Some Operations and Properties of Neutrosophic Cubic Soft Set

Abstract
In this paper we define some operations such as P-union, P-intersection, R-union, R-intersection for neutrosophic cubic soft sets (NCSSs). We prove some theorems on neutrosophic cubic soft sets. We also discuss various approaches of Internal Neutrosophic Cubic Soft Sets (INCSs) and external neutrosophic cubic soft sets (ENCSs). We also investigate some of their properties.

Keywords: Neutrosophic cubic soft set; Neutrosophic soft set; Cubic set; Internal neutrosophic Cubic soft set; External neutrosophic cubic soft set

Introduction
Neutrosophic set [1] grounded by Smarandache in 1998, is the generalization of fuzzy set [2] introduced by Zadeh in 1965 and intuitionistic fuzzy set [3] by Atanassov in 1983. In 1999, Molodstov [4] introduced the soft set theory to overcome the inadequate of existing theory related to uncertainties. Soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory [2], rough set theory [5], probability theory for dealing with uncertainty. The concept of soft set theory penetrates in many directions such as fuzzy soft set [6-9], intuitionistic fuzzy soft set [10-13], interval valued intuitionistic fuzzy soft set [14], neutrosophic soft set [15-18], interval neutrosophic set [19,20]. In 2012, Jun et al. [21] introduced cubic set combining fuzzy set and interval valued fuzzy set. Jun et al. [21] also defined internal cubic set, external cubic set, P-union, R-union, P-intersection and R-intersection of cubic sets, and investigated several related properties. Cubic set theory is applied to CI-algebras [22], B-algebras [23], BCK/BCI-algebras [24,25], KU-Algebras [(26,27), and semi-groups [28]. Using fuzzy set and interval-valued fuzzy set Abdullah et al. [29] proposed the notion of cubic soft set [29] and defined internal cubic soft set, external cubic soft set, P-union, R-union, P-intersection and R-intersection of cubic soft sets and investigated several related properties. Ali et al. [30] studied generalized cubic soft sets and their applications to algebraic structures. Wang et al. [31] introduced the concept of interval neutrosophic soft set. In 2016, Ali et al. [32] presented the concept of neutrosophic cubic set by combining the concept of neutrosophic set and interval neutrosophic set. Ali et al. [32] mentioned that neutrosophic cubic set is basically the generalization of cubic set. Ali et al. [32] also defined some new type of internal neutrosophic cubic set (INCSs) and external neutrosophic cubic set (ENCSs) namely, \[ \frac{1}{3} \text{INCS} + \frac{1}{3} \text{ENCS} + \frac{1}{3} \text{ENC} \]. Ali et al. [32] also presented a numerical problem for pattern recognition. Jun et al. [33] also studied neutrosophic cubic set and proved some properties. In 2016, Chinnadurai et al. [34] introduced the neutrosophic cubic soft sets and proved some properties.

In this paper we discuss some new operations and new approach of internal and external neutrosophic cubic soft sets, and P-union, R-union, P-intersection, R-intersection. We also prove some theorems related to neutrosophic cubic soft sets.

Rest of the paper is presented as follows. Section 2 presents some basic definition of neutrosophic sets, interval-valued neutrosophic sets, soft sets, cubic set, neutrosophic cubic sets and their basic operation. Section 3 is devoted to presents some new theorems related to neutrosophic cubic soft sets. Section 4 presents conclusions and future scope of research.

Preliminaries
In this section, we recall some well-established definitions and properties which are related to the present study.
Definition 1: Neutrosophic set [1]

Let U be the space of points with generic element in U denoted by u. A neutrosophic set \( \lambda \) in U is defined as \( \lambda = \{<u, t^\lambda(u), i^\lambda(u), f^\lambda(u) \rangle : u \in U\} \), where \( t^\lambda(u) : U \rightarrow [0, 1] \), \( i^\lambda(u) : U \rightarrow [0, 1] \), and \( f^\lambda(u) : U \rightarrow [0, 1] \), and \( t^\lambda(u) + i^\lambda(u) + f^\lambda(u) \leq 3 \).

