

$L_e + L_\mu - L_\tau - L_s$ Symmetry and a Mixed 2+2 Scenario for Neutrino Oscillations

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Abstract

Recent results from SuperKamiokande and SNO experiments have set severe constraints on possible mixings of a light sterile neutrino, $\nu_s$, with the three active species required for a simultaneous explanation of the solar, atmospheric and LSND neutrino oscillation data. A consistent scheme has emerged from a global analysis of the data wherein two of the neutrinos are nearly degenerate with a mass of order 1 eV, that mix significantly with the two lighter states. We present realizations of such a mixed 2+2 oscillation scenario based on $L_e + L_\mu - L_\tau - L_s$ symmetry ($L_i$ stands for the $i$th lepton number). Breaking of this lepton number symmetry by a small mass term for $\nu_s$ leads to the required large mixings for both the atmospheric and the solar neutrino oscillations. Sum rules for the neutrino oscillation parameters are derived within this scheme, and are shown to be consistent with present data. These models predict $U_{e3} \simeq 0.02 - 0.03$, which can serve as a test of this idea. We also present gauge models based on mirror extensions of the Standard Model that naturally lead to a light sterile neutrino with the required mixing pattern.
I. INTRODUCTION

Neutrino oscillation data from three different classes of experiments, viz., atmospheric [1], solar [2–5], and LSND [7], provide conclusive evidence in favor of non-zero neutrino masses and mixings and thus for physics beyond the Standard Model. In order to understand the specific nature of new physics, one has to determine the detailed pattern of mixings and masses that fit various observations. As more and more experimental data become available, these details are getting less fuzzy [8] and perhaps more interestingly, some simple possibilities are being either heavily disfavored or eliminated. In this note, we study the nature of new physics and possible new symmetries implied by the three pieces of observation taken simultaneously.

Broadly speaking, since the three different types of experiments are sensitive to three different scales of oscillation lengths, or equivalently three different values of mass splittings $\Delta m^2$, a simultaneous explanation of all data requires that we go beyond the conventional framework of three known light neutrinos ($\nu_e, \nu_\mu, \nu_\tau$) and postulate the existence of a fourth light sterile neutrino, $\nu_s$. This is the four neutrino picture that we will seek to understand in this paper. Needless to say that the need for the sterile neutrino will be somewhat tentative until the LSND results are confirmed by new experiments such as the MiniBOONE experiment at Fermilab.

There are three classes of models that can potentially explain all the data: (i) 2+2 scenario, (ii) 3+1 scenario and (iii) the mixed 2+2 scenario.

2+2 scenario:

In this scenario [9,10], two of the neutrino are nearly degenerate and have masses approximately equal to $\sqrt{\Delta m^2_{LSND}} \sim 1$ eV, while the remaining two states are much lighter. The two heavy eigenstates are generally chosen to consist mainly of the near-maximally mixed weak eigenstates $\nu_\mu$ and $\nu_\tau$ with their mass difference squared of order $\Delta m^2_{atm} \approx 3 \times 10^{-3}$ eV$^2$. Clearly this is designed to explain the atmospheric neutrino data via $\nu_\mu - \nu_\tau$ oscillation. The light eigenstates are assumed to contain mostly $\nu_e$ and $\nu_s$ with a mass difference squared equal to the solar splitting $\Delta m^2_\odot$. There is a small mixing between the two pairs which provides an explanation of the LSND results.

3+1 scenario:

In this scheme [11], the three active neutrinos $\nu_{e,\mu,\tau}$ have masses $\leq \sqrt{\Delta m^2_{atm}} \approx 0.05$ eV, while $\nu_s$ is heavier with a mass of order $\sqrt{\Delta m^2_{LSND}} \sim 1$ eV. The solar and atmospheric neutrino oscillations then involve only the active neutrinos and LSND data is explained by indirect oscillations involving the sterile neutrino [12]. The goodness of fit to the LSND data, while acceptable, is not great [13]. Only certain values of the $\Delta m^2_{LSND}$ are allowed and that too at the 95% confidence level in this scheme.

Mixed 2+2 scenario:

This scenario is a variation of the 2+2 scheme which is necessitated by recent solar neutrino data from SNO experiment in conjunction with SuperKamiokande data. Prior to the SNO results, the 2+2 scenario where solar neutrino oscillations were assumed to involve only the $\nu_e$ and $\nu_s$ was considered to give a good fit to the data although the fit to the energy
distribution was not as good as the fit to the rates. (The 3+1 scenario gave a better fit to the energy spectrum as well as the rates but could fit LSND data only at the 95% CL.)

