Occurrence of Warm Freezing Rain: Observation and Modeling Study

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Abstract Freezing rain has been normally considered to be composed of supercooled raindrops when the 2 m air temperature (hereafter $T_w$) is below freezing. However, according to a statistical survey of freezing rain observations in China from 2000 to 2019, we find that there were 656 events that occurred at $T_w$ greater than 0°C (hereafter warm freezing rain and denoted by WFR), which account for 7% of the total freezing rain observations. Additionally, nearly 3% (266 observations) of freezing rain events occurred when the near-surface wet-bulb temperature was greater than 0°C. The modeling and sensitivity experiments on the nonequilibrium raindrop temperature (hereafter $T_r$) show that the temperature difference between raindrops and the atmosphere is the main cause of WFR. The magnitudes of the $\Delta T_w$ (difference between raindrop temperature $T_r$ and air temperature $T_w$) and $\Delta T_{aw}$ (difference between $T_r$ and wet-bulb temperature $T_{aw}$) are determined by the raindrop diameter $D$, temperature lapse rate $\Gamma$, and relative humidity $RH$. Increases of $D$ and $\Gamma$, and a decrease of $RH$ enhance $\Delta T_w$ and $\Delta T_{aw}$ and thus the occurrence of WFR. Further simulations of 4 idealized and 370 real sounding profiles reveal that either the $T_w$ or the $T_{aw}$ cannot properly distinguish the WFR events. When considering the temperature difference between raindrops and the atmosphere, the WFR can form by the “melting of solid hydrometeors” or “supercooled warm rain process.” This study can also deepen our understanding of the conditions of WFR and freezing rain formation at different altitudes.

Plain Language Summary It has been normally thought that freezing rain occurs when the air temperature or wet-bulb temperature is below 0°C. However, in our analysis of 9,312 freezing rain events in China over a 20-year period, nearly 7% of freezing rain events occurred when the air temperature >0°C (called warm freezing rain [WFR]). Even use of the wet-bulb temperature cannot explain all of the WFR events. Therefore, we developed a theoretical model to explain the formation mechanism of the WFR and analyze the key influencing factors. It is found that the temperature difference between the raindrop and the surrounding atmosphere is the main cause of WFR. An increase of raindrop diameter, an increase of atmospheric temperature lapse rate, and a decrease of relative humidity enhance the temperature difference between raindrops and the atmosphere, favoring the occurrence of WFR.

1. Introduction

Freezing rain is supercooled liquid precipitation near the surface, that is (raindrop temperatures are below 0°C), posing one of the major winter weather hazards in countries such as the United States, Canada, Sweden, Russia, and China (Adhikari & Liu, 2019; Andersson & Chapman, 2011; J. V. Cortinas et al., 2004; Deng et al., 2012; Sanders & Barjenbruch, 2016). Long-term persistence of freezing precipitation can not only damage the electricity system, transportation, and public infrastructure by causing tower collapses, line freezing, and poor communication but also result in economic losses and human casualties (Kiessling et al., 2014). For example, from January to February 2008, a long-term and wide-ranging freezing rain event occurred in southern China that caused direct economic losses of $18.7$ billion (Zhao et al., 2008). Therefore, studies on the microphysical process of freezing rain can deepen our understanding of freezing rain formation and provide an important scientific basis for accurate forecasting, thus reducing economic losses caused by freezing rain.

Numerous studies have been conducted on the formation process of freezing rain and two microphysical mechanisms have been proposed: “melting mechanism” (Rauber et al., 1994, 2001; Tobin & Kumjian, 2017) and “supercooled warm rain mechanism” (Huffman & Norman, 1988; Roberts & Stewart, 2008). In the “melting
mechanism,” snow or ice crystals that formed in the upper air completely melt into raindrops as they fall through a warm layer (air temperature $T_r > 0^\circ\text{C}$). Subsequently, the raindrops fall into a subfreezing layer ($T_r < 0^\circ\text{C}$) and become supercooled raindrops. In the “supercooled warm rain mechanism,” freezing rain grows by collision and coalescence of supercooled cloud droplets and raindrops, and the air temperature is maintained below 0°C as the raindrops fall.

Based on these two mechanisms, several parameterization schemes have been developed to predict the freezing rain, including Huffman and Norman (1988), Ramer (1993), Czys et al. (1996), Zerr (1997), Bourgouin (2000), Thériault and Stewart (2010), Schuur et al. (2012), Benjamin et al. (2016), Reeves et al. (2016), and Kämäräinen et al. (2017). Some of the above schemes only consider the “melting mechanism” of freezing rain. For example, Czys et al. (1996) established a diagnostic model based on the “melting mechanism.” In this model, freezing rain is determined when snowflakes or ice crystals completely melt in the melting layer and fall into the subfreezing layer near the ground. Bourgouin (2000) developed a freezing rain diagnosis method by considering the energy area enclosed by the $T_a$ profile and 0°C isotherm, which has been operationally used in the National Weather Service (NWS; Birk et al., 2021). Some other schemes considered both the “melting mechanism” and “supercooled warm rain mechanism” for predicting freezing rain, such as the methods developed by Ramer (1993) and Huffman and Norman (1988). In addition, the refreezing process of mixed-phase particles can also affect the formation of freezing rain (Barszcz et al., 2018; Thériault & Stewart, 2010). Cholette et al. (2020) reduced the overestimation of the freezing rain rate in the WRF (Weather Research and Forecasting) model by adding the prognostic equation for the liquid fraction of mixed-phase particles to the Predicted Particle Properties (P3) schemes (Morrison & Milbrandt, 2015). Hanesiak and Stewart (1995) suggested that supercooled raindrops in the refreezing layer would refreeze through collision with ice crystals, thus reducing the amount of freezing rain near the ground. By adjusting the temperature threshold for the collision of raindrops with ice crystals in the MY2 (Milbrandt & Yau, 2005) scheme, Barszcz et al. (2018) significantly improved the accuracy of freezing rain forecasting in Canada’s 2.5-km numerical weather prediction model.

The above studies provide theoretical support for the formation of freezing rain. However, these approaches all use the air temperature ($T_a$) or wet-bulb temperature ($T_w$) to determine the precipitation types instead of the raindrop temperature ($T_r$). For example, some methods assume that the surface air temperature below 0°C is the necessary condition for freezing rain formation (Benjamin et al., 2016; Bourgouin, 2000; Czys et al., 1996; Kämäräinen et al., 2017), while others use $T_r$ to distinguish between freezing rain and rain (Birk et al., 2021; Ramer, 1993; Reeves et al., 2016; Schuur et al., 2012; Thériault & Stewart, 2010).

