Holographic QCD predictions for production and decay of pseudoscalar glueballs

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The top-down holographic Witten-Sakai-Sugimoto model for low-energy QCD, augmented by finite quark masses, has recently been found to be able to reproduce the decay pattern of the scalar glueball candidate $f_0(1710)$ on a quantitative level. In this Letter we show that this model predicts a narrow pseudoscalar glueball heavier than the scalar glueball and with a very restricted decay pattern involving $\eta$ or $\eta'$ mesons. Production should be either in pairs or in association with $\eta(\prime)$ mesons. We discuss the prospect of discovery in high-energy hadron collider experiments through central exclusive production by comparing with $\eta(\prime)$ pair production.

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INTRODUCTION

Quantum chromodynamics, the established theory of the strong interactions, predicts the existence of flavor singlet mesons beyond those required by the quark model, because in the absence of quarks gluons by themselves can form bound states. However, the status of such “gluonium” or “glueball” states in the observed meson spectrum is still unclear and controversial.

In 1980, an isoscalar pseudoscalar with mass around 1.44 GeV which is copiously produced in the gluon-rich radiative decays of $J/\psi$ was proposed as the first glueball candidate. Originally named $\epsilon(1440)$, this is now listed by the Particle Data Group as the two states $\eta(1405)$ and $\eta(1475)$. Together with $\eta(1295)$, this indeed would give rise to a supernumerary state beyond the first radial excitations of the $\eta$ and $\eta'$ mesons, with $\eta(1405)$ singled out as glueball candidate.

The situation thus appears to be analogous to the case of the scalar glueball, which is generally considered to give rise to a supernumerary state in the set of isoscalar scalar resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$, where only two are expected from the quark model (namely linear combinations of $uu + dd$ and $ss$). Here the discussion is divided on the question which of the two heavier resonances has the larger glueball contribution.

However, only the case of the scalar glueball candidates is supported by existing lattice QCD calculations which consistently find that the lowest-lying glueball state has mass around 1.7 GeV and quantum numbers $J^{PC} = 0^{++}$. The lowest-lying pseudoscalar glueball state is instead found to have a mass around 2.6 GeV, somewhat higher than the $2^{++}$ tensor glueball with mass around 2.4 GeV. Most lattice results have been obtained in the quenched approximation, i.e. without dynamical quarks, but recent unquenched lattice calculations have found no evidence for significant unquenching effects, which however should be expected if the pseudoscalar glueball were to mix strongly with radi-
ball state with a very restricted decay pattern which will be a conspicuous feature as long as mixing with quarkonia is small. The specific interactions also suggest that the pseudoscalar glueball may be difficult to produce in radiative charmonium decay, but could be a very interesting object for glueball searches in central exclusive production (CEP) experiments at sufficiently high energies.

**EFFECTIVE LAGRANGIAN FOR PSEUDOSCALAR GLUEBALL INTERACTIONS**

The WSS model is an extension of the Witten model for nonsupersymmetric and nonconformal low-energy QCD based on D4 branes in type-IIA supergravity compactified on a circle and subjected to a consistent truncation of Kaluza-Klein states. It possesses an interesting spectrum of glueball states with \( J^{PC} = 0^{++}, 2^{++}, 0^{-+}, 1^{-+}, 1^{--} \) whose mass scale is set by the Kaluza-Klein mass \( M_{KK} \). Sakai and Sugimoto showed that \( N_f \ll N_c \) chiral quarks can be added through probe D8 and anti-D8 branes separated on the compactification circle, which introduces a purely geometric realization of nonabelian chiral symmetry breaking \( U(N_f)_L \times U(N_f)_R \to U(N_f) \). The resulting effective chiral theory involves Goldstone pseudoscalars and a tower of vector and axial vector mesons.

Fixing \( M_{KK} \) through the experimental value of the \( \rho \) meson mass and varying the 't Hooft coupling \( \lambda = 16.63 \ldots 12.55 \) such that either the pion decay constant (as originally done in the string tension in large-\( N_c \) lattice simulations) is matched leads to quantitative predictions which are in the right ballpark when extrapolated to \( N_c = 3 \) QCD. In particular, it reproduces remarkably well the observed hadronic decay rates of the \( \rho \) and the \( \omega \) mesons, which motivates the use of the WSS model also as a model for glueball decay. In Ref. [23] we argued, however, that the lightest scalar glueball mode considered in Ref. [31] which comes from an “exotic polarization” of the dual graviton along the compactified direction (denoted by \( G_E \) in the following) should be discarded and that instead the predominantly dilatonic mode (\( G_D \)) be identified with the glueball ground state.

