Precoder Design for Correlated Data Aggregation via Over-the-Air Computation in Sensor Networks

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Abstract—Over-the-air computation (AirComp) enables efficient wireless data aggregation in sensor networks by simultaneous processing of calculation and communication. This paper proposes a novel precoder design for AirComp that incorporates statistical properties of sensing data, spatial correlation and heterogeneous data correlation. The proposed design of the precoder requires no iterative processes so that it can be realized with low computational costs. Moreover, this method provides dimensionality reduction of sensing data to reduce communication costs per sensor. We evaluate performance of the proposed method in terms of various system parameters. The results show the superiority of the proposed method to conventional non-iterative methods in cases where there are a large number of sensors and where the number of receive antennas at the aggregator is less than that of the total transmit antennas at the sensors.

Index Terms—Over-the-air computation, wireless sensor networks, wireless data aggregation, dimensionality reduction

I. INTRODUCTION

In the fifth or more generation communication systems, one of the core technologies is to connect a large number of Internet-of-Things (IoT) devices that have abilities of sensing, computation, and wireless communication and to utilize their sensing data for many practical applications [1], [2]. There are a lot of active applications of sensor networks composed of the sensing devices, such as in agriculture [3] and in environmental monitoring [4], [5]. In centralized data processing, sensing data of the distributed devices are collected via wireless communication at an aggregator that performs calculations to achieve desired actions for the applications. This procedure is called wireless data aggregation. It is desirable to achieve the wireless data aggregation with low latency for immediate response to demands in large-scale IoT networks.

The idea of over-the-air computation (AirComp) was first investigated in the field of information theory [6] and has been gathering much attention from a signal processing perspective in sensor networks and wireless data aggregation [2], [7]. AirComp enables fast wireless data aggregation by jointly receiving transmitted signals and calculating some function value of the sensing data. This is achieved by simultaneous transmission at sensor nodes over the same frequency band and by obtaining the sum of the transmitted signals with the analog-wave superposition property of wireless multiple-access channels (MAC). It is different from classical data aggregation settings such as time-division based schemes where all the data are separately received and then the function value is calculated at the aggregator. In AirComp, efficient processes of sensing data can be achieved with low latency and the required bandwidth does not depend on the number of sensor nodes so that it is suitable for large-scale IoT networks.

AirComp in sensor networks occurs aggregation errors, i.e., errors between the actual sum of transmitted signals from the sensors and the aggregated data on the air, due to different channel coefficients of the sensors and additive noise at the aggregator. To reduce the aggregation errors, scaling coefficients or matrices are applied as precoders to the transmit signals for the sensors. The optimization of them has been tackled in various contexts [8]–[13].

For designing an efficient precoder for AirComp, channel state information, transmit power constraints, and statistical properties of data at the sensors should be taken into consideration. Especially in sensor networks, sensing data, for example, temperature, humidity, amount of chemicals, and soil conditions, have spatial correlation and heterogeneous data correlation (correlation among data types) in general [14], [15]. Such correlation is often used in signal processing to improve system performance [16], [17]. However, the spatial correlation among sensors is ignored in many precoder design methods [8], [11]–[13] or eliminated within the calculation process [10].

Motivated by the fact, we propose a novel precoder design for AirComp in wireless data aggregation that introduces correlation among sensors and data types, i.e., both the spatial correlation and heterogeneous data correlation. We construct an optimization problem for designing the precoder by explicitly using the correlation. This method can be applied to general cases where each sensor transmits multiple values as a vector, which is not considered in conventional methods such as [9]–[11]. Moreover, we derive a closed-form solution without iterative procedures so that it achieves lower computational costs than the conventional method that requires solving convex programming problems for each iteration [8].

