Extension of Standard Model with a Complex Singlet and Iso-Doublet Vector Quarks

Neda Darvishi
Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland
E-mail: neda.darvishi@fuw.edu.pl

Abstract. In this paper the extension of the SM by a neutral complex scalar singlet with a nonzero vacuum expectation value and a heavy vector quark pair is considered. This model provides an extra source of spontaneous CP violation. The focus of this article is to obtain the rate of baryon number generation. We show that the considered model provides a strong enough first-order electroweak phase transition to suppress the baryon-violating sphaleron process.

1. Introduction
Among the various scenarios that try to explain the baryon asymmetry of the Universe (BAU), the electroweak (EW) baryogenesis is one of the most interesting scenarios [1, 2, 3, 4]. All the necessary conditions for a successful baryogenesis according to the Sakharov’s criteria [5] (i.e. baryon number violation, C and CP violation and departure from thermal equilibrium) can be found in the Standard Model (SM). It has become apparent that the SM of EW interactions is unable to account for the observed magnitude of the BAU for two reasons: (i) The EW phase transition (EWPT) is not strongly first-order and therefore, any baryon asymmetry generated during the EWPT would be washed out by unsuppressed baryon violating processes in the broken phase [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. (ii) There is not enough CP violation from the CKM matrix to generate the baryon asymmetry [17]. In order to address this important issues, various extensions of the SM are considered [16, 18, 19, 20, 21, 22]. In the present work, we obtain the rate of baryon number generation in the model with a neutral complex scalar singlet \( \chi \), which accompanies the SM-like Higgs doublet \( \Phi \), and a pair of iso-doublet vector quarks \( V_L + V_R \). This kind of extension of the SM was discussed in the literature with various motivations, e.g. [23, 24, 25, 26, 27, 28, 29, 30, 31]. We consider the potential with a softly broken global U(1) symmetry, see [32, 33, 34, 35]. This model provides a strong enough first-order EW phase transition via the soft breaking terms in the potential, to suppress the baryon-violating sphaleron process. On the other hand, the mixing of SM quarks with the heavy vector quark pair results in appearance of an additional CP violation. The issue of the CP violation due to a complex singlet with a complex VEV has been previously discussed, see [32, 34, 35].

The Yukawa Lagrangian acquires additional quark mass terms because of the presence of an iso-doublet vector quark and a complex singlet. Diagonalizing the quark mass matrix results in nondiagonal terms coming from the kinetic terms, with appearance of new terms which are functions of the time-dependent phase (complex singlet VEV \( \phi \)). These terms lead to the generation of baryon asymmetry.
The content of this paper is as follows. In section (2), we present a general structure of the model and its constrained version. In section (3), the necessary conditions for strong enough first order EWPT in the model will be verified. We obtain the rate of baryon number generation in the section 4. Finally, the section 5 contains our conclusion.

2. Lagrangian of the model

The full Lagrangian of this model is given by

$$\mathcal{L} = \mathcal{L}_{SM}^{gf} + \mathcal{L}_{scalar} + \mathcal{L}_{Y}^{Y}(\psi f, \Phi) + \mathcal{L}_{Y}^{Y}(V q, \chi),$$

where $\mathcal{L}_{SM}^{gf}$ describes the pure gauge boson terms as well the SM boson- SM fermion interaction, $\mathcal{L}_{scalar}$ describes the scalar sector of the model with one SU(2) doublet $\Phi$ and a neutral complex scalar (spinless) singlet $\chi$. $\mathcal{L}_{Y}^{Y}(\psi f, \Phi)$ and $\mathcal{L}_{Y}^{Y}(V q, \chi)$ represent the Yukawa interaction of $\Phi$ with SM fermions and the Yukawa interaction of singlet scalar with heavy iso-doublet vector quarks and SM quarks.

We consider a pair (left and right handed) of heavy iso-doublet vector quarks, $V_L + V_R$, with $V_L$ and $V_R$ having the same transformation properties under the gauge group of the SM, and where $V_L$ transforms in the same way as a standard model quark doublet $Q_L$. Essentially, the case of iso-singlet vector quarks have the same result as the case of iso-doublet vector quarks, while their transformation are similar to $u_R$ or $d_R$ with respect to the SM gauge group [36].

