Time Reversal in Neutrino Oscillations in Matter

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Abstract
We estimate the time reversal violations for neutrino oscillations in matter for typical experimental energies and baselines. We examine the present status of experiments on neutrino oscillations, propose experiments for TRV, and discuss the future.

1 Introduction
T and CP violations in neutrino oscillations have long been of interest. For a review of CP and T violations for neutrino oscillations in vacuum see, e.g., Ref\textsuperscript{[1]}. More than three decades ago the effects of interactions in matter for neutrino oscillations were estimated\textsuperscript{[2, 3]}. The effects of matter on T reversal in neutrino oscillations have been discussed by a number of authors\textsuperscript{[4, 5]}. We now briefly review the formalism for T, CP and CPT-violating probability differences in neutrino oscillations. Defining the transition probability from neutrino a to neutrino b as \( P(\nu_a \rightarrow \nu_b) \), with flavor a and b = e, \( \mu \), or \( \tau \). the T, CP and CPT probability differences are

\[
\begin{align*}
\Delta P_T^{ab} &= P(\nu_a \rightarrow \nu_b) - P(\nu_b \rightarrow \nu_a) \\
\Delta P_{\text{CP}}^{ab} &= P(\nu_a \rightarrow \nu_b) - P(\bar{\nu}_a \rightarrow \bar{\nu}_b) \\
\Delta P_{\text{CPT}}^{ab} &= \Delta P_{\text{CP}}^{ba} - \Delta P_T^{ba},
\end{align*}
\]

(1)
where $\bar{\nu}_{a,b}$ is an antineutrino with flavor (a,b). Since anti-neutrino oscillations differ from neutrino oscillations due to matter effects, even though the CPT theorem holds in vacuum, $\Delta P_{ab}^{\text{CPT}} \neq 0$ for neutrino/antineutrinos in matter, which we call effective CPT violation. See Ref[6] for references to earlier work on effective CPT violation.

Since the CPT theorem does not hold for neutrinos traversing matter, the relation between T and CP in vacuum does not hold, and TR violation must be derived separately from CP violation. See Ref[8] for our recent study of CPV. One objective of the present work is to estimate TRV due to matter effects for some experiments measuring neutrino oscillation. This is done in section 2. The main objective of the present work is to propose an experiment to test T reversal violation. In section 3 the present status of neutrino oscillation experiments is reviewed. In section 4 the probability of electron to muon conversion is estimated for a typical experiment, which forms the basis for proposed experiments.

\section{Background and Estimates of TRV}

In this section we review the concepts and methods to calculate the neutrino transition probabilities from which one obtains time reversal probabilities associated with neutrino oscillations. We use the notation of Ref[5], most of which is standard. In the unitary transformation, $U$, defined below, the basic CP phase is $\delta_{CP}$. As shown in Ref[5], for uniform symmetric matter, which we assume, T reversal violation (TRV) vanishes if $\delta_{CP}=0$. See Ref[9] for a discussion of $\delta_{CP}$, which is not well known. We use a value consistent with those used in other studies.

Neutrinos (and antineutrinos) are produced as $\nu_e, \nu_\mu, \nu_\tau$ together with the named charged leptons. However, neutrinos of definite masses are $\nu_\alpha$, with $\alpha = 1, 2, 3$. The two forms are connected by a 3 by 3 unitary transformation \[ U = \left( \begin{array}{ccc} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{array} \right), \]

similar to the CKM matrix for quarks. We have neglected the Majorana phases and have used the usual short-hand notation $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. We have

$$\nu_a = U\nu_\alpha.$$  \hspace{1cm} (2)
As in Eq(1), the TRV probability differences are defined as
\[ \Delta P_{ab}^T = P(\nu_a \rightarrow \nu_b) - P(\nu_b \rightarrow \nu_a) \].

