A phonon laser in ultra-cold matter

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Abstract – We show the possible excitation of a phonon laser instability in an ultra-cold atomic gas confined in a magneto-optical trap. Such an effect results from a negative Landau damping of the collective density perturbations in the gas, leading to the coherent emission of phonons. This laser instability can be driven by a blue-detuned laser superimposed to the usual red-detuning laser beams which usually provide the cooling mechanism. Threshold conditions, instability growth rates and saturation levels are derived. This work generalizes, on theoretical grounds, the recent results obtained with a single-ion phonon laser, to an ultra-cold atomic gas, where real phonons can be excited. Future phonon lasers could thus adequately be called phasers.

The last decade the concept of phonon laser has been studied in several different systems, e.g. quantum wells [1], ions [2], nanomechanics [3], and nanomagnets [4]. Furthermore, very recently the possibility to excite a Doppler instability and create a phonon laser with a single trapped atom or ion [5,6] has been discussed. Coherent emission of sound waves has also been demonstrated to be a reality in microcavity systems [7] and in doped superlattices complexes [8]. Here we show that it is possible to generalized the single-trapped-atom configuration to the case of a large ensemble of atoms, thus enabling collective phonon excitations. In particular, the excitation of a phonon laser instability in an ultra-cold atomic gas confined in a magneto-optical trap (MOT) is described. Such an effect results from a negative Landau damping of the collective density perturbations in the gas, leading to the coherent emission of phonons. In contrast with the single-ion case, here the phonon frequency is not determined by the single-atom oscillation frequency due to the parabolic confining potential, but by the boundary conditions of the internal oscillations of the gas. It therefore corresponds to a collective oscillation, instead of a single-particle effect. In that respect, it is closer to the paradigm of an optical laser, where the photon modes correspond to internal vibrations of the optical cavity, thus making the name phaser a natural choice for such collective modes. The acoustic oscillations can also, in principle, be coupled to the outside world, by mechanical or electromagnetic means, thus providing a source of coherent acoustic radiation.

Collective processes in ultra-cold gas clouds have been considered by several authors in recent years. These processes result from the existence of an effective atomic charge in the gas cloud [9], which create collective forces of the Coulomb type [10]. It was also shown that plasma-acoustic waves can be excited in the gas, which are similar to the usual sound waves but with a low-frequency cut-off [11]. These waves can provide the physical support of our laser instability. Nonlinear collective instabilities have also recently been identified [12].

Like in the case of a single trapped ion, the collective laser instability can be driven by a blue-detuned laser, superimposed to the usual red-detuning laser beams which usually provide the cooling mechanism. But in contrast with the single-ion phonon laser, real phonons are excited in the medium and can eventually be coupled to an external system, just like photons produced in current optical laser cavities. The dynamical behavior of the ultra-cold atomic gas will be described by a wave kinetic equation, where the atom recoil effects due to the emission and absorption of both photons and plasmons are retained [13]. This equation corresponds to a straightforward generalization of well-known quantum kinetic equations [14,15], which include a collective (or mean-field) potential. The instability growth rates are derived. Saturation of the phonon laser instability is
also described with the help of a quasi-linear version of the wave kinetic equation. Threshold conditions for the occurrence of the phonon laser instability are also established.

This work therefore extends a recently published concept of single-ion phonon laser [6] into the collective regime, by considering an ensemble of ultra-cold atoms confined in a MOT. In contrast to previous results, real phonons can be excited and eventually coupled to external environments.

The starting point for our analysis is provided by the quantum wave kinetic equation for the ultra-cold gas [13–15]. This equation describes the space and time evolution of the Wigner function $W(r, v, t)$ associated with the kinetic state of the atoms in the cloud, where $r$ and $v$ represent the position and the velocity, respectively. It is well known that this quantity provides the quasi-probability distribution since it can take negative values. Here we retain the exact quantum description of the system, which means that all the recoil effects associated with the atom interaction with electromagnetic and acoustic fields are taken into account. The evolution equation for the quantum quasi-probability distribution is

$$\frac{dW}{dt} = -if(W) + \tilde{g} \left[ W(k_b)^{-} \right] - W(k_b)^{+} \right]$$

$$+ \sum_{n=1}^{6} g_n \left[ W(k_n)^{-} - W(k_n)^{+} \right],$$

where the total time derivative $dW/dt \equiv (\partial/\partial t + v \cdot \nabla)$, $k_b$ represents the blue-detuned pump laser wave vector and $k_n$ labels the wave vectors of the red-detuned laser beams. The radiation coupling coefficient, which is the mediator of photon recoil, is given by $\tilde{g} = -(E_k/\hbar) \text{Im}(d_{21} \bar{\rho}_{21})$ (the same definition holds for the six red-detuned laser beams $g_n$, where $d_{21}$ is the dipole matrix element of the radiative transition between the internal atomic states 1 and 2, and $\bar{\rho}_{21}$ is the density matrix element, assumed nearly at equilibrium with the radiation field). This equation is derived following the well-known Wigner-Moyal procedure and it is formally similar to the kinetic equations considered in refs. [13–15]. It states that the changes in the distribution $W(r, v, t)$ are due to three different factors. The first is the external force term, associated with the collective mean-field potential as defined by

