Low Energy Dynamical Supersymmetry Breaking Simplified

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We present a model in which supersymmetry is dynamically broken at comparatively low energies. Previous efforts to construct simple models of this sort have been hampered by the presence of axions. The present model, which exploits an observation of Bagger, Poppitz and Randall to avoid this problem, is far simpler than previous constructions. Models of this kind do not suffer from the naturalness difficulties of conventional supergravity models, and make quite definite predictions for physics over a range of scales from 100’s of GeV to 1000’s of TeV. Thus “Renormalizable Visible Sector Models” are a viable alternative to more conventional approaches. Our approach also yields a viable example of hidden sector dynamical supersymmetry breaking.
1. Introduction

If supersymmetry is truly to provide a resolution of the hierarchy problem, it is necessary that it be dynamically broken. Yet, while various mechanisms for dynamical supersymmetry breaking (DSB) are known, there does not yet exist any particularly compelling particle physics model. Most models of supersymmetry breaking assume breaking at a scale of order $M_{int} = \sqrt{m_{3/2} M_p}$, with the gravitino mass $m_{3/2}$ of order the weak scale, and simply put in soft supersymmetry-breaking parameters by hand. Moreover, in these theories, the superpotential and Kahler potential cannot be the most general compatible with symmetries. In the context of string theory, a number of models have been constructed. However, explicit models which actually do break supersymmetry have other difficulties, such as a non-vanishing cosmological constant and large flavor changing neutral currents.

An alternative possibility is that supersymmetry is broken at a low scale, within a few orders of magnitude of the weak scale. In such a model, gauge interactions can serve as the “messengers” of supersymmetry breaking, giving rise to a high degree of degeneracy among squarks and sleptons. However, past efforts to build such models have met a number of obstacles. The general strategy has been to take some model which exhibits DSB, and to gauge some global symmetry, identifying this with a subgroup of the standard model gauge group. However, this typically leads to difficulties with asymptotic freedom. In [1], this problem was avoided by identifying the global symmetry with a new gauge symmetry, carried both by “supersymmetry breaking sector” fields and by “messenger sector” fields which also carry standard model quantum numbers. A second problem is the appearance of axions associated with spontaneously broken R symmetries. As explained in [2], the appearance of spontaneously broken R symmetries is generic to models of dynamical supersymmetry breaking. The R axion in these models is not seen in terrestrial experiments, due to its large decay constant. However, it could be emitted by red giants and supernovae, leading to unrealistic cooling rates. To avoid this difficulty, it seemed necessary to introduce additional gauge groups, whose sole purpose was to give mass to the axion. The resulting models were quite unwieldy, with extremely large groups and representations, and suffered from several naturalness and fine-tuning difficulties.

Recently, however, Bagger, Poppitz and Randall [3] have pointed out that the R axion
is never a problem for astrophysics. They noted that in the framework of a supergravity
theory, the $R$ symmetry is necessarily explicitly broken, and the $R$ axion obtains a mass
of order
\[ m_a^2 \sim \frac{|F|^{3/2}}{M_p}. \quad (1.1) \]
Here, $F$ is the Goldstino decay constant (the expectation value of the $F$-component of
some hidden sector field). In models with radiative generation of squark and slepton
masses, this is typically of order $(100 \text{ TeV})^2$, so the axion mass is of order 10 MeV or
larger. This term originates from the expectation value of the superpotential required to
cancel the cosmological constant; such contributions can also arise from other dimension-5
$R$ symmetry breaking operators \[2\]. This mass is large enough to suppress the production
of these particles in red giants and supernovae \[4\].

With the R axion problem disposed of, one may be able to construct simpler and more
compelling models of dynamical supersymmetry breaking. This is the goal of the present
work. We will outline a general strategy for model building, and apply it to some particular
examples. The models we will describe will suffer from none of the fine-tuning problems
of earlier work. We will require that some parameters be small, but will argue that this
is technically natural. The phenomenology of the models will be quite rich. At scales
comparable to the weak scale, one will have the spectrum of the minimal supersymmetric
standard model, with superpartner masses roughly proportional to gauge couplings, and
possibly with an additional singlet and other fields. At higher scales, however, there will
be additional fields, including a vector-like set of messenger quarks and leptons. Finally,
at a still higher scale, one will find the supersymmetry breaking sector itself.

Probably the simplest model of dynamical supersymmetry breaking is that based on
the group $SU(3) \times SU(2)$, and we will illustrate our considerations with this theory. In
the next section, we will review some of the essential features of this model. In section
3, we will gauge a $U(1)$ symmetry, and couple additional fields to the model, allowing
for feed-down of supersymmetry breaking to ordinary fields. We will compute the leading
contributions to squark and slepton masses. We also discuss how this model could be used
in the hidden sector, giving small but possibly adequate masses to the gauginos.

In section 4, we will take up the problem of $SU(2) \times U(1)$ breaking. We will show
that this breaking will require the introduction of additional fields into the model. Several examples involving additional singlet fields and compatible with all existing constraints will be worked out in detail. All of these models will require that some couplings be small (of order \((\alpha_2/\pi)\) or \((\alpha_2/\pi)^2\)). Such numbers, of course, are not unfamiliar in the framework of the standard model, and will be seen to be natural in the sense of 't Hooft [3].

In section 5, we will discuss some experimental signals of low energy supersymmetry breaking, such as terrestrial gravitino production. Finally, in section 6 we present some final remarks and conclusions. We will argue that, from perspectives such as naturalness, these models are at least as successful as more conventional intermediate scale theories. Indeed, they solve the problems of flavor changing neutral currents far more easily than such theories. They thus represent, in our view, a viable alternative to conventional models, and should be taken seriously.

