Constraints from solar and reactor neutrinos on unparticle long-range forces

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Abstract. We have investigated the impact of long-range forces induced by unparticle operators of scalar, vector and tensor nature coupled to fermions in the interpretation of solar neutrinos and KamLAND data. If the unparticle couplings to the neutrinos are mildly non-universal, such long-range forces will not factorize out in the neutrino flavour evolution. As a consequence large deviations from the observed standard matter-induced oscillation pattern for solar neutrinos would be generated. In this case, severe limits can be set on the infrared fixed point scale, $\Lambda_u$, and the new physics scale, $M$, as a function of the ultraviolet ($d_{UV}$) and anomalous ($d$) dimension of the unparticle operator. For a scalar unparticle, for instance, assuming the non-universality of the lepton couplings to unparticles to be of the order of a few per mil we find that, for $d_{UV} = 3$ and $d = 1.1$, $M$ is constrained to be $M > \mathcal{O}(10^9)$ TeV ($M > \mathcal{O}(10^{10})$ TeV) if $\Lambda_u = 1$ TeV ($10$ TeV). For given values of $\Lambda_u$ and $d$, the corresponding bounds on $M$ for vector (tensor) unparticles are $\sim 100$ ($\sim 3/\sqrt{\Lambda_u}$/TeV) times those for the scalar case. Conversely,
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these results can be translated into severe constraints on universality violation of the fermion couplings to unparticle operators with scales which can be accessible at future colliders.

**Keywords:** solar and atmospheric neutrinos, neutrino properties

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## 1. Introduction

We are accustomed to describing fundamental interactions in Nature in terms of quantum fields which manifest themselves as particles. Our most successful description of all the available experimental data today is given by the Standard Model (SM) which is based on such quantum fields. Extensions of the Standard Model, such as supersymmetric, grand-unified, or extradimensional models, also rely on these ideas and give rise to a plethora of new particles that may be discovered soon at the LHC. However, other types of quantum fields may be present in Nature.

Recently, Georgi [1, 2] proposed a novel scenario where no new particle states emerge but where scale invariant fields with non-trivial infrared (IR) fixed point can be part of physics above the TeV scale. If these fields couple to SM operators they can give rise to effective interactions at lower energies that may have impact on particle phenomenology. He denominated this scale invariant matter coupled to SM fields at lower energies unparticle, as it is not constrained by a dispersion relation like normal particles.

Since then there have been a number of papers in the literature devoted to mapping the unusual observable consequences of unparticles to experiments. QED bounds to unparticle interactions have been obtained in [3]. Signals of unparticles have been studied in past and future collider experiments [4, 5]. Possible unparticle contributions to $CP$ violation [6], deep inelastic scattering [7], lepton flavour violating processes [8] as well as to hadron mixing and decay [9], neutrino interactions [10, 11] and nucleon decay [12] have been investigated. Several cosmological and astrophysical bounds have also been devised [13–15].

Another striking spin-off of the unparticle idea is the advent of long-range forces operating at macroscopic distances and generally governed by a non-integral power law.
This unique behaviour was first pointed out in [16], in connection with microscopic spin-dependent interactions between electrons and has also been investigated in connection with deviations of the Newtonian gravitational inverse squared law mediated by a tensor-like ungravity interaction [17]. Limits on vector-like unparticle interactions coupled to baryon and lepton flavour numbers were established in [18] by comparing with results from torsion-balance experiments.

Neutrino oscillation has been shown to be a very powerful tool in constraining new dynamics. Being an interference phenomenon it can be sensitive to very feeble interactions as their effect enters linearly in the neutrino propagation. Furthermore the effect of new fermionic interactions can be enhanced when a neutrino travels in regions densely populated by those fermions. In this way it has been shown that neutrino oscillation data can place stringent bounds on long-range forces coupled to lepton flavour numbers [26]–[28]. Therefore it is expected that they can provide very strong limits also on the couplings of unparticles to fermionic currents.

Assuming that scale invariance is not broken in the IR we derive in this paper, from solar and reactor neutrino experimental data, limits on scalar, vector and tensor unparticle operators which mediate long-range interactions through their couplings to fermionic flavour-conserving currents. In section 2 we briefly present the formalism of unparticle dynamics relevant to the present study. We present the effects of the induced long-range forces in solar neutrino oscillations in section 3. We derive the bounds from the non-observation of such long-range effects from the global analysis of solar and KamLAND data in section 4 and in the final section we discuss our conclusions.

2. Unparticle formalism in brief

We will consider hypothetical fields of a hidden sector of a theory with a non-trivial IR fixed point which can interact with SM fields at very high energies by the exchange of a particle with a large mass $M$ as proposed in [1]. So the ultraviolet (UV) couplings of these hidden sector operators $O_{UV}$ of dimension $d_{UV}$ with SM operators $O_{SM}$ of dimension $d_{SM}$ take the form

$$\frac{1}{M^{d_{UV}+d_{SM}-4}} O_{UV} O_{SM}. \tag{1}$$

Scale invariance emerges at a lower energy scale $\Lambda_u$. As the hidden sector runs towards the IR fixed point, these couplings can be so to speak *sequestered*; their scaling dimension is increased by the anomalous dimension $d$ from the hidden sector dynamics. From this point on, the hidden sector operator matches onto an unparticle operator $O_U$ whose interactions with the SM operators are described by the non-renormalizable effective Lagrangian

$$C_u \frac{\Lambda_u^{d_{UV}-d}}{M^{d_{UV}+d_{SM}-4}} O_{SM} O_U, \tag{2}$$

where $C_u$ is a dimensionless constant of order 1.

