A Semi-Theoretical Model for Water Condensation: Dew Used in Conservation of Earthen Heritage Sites

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Abstract: Dew is a common but important phenomenon. Though water is previously considered to be a threat to earthen heritage sites, artificial dew is showing potential in relic preservation. A model of dew prediction on earthen sites will be essential for developing preventive protection methods, but studies of dew formation processes on relics are limited. In this study, a two parameter model is proposed. It makes approximations according to the features of earthen heritage sites, assuming that a thin and steady air layer exists close to the air–solid interface. This semi-theoretical model was based on calculations of the mass transfer process in the air layer, and was validated by simulations of laboratory experiments (R > 0.9) as well as field experiments. Additionally, a numerical simulation, performed by the commercial software COMSOL, confirmed that the difference between fitting parameter δ and the thickness of assumed mass transfer field was not significant. This model will be helpful in developing automatic environmental control systems for stabilizing water and soluble salts, thus enhancing preventive protection of earthen heritage sites.

Keywords: dew; heritage conservation; earthen heritage sites; semi-theoretical model; mass transfer; numerical simulation

1. Introduction

Dew is a common phenomenon and is becoming increasingly important nowadays [1–4]. It forms out of water in the air once the temperature of a solid surface in contact with the air drops below the dew point.

In the conservation of cultural relics, dew has long been considered as a potential cause of damage. For example, in porous structures such as bricks, murals, sculptures and earthen sites, repeated condensation and evaporation of water can activate soluble salts. With water transportation, salts may migrate, crystallize and swell in pores, causing stress and resulting in destruction [5,6]. In addition, dew may cause corrosion of metals and worsen biological corrosion [7].

Extensive dew and water evaporation are especially serious problems for earthen heritage sites. Various kinds of materials have been developed to consolidate soil surface, such as potassium–silica solution, iron-based bioproducts, calcium hydroxide nanoparticles and organosilicones [8–12]. However, there is no successful case reported in humid regions after long-term preservation. The reason is clear: no matter how high the strength would be, it is just a matter of time before the periodic migration and crystallization of soluble salts destroy the structures from the inside [13]. Inappropriate materials may even cause preservation damage [14]. In the Kuahuqiao Museum, Hangzhou, China, the exhibition area is below the water table. Soluble salts have caused damage on both consolidated and unconsolidated soil surfaces (Figure 1), with diseases rates of 30.54% and 13.65%, respectively [15].
Artificial dew is a more promising method for preserving earthen heritage sites [16], though natural dew have caused problems. In recent years, preventive protection methods have received more and more recognition in the conservation of relics [10,17], and the key is minimizing potential risks by controlling and stabilizing surrounding environments. However, the example of the Kuahuqiao Museum shows that air conditioning without stabilizing inner water cannot stop salt damage. [15] As an improvement, it is promising to produce artificial dew on soil surface, using an automatic monitoring and controlling system of environment. By controlling dew amount and position, equilibrium of water potential can be built to stabilize water inside soil and prevent migration of salts. This preventive protection method will be gentle and can provide a fundamental solution for salt destruction.

In a previous study, artificial dew was produced on a soil surface. [16] Experiments in lab showed that dew formation was affected by temperature and humidity, while in situ experiments showed that dew could take salts downward. However, a rational model of dew formation rate was absent, which is essential for developing an environmental control system.

Dew prediction is a common problem but has not been well studied in terms of conserving relics. Up to now, many models have been developed to predict the duration and amount of water condensation on soil [18–22] and have been well summarized [23]. Most of them were developed using energy and mass transport equations, or were based on assessments of latent-heat flux in the atmosphere [24]. However, none of them were developed for relic conservation uses. Alterations and simplifications can be made to produce a new model according to the features of earthen heritage sites.

In this study, we report the construction of a semi-theoretical model for preservation of earthen sites. The model is based on the mass transfer process in an assumed steady air layer, and the main driven force is the differential pressure of water vapor. The model contains three measurable variables and two fitting parameters. Simulations of dew formation in both laboratory and field experiments from published studies are used to confirm its validity and application potential.