2. Review of the 3-2 Model

The minimal model with calculable dynamical supersymmetry breaking is the 3-2 model, based on the gauge group \(SU(3) \times SU(2)\) [6]. It is natural, then, to use this theory to construct a viable theory of DSB. Here we will recall some of the basic features of this model. More detail is presented in [6] and [3]. The chiral superfields of the model are denoted by

\[
Q = (3, 2)_{1/3}; \bar{U} = (3, 1)_{-4/3}; \bar{D} = (3, 1)_{2/3}; L = (1, 2)_{-1}.
\]  

(2.1)

Here, the numbers in parenthesis refer to the \(SU(3) \times SU(2)\) representation, while the subscript refers to the quantum numbers under a global \(U(1)\) symmetry of the model, which we will later wish to gauge. (This will require at least one additional field to cancel the anomaly.) We will refer to this symmetry as “messenger hypercharge,” or simply as hypercharge, but it should not be confused with the usual hypercharge of the standard model.

The most general renormalizable superpotential consistent with these symmetries is

\[
W = \lambda Q L \bar{D}.
\]  

(2.2)
In addition to hypercharge, this model also has a non-anomalous R symmetry. In the limit of vanishing $\lambda$, the theory has flat directions in which the gauge symmetry is completely broken. The spectrum consists of massive vector multiplets, with mass of order $g_i v$ (here $g_i$ denotes the $SU(3)$ or $SU(2)$ gauge coupling, and $v$ denotes a generic expectation value), and three massless chiral multiplets. These multiplets may be represented by the gauge invariant combinations,

$$X_1 = \bar{D}QL; \quad X_2 = \bar{U}QL; \quad X_3 = \det Q\bar{Q}$$

where, in the last expression,

$$\bar{Q} = (\bar{U} \quad \bar{D}).$$

and the determinant is in flavor space. The actual massless states can be found by expanding these fields about their vev’s. Note that $X_1$ and $X_3$ are neutral under the $U(1)$, while $X_2$ carries charge $-2$. If the $SU(3)$ coupling is larger than the $SU(2)$ coupling, this model reduces to supersymmetric QCD with three colors and two flavors; in this theory, it is well-known that instantons generate a superpotential (we follow the convention of [3]),

$$W_{np} = \frac{2\Lambda^7_3}{\det(Q\bar{Q})}.$$  

For small $\lambda$, we expect that all of the vev’s are large, so that the gauge couplings are effectively weak. Thus one can analyze the theory simply by minimizing the superpotential and the $D$-terms. By simple scaling arguments, the vev’s of the scalar fields all scale as $v \sim \Lambda_3/\lambda^{1/7}$. One finds that supersymmetry is broken. One can compute the spectrum numerically [3], but it is not hard to guess its main features. There are three massless states. Apart from the Goldstino and the axion associated with the breaking of R symmetry, the fermion in the charged multiplet, $X_2$ remains massless – this is necessary to satisfy anomaly constraints. The charged scalar and the other three real scalars gain mass squared of order $\lambda^2 v^2$. The vector multiplets still have masses of order $g_i v$, but are slightly split.

Before closing this section, it is worthwhile to mention some other reasonably simple models which exhibit dynamical supersymmetry breaking, which we will use to illustrate some aspects of model building. There are two general criteria for a model to break supersymmetry dynamically. First, the classical theory should not possess any flat directions.
Second, the model should contain global symmetries which are expected to be spontaneously broken non-perturbatively. Another model which satisfies these two criteria is a theory with gauge group $SU(5)$ with a single $\tilde{5}$ and $10$ \[7\]. It is easy to see that this model possesses no flat directions. By an $SU(5)$ transformation, one can always take the $\tilde{5}$ to have an entry in only the first component. But there is no way that one can cancel the resulting $D$ term with the $10$. This theory also contains a global $U(1)$ which can be gauged by adding a single new field carrying the $U(1)$ to cancel the anomaly.

Unfortunately, in this model, there is no small parameter which permits systematic computations. As a result, one can only guess what happens. However, it is almost certain that some of the global symmetry of the model is spontaneously broken and that supersymmetry is broken.

This model admits a set of generalizations \[6\]. As an example, consider a model with gauge group $SU(7)$, an antisymmetric tensor, $A_{ij}$, and three $7$'s. Before adding a superpotential, this model possesses flat directions in which $SU(7)$ breaks to $SU(5)$ with a $\tilde{5}$ and $10$. As a result of supersymmetry breaking in this lower energy theory, the flat directions are lifted; this cannot, however, be nicely described in terms of an effective superpotential. One can add a tree level potential which lifts the flat directions,

$$ W = A_{ij} \tilde{\tau}_i^1 \tilde{\tau}_j^2. \quad (2.6) $$

The resulting theory is expected to have broken supersymmetry with a good ground state. Note that the model has an $SU(2) \times U(1)^2$ global symmetry which may be of use; for instance a $U(1)$ subgroup of the $SU(2)$ may be gauged without the need for additional fields to cancel anomalies.

