The $\Delta(1232)$ Resonance in Chiral Effective Field Theory

Vladimir Pascalutsa

ECT* Trento, Villa Tambosi, Villazzano (TN), I-38050, Italy

Abstract
I discuss the problem of formulating the baryon chiral perturbation theory ($\chi$PT) in the presence of a light resonance, such as the $\Delta(1232)$, the lightest nucleon resonance. It is shown how to extend the power counting of $\chi$PT to correctly account for the resonant contributions. Recent applications of the resulting chiral effective-field theory to the description of pion production reactions in $\Delta$-resonance region are briefly reviewed.

1 Introduction
A quantitative description of the low-energy physics of nucleons and pions based on the underlying theory of the strong interaction, quantum chromodynamics (QCD), is still lacking, and the Millennium prize for a solution of this problem is still outstanding. Lattice QCD holds a promise to solve the problem one day by a sheer brute force, through a Monte-Carlo simulation of QCD in discrete Euclidean spacetime. In the absence of such a solution, the most appropriate framework for the description of the low-energy strong interaction is Chiral Perturbation Theory ($\chi$PT), an effective field theory of QCD written directly in terms of the hadronic degrees of freedom [1].

The main guiding principle in the construction of $\chi$PT is the chiral symmetry of massless QCD Lagrangian and the pattern of its breaking, which allows to organize the effective Lagrangian in powers of derivatives of the Goldstone boson fields – the pions, schematically:

$$\mathcal{L}(\pi, N, \ldots) = \sum_n \mathcal{L}^{(n)} = \sum_n O_n(c_i) \frac{\partial^n \pi}{\Lambda^n}$$

where $O_n$ are some field operators which may contain pion fields but not their derivatives. The all-possible field operators, constrained by chiral and other symmetries, appear with the free parameters, $c_i$, the so-called low energy constants (LECs). The mass scale $\Lambda$ is the heavy scale which sets the upper limit of applicability of $\chi$PT and is believed to be of order of 1 GeV, the scale of spontaneous chiral symmetry breaking that leads to the appearance of the Goldstone bosons.

This expansion of the Lagrangian translates into a low-energy expansion of the $S$-matrix:

$$S = \sum_n A_n(c_i) \frac{p^n}{\Lambda^n}$$

where $A$’s are amplitudes which depend on LECs, and $p$ denotes the typical momentum of the particles. If an analogous expansion could be obtained directly from QCD, it would be equivalent to the $\chi$PT one,

---

"Based on a seminar given at Erice School "Quarks in Hadrons and Nuclei", 29th Course, 16–24 Sep 2007, Sicily. Supported by the European Community-Research Infrastructure Activity under FP6 contract RII3-CT-2004-506078."
provides the LECs are matched to the QCD coupling, i.e., $c_n = c_n(\Lambda_{QCD})$. At present however the best one can do is to match the LECs to experimental data, and hope they take reasonable (natural) values, such that this above expansion is convergent.

One case where the convergence of the $\chi$PT expansion is immediately questioned is the case of hadronic bound states and resonances. In the presence of a bound state or a resonance the low-energy expansion of the $S$-matrix goes as:

$$S \sim \sum_n A_n \left( \frac{p}{\Delta E} \right)^n,$$

where $\Delta E$ is the excitation (binding) energy of the resonance (bound state). Thus, the limit of applicability of $\chi$PT is limited not by $\Lambda \sim 1 \text{ GeV}$ but by the characteristic energy scale $\Delta E$ of the closest bound or excited state. Furthermore, in the vicinity of a bound state or a resonance the $S$-matrix has a pole, which cannot be reproduced in a purely perturbative expansion in energy that is utilized in $\chi$PT.

This problem arises in various contexts, ranging from pion-pion scattering[2] to halo nuclei[3]. Here I shall consider the case of the $(1232)$, which is an ideal study case for the problem of resonances in $\chi$PT. It is relatively light, with the excitation energy of $\Delta \equiv M_\Delta - M_N \approx 300 \text{ MeV}$, elastic, and well separated from the other nucleon resonances. It is also a very prominent resonance and plays an important role in many processes, including astrophysical ones, e.g., it is responsible for the the so-called GZK cutoff (damping of the high-energy cosmic rays by the cosmic microwave background).

