Biases in the inferred mass-to-light ratio of globular clusters: no need for variations in the stellar mass function

Rosemary L. Shanahan & Mark Gieles
1 Institute for Astronomy, University of Edinburgh, Royal Observatory, Blackford Hill, Edinburgh EH9 3HJ, UK.
2 Department of Physics, University of Surrey, Guildford, GU2 7XH, UK.

Accepted 2014 December 22. Received 2014 December 20; in original form 2014 November 19

ABSTRACT
From a study of the integrated light properties of 200 globular clusters (GCs) in M31, Strader et al. found that the mass-to-light ratios are lower than what is expected from simple stellar population (SSP) models with a ‘canonical’ stellar initial mass function (IMF), with the discrepancy being larger at high metallicities. We use dynamical multi-mass models, that include a prescription for equipartition, to quantify the bias in the inferred dynamical mass as the result of the assumption that light follows mass. For a universal IMF and a metallicity dependent present day mass function we find that the inferred mass from integrated light properties systematically under estimates the true mass, and that the bias is more important at high metallicities, as was found for the M31 GCs. We show that mass segregation and a flattening of the mass function have opposing effects of similar magnitude on the mass inferred from integrated properties. This makes the mass-to-light ratio as derived from integrated properties an inadequate probe of the low-mass end of the stellar mass function. There is, therefore, no need for variations in the IMF, nor the need to invoke depletion of low-mass stars, to explain the observations. Finally, we find that the retention fraction of stellar-mass black holes (BHs) is an equally important parameter in understanding the mass segregation bias. We speculatively put forward that kinematical data of GCs can in fact be used to constrain the total mass in stellar-mass BHs in GCs.

Key words: globular clusters: general – open clusters and associations: general.

1 INTRODUCTION
The stellar initial mass function (IMF) plays a vital role in astrophysics and it is an ongoing debate whether the IMF is universal for all metallicities (Bastian et al. 2010, van Dokkum & Conroy 2010, Geha et al. 2013). Globular clusters (GCs) are important test-beds for studies of the IMF and the present day mass function (MF) because they are, in most cases, single aged stellar populations with a single [Fe/H], and the kinematics of the stars within them is due to the gravity of the stars only. Because most of the GC mass is in low-mass stars, whereas the luminosity is due to the high-mass stars, the GC mass-to-light ratio \( T_V \equiv M/L_V \) is considered to be a good probe of the shape of the stellar MF. Here \( L_V \) is the \( V \)-band luminosity and \( M \) is independently found from a measurement of the cluster’s radius and velocity dispersion. Recent observational advances have allowed us to measure the structural parameters and kinematical properties of GCs in external galaxies up to distances of several tens of Mpc (Smith & Gallagher 2001, Bastian et al. 2004, Brodie & Strader 2006), spanning a large range of environments, thereby potentially increasing the applicability of \( T_V \) as a probe of the IMF.

For Galactica GCs, a slight positive correlation between \( T_V \) and \( M \) was found by early work by Mandushev et al. (1991) and Pryor & Meylan (1993). A more significant correlation was found from a larger sample of GCs by Kimmig et al. (2014). Mclaughlin & van der Marel (2005) found an average \( \langle T_V \rangle \approx 1.45 \) for Milky Way GCs, which is slightly lower than the SSP prediction of \( T_V \approx 2 \), appropriate for low metallicities. This small difference and the trend with \( M \) has been attributed to the preferential escape of low-mass stars over the tidal boundary, reducing \( M \), but hardly affecting \( L_V \) (e.g. Kruijssen 2008).

Strader et al. (2011), hereafter S11) published the \( T_V \) values of 200 GCs in M31, spanning a range of metallicity of \( -2 < [\text{Fe}/\text{H}] < 0 \). They found that at low metallicities the average \( T_V \) agrees well with the values from simple stellar population (SSP) models, assuming a Kroupa IMF, albeit with a scatter of a factor of two. For increasing metallicities, a growing difference between the dynamical \( T_V \) and the SSP models was found, with \( T_V \) falling below the SSP values, by up to a factor of 3 at \( [\text{Fe}/\text{H}] \approx 0 \). Similar results were recently obtained for Milky Way GCs (Kimmig et al. 2014). These results are hard to explain by the preferential escape of low-mass stars, because it implies that almost all metal-rich clusters have lost a significant fraction (\( \sim 60\% - 70\% \)) of their initial mass.

