Improvement of plant performance using Closed loop Reference Model Simple Adaptive Control for Micro Air Vehicle

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Abstract In this paper, we present a novel idea to improve the transient performance of the existing Simple Adaptive Control architecture, without requiring high adaptation gains. Improvement in performance is achieved by incorporating the closed loop reference model based on the output feedback to the Simple Adaptive Control architecture. In this proposed scheme, the reference model dynamics is driven by the desired command as well as the error signal between the plant output and the reference model output. It is shown that the modified control architecture improves the system performance without any additional control efforts, which is then validated through simulations of the lateral model dynamics of Micro Air Vehicle.

Keywords Simple Adaptive Control · Closed loop reference model · Micro Air Vehicle

1 Introduction

Micro Air Vehicle (MAV) dynamics is highly complex due to the coupling of longitudinal and lateral modes, propeller induced non-symmetric flow effects and motor counter torque effects, etc. The nominal velocity of MAVs is comparable to the wind velocity; hence MAVs are susceptible to wind gusts. MAV dynamics is highly uncertain due to uncertain system parameters and large disturbances from the environment; therefore, the control system of MAV should have high disturbance rejection property. If a single fixed controller is to guarantee the stability at all operating conditions, the performance is compromised. The adaptive controller adapts itself to the realistic situation by adjusting the appropriate control parameters [13,9,15]; thereby making it suitable for MAV control system design.

Simple Adaptive Control (SAC) is a direct adaptive control which is implemented using output feedback. Furthermore, SAC does not need persistent excitation and the reference model being of the same order as that of the plant [12, 2,4]. Since MAV states are not available accurately, therefore, SAC is a good candidate for MAV controller design.

In adaptive systems, the objective of asymptotic tracking can be achieved using the conventional Lyapunov method. However, the plant performance during the transient phase may not be satisfactory [9,15]. The transient performance can be improved by increasing the adaptation gain, but this leads to an increase in the frequency of the control signal; which may drive the overall system to instability. Further, this increase in the frequency of control input may not be desirable due to actuator bandwidth limitations and actuator fatigue. To improve the transient performance, the idea of modifying the reference model is considered by the introduction of the error feedback term to the reference model dynamics in standard Model Reference Adaptive Control (MRAC) structure[13,18,6,19]. The reference model is made closed loop in nature by error feedback instead of traditional open loop reference model architecture. The main motivation is to drive the reference model towards the plant according to the error between the outputs of the plant and reference model. The driving of the reference model towards the plant reduces the magnitude and bandwidth of the control efforts required for the plant to follow the reference model.

The novelty of this paper is in extending the concept of closed loop reference model to existing SAC architecture by introducing output error feedback term to the reference model dynamics. We denote the new architecture as Closed loop reference model Simple Adaptive Control (CL-
SAC[10]. It is found that with a proper selection of output error feedback gain, CL-SAC structure improves the transient performance without any additional control efforts. The new output error feedback term increases the damping in reference model dynamics, thus making it faster to learn the plant parameters. The improvement in performance is validated through the simulation of MAV lateral model dynamics.

The rest of the paper is organized as follows: Section 2 describes the existing SAC architecture. The new CL-SAC structure is described in section 3. Performance improvement of CL-SAC is shown through analysis in section 4. Simulation of lateral dynamics of MAV is performed in section 5. Section 6 summarizes the result.

2 SAC Architecture

SAC is a special adaptive control methodology based on model following concept, where a higher dimensional plant can track the output of a lower order reference model with a lower order controller. It has been successfully applied in missiles [1], spacecraft [14], boost converters [11], space manipulators [21], quadcopter [20] etc. Its reference model represents the input-output behaviour of the plant, and it is sufficient to generate the desired command to be tracked. This methodology requires that the plant is “W-almost strictly passive” and the plant transfer function is “W-almost strictly positive real (WASPR)”; therefore it is stabilizable through some positive definite output feedback [3]. SAC architecture assumes that the primary stability properties of the plant are available. The basic stability information about the plant can be used to make the plant WASPR with parallel feedforward configuration[16]. Consider the plant to be linear time-invariant \( m \times m \) square systems as

\[
\dot{x}_p = A_p x_p + B_p u_p \tag{1}
\]

\[
y_p = C_p x_p \tag{2}
\]

where \( x_p \in \mathbb{R}^r \), \( u_p \in \mathbb{R}^m \), \( y_p \in \mathbb{R}^m \) are the plant states, inputs and outputs respectively.

