LONG-TERM EVOLUTION OF PROTOSTELLAR AND PROTOPLANETARY DISKS. II. LAYERED ACCRETION WITH INFALL

Zhaohuan Zhu\textsuperscript{1}, Lee Hartmann\textsuperscript{1}, and Charles Gammie\textsuperscript{2,3}

\textsuperscript{1} Department of Astronomy, University of Michigan, 500 Church Street, Ann Arbor, MI 48109, USA; zhuzh@umich.edu, lhartm@umich.edu
\textsuperscript{2} Department of Astronomy, University of Illinois Urbana-Champaign, 1002 West Green Street, Urbana, IL 61801, USA; gammie@illinois.edu
\textsuperscript{3} Department of Physics, University of Illinois Urbana-Champaign, Urbana, IL 61801, USA

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ABSTRACT

We use one-dimensional two-zone time-dependent accretion disk models to study the long-term evolution of protostellar disks subject to mass addition from the collapse of a rotating cloud core. Our model consists of a constant surface density magnetically coupled active layer, with transport and dissipation in inactive regions only via gravitational instability. We start our simulations after a central protostar has formed, containing $\sim 10\%$ of the mass of the protostellar cloud. Subsequent evolution depends on the angular momentum of the accreting envelope. We find that disk accretion matches the infall rate early in the disk evolution because much of the inner disk is hot enough to couple to the magnetic field. Later infall reaches the disk beyond $\sim 10$ AU, and the disk undergoes outbursts of accretion in FU Ori-like events as described by Zhu et al. If the initial cloud core is moderately rotating, most of the central star’s mass is built up by these outburst events. Our results suggest that the protostellar “luminosity problem” is eased by accretion during these FU Ori-like outbursts. After infall stops, the disk enters the T Tauri phase. An outer, viscously evolving disk has a structure that is in reasonable agreement with recent submillimeter studies and its surface density evolves from $\Sigma \propto R^{-1}$ to $R^{-1.5}$. An inner, massive belt of material—the “dead zone”—would not have been observed yet but should be seen in future high angular resolution observations by EVLA and ALMA. This high surface density belt is a generic consequence of low angular momentum transport efficiency at radii where the disk is magnetically decoupled, and would strongly affect planet formation and migration.

Key words: accretion, accretion disks – stars: formation – stars: pre-main sequence

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1. INTRODUCTION

In the picture of low-mass star formation from large, cold protostellar clouds, any small initial rotation of the cloud will lead to the formation of an accretion disk to conserve angular momentum; thus much, if not most, of the mass of low-mass stars is probably accreted from disks. However, if disks transport the infalling mass steadily to the central star, accretion luminosities are considerably higher than typically observed protostellar luminosities (Kenyon et al. 1990, 1994; Enoch et al. 2007). We find that disk accretion matches the infall rate early in the disk evolution because much of the inner disk is hot enough to couple to the magnetic field. Later infall reaches the disk beyond $\sim 10$ AU, and the disk undergoes outbursts of accretion in FU Ori-like events as described by Zhu et al. If the initial cloud core is moderately rotating, most of the central star’s mass is built up by these outburst events. Our results suggest that the protostellar “luminosity problem” is eased by accretion during these FU Ori-like outbursts. After infall stops, the disk enters the T Tauri phase. An outer, viscously evolving disk has a structure that is in reasonable agreement with recent submillimeter studies and its surface density evolves from $\Sigma \propto R^{-1}$ to $R^{-1.5}$. An inner, massive belt of material—the “dead zone”—would not have been observed yet but should be seen in future high angular resolution observations by EVLA and ALMA. This high surface density belt is a generic consequence of low angular momentum transport efficiency at radii where the disk is magnetically decoupled, and would strongly affect planet formation and migration.

Over the past decade, disk angular momentum transport by the magnetorotational instability (MRI; Balbus & Hawley 1998) and gravitational instability (GI; Durisen et al. 2007) has become much better understood. It seems likely that both types of instabilities need to be considered to understand protostellar accretion. At early evolutionary stages, the disk is likely to be quite massive with respect to the central protostar, suggesting that GI may be important; in addition, these disks are so cold that thermal ionization cannot sustain the MRI, which requires coupling of the magnetic field to the mostly neutral gas through collisions (e.g., Reyes-Ruiz & Stepinski 1995). On the other hand, external ionizing sources (cosmic and/or X-rays) can provide the necessary ionization for MRI activation up to a limiting surface density, ionizing the outer surfaces of the disk and leading to accretion in an “active layer,” leaving a “dead zone” in the midplane (Gammie 1996). In addition, at high accretion rates, the inner disk can become thermally ionized, as in FU Ori outbursts (Zhu et al. 2007, 2009b).

Mismatches in the transport rate between the GI and MRI can lead to outbursts of accretion (Armitage et al. 2001; Zhu et al. 2009b, 2009c). In Zhu et al. (2010, Paper I), we constructed one-dimensional, time-dependent disk models to study the protostellar unsteady accretion. These models assume both thermally activated and layered MRI-driven accretion along with a local treatment of the GI. We further adopted steady mass addition at a specified outer disk radius to drive the system. We found that accretion in protostellar disks is unsteady over a wide range of parameters because of a mismatch between GI transport in the outer disk and MRI transport in the inner disk. Our results for outburst behavior in these one-dimensional models were sufficiently comparable to our previous two-dimensional simulations of outbursts (Zhu et al. 2009c), confirming the utility of the faster one-dimensional simulations to explore wider ranges of parameter space.

In this paper, we extend our results from Paper I to study long-term disk evolution from the protostellar phase to the T Tauri phase, using a more self-consistent treatment for infall from a collapsing, rotating cloud core. This allows us to study the effect of the initial core rotation and different disk accretion configurations (layered accretion, GI-only accretion, and constant-$\alpha$ accretion) on the disk formation and evolution. Rice et al. (2010) have also investigated somewhat similar one-dimensional models but do not assume layered accretion.

Our models treat the same phase of evolution as Vorobyov & Basu (2005, 2006, 2007) investigated with two-dimensional
models. They also argue that protostellar accretion is generally non-steady, but for a different reason, specifically the accretion of clumps created by gravitational instabilities. Kratter et al. (2010) have also investigated the protostellar accretion phase with three-dimensional numerical simulations. Although our treatment of the core collapse and the GI is much more schematic than used in the above investigations, we are able to treat radiative cooling more realistically, and can follow disk evolution to much smaller radii, where the (thermally activated) MRI can become important. Furthermore, our two-zone disk model allows us to study the effects of different disk accretion configurations efficiently. The variety of disk structures predicted by our simulations with different initial cloud core rotations and disk accretion mechanisms can be tested by future EVLA and ALMA observations, which will help us to better understand disk accretion and planet formation.

We describe our one-dimensional, two-zone model with self-consistent infall in Section 2. In Section 3, we explore the behavior of the disk models with infall. In Section 4, we discuss the implications of our results and summarize our conclusions in Section 5. We defer all derivations to the appendices.

2. 1D2Z MODELS WITH INFALL

The one-dimensional two-zone (1D2Z) disk model has been introduced in Paper I. It consists of two vertically averaged layers (the surface layer and the “dead zone”) evolving independently. The surface density evolves according to the mass and angular momentum conservation equations in cylindrical coordinates,

\[ 2\pi R \frac{\partial \Sigma}{\partial t} - \frac{\partial M_r}{\partial R} = 2\pi \alpha M \right\}_{(R,t)}, \tag{1} \]

\[ 2\pi R \frac{\partial \Sigma}{\partial t} \left( \Sigma R^2 \Omega \right) - \frac{\partial}{\partial R} \left( \dot{M}_r R^2 \Omega \right) = 2\pi \frac{\partial}{\partial R} \left( R^2 W_{\phi,i} \right) + 2\pi \Lambda \right\}_{(R,t)}, \tag{2} \]

where \( \Sigma \) is the total surface density, \( \Omega \) is the angular frequency, \( \dot{M}_r = -2\pi \Sigma R v_i \) is the radial mass flux in the disk, \( W_{\phi,i} = R \Sigma v_i R \Omega dR \) is the viscosity, and \( i \) denotes “a” (active layer) or “d” (dead zone). \( 2\pi \alpha M \) and the terms \( 2\pi \Lambda \) are the mass and angular momentum flux brought by the infalling matter into the disk (Cassen & Moosman 1981). Assuming instantaneous centrifugal balance, Equations (1) and (2) can be simplified to

\[ \dot{M}_i = 6\pi R^{1/2} \frac{1}{\dot{M}} \left( R^{1/2} \Sigma v_i \right) + \frac{2\pi R^2 \Sigma \partial M_r}{\dot{M} R^{1/2}} \right\}_{(R,t)}, \tag{3} \]

where \( \dot{M}_i \) is the sum of the mass of the central star (\( M_\ast \)) and the disk mass within \( R \), using the approximation of the gravitational potential adopted by Lin & Pringle (1990). The first term in Equation (3) represents disk accretion due to viscosity; the second term is due to the central star mass changing with time, which can be also derived by adiabatic invariance of the angular momentum; and the third term is due to the infalling matter (Cassen & Moosman 1981). Since the infalling matter only falls onto the surface of the disk, we assume \( g_d \) and \( \Lambda_d \) are 0; thus, \( g_a \) and \( \Lambda_a \) can be written as \( g \) and \( \Lambda \) for short. The effect of the infalling matter onto the disk is limited to just two free functions: \( g(R,t) \) and \( \Lambda(R,t) \).

