Systematics of the heavy flavor hadronic molecules

Kan Chen¹, Rui Chen², Lu Meng³, Bo Wang⁴,⁵, * and Shi-Lin Zhu¹†

¹School of Physics and Center of High Energy Physics, Peking University, Beijing 100871, China
²Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, Department of Physics and Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha 410081, China
³Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany
⁴School of Physical Science and Technology, Hebei University, Baoding 071002, China
⁵Key Laboratory of High-precision Computation and Application of Quantum Field Theory of Hebei Province, Baoding 071002, China

With a quark level interaction, we give a unified description of the loosely bound molecular systems composed of the heavy flavor hadrons (D, D′), (Λc, Σc, Σc′), and (Ξc, Ξc′, Ξc′′). Using the Pc states as inputs to fix the interaction strength of light quark-quark pairs, we reproduce the observed Pc states and predict another narrow Tcc⁺ state with quantum numbers [D∗⁺D∗⁻]I=1/2 J=1/2. If we require a satisfactory description of the Tcc⁺ and Pc states simultaneously, our framework prefers the assignments of the Pc(4440) and Pc(4457) as the [ΣcD∗⁺]I=1/2 J=1/2 and [ΣcD∗⁺]I=3/2 J=3/2 states, respectively. We propose the isospin criterion to explain naturally why the experimentally observed Tcc⁺, Pc, and Pcs molecular candidates prefer the lowest isospin numbers. We also predict the loosely bound states for the bottom di-hadrons.

I. INTRODUCTION

The conventional mesons (qq) and baryons (qqq) have been extensively discussed at the birth of quark model [1–3], and have become the main part of hadron spectrum nowadays [4]. Besides conventional mesons and baryons, quantum chromodynamics (QCD) also allows the existence of hadrons with more complicated configurations, such as qqqq, qqqqqqq, etc.. In the past decades, many XYZ states have been observed [5–12] since the discovery of Χ(3872) in 2003 [13]. Some of these states are below the thresholds of di-hadrons from several to several tens MeVs. The molecular explanations were widely proposed to understand their underlying structures.

In 2015, the LHCb Collaboration reported two structures Pc(4380) and Pc(4450) in the J/ψp mass spectrum [14]. Their masses are consistent with the predictions of the hidden-charm pentaquarks [15–17]. In 2019, the LHCb Collaboration updated their analyses with larger data samples and found that the Pc(4450) consists of two Pc states, i.e., Pc(4440) and Pc(4457) [18]. Besides, they further propose a new near threshold hidden-charm pentaquark Pc(4312). These important results from LHCb provide strong evidences for the existence of the hidden-charm molecular pentaquarks [19–29]. Thus, it is desirable to see whether there exist the hadronic molecules in other heavy flavor di-hadron systems.

If enlarging the flavor symmetry group to SU(3), one may expect that the ΞcD(∗), Ξc∗D(∗), and Ξc∗D(∗) systems may also form molecular bound states. The Pc states were investigated in Refs. [15, 30–34] and the most promising production channel Ξc⁻ → J/ψAK was suggested in Refs. [31, 35]. Later, the LHCb reported the evidence of Pcs(4459) in the J/ψΛ invariant mass spectrum [36], which agrees very well with the prediction from the chiral effective field theory in Ref. [34]. However, this state still needs further confirmation due to limited data samples at present [36]. Very probably, the Pc and Pcs pentaquarks share a very similar binding mechanism.

Very recently, the LHCb reported a very narrow structure Tcc⁺(3875) in the D⁰D⁰π⁺ spectrum [37, 38]. Its mass is slightly below the D∗⁺D⁰ threshold about 300 keV. This signal tends to confirm the predictions of the DD⁺ molecular state with quantum numbers I(JP) = 0(1⁺) [39]. The doubly heavy tetraquark states have been extensively studied and debated in the literatures [39–65]. One can refer to Ref. [6] for a review of the QQ̅q̅q̅ system. This inspiring discovery also stimulated a series of theoretical studies [66–83].

