Stability for a kind of networked control systems with time delays and uncertainties

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Abstract. In this paper, we studied stability for a class of Networked Control Systems (NCSs) with time-delays and uncertainties. At the beginning, the concepts of Linear Matrix Inequality (LMI) and Lyapunov stability are given by definitions. Then the article guaranteed the closed-loop function and the controller gain $K$ is obtained by solving matrix inequalities. Finally, numerical examples are employed to verify the efficiency of the proposed methods.

1. Introduction
Networked Control Systems (NCSs) [1-5] is the combine of traditional control systems with emerging network so that all of the information can transfer through the systems quickly from controller to actuator by network and advanced sensor technology. NCSs can make systems in very long distance to be controlled quickly. But the introduction of the network also will bring the problems such as time-delays, packet loss, packet sequence error and other issues and coupled with the uncertainty in the system modeling, outside interference and other issues, making the analysis and synthesis of such combined systems becomes extremely difficult.

J Yan and Y Xia [6] concerned with the quantized stabilization of linear discrete systems with packet dropout. Based on the zoom strategy and Lyapunov theory, for a given packet dropout rate, a sufficient condition is given for closed-loop system to be mean square stable. M Li and Y Chen [7] studies the tracking control of dc permanent magnet motor (DC-PMM) in networked control systems (NCSs) with random delay and packet dropouts in feedback channel (sensor to controller) and the feedforward channel (controller to actuator). A sliding mode predictive tracking control algorithm based on the theory of pseudo partial derivative (PPD) is designed to compensate random delay and packet dropouts in process of motor control. H Ge, D Yue, D Song and Y Yang [8] studied the design of double closed-loop security control structure. This structure is divided into two parts: sensor-controller-actuator loop completes the transmission of control signal output and measurement signal feedback. During the transmission process, the effects of uncertainties, perturbations, faults and attacks are coupled. Further, controller-detector loop is adopted to detect the faults and attacks due to actual effect on NCSs. W Zhang, MS Branicky and SM Phillips [9] summarize the fundamental issues in NCSs and examine them with different underlying network-scheduling protocols. Then present NCS models with network-induced delay and analyze their stability using stability regions and a hybrid systems technique. Following that, discussed methods to compensate network-induced delay and present experimental results over a physical network. and modeled NCSs with packet dropout and multiple-packet transmission as asynchronous dynamical systems and analyze their stability. D Yue, QL Han and C Peng [10] concerned with the controller design of networked control systems (NCS). A new model of the NCSs is provided under consideration of both the network-induced delay and the data packet dropout in the transmission. In terms of the given model, a controller design method is proposed based on a delay-dependent approach. The feedback gain of a memoryless controller and the
maximum allowable value of the network-induced delay can be derived by solving a set of linear
matrix inequalities. H Lin, H Su, Z Shu and Y Xu [11] investigate the optimal estimation problem in
lossy networked control systems where the control packets are randomly dropped without
acknowledgment to the estimator. Most existing results for this setup are concerned with the design of
controller, while the optimal estimation and its performance evaluation have been rarely treated. Show
that unlike many other cases such as intermittent observations or TCP-like systems, the system state
follows a Gaussian mixture distribution with exponentially increasing terms, which leads to a
Gaussian sum filter-based optimal estimation. Develop an auxiliary estimator method to establish
necessary and sufficient conditions for the stability of the mean estimation error covariance matrices.
It is revealed that the stability is independent of the packet loss rate, and is not affected by the lack of
acknowledgment.

In this paper, $R$, $R^+$ and $Z^+$ denote the field of real numbers, the set of non-negative real, and the
set of non-negative integers. The notation $P > 0$ means that $P$ is a real symmetric and positive
definite; the superscript “$T$” stands for matrix transposition; $R^n$ denotes the n-dimensional Euclidean
space. $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote the minimum and maximum eigenvalues of matrix $P$, respectively.

