Operating modes and electrothermal model of an induction crucible furnace with a conductive crucible

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Abstract. The mathematical electrothermal model based on a finite element method in the universal ELCUT package is developed for calculation of electric and thermal values of an induction crucible furnace. Electric characteristics depending on the frequency, temperature and current of an inductor are studied. The nonstationary thermal task is solved. Two reference duties of the furnace are allocated and investigated: the cold mode when the crucible magnetic and temperature is lower Curie points and the hot mode when the melted molten metal, a crucible unmagnetized, temperature of a crucible is higher than a Curie point. The experimental stand consisting of the high-pitched adjustable transistor power supply, an inductive crucible furnace with a steel conductive crucible, a microprocessor control system and the system of acquisition is developed. The experiment on the laboratory furnace with a steel crucible with loading from aluminum alloy is made. Results of an experiment are comparable to results of mathematical model operation in an ELCUT package, the adequacy of results of a theoretical research is confirmed. Dependenes of the fissile resistance and inductance on frequency, temperature and current in the form of surfaces in the environment of Simulink/Matlab are received. Results of the solution of a thermal task showed that the melted metal heats up evenly, temperatures of a crucible and metal are equal. The conducted theoretical and pilot researches form a basis for creation of structural electrothermal model of an induction crucible furnace of the increased frequency in the environment of Simulink/Matlab.

1. Introduction

Induction crucible furnaces (ICF) with conducting Crucible (PT) are used for melting of non-ferrous metals and alloys, to which special requirements on cleanliness, homogeneity and accuracy of chemical composition are presented. Depending on the physical properties of the crucible material there are also ICF refractory and conducting crucible [1 – 14].

The ICF with PT is used for melting of magnesium and copper alloys, gold, uranium, as well as conductive materials. In the same way ICF with PT (steel, graphite or graphite-chamotte) in an electromagnetic field heats up both a crucible and loading if it is electrically wired [15 – 19].

In the work [18] the method of calculating electrical characteristics ICF with PT was developed, electrical characteristics ICF with non-magnetic PT were calculated and investigated. The calculation ICF of the ferromagnetic crucible presents a much more complex task. ICF with ferromagnetic (steel) crucible is used for melting of magnesium and its alloys as it does not enter into chemical compounds with them. Magnesium and its alloys are under a layer of flux due to the fact that the naked liquid metal instantly ignites.

In Fig. 1 The sketch of the "inductor-load" system (hereinafter and – h) of the laboratory furnace for magnesium melting with the capacity of 2.5 kW with dimensions is shown. Loading of ICF and PT form a steel crucible and magnesium.

The main difficulty of calculating the furnace is that the electrical characteristics of the material loading vary depending on the temperature of T and the magnetic field strength H [9 – 14]. It is possible to
distinguish two characteristic modes, cold mode, when the crucible is magnetic and the temperature is below the Curie point and the hot mode when the magnesium is liquid, the crucible is non-magnetic, the crucible temperature is above the Curie point. So \( \mu = \xi_1(H, t) \), \( \rho = \xi_2(t) \), where \( \mu \)-relative magnetic permeability, \( \rho \)-specific electrical resistance, \( \xi_1 \), \( \xi_2 \)-functional dependencies. At loading temperature above the melting point and \( \Delta T / \delta_2 \leq 1.3 \), where \( \delta_2 \)-the depth of penetration of the electromagnetic wave in the wall of the crucible thick \( \Delta T \), loading can be considered as a layered body. Before the melting point the load is a single-layered body, because the magnesium in the crucible is in the form of a lump charge, which has high electrical resistance. In this regard, it is quite difficult to solve the problem analytically, so it is necessary to use numerical methods.

