A TECHNIQUE OF REMOVING LARGE-SCALE VARIATIONS IN ASTRONOMICAL OBSERVATIONS

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ABSTRACT

In many astrophysical systems, smoothly-varying large-scale variations coexist with small-scale fluctuations. For example, magnetic fields in molecular clouds may consist of smoothly-varying mean components and shorter-scale turbulence components (see, for example, Girart et al. 2006; Hildebrand et al. 2009; Houde et al. 2009). Velocity fields in molecular clouds also exhibit large-scale gradients, as well as small-scale turbulent fluctuations (Imara & Blitz 2011). The separation of signals into large-scale and small-scale ones is not limited to spatial fluctuations. In observations of the redshifted 21 cm lines, we may separate smoothly varying large-scale foreground components and fast-fluctuating small-scale cosmological components in frequency space (Morales et al. 2006; Cho et al. 2012). We may also separate time-series data into two components.

If large-scale variations and small-scale fluctuations coexist, the large-scale components sometimes make it difficult to obtain information on small-scale fluctuations. For many applications, it is necessary to accurately measure small-scale fluctuations. For example, we could constrain turbulence parameters by observing the standard deviation, skewness, or kurtosis of column density (e.g., Burkhart et al. 2009). If there is no large-scale variation of column density, it may be straightforward to obtain those quantities from observations. However, when there are large-scale variations, they will certainly affect all those quantities.

The situation is similar for centroid velocity (for an optically thin line), which is equal to the intensity-weighted average velocity (see Section 2.2 for mathematical definition). Centroid velocity contains information on turbulence velocity field and therefore has been used to diagnose properties of interstellar turbulence (von Hoerner 1951; Dickman & Kleiner 1985; Kleiner & Dickman 1985; O’Dell & Castañeda 1987; Miesch & Bally 1994; Esquivel et al. 2007). Therefore accurate measurements of the small-scale centroid velocity fluctuations is important for the study of interstellar turbulence.

Centroid velocity is also important for measurement of interstellar magnetic fields (Cho & Yoo 2016; González-Casanova & Lazarian 2017). The Chandrasekhar-Fermi method (Chandrasekhar & Fermi 1953) is a popular and simple technique to obtain strengths of interstellar magnetic fields projected on the plane of the sky, which makes use of polarized emission in FIR/sub-mm wavelengths from magnetically aligned grains (Gonatas et al. 1990; Lai et al. 2001; Di Francesco et al. 2001; Crutcher et al. 2004; Girart et al. 2006; Curran & Chrysostomou 2007; Heyer et al. 2008; Mao et al. 2008; Tang et al. 2009; Sugitani et al. 2011; Pattle et al. 2017). The Chandrasekhar-Fermi method is based on the following assumption: If the mean magnetic field is strong, wandering of magnetic field lines is small and, therefore, variation of polarization angle is small. Cho & Yoo (2016) showed that, if there are N independent eddies along the line of sight (LOS), the variation of polarization angle (δφ) is reduced by ~√N due to random averaging effect. If we use the Chandrasekhar-Fermi method, the reduction in δφ results in overestimation of magnetic field strength by a factor of √N. Cho & Yoo (2016) suggested that the standard deviation of centroid velocity divided by average line width can tell us about √N (see Cho (2017) for a heuristic explanation for this). Therefore accurate measurements of the small-scale centroid velocity fluctuations is important for the application of the Chandrasekhar-Fermi method. Since large-scale variations in the LOS velocity can severely affect the standard deviation of centroid velocity, it is necessary to remove the large-scale LOS velocity variations.

Fitting is frequently used to remove large-scale variations. For example, magnetic fields in molecular cores frequently show an hour-glass morphology (Schleming 1998, Houde et al. 2004). As explained in the previous paragraph, the Chandrasekhar-Fermi method requires measurement of δφ. However the large-scale magnetic

1 In fact, the method assumes tan δφ ~ δb/B0 (⇒ MA), where δφ is the variation of polarization angle, δb is the strength of fluctuating magnetic field, B0 is the strength of the mean magnetic field projected on the plane of the sky, and MA is the Alfvén Mach number.
morphology impedes accurate measurement of the quantity, which makes it difficult to apply the Chandrasekhar-Fermi method. To model the hour-glass shape large-scale magnetic fields, a fitting function of the form \( x = g + gCy^2 \), where \( g \) and \( C \) are constants, has been successfully used (Girart et al. 2006; Sugitani et al. 2010). However, in many cases, fitting requires knowledge on the large-scale variations \textit{a priori}.

