Neutralino Proton Cross Sections For Dark Matter In SUGRA and D-BRANE Models

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Abstract

Neutralino proton cross sections are examined for models with R-parity invariance with universal soft breaking (mSUGRA) models, nonuniversal SUGRA models, and D-brane models. The region of parameter space where current dark matter detectors are sensitive, i.e. $1 \times 10^{-6}$ pb, is examined. For mSUGRA models, detectors are sampling parts of the parameter space for $\tan\beta \approx 25$. The nonuniversal models can achieve cross sections that are a factor of 10-100 bigger or smaller than the universal one and in the former case sample regions $\tan\beta \approx 4$. The D-brane models considered require $\tan\beta \approx 15$. The inclusion of CP violating phases reduces the cross section by a factor of $\sim 2-3$ (but also requires considerable fine tuning at the GUT scale). The expected particle spectra at accelerators are examined and seen to differ for each model. Three new regions of possible coannihilation are noted.

I. INTRODUCTION

Supersymmetric theories with R-parity invariance predict the existence of a dark matter candidate, the lightest supersymmetric particle (the LSP). Accelerator and cosmological constraints then imply that the LSP is the lightest neutralino, $\tilde{\chi}_1^0$, and that this particle is mainly gaugino. For heavy nuclei, $\tilde{\chi}_1^0$-nucleus scattering is dominated by the spin independent part of the amplitude where the neutron ($n$) and proton ($p$) amplitudes are approximately equal. This allows one to extract from the data the spin independent cross section $\sigma_{\tilde{\chi}_1^0-p}$. Current dark matter experiments are now sensitive enough to probe a significant part of the SUSY parameter space. Thus DAMA and CDMS are sensitive to $\sigma_{\tilde{\chi}_1^0-p}$ in the range of $(1 - 10) \times 10^{-6}$ pb, and there will be perhaps a factor of 10 improvement in the near future. We ask here then what part of the SUSY parameter space can be tested for the range

$$0.1 \times 10^{-6} \text{ pb} \leq \sigma_{\tilde{\chi}_1^0-p} \leq 10 \times 10^{-6} \text{ pb}.$$ (1)

We do this by examining the maximum theoretical cross section that lies in this domain by varying the SUSY parameters (e.g. $\tan\beta$, $m_{\tilde{\chi}_1^0}$, etc). In the following, we consider three supergravity models: mSUGRA with universal soft breaking parameters, SUGRA...
models with non-universal soft breaking in both the Higgs [12–17] and third generation squark/slepton sector [14,15,17], and a D-brane model which allows for a specific pattern of nonuniversal gaugino, squark and slepton masses [18].

In our analysis [19], we let the SUSY soft breaking parameters $m_\tilde{g}$ (gluino mass), $m_0$ (scalar mass), $A_0$ (cubic soft breaking mass) and $\tan\beta = \langle H_2 \rangle / \langle H_1 \rangle$ range over the following domain:

$$m_\tilde{g}, m_0 \leq 1 \text{ TeV}; \; |A_0/m_0| \leq 5; \; 2 \leq \tan\beta \leq 50.$$ (2)

In order that the analysis be accurate for this domain, we include the following in the calculations; (i) we run the full 1-loop renormalization group equations (RGE) from the GUT scale $M_G = 2 \times 10^{16}$ GeV to the t-quark mass $m_t=175$ GeV, iterating to get a consistent SUSY mass spectrum for a fixed set of GUT scale mass parameters. (ii) L-R mixing in sfermion mass matrices are included (which is important for large $\tan\beta$ for the third squark and slepton generations). (iii) One loop corrections to the Higgs mass matrix is included. (iv) One loop corrections to the mass $m_b$ is included (which is also important for large $\tan\beta$ where it produces a significant correction to the relation between $\lambda_b$ (the b-Yukawa coupling constant) and $m_b$. (v) QCD RGE corrections are included for contributions dominated by light quark masses. (vi) Leading order (LO) and approximate NLO corrections [20,9] to the $b \to s\gamma$ decay rate are included. We do not impose $b-\tau$ unification at $M_G$ (as is done in [16]), as this constraint is sensitive to possible unknown GUT scale physics.