Definition 2: Interval neutrosophic set [31]

Let U be the space of points with generic element in U denoted by u. An interval neutrosophic set A in U is characterized by truth-

Definition 3: Neutrosophic cubic set [32]

A neutrosophic cubic set in U defined as \( \tilde{\lambda} = \{<u, t^\lambda(u), i^\lambda(u), f^\lambda(u) \rangle : u \in U\} \), where \( t^\lambda(u), i^\lambda(u), f^\lambda(u) \subseteq [0, 1] \) and A is defined as

A=\( \{<u, t^\lambda(u), i^\lambda(u), f^\lambda(u) \rangle : u \in U\} \).

Definition 4: Soft set [4]

Let U be the initial universe set and E be the set of parameters. Then soft set \( F_k \) over U is defined by \( F_k = \{<u, F(e)> : e \in K, F(e) \subseteq P(U)\} \).

Where \( F : K \rightarrow P(U) \), \( P(U) \) is the power set of U and \( K \subseteq E \).

Definition 5: Neutrosophic cubic soft set [34]

A soft set \( \tilde{F}_k \) is said to be neutrosophic cubic soft set iff \( \tilde{F}_k \) is the mapping from K to the set of all neutrosophic cubic sets in U (i.e., \( C\tilde{\lambda}(U) \)),

i.e. \( \tilde{F}_k : K \rightarrow C\tilde{\lambda}(U) \), where K is any subset of parameter set E and U is the initial universe set.

Neutrosophic cubic soft set is defined by

\[ \tilde{F}_k = \{<u, A(e), \lambda(e)> : e \in K, u \in U\} \],

Where, \( A(e) \) is the interval valued neutrosophic soft set and \( \lambda(e) \) is the neutrosophic soft set.

Definition: Internal neutrosophic cubic soft set (INCSS)

A neutrosophic cubic soft set \( \tilde{F}_k \) is said to be INCSS if for all \( e_i \in K \)

\[ T_{\lambda(e_i)}(u) \subseteq T_{\lambda(e_i)}(u) \subseteq T_{\lambda(e_i)}(u) \],

\[ F_{\lambda(e_i)}(u) \subseteq F_{\lambda(e_i)}(u) \subseteq F_{\lambda(e_i)}(u) \],

\[ F_{\lambda(e_i)}(u) \subseteq F_{\lambda(e_i)}(u) \subseteq F_{\lambda(e_i)}(u) \], for all \( u \in U \).

Definition: External neutrosophic cubic soft set (ENCSS)

A neutrosophic cubic soft set \( \tilde{F}_k \) is said to be ENCSS if for all \( e_i \in K \)

\[ T_{\lambda(e_i)}(u) \subseteq T_{\lambda(e_i)}(u) \subseteq T_{\lambda(e_i)}(u) \],

\[ F_{\lambda(e_i)}(u) \subseteq F_{\lambda(e_i)}(u) \subseteq F_{\lambda(e_i)}(u) \],

\[ F_{\lambda(e_i)}(u) \subseteq F_{\lambda(e_i)}(u) \subseteq F_{\lambda(e_i)}(u) \], for all \( u \in U \).

Some theorem related to these topics

Theorem 1

Let \( \tilde{F}_k \) be a neutrosophic cubic soft set in U which is not an ENCSS. Then there exists at least one \( e_i, e \in K \subseteq E \) for which there exists some \( u \in U \) such that

\[ T_{\lambda(e_i)}(u) \notin \{T_{\lambda(e_i)}(u) \cup T_{\lambda(e_i)}(u) \},

\[ F_{\lambda(e_i)}(u) \notin \{F_{\lambda(e_i)}(u) \cup F_{\lambda(e_i)}(u) \},

\[ F_{\lambda(e_i)}(u) \notin \{F_{\lambda(e_i)}(u) \cup F_{\lambda(e_i)}(u) \}, \text{ for all } u \in U \).

Hence proved.

Definition

Let \( \tilde{F}_k \) and \( \tilde{G}_{k_2} \) be two neutrosophic cubic soft sets in U and \( K_1, K_2 \) be any two subsets of K. Then, we define the following:
1. \( \tilde{F}_{K_1} = \tilde{G}_{K_2} \) if \( K_1 = K_2 \) and \( \tilde{F}(e) = \tilde{G}(e) \) \( \forall e \in K \) 
   \( \Rightarrow A(e) = B(e) \) and \( \lambda(e) = \lambda(e) \) \( \forall u \in U \).

