Electron Energy Spread in Laser Cooling Process

A. Kolchuzhkin, A. Potylitsyn, and S. Strokov

Tomsk Polytechnic University, Tomsk, Russia, 634034

The problem of electrons energy spread in the linear back Compton scattering process has been considered in the paper. The adjoint kinetic equation for the electrons passing through a laser flash has been obtained and used to get the equations for the mean energy and the variance of the energy distribution. The equations for the distribution moments have been obtained and solved by the iteration method in the continuous slowing down approximation with approximate description of the energy loss fluctuation. It has been shown that the variance of the energy distribution as a function of the electron - photon collisions number in laser flash has a maximum for the electron beam with small incident energy spread. The beam energy spread damping in the laser-electron storage ring has been studied. The data of approximate analytical calculations are compared with the results of the Monte Carlo simulation.

PACS numbers: 07.85.Fv; 13.60.Fz; 24.10.Lx; 41.75.Ht
Keywords: compton back-scattering, multiple energy loss, laser cooling, kinetic equation, Monte Carlo simulation.

I. INTRODUCTION

The interaction of a high energy electron beam with an intense laser flash is now considered in such projects as compact x-ray sources, γ − γ colliders, diagnostics of low emittance electron beams, laser cooling and others [1]. The laser flash intensity in these projects may be so high that an electron can undergo several successive collisions passing through a photon bunch [2, 3]. It means the number of scattered photons is larger than one. Taking into account the discrete character of the Compton back-scattering process, one can see an analogy between the electron passage through a laser flash and through a condensed matter. The stochastic theory of particles penetration through a matter can be found in the book [4]. The model of multiple bremsstrahlung process which leads to emission of a few photons by a single electron was developed in [5]. Here we consider the multiple Compton back-scattering process using the same approach. For simplicity we use the approximation of an uniform photon concentration in the laser flash:

\[ n_L = \frac{A}{\omega_0} \frac{1}{\pi r_L^2 l_L} = \text{const} \]

where \( A \) is the total energy of the laser flash; \( \omega_0 \) is the photon energy, \( r_L \) and \( l_L \) are radius and length of the photon bunch. In other words our approximation corresponds to the short laser pulse (\( l_L \ll z_R, z_R \) being the Rayleigh length).

The kinetic equations have been obtained for monoenergetic electrons and for electrons with incident energy spread. These equations are transformed into the equations for the distributions moments. The method for an approximate solution of the equations has been derived for the mean energy and for the variance of the energy distributions. It has been shown that the energy distribution variance as a function of the target thickness is a curve with a maximum if the incident energy spread is small.

It is shown that the equation for calculation of the variance can be used to study the variation of the energy spread in the laser-electron storage ring (LESR) where an electron beam repeatedly encounters the laser pulses. It has been shown that the variation of the energy spread in this case depends on the incident value of the variance. The energy spread damping takes place if the incident energy spread is larger than some limit value.

The results of analytical calculations are compared with the data of Monte Carlo simulation.

II. TRANSPORT EQUATIONS

Let us consider electrons with an incident energy \( \varepsilon_0 \gg mc^2 \) travelling through a bunch of photons with an energy \( \omega_0 \). The process of an electron transport in a light target is a random process where both the collisions number along the path \( l \) and the energy loss in each collision are random quantities. Therefore, the energy of electron \( \varepsilon \) passing a laser flash is random quantity too.

The angular deflection of an electron in the Compton back-scattering process is small (\( \sim \frac{2\omega_0}{mc^2} \)) and may be neglected (the straight-ahead approximation). In this approximation the probability density function \( P_m(\varepsilon|\varepsilon_0, l) \) (the subscript "m" means "monoenergetic") describing the energy distribution of electrons travelling a path \( l \) in the photon bunch obeys the adjoint balance equation (the Kolmogorov-Chapman equation) [4, 5]:

\[
P_m(\varepsilon|\varepsilon_0, l) = (1 - s\Sigma(\varepsilon_0))P_m(\varepsilon|\varepsilon_0, l - s) \\
+ s\Sigma(\varepsilon_0) \int_{\omega_0}^{\omega_{\max}} \frac{\Sigma(\omega|\varepsilon_0)}{\Sigma(\varepsilon_0)} P_m(\varepsilon|\varepsilon_0 - \omega, l) d\omega ,
\]

