Extrinsic and intrinsic ratchet response of a quantum dissipative spin-orbit medium

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Traditionally the charge ratchet effect is considered as a consequence of the extrinsic spatial asymmetry engineered by external asymmetric periodic potentials. Here we demonstrate that electrically and magnetically driven dissipative systems with spin-orbit interactions represent an exception from this standard idea. The charge and spin ratchet currents appear just due to the coexistence of quantum dissipation with the intrinsic spatial asymmetry of the spin-orbit coupling. The extrinsic spatial asymmetry is inessential.

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A system of charged particles driven by a time-dependent external force may exhibit a net charge current even if the force is periodic and unbiased. This so-called charge ratchet effect [1, 2, 3, 4, 5, 6] is used e.g. in nano-generators of direct currents. If transport involves the spin degree of freedom, the concept of a spin ratchet [7, 8, 9, 10] emerges as a natural analog of the charge ratchet notion. For systems with spin-orbit interactions the spin ratchet effect may have been expected because it could be rooted in an asymmetric excitation of spin dynamics by the orbital dynamics induced by an electric field. Such an expectation is based on the intrinsic spatial asymmetry inherent to systems with spin-orbit interactions. For example the Rashba [11] and Dresselhaus [12] spin-orbit Hamiltonians for semiconductor heterostructures are obviously not invariant with respect to the real space inversion. For electrically driven coherent and dissipative systems with Rashba spin-orbit interaction (RSOI) the spin ratchet mechanism has indeed been confirmed [7, 9, 10]. Even for symmetric periodic potentials the spin ratchet effect exists [2] just due to the intrinsic spatial asymmetry of RSOI. However, the charge ratchet effect is absent in both the coherent and dissipative cases for symmetric periodic potentials. This could deepen the impression that without the extrinsic asymmetry a system will never respond to external fields via the charge ratchet mechanism and systems with spin-orbit interactions like all other systems obey this habitual rule. The present work reveals that this is a delusion and in reality systems with spin-orbit interactions provide a unique opportunity to answer the fundamental questions related to the role of symmetries in ratchet phenomena in general.

In this Letter we show that the extrinsic asymmetry, usually required as a key property of particle ratchets, is not necessary as the intrinsic Rashba asymmetry alone is sufficient if a dissipative system is driven by both electric and magnetic fields. Specifically, it is found that the charge and spin ratchet effects in this case exist for symmetric periodic potentials and stem just from the simultaneous presence of dissipation and the real space asymmetry of the Rashba electrons. We also find that at low temperatures the ratchet charge current in the system is unusual. Its queerness consists in the fact that this current, in contrast to early predictions for systems without spin-orbit interactions [13, 14], appears even when only one energy band provides electrons for transport and no harmonic mixing is present in the driving fields. This charge current is of pure spin-orbit nature and, as a result, it disappears when the spin-orbit coupling strength vanishes. Therefore such spin-orbit charge currents can be controlled by the same gate voltage which controls the strength of the spin-orbit coupling in the system. This could be very attractive from an experimental point of view since measurements of charge currents are experimentally better controlled than measurements of spin currents.

An archetype of the device under investigation is shown in Fig. 1. In this system non-interacting electrons are confined in a quasi-one-dimensional (quasi-1D) periodic structure obtained by appropriately placed gates applied to a two-dimensional electron gas (2DEG) with RSOI. The system interacts with an external environment (or bath): the longitudinal orbital degree of freedom of each electron is coupled to orbital degrees of freedom of the external environment. This coupling is the source of dissipation in the system. The electrons are driven by longitudinal electric and transverse in-plane magnetic homogeneous fields which are time-periodic functions with zero mean value.