Currently in the field of electrical power and automation are relevant issues, as all this leads to a reduction in production costs, improving the quality of products. The main problems in the field of process automation in the ICF with PT are: control of the melt temperature of the metal in the furnace, maintenance of the set capacity of the furnace during smelting, limitation of the output current of the power supply, automatic coordination of the parameters of the inductor and frequency converter with high speed and accuracy without the use of winding elements and complication of the design of the inductor. To solve these problems, it is necessary to use modern methods of design, which in turn require the development of appropriate thermal models ICF.

2. Method of calculation of electrical and thermal characteristics, etc. with ferromagnetic PT.

The thermal task is a related task. The results of solving the problem of the magnetic field of variable currents serve as the initial data for solving the problem of non-stationary heat transfer in ELCUT (Fig. 2).

In the case of calculating hot mode, when the crucible non-magnetic outer cycle is not. The algorithm in the case of cold mode calculation contains the following nonlinearity: Specific electrical resistance of \( \rho \), relative magnetic permeability \( \mu \), dependence of induction \( B \) from magnetic field \( H \) in electrical task, thermal conductivity \( \lambda \), specific heat capacity \( c \) – in the solution of thermal problem. Accordingly, the algorithm in Figure 2. contains twofold iteration cycle. When calculating the hot mode, when the metal is liquid, reached the point of Curie, \( \mu = 1 \) Inner loop is absent, only the outer loop remains.

To solve this problem, a mathematical model was developed using finite element method in the ELCUT package. In Fig. 3 the calculation area is shown, it is divided by the finite elements. The model allows, changing properties of blocks, to carry out calculations for cases of a steel crucible without magnesium and a crucible with metal. On the axis of symmetry (Gr. 1 e in Fig. 3) the boundary condition of Neumann is made \( dH/dr = 0 \) (absence of tangential component of magnetic field tension), on external borders (Gr. 2, 3, 4) is specified – zero boundary condition Dirichlet \( a = 0 \) (it is supposed to be equal to zero value magnetic potential, i.e. the field is localized within the calculation area).

Assumptions and limitations of electrical task:

- When calculating the cold mode, it was not taken into account that the loading is lump, i.e. it is regarded that the crucible is not completely transparent
- The system «inductor – loading» is axisymmetric and cylindrical.

Assumptions and limitations of the thermal problem:

- The calculation area is limited by three radii of the system and «inductor – loading» on the r-axis and two radii of the system and «inductor – loading» on the z-axis;
In the solution of the thermal problem the radiation inside the furnace was not taken into account.

The analysis of the magnetic field of variable currents consists in calculation of an electric and magnetic field, activated by the applied variables (sinusoidally changing in time) currents or an external variable field.

Changing the field in time is supposed to be sinusoidal. All field components and electrical currents are changed as:

\[ z = z_0 \cdot \cos(\omega t + \phi_z), \]

where \( z_0 \) – amplitude (maximum) value \( z \), \( \phi_z \) – phase angle, \( \omega \) – angular frequency.

The full current in the conductor can be considered as the sum of the exterior current caused by the externally applied voltage, and the eddy current induced by the variable magnetic field

\[ j = j_{ext} + j_{eddy}. \]

The equation for the axisymmetric case will be written as

\[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial A}{\partial r} \right) \right) + \frac{\partial}{\partial z} \left( \frac{1}{\mu_z} \frac{\partial A}{\partial z} \right) - i \omega \gamma A = -j_{eddy}. \]

To solve a non-linear problem, exploring a cold melting mode, the ELCUT uses the Newton-Raphson method, also known as the tangent method. This is an iterative numerical method of finding the root (zero) of a given function. The search for a solution is done by constructing consistent approximations.
and based on the principles of simple iteration. The method has quadratic convergence. The solution of linear problem on each iteration of this method is carried out using the iterative method of conjugate gradients with the preconditioning of the matrix by the method of decomposition of the region. Acceleration of the decision-making process is achieved by coincidence with the necessary accuracy of the linear problem solution with a preliminary assessment of the accuracy that can be achieved in this iteration of the Newton-Raphson method [15 – 22].

