What can we learn from $B \to a_1(1260)(b_1(1235))\pi(K)$ decays?

Wei Wang$^a$, Run-Hui Li$^{b,a}$ and Cai-Dian Lü$^{a,c}$

$^a$ Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, P.R. China
$^b$ Physics Department, Shandong University, Jinan 250100, P.R. China
$^c$ Theoretical Physics Center for Science Facilities, CAS, Beijing 100049, P.R. China

We investigate the $B \to a_1(1260)(b_1(1235))\pi(K)$ decays under the factorization scheme and find many discrepancies between theoretical predictions and the experimental data. In the tree dominated processes, large contributions from color-suppressed tree diagrams are required in order to accommodate with the large decay rates of $B^- \to a_0^0\pi^-$ and $B^- \to a_1^-\pi^0$. For $\bar{B}^0 \to (a_1^+, b_1^+)K^-$ decays which are both induced by $b \to s$ transition, theoretical predictions on their decay rates are larger than the data by a factor of 2.8 and 5.5, respectively. Large electro-weak penguins or some new mechanism are expected to explain the branching ratios of $B^- \to b_0^0K^-$ and $B^- \to a_1^-\bar{K}^0$. The soft-collinear-effective-theory has the potential to explain large decay rates of $B^- \to a_0^0\pi^-$ and $B^- \to a_1^-\pi^0$ via a large hard-scattering form factor $\zeta^{B\to a_1}_{J=a_1}$. We will also show that, with proper charming penguins, predictions on the branching ratios of $\bar{B}^0 \to (a_1^+, b_1^+)K^-$ can also be consistent with the data.

PACS numbers: 13.25.Hw,14.40.Cs

I. INTRODUCTION

Since the first measurement on $B^0/\bar{B}^0 \to a_1^\pm(1260)\pi^\mp$ decays reported by BaBar and Belle collaborations [1, 2, 3], many charmless $B$ decays into a pseudo-scalar and an axial-vector meson have been observed. Among the 18 $B \to a_1(1260)(b_1(1235))\pi(K)$ decay channels, 10 of them have been measured with large branching ratios. Besides decay rates, direct CP asymmetries in some $B \to (a_1, b_1)K$ channels and time-dependent CP asymmetries in $B^0/\bar{B}^0 \to a_1^\pm\pi^\mp$ and $B^0/\bar{B}^0 \to b_1^\pm\pi^\mp$ were also studied in the two B factories [4, 5, 6, 7, 8, 9]. Without any doubt, these results are helpful to investigate production mechanisms of axial-vectors in $B$ decays, extract hadronic parameters such as strong phases in $B \to AP$ decays and probe the structures of axial-vectors.

Charmless two-body $B \to AP$ decays have received considerable theoretical efforts [10, 11, 12, 13, 14]. Among these predictions, many of them are not consistent with each other:

\footnote{In the following, we will use $a_1(b_1)$ to denote the $a_1(1260)(b_1(1235))$ meson for simplicity.}
most predictions by Calderón, Munoz and Vera [12] are larger than predictions given by Laporta, Nardulli and Pham [11] and the QCD factorization (QCDF) approach. Predictions on $B \to a_1 \pi$ by Laporta, Nardulli and Pham (using the second sets of form factors) are very close to results in the QCDF approach. However there are large discrepancies in other predictions (See Ref. [14] for a detailed comparison between these theoretical predictions). Many results of the QCDF approach agree with the experimental data, but there still exist some deviations.

In the present paper, we intend to analyze the 18 $B \to AP$ decays with the help of experimental data. We try to check whether these problems can be removed in the perturbative QCD (PQCD) approach and the soft-collinear-effective-theory (SCET). Another objective is to extract the $B \to A$ form factors through $\bar{B}^0 \to a_1^+ \pi^-$ and $\bar{B}^0 \to b_1^+ \pi^+$ decays.

II. NAIVE FACTORIZATION APPROACH

The effective Hamiltonian describing $b \to D(D = d, s)$ transitions are given by [15]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{q=u,c} V_{qd} V_{qD}^* \left[ C_1 O_1^q + C_2 O_2^q + \sum_{i=3}^{10} C_i O_i \right] \right\} + \text{H.c.,} \quad (1)$$

where $V_{qd(D)}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Functions $O_i$ are the local four-quark operators, while functions $C_i$ are the corresponding Wilson coefficients. It is convenient to define combinations $a_i$ of the Wilson coefficients:

$$a_1 = C_2 + C_1/3, \quad a_2 = C_1 + C_2/3,$$

$$a_i = C_i + C_{i+1}/N_c \quad (i = 3, 5, 7, 9),$$

$$a_i = C_i + C_{i-1}/N_c \quad (i = 4, 6, 8, 10). \quad (2)$$

There exist a hierarchy for the Wilson coefficients:

$$a_1 \gg \max\{a_2, a_{3-10}\}. \quad (3)$$

For tree-dominated processes $B^0/\bar{B}^0 \to a_1^+ \pi^+$, the factorization formulae can be written as:

$$\mathcal{A}(\bar{B}^0 \to a_1^+ \pi^-) = \frac{G_F}{\sqrt{2}} m_B^2 f_{\pi} V_{0B}^* \left\{ V_{ub} V_{ud}^* \left[ a_1 + a_4 + a_{10} + r_\pi (a_6 + a_8) \right] \right\} + V_{cb} V_{cd}^* [a_4 + a_{10} + r_\pi (a_6 + a_8)], \quad (4)$$

$$\mathcal{A}(\bar{B}^0 \to \pi^+ a_1^-) = \frac{G_F}{\sqrt{2}} m_B^2 f_{a_1} f_{+B^+\pi} \left\{ V_{ub} V_{ud}^* [a_1 + a_4 + a_{10}] + V_{cb} V_{cd}^* [a_4 + a_{10}] \right\}, \quad (5)$$

where $r_\pi = 2m_\pi^2/m_B$ with $m_\pi^2$ the chiral scale parameter for pion. The CKM matrix elements for tree operators $|V_{ub} V_{ud}| \sim 4 \times 10^{-3}$ have the same order magnitude with those for penguin
Furthermore, in the hierarchy of the Wilson coefficients, penguin contributions from the operators $O_{3-10}$ are small compared with those from tree operators. Thus penguin contributions can be neglected in the study of branching ratios (but crucial to CP asymmetries). Combined with the $\bar{B}^0 \to \pi^+\pi^-$ data \[16\]

$$\text{BR}(\bar{B}^0 \to \pi^+\pi^-) = (5.16 \pm 0.22) \times 10^{-6},$$

we arrive at the $a_1$ meson decay constant and $B \to a_1$ form factor:

$$f_{a_1} = [2.02 \pm 0.26 \pm 0.04 + O(\left(\frac{a_{3-10}}{a_1}\right))]f_{\pi}, \quad V_0^{B \rightarrow a_1} = (1.55 \pm 0.28 \pm 0.03 + O(\left(\frac{a_{3-10}}{a_1}\right)))f_+^{B \rightarrow \pi} \tag{7}$$

where the uncertainties are from the experimental results for branching ratios. As a rough estimation, we take $f_{\pi} = 131$ MeV and $f_+^{B \rightarrow \pi} = 0.25$ which corresponds to $f_{a_1} = (265 \pm 34 \pm 6)$ MeV and $V_0^{B \rightarrow a_1} = 0.39 \pm 0.07 \pm 0.01$. These results are well consistent with predictions based on the PQCD approaches \[17\] and light-cone sum rules \[18,19\].

