\(\kappa\)-Symmetry and Bogomol’ny Bound

Dimitri Polyakov\(^\dagger\)

\textit{L.D.Landau Institute for Theoretical Physics}
\textit{Kosygina 2, 117940, Moscow V-334, Russia}\(^\ddagger\)

Abstract

By applying the twistor-like transform to the GS superstring theory we fix the local fermionic gauge symmetry, known as \(\kappa\)-symmetry. Fixing the \(\kappa\)-symmetry in the GS superstring theory and the \(\mathcal{N} = 1\) worldsheet supersymmetry in the NSR theory, proves the relation between these theories, conjectured earlier; the GS fermionic variable \(\theta\) is shown to be related to the NSR worldsheet fermionic variable \(\psi\) through the twistor field \(\Lambda\). Upon fixing the \(\kappa\)-symmetry, we derive the gauge-fixed expressions for the supercharges (corresponding to spinor representations of \(\text{SO}(8)\)) and analyze the gauge-fixed space-time superalgebra. The analysis relates the \(\kappa\) invariance to the Bogomol’ny-type condition, with the former appearing as a space-time superpartner of the latter. The BPS saturation appears to be crucial for the non-perturbative formulation of the \(\kappa\)-symmetry. The relation to the RR charges of p-brane solutions of the \(D = 10\) supergravity is discussed.

August-September 96

\(^\dagger\) cfctconf@itp.ac.ru,polyakov@pion.rutgers.edu

\(^\ddagger\) after the 1st of October, 1996: IHES, 35, Route de Chartres, 91440 Bures-sur-Yvette, France
1. Introduction

The recent significant achievements in the area of string-string dualities and their both actual and potential relevance to the understanding of superstring theory in general, make the study of the dynamics of extended objects, such as p-branes [1], particularly important. The problem of the quantization of p-branes remains unresolved by now, in particular because of complicated non-linear worldvolume equations of motion. One problem is that, contrary to the case of string theory, it is not possible to use the conformal gauge for a worldvolume metric in case of \( p > 1 \), which crucially simplifies the worldsheet equations of motion in string theory. Apart from this apparent difficulty, the known classical formulations of super p-branes are Green-Schwarz ones rather than NSR. At the same time, even the problem of the covariant quantization of superstring theory in the GS formulation is still not solved consistently, in spite of certain important steps in this direction made in \([2,3,4,5]\). An important breakthrough in the study of extended objects has been made in the work by Polchinski, \([6]\) which used the idea of relating the \( p > 1 \) extended objects to strings with mixed Dirichlet-Neumann boundary conditions (D-branes). D-branes were shown to break one half of the supersymmetries in the theory, i.e. to satisfy the BPS saturation condition. The important computation made in this paper has shown that these BPS-saturated objects carry a complete set of intrinsic Ramond-Ramond electric and magnetic charges, satisfying the Dirac’s quantization condition. It should be noted, however, that the D-brane description of a p-brane dynamics is the approximate one, true only up to massless excitations, propagating on the D-brane, which become the collective coordinates for the transverse fluctuations. The question of whether the Dirichlet string - p-brane correspondence remains true when the massive modes are taken into account, is unclear and requires the knowledge of consistent quantization of p-brane theories. One particular physical example of D-brane as a soliton carrying RR charges has been considered in \([7]\).

2. \( \kappa \)-symmetry and the twistor-like formalism

Let us now return to the question of the covariant quantization of a GS superstring, mentioned above. The main obstacle to the quantization is that the theory contains a specific constraint relating the canonical momenta \( P_X \) and \( P_\theta \), the canonical conjugates of the target space superpartners \( X^m \) and \( \theta^\mu \). Because of this constraint, the straightforward quantization procedure using the Dirac’s brackets leads to extremely cumbersome expressions which cannot be resolved without breaking a general covariance. This problem has a very important and subtle connection with a specific local fermionic gauge symmetry, known as the \( \kappa \)-symmetry, which is present in the GS formulation of superstring theory,
as well as in super p-brane theories. This gauge symmetry, potentially very important for the consistent quantization of both GS superstrings and super p-branes, has a delicate relation to the local worldsheet (worldvolume) supersymmetries and plays an important role in insuring the supersymmetry in the target space. Namely, in the GS superstring theory it eliminates 8 out of 16 space-time fermionic degrees of freedom, thus insuring that 8 physical bosonic coordinates have the equal number of superpartners in the target space. For a GS superstring with the action [3]:

\[
S = -\frac{1}{2\pi} \int d^2\tau \sqrt{h} h^{ij} \Pi_i^m \Pi_j^m + \frac{1}{\pi} \int d\tau (-i\epsilon^{ij} \partial_i X^m \theta_{\mu} \Gamma_{m}^{\mu\nu} \partial_j \theta_{\nu}),
\]

where \( \Pi_i^m = \partial_i X^m - i\theta_{\mu} \Gamma_i^{mn} \partial_n \theta_{\nu}, m = 1, \ldots, 10, \mu = 1, \ldots, 16, i = 1, 2; \) the \( \kappa \)-transformations have the form:

\[
\begin{align*}
\delta \theta_{\mu} &= 2i \Gamma_{i\mu}^m \Pi_i \kappa^i, \\
\delta X^m &= i\theta_{\mu} \Gamma_i^{mn} \delta \theta_{\nu} \\
\delta (\sqrt{h} h^{ij}) &= -16\sqrt{h} P_{+}^{ij} \kappa^i \partial_k \theta_{\mu}
\end{align*}
\]

with \( P_{\pm}^{ij} = \frac{1}{2} (\epsilon^{ik} \pm \epsilon^{jk}) \) This may be generalized to the case of a superstring propagating in a curved target space, coupled to SO(32) YM background. The action then has the form [4]

\[
S = \int d^2\tau (-\frac{1}{2} \sqrt{h} h^{ij} F_i^a F_j^a + \frac{1}{2} \epsilon^{ij} \partial_i Z^M \partial_j Z^N B_{NM} + \\
\frac{1}{2} \psi^I \gamma^i D_i \Psi^J)
\]

where \( Z^M = (X^m, \theta^\mu) \) denotes the superspace coordinates, \( A = (a, \alpha) \) labels the tangent superspace, \( D_i \Psi^I = (\partial_i \delta^{IJ} + \partial_i Z^N A_{IJ}^M) \Psi^J \), the Kalb - Ramond 2-form \( B = B_{NM} dZ^M \wedge dZ^N, \psi^I \) are heterotic fermions, \( A_M \) is the YM superfield and \( E_i^a \) is the pulled back supervielbein: \( E_i^A = \partial_i Z^M E_M^A \). The \( \kappa \)-symmetry for this action has the form:

\[
\begin{align*}
\delta Z^M &= \kappa_{\alpha} P_{i}^{\alpha} E_i^a \Gamma_\alpha^{\beta} E_j^M \\
\delta \psi^I &= -\delta Z^M A_{IJ}^M \psi_J \\
\epsilon_{i\alpha} \delta \epsilon_{j} &= P_{+i}^m P_{+j}^n S_m^\alpha \kappa_{n\alpha}
\end{align*}
\]

where \( S_m^\alpha \) is determined by the \( \kappa \)-invariance of the action. It is important that the \( \kappa \)-invariance condition, along with fixing \( S_m^\alpha \), also imposes the following constraints on the
space-time supertorsion and YM superfield strengths [9,10,11]:

