An alternative method for centrifugal compressor loading factor modelling

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Abstract: The loading factor at design point is calculated by one or other empirical formula in classical design methods. Performance modelling as a whole is out of consideration. Test data of compressor stages demonstrates that loading factor versus flow coefficient at the impeller exit has a linear character independent of compressibility. Known Universal Modelling Method exploits this fact. Two points define the function – loading factor at design point and at zero flow rate. The proper formulae include empirical coefficients. A good modelling result is possible if the choice of coefficients is based on experience and close analogs. Earlier Y. Galerkin and K. Soldatova had proposed to define loading factor performance by the angle of its inclination to the ordinate axis and by the loading factor at zero flow rate. Simple and definite equations with four geometry parameters were proposed for loading factor performance calculated for inviscid flow. The authors of this publication have studied the test performance of thirteen stages of different types. The equations are proposed with universal empirical coefficients. The calculation error lies in the range of plus to minus 1,5%. The alternative model of a loading factor performance modelling is included in new versions of the Universal Modelling Method.

Nomenclature

- \( b \) width of channel
- \( c_u \) tangential component of absolute flow velocity
- \( D \) diameter
- \( h_f \) theoretical head, i.e. the head transferred to gas by impeller blades
- \( K_{pd} \) empirical coefficient of velocity diagram
- \( K_{\mu} \) empirical coefficient of viscosity influence
- \( l \) length of blade
- \( l_t \) cascade solidity
- \( \bar{m} \) mass flow rate
- \( M_u \) blade Mach number
- \( N_i \) internal power, i.e. all power transferred to gas by an impeller
- \( p \) pressure
- \( r \) radius
- \( u \) tangential velocity
\( \chi_{x_0} \) empirical coefficient
\( \dot{w} \) relative velocity
\( \Gamma \) flow circulation
\( y \) empirical coefficient
\( \varpi \) number of blades, empirical coefficient
\( \beta \) angle between the relative speed and the reverse district direction
\( \beta_{bl} \) blade angle
\( \beta_{df} \) non-dimensional coefficient of disk friction
\( \beta_{lk} \) non-dimensional leakage coefficient
\( \beta_{T} \) angle of a loading factor performance inclination
\( \varphi \) flow coefficient
\( \eta \) polytrophic efficiency
\( \lambda \) friction coefficient
\( \omega \) rotation speed
\( \Phi \) flow rate coefficient
\( \psi_{T} \) loading factor.

**Subscripts**

0 impeller inlet
1 impeller blade row inlet
2 impeller outlet
ni non incidence
inl inlet
inv inviscid flow
bl blade
boud boundary layer
imp impeller
des design
max maximum
pd center of pressure

### 1. Aim of the work

For flow path design, an instrument is necessary for calculation of centrifugal compressor gas dynamic performance. The authors use the primary design procedure described in [1]. The Universal Modelling Method presented in [2] is applied to calculate performance of a primary design and of possible better candidates. Calculated non-dimensional performance of a stage is presented in Figure 1.
Figure 1. Calculated non-dimensional performance of a centrifugal compressor stage

Efficiency $\eta$ and work coefficient $\psi_i$ are necessary for calculation of polytrophic coefficient and pressure ratio:

$$\psi_p = \psi_i \times \eta.$$  \hspace{1cm} (1)

$$\pi = \left(1 + (k-1)\psi_iM_p^2\right)^{\frac{k}{k-1}}.$$  \hspace{1cm} (2)

Efficiency prediction is possible with new versions of the Universal Modelling PC programs without special skill and experience [2-4]. Prediction of the work coefficient performance $\psi_i = f(\Phi)$ still requires experience and intuition. The authors’ aim is to offer an alternative way of the work coefficient performance calculation that is simple and precise. Presented work is based on ideas and results of Y. Galerkin and K. Soldatova who had studied loading factor performances of impellers with inviscid flows.