The surface densities \( \Sigma_a \) and \( \Sigma_d \) exchange mass as described in Paper I to maintain \( \Sigma = \Sigma_a = \Sigma_d \), where \( \Sigma_a \) is the maximum MRI active layer surface density and is assumed to be a constant throughout the disk (Gammie 1996). We solve Equations (3) and (1) sequentially: \( M_i \) is calculated with the current disk temperature and \( \Sigma_a \), and then it is inserted into Equation (1) to update \( \Sigma \) at the next time step.

The disk temperature is determined by the balance between the heating and radiative cooling. Here, the external temperature \( T_{\text{ext}} \), which represents the heating effect of the irradiation from the central star, is assumed to be

\[ T_{\text{ext}} = \frac{f L}{4\pi R^2 \sigma}, \tag{4} \]

where \( L \) is the total luminosity of the star and changes with the central star mass by

\[ \frac{L}{L_\odot} = \frac{8M_\ast}{9M_\odot} + \frac{1}{9}, \tag{5} \]

which is a fit to the T-Tauri star birthline from Kenyon & Hartmann (1995). The change of luminosity has little effect on the disk evolution in our simulations. \( \sigma \) is the Stefan–Boltzmann constant, and \( f \) is the coefficient, which accounts for the non-normal irradiation from the central star at the disk’s surface. We assume \( f = 0.1 \) in this paper. The viscous heating rate of the disk is

\[ Q_{\text{visc}} = \frac{3}{2} W_{\phi \phi} \Omega, \tag{6} \]

where the stress \( W_{\phi \phi} = 3/2 \Sigma v_i \Omega \) assuming Keplerian rotation. Both gravitational and MRIs are considered. The anomalous viscosity \( v_i \) is

\[ v_i = c_s^2 \frac{\alpha}{\Omega}, \tag{7} \]

where \( c_s \) are the sound speed of the active layer and the dead zone, \( \alpha = \alpha_Q + \alpha_M \) and

\[ \alpha_Q = e^{-Q^4}, \tag{8} \]

The Toomre parameter \( Q \) is calculated using the disk central temperature \( T_d \) and the total surface density \( (\Sigma_a + \Sigma_d) \). The form of \( \alpha_Q \) is motivated by a desire to make gravitational torques become important only when \( Q \lesssim 1.4 \), as indicated by global three-dimensional simulations (e.g., Boley et al. 2006). The parameter \( \alpha_M \) is set to be 0.01 when the disk temperature is above the preset critical MRI temperature for thermal activation \( T_M = 1500 \) K (assumed in this paper) or at the surface active layer. Since the rise time of MRI is about the orbital timescale \( T_M \) of \( \alpha_M \) whenever the temperature is above the critical temperature \( T_M \). The effects of different values of \( \alpha_M \), \( T_M \), and \( \Sigma_o \) on outbursts are discussed in Paper I. The treatment of radiative cooling is the same as in Paper I.

Recent observations suggest that the density structure of protostellar molecular cores, when circularly averaged, can be approximated by static Bonnor–Ebert spheres (Alves et al. 2001; di Francesco et al. 2007; Ward-Thompson et al. 2007). Though real clouds have more complex structure (e.g., Benson, 2001; di Francesco et al. 2007; Ward-Thompson et al. 2007).
& Myers 1989), resulting in more complicated patterns of infall (Kratter et al. 2010), we adopt this simplified structure to limit the number of model parameters. This leads us to use an approximate model of infall corresponding to that expected from a critical Bonnor–Ebert sphere, which exhibits similarities to that of the collapse of the singular isothermal sphere (SIS; Terebey et al. 1984).

Analytical and numerical studies (Foster & Chevalier 1992; Henriksen et al. 1997; Gong & Ostriker 2009) have shown that the collapse of the critical Bonnor–Ebert sphere can be divided into two stages. In the first stage, the roughly constant-density inner region collapses to the central regions nearly simultaneously; then in the second stage, infall from the remaining cloud core ($r \propto r^{-2}$ region) is at a rate similar to $c_{sc}/G(r_c)$, is the sound speed at the cloud core temperature) of the SIS collapse model (Shu 1977). We assume that our simulation starts at the end of the first stage of very rapid collapse of low-angular momentum material, so the central star initially has the mass of the inner flat region of the Bonnor–Ebert sphere, and the collapse of the remaining cloud follows the SIS solution (Shu 1977).

We use a two-component density profile to represent the Bonnor–Ebert sphere as in Henriksen et al. (1997). The two density components are

$$\rho = \rho_c \text{ at } \xi < \sqrt{2},$$

$$\rho = 2\rho_c \xi^{-2} \text{ at } \sqrt{2} < \xi < 6.5,$$

where $\rho_c$ is the central density, $r$ is in the normalized unit $\xi = r/(c_{sc}/\sqrt{4\pi G \rho_c})^{1/2}$, $c_{sc}$ is the sound speed of the cloud, and $\rho = 2\rho_c \xi^{-2}$ is the density profile of the SIS model. $\xi = 0.5$ corresponds to the critical Bonnor–Ebert sphere radius, and the radius of the flat density region $\xi = \sqrt{2}$ corresponds to $r_{ic} = (c_{sc}/2\pi G \rho_c)^{1/2}$. The total mass of each component is

$$M(r < r_{ic}) = \frac{0.27 c_{sc}^3}{G^{3/2} \rho_c^{1/2}}$$

$$M(r_{ic} < r < r_{cr}) = \frac{2.88 c_{sc}^3}{G^{3/2} \rho_c^{1/2}}.$$

The total mass with this simplified density profile is $3.1 c_{sc}^3/G^{3/2} \rho_c^{1/2}$, which is close to the mass of a real critical Bonnor–Ebert sphere $4.2 c_{sc}^3/G^{3/2} \rho_c^{1/2}$. Since the flat inner region has $10\%$ mass of the $r^{-2}$ region, we assume the flat region has $0.1 M_\odot$ and the rest of the Bonnor–Ebert sphere has $1 M_\odot$.

The rest of the $1 M_\odot$ cloud core beyond $r_{ic}$ collapses in a manner similar to that of the SIS, in which case the inner regions collapse first due to their shortest free-fall times and this free-fall zone extends outward linearly with time ($m_0 c_{sc} t$, where $m_0 = 0.975$; Shu 1977; Terebey et al. 1984). Thus, the mass infall rate is $M_{in} = m_0 c_{sc} t$, which is constant during the collapse. At time $t$, all the material collapsing to the center was originally at the cloud radius $r_0 = (m_0/2) c_{sc} t + r_{ic}$, where the addition of $r_{ic}$ is the radius of the flat density region in the Bonnor–Ebert sphere is due to the fact that we define $t = 0$ as the time when the flat region in the Bonnor–Ebert sphere has already collapsed to the central star. If the protostellar cloud core is initially in uniform rotation with angular velocity $\Omega_c$, the material falling in from different directions will have different angular momenta and arrive at the midplane at differing radii within the so-called centrifugal radius (Cassen & Moosman 1981),

$$R_c = r_0^4 \frac{\Omega_c^2}{G M_c},$$  

where $M_c$ is the central star mass. Assume $M_c$ increases with $t$, $R_c \propto t^{1.6}$, similar to that found by Terebey et al. (1984) for the SIS collapse.

We use the Cassen & Moosman (1981) solution for the infall to the disk at a given time from a spherical cloud initially in uniform rotation. We start the infall from the transition radius $r_{ic}$ at $t = 0$. Then the mass flux $g(R, t)$ and the angular momentum $\Lambda$ brought by the infalling matter for a uniformly rotating SIS are

$$g(R, t) = \frac{M_{in}}{4\pi R_c} \left(1 - \frac{R}{R_c}\right)^{-1/2},$$

$$\Lambda(R, t) = g(R, t) R \left(\frac{G M_c}{R_c}\right)^{1/2} \text{ if } R \leq R_c,$$

and

$$g(R, t) = 0, \text{ if } R > R_c.$$

where $M_{in} = (m_0 c_{sc} t/G)$ is the infall rate and $R_c$ is the centrifugal radius defined in Equation (13). Inserting Equations (14)–(17) into Equations (1)–(3), we can simulate the disk evolution under the infall.

