Two-step measurement of the concurrence for hyperentangled state

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Abstract We describe an efficient way of measuring the concurrence for the hyperentanglement. In this protocol, the hyperentangled state is encoded in both the polarization and the momentum degrees of freedom. We show that measurement of the concurrences of both polarization and momentum entanglements can be conversed into the measurement of the total probability of picking up the odd-parity state and can be measured directly. This protocol requires the weak cross-Kerr nonlinearity to construct the quantum nondemolition measurement and does not require the sophisticated controlled-not gate operation. It is feasible in future experimental technology.

Keywords Hyperentangled state · Concurrence · Cross-Kerr nonlinearity

1 Introduction

Entanglement is a key phenomenon in quantum information processing and is required in nearly all quantum communication and computation protocols [1], such as quantum teleportation [2–5], dense coding [6], quantum key distribution (QKD) [7–10] and
quantum secure direct communication [11, 12], and other protocols [13–16]. Employing such protocols requires exact information of the entanglement. Hence, the quantification of entanglement is an important topic for both theoretical and experimental study. Bennett et al. [17] proposed the concept of the entanglement of formation to quantify entanglement. For a two-qubit pure state $|\Psi\rangle$, the entanglement of formation can be exactly quantified by the concurrence, which is expressed by [17–19]

$$C = |\langle \Psi^* | \sigma_y \otimes \sigma_y | \Psi \rangle|.$$  

Here, $\langle \Psi^* |$ is the complex conjugate of $|\Psi\rangle$ and $\sigma_y$ is the second Pauli matrix. For an arbitrary two-qubit pure state described as $|\varphi\rangle = \alpha|00\rangle + \beta|11\rangle + \gamma|10\rangle + \delta|01\rangle$, the concurrence is defined as $C(|\varphi\rangle) = 2|\alpha\beta - \gamma\delta|$. Here, $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. On the other hand, if $\gamma = \delta = 0$ and the two-qubit state is simplified to a partially entangled state, $|\varphi'\rangle = \alpha|00\rangle + \beta|11\rangle$, the concurrence can be obtained as $C(|\varphi'\rangle) = 2|\alpha\beta|$, with $|\alpha|^2 + |\beta|^2 = 1$.

Walborn et al. [20] demonstrated the detection of the concurrence with linear optics. Romero et al. [21] described the approach for measuring the concurrence of an atomic-qubit pure state. In their protocol, they require the controlled-not (CNOT) gate between two atoms to complete the task. In 2008, the protocol for measuring the concurrence in a cavity QED system was proposed by Lee et al. [22], who used the atoms as the flying qubits to perform the measurement. Recently, by employing the cross-Kerr non-linearity, Cao et al. proposed two different methods for measuring the concurrence [23, 24]. There are some other works for concurrence, such as the indirect approach of using tomography to reconstruct the density matrix and from that to determine the concurrence [25], and measuring the concurrence in a microscopic two-level system and a macroscopic superconducting resonator [26]. Recently, we also described an approach of measuring the atomic entanglement assisted by single photons [27].

Hyperentanglement is the simultaneous entanglement in more than one degree of freedom and has been widely studied in recent years [28, 29]. Hyperentanglement can be used to complete the Bell-state analysis [30–35] and to perform the entanglement purification and distillation [36–40] and entanglement concentration [41–43], and so on [44–46]. Interestingly, Walborn et al. [20] showed that hyperentanglement can also be used to detect the concurrence of the polarization entanglement. Their protocol involves first producing the hyperentanglement in both polarization and momentum (spatial mode) degrees of freedom and subsequently performing a CNOT operation in one of the photons between the momentum and polarization degrees of freedom. Finally, the concurrence of the polarization entanglement can be measured by detecting both the photons.

Recent works for detecting the concurrence have all focused on the entanglement in a single degree of freedom. Though Walborn et al. reported the detection of the concurrence with hyperentanglement, it can only be used to measure the concurrence of polarization entanglement. They did not provide a complete description for hyper-entanglement. In this paper, we will describe an alternative way for definition, quantifying the concurrence, and provide an effective protocol for measuring the concurrence of hyperentanglement. We show that the concurrence in each degree of freedom can be
measured independently. We consider the weak cross-Kerr nonlinearity to construct the quantum nondemolition (QND) measurement.

This paper is organized as follows: In Sect. 2, we provide an alternative definition for the concurrence of a hyperentangled state. In Sect. 3, we explain our protocol with a simple example that of the hyperentangled state encoded in the polarization and momentum entanglements. Both polarization and momentum entangled states are pure partially entangled states. In Sect. 4, we prove that the concurrence of arbitrary pure hyperentangled state can also be measured. Section 5 provides the discussion and conclusion.

