Meson correlators above deconfinement

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Abstract. We review recent progress in studying spectral functions for mesonic observables at finite temperatures, by analysis of imaginary time correlators directly calculated on isotropic lattices. Special attention is paid to the lattice artifacts present in such calculations.

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1. Introduction

Spectral functions of mesonic operators play an important role in finite temperature QCD. Many experimental results in high energy heavy ion collisions (e.g. low invariant mass dilepton enhancement, anomalous $J/\psi$ suppression etc.) can be understood in terms of medium modifications of meson spectral functions [1].

For a long time it has been believed that lattice QCD is suitable only for calculation of static quantities at finite temperatures, such as the transition temperature, equation of state, screening lengths etc. However, it was shown by Asakawa, Hatsuda and Nakahara that using the Maximum Entropy Method one can in principle reconstruct also meson spectral functions. The method was successfully applied at zero temperature [2, 3, 4], and also at finite temperature [5, 6, 7, 8, 9, 10, 11]. Though systematic uncertainties ‡ speaker at the conference
in the spectral function calculated on lattice are not yet completely understood, it was shown in Ref. [5] that precise determination of the imaginary time correlator can alone provide constraints on the spectral function at finite temperature.

In this contribution we are going to review recent results on meson correlators and spectral functions in finite temperature QCD.

2. Meson spectral functions

The spectral function for an operator is directly related to the Fourier transforms of real time correlators of the operator. Defining the real time propagators

\[ D^<(t) = \langle \hat{O}(0) \hat{O}(t) \rangle, \quad D^>(t) = \langle \hat{O}(t) \hat{O}(0) \rangle, \]

for a suitable operator \( O \), the spectral function \( \sigma(\omega) \) is defined as

\[ \sigma(\omega) = \frac{D^>(\omega) - D^<(\omega)}{2\pi}, \]

(2)

where \( D^{>,<}(\omega) \) are the Fourier transforms of the correlators in Eq. (1). The spectral function encapsulates the properties of the channel corresponding to operator \( O \): a stable mesonic state will show up as a \( \delta \) function and an unstable state will be present as a broad (Breit-Wigner) peak. Also, other quantities of experimental interest can be directly extracted from spectral functions: for example, the rate of emission of thermal dileptons can be expressed in terms of the spectral function of the vector current \( J_\mu = \bar{q} \gamma_\mu q \)

\[ \frac{dW}{dp_0 d^3p} = \frac{5\alpha_{em}^2}{2T\pi^2 p_0^3 (e^{p_0/T} - 1)} \sigma_V(p_0, \vec{p}). \]

(3)

In lattice studies of finite temperature systems in equilibrium, on the other hand, one calculates the imaginary time or Matsubara correlator

\[ G(\tau) = \langle \hat{O}(\tau) \hat{O}(0) \rangle, \]

(4)

where the Euclidean time \( \tau \in (0, \beta = 1/T) \). The imaginary time correlators is an analytic continuation of the real time correlator \( G(\tau) = D^>(-i\tau) \). This and the periodicity in \( \tau \) lead to an integral representation for \( G(\tau) \) in terms of the spectral function,

\[ G(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T) K(\tau, \omega, T) \]

\[ K(\tau, \omega) = \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}. \]

(5)

Here we consider correlators of point meson operators \( \bar{q} \Gamma q \), with \( \Gamma = 1, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5 \) for scalar, pseudo-scalar, vector and axial-vector channels. We will also restrict the discussion to the case of zero spatial momentum.

Equation (6) is valid in the continuum. It is not obvious that it holds also for the correlators calculated on the lattice. However, it was shown in that an integral representation like in Eq. (6) can be also defined for the lattice correlator in the limit of...
the free field theory, with the only modification being that the upper integration limit becomes finite $\omega_{\text{max}} \simeq 4 a^{-1}$ [13]. The effect of the discretization is contained solely in the spectral function [13].

In extracting $\sigma(\omega)$ from the lattice measurements of $G(\tau)$ using Eq. (6), the obvious problem is that we have to reconstruct several hundred degrees of freedom (needed for a reasonable discretization of the integral in Eq. (6) with $O(10)$ data points on $G(\tau)$. This ill-posed problem can be solved using the Maximum Entropy Method (MEM) [2]. In the Maximum Entropy Method one searches for a spectral function which maximizes the conditional probability $\exp\left(-\chi^2/2 + \alpha S\right)$, where $\chi^2/2$ is the standard likelihood function for the data and

$$S = \int_0^{\infty} d\omega \left[ \sigma(\omega) - m(\omega) - m(\omega) \ln \frac{\sigma(\omega)}{m(\omega)} \right]$$

(7)

is the Shannon-Jaynes entropy which parametrizes prior knowledge such as positivity of the spectral functions and informations about the asymptotic form of the spectral function. Finally $\alpha$ is a real and positive parameter, which is integrated out to obtain the most probable spectral function. The Shannon-Jaynes entropy depends on one real function $m(\omega)$ called the default model.

3. Numerical results for the correlators and spectral functions

Here we will mostly discuss finite temperature spectral functions for heavy quarkonia. The behavior of heavy quarkonia at finite temperature has been of great interest ever since the suggestion of Matsui and Satz that the disappearance of $J/\psi$ can act as a signal of onset of deconfinement, since the screening in a quark-gluon plasma will lead to the dissolution of charmonium bound states [14]. The problem of the $J/\psi$ suppression was addressed using potential models [14, 15, 16], and resulted in estimates that the $J/\psi$ will dissolve around $1.1 T_c$ [15, 16]. More recent calculations, using lattice results for internal energy of a static $\bar{q}q$ pair [17], as the potential, lead to a much higher dissociation temperature $> 1.5 T_c$ for the $J/\psi$ [18, 19].

