Multi-point optimization on the diffuser of an axial flow pump

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Abstract. For many pump applications, it is necessary to satisfy the performance requirements in more than one operating point. The conventional single-point design method which would cause a sharp decrease in the off-design point cannot fully meet such requirement. In this paper, an approach of the pump diffuser optimization is used to satisfy the performance in two points simultaneously. The three coefficient of the quadratic polynomial which is used to control the three inlet blade angles corresponding to the hub, shroud and the stream surface between (span wise=0.5) are selected as design parameters. Head, efficiency and power of the pump in the two selected point are selected as objective functions. The objective functions in the two selected points are in relations of trade-off. Design of experiments (DOE), steady CFD simulation, response surface method (RSM), Neighborhood Cultivation Genetic Algorisms (NCGA) are used to solve this problem. The DOE theory is applied to reduce the number of tests, three-dimensional simulations are performed to predict the pump performance, the RSM (response surface method) is used to correlate the pump performance to the intermediate variable, NCGA is used to search the pareto solutions along the response surface. The multi-point design optimization method is proved effective in searching the pareto solutions to satisfy the given requirements.

1. Introduction
Axial flow pumps are widely used in industry and agriculture, in many cases of its applications, to satisfy the performance requirements in only point is far from enough. The conventional single-point design theory which would cause a sharp decrease in the off-design point is not applicable in this circumstance. The performance in high flow rate condition is easily guaranteed in the axial flow pump, while that in low flow rate condition is hardly attained with excessive power and low head at the same rotational speed. Joseph [1] developed a meanline pump-flow modeling method to predict the performance of pumps at off design operating conditions. Sun et al. [2] applied CFD model to predict off-design performance of diffuser pumps. Cheah et al. [3] investigated the complex internal flow in a centrifugal pump with standard k-ε model, flow separation is observed at off-design point.

As a result, a new method for multi-point design optimization is needed to solve this problem. The multi-objective design optimization method has been applied in the field of aerodynamics by some researchers [4-8]. In the application of pumps, Neumann [9] established the relationship between geometry and performance of a centrifugal pump by means of loss analysis inside the pump. Takayama Y et al. [10] tried to optimize the meridional profile of the impeller and diffuser of a mixed-flow pump to meet the demands on efficiency, shut-off shaft power and instability characteristics. Rong Xie et al [11] optimized the shape of the impeller of a mixed flow pump to improve the
efficiency. As described above, there are few articles about the multi-point design optimization for the diffuser of a mixed-flow pump.

The axial flow pump to be optimized in this paper is used as a circulating pump, which mainly operated at the point of 5000m$^3$/h, while the performance in the capacity of 3750m$^3$/h must be satisfied in certain circumstances. DOE (design of experiment), CFD, RSM (response surface model) and NCGA (Neighborhood Cultivation Genetic Algorisms) are applied in the optimization. The method presented in this paper helps designers to design the qualified diffusers for axial-flow pump or mixed-flow pump in a short time.

2. Multi-point design optimization

The two-point design problem settled in this paper can be described as the two-point design of a pump with a fixed impeller, position and diameter of the inlet and outlet, which result in a 90 degree bend following the exit of the diffuser. The 90 degree bend causes hydraulic loss and has a significant influence on the performance of the pump.

2.1. Design parameters and objective functions of the optimization

The shape of a diffuser for an axial flow pump is defined by a meridional profile and several blade sections at the radial uniformly distributed stream surfaces. The blade section is determined by inlet blade angle and outlet blade angle. With the meridional profile of the diffuser designed in this article fixed and outlet blade angle set at 90 degree, the inlet blade angles of the three blade sections corresponding to the hub, shroud and the stream surface between(span wise=0.5) are selected to control the shape of the diffuser. A quadratic polynomial is applied to define the three inlet blade angles:

$$\beta = Ax^2 + Bx + C$$

- $10 \leq A \leq 10$
- $0 \leq B \leq 25$
- $20 \leq C \leq 60$

$Hub: x = 0$

$Middle: x = 0.5$

$Shroud: x = 1$

As a result, the coefficient A, B, and C are selected as the design parameters, while the three blade angles ($\beta_1$, $\beta_2$ and $\beta_3$ in degree) are considered as intermediate variable.