Still one other model of potential interest possesses gauge group $SU(5)$, two $10$'s and two $\tilde{5}$'s and the most general superpotential allowed by the symmetries. Again this model has no flat directions, and an $SU(2) \times U(1)^2$ global symmetry. Unlike the previous case, for small value of the superpotential coupling, the ground state is completely weakly coupled, and everything is calculable in principle. However, actually minimizing the potential is quite difficult.\[1\]

1 We thank E. Poppitz and L. Randall for a discussion of their efforts on this problem.
3. Feeding Down DSB

Let us focus on the 3-2 model, and consider how supersymmetry breaking might be fed down to ordinary fields. No renormalizable couplings to ordinary fields can appear in the superpotential (this is true even if we add a singlet), so we will try to take advantage of the hypercharge symmetry. We don’t want to identify this symmetry with any conventional (global or local) symmetry of the standard model. One reason for this is that at one loop, the $D$ term for this $U(1)$ receives a non-vanishing contribution. We will estimate this $D$-term shortly. But there is another reason, which is more generic. It applies even to models in which one does not generate a low order $D$ term. (examples of such models will be discussed later). Consider some general model which breaks supersymmetry, and identify a global $U(1)$ symmetry of the supersymmetry breaking sector with a gauge symmetry carried by ordinary quarks such as ordinary hypercharge or (a now gauged) $B - L$. Squarks will then gain mass, typically at one or two loops. In the hidden sector, $R$ symmetry is spontaneously broken, so gauginos can gain mass. But gluinos, which do not carry the $U(1)$ charge, can gain mass at best at one higher order in the loop expansion than squarks. For example, if squark masses squared arise at two loops, gluino masses arise at three loops. As a result, gluino masses will probably be unacceptably small\textsuperscript{2}. This argument does not apply if one can gauge an $SU(3)$ symmetry of the supersymmetry breaking sector and identify it with color. However, then one must consider rather large gauge groups (which usually entails loss of asymptotic freedom) or consider complicated structures such as that of [1].

Since we will focus here on simple supersymmetry breaking sectors, with only $U(1)$’s which can be gauged, we will adopt a different strategy. We will communicate supersymmetry breaking to the ordinary fields through another set of “messenger” fields. The messengers will include quarks and leptons ($q$ & $\ell$) which are vector-like with respect to ordinary gauge interactions. These vector-like quarks and leptons couple to gauge singlet chiral fields whose scalar and auxiliary components gain expectation values at the same order of perturbation theory, as a result of their interactions with fields carrying messen-

\footnote{The possibility that very light gluinos might still be allowed has been much discussed in the recent literature, \emph{e.g.} in [8].}

6
ger hypercharge. It is, of course, necessary to make sure that messenger hypercharge is anomaly free. As a result, ordinary squark, slepton, and gaugino masses will be of the same order. (Another possibility, which we will not explore further in this paper, occurs in models where a D term for the messenger gauge group is not generated at low order. Then the new vector-like quarks and leptons may be able to also carry the messenger gauge group.) $SU(2) \times U(1)$ breaking will involve couplings to (possibly additional) singlet fields, in conjunction with the usual radiative mechanism for generating negative Higgs mass via the top quark Yukawa coupling.

Before going on to construct models, it is helpful to study the Fayet-Iliopoulos D term generated for messenger hypercharge in this model. Its sign is relevant to model building efforts. This $D$ term is easily estimated by considering in somewhat greater detail the form of the spectrum. For zero $\lambda$, the 3-2 model possesses flat directions. In these flat directions there is one massless chiral field charged under the $U(1)$. One can think of this in terms of the gauge-invariant object,

$$X_2 = \bar{U}QL.$$  \hspace{1cm} (3.1)

It is the fermionic component of this field which is the massless fermion in the true vacuum at non-zero $\lambda$. The complex scalar gains a mass squared of order $\lambda^2 v^2$. Now it is tempting to compute the Fayet-Iliopoulos term by noting that the leading, quadratic divergence, is cancelled provided $\text{Tr}(Y)=0$, and then assuming the first, subleading term is dominated by the light charged scalar. The result is logarithmically divergent. One might want to identify the cutoff with the masses of the vector multiplets, some of which are charged under the $U(1)$.

In fact, this estimate is correct. It follows from sum rules for the spectrum in this model. One can derive the sum rule relevant to the present circumstance by the following considerations. Work in terms of component fields (rather than superfields) and choose ’t Hooft-Feynman gauge. This has the advantage that the scalar fields appearing in the vector multiplets are then complex fields. Expand the superpotential about the minimum in the form

$$W = W_0 + \gamma_{ij} \phi_i^- \phi_j^+ + \ldots$$  \hspace{1cm} (3.2)
where $W_o$ denotes the part of the superpotential involving the neutral fields, and we have explicitly exhibited the part contributing to the masses of charged fields. (There are three fields of charge $+2$ and four of charge $-2$. One can, however, project these fields onto the zeroth order massive states). The actual scalar mass matrix has two pieces. There is a piece of the form $m_{ij}^2 \phi_i^* \phi_j$. There is also a piece of the form $m_{ij}^2 \phi_i \phi_j$. This piece, however, makes a contribution to the $D$-term down by $\lambda^2/g^2_i$ compared with that above. So we can take the mass matrix to be:

$$
\begin{pmatrix}
M_V^2 + \gamma^\dagger \gamma & 0 \\
0 & M_V^2 + \gamma^* \gamma^T
\end{pmatrix}. 
$$

(3.3)

Note that the upper block, which gives the masses of the fields with charge $+2$, is $3 \times 3$, while the lower block, which gives the masses of fields of charge $-2$, is $4 \times 4$.

