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Thermal Effects of Rotation in Random Classical Zero-Point Radiation.

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Abstract

The rotating reference system \( \{ \mu_\tau \} \), along with the two-point correlation functions (CFs) and energy density, is defined and used as the basis for investigating thermal effects observed by a detector rotating through random classical zero-point radiation. The reference system consists of Frenet-Serret orthogonal tetrads \( \mu_\tau \). At each proper time \( \tau \) the rotating detector is at rest and has a constant acceleration vector at the \( \mu_\tau \).

The two-point CFs and the energy density at the rotating reference system should be periodic with the period \( T = \frac{2\pi}{\Omega} \), where \( \Omega \) is an angular detector velocity, because CF and energy density measurements is one of the tools the detector can use to justify the periodicity of its motion. The CFs have been calculated for both electromagnetic and massless scalar fields in two cases, with and without taking this periodicity into consideration. It turned out that only periodic CFs have some thermal features and particularly the Planck’s factor with the temperature \( T_{rot} = \frac{\hbar \Omega}{\pi k_B} \) (\( k_B \) is the Boltzman constant).

Regarding to the energy density of both electromagnetic and massless scalar field it is shown that the detector rotating in the zero-point radiation observes not only this original zero-point radiation but, above that, also the radiation which would have been observed by an inertial detector in the thermal bath with the Plank’s spectrum at the temperature \( T_{rot} \). This effect is masked by factor \( \frac{\gamma^2}{3} (4\gamma^2 - 1) \) for the electromagnetic field and \( \frac{\gamma^2}{3} (4\gamma^2 - 1) \) for the massless scalar field, where \( \gamma = (1 - (\Omega r_c)^2)^{-1/2} \). Appearance of these masking factors is connected with the fact that rotation is defined by two parameters, angular velocity and the radius of rotation, in contrast with a uniformly accelerated linear motion which is defined by only one parameter, acceleration \( a \).

Our calculations involve classical point of view only and to the best of our knowledge the results have not been reported in quantum theory yet.
1 Introduction

This article is focused on investigations of the thermal effects connected with rotation of a classical detector moving through a random classical zero-point field. The case of a linear uniformly accelerated detector has been investigated and used in many works [1, 2, 14, 8]. The rotation, to the best of our knowledge, has not been considered in random classical radiation. It was studied in quantum case and only for the massless scalar field, particularly in connection with rotating vacuum puzzle [9, 18, 33] and different coordinate mappings.

Our approach in CF calculation is very close but not identical to the method developed in [1, 2] for uniformly accelerated detectors and also comes from the ideas developed in [15] and [32].

The observations made by the rotating detector are described in the non-inertial reference system consisting of instantaneous inertial reference frames and mathematically defined as tetrads at each moment of the detector proper time. Along with such reference system the two-point correlation functions of the electromagnetic and scalar massless field and one-point energy density of these fields are defined and analyzed for zero-point radiation.

The article is organized as follows. In section 2 we explicitly show the relationship between the components of the electromagnetic field measured at a tetrad and in the laboratory coordinate system. In section 3 the Frennet-Seret orthogonal tetrads connected with a rotating point-like detector are defined and its merits for our problems compared with the Fermi-Walker tetrads are discussed. The CFs of the electromagnetic field at the rotating detector moving through the classical zero-point electromagnetic filed are considered in section 4 and their expressions are presented in terms of elementary functions. In section 5 these CFs and the energy density measured by the rotating detector are analyzed taking into consideration periodicity of these functions and then using the Abel-Plana summation function. Due to the periodicity of the motion, the observer rotating through a zero-point electromagnetic radiation sees the energy density, which would have been observed by an inertial observer moving in a thermal bath at the temperature \( T_{\text{rot}} = \frac{\hbar \Omega}{2 \pi k} \), and multiplied by the factor \( \frac{2}{3}(4\gamma^2 - 1) \). For the classical massless scalar zero-point field the CF, energy density, and their thermal properties connected with the periodicity are investigated in the section 6. Calculation details are presented in Appendix.
Tetrads and Measurements by a Rotating Detector

We will start from the case of random classical electromagnetic field. A local inertial system of a rotating point-like detector (observer) is associated with an orthogonal tetrad $\mu$ of four vectors $\mu^i(a), \ a = 1, 2, 3, 4$, defined at its location. The tensor of electromagnetic field, $F_{ik}$, defined at this point, may be resolved along these frame vectors according to the following formula

$$F_{ik} = \mu^i(a) \mu^j(k) F_{(ab)}(\mu). \quad (1)$$

The components

$$F_{(ab)}(\mu) = \mu^i(a) \mu^j(b) F_{ik}. \quad (2)$$

are scalars under coordinate transformations. But they clearly transform upon change of orthogonal tetrads $\mu$ according to the local (that is defined at the same point) Lorentz transformations. The electric and magnetic fields, $E(a)$ and $H(b)$, measured by a rotating detector are defined in terms of the components $F_{(ab)}$:

$$(E(a)(\mu)) = \left(F_{(41)}(\mu), F_{(42)}(\mu), F_{(43)}(\mu)\right), \quad \left(H(a)(\mu)\right) = \left(F_{(23)}(\mu), F_{(31)}(\mu), F_{(12)}(\mu)\right). \quad (3)$$

They do not depend on the coordinate system and are connected with a tetrad.

If the tetrad vectors $\mu^i(a), \ a = 1, 2, 3, 4$, of a reference frame $\mu$ and the electromagnetic field state $F_{ik}$ are specified in the laboratory coordinate system then according to (2) the electric and magnetic fields at a rotating detector can be expressed in terms of the electric and magnetic fields in the laboratory coordinate system. It will be done in the next sections to calculate CFs.

The Definition of the Local Reference Frame (Tetrad) for a Rotating Detector

There are different ways to define orthogonal tetrads [32], [17]. Tetrads with the time component proportional to 4-vector velocity of an observer define reference frames in which the observer is at rest. There are two types of such tetrads. The so called not rotating tetrads are described by the Fermi-Walker equations and are used to explain the Thomas procession [25]. Frenet-Serret orthogonal
tetrad gives an example of so-called rotating tetrads and may also be used for observations [27].

In this publication, the reference frames $\mu_\tau$ are defined as Frenet-Serret orthogonal tetrads on the world line of the rotating detector. For the rotating detector, with 4-vector velocity

$$U^i = c \left(-\beta\gamma \sin \alpha, \beta\gamma \cos \alpha, 0, \gamma\right), \quad \alpha = \Omega \gamma \tau, \quad \gamma = \left(1 - \frac{(\Omega\alpha)^2}{c^2}\right)^{1/2},$$  

(4)

the four vectors of a Frenet-Serret orthogonal tetrad have been found to be (Appendix A)

$$\mu^i_1 = (\cos \alpha, \sin \alpha, 0, 0),$$

$$\mu^i_2 = (-\gamma \sin \alpha, \gamma \cos \alpha, 0, \gamma),$$

$$\mu^i_3 = (0, 0, 1, 0),$$

$$\mu^i_4 = \frac{U^i}{c}. \quad (5)$$

In the local reference frames, defined by these tetrads, the detector is at rest at any proper time $\tau$ because

$$U_{(a)} = \mu^i_{(a)} U_i = \mu^i_{(a)} U^k g_{ik} = (0, 0, 0, -c).$$

(6)

and 4-vector acceleration of the detector in it

$$\dot{U}_{(a)} = \mu^i_{(a)} \dot{U}_i = \mu^i_{(a)} \dot{U}^k g_{ik} = (-a\Omega^2 \gamma^2, 0, 0, 0), \quad g_{ik} = \text{diag}(1, 1, 1, -1)$$

(7)

is constant in both magnitude and direction

$$\dot{U}_{(a)} = (-a\Omega^2 \gamma^2, 0, 0, 0) = (-a\Omega^2 \gamma^2, 0, 0, 0).$$

(8)

Unlike Frenet-Serret tetrads, Fermi-Walker tetrads do not have this feature. In the reference frame associated with a Fermi-Walker tetrad, the 4-vector acceleration is not constant in neither direction nor magnitude

$$\dot{U}_{(a)} = e_{(a)} \dot{U}_l = (-a\Omega^2 \gamma^2 \cos \alpha \gamma, -a\Omega^2 \gamma^2 \sin \alpha \gamma, 0, 0).$$

(9)

( Fermi-Walker’s tetrad vectors $e_{(a)}$ are given in Appendix A). The acceleration depends on $\tau$ and at each $e_\tau$ is different.

