CONSTRAINING THE BULK LORENTZ FACTOR OF GAMMA-RAY BURST OUTFLOW IN THE MAGNETIC-DOMINATED JET MODEL

Zhe Chang1,2, Hai-Nan Lin1, and Yunguo Jiang1,2

1 Institute of High Energy Physics, Chinese Academy of Sciences, 100049 Beijing, China; changz@ihep.ac.cn, linhn@ihep.ac.cn, jiangyg@ihep.ac.cn
2 Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, 100049 Beijing, China

Received 2012 May 16; accepted 2012 September 14; published 2012 October 29

ABSTRACT

Recent observations by the Fermi-LAT showed that there are delayed arrivals of GeV photons relative to the onset of MeV photons in some gamma-ray bursts (GRBs). In order to avoid a large optical depth, the minimal value of the Lorentz factor has been estimated to be higher than 1000 in some of the brightest bursts. In this paper, we present a detailed calculation of the time delay between the MeV and GeV photons in the framework of the magnetic-dominated jet model. We find that the time delay strongly depends on the saturated bulk Lorentz factor of the jet. Inspired by this fact, we use this model to calculate the Lorentz factors of the four brightest Fermi bursts. The results indicate that the Lorentz factors are much smaller than those obtained from the “single-zone” scenario. The short burst GRB 090510 has a minimal Lorentz factor of 385, while the three long bursts, GRB 080916c, GRB 090902b, and GRB 090926, have almost the same Lorentz factors with an average value near 260. Another interesting result is that, for long bursts, GeV photons are emitted after the bulk Lorentz factor saturates. For the short GRB, however, MeV and GeV photons are emitted at the same phase, i.e., either in the expansion phase or in the coasting phase.

Key words: gamma-ray burst: general – ISM: jets and outflows

Online-only material: color figure

1. INTRODUCTION

It is well known that photons with energy higher than $m_{e^+}c^2 \approx 0.511$ MeV in a local jet frame may annihilate into electron–positron pairs. The large optical depth of $\gamma\gamma$ annihilation, as well as Compton scattering, will retrain these photons and keep them from escaping the jet. However, the observed gamma-ray burst (GRB) spectra often peak in the MeV range and sometimes extend to the GeV range. This is the so-called compactness problem. Since the optical depth is proportional to the inverse of the bulk Lorentz factor ($\Gamma$) of the jet, the “compactness problem” can be solved if we assume that the GRB outflow is moving with a large $\Gamma$ (Rees 1966; Piran 1999). The requirement of the thin optical depth for the observed high energy photons sets a lower limit on $\Gamma$ (Lithwick & Sari 2001). The Burst and Transient Source Experiment data indicate that the Band-like spectra of most bursts have a thermal component at the prompt phase (Ryde 2004), and the measurement of the temperature allows us to constrain both $\Gamma$ and the initial size $r_0$ of the flow (Pe’er et al. 2007).

The investigation of GRBs has entered into a new epoch since the launch of the Fermi satellite in 2008 June. The Fermi-LAT instrument has observed several GRBs with photon energy as high as tens of GeV. Within the framework of the simplified “single-zone” model, the GeV photons set a very large lower limit on $\Gamma$ (Lithwick & Sari 2001; Soderberg & Ramirez-Ruiz 2003; Razzake et al. 2004; Granot et al. 2008; Abdo et al. 2009a, 2009b, 2009c; Ackermann et al. 2010, 2011; Ghisellini et al. 2010). For example, Abdo et al. (2009c) showed that the minimal Lorentz factor of GRB 080916c outflow was $\Gamma_{\text{min}} \approx 900$. Ackermann et al. (2010) analyzed the spectra of GRB 090510 and showed that $\Gamma_{\text{min}} \approx 1200$. Ghisellini et al. (2010) estimated the decelerating time of GeV emissions and obtained a Lorentz factor as large as 2000 for GRB 090510. However, an efficient physical mechanism to boost the outflow to such a large $\Gamma$ is still unclear.

An interesting feature of the Fermi observations is that GeV photons often arrived seconds later than MeV photons (Abdo et al. 2009a, 2009b, 2009c; Ackermann et al. 2010, 2011). The explanation of this phenomenon should include both the intrinsic emission mechanism and the traveling process (Chang et al. 2012). In some quantum gravity theories, photons can interact with the quantum fluctuation of the space time, so high energy photons travel slower than low energy ones. Although this effect is very small, it can cause a detectable time difference after photons travel a cosmological distance (Gambini & Pullin 1999; Ellis et al. 2008, 2011). Such Lorentz invariance violation (LIV) effects lead to a natural time delay between GeV and MeV photons (Schaefer 1999; Abdo et al. 2009b; Boggs et al. 2004; Nemiroff et al. 2012). As it was shown by Abdo et al. (2009b) and Chang et al. (2012), the LIV effect is very small, so we will neglect this effect in the following.

Without considering LIV effects, the delayed arrival of GeV photons can also be explained by several GRB models (Duran & Kumar 2011; Bošnjak & Kumar 2012). Duran & Kumar (2011) assumed that photons are emitted by electrons via synchrotron radiation; it takes more time for electrons to be accelerated to a large Lorentz factor in order to radiate GeV photons. Bošnjak & Kumar (2012) used the magnetic jet model, which was initially introduced by Drenkhahn (2002) and Drenkhahn & Spruit (2002), to account for this phenomenon. According to this model, the optical depth is larger for high energy photons than for low energy photons. GeV photons can only escape at a larger radius where the optical depth is below unity.