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In addition, this paper deals with a possible scenario in typical sensor networks. Sensing devices usually have tiny batteries so that it is important to reduce power consumption related to wireless communication. The conventional methods [12] and [13] require square or vertically long precoding matrices to avoid rank deficiency of zero-forcing-based precoders, which leads to dimensionality expansion of data. This procedure requires high communication costs for the sensors. On the other hand, the proposed precoder enables dimensionality reduction [18] of sensing data, which can reduce communication costs. We conduct computer experiments focused on a situation where dimensionality reduction is performed by the precoder.

II. PRELIMINARIES

A. Notations

In the rest of the paper, we use the following notation. Superscripts $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and the Hermitian transpose, respectively. The zero vector, zero matrix and identity matrix are represented as $\mathbf{0}$, $\mathbf{O}$, and $\mathbf{I}$, respectively. $\ell_2$-norm is $\| \cdot \|$. The complex circularly symmetric Gaussian distribution $CN(\mathbf{0}, \mathbf{\Sigma})$ has mean vector $\mathbf{0}$ and covariance matrix $\mathbf{\Sigma}$. The expectation and trace operators are $\mathbb{E}[\cdot]$ and $\text{Tr}[\cdot]$, respectively. We denote the set of complex block diagonal matrices with $k$ diagonal blocks of $m \times n$ matrices as $\mathbb{B}_{m \times n}^k$. The function $(\alpha)^+$ for $\alpha \in \mathbb{R}$ denotes $\max(0, \alpha)$. Hadamard product is represented as $\otimes$, which is the elementwise multiplication of matrices.

B. System Model

Assume a wireless data aggregation system with a single aggregator with $r$ receive antennas and $K$ sensor nodes with $m$ transmit antennas per node as illustrated in Fig. 1.

Let $d_k \in \mathbb{R}^n$ ($k = 1, \ldots, K$) be a vector composed of $n$ measurements at $k$th node and $d_{k\ell} \in \mathbb{R}$ ($\ell = 1, \ldots, n$) be $\ell$th element of the vector. The size should be set to $m < n$ to apply dimensionality reduction, but the following method is not limited to this setting.

In many applications of sensor networks, the objective of the aggregator is to obtain some function value of sensors’ raw measurements. For example, arithmetic mean, weighted sum, or Euclidean norm is used as the function. Such functions can be represented by combination of pre- and post-processing functions of the measurements and have been named nomographic functions [7], [13]. The nomographic function $f_\ell(\cdot) : \mathbb{R} \to \mathbb{R}$ is applied to each $\ell$th element of the vectors and given by

$$f_\ell(d_1, \ldots, d_K) = \psi_\ell \left( \sum_{k=1}^K \varphi_{k\ell}(d_{k\ell}) \right),$$

(1)

where $\psi_\ell(\cdot) : \mathbb{R} \to \mathbb{R}$ is a post-processing function and $\varphi_{k\ell}(\cdot) : \mathbb{R} \to \mathbb{R}$ is a pre-processing function, respectively. For example, the elementwise weighted sum $f_\ell(d_1, \ldots, d_K) = \sum_{k=1}^K \omega_{k\ell} d_{k\ell}$ can be represented by the pre-processing function $\varphi_{k\ell}(\chi) = \omega_{k\ell} \chi$ and the post-processing one $\psi_\ell(\chi) = \chi$.

From the form of the nomographic function (1), the aggregator wants to know the sum $\sum_{k=1}^K \varphi_{k\ell}(d_{k\ell})$ via communication with the nodes. We redefine the pre-processed local function values at node $k$ as a vector $x_k = [\varphi_{k1}(d_{k1}), \ldots, \varphi_{kn}(d_{kn})]^T \in \mathbb{R}^n$ and then the elementwise sum can be summarized as $s = \sum_{k=1}^K x_k \in \mathbb{R}^n$. Assume that the further summarized vector $x = [x_1^T, \ldots, x_K^T]^T \in \mathbb{R}^{nK}$ follows $\mathbf{x} \sim CN(0, \mathbf{K})$. The covariance matrix $\mathbf{K} = \mathbb{E}[xx^H] \in \mathbb{R}^{nK \times nK}$ is positive definite and known to the aggregator. It includes information on the spatial correlation as the non-block diagonal elements and on the heterogeneous data correlation as the non-diagonal elements of each block.