Various LEP, Tevatron and CLEO accelerator bounds limit the SUSY parameter space. For the light chargino $\tilde{\chi}^\pm_1$ and light higgs $h$ we require

$$m_{\tilde{\chi}^\pm_1} > 94 \text{ GeV}; \; m_h > 95 \text{ GeV},$$ (3)

and for the $B \to X_s\gamma$ branching ratio we use [21]:

$$1.8 \times 10^{-4} \leq B(B \to X_s\gamma) \leq 4.5 \times 10^{-4}.$$ (4)

The b-quark mass used is $m_b(m_b)$=4.0-4.5 GeV [22] and Tevatron bounds [23] on $m_\tilde{g}$ and $m_\tilde{q}$ are imposed.

The theoretical analysis determines the $\tilde{\chi}^0_1 - q$ scattering cross section, and in order to relate this to the proton cross section two parameters $\sigma_0$ and $\sigma_{\pi N}$ enter [22,24], where

$$\sigma_{\pi N} = \frac{1}{2}(m_u + m_d)\langle p|\bar{u}u + \bar{d}d|p\rangle$$

$$\sigma_0 = \frac{1}{2}(m_u + m_d)\langle p|\bar{u}u + \bar{d}d - 2\bar{s}s|p\rangle.$$ (5)

as well as the quark mass ratio $r = m_s/\frac{1}{2}(m_u + m_d)$. We use here $\sigma_{\pi N} = 65$ MeV, in accord with recent analysis [26] and $\sigma_0 = 30$ MeV [25]. The quark mass ratio is given in [27] as $r=24.4<1.5$.

Theory allows one to calculate the mean $\tilde{\chi}^0_1$ relic cold matter (CDM) of the universe, parametrized by $\Omega_{\tilde{\chi}^0_1} h^2$, where $\Omega_{\tilde{\chi}^0_1} = \rho_{\tilde{\chi}^0_1}/\rho_c = 3H_0^2/8\pi G_N$, $H_0=H_0/(100\text{km/sMpc})$. The combined world average for $H_0$ is [28] $H_0 = (71 \pm 3 \pm 7)$ km s$^{-1}$ Mpc$^{-1}$. A number of astronomical analyses of matter density $\Omega$ gives [29,30] $\Omega_m \approx 0.3$, and using the baryon density $\Omega_B = 0.05$, we assume that
\[ \Omega_{\tilde{\chi}_1^0} = 0.25 \pm 0.10 \] . In view of possible systematic errors that may exist, we chose here a 2 std range for \( \Omega_{\tilde{\chi}_1^0} h^2 \):

\[ 0.02 \leq \Omega_{\tilde{\chi}_1^0} h^2 \leq 0.25 \] .

The theoretical formula for \( \Omega_{\tilde{\chi}_1^0} h^2 \) is given by [31]:

\[ \Omega_{\tilde{\chi}_1^0} h^2 = 2.48 \times 10^{-11} \left( \frac{T_{\tilde{\chi}_1^0}}{T_\gamma} \right)^3 \left( \frac{T_\gamma}{2.73} \right)^3 \int_0^{2\pi} dx \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle \] (7)

where \( T_{\tilde{\chi}_1^0} \) is the freeze out temperature \( T_f (x_f = kT_f/m_{\tilde{\chi}_1^0}) \), \( N_f \) is the number of degrees of freedom at freezeout, \( (T_{\tilde{\chi}_1^0}/T_\gamma)^3 \) is the reheating factor, \( \sigma_{\text{ann}} \) is the annihilation cross section, \( v_{\text{rel}} \) is relative velocity, and \( \langle \ldots \rangle \) means thermal average.

Eq. (6) puts a strong constraint on the SUSY parameter space and this then affects the \( \tilde{\chi}_1^0 - p \) cross section. From Fig. 1 one sees that (i) \( \sigma_{\text{ann}} \) decreases with increasing \( m_0 \) and \( m_{\tilde{\chi}_1^0} \) (thus increasing \( \Omega_{\tilde{\chi}_1^0} h^2 \)) and (ii) if \( 2m_{\tilde{\chi}_1^0} \) is near (but below) \( m_h \), \( m_H \), \( m_A \) rapid annihilation occurs through s-channel pole (thus decreasing \( \Omega_{\tilde{\chi}_1^0} h^2 \)). While LEP has now eliminated most of the parameter space where \( 2m_{\tilde{\chi}_1^0} \approx m_h \), both \( H \) and \( A \) become light for large \( \tan \beta \) making this effect latter significant in that domain.