In this paper, we propose a technique to remove large-scale variations. Our main goal is to obtain the standard deviations of small-scale quantities. Nevertheless, our technique can be also used to filter out large-scale variations and retrieve small-scale maps. Our technique requires multi-point structure functions and does not rely on fitting method. We first describe theoretical backgrounds of our technique and numerical methods for testing our technique in Section 2. We present our results in Section 3. We give discussions and summary in Sections 4.

2. THEORETICAL CONSIDERATIONS AND NUMERICAL METHODS

2.1. Removal of large-scale variation with multi-point structure functions

Suppose that a quantity \( Q \) in real space exhibits a large-scale variation, as well as small-scale fluctuations (see Figure 1):

\[
Q(x) = Q_L(x) + Q_S(x).
\]

We assume the spatial average \( \langle Q_S(x) \rangle \) is zero when we calculate the average on scales larger than the small-scale correlation length \( l_S \):

\[
\langle Q_S(x) \rangle = 0 \quad \text{(if scale} > l_S). \tag{2}
\]

Our goal is to remove the large-scale variation \( Q_L(x) \) and obtain the standard deviation of the small-scale fluctuation \( \sigma_Q \). In this subsection, we show that multi-point structure functions, rather than the conventional 2-point structure function, can effectively remove the large-scale variation.

2.1.1. Two-point structure function

In many problems, the usual 2-point second-order structure function for a variable \( Q \),

\[
SF_{2}^{2pt}(r) = \langle |Q(x + r) - Q(x)|^2 \rangle \text{ avg. over } x, \tag{3}
\]

is frequently used to diagnose structures on different scales. In fact, it is related to power spectrum \( \xi(k) \). However, we should be careful when we use \( SF_{2}^{2pt} \). In the presence of a large-scale variation, it may fail to reveal small-scale structures correctly. If we select two points, \( x \) and \( x + r \), as in Figure 1(a), the difference of the large-scale quantity \( \langle |\Delta^2_{L}| \rangle \equiv \langle |Q_L(x) - Q_L(x + r)|^2 \rangle \) can be larger than that of the small-scale quantity \( \langle |\Delta^2_{S}| \rangle \equiv 2|Q(x + r) - Q(x)|^2 \)

\[
2|Q(x + r) - Q(x)|^2 \] (1)

\[
\langle |\Delta^2_{L}| \rangle \approx 2|Q(x + r) - Q(x)|^2 = \langle |\Delta^2_{S}| \rangle \] (2)

\[
\langle |\Delta^2_{L}| \rangle = \langle |Q_L(x) - Q_L(x + r)|^2 \rangle \] (3)

\[
\langle |\Delta^2_{S}| \rangle = \langle |Q_S(x + r) - Q_S(x)|^2 \rangle \] (4)

\[
\langle |\Delta^2_{L}| \rangle \approx 2|Q(x + r) - Q(x)|^2 \] (5)

\[
\langle |\Delta^2_{S}| \rangle = \langle |Q_S(x + r) - Q_S(x)|^2 \rangle \] (6)

\[
\langle |\Delta^2_{L}| \rangle = \langle |Q_L(x) - Q_L(x + r)|^2 \rangle \] (7)

\[
\langle |\Delta^2_{S}| \rangle = \langle |Q_S(x + r) - Q_S(x)|^2 \rangle \] (8)

Figure 1. Large-scale variation and the 2-point and the 3-point structure functions. (a) The large-scale variation dominates the quantity \( \Delta^2_{L} \) if the separation \( r \) is large enough. (b) It is possible that the small-scale fluctuations can dominate the quantity \( \Delta^2_{S} \) even if \( r \) is large. (c) The behavior of a second-order structure function in the absence of a large-scale variation. (d) The behavior of a multi-point second-order structure function in the presence of large-scale variations. If the structure function successfully removes the large-scale effect, we will have an extended flat part (‘plateau’) on scales larger than \( l_S \).