Each node multiplies a linear precoder $A_k \in \mathbb{C}^{m \times n}$ by its own pre-processed vector $x_k$ for reducing aggregation error, that is, the node $k$ transmits

$$c_k = A_k x_k \in \mathbb{C}^m$$

(2)

to the aggregator. The size $m$ is assumed to be smaller than that of $x_k$ for dimensionality reduction. The aggregated signal through MAC is given by

$$y = H_1 c_1 + \ldots + H_K c_K + n$$

$$= \left( \sum_{k=1}^K H_k A_k x_k \right) + n \in \mathbb{C}^r,$$

(3)

where $H_k \in \mathbb{C}^{r \times m}$ is a channel matrix between node $k$ and the aggregator, and $n \in \mathbb{C}^r$ is the additive noise that follows $n \sim CN(0, \mathbf{S})$. The positive definite covariance matrix $\mathbf{S} \in \mathbb{C}^{r \times r}$ represents correlation of the noise vector and is known to the aggregator. The model (3) is summarized as

$$y = H A x + n,$$

(4)

where

$$H = [H_1, \ldots, H_K] \in \mathbb{C}^{r \times mK}$$

and

$$A = \begin{bmatrix} A_1 & O \\ & \ddots & \ddots \\ & & A_K \end{bmatrix} \in \mathbb{C}^{mK \times nK}.$$

The aggregator is assumed to know information of the statistical properties of the transmit signal $x$ and the noise $n$, the received signal $y$, and the channel matrices $\{H_k\}_{k=1}^K$. 

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III. PROPOSED METHOD
In this section, we describe how to design the precoder $A$ by a non-iterative procedure while including the correlation of data.

A. Optimization Problem for Design of Proposed Precoder
The objective at the aggregator is to obtain the sum
\[ s = \sum_{k=1}^{K} x_k = Qx, \]  
where $Q = [I, \ldots, I] \in \mathbb{R}^{n \times nK}$, as correctly as possible by using the available information because the received signal is distorted by the channel and noise. For this objective, we assume that the aggregator employs a linear MMSE estimate [19]
\[ \hat{s} = Wy = QKA^H H^H (HAKA^H H^H + S)^{-1} y. \]  
The matrix $W = QKA^H H^H (HAKA^H H^H + S)^{-1} \in \mathbb{C}^{n \times r}$ is determined by minimizing the MSE:
\[ \mathbb{E}[\|s - \hat{s}\|^2] = \mathbb{E}[\|WHAx + Wn - Qx\|^2]. \]  
In this paper, we explore a matrix $A$ that minimizes $\mathbb{E}[\|s - \hat{s}\|^2]$ of the linear MMSE estimation. Moreover, the limited power of the sensor nodes should be also taken into consideration for the design of the matrix $A$. The total transmit power of the nodes is $\sum_{k=1}^{K} \mathbb{E}[\|A_k x_k\|^2] = \mathbb{E}[\|Ax\|^2]$. Therefore, we consider the optimization problem:
\[ (P1) \quad \min_{A} \mathbb{E}[\|s - \hat{s}\|^2] \quad \text{s.t.} \quad \mathbb{E}[\|Ax\|^2] = P_0, \quad A \in \mathbb{B}_K^{m \times n}, \]  
where the first constraint means that the total transmit power is set to $P_0(>0)$ and the second constraint means that $A$ has a block diagonal structure. The problem $P1$ can be rewritten by substituting (6) into the cost function and results in an optimization problem of matrix function:
\[ (P2) \quad \min_{A} \text{Tr} \left[ Q \left( K^{-1} + A^H H^H S^{-1} H A \right)^{-1} Q^T \right] \quad \text{s.t.} \quad \text{Tr}[AKA^H] = P_0, \quad A \in \mathbb{B}_K^{m \times n}. \]  
The problem is nonconvex and difficult to solve in general. To make matters worse, the matrix $A$ to be optimized has a block diagonal structure, which complicates the optimization process.

