INTRODUCTION

Electrical discharges that occur during the breakdown of all gaseous, liquid and solid dielectrics are recognized as having a strong tendency to develop into complex branched structures.

With the confirmation of the fractal nature of electrical discharges, stochastic models were introduced to describe the formation of branched discharge patterns. By combining the potential theory with probabilistic selection, these models were able of produce complex branched structures that resemble actual electrical discharges. The work presented in this project utilizes the formation of lightning discharges in two dimensional (2D) domain.

METHODOLOGY

Lightning discharge simulations presented in this project are based on a generalized version of the original Dielectric Breakdown Model developed by Niemeyer. DB model is defined in discrete coordinate space. Figure 1 shows a sample square lattice (in its initial state) used to produce lightning discharge patterns in 2D. Discharge pattern is indicated by black circles connected with thick lines. Dashed lines which connect each discharge point (black circle) with a neighboring charge-free lattice point (white circle) represent possible breakdown links.

The potential of the lattice points in the upper boundary of the lattice (which simulates the cloud base) are fixed to $\phi = 1$ while lattice points in the lower boundary (which simulates the ground plane) are fixed to $\phi = 0$ as for the left and right boundaries, ($\phi = 0$). Potential of the remaining charge-free lattice points (white circles) are defined by the discrete Laplace equation under the boundary conditions imposed by the cloud base, ground plane and the discharge structure developed up to that time.
The central lattice point of the upper boundary is capable of initiating the discharge. During each iteration of the algorithm, a new breakdown link (dashed line segment) is chosen and added to the existing discharge pattern. The new breakdown links are chosen randomly according to a weighted probability function as described below.

Let $p$ denote a lattice point connected to the discharge pattern (black circle) while $q$ represents an adjacent charge-free lattice point (white circle). Magnitude of the component of electric field vector (local electric field) at $p$ pointing in the direction of $pq$ is approximated by;

$$E_{pq} = \frac{\phi_p - \phi_q}{d}$$  \hspace{1cm} (1)

*Where, $d$ is the length of a dashed line segment.*

The probability of choosing a particular $pq$ dashed line segment is proportional to a power of the local electric field associated with it.

$$P_{pq} = \frac{E_{pq}^{\eta}}{\sum_l E_{max}^{\eta}_{pq}}$$ \hspace{1cm} (2)

*Where $\eta$ is weighting exponent.*

A random number, $\tau \in [0,1]$ is generated and $q$ point is being selected from the set of adjacent point difference values, $|P_{pq} - \tau|$ by getting the minimum value. When a new breakdown link is selected and a new lattice point is connected to the discharge pattern, the electric potential $\phi_q$ of the new discharge point $q$ is set to 1.

Each time a new breakdown link is chosen, the newly added discharge point becomes part of the boundary conditions. Therefore, potential at charge-free lattice points have to be recalculated under the new boundary conditions by solving Laplace equation;

$$\nabla \phi = 0$$  \hspace{1cm} (3)

Since Laplace equation is linear, the value of $\phi$ at any point is the average of those around that point.

$$\phi_{i,j} = \frac{1}{4}(\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1})$$ \hspace{1cm} (4)

*Where $i$, $j$ and $k$ represent discrete lattice coordinates.*

To increase the speed of convergence, the system of equations generated by equation 4 was solved by the technique, *successive over-relaxation.* Given the appropriate boundary conditions, potential of the lattice points are computed by iterating equation 5 over the lattice (except at boundaries where the potential is fixed) until convergent results are obtained.

$$\phi_{i,j}^n = \phi_{i,j}^{n-1} + \frac{\omega}{4} \delta_{i,j}$$ \hspace{1cm} (5)

*Where*

$$\delta_{i,j} = \phi_{i+1,j}^{n-1} + \phi_{i-1,j}^{n} + \phi_{i,j+1}^{n-1} + \phi_{i,j-1}^{n}$$  \hspace{1cm} (6)

*Good - we will talk about this later in the course.*
Here, the superscript \( n \) denotes the current iteration cycle while \( (n-1) \) denote the previous cycle. The over-relaxation parameter \( \omega \) (1 \( \leq \omega \leq 2 \)) controls the speed of convergence. \( \omega \approx 1.6 \) considerably increases the speed of convergence of the whole process. Finally, image were save into the user define main directory on the windows computer and simulated using VirtualDub-1.10.4 software.

**RESULT**

The exponent \( \eta \) in DB model parameterizes the relationship between the local electric field and the probability of growth of the discharge pattern. The overall appearance of the pattern is strongly related to the exponent \( \eta \). Different value of \( \eta \) were carried out in order evaluate the effect of \( \eta \) on the shape. According to the results, realistic shape for lightning discharges were observed for \( \eta =3, 4 \) and 5 (see below).

**Figure 2:** simulated lighting channels for \( \eta = 0 \)  
**Figure 3:** simulated lighting channels for \( \eta = 1 \)  
**Figure 4:** simulated lighting channels for \( \eta = 2 \)  
**Figure 5:** simulated lighting channels for \( \eta = 3 \)
CONCLUSION

In this work, stochastic dielectric breakdown model was applied to 2D domains to simulate lightning discharges. The adaptive grid solver can greatly decrease the memory footprint and running time of the simulation. Realistic thundercloud charge distribution are more complex than this model. Furthermore, instead of potential layer by introducing charge layer, some of the physical lighting phenomenon can be obtain and using combination of several model, instead of one model might be helpful to understand the realistic nature of lighting propagation. My future work is to extend this model to 3D domain.

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