An alternative interpretation is that the IMF gets more bot-
tom light (fewer low-mass stars) with increasing [Fe/H]. Before accepting the possibility of a varying IMF, we need to establish that there are no metallicity dependent biases as the result of dynamical evolution of GCs in the $\Upsilon_V$ measurements. A possible bias is the effect of mass segregation on the measurements of the GC radius and velocity dispersion. Most GCs are older than their half-mass relaxation time-scale $\tau_{1/2}$ (e.g. Hénon [1961] Gieles et al. [2011]). This means that they had time to evolve towards equipartition and, as a result, the most massive objects (stellar remnants and turn-off stars) are more centrally concentrated and move with lower velocities. For resolved GCs this effect can be taken into account by fitting multi-mass models (e.g. Paust et al. [2010] Sollima et al. [2012]), which are distribution function based models that include an approximate prescription for equipartition (Da Costa & Freeman 1978; Gunn & Griffin 1979, hereafter GG79). However, in extragalactic samples where the individual stars are not resolved, this is harder to do (although not impossible), and it is therefore often assumed that light follows mass.

For a universal IMF, the stellar mass function at an age of 12 Gyr does depend on the metallicity of the stars because of differences in stellar evolution, and as a result we may expect that clusters with different metallicities, have different observable properties. The idea that a metallicity dependent bias in the observationally derived cluster properties may exist is not new. Jordán (2004) considered multi-mass King models, with mass bins appropriate for different metallicities. He showed that as a result of mass segregation, clusters with the same half-mass radius $r_h$ and the same age, have smaller half-light radii in projection, $r_{hp,L}$, at higher metallicities. This is because at high [Fe/H] the turn-off mass is higher and as a result these stars and the bright evolved stars, which have similar masses, are more centrally concentrated than in metal-poor clusters. Jordán proposed that this effect is responsible for the observed size difference between metal-rich and metal-poor GCs (e.g. Larsen et al. [2001]). An apparent size difference between metal-rich and metal-poor GCs was also found in N-body simulations (Sippel et al. 2012) and Monte Carlo simulations (Downing 2012) of clusters with different metallicities.

In this Letter we use multi-mass King models to quantify the mass segregation bias in $\Upsilon_V$ as a function of [Fe/H] and for different assumptions for the retention of remnants. In §3 we model stellar mass functions for different [Fe/H] and describe the dynamical models. In §4 we present the results of a comparison to the data by S11 and our conclusions and discussion are presented in §5.

## 2. DESCRIPTION OF THE MODELS

### 2.1 Stellar mass functions at an age of 12 Gyr

Here we describe how we compute the stellar mass components for clusters with different metallicities. We create $N_p = 10^5$ star particles with initial masses $m_i$ equally spaced in $\log(m_i)$ between a lower mass of $m_{lo} = 0.2 \, M_\odot$ and an upper mass of $m_{up} = 100 \, M_\odot$. We consider a power-law (‘canonical’) IMF

$$\frac{dN}{dm_i} = A m_i^{-\alpha},$$

where $\alpha = 2.35$ and $A$ is a constant that normalises the total number of stars in the cluster: $A = (1 - \alpha)/(m_{up}^{1-\alpha} - m_{lo}^{-\alpha})$, for $\alpha \neq 1$. Every star particle is given a weight $w = A m_i^{1-\alpha} (\ln m_{up} - \ln m_{lo})/N_p$, such that $\sum_{j=1}^{N_p} w_j = 1$ and $\sum_{j=1}^{N_p} w_j m_{i,j} \approx 0.68 \, M_\odot$ is the mean mass of the IMF, which is similar to the value for more realistic IMFs that roll over at low masses to flatter slopes and are truncated at $m_{lo} = 0.1 \, M_\odot$. For our purpose this suffices, because the behaviour of the multi-mass models is sensitive only to the total mass in low-mass stars, and insensitive to the mean mass of the low-mass stars (see Section 5 of GG79). Tests confirm that our results are insensitive to this choice.

