Abstract. In a recent paper, Lin, Ruberman and Saveliev proved a splitting formula expressing the Seiberg-Witten invariant $\lambda_{SW}(X)$ of a smooth 4-manifold with rational homology of $S^1 \times S^3$ in terms of the Frøyshov invariant $h(X)$ and a Lefschetz number in reduced monopole Floer homology. In this note we observe that a similar splitting formula holds in reduced instanton Floer homology.

1. Introduction

Let $X$ be a smooth, oriented spin 4-manifold that has integral homology of $S^1 \times S^3$. A smooth invariant of $X$, denoted by $\lambda_{SW}(X)$, was defined by Mrowka, Ruberman and Saveliev [MRST11] as a signed count of Seiberg–Witten monopoles plus an index-theoretic correction term. Recently, Lin, Ruberman and Saveliev [LRS18] extended the definition of $\lambda_{SW}(X)$ to all smooth, oriented spin 4-manifolds with rational homology of $S^1 \times S^3$ and proved a splitting formula relating $\lambda_{SW}(X)$ to the Frøyshov invariant $h(Y, s)$, where $Y$ is an embedded rational homology 3-sphere generating $H_3(X; \mathbb{Z})$ (assuming such exists) and $s$ is the induced spin structure. It is proved in [Frø10] that $h(Y, s)$ is an invariant of $X$, denoted by $h(X)$. The splitting formula relating these two invariants reads [LRS18, Theorem A]

$$
\lambda_{SW}(X) + h(X) = -\text{Lef}(W_* : HM^{\text{red}}(Y, s) \to HM^{\text{red}}(Y, s)),
$$

where $W$ is the spin cobordism from $Y$ to itself obtained by cutting $X$ open along $Y$ and $HM^{\text{red}}(Y, s)$ is the reduced monopole Floer homology.

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In this note we show that a similar splitting formula holds when reduced monopole Floer homology is replaced by reduced instanton Floer homology, $\lambda_{\text{SW}}(X)$ is replaced by the Furuta–Ohta invariant $\lambda_{\text{FO}}(X)$, as defined in [FO93], and the monopole Frøyshov invariant $h(Y)$ is replaced by the instanton Frøyshov invariant $h(Y)$ (that the same notation is used for both Frøyshov invariants may be confusing, although the two invariants are known to coincide in all calculated examples).

More precisely, let $X$ be a $\mathbb{Z}[\mathbb{Z}]$-homology $S^1 \times S^3$, that is, a smooth 4-manifold such $H_*(X; \mathbb{Z}) = H_*(S^1 \times S^3; \mathbb{Z})$ and $H_*(\tilde{X}; \mathbb{Z}) = H_*(S^3; \mathbb{Z})$, where $\tilde{X}$ is the universal abelian cover of $X$. The moduli space $\mathcal{M}^s(X)$ of irreducible anti-self dual connections on a trivial $SU(2)$ bundle over $X$ is compact and has formal dimension zero; in fact, it coincides with the moduli space of flat $SU(2)$ connections on $X$. With an appropriate choice of orientations and admissible perturbations, the Furuta–Ohta invariant is defined as a signed count $\lambda_{\text{FO}}(X) = \frac{1}{4} \#\mathcal{M}^s(X)$, see Ruberman–Saveliev [RS04b] for details.

**Theorem A.** Let $X$ be a $\mathbb{Z}[\mathbb{Z}]$-homology $S^1 \times S^3$ and suppose that there exists an embedded integral homology sphere $Y$ which generates $H_3(X; \mathbb{Z})$. Cut $X$ open along $Y$ to obtain a homology cobordism $W$ from $Y$ to itself, and denote by $W_*$ and $\hat{W}_*$ the induced homomorphisms in, respectively, the instanton Floer homology and the reduced instanton Floer homology of $Y$. Then the quantity

$$h(X) = \frac{1}{2} (\text{Lef}(\hat{W}_*) - \text{Lef}(W_*))$$

(1)

defined in terms of the Lefschetz numbers is independent of $W$ and coincides with the instanton Frøyshov invariant $h(Y)$. Moreover, the following splitting formula holds

$$\lambda_{\text{FO}}(X) + h(X) = \frac{1}{2} \text{Lef}(\hat{W}_*: \hat{HF}^*(Y) \longrightarrow \hat{HF}^*(Y)).$$

