Thermo magnetic response on flexural vibration of nanofibers using Timoshenko beam theory

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Abstract: In this study, the flexural vibration of nanofibers with surface effect in the presence of thermo magnetic field is studied using Modified Timoshenko beam equation. The frequency equations are obtained for nanofiber under surface effect. Numerical calculations are performed and the characteristics of dispersion curves are analyzed. Graphs are drawn for phase velocity with increasing wave number at different radius of nanofibers. The investigation shows that as the wave number increases the corresponding phase velocities increases at different radius. The results are used in designing nanodevices and analysis.

1. Introduction
Nanofibers are fibers of diameter 100nm or less. The nanofibers are used in several fields namely medical, chemical, mechanical and electrical due to its properties. Wu et al. [1] developed the surface rippling mechanism of nanofibers with axial tension. Wu et al. [2] analyzed the vibration of nanofibers with the influence of surface energy using nonlinear elasticity. Kong et al. [3] studied the vibration of cylindrical shaped nanofibers which is distributed randomly. Vibration of carbon nanotubes reinforced by electro spinning was investigated by Wan et al. [4]. Narendar et al. [5] analyzed the magnetic effect on vibration of carbon nanotubes with single walled under the influence of elastic medium. Lim and Yang [6] developed the nonlocal elastic stress theory on the vibration of carbon nanotubes.

Yuya and Turner [7] explored wave propagation of nanofibers with free ends. Results obtained by this method are used in experiments of electro spun nanofibers. Yuya et al. [8] reported a method for calculating Young’s modulus of a nanofiber. Ponnusamy and Amuthalakshmi [9-11] investigated the thermo magnetic effect on carbon nanotubes and nanoplate with nonlocal effect using Euler – Bernoulli and Timoshenko beam theory.

This paper studies flexural vibrations of nanofiber under the effect of thermo magnetic field using Timoshenko beam theory. Dynamic equation of nanofiber is derived by applying Lorentz’s force and axial force due to thermal effect. Dispersion relation in terms of wave number and phase velocities are derived using thermo magnetic field.
2. Governing Equation of Nanofiber

A nanofiber under the shear and rotation effect is considered and its elementary part of flexural vibration of nanofiber with the influence of magnetic effect is shown in Figure 1.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{Elementary part of nanofiber in flexural wave propagation}
\end{figure}

The equation of motion in vertical direction is

\[ Q - \left( Q + \frac{\partial Q}{\partial x} \right) + \left( EA\varepsilon_0 + 2\pi R\gamma + N_i \right) \frac{\partial^2 w}{\partial x^2} dx + q dx = \rho A \frac{\partial^2 w}{\partial t^2} dx \]  

(1)

where \( E, Q, A, \varepsilon_0, R, \gamma, \rho \) and \( w \) respectively denote Young’s modulus, shear force, cross-sectional area, prestrain, radius of nanofiber, surface tension or stress, mass density and deflection of the nanofiber and \( N_i \) is the force due to thermal effect given by

\[ N_i = -\alpha_x E A T \]  

(2)

where \( \alpha_x \) is the coefficient of thermal expansion and \( T \) is change in temperature.

Summing the moments at the centre of element about an axis perpendicular to \( x, y \) plane, the following equation is obtained

\[ -M + \left( M + \frac{\partial M}{\partial x} dx \right) - \frac{1}{2} Q dx - \frac{1}{2} \left( Q + \frac{\partial Q}{\partial x} dx \right) + (EA\varepsilon_0 + 2\pi R\gamma) \frac{\partial w}{\partial x} = \rho I \frac{\partial^2 \phi}{\partial t^2} \]  

(3)

where \( M \) the bending moment, \( I = \int_A w^2 dA = \pi R^4/4 \) is the inertial force of the nanofiber cross-section and \( \phi \) is the angle made of pure bending moment.