\[
T^{c}_{\alpha\beta} = 2\Gamma^{c}_{\alpha\beta}
\]

\[
T_{\alpha[bc]} = u_{[b}^{\beta}\Gamma_{c]\beta\alpha} + \eta_{bc} v_{\alpha}
\]

\[
H_{\alpha\beta\gamma} = 0, \quad H_{a\alpha\beta} = -2(\Gamma_{a})_{\alpha\beta},
\]

\[
H_{ab\alpha} = 2(\Gamma_{ab})_{\alpha\beta}\psi_{\alpha} + 2u_{[a}^{\beta}\Gamma_{b]\beta\alpha}
\]

\[
F_{IJ}^{IJ} = 0, \quad F_{a\alpha}^{IJ} = (\Gamma_{a})_{\alpha\beta}\chi^{\beta I J}
\]

\[
S_{m}^{\alpha} = -4e_{m}^{\alpha} + 2E_{m}^{\alpha}(-u_{a}^{\alpha} + \Gamma^{a\beta} v_{\alpha}) - \frac{1}{2} \psi^{I} \gamma_{I J} \chi^{\alpha J}
\]

The generalization to a case of super p-branes is straightforward (see, for example, [9,12]). The background constraints required by the \(\kappa\)-symmetry were shown to be consistent with those of the pure \(N = 1, D = 10\) supergravity, in the absence of the YM sector, ignoring the \(\kappa\)-symmetry anomalies [13]. Let us now return to the case of a GS superstring in a \(D = 10\) flat target space and the related problems of quantization. As we mentioned above, the \(\kappa\)-symmetry removes a half of the fermionic degrees of freedom in the space-time. However, the exact explanation, showing how it happens precisely, has not been given so far. At the same time, this question is crucial for the gauge fixing in the GS theory. In the section 4 we will give an answer to that by using the twistor-like formalism [9,10,11,12]. The straightforward attempt to fix the gauge by using the standard scheme does not lead to a complete success because one has to introduce an infinite number of ghosts and to deal with regularizing infinite sums; moreover, this approach still leaves a residual gauge symmetry [14]. These difficulties are related to the fact that there is no conserved Noether charge corresponding to the \(\kappa\)-symmetry and this symmetry is infinitely reducible, in terms of the BV formalism. The alternative approach, known as twistor-like formalism, which we will particularly use in this paper, consists of trading the \(\kappa\)-symmetry for an extended local worldvolume supersymmetry; the procedure involves the elevation of a \((p+1)\)-dimensional worldvolume into a superspace with a necessary number of fermionic components which replace the ones of the gauge \(\kappa\)-parameter. The idea is that the resulting action with an extended local worldvolume supersymmetry would reproduce the background constraints (5) and have the equal number of the gauge parameters. Such an action may indeed be constructed and shown to be classically equivalent to the original one with the \(\kappa\)-symmetry. This construction has been performed in details in [9] for the cases of massive
superparticles, superstrings and super p-branes. The important element of the construction is the twistor-like identity:

$$E^\bar{\alpha}_a \Gamma_{\bar{\alpha} \bar{\beta}} E^\beta_b = \Gamma^a_{\alpha \beta} E^\bar{a}_a$$  \hspace{1cm} (6)

where we have modified our notations: all the indices with the “bar” now refer to the target superspace while those without the bar label the superworldvolume. These notations will be used from now on and, basically, they follow the reference [9]. The matrix $E^\bar{A}_A = E^M_A \partial_M \bar{Z}^\bar{M} E^A_M$, where $E^M_A$ and $E^\bar{M}_\bar{A}$ are supervielbeins in the superworldvolume and the target superspace respectively. Particularly, in the superstring case (the case we will be interested in) the $\kappa$-symmetry contains 8 gauge parameters; therefore, the appropriate superstring theory with the $\kappa$-symmetry replaced with an extended worldsheet supersymmetry, must in fact have the $N = 4$ supersymmetry on the worldsurface (so that the numbers of gauge parameters in both theories are equal). The appropriate action with the $N = 4$ worldsheet supersymmetry is

$$S = \int d^2 \tau d^8 \theta (P^\alpha_{\bar{a}} + P^{M_1 M_2}_{\bar{a}} (\tilde{B}_{M_1 M_2} - \partial_{M_1} Q_{M_2}))$$  \hspace{1cm} (7)

where $P^\bar{A}_{\bar{a}}, P^{M_1 M_2}_A$ and $Q_M$ are Lagrange multiplier superfields and the 2-form

$$B_{M_1 M_2} = (-1)^{M_1(M_2+M_2)} \partial_{M_1} \bar{Z}^\bar{M}_1 \partial_{M_2} \bar{Z}^\bar{M}_2 B_{\bar{M}_1 \bar{M}_2} -$$

$$\frac{i}{32} \Gamma_{c_2}^{\alpha \beta} (E^c_{M_2} E^{c_1}_{M_1} H_{\alpha \beta c_1} - E^c_{M_1} E^{c_1}_{M_2} H_{\alpha \beta c_1});$$

$$H_{\alpha \beta c_1} = E^\bar{A}_a E^B_b E^C_c H_{CBA}$$  \hspace{1cm} (8)

is constructed so that it is closed on the worldsheet. The fermionic superworldsheet index $\alpha$ may be represented as $\alpha = \tilde{\alpha} r$ where $\tilde{\alpha} = 1, 2$ is the worldsheet spinor index and $r$ labels the automorphism group of the $N = 4, d = 2$ supersymmetry; the matrix $\Gamma_{\alpha \beta}^a = \gamma_{\alpha \beta}^a \eta_{rs}$, where $\gamma^a$ are 2d gamma-matrices and $\eta$ is an invariant $4 \times 4$ tensor of the automorphism group (which is SU(2) in our case). Furthermore, the following constraints should be imposed on the worldsurface supertorsion:

$$T^a_{\alpha \beta} = -2i (\Gamma^a)_{\alpha \beta}$$

$$T^a_{b \alpha} = 0, T^a_{b \alpha} = 0, T^{\gamma}_{\alpha \beta} = 0$$  \hspace{1cm} (9)

Let us make the final remark before explaining the idea behind the procedure of fixing the $\kappa$-symmetry gauge. The fact that the $\kappa$-symmetry removes 8 fermionic degrees of freedom implies that only a half of the total number of supersymmetries (8 out of 16) actually
present in the theory. This becomes especially clear given the relation between GS and NSR formulations of superstring theory, discussed in [13,2,3,4]. The generator of a space-time supersymmetry in the NSR theory [16]:

\[ Q_{\bar{\mu}} = \oint dz \frac{i}{2\pi} e^{-1/2 \phi} \Sigma \bar{\mu}, \]

where \( \phi \) is a bosonized superconformal ghost, \( \Sigma \) is a spin operator for matter fields, \( - \) has 16 components while the actual number of supersymmetries in the space-time is equal to 8. This leads to the following questions: 1. One may think of the connection between \( \kappa \)-invariance and BPS saturation condition, because of the common property of the BPS saturated extended objects (such as D-branes) and the ones with the \( \kappa \)-symmetry to have a half of the supersymmetries eliminated. We will show that this relation between the \( \kappa \)-symmetry and the Bogomol’ny bound does indeed exist, with the BPS saturation condition being a target-space superpartner of the \( \kappa \)-invariance. 2. The expression for the target space supercharge \( Q_{\bar{\mu}} \) must be modified upon fixing the \( \kappa \)-symmetry in order to generate 8 supersymmetries instead of 16. We will obtain the expression for the gauge-fixed space-time SUSY generator (we will call it the “\( \kappa \)-projected supercharge. 3. Given the property of the D-branes (which are BPS saturated extended objects) to carry a set of RR charges, we will discuss the relation of the \( \kappa \)-symmetry to the RR charges of the extended objects. The p-form central charges in the space-time supersymmetry algebra, related to p-brane solutions of the corresponding low-energy effective theories, will appear in the O.P.E. between spin operators for matter fields.

