Resonant spin Hall conductance in quantum Hall systems lacking bulk and structural inversion symmetry

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Following a previous work [Shen, Ma, Xie and Zhang, Phys. Rev. Lett. 92, 256603 (2004)] on the resonant spin Hall effect, we present detailed calculations of the spin Hall conductance in two-dimensional quantum wells in a strong perpendicular magnetic field. The Rashba coupling, generated by spin-orbit interaction in wells lacking bulk inversion symmetry, introduces a degeneracy of Zeeman-split Landau levels at certain magnetic fields. This degeneracy, if occurring at the Fermi energy, will induce a resonance in the spin Hall conductance below a characteristic temperature of order of the Zeeman energy. At very low temperatures, the spin Hall current is highly non-ohmic. The Dresselhaus coupling due to the lack of structure inversion symmetry partially suppresses the spin Hall resonance. The condition for the resonant spin Hall conductance in the presence of both Rashba and Dresselhaus couplings is derived using a perturbation method. In the presence of disorder, we argue that the resonant spin Hall conductance occurs when the two Zeeman split extended states near the Fermi level becomes degenerate due to the Rashba coupling and that the the quantized charge Hall conductance changes by $2e^2/h$ instead of $e^2/h$ as the magnetic field changes through the resonant field.

I. INTRODUCTION

Spintronics, which exploits electron spin rather than charge to develop a new generation of electronic devices, has emerged as an active field in condensed matters because of both the underlying fundamental physics and its potential impact on the information industry.\textsuperscript{1,2,3} One key issue in spintronics is the generation and efficient control of spin current. Spin-orbit interaction of electrons exists extensively in metals and semiconductors and mix spin states. It provides an efficient way to control the coherent motion of electron spins. Recently it is proposed theoretically that an electric field may generate a spin current in hole-doped semiconductors and in two-dimensional electron gases (2DEG) in heterostructures with spin-orbit coupling due to the spin helicity and the noncollinearity of the velocity of the single particle wave function.\textsuperscript{1,5,6} Studies of this intrinsic spin Hall effect has evolved into a subject of intense research.\textsuperscript{7,8,9,10,11,12,13} The spin Hall effect in a paramagnetic metal with magnetic impurities has also been discussed, in which a transverse spin imbalance will be generated when a charge current circulates.\textsuperscript{14,15,16,17} We also note that the spin chirality in systems with strong spin-orbit interaction may induce a pure spin current.\textsuperscript{18}

Over the past two decades, remarkable phenomena have been observed in the 2DEG, most notably, the discovery of integer and fractional quantum Hall effect.\textsuperscript{19,20,21} Research in spin transports provides a good opportunity to explore spin physics in the 2DEG with spin-orbit couplings. The spin-orbit coupling leads to a zero-field spin splitting, and it competes with the Zeeman spin splitting when a perpendicular magnetic field is applied. The result can be detected as beating in Shubnikov-de Haas oscillations.\textsuperscript{22,23}

Very recently we have studied the spin Hall effect in the 2DEG with spin-orbit coupling in a strong perpendicular magnetic field, and predicted a resonant spin Hall effect caused by the Landau level crossing near the Fermi energy.\textsuperscript{24} In this paper we present detailed calculations of the problem. We analyze symmetries in systems with the Rashba and/or Dresselhaus couplings. By using linear response theory, we calculate the spin Hall conductance $G_s$, including its magnetic field and temperature dependences for realistic parameters of InGaAs/InGaAlAs. The nonlinearity in the electric field of the spin Hall current near resonance is also studied beyond the linear response theory. The resonance is a low temperature property, which shows up at a characteristic temperature of the order of the Zeeman energy $E_Z$. The peak of the resonance diverges as $1/\max(k_B T, eE_l b)$ ($l_b$: the magnetic length), and its weight diverges as $-\ln T$ at low $T$ and at $E \to 0$. Near the resonant magnetic field $B_0$, $G_s \propto 1/|B - B_0|$. The resonance arises from the Fermi level degeneracy of the Zeeman-split Landau levels in the presence of the spin-orbit coupling. Among the two types of the spin couplings we consider, the Rashba coupling reduces the Zeeman splitting and is the interaction responsible for the resonance. The Dresselhaus coupling further separates the Zeeman splitting and suppresses the resonance. The resonant condition in the presence of both Rashba and Dresselhaus couplings is derived within a perturbation theory, which is accurate for small ratio of the Zeeman energy to the cyclotron frequency.