To study the effect of mass segregation in a system with a bottom light IMF, or a cluster that is depleted in low-mass stars because of dynamical evolution, we also consider a double power-law (‘flat’) IMF

$$\frac{dN}{dm_i} = \begin{cases} B, & m_{lo} < m_i \leq 1 \, M_\odot, \\ A m_i^{-\alpha}, & 1 \, M_\odot < m_i < m_{up}, \end{cases}$$

where $B = A$ to make the function continuous. We normalise the high-mass end in the same way as for the ‘canonical’ IMF (i.e. the same $A$), so the reduction in mass due to flattening can be found straightforwardly.

We evolve every star particle to an age of 12 Gyr using SSE (Hurley et al. 2000) for 11 metallicities, linearly spaced in $-2 \leq [Fe/H] \leq 0$. We split the data into bins corresponding to the stellar types at 12 Gyr: main sequence stars (MS), evolved stars (EV), white dwarfs (WD), neutron stars (NS) and black holes (BH). The logarithmic spacing in $m_i$ and the fast SSE code allow us to efficiently model MFs of old stellar populations that are well sampled at all evolutionary stages and hence present day masses $m$.

The resulting MFs for a metal-rich and a metal-poor cluster with a ‘canonical’ IMF are shown in Fig. 1. From this figure we see that at [Fe/H] = −2 the turn-off mass is slightly lower (0.8 $M_\odot$) than at [Fe/H] = 0 (1 $M_\odot$) and that the average mass of WDs is higher at low [Fe/H] (0.77 $M_\odot$) than at high [Fe/H] (0.65 $M_\odot$). Also, the contribution of remnants to the total mass is higher at low [Fe/H]. Although these differences are small, they are important for the distribution of the visible stars, as we will show in the next section.

For our dynamical models we create 13 mass components: we split the MS stars into 5 bins that are equally spaced in $\log(m)$, the WDs into 5 bins that are equally spaced in $m$, the EV stars, NSs and, finally, the BHs. With the [Fe/H] dependent mass functions in place, we can now describe the dynamical multi-mass models.

### 2.2 Multi-mass models

In this section we describe the dynamical models that we use to quantify the effect of mass segregation on the inferred dynamical mass from observations. For a self-gravitating system in virial equilibrium, the mass $M$ can be written as a function of the mass-weighted mean-square velocity of the stars, $<v^2>$, and the virial radius of the cluster, $r_v$, as $M = 2<v^2>/G$. Here $r_v$ is defined as $r_v = GM^2/(2U)$, where $G$ is the gravitational constant and $U$ is the total potential energy of the cluster. Our focus is on extragalactic cluster samples for which individual stars are not resolved, so we write the virial relations in terms of the luminosity-weighted mean-square velocity in projection $<v^2>_p$, and the radius containing half the total luminosity in projection $r_{hp,L}$ (or, ‘effective radius’).
The mass-to-light ratio of globular clusters

\[ M = \frac{\eta}{\eta_r} \frac{\langle v^2 \rangle_{\text{hp}} L}{\sigma} \]  

Figure 1. Mass functions for two of our evolved stellar populations: one metal poor (top panel), one metal rich (bottom panel). Each distribution is split into the five stellar types: main sequence stars (MS), evolved stars (EV), white dwarfs (WD), neutron stars (NS), and black holes (BH). The numbers are the fraction of the total mass in each component.

\[ \frac{dn}{dm} = n,m - 2 \]

[Fe/H] = -2

[Fe/H] = 0

Stars. For a typical mass-luminosity relation for MS stars we estimate that about 70% of the MS luminosity is emitted by stars in the mass range of EV stars \((\gtrsim 0.65 \text{ M}_\odot)\). We therefore assume that the dynamical properties of EV stars are the ones that one infers from integrated light studies.