3. Gauge fixing in the NSR superstring theory

Before explaining the procedure of fixing the \( \kappa \)-symmetry gauge in the GS theory, let us illustrate the method on a much more elementary example, namely, fixing the gauge symmetry (N=1 worldsheet supersymmetry) in the NSR theory. In this case fixing the gauge worldsheet supersymmetry eliminates 2 out of 10 worldsheet fermions, thus leaving the appropriate number of physical fermionic degrees of freedom on the worldsheet. The relation implementing the gauge fixing is

\[ S^a = 0 \] (10)

where \( S^a = \frac{1}{2} \gamma^b \gamma^a \psi^\bar{m} \partial_b X_{\bar{m}} \) is the worldsheet supercurrent. In the components, this condition may be written as

\[ S_\pm = \psi^\mu_{\pm} \partial_\pm X_{\bar{\mu}} = 0 \] (11)

Geometrically, this means that \( \psi_{\pm}^{||} = 0 \), i.e. the worldsheet supersymmetry insures that the components of \( \psi \) parallel to the worldsurface vanish. Thus the gauge fixing leaves 8 physical worldsheet fermions out of 10, orthogonal to the worldsheet. Let us now introduce 10 basic vectors \((e^a, n^i)\) in the space-time, \( a = 1, 2, i = 1, ..., 8 \) with \( e^a \) tangent and \( n^i \) orthogonal
to the world surface. We choose: \((e^a, e^b) = \delta_{ab}, (n^i, n^j) = \delta_{ij}, (e^a, n^i) = 0\). The following relations take place:

\[
\psi^\mu = \varphi^a e_a^\mu + \varphi^i n_i^\mu \\
\partial_\pm X^\mu = \eta_{\pm}^a e_a^\mu \\
\partial_\pm e^a = A_{\pm}^{ab} e_b + B_{\pm}^{ai} n_i \\
\partial_\pm n^i = C_{\pm}^{ij} n_j - B_{\pm}^{ai} e_a
\]

(12)

Here \(v\) is a zweibein, \(A, B, C\) are the components of a 10d spin connection (with \(A\) being a 2d spin connection). Note that, for a superstring propagating in a flat space-time, one may choose \(B, C = 0\). Fixing the conformal gauge: \(h_{ab} = e^\varphi \eta_{ab}\) for the worldsheet metric with \(\varphi\) being a conformal factor and \(\eta_{ab}\) the 2d Minkowski metric, one may choose the zweibeins: \(v_1^\pm = \pm \frac{i}{\sqrt{2}} e^\varphi \), \(v_2^\pm = \frac{i}{\sqrt{2}} e^\varphi\). The gauge fixing condition \(S^\pm = 0\) then gives:

\[
\varphi^a_\pm = e^{ab} \varphi^{b}_\pm
\]

(13)

Now, let us substitute this all into the NSR superstring action:

\[
I_{NSR} = \int d^2 \tau \eta_{\pm} \eta^\mu \eta_\nu (\partial X^\mu \partial X^\nu + \psi^\mu \partial \psi^\nu + \psi^\nu \partial \psi^\mu) \\
(\psi) (14)
\]

where \(\eta_{\pm}\) is the Minkowski metric in a 10d target space with the signature :\((-+, +...+\)). The second term in the \(I_{NSR}\) becomes:

\[
\psi^\mu \partial \psi^\nu \eta_{\pm} \eta^\mu \eta^\nu = \varphi^1_\varphi^2 (A_{12}^1 + A_{21}^2) + \\
\varphi^i_\varphi^a (B_{ai}^a - B_{ai}^a) + \varphi^i_\varphi^i + \varphi^i_\varphi^i + \\
(\varphi^1_\varphi^i - \varphi^2_\varphi^i + \varphi^i_\varphi^i)
\]

(15)

The first three terms in this expression vanish automatically the 4th and 5th cancel due to (18) and only the last term remains. Analogously, the constraint \(S^+ = 0\) leads to

\[
\psi_+^\mu \partial \psi_+^\nu = \varphi_+^i \varphi_+^i
\]

(16)

Apart from that, the reparametrizational invariance condition meaning \(T = 0\), \(T\) is the stress-energy tensor, removes 2 out of 10 bosons from the theory, leaving only the transverse ones. Therefore, the gauge-fixed NSR action:

\[
I_{NSR-g.f.} = \int d^2 \tau (\partial X^i \partial X^i + \varphi_+^i \partial \varphi_+^i + \varphi_-^i \partial \varphi_-^i) \\
(17)
\]
consists of the “transverse” worldsheet bosons and fermions only. Next, we are going to generalize the method developed in this section to fix the gauge in the NSR theory, to the case of the $\kappa$-symmetry, in order to fix the gauge in the GS action. The important step will be to use the twistor-like formalism replacing the GS action with the classically equivalent $N = 4$ worldsheet supersymmetric action (6)(?) and to find the corresponding 4 worldsheet supercharges $Q^r_\pm, r = 1,\ldots, 4$. Then, the gauge fixing will be determined by the constraints

$$Q^r_\pm = 0,$$

classically equivalent to the $\kappa$-invariance condition. These constraints will eliminate 8 out of 16 components of $\theta^\bar{\mu}$, leaving the “gauge-fixed” GS fermionic variable $\theta^i, i = 1,\ldots, 8$. We will see the remarkable relation of this variable to the orthogonal worldsheet fermions $\varphi^i$ in the NSR formalism, thus coming to the conclusion that the relation between NSR and GS formulations of superstring theory is insured by the $\kappa$-symmetry.

4. Gauging away the $\kappa$-symmetry

Let us consider the action (7) and the associated supertorsion constraints (9) on the worldsheet. The local $N = 4$ supersymmetry transformations, the part of the worldsheet superdiffeomorphisms, preserving the constraints (9) and leaving the action (7) invariant are given by:

$$\delta \tau^a = -\frac{i}{2} \gamma^a \tilde{\eta}^{rs} \theta^r \tilde{D}^s \lambda$$

$$\delta \theta^r_{\bar{\alpha}} = -\frac{i}{2} \tilde{D}^r \lambda$$

Here $\tau^a$ are worldsheet coordinates, $\gamma^a$ are 2d gamma-matrices, $\eta^{rs}$ is a $4 \times 4$ SU(2) invariant tensor, $\lambda$ is an arbitrary infinitesimal superfield. This is the 2d generalization of the worldline superdiffeomorphisms of [9]: The equations of motion corresponding to the action (7) are given by:

$$E^a_{\bar{\alpha} \tau} = 0$$

$$\partial_{M_1} \tilde{B}_{M_2 M_3} + \text{cyclic}[M_1 M_2 M_3] = \tilde{H}_{M_1 M_2 M_3} = 0$$

$$\partial_{M_1} P^{M_1 M_2} = 0$$

The straightforward computation, using (7), (19), taking into account (20) and transformation properties of supertensors gives the following expression for the supercharges:

$$Q^r_{\bar{\alpha}} = \int d^2 \tau d^8 \theta (\gamma^a_{\bar{\alpha} \beta} \eta^{rs} \tilde{P}^a_{\bar{\alpha} \beta} E^r_a)$$
Analogously to (10), the $\kappa$-invariance condition, written in terms of the $N = 4$ worldsheet supersymmetry, is given by

$$S^r_\alpha = \int d^8\theta \gamma^a_{\tilde{\alpha} \tilde{\beta}} \eta^{rs} \bar{P}^s_{\tilde{\alpha} \tilde{\beta}} \bar{E}^\alpha = 0 \quad (22)$$

