Switching the sign of photon induced exchange interactions in semiconductor microcavities with finite quality factors

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We investigate coupling of localized spins in a semiconductor quantum dot embedded in a microcavity with a finite quality factor. The lowest cavity mode and the quantum dot exciton are coupled forming a polariton, whereas excitons interact with localized spins via exchange. The finite quality of the cavity $Q$ is incorporated in the model Hamiltonian by adding an imaginary part to the photon frequency. The Hamiltonian, which treats photons, spins and excitons quantum mechanically, is solved exactly. Results for a single polariton clearly demonstrate the existence of a resonance, sharper as the temperature decreases, that shows up as an abrupt change between ferromagnetic and antiferromagnetic indirect anisotropic exchange interaction between localized spins. The origin of this spin-switching finite-quality-factor effect is discussed in detail remarking on its dependence on model parameters, i.e., light-matter coupling, exchange interaction between impurities, detuning and quality factor. For parameters corresponding to the case of a (Cd,Mn)Te quantum dot, the resonance shows up for $Q \approx 70$ and detuning around 10 meV. In addition, we show that, for such a quantum dot, and the best cavities actually available (quality factors better than 200) the exchange interaction is scarcely affected.

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I. INTRODUCTION

Artificial control of direct exchange interactions, which occur at length scales of one lattice spacing, is hardly possible with current day technologies. In contrast, there is a number of proposals to control Indirect Exchange Interactions (IEI) of spins sitting several nanometers away, that take advantage of the optical and electrical manipulation of the intermediate fermions afforded in semiconducting hosts. Local spins could be provided by nuclei, by electrons bound to donors, or $d$ electrons of magnetic impurities. A variety of phenomena, like the reversible modification of the Curie temperature and coercive fields in (III,Mn)VI and (II,Mn,N)VI semiconductors, the induction magnetic order in otherwise paramagnetic (II,Mn)VI semiconductor quantum dots and the entanglement of donor spins in (II,Mn)VI quantum wells have been observed experimentally thanks to the artificial manipulation of IEI. Such a control is also required in the implementation of quantum computation using localized spins in solids, since two qubit operations require exchange interactions between distant spin pairs.

Laser induced IEI in semiconductor dots with magnetic impurities can be tuned by changing the laser frequency, intensity and polarization. For frequencies below the dot gap, an optical coherence between valence and conduction bands is induced, capable of mediating exchange interaction between localized spins. This is commonly known as an Optical RKKY (ORKKY) exchange interaction. Above threshold (frequencies higher than the dot gap) real carriers (electron-hole pairs) are responsible for the resulting RKKY-like exchange interaction. An interesting effect has recently been observed in a system with two localized spins interacting with one itinerant exciton. By solving exactly this simple case, the authors found that, as the laser energy approaches a resonance related to excitons bound to impurities, the induced coupling between spins increases and may switch from ferromagnetic to antiferromagnetic. On the other hand, the hybridization of the localized and itinerant electrons has been recently introduced concluding that it may produce, for certain dot geometries, a change of sign in the spin exchange interaction. Both works point to the same direction: the possibility of controlling the switching of the exchange interaction to and fro between ferromagnetic (F) to antiferromagnetic (AF).

Motivated by late experimental results, we have recently proposed a system where optical exchange interaction is greatly enhanced: a semiconductor micropillar cylindric cavity, made of CdTe with inclusions of (Cd,Mn)Te quantum dots. Fine tuning of the cavity modes has been recently achieved, for instance, by using length tunable microcavities or photonic crystal membrane nanocavities. Detuning can also be varied by making use of the temperature dependence of the exciton transition. The Mn spins are exchange-coupled both to electrons and holes confined in the quantum dot. In turn, quantum dot electron hole pairs (excitons) are coupled to the confined
Pillar microcavities based on CdMnTe/CdMgTe heterostructures have been fabricated featuring strong coupling between 2D excitons with 0D photons with a Rabi energy as high as 16 meV, which can hardly be obtained with cw lasers. The strong coupling regime between InGaAs quantum dot excitons and 0D photons has also been achieved, with a Rabi energy of approximately 0.1 meV. This model system, where both photons and fermions have a zero dimensional density of states, permits the exact diagonalization of the Hamiltonian, considering all degrees of freedom fully quantum mechanically. It turns out that confinement of both the light and the intermediate fermions yields an enhancement of the ORKKY interaction that results to be strongly anisotropic.

