Interior Structure of Solar-like Star τ Ceti

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Abstract. τ Ceti, a G8-V class star, has similar physical properties with our sun, even though most values are smaller and not as active as the sun. It has also been widely reviewed that this metal-poor population II star has terrestrial planetary systems, some of which are in the Habitable Zone. This paper aims to build the interior structure of τ Ceti through modeling experiments with Modules for Experiments in Stellar Astrophysics (MESA) program. The work began with the determination of several fundamental parameters obtained from previous observations of both interferometry and spectroscopy on several references, and also from basic calculations which are then used as input to build the model in MESA. Structural modeling has been carried out in accordance with current star conditions, which is in the main sequence phase. Finally, the result various physical parameters so-called stellar interior structure such as mass, luminosity, pressure, temperature, radius, and age, as well as zonal division of nuclear core (R < 0.23 R☉), radiative (0.23 < R < 0.54), and convective zone (0.54 < R < 0.775) were obtained.

1. Introduction

The stars, that have similarities to the sun always be interesting targets to study. One of them is Tau Ceti (τ Ceti), with other names HD 10700, HIP 8102, HR 509, SAO 147986. It has a convective veil that shows oscillations like the sun [1], so it can become one of the unique laboratories for studying the dynamic evolution of planetary systems as an alternative representation of the Solar System evolution [2]. It is also well known that this yellow-orange colored star is the evidence for the planetary system categorized as Super Earth whose findings were presented in detail at some references (e.g [3][4]). It is a G-class star at the main sequence (G8V) in population II-poor metal, with apparent magnitude of 3.50 ± 0.01 and absolute magnitude of 5.69 ± 0.01 [5]. The star lied in RA 01h44m04.09s and DEC -15°56′14.9″ (J2000.0), with galactic coordinates 173.10076° in longitude and -7T3.43975° in latitude. The distance is 3.65 ± 0.01 pc [2] and has a fairly high proper motion of -1729.726 mas/year as well as the revised parallax measured by Hipparcos is 273.96 ± 0.17 mas. The sporadic period at Ca II, the rotation period τ Ceti of 34 days and almost without rotational modulation, causing this star to appear likely to be pole-on and thought to be undergoing a phase such a Maunder minimum of the Sun ([1] and references therein).

With the uniqueness, τ Ceti becomes the target of spectroscopy and interferometry. Moreover, with the planetary system found in this star, understanding fundamental parameters is certainly important to further investigate the abundance of elements contained therein. Recently, most of the references about this star are related to observations of dust disks (e.g. [6][4]) associated with exoplanet studies using the establish physical parameters. However, there are also recent studies that focus on exoplanets starting with determination of some fundamental parameters such as effective temperature (Teff), surface
gravity, and abundance of [Fe/H] through spectroscopy (e.g. [7]) before investigating their planetary systems.

The purpose of this research is to build the static structure of τ Ceti through modeling experiments with Modules for Experiments in Stellar Astrophysics (MESA) program. The static model is important for understanding the physical properties of the interior of this star as well as for estimating the interior zonal division of τ Ceti. As comparison, zonal division of the Sun is R < 0.2 R⊙ for the nuclear core, 0.2 < R⊙ < 0.7 for radiative zone, and 0.7 < R⊙ < 1.0 for the convective zone. To begin with, some physical properties have been reviewed and selected as the main input parameter for the MESA program. This program is developed by Bill Paxton [8] written in the Fortran language with combination of many numerical and physical modules in the simulation of star evolution. It has the ability for modeling structure and evolution of stars with a large range of mass and metallicities, so it is suitable for τ Ceti small mass and lack of metal abundance.