The SNO results have had a major impact on these conclusions. It showed that there is a clear gap between the neutrino flux measured by the SuperKamiokande experiment and the charged current flux measured by the SNO experiment. If the solar neutrinos oscillate only to the sterile neutrinos as is assumed in the original 2+2 scenario, the fluxes measured by SNO and SuperKamiokande should be equal. The observed gap in the flux would appear to exclude the 2+2 scenario. The mixed 2+2 scenario circumvents this problem. In this scenario, solar \( \nu_e \) is assumed to oscillate into a linear combination of \( \nu_s \) and \( \nu_{\mu,\tau} \). This would allow an understanding of the gap in the flux measured at SuperKamiokande and SNO, since the \( \nu_{\mu,\tau} \) components in solar neutrinos will contribute to neutral current cross section at SuperKamiokande, but will not play any role in the charged current cross section at SNO. The challenge then is to achieve such a mixed oscillation scenario without upsetting the other constraints from accelerator data on the one hand and the atmospheric neutrino oscillation data on the other. Atmospheric neutrino data do set severe constraints on such a mixed scheme since that prefers \( \nu_\mu - \nu_\tau \) oscillations with only a limited amount of \( \nu_s \) component allowed.

In a recent paper, Gonzales-Garcia et al. have studied this issue in quantitative detail. They proposed a 4 \times 4 neutrino mixing matrix where \( U_{e3} \approx U_{e4} \approx 0 \) was assumed in order to satisfy bounds from reactor neutrino data, especially from the CHOOZ and PALO-VERDE experiments. Here the two heavy eigenstates with nearly degenerate mass of order 1 eV are denoted by \( \nu_3 \) and \( \nu_4 \), while \( \nu_1 \) and \( \nu_2 \) denote the lighter eigenstates. There is substantial mixing between the heavy and the light sectors. Solar neutrinos can oscillate into \( \nu_{\mu,\tau} \) with a probability proportional to \( (1 - |U_{s1}|^2 - |U_{s2}|^2) \) where \( U_{s1} \) and \( U_{s2} \) parametrize the non-negligible sterile neutrino component in the two light eigenstates \( \nu_1 \) and \( \nu_2 \). This helps reconcile the SNO and the SuperKamiokande solar neutrino data. On the other hand, this mixing pattern allows atmospheric \( \nu_\mu \)'s to oscillate into sterile neutrinos with a probability of \( 2(|U_{s1}|^2 + |U_{s2}|^2)(1 - |U_{\mu3}|^2 - |U_{\mu4}|^2) \). The limit on the sterile neutrino fraction in the atmospheric data therefore tends to reduce the amount of the needed active neutrino fraction in the solar neutrino data. A delicate balance is needed and the authors of Ref. found a fit to both the solar (including SNO data) and atmospheric data for a range of values of the mixing angles consistent with all other observations. This mixing pattern is an interesting way to accommodate the LSND data for a wider range of \( \Delta m^2_{LSND} \) than the 3+1 scenario and in fact this delicate balance means that this mixing pattern can be tested once the data improves and as new experiments give results.

In this paper, we explore theoretical scenarios that can lead to mixed 2+2 scenarios with mixing patterns close to the one just described. We find that this can happen if there is an approximate leptonic symmetry \( L_e + L_\mu - L_\tau - L_s \) in the Majorana mass matrix for the four neutrinos in the context of a seesaw mechanism for neutrino masses. If this symmetry is broken by a mass term for \( \nu_s \), the large mixing angles needed for both solar and atmospheric neutrino oscillations will result. We show that this scenario predicts certain sum rules relating neutrino oscillation parameters which are in agreement with the data currently. We then show that if the Standard Model is duplicated to have a mirror sector, the lightest mirror neutrino can play the role of the light sterile neutrino with the required mixing properties. The desired leptonic symmetry emerges at low energies, if the original
We have organized this paper as follows: In section II, we describe the phenomenology of the mass matrix with the approximate leptonic symmetry described above; in section III, we study the phenomenological implications and numerical fits to data for this mass matrix and derive certain sum rules for oscillation parameters. In section IV, we present a gauge model based on the existence of a mirror sector that leads naturally to a light sterile neutrino with the desired mixing pattern. In section V we conclude.

II. APPROXIMATE $L_E + L_\mu - L_\tau - L_S$ SYMMETRY AND THE MIXED (2+2) NEUTRINO OSCILLATION SCHEME

Consider the four–neutrino system ($\nu_e, \nu_\mu, \nu_\tau, \nu_s$). Let the mass matrix be of the form:

$$
M_\nu = \begin{pmatrix}
0 & 0 & \epsilon_2 & \epsilon_1 \\
0 & 1 & a & 0 \\
\epsilon_2 & 1 & 0 & 0 \\
\epsilon_1 & a & 0 & \delta
\end{pmatrix} m_0 .
$$