Our starting point will be the Lagrangian which contains the effective flavour diagonal interactions which satisfy the SM gauge symmetry, between scalar, vector and tensor unparticles and the SM fermions $f$:

$$\mathcal{L} = - \sum_f \frac{\lambda_S^f}{\Lambda_u^{d-1}} \bar{f} f O_U - \sum_f \frac{\lambda_V^f}{\Lambda_u^{d-1}} \bar{f} \gamma_\mu f O_U^\mu - \sum_f \frac{\lambda_T^f}{2 \Lambda_u^{d}} (\bar{f} \gamma_\mu \partial_\nu f - \partial_\nu \bar{f} \gamma^\mu f) O_U^{\mu \nu}, \tag{3}$$
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where the unparticle operators are taken to be Hermitian. Additional four-fermion contact interactions could also arise as discussed in [19]. In the absence of cancellations they will add up to the effects discussed here making the unparticle effects larger. In what follows we will conservatively neglect the contributions from contact interactions.

The natural size of the $\lambda_{S,V,T}^f$ coefficients is

$$\lambda_{S,V,T}^f = C_{S,V,T}^f \left( \frac{\Lambda_u}{M} \right)^{d_{UV}-1}, \quad (4)$$

where the $C_{S,T,V}^f$ are again of order 1.

Scale invariance determines the propagation properties of these operators up to a normalization factor. Using the same convention as in [2,4], including the corrections given in [19], we obtain that the propagators corresponding to scalar, vector and tensor unparticle operators are correspondingly

$$\Delta(P^2) = \frac{A_d}{2\sin(d\pi)}(-P^2)^{d-2}, \quad (5)$$

$$[\Delta(P^2)]_{\mu\nu} = \Delta(P^2) \pi_{\mu\nu}, \quad (6)$$

and $$[\Delta(P^2)]_{\mu\nu,\rho\sigma} = \Delta(P^2) T_{\mu\nu,\rho\sigma}, \quad (7)$$

$$\pi_{\mu\nu} = -g_{\mu\nu} + aP_{\mu}P_{\nu}, \quad (8)$$

where $a = 1$ for transverse $O^\mu_U$ and $a = (2(d-2))/(d-1)$ in conformal field theories [19].

For transverse and traceless $O^\mu_U$ operators

$$T_{\mu\nu,\rho\sigma} = \frac{1}{2}\left\{ \pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\nu\rho} - \frac{2}{3} \pi_{\mu\nu}\pi_{\rho\sigma} \right\}, \quad (9)$$

while in conformal field theories

$$T_{\mu\nu,\rho\sigma} = \frac{1}{2}\left( g_{\mu\rho}g_{\nu\sigma} + \mu \leftrightarrow \nu \right) + \left[ \frac{4 - d(d + 1)}{2d(d - 1)} \right] g_{\mu\nu}g_{\rho\sigma}$$

$$- \frac{2(d - 2)}{d} \left( g_{\mu\rho}\frac{k_{\nu}k_{\sigma}}{k^2} + g_{\mu\sigma}\frac{k_{\nu}k_{\rho}}{k^2} + \mu \leftrightarrow \nu \right)$$

$$+ \frac{4(d - 2)}{d(d - 1)} \left( g_{\mu\nu}\frac{k_{\rho}k_{\sigma}}{k^2} + g_{\rho\sigma}\frac{k_{\mu}k_{\nu}}{k^2} \right)$$

$$+ \frac{8(d - 2)(d - 3)}{d(d - 1)} \frac{k_{\mu}k_{\nu}k_{\rho}k_{\sigma}}{(k^2)^2} \right]. \quad (10)$$

In the equations above

$$A_d = \frac{16\pi^2}{(2\pi)^{2d}} \frac{\Gamma(d + 1/2)}{\Gamma(d - 1)\Gamma(2d)}. \quad (11)$$

These massless unparticle operators induce long-range forces between fermions. In particular, the effective Lagrangian describing the long-range force generated by a static distribution of fermions $f$ with number density $n_f(r)$, assumed to be spherically symmetric.
and of radius $R$, on a relativistic fermion $f'$ is given by

$$\mathcal{L} = \left[ C^L_{S} C^L_{S} \frac{f}{4\pi} \bar{f} \gamma^0 f' - C^L_{V} C^L_{V} \frac{f}{4\pi} \bar{f} \gamma^0 f' + C^L_{T} C^L_{T} \frac{m f E'}{4\pi} \Lambda_3^2 B \bar{f} \gamma^0 f' \right] W_f(r),$$

(12)

where $B = 2/3$ for a transverse and traceless unparticle tensor operator, while in conformal field theories $B = 1 - ([4 - d(d + 1)]/(4d(d - 1))$. The potential function, irrespectively of the Lorentz structure of the unparticle operator, can be shown to have a universal form as a function of the distance $r$ from the centre of the matter distribution.