2. Problem Formulation
In this paper, we focus on the NCSs with time-delays and uncertainties.

Consider a class of the NCSs described by

$$\dot{x}(t) = (A + \Delta A)x(t - \tau) + f(x, t) + d_1(t)$$

(1)

We can have

$$\dot{\hat{x}}(t) = (A + \Delta A)\hat{x}(t - \tau) + f(\hat{x}, t) + d_2(t) + Bu$$

(2)

$$u = K(\hat{x} + x)$$

(3)

Where $x(t - \tau) \in R^n$ is the state, $K \in R^n$ is the controlled output. $\tau$ is time-delays of
Networked Control Systems and $\Delta A$ is unknown matrice representing time-varying parametric
uncertainties.

$d(t)$ is the outside disturbance. $f(t)$ is the function which satisfies Lipschitz condition:

$$\|f(x, t) - f(y, t)\| \leq L\|x - y\|$$

(4)

Assume $e = \hat{x} + x$

with (1)–(3) we can have

$$e = (A + BK)e + \Delta A(t)e + f(\hat{x}, t) + f(x, t) + d_2(t) + d_1(t) + \Delta A(t)x(t - \tau)$$

(5)

So we can have

$$J = \int_0^\tau (e^T Qe + u^T Ru)dt, \|e\| \geq \frac{M_2}{\sqrt{r}}$$

(6)

Where $Q$, $R$ is positive definite matrix,$r$ is positive constants which respect to

$$M_2^2 = d^2 + \|E_1 - E_2\|^2 + 4p^2M_1^2.$$
Definition 1. Given matrix $D, E, F$, when $F^T(t)F(t) \leq I$, we can have $\dot{V} \leq -\varepsilon V$.

3. Stability Of Networked Control Systems

Theorem 1. Consider system (1) and (2), if matrix $A, B$ are controllability matrix, assume the following condition:

$$
P(A + BK) + (A + BK)^T P + 3P^2 + (L^2 + r)I + E_2^T E_2 + 2PDD^T P + Q + K^T PK < 0
$$

(7)

Which $P$ is positive definite matrix, the state controller $u(t) = Ke(t)$ can make system stable and disturbance respect to $r_m$

Proof.

Assume lyapunov function

$$
V = e^T Pe
$$

(8)

We can have

$$
V = e^T [P(A + BK) + (A + BK)^T P]e + 2e^T P [f(x, t) + f(x, t)] + 2e^T P [d_2(t) + d_1(t)] + 2e^T P [\Delta A(t) - \Delta A(t)]
$$

(9)

With the Theorem 1 we can have

$$
2e^T P \Delta A(t) e = 2e^T P Df(t) E_2 e \leq e^T PDD^T Pe + e^T E_2^T E_2 e
$$

(10)

$$
2e^T P [f(x, t) + f(x, t)] \leq 2e^T P^2 e + 2e^T P d_2(t) + d_1(t) \leq 4e^T P^2 e + 4M_2^2
$$

(11)

$$
2e^T P d_2(t) \leq e^T P^2 e + d_2^2
$$

(12)

$$
2e^T P \Delta A(t) \leq e^T PDD^T Pe + x^T [E_1 - E_2]^2 [E_1 - E_2] x \leq e^T PDD^T Pe + \|E_1 - E_2\|^2 M_2^2
$$

(13)

With the inequality (7) and (10)–(13), we can confirm:

$$
V \leq e^T [P(A + BK) + (A + BK)^T P + 3P^2 + (L^2 + r)I + E_2^T E_2 + 2PDD^T P] e - r \left[\|e\|^2 - M_2^2 \frac{r}{r} \right] = 0
$$

If $\|e\|^2 > M_2^2 \frac{r}{r}$, we can have

$$
V \leq e^T (-Q - K^T RK) e(t) - r \left[\|e(t)\|^2 - M_2^2 \frac{r}{r} \right] \leq e^T (-Q - K^T RK) e(t) < 0
$$

