Multifactorial modelling of high-temperature treatment of timber in the saturated water steam medium

D B Prosvirnikov, R G Safin, D F Ziatdinova, N F Timerbaev and V A Lashkov
Kazan National Research Technological University, 72, Karla Marksa st., Kazan, 420015, Russia

E-mail: prosvirnikov_dmi@mail.ru

Abstract. The paper analyses experimental data obtained in studies of high-temperature treatment of softwood and hardwood in an environment of saturated water steam. Data were processed in the Curve Expert software for the purpose of statistical modelling of processes and phenomena occurring during this process. The multifactorial modelling resulted in the empirical dependences, allowing determining the main parameters of this type of hydrothermal treatment with high accuracy.

1. Introduction
Timber is an anisotropic substance, therefore, the physico-chemical processes occurring in it are not always amenable to a precise mathematical description. In this case performance of a natural experiment assists in assessing the influence of different factors of the studied process on the properties and behavior of timber. However, modeling of physical processes with known techniques, such as, for example, planning of multifactorial experiment with the subsequent derivation of the equations of the polynomial nature, have a number of disadvantages. The lack of physical meaning of such equations and impossibility of their usage for similar processes are the major disadvantages.

Statistical modeling [1] uses a biotechnological exponential law as a basis for constructing mathematical models of real physical processes. This law takes into account the impact of various factors and does not require their fixation in the course of the experiment for the purpose of obtaining data that would allow an adequate description of the process.

Implementation of modeling in the Curve Expert software allows choosing the right mathematical relationship for the experimental data and assessing the adequacy of the resulting model with high reliability. Thus, the process of high-temperature treatment of timber in a saturated steam medium [2 – 4] was modeled in a multidimensional way. The main adjustable process parameters are: heating temperature of the material, coefficient of thermal and mass conductivity. The simulation results are given below.

2. The temperature dynamics of the material layers at a high temperature processing with saturated steam
For multidimensional modeling of temperature dynamics \( t \) (ºC) of material layers at treatment the experiments on the influence of heating time \( \tau \) (from 0 to 12 minutes, a step is 1 minute) and moisture content of timber \( W \) (60, 100, 140 %) in three layers of the processed material were conducted. Initial data of experimental studies are presented in paper [5]. Thus, there are three
influencing variables: \( \tau \) – time of wood heating, min; \( W \) – moisture content of the wood at the initial state, \%; \( I \) – a serial number of the layer. An indicator or dependent factor is the temperature of layer \( t \). Initially single-factor regularities were identified (Figure 1).

The indicator of the dynamics at the single-factor modeling changes in the following way:

\[
t = 20.44 \exp(0.64 \tau^{0.19}) + 3.96 \cdot 10^6 \tau^{5.13} \exp(-13.38 \tau^{0.23})
\]  

(1)

The correlation index in this case is 0.864.

The temperature change depending on the humidity is described by expression (2):

\[
t = 124.15 \exp(-0.003W) + 0.04W^{1.38}
\]  

(2)

The correlation index is 0.033, which is much less than the required level of adequacy of 0.3. Therefore, this factor on the multifactorial modeling is not considered. Under the influence of the layer number code rate \( t \) changes according to the equation (3):

\[
t = 144.06 \exp(-0.002I) - 14.14 \cdot I^{1.23}
\]  

(3)

The correlation index in this case is 0.248.

![Figure 1. Single-factor influence of parameters on heating temperature](image)

From two remnant factors (\( \tau \) and \( I \)) biotechnical regularities [6] for moisture content \( W = 60\% \) were obtained (Figure 2).
For each single-factor modeling the following dependences are composed:

- the influence of heating time on temperature (correlation index is 0.877):

\[
t = 20.33 \exp(0.0001t^{3.02}) + 20.25 \cdot t^{1.22} \exp(-0.1t^{0.94})
\] (4)

- the influence of layer number on temperature (correlation index equals 0.277):

\[
t = 147.73 \exp(-0.001I) - 16.45 \cdot I^{1.17}
\] (5)