However, the evaporation of raindrops causes $T_r$ to be lower than $T_a$ in a subsaturated atmosphere. In addition, $T_a$ is the lowest temperature that can be reached under current ambient conditions by the evaporation of water only (Dunlop, 2008). The value of $T_r$ only approaches that of $T_a$ when the raindrop and atmosphere reach the dynamic equilibrium (Caplan, 1966). Obviously, replacing $T_a$ with $T_a$ or $T_w$ in the freezing rain diagnosis method is only an idealized assumption. Since the time scale of changes in the atmospheric conditions is often shorter than the relaxation time scale of raindrops, the raindrop cannot maintain an equilibrium state when they are falling in the real atmosphere (Tardif & Rasmussen, 2010). Therefore, the variation of $T_r$ always lags behind that of $T_a$ and $T_w$ (Caplan, 1966; Gosnell et al., 1995; Wu, 1991). Some observational studies have verified this theory. For example, the observational study by Anderson et al. (1998) showed that the mean difference between $T_r$ and $T_a$ is 0.4°C when raindrops are falling. Further studies have shown that the temperature difference between raindrops and the atmosphere depends upon many factors, including season, time of day, and other meteorological conditions (Khain et al., 2011; Lee & Feingold, 2010; Salamalikis et al., 2016).

In this study, we first analyzed the freezing rain observed in China over the last two decades (from 2000 to 2019). A total of 9,312 freezing rain observations were identified, 93% of which occurred when the near-surface air temperature (air temperature at 2 m above ground level, hereafter $T_{aw}$) was ≤0°C. However, nearly 7% (656 observations) of the freezing rain could not be explained by air temperature and occurred at $T_{aw} > 0^\circ\text{C}$. We refer to this phenomenon as warm freezing rain (hereafter WFR). Additionally, nearly 3% (266 observations) of the freezing rain events occurred when the surface wet-bulb temperature (hereafter $T_{aw}$) was >0°C. Some researchers have noticed this phenomenon (Bernstein, 2000; J. Cortinas, 2000; J. V. Cortinas et al., 2004; Houston & Changnon, 2007; Roberts & Stewart, 2008), but the WFR formation has never been further studied. To explain the formation of WFR, we then present a theoretical model by relaxing the equilibrium assumption of $T_r$ to consider the nonequilibrium heat exchange between raindrops and the ambient atmosphere. Several sensitivity studies have
been conducted using this model to examine the effects of key meteorological variables on $T_r$, including raindrop diameter ($D$), temperature lapse rate ($\Gamma$), and relative humidity (RH). Finally, the formation of WFR and the corresponding meteorological conditions are discussed.

2. Observational Analysis

The observation data were obtained from the China Meteorological Administration (CMA). They include radiosonde data from 89 sounding stations across China at 00:00 and 12:00 UTC, and surface temperature, dewpoint temperature, and weather phenomena data from 2,168 ground stations at 00:00, 06:00, 12:00, and 18:00 UTC from January 2000 to December 2019. The spatial distributions of the sounding and ground stations are shown in Figure 1.

The $T_w$ was measured with standard mercury thermometers (X. Q. Wen et al., 2008) by trained meteorologists at the ground stations. The precision of this thermometer was 0.1°C and the measurement error of the instrument was ±0.2°C. The observers read the temperature data manually, and the reading was accurate to 0.1°C without the estimation rounding method. In addition, when rain was observed to form the ice glaze on the ground or objects, it was recorded as freezing rain (CMA, 2011).

These data were subjected to quality control and homogeneity tests by trained meteorologists following the CMA procedure (S. H. Zhou, 2000). We selected the ground observations that corresponded to the freezing rain (WMO weather code: 24, 56, 57, 66, 67) and excluded some of the missing values. The $T_w$ was calculated using the method described by Sadeghi et al. (2013), and the error in the calculated $T_w$ was only −0.1°C when the air temperature was near 0°C. In addition, to screen the sounding profiles when freezing rain occurred, the weather phenomena observed by ground stations at 00:00 and 12:00 UTC were selected to match with the observed profiles at sounding stations. We constrained the vertical profiles by requiring that the sounding stations and corresponding surface observational sites were at the same locations.

For convenient discussion, one freezing rain event was defined as one observational record of freezing rain. Figure 2a shows the frequency distribution of $T_w$ when freezing rain occurred at the ground stations in China from 2000 to 2019. A total of 9,312 freezing rain observations were recorded over a 20-year period, 90% (8,394) of which occurred when $T_w$ was between −6°C and 0°C. This statistic is consistent with the results of J. Cortinas (2004) and Y. Zhou et al. (2017). However, nearly 7% (656) of the freezing rain occurred when the $T_w$ was greater than 0°C. We refer to this phenomenon as WFR. In addition, nearly 3% (266 observations) of freezing rain occurred when the $T_w$ was greater than 0°C (Figure 2b), and half of them occurred outside the error range.
of the instrument (±0.2°C) or the calculated wet-bulb temperature itself (∼−0.1°C), meaning that the wet-bulb temperature cannot be used to explain all WFR events. The frequency of such WFR events gradually decreased as the $A_A$ and the $A_W$ increased (Figures 2c and 2d).

Some observational studies have shown that the meteorological characteristics and formation mechanisms of freezing rain change with the station altitude (Carrière et al., 2000). To examine whether the station altitude affects the formation of WFR, the frequency and proportion of WFR observations at stations with different altitudes were analyzed. The results show that the frequency of WFR decreases with increasing station altitudes (Figure 3). Over the entire study period, 360 WFR events were observed at ground stations with station altitudes between 0 and 500 m, accounting for 18% of the total number of freezing rain events in this station altitude range. The ground stations at altitudes >2,000 m only observed 41 WFR events, representing 3% of the total number of freezing rain events at this station altitude. It is obvious that the frequency and proportion of WFR events occurring at low station altitudes are significantly higher than those at high station altitudes, which implies that the station altitude has a significant impact on the spatial distribution of WFR events.

The formation of freezing rain is closely related to the vertical profile of air temperature. We screened the vertical profiles of WFR over a 20-year period by requiring that the sounding stations and corresponding surface observational sites were at the same locations. Since the number of sounding stations in China was significantly lower than that of ground stations, only 25 vertical profiles with WFR were screened. Although all of these profiles contain a melting layer or liquid cloud (cloud top temperature > −10°C), their $A_A$ were greater than 0°C. Additionally, we found that both the $A_A$ and the $A_W$ were greater than 0°C in six WFR profiles. Thus, the WFR could not be perfectly explained by the wet-bulb temperature or air temperature, calling for new and different explanations.

Meanwhile, some researchers have attributed the WFR events to ground temperature <0°C, as the air temperature is reported at 2.0 m height above the ground level (Bernstein, 2000; Roberts & Stewart, 2008). Indeed, a raindrop ($T_r > 0°C$) may gradually freeze when the ground or object temperature is less than 0°C. However, if a raindrop is supercooled ($T_r < 0°C$), it will freeze much faster than an above freezing raindrop once it strikes a below freezing object, which is more consistent with the definition of freezing rain (Ofcm, 2005). Therefore, in order to explain the formation of WFR, the raindrop temperature and their influencing factors within the WFR events are necessary to be calculated.
3. Theoretical Model and Sensitivity Studies

3.1. Model Description

A theoretical model for describing $T_s$ is presented here to quantitatively investigate the formation of WFR. In this model, the equilibrium assumption is relaxed and considerations for the nonequilibrium heat exchange between raindrops and the atmosphere are included. The model contains some simplifying assumptions used in previous studies (Caplan, 1966; Smorodin et al., 2014; Tardif & Rasmussen, 2010). First, raindrops are assumed to be spherical with a uniform temperature from the drop center to the surface. Second, it was assumed that the raindrop sizes do not increase by the collision–coalescence process and do not decrease by break up during their fall from the cloud base. This assumption avoids the calculation errors of raindrop temperature due to the large transient change in raindrop sizes.