(14)

Then we can have

$$
-\int_0^t e^T (Q + K^T RK) e dt < e^T Pe(t) - e^T Pe(0)
$$

(15)

Where $P$ is positive definite matrix, so

$$
-\int_0^t e^T (Q + K^T RK) e dt > -e^T Pe(0)
$$

(16)
If \( \left\| e \right\| > \frac{M_2}{\sqrt{r}} \)
we can have:
\[
\int_0^\infty e(t)^T (Q + K^T RK)e(t) dt \leq e(0)^T Pe(0) \leq \lambda_{\max}(P) \left\| e(0) \right\|^2
\]  \( (17) \)

2) If \( \left\| e \right\| \leq \frac{M_2}{\sqrt{r}} \)
we can have:
\[
e(t)^T (Q + K^T RK)e(t) \leq \lambda_{\max}(Q + K^T RK) \frac{M_2^2}{r}
\]  \( (18) \)

Theorem 2. Consider system (1)–(2), assume matrix \( X \succ 0 \), matrix \( W \) satisfies:
\[
\begin{pmatrix}
AX + XA^T + BW + W^TB^T + 3I & \sqrt{2}D & XE_2^T & X + \sqrt{\delta^2 X} & X & W^T \\
\sqrt{\delta^2 D^T} & -I & 0 & 0 & 0 & 0 \\
E_2X & 0 & -I & 0 & 0 & 0 \\
\sqrt{\delta^2 X} & 0 & 0 & -I & 0 & 0 \\
X & 0 & 0 & 0 & -Q^{-1} & 0 \\
W & 0 & 0 & 0 & 0 & -R^{-1}
\end{pmatrix} < 0
\]  \( (19) \)

then \( \bar{K} = W^T X^{-1} \) can guarantee the system are stable with
\[
\lim_{t \to \infty} \left\| \frac{1}{e(t)} \right\| \leq \frac{M_2}{\sqrt{r}}
\]

Proof.
With the inequality (7), we can finally confirm:
\[
\begin{pmatrix}
P(A + BK) + (A + BK)^T P + 3P^2 & \sqrt{2}PD & E_2^T & \sqrt{\delta^2 X} & I & K^T \\
\sqrt{2}P & -I & 0 & 0 & 0 & 0 \\
E_2 & 0 & -I & 0 & 0 & 0 \\
\sqrt{\delta^2 X} & 0 & 0 & -I & 0 & 0 \\
I & 0 & 0 & 0 & -Q^{-1} & 0 \\
K & 0 & 0 & 0 & 0 & -R^{-1}
\end{pmatrix} < 0
\]  \( (20) \)

4. Numerical Examples
In this section, we give example as follows to verify the section.
Example. Consider the NCSs(1),
\[
\begin{bmatrix}
x_1(x) \\
x_2(x)
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-(\beta + \Delta \beta_i(t)) & -(\sigma + \Delta \sigma_i(t))
\end{bmatrix} \begin{bmatrix}
x_1(x) \\
x_2(x)
\end{bmatrix} + \begin{bmatrix}
-x_1(x) \\
F \sin(\omega t)
\end{bmatrix} + d_i(t)
\]
where
\[
d_i(t) = [0.28 \sin(0.5t), -0.2 \cos(5t)]^T,
\]
\[
\Delta \beta_i(t) = 0.1 \cos(2t),
\]
\[ \Delta \sigma_1(t) = -0.11 \cos(2t). \]

