COLOR DISPERSION AND MILKY-WAY-LIKE REDDENING AMONG TYPE Ia SUPERNOVAE

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ABSTRACT

Past analyses of Type Ia supernovae have identified an irreducible scatter of 5%–10% in distance, widely attributed to an intrinsic dispersion in luminosity. Another equally valid source of this scatter is intrinsic dispersion in color. Misidentification of the true source of this scatter can bias both the retrieved color–luminosity relation and cosmological parameter measurements. The size of this bias depends on the magnitude of the intrinsic color dispersion relative to the distribution of colors that correlate with distance. We produce a realistic simulation of a misattribution of intrinsic scatter and find a negative bias in the recovered color–luminosity relation, $\beta$, of $\Delta \beta \approx -1.0$ ($\sim 33\%$) and a positive bias in the equation of state parameter, $w$, of $\Delta w \approx +0.04$ ($\sim 4\%$). We re-analyze current published datasets with the assumption that the distance scatter is predominantly the result of color. Unlike previous analyses, we find that the data are consistent with a Milky-Way-like reddening law ($R_V = 3.1$) and that a Milky-Way dust model better predicts the asymmetric color–luminosity trends than the conventional luminosity scatter hypothesis. We also determine that accounting for color variation reduces the correlation between various host galaxy properties and Hubble residuals by $\sim 20\%$.

Key words: dark energy – supernovae: general

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1. INTRODUCTION

Since the initial discovery of evidence for cosmic acceleration (Riess et al. 1998; Perlmutter et al. 1999), there has been a concerted effort to discover increasingly larger samples of Type Ia supernovae (SNe Ia) and also to probe the systematic uncertainties in the current samples. SN Ia measurements are still the optimal method to measure the equation of state of dark energy $w = P/\rho c^2$ (where $P$ is pressure and $\rho$ is density) since SNe Ia are high precision and can be observed in a redshift range where dark energy is dominant. In order to optimize the use of SNe Ia as standard candles to determine distances, the majority of the SN Ia light-curve fitters (e.g., MLCS2k2, Jha et al. 2007; SALT2, Guy et al. 2007; SIFTO, Conley et al. 2008; CMAGIC, Wang et al. 2009; SNooPY, Burns et al. 2011; BAYESN, Mandel et al. 2011) include two corrections to the observed peak magnitude of the SN: one using the width/slope of the light curve and the other using the color of the light curves. The width/slope correction for various light-curve fitters all account in some manner for the “Phillips relation” (Phillips 1993), be it the spectral adaptive light-curve template method (SALT2, SIFTO), the multicolor light-curve shape method (MLCS2k2), the color–magnitude intercept method (CMAGIC), a Bayesian hierarchical method (BayesN), or the $\Delta m_{15}(B)$ method itself (SNooPY).

A more fundamental difference between these fitters is how to interpret heterogeneous SN Ia colors. Some light-curve fitters like SALT2 or SIFTO are used to find an empirical relation between color and luminosity, called $\beta$, while MLCS2k2 assumes that the color–luminosity relation follows the Milky-Way (MW) reddening law. Regardless of the approach, the corrections reduce the dispersion in distance to $\sim 5\%-10\%$, which is assumed to be intrinsic but may result from unmodeled effects (e.g., Kim et al. 2013; Chotard et al. 2011). To ensure that all cosmology fits have reasonable parameter errors ($\chi^2 \sim 1$), an intrinsic scatter of 0.05–0.15 mag is added in quadrature to the distance modulus uncertainties.

Historically, this irreducible scatter in the distance modulus residuals relative to a best-fit cosmology, also called “Hubble residual scatter,” has been attributed to random, achromatic variations in the luminosity. This “luminosity variation” is the variation in SN Ia brightness that does not correlate with distance and is independent of the light-curve corrections. While it is relatively simple to apply the assumption that Hubble residual scatter is due to luminosity variation, there is little, if any, direct evidence to support this claim. Variations of color and shape in the SNe light curves, which do not correlate with distance in the way expected by light-curve fitters, present equally plausible alternatives. We denote color variation as the variation in colors for a fixed distance, which can be observed as the variation in distances at a fixed color. (Marriner et al. 2011, hereafter M11) present a mathematical formalism for assigning this “intrinsic scatter” of SN Ia to either the luminosity, color, or stretch variation, or a combination of the three. Here, we call this scatter the “residual scatter,” or $\sigma_r$, as it is needed to explain the scatter in the Hubble residuals. (Kessler et al. 2013, hereafter K13) explore various sources of this residual scatter and show that misidentifying the source of scatter biases the recovery of $\beta$ by up to 10% and $w$ up to 5% when assuming a broad, fixed color distribution. We revisit this assumption here.

There is evidence to suggest that color variation is non-negligible. Foley et al. (2011) and Chotard et al. (2011) both examine the relations between silicon and calcium features of SN Ia spectra and SN Ia color and conclude there must be color variation on the order of what is needed to explain the Hubble residual scatter. Jha et al. (2007) analyzed +35 day nebular colors (Lira 1995), a phase when the light curve shape-dependent color variation is minimized, and found there must be a similarly high amount of color variation. There is also preliminary evidence that color variation may partly account for the trends
between Hubble residuals and host galaxy properties (Childress et al. 2013).

In this paper, we use publicly available data from the Sloan Digital Sky Survey (SDSS) (Holtzman et al. 2008), SNLS3 (Guy et al. 2010), and nearby samples (see Conley et al. 2011 for a review and re-analysis) and the SNANA simulator (Kessler et al. 2009a) and explore how misattribution of the source of residual scatter and ignorance of the underlying color distribution affect \( \beta \) and \( w \) estimation. In Section 2, we present an analysis of the different components of the observed color and explain how the bias in the SN color–luminosity relation depends on both the source of residual scatter and the underlying distribution of color. In Section 3, we show that if color variation causes the residual scatter, the empirical relation between SN Ia distance and color is well represented by a MW reddening law. In Section 4, we discuss implications of different models for SN Ia color, including effects on \( w \) recovery, how the distance residual bias introduces what appears to be \( \beta \) evolution, and how host galaxy luminosity correlations can be partly explained by the bias. Our discussion and conclusions are in Sections 5 and 6.

2. THE DEPENDENCE OF THE COLOR–LUMINOSITY RELATION ON THE SOURCE OF SCATTER

2.1. The Different Sources of Residual Scatter

In order to understand the color–luminosity relation of SN Ia, we must define the different components of SN Ia color and how each component is treated by light-curve fitters to determine distances. For most of the analysis in this paper, we employ the SALT2 light-curve fitter as its empirical framework easily allows for different assumptions and it is one of the most widely used light-curve fitters.