(3)

It is convenient to use the time evolution matrix, \( S(t, t_0) \) to derive \( \Delta P_{ab}^T \):
\[ |\nu(t)\rangle = S(t, t_0)|\nu(t_0)\rangle \]  
\[ i\frac{d}{dt}S(t, t_0) = H(t)S(t, t_0) \],

(5)

with \( H(t) \) the Hamiltonian. In the vacuum
\[ S_{ab}(t, t_0) = \sum_{j=1}^{3} U_{aj} e^{iE_j(t-t_0)} U_{bj}^* \]  

(6)

Since neutrinos travel through matter, we must take into account forward charged current neutrino electron scattering in the earth. The potential which describes the interaction is
\[ V = \sqrt{2}G_F n_e \],

(7)

where \( G_F \) is the universal weak interaction Fermi constant, and \( n_e \) is the density of electrons in matter. Using the matter density \( \rho = 3 \) gm/cc, the neutrino-matter potential is \( V = 1.13 \times 10^{-13} \) eV. Note that \( V \rightarrow -V \) for antineutrinos, the source of effective CPT violation in matter.

The TRV electron-muon probability difference, which is the main topic of the present work, is obtained from
\[ \Delta P_{e\mu}^T = |S_{21}|^2 - |S_{12}|^2 \]

(8)

With the \( V \) included one finds
\[ S_{12} = c_{23}\beta - i s_{23} a A_a \]  
\[ S_{21} = -(c_{23}\beta + i s_{23} a C_a) \]  
\[ a = s_{13}(\Delta - s_{12}\delta) \],

(9)

with \( \delta = \delta m_{12}^2/(2E), \Delta = \delta m_{13}^2/(2E) \). Note that \( \delta \ll \Delta \). We set the CP phase \( \delta_{CP} = 90^\circ \).

With the approximations \( V \leq \delta \ll \Delta \),
\[ A_a \simeq f(t, t_0)I_\alpha * (t, t_0) \]  
\[ I_\alpha * (t, t_0) = \int_{t_0}^{t} dt' \alpha^\ast(t', t)f(t', t) \]  
\[ \alpha(t, t_0) = \cos \omega(t - t_0) - i \sin(2\theta) \sin \omega(t - t_0) \]  
\[ f(t, t_0) = e^{-i\Delta(t-t_0)} \]  
\[ 2\omega = \sqrt{\delta^2 + V^2 - 2\delta V \cos(2\theta_{12})} \]  
\[ \beta = -i \sin 2\theta \sin \omega L \]  
\[ C_a = A_a \].

3
The angle $\theta$ is defined by

$$\cos(2\theta) = \frac{\delta \cos(2\theta_{12}) - V}{2\omega}.$$  \hfill (11)

From $\Delta P_{e\mu} = |S_{21}|^2 - |S_{12}|^2$ it follows that

$$\Delta P_{e\mu} = -2s_{13}s_{23}c_{23}(\Delta - s_{12}\delta)\text{Im}[e^{-i\delta_{CP}}\beta^*(A_a - C_a^*)].$$ \hfill (12)

With the approximations $(V, \delta, \omega \ll \Delta)$, using integration by parts it follows (see Ref[5]) that (with $t - t_0 \simeq L$) $I_{a*} = i(1 - \cos\omega Le^{-i\Delta L})/\Delta + O(1/\Delta^2)$. Note that both $\beta$ and $A_a - C_a^*$ are purely imaginary, so $\text{Im}[\beta^*(A_a - C_a^*)] = 0$. Therefore there is no TRV for our study of uniform matter if $\delta_{CP} = 0$. We choose $\delta_{CP} = 90^\circ$, so $e^{-i\delta_{CP}} = -i$ to simplify our calculation. Using these approximations it follows that[5]

$$A_a - C_a^* = 2i\text{Im}[A_a] \simeq i\frac{2}{\Delta}[\cos\Delta L - \cos\omega L],$$ \hfill (13)

where $L$ is the baseline length for the neutrino experiment.

