Comparative Study on Prediction Effects of Short Fatigue Crack Propagation Rate by Two Different Calculation Methods

Bing Yang, Zhen Liao, Yahang Qin, Yayun Wu, Sai Liang, Shoune Xiao, Guangwu Yang, Tao Zhu

State Key Laboratory of Traction Power, Southwest Jiaotong University, Chengdu, 610031, China

Email: yb@swjtu.cn

Abstract. To describe the complicated nonlinear process of the fatigue short crack evolution behavior, especially the change of the crack propagation rate, two different calculation methods are applied. The dominant effective short fatigue crack propagation rates are calculated based on the replica fatigue short crack test with nine smooth funnel-shaped specimens and the observation of the replica films according to the effective short fatigue cracks principle. Due to the fast decay and the nonlinear approximation ability of wavelet analysis, the self-learning ability of neural network, and the macroscopic searching and global optimization of genetic algorithm, the genetic wavelet neural network can reflect the implicit complex nonlinear relationship when considering multi-influencing factors synthetically. The effective short fatigue cracks and the dominant effective short fatigue crack are simulated and compared by the Genetic Wavelet Neural Network. The simulation results show that Genetic Wavelet Neural Network is a rational and available method for studying the evolution behavior of fatigue short crack propagation rate. Meanwhile, a traditional data fitting method for a short crack growth model is also utilized for fitting the test data. It is reasonable and applicable for predicting the growth rate. Finally, the reason for the difference between the prediction effects by these two methods is interpreted.

Keywords: short fatigue crack; replica test; crack propagation rate; genetic algorithm; date fitting

1. Introduction
Fatigue fracture is one of the main failure modes for metal structures and components in engineering field. The entire fatigue life is spent in the crack initiation and the crack propagation, in which the latter
can be divided into two stages, i.e. the long crack propagation and the short crack propagation. The initiation and propagation of short cracks can account for a large portion of fatigue life, so it’s important of achieving a more precise understanding of short crack initiation and propagation process in fatigue-related components [1]. Propagation law for long crack is based on the linear elastic fracture mechanics (LEFM), while propagation law for short crack which usually will be affected by many uncertain factors is different from that for long crack. Therefore, the evolution behavior of short crack, including the fretting fatigue crack, is a complex process [1-4]. The actual propagation rate of short crack is greater than that calculated by the theory of LEFM. Dangerous conclusions may be obtained when the LEFM is directly applied to describe the behavior of short crack.

Neural network, which is provided with the ability of strong non-linear processing, can provide a new way to solve various nonlinear dynamics problems. In recent years, neural network has been widely used in solving the complex nonlinear problems in materials science [5]. Zhang put forward the Wavelet Neural Network (WNN) on the basis of the fast decay and the nonlinear approximation ability of wavelet analysis and the self-learning ability of Neural Network [6]. Meanwhile, the Wavelet Neural Network can be trained by various algorithms, in which the gradient method is the commonly used one. However, the computing speed is slow and easy to fall into local extreme value while using this method. Instead, genetic algorithm is a common global optimization method which can effectively avoid the influence of local extreme value [7]. Therefore, the genetic wavelet neural network combined with the advantages of genetic algorithm and wavelet neural network can obviously improve the performance of the wavelet neural network [8]. At present, the method has been successfully applied in many aspects, such as speech recognition [9], fault diagnosis [10], GPS positioning [11], Multi-Sensor Information Fusion Technique [12] and so on.

In present study, the short fatigue crack propagation rate model based on genetic wavelet neural network is established. The predicting results of this model are compared with that of a multi-microstructural barriers model. The comparison results show that these two different models can both reflect the trend of the short fatigue crack propagation rate reasonably and effectively. However, the prediction accuracy of the model based on genetic wavelet neural network is higher while the multi-microstructural barriers model can indicate the periodic deceleration behavior of growth rate during the whole short fatigue crack propagation process.

2. Experimental details
Test material applied in present work is LZ50 axle steel, which is originated from an RE2B type railway axle. Test specimens were machined into a smooth axial funnel shape, while the axial of specimens were consistent with the axial of the RE2B axle. The minimum diameter of the specimen was 8mm while the arc segment of the specimen was polished to a mirror effect as shown in Figure 1. Test results of mechanical properties and fatigue limit at room temperature are listed in Table 1. Metallographic test of LZ50 steel showed that the material has the typical banded structure (Figure 2a) due to the forging process in the manufacture of the axle and “ferrite and lamellar pearlite” structure (Figure 2b).

