Unconventional Color Superconductor

Mei Huang

Institute of High Energy Physics, Chinese Academy of Sciences,
Beijing 100049, China
E-mail: huangm@mail.ihep.ac.cn

Abstract. Superfluidity or superconductivity with mismatched Fermi momenta appears in many systems such as charge neutral dense quark matter, asymmetric nuclear matter, and in imbalanced cold atomic gases. The mismatch plays the role of breaking the Cooper pairing, and the pair-breaking state cannot be properly described in the framework of standard BCS theory. I give a brief review on recent theoretical development in understanding unconventional color superconductivity, including gapless color superconductor, the chromomagnetic instabilities and the Higgs instability in the gapless phase. I also introduce a possible new framework for describing unconventional color superconductor.

1. Introduction

The research of charge neutral cold dense quark matter has driven the color superconductivity theory into a new territory beyond standard BCS theory. The requirement of charge neutrality condition induces a substantial mismatch between the Fermi surfaces of the pairing quarks, which reduces the available phase space for Cooper pairing, and eventually breaks the Cooper-pairing state. Superfluidity or superconductivity with mismatched Fermi momenta also appears in other systems, e.g., electronic superconductor in a strong external magnetic field, asymmetric nuclear matter, and in imbalanced cold atomic systems. However, it has been an unsolved old problem how a BCS superconductivity is destroyed as the mismatch is increased.

The results based on conventional BCS framework brings us puzzles. For example, in the standard BCS framework, it was found that at moderate mismatch, homogeneous gapless superconducting phases [1][2] can be formed. However, gapless superconducting phases exhibit anti-Meissner screening effect, i.e., chromomagnetic instability [3][4], which is in contradict with the Meissner effect in standard BCS superconductivity.

I give a brief report on recent theoretical development of understanding the puzzles in g2SC phase. Firstly, I explain that the gapless phase is the result under the conventional BCS framework in mean-field approximation, then I introduce a possible proper new framework.
2. Mismatch induced gapless phase in mean field approximation

In the mean-field approximation, the order parameter of 2SC phase takes a constant and has the form of $\Delta(x) = |\Delta(x)| = \Delta$, and the free energy for $u, d$ quarks in $\beta$-equilibrium has the form [1]:

$$\Omega_M = \frac{\Delta^2}{4G_D} - \sum_A \int \frac{d^3p}{(2\pi)^3} \left[ E_A + 2T \ln \left( 1 + e^{-E_A/T} \right) \right].$$ (1)

The sum over $A$ runs over all (6 quark and 6 antiquark) quasi-particles.

With the increase of mismatch, the ground state will be in the gapless 2SC phase when $\Delta < \delta\mu$. The gapless phase is in principle a metastable Sarma state [5], i.e., the free energy is a local maximum with respect to the gap parameter $\Delta$, and one has

$$\left( \frac{\partial^2 \Omega_M}{\partial \Delta^2} \right)_{\bar{\mu}, \delta\mu} = \frac{4\mu^2}{\pi^2} \left( 1 - \frac{\delta\mu}{\sqrt{\delta\mu^2 - \Delta^2}} \right) < 0$$ (2)

in the gapless phase.

As we already knew that g2SC phase exhibits anti-Meissner screening effect or chromomagnetic instability [3], which is in contradic with the Meissner effect in standard BCS superconductivity.

3. The new theoretical framework

In recent years, there have been lots of effort trying to understand the puzzle of anti-Meissner effect, here I give a brief report from my own point of view.

In the 2SC phase, the color symmetry $G = SU(3)_c$ breaks to subgroup $H = SU(2)_c$. The full order parameter of the 2SC phase is characterized by

$$\Delta(x) = \exp \left[ i \left( \sum_{a=4}^{8} \varphi_a(x)T_a \right) \right] (0, 0, \Delta + H(x)), \quad (3)$$

where $\varphi_a(a = 4, \cdots, 8)$ are five Nambu-Goldstone diquarks describing the phase fluctuation of the order parameter, and $H(x)$ is the Higgs field describing the spatial fluctuation of the order parameter.