The mass terms in the presence of the complex singlet are:

$$\mathcal{L}_{Y}^{Y}(V q, \chi) = \lambda_V \chi \bar{Q}_L V_R + M\bar{V}_L V_R + h.c,$$

where $M$ is the mass of vector quarks (which is considered to be heavy) and $\lambda_V$ is the coupling between the singlet scalar, vector quarks and the SM quarks. In general the couplings of all three generations of SM quarks to vector quarks should be considered, nevertheless, since only one linear combination of these will actually mix with the vector quarks, we consider only one generations of SM quarks (the heaviest), $Q_L$.

The SM-like Higgs boson in the model predominantly consists of a neutral CP-even component of the $\Phi$ doublet and its mass is $\sim 125$ GeV. There are two other neutral Higgs like particles, see discussion in [33, 34, 35].

We assume $\Phi$ and $\chi$ fields have non-zero vacuum expectation values (VEV), $v$ and $we^{i\xi}$, respectively ($v, w, \xi \in \mathbb{R}$). We use the following field decomposition ,

$$\Phi = \left( \frac{1}{\sqrt{2}}(\phi_1 + i\phi_4) \right), \chi = \frac{1}{\sqrt{2}}(\phi_2 + i\phi_3),$$

where

$$we^{i\xi} = w \cos \xi + iw \sin \xi = w_1 + iw_2.$$  

Masses of the gauge bosons are given by the VEV of the doublet, e.g $M_W^2 = g^2 v^2/4$ for the $W$ boson.

The scalar potential of the model can be written as follows [33, 34, 35]

$$V = V_D + V_S + V_{DS},$$

with the pure doublet and the pure singlet parts (respectively $V_D$ and $V_S$) and the mixed term $V_{DS}$. The SM part of the potential is presented by $V_D$:

$$V_D = -\frac{1}{2}m_{11}^2 \Phi^\dagger \Phi + \frac{1}{2} \lambda (\Phi^\dagger \Phi)^2.$$
The potential for a complex singlet is equal to,

\[
V_s = -\frac{1}{2} m_s^2 \chi^* \chi - \frac{1}{2} m_4^2 (\chi^2 + \chi^*)^2 + \lambda_1 (\chi^* \chi)^2 + \lambda_2 (\chi^* \chi)(\chi^2 + \chi^3) + \lambda_3 (\chi^4 + \chi^*)^2 \\
+ \kappa_1 (\chi + \chi^*) + \kappa_2 (\chi^3 + \chi^*) + \kappa_3 (\chi + \chi^*) (\chi^* \chi).
\]

(7)

The doublet-singlet interaction term is,

\[
V_{DS} = \Lambda_1 (\Phi^\dagger \Phi)(\chi^* \chi) + \Lambda_2 (\Phi^\dagger \Phi)(\chi^2 + \chi^*)^2 \\
+ \kappa_4 (\Phi^\dagger \Phi)(\chi + \chi^*).
\]

(8)

There are three quadratic \((m_i^2)\), six dimensionless quartic \((\lambda_i \Lambda_i)\) and four dimensionful parameters \(\kappa_i, \, i = 1, 2, 3, 4\), describing linear \((\kappa_1)\) and cubic terms \((\kappa_2, \kappa_3)\) and \(\kappa_4\). The linear term \(\kappa_1\) can be removed by a translation of the singlet field. To simplify the model, we apply a global U(1) symmetry [33, 34, 35].

\[
U(1): \, \Phi \to \Phi, \, \chi \to e^{i\alpha} \chi.
\]

(9)

