Minimizing the footprint of your laptop
(on your bedside table)

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1 Introduction

I often work on my laptop in bed. When needed, I park the laptop on the bedside table, where the computer has to share the small available space with a lamp, books, notes, and heaven knows what else. It often gets quite squeezy.

Being regularly faced with this tricky situation, it finally occurred to me to determine once and for all how to place the laptop on the bedside table so that its “footprint” - the area in which it touches the bedside table - is minimal. In this note I give the solution of this problem, using some very pretty elementary mathematics.

2 Mathematical laptops and bedside tables

We assume that both the laptop and the bedside table are rectangular, and we will refer to these rectangles as the laptop and the table. We further assume that the center of gravity of the laptop is its midpoint. Finally, without loss of generality we may assume that the laptop is 1 unit wide.\textsuperscript{1}

We are considering all placements of the laptop such that it will not topple off the table; these are exactly the placements for which the midpoint of the laptop is also a point of the table. We are then interested in determining for which of these placements the footprint of the laptop is minimal.

\textsuperscript{1}That is, the shorter side of the laptop is 1 unit in length. And, if the laptop is square then it is a unit square. Yes, yes, only a mathematician would consider the possibility of a square laptop, but bear with me. As will become clear, considering square laptops provides an elegant key to our problem.
the laptop is of minimal area; here, the footprint is the common region of the laptop and the table.

In all reasonable circumstances, the optimal answer to this problem will always resemble the arrangement in Figure 1. This optimal placement is characterized by the fact that the midpoint of the laptop coincides with one of the corners of the table and the footprint is an isosceles right triangle.

![Figure 1](image)

Figure 1: No (stable) placement of your laptop on a bedside table has a smaller footprint.

The proof is divided into two parts. First, we consider those placements for which the midpoint of the laptop coincides with one of the corners of the table: we prove that among such placements our special placement has smallest footprint area. Then, we extend our argument, proving that any placement for which the laptop midpoint is not a table corner must have a greater footprint area.

3 Balancing on a corner

We begin by considering a right-angled cross through the center of a square, as illustrated in the left diagram in Figure 2. Whatever its orientation, the cross cuts the square into four congruent pieces. This shows that if we place a unit square laptop on the corner of a sufficiently large table, its footprint will always have area 1/4, no matter how the square is oriented; see the diagram on the right.

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2 “Reasonable circumstances” means in reference to laptops and tables of relative dimensions close to those of the real items. In the nitty gritty of this note we’ll specify the exact scope of our solution, and also what happens in some unrealistic but nevertheless mathematically interesting scenarios.
Next, consider a non-square laptop with its midpoint on the corner of a large table, as in Figure 3. We regard the footprint as consisting of a blue part and a red part, as shown. Our previous argument shows that, as we rotate the laptop, the area of the blue part stays constant. On the other hand, the red part only vanishes in the special position shown on the right. We conclude that this symmetric placement of the laptop uniquely provides the footprint of least area.

These arguments required that the table be sufficiently large. How large? The arguments work as long as the rotated square never pokes off another side of the table. So, since the short side of the laptop has length 1, we only require that the shortest side of the table be at least of length $1/\sqrt{2}$; see Figure 4.
4 In the corner is best

We now want to convince ourselves that the minimal footprint must occur for one of these special placements over a table corner.

We start with a table that is at least as wide as the diagonal of the square inscribed in our laptop; see the left diagram in Figure 5. Place the laptop anywhere on the table. Now consider a cross in the middle of the laptop square, and with arms parallel to the table sides.

Figure 5: If the table contains the square highlighted on the left, then at least one of the quarters of the square on the right is contained in the footprint of the laptop.

As we saw above, the cross cuts the square into four congruent pieces. Furthermore, wherever the laptop is placed and however it is oriented, at least one of these congruent pieces will be part of the footprint: this is a consequence of our assumption on the table size. Finally, unless the midpoint is over a corner of the table, this quarter-square region clearly cannot be the full footprint.

