Multi-criteria collaborative filtering recommender by fusing deep neural network and matrix factorization

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Abstract

Recommender systems have been an efficient strategy to deal with information overload by producing personalized predictions. Recommendation systems based on deep learning have accomplished magnificent results, but most of these systems are traditional recommender systems that use a single rating. In this work, we introduce a multi-criteria collaborative filtering recommender by combining deep neural network and matrix factorization. Our model consists of two parts: the first part uses a fused model of deep neural network and matrix factorization to predict the criteria ratings and the second one employs a deep neural network to predict the overall rating. The experimental results on two datasets, including a real-world dataset, show that the proposed model outperformed several state-of-the-art methods across different datasets and performance evaluation metrics.

Keywords: Collaborative filtering, Deep learning, Deep neural network, Matrix factorization, Multi-criteria, Recommender system

Introduction

In the era of big data, while the World Wide Web keeps growing exponentially, the size and complexity of many websites (Google, YouTube, Netflix, Amazon, and others) grow along with it, making it increasingly difficult and time-consuming for the users of these websites to find the item (movie, music, restaurant, book, and product in general) they are looking for. Recommender systems (RSs) provide personalized suggestion for items that the user might like [1]. Exploiting the information from users’ ratings can be useful to solve one of the problems recommender systems suffer from, predicting users’ preferences about an item using a single rating. This is a clear limitation, since the user who makes a choice might take into consideration more than one aspect of the item. For an example, in a movie recommender system, some users may like a movie based on its plot, direction, or conflict, while others may like the same movie but for its acting, characters, or any other attribute of that movie. So, Multi-Criteria Recommender System (MCRS) that utilizes multi-criteria ratings to evaluate different attributes of the item can improve the accuracy of the recommendations [2]. RSs use many techniques, Collaborative Filtering (CF) is the most commonly used, and it makes recommendations based...
only on interactions between users and items such as ratings [3]. Matrix Factorization (MF), is the most popular CF techniques, which maps users and items into a joint latent space, using a vector of latent features to represent a user or an item [4]. Then the interaction of a user on an item is obtained from the inner product of their latent vectors. Although MF is effective, the simple choice of the interaction function is insufficient to model the complex relation between the users and items.

Deep Learning (DL) has achieved immense results in many research fields, such as speech recognition, natural language processing, computer vision, and recently in recommender systems, where Deep Neural Networks (DNNs) have proved their capability of modeling nonlinear users and items relationships and approximating any continuous function [5].

Therefore, it is convenient to fuse the DNN with MF to formulate a more general model that makes use of both the nonlinearity of DNN and the linearity of MF to enhance recommendation accuracy.

The objective of this paper is just to propose a multi-criteria collaborative filtering recommender by fusing DNN and MF, to improve collaborative filtering performance. Our model consists of two parts: in the first part, we get the users and items’ features and feed them as an input to a fused model of a DNN and MF to predict the criteria ratings, and we use a deep neural network to predict the overall rating in the second part.

By doing experiments on two datasets including a real-world dataset, it can be observed that our proposed model achieves significant improvement compared to the other state-of-the-art methods.

The main contributions of this work are as follows:

1. We present a multi-criteria collaborative filtering recommender by fusing DNN and MF, to combine the non-linearity of DNN and linearity of MF in a multi-criteria RS for modeling user-item latent structures.
2. We do comprehensive experiments on two datasets including a real-world dataset to show the efficiency of our model and the importance of using multi-criteria and deep learning in collaborative filtering.

The rest of this paper is organized as the following: In Sect. 2, we survey the related work. We will give a detailed overview of the system in Sect. 3. Section 4 presents the experimental evaluations and discussion and in Sect. 5, we provide a brief conclusion and potential future work.

Related work

Multi-criteria recommendation techniques can be divided into two general classes: memory-based and model-based methods. In memory-based methods, the similarity can be computed in two ways: the first approach calculates the similarity on each criteria rating separately using the traditional way and then aggregates the values into a single similarity using an aggregation method such as average [6], worst-case [6], and weighted sum of individual similarities [7]. The second approach uses the multidimensional distance metrics (Euclidean, Manhattan, and Chebyshev distance metrics) to calculate the distance between multi-criteria ratings [6]. Model-based methods use the user-item
ratings to learn a predictive model and later use this model to predict unknown ratings, like Probabilistic Modeling [8, 9], Multilinear Singular Value Decomposition (MSVD) [10], Support Vector Regression (SVR) [11], and aggregation-function-based [6], this approach assumes that there is a relation between the item’s overall rating and the multi-criteria ratings and it is not independent of other ratings.