In the magnetic reconnection model, the Band-type spectra can be produced from the photosphere through magnetic dissipation (Giannios 2006, 2008; Giannios & Spruit 2007). Koers & Giannios (2007) first considered the neutron effects in this model, which was later used by Mészáros & Rees (2011) to interpret the GeV time delay. In the magnetic-dominated but baryon-loaded model (Koers & Giannios 2007; Beloborodov 2010; Mészáros & Rees 2011), MeV photons can
The Astrophysical Journal, 759:129 (6pp), 2012 November 10

CHANG, LIN, & JIANG

escape the plasma at the photosphere radius, which corresponds to the prompt emission. However, GeV photons are produced by the nuclear inelastic collisions between protons and neutrons at a larger radius. In such a two-zone scenario, the strong constraint on the bulk Lorentz factor can be loosened (Hascoet et al. 2011; Zhao et al. 2011; Zou et al. 2011). As pointed out by Zhao et al. (2011), the optical depth depends not only on the energy but also on the emission angle. An average Lorentz factor of $\Gamma_{\text{min}} \approx 600$ can be estimated for GRB 080916c, GRB 090510, and GRB 090902b in the two-zone model. In a similar way, Zou et al. (2011) assumed that the GeV photons were emitted at a larger radius than the MeV photons, and gave an analytical formula for $\Gamma_{\text{min}}$ by calculating the optical depth of a GeV photon going through the MeV photon shell.

In this paper, we use the magnetic-dominated jet model discussed by Koers & Giannios (2007) and Mészáros & Rees (2011) to constrain the bulk Lorentz factor of GRB outflows. We show that the Lorentz factor of the short burst GRB 090510 can be as small as 385, while that of the three long bursts converge to about 260. The rest of the paper is organized as follows. In Section 2, we briefly illustrate the magnetic-dominated jet model and the producing mechanism of MeV and GeV photons. In Section 3, we use the delayed arrival of GeV photons in four *Fermi* bursts to calculate the bulk Lorentz factor of the GRB outflow. In Section 4, we discuss the validity of this model. Finally, conclusions are given in Section 5.

2. THE MAGNETIC-DOMINATED JET MODEL

The hydrodynamics of the GRB outflow depend strongly on its geometry structure. In the magnetic-dominated jet model, the Lorentz factor of the outflow increases with a of radius of roughly (Drenkhahn 2002; Drenkhahn & Spruit 2002; Metzger et al. 2011; Granot et al. 2011)\(^3\)

$$\Gamma(r) \cong \begin{cases} (r/r_0)^{1/3} & \text{for } r < r_{\text{sat}}, \\ \eta & \text{for } r > r_{\text{sat}}, \end{cases}$$

(1)

where $r_0$ is assumed to be the base of the outflow and $r_{\text{sat}}$ is the saturation radius. $\eta$ denotes the ratio of the magnetic energy density to the baryon rest mass energy density at $r_0$ initially.

The injected baryons include both protons and neutrons. Initially, the neutron–proton jet accelerates as a single fluid where neutrons and protons have elastic collisions. When the $n-p$ collision timescale is longer than the expansion timescale, the neutron component will coast with a terminal bulk Lorentz factor $\Gamma_n$, at a characteristic radius, while the proton component is still accelerated. Thus, the neutron component is embedded in a faster proton flow, and the jet becomes a compound flow (Beloborodov 2010).

\(^3\) The bulk Lorentz factor of the flow in the magnetic reconnection model originally took the form $\Gamma(r) \approx n_p(r/r_0)^{1/3}$ for $r < r_{\text{sat}}$, and $\Gamma(r) \cong \eta$ for $r > r_{\text{sat}}$ (Drenkhahn 2002; Drenkhahn & Spruit 2002), where they considered that the flow starts with the Alfven speed at the initial radius $r_0$. A compact form $\Gamma(r) \equiv (r/r_0)^{1/3}$ was taken by Koers & Giannios (2007), where $r_0$ is a length scale defined by a specific combination of the parameters. However, a Poynting jet can also be accelerated efficiently without a reconnection process (Granot et al. 2011), where $\Gamma$ also takes the form $\Gamma \sim n_p^{1/3}(r/r_0)^{1/3}$, but $r_0$ denotes the width of the magnetic shell. Beloborodov & Kumar (2012) assumed that the format in Equation (1) is valid at least in the interval of the Thomson and pair-production-photosphere radii, and $r_0$ is roughly the same order of the radius where the jet is launched. Since it was unphysical for the jet to accelerate to a high speed instantaneously, the Lorentz factor at the base $r_0$ was taken to be of order unity. In the present work, we take the idea of Bolshnjak and Kumar, and write $\Gamma$ as in Equation (1).

