AN INDEPENDENT DERIVATION OF THE OXFORD JET KINETIC LUMINOSITY FORMULA

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ABSTRACT

This Letter presents a theoretical derivation of an estimate for a radio source jet kinetic luminosity. The expression yields jet powers that are quantitatively similar to a more sophisticated empirical relation published by C. Willott, K. Blundell, and S. Rawlings at Oxford. The formula allows one to estimate the jet kinetic luminosity from the measurement of the optically thin radio lobe emission in quasars and radio galaxies. Motivated by recent X-ray observations, the derivation assumes that most of the energy in the lobes is in plasma thermal energy with a negligible contribution from magnetic energy (not equipartition). The close agreement of the two independent expressions makes the veracity of these estimates seem very plausible.

Subject headings: galaxies: jets — quasars: general

1. INTRODUCTION

The purpose of this Letter is to discuss estimates of the power transported by the radio jets in quasars and radio galaxies. An accurate estimate of the jet power is of fundamental physical interest, since it can be used to quantify the power emerging from the central engine of the radio source. In actuality, the radio luminosity is merely an indirect measure of the energy transported through the jets from the central engine that is not readily interpretable. Most of the energy flux is in mechanical form (kinetic luminosity)—the particles and fields necessary to produce the synchrotron luminosity that is detected in the radio lobes. The radiation losses, manifested as radio emission from the jet, are merely the waste energy of this kinematic flow.

Surprisingly, the most difficult methods of estimating jet power rely on observations of the jets themselves. Due to significant Doppler enhancement in relativistic jets, the synchrotron radio emission is a poor indicator of intrinsic jet power. For example, Cygnus A has extremely powerful radio lobes and faint radio jets. Most of the energy in the jets is not radiated away but is transported to the lobes in the classical FR II double-lobe morphology. Even the inclusion of observations of high-energy emission such as optical or X-ray (inverse Compton) in one’s analysis of jet energetics does not tightly constrain the bulk jet flow. If the resolution is poor at high frequency (as is often the case), then one cannot necessarily associate the plasma emitting the high-frequency photons with the radio-emitting plasma. If one has high-resolution images, then the high-frequency emission can be detected in enhancement regions or knots in the jets. One can use this information to get an estimate of the plasma conditions within the dissipative knot, but this does not necessarily constrain the plasma state in the bulk of the jet. Furthermore, there are still ambiguities with the Doppler factor that affect the estimates quite dramatically.

The Doppler enhancement of relativistic flows in jets is a crucial parameter since the total luminosity of an unresolved jet scales as the Doppler factor to the fourth power and to the third power for a resolved cylindrical jet (Lind & Blandford 1985). This is the reason why the implementation of 5 GHz flux densities, as is common in studies of the radio loudness of large quasar samples, is a poor indicator of the true intrinsic kinetic luminosity of the jets. More specifically, the majority of core-dominated blazar-like quasars have incredibly strong 5 GHz flux densities from emission on the subkiloparsec scale, yet they have weak or moderate radio lobe emission (Punsly 1995). This is interpreted as the jet being of modest kinetic luminosity (at most) because there is not a large amount of hot plasma and gas that has been transported through the jets to the radio lobes. The 5 GHz flux only represents the dissipation of hot plasma, and it has been extremely Doppler-boosted. An estimate of kinetic luminosity based on 5 GHz flux density can be off by 4 or more orders of magnitude for a core-dominated blazar.

A far better way to estimate the kinetic luminosity from a jet is to study the isotropic properties of the material ejected from the ends of the jets in the radio lobes. The radiation from the lobe material is generally considered to be of low enough bulk velocity so that Doppler enhancement is not much of an issue. The basic idea is that lobe expansion is dictated by the internal dynamics of the lobes and the physical state of the enveloping extragalactic gas. X-ray observations can indicate a bremsstrahlung spectrum of the surrounding gas that can be used to find the pressure of the extragalactic medium. X-rays also provide information on the working surfaces at the end of the lobes, “the hot spots.” One can associate the X-ray emission with inverse Compton radiation from the hot spots, and the radio luminosity is the synchrotron emission from the hot spots. This constrains the plasma state within the luminous hot spots. However, most of the energy stored in the lobes is in the large diffuse regions of radio emission that constitute the majority of the large volume of the radio lobes. It is the enormous volume of synchrotron-emitting plasma within the lobes (∼104–105 kpc3) that is the most direct indicator that the jets must be supplying huge quantities of hot plasma and magnetic field energy to the lobes. One can also use the curvature of the lobe synchrotron radio spectra to estimate parameters in the diffuse lobe gas—this is known as spectral ageing. Of course, all of these plasma state estimations are most accurate when applied to situations in which one has deep X-ray and radio data of a relaxed classical double-lobe structure. This only occurs in a few instances, so such detailed analyses are not compatible with large sample studies. Motivated by these limitations, this Letter presents two techniques for estimating jet energy based on partial information on the lobe parameters.
The two methods involve different assumptions and have different ambiguities.