The distance modulus \( \mu \) determined by SALT2 for each SN Ia is expressed as

\[
\mu = m_B - M_0 + \alpha x_1 - \beta c,
\]

where \( m_B, x_1, \) and \( c \) are the individual fit parameters representing the rest-frame \( B \)-band peak brightness, stretch of the light curve, and color of the SN, respectively. \( M_0, \alpha, \) and \( \beta \) are parameters that represent the absolute magnitude of a SN Ia, the slope of the stretch–luminosity relation, and the slope of the color–luminosity relation, respectively. We follow M11 since this work accounts for residual scatter in any of the SALT2 fit parameters and has the advantage of separating the determination of the SALT2 nuisance parameters from a specific cosmology (see Appendix A). The error of \( \mu \) is assumed to be the quadrature sum of the “noise,” \( \sigma_\text{r} \), and residual scatter applicable to the model, \( \sigma_r \), such that \( \sigma^2_\mu = \sigma^2_\text{r} + \sigma^2_r \).

The observed color, \( c_{\text{obs}} \), can be expressed as

\[
c_{\text{obs}} = c_{\text{mod}} + c_r + c_n,
\]

where \( c_{\text{mod}} \) is the model color that is the component of SN color that is linearly correlated with luminosity by \( \beta \). The residual color \( c_r \) is the random color component uncorrelated with luminosity and \( c_n \) is the noise of the color measurement. Conventionally, \( c_r = 0 \) and the total residual scatter \( \sigma_r \) are given as a single number that represents only the residual scatter in the peak \( B \)-band luminosity of SN Ia, \( m_B \), as it correlates directly with \( m_B \) or \( M_0 \). It is important to note that a component of \( c_n \) is the uncertainty of the SALT2 template, which includes color variation (Guy et al. 2010; K13). The amount of color variation in their model is propagated by Conley et al. (2011) as a systematic uncertainty (EXPOL and SIGMOID models), although they find the uncertainty to have a negligible effect on measuring cosmological parameters. Declaring \( c_r = 0 \) represents the assumption that the intrinsic scatter typically added to each SN’s uncertainty contains no color variation. M11 address this assumption and allow the residual scatter to represent the residual scatter in \( m_B, c, x_1 \) or combinations of the three. More generally, this residual scatter for each SN is

\[
\sigma^2_r = \sigma^2_m + \alpha^2 \sigma^2_{x_1} + \beta^2 \sigma^2_c - 2 \alpha \beta \Sigma_{m,x_1,c}.
\]

\( \Sigma \) represents the \( 3 \times 3 \) residual scatter matrix in \( m_B, x_1, \) and \( c \) and \( \sigma^2_{m}, \sigma^2_{x_1}, \sigma^2_{c} \) are its diagonal components. It is important to note that since \( c_r \) includes \( \alpha, \beta, \) and terms, these coefficients play a role in not only correcting the distances but also propagating the uncertainty of each distance.

To understand the consequences of incorrectly attributing the source of the observed distance scatter, we analyze publicly available data from SDSS, SNLS3, and nearby samples. We include 91 SDSS SNe Ia (Holtzman et al. 2008) and 241 SNLS3 SNe Ia (Guy et al. 2010) and a nearby sample comprised of 186 SNe Ia from a variety of sources, most of which are described in Conley et al. (2011). The only additions to the Conley et al. (2011) set are 67 SNe Ia from the CfA4 sample (Hicken et al. 2009a) and 34 additional CSP SNe Ia (Carnegie Supernova Project; Contreras et al. 2010). To fit the light curves, we use the SNANA SALT2 light-curve fitter and its provided files for defining the filter transmission functions.

In Figure 1, we show the dependence of \( \beta \) from the full dataset on the fraction of the residual scatter assumed to result from color. We ignore stretch here as luminosity and stretch variation affect the color–luminosity relation in a similar manner. We
find that as the contribution of color to the residual scatter component increases, so does the value of \( \beta \) recovered. Because past analyses have assumed \( \sigma_c = 0 \), they found the lowest possible value of \( \beta \).Attributing the residual scatter to color changes the retrieved value of \( \beta \) from 3.2 to 3.7. Interestingly, we find that \( \beta \) from the full sample is relatively close to the MW-like extinction law of \( \beta = 4.1 \) when the residual scatter is entirely attributed to color variation, a plausible but unproven possibility. In Figure 1, the SNLS3, SDSS, and nearby samples are combined, although we also find that the dependence of \( \beta \) on the source of the residual scatter to be somewhat different for each survey. When residual scatter is attributed entirely to color, we find \( \beta = 3.80 \pm 0.161 \), \( \beta = 3.65 \pm 0.124 \), and \( \beta = 3.20 \pm 0.102 \) for the nearby, SNLS3, and SDSS samples, respectively. For the nearby sample, the value of \( \beta \) is \( <3\sigma \) from the MW-like extinction value. When residual scatter is attributed entirely to luminosity, we find \( \beta = 3.34 \pm 0.171 \), \( \beta = 3.02 \pm 0.164 \), and \( \beta = 2.91 \pm 0.210 \) for the nearby, SNLS3, and SDSS samples, respectively. An explanation for these differences will be presented in the following section.

### 2.2. Knowledge of the Color Distribution

To quantify the bias in the recovery of \( \beta \) when residual color scatter is ignored, we must understand the consequences of the previous assumption that \( \sigma_c = 0 \) in Equation (3). We define, \( \beta_{\text{mod}} \) as the color–luminosity relation if there is no color noise or color variation \( \sigma_{\text{obs}} = \sigma_{\text{mod}} \) and \( \beta_{\text{obs}} \) as the color–luminosity relation if there is non-zero color noise and/or color variation \( \sigma_{\text{c, obs}} \neq 0 \). For a distribution of modeled colors defined by a Gaussian of width \( \sigma_{\text{mod}} \), we expect the bias in \( \beta \) recovery to be (see Appendix B for a review):

\[
\frac{\sigma^2_{\text{obs}}}{\beta^2_{\text{mod}}} \approx \frac{\sigma^2_{\text{mod}}}{\beta^2_{\text{mod}}} + \frac{\sigma^2_c}{\beta^2_{\text{mod}}} + \frac{\sigma^2_{\text{cn}}}{\beta^2_{\text{mod}}}.
\]  

(4)

If \( \sigma^2_c + \sigma^2_{\text{cn}} \ll \sigma^2_{\text{mod}} \), then \( \beta_{\text{obs}} \approx \beta_{\text{mod}} \), as assumed in past analyses. However, if \( \sigma^2_c + \sigma^2_{\text{cn}} \sim \sigma^2_{\text{mod}} \), then \( \beta_{\text{obs}} < \beta_{\text{mod}} \). Therefore, the change in \( \beta_{\text{obs}} \) from the true color–luminosity correlation \( \beta_{\text{mod}} \) is dependent not only on the presence of residual color scatter but also its size in comparison with the underlying color distribution of SNe Ia, \( \sigma_{\text{mod}} \).

