Soft Supersymmetry Breaking, Scalar Top-Charm Mixing and Higgs Signatures

J. L. Díaz-Cruz\textsuperscript{a}, H. J. He\textsuperscript{b}, C.–P. Yuan\textsuperscript{c}

\textsuperscript{a} Lawrence Berkeley National Laboratory, Berkeley, California 92710, USA
\textsuperscript{b} The University of Texas at Austin, Austin, Texas 78712, USA
\textsuperscript{c} Michigan State University, East Lansing, Michigan 48824, USA

Abstract

The squark mass-matrix from the soft supersymmetry (SUSY) breaking sector contains a rich flavor-mixing structure that allows $O(1)$ mixings among top- and charm-squarks while being consistent with all the existing theoretical and experimental bounds. We formulate a minimal flavor-changing-neutral current scheme in which the squark mixings arise from the non-diagonal scalar trilinear interactions. This feature can be realized in a class of new models with a horizontal $U(1)_H$ symmetry which generates realistic quark-mass matrices and provides a solution to the SUSY $\mu$-problem. Finally, without using the mass-insertion approximation, we analyze SUSY radiative corrections to the $H^{\pm}bc$ and $h^{0}tc$ couplings, and show that these couplings can reveal exciting new discovery channels for the Higgs boson signals at the Tevatron and the LHC.

PACS numbers: 12.60.-i, 12.15.-y, 11.15.Ex
1. Introduction

Weak-scale supersymmetry (SUSY) [1], as a leading candidate for new physics beyond the standard model (SM), sensibly explains electroweak symmetry breaking, but leaves the understanding of flavor sector as a major challenge. Being a new fundamental space-time symmetry between fermions and bosons, SUSY necessarily extends the SM flavor structure to include superpartners for all fermions and thus adds further puzzles to the flavor physics. To be consistent with the experimental data, supersymmetry has to be broken. The breaking of the SUSY lifts the mass spectrum of the superpartners and is parametrized by the soft-breaking Lagrangian of the Minimal Supersymmetric SM (MSSM) with a large number of free parameters. The soft breaking sector is often problematic with low-energy flavor changing neutral current (FCNC) data without making specific assumptions about its free parameters. One of the most popular assumptions is the proportionality of the scalar trilinear $A$-terms to the fermion Yukawa couplings. This is however not a generic feature from the string-theory constructions, and certain forms of non-diagonal $A$-terms and their interesting implications were studied recently [2, 3].

In this work, we focus on the flavor-mixings of three-family squarks which originate from the scalar mass and the trilinear interaction terms. We observe that the current data mainly suppress the FCNCs associated with the first two family squarks (and in some cases, with the first and third families), but allow the flavor-mixings between the second- and third-family squarks, the scharm ($\tilde{c}$) and stop ($\tilde{t}$), to be as large as $O(1)$ [4]. Furthermore, the $O(1)$ $\tilde{t} - \tilde{c}$ mixings arising from the non-diagonal $A$-term are consistent with the theoretical bounds derived from avoiding the charge-color breaking (CCB) as well as maintaining the vacuum stability (VS) [5]. Taking a bottom-up approach, we first formulate a minimal SUSY FCNC scenario, called the Type-A models, in which all the observable FCNC effects come from the non-diagonal trilinear $A$-term in the $\tilde{c} - \tilde{t}$ sector. Then, using the simplest horizontal $U(1)_H$ symmetry (à la Froggatt-Nielsen [6]), we construct a class of new models, called the Type-B models which not only exhibit similar flavor-mixings in the $\tilde{t} - \tilde{c}$ sector but also generate realistic quark-mass/mixing pattern and provide a solution to the SUSY $\mu$-problem. Such minimal FCNC schemes can reduce the general $6 \times 6$ squark-mass-matrix down to a $4 \times 4$ or $3 \times 3$ matrix involving only the $\tilde{c} - \tilde{t}$ sector, and therefore simplify the exact squark mass-diagonalization and rotations without invoking the crude mass-insertion approximation. This allows quantitative understanding and predictions of the relevant FCNC signatures from the squark sector, and thus provides reliable probes to the fundamental SUSY flavor structure encoded in the soft-breaking Lagrangian.

As we will show, exploring the SUSY flavor sector is also important for revealing exciting new discovery signatures from the weak-scale supersymmetry, in addition to probing the mechanism of soft SUSY breaking. Applying the minimal FCNC schemes, we analyze the SUSY radiative corrections to the $H^{\pm}bc$ coupling and show that this flavor-mixing coupling can be significant to provide new discovery signals of charged Higgs via charm-bottom fusion [7] at the on-going Fermilab Tevatron Collider and the CERN Large Hadron Collider (LHC). Then, we further study the flavor changing top-decays into the charm-quark and lightest neutral Higgs boson in our schemes, and their observability at the LHC.

