Space-time sensors using multiple-wave atom levitation

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The best clocks to date control the atomic motion by trapping the sample in an optical lattice and then interrogate the atomic transition by shining on these atoms a distinct laser of controlled frequency. In order to perform both tasks simultaneously and with the same laser field, we propose to use instead the levitation of a Bose-Einstein condensate through multiple-wave atomic interferences. The levitating condensate experiences a coherent localization in momentum and a controlled diffusion in altitude. The sample levitation is bound to resonance conditions used either for frequency or for acceleration measurements. The chosen vertical geometry solves the limitations imposed by the sample free fall in previous optical clocks using also atomic interferences. This configuration yields multiple-wave interferences enabling levitation and enhancing the measurement sensitivity. This setup, analogous to an atomic resonator in momentum space, constitutes an attractive alternative to existing atomic clocks and gravimeters.

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The light-matter interaction enables the exchange of momentum between an electromagnetic field and individual atoms: each atom emitting or absorbing a photon experiences simultaneously a change of internal level and a recoil reflecting momentum conservation. This well-controlled momentum transfer can be used to engineer correlations between the motional and the internal atomic state. This is the principle underlying Bordé-Ramsey atom interferometers \cite{1,2,3}, which are the building blocks of our system. Such interferometers consist in the illumination of moving two-level atoms with a first pair of light pulses separated temporally and propagating in the same direction [Fig.\textsuperscript{1}], followed by a second pair of pulses coming from the opposite direction. Each pulse operates a $\pi/2$-rotation on the vector representing the atomic density matrix on the Bloch sphere: applied on a given internal state, it acts as an atomic beamsplitter by creating a quantum superposition of two atomic states with distinct internal levels and momenta. Horizontal Bordé-Ramsey interferometers have been used to build optical clocks \cite{4}. This system presents, however, two drawbacks: the free fall of the atoms through the transverse lasers probing their transition limits the interrogation time and induces undesirable frequency shifts \cite{5}. This led the metrology community to privilege atomic clocks \cite{6} built around atomic traps \cite{7}, able to control the sample position. Such systems have become sufficiently accurate to probe fundamental constants \cite{8}. We propose instead to circumvent these limitations with a multiple-wave atom interferometer \cite{9} in levitation, which comprises a succession of vertical Bordé-Ramsey atom interferometers. This strategy combines the best aspects of optical clocks based on atom traps and on atom interferometers: it prevents the sample free fall without using optical potentials likely to cause spurious frequency shifts. The recent experimental achievement \cite{10} of a sustainable levitation of coherent atomic waves with synchronized light pulses \cite{11} strongly supports the feasibility of this method.

Our purpose is to provide a controlled vertical momentum transfer to the atoms, eventually enabling their levitation, through the repetition of a four vertical $\pi/2$-pulse sequence. Momentum kicks are achieved by performing two successive population transfers with vertical pulses propagating in opposite directions: starting from the adequate atomic state, one obtains successively the absorption of an upward photon followed by the emission of a downward one, imparting a net upward momentum to the atoms. This leads us to consider the point illustrated in Fig.\textsuperscript{1} when do two time-separated $\pi/2$-pulses realize a full population transfer? To achieve this, one must indeed compensate the phase induced by the external atomic motion in the time-interval through fine-tuned laser phases. This phase adjustment is at the heart of our proposal, since it provides the resonance.
condition serving for the laser frequency stabilization in the clock operation. The $\pi/2$-pulse sequence then yields a conditional momentum transfer, controlled by this resonance condition, which distinguishes this process from atomic Bloch oscillations. Long $\pi$-pulses, realizing conditional population transfers, could also perform such levitation. Yet there are several benefits in privileging $\pi/2$-pulses: the atomic illumination time is drastically reduced, the pulses address a broader distribution of atomic momenta, and a better sensitivity is obtained through a wider interferometric area.