However, a non-zero VEV of \(\chi\) would lead to a spontaneous breaking of this symmetry and appearing of massless Nambu-Goldstone scalar particles, what is not acceptable. We keep U(1) soft-breaking terms to solve this problem. Therefore, we consider a potential with U(1) soft-breaking terms, where the singlet cubic terms \(\kappa_2, \kappa_3\) and \(\kappa_4\) and the singlet quadratic term \(m_4^2\) are kept. Using following notation, \(\lambda_3 = \lambda_{s1}, \Lambda = \Lambda_1\), we get the potential in the following form

\[
V = -\frac{1}{2} m_{11}^2 \Phi^\dagger \Phi + \frac{1}{2} \lambda (\Phi^\dagger \Phi)^2 + \Lambda (\Phi^\dagger \Phi)(\chi^* \chi) \\
- \frac{1}{2} m_s^2 \chi^* \chi + \lambda_4 (\chi^* \chi)^2 + \kappa_4 (\Phi^\dagger \Phi)(\chi + \chi^*) \\
- \frac{1}{2} m_4^2 (\chi^* \chi^2 + \chi^* \chi^3) + \kappa_2 (\chi^3 + \chi^3) + \kappa_3 (\chi + \chi^*) (\chi^* \chi).
\]

(10)

The potential is symmetric under the \(\chi \to \chi^*\) transformation and all parameters are real. We express the complex singlet \(\chi\) in terms of its real and imaginary parts, \(\chi = (\phi_2 + i\phi_3)/\sqrt{2}\), so we have

\[
V = -\frac{1}{2} m_{11}^2 \Phi^\dagger \Phi + \frac{1}{2} \lambda (\Phi^\dagger \Phi)^2 - \frac{\mu_2^2}{4} \phi_2^2 \\
- \frac{\mu_3^2}{4} \phi_3^2 + \frac{1}{2} \Lambda (\Phi^\dagger \Phi)(\phi_2^2 + \phi_3^2) \\
+ \frac{1}{4} \lambda_4 (\phi_2^2 + \phi_3^2)^2 + \frac{1}{\sqrt{2}} \kappa_2 (\phi_2^3 - 3\phi_2 \phi_3^2) \\
+ \frac{1}{\sqrt{2}} \kappa_3 (\phi_2^3 + \phi_2 \phi_3^2) + \sqrt{2} \kappa_4 (\Phi^\dagger \Phi) \phi_2.
\]

(11)

The extremum conditions lead to the following constraints,

\[
v(-m_{11}^2 + v^2 \lambda + 2\sqrt{2} w_1 \kappa_4 + \Lambda w^2) = 0,
\]

(12)
\[ w_1(-\mu_1^2 + v^2\Lambda + 2w^2\lambda_s) + \sqrt{2}[3(w_1^2 - w_2^2)\kappa_2 + (3w_1^2 + w_2^2)\kappa_3] + v^22\kappa_4 = 0, \]  
\[ w_2(-\mu_2^2 + v^2\Lambda + 2w^2\lambda_s + 2\sqrt{2}w_1(-3\kappa_2 + \kappa_3)] = 0, \]

where the parameters \( \mu_1^2 \) and \( \mu_2^2 \) are defined as

\[ \mu_1^2 = m^2 + 2m^4, \quad \mu_2^2 = m^2 - 2m^4. \]

We consider the CP violating vacua, i.e. \( v, w_1 \) and \( w_2 \) different from zero. To have a stable minimum the parameters of the potential need to satisfy the positivity conditions. These conditions read as follows:

\[ \lambda, \lambda_s > 0, \quad \Lambda > -\sqrt{2}\lambda\lambda_s. \]  

In addition, the unitarity and perturbativity conditions limit the quartic terms \( \lambda, \lambda_s \) and \( \Lambda \) to be below \( 4\pi \).