2. Minimal Supersymmetric FCNC Models

2.1. Type-A Minimal SUSY FCNC Models with Non-Diagonal $A$-Term

The MSSM soft-breaking squark-sector contains the following quadratic mass-terms and trilinear...
\[ -\tilde{Q}_i^\dagger (M_Q^2)_{ij} \tilde{Q}_j - \tilde{U}_i^\dagger (M_U^2)_{ij} \tilde{U}_j - \tilde{D}_i^\dagger (M_D^2)_{ij} \tilde{D}_j + (A^u_u \tilde{Q}_i H_u \tilde{U}_j - A^d_d \tilde{Q}_i H_d \tilde{D}_j + \text{c.c.}), \]  
\tag{1}

with \( i, j = 1, 2, 3 \) being the family indices. This gives a generic 6 \times 6 mass matrix,

\[
\begin{pmatrix}
M_{LL}^2 & M_{LR}^2 \\
M_{LR}^2 & M_{RR}^2
\end{pmatrix},
\]
\tag{2}

in the up-squark sector, where

\[
\begin{aligned}
M_{LL}^2 &= M_Q^2 + M_u^2 + \frac{1}{6} \cos 2\beta (4m_w^2 - m_z^2), \\
M_{LR}^2 &= M_U^2 + M_u^2 + \frac{2}{3} \cos 2\beta \sin^2 \theta_w m_z^2, \\
M_{RR}^2 &= A_u v \sin \beta / \sqrt{2} - M_u \mu \cot \beta,
\end{aligned}
\tag{3}

\]

with \( m_w, z \) the masses of (\( W^\pm, Z^0 \)) and \( M_u \) the up-quark mass matrix. For convenience, we will choose the super Cabibbo-Kobayashi-Maskawa (CKM) basis for squarks so that in Eq. (3), \( A_u \) is replaced by \( A'_u = K_{UL} A_u K^\dagger_{UR} \) and \( M_u \) by \( M^\text{diag}_u \), etc, with \( K_{UL, R} \) the rotation matrices for diagonalizing \( M_u \) to \( M^\text{diag}_u \). In our minimal Type-A scheme, we consider all large FCNCs to solely come from non-diagonal \( A'_u \) in the up-sector, and those in the down-sector to be negligible, i.e., we define, at the weak scale,

\[
A'_u = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & x \\
0 & y & 1
\end{pmatrix} A,
\tag{4}
\]

where, \( x \) and \( y \) can be of \( O(1) \), representing a naturally large flavor-mixing in the \( \tilde{t} - \tilde{c} \) sector. Such a minimal scheme of SUSY FCNC is compelling as it is fully consistent with the stringent CCB/VS theory bounds \( [3] \) as well as the existing data \( [3] \). (In the case that the CKM matrix is generated from the down-quark sector alone\( [3] \), \( A'_u \) simply reduces back to \( A_u \).) Similar pattern may be also defined for \( A_d \) in the down-sector, but the strong CCB/VS bounds permit \( O(1) \tilde{b} - \tilde{s} \) mixings only for very large \( \tan \beta \) because \( m_b \ll m_t \). Thus, to allow a full range of \( \tan \beta \) value, we consider an almost diagonal \( A_d \). Moreover, indentifying the non-diagonal \( A_u \) as the only source of the observable FCNC phenomena in the Type-A models implies the squark-mass-matrices \( M^2_{Q, U} \) in Eqs. (3)-(5) to be nearly diagonal. For simplicity, we define

\[
M_{LL}^2 \simeq M_{LR}^2 \simeq \frac{m_0}{\sqrt{3}} I_{3 \times 3},
\tag{5}
\]

with \( m_0 \) a common scale of scalar-masses \( [3] \).

Within this minimal Type-A scheme, we observe that the first family squarks \( \tilde{u}_{L,R} \) decouple from the rest in (3) so that the 6 \times 6 mass-matrix is reduced to 4 \times 4,

\[
\begin{pmatrix}
\tilde{m}_0^2 & 0 & 0 & A_x \\
0 & \tilde{m}_0^2 & A_y & 0 \\
0 & A_y & \tilde{m}_0^2 & X_t \\
A_x & 0 & X_t & \tilde{m}_0^2
\end{pmatrix},
\tag{6}
\]

for squarks (\( \tilde{c}_L, \tilde{c}_R, \tilde{t}_L, \tilde{t}_R \)), where

\[
A_x = x \tilde{A}, \quad A_y = y \tilde{A}, \quad \tilde{A} = Av \sin \beta / \sqrt{2}, \quad X_t = \tilde{A} - \mu m_t \cot \beta.
\tag{7}
\]
In Eq. (8), we ignore tiny terms of $O(m_c)$ or smaller. The reduced squark mass matrix \( \mathbf{B} \) has 6 zero-entries in total and is simple enough to allow an exact diagonalization. Especially, for the cases of (i) \( x \neq 0, y = 0 \), (called Type-A1) and (ii) \( x = 0, y \neq 0 \), (called Type-A2), one more squark, \( \tilde{c}_R \) (in Type-A1) or \( \tilde{c}_L \) (in Type-A2), decouples, and the Eq. (8) further reduces to a 3 \( \times \) 3 matrix. We have worked out the general diagonalization of 4 \( \times \) 4 matrix (8) for any \((x, y)\). The mass-eigenvalues of the eigenstates \((\tilde{c}_1, \tilde{c}_2, \tilde{t}_1, \tilde{t}_2)\) are,

\[
M^2_{\tilde{c}_{1,2}} = \tilde{m}_0^2 + \frac{1}{2}|\omega_+ - \omega_-|, \\
M^2_{\tilde{t}_{1,2}} = \tilde{m}_0^2 + \frac{1}{2}|\omega_+ + \omega_-|,
\]

where \( \omega_{\pm} = X_t^2 + (A_x \pm A_y)^2 \). From (8), we can deduce the mass-spectrum of the stop-scharm sector as

\[
M_{\tilde{t}_1} < M_{\tilde{c}_1} < M_{\tilde{c}_2} < M_{\tilde{t}_2}.
\]