To obtain the resonance condition, we consider a dilute sample of two-level atoms evolving in the gravity field, taken as uniform, according to the Hamiltonian $H = \frac{p^2}{2m} + mgz$. It is initially in the lower state and described by the Gaussian wave-function

$$\psi_a(r_t, t_0) = \frac{e^{-\frac{1}{2} \left( \frac{(x-x_0)^2}{w^2_0} + \frac{(y-y_0)^2}{w^2_0} + \frac{(z-z_0)^2}{w^2_0} \right) + \frac{i}{\hbar} p_a(t_0) \cdot (r - r_0)}}{\sqrt{\pi w_0^2}}.$$  

After two $\pi/2$-pulses, performed at the times $t_i$ and $t_f = t_i + T$, the initial wave-packet has been split into four packets following two possible intermediate trajectories. The excited state wave-function receives two wave-packet contributions coming from each path and associated with the absorption of a photon at times $t_i$ and $t_f$, of common central momentum $p_f = p_i + \hbar k + mgT$ and respective central positions $r_{f,a}$ and $r_{f,b}$. These wave-packets acquire a phase $S_{a,b}/\hbar$ reflecting the action on each path and a laser phase $\phi_{a,b}$ evaluated at their center for the corresponding interaction time. Both contributions to the excited state are phase-matched if the following relation is verified

$$-p_f \cdot r_{f,a} + S_a + \hbar \phi_a = -p_f \cdot r_{f,b} + S_b + \hbar \phi_b.$$  

The terms $p_f \cdot r_{f,a,b}$ reflect the atom-optical path difference between both wave-packets at their respective centers. The central time $t_c = (t_i + t_f)/2$ is used as phase reference for the two successive pulses. The phases $\phi_b$ and $\phi_a$ provided respectively by the first and the second $\pi/2$-pulse, read $\phi_{a,b} = k \cdot r_{f,a,b} - \omega_{1,2}(t_f, t_i - t_c) + \phi_{1,2}$. The action is given by $S_{a,b} = mgT^2/3 + p_{a,b} \cdot gT^2 + (p_{a,b}^2/2m - mgz_1 - E_{a,b})T$. Condition (1) is fulfilled if the frequencies $\omega_{1,2}$ of the first and second pulses are set to their resonant values

$$\omega_{1,2} = \frac{1}{\hbar} \left( E_b + \frac{(p_{1,2}^2 + \hbar k)^2}{2m} - E_a - \frac{p_{1,2}^2}{2m} \right)$$  

with $p_1 = p_i$ and $p_2 = p_i + mgT$, and if the constant phases $\phi_{1,2}$ satisfy $\phi_{1} = \phi_{2}$. If these conditions are fulfilled, and if the sample coherence length $w$ is much larger than the final wave-packet separation $|r_{f,a} - r_{f,b}|$ - or equivalently if the Doppler width $k \Delta p/m$ experienced by the travelling atoms is much smaller than the frequency $1/T$ - one obtains an almost fully constructive interference in the excited state. The succession of two $\pi/2$-pulses then mimics very efficiently a single $\pi$-pulse, the quantum channel to the lower state being shut off by destructive interferences. A key point is that condition (1), expressing the equality of the quantity $I = -p_f \cdot r_f + S + \hbar \phi$ for both paths, is independent of the initial wave-packet position. This property allows one to address simultaneously the numerous wave-packets generated in the $\pi/2$ pulse sequence.

Applying a second sequence of two $\pi/2$ pulses with downward wave-vectors, one obtains a vertical Bordé-Ramsey interferometer bent by the gravity field sketched on Fig. 2. Starting with a sufficiently coherent sample in the lower state, and with well adjusted frequencies and ramp slopes, the previous discussion shows that a net momentum transfer of $2\hbar k$ is provided to each atom during the interferometric sequence. For the special interpulse duration

$$T := T^0 = \frac{\hbar k}{mg},$$  

these atoms end up in the lower state with their initial momentum. Two major benefits are then expected. First, the periodicity of the sample motion in momentum gives rise to levitation. Second, only two frequencies, given by (2), are involved in the successive resonant pairs of $\pi/2$ pulses. In particular, the first and the fourth pulse of the Bordé Ramsey interferometer, as well as the second and the third one, correspond to identical resonant frequencies: $\omega_1^0 = \omega_2^0$ and $\omega_3^0 = \omega_4^0$.

