Weighted Calibration Algorithm of Multiple Laser Scanning Sensors Based on Measurement Error Correction

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Abstract. The calibration of pose transformation relationship among multiple laser scanning sensors or between one sensor and a reference coordinate system is of great importance to the accuracy of 3D laser scanning system. In some applications, such as section inspection of Metro tunnels, the sparsity of measuring points and high demand for accuracy make the calibration process very difficult. This paper proposes an automatically calibration algorithm, named GA-ICP-WHT, based on an improved ICP algorithm with point-wise measurement error corrections. The algorithm consists of stages: coarse registration with GA and fine registration with ICP and weighted Hong-Tan algorithm. Mathematical simulation and application experiments show that the proposed algorithm is accurate and robust for sparse point cloud registration.

1. Introduction
The full section data are the basic data for assessing the deformation degree of Metro tunnels. At present, the light propagation time principle-based laser range finder [1] has been widely used in periodic and rapid tunnel inspection. Such 3D laser scanning system, installed on the railway vehicle needs several laser range finders to complete the full section inspection of the tunnel. The calibration problem of the scanning system, namely, how to convert the coordinate system of the laser range finder itself into the extrinsic coordinate system of railway line, will be encountered.

Assume the coordinate system of the laser range finder is $O_{UVW}$, while the extrinsic coordinate system is $O_{XYZ}$. The conversion between two coordinate systems may be achieved through the rotation and translation of the coordinate system of the laser range finder along three axes of extrinsic coordinate system:

$$
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} = 
\begin{bmatrix}
\cos \alpha \cos \beta - \sin \alpha \sin \beta & \sin \alpha \cos \beta + \cos \alpha \sin \beta & -\cos \beta \\
-\sin \alpha \cos \beta & \cos \alpha \cos \theta & \sin \theta \\
\cos \alpha \sin \beta + \sin \alpha \sin \beta \cos \beta & \sin \alpha \sin \beta - \cos \alpha \sin \beta \cos \beta & \cos \theta \cos \beta
\end{bmatrix}
\begin{bmatrix}
U \\
V \\
W
\end{bmatrix} + 
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix}
$$

(1)

where $\alpha$, $\beta$ & $\theta$ represent the deflection angle for the coordinate system of the laser range finder rotating along three axes of extrinsic coordinate system respectively, while $\Delta x$, $\Delta y$ & $\Delta z$ separately represent the translation of the origin of the coordinate system of the laser range finder relative to the origin of extrinsic coordinate system. Therefore, the equation may be noted as follows:

$$
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} = R \begin{bmatrix}
U \\
V \\
W
\end{bmatrix} + \begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix}
$$

(2)

At present, the commonly used calibration method is the standard template method [2] and the 1D coordinate method [3]. Both of the calibration methods suffer from the problems of cumbersome steps,
inconvenient operation and low accuracy. This paper adopts the point registration technique which is widely used in the field of the computer vision into the sensor calibration, and proposes an automatically calibration algorithm based on an improved Iterative Closest Points (ICP) algorithm\cite{4} with point-wise measurement error corrections.

Our contribution includes two aspects: firstly, we propose an error objective function using point-to-tangent-plane distance for sparse calibration point set, which solves the problem of large point-to-point distance error between two sparse point sets. Secondly, we analyze the distance measurement error model as a function of the angle between the incident laser beam and the tangent plane of the measured surface, and propose a weighted correction method based on the measurement error model, which greatly improves the calibration accuracy. Numerical simulation and application experiments show that the algorithm has good robustness, versatility, suitable for point cloud registration with a small number. The registration error can achieve an order of magnitude lower than the measurement error of laser range finder.