This matrix has an approximate $U(1)$ symmetry which can be identified as $L_e + L_\mu - L_\tau - L_s$ where $L_i$ stands for the $i$th lepton number. This $U(1)$ symmetry is not exact, it is broken by the $\nu_s$ mass term ($\delta m_0 (\nu_s \nu_s)$) in Eq. (1). This breaking will turn out to be small, in fact $\delta$ will be the smallest entry in Eq. (1). In the limit $\delta \to 0$, $M_\nu$ of Eq. (1) will collapse effectively to a $2 \times 2$ matrix, with the four neutrino states forming two Dirac fermions. The mass matrix in the $\delta \to 0$ is given by

$$
M_\nu^0 = (\nu_e \quad \nu_\mu)^T \begin{pmatrix}
\epsilon_1 & \epsilon_2 \\
a & 1
\end{pmatrix} m_0 \begin{pmatrix}
\nu_s \\
\nu_\tau
\end{pmatrix} .
$$

Note that due to the $L_e + L_\mu - L_\tau - L_s$ symmetry, mixing occurs only between ($\nu_e, \nu_\mu$) and ($\nu_\tau, \nu_s$) in this limit. We shall be interested in the case where $a \sim 1 \gg \epsilon_1, \epsilon_2$. The two nonzero mixing angles are then

$$
\tan \theta_{\tau \nu_s} \simeq a ,
$$

$$
\theta_{\nu_e \mu} \simeq \frac{\epsilon_1 a + \epsilon_2}{1 + a^2} .
$$

The two Dirac neutrinos will have masses given by

$$
m_h \simeq \sqrt{1 + a^2} m_0 ,
$$

$$
m_l \simeq \frac{|\epsilon_1 - \epsilon_2 a|}{\sqrt{1 + a^2}} m_0 .
$$

The mass splitting relevant for the LSND experiment is $\Delta m_{LSND}^2 \simeq m_h^2 - m_l^2 \simeq (1 + a^2) m_0^2$. We shall choose $m_h \simeq 1$ eV so as to explain the LSND experiment, along with $\epsilon_{1,2} \simeq (2 - 3) \times 10^{-2}$ to fulfill the LSND mixing angle requirement. At this stage, no mass splitting other than that for LSND is induced. We can allow for the possibility that either $\epsilon_1$ or $\epsilon_2$ is
zero due to some flavor symmetry. These special cases will reduce the number of parameters by one. We shall keep both terms to be nonzero to be general, but in Sec. III, we shall also discuss these two special cases.

Now, let us include the effects of $L_e + L_\mu - L_\tau - L_s$ symmetry breaking through the mass term $\delta$ (the (4,4) entry of Eq. (1)). This mass term, which breaks the symmetry rather economically, will serve several purposes. It will induce two more mass splittings, to be identified with the atmospheric mass splitting ($\Delta m^2_{atm}$) and the solar mass splitting ($\Delta m^2_{\odot}$). At the same time the $\delta$ term will lead to maximal or near maximal mixing between the two would be Dirac states both in the atmospheric neutrino sector and in the solar neutrino sector.

On the theoretical side, we envision that the global $L_e + L_\mu - L_\tau - L_s$ symmetry is broken by some high scale physics. For example, quantum gravity is suspected to break all global symmetries, so that could be the source of the $\delta$ term. It is quite natural to assume that the symmetry breaking effects show up first in the $\nu_s$ mass term, which is a complete singlet of the Standard Model. In fact, in the explicit gauge models that we have constructed (see Sec. IV), $\delta$ arises through quantum gravity, and the analogous symmetry breaking effects are negligible in all other entries of Eq. (1).

Including the $\delta$ term, the eigenvalues of $M_\nu$ of Eq. (1) can be computed in the approximation $1 \sim a \gg \epsilon_{1,2} \sim \delta$. Neglecting quadratic terms in $\epsilon_{1,2}$ and $\delta$, these masses are:

$$m_4 \simeq \left( \sqrt{1 + a^2} + \frac{a^2 \delta}{2(1 + a^2)} \right) m_0,$$

$$m_3 \simeq \left( -\sqrt{1 + a^2} + \frac{a^2 \delta}{2(1 + a^2)} \right) m_0,$$

$$m_2 \simeq \left( \frac{\delta + X}{2(1 + a^2)} \right) m_0,$$

$$m_1 \simeq \left( \frac{\delta - X}{2(1 + a^2)} \right) m_0.$$  \hfill (5)

Here we have defined

$$X \equiv \sqrt{\delta^2 + 4(1 + a^2)(\epsilon_1 - a \epsilon_2)^2}$$  \hfill (6)

for convenience. We have tacitly assumed all parameters of $M_\nu$ to be real for simplicity. For $\delta$ positive, we have $m_1 \leq m_2 \leq m_3 \leq m_4$. If $\delta$ is negative, we can rearrange the labels, $m_1 \leftrightarrow m_2$ and $m_3 \leftrightarrow m_4$, so that the hierarchy $m_1 \leq m_2 \leq m_3 \leq m_4$ is maintained.