The total heat of the raindrop ($Q_T$) is determined by the heat transfer between raindrops and atmosphere by evaporation ($Q_e$), conduction ($Q_c$), and radiation ($Q_r$), which can be described by the following equation (Abraham et al., 1972):

$$\frac{dQ_T}{dt} = \frac{dQ_e}{dt} + \frac{dQ_c}{dt} + \frac{dQ_r}{dt}$$  \hspace{1cm} (1)

where the components can be expressed as (Pruppacher & Klett, 2012)

$$\frac{dQ_e}{dt} = -L_e \frac{2\pi DF_b D_s}{K_v} \left[ \frac{e_{sat}(T_r(t))}{T_r(t)} - \frac{RH e_{sat}(T_a(t))}{100} \right]$$  \hspace{1cm} (2)

$$\frac{dQ_c}{dt} = -2\pi \frac{\varepsilon}{\sigma} (T_r^4(t) - T_s^4(t))$$  \hspace{1cm} (3)

$$\frac{dQ_r}{dt} = -\pi D^2 \varepsilon \sigma (T_r^4(t) - T_s^4(t))$$  \hspace{1cm} (4)

$$\frac{dQ_r}{dt} = \frac{1}{6} c_a \pi D^3 \rho_a \frac{dT_r(t)}{dt}$$  \hspace{1cm} (5)

Based on Equations 1–5, the temporal change rate of $T_s$ is described by
\[
\frac{dT}{dt} = \frac{12F_b}{D^2 \rho_w C_w} \left[ \frac{L_v D_v F_v}{R_v^2 F_h} \left( \frac{R H \varepsilon_{sat}(T_a(t))}{100} - \frac{\varepsilon_{sat}(T_r(t))}{T_r(t)} \right) \right. \\
\left. - k_v (T_r(t) - T_v(t)) - \frac{D}{2T_h} \varepsilon \sigma (T_r^4(t) - T_v^4(t)) \right]
\] (6)

The variations of raindrop mass and size were expressed as

\[
\frac{dm_v}{dt} = \frac{2\pi DF_i D_v}{R_v^2} \left[ \frac{R H \varepsilon_{sat}(T_a(t))}{100} - \frac{\varepsilon_{sat}(T_r(t))}{T_r(t)} \right]
\] (7)

\[
\frac{dD}{dt} = \frac{4F_i D_v}{D R_v^2 \rho_w} \left[ \frac{R H \varepsilon_{sat}(T_a(t))}{100} - \frac{\varepsilon_{sat}(T_r(t))}{T_r(t)} \right]
\] (8)

In addition, the raindrop fall velocity \(V_r(D)\) was estimated as follows (Atlas et al., 1973; Best, 1950):

\[
V_r(D) = C_1 - C_2 \exp \left( -\frac{C_3 D}{2} \right)
\] (9)

The initial phases of the particles are assumed to be raindrops when the cloud top air temperature is \(\geq -10^\circ C\), and vice versa for snowflakes. If the particles are solid, the parameterization scheme proposed by Zerr (1997) is used to calculate the melting rate of the solid/mixed particles:

\[
\frac{dm_s}{dt} = -\frac{4\pi f_i C_i}{L_m} \left[ k_v (T_a(t) - T_v(t)) + \frac{D_i M_w L_v}{R_v^2} \left( \frac{R H \varepsilon_{sat}(T_a(t))}{100} - \frac{\varepsilon_{sat}(T_a(t))}{T_a(t)} \right) - \frac{\varepsilon_{sat}(T_0)}{T_0} \right]
\] (10)

It is worth mentioning that the evaporation process of the melting snowflakes can significantly reduce the 3%-10% particle mass (Neumann, 2017), which cannot be neglected for some small snowflakes. Therefore, once the melting process has started, the evaporation mass of the melting portions \(m_{sw}\) was calculated based on Equation 7 (Neumann, 2017).

The initial mass of the snowflakes is set to the equivalent mass of raindrops, and the initial size of the snowflakes is expressed as follows:

\[
D_0 = D_r \sqrt{\frac{\rho_w}{\rho_s}}
\] (11)

The snowflake is assumed to be a spherical mixture of liquid water particles wrapped in snow crystals during the melting process. The size of the melting snowflakes is expressed as

\[
D(t) = \sqrt[3]{\frac{6}{\pi} \left( \frac{(m_{j0} - m_j(t) - m_{sw}(t))}{\rho_w} + \frac{m_j(t)}{\rho_s} \right)}
\] (12)

The fall velocity of the melting snowflake is estimated as follows (Frick et al., 2013):

\[
V_m = V_i + (V_r - V_i) \times X(LWF)
\] (13)

where LWF represents the liquid water fraction of the mixed particle:

\[
LWF = (m_{j0} - m_j(t)) / m_{j0}
\] (14)

\(X\) is the empirical function associated with LWF:

\[
X(LWF) = 0.246 LWF + (1 - 0.246) LWF^7
\] (15)

\(V_i\) represents the fall velocity of dry snowflakes (Langleben, 1954):

\[
V_i = 2.34(100D)^{0.3}
\] (16)

\(V_r\) represents the terminal velocity of a raindrop with the same mass as the snowflake.
The initial height of raindrop generation is defined as the height at which the solid particles completely melt. The particle temperature of the snowflakes during the melting process is assumed to be 0°C. When the snowflakes completely melt into raindrops, the model assumes the initial $T_r = 0°C$ and calculates the variation of $T_r$ with the falling distance. The time step of the model is set to 0.1 s. The variables, symbols, and empirical parameters in the above equations are summarized in Appendix A Table A1.

### 3.2. Sensitivity Experiments

In fact, the atmospheric conditions surrounding falling raindrops are changing, which prevents raindrops from reaching and maintaining a steady equilibrium state instantaneously. In principle, $T_r$ is not equal to $T_s$ or $T_o$ when the raindrops fall in the real atmosphere. It is possible for the $T_r$ to be less than 0°C while the $T_s$ or $T_o$ is greater than 0°C when the raindrops fall to the ground, thus resulting in WFR. To better understand the formation mechanism of WFR and explain the frequency distribution of the WFR, the following sensitivity experiments were conducted.

#### 3.2.1. Experiment Design

To simplify the sensitivity study, the model is constrained with the following conditions: (a) the top of the model is set at 1 km height because the cloud base height of winter precipitation is usually less than 1 km (Zhang et al., 2016). (b) The raindrops remain stable in geometric shape (spherical) and no collision–coalescence process occurs among raindrops during falling. (c) Hydrostatic atmosphere and surface pressure of 1,013.25 hPa are assumed for the atmospheric pressure profile and the $T_m$ is set as 0.5°C. (d) Observations have shown that the spectral width of the raindrop size distribution is 4 mm (Huang et al., 2018) and the RH is usually greater than 80%. Thus, we set the default raindrop size as half of the spectral width (2 mm) of the raindrop size and set the default RH to 80%. In addition, the default value of $\Gamma$ is set to 5°C/km, which is taken in the range of the wet adiabatic lapse rate (−4−9.8°C/km). Since the idealized sensitivity experiments do not involve the melting process of snowflakes, the default value of the initial $T_r$ was equal to $T_o$.