In order to quantify this bias in \( \beta \) recovery and verify Equation (4), we simulate SNe Ia samples with different magnitudes of residual color scatter \( \sigma_{\text{c, obs}} \) and widths of Gaussian, color model distributions \( \sigma_{\text{mod}} \). Any simulation must replicate the observed color distribution of the real data \( \sigma_{\text{c, obs}} \). Because \( \sigma^2_{\text{c, obs}} = \sigma^2_{\text{c, mod}} + \sigma^2_{\text{c,n}} + \sigma^2_{\text{c,r}} \), increasing the magnitude of \( \sigma^2_{\text{c,r}} \) in the simulations requires that \( \sigma^2_{\text{c,mod}} \) is decreased. The main divergence between this analysis and that of K13 is that K13 do not change \( \sigma_{\text{c,mod}} \) when they vary \( \sigma_{\text{c,n}} \). In Table 1, we show \( \sigma_{\text{c,mod}} \) and \( \sigma_{\text{c,n}} \) for each sample. After assuming the magnitude of color scatter \( \sigma_{\text{c,n}} \), we find \( \sigma_{\text{c,mod}} \).

For the simulations, we use the SNANA (Kessler et al. 2009a) simulator, which allows a user to incorporate information such as actual weather history, point spread function characteristics, spectroscopic follow-up strategies, and underlying distributions of color and stretch, all toward mimicking a true SN survey. In order to simulate color or luminosity scatter, we follow the SNANA procedure that adds random magnitude offsets generated for each SN and observed filter to each light-curve point measured in that filter. This process is called “color smearing” and for simulating luminosity scatter, the additional, random magnitude offset is the same for all filters, while for color scatter, the additional, random magnitude offset is different for each filter.

To estimate the largest bias possible in the recovery of \( \beta \), we simulate SN samples with scatter entirely due to residual color variation \( \sigma_{\text{c,n}} = 0.04 \) but in the recovery of \( \beta \) we assume \( \sigma_{\text{c,n}} = 0 \) and therefore misattribute all of the scatter to luminosity variation \( \sigma_{\text{m,n}} \). The simulations used for this exercise have the characteristics of the SNLS3 survey (e.g., weather, cadence, seeing, \( \sigma_{\text{c,n}} \approx 0.035 \) ) and we fix the simulation input \( \beta = 4.1 \) so that the color–luminosity relation is consistent with extinction similar to that in the MW. It is important to note that the SALT2 extinction law and MW extinction law are not the same, although they are in best agreement when \( \beta = 4.1 \). We show, in Figure 2, how the magnitude of the bias in the recovery of \( \beta_{\text{obs}} \) depends on the relative size of the simulated color variation \( \sigma_{\text{c,n}} \) to the width of the color distribution \( \sigma_{\text{mod}} \). The trend seen from the simulations is in decent agreement with what is predicted from Equation (4) and discrepancies are likely due to covariances between the light-curve fit parameters. While we chose to input \( \beta = 4.1 \) in the simulation, the trend seen in Figure 2 would be similar for different input \( \beta \) values.

To best estimate this bias in \( \beta_{\text{obs}} \), for the individual SDSS, SNLS3, and nearby samples, we take the \( \sigma_{\text{c,mod}} \) value for each sample in the color-only case in Table 1. We find from Figure 2 that for the color-only case we recover on average \( \beta_{\text{obs}} \approx 3.1 \) for these samples, 1 lower than the input value and consistent with the value of \( \beta \) seen in the literature (Conley et al. 2011). Therefore, we find that the value of \( \beta \) regularly quoted as disproving the hypothesis that colors of SNe Ia follow a MW reddening law falls out naturally from a simulation that has two simple assumptions: the true color–luminosity relation follows the MW reddening law and the Hubble residual scatter is due to color variation but misattributed to luminosity variation.

We also offer three explanations of why there should be disagreement in \( \beta \) between these SNe samples. The first explanation is that since the \( \sigma_{\text{c,obs}} \) distributions of these three samples are different, then for the same \( \sigma_{\text{c,n}} \) value we would expect that \( \sigma_{\text{c,mod}} \) is different for each sample and therefore \( \beta_{\text{obs}} \) should vary. This claim is reasonable as the nearby sample contains more SNe Ia with higher extinction values than the SDSS or SNLS3 samples and should have a higher \( \sigma_{\text{c,mod}} \) value.

### Table 1

| Survey | \( \sigma_{\text{c,obs}} \) (Obs.) | \( \sigma_{\text{c,n}} \) (Assumed) | \( \sigma_{\text{mod}} \) (Assumed) | Components |
|--------|----------------|----------------|----------------|-----------|
| SNLS3  | 0.087          | 0.043          | 0.140          | Lum.      |
| SDSS   | 0.076          | 0.039          | 0.140          | Lum.      |
| Nearby | 0.094          | 0.032          | 0.140          | Lum.      |

**Notes:** The different components of the observed color distribution given various assumptions of the residual scatter model, \( \sigma_{\text{c,obs}} \) and \( \sigma_{\text{c,n}} \) are found from the real data. The scatter source is assumed and \( \sigma_{\text{mod}} = \sqrt{\sigma^2_{\text{c,obs}} - \sigma^2_{\text{c,n}} - \sigma^2_{\text{c,r}}} \).

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4 Scolnic et al. (2013) and Rest et al. (2013) show that this value may be even higher and depends on the peculiar velocity uncertainty attributed. They find \( \beta = 4.08 \pm 0.21 \) with a velocity uncertainty of 150 km s\(^{-1}\), although in this analysis we assume the velocity uncertainty is 360 km s\(^{-1}\)
(mean of \(c_{\text{obs}}\): \(-0.005 \pm 0.002\); mean of \(c_{\text{obs}}\) for higher-\(z\) samples: \(-0.028 \pm 0.002\)). The second explanation is that the SNe found for the low-\(z\) sample were discovered by galaxy-targeted searches while SNe found in the SNLS and SDSS surveys were discovered by rolling searches. It is therefore possible the low-\(z\) sample may suffer from selection biases so that a higher \(\beta\) is found (Sullivan et al. 2010, hereafter MS10). A third explanation is that the low-\(z\) training sample does not extend to blue colors and that the higher-\(z\) SNe may have biased colors by allowing for colors bluer than those found in the low-\(z\) sample. In the entire low-\(z\) sample, there is only one SN with a color bluer than \(c = -0.12\) by more than \(1\sigma\), while for the SDSS+SNLS sample, there are 17. These SNe have strong leverage on the value of \(\beta\) found. Future work should pursue the nature of any low-\(z\) SNe found with very blue colors.

