D-branes in B Fields

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Abstract

The RR Page charges for the D2-, D4-, D6-brane in B fields are constructed explicitly from the equations of motion and the nonvanishing (modified) Bianchi identities by exploiting their properties — conserved and localized. It is found that the RR Page charges are independent of the background B fields, which provides further evidence that the RR Page charge should be quantized. In our construction, it is highly nontrivial that the terms like $B \wedge B \wedge B$, $B \wedge B \wedge F$, $B \wedge F \wedge F$ from different sources are exactly cancelled with each other.

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1. Introduction

In IIA string theory, a D2p-brane is coupled to the RR gauge fields \[ \mathcal{F} \], the coupling of the RR gauge fields to the pullback of the spacetime B field and U(1) gauge field strength F on D2p-brane seems to demand the integral \( \int (B + F) \) to be quantized. However, in \[ 2 \], Bachas, Douglas and Schweigert found that in WZW model the RR D0-brane charge on the stable spherical D2-brane takes irrational value due to the integral \( \int B \), and suggested only \( \int F \) can be interpreted as properly defined RR D0-brane charge.

Up to now, three different approaches have been proposed to resolve this puzzle. In \[ 3 \], Taylor argued that the D0-brane charge arising from the integral over the D2-brane of the pullback of the B field is cancelled by the bulk contributions, but in his calculation it was implicitly assumed that the gauge field \( C^{(1)} \) is constant. Also his analysis is hard to be generalized to D2p-branes \( (p > 1) \). In \[ 4 \], Alekseev, Mironov and Morozov observed that if one rewrites the WZ action in terms of the gauge invariant field strength \( G^{(2p+2)} = dC^{(2p+1)} - C^{(2p-1)} \wedge H \), the resulting RR brane charges are independent of B fields. However, in their approach when the RR gauge fields \( C^{(2p+1)} \) are not constant they have to deal with some nonlocal transformation, and treat RR gauge fields \( C^{(2p+1)} \) as independent fields even though in IIA string theory the RR gauge field \( C^{(2p+1)} \) and \( C^{(7-2p)} \) are dual to each other. In \[ 5 \], Marolf suggested that due to the presence of Chern-Simons term in IIA supergravity, there are three notions of charge: 1) ‘brane source charge’ is gauge invariant and localized, but not conserved; 2) ‘Maxwell charge’ is conserved and gauge invariant, but not localized; 3) ‘Page charge’ is conserved, localized and invariant under small gauge transformation. Since brane source charge and Page charge were defined separately in \[ 5 \], it is unclear how to relate one to other which is crucial to the questions whether it is possible for us to consistently define the general RR Page charges for a D2p-brane from brane source charges and whether Page charges are independent of B fields or not\[ \footnote{Here we mean that when } p = 2, 3, \text{ we have D4-, D6-brane.} \footnote{In general, the definition of RR Page charge should be compatible with T-duality, and quantization of the Page charge for a D2p-brane follows from T-duality. T-duality implies Page charge quantization.} \]
In the present paper, we study how to consistently construct B-independent RR charges of D2p-branes with B fields in our new frame. Since D-branes are sources of RR gauge fields, we consider IIA supergravity plus D-brane sources, from which we derive the equations of motion and the nonvanishing (modified) Bianchi identities that define the duals of brane source currents for D2p-branes. Exploiting the relation $d A_p \wedge B_q = d(A_p \wedge B_q) - (-1)^p A_p \wedge dB_q$, we insert the equations for the duals of brane source currents for D(2p+2n)-branes ($n > 1$) into that for the D2p-brane iteratively. The new feature is that the resulting equations can be recast into the form whose left sides of equations are exterior derivative and the right sides are localized objects, thus the right side localized objects can be identified as Page charges because of their conservation and locality. The results also show that the Page charges for D2p-branes can be consistently defined from the brane source charges. Inserting the brane source charges into the expression of the Page charges, we find that all the Page charges are independent of the background B fields. In our explicit construction, it is highly nontrivial that the B-dependent terms like $B \wedge B \wedge B$, $B \wedge B \wedge F$, $B \wedge F \wedge F$ from different sources are exactly cancelled with each other.

