MAGNETICALLY ARRESTED DISK: AN ENERGETICALLY EFFICIENT ACCRETION FLOW

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ABSTRACT

We describe an accretion model around a black hole which behaves like a practical analog of an idealized engine originally proposed by Geroch and Bekenstein. The Geroch-Bekenstein engine has nearly 100% efficiency in converting rest mass of accreting matter into useful energy at infinity. Based on recent 3D MHD simulations of black hole accretion, we assume that the accreting gas drags in a strong poloidal magnetic field to the center and that the accumulated field disrupts the axisymmetric accretion flow at a relatively large radius. Inside this radius, the gas is forced to accrete as discrete blobs or streams with a velocity much less than the free-fall velocity. Almost the entire rest mass of the gas is released as heat, radiation and mechanical/magnetic energy. Except for a fraction of the radiation which falls into the black hole, the whole rest mass energy of accreted matter flows out to infinity. Even for a non-rotating black hole, the efficiency is of order 50% or higher.

Subject headings: accretion, accretion disks — black hole physics — galaxies: active — galaxies: nuclei — MHD — quasars:general
1. INTRODUCTION

Following a suggestion by Geroch (colloquium, Princeton Univ., Dec. 1971), Bekenstein (1972) described an engine that makes use of the extreme gravitational potential of a black hole (BH) to convert mass to energy with nearly perfect efficiency. The engine works by slowly lowering a mass $m$ into the BH potential using a strong wire. As the mass is lowered, the energy $E$ as measured at infinity goes down relative to the initial energy $E_0 = mc^2$. The change in energy is equal to the mechanical work done by the wire back at the engine. For a non-rotating BH, if the mass is lowered from an initially large radius down to a final radius $R$, and if the mass is then allowed to fall freely into the BH, the efficiency of the engine is given by

$$
\eta = \frac{E_0 - E(r)}{mc^2} = \left[ 1 - \left( 1 - \frac{1}{r} \right)^{1/2} \right].
$$

Here $r = R/R_S$ is the dimensionless radius in Schwarzschild units, where $R_S = 2GM/c^2$ and $M$ is the black hole mass. As $r \to 1$ one converts nearly all the rest mass energy of $m$ into useful work, and $\eta \to 1$.

The Geroch-Bekenstein engine is not believed to occur naturally in astrophysical systems. Apart from the very special geometry required, no realistic wire is strong enough to withstand the extreme forces that are present near a BH (Gibbons 1972). Astrophysical BHs do convert mass to energy, but they do it via accretion flows which generally have modest efficiencies. The radius below which the gas in an accretion flow free-falls is given by the radius of the innermost stable circular orbit (ISCO). Since this radius is usually located at a few $R_S$, the efficiency is not very large. A standard thin accretion disk around a Schwarzschild BH, for instance, has a radiative efficiency of only $\eta = 0.057$; the remaining 94.3% of the rest energy of the gas disappears into the BH. An accretion flow around a rotating BH is more efficient; for a limiting Kerr BH with $(a/M)_{\text{lim}} \approx 0.998$, $\eta \approx 0.30$ (Thorne 1974).

We describe in §2 a kind of accretion flow that has been seen in computer MHD simulations and that we believe should occur naturally in many BH systems. We argue in §3 that the flow has a high efficiency, and we conclude in §4 with a brief discussion.

2. Magnetically Arrested Disk

Figure 1a shows our basic idea. We assume that a significant amount of poloidal magnetic flux is present in the vicinity of the BH, and that the magnetic field is dynamically dominant. The field collects at the center as a result of the cumulative action of the accretion flow, as explained below, and the field is prevented from escaping by the continued inward
pressure of accretion. The field disrupts the accretion flow at a magnetospheric radius $r_m$, which lies well outside the event horizon of the BH. For $r > r_m$, the flow is essentially axisymmetric, as in any standard accretion flow. However, for $r < r_m$, the flow breaks up into blobs or streams, and the gas fights its way towards the BH by a process of magnetic interchanges and reconnection. The velocity of the gas in this region is much less than the free-fall velocity $v_{ff}$. We call such a disrupted accretion flow a “magnetically arrested disk” or MAD for short. (Despite its name, we believe the idea is quite sane.)

The key features of the model may be summarized as follows: (i) There is dynamically important poloidal magnetic field close to the BH. (ii) The field accumulates as a result of flux-freezing and the dragging action of the accretion flow (as against the case of a pre-existing field on a magnetized star, as shown in Fig. 1b). (iii) The gas velocity is much smaller than the free-fall velocity.