3. TWO DEGENERATE MODELS OF SUPERNOVA COLOR?

3.1. A Physical Color Model

In the previous section, we showed that if \(\beta = 4.1\) and the residual scatter is due to color variation but misattributed to luminosity, then an analysis would find \(\beta \approx 3.1\), in agreement with past studies. Now, we explore the physical assumption that the model color may solely be due to reddening. In this approach, the observed color, \(c_{\text{obs}}\), can be expressed as

\[
c_{\text{obs}} = c_{\text{dust}} + c_{\epsilon} + c_{\text{int}},
\]

where the equation is of the same form as Equation (2) but \(c_{\text{mod}}\) is now replaced with \(c_{\text{dust}}\). MLC2k2, which is founded on astrophysical assumptions, denotes an unreddened color to have \(A_V = 0\), which, according to Kessler et al. (2009b), is roughly analogous to a \(c \approx -0.10\) for SALT2.\(^5\) For this physical model of color, we would therefore expect \(c_{\text{dust}} \geq -0.10\) and that the residual color scatter explains colors bluer than \(c_{\text{obs}} = -0.10\).

The assumption that \(c_{\text{mod}}\) is solely due to reddening has not only physical but also empirical motivations. K13 found that the model distribution of color is best described by an asymmetric Gaussian (see Appendix B for an explanation) with a bluedward standard deviation that is significantly shorter than the redward standard deviation. A smaller blue range likely implies that most of the color of a SN is due to reddening, rather than some other color-related property of the SN. For the 306 SNe 1a in the combined SDSS+SNLS3+nearby sample with small statistical color errors (\(\sigma_c < 0.04\)), only 35 SNe have colors bluer than the \(A_V\) cutoff of \(c = -0.1\) (\(\sim 11\%\)). A residual scatter of \(\sigma_c = 0.04\) is large enough to replicate in a simulation the blue side of the observed distributions of color given an input cutoff at \(c = -0.1\).

We wish to compare how the output color distributions from simulations of two significantly different models of color match the data. For the first model, color is due to dust and random variation; in the second model, we take the conventional SALT2 approach that there is no color variation. For the Color Variation model, we create a simulation with \(\beta_{\text{mod}} = 4.1\), a model distribution that follows a reddening-only, one-sided Gaussian (\(c_{\text{dust}-\text{min}} = -0.1, \sigma_{c_{\text{dust}+}} = 0.12\)) and residual scatter due to color variation. The standard deviation of the one-sided Gaussian is set so that the output color distribution of this simulation best matches the data and is roughly equal to \(2 \times \sigma_{c_{\text{mod}}}\) from Table 1. For the Luminosity Variation model, we create a simulation with \(\beta_{\text{mod}} = 3.1\) and residual scatter due to luminosity variation and the \(c_{\text{mod}}\) distribution is taken from the default SNANA distribution described in K13.

In the previous section, we simulated samples using the M11 “color-smearing” method that added magnitude variations to each filter. To allow for easier reproducibility of our analysis, we follow the K13 method in which \(\sigma_r\) depends on wavelength. For the Luminosity Variation case, we take the SALT2 scatter model (hereafter called Guy10), which claims that scatter is relatively independent of wavelength and therefore that the residual scatter is dominated by luminosity. For the Color Variation case, we follow Chotard (2011), which presents two different color-dominated scatter models in which the scatter has a strong wavelength dependence. Of the two Chotard models, we find that the model denoted as C11_0 in K13 is better at reproducing the data. To further clarify the difference between the Guy10 and C11 models, K13 show that the Guy10 model is composed of \(\sim 70\%\) luminosity variation and \(\sim 30\%\) color variation, while the C11 model is composed of 25% luminosity variation and 75% color variation.

In Figure 3, we present the color distributions of our two models: the Luminosity Variation Guy10 model that is typically used in SALT2 and the color variation-dominated C11 model in which model color is solely due to reddening, called the MW model. We find that the observed color distributions of both models reproduce the data. For the MW model, when residual scatter is attributed to luminosity, we find a \(\beta\) value near 3.1 and a similar intrinsic dispersion value as seen in the data of \(\sigma_m = 0.12\). This result is interesting because we find that a simulation with the physical, MW reddening model of color yields the same global fit parameters as the real data themselves.

\(^5\) From Figure 8 in Hicken et al. (2009b), \(c\) appears to be \(\approx -0.05\) for \(A_V = 0\), although there is a significant amount of scatter.
Figure 3. $c_{\text{obs}}$ distribution for SNLS3 simulations with the SALT2 Luminosity Variation model and the “MW” Color Variation model. The input ($c_{\text{mod}}$) distribution and observed distribution are shown for each simulation, as well as the true SNLS3 observed color distribution. The parameters of the input distribution for each simulation are given.

(A color version of this figure is available in the online journal.)

and this physical model of color has one fewer free parameters and is thus favored by Occam’s Razor. The result is in agreement with the results of Figure 2, although in this case we have an asymmetric $c_{\text{mod}}$ that is tied to a physical understanding of color.

While we find that simulations with a MW-like reddening model can produce the same color distribution as the conventional SALT2 model, we ultimately wish to resolve which model is more accurate. So far, we have observed that the Color Variation model with $\beta = 4.1$ and the Luminosity Variation model with $\beta = 3.1$ are degenerate as they both yield $\beta$ values of $\approx 3.1$ when scatter is attributed to luminosity. We hope to break this degeneracy. In Figure 3, we observe that the MW model assumes that blueward of $c = -0.1$, the color results from noise and residual color scatter. These bluer colors would not be expected to correlate with luminosity in the same way as the redder colors. The conventional SALT2 Luminosity Variation model assumes that blue and red colors would be identically correlated with distance.

To test these two predictions, we analyze the Hubble residuals after including a color-correction with $\beta = 3.1$ for all our samples. In Figure 4, we show that in the SNLS3+SDSS+nearby data, there are effectively different color–luminosity relations for $c > 0$ and $c < 0$. The conventional SALT2 model predicts that $\beta$ should be unchanged over the color range, while the MLCS2k2 MW model predicts a bifurcation because of the $A_V = 0$ cutoff and the long exponential reddening tail. The color–luminosity relations shown are found using simple, linear fits to the data. We see that the bifurcated slopes of the real data appear to match those from the MW prediction ($<1\sigma$ differences) significantly better than the conventional SALT2 prediction ($2\sigma$–$3\sigma$ differences). By analyzing each color range independently, we may break the degeneracy between the physical model of color and the conventional SALT2 model and find that the physical model is empirically optimal. The inconsistent $\beta$ values found for blue and red colors are also seen in Suzuki et al. (2012); they find a $\Delta \beta = 1.48 \pm 0.36$ between SNe Ia with $c > 0.05$ and $c < 0.05$.

