Serving the flow of requests when flying around ground objects by aircraft in the “air taxi” mode

V Goncharenko1,2,*, A Lebedev1, D Mikhaylin1 and A Rumakina1
1Department of Automatic and Intelligent Control Systems, Moscow Aviation Institute (National Research University), 4 Volokolamskoe Highway, 125993, Moscow, Russian Federation
2V A Trapeznikov Institute of Control Sciences of Russian Academy of Sciences, 65 Profsoyuznaya Street, 117997, Moscow, Russian Federation
*E-mail: goncharenkovi@mai.ru

Abstract. The scientific work solves the problem of organizing a route group flight of aircraft, when pre-flight planning is carried out only for the beginning of the flight, and then operational planning is carried out by analogy with a taxi. The peculiarity of the problem being solved is to ensure the required planning efficiency during the flight of the aircraft, taking into account the safety of their flights without crossing routes. As a result of solving the problem, an approach was substantiated to solving the problem of target distribution of vacant aircraft between incoming orders, taking into account all the factors and constraints under consideration; the effectiveness of the chosen approach was assessed. A minimax scheduling criterion is proposed. The criterion at the same time provides: minimum time for completing the flight task; minimization of the total path length and non-intersection of the flight sections, which increases the efficiency and safety of flight in “peak” dispatch modes. The solution to the problem is proposed using the queuing theory, according to which the considered system belongs to the class of multichannel queuing systems with waiting.

1. Introduction
Currently, civil aviation carries out air traffic according to a pre-drawn up flight plan, or in accordance with a prepared flight task before the next flight. At the same time, there are a number of practically important tasks of a route group flight, when flight plan is carried out only for the beginning of a group flight, and then new orders for service are received during the flight [1].

A special prospect for manned light aviation is presented by the “air taxi” mode, when there is no specified demand between the airports, which is satisfied according to the schedule, but requests are received for flights to points whose composition is unknown in advance and is of a random. For manned aviation, the service of one order is understood as a departure from one designated point to another. The regions of Russia such as the Far East, Arctic and Siberia, where the movement of ground transport is extremely difficult, and the local non-systematic air traffic over short distances is extremely necessary, will have a huge need to organize an “air taxi” mode.

The mode under consideration is of similar importance for unmanned aviation, in particular, intended for the delivery of goods and observation of both stationary and mobile objects. In the near future, such a regime will be in demand in order to transport goods by the banking sector, the Post of Russia and oil and gas companies (especially in hard-to-reach regions of the Russia). For unmanned aviation, servicing one order means reaching the next point and, in particular, observing it at a given
speed and altitude. Thus, the organization of a group flight mode when servicing requests that arrive not so much before departure as “on call” in during the flight is an urgent and promising task of air traffic control. In works [2-5] on the organization of group movement of aircraft, the above-mentioned features are not taken into account.

In our scientific work, the problem of air traffic control is solved, the complexity of which lies in ensuring the required planning efficiency during the flight of the aircraft group and obtaining the results of the selected flights that guarantee their safety when the routes do not cross. A fundamental feature is also either the possible absence of applications in the presence of vacant aircraft, or the lack of these aircraft when these orders appear [6-8]. Optimization of the solution to the problem of planning a group flight of an aircraft in the air taxi mode is the purpose of research in this work.

2. Proposed approach to solving the planning problem

Consider the following options for target distribution of incoming orders between vacant aircraft. As the simplest single-criterion decision, you can use the condition of preference for the highest priority orders with importance proportional to the waiting time \( \tau_j \) of the order according to the principle “the order was a first, the order will be served a first”:

\[
J_1 = \max_{j=1,...,S} \tau_j
\]  

(1)

Then, for the priority order, the nearest vacant aircraft is assigned according to the criterion

\[
J_2 = \min_{k=1,...,N} \tau_{kj}
\]  

(2)

where \( k \) is the number of the aircraft.

The advantage of this option is obvious – it minimizes the service time of the most important orders. However, there is a significant lack – among the chosen ones there may be points with a significant distance from the aircraft group, which in general reduces the efficiency of the group flight.

To overcome this drawback, as a second option, it is possible to use a criterion that takes into account both the importance of the \( \tau_j \) selected ground point and its distance \( R_j \) from the UAV (unmanned aerial vehicle) group:

\[
J_3 = \max_{j=1,...,S} \frac{\tau_j}{R_j}
\]  

(3)

where \( R_j = \frac{1}{N} \sum_{k=1}^{N} r_{kj} \) – is the average distance of the \( j \)-th point from the flying UAV group, \( r_{ij} \) – the distance between the given service points.

After selecting the priority order according to criterion (3), the nearest aircraft is assigned for it according to criterion (2), and not only one of the vacant, but also of those occupied by the aircraft that are nearby.