To perform a quantitative analysis of the ratchet effects we model the system by the Hamiltonian \( \hat{H}(t) = \hat{H}_0 + \hat{H}_D(t) + \hat{H}_B \), where \( \hat{H}_D(t) \equiv -eE(t)\hat{x} - g\mu_B H(t)\hat{\sigma}_z \) is the driving term, \( \hat{H}_B \) is the bath term of the Caldeira-Leggett model [13, 14] taking into account the orbital coupling between the electron longitudinal degree of freedom, \( \hat{x} \), and orbital degrees of freedom of the bath. All properties of the bath are encapsulated in its spectral density \( J(\omega) \).
FIG. 1: (Color online) A 2DEG with RSOI of strength $\alpha = 9.94 \times 10^{-12}$ eV·m is obtained by a gate voltage applied to an InGaAs/InP heterostructure using the "Back Gate". The electron effective mass is $m = 0.037m_0$ with $m_0$ being the free electron mass and the effective gyrosopic factor is $g^* = -15$. A parabolic confinement of strength $\hbar \omega_0 = 0.225$ meV forms in the 2DEG a quasi-one-dimensional electron gas (Q1DEG). The system is driven using a harmonic confinement of strength $\omega^2 x^2/2 + U(x)\left(1 + \gamma z^2/L^2\right)$. In this model it is assumed that the 2DEG is in the $z$-axis and the quasi-1D structure is formed along the $x$-axis using a harmonic confinement of strength $\omega_0$ along the $z$-axis. The electron spin $g$-factor is denoted as $g$ and $\mu_B$ is the Bohr magneton. The super-lattice period is $L$, $U(x + L) = U(x)$. The parameters $k_{so} = a m^2/2 \hbar^2$ and $\gamma$ characterize the strength of the spin-orbit and orbit-orbit couplings, respectively. We consider the additional possibility of coupling between the longitudinal and transverse orbital degrees of freedom of the electrons since it is responsible for the existence of the ratchet transport in the system under appropriate combinations of the driving fields.

The electric driving is given by the vector $\mathbf{E}(t) = (E(t), 0, 0)$ while the magnetic driving is $\mathbf{H}(t) = (0, 0, H(t))$. We consider the time dependence $e^H(t) \equiv F \cos(\Omega(t - \tau_0)), H(t) \equiv H \cos(\Omega(t - \tau_0))$. The vector potential is chosen using the Landau gauge $\mathbf{A}(t) = (-H(t)y, 0, 0)$. Since $y = 0$ in the 2DEG, the vector potential is not explicitly present in the model.

To study the ratchet effects at low temperatures when only the lowest energy band of the super-lattice is populated with electrons we calculate the charge and spin currents averaged over one driving period. These currents in the long time limit provide the stationary ratchet response of the system. The common eigenstates of $\hat{x}$ and $\hat{\sigma}_z$ represent a convenient basis to obtain this response. Because of the discrete eigenvalue structure of $\hat{x}$ (see below) the basis is called the $\sigma$-discrete variable representation ($\sigma$-DVR) basis. The eigenstates are denoted as $|m, j, \sigma\rangle$, where $m = 0, \pm 1, \pm 2, \ldots$, and $j$ and $\sigma$ are the transverse mode and spin quantum numbers, respectively. In the $\sigma$-DVR basis the averaged charge and spin currents have a simple form:

$$J_C = -e \lim_{t \to \infty} \sum_{m,j,\sigma} \frac{dx_{m,j}}{dt} P_{m,j,\sigma}(t),$$
$$J_S = \lim_{t \to \infty} \sum_{m,j,\sigma} \sigma x_{m,j} \frac{d}{dt} P_{m,j,\sigma}(t).$$

In Eq. \(P_{m,j,\sigma}(t)\) is the averaged population at time $t$ of the $\sigma$-DVR state $|m, j, \sigma\rangle$, the quantities $x_{m,j} = mL + d_j \left(-L/2 < d_j \leq L/2\right)$ and $\sigma$ are eigenvalues of $\hat{x}$ and $\hat{\sigma}_z$ corresponding to their common eigenstate $|m, j, \sigma\rangle$. Additionally, the $\sigma$-DVR basis allows the path integral formalism to handle the magnetic driving on an equal footing with the standard electric driving since in this basis the whole driving Hamiltonian, $\hat{H}_0(t)$, is diagonal.

In the long time limit the populations $P_{m,j,\sigma}(t)$ come from a master equation which is in this case Markovian.

Before starting a rigorous exploration one can already anticipate that the magnetic field driving brings a whiff of fresh physics because the spin dynamics can be controlled directly and not only through the spin-orbit interaction mediating between the electric field and electron spins.

An analytical treatment of this rather complicated problem is possible when the dynamics of $P_{m,j,\sigma}(t)$ is treated within the first two transverse modes, i.e., $j = 0, 1$.