(2)

We refer the reader to Frøyshov [Frø02] for the definitions of the reduced instanton Floer homology and the instanton Frøyshov invariant; see also Section 2 and Section 3. Note that the splitting formula (2) follows easily from (1) and the formula $\lambda_{\text{FO}}(X) = 1/2 \text{Lef}(W_*)$ of [RS04b].
To prove that $h(X)$ is a well-defined invariant, we will show that it coincides with the instanton Frøyshov invariant $h(Y)$ for any choice of $Y$. When $W$ is a product cobordism $Y \times [0,1]$, both homomorphisms $W_*$ and $\hat{W}_*$ are identity maps, and (1) reduces to the Frøyshov formula

$$h(X) = h(Y) = \frac{1}{2}(\chi(\hat{HF}_*(Y)) - \chi(HF_*(Y))).$$

We will prove the general case of (1) in this paper using properties of the special boundary maps of Frøyshov [Frø02].

**Example 1.** Let $Y$ be the Brieskorn homology 3-sphere $\Sigma(2, 7, 13)$ oriented as the link of a complex surface singularity, and $\tau$ the involution on $\Sigma(2, 7, 13)$ induced by complex conjugation. The mapping torus of $\tau$ is a smooth 4-manifold $X_\tau = [0,1] \times Y/(0,x) \sim (1, \tau(x))$ and a $\mathbb{Z}[\mathbb{Z}]$-homology $S^1 \times S^3$. The mod 8 graded instanton Floer homology of $Y$ is given by (see [FS90])

$$HF_*(Y) = (0, \mathbb{Z}^4, 0, \mathbb{Z}^2, 0, \mathbb{Z}^4, 0, \mathbb{Z}^2)$$

and the reduced instanton Floer homology

$$\hat{HF}_*(Y) = (0, \mathbb{Z}^2, 0, \mathbb{Z}^2, 0, \mathbb{Z}^2, 0, \mathbb{Z}^2)$$

is completely determined by $HF_*(Y)$ and the invariant $h(Y)$ which was computed by Frøyshov [Frø04] to be $h(Y) = 2$. In this case the Furuta–Ohta invariant $\lambda_{FO}(X_\tau)$ equals the Neumann–Siebenmann $\overline{\pi}$–invariant; see [RS04a], and a closed form expression is given by Saveliev [Sav00, Theorem 6.28, p.147] as $-b_1 + b_3$, where $b_i$ is the rank of $HF_i(Y)$. Therefore, $\lambda_{FO}(X_\tau) = -2$. On the other hand, let $W$ be the homology cobordism obtained by cutting $X_\tau$ open along $Y$. According to [RS04a Proposition 9.2], the induced map $W_* = \tau_* : HF_k(Y) \to HF_k(Y)$ is the identity for $k \equiv 1 \pmod{4}$ and minus the identity for $k \equiv -1 \pmod{4}$. It follows that $\text{Lef}(W_*) = -4$ and $\text{Lef}(\hat{W}_*) = 0$ hence

$$h(X_\tau) = \frac{1}{2}(\text{Lef}(\hat{W}_*) - \text{Lef}(W_*)) = 2,$$

which matches $h(Y)$ and hence confirms the splitting formula $\lambda_{FO}(X_\tau) + h(X_\tau) = 0 = 1/2 \text{Lef}(\hat{W}_*)$. 