B. Policy
This paper proposes a closed-form solution of the non-convex problem $P2$ along with the idea in the conventional methods [20], [21] proposed in a different context from AirComp. We first consider the problem:
\[ (P2') \quad \min_{A} \text{Tr} \left[ Q \left( K^{-1} + A^H H^H S^{-1} H A \right)^{-1} Q^T \right] \quad \text{s.t.} \quad \text{Tr}[AKA^H] = P_0. \]  
This is a relaxed problem of $P2$ where the block diagonal constraint is omitted. The problem $P2'$ is known to have an optimal solution when the number of nodes is $K = 1$ [20], [21]. We obtain the solution $\hat{A}$ of the problem $P2'$ by using diagonalization of some matrices as employed in [20] and [21]. Next, a block diagonal matrix $\hat{A}_{bd}$ is derived by removing non-block diagonal elements from $\hat{A}$. This approach of the block diagonalization has been employed in [22] for not-AirComp settings. Finally, we obtain the solution $\hat{A}$ by scaling the norm of $\hat{A}_{bd}$ to satisfy the power constraint $\text{Tr}[AKA^H] = P_0$.

The proposed procedure causes a loss of optimality when $K > 1$. However, it gives us a closed formula of the precoding matrix and allows derivation in lower computational costs than the related work [8]. We show the validity of the proposed method via experimental verification in Sect. IV.

C. Derivation of Non-Block diagonal matrix $\hat{A}$
This section derives non-block diagonal solution $\hat{A}$ by diagonalization of matrices in the objective function. We describe eigenvalue decomposition of the matrices $K$ and $H^H S^{-1} H$ as
\[ K = U \Delta U^T \in \mathbb{R}^{nK \times nK}, \]  
\[ H^H S^{-1} H = V \Lambda V^H \in \mathbb{C}^{mK \times mK}, \]  
respectively, where $U \in \mathbb{R}^{nK \times nK}$, $V \in \mathbb{C}^{mK \times mK}$ are unitary matrices, and $\Delta \in \mathbb{R}_{+}^{nK \times nK}$, $\Lambda \in \mathbb{C}^{mK \times mK}$ are diagonal matrices. The diagonal elements are represented as
\[ \Delta = \begin{bmatrix} \delta_1 & 0 & \cdots & 0 \\ 0 & \delta_2 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \delta_{nK} \end{bmatrix}, \]  
\[ \Lambda = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \lambda_{mK} \end{bmatrix}, \]  
respectively. Without loss of generality, we assume $\delta_1 \geq \delta_2 \geq \cdots \geq \delta_{nK} \geq 0$ and $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{mK} \geq 0$. When $r < mK$, the matrix $\Lambda$ has the property $\lambda_{r+1} = \cdots = \lambda_{mK} = 0$. We further assume that the matrix variable $A$ is decomposed as
\[ A = V \Phi U^T, \]  
\[ \Phi = \begin{bmatrix} \phi_1 & O & \cdots & O \\ \cdots & \cdots & \cdots & \cdots \\ O & \cdots & \cdots & \phi_{mK} \end{bmatrix}, \]  
where $\{\phi_j\}_{j=1}^{mK}$ are scalar parameters and satisfy $|\phi_j|^2 \geq 0$. That decomposed formulation is motivated by the work [20], [21]. Therefore, the solution of the problem $P2'$ is assumed to be fully determined by the $mK$ parameters.

From these diagonalized representations, we can solve the problem $P2'$ in terms of the parameters $\{\phi_j\}_{j=1}^{mK}$ as summarized in Theorem 1.

