Frictional drag between non-equilibrium charged gases

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The frictional drag force between separated but coupled two-dimensional electron gases of different temperatures is studied using the non-equilibrium Green function method based on the separation of center-of-mass and relative dynamics of electrons. As the mechanisms of producing the frictional force we include the direct Coulomb interaction, the interaction mediated via virtual and real TA and LA phonons, optic phonons, plasmons, and TA and LA phonon-electron collective modes. We found that, when the distance between the two electron gases is large, and at intermediate temperature where plasmons and collective modes play the most important role in the frictional drag, the possibility of having a temperature difference between two subsystems modifies greatly the transresistivity.

I. INTRODUCTION

Decades after the original proposal, and after the first successful measurement of the frictional drag effect between separated but coupled electron subsystems is still attracting a lot of interest. In part this is due to the possibility of a direct view of interactions between charged carriers, which play a key role in central questions of condensed matter physics like superconductivity and quantum Hall effects. After the initial theories based on the direct Coulomb drag mechanism, the electron-electron interactions mediated by other channels, such as phonon exchange, and electron-phonon collective modes, have been explored as sources of frictional drag at low temperatures. However, recent experiments showed that further theoretical developments are still necessary before obtaining a quantitative view of this problem. At intermediate temperatures, the importance of plasmons has been predicted theoretically by Flensberg et al. and confirmed experimentally by Hill et al. but the discrepancies between theoretical and experimental results are significant. Güven and Tanatar made a detailed consideration of the optical phonon contribution to the plasmon drag, and obtained an improvement on the theoretical result for the case where optical phonons are of little importance on the drag force due to difficulty of exciting them at temperatures where plasmons dominate. At the same time some studies concluded that the RPA should be substituted by more detailed many-body theories when estimating the frictional force. Besides those studies focusing on the mechanism of frictional drag between 2D-2D electron-electron Fermi gas subsystems, intensive theoretical and experimental works have been done on a 2D-2D electron-hole Fermi gas, and also in other cases.

Most works on Coulomb drag are made in the linear regime, in such a way that the temperature is assumed to be the same for the two subsystems, and equal to that of the heat bath. It is well-known, however, that hot electron effects occur in semiconductor microstructures, where electrons can be easily heated or cooled by very low electric voltage or photon excitations. In non-equilibrium situations, as it is the case in a Coulomb drag experiment, there is no reason to assume, from the very beginning, the two system at the same temperature of the heat bath. The question of thermalization of non-equilibrium systems is of interest also in several other fields in physics. In this paper we study the frictional drag in the non-equilibrium regime resulting from the direct Coulomb interaction, the e-e interaction mediated by virtual and real TA and LA phonons, optic phonons, plasmons, and TA and LA phonon-electron collective modes. The frictional drag is described by a non-equilibrium Green’s function method, which has been applied successfully in the development of quantum transport theory in semiconductors. We believe that allowing in the calculation different temperatures for the two systems, what is the central point in the present work, a new viewpoint is brought to this developing area.

In the Lei-Ting formalism used to describe non-linear transport in semiconductor structures, the balance equations are established by separating the center-of-mass from the relative motion. The Hamiltonian becomes $H = H_0 + H_E + H_I = H_c + H_E + H_r + H_I$, where $H_c$ corresponds to the motion of the center-of-mass, $H_E$ corresponds to the time developing part due to external field, $H_r$ is the Hamiltonian of relative electron system, and $H_I$ represents the scattering by impurities and phonons. The last term is the one which causes the dissipation of momentum and energy.