2. Data and Methods

2.1. Fundamental Parameters

Spectroscopic observations to obtain Teff have been conducted by the previous author [9]. Other references (e.g [10], [11], [12]) made interferometric observations to determine the radius and some physical parameters. In addition, measurements of star oscillations to determine the average density and the mass of τ Ceti carried out [1] in which the result was widely used as several references, e.g. to make a model recalculate the fundamental parameters through combining non-asteroseismological observation data [5]. After reviewing several references, some parameters have been chosen (see table 1) mostly based on data acquisition methods in which direct observation is more favorable than model-based or combining both of them. Finally, parameters mostly from Teixeira et al (2009) [1] and Pagano et al (2015) [7] were adopted (see table 1). Some basic calculation is also used, especially to determine XYZ fraction data with equation (1)(2) [13] with error value calculated in equation (3).

\[
\log Z = 0.977 \frac{[Fe]}{H} - 1.699 \tag{1}
\]

In equation (1), using the value of \( \frac{[Fe]}{H} \) = -0.49 ± 0.0.8 from spectroscopy measurement [7], Z is determined.

\[
\frac{[Fe]}{H} = \log \left( \frac{Z}{X} \right) - \log \left( \frac{Z}{X} \right)_{\odot} \tag{2}
\]

Then, derived from equation (2), with solar value of Z⊙ = 0.7 and X⊙ = 0.02 [13], X is obtained as well as Y because X+Y+Z = 1.

\[
\sigma(\mu) = \frac{1}{\sqrt{\Sigma 1/\sigma_i}} \tag{3}
\]

The error range of Z is very important for this modeling experiment because as the main input, it is very sensitive to the result, therefore error range is also calculated with equation (3) and gives the value Z = 0.00642 ± 0.0012.

| Physical Parameter | value | Reference |
|--------------------|-------|-----------|
| M [M⊙]            | 0.783 ± 0.012 | [1]       |
| R [R⊙]            | 0.793 ± 0.004 | [1]       |
| Log g [dex]       | 4.55 ± 0.06 | [7]       |
3. Results and Discussions

As been shown in table 3, overall result values are still in the error range of the predetermined fundamental parameters in table 1, even though the radius and luminosity are slightly smaller than the selected reference. Nevertheless, the radius value obtained is still within the range of errors from other references, that obtained radius of $0.773 \pm 0.004$ (int) $\pm 0.002$ (ext) $R_\odot$ from interferometry observation \cite{10}.

### Table 3

| Physical properties | Range experiment | Appropriate input |
|--------------------|------------------|------------------|
| M $[M_\odot]$      | $0.783 \pm 0.012$ | 0.775            |
| Metal Fraction (Z) | $0.00642 \pm 0.0012$ | 0.00783       |
| $T_{\text{eff}}$ [K] | $5373 \pm 53$ | 5410             |

Several basic parameters from the observations that have been reviewed (see table 1) are used to limit the model so that the results can approach the actual state or fit the current age of the star. To obtain the desired input values before running this static model, several parameters were selected on experiment modeling in MESA program, and given in table 2. This experiment was carried out with several scenarios by varying those parameters within the range of its uncertainty. In this case, initial mass and metallicity were used as the main input parameters, while the effective temperature was selected as parameter control of stopping condition. For this star, the most suitable results were given by setting up the effective temperature at 5410 K which is appropriate because it is still within the error range. Finally, by selecting the initial mass value approaching the lower limit of 0.775 $M_\odot$ and setting the upper limit metallicity value of 0.00783, desired values of fundamental parameters that appropriate with references in table 1 were obtained (table 3). The program runs the data from the pre-main sequence to the current conditions which are in the main sequence phase.

2.2. Static Model with MESA Program

In determining the static structure of the star, at least four of the following equations must be solved: mass conservation, hydrostatic equilibrium, equation of state, and energy generation. The MESA program can solve them simultaneously so that the mass profile, density, temperature, pressure, luminosity, the mass fraction of several elements, and also the energy transport mechanism can be obtained, respectively with respect to the radius of the star\cite{14}.

| Physical properties | Range experiment | Appropriate input |
|--------------------|------------------|------------------|
| $T_{\text{eff}}$ [K] | $5373 \pm 53$ | 5410             |
| L ($L_\odot$) | $0.488 \pm 0.010$ | 0.775            |
| [Fe/H] | $-0.49 \pm 0.0.8$ | 0.00783       |
| X | 0.718353 | Calculation |
| Y | 0.275005 | Calculation |
| Z | 0.00642 | Calculation |
| $V \sin i$ [km/s] | 2.4 $\pm$ 0.4 | 0.775            |
| Age [Gyr] | 10 $\pm$ 0.5 | 0.00783       |

