CMB bounds on tensor-scalar-scalar inflationary correlations

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Abstract. The nonlinear interaction between one graviton and two scalars is enhanced in specific inflationary models, potentially leading to distinguishable signatures in the bispectrum of the cosmic microwave background (CMB) anisotropies. We develop the tools to examine such bispectrum signatures, and show a first application using WMAP temperature data. We consider several $\ell$-ranges, estimating the $g_{tss}$ amplitude parameter, by means of the so-called separable modal methodology. We do not find any evidence of a tensor-scalar-scalar signal at any scale. Our tightest bound on the size of the tensor-scalar-scalar correlator is derived from our measurement including all the multipoles in the range $2 \leq \ell \leq 500$ and it reads $g_{tss} = -48 \pm 28$ (68\%CL). This is the first direct observational constraint on the primordial tensor-scalar-scalar correlation, and it will be cross-checked and improved by applying the same pipeline to high-resolution temperature and polarization data from \textit{Planck} and forthcoming CMB experiments.

Keywords: non-gaussianity, CMBR experiments, inflation, primordial gravitational waves (theory)

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1 Introduction

Gaussianity of primordial density fluctuations is a natural prediction of simple early Universe scenarios such as a single-field slow-roll inflation based on Einstein gravity [1, 2]. Conversely, a departure from Gaussianity indicates other, nonsimple scenarios, including e.g. the existence of multiple fields, nontrivial particle productions, the modification of Einstein gravity, and so on (see refs. [3–7] for some reviews).

In light of this, primordial non-Gaussianity (NG) is a key inflationary indicator and therefore has been widely and deeply investigated from both the theoretical and the observational side. NG features, generated at primordial stages, are directly imprinted on late-time observables such as the cosmic microwave background (CMB) fluctuations, galaxy clustering, 21-cm anisotropies, and so on.

Among these observables, we focus here on the CMB. A large set of NG primordial scenarios has already been tested with CMB data [8–10]. Recently, the Planck team has produced the most stringent constraints to date on the main NG inflationary shapes, namely local, equilateral and orthogonal (LEO), using both temperature and E-mode polarization data [10]. Such constraints, derived from the CMB bispectrum, indicate no evidence of these NG signatures at $\sigma > 2\sigma$ level [10]. Planck NG studies for these shapes have also been extended to CMB trispectra such of the $g_{NL}$ and $\tau_{NL}$-type NGs. Moreover, a vast amount of additional models, beyond LEO shapes, have been considered, finding data consistent with Gaussianity in all cases. Interestingly, some evidence of oscillatory bispectra at specific frequencies was found in the data [10–12]. However, the statistical significance of such signals vanish when accounting for “look elsewhere” effects, due to the fact that the analysis is blindly scanning over a large range of frequencies.

The analysis mentioned above is thorough and includes a very large number of models. Nonetheless, it is mostly based on the auto-bispectrum of scalar modes, with some analysis of tensor bispectra [10, 13]. Differently from the scalar case, tensor NG also generates CMB bispectra including B-mode polarization [14–18]. Moreover, nonvanishing signals can arise also for odd $\ell_1 + \ell_2 + \ell_3$ configurations in the temperature or E-mode bispectra. This is sourced by parity violation in the tensor sector [14, 16, 19–21].

Interestingly, nonlinear dynamics during inflationary stages can however also produce nonvanishing scalar-tensor couplings. Depending on the model under study, scalar-tensor
signals can be amplified, up to detectable levels \cite{22}.\footnote{See \cite{23, 24} for the inflationary models realizing sizable tensor auto-bispectra.} Such mixed bispectra are currently unconstrained and this motivates the present work, where we will develop a general pipeline for the tensor-scalar-scalar bispectrum estimation, considering an interesting bispectrum shape realized in massive gravity \cite{22}, and show its first application, using WMAP data \cite{8, 25}. This model generates a squeezed-type CMB bispectrum and allows for potential high signal-to-noise even at WMAP resolution, see \cite{22, 26}. In general, CMB shapes arising from tensor primordial bispectra display a complex, nonseparable $\ell$ dependence, which makes a brute-force estimation approach numerically unfeasible. To deal with this issue, we will adopt the so-called separable modal decomposition technique \cite{27, 28}. As we will discuss in the following, our pipeline was thoroughly validated using NG simulations, generated as part of the analysis. The size of the tensor-scalar-scalar coupling was then estimated for several $\ell$-ranges, to check the scale dependence of the constrained values. Although, at the end of the analysis, we do not find any significant signal, we obtain meaningful observational bounds on the tensor-scalar-scalar coupling.