In order to obtain the equation for the density matrix, we take as initial state an equilibrium state of a system free of the interactions $H_I$ and $H_E$. The real system is obtained by adiabatically switching on $H_I$ and $H_E$ at the infinite past time, till the present non-equilibrium state. Then, the final state density matrix of the relative
independent subsystems (electron gases in well 1 and 2, and phonons) in the Hamiltonian $\mathcal{H}$ and $\gamma$ becomes:

$$\mathcal{T}_t = \langle \varphi^b A_t \rangle = \text{Tr} \{ \varrho_0 A_t \} - i \int_{-\infty}^{t} dt \text{Tr} \{ \varphi^b [A_h^b(t-t_1), h_{1t_1}^b] \},$$

(1)

where, $O^b(t) = e^{iH_b t} $ and $O^b_0(t) = e^{iH_0 t}$. Instead of approximating $\varphi^b$ by $\varrho_0$, as in the standard version of Lei-Ting’s balance equations, we argue that, in cases where $h_1$ plays an important role in the ensemble averaging process, it is preferable to use $A^b$ and $h^b$ in place of $A^b_0$ and $h^b_0$. The validity of this approximation is the same as in the standard version, as we can see by considering the Hamiltonian $h_0 = h - h_1$ and expanding $\varrho_0$ in terms of $\varphi^b$. The average value of a physical property $A_t$ becomes:

$$\mathcal{T}_t = \langle \varphi^b A_t \rangle = \text{Tr} \{ \varrho_0 A_t \} - i \int_{-\infty}^{t} dt \text{Tr} \{ \varphi^b [A_h^b(t-t_1), h_{1t_1}^b] \}$$

(2)

Next, we consider a system of two coupled, but separated, quantum wells (no particle exchange between them being allowed), with carrier densities and electron temperatures $n_1$ and $T_{e1}$ in one well, and $n_2$ and $T_{e2}$ in the other. As a general case, it is assumed that the two subsystems have drift velocities $v_1$ and $v_2$, and applied electric fields $E_1$ and $E_2$, respectively. The temperature of the phonon reservoir is $T_p$. After separating the center-of-masses (located at positions $\mathbf{R}_1$ and $\mathbf{R}_2$) from the relative coordinates, the Hamiltonian becomes:

$$H = en_1 \mathbf{R}_1 + en_2 \mathbf{R}_2 + H_{1C} + H_{2C} + H_{\epsilon 1} + H_{\epsilon 2} + H_{\epsilon 12} + H_{ph} + H_{e-ph} + H_{ei},$$

(3)

where $H_{1C} = \frac{p_1^2}{2m_1}$, $H_{2C} = \frac{p_2^2}{2m_2}$, $H_{\epsilon 1} = \sum_k \varepsilon_k c_k^\dagger c_k + \frac{1}{2} \sum_{q_1} U_C(q_1) \rho_{q_1}^1 \rho_{-q_1}^1$, $H_{\epsilon 2} = \sum_p \varepsilon_p d_p^\dagger d_p + \frac{1}{2} \sum_{q_2} U_C(q_2) \rho_{q_2}^2 \rho_{-q_2}^2$, and $H_{\epsilon 12} = \sum_{q} U_d(q) \rho_{q}^1 \rho_{q}^2 e^{i\varepsilon(q)(R_1-R_2)}$. The density operator in system 1 is $\rho_1^1 = \sum_k c_k^\dagger c_k$, and, for system 2, $\rho_2^2 = \sum_k d_k^\dagger d_k$. The final three terms, $H_{ph}$, $H_{e-ph}$, and $H_{ei}$, are defined as usual $U_C(q)$ and $U_d(q)$ are the intra- and inter-well interactions. For the sake of simplifying the formalism, we start by considering the interaction only through the Coulomb electron-electron (e-e) potential.

As in the original Lei-Ting balance equation formalism, we choose an initial equilibrium system composed of independent subsystems (electron gases in well 1 and 2, and phonons) in the Hamiltonian $H_r = H_{\epsilon 1} + H_{\epsilon 2} + H_{ph}$, with the initial density matrix $\varrho_0 = \varrho_{10}\gamma + \varrho_{02} = e^{iH_r/T_r} e^{H_{12}/T_{12}} e^{H_{e1}/T_{e1}}$. For the density matrix of the final relative electrons of the Hamiltonian $H_r + H_f = H_{\epsilon 1} + H_{\epsilon 2} + H_{e-ph} + H_{ei}$, we use $\varrho = e^{iH_r/T_r} e^{H_{12}/T_{12}} e^{H_{e1}/T_{e1}} e^{H_{e2}/T_{e2}}$, which we assume to be close to the steady state of the relative electrons. The reason why we include $H_{12}$ into the density matrix is that the e-e interaction, which is very special and much stronger than other interactions such as electron-impurity and electron-phonon interaction, should give important contributions to the ensemble average for the final state of the relative electrons. The present choice accelerates the convergency.

