Quantum electrodynamics of composite fermions.

Serge B. Afanas’ev

Sankt–Petersburg, Russia
e-mail: serg@ptslab.ioffe.rssi.ru

Abstract

The quantum Dirac–like equation and the QED vertex operator for a composite particle are suggested. The vertex operator and the fermionic propagator are connected by the QED Ward identity. It is shown that all of the Feynman QED–integrals for a three–component particle are finite with some suggestion about large momentum dependence of the vertex operator. This dependence is in agreement with the component counting rules.

The ultraviolet divergences appearance in calculations is one of the most difficult problem of the quantum field theory. At the present time many methods of ultraviolet divergences elimination are developed. Thus the ultraviolet divergences are not in the superstring theory, and this fact is connected with the finite sizes of superstrings as the space–time objects. In this work the quantum electrodynamics of a composite particle with the vertex operator and the motion equation different from the Dirac–Feynman–Schwinger pointlike QED ones is considered.

Pointlike QED based on the Dirac equation

$$(\hat{p} - m)\psi = 0$$

(1)

The general form of the vertex operator consequential from requirements of gauge and relativistic invariances is

$$\Gamma^\mu(q^2) = f(q^2)\gamma^\mu - \frac{1}{2m}g(q^2)\sigma^{\mu\nu}q_\nu,$$

(2)

where $q_\mu$ is the photon momentum.

The calculations with pointlike form factors in each exact vertex $f(q^2) = 1, g(q^2) = 0$ yield the ultraviolet divergences caused by momentum dependence of integrand in the large momentum region.

It is noted that allowing the momentum dependence of form factors $f(q^2)$ and $g(q^2)$ results in eliminating ultraviolet divergences of the Feynman integrals in majority of QED–calculations if $f(q^2)$ and $g(q^2)$
are decreasing functions of momentum in the large momentum region. Asymptotical momentum dependence of composite particle form factors is described with functions \( f(q^2) \sim (1/q^2)^{n-1} \), where \( n \) is a number of components [1]. In agreement with the components counting rules the suggestion about leptons substructure leads to elimination of all ultraviolet divergences, besides the polarization operator divergence and divergences in other Feynman diagrams in which all of external lines are photon. Leptons substructure and the upper limits of leptons sizes are discussed in [2, 3].

However, \( q^2 \)-dependence of the form factors leads to the contradiction with the Ward identity:

\[
G^{-1}(p + q) - G^{-1}(p) = q_\mu \Gamma'^\mu(p, p + q; q)
\]

(3)
because the one puts very rigid conditions on the form of the fermionic propagator. The identity (3) cannot be satisfied if form factors depend on \( q^2 \) only.

Suppose that the vertex operator can depend on the lepton momentum as well as the photon momentum. Obviously, \( p \)-dependence of the vertex operator leads to a modification of the particle propagator, as the Ward identity (3) shows.

Consider the following system consisting of the composite particle quantum relativistic motion equation:

\[
(f(p^2) \hat{p} - m) \psi = 0
\]

(4)
and the vertex operator in the form

\[
\Gamma'^\mu(p, p + q; q) = h_1(p^2, q^2)\gamma'^\mu + h_2(p^2, q^2)\gamma_\nu p'^\nu q'^\mu
\]

(5)
In (3) the ”spin term” is omitted as it does not take a contribution to the Ward identity due to antisymmetric spin tensor presence.

Define the fermionic propagator as the Green function of the equation (4):

\[
G(p) = \frac{f(p^2) \hat{p} + m}{f^2(p^2)p^2 - m^2}
\]

(6)
Let us suppose that \( f(p^2) \) is the decreasing function of \( p^2 \) and

\[
f(p^2)|_{p^2=0} = 1
\]

(7)
The equation (4) and the propagator (6) under the $p^2 \to 0$ conditions have pointlike QED – forms. If

$$f(p^2)|_{p^2 \to \infty} \sim \frac{1}{p^{\alpha+1}}, \alpha > 0$$  \hspace{1cm} (8)

then the propagator (6)

$$G(p)|_{p^2 \to \infty} \to -\frac{1}{m}$$  \hspace{1cm} (9)

Poles of the propagator (6) are yielded by the equation

$$f^2(p^2) \cdot p^2 = m^2$$  \hspace{1cm} (10)

As is shown below, the function $f(p^2)$ is identical to the elastic form factor in the small momentum region. Therefore it may be noted that $|f(p^2)| \ll 1$ in the region $p^2 \gg < r^2 >^{-1}$ where $< r^2 >$ is the average composite particle size squared. Thus if $m^2 < r^2 > \ll 1$ then the propagator poles are $p^2 \approx m^2$ and $p^2 \sim < r^2 >^{-1}$. In the case $m^2 < r^2 > \sim 1$ both poles are in the region $p^2 \sim m^2$. If $m^2 < r^2 > \gg 1$ then poles of propagator are absent.

Solutions of the equation (4) for a particle without external field are not eigenstates of the momentum operator. Thus the solutions of (4) are some linear combinations of the plane waves.

