Constraining the vector interaction strength of QCD

Jan Steinheimer and Stefan Schramm
Frankfurt Institute for Advanced Studies, Johann Wolfgang Goethe Universität, Ruth-Moufang-Str. 1, 60438 Frankfurt am Main, Germany

Abstract
We show how repulsive interactions of deconfined quarks as well as confined hadrons can be constrained in a straightforward way by model comparisons of baryon number susceptibilities with lattice QCD results. We also discuss implications for earlier constraints, extracted from the curvature of the transition line of QCD and compact star observables. Our results clearly point to a strong vector repulsion in the hadronic phase and near-zero repulsion in the deconfined phase.

Keywords: QCD vector interaction strength, baryon number susceptibilities

1. Introduction

Quantum Chromo Dynamics (QCD) at extreme conditions of temperature and/or density is a central topic of many experimental and theoretical investigations. Especially the transition from the hadronic to the quark-gluon phase is a key region for studies of hot matter in ultra-relativistic heavy-ion collisions as well as for the stability of hybrid stars consisting of hadrons and quarks. Here, the role of the repulsive vector interaction in QCD has become a much discussed topic in recent literature. It is not only important for the general understanding of the strong interaction but also has concrete implications for effective models of QCD regarding the location of the hadron-quark transition [1, 2, 3]. In heavy-ion phenomenology the vector repulsion is a possible explanation for the observed splitting in particle and anti-particle azimuthal momentum asymmetry [4], whereas in nuclear astrophysics the possibility of stars with quark matter core depends strongly on the existence of a quark vector repulsion [5, 6]. It is therefore of great interest to possibly constrain the strength of the hadronic and quark repulsive interaction from QCD itself. In this work we propose that such an independent determination has been provided by the conserved charge susceptibilities evaluated with lattice QCD at \( \mu_B = 0 \). These susceptibilities quantify the fluctuations of the conserved charges of QCD, in particular the net baryon number. These susceptibilities can provide constraints on the interaction of particles, as a repulsive interaction of baryons would have direct impact on the magnitude of the fluctuations. In the following we will describe how we combine a hadronic and an effective quark model in order to construct an equation of state that gives a correct description of the two phases of QCD, the confined and deconfined phase connected by a smooth cross-over. We will then use this combined equation of state to investigate the sensitivity of the baryon number susceptibilities on possible hadronic and quark repulsive interactions. In particular we want to understand the role of the repulsive interaction in the two separate phases and relate our results to recent attempts to constrain the vector interaction strength [7, 8, 9].

2. The Combined Equation of State

In the hadronic phase we use the parity doublet model for the baryon octet and add all hadronic resonances, with masses up to 2.2 GeV, to the thermodynamic potential \( \Omega_{\text{tot}} \), in order to correctly describe the QCD thermodynamics below \( T_c \). In the parity doublet model positive and negative parity states of the baryons are grouped in doublets. Their masses are generated by a coupling to the chiral field \( \sigma \). The effective masses of the nucleon and its chiral partner then become:

\[
m^*_\pm = \sqrt{(g^{(1)}_\sigma \sigma)^2 + m^2_0 \pm g^{(2)}_\sigma \sigma},
\]

where \( g^{(1)}_\sigma \) and \( g^{(2)}_\sigma \) are the scalar coupling parameters of the model. In the chirally restored phase, for vanishing \( \sigma \), their masses are degenerate and identical to \( m_0 \) [10, 11]. Within this approach we ensure a good description of well-known properties of nuclear matter properties by adjusting the baryonic attractive scalar and repulsive vector interaction strength.

To describe the transition from the confined hadronic phase to a deconfined quark phase we include explicitly the contributions of the quarks and gluons in the thermodynamic potential, as discussed in detail in [11]. This generates a smooth cross-over chiral and deconfinement transition at small chemical potential and high temperatures, and a first-order transition in the case of cold, dense systems.
like compact stars. The quarks and gluons are incorporated in a similar way as described in so-called Polyakov loop extended quark models \cite{12 13 14 15 16}. In our implementation we add the thermal contribution of the quarks to the thermodynamic potential $\Omega_{\text{tot}}$:

$$\Omega_q = -T \sum_{i \in Q} \frac{\gamma_i}{(2\pi)^3} \int d^3 k \ln \left( 1 + \Phi \exp \frac{E_i^q - \mu_i^q}{T} \right)$$

(1)

where we sum over all three quark flavors. $\gamma_i$ is the corresponding degeneracy factor, $\Phi$ is the Polyakov loop, $E_i^q = \sqrt{m_i^q + p^2}$ the energy and $\mu_i^q = \mu_i - g_i^Q \omega$ the effective chemical potential of the quarks. For the anti quarks we have to use the conjugate of the Polyakov loop $\Phi^*$ and $\mu_i^{q*} = -\mu_i^q$. The effective mass $m_i^* = m_0 + g_5^q \sigma + g_5^\sigma \zeta$ of the quarks is generated through a coupling to the scalar fields $\sigma$ and $\zeta$, which correspond to the non-strange and strange scalar quark condensates, respectively. Here we chose coupling values of $g_5^Q = 1.8$ and, following SU(3) relations, $g_5^\sigma = \sqrt{2} g_{q5}\sigma$ in order to enable a smooth transition between the hadronic and quark part of the EoS.