This is why we use Frenet-Serret tetrads for the case of the rotation, not Fermi-Walker ones. In this formalism rotation is becoming similar to a linear uniformly accelerated motion, with a constant acceleration vector in an instantaneous rest reference frame.
4 Correlation Functions for Electromagnetic Field at a Rotating Detector

4.1 Electric and Magnetic Fields in a Frenet-Serret tetrad

Following formulas (2, 3, 5), the electric $E_{(k)}(\mu|\tau)$ and magnetic $H_{(k)}(\mu|\tau)$ fields in the Frenet-Serret reference frame $\mu_\tau$ at the proper time $\tau$ of the rotating detector can be given in terms of electric and magnetic fields in the laboratory coordinate system:

\[
E_{(1)}(\mu|\tau) = F_{(11)}(\mu|\tau) = E_1 \gamma \cos \alpha + E_2 \gamma \sin \alpha - H_3 \beta \gamma,
\]

\[
E_{(2)}(\mu|\tau) = F_{(42)}(\mu|\tau) = E_1 (-\sin \alpha) + E_2 \cos \alpha,
\]

\[
E_{(3)}(\mu|\tau) = F_{(43)}(\mu|\tau) = E_3 \gamma + H_1 \beta \gamma \cos \alpha + H_2 \beta \gamma \sin \alpha,
\]

\[
H_{(1)}(\mu|\tau) = F_{(23)}(\mu|\tau) = H_1 \gamma \cos \alpha + H_2 \gamma \sin \alpha + E_3 \beta \gamma,
\]

\[
H_{(2)}(\mu|\tau) = F_{(31)}(\mu|\tau) = H_1 (-\sin \alpha) + H_2 \cos \alpha,
\]

\[
H_{(3)}(\mu|\tau) = F_{(12)}(\mu|\tau) = H_3 \gamma + E_1 (\beta \gamma \cos \alpha) + E_2 (-\beta \gamma \sin \alpha),
\]

where $\gamma = 1/(1 - \beta^2)^{1/2}$, $\beta = v/c$, $v = \Omega a$, $\alpha = \Omega \gamma \tau = \Omega t$, $\Omega$ is an angular velocity of the rotating detector, and $a$ is the radius of the rotation circle.

In these equations $E_k$ and $H_k$ are the electric and magnet field components of the random zero-point radiation in the laboratory coordinate system at the location of the rotating detector at its proper time $\tau$ [1]:

\[
\vec{E}(\tau) = \sum_{\lambda=1}^{2} \int d^3k \epsilon(\vec{k}, \lambda) h_0(\omega) \cos[\vec{k}\vec{r}(\tau) - \omega \gamma \tau - \Theta(\vec{k}, \lambda)],
\]

\[
\vec{H}(\tau) = \sum_{\lambda=1}^{2} \int d^3k \epsilon(\vec{k}, \lambda) h_0(\omega) \cos[\vec{k}\vec{r}(\tau) - \omega \gamma \tau - \Theta(\vec{k}, \lambda)],
\]

where the $\theta(\vec{k}, \lambda)$ are random phases distributed uniformly on the interval $(0, 2\pi)$ and independently for each wave vector $\vec{k}$ and polarization $\lambda$ of of a plane wave, and

\[
\pi^2 h_0^2(\omega) = (1/2) \hbar \omega, \quad \vec{r}(\tau) = (a \cos \Omega \gamma \tau, a \sin \Omega \gamma \tau, 0).
\]

Using formulas (10) we can calculate all two-field correlation functions $\langle E_{(a)}(\mu_1|\tau_1)E_{(b)}(\mu_2|\tau_2)\rangle$, $\langle E_{(a)}(\mu_1|\tau_1)H_{(b)}(\mu_2|\tau_2)\rangle$, and $\langle H_{(a)}(\mu_1|\tau_1)H_{(b)}(\mu_2|\tau_2)\rangle$, $a, b = 1, 2, 3$ of the electromagnetic field at the rotating detector. Here $\langle \rangle$ means averaging over random phases $\theta(\vec{k}, \lambda)$ [1]. We will start with $\langle E_{(1)}(\mu_1|\tau_1)E_{(1)}(\mu_2|\tau_2)\rangle$.
4.2 The Correlation Function \( \langle E(1)(\mu_{1}\mid\tau_{1})E(1)(\mu_{2}\mid\tau_{2}) \rangle \): General Expression

As an example let us first treat with

\[
I_{(11)} = \langle E(1)(\mu_{1}\mid\tau_{1})E(1)(\mu_{2}\mid\tau_{2}) \rangle. \tag{13}
\]

In this expression \( \mu_{1} \) and \( \mu_{2} \) are two reference frames (tetrads) on the circle of the rotating detector at the proper times \( \tau_{1} \) and \( \tau_{2} \) respectively. Then from (10) follows

\[
I_{(11)} = \langle E_{1}(\tau_{1})E_{1}(\tau_{2}) \rangle \gamma^{2} \cos \alpha_{1} \cos \alpha_{2} + \langle E_{1}(\tau_{1})E_{2}(\tau_{2}) \rangle \gamma^{2} \cos \alpha_{1} 
+ \langle E_{2}(\tau_{1})E_{1}(\tau_{2}) \rangle \gamma^{2} \sin \alpha_{1} \cos \alpha_{2} + \langle E_{1}(\tau_{1})H_{3}(\tau_{2}) \rangle (-1) \gamma^{2} \cos \alpha_{1} 
+ \langle H_{3}(\tau_{1})E_{1}(\tau_{2}) \rangle (-1) \beta \gamma^{2} \cos \alpha_{2} + \langle E_{2}(\tau_{1})E_{2}(\tau_{2}) \rangle \gamma^{2} \sin \alpha_{1} \sin \alpha_{2} + \langle E_{2}(\tau_{1})H_{3}(\tau_{2}) \rangle (-1) \beta \gamma^{2} \sin \alpha_{1} + \langle H_{3}(\tau_{1})E_{2}(\tau_{2}) \rangle (-1) \beta \gamma^{2} \sin \alpha_{2} + \langle H_{3}(\tau_{1})H_{3}(\tau_{2}) \rangle. \tag{14}
\]