$$f(W) = \frac{1}{\hbar} \int V(k) \left[ W(k)^{-} - W(k)^{+} \right] e^{i k \cdot r} \frac{dk}{(2\pi)^{3}}.$$  \hspace{1cm} (2)

Here and above, $W(k)^{(\pm)} \equiv W(v \pm \hbar k/2M)$, and $M$ is the mass of the atom. We have used a spatial Fourier decomposition in the integral in (2). This collective force describes the atom-atom interactions inside the gas and is determined by the potential $V$ (see footnote $^1$), formally analogous to an electrostatic potential, governed by the Poisson equation

$$\nabla^2 V = -Q n \equiv -Q \int W(r, v, t) dv.$$  \hspace{1cm} (3)

Here $Q$ is determined by the effective charge of the atoms first discovered by Walker et al. [9], and results from the exchange of photons between nearby atoms, as discussed in detail elsewhere [10,11]. It depends on the intensity of the incident cooling and pumping laser beams. The third term in eq. (1) is due to the red-detuned cooling laser beams with electric fields of the form $E(r, t) = \sum_{n} E_n \exp(i k_n \cdot r - i \omega_n t)$, corresponding to the same frequency $\omega_n$, which is nearly equal but slightly lower than the frequency of the atomic transition used for laser cooling, and wave numbers $|k_n| \approx \omega_n/c$. The coupling coefficients $g_n$ are associated with the red-detuned laser beams. Finally, the second source term in eq. (1), is due to the blue-detuned laser pump field, $E(r, t) = \sum_{n} E_n \exp(i k_b \cdot r - i \omega_b t)$, with frequency $\omega_b > \omega_n$. This pump field will produce a population inversion in the velocity state, leading to an instability of the acoustic waves. We remark that these acoustic waves have some special features which make them clearly distinct from common acoustic oscillations. This will become apparent later on.

It is obvious from eq. (1) that the first term couples the atomic states with the spectrum of possible low-frequency oscillations of the collective potential $V(k)$, with $|k| \ll |k_b| \approx |k_n|$, and the second term couples the atoms with the blue-detuned pumping field. This means that the recoil energy is much less that the energy of the blue-detuned laser mode.

In order to simplify the above description we consider an one-dimensional model, by assuming $v = u(k/k) + v_{\perp}$, and integrate over the perpendicular velocities with respect to the pump laser beam. In the low-frequency spectrum of the collective potential $V(k)$, we retain spectral components propagating in the same direction, $k \parallel k_b$. We define the quasi-distribution for the parallel velocities as $G(u) = \int W(u, v_{\perp}) dv_{\perp}$. The resulting evolution equation is

$$\frac{dG}{dt} = -if(G) + \tilde{g} \left[ G(k_b)^{-} - G(k_b)^{+} \right] + \sum_{n=1}^{6} \nu_n [G_0 - G],$$

where $\nu_n = g_n u_n \cdot e_b$ is a phenomenological viscosity coefficient and $u_n$ and $e_b$ represent the directions of the cooling (red-detuned) and pump (blue-detuned) laser beams. This quantity depends on the angle between the pumping and the cooling beams, and it coincides with $g_n$ when, for a given $n$, we have $k_n \parallel k_b$. For order of

$^1$Under common experimental conditions, the collective (or many-body) interactions dominate over the two-body collisions. This leads to the collective Coulomb-like force mentioned above, and stays valid as long as we have many atoms inside a Debye sphere.
magnitude estimates, we can use the average viscosity coefficient \( \nu = \langle \nu_s \rangle = -(E_0/\hbar) \text{Im}(d_{21} \tilde{\rho}_{21}) \), where \( E_0 \) is the average magnitude (computed over the six cooling beams) of the incident electric field, as defined in the absence of the pumping laser beam [10,11]. The distribution \( G_0(u) \) is that of the laser cooled gas, as obtained in the absence of the pumping laser. This last term in (4) plays the role of spontaneous emission by depopulating the high-velocity states driven by \( g \), as shown below. Because the laser pump beam is blue shifted, it will lead to negative viscosity, thus pumping the atom states to the upper \( G(k)_{(+)} \) level.