For a detailed study we derive the charge and spin currents assuming weak coupling between neighboring $\sigma$-DVR states. We obtain:

$$J_C = -e \sum_{m,j,\sigma} \Delta_{\sigma}^2 \left| \Delta_{\sigma}^0 \right|^2 \left( I_{\sigma}^{01,1} I_{\sigma}^{10,1} - I_{\sigma}^{01,f} I_{\sigma}^{10,f} \right),$$
$$J_S = \sum_{m,j,\sigma} \sigma \Delta_{\sigma} \left( \Delta_{\sigma}^2 I_0 I_{\sigma}^{01,1} I_{\sigma}^{10,1} - \Delta_{\sigma}^2 I_0 I_{\sigma}^{01,f} I_{\sigma}^{10,f} \right),$$

where $\Delta_{\sigma}^2 = \langle m + 1, j, \sigma|\hat{H}_0|m, j, \sigma\rangle$ are the hopping matrix elements of the Hamiltonian of the isolated system, Eq. (1), $I_0 \equiv |\Delta_{\sigma}^0|^2 (I_{\sigma}^{01,1} + I_{\sigma}^{10,1}) + |\Delta_{\sigma}^2|^2 (I_{\sigma}^{01,f} + I_{\sigma}^{10,f})$, and $\uparrow, \downarrow$ stand for $\sigma = 1, -1$, respectively. The effects of both the driving fields and quantum dissipation are in the integrals

$$I_{\sigma}^{j,j'}(\Delta) \equiv \frac{1}{h^2} \int_{-\infty}^{\infty} d\tau e^{-\frac{i}{\hbar}Q(\tau; j, j') + \frac{i}{\hbar} (\epsilon_\uparrow - \epsilon_\downarrow)} \times J_0 \left[ \pm 2F \left| L \right| + 2g_B H (\sigma - \sigma') \right. \left. \sin \left( \frac{\Omega \tau}{2} \right) \right].$$
where $Q[\tau; J(\omega), T]$ is the twice integrated bath correlation function \cite{10} whose dependence on $\tau$ is fixed by the bath spectral density $J(\omega)$ and temperature $T$, $\varepsilon^i_a \equiv \langle m, j, \sigma | H_0 | m, j, \sigma \rangle$ are the on-site energies of the isolated system, and $J_0(x)$ is the Bessel function of zero order.

Remarkably, Eq. \cite{3} tells us that at low temperatures the ratchet charge and spin transport in the system exists just because of spin flip processes. Whereas it looks natural for the spin current, it is a quite unexpected and important result for the charge current. This current emerges because the magnetic driving changes the charge dynamics. In this case the spin-orbit interaction plays a role inverse to the one which it plays for the electric driving: the magnetic field exciting spin dynamics induces orbital dynamics through the spin-orbit interaction. The corresponding charge flow, originating just due to the spin-orbit interaction, is finite even when only one energy band contributes to transport.

The situation, however, is highly non-trivial and the final conclusions about the existence of the ratchet charge and spin flows cannot be based only on the presence of spin-orbit interactions. There are also external time-dependent fields driving the system and internal quantum dissipative processes. The mutual driving-dissipation effect is incorporated in the integrals, Eq. \cite{1}. Therefore, a further analysis is required: one should additionally take into consideration the properties of the integrals from Eq. \cite{1} and the properties of the static periodic potential with respect to the spatial inversion symmetry.

There are twelve different cases, shown in Table I, to check whether the charge and spin ratchet effects can take place in the corresponding physical situations. Only those four of them which are given by the row with $F \neq 0$, $H = 0$ have been studied up to now and discussed in Refs. \cite{9, 10}. The other eight possibilities have not been investigated so far.

| $\gamma = 0$ | $\gamma \neq 0$ |
|-----------------|------------------|
| $U(x) \neq U(-x)$ | $U(x) \neq U(-x)$ |
| $J_{\sigma} = 0$ | $J_{\sigma} = 0$ |
| $J_{\bar{\sigma}} = 0$ | $J_{\bar{\sigma}} = 0$ |
| $J_{\sigma} \neq 0$ | $J_{\sigma} \neq 0$ |
| $J_{\bar{\sigma}} \neq 0$ | $J_{\bar{\sigma}} \neq 0$ |

The principal feature of the physics taking place when $F \neq 0$ and $H \neq 0$ is that the existence of the ratchet effects is not dictated only by properties of the isolated system as in Refs. \cite{9, 10}. The physical picture is now more intricate. In the charge and spin currents one cannot find clear traces of either driving and dissipation or the isolated system. The two imprints are not separable and the charge and spin ratchet mechanisms are determined by the whole system-plus-bath complex.