The minimal valance quark components for the Pc, Pcs, and Tcc⁺ states are ccud, ccuds, and ccud, respectively. Such states can not be accommodated within the conventional quark model and thus give us a golden platform to study the structures and dynamics of the multiquark states.

We propose the following picture to understand the above heavy flavor di-hadron systems. The heavy quark (c or b) behaves like a static color triplet source in each hadron. In the heavy quark limit, its velocity v does not change with time. The non-relativistic property of heavy quarks is stimulative to stabilize a molecular system [84–86]. On the other hand, if the particular combinations of the light quark components within the two heavy hadrons can coincidentally provide the attractive force, then the residual strong interaction that mainly comes from their light degrees of freedom (d.o.f) may render this system to be bound. This picture is similar to a hydrogen molecule in QED, where the two electrons are shared by the two protons and the residual electromagnetic force binds this system. Therefore, the question of which heavy flavor di-hadron can form a bound state becomes which kind of light quark combinations in the di-hadron system can provide the enough attractive force.

In this work, we adopt the quark level interaction to address the above question. We relate the heavy flavor di-hadron effective potentials to their matrix elements in the flavor and spin spaces of their light d.o.f, so that we can qualitatively

* wangbo@hbu.edu.cn
† zhushi@pku.edu.cn

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determine which heavy flavor di-hadron is possible to form a bound state.

This paper is organized as follows. In Sec. II, we introduce our theoretical framework. Then we present our numerical results and discussions in Sec. III. In Sec. IV, we conclude this work with a short summary.

II. THEORETICAL FRAMEWORK

We consider the two-body systems that are the combinations of the ground-state hadrons \((\bar{D}, D^*)\), \((\Lambda_c, \Sigma_c, \Sigma_c^*)\), and \((\Xi_c, \Xi_c^*, \Xi_c^*)\). We list the physical allowed heavy flavor meson-meson, meson-baryon, and baryon-baryon systems \([H_1 H_2]_f\) in Table I, where \(H_1\) and \(H_2\) denote the considered heavy flavor hadrons, while \(J\) and \(I\) are the total angular momentum and total isospin of the di-hadron system, respectively. If \(H_1\) and \(H_2\) are the general identical particles \((e.g., \bar{D} D\) and \(D^* D^*)\), then the quantum numbers of the \([H_1 H_2]_f\) (fermion or boson) systems must satisfy the following selection rule

\[ L + S_{\text{tot}} + I_{\text{tot}} + 2i = \text{Even number}, \quad (1) \]

where \(L\) is the orbital angular momentum between \(H_1\) and \(H_2\) \((L = 0\) for the \(s\)-wave case in our calculations), while \(S_{\text{tot}}\) and \(I_{\text{tot}}\) are the total spin and total isospin of the general identical di-hadron system, respectively. \(i\) denotes the isospin of \(H_1\) or \(H_2\), \(e.g., i = 1/2\) for the \(D\) meson, and \(i = 1, 2\) for the \(\Sigma_c\) and \(\Xi_c\) baryons, respectively. The \(2s\) \((s\) is the spin of \(H_1\) or \(H_2\)) is omitted from Eq. (1) since it must be an odd or even number for the two-body systems of the identical fermions or bosons, respectively.