2. Scheme of work coefficient modelling

In accordance with the scheme proposed previously in [5] the head transmitted to gas by an impeller $h_i$ consists of three parts. The main part of engine’s mechanical head $h_i$ is transferred to gas by blades of the impeller. Two additional parts appear due to parasitic losses. The part of the head $h_{ic}$ is lost in labyrinth seals. The friction on outer surfaces of hub and shroud transmits additional mechanical head $h_{df}$ but this head does not increase gas pressure:

$$h_i = h_f + h_{df} + h_{ic}.$$  \hspace{1cm} (3)

Non-dimensional presentation of this equation:

$$\psi_i = \psi_f \left(1 + \beta_{df} + \beta_{ic}\right).$$  \hspace{1cm} (4)
For an impeller with large flow coefficient \( \Phi_{des} \approx 0.15 \) the sum \( \beta_{df} + \beta_{bk} \) is less than 0.01. For an impeller with small flow coefficient \( \Phi_{des} \approx 0.015 \) this sum is about 0.055-0.065. It is not large part of a head coefficient. Semi-empirical formulae in [1] and CFD-calculations [9] are good instruments for modelling of these coefficients. Therefore the main problem is modelling of a loading factor.

As a rule there is no velocity tangential component at an impeller inlet in industrial compressors. Then impeller blades transfer the head to a gas in accordance with the Euler equation:

\[
h_T = c_{u_2}u_z.
\]

This head is presented by the non-dimensional loading factor:

\[
\psi_T = \frac{c_{u_2}}{u_z}.
\]

The modelling of a loading factor performance is facilitated by the fact that it is a linear function – Figure 2.

![Figure 2. Linear function of a loading factor for ideal and real impellers](image)

The linear nature of it is evident for an “ideal” impeller with infinite number of infinitely thin blades:

\[
\psi_{T,\infty} = 1 - \phi_2' \cot \beta_{bl2}
\]

The similar equation is valid for real impellers:

\[
\psi_T = 1 - \phi_2' \cot \beta_2.
\]

The flow angle \( \beta_2 \) is not constant versus flow coefficient. Anyway test data for industrial compressors demonstrates practically a linear characteristic of work input versus flow rate [5]. Model stages’ tests demonstrated linear characteristic of the function \( \psi_T = f(\phi_2') \) independent of blade Mach number [10].
The existing way of modelling [2-4] exploits this fact. Two values of a loading factor are necessary to determine a linear performance \( \psi_T = f(\phi^I) \).

In the method described in [1] these two values are a loading factor at a design flow rate \( \psi_{T_{\text{des}}} \) and at zero flow rate \( \psi_{T_{0}} \). The design flow rate corresponds to non-incidence inlet of the critical streamline. This condition is \( \beta_{\text{av}} = \beta_{\text{bl}} \). Exit and inlet velocity triangles demonstrates influence of blades’ load and blockade on a critical streamline direction – Figure 3.

![Figure 3. Inlet and exit velocity triangles at design flow rate](image)

The absolute velocity \( c_i \) is accelerating near a blade cascade inlet due to a blade blockade: \( c_i' = c_i / \tau_1 \). A critical streamline turns its direction to a suction side of a blade where pressure is less than on a pressure side. Symbols “plus/minus” in the Figure 3 demonstrate difference of pressures on blade surfaces. A critical streamline obtains a tangential velocity \( \Delta c_{u1} \). The scheme of the \( \Delta c_{u1} \) calculation is based on the idea of representing a blade by a swirl with the circulation \( \Gamma_{\text{av}} = 2 \pi r c_{u2}/z \). The swirl induces velocities depending of distance from its center. The equation to calculate tangential velocity induced on the radius \( r_1 \) where the blades start is presented in non-dimensional way:

\[
\Delta \bar{c}_{u1} = \frac{\psi_{T_{\text{des}}}}{z(\bar{r}_{pe} - \bar{r}_1)}.
\]  

(9)

Here \( \bar{r}_{pe} = r_{pe} / r_2 \) is a relative distance from an impeller axis to a center of a velocity diagram, Figure 4.