The paper is organized as follows. In Section II we introduce the Hamiltonian of the system under consid-
eration and analyze its symmetries. In Section III, we study the spin Hall current for systems with only Rashba or only Dresselhaus coupling. In Section IV, we consider systems with both Rashba and Dresselhaus couplings. By treating the couplings as small parameters, we develop a perturbation method to derive the resonance condition. The paper is concluded with a summary and discussions in Section V.

II. MODEL HAMILTONIAN AND SYMMETRY

A. Spin orbit coupling and model Hamiltonian

As an introduction, we start with the three-dimensional (3D) spin-orbit interaction known for III-V compounds such as GaAs and InAs, which is of the form:

\[ V_{so}^{3D} = \alpha_0 K(p) \cdot \sigma + \beta_0 E \cdot (p \times \sigma) \]  (1)

where \( \sigma_\mu \) (\( \mu = x, y, z \)) are the Pauli matrices for spin of electrons, \( p \) is the momentum of the charge carrier, and

\[ K_\mu(p) = \sum_{\nu,\delta} p_\nu p_\delta \sigma_{\mu,\nu,\delta} \]  (2)

In Eq. (1), the first term is the Dresselhaus coupling which originates from the lack of bulk inversion symmetry while the second term is the Rashba coupling which arises from the lack of structure inversion symmetry. The effective field \( E \) is induced by the asymmetry of the external voltage to the system. In quantum wells, by neglecting the weak interband mixing and retaining the linear contribution of \( p \) parallel to the \( x - y \) plane, the spin-orbit interaction in 3D is reduced to an effective one in 2D,

\[ V_{so}^{2D} = H_{so}^D + H_{so}^R \]  (3a)

\[ H_{so}^D(\alpha) = \frac{\alpha}{\hbar} (\sigma_x p_x - \sigma_y p_y) \]  (3b)

\[ H_{so}^R(\beta) = \frac{\beta}{\hbar} (\sigma_y p_x - \sigma_x p_y) \]  (3c)

where \( \alpha = -\alpha_0 \hbar \langle p_x^2 \rangle \) and \( \beta = \beta_0 \hbar \langle E_z \rangle \), with the average taken over the lowest energy band of the quasi-2D quantum well. The Rashba coupling can be modulated up to fifty percent by a gate voltage perpendicular to the plane. In some quantum wells such as GaAs the two terms are usually of the same order of magnitude, while in narrow gap compounds like InAs the Rashba coupling dominates. Experimentally the relative strength of the Rashba and Dresselhaus couplings can be extracted from photocurrent measurements.

In this paper we consider a spin-1/2 particle of charge \(-e\) and effective mass \( m \) confined by a semiconductor quantum well to a 2D \( x - y \) plane of length \( L_x \) and width \( L_y \). The particle is subjected to a spin-orbit interaction \( V_{so}^{2D} \). A perpendicular magnetic field \( B = -B \hat{z} = \nabla \times A \) and an electric field \( E = E \hat{y} \) along the \( y \)-axis are applied as shown in Fig. 1. Both electron-electron interaction and impurities will be neglected in our study. The Hamiltonian reads

\[ H = H_0 + e E y, \]

\[ H_0 = \frac{1}{2m} \left( p + \frac{e}{c} A \right)^2 - \frac{1}{2} \gamma g_\mu_B B \sigma_z + V_{so}^{2D}(A) \]  (4)

where \( g_\mu \) is the Landé g-factor, and \( \mu_B \) is the Bohr magneton. In \( V_{so}^{2D}(A) \) the momentum \( p \) is replaced by the canonical momentum, \( p = p + \frac{e}{c} A \). We choose the Landau gauge \( A = y B \hat{z} \) and consider periodic boundary condition in the \( x \) direction, hence \( p_x = k \) is a good quantum number.