| Table I. The allowed heavy flavor di-hadron systems \([H_1 H_2]_f\) that are considered in this work, where the quantum numbers of the general identical systems are constrained by Eq. (1). |
|----------------------------------|
| **Meson-meson**               |
| \([\bar{D} D]_f\)              |
| \([\bar{D} \bar{D}^*]_f^{0,1}\) |
| \([\bar{D}^* D^*]_f^{0,1}\)   |
| \([\Lambda_c \bar{D}]_f^{1,1}\) |
| \([\Lambda_c \bar{D}^*]_f^{1,1}\) |
| \([\Sigma_c \bar{D}]_f^{1,1}\) |
| \([\Sigma_c \bar{D}^*]_f^{1,1}\) |
| \([\Xi_c \bar{D}]_f^{1,1}\)   |
| \([\Xi_c \bar{D}^*]_f^{1,1}\) |
| **Baryon-meson**               |
| \([\Xi_c^0]_f^{0,1}\)        |
| \([\Xi_c^0]_f^{0,1}\)        |
| \([\Xi_c^0]_f^{0,1}\)        |
| \([\Xi_c^0]_f^{0,1}\)        |
| \([\Xi_c^0]_f^{0,1}\)        |
| \([\Xi_c^0]_f^{0,1}\)        |
| **Baryon-baryon**             |
| \([\Sigma_c^* \Sigma_c^*]_f^{1,1}\) |
| \([\Xi_c^0]_f^{0,1}\)        |
| \([\Xi_c^0]_f^{0,1}\)        |
| \([\Xi_c^0]_f^{0,1}\)        |
| \([\Xi_c^0]_f^{0,1}\)        |
| \([\Xi_c^0]_f^{0,1}\)        |
| \([\Xi_c^0]_f^{0,1}\)        |
| \([\Xi_c^0]_f^{0,1}\)        |

As discussed in the introduction, we assume that the interactions in a heavy flavor di-hadron systems are mainly from the interactions of their light quark components. We neglect the corrections from the heavy d.o.f and study the residual strong interactions induced from their light d.o.f. Then the heavy flavor meson-meson, meson-baryon, and baryon-baryon can be studied simultaneously in the same formalism by checking the interactions with all possible quantum numbers.

We focus on the two-body \(s\)-wave interactions among the considered ground heavy flavor mesons/baryons. One can formulate the corresponding quark level Lagrangians as \([34, 87, 88]\)

\[ \mathcal{L} = g_s \bar{q} S q + g_a \bar{q}_i \gamma_5 A^\mu q, \quad (2) \]

where \(q = (u, d, s)\), \(g_s\) and \(g_a\) are two independent coupling constants. They encode the nonperturbative dynamics between light quarks of two color singlet hadrons and can be determined from the experimental data.

From Eq. (2), we can see that the systems listed in Table I can only couple to the isospin triplet and isospin singlet fields. The systems that can couple to the strange isospin doublet fields are not considered in the present work. Then the fictitious scalar field \(S\) and axial-vector field \(A^\mu\) reduce to the form

\[ S = S_3 \lambda^i + S_1 \lambda^8, \quad (3) \]
\[ A^\mu = A^3_3 \lambda^i + A^1_1 \lambda^8, \quad (4) \]

where \(\lambda^i\) \((i = 1, 2, 3)\) and \(\lambda^8\) are the generators of SU(3) group. \(S_3\) \((A^3_3)\) and \(S_1\) \((A^1_1)\) denote the isospin triplet and isospin singlet fields, respectively.

The effective potential of light quark-quark interactions can be deduced from Eq. (2), and we have

\[ V_{qq} = \bar{g}_s \left( \lambda^1_1 \lambda^8_2 + \lambda^1_2 \lambda^2_1 \right) + \bar{g}_a \left( \lambda^1_1 \lambda^8_2 + \lambda^1_2 \lambda^2_1 \right) \sigma_1 \cdot \sigma_2, \quad (5) \]

where the effective potential \(V_{qq}\) is reduced to the local form when we integrate out the exchanged spurions (which is analogous to the resonance saturation model [89]). The redefined coupling constants are \(\bar{g}_s = g_s^3/m_N^3\) and \(\bar{g}_a = g_a^3/m_N^3\). Then the heavy flavor di-hadron effective potential from the interactions of their light quark components can be written as

\[ V_{[H_1 H_2]_f} = \left[h_{[H_1 H_2]_f} \lambda_{[V qq]} h_{[H_1 H_2]_f} \right], \quad (6) \]

where \([H_1 H_2]_f\) denotes the quark-level spin-flavor wave function of \(H_1 H_2\) system with total isospin \(I\) and total angular momentum \(J\), which is the direct product of spin and flavor wave functions

\[ \left[h_{[H_1 H_2]_f} \right] = \sum_{m_1, m_2} C_{I_1}^{I_2} \phi_{I_1, m_1}^{I_2, m_2} \phi_{H_1}^{H_2}, \quad (7) \]