Many efforts have been made to improve MF, Koren [12] merged MF and neighborhood models. Wang et al. [13] combined MF with topic models of item content. Rendle [14] introduced Factorization Machines that combine Support Vector Machines with factorization models. He et al. [15] proposed Neural Matrix Factorization (NeuMF) model that changed the linearity nature of MF by combining it with Multi-Layer Perceptron (MLP).

There are very few researches on applying deep learning to Collaborative Filtering model. Salakhutdinov et al. [16] proposed restricted boltzmann machines model to learn the item ratings correlation, thereafter Georgiev et al. [17] expanded the former work by adding both user–user and item–item correlations. Ouyang et al. [18] used autoencoder in autoencoder-based collaborative filtering to model users ratings on items. He et al. [15] introduced neural collaborative filtering model that uses MLP to learn the interaction function. Nassar et al. [19] presented deep multi-criteria collaborative filtering (DMCCF) model which is the only attempt in applying deep learning and multi-criteria to collaborative filtering. The model follows the aggregation-function-based approach, where they used a deep neural network to predict the criteria ratings and another DNN to learn the relationship between the criteria ratings and the overall rating in order to predict the overall rating.

**Method**

The proposed model is based on the model that Nassar et al. [19] presented. It contains three steps:

a. Predict criteria ratings $r_1, r_2, \ldots, r_k$ using a DNN.

b. Learn aggregation function $f$, which represents the relationship between the criteria ratings and the overall rating using a DNN.

c. Predict overall ratings using the predicted criteria ratings and the aggregation function $f$.

In our model, we used a fused model of DNN and MF to predict the criteria ratings $r_1, r_2, \ldots, r_k$ in the first step, while in the second step, we kept using a DNN to learn aggregation function $f$, as illustrated in Fig. 1.

**Criteria ratings model**

This model is used to predict the criteria ratings for a user on an item. In this model, we will fuse MF and DNN as He et al. [15] proposed in their NeuMF framework. MF that applies a linear kernel to model the latent feature interactions, and DNN that uses a non-linear kernel to learn the interaction function from data.

1. MF
We can use the item ID or user ID as a feature since it is unique, but the ID is a categorical feature.

Therefore, we converted the IDs into embedding vectors initialized with random values that adjusted to minimize the loss function during the model training.

In the input layer, the input vector $x$ is given by:

$$ x = v_u \odot v_i $$

(1)

where $v_u$ and $v_i$ are user and item embedding vectors and $\odot$ is the element-wise product of vectors. The formula of the output layer is as follows:

$$ y_{ui} = a_{out} \left[ w^T (v_u \odot v_i) \right] $$

(2)

where $a_{out}$ and $w^T$ are the activation function and weights of the output layer.

2. DNN

The input vector $x$ is given by:

$$ x = \text{Concatenate}(v_u, v_i) = \begin{bmatrix} v_u \\ v_i \end{bmatrix} $$

(3)

where $v_u$ and $v_i$ are user and item embedding vectors.

Then followed by a number of dense Rectified Linear Units (ReLU) layers, we choose ReLU activation function because it is the most efficient [20]. The formulation of which is given as follows:

$$ \text{ReLU}(z) = \max(z, 0) $$

(4)

The output of a hidden layer $l$ is formulated as:

$$ h_l = \text{ReLU} \left( W_l h_{l-1} + b_l \right) $$

(5)
where $W_l$ and $b_l$ are the weight matrix and bias vector for the layer $l$ and $h_1 = x$.

In the output layer, we have:

$$y_{ui} = ReLU(W_L h_{L-1} + b_L)$$

where $L$ is the number of layers.

3. Fusion of MF and DNN

For more flexibility to the fused model, we let DNN and MF learn separate embeddings, as shown in Fig. 2, and then we combine them by concatenating the last hidden layer. The outputs of the DNN and MF denoted as $o^D$, $o^M$ respectively, are given as follows:

$$o^D = ReLU(W_L ReLU(W_{L-1} \ldots ReLU(W_2 [v_D^u v_D^i] + b_2) \ldots + b_{L-1})) + b_L$$

$$o^M = v_M^u \odot v_M^i$$

where $v_D^u$ and $v_D^i$ are user embedding vectors for DNN and MF part, and $v_D^i$ and $v_M^i$ are item embedding vectors.