2. Theory

This model assumes that there is a steady air layer close to the air–solid interface (Figure 2). It is thin and not disturbed by turbulent flows in the air. Water vapor would pass this layer before condensation on a solid surface. The mass flow of water vapor in this thin layer is driven mainly by the differential pressure of water vapor, while slightly affected by other factors such as temperature, wind, salt content, soil suction, height and vegetation [25,26].
A sketch of moisture transportation with the difference of water vapor pressure (\( pv \)) in a steady air layer. \( N_{\text{diff}} \) is the diffusion rate of water vapor and \( \delta \) is the thickness of the mass transfer field.

For earthen heritages under preservation, their environments are supposed to be stable. For example, wind should be minimized, vegetation should be removed, desalting should be conducted, heat of sunlight should be controlled and variations of soil properties at a site are often limited. As a result, many factors in other models can be neglected in this model, and the assumption of a steady air layer is reasonable.

The model solves for the mass flow in the air layer. Firstly, differential pressure of water vapor should be the main driving force of water condensation. This will cause molecular diffusion. The process can be described using Fick’s laws of diffusion and their derivatives [27]:

\[
N_{\text{diff}} = \frac{D}{RT_a \delta} (pv_1 - pv_2). 
\tag{1}
\]

Here, \( N_{\text{diff}} \) (mol m\(^{-2}\) s\(^{-1}\)) is the diffusion rate of water vapor, \( D \) (m\(^2\) s\(^{-1}\)) is the molecular diffusivity of water vapor, \( R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1} \) is the ideal gas constant, \( T_a \) (K) is the absolute temperature of interface, \( pv_1 \) (Pa) is the partial pressure of water vapor in the atmosphere, \( pv_2 \) (Pa) is the partial pressure of water vapor upon the soil surface and \( \delta \) (m) is the thickness of the mass transfer field.

Then, alterations to Equation (1) must be made to represent the influence of temperature, wind, etc. Therefore, parameter \( a \) is added:

\[
N = \frac{D}{RT_a \delta} (pv_1 - pv_2 + a) 
\tag{2}
\]

Here, \( N \) is the total rate of dew formation. \( D, pv_1, \) and \( pv_2 \) can be easily calculated using the three variables: soil surface temperature \( T_s \), air temperature \( T_a \) and air relative humidity \( RH \). All variables are easy to measure.

The diffusivity of water vapor in atmosphere near the soil surface can be computed by [28]:

\[
D = D_0 \left( \frac{T_a}{273.16} \right)^{1.75} 
\tag{3}
\]

where \( D_0 = 2.29 \times 10^{-5} \text{ m}^2\text{s}^{-1} \).
A simple fitting equation deduced from the Clausius–Clapeyron equation is used to calculate the saturated vapor pressure of water ($p_s$):

$$\ln p_s = A + \frac{B}{T} + C \ln T$$  \hspace{1cm} (4)

Here, $p_{s,T}$ is the saturated vapor pressure of water at temperature $T$. Over the temperature range 273–323 K, $A = 58.431$, $B = -6750.4$ and $C = -4.8668$ [29].

According to the definition of relative humidity, the vapor pressure in the atmosphere ($p_{v1}$) is given by:

$$p_{v1} = RH \times p_{s,Ta}$$  \hspace{1cm} (5)

where $RH$ is the relative humidity in atmosphere and $T_{a}$ is the air temperature.

At the air–soil interface, water vapor condenses and $p_{v2}$ can be calculated considering phase equilibrium. For desalinized soil, the vapor pressure on the soil surface equals the saturated vapor pressure:

$$p_{v2,soil} = p_s$$  \hspace{1cm} (6)

Here, $T_{s}$ is the surface temperature of soil. If soluble salts were added to the soil, the condensed water would form a solution, and $p_{v2}$ would be reduced according to Raoult’s law. In the phase equilibrium of saturated Na$_2$SO$_4$ solution and water vapor, relative humidity of air is a function of temperature. For the temperature range 0–20 °C, the following linear equation was used [30]:

$$RH_{interface} = \frac{p_{v2}}{p_s} = 7.8474 \times 10^{-3} (T_s - 273.15) + 0.60511$$  \hspace{1cm} (7)

Then, $p_{v2}$ can be given using the combination of Equations (4) and (7). If the soil property is unknown, Equation (6) can be used. However, this may cause deviations because of different soluble salts.

