Are galaxies extending?

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It is suggested that the recently observed size evolution of very massive compact galaxies in the early universe can be explained, if dark matter is in Bose Einstein condensate. In this model the size of the dark matter halos and galaxies depends on the correlation length of dark matter and, hence, on the the expansion of the universe. This theory predicts that the size of the galaxies increases as the Hubble radius of the universe even without merging, which agrees well with the recent observational data.

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Dark matter (DM) and dark energy are two of most important unsolved puzzles in modern physics and cosmology. Identification of one DM particle species by a direct detection experiment such as LHC or DAMA is not enough to fully solve the dark matter problem, because there can be multiple species of DM and we have to explain the observed distribution of DM in the universe. Although the cold dark matter (CDM) with the cosmological constant (i.e., ΛCDM) model is popular and remarkably successful in explaining the large scale structure of the universe, it seems to encounter problems on the scale of galactic or sub-galactic structures. Numerical simulations with ΛCDM model usually predict a cusped central halo DM density and too many satellite galaxies compared to astronomical observations. On the other hand, it is known that the cold dark matter model based on Bose Einstein condensate (BEC) or scalar field dark matter (SFDM) can alleviate these problems and well explain the observed rotation curves of galaxies. In this model the galactic halos are like gigantic atoms where cold boson DM particles are condensed in a single macroscopic wave function ψ(r). (The idea of giant atoms as hypothetical stars goes back to Kaup and Ruffini and developed by Schunck and others. Similar halo DM ideas were suggested by many authors). This BEC/SFDM model, now often known as the fuzzy DM model, is a variant of the CDM model. It is more about the state of DM particles rather than the DM particle itself.

The recent observations of the size evolution of massive galaxies even deepen mysteries of DM and formation of galaxies. In using the combined capabilities of earth-bound telescopes and the Hubble space telescope, the size evolution of 831 very massive galaxies since z ~ 2 is investigated. It is observed that massive quiescent galaxies at z ~ 3 have a very small median effective radius r = 0.9 kpc. According to the observations, very massive (≥ 10^{11} M_\odot) galaxies have a factor of about 5 smaller size in the past (z ~ 2) than their counterparts today. These compact but massive galaxies are puzzling in the context of ΛCDM model, because the size evolution is usually attributed to a hierarchical merging process of small galaxies to form a larger galaxy. Thus, it is expected that small early galaxies have small mass too. The finding of compact but very massive early galaxies which have disappeared calls this interpretation in question, since these galaxies had almost reached the maximum mass limit observed today and might not have been experienced significant merging after z ~ 2. Various mechanisms depending on visible matter could change the size-mass relation by a factor of ~ 2 but not a factor of ~ 5.

In this paper, it is suggested that this galaxy size evolution problem can be also solved in the BEC/SFDM model, if the correlation length of the DM condensate is time dependent. First, let me briefly review the BEC/SFDM model. In 1992, to explain the observed galactic rotation curves, Sin suggested that galactic halos are astronomical objects in BEC of ultra light (mass m ≈ 10^{-24} eV) DM particles such as pseudo Nambu-Goldstone boson (PNGB). In this model the cold boson DM particles are condensed in a single macroscopic wave function ψ(r) and the quantum mechanical uncertainty principle prevents the halos from self-gravitational collapse, while in usual CDM models DM particles move independently and incoherently. ψ(r) satisfies the non-linear Schrödinger equation with the Newtonian gravity;
From this one can also obtain the minimum mass for galaxies $M \rightarrow \infty$, and the size of DM halos $\sigma_{\text{halos}}$ are giant boson stars (boson halos \cite{18, 43, 44}) described by a complex scalar field $\phi$ and $V_{\text{DM}}$ is the energy of each DM particle. The gravitational potential is given by

$$V(r) = \int_0^r dr' \frac{Gm}{r'^2} \int_0^r dr'' 4\pi r''^2 (\rho_{\text{vis}}(r'') + \rho_{\text{DM}}(r'')) + V_0,$$

where DM density $\rho_{\text{DM}}(r) = M_0 |\psi(r)|^2$ and $\rho_{\text{vis}}$ is the visible matter (i.e., stars and gas) density. $M_0$ is a mass parameter and $V_0$ is a constant. According to the model, the condensation of DM particles of which huge Compton wavelength $\lambda_c = 2\pi \hbar/mc \sim 10$ pc, i.e. $m \simeq 10^{-24}$ eV, is responsible for the halo formation.