The physical parameters of the task, described by functional dependencies, are defined in the form of a table, the columns of which are linked by the implications of two physical values, induction \( B \) and the intensity of the magnetic field \( H \). In Fig. 4 shows the graph of dependence \( B(H) \) for steel 20, by which a table is formed in the ELCUT package for further use in the calculation.

The thermal conductivity equation in the following form is used to solve the non-stationary heat problem [13]:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( \lambda(T) r \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left( \lambda(T) \frac{\partial T}{\partial z} \right) = -q(T) - c(T) \rho \frac{\partial T}{\partial t},
\]

where \( T \) – temperature, \( t \) – time, \( \lambda(T) \) – thermal conductivity as a function of temperature represented by a cubic spline, \( q \) – Specific power of heat emission, given by a cubic spline function of temperature, \( c(T) \) – specific heating capacity, specified Cubic spline temperature function, \( \rho \) - density.

On the external borders of the calculation area (Gr. 2 – 4 T, see fig. 3) the known value of the temperature is specified \( T_0 \), the boundary condition of the first kind is set.

On the plane of symmetry, the boundary condition of the second kind is specified (Gr. 1 T, see fig. 3):

\[
F_n = -q_s,
\]

where \( F_n \) is a normal component of the heat flux density vector, \( q_s \) is the heat flow across the border.

On external borders of a sole, an arch and a lateral wall (thermal insulation of a crucible) the boundary condition of convection which is called the boundary condition of the third kind (Gr. 5 T, see fig. 3) is specified. It describes the convective heat transfer and is defined in the following way:

\[
F_n = \alpha(T - T_0),
\]

where \( \alpha \) is the coefficient of heat return, \( T_0 \) is the ambient temperature.

In fig. 5 the calculated temperature distributions of the furnace at different stages of smelting are presented: At the beginning, when the magnesium is solid, in the middle and at the end of the melting, when the magnesium is completely melted.
3. Method of calculation of electrical and thermal characteristics, etc. with ferromagnetic PT.

Using the developed calculation method as an example in Fig. 6 shows the dependence of the active resistance $R$ system and «inductor – loading» of $t$ and inductance $L$ system and «inductor – loading» of $t$ at different frequencies $f$, obtained in the package ELCUT at the current inductor $I_t = 50$ A and the average value $H$ on the outer surface of the crucible $H = 7.7$ kA/M.

During the theoretical study it was established that after reaching the Curie point, when the crucible loses its magnetic properties, and magnesium becomes molten, there is a significant decrease in the power of heat dissipation in the crucible at the same current and frequency due to the sharp decrease of $R$. This is explained that after the Curie point the depth of penetration into the conductive crucible increases significantly, and the load becomes double. For example, the depth of penetration of the electromagnetic wave (ECAs) into the crucible wall thickness of 3 mm on the current frequency of the inductor 22 kHz is about 3.2 mm and therefore it leads to the fact that there is a reverse eca, reflected from the boundary of the Environment section with different values of $\rho$ and $\mu$, and recedes $R$ of the system «inductor – loading» [18].

The obtained dependences $R$ and $L$ are used in the construction of the thermal model of induction system of elevated frequency.

In fig. 7 the dependences of $R$ and $L$ on $f$ and $T$ for current $I = 50$ A are presented in the form of surfaces constructed with the help of the Lookup Table n-D in the Simulink/Matlab environment [22].
4. Experimental studies.

To check the adequacy of the obtained mathematical model, an experimental verification was carried out at the laboratory - industrial stand on the basis of ICF with PT available at the department PSIEE (fig. 8). At carrying out of the experimental research the steel crucible from structural steel and aluminium billets simulating a lump furnace in the form of rods (fig. 9 a) have been prepared. To carry out the smelting process (fig. 9 b) of magnesium, it was decided to replace metal similar to it for all the physical and thermal properties, and aluminum was chosen, as magnesium is oxidized and ignited in the open air. The crucible geometry and load are completely identical to the geometric model in mathematical modelling. In table. 1 the results of electrical calculations and experiment are presented.