$$W_f(r = xR) = \left( \frac{\Lambda_u}{M} \right)^{2(d_{UV} - 1)} \frac{2}{\pi^{2(d - 1)}} \frac{\Gamma[d + 1/2] \Gamma[d - 1/2]}{\Gamma[2d]} \frac{1}{(R \Lambda_u)^2(d - 1)} \frac{1}{r^3} \left\{ \begin{array}{ll}
\frac{1}{3 - 2d} \int_0^1 y n_f(y) \left[ (x + y)^{3-2d} - |x-y|^{3-2d} \right] \, dy & \text{if } d \neq \frac{3}{2} \\
\frac{1}{2} \int_0^1 y n_f(y) \ln \left[ \frac{(x + y)^2}{(x-y)^2} \right] & \text{if } d = \frac{3}{2}.
\end{array} \right.$$  

(13)

For $d \to 1$ one recovers the Coulomb-type potential characteristic of a massless particle exchange. This potential function is always finite for $d < 2$ whereas for $d > 2$ there are infinite contributions that appear when $x = y$. In order to obtain a finite answer for $d > 2$ this infinite contribution has to be cancelled out by the introduction of some counterterm. In principle, this could be done by means of the effective four-fermion contact interactions whose couplings gets renormalized. Technically this is equivalent to removing the point $x = y$ from the integration in equation (13). In principle a finite contact contribution to the potential can remain. As mentioned above, in what follows we will neglect this contribution. In practice we will only be quantitatively discussing the results for $d < 2$, since, as we will see below, the effects become very feeble for larger dimensions.

If the theory is not only scale invariant but also conformally invariant, the dimension $d$ of the unparticle operator is bounded from unitarity constraints [19]–[21]. For scalar, vector and symmetric tensor operators, $d > 1, 3$ and 4, respectively. We will relax this constraint in our phenomenological analysis.

One must also notice that the Higgs coupling to scalar unparticles generally breaks scale invariance at a scale close to the electroweak scale [22]–[25]. Therefore for the scalar unparticle effects to be relevant at solar and reactor energies ($\sim$MeV) discussed here, one must assume the Higgs–scalar unparticle couplings to be suppressed. For vector and tensor unparticles, the Higgs only couples to higher dimension operators which relaxes the bound on the scale of scale invariance breaking.

### 3. Unparticle effects in neutrino oscillations in the Sun

Under the assumptions discussed in the previous section, unparticles mediate a long-range force coupled to fermions in the way described by equation (12). Such force will affect neutrino oscillations in matter in a similar fashion to the leptonic forces discussed in [26].

In our description of the effect of unparticles in the evolution of neutrinos in matter we have to take into account that conventional mass-induced neutrino oscillations describe very well atmospheric [37], K2K [38], MINOS [39], solar [29]–[34] and reactor [35,40]...
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neutrino data as long as all three neutrino flavours participate in oscillations [41]. The mixing among neutrino mass eigenstates and flavour eigenstates is encoded in the 3 × 3 lepton mixing matrix that in the standard parametrization takes the form

\[
U = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix} \begin{pmatrix}
c_{13} & 0 & s_{13}e^{i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{-i\delta} & 0 & c_{13}
\end{pmatrix} \begin{pmatrix}
c_{21} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

(14)

where \(c_{ij} \equiv \cos \theta_{ij}\) and \(s_{ij} \equiv \sin \theta_{ij}\).

The neutrino oscillation data are consistent with the following hierarchy of the neutrino mass squared differences:

\[
\Delta m_{21}^2 = \Delta m_{31}^2 \ll |\Delta m_{32}^2| \simeq |\Delta m_{21}^2| = \Delta m_{\text{atm}}^2.
\]

(15)

Recent KamLAND [35] and MINOS [39] results lead to \(\Delta m_{21}^2/|\Delta m_{31}^2| \approx 0.03\) [41], so for KamLAND and solar neutrinos, the atmospheric neutrino oscillation scale contributions are completely averaged and the interpretation of these data in the three-neutrino oscillation framework depends basically on \(\Delta m_{21}^2, \theta_{12}\) and \(\theta_{13}\). In contrast, atmospheric and long baseline neutrino oscillation experiments are driven mostly by \(\Delta m_{21}^2, \theta_{23}\) and \(\theta_{13}\). Moreover, the non-observation of oscillations in the CHOOZ reactor experiment [40] implies that \(\theta_{13}\), the mixing angle connecting solar and atmospheric neutrino oscillation channels, must be very small, i.e., \(\sin^2 \theta_{13} \leq 0.041\) at 3σ [41]. These considerations make that the 3ν oscillations effectively factorize into 2ν oscillations of the two different subsystems: solar plus reactor, and atmospheric plus long baseline.