The cross-section of the $n-p$ collision is $\sigma_{\text{nuc}} \approx \sigma_p(c/v_{\text{rel}})$, where $\sigma_p \approx 3 \times 10^{-26}$ cm$^2$, and $v_{\text{rel}}$ is the relative speed of $p$ to $n$. When $v_{\text{rel}} \rightarrow c$, the collision is inelastic. This occurs when the comoving expansion time $t'_p \approx r/2\Gamma$ becomes shorter than the comoving collision time $t'_\text{nuc} \approx 1/n_p\sigma_p c$. Here $n_p = L x/4\pi r^2 m_p c^3 \eta \Gamma$ is the comoving proton number density, $L$ is the isotropic equivalent luminosity, and $x = n_p/(n_p + n_n)$ is the proton fraction of the baryon density. This gives a characteristic radius of $r_p/r_0 = \eta_p^2 x/2\eta^2$, where $\eta_p \equiv (L \sigma_p/4\pi m_p c^3 r_0)^{1/6} \approx 1.32 \times 10^2 r_{54}^{1/6} r_{0.7}^{-1/6}$. Here we have adopted the $Q = Q_0 \times 10^p$ convention. Making use of Equation (1), one obtains

$$r_p = \begin{cases} \eta_p^3 (x\eta/2\eta)^{3/5} & r < r_{\text{sat}}, \\ \eta_p^6 x/2\eta^3 & r > r_{\text{sat}}, \end{cases}$$

(2)

The pion production by the inelastic collisions is inevitable. A certain fraction of energy is carried away by neutrinos, which is an important prediction of the baryon-loaded jet model. The $\pi^0$ decay gives primary injected GeV photons. However, these photons undergo $\pi^0$ cascades and cannot escape the opaque jet. Interactions in the plasma are complex; more details can be found in Beloborodov (2010).

Suppose the final components in the jet contain photons with a Band-like spectrum, and the peak energy is around MeV. These photons start to be emitted when $\tau_p = n_p \sigma_T r/2\Gamma \sim 1$, which gives the Thomson photosphere radius, i.e., $r_T/r_0 = \eta_T^2/2\eta^2$, where $\sigma_T \approx 6.65 \times 10^{-25}$ cm$^2$ is the Thomson cross-section, and $\eta_T \equiv (L \sigma_T/4\pi m_p c^3 r_0)^{1/6} \approx 2.22 \times 10^2 r_{54}^{1/6} r_{0.7}^{-1/6}$. Using Equation (1) for $\Gamma$, one obtains

$$r_{\text{ph}} = \begin{cases} \eta_T^3 (x\eta/2\eta)^{3/5} & r < r_{\text{sat}}, \\ \eta_T^6 x/2\eta^3 & r > r_{\text{sat}}, \end{cases}$$

(3)

The simulation of magnetohydrodynamics shows that the jet can form a conical structure (Tchekhovskoy et al. 2008). After the jet exits the stellar envelope, the inner jet runs faster than the outer sheath. Thus, the Lorentz factor tapers off toward the edges. In such a structure, the neutrons from the outer sheath can drift into the inner core. The relative radial Lorentz factor ratio between neutrons and baryons is larger than 1, which ensures that the collisions are inelastic (Mészáros & Rees 2011). Suppose the jet has an open angle $\theta$, the transverse pion optical depth can be expressed as $\tau_{\pi,\perp} \approx n_p^2 \sigma_T r / \eta T = \eta_T^2 (r/r_0)^2 (x\theta/\eta)$. The jet becomes transversely optically thin ($\tau_{\pi,\perp} = 1$) at $r_{\pi,\perp}$, which is defined as

$$r_{\pi,\perp} = (r_0/\eta T)^{1/2} \eta T. \eta.$$

(4)

The dynamical evolution of the jet depends on $\eta$. If $\eta$ is large, for instance $\eta \approx 600 n_{600}$, the saturation radius $r_{\text{sat}}$ may be larger than $r_{\pi,\perp}$, $r_{\text{ph}}$, and $r_{\pi,\perp}$. Making use of $\Gamma = (r/r_0)^{1/3}$, one obtains the following characteristic radii:

$$r_p \approx 4.04 \times 10^{12} r_{54}^{3/5} r_{0.7}^{2/5} \eta_{600} \text{ cm},$$
$$r_{\text{ph}} \approx 3.98 \times 10^{13} r_{54}^{1/5} r_{0.7}^{2/5} \eta_{600} \text{ cm},$$
$$r_{\pi,\perp} \approx 4.41 \times 10^{14} L_{54} n_{500} r_{0.5}^{2/5} \theta^{2} \text{ cm},$$
$$r_{\text{sat}} \approx 2.16 \times 10^{15} r_{0.7} \eta_{600} \text{ cm}.$$
On the other hand, if $\eta$ is small, $r_{\text{sat}}$ may become smaller than $r_{\pi}$, $r_{\gamma\gamma}$, and $r_{\pi,\perp}$. In this case, one obtains the corresponding radii:

\[
\begin{align*}
    r_{\gamma\gamma}(E, r) &\approx \frac{2 \times 10^5}{40 \beta - 1} r_{12} L_{54} \left(\frac{E}{10 \text{ GeV}}\right)^{-\beta - 1} \eta^{2\beta}_{000}, \\
    r_{\pi,\perp} &\approx 2.50 \times 10^{13} E^{3/2} \eta^{-5}_{000} L_{54} \text{ cm}.
\end{align*}
\]

(7)

Or equivalently, $r_{\gamma\gamma}(E) \approx 1.94 \times 10^{17} E^{3/2} \eta_{000}^{-5} L_{54} \text{ cm}$ for the $\eta \sim 100$ case. Here, $E$ is the observed photon energy in units of GeV. Thus, at $r_{\pi,\perp}$, multi-GeV photons will be copiously produced by the transverse indrift neutrons colliding with jet core baryons. In the radius range $r_{\pi,\perp} < r < r_{\gamma\gamma}$, these photons will annihilate into electron–positron pairs due to the large optical depth. Only beyond the radius $r_{\gamma\gamma}$ can the produced GeV photons escape without obstructions.