### 3. The electroweak phase transition

The last requirement in Sakharov’s criteria, the departure from thermal equilibrium, is an important ingredient to explain the baryon asymmetry within the SM of particle physics. The condition for the first-order EWPT with two degenerate minima at critical temperature, \( T_c \), is [11, 37, 38],

\[ \frac{v_{T_c}}{T_c} \geq 1, \]  

where, \( v_{T_c} \) is VEV at critical temperature, \( T_c \). The one loop thermal corrections to the effective potential at finite temperature \( T \) are (see for review, ref. [38]),

\[ \Delta V_{thermal} = \sum_i \frac{n_i T_4^4}{2\pi^2} I_{B,F} \left( \frac{m_i^2}{T^2} \right), \]  

where

\[ I_{B,F}(y) = \int_0^\infty dx x^2 \ln \left[ 1 \mp e^{-\sqrt{x^2+y}} \right], \]

the minus and plus sign corresponds to the bosons and the fermions, respectively. In Eq. (17) \( m_i \) is the field-dependent mass and \( n_i \) is the number of degrees of freedom, (see Appendix A and [Appendix B]). Since the barrier is produced at tree-level, it is sufficient to include the high temperature expansion in the one-loop thermal potential (i.e. only keeping \( T^2 \) terms) [39, 40]. Therefore, the one-loop thermal potential using the high temperature approximation is given by,

\[ V(T) = \frac{1}{2}m_{\Phi_1}^2\Phi_1^2 + \frac{1}{4}\lambda(\Phi^4) + \frac{m_{\Phi_2}^2}{4}\phi_2^2 + \frac{m_{\Phi_3}^2}{4}\phi_3^2 \]

\[ + \frac{1}{4}\lambda(\Phi^4)(\phi_2^2 + \phi_3^2) + \frac{1}{4}\lambda_s(\phi_2^2 + \phi_3^2)^2 \]

\[ + \kappa_2 \frac{1}{\sqrt{2}}(\phi_2^3 - 3\phi_2\phi_3^2) + \kappa_3 \frac{1}{\sqrt{2}}(\phi_3^3 + \phi_2\phi_3^2) \]

\[ + \sqrt{2}\kappa_4(\Phi^4)\phi_2 + \kappa_{34} \frac{T^2}{3}\phi_2, \]  

\[ \text{Eq. (19)} \]
where,
\[ \overline{m}_{11}^2 = -m_{11}^2 + (3\lambda + \Lambda + \frac{2m_W^2 + m_Z^2 + 2m_h^2}{2v^2}) T^2, \]
\[ \frac{1}{2}\overline{m}_1^2 = -\frac{1}{2}m_1^2 + (\Lambda + 2\lambda_s) \frac{T^2}{3}, \]
\[ \frac{1}{2}\overline{m}_2^2 = -\frac{1}{2}m_2^2 + (\Lambda + 2\lambda_s) \frac{T^2}{3}, \]
\[ \kappa_{34} = \sqrt{2}(\kappa_3 + \kappa_4). \]

The extremum conditions of the effective potential Eq. (17) at temperature \( T \), with respect to the fields \( \phi_1, \phi_2 \) and \( \phi_3 \) are
\[ v(\overline{m}_{11}^2 + \lambda v^2 + \Lambda w^2 + 2\sqrt{2}\kappa_4 w_1) = 0, \]
\[ w_1(\overline{m}_1^2 + \lambda v^2 + 2\lambda_s w^2) + \sqrt{2}[3\kappa_2(w_1^2 - w_2^2) + \kappa_3(3w_1^2 + w_2^2) + \kappa_4v^2] + \frac{2}{3}\kappa_{34}T^2 = 0, \]
\[ w_2[\overline{m}_2^2 + \lambda v^2 + 2\lambda_s w^2 + 2\sqrt{2}(-3\kappa_2 + \kappa_3)w_1] = 0. \]

The solution of the Eqs. (21), (22) and (23) at extreme temperatures is as follows,
\[ v = 0, \quad w_1 \approx -\frac{\kappa_{34}}{2\lambda_s + \Lambda}, \quad w_2 \approx 0. \]

At extreme temperatures the scalar component \( \phi_3 \) and vector quarks decouple from the model because their contribution to the finite temperature effective potential is Boltzmann suppressed and therefore, the potential is similar to the SM plus a real singlet [31].

Now, we will scan over the parameter space of the model under the condition for strong first order EWPT,
- \( V_{\text{eff}}(v_{Tc}, T_c) = V_{\text{eff}}(0, T_c) \) (i.e. two degenerate minima at critical temperature, \( T_c \)),
- \( v_{Tc}/T_c \geq 1 \).