The control objective is to ensure that the plant output tracks the commanded bounded reference signal while tracking the output of the specified reference model. The reference model is designed as per system response specification and is driven by reference signal to generate the desired command for the plant. In basic SAC architecture, the following reference model is considered

\[
\dot{x}_m = A_m x_m + B_m u_m \tag{3}
\]

\[
y_m = C_m x_m \tag{4}
\]

where \( x_m \in \mathbb{R}^r \), \( u_m \in \mathbb{R}^m \), \( y_m \in \mathbb{R}^m \) are the reference model states, reference signals and reference model outputs respectively. \( A_m \) is chosen as a Hurwitz matrix and the reference model assumed to be bounded input bounded output (BIBO) stable. The reference model dimension \( n_m \) can be less than plant dimension \( n_p \). However, the output dimension of both plant and reference model should be the same, as the plant output has to track the reference model output. The output tracking error is defined as

\[
\varepsilon_y = y_m - y_p. \tag{5}\]

The control law is defined based on the command generator tracker concept [5] as

\[
u_p = K(t) r \tag{6}\]

where \( K(t) = [K_e(t), K_e(t), K_u] \) and

\[
r = \begin{bmatrix}
    e_y \\
    x_m \\
    u_m
\end{bmatrix}
\]

\[
K_e(t) = K_{e_0}(t) + K_{e_0}(t)
\]

\[
K_{e_0}(t) = e_o x^T_m T_{e_0}; \quad K_{e_0}(t) = e_o x^T_m T_{e_0} - \sigma I_e
\]

\[
K_u(t) = K_{pu}(t) + K_{iu}(t)
\]

\[
K_{pu}(t) = e_o u^T_m T_{pu}; \quad K_{iu}(t) = e_o u^T_m T_{iu}
\]

\[
T_{e_0}, T_{e_0}, T_{iu}, T_{iu}, T_{pu}, T_{iu} \text{ are the time invariant weighing matrices of appropriate dimension, } K_{e_0}(0) = K_{e_0}, K_{iu}(0) = K_{iu}, K_{iu}(0) = K_{iu}, \sigma \text{ is a positive scalar adjusted such that it does not allow the integral gain to increase without bound.}
\]

The block diagram of SAC architecture is given in Fig.1.
After deducting (1) from (7), the error differential equation is

\[ \dot{e}_x = (A_p - B_p \hat{K}_s C_p)e_x - B_p(K(t) - \hat{K})r \]  

where \( \hat{K}(t) = [\hat{K}_s, \hat{K}_e, \hat{K}_u] \). The following Lyapunov function is considered for stability of error dynamics.

\[ V(t) = e_x^T P e_x + \text{trace}[W((K(t) - \hat{K})\Gamma^{-1}(K(t) - \hat{K})^T)] \]  

where \( P \) and \( Q \) are positive definite symmetric matrices, \( W \) is a positive definite matrix and these matrices are the solution of the following two equations.

\[ P(A_p - B_p \hat{K}_s C_p) + (A_p - B_p \hat{K}_s C_p)^T P = -Q \]  

\[ PB_p = C_p^T W^T. \]

The conditions (14) and (15) are known as W-ASPR conditions. If the plant is minimum phase and the eigenvalues of its input output matrix product \( (C_p B_p) \) are located in the right half plane, then it satisfies the W-ASPR conditions. Real world plants are not necessarily W-ASPR, but they can be made W-ASPR after augmenting them with parallel feedforward configurations (PFC). The augmented plant \( P(s) + D(s) \) is W-ASPR if it is relative degree of zero or one, where \( D(s) = \frac{1}{\Gamma(s)} \) and \( C(s) \) stabilizes the plant \( P(s) \) through feedback configuration. In the case of W-ASPR plant, the derivative of candidate Lyapunov function (13) along the trajectory of error dynamics (12) reduces to

\[ \dot{V}(t) = -e_x^T Q e_x \leq 0. \]  