In general one sees that the bounds of Eq. (6) exert a strong influence on the allowed SUSY parameter space, and this in turn affects the size of the \( \tilde{\chi}_1^0 - q \) cross section. From Fig. 2 one has that (i) \( \sigma_{\tilde{\chi}_1^0 - p} \) becomes large for light (first generation) squarks (i.e. small \( m_0 \)) and light Higgs bosons, which is just the region of rapid early universe annihilation. Thus the lower bound on \( \Omega_{\tilde{\chi}_1^0} h^2 \) can produce an upper bound on \( \sigma_{\tilde{\chi}_1^0 - p} \). (ii) We also note that \( \sigma_{\tilde{\chi}_1^0 - p} \) increases with \( \tan \beta \).

II. MSUGRA MODELS

The mSUGRA model with radiative breaking of \( SU(2) \times U(1) \) depends upon four parameters and one sign. These may be taken as following: \( m_0 \), the universal scalar mass at \( M_G \); \( m_{1/2} \), the universal gaugino mass at \( M_G \) (or alternatively one may use \( m_{\tilde{\chi}_1^0} \) or \( m_{\tilde{\gamma}} \) at the electroweak scale since these scale with \( m_{1/2} \) i.e. \( m_{\tilde{\chi}_1^0} \approx 0.4m_{1/2} \) and \( m_{\tilde{\gamma}} \approx 2.8m_{1/2} \)); \( A_0 \), the universal soft breaking mass at \( M_G \); \( \tan \beta = \langle H_2 \rangle / \langle H_1 \rangle \); and the sign of \( \mu \), the Higgs mixing parameter in the superpotential \( (W_\mu = \mu H_1 H_2) \). (With our choice of sign in \( W_\mu \), the \( b \to s\gamma \) constraint eliminates most of the \( \mu > 0 \) parameter space.)

We proceed by varying the above parameters over the allowed space subject to all the constraints discussed above. The maximum value of \( \sigma_{\tilde{\chi}_1^0 - p} \) as a function of \( m_{\tilde{\chi}_1^0} \) is given in Fig. 3. We see that current experiments sensitive to \( \sigma_{\tilde{\chi}_1^0 - p} > 1 \times 10^{-6} \) pb are probing only large \( \tan \beta \) regime for this model, i.e. \( \tan \beta \approx 25 \).

To obtain further insight as to the parameter space being probed by current experiments, we show in Fig. 4 the value of \( \Omega_{\tilde{\chi}_1^0} h^2 \) when \( \sigma_{\tilde{\chi}_1^0 - p} \) takes on its maximum value for the characteristic case of \( \tan \beta = 30 \). We see that \( \Omega_{\tilde{\chi}_1^0} h^2 \) increases as \( m_{\tilde{\chi}_1^0} \) increases (as expected from the general discussion above) from roughly the minimum to maximum value allowed by Eq.(6). Thus an accurate determination of \( \Omega_{\mu} h^2 \) as might be expected from the MAP sattelite, would significantly help narrow the allowed SUSY parameter space.
One notes in Fig. 3 that the larger tan\(\beta\) cross section sustain with increasing \(m_{\chi_1^0}\) more than the smaller tan\(\beta\) do. This is due to the fact noted above that \(m_H\) and \(m_A\) become lighter as tan\(\beta\) increases increasing \(\sigma_{\chi_1^0-p}\). This is shown in Fig. 3 for the two cases of tan\(\beta=30\) and tan\(\beta=50\). In contrast, \(m_h\) is relatively heavy, i.e. for tan\(\beta=30\) one has that \(m_h\) increases monotonically with \(m_{\chi_1^0}\) from 112 GeV to 128 GeV. Since \(m_0\) is not large when \(\sigma_{\chi_1^0-p}\) is at maximum, the squarks lie below and close to the gluino where \(m_\tilde{g} \simeq 7 m_{\chi_1^0}\).