Below we rewrite the Hamiltonian in terms of lowering and raising operators. For each \( k \), we introduce the lowering operator

\[ a_k = \frac{1}{\sqrt{2l_y}} \left[ y + \frac{e}{c B} (k + i p_y) \right] \]

and the corresponding raising operator \( a_k^\dagger = (a_k)^\dagger \), with the magnetic length \( l_y = \sqrt{\hbar e B/c} \). and \( a^\dagger \) satisfy the commutations \( [a_k, a_{k'}^\dagger] = \delta_{kk'}, \) and \( [a_k, a_{k'}] = 0 \). In terms of \( a_k \) and \( a_k^\dagger \), we have

\[ H_0/\hbar \omega = a_k^\dagger a_k + \frac{1}{2} (1 - g \sigma_z) + i \sqrt{2} \eta_R (a_k^\dagger \sigma_- a_k + \sigma_- a_k^\dagger \sigma_+) \]

\[ + \sqrt{2} \eta_D (a_k^\dagger \sigma_+ a_k^\dagger \sigma_- + a_k^\dagger \sigma_+ a_k) \]  (5)

where \( \omega = e B/mc \) is the cyclotron frequency, \( \sigma_\pm = (\sigma_x \pm i \sigma_y)/2 \), and \( g = g_\mu m/2m_c \) is twice of the ratio of the Zeeman energy to the cyclotron frequency \( m_c \) (free electron mass). \( \eta_R = \beta ml_b/h^2 \) and \( \eta_D = \alpha ml_b/h^2 \), both inversely proportional to \( \sqrt{B} \) are the dimensionless Rashba and Dresselhaus coupling respectively.

The velocity operator plays an important role in study of transport properties including the spin Hall conductance. The velocity operator of a single particle is \( v = [\tau, H]/i\hbar \) \((\tau = x, y)\), from which we obtain

\[ v_x = \frac{\hbar}{\sqrt{2ml_y}} \left[ a_k^\dagger + a_k + \sqrt{2} \eta_D \sigma_x + \sqrt{2} \eta_R \sigma_y \right], \]  (6a)

\[ v_y = \frac{ih}{\sqrt{2ml_y}} \left[ a_k^\dagger - a_k + i \sqrt{2} \eta_D \sigma_y + i \sqrt{2} \eta_R \sigma_x \right] \]  (6b)

Comparing this with the standard expression of velocity for a charged particle in a magnetic field, \( v = \)
the Rashba and Dresselhaus couplings are interchanged. At the symmetric point \( \eta = -1 \) only Rashba coupling and an opposite sign in Dresselhaus coupling can be mapped on to a system with Rashba coupling only. Therefore a system with Rashba coupling \( \beta \) can be obtained by replacing \( \eta \) with \( -\eta \). This transformation is equivalent to the flip of the external magnetic field \( B \rightarrow -B \). Therefore, a system of hole carriers has the same physical properties as the corresponding electron system except for possible directional changes in the observables.

H0 can be solved analytically in the systems with only Rashba or only Dresselhaus coupling. An analytical solution is currently not available for \( H_0 \) with both couplings \( \alpha, \beta \). In the next section, we shall discuss the charge and spin Hall conductance of the electron system with a pure Rashba coupling. The results can be mapped easily onto the system with a pure Dresselhaus coupling and to the hole system in semiconductors by using the symmetries discussed above.

III. SYSTEMS WITH PURE RASHBA COUPLING

In this section we focus on systems with the Rashba coupling only. After a brief review of the single particle solution in the absence of an electric field, we will discuss the spin Hall conductance by using linear response theory in IIIIB, and its non-linear effect and scaling behavior near the resonance in IIIC. We will use the interchange symmetry to comment on the Dresshau coupling system.

A. Single particle solution

The single particle problem of \( H_0 \) with \( \eta_D = 0 \) can be solved. The Rashba coupling hybridizes a spin down state in the \( \eta_R \) Landau level with a spin-up state in the \( (\eta_R + 1) \) Landau level, and the eigenenergies are given by

\[
\epsilon_{ns}^R = \hbar \omega \left( n + \frac{s}{2} \sqrt{(1-g)^2 + 8n^2} \right)
\]

with \( s = \pm 1 \) for positive integer \( n \), and \( \epsilon_{0,+} = \hbar \omega (1-g)/2 \). There is a large degeneracy \( N_\eta = L_x L_y/(2\pi \hbar^2) \) to each eigenenergy. The corresponding eigenstates are given by

\[
|n, k, s\rangle = \left( \begin{array}{c} \cos \theta_n \phi_{nk} \\ i \sin \theta_n \phi_{n-1k} \end{array} \right)
\]

where \( \phi_{nk} \) is the eigenstate of the \( n \) Landau level with \( p_x = k \) in the absence of the spin-orbit coupling. \( \theta_{0,+} = 0 \), and tan \( \theta_{ns} = -u_n + s \sqrt{1 + u_n^2} \) for \( n \geq 1 \), with \( u_n = (1-g)/\sqrt{8n\eta^2} \).