2 Concurrence of the hyperentanglement

Current hyperentangled states usually include polarization–momentum entangled state, polarization–time-bin entangled state, polarization–spatial modes–energy-time entangled state, and so on. With consideration of an enlarged Hilbert space, the hyperentangled state can be described as the product of \( N \) Bell states as the form \([29,47]\)

\[
|\Upsilon\rangle = |\Theta_1\rangle \otimes |\Theta_2\rangle \ldots |\Theta_N\rangle. \tag{2}
\]

Here, \( N \) denotes the degrees of freedom. We express the concurrence of the Bell state \(|\Theta_i\rangle\) as \( C_i \) \((i = 1, 2, \ldots N)\). Hence, we can obtain the total concurrence of the hyperentangled state as

\[
C_{\text{hyper}} = \sum_{i=1}^{N} C_N. \tag{3}
\]

From the above definition, if all the \(|\Theta_i\rangle\) are the maximally Bell states, we can get \( C_{\text{hyper}} = N \).

3 Measurement of the concurrence for partially hyperentangled state

Before we start to explain our protocol, we first introduce the key element of our protocol, say the cross-Kerr nonlinearity. The cross-Kerr nonlinearity can be used to perform the QND measurement, which has been widely used in quantum information processing such as the construction of the CNOT gate \([48]\), the performance of the Bell-state analysis \([32,49]\), entanglement purification and concentration \([37,50–56]\), quantum cloning \([57]\), an arbitrary single-photon polarization state transmission \([58]\), among others \([59–68]\). The Hamiltonian of a cross-Kerr nonlinear interaction is \( H = \hbar \chi a_s^{\dagger} a_s a_p^{\dagger} a_p \) \([48]\). Here, \( a_s^{\dagger}, a_s, a_p^{\dagger}, a_p \) are the creation and destruction operators of the signal (probe) mode. As shown in Fig. 1, we consider that a signal state \( \mu |0\rangle + \nu |1\rangle \) is in the \( a_1 \) spatial mode. This signal state combined with the coherent state \(|\tau\rangle_A\) couples with the cross-Kerr material. Here, \(|0\rangle\) and \(|1\rangle\) are the photon numbers. The whole system will evolve as

\[
(\mu |0\rangle + \nu |1\rangle)|\tau\rangle_A \rightarrow \mu |0\rangle|\tau\rangle_A + \nu |1\rangle|\tau e^{i\theta}\rangle_A. \tag{4}
\]
Here, $\theta = \chi t$ with $t$ being the interaction time. This expression implies the coherent state picks up a phase shift $\theta$, which is directly proportional to the photon number of the signal state. Therefore, through the measurement of the phase of the coherent state, one can obtain the information of the photon number of the signal state. This is called the QND measurement.

We first describe the approach for measuring the concurrence of the momentum (spatial mode) entanglement. Suppose that the hyperentangled state can be described as follows

$$|\psi\rangle = (\alpha_1 |H_a\rangle |H_b\rangle + \beta_1 |V_a\rangle |V_b\rangle) \otimes (\alpha_2 |a_1\rangle |b_1\rangle + \beta_2 |a_2\rangle |b_2\rangle). \tag{5}$$

From Eq. (5), the two photons are distributed to Alice and Bob, respectively. The two photons are in the spatial modes $a_1b_1$ or $a_2b_2$, respectively. On the other hand, the polarization of the two photons is $|HH\rangle$ or $|VV\rangle$, respectively. The state $|\psi\rangle$ entangles in spatial mode and polarization degrees of freedom, simultaneously. Here, $|\alpha_1|^2 + |\beta_1|^2 = 1$, and $|\alpha_2|^2 + |\beta_2|^2 = 1$. $a_1$, $b_1$, $a_2$, and $b_2$ are the different spatial modes, as shown in Fig. 1. $|H\rangle$ is the horizontal polarization photon, and $|V\rangle$ is the vertical polarization photon. Such a state can be generated with a spontaneous parametric down-conversion source. As described in Ref. [36], the pump pulse of ultraviolet light passes through a $\beta$-barium borate crystal (BBO). A correlated pair of photons is generated with the probability $p$ in the spatial modes $a1$ and $b1$. Further, $p_2$ is the probability that two correlated pairs of photons are generated in the spatial modes $a1$ and $b1$. The pulse can also be reflected by the mirror and traverse the crystal a second time, producing another correlated pair in the spatial modes $a2$ and $b2$ with the same probability $p$. If $p \ll 1$, the $p_2$ can be omitted. In an ideal case, one can generate the hyperentanglement as shown in Eq. (5). In addition, Ref. [20] also provided another efficient method of generating the hyperentanglement.

