Method to improve accuracy of positioning object by eLoran system with applying standard Kalman filter

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Abstract. This article reports on a novel method to improve the accuracy of positioning an object by a low frequency hyperbolic radio navigation system like an eLoran. This method is based on the application of the standard Kalman filter. Investigations of an affection of the filter parameters and the type of the movement on accuracy of the vehicle position estimation are carried out. Evaluation of the method accuracy was investigated by separating data from the semi-empirical movement model to different types of movements.

1. Introduction
Since 1958 the ground-based low frequency hyperbolic radio navigation system (LFHRNS) "Chaika" operates on the territory of Russia. This system is an analogue of the eLoran system and based on measuring the delay of pulses received from the chain of transmitting stations. In the limits of working zones, taking into account a systematic and random error, the system provides a position accuracy of 20-1500 m. Despite the impressive measurement error, in comparison with the global navigation satellite systems (GNSS), LFHRNS is an active reserve of satellite navigation [1].

To date, ground-based radio navigation is an actual research object [2]. The method of rapid detection of LFHRNS signals with intensive narrow-band noise is developed in [3]; in [4] to improve the accuracy of LFHRNS, additional secondary factors (ASF) are calculated; in [5] a new method for processing highly noisy LFHRNS signals is proposed; in [6], [7] the LFHRNS information is processed jointly with the GPS (Global Positioning System). Integration of the two systems significantly improves the accuracy of the radio navigation system in the short-time absence of satellite navigation signals.

In this paper, the efficiency of applying the Kalman filter in the LFHRNS positioning problem estimated. An algorithm for increasing the accuracy of object positioning based on the nature (type of trajectory, speed) of the object movement is proposed.

2. Formulation of the problem
The purpose of this paper is to obtain optimal estimates of the vehicle coordinates. The difference (discrete) form of equation that determines the model of the motion of an object in the one-dimensional case is:

\[ x_k = x_{k-1} + v_{k-1} \Delta t + (a_{k-1} \Delta t^2) / 2, \]  

(1)
where $\Delta t$ – sampling interval (in this case equal to 1 s).

Obviously, to determine coordinates $x_k$ of an object at any time, it is necessary to know its coordinates $x_{k-1}$, speed $v_{k-1}$ and acceleration $a_{k-1}$ at the previous moment of time. Let us assume that in the measurement interval, the parameters of the motion of the object are unchanged.

3. **Kalman filter**

The Kalman filter (KF) is an effective recursive filter that estimates the state vector of a dynamic system for incomplete and noisy measurements.

The Kalman filter is quite versatile and is used in engineering and economic applications: from radar data processing and machine vision systems to macroeconomic model parameters estimation [8], [9].

In this paper, let us briefly discuss the explanation of the data used in the filter. The principle of operation features and varieties of the filter are well described in [10].

The system’s state vector includes the real coordinates of the vehicle on the plane (X and Y components). The dynamics of the system’s state vector is described by the following expression:

$$
\begin{align*}
    \mathbf{x}(t_k) &= 
    \begin{pmatrix}
        1 & 0 \\
        0 & 1 
    \end{pmatrix}
    \begin{pmatrix}
        X(t_{k-1}) \\
        Y(t_{k-1}) 
    \end{pmatrix}
    \begin{pmatrix}
        \eta_X(t_{k-1}) \\
        \eta_Y(t_{k-1}) 
    \end{pmatrix} 
\end{align*}
$$

(2)

where $x(t_k)$ – system’s state vector at moment $t_k$; $X(t_{k-1})$ and $Y(t_{k-1})$ – real coordinates of the vehicle at moment $t_{k-1}$; $\eta_X(t_{k-1})$, $\eta_Y(t_{k-1})$ – process noise – uncorrelated, random values with zero mean, normal distribution and standard deviation equal to $\sigma_Q$. It can be seen from expression (2) that the evolution matrix of the system is an identity matrix, and the control is absent.

The observation vector is the coordinates of the vehicle obtained with the LFHRNS receiver:

$$
\begin{align*}
    X_{LFHRNS}(t_k) &= X(t_k) + \zeta_X(t_k), \\
    Y_{LFHRNS}(t_k) &= Y(t_k) + \zeta_Y(t_k), 
\end{align*}
$$

(3) (4)

where $\zeta_X(t_k)$ and $\zeta_Y(t_k)$ – measurement noise – uncorrelated, random values with zero mean, normal distribution and standard deviation equal to $\sigma_R$.