Example 2. Let \( W \) be the Akbulut cork as in [RS04b, Section 9.3]. That is, \( W \) is a smooth contractible manifold with boundary an integral homology sphere \( \Sigma \) that can be embedded into a blown up elliptic surface \( E(n) \# \mathbb{CP}^2 \) in such a way that cutting it out and re-gluing by an involution \( \tau : \Sigma \to \Sigma \) changes the smooth structure on \( E(n) \# \mathbb{CP}^2 \) but preserves its homeomorphism type. The instanton homology groups are

\[
HF_*(\Sigma) = (0, \mathbb{Z}, 0, \mathbb{Z}, 0, \mathbb{Z})
\]

and, since \( \Sigma \) is homology cobordant to zero, \( h(\Sigma) = 0 \) and \( \widehat{HF}_*(\Sigma) = HF_*(\Sigma) \). Let \( X_{\tau} \) be the mapping torus of \( \tau \) then it follows from [RS04b] that \( W_* = -\text{Id} \) and \( \lambda_{\text{FO}}(X_{\tau}) = 2 \), which again confirms the splitting formula.

2. Reduced Instanton Floer (Co)homology

In this section, we recall the definition of the reduced instanton Floer homology (cohomology) groups; the reader is referred to Frøyshov [Frø02] for all the details. We work throughout with real coefficients and the orientation conventions of [Frø02] and use the canonical identification \( HF_q(Y) = HF^{5-q}(Y) \).

The instanton Floer cohomology \( HF^*(Y) \) is defined as homology of the mod 8 graded instanton cochain complex \( CF^*(Y) \) generated by the irreducible flat \( SU(2) \) connections on \( Y \), with respect to the boundary map \( d : CF^*(Y) \to CF^{*+1}(Y) \) given by a signed count of anti-self dual connections on \( \mathbb{R} \times Y \) of finite energy. The definition of the reduced instanton cohomology further employs special boundary maps \( \delta_0 \) and \( \delta'_0 \), which are defined as follows. Let \( \theta \) be the trivial flat connection on a trivialized \( SU(2) \) bundle over \( Y \). Define

\[
\delta : CF^4(Y) \to \mathbb{R} \quad \text{by} \quad \delta(\alpha) = \# \bar{\mathcal{M}}(\theta, \alpha),
\]

where \( \mathcal{M}(\theta, \alpha) \) is the moduli space of anti-self dual connections on \( \mathbb{R} \times Y \) limiting to \( \theta \) at \( +\infty \) and \( \alpha \) at \( -\infty \), and \( \bar{\mathcal{M}}(\theta, \alpha) \) its quotient by translations. Define

\[
\delta' : \mathbb{R} \to CF^1(Y) \quad \text{by} \quad \delta'(1) = \sum_{\beta} \# \bar{\mathcal{M}}(\theta, \beta) \cdot \beta,
\]
with the summation extending over the generators $\beta \in CF^1(Y)$. The maps $\delta$ and $\delta'$ satisfy equations $d\delta = 0$ and $d\delta' = 0$, thereby inducing maps in cohomology,

$$\delta_0 : HF^4(Y) \rightarrow \mathbb{R} \quad \text{and} \quad \delta'_0 : \mathbb{R} \rightarrow HF^1(Y).$$

The special boundary maps $\delta_0$ and $\delta'_0$ are further included into the sequence of maps

$$\delta_n = [\delta v^n] : HF^{4-4n}(Y) \rightarrow \mathbb{R} \quad \text{and} \quad \delta'_n = [v^n \delta'] : \mathbb{R} \rightarrow HF^{1+4n}(Y)$$

using the homomorphism $v : CF^*(Y) \rightarrow CF^{*+4}(Y)$ defined in [Frø02]. It follows from the cochain homotopy formula [Frø02 Theorem 4(ii)] that either $\delta_0$ or $\delta'_0$ must vanish; moreover, if $\delta_0 = 0$ then $\delta_n = 0$ for all $n$, and similarly for $\delta'_n$.