2 Power counting for the $\Delta$ resonance

Let me start with a simple example: Compton scattering on the nucleon. The total cross-section of this process, as the function of photon energy $\omega$, is shown in Fig. 1. In this case we are able to examine the entire energy range, starting with the soft-photon limit $\omega \approx 0$, through the pion production threshold $\omega \approx m_\pi$ and into the resonance region $\omega \sim \Delta$. At energies up to around the pion production threshold the cross section shows a smooth behavior which can reproduced by a low energy expansion. In this region the $\Delta$-resonance can be “integrated out", as its tail contribution can be mimicked by the terms already present in the $\chi$PT Lagrangian with nucleons only[5].

Higher in energy, however, the rapid energy variation induced by the resonance pole is not reproducible by a naive low-energy expansion. Obviously, to describe this behavior it is necessary to introduce the $\Delta$ as an explicit degree of freedom, hence include a corresponding field in the effective chiral Lagrangian. The details of how this is done have recently been reviewed in[6].

Once the $\Delta$ appears in the Lagrangian the question is how to power-count its contributions. In $\chi$EFT with pions and nucleons alone the power-counting index of a graph with $L$ loops, $N_\pi$ ($N_N$) internal pion (nucleon) lines, and $V_k$ vertices from $k$th-order Lagrangian is found as

$$n_{\chi\text{PT}} = 4L - 2N_\pi - N_N + \sum_k kV_k.$$  

What about the graphs with the $\Delta$, such as those depicted in Fig. 2? Their power counting turns out to be dependent on how one weighs the excitation energy $\Delta$ in comparison with the other mass scales of the theory. In this case we have the soft momentum $p$ (or, $\omega$), the pion mass $m_\pi$, and heavy scales which we collectively denote as $\Lambda$.

The Small Scale Expansion[7] (SSE) counts all light scales equally: $p \sim m_\pi \sim \Delta$. The small parameter is then: $\epsilon = \{p/\Lambda, m_\pi/\Lambda, \Delta/\Lambda\}$. An unsatisfactory feature of such a counting is that the $\Delta$-resonance contributions are always estimated to be of the same size as the nucleon contributions. As we have seen from Fig. 1 in reality the resonance contributions are suppressed at low energies while being dominant in the resonance region. Therefore, the power counting overestimates the $\Delta$-contributions at lower energies and underestimates them at the resonance energies.
Table 1: The counting for the nucleon, one-Delta-reducible (ODR), and one-Delta-irreducible (ODI) propagators in the two different expansion schemes. The counting in the $\delta$-expansion depends on the energy domain.

|           | $\epsilon$-expansion | $\delta$-expansion |
|-----------|------------------------|---------------------|
|           | $p/\Lambda \chi_{SB} \sim \epsilon$ | $p \sim m_{\pi}$ | $p \sim \Delta$ |
| $S_N$     | $1/\epsilon$          | $1/\delta^2$       | $1/\delta$       |
| $S_{ODR}$ | $1/\epsilon$          | $1/\delta$         | $1/\delta^3$     |
| $S_{ODI}$ | $1/\epsilon$          | $1/\delta$         | $1/\delta$       |

A more adequate power counting is achieved by separating out the resonance energy, e.g., maintaining the following scale hierarchy, $m_{\pi} \ll \Delta \ll \Lambda$, within the power-counting scheme. In the so-called “$\delta$ expansion” the counting depends on the energy domain: in the low-energy region ($p \sim m_{\pi}$) and the resonance region ($p \sim \Delta$), the momentum counts differently. This dependence most significantly affects the counting of the one-Delta-reducible (ODR) graphs. The first row of graphs in Fig. 2 illustrates examples of the ODR graphs for the Compton scattering case. These graphs are all characterized by having a number of ODR propagators, each going as

$$S_{ODR} \sim \frac{1}{s - M_{\Delta}^2} \sim \frac{1}{2M_{\Delta} p - \Delta},$$

where $s = M_N^2 + 2M_N \omega$ is the Mandelstam variable, and the soft momentum $p$ in this case given by the photon energy. In contrast, the nucleon propagator in analogous graphs would go simply as $S_N \sim 1/p$. Therefore, in the low-energy region, the $\Delta$ and nucleon propagators would count respectively as $O(1/\delta)$ and $O(1/\delta^2)$, the $\Delta$ being suppressed by one power of the small parameter as compared to the nucleon. In the resonance region, the ODR graphs obviously all become large. Fortunately, they all can be subsumed, leading to “dressed” ODR graphs with a definite power-counting index. Namely, it is not difficult to see that the resummation of the classes of ODR graphs results in ODR graphs with only a single ODR propagator of the form

