SQM studied in the Field Correlator Method

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By using the recent nonperturbative equation of state of the quark gluon plasma derived in the formalism of the Field Correlator Method, we investigate the bulk properties of the strange quark matter in beta-equilibrium and with charge neutrality at $T = p = 0$. The results show that the stability of strange quark matter with respect to $^{56}Fe$ is strongly dependent on the model parameters, namely, the gluon condensate $G_2$ and the q$\bar{q}$ interaction potential $V_1$. A remarkable result is that the width of the stability window decreases as $V_1$ increases, being maximum at $V_1 = 0$ and nearly zero at $V_1 = 0.5$ GeV. For $V_1$ in the range $0 \leq V_1 \leq 0.5$ GeV, all values of $G_2$ are lower than $0.006 - 0.007$ GeV$^4$ obtained from comparison with lattice results at $T_c (\mu = 0) \sim 170$ MeV. These results do not favor the possibilities for the existence of (either nonnegative or negative) absolutely stable strange quark matter.

Keywords: Strange quark matter; Nonperturbative equation of state.

I. INTRODUCTION

Since the works of A. R. Bodmer [1] and E. Witten [2], the existence of strange quark matter (SQM) has been largely investigated. SQM is a particular form of matter comprising roughly equal amounts of $u$, $d$ and $s$ quarks. It is presumed that SQM has been produced at extreme conditions of high temperatures and densities in the beginning of the universe and/or latter at low temperatures and high densities in compact stellar interiors (e.g., neutron stars and quark stars). In this regard, it is of importance to mention the pioneer work of N. Itoh, who investigated the possibilities for the existence of hypothetical quark stars [3].

According to the Bodmer-Witten conjecture, SQM might be more stable than ordinary

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nuclear matter. In a pioneer work, E. Farhi and R. L. Jaffe \cite{4} investigated SQM in equilibrium with respect to weak interactions, at zero temperature and pressure, in the context of the MIT Bag Model \cite{5}. In this model, quarks enter in the respective equation of state (EOS) as a free quark gas with a Fermi-Dirac distribution. The confinement is represented by a bag enclosing the free quarks with a constant $B$ which gives the vacuum energy density difference between the confined and deconfined phases. Improvement of the model is obtained by the inclusion of corrections to first order in the QCD coupling constant in the $\alpha_c < 1$ (perturbative) regime (see \cite{4} and references therein). Until now, SQM properties in dense nuclear matter and compact stars interiors have mostly been considered in the framework of the MIT Bag Model \cite{4, 6–20}. Applications of the Nambu-Jona-Lasinio \cite{21, 22} quantum field theoretical approach have also been done to describe quark matter properties in compact stars interiors \cite{10, 11, 23–27}. In alternative investigations, in which quark masses are density dependent, SQM is also taken as a free Fermi gas mixture of quarks and anti-quarks \cite{28, 29}. These approaches are used to describe $u$, $d$ and $s$ quark matter at zero and (not so high) nonzero temperatures and large density regions where the approximation of free quarks can be considered. However, this is not so at all densities and (low) temperatures. Quarks strongly interact subjected to a potential that is large when the $q\bar{q}$ distances are large. In this case, nonperturbative methods must be considered.

QCD, the fundamental theory of strong interactions, due to its nonlinearity has been taken as an inappropriate theory for the purposes of practical applications as, for instance, the calculation of an EOS to describe quark matter at all finite temperatures and densities, including nonperturbative effects of confinement. Asymptotically, the description of quark matter becomes simple, but at low temperatures and (or moderate) densities the attempts to obtain an EOS including the confinement have appeared as a difficult task to be achieved. However, since 1987, the investigations of Yu. A. Simonov \cite{30} and H. G. Dosch \cite{31–33} have resulted in the construction of an important method based on vacuum field correlators functions that is being continuously developed up to now.

Recently the nonperturbative EOS of quark-gluon plasma was derived in the framework of the Field Correlator Method (FCM) \cite{34}, also called Stochastic Vacuum Model. FCM (for a review see \cite{35} and references therein) is a nonperturbative approach which naturally includes from first principles the dynamics of confinement in terms of color electric and color magnetic correlators. The parameters of the model are the gluon condensate $G_2$ and the
qq interaction potential $V_1$ which govern the behavior of the EOS, at fixed quark masses and temperature. FCM has been used to describe the quark-gluon plasma dynamics and phase transition \cite{36,39}. An important feature of the model is that it covers the entire phase diagram plane, from the large temperature and small density region to the small temperature and large density region. In the connection between FCM and lattice simulations, the critical temperature at $\mu_c = 0$ turns out to be $T_c \sim 170$ MeV for $G_2 \simeq 0.006 - 0.007$ GeV$^4$ \cite{36,37}.

In a previous work, the existence of stable SQM on strange star surfaces (see Sec.3.3 in \cite{40}) was shortly considered in the FCM framework. In the present work, we perform a more detailed investigation of SQM on the same line. The system is a gas of $u$, $d$ and $s$ quarks and gluons subjected to the interaction potential $V_1$. The vacuum energy density difference between confined and deconfined phases, $\Delta|\varepsilon_{vac}|$, is a nonperturbative quantity expressed in terms of $G_2$. The main parameters of our calculation are $\Delta|\varepsilon_{vac}|$ (or $G_2$) and $V_1$, and the strange quark mass $m_s$ (assuming $m_u = m_d = 0$). We first investigate the energy per baryon, from which we obtain the stability window with respect to the $^{56}Fe$ nucleus. Strangeness, hadronic electric charge and density are also addressed.