We also need to recall the relation between the special boundary maps and the homomorphisms in Floer homology induced by negative definite cobordisms. Let $W$ be a connected Riemannian 4-manifold with two cylindrical ends, $\mathbb{R}_- \times Y_1$ and $\mathbb{R}_+ \times Y_2$, and assume that $W$ has negative definite intersection form, $H_1(W; \mathbb{Z}) = 0$, and both $Y_1$ and $Y_2$ are integral homology spheres. Then $W$ induces a degree preserving cochain homomorphism

$$W^* : CF^*(Y_1) \rightarrow CF^*(Y_2) \quad \text{by} \quad W^*(\alpha) = \sum_\beta \# \mathcal{M}(W; \beta, \alpha) \cdot \beta,$$

where the summation extends over the generators $\beta \in CF^*(Y_2)$ with the same index as $\alpha$, and $\mathcal{M}(W; \beta, \alpha)$ is the zero-dimensional part of the moduli space of anti-self dual connections on $W$ limiting to $\alpha$ at $-\infty$ and to $\beta$ at $+\infty$. Frøyshov [Frø02 Theorem 7] shows that $\delta_0 W^* = \delta_0$ and $W^* \delta'_0 = \delta'_0$ and, moreover, that there exist integers $a_{ij}$ and $b_{ij}$ such that

$$\delta_n W^* = \delta_n + \sum_{i=0}^{n-1} a_{in} \delta_i, \quad \text{(3)}$$

$$W^* \delta'_n = \delta'_n + \sum_{i=0}^{n-1} b_{in} \delta'_i. \quad \text{(4)}$$

A straightforward grading count shows that $a_{in} = 0$ and $b_{in} = 0$ whenever $i$ and $n$ have opposite parity.
We are now ready to define the reduced instanton cohomology groups. Let \( B^* \subseteq HF^*(Y) \) denote the linear span of the vectors \( \delta_n'(1) \) for all \( n \) so that \( B^1 = \text{span}\{\delta_{2k}^2(1)\} \) and \( B^3 = \text{span}\{\delta_{2k+1}^2(1)\} \) with \( k \geq 0 \), while \( B^q = 0 \) for \( q \neq 1, 5 \pmod{8} \). In addition, let

\[
Z^* = \bigcap_n \ker(\delta_n) \subseteq HF^*(Y)
\]

so that

\[
Z^0 = \bigcap_{k \geq 0} \ker(\delta_{2k+1}) \subseteq HF^0(Y) \quad \text{and} \quad Z^4 = \bigcap_{k \geq 0} \ker(\delta_{2k}) \subseteq HF^4(Y),
\]

while \( Z^q = HF^q(Y) \) for \( q \neq 0, 4 \pmod{8} \). The reduced Floer cohomology groups are then defined as

\[
\widehat{HF}^*(Y) = Z^*/B^q.
\]

For any negative definite cobordism \( W \) as above, the map \( W^* \) leaves the subspaces \( Z^q \) and \( B^q \) invariant and induces a map \cite[Theorem 8]{Fr02}

\[
W^* : \widehat{HF}^*(Y_1) \longrightarrow \widehat{HF}^*(Y_2).
\]

Finally, we mention that both instanton Floer cohomology \( HF^*(Y) \) and reduced instanton Floer homology \( \widehat{HF}^*(Y) \), which \textit{a priori} have a mod 8 grading, are 4-periodic; see \cite[Theorem 2 and Corollary 3]{Fr02}.

3. Proof of Theorem A

Recall that the (instanton) Frøyshov invariant \( h(Y) \) is defined by the formula

\[
h(Y) = \frac{1}{2} (\chi(HF^*(Y)) - \chi(\widehat{HF}^*(Y))).
\]

The theorem will be proved as soon as we show that \( h(X) = h(Y) \) for any choice of \( Y \). Using the aforementioned 4-periodicity in Floer homology, we only have two cases to consider, depending on which one of the special boundary maps, \( \delta_0 \) or \( \delta_0' \), vanishes.