Now let us examine the computation of the $D$ term. Starting with the expression

$$
\langle D \rangle = \left( \frac{g_v^2}{2\pi} \right)^4 \sum_i y_i \int \frac{d^4k}{k^2 + m_i^2},
$$

(3.4)

the leading divergence cancels. The subleading term is given by

$$
\langle D \rangle = \frac{g_v^2}{16\pi^2} \sum_i y_i m_i^2 \ln(\Lambda^2/m_i^2). 
$$

(3.5)

The divergent part is easily seen to cancel, in view of the structure of the mass matrix described above. It is proportional to

$$
Tr(M_V^2 + \gamma^\dagger \gamma) - Tr(M_V^2 + \gamma^* \gamma^T) = 0.
$$

(3.6)

In eqn. (3.3), there is a piece proportional to $m_\ell^2 \ln(\Lambda^2/m_\ell^2)$. (Recall $m_\ell^2$ is the mass of the light charged field, of order $\lambda^2 v^2$). In view of the cancellation of infinities we have just noted, $\Lambda$ in this term must be replaced by $M_V$, where $M_V$ is some typical vector mass, of order $g^2 v^2$ (where $g$ is the $SU(3)$ or $SU(2)$ gauge coupling). Putting this together, we have with logarithmic accuracy that the coefficient of the $D$ term is given by

$$
\xi^2 = \frac{g_v^2}{16\pi^2} m_\ell^2 \ln(g^2/\lambda^2). 
$$

(3.7)

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3 It is easy to check that there are no terms of order $M_V^2 \ln(M_V^2)$ or $M_V^2$. This follows from the form of the mass matrix in eqn. (3.3).
Note that if we cancel the anomaly of this model by adding a field, $E$, of charge +2, the sign here is such that this field acquires a positive mass on account of the $D$ term, and the $D$ term has a non-zero expectation value.

We can use this result to build models. As explained earlier, we would like to have a gauge singlet field obtain an expectation value. So we introduce the following fields, with their corresponding U(1) charges in parenthesis:

$$E(+2), P(+1), N(-1), S(0).$$

In addition, we include a set of vector-like quarks and leptons. We take these to have conventional $SU(3) \times SU(2) \times U(1)$ quantum numbers, e.g.,

$$q(3,1)_{-2/3}, \bar{q}(\bar{3},1)_{2/3}, \ell(1,2)_1, \ell(1,2)_{-1}.$$  

For the superpotential we take:

$$W = \lambda_1 PNS + \frac{\lambda_2}{2} EN^2 + \frac{\lambda_3}{3} S^3 + k_1 S\bar{q}q + k_2 S\ell\bar{\ell}.$$  

What are the dynamics of this model? On account of the D-term, the field $N$ obtains an expectation value. This leads to a mass for several of the fields. In particular, the $S$ and $P$ fields pair to gain mass. Note that the sign of the $D$ term is relevant here. Had the $D$-term had the opposite sign, some linear combination of the $P$ and $E$ fields would have obtained a vev. There would have been a massless chiral field at lowest order. At next order, if this field received a negative mass squared, it would have gotten a vev, inducing as well the vev for the $S$ field which is needed to give the $q$ and $\ell$ fermions mass. With the given sign of the $D$ term, it appears to be more difficult to give the $S$ field the necessary vev. However, suppose that the coupling $\lambda_1$ is very small. In this case, corrections to the $S$, $P$ and $E$ masses from gauge field exchanges (at two loop order) can be important. These corrections are easily estimated. Just as we have argued that the light charged

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4 An alternative charge assignment, which allows the new $SU(3) \times SU(2) \times U(1)$ gauge groups to be simply unified, is to take the $P$ and $N$ fields to have charges $\pm 4$. With a suitable superpotential and additional singlets this also leads to a satisfactory model.
scalar makes the most important contribution to the D term, so this field can be argued to make the most important contribution here. In ref. [1], it was shown that the two loop contribution of fig. [1] to the mass of a field of charge $y$, to first order in the supersymmetry breaking mass shifts, can be written as $y^2 \tilde{m}^2$, where

$$\tilde{m}^2 = 8 \left( \frac{g_Y^2}{16\pi^2} \right)^2 \sum_i (-1)^F y_i^2 m_i^2 \ln(\Lambda^2/m_i^2). \quad (3.11)$$

here the sum is over the fields appearing in the diagram; $y_i$ are their U(1) charges and $m_i^2$ their masses. Again, we can consider the contributions of the different states. The same sum rule we used before shows that the leading divergent term cancels; the subleading terms give a result equal to

$$\tilde{m}^2 = -32 \left( \frac{g_Y^2}{16\pi^2} \right)^2 m_\ell^2 \ln \left( \frac{g_\ell^2}{\lambda^2} \right) \quad (3.12)$$

where $m_\ell$ is the mass of the light charged field of the 3 − 2 model, and $\lambda$ is the coupling appearing in the superpotential of that model.

With this correction, the full potential of the model is

$$V = \sum \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \frac{1}{2} g_Y^2 (\xi^2 + 2|E|^2 + |P|^2 - |N|^2)^2 + \tilde{m}^2 (4|E|^2 + |P|^2 + |N|^2). \quad (3.13)$$

Now if $\lambda_1$ is small enough, the negative mass term will lead to vev’s for $P$, $N$ and $E$; this in turn will drive vev’s for $S$ and $F_S$. One might imagine that $\lambda_1$ would have to be quite small, of order $g_y/(4\pi)$, but in practice, it turns out that $\lambda_1$ does not need to be especially small. For example, taking

$$g_y = 1, \quad \lambda_1 = 0.2, \quad \lambda_2 = 0.3, \quad \lambda_3 = 0.3, \quad (3.14)$$

we find the potential minimized at

$$n = -2.44\xi, \quad p = 1.40\xi, \quad s = 1.28\xi, \quad e = 1.27\xi, \quad F_S = -0.188\xi^2. \quad (3.15)$$

It will be convenient to work in a range of parameters for which $F_S$ is relatively small, $(F_S \ll k_{(1,2)}\langle S \rangle^2)$. This is achieved, for example, if $\lambda_3$ is small.
Now that $S$ and $F_S$ have expectation values, the stage is set to give masses to squarks, sleptons, and gauginos. Gaugino masses will arise through the one loop diagrams of fig. 2.