### III. NONUNIVERSAL MODELS

The possibility of nonuniversal soft breaking significantly changes the region of parameter space being accessed by current dark matter experiments. We consider here the case where nonuniversal soft breaking masses are allowed both in the Higgs and third generation sectors. A general parametrization at \(M_G\) then

\[
\begin{align*}
m_{H_1}^2 &= m_0^2(1 + \delta_1); \\
m_{H_2}^2 &= m_0^2(1 + \delta_2); \\
m_{Q_1}^2 &= m_0^2(1 + \delta_3); \\
m_{u_R}^2 &= m_0^2(1 + \delta_4); \\
m_{e_R}^2 &= m_0^2(1 + \delta_5); \\
m_{b_R}^2 &= m_0^2(1 + \delta_6); \\
m_{i_L}^2 &= m_0^2(1 + \delta_7).
\end{align*}
\]

(8)

Here \(m_0\) is the universal mass of the first two generations, and \(\delta_i\) are the deviations for the Higgs and third generation. We assume here that \(-1 \leq \delta_i \leq 1\). (Note that for \(SU(5)\) one would have \(\delta_3 = \delta_4 = \delta_5\) and \(\delta_6 = \delta_7\).)

In order to understand the effects that occur in this more complicated situation we first note that \(\tilde{\chi}_1^0\) is a mixture of gaugino and higgsino part: \(\tilde{\chi}_1^0 = \alpha \tilde{W}_3 + \beta \tilde{B} + \gamma \tilde{H}_1 + \delta \tilde{H}_2\). Further, the spin independent cross section arises due to the interference between the gaugino and higgsino parts of \(\tilde{\chi}_1^0\), causing \(\sigma_{\tilde{\chi}_1^0-p}\) to increase with increasing interference. The SUSY parameter that to a large extent controls the amount of interference is \(\mu^2\), interference increasing (and hence \(\sigma_{\tilde{\chi}_1^0-p}\) increasing) as \(\mu^2\) decreases, and interference decreases as \(\mu^2\) increases. The value of \(\mu^2\) at the electroweak scale in terms of the GUT scale parameter is determined by the RGEs. In general, one must solve these numerically. However, one can get a qualitative understanding of the effects of the \(\delta_i\) from an analytic solution which is valid for low and intermediate tan\(\beta\) (14):

\[
\begin{align*}
\mu^2 &= \frac{t^2}{t^2 - 1} \left[\left(\frac{1 - 3D_0}{2} - \frac{1}{t^2}\right) + \left(\frac{1 - D_0}{2}(\delta_3 + \delta_4) - \frac{1 + D_0}{2}\delta_2 + \frac{\delta_1}{t^2}\right)\right] m_0^2
\end{align*}
\]

(9)

Here \(t \equiv \tan\beta\), and \(D_0 \simeq 1 - (m_A/200\sin\beta)^2\). In general \(D_0\) is small i.e \(D_0 \leq 0.23\). Eq. (9) shows that it is necessary to consider both the squark nonuniversalities (\(\delta_3\) and \(\delta_4\)) as well as the Higgs (\(\delta_1\) and \(\delta_2\)) since they produce effects of comparable size.

One sees from Eq. (9) that \(\mu^2\) will be significantly reduced (and hence \(\sigma_{\tilde{\chi}_1^0-p}\) increased) if one chooses \(\delta_3, \delta_4, \delta_1 < 0\) and \(\delta_2 > 0\). (The reverse will be the case for the opposite choice of signs.) This effect can be seen in Fig. 3, where the maximum value of \(\sigma_{\tilde{\chi}_1^0-p}\) is plotted for tan\(\beta = 7\) for the nonuniversal model (upper curve) and mSUGRA (lower curve). One sees that with the choice \(\delta_3, \delta_4, \delta_1 < 0\) and \(\delta_2 > 0\) one can increase the cross section by a factor of 10 to 100. As a consequence, there are regions of parameter space where
the nonuniversal models can be probed to much lower tan\(\beta\). This is exhibited in Fig. 7, where the maximum cross section is given for tan\(\beta\)=7 and tan\(\beta\)=5. One sees that current experiments are probing tan\(\beta\) as low as tan\(\beta\)\(\simeq\)4. Since these experiments have also excluded the region \(\sigma_{\tilde{\chi}^0 - p} \gtrsim 10 \times 10^{-6}\) pb, one finds that part of the parameter space for tan\(\beta\)\(\simeq\)12 is already experimentally excluded for these models. Of course, the reverse choice of signs for \(\delta_i\) will lower the cross section for any value of tan\(\beta\), leaving such parts of the parameter space mostly as yet unexplored experimentally.