As shown in Fig. 1, we choose two copies of hyperentangled states in each round. One is $|\psi\rangle_1$ in the spatial modes $a_1, b_1, a_2, b_2$, and the other is $|\psi\rangle_2$ in the spatial
modes $a3$, $b3$, $a4$, and $b4$. The states $|\psi\rangle_1$ and $|\psi\rangle_2$ combined with two coherent states evolve as

$$|\psi\rangle_1 \otimes |\psi\rangle_2 \otimes |\tau\rangle_\Lambda \otimes |\tau\rangle_B = (\alpha_1 |H_a\rangle |H_b\rangle + \beta_1 |V_a\rangle |V_b\rangle)$$
$$\otimes \left( \alpha_2 |a1\rangle |b1\rangle + \beta_2 |a2\rangle |b2\rangle \right) \otimes \left( \alpha_1 |H_a\rangle |H_b\rangle + \beta_1 |V_a\rangle |V_b\rangle \right)$$
$$\otimes (\alpha_2 |a3\rangle |b3\rangle + \beta_2 |a4\rangle |b4\rangle) \otimes |\tau\rangle_\Lambda \otimes |\tau\rangle_B$$

$$\rightarrow (\alpha_2^2 |a1\rangle |a3\rangle |b1\rangle |b3\rangle |\tau\rangle_\Lambda \otimes |\tau\rangle_B + \alpha_2 \beta_2 |a1\rangle |a4\rangle |b1\rangle |b4\rangle |\tau e^{i\theta}\rangle_\Lambda \otimes |\tau e^{i\theta}\rangle_B$$
$$+ \alpha_2 \beta_2 |a2\rangle |a3\rangle |b2\rangle |b3\rangle |\tau e^{-i\theta}\rangle_\Lambda \otimes |\tau e^{-i\theta}\rangle_B + \beta_2^2 |a2\rangle |a4\rangle |b2\rangle |b4\rangle |\tau\rangle_\Lambda \otimes |\tau\rangle_B$$

$$\otimes (\alpha_1 |H_a\rangle |H_b\rangle + \beta_1 |V_a\rangle |V_b\rangle) \otimes (\alpha_1 |H_a\rangle |H_b\rangle + \beta_1 |V_a\rangle |V_b\rangle).$$

The distinct feature of the hyperentanglement is that the different degrees of freedom are independent. That is, one can operate each degree of freedom independently. From Fig. 1, the cross-Kerr nonlinearities couple with the spatial modes of the photons. In this way, such an operation does not affect the polarization of the photons. For example, the first item $|a1\rangle |a3\rangle |b1\rangle |b3\rangle$ indicates that the photons are in the spatial modes $a1$, $a3$, $b1$, and $b3$. If we consider the polarization part, the first item will be fully described as $|a1\rangle |a3\rangle |b1\rangle |b3\rangle \otimes (\alpha_1 |H_a\rangle |H_b\rangle + \beta_1 |V_a\rangle |V_b\rangle) \otimes (\alpha_1 |H_a\rangle |H_b\rangle + \beta_1 |V_a\rangle |V_b\rangle).$ Owing to this, the coherent states pick up no phase shift. From Eq. (6), if both coherent states pick up the phase shift $\theta$, the state becomes

$$|\Phi\rangle_1 = |a1\rangle |a4\rangle |b1\rangle |b4\rangle \otimes (\alpha_1 |H_a\rangle |H_b\rangle + \beta_1 |V_a\rangle |V_b\rangle)$$
$$\otimes (\alpha_1 |H_a\rangle |H_b\rangle + \beta_1 |V_a\rangle |V_b\rangle),$$

with the probability $P_m = |\alpha_2 \beta_2|^2$. Such a measurement can be completed by a general homodyne–heterodyne measurement [48]. The subscript $m$ refers to the momentum entanglement.

On the other hand, if both the coherent states pick up the phase shift of $-\theta$, the state becomes

$$|\Phi\rangle_2 = |a2\rangle |a3\rangle |b2\rangle |b3\rangle \otimes (\alpha_1 |H_a\rangle |H_b\rangle + \beta_1 |V_a\rangle |V_b\rangle)$$
$$\otimes (\alpha_1 |H_a\rangle |H_b\rangle + \beta_1 |V_a\rangle |V_b\rangle),$$

with the same probability of $|\alpha_2 \beta_2|^2$.

Otherwise, if both coherent states pick up no phase shift, the state becomes

$$|\Phi\rangle_3 = (\alpha_2^2 |a1\rangle |a3\rangle |b1\rangle |b3\rangle + \beta_2^2 |a2\rangle |a4\rangle |b2\rangle |b4\rangle)$$
$$\otimes (\alpha_1 |H_a\rangle |H_b\rangle + \beta_1 |V_a\rangle |V_b\rangle) \otimes (\alpha_1 |H_a\rangle |H_b\rangle + \beta_1 |V_a\rangle |V_b\rangle),$$

with the probability of $|\alpha|^4 + |\beta|^4$. From Eqs. (7) and (8), we can obtain the concurrence of the momentum entanglement as

$$C_m = 2|\alpha_2 \beta_2| = 2\sqrt{P_m},$$

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Interestingly, the above description implies that the polarization entanglement is not affected during the operation on the momentum entanglement. Therefore, we can measure the concurrence of the polarization entanglement in the next round.

