Saturation of Coulomb sum rules in the $^6\text{Li}$ case

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The Coulomb sums $S_L(q)$ of the $^6\text{Li}$ nucleus have been obtained from electron scattering measurements at 3-momentum transfers $q = 1.125 \pm 1.625$ fm$^{-1}$. It is found that at $q > 1.35$ fm$^{-1}$ the Coulomb sum of the nucleus becomes saturated: $S_L(q) = 1$. 
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1 Introduction

The Coulomb sums (CS) are obtained from the treatment of data on electron scattering by atomic nuclei and can be used as an experimental data representation convenient for investigating some problems of the nuclear structure and the properties of intranuclear nucleons (e.g., see refs. [1][2][3]). The experimental CS were obtained for the most part at Saclay, Bates and SLAC Laboratories [2][4][5][6].

According to the sum rules, at sufficiently high momentum transfers the CS must be a constant quantity equal to 1. The experimental data show that with an increasing momentum transfer the CS value of each of the nuclei studied also increases, and beginning with $q = 1.7 \div 2$ fm$^{-1}$ it becomes constant, just as predicted by the sum rules. However, for the nuclei with the atomic weight $A \geq 4$ at $q > 2$ fm$^{-1}$ the experimental CS values are less than 1 (undersaturation of Coulomb sum rules). To illustrate the CS behavior, fig. 1 shows the CS values for the $^4\text{He}$ nucleus.

The present paper is concerned with the CS of $^6\text{Li}$, i.e., the nucleus that is clustered and non-spherical.

2 Terms and formulas

One calls the Coulomb sum the zero moment of the longitudinal response function (RF) of the atomic nucleus.

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The scattered electrons are momentum analyzed by the spectrometer SP-95 with a solid angle of $2.89 \times 10^{-3}$ sr and a dispersion of 13.7 mm/percent. In the focal plane of the spectrometer, the electrons are registered by eight scintillation detectors, each having an energy acceptance of 1.4%, and then arrive at organic-glass Cherenkov radiators. The pulses from photomultipliers of scintillation detectors and Cherenkov detectors are registered by a coincidence circuit with a time resolution of 9 ns.

In spectral measurements, the background of accidental coincidences of pulses from the scintillation/Cherenkov detectors was about or less than 1% of the effect value and was taken into account, while the background measured in the absence of the target was one order of magnitude lower. In the measured spectra, according to our calculations and a few measurements with positrons, the background contributed by $e^{-}e^{+}$ pairs from the target is insignificant if present at all.

Before and after measuring each spectrum of electrons scattered by $^{6}$Li, the peak of elastic electron scattering by $^{12}$C was measured. Using the data of the measurements after their correction for the radiation effects, the squared form factor $F_{2}^{2}(q)$ values of the ground state of the $^{12}$C nucleus were found. These values were used for normalizing our measured data for $^{6}$Li. Namely, the data normalization factor was found as $k_{abs} = F_{2}^{2}(q)/F_{2}^{2}(q)$, where $F_{2}^{2}(q)$ stands for the squared form factor of the $^{12}$C ground state measured in [15] with a systematic error of 0.4%. Then, with the use of equations from ref. [16], the spectra of inelastic electron scattering by $^{6}$Li were corrected for the radiation effects. Since the RF are determined by inelastic electron scattering, the contribution from the elastic scattering peak was subtracted from the $^{6}$Li($e,e'$) spectra. Due to the fact that in our present measurements the energy resolution in the neighborhood of the elastic scattering peak was between 1.8 and 3.6 MeV, and the first excited-state energy of $^{6}$Li was 2.18 MeV, then to subtract the elastic scattering contribution from the spectrum of scattered electrons, we have used the form factors of both the mentioned excited state and the ground state measured in ref. [17]. The inelastic scattering cross sections were divided by the corresponding Mott cross sections and were averaged within 2 MeV intervals. In the group of spectra measured at $\theta = 160^\circ$, the data were interpolated to the $(q, \omega)$ values that corresponded to the spectra taken at small scattering angles. At those $(q, \omega)$ values and with the use of eq. [2] the data were separated into $R_{L}(q, \omega)$ and $R_{T}(q, \omega)$ values. The obtained $R_{L}(q, \omega)$ values were interpolated to the fixed 3-momentum transfer values: $q_{c} = 1.125, 1.250, 1.375, 1.500, 1.625$ fm$^{-1}$. The interpolation technique used here as well as some
other additional details of the measurements and data processing have been described in papers [18, 19]. Figure 2 shows the derived $R_L(q_c, \omega)$ values.