To leading order in $F_S$, the resulting mass is

$$m_{\tilde{g}} = \frac{g_i^2}{16\pi^2} \frac{F_S}{S}. \quad \text{(3.16)}$$

Squark and slepton masses squared arise at two loops, and thus the masses are of the same order as gaugino masses. Their evaluation is somewhat more complicated, involving the same set of diagrams as in fig. 1, where now the gauge fields are those of conventional $SU(3) \times SU(2) \times U(1)$, rather than those of $U(1)_Y$. We can determine the mass again by repeating the computation ref. [1], working carefully to second order in the supersymmetry breaking mass shifts. In the present case, the fields appearing in the loops are $q$ and $\bar{q}$, or $\ell$ and $\bar{\ell}$. A straightforward computation gives

$$\tilde{m}^2 = \sum_a 2C_F^{(a)} \left( \frac{g^{(a)^2}}{16\pi^2} \right)^2 \frac{F_S^2}{S^2}. \quad \text{(3.17)}$$

Here $a$ denotes the gauge group (so, for example, quark doublets obtain a contribution from $SU(3)$, $SU(2)$ and $U(1)$ gauge field exchange).

Note, in particular, that these contributions are positive and that they depend only on the gauge quantum numbers of the fields. Note also that no Fayet-Iliopoulos $D$ term is generated for ordinary hypercharge at low orders. This is because, before worrying about ordinary squarks and sleptons, the model has a left-right symmetry which exchanges $q$ and $\bar{q}$ and $\ell$ and $\bar{\ell}$. As a result, the first potential contributions to the $D$ term appear at three-loop order, and are harmless. This is a significant improvement over the model of ref. [1], where equality of certain gauge couplings had to hold to a good approximation to avoid such $D$ terms.

The 3–2 model has particular appeal because of its simplicity. The approach which we have adopted here to feeding down supersymmetry breaking to ordinary fields is probably the simplest one available in this case. No “unnatural acts” were required here. One coupling constant had to be reasonably small, but this is perfectly consistent with ’t Hooft’s notion of naturalness.
The most popular approach to communicating supersymmetry breaking is to have supersymmetry broken at a relatively high scale, around $10^{11}$ GeV, and then use super-gravitational and other Planck scale couplings to feed supersymmetry breaking to the visible sector. An early attempt to use the 3-2 model in this way was discarded because ordinary gaugino masses could only arise through operators such as

$$\int d^2 \theta \frac{O(1)}{m_P^3} QLD \tilde{W}_\alpha \tilde{W}^\alpha,$$

leading to gaugino masses suppressed by two powers of $m_P$ relative to the weak scale. With gauged hypercharge and the additional $P, N, S$ and $E$ fields however one could have the operator

$$\int d^2 \theta \frac{O(1)}{m_P} S \tilde{W}_\alpha \tilde{W}^\alpha,$$

leading to gaugino mass terms of order $\alpha_Y/\pi$ times the squark and slepton masses. This could be acceptable provided $\alpha_Y$ is not too small.\footnote{It is also necessary to understand the smallness of the $\mu$ parameter in this framework. $SH_uH_d$ must be forbidden in the superpotential, perhaps by a discrete symmetry acting on the Higgs. One probably wants this to be an R symmetry so that the coupling $S^\dagger H_U H_D$ will be allowed in the Kahler potential, generating a $\mu$ parameter of order $m_{3/2}$.}

Alternatively, the 3-2 model could serve as a hidden sector with communication of supersymmetry breaking done by ultraheavy Grand Unified Theory (GUT) mass fields which couple to the field $S$ and carry ordinary gauge quantum numbers. Such a model would be similar to the MSSM, however the squark, slepton and gaugino masses would arise from ordinary gauge interactions and be calculable as in the visible sector model. Degeneracy of the squarks and sleptons could be upset by Planck scale physics, leading to excessive flavor changing neutral currents (FCNC) unless the ultraheavy masses are much less than $O((\alpha_Y/\pi)(\alpha_{\text{GUT}}/\pi) M_P)$. It is interesting to ask whether other supersymmetry breaking models might give different possibilities for communication of supersymmetry breakdown. We have explained why it is probably necessary to insulate the supersymmetry breaking sector from the visible sector (loss of asymptotic freedom and suitable gluino mass). But, as we have already mentioned, there do exist models in which it is not necessary to introduce spectators to
cancel anomalies. Consider the $SU(7)$ model. Here we can gauge a $U(1)$ which rotates the $7$ and $\bar{7}$ appearing in the superpotential of eqn. (2.6) by opposite phases. In this model, rather than introducing three fields, $E$, $P$ and $N$, we can simply introduce two fields of opposite charge, $\phi^+$ and $\phi^-$, and a singlet, $S$, with couplings

$$\lambda_1 S\phi^+\phi^- + \lambda_2 S^3 + +k_1 S\bar{q}q + k_2 S\bar{\ell}\ell. \quad (3.20)$$

Here $S, q, \bar{q},$ etc. will play a similar role in feeding down supersymmetry breaking as the corresponding fields in the $3 - 2$ case. Now, however, the model has a discrete symmetry which interchanges $\phi^+$ and $\phi^-$, as well as $\bar{7}_1$ and $\bar{7}_2$. If this symmetry is not spontaneously broken, then, because the $D$ term for the $U(1)$ is odd under this symmetry, there can be no Fayet-Iliopoulos term. Because of the strongly coupled nature of the theory, we cannot compute the masses generated at two loops for $\phi^+$ and $\phi^-$. However, as long as they are non-zero, we can obtain an acceptable model. If the masses are negative, the minimum of the potential, for a range of parameters, has a non-vanishing $\langle S \rangle$ and $\langle F_S \rangle$. If the masses are positive, one loop corrections induce a negative mass for $S$, which in turn leads to non-zero $\langle S \rangle$ and $\langle F_S \rangle$. The rest of the story of feed-down and $SU(2) \times U(1)$ breaking then proceeds as in the $3 - 2$ case. This is, of course, not the only alternative model, but we suspect that the strategies we have used here are rather general. We favor the $3 - 2$ model because of its simplicity.