This paper is organized as follows: section 2 illustrates the primordial tensor-scalar-scalar primordial correlator constrained in this paper and its signature in the CMB temperature bispectrum, section 3 discusses the analysis of WMAP data and derives experimental constraints on the model, and section 4 concludes this paper with a summary of its main results.

2 Temperature bispectrum from a tensor-scalar-scalar correlator

In this paper we constrain the cross-bispectrum of primordial tensor and curvature perturbations, parametrized as \cite{2, 26}

$$\left< \gamma_{k_1}^{(\lambda_1)} \zeta_{k_2} \zeta_{k_3} \right> = (2\pi)^3 \delta^{(3)} \left( \sum_{n=1}^{3} k_n \right) e_{ij}^{(-\lambda_1)}(\hat{k}_1)\hat{k}_{2i}\hat{k}_{3j} \frac{16\pi^4 g_{tss} A_s^2}{k_1^3 k_2^3 k_3^3} \frac{I_{k_1 k_2 k_3}}{k_1},$$

(2.1)

where $A_s$ denotes the amplitude of the primordial scalar power spectrum, and

$$I_{k_1 k_2 k_3} = -k_t + \frac{k_1 k_2 + k_2 k_3 + k_3 k_1}{k_t^2} + \frac{k_1 k_2 k_3}{k_t^2}$$

(2.2)

with $k_t \equiv k_1 + k_2 + k_3$. The polarization tensor $e_{ij}^{(±2)}$, obeying $e_{ii}^{(λ)}(\hat{k}) = \hat{k}_i e_{ij}^{(λ)}(\hat{k}) = 0$, $e_{ij}^{(λ)*}(\hat{k}) = e_{ij}^{(−λ)}(\hat{k}) = e_{ij}^{(λ)}(−\hat{k})$ and $e_{ij}^{(λ)}(\hat{k}) e_{ij}^{(λ)}(\hat{k}) = 2\delta_{λ−λ'}$, is used for the decomposition of the primordial gravitational wave into the helicity ($λ = ±2$) basis as

$$\gamma_{ij}(x) \equiv \frac{δ_{ij}^{TT}}{a^2} = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{x}} \sum_{λ=±2} \gamma_{k}^{(λ)} e_{ij}^{(λ)}(\hat{k}).$$

(2.3)

This type of bispectrum is actually realized in the simplest, single-field slow-roll inflationary models, based on Einstein gravity. Its amplitude, in this case, is as usual determined by slow-roll parameters, namely, $|g_{tss}| \simeq \epsilon$. This makes the signal undetectably small \cite{2}. By contrast, ref. \cite{22} has recently shown that, by introducing nonzero mass of gravitons, a nontrivial nonlinear coupling is induced, resulting in a sizable enhancement of the tensor-scalar-scalar signal: $|g_{tss}| \simeq \epsilon \lambda_{sst}$, with $\lambda_{sst}$ characterizing the coupling strength.\footnote{This model actually induces an additional term in eq. (2.1). However, such is suppressed by the slow-roll parameter, hence negligible in a phenomenological analysis \cite{22}.}
Figure 1. Three-dimensional representation of the CMB temperature bispectrum from the tensor-scalar-scalar correlator (2.1) in the tetrahedral domain. We here plot $b_{\ell_1 \ell_2 \ell_3}$ normalized with a constant Sachs-Wolfe template [27] to highlight the dominant configurations. Three axes correspond to $\ell_1$, $\ell_2$ and $\ell_3$, respectively. Dense red (blue) color represents remarkable positive (negative) signal.

For large enough $\lambda_{\text{sat}}$, the enhancement can produce detectable signals using current CMB datasets. It was in fact shown that Planck temperature and E-mode polarization data allow achieving a $g_{\text{tss}} \sim 1$ sensitivity [22, 26], with a further an-order-of-magnitude improvement possible using B-mode information [22, 29]. In this paper we will develop a tensor-scalar-scalar CMB bispectrum estimation pipeline, test it and use it for a preliminary analysis of WMAP data, reaching a sensitivity level $g_{\text{tss}} \sim 10$.

