Hypercritical accretion phase and neutrino expectation in the evolution of Cassiopeia A

N. Fraija1† and C. G. Bernal2‡

1Instituto de Astronomía, Universidad Nacional Autónoma de México, Circuito Exterior, C.U., A. Postal 70-264, 04510 México D.F., México
2Instituto de Física, Universidade Federal Fluminense, Av. Gal. Milton Tavares de Souza s/n, Gragoata, 24210-346 Niterói, Brazil

Accepted 2015 May 1. Received 2015 May 1; in original form 2015 March 19

ABSTRACT

Cassiopeia A, the youngest supernova remnant known in the Milky Way, is one of the brightest radio sources in the sky and a unique laboratory for supernova physics. Although its compact remnant was discovered in 1999 by the Chandra X-Ray Observatory, nowadays it is widely accepted that a neutron star lies in the centre of this supernova remnant. In addition, new observations suggest that such a neutron star with a low magnetic field and evidence of a carbon atmosphere could have suffered a hypercritical accretion phase seconds after the explosion. Considering this hypercritical accretion phase, we compute the neutrino cooling effect, the number of events and neutrino flavour ratios expected on Hyper-Kamiokande Experiment. The neutrino cooling effect (the emissivity and luminosity of neutrinos) is obtained through numerical simulations performed in a customized version of the FLASH code. Based on these simulations, we forecast that the number of events expected on the Hyper-Kamiokande Experiment is around 3195. Similarly, we estimate the neutrino flavour ratios to be detected considering the neutrino effective potential due to the thermal and magnetized plasma and thanks to the envelope of the star. It is worth noting that our estimates correspond to the only trustworthy method for verifying the hypercritical phase and although this episode took place 330 years ago, at present supernova remnants with these similarities might occur thus confirming our predictions for this phase.

Key words: hydrodynamics – magnetic fields – neutrinos – plasmas – stars: neutron – supernovae: individual: Cassiopeia A.

1 INTRODUCTION

At a distance of \( d_L = 3.4^{+0.3}_{-0.1} \) kpc, Cassiopeia A (Cas A) is one of the closest and youngest known supernova remnants (Reed et al. 1995; Chang & Bildsten 2004). It was related to the supernova observed by John Flamsteed in 1680 (Ashworth 1980). As was reported by Krause et al. (2008), Cas A supernova remnant was produced in a IIb type supernova explosion. The progenitors of such explosions, as was shown by Chevalier (2005), are red supergiants which already have lost their hydrogen envelope via a powerful stellar wind. Then, it is inferred that such progenitors with masses of the order of 20 M\(_\odot\) must have a helium core of 6 M\(_\odot\) and an iron core of 1.45 M\(_\odot\). This scenario should produce a neutron star with 1.4 M\(_\odot\). It seems to be the case of Cas A.

Studying the atmosphere surrounding the neutron star of Cas A, Ho & Heinke (2009) found evidence of carbon in the composition of this atmosphere. It suggests that a strong accretion could have occurred after the neutron star formation. In the core-collapse framework, other pieces of evidence related to the low strength of magnetic field, (i) the non-detection of the spectral features related to the electron cyclotron resonance in the Chandra energy range (which is usually associated with \( B \approx (1-5) \times 10^{11} \) G), (ii) the absence of a visible pulsar wind nebula and (ii) no samples of magnetospheric activities (such as radio or \( \gamma \)-ray fluxes) from the direction of Cas A, as found in pulsars with \( B \geq 10^{12} \) G, suggest that the strong magnetic field was buried because of a strong accretion (Muslimov & Page 1995). If the core-collapse paradigm is correct, the shock wave sweeps the outer layers of the progenitor until it encounters a discontinuity in density. At this point, a reverse shock is generated leading to a fall-back episode which allows the deposition of large amounts of material on the newborn neutron star surface. As a consequence of the fall-back episode, the magnetic field is submerged within the new crust formed during such episode (Bernal, Page & Lee 2013).

In the hypercritical phase, super-Eddington accretion gives rise to the material of core collapse to become dense enough so that photons are trapped. Therefore, the gravitational energy is carried out due to neutrino emission in the formation of the newborn

---

*E-mail: nifraija@astro.unam.mx (NF); astrogio@if.uff.br (CGB)
†Luc Binette-Fundación UNAM Fellow.

© 2015 The Authors
Published by Oxford University Press on behalf of the Royal Astronomical Society
neutron star. When the accretion rate $\dot{M}$ exceeds by several orders of magnitude the Eddington accretion rate $\dot{M}_{\text{Edd}} \sim 10^{18}$ g s$^{-1}$ so-called Eddington limit, then it is denoted as hypercritical accretion (Blondin, 1986), and the accretion rate required is higher than $\sim 10^5 \dot{M}_{\text{Edd}}$.

In the hypercritical accretion episode (seconds after the supernova explosion), due to the inverse beta decay, electron–positron annihilation ($e^- + e^+ \rightarrow Z \rightarrow v_i + \bar{v}_i$) and nucleon–nucleon bremsstrahlung ($N + N \rightarrow N + N + v_i + \bar{v}_i$) for $j = e, v, r$, thermal neutrinos (with energies in the range $5 \leq E_i \leq 30$ MeV) will be produced at the core and then oscillate in their propagation through the collapsing star. The resonant conversion (from one flavour to another) and the properties of these neutrinos will get modified due to neutrino effective potential generated by the magnetized and thermal plasma (Sahu, Fraija & Keum 2009a, b), and later by the collapsing and surrounding material of the progenitor. It is worth noting that, although neutrino cannot couple directly to the magnetic field, its effect can be felt through the coupling to charge particles in the thermal plasma. Therefore, the values of neutrino flavour ratios created at the core will be different from the values on the surface of the star and on Earth.