In the model output layer, we predict the user criteria ratings $r_1, r_2, \ldots, r_k$ using Eq. (9) and (10):

$$y_{ui} = ReLU(W_\begin{bmatrix} o^D \\ o^M \end{bmatrix} + b)$$

$$[r_1, r_2, \ldots, r_k]^T = y_{ui}$$
Overall rating deep neural network

The overall rating DNN is used to learn the aggregation function \( f \), which represents the relationship between the overall rating \( r_0 \) and the criteria ratings \( r_1, r_2, \ldots, r_k \), in order to predict the overall rating:

\[
r_0 = f(r_1, r_2, \ldots, r_k)
\] (11)

In the input layer, the input vector is the criteria ratings \( r_1, r_2, \ldots, r_k \) for user \( u \) and item \( i \), as shown in Fig. 2. We normalize the continuous features \( r_1, r_2, \ldots, r_k \) because the DNNs are sensitive to the inputs scaling and distribution [21]. The normalization of a sample \( r_i \) is calculated as:

\[
z_i = \frac{r_i - m_i}{s_i}
\] (12)

with \( m_i \) the mean of the training samples for rating \( r_i \) and \( s_i \) the standard deviation of the training samples for rating \( r_i \). The input vector becomes:

\[
x = [z_1, z_2, \ldots, z_k]^T
\] (13)

Then followed by a number of hidden layers, the output of a hidden layer is given again by Eq. (5).

In the output layer, we predict the overall rating \( r_0 \) in Eq. (6), where:

\[
r_0 = y_{ui}
\] (14)

We used Adam optimizer [22] and MAE loss function in both parts.

Recommendation

After the finishing of the model training, where each part was trained individually without knowing each other. We can use the model to predict the user’s overall rating on the new items.

The recommendation process happens as shown in Fig. 1, for each user \( u \) and item \( i \) pair we:

a) Get the user ID and the item ID, and use them as an input for the Criteria Ratings model, to compute the criteria ratings \( r'_1, r'_2, \ldots, r'_k \).

b) Normalize the previously computed criteria ratings \( r'_1, r'_2, \ldots, r'_k \) and use them as an input for the Overall Rating DNN, to compute the overall rating \( r'_0 \).

c) Recommend the user the items as in traditional recommender systems using the overall rating \( r'_0 \).
Results and discussion

Dataset
We evaluated our model on two multi-criteria rating datasets, a real-world TripAdvisor\(^1\) dataset and a Movies\(^2\) dataset.

1. TripAdvisor dataset

Multi-criteria rating dataset for hotels. It includes an overall rating and seven criteria ratings Value, Rooms, Location, Cleanliness, Check in/front desk, Service, and Business service, the rating range between 1 and 5. Table 1 demonstrates the statistics of the dataset and Table 2 the distribution of values for the different criteria ratings and the overall rating.

2. Movies dataset

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\(^1\) The dataset is crawled from the TripAdvisor website: https://www.tripadvisor.com/.

\(^2\) https://github.com/an888ha/multi_criteria_recommender_system/blob/master/data_movies.txt.
Multi-criteria rating dataset for movies, available on GitHub. It contains four criteria ratings and an overall rating, the rating range between 1 and 13. Tables 3 and 4 demonstrate the dataset statistics and the distribution of the different criteria ratings and the overall rating.

### Evaluation

To evaluate the performance of our model we used the same metrics used in [19].

a. Mean Absolute Error (MAE) [23]

\[
MAE = \frac{1}{M} \sum_{u,i} |r_{ui} - \tilde{r}_{ui}|
\]  

(15)

where \( r_{ui} \) the true rating of user \( u \) for item \( i \), \( \tilde{r}_{ui} \) the predicted rating, and \( M \) is the size of the test set.

b. F-measure (\( F_1 \) and \( F_2 \)) [23]

\[
F_1 = \frac{2PR}{P + R}
\]  

(16)

\[
F_2 = \frac{5PR}{4P + R}
\]  

(17)

where \( P \) the precision and \( R \) the recall.

c. Fraction of Concordant Pairs (FCP) [24]

\[
n_c^u = |\{(i,j)|\tilde{r}_{ui}, \tilde{r}_{uj} and r_{ui} > r_{uj}\}|
\]  

(18)

where \( n_c^u \) the number of concordant pairs for user \( u \), and we calculate \( n_d^u \) the number of discordant pairs for user \( u \) in a similar way.

\[
n_c = \sum_u n_c^u
\]  

(19)

\[
n_d = \sum_u n_d^u
\]  

(20)
d. Mean Average Precision (MAP) [23] is the average precision for all users.
e. Mean Reciprocal Rank (MRR) [25] is the average of the first relevant item rank for all users.

Settings
We used Keras\textsuperscript{3} with TensorFlow\textsuperscript{4} as a backend to implement our model.

1. Criteria Ratings Model Settings

We conducted several experiments to find the optimal parameters for DNN and MF.

