Dynamic Mechanical Properties of Magnetorheological Elastomers: Experiment and Modelling

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Abstract. As a kind of smart material, magnetorheological elastomer (MRE) is composed of magnetizable particles dispersed in an elastomer matrix. Because adjusting an external magnetic field can continuously, rapidly and reversibly control the dynamic properties, there has been increasing research on MRE for mitigation of unwanted vibrations. In this paper, the dynamic mechanical analysis tests were performed to investigate the influence of strain, frequency and magnetic field on the controllable dynamic properties. The storage modulus and loss modulus of MRE were analyzed with frequencies from 1 to 50 Hz, strain amplitudes from 1 to 6\% and magnetic field intensities from 0 to 500 mT. The results show that the storage modulus decreases with increasing strain amplitude, increases with frequency and magnetic strength, and remains constant when the magnetic saturation occurs. Furthermore, the dependence of loss modulus on frequency lies on the matrix material of MRE. Based on the experimental results, a fractional derivative model was developed to describe the viscoelastic properties and the controllable dynamic properties. The comprehensive study of mechanical property characterisation is a guarantee for constitutive models to accurately describe the dynamic behaviour of MRE, which is an essential step towards the application for vibration control.

1. Introduction
As a composite material, MRE consists of magnetisable particles and non-magnetic matrix. MRE can be classified into two types [1]: anisotropic MRE which have a directed particle orientation attributed to the application of magnetic field during the solidification, and isotropic MRE which can be characterized by a random distribution of magnetic particles as the curing progress is without magnetic field. When a magnetic field is applied, the magnetic particles tend to align along the magnetic field direction, as a result, the composite exhibits field dependent properties within a pre-yield regime [2]. Because the dynamic stiffness can be selectively tuned by controlling magnetic fields [3] and finely tailored by adjusting the isotropic/anisotropic ratio [4], MRE is highly suitable for semi-active vibration control systems. Due to the combination of the versatility of active control and the reliability of passive control, semi-active (adaptive-passive) vibration control devises have attracted considerable intention over the past decades, and increasing effort has been devoted to the possibility of incorporating smart materials into semi-active vibration control devices [5]. Among these suited smart materials, MRE is distinctive due to the controllable rheological property by adjusting an external
magnetic field. There has been increasing research on MRE based devices to commercialize and industrialize its application in varies fields, such as the automotive industry and civil engineering [6,7]. Among all the influence factors on the dynamic properties of MRE, temperature can lead to transition behaviour at about 50°C. The storage modulus exhibits two different trends with the temperature variation: firstly decreases rapidly and then increases slightly with increasing temperature [8]. In this paper, an experimental research on mechanical property characterisation of MRE was carried out at a stable temperature to investigate the influence of strain, frequency and magnetic field on the controllable dynamic properties. This experimental study will be an important basis for the establishment of a fractional derivative model, which is capable to describe the viscoelastic properties and the controllable dynamic properties with strain, frequency and magnetic field strength as independently continuous variables. Accurate models are indispensable for effective dynamical analysis to realise the application of MRE, therefore, dynamical modelling is important as the basis for the progress of accomplishing the application of MRE.

2. Background theory
When sinusoidal loads are applied to MRE materials, because of viscoelasticity the response will not be instantaneous. The resulting stress will lag behind the input load by an angle $\phi$ called loss angle whose range is $0<\phi<90^\circ$. In viscoelastic materials, some of the deformation energy can be stored and recovered, whilst the remainder is dissipated as heat during each cycle. The storage modulus $M'\prime$ represents the ability of viscoelastic material to store the energy due to deformation, which contributes to the material stiffness. The loss modulus $M''\prime$ indicates the ability of viscoelastic material to dissipate the energy of deformation. They can be defined as:

$$\sigma = \varepsilon_0 (M'\prime \sin \omega t + M''\prime \cos \omega t)$$

(1)where $M$ can be either the shear modulus $G$ or the Young’s modulus $E$. It is usually convenient to express the modulus as a complex quantity. Then the dependence of the in-phase and out-of-phase stress on the strain can be presented using the complex modulus $M^\ast$:

$$M^\ast = M'\prime + iM''\prime.$$  

(2)The ratio between the loss and storage moduli is another widely used term for viscoelastic materials:

$$\tan \phi = \frac{M''\prime}{M'\prime}$$

(3)where $\tan \phi$ is called the loss factor, which can be used for describing the efficiency of damping caused by the viscoelastic material.