The time delay for a photon with energy $E$ relative to the onset time of MeV photons equates to the time it takes for the jet to propagate from $r_{\gamma\gamma}$ to $r_{\gamma\gamma}(E)$.

\[
\Delta t = (1 + z) \int_{r_{\gamma\gamma}}^{r_{\gamma\gamma}(E)} \frac{dr}{2\Gamma^2 c},
\]

(9)

where $z$ is the redshift. The explicit formula for $\Delta t$ depends on the order of $r_{\gamma\gamma}$, $r_{\text{sat}}$, and $r_{\gamma\gamma}$. From Equations (5), (6), and (8), one can obtain $r_{\gamma\gamma} \lesssim r_{\gamma\gamma}$ for the generic $E \gtrsim 10 \text{ GeV}$ and $\eta \gtrsim 100$. Using Equation (1) for $\Gamma$, one obtains the formats of

| GRB  | $E_{\text{iso},54}$ | $T_{90}$ | $z$ | $E_{\text{high}}$ | $\Delta t_{\text{obs}}$ |
|------|---------------------|---------|----|-------------------|----------------------|
| 080916c | 8.8 | 66 | 4.35 | 13.22 | 12.94 |
| 090510 | 0.11 | 0.6 | 0.90 | 31.0 | 0.20 |
| 090902b | 3.7 | 22 | 1.82 | 11.16 | 9.5 |
| 090926 | 2.2 | 13 | 2.11 | 19.6 | 21.5 |

Notes. $E_{\text{iso},54}$ is the isotropic equivalent energy in unit of $10^{54}$ erg. $T_{90}$ is 90% of the GRB duration time in units of seconds. $z$ is the GRB redshift. $E_{\text{high}}$, is the highest energy of photons for each burst in units of GeV. $\Delta t_{\text{obs}}$ is the observed time delay between the highest energy photon relative to the onset of 100 MeV photons. The data were taken from Chang et al. (2012).

$\Delta t$ for three different cases:

\[
\Delta t \approx \begin{cases} 
    \frac{3(1+z)\eta_{00}}{2c} \left( \frac{r_{\gamma\gamma}}{r_0} \right)^{1/3} - \left( \frac{r_{\gamma\gamma}}{r_0} \right)^{1/3}, & r_{\gamma\gamma} < r_{\gamma\gamma}, \\
    \frac{3(1+z)\eta_{00}}{2c} \left( \frac{r_{\gamma\gamma}}{r_0} \right)^{1/3} - \left( \frac{r_{\gamma\gamma}}{r_0} \right)^{1/3}, & r_{\gamma\gamma} < r_{\gamma\gamma}, \\
    + \frac{3(1+z)\eta_{00}}{2c} \left( \frac{r_{\gamma\gamma}}{r_0} \right)^{1/3} - \left( \frac{r_{\gamma\gamma}}{r_0} \right)^{1/3}, & r_{\gamma\gamma} < r_{\gamma\gamma}.
\end{cases}
\]

(10)

A phenomenological illustration of the magnetic-dominated jet model is depicted in Figure 1. A magnetic-dominated but baryon-loaded jet is launched from a progenitor at the initial radius $r_0$. The bulk Lorentz factor of the jet evolves as Equation (1). MeV photons are produced at the radius $r_{\pi}$ by nuclear collisions, $\pi_0$-decay, electron–positron annihilation, magnetic dissipation, and synchrotron radiation, etc., but these photons can only escape at the photosphere radius $r_{\gamma\gamma}$, where the Thomson optical depth decreases to below unity. Thus, multi-MeV photons are emitted at $r_{\gamma\gamma}$ and lead to the observed Band-type spectra (Veres & Meszaros 2012). The GeV photons are assumed to produce inverse Compton radiation, etc. at $r_{\pi,\perp}$ by transverse drift nuclear collisions, but these GeV photons are only able to escape at a larger radius $r_{\gamma\gamma}$ due to the large optical depth at $r_{\pi,\perp}$. The time it takes for the jet to propagate from $r_{\gamma\gamma}$ to $r_{\gamma\gamma}$ naturally leads to the GeV time delay relative to the onset of MeV photons.

### 3. CONSTRAINTS ON THE LORENTZ FACTORS

From Equations (5), (6), (8), and (10), the time delay $\Delta t$ strongly depends on the terminal bulk Lorentz factor $\eta$. Inspired by this fact, we make use of the magnetic-dominated jet model discussed above to calculate $\eta$ for four Fermi bursts, GRB 080916c, GRB 090510, GRB 090902b, and GRB 090926, respectively.

The observed parameters that are necessary for the calculation are listed in Table 1. Note that GRB 090510 is a short burst, while the other three are long bursts. In Table 1, $E_{\text{high}}$ was taken to be the energy of the most energetic photon in each GRB. One exception is that the second energetic photon with $E_{\text{high}} = 11.16 \text{ GeV}$ in GRB 090902b was chosen, while the most energetic 33.4 GeV photon arriving at 82 s was excluded. This is because the isolated photon was far from the other GeV photons, and it is quite possible that this individual event happened when the jet encountered the interstellar medium.

There are several uncertain parameters, such as the initial radius $r_0$, the jet open angle $\theta$, and the ratio of proton number density to that of baryons $x$. Long bursts usually have time
variability $\delta t \approx 10$ ms, and thus the initial radius is taken to be $r_0 \approx c\delta t \approx 10^8$ cm for long bursts. The value of $r_0$ for short bursts is usually assumed to be smaller than that of the long bursts, and we set $r_0 \approx 10^7$ cm for short burst GRB 090510. A nominal value of jet open angle $\theta$ is taken to be 0.01, and the proton fraction of the baryon density $x$ is approximately 0.5, i.e., $\theta_{-2} \approx 1$ and $x_{0.5} \approx 1$ (Mészáros & Rees 2011). When $x$ approaches zero, this model reduces to the magnetic jet model without the loaded baryons (Bošnjak & Kumar 2012).

