Combining Lattice QCD Results with Regge Phenomenology in a Description of Quark Distribution Functions

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Abstract:
The most striking feature of quark distribution functions transformed to the longitudinal distance representation is the recognizable separation of small and large longitudinal distances. While the former are responsible for the average properties of parton distributions, the latter can be shown to determine specifically their small-\(x\) behavior. In this paper we demonstrate how the distribution at intermediate longitudinal distances can be approximated by taking into account constraints which follow from the general properties of parton densities, such as their support and behavior at \(x \to 1\). We show that the combined description of small, intermediate, and large longitudinal distances allows a good approximation of both shape and magnitude of parton distribution functions. As an application we have calculated low-virtuality C even and odd (valence) u and d quark parton densities of the nucleon and the C-even transversity distribution \(h_1(x)\), combining recent QCD sum rules and lattice QCD results with phenomenological information about their small-\(x\) behavior.

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More than twenty five years ago the discovery of point-like constituents, partons, in the deep-inelastic-scattering revolutionized our understanding of the nucleon structure. Although since then the Quantum Chromodynamics (QCD) has become an established theory of strong interactions, derivation of parton distribution functions from the first principles still remains a challenge. The standard QCD analysis based on the Operator Product Expansion (OPE) relates moments of parton distributions at a given scale to the nucleon matrix elements of local twist-2 operators. The evolution of parton distributions, due to the renormalisation scale dependence of twist-2 quark and gluon operators, is completely understood within QCD perturbation theory. Hence, the whole problem can be reduced to the sufficiently accurate, first-principles computation of parton distributions at low virtuality, which in practice has proven to be quite difficult. For example, QCD sum rules calculations of parton distributions have been only moderately successful - the results for moments beyond the lowest non-trivial one have typically very low accuracy - and the reliable, state-of-art lattice QCD predictions for the lowest two moments of C-even quark distribution functions have become available only recently.

For more than twenty years, the intrinsic complication of QCD has prevented us from obtaining its exact solutions in the non-perturbative domain. This explains the important role played by phenomenological investigations and indeed since the discovery of partons there have been many attempts to model parton distribution functions, for example in terms of non-relativistic potential models, relativistic bag models, or soliton models. Methodologically, all these calculations can be understood as some effective low-energy theories of hadronic structure. All of them have either semi-classical or quantum mechanical character and they do not correctly take into account mechanisms specific to a quantum field theory, such as e.g. the existence of quarks and gluon loops. Moreover, despite the current wisdom that quarks are the most natural effective degrees of freedom in this context, one can argue from the very beginning that there might be a problem with such a choice in the case of low virtuality parton distribution functions. Certainly, the SU(6) algebraic properties of the hadronic spectrum suggest that quarks are very useful effective degrees of freedom as far as static hadronic properties, such as masses or electroweak parameters are concerned, but the same does not have to be true for structure functions, which are related to the infinite series of twist-2 matrix elements. On the other hand, quark models as an effective low-energy description cannot be considered capable to reproduce all of them with a reasonable accuracy. The variety of successful models of hadronic static properties shows that the latter are not very sensitive to details of phenomenological approximation, and therefore can be well understood with the help of an effective, low-energy description. In the case of structure functions the situation is less clear. It is not easy to construct a model which satisfactory reproduces the overall magnitude and the $x$-dependence of flavor-singlet structure functions without having to incorporate some additional assumptions which sometimes are difficult to justify. For example, one starts with a calculation which produces a valence-like distribution which is subsequently perturbatively evolved to higher scales, where DIS measurements are performed. Unfortunately, consistency with the data often requires that the starting
point of the evolution has to be very low, deeply in the non-perturbative domain where the perturbative approach to the evolution is certainly controversial.

