Noise robust target identification based on the wave-coefficients-2dimension case

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Abstract
To correctly identify a remote target in a noisy environment is very challenging. Usually, the accuracy of target recognition degrades under the condition of a low signal-to-noise ratio (SNR). Radar target identification based on wave-coefficients (WCs) is proposed and seems to be promising. We introduce the WCs of two-dimensional (2D) targets in this paper. The main problem of WC based target identification is that the extraction of wave-coefficients is an ill-posed problem. Thus the recognition results yielded in the presence of noise is not reliable. The regularization algorithm is exploited to extract the wave-coefficients, the regularization parameter is determined by the L-curve method. Simulation results using 4 2D targets show that the proposed scheme is effective and high target recognition rate is achieved in cases of low SNR scenarios.

Keywords: wave-coefficient, target’s identification, tikhonov regularization, L-curve method, 2D targets

(Some figures may appear in colour only in the online journal)
1. Introduction

Radar target identification (RTID) refers to the technique where an unknown target can be identified from received signals. This technique is of essential importance in radar engineering and has received a considerable amount of attention. A key issue in the design of an RTID system is the selection of proper features extracted from the raw echoes received. Various features are utilized to realize target recognition. Such as high resolution range profiles (HRRPs) [1, 2], synthetic aperture radar (SAR) images [3], and complex natural frequencies (CNFs) [4, 5]. HRRP describes the scattering centers distribution along the radial distance and provides geometrical and physical characters of the targets. Furthermore, HRRPs are easily obtained. Thus HRRP-based target recognition has been attracting a great amount of research attention. However, HRRP is extremely sensitive to the relative orientation between the radar and target; this is mostly due to its scattering center model assumption which is only valid in high frequency ranges [6]. A lot of work is done to relax its sensitivity to the azimuth angle [7, 8]. SAR and ISAR images are usually used for vessel target identification [9]. Obtaining SAR and ISAR images are usually time-consuming and sometimes difficult. The CNFs, also termed poles, are the only aspect independent target feature reported so far [10]. It is well known that the shape, size, structure and material properties of a target determine the CNFs. Notably, the target is uniquely characterized by a complete set of its CNFs. However, CNF-based identification schemes alone will not be sufficient for many targets of interest due to the low energy problem [11, 12]. Three-dimensional (3D) aerospace target recognition based on a new target feature termed ‘wave-coefficient’ (WCs) is proposed in [13]. The WCs method has the advantage of flexible application; theoretically, it can be applied to any frequency range with increasing sensitivity to the aspect angle when used in the higher frequency range. If the operating frequency is appropriately chosen, the shape of the WC does not vary much, even if the aspect angle changes considerably. Therefore the WCs for all candidate targets form an acceptable set of parameters by which the targets can be identified. Besides, it is very convenient to obtain credible WCs as only the data collected for a single aspect is required. Therefore, WCs might be an advisable feature vector for radar target identification. It is important to note that the extraction of WCs involves solving linear equations with a highly ill-conditioned coefficient matrix. Thus, the recognition results are not reliable when the received signals are contaminated by noise. Currently, the received target echoes contain noises and clutters in practice. Therefore, reliable recognition in a noisy circumstance is a hot research topic in the radar target recognition community [2, 5].

In this paper, the WC of a two-dimensional (2D) target is introduced. We investigate 2D target recognition based on WCs and we particularly focus on noise-robust feature extractions, namely, WCs extraction in a noisy environment. We say the algebraic problems $Ax = b$ and $\min \| Ax - b \|$ are discrete ill-posed problems if matrix $A$ is ill-conditioned. A typical ill-posed problem is the linear Fredholm integral equation of the first kind. To name a few, other examples can be found in a wide variety of applications: mathematical physics [14], geodesy [15], computerized tomography [16], and meteorology [17]. Most numerical schemes for solving discrete ill-posed problems attempt to overcome the problems due to the large condition number of $A$ by replacing the problem with a ‘nearby’ well-conditioned problem whose solution approximates the required solution. Such schemes are called regularization methods which include the choice of the regularization parameter; this parameter was determined by the so-called L-curve technique proposed by Hansen [18]. The L-curve method soon attracted a lot of attention as it seems mathematically attractive. In this study, our aim is to expand the WC concept to 2D target and implement the Tikhonov regularization method to extract the WCs. The regularization parameter is determined by the L-curve method. The performance of
the proposed approach is evaluated by simulating four 2D targets in free space under different SNRs.