From this, and using $\delta \ll \Delta$ it follows that

$$\Delta P_{e\mu} \simeq -4s_{13}s_{23}c_{23}\sin\omega L\sin2\theta(\cos\Delta L - \cos\omega L)$$

$$\simeq 0.374\sin\omega L\sin2\theta(\cos\omega L - \cos\Delta L).$$ \hfill (14)

where we have used $L=t$, with the neutrinos having approximately the speed of light. We parameters $s_{13} = .187$, and $s_{23} = c_{23} = .707$, $\theta_{12} = 32^\circ$. With $E=1$ GeV, $\delta = 3.8 \times 10^{-14}$ eV, $\Delta = 1.2 \times 10^{-12}$ eV, $V=11.3 \times 10^{-14}$ eV, $\omega = 0.5 \times 10^{-13}$ eV, $\sin2\theta = 0.342$. Therefore $4s_{13}s_{23}c_{23}\sin2\theta=0.128$. We use standard units with $1\text{m}=5 \times 10^6/\text{eV}$. Similar relationships for $\Delta P_{e\mu}$ have been used by a number of authors studying neutrino oscillations.

With these parameters, using Eq(14), we find the magnitude of the time reversal violation as a function of $L$ shown in Fig. 1. For $E=1$ GeV and the MINOS baseline, as in Fig. 2, $\Delta P_{e\mu}$ is approximately 3 %, which could be attained in future experiments if there were both $\nu_e$ and $\nu_\mu$ beams.

Among many experiments studying neutrino oscillation, the MINOS experiments have covered a wide range of energies[20]. Since it can only measure $\nu_\mu \rightarrow \nu_e$, it cannot measure TRV, but we evaluate it for possible future experiments. We now apply Eq(14) to evaluate $\Delta P_{e\mu}$ for the parameters relevant to MINOS. The baseline is $L = 735$ km, and the energy range 3 to 18 GeV.
With the other parameters in Eq(14) the same as those used to obtain Fig. 1, our results are shown in Fig. 2.

Figure 1: $\Delta P^T_{e\mu}$ with $E=1$ GeV

Figure 2: $\Delta P^T_{e\mu}$ for $L=735$ km
3 Present Status of Neutrino Oscillation Experiments

The most likely tests of TRV are those for electron and muon neutrinos or antineutrinos. At low energies the conversion of electron antineutrinos has been deduced through the disappearance of antineutrinos produced by reactors \[9\]. The experiments were carried out in Japan \[11\]. They are based on reactor electron antineutrinos with a mean energy of 3 MeV sent to a detector about 180 km away. The observation of electron neutrino disappearance and oscillations was confirmed.

There are, at present, no definitive muon antineutrino oscillation experiments to electron antineutrinos. There are some indications for an excess of electron neutrinos, over background, in the MINOS experiment, which sends muon neutrinos of a few GeV from Fermilab and are detected at the Soudan mine, 735 km away \[12\]. Similar experiments have been carried out by MiniBooNE for both neutrinos and antineutrinos. They report an excess of events in the region \(475 \leq E \leq 1250\) MeV, \[13\] which are consistent with \(\bar{\nu}_\mu \to \bar{\nu}_e\) oscillations for \(0.1 \leq \Delta m^2 \leq 1.0eV^2\). Fits to MiniBooNE and LSND data \[14\] find strong evidence for at least one sterile neutrino\[15,16,17,18\].

4 Proposed Experiment for TRV

Since there are no sources of high energy electron neutrinos or antineutrinos, a TRV test via \(\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu\) is not possible at the present time. In the present work we estimate the probability \(P(\nu_e \to \nu_\mu)\), as a guide for future experiments. We carry out two sets of calculations, one with a fixed baseline as a function of energy, and the other with a fixed energy as a function of baseline