Figure 7. Schedule of active resistance of $R$ (a) and inductance $L$ (b) from temperature $T$ and frequency $f$ at current $I = 50 \ A$

Figure 8. Functional diagram of the laboratory stand.

Figure 9. Prepared materials for Experiment (a) and smelting process (b)
Table. 1. The main results of calculation of ICF with CC without metal and with copper

| Parameter                      | Experiment for Al | Calculation in ELCUT for Al | Calculation in ELCUT for Mg |
|--------------------------------|-------------------|----------------------------|----------------------------|
| Temperature $T$, °C            | 27.3 400 700      | 27 400 700                  | 27 400 700                  |
| The current of the inductor $I_i$, A | 73.9 71.0 68.8 | 73.9 71.0 68.8 | 73.9 71.0 68.8 |
| Voltage inductor $U_i$, B      | 22.5 25.2 25.4    | 26.1 38.4 42.0              | 26.1 38.5 42.1              |
| Frequency $f$, kHz             | 21.34 19.25 18.73 | 21.34 19.25 18.73           | 21.34 19.25 18.73           |
| Power in the loading $P$, watt  | 581 902 857       | 542 870 764                 | 542 871 765                 |
| $\cos \varphi$                 | 0.27 0.32 0.28    | 0.28 0.34 0.30              | 0.28 0.33 0.29              |

The results of the experiment coincide with the calculation results, the maximum deviation compounds not more than 10%. The results of the calculation in ELCUT for Al differ from the calculation for Mg by no more than 0.25%.

Comparing experimental and theoretical curves of heating it is established, that in experiment the growth of temperature to 650 °C occurs faster, has more exponential character, further speed and then the growth of temperature decreases, in consequence of that the magnetic properties of the crucible and loading are changing, the Curie point is approaching. In the course of the theoretical study it is established that the temperature grows almost linearly. The maximum deviation of curves compounds not more than 30%.

5. Conclusion

It is established, that at constant frequency $f$ active resistance $R$ increases with a temperature rise to a point of Curie. However, at a frequency of $f = 10$ kHz, at 200 °C the reduction of active resistance begins. The frequency affects the growth rate of the active resistance. When passing the Curie point there is a sharp decrease in resistance, for example, for $f = 22$ kHz at 10 times.

It is established, that at the change of frequency $f$ from 10 kHz to 100 kHz at the same temperature active resistance grows. For example, for $f = 200$ °C at 0.14 ohm (in 2.5 times).

It is established, that at a constant frequency $f$, inductance $L$ increases with a temperature rise, to a point of Curie. In this case, the frequency effects on the rate growth of inductance (the lower is the frequency, the faster is the growth). Passing the Curie point there is a sharp decrease in inductance. For example, for the frequency of 22 kHz in 2.2 times.

As a result of solution of thermal problem in the package ELCUT it is established, that on a section uniformity of heating fluidize metal is good. The temperature of the fluidize metal and crucible is about 760 °C. Comparing theoretical and experimental heating curves, the maximum difference compounds not more than 30%. Thermal losses are about 350 watts.

The results of the experiment coincide with the calculation results with an inaccuracy not exceeding 10%. The results of the calculation in ELCUT for Al differ from the calculation for Mg by no more than 0.25%.

The process of magnesium melting is fully reproduced on the physical model, and the method of experiment is developed.