The \( \langle \rangle \) expressions can be calculated using (11), the relationships

\[
\langle \cos \theta(\vec{k}, \lambda) \cos \theta(\vec{k}', \lambda') \rangle = \langle \sin \theta(\vec{k}, \lambda) \sin \theta(\vec{k}', \lambda') \rangle = \frac{1}{2} \delta_{\lambda} \phi k \phi (\vec{k} - \vec{k}'),
\]

\[
\langle \cos \theta(\vec{k}, \lambda) \cos \theta(\vec{k}', \lambda) \rangle = 0,
\]

\[
\sum_{\lambda=1}^{2} \epsilon_{i}(\vec{k}, \lambda) \epsilon_{i}(\vec{k}', \lambda) = \delta_{ij} - k_{i} k_{j}/k^2 = \delta_{ij} - \vec{k}_{i}\vec{k}_{j}, \tag{15}
\]

from (11) and variable change in the integrands

\[
\dot{k}_{x} \cos \alpha + \dot{k}_{y} \sin \alpha = \dot{k}'_{x}, \quad -\dot{k}_{x} \sin \alpha + \dot{k}_{y} \cos \alpha = \dot{k}'_{y}, \quad \alpha = \frac{\alpha_{1} + \alpha_{2}}{2} = \frac{\Omega \gamma (\tau_{2} + \tau_{1})}{2}, \tag{16}
\]

\[
\dot{k}_{i} = k_{i}/k, \quad i = x, y, z. \tag{17}
\]

In the final integrand the prime symbol of the dummy variable \( k' \) is omitted for simplicity

\[
\langle E_{1}(\tau_{1})E_{1}(\tau_{2}) \rangle = \int d^{3}k R + (-\cos^{2} \alpha) \int d^{3}k \dot{k}_{x}^{2} R + (-\sin^{2} \alpha) \int d^{3}k \dot{k}_{y}^{2} R,
\]

\[
\langle E_{1}(\tau_{1})E_{2}(\tau_{2}) \rangle = \langle E_{2}(\tau_{1})E_{1}(\tau_{2}) \rangle = \frac{-\sin 2\alpha}{2} \int d^{3}k \dot{k}_{x}^{2} R + \frac{\sin 2\alpha}{2} \int d^{3}k \dot{k}_{y}^{2} R,
\]

\[
\langle E_{1}(\tau_{1})H_{3}(\tau_{2}) \rangle = \langle E_{2}(\tau_{1})H_{3}(\tau_{2}) \rangle = -\cos \alpha \int d^{3}k \dot{k}_{y} R,
\]

\[
\langle E_{2}(\tau_{1})E_{2}(\tau_{2}) \rangle = \int d^{3}k R + (-\sin^{2} \alpha) \int d^{3}k \dot{k}_{x}^{2} R + (-\cos^{2} \alpha) \int d^{3}k \dot{k}_{y}^{2} R,
\]

\[
\langle E_{2}(\tau_{1})H_{3}(\tau_{2}) \rangle = \langle E_{3}(\tau_{1})H_{3}(\tau_{2}) \rangle = (-\sin \alpha) \int d^{3}k \dot{k}_{y} R,
\]

\[
\langle H_{3}(\tau_{1})H_{3}(\tau_{2}) \rangle = \int d^{3}k \dot{k}_{x}^{2} R + \int d^{3}k \dot{k}_{y}^{2} R. \tag{18}
\]
and
\[ R = h_0^2(\omega) \frac{1}{2} \cos k F, \quad F = c\gamma(\tau_2 - \tau_1)[1 - \hat{k}_y \frac{v \sin \delta/2}{c}], \quad \delta = \alpha_2 - \alpha_1 = \Omega\gamma(\tau_2 - \tau_1). \] (19)

After some calculations we come to the following expression for \( I \)
\[ I_{(11)} = \langle E_{(1)}(\mu_1|\tau_1)E_{(1)}(\mu_2|\tau_2) \rangle = \gamma^2 \cos \delta \int d^3 k \ h_0^2(\omega) \frac{1}{2} \cos k F + 2\beta \gamma^2 \cos \frac{\delta}{2} \int d^3 k \ \hat{k}_y \ h_0^2(\omega) \frac{1}{2} \cos k F + \gamma^2 \beta^2 \cos^2 \frac{\delta}{2} \int d^3 k \ \hat{k}_x^2 \ h_0^2(\omega) \cos k F + \gamma^2 [\beta^2 + \sin^2 \frac{\delta}{2}] \int d^3 k \ \hat{k}_y^2 \ h_0^2(\omega) \frac{1}{2} \cos k F. \] (20)

This function clearly depends only on the proper time interval \( \tau_2 - \tau_1 \) and does not dependent of the central proper time \( (\tau_1 + \tau_2)/2 \) that is
\[ I_{(11)} = I_{(11)}(\tau_2 - \tau_1). \] (21)

General expressions for other CFs can be found in Appendix C. They have the same properties and also depend only on the proper time interval \( \tau_2 - \tau_1 \).

4.3 The Correlation Function \( \langle E_{(1)}(\mu_1|\tau_1)E_{(1)}(\mu_2|\tau_2) \rangle: \) Final Expression

The CF \( \langle E_{(1)}(\mu_1|\tau_1)E_{(1)}(\mu_2|\tau_2) \rangle \) can be represented in terms of elementary functions. Completing integration of (20) over \( k \) and then over \( \phi \) we come to the expression:
\[ \langle E_{(1)}(\mu_1|\tau_1)E_{(1)}(\mu_2|\tau_2) \rangle = \frac{3hc}{2\pi^2[\epsilon(t_2 - t_1)]}\gamma^2 \{[2\pi \cos \delta]\int_0^\pi d\theta \frac{\sin \theta}{(1 - k^2 \sin^2 \theta)^{7/2}} \]
\[ + [3\pi k^2 \cos \delta - 2\pi \cos(\delta/2) + 2\pi \beta^2 - 8\pi \beta k \cos(\delta/2) + \pi] \int_0^\pi d\theta \frac{\sin^3 \theta}{(1 - k^2 \sin^2 \theta)^{7/2}} \]
\[ + [-3\pi k^2 \cos^2(\delta/2) + 3\pi \beta^2 k^2 - 2\pi \beta k^3 \cos(\delta/2) + 4\pi k^2] \int_0^\pi d\theta \frac{\sin^5 \theta}{(1 - k^2 \sin^2 \theta)^{7/2}} \} \] (22)

(see Appendix C for details). The integrals over \( \theta \) in this expression are:
\[ \int_0^\pi d\theta \frac{\sin \theta}{(1 - k^2 \sin^2 \theta)^{7/2}} = \frac{2}{5(1 - k^2)} + \frac{8}{15(1 - k^2)^2} + \frac{16}{15(1 - k^2)^3}, \] (23)
\[ \int_0^\pi d\theta \frac{\sin^3 \theta}{(1 - k^2 \sin^2 \theta)^{7/2}} = \frac{4}{15(1 - k^2)^2} + \frac{16}{15(1 - k^2)^3}, \] (24)
\[ \int_0^\pi d\theta \frac{\sin^5 \theta}{(1 - k^2 \sin^2 \theta)^{7/2}} = \frac{16}{15(1 - k^2)^3} \] (25)

(see formulas [29], 1.5.23, 1.2.43) and the constant (not a wave vector \( \vec{k} \)) \( k = -\frac{\omega \sin \delta/2}{c}. \)
So the CF \( \langle E(1)(\mu_1|\tau_1)E(1)(\mu_2|\tau_2) \rangle \) can be represented in terms of elementary functions. Other CFs can also be expressed in terms of elementary functions.

In this form the CFs do not display thermal features. In the next section we will show that the CFs should be modified to take their periodicity into consideration. The periodic CF have thermal features.