The objective functions are the head, power and efficiency at the two selected volume flow rate: 5000m$^3$/h and 3750m$^3$/h, as shown in table 1.

| Q(m$^3$/h) | Symbols | Remarks |
|-----------|---------|---------|
| 5000      | $P_1$   | Minimum |
|           | $\eta_1$| Maximum |
| 3750      | $P_2$   | Minimum |
|           | $\eta_2$| Maximum |

2.2. Design optimization method

The flow chart of the optimization process is showed in figure 1.

First of all, the appropriate design parameters are selected, after that, the DOE theory is used to select the combination of design parameters for analysis, the corresponding geometries are built by UG. Next, the CFD analysis are performed by the commercial software FLUENT. Finally, the response surface model is obtained and the optimum geometric model is obtained through searching
along the response surface by the optimization algorithm NCGA. The response surface model and the search of the optimum geometry is accomplished by the commercial software ISIGHT.

![Flow chart of the optimization design](image1)

**Figure 1.** Flow chart of the optimization design

3. Design of experiments and Numerical simulation

The DOE theory is applied in this study to investigate the effects of selected parameters efficiently. The three design parameters were configured on 3 levels which result in 27 kinds of combinations. The L16 orthogonal table was introduced to reduce the test number to 16. In the post processing, the importance of different design parameters can be valued schematically in main effects graphs and pareto graphs.

![Geometry and computational grid](image2)

**Figure 2.** Geometry and computational grid

Three-dimensional steady simulations were performed to obtain pump performances parameters (head, power and efficiency) of the axial flow pump with different diffusers at the two selected volume flow rates. An example of geometry and computational grid is shown in figure 2.  

The MRF model is applied in the cell zone conditions, the turbulence model is set as standard k-ε model. The SIMPLEC strategy is applied to deal with velocity–pressure coupling. The boundary conditions for the inlet is set as velocity-inlet in which the velocity is kept constant and the pressure will rise to provide the prescribed velocity distribution, i.e. normal to the boundary. The pressure-outlet is used in the outlet boundary condition to maintain a constant static pressure at the outlet.

4. Results and analysis
Fourteen sets of pump performance were obtained after CFD analysis. Then the 14 sets of data was imported ISIGHT to perform optimization. The workflow of the optimization is represented schematically in figure 3. The response surface method was first used to correlate the pump performance to the intermediate variable, the calculator component named ‘Quadratic’ was used to correlate the design parameters to the intermediate variables. Finally, NCGA was performed to search the optimal solutions by changing the inputs and comparing the outputs in the optimization component without need for new diffuser design and numerical simulations.

The $R^2$ coefficient and $R_{adj}^2$ coefficient was applied to evaluate precision of the approximations. In this approximation, $R^2$ (0.997~1) and $R_{adj}^2$ (0.995~0.999) were considered of high acceptance. Once the response surface is obtained, it is convenient and fast to find the suitable design parameters to satisfy new demands.

The pareto graphs shown in Figure 4 reveal the effects of design parameters on the pump performances. The blue bar represents positive effect while the red one represents negative effect. As can be seen in figure 4, the performance parameters in the two selected flow rates are in relations of trade-off. Among the three design parameters, the parameter C or terms related to it have the most significant influence on the response, while the parameter A has minimal effects on the response, the effects of parameter B lies between.
As a matter of fact that in 90% of the running time the axial flow pump is operating at the capacity of 5000 m³/h, so the weight of P₁(η₁) is set at 0.9 and that of P₂(η₂) is set at 0.1. The population size of NCGA is 100, number of generation is 100. Three optimization were performed, the objectives of them are list below:

1). Without constraining H₁ and H₂, the optimization to find the minimum of P₁ and P₂ is performed, its pareto front is shown in figure 5 (a).

2). Without constraining H₁ and H₂, the optimization to find the maximum of η₁ and η₂ is performed, its pareto front is shown in figure 5 (b).

3). The head in the two selected volume flow rates were constrained as required in section 2, the objective is to find the minimum of P₁ and P₂. The pareto front is shown in figure 5(c).