(1)
where $\Sigma(\varepsilon_0)$ and $\Sigma(\omega; \varepsilon_0)$ are the total and differential macroscopic cross-sections of the Compton scattering, $s$ is a small part of $l$, and $\omega$ is the energy of scattered photon ($0 \leq \omega \leq \omega_{\text{max}}$),

$$\omega_{\text{max}} = \varepsilon_0 \frac{x}{1 + x}$$

is the maximum value of the scattered photon energy,

$$\xi = \frac{4 \omega_0 \varepsilon_0}{(mc^2)^2},$$

$$\Sigma(\varepsilon_0) = 2n_L \sigma(\varepsilon_0),$$

$$\Sigma(\omega; \varepsilon_0) = 2n_L \frac{d\sigma(\omega; \varepsilon_0)}{d\omega},$$

$\sigma$ and $\frac{d\sigma}{d\omega}$ are the total and differential cross-sections. $n_L$ is the concentration of laser photons in the bunch.

Note that $\Sigma(\varepsilon_0)$ is the mean number of collisions of an electron per unit path length and $\Sigma(\omega; \varepsilon_0)$ is the mean number of an electron collisions with the energy loss in unit interval about $\omega$ per unit path length.

The first term in the right side of Eq. (1) corresponds to the electrons, which passes the path $s$ without collisions, and $1 - s \Sigma(\varepsilon_0)$ is corresponding probability. These electrons have to lose the energy $\varepsilon_0 - \varepsilon$ along the rest path $l - s$.

The second term corresponds to the electrons, which undergo the first scattering passing the path $s$, and $s \Sigma(\varepsilon_0)$ is the corresponding probability. The energy of the electron after the first scattering equals $\varepsilon_0 - \omega$, where $\omega$ is a random energy of the scattered photon, and $\Sigma(\omega; \varepsilon_0)/\Sigma(\varepsilon_0)$ is the probability density function of $\omega$. In the limit $s \to 0$ Eq. (1) gives the adjoint integro-differential equation for the function $P_m(\varepsilon | \varepsilon_0, l)$:

$$\frac{\partial}{\partial \omega_{\text{max}}} P_m(\varepsilon | \varepsilon_0, l) + \Sigma(\varepsilon_0) P_m(\varepsilon | \varepsilon_0, l)$$

$$- \int_0^{\omega_{\text{max}}} \Sigma(\omega; \varepsilon_0) P_m(\varepsilon | \varepsilon_0 - \omega, l) d\omega = 0$$

(2)

with boundary condition

$$P_m(\varepsilon | \varepsilon_0, l)|_{l=0} = \delta(\varepsilon - \varepsilon_0),$$

$\delta(\varepsilon - \varepsilon_0)$ being the Dirac $\delta$-function.

If the electrons incident on the light target have an energy distribution $P_0(\varepsilon)$, the energy distribution of electrons behind the bunch is the convolution of $P_0(\varepsilon)$ and $P_m(\varepsilon | \varepsilon_0, l)$:

$$P_c(\varepsilon | l) = \int_\varepsilon^\infty P_0(\varepsilon') P_m(\varepsilon | \varepsilon', l) d\varepsilon'$$

(3)

(Eq. (2) can be transformed into the equation for the distribution moments

$$\bar{\varepsilon}^k_m(\varepsilon_0, l) = \int \varepsilon^k P_m(\varepsilon | \varepsilon_0, l) d\varepsilon.$$}

The equation for $\bar{\varepsilon}^k_m(\varepsilon_0, l)$ is

$$\frac{\partial}{\partial l} \bar{\varepsilon}^k_m(\varepsilon_0, l) + \Sigma(\varepsilon_0) \bar{\varepsilon}^k_m(\varepsilon_0, l)$$

$$- \int_0^{\omega_{\text{max}}} \Sigma(\omega; \varepsilon_0) \bar{\varepsilon}^k_m(\varepsilon_0 - \omega, l) d\omega = 0.$$

(4)

The boundary condition for the moments is $\bar{\varepsilon}^k_m(\varepsilon_0, l)|_{l=0} = \varepsilon_0^k$.