Direct lattice investigations of heavy quarkonia at finite temperatures, by studying the Matsubara correlators and extracting the spectral function from them using MEM, were performed only recently. Calculations have been done using anisotropic [9, 10] as well as fine isotropic [11] lattices in quenched approximation (i.e., for a purely gluonic plasma). A check of the cutoff dependence of the results by using several lattice spacings, which turns out to be quite important, has been done so far only in Ref. [11]. The extracted spectral functions at zero temperature reproduced the properties of the ground state mesons, but revealed structures (lattice artifacts) at higher energies whose positions scale like $a^{-1}$ [3, 4, 11]. Due to these structures, the high energy part of the spectral functions in the interacting theory look very different from that in the free theory.

We first discuss the spectral functions at low temperatures where the large extent of the imaginary time direction allows a reliable extraction of the spectral function from
the Matsubara correlators. We have used the continuum free meson spectral function of the form $m_0 \omega^2$ as a default model. It was checked that the result does not depend on the choice of the default model [11]. In Fig. 1 we show charmonium spectral functions below deconfinement at two different lattice spacings, corresponding to temperatures of $0.75 T_c$ and $0.9 T_c$ [11]. The ground state peaks in the pseudo-scalar and vector channels correspond to $\eta_c$ and $J/\psi$ states, while those in the scalar and axial-vector channels correspond to $\chi_{c0}$ and $\chi_{c1}$ states, respectively. While the ground state peaks reproduce the expected masses the positions of the other peaks scales roughly as $1/a$, i.e. these peaks are lattice artifacts.

Now let us discuss what happens at finite temperature. In this case the reconstruction of the spectral function becomes more difficult as not only the number of available time-slices is reduced but also the extent of the imaginary time direction $(= 1/T)$ may be quite small (see discussion in Ref. [9] on this point). While the former limitation can be relaxed using larger anisotropies, the latter will always remain. Therefore it is useful to inspect the temperature dependence of the correlators first. The temperature dependence of the correlators is due both to the explicit temperature dependence of the corresponding spectral functions and to the temperature dependence of the integration kernel in Eq. (6). To factor out the trivial temperature dependence due to the kernel, we construct the model correlators

$$G_{rec}(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T^*) K(\tau, \omega, T),$$

with $T^*$ being some temperature below deconfinement. If the spectral function has no significant temperature dependence also above deconfinement, we would expect
$G/G_{rec} \simeq 1$ also at temperatures above $T_c$. In Fig. 2 we show $G/G_{rec}$ for different channels at $1.5T_c$. As one can see this ratio is not very far from unity for pseudo-scalar and vector channels, while for the scalar and axial-vector channel large changes are visible. In fact, for the scalar and axial-vector channels large changes are seen already at $1.1T_c$ [11]. This suggests that the 1S state charmonia ($\eta_c$ and $J/\psi$) may survive deconfinement with little modifications of their properties, while 1P state charmonia ($\chi_{c0}$ and $\chi_{c1}$) are dissolved or strongly modified by the medium right after deconfinement. Notice that Fig. 2 shows the ratio $G/G_{rec}$ for two different lattice spacings, with very little lattice spacing dependence observed in the results.

To study the problem in more detail one needs to extract the spectral functions from the measured correlators. Because of the limited extent of the time direction this becomes quite difficult at high temperatures, as the result gets sensitive to the default model. We have used the high energy part of spectral function below $T_c$ as part of the default model. This is justified as the high energy part of the extracted spectral function is dominated by lattice artifacts, and also, the effects of moderate temperatures should not be significant at very high energies. In Fig. 3 we show the spectral functions extracted for the vector and scalar channels, at temperatures of $1.5 - 3 \ T_c$. The figure confirms the expectation based on the analysis of the correlator; the $J/\psi$ is seen to exist up to temperatures $2.25T_c$, while the $\chi_{c0}$ state is dissolved already at $1.1T_c$. The situation is similar for the pseudo-scalar and axial vector spectral functions. The results for the survival of the 1S states till quite high temperatures are confirmed by calculations done on anisotropic lattices [9] [10], though there is some controversy about the temperature where $J/\psi$ dissolves and the way it disappears. In Ref. [10] it was found that the 1S charmonia abruptly disappear at $1.7T_c$, while the spectral functions shown above suggest a gradual dissolution with resonance structure surviving till $2.25T_c$.

4. Conclusions

We have discussed lattice calculations of the charmonium spectral functions at finite temperature. Despite the uncertainties present in the current analysis we can conclude that $J/\psi$ and $\eta_c$ survive in the gluonic plasma up to quite high temperatures, possibly to temperatures as high as $2.25T_c$. Moreover, the properties of the 1S charmonia seems to
be only very mildly affected by the deconfinement transition. The 1P states charmonia on the other hand are dissolved right after deconfinement. This conclusion is supported both by the explicit calculations of the spectral functions using MEM and by the analysis of the correlators which is much more robust. The spectral functions calculated on the lattice show strong lattice artifacts at high energies. However the properties of the first peak, which corresponds to the physical state, are not strongly affected by these artifacts. This conclusion is also supported by direct analysis of meson correlators at two different lattice spacings.

Finally we note that meson spectral functions have also been analyzed for the case of light quarks [5, 6, 8]. Contrary to the case for the heavy quarks, the spectral functions for light mesonic observables show quite strong temperature dependence. However, also here one sees resonance like structures at low energies and no sign of free quark propagation. The lattice artifacts at high energy are present also in this case but were not studied in detail yet. This would be absolutely necessary for drawing physical conclusions with confidence.

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