If the previous conditions are fulfilled, the repetition of the interferometer sequence gives rise to a network of levitating paths - detailed in Fig. 2 - reflecting the diffusion of the atomic wave in the successive light pulses. The same laser field is used to levitate the sample and to
perform its interrogation, generating a clock signal based on either one of the two frequencies $\omega^0_0, \omega^0_0$. Our measurement indeed rests on the double condition (2, 3), which must be fulfilled to ensure this periodic motion: should the parameters $(T, \omega_0, \omega_0)$ differ from their resonant values $(T^0, \omega^0_0, \omega^0_0)$, the outgoing channels would open again and induce losses in the levitating cloud, which can be tracked by a population measurement. We expect multiple wave interference to induce a narrowing of the resonance curve associated with the levitating population around this condition.

We have investigated this conjecture through a numerical simulation. The considered free-falling sample is taken at zero temperature, sufficiently diluted to render interaction effects negligible, and described initially by a macroscopic Gaussian wave-function. Its propagation in-between the pulses is obtained by evaluating a few parameters: central position and momentum following classical dynamics, widths satisfying $w^2_{x,y,z}(t) = w^2_{x,y,z}(t_0) + \frac{\hbar}{m}(t-t_0)^2$, and a global phase proportional to the action on the classical path. Interaction effects may be accounted for perturbatively with a generalized ABCD matrix propagation formalism. The diffusion of atomic packets on the short light pulses is efficiently modeled by a position-dependent Rabi matrix evaluated at the packet center. While the evolution of each wave-packet is very simple, their number-doubling by multiple wave interferences sharpen the symmetric “levitating” central fringe fast enough to limit the effect of the asymmetric background of falling fringes. Considering a shift $\delta T$ from the resonant duration $T_0$, one obtains also a central fringe narrowing as the number of pulses increases and thus an improved determination of acceleration $g$ through condition (3). Multiple wave interferences thus improve the setup sensitivity in both the inertial and frequency domains.

To keep the sample within the laser beam diameter, it is necessary to use a transverse confinement, which may be obtained by using laser waves of spherical wavefront for the pulses. In contrast to former horizontal clocks, the atomic motion is here collinear to the light beam, which reduces the frequency shift resulting from the wave-front curvature. A weak-field treatment, to be published elsewhere, shows that this shift is proportional to the ratio $\Delta \omega_{\text{curv.}} \propto k(v^2_{\perp})/g$, involving the average square transverse velocity $v^2_{\perp}$ and the field radius of curvature $R$ at the average altitude of the levitating cloud. Let us note that our proposal implies technological issues which must be solved to achieve accurate measurements, but they are no more challenging than those of current atomic clocks and sensors. The final population in a given internal state can be monitored by using a time-of-flight absorption imaging with a resonant horizontal laser probe.

The atomic transition used in this setup should have level lifetimes longer than the typical interferometer duration (ms). Possible candidates are the Ca, the Sr, the Yb, and the Hg atoms, which have a narrow clock transition in their internal structure. These atoms should be cooled at a temperature in the nano-Kelvin range, preferably in a vertical cigar-shaped condensate, in order to guarantee a sufficient overlap of the interfering wave-packets and preserve a significant levitating atomic population. We consider a cloud of coherence length $w = 100 \mu m$ much larger than the wave-packets separation $2\hbar \approx 28$ ms. Fig. 3 shows the levitating and the falling atomic population in the lower state as a function of the frequency shift $\delta \omega$ from the resonant frequencies $\omega^0_0$. It reveals a fully constructive interference in the levitating arches when resonance conditions are fulfilled, as well as the expected narrowing of the central fringe associated with the levitating wave-packets. Falling wave-packets yield secondary fringe patterns with shifted resonant frequencies, which induce an asymmetry in the central fringe if the total lower state population is monitored. This effect, critical for a clock operation, can nonetheless be efficiently attenuated by limit the detection zone to the vicinity of the levitating arches. This strategy improves as the levitation time increases: the main contribution to the “falling” background comes then from atoms with a greater downward momentum and thus further away from the detection zone. Besides, multiple wave interferences sharpen the symmetric “levitating” central fringe fast enough to limit the effect of the asymmetric background of falling fringes. Considering a shift $\delta T$ from the resonant duration $T_0$, one obtains also a central fringe narrowing as the number of pulses increases and thus an improved determination of acceleration $g$ through condition (3). Multiple wave interferences thus improve the setup sensitivity in both the inertial and frequency domains.