We consider the case where the electron temperature in well 2 is the same as that of the lattice, $T_p = T_{e2} = T$, and $\gamma = T_{e1}/T$. By introducing a new interaction representation corresponding to the virtual Hamiltonian $\tilde{H} = \gamma H_{\epsilon 1} + H_{\epsilon 2} + H_{ph} + \sqrt{\gamma} H_{e12}$, where operators are expressed as $\tilde{A}(t) = e^{iH_1 T} O e^{-iH T}$, the Liouville equation to the first order becomes:

II. THEORY

The Hamiltonian is written as $h = h_0 + h_1$, with ensemble average density matrices $\varrho^b$ and $\varrho_0$ corresponding to $h$ and $h_0$, respectively. The ensemble average of an explicitly time dependent Heisenberg operator $A_t$ is:

$$\mathcal{T}_t = \langle \varrho^b A_t \rangle = \text{Tr} \{ \varrho_0 A_t \} - i \int_{-\infty}^{t} dt \text{Tr} \{ \varphi^b [A_h^b(t-t_1), h_{1t_1}^b] \},$$

(1)

However, this technique has not yet been employed in the electron-electron frictional drag phenomena, mostly because of the complexity of the many-body effect due to the inter-subsystem electron-electron interaction. We noticed that the $H_f$ is important for configuring the ensembles of the system for this problem, so higher order terms must be considered. On the other hand, it is not convenient to include those higher order terms in the usual version of the formalism. In this paper, we propose a way to circumvent this difficulty in the case of the frictional drag problem.
\[ q^h = q_0 - i \int_{-\infty}^{t} dt e^{iH(t_1-t)}e^{-i\hat{H}(t_1-t)}[\hat{H}_{tt}(t_1-t), \hat{q}]e^{i\hat{H}(t_1-t)}e^{-i\hat{H}(t_1-t)} \]

By approximating \( e^{i\hat{H}(t_1-t)}e^{-i\hat{H}(t_1-t)} \approx e^{-i(\gamma_1)\hat{H}_{tt}(t_1-t)} \) the ensemble average becomes:

\[ \overline{A_t} = \langle q^h A_t \rangle = \int_{-\infty}^{t} dt_1 \theta(t-t_1) \text{Tr} \left\{ \hat{q} \hat{A} \gamma^{-(t_1-t)}(t_1), \hat{H}_{tt}(t_1) \right\} \]

where \( \hat{A} \gamma^{-(t_1-t)}(t_1) = e^{i\hat{H}t}e^{-i(\gamma_1)\hat{H}_{tt}t} \hat{A} e^{i(\gamma_1)\hat{H}_{tt}t}e^{-i\hat{H}t} \).

The time derivative operator of the total momentum for the electron gas in well 2 is:

\[ \dot{\mathbf{P}}_2 = -i[\mathbf{P}_2, H] = -i\nabla_{R_2} H = \sum_q i\mathbf{q} U_d(q) \rho_d(\mathbf{q}) \rho_2^{-2}(\mathbf{q}) e^{-i\mathbf{q} \cdot (\mathbf{R}_1 - \mathbf{R}_2)} - en_2 \mathbf{E}_2 + \dot{\mathbf{P}}_{2,\text{ep}} + \dot{\mathbf{P}}_{2,\text{ei}} \]