The layout of the paper is as follows. In section 2, we consider a D2-brane in B-fields in order to illustrate our method. The brane source currents for D2-, D0-branes are derived from the equations of motion. The explicit expression for RR Page charges is obtained by exploiting their properties — conserved and localized, the result is consistent with those in [3] and [4]. In section 3, we study a D4-brane in B fields, which is bound state of D4/D2/D0-branes. With additional D4-brane source defined by the nonvanishing of modified Bianchi identity, the RR Page charges for D2-, D0-branes living on the D4-brane are constructed by combining the equations of motion and the nonvanishing of the modified Bianchi identity. Late we discuss a D6-brane in B fields, where we find that it is highly nontrivial that the RR Page charges are independent of B fields. In section 4, we present our summary and discussion.

in the systems with sufficient translational symmetry [5].
2. D2-brane in B fields

The IIA supergravity in ten dimensions can be obtained from \( N = 1, D = 11 \) supergravity by dimensional reduction, the action for IIA supergravity is

\[
S_{IIA} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left( R + 4\partial\mu\phi\partial^\mu\phi - \frac{1}{2} |H|^2 \right) - \frac{1}{2} (|F^{(2)}|^2 + |\tilde{F}^{(4)}|^2) \right\} - \frac{1}{4\kappa_{10}^2} \int B \wedge F^{(4)} \wedge F^{(4)}
\]

(1)

where \( \kappa_{10}^2 \) is the gravitational coupling in ten dimensions, and the field strengths \( H, F^{(2)}, F^{(4)}, \) and \( \tilde{F}^{(4)} \) are defined as

\[
H = dB, \quad F^{(2)} = dC^{(1)}, \quad F^{(4)} = dC^{(3)}, \quad \tilde{F}^{(4)} = dC^{(3)} - C^{(1)} \wedge H
\]

(2)

The above action can be rewritten as

\[
S_{IIA} = \int d^{10}x \sqrt{-G} e^{-2\phi} \left( R + 4\partial\mu\phi\partial^\mu\phi \right)
\]

\[
+ \frac{1}{2} \int \left\{ - e^{-2\phi} dB \wedge *dB + dB \wedge dC^{(1)} \wedge *dC^{(1)} \right. \\
+ \left. (dC^{(3)} - C^{(1)} \wedge dB) \wedge (dC^{(3)} - C^{(1)} \wedge dB) \\
- B \wedge dC^{(3)} \wedge dC^{(3)} \right\}
\]

(3)

where for simplicity we have chosen \( 2\kappa_{10}^2 = 1 \), that is, the kinetic term is canonical.

Consider a D2p-brane in the background of B fields, the D2p-brane world-volume action is

\[
S_{D-brane} = S_{BI} + S_{WZ}
\]

(4)

The Born-Infeld part of the action is

\[
S_{BI} = -T_{2p} \int e^{-\phi} \sqrt{-det(G_{ab} + B_{ab} + F_{ab})}
\]

(5)

where \( G_{ab}, \phi, B_{ab} \) are the pullback of spacetime metric, dilaton and Neveu-Schwarz two form B to the D2p-brane world-volume, \( F_{ab} \) is the field strength of U(1) gauge field living on the brane.
The Wess-Zumino (WZ) terms couple the brane to the spacetime RR gauge field through

$$S_{WZ} = \int \left( \sum_i C^{(i)} \right) \wedge e^{B+F}$$

and in the D2-brane case, it is reduced to

$$S_{WZ}^{D2} = \int \left\{ C^{(3)} + C^{(1)} \wedge (B+F) \right\}$$

which means there are D0-branes living on D2-brane.