Recently, Fraija, Bernal & Hidalgo-Gaméz (2014) revisited Chevalier’s model in a numerical framework to focus on the neutrino cooling effect on the SN1987A fall-back dynamics. They estimated the number of neutrinos that must have been seen on the Super-Kamiokande neutrino experiment, hours after the ~20 events detected from the direction of SN1987A. In the current work, we consider that Cas A supernova remnant underwent the hypercritical accretion episode in its early life as neutron star. We present a numerical study of hypercritical accretion of matter on to the newborn neutron star. We present this approach and due to the high dependence of the pair annihilation process on pressure near the stellar surface, the neutrino emissivity can be written as

$$\dot{\nu}_i = 0.97 \times 10^{25} \left( \frac{T}{\text{MeV}} \right) \text{erg s}^{-1} \text{cm}^{-3},$$

respectively. At first approach and due to the high dependence of the pair annihilation process on pressure near the stellar surface, the neutrino emissivity can be written as

$$\dot{\nu}_i = 1.83 \times 10^{-34} p^{-2.25} \text{erg cm}^{-3} \text{s}^{-1},$$

where the total pressure $p$ includes (besides the gas pressure) the pressure due to electron/positron pairs which is defined by

$$p_{e^-e^+} = \frac{11}{4} \frac{4}{3} a T^4,$$

with $a$ the radiation constant. It is worth noting that, in the hypercritical phase, the magnetic field may be submerged and confined into the new crust (formed by the piled material). The strength of the magnetic field could reach values in the range of $10^{15} \leq B \leq 10^{17}$ G. Also, the density and pressure of radial profiles can be described as power laws, whereas the mean velocity is null in this region.

### 2.2 Quasi-hydrostatic envelope

The reverse shock induces the hypercritical accretion on to the newborn neutron star surface through the fall-back episode. This accreted material will bounce against the surface of the newly born neutron star, building a new expansive shock. This expansive shock builds an envelope in quasi-hydrostatic equilibrium, with free-falling material raining over it. The structure of such envelope in quasi-hydrostatic equilibrium is given by (Chevalier 1989)

$$p_{\text{qhe}} = \rho_s \left( \frac{r_s}{r} \right)^3, \quad p_{\text{qhe}} = p_s \left( \frac{r}{r_s} \right)^4, \quad v_{\text{qhe}} = v_s \left( \frac{r}{r_s} \right)^{-1},$$

where the subscript $s$ refers to the value of density ($\rho_s$), pressure ($p_s$) and velocity ($v_s$) at the shock front. Given the typical values of the neutron star: a mass $M = 1.4 M_\odot$, an accretion rate $\dot{M} = 500 M_\odot \text{yr}^{-1}$ and a radius $r_{\text{ms}} = 10^6$ cm, the shock radius is

$$r_s \simeq 7.73 \times 10^8 \text{ cm} \left( \frac{M}{1.4 M_\odot} \right)^{-0.04} \left( \frac{r_{\text{ms}}}{10^6 \text{ cm}} \right)^{1.48} \times \left( \frac{\dot{M}}{500 M_\odot \text{yr}^{-1}} \right)^{-0.37}.$$

The pressure $p_s$ and density $\rho_s$ are determined by the strong shock condition $p_s = 6 \rho_s v_s^2$ and $\rho_s = 7 \rho_0$, respectively and $v_s$ by the
mass conservation $v_r = -\frac{1}{2} v_0$. Here $\rho_0$ and $v_0$ are the density and velocity just outside the shock, respectively. In this case, the radial distance of the accretion shock that is controlled by the energy balance between the accretion power and the integrated neutrino losses is given by

$$\frac{G M \dot{M}}{R} = \int_R^\infty \dot{e}_v(r) dr. \quad (7)$$

Chevalier (1989) approached the size of the envelope as $R/4$ for the pressure profile in an atmosphere in quasi-hydrostatic equilibrium. With this, and using the analytical formula of Dicus (1972) for the neutrino emissivity, the shock radius can be calculated through the energy balance as

$$\frac{G M \dot{M}}{R} = 4\pi R^2 \left( \frac{R}{4} \right) \dot{e}_v. \quad (8)$$

Then, the high pressure near the neutron star surface becomes

$$p_{ms} \approx 1.86 \times 10^{-12} \text{dyn cm}^{-2} \dot{M} r_s^{3/2}. \quad (9)$$

It allows the pair neutrino process to be the dominant mechanism in the neutrino cooling. From the strong shock conditions and equation (5), we get

$$\rho_{\phi e}(r) = 7.7 \times 10^7 \left( \frac{r}{r_s} \right)^3 \text{g cm}^{-3}. \quad (10)$$

where the quasi-hydrostatic envelope radius lies in the range $(r_{ms} + r_s) \leq r \leq r_s$.

### 2.3 Free-fall zone

In this case, material at $r_s \leq r \leq r_0$ begins to fall with the velocity and density profiles given by

$$v_{ff}(r) = \sqrt{\frac{2GM}{r}} \quad \text{and} \quad \rho_{ff}(r) = \frac{\dot{M}}{4\pi r^2 v_{ff}(r)}. \quad (11)$$

Taking the typical values for such object, $M = 1.4 M_\odot$ and $\dot{M} = 500 M_\odot \text{yr}^{-1}$, the velocity is written as

$$v_{ff}(r) = 7.81 \times 10^4 \left( \frac{r}{r_0} \right)^{-1/2} \text{cm s}^{-1}, \quad (12)$$

and the density of material in free-fall as

$$\rho_{ff}(r) = 5.74 \times 10^{-2} \left( \frac{r}{r_0} \right)^{-3/2} \text{g cm}^{-3}, \quad (13)$$

where $r_0 = 6.3 \times 10^{10} \text{cm}$.

### 2.4 External layers

By considering a supergiant profile (typical for a pre-supernova), the density of the outer layers can be described as (Chevalier & Soker 1989)

$$\rho_{ul}(r) = 3.4 \times 10^{-5} \text{g cm}^{-3} \times \begin{cases} \left( \frac{r}{r_s} \right)^{n_1}, & r_s < r < r_h, \\ \left( \frac{r}{r_s} \right)^{n_1} \left( \frac{r_h}{r} \right)^{n_2}, & r > r_h, \end{cases} \quad (14)$$

with $r_h = 10^{12} \text{cm}, R_s \simeq 3 \times 10^{12} \text{cm}, n_1 = 17/7$ and $n_2 = 5$.

### 3 THERMAL NEUTRINO EXPECTATION

During the hypercritical accretion episode, a copious amount of neutrinos in the energy range of 1–30 MeV is produced. The processes of neutrino emission involved in the episode are as follows.