For example, suppose that after the measurement, the initial state becomes $|\Phi\rangle_1$. $|\Phi\rangle_1$ can be rewritten as

$$|\Phi\rangle_1 = |a1\rangle|a4\rangle|b1\rangle|b4\rangle \otimes (\alpha_1|H_a\rangle|H_b\rangle + \beta_1|V_a\rangle|V_b\rangle)$$

$$= (\alpha_1|H_a\rangle|a1\rangle|H_b\rangle|b1\rangle + \beta_1|V_a\rangle|a1\rangle|V_b\rangle|b1\rangle) \otimes (\alpha_1|H_a\rangle|a4\rangle|H_b\rangle|b4\rangle + \beta_1|V_a\rangle|a4\rangle|V_b\rangle|b4\rangle)$$

$$= \alpha_1^2|H_a\rangle|H_a\rangle|H_b\rangle|H_b\rangle + \beta_1^2|V_a\rangle|V_a\rangle|V_b\rangle|V_b\rangle$$

$$+ \alpha_1 \beta_1 (|H_a\rangle|V_a\rangle|H_b\rangle|V_b\rangle + |V_a\rangle|H_a\rangle|V_b\rangle|H_b\rangle). \quad (11)$$

This expression implies that one photon pair is in the spatial modes $a1$ and $b1$, and the other is in $a4$ and $b4$. $|H_a\rangle|a1\rangle$ implies that the $|H\rangle$ polarized photon is in the spatial mode $a1$, and it can be rewritten as $|H_a\rangle$. $|V_a\rangle|a1\rangle$ can also be rewritten as $|V_a\rangle$. Then, we let the photons in the $a1$ and $a4$ spatial modes pass through the setup shown in Fig. 2. In Fig. 2, the two spatial modes are denoted as $a1$ and $a2$, respectively. The setup includes the polarization beam splitter (PBSs), which can transmit the $|H\rangle$ polarization photon and reflect the $|V\rangle$ polarization photon. The state $|\Phi\rangle_1$ combined with $|\tau\rangle$ can evolve as

$$|\Phi\rangle_1 \otimes |\tau\rangle = (\alpha_1^2|H_a\rangle|H_a\rangle|H_b\rangle|H_b\rangle + \beta_1^2|V_a\rangle|V_a\rangle|V_b\rangle|V_b\rangle \otimes |\tau\rangle$$

$$+ \alpha_1 \beta_1 (|H_a\rangle|V_a\rangle|H_b\rangle|V_b\rangle + |V_a\rangle|H_a\rangle|V_b\rangle|H_b\rangle) \otimes |\tau\rangle$$

$$\rightarrow \alpha_1^2|H_a\rangle|H_a\rangle|H_b\rangle|H_b\rangle|\tau\rangle + \beta_1^2|V_a\rangle|V_a\rangle|V_b\rangle|V_b\rangle|\tau\rangle$$

$$+ \alpha_1 \beta_1 (|H_a\rangle|V_a\rangle|H_b\rangle|V_b\rangle|\tau e^{-2\theta}\rangle$$

$$+ |V_a\rangle|H_a\rangle|V_b\rangle|H_b\rangle|\tau e^{2\theta}\rangle. \quad (12)$$
Concurrence measurement

Obviously, if the coherent state picks up the phase shift $\pm 2\theta$, the state in Eq. (11) becomes

$$|\Phi\rangle_4 = \frac{1}{\sqrt{2}}(|H_{a1}\rangle|V_{a4}\rangle|H_{b1}\rangle|V_{b4}\rangle + |V_{a1}\rangle|H_{a4}\rangle|V_{b1}\rangle|H_{b4}\rangle).$$

(13)

with the probability of $P_p = 2|\alpha_1\beta_1|^2$. In this way, we can obtain the concurrence of the polarization entanglement as

$$C_p = 2|\alpha_1\beta_1| = \sqrt{P_p}.$$  

(14)

Here, the subscript $p$ refers to the polarization. It should be noted that measurement of the coherent state requires a different approach that makes the phase shift $\pm 2\theta$ undistinguished. This measurement can be achieved by choosing the local oscillator phase $\pi/2$ offset from the probe phase and is called an $X$ quadrature measurement [48]. In this way, the total concurrence of the hyperentanglement is

$$C_{\text{hyper}} = C_m + C_p = 2\sqrt{P_m} + \sqrt{P_p}.$$  

(15)

Actually, if we obtain the state in Eq. (11), then the concurrence of the polarization part can also be simplified by another method. We only need to measure the two photons in the $a1$ and $a4$ modes in the $|H\rangle$ and $|V\rangle$ basis, respectively. From Eq. (11), the probabilities of obtaining the state $|H_{a1}\rangle|V_{a4}\rangle$ or $|V_{a1}\rangle|H_{a4}\rangle$ are both $P_p' = |\alpha_1\beta_1|^2$.

We obtain $C_p = 2\sqrt{P_p'}$ and $C_{\text{hyper}} = 2\sqrt{P_m} + 2\sqrt{P_p'}$.