For photon frequencies below threshold, and at sufficiently low temperatures, strong ferromagnetic coupling shows up without a significant increase in exciton density. In addition we found that the interaction mediated by photon-polaritons is ten times stronger than the one induced by a classical field for equal Rabi splitting.

The present work is addressed to investigate the consequences of having a real cavity with a finite quality factor. Calculations for the case of a (Cd,Mn)Te quantum dot and realistic values of the quality factor ($Q > 200$, see Ref. [17]) give results very close to those reported in our previous work [15]. However, at smaller $Q$, a sharp resonance shows up that is manifested by an abrupt change of the exchange interaction between impurities from ferromagnetic to antiferromagnetic and back to ferromagnetic. The resonance becomes sharper as the temperature decreases and may appear at considerably large positive values of detuning. The switching in sign of the spin-spin interaction is somewhat similar to that reported in Refs. [13,14].

II. MODEL HAMILTONIAN

A. Hamiltonian

The dot that confines conduction and valence band electrons has intra-band level spacing larger than all the other inter-band energy scales, so that we only keep the lowest orbital level in each band. These levels have a twofold spin degeneracy. The electric field of the lowest cavity mode lies in the plane perpendicular to the axis of the cylinder. Thus, there are two degenerate cavity modes, associated to the two possible polarization states in that plane. Their energy $\hbar \omega$ is supposed to be close to the quantum dot band gap $E_g$. $\hbar \Delta \omega$ is the inverse of the photon lifetime and is a characteristic of the microcavity. The microcavity quality factor is commonly defined as

$$Q = \omega/\Delta \omega.$$  

The light-matter coupling Hamiltonian is:

$$\mathcal{H}_g = \hbar g \sum_{\lambda,\sigma} \left( b^\dagger_\lambda b_\lambda \right) \left[ c^\dagger_\sigma v^\dagger_\sigma + v^\dagger_\sigma c_\sigma \right] \delta_{\lambda,\lambda},$$  

where we have assumed that there is purely heavy holes. This assumption leads to the standard spin selective coupling that associates photon polarization and fermion spin degrees of freedom. This kind of coupling breaks spin rotational invariance and privileges the axis of the cavity, $\hat{z}$.

The exchange interaction between the fermions and the spin $M = 5/2$ of the Mn impurities (results for spin 1/2 impurities will also be presented) reads:

$$\mathcal{H}_J = \sum_{I,J} J_{IJ} \vec{M}_I \cdot \vec{S}_J (\vec{x}_I)$$  

where $\vec{S}_f(\vec{r}_I)$ stands for local spin density of the \( f = v, c \) electron and $\vec{M}_I$, is the Mn spin located at $\vec{r}_I$. \((I = 1, 2, \) two impurities will be hereafter considered\). The electron spin density is,

$$\vec{S}_f(\vec{r}_I) = \frac{1}{2}|\psi_f(\vec{r}_I)|^2 c^\dagger_{\sigma} \vec{c} \tau_{\sigma, \sigma'} c_{\sigma'} , \quad (6)$$

where $\vec{c}$ are the Pauli matrices. The strength of the interaction depends both on the exchange constant of the material $J_f$ and on the localization degree of the carrier, $|\psi_f(\vec{r}_I)|^2$.

The eigenstates of this Hamiltonian can be classified using the total matter spin $\vec{\Sigma}_T = \vec{S}_v + \vec{S}_c + \vec{M}_1 + \vec{M}_2$, and its $z$-component $\Sigma_z^T = M_z^1 + M_z^2 + S_z^c + S_z^v$.

We define a spin-spin correlation:

$$\langle \vec{M}_1 \cdot \vec{M}_2 \rangle = \frac{1}{Z} \sum_i \langle \Phi_i | \vec{M}_1 \cdot \vec{M}_2 | \Phi_i \rangle e^{-E_i / k_B T} \quad (7)$$

where $Z$ is the partition function and the sum runs over the eigenstates $\Psi_i$ of the Hamiltonian having a real part of the energy $E_i$.