$$S_{ODR}^* = \frac{1}{S_{ODR} - \Sigma} \sim \frac{1}{p - \Delta - \Sigma},$$

where $\Sigma$ is the $\Delta$ self-energy. The expansion of the self-energy begins with $p^3$, and hence in the low-energy region does not affect the counting of the $\Delta$ contributions. However, in the resonance region the self-energy not only ameliorates the divergence of the ODR propagator at $s = M_{\Delta}^2$ but also determines power-counting index of the propagator. Defining the $\Delta$-resonance region formally as the region of $p$

$$|p - \Delta| \leq \delta^3 \Lambda,$$

we deduce that an ODR propagator, in this region, counts as $O(1/\delta^3)$. Note that the nucleon propagator in this region counts as $O(1/\delta)$, hence is suppressed by two powers as compared to ODR propagators. Thus, within the power-counting scheme we have the mechanism for estimating correctly the relative size of the nucleon and $\Delta$ contributions in the two energy domains. In Table 1 we summarize the counting of the nucleon, ODR, and one-Delta-irreducible (ODI) propagators in both the $\epsilon$- and $\delta$-expansion.

In the following I will show two applications of the $\delta$ expansion to the calculation of processes in the $\Delta$-resonance region.
3 Pion electroproduction

The pion electroproduction on the proton in the Δ-resonance region has been under an intense study at many electron beam facilities, most notably at MIT-Bates, MAMI, and Jefferson Lab. The primary goal of these recent experiments is to measure electromagnetic $N \to \Delta$ transition, which comes in three different multipoles: $M_1$, $E2$, and $C2$. On the theory side, these form factors have been studied in both the SSE\textsuperscript{[8, 9]} and the $\delta$-expansion\textsuperscript{[10, 11]}.

The $N \to \Delta$ transition can be induced by a pion or a photon. The corresponding effective Lagrangians are written as:

\begin{align}
\mathcal{L}_{N\Delta}^{(1)} &= \frac{i h_A}{2 f_\pi M_\Delta} \bar{\gamma}^{\mu\nu\lambda} \partial_{\nu} \Delta_{\mu} \partial_{\lambda} \pi^a + \text{H.c.}, \\
\mathcal{L}_{N\Delta}^{(2)} &= \frac{3 i e g_M}{2 M_N (M_N + M_\Delta)} \bar{T}^{\mu} (\partial_{\nu} \Delta_{\mu}) \hat{F}^{\mu\nu} + \text{H.c.}, \\
\mathcal{L}_{N\Delta}^{(3)} &= \frac{-3 e}{2 M_N (M_N + M_\Delta)} \bar{T}^{\mu} \gamma_5 \left[ g_E (\partial_{\nu} \Delta_{\mu}) + \frac{i g_C}{M_\Delta} \gamma^{\alpha} (\partial_{\alpha} \Delta_{\nu} - \partial_{\nu} \Delta_{\alpha}) \partial_{\mu} \right] \hat{F}^{\mu\nu} + \text{H.c.},
\end{align}

where $N$, $\Delta$, $\pi$ stand respectively for the nucleon (spinor, isodoublet), $\Delta$-isobar (vector-spinor, isosquartet), pion (pseudoscalar, isovector) fields; $F^{\mu\nu}$ and $\hat{F}^{\mu\nu}$ are the electromagnetic field strength and its dual, $T^{\mu}$ are the isospin-1/2-to-3/2 transition ($2 \times 4$) matrices. The coupling constants $h_A$, $g_M$, $g_E$, and $g_C$ are the LECs describing the $N \to \Delta$ transition at the tree level.

We consider now the pion electroproduction on the nucleon to NLO in the $\delta$ expansion. Since we are using the one-photon-exchange approximation\textsuperscript{[8]} the pion photoproduction can be viewed as the particular case of electroproduction at $Q^2 = 0$. The pion electroproduction amplitude to NLO in the $\delta$-expansion, in the resonance region, is given by the graphs in Fig.\textsuperscript{[3]} where the shaded blob in the 3rd graph denotes the NLO $\gamma N \Delta$ vertex. The 1st graph in Fig.\textsuperscript{[3]} enters at the LO, which here is $O(\delta^{-1})$. All the other graphs in Fig.\textsuperscript{[3]} are of NLO = $O(\delta^0)$.