The estimation of the system’s state vector at the predict phase has the form:

$$
\begin{align*}
    \mathbf{x}_{est}(t_k) &= 
    \begin{pmatrix}
        1 & 0 \\
        0 & 1 
    \end{pmatrix}
    \begin{pmatrix}
        X_{est}(t_{k-1}) \\
        Y_{est}(t_{k-1}) 
    \end{pmatrix} 
\end{align*}
$$

(5)

where $x_{est}(t_k)$ – estimation of the system’s state vector at the predict phase at moment $t_k$; $X_{est}(t_{k-1})$, $Y_{est}(t_{k-1})$ – the coordinate estimates obtained in the update phase at moment $t_{k-1}$.

In this problem, it is required to determine the value $\sigma^2_Q$, at which the filter will give out optimal estimates of the vehicle coordinates for each type of motion.

For the convenience, let us introduce parameter $K_{ri}$ – coefficient of relative instability, equal to the ratio of process noise variance $\sigma^2_Q$ to the measurement noise variance $\sigma^2_R$. The $K_{ri}$ is written as follows:

$$
K_{ri} = \frac{\sigma^2_Q}{\sigma^2_R}. 
$$

(6)

The coefficient of relative instability is a dimensionless quantity that takes values in the interval from zero to infinity. A coefficient equal to zero is equivalent to a still object.

3.1. **Criterion of optimality**

An auxiliary parameter for determining the optimality criterion is the estimation of the standard deviation (SD) of position determination, calculated by the formula:
\[
\sigma_k = \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} (x_{est}(t_k,j) - X(t_k))^2 + \frac{1}{n-1} \sum_{j=1}^{n} (y_{est}(t_k,j) - Y(t_k))^2}.
\]  

(7)

In equation (7) \(X_{est}(t_k,j)\) and \(Y_{est}(t_k,j)\) – vehicle coordinate estimations at moment \(t_k\) on \(j\)-th pass at the output of KF, \(n\) – number of passes (not less than 100). This characteristic allows determining the time during which the steady value of the SD estimate comes.

The value of \(\sigma_k\) is necessary for calculating the mean value of the SD over the ensemble of intervals:

\[
\sigma_T = \frac{1}{m} \sum_{k=1}^{m} \sigma_k
\]

(8)

where \(m\) – the number of obtained estimates of the vehicle coordinates by one realization.

The value of \(\sigma_T\) is the criterion for the optimality of the vehicle coordinate estimates issued by the KF, as well as the accuracy characteristic of the method proposed in this paper.

4. Method of modelling and analysis of result

In this paper, the results of LFHRNS measurements are modelled according to equations (3) and (4). In subsection 4.1 and section 5 coordinates received by the GPS receiver taken as the true location of the object, because the error of the GPS receiver was negligible compared to the LFHRNS error. Random values of \(\zeta_X(t_k)\) and \(\zeta_Y(t_k)\) from equations (3) and (4) are taken with a standard deviation of 10, 20, 40, 80 and 160 m. The time sampling is 1 s.

4.1. Application of Kalman filter on the real section of the route through the city

Before considering individual types of motion, let us apply KF to the real data obtained by the GPS receiver with the addition of noise. This will give us a preliminary assessment of the effectiveness of the filtration.

Figure 1 shows a single pass of the path described above. The dots represent the coordinates received by the LFHRNS receiver (imitation data are meant) at a frequency of 1 Hz and a SD of 160 m. The green line shows the data on the location of the vehicle processed by the Kalman filter with \(K_{ri} = 0.06\).

The choice of value \(K_{ri} = 0.06\) in Figure 1 is not accidental. It is justified by the study of the dependence of the SD of the determination of the location at the output of the KF from \(K_{ri}\) for different values of the LFHRNS noise. Figure 2 shows the graphs of the research results.

With the increase in the coefficient of relative instability, the integral SD decreases to its minimum, then rises to the level of the SD noise of measurements. This is happen because smaller \(K_{ri}\) corresponds to a more inert object. Because of that, the filter delay can take on values greater than the measurement noise. With increasing \(K_{ri}\), the object acquires a greater degree of mobility (freedom), the filter begins to "trust" the measurements more, and eventually the filtered data repeats the unprocessed
data. Applying KF, the authors were able to reduce the integral SD of the trace \((\sigma_t)\), and establish the dependence of the filtering efficiency on the SD noise of measurements. The integral SD of position determination has decreased from 160 to 65 m (by 60%), from 80 to 40 m (by 50%), from 40 to 24 m (40%), from 20 to 14 m (30%) and from 10 to 8.4 m (16%).

Next, let us consider in more detail the different types of motion. In modelling the LFHRNS data, let us confine ourselves to noise having a maximum SD equal to 160 m.

4.2. Still object. Movement with low speed
Having a priori information about the mode of movement of the object, namely the "no-motion" mode or moving at a low speed, it is possible to significantly increase the positioning accuracy. Such information can provide a car speedometer or accelerometer readings. In the "no-motion" mode of movement, the results of the LFHRNS measurement were taken as a random value with zero mathematical expectation and a standard deviation of 160 m. A total of 500 points generated. The measurements processed in two ways: a Kalman filter with various \(K_i\) and a method of moving average (MA) the measurements. Evaluation of the results of these methods showed that their effectiveness is the same in the first eight measurements. With a further increase in the number of measurements in the "no-motion" mode, the efficiency of the KF decreases with respect to averaging.