Because the new couplings are flavour diagonal, their effects in the evolution of atmospheric neutrinos do not change the hierarchy (15) [28]. Thus the 2ν oscillation factorization still holds and as long as we neglect the small \(\theta_{13}\) we can perform the analysis of solar and reactor neutrinos in an effective two-neutrino mixing scenario. In this framework, one can easily read from equation (12) the effect of the long-range unparticle force in the evolution of solar neutrinos as

\[
i \frac{d}{dr} \begin{pmatrix}
\nu_e \\
\nu_a
\end{pmatrix} = \left\{ \frac{1}{2E_{\nu}} \left[ V_{\theta_{12}} \begin{pmatrix}
m_1 \\
0 \\
0
\end{pmatrix} U_{\theta_{12}}^\dagger - \begin{pmatrix}
M_{\text{UNP}}(r) & 0 \\
0 & 0
\end{pmatrix} \right]^2 \\
+ \begin{pmatrix}
V_{\text{CC}}(r) + V_{\text{UNP}}(r) & 0 \\
0 & 0
\end{pmatrix} \right\} \begin{pmatrix}
\nu_e \\
\nu_a
\end{pmatrix},
\]

(16)

where \(E_{\nu}\) is the neutrino energy, \(m_{1,2}\) are the neutrino masses in vacuum, \(U_{\theta_{12}}\) is the matrix of mixing between neutrino flavour and the vacuum mass eigenstates, and \(\nu_a = c_{23}\nu_\mu + s_{23}\nu_\tau\). \(V_{\text{CC}}(r) = \sqrt{2}G_F n_e(r)\) is the Mikheyev–Smirnov–Wolfenstein (MSW) matter potential.

In equation (16) \(M_{\text{UNP}}\) and \(V_{\text{UNP}}(r)\) depend on the Lorentz structure of the unparticle operator. The function \(W\) will enter either as an extra mass term (scalar unparticle) or as an addition to the MSW potential (vector or tensor unparticle). For scalar unparticles,

\[
M_{\text{UNP}} \equiv M_S(r) = \frac{C_S^e(C_S^{\nu e} - C_S^{\nu a})}{4\pi} W(r) \simeq \alpha_S W(r), \quad V_{\text{UNP}}(r) = 0,
\]

(17)

while for vector unparticles,

\[
V_{\text{UNP}}(r) \equiv V_V(r) = \frac{C_V^e(C_V^{\nu e} - C_V^{\nu a})}{4\pi} W(r) \simeq \alpha_V W(r), \quad M_{\text{UNP}} = 0,
\]

(18)
and for tensor unparticle operators,

$$V_{\text{UNP}}(r) \equiv -V_T(r) = -\frac{C^e_T(C^{\nu e}_T - C^{\nu n}_T)}{4\pi} \frac{m_e E_\nu}{\Lambda^2_e} B W(r) \simeq -\alpha_T \frac{m_e E_\nu}{\Lambda^2_e} W(r),$$

$$M_{\text{UNP}} = 0,$$  \hspace{1cm} (19)

where $C^{\nu e}_{S,V,T} = C^{\nu e}_{23} C^{\nu e}_{S,V,T} + s^2_{23} C^{\nu e}_{S,V,T}$. In all cases, $W(r)$ is given in equation (13) with

$$n_f(r) = n_e(r) \left[ 1 + \frac{C^{S,V,T}_{S,V,T}}{C^e_{S,V,T}} R_p + \frac{C^{\nu n}_{S,V,T}}{C^e_{S,V,T}} \frac{Y_n(r)}{Y_e(r)} R_n \right],$$

(20)

with $R_{p,n} = 1$ for scalar and vector unparticles and $R_{p,n} = m_{p,n}/m_e$ for tensor unparticles. $Y_f(r)$ is the relative number density of fermion $f$, it is different for neutrons to that for protons and electrons because of the change in composition along the Sun radius. Because of this composition change, not only the normalization but also the $W(r)$ profile (in $r$) has some dependence on the unparticle couplings. We have neglected this small unparticle effect in the profile of the potential and absorbed it in the definition of effective $r$ average couplings:

$$\alpha_{S,V,T} = \frac{C^e_{S,V,T} (C^{\nu e}_{S,V,T} - C^{\nu n}_{S,V,T})}{4\pi} \left[ 1 + \frac{C^{p}_{S,V,T}}{C^e_{S,V,T}} R_p + \frac{C^{\nu n}_{S,V,T}}{C^e_{S,V,T}} \frac{Y_n(r)}{Y_e(r)} R_n \right],$$

(21)

where $\langle Y_n(r)/Y_e(r) \rangle$ stands for the average of the relative number densities along the neutrino trajectory.

From equation (21) we see that, as expected, the unparticle effect on the solar neutrino evolution does not factorize out as long as the unparticle couplings to neutrinos are non-universal. As a matter of fact, under the same assumption, solar neutrinos may also invisibly decay on their way from the Sun to the Earth as described in [11]. In what follows we will neglect the decay and concentrate on the effects due to the long-range forces, which, as we will see, turn out to be more constraining.

In figure 1 we show the function $W(r)$ for the Sun as a function of the distance from the centre in units of $R_\odot$. As seen in the figure, the dimension $d$ acts as a kind of range of the potential. Inside the Sun and up to 0.1 $R_\odot$, $W$ is constant, independently of $d$. We notice that for any value of $d < 1.7$, there is always a value of $M$ for which, at any $r$, the $W(r)$ function is always larger than the corresponding one for a $d = 1$ long-range force of range $\lambda = 0.1$.