(i) Electron–positron annihilation process: $e^\pm$ pairs are annihilated to create neutrino–antineutrino pairs ($e^- + e^+ \rightarrow \nu + \bar{\nu}$).

(ii) Photoneutrino process: a photon is Compton scattered to produce a neutrino–antineutrino pair ($\gamma + e^\pm \rightarrow e^\pm + \nu + \bar{\nu}$).

(iii) Plasmon decay process: a photon propagating within an electron plasma (plasmon) is spontaneously transformed into a neutrino–antineutrino pair ($\gamma \rightarrow \nu + \bar{\nu}$).

(iv) Bremsstrahlung process: A neutrino–antineutrino pair is created by either an electron–nucleon interaction ($e^- + N \rightarrow e^- + N + \nu + \bar{\nu}$) or nucleon–nucleon interaction ($N + N \rightarrow N + N + \nu + \bar{\nu}$).

#### 3.1 Number of expected neutrinos

The expected number of neutrinos can be estimated requiring the neutrino luminosity of the hypercritical phase $L = 4\pi d_L^2 \dot{E} dN/dE$ and the effective volume of the neutrino detector $V$. The expected event rate can be written as

$$N_e = V N_A \rho_N \int \frac{dN}{dE} dT, \quad (15)$$

where $N_A = 6.022 \times 10^{23} \text{g}^{-1}$ is Avogadro’s number, $\rho_N = 2/18 \text{g cm}^{-3}$ is the nucleons density in water (Mohapatra & Pal 2004), $\sigma_{cp}^{\nu\bar{\nu}} \simeq 9 \times 10^{-44} E_{\nu}^2 / M e V^2$ is the cross-section (Bahcall 1989) and $dT$ is the detection time of neutrinos. The number of events (equation 15) is

$$N_e \simeq \frac{t}{(E_{\nu})} V N_A \rho_N \sigma_{cp}^{\nu\bar{\nu}} (E_{\nu})^2 \left( \frac{dN}{dE_{\nu}} \right), \quad (16)$$

where we have assumed an average energy $(E_{\nu}) \simeq E_{\nu}$ during a period of time $t$ (Mohapatra & Pal 2004).

#### 3.2 The Hyper-Kamiokande experiment

The Hyper-Kamiokande detector will be the third-generation water Cherenkov in Kamioka, designed for a vast variety of neutrino studies. At 8 km south of its predecessor Super-Kamiokande, the Hyper-Kamiokande detector will be located in the Tochibara mine of the Kamioka Mining and Smelting Company, near Kamioka town in the Gifu Prefecture, Japan. This detector consists of two separate caverns, each having an egg-shaped cross-section of 48 m wide, 54 m tall and 250 m long. The entire array consists of 99,000 photomultiplier tubes (PMTs), uniformly surrounding the region. It will have a total (fiducial) mass of 0.99 (0.56) million metric tons, approximately 20 (25) times larger than that of Super-Kamiokande. Among the physical potentials of this detector is the detection of astrophysical neutrinos from the supernova remnant and the studies of neutrino oscillation parameters (Abe et al. 2011; Hyper-Kamiokande Working Group 2014).

#### 3.3 Neutrino oscillation

The properties of these neutrinos are modified when they propagate in matter. In the following subsections, we will describe the dynamics of neutrinos going through matter.
3.3.1 Two-neutrino mixing

The neutrino oscillations for two-neutrino mixing \((v_\alpha \leftrightarrow v_\mu, \tau)\) are determined by the evolution equation in matter given by

\[
\frac{d}{dt} \begin{pmatrix} v_\alpha \\ v_\mu \end{pmatrix} = -i \begin{pmatrix} \delta m^2_i / 4E_v \\ -2 \cos 2\theta & \sin 2\theta \end{pmatrix} \begin{pmatrix} v_\alpha \\ v_\mu \end{pmatrix},
\]

(17)

where \(V_{\text{eff}}\) is the neutrino effective potential generated by the medium, \(E_v\) is the neutrino energy, \(\delta m^2\) is the neutrino mass difference and \(\theta\) is the neutrino mixing angle. From the conversion probability \(P_{v_\alpha \rightarrow v_\beta} = |\langle v_\beta(t) | v_\alpha(t = 0) \rangle|^2 = \delta_{\alpha\beta} - 4 \sum_{j \neq \alpha} U_{\alpha j} U_{\beta j}^* \sin^2 \left(\delta m_{\alpha j}^2 L / 4 E_v\right)\), where \(U_{\alpha j}\) is once again the three-neutrino mixing (Gonzalez-Garcia & Nir 2003; Akhmedov et al. 2004; Gonzalez-Garcia & Maltoni 2008; Gonzalez-Garcia 2011). Here the sin term in the probability has been averaged to \(\sim 0.5\) for distances \((L)\) longer than the Solar system (Learned & Pakvasa 1995).

4 RESULTS

4.1 Plasma dynamics on the stellar surface

We carry out hydrodynamic (HD) numerical simulations of the hypercritical accretion phase in a two-dimensional spherical mesh, using the FLASH method developed by Fryxell et al. (2000). This multi-physics Eulerian and parallelized code was designed to figure out many problems in high-energy astrophysics, especially those related to the dynamics of plasmas around neutron stars. Recently, using a customized version of such numerical code, Fraija et al. (2014) presented numerical simulations of strong matter accretion on to the newborn neutron star to compute the size of the neutrinosphere, the emissivity and the luminosity of neutrinos in the SN1987A scenario. Requiring adequate parameters to describe the Cas A scenario, we will follow a similar procedure to that performed for SN1987A.