Theorem 1: The solution of the problem $P2'$ is given as the following water-filling problem
\[ |\phi_j|^2 = \begin{cases} \frac{1}{\delta_j \lambda_j} \left( \sqrt{\frac{\delta_j \lambda_j R_j}{\mu}} - 1 \right)^+, & (j = 1, \ldots, \min(r, mK)), \\ 0, & (j = \min(r, mK) + 1, \ldots, mK), \end{cases} \]  
where $R_j = \sum_{i=1}^{n} |(QU)_{ij}|^2$, $(QU)_{ij}$ is the $(i,j)$th element of $QU$, and $\mu \in \mathbb{R}$ is determined to satisfy the power constraint in the problem. For the case of $j = 1, \ldots, \min(r, mK)$,
if the elements in (·)⁺ in the right-hand side of (14) are nonnegative for all $j$, the solution is given by
\[
\hat{\delta}_j = \left( \frac{1}{\delta_j \lambda_j} \left( \sqrt{\delta_j \lambda_j R_j} P_0 + \frac{\min_{j \in [r,mK]} \frac{1}{\lambda_j}}{\sum_{j=1}^{mK} \sqrt{\delta_j \lambda_j R_j}} - 1 \right) \right). 
\] (15)

**Proof:** We represent the cost function of the problem $\mathcal{P}2'$ as $f((\phi_j)_{j=1}^{mK})$. The function can be rewritten as
\[
f((\phi_j)_{j=1}^{mK}) = \text{Tr} \left[ QU (\Delta^{-1} + \Phi^H \Phi)^{-1} U^T Q^T \right] 
\] (16)

by using (11)–(13). Note that the matrices $Q$ and $U$ are known at the aggregator and then it is possible to calculate the trace in (16) directly. The result of expanding the equation is
\[
f((\phi_j)_{j=1}^{mK}) = \min_{j=1}^{mK} \frac{\delta_j R_j}{1 + \delta_j \lambda_j |\phi_j|^2} + \sum_{j=\min(r,mK)+1}^{nK} \delta_j R_j. 
\] (17)

On the other hand, the transmit power $\text{Tr}[\hat{A}K\hat{A}^H]$ in the problem $\mathcal{P}2'$ is also given by
\[
\text{Tr}[\hat{A}K\hat{A}^H] = \text{Tr}[\Phi \Delta \Phi^H] = \sum_{j=1}^{mK} \delta_j |\phi_j|^2. 
\] (18)

In order to obtain the solution of the problem $\mathcal{P}2'$, we set the following Lagrangian function
\[
\mathcal{L}((\phi_j)_{j=1}^{mK}, \mu) = f((\phi_j)_{j=1}^{mK}) - \mu \left( P_0 - \sum_{j=1}^{mK} \delta_j |\phi_j|^2 \right), 
\] (19)

where $\mu$ is the Lagrange multiplier. The condition in this case is $\frac{\partial \mathcal{L}}{\partial |\phi_j|^2} = 0$. By solving $\frac{\partial \mathcal{L}}{\partial |\phi_j|^2} = 0$ in terms of $|\phi_j|^2$, we can obtain
\[
|\phi_j|^2 = \left\{ \begin{array}{ll} \frac{1}{\delta_j \lambda_j} \left( \sqrt{\delta_j \lambda_j R_j} \mu \right)^+ - 1, & (j = 1, \ldots, \min(r,mK)), \\ 0, & (j = \min(r,mK)+1, \ldots, nK), \end{array} \right. 
\] (20)

where the function $(·)^+$ is applied because $|\phi_j|^2 \geq 0$.

