Infrared Fixed Point in the Strong Running Coupling: Unraveling the $\Delta I = 1/2$ puzzle in $K$-Decays*

R. J. Crewther and Lewis C. Tunstall

1CSSM and ARC Centre of Excellence for Particle Physics at the Tera-scale, School of Chemistry and Physics, University of Adelaide, Adelaide SA 5005 Australia

In this talk, we present an explanation for the $\Delta I = 1/2$ rule in $K$-decays based on the premise of an infrared fixed point $\alpha_{IR}$ in the running coupling $\alpha_s$ of quantum chromodynamics (QCD) for three light quarks $u, d, s$. At the fixed point, the quark condensate $\langle \bar{q}q \rangle_{vac} \neq 0$ spontaneously breaks scale and chiral $SU(3)_L \times SU(3)_R$ symmetry. Consequently, the low-lying spectrum contains nine Nambu-Goldstone bosons: $\pi, K, \eta$ and a QCD dilaton $\sigma$. We identify $\sigma$ as the $f_0(500)$ resonance and construct a chiral-scale perturbation theory $\chiPT$ for low-energy amplitudes expanded in $\alpha_s$ about $\alpha_{IR}$. The $\Delta I = 1/2$ rule emerges in the leading order of $\chiPT$ through a $\sigma$-pole term $K_S \to \sigma \to \pi\pi$, with a $g_{K_S\sigma}$ coupling fixed by data on $\gamma\gamma \to \pi^0\pi^0$ and $K_S \to \gamma\gamma$. We also determine $R_{IR} \approx 5$ for the nonperturbative Drell-Yan ratio at $\alpha_{IR}$.

PACS numbers: 12.38.Aw, 13.25.Es, 11.30.Na, 12.39.Fe

Keywords: Nonperturbative QCD, Infrared fixed point, Dilaton, Chiral lagrangians, Nonleptonic kaon decays

I. THE $\Delta I = 1/2$ PUZZLE

Amidst the rich phenomenology of $K$-mesons lies a severe problem—so old that new solutions are rarely attempted—associated with the strangeness-changing $|\Delta S|=1$, nonleptonic decays of the short- and long-lived states

\[ K_S \to \pi\pi, \quad K_L \to \pi\pi\pi. \tag{1} \]

Experimentally, there is a large enhancement of the isospin-$\frac{1}{2}$ decays. This phenomenon is particularly striking in the $S$-wave $\pi\pi$ mode, where the measured rates \[ \Gamma \] exhibit the ratios

\[ \gamma_{+-} = \frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K^+ \to \pi^+\pi^0)} \simeq 463, \quad \gamma_{00} = \frac{\Gamma(K_S \to \pi^0\pi^0)}{\Gamma(K^+ \to \pi^+\pi^0)} \simeq 205, \tag{2} \]

which are in enormous disagreement with the naive expectations $\gamma_{+-} \sim O(1) \sim \gamma_{00}$ from perturbative electroweak calculations. It is useful to translate the above in terms of isospin amplitudes $A_I$ for the final $\pi\pi$ state. The $I = 1$ state is forbidden by Bose symmetry and thus the transition amplitudes can be parametrized as \[ A \]

\[ A(K_S \to \pi^+\pi^-) = \frac{2}{\sqrt{3}} A_0 e^{i\delta_0} + \frac{2}{\sqrt{3}} A_2 e^{i\delta_2}, \tag{3} \]
\[ A(K_S \to \pi^0\pi^0) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} - \frac{2}{\sqrt{3}} A_2 e^{i\delta_2}, \tag{4} \]
\[ A(K^+ \to \pi^+\pi^0) = \sqrt{\frac{3}{2}} A_2 e^{i\delta_2}, \tag{5} \]

where the $\pi\pi$-scattering phase shifts $\delta_I$ arise as a consequence of Watson’s theorem. Comparison with the data in \[ A \] leads to the $\Delta I = 1/2$ rule for kaons\[ A \]

\[ \Re |A_0/A_2| \simeq 22, \tag{6} \]
whose origin remains a mystery despite five decades of theoretical investigation.