Now we come to the two channels $B^- \to a^0_1\pi^-$ and $B^- \to a^+_1\pi^0$ whose factorization formulae are given by:

$$\sqrt{2}A(B^- \to \pi^- a^0_1) = \frac{G_F}{\sqrt{2}}m_B^2f_\pi V_0^{B \rightarrow a_1} \left\{ V_{ub}V_{ud}^*[a_1 + a_4 + a_{10} + r_\pi(a_6 + a_8)] + V_{cb}V_{cd}^*[a_4 + a_{10} + r_\pi(a_6 + a_8)] \right\} + \frac{G_F}{\sqrt{2}}m_B^2f_{a_1}f_+^{B \rightarrow \pi} \left\{ V_{ub}V_{ud}^*[a_2 - a_4 + \frac{1}{2}a_{10}] + V_{cb}V_{cd}^*[-a_4 + \frac{1}{2}a_{10}] \right\} \tag{8}$$

$$\sqrt{2}A(B^- \to \pi^0 a^+_1) = \frac{G_F}{\sqrt{2}}m_B^2f_\pi V_0^{B \rightarrow a_1} \left\{ V_{ub}V_{ud}^*[a_2 - a_4 + \frac{1}{2}a_{10} + r_\pi(-a_6 + \frac{1}{2}a_8)] + V_{cb}V_{cd}^*[-a_4 + \frac{1}{2}a_{10} + r_\pi(-a_6 + \frac{1}{2}a_8)] \right\} + \frac{G_F}{\sqrt{2}}m_B^2f_{a_1}f_+^{B \rightarrow \pi} \left\{ V_{ub}V_{ud}^*[a_1 + a_4 + a_{10}] + V_{cb}V_{cd}^*[a_4 + a_{10}] \right\}. \tag{9}$$

Because of the small values of $a_{3-10}$, the penguin contributions can be safely neglected:

$$\sqrt{2}A(B^- \to \pi^- a^0_1) = \frac{G_F}{\sqrt{2}}m_B^2V_{ub}V_{ud}^*[a_1 f_\pi V_0^{B \rightarrow a_1} + a_2 f_{a_1} f_+^{B \rightarrow \pi}], \quad \tag{10}$$

$$\sqrt{2}A(B^- \to \pi^0 a^+_1) = \frac{G_F}{\sqrt{2}}m_B^2V_{ub}V_{ud}^*[a_2 f_\pi V_0^{B \rightarrow a_1} + a_1 f_{a_1} f_+^{B \rightarrow \pi}]. \tag{11}$$

Furthermore, in the hierarchy of $a_2 \ll a_1$, branching ratios are required to satisfy the following relation:

$$\text{BR}(\bar{B}^0 \to a^+_1\pi^-) = 2\text{BR}(B^- \to \pi^- a^0_1), \quad \text{BR}(\bar{B}^0 \to \pi^+ a^+_1) = 2\text{BR}(B^- \to a^-_1\pi^0). \tag{12}$$

But the experimental data shows:

$$\text{BR}(B^- \to \pi^0 a^+_1) > \text{BR}(\bar{B}^0 \to a^-_1\pi^+), \quad \text{BR}(B^- \to \pi^- a^0_1) > \text{BR}(\bar{B}^0 \to a^+_1\pi^-), \tag{13}$$
which is dramatically different. This situation is very similar with that in $B \to \pi \pi$ decays: the branching ratio of $B^- \to \pi^0 \pi^-$ is measured with almost equal magnitude with $\mathcal{BR}(\bar{B}^0 \to \pi^- \pi^+)$ but it is expected as one half of $\mathcal{BR}(\bar{B}^0 \to \pi^- \pi^+)$. To solve these problems, an efficient way is to enhance the color-suppressed contribution which is proportional to $a_2$. For example, if the Wilson coefficient $a_2$ can be enhanced to 0.5, the branching ratios of $\mathcal{BR}(\bar{B}^- \to \pi^0 a_1^-)$ and $\mathcal{BR}(\bar{B}^- \to \pi^- a_0)$ are predicted as $20.0 \times 10^{-6}$ and $16.7 \times 10^{-6}$, where we have utilized the experimental data on branching ratios of $\bar{B}^0/\bar{B}^0 \to \pi^\pm a_1^\mp$. And these results are well consistent with the experimental data.

The decay constant of $b_1$ vanishes because of the G-parity, thus $\bar{B}^0 \to \pi^0 b_1^-$ is factorization-suppressed and only the $\bar{B}^0 \to \pi^- b_1^+$ decay survives. From the experimental results collected in table II we can infer that the form factors of $B \to a_1$ and $B \to b_1$ are almost equal in magnitude at maximally recoiling: $|V_{ub}^B| (q^2 = 0)| \simeq |V_{ub}^B| (q^2 = 0)| \simeq 0.35$. One should be careful that the two form factors have different signs, if we use LCDAs of $a_1$ and $b_1$ evaluated by the QCD sum rules. The absolute value of these form factors can be checked by the future measurements on semi-leptonic $B \to A$ decays such as $\bar{B}^0 \to (a_1^+, b_1^+) l^- \bar{\nu}$.