\((1)\) Teixeira et al (2009)  
\((7)\) Pagano et al (2015)  
\((9)\) Pavlenko et al (2013)  
\((11)\) Di Folco et al (2004)
Table 3. Output of τ Ceti parameters in MESA program using selected main input parameters.

| Physical properties | MESA output |
|---------------------|-------------|
| M [M☉]              | 0.775       |
| R [R☉]              | 0.772       |
| Log g [dex]         | 4.55        |
| T eff [K]           | 5417.018    |
| L [L☉]              | 0.461       |
| Age [Gyr]           | 9.6         |

From the calculations performed by MESA, a diagram can be constructed showing the relationship between the radius (R) with other physical parameters such as mass (M), temperature (T), density (ρ), pressure (P), and luminosity (L), mass fraction, and also transport energy mechanism that will be illustrated and discussed below.

### 3.1. Mass-Density and Temperature-Pressure relation

Figure 1 shows a profile of the mass and density of τ Ceti as a function of radius, with an increasing trend of mass (green line) and a decreasing trend of density (red line). A significant increase in mass occurs at 0.1 <R☉ <0.4, then enhances smoothly to the outer radius of the star. Mass values that tend to be constant above a radius of 0.6 R☉ indicate the amount of material after this radius is not significant enough to contribute to the total mass of this star. On the other hand, the density profile has the highest value at around 173,488 g/cm³, which is slightly higher compared to the Sun's core density of 150 g/cm³. This is consistent with the results of the measurement of the average density value through interferometry observations for τ Ceti stars of 2.21 g/cm³ [1], while the average density of the Sun is below, 1.4 g/cm³. The density of this star then decreases sharply from the center to a radius of about 0.3 R☉, then it decreases more slowly until it tends to be constant towards the outer part of the star with a range of values of 0.2 - 0.135 g/cm³, indicating that material near the surface or photosphere of the star consists of more light elements.

![Figure 1. Profile of mass (green line) and density (red line) distribution vs radius of τ Ceti](image1)

![Figure 2. Temperature (green line) and Pressure (red line) profile distribution vs radius of τ Ceti](image2)

Similar to the density profile, the pressure (green line) and temperature (red line) profiles in Figure 2 also decrease in radius. The pressure at the center shows the highest value of about 2.313 x 10¹⁷ dyne/cm² and decreases up to R < 0.2 R☉. Then it slowly decreases at 0.2 < R☉ <0.4 and quite stable around 2.3 x 10¹⁴ dyne/cm² to the surface. Unlike the density value, the core pressure value is slightly
smaller compared to the core pressure of the Sun of $2.65 \times 10^{17}$ dyne/cm$^2$. As seen at the green line, the central temperature of τ Ceti is around $1.35 \times 10^7$ K, with photosphere temperatures at 5417 K. Highest temperatures in the nucleus indicate that the gas is fully ionized, while lower temperatures in the further layer indicate only partial ionization is occurred [15]. The core temperature is also lower than the Solar of around $1.5 \times 10^7$ K.

3.2. Luminosity – Energy Generation

![Figure 3. Luminosity profile distribution (blue line) and energy generation (red line) vs radius of τ Ceti](image)

The star's luminosity profile shown in Figure 3 sharply rises with the increasing of the radius. The sharp increase occurs around $< R_\odot < 0.23$, then it tends to be constant to the outer radius. The resulting luminosity value of 0.46 L$_\odot$ corresponds to its mass which is also smaller than the sun. In addition to that, the plot that is shown in this frame also displays energy generation (dL/dr) with respect to distance. From this graph, it can be estimated that the star's nuclear core is located at a radius of $<0.23$ R$_\odot$.