Three physical processes are assumed to occur in parallel, as described in fig. 1. We can see a very strong resemblance with the usual three-level laser model. First, the red-detuned laser beams, will create a very low-temperature quasi-distribution \( G(u) \), which corresponds to our ground state. Second, the blue-detuned pump laser field will excite high-velocity atomic states around the velocity \( u + \hbar k_b / M \), due to the absorption of photon momentum, therefore populating the upper level for the phonon laser transition. This high-velocity distribution corresponds to a kind of particle beam, thus creating a population inversion in the centre-of-mass states. It will then lead to a negative Landau damping of acoustic-like oscillations with frequency \( \omega \) and wave number \( k \), which will result in the coherent emission of phonons. Due to phonon emission, a third and intermediate-velocity state will be populated. The temporal evolution of the phonon field will be dictated by a complex frequency \( \omega = \omega_r + i(\gamma_k - \nu/2) \). We assume that \( \dot{G}(u) = \dot{G}(u) + \dot{G}(u) \), where \( \dot{G}(u) \) is the equilibrium distribution, and \( \dot{G}(u) \) is the perturbation, which is assumed to evolve as \( \exp(\mathbf{k} \cdot \mathbf{r} - i \omega t) \). At this point, \( \mathbf{k} \) is a generic phonon wave vector, but its precise choice is determined by the experimental conditions, as explained below. By linearizing eqs. (1) and (3), it is possible to show that the instability growth rates are determined by [13]

\[
\gamma_k = -\frac{\pi Q \omega_r}{2 \hbar k} (\dot{G}(+) - \dot{G}(-))_r,
\]

where the population difference is calculated for the resonant parallel velocity \( u_r = k/\omega_r \), and the acoustic oscillations obey the dispersion relation

\[
\omega_r^2 = \omega_p^2 + k^2 u_s^2 + \frac{\hbar^2 k^4}{4M^2}.
\]

This dispersion relation describes a modified form of acoustic oscillations. The first term on the right-hand side of eq. (6) corresponds to a plasma frequency cut-off, the second term is the usual acoustic dispersion term, and the last term is a purely quantum correction due to the atom recoil. The plasma frequency \( \omega_p \) and the ion sound speed \( u_s \) are determined by

\[
\omega_p = \sqrt{\frac{Qn_0}{M}}, \quad u_s = \frac{1}{n_0} \int G(u)u^2 du,
\]

where \( n_0 \) is the unperturbed gas density. By comparing this definition of the plasma frequency of the neutral gas with the usual definition valid for an electron-ion plasma, we conclude that the effective charge of the neutral atoms inside the ultra-cold gas is \( \sqrt{eQ} \), as first considered by [9]. It is clear that we will have an acoustic-wave growth if the inverse Landau damping is positive \( \gamma_k > 0 \), and if it is large enough to compensate for the wave losses, i.e. \( \gamma_k > \nu \). We can then write the threshold condition as

\[
|G(+) - G(-)|_r > \frac{2\nu \hbar k^3}{\pi Q \omega_r}.
\]

It should be noticed here that such an instability remains in the classical limit, i.e., even when the recoil temperature \( T_{rec} = h^2 k_b^2 / 2M \) is much smaller than the Doppler cooling temperature \( T_D = \hbar \gamma \). In this limit, we can develop \( G \) around \( G(u) \), as \( G \simeq G(u) \pm (\hbar k / M)(\partial G / \partial u) \). The threshold condition now reads

\[
\frac{\partial G}{\partial u} > \frac{2M k^2}{\pi Q \omega_r}.
\]

The population inversion is now represented by the derivative of the atom distribution at the resonant velocity \( u_r = \omega_r / k \). The resemblance with a three-level laser will be somewhat lost, but the physical principle stays the same.

Next, we discuss the instability saturation. The acoustic-wave growth will tend to decrease the population difference between the upper and lower kinetic energy levels. This process is approximately described by the quasi-linear equations [13]

\[
\frac{d\dot{G}}{dt} = \frac{|V(\mathbf{k}, t)|^2}{\hbar^2} \left[ \dot{G}(u - \hbar k / M) - \dot{G}(u + \hbar k / M) \right].
\]

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The equidistance of the atom velocity levels with respect to the resonant velocity $u$ is a clear signature of the phonon recoil effect, associated with the emission and absorption of resonant phonons by the atoms [13]. In order to be specific, we assume that the mean distribution $G(u)$ is divided into three distinct regions of spectral interest, centered around $u_0 = 0$, $u_1 = h(k_0 - k)/M$ and $u_2 = h k_0 / M$. We have $G(u) = G_0(u) + G_1(u) + G_2(u)$. For the extreme case of $G_j(u) = G_{j0} \delta(u - u_j)$, with $j = 0, 1, 2$, the above quasi-linear equation would be reduced to simple balance equations relating the three-velocity-level populations. The resonant atom velocity defined in eq. (5) for the inverse Landau damping will then be exactly equidistant from $u_1$ and $u_2$. Integrating over the parallel velocity, and defining the population difference $\Delta G = \int [G_2(u) - G_1(u)] du$, for the atoms centered around the upper and intermediate parallel velocities $u_1$ and $u_2$, we can easily establish the evolution equation

$$\frac{d}{dt} \Delta G = -\frac{V(k, \omega)}{\hbar^2} \Delta G.$$  \hspace{1cm} (11)