The effective potential $U(\Phi, \Phi^*, T)$, which controls the dynamics of the Polyakov-loop, is also included in the thermodynamic potential. In our approach we adopt the ansatz proposed in \cite{14}:

$$U = -\frac{1}{2} a(T) \Phi \Phi^* + b(T) \ln[1 - 6 \Phi \Phi^* + 4(\Phi^3 \Phi^{*3}) - 3(\Phi^* \Phi)^2]$$

(2)

with $a(T) = a_0 T^4 + a_1 T_0 T^3 + a_2 T_0^2 T^2$, $b(T) = b_0 T_0^2 T$. The values of the parameters $a_0 = 3.51$, $a_1 = -8.2$, $a_2 = 14.8$, $b_0 = -1.75$ and $T_0 = 156$ MeV where adjusted to get a reasonable description of the interaction measure from lattice QCD \cite{14} and have the correct asymptotic value for free massless gluons.

To suppress hadrons when deconfinement is realized we adopt an ansatz first used in \cite{18}, where we introduced an excluded volume for the hadrons (not the quarks), which very effectively removes the hadrons once the free quarks give a significant contribution to the pressure. The modified chemical potential of the $i$-th hadronic species then becomes $\mu_i^h = \mu_i^q - v_i P$, where $P$ is the sum over all partial pressures and $v_i$ the volume parameter of the hadrons. This parameter is usually set to be a fixed value, but in this work we have chosen to make it explicitly dependent on the temperature. Such a dependence has the advantage that we can better match the combined equation of state to available lattice data. All densities ($\bar{\epsilon}, \bar{\rho_i}$ and $\bar{s}_i$) have to be multiplied by a volume correction factor $f$, which is the ratio of the total volume $V$ and the reduced volume $V’$, not being occupied. The temperature dependence of the volume parameter in this work is:

$$v_i(T) = v_i \cdot f(T)$$

(3)

with a simple sigmoid function $f(T) = 1/[1 + \exp(-(T - \tau)/\delta T)]$, where $\tau = 156$ MeV corresponds to the transition temperature and $\delta T = 8$ MeV is the width of the transition. The asymptotic excluded volume $v_0$ corresponds to a hadronic radius of $r = 0.84$ fm. Such a dependence leads to a negligible correction at low temperature, as suggested from lattice results, and a strong suppression of hadrons in the deconfined phase. The physical interpretation of such a large volume can be understood as an expected significant phase space broadening of the hadronic states around $T = \tau$.

By construction of the Polyakov Loop potential, one usually observes an appearance of free quarks even at temperatures considerably lower than $T_{PC}$. Even though their contribution to the thermodynamic quantities is very small compared to the hadronic contribution, we suppress the quarks in the confined phase in order to study the separate impact of hadrons and quarks on the susceptibilities and phase transition more clearly. To achieve this we also introduce an explicitly temperature-dependent mass term $\delta m_h^0$, which is added to the quark mass $m_q^0$:

$$\delta m_h^0(T) = m_0 \cdot [1 - f(T)]$$

(4)

with corresponding values of $m_0 = 400$ MeV, $\tau = 130$ MeV and $\delta T = 3$ MeV. Such a dependence essentially suppresses any contribution of the quarks below a temperature of $T \approx 130$ MeV. The resulting interaction measure $\langle (\epsilon - 3p)/T^4 \rangle$ is shown in figure 1 and compared with available lattice data. By adjusting the above mentioned parameters we obtain a very good description of the Interaction measure.