According to the theoretical model of $T_r$ presented in Section 3.1, the evaporation, conduction, and radiation heat transfer of the raindrops and thus $T_r$ are affected by the initial raindrop diameter ($D_r$), temperature lapse rate ($\Gamma$), and relative humidity ($RH$). Therefore, four sensitivity experiments are further conducted (Table 1) to explore the sensitivities in addition to the control experiment. Briefly, the control experiment is to investigate the variability of $T_r$ when the raindrops fall from 1 km height to the ground by fixing $D_r$, $\Gamma$, $RH$ at the default values of 2 mm, 5°C/km, and 80%, respectively. Experiment 1 examined the sensitivity to $D_r$ by fixing the $\Gamma$, $RH$, and initial $T_r$ at their default values but varying $D_r$ from 0.5 to 4 mm, with a diameter interval of 0.1 mm. Experiment 2 examined the sensitivity to $\Gamma$ by fixing $D_r$, $RH$, and initial $T_r$ at their default values but varying the $\Gamma$ from −10°C/km to 10°C/km, with an interval of 0.5°C/km. Experiment 3 examined the sensitivity to $RH$ by fixing $D_r$, $\Gamma$, and initial $T_r$ at their default values but varying the RH from 50% to 100%, with an interval of 5%. Experiment 4 examined the sensitivity to initial $T_r$ by fixing $D_r$, $\Gamma$, and $RH$ at their default values but varying the initial $T_r$ from −5.5°C to −3.5°C, with an interval of 1°C.

### Table 1

| Sensitivity parameter | Variation range | Default value |
|-----------------------|-----------------|---------------|
| CTRL                  | −               | $D = 2$ mm; $\Gamma = 5$°C/km; $RH = 80$%; initial $T_r = T_o$ |
| Ex 1                  | $D$             | $RH = 80$%; $\Gamma = 5$°C/km; initial $T_r = T_o$ |
| Ex 2                  | $\Gamma$        | $D = 2$ mm; $RH = 80$%; initial $T_r = T_o$ |
| Ex 3                  | $RH$            | $D = 2$ mm; $\Gamma = 5$°C/km; initial $T_r = T_o$ |
| Ex 4                  | initial $T_r$   | $D = 2$ mm; $\Gamma = 5$°C/km; $RH = 80$% |

Note. CTRL and Ex refer to the controlled experiment and sensitivity experiment, respectively.
with height as a raindrop falls from 1 km height in the atmosphere (assuming that the initial $D = 2$ mm, $RH = 80\%$, and $\Gamma = 5^\circ C/km$). The $T_w$ is less than $T_a$ in a subsaturated atmosphere. When the raindrop starts to fall, the $T_r$ gradually increases, and the $T_a$ and $T_w$ increase with the falling distance. Thus, the $\Delta T_w$ (difference between $T_w$ and $T_r$) gradually decreases, while the $\Delta T_a$ (difference between $T_a$ and $T_r$) increases with the falling distance. The raindrop reaches the equilibrium state transiently when it falls 67 m (defined as the equilibrium distance) from the model top, at which point the $T_r$ is close to $T_w$ (Caplan, 1966). Subsequently, the $T_r$ gradually increases with the falling distance and its variation rate ($dT_r/dt$) is slightly less than that of $T_a$ ($dT_a/dt$) and $T_w$ ($dT_w/dt$), which causes the $\Delta T_w$ and $\Delta T_a$ to gradually increase with the falling distance.

![Figure 4](image42x722_to_82x748)

**Figure 4.** (a) Variation in the $T_r$, $T_a$, and $T_w$ with falling distance of the raindrop; (b) variation in the $Q_r$, $Q_a$, $Q_h$, and $Q_e$ with falling distance.

### 3.2.2. Results

#### 3.2.2.1. Control Experiment

Figure 4a shows the variations in $T_r$, $T_a$, and $T_w$ with height as a raindrop falls from 1 km height in the atmosphere. From the cloud base to the equilibrium distance, the $Q_r$ is much greater than the $Q_a$ and $Q_w$, and the total heat change of raindrops is negative due to the strong raindrop evaporation. Therefore, the $dT_r/dt$ is negative, and the $\Delta T_{ar}$ rapidly increase and $\Delta T_{aw}$ decrease with the falling distance. With the increase of $\Delta T_{aw}$, the $Q_h$ rapidly increases and the $Q_e$ decreases with the falling distance. When the $Q_h$ is sufficient to compensate for the $Q_e$, the raindrops reach the equilibrium state transiently and the $T_r$ is close to the $T_a$ (Tardif & Rasmussen, 2010). After that, the heat exchange between the raindrop and atmosphere becomes nonequilibrium since the time scale of change for $T_a$ and $T_w$ is shorter than the relaxation time scale of raindrops. The $dT_r/dt$ slightly lags behind the $dT_a/dt$ and $dT_w/dt$, which means that the $T_r$ gradually deviates from the $T_a$ and both the $\Delta T_w$ and $\Delta T_{aw}$ gradually increase with the falling distance. Furthermore, the $Q_r$ is much less than the $Q_e$ and $Q_h$ and can be neglected in the analysis.

#### 3.2.2.2. Effects of Raindrop Diameter

Figure 5a shows the influence of different raindrop diameters on $T_r$. As can be seen from Figure 5a, $D$ largely influences the equilibrium distance of raindrops. Until the raindrops reach the transient equilibrium state, a larger $D$ leads to a decrease in the variation rate of $T_r$, and a gradual increase in the equilibrium distance of raindrops. For example, the equilibrium distance is only 33 m for small raindrops ($D = 1$ mm), while it increases to 91 m for large raindrops ($D = 4$ mm). Furthermore, both the $\Delta T_{aw}$ and $\Delta T_{ar}$ increase with increasing $D$ when the raindrops fall beyond the equilibrium distance.

Figure 5b shows the variation in $\Delta T_{aw}$ and $\Delta T_{ar}$ according to different values of $D$ when the raindrops reach the surface. The $\Delta T_{aw}$ and $\Delta T_{ar}$ are nearly linearly proportional to $D$, with an increase of 1 mm in $D$ leading to a rise.
of 0.4°C in $\Delta T_{aw}$ and $\Delta T_{aw}$. This phenomenon is due to smaller raindrops having a larger surface-to-volume ratio and faster heat transfer between their surface and the atmosphere.

In addition, due to the evaporation process of raindrops or melting snowflakes, $D$ will gradually decrease with the falling distance. Thus, the temperature difference between raindrops and the atmosphere will gradually decrease with the falling distance. However, the decrease of $\Delta T_{aw}$ and $\Delta T_{aw}$ can be neglected due to the assumptions of a short falling distance (1 km), a high RH (80%), and a relative large raindrop size (2 mm).