The difficulty presumably arises from the nonperturbative nature of quantum chromodynamics (QCD) at low energies $\mu \ll m_{b,c}$. Where confinement reigns and chiral symmetry for the light quarks $u,d,s$ is believed to be spontaneously broken by the formation of a quark condensate

$$\lim_{m_q \to 0} \langle \bar{q}q \rangle_{\text{vac}} \neq 0, \quad q = u, d, s.$$  \hfill (7)

Below the chiral symmetry-breaking scale $\Lambda_{SB} \approx 1$ GeV, great progress has been made in charting the low-energy structure of QCD through the use of chiral $SU(3)_{L} \times SU(3)_{R}$ perturbation theory $\chi$PT. The method relies on an alternative expansion parameter to the strong coupling $\chi$ leading order of $\Lambda_{SB}$ structure of QCD through the use of chiral $SU(3)$, SU(2), and SU(1) for $\Delta I=0,1,2$ puzzle: why is $M_{\pi}/M_{\rho}$ unreasonably large ($\approx 0.2$) when compared with simple quark-model estimates of a $27$ operator

$$Q_{27} = J_{13}J_{31} + \frac{3}{2}J_{23}J_{11},$$  \hfill (13)

the nonleptonic decays are described by the compact expression

$$\mathcal{L}_{\text{weak}} = g_{8}Q_{8} + g_{27}Q_{27} + \text{h.c.}.$$  \hfill (14)

In general, the low-energy coefficients $g_{i}$ are not fixed by symmetry arguments alone. This constitutes the essence of the $\Delta I = 1/2$ puzzle: why is $|g_{8}/g_{27}|$ unrealistically large ($\approx 22$) when compared with simple quark-model estimates for $\Delta I = 3/2$ $K$-decay amplitudes? Explaining this 'octet dominance' has been tackled with a variety of techniques including the many color $N_{c}$ limit [7], QCD sum rules [12,13], and direct evaluation on the lattice [14,15], each with varying degrees of success.

II. INFRARED FIXED POINT IN THE STRONG RUNNING COUPLING

The goal of this talk is to present an alternative line of investigation [19] and argue that the $\Delta I = 1/2$ rule for $K$-decays (Eq. (13)) is intimately linked to the behavior of $\alpha_{s}$ in the asymptotic infrared limit. As is well known,
FIG. 1: Varieties of asymptotic behavior for the QCD \( \beta \)-function with three light quarks \( u, d, s \). The dashed line shows the case when the strong running coupling \( \alpha_s \) undergoes continued growth with decreasing scale \( \mu \) (item (i) in text), while the solid line shows \( \alpha_s \) flowing to an infrared fixed point \( \alpha_{\text{IR}} \) (item (ii)). For completeness, we also include the case where \( \alpha_s \) diverges at a finite value of \( \mu \) (dotted line), but we emphasize that this scenario is of no physical relevance since it produces poles in Green’s functions in the spacelike momentum region (i.e. a tachyon or “Landau ghost”).

the running of \( \alpha_s \) with scale \( \mu \) is governed by the QCD \( \beta \)-function. At low energies \( \mu \ll m_{t,c,b} \), heavy quarks \( t, b, c \) decouple from the theory, and thus there are two logical possibilities\(^2\) for the resulting three-flavor theory (Fig. 1):

i  **Growth without bound.** If the integral

\[
\int_{\alpha_0}^{\alpha_s} \frac{dz}{\beta(z)} \tag{15}
\]

is divergent, then the solution to the renormalization group equation

\[
\ln \frac{\mu}{\mu_0} = \int_{\alpha_0}^{\alpha_s} \frac{dz}{\beta(z)} \tag{16}
\]

implies that as \( \mu \) decreases, \( \alpha_s \) grows in value, becoming infinite in the infrared limit \( \ln \frac{\mu}{\mu_0} \to -\infty \). Analyses based on lattice simulations\(^2\) suggest that this scenario occurs for \( N_f = 0 \) quark flavors (pure Yang-Mills).

