Ultra-high energy cosmic rays threshold in Randers-Finsler space

Zhe Chang and Xin Li

Institute of High Energy Physics
Chinese Academy of Sciences
P. O. Box 918(4), 100049 Beijing, China

Abstract

Kinematics in Finsler space is used to study the propagation of ultra high energy cosmic rays particles through the cosmic microwave background radiation. We find that the GZK threshold is lifted dramatically in Randers-Finsler space. A tiny deformation of spacetime from Minkowskian to Finslerian allows more ultra-high energy cosmic rays particles arrive at the earth. It is suggested that the lower bound of particle mass is related with the negative second invariant speed in Randers-Finsler space.

PACS numbers: 03.30.+p, 11.30.Cp, 98.70.Sa

1changz@mail.ihep.ac.cn
2lixin@mail.ihep.ac.cn
Decades ago, Greisen, Zatsepin and Kuz’min (GZK) [1] discussed the propagation of the ultra-high energy cosmic rays (UHECR) particles through the cosmic microwave background radiation (CMBR) [2]. Due to photopion production process by the CMBR, the UHECR particles will lose their energies drastically down to a theoretical threshold (about $5 \times 10^{19}$ eV). That is to say, the UHECR particles which their energy beyond the threshold can not be observed[3]. This strong suppression is called GZK cutoff. However hundreds of events with energies above $10^{19}$ eV and about 20 events above $10^{20}$ eV have been observed[4].

To explain this puzzle, one general accepted hypothesis is that the Lorentz Invariance (LI) is violated[5]. The violation of the LI and the Planck scale physics have long been suggested as possible solutions of the cosmic rays threshold anomalies[5]. LI is one of the foundations of the Standard model of particle physics. Coleman and Glashow have set up a perturbative framework for investigating possible departures of local quantum field theory from LI[6, 7]. In a different approach, Cohen and Glashow suggested [8] that the exact symmetry group of nature may be isomorphic to a subgroup $\text{SIM}(2)$ of the Poincare group. The mere observation of ultra-high energy cosmic rays and analysis of neutrino data give an upper bound of $10^{-25}$ on the Lorentz violation[9].

In fact, Gibbons, Gomis and Pope[10] showed that the Finslerian line element $ds = (\eta_{\mu\nu}dx^\mu dx^\nu)^{(1-b)/2}(\eta_{\rho\sigma}dx^\rho)^b$ is invariant under the transformations of the group $\text{DISIM}_b(2)$. The very special relativity is a Finsler geometry.

Recently, we proposed a gravitational field equation in Berwald-Finsler space [11].
The asymmetric term in field equation violated LI naturally. A modified Newton’s gravity is obtained as the weak field approximation of the Einstein’s equation in Berwald-Finsler space\cite{12}. The flat rotation curves of spiral galaxies can be deduced naturally without invoking dark matter in the framework of Finsler geometry.

In this Letter, we use the kinematics in Randers-Finsler space to study the propagation of the UHECR particles through CMBR. We obtain a deformed GZK threshold for the UHECR particles interacting with soft photons, which depends on an intrinsic parameter of the Randers-Finsler space\cite{13}.

Denote by $T_x M$ the tangent space at $x \in M$, and by $TM$ the tangent bundle of $M$. Each element of $TM$ has the form $(x, y)$, where $x \in M$ and $y \in T_x M$. The natural projection $\pi : TM \rightarrow M$ is given by $\pi(x, y) \equiv x$. A Finsler structure\cite{14} of $M$ is a function

$$F : TM \rightarrow [0, \infty).$$

The Finsler structure $F$ is regularity ($F$ is $C^\infty$ on the entire slit tangent bundle $TM\setminus 0$), positive homogeneity ($F(x, \lambda y) = \lambda F(x, y)$, for all $\lambda > 0$) and strong convexity (the $n \times n$ Hessian matrix $g_{ij} \equiv \frac{\partial^2}{\partial y^i \partial y^j} \left( \frac{1}{2} F^2 \right)$ is positive-definite at every point of $TM\setminus 0$).

It is convenient to take $y \equiv \frac{dx}{d\tau}$ being the intrinsic speed on Finsler space.

In 1941, G. Randers\cite{15} studied a very interesting class of Finsler manifolds. The Randers metric is a Finsler structure $F$ on $TM$ with the form

$$F(x, y) \equiv \sqrt{\eta_{ij} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} + \eta_{ij\kappa^i} d\tau}.$$  \hspace{1cm} (1)
The action of a free moving particle on Randers space is given as

\[ I = \int_s^r \mathcal{L} d\tau = m \int_s^r F \left( \frac{dx}{d\tau} \right) d\tau. \] (2)

Define the canonical momentum \( p_i \) as

\[ p_i = m \frac{\partial F}{\partial \left( \frac{dx^i}{d\tau} \right)}. \] (3)

Using Euler’s theorem on homogeneous functions, we can write the mass–shell condition as

\[ \mathcal{M}(p) = g^{ij} p_i p_j = m^2. \] (4)