4. Ordinary matter and $SU(2) \times U(1)$ Breaking

The minimal “ordinary” sector consists of the usual fields of the minimal supersymmetric standard model (MSSM). The famous “mu problem” of the MSSM, i.e. how to give the Higgs fields a weak scale supersymmetric mass term, shows up here as well. We do not include an $H_u H_d$ term in the superpotential, since our philosophy is that all masses should arise through dimensional transmutation. In [1], this problem was dealt with by introducing a singlet, $S'$, with couplings to the Higgs doublets, and to an additional pair of vector-like quarks and leptons. These extra fields were required in order to generate sufficiently large negative mass for the singlet. The model had several virtues: the superpotential could be taken to be the most general compatible with certain discrete symmetries;
there were no new sources of CP violation beyond the KM phase, and the model predicted
an interesting set of states beyond the MSSM at weak-scale energies.

In the present case, we might hope to break $SU(2) \times U(1)$ in a simpler fashion,
exploiting the singlet $S$ we have already introduced. With a coupling $\lambda_h H_d H_u S$, the vev
of $S$ could provide a “mu-term” type mass for the Higgs. However, unless there are delicate
cancellations between different terms, one cannot obtain an acceptable spectrum this way.
In order that higgsino masses be comparable to the $Z$ mass, we require

$$\lambda_h \langle S \rangle \sim m_Z. \quad (4.1)$$

In addition, there are terms in the Higgs potential of the form

$$\lambda_h \langle F_S \rangle H_u H_d. \quad (4.2)$$

These also shouldn’t be too much larger than $m_Z^2$. So we require that

$$\langle F_S \rangle / S^2 \sim \lambda_h. \quad (4.3)$$

On the other hand, the two loop contribution to the Higgs mass, eqn. (3.17), should not
be much smaller than $m_Z^2$, and this is incompatible with these two conditions. At best, an
acceptable spectrum can be obtained only by appreciable fine tuning.

While this simplest possibility seems not to work, we have found several viable ap-
proaches to $SU(2) \times U(1)$ breaking. One is to include another singlet field $S'$, and to add
to the superpotential

$$\lambda_h H_u H_d S' + \frac{\lambda_{S'}}{3} S'^3 + k_3 S'^2 S. \quad (4.4)$$

Think of $k_3$ as the small parameter in this lagrangian, while the other couplings are of order
one. For small $k_3$ and real $F_S$, the imaginary part of $S'$ obtains a negative mass-squared,
k_3 F_s. So $S'$ obtains a vev:

$$S'^2 \sim \frac{k_3 F_s}{\lambda^2}. \quad (4.5)$$

Note that in this estimate, we can neglect the term cubic in $S'$; it is down by $\sqrt{k_3}$. Note
also that $F_{S'} \sim \frac{k_3}{\lambda} F_s$. In particular, in terms of powers of $k_3$, $F_{S'}$ is comparable to $S'^2$. 14
Now if \( k_3 \sim (\frac{m_w}{\pi})^2 \), the \( H_u H_d \) term in the Higgs potential is comparable to the terms coming from top quark exchange and the higgsino mass is comparable to the Higgs mass.

The main problem with this idea is that \( k_3 \), more or less by accident, must be of the correct order of magnitude. Note, however, that it is natural for \( k_3 \) and the other couplings we have omitted to be small. For example, suppose we have an approximate symmetry under which

\[
S' \rightarrow e^{2\pi i/3} S', \quad H_u H_d \rightarrow e^{4\pi i/3} H_u H_d; \quad S \rightarrow S.
\]

(4.6)

This symmetry explains the smallness of the couplings \( SS'^2 \) and \( SH_u H_d \) which violate the symmetry by the same amount, and \( S'S^2 \) which violates the symmetry by a different amount.

Another fairly simple alternative, which does not require any small nonzero couplings, is to have \( k_3 \) be zero but include couplings

\[
k_4 S'q\bar{q} + k_5 S'\bar{\ell}\ell.
\]

(4.7)

Now a suitably small vev for \( S' \) will be induced radiatively at one loop provided \( S \) gets a vev and the couplings \( k_4 \) and \( k_5 \) are sufficiently small. This vev is easily computed; ignoring \( k_5 \), one finds

\[
S'^3 = \frac{1}{32\pi^2} \frac{k_4 F_S^2}{k_1 S}.
\]

(4.8)

However it is difficult to use symmetry arguments to explain the omitted couplings such as \( S'S^2 \), which would lead to a large \( S' \) vev. At best, approximate symmetries such as those we have described earlier tend to keep these couplings naturally as small as \( k_4 \) and \( k_5 \), and this is perhaps barely small enough.