5 The Spectrum of the Random Classical Electromagnetic Radiation Observed by a Rotating Detector.

5.1 Periodicity of the Correlation Functions.

The two-field CFs at a rotating detector should be periodic because CF measurements is one of the tools the detector can use to justify the periodicity of its motion. Mathematically it means that

\[
I_{(11)}(t_2 - t_1) = I_{(11)}(\tau_2 - \tau_1) + \frac{2\pi n}{\Omega}
\]

or

\[
I_{(11)}((t_2 - t_1) + 2\pi \gamma n) = I_{(11)}((\tau_2 - \tau_1) + 2\pi \gamma n)
\]

Here \( \Omega = \frac{2\pi}{T} \) is an angular velocity of the rotating detector and \( n = 0, 1, 2, 3, \ldots \). Using (20) it is easy to show after some trigonometrical transformations that the CF is periodic \([05.03.06]\) if in its integrand

\[
\omega = \Omega n.
\]

It means that the rotating detector observes not entire random electromagnetic radiation but only the discrete part of it. The discrete spectrum is the same as a rotating electrical charge radiates \([16](39.29)\).

The final expression (22) for \( I_{(11)} \) can not be used to analyze the periodicity consequences because the integration over entire continuous spectrum of \( \omega \) has already been done in it. It is why we have used the expression (20), before the integration over \( \omega \).

Let us now consider the periodic correlation function \( I_{(11)} \) for the discrete spectrum. There are two ways to do that. The first one is simpler, just to modify the formula (20) for \( I_{(11)} \) for the discrete spectrum. It will be described below in the next section. The second one is identical with the approach
we have used above for the continuous spectrum but with the modified equations (11) and relationships (15) for the discrete spectrum. It is described in Appendix D.

5.2 The Correlation Function \( \langle E_{(1)}(\mu_1|\tau_1)E_{(1)}(\mu_2|\tau_2) \rangle \) with the Discrete Spectrum: General Expression

The integrands in (20) can be changed according to an obvious relationship

\[
\int d^3k \left[ \frac{1}{2} \hbar_0^2(\omega) \cos kF \right] = \frac{\hbar k_0^4}{4\pi^2} \int dO \left[ S \right],
\]

where

\[
S = \int d\kappa \, \kappa^3 \cos \kappa F_d, \quad dO = d\theta d\phi \sin \theta, \quad \kappa = \frac{k}{k_0},
\]

and

\[
F_d = k_0 F = \delta \left[ 1 - \kappa \frac{v \sin \delta / 2}{\delta / 2} \right].
\]

The expressions in [ ] are 1, \( \hat{k}_y \), \( \hat{k}_z^2 \), or \( \hat{k}_y^2 \) and do not depend on \( \kappa = k/k_0 \).

For the discrete spectrum case the integration in (29) over \( \kappa \) should be changed to summation over \( n \). So the only term to be changed is \( S \). It becomes (in new notations)

\[
S_d = \sum_{n=0}^{\infty} n^3 \cos nF_d.
\]

Then the periodical CF, corresponding to (20), with the discrete spectrum can be given in the form

\[
\langle E_{(1)}(\mu_1|\tau_1)E_{(1)}(\mu_2|\tau_2) \rangle_d = \frac{\hbar k_0^4}{4\pi^2} \left\{ \gamma^2 \cos \delta \int dO \, S_d + 2\beta \gamma^2 \cos \frac{\delta}{2} \int dO \, \hat{k}_y \, S_d + \gamma^2 [\beta^2 - \cos^2 \frac{\delta}{2}] \int dO \, \hat{k}_z^2 \, S_d + \gamma^2 [\beta^2 + \sin^2 \frac{\delta}{2}] \int dO \, \hat{k}_y^2 \, S_d \right\}.
\]

The physical sense of the correlation functions with the discrete spectrum can be understood if we use the Abel-Plana formula to analyze \( S_d \). It will be done in next sections.

5.3 The Abel-Plana Formula.

The Abel-Plana summation formula [5, 23, 11] has the form:

\[
\sum_{n=0}^{\infty} f(n) = \int_{0}^{\infty} f(x) \, dx + \frac{f(0)}{2} + i \int_{0}^{\infty} dt \frac{f(it) - f(-it)}{e^{2\pi t} - 1}.
\]
Having utilized this formula for (32) with \( f(n) = n^3 \cos nF_d \) and following [23] we come to the following expression for \( S_d \)

\[
\Omega^4 S_d = \int_0^\infty d\omega \omega^3 \cos(\omega \tilde{F}) + \int_0^\infty d\omega \frac{2\omega^3 \cosh(\omega \tilde{F})}{e^{2\pi \omega/\Omega} - 1}, \quad \tilde{F} = \frac{F_d}{\Omega},
\]

or after integration to:

\[
S_d = \frac{6}{F_d^4} + \left[ \frac{3 - 2\sin^2(F_d/2)}{8\sin^4(F_d/2)} - \frac{6}{F_d^4} \right].
\]

So now \( S_d \) is broken into divergent and convergent parts and both expressions are very similar to [1] (74). (Another form of \( S_d \) is given in Appendix E.)

Let us compare the expression (35) for \( S_d \) with the expression [1] (74)

\[
\int_0^\infty d\omega \omega^3 \coth\left(\frac{\hbar \omega}{2kT}\right) \cos \omega t = \int_0^\infty d\omega \omega^3 \cos \omega t + \int_0^\infty d\omega \frac{2\omega^3}{e^{\hbar \omega/kT} - 1} \cos \omega t
\]

which is the Fourier component of the spectral function \( \frac{1}{2} \hbar \omega \coth\left(\frac{\hbar \omega}{2kT}\right) \) of the electromagnetic radiation with Planck’s spectrum at the temperature \( T \), with the zero-point radiation included.

The right sides of these expressions are very similar and have the Planck’s factor \( 1/(\exp^{\hbar \omega/kT} - 1) \) if we define in (35) a new constant, a rotation temperature, \( T_{\text{rot}} \) according to

\[
T_{\text{rot}} = \frac{\hbar \Omega}{2\pi k_B}.
\]

The Planck’s factor is a sign that some thermal effect accompany the detector rotation in the random classical zero-point electromagnetic radiation. There is also a significant distinction between them though. In the first expression \( \tilde{F} = t(1 - \hat{k}_y) \frac{\sin \delta/2}{\delta/2} \) and \( \cosh \) are used instead of \( t \) and \( \cos \) respectively in (37). The coefficient \( F_d \) depends on both \( \theta \) and \( \phi \) because \( \hat{k}_y = \sin \theta \sin \phi \). It means that the thermal radiation observed by the rotating detector differs from the Planck’s radiation and it is anisotropic.

For \( \tilde{F} = 0 \) and \( t=0 \), when two observation points, \( \tau_1 \) and \( \tau_2 \), coincide the right sides of both expressions are identical. This remarkable resemblance brings up the idea that the energy density, one-observation-point quantity, of the random classical electromagnetic radiation measured by a detector, rotating through a zero point radiation, has the Planck spectrum at the temperature \( T_{\text{rot}} \) (38). This issue will be discussed in the next section.
5.4 The Energy Density of Random Classical Electromagnetic Radiation Observed by a Rotating detector and Planck’s Spectrum

Using (10) the energy density measured by the rotating observer at a reference frame $\mu$

$$w(\mu) = \frac{1}{8\pi} \sum_{a=1}^{3} \left( \langle E_{a}^{2}(\mu|\tau) \rangle + \langle H_{a}^{2}(\mu|\tau) \rangle \right)$$  \hspace{1cm} (39)$$
can be given in terms of electric and magnetic fields measured in the laboratory coordinate system

$$w(\mu) = \frac{1}{4\pi} \left\{ \left[ \langle E_{1}^{2} \rangle + \langle E_{3}^{2} \rangle \right] \gamma^{2} (1 + \beta^{2}) + \langle E_{2}^{2} \rangle \right\},$$  \hspace{1cm} (40)$$
where as we will show below $\langle E_{i}^{2} \rangle = \langle H_{i}^{2} \rangle$, $i = 1, 2, 3$.

