Fast Automatic Matching Method on Multi-view Point Clouds For Rail Tanker
Tao Fu 1,2, Weijun Liu 1 and Jibin Zhao 1
1Shenyang Institute of Automation, Chinese Academy of Science Shenyang, Liaoning Province, China
2Graduate University of the Chinese Academy of Sciences Beijing, China
futao@sia.cn, wjliu@sia.cn, jbzhao@sia.cn

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Abstract. In this paper, a novel method is proposed for automatically matching the measured data of a rail tanker. The spindle orientation of point cloud is determined by PCA, and then any relative point pair is well matched according to the spatial topology relations as the complexity reduction point of view, finally the data is mapped to a uniform coordinate system with the Quaternion method. The experimental results demonstrated that the proposed method achieves simplicity of implementation with robustness, and especially suitable for positioning the work piece of the rail tanker and the multi-view data fusion.

1.Introduction
Railway Tanker is a test of the compulsory verification measurement instrument. It is also a closely transportation equipment related to railway safety. In the measurement of three-dimensional objects such as railway tanker, they are in need of many sub-block measures because of the limit of measurement area. The surface is often divided into multiple partial overlapping sub-regional. A number of independent point cloud data measured from different perspectives can be called multi-view point cloud. Reverse objects need to restore the shape of the object from independent point cloud. So matching technique can be used. Since there is no clear corresponding relationship between the overlap regions, matching is still a very difficult problem.

Domestic and foreign scholars have done a lot of research aimed to matching problem. Besl and McKay[1] proposed Iterative Closest Point (referred to as ICP) algorithm is the most widely used. The idea of this algorithm is to find the corresponding point in two matching models. And then you can adopt pure Newton optimization method to get the minimum distance point to find a suitable rigid transformation. ICP algorithm does not specify the corresponding features, but requires good initial input data, after iteration to get a global optimal result. Then how to search for the correct corresponding point is the key to converge to global optimal solution. Ying Liang Ma and Hewitt[2] proposed that a point can be projected onto the surface of the model to find the closest point. In addition, some scholars also presented some matching algorithms based on features[3-5]. But we must accurately estimate as much as possible for each measuring point of the curvature of features. Although the matching algorithm based on reference point is convenient and simple, it needs to manually set the reference point.

On the basis of the above work, the paper makes a further study to fast matching method of the railway tanker. A more simple and fast matching algorithm will be proposed. The algorithm is successfully reducing the search scope of points and cumbersome iterative calculation, and raises operational efficiency. Ultimately, it achieves the measurement data’s fast matching of the railway tanker.

2.Measurement data of tanker acquisition
Three-dimensional computer vision system is able to obtain image information from camera to restore the 3-D objects in the real world. This article has been using a calibrated digital camera to rail tanker for information acquisition, and then get 3-D model of the tanker based on the basic principles
of computer vision. In the measurement process, we use two types of feature points: coding point and identification points, shown in Fig.1. Coding points is mainly used for stereo matching sequence of images and restoring three-dimensional coordinates of points. This paper discusses the fast automatic matching method for piecewise point clouds of a tanker.

Fig.1 Coding points and identification points for 3-D measurement

3. Axis direction calculation

The measuring point cloud of the railway tanker has thousands and even tens of thousands of points. For large point cloud data, we first used the method based on principal component analysis (PCA) [6] to get its principal axis, and further to its endpoints, and then selected the surrounding data points from the endpoint part to find the corresponding points. This can greatly improve the search efficiency.

Fig.2 The principal axis of the rail tanker

Suppose \( \{ p_i \}_{i=1}^n \) is a measurement datum for the tanker, its covariance matrix can be expressed as follows:

\[
H = \frac{1}{m-1} \sum_{i=1}^{m} (p_i - \mu_p)(p_i - \mu_p)^T
\]

(1)

\[
\mu_p = \frac{1}{m} \sum_{i=1}^{m} p_i
\]

(2)

Where \( H \) is a 3\times3 real symmetric matrix and \( \mu_p \) is the centroid of \( \{ p_i \}_{i=1}^n \). Suppose \( \nu(\nu^T\nu=1) \) is a 3-D unit column vector, the projection of vector \( \{ p_i - \mu_p \} \) in the direction \( \nu \) is \( \{ \beta_i : \nu^T(p_i - \mu_p) \} \) and the average of \( \{ \beta_i^2 \}_{i=1}^n \) is

\[
D = \nu^T H \nu
\]

(3)
Where $D$ is the function of the unit vector $v$. When $v$ takes a particular direction, $D$ can obtain the maximum value, that is the squares of $\{\beta_i\}$ the projection of $\{p_i - \mu_s\}$ in this direction reaches its maximum. And on the tanker, its center axis coincides with the direction. Next, we shall prove that the direction of the feature vector which the largest eigenvalue of matrix $H$ coincides with the center axis of the railway tanker. First, we can construct the Lagrange function as follows:

$$\varphi = v^T Hv - \lambda (v^T v - 1)$$

(4)