3. Layered Accretion with Infall

In all cases, we start with $M_c = 0.1 M_\odot$ and a cloud mass of $1 M_\odot$. We adopt a temperature of $20 \text{ K}$ for the envelope. This parameter enters in setting the infall rate $M_{in} = c_{sc}^3/G = 4.2 \times 10^{-6} M_\odot \text{ yr}^{-1}$; thus the infall lasts $2.4 \times 10^5 \text{ yr}$, roughly consistent with observations (Kenyon et al. 1990, 1994; Evans et al. 2009). This leaves only one parameter to vary, the initial angular velocity of the cloud $\Omega_c$, which affects the maximum centrifugal radius onto the resulting disk. For a SIS core,

$$R_{max} \approx \frac{G M_c^3 \Omega_c^2}{16 \pi^8 c_{sc}^8},$$

where we have assumed that the central star mass at the end of infall is approximately the cloud core mass ($M_{co}$); thus

$$R_{max} \approx 12 \text{ AU} \left(\frac{\Omega_c}{10^{-14} \text{ rad s}^{-1}}\right)^2.$$

Since $R_{max}$ directly constrains the disk size after the infall, it is the most important parameter determining the initial disk properties. All the model parameters we explore in the paper are summarized in Table 1.

3.1. Fiducial Model

We take as our fiducial model one with $\Sigma_A = 100 \text{ g cm}^{-2}$, $\alpha_M = 0.01$, and $\Omega_c = 10^{-14} \text{ rad s}^{-1}$ so that $R_{max} = 12 \text{ AU}$.

Figure 1 shows the fiducial model’s mass accretion rate and the central star and disk masses as a function of time. The overall evolution can be divided into three distinct stages. Initially there is a phase of quasi-steady disk accretion, with the accretion rate matching the infall rate ($M_{in} = 4.2 \times 10^{-6} M_\odot \text{ yr}^{-1}$). This occurs because the infall is to small radii where the disk

\footnotetext[5]{In this paper, $R$ denotes the cylindrical radius in the disk while $r$ denotes the radius in the cloud core.}
The disk structure during the outburst stage is illustrated in Figure 2, shown at the time when the centrifugal radius ($R_c$) is 10 AU (labeled by the triangle). Although the infall ends at $R_c$, the disk extends far beyond $R_c$ due to the outward mass transfer by the active layer. With the continuous infall, the disk is marginally gravitationally stable within $R_c$, and the energy dissipation due to the GI has significantly heated the disk so that the outburst will be triggered at 2 AU (the upper right panel shows that $T_c$ is approaching 1500 K). At this time when infall ends, the region between ~1 AU and ~20 AU is still marginally gravitationally unstable, although the disk within 1 AU has been depleted by the previous outburst (Figure 3).

The innermost disk ($R < 0.1$ AU) is purely MRI active due to the high temperature there produced by viscous heating and irradiation. Due to the efficient mass transport by the MRI, this inner region can be depleted rapidly, but is limited by the accretion from the outer active layer. Eventually, a balance is achieved, and the inner disk behaves like a constant-$\alpha$ disk with mass accretion rate equal to the mass accretion rate of the innermost active layer (e.g., Gammie 1996).

In the standard layered accretion model, the disk mass accretion rate is set as the active layer accretion rate at the dead zone inner radius (e.g., Gammie 1996). We use the modification of the accretion rate including irradiation derived by Hartmann et al. (2006; see also Appendix A),

$$\dot{M} = 6.9 \times 10^{-9} \frac{R}{0.2 \text{ AU}} \left( \frac{M_c}{1 \text{M}_\odot} \right)^{-1/2} \left( \frac{\alpha M}{10^{-2}} \right) \left( \frac{\Sigma_A}{100 \text{ g cm}^{-2}} \right) \left( \frac{L_*}{L_\odot} \right)^{1/4} \left( \frac{f}{0.1} \right)^{1/4} \text{M}_\odot \text{ yr}^{-1},$$

where $f$ is as defined in Equation (4). At the sublimation radius, $f$ may be significantly larger than 0.1. With our numerical inner boundary 0.2 AU, the simulation shows the disk has a mass accretion rate of $10^{-8}$M$_\odot$ yr$^{-1}$ in the quiescent state, which agrees with the above estimate.

As discussed in Paper I, we find unstable behavior at the dead zone inner radius because of the rapid change in $\alpha_M$. This accounts for the modest but significant variability seen during the quasi-steady state. 6 This instability cannot be reliably

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### Table 1

| Cloud $\Omega$ | $\Sigma_A$ (g cm$^{-2}$) | $\alpha_M$ | $R_D$ 0.3/1 Myr (AU) | $M_{aM}^a$ (M$_\odot$) | $M_{aM}^b$ (M$_\odot$) | $M_{aM}^c$ (M$_\odot$) |
|---------------|--------------------------|-----------|----------------------|---------------------|---------------------|---------------------|
| $2 \times 10^{-14}$ | 100 | 0.01 | 40/13 | 0.29/0.19 | 0.18/0.14 | 0.47/0.33 |
| $2 \times 10^{-14}$ | 10 | 0.01 | 127/66 | 0.42/0.33 | 0.05/0.09 | 0.48/0.42 |
| $2 \times 10^{-14}$ | 0 | 0.01 | ... | 0.47/0.41 | ... | 0.47/0.41 |
| $1 \times 10^{-14}$ | 100 | 0.01 | 24/7.6 | 0.17/0.1 | 0.07/0.06 | 0.24/0.16 |
| $1 \times 10^{-14}$ | 10 | 0.01 | 47/34 | 0.32/0.23 | 0.008/0.022 | 0.32/0.26 |
| $3 \times 10^{-15}$ | 100 | 0.01 | 4.6/2.4 | 0.05/0.04 | 0.006/0.007 | 0.057/0.049 |
| $3 \times 10^{-15}$ | 10 | 0.01 | 5.5/4.6 | 0.06/0.06 | 0.0004/0.001 | 0.063/0.063 |

**Notes.**

a The disk mass within $R_D$.

b The disk mass beyond $R_D$.

c The total disk mass.
treated in our model, due to the complexity of the processes involved including MRI activation and dust sublimation; thus, the precise nature of the variations in accretion during this phase is uncertain.

A massive, marginally gravitationally unstable dead zone remains well after the end of infall (upper left and lower right panels in Figure 3). This dead zone is maintained because the active layer mass accretion rate increases with radius (Gammie 1996). From the mass conservation equation one can derive the time dependence of the total surface density \( \Sigma = \Sigma_A + \Sigma_d \),

\[
\frac{d\Sigma}{dt} = -\frac{\partial}{\partial R} \left( \frac{M}{2\pi} \right) \Omega^2 R \left( \frac{M}{1\ M_\odot} \right)^{-1/2} \left( \frac{\alpha M}{10^{-2}} \right) \left( \frac{\Sigma_A}{100 \ g \ cm^{-2}} \right) \times \left( \frac{L_\alpha}{L_\odot} \right)^{1/4} \left( \frac{f}{0.1} \right)^{1/4} \ g \ cm^{-2} \ yr^{-1}.
\]

Thus \( \Delta \Sigma / \Delta t > 0 \) and \( \propto R^{-1} \). The disk surface density thus increases linearly with time, more rapidly at smaller radii. When the surface density increases to the extent that the disk becomes gravitationally unstable, self-gravity transfers the excess mass and self-regulates the disk to an approximate \( Q \sim 1 \) state. From Equation (21) one can see that it only takes \( 10^5 \) yr for the disk at 10 AU to become gravitationally unstable by layered accretion. Thus, active-layer accretion can maintain a dead zone that is marginally gravitationally unstable well after infall stops.

A sharp density drop appears at the outer radius of the dead zone \( R_D \) (also shown in Reyes-Ruiz 2007 and Matsumura et al. 2009) and \( R_D \) starts at the maximum centrifugal radius after the infall and gradually moves inward with time, so that the dead zone becomes increasingly narrow as the system evolves (see Appendix A). In the outer disk beyond \( R_D \) where only the active layer is present, the disk surface density evolves from a \( \Sigma \propto R^{-1} \) distribution to an asymmetric \( \Sigma \propto R^{-1.5} \) distribution at large times. This result can be understood by modeling it as a pure \( \alpha \) disk but with a fixed \( \Sigma \) inner boundary, as discussed in Appendix A.

3.2. Varying Core Rotation

The initial disk size is constrained by the maximum infall radius \( R_{\text{max}} \). We next consider the results for \( \Omega_c = 3 \times 10^{-15} \) rad s\(^{-1}\) and \( 2 \times 10^{-14} \) rad s\(^{-1}\).

