Materials with a stochastic microstructure, like foams, typically exhibit low mechanical stiffness, whereas lattices with a designed microarchitecture often show notably improved stiffness. These periodic architected materials have previously been designed by rule, using the Maxwell criterion to ensure that their deformation is dominated by the stretching of their struts. Classical designs following this rule tend to be anisotropic, with stiffness depending on the load orientation, but recently, isotropic designs have been reported by superimposing complementary anisotropic lattices. We have designed stiff isotropic lattices de novo with topology optimization, an approach based on continuum finite element analysis. Here, we present results of experiments on these lattices, fabricated by additive manufacturing, that validate predictions of their performance and demonstrate that they are as efficient as those designed by rule, despite appearing to violate the Maxwell criterion. These findings highlight the enhanced potential of topology optimization to design materials with unprecedented properties.

Materials with a stochastic microstructure, like foams, typically exhibit low mechanical stiffness, whereas lattices with a designed microarchitecture often show notably improved stiffness. These periodic architected materials have previously been designed by rule, using the Maxwell criterion to ensure that their deformation is dominated by the stretching of their struts. Classical designs following this rule tend to be anisotropic, with stiffness depending on the load orientation, but recently, isotropic designs have been reported by superimposing complementary anisotropic lattices. We have designed stiff isotropic lattices de novo with topology optimization, an approach based on continuum finite element analysis. Here, we present results of experiments on these lattices, fabricated by additive manufacturing, that validate predictions of their performance and demonstrate that they are as efficient as those designed by rule, despite appearing to violate the Maxwell criterion. These findings highlight the enhanced potential of topology optimization to design materials with unprecedented properties.

INTRODUCTION
Architected materials are ubiquitous in nature and engineering materials alike. For example, wood, bone, and sponge are biological architected materials with low weight and superior mechanical properties (1). Lightweight architected materials are often desirable for a wide range of engineering applications such as heat exchange (2), catalysis (3), mechanical damping (4–6), and biocompatibility (7). Formally, architected materials can be categorized into periodic and stochastic (non-periodic) structures; in both cases, their performance depends strongly on their relative density (the inverse of porosity) and the constituent material from which the architecture is formed. For most architected materials, the mechanical performance drops rapidly with decreasing relative density (increasing porosity). This loss of performance results from the quadratic or even stronger scaling relationship between the elastic modulus and relative density, that is

\[ E/E_s \propto (\rho/\rho_s)^n \]  

where \( E \) and \( \rho \) are, respectively, the Young’s modulus and mass density of the architected material, \( s \) denotes the respective value of the constituent solid’s material property, and \( n \) is the scaling power index (\( 1 \)).

Young’s modulus measures the stiffness of a material under uniaxial loading. In principle, the high scaling power index, \( n \), is due to inefficient “bend-dominated” deformation mechanisms within the architecture of conventional architected materials; as a result, \( n \approx 2 \) for most periodic structures, and \( n \approx 3 \) for most stochastic structures (4). As one example, the Young’s modulus of a recently reported ultralight silver aerogel (8) decreases from \( \sim 1.7 \times 10^4 \) to \( \sim 78 \) Pa (by three orders of magnitude) when the density is decreased from 48 to 4.8 mg/cm\(^3\) (by one order of magnitude). This inherent trade-off between light weight and mechanical performance poses a challenge in developing low-density architected materials where structural rigidity is also required.

To achieve the goal of high structural efficiency, i.e., maximized stiffness with minimized weight, a class of architected materials has been designed to use a fundamentally different, “stretch-dominated” deformation mechanism (9). For these materials, a linear scaling of \( n \approx 1 \) between stiffness and relative density can be achieved, i.e., reducing their weight by half costs only half the stiffness (9). Hereinafter, we will describe this property simply as linear scaling. A lattice can be shown to be stretch dominated, and, thus, to have linear scaling, by application of the Maxwell criterion (10). Assuming that its struts are connected by frictionless, zero-volume pin joints that allow free rotation between the struts, then, following (11), in three dimensions, this criterion requires

\[ r - m = s - 3j + k \]  

where \( r \) is the number of redundant struts (redundant from the standpoint of making the lattice statically determinant), equivalent to the number of self-stressed states; \( m \) is the number of extension-free mechanisms in the lattice; \( s \) is the number of struts; \( j \) is the number of joints; and \( k \) is the number of kinematic constraints, e.g., fixed motions of joints. Demonstration that \( m = 0 \) and \( r > 0 \) is sufficient to show stretch dominance under these assumptions (12).