From Eq. (5) the three relevant mass splittings are found to be

$$\Delta m^2_{LSND} \simeq m_4^2 - m_2^2 \simeq (1 + a^2) m^2_0,$$

$$\Delta m^2_{atm} \simeq m_4^2 - m_3^2 \simeq \left( \frac{2a^2 \delta}{1 + a^2} \right) m^2_0,$$

$$\Delta m^2_{\odot} \simeq m_2^2 - m_1^2 \simeq (2\delta X) m^2_0.$$  \hfill (7)

The leptonic mixing matrix $U$ is given by (to linear order in $\delta$ and $\epsilon_{1,2}$)
where we have ignored the contribution proportional to the smaller \( \Delta m^2 \) mass eigenstates.

Here the entries in the first row are \( U_{ei}, i = 1 - 4 \), the ones in the second row are \( U_{\mu i} \), the third row entries are \( U_{\tau i} \) and the last row entries are \( U_{si} \). Here we have used the definition

\[

\nu_a = \sum_{k=1}^{4} U_{ak} \bar{\nu}_k,
\]

where \( a = (e, \mu, \tau, s) \) denote the four flavors and \( k = 1 - 4 \) denote the mass eigenstates.

### III. NUMERICAL FITS

With the mass splittings and the mixing matrix entries in hand, we can now confront the model with oscillation data from LSND, solar neutrino and atmospheric neutrino experiments.

#### A. LSND experiment

From the expression for \( \Delta m^2_{\text{LSND}} \) given in Eq. (7), we see that \( (1 + a^2)m_0^2 \approx (0.2 - 6) \) eV\(^2\) in order to explain the positive results seen by the LSND collaboration. The \( (\nu_\mu - \nu_e) \) oscillation probability relevant for LSND is given by

\[
P_{\nu_\mu \rightarrow \nu_e} (\text{LSND}) \approx 4 |U_{e3}^* U_{\mu 3} + U_{e4}^* U_{\mu 4}|^2 \sin^2 \left( \frac{\Delta m^2_{\text{LSND}} L}{4E} \right), \tag{9}
\]

where we have ignored the contribution proportional to the smaller \( \Delta m^2 \) and \( \Delta m^2_{\text{atm}} \) and assumed that \( m_3 \approx m_4 \). Comparing Eq. (9) with the matrix elements of Eq. (8), we make the identification

\[
\sin^2 2\theta_{\text{LSND}} \approx 4 |U_{e3}^* U_{\mu 3} + U_{e4}^* U_{\mu 4}|^2 \approx 4 \left[ \frac{a \epsilon_1 + \epsilon_2}{(1 + a^2)} \right]^2. \tag{10}
\]

Here \( \sin^2 2\theta_{\text{LSND}} \approx 3 \times 10^{-3} \) is the mixing parameter usually quoted in the two flavor oscillation analysis. The experimental observation can be explained by choosing \( (a \epsilon_1 + \epsilon_2)/(1 + a^2) \approx 0.03 \). Note that for this analysis, the breaking of \( L_\mu + L_\tau - L_s \) symmetry is not significant, we could have obtained identical results by performing the two flavor oscillation using Eq. (2) in the exact limit of this symmetry.

Note that the effective mixing parameter \( U_{e3} \), normally discussed in the three neutrino oscillation scenario, is given in our model by \( |U_{e3}^{\text{eff}}|^2 = (|U_{e3}|^2 + |U_{e4}|^2) \) since \( m_3 \approx m_4 \). Thus \( |U_{e3}^{\text{eff}}| \approx \theta_{\text{LSND}} \approx (0.02 - 0.03) \). This is a definite prediction of the model that can be used as one of its tests.
B. Solar and atmospheric neutrino oscillations

We shall follow the global analysis of solar and atmospheric neutrino oscillation data carried out in Ref. \[14\] including the recent SNO results. In that paper it was assumed that \(U_{e3} = U_{e4} = 0\). This approximation holds to a high degree of accuracy in our case, since \(U_{e3} \simeq U_{e4} \simeq \theta_{\text{LSND}} / \sqrt{2} \simeq 0.02\). Such a small mixing of \(\nu_e\) with the heavier mass eigenstates is insignificant for the analysis of solar and atmospheric neutrino oscillations.