4.1 \(P(\nu_e \to \nu_\mu)\) for \(L=735\) km

In this subsection we use the formalism of Ref\[9\], with \(P\nu_e \to \nu_\mu\) given as four terms: \(P = P_0 + P_{\cos\delta_CP} + P_{\sin\delta_CP} + P_3\). For the calculation of the \(P\nu_e \to \nu_\mu\) transition probability, with our choice of \(\delta_{CP}\) (see below), the expression is somewhat simpler than for the time evolution method used in section 2. Also, we use the MINOS baseline, as in section 2; and use parameters \(\delta m^2_{31} = 2.4 \times 10^{-3}eV^2\), \(\delta m^2_{21} = 7.6 \times 10^{-5}eV^2\); and \(\theta_{23} = \pi/4\), \(\theta_{12} = \pi/5.4\), \(\theta_{13} = 0.188\) rad. Using the notation \(A = 2VE\), with \(V\) defined in Eq(8), and \(A = A/\delta m^2_{31}\), we find (with \(\Delta_L \equiv \Delta L/2\)
\[ P_0 = \sin^2\theta_{23} \sin^2(2\theta_{13}) \sin^2[(\hat{A} - 1)\Delta L]/(\hat{A} - 1)^2, \quad (15) \]
\[ P_{\cos\delta_{CP}} = \delta m_{21}^2 \cos\delta_{CP} \cos\theta_{13} \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \cos\Delta_L \sin[\hat{A}\Delta L] \sin[(1 - \hat{A})\Delta L]/(A(1 - \hat{A})) , \quad (16) \]
\[ P_3 = \delta m_{21}^4 \cos^2\theta_{23} \sin^2(2\theta_{12}) \sin^2(\hat{A}\Delta L)/A^2 , \quad (17) \]
\[ P_{\sin\delta_{CP}} = \delta m_{21}^2 \sin\delta_{CP} \cos\theta_{13} \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \sin\Delta_L \sin[\hat{A}\Delta L] \sin[(1 - \hat{A})\Delta L]/(A(1 - \hat{A})) . \quad (18) \]

As in section 2 we use \( \delta_{CP} = 90^\circ \), so \( \sin(\delta_{CP})=1.0, \cos(\delta_{CP})=0.0 \), and therefore \( P_{\cos\delta_{CP}} = 0 \). For our estimates of \( P_{\nu_e \rightarrow \nu_\mu} \) we use the MINOS baseline, \( L=735 \) km, as in section 2.

Because we are not proposing a direct time reversal experiment, e.g., a test of the equality \( P(\nu_e \leftrightarrow \nu_\mu) \), but rather two experiments which compare these two conversions over the same length \( L \) at the same energy, we must take all the terms of the probability of conversion into account. Note that the \( P_{\sin\delta_{CP}} \) term is positive for antineutrinos, but negative for neutrinos.

From the parameters given above and Eqs(17-20) one finds

\[
\begin{align*}
P_0 &\simeq 0.0682, \quad P_{\cos\delta_{CP}} = 0.0, \\
P_3 &\simeq 0.00073, \quad P_{\sin\delta_{CP}} \simeq 0.0186. \\
P &\simeq 0.0875.
\end{align*}
\]

These terms are small because of the small size of \( \theta_{13} \).

In Fig.3 we give the total \( P_{\nu_e \rightarrow \nu_\mu} \) and in Fig. 4 the partial probabilities of Eqs(16-29) for \( L=735 \) km for energies appropriate for current experiments.

![Figure 3: Overall probability, P, for \( \nu_e \rightarrow \nu_\mu \) for \( L=735 \) km and energies \( E=0.5 \) to 5 GeV](attachment:image.png)
The partial probabilities are shown in Fig. 4. Since $P_{\cos \delta_{CP}} = 0$ we do not show that term.

Figure 4: The partial probabilities for $\nu_e \to \nu_\mu$ for L=735 km and energies $E=0.5$ to $5$ GeV
4.2 $\mathcal{P}(\nu_e \to \nu_\mu)$: Accelerator, Reactor Experiments

In this subsection we find the probability $\mathcal{P}(\nu_e \to \nu_\mu)$ with the baseline and energy parameters corresponding to the experimental setups MiniBooNE\cite{19}, MINOS\cite{20}, JHF-Kamioka\cite{21} and CHOOZ\cite{22}. MiniBooNE, MINOS, and JHF-Kamioka have muon neutrino or muon neutrino and muon antineutrino beams, while CHOOZ has electron antineutrino beams. The goal of this study is to estimate the neutrino conversion probabilities over a large range of baselines and energies in order to provide guidance for possible TRV experiments in the future.

The results are shown in Fig.5. Note that CHOOZ, with about a 1 km baseline and a very low energy has a very large probability. There is a problem, however, in identifying the neutrinos or antineutrinos at low energies.

