Rossby rogons in atmosphere and in the solar photosphere

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Abstract – The generation of Rossby rogue waves (Rossby rogons), as well as the excitation of bright and dark Rossby envelope solitons are demonstrated on the basis of the modulational instability (MI) of a coherent Rossby wave packet. The evolution of an amplitude-modulated Rossby wave packet is governed by a one-dimensional (1D) nonlinear Schrödinger equation (NLSE). The latter is used to study the amplitude modulation of Rossby wave packets for fluids in Earth’s atmosphere and in the solar photosphere. It is found that an amplitude-modulated Rossby wave packet becomes stable (unstable) against quasi-stationary, long-wavelength (in comparison with the Rossby wavelength) perturbations, when the carrier Rossby wave number satisfies $k^2 < 1/2$ or $\sqrt{2} + 1 < k^2 < 3$ ($k^2 > 3$ or $1/2 < k^2 < \sqrt{2} + 1$). It is also shown that a Rossby rogon or a bright Rossby envelope soliton may be excited in the shallow-water approximation for the Rossby waves in solar photosphere. However, the excitation of small- or large-scale perturbations may be possible for magnetized plasmas in the ionospheric \(E\)-layer.

Introduction. – The existence of rogue waves in oceanography [1] and in nonlinear optics [2] has been known. Furthermore, the appearance of such localized wave packets in atmosphere [3] as well as in plasmas [4] has been a topic important research during the last few years. The physics of Rossby waves in shallow rotating fluids placed in a gravitational field is of interest owing not only to its numerous theoretical as well as experimental studies [5–8], but also to many important applications in astrophysics, space physics, physics of ocean and planetary atmosphere [9].

The theory of magnetized Rossby waves in weakly ionized \(E\)-layer plasmas has been developed by Kaladze et al. [10]. They derived a generalized Charney-Obukhov wave equation which contains both the scalar and vector nonlinearities. The latter has been shown to balance the wave dispersion for the self-organization of solitary structures in magnetized plasmas. Recently, Kaladze and Wu [11] had shown that Charney-Obukhov model can describe only the propagation of small-scale Rossy wave perturbations in the solar photosphere. On the other hand, the amplitude modulation of drift wave packets in a nonuniform magnetoplasma has been studied by Shukla and Misra [12] owing to its important applications in laboratory and space environments. It was shown that such nonuniform magnetoplasmas with equilibrium electron density, electron temperature, and magnetic field inhomogeneities can support the existence of drift rogons, as well as bright and dark drift wave envelope solitons due to the amplitude modulation of a constant amplitude drift wave packet.

In this letter, we study the amplitude modulation of a coherent Rossby wave packet which may propagate in Earth’s atmosphere and in the solar photosphere. In contrast to the two-dimensional Charney-Obukhov wave equation [10,11] where the Jacobian (vector nonlinearity) plays important roles, we consider the one-dimensional propagation of Rossby waves along one of the horizontal directions (the \(x\)-direction). The model equation is considered in two different cases: one for fluids in the solar photosphere [11] and the other for weakly ionized
E-layer plasmas [10]. We show that due to the modula-
tional instability (MI) of Rossby waves, small-scale pertur-
bations may develop into Rossby wave rogons or bright
envelope Rossby solitons, which may be excited in the
solar photosphere. However, both the small- and large-
scale perturbations of the Rossby waves may lead to the
formation of rogons as well as bright and dark Rossby
wave envelope solitons in magnetized ionospheric E-layer
plasmas.

Evolution equation and modulational instabil-
ity. – We begin our discussion by considering the follow-
ning one-dimensional Charney-Obukhov equation which
describes the dynamics of solitary Rossby waves (for the
two-dimensional wave equation, see, e.g., refs. [10,11]):

$$\frac{\partial}{\partial t} \left( h \cdot \frac{\partial^2 h}{\partial x^2} \right) + v_R \frac{\partial h}{\partial x} + v_R h \frac{\partial h}{\partial x} = 0, \quad (1)$$

where $h = \delta H/H_0 < 1$, $\delta H$ is the perturbation of the total
depth of the fluid and $H_0$ is the uniform thickness in
its stationary state. Also, $v_R = -\beta \delta^2/\epsilon c_g$ is the Rossby
velocity normalized by the gravity wave speed $c_g = \sqrt{gH_0}$
($g$ is the acceleration due to gravity), $\beta = (\partial f/\partial y$ at
$y = 0$) is the so-called Rossby parameter with $f$ denoting
the Coriolis parameter. The space $x$ and time $t$ are
normalized by the Rossby radius $r_R$ and $1/f_0 \equiv r_R/c_g$,
respectively. Equation (1) may be considered as a model
for the propagation of only small-scale perturbations in
solar photosphere [11] or for solitary waves in ionospheric
magnetized E-layer plasmas.