So far, we have fully described our protocol with a simple example. The total protocol can be divided into two steps. The first step is to measure the momentum entanglement, and the second step is to measure the polarization entanglement. In our protocol, the concurrences have been transformed to the probability of picking up the odd-parity states such as $|a1\rangle|a4\rangle$, $|b1\rangle|b4\rangle$ in spatial modes and $|H\rangle|V\rangle$ and $|V\rangle|H\rangle$ in polarization entanglement. In the measurement of the polarization part, we can measure the parity of the state with the QND or measure it directly. In order to complete the exact measurement of the concurrence, we should perform the process over many rounds and consume many photon pairs shown in Eq. (5). This protocol can be realized on the fact that the hyperentangled states of the form of Eq. (5) in two degrees of freedom can be operated independently. Hence, if we manipulate the momentum entanglement, we leave the polarization entanglement unchanged. Certainly, if we operate the polarization entanglement, the momentum entanglement does not change either. This advantage essentially provides us an effective approach for performing this protocol. In order to measure the phase shift of the coherent state, we adopt two different measurement techniques. In the first step, we adopt the general homodyne–heterodyne measurement to pick up the phase shift $\theta$, while in the second step, we use the $X$ quadrature measurement to make the $\pm 2\theta$ undistinguished. Actually, in the first step, we can also make the $\pm \theta$ undistinguished, with the total probability of $2|\alpha_2\beta_2|^2$. However, after this measurement is performed, the system in the momentum degrees of freedom is still entangled. Before measuring the polarization entanglement,
we should add another QND to eliminate the momentum entanglement. In our protocol, after the state picks up the phase shift \( \theta \), or \(-\theta\), the spatial modes of the photons are essentially deterministic. They can start the second step directly.

4 Measurement of the concurrence for arbitrary pure hyperentangled state

In the above section, we have explained the method for measuring the concurrence of the hyperentangled state with four different coefficients. This method can be extended to measure the concurrence of an arbitrary pure hyperentangled state of the form

\[
|\phi\rangle = |\phi\rangle_p \otimes |\phi\rangle_m = (\alpha_1|H_a\rangle|H_b\rangle + \beta_1|V_a\rangle|V_b\rangle + \gamma_1|H_a\rangle|V_b\rangle + \delta_1|V_a\rangle|H_b\rangle) \\
\otimes (\alpha_2|a1\rangle|b1\rangle + \beta_2|a2\rangle|b2\rangle + \gamma_2|a1\rangle|b2\rangle + \delta_2|a2\rangle|b1\rangle).
\]  
(16)

Here, \(|\alpha_1|^2 + |\beta_1|^2 + |\gamma_1|^2 + |\delta_1|^2 = 1\) and \(|\alpha_2|^2 + |\beta_2|^2 + |\gamma_2|^2 + |\delta_2|^2 = 1\). Following the same principle as in the measurement above, we first describe the momentum entanglement in the form

\[
|\phi\rangle_m = \alpha_2|a1\rangle|b1\rangle + \beta_2|a2\rangle|b2\rangle + \gamma_2|a1\rangle|b2\rangle + \delta_2|a2\rangle|b1\rangle.
\]  
(17)

As the entanglement in each degree of freedom can be operated independently, we omit the polarization entanglement in the following description for simplicity. The first step is similar to that in the previous section. From Fig. 1, we let the four photons pass through both the two QNDs. If both the coherent states pick up the phase shift \( \pm \theta \), the state becomes

\[
|\phi\rangle_1 = \frac{\alpha_2 \beta_2}{\sqrt{2(|\alpha_2 \beta_2|^2 + |\gamma_2 \delta_2|^2)}}(|a1\rangle|a4\rangle|b1\rangle|b4\rangle + |a2\rangle|a3\rangle|b2\rangle|b3\rangle) \\
+ \frac{\gamma_2 \delta_2}{\sqrt{2(|\alpha_2 \beta_2|^2 + |\gamma_2 \delta_2|^2)}}(|a1\rangle|a4\rangle|b2\rangle|b3\rangle + |a2\rangle|a3\rangle|b1\rangle|b4\rangle).
\]

The total probability is \( P_{1m} = 2(|\alpha_2 \beta_2|^2 + |\gamma_2 \delta_2|^2) \).