The chiral WSS model correctly incorporates the chiral anomaly and the Witten-Veneziano mechanism for giving mass to the flavor singlet pseudoscalar \( \eta_0 \) with \( \mp \frac{1}{2} m_0^2 \pi^2 N_c \). Introducing explicit quark mass terms in the effective Lagrangian such that physical pion and kaon masses are matched leads to \( \eta \) and \( \eta' \) masses that agree with real QCD to within \( \lesssim 10\% \) [24]. As mentioned above, the flavor-asymmetric decay pattern observed for the scalar glueball candidate \( f_0(1710) \) can be reproduced quantitatively with \( G_D \), if the (as yet undetermined) parameter for scalar glueball couplings to explicit quark mass terms is chosen such that the rate of decay into mixed \( \eta \eta' \) pairs remains small.

The interaction Lagrangian of the pseudoscalar glueball is the same for both, the chiral and the massive version of the WSS model. The pseudoscalar glueball modes are provided by a Ramond 1-form field \( C_1 \) which plays the central role in producing the Witten-Veneziano mass \( m_0 \). Following the notation of Ref. [23], the action for \( C_1 \) is given by \( S_{C_1} \propto \sqrt{-g} F_{2}^{2} \). Cancellation of the \( U(1)_{A} \) anomaly requires that \( F_{2} \) is a gauge invariant combination of \( F_{2} = dC_{1} \) and the field \( \eta_0 = \int_{-\infty}^{z} d x A_{2}(z, x) \) with \( z \) parametrizing the radial extent of the joined D8 and anti-D8 branes on which the flavor gauge field \( A \) lives.

Inserting a mode expansion of the Ramond 1-form field \( C_1 \) with 4-dimensional pseudoscalar glueball fields \( G^{(n)}(x), n = 1, \ldots, \) together with scalar and tensor glueball fields entering through the metric in \( S_{C_1} \), leads to the effective 4-dimensional Lagrangian

\[
\mathcal{L}_{G^{(n)}}^{\text{eff}} = -\frac{1}{2} \partial_{\mu} G^{\mu} \partial_{\nu} G^{\nu} + \frac{1}{2} \partial_{\mu} \tilde{G}^{\mu} \partial_{\nu} \tilde{G}^{\nu} - \frac{1}{2} m^{2}_{0} \eta_{0}^{2} + \mathcal{L}_{\eta_{0}^{2}} + \mathcal{L}_{\eta_{0} G^{(n)}} + \mathcal{L}_{G^{2} G^{(n)}} + \mathcal{O}(G_{D,E,T}^{2}) \tag{1}
\]

(suppressing the summation over the mode number index \( n \)). Here \( O(G_{D,E,T}^{2}) \) denotes higher-order interactions involving terms quadratic in \( G, \eta_0 \) and quadratic or higher in the glueball fields arising from metric fluctuations (the tensor glueball field \( T^{\mu\nu} \) appears at most linearly, but also has interactions involving arbitrarily high powers of the scalar glueball field).

The mass of the lowest pseudoscalar glueball mode \( M_p \approx 1.885M_{KK}, \) which like in lattice QCD results is above the mass of the scalar and tensor glueballs with \( M_D = M_T \approx 1.567M_{KK}. \) With \( M_{KK} = 949 \text{ MeV from having matched the mass of the } \rho \text{ meson, } M_D \approx 1487 \text{ MeV and } M_T \approx 1789 \text{ MeV, but in the eventual applications we shall leave } M_p \text{ a free parameter and either keep } M_D \text{ at } 1.5 \text{ GeV which approximately matches the mass of } f_0(1500) \text{ or artificially raise its mass to the mass of } f_0(1710). \)