A loading factor at design flow rate is calculated on the same principle. The velocity triangle is shown above in the Figure 2. In accordance with Kutta-Zoukovsky postulate a critical streamline leaves blades in direction of an angle \( \beta_{\text{bl}} \). Due to a blade load a pitch-averaged velocity got a tangential velocity component:

\[
\Delta \bar{c}_{u2,\text{des}} = -K_{\psi} \frac{\psi_{T_{\text{des}}}}{z(1-\bar{r}_{pe})}.
\]  

(10)
The equation for a loading factor is:

\[
\psi_{T,\text{des}} = \frac{1 - \varphi^*_{T,\text{des}} \tan \beta_{bl2}}{1 + K'_{\mu} \frac{1}{zo(1 - \rho_c)}}. \tag{11}
\]

Figure 4. Velocity diagram at design flow rate (inviscid flow)

The empirical coefficient \( K'_{\mu} > 1 \) represents influence or real viscous character of flow. Its value for different types of impellers may be between 1.5-2.3. There is no satisfactory correlation with an impeller configuration. The close analog is necessary. The alternative is to model a loading factor performance by values of \( \psi_{T,0} \) and angle \( \beta_T \) that are shown in Figure 2:

\[
\psi_T = \psi_{T,0} - \varphi^* \tan \beta_T. \tag{12}
\]

The aim of this work is to model loading factor performance of impellers in real viscous flow on the basis of several impeller geometry parameters.

Test data for model stages and factory test data of several pipeline compressors were reduced. The information on the objects of modelling is presented in Table 1. All stages and compressors were tested at \( M_u = 0.60 \) or 0.80. Model stage names mean the following. For example, 0.0604-0.527-0.290 means that the design flow rate coefficient of the stage is \( \Phi_{\text{des}} = 0.0604 \), design loading factor is \( \psi_{T,\text{des}} = 0.527 \), hub ratio is \( D_h / D_2 = 0.29 \). Names of compressors are given by their manufacturers on their own principles. Data on compressors are taken from [4]. Symbol * means that 2D impeller has an arc blade mean line. Mean lines are designed by the Method presented in [1] in other cases. In columns 3-9 geometry parameters of impellers are presented, they are included in the presented below equations for loading factor performances modelling.

| Parameter/ stage | \( b_i / b_1 \) | \( z \) | \( D_1 / D_2 \) | \( 1/t \) | \( \beta_{bl1}^0 \) | \( \beta_{bl2}^0 \) | \( \delta_{bl} \) | \( K'_{\mu} \) | \( X_{\psi,0} \) |
|------------------|----------------|------|----------------|--------|----------------|----------------|-------------|-----------|-----------|
| 1                | 0.0373-0.482-0.373* | 0.754 | 15 | 0.514 | 3.423 | 25 | 30.3 | 0.0133 | 2.0 | 2.1 |
| 2                | 0.0455-0.539-0.350* | 0.589 | 13 | 0.565 | 2.40 | 25 | 34 | 0.02 | 1.05 | 2.3 |
| 3                | 0.0480-0.49-0.290  | 0.573 | 11 | 0.534 | 2.461 | 23 | 30 | 0.014 | 1.65 | 1.3 |
The sample of data on model stages from the “IDENT” program is presented in Figure 5. The dimensions that are necessary for calculation of efficiency and a loading factor performances are presented in the database. The measured values of these parameters are black figures and lines. The efficiency calculated according to the 6th version of model with a universal set of coefficients are red figures and lines. The values \( T_X \) (the coefficient for an empirical equation for \( T_Y \) calculation) and \( K_\mu \) are picked up individually for each stage for the best compliance to the measured performances. These values are presented in columns 10, 11 in the table 1. The program calculates also values \( T_B \) and \( T_Y \) corresponding to preset values \( T_X \) and \( K_\mu \).

![Figure 5](image)

**Figure 5.** An example of information from the database of the IDENT program.

The authors have studied all impeller geometry parameters as possible arguments in functions \( \beta_T, \psi_{T0} = f(\varphi, \bar{F}) \), where \( \bar{F} \) - symbol of geometry parameters that define an impeller flow path. Main dimensions of a typical industrial impeller in meridian plane are shown in Figure 6.