The eigenenergies for the system with Dresselhaus coupling only can be obtained by replacing \( \eta_R \) by \( \eta_D \) and \( g \) by \( -g \).

\[
\epsilon_{ns}^D = \hbar \omega \left( n + \frac{s}{2} \sqrt{(1+g)^2 + 8n^2} \right)
\]

The energy spectra versus \( \eta_R \) or \( \eta_D \) are plotted in Fig. 2. In the absence of the spin-orbit coupling, the Zeeman energy splits the degenerate \( \eta_R \) Landau levels of spin-up and spin-down electron states into two nearby ones with the lower level for spin-up and the higher level for spin-down. As \( \eta_R \) increases from zero, the energy of the \( \eta_R \) Landau level state of spin-down is lowered because of its hybridization with the spin-up state at the \( (\eta_R + 1) \) Landau level due to the Rashba coupling. The Rashba interaction competes with the Zeeman energy and there is an energy crossing at certain values of \( \eta_R \) or the magnetic fields as we can see in Fig. 2(a). The spin Hall resonance we examine is closely related to this level crossing. The energy level diagram in Fig. 2(b) for the Dresselhaus coupling has different features. In that case, a spin-up state, which is at the lower level due to the Zeeman splitting, mixes with a spin-down state at a higher Landau level, which separate further the Zeeman splitting, thus there is no resonance in the spin Hall current.
the presence of an electric field $E\hat{y}$ is not analytically solvable, since the Landau levels mixing no longer truncates. After a replacement of $y \rightarrow y + eE/m_0\omega^2$ in the operator $a_k$ by $\tilde{a}_k = a_k + \frac{eE_b}{\sqrt{2}\hbar \omega}$, the Hamiltonian of the system in the presence of the electric field reads, apart from a constant,

$$H = H_0(E) + H'$$

where $H_0(E)$ is the one in Eq. \ref{eq:Hamiltonian} by replacing $a_k$ by $\tilde{a}_k$ and $H' = -eE_b\eta_R\sigma_y$. We now consider $H'$ as a perturbative Hamiltonian to study the charge and spin Hall currents. Up to the first order in $E$, we obtain

$$\langle j_{c,s}\rangle_{nks} = \langle j^{(0)}_{c,s}\rangle_{nks} + \langle j^{(1)}_{c,s}\rangle_{nks}$$

(17)

where the superscript refers to the $0^{th}$ order and $1^{st}$ order in the perturbation in $H'$, and

$$\langle j^{(0)}_{c,s}\rangle_{nks} = \langle n, k, s| j_{c,s} |n, k, s\rangle$$

$$\langle j^{(1)}_{c,s}\rangle_{nks} = \sum_{n', s'} \langle n, k, s| j_{c,s} |n', k', s'\rangle \langle n', k', s' | j_{c,s} |n, k, s\rangle$$

In the above equations, the summation is over the complete and orthogonal set of eigenstates $\{|n, k, s\rangle\}$ corresponding to the spectra $\{\epsilon^n_{n,s}\}$ in Eq. \ref{eq:energy_levels}, and $n' = n \pm 1$ since the matrix elements vanish for other values of $n'$. The charge Hall conductance is found to be independent of $\eta_s$ since spin is not a conserved quantity in the presence of spin-orbit coupling $G_c = \nu e^2/h$, with $\nu = N_c/N_\phi$ being the filling factor. Within the perturbation theory, the spin Hall conductance $G_s$ can be divided into two parts. The part arising from the $0^{th}$ order in $H'$ is found to be the product of the spin polarization $\langle S^z \rangle$ per electron and the Hall conductance $G_c$, divided by the electron charge $(-e)$,

$$G_s^{(0)} = - \langle S^z \rangle G_c/e. \quad (18)$$

The expectation value of the spin polarization per electron is,

$$\langle S^z \rangle = \frac{1}{N_e} \frac{\hbar}{2} \sum_{nks} \langle n, k, s| \sigma_z |n, k, s\rangle f_{nks}$$

$$= \frac{1}{N_e} \frac{\hbar}{2} \sum_{nks} \cos 2\theta_{n,s} f_{nks}. \quad (19)$$