As the derivative of the candidate Lyapunov function is non-positive, the error dynamics is stable. In this case, \( V(t) \) is lower bounded, \( V(t) \) is negative semi definite and \( V(t) \) is uniformly continuous in time. There for using “Lyapunov-like lemma” [17], it can be concluded that \( \lim_{t \to \infty} V(t) = 0 \). Hence the error between the plant and the ideal trajectory vanishes asymptotically. Therefore, \( C_p x_p^* = C_p x_p = C_m x_m \). So, the plant output follows the reference model output.

### 3 CL-SAC architecture

The closed loop reference model gives better performance than the open loop part in MRAC architecture [13, 7, 22]. Closed loop reference model structure is based on the concept that driving the reference model towards the plant will reduce the control efforts required for the plant to track the reference model. The similar modification in the reference model is made in the basic SAC architecture to improve the transient performance of the original structure. In this paper, the modification of the reference model is based on the output error and it is applicable to general MIMO square systems. In CL-SAC, the reference model is not only driven by the reference signal but also by the output error between the plant and the reference model. In this case, the open loop reference model is

\[ \dot{x}_m = A_m x_m + B_m u_m. \]  

The modified closed loop reference model is considered as follows

\[ \dot{x}_{mo} = A_m x_{mo} + B_m u_m - L_e (y_{mo} - y_p) \]  

\[ y_{mo} = C_m x_{mo} \]

The value of \( L_e \) is chosen such that the reference model and subsequent error dynamics are BIBO stable. This reference model is termed as closed loop reference model and earlier reference model in equation (3)-(4) is denoted as open loop reference model. In this case output tracking error is

\[ e_{my} = y_{mo} - y_p. \]  

The controller structure is defined similar to the basic SAC architecture, where reference model states and outputs are calculated from the modified reference model dynamics (18) and (19).

\[ u_p = K(t)r \]  

where,

\[ K(t) = [K_s(t), K_e(t), K_u(t)] \]

\[ r = [e_{my}, x_{mo}, u_m]^T \]

\[ K_{pe}(t) = K_{pe}(t) + K_{le}(t); \]

\[ K_{pke}(t) = e_{my}^T \Gamma_{pke}; \quad K_{le}(t) = e_{my}^T \Gamma_{le} - \sigma K_{le} \]

\[ K_{pke}(t) = e_{my}^T \Gamma_{pke}; \quad K_{le}(t) = e_{my}^T \Gamma_{le}; \]

\[ K_{pke}(t) = K_{pke}(t) + K_{le}(t); \]

\[ K_{pke}(t) = e_{my}^T \Gamma_{pke}; \quad K_{le}(t) = e_{my}^T \Gamma_{le}. \]

The initial conditions are kept the same as that of SAC architecture. The basic block diagram of CL-SAC architecture is given in Fig. 2.
3.1 Output model following

In this section, the condition for output model following for constant step input is revisited as the reference model dynamics is modified. The ideal trajectories are the linear functions of model states and model inputs.

\[
x^*_p = S_{11}x_{m0} + S_{12}u_m \\
u^*_p = S_{21}x_{m0} + S_{22}u_m.
\]

(22)

(23)

From (7), (9), (22) and (23) ideal trajectory dynamics can be written as,

\[
\begin{bmatrix}
\dot{x}_p^* \\
\dot{y}_p^*
\end{bmatrix} =
\begin{bmatrix}
A_p & B_p \\
C_p & 0
\end{bmatrix}
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
x_{m0} \\
u_m
\end{bmatrix}
\]

(24)

Let the reference command \( u_m \) be restricted for a constant step input. After differentiating (22) gives

\[
\dot{x}_p^* = S_{11}x_{m0}
\]