An analysis of the atomic motion in momentum space, sketched in the energy-momentum diagram of Fig. 3, is especially enlightening. In this picture, the total energy...
accounts for the rest mass, the kinetic and the gravi-
tational potential energy. It is a parabolic function of
the momentum. Each star stands for a specific wave-
packet, the motion of which between the light pulses is
represented by horizontal dashed arrows, in accordance
with energy conservation. For the duration \( T := T^0 \),
and for a sufficiently coherent atomic sample, Fig. 1 re-
veals that the atomic motion in momentum is periodic
and bounded between two well-defined values associated
with the photon recoil. The momentum confinement is
here provided by destructive interferences which shut off
the quantum channels going out of this bounded mo-
momentum region. This remarkable property suggests an
analog to an atomic resonator in momentum space.
Following this picture, we have computed the lower-state
wave-function after \( N \) resonant pulse sequences of dura-
tion \( 2T_0 \), considering only the vertical axis with no loss of
generality. Each wave-packet ends up at rest, and with
a momentum dispersion \( \Delta p_f \). Applying the phase re-
lation (1) successively between the multiple arms, one
obtains:

\[
\psi_a(p, t_0 + 2NT_0) = C_N e^{i\frac{\pi^2}{2N\gamma} \sum_{\text{Paths}} e^{-i\pi p}}
\]

\( C_N \) is a complex number, and the endpoints \( z_f \) are
the endpoints of the resonant paths drawn on Fig. 2 on
which the sum is performed. By labelling these paths
with the instants of momentum transfer, this sum ap-
pears up to a global phase as an effective canonical par-
tition function of \( N \) independent particles, with
\( Z_1 = 2 \cos^2(kT\rho/2\pi) \) the one-particle partition function. This
yields a wave-function of the form \( \psi_a(p, t_0 + 2NT_0) =
\]

\[
C_N e^{i\phi(p,N)} e^{i\frac{\pi^2}{2N\gamma} \cos^2 N(p/p_m)}, \text{ with } p_m = 2m/kT_0
\]

As \( N \to +\infty \), multiple wave interferences thus yield an ex-
ponential momentum localization, scaled by the momentum \( p_m \),
around the rest value \( p = 0 \). The diffusion in altitude observed in the network of paths of Fig. 2 reflects
a back-action of this localization.

To summarize, we have proposed a space-time atomic
sensor achieving the levitation of an atomic sample
through multiple wave interference effects in a series
of vertical \( \pi/2 \) light pulses. The sensitivity of this
levitation towards a double resonance condition can be
used to realize a frequency or an acceleration
measurement, with a sensitivity improving with the
number of interfering wave-packets. The sample needs
to be cooled at a nanoKelvin temperature in order to
yield the desired interference effects. At resonance,
constructive multiple wave interferences then maintain
the full atomic population in suspension in spite of the
great number of non-levitating paths. For a sufficiently
diluted cloud, transverse confinement may be provided
by the wave-front of spherical light pulses. In this
system, light shifts are due only to a resonant light field
and thus expected to be small. This proposal opens
promising perspectives for the development of cold atom
gravimeters [17] and optical clocks [4, 5, 6, 7]. It may
also be turned into an atomic gyrometer [2, 10] by using
additional horizontal light pulses and exploiting the
transverse wave-packet motion.