2. If \( \tilde{F}_{K_1} \) and \( \tilde{G}_{K_2} \) are two NCSSs in \( U \) and \( K \), \( \tilde{F}_{K_1} \cap \tilde{G}_{K_2} \) iff the following conditions are satisfied:
   i. \( K_1 \subseteq K_2 \) and 
   ii. \( \tilde{F}(e) \subseteq \tilde{G}(e) \) for all \( e \in K_1 \) iff 
      \[ A(e) \subseteq B(e) \text{ and } \lambda(e) \leq \lambda(e) \] 
      \( \forall u \in U \) corresponding to each \( e \in K_1 \).

Definition

Let \( \tilde{G}_{K_1} \) and \( \tilde{G}_{K_2} \) be two NCSSs in \( U \) and \( K_1, K_2 \) be any two subsets of parameter set \( K \). Then we define the R-order as \( \tilde{F}_{K_1} \cap \tilde{G}_{K_2} = H_{K_3} \), where \( K_3 = K_1 \cup K_2 \).

\[ \tilde{H}(e) = \tilde{F}(e), \text{ if } e \in K_1 - K_2 \]
\[ = \tilde{G}(e), \text{ if } e \in K_2 - K_1 \]
\[ = \tilde{F}(e) \cup \tilde{G}(e), \text{ if } e \in K_1 \cap K_2 \]

Hence the proof.

Definition: Compliment

The compliment of \( \tilde{F}_K \) defined by \( \tilde{F}_K \) is defined by 
\[ \tilde{F}_K(e) = \{ u : A(e) > u \} \cup \{ u : B(e) < u \} \] 
\( \forall u \in U, e \in K \).

Some properties of P-union and P-intersection

\[ \tilde{F}_{K_1} \cup \tilde{G}_{K_2} = \tilde{G}_{K_2} \cup \tilde{F}_{K_1} \]

Proof 1:

\[ \tilde{F}_{K_1} \cup \tilde{G}_{K_2} = \tilde{H}_{K_3} \text{ where } K_3 = K_1 \cup K_2 \]
\[ \tilde{H}(e) = \tilde{F}(e), \text{ if } e \in K_1 - K_2 \]
\[ = \tilde{G}(e), \text{ if } e \in K_2 - K_1 \]
\[ = \tilde{F}(e) \cup \tilde{G}(e), \text{ if } e \in K_1 \cap K_2 \]

Here,
\[ \tilde{F}_{K_1} \cup \tilde{G}_{K_2} = \{ u : \min(A(e), B(e)) \}, \min_{\lambda} \lambda(e) > \forall u \in U, \forall e \in K_1 \cap K_2 \]

Hence the proof.

Definition: R-union and R-intersection

Let \( \tilde{F}_{K_1} \) and \( \tilde{G}_{K_2} \) be two NCSSs over \( U \). Then R-union is denoted as
\[ \tilde{F}_{K_1} \cup R \tilde{G}_{K_2} = \tilde{N}_{K_3}, \text{ where } K_3 = K_1 \cup K_2 \text{ and } \tilde{N}_{K_3} \text{ is defined as} \]
\[ \tilde{N}_{K_3} = \{ u : \max_{\lambda} \lambda(e) \}, \max_{\lambda} \lambda(e) > \forall u \in U, \forall e \in K_1 \cap K_2 \}

Here \( \tilde{F}_{K_1} \cap R \tilde{G}_{K_2} \) defined as
\[ \tilde{F}(e) \cap \tilde{G}(e), \text{ if } e \in K_1 - K_2 \]
\[ = \tilde{G}(e), \text{ if } e \in K_2 - K_1 \]
\[ = \tilde{F}(e) \cap \tilde{G}(e), \text{ if } e \in K_1 \cap K_2 \]

Hence the proof.

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Theorem 3

Let $U$ be the initial universe and $I, J, L, S$ any four subsets of $E$, then for four corresponding neutrosophic cubic soft sets $F_1, G_1, H_1, T_1$ the following properties hold.

i. If $F_1 \subseteq G_1$ and $G_1 \subseteq H_1$, then $F_1 \subseteq H_1$.