We consider \( T_c \) to be smaller than 250 GeV and \( v_{Tc} \) below its zero-temperature value \( v_0 = 246 \) GeV. In our scanning the following regions for parameters space of the model fulfilling positivity and unitarity conditions is considered (see [34]), namely:
\[ \Lambda \in [-0.25, 0.25], \quad \lambda_s \in [0, 1], \quad \rho_{2,3,4} \in [-1, 1], \quad \xi \in [0, \pi], \]
\[ m_{11}^2, \mu_1^2, \mu_2^2 \in [-90000, 90000]\text{GeV}^2, \]
\[ m_1^2, m_2^2 \in [0.2, 0.3]. \]

The model has two additional Higgs neutral scalars \( M_{h_2} \) and \( M_{h_3} \), that we take to be [33, 34]
\[ M_{h_3} \geq M_{h_2} > 150 \text{ GeV}. \]
It is shown that these ranges of parameters are in agreement with LHC data and measurements of the oblique parameters, in ref \[34\].

The results of our scanning are shown in Fig.1. In the Fig.1(a), the allowed region of $v_{T_c}/T_c$ as a function of $T_c$ is presented. Within an interval $T_c \in [100, 200]$ the ratio $v_{T_c}/T_c$ can reach 2.5. The Fig.1(b) shows that the ratio $v_{T_c}/T_c \geq 1$ is possible for $|\rho_3| > 10^{-3}$.

Throughout above discussions, we can concludes that a strong enough first-ordered EWPT is possible in this model and hence, a successful BAU can be achieved \[38\].

4. Baryogenesis

In this section we describe the baryon asymmetry resulting from a mixing of the SM quarks and heavy vector quarks provided by CP violation vacuum \[41, 36\]. To generate baryon asymmetry, the phase of the singlet VEV should be time-dependent, otherwise, such constant phase can be easily rotated away with the redefinition of the $V_L$ and $V_R$ \[36\]. Diagonalizing the quark mass matrix from Eq.(2) results in some non-diagonal kinetic terms dependent on mass eigenstate, $Q'_L$ and $V'_L$. In addition, a couple of time-dependent terms appear in the Lagrangian (see Appendix C), namely

$$\overline{Q}'_L i\gamma^\mu \partial_\mu Q'_L + \overline{V}'_L i\gamma^\mu \partial_\mu V'_L + \Delta \mathcal{L}_k + \text{const.} \tag{28}$$

As mentioned above, the CP violation disappears for a constant phase, therefor the following kinetic term, that are not constant needs to be considered,

$$\Delta \mathcal{L}_k = -\frac{\lambda^2 v^2}{M^2} \xi (\overline{Q}'_L \gamma^0 Q'_L - \overline{V}'_L \gamma^0 V'_L). \tag{29}$$

Such terms result in increasing the baryon density of the universe. The amount of BAU is calculated via the following relation,

$$n_B = -\mathcal{N}_f \int \frac{\Gamma_{sph}(T)}{2T} \mu_B dt, \tag{30}$$
where \( N_f \) is the number of flavors in the model. The sphaleron rate is defined as \( \Gamma_{sph} = K(\alpha_W T)^4 \), in the symmetric phase. \( K \) is the numerical factor reflecting the uncertainty in the estimate of the transition rate between vacua of different \( B + L \) value, that is estimated to be between 0.1 and 1 \([12]\). The chemical potential, \( \mu_B \), is associated with the baryonic charge. The chemical potential for the third generation is given by \([36]\),

\[
\mu_B = -\frac{5}{6} \frac{\lambda_V^2 w^2}{M^2} \delta \xi.
\]

(31)

Since by assumption \( M \geq T \), the sphaleron fluctuations can not produce \( V \) quark pair, therefore we get the number density of baryons \( n_B \) at the temperature \( T \) as follows

\[
n_B = \frac{5K\alpha_W^4}{2} \frac{\lambda_V^2 w^2}{M^2} \delta \xi T^3,
\]