Then, we let the photons in the spatial modes \( b1, b2, b3, \) and \( b4 \) pass through the two beam splitters (BSs) shown in Fig. 3, which will make

\[
|b1\rangle \rightarrow \frac{1}{\sqrt{2}}(|c1\rangle + |c2\rangle), \quad |b2\rangle \rightarrow \frac{1}{\sqrt{2}}(|c1\rangle - |c2\rangle), \\
|b3\rangle \rightarrow \frac{1}{\sqrt{2}}(|c3\rangle + |c4\rangle), \quad |b4\rangle \rightarrow \frac{1}{\sqrt{2}}(|c3\rangle - |c4\rangle).
\]

(19)

After passing through the BSs, the state \( |\phi\rangle_1 \) becomes

\[
|\phi\rangle'_1 = \frac{\alpha_2 \beta_2 + \gamma_2 \delta_2}{2\sqrt{2(|\alpha_2 \beta_2|^2 + |\gamma_2 \delta_2|^2)}}(|a1\rangle|a4\rangle + |a2\rangle|a3\rangle) \otimes (|c1\rangle|c3\rangle - |c2\rangle|c4\rangle) \\
+ \frac{\alpha_2 \beta_2 - \gamma_2 \delta_2}{2\sqrt{2(|\alpha_2 \beta_2|^2 + |\gamma_2 \delta_2|^2)}}(|a1\rangle|a4\rangle - |a2\rangle|a3\rangle) \otimes (|c1\rangle|c4\rangle - |c2\rangle|c3\rangle).
\]

(20)
Concurrence measurement

Fig. 3 Schematic of measurement of the momentum entanglement. In the last step, we should determine the spatial modes of the state to perform further measurement. The BS represents the 50:50 beam splitter.

From Eq. (20), if the phase shift is $\theta_1 + \theta_4$, the state in Eq. (20) will become

$$|\phi''\rangle_1 = \frac{1}{\sqrt{2}} (|a1\rangle|a4\rangle - |a2\rangle|a3\rangle) \otimes |c1\rangle|c4\rangle. \quad (21)$$

The probability is $P_{2m} = \frac{\left|\alpha_2\beta_2 - \gamma_2\delta_2\right|^2}{4(|\alpha_2\beta_2|^2 + |\gamma_2\delta_2|^2)}$. If the coherent state picks up the phase shift $\theta_2 + \theta_3$, the state in Eq. (20) will become

$$|\phi'''\rangle_1 = \frac{1}{\sqrt{2}} (|a1\rangle|a4\rangle - |a2\rangle|a3\rangle) \otimes |c2\rangle|c3\rangle, \quad (22)$$

with the same probability. The total probability of obtaining the state $|\phi''\rangle_1$ or $|\phi'''\rangle_1$ is

$$P_m = P_{1m}P_{2m} = 2 \left( |\alpha_2\beta_2|^2 + |\gamma_2\delta_2|^2 \right) \frac{|\alpha_2\beta_2 - \gamma_2\delta_2|^2}{4(|\alpha_2\beta_2|^2 + |\gamma_2\delta_2|^2)} = \frac{1}{2} |\alpha_2\beta_2 - \gamma_2\delta_2|^2. \quad (23)$$

Therefore, we can obtain the concurrence

$$C(|\phi\rangle_m) = 2|\alpha_2\beta_2 - \gamma_2\delta_2| = 2\sqrt{2P_m}. \quad (24)$$

In the next step, we will describe the measurement of the concurrence of the polarization entanglement. However, before we start the second step of the protocol, we should first decide the spatial mode of the whole system. We take the state $|\phi'''\rangle_1$ as an example. If we consider the polarization entanglement, the whole state $|\phi''''\rangle_1$ should be rewritten as

$$|\psi\rangle_1 = (\alpha_1|H_a\rangle|H_c\rangle + \beta_1|V_a\rangle|V_c\rangle + \gamma_1|H_a\rangle|V_c\rangle + \delta_1|V_a\rangle|H_c\rangle)$$
$$\otimes (\alpha_1|H_a\rangle|H_c\rangle + \beta_1|V_a\rangle|V_c\rangle + \gamma_1|H_a\rangle|V_c\rangle + \delta_1|V_a\rangle|H_c\rangle)$$
$$\otimes \frac{1}{\sqrt{2}} (|a1\rangle|a4\rangle - |a2\rangle|a3\rangle) \otimes |c1\rangle|c4\rangle. \quad (25)$$
Equation (25) indicates that the two photons in Bob’s location are deterministic in $c_1$ and $c_4$, whereas the photons in Alice’s location are still entangled in the spatial modes. Therefore, we let the photons in the $a_1$, $a_2$, $a_3$, and $a_4$ modes pass through the setup shown in Fig. 3 by removing the two BSs. If the phase shift is $\theta_1 + \theta_4$, $|\psi_1\rangle$ becomes

$$|\psi_1\rangle = (\alpha_1 |H_{a_1}\rangle |H_{c_1}\rangle + \beta_1 |V_{a_1}\rangle |V_{c_1}\rangle + \gamma_1 |H_{a_1}\rangle |V_{c_1}\rangle + \delta_1 |V_{a_1}\rangle |H_{c_1}\rangle)$$

$$\otimes (\alpha_1 |H_{a_4}\rangle |H_{c_4}\rangle + \beta_1 |V_{a_4}\rangle |V_{c_4}\rangle + \gamma_1 |H_{a_4}\rangle |V_{c_4}\rangle + \delta_1 |V_{a_4}\rangle |H_{c_4}\rangle).$$