We now derive the governing nonlinear equation for
amplitude-modulated Rossby wave packets using the stan-
dard reductive perturbation technique [13–15]. In the
latter, one stretches the space and time variables as $\xi = \epsilon(x - v_y t)$, $\tau = \epsilon^2 t$, where $\epsilon$ is a small parameter ($0 < \epsilon \ll 1$)
representing the weakness of perturbation and $v_y$ the
Rossby wave group velocity to be obtained shortly. In the
modulation of a plane Rossby wave as the carrier wave
with the wave number $k$ and the frequency $\omega$, the variable $h$
can be expanded as [12]

$$h = \sum_{n=1}^{\infty} \epsilon^n \sum_{l=-\infty}^{\infty} h_{l,1}^{(n)}(\xi, \tau) \exp[i(kx - \omega t)], \quad (2)$$

where $h_{l,1}^{(n)} = h_{-l,1}^{(n)*}$ is the reality condition and the asterisk
denotes the complex conjugate.

Substituting the expansion (2) and the aforementioned
stretched coordinates into eq. (1), and equating different
powers of $\epsilon$, we obtain for $n = l = 1$ (coefficient of $\epsilon$) the
following linear dispersion relation for the Rossby waves:

$$\omega = \frac{v_R k}{1 + k^2}, \quad (3)$$

For $n = 2$, $l = 1$, i.e., for the second-order first-harmonic
modes, we obtain an equation in which the coefficient of $h_1^{(2)}$
vanishes due to the usage of the Rossby wave
dispersion relation. Then, the coefficient of $\partial h_1^{(1)}/\partial \xi$, while
equating to zero, gives the group velocity

$$v_g = \frac{v_R(1 - k^2)}{(1 + k^2)^2}. \quad (4)$$

Proceeding in this way we obtain, from the coefficients of $\epsilon^2$ and $\epsilon^3$, the second- and zeroth-order harmonic modes
for $n = l = 2$ and $n = 2$, $l = 0$ as

$$h_2^{(2)} = \frac{v_R k}{\omega(1 + 2k^2) - 2v_R k} |h_1^{(1)}|^2, \quad h_0^{(2)} = \left( \frac{v_R}{v_g - v_R} \right) |h_1^{(1)}|^2. \quad (5)$$

Finally, for $n = 3$, $l = 1$, we obtain an equation in
which the coefficients of $h_1^{(3)}$ and $\partial h_1^{(2)}/\partial \xi$ vanish by the
Rossby wave dispersion relation (3) and the Rossby wave
group velocity dispersion (4), respectively. In the reduced
equation, we substitute the expressions for $h_0^{(2)}$ and $h_2^{(2)}$
from eq. (5) to obtain the following nonlinear Schrödinger
equation (NLSE):

$$i \frac{\partial h}{\partial \tau} + P \frac{\partial^2 h}{\partial \xi^2} + Q |h|^2 h = 0, \quad (6)$$

where we redefine $h \equiv h_1^{(1)}$, which may be of the order of
unity or less, so that in the expansion $h = \epsilon h_1^{(1)} + \cdots$
(eq. (2)), $\epsilon \ll 1$.

In eq. (6), the group velocity dispersion and the nonlinear
coefficients are, respectively,

$$P = \frac{1}{2} \frac{\partial^2 \omega}{\partial k^2} = \frac{v_R k(k^2 - 3)}{(1 + k^2)^3}, \quad (7)$$

and

$$Q = \frac{v_R(\sqrt{2} - 1 - k^2)(\sqrt{2} + 1 + k^2)}{k(3 + k^2)(1 - 2k^2)}, \quad (8)$$

It has been found that in the atmosphere, solar photosphere
as well as in the ocean, the large-scale flows can be
caused by the nonlinear interaction of different types of
small-scale linear-wave modes, e.g., Rossby waves. The
modulation of slowly varying wave amplitudes occur due to
self-interaction of these carrier wave modes. These
phenomena, however, are relevant to the MI which consti-
tutes one of the most fundamental effects associated with
the propagation of wave packets in nonlinear media. Such
MI signifies the exponential growth or decay of a small
perturbation of the waves. The gain eventually leads to
self-amplification of the sidebands, which break up the other-
wise uniform wave, and lead to energy localization via
the formation of localized structures. Thus, the system’s
evolution is then governed through the MI, which acts as
a precursor for the formation of rogons or bright enve-
lope solitons. However, in absence of the MI, the stable
wave packet can propagate in the form of a dark envelope
soliton.

