Dark matter, MOND or non-local gravity?

F. Darabi
Department of Physics, Azarbaijan University of Tarbiat Moallem, Tabriz 53741-161, Iran

f.darabi@azaruniv.edu

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ABSTRACT

We propose a Machian model of gravitational interaction at galactic scales to explain the rotation curves of these large structures without the need for dark matter or MOND.

Subject headings: Rotation curves; dark matter; MOND; Mach.
1. Introduction

It is well known that classical Newtonian dynamics fails on galactic scales. There is astronomical and cosmological evidence for a discrepancy between the dynamically measured mass-to-light ratio of any system and the minimum mass-to-light ratios that are compatible with our understanding of stars, of galaxies, of groups and clusters of galaxies, and of superclusters. It turns out that on large scales most astronomical systems have much larger mass-to-light ratios than the central parts. Observations on the rotation curves have turn out that galaxies are not rotating in the same manner as the Solar System. If the orbits of the stars are governed solely by gravitational force, it was expected that stars at the outer edge of the disc would have a much lower orbital velocity than those near the middle. In fact, by the Virial theorem the total kinetic energy should be half the total gravitational binding energy of the galaxies. Experimentally, however, the total kinetic energy is found to be much greater than predicted by the Virial theorem. Galactic rotation curves, which illustrate the velocity of rotation versus the distance from the galactic center, cannot be explained by only the visible matter. This suggests that either a large portion of the mass of galaxies was contained in the relatively dark galactic halo or Newtonian dynamics does not apply universally.

The dark matter proposal is mostly referred to Zwicky (1957) who gave the first empirical evidence for the existence of the unknown type of matter that takes part in the galactic scale only by its gravitational action. He found that the motion of the galaxies of the clusters induced by the gravitational field of the cluster can only be explained by the assumption of dark matter in addition to the matter of the sum of the observed galaxies. Later, It was demonstrated that dark matter is not only an exotic property of clusters but can also be found in single galaxies to explain their flat rotation curves.

The second proposal results in the modified Newtonian dynamics (MOND), proposed
by Milgrom, based on a modification of Newton’s second law of motion (Milgrom, 1983). This well known law states that an object of mass $m$ subject to a force $F$ undergoes an acceleration $a$ by the simple equation $F = ma$. However, it has never been verified for extremely small accelerations which are happening at the scale of galaxies. The modification proposed by Milgrom was the following

$$F = m\mu\left(\frac{a}{a_0}\right)a,$$

$$\mu(x) = \begin{cases} 1 & \text{if } x \gg 1 \\ x & \text{if } |x| \ll 1, \end{cases}$$

where $a_0 = 1.2 \times 10^{-10} m s^{-2}$ is a proposed new constant. The acceleration $a$ is usually much greater than $a_0$ for all physical effects in everyday life, therefore $\mu(a/a_0) = 1$ and $F = ma$ as usual. However, at the galactic scale where $a \sim a_0$ we have the modified dynamics $F = m\left(\frac{a^2}{a_0}\right)$ leading to a constant velocity of stars on a circular orbit far from the center of galaxies.

Another interesting model in this direction has been recently proposed by Sanders. In this model, it is assumed that gravitational attraction force becomes more like $1/r$ beyond some galactic scale (Sanders, 2003). A test particle at a distance $r$ from a large mass $M$ is subject to the acceleration

$$a = \frac{GM}{r^2}g(r/r_0),$$

where $G$ is the Newtonian constant, $r_0$ is of the order of the sizes of galaxies and $g(r/r_0)$ is a function with the asymptotic behavior

$$g(r/r_0) = \begin{cases} 1 & \text{if } r \gg r_0 \\ r/r_0 & \text{if } r \ll r_0. \end{cases}$$

Dark matter as the manifestation of Mach principle has also been considered as one of the solutions for the dark matter problem. According to Mach principle the distant mass
distribution of the universe has been considered as being responsible for generating the local inertial properties of the close material bodies. Borzeszkowski and Treder have shown that the dark matter problem may be solved by a theory of Einstein-Mayer type (Borzeszkowski and Treder, 1998). The field equations of this gravitational theory contain hidden matter terms, where the existence of hidden matter is inferred solely from its gravitational effects. In the nonrelativistic mechanical approximation, the field equations provide an inertia-free mechanics where the inertial mass of a body is induced by the gravitational action of the cosmic masses. From the Newtonian point of view, this mechanics shows that the effective gravitational mass of astrophysical objects depends on $r$ such that one expects the existence of new type of matter, the so called dark matter.

2. The model

We introduce a new interpretation of Mach principle by which a particle with the mass $m$ and at the radial position $r$ interacts gravitationally with the matter $M(r)$ encompassed by the region of radius $r$ as follows

$$V = -\frac{GmM(r)}{R},$$

(1)

where $R$ is the radius of the galactic disc. This is a non-local interaction of the particle with the mass distribution inside the galactic disc.

The motivation for taking this type of gravitational potential at galactic scale is the main observation that in the rotation curve of galaxies the linear curve turns into a flat one in a rather sudden way. So, this behavior may be explained if we interpret it as a result of a rather sudden change in the mass distribution of that structure. In fact, this is really the case because the turning region of the curve from linear to flat case corresponds to the region in which the central massive disc of the structure with the typical radius $R$ turns
into the outer void region without mass.

A particle of gravitational mass \( m \) located at a distance \( r < R \) from the center of the galaxy will acquire the total energy

\[
E = \frac{1}{2}mv^2 - \frac{GmM(r)}{R},
\]

where \( v \) is the circular velocity around the center of galaxy. If we roughly assume the mass \( M \) of the galaxy is uniformly distributed over a disc of radius \( R \) then \( M(r) = \sigma \pi r^2 \), where \( \sigma \) is the constant surface mass density. Newton’s law is then written as

\[
\frac{mv^2}{r} = \frac{Gm}{R}2\sigma \pi r,
\]

which results in a linear behavior

\[
v = r \sqrt{\frac{G}{R}2\sigma \pi},
\]

and zero total energy, \( E = 0 \).

At distances \( r > R \) where \( M = \text{Const} \), the potential energy as well as total energy become constant

\[
E = \frac{1}{2}mv^2 - \frac{GmM}{R},
\]

which results in a constant velocity

\[
v = \sqrt{\frac{2}{m}(E + \frac{GmM}{R})}.
\]
3. Concluding remarks

In conclusion, we obtained a linear rotation curve for $r < R$ which turns into a flat curve for $r > R$, $R$ being the radius of the galaxy disc. It is interesting to note that since there is no sharp demarcation between the massive disk and the outer void space, there is no sharp turning point in the rotation curve extending from $r < R$ to $r > R$. In fact, the curve turns in a rather gentle way since the surface mass density $\sigma$ does not change suddenly, rather it decays smoothly over the turning region in passing from massive part toward the outer void region. Therefore, one may define an effective characteristic radius of the disc, namely $R_{\text{eff}}$, for each galaxy so that its substitution in Eqs. (4), (6) would lead to rotation curves in good agreement with observations.

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REFERENCES

Zwicky, F., 1957 Morphological Astronomy

Milgrom, M., 1983 AJ, 270, 365

Sanders, R., 2003 Mod. Phys. Lett., 18, 1861

Borzeszkowski, H. H. v., & Treder, H. J., 1998 Foundations of Physics., 28, 273