HARD EXCLUSIVE SCATTERING IN QCD

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ABSTRACT

We review the theory of hard exclusive scattering in Quantum Chromodynamics. After recalling the classical counting rules which describe the leading scale dependence of form factors and elastic reactions at fixed angle, the pedagogical example of the pion form factor is developed in some detail in order to show explicitly what factorization means in the QCD framework. The inclusion of transverse degrees of freedom leads to the discussion of Sudakov effects which are crucial for protecting the calculation from dangerous infrared regions. The picture generalizes to many hard reactions; a strategy to extract distribution amplitudes from future data is sketched. We discuss also the particular case of hadron-hadron collisions where the independent scattering mechanism dominates asymptotically and where a different factorization formula applies. We briefly present the concepts of color transparency and nuclear filtering and critically discuss the few present data on this subject.
We consider here *exclusive processes*, that is interactions resulting in a final state where all particles are identified. Using a perturbative expansion to study these reactions may *a priori* be foreseen if a large momentum transfer appears: this is what is called a *hard* reaction. Let us take as an example Compton scattering on a nucleon. The process is

\[ \gamma(k) + N(p) \rightarrow \gamma(k') + N(p'), \tag{1} \]

and we are interested in the polarized or unpolarized differential cross section

\[ \frac{d\sigma}{dt}(s, t), \quad s = (k + p)^2, \quad t = (k - k')^2, \tag{2} \]

for large values of \(|t| \sim s\) (large means much bigger than hadronic or confinement scales). In the ultra-relativistic limit, one may neglect the nucleon mass, and kinematics simplifies. In the center of mass frame, one has

\[ E = \sqrt{s}, \quad \sin^2 \frac{\theta}{2} = -\frac{t}{s}, \tag{3} \]

where \(E\) is the energy of any particles and \(\theta\) is the angle between the incoming and outgoing photons.

These reactions have first been shown to obey scaling laws, their energy dependence at fixed large angle being described by the so-called *counting rules* \[1\]. These pre-QCD studies are based on dimensional arguments and are not specific of QCD. We will see later how they are realized (and slightly modified) in the framework of QCD. It is very instructive to first follow their derivation which leads to the correct physical picture of hard exclusive reactions.

## 1 Counting rules

There are two standard ways to present them: a reasonning which emphasises the space-time structure of these reactions and a dimensional argument. They both are based on the hypothesis that the elementary mechanism is hard, that is that *all* elementary constituents undergo a large momentum transfer during the short time process. A rigourous proof of the validity of this hypothesis from field theory is not easy. We shall go back to this point.
1.1 Space-time picture; the example of the electromagnetic form factor

The simplest exclusive quantity is the pion form factor. Let us consider the process $e^-\pi^+ \rightarrow e^-\pi^+$. The electromagnetic interaction is mediated by virtual photon exchanges; effects due to the exchange of more than one photon, of order $\alpha_{\text{em}}$ relatively to the exchange of one photon, are negligible, and one thus limits the discussion to the process of Figure 1.

Figure 1: The pion electromagnetic form factor

The pion is a pseudoscalar particle. If it were elementary, the scattering cross section would equal

$$\frac{d\sigma}{dt}\bigg|_{\text{point}} = \frac{4\pi\alpha^2}{t^2} \frac{(s - m^2 - M^2)^2 + t(s - m^2)}{(s - (m + M)^2)(s - (m - M)^2)} ,$$

(4)

where $s = (k + p)^2$ and $t = (k - k')^2 = -Q^2 \leq 0$. $m$ and $M$ are the electron and pion masses.

The pion is however composite and the cross section writes

$$\frac{d\sigma}{dt} = |F_\pi(Q^2)|^2 \frac{d\sigma}{dt}\bigg|_{\text{point}} ,$$

(5)

which defines the pion form factor $F_\pi$. It measures the ability of the pion to stay itself when being collided by an electron. It is thus a quantity much sensitive to confinement mechanisms.

The physics deals with the restauration of the meson integrity after the violent shock of a high-energy electron with one of the quarks. At the limit $Q^2 = 0$, the meson structure is not resolved, and $F_\pi(0) = 1$.

The parametrization of Eq.(5) is derived by writing the matrix element $S$ under the form

$$\langle e\pi|S|e\pi \rangle = \int d^4 x d^4 y \langle \pi|J^\mu(x)|\pi \rangle \langle 0|T(A_\mu(x)A_\nu(y))|0\rangle \langle e|j^\nu(y)|e \rangle ,$$

(6)

where $J^\mu$ and $j^\nu$ are respectively quark and electron electromagnetic currents. One thus isolates the matrix element where the pion structure plays a role

$$\langle \pi^+(p')|J^\mu(x)|\pi^+(p) \rangle = \langle \pi^+(p')|J^\mu(0)|\pi^+(p) \rangle e^{i(p'-p)x}$$

(7)
As the pion is a (pseudo-)scalar particule, the most general parametrization of such a 4-vector must be written with the help of the 4-vectors \((p + p')^\mu\) and \((p' - p)^\mu\) weighted by functions of \(Q^2\), the only scalar present in the problem (ignoring the pion mass, \(m_\pi\)). Since the electromagnetic current is conserved: \(\partial_\mu J^\mu = 0\), the term in \((p' - p)^\mu\) must vanish. We thus have

\[
\langle \pi^+(p')|J^\mu(0)|\pi^+(p)\rangle = e_\pi (p + p')^\mu F_\pi(Q^2). \tag{8}
\]

Note that the hermiticity of the current leads to the reality of the form factor (for a space-like transition).

Let us now derive the \(Q^2\) dependence of the pion form factor \(F_\pi(Q^2)\) (for large values of this variable) by a careful examination of the way this process takes place. To do this, it turns out to be useful to consider, in the center of mass frame of the reaction, the case, illustrated on Figure 2, where the final electron emerges at an angle of 180° with respect to the initial electron.

![Figure 2: The space-time picture of the process \(e^- \pi^+ \rightarrow e^- \pi^+\)](image)

In its rest frame, the pion is represented as a collection of partons, quarks and gluons, \textit{grosso-modo} uniformly spreaded in a sphere of radius \(R\) (typically the pion charge radius, around 0.5 fm). In the reaction center of mass frame, the longitudinal dimension is Lorentz-contracted to \(R/\gamma\) with \(\gamma = Q/2M\). The transverse dimensions are on the other hand not affected by Lorentz-contraction. At time 0, the electron hits one of the partons, the so-called \textit{active} parton, and both change directions. For the whole process to be elastic, \textit{all} other partons must be alerted before the moment \(t \approx 1/Q\) to form the emerging pion (also contracted in this frame). The motion of the active parton after the collision is \(z(t) = -t, \ x(t), y(t) = 0\) whereas the motion of a \textit{spectator} parton is \(z(t) = t + z_0, \ x(t) = x_0, \ y(t) = y_0\) (one has...
$-1/Q \lesssim z_0 \lesssim 1/Q$ and $-R \leq x_0, y_0 \leq R$). Between the moments 0 and $1/Q$ a spectator parton can receive and respond to a physical signal emitted by the active parton at time 0 only if the interval $\Delta = t^2 - (t + z_0)^2 - x_0^2 - y_0^2$ is positive, that is if the spectator is at a distance $\sqrt{x_0^2 + y_0^2} \lesssim 1/Q$ in the transverse plane. One thus counts the probability to find spectator partons in a transverse disc of radius $1/Q$, in the initial as well as in the final state. One gets

$$F^2_\pi \propto \left( \frac{\pi Q^{-2}}{\pi R^2_\pi} \right)^{n_{in} - 1 + n_{out} - 1}. \quad (9)$$

Since a pion contains at least a valence quark and antiquark, we get a minimal contribution scaling like $1/Q^2$. Adding for instance one gluon to the valence in the initial state, without changing the final state, yields a contribution scaling like $1/Q^3 \ldots$ These contributions diminish relatively to the valence state contribution as energy increases.

This most important feature of the study of form factors at large transfer will be generalized to other exclusive reactions: *when the interaction is at short distance, the valence contributes in a dominant way in terms of scaling law*. Moreover, and this will be crucial for the phenomenon of *color transparency*, the hadron configurations which contribute have small ($O(1/Q)$) transverse sizes.

Let us summarize: asymptotically, one predicts for the energy dependence of pion and nucleon form factors, a power-law fall off:

$$F_\pi(Q^2) \propto \frac{1}{Q^2} \quad F_N(Q^2) \propto \frac{1}{Q^4}. \quad (10)$$

In the proton case, there are two form factors and the reasoning developed here does not allow to distinguish them. In fact, if one separates the form factors with respect to their degree of helicity conservation, one shows that the above counting rule applies only for helicity conserving processes (and thus for the magnetic form factor $G_M$), but that an additional power suppression affects $G_E$.