The remainder of the paper is organized as follows: in section 2, we give a brief introduction to 2D target WCs. Section 3 introduces the Tikhonov regularization and the L-curve method. We investigate several important properties of the WCs extracted by means of the Tikhonov regularization algorithm in section 4. Then, we study the identification performances in a Gaussian noise scenario in section 5. Finally, a concluding remark is given in section 6.

2. The conception of the WCs of a 2D target

Consider that a TM wave is impressing on a 2D conducting cylinder. The scattered field only contains z-component and can be expressed as

\[ E_z^i(k, \rho, \phi_{im}) = - \sum_{n=-\infty}^{\infty} (-j)^n \frac{J_n(ka)}{H_n^{(2)}(ka)} H_n^{(2)}(kp)e^{i\phi_{im} - \phi_n} \]  

where \( \phi_{im} \) is the incident angle, \( \phi \) denotes the observed angle, \( a \) represents the radius of the cylinder, \( k = \omega/c \) is the wave number, \( \rho \) denotes the distance between the radar and target, \( J_n(\chi) \) represents the Bessel function of the first kind of order \( n \), \( H_n^{(2)}(\chi) \) is the Hankel function of the second kind of order \( n \). If the target were with arbitrary cross-section, as shown in figure 1, we assume the scattered field may be written in a similar way as a superposition of cylindrical wave-modes:

\[ E_z^i(k, \rho, \phi_{im}) = - \sum_{n=-\infty}^{\infty} (-j)^n B_n(\phi_{im}) \frac{J_n(ka_e)}{H_n^{(2)}(ka_e)} H_n^{(2)}(kp)e^{i\phi_{im} - \phi_n} \]  

in which \( B_n(\phi_{im}) \) represents the wave-coefficients to be extracted, and \( a_e \) denotes the equivalent radius that is defined by \( a_e = s/(2\pi) \) in this work, with \( s \) being the circumference of the target. The wave-coefficients are supposed to be dependent on the incident angle \( \phi_{im} \), but not sensitive to the angular frequency \( \omega \). Radar target identification is usually a far field problem, which means \( kp \gg 1 \), thus we have the asymptotic formula

\[ H_n^{(2)}(\chi) = \sqrt{\frac{2}{\pi \chi}} e^{-j(\chi - i\pi/2 - \pi/4)} \]
Consider the backscattering \((\phi - \phi_m = \pm \pi)\) circumstance, so (2) can be written as

\[
\tilde{E}_z^i(k, \phi_m) = -\sum_{n=-N}^{N} B_n(\phi_m) \sqrt{\frac{2}{\pi}} \frac{J_0(ka_c)}{H_n^{(2)}(ka_c)} e^{j\pi/4}
\]

where \(N\) is the truncation term. \(\tilde{E}_z^i(k, \phi_m)\) is the backscattered far-field and is given by

\[
\tilde{E}_z^i(k, \phi_m) = E_z^{\ast}(k, \phi_m)e^{j\beta_s}.
\]

The wave-coefficients may be extracted by many means, a way termed ‘Galerkin match’ is suggested in this study. We define \(F_m(k) = \frac{J_m(ka_c)}{H_m^{(2)}(ka_c)}\), multiply both sides of (4) by \(F_m^{\ast}(k)\), \(m = -M, -M+1, \cdots, M\). Here, \(M \geq N\), \(^*\) denotes the complex conjugate, and integrate over the frequency band. We have

\[
\sum_{n=-N}^{N} C_{mn}B_n = g_m
\]

where

\[
C_{mn} = \sqrt{\frac{2}{\pi}} e^{j\pi/4} \int_{k_{\text{min}}}^{k_{\text{max}}} F_m^{\ast}(k) \cdot F_n(k)dk
\]

\[
g_m = \int_{k_{\text{min}}}^{k_{\text{max}}} F_m^{\ast}(k) \cdot \tilde{E}_z^i(k)dk
\]

\(\tilde{E}_z^i(k)\) would in general be obtained from measurements. However, in this paper, we obtain it by numerical computation, and the method of moments (MoM) is exploited. The scattered field is rigorously expressed by the induced surface current on the target’s surface as

\[
\tilde{E}_z^i(k, \phi) = -\frac{\eta_0}{4} e^{j\pi/4} \frac{k}{2\pi} \int_{C} J_c(\rho') e^{j\beta_s'} dC
\]

\(\eta_0\) is the free space wave impedance, \(J_c(\rho')\) represents the induced current on the surface of the target, \(\hat{e}_s\) denotes the field point unit vector and \(\rho'\) is the source point vector. The wave-coefficients are obtained through solving (6) by the method of least square (LS). The corresponding formulation is

\[
\min_B \|CB - g\|_2
\]

The solution is straightforwardly given by

\[
B = (C^T C)^{-1} C^T g.
\]

However, in this work, matrix \(C\) is ill-conditioned, thus the seeking of the inverse matrix is quite unstable. The solutions will be hopelessly distorted even if the received signals are contaminated with a little noise. Hence, some sort of regularization of the problem is needed to diminish the influence of the noise.