According to the Dynamic Mechanical Analysis directions in this study, the complex modulus $M^\ast$ can be obtained by calculating the ratio of the stress range to the strain range. The loss angle $\phi$ is relevant to the energy dissipation per volume within an oscillatory cycle, which is the area enclosed by the hysteresis loop and can be calculated. The storage modulus $M'$, loss modulus $M''\prime$ and loss angle $\phi$ can be defined as:

$$M' = M'\prime \cos \phi$$

$$M'' = M'\prime \sin \phi$$

$$\phi = \arcsin \frac{E_{\text{loop}}}{\pi A_{\text{strain}} A_{\text{stress}}}$$

(4)

where $A_{\text{strain}}$ stands for the strain amplitude, $A_{\text{stress}}$ stands for the stress amplitude and $E_{\text{loop}}$ is the energy enclosed by the hysteresis loop where the numerical integration can reduce the effects of measurement noise and take into account waveform distortion.

A fractional element is introduced to describe the characteristics of viscoelastic materials, which perform between viscous and elastic behavior [9]. The relationship between stress and strain can be expressed in the following form,

$$\tau_v = G_1 \tau_0^\alpha \mathcal{D}^{\alpha} \gamma_v(t) \quad 0 < \alpha < 1$$

(5)

where $G_1$, $\tau_0$ and $\gamma_v$ are the shear modulus, the shear stress and the shear strain of fractional element, respectively, $\tau_0$ denotes the relaxation time constant, $D$ denotes the derivative operator, $\alpha$ is a fractional order. In the extreme cases of $\alpha=1$ and $\alpha=0$, the fractional element turns into a Newton dashpot and a Hook spring, respectively.
Based on the experimental results of dynamic mechanical analysis tests, a fractional derivative model is developed as shown in Figure 1, and the Fourier transform is used to obtain the storage modulus $G'$ and the loss modulus $G''$

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G'(\omega) = G_m + \frac{G_0 G_1^2 \tau_0^2 \omega^{2\alpha} + G_0^2 G_1^2 \tau_0^2 \omega^{2\alpha} \cos \frac{\alpha \pi}{2}}{G_0^2 + G_1^2 \tau_0^2 \omega^{2\alpha}} \quad G''(\omega) = \frac{G_0^2 G_1 \tau_0^\alpha \omega^\alpha \sin \frac{\alpha \pi}{2}}{G_0^2 + G_1^2 \tau_0^2 \omega^{2\alpha}} \quad (6)
$$

where $G$ is the equivalent shear modulus, $G_0$ and $G_m$ are the shear moduli of the Hook spring and the nonlinear spring, respectively, $\omega$ is the angular frequency of the sinusoidal load.

3. Experimental setup

The anisotropic MRE samples were comprised of the silicone rubber (Wacker Chemie AG, Germany) and micron-sized iron particles (Sigma-Aldrich, US). The simplified manufacturing procedure can be illustrated by three steps: firstly Elastosil A and Elastosil B were mixed with a ratio of 10:1 by volume, and then the carbonyl iron powders sized up to 9 μm were added with a particle volume concentration of 30%, which is considered to be able to generate the maximum MR effect [10]; secondly, the mixture was blended thoroughly and placed in a vacuum chamber for 20 minutes to remove air bubbles trapped inside; finally the mixture was put into square aluminium moulds 21.8 × 21.8 × 6.5 mm³ and cured for 16 hours at room temperature to solidify with an external magnetic field of 290 mT produced by the cylindrical grade N42 neodymium permanent magnets (E-magnets, UK). According to the BS ISO 4664-1:2011 for shear modes, the DMA tests of MRE samples were performed with Instron E1000 Electro plus, which is able to apply and control the harmonic shear strain, as shown in Figure 2. The influence of magnetic field on the dynamic properties was obtained by the use of cylindrical grade N42 neodymium permanent magnets (E-magnets, UK), meanwhile the force and displacement were tracked by Instron E1000 Electro plus during tests. The dynamic properties of MRE were measured with frequencies from 1 Hz to 50 Hz, strain amplitudes from 1% to 5%, and magnetic field intensities from 0 to 500 mT. And all the sets of tests were carried out at room temperature (about 25°C) with three pairs of MRE samples independently.