Similar trends between Hubble residuals and color are seen in MS10 and Ganeshalingam et al. (2013). We may test whether this trend can be due to a covariance between $\mu$ and $c$ by finding the best-fit slope of $\Delta \mu + 3.1c$ for blue and red colors separately, rather than finding the remaining trend from Hubble residuals ($\Delta \mu$) once a single $\beta$ has been found (Julien Guy 2013, private communication). When doing so, we find a difference in $\beta$ of $+0.55 \pm 0.32$. However, we also find a difference in luminosity offsets $M_0$ between these subsamples of $0.16 \pm 0.045$ mag.
These results indicate a large discrepancy between blue and red colors. MS10 find $\beta$ for blue and red colors with SiFTO, which gives similar results to SALT2 (Conley et al. 2011). They find that $\beta = 3.981 \pm 0.416$ for $C < 0$ and $\beta = 3.826 \pm 0.199$ for $C > 0$ and a difference in $M_0$ of $\sim 0.14$ mag between these two subsamples. Interestingly, the values of $\beta$ found by MS10 for each half of the color range analyzed separately are consistent with a MW-like extinction law, even though their $\beta$ for the full sample is $3.084 \pm 0.099$, which is inconsistent with a MW-like extinction law. More work must be done on this topic to better explain this inconsistency. We also point out that the trend between Hubble residuals and color shown here (bluer objects are fainter) is opposite to the trend due to a selection bias (bluer objects are brighter).

We note that while the C11 color model is used for this test, the simple color-smearing model in SNANA produces similar results. We also find that simulating the C11 model with a more appropriate underlying color distribution improves the consistency between the simulations and the SNLS data of the $B - V - c$ distributions. This diagnostic is used by K13 to test different intrinsic variation models. The comparison, given as a ratio of the Monte Carlo to the data ($R_{MC/Data}$) of the $B - V - c$ distributions for $\Delta c$ is 1.1, where unity represents statistically similar distributions.

### 3.2. A Bayesian Approach for Analyzing Supernova Color

So far, when we have compared the $\beta$ values from these two approaches toward SN Ia color, we have assumed, correct or otherwise, that the Hubble scatter from each model is due to luminosity. We have done this in order to explore the biases that would be present in other SN Ia analyses, if that conventional assumption is correct. We may also attempt to analyze the data when we assume that the Hubble residual scatter is entirely due to color variation and that there is a dust cutoff of $c > -0.1$. Unfortunately, there is no formalism to incorporate any kind of Bayesian prior in SALT2. We introduce here a simple Bayesian algorithm applied to SALT2 (hereafter called BALT) that allows for the possibility that color follows the physical model outlined above. Once SALT2 finds a color from the light-curve fit, we apply a Bayesian prior (Riess et al. 1996) to the color such that

$$c_B = \frac{1}{P} \int c e^{-\left(c-c_{obs}\right)/2\sigma_{c}^{2}} e^{-\left(c-c_{obs}\right)^2/\tau_S} \partial c,$$

where $c_B$ is the corrected color, $c_{obs}$ is the color from the light-curve fit and $\sigma_c$ is the noise from the color measurement, and $P$ is a normalization constant. The second part of Equation (6) describes the expected distribution of the model color as shown in the bottom panel of Figure 3 where $\tilde{c}$ is the blue cutoff of the distribution ($\bar{A}_V = 0$). $\tau_S(z)$ describes the shape of the one-sided Gaussian due to extinction for a given redshift $z$ for each survey $S$; the dependence of $\tau$ on survey and redshift allows selection effects to be modeled (following Kessler et al. 2009b). To correct the color, we find the color such that $P(c_B)$ is maximized. Here, we do not include any additional error into the color error from the Bayesian prior. From the previous subsection, we expect that $\epsilon = -0.1$ and $\tau(z = 0) = \sigma_{\text{data}} = 0.11$. At higher redshifts, $\tau$ decreases since only SNe with bluer colors are discovered and followed up. $\tau(z > 0)$ may be determined from simulations with an input $\tau(z = 0)$. We estimate the uncertainty of $\sigma_{\text{data}}$ by varying this value for the model distributions in the simulations and observing how well the simulated $c_{obs}$ distribution compares with the data. Doing so, we find $\tau(z = 0) = 0.11 \pm 0.02$.

If Equation (6) is applied to each SN color of the SDSS+SNLS3+nearby sample, we find a very significant reduction in the total $\chi^2$ of the sample and the intrinsic dispersion needed to bring $\chi^2$ to unity. Following the conventional SALT2 approach, where scatter is attributed to luminosity, we determine that the total $\chi^2/N = 801.6/518$ and $\sigma_c = 0.09$. With the BALT approach, setting $\beta = 4.1$, the total $\chi^2/N = 592.3/519$ and $\sigma_c = 0.05$. Interestingly, if we exclude the nearby sample, the total $\chi^2 = 1.03$ for the BALT approach whereas $\chi^2 = 1.56$ for the conventional SALT2 approach. Part of the reason that the total intrinsic dispersion ($\sim 0.01$ mag) is so low after the BALT correction for the SDSS and SNLS3 samples is that due to selection effects, the model color range of the SNe that are followed up is very narrow. We note that for this sample, simply forcing all SNe with $c < -0.1$ to have $c = -0.1$ reduces the $\chi^2$ to 1.26.

The main argument against correcting the colors a posteriori is that it ignores covariances between color and the other fit parameters; however, from Guy10 we expect those covariances to be small. There is currently work being done on a sophisticated Bayesian hierarchical approach to SALT2 (March 2012) and the BALT algorithm applied here shows the promise of this approach. The conventional method of

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6. For $z = [0.0, 0.2, 0.4, 0.6, 0.8, 1.0]$, we find:

$\tau_{SDSS} = [0.11, 0.085, 0.055, - , - , -]$ and

$\tau_{SNLS3} = [0.11, 0.11, 0.105, 0.085, 0.07]$. 