And the system (2):
\[
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} = \begin{bmatrix}
    0 & 1 \\
    -(\beta + \Delta \beta_2(t)) & -\Delta \sigma_2(t)
\end{bmatrix} \begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} - f(\tilde{x}_1) + d^2(\tilde{t}) + BK \begin{bmatrix}
    \tilde{x}_1 + x_1 \\
    \tilde{x}_2 + x_2
\end{bmatrix} + \Delta \beta_2(t) - \sigma - \Delta \sigma_2(t) + F \sin(\omega t)
\]

where
\[
d_2(t) = [-0.1 \cos(2t), 0.26 \sin(5t)]^T,
\]
\[\Delta \beta_2(t) = 0.13 \cos(2t), \]
\[\Delta \sigma_2(t) = 0.098 \cos(2t)\]

We can have
\[
A = \begin{bmatrix}
    0 & 1 \\
    -\beta & -\sigma
\end{bmatrix},
\]
\[
f(x, t) = \begin{bmatrix}
    -f(x_1) \\
    F \sin(\omega t)
\end{bmatrix}
\]
\[
\Delta A_1(t) = \begin{bmatrix}
    0 & 1 \\
    -\Delta \beta_1(t) & -\Delta \sigma_1(t)
\end{bmatrix}
\]
\[
\Delta A_2(t) = \begin{bmatrix}
    0 & 1 \\
    -\Delta \beta_2(t) & -\Delta \sigma_2(t)
\end{bmatrix}
\]
\[
\|f(t) + d_2(t)\| \leq \sqrt{(0.28 + 0.1)^2 + (0.2 + 0.26)^2} < 0.6
\]
\[
P = \begin{bmatrix}
    0 & 1 \\
    1 & 0
\end{bmatrix},
\]
\[
F(t) = \begin{bmatrix}
    0 & 0 \\
    0 & \cos(2t)
\end{bmatrix}
\]
\[
E_1 = \begin{bmatrix}
    0 & 0 \\
    -0.1 & 0.11
\end{bmatrix},
\]
\[
E_2 = \begin{bmatrix}
    0 & 0 \\
    -0.13 & -0.098
\end{bmatrix}
\]

With (13)–(15), we can also have
\[
\dot{e} = (A + BK)e + \Delta A_2(t)e + g(x, t) + g(\tilde{x}, t) + d_2(t) + \\
+ d_1(t) + [\Delta A_1(t) - \Delta A_2(t)]x
\]
\[
A = A + \begin{bmatrix}
    -\frac{a + b}{2} & 0 \\
    2 & 0
\end{bmatrix} = \begin{bmatrix}
    -\frac{a + b}{2} & 0 \\
    -\beta & -\sigma
\end{bmatrix},
\]
\[
g(\tilde{x}, t) + g(x, t) = \begin{bmatrix}
    -k & 0 \\
    0 & 0
\end{bmatrix}(\tilde{x} + x)
\]

Finally we can have:
\[ \|g(\dot{x}, t) + g(x, t)\| = \left\| \begin{bmatrix} -k & 0 \\ 0 & 0 \end{bmatrix} \right\| \leq \left\| \begin{bmatrix} a - b \\ 0 \\ 0 \end{bmatrix} \right\| = L \|e\| \]

where \( \frac{a - b}{2} < k < -\frac{a - b}{2} \).

Then computing the parameters, we have

\[ k_1 = (15, 02 \quad 5, 23), \quad P_1 = \begin{pmatrix} 1.12 & 0.15 \\ 0.59 & 0.92 \end{pmatrix} \]

\[ k_2 = (10, 00 \quad 5, 00), \quad P_1 = \begin{pmatrix} 1.12 & 0.13 \\ 0.51 & 2.26 \end{pmatrix} \]

Finally, we can confirm that NCSs(1-2) is finite-time stable, the simulation results are presented and we can see in Fig. 1.

![Time response of NCSs](image)

**Figure 1.** Time response of NCSs

5. Conclusions

In this paper, we discuss the problem of stability problem of NCS. Some sufficient conditions have been provide for stability of NCS, the result showed that we can guarantee the system to be stable by using LMI. However, this article does not take into account the data-rate of the NCSs. A challenging and interesting future research topic is how to extend the results in this paper with these issues.

6. References

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