In the case where the circuit of the second well is opened (without current), the phonon frictional force \( \dot{\mathbf{P}}_{2,\text{ep}} \) and the impurity frictional force \( \dot{\mathbf{P}}_{2,\text{ei}} \) disappear, together with the derivative of total moment of the second well. In the subsequent equations we make \( \mathbf{R}_2 = 0 \). The frictional force acting on the electron gas in the well 2 is balanced by the electric force:

\[ en_2 \mathbf{E}_2 = i \sum_q \mathbf{q} \int_{-\infty}^{t} dt_1 (-i) \theta(t-t_1) \sum_{k,p} \sum_Q e^{-i\mathbf{q} \cdot [\mathbf{R}_1(t) - \mathbf{R}_1(t_1)] + i\mathbf{Q} \cdot \mathbf{R}_1(t_1) e^{i(1-\gamma)(\varepsilon_{k+q} - \varepsilon_k)(t-t_1)}} \times \text{Tr} \left\{ \hat{q} \hat{U}_d(q) \hat{c}^\dagger_{\mathbf{k}+\mathbf{q}}(t) \hat{c}(t) \hat{d}^\dagger_{\mathbf{p}-\mathbf{q}}(t) \hat{d}_{\mathbf{p}}(t), U_d(\mathbf{Q} - \mathbf{q}) \rho_{\mathbf{Q} - \mathbf{q}}^1(t_1) \rho_{\mathbf{Q} - \mathbf{q}}^2(t_1) \right\}. \]

The above expression can be simplified if (i) the velocity fluctuations are neglected by making \( \mathbf{R}_1(\tau) = \mathbf{v}_d \tau \), where \( \mathbf{v}_d \) is the drift velocity of the electron gas in well 1, and (ii) we consider zero frequency drag, where the imaginary part of the frictional force is zero. Furthermore, the system satisfies the time invariance transformation in steady state with \( \mathbf{Q} = 0 \). Then we have

\[ en_2 \mathbf{E}_2 = -Im \sum_{k,p,q} \mathbf{q} \Gamma^R(k, p, \mathbf{q}, \Omega_0) \]

where \( \Omega_0 = qv_d + (1-\gamma)(\varepsilon_{k+q} - \varepsilon_k) \), and \( \Gamma^R(k, p, \mathbf{q}, \Omega) \) is the Fourier transform of the retarded Green’s function

\[ \Gamma^R(k, p, \mathbf{q}, \tau) = -i\theta(\tau) \text{Tr} \left\{ \hat{q} \hat{U}_d(q) \hat{c}^\dagger_{\mathbf{k}+\mathbf{q}}(\tau) \hat{c}(\tau) \hat{d}^\dagger_{\mathbf{p}-\mathbf{q}}(\tau) \hat{d}_{\mathbf{p}}(\tau), U_d(\mathbf{Q} - \mathbf{q}) \rho_{\mathbf{Q} - \mathbf{q}}^1(0) \rho_{\mathbf{Q} - \mathbf{q}}^2(0) \right\} \]

The later can be obtained by analytical continuation from its Matsubara counterpart:

\[ \Gamma^R(k, p, \mathbf{q}, \Omega) = \lim_{i\omega_n \rightarrow \Omega + i\delta^+} \Gamma^M(k, p, \mathbf{q}, i\omega_n) \]

After a series of calculation to expand \( \Gamma^M(k, p, \mathbf{q}, i\omega_n) \) in the equilibrium representation without inter-subsystem interaction \( \hat{H}_{tt} \), we obtain a diagram, where the renormalization is made with the polarization function of well 1 and well 2, rather than inter-subsystem interaction. This diagram is similar to the result of the Memory Function method, but with different electron temperature in different subsystems. Following the suggestion of L. Zheng and A. H. MacDonald, we adopt the renormalization of the Coulombic interaction between electron gases 1 and 2, as shown in Fig. 1. The drag resistivity then becomes:

\[ R_d = \frac{E_2}{en_1 v_d} = -\frac{1}{\pi n_1 n_2 e^2 v_d} \sum_q \int_{-\infty}^{\infty} d\omega \left| \frac{U_d(q)}{\epsilon_T(q, \omega)} \right|^2 \hat{\Pi}_1(q, \omega, T_1) \hat{\Pi}_2(q, \omega, \omega_0, T) \left[ n \left( \frac{\omega + \omega_0}{T} \right) - n \left( \frac{\omega}{T_1} \right) \right]. \]
\[
\epsilon_T(q, \omega) = [1 - U_C(q) \hat{\Pi}^1(q, \omega, T_1)] [1 - U_C(q) \hat{\Pi}^2(q, \omega, T_2)] - U_d^2(q) \hat{\Pi}^1(q, \omega, T_1) \hat{\Pi}^2(q, \omega, T_2)
\] (12)