When \(\sigma_{\tilde{\chi}^0 - p}\) takes on its maximum value for this model, the light Higgs becomes quite light lying just above the LEP 200 limit. Thus for tan\(\beta\)=7 one finds 100 GeV \(\leq m_h \leq 107\) GeV which would make the h relatively easy to find at the LHC and the Tevatron RUN II. In contrast the squark can become quite heavy. This can be seen from Eq. (9) where the negative nonuniversal term dominates the \(m_0^2\) contribution, and hence making \(m_0^2\) large reduces \(\mu^2\), making \(\sigma_{\tilde{\chi}^0 - p}\) larger. Thus the first two generation squarks for this situation have masses in the range 600 GeV \(\leq m_{\tilde{q}} \leq 1200\) GeV, and lie above the gluino. Large values of \(m_{\tilde{q}}\) tend to suppress proton decay amplitudes of GUT models and would help relieve the tension between a large \(\sigma_{\tilde{\chi}^0 - p}\) and a small p decay rate.

IV. D-BRANE MODELS

Recent advances in string theory based on Dp-branes (manifolds of \(p+1\) dimensions) has led to a revival of phenomenologically motivated string models. One class of such models making use of type IIB orientifolds [32] allows one to put the Standard Model gauge group on 5-branes, manifolds of six dimensions, of which four are usual Minkowski space and two are compactified on a torus.

An interesting model of this class puts \(SU(3)_c \times U(1)_Y\) on one set of 5 branes, \(5_1\) and \(SU(2)_L\) on a second intersecting set \(5_2\) [18]. Strings starting on \(5_2\) and ending on \(5_1\) have massless modes carrying the joint quantum numbers of the two branes (presumably the SM quark and lepton doublets and the Higgs doublets), while strings beginning and ending on \(5_1\) have modes carrying \(SU(3)_C \times U(1)_Y\) quantum numbers (the right handed quark and lepton states). This leads to a model with a unique set of nonuniversal gaugino and squark and slepton masses at \(M_G\) parametrized as follows:

\[
\begin{align*}
\tilde{m}_1 &= \tilde{m}_3 = -A_0 = \sqrt{3}\cos\theta_b \Theta e^{-i\alpha_1} \frac{m_3}{2} \\
\tilde{m}_2 &= \sqrt{3}\cos\theta_b (1 - \Theta^2_1)^{1/2} \frac{m_3}{2}
\end{align*}
\]  

and

\[
\begin{align*}
m^2_{12} &= (1 - 3/2\sin^2\theta_b) m^2_{3/2} \\
m^2_1 &= (1 - 3\sin^2\theta_b) m^2_{3/2}
\end{align*}
\]  

Here \(\tilde{m}_i\), \(i=1,2,3\) are three gaugino masses at \(M_G\), while \(m^2_{12}\) are \(q_L, l_L, H_1, H_2\) sfermion and Higgs (mass)\(^2\) and \(m^2_1\) are \(\tilde{u}_R, \tilde{d}_R, \tilde{e}_R\) sfermion (mass)\(^2\). In addition, one has the B soft breaking mass and the \(\mu\) parameter at the GUT scale

\[
B_0 = |B_0|e^{i\theta_B}; \quad \mu_0 = |\mu_0|e^{i\theta_\mu}
\]  

\[\text{5}\]
where $\alpha_1$, $\theta_{0B}$ and $\theta_{0\mu}$ are possible CP violating phases. As can be seen from Eqs.(9,10), the parameters $\theta_b$ and $\Theta_1$ are restricted to $\sin\theta_b \leq 1/\sqrt{3}$ ($\theta_b \leq 0.615$) and $\Theta_1 \leq 1$.