2.1.2. Multi-point structure functions

If we use multi-point structure functions, we can remove substantial amount of the large-scale effects. Let us consider difference of \( Q \) constructed with 3-points:

\[
\Delta^3_{Q} = Q(x - r) - 2Q(x) + Q(x + r) \quad \text{(see Figure 1(b)). It is trivial to show that } \Delta^3_{Q} \text{ can exactly eliminate a large-scale variation that has a constant slope. If the large-scale variation is so smooth that } \Delta^3_{Q} \text{ can exactly eliminate a large-scale variation that has a constant slope,} \]

\[
\Delta^2_{L} < \Delta^3_{Q} < \Delta^2_{L} \]

As in Figure 1(b), then the 3-point structure function

\[
SF_{3}^{3pt} = \frac{1}{3} \langle (Q(x - r) - 2Q(x) + Q(x + r))^2 \rangle \] (9)

can capture small-scale fluctuations correctly.

We can also construct 4-point and 5-point second-order structure functions as follows:

\[
SF_{2}^{4pt} = \frac{1}{10} \langle (Q(x - r) - 3Q(x) \quad \text{+3Q(x + r) - Q(x + 2r)|}^2 \rangle \tag{10}
\]

\[
SF_{2}^{5pt} = \frac{1}{10} \langle (Q(x - r) - 3Q(x) \quad \text{+3Q(x + r) - Q(x + 2r)|}^2 \rangle \quad \text{+3Q(x + r) - Q(x + 2r)|}^2 \rangle \tag{11}
\]

\[
SF_{2}^{5pt} = \frac{1}{10} \langle (Q(x - r) - 3Q(x) \quad \text{+3Q(x + r) - Q(x + 2r)|}^2 \rangle \quad \text{+3Q(x + r) - Q(x + 2r)|}^2 \rangle \tag{12}
\]
Removing Large-scale Velocity

2.2. Structure functions

2.2.1. Simple sinusoidal large-scale variations

We generate data that contain both small-scale fluctuations and a large-scale variation from the following procedure. We take turbulence data as small-scale fluctuations.

First, we generate 3D turbulence data from a direct numerical simulation of isothermal supersonic magnetohydrodynamic (MHD) turbulence, which contain only small-scale fluctuations. The computational domain is a cubic box of size $2\pi$ ($\equiv L$) and consists of $512^3$ grid points. The simulation is identical to the model ‘KF20’ in Cho & Yoo (2016). The driving scale is about 20 times smaller than the size of the computational domain, which means that the typical size of largest energy-containing eddies is about 20 times smaller than the size of the computational box. The sonic and the Alfvénic Mach numbers are $\sim 7$ and $\sim 0.7$, respectively. The fluid velocity is zero ($v = 0$), density is one ($\rho_0 = 1$), and the Alfvén speed of mean field is one ($B_0/\sqrt{4\pi\rho_0} = 1$) at $t=0$. Further description of the code can be found in Cho & Yoo (2016).

Our goal is to obtain the magnitudes of small-scale fluctuations of column density and velocity centroid. Since these fluctuations are related to 3D density and velocity (see Equations (11) and (12)), we plot time evolution of $v^2$ and $(\delta \rho)^2$ in the left panel of Figure 2. Where $v$ is the 3D velocity and $\delta \rho (\equiv \rho - \rho_0)$ is the fluctuating 3D density. The data we use are taken at $t=6$, at which the r.m.s. velocity is $\sim 0.7$ and $\delta \rho \sim 1.6$. The right panel of Figure 2 shows spectra of the 3D velocity $(E_v(k))$ and density $(E_\rho(k))$. They have peaks at $k \sim 20$, which corresponds to the average driving wavenumber.

Second, using the data, we calculate column density (Equation (11)) and centroid velocity (Equation (12)). The LOS is along the $z$-direction and is perpendicular to the mean magnetic field. The standard deviations of column density and centroid velocity (without a large-scale variation) along the LOS are

$$\sigma_\Sigma \approx 90, \quad \text{and} \quad \sigma_{V_c} \approx 0.084, \quad (13)$$

and, therefore, we have

$$2(\sigma_\Sigma)^2 \approx 1.6 \times 10^4, \quad \text{and} \quad 2(\sigma_{V_c})^2 \approx 0.014 \quad (14)$$

(see Table 1).