Since D-branes are sources of RR gauge field, we consider IIA supergravity + the coupled D2-brane, and see how D2-, D0-branes induce $C^{(3)}$, $C^{(1)}$ gauge field, which can be described by the equations of motion. As brane source charge arises from varying the brane action with respect to the gauge field, from the total action $S_T = S_{IIA} + S_{D2-brane}$ we get the equations of motion for $C^{(1)}$ and $C^{(3)}$:

$$d \ast F^{(2)} - H \wedge \tilde{F}^{(4)} = - \ast j_{D0}^{bs}$$

$$d \ast \tilde{F}^{(4)} - H \wedge \tilde{F}^{(4)} = - \ast j_{D2}^{bs}$$

where we have replaced the second term $H \wedge F^{(4)}$ in the left side of the second equation by $H \wedge \tilde{F}^{(4)}$ due to the fact $H \wedge H = 0$. The brane source currents for D2-, D0-branes are given by

$$(j_{D2}^{bs})^{\mu\nu\rho}(x) = \int dX^\mu(\xi) \wedge dX^\nu(\xi) \wedge dX^\rho(\xi) \delta^{10}(x - X(\xi))$$

$$(j_{D0}^{bs})^{a}(x) = \frac{1}{2} \int d^2\xi \epsilon^{abc} \partial_a X^\nu(\xi)[B_{\nu\rho}(X(\xi))\partial_b X^\nu(\xi)\partial_c X^\rho(\xi) + F_{ab}(\xi)]\delta^{10}(x - X(\xi))$$

where $x^\mu$ are ten-dimensional coordinates, $X^\nu(\xi)$ are the embedding coordinates of D2-brane in ten dimensions, and $\xi^a$ are three-dimensional brane worldvolume coordinates.

The brane source charge for D0-brane is

$$Q_{D0}^{bs} = \int \ast j_{D0}^{bs} = \int_{V_2} (B + F)$$

3Since we choose different sign for the WZ term, there is minus sign difference between our [8] and that in [3], similarly for the definition of the nonvanishing Bianchi identities.
where $V_2$ is two-dimensional space volume of D2-brane.

Since we only consider D2-, D0-brane without other brane sources, we have $dF^{(2)} = 0$, $dF^{(4)} = 0$, $dH = 0$, $H \wedge H = 0$. Under these conditions, we see $d \ast j^{bs}_{D2} = 0$, $d \ast j^{bs}_{D0} \neq 0$, that is, $Q^{bs}_{D2}$ is conserved while $Q^{bs}_{D0}$ nonconserved, however, when $H = dB = 0$, $Q^{bs}_{D0}$ is still conserved.

By rewriting the second term $H \wedge \ast \tilde{F}^{(4)}$ in the first equation in (8) as $d(B \wedge \ast \tilde{F}^{(4)}) - B \wedge d \ast \tilde{F}^{(4)}$ and then inserting the second equation of (8) into $B \wedge d \ast \tilde{F}^{(4)}$, we arrive at

$$d \{ - \ast F^{(2)} + B \wedge \ast \tilde{F}^{(4)} - \frac{1}{2} B \wedge B \wedge \tilde{F}^{(4)} \} = \ast j^{bs}_{D0} - B \wedge \ast j^{bs}_{D2}$$

which shows that we can indeed recast two equations in (8) into the new equation whose left side is written as exterior derivative and the right side as localized object. In the above derivation, we have assumed that the D2-brane stays in the region where the relation $H = dB$ holds. According to the properties of the RR Page current, Eq.(12) indicates that we can define consistently

$$\ast j^{Page}_{D0} = \ast j^{bs}_{D0} - B \wedge \ast j^{bs}_{D2}$$

and $j^{Page}_{D0}$ is indeed conserved and localized. The RR Page charge takes

$$Q^{Page}_{D0} = \int V_2 \ast j^{Page}_{D0} = \int F$$

which is consistent with [3] and [4]. In [2] and [5], it was argued that the RR Page charge for D0-branes in spherical D2-brane defined by $\int_{S^2} F$ should be quantized. In the context of WZW model, it has been shown that the integral $\int_{S^2} F$ is quantized indeed [6]. Other related discussion on this topic can also be found in [7], [8] and [9].

