Syndetic Model of Fundamental Interactions

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Abstract

The standard model of quarks and leptons is extended to connect three outstanding issues in particle physics and astrophysics: (1) the absence of strong $CP$ nonconservation, (2) the existence of dark matter, and (3) the mechanism of nonzero neutrino masses, and that of the first family of quarks and leptons, all in the context of having only one Higgs boson in a renormalizable theory. Some phenomenological implications are discussed.
With the 2012 discovery of the 125 GeV particle at the Large Hadron Collider (LHC), and the likelihood of it being the one physical neutral Higgs boson \( h \) of the standard model (SM) of quarks and leptons, it appears that the SM is essentially complete. Nevertheless, there are at least three loose ends. (1) The SM predicts a source of strong \( CP \) nonconservation (the neutron electric dipole moment) which is not observed. (2) The SM does not have a suitable candidate for the dark matter (DM) of the Universe. (3) The SM does not specify a unique fundamental (renormalizable) mechanism for nonzero neutrino masses, which are required for neutrino oscillations. Whereas well-known piecemeal solutions of each problem exist, it is worth exploring the possibility that these three issues are in fact interconnected. I propose in the following a syndetic model with this idea in mind.

| field | \( SU(3) \times SU(2) \times U(1) \) | \( U(1)_{PQ} \) |
|-------|-----------------------------------|----------------|
| \( (t, b)_L, (c, s)_L \) | (3,2,1/6) | 0 |
| \( (u, d)_L \) | (3,2,1/6) | 2 |
| \( t_R, c_R, u_R \) | (3,1,2/3) | 0 |
| \( b_R, s_R, d_R \) | (3,1,–1/3) | 0 |
| \( (\nu_\tau, \tau)_L, (\nu_\mu, \mu)_L, (\nu_\tau, e)_L \) | (1,2,–1/2) | 0 |
| \( \tau_R, \mu_R \) | (1,1,–1) | 0 |
| \( e_R \) | (1,2,–1/2) | –2 |
| \( (\phi^+, \phi^0) \) | (1,2,1/2) | 0 |

Table 1: Field content of proposed model.
Under the proposed well-known anomalous $U(1)_{PQ}$ symmetry [3], the SM fermions transform as shown in Table 1. The new particles all have $PQ$ charges. They include a heavy singlet quark $Q$ of charge $-1/3$ and three heavy neutral singlet fermions $N_{1R}, N_{2R}, N_{3R}$, as well as one scalar doublet $(\eta^+, \eta^0)$, one scalar singlet $\chi^+$ and one scalar singlet $\chi^0$. Another scalar singlet $\zeta^0$ with two units of $PQ$ charge contains the axion [4, 5] as $U(1)_{PQ}$ is spontaneously broken. This solves the strong $CP$ problem. A residual discrete $Z_2$ symmetry also exists in this case, and acts as a good symmetry for cold dark matter, as was pointed out recently [6]. Since some of the SM quarks as well as the heavy $Q$ transform under $U(1)_{PQ}$, this model is a hybrid of the two well-known examples of realistic axion models: one where only $Q$ transforms [7, 8] and one where there is no $Q$ but the SM quarks transform [9, 10].

The important distinction in the present proposal is that the SM Higgs doublet couples at tree level only to the second and third families of quarks and charged leptons, i.e. they do not transform under $U(1)_{PQ}$. On the other hand, the first family becomes massive only through interactions with $Q$ and $N$ in the presence of $U(1)_{PQ}$ symmetry, thus linking their origin of mass through the one Higgs boson $h$ of the SM with dark matter [11] as well as the solution of the strong $CP$ problem [6]. As for neutrinos, they will acquire nonzero Majorana masses, using the scotogenic mechanism [12], from the Greek scotos meaning darkness.

Under the exactly conserved discrete $Z_2$ symmetry, all SM particles are even. The dark sector consists of particles odd under $Z_2$, namely $Q, N_1, N_2, N_3, (\eta^+, \eta^0), \chi^+, \chi^0$. Their only direct interactions with $\zeta^0$ are

$$f_Q\zeta^0\bar Q_R Q_L, \quad f^\epsilon_{ij}\zeta^0 N_{iR} N_{jR}, \quad \lambda_\chi\zeta^0\chi^0\chi^0. \quad (1)$$

Thus $Q$ and $N$ are expected to acquire large masses from the vacuum expectation value of $\zeta^0$, whereas the mass-squared matrix spanning $(\chi_R, \chi_I)$ where $\chi^0 = (\chi_R + i\chi_I)/\sqrt{2}$ is of the form

$$M^2_\chi = \begin{pmatrix} m^2_\chi + \mu_\chi \langle \zeta^0 \rangle & 0 \\ 0 & m^2_\chi - \mu_\chi \langle \zeta^0 \rangle \end{pmatrix}. \quad (2)$$
This is suggestive of having $\chi_I$ as the lightest particle of odd $Z_2$ and thus a DM candidate.