The FLASH code uses the piecewise parabolic method (PPM) solver to solve the whole set of HD equations for a compressible gas dynamics in one, two or three spatial dimensions. This directionally split method makes use of a second-order Strang time splitting, which uses a finite-volume spatial discretization of the Euler equations together with an explicit forward time difference. Time-advanced fluxes at cell boundaries are computed using the numerical solution to a Riemann shock-tube problem in each boundary. Initial conditions for each Riemann problem are determined by assuming the non-advanced solution to be piecewise constant in each cell. Requiring the Riemann solution, the effect of explicit non-linearity is introduced into the differential equations. It allows the calculation of sharp shock fronts and contact discontinuities without introducing significant nonphysical oscillations into the flow. Since the value of each variable in each cell is assumed to be constant, this method is limited to first-order accuracy in both space and time. PPM improves on Godunov’s method by representing the flow variables with piecewise parabolic functions. It also uses a monotonicity constraint rather than artificial viscosity to control the oscillations near discontinuities. The Euler equations can be written in conservative form as

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0
\]

(25)

\[
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) + \nabla P = \rho \mathbf{g}
\]

(26)

\[
\frac{\partial \rho E}{\partial t} + \nabla \cdot [\mathbf{v} (\rho E + P)] = \rho \mathbf{v} \cdot \mathbf{g}
\]

(27)

where the equations (25), (26) and (27) represent the conservation of mass, momentum and total energy, respectively. In this context, \(\rho\) is the fluid density, \(\mathbf{v}\) is the fluid velocity, \(P\) is the pressure, \(\mathbf{g}\) is the
acceleration due to gravity, $t$ is the time coordinate and $E$ represents the sum of the internal energy $\epsilon$ and kinetic energy per unit mass

$$E = \epsilon + \frac{1}{2} |\mathbf{v}|^2.$$  

The pressure is obtained from the energy and density using the equation of state. In this case, we use a customized version of the Helmholtz unit of the FLASH code which includes contributions from the nuclei, $e^\pm$ pairs, radiation and the Coulomb correction. This routine is appropriate for addressing astrophysical phenomena in which electrons and positrons may be relativistic and/or degenerated, and the radiation may significantly contribute to the thermodynamic state. The pressure and internal energy are calculated considering all components

$$P_{\text{tot}} = P_{\text{rad}} + P_{\text{ion}} + P_{\text{ele}} + P_{\text{pos}} + P_{\text{coul}},$$

$$\epsilon_{\text{tot}} = \epsilon_{\text{rad}} + \epsilon_{\text{ion}} + \epsilon_{\text{ele}} + \epsilon_{\text{pos}} + \epsilon_{\text{coul}}.$$  

Here the subscripts ‘rad’, ‘ion’, ‘ele’, ‘pos’ and ‘coul’ represent the contribution from radiation, nuclei, electrons, positrons and corrections for Coulomb effects, respectively. The radiation portion assumes a blackbody in local thermodynamic equilibrium, the ion portion (nuclei) is treated as an ideal gas with $y = 5/3$, and the electrons and positrons are treated as a non-interacting Fermi gas. That is, the number densities of free electrons $N_{\text{ele}}$ and positrons $N_{\text{pos}}$ in the non-interacting Fermi gas formalism are given by

$$N_{\text{ele}} = \frac{\sqrt{2}}{\pi^3} m_e^{3/2} \left[ F_{1/2}(\eta_{\text{ele}}, \beta) + F_{3/2}(\eta_{\text{ele}}, \beta) \right],$$

$$N_{\text{pos}} = \frac{\sqrt{2}}{\pi^3} m_e^{3/2} \left[ F_{1/2}(\eta_{\text{pos}}, \beta) + \beta F_{3/2}(\eta_{\text{pos}}, \beta) \right],$$

where $m_e$ is the electron rest mass, $\beta = T/m_e$ is the normalized temperature, $\eta_{\text{ele}} = \mu/T$ is the normalized chemical potential for the electrons, $\eta_{\text{pos}} = -\eta_{\text{ele}} - 2/\beta$ is the normalized chemical potential for the positrons and $F_{1/2}(\eta, \beta)$ is the Fermi–Dirac integral given by

$$F_{1/2}(\eta, \beta) = \int_0^\infty \frac{x^{1/2}(1 + 0.5 \beta x)^{1/2}}{\exp(x - \eta) + 1} dx.$$  

Because the electron rest mass is not included in the chemical potential, the positron chemical potential must have the form given by $\eta_{\text{pos}}$. For complete ionization, the number density of free electrons in the matter is

$$N_{\text{ele,matter}} = \frac{Z}{A} N_0 \rho = \frac{Z}{A} N_{\text{ion}},$$

and charge neutrality requires

$$N_{\text{ele,matter}} = N_{\text{ele}} - N_{\text{pos}}.$$  

Solving these equations with a standard one-dimensional root-finding algorithm, we determine $\eta$, and the Fermi–Dirac integrals can be evaluated given the pressure, specific thermal energy and entropy due to the free electrons and positrons. Full details of the Helmholtz equation of state are provided in Timmes & Swesty (2000).

As was pointed out, neutrino energy losses are dominated by the annihilation processes which involve the formation of neutrino–antineutrino pairs through the $e^\pm$ annihilation near the stellar surface ($e^+ + e^- \rightarrow \nu + \bar{\nu}$). In addition, we include other relevant neutrino processes present in such regime, which are implemented in a customized module and described in Itoh et al. (1996). We ignore neutrino absorption and heating, and nuclear reactions.

Recently, Bernal, Lee & Page (2010) and Bernal et al. (2013) showed that, regardless of its initial configuration, the magnetic field is buried and submerged under the stellar surface during the hypercritical phase. Therefore, as the magnetic field is submerged within the surface, it does not play an important role in the dynamics of quasi-hydrostatic envelope, although the dynamics itself of the envelope is relevant. Here, we do not take into account the magnetic field and just focus on the HD near the stellar surface. The HD numerical simulations of the hypercritical accretion regime are carried out in a two-dimensional spherical mesh ($r, \theta$), where the radial component $r$ lies in the range $10^6 \leq r \leq 3 \times 10^7$ cm. Due to the symmetry of the problem, we only simulate a quarter of the total domain ($\pi/4 \leq \theta \leq 3\pi/4$) which has $2048 \times 2048$ effective zones as spatial resolution. It is worth noting that it corresponds to the maximum level resolution which is equal to 6 for this work.