This application of the FCM to the study of the bulk properties of SQM (not considered before) shows the role of the method to provide alternative indications for its parameters. In \cite{40} we have shown the importance of the comparison of the FCM calculations with astrophysical observations of some strange star candidates. Similarly, in the present work, the main purpose is the relevance of the FCM predictions for the SQM properties to be compared with lattice simulations and/or the results at RHIC and LHC experiments.

This paper is organized as follows. In Sec. II we summarize the theoretical framework of the FCM and show the equations to be used in our calculation. In Sec. III we show the results and in Sec. IV we give the final remarks and conclusions.

II. BASICS EQUATIONS

In the FCM approach, the confined-deconfined phase transition is dominated by the nonperturbative correlators \cite{35}. The dynamic of deconfinement is described by Gaussian (quadratic in $F_{\mu\nu}^a F_{\mu\nu}^a$) colorelectric and colormagnetic gauge invariant Fields Correlators $D^E(x)$, $D_1^E(x)$, $D^H(x)$, and $D_1^H(x)$. The main quantity which governs the nonperturbative dynamics of deconfinement is given by the two point functions (after a decomposition is
made)
\[
g^2 \left\langle \hat{r}_f \{E_i(x)\Phi(x,y)E_k(x)\Phi(y,x)\} \right\rangle_B = \delta_{ik} [D^E + D_1^E + u_i^2 \frac{\partial D_1^E}{\partial u_2^2}] + u_i u_k \frac{\partial D^E}{\partial u_2} \quad (1)
\]
\[
g^2 \left\langle \hat{r}_f \{H_i(x)\Phi(x,y)H_k(x)\Phi(y,x)\} \right\rangle_B = \delta_{ik} [D^H + D_1^H + u_i^2 \frac{\partial D_1^H}{\partial u_2^2}] - u_i u_k \frac{\partial D^H}{\partial u_2} \quad (2)
\]
where \( u = x - y \) and
\[
\Phi(x, y) = P \exp \left[ \int_x^y A_\mu dx^\mu \right] \quad (3)
\]
is the parallel transporter (Schwinger line) to assure gauge invariance.

In the confined phase (below \( T_c \)), \( D^E(x) \) is responsible for confinement with string tension \( \sigma^E = (1/2) \int D^E d^2x \). Above \( T_c \) (deconfined phase), \( D^E(x) \) vanishes while \( D_1^E(x) \) remains nonzero being responsible (together with the magnetic part due to \( D^H(x) \) and \( D_1^H(x) \)) for nonperturbative dynamics of the deconfined phase. In lattice calculations, the nonperturbative part of \( D^E(x) \) is parametrized in the form \( 35, 41 \)
\[
D_1^E(x) = D_1^E(0)e^{-|x|/\lambda} \quad (4)
\]
where \( \lambda = 0.34 \text{ fm} \) (full QCD) is the correlation length, with the normalization fixed at \( T = \mu = 0 \),
\[
D^E(0) + D_1^E(0) = \frac{\pi^2}{18} G_2 \quad (5)
\]
where \( G_2 \) is the gluon condensate \( 42 \).

The generalization of the FCM at finite \( T \) and \( \mu \) provides expressions for the thermodynamics quantities where the leading contribution is given by the interaction of the single quark and gluon lines with the vacuum (called single line approximation (SLA)). As in \( 40 \), from \( 34 \) and standard thermodynamical relations \( 43 \), we here explicitly rewrite in more convenient forms (for our purposes) the expressions (for one quark system, \( N_f = 1 \)) for the pressure
\[
p_q^{SLA} = \frac{1}{3} \frac{2N_c}{(2\pi)^3} \int d^3k \frac{k^2}{E} \left[ f_q^{SLA}(T, J_1^E, \mu_q) + \bar{f}_q^{SLA}(T, J_1^E, \mu_q) \right] \quad (6)
\]
energy density
\[
\varepsilon_q^{SLA} = \frac{2N_c}{(2\pi)^3} \int d^3k \left[ E - T \frac{\partial J_1^E}{\partial T} + \mu_q \frac{\partial J_1^E}{\partial \mu_q} \right] \left[ f_q^{SLA}(T, J_1^E, \mu_q) + \bar{f}_q^{SLA}(T, J_1^E, \mu_q) \right] \quad (7)
\]
and include the number density of the quark system

\[
\begin{align*}
    n_q^{SLA} &= \frac{2N_c}{(2\pi)^3} \int d^3k \left[ f_q^{SLA}(T, J_1^E, \mu_q) - \bar{f}_q^{SLA}(T, J_1^E, \mu_q) \right] \\
    &\quad - T \frac{\partial J_1^E}{\partial \mu_q} \frac{2N_c}{(2\pi)^3} \int d^3k \left[ f_q^{SLA}(T, J_1^E, \mu_q) + \bar{f}_q^{SLA}(T, J_1^E, \mu_q) \right],
\end{align*}
\]

(8)

where

\[
\begin{align*}
    f_q^{SLA}(T, \mu_q, J_1^E) &= \frac{1}{e^{\beta(E + T J_1^E - \mu_q)} + 1} \quad \text{and} \quad \bar{f}_q^{SLA}(T, \mu_q, J_1^E) = \frac{1}{e^{\beta(E + T J_1^E + \mu_q)} + 1}
\end{align*}
\]

(9)

\( (q = u, d, s), E = \sqrt{k^2 + m_q^2}, \beta = 1/T, \) and \( J_1^E \equiv V_1/2T \) is the exponent of the Polyakov loop in the fundamental representation, where

\[
V_1 = \int_0^\beta d\tau (1 - \tau T) \int_0^\infty \xi d\xi D_1^E(\sqrt{\xi^2 + \tau^2}).
\]

(10)

is the large distance static q\bar{q} potential \[34, 35, 41\].