The attention is drawn to a relatively small (with regard to errors) scatter of the experimental points. This is due to the fact that each of the points is found through two interpolations of the observed data, each interpolation smoothing the experimental data sequences. Note that the data smoothing can also be observed for the experimental $R_L(q_c, \omega)$ values obtained by other authors (e.g., see refs. [4, 8]).

Relatively high $R_L(q_c, \omega)$ values in the region of low $\omega$ are explained by the contribution from the excitation of low-lying nuclear states, whose peaks have merged because of a low energy resolution of measurements.

As regards the analysis of experimental RF, it would be of great interest to compare them with current theoretical calculations. However, by now, the modern calculations of RF have been made only for the nuclei with $A \leq 4$ (e.g., see ref. [20]), and for heavier nuclei these calculations are only projected.

To calculate the CS (eq. 1), it is necessary to extrapolate the RF to the high-energy transfer region. For this purpose, it is common practice to use an exponential or a power function (e.g., see refs. [6] and [4], respectively). The exponential function, like the power function in some cases, as applied to the RF, is considered as empirical. However, the RF extrapolation with the power function has been substantiated in theoretical papers [2, 21], and, as applied to the experimental RF values, it has been analyzed in ref. [23]. According to refs. [2, 21], the extrapolation power function has the form

$$R^\alpha(q, \omega \to \infty) = C_\alpha(q) \cdot \omega^{-\alpha},$$

where $C_\alpha(q)$ is the fitting parameter, $\alpha$ is either the fitting or the calculated parameter, $\omega' = \omega - q^2/(2AM)$ is the c.m.s. energy transfer.

According to the calculations of ref. [21], the parameter $\alpha$ is equal to 2.5 and is independent of the momentum transfer. With the use of the free parameters $C_\alpha(q)$ and $\alpha$, from the fit of the function $R^\alpha(q, \omega \to \infty)$ to the experimental RF of $^6$Li, obtained in the high $\omega$ region, we have found $\alpha = 2.56 \pm 0.06$. Then, using this $\alpha$ value for each $R_L(q_c, \omega)$ we calculated $C_\alpha(q)$. Using eq. 1, the experimental $R_L(q_c, \omega)$ and the function $R^\alpha(q, \omega \to \infty)$ with the parameters found, we have obtained the Coulomb sum $S_L(q)$ values. Besides, the Coulomb sums $S_{L, c}'(q)$ were obtained through the use of exponential extrapolation of the type

$$R^\beta(q, \omega \to \infty) = C_\beta(q) \cdot e^{-\beta(q) \cdot \omega'},$$

with the fitting parameters $C_\beta(q)$ and $\beta(q)$. Here, unlike the RF extrapolation with the power function, the values of the two parameters $C_\beta$ and $\beta$ are dependent on the momentum transfer.
The $S_L(q)$ and $S'_L(q)$ values found here are presented in table [1] and fig. 3.

![Graph](image)

**Fig.3. Coulomb sum of the $^6$Li nucleus.** Triangles show the CS values from ref. [26]; full circles and open circles show the present data obtained with the use of power and exponential extrapolations, respectively. The data denoted by full circles include minor errors (statistical only) and major errors (statistical plus systematic).

### 4 Significance of some corrections and the errors for the CS

In our present measurements the targets comprise 9.5% $^7$Li by weight. Earlier, we have made preliminary processing of measurements on the targets consisting of 93.8% $^7$Li and 6.2% $^6$Li by weight [24] [25].

![Graph](image)

**Fig.4. Longitudinal response functions of lithium isotopes.** Full circles show the data of the present work for $^6$Li; open circles - the data for $^7$Li taken from ref. [24].