Figure 4 shows the accretion behavior of these models during the first 2 Myr. In the slowly rotating case, all of the mass is accreted in a quasi-steady fashion, and the remaining disk mass is very small, whereas the collapse of the rapidly rotating core results in very little quasi-steady accretion, an extended period of outbursts lasting well beyond the end of infall, and a very massive disk.

The differing behavior of the three models can be understood with reference to Figure 5, which is a modification of the \( M \) versus \( R \) plane discussed by Zhu et al. (2009b). For the slowest rotating core model, \( R_{\text{max}} \sim 1 \) AU (Equation (19)). Reference to Figure 5 shows that this radius is just within the maximum radius \( R_M \) for steady MRI-driven accretion at \( M = 4 \times 10^{-6} \ M_\odot \) yr\(^{-1}\). In this region, the disk can maintain a temperature greater than the critical value for MRI activation; the disk then accretes on to the star as a typical \( \alpha \)-disk, with modest oscillations due to the instability at the inner disk edge.
Figure 4. Disk mass accretion rates and the mass of the central star and disk with time for different core rotations. The upper two panels show the simulation with \( \Omega_c = 2 \times 10^{-14} \text{ rad s}^{-1} \), while the lower ones with \( \Omega_c = 3 \times 10^{-15} \text{ rad s}^{-1} \). The solid curves in the right panels show the mass of the central star with time while the dotted curves show the mass of the disk (dotted curve in the lower right panel is too close to 0 and hard to be seen).

Figure 5. Unstable regions in the \( R - \dot{M} \) plane for an accretion disk around the 1 \( M_\odot \) central star. The shaded region in the upper left shows the region subject to classical thermal instability. The shaded region in the middle shows where the central temperature of steady GI models exceeds an assumed MRI trigger temperature of 1400 K, thus subject to MRI–GI instability. The lines labeled \( R_M \) and \( R_Q \) are the maximum MRI and minimum GI steady accretion radius (the boundaries of the shaded region, detailed in Zhu et al. 2009b). The shaded region on the right shows where the GI disk will fragment (see Appendix B for details). The fragmentation limit agrees with Cossins et al. (2010).

Figure 6. Disk surface density with radii for different rotation cores as shown in Figure 4 at 0.3 (solid curves) and 1 Myr (dashed curves).

3.3. Different Disk Configurations

The ionization structure of disk surface layers depends on the dust size spectrum and the flux of ionizing radiation and cosmic rays, and is therefore poorly constrained. Angular momentum transport in magnetically decoupled disks is also poorly understood, although recent work by Lesur & Papaloizou (2009), following Petersen et al. (2007), strongly suggests that a nonlinear instability driven by baroclinicity and radiative diffusion may give rise to hydrodynamic turbulence and angular momentum transport. Therefore, we have investigated disks with a reduced active layer (Section 3.3.1) and a residual dead zone viscosity (Section 3.3.2).
show that disks with \( \Sigma \) 2 Myr to transport mass from 50 AU to the central star; see also \( \alpha \) (solid curves) and constant-

\( \Sigma \) Figure 7. Disk surface density distributions at 1 Myr for layered models

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the star. In addition, with a smaller \( \Sigma \) fiducial model; thus, the infall mass takes longer to accrete onto

\( \Omega \) (A color version of this figure is available in the online journal.)

\( T \) R edge roughly at

\( Q \) evolves toward

\( \alpha \)M

(we consider disks with lower active-layer surface densities

\( \alpha \) right panel shows the disk density distribution at 0.3 (solid curve) and 1 Myr (dashed curve). The lower

\( \alpha \)A\alphaM

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disk.

If \( \alphaM\SigmaA \) is lower, the disk accretion rate is lower than our

\( \Sigma \) the active layer, with

\( \alpha \)A

the operation of the MRI with the ions turbulent mixing from

\( \alpha \) right panel shows the disk

\( \Sigma \) leads to a less massive outer
disk.

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disk.
argument. Compared with a constant-$\alpha$ disk having the same mass accretion rate, the layered disk has a surface density higher than the constant-$\alpha$ disk by a factor of $\alpha_a/\alpha_d$. If we assume the outer pure MRI active disk and the inner layered disk have the same mass accretion rate, and the outer disk only has $\nu_a$ while the inner dead zone is dominated by $\nu_d$, the disk’s surface density will change by a factor of $\alpha_a/\alpha_d$ within the dead zone outer radius $R_D$, which is shown in Figure 9. Also this density change at $R_D$ is more gradual (Equation (22) is a smooth function of $R$) than the zero dead zone viscosity case. This can have a significant impact on planet migration (Section 4.6).

4. DISCUSSION

4.1. Dead Zones and Star Formation by Disk Accretion

The main result of our work is that star formation through disk accretion, assuming a sufficiently large initial disk that the MRI is not thermally activated throughout the disk, generally results in a dense belt of material—the “dead zone”—which has implications for disk structure and planet formation.

To demonstrate this more clearly, we show the results for a constant-$\alpha$ disk for the same infall model. By comparison with layered disks in Figure 7, constant-$\alpha$ disks have similar outer disk but lack the massive dead zone. Constant-$\alpha$ disks respond to infall just as layered disks do: a faster rotating core leads to a more massive disk since the infall mass onto larger radii takes longer to accrete to the central star. Our models show that with the same $\alpha$ in constant-$\alpha$ models, cores with $\Omega_c = 2 \times 10^{-14}$ rad s$^{-1}$ lead to disks 10 times more massive than the disks produced by cores with $\Omega_c = 3 \times 10^{-15}$ rad s$^{-1}$ at 1 Myr. However, even if we allow constant-$\alpha$ disks to have different values of $\alpha$ (Figure 10), all these models produce surface densities far in excess of 100 g cm$^{-2}$ over large areas of the disk at the end of infall (lower left panel of Figure 10). Thus, a reduced angular momentum transport efficiency associated with the failure of the MRI at $\Sigma > 100$ g cm$^{-2}$ implies the formation of a dense belt or dead zone in the disk.

Another feature of the constant-$\alpha$ model is that, although the core rotation determines the disk mass, the disk surface density shape depends most strongly on $\alpha$. This can be simply explained by the similarity solution of the constant-$\alpha$ disk where the shape of the disk is determined by the scaling radius ($\text{Hartmann et al. 1998}$), which is proportional to $t$ and $\alpha$. Thus as long as $t$ and $\alpha$ are the same, the disk surface density has similar shape. We also note that, even for a constant-$\alpha$ disk, the disk is gravitationally unstable at the infall phase with a reasonable set of parameters ($\alpha = 0.01$ and $\Omega_c = 2 \times 10^{-14}$ rad s$^{-1}$; Figure 10).

4.2. Relation to Previous Work

As mentioned in Section 1, previous investigations by Vorobyov & Basu (2006, 2007) and Kratter et al. (2008, 2010) have considered the evolution of disks formed by infall from rotating cloud cores with angular momentum transport by GI. Vorobyov & Basu find outbursting behavior, as we do, but for different reasons; specifically, in their models time-dependent accretion is driven by gravitational clumping, which we cannot treat in one dimension. However, with more realistic energy equations, it is harder to lead to strong gravitational clumping within 50 AU (Boley et al. 2009; Appendix B). Kratter et al. (2010) studied more generalized models of infall; they found that the disk is massive and can fragment to binaries and multiple stars under conditions such as their analytic estimate (Kratter et al. 2008). However, for single star formation, the accretion is smooth without strong gravitational clumping.

Unlike these investigations, we study slowly rotating cloud cores forming single stars without disk fragmentation, focusing on disk evolution after the central core collapse. We treat the GI as a local phenomenon, which may be problematic if the
disk is very massive (Lodato & Rice 2005). However, during later stages when the disk is less massive, a local treatment appears to be reasonable (Lodato & Rice 2004; Cossins et al. 2009). In addition, we are able to treat radiative cooling more realistically; the treatment of thermal physics plays a crucial role in GI transport (e.g., Durisen et al. 2007) and MRI–GI outbursts (Paper I).

Lodato & Rice (2005) found recurrent episodes of GI in a massive disk. Boley & Durisen (2008) considered three-dimensional simulations of gravitationally unstable disks with accurate radiative transfer and also suggested that high mass fluxes could result from the rapid onset of GI, producing something like an FU Ori outburst. The question is whether this behavior is a transient due to an initial condition, which may or may not be realiziable as part of a natural evolutionary sequence.

Rice et al. (2010) also considered the evolution of disks formed by infall with GI as the transport mechanism. They also find that massive disks result, which leads them to suggest that another mechanism of viscosity must be operating to drain the disk. We address this possibility in Section 4.5.