The equation for the moments of the distribution $P_c(\varepsilon | l)$ describing transport of electrons with an incident energy spread can be obtained from Eq. (3):

$$\bar{\varepsilon}^k_c(l) = \int_0^\infty \varepsilon^k P_c(\varepsilon | l) d\varepsilon = \int_0^\infty P_0(\varepsilon') \bar{\varepsilon}^k_m(\varepsilon', l) d\varepsilon'.$$

(5)

It is seen that they are expressed in terms of the moments for the monoenergetic electrons and the incident energy distribution.

If the energy loss of an electron in one collision is small, the integro-differential equation for the moments can be transformed by the Taylor expansion of integrands:

$$\bar{\varepsilon}^k_m(\varepsilon_0 - \omega, l) \approx \varepsilon_0^k - \omega \frac{\partial}{\partial \varepsilon_0} \bar{\varepsilon}^k_m(\varepsilon_0, l)$$

$$+ \frac{1}{2} \omega^2 \frac{\partial^2}{\partial \varepsilon_0^2} \bar{\varepsilon}^k_m(\varepsilon_0, l).$$

This gives the partial differential equation:

$$\frac{\partial}{\partial l} \bar{\varepsilon}^k_m(\varepsilon_0, l) + \beta(\varepsilon_0) \frac{\partial}{\partial \varepsilon_0} \bar{\varepsilon}^k_m(\varepsilon_0, l)$$

$$- \frac{1}{2} \gamma(\varepsilon_0) \frac{\partial^2}{\partial \varepsilon_0^2} \bar{\varepsilon}^k_m(\varepsilon_0, l) = 0.$$

(6)

The quantities $\beta(\varepsilon_0)$, and $\gamma(\varepsilon_0)$ in (3) are the moments of the macroscopic differential cross-section:

$$\beta(\varepsilon_0) = \int_0^{\omega_{\text{max}}} \omega \Sigma(\omega; \varepsilon_0) d\omega,$$

$$\gamma(\varepsilon_0) = \int_0^{\omega_{\text{max}}} \omega^2 \Sigma(\omega; \varepsilon_0) d\omega.$$
Two first terms in Eq. (3) correspond to the continuous slowing down approximation, whereas the third term gives approximate description of the energy loss fluctuations.

The Eq. (3) written for \( k = 1, 2 \) can be transformed into the equation for the variance

\[
\sigma_m^2(\varepsilon_0, l) = \bar{\varepsilon}_m^2(\varepsilon_0, l) - \overline{\varepsilon_m^2}(\varepsilon_0, l).
\]

The equation has a form

\[
\frac{\partial}{\partial l} \sigma_m^2(\varepsilon_0, l) + \beta(\varepsilon_0) \frac{\partial}{\partial \varepsilon_0} \sigma_m^2(\varepsilon_0, l) - \frac{1}{2} \gamma(\varepsilon_0) \frac{\partial^2}{\partial \varepsilon_0^2} \sigma_m^2(\varepsilon_0, l) = \gamma(\varepsilon_0) \left( \frac{\partial}{\partial \varepsilon_0} \bar{\varepsilon}_m(\varepsilon_0, l) \right)^2.
\]

The Eq. (3) for the energy distribution moments in the case of the electron beam with an incident energy spread can be simplified if the distribution \( P_0(\varepsilon) \) is a symmetric function with maximum at a point \( \varepsilon_0 \) and small variance

\[
\sigma_0^2 = \int_{-\infty}^{\infty} (\varepsilon - \varepsilon_0)^2 P_0(\varepsilon) d\varepsilon.
\]

In this case the Taylor expansion of the function \( \bar{\varepsilon}_m(\varepsilon, l) \) in (3) makes it possible to express the moments of the distribution \( P_\varepsilon(\varepsilon|l) \) in terms of the moments \( \bar{\varepsilon}_m(\varepsilon, l) \) corresponding to monoenergetic electrons:

\[
\bar{\varepsilon}_m(\varepsilon, l) = \bar{\varepsilon}_m(\varepsilon_0, l) + \frac{\sigma_0^2}{2} \frac{\partial^2}{\partial \varepsilon_0^2} \bar{\varepsilon}_m(\varepsilon_0, l).
\]