The amplitude modulation of Rossby waves in the
Earth’s atmosphere due to the action of earthquake’s elec-
 tromagnetic radiation has been considered by Tsintsadze

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et al. [16]. The generation of different stable and unstable branches of oscillations (depending on the intensity of the pumping waves) by seismic activity has been reported there. It has been predicted that such oscillation branches along with energetically reinforcing Rossby solitary vortical anticyclone structures may serve as precursors to earthquakes. However, in our nonlinear theory to be presented shortly we have considered the amplitude modulation of Rossby waves that may propagate in weakly ionized \( E \)-layer magnetoplasmas as well as in fluid media in the solar photosphere. We show that the amplitude modulation of a coherent Rossby wave in these environments leads to the localization of bright envelope solitons or the emergence of Rossby rogueons. The latter may emerge in the planetary atmosphere as well as in the solar photosphere, and can play the roles of precursors of certain extraordinary natural phenomena (earthquakes, magnetic storms, volcano eruptions or man-made activities etc.)

We now consider the amplitude modulation of a plane Rossby wave solution of eq. (6) of the form \( h = h_0 e^{-iQ\tau} \), where \( \Omega_0 = -Qh_0 \) and \( h_0 \) is a constant. The Rossby wave amplitude is then modulated against a plane-wave perturbation with frequency \( \Omega \) and wave number \( K \) as 
\[
h = \left( h_0 + h_1 e^{iK_2\xi - i\Omega\tau} + h_2 e^{-iK_2\xi + i\Omega\tau}\right) e^{-iQ\theta},
\]
where \( h_{1,2} \) are real constants. Since the perturbations are assumed to be small and nonzero, we obtain from eq. (6) the following dispersion relation for the modulated Rossby wave packets [12]:
\[
\Omega^2 = (PK^2)^2 \left( 1 - \frac{K_2^2}{K^2} \right),
\]
where \( K_c = \sqrt{2|Q/P|h_0} \) is the critical value of \( K \), such that the MI sets in for \( K < K_c \), and the wave will be modulated for \( PQ > 0 \). On the other hand, for \( K > K_c \) the Rossby wave is said to be stable (\( PQ < 0 \)) against the modulation. The instability growth rate is given by
\[
\Gamma = |P|K^2 \sqrt{\frac{K_2^2}{K^2} - 1}.
\]

Clearly, the maximum value of \( \Gamma \) is achieved at \( K = K_c/\sqrt{2} \), and is given by \( \Gamma_{\text{max}} = |Q|\Psi_0^2 |P|^2 \).

Inspecting on the expressions for \( P \) and \( Q \) given by eqs. (7), (8), we find that \( PQ \geq 0 \) according as \( (k^2 - 3)(\sqrt{2} + 1 - k^2)(1 - 2k^2) \geq 0 \). So, \( PQ > 0 \) for either \( k^2 > 3 \) or \( 1/2 < k^2 < \sqrt{2} + 1 \), and \( PQ < 0 \) according to which \( k^2 < 1/2 \) or \( \sqrt{2} + 1 < k^2 < 3 \). We find that for the MI to occur and for a fixed \( v_R \), as \( k \) increases (\( > \sqrt{3} \)), the value of \( \Gamma \) also increases with a cut-off at higher wave number of modulations. A similar trend occurs when \( k \) also lies in \( 1/2 < k^2 < \sqrt{2} + 1 \) for which \( PQ > 0 \). This indicates that for long-wavelength perturbations with \( K < 1 \), the MI growth rate can be controlled for Rossby waves with wave numbers \( k \) close to either \( 1/\sqrt{2} \) or \( \sqrt{3} \). Thus, we conclude that the Rossby wave packets may be stable or unstable against the modulation at small- \( (k > 1) \), as well as for large-scale \( (k < 1) \) perturbations. As is evident from ref. [11] that the model (1) describes only the propagation of small-scale low-frequency perturbations in the solar photosphere. However, eq. (1) may be used to describe both the small as well as large-scale phenomena for weakly ionized \( E \)-layer plasmas [10]. In our numerical investigation below, we will consider those cases separately, and show that in both the cases, the Rossby wave packet may propagate as a rogueon, as well as bright or dark envelope solitons.