The constants \(C_{I_1}^{I_2}\) are Clebsch-Gordan (CG) coefficients. \(\phi_{H_1}^{H_2}\) are the quark-level (flavor, spin) wave functions for the \(H_1\) and \(H_2\) states.
From Eqs. (5) and (6), we can see that the residual strong interaction of a specific $[H_1 H_2]^j$ system can be divided into four parts, i.e., the scalar type ($\lambda^j \lambda^j$), the isospin related type ($\lambda^j \lambda^j$), the spin related type ($\lambda^j \lambda^j$), and the isospin-spin related type ($\lambda^j \lambda^j$) interactions. In Table II, we present the matrix elements of these four types of operators for the considered di-hadron systems in Table I.

After we obtain the effective potential of the $[H_1 H_2]^j$ system, we need to check whether this system can form a bound state. This can be achieved by solving the following Lippmann-Schwinger equation (LSE),

$$T(p', p) = V(p', p) + \int \frac{d^3q}{(2\pi)^3} \frac{V(p', q) T(q, p)}{E - \frac{q^2}{2\mu} + i\epsilon},$$

where $m_\mu$ is the reduced mass of the $H_1$ and $H_2$. $p$ and $p'$ represent the momentum of the initial and final states in the center of mass frame, respectively.

Here, we introduce a hard regulator to exclude the contributions from higher momenta $[73, 90]$

$$V(p, p') = V_{[H_1 H_2]^j} \Theta(\Lambda - \theta) \Theta(\Lambda - p'),$$

where $\Theta$ is the step function. The amplitude $T(p', p)$ is a function of $p'$, $p$, and binding energy $E$ with a separable form

$$T(p', p) = \beta(E) \Theta(\Lambda - p') \Theta(\Lambda - p).$$

Then the LSE can be reduced to an algebraic equation

$$\beta(E) = \frac{V_{[H_1 H_2]^j}}{1 - \frac{V_{[H_1 H_2]^j}}{G}},$$

with

$$G = \frac{m_\mu}{\pi^2} \left[ -\Lambda + k \tan^{-1} \left( \frac{\Lambda}{k} \right) \right], k = \sqrt{-2m_\mu E}.$$ 

We can search for the pole position of Eq. (11) to obtain the binding energy of the $[H_1 H_2]^j$ system.

### III. NUMERICAL RESULTS

#### A. The results of the $P_\alpha$, $P_{\pi\pi}$, and $T_{\pi\pi}$ states

We first use the masses of the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ in Ref. [18] to fix the parameters in our model. In our previous work [29], we suggested that the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ have the assignments $[\Sigma^* D]^1_{1/2}$, $[\Sigma^* D]^1_{1/2}$, and $[\Sigma^* D]^1_{3/2}$, respectively. With the matrix elements listed in Table II, we can easily read out the effective potentials for these three $P_c$ states,

$$V_{P_c(4312)} = -\frac{10}{3} \bar{g}_s,$$

$$V_{P_c(4440)} = -\frac{10}{3} \bar{g}_s + \frac{40\bar{g}_a}{9},$$

$$V_{P_c(4457)} = -\frac{10}{3} \bar{g}_s - \frac{20\bar{g}_a}{9}.$$  

There exist three undetermined parameters in Eq. (11), the light quark-quark coupling constants $\bar{g}_s$, $\bar{g}_a$, and the momentum cutoff $\Lambda$. We use the experimental mass of the $P_c$ states [18] to precisely extract these three parameters. The solutions are $\bar{g}_s = 11.739$ GeV$^{-2}$, $\bar{g}_a = -2.860$ GeV$^{-2}$, and $\Lambda = 0.409$ GeV. In our convention, a positive (negative) $V_{[H_1 H_2]^j}$ means a(n) repulsive (attractive) interaction. Once we determine the signs of the $\bar{g}_s$ and $\bar{g}_a$, we can directly find out whether the considered systems have repulsive or attractive forces from the values in Table II.