The author and Koh \cite{9, 42} generalized Sin’s BEC model in the context of quantum field theory and the general relativity and suggested that DM can be described as a coherent scalar field (i.e., SFDM). In this model the BEC DM halos are giant boson stars (boson halos \cite{18, 43, 44}) described by a complex scalar field $\phi$ having a typical action

$$S = \int \sqrt{-g}d^4x \left[ -\frac{R}{16\pi G} - \frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial^\nu \phi - U(\phi) \right]$$

with a potential $U(\phi) = \frac{\mu^2}{2} |\phi|^2 + \frac{\lambda}{4} |\phi|^4$. In this paper, the case with $\lambda = 0$ will be considered for simplicity. The spherical symmetric metric is $ds^2 = -e^{\sigma(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2 d\Omega^2$, where $r$ is the radial coordinate.

In the BEC/SFDM theory, due to the uncertainty principle $\Delta x \Delta p \geq \hbar/2$, the characteristic length scale of a BEC DM halo $\xi$ is inversely proportional to the mass of DM particles, i.e., $\xi \sim \Delta x \sim \hbar/\Delta p \sim \hbar/m \Delta v \sim 10^{-3} c \Delta v / m$, where $\Delta v$ is the velocity dispersion of DM particles and $c$ is the light velocity. This $\xi$ should be comparable with the characteristic length scale of the DM condensate. (For DM halos or boson stars, the correlation length or de Broglie wavelength of the condensate is more suitable for the length scale than the Compton wavelength $\lambda_c \sim 1/m$ \cite{9, 15}.) From this one can also obtain the minimum mass for galaxies $M_c = \frac{\hbar^2}{G\sigma_{\text{halos}}} \simeq 10^7 M_\odot$, which is recently observed \cite{46}.

The temperature-dependent correlation length of DM is about the thermal de Broglie length $\xi(T) \sim 2\pi \hbar / kT$ of the DM condensate \cite{47}, which is an increasing function of the time. This leads to a surprising possibility that the sizes of a DM halo and a galaxy embedded in it are slowly increasing functions of the time even without merging. The DM halo provides a potential well to trap the visible galactic matter such as stars and gas. The extension of the halo induces the formation of a visible part of the galaxy. To see this consider the Newtonian limit of the equations of motions from the action in Eq. (3), or, Eq. (1), which can be written in the dimensionless form as

$$\begin{cases} \nabla^2 V = (\sigma^2 + \rho_{\text{vis}}) \\ \nabla^2 \sigma = 2(V - E)\sigma \end{cases} \tag{4}$$

Here $\sigma = \sqrt{4\pi Ge^{-iE\psi}/\hbar^2}$ is a dimensionless form of the wave function $\psi$ (or the scalar field $\phi$) and $r \rightarrow mc^2/h$, $t \rightarrow mc^2/h$ \cite{9} and all other quantities are dimensionless too from now on. Since galaxies are dominated by DM, we will ignore $\rho_{\text{vis}}$ and assume that visible matter passively moves inside the potential well $V$ of the DM halo. The equations above have approximate solutions \cite{47}

$$\begin{cases} \sigma(r) \simeq \sigma(0) \left( 1 + \frac{(V(0) - E)r^2}{6} \right) \\ V(r) \simeq V(0) + \frac{\sigma(0)^2 r^2}{6} \end{cases} \tag{5}$$