Something very analogous to this model has been seen in 3D computer MHD simulations of radiatively inefficient accretion flows around BHs (Igumenshchev & Narayan 2002; Igumenshchev, Narayan & Abramowicz 2003; see especially Model B in the latter paper). In the simulations, frozen-in magnetic flux is dragged to the center by the accreting gas, causing a substantial increase in the magnetic pressure and a severe disruption of the originally axisymmetric flow. Because of the strong mechanical resistance offered by the magnetic field, the radial, poloidal and azimuthal components of the gas velocity are all found to be small compared to $v_{ff}$.

We assume that the results obtained for radiatively inefficient accretion flows apply also to radiatively efficient accretion disks. There are two caveats in this connection. First, thin disks are sometimes cool enough to be mostly neutral, in which case the magnetic field can slip through the gas via ambipolar diffusion. Second, there could be substantial field slippage even in a fully ionized thin disk, if the anomalous magnetic diffusivity is large enough (Bisnovatyi-Kogan & Ruzmaikin 1976; Lovelace, Romanova & Newman 1994; Lubow, Papaloizou & Pringle 1994). We ignore these complications and uncertainties, and assume that a field configuration of the form shown in Fig. 1a does occur in thin disks.

Magnetically disrupted disks are known in another context, namely accretion onto magnetized neutron stars and white dwarfs. A strong field anchored in the star disrupts the accretion flow at a magnetospheric radius $r_m$, as shown in Fig. 1b. However, the topology of the field is very different from the case shown in Fig. 1a. In a magnetized star, the gas inside $r \sim r_m$ is able to flow in freely along field lines down to the magnetic poles of the star. In a MAD system, on the other hand, there is no field line connecting gas at radius $r_m$ down to the BH horizon. The only way for gas to move inward is via diffusion through the strong magnetic field. This results in a low velocity and a high energy efficiency (§3).
The MAD model requires that enough magnetic flux must collect near the BH to disrupt the flow. This condition is easily satisfied in at least some situations. Consider spherical Bondi accretion onto a supermassive black hole from a surrounding magnetized medium. Let the external medium have a sound speed \( c_s \) and some ambient value of the plasma \( \beta \equiv \rho c_s^2 / (B^2 / 8\pi) \). As the gas free-falls inside the accretion radius \( R_a = 2GM/c_s^2 \), its potential energy per unit volume increases as \( R^{-5/2} \), whereas the energy density of the frozen-in poloidal field that co-moves with the accreting gas increases as \( R^{-4} \) (Igumenshchev & Narayan 2002). Thus, by the time the gas falls to a radius \( R \sim R_a \beta^{-2/3} \), the magnetic energy density equals the gas energy density. Inward of this radius, the radial free-fall is arrested. For a reasonable \( \beta \sim 100 \) (field strength equal to a tenth of equipartition) and \( R_a \sim 10^7 R_S \) (corresponding to a sound speed of 100 km s\(^{-1}\)), the flow is arrested at \( \sim 10^5 R_S \), which is very far from the BH. The details are a little different if the infalling gas has enough angular momentum to form a rotating accretion disk, but one again finds a large disruption radius.

If the accreting gas has alternating signs of the magnetic field, then field can be destroyed by reconnection and accretion can continue without forming a MAD configuration. It is thus necessary that the inflowing gas should have the same sign of \( B_z \) (where \( z \) is taken to be parallel to the rotation axis of the flow) for an extended period of time. This is likely to be satisfied for accretion from an external medium. In the local ISM near the sun, the magnetic field strength is a few \( \mu \)G and the coherence scale of the field is a few hundred pc (at least). The magnetic flux in a coherent magnetic patch is thus \( \sim 0.1 \) pc\(^2\)G. The flux in a spatially coherent patch in the nucleus of a galaxy is uncertain, but possibly comparable (e.g., Lang, Morris & Echevarria 1999). If a flux of this magnitude is dragged by accretion to the vicinity of a supermassive BH, it will disrupt the accretion disk out to \( 100R_S \) or more. Therefore, by the time the BH accretes gas from a magnetically coherent volume in the surrounding medium, a MAD configuration is very likely to develop. With continued accretion, the flux will random walk, as the field in successive coherence volumes will be uncorrelated, but the overall picture should not change.

In a differentially-rotating accretion disk, additional magnetic field will be generated via the magnetorotational instability (Balbus & Hawley 1991), but the field will on average have zero net \( B_z \). This may lead to stochastic MAD-like behavior (e.g., Livio, Pringle & King 2003), with the field reversing on short time scales. Our model is concerned with the original field that comes in with the gas from the external medium. The direction of this primordial flux will be preserved as the gas flows in, and the field near the BH will not reverse until gas from an entire coherence scale is accreted. Regardless of whether we have a stochastic or long-lived MAD configuration, the energy efficiency should be high.
For an accretion disk in a binary system, the relevant quantities are the strength of the field in the outer layers of the donor star and the coherence time of the field in the star, i.e., how long the star retains a given sign of $B_z$ before the field reverses. This topic is beyond the scope of the paper.