If the arguments in the right-hand side of (20) become nonnegative for all indeces $j$, another relation can be derived by substituting (20) into $\frac{\partial \mathcal{L}}{\partial |\phi_j|^2} = 0$ and we then have
\[
\frac{1}{\sqrt{\mu}} = \frac{P_0 + \sum_{j=1}^{\min(r,mK)} \frac{1}{\lambda_j}}{\sum_{j=1}^{\min(r,mK)} \sqrt{\delta_j \lambda_j R_j}}. 
\] (21)

From (20) and (21), we can obtain the solution $\hat{\phi}_j$.

If there exists index $j$ where the argument in the right-hand side of (20) becomes negative, the Lagrange multiplier $\mu$ is determined by the well-known water-filling algorithm (Sect. 3.E in [23]) to satisfy the power constraint.\[ \square \]

We can obtain the matrix $\hat{A}$ by using the solution $\{\hat{\phi}_j\}_{j=1}^{mK}$ and constructing from (13).

**D. Block Diagonalization**

The problem we should solve is $\mathcal{P}2$ and the matrix $A$ must have block-diagonal structure. In this paper, we omit the non-block diagonal elements of the matrix $A$ obtained in the previous section and then rescale to the constrained power.

Let $M \in \mathbb{B}_{K}^{m \times n}$ be a masking matrix composed of $K \times K$ blocks where the diagonal blocks are the matrices whose components are all 1 and the non-diagonal blocks are all 0. The block diagonalized matrix $\hat{A}_{bd}$ can be represented as the elementwise multiplication of $M$ and $A$, i.e.,
\[
\hat{A}_{bd} = M \otimes \hat{A}. 
\] (22)

Note that the matrix $\hat{A}$ satisfies the power constraint $\text{Tr}[\hat{A}K\hat{A}^H] = P_0$ because of the constraint of the problem $\mathcal{P}2'$ but the block diagonalized matrix $\hat{A}_{bd}$ does not. Therefore, we rescale the norm of the matrix $\hat{A}_{bd}$ to satisfy the power constraint. We then obtain the final solution
\[
\hat{A} = \sqrt{\frac{P_0}{\text{Tr}[\hat{A}_{bd}K\hat{A}_{bd}^H]}} \hat{A}_{bd}. 
\] (23)

The proposed method does not include iterative processes and the main factor of the computational costs is eigenvalue decomposition (11), which requires $O((nK)^3)$. However, this complexity is much less than the conventional method [8].

**IV. Simulation Results**

Performance with the proposed precoder was evaluated via computer simulations. We evaluated influence of system parameters on the averaged and normalized squared error, i.e., $\sum_{i=1}^{T} \sum_{z=1}^{Z} ||s_{iz} - \hat{s}_{iz}||^2/(nK \times T \times Z)$, where $T$ is the number of generations of $H$, $Z$ is the number of generations of $x$ for a single generation of $H$, and $s_{iz}$ and $\hat{s}_{iz}$ are corresponding instances. We set to $T = 10$ or more and $Z = 100$. Specifically, the simulation results are examined in terms of the performance dependency on

- Data compression ratio $mK/nK$: ratio of the total number of transmit antennas and that of measurements,
- Communication compression ratio $r/mK$: ratio of the number of receive antennas and the total number of transmit antennas,
- The number of nodes $K$,
- Signal-to-noise ratio (SNR) (dB).

In this paper, we define the SNR as $10 \log_{10}(P_0/\text{Tr}[S])$ (dB). The length of the measurement was set to $n = 8$ and the total transmit power was $P_0 = 10$ in all the simulations. The covariance matrices $K$ and $S$ had correlated formulations and the elements were determined as
\[
[K]_{ij} = 0.8 |i-j|, \quad [S]_{ab} = 0.5 |a-b|/r, 
\] for $i, j = 1, \ldots, nK$ and $a, b = 1, \ldots, r$, respectively. The instances $x$ and $n$ were randomly generated from the distributions $CN(0, K)$ and $CN(0, S)$, respectively. Each element of the channel matrix $H$ was identically and independently generated by $CN(0, 1)$. Moreover, we compared the performance of the proposed method with the following four schemes:
1) ignoring correlation: the proposed method that ignores spatial correlation, namely, ignores non-block diagonal elements of $K$ when designing the precoder,