In Fig. 2 we show the resulting density and velocity profiles for a metal-rich and a metal-poor cluster without BHs. For both clusters the EV stars are more centrally concentrated and move with lower velocities than the MS stars which implies that both \(\eta_1 < 1\) and \(\eta_2 < 1\). The underestimation of \(M\) is more severe for the metal-rich cluster than for the metal-poor cluster. This is because at high [Fe/H], the turn-off mass is higher, and the remnants are less massive (see Fig. 1), allowing the bright stars to segregate further to the centre of the cluster (see also Jordán et al. 2004, Sippel et al. 2012).

We now need to make a choice for the dimensionless central potential \(W_0\). We choose \(W_0\) such that the evolved stars have a ratio of the core radius \(r_0\) (or King radius) over \(r_{\text{hp}}\) that is equal to what is found for the M31 GCs by S11. We obtained the values for \(r_0/r_{\text{hp}}\) of the GCs in M31 from S11 and found that there is no correlation between \(r_0/r_{\text{hp}}\) and [Fe/H]. We therefore adopt 3 values: the mean observed value and the values corresponding to the mean and the plus and minus 1σ standard deviation. Because \(r_0\) for the individual components has a different meaning then \(r_{\text{hp}}\) in single mass models because, we decide to define \(r_0\) as the distance to the centre where the density of EV stars has dropped to 1/3 of the central density, as is the case in single mass models with \(W_0 \gtrsim 5\). Finally, we use \(r_{\text{hp}} = 0.75 \text{r}_{\text{EV}}\), where \(r_{\text{EV}}\) is the (3D) half-mass radius of the EV stars.

Using these three \(r_0/r_{\text{hp}}\) values, we produce ten models with \(W_0\) ranging from 6 to 24, for each [Fe/H]. We then interpolate to obtain the values of \(W_0\) to obtain the desired \(r_0/r_{\text{hp}}\) value. A sample of the \(W_0\) values used can be found in Table 1.

3 RESULTS

For each [Fe/H], \(r_0/r_{\text{hp}}\) and different remnant fractions, we calculate the ratio of the mass as it would be derived from the evolved stars, \(M_{\text{obs}}\), over the true mass \(M\), i.e. \(\eta_1, \eta_2\) (equation 1). In this calculation, we used exclusively the contribution from the evolved stars to derive \(M_{\text{obs}}\), in order to be consistent with the observations. For the models with a ‘canonical’ IMF \(M_{\text{obs}}/M = M_{\text{obs}}/M_{\text{SSP}}\), and for the models with a ‘flat’ IMF we multiplied \(M_{\text{obs}}/M\) stars.

\[ \eta = \frac{\langle v^2 \rangle_{\text{hp}}}{\langle v^2 \rangle_{\text{hp}}}, \quad \eta_1 = \frac{\langle v^2 \rangle_{\text{hp}}}{\langle v^2 \rangle_{\text{hp}}}, \quad \eta_2 = \frac{\langle v^2 \rangle_{\text{hp}}}{\langle v^2 \rangle_{\text{hp}}} \]  

\[ \langle v^2 \rangle_{\text{hp}} = \frac{m}{m_r} \frac{v_{\text{rms}}^2}{\sigma_r^2} + \phi \]

\[ \sigma_r^2 = \langle v^2 \rangle_\text{hp} - \langle v^2 \rangle_\text{hp} = \phi \]

\[ \sigma_r = \sqrt{\sigma_r^2} \]

\[ \bar{E}^{\text{hp}} = \bar{E}^{\text{hp}} + \phi \]

\[ \bar{E} = \frac{1}{2} \bar{v}^2 + \phi, \quad \phi = \text{the (lowered) specific potential and } \sigma_r \]

\[ f_j(E) = A_j \left( \frac{E}{\sigma_j^2} \right)^{3/2} \left( \frac{E}{\sigma_j^2} \right)^{1/2} \]

\[ \sigma_j^2 = \frac{1}{3} \bar{v}^2 + \phi \]

From the SSP models we find that about 65% of the V-band light comes from EV stars and 35% from the MS.
Table 1. A sample of the $W_0$ values used for different [Fe/H] and the different assumptions for retention of NSs and BHs for a 'canonical' IMF.