The work was supported by the Ministry of Education and Science of the Russian Federation (an initial scientific project 8.9608.2017 / BCh).
References
[1] Weinberg A M 1967 Induction flowing furnaces (Moscow: Energoatomizdat) p. 410
[2] Farbman S A and Kolobnev I F 1968 Induction furnaces for melting of metals and alloys. (Moscow: Metallurgy) p. 496
[3] Slukhotsky A E 1981 Installations of induction heating (Leningrad: Energoizdat) p. 328
[4] Sassa K, Kuwabara M, Yasuda T, Asai S Experimental measurements and theoretical analysis of induction heating by use of a conductive crucible
[5] Patidar B, Hussain M, Jha S, Sharma A, Tiwari A. 2017 Materials Science and Technology The characteristics of TiAl-based alloys melted in graphite crucibles (United Kingdom) p 1-9
[6] Patidar B, Hussain M, Jha S, Sharma A, Tiwari A. 2017 J 27 IET Electric Power Applications Analytical, numerical and experimental analysis of induction heating of graphite crucible for melting of non-magnetic materials p 342-351
[7] Labridis D, Dokopoulos P 1989 Calculation of Eddy Current Losses in Nonlinear Ferromagnetic Materials IEEE Trans. Magn., Vol. 25 p 2665-2669
[8] Clain S J, Rappaz M, Swierkosh and R Touzani 1993 Numerical Modelling of Induction Heating for Two-dimensional Geometries, ” Math. Models Methods Appl. Sci., Vol.3, no. 6, p 805-822
[9] Marchand C, Foggia A J 1983 vol. 19 IEEE Trans. Magn 2D finite element program for magnetic induction heating. p 2647-2649
[10] Massé P, Morel B, and Breville T, J 1989 Vol. 21 IEEE Trans. Magn A Finite Element Prediction Correction Scheme for Magneto-thermal Coupled Problem during Curie Transition p 181-183
[11] Meyer, J.-L., N. El-Kaddah, and J. Szekely, J 1987 Vol. 23 IEEE Trans. Magn A New Method for Computing Electromagnetic Force Fields in Induction Furnaces p 1806-1810
[12] Miyoshi T, Sumiya M, and Omori H, J 1987 Vol. 23 IEEE Trans. Magn Analysis of An Induction Heating System by the Finite Element Method Combined with A Boundary Integral Equation p 1827-1832
[13] Skoczkowski T P, Kalus M F J 1989 Vol. 25 IEEE Trans. Magn The Mathematical Model of Induction Heating of Ferromagnetic Pipesgn p 2745-2750, 1989.
[14] Ter Maten EJW, Melissen JBM J 1992 Vol. 28 IEEE Trans. Magn Simulation of Inductive Heating p 1287-1290
[15] Kuvaldin A B, Fedin M A, Kuleshov A O, Zhmurko, I Y J Materials Science Forum 906 2017 Development of relay control systems of power and temperature mode of induction crucible furnaces with use of physical modeling (Moscow: Energoatomizdat) p 8 – 15
[16] Fedin M A, Kuvaldin A B, Kuleshov A O, Generalov I M 2016 Experimental research of physical model of the induction crucible furnace and the development of control system 11th International Forum on Strategic Technology (IFOST): Proceedings of IFOST-2016, Part 2. p 68 – 72.
[17] Fedin M A, Kuvaldin A B, Kuleshov A O, Zhmurko I Y, Akhmetyanov S V 2018 Calculation and research of electrical characteristics of induction crucible furnaces with unmagnetized conductive crucible IOP Conference Series: Earth and Environmental Science Volume 115, Issue 1, Article number 012050.
[18] Fedin M.A., Kuvaldin A.B., Kuleshov A.O. 2017 J. 3(2017) Vestnik MPEI The choice of a calculation procedure and a research of electric characteristics of induction crucible furnaces with a conductive crucible p 77-86
[19] Kuvaldin A B 1988 Induction heating of ferromagnetic steel (Moscow: Energoatomizdat) p 200
[20] Volkov E A 2003 Numerical methods (Moscow: Fizmatlit) p 400
[21] Website of the ELCUT company [Ofits. website]. URL: https://elcut.ru/ (date of the address 31.03.2018)
[22] Website of the MathWorks company [Ofits. website]. URL: https://mathworks.com/ (date of the address 1.04.2018)