3.3. Abundances

Figure 4 shows the abundance profiles of the elements H, $^4$He, and Metal of τ Ceti. The abundance profile indicates the combustion reaction of hydrogen into helium in the star's core. The hydrogen combustion reaction is a fusion reaction of four $^1$H atoms into $^4$He nuclei, indicating that the star is still in the main sequence. The abundance of $^3$He in this Figure shows that $^3$He production is one of the stages in the proton-proton chain (p-p chain) formed from deuterium and $^1$H. The $^3$He profile peaked at 0.23 R$_\odot$ with a fraction value of 0.006. Above all, the metal fraction tends to be constant only around 0.00783 because this star is a poor metal as mentioned in several references.
From Figure 5, it can also be seen that elements C and N also experience changes in the radius above 0.1 R⊙ and element O shows a very small increase, while elements Mg and Ne tend to be constant. This indicates that in addition to the PP chain, there is C combustion in the energy generation reaction via a catalyst in the CNO cycle, but only in a much smaller or not dominant in percentage. This can be seen from the slight changes in small fraction value, and it is suitable for small mass stars such as τ Ceti which the core temperature is also not high enough to undergo CNO cycle. It takes at least 1.3 - 1.5 M⊙ and a core temperature of more than 1.5 X 10⁷ K for the CNO cycle to be quite dominant [15]. Thus, it can be concluded that the more significant and dominant reaction in the star core τ Ceti is the PP cycle whose core temperature is below 1.5 X 10⁷ K.

3.4. Energy Transport Mechanism
In the interior of a star, the mechanism of energy transport from hot regions to cooler regions occurs in two main ways, radiation and convection. Figure 6 shows the energy transfer mechanism that can be described from the dlnP/dlnT plot of the radius where the effective energy transport is indicated by values > 2.5 [15], therefore a radiative zone division is obtained up to about 0.54 R⊙, then the convective shell is located above that radius.

Figure 4. Mass fraction (H, He, and Metal) profile distribution vs radius of τ Ceti
Figure 5. The distribution profile of other elements mass fraction vs radius of τ Ceti

Figure 6. Transport energy mechanism profile vs radius τ Ceti.
Figure 7. Convective velocity vs radius τ Ceti.
The graph also corresponds to the material flow velocity shown in the convective velocity curve in Figure 7 which shows an increase in the radius above 0.54 $R_\odot$, followed by a sharp increase due to the increase in convection flow velocity towards the surface. The estimated scheme for the division of core, radiative, and convective regions from $\tau$ Ceti is shown in Figure 8.

3.5. Interior Structure Scheme

![Figure 8. Interior model of $\tau$ Ceti](image)

The interior division scheme of $\tau$ Ceti (Figure 8) shows that this star has a nuclear nucleus of up to 0.23 $R_\odot$ based on both the Luminosity plot and energy generation (dLr/dr) results (Figure 3) and the area of the combustion reaction shown from Figure 4 & Figure 5. Area at $0.23 < R_\odot < 0.54$ is the radiative zone, and the region at $0.54 < R_\odot < 0.775$ is the convective zone according to the dlnP/dlnT plot (Figure 6), which has been confirmed by its convective velocity curve (Figure 7).

4. Summary

Modeling with MESA is quite appropriate to describe the interior structure of $\tau$ Ceti as a star with the main sequence profile similar to the sun with lower activity and older age (9.6 Gyr). The value of Mass and Luminosity increases to a certain radius and then constant with increasing distance from the core. This is inversely proportional to the value of density, temperature, and pressure which decreases with increasing radius from the core. Comparing the Sun, $\tau$ Ceti has a smaller mass, pressure, temperature, luminosity, and metallicity but has a greater density, especially in the core area. Hydrogen combustion into Helium which is still dominant indicates that $\tau$ Ceti is the main sequence star whose energy generation is dominated by PP reaction. Finally, according to the results model given by MESA program that has been analysed through several plots, a zonal division is obtained. Nuclear Core of $\tau$ Ceti is estimated to be at $R < 0.23 R_\odot$, radiative zone at $0.23 < R_\odot < 0.54$, and convective zone at $0.54 < R_\odot < 0.775$.

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