With the help of the formulas from Appendix D and using the technique, described above for a discrete spectrum we come to the following expressions

$$\langle E_{i}^{2} \rangle = \langle H_{i}^{2} \rangle = \frac{k_{0}^{3} h c}{2\pi^{2}} \int d\Omega (1 - \hat{k}_{i}^{2}) \sum_{n=0}^{\infty} n^{3}$$  \hspace{1cm} (41)$$
for $i = 1, 2, 3$ and finally after integration over $\theta$ and $\phi$ :

$$w(\mu) = \frac{(4\gamma^{2} - 1)}{3} \frac{\hbar}{c^{3}\pi^{2}} \int d\Omega (1 - \hat{k}_{i}^{2}) \sum_{n=0}^{\infty} n^{3}$$  \hspace{1cm} (42)$$
If we take into consideration the Abel-Plana formula (35) for $F_{d} = 0$ this expression can be given in a form more convenient for physical interpretation

$$w(\mu) = \frac{2 (4\gamma^{2} - 1)}{3} w_{em}(T_{\text{rot}}),$$  \hspace{1cm} (43)$$
where

$$w_{em}(T_{\text{rot}}) = \frac{\hbar}{c^{3}\pi^{2}} \left( \int_{0}^{\infty} d\omega \frac{1}{2} \omega^{3} + \int_{0}^{\infty} d\omega \frac{\omega^{3}}{e^{\hbar\omega/k T_{\text{rot}}} - 1}. \right) \hspace{1cm} (44)$$

$w_{em}(T)$ is the energy density of the electromagnetic field observed by an inertial detector in Planck’s spectrum at the temperature $T$. It consists of two parts. The first one, divergent, is a zero-point energy density and the second one, convergent, is a part of the energy density due to a temperature $T_{\text{rot}}$.

Thus the detector rotating in the zero-point radiation under the temperature $T = 0$ observes not only original zero-point radiation but also the radiation determined by the parameter $T_{\text{rot}}$. We can
interpret \( T_{\text{rot}} \) now as the temperature associated with the detector rotation.

Indeed, after integration over \( \omega \) the second convergent term is

\[
w_T(\mu) = \frac{2(4\gamma^2 - 1)}{3} w_{\text{black}},
\]

where

\[
w_{\text{black}} = 4 \pi^2 k_B^4 \frac{T_{\text{rot}}^4}{60(\hbar c)^2} \quad T_{\text{rot}}^4 = \frac{4\sigma c T_{\text{rot}}^4}{3}
\]

is the energy density of the black radiation at the temperature \( T_{\text{rot}} \) \([20], (60,14)\), \( k_B \) is the Boltzmann constant, and \( \sigma \) is the Stefan-Boltzmann constant.

So, due to periodicity of the motion, the energy density observed by a detector rotating through a zero-point radiation and the energy density observed an inertial detector in a thermal bath at the temperature \( T_{\text{rot}} = \hbar \Omega \frac{2\pi}{\hbar} \) are connected by the formula (45). The factor \( \frac{2}{3}(4\gamma^2 - 1) \) comes from integration in (41) over angles and is a consequence of anisotropy of the electromagnetic field measured by the rotating observer.

6 Classical Massless Zero-Point Scalar Field at Rotating Detector.

6.1 Correlation Function and Tetrads.

The field \( \psi_s(\mu_\tau|\tau) \) in a tetrad \( \mu_\tau \) has the same form as in the laboratory coordinate system, \( \psi_s(\tau) \), taken in the location of the tetrad, because it is a scalar. Then the correlation function measured by an observer rotating through a classical massless zero-point scalar field radiation has the form:

\[
\langle \psi_s(\mu_1|\tau_1)\psi_s(\mu_2|\tau_2) \rangle = \langle \psi_s(\tau_1)\psi_s(\tau_2) \rangle,
\]

where \[1\]

\[
\psi_s(\tau_i) = \int d^3k_i f(\omega_i) \cos \{\vec{k}_i \vec{r}(\tau_i) - \omega_i \gamma \tau_i - \theta(k_i)\}, \quad i = 1, 2
\]

and

\[
\langle \cos \theta(\vec{k}_1) \cos \theta(\vec{k}_2) \rangle = \langle \sin \theta(\vec{k}_1) \sin \theta(\vec{k}_2) \rangle = \frac{1}{2} \delta^3(\vec{k}_1 - \vec{k}_2), \quad f^2(\omega) = \frac{\hbar c^2}{2\pi^2 \omega}
\]

\[
\langle \cos \theta(\vec{k}_1) \sin \theta(\vec{k}_2) \rangle = 0, \quad \vec{r}(\tau_i) = (a \cos \Omega \gamma \tau_i, a \sin \Omega \tau_i, 0).
\]


Using these expressions and substitution (16) in the integrand we get the expression:

\[
\langle \psi_s(\mu_1|\tau_1)\psi_s(\mu_2|\tau_2) \rangle = \int d^3k f^2(\omega) \frac{1}{2} \cos kF.
\] (50)

After integration over positive \( k = \omega/c \), it becomes:

\[
\langle \psi_s(\mu_1|\tau_1)\psi_s(\mu_2|\tau_2) \rangle = -\frac{\hbar c}{4\pi^2} \int_0^\pi d\theta \sin \theta \int_0^{2\pi} d\phi \frac{1}{|E \sin \phi - B|^2},
\] (51)

where \( B = \gamma c \), \( E = 2a \sin \theta \), and \( \tau = \tau_2 - \tau_1 \).

Because \( B - |E| = c\gamma \frac{\sin \theta}{\sin \frac{\pi(\gamma \tau / T)}{\pi(\gamma / T)}} > c\gamma (1 - v/c) > 0 \), and using (29) we obtain:

\[
\int_0^{2\pi} d\phi \frac{1}{|E \sin \phi - B|^2} = \frac{2\pi B}{(B^2 - E^2)^{3/2}}.
\] (52)

Having integrated over \( \theta \) we come to the final expression for the CF of the random classical massless scalar field at the rotating detector moving through a zero point fluctuating massless scalar radiation:

\[
\langle \psi_s(\mu_1|\tau_1)\psi_s(\mu_2|\tau_2) \rangle = -\frac{\hbar c}{\pi} \frac{1}{(\gamma (\tau_2 - \tau_1)c)^2 - 4r^2 \sin^2 \frac{\Omega \gamma (\tau_2 - \tau_1)}{2}}.
\] (53)

This correlation function received in the classical approach is identical to the positive frequency Wightman function [9] (3) up to a constant.

In the scalar field CF is periodical for the same reasons it is periodical in the electromagnetical fields. We have not yet taken into consideration the periodicity of this CF connected with the rotation of the detector in the scalar field. This issue is investigated below.

### 6.2 Periodicity of the Correlation Function, Abel-Plana formula, and the Planck’s Factor.

The equation (50) can be given in the form

\[
\langle \psi_s(\mu_1|\tau_1)\psi_s(\mu_2|\tau_2) \rangle = \frac{\hbar c k_0^2}{4\pi^2} \int dO \int d\kappa \kappa \cos \kappa F_d.
\] (54)

If this function is periodic its spectrum should be \( \kappa = \frac{ck}{c_0} = n = 0, 1, 2, \ldots \) and then

\[
\langle \psi_s(\mu_1|\tau_1)\psi_s(\mu_2|\tau_2) \rangle_d = \frac{\hbar c k_0^2}{4\pi^2} \int dO \sum_{n=0}^\infty n \cos n F_d.
\] (55)

Abel-Plana summation formula in this case is

\[
\sum_{n=0}^\infty n \cos n F_d = \int_0^\infty dt t \cos t F_d - \int_0^\infty dt \frac{2t \cosh t F_d}{e^{2\pi t} - 1}.
\] (56)
or
\[
\Omega^2 \sum_{n=0}^{\infty} n \cos nF_d = \int_0^\infty d\omega \cos \omega \tilde{F} = \int_0^\infty d\omega \frac{2\omega \cosh \omega \tilde{F}}{e^{\hbar \omega/2kT_{rot}} - 1}, \tag{57}
\]
where \( T_{rot} \) is defined in (38).