Before the test, the central arc surface of the specimen was etched with 4% nitric acid alcohol solution so as to expose the metallographic structures. It has been proved that the short crack behavior was not affected by surface etching in previous research because the etching depth is smaller than the roughness of finest engineering surface [13]. All replica tests were carried out on MTS 809 type axial
tension and torsion fatigue testing machine following ASTM E647-11 Standard Test Method for Measurement of Fatigue Crack Propagation Rates. The tests were performed under sinusoidal loading wave with a frequency of 15 Hz in air and at room temperature. The stress amplitude was controlled to be 225 MPa and the stress ratio was 0.1. In the loading process, the test was interrupted at predetermined cycle number. Then, cellulose acetate film softened by acetone was pasted on the specimen arc surface. Once the film was dry, it was peeled off and saved by two glass slides. Above steps were repeated until the specimen was finally fractured. To obtain the evolution information of the short fatigue crack as much as possible, the given cyclic interval was relatively small to ensure that the effective replicating times for each specimen was no less than 10 times. In the meantime, a tensile stress of 10 MPa was retained when the fatigue machine was stopped temporarily to keep the crack in open state during surface replicating process [14].

Figure 1. Schematic of shape and dimension of the specimen for fatigue test (unit: mm)

Figure 2. Metallographic test photograph of LZ50 axle steel

| Test results of mechanical properties and fatigue limit |
|-----------------------------------------------|
| Tensile Strength/MPa | Yield Strength/MPa | Elongation | Reduction of area | Fatigue limit/MPa |
|----------------------|-------------------|------------|-------------------|------------------|
| 674                  | 342               | 20.79%     | 40.38%            | 264.88           |
The dried replica films were examined step by step according to the “reverse observation method” [15] from failure to crack initiation using Olympus OLS4100 laser microscope. Ultimately, cracks information, i.e., the crack length, crack angle and number of cracks, of each specimen at every replicating step were obtained. The “effective short fatigue crack criterion” proposed by Zhao [15] has established the basic frame for the study of short fatigue crack behavior. Based on this criterion, the original data of dominant effective short fatigue crack (DESFC) size at each replica were obtained and the DESFC growth rates with corresponding DESFC size were calculated and illustrated in Figure 3.

![Figure 3](image)

**Figure 3.** DESFC growth rate of LZ50 axle steel with corresponding DESFC size

It can be seen from Figure 3 that the DESFC growth rates of all specimens are different to a certain extent, but the growth of DESFC decreases twice during the propagation process. Combined with the work in reference [16], conclusions can be drawn that the reason for these two significant decreases of DESFC growth rate lies in the restraints of ferrite grain boundary and the rich pearlite banded structure.

### 3. Genetic Wavelet neural network model

#### 3.1 Wavelet Neural Network

Wavelet neural network is a feed-forward neural network combined with wavelet analysis and neural network, of which the essence is using wavelet basis function instead of the neural network hidden layer nodes as incentive function, and expressing the signal of the hidden layer through linear superposition of wavelet [11]. In this study, the wavelet neural network with three layers is shown in Figure 4.

The wavelet neural network output can be expressed as,

\[
f(x) = \sum_{j=1}^{m} \omega_j \phi \left[ \sum_{i=1}^{n} v_{ij} x_i - b_j \right] / a_j \tag{1}
\]

Where \(x_i\) is the node of \(i\) in the input layer, \(v_{ij}\) is the connection weight value of the node of \(i\) in the input layer and the node of \(j\) in the hidden layer, \(\omega_j\) is the connection weight value of the node of \(j\) in the hidden layer and the node in the output layer, \(a_j\) and \(b_j\) are the dilation factor and translation factor of wavelet basis function respectively.
The hidden layer is Morlet wavelet basis function,

$$\psi(x) = \cos(1.75x) \cdot \exp\left(-\frac{x^2}{2}\right)$$  \hspace{1cm} (2)

The output layer is Sigmoid function,

$$f(x) = \frac{1}{1+\exp\left(-\frac{x^2}{2}\right)}$$  \hspace{1cm} (3)

Network learning objective function can be defined as selecting a set of training parameters in the sample set so as to ensure the minimum sample variance.

$$E = \frac{1}{N} \sum_{t=1}^{N} (f(X_t) - f^*(X_t))^2$$  \hspace{1cm} (4)

Where $X_t$ is the training samples for group of $t$, $f(X_t)$ and $f^*(X_t)$ are the corresponding actual and expected output respectively when the training samples of $t$ as the input.

