Fuzzy Adaptive PID Control for Missile Guidance

Yuhao Xie 1,a and Zihan Su 2

1Virginia Polytechnic Institute and State University, Virginia, United States
2Harbin University of Science and Technology, Heilongjiang Province, China
a yuhao14@vt.edu

Abstract. A fuzzy guidance law in the command control system is proposed for the missile. The performance of the designed guidance low is compared with the widely used guidance and navigation method, known as the PID control. The simulation results illustrate the effectiveness of the fuzzy adaptive PID control method on missile guidance.

1. Introduction

The design of the missile guidance laws against the target is challenging for present day. The guidance law is the algorithm by which the desired geometrical rule is implemented. It is often based on the position and velocity of the target relative to the guided object. The missile guidance system is a closed loop. There is at least one input, that is, the target acceleration. And, there is one major output: the missile – target relative separation called the miss-distance. [1] When designing homing missile guidance, the efficiency of missile approaching target is required.

The fundamental measure of guidance system performance is generally considered to be the rms miss distance. Fuzzy reasoning are rule-based systems that use fuzzy linguistic variables to model a human’s rule-of-thumb approach to problem solving. Fuzzy reasoning includes rules to direct the decision process and membership functions to convert linguistic variables like high and low into precise numeric values a computer needs. (Paul G. Gonsalves and Alper K. Caglayan)

The seminal work of [2] provided for the first real application of fuzzy control. For now, there are numerous applications of fuzzy logic and fuzzy control. (Mamdani, E.H.. Application of Fuzzy Algorithms for Control of Simple Dynamic Plant) In [3], the effect of noise on missile guidance system is investigated. In order to improve the clearance of the signal, the digital fading memory filter is introduced. It is considered to be more advanced than Kalman filters for such application. (K. Raj and I. Ganesh, “Estimation of Line-of-Sight Rate in a Homing Missile Guidance Loop using Optimal Filters,” IEEE ICCSP, 2015.) In [4], research on using the LOS angular rate as well as the distance between missile and target in order to be used with missile guidance system is proposed by formulating nonlinear equations for PNG. In that study, the target acceleration is unknown and estimated by a specialized observer. The stability of the control system has been proved by measuring the miss distance. (D. Viswanath, S. Krishnaswamy and D. Deb, “IEEE INDICON, 2015.)

This paper differs from prior work in following main aspects. The basic proportional guidance system is built. The proportional guidance system through its advantages and disadvantages are discussed. In order to cancel the noise and make the system more efficient, the PID control is introduced into several experiments. It is found that PID control system is more accurate than proportional guidance system. At last, the fuzzy PID control is employed to guide the missile. Because this method not only has accurate missile guidance but also has high efficiency during real-time control.
The rest of the paper is organized as follows: Section II mainly discuss the missile target intercept geometry. Section III introduce the benefit and disadvantages of proportional guidance system. Section IV shows how the fuzzy PID control works for missile guidance system. Section V is the simulation result which are both proportional guidance system and fuzzy PID system. In that section, the comparison is made to illustrate the reason why we choose fuzzy PID control system.

2. Nomenclature

- \( R \): the distance between missile and target
- \( \sigma_t \): angle between velocity vector of target and base line.
- \( \sigma \): angle between velocity vector of missile and base line.
- \( q_t \): angle between the target position vector and the base line.
- \( q_m \): angle between the missile position vector and the base line.
- \( \eta \): angle between missile and target line.
- \( \eta_t \): angle between target velocity vector and target line
- \( K \): scale factor

3. Missile-Target Intercept Geometry

As an example of the application of this method of nonlinear guidance systems, to further optimize the missile flight time. For this study, we developed and used with a traditional proportional navigation algorithm. The following assumptions are set for the Missile-target engagement geometry:[5]

- The missile’s out of plane motion and air density changes are ignored.
- The missile is a symmetric rigid body.
- The earth is the inertial frame of reference, i.e. the atmosphere is fixed w.r.t. the earth and the coordinate system Xe Ye Ze fixed to the earth.
- Atmospheric winds are assumed zero.
- The missile drag polar has a parabolic shape.

A relative motion equation is created by the relative motion between missile and guidance station. We assume guidance station at point C and missile at point M. The relationship is shown as figure 1:

![Figure 1. Missile-Target intercept geometry.](image)

From Fig1, the equation of relative motion can be established. By depositing missile velocity vector (\( \mathbf{v} \)) and target velocity vector (\( \mathbf{V}_t \)) along normal line, there is:

\[
\frac{dr}{dt} = V_t \cos \eta_t - V \cos \eta
\] (1)
The normal term of the target line $V \sin \eta$ makes the target line rotate around the target counter clockwise wisely. That makes target line $q$ increase. The component $V \sin \eta_r$ makes the target line rotate around missile clockwisely and decreases the target angle $q$. Thus, there is

$$\frac{dq}{dt} = \frac{1}{r}(V \sin \eta - V \sin \eta_r)$$

(2)

Considering the relationship between each component, the following equations are deduced:

$$\begin{cases}
\frac{dr}{dt} = V_t \cos \eta_r - V \cos \eta \\
\frac{dq}{dt} = \frac{1}{r}(V \sin \eta - V \sin \eta_r) \\
q = \sigma + \eta \\
q_r = \sigma_r + \eta_r \\
\epsilon_r = 0
\end{cases}$$

(3)

4. **Design of Proportional Guidance**

Proportional guidance has many advantages, such like high sensitivity, small hysteresis, rapid responsibility over time. Since the rapid and efficient control and response is needed for missile guidance, proportional guidance is used in large number of situations. The control law is:

$$\epsilon : \frac{d\sigma}{dt} = K \frac{dq}{dt}$$

(4)

However, the method of proportional guidance behaves less stable as the scale factor $K$ increases. Proportional guidance is not able to decrease the steady state error which is a disadvantage.

