Analytical and experimental study for characteristic analysis of permanent magnet linear synchronous machines with horizontally magnetized PMs

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ABSTRACT
This paper deals with analytical and experimental study of permanent magnet linear synchronous machines with horizontally magnetized permanent magnets. Six subdomains are defined as analysis domains considering end effects, slotting effects, and flux pass iron poles. The governing equations for each domain are derived from the Maxwell equation and the constitutive relations in electromagnetic theory, and the second-order differential equations are calculated by using the Sturm-Liouville theory. By applying the boundary conditions, analytical solutions in each area are derived, and electromagnetic performance and generating characteristics can be predicted based on the derived analytical solutions. The proposed analytical method is validated by comparison with finite element and experimental results.

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I. INTRODUCTION
Permanent magnet linear synchronous machines (PMLSMs) are widely used where linear and reciprocating motions are required due to their high efficiency and force density as well as good servo performance. For these reasons, PMLSMs have been widely used in various industrial applications. A comparison study on the performance of PMLSM with magnet arrays showed that horizontal magnetization is better than vertical magnetization in terms of efficiency and weight. However, there are few studies on the field analysis method to predict the relationship between design parameters and performance of PMLSM with horizontal array.

To develop analysis techniques for PMLSM with horizontal magnetization, this study proposes the analytical approach to performance analysis of PMLSMs with horizontally magnetized permanent magnets (PMs) using subdomain method. The governing equations for each subdomain were derived from the electromagnetic theory, and the second-order partial differential equations were calculated by using the Sturm-Liouville theory. By applying the appropriate boundary conditions, analytical solutions were derived, and electromagnetic performance and generating characteristics can be predicted based on the analytical solutions. The validity of the proposed analytical method is verified by comparing the finite element (FE) method with the experimental results.

II. ANALYTICAL MODELING FOR PMLSM WITH HORIZONTALLY MAGNETIZED PM
A. Analytical model

Figure 1 illustrates the topology of PMLSM with horizontally magnetized PM. Unlike the general PMLSM, there is no back core to make the path of the magnetic flux, which has the advantage of...
reducing the weight of the mover. The simplified model of PMLSM with horizontally magnetized PM requires several assumptions:

- The subdomain is a two-dimensional Cartesian coordinate system.
- The materials are linear and homogeneous.
- The core material (iron) is infinite permeable.

The analysis domain and main parameters are defined as in Fig. 2.

**B. Analytical solutions**

As mentioned above, the governing equations of PMLSM with horizontally magnetized PMs can be defined for each region from Maxwell’s equations and constitutive relationships as follows:

\[
\begin{align*}
\nabla^2 A_{Iz} + \nabla \times \left( \mu_0 \nabla \times A_{c} \right) &= \frac{n I_y}{2} + \sum_{n=1}^{\infty} \left( A_{n}^{Iz} e^{j \omega_n (y-y_1)} + B_{n}^{Iz} e^{-j \omega_n (y-y_1)} \right) \\
\n\nabla^2 A_{mz} + \nabla \times \left( \mu_0 \nabla \times A_{c} \right) &= \frac{n I_y}{2} + \sum_{n=1}^{\infty} \left( A_{m}^{mz} e^{j \omega_n (y-y_1)} + B_{m}^{mz} e^{-j \omega_n (y-y_1)} \right) \\
\n\end{align*}
\]

(1)

Using the method of separation of variables, the governing equations in (1) can be expressed as the following general solutions:

\[
A_{Iz} = A_{Iz}^{0} + B_{Iz}^{0} y + \sum_{n=1}^{\infty} \left( A_{n}^{Iz} e^{j \omega_n (y-y_1)} + B_{n}^{Iz} e^{-j \omega_n (y-y_1)} \right) \\
\times \cos(\omega_n (x-x_1)) i_z
\]

(2)

\[
A_{mz} = A_{mz}^{0} + B_{mz}^{0} y + \sum_{n=1}^{\infty} \left( A_{m}^{mz} e^{j \omega_n (y-y_1)} + B_{m}^{mz} e^{-j \omega_n (y-y_1)} \right) \\
\times \cos(\omega_n (x-x_1)) i_z
\]

(3)

\[
A_{Iz}^{Q}\|z = A_{Q}^{0} + B_{Iz}^{Q} y + \sum_{n=1}^{\infty} \left( A_{n}^{Q} e^{j \omega_n (y-y_1)} + B_{n}^{Q} e^{-j \omega_n (y-y_1)} \right) \\
\times \cos(\omega_n (x-x_1)) i_z
\]

(4)

\[
A_{mz}^{Q}\|z = A_{mz}^{0} + B_{mz}^{Q} y + \sum_{n=1}^{\infty} \left( A_{m}^{mz} e^{j \omega_n (y-y_1)} + B_{m}^{mz} e^{-j \omega_n (y-y_1)} \right) \\
\times \cos(\omega_n (x-x_1)) i_z
\]

(5)

\[
A_{z}^{IV} = A_{z}^{IV} + B_{z}^{IV} y + \sum_{k=1}^{\infty} \left( A_{n}^{IV} e^{j \omega_n (y-y_1)} + B_{n}^{IV} e^{-j \omega_n (y-y_1)} \right) \\
\times \cos(\omega_n (x-x_1)) i_z
\]