Proof: If $F_1 \Rightarrow G_1 \Rightarrow H_1$ (by definition)

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$F_1 \subseteq H_1$.

Hence the proof.

ii. If $F_1 \subseteq G_1$, then $(G_1^*)^c \subseteq (F_1^*)^c$ if $I = J$.

Proof: If $I = J$, then $F_1 \subseteq G_1$.

$F_1 \subseteq G_1 \Rightarrow F_1 \subseteq G_1$ (by definition)

$F_1 \subseteq G_1$.

Hence the proof.

Theorem 5

Let $F_1$ and $G_1$ be any two INCSSs then $F_1 \cup G_1$ is an INCSS.

$F_1 \cap G_1$ is an INCSS.

Proof

Since $F_1$ and $G_1$ are INCSSs, so for $F_1$ we have

$T_{A \cup B}(u) \leq T_{A \cup B}(u)$

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And

$\forall u \in U$, $\forall e_i \in I$, $\forall e_j \in J$.

Hence the proof.
Now by the definition of P-union $\overline{F_i} \cup e_i \overline{G_i}$, is an INCSS.

$$\overline{F_i} \cup e_i \overline{G_i} = \overline{H_k},$$
where $K \in I \cap J$.

ii. Now, $\overline{H_k} = \overline{F_i} \cup e_i \overline{G_i}$ and by definition,

$$\overline{F_i}(u), \overline{G_i}(u) = \min\{\min_T(A(u), B(u)), \text{r min}_{\min_T(A(u), B(u))} u \in U, \forall e_i \in K\}.$$

Since $\overline{G_i}$ and $\overline{G_i}$ are INCSS then we have for $\overline{H_k}$,

$$T_{\overline{H_k}}(u) \leq T_{\overline{H_k}}(u) \leq T_{\overline{H_k}}(u), \quad \forall u \in U, \forall e_i \in I,$$

And for $\overline{G_i}$,

$$T_{\overline{H_k}}(u) \leq T_{\overline{H_k}}(u) \leq T_{\overline{H_k}}(u), \quad \forall u \in U, \forall e_i \in J.$$

Also, for $\overline{G_i}$,

$$T_{\overline{H_k}}(u) \leq T_{\overline{H_k}}(u) \leq T_{\overline{H_k}}(u), \quad \forall u \in U, \forall e_i \in I.$$

For $\overline{G_i}$ we have,

$$T_{\overline{H_k}}(u) \leq T_{\overline{H_k}}(u) \leq T_{\overline{H_k}}(u), \quad \forall u \in U, \forall e_i \in J.$$

Also, for $\overline{G_i}$, we have

$$T_{\overline{H_k}}(u) \leq T_{\overline{H_k}}(u) \leq T_{\overline{H_k}}(u), \quad \forall u \in U, \forall e_i \in K.$$

From $\overline{H_k}$ and $\overline{G_i}$, we get

$$\min\{T_{\overline{H_k}}(u), T_{\overline{H_k}}(u)\} \leq \max\{T_{\overline{H_k}}(u), T_{\overline{H_k}}(u)\},$$

$$\min\{I_{\overline{H_k}}(u), I_{\overline{H_k}}(u)\} \leq \max\{I_{\overline{H_k}}(u), I_{\overline{H_k}}(u)\}$$

and

$$\min\{F_{\overline{H_k}}(u), F_{\overline{H_k}}(u)\} \leq \max\{F_{\overline{H_k}}(u), F_{\overline{H_k}}(u)\} \quad \forall e_i \in K.$$

Also given that

$$\max\{T_{\overline{H_k}}(u), T_{\overline{H_k}}(u)\} \leq \min\{T_{\overline{H_k}}(u), T_{\overline{H_k}}(u)\},$$

$$\max\{I_{\overline{H_k}}(u), I_{\overline{H_k}}(u)\} \leq \min\{I_{\overline{H_k}}(u), I_{\overline{H_k}}(u)\}$$

and

$$\max\{F_{\overline{H_k}}(u), F_{\overline{H_k}}(u)\} \leq \min\{F_{\overline{H_k}}(u), F_{\overline{H_k}}(u)\} \quad \forall e_i \in K.$$

Now, $\overline{F_i} \cup e_i \overline{G_i} = \overline{H_k}$, where $K = I \cup J$ and

$$\overline{H_k} = \overline{F_i}(e_i) \cup e_i \overline{G_i}(e_i),$$

if $e_i \in I$.