The exception of a very small impact parameter (the moisture content of the raw material) increased the adequacy of single-factor formulas. To strengthen the validity of the outcome equation of the factors influence on temperature it is necessary to consider remnants from the equations (5) and to place the second explanatory variable (temperature). After identifying [1] we will obtain (Figure 3) the equation:

\[
\varepsilon_t = -96.29 \exp(-0.21t) + 1274.75 \cdot t^{2.36} \exp(-5.97t^{0.15})
\] (6)

The correlation index of two items turned out to be equal to 0.9609. At the ends of curves the points have minimal deviation. Thus, the remnants show the influence of some third (unknown) strong factor. This additional factor may well be an attempt to secure the values of so-called constant parameters by the method of Gauss-Seidel during experimenting.

A general two-factor formula takes the following form:
Similarly biotechnical regularities [6] for material moisture $W = 100\%$ were obtained:

$$t_{W=100\%} = 122.42 \exp(-0.22I) + 61.3 \cdot I^{1.57} \exp(-0.73I) - 96.78 \exp(-0.28r)$$

$$+ 13236.1r^{2.47} \exp(-8.17r^{0.13})$$

(8)

for material moisture $W = 140\%$:

$$t_{W=140\%} = 167.27 \exp(-0.04I) + 28.43 \cdot I^{1.06} \exp(-0.23I) - 98.68 \exp(-0.33r)$$

$$+ 1120.7r^{2.23} \exp(-5.53r^{0.16})$$

(9)

When estimating errors of equations (7) - (9) it was revealed that the maximum relative error is shifted to the side of low temperatures in the third layer of the heated material. This fact means that it is necessary to increase the measurement accuracy of layers thickness.

### 3. Thermal conductivity of timber at high temperature treatment with saturated water steam

For multifactorial modeling of the dynamics of timber thermal conductivity $\lambda$ (W/m·K) at high temperature treatment of the material layers with saturated water steam, the experiments on the effect of heating temperature $t$ (from 40 to 240 °C, a step is 20 °C) and humidity of timber $W$ (70, 100, 130%) for softwood (pine) and hardwood (aspen) were conducted. The initial experimental studies are presented in work [5]. Thus, two influencing variables are known: $t$ – temperature of heating wood, °C; $W$ – moisture content at the initial state, % for each of the woods. At the same time a wood of timber refers to qualitative factors. An indicator or a dependent factor is the thermal conductivity of timber $\lambda$. First, single-factor regularities, which were arranged according to the correlation coefficient ascending, were identified (Figure 4). Then the influencing parameters were added sequentially in the remnants of the previous models.

![Figure 4. Single-factor influence of parameters on the thermal conductivity index](image)

**Figure 4.** Single-factor influence of parameters on the thermal conductivity index

For softwood (pine) of timber the following single-factor regularities were obtained separately (Figure 4):

- effect of the temperature on the conductivity index
  
  $$\lambda = 0.39 \exp(-0.02t^{1004}) + 0.002t^{1.29} \exp(-0.006t)$$

(10)
- influence of timber moisture on thermal conductivity

\[ \lambda = 0.25 \exp(-0.001t^{1.005}) + 0.002W^{1.02} \]  

Comparing correlation coefficients, we notice that for the influence of humidity the coefficient is 0.6865, and for the effect of temperature it is 0.6915. So we take remnants \( \varepsilon_W \) from function \( \lambda = f(W) \) and substitute the values of influencing parameter \( t \). After structural-parametric identification we obtain the equation of the following type (Figure 5):

\[ \varepsilon_W = -0.15 \exp(-0.004t^{0.99}) + 0.0004t^{1.44} \exp(-0.003t) \]  

The correlation coefficient of two summands turned out to be equal to 0.9514.