The pressure and energy density of gluons are given by

\[
\begin{align*}
    p_{gl}^{SLA} &= \frac{(N_c^2 - 1)}{3} \frac{2}{(2\pi)^3} \int d^3k \frac{k}{e^{\beta(k + TJ_1^E)} - 1} \\
    \varepsilon_{gl}^{SLA} &= 3 p_{gl} - T^2 \frac{\partial \tilde{J}_1^E}{\partial T} (N_c^2 - 1) \frac{2}{(2\pi)^3} \int d^3k \frac{1}{e^{\beta(k + TJ_1^E)} - 1},
\end{align*}
\]

(11)

and

(12)

where \( \tilde{J}_1^E = \frac{9}{4} J_1^E \) is the exponent of the Polyakov loop in the adjoint representation. In Eqs. (9), (11) and (12), when \( V_1 = 0 \) we recover the ordinary Fermi and Bose gases. In order to give Eqs. (6)-(12) in its most general forms, it was assumed that \( V_1 \) is, in principle, a function of temperature and chemical potential. However, according to the parametrization given by Eq. (10), \( V_1 \) does not depend on the chemical potential. As pointed out in [36], the expected \( \mu \)-dependence of \( V_1 \) should be weak for values of \( \mu \) much smaller than the scale of vacuum fields (which is of the order of \( \sim 1.5 \text{ GeV} \)) and is partially supported by the lattice simulations [44]. As in [40, 45, 46], we take \( V_1 \) independent of the chemical potential, so \( \partial J_1^E / \partial \mu_q = 0 \) in Eqs. (7) and (8).

In order to take into account the presence of electrons to keep the quark matter in \( \beta \)-equilibrium and with charge neutrality, we also include the equations for the pressure, energy density and number density of electrons given by

\[
p_e = \frac{1}{3} \frac{2}{(2\pi)^3} \int d^3k \frac{k^2}{E_e} [f_e(T, \mu_e) + \bar{f}_e(T, \mu_e)],
\]

(13)
\epsilon_e = \frac{2}{(2\pi)^3} \int d^3k \, E_e[f_e(T, \mu_e) + \bar{f}_e(T, \mu_e)] , \quad (14)

n_e = \frac{2}{(2\pi)^3} \int d^3k [f_e(T, \mu_e) - \bar{f}_e(T, \mu_e)] , \quad (15)

where

\[ f_e(T, \mu_e) = \frac{1}{e^{\beta(E_e - \mu_e)} + 1} , \quad \bar{f}_e(T, \mu_e) = \frac{1}{e^{\beta(E_e + \mu_e)} + 1} \quad (16) \]

and \( E_e = \sqrt{k_e^2 + m_e^2} \).

The composition of SQM is maintained in \( \beta \)-equilibrium with respect to weak interactions and in electric charge neutrality. The weak interactions reactions are given by

\[ d \rightarrow u + e + \bar{\nu}_e \quad (17) \]

and

\[ s \rightarrow u + e + \bar{\nu}_e . \quad (18) \]

As pointed out in [4], the neutrino gas is so dilute that it play no role in the dynamics of the system. So, by neglecting the neutrino chemical potential, the chemical equilibrium equations are given by

\[ \mu_d = \mu_u + \mu_e \quad (19) \]

and

\[ \mu_s = \mu_d . \quad (20) \]

The overall charge neutrality requires that

\[ \frac{1}{3} (2n_u^{SLA} - n_d^{SLA} - n_s^{SLA}) - n_e = 0 . \quad (21) \]

By numerically solving Eqs. (19)-(21), for each value of the input total density

\[ n = n_u^{SLA} + n_d^{SLA} + n_s^{SLA} + n_e , \quad (22) \]

the unknown chemical potentials \( \mu_u, \mu_d, \mu_s \) and \( \mu_e \) are determined for fixed values of \( T, G_2 \) and \( V_1 \). However, our calculation here is slightly different from that in [40], as explained below.

The total pressure and energy density of the quark-gluon system, including electrons are given by

\[ p = p_g^{SLA} + \sum_{q=u,d,s} p_q^{SLA} - \Delta|\epsilon_{vac}| + p_e , \quad (23) \]
\[ \varepsilon = \varepsilon_{\text{SLA}}^g + \sum_{q=u,d,s} \varepsilon_{\text{SLA}}^q + \Delta|\varepsilon_{\text{vac}}| + \varepsilon_e, \]  

(24)

where

\[ \Delta|\varepsilon_{\text{vac}}| = \frac{11 - \frac{2}{3}N_f}{32} \Delta G_2, \]  

(25)

is the vacuum energy density difference between confined and deconfined phases in terms of the respective difference between the values of the gluon condensate, \( \Delta G_2 = G_2(T < T_c) - G_2(T > T_c) \sim \frac{1}{2} G_2 \) \[^{36, 37}\], and \( N_f \) is the number of flavors.

We follow the same line of \[^{4}\] to investigate the behavior of the SQM at zero temperature\(^1\) and total pressure by solving the above equations for constant values of the energy per baryon, \( E/A = \varepsilon/n_A \), where \( n_A = (n_u + n_d + n_s)/3 \) is the baryon number density. We perform our calculation in the \( m_u = m_d = 0 \) approximation\(^2\) (in Sec. \[^{III}\] larger values of \( m_u \) and \( m_d \) are speculated in order to look for strangeness excess and negative electric charge possibilities). By using the constant \( E/A \) constraint in the above equations, \( m_s \) and \( \Delta|\varepsilon_{\text{vac}}| \) (or \( G_2 \)) are determined for fixed values of \( V_1 \) in the range \( 0 \leq V_1 \leq 0.5 \text{ GeV} \) (where \( V_1 = 0.5 \text{ GeV} \) is the value of \( V_1 \) at \( T = T_c \) obtained from lattice investigations \[^{47}\]). As in \[^{40, 45, 46}\], our calculation here is made for \( V_1 \) constant, so \( T^2 \partial J_{E} / \partial T = -V_1/2 \) in Eq. (7).