As was mentioned in Section 2, the explicit formula for $\Delta t$ depends on the order of $r_{\text{ph}}, r_{\text{sat}},$ and $r_{\gamma\gamma},$ which was not previously known. Thus, a self-consistent calculation should be taken.

**Case I.** First, we consider the $r_{\text{sat}} > r_{\gamma\gamma} > r_{\text{ph}}$ case (see the first formula in Equation (10)). The calculated saturation Lorentz factor and characteristic radii are listed in Table 2. The characteristic radii of the short burst GRB 090510 follow the order $r_{\text{sat}} > r_{\gamma\gamma} > r_{\text{ph}},$ which is self-consistent. However, for three long bursts, the results indicate that $r_{\text{sat}} < r_{\gamma\gamma},$ which are in contradiction with the assumption. The short burst GRB 090510 has a Lorentz factor of about 720, which is much lower than the predictions of one-zone models. For instance, Abdo et al. (2009a) presented that the bulk Lorentz factor of GRB 090510 was as large as 1200. Our results indicate that GeV photons in the short burst are emitted before the Lorentz factor of the jet is saturated.

**Case II.** Then, we consider the $r_{\text{sat}} < r_{\text{ph}} < r_{\gamma\gamma}$ case (see the second formula in Equation (10)). The results are given in Table 3. For all four bursts, we have $r_{\gamma\gamma} < r_{\text{ph}}.$ Thus, the transverse nuclear collisions happen inside the photosphere, and GeV photons are converted to the $e^\pm$ cascades. The overlap of the regimes that produce GeV and MeV photons is possible, but GeV photons are attenuated until $r_{\gamma\gamma}.$ The spectrum of GRB 090510 is fitted well by the Band function plus a power-law component that dominates in the band above 30 MeV (Zhao et al. 2011), which can be explained well by the magnetic-dominated jet model. If these arguments are true, the bulk Lorentz factor of GRB 090510 is further reduced to 385. In Table 3, one also notes that for GRB 080916c, $r_{\text{ph}} < r_{\text{sat}},$ which is not self-consistent. The bulk Lorentz factor of GRB 090926b and GRB 090926c in this case are calculated to be 245 and 252, respectively.

**Case III.** Finally, we consider the $r_{\text{ph}} < r_{\text{sat}} < r_{\gamma\gamma}$ case (see the third formula in Equation (10)). The results are listed in Table 4. The data for GRB 090510 are absent, because any value of $\eta$ cannot fit $\Delta t_{\text{obs}} = 0.2$ s by this formula. The minimal value of $\Delta t$ is 0.34 s located at $\eta_{600} \approx 1.$ The data of the three long bursts fit well in this case. The bulk Lorentz factors are 270, 252, and 258 for GRB 080916c, GRB 090926b, and GRB 090926c, respectively.

| GRB     | $\eta_{600}$ | $r_{\text{ph}}$/cm | $r_{\gamma\gamma}$/cm | $r_{\text{sat}}$/cm | $r_{\theta}$/cm | $r_{\pi\pi}$/cm |
|---------|--------------|---------------------|-----------------------|---------------------|----------------|----------------|
| 080916c | 0.39         | $5.24 \times 10^{13}$ | $1.78 \times 10^{15}$ | $1.28 \times 10^{15}$ | $5.33 \times 10^{12}$ | $1.51 \times 10^{14}$ |
| 090510  | 1.20         | $3.24 \times 10^{13}$ | $3.73 \times 10^{15}$ | $3.13 \times 10^{15}$ | $6.74 \times 10^{13}$ | $1.05 \times 10^{14}$ |
| 090902b | 0.32         | $6.80 \times 10^{13}$ | $6.78 \times 10^{14}$ | $6.90 \times 10^{14}$ | $2.32 \times 10^{14}$ | $1.05 \times 10^{14}$ |
| 090926c | 0.26         | $7.73 \times 10^{13}$ | $7.73 \times 10^{14}$ | $7.84 \times 10^{14}$ | $2.87 \times 10^{14}$ | $2.87 \times 10^{14}$ |

Notes. We choose $r_{\pi\pi} = 1$ for short burst GRB 090510 and $r_{\pi\pi} = 10$ for the other three long bursts. The characteristic radii of the three long bursts are not self-consistent.

| GRB     | $\eta_{100}$ | $r_{\text{ph}}$/cm | $r_{\gamma\gamma}$/cm | $r_{\text{sat}}$/cm | $r_{\theta}$/cm | $r_{\pi\pi}$/cm |
|---------|--------------|---------------------|-----------------------|---------------------|----------------|----------------|
| 080916c | 2.58         | $1.37 \times 10^{15}$ | $1.09 \times 10^{15}$ | $1.72 \times 10^{15}$ | $5.02 \times 10^{12}$ | $1.36 \times 10^{15}$ |
| 090510  | 3.86         | $6.31 \times 10^{13}$ | $7.16 \times 10^{15}$ | $5.75 \times 10^{14}$ | $1.91 \times 10^{14}$ | $1.25 \times 10^{15}$ |
| 090902b | 2.45         | $1.48 \times 10^{15}$ | $1.13 \times 10^{15}$ | $1.47 \times 10^{15}$ | $5.96 \times 10^{12}$ | $1.81 \times 10^{14}$ |
| 090926c | 2.52         | $1.62 \times 10^{15}$ | $1.60 \times 10^{15}$ | $5.88 \times 10^{12}$ | $1.77 \times 10^{14}$ | $1.77 \times 10^{14}$ |