![Figure 5. Equation diagrams of sequential influence of two parameters on the thermal conductivity coefficient of pine](image)

At critical point temperature \( t^* = 140 \, ^\circ C \) a minimal value of the correlation index results. When increasing or decreasing the temperature from 140 \, ^\circ C remnants increase (Fig. 5). This, apparently, reveals the shortcomings of fixing the non-major factors. Then the overall two-factor equation takes the form:

\[ \lambda_{\text{pine}} = 0.25 \exp(-0.001t^{1.005}) + 0.002W^{1.02} - 0.15 \exp(-0.004t^{0.99}) + 0.0004t^{1.44} \exp(-0.003t) \]  

Similarly, a two-factor dependence of the thermal conductivity of aspen timber on feedstock moisture and temperature of hydrothermal treatment was obtained:

\[ \lambda_{\text{aspen}} = 0.33 \exp(-0.0004W) + 0.002W^{1.02} \exp(-0.001W) - 0.17 \exp(-0.004t^{0.99}) + 0.0004t^{1.47} \exp(-0.004t) \]  

The calculations in the software environment of MS Excel showed that the maximum relative error is equal for pine to 10.76%, for aspen - to 9.11%.

3. The coefficient of mass conductivity of timber at heat treatment with saturated water steam.

The change in the coefficient of mass conductivity \( k_m (\text{m}^2/(\text{sec} \cdot \%)) \) of the timber at high-temperature treatment with saturated water steam was modeled. In this example, the number of values of the influencing variable is equal to 18, according to data from the work [5], and that is enough for the obtaining of additional wave components to the trend. The influencing variables are: heating time \( \tau \) (from 10 to 180 minutes, a step is 10 min), treatment temperature \( t \) (60, 140, 220 ° C), medium acidity \( pH \) (7.0, 3.0, 2.4). Thus, only the process time has gained a sufficient number of measurements, and the rest two factors were taken into account with the minimum acceptable number of repetitions. As a result we should expect a non-equilibrium statistical model, when the factors will be identified by few measurements according to the elementary regularities. Initially the single-factor regularities were identified (Figure 6).
The influence of medium acidity is determined by the regularity of the following type (the correlation coefficient is equal to 0.275):

\[ k_m = 1.62 \times 10^8 \exp(-16.61pH^{0.03}) \]  

(15)

The time influence is determined by the equation of the following type (the correlation coefficient is equal to 0.064):

\[ k_m = 1555.95\exp(-0.006\tau^{0.74}) - 1524.13\tau^{0.006}\exp(-0.007\tau^{0.72}) \]  

(16)

The temperature influence is characterized by the following formula (the correlation coefficient is equal to 0.592):

\[ k_m = 1.58\exp(0.001t^{1.55}) \]  

(17)

After that the temperature as an explanatory variable was added to the remnants after the model (15). After identification of the remnants we obtained the following (Figure 7) formula:

\[ \varepsilon_{pH} = -2.71 + 2.06 \times 10^{-5} t^{2.79} \]  

(18)

Figure 7. Equation diagrams of the subsequent influence of two parameters on the mass conductivity coefficient
Again, we take remnants $\varepsilon_{\text{pH}, \beta}$ and place time $\tau$ as an influencing variable. Again we identify and obtain the following formula:

$$
\varepsilon_{\text{pH}, \beta} = 25.52 \exp(-0.18\tau^{0.74}) - 2.34 \cdot 10^{-5} \tau^{3.36} \exp(-0.02\tau^{0.97})
$$

The remnants in Figure 8 show that for small time values it is necessary to increase the measurements accuracy sharply. A number of the initial data with variable dispersion of the actual distribution appears with the same accuracy of time measurement.

![Figure 8](image)

**Figure 8.** Diagrams of the subsequent influence of three parameters on mass conductivity coefficient

A general three-factor model obtains the formula of the following view:

$$
k_m = 1.62 \cdot 10^8 \exp(-16.61 \text{pH}^{0.03}) - 2.71 + 2.06 \cdot 10^{-5} \tau^{2.79} + 25.52 \exp(-0.18\tau^{0.74}) - 2.34 \cdot 10^{-6} \tau^{3.36} \exp(-0.02\tau^{0.97})
$$

4. Conclusion

For the time being the experiments are carried out according to well-known methods of Gauss-Seidel, when the researchers are trying to modify every single factor at fixed values of other influencing factors. For multifactorial modeling it turns out that secondary factors change very little during the experiment – just in three values. The authors have shown the possibility of multifactorial modeling based on the measurements obtained by the present method of Gauss-Seidel.

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