### 1.2 Dimensional argument: the example of Compton scattering

The dimensional argument to get the scaling law for exclusive processes is quite general, but will be explained on the specific process

$$\gamma + N \to \gamma + N. \quad (11)$$

In the ultra relativistic limit, the differential cross section writes

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} |M|^2, \quad (12)$$
which we will consider for a fixed ratio \(-t/s = O(1)\). To find the scaling law of this reaction, we must identify the \(s\) power dependence of the amplitude \(\mathcal{M}\).

This amplitude \(\mathcal{M}\) is calculated from Feynman rules and one may \textit{a priori} identify the dimensions, in energy units, of the different quantities entering these rules:

\begin{itemize}
  \item an external spinor has a dimension \(1/2\),
  \item an external vector \(0\),
  \item a fermion propagator \(-1\),
  \item a boson propagator \(-2\),
  \item a boson-fermions vertex \(0\),
  \item a 3 gluon vertex \(1\),
  \item a 4 gluon vertex \(0\).
\end{itemize}

Let us now construct a \textit{tree level and connex} graph for the process at the level of elementary particles (see Fig. 3) and count the dimension obtained: we get \(-4\). One easily sees that this result does not depend on an eventual insertion of additional loops; for any connex graph, the dimension only depends on the number of external particles, \(N\), and is equal to \(4 - N\).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{A connex graph contributing to Compton scattering}
\end{figure}

When calculating \(\mathcal{M}\), one must find out the momenta carried by each line and compute the scalar products between these various momenta. One easily sees that distributing to each quark or gluon a \textit{finite} fraction of its parent hadron momentum, leads all particles to undergo a large momentum transfer (if, of course, the global transfer is sufficiently large). Then all scalar products are of order \(s\), which is the unique dimensionful scale in the kinematics studied.

In the conventions where spinors are normalized by \(\bar{u}u = 2m\), the overall dimension of \(\mathcal{M}\) vanishes. We indeed must add a rule to the above list to precise how a hadron exhibits its quark - gluon content and to quantify the transition from the hadron to a \(n\) parton system. This transition,

\[ |\text{Hadron}\rangle \leftrightarrow f_{H,n}\langle n \text{ partons}|, \]
introduces a constant $f_{H,n}$ of dimension $n - 1$ which should be independent of the particular hard reaction studied and comes from confinement physics. The natural energy scale, $M$, for the constants $f_{H,n}$ should thus be $s$-independent. Taking these hadron-partons transitions into account, we find

$$M = f_{H,n} \sqrt{s}^{4-n-1-n'-1} f_{H,n'},$$

which is dimensionless as it should be.

The large angle and high energy behaviour of the Compton differential can thus be written as

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2} f\left(\frac{t}{s}\right),$$

for the transition between *valence* states. The above study indicates indeed that the sub-process $qqqg\gamma \rightarrow qqq\gamma$ contributes to the cross section as

$$\frac{f^2_{N,\text{val}} + g^2_{N,\text{val}}}{s^2},$$

which is negligible at high energy.

The amplitude at large transfer is thus separated as

$$A(s, t/s) = A^{\text{LT}} \left(1 + O(M/\sqrt{s})\right), \quad t/s = O(1)$$

where the so-called “Leading Twist” contribution $A^{\text{LT}}$, yields the lowest power fall-off in $s^{-1}$.

The QCD analysis presented in section 2 will strengthen the argument presented here and develop a consistent way of calculating the leading contribution. It will however be important to phenomenologically verify that the scaling laws, and thus the dominance of valence states, are verified at accessible energies, and this for each physical process under study.

Before studying in more details the pedagogical case of the electromagnetic meson form factor at large $Q^2$, let us digress to an important exception to the counting rules, the so-called “Landshoff” process of multiple or independent scattering.

### 1.3 The exceptional case of independent scattering

This independent scattering process \[^2\] does not appear in electromagnetic form factors but in elastic scattering of hadrons; it is represented on Figure 4 for the case of $\pi$-$$\pi$$ elastic scattering. The power counting of this process goes as follows. The outgoing beams of quarks must coincide in direction well enough to make hadrons in the final state; any discrepancy is set by the wave

\[^3\] $M$ will in the following represent a low energy scale, which can be the QCD constant, $\Lambda_{QCD}$, the $\rho$ meson mass or a typical internal transverse momentum for meson constituents, $\sqrt{\langle k^2_T \rangle}$, i.e. a few hundreds of MeV.
functions, which are defined to have small relative \( k_T \). The allowed \( k_T \) values are much smaller than the beam energies, so we can approximate them as almost zero. Because each independent on–shell quark-quark scattering amplitude scales like \( g^2 \bar{u}u\bar{u}u/t \) the independent scattering matrix element scales like

\[
[g^2 \bar{u}u\bar{u}u/t]^{n/4} \sim g^{n/2}
\]

up to logarithmic corrections. The scaling behaviour of the desired elastic scattering cross section comes then from the integration region constraint on 4-momenta set by: \( \delta^4(k^1 + k^2 - k^3 - k^4) \). There are three large momenta for each scattering, and one out-of-plane transverse momentum. This component of the transverse momentum is not as big as \( \sqrt{s} \) but instead depends sensitively on what the hadronic wave function allows. It should be of order \( C < k_T^2 >^{1/2} \) in the state’s wave function, which for purposes of counting is the same as \( C/ < b^2 >^{1/2} \), \( b \) being a transverse space separation.

\( \delta(p - p') \sim s^{-1/2} \delta(x - x') \),

where \( x \) and \( x' \) are dimensionless scaling variables. The overall probability amplitude for a pair of quarks to coincide in final state direction to make a hadron scales like the product of the delta functions of momentum, namely like \( C < b^2 >^{1/2} (s)^{-3/2} \). Using Eq. (12), one finds

\[
\frac{d\sigma}{dt} \propto < b^2 > s^{-5},
\]

for meson-meson scattering. As \( s \to \infty \), this beats the quark-counting process, which for meson-meson scattering goes like \( s^{-6} \).

\(^4\)In perturbation theory it is necessary to separate bound state properties of the wave function from effects of gluon exchange. To avoid double counting the gluon exchange which produces large \( k_T \), the bound state wave functions should have large \( k_T \) tails subtracted.
Consider next proton-proton scattering. The argument goes the same way, but requires another quark-quark scattering to coincide with the first ones. This adds three more delta functions of big momenta, so the amplitude-squared is smaller by \( s^{-3} \). Independent \( pp \rightarrow pp \) scattering thus has

\[
\frac{d\sigma}{dt} \propto (\langle b^2 \rangle)^2 s^{-8}. \tag{21}
\]

This again beats the quark-counting process, which (recall) goes like \( s^{-10} \).

How did this process manage to evade the power counting of the quark-counting process? It is easy to show that the number of gluons and internal propagators is fewer than the one assumed in the quark-counting induction; the topologies of the low order diagrams are not the same. Because both quark counting and independent scattering were studied before QCD was established, early discussion focused on comparisons with data. At first it seemed as if \( p-p \) scattering went like \( s^{-10} \), creating a puzzle to explain the absence of the much bigger \( s^{-8} \). One argument was made that quark counting diagrams might be more numerous and would dominate for that reason. However, when compared at the same order of perturbation theory, the independent scattering graphs are myriad and re-emerge inside the quark counting diagrams. This happens because internal gluons can become “soft”: a diagram with a soft gluon scales with the same power of \( s \) as if this gluon was absent. The upshot is that many quark counting diagrams contain a region indistinguishable from independent scattering with a soft gluon. Independent scattering cannot in any sense be “absent”. Similarly, if “soft” gluons attached to an independent scattering diagram should receive enough momentum to be counted as “hard”, the diagram may merge into the quark-counting set. It was finally realized [3] that these physically distinct processes actually boil down to different integration regions found in the one theory of QCD.

The independent scattering process had a confused history as these subtleties were only gradually appreciated. Closely related (and as much confused) is the issue of “Sudakov effects”, at first thought to suppress the independent scattering regions, but which were subsequently shown actually to force the independent scattering to dominate in the limit of \( s \rightarrow \infty \). We believe that there is rather convincing evidence that independent scattering region of QCD has been observed and plays a major role in color transparency. However, the subject is unsettled, and the interplay of the independent scattering regions and the quark counting regions is currently a subject of active investigation.
2 Calculating the pion form factor

One now wants to really calculate from QCD the pion form factor at large transfer \[4\]. This leads us to precise first the hadron wave function and the Born hard amplitudes, then the radiative corrections to see if a sensible picture emerges where a non perturbative object sensitive to confinement dynamics factorizes from a hard scattering amplitude controlled by a perturbative expansion which is renormalization group improved. This factorization which is crucial for a consistent understanding of future experimental data may be pictorially described as in Fig.5.

Figure 5: Factorization of a hard exclusive process : $X \ast T_H \ast X'$

We restrict here to the pedagogical case of the $\pi$ meson form factor but the technique is applicable to all hard exclusive reactions.