3. Tikhonov regularization and the L-curve

Well-known regularization methods are the truncated singular value decomposition (TSVD) and Tikhonov regularization. A key issue of these regularization methods is the choice of the regularization parameter. This paper aims to implement the Tikhonov regularization to extract the WCs and the regularization parameter is determined by the L-curve.
3.1. Overview of Tikhonov regularization

As mentioned above, when discrete ill-posed problems $Ax = b$ subject to $\min \|Ax - b\|$ are solved numerically, some kind of regularization is required to ensure that the regularized solution $x$ is not too sensitive to the perturbations in $b$, and has an appropriately small seminorm $\|Lx\|$. Tikhonov regularization [19] is one of the best known regularization methods, its solution is defined as the solution to the following least squares problem

$$\min_x \{ \|Ax - b\|^2 + \lambda \|Lx\|^2 \}$$

(12)

where $\|\cdot\|$ denotes the Euclidian 2-norm, $\lambda$ is a positive constant used to control the size of the solution vector, and $L$ is a matrix that defines a (semi) norm on the solution through which the ‘size’ is measured [15]. For the Tikhonov regularization, we have $\sigma(\lambda) = \|Ax - b\|_2$ and $\eta(\lambda) = \|x\|_2$. For simplicity, $L$ is chosen to be the identity matrix, namely, the standard form Tikhonov problem is considered. The solution to (12) solves the problem

$$(A^TA + \lambda L^TL)x = A^Tb.$$  

(13)

As $\lambda$ increases, the (semi) norm of the solution vector decreases monotonically while the residual $\sigma$ increases monotonically. It is worth mentioning that the Tikhonov regularization method is only suitable for a real number problem; however, the matrix $[C]$ in (6), the coefficient $[B]$ and the simulated signals $[g]$ are complex numbers. Problem (6) needs to be converted into a real number problem, and thus becomes

$$C_{aug}B_{aug} = g_{aug}$$

(14)

where

$$C_{aug} = \begin{bmatrix} C_{re} & -C_{im} \\ C_{im} & C_{re} \end{bmatrix}$$

(15)

$$B_{aug} = \begin{bmatrix} B_{re} \\ B_{im} \end{bmatrix}$$

(16)

$$g_{aug} = \begin{bmatrix} g_{re} \\ g_{im} \end{bmatrix}$$

(17)

The subscript $re$ and $im$ in the formulae signify the real and imaginary part of the corresponding matrix.

3.2. The L-curve method

The L-curve was first proposed by Hansen in [20]. It is a parametric plot of $(\sigma, \eta)$. The underlying idea is that a good method for determining the regularization parameter for discrete ill-posed problem must incorporate information about the solution size in addition to containing information about the residual size. This is quite natural as we are striking an appropriate balance in keeping both of these values small. The L-curve has a distinct L-shaped corner, and this corner corresponds to a good balance between minimization of the sizes of the residual and the solution; notably, the corresponding regularization parameter $\lambda$ is the one we choose. There are two approaches of viewing the problem of corner location. The first is searching for the point on the curve nearest to the origin. This is apparently understandable if we learn by heart that an ideal L-curve consists of a vertical segment and a horizontal line. The second is fitting the discrete point with some curves and finding the point
on the L-curve where the curvature is maximum. We use the second one in this work. We fit a cubic spline curve to \((\sigma, \eta)\), then the curvature is given by

\[
\kappa = \frac{|\eta''|}{(1 + \eta'^2)^{3/2}}.
\]  

(18)

Here, the notation \('\) represents differentiation with respect to \(\sigma\). The regularization parameter is selected as the one corresponds to the maximum curvature.

4. Simulation results

Two assumptions are made in order to simplify the target identification process in this paper. First, we assume that a target has been detected and the backscattered signal contains only one target. Second, the unknown target is presumed to be among a set of candidate targets, we know \(a\ priori\) WCs of them.

4.1. Simulation setup

Four cylinders are chosen as the known targets, and their cross-sections are shown in figure 2. For convenience, the targets are tagged NO.1, NO.2, NO.3, and NO.4. The six edges of target NO.1 and target NO.4 have the same side-length. All the edges of target NO.2 have the same length as well. The right side of target NO.3 is a semi-circle with a 1 meter long radius, and the left side is an isosceles triangle. Target No.1 and NO.4 have the same circumference of \(3\pi\)
Figure 3. The L-curve for the Tikhonov regularization for target NO.4 at $\phi_{in} = 20^\circ$. The number marked is the optimal regularization parameter.