Cancelling terms in Eqs. (1) and (3), we obtain the basic equations for flexural vibration of nanofiber with influence of thermal field as

\[ \rho A \frac{\partial^2 w}{\partial t^2} + \frac{\partial Q}{\partial x} - \left( EA\varepsilon_0 + 2\pi R\gamma + N_i \right) \frac{\partial^2 w}{\partial x^2} - q = 0 \]  

(4)

\[ \rho I \frac{\partial^2 \phi}{\partial t^2} + Q - \left( EA\varepsilon_0 + 2\pi R\gamma \right) \frac{\partial w}{\partial x} - \frac{\partial M}{\partial x} = 0 \]  

(5)

The bending force \( M(x,t) \) and shear force \( Q(x,t) \) are taken from Graff [12] as

\[ M = \int_A w \sigma dA \]  

(6)

\[ Q = \int_A \tau dA \]  

(7)

where \( \sigma \) and \( \tau \) are the normal and shear stress respectively.
A longitudinal magnetic field $H(x,0,0)$ is applied along with magnetic permeability $\eta$ on the nanofiber, therefore a Lorentz force originates on nanofiber. Hence the pressure exerted on nanofiber due to Lorentz force can be expressed as Karilicic et al. [13] as

$$q = \int_A f_x dA = \eta A H_i \frac{\partial^2 w}{\partial x^2}$$

where $f_x$ denotes force due to Lorentz effect along $z$ direction.

By Hooke’s law generalization, the basic relation between the stress and strain in one-dimensional case are given by

$$\sigma = E \epsilon$$

$$\tau = G \gamma_0$$

where $G = E/2(1+\nu)$ denotes shear modulus, $\epsilon$ and $\gamma_0$ denote the normal and shear strain given by

$$\epsilon = w \frac{\partial \phi}{\partial x}$$

$$\gamma = \phi - \frac{\partial w}{\partial x}$$

Substituting Eqs. (11) and (12) in Eqs. (9) and (10) and then using Eqs. (6) – (7) in the resulting equation, the equation for bending force and shear force can be obtained as follows

$$M = EI \frac{\partial \phi}{\partial x}$$

$$Q = G \left( \phi - \frac{\partial w}{\partial x} \right) \beta A$$

where $\beta = 6 + 12\nu + 6\nu^2/7 + 12\nu + 4\nu^2$ is the coefficient of shear that depends on the shape of cross-section. Using Eqs. (13) - (14) in Eqs (4) - (5), we get

$$\rho \frac{\partial^2 w}{\partial t^2} + \beta G \left( \frac{\partial \phi}{\partial x} - \frac{\partial^2 w}{\partial x^2} \right) \left( E\epsilon_0 + \frac{2\gamma}{R_0} - \alpha ET - \eta H_i \right) \frac{\partial^2 w}{\partial x^2} = 0$$

$$\rho I \frac{\partial^2 \phi}{\partial t^2} + \beta AG \left( \phi - \frac{\partial w}{\partial x} \right) \left( E\epsilon_0 + 2\pi R_0 \gamma \right) \frac{\partial^2 \phi}{\partial x^2} - EI \frac{\partial^2 \phi}{\partial x^2} = 0$$

3. Solution of the Problem

Assume the solution of the nanofiber as

$$w(x,t) = A \exp ik(x-ct)$$

$$\phi(x,t) = B \exp ik(x-ct)$$

Where $\hat{A}$ and $\hat{B}$ are amplitudes, $k$ the wave number and $c$ denotes the phase velocity. Substituting Eq. (17) in Eqs. (15) – (16), the equation in terms of amplitudes are obtained as follows

$$\left( -\rho c^2 + \beta G + E\epsilon_0 + \frac{2\gamma}{R} - \alpha ET - \eta H_i \right) k^2 A + i\beta GkB = 0$$

$$-iA \left( \beta G + E\epsilon_0 + \frac{2\gamma}{R} \right) k^2 A - \left( -\rhoIk^2 c^2 + \beta GA + ELk^2 \right) B = 0$$
Eqs. (18) and (19) are homogeneous equation. On solving them a trivial solution is obtained. If the determinant of the coefficient matrix is zero, then a non-trivial solution can be obtained as follows

\[
\left[ \begin{array}{c}
-\rho c^2 + \beta G + E \varepsilon_0 + \frac{2\gamma}{R} - \alpha_s ET - \eta H_s^2 \\
-ikA \left( \beta G + E \varepsilon_0 + \frac{2\gamma}{R} \right) - \rho ik^2 c^2 + \beta GA + EIk^2 \end{array} \right] i\beta Gk = 0
\]  

(20)

On solving the determinant given in Eq. (20), a dispersion relation is obtained as follows

\[
a_{i1}c^4 + a_{i2}c^2 + a_{i3} = 0
\]