- For DNN, we randomly initialized the DNN parameters using a normal distribution with mean of 0 and standard deviation of 0.05. We used Adam optimizer, with 0.001 learning rate and parameters values as provided in [22].

For TripAdvisor dataset, we used batch size of 512 and set epochs to 2. We set each of user and item embedding vector size to 64, and we selected the $[128 \rightarrow 64 \rightarrow 32 \rightarrow 16 \rightarrow 8]$ hidden layers. For Movies dataset, we used batch size of 64 and set epochs to 4. We set each of user and item embedding vector size to 32, and we selected the $[64 \rightarrow 32 \rightarrow 16 \rightarrow 8]$ hidden layers.

- For MF, we tried to find the optimal embedding vector size. As shown in Fig. 3, for TripAdvisor dataset, we set user and item embedding vector size to 16, and for Movies dataset, we set them to 8.

- In the model output layer, for TripAdvisor dataset, there are 7 neurons, equal to the number of the criteria ratings, and 4 neurons for Movies dataset.

\begin{equation}
\text{FCP} = \frac{n_c}{n_c + n_d}
\end{equation}

\text{(21)}

Fig. 3 The impact of embedding vector size in MF model for both datasets

\textsuperscript{3} https://github.com/keras-team/keras.
\textsuperscript{4} https://github.com/tensorflow/tensorflow.
We initialized randomly the DNN parameters like the previous DNN, using a normal distribution with mean of 0 and standard deviation of 0.05. We also used Adam optimizer with 0.001 learning rate and the same parameters values. For TripAdvisor dataset, we set the epochs to 50, and for Movies dataset, we set the epochs to 100. While for both datasets, we set the batch size to 512. We used $[64 \rightarrow 32 \rightarrow 16 \rightarrow 8]$ hidden layers. Finally, in the output layer, there is 1 neuron, for the overall rating.

### Results

We used fivefold cross-validation method, to split the data randomly into 20% test set and 80% training set including a 1% validation set. We repeated this process 5 times and calculated the mean value of each metric. We compared the performance of our model to the DMCCF model [19], then to a single DNN that predicts the overall rating directly where the finest results of this DNN were acquired from settings shown in Table 5. In addition, we compared our model to a number of famous single rating recommendation methods such as SVD [26], SVD++ [12], Baseline Estimates [27], and SlopeOne [28].

This comparison was done on the overall rating. The results are illustrated in Table 6. We can see that our model achieves the best performance on both datasets, significantly outperforming each of DMCCF model, single DNN, and the other state-of-the-art methods on all the evaluation metrics. This indicates the high expressiveness of our model by fusing the non-linear DNN and linear MF models to capture the methods. F1 and F2 of our model are better than the other methods. Our model surpasses the other models in FCP and it exceeds user-item interactions. According to the results, in MAE, our model excelled the other compared methods at MAP. In MRR, our model is the best.

### Conclusion and future work

In this paper, we proposed multi-criteria collaborative filtering recommender by fusing DNN and MF. The model consists of two parts: in the first part, we get the users and items’ features and feed them as an input to a fused model of a DNN and MF to predict the criteria ratings, and in the second part, we use a DNN to predict the overall rating. Then we demonstrated the efficiency of our proposed model, where the experimental results showed that our model significantly outperformed the other methods on both datasets for all the evaluation metrics, which proved that the application of deep

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**Table 5 Single DNN settings**

| Dataset   | Parameter                              | Value |
|-----------|----------------------------------------|-------|
| TripAdvisor | User and item embedding vector size | 256   |
|           | Hidden layers                          | $[512 \rightarrow 256 \rightarrow 128 \rightarrow 64 \rightarrow 32 \rightarrow 16 \rightarrow 8]$ |
|           | Output layer                           | 1     |
|           | Batch size                             | 512   |
| Movies    | User and item embedding vector size    | 128   |
|           | Hidden layers                          | $[256 \rightarrow 128 \rightarrow 64 \rightarrow 32 \rightarrow 16 \rightarrow 8]$ |
|           | Output layer                           | 1     |
|           | Batch size                             | 64    |
### Table 6 Evaluation results