Case 1. Suppose that \( \delta_0' = 0 \) and hence \( \delta_n' = 0 \) for all \( n \). This implies that \( B^1 = 0 \) and \( \widehat{HF}^1(Y) = Z^1 = HF^1(Y) \). Thus the reduced theory
differs from the unreduced one only in degree 0 mod 4, and
\[ h(X) = \frac{1}{2} (\text{Lef}(W^*) - \text{Lef}(\hat{W}^*)) \]
\[ = \text{Tr} (W^* : HF^0(Y) \to HF^0(Y)) - \text{Tr} (\hat{W}^* : \hat{HF}^0(Y) \to \hat{HF}^0(Y)). \]

Note that, since \( B^0 = 0 \), we have
\[ \hat{HF}^0(Y) = \mathbb{Z}^0 = \bigcap_{k \geq 0} \ker(\delta_{2k+1}). \]

We claim that \( h(X) = \dim(HF^0(Y)/Z^0) \); this will imply that \( h(X) \) equals the Frøyshov invariant \( h(Y) = \dim HF^0(Y) - \dim \hat{HF}^0(Y) \).

To prove the claim, we first observe that, since \( \delta_1 W^* = \delta_1 \), we have a commutative diagram of exact sequences
\[
\begin{array}{c}
0 \longrightarrow Z_1^0 \longrightarrow HF^0(Y) \xrightarrow{\delta_1} \mathbb{R} \\
| & | \quad | & | \quad | & |
\downarrow \hat{W}_1^* \quad \downarrow W^* \quad \downarrow \text{Id}
\end{array}
\]
\[
\begin{array}{c}
0 \longrightarrow Z_1^0 \longrightarrow HF^0(Y) \xrightarrow{\delta_1} \mathbb{R} \\
\end{array}
\]
where \( Z_1^0 = \ker(\delta_1) \) and \( \hat{W}_1^* \) is the restriction of \( W^* \) to \( Z_1^0 \). It follows that \( \text{Tr}(W^*) - \text{Tr}(\hat{W}_1^*) = 1 \) if \( \delta_1 \neq 0 \) and \( \text{Tr}(W^*) - \text{Tr}(\hat{W}_1^*) = 0 \) if \( \delta_1 = 0 \).

Next we use relation (3) to define a sequence of \( W^* \)-invariant subspaces
\[ Z_k^0 = \ker(\delta_1) \cap \ldots \cap \ker(\delta_k) \subseteq HF^0(Y) \]
for \( k \) odd, and construct the following commutative diagrams with exact rows,
\[
\begin{array}{c}
0 \longrightarrow Z_k^0 \longrightarrow Z_{k-2}^0 \xrightarrow{\delta_k} \mathbb{R} \\
| & | \quad | & | \quad | & |
\downarrow \hat{W}_k^* \quad \downarrow \hat{W}_{k-2}^* \quad \downarrow \text{Id}
\end{array}
\]
\[
\begin{array}{c}
0 \longrightarrow Z_k^0 \longrightarrow Z_{k-2}^0 \xrightarrow{\delta_k} \mathbb{R} \\
\end{array}
\]
where \( \hat{W}_k^* \) denotes the restriction of \( W^* \) to subspace \( Z_k^0 \). It follows that \( \text{Tr}(\hat{W}_k^*) - \text{Tr}(\hat{W}_{k-2}^*) = 1 \) if \( \delta_k \neq 0 \) and \( \text{Tr}(\hat{W}_k^*) - \text{Tr}(\hat{W}_{k-2}^*) = 0 \) if \( \delta_k = 0 \).

Apply induction to the sequence of subspaces
\[ Z_m^0 \subseteq \ldots \subseteq Z_3^0 \subseteq Z_1^0 \subseteq HF^0(Y) \]
to conclude that
\[ \text{Tr}(W^*) = \dim(HF^0(Y)/Z_m^0) + \text{Tr}(\hat{W}_m^*) \]

for all odd $m$. Since $Z^0 = Z^0_m$ and $\hat{W}^* = \hat{W}^*_m$ for all sufficiently large odd $m$, the claim follows.