Then
\[
\langle \psi_s(\mu_1|\tau_1)\psi_s(\mu_2|\tau_2) \rangle_d = \frac{\hbar}{4\pi^2 c} \int dO \left\{ \int_0^\infty d\omega \cos \omega \tilde{F} - \int_0^\infty d\omega \frac{2\omega \cosh \omega \tilde{F}}{e^{\hbar \omega/2kT_{rot}} - 1} \right\}. \tag{58}
\]

The expression in \{ \} is similar to the right side of the expression [11, (27) for the correlation function of the detector at rest in Planck’s spectrum at the temperature \( T \)
\[
\int_0^\infty d\omega \omega \coth \frac{\hbar \omega}{2kT} \cos \omega t = \int_0^\infty d\omega \cos \omega t + \int_0^\infty d\omega \frac{2\omega \cos \omega t}{e^{\hbar \omega/2kT} - 1}. \tag{59}
\]
The appearance of the Planck’s factor \((e^{\hbar \omega/2kT_{rot}} - 1)^{-1}\) shows the similarity between the radiation observed at the rotating detector in the massless scalar zero-point field and the radiation spectrum observed by an inertial observer placed in a thermostat filled up with the radiation at the temperature \( T = T_{rot} \).
But there is also distinction between them. \( F \) and \( \cosh \) are used in the first expression whereas \( t \) and \( \cos \) are used in the second expression respectively. The \( \tilde{F} \) is a function of \( \theta \) and \( \phi \). It means that a thermal radiation observed by the rotating detector moving in the massless scalar zero-point radiation is anisotropic.

The resemblance between both expressions becomes closer if \( t = 0 \) and \( \tilde{F} = 0 \) and two points of an observation agree. Both expressions are identical. But in the case of one-point observation which occurs when \( \tilde{F} = 0 \) it is better to consider the energy density of the scalar massless field. This brings us to the next section.

### 6.3 The Energy Density of Random Classical Massless Scalar Field Observed by a Rotating Detector and Planck’s Spectrum.

Let us consider the energy density \( \langle T_{(44)}(\mu) \rangle \) of the massless scalar field at the detector rotating through the zero-point massless scalar field. It can be expressed in terms of the tensor of energy-momentum \( T_{ik} \) at the location of the detector in the laboratory coordinate system [32] as
\[
\langle T_{(44)}(\mu) \rangle = \mu_{(4)}^i \mu_{(4)}^k \langle T_{ik} \rangle. \tag{60}
\]
where $\mu^i_\alpha$ are tetrads. The energy-momentum tensor is \[T_{ik} = \psi_{i,\psi_{,k}} - \frac{1}{2} \eta_{ik} \eta^{rs} \psi_{,r}s, \quad \eta_{ik} = \eta_{ik} = \text{diag}(1,1,1,-1)\] (61)

Using (48), (49), and Frenet-Serret tetrads it is easy to show that

$$\langle T_{11} \rangle = \langle T_{22} \rangle = \langle T_{33} \rangle = \frac{1}{3} \langle T_{44} \rangle = \frac{\hbar c}{\pi} \int dk k^3 = \frac{\hbar \Omega^4}{\pi c^3} \int d\kappa \kappa^3$$ (62)

and

$$\langle T_{(44)}(\mu) \rangle = \frac{4\gamma^2 - 1}{3} \langle T_{44} \rangle = \frac{4\gamma^2 - 1}{3} \frac{\hbar \Omega^4}{\pi c^3} \int d\kappa \kappa^3$$ (63)

This expression with periodical features taken into consideration has the following form

$$\langle T_{(44)}(\mu) \rangle_d = \frac{4\gamma^2 - 1}{3} \frac{\hbar}{\pi c^3} \Omega^4 \sum_{n=0}^{\infty} n^3$$ (64)

or

$$\langle T_{(44)}(\mu) \rangle_d = \frac{4\gamma^2 - 1}{3} \frac{\hbar}{\pi c^3} 2 \left( \int_0^\infty d\omega \frac{1}{2} \omega^3 + \int_0^\infty d\omega \frac{\omega^3}{e^{\hbar \omega/kT} - 1} \right)$$ (65)

Let us compare this expression and the expression for the energy density of the massless scalar field with the Planck’s spectrum of random thermal radiation at the temperature $T$ along with the zero-point radiation in an inertial reference frame

$$\langle T_{44} \rangle_T = \frac{1}{2} \left( (\frac{\partial \psi_T}{\partial (ct)})^2 + (\frac{\partial \psi_T}{\partial x})^2 + (\frac{\partial \psi_T}{\partial y})^2 + (\frac{\partial \psi_T}{\partial z})^2 \right),$$ (66)

where $\psi_T = \int d^3k f_T(\omega) \cos \left[ \vec{k} \cdot \vec{r} - \omega t - \theta(\vec{k}) \right]$ (67)

and

$$f^2_T(\omega) = \frac{\omega^2}{\pi^2} \frac{\hbar}{\omega} \left[ \frac{1}{2} + \frac{1}{\exp(\hbar \omega/kT) - 1} \right].$$ (68)

It is easy to show that

$$\langle T_{44} \rangle_T = \frac{3\hbar}{\pi c^3} \left( \int_0^\infty d\omega \frac{1}{2} \omega^3 + \int_0^\infty d\omega \frac{\omega^3}{e^{\hbar \omega/kT} - 1} \right),$$ (69)

and

$$\langle T_{(44)}(\mu) \rangle_d = \frac{2(4\gamma^2 - 1)}{9} \langle T_{44} \rangle_T,$$ (70)
So, due to periodicity of the motion, an observer rotating through a zero point radiation of a massless random scalar field should see the same energy density as would see an inertial observer moving in a thermal bath at the temperature $T_{rot} = \frac{\hbar \Omega}{2\pi k}$ when multiplied by the factor $\frac{2}{9}(4\gamma^2 - 1)$. This factor comes from integration over angles and is a consequence of anisotropy of the scalar field measured by an observer with velocity $\beta$.

7 Discussion

The thermal effects of non inertial motion investigated in the past for uniform acceleration through classical random zero-point radiation of electromagnetic and massless scalar field are shown to exist for the case of rotary motion also.

The rotating reference system $\{\mu_\tau\}$, along with the two-point correlation functions (CFs) and energy density, is defined and used as the basis for investigating effects observed by a detector rotating through random classical zero-point radiation. The reference system consists of Frenet-Serret orthogonal tetrads $\mu_\tau$. At each proper time $\tau$ the rotating detector is at rest and has a constant acceleration vector at the $\mu_\tau$.

The two-point CFs and the energy density at the rotating reference system should be periodic with the period $T = \frac{2\pi}{\Omega}$, where $\Omega$ is an angular detector velocity, because CF and energy density measurements is one of the tools the detector can use to justify the periodicity of its motion. The CFs have been calculated for both electromagnetic and massless scalar fields in two cases, with and without taking this periodicity into consideration. It turned out that only periodic CFs have some thermal features and particularly the Planck’s factor with the temperature $T_{rot} = \frac{\hbar \Omega}{2\pi k_B} (k_B$ is the Boltzman constant). Mathematically this property is connected with the discrete spectrum of the periodic CFs and its interpretation based on the Abel-Plana summation formula.