---

**Figure 4.** Relation between Hubble residuals and color for the full SNLS3+SDSS+nearby sample, a simulated sample based on MW-like extinction, and the conventional empirical model. The parameters of the two simulations are shown in Figure 4. The slopes of the trend of Hubble residuals with color variation model, and the conventional empirical model. The parameters of the two simulations are shown. Interestingly, the values of $\beta$ found by MS10 for each half of the color range analyzed separately are consistent with a MW-like extinction law, even though their $\beta$ for the full sample is $3.084 \pm 0.099$, which is inconsistent with a MW-like extinction law. More work must be done on this topic to better explain this inconsistency. We also point out that the trend between Hubble residuals and color shown here (bluer objects are fainter) is opposite to the trend due to a selection bias (bluer objects are brighter).
Figure 5. Top: statistical constraints for the 68% and 95% confidence levels on ($\Omega_m$, $w$) from simulations using the conventional Luminosity Variation model and the MW dust + Color Variation model, including priors from cosmic microwave background and baryon acoustic oscillation (BAO) observations. In both cases, the analysis assumes that residual scatter is due entirely to luminosity variation. We assume a flat universe and constant dark energy equation of state. Bottom: the difference between observed distance and simulated distance for simulations based on the MW dust + color model ($\beta = 4.1$, $\sigma_m = 0.04$). The observed distances have been derived by making the assumption that residual scatter is due to luminosity variation ($\beta = 3.1$, $\sigma_m = 0.11$). The distances include a Malmquist correction.

(A color version of this figure is available in the online journal.)

not applying a prior to the color distribution is similar to applying a flat Bayesian prior, which itself may bias the analysis.

4. CONSEQUENCES OF DIFFERENT COLOR MODELS

4.1. Cosmological Implications

So far, we have shown that two prominent approaches for handling SN color are degenerate and we have introduced a method to break this degeneracy. Now we ask to what degree do we bias our measurement of $w$ when we make an incorrect assumption about the nature of SN color and the source of SN scatter?

To address this question, we use the SNANA program and its default simulation inputs to simulate the nearby, SDSS, and SNLS3 sample (see K13 for a review). We simulate two types of samples: one with the Color Variation model and the other with the Luminosity Variation model, as described in the previous section. For each survey, we simulate 10 samples with 5000 SNe Ia so as to remove any statistical fluctuations between the simulated samples. Then we analyze all simulations attributing Hubble residual scatter to luminosity variation. Following K13, these analyses include an empirically determined Malmquist correction, which corrects for selection effects and any biases introduced by the light-curve fitter.

In Figure 5 (bottom), we show the bias in the distance modulus for the simulation based on the Color Variation model for each survey. We find that the bias is up to $\sim$0.02 mag for the nearby sample, $\sim$0.01 mag for the SDSS sample, and $\sim$0.005 mag for the SNLS3 sample. Translating the bias in distances into an effect on retrieved cosmology depends on the priors used. K13 use priors from both Wilkinson Microwave Anisotropy Probe (WMAP; Komatsu et al. 2009) and SDSS-BAO (Eisenstein et al. 2005) and we follow that method here, although we remark that the overlap between the statistical contours from these priors and that from different SNe analyses may hide inconsistencies between the SNe constraints. In Figure 5 (top), we present statistical cosmology constraints from two of our different simulations of a combined SNLS3+SDSS+nearby sample (5000 SNe in each sample).

These simulations reflect that over our multiple large simulations, we find an average bias in $w$ for the color variation simulation of $\sim +0.037$ for the full combined sample. The results for the SDSS and SNLS3 samples individually are shown in Table 2.

While the biases in $\beta$ are significantly larger than that in K13, the biases in $w$ found here and in K13 ($\Delta w \approx 1\%-2\%$) are more similar. The relative agreement between these studies is expected as both are probing the degeneracy of various color models. The reason that the bias found here may be up to $\times 2$ larger, although still small, is likely due to the asymmetry in the color distribution.

We also may compare the difference in $w$ when we apply the BALT color method to when we apply the conventional SALT2 luminosity method. From the combined real SNLS3+SDSS+nearby sample, when we attribute the residual scatter to luminosity variation, we find $w = -0.943 \pm 0.056$. When we apply the BALT method, we find $w = -0.995 \pm 0.049$. The difference of $\Delta w = +0.052$ is similar to that predicted from our simulations for the full sample.

4.2. Host Galaxy Properties

Since we have seen that the biases in $\beta$ and SN distances depend on the width of the color distributions, we now ask whether modest correlations between various host galaxy properties and Hubble residuals (e.g., Kelly et al. 2010; MS10; Gupta et al. 2011) may be a result of these biases. This question is further motivated by recent findings of correlations between host galaxy properties and SN colors (Childress et al. 2013).

To explore this question, we find the widths of the color distributions for SNe in both high and low specific star formation rate (sSFR) hosts, using sSFR values from MS10. We analyze the sSFR property, rather than mass, for this exercise as there is a clearer difference in the widths of the observed SN color distribution of subsamples split by sSFR than by mass. Mass subsamples appear to have different average colors although similar widths, which suggests that only the centers of their underlying color distributions may be different (see the Discussion). Following Section 2.2, we derive the width of the model color distribution for the high sSFR hosts (log sSFR

| Sample            | $\Delta \beta$ | $\Delta w$ |
|-------------------|---------------|------------|
| SNLS3+SDSS+Nearby | $-0.92$       | $+0.037$   |
| SNLS3 only        | $-0.90$       | $+0.042$   |
| SDSS only         | $-1.05$       | $+0.023$   |

Notes. The biases in $\beta$ and $w$ due to misattributing the source of scatter to luminosity variation for simulations with color variation, $w$ is found after including priors from SDSS-BAO and WMAP.
split \((\text{yr}^{-1}) > -9.7\) in the SNLS3+SDSS+nearby sample to be \(\sigma_{\text{resid}} = 0.063\) mag and the width for the low sSFR hosts (LOG sSFR split \((\text{yr}^{-1}) < -9.7\)) to be \(\sigma_{\text{resid}} = 0.051\) mag. We note the significance of this difference is only \(\sim 1.5\sigma\). From Figure 2, we extrapolate that the difference in \(\beta\) from these two samples should be \(\Delta \beta \approx -0.36\), roughly half the difference seen in MS10 \((\Delta \beta \approx -0.75)\). To find the difference in Hubble residuals between these two subsamples, we simulate the two subsamples with the derived parameters for the model color distributions. We determine that the difference between the Hubble residuals when these two samples are combined is \(0.018 \pm 0.003\) mag. If the bias in \(\beta\) was similar to that seen in MS10, the difference in Hubble residuals would be \(0.036 \pm 0.007\) mag. While this difference is statistically significant, it only comprises a fraction of the difference in Hubble residuals dependent on host galaxy properties seen in most studies \((\sim 0.07-0.08\) mag; Childress et al. 2013). More work must be done to further understand the difference in the values of \(\beta\) found in MS10. A promising path forward may be to involve the metallicity of the host galaxies, as this appears to have the strongest correlation with SN color (Childress et al. 2013, Figure 8).