Flavor structures of $\bar{B}^0 \to a_1^+ K^-$ and $\bar{B}^0 \to a_1^+ K^-$. The branching ratio of $\bar{B}^0 \to \pi^+ K^-$ has been measured as $[14]$: $\mathcal{BR}(\bar{B}^0 \to \pi^+ K^-) = (19.4 \pm 0.6) \times 10^{-6}$, (15)

which implies:

$$\mathcal{BR}(\bar{B}^0 \to a_1^+ K^-) = 45.9 \times 10^{-6}, \quad \mathcal{BR}(\bar{B}^0 \to b_1^+ K^-) = 41.0 \times 10^{-6}. \quad (16)$$

Comparing with the experimental measurements in table II we see that our theoretical prediction on $\mathcal{BR}(\bar{B}^0 \to a_1^+ K^-)$ is 2.8 times larger while the prediction on $\mathcal{BR}(\bar{B}^0 \to b_1^+ K^-)$ is 5.5 times larger. This discrepancy should be clarified by the theoretical studies with next-to-leading order corrections and improved experimental measurements.
Besides $B^0 \to (a_1^+, b_1^+)K^-$ decays, $B^- \to a_1^- K^0$ and $B^- \to b_1^0 K^-$ decays are also measured by experimentalists whose factorization formulae are:

$$A(B^- \to a_1^- K^0) = \frac{G_F}{\sqrt{2}} m_B^2 f_K V_{b \bar{s}}^{B^- \to a_1} \left\{ V_{u \bar{s}} V_{u \bar{s}}^* [a_4 - \frac{1}{2} a_{10} + r_K (a_6 - \frac{1}{2} a_s)] + V_{c \bar{c}} V_{c \bar{s}}^* [a_4 - \frac{1}{2} a_{10} + r_K (a_6 - \frac{1}{2} a_s)] \right\},$$  \hspace{1cm}(17)$$

$$\sqrt{2} A(B^- \to b_1^0 K^-) = \frac{G_F}{\sqrt{2}} m_B^2 f_K V_{b \bar{s}}^{B^- \to b_1} \left\{ V_{u \bar{s}} V_{u \bar{s}}^* [a_4 + a_{10} + r_K (a_6 + a_s)] + V_{c \bar{c}} V_{c \bar{s}}^* [a_4 + a_{10} + r_K (a_6 + a_s)] \right\}.$$ \hspace{1cm}(18)

In these $b \to s$ transitions, the CKM matrix elements for penguin operators are $|V_{c \bar{c}} V_{c \bar{s}}^*| \sim 40 \times 10^{-3}$ and those for tree operators are $|V_{u \bar{s}} V_{u \bar{s}}^*| \sim 0.8 \times 10^{-3}$. Recalling the values for the Wilson coefficient combinations: $a_1 \sim 1$ and $a_4 \sim a_6 \sim -0.03$, we can see that contributions from tree operators with the coefficient $a_1$ are smaller than that from penguin operators at least by a factor of 2 in magnitude. In order to characterize the contribution from tree operators and symmetry breaking effects between $B^-$ and $\bar{B}^0$ mesons, it is useful to define the two ratios:

$$R_1 \equiv \frac{BR(B^- \to a_1^- K^0)}{BR(\bar{B}^0 \to a_1^+ K^-)} \times \frac{\tau_{B^0}}{\tau_B}, \hspace{1cm} R_2 \equiv \frac{BR(B^- \to b_1^0 K^-)}{BR(\bar{B}^0 \to b_1^+ K^-)} \times \frac{\tau_{B^0}}{\tau_B},$$ \hspace{1cm}(19)

where $\tau$ is the lifetime of $B$ meson. Neglecting tree operators and electro-weak penguins, the ratios obey the limit:

$$R_1 = 1, \hspace{1cm} R_2 = 0.5,$$ \hspace{1cm}(20)

which are quite different from the experimental results:

$$R_{1\text{exp.}} = 2.00 \pm 0.59, \hspace{1cm} R_{2\text{exp.}} = 1.15 \pm 0.34.$$ \hspace{1cm}(21)

The difference between the two channels in the ratio $R_1$ is the tree operator and electroweak penguin operators. Since the contribution of tree operator is smaller than QCD penguins and the two kinds of amplitudes are perpendicular with each other due to the CKM angle $\gamma$ close to 90°, the tree operator can not change the branching ratio of $\bar{B}^0 \to a_1^+ K^-$ too much. Thus this does not improve theoretical predictions on $R_1$. Large electroweak penguins may help us to diminish the large deviation for $R_1$. In the $\bar{B}^0 \to b_1^+ K^-$ and $B^- \to b_1^0 K^-$ decays, the factorization formulae are exactly the same since the $b_1$ decay constant vanishes. Thus in order to explain the large ratio $R_2$, one needs some mechanism beyond factorization to enhance the ratio of $R_2$ by roughly 2.5.

In the above, we have analyzed the charmless non-leptonic $B \to AP$ data under the factorization approach. The decay constant of $a_1$ meson and $B \to a_1, b_1$ form factors $V_0$ are
extracted from the $B^0 \to a_1\pi$ ad $B^0 \to b_1\pi$ decays. The form factors are consistent with
the predictions evaluated in light-cone-sum-rules and the PQCD approach. But there exist
several problems which can be summarized as:

- The Wilson coefficient combination $a_2$ needs to be enhanced to $a_2 = 0.5$ in order to
  solve the problem in $B^- \to a^-_1\pi^0$ and $B^- \to a^0_1\pi^-$. 

- Since the form factor $B \to a_1$ and $B \to b_1$ are almost equal in magnitude, the $\bar{B}^0 \to a_1^+K^-$
  and $\bar{B}^0 \to b_1^+K^-$ decays should possess similar and large branching ratios.
  Compared with the experimental data, theoretical predictions needs to be reduced by
  the factors of 2.8 and 5.5, respectively.

- $B^- \to a^-_1\bar{K}^0$ and $B^- \to b^0_1K^-$ are related to $\bar{B}^0 \to (a^+_1, b^+_1)K^-$ through relations given
  in Eq. (19) which also have large deviations from the data.

### III. THE SOFT-COLLINEAR EFFECTIVE THEORY

The recent development of SCET makes the analysis of $B \to M_1M_2$ decays on a more rig-
orous foundation. The SCET is a powerful method to systematically separate the dynamics
at different scales: hard scale $m_b$ ($b$ quark mass), hard intermediate scale $\mu_{hc} = \sqrt{m_b\Lambda_{QCD}}$,
soft scale and to sum large logs using the renormalization group technics. Integrating out
the hard fluctuations, we arrive at the intermediate effective theory- SCET, where the
factorization formulae for $B \to M_1M_2$ decays to leading power in $\lambda \equiv \sqrt{\Lambda_{QCD}/m_b}$ are given
by:

$$A(B \to M_1M_2) = \frac{G_F}{\sqrt{2}}m_B^2 \left\{ f_{M_1} \int du \phi_{M_1}(u)T_1(u)\zeta^{B \to M_2} \\
+ f_{M_1} \int du \phi_{M_1}(u) \int dz T_{1J}(u,z)\zeta^{B \to M_2}_{J}(z) + (1 \leftrightarrow 2) \right\}, \quad (22)$$

where functions $\zeta$ and $\zeta_{J}$ also enter into the heavy-to-light form factors. $T_1(u)$ and $T_{1J}(u,z)$
are hard kernels which can be calculated using perturbation theory. With the hard-collinear
fluctuation integrated out, the final effective theory-SCET$_{II}$ is obtained where the function
$\zeta_{J}$ can be factorized into convolutions of LCDAs with hard kernels:

$$\zeta^{B \to M_2}_{J}(z) = \phi_B(\omega) \otimes J(z,\omega,v) \otimes \phi_{M_2}(v). \quad (23)$$

