Value-at-Risk Modeling on Stock Return with Exogenous Variables using ARMAX-GARCHX Approach

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Abstract. Stock is one of investment instrument that has high risk with possible high return or loss. Quantifying the risk associated with stock return is important work for financial institution to optimize their portfolios. To do that analysis, Value-at-Risk method becomes very popular and frequently applied recently. This work applies Value-at-Risk modeling using ARMAX-GARCHX approach that considers exogenous variables having significant effect to the volatility of return. Moreover, in this research the risk is estimated based on observations spanning in moving windows. There are three kinds of windows size, i.e. 250, 375, and 500 transaction days. Applying the proposed method to stock return of top four (in market share) companies in construction and building subsector at Indonesian stock exchange (IDX), at the period of tax amnesty, the empirical results show that the Value-at-Risk estimation using windows size 500 days perform better than ones obtained from the shorter windows.

1. Introduction

Quantitative risk modeling becomes very important to measure the risk level of certain company. The modeling can be employed on the basis of cross section data such as predicting default probability using classification methods [1] or based on time series analysis, e.g. to model daily stock return. As a financial instrument, stocks have advantages and risks. Therefore, investors will surely choose to invest their funds in companies that give senses of risk to their investments. At certain level of risk, investors have high expectation of returns. The return is an indicator of increased (or decreased) prosperity for shareholders [1].

In the revision of the Law on General Provisions and Tax Procedures, the Indonesian government has applied the tax amnesty at the end of June 2016. Companies belong to construction; infrastructure and materials sectors are expected to receive direct impact of tax amnesty policy as the government finances various infrastructure projects using money collected from tax penalty and incoming repatriation funds. PT. Waskita Karya Tbk (WSKT), PT. Wijaya Karya Tbk (WIKA), PT. Adhi Karya Tbk (ADHI), and PT. Pembangunan Perumahan Tbk (PTPP) have most liquid shares and have large market capitalization in the subsector in last several years.

One of the most commonly used methods for estimating risk is Value-at-Risk (VaR), i.e. a measurement tool used to calculate the size of the worst losses that occur in the investment with a
certain level of confidence [2]. One approach that can be used to measure VaR values for data with high volatility is Autoregressive Moving Average (ARMA) [3] and Autoregressive Conditional Heteroscedasticity (ARCH) approach. The generalization of ARCH model is Generalized ARCH abbreviated as GARCH introduced by Bollerslev [4]. The GARCH with exogenous variables (GARCHX) considers the exogenous variables affecting the volatility.

This research focuses on quantifying the risk of stock returns of four aforementioned companies using VaR with ARMAX-GARCHX approach. The exogenous variables involved are two macroeconomic indicators, i.e. daily return of exchange rate of Indonesian rupiah against US dollar (IDR/USD) and daily return of Composite Stock Price Index (IHSG) from IDX. These two indicators significantly affect the stock return based on capital asset pricing model (CAPM).

2. Experimental Details

The data used in this work were downloaded from YAHOO finance, i.e. finance.yahoo.com, to obtain the daily close price of four companies and daily close price IHSG as well as official website of Bank Indonesia, i.e. bi.go.id, to obtain daily (IDR / USD) exchange rate and the interest rate of securities issued by the government so-called Bank Indonesia Certificate (SBI). The SBI is employed to estimate CAPM. The period of stock price and exchange rate data is from December 19, 2012 to October 31, 2016, while the SBI data span from December 2012 to July 2016. This research uses the concept of moving window for estimating VaR using ARMAX-GARCHX approach. One window consists of 250, 375, or 500 transaction days.

3. Result and Discussion

This study shows Stock return at time $t$ is the profit or loss obtained by investors computed as $R_t = (P_t - P_{t-1})/(P_{t-1})$, where $P_t$ is daily closing price at time $t$. The CAPM is used to test the effect of the market ($\bar{r}_m$ which is represented by return of IHSG) on the return of an asset ($\bar{r}_t$). The CAPM formulae is $\bar{r}_t = \mu + \beta (\bar{r}_m - \bar{r})$, with $\bar{r}_t$ is return from risk free investment and $\beta$ so-called Beta is the risk measure of corresponding asset.