If we do T-duality along other extra four dimensions, the D2/D0 bound state turns into D6/D4 one, and Eq.(13) becomes

$$\ast j^{Page}_{D4} = \ast j^{bs}_{D4} - B \wedge \ast j^{bs}_{D6}$$

which means we define the RR Page current for D4-branes living on the D6-brane in the way of compatible with T-duality\(^4\).

\(^4\)Here we should mention that (15) is obtained just by the use of T-duality as suggested by Marolf.
3. D4-, D6-brane in B fields

At first, we consider a D4-brane in B fields, which is D4/D2/D0 bound state. The WZ term for D4-brane in the background of B fields is

\[ S_{WZ}^{D4} = \int \left\{ C^{(5)} + C^{(3)} \wedge (B + F) + \frac{1}{2} C^{(1)} \wedge (B + F) \wedge (B + F) \right\} \]  \tag{16} 

Besides the equations of motion (8), there is the nonvanishing of modified Bianchi identity which describes the hodge dual of the D4-brane source current [3]

\[ d\tilde{F}^{(4)} + F^{(2)} \wedge H = - \ast j^{bs}_{D4} \]  \tag{17} 

If we rewrite IIA supergravity in terms of the field \( C^{(5)} \) dual to \( C^{(3)} \), the modified Bianchi identity for \( C^{(3)} \) becomes an equation of motion for \( C^{(5)} \), and the brane source current can be derived by variation of the WZ term for D4-brane with respect to \( C^{(5)} \). From Eq.(16), we know the brane source charges for D2-, D0-branes in the D4-brane are

\[ Q^{bs}_{D2} = \int \ast j^{bs}_{D2} = \int (B + F) \] 
\[ Q^{bs}_{D0} = \int \ast j^{bs}_{D0} = \int \frac{1}{2} (B + F) \wedge (B + F) \]  \tag{18} 

In the presence of D4-brane, \( dF^{(4)} \) is no longer equal to zero, then \( Q^{bs}_{D2} \) is nonconserved. Without D6-brane, we have \( dF^{(2)} = 0 \), so \( Q^{bs}_{D4} \) is still conserved. Combining Eq.(8) and Eq.(17), we have

\[ d\left\{ - \ast \tilde{F}^{(4)} + B \wedge \tilde{F}^{(4)} + \frac{1}{2} B \wedge B \wedge F^{(2)} \right\} = \ast j^{bs}_{D2} - B \wedge \ast j^{bs}_{D4} \]  \tag{19} 
\[ d\left\{ - \ast F^{(2)} + B \wedge \ast \tilde{F}^{(4)} - \frac{1}{2} B \wedge B \wedge \tilde{F}^{(4)} - \frac{1}{6} B \wedge B \wedge B \wedge F^{(2)} \right\} = \ast j^{bs}_{D0} - B \wedge \ast j^{bs}_{D2} + \frac{1}{2} B \wedge B \wedge \ast j^{bs}_{D4} \]  \tag{20} 