2.1 Description of the pion

Let us specify the kinematics. In the Breit frame the momenta are written as:

$$q = \begin{pmatrix} 0 \\ 0 \\ 0 \\ Q \end{pmatrix}, \quad p = \begin{pmatrix} Q/2 \\ 0 \\ 0 \\ -Q/2 \end{pmatrix}, \quad p' = \begin{pmatrix} Q/2 \\ 0 \\ 0 \\ Q/2 \end{pmatrix};$$

where the pion energies

$$E_{\pi} = \frac{Q}{2} \left( \sqrt{1 + \frac{4m_{\pi}^2}{Q^2}} \right)$$

have been approximated by $Q/2$.

To describe the pion in its valence state, one introduces the Bethe-Salpeter (BS) amplitude \[5\]

$$\langle 0 | T (q_{\omega\alpha i}(y) \ P_{ij}(y, 0) \ \bar{q}_{d\beta j}(0)) | \pi^+(p) \rangle,$$

where $u$ and $\bar{d}$ are the flavours of the valence quarks of $\pi^+$, $\alpha$ and $\beta$ are Dirac indices and $i, j$ are color indices. The $P_{ij}$ operator is necessary to have an amplitude invariant under local gauge transformations; when $q(y)$ transforms to $U(y) \ q(y)$, $P(y, 0)$ transforms to $U^{-1}(y) \ P(y, 0) \ U(0)$,
compensating the quark and antiquark variations. The BS amplitude is the relativistic generalisation of the Shrödinger wave function describing the bound state of a quark antiquark pair \[^6\]. One may interpret it as the probability amplitude of finding in a $\pi^+$ a $u$ quark at point $y$ and a $\bar{d}$ antiquark at the origin.

One often prefers to work in momentum space and defines the Fourier transform of the BS amplitude as

$$\int d^4y e^{ik \cdot y} <\cdot> \equiv X_{\alpha\beta}(k, p - k)$$

where $k$ is the quark momentum and, by momentum conservation, $p - k$ is the antiquark momentum.

To discuss the properties of this amplitude, it is convenient to introduce light-cone coordinates defined as:

$$\begin{cases} k^+ = \frac{1}{\sqrt{2}}(k^0 - k^3) \\ k^- = \frac{1}{\sqrt{2}}(k^0 + k^3) \end{cases}$$

The scalar product of two 4--vectors $A$ and $B$ is then

$$A \cdot B = A^+ B^- + A^- B^+ - A_\perp B_\perp.$$  \hspace{1cm} (26)

In our case, we thus have (listing $p = [p^+, p^-, p^1, p^2]$)

$$p = [Q/\sqrt{2}, 0, 0, 0], \quad p' = [0, Q/\sqrt{2}, 0, 0],$$

and we parametrize the internal momenta as $k = [xQ/\sqrt{2}, k^-, k_\perp]$, where $x$ is the light-cone fraction carried by the quark inside the pion. The antiquark then carries the fraction $1 - x = \bar{x}$. The final pion is treated similarly, with $+$ and $-$ components exchanged: $k' = [k'^+, x'Q/\sqrt{2}, k'_\perp]$ and so on.

In terms of these variables, the $k^\mu$ regions favored by the amplitude $X(k, p - k)$ are simply written as:

$$k_\perp^2 \lesssim M^2, \quad |k^-| \lesssim M^2/Q.$$  \hspace{1cm} (28)

### 2.2 The hard scattering at the Born level

The matrix element of Figure 5 is written as the convolution

$$\int \frac{d^4k}{(2\pi)^4} \frac{d^4k'}{(2\pi)^4} X(k) T^\mu_H(k, k') X^\dagger(k').$$

At the lowest order in the QCD coupling constant, $g$, one finds 4 Feynman diagrams. One is drawn on Figure 6 and the 3 others are easily deduced by attaching successively the photon to the points 2, 3 and 4.
Let us first evaluate the gluon squared momentum, which is in Feynman gauge, the denominator of the gluon propagator. We have

\[(p' - k' - p + k)^2 = -\bar{x}'xQ^2 - \sqrt{2}Q(k^-\bar{x}' + k'^-x) - 2k^-k'^+ -(k_\perp - k'_\perp)^2\]

\[O(Q^2) \quad O(M^2) \quad O(M^4/Q^2) \quad O(M^2)\]

(30)

where typical orders of magnitude indicated refer to the momentum regions favored by the amplitudes \(X(k)\) and \(X(k')\). Restricting to leading terms in \(Q\), we may forget terms of order \(M^2\). So, in particular, we write

\[(p' - k' - p + k)^2 \approx -\bar{x}'xQ^2/2.\]

(31)

The same analysis may be repeated for the other quantities present in the hard amplitude \(T_H^\mu\), leading to

\[T_H^\mu(k, k') \approx T_H^\mu\left(x, x' \frac{Q}{\sqrt{2}}, \frac{x'Q}{\sqrt{2}}\right).\]

(32)

Figure 6: Born Graph for the pion form factor; the 3 other graphs are deduced by attaching the photon to the points 2, 3 and 4. Propagators joining Bethe-Salpeter amplitudes to the vertices are absorbed, by definition, in these amplitudes.

We may then express the convolution of equation (30) under the form

\[\int dx dx' \left(\frac{Q}{2\sqrt{2\pi}} \int \frac{dk^-dk^\perp}{(2\pi)^3} X(k)\right) T_H^\mu\left(x, x' \frac{Q}{\sqrt{2}}, x' \frac{Q}{\sqrt{2}}\right) \int \frac{dk'^+dk'^\perp}{(2\pi)^3} X^\dagger(k'),\]

(33)

and the object needed to describe the pion in this reaction is in fact much simpler than the amplitude \(X\) since one may integrate over three components of the internal momentum.

A first simplification comes from the integration over the \(k^-\) (for the outgoing pion over the \(k'^+\) variable). In terms of the conjugated variable \(y^+\), this means that one only needs the Bethe-Salpeter amplitude at \(y^+ = 0\), which is called the light cone wave function, usually noted as \(\psi(x, k_\perp)\) [7]. A useful property of this wave function is that the support in the \(x\) fraction
is limited, as $0 \leq x \leq 1$. This limitation to light cone fractions $x$ between 0 and 1 may be recovered by writing $X$ under the form

$$X(k, p - k) = \frac{f(k)}{[k^2 - m^2 + i\varepsilon] \left((p - k)^2 - m^2 + i\varepsilon\right)},$$

and by evaluating the integral over $k^-$ from $-\infty$ to $+\infty$ by a Cauchy contour. One then obtains a non zero contribution only if the two poles are on opposite sides of the real axis. These poles are at

$$\begin{cases}
k_1^- = \frac{\sqrt{2}(k^2 + m^2)}{xQ} & -i\varepsilon \text{ sgn}(x) \\
k_2^- = -\frac{\sqrt{2}(k^2 + m^2)}{xQ} & +i\varepsilon \text{ sgn}(1 - x)
\end{cases}$$

so that the integral yields a factor

$$\theta(x)\theta(1 - x);$$

the $x$ integral is thus limited to the interval $[0, 1]$.

The Dirac structure of the amplitude $X(k)$ integrated over $k^-$ and $k_\perp$ is easy to extract and one finds

$$M_{\alpha\beta}(x, p) = \frac{1}{4} \gamma^5 \tilde{p} \cdot \varphi(x)|_{\alpha\beta}.$$  

This Dirac structure corresponds to the combination of spinors ($\uparrow$ and $\downarrow$ denote respectively the helicity states $+$ and $-$)

$$\frac{1}{4} \gamma^5 \tilde{p}_{\alpha\beta} = \frac{1}{2\sqrt{2xx}} \frac{1}{\sqrt{2}} (u_\alpha(xp, \uparrow) \bar{v}(\bar{x}p, \downarrow) - u_\alpha(xp, \downarrow) \bar{v}(\bar{x}p, \uparrow)),$$

i.e. one recovers the pion spin wave function in the quark model $\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$.  

The function $\varphi(x)$ is called distribution amplitude; it “measures” how the pion momentum is distributed between the valence quark and antiquark when their transverse separation vanishes. This is the non perturbative amplitude connecting long distance physics of strong interaction to short distance hard processes.