ii  **Infrared fixed point at finite coupling.** In this scenario,

\[
\int_{\alpha_0}^{\alpha_{\text{IR}}} \frac{dz}{\beta(z)} \tag{17}
\]

diverges because of a simple zero in \( \beta(z) \) at an infrared fixed point \( z = \alpha_{\text{IR}} \). From a purely phenomenological point of view, this situation is perhaps the most interesting for it implies that the gluonic term \( \sim G_{\mu\nu}^a G^{a\mu\nu} \) in the trace anomaly of the energy-momentum tensor \( \theta_{\mu\nu} \),

\[
\theta_{\mu\nu} = \frac{\beta(\alpha_s)}{4\alpha_s} G_{\mu\nu}^a G^{a\mu\nu} + \left( 1 + \gamma_m(\alpha_s) \right) \sum_{q=u,d,s} m_q \bar{q}q \tag{18}
\]

vanishes at the fixed point \( \alpha_{\text{IR}} \) and thus \( \theta_{\mu\nu} \) becomes traceless in the chiral limit

\[
\theta_{\mu\nu} \big|_{\alpha_s=\alpha_{\text{IR}}} = \left( 1 + \gamma_m(\alpha_{\text{IR}}) \right) \left( m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s \right) \to 0 \ , \ SU(3)_L \times SU(3)_R \text{ limit} . \tag{19}
\]

Naively, one might consider this scenario a phenomenological disaster: does a traceless \( \theta_{\mu\nu} \) not imply invariance under scaling transformations \( \xi : x \to e^\xi x \) and hence a continuous mass spectrum? In QCD, the answer is in the negative since the attendant strong gluon fields still drive quark condensation \( \langle \bar{q}q \rangle_{\text{vac}} \neq 0 \), from which one concludes that the scale—and of course, chiral—symmetry is in fact *spontaneously broken*.\(^3\) Goldstone’s theorem and approximate chiral-scale symmetry then implies that there are nine pseudo-NG bosons in the low-lying spectrum: the usual pseudoscalar octet \( \pi, K, \eta \) and a 0++ QCD dilaton \( \sigma \), whose mass is (mostly) set by the explicit breaking term \( \sim m_s \) in (19).

\(^2\) The analogous case for QED is discussed in Ref. 21.

\(^3\) The idea that \( \langle \bar{q}q \rangle_{\text{vac}} \) may act as a chiral-scale condensate was considered prior to the advent of QCD in Refs. 22, 23.
It is worth emphasizing that it is unclear from the literature which scenario is actually realized in QCD, and in particular, how sensitive the results are to the number of active quark flavors. Part of the problem resides in the fact that for each calculation, a nonperturbative definition for $\alpha_s$ must be chosen, and thus comparing results from different approaches is rarely straightforward. Popular choices include the Schrödinger functional method on the lattice, 24, 25 solutions to Dyson-Schwinger equations 26, 27 for three-gluon and ghost- or quark-gluon vertices, the method of effective charges 28, 29, and AdS/CFT inspired models 30.

Despite the lack of consensus, we take the view that an infrared fixed point in QCD (item (ii)) should be taken seriously, and the remainder of this talk concerns the phenomenological implications which arise from this scenario.

### III. THE LOWEST QCD RESONANCE AS A QCD DILATON

While it may seem dramatic to introduce a new NG boson into the low-energy spectrum of QCD, there is in fact a very natural candidate for $\sigma$: the $f_0(500)$ resonance, whose mass and width have been determined with remarkable precision through an analytic continuation of the Roy equations 31,

$$M_{f_0} = 441^{+16}_{-8} \text{ MeV}, \quad \Gamma_{f_0} = 544^{+18}_{-25} \text{ MeV}, \quad |g_{f_0 \pi \pi}| = 3.31^{+0.35}_{-0.15} \text{ GeV}.$$  

Since this pioneering work, many studies (see e.g. Fig. 8 in Ref. 32) have extracted $f_0$ pole parameters consistent with (20), to the extent that the Particle Data Group 1 has recently updated their listing to account for the greatly reduced uncertainties. 5

A. The $f_0(500)$ and $\chi_{PT}^3$: Problems with $SU(3)_L \times SU(3)_R$?

In all determinations of the kind given by Ref. 31, the real part of the $f_0$ pole is found to be $\lesssim m_K$. This places $f_0$ right in the middle of the NG sector $\pi, K, \eta$ and encourages us to revisit the observation 34 that $\chi_{PT}^3$ works well except for $0^{++}$ amplitudes with $O(m_K)$ extrapolations in momentum. Since $f_0$ is a broad flavor singlet coupled strongly to $\pi, K, \eta$, it should dominate low-energy scattering. However, this leads to the following problem: $f_0$ cannot contribute to the LO terms $A_{LO}$ in (8) so the dominant effect must be generated at next to leading order (NLO). How does this square with the expectation that NLO terms are $\lesssim 30\%$ the LO prediction?