Since $(u, d)_L$ has PQ charge 2, it does not couple to $\Phi$. Hence $u$ and $d$ quarks are massless at tree level. However, the mixing of $\eta^+$ with $\chi^+$ and $\eta^0$ with $\chi^0$ through $\langle \phi^0 \rangle$ allows both to acquire small masses in one loop as shown.

![Figure 1: One-loop generation of $u$ quark mass.](image1)

![Figure 2: One-loop generation of $d$ quark mass.](image2)

The resulting $3 \times 3$ mass matrix linking $\bar{q}_L$ to $q_R$ for either sector is of the form

$$
\mathcal{M}_q = \begin{pmatrix}
m_{11} & m_{12} & m_{13} \\
0 & m_{22} & m_{23} \\
0 & 0 & m_{33}
\end{pmatrix},
$$

(3)

where $m_{33}, m_{22}, m_{23}$ are tree-level, and $m_{11}, m_{12}, m_{13}$ are one-loop radiative masses. Since the mass eigenvalues in each quark sector are hierarchical, and the (13) and (23) mixing
angles are very small, each mass matrix is diagonalized on the left by

\[
U = \begin{pmatrix}
1 & 0 & -\epsilon_{13} \\
0 & 1 & -\epsilon_{23} \\
\epsilon_{13} & \epsilon_{23} & 1
\end{pmatrix}
\begin{pmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

(4)

where \( \epsilon_{13} = m_{13}/m_{33} \), \( \epsilon_{23} = m_{23}/m_{33} \), and \( \tan \theta = m_{12}/m_{22} \). The mass eigenvalues are then

\[
m_t(m_b) \simeq m_{33}, \quad m_e(m_u) \simeq \sqrt{m_{22}^2 + m_{12}^2}, \quad m_u(m_d) \simeq m_{11} \cos \theta,
\]

(5)

and the quark mixing matrix \( U_u^\dagger U_d \) has the elements

\[
V_{us} \simeq \sin(\theta_u - \theta_d),
\]

(6)

\[
V_{ub} \simeq \cos \theta_u (\epsilon_{13}^u - \epsilon_{13}^d) + \sin \theta_u (\epsilon_{23}^u - \epsilon_{23}^d),
\]

(7)

\[
V_{cb} \simeq \cos \theta_u (\epsilon_{23}^u - \epsilon_{23}^d) - \sin \theta_u (\epsilon_{13}^u - \epsilon_{13}^d).
\]

(8)

Radiative electron and Majorana neutrino masses are also generated. In contrast to

\[
\begin{align*}
\phi^0 & \rightarrow e^L & N_R & N_R & e^R \\
\eta^+ & \rightarrow e^L & N_R & N_R & e^R \\
\chi^+ & \rightarrow e^L & N_R & N_R & e^R \\
\zeta^0 & \rightarrow e^L & N_R & N_R & e^R
\end{align*}
\]

Figure 3: One-loop generation of electron mass.

Eq. (3), the resulting 3 \( \times \) 3 charged-lepton mass matrix linking \( \bar{l}_L \) to \( l_R \) is of the form

\[
\mathcal{M}_l = \begin{pmatrix}
m_{11} & 0 & 0 \\
m_{21} & m_{22} & 0 \\
m_{31} & m_{32} & m_{33}
\end{pmatrix},
\]

(9)
Figure 4: One-loop generation of Majorana neutrino mass.

where \( m_{33}, m_{22}, m_{32} \) are tree-level, and \( m_{11}, m_{21}, m_{31} \) are one-loop radiative masses. As a result, it is diagonalized on the left by

\[
U = \begin{pmatrix}
1 & -(m_e/m_\mu)\epsilon_{21} & -(m_e/m_\tau)\epsilon_{31} \\
(m_e/m_\mu)\epsilon_{21} & 1 & -(m_\mu/m_\tau)\epsilon_{32} \\
(m_e/m_\tau)\epsilon_{31} & (m_\mu/m_\tau)\epsilon_{32} & 1
\end{pmatrix},
\]

(10)

where \( \epsilon_{21} = m_{21}/m_{22}, \epsilon_{31} = m_{31}/m_{33}, \epsilon_{32} = m_{32}/m_{33}, \) and \( m_\tau \simeq m_{33}, m_\mu \simeq m_{22}, m_e \simeq m_{11}. \)

As for the \( 3 \times 3 \) neutrino mass matrix, it is all radiative as in the original scotogenic model \[12\]. Without the imposition of a flavor symmetry, there are no specific predictions of mixing angles or masses.

The radiative mass entries of Eqs. (3) and (9) are of the form \[11, 13\]

\[
m = \frac{f_\eta f_\chi \mu v}{16\pi^2 \sqrt{2} M (x_1 - x_2)} \left( \frac{x_1 \ln x_1}{x_1 - 1} - \frac{x_2 \ln x_2}{x_2 - 1} \right),
\]

(11)

where \( f_\eta \) and \( f_\chi \) are the respective Yukawa couplings in Figs. 1, 2, and 3, \( \mu \) is the corresponding trilinear scalar coupling to \( \phi^0 \), \( v/\sqrt{2} = 174 \) GeV is the vacuum expectation value of \( \phi^0 \), \( M \) is either the mass of \( Q \) or \( N \), \( x_{1,2} = m_{1,2}^2/M^2 \), and \( m_{1,2}^2 \) are the eigenvalues of the \( (\eta, \chi) \) mass-squared matrix.