2) Communicate-then-Compute: method not specifically designed for AirComp by using MSE regarding $x$, where the optimization is done in the same manner as [20], [21] and then applied block diagonalization in Sect. III-D,

3) Random: method using a random matrix as $A$ where each element is identically and independently generated by $CN(0,1)$ and normalized to satisfy the power constraint. Such a dimensional reduction is typically employed in some estimation methods [24],

4) and Huh et al.: method solving convex programming problems in each iteration for deriving the matrices $A_k$ from the problem $P1$ [8]. The spatial correlation is ignored so that the problem $P1$ is divided into $K$ problems. The method requires higher computational cost than the proposed and other methods to obtain solution with sufficient accuracy. It is regarded as a baseline.

The number of iterations in the method [8] was set to 10.

Fig. 2 shows the evaluation with respect to the data compression ratio. The system parameters were set to $(K, r, \text{SNR(dB)}) = (30, 5m, 25)$. From the figure, the iterative method of Huh et al. achieves the lowest error at the cost of longer running time. The performance gap between the method of Huh et al. and ours could be due to the loss of optimality of the proposed method when $K > 1$. The proposed method shows the best performance at any data compression ratio among the non-iterative methods.

The key feature of the proposed method is revealed from viewpoints of communication compression ratio and the number of nodes, shown in Fig. 3 and Fig. 4, respectively.

The evaluation in terms of communication compression ratio is shown in Fig. 3, where the system parameters were set to $(m, K, \text{SNR(dB)}) = (2, 30, 25)$. The MSE curves of the proposed and communicate-then-compute methods appear to have two modes. In the region $1 \leq r/mK$, i.e., when the number of the receive antennas $r$ is sufficiently large, the proposed method shows higher error than the other methods. This may be caused by the suboptimality of the proposed method. On the other hand, in the region $r/mK < 1$, the proposed method achieves lower errors than the other methods in most cases.

In Fig. 4, we evaluated the influence of the number $K$ of nodes on normalized MSE. The system parameters were set to $(m, r, \text{SNR(dB)}) = (2, 16, 25)$. In all the methods, the smaller the number of nodes, the lower the estimation error. The error of the proposed method is the lowest when $K \geq 20$ and the performance difference from the other methods becomes larger with increase of $K$. Compared with the method ignoring correlation, we can see that the spatial correlation should be exploited for the precoder design especially when there are a large number of nodes in a network.

These results indicate that the proposed method is suitable in situations where the number of receive antennas are limited but the number of nodes is increasing. The situations are nothing short of typical IoT environments.

Finally, we evaluated performance of the proposed method in different SNR. The system parameters were set to $(m, r, K) = (2, 16, 30)$. From Fig. 5, the performance of the proposed method is the best among the methods at any SNR.

V. CONCLUSIONS

This paper has proposed a novel precoder design for Aircomp in wireless data aggregation that explicitly employs spatial correlation and heterogeneous data correlation. The correlation appears in typical applications of sensor networks such as environmental monitoring and the appropriate use helps reduce aggregation errors that occurred in AirComp.

The proposed method includes no iterative procedure so that it does not require high computational costs. This is motivated by the idea of matrix diagonalization proposed in a different context from AirComp. Furthermore, this method provides...
Simulation results on synthetic data showed that the performance of the proposed method including correlation is better than the method ignoring correlation and the other non-iterative methods, especially when there are a lot of nodes in a network and when the number of receive antennas at the aggregator is less than the total number of transmit antennas at the sensors. In other words, the proposed method achieves better performance in typical large-scale IoT environments.

Future work includes the extension of the proposed method involving more sophisticated operations of block diagonalization and simulation on real datasets.

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