| Dataset          | Metric   | SVD         | SVD++        | SlopeOne     | Baseline estimates | Single DNN      | DMCCF [19]   | Our         |
|------------------|----------|-------------|--------------|--------------|--------------------|-----------------|--------------|-------------|
| TripAdvisor      | MAE      | 0.8071 ± 0.0043 | 0.8222 ± 0.0071 | 0.9160 ± 0.0053 | 0.8094 ± 0.0019 | 0.7855 ± 0.0153 | 0.7552 ± 0.0050 | 0.7434 ± 0.0068 |
|                  |         |             |              |              |                    |                 |              |             |
|                  | F1       | 0.8370 ± 0.0035 | 0.8901 ± 0.0035 | 0.8666 ± 0.0018 | 0.8391 ± 0.0020 | 0.8572 ± 0.0024 | 0.8769 ± 0.0043 | 0.8811 ± 0.0031 |
|                  | F2       | 0.7951 ± 0.0031 | 0.7991 ± 0.0057 | 0.9248 ± 0.0011 | 0.7986 ± 0.0029 | 0.9176 ± 0.0057 | 0.9295 ± 0.0014 | 0.9319 ± 0.0024 |
|                  | FCP      | 0.5024 ± 0.0210 | 0.5092 ± 0.0056 | 0.5096 ± 0.0061 | 0.5025 ± 0.0117 | 0.4957 ± 0.0102 | 0.5126 ± 0.0160 | 0.5164 ± 0.0048 |
|                  | MAP      | 0.3935 ± 0.0057 | 0.3483 ± 0.0049 | 0.3006 ± 0.0026 | 0.3881 ± 0.0023 | 0.4481 ± 0.0091 | 0.4643 ± 0.0072 | 0.4836 ± 0.0047 |
|                  | MRR      | 0.7272 ± 0.0054 | 0.7513 ± 0.0018 | 0.7502 ± 0.0037 | 0.7374 ± 0.0028 | 0.7244 ± 0.0092 | 0.7664 ± 0.0039 | 0.7693 ± 0.0036 |
| Movies dataset   | MAE      | 2.1072 ± 0.0168 | 2.1594 ± 0.0050 | 2.1798 ± 0.0112 | 2.1333 ± 0.0069 | 2.0613 ± 0.0347 | 2.0474 ± 0.0119 | 2.0188 ± 0.0181 |
|                  | F1       | 0.8061 ± 0.0049 | 0.7985 ± 0.0010 | 0.7860 ± 0.0026 | 0.8107 ± 0.0031 | 0.8344 ± 0.0106 | 0.8414 ± 0.0041 | 0.8563 ± 0.0036 |
|                  | F2       | 0.7608 ± 0.0062 | 0.7553 ± 0.0023 | 0.7325 ± 0.0045 | 0.7699 ± 0.0045 | 0.8256 ± 0.0287 | 0.8383 ± 0.0049 | 0.8442 ± 0.0056 |
|                  | FCP      | 0.6022 ± 0.0107 | 0.5985 ± 0.0029 | 0.6251 ± 0.0098 | 0.6233 ± 0.0027 | 0.6226 ± 0.0055 | 0.6258 ± 0.0083 | 0.6289 ± 0.0029 |
|                  | MAP      | 0.1910 ± 0.0056 | 0.2144 ± 0.0035 | 0.2089 ± 0.0067 | 0.1612 ± 0.0027 | 0.2136 ± 0.0143 | 0.2345 ± 0.0109 | 0.2458 ± 0.0090 |
|                  | MRR      | 0.6651 ± 0.0091 | 0.6502 ± 0.0048 | 0.6085 ± 0.0048 | 0.6721 ± 0.0052 | 0.6922 ± 0.0145 | 0.7061 ± 0.0078 | 0.7092 ± 0.0049 |

Evaluation results for our model against the other algorithms. F1, F2, FCP, MAP, and MRR the higher the better while MAE the lower the better.
learning and multi-criteria in collaborative filtering is a successful attempt and it can be enhanced using different deep learning techniques or by building more complex models.

In future work, we will study the use of different deep learning techniques, such as Autoencoder, Convolutional Neural Network (CNN), and Recurrent Neural Network (RNN) in recommendation systems and attempt to further improve the performance of our model, we will also try other feature representation methods precisely to solve the cold start problem by using user and item content features.

Abbreviations
CF: Collaborative filtering; DL: Deep learning; DMCCF: Deep multi-criteria collaborative filtering; DNN: Deep neural network; FCP: Fraction of Concordant Pairs; MAE: Mean Absolute Error; MAP: Mean Average Precision; MCRS: Multi-Criteria Recommender System; MF: Matrix factorization; MLP: Multi-layer perceptron; MRR: Mean Reciprocal Rank; NeuMF: Neural Matrix Factorization; ReLU: Rectified Linear Unit; RS: Recommender system.

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Authors’ contributions
All the corresponding authors contributed equally to the conduct of the present study. All authors read and approved the final manuscript.

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Availability of data and materials
The datasets used during the current study are available in the Datasets repository, https://github.com/nournassar/Datasets.

Competing interests
The authors declare that they have no competing interests.

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