High-Squint FMCW SAR Imaging via Wavenumber Domain Algorithm

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1. Introduction

Synthetic aperture radar (SAR) is an active microwave imaging system usually mounted on a satellite or airplane to achieve high-resolution earth observation [1–3]. Compared to typical optical imaging, it has all-time and all-weather surveillance capability and hence has been widely used in early-warning, marine surveillance, and disaster monitoring applications [4]. Compared with a conventional pulse mode-based SAR system, frequency modulation continuous-wave (FMCW) SAR has shown good potential in minimal transmission power and weight reduction of radar systems and hence has been widely used and has become a common technique in modern short-range high-resolution earth observation. Different from the conventional pulsed mode, the stop-to-go approximation is not valid in FMCW SAR. Thus, the typical SAR imaging methods need to be modified to adapt the continuous-wave scheme in practical data processing. In this letter, a wavenumber domain algorithm (WDA) is derived and used for the FMCW SAR imaging under squint and high-squint cases. With the help of the exact scene recovery ability of the proposed WDA, we can achieve the accurate recovery of the considered scene even when using the collected high-squint data and hence obtain the well-focused high-resolution image of the considered scene. Experimental results with simulated and airborne data verify the effectiveness of the proposed method.
well-examined high-squint data-based high-resolution SAR imaging ability. In 2010, Wang et al. introduced WDA to the side-looking FMCW SAR data processing [9, 18]. With the help of an accurate focus function, WDA has also achieved the high quality scene recovery under the FMCW SAR imaging mode.

In this letter, an accurate representation of the WDA algorithm is derived for the FMCW SAR image focusing in squint and high-squint cases. Experimental results show that the proposed method can achieve accurate recovery of the considered scene even when using the collected high-squint data, which means that it can be regarded as a general technique for FMCW SAR imaging in any squint angle. The rest of this letter is organized as follows. Section 2 provides an introduction of the FMCW SAR imaging model with the squint angle. In Section 3, we show the detailed processing procedures of the proposed WDA method for FMCW SAR imaging under squint and high-squint cases. In Section 4, the proposed WDA-based high-squint FMCW SAR method is validated by simulated data to show its advantages in high-resolution image focus. Experimental results via real airborne data as well as a performance analysis are shown in Section 5. Finally, conclusions are drawn in Section 6.

2. FMCW SAR Imaging

In FMCW SAR, let $s(t_c) = \exp \{j2\pi f_0 t_c \} \cdot \exp \{j\pi K_r t_c^2 \}$ denote the transmitted signal, where $f_0$ is the carrier frequency, $K_r$ is the chirp rate of frequency modulated signal, and $t_c$ is the range time. For a single point target $P(t_r^0 ; r_0)$ with zero Doppler time $r_0$ and closest slant range $r_0$ between $P(t_r^0 ; r_0)$ and platform trajectory along the line of sight (LOS) direction, we can express the received backscattered energy as

$$g(\tau, t_r, t_0 ; r_0) = \sigma(t_0 ; r_0) \cdot \exp \{j2\pi f_0 (t_r - \tau_c) \} \cdot \exp \{j\pi K_r (t_r - \tau_c)^2 \},$$

(1)

where $\tau_c$ is the round-trip delay time of the wave propagation. Using the reference signal

$$g_{\text{ref}}(t_c) = \exp \{j2\pi f_0 (t_r - \tau_c) \} \cdot \exp \{j\pi K_r (t_r - \tau_c)^2 \},$$

(2)

for the dechirp process in the FMCW SAR system, where $r_s$ is the reference slant range for the dechirp-on-receive operation, and $\tau_c = 2\alpha r_s / c$ with the “Doppler factor” $\alpha = c^2/(c^2 - \nu^2)$. Then, the received dechirp signal can be expressed as

$$g_{\text{de}}(\tau, t_r, t_0 ; r_0) = g(\tau, t_r, t_0 ; r_0) \cdot g_{\text{ref}}(t_c)$$

$$= \sigma(t_0 ; r_0) \cdot \exp \{-j2\pi f_0 (\tau - \tau_c) \} \cdot \exp \{-j2\pi K_r (\tau - \tau_c) (t_r - \tau_c) \} \cdot \exp \{j\pi K_r (t_r - \tau_c)^2 \}. \tag{3}$$