Our treatment differs from all of the above in including MRI transport with layered accretion. This results in our outer disks tending to evolve viscously rather than through GI, so that mass accretion can continue even after the disk is gravitationally stable. Perhaps most importantly, our one-dimensional model allows us to treat the very innermost disk at the same time considering the outer disk evolution, something that is extremely difficult to do with two- or three-dimensional codes. This allows us to demonstrate behavior not seen in the other simulations; specifically, the onset of MRI-driven accretion in the inner disk, which tends to dominate the outburst behavior—or, as we have shown, result in a phase of quasi-steady disk accretion while low angular momentum core material is infalling.

At high accretion rates, it appears almost certain that the MRI should operate, as temperatures become high enough to sublimate dust, which might otherwise absorb charges. This notion is supported by observations of FU Ori (Zhu et al. 2007, 2008), which indicate that the disk can be hot enough to eliminate dust out to radial scales of order 1 AU. Thus, we argue that inclusion of the MRI is important for any understanding of accretion onto central protostars.

The behavior of non-thermally activated MRI by X-ray or cosmic-ray radiation is much more uncertain, but its inclusion can also have important effects not seen in pure GI treatments. In our layered simulations, during the T Tauri phase the GI has limited effects on the long-term disk evolution, and the disk mass accretion rate is controlled by the active layer, as discussed in Section 3. To confirm this, we have run the same simulation as before but with the $\alpha Q$ from Armitage et al. (2001), which is based on Lin & Pringle (1987, 1990). We found that the different forms of $\alpha Q$ have little effect on the long-term evolution of a layered disk; the T Tauri disk evolution is more determined by the active layer. In both treatments of $\alpha Q$, the layered accretion leads to a relatively massive “inner belt” within 30 AU and a viscous disk beyond 30 AU (Figure 3).

4.3. Outbursts and Luminosity Problem

Our model predicts that protostars will undergo FU Ori-like outbursts of rapid mass accretion to accumulate a significant amount of their mass, helping to solve the luminosity problem (Section 1). Here, we look at the question of luminosity evolution more closely.

The largest current survey of star-forming regions bearing upon this issue is that of the c2d Spitzer Legacy project (e.g., Enoch et al. 2009; Evans et al. 2009), which strongly suggests that protostellar accretion is time variable, with prolonged periods of low accretion rates. Evans et al. (2009) estimate that half of the mass of a typical low-mass star is accreted during only ∼7% of the Class I lifetime, which they estimate as ∼0.5 Myr.

In our fiducial model, roughly half of the mass is accreted quasi-steadily during the first $\sim 10^5$ yr, with the other half being accreted during outbursts within the next ∼1.5 $\times 10^5$ yr. This agrees with the c2d observations that half of the mass is accreted during high-mass accretion rate stage. However, the outburst behavior (episodic accretion) for the individual protostar does depend on the initial rotation of the cloud core. The $\Omega_\star = 2 \times 10^{-14}$ case accretes almost entirely via outbursts, while the low-rotation case accretes essentially all of the stellar mass quasi-steadily.

Evans et al. (2009) found that 59% of Class 0/I stars have bolometric luminosities smaller than 1.6 $L_\odot$. However, in our fiducial model, 30% of the Class 0/I phase is in the quasi-steady accretion phase with luminosity $\sim 13 L_\odot$ ($GM_\star M/R_\star$ by assuming $M = 4 \times 10^{-9} M_\odot$ yr$^{-1}$ and $M_c/R_\star = M_\odot/10 R_\odot$), much larger than 1.6 $L_\odot$. One possible solution of this problem is that the extinction toward the protostar during this quasi-steady phase is high, with $A_V \gtrsim 400$ using our model parameters; it might thus be difficult to identify these objects in such early stages of evolution.

Another possible solution is that the oscillation of the thermal MRI region (Wünsch et al. 2006) can lead to additional variations in the mass accretion rate and accretion luminosity (which is the reason we call the first steady accretion phase “quasi-steady”); variations are clearly seen in Figure 1 during the first $10^5$ yr—see Section 3.1. We have avoided concentrating on this feature because it depends on the complex physics at the inner edge of the dead zone, which is unclear to us now. Nevertheless, this potentially unstable behavior may be important in understanding protostellar accretion in general and may even provide a mechanism for explaining EXor outbursts (Herbig 1977). Further investigation of this problem with a realistic calculation of the onset of the MRI is needed.

Some other limitations of our models are that the infall is axisymmetric with a constant infall rate and that the GI is treated as a local effect. If the infall has a complex pattern and the GI has a strongly global character, the angular momentum transport during the infall phase could be very efficient (Larson 1984) and disk accretion may be more variable (Vorobyov 2009) or even form warped disk (Bate et al. 2010).

There is also a distinction between the model predictions for outbursts during the infall phase and those which occur after infall ends, during the T Tauri phase. Though they are both triggered by mass accumulation, this is caused by GI during the infall phase, which is independent of the layered structure (Paper I), while the outbursts during the T Tauri phase are triggered by the mass accumulation from active layer accretion, which is sensitive to the assumed value of $\Sigma_c$.

4.4. Rotation and Fragmentation

Our most rapidly rotating model formally predicts FU Ori-like outbursts during the T Tauri phase, driven by active layer accretion. However, this T Tauri phase outburst behavior is uncertain because it sensitively depends on active layer properties and the massive disks which would produce such behavior fragment into multiple stellar systems with individually smaller
disks (Rapidly rotating cores tend to produce large disks that are subject to gravitational fragmentation at $R > 50$ AU in a realistic disk; see Rafikov 2009; Cossins et al. 2010; see Appendix B.).

Finally, we consider possible values of envelope angular momentum. Typical observational estimates of the ratio of rotational kinetic to gravitational energy span a range of 3 orders of magnitude (e.g., Caselli et al. 2002), and thus are difficult to apply to our simulations. Moreover, the total angular momentum is extremely sensitive to assumptions about the distribution of mass with radius as well as the (not necessarily uniform) angular velocity; as cores are generally not Bonnor–Ebert spheres, or even axisymmetric about the rotation axis (Tobin et al. 2010), making it difficult to compare with observations. As mentioned above, cores with more angular momentum than our fastest rotating model are likely to fragment into multiple systems. Whether significant numbers of slowly rotating cores exist which can produce diskless T Tauri stars, or T Tauri systems with very low-mass disks, will require more detailed observational analyses of the complex geometries of starless cores.

4.5. Dead Zones: Do They Exist?

We have shown that a natural consequence of our layered model is the formation of dead zones that may remain relatively massive for more than $10^6$ yr (see also Vorobyov & Basu 2006, 2007; Rice et al. 2010). This stands in contrast with purely viscous models of disk evolution, as shown in Figure 7. An advantage of highly viscous models is that they provide an explanation for disk accretion rates during the T Tauri phase (Hartmann et al. 1998; Dullemond et al. 2006), whereas the layered accretion model has some difficulties in explaining T Tauri accretion rates (Hartmann et al. 2006). On the other hand, the pure viscous models are unable to explain FU Ori outbursts, which require large mass reservoirs at distance scales on the order of an AU (Armitage et al. 2001; Zhu et al. 2007, 2008, 2009b).

At present the best that can be said is that the MRI is likely to operate at $r < 0.1$ AU, and it is unlikely to operate at larger radii where the temperature is lower. Even the standard form of the minimum-mass solar nebula (MMSN; Weidenschilling 1977), let alone the recent amplification suggested by Desch (2007; Figure 11), indicates surface densities well in excess of those thought to be penetrated by cosmic rays, or even stellar X-rays. At any rate, the associated change (presumably decrement) in angular momentum transport efficiency where the MRI shuts off is likely to give rise to a feature in the surface density.

At first sight, the surface densities of the dead zones in our evolutionary models seem unreasonably large in comparison with previous models. However, Terquem (2008) has shown that, even for a constant mass accretion rate layered model, the dead zone is very massive. Reyes-Ruiz (2007) has also shown that a massive disk still exists even if the dead zone has some residual viscosity. Some of our models do not predict values of $\Sigma$ much higher than that of the Desch (2007) model (Figure 11). If the dead zone has some residual viscosity with $\alpha > 10^{-4}$, the dead zone mass can be significantly reduced but still relatively massive compared with a constant-$\alpha$ disk (Figure 12). Furthermore, some exoplanetary systems have masses substantially larger than that of Jupiter, suggesting that more than minimum mass solar nebulae are required, especially if migratory loss of solid bodies is significant.