$J(z,\omega,v)$ is the hard kernel and $\phi_B$ and $\phi_{M_1,M_2}$ are the light-cone distribution amplitudes
(LCDAs). With our knowledge on these LCDAs, one can predict the decay amplitude by
convoluting the LCDAs with the perturbatively calculated hard kernels. But there is another
alternative way for phenomenological studies: one can fit experimental results, including
branching ratios and CP asymmetries, to determine essential non-perturbative inputs. Note that in this way, no expansions in $\alpha_s(\sqrt{m_b \Lambda_{QCD}})$ are needed and thus the exploration of the convergence is spontaneously avoided. This method is especially useful at tree level: $T_1(u)$ is a constant and $T_{1J}(u, z)$ is a function of one argument $u$. It leads to a rather simple form for decay amplitudes:

$$\mathcal{A}(B \rightarrow M_1M_2) = \frac{G_F}{\sqrt{2}} m_B^2 \left\{ f_{M_1} T_1 \zeta^{B \rightarrow M_2} + f_{M_2} \int du \phi_{M_2}(u) T_{1J}(u) \zeta^{B \rightarrow M_2} + (1 \leftrightarrow 2) \right\},$$

where the functions $\zeta^{B \rightarrow M_2}$ and

$$\zeta^{B \rightarrow M_2}_J = \int dz \zeta^{B \rightarrow M_2}_J(z)$$

are treated as non-perturbative parameters to be fitted from the data. With the help of the flavor SU(3) symmetry, the $B \rightarrow AP$ decays involve only 6 parameters:

$$\zeta^{B \rightarrow P}, \quad \zeta^{B \rightarrow P}_J, \quad \zeta^{B \rightarrow P_1^0}, \quad \zeta^{B \rightarrow P_1^+}, \quad \zeta^{B \rightarrow P_1^-},$$

which contribute to the $B \rightarrow P$ and $B \rightarrow A$ form factors.

Including the non-perturbative contributions from loop diagrams involving $c \bar{c}$, the SCET can successfully explain most of $B \rightarrow PP$ and $B \rightarrow VP$ decays. This phenomenological approach has many important features. In $b \rightarrow d$ transitions such as $\bar{B}^0 \rightarrow \pi^+\pi^-$, tree operators provide the dominant contributions and contributions from charming penguins and penguin operators are sub-leading. From the experience in $B \rightarrow PP$ and $B \rightarrow VP$ phenomenological study, we know that the hard-scattering form factor $\zeta_J$ is potentially large. Furthermore, as we have shown in Ref. 27, the corresponding Wilson coefficient is of order 1 which amounts to a large effective Wilson coefficient $a_2$. Here we take $\bar{B}^0 \rightarrow a_1^- \pi^+$ and $B^- \rightarrow a_1^- \pi^0$ as an example: if hard-scattering form factors are equal with soft form factors for pion and $a_1$ meson: $\zeta = \zeta_J$, the effective Wilson coefficient equals to $a_2 \approx \frac{\zeta_J}{\zeta_J + \zeta_J} = 0.5$. Thus it is easy to solve the problems in $B \rightarrow a_1 \pi$ decays under the SCET framework.

For decays induced by $b \rightarrow s$ transition, since tree operators are suppressed by the CKM matrix elements $|V_{ub}V^*_{us}|/(V_{cb}V^*_{cs})| \sim 0.02$ and penguin operators have smaller Wilson coefficients $\max[C_{3-10}] \ll \alpha_s(2m_c)C_1$, charming penguins play a significant role. Due to the non-perturbative nature, charming penguins are totally unknown from perturbation theory and needs to be extracted from data. This stuff depends on the three involved mesons: $B$ meson, recoiling meson and emitted meson. Thus in order to predict physical observables, too many parameters for charming penguins are required. An efficient way to reduce the independent inputs is to utilize the flavor SU(3) symmetry and as a result only 8 parameters for charming penguins in $B \rightarrow AP$ decays are left. But even so, due to the lack of data, one
can always obtain proper branching ratios of $B^0 \to b^+_1 K^-$ and $B^0 \to a^+_1 K^-$ by adjusting charming penguins. Despite of that, there is another deficit: since the inputs, form factors and charming penguins, have been assumed to respect the SU(3) symmetry, large deviations of the ratios shown in Eqs. (20) and (21) can not be eliminated by the SCET either.

IV. THE PERTURBATIVE QCD APPROACH

There is another commonly-accepted approach to handle hadronic $B$ decays: the perturbative QCD approach \cite{28, 29, 30}. The basic idea of the PQCD approach is that it takes into account the transverse momentum of the valence quarks in hadrons. Decay amplitudes and form factors can be written as convolutions of wave functions with perturbatively hard kernels integrated over the longitudinal and transverse component. When considering radiative corrections, one encounters double logarithm divergences when soft and collinear momenta overlap. These large double logarithm can be resummed into the Sudakov factor. Loop corrections to the weak decay vertex also give rise to double logarithms in the threshold region. Resummation of this type of double logarithms leads to the Sudakov factor $S_t$. This factor decreases faster than any power of $x$ as $x \to 0$ and changes the behavior at the end-point region. The Sudakov factor and threshold resummation make the PQCD approach more self-consistent. This approach have successfully explained the $B \to \pi\pi$ and $B \to \pi K$ decay rates and CP asymmetries \cite{31} together with the proper polarizations in $B \to VV$ decays \cite{32}.