Among the stretch-dominated designs, the octet truss periodic structure (13) is arguably the best known and most studied (9, 14). Although its linear scaling has been predicted for decades, it was not until recently that this property was successfully demonstrated through the state-of-art additive manufacturing methods (6). However, a study of the octet truss architecture shows that it is highly anisotropic (6). That is, at a given relative density, its stiffness when loaded normal to a unit cell face can be \( \sim 40\% \) lower than when it is loaded along the unit cell’s spatial diagonal (6), which may be suboptimal for many structural applications. Recognizing the potential shortcomings due to anisotropy, others have recently designed isotropic lattice architectures by superimposing two or more complimentary stretch-dominated anisotropic trusses, e.g., a pair for which the stiffest direction of one unit cell coincides with the least stiff direction of the other, and vice versa (12,15–17), such that they tend to balance each other. By carefully selecting the relative densities, and, thus, the relative stiffness, of each constituent lattice, the combination can be made isotropic. A common characteristic of these designs is groups of struts of different diameters, reflecting the different relative densities of each lattice needed to achieve isotropy.

A different approach to creating stiff isotropic lattices is to design them de novo using topology optimization. In this numerical methodology, an initial design is updated iteratively to improve its performance as

1Engineering Directorate, Lawrence Livermore National Laboratory, Livermore, CA, USA. 2Department of Mechanical and Industrial Engineering, University of Massachusetts Amherst, Amherst, MA, USA. 3Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, USA.

*Corresponding author. Email: wenchen@umass.edu (W.C.); watts24@llnl.gov (S.W.)
defined by an objective function, e.g., its Young’s modulus, while respecting various constraints, e.g., on its relative density and anisotropy (18, 19). The mechanical performance is evaluated numerically using continuum finite element analysis, in contrast to the closed-form analytical expressions typically used in Maxwell criterion design (12), allowing the joints between struts to be modeled at their true physical size and composed of the same solid material as the struts. The manner in which the design is described and how it is updated depend on the choice of parameterization of the topology optimization problem (18).

A recent parameterization (20) allows one to guarantee a priori that the design will consist of straight cylindrical struts, i.e., that it will be in the same class of truss lattices under consideration. These lattices are known to be compatible with current additive manufacturing processes (4, 6); hence, architectures designed with this topology optimization parameterization can be expected to be manufacturable. A full discussion of the topology optimization problem—including the strut parameterization, typical randomly generated initial designs, the applied constraints, and the convergence criteria—may be found in (20).

Two unit cell designs with struts of only a single diameter, respectively with an octahedral and rectified cubic (ORC) structure and an oblate and quasi-spherical octahedral (OQSO) structure, are used (20). Lattices of these new unit cells are our focus herein, along with the octet truss (13), which serves as a benchmark of Maxwell criterion design; these lattices are shown in Fig. 1. We report on numerical simulations and experimental testing on the additively manufactured specimens shown in Fig. 1 to consider their directional stiffness performance. Unexpectedly, we found a linear scaling behavior in these new designs, despite the fact that their inherent deformation mechanisms are not fully stretch dominated when they are considered from a Maxwell criterion standpoint. That is, the ORC and OQSO structures are effectively stretch dominated, in the sense that they display linear scaling, regardless of whether an actual stretch-dominated deformation is present at the microscale. Our findings offer a fundamentally novel insight into designing lightweight, isotropic, and stiff truss architected materials and complement the design strategy beyond the classical Maxwell criterion.

Static equilibrium in deformable bodies requires that loads applied to the body’s surface must be balanced by internal stresses. Under the assumption of linear elasticity, the stress σ at any point x in the body depends linearly on the strain ε (the linearized, symmetric gradient of the displacement vector field u) at x, with this linear relationship described by the fourth-order elasticity tensor C. That is, σ = Cε. So long as linear elasticity is a valid model, this relationship holds regardless of the length scale and regardless of the material type, e.g., metal, ceramic, or polymer.

