Spin-rotor Interpretation of Identical Bands and Quantized Alignment in Superdeformed $A \approx 190$ Nuclei

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Abstract

The “identical” bands in superdeformed mercury, thallium, and lead nuclei are interpreted as examples of orbital angular momentum rotors with the weak spin-orbit coupling of pseudo-$SU(3)$ symmetries and supersymmetries.

21.60.Fw, 21.60.Ev, 23.20.Lv, 27.80.+w
I. INTRODUCTION

Considerable excitement in nuclear structure science has been generated by the observation of cascades of $\gamma$-ray transitions in superdeformed (SD) rotational bands with “identical” energies. The initial observation was in pairs of $A \approx 150$ nuclei ($^{151}$Tb$^*$-$^{152}$Dy) and ($^{150}$Gd$^*$-$^{151}$Tb) where the $\gamma$-ray energies in the pairs were identical within 1-3 keV. This was followed shortly by the observation of excited superdeformed bands in $^{194}$Hg with $\gamma$-ray energies related to those of the only known SD band in $^{192}$Hg: one of these bands has $\gamma$-ray energies identical to those in SD $^{192}$Hg, the other has energies which occur at the arithmetic mean or “midpoint” values. In the following “identical” will refer to $\gamma$-ray transition energies which are simply related to those in a reference band, and will include “quarter points”, as well as midpoint and equal energies. This paper will propose a new rotational coupling scheme in which the observed pattern of $\gamma$-ray transition energies is predicted. Preliminary, but incomplete, reports of this work have been previously presented.

Since the initial observations and with the advent of the largest arrays, Eurogam and Gammasphere, a large number of SD bands have been identified and the earlier candidates have been more firmly established and extended to both lower and higher $\gamma$-ray energies. The data which we shall discuss are related to $^{192}$Hg and $^{193}$Tl SD reference configurations. Related to the $^{192}$Hg reference are the yrast SD band in $^{194}$Pb, $^{191}$Hg* SD-2 and 3, $^{193}$Hg SD-3 and 4, and $^{194}$Hg* SD-2 and 3. Related to the $^{193}$Tl reference are 4 SD bands in $^{194}$Tl (SD 1-4) and 2 SD bands in $^{194}$Pb* (SD-2a and 2b). The available data have recently been compiled and we have adopted the energies and nomenclature of Ref. [12], supplemented with the most recent available data from Refs. [9–11]. We adopted the energies and nomenclature of the evaluated date reported in ref. [12] because some of the bands have been observed by several groups, all with their own system to name the bands. In ref. [12] the bands are uniformly labeled by an arabic number, with the lowest being the first, usually strongest, band observed. Adjacent numbers could be signature partners, but no such assumption of physical properties are proposed in ref. [12].
One of the main differences between the $A \approx 150$ and $A \approx 190$ SD excitations is that the $A \approx 190$ SD $\gamma$-ray cascades extend to relatively low $E_\gamma$, typically $< 250$ keV for the data in Figs. 1-4. The regular behavior of the SD cascades and the low transition energies imply that the cascades extend down to low angular momentum and suggest that reliable spin assignments \[13\] can be made through comparison with the predictions of the quantum rotor. For the bands displayed in Figs. 1-4, the extracted $J_f$ values are within $0.1\hbar$ of integer (for even-even) or half-integer (for odd-$A$) values. Uncertainties in $J_f$ values are larger for the odd-odd cases and the weak $^{194}\text{Pb}^*$ SD bands.

There are two new aspects of these SD excitations which are not expected. First, the $\gamma$-ray energies in neighboring nuclei are directly related to those of $^{192}\text{Hg}$ or $^{193}\text{Tl}$. This means that these rotational structures have identical moments of inertia, in itself an unexpected result. The second new and unexpected result is the value of the difference in angular momenta between one SD band and the reference, which was summarized in Fig. 3 of Ref. \[2\]b. This additional angular momentum, or alignment, sets in at moderate $\gamma$-ray energies and saturates at $1.00(4)\hbar$ for the bands displayed in Figs. 1 and 2, and is very close to $1\hbar$ or $0\hbar$ for four $^{194}\text{Tl}$ SD bands with respect to $^{193}\text{Tl}$. This observation of $1\hbar$ of alignment was not expected, and has led to intense scrutiny of the methods used to extract the spins. We stand by our spin assignments; a full discussion is given in Ref. \[13\].