| [Fe/H] | $r_0/r_{upp.L}$ | All BH & NS | No BH | No BH & NS |
|--------|-----------------|-------------|--------|------------|
| +0.45  | 15.96           | 8.86        | 8.41   |            |
| $-2$   | 0.29            | 17.5        | 11.1   | 10.3       |
|       | 0.13            | 19.8        | 14.2   | 12.9       |
| 0      | 0.45            | 23.0        | 10.8   | 10.5       |
|       | 0.29            | 24.4        | 13.4   | 12.4       |
|       | 0.13            | 25.8        | 16.6   | 15.1       |

by $M/M_{SSP} \simeq 0.41$, slightly dependent on [Fe/H], to obtain $M_{obs}/M_{SSP}$. The results are shown in Fig. 7. The three panels show the results of three assumptions for the remnant retention fraction, from left to right: all BHs and all NSs retained, no BHs retained and all NSs retained, no BHs and no NSs retained. The results for a model in which all BHs are retained and no NSs are retained were found to be almost identical to the model in which all NSs and all BHs were retained.

We divided the inferred $\Upsilon_V$ of each GC in M31 by its corresponding value of the SSP model (Fig. 1 of S11), based on a Kroupa IMF, to obtain $M_{obs}/M_{SSP}$ for M31 GCs, where $M_{SSP}$ is the present day mass of a stellar population with a 'canonical' IMF. We note that there is a slight inconsistency in the comparison between our models and the data because the $\Upsilon_V$ values of the SSP models are based on a single value for the retention fraction of NSs and BHs. However, as we show in Fig. 7 the total mass in NSs and BHs never exceeds 9% of the present day $M$, so this effect is small.

If all NSs and BHs are retained (left panel), there is hardly any bias because of mass segregation and the correct $M$ is inferred from the EV stars. Without NSs and BHs (right panel), $0.25 \leq M_{obs}/M_{SSP} \leq 0.5$ for both MFs. The mass segregation term ($\eta_r, \eta_p$) is higher when the mass function is flatter, while $M/M_{SSP}$ is smaller. The two effects appear to roughly cancel, such that the resulting ratio $M_{obs}/M_{SSP}$ is similar. This means that a low $M_{obs}/M_{SSP}$ can not be interpreted as a flattening of the MF, nor the IMF, because a mass segregated model with a 'canonical' IMF gives similar results.

The two extremes of the retention fraction models (i.e. all the BH and NS retained, and none of the BH and NS retained) encompass the majority of the GC values, and it is possible to interpret the spread in the data as a spread in BH retention.

The [Fe/H] dependence in the middle and right panels can be explained with some simple stellar evolution arguments. From Fig. 4 we see that at lower metallicities the turn-off mass is lower and there is more mass in remnants, and the average mass of white dwarf mass and black holes is higher (see also the discussion in Sippel et al. 2012). As a result, in a low-metallicity cluster, the remnants are more efficient in pushing the evolved stars out, which increases their velocities and their half-light radius, such that $\eta_r$ and $\eta_p$ are closer to unity.

These results suggest that the discrepancy between the observed GC masses of S11 and the SSP models, can be explained by metallicity dependent mass segregation effects and a spread in the retention fraction of NSs and BHs. Our results suggest that the low-mass end of the MF can not be constrained from integrated light properties.

4 CONCLUSIONS AND DISCUSSION

In this study we show that the discrepancy between the observed mass-to-light ratio $\Upsilon_V$ and simple stellar population (SSP) models as found in a sample of 200 globular clusters (GCs) in M31 by S11, can be explained by mass segregation. We modelled GCs with a range of metallicities and with different remnant retention fractions. Our key results are:

- For clusters without stellar-mass black holes (BHs) and neutron stars, the dynamical mass derived from evolved stars ($M_{obs}$) of a mass segregated cluster underestimates the true mass $M$, and this bias is stronger at high [Fe/H] (factor of $\sim 4$) than at low [Fe/H] (factor of $\sim 2$). This is due to the higher turn-off mass and the lower white dwarf masses at high [Fe/H] which cause the evolved stars to be more centrally concentrated and move with lower velocities, compared to turn-off stars in low [Fe/H] clusters with the same mass density profile (see also Jordán 2004; Sippel et al. 2012).