In the PQCD approach, the predicted $B \to a_1$ form factor \cite{17} is consistent with the one derived from the data, thus our PQCD prediction on $\mathcal{BR}(\bar{B}^0 \to a^+_1 \pi^-)$ is in good agreement with the data. But due to the small value of $a_2$, the color-suppressed contribution is too small to explain the large decay rates of $B^- \to a_1^-\pi^0$ and $B^- \to a_0^0\pi^-$. The investigations of next-to-leading order corrections in Ref. \cite{33} show that the branching ratio of $B^- \to \pi^-\pi^0$ is enhanced by the factor $4.0/3.5$ while $\bar{B}^0 \to \pi^+\pi^-$ is reduced by $6.5/7.0$. But even if we assume the same $k$ factor for $B \to a_1\pi$ decays, the PQCD predictions on $B^- \to a_1^-\pi^0$ and $B^- \to a_0^0\pi^-$ are still smaller than the data. The PQCD prediction on the $B \to b_1$ form factor is large, thus the branching ratio of $\bar{B}^0 \to b^+_1\pi^-$ is $2$ times larger than the experimental data and the QCDF results. From the factorization formulae of $B \to PP$ decays given in the literature \cite{30}, one can see that the contributions from hard spectator scattering diagrams are small due to the cancelation between two diagrams where a gluon is attached to either the positive quark or the anti-quark in the emitted hadron. But if the emitted meson is a $P$-wave meson and the twist-2 LCDA is anti-symmetric (like a scalar or an axial-vector meson with quantum number $^{2S+1}L_J = 1 P_1$), the two diagrams give constructive contributions to make them sizable. For example, the large hard spectator scattering contributions to
$B^- \to b_0^0 \pi^-$ make $\mathcal{BR}(B^- \to b_0^0 \pi^-) > \frac{1}{2} \mathcal{BR}(B^0 \to b_1^+ \pi^-)$. Moreover, annihilation diagrams play an important role in the PQCD approach which often enters into decay amplitudes as imaginary. It provides the dominant strong phase which are essential to explain the large CP asymmetries. Thus unlike the situation in the QCDF approach, annihilation diagrams do not cancel with emission diagrams in $\bar{B}^0 \to b_1^+ K^-$ which results in much larger predictions on branching ratios of $\bar{B}^0 \to b_1^+ K^-$. Similar as the factorization approach, there are large differences between the PQCD approach predictions on ratios $R_{1,2}$ and those extracted from the data.

V. NUMERICAL RESULTS

In the PQCD framework and SCET framework, we calculate the decay rates, direct CP asymmetries and time-dependent CP asymmetries shown in table I, II, III and IV. We have adopted the same conventions with Ref. [14] for observables in time-dependent decay widths of $B \to a_1^+ \pi^+$ and $B \to b_1^+ \pi^+$.

In the SCET calculation, we use the following values for the 14 inputs:

\begin{align}
\zeta_{B^- \to \pi} &= 0.12, \quad \zeta_{B^- \to a_1} = 0.17, \quad \zeta_{B^- \to b_1} = -0.16, \\
|A_{cc}^{3P_1 P_P}| &= 40 \times 10^{-4}, \quad \arg[A_{cc}^{3P_1 P_P}] = 160^\circ, \\
|A_{cc}^{1P_1 P_P}| &= 40 \times 10^{-4}, \quad \arg[A_{cc}^{1P_1 P_P}] = 155^\circ.
\end{align}

We should point out that this set of inputs is presented by hand instead of any reasonable way. To test the sensitivities on these parameters, we show the first uncertainty in numerical results by varying the form factors by 0.03, 20% for magnitudes of charming penguins and 20° for the phases. The second uncertainty is from CKM matrix elements. In the PQCD calculation, we have used the same inputs as those in Ref. [17, 36, 37]. The theoretical uncertainties are from: (i) the hadronic inputs: decay constants of $B$ meson, and shape parameters in the wave function of $B$ meson; (ii) $\Lambda_{QCD}$, the hard scale $t$ and the threshold resummation parameter $c$; (iii) the CKM matrix elements $V_{ub}$ and $\gamma$ angle. The factorization formula for each type of diagrams in $B \to AP$ decays are the same with those in $B \to PP$.

---

\[ \text{The } A_{a_1^\pm \pi^\mp} \text{ in the present paper correspond to } A_{a_1^\mp \pi^\pm} \text{ defined in Ref. [16, 34, 35].} \]
TABLE I: Theoretical predictions and experimental results on branching ratios (in unit of $10^{-6}$) of $B \to a_1(b_1)\pi(K)$ decays. The QCDF predictions are quoted from Ref. [14]. In the PQCD approach, the uncertainties are from: (i) the hadronic inputs: decay constants of $B$ meson, and shape parameters in the wave function of $B$ meson; (ii) $\Lambda_{QCD}$, the hard scale $t$ and the threshold resummation parameter $c$; (iii) the CKM matrix elements $V_{ub}$ and $\gamma$ angle. In the SCET framework, the uncertainties are from: (i) hadronic parameters: form factors and charming penguins; (ii) the CKM matrix elements.

