Interaction and heat exchange in two-component relativistic fluid composite

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Abstract

A model of two-component relativistic fluid is considered, and the thermal nature of coupling between the fluid constituents is outlined. This thermal coupling is responsible for non-ideality of the fluid composite where the components are not fully independent. The interaction between particles is reflected only in the equation of state of each component, but it deals nothing with the coupling between the fluid components and does not influence the hydrodynamic motion. A general form of two-fluid decomposition is formulated for arbitrary interacting system.

1 Introduction

Two-component and multi-component relativistic fluid systems are often considered in various problems of nuclear physics and astrophysics. As a rule, the matter is described in the frames of ideal fluid mechanics. However, the coupling between fluid components may give rise to non-ideality of the fluid composite. Relativistic hydrodynamics of non-ideal multi-component fluid [1] involves complicated equations but it allows to operate with more precise theory. Particularly, the theory of two-component relativistic fluid has found important applications to hydrodynamics of relativistic superfluidity [2, 3] and nuclear hydrodynamics of proton-neutron systems [4]. The stress-energy
tensor of a two-component fluid [3]

\[ T_{\nu\sigma} = n_\nu \mu_\sigma + s_\nu \Theta_\sigma + P g_{\nu\sigma} \]  

includes the particle number current \( n_\nu \) and the flux of chemical potential \( \mu_\sigma \) corresponding to the first component, and the particle number current \( s_\nu \) and the flux of chemical potential \( \Theta_\sigma \) corresponding to the second component. In a relativistic superfluid the variables \( n_\nu \) and \( \mu_\sigma \) belong to the "cold" component, while the entropy density current \( s_\nu \) and the temperature flux \( \Theta_\sigma \) belong to the "warm" component.

The vectors in (1) are not independent but obey relationship

\[ \begin{pmatrix} n_\nu \\ s_\nu \end{pmatrix} = \begin{pmatrix} F & Q \\ Q & G \end{pmatrix} \begin{pmatrix} \mu_\nu \\ \Theta_\nu \end{pmatrix} \]  

with the coefficients

\[ F = \frac{1}{\mu} \frac{\partial P}{\partial \mu} \quad G = \frac{1}{\Theta} \frac{\partial P}{\partial \Theta} \quad Q = \frac{1}{z} \frac{\partial P}{\partial z} \]  

because the pressure \( P \), in general, depends on three variables

\[ \mu^2 = -\mu_\nu \mu_\nu \quad \Theta^2 = -\Theta_\nu \Theta_\nu \quad z^2 = -\mu_\nu \Theta_\nu \]  

Substituting (3) in (1) we write the stress-energy tensor in the form

\[ T_{\nu\sigma} = F \mu_\nu \mu_\sigma + G \Theta_\nu \Theta_\sigma + 2Q \mu_\nu \Theta_\sigma + P g_{\nu\sigma} \]  

The vanishing coefficient \( Q = 0 \) will imply ideality of the two-fluid system whose stress-energy tensor (3) will be written in the form

\[ T_{\nu\sigma} = \mu \mu_\nu \mu_\sigma + \Theta \Theta_\nu \Theta_\sigma + P g_{\nu\sigma} \]  

or

\[ T_{\nu\sigma} = \mu u_\nu u_\sigma + \Theta v_\nu v_\sigma + P g_{\nu\sigma} \]  

It is a model of two ideal fluids without coupling. Velocities \( u_\nu \) and \( v_\nu \) can be different but there is no interference between them.

So, non-zero coefficient \( Q \) implies that the motion of fluid components is not fully independent. The pressure \( P \) depends on the cross term \( z \). This fact reflects dependence on the relative velocity between the fluid components

\[ w = \sqrt{1 - \frac{\mu^2 \Theta^2}{z^4}} \]
In the light of (3), it results in non-zero coefficient $Q$ that implies coupling between the components and non-ideality of the two-fluid system: vectors $n_\nu$ and $\mu_\nu$ are not collinear (as well as $s_\nu$ and $\Theta_\nu$).