The equations (3) written for \( k = 1, 2 \) can be easily transformed into the equation for the variance

\[
\sigma_m^2(l) = \bar{\varepsilon}_m^2(l) - \overline{\varepsilon_m^2}(l).
\]

The equation is

\[
\sigma_m^2(l) = \sigma_m^2(\varepsilon_0, l) + \frac{\sigma_0^2}{2} \left( \frac{\partial^2}{\partial \varepsilon_0^2} \sigma_m^2(\varepsilon_0, l) + 2 \left( \frac{\partial}{\partial \varepsilon_0} \bar{\varepsilon}_m(\varepsilon_0, l) \right)^2 \right).
\]

It describes the variance of the energy distribution for the electron beam with an incident variance \( \sigma_0 \) in terms of stochastic characteristics for monoenergetic electrons.

**III. COMPTON SCATTERING CROSS-SECTIONS AND INTERACTION COEFFICIENTS**

In this paper we restrict our consideration to the linear Compton scattering. The differential cross-section of this process for relativistic electrons is

\[
\frac{d\sigma(y; x)}{dy} = \frac{2\pi r_0^2}{x} \times \left( 1 - y + \frac{1}{1 - y} - \frac{4y}{x(1 - y)} + \frac{4y^2}{x^2(1 - y)^2} \right).
\]

where \( y = \frac{\omega}{\varepsilon_0} \) and \( r_0 = \frac{e^2}{mc^2} \) is the classical radius of electron [2].

In the energy region of our interest the invariant dimensionless parameter \( x \) is small \( (x \ll 1) \) and the interaction coefficients \( \beta(\varepsilon_0) \), and \( \gamma(\varepsilon_0) \) are described by the approximate formulas

\[
\beta(\varepsilon_0) \approx \frac{\Sigma_0}{2} x \varepsilon_0 x
\]

\[
\gamma(\varepsilon_0) \approx \frac{7}{20} \Sigma_0 x^2 \varepsilon_0^2 x^2,
\]

where \( \Sigma_0 = 2n_L \sigma_T \), \( \sigma_T = \frac{8}{3} \pi r_0^2 \) being the Thomson cross-section.

The quantities \( \beta(\varepsilon_0) \) and \( \gamma(\varepsilon_0) \) are the mean energy loss and the mean squared energy loss of an electron per unit path length.

**IV. SOLUTION OF EQUATIONS FOR MONOENERGETIC ELECTRONS**

The partial differential equations (3), (4) with the interaction coefficients (11), (12) describe the mean energy and the variance of the energy distribution for incidentally monoenergetic electrons passing a path length. The equations can be solved by the iteration method. In the first approximation, where the terms with the second derivatives are neglected, the solutions are

\[
\bar{\varepsilon}_m(\varepsilon_0, l) = \frac{\varepsilon_0}{1 + \frac{2}{\pi} \varepsilon_0 x}
\]

\[
\sigma_m^2(\varepsilon_0, l) = \frac{7}{20} \varepsilon_0^2 x^2 \pi^2 x^2
\]

where \( \varepsilon_0 = \sum_{l} \) is the mean number of collisions in the light target [3].

The mean value (13) is a monotonically decreasing function of variable \( l \). The Taylor expansions show that for small \( l (\pi x \ll 1) \)

\[
\bar{\varepsilon}_m(\varepsilon_0, l) \approx \varepsilon_0 \left( 1 - \frac{\pi x}{2} \right)
\]

and for large \( l (\pi x \gg 1) \)

\[
\bar{\varepsilon}_m(\varepsilon_0, l) \approx \frac{2\varepsilon_0}{\pi x} \left( 1 - \frac{2}{\pi x} \right).
\]

Similarly, it can be shown that the variance (14) grows with \( l \) for thin targets:

\[
\sigma_m^2(\varepsilon_0, l) \approx \frac{7}{20} \pi^2 x^2 \varepsilon_0^2 x^2 \left( 1 - 2 \pi x \right)
\]
and decreases for large $l$:

$$\sigma_m^2(\varepsilon_0, l) \approx \frac{28}{5} \frac{\varepsilon_0^2}{l^2 x^2 (1 - \frac{8}{3} \frac{\varepsilon_0}{l})}. $$