| channel | QCDF      | PQCD       | SCET       | Exp.       |
|---------|-----------|------------|------------|------------|
| $B^- \to a_1^0 \pi^0$ | $14.4^{+1.4+3.5+2.1}_{-1.3-3.2-1.9}$ | $8.1^{+1.4+2.1+0.7}_{-2.7-1.2-0.9}$ | $19.0^{+5.1+1.8}_{-4.7-1.7}$ | $26.4 \pm 5.4 \pm 4.1$ |
| $B^- \to a_1^0 \pi^-$ | $7.6^{+0.3+1.7+1.4}_{-0.3-1.3-1.0}$ | $6.7^{+2.9+2.8+0.5}_{-2.2-1.7-0.7}$ | $17.2^{+4.7+1.7}_{-4.3-1.6}$ | $20.4 \pm 4.7 \pm 3.4$ |
| $B^0 \to a_1^- \pi^+$ | $23.4^{+2.3+6.2+1.9}_{-2.2-5.5-1.3}$ | $15.7^{+8.3+5.9+1.2}_{-5.6-3.6-1.7}$ | $17.0^{+5.8+1.6}_{-5.2-1.4}$ | $21.0 \pm 5.4$ |
| $B^+ \to a_1^+ \pi^-$ | $9.1^{+0.2+2.1+1.7}_{-0.2-1.8-1.1}$ | $12.7^{+5.6+6.2+0.9}_{-4.4-3.8-1.3}$ | $10.7^{+2.5+1.0}_{-2.4-0.9}$ | $12.2 \pm 4.5$ |
| $B^0/B^0 \to a_1^0 \pi^0$ | $0.9^{+0.1+0.3+0.7}_{-0.1-0.2-0.3}$ | $0.12^{+0.0+0.0+0.0}_{-0.0-0.0-0.0}$ | $5.5^{+1.7+0.6}_{-1.5-0.6}$ | $31.7 \pm 3.7$ |
| $B^- \to a_1^0 K^-$ | $13.9^{+0.9+9.5+12.9}_{-0.9-5.1-4.9}$ | $15.4^{+7.8+10.1+2.4}_{-7.4-5.5-2.5}$ | $10.5^{+3.3+1.8}_{-2.9-1.5}$ | $34.9 \pm 5.0 \pm 4.4$ |
| $B^- \to \bar{a}_1^0 \bar{K}^0$ | $21.6^{+1.2+16.5+23.6}_{-1.1-8.5-11.9}$ | $25.5^{+12.9+18.0+3.7}_{-9.2-10.2-3.9}$ | $15.5^{+5.8+2.5}_{-5.0-2.1}$ | $16.3 \pm 2.9 \pm 2.3$ |
| $B^+ \to a_1^+ K^-$ | $18.3^{+1.0+14.2+21.1}_{-1.0-7.2-7.5}$ | $20.6^{+10.2+14.6+3.2}_{-7.3-8.5-3.3}$ | $15.8^{+5.6+2.7}_{-4.9-2.3}$ |
| $B^0 \to \bar{a}_1^0 \bar{K}^0$ | $6.9^{+0.3+6.1+9.5}_{-0.3-2.9-3.2}$ | $8.0^{+3.9+6.4+1.2}_{-2.8-3.4-1.2}$ | $6.3^{+2.5+1.0}_{-2.1-0.8}$ |
| $B^- \to b_1^0 \pi^0$ | $0.4^{+0.0+0.2+0.4}_{-0.0-0.1-0.2}$ | $1.0^{+0.2+0.3+0.1}_{-0.2-0.2-0.2}$ | $2.0^{+0.8+0.2}_{-0.6-0.2}$ | $< 3.3^a$ |
| $B^- \to b_1^0 \pi^-$ | $9.6^{+0.3+1.6+2.5}_{-0.3-1.6-1.2}$ | $5.1^{+3.3+3.1+0.3}_{-3.0-1.9-1.5}$ | $5.0^{+1.3+0.5}_{-1.0-0.4}$ | $6.7 \pm 1.7 \pm 1.0$ |
| $B^0 \to b_1^- \pi^+$ | $0.9^{+0.1+0.1+0.3}_{-0.0-0.1-0.1}$ | $1.4^{+0.4+0.4+0.1}_{-0.4-0.2-0.2}$ | $0.6^{+0.3+0.1}_{-0.2-0.1}$ |
| $B^0 \to b_1^+ \pi^-$ | $11.2^{+0.3+2.8+2.2}_{-0.3-2.4-1.9}$ | $18.7^{+9.6+8.2+1.3}_{-6.4-4.5-1.9}$ | $7.7^{+2.1+0.7}_{-1.9-0.7}$ | $< 3.3^a$ |
| $B^0/B^0 \to b_1^0 \pi^0$ | $11.4^{+0.4+2.9+2.5}_{-0.3-2.5-2.0}$ | $20.2^{+9.9+8.1+1.4}_{-6.9-4.9-2.1}$ | $8.3^{+2.1+0.7}_{-1.9-0.7}$ | $10.9 \pm 1.2 \pm 0.9$ |
| $B^- \to b_1^0 K^-$ | $6.3^{+0.5+5.0+6.1}_{-0.5-5.2-5.2}$ | $24.9^{+1.9+14.9+3.7}_{-7.8-9.3-3.9}$ | $4.6^{+1.9+0.7}_{-1.5-0.6}$ | $9.1 \pm 1.7 \pm 1.0$ |
| $B^- \to b_1^- \bar{K}^0$ | $14.0^{+1.3+11.5+13.9}_{-1.2-9.2-13.9}$ | $55.0^{+23.0+33.5+8.0}_{-17.0-21.2-8.3}$ | $8.6^{+3.8+1.4}_{-3.2-1.2}$ | $9.6 \pm 1.7 \pm 0.9^a$ |
| $B^0 \to b_1^+ K^-$ | $12.1^{+1.0+9.7+12.3}_{-0.9-9.0-15.2}$ | $42.9^{+1.7+26.9+6.6}_{-13.4-16.9-6.9}$ | $8.5^{+3.5+1.3}_{-2.8-1.1}$ | $7.4 \pm 1.0 \pm 1.0$ |
| $B^0 \to b_1^- \bar{K}^0$ | $7.3^{+0.5+5.4+6.7}_{-0.5-2.8-6.5}$ | $23.3^{+1.0+16.5+3.5}_{-6.8-8.8-3.6}$ | $4.0^{+1.8+0.7}_{-1.4-0.6}$ | $< 7.8^a$ |

^a The experimental data is obtained on the assumption that the daughter decay $b_1 \to \pi \omega$ has a branching ratio $BR = 1$. 
decays which can be found in the literature. Because of the same flavor structures, the hard spectator scattering diagrams often accompany with the factorizable diagrams. One only needs to consider the flavor structure for factorizable diagrams and to use meson matrices by evaluating the master equations. For the CKM matrix elements, we use the updated global fit results from CKMfitter group:

\[ V_{ud} = 0.97400, \; \; V_{us} = 0.22653, \; \; |V_{ub}| = (3.57^{+0.17}_{-0.15}) \times 10^{-3}, \]

\[ V_{cd} = -0.22638, \; \; V_{cs} = 0.97316, \; \; V_{cb} = (40.5^{+3.2}_{-2.9}) \times 10^{-3}, \]

\[ \beta = (21.7^{+0.017}_{-0.017})^\circ, \; \; \gamma = (67.6^{+2.8}_{-4.5})^\circ. \]  

Predictions in the QCDF approach are also collected in the tables to make a compari-

---

3 There still exist two differences between the factorizable emission diagrams of \( B \rightarrow AP \) and \( B \rightarrow PP \) decays: the axial-vector meson cannot be generated by the scalar or pseudo-scalar current, thus the chirally enhanced penguins vanish; due to the vanishing decay constant, \( b_1 \) cannot be factorized from the \( B \) meson and the recoiling meson.
son[14] In the QCDF approach, $a_2$ (to be precise, $\alpha_2$) is much smaller than 0.5, thus their amplitude from color-suppressed tree diagrams is not large enough to resolve the problem in $B^0\bar{B}^0 \to a_\pm^0\pi^\mp$ and $B^0 \to (a_1^0\pi^0, a_0^0\pi^-)$ decays. Their prediction on the branching ratio of $\bar{B}^0 \to a_1^+K^-$ is compatible with the data. For $B \to b_1K$, they found that decay rates are sensitive to the interference between emission diagrams and annihilation diagrams. The small decay rate of $\bar{B}^0 \to b_1^+K^-$ arises from the destructive interference between emission diagrams and annihilations, thus the prediction on branching ratio $\bar{B}^0 \to b_1^+K^-$ is basically consistent with the data. But their predictions on four ratios of branching fractions $R_{1-4}$ ($R_3$ and $R_4$ are related to ratios $R_1$ and $R_2$ defined in the present paper; their ratios $R_1$ and $R_2$ characterize the magnitude of color-suppressed contributions in $B \to a_1\pi$ decay modes.) deviate from experimental data.