**B. Model parameters**

The value of the light-matter interaction $g$ depends on the amplitude of the cavity mode in the dot and plays the same role than the Rabi energy $\Omega$ in the case of a photoexcited semiconductor. We take $g = 5 \text{ meV}$ which is within the range of Rabi splittings reported in the literature for CdTe nanopillars (larger values have been reported experimentally). A key quantity of the model is the detuning $\delta = E_g - \hbar \omega$. As this can be varied experimentally, it will be one of the main variables in our analysis.

We consider a hard wall quantum dot, with lateral dimensions $L \approx 10 \text{ nm}$ and total volume $\approx 1200 \text{ nm}^3$. In such a dot, a realistic value for the exchange interaction between a conduction (valence) band electron and a Mn spin is $J_{c}^{\text{max}} = -0.1 \text{ meV}$ ($J_{v}^{\text{max}} = -0.5 \text{ meV}$). The band gap in (Cd,Mn)Te is the largest energy scale (we take $E_g = 2 \text{ eV}$). Therefore the effect of the terms that do not conserve the number of excitons plus photons is negligible and they can safely be removed. This permits to work in subspaces with $N$ excitations. Here we consider the coupling between two Mn impurities, in presence of $N = 1$ polaritons. For $N = 1$ excitation the ground state manifold is mainly photonic for $\delta >> 0$, mainly excitonic for $\delta << 0$ and it is a compensated mixture around $\delta = 0$.

Aiming to attain a full understanding of the dependence of the results on the model parameters, the realistic values for the exchange interaction and that chosen for the light-matter coupling $g$, were not always used in the calculations discussed hereafter.
III. RESULTS

In the following, we present results obtained by varying either $\Delta \omega$ (and, thus, the quality factor) while keeping constant the photon frequency (subsection A) or the detuning $\delta$ (subsection B). The latter will be varied by changing the photon frequency, the most feasible way in present experimental set ups. Note that changing detuning over a narrow range around threshold ($\delta = 0$) will also slightly modify the quality factor. Identical results are obtained if, instead, the dot gap is varied, a procedure that keeps constant $Q$.

A. Switching the sign of the exchange interaction by varying the photon lifetime

Figs. 1 and 2 depict results for total $\langle M_1 \cdot M_2 \rangle$, out-of plane $\langle M^z_1 M^z_2 \rangle$ and in-plane $\langle M^p_1 M^p_2 \rangle = \langle M^x_1 M^x_2 + M^y_1 M^y_2 \rangle$ spin-spin correlations versus the inverse of the quality factor $Q$ for spin 1/2 and 5/2, respectively, and a temperature of $T = 0.001$ K (the rest of the model parameters are given in the captions). At large quality factors the spin-spin correlation is, in both cases, maximum and fully out-of-plane. In addition, for quality factors larger than 100 no differences are noted with respect to the case of an ideal cavity. Abrupt changes in the character of the effective exchange interaction between impurities are noticed for $Q^{-1} \approx 0.013$ for both spin 1/2 and 5/2 impurities. The spin-spin correlation abruptly changes from ferromagnetic to antiferromagnetic and back to ferromagnetic over a narrow range of $Q$. Ferromagnetic correlations occurring at small $Q$ are fully in-plane. At the very low temperature as shown in Fig. 1, the spin-spin correlation reaches always its maximum (ferro or antiferro) values. The range of quality factor over which this switching of the sign of the exchange interaction occurs, is narrower for spin 1/2 impurities. Results for a higher temperature $T = 0.1$ K for spin 5/2 impurities are shown in Fig. 3. Now, the ideal cavity (infinite quality factor) limit is reached only at quality factors larger than 2000. In addition, the abrupt switching observed at the lower temperature is appreciably smeared. Moreover, although the maximum antiferromagnetic correlation occurs at roughly the same quality factor, it is sharply weakened. Altogether, the spin correlation decreases steadily with the inverse of the quality factor. This decrease is in fact what one should expect at any temperature. However at very low temperatures the spin correlation remains constant up to rather low values of $Q$ for the reasons discussed hereafter.