Have high surface density regions been ruled out by recent interferometric observations of submillimeter dust emission in disks, such as those of Andrews et al. (2009)? To examine this possibility, in Figure 12 we compare various models (our fiducial model, fiducial model with a residual dead zone viscosity, and a constant-$\alpha$ accretion model) with the suite of disk structures inferred by Andrews et al. (2009). The agreement between the Andrews et al. results and our models in the outer disk is not...
surprising, because they adopted viscous disk solutions like ours to model the observed interferometric visibilities. If we adopt the same opacity as Andrews et al., we find that there is no strong signal in the disk surface brightness distribution at the edge of the dead zone because it is optically thick and the temperature distribution is dominated by irradiation from the central star (the brightness temperatures for the layered model and the constant-\(\alpha\) model are the same at 1 mm in Figure 13). The model does predict a possible feature at \(\lambda \sim 1\) cm where the outer disk becomes optically thin (Figure 13); this might be testable with the EVLA. The high surface densities in the dead zone may lead to rapid coagulation of the solid particles, however, reducing the optical depth contrast between the dead zone and the MRI-active outer disk region.

Our results suggest the potential importance of dead zones for simply providing a large mass reservoir for solid body formation. Higher density regions may be problematic in that timescales for radial drift of small bodies and Type I migration become much shorter; on the other hand, this may be offset by having more material to start with, and other features of the dead zone may help provide “traps” where planetesimal can form efficiently (Rice et al. 2004) and migration can be halted or even reversed (see the following section).

In this connection the existence of the so-called “transition” disks is worth noting, where outer, optically thick disks surrounding T Tauri stars have very large inner disk holes—either partially or fully evacuated (e.g., Espaillat et al. 2007). In some cases, these large inner disk holes may be the result of tidal torques from companion (binary) stars, but in others giant planets may be the cause of the inner disk clearing. In the latter case, it may be necessary to have multiple giant planets form nearly simultaneously over a range of radii in order to explain the sizes of these disk holes. The dead zones of our models, with relatively large surface densities over a significant range of radii, may be able to promote the necessary runaway growth.

\[
F = \frac{\Sigma_{Q=1}}{\Sigma_A} = \frac{c_s \Omega}{\pi G \Sigma_A} = \left( \frac{45 \text{ AU}}{R_D} \right)^{7/4} \left( 100 \text{ g cm}^{-2} \right) \left( \frac{100 \text{ g cm}^{-2}}{\Sigma_A} \right). \tag{23}
\]

Here, we have assumed the temperature profile as \(T = 200 \text{ K} (R/1 \text{ AU})^{-1/2}\), thus, as the dead zone moves inward (\(R_D\) becomes smaller), \(F\) increases. However, the density jump width, which is the last parameter required to calculate the torque on the planet, cannot be constrained by our one-dimensional simulation.

4.7. MMSN and Planet Formation in the Layered Disk

Although at the early stage the inner disk is massive, at later stage (~Myr) the outer disk (beyond 10 AU) is comparable to the MMSN from Weidenschilling (1977; Figure 11). Due to the boundary effect at the dead zone outer radius \(R_D\) (Section 3.1 and Appendix), the outer disk evolves toward \(\Sigma \propto R^{-1.5}\) as in
the standard MMSN. Furthermore, if the dead zone is massive, $R_D$ moves inward very slowly (Section 4.5); then $\Sigma \propto R^{-1.5}$ lasts for a long time.

If planets form in a massive dead zone, they may be lost by inward migration; however, some may be trapped at the inner boundary (Kretke et al. 2009) or outer boundary (Matsumura et al. 2007, 2009) of the dead zone.

5. CONCLUSIONS

In this paper, we have constructed a 1D2Z accretion disk model to study disk formation and long-term evolution under the collapse of a Bonnor-Ebert rotating core. The model evolution can be divided into three stages. At the early stage, when the mass falls to the inner disk within AU scale, the MRI can be sustained in the inner disk and efficiently and steadily transfers the infalling mass to the central star. Later, when the mass falls beyond AU scale, the disk goes to the outburst stage due to the accretion rates’ mismatch by the MRI and GI as described in Paper I. After the infall completes, the disk enters the T Tauri phase and evolves on its own. Cores with higher initial rotation end up with a more massive disk and more disk episodic accretion events (outbursts). As long as the initial cloud core does not rotate extremely slowly to form a tiny disk ($R_{\text{max}} \sim 1$ AU), more than half of the star mass is built up by outbursts, which eases the “luminosity” problem.

Disks exhibit a variety of behavior during the T Tauri phase. For a disk with accretion sustained only by GI, the disk evolves toward a $Q = 1$ disk and the disk truncates at a radius slightly larger than the maximum centrifugal radius of the infall. If the disk has an active layer at the surface, however, the active layer can extend to a much larger radius and a sharp density drop develops at a characteristic radius $R_D$ that separates the marginally gravitationally stable dead zone and the MRI active but gravitationally stable outer disk. The density jump at $R_D$ may be observable by the EVLA and ALMA. The formation of a dense belt of material is associated with the failure of magnetically driven transport due to low ionization at intermediate radius in the disk; the only ways to avoid this are (1) if there is a separate, equally efficient hydrodynamic transport mechanism or (2) if for some reason the MRI fails in the outer disk as well, perhaps due to dynamo failure.

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APPENDIX A
LAYERED ACCRETION

For a layered disk dominated by viscous heating, Gammie (1996) showed that the central temperature and the mass accretion rate at the radius $R$ are

$$T_c = 290 \left( \frac{R}{1 \text{ AU}} \right)^{-3/5} \left( \frac{M_c}{1 M_\odot} \right)^{1/5} \left( \frac{\alpha M}{10^{-2}} \right)^{2/5} \times \left( \frac{\Sigma_A}{200 \text{ g cm}^{-2}} \right)^{4/5} \text{ K},$$

$$\dot{M} = 1.17 \times 10^{-7} \left( \frac{R}{1 \text{ AU}} \right)^{9/10} \left( \frac{M_c}{1 M_\odot} \right)^{-3/10} \left( \frac{\alpha M}{10^{-2}} \right)^{7/5} \times \left( \frac{\Sigma_A}{200 \text{ g cm}^{-2}} \right)^{9/5} M_\odot \text{ yr}^{-1},$$

where the Bell & Lin (1994) opacity has been used.

In the irradiation-dominated limit, the disk temperature is the external temperature (Equation (4)); $\sigma T^4 = f L_b/4\pi R^2$. In this case, the disk temperature and mass accretion rate with radii are

$$T_c = 221 \left( \frac{L_b}{L_\odot} \right)^{1/4} \left( \frac{R}{1 \text{ AU}} \right)^{-1/2} \left( \frac{f}{0.1} \right)^{1/4} \text{ K},$$

$$\dot{M} = 6.9 \times 10^{-8} \left( \frac{R}{1 \text{ AU}} \right) \left( \frac{M_c}{1 M_\odot} \right)^{-1/2} \left( \frac{\alpha M}{10^{-2}} \right) \times \left( \frac{\Sigma_A}{200 \text{ g cm}^{-2}} \right) \left( \frac{L_b}{L_\odot} \right)^{1/4} \left( \frac{f}{0.1} \right)^{1/4} M_\odot \text{ yr}^{-1}.$$  \hspace{1cm} (A3)

Comparing Equations (A4) and (A2), we see that the disk is irradiation dominated if the active layer has surface density ($\Sigma_A < 40 \text{ g cm}^{-2}$), or $\alpha M$ is smaller than 0.002, or the luminosity is significantly larger than the stellar radiation ($L > 10 L_\odot$).

A.1. Dead Zone

In either of the above cases the layered disk accretion rates increase nearly linearly with radius, which results in piling up mass in the dead zone at small radii. Using the mass conservation equation

$$R \frac{\partial \Sigma}{\partial t} = - \frac{\partial}{\partial R} \left( \frac{M}{2\pi} \right),$$

\hspace{1cm} (A5)

$\partial \Sigma / \partial t \propto R^{-1.5}$. Thus, the layered disk surface density increases linearly with time and it increases more rapidly at smaller radii. If we assume the dead zone has zero residual viscosity (non-zero residual viscosity has been discussed in Section 3.3.2) and active layer viscosity $\nu_a = k R^4$, disk evolution becomes

$$\frac{\partial \Sigma}{\partial t} = 3n \left( n + \frac{1}{2} \right) k \Sigma_A R^n t^{-2},$$

\hspace{1cm} (A6)

Thus

$$\Sigma(R, t) = 3n \left( n + \frac{1}{2} \right) k \Sigma_A R^{n-2} t + C,$$

\hspace{1cm} (A7)

which increases linearly with time due to the layered accretion, as shown by Matsumura et al. (2007).

The dead zone starts at 0.1 AU and ends at the radius where the disk surface density is smaller than $\Sigma_A$ (the dead zone outer radius $R_D$), leaving only the MRI active layer ionized by cosmic and/or X-rays at larger radii. This pure MRI active outer disk beyond $R_D$ evolves like a constant-$\alpha$ viscous disk. However since the active layer of the inner disk accretes mass inward, the dead zone is gradually depleted so that $R_D$ moves inward with time.