(32)

where \( \delta \xi \) is the total change of the phase \( \xi \). The BAU is determined via the ratio of the baryon number to the entropy \([33]\). The entropy density is defined as

\[
s = \frac{2\pi^2}{45} g^* T^3,
\]

(33)

therefore, the ratio of the baryon number to the entropy in this model is given by,

\[
\frac{n_B}{s} = \frac{225K\alpha_W^4}{4\pi^2 g^*} \frac{\lambda_V^2 w^2}{M^2} \delta \xi,
\]

(34)

where \( \alpha_W = 3.4 \times 10^{-2} \) is the \( SU(2) \) gauge coupling and \( g^* \sim 100 \) is the effective number of degrees of freedom in the thermal equilibrium.

The observed value for \( n_B/s \) ratio is \([43,45]\)

\[
\frac{n_B}{s} = 8.7 \pm 0.3 \times 10^{-11}.
\]

(35)

Comparing Eq.(34) and Eq.(35), one concludes

\[
K \frac{\lambda_V^2 w^2}{M^2} \delta \xi = 1.14 \pm 0.3 \times 10^{-3}.
\]

(36)

The numerical analysis has been performed via scanning over the relevant parameters in the following range,

\[
M \in [0.3, 13] \text{ TeV},
\]

\[
\lambda_V \in [0, 1],
\]

\[
w \in [2, 400] \text{ GeV},
\]

\[
\delta \xi \in [0, \pi],
\]

(37)

taking \( K = 1 \). Figure 2 illustrates the parameter space allowing the generation of observed BAU for the \( n_B/s \) ratio within 2\( \sigma \). The results were obtained from scanning in the ranges given by Eq.(37) with the central value of Eq.(35). Also, we have used our results from ref. \([34]\) for the range of \( w \). Fig.2(a) demonstrates the region of parameters \( (\delta \xi, w) \), while Fig.2(b) shows the \( (w,M) \) points. Since \( M \) and \( w \) are independent parameters, their correlation is a direct consequence of the constraint (36). Based on these results, we conclude that our model provides successful BAU.
Figure 2. The allowed region for $\delta \xi$, $w$ and $M$ in the Eq.(35) of the given ranges, i.e. Eq.(37), for acceptable BAU within $2\sigma$. (a) is the correlation of $\delta \xi$ and $w$. (b) is the correlation of $w$ versus $M$.

5. Conclusion
In the present work the possibility of the first order EWPT for the extended SM with complex singlet and a pair of iso-doublet vector quarks is investigated, showing that such process is strong enough to generate BAU in the presence of non zero value of cubic terms, $\kappa_2$, $\kappa_3$ and/or $\kappa_4$. Afterwards, the parameter space of the model for the valid regions of BAU is scanned, concluding that this model with a heavy iso-doublet vector quark pair could successfully predict an acceptable value for BAU.

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Appendix A. The field-dependent mass $m_i$
The field-dependent mass $m_i$ of gauge bosons, Goldstone boson, $m_{\phi_1}$, $m_{\phi_2}$ and $m_{\phi_3}$, which is used in Eq.(17) are given by,

$$
M_W^2 = \frac{g^2 \phi_1^2}{4}, \quad M_Z^2 = (g^2 + g'{}^2) \phi_1^2 \frac{1}{4},
$$
$$
m_G^2 = \lambda \phi_1^2 + \Lambda (\phi_2^2 + \phi_3^2) + 2\sqrt{2} \kappa_4 \phi_2,
$$
$$
m_{\phi_1}^2 = 3 \lambda \phi_1^2 + \Lambda (\phi_2^2 + \phi_3^2) + 2\sqrt{2} \kappa_4 \phi_2,
$$
$$
m_{\phi_2}^2 = 3 \lambda_s \phi_2^2 + \lambda_s \phi_3^2 + 3\sqrt{2}(\kappa_2 + \kappa_3) \phi_2 + \frac{1}{3} \Lambda \phi_2^2,
$$
$$
m_{\phi_3}^2 = 3 \lambda_s \phi_3^2 + \lambda_s \phi_2^2 + \sqrt{2}(-3\kappa_2 + \kappa_3) \phi_2 + \frac{1}{3} \Lambda \phi_2^2.
$$

(A.1)

$n_i$ is the number of degrees of freedom in Eq.(17) as,

$n_W = 6$, $n_Z = 3$, $n_G = 3$, $n_{\phi_1,\phi_2,\phi_3} = 1$, $n_t = 12$.