- The presence of BHs has a large effect on the derived $M_{obs}$, in that its value is closer to the real $M$ for higher retention fractions. Despite the fact that a large fraction of BHs probably gets ejected from GCs as the result of supernova kicks, it is important to consider the BHs in dynamical mass modelling of GCs. A small remaining BH population can survive for as long as 10 half-mass relaxation times ($\tau_{rh}$, Breen & Heggie 2013). The recent discovery of two BH candidates in M22 (Strader et al. 2012) confirms the need to consider BHs when deriving $\Upsilon_V$.

- For clusters with a 'flat' mass function, we find that the effect of mass segregation on $M_{obs}/M$ is less important, such that the ratio $M_{obs}/M_{SSP}$, with $M_{SSP}$ is the present day mass for a canonical IMF, is similar to $M/M_{SSP}$ models with a 'canonical' IMF. This makes $\Upsilon_V$ insensitive to the low-mass end of the MF.

S11 find that the discrepancy between $\Upsilon_V$ and the SSP models is more important for clusters with lower mass and for clusters with shorter $\tau_{rh}$, which was also found for GCs in the Milky Way (Mandushev et al. 1991; Kimmig et al. 2014) and Centaurus A (NGC 5128, Rejkuba et al. 2007). This is consistent with the scenario put forward in this Letter, because mass segregation is more important for clusters with shorter $\tau_{rh}$, i.e. those with lower mass. Zaritsky et al. (2014) report a bi-modality in $\Upsilon_V$, after all clusters have been aged to a common age of 10 Gyr ($\Upsilon_V|_{10}$). They determine dynamical masses from integrated light properties of a sample of star clusters with different ages and metallicities and in different galaxies. The authors interpret their result as evidence for two distinct IMFs. Although we do not discuss the mass segregation bias in their sample in detail here, we caution to invoke the need of a varying IMF in a study based on integrated light measurement, before biases as a result of mass segregation have been fully quantified.

The reduced $\Upsilon_V$ of MW GCs (McLaughlin & van der Marel 2005) and other galaxies has been explained by the depletion of low-mass stars as the result of dynamical ejections (Kruijssen 2008). This effect is also more important for clusters with shorter $\tau_{rh}$/lower mass, therefore degenerate with the mass segregation bias reported here. We note that preferential depletion of low-mass stars requires mass segregation, but not vice versa and it is therefore necessary to consider the biases due to mass segregation together with the effect of the depletion of low-mass stars. Here we showed that the two effects have opposing effects on $M_{obs}/M_{SSP}$, such that $\Upsilon_V$ is insensitive to the slope of the MF.

The $\Upsilon_V$ values of young (few 100 Myrs) massive ($\gtrsim 10^6 M_\odot$) clusters in merger remnants are consistent with the predictions of...
SSP models with a ‘canonical’ IMF (Bastian et al. 2006). Although these values were derived from integrated light properties, these clusters are significantly younger than their two-body relaxation time scale, such that mass segregation is likely not important yet.

Finally, the strong dependence of $M_{\text{obs}}/M$ on the retention fraction of stellar mass BH opens the exciting opportunity to use kinematic data of GCs to determine the total mass in stellar mass BHs, thereby placing complementary constraints on BH retention in Milky Way GCs.

ACKNOWLEDGEMENTS

This work was carried out as part of the ISIMA programme on Gravitational Dynamics held in July 2014 at CITA, Toronto. We acknowledge Pascale Garaud for the organisation, the financial support and for providing a stimulating environment. MG thanks the Royal Society for financial support in the form of a University Research Fellowship (URF) and the European Research Council (ERC-StG-335936, CLUSTERS). RLS acknowledges the support of the Science and Technology Facilities Council (STFC) via the award of an STFC Studentship. RLS thanks Anna Lisa Varri for her support and guidance throughout this work. We thank Miklos Peuten, Antonio Sollima, Anna Lisa Varri and Alice Zocchi for discussions during the ‘Gaia Challenge’ meeting and Diederik Kruijssen, Vincent Hénault-Brunet, Jay Strader and Nate Bastian for comments on the manuscript. Finally, we thank the referee for constructive comments that helped to improve the paper.

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