Let us now precise a little bit the color algebra involved here. A useful way to simplify this matter is to choose for a pion of momentum along the $+$ direction, axial gauges with axis along the $-$ direction (fixing $A^+ = 0$). In these gauges, one has $P_{ij}(y, 0) = \delta_{ij}$ and the color component for the quark-antiquark pair is simply $\delta_{ij}/3$. This fact partly explains the interest of light-cone gauges in the study of hard processes. For another gauge choice, an explicit form of $P_{ij}(y, 0)$ is necessary, but we will not pursue this here. Note however that $P_{ij}(y, 0)$ may

\footnote{$\theta$ is the function defined by: $\theta(x) = \begin{cases} 0, & x < 0, \\ 1, & x > 0. \end{cases}$}
be perturbatively analyzed and gauge invariance preserved order by order in the perturbative expansion. At zeroth order, one has

\[ P_{ij}(y,0) = \delta_{ij} + O(g). \]  

(39)

We are now able to calculate the graph of Figure 5 with a new Feynman rule for the pion

\[ \frac{1}{3} \delta_{ij} \frac{1}{4} \gamma^5 \not{p} |_{\alpha\beta} \varphi(x), \]  

(40)

and a loop integral \( \int_0^1 dx \). The amplitude of the process may thus be written as

\[ \int_0^1 dx \int_0^1 dx' \varphi(x) \langle T^\mu_H(x,x') \rangle \varphi^*(x') \]  

(41)

where the hard process is evaluated on the spin and color components written above. Color algebra leads to the trace

\[ \frac{1}{3} \delta_{ij} T_{jk} \frac{1}{3} \delta_{kl} T_{li} = \frac{4}{9}, \]  

(42)

and the amplitude neglecting quark masses is

\[ \int_0^1 dx \int_0^1 dx'(-) \frac{C_F}{3} T_F \left\{ e_u \gamma^\mu \frac{1}{4} \gamma^5 \not{p} g g^\alpha \frac{1}{4} \gamma^5 \not{p}' g g^\beta \not{p}' - \bar{x}p \right\} \frac{-\eta_{\alpha\beta}}{-\bar{x}x'Q^2} \varphi(x) \varphi^*(x') \]

\[ = e_u p^\mu \frac{C_F g^2}{6Q^2} \left| \int_0^1 dx \frac{\varphi(x)}{\bar{x}} \right|^2. \]  

(43)

The graph with the photon attached to point 2 leads to the same expression replacing \( p^\mu \) by \( p'^\mu \). The two other graphs are identical to the two first ones after exchanging \( e_u \leftrightarrow -e_d \) and \( \bar{x} \leftrightarrow x \) in the integrand denominator. Charge conjugation invariance and isospin symmetry lead to the relation \( \varphi(x) = \varphi(\bar{x}) \), so that one can factorize the term \( (e_u - e_d)(p + p')^\mu \) expected in Eq. (8) and isolate the form factor expression

\[ F_\pi(Q^2) = \frac{C_F g^2}{6Q^2} \left| \int_0^1 dx \frac{\varphi(x)}{\bar{x}} \right|^2. \]  

(44)

Let us stress that we recover the scaling law in \( Q^{-2} \) predicted by the counting rules.

The pion lifetime fixes a constraint on the valence wave function of the pion. The process is described on Figure 7.

Figure 7: pion weak decay.
As in the form factor case, one may isolate the weak transition at the quark level, under
the form of the matrix element of the electroweak current \([9]\). One gets

\[
\langle 0 | \bar{q}_d(0) \gamma^\mu (1 - \gamma^5) q_u(0) | \pi^- (p) \rangle = f_\pi p^\mu ,
\]

(45)

where the decay constant, \(f_\pi\), is in this parametrization equal to 133MeV.

The BS amplitude at the origin may then be written as

\[
\langle 0 | T (q_{uai}(0) \bar{q}_{d\beta j}(0)) | \pi^- (p) \rangle = \int_0^1 dx \frac{Q}{2\sqrt{2}\pi} \int \frac{dk^- \, dk_\perp}{(2\pi)^3} X(k) ,
\]

(46)

that one multiplies by the tensor \([\gamma^\mu (1 - \gamma^5)]_{\beta \alpha \delta j i}\) to get

\[
- \langle 0 | \bar{q}_d(0) \gamma^\mu (1 - \gamma^5) q_u(0) | \pi^- (p) \rangle = Tr \left( \frac{\gamma^5}{4} \gamma^\mu (1 - \gamma^5) \right) \frac{\delta_{ij}}{3} \int_0^1 dx \, \varphi(x) ,
\]

(47)

where it can be noted that the componant \(\varphi'\) does not survive to the projection. One gets

\[
p^\mu \int_0^1 dx \, \varphi(x) = f_\pi p^\mu ,
\]

(48)

which fixes the normalization of the distribution amplitude.

### 2.3 Radiative corrections

It is important, when calculating a quantity in any field theory, and in particular in perturbative
QCD, to keep track of radiative corrections and control them so that the picture obtained at
lowest order survives their inclusion. The ultraviolet regimes does not a priori cause much prob-
lem since the theory is known to be renormalizable. In fact, the subtractions to be taken into
account are automatically taken care of when correctly treating quark and gluon propagators
on the one hand, and the running coupling constant on the other hand.

The infrared regions in the loop calculations must be very carefully scrutinized. In the
specific process studied here, one finds in a \(n\) loops diagram corrections of order

\[
\frac{\alpha_s(Q^2)}{Q^2} \left[ \frac{\alpha_s(Q^2) \ln \frac{Q^2}{M^2}}{\alpha_s(Q^2)} \right]^n ,
\]

(49)

which, since \(\alpha_s(Q^2) \propto (\ln Q^2/\Lambda^2)^{-1}\) is of the same order as the tree level process! One has
to resum these large logarithms in the distribution amplitude to recover the predictibility of
the formalism. This is factorization since then the process may be written as the convolution
illustrated by Figure 5:

\[
F_\pi = \varphi \ast T \ast \varphi^* \label{50}
\]

where:
$T$ is a hard amplitude that one can evaluate within perturbative QCD; namely, higher order corrections to $T$ are of order $\alpha_s^n(Q)$, and thus sufficiently small at sufficiently large transfer;

all large logarithms are absorbed in $\varphi$; the distribution $\varphi$, which represents the wave function evolves with the scale $Q$ characteristic of the virtual photon probe. This stays an essentially non perturbative quantity expressing the way confined valence quarks share the hadron momentum when they interact at small distance in an exclusive process.

Let us now examine how leading logarithms are resummed in the distribution $\varphi_{LL}$. It turns out that it is most interesting to choose to work in a gauge which is different from the Feynman gauge, namely an axial gauge, with axis $n^\mu$, fixing the condition on gluon fields $A^\mu$ as: $n.A = 0$. The leading corrections have then the form illustrated on Figure 8.

One may show that the graph summation yields

$$\varphi_{LL}(x, Q) = \varphi_0(x) + \kappa \int_0^1 du V_{q\bar{q} \to q\bar{q}}(u, x) \varphi_0(u)$$

$$+ \frac{\kappa^2}{2!} \int_0^1 du V_{q\bar{q} \to q\bar{q}}(u, x) \int_0^1 du' V_{q\bar{q} \to q\bar{q}}(u', u) \varphi_0(u') + \ldots$$

(51)

where $\kappa$ contains large collinear logarithms under the form

$$\kappa = \frac{1}{\beta_1} \ln \frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \left( \beta_1 = \frac{1}{4} \left( 11 - \frac{2}{3} n_f \right) \right);$$

(52)

and $V_{q\bar{q} \to q\bar{q}}$ is a characteristic kernel describing the splitting of the valence distribution of the pion

$$V_{q\bar{q} \to q\bar{q}}(u, x) = \frac{2}{3} \left\{ \bar{x} \frac{1}{u} \left( 1 + \frac{1}{u - x} \right)_+ \theta(u - x) + \frac{x}{u} \left( 1 + \frac{1}{x - u} \right)_+ \theta(x - u) \right\},$$

(53)

The $(\cdot)_+$ distribution comes from the compensation of infrared divergences (here in the limit $u \to x$) between graphs b and c of Figure 8. This is a consequence of the colour neutrality of a hadron.
integro-differential form
\[
\frac{\partial \varphi}{\partial \kappa} = \int_0^1 du V(u, x) \varphi(u, Q),
\]
(54)
the general solution of which is known as
\[
\varphi(x, Q) = x(1 - x) \sum_n \phi_n(Q) C_n^{(3/2)}(2x - 1);
\]
(55)
where Gegenbauer polynomials $C_n^{(m)}$ are such that
\[
\int_0^1 du (1 - u) V(u, x) C_n^{(3/2)}(2u - 1) = A_n x(1 - x) C_n^{(3/2)}(2x - 1),
\]
(56)
with $A_n$ coefficients which depend on $n$. Injecting this solution in the equation, one gets
\[
\phi_n(Q) = \phi_n(\mu) e^{A_n \kappa} = \phi_n(\mu) \left( \frac{\alpha_S(\mu^2)}{\alpha_S(Q^2)} \right)^{A_n/\beta_1},
\]
(57)
where the exponents in the expansion begin with
\[
\frac{A_0}{\beta_1} = 0, \quad \frac{A_2}{\beta_1} = -0, 62, \ldots
\]
(58)
Odd terms disappear since the distribution is symmetric in the interval $[0, 1]$.