From the point of view of the potential model, the spin-spin interaction is suppressed by a factor of $1/(m_{a(\tau)} m_{\bar{D}(\bar{c})})$, which is roughly consistent with our obtained ratio $|\bar{g}_a|/|\bar{g}_s| \approx 0.24$.

The cutoff $\Lambda$ is smaller than the masses of the ground scalar or axial-vector mesons, which are regarded as the hard scales and integrated out in the effective field theory. In principle, there may exist contributions from the pion-exchange interactions. Although the two ground heavy meson/baryons can easily couple to the pion field via the $p$-wave interactions, our calculations based on the chiral effective field theory [23, 29, 34, 88, 91] showed that the magnitude of the one-pion-exchange (OPE) interaction is comparable to that of the next-to-leading order two-pion-exchange (TPE) interaction. The OPE and TPE have considerable corrections to the binding energies of the bound states, but they are not the main driving force of the formation of the bound states. In this work, we do not include the pion exchange dynamics.

In Table III, we list the masses of the experimentally observed molecular candidates and the results from our model. The center values of the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ are used as inputs to determine the values of $\bar{g}_s$, $\bar{g}_a$, and $\Lambda$. We also predict a $[\Sigma^* D]^1_{1/2}$ molecular state with the mass 4376.2 MeV, which may correspond to the observed $P_c(4380)$ state [14]. The bound state $[\Sigma^* D]^1_{3/2}$ is also obtained in different models [19, 20, 22, 24–26, 28, 29].

We further adopt our picture to study the recently observed $T_{\pi\pi}^+$ state. We assign the $T_{\pi\pi}^+$ as the $[DD]^1_1$ molecular state and calculate its mass with the same parameters extracted from the $P_c$ states. As shown in Table III, our approach gives a rather good description of the mass of $T_{\pi\pi}^+$. This nice agreement indicates that neglecting the corrections from the heavy degrees of freedom is a fairly good approximation in this case. The heavy flavor meson-meson and baryon-baryon systems share the same binding mechanism that is dominated by their light degrees of freedom.

The $P_c(4459)$ is close to the threshold of $\Xi_c^0 \bar{D}^*$, which can be assigned as a $[\Xi_c^0 D^*]^0_{1/2}$ or $[\Xi_c^0 D^*]^0_{3/2}$ molecular state [34]. The spin of the light diquark in the $\Xi_c$ baryon is 0, so the $\Xi_c^0 \bar{D}^*$ system has the vanishing spin-spin interaction from its light d.o.f. The $[\Xi_c D]^1_{1/2}$ and $[\Xi_c D]^1_{3/2}$ states are degenerate in our formalism, as can be seen from Table II. The inclusion of the spin-spin interaction from heavy degrees of freedom or pion-exchange shall distinguish these two states, which is beyond the scope of the present work. However, from a serious calculation within the framework of chiral effective field theory [34], the mass gaps induced from the spin-spin...
TABLE II. The matrix elements of the operators $O_1 (\lambda_1 \lambda_1^* \sigma_1 \cdot \sigma_2)$, $O_2 (\lambda_2 \lambda_2^* \sigma_1 \cdot \sigma_2)$, $O_3 (\lambda_1 \lambda_1^* \sigma_3 \cdot \sigma_2)$, and $O_4 (\lambda_1 \lambda_1^* \sigma_1 \cdot \sigma_2)$ for the considered heavy flavor hadron-hadron systems ($[H, H]^J_2$) listed in Table I.