Third, after calculating column density and centroid velocity, we add simple large-scale variations. The large-scale variations have the sinusoidal form

$$Q(x) = A_Q \sin [k(x - \pi)], \quad k=1/2, 5/2, 9/2, \quad (15)$$

where $0 < x < 2\pi$ and $Q$ is either column density or centroid velocity. The corresponding wavelengths of the large-scale variations are $\lambda (= 2\pi/k) = 2L, 2L/5, \text{and} 2L/9$, respectively. The amplitude $A$ is 1024 for column density (i.e., $A_\Sigma=1024$) and 1.0 for centroid velocity (i.e., $A_{V_c}=1.0$), which are $\sim 10$ times larger than the amplitudes of the corresponding small-scale fluctuations. We list properties of the turbulence data, including standard deviations of small-scale fluctuations ($\sigma_\Sigma$ and $\sigma_{V_c}$), in Table 1 (see Model KF20).

3 Note that, if the large-scale gradient is very small, even the 2-point structure function can also show a similar behavior.

4 Note that the small-scale data are not necessarily turbulence data. We use existing small-scale turbulence data for simplicity.
2.2.2. More complicated turbulent large-scale variations

In the previous subsection, we considered idealistic large-scale variations. To see if our technique works also for more complicated large-scale fluctuations, we take large-scale turbulence data as the large-scale variations. To be specific, we use data of isothermal turbulence driven at two different spatial scales simultaneously. The driving wavenumbers are near \( k \sim 2.5 \) and \( k \sim 20 \). Since the two driving scales are well separated, we can assume that the large-scale driving (i.e., driving near \( k \sim 2.5 \)) generates large-scale variations, while the small-scale driving (i.e., driving near \( k \sim 20 \)) creates small-scale fluctuations. We want to remove the former and retain the latter. The sonic Mach number is around unity and the numerical resolution is \( 512^3 \). The numerical setups for the simulation are virtually identical to those of the Run CS-L1.0_S2.0 in Yoo & Cho (2014), but the numerical resolution for the current run is higher. We list properties of turbulence in Table 1 (see Model L1.0_S2.0).

Since turbulence is driven at small and large scales simultaneously, it is not easy to define which are small-scale fluctuations and which are large-scale ones. Nevertheless, since our goal is to retrieve small-scale fluctuations, it is necessary to have rough estimates about the magnitudes of small-scale fluctuations. We calculate the standard deviations of the small-scale fluctuations, \( \sigma_\Sigma \) and \( \sigma_{V_c} \), from the following procedure. First, we perform Fourier transformation of the real-space data and obtain wavevector-space data. Second, we filter out large-scale data. To be specific, we set the Fourier amplitudes to zero when \( k < 10 \) and retain the data when \( k \geq 10 \). We take \( k = 10 \) because the 3D spectra of velocity and density show different behaviors for \( k < 10 \) and \( k > 10 \) (see Section 3.2 for details). Third, we transform the filtered data back to real space. Fourth, we calculate \( \sigma_\Sigma \) and \( \sigma_{V_c} \) from the (filtered) real-space data. The resulting \( \sigma_\Sigma \) and \( \sigma_{V_c} \) are

\[
\sigma_\Sigma \approx 16, \quad \text{and} \quad \sigma_{V_c} \approx 0.041, \quad (16)
\]

which give

\[
2(\sigma_\Sigma)^2 \approx 510, \quad \text{and} \quad 2(\sigma_{V_c})^2 \approx 0.033 \quad (17)
\]

(see the data for KF2.5_20 in Table 1).
the small-scale driving become clearly visible for $k \gtrsim 15$. The behavior of the spectra of column density and centroid velocity (upper-right panel) is also similar. They decrease as the wavenumber increases for $k < 10$, become flat for $10 \lesssim k \lesssim 20$, and decrease again after $k \sim 20$. We may assume that the flat and decreasing spectra for $k \gtrsim 10$ are due to small-scale fluctuations.

