Limitations of PLL simulation: hidden oscillations in MatLab and SPICE

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Abstract—Nonlinear analysis of the phase-locked loop (PLL) based circuits is a challenging task, thus in modern engineering literature simplified mathematical models and simulation are widely used for their study. In this work the limitations of numerical approach is discussed and it is shown that, e.g. hidden oscillations may not be found by simulation. Corresponding examples in SPICE and MatLab, which may lead to wrong conclusions concerning the operability of PLL-based circuits, are presented.

I. Introduction

The phase-locked loop based circuits (PLL) are widely used nowadays in various applications. PLL is essentially a nonlinear control system and its rigorous analytical analysis is a challenging task. Thus, in practice, simulation is widely used for the study of PLL-based circuits (see, e.g. [1]–[4]). At the same time, simulation of nonlinear control system may lead to wrong conclusions, e.g. recent work [5] notes that stability in simulations may not imply stability of the physical control system, thus stronger theoretical understanding is required.

In this work the two-phase PLL is studied and corresponding examples, where simulation leads to unreliable results, is demonstrated in SPICE and MatLab.

II. PLL operation

Typical analog PLL consists of the following elements: a voltage-controlled oscillator (VCO), a linear low-pass filter (LPF), a reference oscillator (REF), and an analog multiplier ⊙ used as the phase detector (PD). The phase detector compares the phase of VCO signal against the phase of reference signal; the output of the PD (error voltage) is proportional to the phase difference between its two inputs. Then the error voltage is filtered by the loop filter (LPF). The output of the filter is fed to the control input of the VCO, which adjusts the frequency and phase to synchronize with the reference signal.

Consider a signal space model of the classical analog PLL with a multiplier as a phase detector (see Fig. 1).

Fig. 1: Operation of classical phase-locked loop for sinusoidal signals

Suppose that both waveforms of VCO and the reference oscillator signals are sinusoidal [1] (see Fig. 1). The low-pass filter passes low-frequency signal \( \sin(\theta_1(t) - \theta_2(t)) \) and attenuates high-frequency signal \( \sin(\theta_1(t) + \theta_2(t)) \).

The averaging under certain conditions [6], [7], [9] and approximation \( \varphi(t) \approx \sin(\theta_1(t) - \theta_2(t)) \) allow one to proceed from the analysis of the signal space model to the study of PLL model in the signal’s phase space. Rigorous consideration of this point is often omitted (see, e.g. classical books [12], p.12,p15-17, [13], p.7) while it may lead to unreliable results (see, e.g. [14], [15]).

One of the approaches to avoid this problem is the use of two-phase modifications of PLL, which does not have high-frequency oscillations at the output of the phase detector [16].

III. Two-phase PLL

Consider two-phase PLL model in Fig. 2.

Fig. 2: Two-phase PLL

Here a carrier is \( \cos(\theta_1(t)) \) with \( \theta_1(t) \) as a phase and the output of Hilbert block is \( \sin(\theta_2(t)) \). The VCO generates oscillations \( \sin(\theta_2(t)) \) and \( \cos(\theta_2(t)) \) with \( \theta_2(t) \) as a phase. Fig. 3 shows the structure of phase detector (complex multiplier). The phase detector consists of two

\[ \varphi(t) = \sin(\theta_1(t) - \theta_2(t)) \]

\[ \varphi(t) = \sin(\theta_1(t) + \theta_2(t)) \]

\[ \varphi(t) = \sin(\theta_1(t)) \cos(\theta_2(t)) - \cos(\theta_1(t)) \sin(\theta_2(t)) \]

Accepted to IEEE 7th International Congress on Ultra Modern Telecommunications and Control Systems, 2015

†‡ Other waveforms can be similarly considered [6], [8]
The relation between input \( \varphi(t) \) and output \( g(t) \) of the loop filter is as follows
\[
\dot{x} = Ax + b \varphi(t), \quad g(t) = c^*x + h \varphi(t),
\]
where \( \omega_2 \) is the VCO free-running frequency (i.e. for \( g(t) \equiv 0 \)) and \( L \) the VCO gain.