To see the size evolution for the most massive galaxies, we need to calculate the DM distribution in their halos for a fixed galaxy mass $M \equiv \int_0^\infty 4\pi r^2 (\sigma(r)^2 + \rho_{\text{vis}}(r))dr \simeq O(\sigma^2 r^3)$. Thus, for a constant $M$ and under the scaling $\xi \rightarrow l \xi$ corresponding to the increase of the correlation length, the other parameters for DM halos scale as $r \rightarrow l r$, $\sigma \rightarrow l^{-3/2} \sigma$ and $V \sim M/r \rightarrow V/l$. This means, under this scaling, the gravitational potential well becomes shallower and the size of DM halos $r_{\text{DM}}$ increases like $l$. We can also assume that the total energy (kinetic + potential) of a star $E_{st}$ measured from $V(0)$ is conserved during the extension. The star orbits around the galactic center within the gravitational potential of the halo $V$. Note that $E_{st}$ is different from the energy of a DM particle, $E$. The visible radius of a galaxy can be defined by an orbital radius of outermost stars $r_s$, which can be defined as the position satisfying the condition $E_{st} = V = V(r = r_s)$, i.e., the position where the kinetic energy of the stars is zero. From Eq. (5) and the energy conservation one can see that $E_{st} = V(r_s) = \sigma(0)^2 r_s^2 / 6$ and $r_s = \sqrt{6E_{st}/\sigma(0)}$. Since $\sigma(0) \rightarrow l^{-3/2} \sigma(0)$, the orbital radius of the star scales as $r_s \rightarrow l^{3/2} r_s$. It means that the visible radius of galaxy follows approximately the $3/2$ power of the size of its dark matter halo. (The extension of the halo also reduces the density of matter...
Since the Hubble radius \( r \sim 4.78 \times 10^{-6} \) and the expected rotation velocity dispersion is \( v_{\text{rot}} \approx O(\sqrt{\sigma}) \approx 0.0022c \approx 650\text{ km/s} \), which and the rotation velocity of stars. This is also observed [40]. Since the observed ‘extension speed’ of the galaxy is very low (\( O(\text{kpc}/10^{10} \text{ yrs}) \)) [40], we can treat the expansion as an adiabatic one. Thus, the temperature is inversely proportional to the scale factor \( R \) of the universe, i.e., \( T \sim 1/R \), as usual.

Collecting all together, we obtain a simple relation between the size parameter \( r_{DM}(z) \) of DM halos and the redshift \( z \):

\[
\frac{r_{DM}(z)}{r_{DM}(0)} = \frac{\xi(z)}{\xi(0)} = \frac{T(0)}{T(z)} = \frac{R(z)}{R(0)} = \frac{1}{1+z}.
\]

Here \( r_{DM}(z) \) denotes the effective radius of a DM halo at \( z \), and \( r_{DM}(0) \) denotes that at present. Thus, the size of visible galaxies \( r* \) evolves as

\[
\frac{r_*(z)}{r_*(0)} = \left( \frac{r_{DM}(z)}{r_{DM}(0)} \right)^{3/2} = \left( \frac{R(z)}{R(0)} \right)^{3/2} = \left( \frac{1}{1+z} \right)^{3/2}.
\]

Since the Hubble radius \( H^{-1}(t) \sim t \) and \( R(t) \sim t^4 \) during the matter dominated era, the size of the most massive galaxies increases as the Hubble parameter at the first order. Fig. 1 shows the observed size evolution \( r(z) \) of the massive galaxies [33, 40] versus the redshift \( z \). The theoretical prediction \( r_*(z) \) in Eq. (7) is well coincident with the observational data. The black dot represents the typical size parameter for 9 galaxies in Ref. [39] for which I used the typical values \( r(z) = 2.3, r_*(z) = 0.9 \text{ kpc}, \) and \( r(0) = 5 \text{ kpc} \) from the paper. For the error bar for the dot, I used the effective radii of the smallest and the largest galaxies in the table 1 of Ref. [39]. Although there are still many theoretical and observational uncertainties, the coincidence between the theoretical prediction and the data for disk-like galaxies is remarkable, especially for high \( z \). The small discrepancy can be attributed to the systematic observational uncertainties and ignoring of possible merging history. It is unclear why disc-like galaxies and spheroid-like galaxies seem to show rather different size evolutions. We need more observations to determine whether the difference is real or not.