The 2-point structure functions ($S_{F_{2}^{pt}}$) in the lower panels do not exhibit plateaus, while structure functions based on 3 or more points clearly show plateaus. The values of the multi-point second-order structure functions at the plateaus are

$$S_{F_{2}^{at plateau}} \sim 800 \quad \text{(for } \Sigma)$$

and

$$S_{F_{2}^{at plateau}} \sim 0.005 \quad \text{(for } V_{c}),$$

which are not far from the estimates for $2(\sigma_{\Sigma})^{2}$ and $2(\sigma_{V_{c}})^{2}$, respectively, in Equation (17). Therefore we can conclude that the multi-point structure functions can also remove complicated large-scale variations reasonably well.

4. DISCUSSIONS AND SUMMARY
Figure 5. The Run K2.5-20. We drive the fluid at $k \sim 2.5$ and $k \sim 20$ simultaneously and generate transonic isothermal turbulence. We regard the structures generated by the large-scale driving (i.e., $k \sim 2.5$) as large-scale variations and the ones by the small-scale driving (i.e., $k \sim 20$) as small-scale fluctuations. Upper-left: Spectra of (3D) $v$ and $\rho$. Upper-right: Spectra of (2D) column density $\Sigma$ and centroid velocity $V_c$. Lower-left: Second-order structure functions for $\Sigma$. Lower-right: Second-order structure functions for $V_c$. Note the plateaus near $k \sim 15$.

Figure 6. Spectra for column density (upper panels) and centroid velocity (lower panels). We use a data cube from the Run K2.5-20. We calculate spectra using either the original maps ($512 \times 512$) or partial maps ($256 \times 256$). The reason we use the partial maps is to include the edge effect. The plots in the first column from the left are the spectra of the original maps (on a grid of $512 \times 512$). The plots in the other columns are the spectra of the partial maps (on a grid of $256 \times 256$) tapered by gaussian windows with different widths (see the standard deviations $\sigma$‘s of the window functions on the panels).
4.1. Spectrum vs. multi-point structure functions

Power spectrum is also a useful tool to study small-scale fluctuations. Indeed, if we can obtain the correct power spectrum, it may be possible to separate large-scale variations and small-scale fluctuations. However, obtaining the correct spectrum is not easy when the data are not periodic. If the data are not periodic, the discontinuity at the edge can severely affect the shape of the power spectrum. To reduce this artifact, a tapering window function is frequently used, which forces the values near the edge converge to zero. While the tapering method should work fine when there are only small-scale fluctuations, it may cause nontrivial effects when there are also large-scale variations.

To demonstrate the effects of tapering window, we calculate power spectra of non-periodic 2D maps using gaussian tapering windows. We make use of the column density and the centroid velocity maps of the Run K2.5,20, the resolution of which is 512 × 512. In order to make the maps non-periodic we divide each map into 4 equal quadrants and take only one of them, the resolution of which is 256 × 256. To be precise, the original periodic maps are define for 0 < x, y ≤ 2π and the new non-periodic maps are defined for 0 < x, y ≤ π. We apply 2D gaussian tapering windows with different widths.

\[ W(x, y) = e^{((x-\pi/2)^2+(y-\pi/2)^2)/(2\pi^2)}, \]  

where \( \sigma = \text{L}/5, \text{L}/10, \text{L}/15 \), to the non-periodic maps and calculate spectra. We plot the results in Figure 6. The upper and lower panels are for column density and centroid velocity, respectively. The far left panels show the spectra of the original maps (with 512 × 512 resolution), which should be identical to the spectra in the upper-right panel of Figure 6. Note that each spectrum has two components - one for \( k \leq 10 \) and the other for \( k \geq 10 \). The spectra in the other columns are the results of 2D gaussian tapering. From left to right, the standard deviation (\( \sigma \)) of the gaussian function decreases. In all the cases with the tapering windows, the small-scale component seems to be marginally visible. Nevertheless it may be difficult to draw any useful information from the spectra.

As we can see in Figure 6, the shape of spectrum changes when the shape of the tapering window changes. It may be possible to get a correct power spectrum if we know a proper shape of the window function. However, there is no way to know the proper shape of the window function a priori. The bottom line is that, although spectrum provides useful information on power distribution as a function of scale, it is not easy to obtain the correct spectrum. On the other hand, the multi-point structure functions do not require any knowledge a priori, which makes them more useful in deriving information on small-scale fluctuations.

