Probing the nuclear equation of state by \(K^+\) production in heavy ion collisions

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The dependence of \(K^+\) production on the nuclear equation of state is investigated in heavy ion collisions. An increase of the excitation function of \(K^+\) multiplicity obtained in heavy \((Au + Au)\) over light \((C + C)\) systems when going far below threshold which has been observed by the KaoS Collaboration strongly favours a soft equation of state. This observation holds despite of the influence of an in-medium kaon potential predicted by effective chiral models which is necessary to reproduce the experimental \(K^+\) yields.

From the very beginning kaons have been considered as one of the best probes to study dense and hot nuclear matter formed in relativistic heavy ion collisions \(^1\). In particular at incident energies below the corresponding production thresholds in free space \(K^+\) mesons are created in the early and high density phase of such reactions and – due to strangeness conservation – are not reabsorbed by the nuclear environment. Furthermore, there exist strong evidences that kaons change their properties inside the nuclear medium as predicted by effective chiral models \(^2,3\). The investigation of a partial restoration of chiral symmetry in dense matter probed by \(K^0\) mesons has strongly stimulated both experimental and theoretical efforts in the recent years \(^4,11\).

The original motivation to study the kaon production in heavy ion reactions at intermediate energies, namely to extract information on the nuclear equation of state (EOS) at high densities is a matter of current debate. Already in the first theoretical investigations by transport models it was noticed that the \(K^+\) yield reacts sensitive on the nuclear equation of state \(^1,2,14\), i.e. it was found to be about a factor 2–3 larger when a soft EOS was applied compared to a hard EOS. At that time the available data \(^1\) already favoured a soft equation of state. However, calculations as well as the experimental data were still burdened with large uncertainties. It was further noticed \(^12\) that the influence of the repulsive momentum dependent part of the nuclear interaction leads to a strong suppression of the kaon abundances. An underprediction of \(K^+\) yields using more realistic momentum dependent forces was due to the fact that the production mechanism is twofold: baryon induced processes \(BB \rightarrow BYK^+\) where the kaon is created via binary baryon–baryon collisions \((B\) stands either for a nucleon or a \(\Delta\)-resonance and \(Y\) for a \(\Lambda\) or a \(\Sigma\) hyperon, respectively) and processes \(\pi B \rightarrow YK^+\) induced by pion absorption. In the early studies on subthreshold \(K^+\) production only the baryon induced channels have been considered \(^12,13\). As shown in \(^14–16\) the pionic channel plays an important role, particular in heavy systems. Taking this fact into account the kaon yield could be explained adopting realistic momentum dependent nuclear interactions \(^17,18\). However, the dependence of the kaon production on the nuclear EOS turned now out to be too small for definite conclusions.

The aim of the present work is to study the question if in the meantime decisive information on the nuclear EOS can be extracted from subthreshold kaon production in heavy ion collisions. There are several reasons why it appears worthwhile to do this: Firstly, there has been significant progress in the recent years towards a more precise determination of the elementary kaon production cross sections \(^17,18\), based also on new data points form the COSY-11 for the reactions \(pp \rightarrow pK^+X\) very close to threshold \(^19\). Secondly, the KaoS Collaboration has performed systematic measurements of the \(K^+\) production far below threshold in heavy \((Au + Au)\) and light \((C + C)\) systems \(^20\). Looking at the ratios built from heavy and light systems possible uncertainties which might still exist in the theoretical calculations should cancel out to a large extent which allows to draw reliable conclusions. Furthermore, far below threshold the kaon production is a highly collective process and a particular sensitivity to the compression of the participant matter is expected.

The present investigations are based on the Quantum Molecular Dynamics (QMD) transport model \(^21\). For the nuclear EOS we adopt soft and hard Skyrme forces corresponding to a compression modulus of \(K=200\) MeV and \(380\) MeV, respectively, and with a momentum dependence adjusted to the empirical optical nucleon-nucleus potential \(^21\). The saturation point of nuclear matter is thereby fixed at \(E_B = -16\) MeV and \(\rho_{\text{sat}} = 0.17\) fm\(^{-3}\) \(^21\). The calculations include \(\Delta\) and \(N^*(1440)\) resonances with \(^21\). The QMD approach with Skyrme interactions is well tested, contains a controlled momentum dependence and provides a reliable description of the reaction dynamics in the SIS energy range, expressed e.g. by collective nucleon flow observables as well as particle production. Also the EOS predicted by microscopic approaches (G-matrix) \(^23\) is similar to the soft version of the Skyrme interaction for densities up to \(2\rho_{\text{sat}}\).

