A Kind of Triple-negative Elastic Metamaterial

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Abstract. In this paper, a kind of triple-negative elastic metamaterial (EMM) is proposed, which means it possesses negative equivalent mass density, negative equivalent bulk modulus, negative equivalent shear modulus in the same frequency range simultaneously, and can realize negative refraction of both transverse and longitudinal waves. Furthermore, the influence of rectangular oscillators and semi elliptical oscillators on the band structure is analysed, and we find that the unit cell with semi elliptical oscillators has a much wider band gap, which has a potential in supressing vibration at a wide frequency range.

1. Introduction
Double-negative elastic metamaterials (EMMs) were proposed by analogy with double-negative electromagnetic metamaterials. In 1967, Russian physicist Victor Veselago first predicted a kind of medium with negative dielectric constant ($\varepsilon$) and negative permeability ($\mu$) simultaneously in an article [1]. He also indicated that the medium had a negative refractive index. John Pendry, a British physicist, first designed structural materials with negative equivalent $\varepsilon$ and negative equivalent $\mu$ in 1996, which made Veselago's predictions come true [2]. At the beginning of 2000, Sheng Ping research group of the University of Hong Kong validated the local resonance of elastic waves in three-dimensional coated spherical arrays by numerical simulation and experiments [3]. This work laid a foundation for the analogy between electromagnetic metamaterials and acoustic metamaterials. A EMM is a well-designed and processed artificial composite structure with special characteristics. It can realize wave focusing [4], anisotropy [5], and refractive index changing with position [6]. There are far-reaching and broad application prospects in engineering fields, such as sub-wavelength waveguide [7], low frequency vibration reduction [8], seismic wave stealth of buildings [9], etc. The design of EMMs is more challenging than that of electromagnetic metamaterials [10]. For now, the mechanism of double-negative EMMs is clear [11]. Negative equivalent mass density ($\rho_{\text{eff}}$) is produced by dipole resonance. Negative equivalent bulk modulus ($E_{\text{eff}}$) is produced by monopole resonance. Negative equivalent shear modulus ($G_{\text{eff}}$) is produced by quadrupole resonance. In addition, many kinds of double-negative EMMs which can realize double negative parameters simultaneously in the same frequency range has been designed [12] [13]. The refractive index of longitudinal (P) waves is negative when $\rho_{\text{eff}}$ and $E_{\text{eff}}$ are both negative simultaneously, and the refractive index of transverse (S) waves is negative when $\rho_{\text{eff}}$ and $G_{\text{eff}}$ are both negative simultaneously. Negative refraction facilitates the propagation control of elastic waves. Moreover, triple-negative EMMs which mean $\rho_{\text{eff}}$, $E_{\text{eff}}$, and $G_{\text{eff}}$ are negative simultaneously in the same frequency range may have significance in elastic wave propagation control. Double-negative EMMs mainly could be used in acoustic devices, because sound waves are P waves usually. However, in mechanical engineering and
civil engineering, etc., there are both P waves and S waves in structural members. So, it is meaningful to study triple-negative EMMs. In this paper, a kind of triple-negative EMMs that can realize three negative parameters in the same frequency range is proposed, which may provide a reference for the design of triple-negative EMMs.

2. Modelling
The two-dimensional unit cell of the triple-negative EMMs we proposed is shown in figure 1. The lattice constant is $a = 6\text{mm}$. A round hole with radius $R$ and four oval holes with major axis $D_1$ and short axis $D_2$ are excavated on the square foam matrix. Half of every elliptical hole is filled with tungsten as a scatterer, and the distance from the vertex of the ellipse to the center of the circle is $D_4$. The blank parts are vacuums. The four tungsten blocks are centrosymmetric, which make it easier to form multiple resonance. The material parameters of foam are $\rho = 115\text{kg/m}^3$, $\lambda = 6 \times 10^6 \text{N/m}^2$, $\mu = 3 \times 10^4 \text{N/m}^2$, and the material parameters of tungsten are $\rho = 19300\text{kg/m}^3$, $\lambda = 1.974 \times 10^{11} \text{N/m}^2$, $\mu = 1.513 \times 10^{11} \text{N/m}^2$, where $\lambda$ and $\mu$ are the lame constants and $\rho$ is the mass density. The band structure and eigenmodes of the unit cell are calculated by COMSOL.

![Figure 1. Unit cell design drawing of the triple-negative metamaterial. Here, $a = 6\text{mm}$, $R = 1\text{mm}$, $D_1 = 1.6\text{mm}$, $D_2 = 0.6\text{mm}$, $D_3 = 0.8\text{mm}$, $D_4 = 0.3\text{mm}$](image)

3. Analysis
The band structure of the calculated $\Gamma X$ direction is shown in figure 2. The 5th band and 6th band are negative, and the negative equivalent parameters of the unit cell appear on these two bands. The 5th negative band is a transverse wave band propagating S waves only and the 6th negative band is longitudinal wave band propagating P waves only, which will be proved later. For the convenience of observation, these two negative bands are enlarged as shown in figure 3. The 5th band curve is represented by line A with a starting point A1, and the 6th band curve is represented by line B with a starting point B1. The 5th negative band starts at 8097.9Hz and ends at 7668.1Hz. The 6th negative band starts at 8154.8Hz and ends at 7812.9Hz. A2 and B2 are the points on line A and line B separately.
Figure 2. Band structure of the unit cell in figure 1 along the ΓX direction