In Eq. (8), the stop \( \tilde{t}_1 \) can be as light as about \( 100 \text{–} 300 \text{ GeV} \) for the typical range of \( \tilde{m}_0 \gtrsim 0.5 \text{–} 1 \text{ TeV} \). The 4 \( \times \) 4 rotation matrix of the diagonalization is given by,

\[
\begin{pmatrix}
\tilde{c}_L \\
\tilde{c}_R \\
\tilde{t}_L \\
\tilde{t}_R
\end{pmatrix} =
\begin{pmatrix}
c_1 c_3 & c_1 s_3 & s_1 s_4 & s_1 c_4 \\
-c_2 s_3 & c_2 c_3 & s_2 c_4 & -s_2 s_4 \\
-s_1 c_3 & -s_1 s_3 & c_1 s_4 & c_1 c_4 \\
s_2 c_3 & -s_2 s_3 & c_2 c_4 & -c_2 s_4
\end{pmatrix}
\begin{pmatrix}
\tilde{c}_1 \\
\tilde{c}_2 \\
\tilde{t}_1 \\
\tilde{t}_2
\end{pmatrix},
\]

with

\[
s_{1,2} = \frac{1}{\sqrt{2}} \left[ 1 - \frac{X_t^2 \mp A_x^2 \pm A_y^2}{\sqrt{\omega_+ \omega_-}} \right]^{1/2}, \quad s_4 = \frac{1}{\sqrt{2}},
\]

and \( s_3 = 0 \) (if \( xy = 0 \)), or, \( s_3 = 1/\sqrt{2} \) (if \( xy \neq 0 \)), where \( s_j^2 + c_j^2 = 1 \).

### 2.2. Type-B Minimal SUSY FCNC Models with a Horizontal \( U(1) \) Symmetry

The minimal Type-A SUSY FCNC schemes with a non-diagonal \( A_u \)-term, as discussed above, are truly economical as they uniquely result from imposing all the stringent theoretical and experimental bounds. Below, we further support such type of FCNC in the \( \tilde{t} - \tilde{c} \) sector by providing theoretically compelling constructions based upon a minimal family symmetry. An attractive approach is to make use of the simplest horizontal \( U(1)_H \) symmetry to generate realistic flavor structure of both quarks and squarks, via proper powers of a single suppression factor \[ \mathcal{H}_1 \mathcal{H}_2 \], and to provide a solution to the SUSY \( \mu \)-problem. For convenience, we define the suppression factor \( \epsilon = \langle S \rangle / \Lambda \) to have a similar size as the Wolfenstein-parameter \( \lambda \) in the CKM matrix, i.e., \( \epsilon \approx \lambda \approx 0.22 \). Here, \( \langle S \rangle \) is the vacuum expectation value of a singlet scalar \( S \), responsible for spontaneous \( U(1)_H \) breaking, and \( \Lambda \) is the scale at which the \( U(1)_H \) breaking is mediated to light fermions. In general, the supermultiplets of three-family quarks/squarks may carry different \( U(1)_H \) charges, as defined in Table 1.

| Table 1. Quantum number assignments under the horizontal symmetry \( U(1)_H \). |
| --- |
| \( Q_1 \) | \( Q_2 \) | \( Q_3 \) | \( \pi_1 \) | \( \pi_2 \) | \( \pi_3 \) | \( \overline{c}_1 \) | \( \overline{d}_2 \) | \( \overline{d}_3 \) | \( H_u \) | \( H_d \) | \( S \) |
| \( h_1 \) | \( h_2 \) | \( h_3 \) | \( \alpha_1 \) | \( \alpha_2 \) | \( \alpha_3 \) | \( \beta_1 \) | \( \beta_2 \) | \( \beta_3 \) | \( \xi \) | \( \xi' \) | \( -1 \) |
It is then straightforward to deduce the following hierarchy structures in the quark mass matrices:

$$M_u^{ij} \sim \frac{v_u}{\sqrt{2}} \lambda^{\alpha_i + h_j + \xi}, \quad M_d^{ij} \sim \frac{v_u}{2 \tan \beta} \lambda^{\beta_i + h_j + \xi'}.$$  \hspace{1cm} (12)

Similarly, in the CKM matrix,

$$(V_{us}, V_{cb}, V_{ub}) \sim \left(\lambda^{h_1 - h_2}, \lambda^{h_2 - h_3}, \lambda^{h_1 - h_3}\right).$$  \hspace{1cm} (13)

Unlike Ref. [9], the key ingredient of our model-buildings is to impose a new condition,

$$\alpha_2 = \alpha_3,$$  \hspace{1cm} (14)

which ensures the mixings between $\tilde{c}$ and $\tilde{t}$ in the squark mass matrix to be naturally of $O(1)$. From the condition (14) and the current data of quark-masses and CKM angles (which can all be counted in powers of $\lambda$), we find an almost unique solution for all quark/squark quantum numbers (cf., Table 2), which is based upon a single $U(1)_H$ symmetry and will be called the minimal Type-B model. In Table 2, we consider $\tan \beta \sim O(1)$ for the down-type quark mass-matrix $M_d$ [cf. Eq. (12)]. The extension to a larger $\tan \beta$ only affects the quantum numbers of $\tilde{d}_j$’s in a trivial way as it just contributes an overall factor $1/\tan \beta \sim \lambda^k$ (with the integer $k \sim 0.66 \log \tan \beta$) to $M_d$ in Eq. (12) and thus simply adds $-k$ to each quantum number of $\tilde{d}_j$ listed in Table 2.