Next examples show the importance of analytical methods for investigation of PLL stability. It is shown that the use of default simulation parameters for the study of two-phase PLL in MatLab and SIMULINK can lead to wrong conclusions concerning the operability of the loop, e.g. the pull-in (or capture) range (see discussion of rigorous definitions in [17], [18]).

IV. SIMULATION IN MATLAB

Consider a passive lead-lag filter with the transfer function \( H(s) = \frac{1}{1+\tau_1 s + \tau_2 s^2} \), \( \tau_1 = 0.0448 \), \( \tau_2 = 0.0185 \) and the corresponding parameters \( A = -\frac{1}{\tau_1+\tau_2} \), \( b = 1 - \frac{\tau_2}{\tau_1+\tau_2} \), \( c = \frac{1}{\tau_1+\tau_2} \), \( h = \frac{\tau_2}{\tau_1+\tau_2} \). The model of two-phase PLL in MatLab is shown in Fig. 4 (see more detailed description of simulating PLL based circuits in MatLab Simulink in [19], [21]).

For the case of the passive lead-lag filter a recent work [22] p.123 notes that “the determination of the width of the capture range together with the interpretation of the capture effect in the second order type-I loops have always been an attractive theoretical problem. This problem has not yet been provided with a satisfactory solution”. Below we demonstrate that in this case a numerical simulation may give wrong estimates and should be used very carefully.

In Fig. 4 we use the block Loop filter to take into account the initial filter state \( x(0) \); the initial phase error \( \theta_\Delta(0) \) can be taken into account by the property initial data of the Intergator blocks. Note that the corresponding initial states in SPICE (e.g. capacitor's initial charge and inductor's initial currents) are zero by default but can be changed manually.

In Fig. 5 the two-phase PLL model simulated with relative tolerance set to “1e-3” or smaller does not acquire lock (black color), but the PLL model in signal’s phase space simulated in MatLab Simulink with standard parameters (a relative tolerance set to “auto”) acquires lock (red color). Here the input signal frequency is 10000, the VCO free-running frequency \( \omega_2^{\text{free}} = 10000 - 178.9 \), the VCO input gain is \( L = 500 \), the initial state of loop filter is \( x_0 = 0.1318 \) and the initial phase difference is \( \theta_\Delta(0) = 0 \).

V. SIMULATION IN SPICE

In this section the previous example is reconstructed in SIMetrix, which is one of the commercial versions of SPICE.

Consider SIMetrix model of two-phase PLL shown in Fig. 6. The input signal and the output of Hilbert block in Fig. 2 are modeled by sinusoidal voltage sources V1 (a
frequency parameter is $1.5915494k$ and $V2$ (a frequency parameter is $1.5915494k$ and a phase is 90) (sin_input and cos_input). A complex multiplier in Fig. 3 is modeled as two arbitrary sources ARB1 and ARB2 with definitions set to $V(N1) \times V(N2)$. To subtract the output signals of multipliers, Voltage Controlled Voltage Source (E3) is used. Phase detector gain (E5) is equal to $\frac{1}{2}$. Loop Filter in Fig. 2 is modeled as a passive lead-lag filter with resistor R2, capacitor C2, and resistor R1. The input gain of VCO (E6) is equal to $-500$. VCO self frequency (DC Voltage Source V3) is set to 9.8211k. Voltage Controlled Voltage Source E2 summarizes a VCO self frequency and a control signal from E6. Resistor R1b1 (10u), capacitor C1 (5G), and amplifier E1(50k) form an integrator. The VCO waveforms are defined by arbitrary blocks ARB3 (with the function $\sin(V(N1))$) and ARB4 (with the function $\cos(V(N1))$).

