INFLUENCE OF $S_0$ TYPE OF ADDENDUM MODIFICATION ON SLIDING PERFORMANCE OF SPUR GEARS

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Abstract. The effect of $S_0$ type of addendum modification on sliding velocity of the involute spur gear has been studied. For the analysis four critical points such as Beginning of Engagement, Low Point of Single Tooth Contact, High Point of Single Tooth Contact and Disengagement point corresponding to pinion are considered. Length of approach and length of recess for modified gears are evaluated. Effects of addendum modification on contact fatigue life and wear prediction are discussed. Product of contact stress and sliding velocity ($\sigma_h \times V_s$) at critical points for varied addendum modification factor $X$ are evaluated, and its effect on sliding velocity has been analysed. Effect of change in value ($\sigma_h \times V_s$) at critical points of contact for modified gear pair on Hertzian fatigue is discussed.

Nomenclature

$m$ Module

$\phi$ Pressure angle

$r_{bp}, r_{bg}$ Base circle radius of pinion and gear respectively

$r_p, r_g$ Pitch circle radius of pinion and gear respectively

$r_{ap}, r_{ag}$ Addendum circle radius of pinion and gear respectively

$r$ Radius at any arbitrary point

$C$ Centre distance between gear pair

$b$ Face width

$\rho_c$ Radius of curvature at arbitrary point $r$

$F$ Tangential load

$J$ Number of teeth pair in contact

$E_1, E_2$ Elastic modulus of pinion and gear respectively
Poisson’s ratio

RPM of pinion and gear respectively

Sliding velocity

Rolling velocity of pinion and gear respectively

Low Point of Single Tooth Contact

High Point of Single Tooth Contact

Beginning of Engagement

Pitch Point

Disengagement Point

1. Introduction

In mechanical systems transfer of power from source to application is very much necessary. Considering all other transmission systems, gears are having their own advantage of constant velocity ratio of transmission [1]. In a situation where pinion has to be designed with the number of teeth lesser than the minimum teeth required to avoid under cutting and interference, addendum modification universally adopted concept and amount of addendum modification is quantified using modification factor X [2]. Positively modified gear teeth will have increased bending strength and better contact and sliding relations [2]. There are two type of addendum modification, S type in which there is a scope for altering centre distance between the operating gear pair and other type is $S_0$, where the sum of modification provide on pinion ($X_1$) and gear ($X_2$) will be equal to zero [2]. One of the most significant research orientations in gear pair design is the selection of addendum modification factor (X) [3]. Transmitted load, contact stresses and sliding velocity determines specific wear rate on working surface of the gear pair [4]. Theoretically, the maximum value of contact stress will be on pitch point. Contact stress developed influences the wear rate of mating teeth surface. Product of contact stress and sliding velocity ($\sigma_h \cdot V_s$) is very much critical at beginning and end of path of contact on the line of action in wear prediction of mating teeth surface. Portion of gear teeth will have sliding velocity opposite to the rolling velocity. The crucial value ($\sigma_h \cdot V_s$) causes Hertzian fatigue [5]. Hassan [6] has carried out contact stress analysis at different points using Hertzian contact stress equation. H. K. Sachidananda [7] has carried sliding velocity analysis for profile modified gears with different amount of addendum modification factor and also by varying number of teeth of pinion and gear. Seok-Chul Hwang [8] carried out contact stress analysis of standard helical and spur gear in the region between LPSTC to HPSTC. From the literature it is evident that majority of the research considers the study of sliding velocity, contact stress for standard and addendum modified spur and helical gears. Also the influence of crucial value ($\sigma_h \cdot V_s$) on wear prediction for addendum modified spur gear. Whereas, research literature relating to influence of $S_0$ type of addendum modification on contact stress, sliding velocity and wear behaviour of involute spur gears is scare. In this work, an attempt has been made to study the sliding velocity, contact stress and influence of $S_0$ type of addendum modification on sliding velocity.

2. $S_0$ Type of Addendum Modification

$S_0$ type of addendum modification is a type of profile modification in which addendum and dedendum of the pinion and gear are disposition with respect to base circle. In some of the gear pair design procedure, the centre distance between the rotating gear pair is fixed by some other design features. In such cases, improved performance through modification cannot be obtained by varying the centre distance. In $S_0$ type of addendum modification even though the addendum and dedendum of the gear and pinion are disposition, pitch and base circle are not affected. The amount of profile modification is
decided by the modification factor “X”. In most general case pinion is provided with positive modification and gear with negative modification. Provided positive modification for pinion and negative modification for gear will increase the bending strength of the pinion and protects pinion teeth from bending fatigue [2]. Weight of pinion gets increased with the increase in modification factor X and weight of gear decreases with the increase in modification factor X. Therefore overall weight of meshing gear pair in contact decreases with the increase in modification factor X.