3. Efficiency

We define the efficiency $\eta$ to be the ratio of the energy that flows out to infinity to the rest energy of the accreting gas (eq.[1]). Unlike a standard thin accretion disk, where the energetically efficient zone lies outside the ISCO and the gas free-falls inside the ISCO, the gas in a MAD has no free-fall region. The arresting action of the magnetic field ensures that the gas moves slowly inward (slow compared to the local free-fall speed) all the way down to the BH horizon, releasing its energy along the way, just as in the Geroch-Bekenstein engine. Thus, essentially the entire rest mass energy of the accreting gas is made available in the form of heat or radiation or mechanical/magnetic outflow. This leads to a high efficiency. The critical question is what fraction of the released energy actually escapes to infinity.

Inside the disruption radius $r_m$, where the gas breaks up into magnetically confined blobs (or streams as in the Igumenshchev et al. 2003 MHD simulations), the energy released will go partly into heating the blobs and partly into heating the surrounding medium, which we call a “corona.” Let us assume that the blobs are optically thick and that nearly all of their heat energy is emitted as radiation. To avoid complications, assume that the radiation comes out isotropically in the rest frame relative to the BH (we can ignore relativistic beaming since the velocity is small). In addition, some of the energy that is deposited in the corona may also be radiated locally and isotropically. Combining both sources of radiation, let us say that a fraction $f_1$ of the energy released at each radius is emitted locally as radiation. We will, for simplicity, assume that $f_1$ is independent of $r$.

For a Schwarzschild BH, a fraction $(1 - \cos \delta)/2$ of isotropically emitted radiation at radius $r$ escapes to infinity if $r < 3/2$ (radii inside the circular photon orbit) and a fraction $(1 + \cos \delta)/2$ escapes if $r > 3/2$, where

$$\cos \delta = \left[ 1 - \frac{27}{4r^2} \left( 1 - \frac{1}{r} \right) \right]^{1/2}. \tag{2}$$

Integrating over the differential energy release $dE(r)/dr$ (eq. [1]) as a function of radius, we find that a fraction $0.4375f_1$ of the rest mass energy of the accreting gas escapes to infinity as radiation, and a fraction $0.5625f_1$ of the rest mass energy falls into the BH. The precise value $0.4375$ depends on the particular assumptions we have made, but the general result,
that nearly half the radiated energy escapes to infinity, is probably quite general.

The remainder of the energy \((1 - f_1)\) goes into the corona and comes out partly as a thermal wind and partly as Poynting flux. Because both of these fluxes flow along the magnetic field lines, all of this energy escapes to infinity. Some of the energy may actually come out as radiation, e.g., beamed from the outflowing wind or created farther out where the wind meets an external medium. Let a fraction \(f_2\) of the rest mass energy of the accreting gas come out in this form of radiation, all of which escapes to infinity. Finally, a fraction \((1 - f_1 - f_2)\) remains as mechanical or magnetic energy and flows out into the external medium.

Energy is, of course, conserved in this process. If we start with a parcel of gas of mass \(m\) around a BH of mass \(M\) and if the gas accretes via a MAD flow as described above, then, after the gas has fallen into the BH, the mass of the hole will increase to \(M + 0.5625f_1m\), and an energy \((1 - 0.5625f_1)mc^2\) will return to infinity, of which \((0.4375f_1 + f_2)mc^2\) will be in the form of radiation and the rest in a non-radiative form. We now define two efficiencies: \(\eta_{\text{rad}}\), the ratio of radiative energy reaching infinity to the rest mass energy of the inflowing gas, and \(\eta_{\text{energy}}\), the ratio of the total energy reaching infinity (radiative, mechanical, magnetic) to the rest mass energy. From the previous discussion, we obtain

\[
\eta_{\text{rad,MAD}} = 0.4375f_1 + f_2, \tag{3}
\]

\[
\eta_{\text{energy,MAD}} = 0.4375f_1 + (1 - f_1) = 1 - 0.5625f_1. \tag{4}
\]

As mentioned in §1, a standard thin accretion disk around a Schwarzschild BH has \(\eta_{\text{rad}} = 0.057\), while Thorne's (1974) limiting Kerr BH has \(\eta_{\text{rad}} \sim 0.30\). Li & Paczyński (2000) have described a model in which a BH alternates between episodes of accretion spin-up and spin-down, giving a net efficiency of \(\eta_{\text{rad}} = 0.436\). Gammie (1999) has discussed a model in which magnetic fields exert a torque on an accretion disk at the ISCO (Krolik 1999) and cause an increase in the radiative efficiency of the disk. He shows that \(\eta_{\text{rad}}\) is nearly 0.2 for a Schwarzschild BH under ideal conditions and about 0.3 – 0.4 for an equilibrium Kerr BH with \(a/M \approx 0.7\). Agol & Krolik (2000) include the effects of returning radiation and show that the maximum efficiency for a BH in spin-equilibrium is 0.36.