When designing at the macroscale, e.g., an airplane wing, it is impractical to resolve the details of the microscale, e.g., the individual struts of an architected material of which the wing may be composed. What is needed to satisfy equilibrium at the macroscale is only the effective response of the microscale, i.e., its properties if we treated it as a homogeneous material. Given the microarchitecture, this homogenized response, encoded in a homogenized elasticity tensor \( C^{h} \), can be computed numerically via asymptotic analysis (19, 21). Then, at the macroscale, the constitutive relationship becomes \( \sigma = C^{h} \), and we can ignore the microstructural detail.

If this relationship does not depend on the load orientation, i.e., if the values of \( \sigma \) after an arbitrary rotation match those of \( C^{h} \) after \( \varepsilon \) is similarly rotated, then the material is isotropic. In this case, the values of elastic moduli such as the Young’s modulus \( E^{h} \) and the shear modulus...
which are scalar descriptors of \( C^h \) that depend on loading type, have a single value that similarly does not depend on the load orientation. Many common natural and engineering materials—including many metals, ceramics, and polymers—are isotropic at the macroscale.

Within the set of anisotropic materials, i.e., those for which \( C^h \) is not isotropic, the subset with behavior closest to the isotropic case is materials with cubic symmetry; this subset includes many common microarchitectures, such as the octet truss (13), the ORC and OQSO trusses (20), and many others (22). Note that some cubic materials can be isotropic, although in general, they are not. For these materials, the representation of \( C^h \) as a matrix now depends on the choice of coordinate axes, and the values of the moduli \( E^h \), \( C^h \), and other moduli now depend on the load orientation. The convention (22, 23) is to use the orthogonal planes of material symmetry in the cubic case to define an orthogonal coordinate system and to report \( C^h \), \( E^h \), etc. in this basis. In crystallographic terminology, this basis is formed by the directions with Miller indices <001>.

**RESULTS**

**Predictions from finite element analysis**

In the standard coordinate system, the ORC and OQSO trusses, respectively, have Young’s modulus \( E^h \) approximately 5 and 38% greater than the octet truss at an equal relative density of 5% and using the same constituent material. At this relative density, both these trusses are additionally more nearly isotropic than the octet truss. The Zener ratio, which measures anisotropy in cubic materials (23, 24), is 1.927 for the octet truss, while it is 1.783 for the ORC truss and 1.000 (perfectly isotropic) for the OQSO truss.

When we compare these three unit cell designs over a range of orientations, as shown in Fig. 2, we find that ORC and OQSO trusses are stiffer than the octet truss over many orientations, not just in the standard <001> orientations. However, the octet is stiffer when loaded on the unit cell vertices (the <111> directions). In addition, we can vary the diameters of the struts in these trusses to vary their relative densities and repeat the homogenization calculations to recompute their values of the Young’s modulus \( E^h \). Our numerical results predict that all three designs exhibit linear scaling over a range of densities from at least 0.5 to 20% and beyond; the ORC and OQSO designs thus retain their higher stiffness over the octet truss over a range of densities and a range of loading directions.

**Stiffness dependence on load orientation**

The experimental values of the homogenized Young’s modulus \( E^h \) that we extracted from compression tests on the two orientations of the unit cells relative to the loading direction are plotted alongside our numerical predications for all orientations in Fig. 2 and are tabulated in table S1. Figure 2 displays the data on a polar graph, where the azimuthal coordinate represents the load orientation (with certain orientations labeled) and the radial coordinate represents the Young’s modulus, \( E^h \), which increases radially outward. The numerical data are also shown on “orientation spheres” with the common color axis representing the
stiffness and the cut plane through the sphere corresponding to the polar plot. Note that the degree of anisotropy in each of the unit cell designs, e.g., as measured by the Zener ratio, depends on the relative density of the unit cell; that is, the anisotropy depends on both the unit cell geometry and its topology. This effect, shown in fig. S1, is attributable to the struts getting thicker at higher relative densities; as they do so, they become stiffer in bending faster than in stretching since their area scales with the second power of their diameter while their area moment of inertia scales with its fourth power. This affects the effective response of the truss. The OQSO truss was designed for isotropy at a relative density of 5% but was built and tested at 20% because of print quality issues associated with the thin struts at the lower volume fraction and, accordingly, is not perfectly isotropic, although it is still less anisotropic than the octet truss. It is likely that a slight rearrangement of the struts within the OQSO unit cell would allow it to recover isotropy at other relative densities, although, in general, a separate configuration would be required for each relative density.