We show in figure 2 the survival probability of solar $\nu_e$ on the sunny face of the Earth as a function of the neutrino energy $E_\nu$ for some values of the parameters. The probability is obtained by numerically solving equation (16) along the neutrino trajectory from its production point in the Sun to its detection point on the Earth. We have verified that for the range of parameters of interest the evolution in the Sun and from the Sun to the Earth is always adiabatic. From the figure we can infer the expected order of magnitude of the bounds that can be derived from the solar and KamLAND analysis which we describe next. For the scalar case we have chosen $m_1 = 0$ so $m_2 = \sqrt{\Delta m^2_{\odot}}$. As shown in [26], the effect of scalar interactions becomes stronger as $m_1$ grows, so we chose $m_1 = 0$ which gives the most conservative constraints.
Figure 1. Potential function $W(r)$ due to the density of protons (or electrons) in the Sun as a function of the distance from the solar centre in units of the solar radius, $R_\odot$, for various values of the dimension $d$ and the mass scale $M$. We take $\Lambda_u = 1$ TeV and $d_{\text{UV}} = 3$. For comparison, we show the value of the $W(r)$ function for a Coulomb-type ($d = 1$) long-range force with range $\lambda = 0.1R_\odot$ and with the same strength at the centre of the Sun.

4. Constraints from solar and reactor neutrino data

We present in this section the results of the global analysis of solar and KamLAND data for the unparticle long-range forces discussed in the previous section. Details of our solar neutrino analysis have been described in previous papers [43,41]. We use the solar fluxes from Bahcall, Serenelli and Basu from 2005 [42]. The solar neutrino data include a total of 118 data points: the gallium [30,31] and chlorine [29] (one data point each) radiochemical rates, the Super-Kamiokande [32] zenith spectrum (44 bins), and SNO data reported for phase 1 and phase 2. The SNO data used consist of the total day–night spectrum measured for the pure $D_2O$ (SNO-I) phase (34 data points) [33], plus the full data set corresponding to the salt phase (SNO-II) [34]. This last one includes the neutral current and elastic scattering event rates during the day and during the night (four data points), and the charged current day–night spectral data (34 data points). It is done by a $\chi^2$ analysis using the experimental systematic and statistical uncertainties and their correlations presented in [34], together with the theoretical uncertainties. In combining with the SNO-I data, only the theoretical uncertainties are assumed to be correlated between the two phases. The experimental systematics errors are considered to be uncorrelated between the phases.
Figure 2. Survival probability of $\nu_e$ in the Sun as a function of the neutrino energy $E_\nu$ for a scalar (upper panel), vector (middle panel) and tensor (lower panel) unparticle force, for various values of the parameters. For all curves we have used $\tan^2 \theta_{12} = 0.44$ and $\Delta m^2_{21} = 7.9 \times 10^{-5}$ eV$^2$.

In the analysis of KamLAND, we neglect the effect of the long-range forces due to the small Earth-crust density in the evolution of the reactor antineutrinos. For KamLAND we have used the observed events as a function of $L_0/E_\nu$, with $L_0$ fixed at 180 km [36] and minimized the $\chi^2$ function

$$\chi^2_{KL} = 2 \sum_{i=1}^{24} \left[ f R^i_{th} - R^i_{ex} + R^i_{ex} \log \left( \frac{R^i_{ex}}{f R^i_{th}} \right) \right] + \left( \frac{1 - f}{0.041} \right)^2,$$

with respect to $\Delta m^2_{21}$, $\tan^2 \theta_{12}$ and $f$. Here $R^i_{ex}$ and $R^i_{th}$ are, respectively, the observed and theoretical (which depends on the oscillation parameters) number of events, for the $i$th bin.

One must notice as well that the Lagrangian given in equation (3) also leads to additional contributions to neutrino neutral current (NC) and electron scattering (ES; this latter only for $\nu_e$) cross-sections at the detectors which we are ignoring in our analysis. The largest modifications are the ones from the vector unparticle contribution because in this case interference between the SM and the unparticle contributions is possible. The characteristic modification to the $\nu_\alpha f \to \nu_\alpha f$ interaction cross-section is

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Figure 3. Bounds from the analysis of solar and KamLAND data on the fundamental parameters \((\Lambda_u, M)\) for a scalar unparticle for \(d = 1.1, 1.3, 1.6\) and \(d_{UV} = 3\). The region below the curves is excluded. We have fixed \(\alpha_S = 10^{-3}\). The dashed lines correspond to the approximation given in equation (23).

\[
\frac{\sigma_{SM}}{\sigma_{SM+UNP}} = 1 + O(\left(\frac{C_V C_F^f}{M^2 G_F}\right)(\Lambda_u/M)^{2d_{UV}-4}(E_\nu m_f/\Lambda_u^2)} d^{-2}.\]

For scalar unparticles there is no interference between the unparticle and the SM contributions to the interaction cross-section and the correction enters only quadratically.

For tensor unparticles there is an additional suppression in the unparticle contribution to the interaction cross-section of \(O\left(\frac{E_\nu}{\Lambda_u}\right)\) due to the derivative present in the fermion operator. For the values of the unparticle parameters under consideration in the present work these modifications are negligible, which justifies our approximation of ignoring them.

As an illustration we show in figure 3 the bounds obtained from our combined analysis of solar and KamLAND data on the fundamental parameters for a scalar unparticle. For the sake of concreteness we have chosen \(\alpha_S > 0\) but similar order of magnitude bounds can be derived for negative \(\alpha_S\).

First we notice that the bounds from the long-range force effects are orders of magnitude stronger than the corresponding ones from the unparticle-induced invisible solar neutrino decay described in [11]. This justifies our approximation of neglecting the decay in the present work.