We also explore the significance of the relations between mass and Hubble residuals after we apply the BALT correction. We find that following the conventional SALT2 approach, attributing residual scatter to luminosity, there is a difference in Hubble residuals of \(0.075 \pm 0.014\) mag for SNe in high- and low-mass hosts. With the BALT correction, we still find a difference of \(0.062 \pm 0.016\) mag even though the reduction in \(\chi^2\) from the BALT method is roughly 10 times the reduction from the host galaxy correction \((\Delta \chi^2 \approx 5\%)\). We note though that for both SALT2 (Childress et al. 2013) and BALT, there is a remaining trend between color and Hubble residuals. If we correct the distance modulus of each SN for this trend, the mass–Hubble residual effect is decreased by \(\sim 0.01\) mag. Therefore, we conclude that accounting for color variation may weaken the trend between host galaxy properties and Hubble residuals, but this reduction alone is not large enough to explain the trend between Hubble residuals and host galaxy mass.

### 4.3. Evolution of the Color–Luminosity Relation with Redshift

Kessler et al. (2009b) found an evolution of the \(\beta\) parameter with redshift and this uncertainty was one of their largest systematic errors. Guy et al. (2010) addressed a possible \(\beta\) evolution and found that the apparent evolution was due to issues with the SALT2 light-curve fitter, partly that the color variation in the template model had been underestimated. However, even with an updated SALT2 model, Guy et al. (2010) still find a \(\beta\) evolution with redshift. Although they later argue that \(\beta\) evolution is not real using comparisons with SIFTO, they claim that neither of the SALT2 models, we assume that the peak of the model color distribution is at the \(A_V = 0\) limit. It is possible that the peak is actually more red than this blue cutoff, as suggested from the analysis of Hicken et al. (2009a). We have also assumed that residual variation in color or luminosity is Gaussian. However, Foley & Kasen (2011) show that the velocity of SNe Ia is tied to color and SNe Ia are not distributed evenly between high and low velocities. One argument against the statement that SN Ia color is consistent with the MW reddening law is that infrared observations of SNe Ia are inconsistent with this law and these SN Ia have a lower Hubble residual scatter than those observed optically. Countering this argument requires further work, although we though the amount departs from 0 by \(2.8\sigma\) in the real data (for SNLS3: \(\partial \beta / \partial z = -2.38 \pm 0.85\)). The magnitude of the \(\beta\) evolution seen here is inconsistent with that seen in Conley et al. 2011 \((\partial \beta / \partial z = 0.588 + / -0.40)\) for the combined SALT2+SIFTO analysis, which is likely due to the results of SIFTO, which suggest positive \(\beta\) evolution. We simulate with two different color variation models: the Color Smear model discussed in Section 2 and the C11 model discussed in Section 3. The difference between these two models is that the magnitude of color variation in the C11 model is higher at the blue end of the SN Ia spectral model, which is sampled at high-\(z\). We find that the C11 model better predicts \(\beta\) evolution.

We also note that when we allow a non-zero \(\partial \beta / \partial z\) for the real data, \(\beta = 4.26 \pm 0.48\), which is near the MW extinction value. This implies that color appears to follow the MW reddening law at low-\(z\) where scatter, noise, and selection effects are weaker. One other possible explanation (M11) for \(\beta\) evolution is that at higher redshifts, the color range decreases and there is less leverage from the tails of the color distribution to help determine \(\beta\). However, we find that if we reduce the color range of the sample, the observed \(\beta\) evolution becomes negligible.

### 5. Discussion

If residual scatter originates from color variation, then understanding the cause of color variation is paramount. Much focus in the last few years has been placed on the relation between host galaxy properties and Hubble residuals. Since host galaxy properties correlate with stretch and color, we stress that a correction to the distance modulus after light-curve corrections are done may not be the ideal method. This approach is analogous to observing the correlation between Hubble residuals and color, finding a property like velocity that correlates with color, and removing the bias by finding a relation between Hubble residual and velocity. A better approach to these scenarios may be to use the host galaxy or velocity information to inform the priors in which the color or stretch values are found.

We have begun to explore how to incorporate Bayesian prior information into the SALT2 light-curve fitter. The BALT approach reduces the intrinsic dispersion in the sample to nearly null, which shows the promise of this approach. Biases from introducing a Bayesian prior still need to be explored, although we reiterate that not including a prior is equivalent to using a flat prior. There are currently fitters, like BayesN (Mandel et al. 2011), that include detailed Bayesian priors already and these offer helpful guidance.

While we have shown how biases in \(\beta\) and \(w\) depend on the width and shape of the model color distribution, there are further complexities to understanding this distribution. For our dust models, we assume that the peak of the model color distribution is at the \(A_V = 0\) limit. It is possible that the peak is actually more red than this blue cutoff, as suggested from the analysis of Hicken et al. (2009a). We have also assumed that residual variation in color or luminosity is Gaussian. However, Foley & Kasen (2011) show that the velocity of SNe Ia is tied to color and SNe Ia are not distributed evenly between high and low velocities.
point to Phillips (2012), who reviews studies of infrared observations of SNe Ia and finds that the majority of the SNe with normal \((E(B-V) < 0.3)\) extinction values are consistent with the standard Galactic value of \(R_V \sim 3\).

Finally, we mention that although the focus of this paper has been on the SALT2 fitter, the biases discussed in this paper will affect any light-curve fitter that assumes that the source of residual scatter among most SNe Ia is due to luminosity variation or ignores the effects of the underlying color distribution. Another potentially fruitful path to characterizing intrinsic scatter is to reduce it through further sub-typing or discovery of additional SN parameters.

6. CONCLUSIONS

In this paper, we have explained how fitting SN Ia distances depends on assumptions about the residual scatter of SN Ia. We have also introduced a discussion of the biases due to ignorance of the model color distribution. We show that the combination of residual scatter due to color and a realistic color distribution will bias \(\beta\) by roughly \(\sim 1\) lower than its true value. We find that a model in which color is solely due to MW-like reddening along with residual color variation can explain the trend between Hubble residuals and color seen in the SNLS3, SDSS, and nearby datasets. We also argue that an empirically-observed correlation of light-curve data contains multiple degenerate parameters and further progress may stem from including astrophysical priors. We have shown one method to include knowledge of the color model as a Bayesian prior and how this approach significantly reduces the intrinsic dispersion in the sample.

Finally, the ultimate goal for SN Ia analysis is to measure cosmological parameters. We find that misattributing the source of residual scatter can bias \(w\) by as much as 4%. This amount has been significantly underestimated in the past. Further improvements in the determination of \(w\) may come from a better understanding of the residual scatter of SN Ia and the true nature of SN Ia color.

We thank Rick Kessler for many useful conversations.