Similarly to the NSR case, this becomes the gauge-fixing condition for the GS superstring theory, which gauges away the $\kappa$-symmetry. Here (22) is the system of 8 equations with 16 variables, i.e. the one that gauges away 8 space-time fermionic components. We will now concentrate on analyzing (22) for a flat target superspace, in order to find the gauge-fixed expression for the $\theta^\mu$. The space-time supervielbeins may be chosen as follows:

$$E^\alpha = \delta^\alpha, \quad E_\mu = \delta_\mu, \quad E^\alpha = \Gamma^\alpha_{\tilde{\alpha} \tilde{\beta}} \bar{E}^\beta$$

and the expansions of the space-time supercoordinates $Z^\bar{M} = (X^\bar{m}, \theta^\mu)$ in $\theta^\mu$ is given by:

$$X^\bar{m} (\tau^a, \theta^\mu) = X^\bar{m} (\tau^a) + \theta^\mu \Gamma^\alpha_{\tilde{\alpha} \tilde{\beta}} \bar{E}^\beta + \ldots$$

where $\Lambda$ is a twistor-like variable, a commuting worldsheet and space-time spinor. Using (23), (24), the expression for the matrix $E^A_\bar{A}$ and performing the integration over $\theta^\mu$, we obtain after the computation:

$$\gamma^a_{\tilde{\alpha} \tilde{\beta}} \eta^{rs} p_\alpha \left[ \partial_a \psi^\alpha_{\bar{s}} + \partial_a \theta^\mu \Gamma^\alpha_{\tilde{\alpha} \tilde{\beta}} \bar{E}^\beta + \partial_a \Lambda^\alpha_{\tilde{\alpha} \tilde{\beta}} \bar{E}^{\bar{s}} \right] = 0 \quad (25)$$

where

$$p_\alpha = (D_{\bar{s}}) \bar{P}^{\bar{s}} \bar{E}^\alpha$$

Several conclusions may now be drawn from the equation (25), which plays the central role in fixing the gauge. Remarkably, it relates the NSR-like variable $\psi$ to the GS-like variable $\theta$, through the twistor-like field $\Lambda$. As we will see, the worldsheet fermions $\psi$'s in (25) are related to the “transverse” fermions $\varphi^i$, introduced earlier in the NSR formalism. The relation between the NSR and GS formulations of superstring theory, which has been discussed earlier in a number of papers, including reference [13], arises as a result of fixing the $\kappa$-symmetry gauge in the GS theory and the $N = 1$ worldsheet supersymmetry in the NSR formulation. Furthermore, as we will show, the equation (25) leads, at the same time,
to some additional constraints, related to Bogomol’ny-type conditions. Consider now the first term in (25), related to the "worldsheet part" of the supercurrent \( S^r_{\tilde \alpha} \). Let us denote

\[ \partial_a \gamma^a_{\tilde \alpha \tilde \beta} \psi^{\tilde \alpha a} = \partial_a \tilde \psi^{\tilde \alpha a}. \]  

(27)

Note that the Lagrange multiplier field \( p_{\tilde \alpha} \) does not in fact describe any new degree of freedom, as was pointed out in [17,9]. Moreover, from the O.P.E arguments:

\[ S(z)S(w) \sim \text{const} \frac{1}{(z-w)^3} + \cdots \]  

\[ \partial \psi(z)\partial \psi(w) \sim \text{const} + \cdots \]  

we deduce \( p(z)p(w) \sim \text{const} + \cdots \) or, in other words, the Lagrange multiplier field \( p_{\tilde \alpha} \) is of conformal dimension 0. Without a loss of generality, we may therefore choose \( p_{\tilde \alpha} \) to be a constant (on the worldsheet) 10-vector. Then, let us further introduce a new fermionic field \( \tilde \varphi \):

\[ p_{\tilde \alpha} \partial_a \tilde \psi^{\tilde \alpha a} = \text{def} \partial_a \tilde \varphi^a_{\tilde \alpha r}. \]  

(28)

The indices \( \tilde \alpha = 1, 2 \) and \( r = 1, \ldots, 4 \) may be unified into one 8-dimensional index \( i = 1, \ldots, 8 \); then the first term of (25) may be written as

\[ \gamma^a_{\tilde \alpha \tilde \beta} \eta_{rs} p_{\tilde \alpha} \psi^{\tilde \beta s} = \partial_+ \tilde \varphi_i^- + \partial_- \tilde \varphi_i^+. \]  

(29)

We will show now that the index \( i \) corresponds to 8-dimensional (namely, the vector) representation of SO(8), i.e. it is SO(8) covariant. Indeed, the SU(2) invariant metric \( \eta_{rs} \) may be chosen as

\[ \eta = I_{2\times2} \otimes \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \]  

(30)

where \( I_{2\times2} \) is the \( 2 \times 2 \) unit matrix and the metric spinor alternating the index \( \tilde \alpha \) is given by \( \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \). Therefore the metric tensor alternating the index \( i \) may be chosen as

\[ \eta_{8\times8} = I_{2\times2} \otimes \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \]  

(31)

which, after the appropriate 2d rotation, becomes

\[ I_{2\times2} \otimes \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} \otimes \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} = -I_{2\times2} \otimes \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \]  

(32)

We see that the index \( i \) is SO(4,4) covariant or, after the appropriate Wick's rotations, SO(8) covariant. Finally, let us redefine:

\[ \tilde \varphi_i^\pm = \varphi_i^\pm, i = 1, \ldots, 4 \]  

\[ \tilde \varphi_i^\pm = i \varphi_i^\pm, i = 5, \ldots, 8 \]  

(33)
Note that the worldsheet fermions \( \varphi^i \) have the same worldsheet properties (conformal dimension \( \frac{1}{2} \)) and transform according to the same 8-dimensional representation of SO(8) as the “transverse” NSR fermions \( \varphi^i \), introduced in the previous section. One should therefore identify them and indeed, as we will see shortly, such an identification insures that the l.h.s. of (25) vanishes, as required by the \( \kappa \)-symmetry. Let us consider next the second and the third terms of (25). Using the same arguments as above, we redefine:

\[
\begin{align*}
\eta_{rs} \gamma^a_{\alpha \beta} \Lambda^{\bar{r} a}_\rho &= \text{def} \tilde{\Lambda}^{ia}_\rho; \\
\tilde{\Lambda}^i &= \Lambda^i, i = 1, \ldots, 4 \\
\tilde{\Lambda}^i &= i\Lambda^i, i = 5, \ldots, 8
\end{align*}
\]

and the equation (25) becomes:

\[
\partial_+ \varphi^i_+ + \partial_- \varphi^i_- + \partial_- \theta_\mu (\Gamma^\mu_\bar{\nu}) \Lambda^{i+}_\bar{\nu} + \partial_+ \theta_\mu (\Gamma^\mu_\bar{\nu}) \Lambda^i_\bar{\nu} + \\
+ \partial_+ \Lambda^{i-}_\mu (\Gamma^\mu_\bar{\nu}) \theta_\bar{\nu} + \partial_- \Lambda^{i+}_\mu = 0
\]

where \( \Gamma^\mu_\bar{\nu} = p_a \Gamma^a_\mu_\bar{\nu} \). In order for the l.h.s. of (25) to vanish we must then have

\[
\partial_\pm \theta_\mu (\Gamma^\mu_\bar{\nu}) \Lambda^{i\mp}_\bar{\nu} + \partial_\pm \Lambda^{i\mp}_\mu \theta_\bar{\nu} = \text{const} \partial_\pm \varphi^i_\pm
\]