To improve the stability of missile guidance system, we focus on decrease the decreasing velocity of angle between missile and target to realize proportional movement between missile and target. By focusing on such aspect, the missile velocity can make missile fast approaching target.

5. **Design of the Fuzzy PID Guidance**

The differential control and integral control are taken into consideration. Integral control aims at decreasing velocity of rotation angle between missile and target to cancel the steady-state error. Differential control aims at restraining the overshoot in guidance system. By taking PID control system under consideration, we expect that PID control system can resolve some control problem that proportional guidance system has.

The PID system we designed follows the following controlling rules:

$$\psi' = K_p q + K_d q' + K_i q^*$$

(5)

In the actual missile guidance environment, missile can be easily distracted by various environmental factors. The way that setting variables manually for PID control system is not an efficient way for real-time control. Thus, we introduce the fuzzy PID control system in this case.

The fuzzy PID guidance system structure is shown below:
The guideline of fuzzy PID guidance system is finding the relationships between three parameters \( (k_p, k_i, k_d) \) and two variables (target line rotational angular velocity \( dq / dt \) and the distance between missile and target \( r \)). During the guidance process, the sensor continuously detects the changes of the parameters \( dq / dt \) and \( r \) and passes them to the fuzzy controller through the transmitter in real time. Then, in terms of the fuzzy control principle, three parameters of PID are modified by the controller.

Fuzzy controller changes those three variables of PID system. Three PID control parameters are adjusted respectively based on the following principles:

• \( \Delta k_p \): When the variable \( r \) is much larger than zero, \( \Delta k_p \) is set to be greater than zero. That corresponds to increasing the proportional gain. There are three circumstances where \( r \) is closed to zero: When \( dq / dt \) is negative, it indicates that the system has overshoot. At this point, \( \Delta k_p \) is set to be less than zero, that is, the proportional action is reduced. When \( dq / dt \) is equal to zero, this is the expected situation. Hence, \( \Delta k_p \) is equal to zero to maintain the gain.

• \( \Delta k_i \): When both \( r \) and \( dq / dt \) are near zero, \( \Delta k_i \) is adjusted to a positive value to eliminate the system steady-state error. Otherwise, the value of it is 0.

• \( \Delta k_d \): When \( r \) and \( q \) are both less than or equal to zero, the system appears, or is about to overshoot. \( \Delta k_d \) is set to be greater than zero to suppress deviation. When \( r \) is much greater than zero, \( \Delta k_d \) is set to be less than zero to make the system respond quickly. If \( r \) and \( dq / dt \) are expected to be zero at the same time, the differential gain variation is set to zero.

6. Simulation Results

Based on Euler discrete method, the guidance system is simulated by computer. For PID control rules, we adopt numerical approximation method, which means that we can set an enough short sampling period and use the summing method instead of the integral operation. The differential algorithm can replace the differential expression of the control law. Thus, the simulated PID equation is discretized into the form of difference, which is convenient for computer simulation.

Ultimately, we compare the three control methods and give the results to the above examples as follow. Where, we select the proportional gain parameter of 40 for proportional guidance, and the initial value of parameters in PID control and fuzzy adaptive control is set as: \( k_p = 35 \), \( k_i = 10 \), \( k_d = 0.01 \).

The following figure is obtained by simulation.

The fuzzy sets of one of the inputs of the bivariate fuzzy adaptive controller \( r \) is \( \{PS, PM, PB\} \), which represents that the parameter is slightly greater than zero, greater than zero and much greater.
than zero, whereas, the other input \( \frac{dq}{dt} \) also has three fuzzy sets: \( \{N, Z, P\} \). Those mean positive, zero, negative. The theoretical domain of \( r \) is \([0, 3000]\), and that of \( \frac{dq}{dt} \) is \([-0.3, 0.3]\). Meanwhile, The fuzzy sets of the control quantities \( (k_p, k_i, k_d) \) are all \( \{N, Z, P\} \), and their theoretical domain are defined as \([-5, 5]\), \([-1, 1]\), \([-0.01, 0.01]\) respectively. Triangle and S-shaped membership functions were selected, and the fuzzy reasoning method was Mamdani’s algorithm. The membership functions of both the inputs and outputs of the system are shown in Figure 3 and Figure 4.

**Figure 3.** The membership function of the inputs.

**Figure 4.** The membership function of the output of the fuzzy controller.

The simulation results of the computer operation are shown in Figure 5 below. We can see that all the three control methods ultimately enable the missile to hit the target. All the process cost approximately 24.28s. Fig 6 shows the variation laws of three control output quantities in the fuzzy system.
In Figure 6, the ratio coefficient shows a downward trend at the beginning. However, ten seconds later, it stabilized at about 35. By contrast, both the integral and differential action parameters were kept very low at first. After about 13 seconds, they increased rapidly to suppress system overshoot.

In Figure 7 and Figure 8, we reflect and compare respectively the changes of two parameters that the three control methods focus on. It can be seen that both PID and fuzzy adaptive algorithms have better control effects than proportional guidance. In addition, the fuzzy adaptive algorithm contributes to the PID tuning process, eventually reducing the system overshoot and the distance between the missile and the target to zero more rapidly.
7. Conclusion
A fuzzy guidance law for the missile in the command control systems is presented in this paper. The new guidance law is developed from proportional navigation guidance and based on fuzzy logic theory. Simulation results show that the proposed guidance law gives smaller miss distance and faster intercept time than the conventional proportional navigation guidance law. From all results obtained, fuzzy guidance law is suggested for the future studies in the missile guidance area.

8. References
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