3. Materials and Methods
3.1. Data Collection

Data including environmental variables and dew amounts were collected from published articles, and simulations were conducted to validate the proposed model. One piece of literature is our previous study [16] and the other is an important study conducted by Jacobs et al. [18,19,31]. Experimental protocols of our previous study are given in the following sections, because they were originally in Chinese.

3.1.1. Laboratory Experiments

In laboratory experiments, soil samples were kept in culture dishes (diameters 5 cm) and were placed in a desiccator in which the relative humidity was stabilized using a saturated salt solution. The desiccator was then placed in an electron oven (DHG-9070, Shanghai Jinghong Laboratory Instrument Co., Ltd., Shanghai, China), and the soil was cooled using circulating water (Figure 3). According to the experimental settings (Table 1), the temperature of the soil surface was always below the dew point. As a result, the mass of the culture dish increased during the experiment; this increase was equal the mass of dew formed. The mass of the culture dish was measured hourly using an analytical balance (accuracy of 0.1 mg).

Soil was collected from the Kuahuqiao Site Museum. It was desalinized before experiment until the conductivity of the dissolution liquid was equal to that of tap water (XRD analysis results for desalinized soil are shown in supplementary materials file “Fig XRD-soil.jpg”). Next, a small amount of sodium sulfate was added, since it is the most common and destructive salt in heritage conservation [6].

In the previous study [16], each cumulative dew curve was linearly fitted to calculate overall dew formation rate. Then, environmental data and dew formation rates were treated using a binary quadratic equation containing six variables. This will certainly cause
overfitting and large deviations beyond tested conditions. In this study, experimental data were reprocessed from the beginning using the proposed semi-theoretical model.

![Figure 3. A photograph (left) and sketch (right) of the laboratory setup [16]. The desiccator (c) is placed in an electron oven. The soil sample (a) is kept in a culture dish and cooled down using a sandwich beaker and circulating water (b). The vapor pressure in the desiccator is controlled by the oven temperature and a saturated salt solution (d).](image-url)

Table 1. Data from published laboratory experiments [16] and simulations using the proposed model. R values of simulations using the proposed model were >0.9 and fitting parameters are given. Optimized thicknesses of the equivalent air layers $\delta_e$ were also listed.