The most sophisticated calculation of the jet kinetic luminosity incorporates deviations from the minimum-energy estimates in a multiplicative factor $f$ that represents the small departures from minimum energy, geometric effects, filling factors, protonic contributions, and the low-frequency cutoff (see Willott et al. 1999 for details). The quantity $f$, it is argued, is constrained to be between 1 and 20. In Blundell & Rawlings (2000), it was further determined that $f$ is most likely in the range of 10–20. Therefore, we choose $f = 15$ in order to convert 151 MHz flux densities, $S_{151}$, to estimates of kinetic luminosity, $Q_{151}$, using equation (12) and Figure 7 of Willott et al. (1999).

$$Q_{151} \approx 1.1 \times 10^{48} (1+z)^{1+a} Z F_{151}^{-0.7} \text{ ergs s}^{-1}. \quad (1)$$

$$Z \approx (3.31–3.65)[(1+z)^{-4} – 0.203(1+z)^{-1} + 0.749(1+z)^{2} + 0.444(1+z) + 0.205]^{-0.125}. \quad (2)$$

The quantity $F_{151}$ is the optically thin flux density from the lobes (i.e., no contribution from Doppler-boosted jets or radio cores) measured at 151 MHz in units of Janskys. The flux density spectral index is defined as $F \sim \nu^{-a}$. We have assumed a cosmology with $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.7$, and $\Omega_{\Lambda} = 0.3$. The expression for $Z$ is from Pen (1999).

In the following, a new estimate of the jet kinetic luminosity is derived that is motivated by the wealth of X-ray data on radio lobes that has been published since Willott et al. (1999) and Blundell & Rawlings (2000). Both the current manuscript and the Willott et al. (1999) derivations rest on the basic premise that $Q = U/T + \text{radiation losses}$, where $U$ is the energy stored in the lobes and $T$ is the elapsed time. In Willott et al. (1999), $U$ is found by assuming that the lobes are near equipartition and that there is uncertainty in the energy from the protonic components and low-frequency cutoffs that are incorporated in the empirical factor $f$ discussed above. Conversely, motivated by the new X-ray data described below, $U$ is computed theoretically in the limit that the lobes are very far from equipartition. In Willott et al. (1999), $T$ is determined by an empirical estimator for the age of the radio source based on its length and head advance speed. Conversely, in this treatment, $T$ is computed from spectral ageing. In spite of the fact that $U$ and $T$ are determined from completely different methods and assumptions, the expressions for $Q$ that are found in equations (1) and (11) yield similar values (to within a factor of 2) for the jet luminosity. This close agreement lends credence to the claim that these formulae are robust estimators of jet kinetic luminosity.

2. MOTIVATION: X-RAY OBSERVATIONS

The minimum-energy condition in the lobes seems to be in conflict with the X-ray data on the surrounding extragalactic gas. This was first noted for Cygnus A (see Punsly 2001 and references therein) and later for 3C 388 in Leahy & Gizani (2001), as well as for a large sample of FR II radio sources in Hardcastle & Worrall (2000) based on ROSAT data. These studies concluded that typically the pressure in the external gas greatly exceeds (by at least an order of magnitude) the lobe pressure associated with the minimum-energy assumption. The general picture that seems to be emerging from the X-ray data of ROSAT, ASCA, XMM, and Chandra is that the hot spot energies seem to slightly exceed the minimum-energy requirement based on inverse Compton calculations of X-rays from the hot spots (see, e.g., Wilson et al. 2001 and Brunetti et al. 2002) but that the lobes themselves are far from equipartition. In Leahy & Gizani (2001), it was deduced that the departure from equipartition was most likely the consequence of a low-energy population of positrons and electrons that was not an extension of the power-law distribution responsible for the radio emission but a low-energy excess. The other possibility is protonic matter, which would also drastically increase the energy content of the lobes over the equipartition estimates. In either case, the field energy is only a few percent of the particle energy in the lobes. For example, high-resolution Chandra data were used to model the X-rays as inverse Compton emission in the lobes in the radio galaxy 3C 219 (Brunetti et al. 2001; Comastri et al. 2003). It was concluded that the particle energy exceeded the magnetic energy by a factor of 60 in the radio lobes. Similarly, the FR II radio galaxy 3C 452 was studied with Chandra in Isobe et al. (2002), who estimate that the energy density in the particles is 27 times that of the magnetic field in the lobes. For most FR II sources, typical magnetic field strengths in the lobes estimated from X-ray data and pressure balance are only a third to a fifth of the minimum-energy value (Hardcastle & Worrall 2000; Leahy & Gizani 2001).