When the temperature fluctuation is multipole expanded in the usual manner, $T(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n})$, the harmonic coefficients of the scalar and tensor modes are expressed, respectively, as

\begin{align}
    a^{(s)}_{\ell m} &= 4 \pi \ell^2 \int \frac{d^3 k}{(2\pi)^3} T^{(s)}(k) \zeta_k Y^*_{\ell m}(\hat{k}), \\
    a^{(t)}_{\ell m} &= 4 \pi \ell^2 \int \frac{d^3 k}{(2\pi)^3} T^{(t)}(k) \sum_{\lambda=\pm 2} \gamma^{(\lambda)}_{\ell m}(\hat{k}),
\end{align}

where $T^{(s)}(k)$ ($T^{(t)}(k)$) is the radiation transfer function of the scalar (tensor) mode. According to ref. [26], applying radiative transfer to eq. (2.1) leads to the CMB angular bispectrum,
Figure 2. Fisher matrix forecasts of 1σ errors on $g_{tss}$ as a function of $\ell_{\text{max}}$ assuming a CVL-level survey and the WMAP one.

$$\langle a_{l_1 m_1}^{(s)} a_{l_2 m_2}^{(s)} a_{l_3 m_3}^{(s)} \rangle = g_{tss} B_{l_1 l_2 l_3}^{(tss)} \left( \frac{\ell_1 \ell_2 \ell_3}{m_1 m_2 m_3} \right),$$

where

$$B_{l_1 l_2 l_3}^{(tss)} = \frac{(8\pi)^{3/2}}{3} \sum_{L_1=|l_1\pm 1|, \ell_1}^{L_2=|l_2\pm 1|, \ell_2} (-1)^{\sum_{n=1}^{3} 2^{l_n+m_n}} m_{L_1} m_{L_2} m_{L_3} h_{\ell_1 L_1} h_{\ell_2 L_2} h_{\ell_3 L_3} \left\{ \frac{\ell_1 \ell_2 \ell_3}{L_1 L_2 L_3} \right\}
\times \int_{0}^{\infty} y^2 dy \int_{0}^{\infty} k_1^2 dk_1 T_{\ell_1}(k_1) j_{L_1}(k_1 y) \left[ \prod_{n=2}^{3} \frac{2}{\pi} \int_{0}^{\infty} k_n^2 dk_n T_{\ell_n}(k_n) j_{L_n}(k_n y) \right]
\times \frac{16\pi^4 A_S^2 I_{k_1 k_2 k_3}}{k_1},
$$

(2.6)

with $h_{l_1 l_2 l_3}^{s_1 s_2 s_3} = \sqrt{\frac{(l_1 + 1)(l_2 + 1)(l_3 + 1)}{4\pi}} \left( \frac{l_1}{s_1} \frac{l_2}{s_2} \frac{l_3}{s_3} \right)$ and $h_{\ell_1 L_1} \equiv h_{\ell_1 L_1}^{0,0,0}$. Because of parity conservation and rotational symmetry in eq. (2.1), nonvanishing signal is confined to the $\ell$-space domain satisfying $\ell_1 + \ell_2 + \ell_3 = \text{even}$ and $|\ell_1 - \ell_2| \leq \ell_3 \leq \ell_1 + \ell_2$.