The order of magnitude of the bounds derived in figure 3 can be quantitatively understood from the comparison shown in figure 1 between the unparticle-induced potential function and that expected from a standard finite-range interaction. As seen in the figure, for any value of \(d \lesssim 2\) the \(W\) function is always larger than the corresponding one for a \(d = 1\) force of range \(\lambda = 0.1 R_\odot\). Consequently, for any value of \(d \lesssim 2\) the bounds
on the unparticle effects must be comparable to or stronger than the limits obtained in [26] for a leptonically scalar, vector, or tensor long-range force with coupling \( \kappa_S, \kappa_V \) and \( \kappa_T \), respectively, and of range \( \lambda = 0.1 R_\odot \) with the identification

\[
\kappa_S \rightarrow \alpha_S \left( \frac{\Lambda_u}{M} \right)^{2(d_{\text{UV}} - 1)} \frac{2}{\pi^{2d - 1}} \frac{\Gamma[d + 1/2]\Gamma[d - 1/2]}{\Gamma[2d]} \frac{1}{(R_\odot \Lambda_u)^{2(d - 1)}} \leq 6.8 \times 10^{-45},
\]

\[
\kappa_V \rightarrow \alpha_V \left( \frac{\Lambda_u}{M} \right)^{2(d_{\text{UV}} - 1)} \frac{2}{\pi^{2d - 1}} \frac{\Gamma[d + 1/2]\Gamma[d - 1/2]}{\Gamma[2d]} \frac{1}{(R_\odot \Lambda_u)^{2(d - 1)}} \leq 4.5 \times 10^{-53},
\]

\[
\kappa_T \rightarrow \alpha_T \frac{m_p}{\Lambda_u^2} B \left( \frac{\Lambda_u}{M} \right)^{2(d_{\text{UV}} - 1)} \frac{2}{\pi^{2d - 1}} \frac{\Gamma[d + 1/2]\Gamma[d - 1/2]}{\Gamma[2d]} \frac{1}{(R_\odot \Lambda_u)^{2(d - 1)}} \leq 2 \times 10^{-61} \text{ eV}.
\]

For comparison we also plot in figure 3 the bounds directly given by equation (23). As seen, the results from the full analysis are in reasonable quantitative agreement with the approximations above.

From equations (23) to (25) we can see that for a given value of \( \Lambda_u \) at the same scale dimension \( d \), the bounds on \( M \) are \( \sim 100 \) times more stringent for vector unparticles. For tensor unparticles the bounds on \( M \) will be \( \sim 3/\sqrt{\Lambda_u/\text{TeV}} \) times those for the scalar case. The bounds tend to get substantially relaxed as \( d \) increases. Consequently, if the low energy scale is of the order \( \Lambda_u \sim \text{TeV} \), one cannot provide very significant constraints on vector and tensor unparticles arising in a full conformal symmetry—for which \( d > 3, 4 \) respectively—from their long-range effects on neutrino oscillation data.

5. Discussion

We have studied the effect of the long-range forces induced by the coupling of unparticle operators to fermions of the SM, which are exerted along the neutrino trajectory from its production point inside the Sun to its detection at the Earth. We have used data from solar neutrino experiments and the newest KamLAND results to show that one can place stringent constraints on the parameters involved in the description of these new interactions.

For a scalar unparticle, for instance, assuming the non-universality of the lepton couplings to unparticles to be of the order of a few per mil, i.e. \( \alpha_s = 10^{-3} \), and \( d_{\text{UV}} = 3 \), we find that the mass \( M \) of the UV exchanged particle is constrained to be \( M > 8 \times 10^8 \text{ TeV} \) \((M > 7 \times 10^8 \text{ TeV})\), for the scaling dimension \( d = 1.1 \), if the lower energy scale is at \( \Lambda_u = 1 \text{ TeV} \) (10 TeV).

In general the bounds derived here are much stronger than the reach at present and near future colliders. They are comparable with those imposed from cosmology and astrophysics and from the effect on the modification of the Newtonian law of gravity. For the sake of comparison we show in table 1 our bounds together with those imposed by BBN [13], and from the compilation in table 2 of [15] of the bounds from astrophysics and fifth-force experiments. In order to make this comparison we have assumed all \( C_d^I \) to be of order 1 but that some violation of universality is allowed at the per mil level.

We can see that for low values of the scale \( d \), the bounds derived in this work are comparable to the most stringent ones, coming from scalar unparticle mediated fifth-force
Table 1. Limits on $M$ from various sources testing for signals from scalar unparticle couplings to fermions. Here $d_{UV} = 3$ was used in deriving all bounds and $\alpha_s = 10^{-3}$ was used for our work.

| Scalar unparticle | $M > \text{(TeV)}$ |
|-------------------|---------------------|
|                   | 1 TeV               | 10 TeV               |
| $d$               | 1.3                 | 1.3                  |
| This work         | $1.3 \times 10^6$   | $9 \times 10^6$      |
| BBN               | $3.2 \times 10^3$   | $5.8 \times 10^3$    |
| Eötvös type       | $1.9 \times 10^6$   | $3.5 \times 10^3$    |
| Energy loss from stars | 800            | $5.5 \times 10^3$   |
| SN 1987A          | 430                 | 26                   |

tests. However, unlike these latter, the bounds from the solar neutrino oscillation effects would apply even if the unparticles do not couple baryons.