Calculating the integral
\[
\int_0^1 dx \varphi(x, Q) = \phi_0(Q) \int_0^1 dx x(1 - x) = \frac{\phi_0}{6} = f_\pi
\]
(59)
one can write down the beginning of the expansion:
\[
\varphi(x, Q) = 6 f_\pi x(1 - x) + (\ln Q^2)^{-0.62} \Phi_2 x(1 - x)[5(2x - 1)^2 - 1] + \ldots
\]
(60)
The pion asymptotic distribution, when $Q \to \infty$, is then
\[
\varphi(x, Q \to \infty) \sim 6 f_\pi x(1 - x).
\]
(61)
This however does not tell us much on the realistic distribution amplitude at accessible energies: the constants $\Phi_2, \ldots, \Phi_n$ are unknown.

This is how far perturbative QCD can lead us about the distribution amplitude $\varphi$; i.e. to understand how strong interactions build a hadron from its valence quarks. To go further, one needs other methods, which are non perturbative by nature. Experiments can guide us to develop new ways since exclusive scattering data may be processed to extract distribution amplitudes. The existing methods, like lattice calculations or QCD sum rules, are still too primitive and rely on too many unchecked hypotheses to be trusted. They however lead to
useful rate estimates. They generally evaluate moments of the distribution amplitude defined as:

$$\int_0^1 dx (2x - 1)^2 \varphi(x, \mu), \ldots$$

(62)

Such a study lead Chernyak and Zhitnitsky [11] to propose the distribution

$$\varphi_{cz}(x, Q^2) = 6 f_{\pi} x (1 - x) \left\{ 1 + [5(2x - 1)^2 - 1] \left( \frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2} \right)^{-0.62} \right\},$$

(63)

with $Q_0 \approx 500$MeV. Figure 9 shows the distribution proposed by Chernyak and Zhitnitski.

Figure 9: The CZ distribution and its evolution with the scale $\mu^2$.

2.4 Transverse Degrees of Freedom

A study of one loop corrections [12] leads to propose that the scale relevant for the running coupling constant $\alpha_S$ is more likely to be the exchanged gluon virtuality $xx'Q^2$ than the photon virtuality $Q^2$. The whole treatment would then be correct only when the gluon is far off mass shell, that is as far as $x$ or $x'$ do not approach 0. However, for intermediate transfers, it turns out that an important part of the amplitude comes from these regions. One should thus reexamine the whole story in the region where gluons may become soft. In this region transverse momentum (or transverse distance) degrees of freedom become important and invalidate the collinear approximation [13]. Let us qualitatively explain the expected modifications.

The elastic interaction of a coloured object (a quark for instance) is suppressed by a Sudakov form factor [14] which quantifies the difficulty of preventing an accelerated charge from radiating. Similarly the elastic interaction of a dipole of transverse size $b$ is strongly suppressed unless $b$ approaches $Q^{-1}$ [3]. The approximation where transverse degrees of freedom are neglected leads to consider the region $b^2 \leq (xx'|q^2|)^{-1}$, which is an unsuppressed region when $xx'$ is of order 1. When $xx' \to 0$, this approximation becomes illegitimate, and one may imagine that taking the transverse size into account should allow, with the help of an associated Sudakov
suppression, to bypass dangerous infrared contribution. We shall come back to this point in section 4.2.

3 Other scattering processes.

The results obtained above for the electromagnetic form factor may be generalized to other hard exclusive processes, with an important difference in the case of hadron - hadron collisions (see section 4). One thus defines a distribution amplitude for the proton and analyzes the magnetic form factor $G_M$ very similarly. One can then consider sharper reactions as real or virtual Compton scattering, which still only depend on the proton structure but where one can vary dimensionless ratio such as angles.

3.1 The proton distribution amplitude

As for the pion case, the valence nucleon wave function can be written \[ \varphi(x, y, z) \] as a combination of definite tensors of colour, flavour and spinor indices with a (unique) proton distribution amplitude $\varphi(x, y, z)$. This distribution amplitude may be written as an expansion quite similar to what was derived above for the pion case but on a different basis of polynomials:

$$\varphi(x_i, Q) = 120 x_1 x_2 x_3 \delta(x_1 + x_2 + x_3 - 1) \times \left[ 1 + \frac{21}{2} \left( \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right)^{\lambda_1} A_1 P_1(x_i) + \frac{7}{2} \left( \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right)^{\lambda_2} A_2 P_2(x_i) + \ldots \right],$$

where the slow $Q^2$ evolution comes entirely from the terms $\alpha_s(Q^2)^{\lambda_i}$, and the $\lambda_i$’s are decreasing numbers:

$$\lambda_1 = \frac{5}{9 \beta_1}, \quad \lambda_2 = \frac{6}{9 \beta_1},$$

and the $P_i(x_j)$’s are Appell polynomials:

$$P_1(x_i) = x_1 - x_3, \quad P_2(x_i) = 1 - 3x_2, \ldots$$

The $A_i$’s are unknown constants and measure the wave function projection on the Appell polynomials:

$$A_i = \int_0^1 dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1) \varphi(x_i) P_i(x_i)$$

3.2 The proton magnetic form factor

One describes the elastic interaction of a proton and an electron

$$e^{-} + p \rightarrow e^{-} + p,$$
with two form factors $F_1$ and $F_2$ (still within the one virtual photon exchange hypothesis)

$$
\langle p', h'|J^\mu(0)|p, h \rangle = e\bar{u}(p', h') \left[ F_1(Q^2)\gamma^\mu + i\frac{\kappa}{2M}F_2(Q^2)\sigma^{\mu\nu}(p' - p)_\nu \right] u(p, h); \quad (69)
$$

$h$ and $h'$ are respectively the incoming and outgoing proton helicities, $u$ and $\bar{u}$ their spinors and $M$ the proton mass. In this decomposition, $e$ is the proton charge and $\kappa = 1.79$ is its anomalous magnetic moment. With these conventions, the two form factors have at zero transfer the values:

$$
F_1(0) = 1, \quad F_2(0) = 1. \quad (70)
$$

Figure 10: $Q^2$ evolution of the proton magnetic form factor

$F_1$ and $F_2$ are respectively called Dirac and Pauli form factors. From the Gordon identity

$$
i(p' - p)_\nu\bar{u}'\sigma^{\mu\nu}u = 2M\bar{u}'\gamma^\mu u - (p + p')^\mu\bar{u}'u, \quad (71)$$

one writes the current matrix element as

$$
\langle p', h'|J^\mu(0)|p, h \rangle = e\bar{u}' \left[ (F_1(Q^2) + \kappa F_2(Q^2))\gamma^\mu - \frac{\kappa}{2M}F_2(Q^2)(p + p')^\mu \right] u, \quad (72)
$$

which leads to define the Sachs form factors which appear in the process cross section; they are the linear combinations

$$
G_M = F_1 + \kappa F_2
$$

$$
G_E = F_1 + \frac{q^2}{4M^2}\kappa F_2. \quad (73)
$$

In the formalism we are presenting here, only the magnetic form factor is accessible. With a proton distribution amplitude deduced from a QCD sum rule analysis à la Chernyak-Zhitnitsky, one obtains the results shown on Figure 10. The slight decrease of $Q^4G_M(Q^2)$ is understood as a manifestation of radiative corrections on top of the counting rules.
3.3 Compton scattering

The perturbative part of the analysis of real [16, 17] or virtual [18] Compton scattering consists in evaluating the 336 topologically distinct diagrams obtained when coupling two photons to the three valence quarks of the proton, two gluons being exchanged. Moreover, there are 42 diagrams with a three-gluon coupling but it turns out that their color factor vanishes.

At lowest order in $\alpha \sim \frac{1}{137}$, Virtual Compton Scattering (VCS) is described as the coherent sum of the amplitudes drawn on Figure 11, namely the Bethe Heitler (BH) process (Fig. 11b) where the final photon is radiated from the electron and the genuine VCS process of (Fig. 11a).

As the BH amplitude is calculable from the elastic form factors $G_{Mp}(Q^2)$ and $G_{Ep}(Q^2)$, its interference with the VCS amplitude is an interesting source of information, different from what real Compton scattering yields. The VCS amplitude depends on three invariants; one usually chooses $Q^2, s, t$ or $s, Q^2/s, \theta_{CM}$.

Each incoming (outgoing) quark carries a light-cone fraction $x$ ($y$) of the + ($-$) component of the parent proton momentum, together with components along the three other directions. When these fractions $x$ or $y$ stay of order 1, it is legitimate to neglect these three other components in the hard process and one gets:

$$A = \varphi_{(uud)} \otimes T_H(\{x\}, \{y\}) \otimes \varphi'_{(uud)}(1 + O(M^2/t)),$$ (74)
Figure 12 shows the few existing data for real Compton scattering on the proton with $-t > 1\text{GeV}^2$ [19]. $s^{6}d\sigma/dt$ is plotted as a function of $\cos\theta_{CM}$ to illustrate the approach to asymptotic scaling laws. If one fits the data with a law in $s^{-\alpha}$, one gets $\alpha = 7.0 \pm 0.4$: that is a $2.5\sigma$ deviation from the counting rule prediction $\alpha = 6$.