The ARIMAX is the development of ARIMA model by adding exogenous variable to the model. This research employs ARMAX (without integrated) because return is stationary in mean (around zero). The general ARMAX model is formulated as $R_t = \beta X_t + \phi_1 R_{t-1} + \cdots + \phi_p R_{t-p} + \alpha_t + \theta_1 \alpha_{t-1} + \cdots + \theta_q \alpha_{t-q}$, where $X_t$ is exogenous variables (metric scale) at time $t$ with its corresponding coefficient $\beta$. More general, the $X_t$ can be replaced by $X_{t-1}$ or $X_{t-p}$ depends on which time lag can give reasonable interpretation or better forecasting. Another alternative of ARIMAX model is employed by Suhartono et al. [5] that imposed ARIMA model on the residual of time series regression. In addition, the multivariate time series model of this approach is proposed by Apriliandara et al. [6]

Today’s conditional variance not only be influenced by the squared residual at past (as imposed in ARCH model) but can also be affected by the residual’s variance at the past as formulated in GARCH model. The GARCH (r, s) model is $\sigma_t^2 = \phi_0 + \sum_{j=1}^{r} \phi_j \sigma_{t-j}^2 + \sum_{j=1}^{s} \lambda_j \sigma_{t-j}^2$, with $\lambda_j \geq 0$, $j=1,2,...,s$. Later, the GARCH with exogenous variables (GARCHX) was introduced by Apergis and Rezitis [7] as well as by Hand and Kristensen [8] formulated as $\sigma_t^2 = \phi_0 + \sum_{j=1}^{r} \phi_j \sigma_{t-j}^2 + \sum_{j=1}^{s} \lambda_j \sigma_{t-j}^2 + \sum_{l=1}^{m} \gamma_l X_{lt}^2$, with $\gamma_l \geq 0$.

The calculation of VaR value at time $t$ using normal distribution approach is formulated as $VaR_t(t) = \mu_t + F^{-1}(t) \sigma_t$, where $F^{-1}(t)$ is the inverse cumulative of standard normal distribution for quantile $t$. In this study, the $\mu_t$ is estimated using ARMAX, while $\sigma_t$ is estimated using GARCHX. Backtesting can be used to measure the accuracy of VaR model compared to the reality of the market. The risk function for backtesting is $I_{t,t} = 1\{r_t < -VaR_{t,t}\}$. The average of indicator function $I_{t,t}$ is compared with the risk level considered ($\tau$).

The empirical results are reported in this section. Some of empirical results are not written in detail here because the limited space available. Some of that information is as follows. Based on the coefficient of variations value, the ADHI shares have greater volatility compared to the other three.
The four companies tend to have negative returns on Monday indicating the occurrence of Monday effect on their stock returns. Throughout the year 2016, the returns are relatively stable and tend to rise. This is one of the possible effects of the tax amnesty policy.

Figure 1. Estimates of beta coefficient in CAPM

In Figure 1 it is known that the value of Beta of CPAM in each window for each company is greater than zero. This means that the fluctuation of return of IHSG has positive relationship with returns of each stock. Moreover, the Beta coefficients for WIKA and ADHI are greater than one. Thus, stock of WIKA and ADHI can be classified as aggressive shares. At the end of December 2012 until the end of December 2014 the Beta coefficient for WSKT and PTPP still greater than one. It means that in that period, the WSKT and PTPP stocks are sensitive to the market changes. However, after that period, the Beta coefficient for WSKT and PTPP become less than one. It means that the company’s sensitivity to market changes has decreased.

The ARMAX model in this study consists of two parts, the first step is to do estimation of ARMAX parameters and test its significance by involving exogenous variables at time \( t \). The second is adding the influence of exogenous variable at time \((t-1)\). Testing and estimation of GARCHX parameters is done simultaneously with ARMAX model. This is done because in modeling ARMAX and GARCHX simultaneously possibly will change the model and also parameter estimation. In the ARMAX model, there are several assumptions that must be fulfilled, i.e. the residual is white noise and follows normal distribution. The residual ARMAX models are not white noise possibly caused by heteroscedasticity. Therefore, the further modeling using GARCHX modeling plays into role.

Calculation of VaR in this research is done using ARMAX and GARCHX approach, where the order of ARMAX-GARCHX model employed in every window is based on order estimated from all data return without windowing, see Table 1.

Table 1a. Estimates of ARMAX-GARCHX for WSKT and WIKA.

| Exogenous | Stock | Model       | Parameter | Estimates | \( |t_{calc}| \) |
|-----------|-------|-------------|-----------|-----------|--------------|
| \( X_{1,t} \) | WSKT  | ARMA(1,0)-X(0,-) GARCH(2,0)-X(0,-) | \( \mu \) | 0.04687 | 30.30 |
|           |       |             | \( \phi_1 \) | 0.80939 | 32.11 |
|           |       |             | \( \beta_1 \) | 0.73585 | 11.99 |
|           |       |             | \( \phi_0 \) | 0.00059 | 5920.71 |
|           |       |             | \( \phi_1 \) | 0.76789 | 13.55 |
|           |       |             | \( \phi_2 \) | 0.23125 | 6.31 |
|           |       |             | \( \beta_1 \) | 0.04655 | 799.85 |
| \( X_{2,t} \) | WSKT  | ARMA(1,0)-X(-,0) GARCH(2,0)-X(-,1) | \( \mu \) | 0.00525 | 260.40 |
|           |       |             | \( \phi_1 \) | -0.16524 | 191.45 |
Table 1b. Estimates of ARMAX-GARCHX for ADHI and PTPP.