Table 2. Quantum number assignments are derived for the minimal Type-B model with $\tan \beta \sim O(1)$.

| $Q_1$ | $Q_2$ | $Q_3$ | $\bar{u}_1$ | $\bar{u}_2$ | $\bar{u}_3$ | $\bar{d}_1$ | $\bar{d}_2$ | $\bar{d}_3$ | $H_u$ | $H_d$ | $S$ |
|------|------|------|----------|----------|----------|--------|--------|--------|------|------|-----|
| 4    | 3    | 0    | 3 - $\xi$| $-\xi$   | $-\xi$   | 4 - $\xi'$| 3 - $\xi'$| 3 - $\xi'$| $\xi$ | $\xi'$| -1  |

There are some slight variations of this minimal Type-B model, by allowing the quantum numbers of $Q_j$’s to have $\xi(\xi')$-dependence, but they all predict the same patterns for the quark masses and mixings. By this construction, we have attempted to simultaneously solve the SUSY $\mu$-problem from the same $U(1)_H$. A dynamical $\mu$-term can originate from $(\kappa/\Lambda^{n-1}) S^n H_u H_d$, with $n = \xi + \xi'$, such that $\mu = \kappa \lambda^{n-1} \langle S \rangle$ is generated at a scale $\langle S \rangle \ll M_{\text{Planck}}$. Hence, a weak-scale value of $\mu$ can be obtained by properly choosing $n$ for a given $\langle S \rangle$. If $U(1)_H$ is not related to the SUSY $\mu$-problem, the minimal Type-B model becomes truly unique, corresponding to a special case of $\xi = \xi' = 0$ in Table 2. With Table 2, we can readily derive the structures of quark/squark mass-matrices. For instance, the up-quark mass-matrix takes the form of

$$M_u \sim \frac{v_u}{\sqrt{2}} \begin{pmatrix} \lambda^7 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^3 & \lambda^3 \\ \lambda^3 & 1 & 1 \end{pmatrix},$$  \hspace{1cm} (15)

for any $\tan \beta \gtrsim 1$, while the squark mass-matrices $M_{LL}^2$ and $M_{RR}^2$ in Eq. (2) are deduced as,

$$M_{LL}^2 \sim \tilde{m}_0^2 \begin{pmatrix} 1 & \lambda & \lambda^4 \\ \lambda & 1 & \lambda^3 \\ \lambda^4 & \lambda^3 & 1 \end{pmatrix}, \quad M_{RR}^2 \sim \tilde{m}_0^2 \begin{pmatrix} 1 & \lambda^3 & \lambda^3 \\ \lambda^3 & 1 & 1 \\ \lambda^3 & 1 & 1 \end{pmatrix},$$  \hspace{1cm} (16)
Eqs. (15) and (16) show a partial quark-squark “alignment” that effectively suppresses the FCNCs between 1st and 2nd(3rd) families, while at the same time provides $O(1)$ mixings in the $\tilde{t} - \tilde{c}$ sector of $M_{RR}^2$. Furthermore, because squarks carries the same $U(1)_H$-charge as quarks, the $\tilde{t} - \tilde{c}$ mixings originating from a non-diagonal $A_u$ term is predicted to share the same hierarchy structure as that in $M_u$ [cf. Eq. (15)]. This, however, does not imply an exact “proportionality” between $A_u$ and $M_u$ because the power-counting of $\lambda$ allows the coefficients in the corresponding entries of $A_u$ and $M_u$ to differ by ratios of $O(1)$. Ignoring the small $O(\lambda^3) \approx 1\%$ terms in $M_u$, we can diagonalize $M_u$ by a $2 \times 2$ rotation of the singlet quarks ($\tilde{t}, \tilde{c}$). Under the super-CKM basis, the off-diagonal block $M_{LR}^2$ of the Eq. (2) becomes

$$M_{LR}^2 = A_u' v \sin \beta / \sqrt{2} - M_u^{\text{diag}} \mu \cot \beta ,$$

where $A'_u = K_{UL} A_u K_{UR}^\dagger = A_u K_{UR}^\dagger + O(\lambda^3)$ and the singlet-quark rotation matrix $K_{UR}$ contains a nontrivial sub-matrix involving only the second and the third family squarks. After neglecting the tiny $O(\lambda^3)$ terms, we can parametrize the minimal $A'_u$ term of the Type-B model as

$$A'_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & y & 1 \end{pmatrix} A ,$$

where the size of the mixing parameter $y$ is naturally of $O(1)$. Thus, under this construction, the squarks ($\tilde{u}_L, \tilde{u}_R, \tilde{c}_L$) decouple and the Eq. (4) greatly reduces to a $3 \times 3$ matrix, which takes the form, under the basis ($\tilde{c}_R, \tilde{t}_L, \tilde{t}_R$),

$$\tilde{M}_{\tilde{c}t}^2[B] = \begin{pmatrix} \tilde{m}_0^2 & A_y & x\tilde{m}_0^2 \\ A_y & \tilde{m}_0^2 & X_t \\ x\tilde{m}_0^2 & X_t & \tilde{m}_0^2 \end{pmatrix} ,$$