where we use the similar way as in the derivation of Eqs.(12) to get Eqs.(19). To derive (20), we first rewrite the second term \( H \wedge \ast \tilde{F}^{(4)} \) in the first equation in (8) as \( d(B \wedge \ast \tilde{F}^{(4)}) - B \wedge d \ast \tilde{F}^{(4)} \) and insert the second equation of (8) into \( B \wedge d \ast \tilde{F}^{(4)} \), then besides the exterior derivative and localized terms we have the term \( B \wedge H \wedge \tilde{F}^{(4)} \) which can
rewrite as \( d\left( \frac{1}{2} B \wedge B \wedge F^{(4)} \right) - \frac{1}{2} B \wedge B \wedge d\tilde{F}^{(4)} \). Inserting the nonvanishing modified Bianchi identity \((17)\) into the term \( \frac{1}{2} B \wedge B \wedge d\tilde{F}^{(4)} \) and exploiting \( dF^{(2)} = 0 \), we get Eq.\((20)\). Eqs.\((19)\) and \((20)\) show that we can consistently define the hodge duals of Page currents

\[
\begin{align*}
* j^{Page}_{D2} &= *j^{bs}_{D2} - B \wedge *j^{bs}_{D4} \\
* j^{Page}_{D0} &= *j^{bs}_{D0} - B \wedge *j^{bs}_{D2} + \frac{1}{2} B \wedge B \wedge *j^{bs}_{D4}
\end{align*}
\]

from the brane source currents, the resulting \( j^{Page}_{D2} \) and \( j^{Page}_{D0} \) are conserved and localized on D4-brane, and the RR Page charges are

\[
\begin{align*}
Q^{Page}_{D2} &= \int *j^{Page}_{D2} = \int F \\
Q^{Page}_{D0} &= \int *j^{Page}_{D0} = \int \left[ \frac{1}{2} (B + F) \wedge (B + F) - B \wedge (B + F) + \frac{1}{2} B \wedge B \right] \\
&= \int \frac{1}{2} F \wedge F
\end{align*}
\]

where we have seen the B-dependent terms \( B \wedge B, B \wedge F \) have been cancelled with each other.

Now we turn to a D6-brane in B fields, the WZ terms for D6-brane is

\[
S^{D6}_{WZ} = \int \left\{ C^{(7)} + C^{(5)} \wedge (B + F) + \frac{1}{2} C^{(3)} \wedge (B + F) \wedge (B + F) \\
+ \frac{1}{6} C^{(1)} \wedge (B + F) \wedge (B + F) \wedge (B + F) \right\}
\]

which describes D6/D4/D2/D0 bound state. In the presence of D6-brane, besides the equations of motion \((8)\) and the modified Bianchi identity \((17)\), we have another nonvanishing Bianchi identity\(5\)

\[
dF^{(2)} = *j^{bs}_{D6}
\]

\(5\)In IIA supergravity, the RR gauge field \( C^{(2p+1)} \) is dual to \( C^{(7-2p)} \). Because of \( * * A_{2p} = - A_{2p} \) in 10D Minkowski spacetime, there are two ways to performing the electromagnetic duality. For example, consider D6-brane RR gauge field \( C^{(7)} \) (for illustration, we assume B fields vanish), we can define either \( *dC^{(7)} = dC^{(1)} \), i.e., \( *dC^{(1)} = -dC^{(7)} \), or \( *dC^{(1)} = dC^{(7)} \), i.e., \( *dC^{(7)} = -dC^{(1)} \). In case 1, the D6-brane source current is described by \( dF^{(2)} = *j^{bs}_{D6} \), and in case 2, it is \( dF^{(2)} = - *j^{bs}_{D6} \). The WZ term does not contain any information about which definition one should use \((10)\). In order to make the definition for RR Page current compatible with T-duality, we have to choose \( dF^{(2)} = *j^{bs}_{D6} \).
Combining two nonvanishing Bianchi identities (17) and (24), we have
\[ - d\left( \tilde{F}^{(4)} + B \wedge F^{(2)} \right) = *j_{D4}^{bs} - B \wedge *j_{D6}^{bs} \] (25)
which shows that we can define the hodge dual of RR Page current for D4-branes living in D6-brane as
\[ *j_{D4}^{Page} = *j_{D4}^{bs} - B \wedge *j_{D6}^{bs} \] (26)
which is conserved and localized. Here we should point out that Eq.(26) is derived from the nonvanishing modified Bianchi identity (17) for D4-brane and the other nonvanishing Bianchi identity (24) for D6-brane. On the other hand, Eq.(15) is obtained from T-duality, which means the ambiguity in the definition of $C^{(7)}$ can be fixed by T-duality.