APPENDIX A
SALT2MU

To determine the nuisance functions \(M_0, \alpha, \beta\), the SN sample is divided into \(>5\) equally sized redshift bins and \(M_0, \alpha, \beta\) are found in each bin to minimize the distance modulus scatter relative to a trial cosmology. This process is done by the routine SALT2mu (M11) and, for each bin, the \(\chi^2\) is minimized where

\[
\chi^2 = \sum_{n=1}^{N} \frac{[\mu_n - \mu_i(z_n, \Omega, w)]^2}{\sigma_n^2 + \sigma_i^2}
\]

\[
= \sum_{n=1}^{N} \left[ m_{\beta n} - M_0 + \alpha x_{1n} - \beta c_n - \mu_0 (z_n, \Omega, w) \right]^2 / \left( \sigma_n^2 + \sigma_i^2 \right). \tag{A1}
\]

For the \(n\)th SN, \(\sigma_n\) is the error from the light-curve fit, \(\sigma_i\) is the total residual scatter, and \(\mu_i\) is a trial cosmology given the matter density of the universe \(\Omega\) and the equation of state parameter \(w\). Since \(M_0\) is allowed to vary and it is degenerate with the cosmology, the determination of \(\alpha, \beta\) will be independent of the cosmology. This is shown in both M11 and K13. Given the best-fit \(\alpha\) and \(\beta\), and that \(\mu_i\) is the best-fit cosmology, the numerator of the expressions in Equation (A1) represents the “Hubble residuals.”

Following K13, we do not include SN-to-SN covariances in the determination of \(\alpha\) and \(\beta\) or the cosmology. As this analysis focuses on an individual systematic uncertainty, we do not include all other systematic uncertainties when finding the bias in \(\alpha, \beta, \) or \(w\). As shown in Conley et al. (2011), we expect the statistical covariances between SNe to be insignificant, as they are \(10^3 \div 10^4\) smaller than that from the systematic covariance matrix, which itself has a \(<1\sigma\) effect on \(\alpha, \beta, \) and \(w\).

APPENDIX B
COLOR DISTRIBUTION

Assuming the true color population of the SNe is a Gaussian, the distribution of the observed colors may be expressed as

\[
e^{-\left(c_{\text{obs}} \sqrt{2(\sigma_{\text{mod}}^2 + \sigma_{\text{ran}}^2)} \right)} = e^{-\left(c_{\text{mod}} \sqrt{2(\sigma_{\text{mod}}^2 + \sigma_{\text{ran}}^2)} \right)} \ast e^{-\left(c_{\text{ran}} \sqrt{2(\sigma_{\text{mod}}^2 + \sigma_{\text{ran}}^2)} \right)} = e^{-\left(c_{\text{ran}} \sqrt{2(\sigma_{\text{mod}}^2 + \sigma_{\text{ran}}^2)} \right)}. \tag{B1}
\]

Therefore, we find that

\[
\sigma_{\text{obs}}^2 = \sigma_{\text{mod}}^2 + \sigma_{\text{ran}}^2 + \sigma_{\text{ran}}^2. \tag{B2}
\]

To find the relation between values of \(\beta\) for different models of color, one must take the square of the derivative of the distance modulus (Equation (1)) with respect to \(c\). So that the distribution of \(c_{\text{obs}}\) from the SNANA Monte Carlo simulations of the SDSS and SNLS3 samples match the data, K13 retroactively derive the true color distribution of \(c_{\text{mod}}\) for both the SNLS3 and SDSS surveys. K13 find that an asymmetric Gaussian is needed to describe both the input stretch and color distribution. Here, we express the function for color and note that stretch can be defined

\[4.260 \pm 0.484, -2.378 \pm 0.847, 3.950 \pm 0.109, -1.407 \pm 0.187, 2.741 \pm 0.313, 1.551 \pm 0.617, 3.185 \pm 0.066, 0.006 \pm 0.122, 3.360 \pm 0.662, -2.287 \pm 3.093, 4.331 \pm 0.091, -4.809 \pm 0.446, 4.113 \pm 0.090, -4.888 \pm 0.430, 3.282 \pm 0.059, -0.462 \pm 0.294.

Notes. \(\beta\) and \(\delta \beta / \delta z\) are determined together given the assumption that the reduced \(\chi^2\) of the sample must be unity and that the residual scatter is attributed to the luminosity term. In the real data samples, the SNLS3 sample has 240 SNe Ia and the SDSS sample has 91 SNe Ia. In the simulation, each sample has 10,000 SNe Ia.

\[\begin{array}{|c|c|c|c|c|}
\hline
\text{Sample} & \text{Data} & \text{Sim. Color (C11)} & \text{Sim. Color (Smear)} & \text{Sim. Lum (Guy10)} \\
\hline
\text{SNLS3} & 4.260 \pm 0.484, -2.378 \pm 0.847, 3.950 \pm 0.109, -1.407 \pm 0.187, 2.741 \pm 0.313, 1.551 \pm 0.617, 3.185 \pm 0.066, 0.006 \pm 0.122, 3.360 \pm 0.662, -2.287 \pm 3.093, 4.331 \pm 0.091, -4.809 \pm 0.446, 4.113 \pm 0.090, -4.888 \pm 0.430, 3.282 \pm 0.059, -0.462 \pm 0.294 \\
\text{SDSS} & 4.260 \pm 0.484, -2.378 \pm 0.847, 3.950 \pm 0.109, -1.407 \pm 0.187, 2.741 \pm 0.313, 1.551 \pm 0.617, 3.185 \pm 0.066, 0.006 \pm 0.122, 3.360 \pm 0.662, -2.287 \pm 3.093, 4.331 \pm 0.091, -4.809 \pm 0.446, 4.113 \pm 0.090, -4.888 \pm 0.430, 3.282 \pm 0.059, -0.462 \pm 0.294 \\
\hline
\end{array}
\]
in the same manner:

\[ e^{-\frac{(\bar{c}_{\text{mod}} - \bar{c}_{\text{mod}})^2}{2\sigma_{\text{mod}}^2}} \bar{c}_{\text{mod}} < \bar{c}_{\text{mod}} \]  
(B3)

\[ e^{-\frac{(\bar{c}_{\text{mod}} - \bar{c}_{\text{mod}})^2}{2\sigma_{\text{mod}}^2}} \bar{c}_{\text{mod}} > \bar{c}_{\text{mod}}. \]  
(B4)

In Equation (B4), \( \bar{c}_{\text{mod}} \) is the peak value of the distribution, \( \sigma_{\text{mod}_{+}} \) is the standard deviation of the colors redder than the mean, and \( \sigma_{\text{mod}_{-}} \) is the standard deviation of the colors bluer than the mean.

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