Static and rotating white dwarfs at finite temperatures

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Static and rotating hot white dwarfs are investigated in Newtonian gravity, employing the well-known Chandrasekhar equation of state. The mass-radius relations of stable white dwarfs are constructed for different temperatures. It is shown that the maximum mass of hot rotating white dwarfs is slightly less than for cold rotating white dwarfs.

Keywords: White dwarfs; finite temperatures; rotation.

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1. Introduction

White dwarfs (WDs) are the final products of the evolution of average and low-mass main sequence stars. They are formed in the cores of red giant stars. Most of the stellar population will end up as a white dwarf star. Therefore, WDs are considered to be the most abundant stellar remnants. The average mass of a white dwarf is around 0.6 solar mass ($M_\odot$) and radius is roughly 10000 km. Correspondingly, their average density is approximately $10^6$ g/cm$^3$.

Unlike neutron stars (NSs), there are only three equations of state (EoS) in the literature for WDs: the Chandrasekhar EoS, the Salpeter EoS, and the Relativistic Feynman-Metropolis-Teller EoS. These equations with some modifications, including finite temperatures, magnetic field etc. are used to describe the cores of WDs and outer crusts of NSs. The physical properties of WDs have been intensively studied both in Newtonian gravity (NG) and general relativity (GR). It has been shown that the effects of GR are crucial to analyze the stability of WDs close to the Chandrasekhar mass limit 1.44 $M_\odot$ and can be neglected for low mass WDs.

In this work, the analyzes performed in Refs. 6–8 are extended to include the effects of rigid rotation in WDs employing the Chandrasekhar EoS at finite-temperatures. The mass-radius ($M-R$) relations for hot static and rotating WDs at Keplerian rate are constructed in NG for simplicity.
2. Formulation of the Problem and Results

The physical characteristics of rigidly rotating WDs at different finite temperatures are studied here in the range of radius from 1000 km to 200000 km. For clarity, the Chandrasekhar EoS is exploited since it is well-known and widely used to describe WDs. According to Chandrasekhar, the total energy density of the WD matter is determined by the energy density of nuclei and the total pressure is given by the degenerate electron gas. The ratio of the average atomic number to the number of proton of the nucleus $A/Z = 2$ is used throughout the paper.

The temperatures of the WD isothermal cores $T = T_c$ are considered here without taking into account the atmosphere of WDs. The effective surface temperature $T_{eff}$ is roughly three order of magnitude less than $T_c$ according to the Koester formula $T_{eff}^4/g = 2.05 \times 10^{-10}T_c^{2.56}$, where $g$ is the surface gravity.

To investigate static WDs the Newtonian hydrostatic equilibrium equations are solved numerically. To construct rotating WDs the Hartle formalism is applied in NG. All rotating WDs are calculated at the Keplerian sequence: $\Omega_{Kep} = \sqrt{GM_{rot}/R_{eq}^3}$, where $\Omega_{Kep}$ is the maximum angular velocity, $G$ is the gravitational constant, $M_{rot}$ is the total rotating mass, and $R_{eq}$ is the corresponding equatorial radius of WDs.

![Fig. 1. Mass-radius relations for static and rotating white dwarfs at finite temperatures $T = [0, 4 \times 10^7, 10^8]$ K in the range of equatorial radius $R_{eq} = (10^3 - 2 \times 10^5)$ km.](image)

By solving the structure equations numerically, $M - R$ relations for static and rotating hot WDs are obtained. The stability of hot rotating WDs has been analyzed in Refs. [12,13]. In Fig. 1 the $M - R$ relations are shown at different temperatures $T = [0, 4 \times 10^7, 10^8]$ K. All solid curves indicate static WDs and dashed curves correspond to rotating WDs. It is evident that the finite-temperature effects are more pronounced for low mass WDs and the effects of rotation are crucial for
smaller in size WDs. Nevertheless both the finite temperature and rotation effects contribute to the radius and the mass of WDs. The comparison of these theoretical curves with the estimated masses and radii of WDs from the Sloan Digital Sky Survey Data Release 4 (see Ref. [14]) is given in Ref. [13].

One can see that for a fixed mass within the range \( R_{eq} = (5 \times 10^{3} - 50 \times 10^{3}) \) km the higher the temperature the larger the radii. The same is true for a fixed radius the higher the temperature the larger the mass.

### Fig. 2

Fig. 2 is magnified Fig. 1 in the range \( R_{eq} = (8 \times 10^{2} - 5 \times 10^{3}) \) km. Within this range for static WDs, the higher temperatures the larger masses. This effect is natural, since due to the temperature, for a fixed central density, the pressure of partially or non-degenerate electrons increases and can sustain more mass. Instead, for rotating WDs, the higher temperatures the smaller masses. This effect is counter-intuitive, since one would expect that the effects observed in static WDs would automatically translate to rotating WDs. However, this is not the case, at least in this range of mass and radius. Here, the temperature has more contribution to the radius than to the mass. Hence, the Keplerian angular velocity is lower for hotter WDs, correspondingly the total rotating mass is less than for colder WDs.

### 3. Conclusion

The properties of static and rotating WDs have been investigated using the Chandrasekhar EoS at finite-temperatures. The structure equations have been solved numerically to construct \( M - R \) relations for hot static and rotating isothermal cores of WDs. The atmosphere of WDs was not considered for simplicity.

It has been shown that temperature affects the masses of WDs at larger radii. At smaller radii the thermal effects are negligible. Rotation affects the masses of cold...
WDs in all density range. For hot WDs the effects of rotation are less noticeable at larger radii and more pronounced at smaller radii. It was found that close to the maximum mass the hotter rotating WDs possess less mass than the colder ones. Since hot WDs are slightly larger in size they can not rotate faster. Therefore, in the range $R_{eq} = (8 \times 10^2 - 5 \times 10^3)$ km hot rotating WDs possess slightly less mass than the cold ones.

In view of the latest observational data on WDs\textsuperscript{15–17} it would be interesting to explore WDs, taking into account the Coulomb and Thomas-Fermi corrections at finite temperatures in the EoS as in Ref.\textsuperscript{6} including the effects of rotation in the structure equations. That will be the issue of future studies.

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