Putting everything together, we can therefore guarantee that our symmetric corner arrangement is optimal if the table is at least as large as the square table in Figure 5. This square table has side length $\sqrt{2}$. 
By refining the arguments above, we now want to show that our solution holds for any table that is at least 1 unit wide. Since our laptop is also 1 unit wide, this probably takes care of most real life laptop balancing problems.

Begin with a circle inscribed in the laptop square, and with the red and the green regions within, as in Figure 6. The regions are mirror images, and are arranged to each have area $1/4$. Note that if the laptop is rotated around its midpoint, either fixed region remains within the laptop.

![Figure 6: Both the red and the green regions have the critical area of 1/4.](image)

Now place the laptop on the table with some orientation. Suppose that the laptop footprint contains a red or a green region, or such a region rotated by 90, 180, or 270 degrees; see Figure 7. Then it is immediate that the footprint area for the laptop in that position is greater than $1/4$.

![Figure 7: The footprint area is at least 1/4, for the laptop midpoint in either the blue or brown region.](image)

In fact the footprint may not contain such a region. However, this will be the case unless the laptop midpoint is close to a table corner, in one of the little blue squares pictured in Figure 7. On the other hand, if the midpoint is in a blue square then the footprint will contain one of the original quarter-squares of area $1/4$; see the diagram on the right side of Figure 7.

At this point we summarize what we have discovered so far.
Theorem 1 Consider a laptop that is 1 unit wide and a table that is at least 1 unit wide. If the laptop is not a square, then the placement of the laptop on the table that gives the smallest footprint is shown in Figure 7. If the laptop is a square, then the minimal area footprints are for placements for which the midpoint of the laptop coincides with a corner of the table.

5 Odds and ends

What if you are the unlucky owner of a really small bedside table? First of all, it is usually not difficult to determine the best placement for a specific laptop/table combination. To get a feel for this, and for what to expect in general, consider Figure 8, where we balance a laptop on square tables of different sizes.

![Figure 8: Balancing a laptop on the corners of squares of different sizes.](image)

Here are some simple observations, applicable both to square and non-square tables:

1. If your table is really tiny, the footprint will always be the whole table, no matter where the laptop is placed. This will be the case if the table diagonal is no longer than half the width of the laptop.

2. Suppose that the (square or non-square) table diagonal is just a little bit longer than half the width of the laptop, ensuring that if part of the table sticks out from underneath the laptop, then this part is a triangle cut off one of the corners of the table. In this case, if a table corner is sticking out and if the laptop midpoint is not at the opposite corner, then it is easy to see that simply translating the laptop to this opposite corner will lower the footprint area. Consequently, the minimal footprint will correspond to one of these special placements. For a square laptop, the minimal footprint occurs when the protruding triangle is isosceles. This is not terribly surprising. What
may be surprising is that this is not at all obvious to prove; a descent into the land of nitty gritty seems unavoidable.

For non-square tables, the optimal placement will not necessarily correspond to an isosceles table corner sticking out. To see this, consider a very thin table. Then the only way a corner can stick out is if the table diagonal is almost perpendicular to the long side of the laptop. This precludes an isosceles triangle part of the table sticking out; see Figure 9.

3. From here on things get even more complicated: all the problems that we mentioned for the last scenario plus many-sided, odd-shaped footprints, no easy way to see why the best placement should be among the placements for which the midpoint of the laptop is one of the corners, etc.

4. From here on our theorem applies.

We end this note with some challenges for the interested reader (in likely order of difficulty):

- Extend our theorem to include all tables that are at least $1/\sqrt{2}$ wide. (The table shown in Figure 4 has these dimensions.)
- Turn Scenario 2 discussed above into a theorem (are there pretty proofs?).
- Prove the Ultimate Laptop Balancing Theorem, that includes everything that your lazy author did and did not cover in this note: arbitrary location of the center of gravity, starshaped laptops and jellyfish-shaped tables, higher-dimensional tables and laptops, etc.

Have Fun, and Good Luck!
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