Note that Eq. (1) contains a mass term for the flavor singlet \( \eta_0 \) [28], but no mixing of the pseudoscalar glueball modes \( G^{(n)} \) with \( \eta_0 \). Terms proportional to \( \eta_0 \bar{G}^{(n)} \) vanish in the unperturbed background geometry, but appear in the presence of metric fluctuations dual to scalar glueballs. In the WSS model, such terms are the only ones which can mediate a decay of pseudoscalar glueballs. Explicitly they read (keeping the exotic glueball mode \( G_E \) for completeness)

\[
\mathcal{L}_{\eta_{0} G^{(n)}} = \bar{d}_{0} \tilde{G} \eta_0 G_D + \bar{d}_{0} \tilde{G} \eta_0 G_E + \partial_{\mu} \tilde{G} \eta_0 \partial^{\mu} G_E + \partial_{\mu} \tilde{G} \eta_0 \frac{\Box - M_{E}^{2}}{M_{E}^{2}} \tilde{G} E \tag{2}
\]

with the numerical results for the coupling constants for the lowest pseudoscalar glueball mode listed in Table [1] (their integral representations will be given elsewhere).
The part of the action which leads to the Witten-Veneziano mass term also gives rise to interactions with scalar glueballs which were obtained (on-shell) in [24]. To linear order in glueball fields the corresponding interaction Lagrangian reads (also including an extra off-shell contribution for the exotic mode $G_E$)

$$\mathcal{L}_{\tilde{G}G} = \frac{1}{2} m_0^2 \eta_0^2 \left( 3d_0 G_D - 5c_0 G_E \right)$$

$$\quad + \frac{1}{2} c_0 m_0^2 \eta_0 \Box - M_E^2 G_E. \quad (3)$$

There are also interaction terms of the form $(\partial \eta_0)^2 G_{D,E,T}$ coming from the DBI action of the D8 branes, which can be found in Ref. [23], as well as natural-parity violating terms $\eta_0 G_2 \tilde{G}$ from Chern-Simons action of the D8 branes, which have been obtained in Ref. [34].

Interaction terms involving pairs of pseudoscalar glueballs and a scalar or tensor glueball are given by

$$\mathcal{L}_{G^2 G} = \tilde{d}_1 \left[ \frac{1}{2} \partial_\mu \tilde{G} \partial^\mu \tilde{G} - \frac{1}{8} \partial_\mu \tilde{G} \partial_\nu \tilde{G} \partial^{\mu \nu} \right] G_D$$

$$\quad + \frac{1}{2} \tilde{d}_2 m_0^2 \tilde{G}^2 G_D + \sqrt{6} \tilde{d}_1 \partial_\mu \tilde{G} \partial_\nu \tilde{G} T^{\mu \nu} + \mathcal{L}_{G^2 G_E}. \quad (4)$$

(The more unwieldy expression $\mathcal{L}_{\tilde{G}^2 G_E}$ will be given elsewhere.)

### DECAY PATTERN OF THE PSEUDOSCALAR GLUEBALL

The only interaction terms arising within the WSS model that are relevant for the decay of pseudoscalar glueballs are contained in [2]. They differ strongly from the leading interaction terms that have been assumed previously in phenomenological models.

Rosenzweig et al. [35, 36] have assumed that the chiral anomaly is not saturated by $\eta_0$ alone, but involves a further physical pseudoscalar field ($G_2$) [37], which couples to the imaginary part of $\log \det \Sigma$, where $\Sigma$ is the matrix of $q\bar{q}$ states (which is unitary in the nonlinear sigma model, involving only the pseudoscalars, but unrestricted in linear sigma models [35] so that it also accommodates scalar mesons). While a natural possibility [37] was to identify $G_2$ with the radial excitation of $\eta_0$, it was proposed to identify $G_2$ with the pseudoscalar glueball instead. Originally used in the context of the glueball candidate $\xi(1440)$, this approach was also adopted in the extended linear sigma model of Ref. [39] for pseudoscalar glueballs with a mass suggested by lattice QCD. The dominant decay mode of a pseudoscalar glueball in this approach turns out to be $KK\pi$ (branching ratio $B \approx 1/2$) followed by $\eta\pi\pi$ ($B \approx 1/6$) and $\eta'\pi\pi$ ($B \approx 1/10$).

Using large-$N_c$ chiral Lagrangians, Gounaris et al. [40] argued that there should be no coupling of the pseudoscalar glueball to $\Im \log \det \Sigma$. Instead, a coupling to $\Im \text{tr} \mathcal{M}_q \Sigma$ was considered so that the pseudoscalar glueball is stable in the limit of massless quarks ($\mathcal{M}_q$ being the quark mass matrix). This again gives a dominant decay mode $KK\pi$, but with $\eta\pi\pi$ being more strongly suppressed (parametrically by a factor $m_\pi^2/m_K^2$).