One very practical aspect of this situation is that the input parton distributions for modern parametrizations of structure functions [11, 12, 13] do not yet have a satisfactory i.e., fundamental, physical interpretation. Typically a simple functional form is evolved to higher $Q^2$ and fitted to the data in this domain. This program has been applied so successfully in practice that the $Q^2$ dependence of structure functions, governed by QCD evolution equations, has become the key factor for our understanding of experimental data, and the major argument which strongly points to QCD as the theory of strong interactions [4]. To the contrary, the magnitude and $x$-dependence of the deep-inelastic cross-section still belong to the much less understood, non-perturbative aspects. In principle, the OPE supplemented by non-perturbative methods such as QCD sum rules, models of the QCD vacuum [14], or ultimately the lattice calculations should allow to predict the cross-section as a whole, but as we have already mentioned, in practice this problem is very difficult.

In a recent paper we have attempted to optimize the applicability of OPE to the problem of hadronic structure functions $q(x, \mu^2)$ [15]. The key observation is that even at the twist-2 level their Bjorken $x$-dependence reflects physics of two very different regimes, which can be identified and separated by the following transformation into the coordinate space,

$$
Q(z, \mu^2) = \int_0^1 du \, q(u, \mu^2) \sin uz,
$$

$$
q(x, \mu^2) = \frac{2}{\pi} \int_0^\infty dz \, Q(z, \mu^2) \sin uz,
$$

(1)

where the distribution function $Q(z, \mu^2)$ has an interpretation of the gauge-invariant correlation function of two quark fields at the light cone, i.e. in QCD it is defined by the nucleon matrix element of a non-local quark string operator [16, 17]. The Ioffe-time $z$, the coordinate variable dual to the Bjorken $x$, is an invariant measure of the longitudinal distance along the light-cone between the quark fields [18, 19]. The Taylor expansion of $Q(z, \mu^2)$ around the origin has coefficients given by moments of the corresponding momentum space parton distribution, or equivalently by the matrix elements of twist-2 local operators [16]. As in the following we are interested in parton distributions at low virtualities, we choose a normalization point $\mu^2$ to be as low as possible from the common sense point of view, somewhere between 1 and, say, 3-4 GeV$^2$.

As it has been found in [15, 20, 21], the experimental data shows completely different behavior of Ioffe-time distributions for small and large longitudinal distances, see Figures 1 and 2 of Ref.[15]. For small $z$, the distribution is almost linear, which means that it can be described by the first one or two terms of its Taylor expansion around the origin, given by the matrix elements of the two local twist-2 operators of the lowest dimension. When the variable $z$ reaches $\sim 5$, the character of the distribution changes and it turns to a almost flat, asymptotic form for $z \geq 10$. In this region the shape of $Q(z)$ is determined
by the small-\(x\) behavior of the corresponding parton densities:

\[
q(x) \sim \frac{1}{x^{1+\alpha}} \iff Q(z) \sim z^{\alpha}.
\]

Note that \(z = 10\) corresponds in the nucleon rest frame to the linear distance of the order of 2 fm. In this region the contribution from valence partons i.e., partons carrying the momentum fraction \(x \geq 0.1\) becomes subleading in comparison with that of wee partons from the domain \(x \leq 0.1\) \cite{21}. For \(z \geq 10\) the corresponding quark-quark correlation function involves fields separated by a distance larger then the electromagnetic nucleon size. Naively, it is tempting to assume that the correlation function should vanish in this region, to the contrary to what can be seen on Figures 1 and 2 of Ref.\cite{15}, where the distribution stays approximately constant, or even rises at large \(z\). This behavior is usually explained by the contribution of the virtual photon transition into a quark-antiquark pair which occurs "outside" the nucleon, but one has to realize that such an interpretation assumes that the target nucleon indeed occupies a well defined space-time volume. We stress that the validity of this assumption depends on the relevant quantum numbers in the process under investigation. In particular, the finiteness of the hadronic electromagnetic radius shows that electromagnetic charge, a C-odd quantity, is distributed in a finite volume associated with the hadronic state. In a C-even process such as the Compton scattering this corresponds to a transition between the valence and wee parton regimes around longitudinal distance of 2 fm in the nucleon rest frame. Qualitatively, degrees of freedom responsible for large-\(z\) behavior of \(Q(z)\) give no important contribution to, say, nucleon electromagnetic formfactor. This observation can explain successful application of quark models to charge-parity odd quantities like charge radii and magnetic moments and their difficulties with parton distribution functions.