The coupling between the fluid components should not be mixed with the inter-particle interaction. The first component can be an ideal gas of free particles, the second component can be an ideal gas of free particles or thermal excitations (quasi-particles), but the fluid components are coupled as soon as the pressure $P$ depends on the relative velocity $w$ (8). It may look as an artifact of a plain mathematical trick without solid physical background: as soon as the relative flow $w$ is introduced, the thermodynamical parameters will contain functional dependence on $w$. However, this idea was successfully developed in the Landau two-fluid model of superfluid helium and it was confirmed in experiments [6]. Although the source of coupling between the ”cold” and ”warm” components of relativistic superfluid is a consequence of interaction between the particles [3], dependence on the relative velocity $w$ is not evident in the field-theoretical approach to interacting many-particle system [2, 7], and it had been a subject of dispute for a while [3, 8].

Researchers consider both variants – either a mixture of two ideal fluids, or a two-fluid composite where the coupling between components implies non-ideality of the whole fluid system. The first model (without coupling) is simple and it is often applied in astrophysics and nuclear physics. The latter model (with coupling) is much more complicated and it promises to give comprehensive description of hydrodynamic motion, although it is rarely used on account of its complexity. Of course, it is very desirable to take into account the role of coupling when we consider a strongly-interacting medium, like superfluid or nuclear matter.

In the present paper we discuss the nature of coupling between the components of a two-fluid composite and emphasize that it arises from the heat exchange between them. The interaction between particles in a quantum many-body system will also result in non-ideality of the continuous medium. It is necessary to clarify what factor plays dominant role in the hydrodynamics of a two-component fluid: the interaction between particles or the thermal contact between fluid components?

Standard relativistic units $c_{\text{light}} = \hbar = k_B = 1$ are used in the paper.
2 Thermal coupling between components

Taking Equation (5), we immediately determine the energy density of a two-component fluid

\[ E = T_0^0 = F\mu_0\Theta_0 + 2Q\mu_0\Theta_0 + P g_0^0 \]  \hspace{1cm} (9)

Non-ideality of the two-fluid composite, is due to non-zero term \( Q \). In a relativistic superfluid this is introduced at the macroscopic level of hydrodynamics \([5]\). However, its appearance is a consequence of internal processes in superfluid system \([9]\) and it implies dependence on the relative velocity between the "cold" and "warm" component \( w \), that is confirmed in experimental research \([6]\). The similar term \( Q \) will appear in the stress-energy tensor of an arbitrary two-component fluid if its components are not fully independent \([1]\). What process is responsible for non-ideality of this two-fluid composite? In order to understand how this non-ideality arises, let us turn from classical description to the quantum level.

For an arbitrary classical fluid system with the stress-energy tensor \([5]\) and the energy density \( E \) \([9]\) we can recognize the Hamiltonian \( \hat{H} \) that determines the ground state \( |\Phi\rangle \) according to the stationary Schrödinger equation

\[ \hat{H} |\Phi\rangle = E_g |\Phi\rangle \]  \hspace{1cm} (10)

with the ground-state energy

\[ E_g = \langle \Phi | \hat{H} | \Phi \rangle = \int T_0^0 d^3r \]  \hspace{1cm} (11)

Let us present the field variables \( \mu_\nu \) and \( \Theta_\nu \) in the form \([10]\)

\[ \hat{\mu}_\nu (x) = \frac{1}{X} \sum_k \left\{ e_\nu (k) \hat{a}_k e^{-ikx} + e_\nu^* (k) \hat{a}_k^\dagger e^{ixk} \right\} \]  \hspace{1cm} (12)

\[ \hat{\Theta}_\nu (x) = \frac{1}{Y} \sum_k \left\{ e_\nu (k) \hat{b}_k e^{-ikx} + e_\nu^* (k) \hat{b}_k^\dagger e^{ikx} \right\} \]  \hspace{1cm} (13)

with a unit polarization vector \( e_\nu \) and form-factors \( X \) and \( Y \), while the annihilation operators \( \hat{a}_k \) and \( \hat{b}_k \) satisfy commutation relations

\[ [\hat{a}_k, \hat{a}_p^\dagger] = \delta_{kp} \hspace{1cm} [\hat{b}_k, \hat{b}_p^\dagger] = \delta_{kp} \]  \hspace{1cm} (14)
and

\[ [\hat{a}_k, \hat{a}_p] = [\hat{b}_k, \hat{b}_p] = [\hat{a}^\dagger_k, \hat{a}^\dagger_p] = [\hat{b}^\dagger_k, \hat{b}^\dagger_p] = 0 \quad (15) \]