In agreement with the considerations of Ref. [40], the WSS model, which also corresponds to a large-$N_c$ chiral Lagrangian, does not lead to a coupling of the pseudoscalar glueball to Im $\log \det \Sigma$. However, its extension to finite quark masses (either through world-sheet instantons [41] or open-string tachyon condensation [42]) does not naturally lead to a coupling to $\Im \text{tr} \mathcal{M}_q \Sigma$, because Ramond fields do not couple directly to fundamental strings. In the WSS model, the only coupling linear in $G$ is to $\eta_0 G$. This suggests that the pseudoscalar glueball should decay dominantly in $\eta'$ and the $f_0$ meson which corresponds to the scalar glueball, or $\eta'$ and decay products of the latter. According to the WSS model, the decay mode $KK\pi$ that is obtained as the dominant one in the approaches mentioned above should instead be strongly suppressed.

When the mass of the pseudoscalar glueball is larger than the mass of the scalar glueball plus the $\eta'$ mass, the scalar glueball can be produced on-shell. The resulting decay width is displayed in Fig. [1] as a function of the pseudoscalar glueball mass for the glueball mode $G_{P}$ with mass $1.5 \, \text{GeV}$ and also when raised in mass to match $f_0(1710)$, which in Ref. [24] we found to be favored by the WSS model [33]. For the latter case, Fig. [2] shows the (not necessarily resonant) dimensionless partial decay widths $\Gamma_{\pi}/M_P$ for $G \rightarrow G\eta' \rightarrow PP\eta'$ where $P = K, \pi, \eta, \eta'$ with the decay pattern for the scalar glueball $G = f_0(1710)$ obtained in Ref. [24]. With $M_P \approx 2.6 \, \text{GeV}$ as predicted by lattice QCD, the pseudoscalar glueball is predicted to be a rather narrow state; for $M_P \lesssim 2.3 \, \text{GeV}$ it would be extremely narrow.
PRODUCTION OF PSEUDOSCALAR GLUEBALLS

While scalar and tensor glueballs couple directly to $q\bar{q}$ mesons, pseudoscalar glueballs do so only through the former in the WSS model. This suggests that pseudoscalar glueballs are not as easily formed in radiative decays of $J/\psi$ as the other glueballs, but they would have to arise from excited scalar or tensor glueballs decaying into $\eta(1250)G$ or $GG$ pairs. The thresholds for these processes are thus above the mass of the $J/\psi$ so that excited $\psi$ mesons or $\Upsilon$ would be required.

Another possibility is central exclusive production (CEP) in high-energy hadron collisions through double Pomeron or Reggeon exchange (corresponding to $GT$ and $(\rho, \omega)$ trajectories; pion and scalar glueball exchanges are subdominant at high energies). The parametric orders of the corresponding amplitudes are shown in Fig. 3. Here production of $G(00)$ occurs only via virtual scalar glueballs, whereas production of $GG$ can additionally proceed through virtual tensor glueballs. Also shown is the possibility of $GG$ production through the natural-parity violating coupling of $\eta_0$ to two tensor glueballs (Pomerons), which is provided by the Chern-Simons part of the action of the $D8$ branes and which was recently studied within the WSS model in Ref. [31].

Associated production of pseudoscalar glueballs with either $\eta(1250)$ or other glueballs is presumably beyond the reach of the older fixed-target experiments searching for glueballs, but seem to be an exciting possibility for the new generation of CEP experiments at the LHC.

Calculation of the corresponding production cross sections within the WSS model could be attempted by employing the techniques used in Ref. [31] for $\eta$ and $\eta'$ production, but will be left for future work. In this Letter we only present results for the ratio of production rates of $G\eta'$ and $GG$ pairs over $\eta/\eta'$ pairs [40], when both are produced through a virtual $G_D$ glueball. This ratio is fixed by the vertices obtained above together with the results obtained in Ref. [24], and the result is shown in Fig. 4 for the range of 't Hooft coupling discussed above. The amplitude $M(G^{*} \to G(2)) \sim \lambda^{-1/2} N_c^{-1}$ is parametrically of the same order as $M(G^{*} \to \eta(2))$ so that the ratio $N(GG)/N(\eta\eta')$ is particularly well determined (at least for fixed meson masses in the scenario of Ref. [24]).

The results in Fig. 4 indicate that CEP of $\eta' G$ is only one order of magnitude below CEP of $\eta'/\eta'$, while above the threshold for $GG$ pairs, production of the latter is even up to one order of magnitude larger than CEP of $\eta'/\eta'$.