In order to understand the origin of the behavior of spin-spin correlations, and in particular of the switching mentioned above, we proceed to analyze how the ground and excited states evolve, in the simple case of spin 1/2 impurities, as the quality factor is decreased. Let us denote $|\lambda; \uparrow \downarrow, 0; M^z_1, M^z_2 \rangle$ and $|0; S^z_v, S^z_c; M^z_1, M^z_2 \rangle$ photonic and excitonic configurations with $z$-components of the two spin impurities $M^z_1$ and $M^z_2$ and either one photon of polarization $\lambda$ (we denote right polarization by $\uparrow$ and left polarization by $\downarrow$) and two electrons in the valence band or no photon and two electrons one in the valence band and other in the conduction band of $z$-component of the spin $S^z_v$ and $S^z_c$, respectively. The spin-selective coupling considered here requires that $S^z_c = \lambda$. At infinite quality factor and in the absence of exchange coupling, the ground state is eight-fold degenerate. Exchange coupling lifts this degeneracy as follows. Two of these states, both doubly degenerate, give ferromagnetic correlation among impurities,
are $\Sigma$

Both states may involve other basis functions albeit with a very small weight. The quantum numbers of these states are $\Sigma_T^{1}=-1$ and $\Sigma_T=1$ for $|\psi_{-1}\rangle$ and $\Sigma_T^{1}=1$ and $\Sigma_T=1$ for $|\psi_{1}\rangle$. They are degenerate with the states corresponding to a linear combination of left polarization $\lambda=\downarrow$ and spin down electron in the conduction band.

The third state is also doubly degenerated and give antiferromagnetic correlations,

$$|\psi_{0}\rangle = a_0 |\uparrow \downarrow, 0; \uparrow \uparrow > + b_0 |\downarrow \uparrow, 0; \downarrow \downarrow >.$$

This state has total spin $\Sigma_{T}=0$. Therefore, left and right photon polarizations occur with equal weight. Symmetric and anti-symmetric combinations of basis functions with opposite photon polarizations are degenerate. The remaining two states are not degenerate and have energies very close to that of $|\psi_{0}\rangle$. In these states the total spin is $\Sigma_{T}\neq 0$, but $\Sigma_{T}=0$. Left and right polarizations are equally mixed, but symmetric and anti-symmetric combinations of the photon polarizations are no longer degenerate, because the state with photon polarization antiparallel to the total spin $\Sigma_{T}$ has lower energy than that having photon polarization parallel to the total spin (see below).

The way the eightfold degeneracy is lifted when the exchange Hamiltonian $H_{J}$ is switched on, is better understood by rewriting $H_{J}$ as,

$$H_{J} = J_{c}(\vec{M}_{1} + \vec{M}_{2}) \cdot (\vec{S}_{e} + \vec{S}_{c}) + (J_{e} - J_{c})(\vec{M}_{1} + \vec{M}_{2}) \cdot \vec{S}_{c}.$$ 

It is clear that this Hamiltonian does not change the energy of states that only involve basis functions in which the two impurities have opposite spins (as is the case of the state of Eq. (9)). Instead the energies of the first two states are modified in first order. Degeneracy is lifted as diagonal elements from the second term in $H_{J}$ give non-zero contributions. If $J_{e} - J_{c} > 0$ (as is the case of the realistic model parameters given above), the two states in which the two impurities have parallel spins antiferromagnetically (ferromagnetically) coupled with the photon and the conduction band electron, reduce (increase) their energy (see Fig. 2a right). Upon switching on the exchange interaction the three states of Eqs. (8) and (9) keep their spin-related twofold degeneracy. This is most clearly seen by writing a secular equation which is valid for the three wavefunctions,

$$[E + \delta - i\Delta \omega] [E - k(J_{c} - J_{e})] - g^2 = 0$$

where the energy $E$ is referred to the dot band gap $E_{g}$, and $k$ runs over the subindexes of the three wave functions, namely, $k = -1, 0, 1$. When exchange coupling is zero the three wavefunctions are degenerate, while degeneracy is lifted when it is switched on. Note that this equation approximately gives the energy of the eigenstates in Eqs. (8) and (9) and those of three additional eigenstates that lie at higher energies.