A third approach, which does not require small parameters, repeats the construction of [1]. In addition to \( S' \), one includes a second set of vector-like quarks and leptons (beyond \( q, \bar{q}, \ell \) and \( \bar{\ell} \), \( q', \bar{q}', \ell' \) and \( \bar{\ell}' \)). One now introduces couplings

\[
S'H_u H_d + S'q'\bar{q}' + S'\bar{\ell}'\ell' + S'^3 + H_d Qq'.
\]

(4.9)

(In this expression \( Q \) now denotes the conventional quark doublets.) This structure can be enforced by a \( Z_3 \) symmetry which rotates ordinary fields and primed fields by \( e^{2\pi i/3} \).
There is a danger, here, of strangeness changing neutral currents if $q'$ is too strongly mixed with the light down quarks. This can also be avoided by suitable (approximate) discrete symmetries.

The primed quark and lepton fields obtain two loop masses just like the ordinary fields. These, in turn, lead to negative masses at one loop for the field $S'$ and for $H_d$. Note that, as in [1], these are comparable to the two loop masses for the weak doublets, because the fields in the loop carry color, and because of color and logarithmic factors in the diagrams. With all couplings of order one, one can readily obtain an acceptable spectrum [3]. Not only is this model the most general consistent with symmetries, but all of the phases in the superpotential, apart from the KM phase, can be removed by field redefinitions, so there are no new sources of $CP$ violation.

All of these approaches result in a low energy theory which is similar to the MSSM, with $SU(2) \times U(1)$ breaking driven by the usual top quark radiative correction, and with an additional light singlet. There are fewer potentially free parameters, however, associated with supersymmetry breaking. In the limit that $F_S$ is small compared with the masses in the messenger sector, the squark, slepton, and gaugino masses depend only on $F_S/S$, gauge couplings, and the ordinary $SU(3) \times SU(2) \times U(1)$ quantum numbers of $q$ and $\ell$, while the higgsino and Higgs masses depend also on $\lambda_h \langle S' \rangle$ and $\lambda_h F_{S'}$. Thus with small $F_S$, once the top mass and weak scale are fixed the superpartner and Higgs masses depend only on two additional parameters. For any value of $F_S$, in all versions of this model the squark and slepton masses naturally come out degenerate, since the leading contributions come from gauge couplings, and do not lead to new sources of FCNC. In general the superpartner masses also depend on $\lambda_{\ell}$ and $\lambda_q$.

Besides the superpartner masses, other supersymmetry-breaking couplings are also nonzero. For instance there will be trilinear scalar couplings; these arise at two loops. Supersymmetry-breaking dimension-4 and higher couplings also arise radiatively. We believe all these supersymmetry breaking couplings to be too small to be phenomenologically interesting.

We have already noted that in the third model, the only source of $CP$-violation are the KM phase (apart from the $\theta$-parameter). In the first two models, all CP violation
may be removed from the superpotential couplings in the messenger and supersymmetry breaking sectors, except for the phase of $k_3$ in the first version, or $k_4$ and $k_5$ in the second. Thus the low energy supersymmetry breaking parameters will be CP conserving, except for one phase in the Higgs sector. Fine-tuning of this phase to $10^{-2} - 10^{-3}$ is required to avoid inducing electric dipole moments for the neutron and for atoms. The usual strong CP problem also still exists.

As we have noted, in the third version, where the $q'$ and $\ell'$ have weak scale masses, there is a danger of flavor changing neutral currents. This problem is not as severe in the first two models, since, provided that the couplings $k_i$ are of order one, the masses of the $q$ and $\ell$ are of order $(16\pi^2/g_2^2)$ times the weak scale. Thus mixing is highly suppressed by the large masses, and is not a problem. However it would still be of interest to study the potential for observing nonstandard FCNC and CP violation in the B meson system.

5. Experimental signatures

We believe that the model-building strategy we have described is rather general. There are a number of predictions for supersymmetry phenomenology which follow in this framework:

1. The masses of squarks and sleptons are governed (apart from the top squark) by their gauge couplings, in accord with eqn. (3.12). Flavor changing neutral currents are not a problem.
2. The masses of the gauginos are related in a well-defined way to those of squarks and sleptons, as can be seen by comparing equations (3.16) and (3.17). For example, when $F_S$ is small the ratio of squark to gluino masses is approximately $\frac{2}{\sqrt{3}}$.
3. The Higgs sector is necessarily more complicated than that of the MSSM, if one insists that all masses arise from dimensional transmutation. One expects at a minimum that there is an additional gauge singlet with weak scale mass.
4. There is new physics at a variety of scales. The fields $q, \bar{q}, \ell$ and $\bar{\ell}$, as well as the fields $P, N$ and $E$ lie at a comparable scale. Finally, the supersymmetry breaking fields of the 3−2 sector lie at energies $1−2$ orders of magnitude larger. So one expects new physics up to scales of order $10^4$ TeV or so.
5. The supersymmetry breaking scale is constrained from below by the need to have the R-axion heavier than 10 MeV, and from above by the cosmological requirement that the gravitino be lighter than 10 keV \[10\] and is predicted to be in the range \(10^5 - 10^7\) GeV (1 eV \(< m_{3/2} < 10\) keV). (This scale depends on the size of the messenger hypercharge coupling and the superpotential couplings.) The gravitino is the lightest supersymmetric partner, and the next to lightest supersymmetric partner is a neutralino (linear combination of neutral gauginos, higgsinos, and gauge singlet fermions) which should decay into a photon and a gravitino with a lifetime in the range \(10^{-13} - 10^{-5}\) sec. (Note that this decay rate is more rapid than would be expected for a process involving gravity because of the goldstino component of the gravitino, and is so uncertain because it is inversely proportional to the fourth power of the supersymmetry breaking scale.) This decay of the neutralino is a distinctive model independent signal for low energy supersymmetry breaking.