These X-ray observations yield valuable information on the energy content of the lobes but are disjoint from the spectral ageing estimates of the lobe advance speeds. In the following estimation of jet kinetic energy, it is assumed, based on the X-ray data presented above, that the energy content of the lobes is purely in particle form to first order (accurate to a few percent). Yet, the notion that spectral ageing provides an estimate of lobe age is retained. By choosing a subequipartition field strength in the lobes, spectral ageing estimates are found to be longer than the corresponding minimum-energy estimates (Alexander & Pooley 1996). Similarly, by setting $t_{\text{sep}} = t_{\text{syn}}$, the subequipartition fields yield lower lobe separation velocities if this spectral ageing argument is viable. Thus, the problem of the large lobe advanced speeds in the minimum-energy assumption is remedied by this modification (Alexander & Pooley 1996). This method of computing jet kinetic luminosity is the lowest order improvement to the minimum-energy estimate and was implemented in Punsly (2001) to study Cygnus A.

3. PARTICLE ENERGY–DOMINATED LOBES

Motivated by the X-ray observations, we proceed to compute the jet power based on the limit that all of the lobe energy is in the hot particles. We also assume that the time to convert a jet energy flux to the stored lobe energy is the time that it has taken the lobes to propagate from the central engine to their current separation, $t_{\text{sep}}$.

3.1. Spectral Ageing

Spectral ageing within the radio lobes is often used to determine the lobe plasma age. The results are predicated on the assumption that the lobe plasma is primarily back-flowing plasma in the sense that jet plasma is deflected backward at the working surfaces in the hot spots to form the lobe plasma. By studying the curvature of the radio spectra at different points within the lobes, one can in principle (if there is no reheating or reionization of the plasma) determine the gradient in the high-energy cutoff of the electron distribution due to synchrotron
cooling and hence the plasma age. The age of the lobe plasma closest to the central engine should be the oldest plasma. Thus, one has an estimate of lobe age and therefore the lobe advance speed. Defining the spectral break frequency as \( \nu_b \), the synchrotron lifetime is expressed in cgs units as (Liu et al. 1992)

\[
t_{\text{syn}} \approx 1.58 \times 10^{12} B^{-3/2} \nu_b^{-1/2}.
\]

### 3.2. The Energy Contained within the Synchrotron-emitting Plasma

Consider a power-law distribution of energetic particles (probably electrons and positrons) expressed in terms of the thermal Lorentz factor, \( \gamma \), for a uniform source in a volume, \( V \). The total number of particles contributing to the synchrotron radiation in the frequency interval \( \nu_1 \leq \nu \leq \nu_2 \) is

\[
N_v = N_0 V \int_{\nu_1}^{\nu_2} \gamma^{-\alpha} d\gamma.
\]

The minimum and maximum Lorentz factors in the expression above are related to lower and upper cutoff frequencies \( \nu_1 \) and \( \nu_2 \), respectively, in the synchrotron spectrum by (Ginzburg 1979)

\[
\gamma_1 = \left( \frac{2 \nu_1 y_1(n)}{3 \nu_b} \right)^{1/2}, \quad \gamma_2 = \left( \frac{2 \nu_2 y_2(n)}{3 \nu_b} \right)^{1/2},
\]

where \( \nu_b = (e B)/(2 \pi m_e c) \) is the cyclotron frequency, and note that

\[
y_1(n) = 2.2, \quad y_2(n) = 0.10, \quad \text{if } n = 2.5;
\]

\[
y_1(n) = 2.7, \quad y_2(n) = 0.18, \quad \text{if } n = 3.0.
\]

