Modeling and simulation of a gauge shaped beam coupled with macro fiber composite for energy harvesting application

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Abstract. This paper presents a novel unimorph piezoelectric cantilever with non-conventional geometry named as gauge shaped beam to investigate stress, Eigen frequencies and electric potential for improved vibration energy scavenging. The harvester is made of macro fiber composite (MFC) as the active piezoelectric layer bonded to a brass substrate. Complete transversely isotropic elastic and piezoelectric properties are assigned to the composite structure in a three-dimensional finite element model using COMSOL Multiphysics. Dynamic behaviour of the proposed structure was simulated and the results obtained were compared with traditional rectangular geometry. The results proved that the gauge shaped beam has a higher stress distribution around the gauge length, enhanced electric potential and lower resonance frequencies than the conventional rectangular structure which makes it better for the design of piezoelectric based vibration energy harvesters.

1. Introduction

In a numerous effort to achieve 100% pollution free environment by most countries of the world, ambient energy scavenging from unconventional sources such as Hydro, solar, wind, and geothermal have attracted keen research interest from various field of studies\cite{1-3}. The aforementioned technologies have been successful in providing green energy in macro scale. But, when dealing with power generation in micro scale, the readily available technology is using chemical batteries. However, batteries have stringent lifetime and there is difficulty associated with their replacement when placed at remote or inaccessible locations. Moreover, they are not biodegradable which make disposal of the depleted chemical cells infeasible. Thus, scavenging mechanical energy in form of vibration using piezoelectric (PZT) transducers has become one of the potential technologies to replace the battery cells. Piezoelectric material bonded to a rectangular cantilever beam as a unimorph or bimorph has been the generic configuration used for designing harvesters\cite{4}. The rectangular cantilever beam serving as the substrate structure has the following advantages: For a given input excitation, it produces average mechanical strain and also it can easily be manufactured through micro-fabrication process. In spite of this, the output from the layered active material is usually below the threshold needed to power the intended systems\cite{5}. This could be attributed to non-optimal utilization of the strain experienced on the shim. Therefore, to enhance the conversion efficiency and thus the energy scavenging abilities of piezoelectric energy harvester (PEH) systems, development of innovative configurations have been considered as one of the premium solutions for optimum vibration based energy harvesting\cite{6-8}. In this paper, a gauge shaped cantilever beam is designed and
simulated with COMSOL Multiphysics 5.3 for the conversion of vibration energy into electrical energy via piezoelectric transducer to achieve a larger strain and higher voltage at lower frequency.

2. Piezoelectricity Theory and Governing Equations
Piezoelectric materials produce charges when subjected to mechanical deformation by applied force or vibration. In addition, it undergoes strain when voltage is applied to the material. Therefore, piezoelectric material exhibit a two way coupling which can be described by constitutive equations in a compacted matrix form as [9];

\[ D = \varepsilon^T \bar{E} + d T \tag{1} \]
\[ S = s^E T + d' \bar{E} \tag{2} \]

Where \( D \) represents the electric displacement, \( \varepsilon^T \) is the dielectric permittivity tensor, \( \bar{E} \) is the applied electric field vector, \( T \) is the stress vector, \( S \) is the strain vector, \( s^E \) is the elastic compliance matrix and \( d' \) is the piezoelectric coefficient or Electro-mechanical coupling factor.

One of the fundamental factors that influences the generation of charges in a piezoelectric cantilevered beam is the amount of stress produced when loaded. Hence, a cantilevered PEH subjected to base excitation in 1st mode of vibration, produces voltage as a function of the stress experienced in the material as[10];

\[ V_{31} = \sigma_{xx} g_{31} f_{xx} \tag{3} \]

Where \( \sigma_{xx} \) is the applied stress, \( g_{31} \) is the effective piezoelectric constant and \( f_{xx} \) is the thickness of PZT layer. Generally, ambient vibration sources exhibit lower frequencies below 200 Hz [11, 12]. In order to harness the mechanical vibration properly, resonant frequency of the PEH should be less than the range. Thus, optimum energy can be harnessed efficiently when harvester is driven at resonant frequency. Due to flexibility, lower resonant frequency and high stress generation cantilever beam structure is more preferred [13].

3. Design of Gauge Shaped and Rectangular beam Harvesters
The proposed gauged shaped harvester was designed using SolidWorks. It is made of three components; a substrate, macro fiber composite (MFC) as the piezoelectric layer and adhesive for bonding the smart material. Figure 1(a) and (b) show the schematics of the harvesters. The dimensions of the substrates are shown in Figure 2(a) and (b) while the dimension (length x width x thickness in mm) of MFC 2807-P2 (28 x 7 x 0.3) from smart material corporation [14] is considered in the design. The adhesive has the same dimension with the MFC patch except its thickness, which is considered to be 0.1mm. It is important to note that the design was aimed at maintaining equal mass for fair comparison.
4. Modeling in COMSOL Multiphysics