The very particular shape of the longitudinal distance distribution suggests a new approach to the problem of reconstruction of parton distributions from available theoretical information. We have already demonstrated \cite{15} that at small \(z\) the parton distributions can be very well described by their first two moments, alias calculable matrix elements of local twist-2 operators. However, the expansion in terms of moments becomes less and less efficient when \(z\) approaches, and passes the transition region towards the large-\(z\) domain. Using the Pade approximation instead of Taylor expansion still requires three, four moments to reach the onset of the asymptotic domain. Note that already the third moment is related to the twist-2 operator with five covariant derivatives, and it is probably fair to say that a reliable computation of its matrix element using present day lattice technology would be very difficult. In fact all the QCD methods available at present are based on the short longitudinal distance expansion and therefore are not well suited to study the large-\(z\) region. In our opinion it is related to the fact that physics of large and small longitudinal distances is different and it should be treated separately. In the previous paper we showed that one can combine the short distance expansion and the assumption about large-\(z\) alias small-\(x\) behavior to obtain the zeroth-order approximation showed as the dotted lines on Figures 1 and 2 \cite{21}. There, \(Q(z)\) has been approximated by

\[Q(z)\]

\[\text{Please note that the corresponding curve depicted by the dotted line on Figure 5 in Ref.\cite{15} is wrong.} \]
two straight lines, representing the small-\(z\) and large \(z\) behavior. Note that such a method clearly ignores the information about the \(z\) dependence in the transition region, and that it results in the distribution functions which are not far from the data in the small and intermediate \(x\) domain, but become negative for large \(x\). It is therefore natural to assume that, given the known behavior of \(Q(z)\) in the small and large \(z\) domains, the transition region could be constrained by the requirement of the correct behavior of the parton distribution at \(x \to 1\). In fact, we have found it difficult to implement directly the positivity constraint on the Bjorken-\(x\) distributions in the Ioffe-time representation. Instead, we imposed conditions that the distribution function vanishes at \(x = 1\), together with at least its two derivatives, in agreement with the QCD counting rules [22] which state that parton distributions should vanish at least as \((1 - x)^3\) at \(x \to 1\). The most obvious idea to use the anticipated large-\(x\) behavior to implement the subleading behavior of \(Q(z)\) in the large and intermediate \(z\) region turned out to be insufficient. Instead, we have chosen to approximate the shape of the Ioffe-time distribution in the small- and intermediate-\(z\) region up to \(z_a\) - the onset of the asymptotic large-\(z\) behavior, or the nucleon boundary, using the formula motivated by the Taylor expansion of \(Q(z)\), i.e.:

\[
W(z) = a_1 z - a_3 z^3 + a_5 z^5 - a_7 z^7 + a_9 z^9 - a_{11} z^{11}.
\] (3)

with the coefficients \(a_1\) and \(a_3\) given by the first two non-trivial moments of the corresponding quark distribution function. We have found that this is the only expansion which can be equally well applied to all three different cases which we consider in this paper. Slightly better results can be obtained when one uses in each case a method which anticipates the shape of the Ioffe-time distribution one wants to reconstruct. Having the point \(z_a = 10\) fixed by the classical nucleon diameter, the continuity of \(Q(z)\) eliminates one from four free parameters of (3), and the other three are determined using three constraints imposed by the expected behavior at \(x \to 1\). Altogether the whole procedure is equivalent to the solution of a system of linear equations which can be easily found with the help of Mathematica. The results are plotted as dashed lines on Figures 1 and 2 in comparison with solid lines which represent the MRS(A) parametrizations [11]. The improved approximation, despite its seemingly "kinematical" character, is indeed quite good as far as both shapes and magnitudes of parton distributions are concerned. Let us summarize below once more all the information which has been required to obtain this result:

- the first two moments, which have been already computed on the lattice, at least in the quenched approximation,