To determine the dead zone size ($R_D$) during the layered disk evolution, we need to know its initial position just after the infall. Initially $R_D$ should be close to $R_{\text{max}}$ inside of which the infall mass lands. However, this is only true if the initial core rotates slowly and $R_{\text{max}}$ is small. If the initial core rotates rapidly and $R_{\text{max}}$ is large, the dead zone size is constrained by the radius where the active layer becomes gravitationally unstable. Generally, the dead zone cannot extend to $R > 50$ AU.
if \( \Sigma_0 = 100 \text{ g cm}^{-2} \) due to the active layer GI. This is because the GI can be very efficient in transporting mass and angular momentum when \( Q < 1 \), and the surface density with \( Q = 1 \) can be considered as an upper limit that the disk surface density cannot exceed during the evolution. For a layered disk dominated by viscous heating, Gammie (1996) has shown that the Toomre \( Q \) parameter at \( R \) is

\[
Q = 1800 \left( \frac{R}{1 \text{ AU}} \right)^{-0.4} \left( \frac{M_c}{1 M_\odot} \right)^{3/4} \left( \frac{\alpha M}{10^{-2}} \right)^{1/2} \left( \frac{\Sigma_A}{\Sigma} \right).
\]

At the outer dead zone radius \( (R_D) \), \( \Sigma = \Sigma_A \); with the condition that \( Q > 1 \),

\[
R_D < 28 \left( \frac{M_c}{1 M_\odot} \right)^{1/3} \left( \frac{\alpha M}{10^{-2}} \right)^{2/9} \text{AU}.
\]

For an irradiation-dominated disk, we derive a similar condition

\[
Q = \frac{c_s \Omega}{\pi G (\Sigma + \Sigma_d)}.
\]

assuming

\[
T(R) = 200 \text{ K} \left( \frac{R}{\text{AU}} \right)^{-1/2}.
\]

Then,

\[
Q(R_D) = \left( \frac{45 \text{ AU}}{R_D} \right)^{7/4} \left( \frac{100 \text{ g cm}^{-2}}{\Sigma_A} \right).
\]

Since \( Q(R_D) > 1 \),

\[
R_D < 45 \left( \frac{100 \text{ g cm}^{-2}}{\Sigma_A} \right)^{4/7} \text{AU}.
\]

In either case, \( R_D \lesssim 50 \text{ AU} \). To test this, we computed a case with a \( \Omega_c = 2 \times 10^{-13} \text{ rad s}^{-1} \text{ core} \). The core mass falls into \( R_{\text{max}} \) as large as 400 AU, but the dead zone beyond 30 AU is quickly cleared by the GI so that \( R_D \) is smaller than 30 AU during the T Tauri phase.

After we know the initial \( R_D \) position, the motion of \( R_D \) can be derived by considering the mass conservation at \( R_D \). For the region extending from \( R_D - \Delta R \) to \( R_D \),

\[
\frac{R \Sigma}{\partial t} + \frac{\Delta(R \Sigma_0 v_R)}{\Delta R} = 0.
\]

Assuming the mass flux (or \( v_R \)) at \( R_D \) is zero, which will be justified in some cases later, and considering the time for the dead zone within \( \Delta R \) to be depleted is \( \Delta t \), we get

\[
R \frac{\Sigma_0}{\partial t} - \frac{\Delta(R \Sigma_0 v_R)}{\Delta R} = 0,
\]

thus

\[
v_D = \frac{\Delta R}{\Delta t} = \frac{\Sigma_d}{\Sigma_0} = \frac{\dot{M}(R_D)}{2 \pi R (\Sigma_0 + \Sigma_d)},
\]

where \( \dot{M} \) is as in Equation (A.4). If we insert \( \dot{M} \) we can derive \( v_D \propto 1/(\Sigma_0 + \Sigma_d) \). Considering \( \Sigma_d + \Sigma_d \) is larger at smaller radii, the speed of \( R_D \) decreases with time when it is moving inward. Finally, when \( R_D \) moves to the very inner radii where \( \Sigma_d \gg \Sigma_d \), \( R_D \) is almost halted.

### A.2. Outer Pure MRI Active Disk Beyond \( R_D \)

Beyond \( R_D \) the disk is purely MRI active with a constant \( \alpha_M \). Its surface density evolution can be solved by

\[
\frac{\partial \Sigma}{\partial t} = \frac{1}{R} \frac{\partial}{\partial R} \left[ 3 R^{1/2} \frac{\partial}{\partial R} \left( v \Sigma R^{1/2} \right) \right] (A17)
\]

with the boundary condition \( \Sigma(R_D) = \Sigma_A \). If \( v = k R^n \), this solution can be simplified by dividing it into two parts

\[
\Sigma(R, t) = \Sigma(R, t) + \Sigma_A \left( \frac{R}{R_D(t)} \right)^{-n-1/2},
\]

where the first term on the right comes from the disk evolution with a zero surface density boundary condition at \( R_D (\Sigma_{R_D} = 0) \), and the second term is the effect of the non-zero boundary surface density at \( R_D \). The first term behaves like the similarity solution with a \( R^{-1} \) part and a power-law decrease part at larger radius, and it eventually decreases to zero with time. The surface density evolution is determined by the competition between the first and the second term. During the infall phase, the disk is massive, and the first term is larger than the second term, thus the disk behaves like a similarity solution. Then after infall stops, as the disk accretes and \( R_D \) moves inward, both of these two terms decrease, and as shown above, \( R_D \) moves inward more slowly. Eventually \( R_D \) can be considered independent of \( t \); the first term goes to zero and the second term dominates so that we have \( \Sigma \propto R^{-1.5} \) if \( n = 1 \), which distinguishes it from the self-similarity solution.

Next, we consider the late stage when \( R_D \) is independent of time analytically by the Green’s function method. If the viscosity \( v \) is a function of radius, Equation (A17) is a linear equation for \( \Sigma \) and can be solved by a Green’s function (Lynden-Bell & Pringle 1974). If \( v = k R^n \), the above equation becomes

\[
\frac{\partial \Sigma}{\partial t} = \frac{1}{R} \frac{\partial}{\partial R} \left[ 3 R^{1/2} \frac{\partial}{\partial R} \left( k \Sigma R^{n+1/2} \right) \right],
\]

and the radial dependence of \( \Sigma \) is a linear combination of the Bessel functions \( J_m \) and \( J_{m-1} \), where \( \mu^2 = 1/(4(2-n)^2) \) (Lynden-Bell & Pringle 1974). However for a fixed \( \Sigma \) inner boundary condition at \( R_D \),

\[
\Sigma(R = R_D, t) = \Sigma_A,
\]

we can substitute \( \Sigma \) with \( \Sigma' \)

\[
\Sigma(R, t) = \Sigma(R, t) + \Sigma_A \left( \frac{R}{R_D} \right)^{-n-1/2},
\]

so that Equation (A19) and the boundary condition (A20) change to

\[
\frac{\partial \Sigma'}{\partial t} = \frac{1}{R} \frac{\partial}{\partial R} \left[ 3 R^{1/2} \frac{\partial}{\partial R} (k \Sigma' R^{n+1/2}) \right]
\]

and

\[
\Sigma'(R = R_D, t) = 0,
\]

which becomes a normal disk evolution equation of \( \Sigma' \) with a zero surface density boundary condition. The solution of \( \Sigma' \) is well studied and this disk expands with limit \( t \to \infty \), \( \Sigma' \to 0 \), thus from Equation (A21), we derive \( t \to \infty \), \( \Sigma \to \Sigma_A(R/R_D)^{-n-1/2} \). Thus as long as the disk evolves long enough the impact of any initial condition will be washed out,
and the boundary term dominates the surface density distribution with \( \Sigma \propto R^{-n-1/2} \).

In the irradiation-dominated case, \( n = 1 \), we transform Equation (A22) by writing \( x = R^{1/2} \) and \( \sigma = \Sigma'R^{3/2} \) (Pringle 1991) to get

\[
\frac{\partial \sigma}{\partial t} = \frac{3k}{4} \frac{\partial^2 \sigma}{\partial x^2}
\]

(A24)

with the boundary condition \( \sigma = 0 \) at \( x = x_D = R_D^{1/2} \). The general solution is then

\[
\sigma(x, t) = \int_{-\infty}^{\infty} A_\lambda e^{-\lambda^2} \sin[\lambda(x - x_D)/c] d\lambda,
\]

(A25)

where \( c^2 = 3k/4 \). \( A_\lambda \) is determined by the initial conditions. Following Pringle (1991), in order to obtain the Green’s function, we set the initial condition with

\[
\sigma(x, t = 0) = \sigma_0 \delta(x - x_1).
\]

(A26)

With the delta function Fourier transform \( \int \exp(2\pi x \lambda i) d\lambda = \delta(x) \) and \( x > x_D \), we derive

\[
A_\lambda = -\frac{\sigma_0}{\pi c} \sin[\lambda(x_D - x_1)/c].
\]

(A27)

Thus, the solution for the initial condition (A26) is

\[
\sigma(x, x_1, t) = \frac{\sigma_0 (-c)}{2\pi c} [\exp(-(x - x_1)^2/(3kt)]
\]

\[
- \exp[-(x + x_1 - 2x_D)^2/(3kt)] dx_1,
\]

(A28)

Finally, the solution \( \sigma(x, t) \) for any initial condition \( \sigma(x, t = 0) = \sigma'(x) \) is

\[
\sigma(x, t) = \int \sigma'(x_1) t^{-1/2} \left[ \exp(-(x - x_1)^2/(3kt) \right]
\]

\[
- \exp[-(x + x_1 - 2x_D)^2/(3kt)] dx_1,
\]

(A29)

and

\[
\Sigma(R, t) = \sigma(R, t) R^{-3/2} + \Sigma_A \left( \frac{R}{R_D} \right)^{-3/2},
\]

(A30)

where the first term is the disk evolution with a zero boundary condition and the second term is the fixed \( \Sigma \) boundary effect. As \( t \to \infty \), \( \Sigma \to \Sigma_A(R/R_D)^{-3/2} \).