3. Results

Because lattice QCD suffers from the sign problem it is difficult to compute the QCD phase structure and thermodynamics at non-zero baryochemical potential $\mu_B$. It
is however possible to infer information on the thermodynamics of QCD at small values of $\mu_B/T$ through a Taylor expansion of lattice results at $\mu_B = 0$ in terms of the chemical potential $\mu$. In the Taylor expansion of the pressure $p = -\Omega$, the coefficients $c_n^B$, which can be related to the baryon number susceptibilities $\chi_n^B$, follow from:

$$
\chi_n^B/T^2 = n! c_n^B(T) = \frac{\partial^n (p(T, \mu_B))/T^n}{\partial (\mu_B/T)^n}
$$

(5)

for $\mu_B = \mu_S = \mu_Q = 0$. As $p(T, \mu_B)$ also depends on the value of the vector field $\omega(T, \mu_B)$ explicitly one can easily see that the susceptibilities have contributions which depend on the derivatives of this field $\partial^n \omega(T, \mu_B)/(\partial \mu_B)^n \neq 0$. It is now interesting and instructive to investigate how large these contributions are and if one can use them to constrain $\omega(T, \mu_B)$ and subsequently $g_V$. In figure 2 we show our results for the baryon number susceptibility as a function of the temperature at $\mu_B = 0$. As for the interaction measure we again obtain a good description of the lattice data, over the whole temperature range, from our combined EoS. The most interesting feature of this figure is the strong dependence of the second-order baryon number susceptibility on the value of the free quark repulsive interaction $g_V^Q$. Below the pseudocritical temperature $T_{PC}$ we observe hardly any change in $\chi_2^B$ due to the repulsive hadronic interaction strength $g_V^B$, but a strong decrease in $\chi_2^B$ above $T_{PC}$ due to a quark repulsive coupling $g_V^Q$.

A similar picture can be draw from figure 3 where the ratio of the fourth order over the second order baryon number susceptibilities is shown. Below $T_{PC}$, in the hadronic phase, we observe only a very small dependence of the susceptibility on the hadronic repulsive interaction strength, even though the dependence appears somewhat stronger for $\chi_4^B/\chi_2^B$ than for $\chi_2^B$. Such a weak dependence of the susceptibilities is understandable when we recall which hadronic degrees of freedom contribute to the susceptibilities. These are, in the case of the baryon number, dominantly nucleons and heavier baryons. Because these hadrons have a large mass, their density, and therefore any repulsive force, will only be significant at very large chemical potentials, comparable to their mass. At such high chemicals potentials, the lower-order susceptibilities will not be the relevant contributors to a Taylor expansion, but rather higher-order terms. To understand the influence of the repulsive interactions of hadrons one therefore would have to evaluate susceptibilities of even higher order. Because the quarks have significantly smaller masses, once chiral symmetry is restored, they contribute strongly also to lower orders of the baryon number susceptibility which allows for a much stricter constraint on the quark repulsive interaction strength. In the deconfined phase, for $T > T_{PC}$ we again observe a significant deviation of the calculated values of $\chi_4^B/\chi_2^B$ from lattice results, whenever we assume a repulsive interaction between the free quarks. As the ratio decreases by a factor of $1/2$ above $T_{PC}$ we can conclude that the relative contribution of the repulsive interaction to $\chi_4^B$ is considerably larger than for $\chi_2^B$.

In several publications (e.g. [1, 2, 3, 5, 6]) it has been argued that a non-zero repulsive quark vector coupling is required by constraining the interior of compact stars and the phase structure of constituent quark mean field models. For example one can show that the newly measured
maximum mass of compact stars of 2 solar masses can only be accommodated for, if one either assumes a very small or no quark content for these stars, or introduces a quite sizable repulsive interaction for quarks. For isospin symmetric matter, it has been shown that constituent-quark based models like the PNJL and PQM model can only accommodate for a "nuclear ground" state (in these models this is approximated by a constituent-quark saturated state) if the quarks have a finite vector interaction strength. We therefore have to ask the question if our results are in contradiction to these earlier attempts on constraining the quark repulsive interaction.

To give a possible answer, in figure 4 we show the behavior of the normalized chiral condensate $\sigma/\sigma_0$, from our quark part of the equation of state, as a function of baryochemical potential, for an arbitrarily fixed temperature $T = 140$ MeV relatively close to, but below, $T_c$. Here the red solid line represents the result for vanishing quark vector coupling. We also show the same curve with finite quark vector coupling as black dashed line. We observe that the transition for finite vector couplings is moved to larger chemical potentials, i.e. the curvature of the transition is decreased, as required by \[1, 2, 3\]. This apparent contradiction with our result, stating that there should be no quark repulsive interaction, can be explained in a very simple way. In PNJL and PQM type models, only one type of vector coupling strength exists, which is the same for light "unconfined" quarks above the transition as well as possible three-quark states that appear in the confined phase. In reality of course this might not hold true. We know that there should be no quarks in the confined phase and we also have strong indications for a considerable repulsive hadronic interaction. To isolate the effect of a possible quark vector interaction, we introduce a chemical dependent quark vector coupling strength $g_V^Q(\mu_B)$ in a schematic way, which we can define such that it disappears at the pseudo critical line. For our result in figure 4 this would correspond to:

$$g_V^Q(\mu_B) = g_V^Q(\mu_B = 0) \cdot (1 + \exp(\mu_B - \mu_B^{PC})/\delta_\mu)^{-1}$$