One alternative (and the one we shall pursue) is to note that if $f_0 = \sigma$ is treated a pseudo-NG boson (dilaton), then all the convergence issues in the $0^{++}$ channel disappear. To that end, we replace $\chi_{PT}^3$ with a model-independent chiral-scale theory $\chi_{PT_\sigma}$ based on expansions in $\alpha_s$ about $\alpha_{IR}$.

### IV. EFFECTIVE LAGRANGIANS FOR APPROXIMATE CHIRAL-SCALE SYMMETRY

A. Strong Interactions

Our task is to construct an effective field theory of approximate scale and chiral $SU(3)_L \times SU(3)_R$ symmetry. For the strong interactions we seek an effective Lagrangian of the form

$$L[\sigma, U, U^+] = L_{d=4}^{\text{inv}} + L_{d>4}^{\text{anom}} + L_{d<4}^{\text{mass}} ;$$

where each term is distinguished by the scaling dimension $d$:

$$\delta_{\xi} \mathcal{L}_d = \partial^{4}(x_{\lambda} \mathcal{L}_d) + (d - 4) \mathcal{L}_d .$$

The rules for constructing Lagrangians of this type were worked out long ago and we refer to Refs. 23, 35–37 for explicit details. The salient features are as follows. For expansions in $\alpha_s$ about $\alpha_{IR}$, we have

$$d_{\text{max}} = 3 - \gamma_m(\alpha_{IR}) < 4 ,$$

---

4 Extrapolations based on fixed-order perturbation theory are likely to introduce non-physical artifacts.

5 See Ref. 32 for a discussion on this point.
while the gluonic operator insertion $\sim G_{\mu\nu}^a G^{a\mu\nu}$ (obtained by acting $\partial/\partial s$ on the Callan-Symanzik equation) implies

$$d_{\text{anom}} = 4 + \beta'(\alpha_{\text{IR}}) > 4. \quad (24)$$

To realize scale invariance as a NG symmetry, a dilaton field $\sigma$ is introduced with nonlinear scaling property $\xi: \sigma \to \sigma + \xi F_\sigma$ so that $e^{\sigma/F_\sigma}$ is covariant and has $d = 1$:

$$\delta_\xi e^{\sigma/F_\sigma} = (1 + x \cdot \partial) e^{\sigma/F_\sigma}. \quad (25)$$

Here $F_\sigma$ is the dilaton decay constant, defined by the coupling of $\theta_{\mu\nu}$ to the vacuum

$$\langle \sigma | \theta_{\mu\nu} | \text{vac} \rangle = \frac{F_\sigma}{3} (q_\mu q_\nu - g_{\mu\nu} q^2). \quad (26)$$

Lagrangian operators of the required scale dimension are then obtained by multiplying operators such as

$$K = \frac{F_\sigma^2}{4} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger), \quad K_\sigma = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma, \quad (27)$$

by appropriate powers of $e^{\sigma/F_\sigma}$. In the LO of $\chi PT_\sigma$ the most general effective Lagrangian for strongly interacting $\pi, K, \eta$ and $\sigma$ mesons is given by

$$\mathcal{L}^{d=4}_{\text{inv}} = \{ c_1 K + c_2 K_\sigma + c_3 e^{2\sigma/F_\sigma} \} e^{2\sigma/F_\sigma},$$

$$\mathcal{L}^{d>4}_{\text{anom}} = \{ (1 - c_1) K + (1 - c_2) K_\sigma + c_4 e^{2\sigma/F_\sigma} \} e^{(2 + \beta')\sigma/F_\sigma}, \quad (28)$$

$$\mathcal{L}^{d<4}_{\text{mass}} = \text{Tr}(M U^\dagger + U M^\dagger) e^{(3 - \gamma_m)\sigma/F_\sigma}. \quad (29)$$

The low-energy constants $c_i$ are not fixed by symmetry arguments, however, the requirement of a stable vacuum in the $\sigma$ direction implies that $c_3$ and $c_4$ are $O(M)$ and not independent. For expansions about $\sigma = 0$, the absence of tadpoles implies that all terms linear in $\sigma$ must cancel:

$$4c_3 + (4 + \beta')c_4 = (\gamma_m - 3) \langle \text{Tr}(M U^\dagger + U M^\dagger) \rangle_{\text{vac}}$$

$$= (\gamma_m - 3) F_\sigma^2 (m_K^2 + \frac{1}{2} m_\pi^2). \quad (30)$$

1. Determining $F_\sigma$

The relationship between $F_\sigma$ and $g_{\sigma NN}$ is deduced by an analogue of the Goldberger-Treiman relation (Fig. 2):

$$\langle N | \theta_\mu^\sigma | N \rangle = M_N = (-m_\sigma^2 F_\sigma) \cdot (-i m_\pi^2)(-i g_{\sigma NN}) = F_\sigma g_{\sigma NN}. \quad (31)$$