| Experiment Conditions | Results of Phase I | Results of Phase II |
|-----------------------|-------------------|-------------------|
| Relative Humidity ($\%$) | Measured Rate ($\mu$kgm$^{-2}$ h$^{-1}$) | Simulated Rate ($\mu$kgm$^{-2}$ h$^{-1}$) | Equivalent Thickness $d_e$/m | Measured Rate ($\mu$kgm$^{-2}$ h$^{-1}$) | Simulated Rate ($\mu$kgm$^{-2}$ h$^{-1}$) | Equivalent Thickness $d_e$/m |
| |  | | | | | |
| 1 | 85.39 | 30.07 | 14.49 | 0.06202 | 0.05676 | 0.05031 | 0.02440 | 0.03511 |
| 2 | 75.31 | 31.05 | 23.60 | 0.03201 | 0.04786 | 0.05676 | 0.03003 | 0.02324 |
| 3 | 73.02 | 33.35 | 21.21 | 0.02740 | 0.02834 | 0.02834 | 0.02834 | 0.02834 |
| 4 | 73.93 | 30.10 | 14.44 | 0.05135 | 0.04340 | 0.02834 | 0.02834 | 0.02834 |
| 5 | 86.03 | 30.79 | 23.60 | 0.04972 | 0.05025 | 0.02834 | 0.02834 | 0.02834 |
| 6 | 84.40 | 33.28 | 21.27 | 0.08520 | 0.07286 | 0.01742 | 0.02604 | 0.03360 |
| 7 | 83.80 | 32.12 | 16.24 | 0.08257 | 0.08853 | 0.02604 | 0.03511 | 0.04851 |
| 8 | 85.39 | 35.03 | 19.45 | 0.05678 | 0.05857 | 0.01742 | 0.02604 | 0.03403 |
| 9 | 75.31 | 31.05 | 25.60 | 0.02604 | 0.01977 | 0.02604 | 0.03511 | 0.02604 |
| 10 | 73.02 | 33.35 | 21.20 | 0.06667 | 0.02511 | 0.01977 | 0.03511 | 0.02355 |
| 11 | 73.93 | 35.07 | 19.41 | 0.07227 | 0.07862 | 0.01977 | 0.03511 | 0.0263 |
| 12 | 88.03 | 30.79 | 23.69 | 0.05032 | 0.05025 | 0.01977 | 0.03511 | 0.0263 |
| 15 | 84.40 | 33.28 | 21.27 | 0.08433 | 0.07286 | 0.01742 | 0.02604 | 0.03511 |
| 14 | 82.80 | 35.16 | 19.30 | 0.09740 | 0.09341 | 0.01742 | 0.02604 | 0.03511 |
| 15 | 87.34 | 35.12 | 19.36 | 0.08233 | 0.06436 | 0.01742 | 0.02604 | 0.03511 |
| 16 | 74.79 | 30.96 | 23.70 | 0.03201 | 0.06436 | 0.01742 | 0.02604 | 0.03511 |
| 17 | 79.18 | 33.62 | 20.72 | 0.04540 | 0.04223 | 0.01742 | 0.02604 | 0.03511 |
| 18 | 70.73 | 35.29 | 19.18 | 0.02707 | 0.07474 | 0.01742 | 0.02604 | 0.03511 |
| 19 | 86.10 | 31.34 | 25.29 | 0.05600 | 0.04786 | 0.01742 | 0.02604 | 0.03511 |
| 20 | 84.89 | 35.60 | 20.39 | 0.06055 | 0.07914 | 0.01742 | 0.02604 | 0.03511 |
| 21 | 80.06 | 34.98 | 19.50 | 0.09126 | 0.09149 | 0.01742 | 0.02604 | 0.03511 |

Fitting parameters:

| $J = 2.308 \times 10^{-2}$ | $x = 19.39$ |
| $J = 2.962 \times 10^{-2}$ | $x = 0.02355$ |

Correlation coefficient $R^2$:

| 0.9182 | 0.9829 |

3.1.2. Field Experiments in the Kuahuqiao Museum

In the previous study, field experiments were conducted to preliminarily prove that artificial dew can take surface salts downward. [16] Monitored environmental variables and cumulative dew amounts were simulated using a new model in this study.

At the Kuahuqiao Site Museum, an 8000-year-old dugout canoe is on display. As preservation methods of the boat and the surrounding earthen site, the museum firstly built a ceiling to prevent rain wash, and then installed an air-conditioning system to prevent microorganism growth and excessive water condensation. Consolidation of soil surface using epoxy resin was also carried out [32]. However, these methods prevented water transportation and salt migration uniaxially. After the construction of ceiling and environmental control system, soluble salts moved upward along with water evaporation, crystallized on soil surfaces, and caused damage, [15] as shown in Figure 1. This suggests that factors such as water transportation must be considered in controlling surrounding environments, as a part of preventive protection.
The experimental setup used is shown in Figure 4. Five 0.5 m × 0.5 m square boards pasted with electrothermal films were spliced and arranged to form a cubic space. A humidifier composed of a water bath, a plastic cover and a fan was placed in the cubic space. The walls were then heated and moisture was produced for 6 h. The temperature of the electrothermal films and the water bath were kept at 45 and 50 °C, respectively. The electrical power was then switched off and the top board removed for 1 h so that the soil could cool off naturally. The protocol was carried out four times. Water content inside the soil (0–5 cm deep) was measured using a soil temperature and humidity measuring instrument.

![Experimental Setup](image)

**Figure 4.** A photograph of the experimental setup ((a), top view, top board removed) and a schematic ((b), side view) [16]. Water will condense on the soil surface where the temperature was below dew point.

3.1.3. Field Experiments in a Desert System

Jacobs et al. [18,19,31] conducted an important study on a desert system. They measured daily dew amounts as well as micrometeorological conditions and constructed a simulation model. Environmental variables and cumulative dew amounts were collected from published figures in their study.