In the above equation, $A_y = y A v \sin \beta / \sqrt{2}$, and the parameter $x = O(1)$ characterizes the mixing of $\tilde{c}_R - \tilde{t}_R$ in the mass matrix $M_{RR}^2$ [cf., Eq. (16)]. For convenience, we define the typical case with $y = 0$ as the Type-B1 scheme, and that with $x = 0$ as the Type-B2 scheme. We note that the Type-B2 model is identical to the Type-A2 model as they have the same form of the non-diagonal $A_u$ and $\tilde{c}_L$ decouples. Also, Type-B2 mass matrix with $y = 0$ in Eq. (19) takes the same structure as that of Type-A1 except that Eq. (19) involves $\tilde{c}_R$ (not $\tilde{c}_L$) and its $x$ originates from $M_{RR}^2$ (not $A_u$). Without losing generality, we will study the physical applications of the typical Type-A1 and -A2 (-B2) models in the following sections.

3. SUSY Radiative Corrections and Higgs Signatures at Colliders

3.1. SUSY Induced $H^\pm bc$ Vertex and $H^\pm$ Production at Hadron Colliders

Different from the TopColor models and the Type-III of two-Higgs-doublet models discussed in Ref. [7], the MSSM Higgs sector has no FCNC at the tree level and the same is true for flavor-changing mixings in the charged sector (except the usual CKM mechanism). Thus, the new flavor-changing effects in the neutral and charged sectors of the MSSM have to be generated radiatively. From Eq. (10) and its resulted Feynman rules, we can compute the dominant SUSY-QCD radiative corrections to the $H^\pm bc$ coupling. It contains vertex corrections [scharm(stop)-sbottom-gluino loop]
and self-energy corrections [scharm(stop)-gluino loop], and can be summarized as,

\[ \Gamma_{H^+\ell\ell} = i \bar{u}_c(k_2) (F_L P_L + F_R P_R) u_b(k_1), \]

\[ F_{L,R} = F_{L,R}^0 + F_{L,R}^V + F_{L,R}^S, \]

where \( P_{L,R} = (1 \mp \gamma_5)/2 \) and the tree-level results are

\[ (F_L^0, F_R^0) = \frac{g V_{cb}}{\sqrt{2} m_w} (m_c \cot \beta, m_b \tan \beta), \]

with \( V_{cb} \) being the CKM mixing matrix element involving \( c \) and \( b \) quarks. The one-loop vertex corrections in the Type-A1 model give,

\[ F_L^V = 0, \]

\[ F_R^V = \frac{\alpha_s}{3\pi} m_{\tilde{g}} \sum_{j,k} \kappa_{jk}^R C_0(m_H^2, 0, 0; m_{\tilde{b}_j}, m_{\tilde{g}}, m_{\tilde{u}_k}), \]

where \( \tilde{u}_k \in (\tilde{c}_2, \tilde{t}_1, \tilde{t}_2), \tilde{b}_j \in (\tilde{b}_1, \tilde{b}_2) \), and \( C_0 \) is the 3-point \( C \)-function of Passarino-Veltman \([10]\). \( \kappa_{jk}^R \) is the product of the relevant \( H^+\tilde{b}_j\tilde{u}_k \), \( \tilde{b}_j\tilde{g}\tilde{b} \) and \( \tilde{u}_k\tilde{g}\tilde{c} \) couplings. The \( \tilde{b}_L - \tilde{b}_R \) mixings are also included, which can be sizable for large \( \tan \beta \). Furthermore, the Type-A1 self-energy corrections yield

\[ F_L^S = 0, \]

\[ F_R^S = \bar{F}_R^0 \frac{\alpha_s s_1}{3\pi} m_{\tilde{g}} \sum_{j=1,2} (-)^{j+1} B_0(0; m_{\tilde{g}}, m_{\tilde{t}_j}), \]

where \( B_0 \) is the 2-point Passarino-Veltman function \([10]\) and \( s_1 \) is given in Eq. \([10]\) for \( y = 0 \), and tree-level \( H^+tb \) couplings \( F_{L,R}^0 = (g V_{tb}/\sqrt{2} m_w)(m_t \cot \beta, m_b \tan \beta) \). In Eqs. \([22]-[23]\), the tiny sub-leading terms suppressed by the powers of \( m_{\tilde{c}}/m_{\tilde{g}} \) have been ignored.

The form factors \( F_{L,R}^{V,S} \) for the Type-A2 and Type-B2 models can be easily obtained from Eqs. \([22]-[23]\) by the exchanges of \( L \leftrightarrow R \) and \( x \rightarrow y \). We note that \( F_L (F_R) \) vanishes in Type-A1 (Type-A2 & -B2) schemes because \( \tilde{c}_R (\tilde{c}_L) \) decouples.