Given that \( \varphi^i \)'s of this equation are identified with the “transverse” worldsheet fermions of (17), the \( \kappa \)-symmetry condition (25) will then be satisfied due to the equations of motion for the \( \varphi^i \)'s: \( \partial_\pm \varphi^i_\pm = 0 \).

Thus, we have found that fixing the local fermionic gauge \( \kappa \)-symmetry in the GS formulation of superstring theory yields the action (17), along with the constraint (36) relating the GS and NSR variables. The fulfillment of (36) leads to some extra conditions which we will analyze in the next section. We will see their relation to the BPS saturation criteria.

5. **NSR-GS relation and p-form charges**

In the paper [15] we analyzed the relation between the NSR and GS formulations of superstring theory in a flat 10d target space. Our conclusion, based on the analysis of the worldsheet and target space properties of the GS space-time fermionic variable \( \theta^\mu \) was that it has to be related to the NSR theory through the relation:

\[
\theta^\mu = e^{\varphi \Sigma^\mu}
\]

where \( \Sigma^\mu \) is the 10d spin operator for matter fields. We have shown that, up to a picture-changing, the relation (37) transforms the GS stress-energy tensor into the stress-energy
tensor of the NSR formulation; also we have shown (37) to be consistent with the GS equations of motion, in terms of the O.P.E. in the NSR formalism; and to reproduce the space-time supersymmetry transformations of the GS theory, up to the operation of picture-changing. The important expressions we used to prove these relations were the one for the conjugate momentum: \( P_{\bar{\mu}} = \frac{i}{\pi} e^{\frac{\phi}{2}} \Sigma_{\bar{\mu}} \); and the one for the space-time supersymmetry generator \( [16] \) in the +\( \frac{1}{2} \)-picture:

\[
: \Gamma_1 \int \frac{dz}{2i\pi} e^{-\frac{\phi}{2}} \Sigma_{\bar{\mu}} := -\frac{1}{2} \int \frac{dz}{2i\pi} \theta_{\bar{\nu}} \Gamma_{\bar{\mu} \bar{\nu}} \partial X_{\bar{m}} \tag{38}
\]

where \( \Gamma_1 =: e^{\phi}(S_{\text{matter}} + S_{\text{ghost}}) \) is the picture-changing operator:

\[
: \Gamma_1 := -\frac{1}{2} e^{\frac{\phi}{2}} \psi_{\bar{\mu}} \partial X_{\bar{\mu}} - \frac{1}{2} e^{2\phi-\chi} \partial \phi b + e^{\chi}c \tag{39}
\]

Analyzing the \( \kappa \)-symmetry in the GS superstring theory and exploring the NSR-GS correspondence, we deduced the expression for the local NSR operator generating the transformations which, given the NSR-GS connection, correspond to the \( \kappa \)-transformations in the GS formulation:

\[
G_{\kappa}^{\bar{\mu}} = e^{-\frac{\phi}{2}} \Sigma_{\bar{\mu}} \tag{40}
\]

Note that both the well-known expression for the space-time supercharge and for the \( \kappa \)-symmetry generator in the NSR picture, correspond to the situation when the gauge freedom corresponding to the \( \kappa \)-symmetry is not yet fixed: they both have 16 components while the gauge-fixed space-time supersymmetry (as well as the \( \kappa \)-symmetry) should involve 8 degrees of freedom only. In order to obtain the proper expressions for these generators in the gauge-fixed theory one should consider the \( \kappa \)-projections, similar to the one performed with \( \theta_{\bar{\mu}} \) in (25). To explore this question in details, let us first consider the NSR formulation of the twistor field \( \Lambda_{\bar{\mu}}^{\pm} \) of (35),(36), analogous to the NSR formulation of the space-time GS fermion \( \theta_{\bar{\mu}} \), which is the worldsheet superpartner of \( \Lambda \). Before doing it, it would be instructive to find the worldsheet superpartner of the field \( e^{\frac{\phi}{2}} \) in the regular NSR theory with the \( N = 1 \) worldsheet supersymmetry. The found expression may then be easily generalized to a case of the \( N = 4 \) worldsheet supersymmetry relevant to our approach. The expression to be found is relevant to the NSR formulation of the doubly supersymmetric approach of [10,11] (the case of \( N=1 \) SUSY both on the worldsheet and in the target space). To proceed with our computations, we will choose to use the picture-changed version of the \( N = 1 \) worldsheet supercharge:

\[
: \Gamma_1 S^{\pm} := \Gamma_1 \int \frac{dz}{2i\pi} (-\frac{1}{2} \psi_{\bar{\mu}} \partial X_{\bar{\mu}} - \frac{1}{2} b \gamma + c \partial \beta + \frac{3}{2} \beta \partial c). \tag{41}
\]
Applying this picture-changed worldsheet supercharge to the GS fermionic field \( \theta^\mu = e^{\frac{2i}{\kappa}} \Sigma^\mu \), we obtain after lengthy computations:

\[
\Lambda^\pm_\mu = \oint \frac{dz}{2i\pi} : \Gamma_1 S^\pm : (w) : \theta_\mu : (z) = \oint \frac{dz}{2i\pi} z - w \left( \frac{1}{4} e^{\frac{2i}{\kappa} - \chi b \Sigma^\mu S^\pm_{\text{matter}}} + \right)
+ \{ Q_{BRST}, \ldots \} = -\frac{1}{4} e^{\frac{2i}{\kappa} - \chi b \Sigma^\mu S^\pm_{\text{matter}}}
\]

The generalization of this expression to our \( N = 4 \) case is quite straightforward, given that the necessary condition (36) for the \( \kappa \)-symmetry is fulfilled. We find that, up to a constant, related to the constant in (36), which will be identified later, the NSR formulation of the twistor field \( \Lambda^i_\mu \) is given by:

\[
\Lambda^i_\mu = (\text{const}) e^{\frac{2i}{\kappa} - \chi b \Sigma^\mu \partial_\mp \varphi^i_\pm}
\]