3.2. Calculations

3.2.1. Simulations Using the Semi-Theoretical Model

Data including environmental variables and cumulative dew amounts were reprocessed using Equations (2)–(7).

First, cumulative dew curves of laboratory experiments were manually divided into two phases, because transition points were observed at similar position of curves. The relationship between cumulative dew and time was estimated using linear least square fitting with Excel software, and the slopes of the fitting lines are the water condensation rates.

Then, simulations of the dew formation rates were conducted using the proposed model. The least squares curve fit routine (lsqcurvefit) with optimization was used to estimate the values of the two parameters. Curve fitting was performed using Matlab R2016b software (MathWorks, Inc., Natick, MA, America) and the codes are shown in the supplementary materials file “phase1.m” and “phase2.m”.

Last, simulations of field experiments were conducted. Dew formation rates at specific points in time were estimated. Corresponding environmental variables were used to estimate the fitting parameters of the field experiment. Equation (6) was used in fitting because salts in soil were not controlled. Combining environmental variable curves with fitting parameters, the dew formation rate curve and cumulative dew amount curve were constructed.
3.2.2. Numerical Simulations of Laboratory Experiments

Numerical simulations were performed to validate the assumption of a thin and steady air layer in the proposed model. In laboratory experiments, soil samples were placed in culture dishes. As a result, an equivalent air layer was constructed in numerical simulations. The simulation is axisymmetric two-dimensional including a fluid domain composed of moist air (Figure 5). Equivalent layer thickness $\delta_e$ was calculated and was compared with the fitting parameter $\delta$ to check consistency.

![Figure 5](image-url)

**Figure 5.** (a) 2D and (b) 3D schematic of the axisymmetric computational domain used in the model and (c) settings of component node. Optimization study was conducted to calculate the thickness $\delta_e$.

Simulations were performed with Comsol Multiphysics version 5.4 (COMSOL, Inc., Stockholm, Sweden), which was installed on a Dell OptiPlex 3060 with Intel Core i5-8500 (3 GHz). Three interfaces including laminar flow, heat transfer in moist air and moisture transport in air were used. Natural convection and moist transportation were solved together as a stationary problem, and optimization studies were conducted to determine the thickness of the mass transfer field.

Input boundary conditions including three environmental variables are shown in Table 1. Relative humidity on the upper surface (RH), temperature on the upper surface ($T_a$) and temperature on the lower surface ($T_s$) are listed. Additionally, the relative humidity on the lower surface (RH$_{interface}$) was calculated using Equation (7).

In the optimization study, the height of the computational domain (thickness of mass transfer field $\delta_e$) was the controlled variable and optimization objective was minimizing the difference between the water vapor flux and the measured dew formation rate ($N$). Mesh and other unspecified settings remained as default. (Figure 5c)

4. Results

4.1. Laboratory Experiments and Numerical Simulations

Cumulative dew curves of laboratory experiments were composed of two phases, both of which were linearly fitted (Figure 6 and supplementary materials file “Figs lab cum dew.pdf”, all R-square values are above 0.99). The transition points between the two phases were about 0.8–1 kg/m$^2$. Around these points, water could be observed on the
Simulations were performed with Comsol Multiphysics version 5.4 (COMSOL, Inc. US) and a steady air layer was assumed. The inner water vapor flux equals the measured dew formation rate. Results are given in Table 1. Simulations of both phases produced satisfying results. In the proposed model, a steady air layer was assumed and the inner water vapor flux equals the measured dew formation rate \( N \). As a result, an equivalent air layer was constructed in a numerical simulation. Once \( N \) and the other boundary conditions (\( RH, Ta, Ts \)) were set, the equivalent thickness of the assumed air layer (denoted by \( \delta_e \)) was determined. Results are also shown in Table 1.

### 4.2. Field Experiments at the Kuahuqiao Site Museum

During the field experiment, humidity, air temperatures and soil temperature were monitored (Figure 7a). The measured cumulative amount of dew (Figure 7b) was estimated using known dry soil density (1.802 g/cm\(^3\)) and the measured moisture content. These original data were cited, and a cumulative dew curve was constructed using Equations (2)–(6) to simulate this field experiment. Simulation was satisfied when \( \delta = 4.58 \times 10^{-3} \) and \( a = 2.91 \times 10^3 \).