The observed bispectrum is given by the sum of all cyclic terms as $B_{\ell_1 \ell_2 \ell_3} = B_{\ell_1 \ell_2 \ell_3}^{(tss)} + B_{\ell_1 \ell_2 \ell_3}^{(s_1 s_2 s_3)} + B_{\ell_1 \ell_2 \ell_3}^{(s_1 s_2 s_3)}$, where $B_{\ell_1 \ell_2 \ell_3}^{(tss)} = B_{\ell_1 \ell_2 \ell_3}^{(s_1 s_2 s_3)} = B_{\ell_1 \ell_2 \ell_3}^{(s_1 s_2 s_3)}$. As it is customary in CMB bispectrum studies, let us introduce a “reduced bispectrum”, $b_{\ell_1 \ell_2 \ell_3}$, defined as $b_{\ell_1 \ell_2 \ell_3} = B_{\ell_1 \ell_2 \ell_3}/h_{\ell_1 \ell_2 \ell_3}$.
Figure 1 displays the shape of $b_{\ell_1\ell_2\ell_3}$ in $\ell$ space. It is clear that the shape under study strongly peaks on squeezed configurations (e.g., $\ell_1 \ll \ell_2 \sim \ell_3$). The signal-to-noise ratio grows in this case like $\propto \ell_{\text{max}}$, with $\ell_{\text{max}}$ denoting the maximum available multipole [26, 30]. Figure 2 shows expected 1σ errors, $\Delta g_{\text{tss}}$, from a Fisher matrix analysis, for an ideal, full-sky cosmic-variance-limited-level (CVL-level) measurement and for a realistic one, including WMAP-level instrumental noise. We see that for a WMAP-like survey, signal-to-noise essentially saturates at $\ell_{\text{max}} \gtrsim 500$. Therefore we will discard multipoles with $\ell > 500$ in the data analysis phase, discussed below.

Because of the similarity between the two shapes (both peaking in the squeezed limit), one may think that the tensor-scalar-scalar bispectrum, discussed here, and the usual local-type shape, parametrized by $f_{\text{NL}}$, are completely degenerate. In fact this is not the case. The correlation coefficient between the two shapes falls rapidly for $\ell_{\text{max}} \gtrsim 100$, due to the difference between their oscillating patterns [22]. This allows to measure $g_{\text{tss}}$ independently of $f_{\text{NL}}$.

3 CMB bispectrum estimations

In this section we measure $g_{\text{tss}}$ with the WMAP temperature data. We will assume that $g_{\text{tss}}$ is small so the NG signal is subdominant. We may therefore employ an optimal bispectrum estimator:

$$
\hat{g}_{\text{tss}} = \frac{1}{N} \sum_{\ell_1\ell_2\ell_3} \left( \frac{\ell_1}{m_1}, \frac{\ell_2}{m_2}, \frac{\ell_3}{m_3} \right) h_{\ell_1\ell_2\ell_3} b_{\ell_1\ell_2\ell_3} \left[ \prod_{n=1}^{3} \frac{a_{\ell_1 m_n}}{C_{\ell_n}} - 3 \frac{a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}}{C_{\ell_1} C_{\ell_2} C_{\ell_3}} \right],
$$

where $C_{\ell}$ is the temperature power spectrum and $N \equiv \sum_{\ell_1\ell_2\ell_3} (b_{\ell_1\ell_2\ell_3})^2 / (C_{\ell_1} C_{\ell_2} C_{\ell_3})$ is a normalization factor. The input CMB coefficients $a_{\ell m}$ are computed from the observed data or simulated maps, and $\langle \cdots \rangle_{\text{MC}}$ is given by an ensemble average of the products of Gaussian simulated maps with realistic experimental features (i.e., beam shape, anisotropic noise and sky cut).

Bispectrum estimation using eq. (3.1) is fully optimal as long as the signal + noise covariance matrix is diagonal. This is of course not true in realistic cases, due to e.g. sky masking and nonstationary noise. To completely restore optimality, in this case, full-inverse covariance weighting of the data is required, rather than using just diagonal covariance elements, as in the formula above. While the numerical problem of producing inverse covariance weighted CMB maps at WMAP or Planck resolution is fully solved nowadays [31–33], it has also been shown, for bispectrum analyses, that just a minimum (∼5%) loss of optimality can be achieved by means of a much simpler, recursive inpainting pre-filtering technique [34, 35]. This is the approach adopted in Planck papers [9, 10], and followed also in the current analysis.

The best-fit value of $g_{\text{tss}}$ is determined from a coadded temperature map, using WMAP 9-years, V and W bands [8, 25]. The results reported in this paper are obtained from foreground-cleaned data, and we checked that (not-foreground-cleaned) raw data also gives compatible values. For Monte-Carlo evaluation of the error on $g_{\text{tss}}$ and to compute the linear correction term in the estimator ($\langle \cdots \rangle_{\text{MC}}$ in the formula above), we use 500 random Gaussian simulations and follow the WMAP-team methodology [36] to include the effects of anisotropic noise and beam shape in the relevant frequency bands. We mask both observations and simulations with the KQ75 mask, characterized by a sky coverage fraction $f_{\text{sky}} = 0.688$. After removing monopole and dipole components, masked regions are inpainted via the recursive
inpainting pre-filtering technique mentioned above. All the data and instrumental information used here are provided by the Lambda website [37].