Expanding around the ground state: $(0, 0, \Delta)$, the free energy of the system takes the expression as $\Omega = \Omega_M + \Omega_{NG} + \Omega_H$, where $\Omega_M$ is the contribution from the mean-field approximation and was given in Eq. [1], $\Omega_{NG}$ and $\Omega_H$ are contributions from the fluctuation of Nambu-Goldstone currents and Higgs field, respectively.

3.1. Nambu-Goldsone current generation and single plane-wave state

The quadratic action of the Goldstone modes in the long wavelength limit can be written down with the aid of the Meissner masses $m_a^2$ evaluated in Ref.[3], it takes the form of

$$\Omega_{NG} = \frac{1}{2} \int d^3\vec{r} \sum_{a=1}^{8} m_a^2 \left[ \vec{A}_a^a - \frac{1}{g} \vec{\nabla} \varphi^a \right] \left[ \vec{A}_a^a - \frac{1}{g} \vec{\nabla} \varphi^a \right] + \text{higher orders} \,. \quad (4)$$
It was found that at zero temperature, with the increase of mismatch, for five gluons with $a = 4, 5, 6, 7, 8$ corresponding to broken generator of $SU(3)_c$, their Meissner screening mass squares become negative [3]. This indicates the spontaneous generation of Nambu-Goldstone currents $\sum_{a=4}^{8} \vec{A}^a \neq 0$ or gluon condensate state $\sum_{a=4}^{8} \vec{A}^a \neq 0$ [9]. The NG current state can also be interpreted as a colored Larkin-Ovchinnikov-Fulde-Ferrel (LOFF) state with the single plane-wave order parameter $\Delta(x) = \Delta \exp(i \sum_{a=4}^{8} \vec{\nabla} \phi^a \cdot \vec{x})$.

The LOFF state [10] was proposed in 1960s to describe a possible ground state of the pair-breaking state of an electronic superconductor when applying a strong external magnetic field. However, it has still not yet been confirmed experimentally in electronic superconducting systems, and it still remains being pursued after more than 40 years. Imbalanced cold atom systems offer another intriguing experimental possibility to understand how Cooper pairing is destroyed. Due to the absence of both the orbital effects and impurities, it seems very promising to search for the LOFF state in imbalanced cold atom systems. However, recent experiments [11] in imbalanced cold atom systems did not show evidence of the formation of the LOFF state, rather indicated a non-uniform state of phase separation state.

Obviously, there is something missing in our understanding of the pairing breaking state if we failed to observe the (LO)FF state in the mismatch regime where the magnetic or superfluid density instability develops.

3.2. Higgs/amplitude instability and spatial inhomogeneity

The free energy from the Higgs field can be evaluated and takes the form of

$$\Omega_H = \frac{T}{2} \sum_n \int \frac{d^3 \vec{k}}{(2\pi)^3} H^*(\vec{k}) \Pi_H(k) H(\vec{k}).$$

(5)

It was found in Ref. [12] that the self-energy of the Higgs field, $\Pi_H(k)$, becomes negative in the gapless phase when $\delta \mu > \Delta$, the same type of instability was also discussed in Refs. [13] [14].

Negative $\Pi_H(k)$ indicates the Higgs mode is unstable and will decay. The numerical results in Ref. [12] shows that $\Pi_H(k)$ reaches its minimum at a momentum, i.e., $k \simeq 4\Delta$, which indicates that a stable state may develop around this minimum, we characterize this momentum as $k_{\text{min}}$. The inverse $k_{\text{min}}^{-1}$ is the typical wavelength for the unstable mode. If mixed phase can be formed, the typical size $l$ of the 2SC bubbles should be as great as $k_{\text{min}}^{-1}$, i.e., $l \simeq k_{\text{min}}^{-1}$, which turns out to be comparable to the coherence length of 2SC, $\xi_0 \simeq \Delta_0^{-1}$. Considering that the coherence length $\xi$ of a superconductor is proportional to the inverse of the gap magnitude, i.e., $\xi \simeq \Delta^{-1}$, therefore, a rather large ratio of $k_{\text{min}}/\Delta$ means a rather small ratio of $l/\xi$. When $l/\xi < 1$, a phase separation state is more favorable.