Consider first the atmospheric neutrino oscillations. The parameter space is determined by \(\{\Delta m^2_{\text{atm}}, |U_{\mu\ell}|^2 / (|U_{\mu\ell}|^2 + |U_{\mu4}|^2), (|U_{\mu1}|^2 + |U_{\mu2}|^2), (|U_{\mu1}|^2 + U_{\mu2}|^2)\}\). Here the mass splitting is given in Eq. (7), which we shall choose to be \(\Delta m^2_{\text{atm}} \simeq (1 - 6) \times 10^{-3} \text{ eV}^2\). The mixing parameter \(|U_{\mu3}|^2 / (|U_{\mu3}|^2 + |U_{\mu4}|^2) \simeq 1/2\) from Eq. (8). This is the leading parameter that controls the disappearance of \(\nu_{\mu}\) through oscillations into either \(\nu_{\tau}\) or \(\nu_{\alpha}\). Our model prediction agrees quite well with the results of the global analysis \[14\]. In particular, our model predicts the deviation of this parameter from 1 to 1%, the parameter \((|U_{\mu3}|^2 + |U_{\mu4}|^2) \simeq 1/(1 + a^2)\) parametrizes the projection of the sterile neutrino component in the atmospheric neutrino oscillations. When this parameter is equal to 1, we have pure \(\nu_{\mu} - \nu_{\tau}\) oscillations, while if it were zero we have pure \(\nu_{\mu} - \nu_{\alpha}\) oscillations. The atmospheric neutrino data prefers \(\nu_{\mu}\) oscillations into mostly \(\nu_{\tau}\), but significant sterile component is also allowed. When combined with the solar neutrino analysis, which prefers this parameter to be small rather than large, Ref. \[14\] obtains \((|U_{\mu1}|^2 + |U_{\mu2}|^2) \simeq 0.21 - 0.5\) as preferred \[14\]. That fixes the parameter \(a = (1 - 2)\). Lastly, the mixing parameter \((|U_{\mu1}|^2 + |U_{\mu2}|^2) \simeq \theta_{\text{LSND}}^2 + a^2 \delta^2 / (1 + a^2)^3\), as can be seen with a little algebra. Numerically this is about 0.001, which is too small to be of significance in atmospheric oscillations. Thus, our model corresponds to the restricted case of \((|U_{\mu1}|^2 + |U_{\mu2}|^2) = 0\) studied in Ref. \[14,17\]. This scenario gives a reasonable global fit with a goodness of fit (GOF) quoted to be 59% \[14\]. Actually, within our scheme this GOF will be somewhat better since one of the parameter \(|U_{\mu3}|^2 / (|U_{\mu3}|^2 + |U_{\mu4}|^2)\) is fixed to be 1/2, which is very close to the central value of the experiments.

Thus, a good fit to atmospheric neutrino oscillation within our scheme requires \(a \simeq (1 - 2)\), and \(\delta = (2 \times 10^{-3} - 3 \times 10^{-2})\), if we make use of \(\Delta m^2_{\text{LSND}} \simeq (0.5 - 1) \text{ eV}^2\). The parameter \((ae_1 + e_2)\) lies in the range \((0.06 - 0.15)\), obtained from fitting the LSND mixing angle. We see that \(\delta, \epsilon_{1,2} \ll 1\) while \(a \sim 1\), consistent with our approximations. Note also that while \(\delta\) is somewhat smaller than \(\epsilon_{1,2}\), it is not much smaller. In particular, \(\delta\) may be comparable to \((\epsilon_1 - ae_2)\) for a wide range of parameters. This observation will be relevant for the solar neutrino mixing angle prediction.

Turning to solar neutrinos, the parameter space is characterized by \((\Delta m^2_{\odot}, |U_{\mu2}|^2 / (1 - |U_{\mu3}|^2), (|U_{\mu1}|^2 + |U_{\mu2}|^2))\). The mass splitting is given in Eq. (7). The first mixing parameter is recognizable as \(\tan^2 \theta_{\odot}\) that is usually used in the two flavor oscillation analysis for \(\nu_e\) disappearance. The second mixing parameter specifies the amount of sterile neutrinos in solar neutrino oscillations. When \((|U_{\mu1}|^2 + |U_{\mu2}|^2) = 0\), \(\nu_e\) oscillates only to active species, when this parameter is 1, \(\nu_e\) oscillates into a pure sterile state. Solar neutrino data prefers \(\nu_e\) oscillating predominantly to an active species, with some sterile admixture allowed.

From Eq. (8), we see that \(\tan^2 \theta_{\odot} \simeq (X - \delta)/(X + \delta)\). If \(\delta \ll X\), this angle will be maximal. However, \(\delta\) and \(X\) may be comparable, so \(\tan^2 \theta_{\odot}\) can deviate significantly from one. Note that the deviation will be to lower values compared to one, a feature that goes
well with oscillation data.