Let $R(\tau)$ denote the instantaneous slant range at time $\tau = t_a + t_r$, with $t_a$ being the azimuth time and $\theta$ represent the squint angle. Then, the slant range at the receiver can be expressed as $R(\tau + \tau_d)$, and

$$\tau_d = \frac{R(\tau) + R(\tau + \tau_d)}{c}, \tag{4}$$

with

$$R(\tau) = \sqrt{\tau_d^2 + \nu^2 (\tau - \tau_0)^2} - 2r_0 \nu (\tau - \tau_0) \sin \theta,$$

$$R(\tau + \tau_d) = \sqrt{\tau_d^2 + \nu^2 (\tau + \tau_d - \tau_0)^2} - 2r_0 \nu (\tau + \tau_d - \tau_0) \sin \theta, \tag{5}$$

where $\nu$ and $c$ are the velocity of the platform and light, respectively. After applying (5) into (4), $\tau_d$ has an exact expression, i.e.,

$$\tau_d = 2a \left[ \frac{R(\tau)}{c} + \frac{\nu^2}{c^2} (\tau - \tau_0) - \frac{r_0 \nu \sin \theta}{c^2} \right]. \tag{6}$$

The last term in (3) is the residual video phase (RVP). After removing the RVP term in the range frequency domain by phase multiplication operation, we have

$$g_{\text{de}}(\tau, t_r, t_0 ; r_0) = \sigma(t_0 ; r_0) \cdot \exp \{-j2\pi f_0 (r_d - \tau_c) \} \cdot \exp \{-j2\pi K_r (\tau - \tau_c) (t_r - \tau_c) \}. \tag{7}$$

Using the replacement $f_s = K_r (t_r - \tau_c)$, (7) can be rewritten as

$$g_{\text{de}}(\tau, f_s, t_0 ; r_0) = \sigma(t_0 ; r_0) \cdot \exp \{-j2\pi (f_0 + f_s) (\tau - \tau_c) \}. \tag{8}$$

Substituting $\tau = t_a + t_r$ and introducing (6) into (8), we have

$$g_{\text{de}}(f_s, f_s' ; t_0 ; r_0) = \sigma(t_0 ; r_0) \cdot \exp \left\{-j2\pi f_0 \cdot f_s \cdot \frac{t_a + t_r + t_0 - \frac{r_0 \nu \sin \theta}{c}}{c^2} \right\}. \tag{9}$$

Then, the Fourier transformation of (9) with respect to $t_a$ is

$$G_{\text{de}}(f_s, f_s' ; t_0 ; r_0) = \int g_{\text{de}}(f_s, f_s', t_a, t_0 ; r_0) \cdot \exp \{-j2\pi f_s t_a \} dt_a$$

$$= \sigma(t_0 ; r_0) \int \exp \{-j2\pi \Phi_1(f_s, f_s', f_0, f_s', t_a, t_0 ; r_0)\} dt_a, \tag{10}$$

with

$$\Phi_1(f_0, f_s, f_0', t_a, t_0, t_0 ; r_0) = 4\pi \alpha (f_0 + f_s)$$

$$\cdot \left[ \frac{R(t_a + t_r)}{c} + \frac{\nu^2}{c^2} (t_a + t_r - \tau_0) - \frac{r_0 \nu \sin \theta}{c^2} \right] + 2\pi f_s t_a. \tag{11}$$
To obtain the desired reference spectrum of a point target, by using the principle of stationary phase (POSP), we set the first-order derivative of $\Phi$ to be zero to obtain the stationary phase center $t_p$ as

$$t_p = r_0 - r_c + \frac{r_0}{v} \cdot h(f, f_r),$$

where

$$h(f, f_r) = \sqrt{1 - \left|\frac{c f_a}{2 f_c (f + f_r) + v/c^2}\right|^2} \sin \theta - \left|\frac{c f_a}{2 f_c (f + f_r) + v/c^2}\right| \cos \theta.$$  

After introducing (12) into (10), we have

$$G_{de}(f_a, f_r, r_0; r_0) = \sigma(r_0; r_0) \cdot \exp \left[-j\Phi(f_a, f_r, r_0; r_0)\right],$$

with

$$\Phi(f_a, f_r, r_0; r_0) = \frac{4\pi a (f_0 + f_r)}{c} r_0 \cdot \sqrt{1 + h(f_a, f_r)^2 - 2h(f_a, f_r) \sin \theta}$$

$$+ \left[\frac{c f_a}{2av_0 + f_r} + \frac{v}{c} h(f_a, f_r)\right] - 2\pi f_s \frac{f_r}{c} - 2\pi f_s r_0$$

$$+ 2\pi f_s r_0 - 4\pi a (f_0 + f_r) \frac{r_0 + f_r}{c} - 4\pi a (f_0 + f_r) \frac{r_0 + f_r}{c} \sin \theta,$$

is used to remove the range-invariant phase in the 2-D frequency domain. After RFM, the signal becomes

$$G_f(f_a, f_r, r_0; r_0, r_{ref}) = G_{de}(f_a, f_r, r_0; r_0) \cdot G_{ref}(f_a, f_r; r_0, r_{ref})$$

$$= \sigma(r_0; r_0) \cdot \exp \left[-j\Phi(f_a, f_r, r_0; r_0, r_{ref})\right],$$

with residual phase

$$\Phi_{ref}(f_a, f_r, r_0; r_0, r_{ref}) = \frac{4\pi a (f_0 + f_r)}{c} (r_0 - r_{ref})$$

$$+ \left[\sqrt{1 + h(f_a, f_r)^2 - 2h(f_a, f_r) \sin \theta} + \frac{c f_a}{2av_0 + f_r} + \frac{v}{c} h(f_a, f_r)\right] - 2\pi f_a r_0,$$