Considering the equation (4) and the vertex operator (5) the Ward identity (3) is:

$$f((p + q)^2)(\hat{p} + \hat{q}) - f(p^2)\hat{p} = q_\mu(h_1(p^2, q^2)\gamma^\mu + h_2(p^2, q^2)\gamma_\nu p^\nu q^\mu)$$  \hspace{1cm} (11)

This equation puts the following conditions on the functions $f(p^2)$, $h_1(p^2, q^2)$ and $h_2(p^2, q^2)$:

$$f((p + q)^2) - f(p^2) = q^2 h_2(p^2, q^2)$$  \hspace{1cm} (12)

$$f((p + q)^2) = h_1(p^2, q^2)$$  \hspace{1cm} (13)

The functions $h_1(p^2, q^2)$ and $h_2(p^2, q^2)$ are exactly determined by the equations (12), (13) and are connected by these equations.
The last equation leads to the condition

\[ h_1(p_1^2, p_2^2) = h_1(p_2^2, p_1^2) \]  \hspace{1cm} (14)

Analyzing the Ward identity (3) with the vertex operator in the form (5), (12), (13) we come to the conclusion that this \( \Gamma^{\mu} \) is orthogonal to \( q_\mu \). Thus the system of the composite particle quantum equation (4) and the vertex operator (5), (12), (13), is gradient-invariant and satisfies the electric charge conservation law.

The vertex operator (5) is analogous to the vertex functions that appear at the investigation of the deep inelastic scattering. There can be suggested that the operator (5) describes the internal processes in a composite system. \( p^2 \)-dependence of the function \( h_1(p^2, q^2) = f(p^2) \) even at the zero momentum transmission describes the internal processes, but not the interaction of a composite particle with the electromagnetic field. This dependence coincides with \( p^2 \)-dependence of the function \( G^{-1}(p^2) \).

Consider the ultraviolet divergences in composite particle QED (4), (5). In this theory the photon propagator has the standard QED–form. In the large momentum region the fermionic propagator is finite. A number of integrations over internal momenta of a diagram with \( N \) vortices, \( N_f \) external fermionic lines and \( N_{ph} \) external photon lines, is \( 2(N - N_f - N_{ph} + 2) \). Thus the index of a diagram \((N, N_f, N_{ph})\) equals

\[ \Omega_{N, N_f, N_{ph}} = 2(N - N_f - N_{ph} + 2) - 2\frac{N - N_{ph}}{2} - N \Gamma \cdot N \]  \hspace{1cm} (15)

where \( N \Gamma \) is the power of momentum in the expression for the vertex operator. Hence

\[ \Omega_{N, N_f, N_{ph}} = N - 2N_f - N_{ph} + 4 - N \Gamma \cdot N \]  \hspace{1cm} (16)

It is known that the most critical divergence is in the polarization operator \((N_{ph} = 2, N_f = 0, N = 2)\). In this case (16) yields:

\[ \Omega_{2,0,2} = 4 - 2N \Gamma \]  \hspace{1cm} (17)

Thus if \( N \Gamma > 2 \) then all of Feynman integrals are finite.
Consider possible large momentum dependence of the vertex operator (5), (12), (13). In the approximation $p^2 \rightarrow 0$ this vertex operator has the following form:

$$\Gamma^\mu \rightarrow h_1(0, q^2)\gamma^\mu$$  \hspace{1cm} (18)

i.e. the vertex operator (2) without ”spin term”. Hence we might suppose that

$$h_1(p^2, q^2)|_{p^2, q^2 \rightarrow \infty} \rightarrow \frac{1}{(p + q)^{2(n-1)}}$$  \hspace{1cm} (19)

in agreement with the components counting rules. $h_2(p^2, q^2)$ has analogous $p^2, q^2$–dependence. Thus with the supposition (19) in the case $n = 3$ the Feynman integrals are finite for all diagrams. In the case $n = 2$ ultraviolet divergence remain in the polarization operator.

This work approach is not consistently one to the Feynman diagram calculations as well as the Feynman rules in pointlike QED where the vertex operator form is consequence of the Dirac equation with the interaction taking into account. It is to be supposed that the equation (2) and the vertex operator (5), (12), (13) could be obtained from multifermionic Green functions consideration.

The information about the momentum dependence of the form factors could be found from concrete models of a composite particle with knowledge of the components interactions. The functions $f(p^2), h_1(p^2, q^2), h_2(p^2, q^2)$ for leptons could be obtained from preon models [3]. Hence this problem is not a pure QED–problem as the electron structure cannot be determined by electrodynamics. Thus the fact of the vertex operator and the lepton propagator being in principle written in the form that allows for calculating QED–quantities without divergences has a principal character only. The QED–calculations with the account of small corrections, which are proportional to $\langle r_{\text{lept}}^n \rangle$, is possible.

The author thanks T. Kinoshita for valuable questions, that enabled deeper look on the problem, S.J. Brodsky for the support of this work and M. Tyntarev and M. Golod for their technical help.
References

[1] S.J. Brodsky, G.R. Farrar, Phys. Rev. Lett. 31, 1153 (1973).

[2] S.J. Brodsky, S.D. Drell, Phys. Rev. D22, 2236 (1980); H. Dehemelt, Nobel Prize lecture (Nobel Foundation, 1990).

[3] J.C. Pati, Phys. Lett. B228, 228 (1989); K.S. Babu, J.C. Pati, Phys. Rev. D48, R1921 (1993); K. Akama, T. Hattori, Phys. Rev. D51, 3895 (1995).