The time resolution of these simulations is $dt \simeq 10^{-7}$ s. We use, for the discretization grids, a block-structured oct-tree based on an adaptive grid used by the PARAMESH library which is included in the FLASH code. If the adaptive mesh refinement grid is used, then the formation of the physical domain starts at the lowest level of refinement. Initial conditions are applied to each block at this level by calling initial condition routines. The grid unit then checks the refinement criteria in the blocks that it has created and also refines these blocks when the criteria are met. It then calls the routines of the initial conditions to initialize the newly created blocks. This process repeats until the grid reaches the required refinement level in the areas marked for refinement.

In the boundary conditions, we impose mass inflow along the top edge of the computational domain. We assume that the accreting matter is not magnetized. Concerning the FLASH code, the parallel structure of blocks means that each processor works independently. If a block is on a physical boundary, the guard cells are filled by calculations since there are not any neighbouring blocks from which to copy values. At the top of the domain, initially we set the velocity in free-fall to be $v_r = \sqrt{2GM/r}$ in all the guard cells, and set a density profile which fixes a constant accretion rate, $\rho = M/4\pi r^2 v_r$, as in the computational domain. The temperature is uniform and set to $T = 10^7$ K. Other variables are calculated with the equation of state. When the shock reaches this boundary, we allow it to leave the computational domain. At the same time, this boundary switches to the quasi-hydrostatic envelope mode, i.e. it is adaptable for consistency with the model. For this purpose, we use the $\rho$, $p$ and $v$ profiles given by the analytical solution (equations 5 and 6). It is justified due to the strong initial transient and short duration of the hypercritical phase. At the bottom, on the neutron star surface, we use a custom boundary condition that enforces the hydrostatic equilibrium. In order to establish this boundary in the problem of hypercritical accretion, we fix the velocities as null in all the guard cells ($v_r = v_\theta = 0$), and copy the density and the pressure of the first zone of the numerical domain. This zone corresponds to the neutron star surface ($\rho = \rho(\text{ic})$, $p = p(\text{ic}) + \rho v^2 + \rho gh$), where $\text{ic}$ is the first zone in the domain. The rest of thermodynamic variables are calculated from the equation of state. In addition, we have implemented periodic conditions along the sides. Periodic (wrap-around) boundary conditions are initially configured in this routine as well. If periodic boundary conditions are set in the $x$-direction, for instance, the first blocks in the $x$-direction are set to have as their leftmost neighbour the blocks that are the last in the $x$-direction, and vice versa. Thus, when the guard cell filling...
is performed, the periodic boundary conditions are automatically maintained.

We focus our study on the dynamics of the initial transient when the reverse shock rebounds off the stellar surface and builds a quasi-hydrostatic equilibrium envelope, instead of following the shock. We start the simulation requiring the free-fall profiles described in Section 2.3 and considering an initial temperature of $10^9$ K. The initial value of temperature was elected because it is close to the value expected from the theoretical model, and then the code finds the correct pressure profile after some steps of simulation. The neutron star mass and the accretion rate, for the Cas A parameters, are chosen as $M \approx 1.4 \, M_\odot$ and $\dot{M} \approx 500 \, M_\odot \, yr^{-1}$, respectively.

Figure 1 shows the time evolution colour maps of density (A) and neutrino emissivity (B) for different times: $t = 0.2$ ms (up) shows the initial transient of the third expansive shock with material falling on to it; $t = 1$ ms (right) illustrates the evolution of the shock in the computational domain resulting in a complex morphology above the stellar surface; $t = 5$ ms (down) exhibits the moment when the shock reaches the external boundary and leaves the computational domain; and finally, $t = 30$ ms (left) shows the quasi-hydrostatic envelope established with a new thin crust on the neutron star surface. The HD instabilities and the rich morphology observed inside the envelope in early times have disappeared. Note that in the new thin crust the neutrino cooling is very effective creating an energy sink that allows the material to be deposited on the surface slowly. That is, in such a region the neutrino emissivity is highly efficient.

In this analysis, not only HD simulations of the hypercritical regime are performed, but also the results of previous magnetohydrodynamic (MHD) simulations for similar parameters of Cas A are presented. Such results show a complete magnetic field submergence under the new formed crust, as was reported in Bernal et al. (2010, 2013). In these papers, to perform the MHD simulations the authors used the Split Eight-Wave solver to solve the whole set of MHD equations. They considered wide accretion columns in 2D and 3D to follow the accretion shock and also a magnetic field loop (in the shape of a hemi-torus) as magnetic initial condition. On the central hemi-circle of the loop, the field had a strength $B_0 = 10^{12}$ G; it was shaped as a Gaussian, i.e. with strength $B(d) = B_0 \exp \left[-(d/R_c)^2\right]$, $d$ being the distance to the loop central hemi-circle and $R_c = 1$ km. The two feet of the loop were centred at $x = -5$ and $+5$ km, and $z = 0$ in the 3D model. When the magnetic field was present, it was put at the bottom boundary in such a form that it was continuous from the guard cell to the physical domain, i.e. the authors anchored the magnetic field on to the neutron star surface and in the rest of the guard cells it was null. On the other hand, Fig. 2 shows radial profiles of density and temperature as a function of the stellar radius. In the left-hand panels, we show the density and temperature profiles obtained with MHD simulations with similar parameters of Cas A (Bernal et al. 2013), whereas in the right-hand panels the analytical approach developed in Chevalier (1989) is shown. Here, [1] represents the Cas A accretion rate, and [10] and [100] are accretion rates one and two orders of magnitude larger, respectively. Note the new crust formed in the hypercritical regime very close to the neutron star surface ($\sim 300$ m). The quasi-hydrostatic envelope lies between the new crust until $\sim 50$ km. The free-fall region is the remaining region, for the Cas A parameters.
function of the stellar radius when the envelope has been well established. It is worth noting that density profiles obtained by MHD and HD are very similar; hence, we show only those densities and temperatures obtained by MHD. This is justified because in this case we were able to follow the evolution of the shock until it formed the quasi-hydrostatic envelope with matter falling over it. In the left-hand panels, we show the density and temperature profiles obtained from simulations for several hyperaccretion rates, in terms of a fiducial accretion rate ($\dot{M} \approx 500 \, M_\odot \, \text{yr}^{-1}$), whereas in the right-hand panels we show the analytical solutions (with the same rates), using the analytical model (Chevalier 1989). Note the excellent agreement between them. The three regions (new crust, envelope and free-fall) are visible and distinct. In this case, label [1] represents the Cas A accretion rate, and labels [10] and [100] are accretion rates one and two orders of magnitude larger, respectively. The envelope is well established in 600 ms for label [1], 300 ms for label [10] and 100 ms for label [100]. A detail comparison between the MHD and HD solvers was reported in Bernal et al. (2010). In the new crust zone, the magnetic field is compressed and amplified, as shown in Fig. 3 (upper-left panel). The magnetic field submergence into the new crust formed in the hypercritical regime allows us to justify the fact that the magnetic field does not play an important role in the subsequent dynamics of the envelope.