To be more illustrative, we perform a numerical study using the shooting method [3]. Fig. 2 shows the result of our numerical study with boundary conditions \( dV/dr(0) = 0, V(\infty) = 0 \) and \( d\sigma/dr(0) = 0 \). We consider 3 cases with the parameters \( \sigma(0) = 5 \times 10^{-7}, V(0) = -3.678 \times 10^{-7}, E = -1.52 \times 10^{-8} \), \( \sigma(0) = 3.21 \times 10^{-7}, V(0) = -2.72 \times 10^{-7}, E = -5.71 \times 10^{-8} \), and \( \sigma(0) = 2.17 \times 10^{-7}, V(0) = -2.04 \times 10^{-7}, E = -6.23 \times 10^{-8} \), respectively, from the top to the bottom for \( \sigma(0) \). With \( E_m = 10^{-7} \) we changed the length scale as \( l = 1, 1.5, 2 \) for 3 cases and obtained \( r_* = 1736, 2861, 4590 \), respectively. The masses \( M = 0.013, 0.0125, 0.0124 \) are similar for all 3 cases. The numerical results support the theoretical argument above.

We need to check the reliability of the Newtonian approximation used to derive the Eqs. (6) and (7). From the action in Eq. (3) and by defining \( \sigma \equiv \sqrt{4\pi Ge^{-i\omega t}} \) one can obtain the dimensionless versions of the scalar field equation and the Einstein equation [48], which can be reduced to the equation in Eq. (3) in the Newtonian limit [3]. Since the typical compact galaxies observed have mass \( M \sim 10^{11}M_\odot \) and size \( R \sim 1\text{ kpc} \), the dimensionless gravitational potential \( V \sim M/R \sim 4.78 \times 10^{-6} \) and the expected rotation velocity dispersion is \( v_{\text{rot}} \approx O(\sqrt{\sigma}) \approx 0.0022c \approx 650\text{ km/s} \), which...
is comparable with the recent observational data $v_{\text{rot}} \simeq 510^{+165}_{-90} \text{km/s}$ by Dokkum et al.\cite{45}. Thus, the compact early galaxies are basically non-relativistic objects and the Newtonian approximation is good for these galaxies.

Schunck et al pointed out that the effect of pressure generated by scalar fields should be included in the rotation curves\cite{19,50}. The rotation velocity given by the circular geodesics is $v_{\text{rot}} = r d\nu/d\epsilon^r/2 \simeq M(r)/r + p_r r^2 e^{\lambda + \nu}/2$, where the second term denotes the contribution of the radial pressure $p_r = \rho - U = 1/2(\omega^2 \sigma^2 e^{-\nu} + \omega^2 e^{-\lambda} - U)$ of the DM field. Since $\sigma \approx O(10^{-7}) \ll 1$ and $v_{\text{rot}} \ll c$ we can use the weak field approximation in Ref\cite{51,52}. A relevant parameter for this approximation is $\epsilon \equiv (1 - \omega^2/m^2)^{1/2} \ll 1$. In this limit $\omega \approx m$, $d/\nu \sim \epsilon$ and $e^{\lambda + \nu} \rightarrow 1$. Thus, we obtain (dimensionless) $p_r \approx 1/2(\sigma^2 + \epsilon^2 \sigma^2 - \sigma^2) = O(\epsilon^2 \sigma^2) \ll \rho \sim O(\sigma^2)$ and neglecting the contribution from $p_r$ in our model (with $U = m^2 \sigma^2/2$) is a good approximation for these galaxies. This is different from the case of the model with massless scalar DM particles, where $U = 0$\cite{19,50}.