- the shape of the longitudinal distance distribution at large-\(z\), which is directly related to the small-\(x\) behavior of the corresponding parton distribution function,

- the onset of the large longitudinal distance regime which, as we have discussed above, should in principle correspond to the nucleon electromagnetic size. Nevertheless, it

due to an error in our computer program.
is somewhat surprising that the data indeed show the onset of the asymptotics at $z \approx 2 \text{ fm}$ in the nucleon rest frame,

- behavior of parton distribution at $x \to 1$ which follows from support properties and perturbative QCD counting rules. In the longitudinal distance representation it allows to construct a solution satisfying simultaneously the positivity requirement, for which we have not been able to find an equivalent condition in terms of $Q(z)$.

The only required piece of information which does not have yet a transparent physical interpretation is normalization at small-$x$. As we have already argued in [13], this information is rather difficult to extract from the standard OPE analysis. In this sense as long as a procedure aimed to determine this missing element has not been proposed, our program cannot be considered to be complete, and we have to rely on phenomenological information. On the other hand it is tempting to suggest that the correct QCD description of parton distribution functions has to rely on our understanding of large $z$ physics as well, and therefore to complete the whole program it is necessary to develop a theoretical framework in which the required information is calculable through a relation to, say, matrix element of a certain, possibly non-local, QCD operator. The first step in this direction has been made recently in [23].

To illustrate the above considerations we have attempted to reconstruct the u and d C-even parton distributions using the recent lattice results for their first two moments [7]. They correspond to the normalization point of about 2 GeV$^2$ in the $\overline{\text{MS}}$ scheme. To predict the corresponding parton distributions we have to combine them with the normalization of small-$x$ data at the same low scale. This information has been extracted from the CTEQ NLO [12] parametrization which has the low starting point at 2.5 GeV$^2$. The difference between LO and NLO parton distributions is in our case irrelevant as it is much smaller than the expected accuracy. The results are compared to the CTEQ parametrizations on Figures 3 and 4. Clearly, the lattice u-quark data overestimate true magnitude of the first two moments. As a consequence, the predicted parton distributions is larger than the real one at intermediate and large values of $x$. The agreement with the d-quark distribution, even if perhaps a bit fortuitous, is certainly impressive.

The same method can be applied to reconstruct the valence quark distributions

$$Q_{\text{val}}(z, \mu^2) = \int_0^1 du q_{\text{val}}(u, \mu^2) \cos uz ,$$

$$q_{\text{val}}(x, \mu^2) = \frac{2}{\pi} \int_0^\infty dz Q_{\text{val}}(z, \mu^2) \cos uz ,$$

from their first moments and large-$z$/small-$x$ asymptotics. As it follows from (4), the value of $Q_{\text{val}}(z = 0)$ is given by the number of valence quarks. At large $z$ the Ioffe time distribution should fall as $z^{-\alpha}$, where, according to the classical Regge theory [24], $\alpha \approx -0.5$. Hence, the C-odd correlation function of quark fields along the light-cone slowly vanishes at large longitudinal distances. As in the case of C-even parton distributions, as long as both the asymptotic behavior at large $z$ and the first non-trivial moment of $q_{\text{val}}$
are known, the corresponding Ioffe time distribution can be interpolated between these two regions. In this case the interpolating formula reads

\[ W(z) = a_0 - a_2 z^2 + a_4 z^4 - a_6 z^6 + a_8 z^8 - a_{10} z^{10}, \tag{5} \]

with \( a_0 \) given by the number of valence quarks and \( a_2 \) known from a non-perturbative calculation of the first non-trivial moment of C-odd quark distribution function, gives much better results. Combining the recent lattice \([7]\) results with the small-\( x \) normalization extracted from the CTEQ parametrization\([12]\) at \( \mu^2 = 2.5 \) GeV\(^2\) we have obtained results presented on Figures 5 and 6. All the coefficients \( a_4 \ldots a_{10} \) come out positive as expected. Due to the fact that the lattice results for the u-quark are much closer to the experimental data, the agreement with the parametrization is better in this case then for the C-even distribution. One can try to improve the d-quark approximation by requiring that more than two derivatives vanish at \( x = 0 \), however sooner or later the interpolation becomes unstable when the order of the corresponding polynomial is too high.