where $\mu_B^{PC}$ is the pseudo-critical chemical potential for a constant $g_V^Q = g_{q\pi}$, and $\delta_\mu = 10$ MeV. The resulting curve of the normalized chiral condensate is shown as short dashed blue line in figure 4. One can clearly see that we obtain a larger pseudo-critical chemical potential, comparable to that with large quark vector repulsion, even though the repulsive strength is vanishing in the 'deconfined' phase. The shift in $T_{PC}$ is therefore determined by the behavior of the interacting matter below the pseudo-critical temperature. In other words the large repulsive vector interaction, required in the PQM and PNJL models might be simply a result of the unsatisfactory description of the confined, or hadronic, phase.

4. Summary

We have shown that lattice results for the baryon number susceptibilities can be used, even to lowest order, to constrain the repulsive vector interaction strength of quarks in the deconfined phase. We find that only a nearly vanishing strength is supported by lattice QCD data. Even a small vector coupling would lead to a systematic deviation of the baryon number susceptibilities, i.e. a maximum as function of temperature at $\mu_B = 0$. Such a behavior is not observed, even when susceptibilities are calculated to very high temperatures [22]. Concerning the repulsive hadronic interaction we find that, due to the large mass of the baryonic hadrons, the lowest order susceptibilities show only a very weak dependence and are not useful to constrain the hadronic repulsive interactions. We also show that earlier constraints on the quark repulsive coupling, using the curvature of the transition line and compact star masses are not as strict as they claim to be. Both can also be accommodated for if one takes into account a realistic repulsive interaction in the confined phase only. For example the curvature of the transition curve is then sensitive to the repulsive interaction in the confined phase and not in the deconfined phase. Also compact stars with large masses can be accommodated for with stiff hadronic equations of state, due to repulsive forces.

Concluding, we believe that the results shown represent a considerable step forward in the understanding of the interactions of deconfined quarks. Furthermore they present a strict set of constraints for effective models of QCD.
5. Acknowledgments

We wish to acknowledge stimulating discussion with Volker Koch. This work was supported by GSI and the Hessian initiative for excellence (LOEWE) through the Helmholtz International Center for FAIR (HIC for FAIR).

References

[1] N. M. Bratovic, T. Hatsuda and W. Weise, Phys. Lett. B 719, 131 (2013)
[2] G. A. Contrera, A. G. Grunfeld and D. B. Blaschke, arXiv:1207.4890 [hep-ph].
[3] O. Lourenco, M. Dutra, T. Frederico, A. Delfino and M. Malheiro, Phys. Rev. D 85, 097504 (2012)
[4] T. Song, S. Plumari, V. Greco, C. M. Ko and F. Li, arXiv:1211.5511 [nucl-th].
[5] T. Klahn, D. Blaschke and R. Lastowiecki, Acta Phys. Polon. Supp. B 5, 757 (2012)
[6] T. Klahn, D. B. Blaschke and R. Lastowiecki, Phys. Rev. D 88, 085001 (2013)
[7] T. Kunihiro, Phys. Lett. B 271, 305 (1991).
[8] L. Ferroni and V. Koch, Phys. Rev. C 83, 045205 (2011)
[9] J. Steinheimer and S. Schramm, Phys. Lett. B 696, 257 (2011)
[10] V. Dexheimer, S. Schramm and D. Zschiesche, Phys. Rev. C 77, 025803 (2008)
[11] J. Steinheimer, S. Schramm and H. Stocker, Phys. Rev. C 84, 045208 (2011)
[12] K. Fukushima, Phys. Lett. B 591, 277 (2004)
[13] K. Fukushima, Phys. Rev. D 77, 114028 (2008) [Erratum-ibid. D 78, 039902 (2008)]
[14] C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D 73, 014019 (2006)
[15] S. Roessner, C. Ratti and W. Weise, Phys. Rev. D 75, 034007 (2007)
[16] B. J. Schaefer, J. M. Pawlowski and J. Wambach, Phys. Rev. D 76, 074023 (2007)
[17] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, arXiv:1309.5258 [hep-lat].
[18] J. Steinheimer, S. Schramm and H. Stocker, J. Phys. G38, 035001 (2011)
[19] C. R. Allton et al., Phys. Rev. D 66, 074507 (2002)
[20] S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti and K. Szabo, JHEP 1201, 138 (2012)
[21] R. Bellwied, S. Borsanyi, Z. Fodor, S. DKatz and C. Ratti, Phys. Rev. Lett. 111, 202002 (2013)
[22] A. Bazavov, H. -T. Ding, P. Hegde, F. Karsch, C. Miao, S. Mukherjee, P. Petreczky and C. Schmidt et al., arXiv:1309.2317 [hep-lat].