Among dimensions shown in Figure 6 only the ratio \( b_r / b_n \) influences functions \( \beta_T, \psi_{T0} = f(\varphi, \bar{F}) \). The most important parameter is a blade exit angle \( \beta_{bl} \). Ratio of a blade length \( l \) to average pitch \( t \) presents in functions too. This ratio is calculated by the formula from [5]:

| Stage No. | \( T_X \) | \( \psi_{T0} \) | \( T_B \) | \( \psi_{T0} \) |
|----------|------------|----------------|-----------|----------------|
| 4        | 0.0604-0.290 | 0.618          | 0.592     | 1.916          |
| 5        | 0.0653-0.290 | 0.657          | 0.570     | 1.967          |
| 6        | 0.0692-0.290 | 0.707          | 0.570     | 2.062          |
| 7        | 0.0685-0.345 | 0.539          | 0.577     | 1.864          |
| 8        | 16-76-1.6   | 0.693          | 0.5765    | 1.987          |
| 9        | 108-5-1*    | 0.587          | 0.5651    | 2.399          |
| 10       | GPA-76-1.7  | 0.614          | 0.583     | 2.227          |
| 11       | 61-1.64     | 0.549          | 0.548     | 2.518          |
| 12       | 650-1.37-76 | 0.711          | 0.570     | 1.967          |
| 13       | 16-76-1.7   | 0.360          | 0.5       | 2.996          |
Most of 2D impellers designed by the authors have either an arc mean line of blades, or a mean line is optimized by analysis of velocity diagrams. Two mean lines and corresponding velocity diagrams are presented in Figure 7.

\[ \frac{1}{t} = \frac{\lg \frac{D_2}{D_1}}{2.73 \sin \left( \frac{\beta_{2c} + \beta_{1c}}{2} \right)} \]  

(13)

\[ \psi_{\tau_0} = 0.964 + 0.002 \left( \frac{\beta_{2e}}{40} \right)^4 - 0.068 \left[ \ln \left( \frac{\ell}{t} \right) \right]^3 - 0.00011 \left[ \ln \left( \frac{b_2}{b_1} \right) \right]^4. \]  

(14)
- for impellers with arc mean line:

\[

\psi_{T_0} = 0.747 + 0.097 \left( \frac{\beta_{M2}}{40} \right)^4 - 0.008 \left[ \ln \left( \frac{l}{t} \right) \right]^7 - 20.247 \left[ \ln \left( \frac{b_2}{b_1} \right) \right]^9.
\]  

(15)

Angle \( \beta_f^0 \):
- for impellers with optimized mean line:

\[

\beta_f^0 = 0.534 \frac{\beta_{M2}}{40} + 0.028 \left( \frac{l}{t} \right)^6 - 3.577 \left[ \ln \left( \frac{b_2}{b_1} \right) \right]^{-1}.
\]  

(16)

- for impellers with arc mean line:

\[

\beta_f^0 = 20.311 \left( \frac{\beta_{M2}}{40} \right)^{-2} + 3.595 \left( \frac{l}{t} \right) - 0.106 \left[ \ln \left( \frac{b_2}{b_1} \right) \right]^4.
\]  

(17)

Figures 8, 9 demonstrate accuracy of approximation.

**Figure 8.** Comparison of the measured and calculated \( \beta_f \) and \( \psi_{T_0} \) for impellers with different \( \beta_{M2} \) (optimized mean line)
Figure 9. Comparison of the measured and calculated $\beta_t$ and $\Psi_{T,0}$ for impellers with different $\beta_{hi2}$ (arc mean line).

Modelling accuracy is satisfactory for an exception of the 1\textsuperscript{st} stage of 6-stage pipeline compressor GPA-76-1.7 (line 10 Table 1). For impellers of this compressor the empirical coefficient is $K_\mu = 2.62$ (eq. 10). It is an unusually large value. The authors have no explanation for this fact. The modelling error in all other cases is within admissible limits. Figure 10 presents the comparison of performance of one of model stages.

Graphics with individual adjustment of the loading factor performance are presented on the left. Right – loading factor is calculated by eq. (14, 16). Efficiency performance is calculated by 6\textsuperscript{th} version of the Math model in both cases. Despite visually noticeable difference of load performances, an error of calculation $\psi_{T, des}$ is 1.2%. It means that an impeller designed by this method will have $+1.2\%$ of head input. It is acceptable for design practice.

Figure 10. Model stage performance comparison (#3, $\beta_{hi2}=79^\circ$). Green – measured, red – calculated. Right – loading factor is calculated by eq. (15, 17).
Conclusion
The authors plan to apply the presented method of a loading factor performance calculation in parallel with the existing method in their future projects. In case of a positive result the new method will be incorporated into the Math models. This method is simple and does not require users’ high experience and corrections with analogs.

Acknowledgment
The reported study was funded by RFBR according to the research project № 16-08-00624 A.

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