We find that the effective \( H^+bc \) couplings \( F_{L,R} \) are typically around 0.03 – 0.1 for \( (x, y) \simeq 0.5 – 0.9 \), \( (A, \tilde{m}_0) \simeq 0.5 – 2 \text{ TeV} \), and \( \tan \beta \simeq 15 – 50 \). Therefore, the production of a charged Higgs boson via the \( b\)-\( c \) quark fusion process can become important in a wide range of the parameter space. In Fig. 1(a), the production cross sections of \( H^\pm \) via \( p\bar{p}/pp \rightarrow H^\pm X \) at the Tevatron and LHC are shown for \( (\mu, m_{\tilde{g}}, \tilde{m}_0) = (300, 300, 600) \text{ GeV}, (A, -A_b) = 1.5 \text{ TeV}, \tan \beta = (15, 50), \) and \( x = 0.75 \) in the Type-A1 model. The dotted and dash curves are the SM production rates of \( cs \rightarrow H^\pm \) and \( cb \rightarrow H^\pm \) (induced by \( V_{cb} \simeq 0.04 \)), respectively, after including the complete next-to-leading order (NLO) SM-QCD corrections \([11]\). (The NLO SM-QCD corrections include the subprocesses with one single gluon in the initial state.) The solid curve is the full \( cb \rightarrow H^\pm \) production rate as a function of the Higgs boson mass after including both the SM-QCD and new SUSY-QCD corrections. As indicated, the SUSY loop corrections can significantly dominate over the CKM-suppressed \( F_{L,R}^0 \) contributions by a factor of \( \sim 2 – 5 \). In Fig. 1(b) and (e), we show the \( K \)-factor, defined as the ratio of \( (F_L^2 + F_R^2) \) over \( (F_L^{02} + F_R^{02}) \), which characterizes the enhancement of the \( H^\pm \) production rate by the SUSY loop contributions.
To study the detection of a $H^\pm$ scalar, we consider its decay modes in the following. For $m_H \lesssim 190$ GeV, the $\tau \nu$ channel dominates, and for $m_H \gtrsim 190$ GeV, $H^\pm$ mostly decays into the $tb$ channel unless $H^\pm$ mass is above the threshold of $W^\pm h^0$, in which case the $W^\pm h^0$ channel (with $W \to \ell \nu$ and $h^0 \to \tilde{b} \tilde{b}$) can become important as well [11]. Fig. 1 suggests that the Tevatron may be sensitive to the $H^\pm$ signals for $m_H$ below $\sim 300$ GeV, with an integrated luminosity of $2 - 20$ fb$^{-1}$ per detector, while the LHC can potentially probe the full mass-range of $H^\pm$ with an integrated luminosity of about $100$ fb$^{-1}$. A detailed Monte Carlo simulation is needed to further quantify the discovery potential of the Tevatron and the LHC, which is beyond the scope of this Letter.

We have also examined the similar SUSY-induced enhancement $K$-factor for the $H^\pm$ production rate in the type-A2 and -B2 models and found that in the low tan $\beta$ ($\lesssim 5 - 10$) region it can reach to about $2 - 5$ for $y = 0.5 - 0.9$. But, as tan $\beta$ increases to above $\sim 15$, the enhancement factor decreases to less than $\sim 1.3$ for Higgs mass below 1 TeV.

3.2. SUSY Induced $h^0tc$ Vertex and Neutral Higgs Signal from Top Decay

It is known that the SM branching ratio of the flavor-changing top decay $t \to ch^0$ is extremely small ($\lesssim 10^{-13} - 10^{-14}$) [12], so that this channel becomes an excellent window for probing new physics [13, 14, 15]. Our minimal SUSY FCNC models predict the branching ratio of the $t \to ch^0$ decay to be substantially above the SM value so that this decay mode becomes observable at the LHC. (Note that this decay channel is always kinematically allowed in the MSSM.)

In our minimal FCNC schemes, the one-loop SUSY QCD induced $tch^0$ coupling can be written as

$$\Gamma_{tch} = i \bar{u}_c(k_2)(F_L P_L + F_R P_R) u_t(k_1),$$

$$F_{L,R} = F_{L,R}^V + F_{L,R}^S,$$

which contains the vertex corrections (from scharm-stop-gluino, stop-stop-gluino and scharm-scharm-gluino, triangle loops), and the self-energy corrections (from stop-gluino and scharm-gluino loops). The one-loop vertex corrections in Type-A1 are,

$$F_L^V = \frac{\alpha_s}{3\pi} \sum_{j,k} \lambda^{L}_{jk} m_t (C_0 + C_{11})(m_h^2, m_{t_1}^2, 0; m_{\tilde{u}_j}, m_{\tilde{g}}, m_{\tilde{u}_k}),$$

$$F_R^V = \frac{\alpha_s}{3\pi} \sum_{j,k} \lambda^{R}_{jk} m_{\tilde{g}} C_0(m_h^2, m_{t_1}^2, 0; m_{\tilde{u}_j}, m_{\tilde{g}}, m_{\tilde{u}_k}),$$