(25)
as \( u_m \) is considered as a constant input. Substituting the value of \( x_{m0} \) from (18) in (25) gives

\[
\dot{x}_p^* = S_{11}(A_mx_{m0} + B_mu_m - L_v(dy_{m} - y_p))
\]

(26)

In perfect tracking case, \( y_p \) is denoted as \( y^*_p \), then (26) can be simplified as

\[
\dot{x}_p^* = (S_{11}A_m - L_vC_m)x_{m0} + S_{11}B_mu_m + S_{11}L_vC_p x_p^*
\]

(27)

Substituting the value of \( x_p^* \) from (22) in (27) gives

\[
\dot{x}_p^* = (S_{11}A_m - L_vC_m + S_{11}L_vC_p S_{11})x_{m0} + (S_{11}B_m + S_{11}L_vC_p S_{12})u_m
\]

(28)

In case of perfect tracking,

\[
y^*_p = y_{m0} = C_px^*_p = C_mx_{m0}.
\]

(29)

Equation (29) and (29) can be combined as

\[
\begin{bmatrix}
\dot{x}_p^* \\
y_p^*
\end{bmatrix} =
\begin{bmatrix}
S_{11}A_m - L_vC_m + S_{11}L_vC_p S_{11} & S_{11}B_m + S_{11}L_vC_p S_{12} \\
C_m & 0
\end{bmatrix}
\]

(30)

From (24) and (30), (30) can be written as

\[
\begin{bmatrix}
S_{11}A_m - L_vC_m + S_{11}L_vC_p S_{11} & S_{11}B_m + S_{11}L_vC_p S_{12} \\
C_m & 0
\end{bmatrix}
\]

\[
= \begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}
\]

The solution of the above matrix equation exist as there are \( (n_p + m)(n_m + m_m) \) number of equations as well as unknowns. This implies that perfect tracking in the case of constant step input is possible in case of modified architecture. If the reference command is generated from the following differential equation,

\[
v_m = A_xv_m; \quad u_m = c_v v_m
\]

(31)

the perfect tracking is expected for following conditions

\[
dim(x_m) + dim(u_m) \geq dim(v_m).
\]

(32)

In case of general tracking command, it can be shown that CL-SAC architecture follows the reference command with bounded tracking error.

3.2 Stability analysis

In this section, we analyze the stability of the system since the reference model dynamics is modified. This modification in the reference model does not affect the error dynamics (12).

\[
e_x = x_p^* - x_p
\]

(33)

\[
e_x = (A_p - B_p\tilde{K}C_p)e_x - B_p(K(t) - \tilde{K})r
\]

(34)

Hence, it can be shown that the error between plant and ideal state trajectory goes down asymptotically. Therefore, the output error between plant and closed loop reference model goes to zero, and hence, the closed loop model reduces to open loop reference model asymptotically. So, the plant output is able to track the reference model output.

The effect of the modified architecture is visible in output error dynamics between the plant output and the reference model output. This is discussed in details in the next section.
Performance analysis

In the case of SAC architecture, the error dynamics between the plant output and the reference model output is obtained as follows:

\[ e_y = C_m \dot{x}_m - C_p \dot{x}_p. \]  

(35)

Using the value of \( \dot{x}_m \) and \( \dot{x}_p \) from (3) and (1) respectively in (35); we get

\[ e_y = C_m(A_m \dot{x}_m + B_m u_m) - C_p(A_p \dot{x}_p + B_p u_p) \]  

(36)

Putting the value of \( u_p \) as per (6) in (36), we get

\[ e_y = C_m(A_m \dot{x}_m + B_m u_m) - C_p(A_p \dot{x}_p + B_p K_e(t) e_y + K_c(t) \dot{x}_m + K_u u_m) \]  

(37)

(38)

Finally, we get

\[ e_y = A_{mn} e_y + \hat{\theta}^T(t) \omega \]  

(39)

where \( A_{mn} \equiv -C_p B_p K_e \),

\[ \hat{\theta}^T(t) = [ -C_p A_p, C_m A_m - C_p B_p K_e(t), C_m B_m - C_p B_p K_u(t) ] \]

and \( \omega = [ \dot{x}_p, \dot{x}_m, u_m ]^T \).