$\langle S^z \rangle$ at $T = 0$ is plotted in Fig. 3(a). The oscillation is due to the alternate filling by electrons of the energy levels with mainly spin-up and spin-down. A jump is visible at $\nu = 12.6$ (corresponding to inverse magnetic field 0.162T$^{-1}$) because of the energy crossing.

The second part in $G_s$ arises from the first order in $H'$,

$$G^{(1)}_s = \frac{e\eta_R}{8\pi\sqrt{2}} \sum_{n,s,n'=n+1,s'} \frac{f_{n,s} - f_{n',s'}}{\epsilon^n_{n,s} - \epsilon^{n'}_{n',s'}}$$

$$\times \left( \sqrt{n} \sin 2\theta_{n,s} \sin^2 \theta_{n',s'} - \sqrt{n'} \cos^2 \theta_{n,s} \sin 2\theta_{n',s'} \right) \quad (20)$$

FIG. 2: (a) Energy levels in unit of $\hbar \omega$ as a function of the dimensionless Rashba coupling $\eta_R$. The parameters are $\beta = 0.9 \times 10^{-11}$ eVm, $n_c = 1.9 \times 10^{16}/m^2$, $m = 0.05 m_e$ and $g_s = 4$, taken from Reference for the inversion heterostructures In 0.53 Ga0.47 As/In0.52 Al0.48 As. (b) same as in (a), but for the Dresselhaus coupling $\eta_D$. 

B. Linear response theory: Spin Hall conductance

We consider the charge and spin Hall currents along the $x$–axis induced by an electric field along the $y$–axis as in Fig. 1. In terms of the velocity operator, the charge and spin-$z$ component current operators are defined by

$$j_c = -ev_x, \quad (11)$$

$$j_s = \frac{\hbar}{4} (\sigma_z v_x + v_x \sigma_z), \quad (12)$$

respectively. We refer readers to Ref. for the discussions on the other spin components. The symmetrized form of the spin current operator guarantees that it is Hermitian. Each single particle state $|\phi_{nks}\rangle$ carries a current $\langle \phi_{nks}|j_{c,s}|\phi_{nks}\rangle$. The average current density carried by the 2DEG is then given by

$$I_{c,s} = \frac{1}{L_x L_y} \sum_{nks} f_{nks} \langle \phi_{nks}|j_{c,s}|\phi_{nks}\rangle \quad (13)$$

where $f_{nks}$ is the Fermi-Dirac distribution function. Note that since spin is not a conserved quantity in the presence of spin-orbit couplings, the spin current defined above and the spin density do not satisfy a continuity equation. Nevertheless, the expectation values of the spin density and the spin current are well defined. The charge and spin Hall conductance are then given by

$$G_c = \frac{I_c^0}{E}; \quad (14)$$

$$G_s = \frac{I_s^0}{E}. \quad (15)$$

Unlike a free electron in an uniform magnetic field, the single particle problem with the spin-orbit coupling in
At $T = 0$, if the two degenerate energy levels (crossing point in Fig. 2(a)) are partially occupied, $G_s^z$ may become divergent. Mathematically, the resonance is given by the condition $2n < \nu < 2n + 1$ for the electron filling factor $\nu$, with $n$ an integer satisfying the equation

$$\sqrt{(1-g)^2 + 8n\eta_R^2} + \sqrt{(1-g)^2 + 8(n+1)\eta_R^2} = 2. \quad (21)$$

From the above condition, for a system with any $\eta_R \neq 0$, $\eta_D = 0$, and $g_s > 0$, there is a unique resonant magnetic field $B_0$ such that the resonant condition is satisfied. By symmetry, we obtain the resonance condition for the system with a pure Dresselhaus coupling, which is given by the solution for $n$ of the equation,

$$\sqrt{(1+g)^2 + 8n\eta_D^2} + \sqrt{(1+g)^2 + 8(n+1)\eta_D^2} = 2. \quad (22)$$

Unlike the pure Rashba coupling case, there is no solution for any $g_s > 0$ in the pure Dresselhaus coupling system. This is because the energy levels $\epsilon_{0,n}'$ and $\epsilon_{n,n}'$, with $n' = n \pm 1$ do not cross over, so the pairs of the crossing levels in Fig. 2(b) correspond to $n' \neq n \pm 1$ and do not contribute to the spin Hall conductance.