The above theoretical predictions have been confirmed numerically. Figure \cite{2} shows the situation with $\gamma = 0$. The superlattice is modeled by the potential $U(x) = V_0 + \sum_{i=1}^2 V_i \cos(2\pi lx/L - \phi_i)$ with $V_0 = -V_1 = 2.6\hbar\omega_0$, $V_2 = 1.9\hbar\omega_0$, $\phi_1 = 1.9$, $\phi_2 = 0$ for the asymmetric case while for the symmetric one $V_0 = -V_1 = 2.6\hbar\omega_0$, $V_2 = 0$, $\phi_1 = \phi_2 = 0$. The period is $L = 2.5\sqrt{\hbar/m\omega_0}$ which gives $k_\omega L \approx 0.368\pi$. The driving frequency of the electric and magnetic fields is $\Omega = \sqrt{3}\omega_0/4$. The bath is Ohmic with the exponential cut-off at $\omega_* = 10\omega_0$: $J(\omega) = \eta \omega \exp(-\omega/\omega_*)$. The viscosity coefficient is $\eta = 0.1$ and the temperature is $k_B T = 0.5\hbar\omega_0$. As theoretically expected the ratchet effects exist even when the periodic potential is symmetric, Figs. 2a and 2c. However, the currents of these intrinsic ratchet effects are much smaller than the corresponding currents of the extrinsic ones, Fig. 2b and 2d. What is surprising in the case when both of the driving fields are present is that the orbit-orbit coupling has a weak effect on the ratchet spin current as it is demonstrated in Fig. 3. At the same time when $H = 0$ the orbit-orbit coupling is responsible for the existence of the pure spin ratchet effect (see Ref. \cite{10}) as one can see in the inset of Fig. 3a. Physically it is explained by the increased contribution from the spin torque to the spin current. When $H \neq 0$, the high-frequency magnetic field flips periodically the electron spins. Since this field is uniform the difference (which is created by the orbit-orbit coupling) between the group velocities of the electrons moving in the center of the wire and closer to its edges is not decisive for the ratchet effect.

In summary, we have shown that the intrinsic spatial asymmetry, i.e., the asymmetry not related to the ratchet potential, of a dissipative system with Rashba spin-orbit interaction can lead to charge and spin ratchet effects when the system is driven by both electric and magnetic fields. The charge ratchet current has been found to have a purely spin-orbit origin. The extrinsic spatial asymmetry, i.e., the asymmetry induced by the ratchet potential, is not critical for the existence of the ratchet effects but its presence amplifies the ratchet currents due to the superposition of the intrinsic and extrinsic ratchet effects. The proposed system could thus be a multifunctional spintronic device which, when appropriately electrically
FIG. 2: (Color online) The charge and spin ratchet currents as functions of the amplitudes of the electric and magnetic fields.

a,b, Spin current for the symmetric and asymmetric cases, respectively. c,d, Charge current for the symmetric and asymmetric cases, respectively. The amplitudes of the electric, $F_L$, and magnetic, $g_B H$, fields are in units of $\hbar \omega_0$. The currents are in units of $L \omega_0$. The orbit-orbit coupling is absent, $\gamma = 0$, but the spin current is finite in the symmetric case when both of the fields are present. The charge current is excited when both the electric and magnetic fields simultaneously drive the system. In the intrinsic ratchet response (a and c) the magnitude of the charge and spin currents is strongly suppressed by the symmetry of the periodic potential while in the extrinsic ratchet response (b and d) the charge and spin currents are enhanced by the spatial asymmetry of the system.

FIG. 3: (Color online) The charge and spin ratchet currents as functions of the magnetic field amplitude. The magnetic amplitude, $g_B H$, is in units of $\hbar \omega_0$. The electric amplitude is fixed, $F_L = \hbar \omega_0$. The solid curves correspond to $\gamma = 0$. The dotted curves correspond to $\gamma = 0.1$. a, Asymmetric case. The inset shows a vicinity of the point $H = 0$ at which the pure spin ratchet response takes place for $\gamma \neq 0$. b, Symmetric case.

programmed by the external periodic gates, works as a spin and/or charge direct current generator.

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