In Fig.\textsuperscript{[4]} the different virtual photon absorption cross sections around the resonance position are displayed at $Q^2 = 0.127$ GeV$^2$, where recent precision data are available. We compare these data with the present $\chi$EFT calculations as well as with the results of SL, DMT, and DUO models\textsuperscript{[15, 14, 16]}. In the $\chi$EFT calculations, the low-energy constants $g_M$ and $g_E$ were fixed from the resonant pion photoproduction multipoles. Therefore, the only other low-energy constant from the chiral Lagrangian entering the NLO calculation is $g_C$. The main sensitivity on $g_C$ enters in $\sigma_{TL}$. A best description of the $\sigma_{TL}$ data at low $Q^2$ is obtained by choosing $g_C = -2.6$. One sees that the NLO $\chi$EFT calculation, within its accuracy, is consistent with the experimental data for these observables at low $Q^2$.

Since the low-energy constants $g_M$, $g_E$, and $g_C$ are fixed to experiment, one can provide a prediction for the $m_{\pi}$ dependence of the $\gamma N \Delta$ transition. The study of the $m_{\pi}$-dependence is crucial to connect to lattice QCD results, which at present can only be obtained for larger pion masses. In Fig.\textsuperscript{[5]} one sees the $m_{\pi}$-dependence of the ratios $R_{EM} = E2/M1$ and $R_{SM} = C2/M1$ and compare them to lattice QCD calculations. The recent state-of-the-art lattice calculations of $R_{EM}$ and $R_{SM}$\textsuperscript{[21]} use a linear, in the quark mass ($m_q \propto m_\pi^2$), extrapolation to the physical point, thus assuming that the non-analytic $m_q$-dependencies are negligible. The thus obtained value for $R_{SM}$ at the physical $m_{\pi}$ value displays a large discrepancy with the experimental result, as seen in Fig.\textsuperscript{[5]}. This $\chi$EFT calculation, on the other hand, shows that the non-analytic dependencies are not negligible. While at larger values of $m_{\pi}$, where the $\Delta$ is stable, the ratios display a smooth $m_{\pi}$ dependence, at $m_{\pi} = \Delta$ there is an inflection point, and for $m_{\pi} \leq \Delta$ the non-analytic effects are crucial. The $m_{\pi}$ dependence obtained here from $\chi$EFT clearly shows that the lattice results for $R_{SM}$ may in fact be consistent with experiment.

\textsuperscript{1}For first analyses of the two-photon-exchange effects in the $\gamma N \to \Delta$ transition see Refs.\textsuperscript{[12, 13].}
4 Radiative pion photoproduction

The radiative pion photoproduction ($\gamma N \rightarrow \pi N \gamma'$) in the $\Delta$-resonance region is used to access the $\Delta^+$ magnetic dipole moment (MDM) \cite{22, 23, 24}. The pioneering experiment\cite{25} was carried out at MAMI in 2002 and a series of dedicated experiments were run in 2005 by the Crystal Ball Collaboration with first results announced at this school \cite{26}.

In this process, the energy flow can be defined by the energies of incoming and outgoing photon. To access the MDM of the $\Delta$ the energy of the incoming photon must be sufficient to excite the resonance, while the emitted photon must be soft. Therefore, in computing this process, one uses a chiral expansion with $\Delta$-isobar degrees of freedom, the $\delta$-expansion, and simultaneously the soft-photon expansion with respect to the energy of the emitted photon. In Ref. \cite{27}, the soft-photon expansion is performed to the next-next-to-leading order, since this is the order at which the MDM first appears, while the chiral expansion is performed to next-to-leading order.