When investigating the effectiveness of the application of KF for motion with low speed and for linear motion, described below, the simulated X and Y coordinates changed according to the following law:

\[
X(t_k) = v_k t_k, Y(t_k) = 0, k = 0..500, \tag{9}
\]

A simulation was performed to estimate integral SD when traveling at speeds of 1, 3, 5 m/s for various processing methods. The results show that the Kalman filter is more efficient at low speeds than the moving average method. The accuracy of the object location at speeds of up to 5 m/s is 3-4 times higher than the “raw” LFHRNS data (35 m - 55 m, instead of 160 m).

4.3. Linear motion
With long straight-line movement of the object, the coordinates obtained from the LFHRNS receiver can show the direction of motion with a certain accuracy. The problem of determining the direction of motion according to the LFHRNS data is solved by the method the linear approximation (first order). With rectilinear motion, there is a certain number of measurements sufficient to evaluate the equation of a straight line of form \(y = ax + b\), which, with a certain accuracy, corresponds to the trajectory of motion in a given section. For uniform rectilinear motion, the filter lag is a systematic error, the value of which can be compensated. Depending on the speed of the object and the selected value of \(K_{ri}\), one can calculate the amount of compensation.

Figure 3 shows the dependence of integral SD on \(K_{ri}\) for different velocities of motion. Figure 4 shows the compensation of the filter lag for uniform motion and small values of \(K_{ri}\) \((6 \times 10^{-3}, 12 \times 10^{-3}, 18 \times 10^{-3})\). For this, by varying the magnitude of the compensation in a known direction for different values of the velocities of motion and \(K_{ri}\), let us determine integral SD.
It can be seen from the graphs in figure 3 that the optimum value of $K_{ri}$ corresponding to the minimum of the integral SD, is proportional to the speed of motion. As the speed of the object increases, the efficiency of the filter decreases, since the fast moving object corresponds to a larger $K_{ri}$, in which the filter more “trusts” the noisy LFHRNS measurements. When moving at a speed of 5 m/s, integral SD is reduced to 55 m (by 65%), and at a speed of 25 m/s – up to 90 m (~ 45%).

It can be seen from the graphs that there are optimal compensation values for various parameters of motion and properties of the object. The correction level is directly proportional to the speed of motion for various $K_{ri}$.

The effectiveness of compensation depends on the accuracy of determining the direction of motion, on which the compensation will be applied. The longer the object moves linearly, the more accurately its direction of motion is determined. An estimate of the required number of measurements to determine an acceptable straight line was made. For each subsequent measurement, starting with the second, by the method of least squares, we determine the line approximating the real trajectory. Next let us add the value corresponding to the optimal compensation, in the established direction of movement to the data from the output of the KF. The criterion for estimating is the value of $\sigma_k$ from (7). As a rule, the vehicle cannot move linearly for a long time, so it is necessary that the algorithm for a smaller number of measurements form an acceptable straight line – for movement speed 15 m/s this condition is satisfied by a filter with $K_{ri} = 18 \times 10^{-3}$. If one projects the compensated data to the calculated straight line, the lateral deviation will constantly decrease, tending to zero. Over time, the system will only have errors along the direction of travel. Figure 5 shows a comparison of the processing methods and the projection of data. Movement speed is $v = 15$ m/s, $K_{ri} = 18 \times 10^{-3}$. Triangles show data processed by the KF, with filter lag compensation. The squares show the KF processed data with compensation for the lag and with the projection onto a straight line calculated from the LFHRNS data. The circles show data calculated by the Kalman filter with compensation for the lag and projected onto a straight line calculated from previous KF data. Figure 6 explains the operation of the KF with a small $K_{ri}$ and the results of compensating the filter lag.
Green squares indicate the coordinates of the object obtained after applying the KF. Magenta circles show coordinates with filter lag compensation projected onto the calculated line. Segments for each time point connect the coordinates of the real path and coordinates of the processed data. The error in locating the object at the output of the Kalman filter has the form:

$$\Delta S = \sqrt{\Delta S^2 + \Delta S^2_{\perp}},$$

(10)

where $\Delta S$ – error component along the direction of travel (filter lag), $\Delta S_{\perp}$ – perpendicular error component. The use of KF significantly accelerates the determination of the direction of motion, which makes it possible to achieve a reduction in the integral SD of a uniformly moving object to 38 m (by 75%) in 30 measurements.