### 3.2.2.3. Effects of Temperature Lapse Rate

Figure 6 shows the variation of $T_i$ with falling distance for three different values of $\Gamma$. When the $\Gamma$ is less than 0°C/km (Figure 6a), the cooling rate of $T_i$ is lower than that of $T_{aw}$ and $T_{aw}$, and thus the $\Delta T_{aw}$ and $\Delta T_{aw}$ gradually decrease with the falling distance. When the $\Gamma$ is equal to 0°C/km (Figure 6b), the $\Delta T_{aw}$ and $\Delta T_{aw}$ vary little with height after the raindrops reach the transient equilibrium state. In addition, even in an isothermal atmosphere, the $T_i$ is still slightly lower than $T_{aw}$ due to a 0.1°C bias in the empirical function itself used to calculate the wet-bulb temperature (Sadeghi et al., 2013). When the $\Gamma$ is greater than 0°C/km (Figure 6c), the heating rate of $T_i$ is slower than that of $T_{aw}$ and $T_{aw}$, which results in $T_i < T_{aw} < T_{aw}$ and $\Delta T_{aw}$ (or $\Delta T_{aw}$) increases with the falling distance (Figure 6c). In general, the positive or negative values of $\Gamma$ determine whether the $\Delta T_{aw}$ and $\Delta T_{aw}$ will increase or decrease with the falling distance after the raindrops reach the transient equilibrium.

Figure 6d shows the variation in the $\Delta T_{aw}$ and $\Delta T_{aw}$ with $\Gamma$ when the raindrops fall to the surface. Both $\Delta T_{aw}$ and $\Delta T_{aw}$ are linearly proportional to $\Gamma$, and the magnitude of $\Gamma$ determines the degree of deviation of $T_i$ from $T_{aw}$ and $T_{aw}$. Both the $\Delta T_{aw}$ and $\Delta T_{aw}$ increase by 0.055°C for each increase of 1°C/km in $\Gamma$. Due to the relaxation time scale of raindrops being longer than the time scale of changes in $T_{aw}$, the cooling/warming rate of $T_i$ is slower than that of $T_{aw}$ and $T_{aw}$. Thus, $\Delta T_{aw}$ and $\Delta T_{aw}$ gradually increase with an increasing $\Gamma$. It is also worth noting that $\Delta T_{aw}$ increases from −0.39°C to 0.6°C when $\Gamma$ increases from −10°C/km to 10°C/km, which indicates that the $\Gamma$ also determines whether the $T_i$ is larger or smaller than the $T_{aw}$ when raindrops reach the surface.

### 3.2.2.4. Effects of Relative Humidity

It is well known that RH influences $T_i$ by affecting the evaporation process of the raindrops (Caplan, 1966). As expected, with the increase of RH, the evaporation effect of raindrops gradually decreases and leads to the decrease of $\Delta T_{aw}$ (Figure 7a). Figure 7b shows the variation of $\Delta T_{aw}$ at the surface is inversely proportional to RH. When the RH increases from 50% to 100%, the $\Delta T_{aw}$ decreases from 3.5°C to 0.29°C.

The effect of RH on $\Delta T_{aw}$ is relatively weak compared to that on $\Delta T_{aw}$, because $T_{aw}$ also considers the effect of evaporation, and thus the $\Delta T_{aw}$ only decreases from 0.41°C to 0.29°C when the RH increases from 50% to 100%. Since the time scale of changes in the atmospheric conditions is often shorter than the relaxation time scale of
raindrops, the difference between the $T_r$ and $T_w$ is still 0.29°C when the RH is 100%. Even if the evaporation of the raindrops stops, the $T_r$ is not equal to $T_w$.

3.2.2.5. Effects of Initial Raindrop Temperature

Figure 8 shows the variation of $T_r$, $T_w$, and $T_o$ with falling distance for different three typical values of $\Gamma$; (d) relationship between $\Delta T_{ar}$, $\Delta T_{aw}$, and $\Gamma$ when the raindrops reach the surface.

**Figure 6.** (a–c) Variation of $T_r$, $T_w$, and $T_o$ with falling distance for different three typical values of $\Gamma$; (d) relationship between $\Delta T_{ar}$, $\Delta T_{aw}$, and $\Gamma$ when the raindrops reach the surface.

3.2.3. Combined Effect of $D$, $\Gamma$, and RH

The above sensitivity experiments demonstrate that the $D$, $\Gamma$, and RH all affect the $T_r$, $\Delta T_{ar}$, and $\Delta T_{aw}$. The combined effect of the three parameters on $\Delta T_{aw}$ is summarized in Figure 9a. On the one hand, the effect of $D$ on $\Delta T_{aw}$ depends upon the magnitude of $\Gamma$. When $\Gamma$ is less than 0°C/km (ambient temperature increases with height),
\( \Delta T_{aw} \) decreases with \( D \) and becomes more sensitive to \( D \) with the decreasing \( \Gamma \). Conversely, when \( \Gamma \) is greater than 0 (ambient temperature decreases with height), \( \Delta T_{aw} \) increases with \( D \). The larger the \( \Gamma \) is, the more sensitive \( \Delta T_{aw} \) is to the change in \( D \). For example, when \( \Gamma = -10 \, ^\circ\text{C}/\text{km} \), \( \Delta T_{aw} \) decreases by about 1°C as \( D \) increases from 0.5 to 3 mm. When \( \Gamma \) increases to 10°C/km, \( \Delta T_{aw} \) increases by about 1.5°C as \( D \) increases from 0.5 to 3 mm. This phenomenon implies that when the surface temperature is greater than 0°C, the possibility of WFR is higher for \( \Gamma > 0 \) than for \( \Gamma < 0 \). On the other hand, the sensitivity of \( \Delta T_{aw} \) to \( RH \) seems to be unaffected by \( \Gamma \). Regardless of the sign and value of \( \Gamma \), \( \Delta T_{aw} \) gradually decreases with increasing \( RH \). Since the weak effect of \( RH \) on \( \Delta T_{aw} \), we only calculate the combined effect of \( D \) and \( \Gamma \) on \( \Delta T_{aw} \) for \( RH = 80\% \) (Figure 9b). The maximum \( \Delta T_{aw} \) can reach over 1.5°C when the large raindrops (\( D = 4 \, \text{mm} \)) fall in the atmosphere with a large \( \Gamma \), which indicates that using the \( T_{aw} \) instead of \( T_{w} \) to determine the precipitation type will introduce some bias. Due to the deviation of the empirical function used to calculate the \( T_{aw} \), the calculated \( T_{w} \) is slightly lower than the \( T_{aw} \) when \( \Gamma = 0 \) and leads to a bias of the contour of \( \Delta T_{aw} = 0 \) to the side of \( \Gamma > 0 \). In addition, for smaller raindrops (\( D < 1.25 \, \text{mm} \)), the \( \Delta T_{aw} \) is maintained between \(-0.5^\circ\text{C}\) and \(0^\circ\text{C}\) regardless of the variation of \( \Gamma \). This phenomenon implies that the possibility of WFR is relatively low when small raindrops fall to the surface with \( T_{aw} > 0^\circ\text{C} \).