An interesting effect which can be studied here is the absorptive (or, imaginary) part of the MDM \cite{28, 29}. It arises due to the unstable nature of the $\Delta$-isobar. In our $\chi$EFT calculation, for instance, we find the following result for the $\Delta^+$ MDM (in the heavy-baryon limit):

\[
\text{Im} \mu_{\Delta^+} = \frac{h^2 M_\Delta}{48\pi f_\pi^2} \sqrt{\Delta^2 - m_\Delta^2} \left(e/2M_\Delta\right).
\]

(9)

The absorptive MDMs quantify the change in the width of the resonance that occurs in an external magnetic field $B$:

\[
\Delta \Gamma = 2 \text{Im} \mu_{\Delta} \vec{B} \cdot \vec{n}_s,
\]

(10)

where $\vec{n}_s$ is the direction of the resonance’s spin. Equivalently, one may look for a change in the lifetime of the resonance: $\Delta \tau / \tau = -2 \text{Im} \mu_{\Delta} \vec{B} \cdot \vec{n}_s \tau$, where $\tau = 1/\Gamma$ is the lifetime. Such a change in the lifetime appears to be extremely small in moderate magnetic fields and is difficult to be observed directly \cite{30}. There is perhaps a possibility to compute the absorptive MDMs of hadron resonances in lattice QCD where the effect of arbitrarily large magnetic fields on the width can be studied.

The $\chi$EFT description of the $\gamma p \rightarrow \pi^0 p \gamma'$ unpolarized cross section was found to be consistent with first experimental data for this process\cite{27}. It appears that, at low energies of the outgoing photon, the dependence of the cross-section and linear-photon asymmetries on the MDM is quadratic, i.e., depends on $|\mu_\Delta|^2$. The asymmetry for a circularly polarized photon beam, however, displays a linear dependence on the $\Delta^+$ MDM. The helicity difference for a circularly polarized photon beam vanishes when approaching the soft-photon limit, with a rate that is proportional to the MDM. Therefore, a dedicated measurement with a circularly polarized photon beam could provides a model-independent extraction of the $\Delta$ MDM. I refer to the recent paper \cite{27} for further details.

5 Summary

In the single-nucleon sector the limit of applicability of chiral perturbation theory is set by the excitation energy of the first nucleon resonance – the $\Delta$(1232). Inclusion of the $\Delta$ in the chiral Lagrangian extends the limit of applicability into the resonance energy region. The power counting of the $\Delta$ contribution depends crucially on how the $\Delta = M_\Delta - M_N$, weighted in comparison to the other mass scales in the problem, in this case the pion mass $m_\pi$ and the scale of chiral symmetry breaking $\Lambda$.

Two different schemes exist in the literature. In the Small Scale Expansion, $\Delta \sim m_\pi \ll \Lambda$, while in the “$\delta$-expansion”, $m_\pi \ll \Delta \ll \Lambda$. The hierarchy of scales used in the $\delta$ expansion provides a more adequate power-counting of the $\Delta$-resonance contributions. It provides a justification for “integrating out” the resonance contribution at very low energies and for a resummation and dominance of resonant contributions in the resonance region. The $\delta$ expansion has already been successfully applied to the calculation of observables for processes such as Compton scattering, pion electroproduction and radiative pion photoproduction in the $\Delta$-resonance region.
References

