The Effect of Strange Quark on the Chiral Symmetry Breaking in Magnetic Background in the framework of logarithmic quark-sigma model

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Abstract

The chiral symmetry breaking in the presence of an external magnetic field is studied in the framework of logarithmic quark-sigma model with three flavors ($u, d, s$). The effective logarithmic mesonic potential is extended to strange quark and is numerically solved in the mean-field approximation. The present results show that the chiral symmetry breaking is sensitive for the strange quark flavour in the presence an external magnetic field. In addition, the effect of the free parameters is studied. A comparison with original sigma model is discussed.

Keywords: Chiral Lagrangian density, Magnetic catalysis, Mean-field approximation
I. INTRODUCTION

The quantum chromodynamic (QCD) theory is an acceptable theory for strong interactions. A direct technique to calculate the hadron properties through using the lattice QCD but this is not easy task, in particular, at finite density. So the investigations are preformed by using effective models such as the Nambu-Jona-Lasinio model and quark sigma model that share the QCD theory in same properties. The linear sigma model is introduced by Gell-Man and Levy [1] to describe pion-nucleon interactions. This model is extended to quark level by Birse and Banerjee [2] to calculate hadron properties at low energy. Birse and Banerjee model [2] has some difficulties that conflict with experimental data. So, the model is extended to avoid these difficulties as in Refs. [3−5]. In addition, the model is extended to include finite temperature as Refs. [6−8].

The study of the influence of external magnetic fields on the fundamental properties of quantum chromodynamic (QCD) theory, confinement and dynamical chiral symmetry breaking is still a matter of great interest theoretical and experimental activities. In Ref. [9], the fact that an external magnetic field enhances the generation of a fermion mass in 3+1 dimensions was first established in the framework of the Nambu-Jona-Lasinio (NJL) model made in Refs. [10,11]. Also, in the context of the (2+1)-dimensional Gross-Neveu model, which was expected to give a simple effective description for certain condensed matter planar systems, the authors of Refs. [12,13] showed that the dynamical generation of nonzero fermion mass takes place in a magnetic field as soon as there an attractive interaction between fermions and antifermions. The symmetric quark matter and quark matter in $\beta$ equilibrium are investigated in the NJL model [14] in the presence external magnetic field at zero temperature [15,16]. The effective quark model that takes into account chiral symmetry [17,18]. It was shown that non-negligible effects on the equation of state and single-particle quark properties. The linear sigma model with two flavors has also applied to determine the critical point temperature in the presence of external magnetic field [19,20]. The effect of the higher-order mesonic interactions on the chiral symmetry breaking is investigated in the presence of an external magnetic field [21].

In Ref. [22], the authors have modified the linear sigma model by including the logarithmic mesonic potential and study its effect on the phase transition at finite temperature. In addition, the comparison with other models is done. On the same hand, the logarithmic
The interactions of quarks via the exchange of $\sigma$- and $\pi$-meson fields are given by the Lagrangian density [2] as follows:

$\mathcal{L}(r) = i\overline{\Psi} \gamma_\mu \partial_\mu \Psi + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi \right) + g \overline{\Psi} (\sigma + i\gamma_5 \tau \cdot \pi) \Psi - U_1(\sigma, \pi), \quad (1)$

with

$U_1(\sigma, \pi) = \frac{\lambda^2}{4} \left( \sigma^2 + \pi^2 - \nu^2 \right)^2 + m^2_\pi f_\pi \sigma. \quad (2)$

$U_1(\sigma, \pi)$ is the meson-meson interaction potential where $\Psi, \sigma$ and $\pi$ are the quark, sigma, and pion fields, respectively. In the mean-field approximation, the meson fields are treated as time-independent classical fields. This means that we replace the power and products of the meson fields by corresponding powers and the products of their expectation values. The meson-meson interactions in Eq. (2) lead to hidden chiral $SU(2) \times SU(2)$ symmetry with $\sigma(r)$ taking on a vacuum expectation value