In the system of imbalanced neutral atoms, the Higgs instability persists and induces spatial non-uniform phase separation state. This explains why imbalanced cold atom experiments did not observe LOFF state rather showed strong evidence of phase
separation. For the 2-flavor quark matter being considered, it was found in Ref. [12] that, the electric Coulomb interaction is not strong enough to compete with the Higgs instability.

4. Summary

I reviewed the recent theoretical development for describing unconventional color superconductor. This review is based on my own understanding, and I apologize if I missed some important work in this field. From my own point of view, I think with the increase of mismatch, it is very essential to consider the contribution from the phase fluctuation and the amplitude fluctuation of the order parameter. The instability from the phase part of the order parameter induces the Nambu-Goldstone currents generation and forms the single plane-wave state, and the instability from the amplitude part induces the spatial inhomogeneity and forms the mixed state. The true ground state of a mismatched pairing state should be determined by the competition of the instabilities.

Acknowledgments

I thank W. Q. Chao, I. Giannakis, D.F. Hou, H.C. Ren, I. Shovkovy, and P.F. Zhuang for collaboration. The work is supported by the Institute of High Energy Physics, Chinese Academy of Sciences (CAS), and CAS key project under grant No. KJCX3-SYW-N2.

References

[1] Huang M, Zhuang P and Chao W, 2003 Phys. Rev. D 67 065015; Shovkovy I and Huang M 2003 Phys. Lett. B 564 205; Huang M and Shovkovy I 2003 Nucl. Phys. A 729 835.
[2] Alford M, Kouvaris C and Rajagopal K 2004 Phys. Rev. Lett. 92 22200; 2005 Phys. Rev. D 71 054009.
[3] Huang M and Shovkovy I 2004 Phys. Rev. D(R) 70 051501; 2004 Phys. Rev. D 70 094030.
[4] Casalbuoni R, Gatto R, Mannarelli M, Nardulli G and Ruggieri M 2005 Phys. Lett. B 605 362; Alford M and Wang Q 2005 J. Phys. G 31 719.
[5] Sarma G 1963 J. Phys. Chem. Solids 24 1029.
[6] Hong D Preprint hep-ph/0506097.
[7] Huang M 2006 Int. J. Mod. Phys. A 21 910; 2006 Phys. Rev. D 73 045007.
[8] Kryjevski A Preprint hep-ph/0508180; Schafer T 2006 Phys. Rev. Lett. 96, 012305; Gerhold A and Schafer T 2006 Phys. Rev. D 73 125022.
[9] Gorbar E, Hashimoto M and Miransky V 2006 Phys. Lett. B 632 305.
[10] Fulde P and Ferrell R 1964 Phys. Rev. 135 A550; Larkin A and Ovchinnikov Yu 1965 Sov. Phys. JETP 20 762.
[11] Zwielein M, Schirozhek A, Schunck C and Ketterle W 2006 Science 311 492; Partridge G, Li W, Kamar R, Liao Y, and Hulet R Preprint cond-mat/0511752; Zwielein M, Schunck C, Schirozhek A, and Ketterle W 2006 Nature 442 54.
[12] Giannakis I, Hou D, Huang M and Ren H 2007 Phys. Rev. D 75 011501(R); 2007 Phys. Rev. D 75 014015.
[13] Hashimoto M 2006 Phys. Lett. B 642 93.
[14] Iida K and Fukushima K 2006 Phys. Rev. D 74 074020.