Conversely, the bounds that we derived in this work can also be converted into severe constraints on universality violation of the neutrino couplings to unparticle operators with scales which can be accessible at future colliders. For example, for $M = \Lambda_u = 1$ TeV and $C_S^f = \mathcal{O}(1)$, the bounds in equation (23) imply that

$$\frac{C_S^{\nu e} - C_S^{\nu a}}{C_S^{\nu e}} \leq 4.5 \times 10^{-38} - 1.5 \times 10^{-9}$$

for $d = 1.1$–1.6. Therefore in the absence of a model justification of why flavour effects on the neutrino couplings to unparticle operators are more constrained than those on unparticle couplings for other fermions, this analysis implies that most probably the fermion couplings to unparticles which can be tested at colliders will be flavour blind.

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References

[1] Georgi H, 2007 Phys. Rev. Lett. 98 221601 [SPIRES] [hep-ph/0703260]
[2] Georgi H, 2007 Phys. Lett. B 650 275 [SPIRES] [0704.2457] [hep-ph]
[3] Liao Y, 2007 Phys. Rev. D 76 056006 [SPIRES] [0705.0837] [hep-ph]
Luо M and Zhu G, 2007 Preprint 0704.3532 [hep-ph]
Sahin I and Sahin B, 2007 Preprint 0711.1665 [hep-ph]
Cakir O and Ozansoy K O, 2007 Preprint 0710.5773 [hep-ph]
Iltan E O, 2007 Preprint 0710.2677 [hep-ph]
[4] Cheung K, Keung W Y and Yuan T C, 2007 Phys. Rev. Lett. 99 051803 [SPIRES] [0704.2588] [hep-ph]
Cheung K, Keung W Y and Yuan T C, 2007 Phys. Rev. D 76 055003 [SPIRES] [0706.3155] [hep-ph]
[5] Chen S L, He X G and Tsai H C, 2007 J. High Energy Phys. JHEP11(2007)010 [SPIRES] [0707.0187] [hep-ph]
Luo M, Wu W and Zhu G, 2007 Preprint 0708.0671 [hep-ph]
Greiner N, 2007 Phys. Lett. B 653 75 [SPIRES] [0705.3518] [hep-ph]
Constraints from solar and reactor neutrinos on unparticle long-range forces

Mathews P and Ravindran V, 2007 Phys. Lett. B 657 198 [SPIRES] [0705.4599] [hep-ph]
Kumar M C, Mathews P, Ravindran V and Tripathi A, 2007 Preprint 0709.2478 [hep-ph]
Cheung K, Li C S and Yuan T C, 2007 Preprint 0711.3361 [hep-ph]
Fox P J, Rajaraman A and Shirman Y, 2007 Phys. Rev. D 76 075004 [SPIRES] [0705.3092] [hep-ph]
Alan A T and Pak N K, 2007 Preprint 0708.3802 [hep-ph]
Alan A T, Pak N K and Senol A, 2007 Preprint 0710.4239 [hep-ph]
Cakir O and Ozansoy K O, 2007 Preprint 0710.5773 [hep-ph]
Sahin I, 2008 Preprint 0802.2818 [hep-ph]
Sahin I, 2008 Preprint 0802.1937 [hep-ph]
Li H F, Li H I, Si Z G and Yang Z J, 2008 Preprint 0802.0236 [hep-ph]
Barger V, Gao Y, Keung W Y, Marfatia D and Senoguz V N, 2008 Preprint 0801.3777 [hep-ph]
Madonia R and Giri A K, 2007 Preprint 0711.3516 [hep-ph]
[6] Chen C H and Geng C Q, 2007 Phys. Rev. D 76 115003 [SPIRES] [0705.0689] [hep-ph]
Cheung K and Geng C Q, 2007 Phys. Rev. D 76 036007 [SPIRES] [0706.0850] [hep-ph]
Mohanta R and Giri A K, 2007 Preprint 0711.3516 [hep-ph]
Bashiry V, 2008 Preprint 0801.1490 [hep-ph]
[7] Ding G J and Yan M L, 2007 Phys. Rev. D 76 075005 [SPIRES] [0705.0794] [hep-ph]
[8] Aliev T M, Cornell A S and Gaur N, 2007 Phys. Lett. B 657 77 [SPIRES] [0705.1326] [hep-ph]
Lu C D, Wang W and Wang Y M, 2007 Phys. Rev. D 76 077701 [SPIRES] [0705.2909] [hep-ph]
Hektor A, Kajiyama Y and Kannike K, 2008 Preprint 0802.4015 [hep-ph]
Itan E O, 2008 Preprint 0802.1277 [hep-ph]
Itan E O, 2008 Preprint 0801.0301 [hep-ph]
Ding G J and Yan M L, 2008 Phys. Rev. D 77 014005 [SPIRES]
[9] Li X Q and Wei Z T, 2007 Phys. Rev. D 76 054038 [SPIRES] [0705.1821] [hep-ph]
Aliev T M, Cornell A S and Gaur N, 2007 J. High Energy Phys. JHEP07(2007)072 [SPIRES] [0705.4542] [hep-ph]
Mohanta R and Giri A K, 2007 Phys. Rev. D 76 057015 [SPIRES] [0705.1234] [hep-ph]
Mohanta R and Giri A K, 2008 Phys. Lett. B 660 376 [SPIRES] [0711.3516] [hep-ph]
Lenz A, 2007 Phys. Rev. D 76 065006 [SPIRES] [0707.1535] [hep-ph]
Aslam M J and Lu C D, 2008 Preprint 0802.0739 [hep-ph]
Chen C H, Kim C S and Yoon Y W, 2008 Preprint 0801.0895 [hep-ph]
Wu Y f and Zhang D X, 2007 Preprint 0712.3923 [hep-ph]
Chen S L, He X G, Li X Q, Tsai H C and Wei Z T, 2007 Preprint 0710.3663 [hep-ph]
Aliev T M and Savci M, 2007 Preprint 0710.1505 [hep-ph]
[10] Li X Q, Liu Y and Wei Z T, 2007 Preprint 0707.2285 [hep-ph]
Zhou S, 2007 Preprint 0706.0302 [hep-ph]
Montanino D, Picariello M and Pulido J, 2008 Preprint 0801.2643 [hep-ph]
Dutta S and Goyal A, 2008 Preprint 0801.2143 [hep-ph]
Balantekin A B and Ozansoy K O, 2007 Phys. Rev. D 76 095014 [SPIRES] [0710.0028] [hep-ph]
[11] Anchordoqui L and Goldberg H, 2007 Preprint 0709.0678 [hep-ph]
He X G and Pakvasa S, 2008 Preprint 0801.0189 [hep-ph]
[12] Davoudiasl H, 2007 Phys. Rev. Lett. 99 141301 [SPIRES] [0705.3636] [hep-ph]
McDonald J, 2007 Preprint 0709.2350 [hep-ph]
[13] Hannestad S, Raffelt G and Wong Y Y Y, 2007 Preprint 0708.1404 [hep-ph]
Das P K, 2007 Preprint 0708.2812 [hep-ph]
Lewis I, 2007 Preprint 0710.4147 [hep-ph]
Collins H and Holman R, 2008 Preprint 0802.4416 [hep-ph]
Kikuchi T and Okada N, 2007 Preprint 0711.1506 [hep-ph]
Chen S L, He X G, Hu X P and Liao Y, 2007 Preprint 0710.5129 [hep-ph]
Alberghi G L, Kamenschik A Y, Tronconi A, Vacca G P and Venturi G, 2007 Preprint 0710.4275 [hep-th]
Constraints from solar and reactor neutrinos on unparticle long-range forces