3.2 Wavelet neural network optimized by genetic algorithm

Genetic algorithm (GA) has the typical characteristics of random search and global optimization, and its application object is not limited only if the questions are solvable. The WNN structure is not easy to determine because the node number of hidden layer is not easy to determine. In the present study, the total number of WNN hidden layer nodes is initially determined by trial and error method firstly, but the convergence speed is slow, and it is difficult to obtain the optimal parameters quickly. Therefore, GA is applied to optimize wavelet neural network, which can effectively overcome the characteristic of the uncertainty, avoid the feedback neural network falling into local optimum, and simplify the network training process to a great extent.

GA optimization of WNN includes population initialization, fitness function, crossover operation and mutation operation. Therefore, GA is used to encode the chromosome parameters of WNN, which is composed of wavelet network structure, weight factor, dilation factor and translation factor. The coding method of the specific chromosome is shown in Figure 5, and the $j$ hidden layer of wavelet neural network is taken as an example to display coding method. $v_{1j}, v_{2j}, \ldots, v_{nj}$ is the connection weight value of the node in the input layer to the hidden layer of $j$, $\omega_j$ is the connection weight value of the hidden layer of $j$ to the input layer, $a_j$ and $b_j$ are the wavelet dilation factor and translation.
factor of $j$ respectively. $C_j \epsilon [0, 1]$ is the hidden layer node characteristic coefficient. The structure of WNN can be determined based on the value of $C_j$. According to the experience, when $C_j \geq 0.5$ the wavelet neurons exist, otherwise, it does not exist [17].

\[ \cdots \ C_j \ v_{1j} \ \cdots \ v_{nj} \ \omega_j \ a_j \ b_j \ \cdots \]

Figure 5. GAWNN real coding

The parameters of the GA chromosome were assigned to the corresponding WNN structure, and then the fitness of each individual in the population was calculated, which could reach or approach the optimal solution. The fitness function can be expressed,

\[ F = \frac{1}{E} \]  
(5)

When the value of $E$ is minimum, that is, when the value of $F$ is the maximum, the WNN structure determined by the parameters of the GA chromosome is the best.

Because the nature of the genetic evolution is dynamic and random, it is necessary to carry out the crossover and mutation operation to ensure the diversity of the population. In order to avoid falling into the local optimal solution of WNN, the value of $P_C$ and $P_m$ is changed according to the fitness function value in the process of operation. This adaptive method is to make the value of the $P_C$ and $P_m$ self-adaption change relative to the variety of the maximum and minimum value of $F_{cmax}$, $F_{mmax}$, $F_{cmin}$ and $F_{min}$ and the mean fitness of population $F_{avg}$.

\[ P_C = \begin{cases} \frac{P_{cmax} - P_{cmin}}{1 + \exp \left( \beta \frac{F_c - F_{cavg}}{F_{cmax} - F_{cavg}} \right)} + P_{cmin} & F_c \geq F_{avg} \\ P_{cmax} & F_c < F_{avg} \end{cases} \]
\[ F_c < F_{avg} \]
(6)

\[ P_m = \begin{cases} \frac{P_{mmax} - P_{mmin}}{1 + \exp \left( \beta \frac{F_m - F_{mavg}}{F_{mmax} - F_{mavg}} \right)} + P_{mmin} & F_m \geq F_{avg} \\ P_{mmax} & F_m < F_{avg} \end{cases} \]
\[ F_m < F_{avg} \]
(7)

Where $F_c$ and $F_m$ are the fitness values of the crossed chromosomes and the variant chromosomes respectively. Considering the influence of GA on the global convergence of GAWNN, the value of adjustment factor $\beta$ can be determined by several experiments.

3.3 Genetic Wavelet neural network model

In this study, the number of wavelet neural network input nodes is set to be $n$ while the output node number is 1. The initial number of nodes in the hidden layer is determined by empirical formula [18]. The final number of nodes in the hidden layer needs to be determined by the characteristic coefficients obtained by GAWNN training. The flow chart is shown in Figure 6.