The RF of $^7$Li found in ref. [24] [25] are close to the RF of $^6$Li found here (see fig. 4), and the $S_L(q)$ values of $^7$Li are not different (to an accuracy of experimental errors) from $S_L(q)$ values of $^6$Li. This implies that the presence of $^7$Li impurity in the $^6$Li target exerts no essential effect on the obtained $S_L(q)$ values of $^6$Li.

*The correction contributions* to the $S_L(q)$ values obtained are as follows:

i) the use of calculation of the electrical form factor of the proton, $G_P(q^2)$, by equations of ref. [13] instead of the traditional dipole formula, gives $(1.2 \div 2.4)\%$;

ii) $\eta$ in eq. [1] makes $(1.4 \div 2.8)\%$;

iii) taking into account the nuclear Coulomb field effect on the momentum transfer of the incident electron is about 1%;

iv) calculation of the electrical form factor of the neutron, $G_n(q^2)$ gives about 0.1%.

*The contributions to the systematic error of $S_L(q)$ values, which come from the errors in:*

a) normalization of data, including the measurement errors of the ground-state form factor of the $^{12}$C nucleus in the present work and in ref. [15] - $(1.2 \div 2.5)\%$;

b) procedure of radiation correction of spectra - 2%;

c) determination of thickness of $^6$Li targets employed in the measurements at $\theta = 160^\circ$ and at $\theta \leq 94^\circ 10'$ - $(0.6 \div 1.3)\%$;

d) interpolation procedures - up to 1%;

e) procedure of determination of the parameter $\alpha$ from the fit of eq. [3] to the experimental RF - $(0.6 \div 0.9)\%$;

*The contributions to the statistical error of $S_L(q)$ values, which come from statistical errors of:*

f) experimental $R_L(q_c, \omega)$ - $(1.7 \div 3.3)\%$;

g) derivation of the parameter $C_\alpha(q)$ in eq. [5] - $(1.2 \div 2.5)\%$.

To determine the contribution from correction iii) and the errors (a, b, c, d, f), eq. [2] was used. The values of systematic $(\Delta S_{L,syst})$ and statistical $(\Delta S_{L,stat})$ errors, given in table [1] are the quadratic sums of the above-mentioned contributions.
The CS values obtained in the present study can be supplemented with the data from ref. [26]. The CS of $^6$Li has been denoted there as $\sigma(q)$, and is related to the present-day definition of the Coulomb sum by the expression $S_L(q) = \sigma(q)/G_\rho^2(q^2)$. The CS values of ref. [26] transformed in this way are shown in fig. 3. It can be seen from the figure that the function $S_L(q)$ for $^6$Li is different from $S_L(q)$ for other nuclei (e.g., see $S_L(q)$ for $^4$He in fig. 1).

Let us consider some special features of the CS for $^6$Li.

5A. The data of Saclay and Bates Laboratories show an increase in the CS up to $q = 1.7 \div 2$ fm$^{-1}$ for the nuclei studied with $A = 4 \div 56$. As it can be seen from fig. 3, at $q \leq 1.4$ fm$^{-1}$ the function $S_L(q)$ for $^6$Li attains the range of constant values.$^1$ Relying on papers [26, 27], it can be demonstrated that if $S_L(q)$ of $^6$Li took on the constant value at higher momentum transfers (as in the case of other nuclei), then the clusterization in this nucleus would be small or absent. However, that is not the case.

5B. For momentum transfers, at which the $S_L(q)$ values are constant to an accuracy of experimental errors, we denote the average CS as $S_{L,0}$. In the range of $q = 1.375 \div 1.625$ fm$^{-1}$ for $^6$Li we have $S_{L,0} = 1.031 \pm 0.016 \pm 0.034$, where the given errors are statistical and systematic, respectively.

The found result shows the CS saturation, that corresponds to the viewpoint of paper [9]. It has been stated there that the CS undersaturation of the nuclei with $A \geq 4$, observed in the Saclay and Bates experiments, was the result of error in the data analysis.

If to take for granted the phenomenon of CS undersaturation ($S_{L,0} < 1$), revealed in the previously investigated nuclei with $A \geq 4$, then $S_{L,0} = 1$ for $^6$Li falls out of the systematics of the effect.