(26)

Otherwise, if the phase shift is $\theta_2 + \theta_3$, $|\psi_1\rangle$ becomes

$$|\psi_1\rangle = (\alpha_1 |H_{a_2}\rangle |H_{c_1}\rangle + \beta_1 |V_{a_2}\rangle |V_{c_1}\rangle + \gamma_1 |H_{a_2}\rangle |V_{c_1}\rangle + \delta_1 |V_{a_2}\rangle |H_{c_1}\rangle)$$

$$\otimes (\alpha_1 |H_{a_3}\rangle |H_{c_4}\rangle + \beta_1 |V_{a_3}\rangle |V_{c_4}\rangle + \gamma_1 |H_{a_3}\rangle |V_{c_4}\rangle + \delta_1 |V_{a_3}\rangle |H_{c_4}\rangle).$$

(27)

After the measurement, the momentum entanglement disappears, and the spatial mode of each photon is deterministic. In this way, we can start the detection of the concurrence of the polarization entanglement. Suppose that both Alice and Bob own the QND as shown in Fig. 2. We first let the four photons pass through the QNDs, respectively. After performing the Hadamard operation, the state $|\psi_1\rangle$ will become

$$|\psi_2\rangle = \frac{\alpha_1 \beta_1}{\sqrt{2(|\alpha_1 \beta_1|^2 + |\gamma_1 \delta_1|^2)}} (|H_{a_1}\rangle |V_{a_4}\rangle |H_{c_1}\rangle |V_{c_4}\rangle + |V_{a_1}\rangle |H_{a_4}\rangle |V_{c_1}\rangle |H_{c_4}\rangle)$$

$$+ \frac{\gamma_1 \delta_1}{\sqrt{2(|\alpha_1 \beta_1|^2 + |\gamma_1 \delta_1|^2)}} (|H_{a_1}\rangle |V_{a_4}\rangle |V_{c_1}\rangle |H_{c_4}\rangle + |V_{a_1}\rangle |H_{a_4}\rangle |H_{c_1}\rangle |V_{c_4}\rangle),$$

(28)

with the probability $P_{1,p} = 2(|\alpha_1 \beta_1|^2 + |\gamma_1 \delta_1|^2)$. In the second round, we first perform the Hadamard operation on the photons in the $a_1$ and $a_4$ modes. The Hadamard operation can be implemented with the quarter-wave plate (QWP) and makes

$$|H\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle), \quad |V\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle).$$

(29)

After performing the Hadamard operation, the state $|\psi_2\rangle$ will become

$$|\psi_3\rangle = \frac{\alpha_1 \beta_1 + \gamma_1 \delta_1}{2\sqrt{2}(|\alpha_1 \beta_1|^2 + |\gamma_1 \delta_1|^2)} (|H_{a_1}\rangle |H_{a_4}\rangle - |V_{a_1}\rangle |V_{a_4}\rangle)$$

$$\otimes (|H_{c_1}\rangle |V_{c_4}\rangle + |V_{c_1}\rangle |H_{c_4}\rangle)$$

$$+ \frac{\alpha_1 \beta_1 - \gamma_1 \delta_1}{2\sqrt{2}(|\alpha_1 \beta_1|^2 + |\gamma_1 \delta_1|^2)} (|H_{a_1}\rangle |V_{a_4}\rangle - |V_{a_1}\rangle |H_{a_4}\rangle)$$

$$\otimes (|H_{c_1}\rangle |V_{c_4}\rangle - |V_{c_1}\rangle |H_{c_4}\rangle).$$

(30)
Finally, we let the photons in $a_1$ and $a_4$ modes pass through the QND in Fig. 3 again and pick up the odd-parity state in a second time. This changes the state $|\psi\rangle'_{3}$ to

$$
|\psi\rangle'_{4} = \frac{1}{2}(|H_{a1}\rangle|V_{a4}\rangle - |V_{a1}\rangle|H_{a4}\rangle) \otimes (|H_{c1}\rangle|V_{c4}\rangle - |V_{c1}\rangle|H_{c4}\rangle),
$$

with the probability $P_{2p} = \frac{|\alpha_2\beta_2 - \gamma_2\delta_2|^2}{4(|\alpha_2\beta_2|^2 + |\gamma_2\delta_2|^2)}$. Therefore, the total probability

$$
P_p = P_{1p}P_{2p} = |\alpha_1\beta_1 - \gamma_1\delta_1|^2.
$$

We can obtain the concurrence

$$
C(|\phi\rangle_p) = |\alpha_2\beta_2 - \gamma_2\delta_2| = 2\sqrt{P_p},
$$

and the total concurrence

$$
C(|\phi\rangle) = C(|\phi\rangle_m) + C(|\phi\rangle_p) = 2\sqrt{2P_m + 2P_p}.
$$