4. Multi-microstructural barriers short crack growth rate model

In early research on the short fatigue crack behavior of LZ50 axle steel, it has been pointed out that the
ferrite grain boundaries and the pearlite banded structure are the main micro-structures which constrain the propagation of short crack [16]. A short fatigue crack growth rate model including multi-microstructural barriers (MMB) has been proposed by introducing a resistance coefficient function to comprehensively consider the effects of ferrite grain boundary and pearlite banded structure on short fatigue crack behavior [16],

$$\frac{da}{dN} = G_o + A[\Delta W_f, a - \Delta W_f \sum_{i=1}^{n} f_i(\Delta d_i) d_i]^m$$  \hspace{1cm} (8)$$

Where a is the size of DESFC, N is the cyclic number, $G_o$ is the minimum growth rate in the first cycle of microscopic scale, $\Delta W_f$ is the total energy density of remote fields, $d_i$ is the characteristic microstructural barrier size, $i$ is the subscript to specify the type of barriers, i.e., $d_1$ for the average equivalent diameter of ferrite grains and $d_2$ for the mean value of intervals between two rich pearlite bands; $A$ and $m$ are corresponding material constants; $f_i(\Delta d_i)$ is the resistance coefficient function, which can reflect the behavior that the closer the crack tip is to the microstructural barrier, the stronger the constraint effect is. Particularly, while the total energy density of remote fields is constant, the model for LZ50 axle steel can be simplified as,

$$\frac{da}{dN} = G_m + A[a - \sum_{i=1}^{n} f_i(\Delta d_i) d_i]^m$$  \hspace{1cm} (9)$$

![Figure 6. Optimization of wavelet neural network with genetic algorithm](image)

5. Predicting results
Genetic neural network model is used to simulate and predict. 90 groups of data were selected from the test results of all the 9 specimens, and a total of 80 groups of data were randomly selected to train the neural network as the learning samples, while the other 10 groups of data were used as the test samples to verify the performance of the network. According to the relationship between the DESFC scale and
the DESFC expansion rate, the network is trained. The crack length, the number of cracks, the applied stress amplitude, the stress ratio, the material fracture toughness and the life fraction are selected as the input node variables. The ESFCs density \( n(N) \) is used as the output node variable, that is to say, the number of neurons in the input layer is 6, while the number of neurons in the output layer is up to 1. The optimization result of the number of neurons in the hidden layer is 12. Error limits \( E_{\text{goal-error}} \) are all \( 10^{-4} \). Meanwhile, the fitting parameters of the fatigue short crack growth rate model as shown in equation (9) based on above 10 groups of data which were applied as the test samples are also calculated and listed in table 2. The fitting curves derived from two different calculation methods are compared with the experimental data as shown in table 3 and figure 7.

It can be seen that both the GAWNN model and the MMB model can reflect the growth trend of short fatigue crack well. In general, the predicted growth rates of the GAWNN method are closer to the test data due to the consideration of synergetic effects of multi-factors. The GAWNN method has smaller absolute values of prediction error at seven of all the ten given DESFC sizes according to test data. However, when the DESFC was initially observed with a crack length of 8.0 \( \mu m \) and propagated at 4.83\( \times 10^{-4} \mu m/\text{cycle} \), and then when it grew to 10.6 \( \mu m \) while the first significant deceleration occurred, and finally when it extended to 131.1 \( \mu m \) according to the second deceleration, the fitting precision of MMB model are much better than or at least the same as that of GAWNN model. To some extent, this is because the MMB model has more clear physical meaning. It takes the influence of microstructural barriers, i.e. the grain boundary of ferrite and the rich pearlite banded structure, into account, so that it can indicate the two significant decreases of crack propagation rate when DESFC propagates to the sizes which are close to the characteristic sizes of microstructural barriers. Moreover, the MMB model can reveal the periodic deceleration behavior of growth rate during the whole short fatigue crack propagation process.

6. Conclusions

Based on the results of replica fatigue short crack test with nine smooth funnel-shaped specimens, the DESFC growth rates are calculated. The DESFC growth rate decreases twice significantly with the increase of DESFC size during the whole propagation process.

Both the short fatigue crack propagation rate model based on genetic wavelet neural network and the multi-microstructural barriers model can describe the change of the short fatigue crack propagation rate reasonably and effectively. The prediction accuracy of the model based on genetic wavelet neural network is relatively higher due to the consideration of synergetic effects of multi-factors while the multi-microstructural barriers model can indicate the periodic deceleration behavior of growth rate during the whole short fatigue crack propagation process due to the physical meaning of its construction.