(A.2)
Appendix B. Evaluation of the integral
The integral Eq. (18) is evaluated as follow,
\[
\frac{\partial}{\partial y} I^{B,F}_{B,F}(y) = \frac{1}{2} \int_0^\infty dx \frac{x^2}{(x^2 + y)^{1/2}} \exp((x^2 + y)^{1/2}) - 1,
\]
(B.1)
\[
I^{B,F}_{B,F}(y)|_{y=0} = \int_0^\infty dx x^2 \ln(1 - e^{-x}) = \frac{\pi^4}{45},
\]
(B.2)
\[
\frac{\partial}{\partial y} I^{B,F}_{B,F}(y)|_{y=0} = \frac{1}{2} \int_0^\infty dx \frac{x}{e^x - 1} = \frac{\pi^2}{12}.
\]
(B.3)

Appendix C. Rotation matrix
As discussed in sec.4 (see the Eq.(28)) the transformation of $Q$ and $V$ with rotation matrix to $Q'$ and $V'$ reads as follows
\[
\begin{bmatrix}
Q' L \\
V' L
\end{bmatrix} =
\begin{bmatrix}
a & b \\
-b^* & a^*
\end{bmatrix}
\begin{bmatrix}
Q_L \\
V_L
\end{bmatrix}
\]
\[
a = \left[ 1 + \left( \frac{\lambda v w}{M} \right)^2 \right]^{-1/2}
\]
\[
b = \left( \frac{\lambda v w}{M} \right) \left[ 1 + \left( \frac{\lambda v w}{M} \right)^2 \right]^{-1/2} e^{-i \xi}
\]
(C.1)