| System | $O_1$ | $O_2$ | $O_3$ | $O_4$ |
|--------|-------|-------|-------|-------|
| $[D D]^1_0$ | 1/2 | 1    | 0     | 0     |
| $[D^* D^*]^1_0$ | 1/2 | 1/2  | -1/2  | -1/2  |
| $[\Lambda, \bar{D}]^1_0$ | 1/2 | 1/2  | -1/2  | -1/2  |
| $[\Sigma, \bar{D}]^1_0$ | 1/2 | 1/2  | -1/2  | -1/2  |
| $[\Sigma^* \bar{D}]^1_0$ | 1/2 | 1/2  | -1/2  | -1/2  |
| $[\Xi, \bar{D}]^1_0$ | 1/2 | 1/2  | -1/2  | -1/2  |
| $[\Xi^* \bar{D}]^1_0$ | 1/2 | 1/2  | -1/2  | -1/2  |
| $[\Lambda, \bar{D}]^3_0$ | 1/2 | 1/2  | -1/2  | -1/2  |
| $[\Sigma, \bar{D}]^3_0$ | 1/2 | 1/2  | -1/2  | -1/2  |
| $[\Sigma^* \bar{D}]^3_0$ | 1/2 | 1/2  | -1/2  | -1/2  |
| $[\Xi, \bar{D}]^3_0$ | 1/2 | 1/2  | -1/2  | -1/2  |
| $[\Xi^* \bar{D}]^3_0$ | 1/2 | 1/2  | -1/2  | -1/2  |

TABLE III. The experimental data [14, 18, 36-38] and our results of the masses and binding energies (BE) for the $T_{cc}^+$, $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4547)$, and $P_c(4657)$. We adopt the isospin averaged masses for the single-charm mesons and baryons [4]. The listed values are all in units of MeV.

| System | Mass (Expt.) | BE (Expt.) | Mass (Our) BE (Our) |
|--------|-------------|------------|---------------------|
| $T_{cc}(3875)^+$ | 3874.8 | -1.0 | 3874.5 | -1.8 |
| $P_c(4312)^+$ | 4311.9 ± 0.7 | 8.9 (input) | 4311.9 | -8.9 |
| $P_c(4380)^+$ | 4380 ± 29 | -6.2 | 4376.2 | -9.1 |
| $P_c(4440)^+$ | 4440.3 | -21.8 | 4440.2 | -21.8 |
| $P_c(4547)^+$ | 4573.0 ± 6.4 | -4.8 (input) | 4573.0 | -4.8 |
| $P_c(4657)^+$ | 4588.8 ± 2.9 | -19.7 | 4468.1 | -10.0 |

interactions are within several MeVs. In this sense, our prediction is consistent with the observed $P_c(4459)$. Indeed, the LHCb collaboration also fitted the data using two resonances with masses 4454.9 ± 2.7 MeV and 4467.8 ± 3.7 MeV [36]. However, the limited data samples cannot confirm or refute the two-peak hypothesis. An updated analysis with more data samples is desired to clarify this issue.

We further swap the assignments of the $P_c(4440)$ and $P_c(4457)$, and regard them as the $[\Sigma_c \bar{D}]^1_3$ and $[\Sigma_c \bar{D}]^1_5$ molecular states, respectively. We can also find a set of solutions that can reproduce the masses of the three $P_c$ states. However, the cutoff $\Lambda$ is at 1.763 GeV, which is far away from the scale of light scalar mesons. Moreover, we can not reproduce the $T_{cc}^+$ state in this case. Thus, we rule out this set of assignments for the $P_c(4440)$ and $P_c(4457)$. In our framework, we can identify the quantum numbers of the $P_c(4440)$ and $P_c(4457)$ states if we require a satisfactory description of the $T_{cc}^+$ and $P_c$ states simultaneously.

B. $T_{cc}^+$ state and other heavy flavor molecular states

In the previous section, we have shown that our framework gives a nice description of the observed $T_{cc}^+$, $P_c$, and $P_{cs}$ states. In the following, we further adopt the fitted parameters $g_s$, $G_s$, and $\Lambda$ to calculate the other heavy flavor di-hadron systems listed in Table I. The effective potentials for these systems can be easily read from Table II. Their calculated masses and binding energies are listed in Table IV.