Hence $\overline{H_k}$ is an INCSS.

Theorem 6

Let $\overline{F_i}$ and $\overline{G_i}$ be any two INCSSS over $U$ having the conditions:

$$\max\{T_{\overline{F_i}}(u), T_{\overline{G_i}}(u)\} \leq \min\{T_{\overline{F_i}}(u), T_{\overline{G_i}}(u)\},$$

$$\max\{I_{\overline{F_i}}(u), I_{\overline{G_i}}(u)\} \leq \min\{I_{\overline{F_i}}(u), I_{\overline{G_i}}(u)\}$$

and

$$\max\{F_{\overline{F_i}}(u), F_{\overline{G_i}}(u)\} \leq \min\{F_{\overline{F_i}}(u), F_{\overline{G_i}}(u)\} \quad \forall e_i \in K.$$

Then $\overline{F_i} \cup e_i \overline{G_i}$ is also INCSS.

Proof

Since $\overline{F_i}$ and $\overline{G_i}$ are INCSSS in U.

So for $\overline{F_i}$ and $\overline{G_i}$, we have

$$\max\{T_{\overline{F_i}}(u), T_{\overline{G_i}}(u)\} \leq \min\{T_{\overline{F_i}}(u), T_{\overline{G_i}}(u)\},$$

$$\max\{I_{\overline{F_i}}(u), I_{\overline{G_i}}(u)\} \leq \min\{I_{\overline{F_i}}(u), I_{\overline{G_i}}(u)\}$$

and

$$\max\{F_{\overline{F_i}}(u), F_{\overline{G_i}}(u)\} \leq \min\{F_{\overline{F_i}}(u), F_{\overline{G_i}}(u)\} \quad \forall e_i \in K.$$

Also for $\overline{G_i}$, we have

$$\max\{T_{\overline{F_i}}(u), T_{\overline{G_i}}(u)\} \leq \min\{T_{\overline{F_i}}(u), T_{\overline{G_i}}(u)\},$$

$$\max\{I_{\overline{F_i}}(u), I_{\overline{G_i}}(u)\} \leq \min\{I_{\overline{F_i}}(u), I_{\overline{G_i}}(u)\}$$

and

$$\max\{F_{\overline{F_i}}(u), F_{\overline{G_i}}(u)\} \leq \min\{F_{\overline{F_i}}(u), F_{\overline{G_i}}(u)\} \quad \forall u \in U, \forall e_i \in J.$$
Now by definition of P-union, $\overline{F}(\cup)G = \overline{F}(\cup)$, where $K = U \cup J$.

Therefore, $\overline{F}(\cup)G = \overline{F}(\cup)$, if $\forall e \in U$.

$= \overline{F}(\cup)$, if $\forall e \in U$.

Then $= \overline{F}(\cup)$, if $\forall e \in U$.

For $e \in I \cup J$ these results are trivial.

Hence the proof.

\section*{Theorem 9}

Let $F_I$ and $G_I$ be any two INCSSs in $U$ such that $\min\{x_{\wedge I}(u), x_{\vee I}(u)\} \leq \min\{x_{\wedge I}(u), x_{\vee I}(u)\}$, $\forall u \in U$. Then $F_I \cup G_I$ is an INCSS.

\section*{Theorem 10}

Let $F = F_I \cup G_I$ be any two INCSSs in $U$ such that $\min\{x_{\wedge I}(u), x_{\vee I}(u)\} \leq \min\{x_{\wedge I}(u), x_{\vee I}(u)\}$, $\forall u \in U$. Then $F_I \cup G_I$ is an INCSS.

Definitions

Let $F = \overline{F}(e') = [\min\{x_{\wedge I}(u), x_{\vee I}(u)\}] \cup \{v \in U, v \notin \forall e \in I \cup J\}$ and $G = \overline{G}(e') = [\min\{x_{\wedge I}(u), x_{\vee I}(u)\}] \cup \{v \in U, v \notin \forall e \in I \cup J\}$ are two INCSSs in $U$. We defined new INCSSs by interchanging the neurofuzzy part of the two INCSSs. We denoted its by $F_I$, $G_I$, and defined by $F_I = \{u, \lambda, \lambda_e \cup \forall u \in U, v \notin \forall e \in I \cup J\}$ and $G_I = \{u, B \lambda, \lambda_e \cup \forall u \in U, v \notin \forall e \in I \cup J\}$ respectively.