### 3.4 A strategy for data analysis

A first way to extract physics from experimental points consists in comparing data with a computation done with distribution amplitudes coming from a theoretical model. Kronfeld and Nižić [17] have for instance calculated real Compton scattering with various distribution amplitudes as shown on Figure 12. One sees that the differential Compton cross section has a high discriminating power with respect to the non perturbative object $\varphi_{(uud)}$ that we want to study.

A less biased way to extract the distribution amplitude from experimental numbers is to write the cross section as a sum of terms

$$A_{i}T_{ij}^{ij}(\theta)A_{j}$$

(75)

where the decomposition of the distribution amplitude on the Appell polynomials (Eq. (67)) has been used and where $T_{ij}^{ij}$ are integrals over $x$ and $y$ variables of the product of the hard amplitude at a given scattering angle $\theta$ by the two Appell polynomials $A_{i}(x)$ and $A_{j}(y)$. The $T_{ij}^{ij}$ are ugly long expressions but they can be numerically handled.

Determining the proton distribution amplitude from experimental data boils down then to the extraction by a maximum of likelyhood method of the $A_{i}$ parameters, amputating the series of Eq. (67) to its first $n$ terms, verifying afterwards that including the term $n + 1$ does not drastically modify the conclusion. One can then explore other reactions, virtual Compton scattering for instance, which must be well described by the same series of $A_{i}$’s.

### 3.5 Other processes

Photo- and electro-production of mesons at large angle will allow to probe distribution amplitudes of $\pi$ and $\rho$ mesons in the same way. The production of the $KA$ final state selects a few hard scattering diagrams. The analysis of these reactions is still to be done if one excepts some works done in the simplifying framework of the diquark model [20].

Heavy particle decays such as $B \rightarrow \pi\pi$ have also been studied in this formalism. We shall not deal here with that interesting physics case [21].
4 Independent scattering formalism

The QCD formalism for the independent scattering process proposed by Landshoff has been established by Botts and Sterman [3], after pioneering work by Mueller [22]. A new factorization property has been derived. An important result is that this mechanism asymptotically dominates pure short distance contributions à la Brodsky-Lepage in hadron-hadron collisions at fixed angle but is sub-dominant in the case of photo- and electro-production reactions.

One generally writes an helicity amplitude for the $\pi\pi \rightarrow \pi\pi$ elastic scattering process of Figure 4 as

$$A = \int \{ \prod_{i=1}^{4} \frac{d^{4}k_{i}}{(2\pi)^{4}} X_{\alpha_{i}\beta_{i}}(k_{i}) \} H(\{k\})H'(\{p - k\})|_{\{\alpha\beta\}} ,$$

(76)

where quark color indices have been skipped, $\{k\}$ denotes $k_{1}, k_{2}, k_{3}, k_{4}$, and only one quark flavor has been kept.

In this equation, $X(k)$ is the Bethe-Salpeter amplitude

$$\int d^{4}y e^{ik.y} \langle 0 | T (q_{\alpha}(y) P(y, 0) \bar{q}_{\beta}(0)) | M(p) \rangle ,$$

(77)

and $H$ and $H'$ are the subprocesses hard amplitudes, i.e. a sum of perturbative QCD graphs. $H$ is conventionally the graph where the quark from meson 1 enters the hard process.

To simplify this expression, we first examine the kinematical regions which dominate the integral, either because of the behaviour of various amplitudes, either because of momentum conservation in the hard diagrams (global momentum conservation being extracted as usual)

$$(2\pi)^{4}\delta(\sum_{i} k_{i}) (2\pi)^{4}\delta(\sum_{i} p_{i} - k_{i}) = (2\pi)^{4}\delta(\sum_{i} k_{i}) (2\pi)^{4}\delta(\sum_{i} p_{i}) ,$$

(78)

4.1 Kinematics

It is interesting to attach to each meson $M_{i}$ a light-cone basis, $(v_{i}, v'_{i}, \xi_{i}, \eta)$. In the center of mass system, one chooses the direction of flight of $M_{1}$ as the axis $\hat{3}$. Denoting $\theta$ the scattering angle of $M_{3}$ measured with respect to $\hat{3}$, one chooses the axis $\hat{1}$ such that the momentum of $M_{3}$ be along $\cos \theta \hat{3} + \sin \theta \hat{1}$. The basis vectors are then

$$v_{1} = v'_{2} = \frac{1}{\sqrt{2}} (\hat{0} + \hat{3}) \quad v'_{1} = v_{2} = \frac{1}{\sqrt{2}} (\hat{0} - \hat{3})$$

$$\xi_{1} = \xi_{2} = \hat{1} \quad \eta = \hat{2}$$

$$v_{3} = v'_{4} = \frac{1}{\sqrt{2}} (\hat{0} + \sin \theta \hat{1} + \cos \theta \hat{3}) \quad v'_{3} = v_{4} = \frac{1}{\sqrt{2}} (\hat{0} - \sin \theta \hat{1} - \cos \theta \hat{3})$$

$$\xi_{3} = \xi_{4} = \cos \theta \hat{1} - \sin \theta \hat{3};$$

neglecting the meson masses in front of $Q = \sqrt{s/2}$, the mesons momenta write simply $p_{i} = Qv_{i}.$
An analysis similar to the one leading from Eq. (30) to Eq. (31) allows one to replace

\[ H(\{k\}) \approx H(\{xQ\}) \]

where \( x_i \) is the momentum fraction of the quark or the antiquark \( i \) entering the diagram \( H \).

An equivalent approximation applies to \( H' \). One also approximates

\[ \delta^{(4)}(k_1 + k_2 - k_3 - k_4) \approx \frac{\sqrt{2}}{|\sin \theta|Q^3} \prod_{i=2}^{4} \delta(x_1 - x_i) \delta(l_1 + l_2 - l_3 - l_4), \]

with \( l_i \) the momentum carried by the quark or antiquark \( i \) along the direction \( \eta \).

This equation shows that all momentum fractions in \( H \) are equal; one denotes \( x \), the unique resulting fraction, and \( \bar{x} = 1 - x \), the fraction which prevails in \( H' \).

One may then rearrange integrals in Eq. (76), by introducing the impact parameter \( b \)

\[ 2\pi \delta(l_i) = \int_{-\infty}^{+\infty} db e^{-i(l_3 + l_4 - l_1 - l_2)b}, \]

and the hybrid wave function of a meson propagating along the + direction,

\[ \mathcal{P}_{\alpha\beta}(x, b) = Q \int \frac{dl}{2\pi} \frac{e^{ib}}{|\sin \theta|} \int \frac{dk^- dk^1}{(2\pi)^3} X_{\alpha\beta}(xQ, k^-, k^1, l), \]

to get

\[ A(s, t) = \frac{\sqrt{2Q}}{2\pi |\sin \theta|} \int_{0}^{1} dx \int_{-\infty}^{\infty} db \left[ H(\{xp\})H'(\{\bar{x}p\})\right] \prod_{i=1}^{4} \mathcal{P}_{\alpha_i\beta_i}(x, b; p_i) Q. \]

Each hard process scales like \( Q^{-2} \), so that the naive scaling law for the reaction amplitude is \( \overline{b}|Q^{-3} \), where \( \overline{b} \) is a typical average between the quark and the antiquark in the valence state of the mesons. This is the exceptional scale dependence discussed in Section 1.3.

Let us stress that in a short distance convolution, we would have written

\[ A' = \prod_{i=1}^{4} \varphi_i(x_i) * T_H(\{x\}), \]

\( T_H \) consisting, at the lowest order in the exchange of three hard gluons for different \( x_i \)'s. In this convolution, one gluon becomes soft when all \( x_i \)'s become equal and \( T_H \) gets an infrared divergence of the type \( \int d^4k/k^4 \) [22].