We check all the physically allowed $D^{(*)}D^{(*)}$ systems and find that there exists another $T_{cc}^+$ state with $[D^* D^*]^0$ assignment. The $T_{cc}^+$ lies about 7 MeV below the $D^* D^*$ threshold. This state has no hidden-charm strong decay channels
TABLE IV. The predicted masses and binding energies (BE) for the charmed di-hadrons ([H1H2]J) in Table I. We adopt the isospin averaged masses for the single-charm hadrons [4]. The values are all in units of MeV.

| System | [D*+D] \( ^0 \) | [Σcc+Ξc] \( ^0 \) | [Σcc+Ξc] \( ^1 \) | [Ξc+D] \( ^0 \) | [Ξc+D] \( ^1 \) | [Ξc+D] \( ^2 \) | [Ξc+D] \( ^3 \) | [Ξc+D] \( ^4 \) | [Ξc+D] \( ^5 \) | [Ξc+D] \( ^6 \) | [Ξc+D] \( ^7 \) | [Ξc+D] \( ^8 \) |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Mass   | 4099.7          | 4501.3          | 4510.1          | 4523.8          | 4327.7          | 4468.1          | 4436.7          | 4564.9          | 4582.1          | 4503.6          | 4628.5          |
| BE     | -7.4            | -25.4           | -15.9           | -2.9            | -8.9            | -10.0           | -9.4            | -22.5           | -5.2            | -9.6            | -26.0           |
| System | [Ξc+D] \( ^0 \) | [Ξc+D] \( ^1 \) | [Ξc+D] \( ^2 \) | [Ξc+D] \( ^3 \) | [Ξc+D] \( ^4 \) | [Ξc+D] \( ^5 \) | [Ξc+D] \( ^6 \) | [Ξc+D] \( ^7 \) | [Ξc+D] \( ^8 \) |
| Mass   | 4638.0          | 4651.3          | 4825.4          | 4903.9          | 4894.3          | 4931.9          | 4958.4          | 4969.3          | 5021.9          | 5035.1          | 4946.5          |
| BE     | -16.5           | -3.2            | -81.7           | -3.2            | -77.4           | -39.8           | -13.3           | -2.4            | -14.4           | -12.2           | -89.8           |
| System | [Ξc+D] \( ^0 \) | [Ξc+D] \( ^1 \) | [Ξc+D] \( ^2 \) | [Ξc+D] \( ^3 \) | [Ξc+D] \( ^4 \) | [Ξc+D] \( ^5 \) | [Ξc+D] \( ^6 \) | [Ξc+D] \( ^7 \) | [Ξc+D] \( ^8 \) |
| Mass   | 4996.1          | 4933.1          | 5042.1          | 5109.0          | 5135.6          | 5210.3          | 5221.7          | 5276.4          | 5290.3          |
| BE     | -40.1           | -5.8            | -6.2            | -6.4            | -4.0            | -14.4           | -3.0            | -15.6           | -1.7            |

due to its c \( \bar{u} \bar{d} \) valance quark component, thus should decay into \( D^0 D^0 \pi^0 \pi^+ \) or \( D^0 D^+ \pi^0 \pi^0 \) final states. In addition, because of the small phase space for \( D^+ \rightarrow D^+ \pi^0 \) (\( D^0 \pi^+ \)) and \( D^0 \rightarrow D^0 \pi^0 \), if the \( T_{cc}^+ \) does exist, similar to the \( T_{cc}^+ \), the \( T_{cc}^- \) should also be a narrow state in a loosely bound molecular picture. We suggest the LHCb Collaboration to look for this state in the future.

The results for the charmed baryon-meson (baryon) systems are also presented in Table IV. The determined \( g_b \) is a positive value and about 3 times larger than the \( g_\bar{d} \). From Eq. (5) we can see that for the lowest isospin di-hadron systems, the isospin-isospin matrix elements are negative and dominate the whole effective potentials of the di-hadron systems. Thus, if a heavy flavor two-body system has a large negative \( \lambda_1 \lambda_2 \) eigenvalue, this two-body system will have an attractive force and may form a bound state. As shown in Table II and IV, this feature is universal for all the studied heavy flavor meson-meson, meson-baryon, and baryon-baryon systems. The \( \lambda_1 \lambda_2 \) reduces to \( \tau_1 \cdot \tau_2 \) in the SU(2) case, and can be calculated with

\[
\tau_1 \cdot \tau_2 = 2 [I(I+1) - I_1(I_1+1) - I_2(I_2+1)].
\]

As shown in Eq. (16), the lowest total isospin generally leads to a negative eigenvalue and corresponds to an attractive force. Our formalism gives a very practical criterion to understand why the currently observed \( T_{cc}^+ \), \( P_c \), and \( P_{cs} \) states all prefer the lowest isospins.