2. Calibration Principle & Error Analysis

2.1. ICP Algorithm
The basic idea of ICP is to realize the best results of registration between point sets through the iterative operation. Assume that the data point set is Q, while the number of data point set is $N_q$, and reference point set is X. Besides, assume that the closest point set searched from the reference point set and corresponding to the measured point set is P, then, $P \in X$. The objective function is represented as follows:

$$
\varepsilon_{\text{ICP}}(p,q) = \sum_{i=1}^{N_q} (p_i - R_q T q_i)^2
$$

(3)

The final purpose of point set registration is to realize the minimization of such objective function. The procedure of ICP algorithm is as follows: (1) Transform the data point set with initial transformation matrix R and Y. (2) Search the closest point pairs in the data point set and the reference point set. (3) Form the cross-covariance matrix through the searched closest point pairs, and calculate the transformation matrix R and Y minimizing the objective function value. (4) Go back to the Step (1) with renewed R and Y. When the registration error does not decrease, or when the iterative times are reached, the iteration ends.

ICP algorithm suffers from three problems. First, it needs a good initial value\cite{5}, otherwise it may fall into the local optimum. Second, during the registration process, the data point set shall be the subset of reference point set. Therefore, a large reference point set is required. Third, in the original ICP algorithm, the contribution of each point to the error function is the same, as shown in Equation 3. However, as we will see in this subsection, the angle between the incident laser beam and the tangent plane of the measured surface will influence the measurement error. Thus, each point in the data set should not be treated equally.

2.2. Uncertainty Analysis of Single Point Position of Laser Range Finder
The measurement error of 3D Laser Range Finder mainly comes from two sources\cite{6}: the direction uncertainty when the laser transmitted from the 3D Laser Range Finder aims at the target point, and the distance error resulted from measuring along the laser incident ray. Assume the measurement point coordinate system of the Laser Range Finder is $O_{\text{ref}}$. As shown in figure 1, the incident direction of Laser sensor is $L_s$, while $L_s$ is located within the surface where x axis and y axis are located, and the angles between the incident direction and Laser Range Finder coordinate system are $\alpha, \beta, \phi$ respectively. The following shall be satisfied:

$$
\alpha + \beta = \pi/2 \quad \text{and} \quad \phi = \pi/2
$$

(4)
Define a scanner coordinate system $O_{scv}$ along the $L_i$ direction and vertical to the $L_i$ space plane. During the measurement process, return the relative angle between the sensor and measured object. The coordinates of measured point under the $O_{scv}$ are as follows:

\[
x = l \cos \alpha \quad y = l \sin \alpha \quad z = 0
\]  

Due to the uncertainty of laser direction, a certain deviation exists between the actual incident direction of laser $L_a$ and the ideal incident direction $L_i$, as shown in figure 2. Along the direction of $L_i$, the positional uncertainty resulted from the theoretical error of Laser Range Finder is $\sigma_a$. Besides, along the direction vertical to $L_i$, the positional uncertainty is $l \sin \sigma_a$. Therefore, the positional uncertainty along the direction of $u, v, w$ axis shall be as follows:

\[
V_{scv} = (l^2 (\sin \sigma_a)^2, l^2 (\sin \sigma_a)^2, \sigma_a^2)
\]  

As per the positional uncertainty of measured point under the coordinate system $O_{scv}$, the positional uncertainty $V_n$ of measured point along the arbitrary direction $n$ under the coordinate system $O_{scv}$ may be obtained by utilizing the linear error propagation law. As shown in Figure 3, assume that the included angle between $n$ and laser incident direction $L$ is $\theta$, and let $v = n \times z$ and $v = n \times z$. The projection of $u, v, w$ on $n$ shall be as follows:

\[
(u, n) = \frac{z \times v}{|z \times v|} = |\sin \theta|
\]

\[
(w, n) = -\cos \theta
\]

\[
(v, n) = 0
\]

Therefore, under the laser measurement point coordinate system, the positional uncertainty of measurement point along the arbitrary direction $n$ shall be as follows:

\[
V_n = (|\sin \theta|, 0, -\cos \theta) V_{scv} (|\sin \theta|, 0, -\cos \theta)^T
\]

When the value $\sigma_a$ is very small, $\sin \sigma_a \approx \sigma_a$. Therefore,

\[
V_n = \sin^2 \theta l^2 \sigma_a^2 + \cos^2 \theta \sigma_a^2
\]

where, $0 \leq \theta < \pi/2$. When $n$ is the laser incident direction $L$, $\theta = 0$. Then, only the radial uncertainty $\sigma_r^2$ along the $L_i$ direction exists.