### 4.2 Dynamical factorization and Sudakov suppression

We already stressed the importance of radiative corrections in processes involving hadrons: to evaluate a cross section with a perturbative treatment of the theory, one must check that the infrared regime is under control.
Taking radiative corrections into account modifies the hard process amplitude, leading to
\[ A(s, t) = \frac{\sqrt{2Q}}{2\pi|\sin\theta|} \int_0^1 dx \ H(xp) \ H'(\bar{xp}) \int_{-1/\Lambda}^{+1/\Lambda} db \ U(x, b, Q) \prod_{i=1}^4 \frac{P_i^{(0)}(x, b)}{Q}, \] (85)
where the \( U \) factor contains the corrections. These corrections turn out to be very important by strongly suppressing the integrand in the region where the impact parameter \( b \) is large in front of the scale \( 1/Q \). This is the Sudakov phenomenon already mentioned above. Then, the Sudakov-resummed amplitude is still dominated by a short distance dynamics. Let us here restrict to leading corrections. In axial gauge, they come from corrections on wave functions. The equation satisfied by \( P \) is
\[ \frac{\partial}{\partial \ln Q} P(x, b, Q) = -\frac{1}{2} \left( \int_{1/b}^{xQ} d\ln \mu' \gamma_K + \int_{1/b}^{xQ} d\ln \mu' \gamma_K \right) P(x, b, Q), \] (86)
where
\[ \gamma_K(\mu') = \frac{C_F}{\beta_1 \ln \mu'}, \] (87)
the solution of which is
\[ P(x, b, Q) = P^{(0)}(x, b) \exp -S(x, b, Q) \] (88)
where
\[ S(x, b, Q) = \left( \frac{c}{4} \ln xQ(u - 1 - \ln u) + x \leftrightarrow \bar{x} \right) \] (89)
with \( u(xQ, b) = -\frac{\ln b}{\ln xQ} \), and \( c = 2C_F/\beta_1 = 32/27 \) for three quark flavours.

The generic form of \( P \) shows the strong suppression of large transverse distances \( b \gg 1/Q \) (Sudakov suppression) and a regime without corrections (\( P \approx P^{(0)} \)) around \( b \sim 1/Q \). In Figure 13 we plot \(-S\) as a function of \( x \) and \( b \).

Figure 13: Exponent of the Sudakov suppression for the wave function \((-S(x, b, Q))\) for \( Q = 2 \) and 6 GeV (\( \Lambda = 200 \) MeV) as a function of the dipole transverse size \( b \) (in fm) and of the momentum fraction \( x \).
One observes that for intermediate values of the energy, the suppression affects only the region of large transverse distances, but with a very rapid decrease toward $-\infty$; at higher energies, however, the correction enforces the process to be dominated by short distances. Remember that it was a completely different mechanism, precisely the physics of the hard subprocess, which was driving exclusive processes in the short distance domain (see section 1).

\textbf{Back to the pion form factor}

This is where we can come back to the problems noted in section 2.5 concerning the study of the pion form factor at accessible energies. One may now envisage to compute the hard scattering without freezing the transverse degrees of freedom and use the $b$-dependence of the wave function [13]. One gets (with some technically justified approximations)

$$T(-xx'q^2, b) \approx \frac{2}{3}\pi\alpha_S(t)C_F K_0(\sqrt{-xx'q^2} b),$$

where $K_0$ is a modified Bessel function.

The interest of this improved approach is that taking radiative corrections grouped in the wave function into account, Eq. (88), and analyzing through the renormalization group the pertinent scale for the coupling $\alpha_S$ in the above expression of $T$, one gets

$$t = \max(1/b, \sqrt{xx'|q^2|}).$$

The Sudakov suppression of large transverse sizes enforces then the form factor to receive sizeable contributions at large transfer ($\gtrsim 5 \text{GeV}^2$), only from the region where $b$ is sufficiently small. The scale $t$ of the perturbative approach then remains large enough in the whole relevant integration domain.

Let us come back to the Landshoff independent scattering process. The $U$ factor in the amplitude is the product of factors $e^{-S}$ coming from the four wave functions, \textit{i.e.}

$$U(x, b, Q) = \exp - (c \ln xQ(u - 1 - \ln u) + x \leftrightarrow \bar{x}).$$

We shall not here explicitly calculate the hard diagrams which are necessary to quantitatively compute the amplitude, but simply show how the $U$ suppression modifies the counting rule found in section 1-3.

Let us thus study the behaviour of the amplitude for $Q \to \infty$. At large $Q$ values, one may analytically evaluate the $b$-integral in Eq. (85)

$$\int_0^{\Lambda^{-1}} db U(b, x, Q),$$

(93)
by a saddle point method. To do this, one approximates the exponent in $U$

$$c \ln xQ \left( -\frac{\ln b}{\ln xQ} - 1 - \ln \frac{\ln b}{\ln xQ} \right) + x \leftrightarrow \tilde{x} \approx 2c \ln \sqrt{xQ} (u - 1 - \ln u)$$ (94)

where $u = -\ln b/\ln \sqrt{xQ}$. Changing then variables, $b \rightarrow u$, one gets

$$\ln \sqrt{xQ} \int_0^{+\infty} du \exp -\ln \sqrt{xQ} (2c(u - 1 - \ln u) + u)$$ (95)

The exponent is maximum for $u_0 = \frac{2c}{2c+1}$ and one gets the approximate value

$$\int_0^{\Lambda^{-1}} db U(b, x, Q) \approx u_0 \sqrt{\frac{\pi \ln Q}{c}} (x\tilde{x}Q^2)^{c \ln u_0}.$$ (96)

The $x$-integration does not modify this behaviour but by logarithms. The effect of radiative corrections is thus to strongly suppress the contribution to the exclusive channel when the impact parameter $b \gg 1/Q$. The independent interactions must be spatially nearby and the scaling law is modified as $Q^{-3} \rightarrow Q^{-3.83}$, that is for the differential cross section $s^{-5} \rightarrow s^{-5.83}$. In the case of proton-proton elastic scattering, the modification is $s^{-8} \rightarrow s^{-9.7}$.[3]

The resulting scaling laws are thus not much different from those deduced for diagrams respecting the counting rules hypothesis ($s^{-10}$ in the $p$-$p$ case). These two types of processes are then able to compete and interfere in some energy interval where their amplitudes are comparable. This is how one can naturally explain [23] the experimentally observed oscillations in the differential cross section as due to the interference of the amplitudes of these two processes, the Sudakov form factor being accompanied by a “chromo-coulomb” phase, which depends logarithmically of the transfer.

## 5 Color transparency

Hard exclusive scattering (with a typical large $Q^2$ scale) selects a very special quark configuration: the minimal valence state where all quarks are close together, forming a small size color neutral configuration sometimes referred to as a mini hadron. This mini hadron is not a stationary state and evolves to build up a normal hadron.

Such a color singlet system cannot emit or absorb soft gluons which carry energy or momentum smaller than $Q$. This is because gluon radiation — like photon radiation in QED — is a coherent process and there is thus destructive interference between gluon emission amplitudes by quarks with “opposite” color. Even without knowing exactly how exchanges of soft gluons and other constituents create strong interactions, we know that these interactions must be turned off for small color singlet objects.
An exclusive hard reaction will thus probe the structure of a \textit{mini hadron}, i.e. the short distance part of a minimal Fock state component in the hadron wave function. This is of primordial interest for the understanding of the difficult physics of confinement. First, selecting the simplest Fock state amounts to the study of the confining forces in a colorless object which is quite reminiscent of the “quenched approximation” much used in lattice QCD simulations, where quark-antiquark pair creation from the vacuum is forbidden. Secondly, letting the mini-state evolve during its travel through different nuclei of various sizes allows an indirect but unique way to test how the squeezed mini-state goes back to its full size and complexity, \textit{i.e.} how quarks inside the proton rearrange themselves spatially to “reconstruct” a normal size hadron. In this respect the observation of baryonic resonance production as well as detailed spin studies are mandatory.

To the extent that the electromagnetic form factors are understood as a function of $Q^2$, \begin{equation}
  e + A \rightarrow e + (A - 1) + p
\end{equation}
experiments will measure the color screening properties of QCD. The quantity to be measured is the transparency ratio $T_r$ which is defined as:
\begin{equation}
  T_r = \frac{\sigma_{\text{Nucleus}}}{Z\sigma_{\text{Nucleon}}}
\end{equation}

At asymptotically large values of $Q^2$, dimensional estimates suggest that $T_r$ scales as a function of $A^4/Q^2$ \cite{24}. The approach to the scaling behavior as well as the value of $T_r$ as a function of the scaling variable determine the evolution from the pointlike configuration to the complete hadron. This highly interesting effect can be measured in an $(e,e'p)$ reaction that provides the best chance for a \textit{quantitative} interpretation. We will not present here the many ideas which have recently emerged in this new field \cite{25}.

5.1 Present Data

Experimental data on color transparency are very scarce but worth considering in detail. The first piece of evidence for something like color transparency came from the Brookhaven experiment on pp elastic scattering at 90° CM in a nuclear medium \cite{24}. These data lead to a lively debate with no unanimous conclusion. The problem is that hadron hadron elastic scattering is not an as-well clear cut case of short distance process as the electromagnetic form factors discussed above. There are indeed infrared sensitive processes (the so-called independent scattering mechanism) which allow not so small protons to scatter elastically. The phenomenon
of colour transparency is thus replaced by a nuclear filtering process: elastic scattering in a nucleus filters away the big component of the nucleon wave function and thus its contribution to the cross-section. Since the presence of these two competing processes had been analysed [23] as responsible of the oscillating pattern seen in the scaled cross-section $s^{10}d\sigma/dt$, an oscillating color transparency ratio emerges (see Figure 14).