(6)

where \( n, e, m, t, \) and \( k \) are harmonic orders of spatial harmonics; \( \omega_n = n \pi / \tau_p, \omega_c = c \pi / \tau_m, \omega_m = m \pi / \tau_m, \omega_l = l \pi / \tau, \) and \( \omega_k = k \pi / \tau; \) \( 2 \tau \) is the width of the periodic region, \( \tau_p \) is the pole pitch, and \( l_{z,\text{sk}} \) is the stack length; \( x_1, x_1, x_3, \) and \( x_4 \) represent the mechanical position of \( \hat{z}^\text{th} \) PM, the end position of the mover, the position of the slot, and the end position of the stator, respectively. Further, \( f_1 \) and \( f_2 \) are the current density components. \( A_{Iz}^{I,II,III,IV,P,Q}, B_{Iz}^{I,II,III,IV,P,Q}, A_{mz}^{I,II,III,IV,P,Q}, B_{mz}^{I,II,III,IV,P,Q} \) and \( B_{z}^{P,Q} \) are unknown coefficients.

From the definition of electromagnetics, magnetic flux density can be expressed using magnetic vector potential.

\[
B_x = \frac{\partial A_x}{\partial x}, \quad B_y = \frac{\partial A_y}{\partial y}
\]

(8)

The undetermined coefficients in the general solution are obtained by applying the interface conditions.

![Fig. 2. Schematic view of the simplified analytical model.](image-url)
As shown in Fig. 3, the reliability of the simplified model and the derived analytical solution can be verified by comparing the flux density results calculated from the FE method.

III. ELECTROMAGNETIC PERFORMANCE ANALYSIS OF PMLSM

The electromagnetic performance of PMLSM can be estimated by calculating electrical circuit parameters such as induced voltage (back-EMF), resistance, inductance based on the derived analytical solution. The calculation of flux linkage to predict induced voltage and inductance can be derived by applying Stokes’ theorem to the definition of magnetic flux.

\[
\phi_{Qj} = \frac{N_c l_{stk}}{A_{coil}} \int_{y_5} \int_{x_j \pm \tau_j/2} A_{Qj}^0 \, dx \, dy \tag{9}
\]

where \(A_{coil}\) is the cross-sectional area of the coil region and \(N_c\) is the number of turns per coil.

Using the connecting matrix, the flux of all slots is used to calculate the flux linkage for each phase. The induced voltage can be calculated from the derived flux linkage and Faraday’s law.

The synchronous inductance can be obtained by adding self and mutual inductance calculated as follows:

\[
L_s = \lambda_a/I_a, \quad L_m = \lambda_a/I_b \tag{10}
\]

The self- and the mutual-inductance can be predicted from the flux linkage calculated by applying current to the stator without magnetization of the PM.

The phase resistance depends on the ambient temperature, the structure of the PMLSM, and the winding method.

\[
R_{ph} = \frac{\rho_c N_{cp} l_c}{A_c} \tag{11}
\]

where \(N_{cp}\) is the number of coils per phase, \(\rho_c\) is resistivity of copper, \(l_c\) is the total length of the coil given by \(l_c = 2l_{stk} + \pi \tau_{cp}\), and \(A_c\) is the cross-sectional area of the conductor. \(\tau_{cp}\) is the coil pitch and \(r_c\) is the radius of the conductor.

| Table I. Comparison of analysis and measurement results for electrical parameters. |
| Symbol | Description | Unit | Analytical | FE | Meas. |
|--------|-------------|------|------------|----|------|
| \(E_0\) | Back-EMF | \(V_{\text{peak}}\) | 9.483 | 9.986 | 9.55 |
| \(L_s\) | Inductance | mH | 1.234 | 1.136 | 1.15 |
| \(R_{ph}\) | Resistance | \(\Omega\) | 0.88 | 0.85 |
Using these parameters, the equivalent electrical circuit for generating characteristic analysis can be derived. 8

IV. RESULTS AND DISCUSSION

The reliability of the proposed method is ensured by comparison with the results of FE method and measurement. Figure 4 shows the experimental system for measuring the electromagnetic performance of PMLSM.

Table 1 shows the analysis and measurement results for induced voltage, phase resistance, and synchronous inductance. The proposed analytical results are in good agreement with the FE method and the measurements. As shown in Fig. 5, the generating characteristics of PMLSM can be evaluated by measuring the voltage and current according to the load resistance.

Figure 6(a)–(c) show the results of generating characteristic analysis of PMLSM according to load resistance. From the comparison of the FE method and the measurements, the proposed method was validated.

V. CONCLUSION

This paper presents an electromagnetic analysis that takes into account the slotting, end, and flux passing iron poles of PMLSM with horizontally magnetized PM. Using some assumptions, a simplified analytical model was defined. The general solutions for each domain were derived from electromagnetic and mathematical theories. The analytical solution was obtained by applying the appropriate boundary conditions and the characteristic analysis was performed based on the derived analytical solution. To verify the proposed analytical method, FE modeling, prototype fabrication, and experimental set-up were developed. The reliability of the proposed method was validated from the comparison with the analytical method, the FE method, and the experiment. The proposed analytical method can be useful for initial design and characteristic analysis of PMLSM with horizontally magnetized PMs.

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