The experimental data agree with our numerical predictions at the two orientations that we tested; this validation suggests that our predicted homogenized elasticity tensors $C^h$ are correct and, thus, that our predictions at other orientations, e.g., the <111> directions, are also correct since these values are extracted from a common $C^h$ tensor. Note also that for the same reason, the agreement between the predicted <001> stiffness for the OQSO truss at 5% relative density and the measured <001> stiffness at 6% suggests that the predicted isotropy is correct. The data confirm the highly anisotropic response of the octet truss and the progressively less anisotropic response of the ORC truss and the OQSO truss. For example, at the tested relative density of 20%, the octet architecture has its highest stiffness of $E_{<001>}$ ≈ 27 MPa along the <111> direction, which is significantly higher than that of $E_{<110>}$ ≈ 22 MPa along the <110> direction and is ~70% higher than the lowest stiffness of $E_{<111>}$ ≈ 16 MPa along the weakest direction of <001> (Fig. 2B). In contrast, the ORC and OQSO optimized architectures show considerably less anisotropy. Specifically, $E_{<110>}$ ≈ 20, 22.5, and 25 MPa were measured for the ORC architecture along the <001>, <110>, and <111> orientations, respectively. More significantly, $E_{<001>}$ ≈ 25, 22.5, and 22.5 MPa were found for the OQSO architecture along these same orientations. The variation about the mean value of the OQSO design is approximately one-third that of the variation of the octet truss. These results show improved isotropy as well as comparable and even enhanced stiffness along certain directions in the topologically optimized architectures relative to the traditional octet architecture. Note that the ORC and OQSO architectures are composed of struts of a single diameter.

This is in contrast with other isotropic truss designs (12, 15–17) that each use struts of different diameters that can vary by 13% (15, 17) or more. A practical ramification is that the proposed new designs can more readily be additively manufactured than other isotropic designs where the different strut sizes can complicate postbuild processing due to, for example, the resulting different diffusion lengths during polymer curing or nonuniform thermal shrinkage during post-sintering of metal or ceramic parts that may cause processing defects (e.g., micro-cracks). Struts of uniform diameter may also serve to reduce stress concentrations at the microscale, leading to higher macroscale failure stress and longer fatigue life.

**Linear scaling**

In addition to the enhanced isotropy and stiffness, we observed that the high stiffness in the optimized architectures relative to the octet truss benchmark is retained over a large range of relative densities, as shown in Fig. 3 and fig. S2 by both experimental measurements and numerical simulation. We found that the relative stiffness, $E_{\text{rel}}$, decreases in direct proportion to decreasing relative density, $p/p_o$, regardless of the architecture; that is, all three considered trusses display linear scaling with a scaling power index $n = 1$. This result is well known for the octet truss (6), but it is new for the ORC and OQSO trusses. Linear scaling holds for each of the unit cells regardless of the load orientation, i.e., the stiffness in each of the <001>, <110>, and <111> directions scales linearly over the range of relative densities studied. Hence, each of the designs is effectively stretch dominated. Despite the qualitative agreement in linear scaling behavior between the experimentally measured and the numerically simulated stiffness, it is worth noting that the experimentally measured stiffness is unexpectedly slightly higher than that of numerically simulated at a similar relative density, which is likely due to the enhanced compliance from the nanoindentation machine during flat punch test and the viscoelastic characteristic of the cured polymers. Although the underlying origin of this variation warrants further study in the future, the main goal of the current study is to investigate the scaling behavior and compare our designed topology against the octet trusses using the same sample preparation protocols and mechanical testing procedures.

**DISCUSSION**

Since the ORC and OQSO trusses are formed of straight cylindrical struts, they are amenable to the same Maxwell criterion analysis used in the design of other isotropic lattices. Following the approach in (11), we form the equilibrium matrix $A$ for the $4 \times 4 \times 4$ lattices that we tested;
this matrix linearly relates forces $f$ applied to the joints of the lattice to tensions $t$ in the struts as

$$At = f$$  \hspace{1cm} (3)$$

and can be formed using only the spatial locations of the joints and their connectivity. The number of self-stressed states, $s$, and the number of mechanisms, $m$, are respectively equal to the dimension of the right and left null spaces of $A$. The Maxwell criterion can therefore be seen as an application of the rank-nullity theorem of linear algebra applied to the equilibrium matrix, as noted in (11).