| Exogenous | Stock | Model | Parameter | Estimates | $t_{calc}$ |
|-----------|-------|-------|-----------|-----------|------------|
| $X_{2,t-1}$ | | | $\beta_2(X_{2,t})$ | 1.48260 | 3211.30 |
| | | | $\varphi_0$ | 0.00001 | 106.63 |
| | | | $\varphi_1$ | 0.76335 | 269.95 |
| | | | $\varphi_2$ | 0.59528 | 139.77 |
| | | | $\gamma_4(X_{2,t-1})$ | 0.04919 | 641.63 |
| $X_{1,t}$, $X_{2,t}$ | ADHI | WKA | GARCH(2,0)-X(0,0) | | |
| | | | $\varphi_0$ | 0.00123 | 1471.60 |
| | | | $\varphi_1$ | 0.09892 | 11.72 |
| | | | $\varphi_2$ | 0.14824 | 43.97 |
| | | | $\beta_1$ | 0.08414 | 6680.70 |
| | | | $\beta_2$ | 0.01316 | 13.09 |
| $X_{2,t-1}$ | | WKA | ARMA(1,0)-X(-,0)(-,1) | | |
| | | | GARCH(2,0)-X(-,-) | | |
| | | | $\mu$ | -0.00118 | 428.32 |
| | | | $\phi_1$ | 0.11449 | 455.88 |
| | | | $\beta_4(X_{2,t-1})$ | -0.41047 | 465.18 |
| | | | $\varphi_0$ | 0.00156 | 525.04 |
| | | | $\varphi_1$ | 0.59348 | 415.49 |
| | | | $\varphi_2$ | 0.18669 | 489.59 |
| | | | $\gamma_4(X_{2,t-1})$ | 0.07109 | 525.99 |

The calculation of the risk is performed using three windows size. Based on Table 2,
Table 2. Loss and profit estimation.

| Model  | Stock | Risk (Loss) | Profit |
|--------|-------|-------------|--------|
|        |       | 250   | 375   | 500   | 250   | 375   | 500   |
|        |       |       |       |       |       |       |       |
| Model 1| WSKT  | -0.0386 | -0.0385 | -0.0387 | 0.0481 | 0.0452 | 0.0432 |
|        | WIKA  | -0.0461 | -0.0439 | -0.0466 | 0.0461 | 0.0439 | 0.0466 |
|        | ADHI  | -0.0402 | -0.0401 | -0.0402 | 0.0731 | 0.0408 | 0.0405 |
|        | PTPP  | -0.0499 | -0.0508 | -0.0538 | 0.0499 | 0.0508 | 0.0538 |
| Model 2| WSKT  | -0.0327 | -0.0334 | -0.0372 | 0.0455 | 0.0509 | 0.0633 |
|        | WIKA  | -0.0499 | -0.0491 | -0.0502 | 0.0706 | 0.0657 | 0.0716 |
|        | ADHI  | -0.0455 | -0.0502 | -0.0511 | 0.0647 | 0.0699 | 0.0689 |
|        | PTPP  | -0.0429 | -0.0434 | -0.0441 | 0.0589 | 0.0625 | 0.0697 |

it can be seen that VaR calculation using ARMAX-GARCHX approach involving exogenous variables at time \( t \) (Model 1) informs that investment risk level in PTPP is the highest compared to the other three companies. This informs that the investment in PTPP shares is more risky than the other three. Meanwhile, the calculation of VaR return by adding the exogenous variable at time \( (t-1) \) on model (Model 2) results in information that WIKA is riskier than other three companies.

Based on the backtesting results, it can be seen that the estimation of the risk and profitability of the stock returns using ARMAX-GARCHX approach with window size 500 transaction days gives the smaller loss and expected shortfall compared to the 250 and 375 windows size. This indicates that the longer the interval window used for model estimation results in more accurate VaR calculation. The VaR estimation using windows size 250 and 375 tends to underestimate of its true value.

4. Conclusion

Based on The CAPM informs that there is significant effect of IHSG on the stock return for each company. This is a strong reason to impose the IHSG as one of the exogenous variable in ARMAX and GARCH model to estimate VaR. In the first model (imposing exogenous variable at time \( t \)), the VaR model calculated from window size 250, 375 and 500 transaction days informs that the risk level of PTPP tend to be higher than other three companies. In the model II (imposing exogenous variable at time \( t \) and \( (t-1) \)), the stock with highest risk is WIKA stock. Among the three windows used for VaR calculation, a window with a length of 500 intervals provides a more accurate VaR estimates compared ones obtained from 250 and 375 transaction days. The mean model (ARMAX part) in ARMAX-GARCHX approach for VaR calculation is redundant causing the decreasing accuracy. Therefore, in subsequent research it is recommended to assume a zero value of conditional mean to provide prediction with higher accuracy.

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