It follows from (13) that the variance $\sigma_m^2(\varepsilon_0, l)$ has a maximum at the point, where

$$\frac{\partial}{\partial \varepsilon_0} \sigma_m^2(\varepsilon_0, l) = 0,$$

or

$$\frac{\sigma_m(\varepsilon_0, l)}{\varepsilon_0} = \frac{3}{4}.$$

The energy spread damping for large $l$ is due to the energy loss $\beta \varepsilon$ per unit path length is proportional to $\varepsilon^2$, therefore, higher energy electrons in a bunch lose more energy than lower energy electrons [8, 9].

Using Eqs. (14) and (13) one can derive the formula

$$\sigma_m^2(\varepsilon_0, l) = \frac{7}{10} \frac{\varepsilon_0}{\varepsilon_0} \left(1 - \frac{\varepsilon_0}{\varepsilon_0}\right),$$

which was earlier obtained in [8].

V. SOLUTION OF EQUATIONS FOR ELECTRON BEAM WITH INCIDENT ENERGY SPREAD

Substitution of Eqs. (13), (14) in Eqs. (8) and (9) makes it possible to calculate the mean energy and the energy distribution for the beam with the incident energy spread $\sigma_0$.

These calculations show that the mean value $\varepsilon_0(l)$ is decreased monotonically with $l$:

$$\varepsilon_0(l) \approx \varepsilon_0 \left(1 - \frac{20}{3} (1 + \frac{\sigma_0^2}{\varepsilon_0^2})\right)$$

for small $l$ and

$$\varepsilon_0(l) \approx \frac{2\varepsilon_0}{\pi x} \left(1 - \frac{2}{\pi x} (1 + \frac{\sigma_0^2}{\varepsilon_0^2})\right)$$

for large $l$.

But the dependence of variance $\sigma_0^2(l)$ on $l$ is determined by the incident energy spread. The Taylor expansion of Eq. (4) in powers of $l$ gives

$$\sigma_0^2(\varepsilon_0, l) \approx \sigma_0^2 + \frac{7}{20} \varepsilon_0^2 \pi x^2 \left(1 - \frac{8\sigma_0^2}{\varepsilon_0^2} (1 - \frac{21x}{20})\right),$$

whereas for large $l$ it is decreased with $l$:

$$\sigma_0^2(\varepsilon_0, l) \approx \frac{28}{5} \frac{\varepsilon_0^2}{\pi x^2} \left(1 - \frac{8}{3} \frac{\varepsilon_0}{l} (1 + \frac{\sigma_0^2}{\varepsilon_0^2} (1 - \frac{10}{1} l))\right).$$

It means that the variance $\sigma_0^2(l)$ has a maximum at some $l$ if $\sigma_0^2 \leq \sigma_{max}^2$, where

$$\sigma_{max}^2 = \frac{7}{40} \frac{\varepsilon_0^2}{x^2} \frac{1}{1 - \frac{21}{20} \frac{l}{x}}. $$

For $\sigma_0 = \sigma_{max}$ this maximum is at the point $l = 0$. If $\sigma_0 > \sigma_{max}$ the variance $\sigma_0^2(l)$ is a decreasing function of $l$.

VI. MULTIPLE CROSSING OF LASER BUNCH

It was pointed out in [3, 8] that effective radiative cooling of an electron beam can be realized in LESR where electrons repeatedly interact with an intense laser pulses. Eq. (9) can be used to study the variation of the energy spread for electrons multiply crossing the light target. If the lost energy is restored by an rf accelerating system after each turn, the variance of energy distribution after n-th turn can be calculated using Eq. (9) with $\sigma_0^2$ replaced by the variance after (n-1)-th turn:

$$\sigma^2(n) = \sigma_m^2(\varepsilon_0, l) + \eta \sigma^2(n-1),$$

where

$$\eta = \frac{1}{2} \left(\frac{\partial^2}{\partial \varepsilon_0^2} \sigma_m^2(\varepsilon_0, l) + 2 \frac{\partial}{\partial \varepsilon_0} \sigma_m^2(\varepsilon_0, l) \right)^2. $$