This analysis, the results of which are tabulated in table S3, shows that both the ORC and OQSO designs contain extension-free mechanisms in their unit cells, as well as in lattices built from these unit cells. Since $m > 0$ in both cases, neither of these trusses is stretch dominated in the classic sense, in contrast to the octet truss, for which $m = 0$ always. However, both the ORC and OQSO trusses exhibit linear scaling, i.e., they are effectively stretch dominated. In the former case, further investigation of the null spaces of $A$ contained in the Supplementary Materials (table S3) shows that the mechanisms in the ORC truss are not activated by the uniaxial compressions that we considered; thus, this result is not fundamentally inconsistent with Maxwell criterion analysis. However, the same argument cannot be applied to the OQSO truss; its mechanisms should be activated by the loadings that we applied, yet it shows linear scaling.

Rather than indicating that our experimental results are erroneous, this result simply highlights the limitations of pin-joint analysis, particularly its assumption of perfectly rotating joints. Namely, the assumption that no torque can be transmitted between struts is overly simplistic. As we have shown, continuum finite element analysis that treats both the struts and joints as deformable material (Fig. 3B) shows that OQSO, as well as octet and ORC, has linear scaling and, thus, behaves in a stretch-dominated fashion for the loads that we considered. This suggests that one must be cautious in concluding that a structure is not stretch dominated; a flowchart illustrating the necessary checks is provided in Fig. 4.

We have investigated the elastic performance of three truss microarchitectures: the octet truss, the ORC truss, and the OQSO truss, the latter two of which were designed by topology optimization. Our numerical results indicate that both of these architectures have a higher stiffness-to-weight ratio than the octet truss as measured by the Young’s modulus in the standard orientation. In addition, both ORC and OQSO architectures are more isotropic than the octet architecture. Our experimental results on additively manufactured lattices validate these numerical findings and further validate our numerical prediction of near-linear scaling of the stiffness with the relative density. This confirms that the proposed ORC and OQSO architectures remain stiffer than the octet truss over a range of volume fractions and load orientations. These results are unexpected from the standpoint of Maxwell criterion analysis but can be understood with finite element analysis using continuum theory, as validated by our results. We suggest a sequence of analyses in increasing order of rigor and cost to confirm the effective stretch dominance of unit cells considered in the future.

**MATERIALS AND METHODS**

**Additive manufacturing of architected materials**

To validate our computational results in the previous section, we additively manufactured lattices composed of a $4 \times 4 \times 4$ array of the ORC truss and OQSO truss unit cell designs, along with the octet truss unit cell to serve as a benchmark. Projection microstereolithography (PµSL) is a microscale additive manufacturing technique that uses a spatial light modulator [a liquid crystal on silicon (LCoS) chip] as a dynamically reconfigurable digital photomask (6). PµSL is capable of fabricating three-dimensional microstructures with complex geometries in a bottom-up, layer-by-layer fashion. Figure 1 shows a schematic of the process. First, a computer-aided design model of the truss was sliced into a series of closely spaced horizontal planes, i.e., those with normal vector [001]. These two-dimensional slices were subsequently digitized as an image and transmitted to an LCoS chip, which projected an image of a 405-nm wavelength through a reduction lens into a bath of photosensitive resin. The exposed material was cured, and the stage on which it rests was subsequently lowered to repeat the process with the next image slice. All structures were built with a photosensitive resin formulation of poly(ethylene glycol) diacrylate (PEGDA) with 1.2 weight % (wt %) photoabsorber (Sudan 1) and 2 wt % photoinitiator [phenylbis(2,4,6-trimethylbenzoyl)phosphine oxide]. The resin was exposed to 110 mJ/cm² for each layer. After printing, the structures were cleaned with acetone and left to dry for 24 hours before mechanical testing. Since all specimens were built from the same constituent material and stereolithography protocol, the relative density has a one-to-one correspondence with the absolute density. Absolute densities matched between different unit cell designs to within manufacturing tolerance.