The above procedure can also be applied to the proton transversity distribution \( h_1^p(x, \mu^2) \) \([24]\). This idea is especially attractive if one considers that the calculation of, say, the first two moments of \( h_1(x) \) is probably possible using the present lattice resources \([20]\), so this structure function can be predicted before it will be measured in the next round of polarized DIS experiments \([27]\). As the lattice QCD predictions are not yet available, we have reinterpreted, following the discussion of \([20]\), the recent QCD sum rules calculation \([28]\) of the \( x \)-dependence of \( h_1^u(x) \) as the calculation of the Taylor expansion of the Ioffe-time distribution. According to Ref.\([28]\) the d-quark contribution is small, so the proton and neutron transversity distributions are approximately equal to \( 4/9 \) and \( 1/9 \) \( h_1^u \), respectively. Using results of \([28]\) it is not difficult to extract corresponding QCD sum rules predictions for the first moment of the C-even u-quark transversity distribution, given by the reduced matrix element of the QCD local operator \( \bar{u}(0) \gamma_5 \sigma^{+\perp} iD^+ u(0) \). The result is finite and free of any infrared singularities:

\[ \int_0^1 du \ u \ h_1^u(u) \approx 0.3 \tag{6} \]

Note that this estimate corresponds to a low virtuality of the order of a few GeV\(^2\). Note also that the singular behavior of C-odd tensor charge \( \int_0^1 du \ h_1^u(u) \), equal to the reduced matrix element of the operator \( \bar{u}(0) \gamma_5 \sigma^{+\perp} u(0) \), present in the OPE of Ref.\([23]\), is related to the prohibited contributions from large t-channel distances. Consequently, it will disappear when bilocal power corrections \([29]\) are properly taken into account. One striking feature of the value of the QCD sum rules result \([3]\) is its magnitude – approximately equal to the magnitude of the first moment of the unpolarized distribution \( \int_0^1 dx u(x, \mu^2) \) at the scale around 2 GeV\(^2\). On the other hand unitarity requires that \( h_1^u(x) \leq u(x) \). Now, because at small \( x \) Regge arguments suggest that \( h_1^u(x) \ll u(x) \), one can suspect

\(^8\)Extrapolating from the large \( z \) domain, i.e., using the expansion in the inverse powers of \( z^{-0.5} \) it is possible to obtain qualitatively very similar results using the QCD sum rules formula for the u-quark Ioffe-time time distribution from Ref.\([20]\).
that the resulting transversity distribution, reconstructed subject to the constraint \( \mathcal{F} \), will violate unitarity at intermediate and large values of \( x \). To determine the behavior at large \( z \) we followed Regge arguments \([28, 30]\) which suggest that the ratio of \( h_1(x)/g_1(x) \) should be constant at small \( x \) and a low normalization scale. Matching the Taylor expansion at small \( z \), with the first coefficient given by \( \mathcal{F} \), to the large \( z \) Regge asymptotics \( \sim z^{-1.3} \), with the magnitude fixed simply by the \( g_1^p \) data at small \( x \), we have obtained a prediction for \( h_1^p(x, \mu^2) \) at low \( \mu^2 \) depicted as the dashed line on Figure 7. As expected, at large and intermediate \( x \) it is larger than the unpolarized distribution represented on Figure 7 by the solid line. Assuming that the result \( \mathcal{F} \) overestimates the true value by 50\% one obtains the dot-dashed line on Figure 7. This prediction is similar in magnitude to the bag-model result of \([31]\). One should also keep in mind that the complicated shape inherent to a C-even Ioffe-time transversity distribution, which is equal to zero at \( z = 0 \), goes over a maximum at \( z \sim 4 - 5 \), and vanishes again for large \( z \), makes any approximation procedure in principle more sensitive to yet unknown higher moments.