Models of this type are of interest in that they are natural possibilities in string theory, and yet it would be difficult to see how such a symmetry breaking pattern could arise in conventional SUGRA GUT models. One aspect of the model is that the cancelations between the gluino and neutralino CP violating phases in the electron and neutron electric dipole moments (EDMs) implied by Eq. (10) allows the CP violating phases to be larger than usual at the electroweak scale and still satisfy the experimental EDM bounds [33]. However, these experimental bounds combined with the requirement of radiative breaking of $SU(2) \times U(1)$ at the electroweak scale, leads to serious fine tuning of parameters at the GUT scale unless $\tan\beta \lesssim 3 - 5$ [34]. Since we will be interested here in large $\tan\beta$ (to investigate the maximum values of $\sigma_{\tilde{\chi}_1^0-p}$) in the following we will first set all the CP violating phases to zero. One then has a model with four parameters and one sign i.e. $m_{3/2}$, $\theta_b$, $\Theta$ and $\tan\beta$, and the sign of $\mu$. (As before, $\mu$ must be negative for most of the parameter space to satisfy the $b \to s\gamma$ constraint.)

Fig. 8 shows $\sigma_{\tilde{\chi}_1^0-p}$ as a function of $m_{\tilde{\chi}_1^0}$, which is parametrized by $\theta_b$, $\Theta_1$, $m_{3/2}$. One has that $\sigma_{\tilde{\chi}_1^0-p}$ increases with increasing $\theta_b$ since by Eq. (10) the squark masses of Fig. 4 decrease. Fig. 9 shows $\sigma_{\tilde{\chi}_1^0-p}$ for $\tan\beta=15$ and $m_{3/2}=200$ GeV. One sees that one needs a large $\tan\beta$, i.e. $\tan\beta \gtrsim 15$ to obtain a cross section within current experimental sensitivities, i.e. $\sigma_{\tilde{\chi}_1^0-p} \gtrsim 1 \times 10^{-6}$ pb.

We next allow the CP violating phases to be non-zero to investigate their effect on $\sigma_{\tilde{\chi}_1^0-p}$ cross section. We impose here the experimental constraint on the electron EDM of $d_e < 4.3 \times 10^{-27}$ ecm at 95% C.L. [35]. (We do not impose the neutron EDM constraint here as there are a number of ambiguities in calculating $d_n$ [34]). Fig. 10 shows that $\sigma_{\tilde{\chi}_1^0-p}$ decreases with increasing phase $2\pi - \alpha_1$, the presence of the phase decreasing $\sigma_{\tilde{\chi}_1^0-p}$, by a factor of two more. As mentioned above, such large phases, while still satisfying the experimental bound on $d_e$, require a serious fine tuning of other parameters. Fig. 11 shows the allowed range $\Delta\phi_\mu$ of the phase $\phi_\mu$ to satisfy the electron EDM for $\tan\beta=15$, $\alpha_1 = 1.75\pi$. One sees that $\phi_\mu$ must be chosen very precisely to satisfy the EDM constraint on $d_e$. If one were also to impose the neutron EDM constraint as well, the fine tuning would be even more severe [35].

V. CONCLUSIONS

We have considered here the question of what part of the SUSY parameter space can be probed by detectors sensitive to the neutralino proton cross section in the range of Eq. (1). In order to examine this we have considered the maximum theoretical cross section as function of $m_{\tilde{\chi}_1^0}$. The answer depends on the particular SUSY model one is considering. Thus for mSUGRA models, one requires $\tan\beta \gtrsim 25$ to achieve current detector sensitivities $\sigma_{\tilde{\chi}_1^0-p} \geq 1 \times 10^{-6}$ pb. In this case, a large $\sigma_{\tilde{\chi}_1^0-p}$ corresponds to relatively heavy Higgs, $m_h \simeq (110 - 130)$ GeV, a light neutralino of $m_{\tilde{\chi}_1^0} \lesssim 120$ GeV (from the relic density constraint) and moderate squark masses (e.g. $m_{\tilde{d}} \simeq (400 - 700)$ GeV lying below but close to the gluino mass.

Nonuniversal SUGRA models can increase or decrease $\sigma_{\tilde{\chi}_1^0-p}$ by a factor of 10 to 100. In
the former case, one can begin to probe the parameter space for \( \tan\beta \gtrsim 4 \) for \( \sigma_{\tilde{\chi}_1^0 - p} \geq 1 \times 10^{-6} \) pb. Here the Higgs is relatively light, \( m_h \simeq (100 \text{--} 110) \text{ GeV} \), while the squarks are heavy (e.g. \( m_{\tilde{q}} \simeq 600 \text{--} 1200 \text{ GeV} \)) and lie well above the gluino. Current data has in fact begun to eliminate part of the parameter space for \( \tan\beta \gtrsim 15 \).