In order to show that this NSR formulation of the twistor field is consistent with the \( \kappa \)-invariance, we have to show that it satisfies (36) (up to picture-changing) in terms of the O.P.E. of the NSR formalism. Thus, we have:

\[
\partial_\mu \Gamma^{\bar{\mu}\nu} \Lambda^i_\nu - \partial \Lambda^i_\mu \Gamma^{\bar{\mu}\nu} \theta_\nu =
\]

\[= (\text{const}) e^{3\phi - \chi \{ b \partial_\nu \varphi^i - \Gamma^{m_1 \ldots m_7} \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_7} +}
\]

\[+ (\partial b \partial_\nu \varphi^i - \frac{1}{2} \partial^2 \varphi^i - \frac{1}{2} \partial \phi \partial \varphi^i ) \Gamma^{m_1 \ldots m_5} \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_5} +
\]

\[+ P^{(3)}(\partial b, \partial \phi, \partial \varphi^i) \Gamma^{\bar{m}_1 \ldots \bar{m}_3} \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_3} + P^{(4)}(\partial b, \partial \phi, \partial \varphi^i) \Gamma^{\bar{m}_1} \psi_{\bar{m}} +
\]

\[e^{3\phi - \chi \Gamma^i \{ b \Gamma^{m_1 \ldots m_{10}} \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_{10}} + (\partial b - \frac{1}{2} \partial \phi ) \Gamma^{m_1 \ldots m_8} \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_8} +
\]

\[+ (\frac{1}{2} \partial^2 b - \frac{1}{4} \partial^2 \phi + \frac{1}{2} \partial \phi \partial b ) \Gamma^{m_1 \ldots m_6} \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_6} +
\]

\[+ G^{(3)}(\partial b, \partial \phi) \Gamma^{m_1 \ldots m_4} + G^{(4)}(\partial b, \partial \phi) \Gamma^{m_1 m_2} \} + e^{3\phi - \chi \varphi^i \partial \Gamma^{m_1 \ldots m_8} \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_8} +
\]

Here \( \Gamma^i \) stands for \( \Gamma^{\bar{m}_1 \ldots \bar{m}_k} \), \( \Gamma^{m_1 \ldots m_k} \) denotes the antisymmetrized product of \( k \) 10d gamma-matrices contracted with \( \Gamma_{\bar{\mu}\nu} \); and \( P^{(j)}(\partial b, \partial \phi, \partial \varphi^i) \) with \( G^{(j)}(\partial b, \partial \phi) \) are the certain polynomials in the derivatives of the fields \( b, \phi \) and \( \varphi \), such that all the terms these polynomials consist of, have the same conformal dimension \( j \) (the example of such a term is \( \partial^2 b \partial \varphi^i - \partial^2 \phi \)). These polynomials appear in the process of computing the O.P.E. between \( \theta \) and \( \Lambda \) (see also [13]). The precise expressions for them are not given here because of their length; however, one may show that the lengthy expression (44) for the operator product
may be rewritten in the much more convenient form (from now on we will suppress the signs $+, -$ in the indices for the sake of brevity):

$$\partial \theta_{\mu} \Gamma^{\mu \nu} \Lambda_{\nu}^i + \partial \Lambda_{\mu}^i \Gamma^{\mu \nu} \theta_{\nu} =$$

$$= \text{const} \{ \Gamma_2 (\partial \phi^i + (\Gamma^{\bar{m}_1 \bar{m}_2} \psi_{\bar{m}_1} \psi_{\bar{m}_2} + \Gamma^{\mu} \partial X_{\mu}) \phi^i) +$$

$$+ \Gamma_1 e^\phi (\Gamma^{\bar{m}_1 \cdots \bar{m}_5} \psi_{\bar{m}_1} \cdots \psi_{\bar{m}_5} \phi^i + \Gamma^{\bar{m}_1 \cdots \bar{m}_4} \psi_{\bar{m}_1} \cdots \psi_{\bar{m}_4} + \Gamma^{\bar{m}_1 \cdots \bar{m}_3} \psi_{\bar{m}_1} \cdots \psi_{\bar{m}_3} \partial \phi^i)$$

$$(45)$$

$$+ e^{2\phi} (\Gamma^{\bar{m}_1 \cdots \bar{m}_9} \psi_{\bar{m}_1} \cdots \psi_{\bar{m}_9} \partial X^i + \Gamma^{\bar{m}_1 \cdots \bar{m}_8} \psi_{\bar{m}_1} \cdots \psi_{\bar{m}_8} \partial \phi^i +$$

$$+ \Gamma^{\bar{m}_1 \cdots \bar{m}_7} \psi_{\bar{m}_1} \cdots \psi_{\bar{m}_7} (\phi^j \partial X_j) \phi^i + \Gamma^{\bar{m}_1 \cdots \bar{m}_6} \psi_{\bar{m}_1} \cdots \psi_{\bar{m}_6} (\phi^j \partial X_j) \partial X^i) \}$$

Here $\Gamma_1$ and $\Gamma_2 =: \Gamma_1 \Gamma_1$ are picture-changing operators. In order to check that this expression is indeed consistent with the $\kappa$-invariance condition (25) we have to show that the entire expression (45) is proportional (up to picture-changing) to $\partial \phi^i$; or, in other words, that the $p$-form terms (with $p = 1, \ldots, 9$) that occured in (45) do vanish somehow. In the next section we will see, from the analysis of the extended space-time superalgebras, that the conditions, necessary for the vanishing of these terms, are closely related to the Bogomol’ny-type criteria and the BPS saturation of the corresponding $p$-brane solutions to the low-energy effective theory. Furthermore, as we will show, the Bogomol’ny condition itself will appear to be the target space superpartner of the $\kappa$-invariance condition (25).

6. New space-time superalgebras and the BPS condition

In the paper [18] the following extended space-time superalgebra has been proposed for the GS superstring theory:

$$\{ Q_{\bar{m}}, Q_{\bar{\nu}} \} = \Gamma_{\mu \nu}^{\bar{m}} P_{\bar{m}}$$

$$[ P_{\bar{m}}, Q_{\bar{\mu}} ] = -(\Gamma_{\bar{m}})^\mu_{\bar{\nu}} T^{\bar{\nu}}$$

(46)

where the new fermionic generator $T^{\bar{\nu}}$ has been introduced. In the references [19, 20] the generalizations of this superalgebra have been found for supermembranes and for super $p$-brane theories. It is not difficult to show now that the generator $T^{\bar{\nu}}$ is related in fact to the NSR formulation of the $\kappa$-symmetry generator. Using the NSR expressions for the space-time supercharge: $Q_{\bar{\mu}} = \oint \frac{dz}{2i\pi} e^{-\phi} \Sigma_{\bar{\mu}}$ and for the momentum in the $-1$-picture: $P_{\bar{m}} = e^{-\phi} \psi_{\bar{m}}$ we compute the commutator

$$[ P_{\bar{m}}, Q_{\bar{\mu}} ] = - \oint \frac{dz}{2i\pi} ( e^{-\phi} \Sigma_{\bar{\mu}} : (z) : e^{-\phi} \psi_{\bar{m}} : (w) ) = \Gamma_{\mu \bar{\nu}}^{\bar{m}} e^{-\phi} \Sigma_{\bar{\nu}} (w)$$

(47)

We see that the result, $T^{\bar{\nu}}$ coincides with the expression for the $\kappa$-generator in the NSR formalism, introduced earlier. Though we would like to stress once again that the NSR expressions we used in (47) refer to the situation before fixing the $\kappa$-symmetry gauge, we can
already make some important conjectures about the relation between the \(\kappa\)-invariance and the BPS condition. Thus, it has been shown in [21,22] that the expression for the anticommutator of two supercharges in (46) may also be modified to include p-form charges, which have been shown to correspond to p-brane solutions of the low-energy effective theory. In particular, in [22] the following extension for the superalgebra has been considered:

\[
\{Q_{\bar{\mu}}, Q_{\bar{\nu}}\} = \Gamma^{\bar{m}}_{\bar{\mu}\bar{\nu}}(P_{\bar{m}} + T_{\bar{m}}) + \Gamma^{\bar{m}_1...\bar{m}_5}_{\bar{m}_1...\bar{m}_5} Z_{\bar{m}_1...\bar{m}_5}
\]