Table 1: Simulation parameters.

| Parameter                     | Value       |
|-------------------------------|-------------|
| Radar center frequency        | 9.6 GHz     |
| Pulse repetition frequency    | 3200 Hz     |
| Bandwidth                     | 500 MHz     |
| Platform height               | 1000 m      |
| Down-looking angle            | 45°         |

where $r_{ref}$ is the reference slant range. After RFM, the range walk continuous motion is compensated to obtain a coarsely focused image.

3.3. Stolt Interpolation. For $G_f(f_a, f_r, r_0; r_0, r_{ref})$, the Stolt interpolation,

$$\langle f_0 + f_r \rangle \left[\sqrt{1 + h(f_a, f_r)^2 - 2h(f_a, f_r) \sin \theta} + \frac{c f_a}{2av_0 + f_r} + \frac{v}{c} h(f_a, f_r)\right] \to f_0 + f_1,$$

can be achieved by replacing the range frequency $f_r$ with a new one $f_1$. Then, the signal phase $\Phi_f$ in 2-D frequency domain becomes linearly depending on the new range frequency variable $f_1$, i.e.,

$$\Phi_f(f_a, f_1, r_0; r_0, r_{ref}) = \frac{4\pi a (f_0 + f_1)}{c} (r_0 - r_{ref}) - 2\pi f_a r_0.$$

3.4. Image Focusing. Finally, we perform 2-D inverse Fourier transform on the signal after Stolt interpolation to obtain the focused image, i.e.,

$$g_f(t, t_r, r_0; r_0, r_{ref}) = w_{r_t}(t - t_0) \cdot w_{r_r}(t_r - \frac{2\alpha(r_0 - r_{ref})}{c}),$$

where $w_{r_t}(\cdot)$ and $w_{r_r}(\cdot)$ are the envelopes in the range and azimuth directions, respectively.

4. Experiment via Simulated Data

In order to validate the derived WDA-based squint and high-squint FMCW SAR imaging method, we first perform the
Figure 1: Continued.
experiment based on the simulated data. The simulated scene includes three point targets, which are located on the near (T1, (Rc −25 m, −25 m)), center (T2, (Rc, 0)), and far ranges (T3, (Rc +25 m, 25 m)). Rc = 1414 m is the center slant range of the considered scene. Simulation parameters are shown in Table 1. Figure 1 shows the WDA-recovered images of three simulated point targets from the collected data with different squint angles. It is seen that when θ = 0°, i.e., the side-looking case, WDA can focus all targets well, which is consistent with the result in [9]. After increasing θ to 30°, we can see that the proposed method can also achieve the accurate recovery of the considered scene. Further increasing the squint angle to 45°, FMCW SAR can be regarded as in the high-squint case. The derived WDA-based method also reconstructs the scene successfully. While for the complicated simulated scene with multitargets, as shown in Figure 2, it is seen that the proposed method also can recover the point targets well even under the high-squint case with a 45° squint angle.
The above experimental results agree with the theoretical analysis, i.e., the derived method achieves the accurate recovery of the considered scene even when using the collected squint and high-squint data. It means that the proposed method in this letter can be regarded as a general technique for both side-looking and squint and high-squint FMCW SAR imaging and can be used further for the practical data processing.

5. Experiment via Real Data

In this section, to further support our viewpoint, the derived method is introduced to the practical airborne FMCW SAR data processing. The WDA-focused images of different surveillance regions are shown in Figures 3 and 4. From Figures 3 and 4, it is seen that WDA can reconstruct the considered scene accurately, such as buildings and the airport runway, which illustrates that the method presented in this letter is effectively used for real FMCW SAR imaging.

6. Conclusion

In this letter, we introduce WDA to FMCW SAR imaging for the cases of squint and high-squint. The exact theoretical derivation of the squint and high-squint FMCW SAR imaging helps us to construct a universal model for WDA-based image focusing, which can make our derived model be used for FMCW SAR imaging from the collected data with any squint angle. Experimental results based on simulated and practical data validate the derived method in image-exact focusing and its potential in real airborne data processing.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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