Borromean supercounterfluidity

Emil Blomquist,1 Andrey Syrwid,1 and Egor Babaev1

1Department of Physics, KTH Royal Institute of Technology, Stockholm SE-10691, Sweden

We demonstrate microscopically the existence of a new superfluid state of matter in a three-component Bose mixture trapped in an optical lattice. The superfluid transport involving co-flow of all three components is arrested in that state, while counterflows between any pair of components are dissipationless. The presence of three components allows for three different types of counterflows with only two independent superfluid degrees of freedom.

The advent of optical lattices [1–6] allowed for highly controllable access to strongly-correlated quantum many-body systems and opened up a way to realize various phases of matter. One of the theoretical predictions was that bosons in optical lattices could host a new type of transport phenomenon called supercounterfluidity [7–16]. That is, having two bosonic fields $\psi_1, \psi_2$ in an ordinary case, one finds a superfluid mixture when $\langle \psi_1 \psi_2 \rangle$ reveals a (quasi) long-range order. This state is predicted to appear [7–16] when the averages of individual fields vanish, $\langle \psi_1^2 \rangle = 0$, but there is a (quasi) long-range order in the composite field $\langle \psi_1 \psi_2^* \rangle \neq 0$. Therefore, in a supercounterfluid phase, individual bosonic species do not exhibit superfluidity, but the transport of particle-hole composites is dissipationless.

A similar type of order was predicted in superconducting systems arising from a different microscopic origin [17–24]. The Andreev–Bashkin effect was studied in various regimes in optical body systems and opened up a way to realize various controllable access to strongly-correlated quantum many-body systems [1–6]. Note that in certain asymmetrical optical lattices there are additional terms responsible for transverse entrainment [32]. However, in this Letter, we will restrict ourselves to square lattices where only the Andreev–Bashkin drag effect is present.

In the simplest case of two components with identical masses, the free-energy density can be cast into the form associated with co- and counter-flows $f = \rho_2 (\nabla \theta_1 + \nabla \theta_2)^2 / 4 + \rho_0 (\nabla \theta_1 - \nabla \theta_2)^2 / 4$. Here $\rho_2 = \rho_k + (\xi - 1) \rho_0 \geq 0$ where $\rho_2 > 0$ describe the prefactor of the standard gradient term, and $\rho_0$—either positive or negative—denotes the drag strength. When the drag is sufficiently strong and negative, the cheapest topological excitations become co-circulating composite vortices, i.e., vortices where both phases $\theta_1, \theta_2$ wind by $2\pi$ around the core [25]. Thermal or quantum fluctuations can then lead to the proliferation of these composite vortices—but not elementary ones—resulting in a phase transition to a supercounterfluid (for a detailed discussion of the principle, see, e.g., Chapter 6 in [25]). The composite vortices do not induce gradients in the phase difference and thus do not disorder the phase difference part of the free energy. The free-energy density of the resulting state therefore only involves the phase stiffness corresponding to the phase difference, i.e., $f = \rho_0 (\nabla \theta_1 - \nabla \theta_2)^2 / 4$. This term can be interpreted as the kinetic free-energy contribution related to the composite particle-hole order parameter $\psi_1 \psi_2^*$. Consequently, only counter-flow dissipationless transport can take place.

In a system with more than two components, states may arise with no direct counterpart among two-component superfluids. Let us therefore consider a two-dimensional $N$-component symmetric quantum system, i.e., components with identical masses and densities, and equal Andreev–Bashkin drag strength $\rho_0$ between each pair. We start with a phase-only approximation assuming identical and homogeneous densities of unit mass particles ($m_\alpha = 1$) in each superfluid component. The corresponding free-energy density reads

$$f = \frac{\rho_2}{2} \sum_\alpha (\nabla \theta_\alpha)^2 + \frac{\rho_0}{2} \sum_{\alpha, \beta} \nabla \theta_\alpha \cdot \nabla \theta_\beta$$

$$= \frac{\rho_0}{2N} \left( \sum_\alpha (\nabla \theta_\alpha)^2 \right) + \frac{\rho_0}{4N} \sum_{\alpha, \beta} (\nabla \theta_\alpha - \nabla \theta_\beta)^2,$$

mixtures. Note that in certain asymmetrical optical lattices there are additional terms responsible for transverse entrainment [32]. However, in this Letter, we will restrict ourselves to square lattices where only the Andreev–Bashkin drag effect is present.
where again \( \rho_\xi = \rho_k + (\xi - 1)\rho_d \geq 0 \). Now, when the drag is strong and negative, i.e., when \( \rho_N \ll \rho_0 \), the cheapest topological excitations are composite vortices where all \( N \) phases wind by \( 2\pi \) with the same orientation. Consequently, the proliferation of the three-component topological defects leads to a state where the sum of three phases is disordered. However, these composite vortices are unable to disorder phase differences and the system retains \( N - 1 \) superfluid modes. Equation (1) therefore reduces to \( f \propto \sum_{\alpha,\beta} (\nabla \theta_\alpha - \nabla \theta_\beta)^2 \) and the corresponding phase is characterized by zero net superflow, i.e., \( \sum_{\alpha} j_\alpha \propto \sum_{\alpha} \partial f / \partial (\nabla \theta_\alpha) = 0 \).