C. Sum rules for oscillation parameters

Now we show that the model under study leads to an interesting sum rule when the oscillation data from LSND, solar, and atmospheric neutrinos are combined. To see this, note that \( (|U_{e1}|^2 + |U_{e2}|^2) \approx 1/(1 + a^2) \) fixes the value of \( a \). The solar oscillation angle \( \tan^2 \theta_\odot \) fixes the value of \( \delta/X \), while the ratio \( \Delta m^2_\odot/\Delta m^2_{LSND} \) fixes the product \( \delta X \). The parameters \( (a, \delta, X) \) are then completely fixed. The sum rule arises by examining the ratio \( \Delta m^2_{atm}/\Delta m^2_{LSND} \approx 2a^2\delta/(1 + a^2)^2 \), which only depends on the same set of parameters. A simple calculation shows the sum rule to be

\[
\Delta m^2_\odot \cos 2\theta_\odot \approx \frac{[\Delta m^2_{atm}]^2}{2\Delta m^2_{LSND}} \left( \frac{1}{(|U_{e1}|^2 + |U_{e2}|^2)(1 - |U_{e1}|^2 - |U_{e2}|^2)^2} \right). 
\] (11)

This sum rule is obeyed by current experiments rather well. As an example, let us choose \( (|U_{e1}|^2 + |U_{e2}|^2) = 0.25 \), \( \Delta m^2_{atm} = 3 \times 10^{-3} \text{ eV}^2 \), \( \Delta m^2_{LSND} = 1 \text{ eV}^2 \), \( \tan^2 \theta_\odot = 0.5 \). Eq. (11) then predicts \( \Delta m^2_\odot = 9.6 \times 10^{-5} \text{ eV}^2 \). This value is nicely consistent with large angle solar neutrino oscillations.

Consider now the special case \( \epsilon_1 = 0, \epsilon_2 \neq 0 \), which may be imposed by some flavor symmetry. In this case, there is an additional sum rule, which may be taken to be a relation for the LSND mixing angle. (This can be seen by noting that \( 4(1 + a^2)(\epsilon_1 - a\epsilon_2)^2 \approx X^2 - \delta^2 \).

\[
\theta_{LSND} \approx \frac{\Delta m^2_{atm}}{4\Delta m^2_{LSND}} \left( \frac{|U_{e1}|^2 + |U_{e2}|^2}{(1 - |U_{e1}|^2 - |U_{e2}|^2)^2} \right) \tan 2\theta_\odot. 
\] (12)

To see how this sum rule compares with experiment, let us take \( \Delta m^2_{atm} = 2 \times 10^{-3} \text{ eV}^2 \), \( \Delta m^2_{LSND} = 0.5 \text{ eV}^2 \), \( (|U_{e1}|^2 + |U_{e2}|^2) = 0.7 \), \( \tan^2 \theta_\odot = 0.75 \). From Eq. (11) this choice will predict \( \Delta m^2_\odot = 4.4 \times 10^{-4} \text{ eV}^2 \), and from Eq. (12), \( \theta_{LSND} = 0.03 \). Both predictions are in good agreement with observations.

In the opposite case, where \( \epsilon_2 = 0, \epsilon_1 \neq 0 \), there is again a sum rule analogous to Eq. (12), given by

\[
\theta_{LSND} \approx \frac{\Delta m^2_{atm}}{4\Delta m^2_{LSND}} \left( \frac{\tan 2\theta_\odot}{(1 - |U_{e1}|^2 - |U_{e2}|^2)^2} \right). 
\] (13)

In this case, \( \theta_{LSND} \) tends to be small, \( \theta_{LSND} \approx 0.013 \) for the same set of parameters as above. So this special case appears to be disfavored by present data, unless LSND mixing angle settles to a much smaller value.

IV. GAUGE MODELS

The mass matrix of Eq. (1) can be obtained from extensions of the Standard Model that incorporate the seesaw mechanism along with an approximate \( L_e + L_\mu - L_\tau - L_\tau \) symmetry. In order to have a naturally light sterile state \( \nu_s \) in the light spectrum, we resort to the idea of mirror universe [18,19]. In this scenario, it is assumed that there is a mirror sector.
to the Standard Model, which is identical to our world in terms of its particle and force content. The symmetry breakings can however be different \[13\]. The main advantage of this extension is that the existence of the extra gauge quantum numbers of the new neutrino species makes it easier to understand the smallness of their masses. For instance one can employ a type II seesaw mechanism, i.e., seesaw via tiny VEV of a superheavy triplet Higgs \[20\] instead of the heavy right handed Majorana neutrinos in both sectors to make all active as well as sterile neutrinos light. The mirror neutrinos, being Standard Model singlets, can play the role of sterile neutrinos and one has a natural way to understand why a singlet fermion, $\nu_s$, may have such a tiny mass.

As mentioned before, the scale of electroweak symmetry breaking in the mirror sector $v'$ is assumed to be different from that in the familiar sector ($v$) \[19\]. This is achieved by incorporating a mirror odd field $\eta$ which has a VEV and asymmetrizes the $SU(2)$ doublet Higgs mass terms in both sectors. (Equivalently, this can be achieved by the soft breaking of the mirror symmetry.) Due to different weak scales, the mirror neutrinos will have different masses compared to the Standard Model ones. If there is mixing between the neutrinos of the two sectors, which is allowed by gauge symmetry, then oscillations between the two types of neutrinos will occur.