The blue points in figure 5 refer to pareto solutions, the black ones refer to feasible solutions.

| Table 2. Pareto solutions validated by CFD |
|-----------------|------|------|------|------|------|------|------|------|------|
| A   | B     | C     | H₁(m) | H₂(m) | P₁(kW) | P₂(kW) | η₁(%) | η₂(%) | P(kW) | η(%) |
|-----|-------|-------|-------|-------|--------|--------|-------|-------|-------|------|
| 1   | -2.1  | 17.2  | 58.1  | 4.71  | 8.74   | 91.84  | 121.57 | 69.8  | 73.3  | 94.81 | 70.11 |
| 2   | -7.6  | 15.9  | 58.1  | 4.78  | 8.84   | 91.94  | 121.35 | 70.7  | 74.3  | 94.89 | 71.08 |
| 3   | -7.0  | 23.0  | 35.4  | 4.50  | 9.20   | 93.88  | 122.13 | 65.2  | 76.9  | 96.70 | 66.34 |
| 4   | -7.6  | 22.9  | 52.9  | 4.74  | 8.94   | 92.08  | 121.87 | 70.0  | 74.8  | 95.06 | 70.54 |
| 5   | -12.4 | 18.9  | 23.25 | 1.58  | 9.32   | 95.52  | 125.10 | 22.5  | 76.0  | 98.48 | 27.8  |

Figure 6. Diffuser models refer to table 2

The pareto solutions selected from the pareto front obtained in the three optimization mentioned above are validated by CFD, the results of them are presented in table 2, from row one to row
three. Row 4 and row 5 refer to the diffusers which are designed in 5000 m$^3$/h and 3750 m$^3$/h with the application of one-point design theory. The last two columns in bold are the objective functions with the weight taken into consideration.

As can be seen in table 2: In row 1, the power in the two selected flow rates is smaller than that in row 4 and 5; in row 2, the objective function $\eta$ which has taken the weights into account is increased after optimization; in row 3, the head obtained from the pareto solutions has satisfied the demands mentioned in section 2. In summary, the optimization design method introduced in this paper is effective. Figure 6 shows the diffuser models refer to table 1.

5. Conclusions
This paper presents a multi-point optimization method for the diffuser of an axial flow pump to satisfy the performance requirement such as head, power and efficiency in more than one point. The three coefficient of the quadratic polynomial which is used to control the three inlet blade angles corresponding to the hub, shroud and the stream surface between (span wise=0.5) are selected as design parameters. The DOE theory is applied to reduce the number of tests. Three-dimensional steady simulations are performed to predict the pump performance, the RSM (response surface method) is used to correlate the pump performance to the intermediate variable, NCGA is used to search the pareto solutions along the response surface. The head, power, efficiency in the two selected volume flow rates are in relations of trade-off. The design parameter C has the most significant effects on the pump performance. Once the response surface is obtained, it is convenient and fast to find the suitable design parameters to satisfy new demands. The multi-point design optimization method is proved effective in searching the pareto solutions to satisfy the given requirements.

References
[1] Veres J P 1994 *Centrifugal and axial pump design and off-design performance prediction* (USA: NASA)
[2] Sun J and Tsukamoto H 2001 *Proc. of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy* 215 191-201
[3] Cheah K W 2007 Numerical flow simulation in a centrifugal pump at design and off-design conditions *International Journal of Rotating Machinery*
[4] Fan H Y, Xi G and Wang S J 2000 *Proc. of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy* 214 187-190
[5] Shahpar S, Giacche D and Lapworth L 2003 Multi-objective design and optimisation of bypass outlet-guide Vanes *ASME Paper GT-2003-38700*
[6] Sugimura K, Obayashi S and Jeong S 2010 *Engineering Optimization* 42 271-293
[7] Demeulenaere A, Ligout A and Hirsch C 2004 Application of multipoint optimization to the design of turbomachinery blades *ASME*
[8] Chen B and Yuan X 2008 Advanced aerodynamic optimization system for turbomachinery *Journal of turbomachinery* 130(2)
[9] Neumann B 1991 *The interaction between geometry and performance of a centrifugal pump.* (UK, Mechanical Engineering Publications)
[10] Takayama Y and Hiroyoshi W 2009 Multi-objective design optimization of a mixed-flow pump *ASME*
[11] Xie R 2010 Numerical Study and Structural Improvement on Mixed-Flow Pump Impeller: Suppression of vortexes in mixed-flow pump impeller by controlling blade profile *Power and Energy Engineering Conference ( Yokohama, Japan, 7-10 December 2010)* pp 1-4