C. Implications for the bottom hadron-hadron systems

In our calculations, we neglect the corrections from the heavy quarks in the charmed two-body systems and obtain a good description of the observed \( T_{cc}^+ \), \( P_c \), and \( P_{cs} \) states. If we adopt the same approximation for the bottom di-hadrons, then the \( (T_{bb}^+, P_b, P_{bs}) \) and \( (T_{bb}^-, P_b, P_{bs}) \) molecular states share the identical effective potentials from their light d.o.f.

In Fig. 1, we present the variation of binding energies for some typical molecular states as their corresponding reduced masses gradually increase. In each system, there exists a critical reduced mass at \( E_{BE} = 0 \), from which the system starts to form a bound state. Then the absolute values of binding energies increase as their reduced masses increase. The increased rate depends on the different types of light quark combinations in the two-body heavy flavor systems. We mark the \( T_{cc}^+, T_{bb}^+, P_c(4312), P_c(4440) \), and \( P_c(4457) \), as well as their bottom partners in Fig. 1. For the rest of the considered bottom meson-meson, meson-baryon and baryon-baryon systems, we list our predictions in Table V. As shown in Fig. 1, due to the large reduced masses of the bottom di-hadron systems, if there exist bound states in the charm di-hadrons, there should also exist the bottom partners with deeper binding energies as well.

![FIG. 1. The variation of binding energies for the \( T_{cc}^+, T_{bb}^+, P_c(4312), P_c(4440) \), and \( P_c(4457) \) states as their reduced masses increase. At \( m_Q = m_b \), we have their bottom partners \( T_{bb}^+, T_{bb}^-, |\Sigma B|^{1/2} \), and \( |\Sigma B|^{1/2} \) respectively.](image-url)

IV. SUMMARY

In this work, we use a quark level effective potential to give a universal description of the heavy flavor hadronic molecules that are composed of the ground \( (D, D^*), (\Lambda_c, \Sigma_c, \Sigma_c') \), and \( (\Xi_c, \Xi_c, \Xi_c') \) hadrons. Based on this quark-level effective Lagrangian, we neglect the contributions from heavy quarks and relate the effective potentials of di-hadrons to their flavor and spin interaction operators of light degrees of freedom.
TABLE V. The predicted masses and binding energies (BE) for the bottom di-hadrons ([H, H] policy) in Table I. We adopt the isospin averaged masses for the single-bottom hadrons [4]. The values are all in units of MeV.

| System | Mass (MeV) | BE (MeV) |
|--------|------------|----------|
| BB | 10584.9 | -19.3 |
| BB̅ | 10621.7 | -31.0 |
| BB̅̅ | 10104.9 | -26.7 |
| BB̅̅̅ | 11094.9 | -18.5 |

In our approach, we only introduce three parameters ̅, ̅, and . They can be well extracted from the observed ̅(5332), ̅(4440), and ̅(4457). We exclude the assignments of ̅(4440) and ̅(4457) as the [̅̅̅] and [̅̅̅̅] states, respectively, due to the poor description of in this case. Our results strongly indicate a very similar binding mechanism between the heavy flavor meson-meson and meson-baryon systems, i.e., they are bound dominantly by the interactions of their light degrees of freedom. We further generalize this similarity to the heavy flavor baryon-baryon systems.

We predict another system with the assignment [̅̅̅̅]. From our calculations, the and are the only two molecular states in the D(1)D(1) systems. We suggest the LHCb to look for this state in the future. We also predict other possible heavy flavor hadronic molecules in the charmed and bottom sectors (e.g., see Tables IV and V).

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