Assume that our world respects $L_e + L_\mu - L_\tau$ and the mirror world respects the analogous $L'_e + L'_\mu - L'_\tau$. If these symmetries were exact, a linear combination of familiar neutrinos, $\nu_e$ and $\nu_\mu$, as well as another one of mirror neutrinos, i.e., $\nu'_e$ and $\nu'_\mu$, will remain massless. We shall identify the second linear combination as $\nu_s$. Such global symmetries are perhaps good only up to quantum gravitational corrections. We assume that they are indeed broken by Planck scale effects. Therefore, once the Planck scale breaking terms are included in the Lagrangian, both the massless neutrino states would pick up small masses. They give a mass to $\nu_s$ of order $\delta m_0 \sim v'^2/M_{Pl}$. If we choose the mirror weak scale, $v' \sim (2 - 4) \text{ TeV}$, one finds $m_{\nu_s} \sim (2 - 8) \times 10^{-3} \text{ eV}$. There will be Planck suppressed corrections to other entries in the neutrino mass matrix as well, but they will not be of much consequence for our purpose. For example, the $\nu_e \nu_e$ entry will be corrected by a negligible amount $\sim 10^{-5} \text{ eV}$.

Let us now discuss how the dominant entries of the neutrino mass matrices arise. In the familiar sector, the dominant entries of $M_\nu$, in units of $m_0$, are the 1 and the $a$ entries. The 1 arises via the seesaw mechanism that we describe now.

It proves convenient to use the type II seesaw formula \[20\] for this purpose, where the tiny VEV of a $B - L = 2$ triplet, rather than a heavy Majorana neutrino, gives small mass to the neutrino. To implement this mechanism \[20\], we introduce into the Standard Model in both sectors, $SU(2)$ triplets $\Delta_L$ and $\Delta'_L$ with hypercharges +2. These fields will have the following Yukawa couplings to the lepton doublets:

$$\mathcal{L}_{\text{Yuk}} = f_{ij}(L_i^T i\tau_2 \vec{\tau}.\Delta_L L_j + L_i^T i\tau_2 \vec{\tau}.\Delta'_L L_j') + h.c.$$  \hspace{1cm} (14)

By itself, Eq. (14) does not break lepton number, but the $\Delta_L$ and $\Delta'_L$ fields will also have gauge invariant couplings to the Standard Model Higgs doublet fields that violate lepton number. The origin of the naturally small VEVs can be seen from the following effective potential, where mirror symmetry is assumed to be softly broken:

$$V(\phi, \phi', \Delta_L, \Delta'_L) = + M^2(\Delta_L^\dagger \Delta_L + \Delta'_L^\dagger \Delta'_L) + (\mu \Delta_L^\dagger \phi \phi' + \mu' \Delta'_L^\dagger \phi' \phi') + ....$$  \hspace{1cm} (15)
Here the $\cdots$ denote higher order polynomial terms that are not relevant for the present discussions. The scales in Eq. (15) is chosen to be high seesaw scales in both the sectors. The VEVs of the triplet fields are given by
\[
v_T \equiv \langle \Delta^0_L \rangle \simeq \frac{\mu v^2}{M^2}; \quad v_T' \equiv \langle \Delta^{0'}_L \rangle \simeq \frac{\mu' v'^2}{M^2}.
\] (16)
The value of $M^2/\mu$ is fixed so that the active species has a mass term (connecting $\nu_\mu$ and $\nu_\tau$) of order 1 eV (the 1 entry). That sets $M^2/\mu \sim 3 \times 10^{13}$ GeV. The mirror seesaw scale $M^2/\mu'$ is not identical, we shall choose it to satisfy the cosmological requirement that during big bang nucleosynthesis, no more than one sterile neutrino species be in thermal equilibrium with the photons. This requires $M^2/\mu' \sim (10^6 - 10^7)$ GeV. The heavy mirror neutrinos will then have a degenerate mass of order $(10 - 100)$ GeV.

The mixing term $a m_0$ of Eq. (1) is also of order 1 eV. This entry involves the mixing of $\nu_\epsilon$ with $\nu_s$ and thus will require the breaking of the two $L_\epsilon + L_\mu - L_\tau$ symmetries to a single $U(1)$. This is achieved by choosing a Higgs field $\chi$, which transforms like a bidoublet, i.e., $\langle 2, 1; 2', 1' \rangle$ under the two gauge groups and carries charge of $(-1, -1)$ under the two leptonic $U(1) \times U(1)'$ symmetries. Its VEV will break the $U(1) \times U(1)'$ to the diagonal $U(1)$ subgroup $\{(L_\epsilon + L_\mu - L_\tau) - (L'_\epsilon + L'_\mu - L'_\tau)\}$. This is equivalent to $L_\epsilon + L_\mu - L_\tau - L_s$ since two of the mirror states are heavy.