One-boson exchange combined with large-$N_c$ arguments \cite{38,39} leads to the estimate $g_{\sigma NN} \simeq 9$ with a large model-dependent uncertainty. We take the central value and obtain $F_\sigma \simeq 100$ MeV.

2. Strong phenomenology: $\pi\pi$-scattering

From $\mathcal{L}$ we obtain the dilaton mass

$$m_\sigma^2 F_\sigma^2 = F_\sigma^2 (m_K^2 + \frac{1}{2} m_\pi^2)(3 - \gamma_m)(1 + \gamma_m) - \beta'(4 + \beta') c_4, \quad (33)$$
and the effective $g_{\sigma\pi\pi}$ coupling

$$\mathcal{L}_{\sigma\pi\pi} = \left\{ (2 + (1 - c_1)\beta')|\partial\pi|^2 - (3 - \gamma_m)m_\pi^2|\pi|^2 \right\} \sigma/(2F_\sigma).$$

(34)

Note the key feature of (34): it is mostly derivative so it has a small effect on $\pi\pi$-scattering in the $SU(2)_L \times SU(2)_R$ limit where $\vartheta = O(m_\pi)$ and $m_\pi$ (and thus $m_\sigma^2$) remains fixed. The vertex for an on-shell amplitude for $\sigma \rightarrow \pi\pi$ is readily obtained,

$$g_{\sigma\pi\pi} = -(2 + (1 - c_1)\beta')m_\sigma^2/(2F_\sigma) + O(m_\pi^2),$$

(35)

and generates part of the broad width for $\sigma$

$$\Gamma_{\sigma\pi\pi} \approx \frac{|g_{\sigma\pi\pi}|^2}{16\pi m_\sigma} \sim \frac{m_\sigma^3}{16\pi F_\sigma^2} \sim 250 \text{ MeV}.$$ 

(36)

Note that $\Gamma_{\sigma\pi\pi} \sim O(m_\sigma^3)$ and thus NLO in the chiral-scale expansion.

B. Weak Interactions and $K_S \rightarrow \pi\pi$

In $\chi PT_\sigma$, the anomalous dimension $\gamma_{mw}$ of $Q_M$ differs from that of $\mathcal{L}_{\text{mass}}$. Consequently, the chiral-scale Lagrangian includes a term $Q_M e^{(3-\gamma_{mw})\sigma}/F_\sigma$ whose $\sigma$ dependence cannot be eliminated by a chiral rotation. Instead, after vacuum alignment, the weak interactions are described in the LO of $\chi PT_\sigma$ by

$$\mathcal{L}_{\text{align}} = Q_8 \sum_n g_{8n} e^{(2-\gamma_{8n})\sigma}/F_\sigma + g_{27} Q_{27} e^{(2-\gamma_{27})\sigma}/F_\sigma$$

$$+ Q_{mw} \left\{ e^{(3-\gamma_{mw})\sigma}/F_\sigma - e^{(3-\gamma_m)\sigma}/F_\sigma \right\} + h.c.,$$

(37)

noting that $Q_8$ represents quark-gluon operators with differing dimensions at $\alpha_{IR}$. As a result, $K_S$ and $\sigma$ mix through the interaction $\mathcal{L}_{K\sigma} = g_{K\sigma} K_S^0 \sigma$, where the effective coupling

$$g_{K\sigma} = (\gamma_m - \gamma_{mw})\text{Re}\{(2m_K^2 - m_\pi^2)\bar{g}_M - m_\sigma^2g_M\} F_\sigma/2F_\sigma,$$