These expressions were obtained by assuming that only the Coulomb interaction is important to the electronic coupling. It is easy to extend this result to the case in which other mechanisms of interaction between electrons become important, just by replacing the inter- and intra-well e-e Coulomb potential by the sum of the corresponding interaction potentials due to different mechanisms. For instance, if phonon mediated electron-electron interaction is to be taken into account, the following substitution should be made:

\[
U_C(q) \rightarrow U_i(q, \omega) = U_C(q) + \sum_{\lambda} D^\lambda_i(q, \omega),
\]

\[
U_d(q) \rightarrow U_{12}(q, \omega) = U_d(q) + \sum_{\lambda} D^\lambda_{12}(q, \omega).
\] (13)

Here \(D^\lambda_i = \int \frac{dq}{2\pi} |M^\lambda(q, \omega)|^2 f_i(q_2)f_j(q_2) \left[ \frac{1}{\omega - \omega_\lambda + i/2\lambda_{ph}} - \frac{1}{\omega - \omega_\lambda - i/2\lambda_{ph}} \right] \) is the interaction potential between electrons in wells \(i\) and \(j\) mediated by phonons of the branch \(\lambda\), as discussed in Ref. [3]. \(M^\lambda(q, \omega)\) is the bulk matrix of electron-phonon interaction, and \(f_i(q_2)\) is the form factor for the electron-phonon interaction in well \(i\). The mean free time of phonons \(\tau^\lambda_{ph}\) equals the mean free path \(l^\lambda_{ph}\) divided by phonon velocity \(v_\lambda\). For small drift velocities, the drag resistivity becomes:

\[
R_d = \frac{1}{2\pi^3 e^2 n_1 n_2} \int_0^{\pi/2} d\theta \cos^2 \theta \int_0^{\infty} q^3 dq \int_0^{\infty} d\omega \frac{U_{12}(q, \omega)}{\epsilon_T(q, \omega)} \left[ \hat{\Pi}^2_{12}(q, \omega, T_2) \right]^2 \left[ \frac{\hat{\Pi}^2_{12}(q, \omega, T_2)}{T_2 \sinh^2(\omega/2T_2)} + 2 \left[ \coth(\omega/2T_1) - \coth(\omega/2T_2) \right] \frac{\partial}{\partial \omega} \hat{\Pi}^2_{12}(q, \omega, T_2) \right].
\] (14)

The real (virtual) phonon contribution comes from the imaginary (real) part of \(D^\lambda_i\) and the contribution of plasmons and electron-phonon collective modes come from the zeroes of the real part of \(\epsilon_T(q, \omega)\). The well-known expression for the linear drag resistivity is recovered if we neglect the anisotropy of the interaction matrix, and assume the same temperature \(T_1 = T_2 = T\) for the electron gases in both wells. Then, the drag resistivity of the two-temperature model becomes the generally accepted linear result:

\[
R_d = \frac{1}{8\pi^2 e^2 n_1 n_2 T} \int_0^{\infty} dq q^3 \int_0^{\infty} d\omega \frac{U_{12}(q, \omega)}{\epsilon_T(q, \omega)} \left[ \frac{\hat{\Pi}^2_{12}(q, \omega)}{\sinh^2(\omega/2T)} \right].
\] (15)