Several remarks on the numerical results in the PQCD approach and SCET approach are in order:

- The predictions on $BR(\bar{B}^0 \to a_1^-\pi^+)$ in both approaches are a bit smaller than experimental data, because the decay constant of $f_{a_1} = 0.238 \text{ GeV}[39]$ is a bit smaller than that extracted from the data.
- As we expected, color-suppressed contributions to $B \to a_1\pi$ decays are large in the SCET framework but small in the PQCD approach: SCET predictions are much larger and consistent with the present data within the uncertainties.
- In the PQCD approach, $\bar{B}^0 \to b_1^-\pi^+$ occur via the so-called hard spectator scattering diagrams, despite of the zero decay constant of $b_1$. In $B^- \to b_1^0\pi^-$, the hard spectator scattering diagrams contributions (tree operators), with a $b_1^0$ meson emitted, are sizable and cancel with color-allowed contribution where the pion is emitted. Thus the branching ratio of $B^- \to b_1^0\pi^-$ is smaller than one half of $BR(\bar{B}^0 \to b_1^+\pi^-)$.
- In the SCET approach, $\bar{B}^0 \to b_1^-\pi^+$ only receive contributions from charming penguins and correspondingly the direct CP asymmetry in this channel is 0. The predicted branching ratio is smaller than the PQCD prediction but larger than the QCDF prediction.
- In the SCET, the direct CP asymmetries in $\bar{B}^0 \to a_1^+K^-$ and $B^- \to a_0^0K^-$ have the same sign and similar size. Moreover, their branching ratios obey the simple relation: $BR(\bar{B}^0 \to a_1^+K^-) = 2BR(B^- \to a_0^0K^-)$. It is also similar for $\bar{B}^0 \to b_1^+K^-$ and $B^- \to b_1^0K^-$: the direct CP asymmetries are equal with each other; the branching ratios also satisfy the relation $BR(\bar{B}^0 \to b_1^+K^-) = 2BR(B^- \to b_1^0K^-)$, where the small deviation arises from the different mass and decay width of $B^0$ and $B^-$ meson.
• As expected, the two ratios $R_1$ and $R_2$ are predicted with large deviations from the data:

$$R_1 = 1.16, \quad R_2 = 0.54, \quad \text{PQCD} \quad (29)$$

$$R_1 = 0.91, \quad R_2 = 0.50, \quad \text{SCET} \quad (30)$$

• Predictions on the observables in time-dependent decay width of $B^0/\bar{B}^0 \to a_{1}^{\pm} \pi^{\mp}$ and $B^0/\bar{B}^0 \to b_{1}^{\pm} \pi^{\mp}$ are basically consistent with the experimental data except the $\Delta S$, the $\alpha_{\text{eff}}$ in $\bar{B}^0 \to a_{1}^{\pm} \pi^{\mp}$ and $A_{b_{1}\pi}$. For $\bar{B}^0 \to b_{1}^{\pm} \pi^{\mp}$ decays, predictions on $\Delta C$ in the two approaches are close to $-1$ and they are consistent with the QCDF prediction and the data. In the SCET framework, the angle $\alpha_{\text{eff}}^+(\bar{B}^0 \to b_{1}^{\pm} \pi^{\mp})$ is equal to $\frac{\pi}{2} - \beta$ which is also a consequence of the vanishing decay constant of $b_{1}$ meson.

VI. SUMMARY

In summary, we have investigated the $B \to a_1(b_1)\pi(K)$ decays under the factorization framework and find large differences between theoretical predictions and experimental data. In tree dominated processes $B \to a_1 \pi$, large contributions from color-suppressed tree diagrams are required. In $\bar{B}^0 \to (a_{1}^{+}, b_{1}^{+})K^-$ decays, theoretical results are larger than data by factors of 2.8 and 5.5 respectively, meanwhile ratios $R_1$ and $R_2$ defined in Eq. (19) are too much larger too. In the PQCD framework, the predicted decay rates of $B \to a_{1}^{\pm} \pi^{\mp}$ are consistent with data. But the other problems can not be resolved. The SCET approach has the potential to resolve the first two problems: if large hard-scattering form factors are allowed, theoretical predictions $BR(B^- \to a_{1}^{-}\pi^0)$ and $BR(B^- \to a_{0}^{0}\pi^-)$ are in good agreement with data; with the help of charming penguins, large branching ratios of $\bar{B}^0 \to (a_{1}^{+}, b_{1}^{+})K^-$ are also pulled down to the same magnitude with the data. However, the two problems on ratios in $b \to s$ transitions remain in the present theoretical methods. These two problems may indicate some new mechanism, from the non-perturbative contributions such as final state interactions or new physics scenarios, which needs further study.

Acknowledgements

This work is partly supported by National Natural Science Foundation of China under the Grant Numbers 10735080, 10625525 and 10525523. We would like to acknowledge H.Y.
TABLE III: Same as Table III but for Time-dependent CP asymmetry parameters in $B^0/\bar{B}^0 \to a_1^\pm \pi^\mp$ and $B^0/\bar{B}^0 \to b_1^\pm \pi^\mp$ decays.

| Observables | QCDF          | PQCD          | SCET          | Exp.           |
|-------------|---------------|---------------|---------------|----------------|
| $A_{a_1^\pi}$ | $0.003^{+0.001}+0.002+0.043$ | $-0.006^{+0.002}+0.000+0.000$ | $0.02^{+0.08}+0.00$ | $0.07^{+0.07}+0.02$ |
| $C$          | $0.02^{+0.00}+0.00+0.11$ | $-0.006^{+0.002}+0.000+0.000$ | $0.10^{+0.08}+0.00$ | $0.10^{+0.07}+0.09$ |
| $\Delta C$  | $0.44^{+0.03}+0.03+0.03$ | $0.11^{+0.03}+0.06+0.01$ | $0.23^{+0.18}+0.00$ | $0.26^{+0.15}+0.07$ |
| $S$          | $-0.37^{+0.02}+0.05+0.09$ | $-0.23^{+0.10}+0.02+0.03+0.09$ | $0.45^{+0.07}+0.08$ | $0.37^{+0.21}+0.07$ |
| $\Delta S$  | $0.01^{+0.00}+0.00+0.02$ | $-0.03^{+0.01}+0.01+0.00$ | $0.02^{+0.04}+0.00$ | $0.14^{+0.21}+0.06$ |
| $\alpha_{eff}^+$ | $(97.2^{+0.3}+1.0+4.7)^\circ$ | $(93.8^{+0.4}+0.7+4.4)^\circ$ | $(103.5^{+2.4}+4.0)^\circ$ | $(78.6^{+7.3})^\circ$ |
| $\alpha_{eff}^-$ | $(107.0^{+0.5}+3.6+6.6)^\circ$ | $(99.8^{+0.5}+1.5+4.2)^\circ$ | $(104.8^{+2.7}+4.0)^\circ$ | |
| $\alpha_{eff}$ | $(102.0^{+0.4}+2.3+5.7)^\circ$ | $(96.8^{+0.4}+1.0+4.3)^\circ$ | $(104.2^{+1.8}+4.0)^\circ$ | |

