On the thermal boundary condition of the wave function of the Universe

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We broaden the domain of application of the recently proposed thermal boundary condition of the wave function of the Universe, which has been suggested as the basis of a dynamical selection principle on the landscape of string solutions.

1. Introduction

The existence of a multiverse of vast solutions to string theory constitutes currently an important challenge: How to select a Universe or a class from the multiverse that will bear significant similarities to ours?

The framework of quantum cosmology provides a methodology to establish a probability distribution for the dynamical parameters of the Universe. In this context, Brustein and de Alwis proposed in, using FRW quantum cosmology, a dynamical selection principle on the landscape of string solutions.

We prove that the thermal boundary condition applied in corresponds to the particular physical situation where the amount of radiation is very large. We then provide a broader and improved analysis of the generalised thermal boundary condition that is independent of such restrictive limit; i.e. we consider an arbitrary amount of radiation consistent with the tunnelling of a closed radiation-filled Universe with a positive cosmological constant.

2. The generalised thermal boundary condition

The thermal boundary condition for the wave function of the Universe states that the Universe emerges from the string era in a thermally excited state above the Hartle-Hawking (HH) vacuum. Furthermore, the primordial thermal bath is effectively described by a radiation fluid whose energy density ρ “depends” on the cosmological constant λ

\[ \rho = \frac{3\tilde{K}}{8\pi G a^4}, \quad \tilde{K} \simeq \frac{\nu}{\lambda^2}. \]

In the previous expressions \( \tilde{K} \) and \( \nu \) are parameters that quantify the amount of radiation, G is the gravitational constant and \( a \) the scale factor. Therefore, the
transition amplitude of a closed radiation-filled FRW Universe\textsuperscript{11,12} to tunnel from the first Lorentzian region ($a < a_-$, see Fig. 1-a) to the larger Lorentzian region ($a_+ < a$, see also Fig. 1-a) within a WKB approximation reads\textsuperscript{7,10}

$$A = \exp(\epsilon 2I), \quad I = \frac{\pi}{2G \nu} g,$$  

(2)

where $\epsilon = \pm 1$ and

$$g = \frac{\nu}{\lambda} \sqrt{1 + m \left[ E(\alpha_{II}) - (1 - m) K(\alpha_{II}) \right]}, \quad \alpha_{II} = \sqrt{2m \left[ 1 + m \right]}, \quad m = \sqrt{1 - 4\tilde{K}\lambda},$$  

(3)

with $K(m)$ and $E(m)$ as complete elliptic integrals of the first and second kind, respectively. Consequently, the thermal boundary condition implies a switch in the usual features of the HH\textsuperscript{4} ($\epsilon = 1$ choice in Eq. (2)) and the tunnelling ($\epsilon = -1$ choice in Eq. (2)) wave functions: The HH wave function, once the thermal boundary condition of\textsuperscript{7} is assumed, favours a non-vanishing cosmological constant; $\lambda \simeq 8.33\nu$, larger than the one preferred by the tunnelling wave function; $\lambda \simeq 4\nu$, (see Fig. 1-b).

Fig. 1. Figure 1-a corresponds to the potential barrier ($V_0 - \tilde{K}$) separating the two Lorentzian regions of a closed radiation-filled FRW Universe. $\tilde{K}_{\text{max}}$ corresponds to the maximum amount of radiation consistent with the tunnelling of the Universe. We will refer to this situation as a large amount of radiation. Figures 1-b and 1-c corresponds to $g$ defined in Eq. (3) for the thermal boundary condition and the generalised thermal boundary condition, respectively.

It turns out that the thermal effect considered in\textsuperscript{7} corresponds to a large amount of radiation where $\tilde{K}$ is close to $\tilde{K}_{\text{max}}$ (see Fig. 1-a); i.e. the turning points $a_-$ and $a_+$ are very close or equivalently the height of the barrier separating both Lorentzian regions is very small.

Next we consider a generalised thermal boundary condition for the wave function\textsuperscript{10} where we will assume instead an arbitrary amount of radiation, consistent with a tunnelling of the Universe; i.e. $\tilde{K} < \tilde{K}_{\text{max}}$ (see Fig. 1-a). Consequently, Eqs. (1)-(3) are replaced by consistent and more general relations where the amount of radiation as measured by $\tilde{K}$ depends also on $\lambda$ and reads

$$\tilde{K} = \frac{4\nu\lambda^{-2}}{(1 + 4\nu\lambda^{-1})^2}.$$  

(4)

Within this broader range, the relevant features is that the transition amplitude as a function of $\nu/\lambda$ will be unlike the one deduced in\textsuperscript{7}. Indeed, this is the case as is shown in Figs. 1-b and 1-c.
Regarding the HH wave function, it now favours a vanishing cosmological constant \( (\nu/\lambda \to \infty) \) and \( \tilde{K} \sim 1/(4\nu) \). This physical case is represented schematically by a star in Fig. 1-c. In this manner, the role of the HH wave function and subsequent transition amplitude is returned to its “original” implication, with the thermal boundary condition being implemented in a fully consistent manner and not restricted to a narrow (perhaps not fully valid) limit.

Concerning the tunnelling wave function, it favours two possible physical situations depicted by a circle and a square in Fig. 1-c. On the one hand, the “circle” option corresponds to a large cosmological constant \( (\nu/\lambda \to 0) \) and a small amount of radiation as measured by \( \tilde{K} (\tilde{K}\lambda \to 0) \). On the other hand, the “square” option implies no tunnelling, that is, \( 4\nu/\lambda \to 1 \) or equivalently \( 4\tilde{K}\lambda \to 1 \); i.e. both turning points coincide. In order to select one of these two possibilities for the tunnelling wave function, we employed the DeWitt’s argument,\(^{13,14}\) since there is a curvature singularity at small scale factors. It turns out that the preferred value of the cosmological constant in this case is a large one. Moreover, this condition implies a small amount of radiation (as measured by the parameter \( \tilde{K} \)) allowing consequently the tunnelling of the Universe.

3. Conclusions
We prove that the thermal boundary condition applied in\(^7\) corresponds to the particular physical situation where the amount of radiation is very large. We then provide a broader and improved analysis of the generalised thermal boundary condition that is independent of such restrictive limit.\(^{10}\)

Acknowledgments
MBL acknowledges the support of a Marcel Grossmann fellowship to attend the meeting. MBL also acknowledges the support of CENTRA-IST BPD (Portugal) as well as the fellowship FCT/BPD/26542/2006.

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