3.1 Modal decomposition

Even if $b_{\ell_1\ell_2\ell_3}, a_{\ell m}, C_\ell$ and $\langle \cdots \rangle_{MC}$ are precomputed, a direct implementation of the estimator (3.1) has a prohibitive computational scaling $O(\ell_{\text{max}}^5)$. It is well-known that this issue can be solved when the theoretical bispectrum template can be written in factorized form. While for some bispectrum templates, e.g. local, are explicitly factorizable (see e.g. ref. [38]), this is not the case in general. The tensor case considered here, in particular, displays a very complex, nonseparable $\ell$ dependence in $b_{\ell_1\ell_2\ell_3}$ (see eq. (2.6) and refs. [14, 16, 20, 39]), so there is no simple solution.

To address this issue, in this paper we rely on the so-called separable modal methodology [27, 28]. In this approach, one considers decompositions of $b_{\ell_1\ell_2\ell_3}$ into products of separable modal eigenfunctions in the tetrahedral $\ell$ space. Suitable bases can be found to achieve high correlation between the decomposed and the original shape, using finite, small sets of eigenmodes. This allows the desired construction of approximate separable templates (to arbitrarily high accuracy, at the price of adding more modes). This methodology enables the measurements of many complex shapes, including nonstandard scalar bispectra (e.g., oscillatory bispectra) [9–12, 27, 28] and tensor-mode shapes, both in even and odd $\ell_1 + \ell_2 + \ell_3$ domains [10, 13, 21].

Following this approach, we decompose our starting template (2.6) as

$$\frac{v_{\ell_1}v_{\ell_2}v_{\ell_3}}{\sqrt{C_{\ell_1}C_{\ell_2}C_{\ell_3}}} b_{\ell_1\ell_2\ell_3} = \sum_n \alpha_n Q_n(\ell_1, \ell_2, \ell_3),$$

(3.2)

where the (real) modal basis is given by

$$Q_{ijk}(\ell_1, \ell_2, \ell_3) = \frac{1}{6} q_i(\ell_1) q_j(\ell_2) q_k(\ell_3) + 5 \text{ perms} \equiv q_i^{(\ell_1)} q_j^{(\ell_2)} q_k^{(\ell_3)},$$

(3.3)

and $v_\ell$ is an arbitrary function introduced to improve convergence by adjusting the overall $\ell$ scaling. For notational convenience, here, the triples $ijk$ in the modal basis are represented with a single index $n$. For the modal elements $q_i(\ell)$, we adopt polynomial eigenfunctions and some specific functions peaking in the squeezed limit, to accelerate convergence in the region where the signal is largest. In the following analysis, we perform the decomposition with 600 modes, as in ref. [10], achieving more than 90% correlation between the original and the expanded shape.

Factorizing eq. (3.1) via eq. (3.2) and the identity

$$\left( \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\
1 & m_2 & m_3 \end{array} \right) h_{\ell_1\ell_2\ell_3} = \int d^2 \hat{n} Y_{\ell_1 m_1}(\hat{n}) Y_{\ell_2 m_2}(\hat{n}) Y_{\ell_3 m_3}(\hat{n}),$$

(3.4)

yields

$$\hat{g}_{\ell ss} = \frac{1}{N} \sum_n \alpha_n \beta_n,$$

(3.5)

where $\beta_n$ are computed from the products of the maps filtered by $q_i(\ell)$ in $\ell$ space

$$M_i(\hat{n}) = \sum_{\ell m} q_i(\ell) \frac{a_{\ell m}}{v_\ell \sqrt{C_\ell}} Y_{\ell m}(\hat{n}),$$

(3.6)
higher-order ones are negligible. A commonly used expression reads

\[ g = \text{theoretical template form of } a \]

which in our case reads:

\[ \text{induce spuriously divergent signals on small } \ell \]