[15] Freitas A and Wyler D, 2007 Preprint 0708.338 [hep-ph]
[16] Liao Y and Lin J Y, 2007 Phys. Rev. Lett. 99 191804 [SPIRES] [0706.1284] [hep-ph]
[17] Goldberg H and Nath P, 2007 Preprint 0706.3898 [hep-ph]
[18] Das S, Mohanty S and Rao K, 2007 Preprint 0709.253 [hep-ph]
[19] Liao Y and Liu J Y, 2007 Phys. Rev. Lett. 99 191804 [SPIRES] [0705.3092] [hep-ph]
[20] Grinstein B, Intriligator K and Rothstein I Z, 2007 Preprint 0708.2735 [hep-ph]
[21] Das S, Mohanty S and Rao K, 2007 Phys. Rev. D 75 043001 [SPIRES] [0709.253]
[22] Fox P J, Rajaraman A and Shirman Y, 2007 Phys. Rev. D 76 075004 [SPIRES] [0709.3092] [hep-ph]
[23] Feng J L, Rajaraman A and Tu H, 2008 Preprint 0801.1534 [hep-ph]
[24] Delgado A, Espinosa J R and Quiros M, 2008 Preprint 0802.2680 [hep-ph]
[25] Delgado A, Espinosa J R and Quiros M, 2007 J. High Energy Phys. JHEP10(2007)005 [SPIRES]
[26] Gonzalez-Garcia M C, de Holanda P C, Masso E and Zukanovich Funchal R, 2007 J. Cosmol. Astropart. Phys. JCAP01(2007)005 [SPIRES] [0707.4309] [hep-ph]
[27] Grifols J A and Masso E, 2004 Phys. Lett. B 579 123 [SPIRES] [hep-ph/0311141]
[28] Joshipura A S and Mohanty S, 2004 Phys. Lett. B 579 123 [SPIRES] [hep-ph/0311141]
[29] Eguchi K et al (KamLAND Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[30] Eguchi K et al (KamLAND Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[31] Aharmim S B et al (SNO Collaboration), 2005 Preprint nucl-ex/0502021
[32] Ayashi Y et al (Super-Kamiokande Collaboration), 2005 Phys. Rev. D 71 112003 [SPIRES]
[33] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[34] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[35] Ahmmed Q R et al (K2K Collaboration), 2005 Phys. Rev. D 71 112003 [SPIRES]
[36] Ahmmed Q R et al (K2K Collaboration), 2005 Phys. Rev. D 71 112003 [SPIRES]
[37] Ahmmed Q R et al (K2K Collaboration), 2005 Phys. Rev. D 71 112003 [SPIRES]
[38] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[39] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[40] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[41] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[42] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[43] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[44] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[45] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[46] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[47] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[48] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[49] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[50] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[51] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[52] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[53] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[54] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[55] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[56] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[57] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[58] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[59] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[60] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[61] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[62] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[63] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[64] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[65] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[66] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[67] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[68] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]
[69] Ahmmed Q R et al (SNO Collaboration), 2005 Phys. Rev. Lett. 94 081801 [SPIRES]