On the other hand, the inverse Landau damping of the phonon mode will also change and, as a consequence, the phonon square amplitude (energy) will evolve according to

$$\frac{d}{dt} |V_k(t)|^2 = \left[ \frac{\pi Q \omega_r}{\hbar^3} \delta G - \nu \right] |V_k(t)|^2,$$  \hspace{1cm} (12)

where $\gamma_k$ is the growth rate defined above. Introducing a new time variable, $\tau$, and a new viscosity coefficient $\tilde{\nu}$, such that $\tau = (\hbar k^3 / \pi Q \omega_r) \Delta G(0)t$, and $\tilde{\nu} = (\tau / t) \nu$, we can rewrite these two equations as

$$\frac{dz}{d\tau} = -a z y, \quad \frac{dy}{d\tau} = (z - \tilde{\nu}) y,$$  \hspace{1cm} (13)

with $z \equiv \Delta G / \Delta G(0)$, $y \equiv |V_k|^2$ and $a = (\pi Q \omega_r / \hbar^3 k^3)$. These coupled evolution equations clearly exhibit the threshold condition $z(0) > 2\nu$, already stated in eq. (8), and saturation for increasing values of time. This is illustrated in fig. 2. We can see that the population difference decreases along with time, due to the increase of the phonon coupling between the two velocity states around $u_1$ and $u_2$. As a consequence, the growth rate slows down, leading asymptotically to saturation. Before a technical discussion of our results, we would like to make the following points: Firstly, it is important to note that the phonons in our ultra-cold gas differ in several ways from classical monochromatic sound in a gas, as for example can be seen clearly from the dispersion relation (6). Secondly, the phonon generation mechanism described here is in direct analogy with the standard laser mechanism. Although one can imagine other means for producing coherent sound waves in an ultra-cold gas, the direct resemblance to the laser is striking.

In our discussion we have only considered a single-phonon mode, corresponding to a given wave number $k$. It is important to discuss the selection mechanisms identifying such a mode inside the phonon spectrum. Several different processes can lead to mode selection. First, we have spontaneous selection of the mode with the largest growth rate. Due to the exponential growth, such a mode can easily detach from the background noise. However, a more effective mechanism is due to the finite size of the gas cloud, which can be confined in a MOT. This leads to a discretization of the phonon mode spectrum, as recently discussed by us [11], and the resulting discrete phonon modes with a well-defined internal structure are called the Tonks-Dattner resonances, in analogy with confined plasma systems, and are related with the finite-size effect of the MOT. Such modes are quantized inside the cloud and will work as a spherical acoustic cavity. This is valid in the limit (already observed experimentally) where the density is almost constant, i.e., for MOTs with an average number of atoms (typically, $N \sim 10^9$, see ref. [16]). The dominant phonon mode in a MOT will be due to the Tonks-Dattner resonance mode with the largest inverse Landau damping. Recent experimental work suggests that such modes can be spontaneously excited in the absence of any pump laser beam [12]. Finally, these modes can be excited by an external source, for example due to amplitude modulation of one of the incident laser beams, and the phonon laser system will operate as an amplifier. Such a phonon laser system, operating as an oscillator or as an amplifier, could adequately be called a phaser (instead of laser, as proposed by others). Few words about the experimental observation of the present phenomenon are in order. The observation of a phonon laser in MOTs could be envisaged by measuring the spectrum of the density fluctuations. If a coherent mode amplification occurs, then a strong peak around a certain mode should appear in the FFT signal. This could be done with both far-field fluorescence and absorption imaging techniques in regular MOTs.
In conclusion, we have shown that a phonon laser instability can be excited in the ultra-cold atomic cloud confined in a MOT, and pumped by a blue-detuned laser beam. Threshold conditions, linear growth rates and nonlinear saturation have been established. We have used a quantum kinetic description, which retains the atom recoil effects associated with the emission and absorption of both photons and phonons. This work generalizes for the first time (to the best of our knowledge) recent proposals for a single-atom phonon laser [5,6] to a large ensemble of atoms, where long-range interactions are present. The present phonon laser configuration is able to produce real phonons in the gas. Due to the acoustic cavity, the phonon modes are determined by the boundary conditions of the gas, in contrast to the single-atom oscillation frequency studied in previous works.

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