**Solutions of the NLSE.** Exact solutions of the NLSE (6) can be presented (see for details, e.g., refs. [17]). Assuming \( h = \sqrt{2} \exp(i\theta) \), where \( \Psi \) and \( \theta \) are real functions, the bright envelope soliton solution of eq. (6) is obtained for \( PQ > 0 \) as [12]
\[
\Psi = \Psi_0 \sech^2 \left( \frac{\xi - U\tau}{W} \right),
\]
\[
\theta = \frac{1}{2P} \left( U\xi + \Omega_0 - \frac{U^2}{2}\tau \right).
\]

This represents a localized pulse traveling at a speed \( U \) with oscillation frequency \( \Omega_0 \) at rest. The pulse width \( W \) is given by \( \Psi_0 \) as \( W = \sqrt{2P/Q}\Psi_0 \), where \( \Psi_0 \) is a constant.

Furthermore, eq. (6) can have a rogue wave solution that is located in a nonzero background and localized both in space and time, given by (for \( PQ > 0 \)) [3,12]
\[
h = h_0 \left[ \frac{4(1 + 2iQ\tau)}{1 + 4Q^2\tau^2 + 2Q^2/P} - 1 \right] \exp(iQ\tau).
\]

On the other hand, for \( PQ < 0 \), the modulationaly stable wave packet will propagate in the form of a dark
The solution (13) represents a localized region of hole (void) traveling at a speed $U$. The pulse width $W_1$ depends on the constant amplitude $\Psi_1$ as $W_1 = \sqrt{2|P/Q|/\Psi_1}$.

Rossby waves in the solar photosphere. – Following ref. [11], we consider the low-frequency oscillations characterized by a small Rossby number, i.e., $\epsilon_\omega \sim (1/f)(\partial/\partial t) \sim (\omega/f) \ll 1$. We also assume that the length scale $L$ of motion is much smaller than the scale of
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Figure 4: (Colour on-line) The evolution of rogons (eq. (12)) for Rossby wave packets in weakly ionized ionospheric $E$-layer plasmas at three different scales: (a) small scale ($k > 1$) with $k = 5$; (b) intermediate scale ($k \sim 1$) with $k = 1.2$; and (c) large scale ($k < 1$) with $k = 0.8$. The corresponding contour plots are shown in (d), (e) and (f), respectively. Here $h_0 = 0.01$ and other parameter values are as in the text.

Figure 5: (Colour on-line) The evolution of dark envelope solitons (eq. (13)) for Rossby wave packets in weakly ionized ionospheric $E$-layer plasmas at two different values of $k$: $k = 1.6$ (upper panel) and $k = 0.02$ (lower panel) corresponding to intermediate ($k \sim 1$) and large-scale ($k < 1$) motions with $\tau = 0$, $\Psi_1 = 0.02$ and $U = 0.2$. The other parameters are as in the text.

Rossby waves in the ionospheric $E$-layer. – The model equation (1) can be considered to describe the Rossby solitary-wave dynamics for ionospheric magnetized $E$-layer plasmas [10]. In this case, the corresponding parameter values will be different (see for details ref. [10]).

variation of $f$, i.e., $\epsilon_\beta \sim (L f'/f) \ll 1$, and that the amplitude of oscillations, $\epsilon_h \sim h \ll 1$. We note that the ratio of the parameters $\epsilon_\omega$ and $\epsilon_h$ gives a typical length scale, $\epsilon_\omega/\epsilon_h \sim r^2_R/L^2$, which shows that if $h > \epsilon_\omega$, then $L$ must be larger than $r^2_R$. In the small-scale dynamics ($k > 1$) assuming $k \sim \delta/r_R$, where $\delta > 1$, we obtain (considering magnitude only) the following estimates for $\omega$ and $v_g$ (in dimensional forms) as

$$\omega \sim \frac{\beta}{k} \frac{\beta r^2_R}{\delta}; \quad v_g \sim \frac{\beta}{k^2} \frac{\beta r^2_R}{\delta^2}. \quad (14)$$