$\langle \sigma \rangle = -f_\pi, \quad (3)$

where $f_\pi = 93$ MeV is the pion decay constant. In Eq. (2), the final term is included to break the chiral symmetry explicitly. It leads to the partial conservation of axial-vector
isospin current (PCAC). The parameters $\lambda^2$ and $\nu^2$ can be expressed in terms of $f_\pi$, sigma and pion masses as,

$$\lambda^2 = \frac{m_\pi^2 - m_\sigma^2}{2f_\pi^2},$$

(4)

$$\nu^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda^2}.$$  (5)

III. THE EFFECTIVE LOGARITHMIC POTENTIAL IN THE PRESENCE OF MAGNETIC FIELD

In this section, the logarithmic mesonic potential $U_2(\sigma, \pi)$ is employed. In Eq. (6), The effective logarithmic potential is extended to include the external magnetic field at zero temperature and chemical potential as follows,

$$U_{\text{eff}}(\sigma, \pi) = U_2(\sigma, \pi) + U_{\text{V accum}} + U_{\text{Matter}},$$

(6)

where

$$U_2(\sigma, \pi) = -\lambda_1^2 (\sigma^2 + \pi^2) + \lambda_2^2 (\sigma^2 + \pi^2)^2 \log \left( \frac{\sigma^2 + \pi^2}{f_\pi^2} \right) + m_\pi^2 f_\pi \sigma,$$

(7)

In Eq. 7, the logarithmic potential satisfies the chiral symmetry when $m_\pi \rightarrow 0$ as well as in the standard potential in Eq. 2. Spontaneous chiral symmetry breaking gives a nonzero vacuum expectation for $\sigma$ and the explicit chiral symmetry breaking term in Eq. 6 gives the pion its mass.

$$\langle \sigma \rangle = -f_\pi.$$  (8)

Where

$$\lambda_1^2 = \frac{m_\pi^2 - 7m_\sigma^2}{12},$$

(9)

$$\lambda_2^2 = \frac{m_\pi^2 - m_\sigma^2}{12f_\pi^2}.$$  (10)

For details, see Refs. [22, 23]. To include the external magnetic field in the present model, we follow Ref. [20] by including the pure fermionic vacuum contribution in the free potential energy. Since this model is renormalizable the usual procedure is to regularize divergent integrals using dimensional regularization and to subtract the ultra violet divergences. This procedure gives the following result

$$U_{\text{V accum}} = \frac{N_c N_f g^4}{(2\pi)^2} (\sigma^2 + \pi^2)^2 \left( \frac{3}{2} - \ln \left( \frac{g^2(\sigma^2 + \pi^2)}{\Lambda^2} \right) \right),$$

(11)
where $N_c = 3$ and $N_f = 3$ are color and flavor degrees of freedom, respectively, and $\Lambda$ is mass scale,

$$U_{\text{Matter}} = \frac{N_c}{2\pi^2} \sum_{f=u,d,s} (|q_f| B)^2 [\zeta^{(1,0)}(-1,x_f) - \frac{1}{2} (x_f^2 - x_f) \ln x_f + \frac{x_f^2}{4}]$$  

(12)

In Eq. 12, we have used $x_f = \frac{\sigma^2 (\sigma^2 + \pi^2)}{(2 |q_f| B)}$ and $\zeta^{(1,0)}(-1,x_f) = \frac{d \zeta(z,x_f)}{dz} \bigg|_{z=-1}$ that represents the Riemann-Hurwitz function, and also $|q_f|$ is the absolute value of quark electric charge in external magnetic field with intense $B$ and the potential in Eq. 12 is extended to include strange quark based on Ref. [25]. In Eq. 6, the effect of the finite temperature and chemical potential is not included in the present model, in which the present model focuses on the study of magnetic catalysis at low energy due to the enhancement in the chiral symmetry breaking in the bound state.

IV. DISCUSSION OF RESULTS

In this section, we study the effective potential of logarithmic sigma model. For this purpose, we numerically calculate the effective potential in Eq. (6). The parameters of the present model are the coupling constant $(g)$ and the sigma mass $(m_\sigma)$. The choice of free parameters of $(g)$ and $(m_\sigma)$ based on Ref. [20]. The parameters are usually chosen so that the chiral symmetry is spontaneously broken in the vacuum and the expectation values of the meson fields. In this work, we consider two different sets of parameters in order to get high and a low value for sigma mass. The first set is given by $\Lambda = 16.48$ MeV which yields $m_\pi = 138$ MeV and $m_\sigma = 600$ MeV. The second set as the first, yielding $m_\sigma = 400$ MeV.