III. RESULTS AND CONCLUSION

The numerical calculation is performed in a model in which the electron system occupies two GaAs quantum wells separated by infinite barriers. This model may underestimate the electron effective mass and the correlation between two wells, but these effects are negligible for the cases discussed in this paper. In addition, it is well known that the RPA is expected to give poor results for low density electron gases, and several improvements beyond RPA have been employed in the literature, as already discussed in Chap. [3]. However, RPA is used in this paper for estimating the many-body effects, since the mechanism leading to the unexpected large measured drag resistivity has not yet been clarified, and also because our aim here is not focused on the improvements or more detailed estimative or corrections beyond RPA. We believe that local field corrections to our result give similar results as discussed in other places. It is worthwhile to mention that effects of the optic phonon directly causing, or mediating, the frictional drag were considered in Refs. [13] and [14]. Here the optic phonon contribution is taken into account just by using the static dielectric constant \(\kappa_s\) as the background dielectric function. An argument in favor of this assumption is that the plasmon energy is much smaller than those of optic phonons, and at the plasma frequency the optic phonon contribution to dielectric function is practically frequency independent.

The contributions of the longitudinal acoustic (LA) and the transversal acoustic (TA) phonons via deformation potential and piezoelectric interaction, on the other hand, are fully included. The mean free times, \(\tau_{LO, ph}\) and \(\tau_{TO, ph}\), are used as fitting parameters for both the LA and the TA branches. The parameters used for GaAs are \(\kappa_s = 12.9\), \(d_1 = 5.31 g/cm^3\), \(\Xi = 14 eV\), \(m^* = 0.067\), \(v_{sl} = 5.29 \times 10^7 m/s\), \(e_{14} = 1.41 \times 10^{-13} V/m\), and \(v_{st} = 2.48 \times 10^3\) for the dielectric constant, density, deformation potential, electron effective mass, longitudinal sound velocity, piezoelectric constant and transversal sound velocity, respectively. The numerical calculation is performed for a structure of two 200˚A wide quantum wells, with a 300˚A barrier between them.
In Fig. 2 (a), the drag resistivity $R_d$ is plotted as a function of the equilibrium temperature in $(T_1 = T_2)$ for a matching electron density of $n = 1.37 \times 10^{11}\text{cm}^{-2}$ in both wells. The calculation is performed in a range of values of $\tau_L$ and $\tau_A$ suitable for analyzing the relative importance of the electron-phonon collective mode for the LA and TA phonons. The solid lines represent results of different values of the phonon mean free paths (the same for LA and TA phonons), namely $l_{ph} = 10^{-2}, 1, 10^{2}\text{cm}$. In the two upper curves (1 cm, 10^2 cm) the electron-phonon collective modes dominate the contribution. In the lower curve (1 cm), however, the main contribution comes from the virtual and real phonon terms. Similarly to previous studies, our theoretical results fit qualitatively well the experimental data from Ref. 4 (filled circles), but several quantitative inconsistencies still remain. For instance, at low temperatures the calculated peak of the acoustic phonon is displaced to the high temperature side. If we allow $l_L$ and $l_T$ to be different and diminish the value of $l_T$ to reduce the contribution of LA phonons, the peak moves to the low temperature side. In dotted line, we show the drag resistivity including only the contributions of the TA phonons ($l_T = 10^2\text{cm}$), together with the Coulomb interaction. Comparing with the experimental result, this seems to indicate that TA phonons dominate the contribution to frictional drag force, which reaches its maximum at about $T \approx 0.5T_F$. However, even though the position of the calculated peak can coincide with the experimentally observed by fitting $l_L$ and $l_T$, the value of the maximum cannot reach the experimental result in the low carrier density case. At intermediate temperatures ($T \geq 0.2T_F$), the plasmon begin to dominate the contribution to frictional drag force, which reaches its maximum at about $T \approx 0.5T_F$, as calculated in Ref. 4. Our conclusion is that the LA phonons enhance the plasmon peak at intermediate temperatures, in the same way as the optic phonon, while the TA phonons have little influence at this temperature range. However, our calculation gives a smaller frictional drag force, and a higher peak temperature than what is observed experimentally.