4.2. Obtaining a small-scale map

Our technique discussed in earlier sections returns only the magnitudes of small-scale fluctuations. In this subsection, we demonstrate our technique can be also used to filter out large-scale variations and obtain a small-scale map. For simplicity, we use the 3-point (SF\textsuperscript{3pt}2) and the 5-point (SF\textsuperscript{5pt}2) second-order structure functions.

Suppose that we have a map of an observable quantity \( Q \) that contains both large-scale variations (\( Q_L \)) and small-scale fluctuations (\( Q_S \)). If SF\textsuperscript{3pt}2 or SF\textsuperscript{5pt}2 shows a plateau near a scale \( r_p \), then we have

\[ Q_L(x) = \frac{[Q_L(x + r) + Q_L(x - r)]}{2} \]  

for SF\textsuperscript{3pt}2 and

\[ Q_L(x) = \frac{4Q_L(x + r) + 4Q_L(x - r) - Q_L(x + 2r) - Q_L(x - 2r)}{6} \]  

for SF\textsuperscript{5pt}2 (see the definitions of SF\textsuperscript{3pt}2 and SF\textsuperscript{5pt}2), where \( x \) is a point on the map, \( r \) is a 2D displacement vector, and \( |r| \sim r_p \). Therefore, the 2-point average

\[ \bar{Q}(x) = \sum_{r_p - \Delta < |r| < r_p + \Delta} \frac{[Q_L(x + r) + Q_L(x - r)]}{2N} \]  

and the 4-point average

\[ \bar{Q}(x) = \sum_{r_p - \Delta < |r| < r_p + \Delta} \frac{4Q_L(x + r) + 4Q_L(x - r) - Q_L(x + 2r) - Q_L(x - 2r)}{6N} \]  

should be very good approximations for \( Q_L(x) \). Here both \( r_p - \Delta \) and \( r_p + \Delta \) should lie in the plateau scale and \( N \) is the number of summation. Note that the multi-point averages are different from the usual (1-point) average with a top-hat window:

\[ \bar{Q}(x) = \sum_{|x - x'| < r_p} \frac{Q_L(x')}{N}. \]

If we calculate a multi-point overage on a scale smaller than the plateau scale, the the value \( \bar{Q}(x) \) contains part of small-scale fluctuations. On the other hand, if we calculate a multi-point overage on a scale larger than the plateau scale, then the value \( \bar{Q}(x) \) loses some information about large-scale fluctuations.

After obtaining an approximate value of \( Q_L(x) \) (i.e., \( \bar{Q}(x) \)), it is trivial to obtain the small-scale value \( Q_S(x) \):

\[ Q_S(x) \approx Q(x) - \bar{Q}(x). \]

We may calculate spectrum of small-scale fluctuations using \( Q_S(x) \).

In Figure 7 we demonstrate that this procedure is indeed working. We apply the multi-point average technique to the column density maps shown in Figure 3 in which we can clearly see that the large-scale variations dominate the small-scale fluctuations. We plot the results for the cases of \( \lambda = 2L \) and \( \lambda = 2L/5 \) in Figure 7. We use \( r_p = 17.5 \) and \( \Delta = 2.5 \) (see Equations (25) and (26)). Note that, while both SF\textsuperscript{3pt}2 and SF\textsuperscript{5pt}2 for \( \Sigma \) have wide plateaus for \( \lambda = 2L \), only SF\textsuperscript{5pt}2 has a reasonably wide plateau near \( r = 17.5 \) for \( \lambda = 2L/5 \) (see Figure 4). We plot the resulting small-scale maps of the usual 1-point average (Equation (27)), the 2-point average (Equation (25)), and the 4-point average (Equation (26)) in the first, the second, and the third column from the left, respectively. The upper panels are for \( \lambda = 2L \) and the lower panels are for \( \lambda = 2L/5 \). As we can see in the contour plots, since the large-scale variation is
smooth enough in the case of $\lambda = 2L$ (upper panels), all 3 averaging methods can remove the large-scale variation quite well. However, in the case of $\lambda = 2L/5$ (lower panels), the usual 1-point average and the 2-point average leave residuals of the large-scale variation on the maps, which means the usual 1-point average and the 2-point average cannot filter out the large-scale variation completely. The result of the usual 1-point average is worse than that of the 2-point average. On the other hand, filtering by the 4-point average does not leave visible residuals on the map (see the lower panel in the third column from the left). These results are not surprising because the 5-point structure function does have a well-defined plateau near $r \sim 17.5$, while the 3-point structure function doesn’t.