6. Some of the new particles predicted are potential dark matter candidates. The R axion is unstable once the R symmetry is broken; for instance it can decay into gravitino pairs. The gravitino could provide an interesting amount of warm dark matter if its mass is in the 10 keV range. The massless charged fermion of the supersymmetry breaking sector could be given a small mass through higher dimension operators such as

\[
\frac{O(1)}{m_P} \bar{UQLE}
\]  

and provide hot dark matter. The other particles of the supersymmetry breaking sector can all decay into these. The \(q\) and \(\ell\) particles could decay by mixing with ordinary quarks and leptons, or could be cold dark matter candidates. The other messenger particles are all unstable since they mix with the neutralinos and Higgs scalars by a small amount.

6. Conclusions

We have presented a renormalizable approach to spontaneous supersymmetry breaking, in which all mass scales arise via dimensional transmutation. Supersymmetric partner
masses are calculable in terms of a few new couplings, and at the weak scale the model resembles the MSSM, but with a constrained parameter space. The absence of observed FCNC and CP violation from the supersymmetry breaking sector is explained by having flavor universal gauge couplings transmit supersymmetry breaking to squarks, sleptons and gauginos. The supersymmetry breaking sector can be as simple as an additional $SU(3) \times SU(2) \times U(1)$ gauge theory with the particle content of one family, and with communication of supersymmetry breaking facilitated by a small number of additional fields including vector like quarks and leptons and one or more gauge singlets. The only important role played by nonrenormalizable supergravitational couplings is to cancel the cosmological constant (by fine tuning) and to give mass to an otherwise troublesome axion. The model is easily made consistent with all terrestrial, cosmological, and astrophysical constraints. The lightest superpartner is the gravitino, which may lead to a distinctive signal in future accelerators such as LEP II. As one would expect for a dynamical model, these theories can readily explain the hierarchy. For example, in the 3-2 model, with the assumption that all couplings are equal at the GUT scale, the supersymmetry breaking scale is in the desired $10^3$ TeV range.

Some readers may be concerned about our liberal use of discrete symmetries, and the associated problem of domain walls. We do not view this problem as serious. Our discussion does not require that these symmetries be exact; if they are broken by dimension five operators generated by Planck scale physics, or by operators generated at lower scales by gauge anomalies, these domain walls will quickly disappear.

In comparison with the conventional MSSM, it is a great advantage to have the supersymmetry breaking sector made explicit so that supersymmetry breaking parameters are calculable. The MSSM can arise from models in which supersymmetry is dynamically broken in a gravitationally coupled “hidden sector”. In fact, the 3-2 model which we have used as our prototypical example can be used as a hidden sector model. Hidden sector models with dynamical supersymmetry breaking have the virtue that they can explain the origin of the hierarchy. If, for example, we take the two family $SU(5)$ model as hidden sector, and assume that the $SU(5)$ coupling is equal to the unified coupling at $M_{GUT}$, we obtain roughly $4 \times 10^{10}$ GeV for the SU(5) scale, i.e. a quite reasonable intermediate
scale value. In addition, these models do not suffer from the conventional Polonyi problem since the hidden sector, by assumption, does not have flat directions. However, there are still potential difficulties with the hidden sector approach, such as the cosmological abundance of gravitinos and flavor changing neutral currents, which simply do not arise when supersymmetry breaking occurs in a renormalizable visible sector theory. Thus we feel that if nature turns out to be supersymmetric, then the possibility of low energy supersymmetry breaking should be taken seriously.

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6 Of course, in the context of string theory, where one expects there may be additional light moduli, there may still be serious cosmological difficulties.
Figure Captions

Fig. 1. Two loop diagrams contributing to scalar masses in various models. Dashed lines denote scalar fields; wavy lines are gauge fields. In (a) and (b), the scalar emits a gauge field which couples, in turn, to fields without a supersymmetric spectrum; in (c) the scalar couples to other scalars through the $D$ term; in (d), it couples to its fermionic partner and a gaugino. The labeling in the figure refers to the contributions to masses of “ordinary” squarks and sleptons. For the $P$, $N$ and $E$ fields it is the hidden sector fields which run in the loop.

Fig. 2. One loop diagram contributing to gaugino masses.
References

[1] M. Dine and A.E. Nelson, Phys. Rev. D47 (1993) 1277.
[2] A.E. Nelson and N. Seiberg, Nucl. Phys. B416 (1994) 46.
[3] J. Bagger, E. Poppitz and L. Randall, Johns Hopkins preprint JHU-TIPAC-940005 (1994).
[4] G. Raffelt, Phys. Rep. 198 (90) 1.
[5] G. ’t Hooft, Lecture given at Cargese Summer Inst., Cargese, France, Aug 26 - Sep 8, 1979. Published in Cargese Summer Inst. 1979.
[6] I. Affleck, M. Dine, and N. Seiberg, Nucl. Phys. B416 (1985) 557.
[7] I. Affleck, M. Dine and N. Seiberg, Phys. Lett. 137B (1984) 187.
[8] G.R. Farrar, Rutgers preprint RU-94-35, HEP-PH-9407401 (1994), and references therein.
[9] J. Bagnasco, SCIPP preprint to appear.
[10] see e.g. T. Moroi, H. Murayama, and M. Yamaguchi, Phys. Lett. 303B (1993) 289 and references therein.
[11] G.D. Coughlan, W. Fischler, K.W. Kolb, S. Raby and G.G. Ross, Phys. Lett. 131B (1093) 59; M. Dine, D. Nemeschansky and W. Fischler, Phys. Lett. 136B (1984) 169.
[12] T. Banks, D.B. Kaplan, A.E. Nelson,Phys. Rev. D49 (1994) 779, B. de Carlos, A. Casas, F. Quevedo and E. Roulet, Phys. Lett. 318B (1993) 447.
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