**A. Quark matter at \( T = 0 \) and constant \( V_1 \).**

For pedagogical purposes, we show the previous equations for the quark system at zero temperature and constant \( V_1 \) for the general case of nonzero quark masses. Zero temperature implies that

\[ f_q^{\text{SLA}}(T, \mu_q, J_{E}^1) \xrightarrow{T \to 0} \Theta(\mu_q - E - T J_{E}^1) \]  

(26)

and Eqs.(6)-(8) lead to

\[ p_q^{\text{SLA}} = \frac{N_c}{3\pi^2} \left\{ \frac{k^3}{4} \sqrt{k^2 + m^2} - \frac{3}{8} m^2 \left[ k_q \sqrt{k^2 + m^2} - m^2 \ln \left( \frac{k_q + \sqrt{k^2 + m^2}}{m} \right) \right] \right\} , \]  

(27)

\(^1\) In reality, we take \( T = 0.001 \text{ GeV} \) in Eqs. (6)-(15) which is a good approximation or, alternatively, by using Eqs. (27) - (29).

\(^2\) For our purposes here, it is irrelevant if electrons are assumed massless or not.
\[
\varepsilon_{q}^{SLA} = \frac{N_c}{\pi^2} \left\{ \frac{k_q^3}{4} \sqrt{k_q^2 + m_q^2} + \frac{m_q^2}{8} \left[ k_q \sqrt{k_q^2 + m_q^2} - m_q^2 \ln \left( \frac{k_q + \sqrt{k_q^2 + m_q^2}}{m_q} \right) \right] + \frac{V_1}{2} \frac{k_q^3}{3} \right\} 
\]

and

\[
n_{q}^{SLA} = \frac{N_c}{\pi^2} \frac{k_q^3}{3},
\]

where

\[
k_q = \sqrt{(\mu_q - V_1/2)^2 - m_q^2}, \quad (q = u, d, s).
\]

When \( V_1 = 0 \), the ordinary Fermi momentum \( k_F \) is recovered.

1. Zero mass approximation

In order to better understand the role of \( G_2 \) and \( V_1 \) in the study of stability of quark matter, it is instructive to apply the above equations to quark matter at zero temperature and pressure, assuming that all quark species are massless particles. This simple case serve to help us to understand the behavior of the constant \( E/A \) curves in Fig. 1 as well as the shrinking of the stability window in panel (a) of Fig. 3 at \( m_s = 0 \). For massless quarks, Eqs. (27)-(29) (with \( N_c = 3 \), by using Eq. (30)), are reduced to

\[
\varepsilon_{q}^{SLA} = \frac{1}{\pi^2} \left\{ \frac{3}{4} \left( \mu_q - \frac{V_1}{2} \right)^4 + \frac{V_1}{2} \left( \mu_q - \frac{V_1}{2} \right)^3 \right\} 
\]

and

\[
n_{q}^{SLA} = \frac{1}{\pi^2} \left( \mu_q - \frac{V_1}{2} \right)^3.
\]

At total zero pressure, the sum of the quark pressures is balanced by the vacuum energy density,

\[
\sum_q \frac{1}{4\pi^2} \left( \mu_q - \frac{V_1}{2} \right)^4 - \Delta|\varepsilon_{vac}| = 0,
\]
and the energy density is

\[ \varepsilon = \sum_q \varepsilon_q + \Delta|\varepsilon_{\text{vac}}| \]

\[ = 3 \sum_q p_q + \frac{V_1}{2\pi^2} \sum_q \left( \frac{\mu_q - V_1}{2} \right)^3 + \Delta|\varepsilon_{\text{vac}}| \]

\[ = \frac{3}{2} V_1 n_A + 4 \Delta|\varepsilon_{\text{vac}}| \tag{35} \]

where (here, in this section) the baryon number density is \( n_A = (n_u + n_d + n_s)/3 \) for SQM and \( n_A = (n_u + n_d)/3 \) for nonstrange quark matter. Notice the presence of the additional term \((3/2)V_1n_A\) with respect to the corresponding expressions in the MIT Bag Model [9].

Also noticed is that the sum of the quark pressures and energy densities are given in terms of \( \Delta|\varepsilon_{\text{vac}}| \) (or \( G_2 \)) and \( V_1 \) (differently from the MIT Bag Model where they are given solely in terms of the bag constant \( B \)). Now, let us particularize the above equations for two and three flavor quark matter.