Three small concaves can be observed in the simulation curve (Figure 7b) in accordance with the fluctuations of environmental variables, but disappeared in the measured curve. This is explained by the delay in measuring dew amounts. Measured dew amounts were calculated using the water content beneath the soil surface while simulated dew amounts were on air–soil surface. When the surface dew formation stopped, free water transferred downward continuously. Thus, fluctuations were smoothed by the mass transfer process in soil.

![Figure 6](cum_dew.pdf), all R-square values are above 0.99). The transition points between the two phases were linearly fitted (Figure 6 and supplementary materials file "Figs lab.pdf"). Cumulative dew curves of laboratory experiments were composed of two phases, both of which were linearly fitted (Figure 6 and supplementary materials file "Figs lab.pdf"). The mass increases of samples are composed by two phases. Both phases were treated linearly to calculate the dew formation rate \( N \).

Results are given in Table 1. Simulations of both phases produced satisfying results. Statistical tests revealed that the equations for both phases I and II explained a high proportion of the variation in the dew formation rate—i.e., both \( R^2 \) values between measured and simulated dew formation rates were >0.81. The unexplained variance can be attributed to systematic error caused by accuracy of the model as well as random error in observation.

In the proposed model, a steady air layer was assumed and the inner water vapor flux equals the measured dew formation rate \( N \). As a result, an equivalent air layer was constructed in a numerical simulation. Once \( N \) and the other boundary conditions (\( RH, Ta, Ts \)) were set, the equivalent thickness of the assumed air layer (denoted by \( \delta_e \)) was determined. Results are also shown in Table 1.

![Figure 6](cum_dew.pdf). The mass increases of samples are composed by two phases. Both phases were treated linearly to calculate the dew formation rate \( N \).
4.2. Field Experiments at the Kuahuqiao Site Museum

During the field experiment, humidity, air temperatures and soil temperature were monitored (Figure 7a). The measured cumulative amount of dew (Figure 7b) was in the confidence intervals of the simulation using the proposed model fitted well. Figure 7. (a) Measured environmental variables [16] and (b) cumulative amount of dew in field experiments at the Kuahuqiao Museum. Simulation using the proposed model fitted well.

4.3. Field Experiments in a Desert System

Data from another field experiment [18,19,31] were also simulated. Using the proposed model, a simulation of the cumulative dew amount of dew was performed and the results were shown in Figure 8. \( p_v^2 \) was calculated using Equation (6) because the species of soluble salts were unknown. Fitting parameters used were \( \delta = 3.82 \times 10^{-2} \) and \( a = 1.25 \times 10^5 \). During the time range 10.0–11.6 h, calculated dew amounts were below zero. They were replaced by zero manually, because the value of water content in soil cannot be negative. This simulation of an outdoor experiment was satisfactory, suggesting that the proposed model is potentially useful beyond heritage preservation.

5. Discussions

5.1. Model Validation Using Numerical Simulation

The validity of the proposed model in laboratory experiments has been preliminarily shown in 4.1, by R values over 0.9. Here, the accordance between \( \delta \) and \( \delta_e \) provides more evidence for the assumptions in the proposed model. The difference between fitting parameter \( \delta \) and thickness of the equivalent air layer \( \delta_e \) was analyzed using a two-tailed \( t \)-test (shown in Table 2, \( \alpha = 0.05 \)). For both phases, \( \delta \) was in the confidence intervals of \( \delta_e \), indicating that there was no significant difference between the two groups. The two parameters were not statistically different.
Table 2. Statistical analysis results with a significance level of 0.05. \( \delta_e \) values in numerical simulations were not statistically different from \( \delta \) in model fitting.

|          | Phase I |          | Phase II |          |
|----------|---------|----------|----------|---------|
|          | Equivalent Thickness \( \delta_e \) | Fitting Parameter \( \delta \) | Equivalent Thickness \( \delta_e \) | Fitting Parameter \( \delta \) |
| \( M \)  | 0.02236 | 0.02309  | 0.02692  | 0.02582 |
| \( SD \) | 1.428 \times 10^{-5} | 0  | 4.330 \times 10^{-5} | 0 |
| \( Df \) | 20 | 0 | 13 | 0 |
| \( p \)  | 0.39  | 0.54  | 0.39  | 0.54  |
| \( t \)  | -0.88 | 0.62  | -0.88 | 0.62  |

Additionally, significant differences can be observed between \( \delta_e \) values in two phases \((p = 0.01)\), indicating that the thickness of the air layer was changed after saturation of soil. This rationalized the division of the two phases of dew formation.