Here it should be noted that if the exhaustion or underexhaustion of Coulomb sum rules is dealt with, it is generally assumed that the $S_L(q)$ value is virtually fully determined by the cross section for quasielastic electron scattering (QES) from intranuclear nucleons. This takes place at $q \geq 2$ fm$^{-1}$. The $S_{L,0}$ plateau of $A < 208$ nuclei (except $^6$Li) is observed at $q$ ranging from $1.7 \div 2$ fm$^{-1}$ to $3.5$ fm$^{-1}$. In the $^6$Li case, this plateau begins at $q = 1.4$ fm$^{-1}$, and in the measured range of momentum transfers ($q = 1.4 \div 1.6$ fm$^{-1}$) the contribution of QES to $S_L(q)$ makes about 90%. It is believed that after reaching the plateau the $S_L(q)$ value of $^6$Li remains constant in the region of high momentum transfers, too, as it is observed in the case of other previously investigated nuclei with $A < 208$.

It appears of interest to consider this case ($^6$Li $S_L(q) = 1$ at $q > 1.6$ fm$^{-1}$) from the standpoint of the hypothesis about undersaturation of the Coulomb sum rules.

The undersaturation of Coulomb sum rules can be explained by the modification of intranuclear nucleons. A prerequisite to the nucleon modification may be the density of medium surrounding the nucleon, i.e., the nucleon density in the nucleus without the contribution from the nucleus under consideration. Since the calculation of this density is qualitatively unobvious, then, to the first approximation, we may restrict our consideration for the $A \geq 4$ nuclei simply to the highest nucleon density in the nucleus, max($\rho(r)$). All the nuclei, for which $S_{L,0} < 1$ have been previously obtained, have max($\rho(r)$) $> 0.16$ fm$^{-3}$ [15, 28, 29, 30], whereas in the $^6$Li case we have max($\rho(r)$) $= 0.15$ fm$^{-3}$ [17]. From the comparison between $S_{L,0}$ and max($\rho(r)$) of the nuclei under consideration it follows that the critical density value, over which nucleon modification takes place,

| $q$, fm$^{-1}$ | $\omega_{max}$, MeV | $S_L$ | $\Delta S_{L,stat}$ | $\Delta S_{L,syst}$ | $S_{L,0}/S_L$ | $S_{L,0}/S_L'$ |
|--------------|----------------------|-----|-------------------|--------------------|-----------------|-----------------|
| 1.125        | 118                  | 0.842 | 0.017            | 0.022              | 0.888           | 0.801           |
| 1.250        | 108                  | 0.919 | 0.018            | 0.026              | 0.117           | 0.857           |
| 1.375        | 110                  | 1.015 | 0.021            | 0.030              | 0.127           | 0.938           |
| 1.500        | 140                  | 1.050 | 0.030            | 0.033              | 0.105           | 0.994           |
| 1.625        | 140                  | 1.056 | 0.043            | 0.039              | 0.130           | 0.990           |

$^1$Note that the special feature of the CS for $^4$Li discussed here could also be seen in the data of ref. [26]. However, in 1977, when that work was published, there was no systematics of the CS data for a number of nuclei (the data appeared only in the eighties) and it was impossible to make any reasonable comparison between the CS of different nuclei.
is \( \rho_c \approx 0.15 \text{ fm}^{-3} \). The hypothesis of the relationship between nucleon modification and nucleon distribution in the nucleus has been described in detail in paper [31].

In conclusion, we note that, as it can be seen from item 5B, of great importance are the experimental data on \( S_L(q) \) of the \(^6\text{Li}\) nucleus at \( q > 1.6 \text{ fm}^{-1} \). As should the experiment at the Jefferson Lab [11] confirm the effect of undersaturation of CS rules, then it would be exceptionally interesting to carry out measurements for obtaining \( S_L(q) \) of the \(^6\text{Li}\) nucleus at high momentum transfers and, possibly, to perform similar measurements on \(^7\text{Li}\) and \(^9\text{Be}\) nuclei, where the nucleon density is relatively low. The results of these experiments would be the basis for drawing important conclusions about nucleon modification in the atomic nucleus.

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