It follows from (16) that for $|\eta| \leq 1$

$$\sigma^2(\infty) = \sigma_m^2(\varepsilon_0, l) (1 + \eta + \eta^2 + \eta^3 + ... ) = \frac{\sigma_m^2(\varepsilon_0, l)}{1 - \eta}. $$

Let us point out that the multiple crossing of the laser bunch leads to the energy spread damping if the incident energy spread $\sigma_0^2 > \sigma^2(\infty)$. In the opposite case ($\sigma_0^2 < \sigma^2(\infty)$) the energy spread is increased.

Limit value of energy spread $\sigma^2(\infty)$ depends on the thickness of laser bunch especially for high energy electrons. For small $l$

$$\sigma^2(\infty) \approx \sigma_{max}^2 \left(1 - \frac{\pi}{2} x \frac{15 + 28\eta}{20 - 21x}\right),$$

whereas for large $l$

$$\sigma^2(\infty) \to \frac{28}{5} \frac{\varepsilon_0^2}{\pi x^2}. $$

It is easy to see that for small $x$

$$\sigma(\infty) \approx \frac{\sigma_{max}^2}{\varepsilon_0} \approx \sqrt{\frac{l}{40}} = \sqrt{\frac{\eta}{10}} \frac{\lambda_0}{\lambda_L},$$

where $\lambda_L$ is the laser radiation wavelength and $\lambda_e = \frac{h}{mc}$ is the Compton wavelength of electron. This result coincides with that one given in [10]. But our approach allows to estimate the number of turns in a LESR to reach the limit value of the energy spread.

VII. NUMERICAL RESULTS

Numerical calculation of the mean energy and the variance of the energy distribution were made for 100 MeV electrons interacting with 1.24 eV laser photons and for 5 GeV electrons interacting with 2.48 eV photons. Eqs.
\( n \) and \( \xi \) were used to calculate the relative energy spread \( \frac{\sigma_{e}(l)}{\bar{\epsilon}} \). The results are given in Fig. 1 for 3 values of the incident energy spread. They agree with the data obtained using Eq. (15) of [1].

It is seen from the figure that the relative energy spread of the electron beam as a function of the collisions number \( n \) is a decreasing function if the incident energy spread is large enough. But for the beams with small incident spread the ”heating” of the beam takes place instead of ”cooling” for small \( l \).

The dependence of the energy distribution variance on the number of turns in LESR is shown in Fig. 2 for two incident energy spreads. The calculations were made for LESR suggested for laser beam cooling in [10]. In this case \( \omega_0 = 1.24 \text{ eV}, \; \varepsilon_0 = 100 \text{ MeV} \), the mean energy of scattered photon equals \( \bar{x} = \frac{1}{2} \varepsilon_0 x = 95 \text{ keV} \), the average ring radius equals 1 m, and the energy loss per turn \( \Delta \omega = 25 \text{ keV} \). The mean number of collisions in laser bunch \( \pi = \frac{\Delta \omega}{\bar{x}} = 0.27 \). It is seen from the figure that the asymptotic value of the energy spread is established in \( \sim 5 \cdot 10^3 \) turns and it takes \( \sim 100 \mu\text{sec} \).

It is seen from Fig. 2 that the energy spread of electrons multiply crossing the light target is decreased or increased depending on its incident value.

The limit value of the relative energy spread \( \sigma(\infty)/\varepsilon_0 \) as a function of the light target thickness is given in Fig. 3. It is seen from the figure that the variance of the energy distribution is decreased with the target thickness increasing.

**VIII. MONTE CARLO SIMULATION**

It is supposed in our Monte Carlo simulation of the back Compton scattering that the incident energy distribution of electrons is Gaussian with given variance \( \sigma^2 \). The number of electron collisions with laser photons is supposed to be random. It is selected from the Poisson distribution with fixed \( \pi \). The simulation of individual collisions is carried out in the electron rest frame using the Klein and Nishina formula with the Lorentz transformation to the lab system. In the same way as in analytical calculations above we neglected the angular deflection of electrons but accounted for the energy decreasing after each collision.