4.4. Curvilinear motion
Let us consider the effectiveness of using the Kalman filter to determine the position in curvilinear motion, which is determined by the mathematical model of motion along a sinusoidal path with an amplitude of 150 m and a period of 300 m with normal noise ($\sigma_R = 160$). The dependence of the integral SD on $K_{ri}$ under sinusoidal curvilinear motion coincided with the results for the rectilinear motion. The graphs, shown in figure 3, describe both rectilinear motion and curvilinear motion. This indicates the universality of the filter with the given parameters. The optimal $K_{ri}$ can be expressed as a linear dependence on the velocity of motion, as well as for rectilinear motion.

The compensation method, considered for rectilinear motion, significantly improves positioning accuracy. The extension of this method to the case of curvilinear motion will allow achieving similar indicators of accuracy improvement. It should be noted that the use of delay compensation for curvilinear motion requires the preliminary construction of approximation lines of a higher order. In this paper, analysis of compensation for delay for curvilinear motion was not performed.

5. Application of the adaptive filtration method
The study of various types of motion made it possible to conclude that at zero speed, the most effective processing method is the averaging of the LFHRNS data, and at speed $\nu > 0$, it is necessary to use a KF with $K_{ri}$, depending on the speed of motion. When moving along a straight line, it is reasonable to compensate lag in estimating the coordinates at the output of the KF. Accounting for all of the above implies the use of an adaptive filtration method (AFM), in which $K_{ri}$ changes depending on the nature of the movement. The adaptive filtering method applied to the route is described in section 4.1. The result of AFM application is a reduction in SD in all considered cases, namely for LFHRNS noise of data from SD 10, 20, 40, 80 and 160 meters. The average decrease in SD after application of AMF is 9% relative to filtration with fixed $K_{ri}$. Table 1 contains data on the accuracy of the positioning of LFHRNS with the use of the Kalman filter and AFM.
Table 1. Evaluation of the efficiency of the data processing method of LFHRNS.

|                      | SD of LFHRNS data, m | 10   | 20   | 40   | 80   | 160  |
|----------------------|----------------------|------|------|------|------|------|
| SD of LFHRNS data after KF, m | 8.4     | 14.0 | 24.0 | 40.0 | 65.0 |
| SD of LFHRNS data after AFM, m | 7.6     | 12.0 | 20.5 | 32.0 | 55.0 |

6. Conclusion

In this article, an estimation of the Kalman filter use in the vehicle-positioning task with reference to the LFHRNS signals was made. The correspondence between the KF parameter $K_r$ and the dynamic characteristics of the vehicle was established. The combination of the most effective filtering methods for each type of motion allowed us to apply the adaptive filtering method. The AFM test on a semi-empirical model showed an increase in the accuracy of the positioning for various noise levels. In general, the error in determining the location after processing the “raw” data reduced by 24 to 66% for the SD noise of measurements of 10-160 m. For the application of AFM, information about the nature of the object movement is required. Obviously, taking into account the model of the real vehicle and using the inertial navigation system together with the LFHRNS will not only enable us to apply the described filtering method, but will also improve the accuracy of position determination.

References

[1] Jonson G W, Swaszek P F and Hartnett R J 2007 An evaluation of eLoran as a backup to GPS Proc. IEEE Int. Conf. on Technologies for Homeland Security pp 95–100
[2] Wang D, Xi X, Pu Y, Liu J and Zhou L 2016 Parabolic equation method for Loran-C ASF prediction over irregular terrain IEEE Antennas Wireless Propag. Lett. 15 734–7
[3] Li S, Wang Y, Hua Y and Gao Y 2013 A fast anti-interference detection method for Loran-C signal J. Xi’an Jiaotong University 47 91–6
[4] Xi X, Zhou L, Zhang J and Pu Y 2013 A new method for Loran-C ASF calculation over irregular terrain IEEE Trans. Aerosp. Electron. Syst 49 1738–43
[5] Gao Y, Yu H, Li S and Yang C 2015 Acquisition method of Loran-C signal based on matched filter Joint Conf. of the IEEE Int. Frequency Control Symp. & the Eur. Frequency and Time Forum pp 265–9
[6] Pichugin S M 2010 Joint use of pulse-phase and satellite radio navigation systems T-Comm 9 118–21
[7] Tong H and Xu H 2010 Horizontal dilution of precision analysis for Loran C/Beidou integrated navigation system 3rd Int. Symp. on Electron. Commerce and Security 167–70
[8] Strid I and Walentin K 2009 Block Kalman filtering for large-scale DSGE models Computational Economics 33 277–304
[9] Andreasen M M 2011 Non-linear DSGE models and the optimized central difference particle filter J. of Econ. Dynamics and Control 35 1671–95
[10] Simon D 2006 Optimal State Estimation. Kalman, H∞, and Nonlinear Approaches (Hoboken: John Wiley & sons) pp 526–27