Fig. 4 shows the real part of the energy of these states versus the inverse of the quality factor $Q^{-1}$ for two sets of values of the light-matter and exchange couplings. Two state crossings are clearly visible (see insets in Fig. 4). At low $Q^{-1}$, the ground state is $|\psi_{-1}\rangle$, whereas for large $Q^{-1}$, state $|\psi_{1}\rangle$ is the one with the lowest energy. Around the value at which maximum antiferromagnetic correlation shows up, the ground state is $|\psi_{0}\rangle$. The question now is why these crossings occur. Adding an imaginary part to the photon frequency increases the energy of all states involving photonic configurations. However, the actual increase depends on how close to the photon energy these states lie. This offers a qualitative explanation of the crossings of Fig. 4.

As regards the quantitative results shown in Fig. 4 the following features are worth commenting. Increasing the exchange coupling enlarges the range of quality factors over which level crossings occur, although the first crossing shows up at roughly the same $Q$. In addition, energy differences increase suggesting a weaker dependence on temperature. Increasing light-matter coupling keeping constant the exchange coupling, does also enlarge energy differences and decreases the $Q$ at which crossings occur. The latter is due to the fact that, as decreasing the quality factor decreases the photon density of states, a larger light-matter coupling is required to produce the same effect. This is an interesting effect which suggests that in order to produce the switching in sign of the exchange interaction in cavities with a sufficiently long photon lifetime, light-matter coupling should be decreased. On the other hand, effects of couplings on energy differences can be trivially understood in terms of the secular equation given above.

An additional feature of the results shown in Fig. 4 is important to comment. For large quality factors the state in which the spin of the impurities is antiparallel to the photon polarization has the lowest energy. However, once
crossings have occurred, the state having the spin impurities parallel to the photon polarization becomes the ground state. This interesting effect opens the possibility of switching the photon polarization as follows. As remarked above the three states are twofold degenerate. This degeneracy can be removed by applying a weak magnetic field. Then, as the magnetic field forces the spin of the impurities to lie along the field, the above crossing will imply an inversion of photon polarization. Note that, if the magnetic field is sufficiently weak, level crossings will not be eliminated. The field, however, cannot be too low, otherwise a very low temperature will impede this effect to show up. The effect can be most easily produced by changing detuning (see below).

B. Varying the photon frequency

In the previous section we have shown how changing the cavity quality factor induces a switching in the sign of the exchange interaction. However, the quality factor is not a parameter that can be easily varied. In this section we show that sign switching can also be triggered by varying detuning. Detuning will be varied by changing the photon frequency. Figs. 5 and 6 show spin-spin correlations and exciton and photon occupation numbers, versus detuning $\delta$, for two values of the quality factor $Q \sim 100$ and $Q \sim 200$, respectively. Note that as $\delta$ is usually varied over a narrow range around $\delta = 0$, the quality factor $Q$ changes only slightly ($\omega$ remains always close to $E_g$). In order to get the switching of the exchange interaction at a sufficiently large quality factor, we take $g = 2.5\text{ meV}$. As such low light-matter couplings excessively reduce the energy differences between the states that matter (see above), we also choose the largest values of the exchange couplings used to obtain the results presented in Fig. 3, namely, $J_e = -0.2\text{ meV}$ and $J_c = -1\text{ meV}$.

As detuning is varied, we go continuously from a pure photonic state ($\delta > 0$) to a pure excitonic state ($\delta < 0$). For a cavity with an infinite quality factor, the exciton and photon occupation numbers vary smoothly with $\delta$. However, the crossover from a photonic to an excitonic state occurs at $\delta = 0$. Finite quality factors modify this behaviour because the light-matter interaction is weakened, shifting also that crossover to positive detuning (see Figs. 5b and 6b). For $Q \sim 100$ (Fig. 5), the crossover occurs over a very narrow window of positive detuning values. For a larger $Q \sim 200$ (see Fig. 6) the effective light-matter coupling is raised up and therefore the transition from the photonic to the excitonic state is less abrupt and occurs at a lower detuning. Note that, on the pure photonic side, correlations (either ferromagnetic or antiferromagnetic) are large, with a negligible exciton population, a fact already observed in our previous work. The latter effect is here enhanced due to the effective reduction of the electron-photon interaction produced by a finite quality factor.