| Table 2. The model parameters of short crack growth rate of specimens |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( d_1 \) (\( \mu m \)) | \( G_{\text{c1}} \) (\( \mu m/\text{cycle} \)) | \( A_1 \) | \( m_1 \) | \( d_2 \) (\( \mu m \)) | \( G_{\text{c2}} \) (\( \mu m/\text{cycle} \)) | \( A_2 \) | \( m_2 \) |
| 10.60 | 5.20\( \times 10^{-8} \) | 2.71\( \times 10^{-10} \) | 2.88 | 131.12 | 5.81\( \times 10^{-7} \) | 6.10\( \times 10^{-12} \) | 1.38 |
Table 3. Growth rate comparison between test data and predicted value by two calculation methods

| DESFC length (μm) | Test data (μm/cycle) | GAWNN model | MMB model |
|-------------------|----------------------|-------------|-----------|
|                   |                      | Predicted value (μm/cycle) | Difference ration (%) | Predicted value (μm/cycle) | Difference ration (%) |
| 8.0               | 4.83×10^{-4}        | 5.10×10^{-4} | 5.59      | 4.74×10^{-4} | -1.86     |
| 9.2               | 2.33×10^{-4}        | 2.2×10^{-4}  | -5.58     | 2.74×10^{-4} | 17.60     |
| 10.6              | 1.05×10^{-4}        | 1.06×10^{-4} | 0.95      | 1.06×10^{-4} | 0.95      |
| 11.9              | 1.27×10^{-4}        | 1.39×10^{-4} | 9.45      | 6.06×10^{-4} | 377.17    |
| 18.7              | 1.87×10^{-4}        | 2.43×10^{-4} | 29.95     | 6.24×10^{-4} | 233.69    |
| 25.7              | 5.08×10^{-4}        | 4.81×10^{-4} | -5.31     | 6.47×10^{-4} | 27.36     |
| 85.4              | 7.35×10^{-4}        | 8.32×10^{-4} | 13.20     | 8.47×10^{-4} | 15.24     |
| 131.1             | 5.81×10^{-4}        | 6.32×10^{-4} | 8.78      | 5.88×10^{-4} | 1.20      |
| 142.7             | 3.84×10^{-3}        | 3.87×10^{-3} | 0.78      | 2.26×10^{-3} | -41.15    |
| 654.6             | 1.06×10^{-2}        | 1.28×10^{-2} | 20.75     | 1.52×10^{-2} | 43.40     |

Figure 7. Fitting effects of two different models to test data

Acknowledgements
Present work is supported by the National Natural Science Foundation of China (51675446 and U1534209), the National Key Research and Development Program of China (2016YFB1200403), and the Opening Project of State Key Laboratory of Traction Power (2015TPL_T13).

Reference
[1] Ferjaoui A, Yue T, Abdel Wahab M and Hojjati-Talemi R 2015 International Journal of Fatigue 73 66
[2] Kumar D, Biswas R, Poh LH and Abdel Wahab M 2017 Tribology International 109 124
[3] Resende Pereira KdF, Bordas S, Tomar S, Trobec R, Depolli M, Kosec G and Abdel Wahab M 2016
[4] Hojjati Talemi R and Abdel Wahab M 2013 *Tribology International* **60** 176
[5] Xu Q, Zhang X H, Han J C, He X D and Pan W 2005 *Materials Science &Technology* **13** 352
[6] Zhang Q H and Benveniste A 1992 *IEEE Transactions on Neural Networks* **3** 889
[7] Lin X Y 2011 *Ordnance Industry Automation* **30** 42
[8] Zhang X L, Gao M J and Gu F C 2003 *Journal of System Engineering* **18** 147
[9] Han Z Y, Wang J and Lun S X 2010 *Computer Science* **37** 243
[10] Liu M R 2009 *The Research on Method of Fault Diagnosis for Analog Circuits Based on Genetic Algorithm, Wavelet Analysis and Neural Networks* (Changsha: Hunan University) 25
[11] Li J P 1997 *Wavelet Analysis and Signal Processing* (Chongqing: Chongqing Publishing Group) 259
[12] Gao M J, Zhao Y and TAN A 2007 *Chinese Journal of Scientific Instrument* **28** 2103
[13] Zhao Y X, Gao Q and Wang J N 2000 *Acta Metallurgica Sinica* **36** 931
[14] Zhao Y X, Gao Q and Wang J N 2000 *Acta Metallurgica Sinica* **36** 937
[15] Zhao Y X, Gao Q and Wang J N 1999 *Fatigue & Fracture of Engineering Materials & Structures* **22** 459
[16] Yang B and Zhao Y X 2012 *International Journal of Fatigue* **35** 71
[17] Hu X H and Chen L 2009 *Journal of Northwest University: Natural Science Edition* **39** 203
[18] Luo Y C, Du H J amd Cui F F 2007 *Modern Electronics Technique* **30** 88