Figure 14: Oscillating transparency ratio for pp elastic scattering at 90° [27].

One way to understand data is to define a survival probability related in a standard way to an effective attenuation cross section $\sigma_{\text{eff}}(Q^2)$ and to plot this attenuation cross section as a function of the transfer of the reaction [28]. One indeed obtains values of $\sigma_{\text{eff}}(Q^2)$ decreasing with $Q^2$ and quite smaller than the free space inelastic proton cross section. The survival probability is even found to obey a simple scaling law in $Q^2/A^{1/3}$ [24]. The SLAC NE18 experiment [29] recently measured the color transparency ratio up to $Q^2 = 7\text{ GeV}^2$, without any observable increase. These data are shown in Fig. 15. This casts doubts on the most optimistic views on very early dominance of pointlike configurations and emphasizes the importance of a sufficient boost to prevent small states to dress-up too quickly, then losing their ability to escape freely the nucleus.

Figure 15: The Transparency ratio as measured at SLAC [29].

The diffractive electroproduction of vector mesons at Fermilab [30] recently showed an
interesting increase of the transparency ratio for data at $Q^2 \approx 7\text{ GeV}^2$. In this case the boost is high since the lepton energy is around $E \approx 200\text{ GeV}$, but the problem is to disentangle diffractive from inelastic events.

5.2 Future prospects

It should by now be obvious to the reader that Color Transparency is just an emerging field of study and that one should devote much attention to get more information on this physics in the near future.

5.2.1 Eva

A second round of proton experiments at Brookhaven has been approved and a new detector named Eva [31] with much higher acceptance has been taking data for about one year. Along with other improvements and increased beam type, this should increase the amount of data taken by a factor of 400 allowing a larger energy range and an analysis at different scattering angles. It would be very interesting to analyze meson-nucleus scattering in similar conditions. It has been predicted [32] that the amount of helicity non conservation seen for instance in the helicity matrix elements of the $\rho$ meson produced in $\pi p \to \rho p$ at $90^\circ$ would be filtered out in a nucleus. Experimental data in free space [33] yield $\rho_{1-1} = 0.32 \pm 0.10$, at $s = 20.8\text{ GeV}^2$, $\theta_{\text{CM}} = 90^\circ$, for the non-diagonal helicity violating matrix element. If the persistence of helicity non-conservation is correctly understood as due to independent scattering processes which do not select mini-hadrons and thus are not subject to color transparency, helicity conservation should be restored at the same $Q^2$ in processes filtered by nuclei. One should thus observe a monotonous decrease of $\rho_{1-1}$ with $A$. 
5.2.2 Hermes

The Hermes detector at HERA is beginning operation. It will enable a confirmation of FNAL data on ρ meson diffractive production at moderate $Q^2$ values and quite smaller values of energies $10 \leq \nu \leq 22$ GeV. This experiment might however suffer from the same weakness as the one from Fermilab since Hermes small luminosity only allows integrated measurements and thus cannot assure that diffractive events are not polluted by inelastic events. It seems difficult to envisage in the near future the detection of the recoiling proton.

5.2.3 ELFE

The 15–30 GeV continuous electron beam ELFE project has been presented elsewhere. Besides the determination of hadronic valence wave functions through the careful study of many exclusive hard reactions in free space, the use of nuclear targets to test and use color transparency is one of its major goals. The $(e,e'p)$ reaction should in particular be studied in a wide range of $Q^2$ up to 21 GeV^2, thus allowing to connect to SLAC data (and better resolution but similar low $Q^2$ data from CEBAF) and hopefully clearly establish this phenomenon in the simplest occurrence. The normal component $P_n$ of the polarization of the recoiling proton will also be measured in order to discriminate against 2-nucleon knockout and to allow a limited but fruitful study of shell effects. The vanishing of $P_n$ is indeed a good signal of the absence of final state interactions.

The measurement of the transparency ratio for photo- and electroproduction of heavy vector mesons, in particular of $\psi$ and $\psi'$ will open a new regime where the mass of the quark enters as a new scale controlling the formation length of the produced meson. ELFE at 30 GeV will be the ideal machine to study these physics.

References

[1] S.J. Brodsky and G.R. Farrar, Phys. Rev. Lett. 31, 1153 (1973); V.A. Matveev, R.M. Muradyan and A.V. Tavkhelidze, Lett. Nuovo Cimento 7, 719 (1973).

[2] P.V. Landshoff, Phys. Rev. D10, 1024 (1974).

[3] J. Botts and G. Sterman, Nucl. Phys. B325, 62 (1989).

[4] G.R. Farrar and D. Jackson, Phys. Rev. Lett. 43, 246 (1979); S.J. Brodsky and G.P. Lepage, Phys. Lett. 87B, 359 (1979); A.V. Efremov and A.V. Radyushkin, Phys. Lett.
94B, 245 (1980); V.L. Chernyak, V.G. Serbo and A. R. Zhitnitsky, Yad. Fiz. 31, 1069 (1980); A. Duncan and A.H. Mueller, Phys. Rev. D21, 1636 (1980).

[5] E.E. Salpeter and H.A. Bethe, Physical Review 84, 1232 (1951).

[6] D. Lurie, *Particles and Fields* (Wiley-Interscience, New York, 1968).

[7] S.J. Brodsky, contribution to these proceedings.

[8] T. Gousset, “Hadron wave function in hard exclusive scattering”, these proceedings.

[9] J.F. Donoghue, E. Golowich and B.R. Holstein, *Dynamics of the Standard Model* (Cambridge University Press, Cambridge, 1992).

[10] R.D. Field, *Applications of Perturbative QCD* (Addison-Wesley, Redwood, 1989).

[11] V.L. Chernyak and A.R. Zhitnitsky, Phys. Rep. 112, 173 (1984).

[12] F.-M. Dittes and A.V. Radyushkin, Sov. J. Nucl. Phys. 34, 293 (1981).

[13] H.-N. Li and G. Sterman, Nucl. Phys. B381, 129 (1992).

[14] V.V. Sudakov, Sov. Phys. JETP 3, 65 (1956).

[15] A.F. Sill *et al.*, Phys. Rev. D28, 860 (1993).

[16] G.R. Farrar and E. Maina, Phys Lett B206, 120 (1988); G.R. Farrar and H. Zhang, Phys Rev Lett 65, 1721 (1990); G.R. Farrar and H. Zhang, Phys Rev D41, 3348 (1990).

[17] A.S. Kronfeld and B. Nižić, Phys Rev D44, 3445 (1991).

[18] G.R. Farrar, K. Huleihel and H. Zhang, Nucl. Phys. B349, 655 (1991).

[19] M.A. Shupe *et al.*, Phys. Rev. D19, 1929 (1979).

[20] P.Kroll, M.Schurmann and W. Schweiger, *Proceedings of Quark Cluster Dynamics*, (Bad Honnef, Germany 1992), edited by K. Goeke (Springer-Verlag, 1992) p 179; M. Anselmino *et al.*, Rev. Mod. Phys.66, 195 (1993).

[21] C.E. Carlson and J. Milana, Phys Rev D49, 5908 (1994).

[22] A.H. Mueller, Phys. Rep. 73, 237 (1981).

[23] B. Pire and J.P. Ralston, Phys. Lett. B117, 233 (1982).
[24] B. Pire and J.P. Ralston, Phys. Lett. B256, 523 (1991).

[25] J.P. Ralston, contribution to these proceedings; for a long list of references, see P. Jain, B. Pire and J.P. Ralston, to be published in Physics Reports; see also L. Frankfurt, G.A. Miller, and M. Strikman, Comments Nucl. Part. Phys. 21, 1 (1992).

[26] A.S. Carroll et al., Phys. Rev. Lett. 61, 1698 (1988).

[27] J.P. Ralston and B. Pire, Phys. Rev. Lett. 65, 2343 (1990).

[28] P. Jain and J.P. Ralston, Phys. Rev. D48, 1104 (1993).

[29] T.A. Armstrong et al., Phys. Rev. Lett. 70, 1212 (1993).

[30] M.R. Adams et al., Phys. Rev. Lett. 74, 1525 (1995).

[31] BNL–Experiment 850; spokesmen A. Carroll and S. Heppelmann.

[32] T. Gousset, B. Pire and J.P. Ralston, to be published in Phys Rev D; J.P. Ralston and B. Pire, in Polarized Collider Workshop (Penn State University, November 1990) edited by J.C. Collins et al. (AIP Conference Proceedings No. 223) p. 228.

[33] S. Heppelmann et al., Phys. Rev. Lett. 55, 1824 (1985)

[34] Hermes proposal, Report DESY–PRC 90/01.

[35] The ELFE Project Conference Proceedings, Vol.44, Italian Physical Society, Bologna, Italy (1993) edited by J. Arvieux and E. DeSanctis; J. Arvieux and B. Pire, Progress in Particle and Nuclear Physics, 30, 299 (1995).