We study the electromagnetic nucleon form factors within the approach based on light-cone sum rules. We include the next-to-leading-order corrections for the contributions of twist-three and twist-four operators and a consistent treatment of the nucleon mass corrections in our calculation. It turns out that a self-consistent picture arises when the three valence quarks carry 40\% : 30\% : 30\% of the proton momentum.

Our work can be split into three essential parts: (i) calculations within LCSR; (ii) derivation of the factorized amplitude at the leading order (LO) up to twist-6 and at the next-to-leading order (NLO) up to twist-4. We calculated 22 coefficient functions at NLO and 20 of them are new ones. To avoid mixing with the so-called evanescent operators, we use the renormalization procedure for operators with open Dirac indices; (iii) study of the corresponding distribution amplitudes. In particular, the light-cone expansion to twist-4 accuracy of the three-quark matrix elements with generic quark positions.

II. LCSRS FOR NUCLEON FORM FACTORS: GENERAL STRUCTURE

The LCSR approach allows one to calculate the form factors in terms of the nucleon (proton) DAs. To this end we consider the correlation function

\[ T_\nu(P,q) = i \int d^4x \epsilon^{\nu q} \langle 0 | T [\gamma_\mu \bar{u}(x) \gamma_\nu u(x)] | P \rangle \]

where \( \eta (0) \) is the Ioffe interpolating current. The matrix element of the electromagnetic current \( J^{em}_{\mu}(x) = e_{\mu \nu} \bar{u}(x) \gamma_\nu u(x) + e_{\mu \nu} d(x) \gamma_\nu d(x) \) taken between nucleon states is conventionally written in terms of the Dirac and Pauli form factors \( F_1(Q^2) \) and \( F_2(Q^2) \):

\[ \langle P' | J^{em}_{\mu}(0) | P \rangle = N(P') \left[ \gamma_\mu F_1(Q^2) - i \frac{\sigma_{\mu \nu} q^\nu}{2m_N} F_2(Q^2) \right] N(P). \] (2)

In terms of the electric \( G_E(Q^2) \) and magnetic \( G_M(Q^2) \) Sachs form factors, we have

\[ G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \quad G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N} F_2(Q^2). \] (3)

We also define a light-like vector \( n_\mu \) by the condition \( q \cdot n = 0, n^2 = 0 \) and introduce the second light-like vector as \( p_\mu = P_\mu - n_\mu m_N^2/(2P \cdot n) \), \( p^2 = 0 \), and \( g_{\mu \nu} = \eta_{\mu \nu} - (p_\mu n_\nu + p_\nu n_\mu)/\langle p n \rangle \). We consider the “plus” spinor projection of the correlation function involving the “plus” component of the electromagnetic current, which can be parametrized in terms of two invariant functions

\[ \Lambda, T_+ = p_+ \left\{ m_N \Omega(Q^2, P^2) + q_+ \partial \mathcal{R}(Q^2, P^2) \right\} N_+ (P), \] (4)
where \( Q^2 = -q^2 \) and \( P^2 = (P-q)^2 \) and \( N^\pm (P) = \Lambda^\pm N(P) \), \( \Lambda^+ = \bar{P}n/(2pn) \), \( \Lambda^- = \bar{P}/(2pn) \). Further, making use of the Borel transformation \((s-P^2)^{-1} \rightarrow e^{-s/M^2}\), one obtains the following sum rules:

\[
2\lambda_1 F_1(Q^2) = \frac{1}{\pi} \int_0^{s_0} ds e^{i(s_M^2-s)/M^2} \text{Im} \mathcal{A}^{\text{QCD}}(Q^2, s), \quad \lambda_1 F_2(Q^2) = \frac{1}{\pi} \int_0^{s_0} ds e^{i(s_M^2-s)/M^2} \text{Im} \mathcal{B}^{\text{QCD}}(Q^2, s).
\]

The correlation functions \( \mathcal{A}(Q^2, P^2) \) and \( \mathcal{B}(Q^2, P^2) \) can be written as a sum:

\[
\mathcal{A} = e_d \mathcal{A}_d + e_u \mathcal{A}_u, \quad \mathcal{B} = e_d \mathcal{B}_d + e_u \mathcal{B}_u.
\]