References
[1] Cohen A G, Kaplan D B and Nelson A E 1993 Ann. Rev. Nucl. Part. Sci. 43 27–70 (Preprint hep-ph/9302210)
[2] Quiros M 1994 Helv. Phys. Acta 67 451–583
[3] Rubakov V A and Shaposhnikov M E 1996 Usp. Fiz. Nauk 166 493–537 [Phys. Usp.39,461(1996)] (Preprint hep-ph/9603208)
[4] Carena M and Wagner C E M 1997 (Preprint hep-ph/9704347)
[5] Sakharov A D 1967 Pisma Zh. Eksp. Teor. Fiz. 5 32–35 [Usp. Fiz. Nauk161,61(1991)]
[6] Anderson G W and Hall L J 1992 Phys. Rev. D45 2685–2698
[7] Carrington M E 1992 Phys. Rev. D45 2933–2944
[8] Dine M, Leigh R G, Huet P, Linde A D and Linde D A 1992 Phys. Lett. B283 319–325 (Preprint hep-ph/9203201)
[9] Dine M, Leigh R G, Huet P Y, Linde A D and Linde D A 1992 Phys. Rev. D46 550–571 (Preprint hep-ph/9203203)
[10] Bochkarev A I, Kuzmin S V and Shaposhnikov M E 1991 Phys. Rev. D43 369–374
[11] Davies A T, Froggatt C D, Jenkins G and Moorhouse R G 1994 Phys. Lett. B336 464–470
[12] Espinosa J R, Quiros M and Zwirner F 1993 Phys. Lett. B307 106–115 (Preprint hep-ph/9303317)
[13] Shaposhnikov M E 1993 Phys. Lett. B316 112–120 (Preprint hep-ph/9306296)
[14] Nelson A E, Kaplan D B and Cohen A G 1992 Nucl. Phys. B373 453–478
[15] Cohen A G and Nelson A E 1992 Phys. Lett. B297 111–117 (Preprint hep-ph/9209245)
[16] Comelli D, Pietroni M and Riotto A 1994 Nucl. Phys. B412 441–458 (Preprint hep-ph/9304267)
[17] Shaposhnikov M E 1987 Nucl. Phys. B287 757–775
[18] Accomando E et al. 2006 (Preprint hep-ph/0608079)
[19] Haber H E and Surujon Z 2012 Phys. Rev. D86 075007 (Preprint 1201.1730)
[20] Ginzburg I F and Krawczyk M 2005 Phys. Rev. D72 115013 (Preprint hep-ph/0408011)
[21] Gunion J F and Haber H E 2005 Phys. Rev. D72 095002 (Preprint hep-ph/0506227)
[22] Davidson S, González Felipe R, Seródio H and Silva J P 2013 JHEP 11 100 (Preprint [1307.6218])
[23] Branco G C, Parada P A and Rebelo M N 2003 (Preprint hep-ph/0307119)
[24] Lebedev O 2011 Phys. Lett. B697 58–62 (Preprint 1011.2630)
[25] Espinosa J R, Gripaios B, Konstandin T and Riva F 2012 JCAP 1201 012 (Preprint 1110.2876)
[26] Gabrielli E, Heikinheimo M, Kannike K, Racioppi A, Raidal M and Spethmann C 2014 Phys. Rev. D89 015017 (Preprint 1309.6632)
[27] Costa R, Morais A P, Sampaio M O P and Santos R 2015 Phys. Rev. D92 025024 (Preprint 1411.4048)
[28] Kozaczuk J 2015 JHEP 10 135 (Preprint 1506.04741)
[29] Jiang M, Bian L, Huang W and Shu J 2016 Phys. Rev. D93 065032 (Preprint 1502.07574)
[30] Alexander-Nunneley L and Pilaftsis A 2010 JHEP 09 021 (Preprint 1006.5916)
[31] Barger V, Langacker P, McCaskey M, Ramsey-Musolf M and Shaughnessy G 2009 Phys. Rev. D79 015018 (Preprint 0811.0393)
[32] Darvishi N 2016 JHEP 11 065 (Preprint 1608.02820)
[33] Bonilla C, Sokolowska D, Darvishi N, Diaz-Cruz J L and Krawczyk M 2016 J. Phys. G43 065001 (Preprint 1412.8730)
[34] Darvishi N and Krawczyk M 2016 (Preprint 1603.00598)
[35] Krawczyk M, Darvishi N and Sokolowska D 2016 Acta Phys. Polon. B47 183 (Preprint 1512.06437)
[36] McDonald J 1996 Phys. Rev. D53 645–654
[37] Kane G L Perspectives on Higgs physics II. Singapore: World Scientific
[38] Quiros M 1999 Proceedings, Summer School in High-energy physics and cosmology: Trieste, Italy, June 29-July 17, 1998 pp 187–259 (Preprint hep-ph/9901312) URL http://alice.cern.ch/format/showfull?
synee=0302087
[39] Dolan L and Jackiw R 1974 Phys. Rev. D9 3320–3341
[40] Espinosa J R, Konstandin T and Riva F 2012 Nucl. Phys. B854 592–630 (Preprint 1107.5441)
[41] Branco G C, Delepine D, Emmanuel-Costa D and Gonzalez F R 1998 Phys. Lett. B442 229–237 (Preprint hep-ph/9805302)
[42] Ambjorn J, Askgaard T, Porter H and Shaposhnikov M E 1991 Nucl. Phys. B353 346–378
[43] Cline J M 2006 Les Houches Summer School - Session 86: Particle Physics and Cosmology: The Fabric of Spacetime Les Houches, France, July 31-August 25, 2006 (Preprint hep-ph/0609145)
[44] (WMAP) S T Http://map.gsfc.nasa.gov/site/citations.html
[45] Ade P A R et al. (Planck) 2014 Astron. Astrophys. 571 A16 (Preprint 1303.5076)