As discussed in the preceding subsections, at high quality factors the AF-F transition is shifted to lower $\delta$. Thus, the $\delta$ value at which the ground state switches from photon–like to exciton–like may be approached. When that switching occurs, the AF peak disappears abruptly (see Fig. 6), revealing the fundamental role played by the photon in inducing correlation between impurity spins. Another important point is that, at large quality factors, the effect of temperature on the AF peak is weakened. This can be understood by remembering that the energy differences between the three states of Eqs. (8) and (9) are proportional to the effective exchange interaction, which is raised up as detuning decreases. As shown in Fig. 6, increasing the temperature up to $T = 0.1\text{ K}$ scarcely affects the AF-F transition; moreover the maximum AF spin-spin correlation is still reached. The width of the frequency window in which the AF correlations rise up can be enlarged by changing the couplings $J_e$ and $J_c$. For $Q = 200$, with $J_e = -0.085\text{ meV}$ and $J_c = 0.1\text{ meV}$ a window as large as 5 mev is obtained. Finally note that, the effect highlighted above, related to a possible change of photon polarization induced by a weak magnetic field, may also appear when detuning is varied. This is definitively proved by the numerical results shown in Fig. 7. For large positive detuning the correlation between the spin of the impurities and the photon polarization is negative. This correlation function changes smoothly over the detuning range where the F-AF-F transition occurs, going through zero as expected for two fully AF correlated impurities. Beyond the transition, the spin-photon polarization correlation becomes positive.

IV. CONCLUDING REMARKS

Summarizing, we have studied the indirect exchange interaction between two spins in a cavity-dot system with one polariton, assuming that the cavity has a finite quality factor. For the best cavities nowadays available (quality factors better than 200), we have shown that our previous results on ideal cavities (infinite cavity factor) remain valid. However, if the quality factor is decreased a switching of the exchange interaction to and from between ferromagnetic and antiferromagnetic may be the case. We were able to demonstrate that this effect occurs for realistic values of the model parameters and does not require excessively low temperatures. The effect can be experimentally proved by varying detuning, a parameter that can be reliably varied in several experimental configurations available nowadays. In addition we have shown that this switching is related to level crossings in such a way that the ground state
changes from one in which the impurity spins are AF correlated to the conduction band electron, to one in which this correlation is just the opposite. We have suggested that this may induce inversion of the photon polarization with the help of a small magnetic field. Although more work is of course required to fully understand this system, we believe that the results presented here clearly illustrate the possibility of switching the sign of the exchange interaction, an effect that has recently attracted a considerable interest.\textsuperscript{13,14}