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**Figure 1.** Laser and scanner coordinate system diagram.

**Figure 2.** Incident direction of laser.

**Figure 3.** Laser measuring point error.

**Figure 4.** Point to plane optimization schemes.
3. GA-ICP-WHT Automatic Calibration Algorithm

At the actual calibration scenario, the acquisition of reference point set is mainly obtained with a total station, and the data point set is received from the laser range finder to be calibrated. The proposed calibration or registration scheme mainly consists of two steps: the coarse registration and fine registration. The coarse registration adopts Genetic algorithm to provide good initial registration value. And the fine registration mainly intends to get the optimal transform parameters between two coordinate systems. The proposed calibration algorithm is called GA-ICP-WHT.

3.1. GA-based Coarse Registration

The genetic algorithm (GA) is a sort of optimal method \[^7\] for simulating the genetics and evolution process of the biology. It is required to consider the possible solution in the problem domain as the individuals and conduct the gene code upon the individuals. During the GA-based coarse registration process, the variables to be optimized are the deflection angles \(\alpha, \beta, \theta\) of the Laser Range Finder coordinate system rotating around the three axes of reference coordination system and the translations \(\Delta x, \Delta y, \Delta z\) of the origin of Laser Range Finder coordinate system relative to the origin of extrinsic reference coordinate system. Define

\[
\omega = [\alpha, \beta, \theta, \Delta x, \Delta y, \Delta z]
\]  

(10)

(1) Population Initialization

It is required to make the binary encoding upon \(\omega\), while the population size is \(L=100\). In order to ensure that the initial point set located at variable positions satisfy the same feasible solution region, it is required to firstly adjust the laser data point set and reference point set gravity to the original position. The scope of deflection angle is: \(\alpha, \beta, \theta \in (-\pi/2, \pi/2)\). The scope of migration is the maximum value of the corresponding coordinate difference of reference point set. Assume that \(p_{1i}, p_{2i}, p_{3i}, i=1,2,...,N_p\) are the coordinate value of reference point set \(P\) on the \(X, Y, Z\) axis, while \(N_p\) is the number of reference point set.

\[
t_j = \max(p_{j}) - \min(p_{j}), j = 1,2,3 \quad \Delta x \in (-t_1, t_1) \quad \Delta y \in (-t_2, t_2) \quad \Delta z \in (-t_3, t_3)
\]  

(11)

Within the threshold value of feasible solution, the initial population \(\omega_i, i=1,2,...,L\), will be randomly generated.

(2) Determination of Fitness Function

The fitness function is the evaluation criteria of individual quality. The registration objective function of coarse registration will select the square sum of the average distance of the corresponding point pair from the measurement point set and reference point set after the coordination transformation, as shown below:

\[
F(\omega) = \frac{1}{N_q} \sum_{i=1}^{N_q} \|q_i - R(\alpha, \beta, \theta)p_i - T(Vx, Vy, Vz)\|^2
\]  

(12)

The better the individual gene is, the smaller the average distance between the corresponding point pairs and the value of fitness function. Therefore, the fitness function takes the following form:

\[
S(\omega) = 1/F(\omega)
\]  

(13)

In order to accelerate the search speed, the corresponding point set will adopt the k-d tree-based closest point search method.

(3) Genetic Operation

The selection shall adopt the roulette wheel selection method, while the crossover shall adopt two randomly selected crossover positions method, while the crossover probability is 0.7. The mutation
shall adopt the randomly selected mutation position to make the mutation, while the mutation probability is 0.02.

(4) Stop Conditions

When the objective function value between two-point sets is less than the threshold value $\delta = 200$, the GA will stop and coarse registration is finished.