The designed 3-D models of the substrates were exported to COMSOL to carry out static analyses. Brass was chosen as the substrate materials and the boundary conditions for the beams such as fix end, free end and applied static force (20N) were set in solid mechanics physics node of the FE model builder. Subsequently, the adhesive and the MFC patch were coupled with the substrate using assembly operation in SolidWorks and then exported again into the FE software to analyse its mechanical and electrical properties. Table 1 shows the physical properties of brass and adhesive used for the simulation. To fully define the MFC patch properties in COMSOL Multiphysics, we need the orthotropic elastic compliance ($S^E$), dielectric piezoelectric constant ($d'$) and dielectric permittivity ($\varepsilon^T$) matrices. Therefore, $S^E$ and $d'$ matrices were populated using manufacturer's data sheet values [14] whereas; the $\varepsilon^T$ was obtained using the mixing rule for the $\varepsilon_{33}^T$ by Deraemake et.al. [15] and a mixing rule for dielectric composites $\varepsilon_{22}^T$ from Jaffe [16]. After that, the top and bottom surfaces of the piezoelectric layers were defined as ground and terminal in boundary conditions of electrostatics physics. The rest of the faces were labelled as zero charge. In the next step, a tetrahedral mesh with extremely fine element size was used for the discretization of the beams. Finally, Eigen frequency and electric potential analysis were performed to determine eigenvalues and voltage output potential of the harvesters.

Table 1. Properties of materials used for simulation

| Parameters                  | Brass         | Adhesive (Epoxy DP-460) |
|-----------------------------|---------------|-------------------------|
| Young’s Modulus (N/m$^2$)   | 100x10$^9$    | 2.7x10$^9$              |
| Poison’s ratio              | 0.34          | 0.4                     |
| Density (Kg/m$^3$)          | 8740          | 1100                    |

5. Simulation Results and Discussion

In this section, the two configurations have been studied to obtain their mechanical and electrical characteristics. Figure 3 (a) and (b) show the static analysis results. From the colour contour of von Mises stress for the gauge shaped beam, a maximum stress of 9.29x10$^8$ N/m$^2$ was observed around the curvature region close to the face where it was fixed.

Figure 3. Von Mises stress analysis of (a) Gauge shaped beam and (b) Rectangular cantilever beam.
On the other hand, the conventional beam has a maximum stress of $7.91 \times 10^8 \text{N/m}^2$ close to its clamped end. Hence, it can be deduced that the energy conversion from mechanical to electrical will be higher when a piezoelectric cantilevered beam experienced larger stress at its resonance frequency. Therefore, it is obvious that the proposed beam harvester has greater tendency of converting its stress into voltage output based on relationship presented in equation (3). It is worth noting that the lower values of stresses ($1.95 \times 10^6 \text{N/m}^2$ and $2.92 \times 10^6 \text{N/m}^2$) exhibited around the free ends have less significance on the energy conversion, which is why the design of partially covered piezoelectric beam harvesters are mostly considered in the literature. Furthermore, an Eigen frequency analysis was conducted to determine resonance frequencies as well as tip displacements of the two configurations. Figure 4 (a) and (b) show the first three vibration modes of the gauged shaped and rectangular cantilever beam harvesters respectively.

The results show that the free end of the gauged shaped harvester was displaced and reached a maximum value of 0.065 mm at a fundamental resonance frequency of 109.15 Hz. Also, maximum tip displacement values of 0.02 mm and 0.08 mm were recorded at second and third resonance frequencies of 583.58 Hz and 834.54 Hz respectively. For the conventional harvester, maximum tip displacement of 0.045 mm, 0.001 mm and 0.015 mm were obtained at 1st, 2nd and 3rd resonance frequencies of 124.92 Hz, 621.07 Hz and 1061.1 Hz respectively. It is important to note that there is decrease in natural frequencies of the proposed harvester over the rectangular beam harvester in all the three modes of vibration considered. This advantage is highly desirable in designing vibration based energy harvester due to the fact that most vibration sources have lower excitation frequencies. The lower resonance frequencies of the gauge shape harvester could be attributed to the designed flange at its free end serving as an integrated tip mass. Figure 5 (a) and (b) show the electric potential of the two configurations at their fundamental resonance frequencies (109.15 Hz and 124.94 Hz). A voltage output ranging between 13.3V to -13.2V was achieved from the proposed harvester while 10V to -10V was produced by the conventional harvester. The enhanced output of the gauge shaped harvester could ascribe to the two flanges at both ends which provide an appreciable strain along its length.
6. Conclusion

In this study, an energy harvester with non-traditional geometry was designed and modelled in COMSOL Multiphysics. The simulation of the proposed geometry (stress, resonant frequency, displacement and electric potential) were analysed and compared with rectangular shaped cantilever beam. The results showed that the proposed shape had a higher stress distribution, lower resonant frequencies and higher electric potential than the conventional beam harvester. Hence, the design of the proposed gauge shaped beam provides a new insight for development of an improved piezoelectric based vibration energy harvester.

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