(\pi^0 K^+) = -0.139 \pm 0.037 \]  

(18)

Indeed, the SM-like fit in Ref. \cite{12} finds A_{CP}(\pi^0 K^0) = -0.12. (We restrict our attention to those fits which include the constraints on CKM phases \cite{13} β = (21.66^{+0.95}_{-0.85})^o and γ = (66.8^{+5.4}_{-3.8})^o.)

On the other hand, in the other two fits of Ref. \cite{12}, ΔI = 1 NP amplitudes are involved, which are much more significant than those in the SM, and very large values of |C| are obtained. This results in a crucial difference with the SM fit: both of these fits predict A_{CP}(\pi^0 K^0) = -0.03. We thus see that it is possible to detect the presence of NP with a precise measurement of A_{CP}(\pi^0 K^0).

Another sum rule involving decay rates instead of CP asymmetries is, in principle, sensitive to NP in ΔI = 1 amplitudes \cite{18} \cite{30}. In terms of branching ratios \[ B(B \rightarrow f) \equiv B(f) \] it is expressed as

\[ 2B(\pi^0 K^+) + 2(\tau_+ / \tau_0) B(\pi^0 K^0) = (\tau_+ / \tau_0) B(\pi^- K^+) + B(\pi^+ K^0) . \]  

(19)

The difference between the left- and right-hand sides is quadratic in ΔI = 1 amplitudes, expected to be small in the Standard Model. A discussion of the terms violating this sum rule can be found in Ref. \cite{25}, which also discusses NP in B \rightarrow \pi K decays.

The experimental branching ratios in Table 1 satisfy the sum rule at the 1.4σ level, with the (left, right)-hand sides giving (46.8 ± 1.8, 43.9 ± 1.2) in units of 10^{-6}. The three fits of Ref. \cite{12} satisfy the sum rule as well or better, as shown in Table 2. A large P_{EW,NP} amplitude in Fit 2 is nearly cancelled by a large C contribution in a delicate way so as to preserve the sum rule.

One may ask if the large value of |C/T| in Fits 2 and 3 is obligatory. Performing the same fits but constraining |C/T| = 0.5, one finds results summarized in Table 2. The number of degrees of freedom with the constraint |C/T| = 0.5 is 3, while without the constraint it is 2. It appears that a large value of |C|, along with a sizable NP amplitude, is responsible for moving the predicted A_{CP}(\pi^0 K^0) away from its Standard Model value of -0.15.

The difference between the violation of the rate sum rule [Eq. (19)] and the asymmetry sum rule [Eq. (11)] in the presence of NP can be seen as follows. Define
Table 2: Comparison of fits of Ref. [12] with and without constraint $|C/T| = 0.5$. Prediction for (a) $A_{CP}(\pi^0 K^0)$, (b) Eq. (19), l.h.s; (c) Eq. (19), r.h.s.

|   | $\chi^2$/d.o.f. | $A_{CP}$ | $\chi^2$/d.o.f. | $A_{CP}$ |
|---|----------------|----------|----------------|----------|
| 1 | 3.7/3          | -0.11    | 3.6/2          | -0.12    |
| 2 | 3.0/3          | -0.12    | 0.4/2          | -0.03    |
| 3 | 3.8/3          | -0.12    | 2.5/2          | -0.03    |

$P_1 \equiv P_{EW,NP}$ and $P_2 \equiv P_{EW,NP}^C$ and assume they are large, neglecting the violation of the two sum rules by small SM contributions (which is a good approximation).

The terms violating the rate sum rule are

$$2[|P_1|^2 + |P_1||P_2|\cos(\delta_1 - \delta_2)\cos(\phi_1 - \phi_2)],$$

(20)

where $\delta_1, \delta_2$ and $\phi_1, \phi_2$ are the strong and weak phases of $P_1$ and $P_2$. This leads to a violation of the rate sum rule [Eq. (19)] in the case $P_1 \neq 0$, $P_2 = 0$ (Fit 2 of Ref. [12]) but to no violation in the case $P_1 = 0$, $P_2 \neq 0$ (Fit 3 of Ref. [12]).

On the other hand, the term violating the asymmetry sum rule is

$$2|P_1||P_2|\sin(\delta_1 - \delta_2)\sin(\phi_1 - \phi_2).$$

(21)

In Ref. [12] it was assumed that $\delta_1 = \delta_2$, so the asymmetry sum rule was not violated by this term. Also, the cases $P_1 = 0$, $P_2 \neq 0$ and $P_1 \neq 0$, $P_2 = 0$ were considered separately. In these cases the sum rule would hold even when $\delta_1 \neq \delta_2$.

The reason for the violation of the asymmetry sum rule in the two fits (Fit 2 and Fit 3) of Ref. [12] without a constraint on $|C/T|$ (seventh column, second and third rows of Table 2), giving $A_{CP} = -0.03$ instead of $-0.12$, is that $C$ was also large in both fits, so the sum rule was violated by contributions

$$4|P_1||C|\sin(\delta_1 - \delta_C)\sin(\phi_1 - \gamma)$$

(Fit 2),

$$2|P_2||C|\sin(\delta_2 - \delta_C)\sin(\phi_2 - \gamma)$$

(Fit 3).

To sum up, the asymmetry sum rule [Eq. (11)] can be violated significantly (when assuming $|C/T|$ is not very large) only by taking $P_1 \neq 0$, $P_2 \neq 0$, $\delta_1 \neq \delta_2$, and $\phi_1 \neq \phi_2$. In other words, violation of the sum rule requires both $P_{EW,NP}$ and $P_{EW,NP}^C$ to be present and both strong- and weak-phase differences of these amplitudes to be non-negligible. One may imagine a situation in which these circumstances hold leading to an asymmetry $A_{CP} = -0.03$ in $B^0 \rightarrow \pi^0 K^0$ as discussed above.

Although we recognize the difficulty of the measurement, we thus urge a reduction of the present experimental error on $A_{CP}(\pi^0 K^0)$, whose world average is $-0.01 \pm 0.10$. The BaBar and Belle contributions to this average are compared in Table 3. The error must be reduced by at least a factor of three in order to
Table 3: Comparison of BaBar [28] and Belle [29] values of $A_{CP}(B^0 \rightarrow \pi^0 K^0)$.

| Source   | $N(B\bar{B})$ (M) | $A_{CP}$       |
|----------|-------------------|----------------|
| BaBar    | 467               | $-0.13 \pm 0.13 \pm 0.03$ |
| Belle    | 657               | $0.14 \pm 0.13 \pm 0.06$ |
| Average  | 1124              | $-0.01 \pm 0.10$ |

determine whether new physics is generating a large $\Delta I = 1$ transition amplitude. An experiment collecting of the order of $10^{10}$ $B\bar{B}$ pairs ought to be able to make the necessary distinction.

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