\section*{II. COUPLING SCHEMES WITH PSEUDO ORBITAL ANGULAR MOMENTA}

The observation of identical bands and integer alignment at moderate $\gamma$-ray energies suggests these rotational excitations are dominated by an integer angular momentum, $L$, with a contribution from the (pseudo-) orbital angular momenta of the valence fermions, and only a small (pseudo-) spin-orbit interaction. While not a new coupling scheme, it is not the usual one for heavy nuclei, in which the single-particle structure of nuclei is assumed to be governed by the total angular momentum, $\vec{j} = \vec{l} + \vec{s}$, rather than $l$ and $s$, separately. For example, in the 50-82 nuclear shell, the orbitals are $g_{7/2}$, $d_{5/2}$, $h_{11/2}$, $d_{3/2}$, and $s_{1/2}$; the $g_{9/2}$
orbital is below the 50 shell gap, and the $h_{11/2}$ negative-parity orbital has come down from the $N = 5$ shell. The normal, in this case positive parity, orbitals in the 50-82 shell have $j = 7/2, 5/2, 3/2, 1/2$; these are exactly the $j$-values for the orbitals in a (pseudo) $N = 3$ shell. In many cases a pseudo-harmonic oscillator \[14\], or pseudo-$SU(3)$ scheme for deformed nuclei, is an appropriate framework in which to discuss nuclear excitations, and it can provide a good description of the observables. We are suggesting that a coupling dominated by the total orbital angular momentum, with relatively little spin-orbit splitting, as is the case for the pseudo-harmonic oscillator, can reproduce the observed pattern of identical SD bands in $A \approx 190$ nuclei. In the following we shall discuss two different coupling schemes in which the integer alignments can be accommodated.

A. Strong Coupling between the Spins

We consider a core-particle model in which the core angular momentum $R$ is coupled with the pseudo-orbital part $\tilde{L}$ of the nucleons outside the core to $\vec{L} = \vec{R} + \vec{\tilde{L}}$, which is subsequently coupled with the spin part $S$ to total angular momentum $\vec{J} = \vec{L} + \vec{S}$. In the first coupling scheme, in which we assume a strong coupling between the spin parts of the angular momenta of the nucleons outside the core, the rotational hamiltonian for identical SD excitations is \[15,16\]

\[
H = a \vec{S} \cdot \vec{S} + b \vec{L} \cdot \vec{L} + c \vec{L} \cdot \vec{S} .
\]  

The eigenvalues and $\gamma$-ray transition energies of this hamiltonian can be written as

\[
E = A_1 S(S + 1) + B_1 L(L + 1) + C_1 J(J + 1) , \\
E_\gamma(J + 1 \rightarrow J - 1) = B_1 (4L + 2) + C_1 (4J + 2) ,
\]  

with parameters $A_1 = a - c/2$, $B_1 = b - c/2$ and $C_1 = c/2$. The $\gamma$-ray energies depend only on $B_1$ and $C_1$, since the $A_1$ term only contributes to the band-head energy. This formulation has been proposed previously to study pseudo-spin symmetries in superdeformed nuclei \[13\].
For a one-fermion \((N_F = 1)\) nucleus, \(S = 1/2\) and \(J = L \pm 1/2\). For a two-fermion \((N_F = 2)\) configuration, \(S = 0\) or \(S = 1\); since \(\vec{J} = \vec{L} + \vec{S}\), a plethora of \(J\) values arises from the vector addition. Generic spectra for \(N_F = 1\) and \(N_F = 2\) systems are illustrated in Figs. 5 and 6, respectively. For the \(N_F = 1\) case three types of spectra are expected. The first two in Figs. 5a and 5b, respectively, are decoupled structures, which in a more traditional framework are bands with decoupling parameters +1 and −1. In both of these cases only one \(\gamma\)-ray cascade would be observed, with \(\gamma\)-ray energies either identical to or at midpoint values to those of the even-even reference, when \(C_1 = 0\). This type of spin-rotor could explain many of the identical bands observed in the \(A \approx 150\) region, and in particular, the \(^{151}\text{Tb}^{*},^{152}\text{Dy}\) pair of SD bands \([1]\). However, the “identical” bands in \(^{191,193}\text{Hg}\) compared to \(^{192}\text{Hg}\) are not examples of decoupled structures, but rather can be understood as examples of the generic spectrum of Fig. 5c.

For the \(N_F = 1\), \(J = L + 1/2\) (Fig. 5c) and \(N_F = 2\), \(S = 1\), \(J = L + 1\) (Fig. 6c) cases, the transition energies are given by:

\[
egin{align*}
N_F = 0 & \quad E_{\gamma} = (B_1 + C_1)(4J + 2) & J = L, \\
N_F = 1 & \quad E_{\gamma} = B_1(4J) + C_1(4J + 2) & J = L + 1/2, \\
N_F = 2 & \quad E_{\gamma} = B_1(4J - 2) + C_1(4J + 2) & J = L + 1. \\
\end{align*}
\]

Although the spectra in Fig. 5 were generated with \(C_1 = 0\), the \(\gamma\)-ray energies can depend on \(C_1\) without breaking the symmetry; it is the \(C_1\) term which is the additional ingredient required to reproduce the observed alignments. When \(B_1 = -2C_1\) the transition energies in Eq. 3 become

\[
egin{align*}
N_F = 0 & \quad E_{\gamma} = B_1(2J + 1) & J = L, \\
N_F = 1 & \quad E_{\gamma} = B_1(2J - 1) & J = L + 1/2, \\
N_F = 2 & \quad E_{\gamma} = B_1(2J - 3) & J = L + 1. \\
\end{align*}
\]

In Fig. 1b we compare the \(\gamma\)-ray transitions in \(^{193}\text{Hg}\) SD-3 and 4 bands to the expectations of a spin-rotor, Eq. 4. With this choice of parameters the observed alignment \(i = 1h\) can be reproduced, and the same value of \(B_1\) is used for the reference and one-fermion bands. The
same quality of agreement would have been observed if we had chosen to compare the data for $^{191}\text{Hg}$ SD-2,3 with the predictions of Eq. 4.

For the $N_F = 2$ case there are a large number of generic spectra. The simplest case occurs for $S = 0$ and the same orbital angular momenta, $L$, for the reference and $N_F = 2$ configuration. This gives identical $E_\gamma$ values, exactly what is observed for $^{192}\text{Hg}$ and $^{194}\text{Pb}$, where the $\gamma$-ray energies are on average within 1 keV for 10 transitions. In addition to $S = 0$, $S = 1$ is allowed for the two-fermion system. That two-particle excitations in which the spins are aligned (with $S = 1$) could be important is in accord with the earlier suggestion that triplet pairing could be used to understand the observed $i = 1\hbar$ alignment. Since two excited bands are observed in $^{194}\text{Hg}$, a strongly coupled spectrum is suggested with $S = 1$ and $J = L + 1$, as displayed in Fig. 6c. With Eq. 4 and the same parameters used to fit the odd-$A$ spectrum, this coupling scheme gives $i = 2\hbar$, as displayed in Fig. 2b, but in contrast to the data. The hamiltonian of Eq. 1 cannot then be used to fit simultaneously the data for the single neutron and two neutron excitations, if we require the same parameters for all nuclei in a multiplet. Rather, when $B_1 = -2C_1$ one gets $i = 1\hbar$ for $N_F = 1$ systems and $i = 2\hbar$ for $N_F = 2$; or when $C_1 = 0$ one gets $i = \hbar/2$ for $N_F = 1$ and $i = 1\hbar$ for $N_F = 2$ systems.

In $^{194}\text{Tl}$ six SD bands have been identified; two of these have zero alignment with respect to the $^{193}\text{Tl}$ reference, and two have $i \approx 1\hbar$, as shown in Fig. 3a. $^{194}\text{Tl}$ is again a two-fermion system, for which we propose $S = 1$. In Fig. 3b we present the expectations for the reference ($N_F = 0$), $N_F = 1$ ($^{193}\text{Tl}$) and $N_F = 2$, $S = 1$ bands. The $\gamma$-ray transitions in Eq. 4 give $N_F = 2$ bands with $1\hbar$ of alignment with respect to the one-fermion core, which reproduces the pattern for SD-1 and 2 bands in $^{194}\text{Tl}$. In contrast, the SD-3 and 4 bands in $^{194}\text{Tl}$ have zero alignment with respect to $^{193}\text{Tl}$, or $1\hbar$ of alignment with respect to an even-even core. As was the case for the two-neutron bands in $^{194}\text{Hg}$, the coupling scheme of Eq. 2, with the same parameters for the $N_F = 1$ and $N_F = 2$ nuclei, cannot reproduce the data for SD-3 and 4 in $^{194}\text{Tl}$.

The coupling scheme of Eq. 2 assumes that the spins of the two fermions couple strongly
to $S = 1$. For identical fermions, the Pauli principle requires that the total wave function is antisymmetric. Therefore, the relative orbital angular momentum of the two identical fermions must be odd. There is also no guarantee that both excited fermions in $^{194}\text{Hg}$ will come from the same oscillator shell as the single fermion in $^{193}\text{Hg}$, for example, and clearly the odd-neutron and odd-proton in $^{194}\text{Tl}$ can be expected to have very different configurations. Therefore, it may not always be correct to assume the strong coupling of the spins of the two fermions.

The level diagram for single-particle configurations at large deformations is a complicated mixture of orbitals from many shells. For example, the neutron orbitals for Hg nuclei in their ground states are mostly from the $N = 5$ oscillator shell; at large deformations one also finds many orbitals from the $N = 6$ shell, as well as “intruder” $j_{15/2}$ configurations from the $N = 7$ shell. Only the isolated high-$j$ $N = 7$ orbitals need be considered as outside of the framework of a symmetry. Therefore, the two-fermion system can either have two particles in the same shell, or each fermion can come from an orbital from a different shell.

**B. Weak Coupling between the Spins**

Again we consider a core-particle model in which the core angular momentum $R$ is coupled with the pseudo-orbital part $\tilde{L}$ of the nucleons outside the core to $\tilde{L} = \tilde{R} + \tilde{L}$. In this case we anticipate that the two fermions are from different oscillator (or pseudo-oscillator) shells. Therefore, the spins are not coupled together, but rather $\tilde{J}_1 = \tilde{L} + \tilde{S}_1$ is the angular momentum involving the spin of one of the fermions, and $\tilde{J} = \tilde{J}_1 + \tilde{S}_2$ is the total angular momentum. In this model the excitation spectrum and $\gamma$-ray transition energies are given by

$$E = B_2 L(L + 1) + C_2 J_1(J_1 + 1) + D_2 J(J + 1),$$

$$E_\gamma(J + 1 \rightarrow J - 1) = B_2 (4L + 2) + C_2 (4J_1 + 2) + D_2 (4J + 2). \quad (5)$$

Again, there will be a large variety of bands arising from the different ways the angular
momenta can be coupled to total $J$. For the generic spectra illustrated in Fig. 7, with $J = J_1 + 1/2$ for the $N_F = 2$ nucleus, the transition energies are

\[
\begin{align*}
N_F = 0 & \quad E_\gamma = (B_2 + C_2 + D_2) (4J + 2) \\
N_F = 1 & \quad E_\gamma = B_2 (4J - 2) + C_2 (4J) + D_2 (4J + 2) \\
N_F = 2 & \quad E_\gamma = B_2 (4J + 2) + C_2 (4J) + D_2 (4J + 2)
\end{align*}
\]

(6)

\[
\begin{align*}
N_F = 0 & \quad E_\gamma = B_2 (2J + 1) \\
N_F = 1 & \quad E_\gamma = B_2 (2J - 1) \\
N_F = 2 & \quad E_\gamma = B_2 (2J - 1)
\end{align*}
\]

(7)

Alignment $i = 1\hbar$ can be obtained in the $N_F = 1$ and $N_F = 2$ systems when $C_2 = -B_2$ and $2D_2 = B_2$. For this case Eq. 6 becomes

A comparison between experiment and the predictions from Eq. 7 for $^{192}$Hg - $^{194}$Hg(SD-2,3) and $^{193}$Tl - $^{194}$Tl(SD-3,4) are shown in Figs. 2 and 3, respectively. The data for $^{194}$Tl actually require two different coupling schemes for this odd-odd nucleus: (i) the orbital angular momenta of both fermions are strongly coupled, and their spins couple to $S = 1$ (Eq. 2); (ii) the spin of the second fermion is weakly coupled to the total angular momentum of the first fermion (Eq. 5). This should not be unexpected since the odd proton is most likely in an $i_{13/2}$ orbital [3], while the odd neutron could be in either an $N = 6$ or $N = 5$ orbital, which have very different radial overlaps with respect to the proton orbit. Different predictions come from these two coupling schemes. Equation 2 arises when both fermions are in the same shell, and a positive-parity band will result, while that of Eq. 5 arises when the fermions are in orbitals from different shells, so that it is quite likely that the parity of the SD band will be negative. A measure of the parities of these SD excitations could further test these predictions.

The cases on which we have focused were identified with the previous generation of large arrays of $\gamma$-ray detectors. In the past two years there has been an explosion of new data with the first results from the larger arrays, Eurogam and Gammasphere. One of these results was the identification [11] of SD excited bands in $^{194}$Pb. As displayed in Fig. 4 the
\( \gamma \)-ray energies and spins of these bands indicate zero alignment with respect to \(^{193}\text{Tl}\). This is another example of the coupling scheme of Eq. 5, which indicates that the two excited protons are probably in orbitals from different major shells.

In the present analysis we have not attempted to superimpose the predictions on the data. The main reason: while the moments of inertia are identical for these nuclei, they are not constant as a function of spin. Rather, the dynamical moments of inertia increase by \( \approx 50\% \) over the measured range of \( \gamma \)-ray energies. This could be reproduced by allowing \( B_1 \) or \( B_2 \), the only free parameters, to have a dependence on spin.

**III. DISCUSSION**

The spin-rotor interpretation of the identical bands and quantized alignment is included in a number of nuclear structure models which involve good rotors and the pseudo-harmonic oscillator. For example, the identical bands in the \( A \approx 150 \) and 190 regions have been proposed as examples of a dynamical supersymmetry \[15,17\]. For a boson-fermion deformed or \( SU(3) \) symmetry, the eigenvalue equations in both Eqs. 2 and 5 can be appropriate. A supersymmetry is a valid description when the same parameters are used for the even core and the one fermion, or two-fermion, systems. The observation of identical behavior in \(^{194}\text{Hg}\*) and \(^{192}\text{Hg}\) is then the first candidate for a two-fermion dynamical supersymmetry. In addition, the spin-rotor is also part of the fermion pseudo-\( SU(3) \) framework \[14\], where again Eq. 2 is valid \[16\] for the one-fermion system and can be extended to two-fermion excitations. However, the coupling scheme of Eq. 5 does not naturally occur in this latter framework \[18\].

The spin-rotor interpretation of the identical SD bands is based on the assumption that the additional particle(s) are in orbitals that can be assigned either harmonic oscillator or pseudo-harmonic oscillator quantum numbers, although it has been recognized that the asymptotic pseudo-harmonic oscillator behavior often better explains the spectroscopic properties at finite deformation. It is accepted \[3\] that the odd-proton in \(^{193}\text{Tl}\) SD bands is in
an $i_{13/2}$ orbital, which is separated from other $N = 6$ orbitals, and therefore these SD bands are not expected to be simply related to the $N_F = 0$ reference, $^{192}$Hg. The SD-3 and 4 bands in $^{193}$Hg have been suggested to come from the $i_{13/2}$ extruder orbital. While such a configuration is outside of the pseudo-harmonic oscillator framework, these SD bands in $^{193}$Hg are observed to be simply related to the $N_F = 0$ reference, $^{192}$Hg SD. The exact ordering of orbitals at these large deformations is sensitive to the parameters used for the $\ell^2$ and spin-orbit terms in the single-particle potential. While most studies have assumed the ordering of single-particle orbitals given in Ref. 9b, no model independent measure of the configurations involved in the SD bands exists. In contrast to the calculations in Ref. 9b, Nilsson calculations using parameters by Åberg predict the $7/2^- [514]$ and $5/2^- [512]$ orbitals to be close in energy to the $9/2^+ [624]$ orbital assigned to SD-3,4 in $^{193}$Hg in Ref. 8.

These $N = 5$ orbitals are $N = 4$ pseudo-spin partners and within the present framework. Given that such critical properties as spin, parity, and excitation energy have not been determined, we shall have to wait for more definitive measures of the spectroscopic properties of these SD bands to test the microscopic basis of the spin-rotor predictions.

**IV. CONCLUSIONS**

In summary, we are able to understand both the $\gamma$-ray energies and extracted alignments of a large number of the superdeformed rotational bands in mercury, thallium, and lead nuclei as examples of quantum rotors in which an orbital angular momentum plays the dominant role, with only a weak dependence on the total angular momentum, which arises from a relatively small spin-orbit interaction. This is a new coupling scheme for heavy nuclei. Traditionally, the total angular momenta carried by the particles dominates the coupling, because of the strong spin-orbit interaction. The spin-rotor scheme arises naturally in models which involve pseudo orbital angular momenta, for example, pseudo-$SU(3)$ models of fermions, or bosons and fermions. We have shown that this coupling scheme is not only valid for one-fermion systems, but also for two-fermion excitations, and the $^{192}$Hg–$^{193}$Hg–
$^{194}\text{Hg}^*$ multiplet, and possibly $^{192}\text{Hg}^* - ^{193}\text{Tl}^* - ^{194}\text{Pb}^*$, could be the first examples of a multi-

fermion supersymmetry.

With the advent of the new, large arrays of high-resolution Ge detectors, such as Eurogam and Gammasphere, there has been an explosion in the number of superdeformed rotational bands which have been identified \cite{20}, and in a large number of these new cases identical bands have been observed. We look forward to these new results and, in particular, the confirmation of spin and parity assignments which will be possible when definitive links between superdeformed and normal excitations have been identified.

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FIGURES

FIG. 1. Average angular momentum in units of $\hbar$ as a function of $E_\gamma$ for (a) SD-3 and 4 in $^{193}\text{Hg}$ (circles) compared to the $^{192}\text{Hg}$ reference (closed triangles); (b) the spin-rotor (Eq. 4) with $N_F = 1$, (circles) compared to the $N_F = 0$, $S = 0$ reference (closed triangles). The parameter $B_1 = 10$ keV. Data are taken from Refs. 4 and 8.

The average angular momentum is the value of $J$ for the $\gamma$-ray transition between levels with $J+1$ and $J-1$.

FIG. 2. Average angular momentum in units of $\hbar$ as a function of $E_\gamma$ for (a) SD-2 and 3 in $^{194}\text{Hg}$ (circles) compared to the $^{192}\text{Hg}$ reference (closed triangles); (b) the spin-rotor (Eq. 4) with $N_F = 2$, $S = 1$ (circles) compared to the $N_F = 0$, $S = 0$ reference (closed triangles), and the $N_F = 2$, $J = J_1 + 1/2$ spectrum (Eq. 7) (squares). The parameters $B_1 = B_2 = 10$ keV. Data are taken from Refs. 4 and 9.

FIG. 3. Average angular momentum in units of $\hbar$ as a function of $E_\gamma$ for (a) SD bands 1-4 in $^{194}\text{Tl}$ (circles and squares) compared to the $^{193}\text{Tl}$ reference (triangles); (b) the spin-rotor (Eq. 4) with $N_F = 2$, $S = 1$ (circles) compared to the $N_F = 0$, $S = 0$ (closed triangles) and $N_F = 1$ (small diamonds) references, and the $N_F = 2$, $J = J_1 + 1/2$ spectrum (Eq. 7) (squares). The parameters $B_1 = B_2 = 10$ keV. Data are taken from Refs. 5 and 10.

FIG. 4. Average angular momentum in units of $\hbar$ as a function of $E_\gamma$ for (a) SD-2a and 2b in $^{194}\text{Pb}$ (circles) compared to the $^{193}\text{Tl}$ reference (triangles); (b) the $N_F = 2$, $J = J_1 + 1/2$ spectrum (Eq. 7) (squares) compared to the $N_F = 0$, $S = 0$ (closed triangles) and $N_F = 1$ (small diamonds) references. The parameter $B_2 = 10$ keV. Data are taken from Refs. 5 and 11.

FIG. 5. Generic spectra for $N_F = 1$, $S = 1/2$ spin-rotors (Eq. 2) with $C_1 = 0$. Type c corresponds to signature partner bands and are compared to data in odd-$A$ candidates. The left-hand spectra are the references.
FIG. 6. Generic spectra for $N_F = 2$, $S = 1$, spin-rotors (Eq. 2) with $C_1 = 0$. Type c corresponds to signature partner bands and are compared to data in two-fermion candidates. The left-hand spectra are the references.

FIG. 7. Generic spectra for $N_F = 1$ and $N_F = 2$, $J = J_1 + 1/2$ spin-rotors (Eq. 5) with $C_2 = D_2 = 0$. The left-hand spectrum is the reference.