The mode of A1, as shown in figure 4 (a), is quadrupole resonance formed by the rotation of four metal tungsten blocks. The quadrupole resonance results in negative equivalent shear modulus, which has an effect on the propagation of P waves. As shown in figure 4 (b), the mode of B1 is monopole resonance, which is formed by four tungsten blocks moving away from the center at the same time. Monopole resonance results in negative equivalent bulk modulus, which has an effect on the propagation of S waves. The mode of B2 is shown in figure 4 (c). The mode is composed of monopole and dipole resonance coupling, which can realize negative $\rho_{\text{eff}}$ and $E_{\text{eff}}$ simultaneously. The mode of A2 is shown in figure 4 (d). The mode is composed of quadrupole and dipole resonance coupling, which can realize negative $\rho_{\text{eff}}$ and $G_{\text{eff}}$ simultaneously.
Figure 4. (a)–(d) Eigenfields of point $A_1$, $B_1$, $B_2$, $A_2$ respectively. Arrows represent field directions, and colour red represents large amplitude, and colour blue represents small amplitude.

(e) Sketch map of hybridization of the monopolar and dipolar mode shown in (c).

(f) Sketch Map of hybridization of the quadrupolar and dipolar mode shown in (d).

In order to further prove that band $A$ is a transverse wave band and band $B$ is a longitudinal wave band, it is necessary to study the response of the two bands to $P$ and $S$ waves. The established verification model is shown in figure 5, with 10 cells connected to form a metamaterial belt. The source line on the left end of the belt is the signal input end, and the probe line is the signal receiving end. Perfectly Matched Layers (PMLs) should be set at both ends of the model to prevent the interference of reflected waves, and periodic conditions should be set to the upper and lower sides of the model. If $P$ waves and $S$ waves are input into the belt separately, we can see that the mode of $S$ wave incidence is quadrupole resonance, and the mode of $P$ wave incidence is monopole resonance, which is consistent with the previous analysis. Besides, in this case, the transmission of the model is calculated, as is shown in figure 6. It can be seen that the $S$ wave transmissibility is higher in the frequency range of the 5th band and the $P$ wave transmissibility is higher in the frequency range of the 6th band. Therefore, it is verified that the 5th band is a transverse wave band and the 6th band is a longitudinal wave band. The transmissibility of $S$ waves and $P$ waves is both very high in the overlapping frequency range of the two bands, so the negative $P_{\text{eff}}$, negative $E_{\text{eff}}$ and negative $G_{\text{eff}}$ can be realized simultaneously in this overlapping frequency range.

Figure 5. Model of a EMM belt composed of ten unit cells.

Figure 6. Transmission of $S$ wave and $P$ wave incident on the model in figure 5. The frequency intervals of the 5th band and 6th band are shown in this figure.
4. Comparison and Discussion
The unit cell proposed in this paper has a wide band gap, which is a peculiarity. Figure 7 shows the structure of a unit cell proposed by Wu et al [11]. It has rectangular oscillators and also can realize three negative parameters in the same frequency range simultaneously. Its band structure is shown in figure 8. The width of the first band gap is almost the same as that of the cell proposed in this paper, and its width is about 2KHz. The second band gap width is about 3kHz, while the second band gap width of the unit cell proposed in this paper is about 4.5khz. In order to make the unit cell with rectangular oscillators produces a wider band gap, the mass of the tungsten block can be reduced by half (L reduced from 1mm to 0.5mm), and the band structure is shown in figure 9. However, it can be seen that there is not any overlapping part of the 5th and 6th band, which means the unit cell loses the characteristic of three negative parameters in the same band simultaneously. The combination of wide band gaps and negative refraction can be used for vibration reduction and isolation. Band gaps prevent elastic waves from propagating at certain frequency ranges, and for the elastic waves that can't be stopped by band gaps, negative refraction can facilitate making the wave propagation path change so that the waves don't propagate to the target. Based on this principle, functional devices that can suppress vibration at a wide frequency range can be fabricated, such as functionally graded materials and directional invisibility cloaks.

Figure 7. A unit cell with rectangular oscillators that can can realize three negative parameters in the same frequency range simultaneously

Figure 8. The band structure of the unit cell shown in figure 7 with L=1mm

Figure 9. The band structure of the unit cell shown in figure 7 with L=0.5mm

5. Conclusion
In conclusion, we have proposed a novel triple-negative EMM with a wide band gap, which may enlighten the design or optimization of triple-negative EMMs for vibration isolation and reduction. Based on studying the eigenmodes and transmission of the triple-negative EMM, we have proved that it can achieve negative equivalent $\rho_{\text{eff}}$, $E_{\text{eff}}$ and $G_{\text{eff}}$ at the same frequency range simultaneously,
which can facilitate the propagation control of both P and S waves. We also find that it has a wide band gap, so we put forward the idea of using both passbands and band gaps to isolate vibration.

6. References

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