The modified dispersion relation in Randers spaces is of the form

\[ m^2 = \eta^{ij} p_i p_j - \eta^{ij} \kappa_i (\mu, M_p) p_j, \] (5)

where we have used the notation

\[ \eta_{ij} = \text{diag}\{1, -1, -1, -1\}, \] (6)

\[ \kappa_i = \kappa\{1, -1, -1, -1\}, \] (7)

and \( \eta^{ij} \) is the inverse matrix of \( \eta_{ij} \). Here \( \kappa \) can be regarded as a measurement of LI violation. We consider the head-on collision between a soft photon of energy \( \epsilon \), momentum \( q \) and a high energy particle \( m_1 \) of energy \( E_1 \), momentum \( p_1 \), which leads to the production of two particles \( m_2, m_3 \) with energies \( E_2, E_3 \) and momentums \( p_2, p_3 \), respectively. By making use of the energy and momentum conservation law and the
modified dispersion relation (5), we obtain the deformed GZK threshold in Randers-Finsler space

\[ E_{\text{th}} = \frac{(m_2 + m_3)^2 - m_1^2}{4(\epsilon - \kappa/2)}. \]  

(8)

Taking roughly the energy of soft photon to be $10^{-3}\text{eV}$, we give a plot for the dependence of the threshold $E_{\text{th}}^N$ on the deformation parameter $\kappa$ in FIG. 1.

We can see clearly that a tiny deformation of spacetime ($\kappa$ with the order of the CMBR) can provide sufficient correction to the primary predicted threshold for the propagation of UHECR particles through the CMBR\[1\]. If the nature of our universe is Finslerian, more UHECR particles should be detected than Greisen, Zatsepin and Kuzmin expected.

Another invariant speed in Randers-Finsler space is expressed as\[13\]

\[ C_2 = \frac{\kappa - 4m}{\kappa + 4m}. \]  

(9)
From the above discussion, we know that the deformation parameter $\kappa$ may be the same order with CMBR. So far as we know that there is no observational evidence for the existence of the second invariant speed $C_2$. Thus, we suppose that the $C_2$ is negative or $C_2$ is beyond the speed of light. The negative condition of the invariant speed $C_2$ deduces that $m \geq \kappa/4$. This gives particle mass a lower bound for massive particle. The condition that $C_2$ is beyond the speed of light deduces that the mass of particle is negative. In such a case, $C_2$ may be corresponded to the speed of Goldstone boson.

Recently, there is a renewed interest in experimental tests of LI and CPT symmetry. Kostelecky\textsuperscript{[16]} has tabulated experimental results for LI and CPT violation in the minimal Standard-Model Extension. Our result would not violate the minimal Standard-Model Extension, since $\kappa$ can be eliminated by a redefinition of the energy and momentum. $\kappa$ is very small, the minor change in energy and momentum can be neglected except for soft photon.

**Acknowledgements**

We would like to thank T. Chen, J. X. Lu, N. Wu, M. L. Yan and Y. Yu for useful discussion. One of us (X. Li) indebt W. Bietenholz for useful discussion on UHECR. The work was supported by the NSF of China under Grant NOs. 10575106 and 10875129.
References

[1] K. Greisen, Phys. Rev. Lett. 16, 748 (1966); G. T. Zatsepin and V. A. Kuzmin, JETP Lett. 4, 78 (1966).

[2] P. G. Roll and D. T. Wilkinson, Phys. Revs. Lett. 16, 405 (1966).

[3] F. W. Stecker, Phys. Rev. Lett. 21, 1016 (1968).

[4] M. Takeda et al., Phys. Rev. Lett. 81, 1163 (1998); M. Takeda et al., Astrophys. J. 522, 225 (1999); N. Hayashida et al., Phys. Rev. Lett. 73, 3491 (1994); D. J. Bird et al., Astrophys. J. 441, 144 (1995); D. J. Bird et al., Phys. Rev. Lett. 71, 3401 (1993); D. J. Bird et al., Astrophys. J. 424, 491 (1994); M. A. Lawrence, R. J. O. Reid, and A. A. Watson, J. Phys. G 17, 733 (1991).

[5] V. A. Kostelecky, Phys. Rev. D. 69, 105009 (2004); R. Aloisio, P. Blasi, P. L. Ghia, and A. F. Grillo, Phys. Rev. D 62, 053010 (2000); O. Bertolami and C. S. Carvalho, Phys. Rev. D 61, 103002 (2000); H. Sato, arXiv: astro-ph/0005218; T. Kifune, Astrophys. J. 518, L21 (1999). W. Kluzniak, arXiv: astro-ph/9905308; R. J. Protheroe and H. Meyer, Phys. Lett. B 493, 1 (2000).

[6] S.R. Coleman and S.L. Glashow, Phys. Lett. B405, 249 (1997).

[7] S.R. Coleman and S.L. Glashow, Phys. Rev. D59, 116008 (1999).

[8] A.G. Cohen and S.L. Glashow, Phys. Rev. Lett. 97 021601 (2006).

[9] G. Battistoni et al., Phys. Lett. B615 14 (2005).
[10] G.W. Gibbons, J. Gomis and C.N. Pope, "General Very Special Relativity is Finsler Geometry", hep-th/0707.2174.

[11] X. Li and Z. Chang, arXiv: gr-qc/0711.1934.

[12] Z. Chang and X. Li, arXiv: gr-qc/0806.2184, to be published in Phys. Lett. B.

[13] Z. Chang and X. Li, Phys. Lett. B. 663, 103 (2008).

[14] D. Bao, S.S. Chern and Z. Shen, An Introduction to Riemann-Finsler Geometry, Graduate Texts in Mathematics 200, Springer, New York, 2000.

[15] G. Randers, Phys. Rev. 59, 195 (1941).

[16] V. A. Kostelecky, arXiv: hep-ph/0801.0287v1.