**Mechanical characterization**

Uniaxial compression tests were performed to evaluate the homogenized Young’s modulus, $E^\gamma$, of the fabricated architectures through nanoindentation on an MTS Nanoindenter XP, equipped with a flat punch stainless steel tip with a diameter of 1.52 mm. A loading-unloading cycle at a strain rate of $10^{-4}$ per second was conducted on each sample to extract $E^\gamma$ from the elastic slope (fig. S3). Bulk PEGDA, cured by ultraviolet cross-linking a solid sample of a dog-bone shape was also fabricated to determine the Young’s modulus of the parent material, $E_n$, for the architectures (fig. S4).
To characterize the dependence of Young’s modulus on load orientation, we fabricated cubic lattices of 4 by 4 by 4 unit cells of the octet, ORC, and OQSO truss architectures at a specific relative density of 20% in orientations [001] and [110] of the unit cell with respect to the build and test direction, as shown in Fig. 1. The overall lattice was 1 mm on each side; each unit cell was 250 μm on each side, and typical strut diameters were in the order of 25 μm. To characterize the stiffness scaling behavior with respect to relative density, each of the considered architectures was fabricated in the [001] orientation over a range of relative densities: 6, 10, 20, and 30%. These relative densities were achieved by uniformly varying the strut diameters while maintaining the 250-μm unit cell size, matching the numerical approach of the previous section. The strut diameter needed to achieve each relative density for each unit cell design was computed analytically for an ideal truss with cylindrical struts by ignoring the overlapping volume at the nodes, as represented in (9, 14). The pertinent formulae are contained in the Supplementary Materials. Mechanical testing was repeated on three specimens for each architecture, orientation, and relative density.

SUPPLEMENTARY MATERIALS

Supplementary material for this article is available at http://advances.sciencemag.org/cgi/content/full/5/9/eaaw1937/DC1

Materials and Methods

Supplementary Text

Fig. S1. Comparison of (an)isotropy at 5 and 20% relative density.

Fig. S2. Scaling behavior of stiffness as a function of relative density with both experimental and numerical simulation results as shown in Fig. 3.

Fig. S3. Representative loading-unloading compression curves by nanoindentation flat punch test.

Fig. S4. Tensile engineering stress-strain curves for the three-dimensional truss architectures was fabricated in the [001] orientation over a range of relative densities: 6, 10, 20, and 30%.

Table S1. A summary of the tensile behavior of bulk PEGDA dog-bone samples.

Table S2. The diameters of the struts in each of the truss microarchitectures additively manufactured for testing.

Table S3. Exploration of the equilibrium matrix \( A \) resulting from an initial stability analysis of the three truss architectures, assuming pinned joints.

Reference (25)

REFERENCES AND NOTES

1. L. J. Gibson, M. F. Ashby, Cellular Solids: Structure and Properties (Cambridge Univ. Press, 1999).

2. L. Valdevit, A. Pantano, H. A. Stone, A. G. Evans, Optimal active cooling performance of metallic sandwich panels with prismatic cores. Int. J. Heat Mass Transf. 49, 3819–3830 (2006).

3. C. Zhu, Z. Qi, A. A. Beck, M. Luneau, J. Lattimer, W. Chen, M. A. Worsley, J. Ye, E. B. Duoss, C. M. Spadaccini, C. M. Friend, J. Biener, Toward digitally controlled catalyst architectures: Hierarchical nanoporous gold via 3D printing. Sci. Adv. 4, eaax9459 (2018).

4. T. A. Schaedler, A. J. Jacobsen, A. Torrents, A. E. Sorensen, J. Lian, J. R. Greer, L. Valdevit, W. B. Carter, Ultralight metallic microlattices. Science 334, 962–965 (2011).

5. L. R. Meza, S. Das, J. R. Greer, Strong, lightweight, and recoverable three-dimensional ceramic nanolattices. Science 345, 1322–1326 (2014).

6. X. Zheng, H. Lee, T. H. Weisgraber, M. Shusteff, J. DeOtte, E. B. Duoss, J. D. Kurtz, M. M. Biener, Q. Ge, J. A. Jackson, S. O. Kucheyev, N. X. Fang, C. M. Spadaccini, Ultralight, ultra stiff mechanical metamaterials. Science 334, 1373–1377 (2014).