Substituting (12), (13) in (9), we find the Hamiltonian

\[ \hat{H} = \int \hat{T}^0 \, d^3 = V \sum_k \left\{ \frac{2F_k}{X^2} \left( \hat{a}^\dagger_k \hat{a}_k + \frac{1}{2} \right) + \frac{2G_k}{Y^2} \left( \hat{b}^\dagger_k \hat{b}_k + \frac{1}{2} \right) + \frac{2Q_k}{XY} \left( \hat{a}^\dagger_k \hat{b}_k + \hat{b}^\dagger_k \hat{a}_k \right) \right\} + O \quad (16) \]

where \( V \) is the volume of the system, \( F_k, G_k \) and \( Q_k \) are Fourier images of \( F(x), G(x) \) and \( Q(x) \), while \( O \) includes the terms with zero average in the light of (14)-(15). Expression

\[ \hat{N}^a_k = \hat{a}^\dagger_k \hat{a}_k \quad (17) \]

is no more than the operator of number of particles with energy

\[ \varepsilon^a_k = \frac{2F_k}{X^2} \quad (18) \]

and momentum \( k \). Expression

\[ \hat{N}^b_k = \hat{b}^\dagger_k \hat{b}_k \quad (19) \]

is the operator of number of particles with energy

\[ \varepsilon^b_k = \frac{2G_k}{Y^2} \quad (20) \]

and momentum \( k \). Quantity

\[ \lambda_k = \frac{2Q_k}{XY} \quad (21) \]

also plays the role of energy. It reveals the fact of coupling between the components because \( \xi_k = 0 \) is equivalent to \( Q = 0 \), and the Hamiltonian of this ideal system

\[ \hat{H}_0 = \sum_k \left\{ \varepsilon^a_k \left( \hat{a}^\dagger_k \hat{a}_k + \frac{1}{2} \right) + \varepsilon^b_k \left( \hat{b}^\dagger_k \hat{b}_k + \frac{1}{2} \right) \right\} \quad (22) \]

corresponds to a two-component ideal fluid with the stress-energy tensor (6) or (7).
The Hamiltonian (22) is already written in a diagonalized form. In order to diagonalize the Hamiltonian of non-ideal fluid (16) let us apply the following transformation

\[
\hat{a}_k = \cos \eta_k \hat{\alpha}_k + \sin \eta_k \hat{\beta}_k \tag{23}
\]
\[
\hat{b}_k = \cos \theta_k \hat{\beta}_k + \sin \theta_k \hat{\alpha}_k \tag{24}
\]

According to (14)-(15), the operators \(\hat{\alpha}_k\) and \(\hat{\beta}_k\) also satisfy commutation relations

\[
\left[\hat{\alpha}_k, \hat{\alpha}_p^\dagger\right] = \delta_{kp} \quad \left[\hat{\beta}_k, \hat{\beta}_p^\dagger\right] = \delta_{kp} \tag{25}
\]
\[
\left[\hat{\alpha}_k, \hat{\alpha}_p\right] = \left[\hat{\beta}_k, \hat{\beta}_p\right] = \left[\hat{\alpha}_k^\dagger, \hat{\alpha}_p^\dagger\right] = \left[\hat{\beta}_k^\dagger, \hat{\beta}_p^\dagger\right] = 0 \tag{26}
\]

Then, the Hamiltonian (16) will be presented in a diagonalized form

\[
\hat{H} = W_0 + \sum_k \left\{ (\varepsilon^a_k \cos^2 \eta_k + \varepsilon^b_k \sin^2 \theta_k + 2 \xi_k \cos \eta_k \sin \theta_k) \hat{\alpha}_k^\dagger \hat{\alpha}_k + \right.
\]
\[
+ \left. (\varepsilon^b_k \sin^2 \eta_k + \varepsilon^a_k \cos^2 \theta_k + 2 \xi_k \sin \eta_k \cos \theta_k) \hat{\beta}_k^\dagger \hat{\beta}_k \right\} \tag{27}
\]

where

\[
W_0 = \frac{1}{2} \sum_k \left\{ \varepsilon^a_k + \varepsilon^b_k \right\} \tag{28}
\]

is the zero-point vibration energy, and parameters of transformation (23)-(24) must satisfy condition

\[
2 \lambda_k \left( \cos \eta_k \cos \theta_k + \sin \eta_k \sin \theta_k \right) + \varepsilon^a_k \sin (2\eta_k) + \varepsilon^b_k \sin (2\theta_k) = 0 \tag{29}
\]