Where $\lambda$ is the Lagrange multiplier. If the function is able to achieve extreme value in the constraints of $v^T v = 1$, it must satisfy $\partial \varphi / \partial v = 0$, that is to satisfy $(H - \lambda I)v = 0$. If this formula has a solution, $H - \lambda I$ must be a singular matrix. In other words, $\lambda$ must satisfy the formula as following:

$$|H - \lambda I| = 0$$

(5)

Obviously, the solution $\lambda$ of the above formula is the eigenvalue of the covariance matrix $H$. Multiplied by the vector $v^T$ on both sides of the equation $(H - \lambda I)v = 0$, it can be $v^T Hv = \lambda$. Thus, the largest eigenvalue $\lambda_{\text{max}}$ of matrix $H$ is the maximum of the average of $\{\beta_i^2\}_{i=1}^m$. On the tanker concerned, the feature vector corresponding to $\lambda_{\text{max}}$ is namely the central axis of tanker.

4. Corresponding points searching

Firstly, as regards the measurement point clouds of a tanker, we choose blocks of them which have a nearest distance with the two endpoints along the direction of the principal axis. Then, we can find the corresponding points for the block of point clouds.

5. Description of point sets

Any set of points is composed of the following three parts:

1. A group of elements containing basic features: point, edge and surface, and they form a set. For example, a set of points shown in Fig.3, consists of four points, six edges and four surfaces.

$$F = \{a_1, \cdots, a_s; l_1, \cdots, l_k; s_1, \cdots, s_l\}$$

![Fig.3 A set of points](image)

(a) a set of points named A  (b) a set of points named B

Fig.3 A set of points

2. Spatial relationships of the group of elements: for example, the element of edge is made up of point. Fig.3 is still an example and the edges can be defined by the relationship of $R_{\text{edge}}$:

$$R_{\text{edge}} = \{(a_1, a_2); (a_1, a_3); (a_1, a_4)\}$$

$$\{(a_2, a_3); (a_2, a_4); (a_3, a_4)\}$$

The surfaces can also be defined by the relationship of $R_{\text{area}}$:

$$R_{\text{area}} = \{(l_1, l_2); (l_1, l_3); (l_1, l_4)\}$$

$$\{(l_2, l_3); (l_2, l_4); (l_3, l_4)\}$$
The nature of the elements mainly refers to geometric properties, such as coordinates of points, length of edge, area of surface and so on.

In this paper, we describe a point set using a kind of hierarchical structure shown in Fig. 4. The top-level is the triangles formed by connecting any 3 points; the middle-level is the three edges of the triangles; the most lower-level is the vertexes of the edges. The structure consists of all the three parts described above.

**Fig. 4 Description point sets using hierarchy**

6. **Corresponding points searching**

In order to obtain corresponding relationship between point sets, we need to design a transformation $h$, which can map the point sets A to B. And then we should compare the similarity of characteristics of the nature of the elements. The transformation with the largest similarity identifies the relationship between the two point sets. For example, we assume that there is a transformation $h_1$ for the point sets A and B and $h_1$ is:

\[
a_1 \rightarrow b_1, a_2 \rightarrow b_2, a_3 \rightarrow b_3, a_4 \rightarrow b_4;
\]

Then the two point sets will have the greatest degree of similarity.

Corresponding point searching is to find the best one in a set of transformation $H = \{h_1, h_2, \ldots, h_n\}$, which makes the two point sets with the greatest degree of similarity. This process is called relationship matching. Concentrated on two points, a very long time may be consumed in finding the best matching because the possible matching point will be a lot and not every point can find its mapping point in the other sets. In order to speed up the calculation[7], this paper presents a new way to search the best corresponding point as follows:

1. Firstly, compare the areas of triangles. We can determine the compatibility of the two surface areas according to the equation (6), where $A_1$ and $A_2$ are the area of the triangles, and $T_A$ (subscript A represents area) is the pre-set threshold area. So we can find all the compatible triangular area.

\[
-T_A < \frac{A_1 - A_2}{\min(A_1, A_2)} < T_A
\]  

(6)

2. Secondly, we perform the equation (7) to find the number of compatible edges $N_{\text{edge}}$ from the compatible area, where $L_1$ and $L_2$ are for the length of the edges of the triangle and $T_L$ (subscript L represents length) is for the pre-set threshold length. If $N_{\text{edge}} = 3$, the two triangles are compatible.

\[
-T_L < \frac{L_1 - L_2}{\min(L_1, L_2)} < T_L
\]  

(7)

3. If we have a compatible triangle, we can record the correspondence of the three pairs of points; if not, the two point sets can not match.

Area threshold and length threshold are the two key parameters to find the best match in this method. They are closely linked with the measurement range, the measurement error and the statistical distribution of error of the measurement system. In practice, they need to be set according to the specific conditions of the equipment. In the matching example of rail tanker, area threshold and length threshold are set at 2.5% and 5%.