Fortunately also shown that this can be cured by using a slightly different expansion kernel, approximately divided into Gaussian and NG contributions as

\[ g = \text{estimator, eq. (3.5), for the shape under study. To this end, we estimate } g \]

Before moving to actual data analysis, we need to perform some validation checks of our estimation process. This makes our bispectrum estimation feasible and fast.

\[ 3.2 \text{ Validation tests with non-Gaussian simulations} \]

To obtain \( \alpha_n \) from \( b_{\ell_1 \ell_2 \ell_3} \), we need to compute another inner product:

\[ \alpha_n = \sum_p \gamma_{np}^{-1} \left\langle \frac{\sqrt{C_{\ell_1} C_{\ell_2} C_{\ell_3}}}{v_{\ell_1} v_{\ell_2} v_{\ell_3}} b_{\ell_1 \ell_2 \ell_3}, Q_p(\ell_1, \ell_2, \ell_3) \right\rangle . \]  

(3.9)

The inner product in \( \gamma_{np} \) can be always written in separable form, reducing its computational cost to \( O(\ell_{\text{max}}^3) \). There is however no way to factorize the inner product in eq. (3.9), since that is where the starting nonseparable shape appears. Computing this product therefore requires \( O(\ell_{\text{max}}^3) \) operations. This is the most time-consuming process in the modal methodology. Note however that this is just a preliminary computation, that needs to be performed only once. Afterwards, the shape is decomposed and only separable quantities enter the actual estimation process. This makes our bispectrum estimation feasible and fast.

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\[ \beta_{n+ijk} \equiv \int d^3n [M_i(\hat{n})M_j(\hat{n})M_k(\hat{n}) - 3 \langle M_i(\hat{n})M_j(\hat{n}) \rangle_{\text{MC}} M_k(\hat{n})] . \]  

(3.7)

Given that \( M_i(\hat{n}) \) is precomputed, this step requires \( O(\ell_{\text{max}}^3) \) operations. In the identical manner, the normalization factor also reduces to \( N = \sum_{n,p} \alpha_n \gamma_{np} \alpha_p \), where \( \gamma_{np} \equiv \langle Q_n, Q_p \rangle \) denotes the inner product of the \( Q \) basis, computed according to

\[ \langle f, g \rangle = \sum_{\ell_1 \ell_2 \ell_3} \frac{b_{\ell_1 \ell_2 \ell_3}}{v_{\ell_1} v_{\ell_2} v_{\ell_3}} f(\ell_1, \ell_2, \ell_3) g(\ell_1, \ell_2, \ell_3) . \]  

(3.8)

To obtain \( \alpha_n \) from \( b_{\ell_1 \ell_2 \ell_3} \), we need to compute another inner product:

\[ \alpha_n = \sum_p \gamma_{np}^{-1} \left\langle \frac{\sqrt{C_{\ell_1} C_{\ell_2} C_{\ell_3}}}{v_{\ell_1} v_{\ell_2} v_{\ell_3}} b_{\ell_1 \ell_2 \ell_3}, Q_p(\ell_1, \ell_2, \ell_3) \right\rangle . \]  

(3.9)

The inner product in \( \gamma_{np} \) can be always written in separable form, reducing its computational cost to \( O(\ell_{\text{max}}^3) \). There is however no way to factorize the inner product in eq. (3.9), since that is where the starting nonseparable shape appears. Computing this product therefore requires \( O(\ell_{\text{max}}^3) \) operations. This is the most time-consuming process in the modal methodology. Note however that this is just a preliminary computation, that needs to be performed only once. Afterwards, the shape is decomposed and only separable quantities enter the actual estimation process. This makes our bispectrum estimation feasible and fast.

\[ 3.2 \text{ Validation tests with non-Gaussian simulations} \]

Before moving to actual data analysis, we need to perform some validation checks of our estimator, eq. (3.5), for the shape under study. To this end, we estimate \( g_{\text{tss}} \) from simulated CMB maps including nonzero \( g_{\text{tss}} \) and check the consistency with input \( g_{\text{tss}} \). In the presence of nonzero (but small) \( g_{\text{tss}} \), the temperature fluctuations can be approximately divided into Gaussian and NG contributions as

\[ a_{\ell m} = a_{\ell m}^{G} + g_{\text{tss}} a_{\ell m}^{NG} . \]