Particularly, at \(\lambda_k = 0\) condition (29) is simplified

\[
\varepsilon^a_k \sin (2\eta_k) + \varepsilon^b_k \sin (2\theta_k) = 0 \tag{30}
\]

and its solution is \(\sin \eta_k = \sin \theta_k = 0\) that implies no more than

\[
\hat{a}_k = \hat{\alpha}_k \quad \hat{b}_k = \hat{\beta}_k \tag{31}
\]

because the Hamiltonian (22) is already given in a diagonalized form.

Thus, condition (29) imposed on transformation (23)-(24) determines a link between parameters \(\eta_k\) and \(\theta_k\). However, it is not enough for their ultimate definition. An additional condition

\[
\varepsilon^0_k = \varepsilon^a_k \left( \cos^2 \eta_k + \sin^2 \theta_k \right) = \varepsilon^b_k \left( \sin^2 \eta_k + \cos^2 \theta_k \right) \tag{32}
\]
determines exact \( \eta_k \) and \( \theta_k \) that allow to present the Hamiltonian (27) in the following universal form

\[
\hat{H} = \hat{H}_{\text{ideal}} + \sum_k \omega_k \left( \hat{\alpha}_k^\dagger \hat{\alpha}_k - \hat{\beta}_k^\dagger \hat{\beta}_k \right)
\]  

(33)

where

\[
\hat{H}_{\text{ideal}} = W_0 + \sum_k \left( \varepsilon_0^k + \Delta_k \right) \left( \hat{\alpha}_k^\dagger \hat{\alpha}_k + \hat{\beta}_k^\dagger \hat{\beta}_k \right)
\]  

(34)

is the Hamiltonian of ideal system, while

\[
\omega_k = \lambda_k \sin (\eta_k - \theta_k)
\]  

(35)

and

\[
\Delta_k = \lambda_k \sin (\eta_k + \theta_k)
\]  

(36)

The energy shift \( \Delta_k \) reflect no qualitative difference from the ideal system with \( Q = 0 \) and \( \lambda_k = 0 \) whose Hamiltonian is

\[
\hat{H}_0 = W_0 + \sum_k \varepsilon_0^k \left( \hat{\alpha}_k^\dagger \hat{\alpha}_k + \hat{\beta}_k^\dagger \hat{\beta}_k \right)
\]  

(37)

So, there is no qualitative difference between two systems with Hamiltonian (34) and (37). They both describe an ideal fluid. As soon as nonzero \( Q \neq 0 \) appears in the stress-energy tensor (5), it implies non-ideality, and the relevant Hamiltonian (33) includes additional nontrivial terms with \( \hat{\alpha}_k^\dagger \hat{\alpha}_k - \hat{\beta}_k^\dagger \hat{\beta}_k \). To understand their role let us appeal to the formalism of thermo field dynamics [11, 12]. Let us introduce the operation of tilde-conjugation

\[
\hat{\tilde{\alpha}}_k = \hat{\beta}_k
\]  

(38)

For operator

\[
\hat{h} = \sum_k \omega_k \hat{\alpha}_k^\dagger \hat{\alpha}_k
\]  

(39)

its tilde-conjugated counterpart will be

\[
\hat{\tilde{h}} = \sum_k \omega_k \hat{\tilde{\alpha}}_k^\dagger \hat{\tilde{\alpha}}_k = \sum_k \omega_k \hat{\beta}_k^\dagger \hat{\beta}_k
\]  

(40)

and we can rewrite the Hamiltonian (33) in the following form

\[
\hat{H} = \hat{H}_{\text{ideal}} + \hat{h} - \hat{\tilde{h}}
\]  