![Image](https://via.placeholder.com/150)

**FIG. 2: (Color online)** The dark matter field $\sigma$ (blue dotted lines), the energy level of visible matter (green thick lines) and the gravitational potential $V$ (red dashed lines) for a galaxy as a function of distance $r$ from the halo center. As the universe expands, the temperature of DM decreases and its correlation length $\xi$ increases. This induces the increase of the size of the visible galaxy ($r_*$) represented by the green lines.

We have shown analytically and numerically that the BEC/SFDM theory could explain the observed size evolution of the most massive galaxies. In our theory the size of galaxies can increase not only by merging or accretion of small galaxies but also by the increase of the length scale of DM halos itself. For small and medium sized galaxies, it is hard to distinguish these two effects. This may explain why the pure size extension without merging is observed only recently and only for the most massive galaxies. Our theory explains how these massive galaxies were so dense in the past and reached the size of the massive galaxies today.

A self-gravitational redshift effect of the DM halo may contribute to the relation in Eq.\cite{6} or Eq.\cite{7}. The observed redshift of the galaxies could be a combination of the cosmological $(1 + z)$ and the gravitational redshift $(1 + z_g)$, i.e., $(1+z)(1+z_g)$\cite{19}. The gravitational redshift parameter for the boson halo is given by $z_g = e^{(\nu(\infty) - \nu(r))/2} - 1 \simeq V$ in the weak gravitation limit. Since $V \simeq O(10^{-9})$, the correction from $z_g$ is negligible for galactic DM halos in this model\cite{33} compared to the cosmological redshift. A time-varying gravitational constant or the gravitational memory in Brans-Dicke models could mimic the extension of the galaxies\cite{53,54}.

In conclusion, the idea that DM is in BEC seems to provide us a new way to explain not only the CDM problems but also the galaxy evolution. From this perspective it is important to determine the exact size evolution of the galaxies by future observation.