Certainly, if we obtain the state in Eq. (30), one can also measure the photons in the $a_1$ and $a_4$ modes in the $|H\rangle$ and $|V\rangle$ basis, respectively. The probability of obtaining $|H_{a1}\rangle|V_{a4}\rangle$ or $|V_{a1}\rangle|H_{a4}\rangle$ is $P'_{2p} = \frac{|\alpha_2\beta_2 - \gamma_2\delta_2|^2}{4(|\alpha_2\beta_2|^2 + |\gamma_2\delta_2|^2)}$. The total concurrence can be rewritten as

$$
C(|\phi\rangle) = C(|\phi\rangle_m) + C(|\phi\rangle_p) = 2\sqrt{2P_m + 2\sqrt{2P_p}}.
$$

### 5 Discussion

So far, we have completely described our protocol. We first described the method for measuring the concurrence of the partially hyperentangled state. Subsequently, we provided the approach for measuring the arbitrary pure hyperentangled state. This protocol can be divided into two steps. In the first step, we perform the measurement of the momentum entanglement. In the second step, we describe the measurement of the polarization entanglement. This protocol depends on the distinct feature that the different degrees of freedom are independent. It essentially ensures that one can manipulate each degree of freedom independently. In the practical operation, momentum entanglement should be measured first, that is, because once the spatial modes of the photon is deterministic, one can easily perform further measurement of polarization entanglement. During the whole procedure, we mainly explain the polarization entanglement measurement after successfully measuring the momentum entanglement. Actually, the first measurement step does not affect the second one. Even if the measurement of the momentum entanglement is a failure, we can also perform the second step, after determining the spatial mode by adding another QND. In this manner, we can improve the practical efficiency.
Generally, in the initial studies on the generation of the hyperentanglement, the Bell inequalities were adopted to characterize the quality of the hyperentanglement. Vallone et al. [69] also introduced the hyperentanglement witness to detect whether a two-particle state is hyperentangled. Unfortunately, as pointed out by Walborn et al., the Bell inequalities and entanglement witness cannot generally provide satisfactory results, because they disclose the entanglement of some quantum states but fail for other states. These approaches are fundamentally different from entanglement measurement which, by definition, quantifies the amount of entanglement in any state. With the help of hyperentanglement, Walborn et al. have successfully determined the polarization entanglement. However, in their experiment, the momentum entanglement acts as the auxiliary resources to perform the CNOT gate. Their protocol cannot describe the complete concurrence for hyperentanglement including both momentum entanglement and polarization entanglement. On the other hand, a direct method for detecting the entanglement would be the quantum state tomographic reconstruction [70]. It requires 15 parameters to reconstruct a two-qubit state. If we consider a two-qubit hyperentangled state in an enlarged Hilbert space, the method requires 255 parameters, which makes it extremely complicated.

Finally, let us briefly discuss the physical mechanism of the QND, which plays an important role in this protocol. First, the QND relies on the assumption that the cross-Kerr effect is independent of the polarization states of the signals. Generally, the selection rules for the transition between energy levels could make the cross-Kerr effect for different polarization states. Rebić et al. [71] showed that the artificial multilevel system in circuit-QED, of two capacitively coupled Cooper-pair boxes coupled to the quantized field in a Co-Planar Waveguide resonator, can make the generation of giant self-Kerr nonlinearities. In their system, the cross-Kerr effect is independent of the linear polarization. We require a weak cross-Kerr nonlinearity for implementing the QND. The requirement for distinguishing between the output coherent state components in Eq. (4) is $\alpha \theta > 1$, where $\alpha$ is the amplitude of the coherent state. Though the normal cross-Kerr nonlinearities are generally weak ($\chi^3 \approx 10^{-22} \text{m}^2 \text{V}^{-2}$ [72]), in order to implement the QND, it is possible to achieve the purpose by using a relatively large amplitude $\alpha$ and/or effectively magnifying the cross-Kerr phase shift with weak measurement [73]. The giant Kerr nonlinearity can also be obtained in a multiple quantum-well structure with a four-level, double $\wedge$-type configuration [74]. Recent work also showed that the Rydberg atom system could generate rather large cross-phase between photons [75]. So far, the debate over the usefulness of photonic quantum information processing based on XPM centers around the feasibility of realizing an ideal single-mode XPM process such as in Eq. (4) [76–79]. For the XPM between a single photon and a coherent state, which is relevant to the present protocol, a nearly to ideal process that generates a small cross-phase $\theta$ can be realized by letting two transversely confined pulses in the respective quantum state pass through each other [61]. The experimental progress in the XPM for quantum light is expected in the near future.

In conclusion, we have described an effective way of measuring the concurrence for a hyperentangled state. We first provide an alternative definition of the concurrence for the hyperentangled state. We show that the concurrence in different
degrees of freedom can be measured independently. We do not require the sophisticated CNOT gate operation and resort to the feasible weak cross-Kerr nonlinearity to perform the parity-check measurement. This approach of characterizing the hyperentanglement may prove extremely useful in future quantum information processing.

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