In Fig. 2(b), we show the dependence of the drag resistivity on the temperature, as in Fig. 2(a), with a different carrier density, $n = 2.66 \times 10^{11}\text{cm}^{-2}$, and several TA and LA mean free paths, namely, $l_{ph} = 10^{-5}, 10^{-2}, 1\text{cm}$. Differently from the previous low density regime, we find a very short phonon mean free path, $l_{ph} = 0.01\text{cm}$, when fitting the experimental result. Since samples differing just by the carrier densities are expected to have the same order of the phonon mean free path, this points to a disagreement between the theoretical and experimental results for the density dependence of the frictional drag force. The derivatives of curves in Fig. 4(a) at the point $T_1/T_2 = 1$ is a direct result of the non-linearity of the correlation function over frequency, confirming that a correct approach to the Coulomb drag effect must contain the non-equilibrium assertion of the present work.

In order to make the above point clearer, we obtain, from Eq. (14), the expression for the resistivity difference $\delta R_d$ resulting from a temperature difference near $T_2 = T$.

$$
\delta R_d = \frac{\delta T}{T} \left( \frac{1}{8\pi^2 e^2 n_1 n_2} \right) \int_0^\infty dq \int_0^\infty d\omega |W_{12}(q, \omega)|^2 \frac{\tilde{H}_2(q, \omega) \tilde{H}_2^*(q, \omega)}{\sinh^2(\omega/2T)} \Theta(q, \omega),
$$

where $\Theta(q, \omega) = \frac{\omega}{\tilde{H}_2(q, \omega)} \frac{\partial}{\partial \omega} \tilde{H}_2(q, \omega) \bigg|_{q=\omega}$. If $\tilde{H}_2(q, \omega)$ is a linear function of $\omega$, then $\Theta(q, \omega) = 1$ and $\delta R_d = \frac{\delta T}{T} R_d$, i.e. $R_d/(T_1T_2)$ is a constant near $T_1 = T_2$. Otherwise, $R_d/(T_1T_2)$ will be sensitive to the temperature difference. The contribution from acoustic phonons comes mainly from $q \approx 2k_F$, where $\tilde{H}_2(q, \omega)$ is a linear function of $\omega$ in the range of the corresponding phonon energy (with the parameters used in this paper). This leads to the fact that the acoustic
peak in Fig. 3 does not show any non-equilibrium trace. On the other hand, the plasmon energy is higher than the energy of the single particle excitation, and the corresponding non-linearity of $\Pi^I(q, \omega)$ invokes strong non-equilibrium character for drag resistivity $R_d/(T_1 T_2)$ due to plasmons.

Fig. 5 shows the drag resistivity $R_d/(T_1 T_2)$ in equilibrium cases (solid curves) as a function of the ratio $n_1/n_2$ of the electron densities at different drag temperatures, $T_2 = 0.04, 0.09, 0.16, 0.24$, and $0.44T_F$ in comparison with the experimental results. The mean free paths of the LA and TA phonons is assumed to be $l_{ph} = 0.01 cm$. We found that the acoustic phonon enhances the drag resistivity at large $n_1/n_2$. It can be expected that the inclusion of local field effects will provide good fittings between theoretical and experimental results, except at temperature $T_2 = 0.44T_F$.

In conclusion, we have studied the momentum transfer rate between two nearby separated electron gases, which are coupled via the Coulomb interaction and phonon mediated interaction. We focused on the non-equilibrium configuration, and a theory was developed to describe the frictional drag force felt by one electron gas as a result of the relative drift of the other, taking into account the possibility of difference on the subsystems temperatures. It was found that a cooler (hotter) driving electron gas greatly enhances (decreases) the frictional force caused by plasmons. This behavior results from the fact the plasmons locate in a region of the $\omega - k$ space where the correlation function shows a nonlinear frequency dependence, and they modifies little the acoustic phonons force because the latter are important mainly in the region of the energy-momentum space where the correlation function is linear with the frequency.

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FIG. 1. Feynmann diagram for the drag resistivity

FIG. 2. Drag resistivity $R_d/T^2$ as functions of temperature $T/T_F$ at electron density (a) $n = 1.37 \times 10^{11} \text{cm}^{-2}$ ($T_F = 57K$), and (b) $n = 2.66 \times 10^{11} \text{cm}^{-2}$ ($T_F = 110K$). Results including the direct Coulomb interaction plus the LA and TA phonons with the same mean free paths $l_{ph}$ are plotted by solid lines. Experimental results according to Ref. 14 are shown as filled circles. The dashed line shows the results when LA phonons are excluded in the calculation to better identify the peak position of TA phonon. The phonon mean free paths are denoted by numbers besides the curves in cm.