Proof

Since $F_I = \{u, \lambda, \lambda_e \cup \forall u \in U, v \notin \forall e \in I \cup J\}$ and $G_I = \{u, B \lambda, \lambda_e \cup \forall u \in U, v \notin \forall e \in I \cup J\}$ are INCSSs, we have

For $F_I$

$T_{I \cup J}(u) \in [\min\{x_{\wedge I}(u), x_{\vee I}(u)\}] \cup \{v \in U, v \notin \forall e \in I \cup J\}$

For $G_I$

$T_{I \cup J}(u) \in [\min\{x_{\wedge I}(u), x_{\vee I}(u)\}] \cup \{v \in U, v \notin \forall e \in I \cup J\}$

Thus $F_I \cup G_I$ is an INCSS.
definition of ENCSSs and INCSSs all the possibility are as under:

\[ T_{v}(a(u)) \leq T_{v}(a(u)) \leq T_{v}(a(u)) \leq T_{v}(a(u)), \]

\[ T_{v}(u) \leq T_{v}(u) \leq T_{v}(u) \leq T_{v}(u), \]

\[ F_{v}(u) \leq F_{v}(u) \leq F_{v}(u) \leq F_{v}(u), \quad \forall e \in I. \]

\[ \text{Case 1} \]

If \( \overline{F}_u = F(u) \), then from (a) and (ii(a)), we have

\[ T_{v}(u) = T_{v}(u), \]

\[ I_{v}(u) = I_{v}(u), F_{v}(u) = F_{v}(u) \text{ and } T_{v}(u) = T_{v}(u), \]

\[ I_{v}(u) = I_{v}(u), F_{v}(u) = F_{v}(u) \text{ and } \forall e \in I \text{ and } \forall u \in U. \]

Thus

\[ T_{v}(u) \leq T_{v}(u) \leq T_{v}(u) \leq T_{v}(u), \]

\[ I_{v}(u) \leq I_{v}(u) \leq I_{v}(u) \leq I_{v}(u), \]

\[ F_{v}(u) \leq F_{v}(u) \leq F_{v}(u) \leq F_{v}(u), \forall e \in I \forall u \in U. \]

\[ \text{Case 2} \]

\[ \overline{F}_u = F(u) \cup \overline{G}(u) \text{ if } e \in I \cap J, \quad \text{then from (a) and (ii(b)), we have} \]

\[ T_{v}(u) = T_{v}(u), \]

\[ I_{v}(u) = I_{v}(u), F_{v}(u) = F_{v}(u) \text{ and } T_{v}(u) = T_{v}(u), \]

\[ I_{v}(u) = I_{v}(u), F_{v}(u) = F_{v}(u) \text{ and } \forall e \in J \forall u \in U. \]

Thus

\[ T_{v}(u) \leq T_{v}(u) \leq T_{v}(u) \leq T_{v}(u), \]

\[ I_{v}(u) \leq I_{v}(u) \leq I_{v}(u) \leq I_{v}(u), \]

\[ F_{v}(u) \leq F_{v}(u) \leq F_{v}(u) \leq F_{v}(u), \forall e \in J \forall u \in U. \]

Hence, if \( e \in I \cap J \), then

\[ \max \{ A_{v}(e) \cup B_{v}(e) \} \leq (A_{v}(e) \cup B_{v}(e)) \leq \max \{ A_{v}(e) \cup B_{v}(e) \} \text{ in all the three cases.} \]

\[ \overline{F}_u \cup \overline{G}_u \text{ is an INCSS in U.} \]

\[ \text{Conclusion} \]

In this paper we have defined some operations such as P-union, P-intersection, R-union, R-intersection for neutrosophic cubic soft sets. We have also defined some operation of INCSSs and ENCSSs. We have proved some theorems on INCSSs and ENCSSs. We have discussed various approaches INCSSs and ENCSSs. We hope that proposed theorems and operations will be helpful to multi attribute group decision making problems in neutrosophic cubic soft set environment.
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