The synchrotron spectral luminosity of the plasma, \( L(\nu) \), is a function of both the particle distribution in momentum space and the magnetic field strength. Integrating the synchrotron power formula over the particle distribution yields (Ginzburg 1979)

\[
U_\nu \approx \frac{2 \times 10^{12} B^{-3/2}}{a(n)(n-2)} L(\nu_b) n^{1/2} [y_1(n)]^{n-1/2}
\]

\[
\times \left\{ 1 - \frac{y_2(n) \nu_b}{y_1(n) \nu_2} \right\}^{n-1/2},
\]

where

\[
a(n) = \frac{\left(2^{n-1/2} \sqrt{3}\right)\Gamma((3n - 1)/12)\Gamma((3n + 19)/12)\Gamma([n + 5]/4)}{8\sqrt{\pi}(n + 1)\Gamma([n + 7]/4)}.
\]

### 3.3. Estimating the Jet Power

Set \( t_{\text{sep}} \) equal to the synchrotron ageing timescale \( t_{\text{syn}} \), associated with the spectral break in the flux density, \( F_\nu \), of the lobe plasma closest to the quasar (the emission just above the spectral break is from the lowest energy electrons that have synchrotron-radiated away their energy and hence the oldest subpopulation of charges that have experienced synchrotron decay in the lobes). By combining \( t_{\text{syn}} \), from equation (3) with the expression for the plasma luminosity, equation (7), one obtains an estimate for the energy stored in the lobes as a function of spectral luminosity, \( U(\nu) \), in the limit of particle energy dominance,

\[
U_\nu \approx \frac{L(\nu_b) [y_2(n) \nu_b]^{1/2} [y_1(n)]^{(n-1)/2} t_{\text{syn}}}{7.9(n-2) a(n) [y_1(n)]^{(n-1)/2}}.
\]

Evaluation of the formula above requires numerous characteristic frequencies that need to be determined. Since expressions that are applicable to sparse data are desired for evaluating large samples, we choose a common set of “typical” parameters for an FR II radio source. First of all, determining \( \nu_b \) requires high-resolution maps of the lobes at a variety of frequencies. These data have been obtained for only a limited number of bright sources. The largest sample of these detailed observations is from Liu et al. (1992). The average rest-frame break frequency from the sample of Liu et al. (1992) is \( \nu_b = 8.9 \pm 7.0 \) GHz. Second, in order to estimate the minimum synchrotron frequency, we note that from Braude et al. (1969) (even though the measurements are likely to be extremely inaccurate) it is clear that many FR II sources are very strong emitters down to frequencies at least as low as 12.6 MHz. Thus, we pick \( \nu_1 = 10 \) MHz in the quasar rest frame. Finally, in order to approximate the total radio luminosity, \( L = \int L(\nu) d\nu \) (including the significant contribution at frequencies above the spectral break), with a single spectral index, a value of \( \nu_b = 100 \) GHz is chosen. Inserting these “typical” frequency values into equation (9), one obtains a simple estimator of lobe power in the limit of particle dominance, and noting that at the spectral break frequency, \( t_{\text{sep}} \approx t_{\text{syn}} \),

\[
Q \approx \frac{U_\nu}{t_{\text{sep}}} + L \approx \frac{[y_1(n)]^{n-1/2}(15.1)^n}{(n-2) a(n)} \times 10^{42} (1 + z)^{1+z} \nu_Z^2 F_{51} \text{ ergs s}^{-1} + L,
\]

where the spectral index \( \alpha = (n - 1)/2 \) has been introduced \([L(\nu) \sim \nu^{-\alpha}]\). It should be noted that the estimates above are very conservative. The existence of a substantial proton component to the lobe gas or an extension of the low-frequency portion of the electron spectrum would increase the energy flux estimates significantly. Observations suggest that \( \alpha \approx 1 \) is a good fiducial value for equation (10) (Kellermann et al. 1969),

\[
Q_{\text{par}} \approx 5.7 \times 10^{44} (1 + z)^{1+z} \nu_Z^2 F_{51} \text{ ergs s}^{-1}, \quad \alpha \approx 1.
\]

### 4. Conclusion

An independent formula for the jet kinetic luminosity estimator in equation (1) is derived in equation (11) and was motivated by different physical assumptions. The two estimates agree to within a factor of 2. This lends credence to the idea that equations (1) and (11) are robust estimators of jet kinetic luminosity when the optically thin extended emission is measured in a deep radio map. The main result of this Letter is that equation (1), although very ambitious in its intent, is likely to be correct to within a factor of a few even if some of the assumptions in its derivation are inaccurate.
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