An explicit form of \( a_{\ell m}^{NG} \) should be determined as the bispectrum \( \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle \) recovers the input theoretical template \( g_{\text{tss}} B_{\ell_1 \ell_2 \ell_3} \left( \frac{\ell_1}{m_1} \frac{\ell_2}{m_2} \frac{\ell_3}{m_3} \right) \), provided that the \( O(g_{\text{tss}}^0) \) contributions and higher-order ones are negligible. A commonly used expression reads

\[ a_{\ell_1 m_1}^{NG} = \frac{1}{6} \prod_{n=2}^{3} \left[ \sum_{\ell_n m_n} \frac{a_{\ell_n m_n}^{G} a_{\ell_n m_n}^{NG}}{C_{\ell_n}} \right] \left( \frac{\ell_1}{m_1} \frac{\ell_2}{m_2} \frac{\ell_3}{m_3} \right) B_{\ell_1 \ell_2 \ell_3} . \]  

(3.10)

However, ref. [40] has shown that, for the squeezed-type bispectrum, this algorithm can induce spuriously divergent signals on small \( \ell_1 \) and hence does not work in practice. It was fortunately also shown that this can be cured by using a slightly different expansion kernel, which in our case reads:

\[ a_{\ell_1 m_1}^{NG} = \frac{1}{2} \prod_{n=2}^{3} \left[ \sum_{\ell_n m_n} \frac{a_{\ell_n m_n}^{G} a_{\ell_n m_n}^{NG}}{C_{\ell_n}} \right] \left( \frac{\ell_1}{m_1} \frac{\ell_2}{m_2} \frac{\ell_3}{m_3} \right) B_{\ell_1 \ell_2 \ell_3}^{(\text{sst})} . \]  

(3.11)
$$(l+1)C_{NG}^l / (2\pi) \times T_0^2 \left[ \mu K^2 \right]$$

**Figure 3.** Angular power spectrum of the NG part of a single realization ($a_{NG}^{\ell m}$), generated from formula (3.11), up to $\ell = 300$ (yellow line), and its theoretical prediction (purple line).

Note that, in the formula above, we have replaced $B_{\ell_1\ell_2\ell_3}$ with $B_{\ell_1\ell_2\ell_3}^{(s s t)}$, i.e. we removed cyclic terms from the original formula, and replaced accordingly the prefactor 1/6 with 1/2. This stabilizes the algorithm against the small-$\ell_1$ divergence, because of the suppression of $B_{\ell_1\ell_2\ell_3}^{(s s t)}$ outside of the squeezed triangles ($\ell_1 \sim \ell_2 \gg \ell_3$).

Also for this algorithm, the straightforward computation of eq. (3.11) requires $O(\ell_{\text{max}}^5)$ operations, which are however in principle reduced to $O(\ell_{\text{max}}^3)$, once separability via modal decomposition is obtained [27, 28]. Unfortunately, expanding $b_{\ell_1\ell_2\ell_3}^{(s s t)}$ (no cyclic terms) turned out to be very difficult. The asymmetry of this kernel led in fact to very slow convergence. Therefore, we decided in the end to rely on the straightforward, brute-force computation of the nonseparable form (3.11). This turned out to be slow but numerically feasible for the angular resolution of this analysis ($\ell < 500$).

To validate our NG estimator, we generate 50 $a_{NG}^{\ell m}$’s up to $\ell = 300$. The yellow line in figure 3 corresponds to the angular power spectrum of the NG part of one of the maps. As expected above, there is no pathological enhancement at small $\ell$’s, and the recovered $C_{\ell}$ are fully consistent with their theoretical expectation (purple line in the plot), which reads:

$$C_{NG}^{\ell_1, \text{th}} = \frac{1}{4} \sum_{\ell_2\ell_3} \frac{\left| B_{\ell_1\ell_2\ell_3}^{(s s t)} \right|^2 + B_{\ell_1\ell_2\ell_3}^{(s s t)}B_{\ell_1\ell_2\ell_3}^{(s s t)}}{(2\ell_1 + 1)C_{\ell_2}C_{\ell_3}}.$$