\( \psi \rightarrow \psi + \psi \) yields a wave-function of the form
\( \sum_{\text{Paths}} \frac{1}{2} \cos 2 \theta \). Each star stands for a specific wave-
packet, the motion of which between the light pulses is
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the full atomic population in suspension in spite of the
great number of non-levitating paths. For a sufficiently
diluted cloud, transverse confinement may be provided
by the wave-front of spherical light pulses. In this

Note added in proof: This system may also work with
ultracold fermionic clouds. As the recent experiment [20],
our system implements a quantum random walk [21], but
here it involves a macroscopic number of atoms propa-
gating in free space.

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[1] C. J. Bordé et al. Phys. Rev. A 30, 1836 (1984).
[2] C. J. Bordé, Phys. Lett. A 140, 10 (1989).
[3] Atom Interferometry, edited by P. R. Berman (Academic
Press, New York, 1997).
[4] G. Wilpers et al., Metrologia 44, 146 (2007).
[5] T. Trebst et al. IEEE Trans. on Inst. and Meas. 50, 535
(2001).

[6] A. D. Ludlow et al., Science 319, 1805 (2008); R. LeTar
gat et al. Phys. Rev. Lett. 97, 130801 (2006); M. Takamoto et al. J. Phys. Soc. Jpn. 75, 104302 (2006).

[7] H. Katori, M. Takamoto, V. G. Palchikov, and V. D. Ovsianikov, Phys. Rev. Lett. 91, 173005 (2003).

[8] S. Blatt et al., Phys. Rev. Lett. 100, 140801 (2008); T. Rosenband et al., Science 319, 1808 (2008).

[9] M. Weitz, T. Heupel, and T. W. Hänsch, Phys. Rev. Lett. 77, 2356 (1996); H. Hinderthür et al., Phys. Rev. A 59, 2216 (1999); T. Aoki, M. Yasuhara, and A. Morinaga, ibid. 67, 053602 (2003).

[10] K. J. Hughes, J. H. T. Burke, and C. A. Sackett, Phys. Rev. Lett. 102, 150403 (2009).

[11] F. Impens, P. Bouyer, and C. J. Bordé, Appl. Phys. B 84, 603 (2006).

[12] C. J. Bordé, in Frequency Standards and Metrology, edited by A. De Marchi (Springer, Berlin, 1989), p. 196; M. Ben Dahan, E. Peik, J. Reichel, Y. Castin, and C. Salomon Phys. Rev. Lett. 76, 4508 (1996); R. Battestì et al., Phys. Rev. Lett. 92, 253001 (2004); M. Fattori et al., Phys. Rev. Lett. 100, 080405 (2008).

[13] This assumption does not induce any loss of generality: the following discussion would also apply to any mode of the Hermite-Gauss basis, and thus to any wave-function by linearity.

[14] This equation can also be interpreted as a generalized optical path in 5D. This formalism is presented in C. J. Bordé, in Proceedings of the Enrico Fermi International School of Physics (Academic Press, New York, 2007), Vol. 168, and Eur. Phys. J. Spec. Top. 163, 315 (2008).

[15] In the configuration of Fig. 2 the two pulse pairs are performed successively, which maximizes the interferometric area for a fixed interferometer duration; one could nonetheless also let a finite time between them.

[16] C. J. Bordé, Metrologia 39, 435 (2002).

[17] A. Peters, K. Y. Chung, and S. Chu, Nature 400, 849 (1999) and Metrologia 38, 25 (2001).

[18] F. Impens and C. J. Bordé, Phys. Rev. A 79, 043613 (2009); F. Impens, e-print arXiv:0904.0150.

[19] See B. Canuel et al., Phys. Rev. Lett. 97, 010402 (2006), and references therein.

[20] Michal Karski et. al., Science 325, 5937 (2009).

[21] Y. Aharonov, L. Davidovich, and N. Zagury, Phys. Rev. A 48, 1687 (1993).