![Schematic representation of S_0 type of addendum modification.](image)

**Figure 1.** Schematic representation of $S_0$ type of addendum modification.

3. **Location of BE, LPSTC, HPSTC and DP**

Beginning of Engagement and Disengagement Point are crucial in lubrication and wear point of view. For a gear pair contact ratio 1.55 indicates, about 55% of the revolution two pair of teeth will be in contact and for remaining 45% of revolution one pair of teeth will be in contact. When contact ratio of gear pair lies in between 1 and 2, LPSTC and HPSTC are the lowest point and highest point on the tooth surface where there will be only one pair of teeth in contact. And one pair of teeth in contact carries entire load at any point in between LPSTC to HPSTC on the gear teeth surface. As load carried by single teeth in contact, wear rate will be maximum in the region between LPSTC and HPSTC compared to any other region on the gear surface.

![Representation of BE, LPSTC, HPSTC and DP and contact force on line of action.](image)

**Figure 2.** Representation of BE, LPSTC, HPSTC and DP and contact force on line of action.

In the fig shown above,
SE is the line of action.
AB is the length of contact.
B and A are the BE and DP of the pinion on line of action respectively.
M and N are the LPSTC and HPSTC of the pinion on line of action respectively.
At any other region other below LPSTC and above HPSTC, the applied load will be shared between two pair of teeth in contact. Therefore LPSTC and HPSTC are considered as the critical points considered for analysis [6] [8].
Using mathematical equations presented in [6], radius at critical points and radius of curvature of both gear and pinion can be calculated.

Theoretically, the maximum contact stress is on pitch point. For a pair of standard gear pair, pitch points lies between LPSTC and HPSTC. For the case of \(S_0\) type of addendum modified gear pair, the location of pitch point depends on modification factor “X”. The contact fatigue life of the gear pair can be increased considerably by locating the pitch point in the double tooth contact region [7]. In modified gear pair as the modification factor varies the position of LPSTC and HPSTC gets altered not the pitch point. In case of \(S_0\) gearing, as modification factor increases, pitch point turns towards double tooth contact region. When modification factor reaches \(X = 0.4\) to \(0.5\) the pitch point lies in double tooth contact region. With this modification the contact fatigue life has been increased.

**Table 1.** Data of mating spur gear considered for analysis.

| Parameters                  | Pinion | Gear  |
|-----------------------------|--------|-------|
| Number of teeth             | 18     | 72    |
| Module (mm)                 | 5      | 5     |
| Pressure angle \(^0\)       | 20     | 20    |
| Pitch diameter (mm)         | 90     | 360   |
| Base diameter (mm)          | 84.5723| 338.289|
| Number of rotation (RPM)    | 1440   | 360   |
| Tangential load (N)         | 1473.8393|     |
| Modulus of elasticity (Mpa) | \(2*10^5\) |     |
| Poissons ratio              | 0.29   |       |

4. **Length of Approach (LA) and Length of Recess (LR)**

Length of approach and length of recess for a pair of modified gear pair differs with modification factor compared to standard gear pair in contact. Length of approach and length of recess can be tabulated using the formulas 1 and 2.

\[
\text{Length of Approach (LA)} = \sqrt{r_{ag}^2 - r_{bg}^2 - r_p \sin \theta} \quad (1)
\]

\[
\text{Length of Recess (LR)} = \sqrt{r_{ap}^2 - r_{bp}^2 - r_p \sin \theta} \quad (2)
\]

![](https://via.placeholder.com/150)

**Figure 3.** Variation of LA and LR with modification factor.

In wear prediction of gear pair LA and LR plays a major role. LA reduces and LR increases with the increase in modification factor X. The approach action of gear teeth is like pushing a piece of chalk across a blackboard, the chalk screeches. Recess action is like pulling the piece of chalk across the blackboard, it glides smoothly. Thus, recess action is always preferable because of lower frictional forces and smoothness of operation [10]. In recess action the abrasive wear of material (scoring) takes place as one tooth slides over the other. The scoring limits the value of LR in turn modification factor.
5. Contact Pressure * Sliding Velocity ($\sigma_h \ast V_s$)

Involute gear pair in contact, the meshing action is a combination of both sliding and rolling. This is because of difference in radius of curvature of mating pairs. At any given point the difference in rolling velocity of driving and driven gear defines the sliding velocity at that point. Gear tooth surface wear rate depends on the contact stresses, sliding velocity and applied load. So the product of contact stresses and sliding velocity ($\sigma_h \ast V_s$) at any point of contact on the gear teeth mating surface is one of the most crucial value which defines the wear behaviour of the gear pair teeth in contact.