It is also shown that energy densities of the electromagnetic and massless scalar fields observed by the rotating detector are respectively

$$w(\mu) = \frac{2}{3} \frac{(4\gamma^2 - 1)}{w_{em}(T_{rot})}$$

and

$$\langle T_{(44)}(\mu) \rangle_d = \frac{2(4\gamma^2 - 1)}{9} \langle T_{44} \rangle_{T_{rot}}.$$
The \( w_{em}(T_{rot}) \) and \( \langle T_{44}\rangle_{T_{rot}} \) are Planck’s energy densities of electromagnetic field and massless scalar field respectively observed by an inertial detector at the temperature \( T_{rot} \) along with their random zero-point radiation.

This thermal effect is masked by factor \( \frac{2}{3}(4\gamma^2 - 1) \) for the electromagnetic field and \( \frac{2}{9}(4\gamma^2 - 1) \) for the massless scalar field, where \( \gamma = (1 - \left(\frac{\Omega c}{r}\right)^2)^{-1/2} \). Appearance of these masking factors is connected with the fact that rotation is defined by two parameters, angular velocity and the radius of rotation, in contrast with a uniformly accelerated linear motion which is defined by only one parameter, acceleration \( a \). As a consequence the thermal effects observed by detectors rotating with the same angular velocity on different circumferences are different. The further detector is from the rotation center the greater energy density it sees. If \( r \to \infty \) then \( w(\mu) \to \infty \) and \( \langle T_{44}(\mu) \rangle_d \to \infty \). Appearance of the masking factors is connected with the fact that rotation is defined by two parameters, angular velocity and the radius of rotation, in contrast with a uniform accelerated linear motion. The latter is defined by only one parameter, acceleration ”\( a \”).

Some of the results discussed in this paper have been obtained in [21] using reference systems consisting of global instantaneous inertial reference frames, not tetrads.

APPENDIX

A Frenet-Serret Orthogonal Tetrads

The Frenet-Serret orthogonal tetrad mentioned in the section 3 can be associated with each point of the time-like world line of a moving detector. The vectors of the tetrad are defined by the equations [32](55):

\[
D\mu^i_{(4)} = b\mu^i_{(1)},
\]

\[
D\mu^i_{(1)} = \tilde{c}\mu^i_{(2)} + b\mu^i_{(4)},
\]

\[
D\mu^i_{(2)} = d\mu^i_{(3)} - \tilde{c}\mu^i_{(1)},
\]

\[
D\mu^i_{(3)} = -d\mu^i_{(2)}
\]

(71)
together with
\[ \mu_i^{(4)}\mu_i^{(4)} = -1, \quad \mu_i^{(1)}\mu_i^{(1)} = \mu_i^{(2)}\mu_i^{(2)} = \mu_i^{(3)}\mu_i^{(3)} = 1. \] (72)

They are also orthogonal:
\[ \mu_i^{(a)}\mu_i^{(b)} = \eta_{(ab)}, \quad \eta_{(ab)} = \text{diag}(1, 1, 1, -1), \quad a, b = 1, 2, 3, 4. \] (73)

In the flat space-time, with metrics \( g_{ik} = \text{diag}(1, 1, 1, -1) \), \( D = \frac{d}{d\tau} \) where \( \tau \) is a proper time of the detector.

Let
\[ \mu_i^{(4)} = \frac{U_i}{c} = \mu_i^{(4)} = (-\beta\gamma \sin \alpha, \beta\gamma \cos \alpha, 0, \gamma), \] (74)

where \( U_i \) is a 4 vector velocity of the rotating detector in the laboratory coordinate system, \( \beta = v/c = \Omega a/c, \gamma = (1 - \beta^2)^{-1/2}, \alpha = \Omega \gamma \tau \), and \( \Omega, a \) are an angular velocity and the circumference radius respectively of the rotating detector. It is easy to check that the set of the 4-vectors (5) is a solution to this equation system, with the coefficients
\[ b = -\beta \Omega \gamma^2, \quad \tilde{c} = \Omega \gamma^2; \quad d = 0. \] (75)

In a Fermi-Walker tetrad [26] (9.148, 4.139, and 4.167), tetrad vectors are defined as
\[ \frac{de_{(a)k}}{d\tau} = (e_{(a)l}\dot{U}_l/c^2 - (e_{(a)l}U_l)\dot{U}_k/c^2 \] (76)

and can be given in the form [26] 4.167
\[ e_{(1)k} = (\cos \alpha \cos \alpha \gamma + \gamma \sin \alpha \sin \alpha \gamma, \sin \alpha \cos \alpha \gamma - \gamma \cos \alpha \sin \alpha \gamma, 0, -i(v\gamma/c) \sin \alpha \gamma), \]
\[ e_{(2)k} = (\cos \alpha \sin \alpha \gamma - \gamma \sin \alpha \cos \alpha \gamma, \sin \alpha \sin \alpha \gamma + \gamma \cos \alpha \cos \alpha \gamma, 0, +i(v\gamma/c) \cos \alpha \gamma), \]
\[ e_{(3)k} = (0, 0, 1, 0), \]
\[ e_{(4)k} = (i\frac{\nu}{c} \gamma \sin \alpha, -i\frac{\nu}{c} \gamma \cos \alpha, 0, \gamma). \] (77)

In [26] 4.167 the metrics is chosen in the form \( g_{ik} = (1, 1, 1, 1) \).

**B Other Correlation Functions of Electromagnetic Field at a Rotating Detector: General Expressions**

The general expression for \( \langle E_{(1)(\mu_1|\tau_1)}E_{(1)(\mu_2|\tau_2)} \rangle \) has been received in the section (4.2). The other diagonal electric field components of the CFs are as follows:
\[
\langle E(2)\vert (\mu_1\vert \tau_1) E(2)\vert (\mu_2\vert \tau_2) \rangle = \langle E(1)\vert E(1) \rangle \sin \alpha_1 \sin \alpha_2 + \langle E(1)\vert E(2) \rangle (-1) \sin \alpha_1 \cos \alpha_2 +
\]
\[
\langle E(2)\vert E(1) \rangle (-1) \cos \alpha_1 \sin \alpha_2 + \langle E(2)\vert E(1) \rangle \cos \alpha_1 \cos \alpha_2
\]
\[
(78)
\]
\[
\langle E(3)\vert (\mu_1\vert \tau_1) E(3)\vert (\mu_2\vert \tau_2) \rangle = -\gamma^2 \frac{v}{c} \cos \alpha_1 \langle E(3)\vert \tau_1 \rangle H_1(\tau_1) - \gamma^2 \frac{v}{c} \cos \alpha_2 \langle H(3)\vert E(2) \rangle \rangle -
\]
\[
\gamma^2 \frac{v}{c} \sin \alpha_1 \langle E(3)\vert \tau_2 \rangle H_2(\tau_1) - \gamma^2 \frac{v}{c} \sin \alpha_2 \langle H(3)\vert E(2) \rangle \rangle + \gamma^2 \frac{v}{c} \cos \alpha_1 \langle H(3)\vert H(2) \rangle \rangle +
\]
\[
\gamma^2 \frac{v}{c} \cos \alpha_2 \langle E(3)\vert H(2) \rangle \rangle
\]
\[
(79)
\]
Is easy to show that they depend on the difference \( \delta = \alpha_2 - \alpha_1 \) only.