Notes. The parameters are the same as in Table 2. The characteristic radii of the GRB 080916c are not self-consistent.

| GRB     | $\eta_{600}$ | $r_{\text{ph}}$/cm | $r_{\gamma\gamma}$/cm | $r_{\text{sat}}$/cm | $r_{\theta}$/cm | $r_{\pi\pi}$/cm |
|---------|--------------|---------------------|-----------------------|---------------------|----------------|----------------|
| 080916c | 0.45         | $4.82 \times 10^{13}$ | $8.68 \times 10^{15}$ | $1.97 \times 10^{15}$ | $4.89 \times 10^{12}$ | $1.31 \times 10^{14}$ |
| 090510  | 0.42         | $5.77 \times 10^{13}$ | $1.20 \times 10^{16}$ | $1.60 \times 10^{15}$ | $5.86 \times 10^{12}$ | $1.77 \times 10^{14}$ |
| 090926c | 0.43         | $5.71 \times 10^{13}$ | $2.50 \times 10^{16}$ | $1.72 \times 10^{15}$ | $5.80 \times 10^{12}$ | $1.74 \times 10^{14}$ |

Notes. The observed data $\delta t_{\text{obs}} = 0.2$ s in GRB 090510 cannot be fitted.

| Case    | 080916c | 090510  | 090902b | 090926c |
|---------|---------|---------|---------|---------|
| $r_{\text{ph}} < r_{\gamma\gamma} < r_{\text{sat}}$ | $\times$ | 720 | $\times$ | $\times$ |
| $r_{\text{sat}} < r_{\text{ph}} < r_{\gamma\gamma}$ | $\times$ | 385 | 245 | 252 |
| $r_{\text{ph}} < r_{\text{sat}} < r_{\gamma\gamma}$ | 270 | $\times$ | 252 | 258 |

Notes. The saturation bulk Lorentz factors for the four Fermi-detected bursts in three different cases. “$\times$” denotes the inconsistent case.
4. DISCUSSION

Besides the time delay, another important feature of GeV emissions is that they last much longer than the sub-MeV photons (Gao et al. 2009; Kumar 2009; Ghirlanda 2010; Ghisellini et al. 2010). For instance, the duration time of the sub-MeV photons is 55 s in GRB 080916c, while photons with energy > 100 MeV last about 1400 s (Kumar 2009). The observed decline of flux can be explained by the synchrotron radiation in the external shock (ES), i.e., \( F_{\nu} \propto t^{(3\beta+2)/4} \nu^{-\beta/2} \) (\( \beta = -2.4 \) for GRB 080916c). The data of the initial 55 s are able to explain the observed X-ray and optical flux of the afterglow one day later. Thus, the GeV emissions have an afterglow origin.

The spectrum and the light curve of the GRB 090510 were also explained by the synchrotron radiation in the ES model (Ghirlanda 2010). Ghisellini et al. (2010) studied the light curves of 11 GRBs detected by the Large Area Telescope (LAT), and concluded that LAT fluxes decay in a common way \( F_{\nu} \propto t^{-1.5} \) for the four brightest GRBs studied in this paper. The LAT fluxes can be interpreted as the fireball emission in the radioactive regime. The spectra of the GeV emissions in some bursts showed a different power law from the Band function. Thus, the spectra and the light curves present strong evidence that GeV emissions have a different origin than sub-MeV emissions. As hinted by Ghisellini et al. (2010), one can divide the “total emission time” of sub-MeV and GeV emissions into two parts: \(^4\) one is the overlap regime where both the sub-MeV and GeV photons are present; another is the regime where only LAT photons exist. The latter can be named as the early afterglow.

Note that our calculation on the GeV time delay is valid in the overlap regime, the GeV emissions in the early afterglow are not discussed. Once the outflow collides with the environment medium, the forward shock can also occur in the magnetic-dominated jet model. Both electrons and protons can be accelerated by the shock and form a power-law spectral distribution. The characteristic frequency of the synchrotron radiation follows \( \nu = \Gamma \gamma_{\text{e,p}} B / 2 \pi m_e c \), and the synchrotron radiation of protons can be ignored compared to that of electrons. The produced photons have the same power-law spectrum with electrons. Since the optical depth is small at so large a radius, photons are emitted immediately. The radiative fireball leads to the long duration time and light curve of GeV photons. Therefore, synchrotron radiation in the ES can explain the GeV emissions of the early afterglow in the magnetic-dominated jet model.

Now we consider the spectra and light curve in the overlap regime. In the neutron-rich environment, the inelastic collisions produce pions, which further decay into photons with a minimal energy of 70\( \Gamma \) MeV (Fan & Piran 2008). In the meantime, the produced neutrons can escape with an observed energy of \( \sim 0.1 \Gamma \) GeV (Beloborodov 2010). However, these high energy neutrinos are difficult to detect from earth. Koers & Giannios (2007) estimated that a less than 1 GRB neutrino event can be detected every year for nominal GRB parameters (\( z = 1 \)). Thus, one cannot exclude the baryon-loaded model by the neutrino argument. If the spectrum of the protons is in power law, the resulted photons will also follow the same distribution. However, these original photons will quickly convert to \( e^\pm \) via a \( \gamma \gamma \) reaction unless they are produced at a large radius where optical depth is below unity. The subsequent processes including Coulomb and Compton interactions are complex. Finally, a Band-like spectrum can form and the radiation becomes the observed photons in a prompt phase (Beloborodov 2010).