**Two flavor**

For a gas of \( u \) and \( d \) quarks, charge neutrality (neglecting the not important contribution of electrons as in [9]) requires that \( n_d^{SLA} = 2 n_u^{SLA} \), from which it follows that \((\mu_d - V_1/2) = 2^{1/3}(\mu_u - V_1/2)\). The two-flavor vacuum energy density (for \( N_f = 2 \) in Eq. (25)) is \( \Delta|\varepsilon_{\text{vac}}|_{ud} = (29/192) G_2 \). Thus, the energy per baryon of the \( ud \) system is

\[ \left( \frac{E}{A} \right)_{ud} = (1 + 2^{4/3})^{3/4} (4\pi^2)^{1/4} (\Delta|\varepsilon_{\text{vac}}|_{ud})^{1/4} + \frac{3}{2} V_1 \]

\[ = 6.441 (\Delta|\varepsilon_{\text{vac}}|_{ud})^{1/4} + \frac{3}{2} V_1 \]

\[ = 4.016 G_2^{1/4} + \frac{3}{2} V_1 . \tag{36} \]

**Three flavor**

The three flavor quark system (SQM) is naturally charge neutral, with \( n_u^{SLA} = n_d^{SLA} = n_s^{SLA} \), \( \mu_u = \mu_d = \mu_s = \mu \), and \( n_e = \mu_e = 0 \). The vacuum energy density (for \( N_f = 3 \)) is \( \Delta|\varepsilon_{\text{vac}}| = (9/64) G_2 \), and the energy per baryon becomes (using the previous notation for the SQM system, without subscripts)

\[ \left( \frac{E}{A} \right) = 3^{3/4} (4\pi^2)^{1/4} (\Delta|\varepsilon_{\text{vac}}|)^{1/4} + \frac{3}{2} V_1 \]

\[ = 5.714 (\Delta|\varepsilon_{\text{vac}}|)^{1/4} + \frac{3}{2} V_1 \]

\[ = 3.499 G_2^{1/4} + \frac{3}{2} V_1 . \tag{37} \]
In Eqs. (36) and (37), for fixed $E/A$, the increase of $V_1(G_2)$ is compensated by the corresponding decrease of $G_2(V_1)$. As shown below in Fig. 1 (at $m_s = 0$ and a given $E/A$) the maximum value of $G_2$ is obtained for $V_1 = 0$.

The energy per baryon of $^{56}Fe$ is 930.4 MeV, so in this simple analysis the stability of SQM relative to iron corresponds to $G_2 < (0.266 - 0.428V_1)^4$. As a result, we obtain $G_2 < 0.005 \text{ GeV}^4$ for $V_1 = 0$, $G_2 < 0.002 \text{ GeV}^4$ for $V_1 = 0.1 \text{ GeV}$, and so on, until a very small value of $G_2 (\lesssim 10^{-5} \text{ GeV}^4)$ for $V_1 = 0.5 \text{ GeV}$. This behavior explains the shrinking of the stability window at $m_s = 0$ shown in panel (a) of Fig. 3.

Finally, from Eqs. (36) and (37) we obtain

$$\frac{(E/A)}{(E/A)_{ud}} = \frac{3^{3/4}(4\pi^2)^{1/4} \left( \Delta|\varepsilon_{vac}| \right)^{1/4} + 1.5 V_1}{(1 + 24/3)^{3/4}(4\pi^2)^{1/4} \left( \Delta|\varepsilon_{vac}|_{ud} \right)^{1/4} + 1.5 V_1} = \frac{5.714 \left( \Delta|\varepsilon_{vac}| \right)^{1/4} + 1.5 V_1}{6.441 \left( \Delta|\varepsilon_{vac}|_{ud} \right)^{1/4} + 1.5 V_1} = \frac{3.499 G_2^{1/4} + 1.5 V_1}{4.016 G_2^{1/4} + 1.5 V_1}.$$  

(38)

It is evident that $(E/A) < (E/A)_{ud}$ (for the same values of $G_2$ and $V_1$ in SQM and ud systems). From the last line of Eq. (38) it follows that $(E/A)/(E/A)_{ud} = 0.87$ for $V_1 = 0$. On the other hand, for the MIT Bag Model, with the correspondences $\Delta|\varepsilon_{vac}| = \Delta|\varepsilon_{vac}|_{ud} \equiv B$ and $V_1 = 0$, we obtain $(E/A)/(E/A)_{ud} = 0.89$ as in [9].

### III. RESULTS

We are concerned with the bulk properties of SQM and concentrate ourselves to investigate the stability with respect to the $^{56}Fe$ nucleus. In our investigation, $m_s$ enters as input parameter and $\Delta|\varepsilon_{vac}|$ (or $G_2$) is determined for fixed values of $E/A$ and $V_1$. By this way, as in [40], but with a different logic, we obtain a scenario for the model parameters, independently of what the results of lattice calculation may be. We discuss the relations between the parameter values required for the SQM stability and the values obtained by comparison with lattice predictions in [36, 37].

In Fig. 1 the constant $E/A$ contours give $m_s$ vs $\Delta|\varepsilon_{vac}|$ (for the purpose of comparison with MIT Bag Model results in [4, 8, 9]) for different values of $V_1$. In order to understand the role the gluon condensate, we use the relation between $\Delta|\varepsilon_{vac}|$ and $G_2$, given by Eq. (25), to plot $m_s$ vs $G_2$ for the same values of $V_1$. The contours are very sensitive to the values of $V_1$,
being shifted towards lower values of $\Delta|\varepsilon_{\text{vac}}|$ and/or $G_2$ as shown for $V_1 = 0$ (panels (a) and (b)) and $V_1 = 0.01$ GeV (panels (c) and (d)). To the right of the $E/A = 0.93$ GeV contour (in reality, 930.4 MeV corresponding to the energy per nucleon of $^{56}F_e$), SQM is unstable with respect to the iron nuclei. The vertical line at the left of each panel is the limit of $\Delta|\varepsilon_{\text{vac}}|$ and/or $G_2$ when $m_s$ becomes large, so the strangeness per baryon goes to zero (see panel (a) in Fig. 2). In this case, there is no distinction between strange and non-strange quark matter. Contours with $E/A < 930.4$ MeV terminate at the crossing with the vertical line of $^{56}F_e$. For $V_1 = 0$, the results shown in panel (a) are numerically equivalent to the ones found in the case of the MIT Bag Model. However, we remark that in the FCM the vacuum energy difference $\Delta|\varepsilon_{\text{vac}}|$ is essentially a nonperturbative quantity given in terms of the gluon condensate. Also shown is the $E/A = 0.939$ GeV contour corresponding to the nucleon mass.