However, this criterion also has its lack – with a significant proximity of the point to the UAV group, the most distant points will be served last.

In connection with the indicated lacks, our scientific work proposes a more perfect minimax dispatch criterion based on a combination of not two, but three sequentially performed operations [9-12]. In particular, in the case when at each planning step the number of claims exceeds the number \( N \) (at least for \( S>d \)), the following actions are provided.

The first operation allows ranking all free orders according to criterion (4):

\[
J_3 = \max_{j=1,...,S} \frac{\tau_j}{R_j}
\]  

(4)

Then, from the ranked general list for subsequent actions, a list of priority orders is formed.
In the second operation, the generated list is ranked according to a different criterion that takes into account the importance and total distance of each ground target from the UAV group:

\[ J_2 = \max_{j=1,\ldots,S} \left( \tau_j \sum_{k=1}^{N} r_{kj} \right) \]  

(5)

In the third operation of the UAV distribution, after the selection of the most distant target with the maximum rank \( J_2 \), the problem of assigning «own» aircraft is solved according to the third criterion of maximum proximity:

\[ J_3 = \min_{k=1,\ldots,N} \tau_{kj} \]  

(6)

The main ones are the second and third operations (5-6), which implement the minimax criterion when choosing the primary target with the maximum service time and designating the nearest UAV that is most convenient for this service. It can be shown that this criterion simultaneously minimizes the total path length \( l \) of the UAV group and excludes the intersection of their routes. Figure 1 shows the layout of the UAV relative to the served targets for \( N = 2 \) and \( \tau_1 = \tau_2 \).

![Figure 1](image)

**Figure 1.** The layout of the UAV relative to the served targets in the Cartesian coordinate system \((x, z)\).

The group flight routes found by the minimax criterion are shown by continuous lines of length \( l_1 \) and \( d_2 \) for two UAVs \((k=1, k=2)\). Let the condition of maximum distance by criterion be given \( J_2 \) in the form of inequality:

\[ d_1 + d_2 > l_1 + l_2 \]  

(7)

Let also, according to the criterion \( J_3 \) for the most distant target with the number \( j = 1 \), the nearest unoccupied UAV with the number \( k = 1 \) is selected, since:

\[ l_1 < l_2 \]  

(8)

But for a convex tetrahedron, condition (8) practically corresponds to the observance of another inequality:

\[ d_2 < d_1 \]  

(9)

Then adding inequalities (8) and (9) with each other, we get the answer:

\[ l_1 + d_2 < l_2 + d_1 \]  

(10)

that is, the sum of the two sides of a tetrahedron is less than the sum of its diagonals, which was required to prove that the necessary condition for the uncrossing of these sides is satisfied, and the total path length is minimized. Otherwise, it is necessary to swap places of observation.

Separately, it should be emphasized that in the particular case of an aircraft “vacant”, it is proposed, instead of the first operation (3) of the minimax criterion, to truncate the number \( N \) used in the aircraft target allocation to the value \( S \), ranking the aircraft according to
\[ J_4 = \min_{k=1,\ldots,N} \left( \frac{1}{S} \sum_{j=1}^{S} r_{kj} \right) \] (11)

In other words, first, the \( N \) most “close” UAVs to a small number of points are formed, then the most distant UAV is selected from them in the third operation - the closest point is assigned. This case requires additional research, and comparison of the proposed option with others will show which one is better.

In optimization, along with the choice of the most efficient planning algorithm, an important place is also occupied by the assignment of the required number of aircraft \([13-16]\). So, in manned aviation, in the absence of a regular schedule of arrivals and departures, but with a known flow of applications, it may be necessary to choose such a number of aircraft to ensure the optimal combination of efficiency and performance of the required set of flights. In unmanned aviation, it is also required to determine the optimal number of UAVs, taking into account the demand for regular observations of ground objects that arises when information is updated.

The solution to this problem is possible using the queuing theory \([6]\), according to which the considered system belongs to the class of multichannel queuing systems with waiting, when \( N \) is the number of channels, in each of which one aircraft serves requests with an average rate \( \mu \), equal to

\[ \mu = \frac{V}{r_{cp}} = qV \] (12)

In turn, the load factor of \( \rho \) one channel is

\[ \rho = \frac{\lambda}{qV} \]; \( \rho < N \) (13)

where \( V \) – is the flight speed of the aircraft, \( \lambda \) – is the reciprocal of the average time between arising requests (the intensity of the arrival of requests), \( q \) – is the reciprocal of the average distance between the points of flight.

Inequality \( \rho < N \) is a necessary constraint under which the average rate of service of claims must be higher than the rate of their arrival.