### 3.3. Effects of More Realistic Temperature Profiles

The sensitivity experiments have shown that \( \Gamma \) determines the \( \Delta T_{aw} \) and \( \Delta T_{aw} \), thus playing a significant role in the formation of WFR. In the real atmosphere, the temperature profile is composed of multiple values of \( \Gamma \). For example, winter precipitation is mainly caused by frontal systems that often contain at least one inverse structure in the atmosphere (Thériault & Stewart, 2010). Furthermore, WFR requires the surface temperature to be greater than 0°C. Therefore, according to the above feature of temperature layers and combinations of different \( \Gamma \) (positive or negative values), four types of idealized temperature profiles are analyzed for the possibility of WFR occurrence (Figure 10). The variations of \( \Gamma \), corresponding to each type of temperature profile are calculated to determine whether WFR would occur. All of the profiles assume that \( D = 2 \, \text{mm} \), \( RH = 80\% \), \( T_{aw} = 1.5^\circ\text{C} \), and \( T_{aw} = 0.34^\circ\text{C} \).

The first type of profile (Figure 10a) is similar to the “melting mechanisms” profile of freezing rain, including an upper warm layer, a lower subfreezing
layer, and a warm near-surface layer. The calculated result shows that $A_A r = -0.4°C$ when the raindrops fall to the ground. Although the $A_A r r$ and $A_A w w$ are greater than 0°C, the raindrops are still in a supercooled state. Therefore, WFR might occur for the first type of profile.

The second and third types of profiles have no subfreezing layer below 1.5 km height. Whether the $\Gamma$ is less (Figure 10b) or greater than 0°C/km (Figure 10c) in the near-surface layer, the $A_A r$ is slightly greater than 0°C when the raindrops fall into the near-surface layer. Therefore, the possibility of WFR is lower for these two types of profiles.

The fourth profile type (Figure 10d) is similar to the freezing rain profile of the “supercooled warm rain process,” which includes an upper cold layer but a weak warm layer near the surface. The falling raindrops remain supercooled and fall into the weak warm layer with $\Gamma > 0$ near the surface. Although the surface temperature and wet-bulb temperature are greater than 0°C, the $A_A r$ near the surface is less than 0°C (−0.6°C) and results in WFR due to the temperature difference between raindrops and the atmosphere.

In summary, the temperature profiles of WFR have two common features: (a) there is a cold layer (or subfreezing layer) above the near-surface layer, which allows the raindrops (or completely melted ice crystals) to remain supercooled before falling into the near-surface layer. (b) $\Gamma$ in the lowest atmosphere needs to be greater than 0°C/km for the raindrops falling into the near-surface layer to maintain a large $A_T w$. It is important to note that these results only apply to idealized conditions. In a real atmosphere, if the refreezing layer is cold or deep enough, the raindrops might be coalescing with crystals or renucleation to form ice pellets, thus reducing the WFR probability. In general, the first and fourth analyzed profiles have a high possibility for WFR formation.

Figure 9. Distribution of (a) $A_T a$ and (b) $A_T w$ for various initial $D$, $\Gamma$, and $RH$ when the raindrops reach the surface (in Figure 8a, the dashed lines represent $A_T a$ when $RH = 75\%$ and the solid lines represent $A_T a$ when $RH = 80\%$; in Figure 8b, initial $RH = 80\%$).
3.4. Evaluation of the Model for Real Borderline Rain/Freezing Rain Events

The above sensitivity experiments should be considered idealized temperature profiles, whereas real temperature profiles are more complex. To test the applicability of the calculated $\Delta T$ for real freezing rain events, the theoretical model (Section 2.2) was applied to the borderline rain/freezing rain events (when the $\Delta T_{\text{st}}$ is near 0°C) observed at the sounding stations over the two decades.

Figures 11a and 11b represent the profiles of the “melting process” and “supercooled warm rain process” in freezing rain events, respectively. The calculated results of the “melting mechanism” (Figure 11a) show that the $\Delta T_{\text{st}} = 0.3°C$ and $\Delta T_{\text{su}} = -0.14°C$. The ice crystals completely melt into raindrops at the bottom of the melting layer (~2,500 m), during which time the $T_r$ is 0°C and the $\Delta T_{\text{su}}$ is close to 0°C. The $\Delta T_{\text{su}}$ gradually increases as the raindrop falling distance increases. Since $\Gamma > 0$ in the near-surface layer, $T_r$ of the raindrops falling to the ground is much lower than $T_a$ and $T_w$. Although the $T_a > 0°C$ near the surface, the falling raindrops remain supercooled ($T_r = -0.3°C$) and form WFR when they are falling into the warm layer near the surface.

The corresponding profile for the “supercooled warm rain mechanism” (Figure 11b) shows that the initial precipitation particles in the cloud remained supercooled. The air temperature above the near-surface layer was below 0°C. The temperature difference between raindrops and the atmosphere is about ±0.9°C. When the raindrops fell into the warm layer near the surface, although $T_a$ and $T_w$ are greater than 0°C near the surface, the raindrops remained supercooled ($T_r = -0.6°C$) and formed WFR.

To test the applicability of the calculated $T_r$ for real freezing rain events, the borderline rain/freezing rain events with $T_{sw}$ between −1°C and 1°C are filtered based on the records from the sounding and ground stations in China. Over the last two decades, a total of 370 vertical profiles of borderline rain/freezing rain events were observed, including 126 freezing rain events and 244 rain events. The theoretical model (Section 2.2) is applied to these sounding profiles to calculate the $T_r$ at surface (assume $D = 2 $ mm).

The distributions of the calculated $T_r$, observed $T_{sw}$, and $T_{su}$ at the surface for the 370 borderline rain/freezing rain events (red or blue dots) are shown in Figure 12. Almost all freezing rain events (including WFR) occurred when $T_r < 0$, but there are 25 and 7 WFR events were missed when using the distinguishing criteria of $T_{sw} < 0$ or
When $T_w < 0$, respectively. Meanwhile, there are 40, 87, or 55 rain events were missed when $T_r$, $T_w$, or $T_r > 0$, respectively. The critical success index (CSI) was used to assess the accuracy of $T_r$, $T_w$, and $T_r$ in distinguishing between rain and freezing rain events. The CSI of $T_r$ is 0.75, which is better than that of $T_w$ (0.68) and $T_r$ (0.53). Overall, although $T_w$ and $T_r$ are more easily obtained, $T_r$ tends to underestimate the real raindrop temperature, resulting in large false alarms of freezing rain events. Meanwhile, $T_w$ cannot distinguish between WFR and rain. The results show that $T_r$ is more accurate than $T_w$ and $T_r$ in distinguishing between rain and freezing rain.

Moreover, the model and sensitivity results can be applied to explain the difference in the frequency of WFR events occurring at different altitude stations as follows.