For the photospheric parameters [11] with $H_0 \approx 500$ km, the solar surface gravity $g \approx 274$ m/s$^2$, the Coriolis parameter $f \approx 5.8$ $\mu$Hz, for which $r_R \approx 2 \times 10^8$ m $> R_\odot$, the solar radius and taking $\delta = 5$, we find from eqs. (14) that $\omega \approx 3.2$ $\mu$Hz, $v_g \approx 1.28 \times 10^3$ m/s. These estimates well agree with those obtained numerically and will be appropriate to explain the observed data [11]. Furthermore, we also find that in the small-scale approximation, $PQ$ is always positive, giving rise to the possibility of the existence of Rossby wave packets in the form of bright envelope solitons or rogons. The profile of the bright envelope solitons at $k = 5$ is shown in fig. 1. Figure 2 shows that the Rossby rogue wave can be generated at small scales. For smaller values of the Coriolis parameter, it is seen that the wave packet gets localized in relatively a small space and time implying that a significant amount of energy is concentrated in a relatively small area in space. Thus, a random perturbation of the Rossby wave amplitude will grow on account of the modulational instability.
from those considered in the case of solar photosphere. For illustration, we consider \( H_0 = 200 \text{ km}, \beta \approx 1.1 \times 10^{-11}/\text{ms}^2 \), \( f \approx 5.2 \times 10^{-5} \text{s} \) at latitude \( \lambda = \pi/6 \) and \( r_R \approx 2.6 \times 10^7 \text{ m} \). Here the Coriolis parameter \( f \) is obtained using the \( \beta \)-plane approximation in the vicinity of the latitude \( \lambda = \lambda_0 \), as \( f \approx 2H_0 \sin \lambda \), where \( \Omega_0 = 50 \mu \text{Hz} \) is the constant rotation frequency \([10]\). The Rossby radius is then obtained from the relation \( r_R = \sqrt{\beta H_0/f} \), where \( g \) is the acceleration due to gravity. We see that the generation of bright envelope (fig. 3) and rogue wave solutions (fig. 4) may be possible at three different scales: i) small scale \((k > 1)\) with \( k = 5 \) for which \( \omega \approx 1.4 \mu \text{Hz} \), \( v_g \approx 1.03 \times 10^3 \text{ m/s} \), ii) intermediate scale \((k \approx 1)\) with \( k = 1.2 \) for which \( \omega \approx 1.4 \mu \text{Hz} \), \( v_g \approx 5.68 \times 10^2 \text{ m/s} \) and iii) large scale \((k < 1)\) with \( k = 0.8 \) for which \( \omega \approx 1.4 \mu \text{Hz} \), \( v_g \approx 1.03 \times 10^3 \text{ m/s} \). From fig. 3, it is also found that the Rossby wave packets propagate as single solitary pulses with reduced amplitudes at intermediate- and large-scale motions. However, the dark envelope solitons (fig. 5) may exist only at intermediate- \((k = 1.61)\) and large-scale \((k = 0.02)\) motions.

**Conclusion.** – We have considered the amplitude modulation of Rossby wave packets that may propagate for fluids in the solar photosphere \([11]\) as well as for weakly ionized magnetized plasmas in the ionospheric \( E \)-layer \([10]\). We show that the generation of rogue waves (rogons) or bright/dark envelope solitons may be possible in these environments. Starting from the one-dimensional Charney-Obukhov equation for the dynamics of Rossby waves, and using the standard reductive perturbation technique, we derive a nonlinear Shr"odinger equation which describes the dynamics of small-amplitude Rossby wave packets. It is found that the wave packets may propagate as rogons or bright envelopes when either \( k^2 > 3 \) or \( 1/2 < k^2 < \frac{1}{\sqrt{2}} + 1 \). On the other hand, the generation of dark envelope soliton is also possible for either \( k^2 < 1/2 \) or \( \sqrt{2}/1 + 1 < k^2 < 3 \). Since the Charney-Obukhov model is valid for only small-scale low-frequency perturbations in the solar photosphere, the existence of dark envelopes may not be possible there, whereas both the bright and dark envelope solitons may be excited in the ionospheric \( E \)-layer plasmas.

It is to be mentioned that, in one-dimensional propagation, the vector or Poisson bracket nonlinearity can be neglected. It can also be neglected in the large-scale dynamics of Rossby waves in which the size \( a \) of the soliton exceeds the Rossby radius \( r_R \), i.e., \( a > r_R \) \([10]\). Since \( \xi \) is normalized by \( r_R \), it is clear from fig. 3 that the soliton width (size) exceeds \( r_R \) and the inequality can be satisfied. Furthermore, in two-dimensional motion, since the vector nonlinearity involves the symmetric derivatives of \( h \) and \( V^2 h \) with respect to \( x \) and \( y \), the contributions from the nonlinear interations of the higher-harmonic modes (for \( n = 2, l = 2; n = 2, l = 1; n = 2, l = 0 \) and \( n = 3, l = 1 \)) get cancelled. In conclusion, the present results should be helpful in identifying modulated Rossby wave packets that may be generated in the solar photosphere, ionospheric \( E \)-layer, as well as laboratory fluids or plasmas.

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