Stability window of the SQM is the region of allowed values of $m_s$ and $\Delta|\varepsilon_{\text{vac}}|$ (or $G_2$) where the energy per particle is lower than the one of $^{56}F_e$ (bounded by the $E/A = 0.93$ GeV contour and the respective vertical line). In the FCM, the stability of SQM depends on the values of $V_1$ and/or $G_2$. For a given value of $E/A$, the higher $V_1$, the lower $G_2$ (cf. Eq.(37) for the case $m_s = 0$). Moreover, even for $V_1 = 0$ (for which the contours present the maximum $G_2$ at $m_s = 0$), the values of $G_2$ within the stability window are lower than $0.006 - 0.007$ GeV$^4$ obtained from lattice data on the critical temperature [36, 37]. The possibility of the SQM be more bound than $^{56}F_e$ is realized only for $G_2 < 0.005$ GeV$^4$. This has also been the case for $m_u = 5$ MeV, $m_d = 7$ MeV and $m_s = 150$ MeV for which we have shown that $G_2 < 0.0041$ GeV$^4$ for the existence of stable SQM in strange star surfaces [40]. Even if we take stability with respect to the nucleon mass ($E/A = 0.939$ GeV), instead of $^{56}F_e$, the values of $G_2$ remain lower than the one in [36, 37].

Fig. 2 shows for the given value of $E/A$ (the same as in [3]) the strangeness per baryon (defined as in [4]) in panel (a) as function of $m_s$ for some values $V_1$. The strangeness per baryon is always lower than unity, going to zero at $m_s \sim 0.3$ GeV (however, strangeness excess might be possible for nonzero $m_u$ and/or $m_d$ as shown in panel (a) of Fig. 5). For the same values of $V_1$, panel (b) shows the decrease of the baryon number density from its maximum at $m_s = 0$ until a constant value around $m_s \sim 0.3$ GeV. We must be aware that for larger values of $V_1$ and at some value of $m_s$, the baryon number density $n_A$ might becomes lower than a critical value (if it exists) at which the phase transition takes place.
However, the determination of such a critical value is not the scope of the present work.

We have also calculated the hadronic electric charge per baryon,

$$\frac{Z}{A} = \frac{2n_u - n_d - n_s}{n_u + n_d + n_s} = \pm \frac{n_e}{n_A}, \quad (39)$$

shown in panel (c). For $m_s = 0$ the equilibrium configuration is given by an equal number of $u$, $d$ and $s$ quarks ($n_u = n_d = n_s$ and $n_e = 0$) with zero electric charge. When $m_s$ and/or $V_1$ grow, the system develops a positive hadronic electric charge. For large $m_s$, the hadronic electric charge per baryon saturates at a constant value which also depends on $V_1$ (cf. panel (b) of Fig. 4). For $V_1 = 0$, this saturation point is $\sim 0.0056$ at $m_s \sim 0.3$ GeV as in [4].

Given that all $E/A < 0.93$ GeV contours are within the stability window, it is instructive to consider some features of SQM at $E/A = 0.93$ GeV of $^{56}Fe$. In panel (a) of Fig. 3 we show that the overall effect of the confining forces is to shift the stability windows towards lower values of $G_2$ for increasing values of $V_1$. The windows not only narrow as $m_s$ grows at a fixed $V_1$, but they also narrow as $V_1$ grows at the same value of $m_s$. In particular, for $V_1 = 0.5$ GeV the vertical line is located at a negligible value of $G_2$ and the stability window width is very small (not visible in the scale of the figure). From the locations (at fixed values of $V_1$) of the constant $E/A$ contours and the respective vertical lines on the horizontal axis at $m_s = 0$, where each window presents its maximum width, we construct two plots $V_1$ vs $G_2$ as shown in panel (b). The region between the dashed and solid curves illustrates the decrease of the stability window width with $V_1$. In panel (c), we show the baryon number density for $0 \leq V_1 \leq 0.5$ GeV. For $V_1 = 0.5$ GeV, it is nearly zero. So, as we have observed above, due to the decreasing of $n_A$ it might happens that a quark-hadron phase transition occurs at some value of $V_1$ and $m_s$.

Fig. 4 shows the strangeness per baryon (in panel (a)) and the hadronic electric charge per baryon (in panel (b)), at the energy per baryon of $^{56}Fe$, as function of $m_s$ for $V_1 = 0$ and $V_1 = 0.5$ GeV. For all values of $V_1$ in the region $0 \leq V_1 \leq 0.5$ GeV and $m_s$ in the region $0 \leq m_s \lesssim 0.35$ GeV, the strangeness per baryon is always less than unity. Depending on the values of $V_1$, the saturation of the hadronic electric charge per baryon is between 0.0056 for $V_1 = 0$ and $\sim 0.006$ for $V_1 = 0.5$ GeV (this variation is not visible in panel (c) of Fig. 2 because of the low values of $V_1$).