where $\tilde{u}_{j,k} \in (\tilde{c}_2, \tilde{t}_1, \tilde{t}_2)$, and $(C_0, C_{11})$ are the 3-point $C$-function of Passarino-Veltman. $\lambda^{L,R}_{jk}$ is the product of the relevant $h - \tilde{u}_j - \tilde{u}_k$ and $\tilde{u}_k - \tilde{g} - t(c)$ couplings, derived from applying the squark-rotation [10]. The Type-A1 self-energy corrections yield,

$$F_L^S = 0,$$

$$F_R^S = \tilde{F}_0 \frac{\alpha_s s_\theta}{3\pi} \frac{m_{\tilde{g}}}{m_t} [B_0(0; m_{\tilde{g}}, m_{t_2}) - B_0(0; m_{\tilde{g}}, m_{t_1})],$$

where $B_0$ is the 2-point Passarino-Veltman function and $s_\theta$ is given in Eq. [11] with $y = 0$. $\tilde{F}_0$ denotes the tree-level $h^0 - t - \tilde{t}$ coupling, and is given by $\tilde{F}_0 = (m_t/v)(\cos \alpha / \sin \beta)$. Again, in Eqs. (25)-(26), we have ignored the tiny sub-leading terms suppressed by the powers of $m_c/m_{t,\tilde{g}}$. 

7
The form factors $F_{L,R}^{V,S}$ in the Type-A2 model can be obtained from (24)-(26) by the exchanges of $L \leftrightarrow R$ and $x \rightarrow y$ everywhere.

For the numerical study, we assume that the only dominant decay mode of the top quark is its SM decay mode, $t \rightarrow bW$. Thus, the decay branching ratio of $t \rightarrow c h^0$ is given by, $\text{Br}[t \rightarrow c h^0] \simeq \Gamma[t \rightarrow c h^0]/\Gamma[t \rightarrow bW]$, with the partial decay width

$$\Gamma(t \rightarrow c h) = \frac{m_t}{16\pi} \left[1 - \frac{m_h^2}{m_t^2}\right]^{\frac{1}{2}} \left(F_L^2 + F_R^2\right),$$

(27)

where the form factors $(F_L, F_R)$ are defined in Eq. (24). As summarized by Table 3, in our minimal SUSY-FCNC schemes, the decay branching ratio $\text{Br}[t \rightarrow c h^0]$ can be as large as $10^{-3} - 10^{-5}$ over a large part of the SUSY parameter space where the mass of the lightest Higgs boson $h^0$ is around 110 – 130 GeV. We see that these decay branching ratios are very sensitive to the mixing parameter $x$ when it varies from 0.5 to 0.9. One reason is that the branching ratio (or decay width) contains, besides other mass-diagonalization effects, a sensitive overall power factor $x^2$ which comes from the stop-scharm mixing induced flavor-changing coupling in the squark-squark-gluino triangle loop and squark-gluino self-energy loop. Another reason is that unlike the usual analyses with mass-insertion approximation, we have performed exact squark mass diagonalization [cf. Eqs. (8)-(11)], so that stops and/or scharms can have significant mass-splittings. For instance, Eq. (8) shows that the mass-splitting between two top-squarks is not just due to the usual left-right mixing from $X_t$, but also arises from the non-diagonal $A$-term, $xA$ (and $yA$). The latter further enhances the mass-splittings and results in a light $\tilde{t}_1$ of mass $\sim 300 – 100$ GeV for $x = 0.5 – 0.9$, and the heavier $\tilde{t}_2$ always has mass above the input $\tilde{m}_0$ (set as 600 GeV in Table 3). The radiative vertex corrections to $tch^0$ coupling are thus dominated by the diagram with $\tilde{t}_1 - \tilde{t}_1 - \tilde{g}$ triangle loop as we have explicitly verified from the 3-point Passarino-Veltman $C$-functions in Eq. (25). This second reason further increases the sensitivity of our decay branching ratios to the mixing parameter $x$. From Table 3, we also see that for moderate mixings with $x \lesssim 0.5$, the branching ratios are generally bounded to around the order of $10^{-5}$, consistent with other studies in the literature [13]. The similar conclusion also holds for our Type-A2 and -B2 models.

Since the LHC with an integrated luminosity of 100 fb$^{-1}$ can produce about $10^8$ $t$ and $\bar{t}$ pairs [14], it can have a great sensitivity to discover this decay channel and test the model predictions, by demanding one top decaying into the usual $bW^\pm$ mode and another to the FCNC $c h^0$ mode. As shown in a recent model-independent Monte Carlo analysis [14], the LHC (100 fb$^{-1}$) can already measure the $\text{Br}[t \rightarrow c h^0]$ down to the level of $4.5 \times 10^{-5}$ at the 95% C.L. The future Linear Collider, with a high luminosity, is also expected to have a good sensitivity to detect this channel.