In the case of CL-SAC architecture, the output tracking error is \( e_{my} \) and the error dynamics is

\[ e_{my} = C_m(A_m x_m + B_m u_m - L_e(y_m - y_p)) - C_p(A_p x_p + B_p u_p). \]  

(40)

Equation (40) can be written as:

\[ e_{my} = A_{mn} e_{my} + \hat{\theta}^T(t) \omega \]  

(41)

where \( A_{mn} = -C_p B_p K_e - C_m L_e \),

\[ \hat{\theta}^T(t) = [ -C_p A_p, C_m A_m - C_p B_p K_e(t), C_m B_m - C_p B_p K_u(t) ] \]

and \( \omega = [ \dot{x}_p, \dot{x}_m, u_m ]^T \).

The plant is able to do bounded tracking of the output of the reference model, so the output error dynamics is stable.

In the case of SAC architecture, since the output error dynamics is stable; there exist a Lyapunov function \( V(t) \) whose derivative along the trajectories of the system in (39) is negative semi-definite:

\[ \dot{V}(t) = -e_y^T Q_1 e_y \leq 0 \]  

(42)

where \( Q_1 = Q_1^T > 0 \). Let’s consider this Lyapunov function in following form

\[ \dot{V}(t) = e_y^T P_1 e_y + f(\hat{\theta}(t), \Gamma) \]  

(43)

where \( \Gamma \) represent the set of gains \( \Gamma_k, \Gamma_e \) etc and \( P_1 = P_1^T > 0 \) is the solution of the following Algebraic Riccati equation.

\[ P_1 A_{mn} + A_{mn}^T P_1 = -Q_1 \]  

(44)

Since, \( \dot{V}(t) \leq 0; \)

\[ \| e_y \|^2 \leq \frac{V(t)}{\lambda_{\text{min}}(P_1)} \leq \frac{V(0)}{\lambda_{\text{min}}(P_1)} \]  

(45)

where \( \lambda_{\text{min}}(P_1) \) is minimum eigenvalue value of \( P_1 \). Since the error dynamics is bounded, \( f(\hat{\theta}(t)) \) is bounded by positive constant \( C \). So, bound for \( V(t) \) can be written as;

\[ V(t) \leq e_y^T P_1 e_y + C \]  

(46)

We have the following inequality,

\[ e_y^T Q_1 e_y \geq e_y^T \lambda_{\text{min}}(Q_1) e_y \geq \frac{\lambda_{\text{min}}(Q_1)}{\lambda_{\text{max}}(P_1)} e_y^T P_1 e_y = \gamma(e_y^T P_1 e_y) \]

Further,

\[ \lambda_{\text{max}}(Q_1) e_y^T P_1 e_y \geq \lambda_{\text{max}}(Q_1) e_y^T P_1 e_y = \gamma(e_y^T P_1 e_y) \]

Therefore,

\[ V(t) = -e_y^T Q_1 e_y \leq -\gamma(e_y^T P_1 e_y). \]  

(47)

Using the value of \( e_y^T P_1 e_y \) from (46) in (47) gives

\[ V(t) + \gamma V(t) \leq \gamma C. \]  

(48)

After integration of \( \dot{V}(t) \) we get,

\[ V(t) \leq V(0)e^{-\gamma t} + C - Ce^{-\gamma t}. \]  

(49)

From (46), it follows that

\[ V(0) - C \leq e_y^T(t) P_1 e_y(t). \]  

(50)

Therefore \( V(t) \) is upper bounded as

\[ V(t) \leq C + e_y^T(t) P_1 e_y(t) e^{-\gamma t}. \]  

(51)

Considering (45), the output error has the following upper bound:

\[ \| e_y \| \leq \sqrt{\lambda_{\text{min}}(P_1)^{-1}(C + e_y^T(0) P_1 e_y(0) e^{-\gamma t})} \]  

(52)