(41)
while the Hamiltonian $\hat{H}_{\text{ideal}}$ is invariant under tilde-conjugation

$$\hat{H}_{\text{ideal}} = W_0 + \sum_k (\varepsilon_k^0 + \Delta_k) \left( \hat{\alpha}_k^\dagger \hat{\alpha}_k + \hat{\alpha}_k^\dagger \hat{\alpha}_k \right) = W_0 + \sum_k (\varepsilon_k^0 + \Delta_k) \left( \hat{\alpha}_k^\dagger \hat{\alpha}_k + \hat{\alpha}_k^\dagger \hat{\alpha}_k \right)$$  \hspace{1cm} (42)

If we introduce thermal doublets

$$\hat{A}_k = \begin{pmatrix} \hat{\alpha}_k \\ \hat{\alpha}_k^\dagger \end{pmatrix} \hspace{1cm} \hat{\bar{A}}_k = \begin{pmatrix} \hat{\alpha}_k^\dagger \\ -\hat{\alpha}_k \end{pmatrix}$$  \hspace{1cm} (43)

which satisfy commutation relations

$$\left[ \hat{A}_k, \hat{\bar{A}}_p \right] = \delta_{kp}$$  \hspace{1cm} (44)

then, the Hamiltonian (41) will be presented so

$$\hat{H} = \hat{H}_{\text{ideal}} + \sum_k \omega_k \left( \hat{\bar{A}}_k \hat{A}_k + 1 \right)$$  \hspace{1cm} (45)

or

$$\hat{H} = \hat{H}'_{\text{ideal}} + \sum_k \omega_k \left( \hat{\bar{A}}_k \hat{A}_k + \frac{1}{2} \right)$$  \hspace{1cm} (46)

where

$$\hat{H}'_{\text{ideal}} = W'_0 + \sum_k (\varepsilon_k^0 + \Delta_k) \left( \hat{\alpha}_k^\dagger \hat{\alpha}_k + \hat{\beta}_k^\dagger \hat{\beta}_k \right)$$  \hspace{1cm} (47)

and

$$W'_0 = W_0 + \frac{1}{2} \sum_k \omega_k = \frac{1}{2} \sum_k \left\{ \varepsilon_k^a + \varepsilon_k^b + \omega_k \right\}$$  \hspace{1cm} (48)

Again, the shift of the zero-point vibration energy (48) reflects no qualitative difference between the Hamiltonians (47) and (37). However, the last term in the Hamiltonian (46) makes essential difference from the ideal system with the Hamiltonian (37).

We can take the Hamiltonian of non-ideal fluid in various forms, applying the tilde-conjugation (38) and thermal doublets (43). However, no transformation can reduce this Hamiltonian to the form which corresponds to the Hamiltonian of an ideal fluid. The last term in (46) is responsible for the coupling between the components. We can consider the Hamiltonian of non-ideal fluid in the form (41), where the last two terms are responsible for the
coupling. In the Hamiltonian taken in the form (33) the last terms, again, include the coupling. Now we can explain the very nature of the coupling between the components of non-ideal fluid. It is resulted from exchange of thermal excitations in the process of absorption of a quantum with the energy $\omega_k$ and momentum $k$ (action of operator $\hat{\alpha}_k$), or in the process of annihilation of a hole with the energy $-\omega_k$ and the same momentum $k$ (action of operator $\hat{\beta}_k$). The excitation and the hole with respect to the zero energy level of the ideal fluid $E'_0$ [48]. If the components are isolated and there is no heat exchange between them, then, $\omega_k \equiv 0$ and it is necessary to be $\hat{h} = \hat{\bar{h}} = 0$ that corresponds to an ideal fluid. Thus, the heat exchange is the source of coupling between the fluid components. The similar interpretation is known in hydrodynamics of superfluid helium where collective motion of thermal excitations is responsible for hydrodynamic behavior of the whole system [6]. Although this link between thermal excitations and macroscopic properties of the continuous medium is intuitively evident, we have proved it in the explicit analysis with formulas (33), (41) and (46) for the first time.

However, it is still unclear what is the role of interaction between particles.