Netlist for the model, generated by SIMetrix, is as follows:

```plaintext
1 *#SIMETRIX
2 V1 sin_Input 0 0 Sine(0 1 1.5915494k 0 → 0)
3 V2 cos_input 0 0 Sine(0 1 1.5915494k → -157.03518u 0)
4 V3 vco_frequency 0 9.8211k
5 R1 C2_N 0 1.85k
6 R2 filter_out PD_output 4.48k
7 X$ARB1 sin_Input vco_cos_output
   ⇐ ARB1_OUT $arbsourceARB1
   ⇐ pinnames: N1 N2 OUT
8 .subckt $arbsourceARB1 N1 N2 OUT
9 B1 OUT 0 V=V(N1)∗V(N2)
10 .ends
11 X$ARB2 cos_input vco_sin_output E3_CN
   ⇐ $arbsourceARB2 pinnames: N1 N2
   ⇐ OUT
12 .subckt $arbsourceARB2 N1 N2 OUT
```

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<sup>4</sup>Zero input response (ZIR) frequency
<sup>5</sup>The same results could qualitatively be obtained using 10K for the resistor and 5 Farad for the capacitor (keeping $RC$ constant)
VI. Mathematical reasoning

Two-phase PLL is described by \(1\), \(3\), and \(2\), which form the following system of differential equations

\[
\dot{x} = Ax + \frac{b}{2} \sin(\theta),
\]

\[
\dot{\theta} = \omega - Lc_x - \frac{Lh}{2} \sin(\theta),
\]

\[
\theta(t) = \theta_1(t) - \theta_2(t), \quad \omega = \omega_1 - \omega_{free}
\]

For a lead-lag filter, described by the transfer function \(H(s) = \frac{1 + s\tau_2}{1 + s(\tau_1 + \tau_2)}\), system \(4\) takes the form

\[
\dot{x} = -\frac{1}{\tau_1 + \tau_2}x + (1 - \tau_2/\tau_1 - \tau_2/\tau_1 + \tau_2) \cdot \frac{1}{2} \sin(\theta),
\]

\[
\dot{\theta} = \omega - \frac{L}{\tau_1 + \tau_2}x - \frac{\tau_2}{\tau_1 + \tau_2} \cdot \frac{L}{\tau_1 + \tau_2} \sin(\theta).
\]

The equilibrium points of \(5\) are defined by the following relations:

\[
x_{eq} = \tau_1/2 \sin(\theta_{eq}), \quad \sin(\theta_{eq}) = 2 \cdot \omega_{eq} \cdot L.
\]

For \(\tau_1 = 0.0448\), \(L = 500\), and \(\omega_{eq} = 178.9\) we get

\[
x_{eq} = 0.016, \quad \theta_{eq} = (-1)^k \cdot 0.7975 + \pi k, \quad k \in \mathbb{N}.
\]

Consider now a phase portrait (where the system’s evolving state over time traces a trajectory \((x(t), \theta(t))\)), corresponding to signal’s phase model (see Fig. 8).

![Phase portrait of the classical PLL with stable and unstable periodic trajectories](image)

The solid blue line in Fig. 8 corresponds to the trajectory with the loop filter initial state \(x(0) = 0.005\) and the VCO phase shift 0 rad. This line tends to the periodic trajectory, therefore it will not acquire lock. All the trajectories under the blue line (see, e.g., a green trajectory with the initial state \(x(0) = 0\)) also tend to the same periodic trajectory.

The solid red line corresponds to the trajectory with the loop filter initial state 0.00555 and the VCO initial...
An oscillation in a dynamical system can be easily localized numerically if the initial data from its open neighborhood lead to a stable equilibrium. In this case PLL acquires lock.

All the trajectories between stable and unstable periodic trajectories tend to the stable one (see, e.g., a solid green line). Therefore, if the gap between stable and unstable periodic trajectories is smaller than the discretization step, the numerical procedure may slip through the stable trajectory. This example demonstrates also the difficulties of numerical search of so-called hidden oscillations, whose basin of attraction does not overlap with the neighborhood of an equilibrium point, and thus may be difficult to find numerically. In this case the observation of one or another stable solution may depend on the initial data and integration step.

VII. Conclusion

The considered example is a motivation for the use of rigorous analytical methods for the analysis of nonlinear PLL models. Rigorous study of the above effect can be done by Andronov’s point transformation method and phase plane analysis. Corresponding bifurcation diagram were given in [49] (see, also [11], [28]).

For Costas loop models, similar effect is shown in [19], [20], [50].

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