\[
\langle E(2)\vert (\mu_1\vert \tau_1) E(2)\vert (\mu_2\vert \tau_2) \rangle = \cos \delta \int d^3 k R + \sin^2 \frac{\delta}{2} \int d^3 k \hat{k}_x^2 R + (-1) \cos^2 \frac{\delta}{2} \int d^3 k \hat{k}_y^2 R.
\]
\[
(80)
\]
\[
\langle E(3)\vert (\mu_1\vert \tau_1) E(3)\vert (\mu_2\vert \tau_2) \rangle = \gamma^2 \frac{v^2}{c^2} \cos \delta \int d^3 k R + \gamma^2 \frac{v}{c} (-2) \cos \frac{\delta}{2} \int d^3 k \hat{k}_y R +
\]
\[
\gamma^2 [1 - \frac{v^2}{c^2} \cos^2 \frac{\delta}{2}] \int d^3 k \hat{k}_x^2 R + \gamma^2 [1 + \frac{v^2}{c^2} \sin^2 \frac{\delta}{2}] \int d^3 k \hat{k}_y^2 R
\]
\[
(81)
\]
The non diagonal components of the correlation function are zeroes:

\[
\langle E(1)\vert (\mu_1\vert \tau_1) E(2)\vert (\mu_2\vert \tau_2) \rangle = \langle E(1)\vert (\mu_2\vert \tau_2) E(2)\vert (\mu_1\vert \tau_1) \rangle = 0,
\]
\[
\langle E(1)\vert (\mu_1\vert \tau_1) E(3)\vert (\mu_2\vert \tau_2) \rangle = \langle E(1)\vert (\mu_2\vert \tau_2) E(3)\vert (\mu_1\vert \tau_1) \rangle = 0,
\]
\[
\langle E(2)\vert (\mu_1\vert \tau_1) E(3)\vert (\mu_2\vert \tau_2) \rangle = \langle E(2)\vert (\mu_2\vert \tau_2) E(3)\vert (\mu_1\vert \tau_1) \rangle = 0
\]
\[
(82)
\]
Similar expressions have been received for the CF with magnetic field components. So all CFs can be given as 3-dimensional integrals over \((k, \theta, \phi)\).

\section{Integral calculations: final expression for \( \langle E(1)\vert (\mu_1\vert \tau_1) E(1)\vert (\mu_2\vert \tau_2) \rangle \).}

All non zero expressions for CFs in the section \([13]\) should be integrated over \(k, \theta\), and \(\phi\). The integral over \(k\) can be easily calculated:

\[
\int_0^\infty dk k^3 \cos \left\{k (2r \sin \frac{\delta}{2} \sin \theta \sin \phi - c(t_2 - t_1))\right\} = \frac{6}{\{2r \sin \frac{\delta}{2} \sin \theta \sin \phi - c(t_2 - t_1)\}^4}
\]
\[
\frac{6}{[c(t_2 - t_1)]^4} \frac{1}{[1 - \frac{v}{c} \sin \delta/2 \sin \theta \sin \phi]^4}. \tag{83}
\]

The integrals over \(\theta\) and \(\phi\) can be represented in terms of elementary functions. Let us show it for \(\langle E_1(\mu_1|\tau_1)E_1(\mu_1|\tau_2)\rangle\)

\[
\langle E_1(\mu_1|\tau_1)E_1(\mu_1|\tau_2)\rangle = \frac{3hc}{2\pi^2 [c(t_2 - t_1)]^2} \gamma^2 \int_0^\pi d\theta
\]
\[
\times \left\{ \cos \delta \sin \theta + (-\cos^2 \frac{\delta}{2} + \frac{v^2}{c^2}) \sin^3 \theta \int_0^{2\pi} d\phi \frac{1}{(1 + b \sin \phi)^4}\right.
\]
\[
+ (-2\frac{v}{c} \cos \frac{\delta}{2}) \sin^2 \theta \int_0^{2\pi} d\phi \frac{\sin \phi}{(1 + b \sin \phi)^4} + \sin^3 \theta \int_0^{2\pi} d\phi \frac{\sin^2 \phi}{(1 + b \sin \phi)^4} \right\}, \tag{84}
\]

We have taken into consideration here that

\[
\hat{k}_x = \sin \theta \cos \phi, \quad \hat{k}_y = \sin \theta \sin \phi, \quad \hat{k}_z = \cos \theta \tag{85}
\]

and used notations \(b \equiv k \sin \theta, \quad k \equiv -\frac{v}{c} \sin \delta/2\). So \(k\) is a constant, not a wave vector.

The next step is to calculate the integral over \(\phi\). Because \([12]\),

\[
\int_0^{2\pi} d\phi \frac{1}{(1 + b \sin \phi)^4} = \frac{\pi(2 + 3b^2)}{(1 - b^2)^{7/2}}, \tag{86}
\]
\[
\int_0^{2\pi} d\phi \frac{\sin \phi}{(1 + b \sin \phi)^4} = -\frac{b\pi(4 + b^2)}{(1 - b^2)^{7/2}}, \tag{87}
\]

and

\[
\int_0^{2\pi} d\phi \frac{\sin^2 \phi}{(1 + b \sin \phi)^4} = \frac{\pi(1 + 4b^2)}{(1 - b^2)^{7/2}}, \tag{88}
\]

the correlation function takes the form \([22]\).

D The second way to receive the \(\langle E_1(\mu_1|\tau_1)E_1(\mu_2|\tau_2)\rangle\) for the Discrete Spectrum.

In the section \([5.2]\) we have obtained the general expression for the CF \(\langle E_1(\mu_1|\tau_1)E_1(\mu_2|\tau_2)\rangle\) for the discrete spectrum.

Here we show that it can be received in a different way. For the discrete spectrum the equations
The unit vector \( \hat{k} \) defines a direction of the wave vector and does not depend on its value, \( n \).

The right side of the first equation in the relation (15) should be modified. We do it in two steps. First we rewrite them in a spherical momentum space [10], p.656 as:

\[
\langle \cos(\theta(\vec{k}_1\lambda_1) \cos(\theta(\vec{k}_2\lambda_2))) \rangle = \langle \sin(\theta(\vec{k}_1\lambda_1) \sin(\theta(\vec{k}_2\lambda_2))) \rangle = \frac{1}{2} \delta_{\lambda_1\lambda_2} \delta^3(\vec{k}_1 - \vec{k}_2) = \frac{1}{2} \delta_{\lambda_1\lambda_2} \frac{2}{k_i^2} \delta(k_1 - k_2) \delta(\hat{k}_1 - \hat{k}_2). \tag{90}
\]

And then, in the case of the discrete spectrum, it takes the form:

\[
\langle \cos(\theta(\vec{k}_{n_1}\lambda_1) \cos(\theta(\vec{k}_{n_2}\lambda_2))) \rangle = \langle \sin(\theta(\vec{k}_{n_1}\lambda_1) \sin(\theta(\vec{k}_{n_2}\lambda_2))) \rangle = \frac{1}{2} \delta_{\lambda_1\lambda_2} \frac{2}{k_0(k_0 n_1)^2} \delta_{n_1 n_2} \delta(\hat{k}_1 - \hat{k}_2). \tag{91}
\]

The equation \( \sum_{\lambda=1}^{2} \epsilon_i(\vec{k}\lambda)\epsilon_j(\vec{k}\lambda) = \delta_{ij} - \hat{k}_i\hat{k}_j \) does not depend on \( n \). The correlation function finally takes the form (33,32).

**E  Another expression for \( S_d \)**

The sum \( S_d \) has been calculated in (36). It can also be given in a different form:

\[
S_d = \frac{6}{F_d^4} + 6 \sum_{n=1}^{\infty} \frac{1}{(2\pi n)^4} \left[ \frac{1}{(1 + F_d/2\pi n)^4} + \frac{1}{(1 - F_d/2\pi n)^4} \right]. \tag{92}
\]

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