Required for calculating the probability of the state \( P_k \) of the system are calculated by the Erlang formula \([6-8]\):

\[ P_k = \frac{\rho^k}{k!} \sum_{i=0}^{k} \frac{P_i}{i!} + \sum_{S=N+1}^{\infty} P_s(N) \] (14)

where \( P_k \) – is the probability that the number \( N \) of occupied aircraft is equal to \( k \), \( P_s \) – are the probability that there are \( S \) customers \((S = N, \ldots, \infty)\) in the queue waiting for assignment, the number \( l \) of free channels. These probabilities allow us to determine the final value of the average number of applications in queue \( S_{cp} \) by the formula:

\[ S_{cp} = \sum_{S=N+1}^{\infty} S \cdot P_s(N) \] (15)

3. **Computational experiment results**

In order to compare various dispatch algorithms and estimate the optimal number of aircrafts, the following block diagram of computer modeling is proposed, shown in figure 2.
According to the presented scheme, the following designations are adopted for modeling:

- \( l = 1, \ldots, n \) is the number of the current operational planning step during discretization;
- \( j = 1, \ldots, n \) is the number of the serviced ground point or order;
- \( k = 1, \ldots, N \) is the number of the aircraft selected for servicing.

The discretization step \( l \) is understood as actions on target allocation at any of the two moments when one of the aircraft is released from service, or when a new order for the required flight. At the same time, at each step, the process of dispatching and performing flights is modeled using blocks 2-6 shown in figure 2.

Accordingly, at the input of the system the process of dispatching and performing flights is modeled using block 1. The picture of the location on a given rectangular territory of a set of ground points, which are a potential source of orders, is simulated. In this case, the points are placed so that the distances between them are subject to Poisson's law, and their number \( n >> N \).

Further, at any step \( k \) (including the first), a “random” moment of the appearance of an order from any of the points at the moment of time is simulated \( t_k \), which also corresponds to the Poisson distribution. Then (taking into account the number \( i \) of the point of the new order, coordinates \( x_i, z_i \) the current location of the aircraft and the determination in block 2 of the number \( d \) of vacant aircraft and the number \( S \) of other orders in the queue) in block 3, the assigned dispatching discipline is implemented, which determines the indication of the selected flight from point \( i \) to item \( j \).

This allows using the kinematic model of flights in block 4 and calculating the lost time for servicing in block 5 to find the intermediate result of adherence to the plan at the \( k \)-step in the form of penalty functions of costs \( E_i \) and delays in servicing \( T_i \) for those aircraft and orders that are involved in servicing, as well as for other “free” orders and aircraft.

After completing the required number of steps in servicing the flow of orders at the end of the simulation, the final value of the criterion \( J_0 \) for a given number \( N \) and the assigned service discipline is estimated, after which the simulation is repeated for other options and similar data.

Thus, in contrast to the generally accepted simulation modeling in the queuing theory, the proposed scheme is capable of assessing not only probabilistic characteristics, but also:
- to introduce various scheduling algorithms into the simulated system;
- simulate the processes of changing the importance of each time request at any planning step;
- to determine the number of free aircraft, or the number of applications waiting for service in the queue, and as a result - to distinguish between the “downtime” and “peak” modes from each other;
- to quantify the advantages of the chosen discipline of servicing applications, when in the “peak” mode the priority points are the aircraft closest to the group, and in the “downtime” mode, preference is given to important, albeit more distant points.
Figure 3. The results of calculating the total aircraft downtime and the total waiting time for order: priority in terms of time of order creation (a), priority in relation to distance and time of order creation (excluding aircraft completing a flight mission) (b), priority in relation to distance and time the creation of the order (taking into account the aircraft completing the flight mission) (c).

To evaluate successful service, two penalty functions are calculated – the average waiting time $\tau_{ср}$ for the start of servicing one request for different values of $N$, as well as the total cost of operating costs, consisting of the cost of the aircraft downtime and the cost of the flight to the designated place of service of the request. In addition, a unified indicator for evaluating the scheduling efficiency is
required, depending on the selected number $N$ of serving aircraft. With an increase in this number, the value of $\tau_{ср}$ decreases, and the value of operating costs increases.

The results of calculating the total downtime of the aircraft and the total waiting time for the order, depending on the selected dispatching criteria and the number of aircraft, are shown in figure 3.

Thus, in contrast to the generally accepted simulation modeling in the queuing theory, the proposed scheme is capable of assessing not only probabilistic characteristics, but also:

- introduce various dispatching algorithms into the simulated system;
- simulate the processes of changing the importance of each time order at any planning step;
- determine the number of vacant aircraft, or the number of orders waiting for service in the queue, and as a result – to distinguish between “vacant” and “peak” modes from each other;
- to quantify the advantages of the chosen discipline of servicing orders, when in the “peak” mode the priority points are the nearest aircraft to the group, and in the “vacant” mode, preference is given to important, albeit more remote points.