Both in the HD and MHD cases, the estimated size of the neutrinosphere (including all the relevant neutrino cooling processes) is $r \approx 3.7 \times 10^5 \, \text{cm} \approx (1/3) R$. The mean value of emissivity in this region is $\bar{\epsilon}_\nu \approx 2.4 \times 10^{30} \, \text{erg} \, \text{s}^{-1} \, \text{cm}^{-3}$. The volume of the neutrinosphere is given by $V \approx (4/3)\pi R^3 = (4/3)\pi \times 10^{38} \, \text{cm}^3$ and then the neutrino luminosity is obtained to be $L_\nu \approx 8.4 \times 10^{48} \, \text{erg} \, \text{s}^{-1}$. This estimate agrees with the numerical value obtained by integrating directly the neutrino luminosity in the whole computational domain. The upper-right panel in Fig. 3 shows the parameter space of temperature and density. Taking into account the Itoh et al. (1996) tables, this panel has been divided into grey zones, highlighting the different regions where neutrino cooling processes dominate. Also, we have drawn lines in white colour to show the region of interest for the Cas A parameters. The bottom panel in Fig. 3 shows the time evolution of the neutrino luminosity integrated in the whole...
Neutrino effective potential is plotted as a function of magnetic field for three values of temperatures (1, 2.5 and 5 MeV). We have used the neutrino energy of 5 MeV.

4.2 Neutrino expectation

Taking into account the values of neutrino luminosity \( L_\nu \approx 8.4 \times 10^{48} \text{ erg s}^{-1} \), effective volume \( V \approx 0.56 \times 10^{12} \text{ cm}^3 \) (Hyper-Kamiokande Working Group 2014) and the average neutrino energy \( \langle E_\nu \rangle \approx 15 \text{ MeV} \), from equation (16) we get that the number of events expected from the hypercritical accretion episode on Hyper-Kamiokande is 3195.

These thermal neutrinos will oscillate when they propagate through the star. On the neutron star surface, the plasma is highly magnetized with \( B \approx 3 \times 10^{12} \text{ G} \) (see Fig. 3, upper-left panel) and thermalized at \( T \approx 3 \text{ MeV} \) (see Fig. 2, bottom panels). The neutrino effective potential at the moderate field limit is given by (Fraija 2014)

\[
V_\text{eff} = \frac{\sqrt{2} G_F m_\nu^2 B}{\pi^2 B_c} \left[ \sum_{l=0}^{\infty} (-1)^l \sinh \left( \frac{m_\nu^2}{m_\nu^2} \left( 1 + 4 \frac{E_\nu^2}{m_\nu^2} \right) K_1(\sigma_l) \right) \right. \\
+ \left. \sum_{n=1}^{\infty} \lambda_n \left( 2 + \frac{m_\nu^2}{m_W^2} \left( 3 - \frac{2 B}{B_c} + 4 \frac{E_\nu^2}{m_\nu^2} \right) K_1(\sigma_n \lambda_n) \right) \right]
\]

\[
-4 \frac{m_\nu^2}{m_W^2} \frac{E_\nu}{m_\nu^2} \sum_{l=0}^{\infty} (-1)^l \cosh \alpha \left\{ \frac{3}{4} K_0(\sigma_l) + \sum_{n=1}^{\infty} \lambda_n^2 K_0(\sigma_n \lambda_n) \right\},
\]

where \( K_0 \) is the modified Bessel function of integer order \( i \), \( \mu \) is the chemical potential, \( \lambda_n^2 = 1 + 2 n B / B_c \), \( \alpha_l = (l + 1) \mu / T \), \( \sigma_l = (l + 1) m_\nu / T \) and \( B_c = 4.4 \times 10^{13} \text{ G} \). As shown in Fig. 4, the effective potential is an increasing function of magnetic field for \( T = 1, 2.5 \) and 5 MeV. The value of the effective potential lies in the ranges: \( 2.3 \times 10^{-11} - 5.1 \times 10^{-10} \text{ eV} \) for \( T = 1 \text{ MeV} \), \( 2.61 \times 10^{-11} - 5.7 \times 10^{-10} \text{ eV} \) for 2.5 TeV and \( 2.64 \times 10^{-11} - 6.3 \times 10^{-10} \text{ eV} \) for 5 MeV when the magnetic field lies at \( 4 \times 10^9 \leq B \leq 4 \times 10^{12} \text{ G} \).

From this figure, one can see that the effective potential is positive; therefore, due to the positivity of the effective potential \( (V_\text{eff} > 0) \) neutrinos can oscillate resonantly. Taking into account the parameters of the two- (Table 1) and three-neutrino (Table 2) mixing, we analyse the resonance condition, as shown in Fig. 5.

From this figure, we can see that temperature is a decreasing function of chemical potential, for the values of temperature in the range \( 0.7 \leq T \leq 5 \text{ MeV} \). We found that the chemical potential is in the range of \( 1.7 \leq \mu \leq 32 \text{ eV} \) for solar (upper-left panel), \( 2.8 \leq \mu \leq 91 \text{ eV} \) for atmospheric (upper-right panel), \( 36 \leq \mu \leq 465 \text{ keV} \) for accelerator (lower-left panel) and \( 187 \leq \mu \leq 6.5 \times 10^3 \text{ eV} \), for three-neutrino (lower-right panel) mixing. It can also be seen from Fig. 5 that the chemical potential achieves the largest value when accelerator

| Parameters | Best fit |
|------------|----------|
| \( \delta m_1^2 \) | \( 7.62 \times 10^{-5} \text{ eV}^2 \) |
| \( \delta m_2^2 \) | \( 2.55 \times 10^{-3} \text{ eV}^2 \) |
| \( \sin^2 \theta_{12} \) | 0.320 |
| \( \sin^2 \theta_{23} \) | 0.613 |
| \( \sin^2 \theta_{13} \) | 0.0246 |

Table 1. The best-fitting values of two-neutrino mixing (solar, atmospheric and accelerator neutrino experiments) (Araki et al. 2005; Shirai & KamiLAND Collaboration 2007; Aharmim et al. 2011; Abe et al. 2011; Miyata et al. & KamiLAND Collaboration 2011; Aharmim et al. 2013).