We further consider the influence of an in-medium
kaon potential based on effective chiral models. The $K^+$ mean field consists of a repulsive vector part $V_\mu = 3/8 f_\pi^2 j_\mu$ and an attractive scalar part $\Sigma_S = m_K - m_K^* = m_K - \sqrt{m_K^2 - \frac{\Sigma_{KS}}{f_\pi^2} \rho_\sigma + V_\mu V^\mu}$. Here $j_\mu$ is the baryon vector current and $\rho_\sigma$ the scalar baryon density and $\Sigma_{KN} = 450$ MeV. Following, in the vector field the pion decay constant in the medium $f_\pi^2 = 0.6 f_\pi^2$ is used. However, the enhancement of the scalar part using $f_\pi^2$ is compensated by higher order contributions in the chiral expansion, and therefore here the bare value is used, i.e. $\Sigma_{KN} / f_\pi^2$. Compared to other chiral approaches the resulting kaon dispersion relation shows a relatively strong density dependence.

The increase of the in-medium $K^+$ mass $\tilde{m}_K$, Eq. (2), with this parameterisation is still consistent with the empirical knowledge of kaon-nucleus scattering and allows to explore in-medium effects on the production mechanism arising from zero temperature kaon potentials. For the kaon production via pion absorption the elementary cross section of $\pi B \rightarrow Y K^+$ the in-medium kaon potential is absorbed into an newly defined optical potential for nucleons

$$U_{\text{opt}}(\rho, \mathbf{k}) = -\Sigma_S + \frac{1}{m_K} k_\mu V^\mu + \frac{\Sigma_S^3 - V_\mu^2}{2m_K} .$$

and leads to a shift of the thresholds conditions inside the medium. To fulfil energy-momentum conservation the optical potential is absorbed into an newly defined effective mass

$$\tilde{m}_K(\rho, \mathbf{k}) = \sqrt{m_K^2 + 2m_K U_{\text{opt}}(\rho, \mathbf{k})}$$

which is a Lorentz scalar and sets the canonical momenta on the mass-shell $0 = k_\mu^2 - \tilde{m}_K^2$. Thus, e.g., the threshold condition for $K^+$ production in baryon induced reactions reads $\sqrt{s} \geq \tilde{m}_B + \tilde{m}_Y + \tilde{m}_K$ with $\sqrt{s}$ the centre-of-mass energy of the colliding baryons. For a consistent treatment of the thresholds the scalar and vector baryon mean fields entering into eq. (2) are determined from two versions of the non-linear Walecka model with $K=200/380$ MeV, respectively. The hyperon field is thereby scaled by $2/3$ which yields also a good description of the $\Lambda$ flow. Since the parameterisations chosen for the non-linear Walecka model yield the same EOS as the Skyrme ones, the overall energy is conserved. The kaon production is treated perturbatively and does generally not affect the reaction dynamics.

In Fig. 1 the $K^+$ excitation function for $Au + Au$ and $C + C$ reactions starting from $0.8 \text{ A-GeV}$ which is far below threshold ($E_{\text{thr}} = 1.58 \text{ GeV}$) are shown. The calculations are performed with in-medium kaon potential and using a hard/soft nuclear EOS and are compared to data from the KaoS Collaboration. For $C + C$ also calculations without kaon potential are shown.

To extract more clear information on the nuclear EOS next we consider the ratio $R$ of the kaon multiplicities obtained in $Au + Au$ over $C + C$ reactions, normalised to the corresponding mass numbers. In Fig. 2 central ($b=0$ fm) collisions are analysed. Going far below threshold $R$ strongly increases when a soft EOS is applied whereas the increase of $R$ is much less pronounced using the stiffer EOS and $R$ even slightly drops at the lowest energy.

Hence, this ratio reflects the higher compression achieved in the heavy system. The $C + C$ system, on the other hand, is too small to develop a significantly larger compression in the case of a soft EOS compared to the hard EOS. Moreover, in the latter case the slightly more energetic binary collisions lead even to a higher $K^+$ yield in the case of a hard EOS at $0.8 \text{ A-GeV}$. Remarkably, this behaviour is seen despite of the presence of an in-medium kaon potential which acts opposite to the EOS effect; a higher compression increases the kaon yield but
also the value of the in-medium kaon mass which, on the other hand, tends to lower the yield again. However, the increase of the in-medium mass goes linear with density whereas the collision rate per volume increases approximately with ρ². E.g. in central Au + Au reactions at 0.8 A-GeV the average density < ρ > at kaon production is enhanced from 1.47 to 1.57 ρsat switching from the hard to the soft EOS. This leads to an average shift of the in-medium mass [8] compared to the vacuum value of 55/61 MeV using the hard/soft EOS. Generally, in C + C densities above 2 ρsat are rarely reached whereas in Au + Au the kaons are created at densities up to three times saturation density. Furthermore, for C + C the density distributions are weakly dependent on the nuclear EOS. The situation changes completely in Au + Au. Here the densities profile shows a pronounced EOS dependence [13]. Moreover, the excess of kaons obtained with the soft EOS originates almost exclusively from high density matter which demonstrates that compression effects are probed.