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1. D.D. Awschalom, D. Loss, and N. Samarth (editors), Semiconductor Spintronics and Quantum Computation (Springer, New York, 2002).
2. D. Loss and D. P. DiVincenzo, Phys. Rev. A \textbf{57}, 120 (1998).
3. A. Imamoglu, D. D. Awschalom, G. Burkard, D. P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, Phys. Rev. Lett. \textbf{83}, 4204 (1999).
4. C. Piermarocchi, P. Chen, L.J. Sham, and D.G. Steel, Phys. Rev. Lett. \textbf{89}, 167402 (2002).
5. J.M. Bao, A.V. Bragas, J.K. Furdyna, and R. Merlin, Nature Mater. \textbf{2}, 175 (2003); J. Bao \textit{et al.}, cond-mat/0406672.
6. T. Calarco, A. Datta, P. Fedichev, E. Pazy, and P. Zoller, Phys. Rev. A\textbf{68}, 012310 (2003).
7. J. Fernández-Rossier, C. Piermarocchi, P. Chen, A.H. MacDonald, and L.J. Sham, Phys. Rev. Lett. \textbf{93}, 127201 (2004).
8. H. Ohno, D. Chiba, F. Matsukura, T. Omiya, E. Abe, T. Dietl, Y. Ohno, and K. Ohtani, Nature, \textbf{408}, 944 (2000). D. Chiba, M. Yamanouchi, F. Matsukura, and H. Ohno, Science, \textbf{301}, 943 (2003).
9. S. Mackowski, T. Gurun, T.A. Nguyen, H.E. Jackson, I.M. Smith, G. Karczewski, and J. Kossut, Appl. Phys. Lett. \textbf{84}, 3337 (2004).
10. G. S. Solomon, M. Pelton and Y. Yamamoto, Phys. Rev. Lett. \textbf{86}, 3903 (2001).
11. A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. A. Smolin, and H. Weinfurter, Phys. Rev. A\textbf{52}, 3457 (1995).
12. D. P. DiVincenzo, D. Bacon, J. Kempe, G. Burkard, and K.B. Whaley, Nature \textbf{408}, 339 (2000).
13. C. Piermarocchi, and G.F. Quinteiro, Phys. Rev. B \textbf{70}, 235210 (2004).
14. G. Ramon, Y. Lyanda-Geller, T.L. Reinecke, and L.J. Sham, Phys. Rev. B \textbf{71}, 121305(R) (2005).
15. G. Chiappe, J. Fernández-Rossier, E. Louis and P. Zoller, Phys. Rev. B\textbf{68}, 245311 (2005).
16. M. Obert, J. Rennert, A. Forchel, G. Bacher, R. Andre, and D.L.S. Dang, Appl. Phys. Lett. \textbf{84}, 1435 (2004).
17. J. P. Reithmaier, G. Sek, A. Loßler, C. Hofmann, S. Kuhn, S. Reitzenstein, L.V. Keldysh, V.D. Kulakovskii, T.L. Reinecke, and A. Forchel, Nature \textbf{432} 197 (2004).
18. A. A. Maksimov, G. Bacher, A. McDonald, V. D. Kulakovskii, A. Forchel, C. R. Becker, G. Landwehr, and L. Molenkamp, Phys. Rev. B \textbf{62}, R7767 (2000); G. Bacher, A.A. Maksimov, H. Schomig, V.D. Kulakovskii, M.K. Welsch, A. Forchel, P.S. Dorożkin, A.V. Chernenko, S. Lee, M. Dobrowolska, and J.K. Furdyna, Phys. Rev. Lett. \textbf{89}, 127201 (2002); L. Besombes, Y. Leger, L. Maingault, D. Ferrand, H. Mariette, and J. Cibert, Phys.Rev.Lett\textbf{93}, 207403(2004).
19. C.E. Finlayson, G. Vijaya Prakash, and J.J. Baumberg, Appl. Phys. Lett. \textbf{86}, 041110 (2005).
20. A. Bodolato, K. Hennessy, M. Atature, J. Dreiser, E. Hu, P.M. Petroff, and A. Imamoglu, Science \textbf{308}, 1158 (2005).
21. A. Kiraz, P. Michler, C. Becher, B. Gayral, A. Imamoglu, L. Zhang, E. Hu, W.V. Schoenfeld, and P.M. Petroff, Appl. Phys. Lett. \textbf{78}, 3932 (2001).
22. A. Barenco and M. A. Dupertuis, Phys. Rev. B\textbf{52}, 2766 (1995).
23. Optical Orientation, edited by F. Meier and B. P. Zakharchenya (North Holland, New York, 1984).
24. We are assuming that the photon lifetime does not depend on the photon frequency, and, thus, the quality factor is linearly related to the latter. Note, however, that this may not always be the case.
FIG. 3: Same as Fig. 1 for spin 5/2 impurities and T=0.1 K.
FIG. 4: Real part of the energies of the states $|\psi_{-1}\rangle$ (thick continuous line) $|\psi_1\rangle$ (broken line) and $|\psi_0\rangle$ (thin continuous line) of Eqs. (8a), (8b) and (9) respectively, versus the inverse of the quality factor $Q^{-1} = \Delta\omega/\omega$. The results correspond to spin 1/2 impurities and $\delta = 10$ meV, and values of the light matter coupling $g$ and valence and conduction band exchange couplings ($J_v$ and $J_c$) given in the Figures. All energies referred to the energy of state $|\psi_0\rangle$. 


FIG. 5: a) Spin correlation functions versus detuning $\delta$ for spin 5/2 impurities. b) Photon and exciton occupation numbers. The results correspond to $Q=100$, $g=2.5$ meV, and valence and conduction band exchange couplings of $J_v = -1$ meV and $J_c = -0.2$ meV, respectively.
FIG. 6: Same as Fig. 4 for $Q=200$

FIG. 7: Spin-spin and photon polarization-total impurity spin correlation functions versus $\delta$ for spin $5/2$ impurities for the parameters of Fig. 5.