$^a$One needs to be careful about the phase of the $B$-meson decay amplitudes $[38]$. For example, the $\bar{B}^0 \to b_1^- \pi^+$ and $B^0 \to b_1^+ \pi^-$ decay amplitudes are determined as:

\[
A(\bar{B}^0 \to b_1^- \pi^+) = V_{ub}V_{ud}^* T + V_{cb}V_{cd}^* P, \quad A(B^0 \to b_1^+ \pi^-) = -[V_{ub}V_{ud}^* T + V_{cb}V_{cd}^* P],
\]

TABLE IV: Mixing-induced CP asymmetries in $\bar{B}^0 \to a_1^0 K_S$ and $\bar{B}^0 \to b_1^0 K_S$ decays.

| Channel     | PQCD          | SCET          |
|-------------|---------------|---------------|
| $\bar{B}^0 \to a_1^0 K_S$ | $0.09^{+0.20}+0.78+0.08$ | $0.48^{+0.11}+0.09$ |
| $\bar{B}^0 \to b_1^0 K_S$ | $0.71^{+0.01}+0.02+0.00$ | $0.85^{+0.05}+0.01$ |
| $B^0 \to a_1^0 K_S$ | $0.67^{+0.02}+0.09+0.09$ | $0.61^{+0.09}+0.09$ |
| $B^0 \to b_1^0 K_S$ | $-0.61^{+0.01}+0.03+0.01$ | $-0.69$ |
Cheng, Y.L. Shen and D.S. Yang for valuable discussions.

[1] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 97, 051802 (2006) [arXiv:hep-ex/0603050].
[2] K. Abe et al. [Belle Collaboration], arXiv:0706.3279 [hep-ex].
[3] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 98, 181803 (2007) [arXiv:hep-ex/0612050].
[4] F. Palombo, arXiv:hep-ex/0703005.
[5] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 99, 241803 (2007) [arXiv:0707.4561 [hep-ex]].
[6] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 99, 261801 (2007) [arXiv:0708.0050 [hep-ex]].
[7] B. Aubert et al. [BABAR Collaboration], arXiv:0709.4165 [hep-ex].
[8] G. B. Mohanty [BABAR Collaboration], arXiv:0711.4956 [hep-ex].
[9] B. Aubert et al. [BABAR Collaboration], arXiv:0805.1217 [hep-ex].
[10] C. H. Chen, C. Q. Geng, Y. K. Hsiao and Z. T. Wei, Phys. Rev. D 72, 054011 (2005) [arXiv:hep-ph/0507012].
[11] V. Laporta, G. Nardulli and T. N. Pham, Phys. Rev. D 74, 054035 (2006) [Erratum-ibid. D 76, 079903 (2007)] [arXiv:hep-ph/0602243].
[12] G. Calderon, J. H. Munoz and C. E. Vera, Phys. Rev. D 76, 094019 (2007) [arXiv:0705.1181 [hep-ph]].
[13] K. C. Yang, Phys. Rev. D 76, 094002 (2007) [arXiv:0705.4029 [hep-ph]].
[14] H. Y. Cheng and K. C. Yang, Phys. Rev. D 76, 114020 (2007) [arXiv:0709.0137 [hep-ph]].
[15] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996) [arXiv:hep-ph/9512380].
[16] E. Barberio et al. [Heavy Flavor Averaging Group (HFAG) Collaboration], arXiv:0704.3575 [hep-ex]. The updated results can be found at http://www.slac.stanford.edu/xorg/hfag.
[17] W. Wang, R. H. Li and C. D. Lu, arXiv:0711.0432 [hep-ph].
[18] K. C. Yang, unpublished; H. Hatanaka and K. C. Yang, arXiv:0804.3198 [hep-ph].
[19] Z. G. Wang, arXiv:0804.0907 [hep-ph].
[20] C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D 70, 054015 (2004) arXiv:hep-ph/0401188.
[21] C. W. Bauer, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D 74, 034010 (2006) arXiv:hep-ph/0510241.
[22] C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D 72, 098502 (2005) [arXiv:hep-ph/0502094].
[23] P. Colangelo, G. Nardulli, N. Paver and Riazuddin, Z. Phys. C 45, 575 (1990).
[24] M. Ciuchini, E. Franco, G. Martinelli and L. Silvestrini, Nucl. Phys. B 501 (1997) 271 [arXiv:hep-ph/9703353].
[25] M. Ciuchini, E. Franco, G. Martinelli, M. Pierini and L. Silvestrini, Phys. Lett. B 515, 33 (2001) [arXiv:hep-ph/0104126].
[26] A. R. Williamson and J. Zupan, Phys. Rev. D 74, 014003 (2006) [Erratum-ibid. D 74, 03901 (2006)] [arXiv:hep-ph/0601214].
[27] W. Wang, Y. M. Wang, D. S. Yang and C. D. Lu, [arXiv:0801.3123] [hep-ph].
[28] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Lett. B 504, 6 (2001) [arXiv:hep-ph/0004004].
[29] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Rev. D 63, 054008 (2001) [arXiv:hep-ph/0004173].
[30] C. D. Lu, K. Ukai and M. Z. Yang, Phys. Rev. D 63, 074009 (2001) [arXiv:hep-ph/0004213].
[31] B. H. Hong and C. D. Lu, Sci. China G 49, 357 (2006) [arXiv:hep-ph/0505020].
[32] J. Zhu, Y.L. Shen, C.D. Lu, Phys. Rev. D72, 054015 (2005); H-n Li, Phys. Lett. B622, 68 (2005); H.W. Huang, Phys. Rev. D73, 014011 (2006)
[33] H. n. Li and S. Mishima, Phys. Rev. D 73, 114014 (2006) [arXiv:hep-ph/0602214].
[34] M. Bona et al. [UTfit Collaboration], JHEP 0507, 028 (2005) [arXiv:hep-ph/0501199].
[35] J. Charles et al. [CKMfitter Group], Eur. Phys. J. C 41, 1 (2005) [arXiv:hep-ph/0406184].

The updated results can be found at http://ckmfitter.in2p3.fr/.
[36] A. Ali, G. Kramer, Y. Li, C. D. Lu, Y. L. Shen, W. Wang and Y. M. Wang, Phys. Rev. D 76, 074018 (2007) [arXiv:hep-ph/0703162].
[37] R. H. Li, C. D. Lu and H. Zou, [arXiv:0803.1073] [hep-ph].
[38] M. Beneke and M. Neubert, Nucl. Phys. B 675, 333 (2003) [arXiv:hep-ph/0308039].
[39] K. C. Yang, Nucl. Phys. B 776, 187 (2007) [arXiv:0705.0692] [hep-ph].