Table 3. $\text{Br}[t \rightarrow c h^0] \times 10^3$ is shown for a sample set of Type-A1 inputs with $(\tilde{m}_0, \mu, A) = (0.6, 0.3, 1.5)$ TeV and Higgs mass $M_{A^0} = 0.6$ TeV. The three numbers in each entry correspond to $x = (0.5, 0.75, 0.9)$, respectively.

| $m_{\tilde{g}}$ | $\tan\beta = 5$ | 20 | 50 |
|-----------------|-----------------|----|----|
| 100 GeV         | (0.011, 0.10, 0.81) | (0.015, 0.19, 4.6) | (0.016, 0.21, 7.0) |
| 500 GeV         | (0.011, 0.09, 0.41) | (0.015, 0.13, 1.0) | (0.016, 0.14, 1.2) |
4. Conclusions

The three-family squark mass-matrix originating from the soft SUSY breaking sector contains a rich flavor-mixing structure. In this work, we have constructed the minimal FCNC schemes for the squark mass-terms and the scalar trilinear interactions which are consistent with the existing experimental and theoretical bounds. We find that the $O(1)$ large mixings among the top- and charm-squarks are allowed. We demonstrate that this feature can be naturally realized in a class of new models with a horizontal $U(1)$ symmetry which also generate realistic quark-mass pattern and solve the SUSY $\mu$-problem. Finally, we systematically analyze the dominant supersymmetric radiative corrections to the $bcH^\pm$ and $tch^0$ couplings in our minimal schemes, without using the mass-insertion approximation. We show that these couplings can be significant to provide new discovery signatures of the charged and neutral Higgs bosons at the Fermilab Tevatron and the CERN LHC.

Acknowledgments

We thank G. L. Kane, D. A. Dicus, T. Han and Y. Nir for valuable discussions and reading the manuscript. This work was supported by U.S. DOE and NSF.

References

[1] See, for instance, recent reviews in “Perspectives on Supersymmetry”, ed. G.L. Kane, World Scientific Publishing Co., 1998; H. E. Haber, Nucl. Phys. Proc. Suppl. 101, 217 (2001), [hep-ph/0103095].

[2] E.g., S. Khalil, J. Phys. G27, 1183 (2001), [hep-ph/0011331]; D. F. Carvalho, M. E. Gomez, S. Khalil, hep-ph/0104292; and references therein.

[3] A. Masiero and H. Murayama, Phys. Rev. Lett. 83, 907 (1999), [hep-ph/9903363].

[4] For a nice review, M. Misiak, S. Pokorski, J. Rosiek, “Supersymmetry and FCNC Effects”, [hep-ph/9703442], in Heavy Flavor II, pp. 795, eds., A. J. Buras and M. Lindner, Advanced Series on Directions in High Energey Physics, World Scientific Publishing Co., 1998, and references therein.

[5] J. A. Casas and S. Dimopolous, Phys. Lett. B387, 107 (1996), [hep-ph/9606237].

[6] C. D. Frogatt and H. B. Nielsen, Nucl. Phys. B147, 277 (1979).

[7] H.-J. He and C.-P. Yuan, Phys. Rev. Lett. 83, 28 (1999), [hep-ph/9810367].

[8] E.g., Y. Nir and N. Seiberg, Phys. Lett. B309, 337 (1993), [hep-ph/9304307].

[9] Y. Nir, M. Leurer, and N. Seiberg, Nucl. Phys. B 309, 337 (1993), [hep-ph/9212278]; Nucl. Phys. B 420, 468 (1994), [hep-ph/9310320].

[10] G. Passarino and M. Veltman, Nucl. Phys. B160, 151 (1979).

[11] C. Balazs, H.-J. He, C.-P. Yuan, Phys. Rev. D60, 114001 (1999), [hep-ph/9812263].
[12] B. Mele, S. Petrarca, and A. Soddu Phys. Lett. B435, 401 (1998), [hep-ph/9805498];
G. Eilam, J.L. Hewett and A. Soni, Phys. Rev. D44, 1473 (1991); D59, 039901 (1999) (E).

[13] For recent reviews, see, e.g., J. Solà, talk at the 5th International Symposium on Radiative Corrections, Carmel, CA, USA, Sept. 11-15, 2000, [hep-ph/0101294]; B. Mele, talk at the Workshop of High Energy Physics and Quantum Field Theory, Moscow, May 27 - June 2, 1999, [hep-ph/0003064]; and references therein.

[14] J. A. Aguilar-Saavedra and G. C. Branco, Phys. Lett. B495, 347 (2000), [hep-ph/0004190].

[15] For a very recent study of $t \to ch^0$ decay in the non-SUSY 2HDM (type-III), see, T. Han, J. Jiang, and M. Sher, Phys. Lett. B516, 337 (2001), [hep-ph/0106277]; and in the $R$-parity-violation SUSY model, see, e.g., G. Eilam, A. Geminter, T. Han, J. M. Yang, and X. Zhang, Phys. Lett. B510, 227 (2001), [hep-ph/0102037]; and references therein. For extensive lists of early studies, see the recent nice reviews in Ref. [13].

[16] For a recent review, see, e.g., M. Beneke, et al., “Top Quark Physics”, in the Report of the 1999 CERN Workshop on SM physics (and more) at the LHC, [hep-ph/0003033].
Figure 1: (a). $H^\pm$ production via $cb$ (and $cs$) fusions at hadron colliders, with sample inputs $\tan\beta = 15$ (50) shown as lower (upper) set of curves, and $x = 0.75$. (b) and (c). Factor $K \equiv (F_L^2 + F_R^2) / (F_L^0 + F_R^0)$ for $H^\pm bc$ vertex, as a function of parameter $x$ and for $\tan\beta = (15, 50)$. 