Each of the functions has a perturbative expansion which we write as

\[
\mathcal{A} = \mathcal{A}^{\text{LO}} + \frac{\alpha_s(\mu)}{3\pi} \mathcal{A}^{\text{NLO}} + \ldots
\]

and similar for \( \mathcal{B} \); \( \mu \) is the renormalization scale. For consistency with our NLO calculation, we rewrite our results in a different form, expanding all kinematic factors in powers of \( m_N^2/Q^2 \). We keep all corrections \( O(m_N^2/Q^2) \) but neglect terms \( O(m_N^4/Q^4) \) etc. which is consistent with taking into account contributions of twist-three, -four, -five (and, partially, twist-six) in the operator product expansion (OPE). After calculations, the NLO corrections read (see all details in [I]).

\[
Q^2 \mathcal{A}_d^{\text{NLO}} = \int [dx_l] \left\{ \sum_{k=1,5} \left[ \nabla_1 h_k(x_l) C_{\text{q}}^{\text{V}1}(x_l, W) + \cdots \right] \right. \]

\[
\left. + \sum_{m=1,2,3} \left[ \nabla_2 h_1(x_l) C_{\text{q}}^{\text{V}2}(x_l, W) + \cdots \right] \right\} + O(\text{twist-5})
\]

and

\[
Q^2 \mathcal{B}_d^{\text{NLO}} = \int [dx_l] \left[ \nabla_1 h_1(x_l) D_{\text{q}}^{\text{V}1}(x_l, W) + \cdots \right] + O(\text{twist-5}).
\]

Notice that \( C_d^{\text{V}2}(x_l, W) = C_d^{\text{V}1}(x_l, W) = 0 \). As an example, we present here only the two simplest coefficient functions:

\[
x_2 C_{\text{V}1}(x_l) = 2\bar{x}_l x_l \left[ 3(L-2)g_1(x_l) + 2(L-1)g_1(x_l, x_l) + g_2(x_l, x_l) \right] + \left[ 2x_l + (4L-3)x_l \right] h_1(x_l) + (3-4L)x_l h_1(x_l)
\]

\[
+ 2\bar{x}_l x_l h_2(x_l) - 2\bar{x}_l x_l h_2(x_l) - 2 \left[ 3(x_l x_l)(2L-3) + 5L-7 \right] h_1(x_l) - 5 \left( h_1(x_l) + (3/x_l) h_2(x_l) - (3/x_l) h_2(x_l) \right),
\]

and

\[
x_2 C_{\text{V}1}(x_l) = 3\bar{x}_l h_1(x_l) - 3x_l h_1(x_l) + 2(3L-10)h_1(x_l) - 2(3L-10)h_1(x_l) + 3h_2(x_l) - 3h_2(x_l)
\]

\[-(6/x_l)(L-3)h_1(x_l) + (6/x_l)(L-3)h_1(x_l) - (3/x_l) h_2(x_l) + (3/x_l) h_2(x_l),
\]

where

\[
g_{nk}(y, x; W) = \frac{\ln^n[1-yW-i\eta]}{(-1+xW+i\eta)^k}, \quad h_{nk}(y, x; W) = \frac{\ln^n[1-xW-i\eta]}{(W+i\eta)^k}
\]

with \( n = 0, 1, 2 \) and \( k = 1, 2, 3 \). For \( n = 0 \) the first argument becomes dummy, i.e \( g_k(x; W) \equiv g_0(k, x; W) \).

### III. RESULTS

In this section, we discuss very shortly our main results. The full and comprehensive analysis and discussion of all input parameters, form factors and DAs can be found in [I]. It is instructive to write down schematically the structure of all our form factors as