The $\chi$ field has couplings to the lepton doublets of type $L_\epsilon \chi L'_\epsilon$. The field $\chi$ also acquires a VEV via seesaw type coupling like the triplets, i.e., $V \ni \mu'' \chi^1 \phi^\dagger + M^2 \chi^1 \chi^1$. Since $a m_0 \sim 1$ eV, we need $\langle \chi \rangle \sim \mu''/M^2 \sim 1$ eV. This can be obtained by choosing $v \sim 200$ GeV, $v' \sim 2$ TeV, $\mu'' \sim M_\chi \sim 10^{14}$ GeV, so that $a m_0 \sim 0.4$ eV. As for the entries $\epsilon_{1,2}$, they will arise in a manner similar to the entries 1 and 2 except that their magnitudes have to be explained by some effective Yukawa couplings being order 0.02. As discussed before, to get a nonzero value of $\delta \simeq (2 \times 10^{-3} - 3 \times 10^{-2})$, we invoke the Planck scale induced $U(1)'$ violating couplings and $v' \simeq 4$ TeV. This is about 25 times larger than $v$. So cosmology of mirror sector will be identical to what has been studied already [21].

Since this model has two heavy sterile neutrinos (with masses in the multigi GeV range), one must address their cosmological implications. Their weak interaction has a strength of $G_F \epsilon \simeq 10^{-3} G_F$ due to the higher mirror symmetry breaking scale. From this, one can calculate their decoupling temperature of the heavy mirror neutrinos by using the out of equilibrium condition as follows. The mirror neutrino interaction rate is given by
\[
G_F^2 \epsilon^2 M^2 (T' M)^{3/2} e^{-\frac{T'}{T}}
\] where $T'$ is the temperature of the mirror sector when the temperature of the visible sector is $T$, and $M$ is the mass of the heavy neutrino. Let us define the ratio of these temperatures to be $\beta = T'/T$. The out of equilibrium condition then gives the following equation:
\[
\frac{n_{\nu'}}{n_\gamma} \simeq \frac{\sqrt{g} x_D}{M_P G_F^2 \epsilon^2 M^3}.
\] (17)
Here $x_D$ stands for the ratio $M/T_D$ where $T_D$ is the decoupling temperature. $x_D$ is determined by the following equation:
\[
G_F^2 \epsilon^2 \beta^3 M^3 x_D^{1/2} e^{-\frac{T'}{T}} \simeq \frac{\sqrt{g}}{M_P}.
\] (18)
This leads to \( x_D \sim 3.6 \). Using this, we see that for \( M = 100 \text{ GeV} \), the contribution of mirror heavy neutrinos to the energy density at the BBN epoch is \( \frac{\rho_{\nu'} \rho_{\nu}}{\rho_{\nu}} = \frac{n_{\nu}(T_D)M}{n_{\nu}(T_D)T_{BBN}} \simeq 0.036 \). Thus, even with both the heavy neutrinos, the total contribution to the energy density during nucleosynthesis is negligible. The lighter mirror neutrino will be in equilibrium during nucleosynthesis, so the model predicts \( N_\nu = 4 \) for nucleosynthesis.

The heavy mirror neutrinos are not stable, they decay dominantly into the light mirror neutrino and a mirror photon. The decay lifetime can be estimated to be \( \tau^{-1}(\nu' \to \nu'+\gamma') \simeq (\alpha/2)m_{\nu'}^3[3\epsilon G_F m_{\nu'} m_{\gamma}^2/(32\pi^2 m_W^2)]^2 \simeq 10^{-4} \text{ sec.} \) for \( m_{\nu'} \sim 100 \text{ GeV} \). This will help eliminate most of the heavy neutrinos from the universe.

The mirror universe hypothesis is one possible framework that explains naturally the existence of a light sterile neutrino. Other approaches based on radiative neutrino mass generation mechanisms \(^{22} \) may also provide realizations of our mass matrix, Eq. (1).

V. CONCLUSIONS

We have presented a simple ansatz for the four neutrino mass matrix which leads to a mixed 2+2 scenario that fits all neutrino oscillation data (solar, atmospheric and LSND). The mass matrix follows from \( L_e + L_\mu - L_\tau - L_s \) symmetry (\( L_i \) is the \( i \)th lepton number). When this symmetry is broken by the small mass of \( \nu_s \), maximal mixings required for explaining the solar and atmospheric neutrino data result. We have derived certain sum rules for neutrino oscillation parameters within this scheme. These sum rules are consistent with present data. The model predicts a small but nonzero value for the effective \( U_{e3} \) entry of the leptonic mixing matrix, which is approximately equal to the LSND oscillation angle, \( \sim (0.02 - 0.03) \). This prediction can be used to test the model. We have also presented a gauge theory realization of the neutrino mass matrix based on the mirror sector hypothesis. This specific realization leads to interesting dark matter cosmology that has been studied previously.

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