(38)

produces a $\Delta I = 1/2$ amplitude $A_{\sigma\text{-pole}}$ (Fig. 3). The value of this coupling can be fixed by data on $\gamma\gamma \rightarrow \pi^0\pi^0$ and $K_S^0 \rightarrow \gamma\gamma$, with the result

$$|g_{K\sigma}| \approx 4.4 \times 10^3 \text{ keV}^2$$

(39)

accurate to about 30% precision. Subject to the uncertainty on $F_\sigma$, we estimate

$$|A_{\sigma\text{-pole}}| \approx 0.34 \text{ keV},$$

(40)

thereby accounting for the large $I = 0 \pi\pi$ amplitude $A_0$ in $\chi PT_\sigma$. It follows that $g_8$ and $g_{27}$ are allowed to have similar magnitudes (with the latter fixed precisely by $K^+ \rightarrow \pi^+\pi^0$) and thus the octet dominance hypothesis is no longer necessary to explain the $\Delta I = 1/2$ rule.
C. Bonus: Drell-Yan Ratio at $\alpha_{\text{IR}}$

Another welcome feature of $\chi_{\text{PT}}$ is that it makes a nonperturbative prediction for the Drell-Yan ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}, \quad (41)$$

at $\alpha_s = \alpha_{\text{IR}}$. This is so because the electromagnetic trace anomaly \cite{40, 41}

$$\theta^{\mu\nu} = \theta^{\mu\nu}_{\text{str}} + \frac{\alpha}{6\pi} R F_{\mu\nu} F^{\mu\nu}, \quad (42)$$

implies an effective $\sigma\gamma\gamma$ coupling

$$L_{\sigma\gamma\gamma} = \frac{1}{2} g_{\sigma\gamma\gamma} \sigma F_{\mu\nu} F^{\mu\nu}. \quad (43)$$

Here $F_{\mu\nu}$ and $\alpha \approx 1/137$ are the electromagnetic field strength and fine-structure constant. Through direct calculation, \cite{43} can be shown to be \cite{19}

$$g_{\sigma\gamma\gamma} = \frac{(R_{\text{IR}} - \frac{1}{2})\alpha}{3\pi F_{\sigma}}. \quad (44)$$

Dispersive analyses \cite{42, 43} of $\gamma\gamma \to \pi^0\pi^0$ indicate that the residue of the $f_0$ pole can be unambiguously extracted from data. Within the uncertainty in the value of $F_{\sigma}$, we use the updated value \cite{44} $\Gamma_{f_0\gamma\gamma} = (1.98 \pm 0.3)$ keV to obtain

$$R_{\text{IR}} \approx 5. \quad (45)$$

V. CONCLUDING REMARKS

We have seen that an infrared fixed point in the three flavor $\beta$-function of QCD leads to an extended NG sector \{\pi, K, \eta, \sigma\}, where the $f_0(500)$ resonance is identified with $\sigma$ as the dilaton of spontaneously broken scale symmetry. Despite a seemingly drastic change to the accepted low-energy structure of QCD, the resulting chiral-scale perturbation theory $\chi_{\text{PT}}$ is rather conservative: $f_0$ pole terms are promoted to leading order in $\chi_{\text{PT}}$ (thereby evading $\chi_{\text{PT}}^3$’s convergence problems in the $0^{++}$ channel), yet the successful leading order predictions of $\chi_{\text{PT}}^3$ are preserved. While our key result is a simple explanation for the $\Delta I = 1/2$ rule in $K$-decays, $\chi_{\text{PT}}$ is certainly a general framework and it would be interesting to examine the consequences of the effective theory for other well studied conundrums such as $CP$-violation, rare kaon decays, and $\eta \to 3\pi$.

Acknowledgements

L. C. T. thanks the organizers and C. Dominguez in particular for a most pleasant workshop and for the opportunity to present this work. L. C. T. has benefited from discussions with Profs. H. Leutwyler, M. Jamin, and P. Minkowski, and thanks them for their useful comments.