Through the above steps, we can find the corresponding points quickly and reliably. For an accurate and reliable measurement system, a pair of points can be found in a number of compatible triangles when the number is greater than one. However, a point can only find one corresponding point.
7. Transformation matrix solution

We assume that the two corresponding point sets are \( P = \{ p_i \in \mathbb{R}^3, i = 1, \ldots, n_p \} \) and \( X = \{ x_i \in \mathbb{R}^3, i = 1, \ldots, n_x \} \) and \( n_p = n_x \). Next, we will use the method of quaternion to obtain the transformation matrix.

Unit quaternion is a vector of the array consisting of four elements.

\[
q = [q_0, q_1, q_2, q_3]^T
\]

Where \( q_0 \geq 0 \) and \( q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \).

The quaternion can generate a \( 3 \times 3 \) rotation matrix

\[
R = \begin{bmatrix}
q_0^2 + q_2^2 - q_3^2 - q_1^2 & 2(q_1q_3 + q_0q_2) & 2(q_2q_0 - q_1q_3) \\
2(q_1q_3 - q_0q_2) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_3q_1 - q_2q_0) \\
2(q_2q_0 + q_1q_3) & 2(q_3q_1 + q_2q_0) & q_0^2 - q_1^2 - q_2^2 + q_3^2
\end{bmatrix}
\]

Set \( q_r = [q_4, q_5, q_6]^T \) be a translation vector. The vector \( q \) with a fully aligned state can be expressed as \( q = [q_r | q_t]^T \). An objective function with least squares is expressed as

\[
F(q) = \frac{1}{n_p} \sum_{i=1}^{n_p} \| x_i - R(q_r)p_i - q_r \|^2
\]

The censored of the measurement points and the target point sets are defined as:

\[
\mu_p = \frac{1}{n_p} \sum_{i=1}^{n_p} p_i
\]
\[
\mu_x = \frac{1}{n_x} \sum_{i=1}^{n_x} x_i
\]

The orthogonal covariance matrix of the point sets is

\[
\Sigma_{px} = \frac{1}{n_p} \sum_{i=1}^{n_p} (p_i - \mu_p)(x_i - \mu_x)^T
\]

The cyclic components of the anti-symmetric matrix \( A_y = (\Sigma_{px} - \Sigma_{yx}) \) can constitute a column vector \( \Delta = [A_{y3}, A_{y1}, A_{y2}]^T \). And the column vector can construct a \( 4 \times 4 \) symmetric matrix denoted \( Q(\Sigma_{px}) \).

\[
Q(\Sigma_{px}) = \begin{bmatrix}
tr(\Sigma_{px}) & \Delta^T \\
\Delta & \Sigma_{px} + \Sigma_{yx} - tr(\Sigma_{px})I_3
\end{bmatrix}
\]

Where \( I_3 \) is a unit \( 3 \times 3 \) matrix. The unit eigenvector \( q_e = [q_0, q_1, q_2, q_3]^T \) which is corresponding with the largest eigenvalue of the matrix \( Q(\Sigma_{px}) \) will be chosen as the optimal rotation matrix. And the translation matrix can be derived by the following formula:

\[
q_r = \mu_x - R(q_e)\mu_p
\]
8. Implementation and results

In this paper, we use the method above to match the two point clouds of the rail tanker.

(a) Original position with a smaller principal axis angle (15°)
(b) Matching results (15°)

(c) Original position with a larger principal axis angle (90°)
(d) Matching results (90°)

Fig.5 Matching example of the rail tanker

(a) and (c) in Fig.5 have the same two point clouds with different angle between the two principal axis, and there is an overlap between them. Firstly, we find the corresponding pairs of points in the two point sets using the method above. And then we can use them to obtain the rotation matrix and the translation vector for the rigid transformation. Finally, the two matrixes can be applied to any of the two point clouds. (b) and (d) in Fig.5 are the matching results. But we haven’t done any homogenization to the overlap. Table 1 shows the distance error between the corresponding points after matching, which is much less than the error of the measurement system. It proves that the proposed method can complete matching for rail tanker and the precision fully meet the actual needs.

Table 1 Matching error of corresponding points

| Corresponding points | a₁-b₁ | a₂-b₂ | a₃-b₃ | a₄-b₄ | a₅-b₅ |
|----------------------|-------|-------|-------|-------|-------|
| ICP(mm)              | 0.0026| 0.0034| 0.0029| 0.0043| 0.0032|
| Proposed method(mm)  | 0.0013| 0.0024| 0.0017| 0.0028| 0.0021|

9. Conclusions

This paper presents a new method to matching for a rail tanker. It can automatically identify the correspondence between point sets; significantly reduce the searching scope, and improves the operational efficiency of data measurement. The searching process for corresponding points mainly uses topological relationship between points, simple computationally and good stably. This method is suitable to matching for three corresponding points and also for more than three points. With the simple implementation and high precision, it is very suitable for applying to matching such as rail tanker.
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