![Fig. 2](image2.png)

**FIG. 2.** Excitation function of the ratio of K⁺ multiplicities obtained in central (b=0 fm) Au + Au over C + C reactions. The calculations are performed without in-medium kaon potential and using a hard/soft nuclear EOS.

To obtain a quantitative picture of the explored density effects in Fig. 3 the baryon densities are shown at which the kaons are created. The energy is chosen most below threshold, i.e. at 0.8 A-GeV and only central collisions are considered where the effects are maximal. dM_K⁺/dρ is defined as

\[ dM_K^+ / d\rho = \sum_i N_{K^+} \frac{dP_i}{d\rho_B(x_i, t_i)} \]  \hspace{1cm} (3)

where \( \rho_B \) is the baryon density at which the kaon \( i \) was created and \( P_i \) is the corresponding production probability. For the comparison of the two systems the curves are normalised to the corresponding mass numbers. Fig. 3 illustrates several features: Only in the case of a soft EOS the mean densities at which kaons are created differ significantly for the two different reaction systems, i.e. \( < \rho / \rho_{sat} > = 1.46/1.40 \) (C + C) and 1.47/1.57 (Au + Au) using the hard/soft EOS. Generally, in C + C densities above \( 2\rho_{sat} \) are rarely reached whereas in Au + Au the kaons are created at densities up to three times saturation density. Furthermore, for C + C the density distributions are weakly dependent on the nuclear EOS. The situation changes completely in Au + Au. Here the densities profile shows a pronounced EOS dependence [13]. Moreover, the excess of kaons obtained with the soft EOS originates almost exclusively from high density matter which demonstrates that compression effects are probed.

![Fig. 3](image3.png)

**FIG. 3.** Kaon multiplicities (normalised to the mass numbers of the colliding nuclei) as a function of the baryon density at the space-time coordinates where the K⁺ mesons have been created. Central (b=0 fm) Au + Au and C + C reactions at 0.8 A-GeV are considered. The calculations are performed with in-medium kaon potential and using a hard/soft nuclear EOS.

The comparison to the KaoS data [24] is finally made in Fig. 4. Here only calculations including the kaon potential are shown since it is already clear from Fig. 1 that without the potential one is not able to reproduce the experimental yields. The calculations are performed under minimal bias conditions with \( b_{max} = 11 \text{ fm} \) for Au + Au and \( b_{max} = 5 \text{ fm} \) for C + C and normalised to the experimental reaction cross sections [24]. Both calculations show an increase of \( R \) with decreasing incident energy down to 1.0 A-GeV. As already seen for the central collisions this increase is much less pronounced.
using the stiff EOS. In the latter case $R$ even drops for 0.8 A-GeV whereas the soft EOS leads to an unrelied increase of $R$. At 1.5 A-GeV which is already very close to threshold the differences between the two models tend to disappear. The overall behaviour of $R$ is found to be quite independent of the various production channels with initial states $i = NN, \pi N, N\Delta, \pi \Delta, \Delta\Delta$. Ratios $R_i$ built separately for the individual channels show in both cases (soft or hard) a similar energy dependence as the total $R$ (except of $R_{\pi\Delta}$ which tends to remain large also at high energies). The transport calculations further demonstrate that the increase of $R$ is not is not caused by a trivial, i.e. EOS independent limitation of phase space at low energy in the small system. This is supported by the fact that the number of collisions which the involved particles encountered prior to the production of a kaon and which is a measure of the collectivity provided by the system does not reach a sharp limit for $C + C$ at low energies. The strong increase of $R$ can be directly related to higher compressible nuclear matter. The comparison to the experimental data from KaoS \cite{24} where the increase of $R$ is even more pronounced strongly favours a soft equation of state.

To summarise, we find that at incident energies far below the free threshold $K^+$ production is a suitable tool to study the dependence on the nuclear equation of state. Using a light system as reference frame there is a visible sensitivity on the EOS when ratios of heavy ($Au + Au$) over light ($C + C$) systems are considered. Transport calculations indicate that the $K^+$ production gets hardly affected by compression/cont Effects in $C + C$ but is highly sensitive to the high density matter ($1 \leq \rho/\rho_{sat} \leq 3$) created in $Au + Au$ reactions. Results for the $K^+$ excitation function in $Au + Au$ over $C + C$ reactions as measured by the KaoS Collaboration, strongly support the scenario with a soft EOS. This statement is also valid when an enhancement of the in-medium kaon mass as predicted by chiral models is taken into account.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Excitation function of the ratio $R$ of $K^+$ multiplicities obtained in inclusive $Au + Au$ over $C + C$ reactions. The calculations are performed with in-medium kaon potential and using a hard/soft nuclear EOS and compared to the experimental range of $R$ (shaded area) given by the data from the KaoS Collaboration \cite{24}.}
\end{figure}

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