We have calculated the spin Hall conductance numerically. $G_s^z$ at $T = 0$ is shown in Fig. 3(b). In addition to the oscillation in $1/B$ similar to that of $\sigma_z$, there is a pronounced resonant peak at the filling $\nu = 12.6$. In Fig. 4, we show $G_s^z$ at several temperatures. The height of the resonance peak increases drastically as the temperature decreases below a few kelvin. In Fig. 5, we show the $T$-dependence of the height of the resonant peak and the two nearby side peaks. The characteristic temperature for the occurrence of the peak can be estimated to be the Zeeman energy $E_Z$, which is about $10K$ at the resonant field for the parameters in Fig. 2. More explicit derivation of this will be given in the next subsection.

### C. Non-ohmic spin Hall current and scaling behavior

In this section we study the non-linear effect of the electric field to the resonant spin Hall current and the scaling behavior. Since the resonance originates from the interference of two degenerate levels near the Fermi energy, we will focus on those two levels to examine the problem. As an example, we shall consider In$_{0.53}$Ga$_{0.47}$As/In$0.53$Ga$_{0.48}$As with the parameters given in Fig. 2, in which case the resonance occurs at the filling factor $\nu = 12.6$ (see Fig. 3(b)) and the relevant two levels are $|1\rangle = |n = 6, k, s = +1\rangle$ and $|2\rangle = |n + 1 = 7, k, s = -1\rangle$. The energy levels below the two levels are assumed to be fully filled, and all levels above the two to be empty. This is valid if $\hbar \omega \gg k_B T$. The Hamiltonian is then, up to a constant, reduced to a $2 \times 2$ matrix,

$$H_{\text{reduced}} = \begin{pmatrix} \Delta \epsilon & \epsilon_0 \\ \epsilon_0 & -\Delta \epsilon \end{pmatrix} \quad (23)$$

where

$$\Delta \epsilon = (\epsilon_{0,+1}^R - \epsilon_{7,-1}^R)/2,$$

$$\epsilon_0 = \langle 2 | H' | 1 \rangle = -eE_k \eta_R \cos \theta_{0,+1} \sin \theta_{7,-1}.$$
the Zeeman energy and the resonant point, $\Delta$ dimensionless magnetic field. Using the identity

$$f_+ + f_- = \delta_\nu = \nu - 2n, \mu \text{ chemical potential, and } i_{\pm} = \langle \Phi_{\pm} | j_{\pm}^z | \Phi_{\pm} \rangle.$$ 

The electric field and temperature dependences of the spin current $i$ and $f$

we obtain the singular part of the spin Hall conductance

$$G_s \approx \frac{-\delta_\nu e}{4\pi} \left[ \frac{E_Z}{(\Delta \epsilon)^2 + v_0^2} \right].$$

(26)

where the factor $f_-(1-f_+)/f_+(f_-+f_+)$ is a slowly varying function of $T$ ranging from 1 at low temperatures to $(1-\delta_\nu/2)/2$ at high temperatures. At low temperatures $G_s$ is given by,

$$G_s \approx -\frac{e}{4\pi} \frac{\delta_\nu}{|b|}.$$ 

(27)

It is only a function of the reduced magnetic field and the excess part of the filling factor from $2n$. At the resonant magnetic field, i.e., $b = 0$, the spin Hall current approaches with lowering temperature to a constant, $I_s = -\frac{|e|}{\pi^2} \delta_\nu E_Z/|\langle e| \eta_B \cos \theta_{n+1} \sin \theta_{n+1,-1} | \rangle$ as can be seen in Fig. 7. Using the resonance condition in Eq. (21), $B_0 \approx 4nm^2c^2\beta^2/\hbar g^3 (n = 6)$ and using the fact that for large $n$, $n$ is proportional to $1/B_0$, the resonant magnetic field $B_0 \propto \beta/\sqrt{g}$ approximately. Thus the resonant spin current is proportional to

$$I_s = -\frac{\delta_\nu e^2}{8\pi^2m^2c^2} \frac{gB_0^2}{\beta} \propto \delta_\nu \beta.$$ 