The line plots is in far right panels show the power spectra. The thick long-dashed lines in the upper and the lower panels denote the spectrum of the original small-scale map of column density (see the upper-left panel of Figure 3 for the original small-scale map). The dashed and the thick solid lines represent the spectra of the small-scale maps obtained by the 2-point and the 4-point average techniques, respectively. That is, they are spectra of the maps in the second and third columns in Figure 7. The spectra represented by the dashed and the thick solid lines do not have significant powers at small wavenumbers (i.e., $k \lesssim 10$). However, the spectrum represented by the dashed line in the lower panel clearly shows a peak near $k \sim 2.5$, which corresponds to the wavenumber of the large-scale variation. Note that the values of $E(k)$ is largest at $k = 2.5$ for the dashed line, which is in agreement with the fact that the residual of the large-scale variation is an outstanding feature of the map in second-lower panel from the left. The thick solid curve in the lower panel also has a peaks near $k \sim 2.5$. But, its value at $k = 2.5$ is not large, which is consistent with the fact that the residual of the large-scale variation is not really visible on the map in the third-lower panel from the left. It is worth noting that the spectra from the 2-point and the 4-point average techniques virtually coincide with the spectrum of the original map when the wavenumber $k$ is large.

4.3. Application to observations

In this paper, we have proposed and tested a technique to remove large-scale variations and obtain magnitudes of small-scale fluctuations. Our technique does not rely on fitting method that requires knowledge on a fitting function a priori. Although we have focused only on column density and centroid velocity in this paper, we can also apply our technique to FIR/sub-mm polarization, redshifted 21 cm observations, or synchrotron emission data. In principle, our technique is applicable to any data that contain large-scale and small-scale fluctuations, if their spatial/temporal/frequency scales are well separated. For example, we can use our technique to separate small-scale fluctuating velocity and large-scale rotational velocity. We can also use our technique to obtain variations of polarization angles in regions where magnetic fields have hourglass morphologies.

4.4. Construction of an n-point structure function

In general, we can construct an n-point second-order structure function as follows:

$$SP^{n-p}_2(r) = \langle |\Delta^n|^2 \rangle$$  \hspace{1cm} (29)

with

$$\Delta^n = \frac{1}{N} \sum_{l=0}^{n-1} (-1)^l \binom{n-1}{l} Q \left( x + \left( \frac{n-1}{2} - l \right) r \right)$$  \hspace{1cm} (30)

with

$$N = \frac{1}{2} \sum_{l=0}^{n-1} \binom{n-1}{l}^2.$$  \hspace{1cm} (31)
we calculate. We also thank Min-Young Lee for useful discussions.

Chandrasekhar, S., & Fermi, E. 1953, ApJ, 118, 113

Burkhart, B., Falceta-Gonçalves, D., Kowal, G., & Lazarian, A. 2009, ApJ, 693, 250

Chandrasekhar, S., & Fermi, E. 1953, ApJ, 118, 113

Here \binom{n}{k} is the binomial coefficient and \(n \pm 1)/2\) can be either \(n/2\) or \(n/2\) \(-1\) if \(n\) is an even number. Note that \(\Delta^n\) is the same as the \(n\)-th order central difference.

4.5. Summary

In summary, we have obtained the following results.

1. We develop a technique that can remove large-scale variations in observable quantities. Our technique relies on multi-point structure functions and gives us magnitudes of small-scale fluctuations (see Equations (7), (9), and (29)).

2. Our technique works fine for a large-scale variation of a simple sinusoidal form. It also works reasonably well for a more complicated turbulent large-scale fluctuations.

3. If a second-order structure function shows a plateau, then the variance of the small-scale fluctuations is equal to the value of the structure function at the plateau divided by two (Equation (10)).

4. Our technique can be used to separate small-scale fluctuations and large-scale variations. We have discussed how to filter out large-scale variations and obtain maps of small-scale fluctuations using multi-point averages (Section 4.2).

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