### 3.2. ICP-WHT based Fine Registration Method

GA based coarse registration can provide a good initial value for ICP algorithm. Here we propose to use Hong-Tan (HT) algorithm to improve the accuracy due to the sparsity of reference data set. HT algorithm was originally proposed to solve the workpiece positioning problem \[8\]. Similar to the ICP algorithm, it is also the process of searching the corresponding points, obtain the transformation matrix and then iterate. In the HT algorithm, however, it adopts the distance from point to tangent plane as the objective function. As shown in figure 4, the objective function is as follows:

$$e_{icp-HT}(p,q) = \sum_{i=1}^{Nq} (p_i - Rq_i - T, n_i)^2$$

(14)

where, $(p_i, q_i)$ is the corresponding point set searched based on the closest point among the reference point set $p$ and laser data point set $q$. The value of optimization object function represents the distance square sum of tangent plane from the transformed $q_i$ to point $p_i$, namely, $(p_i - Rq_i - T, n_i)$ represents the projection of the distance between corresponding points at the normal direction of reference point $p_i$, after the coordinate transformation. However, in the ICP algorithm, the optimization object function which intends to evaluate the registration quality of two-point sets is the distance square sum of corresponding points after the coordinate transformation.

Assume that in the reference point set, the corresponding point $q_i$ in the strict sense is $p_i$. When the HT objective function is adopted, and only if the searched homonymy point $p_i$ is on the direction in parallel to the tangent plane where $p_i$ is located, the objective function value is the same. If the estimation of tangent plane is correct, very high registration precision will be reached.

Therefore, in the actual calibration scenario, the optimization object function of distance square sum from the point to tangent plane shall be adopted. Such measurement is more precise, which is not only free from the dependence upon the very large point set, but also greatly enhances the registration precision. The coordinate conversion matrix for the least square approximation of equation (14) may be obtained by solving with the helical motion-based linearization method. The solution formula is as follows \[9\]:

$$
\begin{pmatrix}
\sum_{i=1}^{N} (p_i \times n_i) y' \\
\sum_{i=1}^{N} n_i (p_i \times n_i) y'
\end{pmatrix} =
\frac{1}{\sum_{i=1}^{N} (p_i \times q_i \times n_i)}
\begin{pmatrix}
\sum_{i=1}^{N} (p_i \times q_i \times n, n_i) \\
\sum_{i=1}^{N} (p_i \times q, n_i)
\end{pmatrix}
$$

(15)

During the analysis of 3D laser range finder provided in section 2, we found that the explicit expression of position uncertainty of laser measurement point in the arbitrary direction is as follows:

$$V_\theta = \sin^2 \theta \sigma_\theta^2 + \cos^2 \theta \sigma_\theta^2$$

(16)

As shown in figure 5, the position error of laser measurement $q_i$ along the direction of normal vector $n_i$ where its corresponding point is located, will affect the distance from point $q_i$ to the corresponding tangent plane, and further have greater impact upon the matching precision. Therefore, in the optimization object function of ICP-HT Algorithm, we propose to add a weighting factor. For the point of greater measurement error on the $n_i$ direction, the smaller weight will be allocated. Otherwise, the larger weight will be allocated. The weight size is as shown below:

$$\varphi_i = 1/V_{n_i} = \left(\sin^2 \theta \sigma_\theta^2 + \cos^2 \theta \sigma_\theta^2\right)^{-1}$$

(17)
Therefore, the optimization object function (15) may be modified as follows:

$$e_{ICP-WHT} = \phi \sum_{i=1}^{Nq} (p_i - Rq_i - T, n_i)^2$$  \hspace{1cm} (18)

Similarly, the coordinate transformation parameter $\omega$ for the least square approximation of equation (19) may be obtained by the linear equation as shown below:

$$A \omega = b$$  \hspace{1cm} (19)

It may be obtained by adopting the Gaussian Elimination method, where

$$A = \sum_{i=1}^{Nq} \begin{bmatrix} (p_i \times n_i)(n_i)^T & (p_i \times n_i)(p_i \times n_i)^T \\ n_i(p_i \times n_i)^T & n_i(p_i \times n_i)^T \end{bmatrix} \in \mathbb{R}^{6 \times 6}, \quad b = \sum_{i=1}^{Nq} \begin{bmatrix} (p_i - q_i, n_i) \end{bmatrix}$$  \hspace{1cm} (20)

The ICP Algorithm optimized and enhanced by introducing the Hong-Tan Algorithm is called the ICP-HT. After finally adding the weight optimization links, the algorithm is called the ICP-WHT.