The self-dual 5-form charge \(Z\) has appeared to be the central charge in the \(N = 1, D = 10\) supertranslation algebra; one could then relate the appropriate magnetic charge (corresponding to the magnetic-type non-singular 5-brane solution) to a contraction of a 5-form \(Z\) with the outer product of 5 translation Killing vectors of the fivebrane solution. The 1-form \(T\) corresponded to the 1-form electric charge carried by the singular elementary string solution. Generalizations of this superalgebra corresponding to other p-brane solutions of the \(N = 1, D = 10\) supergravity have also been considered in these papers. Returning to the space-time superalgebra (46) we see that \(T_{\bar{\nu}}\), identified as the NSR-formulated generator of the \(\kappa\)-transformations may be viewed as the superpartner of the momentum generator in (46), supplemented with p-form charges; while the \(\kappa\)-invariance condition (which may be expressed naively as \(T_{\bar{\nu}} = 0\)), would then correspond to the vanishing of this anticommutator, i.e. to the Bogomol’ny-type condition. Nevertheless, in order to address the problem more accurately, and to develop the above intuitive arguments, one has to fix the \(\kappa\)-symmetry gauge first. Upon fixing the gauge, the space-time supersymmetry generator \(Q_{\bar{\mu}}\) with 16 components will have to be replaced with certain gauge-fixed 8-component supersymmetry while the generator of the \(\kappa\)-transformations will enter the gauge-fixed superalgebra as in (25). We will argue that the p-form terms that occurred in the NSR expressions (44),(45) for the \(\kappa\)-symmetry generator, rewritten in the twistor-like formalism, are related to the p-form charges in the anticommutator of the extended space-time superalgebra (48) (?). The Bogomol’ny-type constraint will then insure that the p-form terms of (45) will cancel, i.e. the \(\kappa\)-invariance condition will be fulfilled. To show that, we have to deduce first the expression for the supercharge \(Q^i, i = 1, ...8\) since the 16-component NSR formulation \(Q_{\bar{\mu}}\) has twice more components than necessary and may not be used after fixing the gauge. The 8-component supercharge must be constructed so that it would reproduce the expression (45) for the \(\kappa\)-symmetry generator in the space-time superalgebra. The p-form terms of (45) should then appear as superpartners of the p-form charges in the
superalgebra. Given these considerations, the expression for the gauge-fixed supercharge should be written as:

\[ Q^i_{SUSY-g.f.} = \oint \frac{dz}{2i\pi} \Gamma_{\bar{m}0} \Lambda^i_{\bar{m}} \theta \bar{\psi} \partial X_{\bar{m}} = \]

\[ = \oint \frac{dz}{2i\pi} \{ \Gamma_2 \partial \phi^i + \Gamma_1 [\partial \phi^i (\Gamma^{\bar{m}_1} \ldots \bar{m}_3 \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_3} + \Gamma^{\bar{m}_1} \bar{\psi}_{\bar{m}_1}) (\Gamma^{\bar{m}_1} \partial X_{\bar{m}_1})] \]

\[ + \partial X^i (\Gamma^{\bar{m}_1} \bar{m}_4 \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_4} + \Gamma^{\bar{m}_1} \bar{m}_2 \psi_{\bar{m}_1} \psi_{\bar{m}_2} (\Gamma^{\bar{m}_1} \partial X_{\bar{m}_1})) + \]

\[ + e^{2\phi} [\partial \phi^i (\Gamma^{\bar{m}_1} \bar{m}_s \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_s} + \Gamma^{\bar{m}_1} \bar{m}_6 \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_6} (\Gamma^{\bar{m}_1} \partial X_{\bar{m}_1})) + \]

\[ + \partial X^i (\Gamma^{\bar{m}_1} \bar{m}_7 \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_7} (\Gamma^{\bar{m}_1} \partial X_{\bar{m}_1}) + \Gamma^{\bar{m}_1} \bar{m}_9 \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_9}) \}\]

This expression for the space-time supersymmetry generator corresponds to the spinor representation \( 8_s \) of SO(8). Beside that, we will need the expression for another gauge-fixed supercharge in the space-time, corresponding to another spinor representation \( 8_c \) of SO(8):

\[ \tilde{Q}^j_{SUSY-g.f.} = \oint \frac{dz}{2i\pi} \{ \Gamma_2 \phi^j + \Gamma_1 e^{\phi} [\phi^j (\Gamma^{\bar{m}_1} \bar{m}_3 \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_3} + \Gamma^{\bar{m}_1} \bar{\psi}_{\bar{m}_1}) (\Gamma^{\bar{m}_1} \partial X_{\bar{m}_1})] + \]

\[ + \partial X^j (\Gamma^{\bar{m}_1} \bar{m}_2 \psi_{\bar{m}_1} \psi_{\bar{m}_2}) + \]

\[ + e^{2\phi} [\phi^j (\Gamma^{\bar{m}_1} \bar{m}_8 \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_8} + \Gamma^{\bar{m}_1} \bar{m}_6 \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_6} (\Gamma^{\bar{m}_1} \partial X_{\bar{m}_1})) + \]

\[ + \partial X^j (\Gamma^{\bar{m}_1} \bar{m}_7 \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_7} + \Gamma^{\bar{m}_1} \bar{m}_9 \psi_{\bar{m}_1} \ldots \psi_{\bar{m}_9}) \}\]

The appropriate anticommutator in the gauge-fixed space-time superalgebra will then be given by:

\[ \{Q^i_{8s}, \tilde{Q}^j_{8c}\} = \gamma^k_{ij} (\Gamma_4 \oint \frac{dz}{2i\pi} (p^k + \sum_{l=1}^{9} Z^{k}_{\bar{m}_1} \ldots \bar{m}_1 \Gamma^{\bar{m}_1} \ldots \bar{m}_1)) \]

where \( p^k =: \Gamma_1 e^{-\phi} \phi^k \sim \partial X^k, k = 1, \ldots 8 \), up to terms not contributing to correlation.
functions, $\gamma^k$ are 8-dimensional gamma-matrices and the p-form charges are given by:

\[
Z^k_{\bar{m}_1} = \Gamma_3 \oint \frac{dz}{2i\pi} e^{\phi} p^k \psi \bar{\psi}^1 (\Gamma^\bar{m} \partial \bar{X}^\bar{m})
\]

\[
Z^k_{\bar{m}_1 \bar{m}_2} = \Gamma_3 \oint \frac{dz}{2i\pi} e^{\phi} \varphi^k \psi \bar{\psi}^2 (\Gamma^\bar{m} \partial \bar{X}^\bar{m})
\]

\[
Z^k_{\bar{m}_1 \ldots \bar{m}_4} = \Gamma_3 \oint \frac{dz}{2i\pi} e^{\phi} \varphi^k \bar{\psi}^3 (\Gamma^\bar{m} \partial \bar{X}^\bar{m})
\]

\[
Z^k_{\bar{m}_1 \ldots \bar{m}_8} = \Gamma_2 \oint \frac{dz}{2i\pi} e^{2\phi} \varphi^k \partial X \bar{X}^\bar{m} \partial \bar{X}^\bar{m} \bar{\psi}^4 \bar{\psi}^5 \bar{\psi}^6 \bar{\psi}^7 \bar{\psi}^8
\]

\[
Z^k_{\bar{m}_1 \ldots \bar{m}_9} = \Gamma_2 \oint \frac{dz}{2i\pi} e^{2\phi} \varphi^k \partial X \bar{X}^\bar{m} \partial \bar{X}^\bar{m} \bar{\psi}^4 \bar{\psi}^5 \bar{\psi}^6 \bar{\psi}^7 \bar{\psi}^8 \bar{\psi}^9
\]