\begin{itemize}
  \item \cite{1} J. Beringer et al. (Particle Data Group), Phys. Rev. D 86, 010001 (2012).
  \item \cite{2} R. E. Marshak, Riazuddin, and C. P. Ryan, Theory of Weak Interactions in Particle Physics (John Wiley & Sons, 1969), Vol. 24, pp. 543–549.
  \item \cite{3} J. A. Cronin, Phys. Rev. 161, 1483 (1967).
  \item \cite{4} C. Bernard, T. Draper, A. Soni, H. D. Politzer, and M. B. Wise, Phys. Rev. D 32, 2343 (1985).
  \item \cite{5} R. J. Crewther, Nucl. Phys. B 264, 277 (1986).
  \item \cite{6} M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108 (1974).
  \item \cite{7} D. Tadic and J. Trampetic, Phys. Lett. B 114, 179 (1982).
  \item \cite{8} A. Buras and J. -M. Gerard, Nucl. Phys. B 264, 371 (1986).
  \item \cite{9} R. S. Chivukula, J. M. Flynn, and H. Georgi, Phys. Lett. B 171, 453 (1986).
  \item \cite{10} W. A. Bardeen, A. J. Buras, and J. -M. Gerard, Phys. Lett. B 180, 133 (1986).
  \item \cite{11} J. Bijnens and B. Guberina, Phys. Lett. B 205, 103 (1988).
\end{itemize}
[12] B. Guberina, A. Pich, and E. de Rafael, *Phys. Lett. B* **163**, 198 (1985).
[13] A. Pich, B. Guberina, and E. de Rafael, *Nucl. Phys. B* **277**, 197 (1986).
[14] A. Pich and E. de Rafael, *Phys. Lett. B* **189**, 369 (1987).
[15] A. Pich and E. de Rafael, *Nucl. Phys. B* **358**, 311 (1991).
[16] T. Blum et al. (RBC and UKQCD Collaboration), *Phys. Rev. D* **84**, 114503 (2011).
[17] T. Blum et al. (RBC and UKQCD Collaboration), *Phys. Rev. Lett.* **108**, 141601 (2012).
[18] P. A. Boyle et al. (RBC and UKQCD Collaboration), *Phys. Rev. Lett.* **110**, 152001 (2013).
[19] R. J. Crewther and L. C. Tunstall, [arXiv:1203.1321 [hep-ph]].
[20] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, 1996), Vol. 2, pp. 130–133.
[21] M. Lüscher, R. Sommer, P. Weisz, and U. Wolff, *Nucl. Phys. B* **413**, 481 (1994).
[22] R. J. Crewther, *Phys. Lett. B* **33**, 305 (1970).
[23] J. Ellis, *Nucl. Phys. B* **22**, 478 (1970).
[24] M. Lüscher, R. Narayanan, P. Weisz, and U. Wolff, *Nucl. Phys. B* **384**, 168 (1992).
[25] S. Aoki et al. (PACS-CS Collaboration), *JHEP* **10**, 053 (2009).
[26] L. von Smekal, A. Hauck and R. Alkofer, *Phys. Rev. Lett.* **79**, 3591 (1997).
[27] C. S. Fischer and R. Alkofer, *Phys. Rev. D* **67**, 094020 (2003).
[28] G. Grunberg, *Phys. Lett. B* **95**, 70 (1980); **110**, 501 (E) (1982).
[29] G. Grunberg, *Phys. Rev. D* **29**, 2315 (1984).
[30] S. Brodsky, G. F. de Teramond, and A. Deur, *Phys. Rev. D* **81**, 096010 (2010).
[31] I. Caprini, G. Colangelo, and H. Leutwyler, *Phys. Rev. Lett.* **96**, 132001 (2006).
[32] M. Albada de and J. A. Oller, *Phys. Rev. D* **86**, 034003 (2012).
[33] J. R. Pelaye, [arXiv:1303.0125 [hep-ph]].
[34] U.-G. Meissner, Comments *Nucl. Part. Phys.* **20**, 119 (1991).
[35] S. Aoki and J. Ellis, *Phys. Rev. D* **184**, 1760 (1969).
[36] C. J. Isham, A. Salam, and J. Strathdee, *Phys. Lett. B* **31**, 300 (1970).
[37] C. J. Isham, A. Salam, and J. Strathdee, *Phys. Rev. D* **2**, 685 (1970).
[38] A. Calle Cordon and E. Ruiz Arriola, *Phys. Rev. C* **81**, 044002 (2010).
[39] R. J. Crewther, *Phys. Rev. Lett.* **28**, 1421 (1972).
[40] M. S. Chanowitz and J. Ellis, *Phys. Lett. B* **40**, 397 (1972).
[41] M. R. Pennington, *Phys. Rev. Lett.* **97**, 011601 (2006).
[42] M. R. Pennington, *Mod. Phys. Lett. A* **22**, 1439 (2007).
[43] J. A. Oller and L. Roca, *Eur. Phys. J. A* **37**, 15 (2008).