4. Numerical Simulation and Error Analysis
Calibration requires a set of reference points and a set of data points. Assuming that the calibrated plane is a rectangular section, the laser range finder outputs a point every 0.5 degree in the range of 180 degrees to obtain 360 discrete sets of data points. At the same time, 35 points were randomly selected from the rectangular section as the reference point set. In the rectangular section, the position distribution of sensor data point set and reference point set is shown in figure 5 (The unit is in mm). In the simulation, the measurement error is added to the data point sets, according to Equation 9 with $\sigma_0 = 0.003$.

The effect after coarse registration is shown in figure 6. Coarse registration improves the precision registration rate and trend. After coarse registration, three different algorithms including ICP, ICP-HT and ICP-WHT were used for fine registration. The registration error was defined as the coincidence degree after the point registration at the same position. The registration error is

$$r_{rms} = \frac{1}{N_q} \sum_{q=1}^{N_q} \| Rq_y + T_y \| - (R_{bd}q_y + T_{bd}) \|$$  \hspace{1cm} (21)

where, $R_y$ and $T_y$ are the real coordinate transformation matrix, $R_{bd}$ and $T_{bd}$ are the coordinate transformation matrix obtained by the calibration algorithm.

**Figure 5.** The points position distribution.  
**Figure 6.** After coarse registration.
Table 1. Three algorithms calibration results.

| Parameter | True value | GA-ICP | GA-ICP-HT | GA-ICP-WHT |
|-----------|------------|--------|-----------|------------|
| α(°)      | 45.00      | 45.16  | 45.04     | 44.99      |
| β(°)      | 30.00      | 29.17  | 29.97     | 30.00      |
| θ(°)      | 60.00      | 60.05  | 60.02     | 60.00      |
| x(mm)     | 3000.00    | 2957.80| 2998.00   | 2999.40    |
| y(mm)     | 1000.00    | 1028.70| 1004.30   | 1001.40    |
| z(mm)     | -100.00    | -54.30 | -99.00    | -100.00    |
| Error(mm) | 0.00       | 112.27 | 4.21      | 1.48       |

The registration results are shown in table 1. It can be found that the ICP-HT algorithm has higher accuracy than the original ICP algorithm. After adding point-wise weighting, the effect of laser point set error on the objective function is fully considered and the registration accuracy is further improved in ICP-WHT. The probability distribution histogram of registration error is shown in figure 7. The mean and variance of registration error are shown in Table 2.

5. Field Verification and Conclusion
The proposed calibration algorithm GA-ICP-WHT has been used in Guangzhou Metro tunnel deformation detection system. In this system, two laser range finders are mounted on the detection vehicle at a certain angle, in order to get the full-section data. The laser range finder is the LMS500 manufactured by the SICK Company, with a measuring range of 190°.

Figure 8. Rectangular tunnel, point distribution before registration and after GA-ICP-WHT.
In order to transform the scan sensor's own coordinate system into the external reference coordinate system of the railway line, a total station is used to collect a certain number of points in the calibrated section as the reference point set. Then GA-ICP-WHT calibration algorithm is used to calibrate the data point set collected by the sensor so as to transform the data point set from the sensor coordinate system to the reference coordinate system. The results are shown in Figures 7 and 8.

In conclusion, the GA-ICP-WHT algorithm is proposed to calibrate the data point set collected by laser scanning sensors, and numerical study and field test show proves the feasibility of the algorithm. The proposed GA-ICP-WHT algorithm can be used to estimate the pose transformation relationship among multiple laser scanning sensors or between one sensor and a reference coordinate system.

6. References
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