(3.12)
Having checked the reliability of the NG part of our 50 simulated maps, we now generate 50 NG realizations with $g_{\text{tss}} = 100$, i.e. we set $a_{\ell m} = a_{\ell m}^{G} + 100a_{\ell m}^{NG}$. We then use our estimator (3.5) to measure $g_{\text{tss}}$, map-by-map. We consider both a CVL-level scenario, with $f_{\text{sky}} = 1$, and a realistic one, with the same noise properties and sky coverage as in WMAP data. All CMB maps used for the latter case were processed in accordance with our WMAP data analysis pipeline described at the beginning of this section. Figure 4 shows the map-by-map comparison of estimated $g_{\text{tss}}$ for these two cases. It is visually apparent there that the CVL-level case and the WMAP-like one fluctuate around $g_{\text{tss}} = 100$ in a very similar way, with the latter case being more scattered than the former, due to the inclusion of mask and noise. The average $g_{\text{tss}}$ from the 50 maps becomes turned out to be 97 in both cases. The error bar on $g_{\text{tss}}$ was derived from 500 Gaussian realizations, obtaining $\Delta g_{\text{tss}} = 36$ (68%CL), in excellent agreement with the Fisher matrix forecast displayed in figure 2. The above results validate our pipeline, showing that the estimator is unbiased and optimal.

3.3 WMAP limits

Having tested the estimator on mock realizations, we moved to actual WMAP data and repeated the estimation procedure described above, to find our $g_{\text{tss}}$ bounds. We checked stability of the results in the $\ell$-domain by repeating the analysis for varying $\ell_{\text{max}}$, from 100 to 500. As a further validation test, we also used the same pipeline to measure the standard $f_{\text{NL}}^{\text{local}}$
Figure 5. Central values and 2σ errors on $g_{tss}$ obtained from the WMAP data as a function of $\ell_{\text{max}}$.

parameter, obtaining fully consistent results with those shown in the literature [8, 36, 41, 42]. Figure 5 shows the limits on $g_{tss}$ as a function of $\ell_{\text{max}}$, indicating no evidence of $g_{tss}$ for $\ell_{\text{max}} \leq 500$, at any scale, at 95%CL. We take the result at $\ell_{\text{max}} = 500$ as our final bound: $g_{tss} = -48 \pm 28$ (68%CL).

One may worry here about the contamination due to secondary sources of temperature NG, which we have not considered so far. In particular, it is widely known that the lensed bispectrum can become an important source of bias in the squeezed limit. However, such lensed signal becomes large when higher multipoles are considered, and almost uncorrelated to the primordial one for $\ell_{\text{max}} \leq 500$ [22]. Therefore, ISW-lensing debiasing is not necessary in our analysis.

4 Conclusions

In this paper, we have studied the inflationary tensor-scalar-scalar three point function, for models characterized by nonzero graviton mass. A nonvanishing CMB temperature bispectrum is one of the predictions of such models, so we have actually tested it, using WMAP 9-year temperature data. The primordial and the induced CMB bispectrum, $b_{\ell_{1}\ell_{2}\ell_{3}}^{(tss)}$, peak in the squeezed limit, in this scenario, and specific Fisher matrix forecasts show that interesting bounds can already be obtained at WMAP angular resolution, which motivated our analysis.

To circumvent nonseparability issues of the primordial shape under study, we have relied on a modal estimation pipeline, which was thoroughly validated on simulated NG
maps, generated as a part of this work. After the preliminary validation stage we measured $g_{tss}$ for several $\ell_{\text{max}}$’s, finding no evidence for a tensor-scalar-scalar signal at all scales. The most stringent bound is obtained with $\ell_{\text{max}} = 500$ and reads $g_{tss} = -48 \pm 28$ (68\%CL).

To the best of our knowledge, this is the first paper reporting observational constraints on the primordial tensor-scalar-scalar bispectrum. While we report no evidence of such a signal at the end of our WMAP analysis, it is worth to point out that more sensitive, higher angular resolution surveys, including polarization information, can lead to a further order of magnitude improvement over the current bound [22, 29], thus extending our detectability window. This encourages follow-up investigations with temperature and polarized data measured in Planck [10] and forthcoming CMB experiments [43–45], as part of our future work. An analysis of the Planck data using the present pipeline, is actually already ongoing within the Planck collaboration.

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