Finally we point out once more that an attempt to "squeeze out" the most relevant information from structure functions can serve as a basis for construction of an effective theory of parton distributions. For example, if only the first two moments are required, one can hope that they can be computed with a reasonable accuracy in low-energy phenomenological approximations to QCD such as quark models or soliton models, with parameters unambiguously determined by comparison with other low-energy observables. Also from the lattice QCD point of view the precise calculation of the first one or two moments although technically very difficult, is perfectly feasible using the state-of-art technology. The small-\( x \) dynamics is much more specific and indeed it has been difficult to describe this region in the framework of a standard low-energy phenomenology. One reason for these difficulties is certainly related to the fact that, looking from the point of view of the longitudinal distance expansion, only quantities like low moments of parton distribution functions which correspond to short-distances can be understood by semi-classical or quantum mechanical models of hadronic structure. Analogous description of the physics of longitudinal distances larger than the nucleon electromagnetic size is much less justified because in this region parton correlation functions are much more sensitive to a proper treatment of these aspects of QCD dynamics which are difficult to represent using effective quark degrees of freedom. In QCD the required information about the normalization of small-\( x \) distributions can be perhaps related \([23]\) to matrix elements of some non-local QCD operators. In this language our discussion can be interpreted as an indication that in fact the required smearing does not have to extend over longitudinal distances larger than \( \sim 2 \) fm in the nucleon rest frame.

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Fig. 1 Consecutive approximations of the u-quark parton distribution $u(x)$, solid line, calculated using the MRS(A) parametrization at $\mu^2 = 4 \text{ GeV}^2$. The first approximation (dotted line) considered in [15] ignores the existence of the transition region between small and large Ioffe-time regimes. The improved approximation (dashed line) is obtained with the help of an interpolation between these two domains, subject to constraints on the behavior of Bjorken $x$ distribution at $x \rightarrow 1$. 
Fig. 2 Consecutive approximations of the d-quark parton distribution $d(x)$, solid line, calculated using the MRS(A) parametrization at $\mu^2 = 4 \text{ GeV}^2$. The first approximation (dotted line) considered in [15] ignores the existence of the transition region between small and large Ioffe-time regimes. The improved approximation (dashed line) is obtained with the help of an interpolation between these two domains, subject to constraints on the behavior of Bjorken $x$ distribution at $x \rightarrow 1$. 
Fig. 3 Reconstruction of the u-quark distribution function using the lattice results for the first two moments [7] and the experimental normalization at small $x$. The solid line is the CTEQ parametrization [12] at $\mu^2 = 2.5 \text{ GeV}^2$. 
Fig. 4 Reconstruction of the d-quark distribution function, dashed line, using the lattice results for the first two moments [7] and the experimental normalization at small $x$. The solid line is the CTEQ parametrization [12] at $\mu^2 = 2.5$ GeV$^2$. 
Fig. 5 Reconstruction of the valence u-quark distribution function using lattice QCD results for the first non-trivial moment and the experimental normalization at small $x$, dashed line. The solid line is the CTEQ parametrization [12] at $\mu^2 = 2.5 \text{ GeV}^2$. 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{Reconstruction of the valence u-quark distribution function using lattice QCD results for the first non-trivial moment and the experimental normalization at small $x$, dashed line. The solid line is the CTEQ parametrization [12] at $\mu^2 = 2.5 \text{ GeV}^2$.}
\end{figure}
Fig. 6 Reconstruction of the valence d-quark distribution function using the lattice QCD results for the first non-trivial moment and the experimental normalization at small $x$. The solid line is the CTEQ parametrization \cite{12} at $\mu^2 = 2.5$ GeV$^2$. 
Fig. 7 Reconstruction of the proton C-even transversity distribution $h_1^p(x)$, dashed and dot-dashed lines, using the QCD sum rules estimates for its first moment [28], and the asymptotics at large $z$ fixed by Regge arguments and comparison with the $g_1^p(x)$ data. The unpolarized $u$-quark distribution $\frac{4}{9} u(x, \mu^2)$ according to the CTEQ parametrization [12] at $\mu^2 = 2.5$ GeV$^2$ is depicted by the solid line.