### 3 Interaction between particles

The pressure $P$ and energy density $E$ of a many-particle system are defined by standard formulas [13]

$$PV = -\Theta \ln Z \quad (49)$$

and

$$EV = \Theta^2 \frac{\partial \ln Z}{\partial \Theta} \quad (50)$$

where the statistical sum $Z$ is determined by formula

$$\ln Z (\varepsilon_p, \mu, \Theta) = \mp \frac{\gamma V}{(2\pi)^3} \int d^3p \ln \left[ 1 \pm \exp \left( \frac{\mu - \varepsilon_p}{\Theta} \right) \right] \quad (51)$$

with the upper and low sign corresponding to fermions and bosons, and $\varepsilon_p$ is the single-particle energy. For a system in motion at a velocity $\vec{w}$ the statistical sum is determined by formula [5]

$$\ln Z (\varepsilon'_p, \mu, \Theta) = \mp \frac{\gamma V}{(2\pi)^3} \int d^3p \ln \left[ 1 \pm \exp \left( \frac{\mu - \varepsilon'_p}{\Theta} \right) \right] \quad (52)$$
and the distribution function is

\[ f_p (\Theta) = \frac{1}{\exp (\varepsilon'_p - \mu) / \Theta \pm 1} \]  \hspace{1cm} (53)

where

\[ \varepsilon'_p = \varepsilon_p - \vec{p} \cdot \vec{w} \]  \hspace{1cm} (54)

is the single-particle energy in the moving reference frame.

Now consider a two-component fluid composite. When there is no relative flow between the components, the statistical sums of each component will be determined by the same formula \((51)\) in the co-moving reference frame, or by the same formula \((52)\) in the laboratory reference frame. What happens if relative motion between the fluid components occurs? Let us consider this two-fluid system in the reference frame co-moving the first component. Then, the statistical sum of the first component is defined by formula \((51)\), that can be written briefly so

\[ Z_a = Z (\varepsilon^a_p, \mu_a, \Theta) \]  \hspace{1cm} (55)

The statistical sum of the second component is calculated, according to \((52)\), in the reference frame co-moving the first component:

\[ Z_b = Z (\varepsilon^b_p - \vec{p} \cdot \vec{w}, \mu_b, \Theta) \]  \hspace{1cm} (56)

where \(\vec{w}\) implies the relative velocity between the first and second components.

The total statistical sum of two-component system is

\[ Z = Z_a Z_b \]  \hspace{1cm} (57)

Hence

\[ \ln Z = \ln (Z_a Z_b) = \ln Z_a + \ln Z_b \]  \hspace{1cm} (58)

and, according to \((49)\) and \((50)\), the pressure and energy density are additive quantities

\[ P = P_a + P_b \]  \hspace{1cm} (59)

\[ E = E_a + E_b \]  \hspace{1cm} (60)

How to complete a two-fluid decomposition for a system of interacting particles? It is convenient to apply the ideology of density functional theory that considers a system of free particles under the action of external
self-consistent field as a model of real interacting system. This ideology is developed in the nuclear mean-field approximation [15] where fluctuations of meson fields are neglected, and particles move independent in the mean fields, which themselves are generated self-consistently by the particles. This mean-field approximation allows to split the statistical sum

$$Z = \tilde{Z}Z^\psi$$

(61)

into a product of the statistical sum of mean fields $\tilde{Z}$ and statistical sum of particles $Z^\psi$ where the latter is defined according to (51). A set of self-consistent mean-field equations determine the effective mass $M^*$ and effective chemical potential $\mu^*$ which depend on the constant of interaction. As a result of (49), (50) and (61), the pressure and energy density of the whole interacting system is

$$P \left( \varepsilon^*_p, \mu_s, \Theta \right) = P_\psi \left( \varepsilon^*_p, \mu_s, \Theta \right) + \tilde{P} \left( \varepsilon^*_p, \mu_s, \Theta \right)$$

(62)

$$E \left( \varepsilon^*_p, \mu_s, \Theta \right) = E_\psi \left( \varepsilon^*_p, \mu_s, \Theta \right) + \tilde{E} \left( \varepsilon^*_p, \mu_s, \Theta \right)$$

(63)

where $P_\psi$ and $E_\psi$ imply contributions of the particles, and $\tilde{P}$ and $\tilde{E}$ are contributions of the mean fields, while

$$\varepsilon^*_p = \sqrt{p^2 + M^*_s}$$

(64)