Without the neutron component, the magnetic-dominated outflow can dissipate energy efficiently (Drenkhahn 2002; Drenkhahn & Spruit 2002). A non-thermal spectrum can be produced by the magnetic reconnection model, and this spectrum is close to the observed prompt GRB emission (Giannios 2006; Giannios & Spruit 2007). This means a broken power-law (Band-like) spectrum can be produced with or without baryons. The luminosity of GeV emissions in the overlap regime rose as \( L \propto r^2 \) in most GRBs (Ghisellini et al. 2010). One exception is GRB 080916c, where the luminosity rose as \( L \propto r^6 \), which was puzzling (Kumar 2009). Our conclusion that the long bursts have the order \( r_{\text{ph}} < r_{\text{sat}} < r_{\gamma \gamma} \) may help to understand this puzzle. In this order, the jet is still in the expansion phase after the prompt emission, which means that \( \Gamma \) increases with time. If the jet is dominated at lower energies, one has \( \Gamma \sim (r/r_0)^{\mu} \) and \( r = 2ac\Gamma^2 \). In this way, \( \mu = 1/4 \) leads to \( \Gamma \propto t^{1/2} \). Since \( L \propto r^2 \Gamma^6 \) (Ghisellini et al. 2010), one can explain the puzzle because the light curve of the GeV emissions in the overlap regime strongly favors the magnetic jet model. Gao et al. (2009) also found that the physical composition of the GRB 080916c is likely magnetic.

The spectra of GeV photons do not evolve with time, and have a flatter component (the slope intermediate between \( \alpha \) and \( \beta \) of the Band function; Ghisellini et al. 2010). This evidence strongly indicates that the GeV photons have a different producing mechanism. The two-component GRB spectra were discussed by Veres & Mészáros (2012) recently, where a dissipative photosphere gives the prompt MeV emission, while GeV emission are produced by inverse Compton scattering. The model

\[ \eta = \eta(t) \]

\[ \eta = \eta(t) \]

Figure 2. Relation between the saturation bulk Lorentz factor \( \eta \) and the initial radius \( r_0 \) for short burst GRB 090510.

(A color version of this figure is available in the online journal.)

\[ \eta = \eta(t) \]

\[ \eta = \eta(t) \]

\[ \eta = \eta(t) \]

\[ \eta = \eta(t) \]
studied in this work also belongs to the two-component case; many possible spectra are able to account for different GRBs. Therefore, the magnetic-dominated jet model can explain many phenomenon of GRBs, such as the GeV time delay, the light curves and the spectra, etc.

5. CONCLUSION

In this paper, we have studied the bulk Lorentz factor of GRB outflow within the framework of a magnetic-dominated jet model. We found that the emission mechanisms of the short and long GRBs are different. The long bursts have a unified bulk Lorentz factor around 260 in both Cases II and III. However, the Lorentz factor of the short burst is 720 in Case I and 385 in Case II. These values are much smaller than that obtained from the “one-zone” scenario. Zhao et al. (2011) calculated the Lorentz factor of GRB 080916c, GRB 090510, and GRB 090902b, and showed that $\eta \sim 600$ could be consistent with observations in the “two-zone” scenario. Their values were still 2–3 times larger than our result for long bursts. They also proposed that the Lorentz factor could even be lowered in the “multi-zone” scenario. The magnetic-dominated jet model discussed here is a kind of “multi-zone” model, where photons with higher energy are emitted at a larger radius. The Lorentz factors for the long bursts that we obtained here were well inside the limits ($\sim 200–400$) given by Zou et al. (2011).

According to the magnetic-dominated jet model, the Lorentz factor depends on the initial radius $r_0$, which is an undetectable parameter. In Case I, only short burst GRB 090510 is self-consistent. As indicated in Figure 2, $\eta_{600}$ is asymptotic to 1.7 when $r_0$ goes to infinity. This means that the maximal Lorentz factor of GRB 090510 is about 1000 in this model. A small $r_0$ leads to a small $\eta$. For instance, if $r_0 = 0.1$, $\eta$ becomes 360. In Case II, $\eta_{600}$ does not depend on $r_0$, because the formulae of $r_{pp}$ and $r_{ph}$ are independent of $r_0$ (see Equations (6) and (8)). In Case III, $\eta_{600}$ depends weakly on $r_0$, since $r_{pp}$ is 2–3 orders of magnitude larger than $r_{ph}$. Thus, $r_{ph}$ can be ignored in the calculation. The strong correlation between $\Delta$ and $\eta$ has an advantage: a small variation of $\eta$ will not lead to a big change of $\eta$.

The time delay of GeV photons relative to the MeV photons can be well explained in the magnetic-dominated jet model. The bulk Lorentz factors of both long and short GRBs are reduced significantly. For GRB 090510, the possible minimal Lorentz factor is 385. For the three long bursts, GRB 080916c, 090902b, and 090926, the Lorentz factors converge to about 260. The Lorentz factor of the short burst is still larger than that of the long bursts. One common feature of the long bursts is that GeV photons are emitted after the bulk Lorentz factor saturates. In contrast, GeV photons in the short burst can be emitted either in the expansion phase or in the coasting phase, and the bulk Lorentz factor in the former case is about one time larger than that in the latter case.