4. Conclusion
Based on the research carried out, the following conclusions can be drawn.

1. The formulation of the problem of target allocation between aircraft of ground objects when servicing them in the “air taxi” mode is formulated.
2. It has been established that the process of dispatching group actions of aircraft should contain different disciplines of service in the normal mode, in the “standby” mode and in the “peak” mode, taking into account the lowest fuel consumption and the waiting time for orders in the queue.
3. When choosing a dispatch algorithm, an approach is proposed based on a combination of sequentially performed ranking operations according to the minimax criterion to ensure the lowest operating costs in the worst conditions in terms of service time.
4. Using the multiplicative form of a single criterion for the effectiveness of group actions of an aircraft, the problem of parametric optimization is posed when choosing the number of aircraft that provides acceptable fuel consumption and the rate of service of the flow of orders.

In our opinion, the application of this approach will be implemented in the near future for the urgent delivery of correspondence, medicines and other materials by the Post of Russia and oil and gas companies in remote regions of our country.

Acknowledgments
The reported study was funded by RFBR, project number 20-08-00652 a.

References
[1] Shibitov A 2020 Urban airmobility 19th Int. Conf. “Aviation and Cosmonautics” (Moscow: Logotip) p 332
[2] Rebrov V A, Rudel'son L E and Chernikova M A 2007 A model of flight request collection and processing in the flight scheduling problem. J. Comput. and Sys. Sc. Int. 46(3) 429 https://doi.org/10.1134/S1064230707030124
[3] Bo R, Lei Y and Lixun H 2007 On path planning for UAVs based on adaptive ant system algorithm. Electronics Optics and Control 6(14) 36
[4] Farzad K, Lozano G and Rassul A 2006 Path planning for UAVs using symbiotic simulation. Proc. 20th annual Europe Optics Simulation and Modelling Conf. (France: Toulouse ) p 215
[5] Evdokimenkov V N, Krasilshchikov M N and Kozorez D A 2019 Development of pre-flight planning algorithms for the functional-program prototype of a distributed intellectual control system of unmanned flying vehicle groups. INCAS Bulletin 11(1) 75 doi: 10.13111/2066-8201.2019.11.S.8
[6] Saaty T L 1961 Elements of queueing theory with applications (New York: McGraw-Hill) p 510
[7] Dunbar W B and Murray R M 2002 Model predictive control of coordinated multi-vehicle formations. Proc. of the 41st Conf. on Decis. and Cont. (New York: IEEE Press) p 4631
[8] Wentzel E S 1972 *Operations research* (Moscow: Owls) p 552
[9] Schouwenaars T, Moor B, Feron E and How J 2001 Mixed integer programming for multi-vehicle path planning. *Proc. of the European Control Conf.* (New York: IEEE Press) p 2603
[10] Zhu R, Sun D and Zhou Z 2005 Cooperation Strategy of Unmanned Air Vehicles for Multitarget Interception. *J Guidance* **28**(5) 1068 https://doi.org/10.2514/1.14412
[11] Goncharenko V I, Zheltov S Y, Knyaz V A, Lebedev G N, Mikhailin D A and Tsareva O Y 2021 Intelligent system for planning group actions of unmanned aerial vehicles when observing ground mobile objects in a given territory. *J. Comput. and Sys. Sc. Int.* **60**(3) 64
[12] Lebedev G N, Mikhailin D A, Tsareva O Yu and Chernyakova M E 2020 Multi-criteria evaluation of efficiency of group flight of aircraft using multiplicative form. *Proc. of VSU, Series: Systems Analysis and Information Technologies* **82**(2) 36 doi:10.20914/2310-1202-2020-1-1-6
[13] Kaluder H, Brezak M and Petrovic I 2011 A visibility graph based method for path planning in dynamic environments. *Proc. of the 34th Int. Convention* (Croatia: IEEE Press) p 717
[14] Kim N and Krylov I 2012 Group use of an unmanned aerial vehicle in observation tasks. *Proc. of the MAI* (Moscow: Logotip) p 11
[15] Merkulov V I, Milyakov D A and Samodov I O 2013 Optimization of the UAV group control algorithm as part of a local network. *Izvestiya SFedU. Technical science* **3** 157
[16] Knyaz V, Zheltov S, Lebedev G, Mikhailin D and Goncharenko V 2019 Intelligent Mobile Object Monitoring by Unmanned Aerial Vehicles. *18th Int. Conf. on Smart Technol.* (Serbia: IEEE Press) p 6