Figure 4. WCs at $\phi_{in} = 20^\circ$ of (a) target NO.1, (b) target NO.2, (c) target NO.3, (d) target NO.4.
meters, and target NO.2 and NO.3 have the same circumference of \(2\pi\) meters. The equivalent radius of this group is set to be the biggest one of all the targets, i.e. 1.5 meters. The frequency range is chosen to be 300 MHz – 600 MHz in this paper.

4.2. Distinction of the WCs

The truncation term is taken to be \(N = 25\), so that the dimension of the coefficients \((B(n): -25 \leq n \leq 25)\) is 51, but actually only 26 are stored due to symmetry \((B(-n) = B(n))\). The L-shaped plot of the regularized solution size versus the size of the corresponding residual is examined at \(\phi_0 = 20^\circ\) of target NO.4. The result is shown in figure 3. It can be seen that the curve indeed has a characteristic L-shaped appearance. Figure 4 presents the WCs of the four targets at the same incident angle.

From figure 4 we can see that the WCs of different targets have significant differences from each other, which indicates that the WCs have obvious target distinctions, and therefore can be exploited for target identification.

4.3. The WC’s sensitivity to aspect angle

The right hand side of equation (6) varies with the incident angles, whereas equation (7) is independent of them. Consequently, WC changes with the incident angles, which are referred
to as target-aspect sensitivity. The correlation coefficient (CC) is used to evaluate the similarity of two feature vectors (namely two WCs in this work). Assume \( B_1 = \{B_1(n) : n = 0, 1, \cdots, N\} \) and \( B_2 = \{B_2(n) : n = 0, 1, \cdots, N\} \) to be two WCs. Then CC is defined as

\[
C(B_1; B_2) = \frac{\sum_{n=0}^{N} B_1(n) \cdot B_2^*(n)}{\sqrt{\sum_{n=0}^{N} |B_1(n)|^2 \cdot \sum_{n=0}^{N} |B_2(n)|^2}}
\]

where the asterisk denotes the complex conjugate. It is clear that \( 0 \leq C \leq 1 \) and \( C = 1 \), if and only if the two feature vectors are in proportion. The CC is an intuitive measure of similarity between the two feature vectors.

To study the WCs’ sensitivity to the target-aspect, an angle matching width (AMW) is introduced, in which the WCs are supposed to be independent of the target aspect. \( \delta \) is a prescribe CC threshold, for a given target \( i_0 \), matching width is defined as \( \text{AMW} = \phi_2 - \phi_1 \), where \( \phi_1 \) and \( \phi_2 \) are respectively the minimum and maximum values that satisfy

\[
C(i_0, \phi_l; \phi_0) \geq \delta, \quad l = 1, 2.
\]

In which \( \phi_0 \) is called the middle angle. Next, the CCs versus aspect angles for \( \phi_0 = 10^\circ \) and \( \phi_0 = 100^\circ \) are plotted in figures 5 and 6, respectively. The azimuth search window is \( \Delta \phi = \pm 6^\circ \). AMW for each target can be derived intuitively from these figures, 1–2 denotes...
the CCs between the WCs of target 2 in the search window and the middle angle WC of target 1, and so forth. Figures 5 and 6 show that the AMW varies from target to target and is relevant to the azimuth angle of the target. Suppose that a matching tolerability of 10% is acceptable, that is \( \delta \) in (20) is set to be 0.90 as we found only a few CCs larger than 0.9 in the examples we considered. In figure 5 the biggest AMW for the four cylinders is greater than 11° and at least 4°, in figure 6 the biggest AMW for the four cylinders is still more than 12° and at least 2°. The results are quite inspiring compared to the HRRPs based target recognition as the corresponding AMW is less than 0.5° [1]. However, this is mostly because the WCs are extracted in comparatively low frequency ranges. It is worth mentioning that the research in [13] manifests AMWs for 3D aircrafts are even larger. When constructing the feature vector database, if we investigate the matching width in all the aspect of interest (because of symmetry, the aspect of interest for target NO.1 is 0°–180°, for target NO.2 is 0°–45°, for target NO.3 is 0°–180°, and for target NO.4 is 0°–30°), the sampling interval \( \delta \phi \) should vary with the azimuth angle. Thus the number of WCs stored in the data base for the targets can be greatly reduced.