Consequently, for \( N > 2 \), the transport properties of the new phase can be understood as a counterflow of two components where the presence of the third symmetric component allows for fluctuations in the type of counterpropagating companions. One would anticipate that this should be reflected in the world lines of the microscopic path-integral formulation. That is, in different regions of the system, one should find different types of particle-hole paired world lines. Moreover, there is no superfluid co-flow of bound \( N \) particle states, while counter-propagation of any two different components is dissipationless. Specifically, for \( N = 3 \) where \( \alpha \in \{ r, g, b \} \), there are three types of counterflows for which there are only two independent degrees of freedom. That implies that we can have a superfluid co-flow of two components as long as their combined flow is counteracted by the flow of the third component, e.g., \( j_r = j_b = j \) and \( j_g = -2j \). Here we can draw a distant analogy to the Borromean rings where three rings are confined while each pair of rings is deconfined, see Fig. 1. Hence we coin this phenomenon Borromean supercounterfluidity.

Below we microscopically demonstrate that such superfluid state exists in a three-component \((N = 3)\) Bose–Hubbard model [33]

\[
\hat{H} = -t \sum_\alpha \sum_{\langle ij \rangle} \hat{b}^\dagger_{i\alpha} \hat{b}_{j\alpha} + \frac{U}{2} \sum_\alpha \sum_i \hat{n}_{i\alpha} (\hat{n}_{i\alpha} - 1) + \frac{U'}{2} \sum_{\alpha,\beta \neq \alpha} \sum_i \hat{n}_{i\alpha} \hat{n}_{i\beta} . \tag{2}
\]

Here \( \hat{b}_{i\alpha} (\hat{b}^\dagger_{i\alpha}) \) is the bosonic annihilation (creation) operator of component \( \alpha \) at site \( i \), and \( \hat{n}_{i\alpha} = \hat{b}^\dagger_{i\alpha} \hat{b}_{i\alpha} \) is the corresponding particle number operator. Greek subscripts label the component type, i.e., \( r \) (red), \( g \) (green), and \( b \) (blue). The parameter \( t \) represents the hopping amplitude, while \( U \) and \( U' \), respectively, are the intra-component and inter-component on-site interaction strengths. We will consider a \( L \times L \) square lattice with unit lattice constant and periodic boundary conditions. We further analyze the two separate cases where either the individual particle-number densities are fixed, i.e., \( n_\alpha = (\sum_i \hat{n}_{i\alpha})/L^2 = 1/3 \), or the total particle-number density is conserved, i.e., \( \sum_\alpha n_\alpha = 1 \), while allowing for fluctuations in \( n_\alpha \).

We numerically investigate the system by utilizing worm-algorithm Monte Carlo [34–38]—a quantum Monte-Carlo method which samples path-integral configurations of the partition function in real space and imaginary time. To extract the numerical values of \( \rho_k \) and \( \rho_d \) appearing in the free-energy density, Eq. (1), we generalize Pollock and Ceperley’s formula [16, 32, 39]: \( \rho_k = T (\omega_k^2)/2 \) and \( \rho_d = T (\omega_d^2)\cdot\omega_d^2)/2 \) where \( T \) is the temperature \((k_B = 1)\) and \( \alpha \neq \beta \). The winding numbers \( \omega_{\alpha} \) encode the net number of times, and in which direction, \( \alpha \)-type particles cross the periodic boundaries. The notation \((\cdot)\) refers to the standard statistical Monte Carlo average.

The calculated coefficients \( \rho_k, \rho_d \), and their ratios \( \rho_d/\rho_k \) are presented in Fig. 2 as functions of \( t/U \) for the interactions strengths \( U = 1 \) and \( U' = 0.9 \). For small \( t/U \) we observe \( \rho_k = \rho_d = 0 \), which indicates a Mottn insulating phase. However, at \( t/U \gtrsim 0.02 \) the system enters the Borromean supercounterfluid phase, where \( \rho_d/\rho_k \) rapidly saturates at the value \(-1/2 \) for which the coefficient \( p_{N=3} \)
When further increasing $t/U$ which presents $\Delta n$ numbers forms bound pairs between two components only, and in the system. Namely, suppose the system spontaneously becomes black. These particle-numbers distributions clearly display a miscible phase which is corroborated by $\Delta n$ for the largest $L$ on logarithmic scales. This shows that $\Delta n \to 0$ as $L \to \infty$. The bottom panels present two representative real-space particle-number distributions, i.e., imaginary-time slices of the world-line configuration, with $L = 60$ for $t/U = 0.02$ (b) and $t/U = 0.05$ (c). The three different components are represented by the colors red, green, and blue, respectively. The darker the color, the fewer the particles occupying the corresponding site such that an empty site becomes black. These particle-numbers distributions clearly display a miscible phase which is corroborated by $\Delta n$ of panel (a). In all cases the values $U = 1$ and $U' = 0.9$ were used.

In conclusion, we have demonstrated microscopically that a strongly correlated three-component bosonic mixture—realizable in optical lattice setups—has a phase with “super” transport phenomenon different from conventional superfluidity. In this phase, the simultaneous co-flow of all three bosonic components is arrested, while the system retains dissipationless counterflows between any pair of components. These three counterflows are not independent but rather described by two superfluid degrees of freedom. At the microscopic level, the types of counterpropagating partners can vary.

Possible realization of these states in optical lattices could be obtained by trapping mixtures of bosonic isotopes of Na and K. Possible ways to detect the Borromean supercounterfluid state experimentally is through tilting the lattice and detecting the ratio between transport of different individual components. However, the more striking signature can be obtained by observing a dramatic change in the system’s rotational response. Namely, a rotating conventional superfluid can be described by introducing a fictitious vector potential $\Theta$, leading to a vortex lattice
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