The MAD model compares favorably with all these models. Even the minimum value of \(\eta_{\text{energy,MAD}}\) is larger than \(\eta\) of all the other models; \(\eta_{\text{rad,MAD}}\) is also large unless \(f_1\) and \(f_2\) are both small. What is interesting is that the large efficiencies are obtained with a generic magnetic field configuration and for a non-rotating BH.
4. Discussion and Conclusions

The analogy between the Geroch-Bekenstein engine and the MAD model is quite close. In the former, a strong wire arrests the falling mass $m$ and lowers it slowly into the BH, while at the same time transporting energy out to infinity. In the MAD model, the poloidal magnetic field plays the role of the wire (all the magnetic field lines go to infinity). The magnetic field arrests the free-fall of the gas, and transports at least part of the energy out to infinity in the form of a wind or an MHD wave or beamed radiation. The rest of the energy is radiated locally, of which about half escapes to infinity and half falls into the BH. The overall energy efficiency of the model is quite impressive (eqs.\[3\],\[4\]): even a non-rotating BH, under fairly generic conditions, has an efficiency comparable to, or perhaps superior to, the most optimistic accretion disk models proposed so far. A problem with Bekenstein’s (1972) original proposal is that no physical wire is strong enough to survive the intense tidal force of a BH (Gibbons 1972). Znajek (1976) showed that a poloidal magnetic field with a topology similar to that in the MAD model can circumvent Gibbons’ argument. Therefore, in principle, there is no limit to the efficiency of a Geroch-Bekenstein engine based on magnetic fields.

Soltan (1982) has shown that, by comparing the integrated luminosity of quasars over redshift with the BH mass density in the local universe, one can estimate the average efficiency of accretion flows in quasars. Elvis, Risaliti & Zamorani (2002) and Yu & Tremaine (2002) have used this method to argue for highly efficient accretion, with $\eta_{\text{rad}} > 0.1$ (see also Merritt & Ferrarese 2001). Considering the many conservative assumptions made by Yu & Tremaine (2002), it would appear that the efficiency may be quite large, perhaps closer to 1 than 0.1. The relevant efficiency when considering the Soltan argument is not $\eta$ as defined in the present paper, but the ratio $\eta'$ of the radiative energy reaching infinity to the mass energy added to the BH. For the MAD model described here,

$$\eta_{\text{rad,MAD}} = \frac{0.4375f_1 + f_2}{0.5625f_1} = 0.7778 + \frac{f_2}{f_1}. \quad (5)$$

Even the minimum radiative efficiency of the MAD model (achieved when $f_2 = 0$) is large enough to satisfy the quasar observations.

In this paper, we have dealt exclusively with the extraction of rest mass energy from accreting gas. With a rotating BH, one can in addition extract energy from the BH (Penrose 1969; Teukolsky & Press 1974; Blandford & Znajek 1977). While not the topic of this paper, we note that a MAD flow is particularly well-suited for energy extraction from a rotating BH. The azimuthal dragging of field lines by the rotating BH will drive a torsional Alfven wave along the poloidal field in the $z$ direction, giving twin energetic jets.
In principle, a MAD flow is not restricted to a BH — it could equally well be present around a star with a surface. In that case, the energy efficiency of a MAD is no different from that for accretion flows of other kinds, where the flow directly interacts with the star surface (e.g., disks with boundary layers, accretion columns in magnetized stars). There should, however, be some observational differences.

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Fig. 1.— (a) Shows the basic elements of the proposed MAD accretion model. An axisymmetric accretion disk is disrupted at a magnetospheric radius $R_m$ by a strong poloidal magnetic field which has accumulated at the center. Inside $R_m$ the gas accretes as magnetically confined blobs which diffuse through the field at a relatively low velocity. Surrounding the blobs is a hot low-density corona. (b) Shows the accretion flow around a magnetized compact star. An axisymmetric disk is disrupted at the magnetospheric radius $R_m$ by the strong stellar field. Inside $R_m$ the gas follows the magnetic field lines and free-falls on the polar caps of the star.