7. K. Arcuate, B. K. Mann, R. B. Wicker, Fabrication of off-the-shelf multilumen poly (ethylene glycol) nerve guidance conduits using stereolithography. Tissue Eng. Part C Methods 17, 27–38 (2010).

8. F. Qian, P. C. Lan, M. C. Freyman, W. Chen, T. Kou, T. Y. Olson, C. Zhu, M. A. Worsley, E. B. Duoss, C. M. Spadaccini, T. Baumann, T. Y.-J. Han, Ultralight conductive silver nanowire aerogels. Nano Lett. 17, 7171–7176 (2017).

9. V. S. Deshpande, N. A. Fleck, M. F. Ashby, Effective properties of the octet-truss lattice material. J. Mech. Phys. Solids 49, 1747–1769 (2001).

10. V. S. Deshpand, M. F. Ashby, N. A. Fleck, Foam topology: Bending versus stretching dominated architectures. Acta Mater. 49, 1035–1040 (2001).

11. S. Pellegrino, C. R. Calladine, Matrix analysis of statically and kinematically indeterminate frameworks. Int. J. Solids Struct. 22, 409–428 (1986).

12. T. Tangme-Dejean, D. Mohr, Elastically-isotropic truss lattice materials of reduced plastic anisotropy. Int. J. Solids Struct. 138, 24–39 (2018).

13. R. B. Fuller, Synergetic building construction, U.S. Patent No. 2,986,241 (1961).

14. M. C. Messner, M. I. Barham, M. Kumar, N. R. Barton, Wave propagation in equivalent continua representing truss lattice materials. Int. J. Solids Struct. 73, 55–66 (2015).

15. M. C. Messner, Optimal lattice-structured materials. J. Mech. Phys. Solids 96, 162–183 (2016).

16. F. W. Zok, R. M. Latture, M. R. Begley, Periodic truss structures. J. Mech. Phys. Solids 96, 184–203 (2016).

17. G. Gurtner, M. Durand, Stiffest elastic networks. Proc. R. Soc. A 470, 20130611 (2014).

18. M. Bendsoe, O. Sigmund, Topology Optimization: Theory, Methods, and Applications (Springer, 2003).

19. G. Allaire, Shape Optimization by the Homogenization Method (Springer, 2002).

20. S. Watts, D. A. Tortorelli, A geometric projection method for designing three-dimensional open lattices with inverse homogenization. Int. J. Numer. Methods Eng. 112, 1564–1588 (2017).

21. O. Sigmund, S. Torquato, Design of materials with extreme thermal expansion using a three-phase topology optimization method. J. Mech. Phys. Solids 45, 1037–1067 (1997).

22. J. B. Berger, H. N. G. Wadley, R. M. McMeeking, Mechanical metamaterials at the theoretical limit of isotropic elastic stiffness. Nature 543, 533–537 (2017).

23. A. F. Bower, Applied Mechanics of Solids (CRC Press, ed. 1, 2009).

24. C. Zener, Elasticity and Anelasticity of Metals (University of Chicago Press, 1948).

25. M. T. Heath, Scientific Computing: An Introductory Survey (McGraw-Hill, ed. 2, 2002).

Acknowledgments: We would like to thank L. B. Bayu Aji and S. O. Kucheyev for the experimental assistance on nanoindentation. Funding: This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory (LLNL) under contract DE-AC52-07NA27344. The authors gratefully acknowledge funding from LLNL LDRD projects numbered 14-SI-005 and 17-SI-005 as well as LLNL-JRNL-759177. Author contributions: Specimens were fabricated by J.A.J., W.L.S., and C.M.S. and tested by W.C. Numerical simulations were conducted by S.W. and D.A.T. Data analysis was performed by W.C., S.W., and D.A.T. Data analysis was performed by W.C., S.W., and D.A.T. Competing interests: The authors declare they have no competing interests. Data and materials availability: All data needed to evaluate the conclusions in the paper are present in the paper and/or the Supplementary Materials. Additional data related to this paper may be requested from the authors.

Citation: W. Chen, S. Watts, J. A. Jackson, W. L. Smith, D. A. Tortorelli, C. M. Spadaccini, Stiff isotropic lattices beyond the Maxwell criterion. Sci. Adv. 5, eaaw1937 (2019).