[1] S. Weinberg, Physica A 96, 327 (1979); J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985).
[2] I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96, 132001 (2006).
[3] P. F. Bedaque, H. W. Hammer and U. van Kolck, Phys. Lett. B 569, 159 (2003); U. van Kolck, Nucl. Phys. A 752, 145 (2005).
[4] V. Pascalutsa and D. R. Phillips, Phys. Rev. C 67, 055202 (2003).
[5] J. Gasser, M. E. Sainio and A. Svarc, Nucl. Phys. B 307, 779 (1988).
[6] V. Pascalutsa, M. Vanderhaeghen and S. N. Yang, Phys. Rept. 437, 125 (2007).
[7] T. Hemmert, B. R. Holstein and J. Kambor, Phys. Lett. B 395, 89 (1997); J. Phys. G 24, 1831 (1998).
[8] G. C. Gellas et al., Phys. Rev. D 60, 054022 (1999).
[9] T. A. Gail and T. R. Hemmert, Eur. Phys. J. A 28, 91 (2006).
[10] V. Pascalutsa and M. Vanderhaeghen, Phys. Rev. Lett. 95, 232001 (2005).
[11] V. Pascalutsa and M. Vanderhaeghen, Phys. Rev. D 73, 034003 (2006).
[12] V. Pascalutsa, C. E. Carlson and M. Vanderhaeghen, Phys. Rev. Lett. 96, 012301 (2006).
[13] S. Kondratyuk and P. G. Blunden, Nucl. Phys. A 778, 44 (2006).
[14] S. S. Kamalov and S. N. Yang, Phys. Rev. Lett. 83, 4494 (1999); S. S. Kamalov, S. N. Yang, D. Drechsel, O. Hanstein, and L. Tiator, Phys. Rev. C 64, 032201(R) (2001).
[15] T. Sato and T.-S.H. Lee, Phys. Rev. C 54, 2660 (1996); ibid. 63, 055201 (2001).
[16] V. Pascalutsa and J. A. Tjon, Phys. Rev. C 61, 054003 (2000); ibid. 70, 035209 (2004); G. L. Caia et al., Phys. Rev. C 70, 032201(R) (2004); ibid. 72, 035203 (2005).
[17] C. Mertz et al., Phys. Rev. Lett. 86, 2963 (2001).
[18] C. Kunz et al., Phys. Lett. B 564, 21 (2003).
[19] N. F. Sparveris et al. [OOPS Collaboration], Phys. Rev. Lett. 94, 022003 (2005).
[20] T. Pospischil et al., Phys. Rev. Lett. 86, 2959 (2001).
[21] C. Alexandrou et al., Phys. Rev. Lett. 94, 021601 (2005).
[22] A. I. Machavariani, A. Fäßler and A. J. Buchmann, Nucl. Phys. A 646, 231 (1999) [Erratum-ibid. A 686, 601 (2001)].
[23] D. Drechsel et al., Phys. Lett. B 484, 236 (2000).
[24] D. Drechsel and M. Vanderhaeghen, Phys. Rev. C 64, 065202 (2001).
[25] M. Kotulla et al., Phys. Rev. Lett. 89, 272001 (2002).
[26] M. Kotulla (for the Crystal Ball @ MAMI Coll.), contribution to these proceedings.
[27] V. Pascalutsa and M. Vanderhaeghen, arXiv:0709.4583 [hep-ph].
[28] V. Pascalutsa and M. Vanderhaeghen, Phys. Rev. Lett. 94, 102003 (2005).
[29] C. Hacker, N. Wies, J. Gegelia, and S. Scherer, Eur. Phys. J. A 28, 5 (2006).
[30] D. Binosi and V. Pascalutsa, arXiv:0704.0377 [hep-ph].
Figure 1: (Color online) Total cross-section of the Compton scattering on the nucleon (proton – red solid curve, neutron – blue dashed curve), as the function of the incident photon lab energy. The curves are obtained in a χEFT calculation[4].

Figure 2: Examples of the one-Delta-reducible (1st row) and the one-Delta-irreducible (2nd row) graphs in Compton scattering.

Figure 3: Diagrams for the $eN \rightarrow e\pi N$ reaction to LO and NLO in the δ-expansion. The dots denote the vertices from the 1st-order Lagrangian, while the circles are the vertices from the 2nd order Lagrangian (e.g., the $\gamma N\Delta$-vertex in the first two graphs is the $g_M$ coupling from $\mathcal{L}^{(2)}$).
Figure 4: The pion angular dependence of the $\gamma^* p \to \pi^0 p$ cross sections at $W = 1.232$ GeV and $Q^2 = 0.127$ GeV$^2$. Dashed-dotted (black) curves: DMT model [14]. Dashed (red) curves: SL model [15]. Dotted (green) curves: DUO model [16]. Solid (blue) curves: $\chi$EFT results [10, 11]. The bands provide an estimate of the theoretical error for the $\chi$EFT calculations. Data points are from BATES experiments [17, 18, 19].

Figure 5: The pion mass dependence of $R_{EM}$ (upper panel) and $R_{SM}$ (lower panel), at $Q^2 = 0.1$ GeV$^2$. The blue circle is a data point from MAMI [20], the green squares are data points from BATES [17, 19]. The three filled black diamonds at larger $m_\pi$ are lattice calculations [21], whereas the open diamond near $m_\pi \simeq 0$ represents their extrapolation assuming linear dependence in $m_\pi^2$. Red solid curves: NLO result when accounting for the $m_\pi$ dependence in $M_N$ and $M_\Delta$; green dashed curves: NLO result of Ref. [10], where the $m_\pi$-dependence of $M_N$ and $M_\Delta$ was not accounted for. The error bands represent the estimate of theoretical uncertainty for the NLO calculation.