(28)

Therefore for a given filling factor, the larger the spin-orbit coupling $\beta$ is, the stronger the spin Hall resonance. The resulted spin Hall conductance diverges at $T = 0$ as

$$G_s \approx -\frac{\delta_\nu e}{4\pi} \frac{E_Z}{|v_0|} = -\frac{\delta_\nu e^2}{8\pi^2m^2c^2} \frac{gB_0^2}{\beta} \frac{1}{E}.$$ 

(29)

At temperatures $k_B T > \sqrt{(\Delta \epsilon)^2 + v_0^2}$,

$$G_s \approx \frac{\delta_\nu (1-\delta_\nu/2)e}{4\pi} \frac{E_Z}{k_B T}.$$ 

(30)
and the integral
\[ \int G_s \text{d}b \rightarrow \frac{\delta_e e^2}{2\pi} \left( \ln \frac{2E_z}{k_B T} \right), \]

This integral reflects the weight of the resonant peak of the spin Hall conductance.

Since the method used in this subsection is beyond perturbation theory, we conclude that the resonance spin Hall conductance we predict is not an artifact of the perturbation method. Instead, the resonance is caused by the interference between the two degenerate energy levels at the Fermi energy.

**IV. SYSTEMS WITH BOTH RASHBA AND DRESSELHAUS COUPLINGS**

In this section we briefly discuss the resonance in the spin Hall conductance in systems with both Rashba and Dresselhaus couplings. The Hamiltonian including the electric potential reads,
\[ H = H_0(E) + H' \]
with \( H' = -eE\hat{b}(\eta_D\sigma_x + \eta_R\sigma_y) \). In this case \( H_0 \) is not solvable analytically. A state \(|n_0, \downarrow\rangle \) (in the basis of the Landau levels with \( n_D = n_R = 0 \)) is coupled to \(|n_0 + 1, \uparrow\rangle \) via the Rashba coupling, which is further coupled to \(|n_0 + 2, \downarrow\rangle \) due to the Dresselhaus coupling. In this way, a Landau level is coupled to infinite number of other Landau levels, and the analytic solution is not available. The problem, however, may be approximately solved by using perturbation theory to treat \( \eta_R \) and \( \eta_D \) as small parameters. This is equivalent to the limit \( B \to \infty \), since \( \eta_R, R \propto 1/\sqrt{B} \). For the sample we consider in the present paper given in Fig. 2, \( \eta_R^2 = 0.004 \ll 1 \) at the resonant field \( B \approx 6.1 \text{ tesla} \). In the absence of the electric field, the single particle energy, up to the second order in \( \eta_R \) and \( \eta_D \), is given by

\[ \frac{\epsilon_{n_0, \downarrow}}{\hbar\omega} = n_0 + 1 - g \frac{2\eta_D^2}{1 + g} - \frac{2(n_0 + 1)\eta_R^2}{1 + g} \]  
\[ \frac{\epsilon_{n_0, \uparrow}}{\hbar\omega} = n_0 + 1 + g \frac{2\eta_D^2}{1 + g} - \frac{2(n_0 + 1)\eta_R^2}{1 - g} \]

Note that the mixed term of \( \eta_R^2 \eta_D \) does not appear in the perturbation to the second order. The two levels become degenerate if the following equation is satisfied,
\[ \frac{g}{2(2n_0 + 1)} = \frac{\eta_D^2}{1 - g} - \frac{\eta_R^2}{1 + g} \]  

It follows that a necessary condition for the resonant spin Hall current is \( \eta_D^2/\eta_R^2 > (1 - g)/(1 + g) \approx 1 \), for \( g \ll 1 \). At \( \eta_D = 0 \) and in the limit \( \eta_R \ll 1 \), Eq. (33) is consistent with Eq. (2) for the resonant condition we derived for the pure Rashba system. Alternatively the resonant magnetic field is
\[ B_0 \approx \frac{2(2n_0 + 1) m^2 c}{g} \frac{\eta_R^2}{\hbar^2} \frac{1}{(\beta^2 - \alpha^2)}. \]  