One part of the above bound is due to initialization error between the plant output and the reference model output, this term decays exponentially. If initialization is done perfectly, this output error bound reduces to,

\[ \| e_y \| \leq \sqrt{\lambda_{\text{min}}(S_1)^{-1}}. \]  

(53)

where \( \lambda_{\text{min}}(P_1) \) satisfies the following inequality [23]:

\[ \lambda_{\text{min}}(P_1) \geq 2\lambda_{\text{max}}^{-1}(S_1) \]  

(54)

where \( S_1 = -(A_{mn} + A_{mn}^T) Q_1^{-1}/2 \) and \( \lambda_{\text{max}}(S_1) \) is the maximum eigenvalue of \( S_1 \).

Therefore, the output error satisfy the following bound:

\[ \| e_y \| \leq \beta \sqrt{\lambda_{\text{max}}(S_1)} \]  

(55)
where $\beta$ is constant depends on value of C. The similar bound for the CL-SAC architecture is

$$\| e_{my} \| \leq \beta \sqrt{\lambda_{\text{max}}(S_2)}$$  \hspace{1cm} (56)

where $S_2 = -(A_{mn} + A_{mn}^T)Q_2^{-1}/2$.

Let $Q_1 = Q_2$ is chosen as Identity matrix.

$$\lambda_{\text{max}}(S_2) = \lambda_{\text{max}}\left( -(A_{mn} + A_{mn}^T) - C_mL_v - (C_mL_v)^T \right)$$

Clearly with proper choice of $L_v$, $\lambda_{\text{max}}(S_2)$ can be made smaller than the $\lambda_{\text{max}}(S_1)$. Therefore, the bound of $e_{my}$ can be made tighter than the $e_r$. Hence, in CL-SAC architecture output tracking error can be made further smaller than SAC architecture, which will improve the transient performance.

4.1 Selection of $L_v$

In [7], some guidelines are available for design of the gain of the error feedback term for the case of MRAC structure. In the case of SAC structure similar analysis is performed. A simplified analysis is done for scalar reference model, i.e the dimension of $n_m, m_m, m$ is one. The response of the open loop reference model in (17) with initial conditions $x_m(0) = x(0)$ is

$$x_m(t) = e^{A_{mn}t}x(0) + \int_0^t e^{A_{mn}(t-\tau)}B_{mn}u_m(\tau)d\tau$$  \hspace{1cm} (57)

The response of the closed loop reference model in (18) with same initial condition as $x_m(0) = x(0)$ is

$$x_m(t) = e^{A_{mn}t}x(0) + \int_0^t e^{A_{mn}(t-\tau)}B_{mn}u_m(\tau)d\tau - L_v\int_0^t e^{A_{mn}(t-\tau)}e_{my}(\tau)d\tau$$  \hspace{1cm} (58)

The difference of response between closed loop reference model in (18) and the open loop reference model in (17) having same initial conditions can be written as

$$\Delta x_{mo}(t) = x_m(t) - x_m(t) = -L_v\int_0^t e^{A_{mn}(t-\tau)}e_{my}(\tau)d\tau$$  \hspace{1cm} (59)

Applying Cauchy Schwartz inequality;

$$\int_0^t e^{A_{mn}(t-\tau)}e_{my}(\tau)d\tau \leq \sqrt{\int_0^t (e^{A_{mn}(t-\tau)})^2d\tau \int_0^t (e_{my}(\tau))^2d\tau}$$  \hspace{1cm} (60)

We have,

$$\int_0^t (e^{A_{mn}(t-\tau)})^2d\tau \leq \frac{1}{\| 2A_m \|}$$  \hspace{1cm} (61)

Also,

$$\int_0^t (e_{my}(\tau))^2d\tau \leq \frac{V(0)}{\lambda_{\text{min}}(P_1)}$$  \hspace{1cm} (62)

Using the inequalities in (4.1) and (62), the inequality in (60) can be simplified as,

$$\Delta x_{mo}(t) \leq \| L_v \| \sqrt{\frac{1}{\| 2A_m \|}} \frac{V(0)}{\lambda_{\text{min}}(P_1)}$$  \hspace{1cm} (63)