Central exclusive production of $\eta'/\eta'$ pairs has been studied in the Durham model in Ref. [17], where its production cross section was estimated. For example, at $\sqrt{s} = 1.96$ TeV this work obtained $\sigma(\eta'/\eta')/\sigma(\eta^0/\eta^0) \sim 10^3 \ldots 10^5$ assuming sufficiently high transverse momentum such that a perturbative approach becomes justified.

Since small transverse momentum is expected to pro-
vide a glueball filter \[\text{[8]}\] and the production of \(\hat{G}\) together with another \(G\) or \(\eta(\prime)\) according to the present model proceeds through virtual scalar glueballs, the kinematical regime of small transverse momentum (small azimuthal angle \(\phi_{B\pi}\)) would be particularly interesting for the search of pseudoscalar glueballs.\[51]\]

To summarize, the WSS model suggests very specific production and decay mechanisms of pseudoscalar glueballs that make them very interesting for CEP experiments at high-energy hadron colliders. All this of course assumes that pseudoscalar glueballs do not mix strongly with \(q\bar{q}\) states. At large \(N_c\), this mixing is suppressed, but it is uncertain whether this feature extends to real QCD. However the smallness of unquenching effects in glueball studies in lattice QCD found in Ref. \[\text{[18]}\] \[\text{[19]}\] could indicate that nearly pure glueballs are possible after all.

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It has been argued that the pseudoscalar sector may be particularly sensitive to unquenching in Ref. \cite{55}, but in a subsequent extension \cite{56} more possibilities were introduced.

In Ref. \cite{55} a unique form of the interaction Lagrangian for extended linear sigma models has been posited, where only the coupling strength is left undetermined, but in a subsequent extension \cite{56} more possibilities were introduced.

E. Gregory, A. Irving, B. Lucini, C. McNeile, A. Rago, \cite{54} E. Gregory, A. Irving, B. Lucini, C. McNeile, and D. H. Rischke, The Glueball in a Chiral Linear Sigma Model with Vector Mesons, Phys.Rev. D84 (2011) 054007, arXiv:1103.3238.

In Ref. \cite{56} a unique form of the interaction Lagrangian \cite{53} gives an upper limit for the experiment \cite{53, 54} (though not by the CLEO collaboration, C. M. Richards, A. C. Irving, E. B. Gregory, and C. McNeile, \cite{56}).

It has been argued that the pseudoscalar sector may be particularly sensitive to unquenching in Ref. \cite{55}, but the estimated effects on the mass were of the order of 15\%, whereas almost 50\% would be needed to bring the latter was observed in the \(\eta(1405)\) resonance is about 40\% of the signal in the \(K\bar{K}\pi\) channel \cite{8}. (We thank Denis Parganilja for discussions of this point.).

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Ref. \cite{2} gives an upper limit for the \(\eta\pi\pi\) channel which is about 40\% of the signal in the \(K\bar{K}\pi\) channel, thus it may actually be consistent with having a single resonance \(\eta(1440)\) comprising \(\eta(1405)\) and \(\eta(1475)\) which appears in \(\gamma\gamma\) reactions, since in radiative \(J/\psi\) decays \(J/\psi \rightarrow \gamma\eta(1405/1475)\) the \(\eta\pi\pi\) channel is only 16\% of the \(K\bar{K}\pi\) channel \cite{3}. (We thank Denis Parganilja for discussions of this point.).

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If we had kept the “exotic” scalar glueball mode \( G \) and raised its mass (which is originally only 855 MeV) to the mass of \( f_0(1500) \) or \( f_0(1710) \), Fig. 1 would look very similar, but the decay width would be about a factor of 10 larger.

A natural-parity violating coupling of \( \eta \) also exists with Reggeons. Fig. 3 gives the parametric order for double Pomeron exchange, which is down by a factor of \( \alpha_s/c \) compared to Reggeons, but becomes dominant at very high energies.

1 The decay rate of \( G \eta \) has a smaller threshold and thus larger phase space but is reduced by a factor \( (\tan \theta_P)^2 \sim 0.1 \).

Strictly speaking, the amplitude \( M(G^* \rightarrow \eta^2) \) contains contributions proportional to \( \lambda^{-1/2} N_c^{-1} m_0^2 \) and \( \lambda^{-1/2} N_c^{-1} m_0^2 \), respectively, with \( m_0^2 \propto \lambda^2 N_f/N_c \). In the scenario of Ref. 21 (corresponding to that of Ref. 25 with \( x = 1 \)) which we employ here, the decay rate of scalar glueballs in massive pseudoscalar mesons is enhanced by a factor which only depends on the physical masses of the mesons.

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