The fact that three long bursts have a common Lorentz factor may imply that the long bursts have the same origin. One prevalent idea is that long GRBs are caused by the collapse of a massive star (such as Wolf-Rayet star; Woosley 1993; Paczyński 1998; Woosley et al. 2006). The short duration time and the large Lorentz factor of the short burst may imply a different kind of central engine mechanism. For instance, short GRBs can originate from the merger of two compact objects (such as a NS–NS binary system and a NS–BH binary system; Goodman 1986; Mészáros & Rees 1992; Zhang 2006).

We are grateful to M. H. Li, X. Li, and S. Wang for useful discussions. This work has been funded in part by the National Natural Science Fund of China under grant Nos. 10875129 and 11075166. The work of Y. G. Jiang is also funded by the China Postdoctoral Science Foundation funded project (grant No. 2012M510548).

REFERENCES

Abdo, A. A., Ackermann, M., Ajello, M., et al. 2009a, ApJ, 706, L138
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2009b, Nature, 462, 331
Abdo, A. A., Ackermann, M., Arimoto, M., et al. 2009c, Science, 323, 1688
Ackermann, M., Ajello, M., Asano, K., et al. 2011, ApJ, 729, 114
Ackermann, M., Ajello, M., Baldini, L., et al. 2012, ApJ, 754, 121
Ackermann, M., Asano, K., Atwood, W. B., et al. 2010, ApJ, 716, 1178
Beloborodov, A. M. 2010, MNRAS, 407, 1033
Boggs, S. E., Wunderer, C. B., Hurley, K., & Coburn, W. 2004, ApJ, 611, L77
Bošnjak, Z., & Kumar, P. 2012, MNRAS, 421, L39
Chang, Z., Jiang, Y. G., & Lin, H. N. 2011, Astropart. Phys., 36, 47
Drenkhahn, G. 2002, A&A, 387, 714
Drenkhahn, G., & Spruit, H. C. 2002, A&A, 391, 1141
Duran, R. B., & Kumar, P. 2011, MNRAS, 412, 522
Ellis, J. R., Mavromatos, N. E., & Nanopoulos, D. V. 2008, Phys. Lett. B, 665, 412
Ellis, J. R., Mavromatos, N. E., & Nanopoulos, D. V. 2011, Int. J. Mod. Phys. A, 26, 2243
Fan, Y. Z., & Piran, T. 2008, Front. Phys. Chin., 3, 306
Gambini, R., & Pullin, J. 1999, Phys. Rev. D, 59, 124021
Gao, W. H., Mao, J. R., Xu, D., & Fan, Y. Z. 2009, ApJ, 706, L33
Giannios, D., Ghisellini, G., & Nava, L. 2010, A&A, 510, L7
Ghisellini, G., Ghirlanda, G., Nava, L., & Celotti, A. 2010, MNRAS, 403, 926
Giannios, D. 2006, A&A, 457, 763
Giannios, D. 2008, A&A, 480, 305
Giannios, D., & Spruit, H. C. 2007, A&A, 469, 1
Goodman, J. 1986, ApJ, 308, L47
Granot, J., Cohen-Tanugi, J., & do Couto e Silva, E. 2008, ApJ, 677, 92
Granot, J., Komissarov, S. S., & Spitkovsky, A. 2011, MNRAS, 411, 1523
Haskoet, R., Daigne, F., Moszkovitch, R., & Vennin, V. 2011, arXiv:1110.6313
Koers, H. B. J., & Giannios, D. 2007, A&A, 471, 395
Kumar, P., & Duran, R. B. 2009, MNRAS, 400, L75
Lithwick, Y., & Sari, R. 2001, ApJ, 555, 540
Mészáros, P., & Rees, M. J. 1992, ApJ, 392, L77
Mészáros, P., & Rees, M. J. 2011, ApJ, 733, L40
Metzger, B. D., Giannios, D., Thompson, T. A., Bucciantini, N., & Quataert, E. 2011, MNRAS, 413, 2031
Nemiroff, R. J., Connolly, R., Holmes, J., & Kostinski, A. B. 2012, Phys. Rev. Lett., 108, 231103
Paczynski, B. 1998, ApJ, 494, L45
Pe’er, A., Ryle, F., Wijers, R. A. M. J., et al. 2007, ApJ, 664, L1
Piran, T. 1999, Phys. Rep., 314, 59
Razzaque, S., Mészáros, P., & Zhang, B. 2004, ApJ, 613, 1072
Rees, J. M. 1966, Nature, 211, 468
Ryde, T. 2004, ApJ, 614, 827
Schaerer, D. 1999, Phys. Rev. Lett., 82, 4964
Soderberg, A. M., & Ramirez-Ruiz, E. 2003, in AIP Conf. Proc. 662, Gamma-ray Burst and Afterglow Astronomy 2001, ed. G. R. Ricker & R. K. Vanderspek (Melville, NY: AIP), 172
Tchekhovskoy, A., McKinney, J. C., & Narayan, R. 2008, MNRAS, 388, 551
Veres, P., & Mészáros, P. 2012, ApJ, 755, 12
Woosley, S. E. 1993, ApJ, 405, 273
Woosley, S. E., & Bloom, J. S. 2006, ARA&A, 44, 507
Zhang, B. 2006, Nature, 444, 1010
Zhao, X., Li, Z., & Bai, J. 2011, ApJ, 726, 89
Zou, Y. C., Fan, Y. Z., & Piran, T. 2011, ApJ, 726, L2