4.3 $\mathcal{P}(\nu_e \to \nu_\mu)$ Matter Effects

An important question is how large are matter effects on neutrino conversion probability. Matter effects are removed by setting $V = 0$. In the notation used in Eqs(17-21), when $V \to 0$

$$A \to 0$$

$$\hat{A} \to 0$$

$$\frac{\sin[\hat{A}\Delta_L]}{A} \to \frac{\Delta_L}{\delta m_{31}^2}. \quad (19)$$

From this one can show that Eqs(17-20) become

$$\mathcal{P}_0 = \sin^2\theta_{23}\sin^2(2\theta_{13})\sin^2(\Delta_L) \quad (20)$$

$$\mathcal{P}_{\cos\delta_{CP}} = \frac{(\delta m_{21}^2/\delta m_{31}^2)\cos\delta_{CP}\cos\theta_{13}\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})}{\cos(\Delta_L)\sin(\Delta_L)\Delta_L}, \quad (21)$$

$$\mathcal{P}_3 = (\delta m_{21}^2/\delta m_{31}^2)^2\cos^2\theta_{23}\sin^2(2\theta_{12})\Delta_L^2, \quad (22)$$

$$\mathcal{P}_{\sin\delta_{CP}} = \frac{(\delta m_{21}^2/\delta m_{31}^2)\sin\delta_{CP}\cos\theta_{13}\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})}{\sin^2(\Delta_L)\Delta_L}, \quad (23)$$

The probabilities $\mathcal{P}(\nu_e \to \nu_\mu)$ with $V=0$ are shown as dashed curves in Fig 5. For the short baselines $L=500$ m and $1.03$ km, the matter effects are so small we do not show the results for $V=0$. 

9
Figure 5: $\mathcal{P}\nu_e \rightarrow \nu_\mu$ for MiniBooNE($L=500\,\text{m}$), MINOS($L=735\,\text{km}$), JHF-Kamioka($L=295\,\text{km}$), and CHOOZ ($L=1.03\,\text{km}$)
5 Conclusions and Proposed Experiments

In our studies of probability differences, we find a rather large $\Delta P_{\mu e}^T$ at 1 GeV with a baseline of 500 to 700 km (Fig.1), somewhat smaller than the present 735 km at MINOS, and for the 735 km baseline $\Delta P_{\mu e}^T \simeq .01$ at about 3.0 GeV (Fig.2), within the MINOS range, if there were both $\nu_e$ and $\nu_\mu$ beams.

Our studies of the probability of electron to muon neutrino conversion with parameters corresponding to MiniBooNE, MINOS, JHF-Kamioka, and CHOOZ find large conversion probabilities at low energies, however, identifying the type of neutrino at low energies is difficult at the present time.

With sufficiently intense antineutrino beams, it might be possible to reach an accuracy of $\approx 20\%$. Over time, improvements will undoubtedly occur.

We conclude with a discussion of specific experiments for future tests of TRV. At the present time, with only indirect evidence for $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ and no firm evidence for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, it is difficult to cull data for a test of TRI. Our first proposed tests might not be of sufficient accuracy to find a TRV, but are, at least steps in the right direction. The tests involve two separate experiments, namely $P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$. We have chosen parameters corresponding to those available at accelerators and reactors at the present time.

A second proposed test we call the L-2L experiment. If $\nu_e \rightarrow \nu_\mu$ with a probability of 10% at 1 GeV for $L=735$ km (see Fig. 3), then after a further distance of 735 km, if TR symmetry holds, 1% of muon neutrinos will have converted back to electron neutrinos. Assume that 90% of electron neutrinos do not convert at $L=735$ km and remain $\nu_e$. Then at 1470 km, 81% of electron neutrinos will remain as such, but there will be an added 1% from the conversion to and from muon neutrinos. The difference is small, but may be measurable. This argument neglects the conversion of electron neutrinos to tau neutrinos. If this is large, it can be used instead of muon neutrinos. This might be a possible future experiment for MINOS, as well as Kamioka and CHOOZ.

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