Let us now consider the question of the strangeness excess. We have performed our calculation assuming that $m_u = m_d = 0$. However, for nonzero $m_u$ and/or $m_d$, strangeness
excess can be obtained which is more sensitive to \( m_d \) than it is to \( m_u \). For the usual values of \( u \) and \( d \) quark masses, \( m_u = 5 \text{ MeV} \) and \( m_d = 7 \text{ MeV} \), the strangeness excess is less than 1\%, but it can be larger for larger \( m_u \) and \( m_d \). As an illustrative example, we have (speculatively) extrapolated \( m_u \) and \( m_d \) beyond its usual values in order to obtain a strangeness excess around 9-14 \% as shown in panel (a) of Fig. 5 for the given quark masses, \( V_1 \) and \( E/A \). Generally speaking, this excess only occurs for low values of \( m_s (\lesssim 0.02 \text{ GeV}) \) and large values of \( V_1 \) which, in turn, correspond to very small values of \( G_2 (\sim 10^{-6} \text{ GeV}^4) \) (the increase of \( u \) and \( d \) quark masses also shifts the \( E/A \) contours towards lower values of \( G_2 \)).

Correspondingly, we also speculate the possibility of negative hadronic electric charge. The change of the hadronic electric charge is more sensitive to \( m_u \) and \( m_s \) than it is to \( m_d \). For large values of \( V_1 \), it can happen that strange quarks are more abundant than the massless \( u \) and \( d \) quarks in the region of small values of \( m_s \). So, negative hadronic electric charge appears to be allowed for large values of \( V_1 \), \( m_u \) and \( m_d \) (as for the case of strangeness excess), but for small \( m_s \), as shown in panel (b). In this case, instead of electrons, a sea of positrons neutralizes the negative hadronic electric charge. We also remark that, at the same values of \( V_1 \), \( m_u \) and \( m_d \), larger values of \( S/A \) and lower (negative) \( Z/A \) are allowed for \( E/A < 0.899 \text{ GeV} \) and \( m_s \) in the region \( 0 \leq m_s \lesssim 0.02 \text{ GeV} \).

Summarizing the above results, strangeness excess and negative hadronic electric charge per baryon are realized only for large values of \( m_u \) and/or \( m_d \) and \( V_1 \) (say, \( V_1 \gtrsim 0.3 \text{ GeV} \)), but for values of \( G_2 \) much lower than the one in \([36, 37]\). For \( m_u = m_d = 0 \) as well as for the usual \( m_u = 5 \text{ MeV} \) and \( m_d = 7 \text{ MeV} \), we observe that the values of the model parameters obtained in the present paper do not favor the existence of neither nonnegative nor negative SQM with energy per baryon lower than the one of the \( ^{56}\text{Fe} \) nucleus.

IV. FINAL REMARKS AND CONCLUSIONS

In this work we have investigated the bulk properties of SQM by using the quark-gluon plasma EOS derived in the FCM nonperturbative approach \([34]\). The important parameters of the model are the gluon condensate \( G_2 \) (which enters the EOS through the vacuum energy difference \( \Delta |\epsilon_{\text{vac}}| \) between confined and deconfined phases) and the large distance q\bar{q} interaction potential \( V_1 \). The results have shown that confinement plays an important role
for the stability of SQM.

We have performed the calculation in the \(m_u = m_d = 0\) approximation and assumed SQM in \(\beta\)-equilibrium and charge neutrality, at zero temperature and pressure. We have been mainly concerned with the absolute stability of SQM with respect to \(^{56}Fe\) nucleus. In order to look for stability windows of SQM, we have started our investigation by drawing contours of constant energy per baryon, \(E/A\), in the \(m_s\) vs \(\Delta|\varepsilon_{\text{vac}}|\) (and/or \(m_s\) vs \(G_2\)) plane for fixed values of \(V_1\). Strangeness and hadronic electric charge have also been considered. Our study revealed remarkable features which we summarize as follows.

The general trend is that the SQM stability is very sensitive to the values of the model parameters responsible by the confining forces. A remarkable aspect is that the behavior of the stability window strongly depends on the values of \(V_1\). For increasing values of this parameter, the constant \(E/A\) contours and also the respective stability windows as a whole are shifted towards lower and lower values of \(\Delta|\varepsilon_{\text{vac}}|\) (and/or \(G_2\)) until to \(\sim 0\) at \(V_1 = 0.5\) GeV. Moreover, the width of the stability windows diminish when \(V_1\) becomes larger. At \(m_s = 0\), it has the maximum width between \(G_2 = 0.003\) GeV\(^4\) and \(G_2 = 0.005\) GeV\(^4\) for \(V_1 = 0\) and a nearly zero width at \(G_2 \sim 0\) for \(V_1 = 0.5\) GeV. This amounts to say that absolutely stable SQM would exists, in principle, for \(0 \leq V_1 \leq 0.5\) GeV (although somewhat problematic at \(V_1 = 0.5\) GeV due to the smallness of the corresponding value of \(G_2\)). However, a striking point is that the values of \(G_2\) are lower than the one in the range 0.006 – 0.007 GeV\(^4\) obtained from comparison with the lattice data at the critical temperature [36, 37]. This point puts a severe restriction for the existence of absolutely stable SQM with respect to \(^{56}Fe\).

We have also calculated the strangeness per baryon and the hadronic electric charge per baryon. As \(m_s\) grows, the strangeness per baryon decreases from \(S/A = 1\) at \(m_s = 0\) to \(S/A = 0\) for some value of \(m_s\) which depends on \(E/A\) and \(V_1\). Correspondingly, the hadronic electric charge per baryon is always nonnegative, rising from \(Z/A = 0\) up to a constant value which depends on the value of \(V_1\). Another remarkable feature (in the \(m_u = m_d = 0\) approximation) is that \(S/A \leq 1\) and \(Z/A \geq 0\) for all values of \(m_s\) within the stability window.