Using these expressions for the p-form charges, taking (36),(45) into account and deducing the const of (43) to be equal to 1, one may show that the the $\kappa$-symmetry generator and, accordingly, the $\kappa$-invariance condition (25),(35) may be written in the form:

\[
\oint G^i_\kappa = \oint \partial \varphi^i + \gamma^i_{kl} \{ Q^k_{\bar{m}_5}, \oint p^l + \sum_{j=1}^{9} Z^l_{\bar{m}_1 \ldots \bar{m}_j} \Gamma^{\bar{m}_1 \ldots \bar{m}_j} \} = 0
\] (53)

Therefore, we see that the necessary condition for the $\kappa$-invariance, the cancellation of the p-form terms, is related to the Bogomol’ny-type condition, applied to the anticommutator (51) of the gauge-fixed space-time superalgebra, i.e. the vanishing of the r.h.s. of the formula (51) which physically means the “charge = momentum” condition, typical for the BPS saturation criteria. This develops the arguments of the (47), applied to the situation prior to the gauge fixing and is in accordance with the above conjecture about the existing connection between the BPS saturation and the $\kappa$-symmetry. We see that if the p-brane solutions of the D=10 supergravity (the low-energy limit of the GS superstring theory) which are related to the p-form charges in the superalgebra, satisfy the BPS saturation condition, one may hope that the $\kappa$-symmetry, formulated classically, may also exist non-perturbatively. Given the fact that the $\kappa$-symmetry is crucial for the formulation of
space-time supersymmetric superstring and super p-brane theories, its formulation on the non-perturbative level would be important to the understanding of the space-time supersymmetry in these theories, when the non-perturbative effects are taken into account. In particular, one needs to develop more precise and quantitative arguments that relate the p-form charges in superalgebras to p-branes. At present, the question of how this relation works, needs the further insight.

7. Conclusion

In our approach the p-form charges and, in particular, the RR charges of p-brane solutions to the low-energy effective theory, appear as a result of the O.P.E. between spin operators for matter fields, contained in the NSR formulations of both space-time fermions $\theta_{\bar{\mu}}$ and the twistor fields $\Lambda_i^{\bar{\mu}}$. In the previous paper [23] we have argued that, in order to take the RR sector into account in the $\sigma$-model approach, one had to supplement the standard $\sigma$-model action with the worldsheet terms

$$I_{RR} \sim \int d^2\tau \Phi F \Phi$$ (54)

where $\Phi$ and $\Phi$ are (0,1) and (1,0) matter-ghost spin operators respectively and $F$ are p-form RR charges, contracted with gamma-matrices (with p - even and odd for the type IIA and IIB theories respectively). Note that the spin operators $\Phi$ and $\Phi$ are 16-component ones, i.e. they are constructed so that the local fermionic gauge symmetry still needs to be fixed. Upon fixing the kappa-symmetry, the 16-component spinors $\Phi^{\bar{\mu}}$ and $\Phi^{\bar{\mu}}$ should be replaced with some 8-component fields, corresponding to the spinor representations $\mathbf{8}_s$ or $\mathbf{8}_c$ of SO(8). One may attempt to relate the gauge-fixed version of the p-form worldsheet RR terms (54) to the p-form charges of (52), in other words (52) should somehow arise from (54) in the process of fixing the gauge, though we do not yet have convincing arguments to show that and hope to do it in the future papers. One should pay the special attention to the p-brane solutions of the D=10 supergravity, coming from the RR sector, particularly, given the fact that the p-brane solutions arising from the NS-NS sector are either the elementary string (which may be identified with the fundamental string), or the geodesically complete 5-brane solution [23], therefore the deep understanding of the RR sector is of the biggest interest. Finally, let us make one concluding remark regarding the twistor-like approach. Generalizing this method to the super fivebrane theory would lead one to consider the theory with the N=1 supersymmetry on the (5+1)-volume (which would replace the $\kappa$-symmetry). Given the established duality between string and fivebrane
theories, it would be interesting to try to study the relation between the $N = 1$ superalgebra on the (5+1)-volume and the $N = 4$ superalgebra on the worldsheet, which appeared previously in our analysis of the string theory, supersymmetric in the space-time. In general, exploring all the questions related to this subject, one inevitably faces the problem of the consistent quantization of super p-brane theories, which still waits for the resolution.

Acknowledgements

The author would like to express his sincere gratitude to the Landau Institute for Theoretical Physics for the hospitality.
References

[1] M.J.Duff, R.Khuri, J.X.Lu, String Solitons, Phys. Rept. 259 (1995), 213-326
[2] N. Berkovits, Nucl. Phys. B395 (1993)
[3] N. Berkovits, Nucl. Phys. B408 (1993)
[4] N. Berkovits, KCL-TH-94-5
[5] N. Berkovits, Nucl. Physics B 379 (1992) 96
[6] J. Polchinski, NSF-ITP-95-122, hep-th/9510017
[7] S. Gubser, A. Hashimoto, I.R. Klebanov, J. Maldacena, Nucl. Phys. B472 (1996), 231
[8] M.B. Green, J.H. Schwarz, E. Witten, Superstring Theory, Cambridge University Press (1987)
[9] E. Bergshoeff, E. Sezgin, Nucl. Phys. B422 (1994) 329, hep-th/9312168
[10] I. Bandos, D. Sorokin, D. Volkov, Phys. Lett. B352 (1995) 269
[11] I. Bandos, D. Sorokin, M. Tonin, P. Pasti, D. Volkov, Nucl. Phys. B446 (1995), 79
[12] E. Sezgin, Aspects of Kappa Symmetry, hep-th/9310126
[13] E. Witten, Nucl. Phys. B266 (1986) 245
[14] R. Kallosh, Phys. Lett. B225 (1989) 49
[15] D. Polyakov, Nucl. Phys. B449 (1995) 159
[16] D. Friedan, S. Shenker, E. Martinec, Nucl. Phys. B271 (1986) 93
[17] A. Galperin, E. Sokachev, BONN-HE-93-05 (1993)
[18] M.B. Green, Phys. Lett. B223 (1989) 157
[19] E. Bergshoeff, E. Sezgin, Phys. Lett. B354 (1995) 256
[20] E. Sezgin, CTP TAMU-49/95, hep-th/9512082
[21] J.A. de Azcarraga, J.P. Gauntlett, J.M. Izquierdo, P.K. Townsend, Phys. Rev. Lett. D63 (1989) 2443
[22] P. Townsend, p-Brane Democracy, hep-th/9507048
[23] D. Polyakov, Nucl. Phys. B468 (1996) 155