$T_w < 0$, respectively. Meanwhile, there are 40, 87, or 55 rain events were missed when $T_r$, $T_w$, or $T_r > 0$, respectively. The critical success index (CSI) was used to assess the accuracy of $T_r$, $T_w$, and $T_r$ in distinguishing between rain and freezing rain events. The CSI of $T_r$ is 0.75, which is better than that of $T_w$ (0.68) and $T_r$ (0.53). Overall, although $T_w$ and $T_r$ are more easily obtained, $T_r$ tends to underestimate the real raindrop temperature, resulting in large false alarms of freezing rain events. Meanwhile, $T_w$ cannot distinguish between WFR and rain. The results show that $T_r$ is more accurate than $T_w$ and $T_r$ in distinguishing between rain and freezing rain.

Moreover, the model and sensitivity results can be applied to explain the difference in the frequency of WFR events occurring at different altitude stations as follows.

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**Figure 11.** Calculated variation of $T_r$ corresponding to two real WFR profiles ($D = 2$ mm).

**Figure 12.** Distribution of the (a) $T_w$ and $T_r$, and (b) $T_w$ and $T_r$ for the borderline rain/freezing rain events.
1. Raindrop size effect. Observations at Nanjing (station elevation of 20 m; L. Wen et al., 2019), Lushan (station elevation of 1,164 m; Huang et al., 2018), and Weining (station elevation of 2,236 m; Zhang et al., 2016) show that the raindrop size distribution for freezing rain events is narrow at high-altitude stations, with the maximum raindrop size is only 1.25 mm and the mean diameter is 0.34 mm. In contrast, the raindrop size distributions are significantly wider at low- and middle-altitude stations (Figure 13a). The maximum sizes of raindrops are 4.1/4.25 mm and the mean diameters are 0.53/0.65 mm for low/middle-altitude stations, respectively. Therefore, the raindrop sizes at high-altitude stations tend to be smaller than those at low- and middle-altitude stations.

According to the sensitivity test, the $\Delta T_w$ and $\Delta T_{wo}$ increase with increasing $D$. Therefore, the temperature difference between raindrops and the atmosphere at high-altitude stations is weaker, which results in a lower probability of WFR occurring compared to that at low- and middle-altitude stations.

To visualize the effects of raindrop size distribution on $T_i$ at different altitude stations, the real drop size distribution (DSD) is fitted using the Gamma function:

$$N(D) = N_0 D^\mu exp(-\lambda D)$$  \hspace{1cm} (17)

where the value of $N_0$, $\mu$, and $\lambda$ at different altitude stations are listed in Table 2.

The drop sizes are partitioned into several bins (set the increments as 0.1 mm) according to the spectral width of the DSD at different altitude stations. The theoretical model (Section 2.2) is applied to the real WFR profile (Figure 10a) and DSDs to calculate the $\Delta T_{wo}$ for each bin when the raindrops fall to the surface. The distribution of $\Delta T_{wo}$ at stations with different altitudes is shown in Figure 13b. Due to the narrow DSDs, the $\Delta T_{wo}$ distribution is relatively narrow and the maximum $\Delta T_{wo}$ is only 0.2°C at high-altitude stations. In contrast, the raindrop size distributions are significantly wider at low- and middle-altitude stations, which result in a wider distribution of $\Delta T_{wo}$, with the maximum $\Delta T_{wo}$ can reach over 1°C. Therefore, the temperature difference between raindrops and the atmosphere at high-altitude stations is weaker, which results in a lower probability of WFR occurring than that at low- and middle-altitude stations.

In addition, the DSDs (Figure 13) indicate that the number of large raindrops is less than that of small raindrops at the same stations. Therefore, with increasing air temperature, the temperature difference between raindrops and the atmosphere is insufficient to keep the temperature of small raindrops below 0°C, resulting in a gradual decrease in the frequency of WFR.

| Station | $N_0$ | $\mu$ | $\lambda$ |
|---------|-------|-------|----------|
| Weining | 1,667 | -2.125 | 9.309 |
| Lushan  | 76    | -2.18  | 1.11 |
| Nanjing | 160   | -3.2   | 0.775 |

Table 2. $N_0$, $\mu$, and $\lambda$ Values at Different Altitude Stations

Figure 13. Characteristics of (a) raindrop size and (b) temperature distributions in freezing rain events at stations at different altitudes. The data in (a) combine the data from the observational studies by L. Wen et al. (2019), Huang et al. (2018), and Zhang et al. (2016).
2. *RH effect.* The temperature difference between the raindrop and atmosphere increases with decreasing RH. The statistical results show some differences in the distributions of average RH in the lower atmosphere (0–500 m) at different altitude stations when freezing rain occurred (Figure 14a). When the station altitude is lower than 500 m, only 20% of the freezing occurred with high RH (>90%). In contrast, when the station altitude is greater than 2,000 m, the frequency of freezing rain events in a high RH atmosphere increases to 50%. Therefore, all else being equal the $\Delta T_w$ at high-altitude stations is smaller than at lower-altitude stations, which leads to a lower probability of WFR at high-altitude stations.

3. *Lapse effect.* Figure 14b shows the distribution of $\Gamma$ in the lower atmosphere (0–500 m) for stations at various altitudes when freezing rain occurred in China from 2000 to 2019. There are large differences in the distribution of $\Gamma$ at different altitude stations. When the station altitude is lower than 1,000 m, almost all freezing rain occurs at $\Gamma > 0$. In contrast, when the station altitude is greater than 1,000 m, nearly 40%–50% of the freezing rain events occurred at $\Gamma < 0$. According to the sensitivity test, the temperature difference between raindrops and atmosphere increases with increasing $\Gamma$. The $\Gamma$ values at low altitudes are significantly higher than that at high-altitude stations when the freezing rain occurred, implying that the $\Delta T_w$ and $\Delta T_a$ at high-altitude stations are weaker and the probability of WFR occurring is relatively lower than that at low-altitude stations.

The above studies also help to explain the difference of freezing rain formation conditions at different altitudes from another perspective. The ice nuclei in the raindrop tend to activate when the $T_r < -5^\circ$C (Meyers et al., 1992; Petters & Wright, 2015), which may lead to a change in precipitation type from freezing rain to ice pellet (Reeves et al., 2016). Since the $\Delta T_{aw}$ and $\Delta T_w$ at high-altitude stations are relatively weak, the air temperature range corresponding to the maintenance of $T_r$ between $-5^\circ$C and $0^\circ$C is wider at high-altitude stations than that at low-altitude stations (Lu et al., 2021), which also leads to a higher frequency and wider temperature range of freezing rain at high-altitude stations than at low-altitude stations.

4. Conclusions and Discussion

Analysis of 20 years of observations across China shows that 656 WFR events occurred, accounting for 7% of the total number of freezing rain events. Even using the wet-bulb temperature cannot explain all WFR events; nearly 3% (266 observations) of freezing rain occurred when the $T_{aw} > 0^\circ$C. The WFR occurrence frequency at low-altitude stations is found to be higher than that at high-altitude stations. Moreover, the frequency of freezing rain gradually decreases as the surface temperature increases, but WFR may still occur at surface temperatures greater than $1^\circ$C.