Sliding velocity = Rolling velocity of pinion - Rolling velocity of gear.

Sliding velocity at any point can be written as [1]

$$V_s = V_{rp} - V_{rg}$$  
(3)

$$V_{rp} = \frac{2 \pi N_p}{60}$$  
(4)

$$V_{rg} = \frac{2 \pi N_g}{60}$$  
(5)

In some points on the gear teeth sliding velocity is negative. The negative sliding velocity indicates direction of sliding velocity is opposite to the rolling velocity of gear. The frictional forces due to sliding are high in the negative velocity region.

Gear pair mating surface is assumed to be of Hertzian contact. This means two mating surfaces of gear are assumed as two cylinders in contact. For a gear pair in contact, contact stresses developed due to Hertzian contact depends on the radius of curvature of mating surfaces in contact point.

As the pinion is provided with positive addendum modification factor, radius of curvature ($r_p$) of pinion gets increased. The increased radius of curvature reduces the contact stresses of the mating gear pairs. The reduced contact stresses retard the pit initiation.

Equation for Hertzian contact stresses will be,

$$\sigma_h = \sqrt{\frac{F[(1/r_1)+(1/r_2)]}{\pi [1-\sigma_1^2]/E_1+ [1-\sigma_2^2]/E_2}}$$  
(6)

The value ($\sigma_h \ast V_s$) will be crucial at beginning and end of engagement of the gear tooth in contact. The negative value of ($\sigma_h \ast V_s$) will allow the lubricating oil to enter into surface cracks of mating teeth and by the hydraulic pressure propagation mechanism the surface crack growth gets accelerated which will cause Hertzian fatigue [7].

Increases radius of curvature due to addendum modification will increase the value($\sigma_h \ast V_s$) in recess region of gear pair. The increased value ($\sigma_h \ast V_s$)can result in squeezing out of lubricating oil from the tooth surface, which will cause metal-to-metal contact of gear teeth. The increased value ($\sigma_h \ast V_s$)will cause scoring (abrasive wear) in the gear teeth. Squeezing of lubricant and scoring in recess of gear teeth will limit the amount of addendum modification [7].

| Contact Region / ($\sigma_h \ast V_s$) | Addendum modification factor |
|--------------------------------------|-----------------------------|
|                                      | 0   | 0.2 | 0.4 | 0.5 | 0.6 |
| BE                                   | -949673.01 | -524723.50 | -325125.52 | -253386.94 | -191851.33 |
| LPSTC                                | -217911.29 | -97287.87 | 6281.11 | 53617.99 | 98111.34 |
| PP                                   | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| HPSTC                                | 76631.90 | 204106.15 | 326036.66 | 385851.77 | 445370.03 |
| DP                                   | 269404.69 | 308529.44 | 345758.58 | 363855.88 | 381659.29 |
6. Conclusion
Theoretically, the contact stress will be maximum at pitch point other than any other point on the teeth surface. By $S_0$ type of addendum modification, the position of LPSTC and HPSTC gets altered corresponding to modification factor $X$. Thus pitch points lie in the region of double tooth contact when modification reaches 0.4 and above for considered gear pair. Thus modification will increase the contact fatigue life of the gear pair. Modification also increases the length of recess by reducing length of approach. Gears are smoother in operation in the recess region than approach region. The critical value $(\sigma_h \ast V_s)$ is negative in BE and LPSTC. Through modification, the value $(\sigma_h \ast V_s)$ comes towards the positive region in BE and LPSTC. This increases Hertzian fatigue life. Meanwhile the positive value $(\sigma_h \ast V_s)$ in HPSTC and DP (recess region) gets increased with increasing modification factor. The increased positive value $(\sigma_h \ast V_s)$ leads to scoring. Thus scoring limits the magnitude of modification factor. For the case of $S_0$ type of addendum modification considering above mentioned parameters the optimal value of modification factor can be considered in between 0.4 to 0.5.

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