(21)

where

\[
a_{i1} = \rho^2 ik^2, \quad a_{i2} = -\rho \left[ \left( \beta G + E \varepsilon_0 + \frac{2\gamma}{R} - \alpha_s ET - \eta H_s^2 \right) ik^2 + \beta GA + EIk^2 \right] \quad \text{and}
\]

\[
a_{i3} = \left( \beta G + E \varepsilon_0 + \frac{2\gamma}{R} - \alpha_s ET - \eta H_s^2 \right) EIk^2 - \beta GA \left( \alpha_s ET + \eta H_s^2 \right).
\]

Eqs.(21) yields two phase velocities in which the lower value denotes the flexural wave and the higher value denotes the transverse shear wave. If magnetic field strength along with temperature terms are neglected the result matches with the result of Wu and Dzenis [14] and it shows the accuracy of the present result.

4. Numerical Results and Discussion

In this paper, the dispersion relation of a nanofiber with surface effect under a thermal and longitudinal magnetic field has been derived using Timoshenko beam theory. The material parameters are taken from Wu and Dzenis [14] as Young’s modulus \( E = 100\, MPa \), Poisson’s ratio \( \nu = 0.5 \), mass density \( \rho = 2000\, kg/m^3 \) and surface tension \( \gamma = 0.05\, N/m \). From Chang [15] the temperature and coefficient of thermal expansion are considered as \( T = 40K \) and \( \alpha_t = -1.6 \times 10^{-6}\, K^{-1} \). The numerical value of longitudinal magnetic field and magnetic permeability are taken from Li et al. [16] as \( H_s = 10^7\, A/m \) and \( \eta = 4\pi \times 10^{-7}\, H/m \). The prestrain \( \varepsilon_0 \) is assumed as zero.

Dispersion curves of a nanofiber in the absence of surface effect under thermal and longitudinal magnetic field for lower value at different nanofiber radius is drawn and is shown in Figure 2. From Figure 2, it is observed that as the nanofiber radius increases the phase velocities decreases. Also it is observed that as the wave number increases the phase velocities increases and finally becomes asymptotic with the horizontal axis.

![Figure 2](image_url)

**Figure 2.** Dispersion curve of nanofiber without surface effect under magnetic and thermal field at different nanoradius.
Dispersion curves of a nanofiber with surface effect under magnetic and thermal field for lower value at distinct nanofiber radius are drawn and represented in Figure 4. From Figure 4, it perceived that as the nanofiber radius increases the phase velocities decreases. Also it is observed that the value of phase velocities with surface effect is higher than without the surface effect.

**Figure 3.** Dispersion curves of a nanofiber with surface effect under magnetic and thermal field at distinct nanoradius

Dispersion curves of a nanofiber with surface effect under magnetic and thermal field at different magnetic field strength is drawn and is represented in Figure 4. Figure 4 perceived that as the magnetic field strength increases the phase velocities also increases.

**Figure 4.** Dispersion curves of nanofiber under magnetic and thermal effect at distinct magnetic field strength
Change in frequency with respect to wave number at distinct nanofiber radius is drawn and is presented in the form of Figure 5, it is perceived that as the wave number increases the frequency increases. Further as the nanofiber radius increases the value of frequency decreases.

![Figure 5. Change in frequency and wave number under magnetic and thermal field at distinct nanofiber radius](image)

Change in phase velocity and wave number at distinct nanofiber radius in absence of magnetic and thermal field is drawn and is presented in the form of Figure 6. From Figure 6, it is perceived as the wave number increases the phase velocities increases. It is also observed that for greater nanofiber radius the phase velocities are also higher and this graph matches exactly with the graph of author Wu and Dzenis [14] and it shows the exactness of the present result.

![Figure 6. Change in phase velocity with wave number at distinct nanofiber radius](image)

5. Conclusions
This study investigates effect of thermal and longitudinal magnetic field on flexural vibrations of nanofiber under surface effect using Timoshenko beam theory. Characteristics of flexural vibrations in
nanofibers are studied through dispersion curves. It is concluded that for each wave number there corresponds two phase velocities. The influence of longitudinal magnetic field increases the phase velocities and the surface effect decreases phase velocities for increasing nanofiber radius. Further the value of phase velocities surface effect is lower than without the surface effect.

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