Again, following (49), (50) and (55)-(57), we determine the pressure and energy of the first component as

$$P_a = P \left( \varepsilon^*_p, \mu^*_a, \Theta \right) = P_\psi \left( \varepsilon^*_p, \mu^*_a, \Theta \right) + \tilde{P} \left( \varepsilon^*_p, \mu^*_a, \Theta \right)$$

(65)

$$E_a = E \left( \varepsilon^*_p, \mu^*_a, \Theta \right) = E_\psi \left( \varepsilon^*_p, \mu^*_a, \Theta \right) + \tilde{E} \left( \varepsilon^*_p, \mu^*_a, \Theta \right)$$

(66)

while the pressure and energy of the second component will be

$$P_b = P \left( \varepsilon^*_p - \vec{p} \cdot \vec{w}, \mu^*_b, \Theta \right) = P_\psi \left( \varepsilon^*_p - \vec{p} \cdot \vec{w}, \mu^*_b, \Theta \right) + \tilde{P} \left( \varepsilon^*_p - \vec{p} \cdot \vec{w}, \mu^*_b, \Theta \right)$$

(67)

$$E_b = E \left( \varepsilon^*_p - \vec{p} \cdot \vec{w}, \mu^*_b, \Theta \right) = E_\psi \left( \varepsilon^*_p - \vec{p} \cdot \vec{w}, \mu^*_b, \Theta \right) + \tilde{E} \left( \varepsilon^*_p - \vec{p} \cdot \vec{w}, \mu^*_b, \Theta \right)$$

(68)

As soon as we know the thermodynamical functions of interacting system (62)-(63), we can define the thermodynamical functions of the two components (65)-(66) and (67)-(68), thus making a two-fluid decomposition (59)-(60).
4 Conclusion

Non-ideality of the two-component superfluid at the macroscopic level is expressed in additional coefficient $Q$ that appears in the stress-energy tensor and reflects its dependence on the relative flow $w$, resulting to coupling between the fluid components. The analysis in the frames of thermo field dynamics has revealed that the Hamiltonian contains irreducible terms responsible for coupling and implying the heat exchange between the components. This effect of thermal contact is taken into account when the relative velocity between the components is introduced in the thermodynamical functions of a two-fluid system.

One can consider the thermodynamical functions independent on the relative velocity $w$, that corresponds to the vanishing coefficient $Q = 0$. This model describes a two-fluid system where the components are in full thermal isolation. Of course, each component can be a strongly-interacting matter rather than ideal gas.

Indeed, a two-fluid model with the heat exchange between the components pertains better to superfluid systems because the ”cold” and ”warm” are not independent, and interference between them is evident. On the other hand, a two- or multi-component nuclear matter can be also described as an ideal two-component fluid if no macroscopic relative flow is expected. It can be a proton-neutron or quark-gluon system in equilibrium where only infinitesimal relative motion between the components is admitted in the form of spin-isospin sound.

The essence of two-fluid model is expressed in the decomposition of statistical sum that results in additive formulas for the pressure and energy density. The statistical sum of the first component is calculated by standard formula, while the statistical sum of the second component is determined by formula in the reference frame co-moving the first component. As a result, the relative velocity between the components $w$ appears in the thermodynamical functions.

When the pressure and energy density of interacting system is given, the two-fluid decomposition is completed immediately according to formulas and .

Both effects of coupling between fluid constituents and interaction between particles and taken into account. The interaction between particles is responsible for non-ideality of the equation of state. It yields the potential energy in addition to the kinetic energy of non-interacting system. In
the frames of mean-field approach it sets the effective mass $M^*$ and effective chemical potential $\mu^*$ that appear in formulas (62)-(68). However, the hydrodynamic non-ideality of this fluid system is not directly associated with the interaction between particles.

Without loss of generality, the two-fluid decomposition (59)-(60) and (65)-(68) can be also applied to superfluid systems, where the potential energy of interaction includes additional anomalous fields responsible for the condensation pressure and energy of the superfluid ground state [16]. For weakly interacting medium, like superfluid helium, the second (or ”warm”) component is often considered as an ideal gas of thermal excitations. However, such concept cannot be applied to superfluid nuclear matter because the contribution of meson fields is not small, and exact two-fluid decomposition (65)-(68) should be developed. It is the subject for further research.

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