Table 2. The best-fitting values of the three-neutrino mixing (Forero, Törtösa & Valde 2012).
parameters are considered, and the smallest one if solar parameters are taken into account.

Recently, Fraija et al. (2014) showed that neutrinos can oscillate resonantly due to the density profiles of the collapsing material surrounding the progenitor. The neutrino effective potential associated with each region is 

\[ V_{\text{eff}} = \sqrt{2} G_F N_A \rho(r) Y_e, \]

with \( Y_e \) the number of electrons per nucleon and \( \rho(r) \) given by equations (10), (13) and (14). We plot the survival and conversion probabilities for the active–active (\( \nu_e, \mu, \tau \leftrightarrow \nu_e, \mu, \tau \)) neutrino oscillations in each region as shown in Fig. 6. From top to bottom panels, we show the oscillation probabilities when neutrinos pass through the new thin crust on the neutron star surface (top), the quasi-hydrostatic envelope (second), the free-fall zone (third) and the external layers (four). One can see that these probabilities vary with neutrino energy (left) and distance (right). Taking into account the oscillation probabilities in each region and in the vacuum (on its path to the Earth), we calculate the flavour ratio expected on the Earth for four neutrino energies (\( E_\nu = 5, 10, 15 \) and \( 20 \) MeV), as shown in Table 3. In this table, we can see a small deviation from the standard ratio flavour 1:1:1. In this calculation, we take into account the fact that for neutrino cooling processes (electron–positron annihilation, inverse beta decay, nucleonic bremsstrahlung and plasmons), only inverse beta decay is the one producing electron neutrino. It is worth noting that our calculations of resonant oscillations were performed for neutrinos instead of antineutrinos, due to the positivity of the neutrino effective potential.

Finally, we estimate the number of initial neutrino bursts expected during the neutron star formation. Considering a temperature \( T \approx 4 \) MeV, a duration of the neutrino pulse \( t \approx 10 \) s, the average neutrino energy \( \langle E_\nu \rangle \approx 15 \) MeV and a total fluence equivalent of Cas A \( \Phi \approx 1.02 \times 10^{52} \bar{v}_e \text{ cm}^{-2} \) (Bahcall 1989; Mohapatra & Pal 2004), then the total number of neutrinos emitted from Cas A would be \( N_{\text{tot}} = 6 \Phi 4\pi d^2 \approx 8.48 \times 10^{57} \). Similarly, we can compute the total radiated luminosity corresponding to the binding energy of the neutron star \( L_\nu \approx N_{\text{tot}} \langle E_\nu \rangle \approx 2 \times 10^{52} \text{ erg s}^{-1} \). Taking into account the effective volume \( V \approx 0.56 \times 10^{52} \text{ cm}^3 \) (Hyper-Kamiokande Working Group 2014), from equation (16) we get \( N_{\text{ev}} \approx 2.29 \times 10^6 \) events that could have been expected during the neutron star formation in a neutrino detector as Hyper-Kamiokande experiment. Comparing the number of neutrinos that would have been expected during the neutron star formation and the hyperaccretion phase, we would get 716.75 events.

![Figure 5](https://academic.oup.com/mnras/article-abstract/451/1/455/1345729)

**Figure 5.** Contour plots of temperature and chemical potential as a function of neutrino energy for which the resonance condition is satisfied. We have required the best-fitting values of the two-neutrino mixing (solar, top left; atmospheric, top right; and accelerator, bottom left) and three-neutrino mixing (bottom right).
Figure 6. Oscillation probabilities of neutrinos as a function of neutrino energy (left) and distance (right) for the three-neutrino mixing parameters. We show the four regions (from top to bottom panels): the new crust on the neutron star surface (top), quasi-hydrostatic envelope (second), free-fall zone (third) and external layers (four).
5 CONCLUSIONS

Although the Cas A supernova event occurred 330 yr ago, its similarity to other core-collapse supernovae allows one to study the early evolution of the protoneutron star inside the supernova remnant. In this work, we have done a study of the hypercritical phase on to the newborn neutron star, in the Cas A scenario, in order to analyse the neutrino cooling effect when a quasi-hydrostatic envelope is formed. This phase is important because it allows one to submerge the magnetic field within the new crust formed by the strong accretion. In this context, after the hyperaccretion has stopped, the magnetic field could diffuse back to the surface and result in a delayed switch-on of a pulsar (Muslimov & Page 1995). Depending on the amount of accreted matter, the submergence could be deep enough that the neutron star may appear and remain unmagnetized for several centuries or millennia (Geppert, Page & Zannias 1999).

Cas A has been classified as one of central compact objects (Pavlov, Sanwal & Teter 2004), defined as X-ray sources with thermal-like spectra observed close to the centres of supernova remnants without any counterparts in radio and gamma wavebands. With blackbody temperatures of hundreds of eV and luminosities in the range $10^{33} - 10^{34}$ erg s$^{-1}$, they present no evidence of a pulsar magnetic field. This scenario was recently revisited by Viganò & Pons (2012) in the context of diffusion of the magnetic field post-hyperaccretion. Following these ideas, we perform HD numerical simulations of the hypercritical phase for the Cas A scenario, focusing on the formation of a thin new crust built in such a phase. We found that the size of this crust is at the same height scale of the neutrinosphere where the neutrino cooling is operative. We show that a very necessary ingredient for forming an atmosphere in quasi-hydrostatic equilibrium is a sink of energy at the bottom of the envelope, which allows the deposition of large amounts of material on to the neutron star’s surface. The copious amount of neutrinos generated on the neutrinosphere, at a very efficient rate, provides this support.