If all fermion masses in a given sector, say \( (d, s, b) \) come from tree-level couplings due to a single Higgs boson, then the diagonalization of that mass matrix automatically diagonalizes
the Higgs Yukawa couplings. However, since some of the mass entries are one-loop effects, this will not be the case, because the corresponding Higgs Yukawa coupling is then not exactly equal to $m/v$, as pointed out recently [13]. As a result, there will be small off-diagonal Higgs Yukawa couplings to quarks, which induces small flavor-changing neutral-current processes. The most sensitive probe of this effect is $K^0 - \bar{K}^0$ mixing in the quark sector and $\mu \to e\gamma$ in the lepton sector.

Since $m_{11}$, $m_{12}$, and $m_{13}$ in Eq. (3) are radiative masses, the corresponding Higgs Yukawa coupling matrix is of the form

$$H_q = \frac{1}{v} \begin{pmatrix}
m_{11}(1 + \delta) & m_{12}(1 + \delta) & m_{13}(1 + \delta) \\
0 & m_{22} & m_{23} \\
0 & 0 & m_{33}
\end{pmatrix},$$

(12)

where $\delta$ is a loop factor computed exactly in Ref. [13]. Let $U_L M_q U_R^\dagger$ be diagonal, then $U_L H_q U_R^\dagger$ will have off-diagonal pieces, i.e. flavor-changing neutral currents. Using Eq. (4) and the fact that $U_R$ has suppressed off-diagonal entries relative to $U_L$, the dominant effective operator for $K^0 - \bar{K}^0$ mixing through Higgs exchange is given by

$$O_2 = \frac{\delta^2 m_s^2 \sin^2 \theta \cos^2 \theta}{v^2 m_h^2} (\bar{d}_L s_R)^2.$$

(13)

Using $v = 246$ GeV, $m_h = 125$ GeV, and $m_s = 55$ MeV, this contribution to the $K_L - K_S$ mass difference is $-1.9 \times 10^{-14}\delta^2$ GeV [14], as compared to the experimental value of $3.484 \times 10^{-15}$ GeV. Allowing for a 10% uncertainty in the SM contribution, this means that $\delta$ may be as large as 0.135.

In the charged-lepton sector, after diagonalizing Eq. (9), flavor violating decays such as $\mu \to e\gamma$ will occur. This is dominated by the radiative transition $l_{2L} \to l_{1R}\gamma$ and its amplitude is proportional to $m_{21} \sim \epsilon_{21} m_\mu$. Its calculation is analogous to that of the muon anomalous magnetic moment given in Ref. [13], i.e.

$$A = \frac{m_{21}}{2m_\mu m_N^2} \left[ \frac{G(x_1) - G(x_2)}{H(x_1) - H(x_2)} \right],$$

(14)
where
\[ G(x) = \frac{2x \ln x}{(x-1)^3} - \frac{x+1}{(x-1)^2}, \quad H(x) = \frac{x \ln x}{x-1}. \] (15)

The branching fraction of \( \mu \rightarrow e\gamma \) is constrained by the current experimental upper bound \[15\] according to
\[ B = \frac{12\pi^3 \alpha \epsilon_{21}^2}{G_F m_N^4} \left[ \frac{G(x_1) - G(x_2)}{H(x_1) - H(x_2)} \right]^2 < 5.7 \times 10^{-13}. \] (16)

For \( m_N < 1 \) TeV, this requires \( \epsilon_{21} \) to be less than about \( 10^{-5} \). This implies that a flavor symmetry, e.g. \( Z_3 \), is desirable in a more complete model to make \( m_{21} \) zero.

At the Large Hadron Collider, the heavy quark \( Q \) may be produced in pairs if kinematically allowed. Consider the decay chain:
\[ Q^{-1/3} \rightarrow u + \eta^-, \quad \eta^- \rightarrow e^- N \text{ or } \mu^- N, \] (17)
then if \( N \) is dark matter, a distinct signature of “2 jets + \( e^\pm + \mu^\mp \) + missing energy” may be observed. This same final state is also possible in the model of Ref. \[11\] and has been analyzed in Ref. \[16\], but the topology here is different and will be studied further elsewhere.

In conclusion, a syndetic model of fundamental interactions has been presented in the context of having one and only one Higgs boson in accordance with the standard model. The difference here is that radiative masses are obtained for the first family of quarks and leptons, as well as for all neutrinos. The \( U(1)_{PQ} \) symmetry is implemented to solve the strong \( CP \) problem, in such a way that an exactly conserved residual \( Z_2 \) discrete symmetry remains to support a candidate particle for cold dark matter. The same \( Z_2 \) symmetry enables the one-loop radiative masses. The new particles required are possibly observable at the Large Hadron Collider in the near future.

This work is supported in part by the U. S. Department of Energy under Grant No. de-sc0008541.
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