In [5], the hodge dual of RR Page current for D4-brane was defined as
\[ - d(\tilde{F}^{(4)} + C^{(1)} \wedge H) = *j_{D4}^{Page} \] (27)
Comparing the Eq.(25) with Eq.(27), we find there is a difference in the definition of the RR Page current, which can be interpreted as gauge dependence of the RR Page charge associated with choosing a surface in $N = 1$ eleven-dimensional supergravity that projects onto the chosen surface in IIA supergravity.

By making use of the equations of motion and two nonvanishing (modified) Bianchi identities, in the way similar to deriving Eqs.(19) and (20) we get the relations
\[ d\left( - *\tilde{F}^{(4)} + B \wedge \tilde{F}^{(4)} + \frac{1}{2} B \wedge B \wedge F^{(2)} \right) = *j_{D2}^{bs} - B \wedge *j_{D4}^{bs} + \frac{1}{2} B \wedge B \wedge *j_{D6}^{bs} \] (28)
\[ d\left( - *F^{(2)} + B \wedge *\tilde{F}^{(4)} - \frac{1}{2} B \wedge B \wedge \tilde{F}^{(4)} - \frac{1}{6} B \wedge B \wedge B \wedge F^{(2)} \right) = *j_{D0}^{bs} - B \wedge *j_{D2}^{bs} + \frac{1}{2} B \wedge B \wedge *j_{D4}^{bs} - \frac{1}{6} B \wedge B \wedge B \wedge *j_{D6}^{bs} \] (29)
If we choose $d\tilde{F}^{(4)} = - *j_{D6}^{bs}$, then we would have $*j_{D4}^{Page} = *j_{D4}^{bs} + B \wedge *j_{D6}^{bs}$, which is incompatible with T-duality.

In [5], the brane source charge and Page charge were defined separately, they did not study how to define Page charge based on the definition for brane source charge, and (27) was given just by definition, but here (26) is derived from the consistent consideration.
Eqs. (28) and (29) indicate that the hodge dual of the RR Page currents for D2-, D0-branes can be defined as

\[ *J_{Page}^{D2} = *j_{bs}^{D2} - B \wedge *j_{bs}^{D4} + \frac{1}{2} B \wedge B \wedge *j_{bs}^{D6} \] (30)

\[ *J_{Page}^{D0} = *j_{bs}^{D0} - B \wedge *j_{bs}^{D2} + \frac{1}{2} B \wedge B \wedge *j_{bs}^{D4} - \frac{1}{6} B \wedge B \wedge B \wedge *j_{bs}^{D6} \] (31)

Recall that for D6/D4/D2/D0 bound state, the brane sources can be read off from the WZ term for D6-brane (23), the corresponding brane source charges are

\[ Q^{bs}_{D4} = \int *j_{bs}^{D4} = \int (B + F) \]
\[ Q^{bs}_{D2} = \int *j_{bs}^{D2} = \int \frac{1}{2} (B + F) \wedge (B + F) \]
\[ Q^{bs}_{D0} = \int *j_{bs}^{D0} = \int \frac{1}{6} (B + F) \wedge (B + F) \wedge (B + F) \] (32)