**Case 2.** Suppose that $\delta_0 = 0$ and hence $\delta_n = 0$ for all $n$. In this case, $\hat{HF}^0(Y) = HF^0(Y)$ and $\hat{HF}^1(Y) = HF^1(Y)/B^1$. The reduced Floer cohomology differs from Floer cohomology only in degree 1 mod 4, and

$$h(X) = \frac{1}{2} \left( \text{Lef}(W^*) - \text{Lef}(\hat{W}^*) \right)$$

$$= \text{Tr} (\hat{W}^* : \hat{HF}^1(Y) \to \hat{HF}^1(Y)) - \text{Tr} (W^* : HF^1(Y) \to HF^1(Y))$$

It follows from the commutative diagram of exact sequences

$$\begin{array}{ccc}
0 & \longrightarrow & B^1 \\
\downarrow W_i^* & & \downarrow W^* \\
0 & \longrightarrow & B^1 \\
\end{array}$$

that $\text{Tr}(W^*) - \text{Tr}(\hat{W}^*) = \text{Tr}(W_1^*)$, where $W_1^*$ is the restriction of $W^*$ to $B^1$. We claim that

$$\text{Tr}(W_1^*) = \dim(B^1) = \dim HF^1(Y) - \dim \hat{HF}^1(Y).$$

This will imply that $h(X)$ equals $h(Y) = \dim \hat{HF}^1(Y) - \dim HF^1(Y)$ so the conclusion will follow.

To prove the claim, use relation (4) to define $W^*$-invariant subspaces $B^1_k = \text{span}\{\delta'_0(1), \ldots, \delta'_k(1)\}$ for $k$ even. We have an increasing sequence of quotient spaces,

$$\hat{HF}^1(Y) = HF^1(Y)/B^1_m \subseteq HF^1(Y)/B^1_{m-2} \subseteq \ldots \subseteq HF^1(Y)/B^1_0$$

corresponding to the sequence $B^1_0 \subseteq \ldots \subseteq B^1_m \subset HF^1(Y)$, where $B^1 = B^1_m$ for some sufficiently large even $m$. We also have the commutative diagrams with exact rows

$$\begin{array}{c}
\mathbb{R} \rightarrow \delta_k \rightarrow HF^1(Y)/B^1_{k-2} \rightarrow HF^1(Y)/B^1_k \rightarrow 0 \\
\downarrow \text{id} \downarrow W_{k-2} \downarrow W_k \\
\mathbb{R} \rightarrow \hat{W}^*(1) \rightarrow \hat{HF}^1(Y)/\hat{B}^1_{k-2} \rightarrow \hat{HF}^1(Y)/\hat{B}^1_k \rightarrow 0 \\
\downarrow \text{id} \downarrow \hat{W}_{k-2} \downarrow \hat{W}_k \\
\end{array}$$
and the relations $\text{Tr}(W_{k-2}^{*}) - \text{Tr}(W_{k}^{*}) = 1$ if $\delta'_{k} \neq 0$ and $\text{Tr}(W_{k-2}^{*}) - \text{Tr}(W_{k}^{*}) = 0$ if $\delta'_{k} = 0$. By induction we obtain

$$\text{Tr}(W_{0}^{*}) = \dim(B_{1}^{1}/B_{0}^{1}) + \text{Tr}(\hat{W}^{*}).$$

Finally, it follows from the relation $W^{*}\delta'_{0} = \delta'_{0}$ and the exact sequence

$$0 \rightarrow B_{1}^{1} \rightarrow HF^{1}(Y) \rightarrow HF^{1}(Y)/B_{0}^{1} \rightarrow 0$$

that $\text{Tr}(W_{0}^{*}) = \text{Tr}(W^{*}) - \dim(B_{0}^{1})$. Therefore, $\text{Tr}(W^{*}) - \text{Tr}(\hat{W}^{*}) = \dim(B^{1})$, which completes the proof of the claim.

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