In Fig. 1, the effective potential is plotted as a function of sigma field for different values of magnetic field $(B)$. By ignoring the $B$ independent one loop term in Eq. (6). At zero magnetic field, we note that qualitative agreement between the effect of light quark field and strange quark field. The difference appears that the effective potential shifts to lower values at lower values of sigma field. Therefore, we deduce that the spontaneous symmetry breaking remains unchanged under at zero magnetic field by including strange quark field. By increasing the external magnetic field as in Fig. (2), we note that the spontaneous symmetry breaking is clearly appeared and the potential has largest two minima values in comparison with their values in the case of light quark. Therefore, the strange quark
increases the generated fermionic mass which leads to increase magnetic catalysis in the present model.

**Fig. 1:** The effective potential is plotted as a function of sigma field for $m_\sigma = 600$ MeV and $g = 4.5$ at zero magnetic field in the chiral limit.
Fig. 2: The effective potential is plotted as a function of sigma field for $m_{\sigma} = 600$ MeV and $g = 4.5$ strong magnetic field ($eB = 0.214 \text{ GeV}^2$) in the chiral limit.

It is important to discuss the effect of free parameters of the model on the chiral symmetry breaking. So, we select two sets of parameters. First set, the change of coupling constant with fixed sigma mass as in Fig. 3. The effective potential is plotted as a function of sigma field for different values of $g$ at dense of magnetic field ($eB = 0.214 \text{ GeV}^2$). The effect of coupling constant $g$ strongly clarify when sigma field increases. We note that effective potential shifts to higher values by decreasing $g$ which means that the energy of the potential increases with decreasing coupling constant $(g)$. In addition, increasing coupling constant $(g)$ enhances the chiral symmetry breaking and the potential has two largest minima values in comparison with their values at $g = 4.5$. Therefore, the generate fermionic mass increases. In the second set of parameters, we fixed the coupling constant $g$ with two values of sigma mass as in Fig. 4.
Fig. 3: The effective potential is plotted as a function of sigma field at $m_\sigma = 600$ MeV and $eB = 0.214$ GeV$^2$ for two values of coupling constant $g$.

We note that qualitative features of effective potential are remains unchanged, in which the effective potential is not sensitive up to $\simeq \pm 175$ MeV and then the potential shifts to higher values by increasing sigma mass. Also, we note that two minima values of the potential increases with decreasing sigma mass. Therefore, the decrease in the sigma mass enhances the chiral symmetry breaking at dense of magnetic field.
**Fig. 4:** The effective potential is plotted as a function of sigma field at coupling constant $g = 4.5$ and $eB = 0.214$ GeV$^2$ for two values of sigma mass.

**Fig. 5:** A comparison between the original sigma model and the logarithmic sigma model at fixed parameters at 4.5 and $eB = 0.214$ GeV$^2$ and $m_\sigma = 600$ MeV.
In Fig. 5, we compare between the original sigma model and the logarithmic sigma model. We fixed all parameters in the two models to show realistic effect on the chiral symmetry breaking. Qualitative features are similar for the two models. In addition, we note that the two minima values of logarithmic potential decreases in comparison with their values in the original sigma model.

V. SUMMARY AND CONCLUSION

In this work, the effect strange quark flavor on the spontaneous chiral symmetry breaking in the presence of magnetic field is studied. In addition, the effect of free parameters of the model is studied in the presence of strong magnetic field when strange quark flavor is included. A comparison with original sigma model is presented, showing qualitative agreement with original sigma model. So, novelty in this work that the effect of strange quark flavor on the spontaneous chiral symmetry breaking is studied in the framework of the logarithmic sigma model. Most previous studies have been concentrated on the original sigma model and NJL model at zero or finite temperature.

We hope to extend the present model to finite temperature and chemical potential which play an important role for studying properties of the universe and neutron star.

VI. REFERENCES

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