4.4. Identification algorithm

For an incoming WC \( X = \{X_n : n = 0, 1, \cdots, N\} \) belongs to an unknown target in a pre-estimated incident direction denoted by \( j_0 \), the CC value for target \( i \) is \( CC = C(i, j_i ; X) \), and the unknown target will be judged to be target \( i_0 \) if the maximum CC value is produced when \( i = i_0 \).

Based on the above decision rule, for a 2D targets situation, the proposed recognition procedure goes as follows:

Step 1: Build the database containing the WCs of K preselected targets at aspects of interest. Denote it by \( B_i(\phi) \) for each target \( i \) with the azimuth \( (\phi) \). The sampling interval \( \delta \phi \) is determined by the WCs’ sensitivity to incident angle \( \phi \). For simplicity, the azimuth angle is uniformly sampled and the sampling interval is fixed at 6° in this paper. Therefore, the feature vector database comprises 31 WCs for target NO.1, 8 for target NO.2, 31 for target NO.3 and 6 for target NO.4.

Step 2: Obtain the input WC \( X(\phi_0) \) of an unknown target and estimate its aspect \( (\phi_0) \) with respect to the radar. Denote it by \( (\hat{\phi}_0) \).

Step 3: Suppose the estimated errors are within \( \pm \Delta \phi \). Find all WCs \( \{B_i(\phi) : \phi_0 - \Delta \phi \leq \phi \leq \hat{\phi}_0 + \Delta \phi\} \) from the database, and compute the CCs \( \{C(X, B_i)/1 \leq i \leq K\} \).

Step 4: Identify the unknown target to be the one that provides the maximum CC.

5. Performance in gaussian noise environment

To investigate the performance of the proposed scheme in a noisy environment, an additive white Gaussian noise (AWGN) model is assumed, and the noise-corrupted received fields on the left-hand side of (5) is taken to be

\[
E_i^l(k_m) = E_i^m(k_m) + \nu_m, \quad m = 1, 2, \cdots, n_{freq}
\]

(21)

where \( n_{freq} \) is the number of frequency points, \( E_i^m(k_m) \) is the noiseless fields, i.e., the theoretical fields received by simulations, and \( \nu_m = \nu_m^R + \mu_m^I \) is a complex white Gaussian noise, i.e., both sequences \( \{\nu_m^R : m = 1, 2, \cdots, n_{freq}\} \) and \( \{\nu_m^I : m = 1, 2, \cdots, n_{freq}\} \) have zero mean values and variances \( \tau^2/2 \) with \( \tau^2 \) being the noise power calculated by
where $P_0$ is the power of the noiseless sequence $\{E_x^{m}(k_m) : m = 1, 2, \cdots, n_{freq}\}$. This paper aims to explore a noise-robust extraction method of the WCs. Two typical WCs at the aspect of $\phi_{in} = 13^\circ$ of target NO.1 and NO.2 are randomly taken as examples to check the performance of the proposed method. Figure 7 presents the WCs of NO.1 and NO.2 targets extracted under different SNRs.

The results show that the WCs extracted by the proposed regularization scheme in low SNRs match well with those extracted in high SNRs, which indicates the effectiveness of the proposed algorithm. In contrast, the conjugate gradient (CG) solver used in [13] fails to yield a stable result until the SNR is higher than about 20 dB while the common linear system solvers are invalid in this problem.

Figure 8 depicts the average recognition rate (the percentage of correct identifications in 1000 trials) versus SNR at two specified incident angles. As shown in figure 8, the recognition performance varies with the target aspect. At the aspect angle of $\phi_i = 10^\circ$ the best recognition performance is achieved for target NO.2, it begins to be correctly identified in every trial at an SNR of 0 dB. The worst is for target NO.4, which achieves a 63% recognition rate at an SNR of −5 dB and an approximate 90% recognition rate at an SNR of 0 dB. At the aspect angle of $\phi_i = 51^\circ$, the recognition rates are greater than 95% for all the targets even at a low SNR of −5 dB, which demonstrate the robustness of the proposed technique.
6. Conclusion

In this paper, we introduce the WC conception of 2D targets and develop a noise-robust approach, namely, the Tikhonov regularization method to extract wave-coefficients in the presence of noise. The regularization parameter is determined by the L-curve method. Numerical simulations of four 2D targets are conducted and encouraging recognition performances were acquired, thus verifying the correctness and efficiency of the proposed method. Further investigation of the WCs based target recognition method, such as using combined multiple polarization WCs, other classifier, and extending the regularization scheme to 3D targets are in progress.

Figure 8. Variation of the recognition performance versus SNR with the testing aspect angle (a) $\phi_{in} = 10^\circ$, (b) $\phi_{in} = 51^\circ$. 
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