\[
\mathcal{F} = \mathcal{F}_0 + \frac{f_N}{\lambda_1} \mathcal{F}_{f_N} + \sum_{i=0,1} \eta_i \mathcal{F}_{\eta_i} + \frac{f_N^2}{\lambda_1} \sum_{i=0}^2 \sum_{j=0}^2 \phi_{ij} \mathcal{F}_{\phi_{ij}}.
\]
Main nonperturbative input in the LCSR calculation of form factors is provided by the normalization constants, $f_N$, $\lambda_1$, and shape parameters of nucleon DAs, $\phi_{ij}$ and $\eta_{ij}$. The existing information, together with our final choices explained below, is summarized in Table I. As it is seen from Table I there only exist quantitative estimates for $f_N/\lambda_1$ and the first-order shape parameters $\phi_{10}$, $\phi_{11}$ of the leading twist-3 DA. The other parameters, in contrast, are very weakly constrained. Experimental data favors larger values of $f_N/\lambda_1$ so that we fix $f_N/\lambda_1 = -0.17$ and also take $\phi_{10} = \phi_{11} = 0.05$ in agreement with lattice calculations and the previous LO LCSR studies [2]. We then make a fit to the experimental data on the magnetic proton form factor $G_M^p(Q^2)$ and $G_E^p/G_M^p$ in the interval $1 < Q^2 < 8.5$ GeV$^2$ with all other entries as free parameters. We did two separate fits for $M^2 = 1.5$ GeV$^2$ and $M^2 = 2$ GeV$^2$ that are referred as ABO1 and ABO2, respectively. The resulting values for the shape parameters are collected in Table I and the corresponding form factors (solid curves for the set ABO1 and dashed for ABO2) are shown in Fig. 1 for the proton (left two panels) and the neutron (right two panels). The ratio $Q^2 F_2^p(Q^2)/F_2^n(Q^2)$ of Pauli and Dirac form factors in the proton is demonstrated in Fig. 2. The quality of the two fits of the proton data is roughly similar, whereas the description of the neutron form factors is slightly worse for ABO2 compared to ABO1. In both fits the neutron magnetic form factor comes out to be 20-30% below the data.

| Model  | Method  | $f_N/\lambda_1$ | $\phi_{10}$ | $\phi_{11}$ | $\phi_{20}$ | $\phi_{21}$ | $\eta_{10}$ | $\eta_{11}$ | Reference |
|--------|---------|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-----------|
| ABO1   | LCSR (NLO) | -0.17          | 0.05        | 0.05        | 0.075(15)   | -0.027(38) | 0.17(15)    | -0.039(5)  | 0.140(16) | this work |
| ABO2   | LCSR (NLO) | -0.17          | 0.05        | 0.05        | 0.038(15)   | -0.018(37) | -0.13(13)   | -0.027(5)  | 0.092(15) | this work |
| BLW    | LCSR (LO)  | -0.17          | 0.0534      | 0.0664      | -           | -           | -           | 0.05        | 0.0325    | [9]       |
| BK     | pQCD     | -              | 0.0357      | 0.0357      | -           | -           | -           | -           | -         | [3]       |
| COZ    | QCDSR (LO) | -              | 0.163       | 0.194       | 0.41        | 0.06        | -0.163      | -           | -         | [4]       |
| KS     | QCDSR (LO) | -              | 0.144       | 0.169       | 0.56        | -0.01       | -0.163      | -           | -         | [3]       |
| HET    | QCDSR (LO) | -              | 0.152       | 0.205       | 0.65        | -0.27       | 0.020       | -           | -         | [6]       |
|        | QCDSR (NLO) | -0.15         | -           | -           | -           | -           | -           | -           | -         | [7]       |
| LAT09  | LATTICE  | -0.083(6)      | 0.043(15)   | 0.041(14)   | 0.038(100)  | -0.14(15)   | -0.47(33)   | -           | -         | [8]       |
| LAT13  | LATTICE  | -0.075(5)      | 0.038(3)    | 0.039(6)    | -0.050(80)  | -0.19(12)   | -0.19(14)   | -           | -         | [9]       |

**TABLE I:** Parameters of the nucleon distribution amplitudes at the scale $\mu^2 = 2$ GeV$^2$. For the lattice results [9] only statistical errors are shown.
IV. CONCLUSIONS

In conclusion, our calculation incorporates the following new elements as compared to previous studies: (i) NLO QCD corrections to the contributions of twist-three and twist-four DAs; (ii) the exact account of “kinematic” contributions to the nucleon DAs of twist-four and twist-five induced by lower geometric twist operators (Wandzura-Wilczek terms); (iii) the light-cone expansion to twist-four accuracy of the three-quark matrix elements with generic quark positions; (iv) a new calculation of twist-five off-light cone contributions; (v) a more general model for the leading-twist DA, including contributions of second-order polynomials.

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