4. Mechanical properties of MRE

Based on the hysteresis loops of anisotropic MRE in Figure 3, it can be seen that the slope of hysteresis loop decreases with increasing strain amplitudes, indicating that the storage modulus decreases with the increasing strain amplitude of the shown applied harmonic load. It can be seen from Figures 4 that the storage modulus and loss modulus increases with magnetic field strength at various excitation frequencies, and when magnetic saturation occurs these moduli will remain constant with the maximum magnetic force between magnetisable particles. Most experimental research on MRE shows that higher than 500 mT the increase of storage and loss moduli gradually slows down with magnetic field strength, and above about 800 mT magnetic saturation occurs in the composite there is no more increase of storage and loss moduli with further increasing magnetic field intensities [2]. It is also obvious that the storage modulus and loss modulus decrease with the strain
amplitude, because when the strain amplitude is 1% the two moduli are higher than them when the strain amplitude is 5%.

**Figure 2.** Experimental setup for MRE samples in shear mode.

**Figure 3.** Dependence of hysteresis loops on strain amplitude.

Figure 5 indicates that the storage modulus increases with frequency, meanwhile the loss modulus initially increases with driving frequency, and then reaches a maximum around 10Hz and stops to decrease with further increasing excitation frequency. Because the loss modulus of silicone rubber shows the same change with frequencies, the matrix material determines the dependence of loss modulus on frequency. Furthermore, the goodness of fit proves the effectiveness of the fractional derivative model. With the use of genetic algorithm (GA) in Matlab optimization toolbox, the parameters of the fractional derivative model can be identified by minimizing the sum of square of error between the experimental data and predicted results of storage modulus and loss modulus.

**Figure 4.** Dependence of storage and loss moduli on frequency and magnetic field under strain amplitude of 1%.

### 5. Conclusions

This study experimentally investigated the dependence of dynamic properties on frequency, strain and magnetic field for MRE in shear mode. The primary experimental results are as follows.

1. Both the storage modulus and loss modulus of MRE decrease with increasing strain amplitudes.
2. Both the storage modulus and loss modulus of MRE increase with magnetic flux density and maintain constant values when the magnetic saturation occurs.

3. The storage modulus of MRE increases as the excitation frequency increases, but the dependence of loss modulus on frequency is determined by the matrix material of MRE: in this study the loss modulus initially increases with increasing excitation frequency (<10Hz) up to a maximum value and then it decreases slightly with further increasing excitation frequency. And finally, a fractional derivative model was developed to describe the dynamic properties of MRE, and the model prediction exhibited good coincidence with experimental results.

6. References

[1] Kaleta J, Krolewicz M and Lewandowski D 2011 Magnetorheological properties of anisotropic and isotropic magnetorheological composites with thermoplastic elastomer matrices Smart Materials and Structures 20(8) 085006

[2] Bose H and Roder R 2009 Magnetorheological Elastomers with High Variability of their Mechanical Properties Journal of Physics: Conference Series 149(1) 012090

[3] Li Y, Li J, Li W and Du H 2014 A state-of-the-art review on magnetorheological elastomer devices Smart Materials and Structures 23(12) 123001

[4] Sapouna K, Xiong Y P and Shenoi R A 2017 Dynamic mechanical properties of isotropic/anisotropic silicon magnetorheological elastomer composites Smart Materials and Structures 26(11) 115010

[5] Ibrahim R A 2008 Resent advances in nonlinear passive vibration isolators Journal of Sound and Vibration 314(3) pp 371-452

[6] Du H, Li W and Zhang N 2011 Semi-active variable stiffness vibration control of vehicle seat suspension using an MR elastomer isolator Smart materials and structures 20(10) 105003

[7] Li Y, Li J, Li W and Samali B 2013 Development and characterization of a magnetorheological elastomer based adaptive seismic isolator Smart Materials and Structures 22(3) 035005

[8] Wan Y, Xiong Y P and Zhang S 2018 Temperature dependent dynamic mechanical properties of Magnetorheological elastomers: experiment and modeling Composite Structures DOI: 10.1016/j.compstruct.2018.04.010

[9] Sasso M, Palmieri G and Amodio D 2011 Application of fractional derivative models in linear viscoelastic problems Mechanics of Time-Dependent Materials 15(4) pp 367-387

[10] Davis L C 1999 Model of magnetorheological elastomers Journal of Applied Physics 85(6) pp 3348-3351

Figure 5. Comparison between measured data and predicted results of storage and loss moduli under strain amplitude of 5%.