The large number \( n_0 \) increases with \( 1/B_0 \) for a specific density of particles. Thus for a certain Rashba coupling the increasing of Dresselhaus coupling will decrease the resonant magnetic field \( B_0 \). The singular part of the spin Hall conductance can be studied by examining the two level system in the presence of an electric field as we described in Section IIIC. At the resonant point and at low temperature,
\[ G_s = \frac{\delta_e e^2 h^2}{8\pi m^2 e^2} \frac{gB_0^2}{\sqrt{q^2 + \beta^2} E} \]

As the Dresselhaus coupling increases from zero, the resonance is shifted to lower magnetic fields, the resonance occurs at higher Landau levels with a weaker resonant strength.

**V. SUMMARY AND DISCUSSIONS**

In summary, we have studied the spin Hall effect in a two-dimensional electron system with spin-orbit couplings in a strong perpendicular magnetic field. In systems with the Rashba coupling dominating over the Dresselhaus coupling, there is a resonant magnetic field at which the spin Hall conductance diverges at low temperature and low electric field. The physics for this resonance is the energy level crossing of the two Landau levels due to the competition of the Zeeman splitting and the Rashba coupling. For a given system, there is a unique resonant magnetic field, at which the two Landau levels become degenerate at the Fermi energy. In this case, some physical properties may show singularity. As studied earlier, the spin polarization will change its sign as the magnetic field is varied passing through the resonant field. Namely the magnetic susceptibility is divergent. The spin Hall conductance is another singular response due to this level crossing. When an infinitesimally weak d.c. electric field is applied in the plane, the two degenerate Landau levels are split accordingly and a finite spin Hall current is induced. The resonance is macroscopic in the sense that a huge number of the states in the same Landau level are involved in the process. We have calculated the temperature and electric field dependences of the resonance. The characteristic temperature for the resonant spin Hall current is of order of the Zeeman energy. As the temperature decreases, the height of the resonance peak diverge like \( 1/T \) and the weight diverges like \( \propto \ln T \). While the spin orbit coupling has a dramatic effect on the spin Hall conductance, the charge Hall conductance is not affected and remains quantized. The spin Hall current is non-linear with the electric field.
at the resonant field. At low temperatures, the spin Hall current rapidly rises linearly with the electric field and saturates at higher electric fields. At $T = 0$, the spin Hall conductance diverges as $1/E$ at resonance. Near the resonant magnetic field $B_0$, it is $\propto 1/(B - B_0)$. Contrary to the Rashba coupling, the Dresselhaus coupling further increases the Zeeman energy splitting to suppress the effect of the Rashba coupling. The strength of the Rashba coupling necessary to surpass the Dresselhaus coupling by in order to have the resonant spin Hall current was estimated by using a perturbation method treating the couplings as small parameters. This is accurate as long as the Zeeman energy is much smaller than then cyclotron frequency. We have assumed no potential disorder in our theory. The effects of disorder in 2DEG with Rashba coupling, especially in a strong magnetic field, is not well understood at this point. Nevertheless, it seems reasonable to assume that the spin-orbit coupling does not change the effects of disorder qualitatively. This is likely to be the case in the presence of a strong magnetic field, which ensures extended states in the Landau levels when the disorder is not sufficiently strong as evidenced by the experimentally observed quantization of the Hall conductance. We then assume that the disorder gives rise to broadening of the Landau level and localization so that the extended states in a Landau level are separate in energy from those in the next one by localized states. Inspection of the spin-orbit coupling shows that Laughlin’s gauge argument still holds, and each Landau level with its extended states completely filled contribute $e^2/h$ to the charge Hall conductance. Thus we conclude that the quantum Hall conductance remains intact with the spin-orbit interaction, except at the special degeneracy point. As the Fermi energy varies across this degenerate extended state, the charge Hall conductance $G_C$ is expected to change by $2e^2/h,$ instead of $e^2/h$ for the other extended levels. This fact can be used experimentally to determine the Rashba interaction induced degeneracy discussed in this paper.

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