5.2. Application Potential and Unexplained Parameters

Setting of the three experiments varied largely. The space of dew formation was centimeter-level in the laboratory experiment, decimeter-level in the Kuahuqiao Museum and open space in desert. The wind causing turbulent flow was zero in the laboratory experiment, strong in the Kuahuqiao Museum (approx. 10 m/s according to product description) and weak in the desert (approx. 2.7 m/s mean annual wind speed [19]). The property of soil was controlled in lab and unknown in situ. Only the water condensation process was tested in our previous study [16], while evaporation was included in desert. As a result, the proposed model has shown application potential in various situations.

The parameters used for model fitting are listed in Table 3. When this model is used to predict dew in a new environment, a preliminary experiment is recommended to determine the values of parameters \( \delta \) and \( a \).

Table 3. Values of fitting parameters used for different experiments.

|          | Laboratory Experiments | Field Experiments in the Kuahuqiao Site | Field Experiments in a Desert System |
|----------|------------------------|----------------------------------------|-------------------------------------|
|          | Phase I | Phase II |                      |                          |
| \( \delta / 10^{-2} \) | 2.309  | 2.583  | 0.458  | 3.82  |
| \( a \)  | -10.03 | -0.04854 | 2.91 \times 10^3 | 1.25 \times 10^3 |

It should be mentioned that the assumption of the proposed model was not fully met in field experiments, where parameter \( a \) were extremely high and the maximum differences of vapor pressure were 5176 and 268 Pa. In these cases, neglected factors such as wind and soil suction, may be the driving force of water condensation. As a result, the physical meaning of \( \delta \) must be reconsidered. It is interesting that simulations using the proposed model still worked well, and the variations of parameter \( \delta \) seem to be explicable. On the one hand, \( \delta \) was much smaller at the Kuahuqiao Site. This is reasonable because the electric fan caused intensely turbulent flows. On the other hand, \( \delta \) was lower in laboratory experiments compared with in a desert; the reason should be the limited space in a desiccator.

Additionally, parameter \( a \), simulating Phase II of laboratory experiments, was close to zero. This can be attributed to the water saturation of soil—the influence of soil suction was prevented after water emerged on the soil surface.

In general, the application potential of the proposed model has been shown, though the physical meanings of parameters in field experiments are obscured. Further study will be needed to give more particular explanations.

6. Conclusions

In this study, a semi-theoretical model is constructed to calculate dew formation rates. This model assumes a steady air layer at air–solid interface and mass transfer in it is calculated. The model contains two fitting parameters: \( \delta \) represents thickness of the
transfer field and $a$ represents overall influence of other factors. Three variables of the model are easy to measure: relative humidity in air ($RH$), air temperature ($T_a$) and the temperature of the soil surface ($T_s$).

The model yielded satisfactory results in fitting laboratory experiments ($R > 0.9$), as well as in fitting published field experiments. Moreover, the physical interpretation of parameter $\delta$ in laboratory experiments was studied using a numerical simulation.

Models such as the one presented here will be useful for preservation of earthen heritage sites. Further research should be conducted to make the model more practical. For example, more accurate interpretations of parameters can be studied. As an expectation, an automatic system using a practical model can be developed to monitor and control surrounding environments of cultural relics. Artificial dew could be produced to prevent soil desiccation in arid regions or salt accumulation in humid regions, while superfluous water condensation can be avoided before it causes damage.

**Supplementary Materials:** The following are available online at https://www.mdpi.com/2073-4441/13/1/52/s1, Figure S1: Fig XRD-soil.jpg, Code S1: phase1.m, Code S2: phase2.m, Figure S2: Figs lab cum dew.pdf.

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