FIG. 3. Drag resistivity as functions of drag temperature $T_2$ at different driving/drag temperature ratio $T_1/T_2$, denoted by numbers besides the curves. The electron densities and phonon mean free paths are (a) $n = 1.37 \times 10^{11} \text{cm}^{-2}$, $l_{ph} = 100cm$ and (b) $n = 2.66 \times 10^{11} \text{cm}^{-2}$, $l_{ph} = 0.01cm$. Experimental results from Ref. 14 are shown as filled circles.

FIG. 4. (a) The normalized drag resistivity $R_d/T_1$ as functions of driving/drag temperature ratio $T_1/T_2$ at fixed drag temperature $T_2$ numbered besides curves, and (b) its derivative at $T_1/T_2 = 1$. Only the direct Coulomb interaction is considered.

FIG. 5. Drag resistivity $R_d/(T_1T_2)$ is shown as function of driving/drag electron density $n_1/n_2$ at different normalized drag temperature $T_2/T_F = 0.04, 0.09, 0.16, 0.24, 0.44$ and driving electron density $n_1 = 2.66 \times 10^{11} \text{cm}^{-2}$. Experimental results from Ref. 14 is shown by filled circles. Solid lines represent results including contributions from all kinds of mechanisms in equilibrium, with TA and LA phonon mean free paths $l_{ph} = 0.01cm$. Dotted lines represents direct Coulomb drag forces (including plasmons) in equilibrium. Dashed lines show the Coulomb drag forces in non-equilibrium cases with $T_1/T_2 = 0.9$.
\[ G^{01}(k+q, i\omega_n + i\omega + i\omega_0) \]

\[ G^{01}(k, i\omega_n) \]

\[ W(q, i\omega + i\omega_0) \]

\[ W(q, i\omega) \]

\[ G^{01}(k, i\omega_m) \]

\[ G^{01}(k+q, i\omega_m + i\omega) \]

X.F. Wang and I.C. da Cunha Lima, Fig. 1
X.F. Wang and I.C. da Cunha Lima, Fig. 2(a)

\[ n = 1.37 \times 10^{11} \text{ cm}^{-2} \]

\[ R_d T^{-2} (10^{-2} \Omega K^{-2}) \]

\[ T/T_F \]
$n = 2.66 \times 10^{11} \text{ cm}^{-2}$
X.F. Wang and I.C. da Cunha Lima, Fig. 3(a)

\[ n = 1.37 \times 10^{11} \text{ cm}^{-2} \]

Graph showing \( \frac{R_d}{(T/T_2)} \left( 10^{-3} \Omega K^2 \right) \) against \( \frac{T_2}{T_F} \) for different values of a parameter, with points and fitted curves for 0.9, 0.95, 1.0, and 1.05.
$n = 2.66 \times 10^{11} \text{ cm}^{-2}$
X.F. Wang and I.C. da Cunha Lima, Fig. 4(a)

\[ n = 2.66 \times 10^{11} \text{ cm}^{-2} \]

\[ R_{d_2} / R_{d_1} = n T_2 / T_1 \]

Temperature markers: 71K, 47K, 23K, 3K
X.F. Wang and I.C. da Cunha Lima, Fig. 4(b)

\[
\frac{(T_2/R_d^0)}{dT_2} \frac{dR_d}{dT_1^{-1}}
\]

\[n = 1.37 \times 10^{11} \text{ cm}^{-2}\]

\[n = 2.66 \times 10^{11} \text{ cm}^{-2}\]
X.F. Wang and I.C. da Cunha Lima, Fig. 5

\[ R_d(T_1, T_2) (m\Omega K^2) \]

\[ n_1 = 2.66 \times 10^{11} \text{ cm}^{-2} \]