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\[\text{[1] J. R. Ellis, Phil. Trans. Roy. Soc. Lond. A361, 2607 (2003).}\]
\[\text{[2] R. Bernabei et al., eprint:0804.2741 (2008).}\]
\[\text{[3] J. P. Ostriker and P. Steinhardt, Science 300, 1909 (2003).}\]
\[\text{[4] P. Salucci, F. Walter, and A. Borriello, Astronomy and Astrophysics 409, 53 (2003).}\]
\[\text{[5] J. F. Navarro, C. S. Frenk, and S. D. M. White, Astrophys. J. 462, 563 (1996).}\]
[6] W. J. G. de Blok, A. Bosma, and S. S. McGaugh, astro-ph/0212102 (2002).
[7] A. Tasitsiomi, International Journal of Modern Physics D 12, 1157 (2003).
[8] S.-J. Sin, Phys. Rev. D 50, 3650 (1994).
[9] J.-W. Lee and I.-G. Koh, Phys. Rev. D 53, 2236 (1996) hep-ph/9507385.
[10] P. Peebles, Astrophys. J. 534, L127 (2000).
[11] V. Sahni and L. Wang, Phys. Rev. D 62, 103517 (2000).
[12] T. Matos and L. A. Urena-Lopez, Class. Quant. Grav. 17, L75 (2000).
[13] T. Matos and L. Arturo Ureña López, Phys. Rev. D 63, 063506 (2001).
[14] A. Arbey, J. Lesgourgues, and P. Salati, Phys. Rev. D 68, 023511 (2003).
[15] C. G. Boehmer and T. Harko, JCAP 0706, 025 (2007).
[16] F. S. Guzman and L. A. Urena-Lopez, Physical Review D 68, 024023 (2003).
[17] R. Ruffini and S. Bonazzola, Phys. Rev. 187, 1767 (1969).
[18] F. E. Schunck and E. W. Mielke, Class. Quant. Grav. 20, R301 (2003).
[19] F. E. Schunck, arXiv:astro-ph/9802258 (1998).
[20] T. Matos, F. S. Guzman, L. A. Urena-Lopez, and D. Nunez, astro-ph/0102419 (2001).
[21] A. Arbey, J. Lesgourgues, and P. Salati, Phys. Rev. D 64, 123528 (2001).
[22] A. Arbey, J. Lesgourgues, and P. Salati, Phys. Rev. D 65, 083514 (2002).
[23] J. Goodman, New Astronomy Reviews 5, 103 (2000).
[24] W. Hu, R. Barkana, and A. Gruzinov, Phys. Rev. Lett. 85, 1158 (2000).
[25] E. W. Mielke and F. E. Schunck, Phys. Rev. D 66, 023503 (2002).
[26] M. Alcubierre et al., Class. Quant. Grav. 19, 5017 (2002).
[27] B. Fuchs and E. W. Mielke, Mon. Not. Roy. Astron. Soc. 350, 707 (2004).
[28] T. Matos, F. S. Guzman, L. A. Urena-Lopez, and D. Nunez, astro-ph/0102419 (2001).
[29] M. P. Silverman and R. L. Mallett, Classical and Quantum Gravity 18, L103 (2001).
[30] U. Nucamendi, M. Salgado, and D. Sudarsky, Phys. Rev. D 63, 125016 (2001).
[31] A. A. Julien Lesgourgues and P. Salati, New Astronomy Reviews 46, 791 (2002).
[32] J. W. Moffat, astro-ph/0602607 (2006).
[33] E. W. Mielke, B. Fuchs, and F. E. Schunck, Proc. of the Tenth Marcel Grossman Meeting on General Relativity, Rio de Janeiro (2003).
[34] P. Sikivie and Q. Yang, arXiv:0901.1106 (2009).
[35] E. W. Mielke and J. A. V. Perez, Phys. Lett. B 671, 174 (2009).
[36] J. Balakrishna and F. E. Schunck, arXiv:gr-qc/9802054 (1998).
[37] J.-W. Lee, arXiv:0801.1442 (2008).
[38] E. Daddi et al., The Astrophysical Journal Letters 631, L13 (2005).
[39] P. G. van Dokkum et al., The Astrophysical Journal Letters 677, L5 (2008).
[40] I. Trujillo et al., Monthly Notices of the Royal Astronomical Society 382, 109 (November 2007).
[41] S. U. Ji and S. J. Sin, Phys. Rev. D 50, 3655 (1994).
[42] J.-W. Lee and I.-G. Koh, Galactic halo as a soliton star, Abstracts, bulletin of the Korean Physical Society, 10 (2) (1992).
[43] P. Jetzer, Phys. Rep. 220, 163 (1992).
[44] T. D. Lee and Y. Pang, Phys. Rep. 221, 251 (1992).
[45] M. Silverman and R. Mallett, General Relativity and Gravitation 34, 633 (May 2002).
[46] J.-W. Lee and S. Lim, arXiv:0812.1342 (2008).
[47] L. A. Ureña López and A. R. Liddle, Phys. Rev. D 66, 083005 (2002).
[48] M. Colpi, S. L. Shapiro, and I. Wasserman, Phys. Rev. Lett. 57, 2485 (1986).
[49] P. G. van Dokkum, M. Kriek, and M. Franx, arXiv:0906.2778 (2009).
[50] F. E. Schunck, Aspects of dark matter in astro and particle physics (World Scientific, Singapore, 1997), pp. 403–408.
[51] R. Friedberg, T. D. Lee, and Y. Pang, Phys. Rev. D 35, 3640 (1987).
[52] V. Silveira and C. M. G. de Sousa, Phys. Rev. D 52, 5724 (1995).
[53] D. F. Torres, A. R. Liddle, and F. E. Schunck, Phys. Rev. D57, 4821 (1998).
[54] The author thank the anonymous referee for pointing out this fact.