Figure 14. Distributions of the (a) RH and (b) $\Gamma$ at different altitude stations during freezing rain over a 20-year period.
Based on the assumption that raindrops cannot maintain an equilibrium state in the real atmosphere, a theoretical model of raindrop temperature is presented by considering the nonequilibrium heat exchange between raindrops and the atmosphere. Sensitivity analysis of the model shows that the $T_r$ is not equal to the $T_o$ or $T_w$ when raindrops fall.

The magnitudes of the $\Delta T_o$ and $\Delta T_w$ are determined by the raindrop diameter ($D$), temperature lapse rate ($\Gamma$), and relative humidity (RH). Meanwhile, the $\Gamma$ also determines whether the $\Delta T_o$ is positive or negative when raindrops reach the surface. An increase in $D$, increase in $\Gamma$, and decrease in RH enhance the $\Delta T_o$ and $\Delta T_w$ and thus prompt the occurrence of the WFR. In addition, the initial $T_r$ does not affect the $T_r$ at the surface when the falling distance of the raindrop is long enough.

The temperature difference between raindrops and the atmosphere is the main cause of WFR. The diagnosis results for ideal temperature profiles showed that WFR can form by the “melting mechanism” or “supercooled warm rain mechanism” when considering the temperature difference between raindrops and the atmosphere. The temperature profiles of WFR have two common features: a cold layer (or subfreezing layer) above the near-surface layer and $\Gamma > 0^\circ C/km$ in the lower atmosphere. The evaluation results of the model show that the calculated $T_r$ is more accurate than $T_o$ and $T_w$ in distinguishing between rain and freezing rain.

Smaller raindrop sizes, $\Gamma$, and a large RH at high-altitude stations result in weaker temperature differences between raindrops and the atmosphere, which leads to less frequent WFR events. Conversely, with decreasing station altitudes, the raindrop sizes gradually increase and RH decreases, which results in the high frequency of WFR events.

The above analyses theoretically explain the occurrence of WFR, which also deepens our understanding of freezing rain formation. Meanwhile, the temperature difference between raindrop and atmosphere also helps to explain the difference of freezing rain formation conditions at different altitudes from another perspective. Since the $\Delta T_o$ and $\Delta T_w$ at high-altitude stations are relatively weak, the air temperature range corresponding to the maintenance of $T_r$ between $-5^\circ C$ and $0^\circ C$ is wider at high-altitude stations than that at low-altitude stations, which also leads to a higher frequency and wider temperature range of freezing rain at high-altitude stations than at low-altitude stations.

It should be noted that various microphysical parameterization schemes commonly neglect the difference between $T_r$ and $T_o$ in the weather forecasting model. Therefore, the existing parameterization schemes cannot properly represent the nonequilibrium heat exchange between raindrops and the atmosphere, which may ignore some WFR or ice pellet events (when the $T_r <$ ice-nucleation temperature but $T_o >$ ice-nucleation temperature). In future work, we will consider adding $T_r$ to a suitable parameterization scheme to evaluate its impact on the numerical models to predict freezing rain and other precipitation type.

Additionally, this research can also help to refine the disaster assessment model when freezing rain occurs. For example, existing ice accumulation prediction models, such as Makkonen (1998), commonly use air temperature instead of raindrop temperature to calculate the freezing efficiency of the raindrop, which does not reflect the ice growth on conductors when the air temperature is above $0^\circ C$. In future, we will add $T_r$ to the ice accretion forecast model to assess the impact of $T_r$ on the freezing efficiency when freezing rain occurs.
Appendix A

Table A1 gives a list of symbols and their descriptions and units used in this paper.

| Symbol | Definition | Value (or reference) | Units |
|--------|------------|----------------------|-------|
| $T_r$  | Raindrop temperature | – | K |
| $T_a$  | Air temperature | – | K |
| $T_w$  | Wet-bulb temperature | – | °C |
| $T_{w,s}$ | Wet-bulb temperature at surface | – | °C |
| $T_{a,s}$ | Air temperature at surface | – | °C |
| $Q_r$  | Total heat change of raindrops | – | J |
| $Q_e$  | Heat change of evaporation | – | J |
| $Q_r$  | Heat change of radiation | – | J |
| $Q_a$  | Heat change of conduction | – | J |
| $F_h$  | Ventilation coefficient for heat | Rasmussen and Heymsfield (1987) | – |
| $F_v$  | Ventilation coefficient for vapor | Rasmussen and Heymsfield (1987) | – |
| $D$    | Diameter of raindrop | – | m |
| $\rho_w$ | Density of water | 1,000 | kg⋅m$^{-3}$ |
| $\rho_r$ | Density of snowflakes | 100 | kg⋅m$^{-3}$ |
| $C_i$  | Capacitance of the melting flake | Hall and Pruppacher (1976) | m |
| $C_w$  | Specific heat capacity of water | 4,200 | J⋅kg$^{-1}$⋅K$^{-1}$ |
| $L_v$  | Latent heat of evaporation | 2,264 | kJ⋅kg$^{-1}$ |
| $D_v$  | Coefficient of diffusion | Hall and Pruppacher (1976) | cm$^2$⋅s$^{-1}$ |
| $R_v$  | Gas constant of vapor | Hall and Pruppacher (1976) | J⋅K$^{-1}$⋅kg$^{-1}$ |
| $\text{RH}$ | Relative humidity | – | % |
| $e_{sat}$ | Saturated vapor pressure | Murray (1966) | hPa |
| $K_a$  | Thermal conductivity of air | Beard and Pruppacher (1971) | J⋅m$^{-1}$⋅s$^{-1}$⋅K$^{-1}$ |
| $\varepsilon$ | Coefficient of black-body radiation | 0.9 | – |
| $\sigma$ | Stefan Boltzmann constant | $5.67 \times 10^{-8}$ | W⋅m$^{-2}$⋅K$^{-4}$ |
| $V_r$  | Raindrop fall velocity | Best (1950); Atlas et al. (1973) | cm⋅s$^{-1}$ |
| $V_s$  | Dry snowflakes fall velocity | Langleben (1954) | cm⋅s$^{-1}$ |
| $V_m$  | Melting snowflakes fall velocity | Frick et al. (2013) | cm⋅s$^{-1}$ |
| $C_1$  | Empirical coefficient | 965 | cm⋅s$^{-1}$ |
| $C_2$  | Empirical coefficient | 1,030 | cm⋅s$^{-1}$ |
| $C_3$  | Empirical coefficient | 1,200 | cm$^{-1}$ |
| $m_r$  | Raindrop mass | – | kg |
| $m_s$  | Snowflake mass | – | kg |
| $m_{i,s}$ | Initial snowflake mass | – | kg |
| $m_{e,s}$ | Evaporation mass of melting snowflakes | Neumann (2017) | kg |
| $D_h$  | Initial snowflake size | – | m |
| LWF   | Liquid water fraction of particle | – | – |
Conflict of Interest
The authors declare no conflicts of interest relevant to this study.

Data Availability Statement
The observational data and code in the study are available at Lu (2021).

Acknowledgments
This study is supported by the National Natural Science Foundation of China (41875176). Y. Liu is supported by the U.S. Department of Energy’s Atmospheric System Research (ASR) program.

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