Figure 15 shows the evolution of the constant \( \alpha = 0.01 \) disks with two different boundary conditions: \( \Sigma(10 \text{ AU}) = 50 \text{ g cm}^{-2} \) (black curves) and \( \Sigma(10 \text{ AU}) = 0 \text{ g cm}^{-2} \) (red curves). The initial conditions are set as

\[
\Sigma(R \leq 50 \text{ AU}, t = 0) = 50 \text{ g cm}^{-2},
\]

(A31)

\[
\Sigma(R > 50 \text{ AU}, t = 0) = 0 \text{ g cm}^{-2}.
\]

(A32)

As shown in Figure 15, the disk with zero boundary condition behaves quite similar to the similarity solution. However, for the \( \Sigma(10 \text{ AU}) = 50 \text{ g cm}^{-2} \) disk, the influence of the boundary becomes significant with a part \( \Sigma \propto R^{-1.5} \) and an outer part decreasing exponentially.

The mass accretion rate at \( R_D \) decreases to zero eventually (\( \Sigma \propto R^{-n-1/2} \)). This can be shown by assuming \( \Sigma = k R^n \) and \( \nu = k R^n \). Inserting these into

\[
\frac{d\Sigma}{dR} = 6\pi R^{1/2} \frac{\partial}{\partial R}(\nu \Sigma R^{1/2}),
\]

(A33)

we find

\[
\dot{M} = 6\pi k g \left( m + n + \frac{1}{2} \right) R^{n+1}.
\]

(A34)

In the asymptotic case as discussed above with \( \Sigma \propto R^{-n-1/2} \), \( \dot{M} = 0 \), which means at \( t \to \infty \), \( \dot{M} \to 0 \) at the inner boundary \( R_D \), so no mass flow in the disk. This can be simply understood because the disk has zero torque with \( \Sigma \propto R^{-n-1/2} \) if \( \nu = k R^n \).

However, note that this \( \Sigma \propto R^{-3/2} \) behavior is only observed for layered disks with small or negligible dead zone viscosity, so that at very late stage \( R_D \) is very small and almost fixed.

### A.3. Residual Dead Zone Viscosity

If the residual dead zone viscosity \( \alpha_d \gtrsim 10^{-4} \), the dead zone can transport the mass at a rate comparable to the accretion rate of the active layer, considering \( M_\text{f} \propto \Sigma_\text{d} \alpha \) and \( \Sigma_\text{d} \gg \Sigma_\text{f} \). If we assume the disk has a constant mass accretion rate after a long period of time, then \( \dot{M} = -2\pi R \Sigma_a v_{Ra} - 2\pi R \Sigma_d v_{Rd} \) is a constant and we can integrate the angular momentum equation to derive

\[
(S_a v_{Ra} + S_d v_{Rd}) R^3 \Omega = (v_a \Sigma_a + v_d \Sigma_d) R^2 d\Omega/dR + C.
\]

(A35)

If we assume at the stellar surface \( R_s \), \( d\Omega/dR = 0 \), we find \( C = -M \Omega R_s^2/2\pi \).

\[
v_a \Sigma_a + v_d \Sigma_d = \frac{M}{3\pi} \left[ 1 - \left( \frac{R_s}{R} \right)^{1/2} \right].
\]

(A36)

Furthermore, if the disk is irradiation-dominated so that the temperature is \( \propto R^{-1/2} \), we have \( v_a = k_a R \) and \( v_d = k_d R \) where \( k_a \) and \( k_d \) are constants and proportional to \( \alpha_a \) and \( \alpha_d \). So

\[
\Sigma_d = \frac{M}{3\pi k_d R} \left[ 1 - \left( \frac{R_s}{R} \right)^{1/2} \right] - \frac{k_a \Sigma_a}{k_d}.
\]

(A37)
If the dead zone transports mass at a rate higher than the active layer, the last term can be neglected for order of magnitude argument. And compared with a constant-\(\alpha\) disk having the same mass accretion rate, the layered disk has a surface density higher than the constant-\(\alpha\) disk by a factor of \(\alpha/\alpha_d\). If we assume the outer pure MRI active disk and the inner layered disk have the same mass accretion rate, and the outer disk only has \(\alpha_d\) while the inner dead zone is dominated by \(\alpha_d\), the disk’s surface density will increase by a factor of \(\alpha_d/\alpha_d\) within the dead zone outer radius \(R_d\), which is shown in Figure 9. Also this density change at \(R_d\) is more gradual (Equation (22) is a smooth function of \(R\)) than the zero dead zone viscosity case.

**APPENDIX B**

**GI DISK FRAGMENTATION RADIUS**

Gammie (2001) has pointed out that when the disk cooling timescale \(t_{cool} \lessapprox 3\Omega^{-1}\) the disk will fragment. By assuming local dissipation, Gammie (2001) has shown that

\[
t_{cool} = \frac{4}{9\gamma(\gamma - 1)\alpha\Omega}.
\]  

(B1)

Thus, the disk will fragment if \(\alpha > 0.06\) (Rice et al. 2005). However, the above fragmentation condition has only been tested for disks without any irradiation. With irradiation-dominated disk, \(t_{cool}\) is hard to be defined and \(\alpha > 0.06\) conditions are used (Rafikov 2009) instead.

For a constant accretion rate we have

\[
\alpha \frac{c^2\Sigma}{\Omega} = \frac{M}{3\pi}.
\]  

(B2)

Combined with \(Q = 1.5\), we derive

\[
\alpha = \frac{MG}{2c^3}.
\]  

(B3)

Thus, the disk will fragment if \(MG/2c^3 > 0.06\).

For a viscous heating dominated \(Q = 1\) disk, at a given mass accretion rate (\(M\)), the relationship between \(T_c\) and \(R\) is given in Equation (23) in Paper I. If we reorganize the equation and assume \(\beta = 0 (\kappa = C\sigma P\beta)\) with \(Q = 3/2\), we got

\[
T_c = 3^{2/(7-2\alpha)}2^{-12/(7-2\alpha)} R^{-9/(7-2\alpha)} \left(\frac{M}{\sigma}\right)^{2/(7-2\alpha)}
\]  

\[
\left(\frac{\Omega}{\mu}\right)^{1/(7-2\alpha)} G^{1/(7-2\alpha)} M^{1/(7-2\alpha)} \pi^{-4/(7-2\alpha)}.
\]  

(B4)

Inserting this into \(MG/2c^3 > 0.06\),

\[
R > 0.5 \times 0.12^{(14-4\alpha)/27} \pi^{-4/9} G^{(11+4\alpha)/27} C^2/9 \sigma^{-2/9} \times \left(\frac{k}{\mu m_h}\right)^{(8-2\alpha)/9} M^{1/3} \pi^{-8+4\alpha/27}.
\]  

(B5)

With our dust opacity \(\alpha = 0.738\) (Zhu et al. 2009b), the critical fragmentation radius \(R \propto M^{-0.19}\). Thus, the fragmentation radius is insensitive to \(M\) in the viscous heating dominated case and \(\sim 100\) AU (Figure 5).

For an irradiation-dominated case (low-mass accretion rate), \(c_j\) in Equation (B3) is determined by the irradiation (Equation (4)), thus

\[
R > 0.06 \left(\frac{MG}{c_j}\right)^{4/3} \left(\frac{k}{\mu m_h}\right)^2 \left(\frac{f L}{4\pi \sigma}\right)^{1/2}.
\]  

(B6)

In this case, the critical fragmentation \(R\) has a sharper dependence on \(M^{-4/3}\) (Figure 5). The GI fragmentation region is outlined in Figure 5, which agrees with Figure 10 of Cossins et al. (2010).

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