Equivalently,

$$\Delta x_{mo}(t) \leq \| L_v \| \sqrt{\frac{1}{\| 2A_m \|}} \frac{e_r(0)^T P_re_r(0) + f(\hat{\theta}(0), \Gamma)}{\lambda_{\text{min}}(P_1)}$$  \hspace{1cm} (64)

The value of $L_v$ is chosen such that the reference model dynamics and subsequent error dynamics is BIBO stable. Clearly from the bound of $\Delta x_{mo}(t)$; the high value of $L_v$ will make the $\Delta x_{mo}(t)$ large, i.e, the response of closed loop reference model will deviate more from the response of open loop reference model. Also, The low value of $L_v$ will reduce the effect of error feedback in the reference model dynamics. The value of $L_v$ needs to be selected as a function of $\Gamma$ optimally to balance both the effects. Mathematically, it can be shown that a high value of $L_v$ will reduce the oscillation in the control signal. In general, if the magnitude of $L_v$ is selected as high as possible and closer to the value of $\Gamma$, the advantage of $L_v$ can be obtained.

5 Simulation

MAV lateral dynamics are considered to verify the improvement in the new architecture. MAV lateral states are lateral velocity ($v$), roll rate ($r$), yaw rate ($t$), roll attitude ($\phi$). Generally, in MAV case, roll rate and yaw rate are available from the gyroscope sensor and the ($\phi$) is estimated from the estimation loop. The lateral dynamics need to track the roll attitude command from the guidance loop. Therefore, the plant output $y_p(\phi)$ has to track the output of the reference model which is driven by the commanded roll angle $(\hat{\phi}_{cmd})$. The roll rate and yaw rate can be feedback to increase the damping in the inner loop. Here, we will consider the tracking and stabilization problem only using the attitude ($\phi$).

5.1 Lateral dynamics-Roll attitude tracking

The lateral model of 150 mm KH2013A MAV is considered for simulation [8]. The photograph of KH2013A MAV is shown in Fig.3. In this MAV, the lateral control is done by rudder input($\delta_5$). The states, control and output variable are as follows:

$$x_p = [v, p, r, \phi]^T; \quad u_p = \delta_5; \quad y_p = \phi.$$

The state matrix, control matrix, output matrix are as follows:

$\begin{bmatrix}
A
B
C
\end{bmatrix}$
Considered. In case of SAC, the following reference model is
considered.
\[ \dot{x}_m = -5x_m + 5\phi_{cmd}; \quad y_m = x_m \]
In case of CL-SAC, the reference model is as follows:
\[ \dot{x}_{mo} = -5x_{mo} + 5\phi_{cmd} - 20(y_{mo} - y_p); \quad y_{mo} = x_{mo}. \]
So in this case, \( A_m = [-5]; \quad B_m = [5]; \quad C_m = [1]; \]
\( u_m = \phi_{cmd}; \quad L_v=20 \) are considered. Also, the dimension of the considered reference model is smaller than the plant model and the reference model output is the same as number of output variable to be tracked. The augmented plant is not W-ASPR. So, parallel feedforward configuration is used to make the plant W-ASPR. In this case the plant transfer function \( T(s) = C_p(sI - A_p)^{-1}B_p \frac{\text{num}(s)}{\text{den}(s)} \), where,
\[ \text{num}(s) = 6.444e05s^2 + 1.119e07s + 9.185e08 \]
\[ \text{den}(s) = s^6 + 104.8s^5 + 6728s^4 + 2.28e05s^3 + 5.18e06s^2 \\
+ 1.4e07s + 6.351e06. \]
\( C(s) \) is chosen such that it stabilized the plant \( T(s) \) and its gain is selected as high as possible while ensuring the stability. In this case, after root locus analysis \( C(s) \) is chosen as: \( \frac{4s + 40}{s} \). The parallel feedforward transfer functions is selected as: \( D(s) = \frac{1}{s} \). The plant with feedforward transfer function \( T(s) + D(s) \) is:
\[ F(s) = \frac{\text{num}(s)}{\text{den}(s)} \]
\[ \text{num}_F(s) = 10s^6 + 1048s^5 + 6.7e04s^4 + 4.8e06s^3 \\
+ 1.2e08s^2 + 4.2e09s + 3.68e10 \]
\[ \text{den}_F(s) = 4s^7 + 459.1s^6 + 3.11e04s^5 + 1.1e06s^4 \\
+ 2.9e07s^3 + 2.6e08s^2 + 5.8e08s + 2.5e08. \]
The root locus of \( T(s) + D(s) \) is shown in Figure 4. Clearly, the augmented plant is W-ASPR.