Fig. 4 shows the energy distributions of 5 GeV electrons for several values of the light target thickness. It should be pointed out the discontinuous of the spectra for small \( \pi \) due to the single scattered electrons. It is seen from the figure that the energy spread increases with the target thickness increasing for small \( \pi \) and decreases for such \( \pi \), which are greater than \( \sigma_{\max}^2 \) determined by Eq. (13).

Fig. 5 and 6 shows the electron energy distributions in LESR for several number of turns.

All results are obtained with statistics more than \( 10^6 \) trajectories.

It should be pointed out that the analytical formulas obtained in the continuous slowing down approximation with approximate consideration of the energy loss fluctuations are inapplicable for highly relativistic electrons if the mean energy loss in one collision is comparable to \( \varepsilon_0 \). The Monte Carlo method has to be used in this case.

Fig. 7 shows the relative energy spread as a function of the light target thickness calculated in continuous slowing down approximation and by the Monte Carlo method for 250 GeV electrons. Fig. 8 shows the energy spectra of these electrons. Notice that the Monte Carlo technique allows to get the results with taking into account the energy dependence of the interaction cross-sections and the lateral distribution of photons in the laser flash.

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[1] Proc. Int. Symp. on New Visions of Laser-Beam Interaction, Nucl. Instr. Meth., No. 1 (2000).
[2] I. Ginzburg, G. Kotkin, V. Serbo, and V. Telnov, Nucl. Instr. Meth. 285 (1983) 47.
[3] V. Telnov, Nucl. Instr. Meth. A 355 (1995) 3.
[4] A. M. Kolchuzhin, V. V. Uchaikin, Introduction to the Theory of Particles Penetration through a Matter, (book in Russian), Moscow, Atomizdat, 1978.
[5] A. Kolchuzhin, A. Potylitsyn, A. Bogdanov, I. Tropin, Physics Let., A 264 (1999) 202.
[6] A. Kolchuzhin, A. Potylitsyn, S. Strokov, V. Ababiy.
[7] W. Feller, An Introduction to Probability Theory and its Application, Vols 12, Wiley, New York, 1971.
[8] V. Telnov, Phys. Rev. Lett., 78 (1997) 4757.
[9] Z. Huang, and R. Ruth, Phys. Rev. Lett., 80 (1998) 976.
[10] Z. Huang, R. Ruth, Proc. 1999 Particle Acc. Conf, N.Y., 1999, p.262.
[11] V. Telnov. Quantum Aspects of Beam Physics, Monterey, California, USA, 1998, p.173.
FIG. 1: The relative energy spread $\sigma_c(l)/\varepsilon_c(l)$ as a function of the target thickness for various initial energy spread.

FIG. 2: The relative energy spread as a function of the number of turns in the LSR, $\varepsilon_0 = 100\, MeV$, $\omega_0 = 1.24\, eV$, and $\pi = 0.27$.
FIG. 3: The limit value of the relative energy spread $\sigma(\infty)/\varepsilon_0$ as a function of the mean collisions number.

FIG. 4: The energy distributions of initially monoenergetic electrons for $\bar{n}=1, 4, 15$ and 40.
FIG. 5: Evolution of the energy distributions $P(\varepsilon)$ for electrons with $\sigma_0 < \sigma(\infty)$ multiply crossing the light target. Solid line is the Gaussian distribution with the variance $\sigma(\infty)$. Number of turns is shown near the curves.

FIG. 6: Evolution of the energy distributions $P(\varepsilon)$ for electrons with $\sigma_0 > \sigma(\infty)$ multiply crossing the light target. Solid line is the Gaussian distribution with the variance $\sigma(\infty)$. Number of turns is shown near the curves.
FIG. 7: The relative energy spread $\sigma_e(l)/\bar{e}_e(l)$ as a function of the target thickness. Solid line - Eqs. (8), (9), points - Monte Carlo simulation.

FIG. 8: The energy distributions of initially monoenergetic electrons for $\bar{n} = 0.1, 0.5, 1$ and 2.