Putting Eqs. (26), (30), (31) and (32) together, we obtain for D4-, D2-branes

\[ Q_{Page}^{D4} = \int *J_{Page}^{D4} = \int F \] (33)

\[ Q_{Page}^{D2} = \int \left( *j_{bs}^{D2} - B \wedge *j_{bs}^{D4} + \frac{1}{2} B \wedge B \wedge *j_{bs}^{D6} \right) \]
\[ = \int \left\{ \frac{1}{2} (B + F) \wedge (B + F) - B \wedge (B + F) + \frac{1}{2} B \wedge B \right\} \]
\[ = \int \frac{1}{2} F \wedge F \] (34)

and for D0-branes

\[ Q_{Page}^{D0} = \int \left( *j_{bs}^{D0} - B \wedge *j_{bs}^{D2} + \frac{1}{2} B \wedge B \wedge *j_{bs}^{D4} - \frac{1}{6} B \wedge B \wedge B \wedge *j_{bs}^{D6} \right) \]
\[ = \int \left\{ \frac{1}{6} (B + F) \wedge (B + F) \wedge (B + F) - \frac{1}{2} B \wedge (B + F) \wedge (B + F) \right. \]
\[ + \frac{1}{2} B \wedge B \wedge (B + F) - \frac{1}{6} B \wedge B \wedge B \right\} \]
\[ = \int \frac{1}{6} F \wedge F \wedge F \] (35)
Eq.(35) shows that the terms $B \wedge B \wedge B$, $B \wedge B \wedge F$, $B \wedge F \wedge F$ are cancelled with each other in the RR Page charge of D0-branes living in D6-brane, which was expected in [3], [4] and [5].

4. Summary and Discussion

So far we have developed new approach to consistently construct B-independent RR charges of D2p-branes in B fields. As D-branes are sources of RR gauge fields, we have induced the equations of motion and the nonvanishing (modified) Bianchi identities from total action for IIA supergravity plus D-brane sources, with which we have defined the duals of brane source currents for D2p-branes. By making use of the relation $dA_p \wedge B_q = d(A_p \wedge B_q) - (-1)^p A_p \wedge dB_q$, we have plugged the equations for the duals of brane source currents for D(2p+2n)-branes into that for the D2p-brane iteratively, and found that the resulting equations can be recast into that the left sides of equations are exterior derivative and the right sides are localized objects. This particular feature indicates that the right side localized objects can be identified as Page charges because of their conservation and locality, which shows that the Page charges for D2p-branes can be consistently defined from the brane source charges. Inserting the brane source charges into the expression of the Page charges, we have shown that all the Page charges are independent of the background B fields which provides further evidence that the RR Page charge should be quantized. In our explicit construction, it is highly nontrivial that the B-dependent terms like $B \wedge B \wedge B$, $B \wedge B \wedge F$, $B \wedge F \wedge F$ from different sources are exactly cancelled with each other. Thus we conclude that the properly defined RR charges of D-branes should be B-independent, and we can identify them as Page charges which are conserved and localized.

The IIA supergravity only contains 2-form $F^{(2)}$ and 4-form $F^{(4)}$ field strength, until now we only considered D2-, D4-, D6-brane in B fields. It would be interesting to study D8-brane in B fields and construct the corresponding RR Page charges for D0-, D2-, D4-, D6-brane.

\[\text{8When B fields are constant, all the brane source charges are conserved, so in that case there is no necessity to construct RR Page charge.}\]
D6-branes which couple to D8-brane via WZ term. In this case, we should turn to massive type IIA supergravity. In the above construction, we have assumed \( H = dB \), however when it does not hold in whole region, for instance, in the near-horizon region of the NS5-brane background where the geometry is \( R^{1,5} \times R_5 \times S^3 \), we cover \( S^3 \) with two open sets \( U = \{ u_\alpha \} \), the RR Page charge defined on each open set with \( B_\alpha \) is conserved. It is interesting to see how two RR Page charges defined in each open set are connected. It is probably related to brane creation phenomenon \[9\]. We hope to discuss the above questions in near future.

**Acknowledgement**

I would like to thank B. Harms, M. Kreuzer, D. Marolf, H.J. Schnitzer and P.K. Townsend for valuable discussion. This work is supported by the Austrian Research Funds FWF under grant No. M597-TPH.

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