\[ \]
The details of implementation in case of CL-SAC are shown in Fig. 5. The parallel feedforward compensator $D(s)$ cannot be implemented parallel to the plant (shown in dotted line in Fig. 5); therefore, the equivalent representation of $D(s)$ is considered through the solid line path.

![Fig. 5 Details of CL-SAC architecture](image)

The value of $\sigma$ is chosen as 5. The value of adaptation gains $\Gamma_{pe}$, $\Gamma_{ie}$, $\Gamma_{ps}$, $\Gamma_{is}$, $\Gamma_{pu}$, $\Gamma_{iu}$ are chosen as 10 in simulation.

The tracking of the commanded square wave by the SAC and the CL-SAC is plotted in Fig.6 and Fig.7 respectively.

![Fig. 6 Reference signal tracking using SAC architecture](image)

![Fig. 7 Reference signal tracking using CL-SAC architecture](image)

From Fig.6 and Fig.7, it is clear that the tracking performance improves in case of CL-SAC as compared to SAC. Fig. 8 compares the control efforts between both architecture.

![Fig. 8 Figure comparing the control input ($u_p$) between SAC and CL-SAC](image)

The control efforts in CL-SAC is also less as compared to that of SAC in Fig.8. Clearly, improvement of the tran-
sient performance in CL-SAC is obtained without extra control efforts. The variation of other states lateral velocity (v), roll rate (p) and yaw rate (r) are plotted in Fig.9, Fig.10 and Fig.11.

From Fig.9, Fig. 10 and Fig.11; it is evident that the variation of other states in CL-SAC structure is smoother than SAC structure. The control gains $K_c$, $K_x$, $K_u$ are plotted in Fig.12, Fig.13 and Fig.14.
The value of $L_v$ needs to be chosen appropriately. The variation in tracking response is investigated with three different values of output error feedback gain; $L_v = 10$, 50 and 100. The variation in tracking performance and control input with the variation of ($L_v$) is shown in Fig.15 and Fig.16. Clearly from Fig.15, the error dynamics between the plant output and the close loop reference model output becomes faster with the increase in the value of ($L_v$), however, it causes poor tracking of the output of the open loop reference model; hence poor tracking of original reference command. The lower value of $L_v$ will have less effect on error feedback term, where the high value of $L_v$ can cause poor tracking of the output of open loop reference model. As output error
Improvement of plant performance using Closed loop Reference Model Simple Adaptive Control for Micro Air Vehicle

Reduced with a higher value of \( L_v \), the control effort also reduces; which is evident from Fig. 16.

5.2 Discussions

From the simulation of lateral model dynamics, it is clear that the roll attitude tracking performance during transient phase is better in the case of CL-SAC as compared to SAC without additional control efforts. So, the output error feedback term in reference model dynamics improves the performance of the system. However, the gain of the output error feedback term needs to be chosen properly as per performance specification and available actuator. The optimum design of \( L_v \) for CL-SAC structure should be investigated in future studies.

6 Conclusions

In this paper new CL-SAC architecture is proposed after modification of SAC architecture with closed loop reference model based on output feedback. The proposed architecture is based on the intuition that output tracking error feedback in the reference model dynamics will help the plant to follow the reference model. It is shown through mathematical analysis and simulations of lateral model dynamics of Micro Air Vehicle that CL-SAC improves the transient performance without additional control requirement.

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