As a signature of this phase, we calculate the number of events expected on Hyper-Kamiokande detector to be 3195. This number of events going through the supernova remnant will oscillate, first, in the thermal and magnetized plasma and, secondly, due to different electron densities from two to four regions and finally, on their paths (vacuum) to the Earth. For the first region, we have used the neutrino effective potential derived in Fraija (2014) which is a function of temperature ($T$), chemical potential ($\mu$), neutrino energy ($E_\nu$) and magnetic field ($B$). We have shown that for a neutrino test of energy 1, 5, 10 and 20 MeV, and parameters considered of temperature and chemical potential in the range of $1 \leq T \leq 5$ MeV, $1 \leq \mu \leq 4.3 \times 10^2$ eV, neutrinos oscillate resonantly, for two- and three-neutrino mixing. In regions from two to four, we have also calculated each effective potential and then analysed their oscillations through these regions. Considering the values obtained in the simulations for Cas A, the neutrino flavour ratios expected on the Earth were computed. Our analysis shows that deviations from 1:1:1 are obtained for neutrino energies of 5, 10, 15 and 20 MeV (see Table 3). Diverse flavour ratios of thermal neutrinos will give us information about the parameters involved in this hypercritical accretion episode.

RCW103, Pup A and Kes 79 (Kaspi 2010) supernova remnants are good candidates to test the validity of the hypercritical phase and the subsequent submergence of the magnetic field, in the early history of these supernovae.

ACKNOWLEDGEMENTS

We are grateful to DGfT-UNAM and to IA-UNAM for allowing us to use their MIZTLI Cluster where all the simulations were performed. The software used in this work was in part developed by the DOE NNSA-ASC OASCR Flash Center at the University of Chicago. This work was supported in part by CONACyT grants CB-2009-1 No. 132400, CB-2008-1 No. 101958, project 128556-F and project 165584. Also, it was supported by PAPIIT project IN106212. Also we thank John Beacom, Dany Page and William Lee for useful discussions. This work was supported by Luc Binette scholarship and the project IG100414. CGB is grateful to CAPES–Brazil for the postdoctoral fellowship received through the Science Without Borders programme.

REFERENCES

Abe K. et al., 2011, Phys. Rev. Lett., 107, 241801
Aharmim B. et al., 2013, Phys. Rev. C, 88, 025501
Ahmedov E. K., Johansson R., Lindner M., Ohlsson T., Schwetz T., 2004, J. High Energy Phys., 4, 78
Araki T. et al., 2005, Phys. Rev. Lett., 94, 081801
Ashworth W. B., Jr, 1980, J. Hist. Astron., 11, 1
Bahcall J. N., 1989, Neutrino Astrophysics. Cambridge Univ. Press, Cambridge
Bernal C. G., Lee W. H., Page D., 2010, Rev. Mex. Astron. Astrofis., 46, 309
Bernal C. G., Page D., Lee W. H., 2013, ApJ, 770, 106
Blondin J. M., 1986, ApJ, 308, 755
Chang P., Bildsten L., 2004, ApJ, 605, 830
Chevalier R. A., 1989, ApJ, 346, 847
Chevalier R. A., 2005, ApJ, 619, 839
Chevalier R. A., Soker N., 1989, ApJ, 341, 867
Dicus D. A., 1972, Phys. Rev. D, 6, 941
Forero D. V., Tórtola M., Valle J. W. F., 2012, Phys. Rev. D, 86, 073012
Fraija N., 2014, ApJ, 787, 140
Fraija N., Bernal C. G., Hidalgo-Gamé A. M., 2014, MNRAS, 442, 239
Fryxell B. et al., 2000, ApJS, 131, 273
Geppert U., Page D., Zannias T., 1999, A&A, 345, 847
Gonzalez-Garcia M. C., 2011, Phys. Part. Nuclei, 42, 577
Gonzalez-Garcia M. C., Maltoni M., 2008, Phys. Rep., 460, 1
Gonzalez-Garcia M. C., Nir Y., 2003, Rev. Mod. Phys., 75, 345
Ho W. C. G., Heinke C. O., 2009, Nature, 462, 71
Hyper-Kamiokande Working Group, 2014, preprint (arXiv:1407.4792)
Itoh N., Hayashi H., Nishikawa A., Kohyama Y., 1996, ApJS, 102, 411
Kaspi V. M., 2010, Proc. Natl. Acad. Sci., 107, 7147
Krause O., Birkmann S. M., Usuda T., Hattori T., Goto M., Rieke G. H., Misselt K. A., 2008, Science, 320, 1195

Table 3. The neutrino flavour ratio in each zone of the collapsing star for $E_\nu = 5, 10, 15$ and $20 \text{ MeV}$.
Learned J. G., Pakvasa S., 1995, Astropart. Phys., 3, 267
Mitsui T., KamLAND Collaboration, 2011, Nucl. Phys. B, 221, 193
Mohapatra R. N., Pal P. B., eds, 2004, Lecture Notes in Physics, Vol. 2, Massive Neutrinos in Physics and Astrophysics. World Scientific Press, Singapore, p. 264
Muslimov A., Page D., 1995, ApJ, 440, L77
Pavlov G. G., Sanwal D., Teter M. A., 2004, in Camilo F., Gaensler B. M., eds, Proc. IAU Symp. 218, Young Neutron Stars and Their Environments. Astron. Soc. Pac., San Francisco, p. 239
Reed J. E., Hester J. J., Fabian A. C., Winkler P. F., 1995, ApJ, 440, 706
Sahu S., Fraija N., Keum Y.-Y., 2009a, J. Cosmol. Astropart. Phys., 11, 24
Sahu S., Fraija N., Keum Y.-Y., 2009b, Phys. Rev. D, 80, 033009
Shirai J., KamLAND Collaboration, 2007, Nucl. Phys. B, 168, 77
Timmes F. X., Swesty F. D., 2000, ApJS, 126, 501
Viganò D., Pons J. A., 2012, MNRAS, 425, 2487

This paper has been typeset from a TEX/LATEX file prepared by the author.