In the attempt to find strangeness excess and negative hadronic electric charge, we have observed that \(S/A > 1\) and \(Z/A < 0\) appear to be allowed only for very large values of \(m_u\) and/or \(m_d\) (beyond the usual ones), but for small \(m_s\) (\(\lesssim 0.02\) GeV) and large \(V_1\) (say,
between 0.3 GeV and 0.5 GeV). However, the corresponding values of $G_2$ remain lower than the one in $\text{[36, 37]}$, as in the case of $m_u = m_d = 0$.

From the above, in the context of the FCM approach, it appears that the values of the model parameters obtained in our investigation do not favor the existence of absolutely stable SQM. Of course the above results depend on the constant $V_1$ assumption, in the present work.

Taking into account the importance of experiments at RHIC and LHC, it is instructive at this point to consider finite temperature effects on the SQM properties. In order to check the influence of nonzero temperatures on the SQM stability at zero pressure, we have applied the same procedure employed for the $T = 0$ case to study the behavior of the stability window shown in Fig. $\text{[31]}$ but for $T \neq 0$. We have taken several values of $T$ up to 30 MeV for constant $V_1$ and for $V_1(T)$ parametrized in $\text{[37]}$ for $T \geq T_c$ as

$$V_1(T) = \frac{0.175 \text{ GeV}}{1.35 \frac{T}{T_c} - 1}, \quad V_1(T_c) = 0.5 \text{ GeV},$$

for $T = T_c, 2T_c, 3T_c$ and some arbitrary values of $T_c$ along the phase diagram transition curve. In both cases, the results are qualitatively analogous to those in Fig. 1 of $\text{[8]}$ and Fig. 3 of $\text{[29]}$. However, $V_1(T)$ is decreasing with the growth of $T$, so the shift of the stability window towards lower values of $G_2$ takes place as $T \to T_c$. For $V_1(T = T_c)$ the result is the same as for constant $V_1 = 0.5 \text{ GeV}$ at $T = 0$ shown in panel (a) of Fig. $\text{[31]}$.

Generally speaking, our results appear to be consistent with the fact that absolutely stable SQM has not been observed up to the present. The experiment with STAR at RHIC has not confirmed the existence of SQM nor proved that it does not exist $\text{[48–50]}$. The low values of $G_2$ with respect to the ones in $\text{[36, 37]}$ would provide a possible explanation for the absence of absolutely stable SQM signature. However, before any conclusion towards the nonexistence of absolutely stable SQM, we must have in mind that our theoretical results should be a consequence of the approximations contained in the development of the FCM nonperturbative EOS. FCM is a robust theoretical approach where the dynamics of confinement is one of the most important aspects of the model. Therefore, we must be aware that the FCM nonperturbative EOS is presently developed in the so called Single Line Approximation, where the confinement dynamics include only single quarks and gluons interactions with the vacuum $\text{[34]}$.

On the other hand, in our calculations, $V_1$ and $G_2$ were taken as $\mu$-independent pa-
rameters. As pointed out in [45, 46], the $\mu$-independence of $V_1$ should be a questionable assumption. Also, in the large density domain, important effects should be related to a possible density dependence of $G_2$ [51]. In our opinion, these aspects are very interesting possibilities to be considered. However, this does not have been the scope of the present work.

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FIGURE CAPTION

**Fig. 1** - Contours of constant $E/A$ of strange quark matter at zero temperature and pressure for two different choices of the parameter $V_1$ ($E/A$ and $V_1$ in GeV units). The vertical dashed line is where the energy per baryon number of the two-flavor quark matter exceeds the one of $^{56}Fe$. Panels (a) and (c): strange quark mass as function of $\Delta\varepsilon_{\text{vac}}$. Panels (b) and (d): strange quark mass as function of $G_2$. The contours labeled (N) stand for the nucleon mass. The 0.93 contours correspond to 930.4 MeV energy per nucleon of $^{56}Fe$.

**Fig. 2** - Panel (a): The strangeness per baryon as function of the strange-quark mass for different choices of $V_1$. Panel (b): As in panel (a), but for the baryon number density. Panel (c): As in panel (b), but for the hadronic electric charge per baryon.

**Fig. 3** - Panel (a): Stability windows of SQM at zero temperature and pressure, bounded by the $E/A = 0.93$ GeV contour of $^{56}Fe$ and its vertical line, for different choices of $V_1$ (in GeV units) labeling each contour. Panel (b): for each value of $V_1$ in panel (a), the respective locations of the $E/A$ contour (solid) and its vertical line (dashed) on the horizontal axis. Panel (c): Baryon number densities at the energy per baryon of $^{56}Fe$ as function of the strange quark mass for different values of $V_1$. For $V_1 = 0.09$ GeV and $V_1 = 0.167$ GeV, we have $n_A = n_0$ at $m_s \sim 0.3$ GeV and $m_s = 0$, respectively, where $n_0 = 0.153$ fm$^{-3}$ is the nuclear saturation number density.

**Fig. 4** - Panel (a): Strangeness per baryon as function of the strange-quark mass for the given values of $V_1$ (in GeV units), all at $E/A = 0.9304$ GeV of $^{56}Fe$. Panel (b): As in panel (a), but for the hadronic electric charge per baryon.

**Fig. 5** - Panel (a): Strangeness per baryon as function of the strange-quark mass for the given nonzero u and d quark masses, $V_1$ and $E/A$ (all in GeV units). The energy per baryon $E/A = 0.899$ GeV was taken for the purpose of comparison with the results in [4]. Panel (b): As in panel (a), but for the hadronic electric charge per baryon.
FIG. 1:
FIG. 2:
FIG. 3:
FIG. 4:
FIG. 5: