Degrees-of-Freedom Region of Time Correlated MISO Broadcast Channel with Perfect Delayed CSIT and Asymmetric Partial Current CSIT

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Abstract—The impact of imperfect CSIT on the degrees of freedom (DoF) of a time correlated MISO Broadcast Channel has drawn a lot of attention recently. Maddah-Ali and Tse have shown that the completely stale CSIT still benefit the DoF. In very recent works, Yang et al. have extended the results by integrating the partial current CSIT for a two-user MISO broadcast channel. However, those researches so far focused on a symmetric case. In this contribution, we investigate a more general case where the transmitter has knowledge of current CSI of both users with unequal qualities. The essential ingredient in our work lies in the way to multicast the overhead interference to boost the DoF. The optimal DoF region is simply proved and its achievability is shown using a novel transmission scheme assuming an infinite number of channel uses.

I. INTRODUCTION

Maddah-Ali and Tse have recently investigated the DoF region of Broadcast Channels with perfect delayed (completely stale) CSIT [1]. They have shown that the per-user optimal DoF is $\frac{3}{2}$ in a two-user setup and proposed a simple transmission scheme (denoted as MAT scheme in the sequel) to achieve that DoF over 3 time slots. In the first and second slot, each user in turn receives the desired signal and overhears the unwanted signal, while in the third slot, the sum of the overhead interference is transmitted to enable the decoding of the desired signal at each receiver.

Those results have recently been extended to a more general setup with perfect delayed CSIT and partial current CSIT [2] [3]. An optimal combining of perfect delayed CSIT and partial current CSIT is obtained to bridge the DoF of $\frac{3}{2}$ with outdated CSIT and the DoF achievable with zero-forcing beamforming with perfect current CSIT. However results in [2] and [3] are limited to a symmetric case where the transmitter has access to both users’ CSI with the same accuracy.

In [4], an outer-bound on the DoF region was given under the settings that the transmitter and receivers have multiple antennas and perfect delayed CSIT. The authors of [5] and [6] studied the same settings but without CSIT. Moreover, In [7][8], the optimal sum DoF of $\frac{3}{2}$ was derived when the CSIT of one user is perfect but it is out-dated for the other user.

In this paper, we further extend the analysis by considering a more general asymmetric scenario where the qualities of current CSI of users’ channels available at the transmitter are unequal. Our contributions are summarized as follows:

1) the transmission schemes derived for the symmetric case are shown to incur a DoF loss in an asymmetric scenario,
2) the DoF region obtained in [2] is extended to the asymmetric scenario and is expressed as a function of two parameters representing the accuracy of CSIT of each user’s channel,
3) a novel transmission scheme (that subsumes the scheme in [2] in the symmetric case) is derived and shown to achieve the DoF region in the limit of an infinite number of channel uses.

At the time of submission, we have been informed of another independent work, also in submission, that addresses the same problem [9]. Interestingly, both works derive the same achievable DoF region using different approaches and its achievability is demonstrated using two different transmission strategies.

The rest of this paper is organized as follows. The system model is introduced in Section II and the DoF region is derived in Section III. The limitations of the transmission schemes designed for symmetric partial current CSIT are discussed in Section IV and a novel transmission scheme achieving the optimal DoF region in the setting of asymmetric partial current CSIT is introduced. Section V concludes the paper.

The following notations are used throughout the paper. Upper letters in bold font represent matrices whereas bold lower letters stand for vectors. However, symbol not in bold represents a scalar. $(\cdot)^T$ and $(\cdot)^H$ represent the transpose and conjugate transpose of a matrix or vector respectively. $h_\perp$ denotes the orthogonal space of channel vector $h$. $E[\cdot]$ refers to the expectation of a random variable, vector or matrix. $\|\cdot\|$ is the norm of a vector. $f(P) \sim P^B$ corresponds to $\lim_{P \to \infty} \frac{\log f(P)}{\log P} = B$, where $P$ refers to SNR throughout the paper and logarithms are in base 2.

II. SYSTEM MODEL

We consider a two-user Broadcast Channel with two transmit antennas and one antenna per user. $h_1$ and $g_t$ are the channel states at time slot $t$ of user 1 and user 2 respectively. Denoting the transmit signal vector as $s_t$, subject to a transmit power constraint $E[\|s_t\|^2] \leq P$, the observations at receiver 1
and 2, $y_t$ and $z_t$ respectively, can be written at time slot $t$ as

$$y_t = h_t^H s_t + \epsilon_{i,1} \tag{1}$$

$$z_t = \tilde{g}_t^H s_t + \epsilon_{i,2}, \tag{2}$$

where $\epsilon_{i,1}$ and $\epsilon_{i,2}$ are unit power AWGN noise. Signal vector $s_t$ is expressed as a function of the symbol vectors for user 1 and user 2, denoted as $u_t$ and $v_t$ respectively. $u_t$ is a two-element symbol vector containing $u_{t,1}$ and $u_{t,2}$. $v_t$ is defined similarly and is composed of $v_{t,1}$ and $v_{t,2}$. The power allocated to symbol $u_{t,1}$ is stated as $P_{u_{t,1}}$, while the rate achieved by $u_{t,1}$ is $R_{u_{t,1}}$. As $u_t = [u_{t,1}, u_{t,2}]^T$, the power and rate of $u_t$ are expressed as $P_{u_t} = P_{u_{t,1}} + P_{u_{t,2}}$ and $R_{u_t} = R_{u_{t,1}} + R_{u_{t,2}}$. These notations are applicable to $v_t$, $v_{t,1}$ and $v_{t,2}$. For the sake of convenience, in a few instances, we denote the rate of each symbol or symbol vector by the pre-log factor (ignoring $\log P$).

The channel state information $h_t$ and $\tilde{g}_t$ are supposed to be mutually independent and identically distributed with zero mean and unit covariance matrix ($E[|h_t^H \tilde{g}_t|^2] = 0$ and $E[|h_t^H \tilde{g}_t|^2] = I_2$). At any given time slot $t$, the transmitter and each user have perfect global knowledge of the channel states from time slot 0 to $t-1$, i.e. $\{h_{0,1}, ..., h_{t-1}\}$ and $\{g_{0,1}, ..., g_{t-1}\}$. Moreover, the transmitter can predict the current channel state of each user. The estimated channels are denoted as $\hat{h}_t$ and $\tilde{g}_t$, with the corresponding error vectors written as $e_t = h_t - \hat{h}_t$ and $e_t = g_t - \tilde{g}_t$. $\hat{h}_t$ and $\tilde{g}_t$ have the variances denoted as $E[\|\hat{h}_t\|^2] = \sigma_1^2$ and $E[\|\tilde{g}_t\|^2] = \sigma_2^2$, respectively. In the general asymmetric scenario under interest, the transmitter has unequal accuracy of the current CSI for each user, with the accuracy of the each current CSIT obtained according to

$$\alpha_k \triangleq \lim_{P \to \infty} -\frac{\log \sigma_k^2}{\log P}, \quad k = 1, 2. \tag{3}$$

In other words, $E[\|\hat{h}_t\|^2] \sim P^{-\alpha_1}$ and $E[\|\tilde{g}_t\|^2] \sim P^{-\alpha_2}$. $\alpha_1$ and $\alpha_2$ are supposed to vary in the range of $[0, 1]$, where $\alpha_2 = 0$ represents no CSIT whereas $\alpha_1 = 1$ stands for perfect CSIT. Throughout the paper, we assume $\alpha_1 \leq \alpha_2$ without loss of generality. Moreover, it is important to note the relationship $E[|h_t^H \tilde{g}_t|^2] \sim P^{-\alpha_1}$ and $E[|\tilde{g}_t^H \tilde{g}_t|^2] \sim P^{-\alpha_2}$.

III. DoF REGION WITH ASYMMETRIC PARTIAL CURRENT CSIT

**Theorem 1:** In the two-user MISO Broadcast Channel with perfect delayed CSIT and asymmetric partial current CSIT, the optimal DoF region is characterized by

$$\{d_1 \leq 1; d_2 \leq 1; d_1 + 2d_2 \leq 2 + \alpha_2; 2d_1 + d_2 \leq 2 + \alpha_1\} \tag{4}$$

A sketch of the proof will be given in the Appendix. The DoF region is illustrated in Figure 1. As shown, the outer-bound is a polygon composed of the points $(1, \alpha_1)$, $(\alpha_2, 1)$ and $(\frac{2+2\alpha_2-\alpha_1}{2+2\alpha_2}, \frac{2+2\alpha_2-\alpha_1}{2+2\alpha_2})$. When $\alpha_1$ is fixed, the intersection point which maximizes the sum DoF moves as $\alpha_2$ increases. However, the intersection point goes outside the valid region as shown by the circle point in Figure 1 when $2\alpha_2 - \alpha_1 > 1$. In this case, the region is only formed by $(1, \alpha_1)$, $(\frac{2+2\alpha_2-\alpha_1}{2+2\alpha_2}, 1)$. Also, the point that achieves the maximum sum DoF returns to the triangle point (see Figure 1) achieved by taking $2\alpha_2 - \alpha_1 = 1$.

When $\alpha_1 = \alpha_2$, the DoF region boils down to that of [2] as shown by the diamond point, suggesting that scheme II in [2] achieves a subset of the asymmetric region. Moreover, when $\alpha_1 = \alpha_2 = 0$, the region further boils down to the region achieved by MUI scheme in [1].

IV. ACHIEVABILITY

A. Limitation of scheme II in [2]

Following the system model defined in Section III we briefly review the transmission scheme II as proposed in [2] and identify the limitations of that scheme in an asymmetric configuration. The transmit signal and the received signal at each user at first time slot write as

$$s_1 = [g_1^H, \tilde{g}_1^H] u_1 + [h_1^H, \tilde{h}_1^H] v_1, \tag{5}$$

$$y_1 = h_1^H s_1 u_1 + h_1^H s_1 u_2 + \eta_{1,1} + \epsilon_{1,1}, \tag{6}$$

$$z_1 = \eta_{1,2} + g_1^H \tilde{h}_1 v_1 u_1 + g_1^H \tilde{h}_1 v_1 u_2 + \epsilon_{1,2}, \tag{7}$$

where $P_{u_1} = P_{v_1} = P$ and $P_{u_2} = P^{1-\alpha_2}$, $P_{v_{1,2}} = P^{1-\alpha_1}$, $\eta_{1,1}$ (resp. $\eta_{1,2}$) represents the overhead interference generated at receiver 1 (resp. 2) at slot 1 and is expressed as

$$\eta_{1,1} = h_1^H \tilde{h}_1^H v_1 u_1 + h_1^H \tilde{h}_1^H v_1 u_2, \tag{8}$$

$$\eta_{1,2} = g_1^H \tilde{g}_1^H u_1 u_1 + g_1^H \tilde{g}_1^H u_2 u_2. \tag{9}$$

By integrating the current CSIT in the preceding procedure to $u_1$, $\eta_{1,1}$ and $\eta_{1,2}$ are received with the power of $P^{1-\alpha_1}$ and $P^{1-\alpha_2}$ respectively since $h_1$ and $\tilde{g}_1$ have the respective quality of $\alpha_1$ and $\alpha_2$. As both overhead interferences have reduced power, the resource can be saved when multicasting them separately and sequentially at slot 2 and 3. The transmission and receive signals at slot 2 and 3 are expressed as

$$s_2 = [\tilde{\eta}_{1,1}, 0]^T + \tilde{g}_2^H u_2 + \tilde{h}_2^H v_2, \tag{10}$$

$$y_2 = h_2^H t_{\tilde{\eta}_{1,1}} + h_2^H \tilde{g}_2^H u_2 + h_2^H \tilde{h}_2^H v_2 + \epsilon_{2,1}, \tag{11}$$

$$z_2 = g_{2,1}^H \tilde{\eta}_{1,1} + g_2^H \tilde{g}_2^H u_2 + g_2^H \tilde{h}_2^H v_2 + \epsilon_{2,2}, \tag{12}$$

Fig. 1: DoF region with asymmetric partial current CSIT.
s_3 = \hat{\eta}_{1,2}^T + \mathbf{g}_3^\perp u_3 + \mathbf{h}_3^\perp v_3, \quad (13)

y_3 = \hat{\eta}_{3,1} y_1 + \mathbf{h}_3^\perp \mathbf{g}_3^\perp u_3 + \mathbf{h}_3^\perp v_3 + \epsilon_{3,1}, \quad (14)

z_3 = \hat{\eta}_3 y_1 + \mathbf{h}_3^\perp \mathbf{g}_3^\perp u_3 + \mathbf{h}_3^\perp v_3 + \epsilon_{3,2}, \quad (15)

where \( \hat{\eta}_{1,1} \) and \( \hat{\eta}_{1,2} \) are quantized versions of overhead interference with the quantization rate of \( 1-\alpha_1 \) and \( 1-\alpha_2 \) respectively. \( u_2 \) and \( v_2 \) are new symbols at slot 2 intended to user 1 and user 2 respectively. Similarly, \( u_3 \) and \( v_3 \) are defined at slot 3. The power allocated to each symbols in (10) and (13) respectively.

B. Building Blocks of a New Transmission Scheme

We identify important building blocks of a new transmission scheme that achieves the asymmetric DoF region of Theorem (1) Building upon scheme II in [2], a more general transmission at a given time slot \( t \) writes as

s_t = [\hat{\eta}_t^v, 0]^T + \mathbf{g}_t^\perp u_t + \mathbf{h}_t^\perp v_t, \quad (16)

where \( \hat{\eta}_t^v \) can be made up of multiple overhead interferences generated at any previous slots \( \tau \leq t \). The power and rate allocated to each symbol are defined for a given quantity \( S \) as shown in Table I that represents a fraction of channel use and whose physical meaning will appear clearer in the sequel. Regarding \( s_t \) and the power allocated to each symbol, the received signal at each user can be written as

\[
y_t = \hat{\eta}_t^u y_t + \hat{\eta}_t^v \mathbf{g}_t^\perp u_t + \mathbf{h}_t^\perp \mathbf{g}_t^\perp u_t + \epsilon_{t,1},
\]

\[
z_t = \hat{\eta}_t^u y_t + \mathbf{g}_t^\perp \mathbf{h}_t^\perp u_t + \mathbf{h}_t^\perp v_t + \epsilon_{t,2},
\]

where \( \hat{\eta}_t^u \) and \( \hat{\eta}_t^v \) are overheard interferences received with the power \( p_{S-\alpha_1} \) and \( p_{S-\alpha_2} \) respectively. Both \( \eta_t^u \) and \( \eta_t^v \) are composed of two symbols

\[
\eta_t^u = \hat{\eta}_t^u + g^t_{u,1} + h^t_{u,2} + \epsilon_{t,1},
\]

\[
\eta_t^v = g^t_{v,1} + h^t_{v,2}.
\]

To save the resource required by retransmission, \( \eta_t^u \) and \( \eta_t^v \) are quantized at the end of slot \( t \) as

\[
\eta_{t,1} = \hat{\eta}_{t,1} + \hat{\eta}_t^u, \quad \eta_{t,2} = \hat{\eta}_{t,2} + \hat{\eta}_t^v,
\]

where \( \hat{\eta}_{t,1} \) and \( \hat{\eta}_{t,2} \) are the quantized overhead interference, \( \hat{\eta}_{t,1} \) and \( \hat{\eta}_{t,2} \) are the quantization errors. According to the Rate-distortion theory [10], we quantize \( \eta_t^u \) and \( \eta_t^v \) using \( R_{\eta_t^u} \) and \( R_{\eta_t^v} \) bits, which are equal to \((1-\alpha_1)\log P + o(\log P)\) and \((1-\alpha_2)\log P + o(\log P)\) respectively, allowing the quantization noise to have the same variances as AWGN. As a result, \( \eta_{t,1} \) and \( \eta_{t,2} \) can be employed to remove the overhead interference completely while providing an additional observation to enable the decoding of all symbols.

Moreover, \( \hat{\eta}_t^v \) decoded with rate \( 1-S \) by treating all the other components as noise at both receivers, can be considered as occupying \( 1-S \) channel use, leaving \( S \) channel use for the new symbols, \( u_t \) and \( v_t \). The number of new symbols transmitted highly depends on the value of \( S \). If \( S> \alpha_2 \), both users will observe an overhead interference. When \( \alpha_1 \leq S \leq \alpha_2 \), only \( u_{t,1} \) is sent to user 1 and will be drawn by the noise in \( z_t \), thus, only one overhead interference \( \eta_{t,1} \) needs to be retransmitted. However, when \( S < \alpha_1 \), only \( u_{t,1} \) and \( v_{t,1} \) are transmitted and they will be drawn by the noise in \( z_t \) and \( y_t \) respectively so that the transmission can finalize without the requirement of overhead interference retransmission.

| Symbols | Power Allocation | Encoding Rate |
|---------|------------------|---------------|
| \( \eta_{t,1} \) | \( p_{S-\alpha_1} \) | \( S \) |
| \( \eta_{t,2} \) | \( p_{S-\alpha_2} \) | \( S \) |

TABLE I: Power and rate allocation

### Symbols
- **\( s_t \)**: A symbol generated at any previous slots \( \tau \leq t \).
- **\( u_t \)**: A new symbol intended for user 1.
- **\( v_t \)**: A new symbol intended for user 2.
- **\( \hat{\eta}_t^u \)**: A quantized version of overhead interference at slot \( t \) with the quantization rate of \( 1-\alpha_1 \).
- **\( \hat{\eta}_t^v \)**: A quantized version of overhead interference at slot \( t \) with the quantization rate of \( 1-\alpha_2 \).

### Power and Rate Allocation

\[
\begin{align*}
\text{Symbols} & \quad \text{Power Allocation} & \quad \text{Encoding Rate} \\
\eta_{t,1} & \quad p_{S-\alpha_1} & \quad S \\
\eta_{t,2} & \quad p_{S-\alpha_2} & \quad S
\end{align*}
\]

### Building Blocks

1. **Building upon scheme II in [2]**: A general transmission at a given time slot \( t \) writes as

\[
s_t = [\hat{\eta}_t^v, 0]^T + \mathbf{g}_t^\perp u_t + \mathbf{h}_t^\perp v_t,
\]

2. **Rate-distortion theory [10]**: We quantify \( \eta_t^u \) and \( \eta_t^v \) using \( R_{\eta_t^u} \) and \( R_{\eta_t^v} \) bits, which are equal to \((1-\alpha_1)\log P + o(\log P)\) and \((1-\alpha_2)\log P + o(\log P)\) respectively, allowing the quantization noise to have the same variances as AWGN. As a result, \( \eta_{t,1} \) and \( \eta_{t,2} \) can be employed to remove the overhead interference completely while providing an additional observation to enable the decoding of all symbols.

3. **Overhead Interference**: The overhead interference is removed at the receiver by treating all the other components as noise at both receivers.
two parts: the first part is the transmission of new symbols, the second part refers to the retransmission of overhead interferences, which can boost the DoF from two aspects: 1) help user remove overhead interference; 2) provide an additional independent observation to enable decoding.

Next, as the limitation of the scheme in [2] lies at time slot 3, we derive the achievable scheme maintaining the transmission in the first two time slots plus the transmission of \( \eta_{2,1} \) at time slot 3. Over those 3–\( \alpha_2 \) channel uses, the DoF achieved by each user is \((d_1, d_2) = \left(\frac{d_1 + d_2}{2}, \frac{d_1 - d_2}{2}\right) = 2\alpha_2 - \alpha_2 = \frac{2}{3} - \frac{1}{3} \alpha_2\).

Moreover, as illustrated in Figure 1, the DoF region varies depending on the location of the intersection point. Obviously, when \( 2\alpha_2 - \alpha_2 \geq 1 \), the intersection point lies on or outside the valid region formed by \( d_1 = 1 \) and \( d_2 = 1 \). The maximum sum DoF is obtained by the point \((\frac{1+x}{2}, 1)\). When \( 2\alpha_2 - \alpha_2 < 1 \), the intersection point is located inside the valid region and achieves the maximum sum DoF. Hence, the achievable schemes are sketched in these two cases respectively.

C. Case I: \( 1 - \Delta \leq \alpha_2 \) Achieving \((\frac{1+x}{2}, 1)\)

Lemma 1: Assume the perfect delayed CSIT and partial current CSIT with \( \alpha_1 \) and \( \alpha_2 \) for each user. Under the condition \( 2\alpha_2 - \alpha_2 \geq 1 \) \((1 - \Delta \leq \alpha_2)\), the achievable DoF is \((d_1, d_2) = (\frac{1+x}{2}, 1)\).

Proof: In addition to the existing two time slots of scheme II in [2] shown as [5] and [10], at time slot 3, the transmission is processed according to the scheme described in section IV-B which is expressed in the following group of equations as

\[
\begin{align*}
\mathbf{s}_3 &= [\hat{y}_{1,2}, 0]^T + \mathbf{g}_u^1 \mathbf{u}_3 + [\hat{\mathbf{h}}_{1,1}^1, \hat{\mathbf{h}}_{3,1}] \mathbf{v}_3, \quad (22) \\
\mathbf{s}_4 &= [\hat{y}_{1,3}, 0]^T + \mathbf{g}_u^1 \mathbf{u}_4 + [\hat{\mathbf{h}}_{1,2}^1, \hat{\mathbf{h}}_{4,2}] \mathbf{v}_4, \quad (23) \\
\mathbf{s}_5 &= [\hat{y}_{1,4}, 0]^T + \mathbf{g}_u^1 \mathbf{u}_5 + [\hat{\mathbf{h}}_{1,3}^1, \hat{\mathbf{h}}_{5,3}] \mathbf{v}_5, \quad (24)
\end{align*}
\]

with the power and rate allocation given in Table II where \( \Delta = \alpha_2 - \alpha_1 \). The power of \( \mathbf{u}_5 \) and \( \mathbf{v}_5 \) are not shown because where \( \eta_{1,1} \) is given as (19) and quantized as \( \hat{\eta}_{1,1} \) via (21). Regarding the transmission flow and power allocation, two virtual channels can be established from slot 3 to 5.

1) Virtual Channel 1: This virtual channel contains the transmission of \( \mathbf{u}_3 \), \( \mathbf{v}_3 \) plus \( \hat{\mathbf{h}}_{1,1} \). At slot 3, \( \alpha_2 \) channel use remains after transmitting \( \hat{\eta}_{1,2} \), so that one symbol, \( \mathbf{u}_3 \), is intended to user 1 and two symbols, \( \mathbf{v}_3,1 \) and \( \mathbf{v}_3,2 \) to user 2. The interference overhead by user 1, \( \eta_{1,1} \), has power \( \mathbf{P}_{\mathbf{v}_3} P^{-\alpha_1} = P^\Delta \), while user 2 overhears nothing since \( \mathbf{u}_3 \) is drawn by the noise. At the end of slot 3, \( \eta_{1,1} \) is quantized as \( \hat{\eta}_{1,1} \) with rate \( R_{\mathbf{v}_3,1} \), and sent using \( \Delta \) channel use at slot 4.

2) Virtual Channel 2: This virtual channel consists of the transmission of \( \mathbf{u}_4 \) and \( \mathbf{v}_4 \) together with \( \hat{\mathbf{h}}_{1,1} \). At slot 4, \( 1 - \Delta \) channel remains for \( \mathbf{u}_4 \) and \( \mathbf{v}_4 \) after transmitting \( \hat{\eta}_{1,1} \). Under the condition that \( 1 - \Delta \leq \alpha_2 \), the transmitter will send only one symbol, \( \mathbf{u}_4 \) to user 1 but two symbols, \( \mathbf{v}_4,1 \) and \( \mathbf{v}_4,2 \) to user 2. Consequently, \( \eta_{1,1} \) is the only overhead interference at slot 4 with power \( P^{1-\alpha_2} \). In each virtual channel, three new symbols and one overhead interference are transmitted. The only difference between these two virtual channels lies in the power allocation. The decodability is sketched next.

Firstly, as aforementioned, the overhead interference must be decoded first to enable the decoding procedure for the new symbols. From \( y_{t+1} \) and \( z_{t+1} \), \( \hat{\eta}_{1,1} \) is decoded by treating all the other symbols as noise. To be specific, at observation \( y_{t+1} \), the rate of \( \hat{\eta}_{1,1} \) is \( I(\hat{\eta}_{1,1}; y_{t+1} | \hat{\mathbf{h}}_{t+1,1}^1) = \log \frac{P_{\mathbf{v}_3}}{P_{\mathbf{v}_4}} \), while it is \( I(\hat{\eta}_{1,1}; z_{t+1} | \hat{\mathbf{h}}_{t+1,1}^1) = \log \frac{P_{\mathbf{v}_4}}{P_{\mathbf{v}_3}} \). As shown in Table II we let \( P_{\mathbf{v}_3} \neq P_{\mathbf{v}_4} \), \( \hat{\eta}_{1,1} \) is decoded with the same rate at both receivers. Moreover, as \( \eta_{1,1} \) is seen by user 1 with power of \( \mathbf{P}_{\mathbf{v}_3} \) and \( P^{-\alpha_1} \) in (25), \( \hat{\eta}_{1,1} \) can completely recover \( \eta_{1,1} \) provided that \( \frac{P_{\mathbf{v}_3}}{P_{\mathbf{v}_4}} = \mathbf{P}_{\mathbf{v}_3} P^{-\alpha_1} \).

Secondly, \( \hat{\eta}_{1,1} \) is employed to remove the overhead interference in (25) and provide an additional independent observation for user 2. Denoting \( y_{t} = y_{t} - \hat{\mathbf{h}}_{1,1}^1 \hat{\eta}_{1,1} \) and \( z_{t} = z_{t} - \hat{\mathbf{h}}_{1,1}^1 \hat{\eta}_{1,1} \) as the signal after decoding and subtracting \( \hat{\eta}_{1,1} \) from \( y_{t} \) and \( z_{t} \) respectively, we write the decoding of \( \mathbf{v}_t \) and \( \mathbf{u}_t \) as

\[
\begin{align*}
\mathbf{y}_t - \hat{\eta}_{1,1} &= \hat{\mathbf{h}}_{1,1}^2 \mathbf{u}_t + \hat{\mathbf{h}}_{1,1} + \mathbf{v}_t + \epsilon_{t,1}, \quad (27) \\
\mathbf{z}_t &= \hat{\mathbf{h}}_{1,1}^2 \mathbf{u}_t + \hat{\mathbf{h}}_{1,1} + \mathbf{v}_t + \epsilon_{t,2}, \quad (26)
\end{align*}
\]

where \( \hat{\eta}_{1,1}, \epsilon_{t,1}, \) and \( \epsilon_{t,2} \) have unit power so that \( \mathbf{u}_t \) is decodable with the rate of \( \log P_{\mathbf{u}_t} \) and \( \mathbf{v}_t \) achieves the rate of \( \log \left( \frac{P_{\mathbf{v}_3}}{P_{\mathbf{v}_4}} \right) \).

Replacing the power stated in Table I the first virtual channel lasts for \( \alpha_2 + \Delta \) channel uses, over which the rates achieved are \( R_{\mathbf{v}_3,1} = \alpha_2 \) and \( R_{\mathbf{v}_4} = \alpha_2 + \Delta \). Similarly, they are \( R_{\mathbf{v}_3,1} = \alpha_2 + \Delta \) and \( R_{\mathbf{v}_4} = 2 - \alpha_2 - \Delta \) during the \( 2 - \alpha_2 - \Delta \) channel uses in virtual channel 2. In all, when we consider these two virtual channels which last for 2 channel uses, the DoF is

\[
(d_1, d_2) = \left( \frac{R_{\mathbf{v}_3} + R_{\mathbf{v}_4}}{2}, \frac{R_{\mathbf{v}_3} + R_{\mathbf{v}_4}}{2} \right) = \left( 1 + \alpha_1, \frac{1}{2} \right) \text{.} \quad (29)
\]
by user 1 with the rate $R_{\eta_{1},1}=R_{\eta_{1},1}$, leading to $P_{u_{1}}=P_{u_{4}}$ and $P_{v_{1}}=P_{v_{4}}$, at slot 6. In this way, the transmission can keep cycling with the same strategy as at slot 3 and 4. Combining with the transmissions prior $u_{3}$ and $v_{3}$ and assume the strategy is repeated for $N$ times, the asymptotic DoF is given by

$$ (d_{1}, d_{2}) = \lim_{N \to \infty} \left( \frac{2 + \alpha_{1} - \alpha_{2} + N \times (1 + \alpha_{1})}{3 - \alpha_{2} + N \times 2} \right) \cdot \ldots \cdot \left( \frac{2 + N}{3 - \alpha_{2} + N \times 2} \right) = \left( \frac{1 + \alpha_{1}}{2}, 1 \right). \quad (30) $$

D. Case II: $1 - \Delta > \alpha_{2}$

**Lemma 2:** Assume the transmitter has perfect delayed CSIT and partial current CSIT with $\alpha_{1}$ and $\alpha_{2}$ for each user. Under the condition $2\alpha_{2} - \alpha_{1} < 1 (1 - \Delta > \alpha_{2})$, the achievable DoF is $(d_{1}, d_{2}) = \left( 2 + 2\alpha_{2} - \alpha_{1}, 2 + 2\alpha_{2} - \alpha_{1} \right)$.\]

**Proof:** To close the gap mentioned in section II-V and C, we derive the achievable scheme from slot 3 (The first two slots are the same as in Case I). The transmission flow is expressed in the following group of equations as

$$ s_{3} = [\hat{\eta}_{3,2}, 0]^{T} + \hat{g}_{3}^{+} u_{3} + [\hat{h}_{3}^{+}, \hat{h}_{3}] v_{3}, \quad (31) $$

$$ s_{4} = [\hat{\eta}_{3,1}, 0]^{T} + \hat{g}_{4}^{+} u_{4} + [\hat{h}_{4}, \hat{h}_{1}] v_{4}, \quad (32) $$

$$ s_{5} = [\hat{\eta}_{4,1}, 0]^{T} + \hat{g}_{5}^{+} u_{5} + [\hat{h}_{5}, \hat{h}_{5}] v_{5}, \quad (33) $$

$$ s_{6} = [\hat{\eta}_{4,2}, 0]^{T} + [\hat{\eta}_{5,1}, 0]^{T} + \hat{g}_{6}^{+} u_{6} + [\hat{h}_{5}^{+}, \hat{h}_{6}] v_{6}, \quad (34) $$

where, $\eta_{1,1}$ and $\eta_{2,2}$, given in (19) and (20), are quantized with rate $R_{\eta_{1,1}}$ and $R_{\eta_{2,2}}$ respectively at the end of each slot via (21). Since the transmission and power allocation at slot 3 and 5 are identical to that in Table II, Table III only provides the power and the encoding rate at slot 4 and 6.

| Symbols   | Power      | Encoding Rate |
|-----------|------------|---------------|
| $\eta_{1,1}$ | $P_{u_{1}} - P_{v_{1}}$ | $1 - \Delta$ |
| $u_{4,2}$   | $P_{u_{4}} - P_{v_{4}}$ | $\Delta$ |
| $u_{4,2}$   | $P_{u_{4}} - P_{v_{4}}$ | $\Delta$ |
| $u_{4,2}$   | $P_{u_{4}} - P_{v_{4}}$ | $\Delta$ |
| $\eta_{1,1}$ | $P_{u_{1}} - P_{v_{1}}$ | $1 - \Delta$ |
| $u_{6}$     | $P_{u_{6}} - P_{v_{6}}$ | $\Delta$ |
| $\eta_{1,1}$ | $P_{u_{1}} - P_{v_{1}}$ | $1 - \Delta$ |

TABLE III: Power and rate allocation for case II (slot 4 and 6).

Regarding the transmission flow and power allocation, three virtual channels are constructed from slot 3 to 6.

1) Virtual Channel 1: This virtual channel consists of the transmission of $u_{3}$, $v_{3}$ and $\eta_{3,1}$. Power of $u_{3}$ and $v_{3}$ are stated in Table II. $v_{3}$ results in $\eta_{3,1}$, which is quantized as $\eta_{3,1}$. The sending of $\eta_{3,1}$ occupies $\Delta$ channel uses at slot 4. This virtual channel is identical to the first virtual channel in case I (see Section IV-C). The decodability can be derived in the same way as (25) and (26). The total amount of channel uses is $\alpha_{2} + \Delta$, over which the rates achieved by the symbols for each user are $R_{u_{3}}=\alpha_{2}$ and $R_{v_{3}}=\alpha_{2} + \Delta$.

2) Virtual Channel 2: This virtual channel is made up of $u_{4}$ and $v_{4}$ and the retransmission of $\hat{\eta}_{4,1}$ and $\hat{\eta}_{4,2}$. At slot 4, the remaining amount of channel use for new symbols is $1 - \Delta$, which is higher than $\alpha_{2}$ so that the transmitter sends two symbols per user, resulting in the overhead interference $\hat{\eta}_{4,1}$ and $\hat{\eta}_{4,2}$ generated at user 1 and user 2 respectively. The observations at each receiver are written as

$$ y_{4} = h_{4}^{H} \hat{\eta}_{4,1} + h_{4}^{H} g_{1}^{H} u_{4} + \eta_{4,1} + \epsilon_{4,1}, \quad (35) $$

$$ z_{4} = g_{4}^{H} \hat{\eta}_{4,1} + g_{4}^{H} h_{4}^{H} v_{4} + \eta_{4,1} + \epsilon_{4,2}. \quad (36) $$

At the end of slot 4, $\hat{\eta}_{4,1}$ and $\hat{\eta}_{4,2}$ are obtained via quantization with rates $\log (P_{u_{4}} P_{v_{4}})$ and $\log (P_{u_{4}} P_{v_{4}})$ respectively, whose pre-log factors are $1 - \alpha_{2}$ and $1 - \Delta - \alpha_{2}$. The decodability of the new symbols in this virtual channel is enabled after decoding $\hat{\eta}_{4,1}$ and $\hat{\eta}_{4,2}$, which are transmitted using part of the channel at slot 4 and 5 respectively. The received signals of user 1 at slot 5 and 6 are expressed as

$$ y_{5} = h_{5}^{H} \hat{\eta}_{4,1} + h_{5}^{H} g_{5}^{H} u_{5} + \eta_{5,1} + \epsilon_{5,1}, \quad (37) $$

$$ y_{6} = h_{6}^{H} \hat{\eta}_{4,2} + h_{6}^{H} g_{6}^{H} u_{6} + \eta_{6,1} + \epsilon_{6,1}. \quad (38) $$

By treating all the other symbols as noise, $\hat{\eta}_{4,1}$ is obtained with the rate of $I (\hat{\eta}_{4,1}; y_{5}|h_{5}^{H}) = \log \frac{P_{u_{5}}}{P_{v_{5}}}$. Revisiting Table II $P_{u_{5}}=P_{u_{3}}=P_{v_{2}}$ so that $\hat{\eta}_{4,1}$ is decodable with rate $1 - \alpha_{2}$ and $\eta_{4,1}$ can be successfully recovered. Similarly, $\eta_{4,2}$ is decoded from $y_{6}$ by treating all the other symbols as noise, among which, $\hat{\eta}_{4,1}$ is the dominant component. Consequently, $\hat{\eta}_{4,2}$ is decoded with the rate of $I (\hat{\eta}_{4,2}; y_{6}|h_{6}^{H}) = \log \frac{P_{u_{6}}}{P_{v_{6}}}$, which meets its distortion rate $1 - \Delta - \alpha_{2}$. Similarly, $\eta_{4,1}$ and $\eta_{4,2}$ are decoded from $z_{4}$ and $z_{6}$ respectively. Denoting $y_{4}=y_{4}-h_{4}^{H} \hat{\eta}_{4,1}$ and $z_{4}=z_{4}-g_{4}^{H} \hat{\eta}_{4,1}$, the decoding formulas for $u_{4}$ and $v_{4}$ are

$$ \begin{bmatrix} y_{4} - \eta_{4,1} \\ \eta_{4,2} \end{bmatrix} = \begin{bmatrix} h_{4}^{H} \\ g_{4}^{H} \end{bmatrix} \begin{bmatrix} \hat{\eta}_{4,1} + \epsilon_{4,1} \\ -\eta_{4,1} \end{bmatrix}, \quad (39) $$

$$ \begin{bmatrix} y_{4} - \eta_{4,1} \\ \eta_{4,2} \end{bmatrix} = \begin{bmatrix} h_{4}^{H} \\ g_{4}^{H} \end{bmatrix} \begin{bmatrix} \hat{\eta}_{4,1} + \epsilon_{4,2} \\ -\eta_{4,1} \end{bmatrix}, \quad (40) $$

where $\eta_{4,1}$, $\eta_{4,2}$, $\epsilon_{4,1}$ and $\epsilon_{4,2}$ have unit power, $u_{4}$ and $v_{4}$ can be decoded with the rate of $\log P_{u_{4}}$ and $\log P_{v_{4}}$, respectively, which equal their encoding rate. As a result, virtual channel 2 lasts for $3 - 2\alpha_{2} = 2\Delta$ channel uses while the rate achieved are $R_{u_{4}}=2-2\Delta-\alpha_{2}$ and $R_{v_{4}}=2-\Delta-\alpha_{2}$.

3) Virtual Channel 3: This virtual channel contains the transmissions of $u_{5}$, $v_{5}$ and $\tilde{\eta}_{5,1}$. As the transmission and power allocation of $u_{5}$ and $v_{5}$ are identical to that at slot 3, $\tilde{\eta}_{5,1}$ is quantized with the rate $\Delta$. After subtracting $\eta_{4,2}$ from (38), $\eta_{5,1}$ is decoded by treating the rest as noise, among which, $u_{6}$ is the dominant. The decoding rate of $\eta_{5,1}$
is $I(\hat{y}_{5,1}; y_6|h_{5,1}, \hat{y}_{4,2}) = \log \frac{P}{\sigma^2}$, whose pre-log factor is $\Delta$. Consequently, $\hat{y}_{5,1}$ can be employed to remove the interference in $y_5$ and provide an additional observation to $z_5$. In this way, $u_5$ and $v_5$ can be decoded using (25) and (26). The rates achieved are $R_{u_5} = \alpha_2$ and $R_{v_5} = \alpha_2 + \Delta$ over the $\alpha_2 + \Delta$ channel.

In all, looking at those three virtual channels which last for 3 channel uses in total, the DoF achieved are expressed as

$$(d_1, d_2) = \left( \frac{R_{u_3} + R_{u_4} + R_{u_5} + R_{v_3} + R_{v_4} + R_{v_5}}{2(\alpha_2 + \Delta) + 3 - 2\alpha_2 - 2\Delta} \right) = \left( \frac{2 + 2\alpha_1 - \alpha_2}{3}, \frac{2 + 2\alpha_2 - \alpha_1}{3} \right).$$

(41)

Combining with the DoF prior to $u_3$ and $v_3$ and cycling those three virtual channels for $N$ times, the achievable DoF is

$$(d_1, d_2) = \lim_{N \to \infty} \left( \frac{2 + N \times 3d_1}{3 - \alpha_2 + N \times 3}, \frac{2 + N \times 3d_2}{3 - \alpha_2 + N \times 3} \right) = \left( \frac{2 + 2\alpha_1 - \alpha_2}{3}, \frac{2 + 2\alpha_2 - \alpha_1}{3} \right).$$

□

E. Case II: $1 - \Delta > \alpha_2$-Achieving ($\alpha_2, 1$)

Point ($\alpha_2, 1$) is achieved under the condition that $1 - \Delta > \alpha_2$ and $\alpha_2 + \Delta$ in case 1 but by reusing the flow described in Section IV-C. The first virtual channel is maintained, where $u_3$ and $v_3$ achieve the rate of $\alpha_2$ and $\alpha_2 + \Delta$ respectively and consume $\alpha_2 + \Delta$ channel uses. However, the second virtual channel is changed by reallocating the power of $u_4$ from $P^{\alpha_2}$ to $P^{\alpha_1}$. In this way, the amount of channel uses in this virtual channel is kept to $2 - \Delta - \alpha_2$, while the rates achieved become $R_{u_4} = \alpha_2$ and $R_{v_4} = 2 - \Delta - \alpha_2$. As a result, the DoF achieved using this 2 channel uses are $(d_1, d_2) = (\alpha_2, 1)$.

F. Achieving $(1, \alpha_1)$

Point $(1, \alpha_1)$ is achieved in a "SC+ZF" manner which has been mentioned in [2]. The transmission scheme is finished in one slot and is expressed as

$$s_t = [x_{c,t}, 0]^T + \tilde{g}_{1} u_t + \tilde{h}_{1} v_t,$$

(42)

where $x_{c,t}$ is intended to user 1 and transmitted with power $P$ superposed to $u_t$ and $v_t$, which are transmitted with power $P^{\alpha_1}$. $u_t$ and $v_t$ are precoded to the orthogonal space of the channel of their unintended user. At each receiver, $x_{c,t}$ is decoded with the rate of $1 - \alpha_1$ using SIC. After subtracting $x_{c,t}$, user 1 decodes $u_t$ with the rate of $\alpha_1$ without any interference. Similarly, user 2 obtains $v_t$ with DoF of $\alpha_1$. Resultantly, point $(1, \alpha_1)$ is achieved.

As a last remark, we note the similarity of this work with [9], where the same DoF region is derived under the same system settings. Their achievable scheme in terminated in three phases. The overhead interferences generated in current phase are accumulated and then split evenly across the transmission in the next phase. The optimal bound is achieved by choosing phase durations which optimally combine the transmission of overhead interference and new symbols. However, in this contribution, we achieved the optimal bound by keeping retransmitting overhead interference to boost up the DoF and making every channel use employed efficiently.

V. CONCLUSION

The optimal DoF region is derived for a two-user MISO broadcast channel when the transmitter has perfect delayed CSI and asymmetric partial current CSI. A novel transmission scheme is motivated and shown to achieve that DoF region. The results boil down to (11, 2, 3) in the symmetric scenario.

APPENDIX-BRIEF PROOF OF OUTER-BOUND

The converse proposed in [2] can be reused to prove the optimal region given in Theorem 1. In that work, genie-aided model is employed to construct a physically-degraded channel. Skipping the same fundamental steps, the only difference lies in deriving the following lower bound

$$E_{\phi} \left( \log \left( 1 + \sum_{i=1}^{m} \lambda_i |\hat{\phi}_i + \tilde{\phi}_i|^2 \right) \right) \geq E_{\hat{\phi}} \left( \log \left( \lambda_1 |\hat{\phi}_1|^2 \right) \right) \approx \log \lambda_1 \sigma^2 = (1 - \alpha_2) \log P.$$

(43)

$\lambda_1 > \lambda_2 > \cdots > \lambda_m$ are eigenvalues of $\tilde{g}, \hat{\phi}_1 + \tilde{\phi}_1$ are eigenvalues of $g, \phi$ is Gaussian distributed as $\mathcal{N}(0, \sigma^2)$. Replacing (43) into its corresponding equations in [2], we have (44). When switching the role of the two users, (45) is obtained.

$$R_1 + 2R_2 \leq (2 + \alpha_2) \log P,$$

(44)

$$2R_1 + R_2 \leq (2 + \alpha_1) \log P.$$

(45)

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