In the D-brane model considered, one requires \( \tan\beta \gtrsim 15 \) for \( \sigma_{\tilde{\chi}_1^0 - p} \geq 1 \times 10^{-6} \) pb. CP violating phases that can appear in such models lower the \( \tilde{\chi}_1^0 - p \) cross section by a factor of two or more. However, there is a serious fine tuning problem for the \( \mu \) phase at \( M_G \) for such large values of \( \tan\beta \).

The fact that in the nonuniversal SUGRA models one can get a large \( \sigma_{\tilde{\chi}_1^0 - p} \) with a small \( \tan\beta \) and large squark mass tends to relieve some of the tension between dark matter cross sections and proton decay. Thus if \( \tilde{\chi}_1^0 \) dark matter were discovered with \( \sigma_{\tilde{\chi}_1^0 - p} \gtrsim 1 \times 10^{-6} \) pb, such models could still have a low proton decay rate since \( \tau_p \) is suppressed for small \( \tan\beta \) and large \( m_{\tilde{q}} \).

In our analysis here, we have neglected the possibility of coannihilation effects since the rapid early universe annihilation produced by such effects generally require raising \( m_0 \) and \( m_{1/2} \) to avoid violating the lower bound of Eq. (6), lowering the \( \tilde{\chi}_1^0 - p \) cross section . (We have been interested here in the maximum values of \( \sigma_{\tilde{\chi}_1^0 - p} \).

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FIG. 1. Early universe annihilation of $\tilde{\chi}_1^0$ through s channel Higgs ($h, H, A$) and $Z$ poles and t channel squark and slepton ($\tilde{f}$) poles.

FIG. 2. Scattering of $\tilde{\chi}_1^0$ by a quark of a nucleon through s channel $\tilde{q}$ and t channel $h, H, A, Z$ poles.
FIG. 3. $(\sigma_{\tilde{\chi}_0^0-p})_{\text{max}}$ vs. $m_{\tilde{\chi}_1^0}$ for (from top to bottom) $\tan\beta = 50$, 40, 30, and 20.

FIG. 4. Value of $\Omega_{\chi_1^0 h^2}$ vs. $m_{\tilde{\chi}_1^0}$ for $\tan\beta = 30$ when $\sigma_{\tilde{\chi}_0^0-p}$ takes on its maximum value.
FIG. 5. $m_H$ when $\sigma_{\tilde{\chi}^0_1-\tilde{\chi}^0_1}$ takes on its maximum value for tan $\beta = 30$ (top curve) and tan $\beta = 50$ (lower curve).

FIG. 6. Maximum $\sigma_{\tilde{\chi}^0_1-\tilde{\chi}^0_1}$ for tan $\beta = 7$ for mSUGRA (lower curve) and nonuniversal model (upper curve).
FIG. 7. Maximum $\sigma_{\tilde{\chi}^0_{1} - p}$ for nonuniversal models for $\tan \beta = 7$ (upper curve) and $\tan \beta = 5$ (lower curve).

FIG. 8. $\sigma_{\tilde{\chi}^0_{1} - p}$ for the D-brane model for $m_{3/2} = 175$ GeV, $\tan \beta = 10$ for $\theta_b = 0.5$ (upper curve), $\theta_b = 0.2$ (lower curve). (The gaps are excluded regions.)
FIG. 9. $\sigma_{\tilde{\chi}_1^0 p}$ for the D-brane model for $m_{3/2} = 200$ GeV, tan$\beta=15$ for $\theta_b=0.5$ (upper curve), $\theta_b=0.2$ (lower curve).

FIG. 10. $\sigma_{\tilde{\chi}_1^0 p}$ for $\theta_b=0.4$, $m_{3/2} = 200$ GeV, tan$\beta=15$ for $\alpha_1 = 0, 1.85\pi, 1.75\pi$ in decreasing order.
FIG. 11. The allowed range $\Delta \phi_\mu$ of the phase $\phi_\mu$ for $\theta_b=0.4$, $m_{3/2} = 200$ GeV, $\tan\beta=15$ for $\alpha_1 = 1.75\pi$. 