Electroweak Interactions: 
Summary

Wolfgang Kilian

_Institut für Theoretische Teilchenphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany_

Abstract. The session on precision studies of electroweak interactions is summarized. The contributions address the bilinear, trilinear, quartic as well as heavy-quark interactions of the electroweak gauge bosons. This makes up a picture of the physics of electroweak symmetry and symmetry breaking which can be investigated with the proposed design of $e^+e^-$ Linear Colliders and detectors.

INTRODUCTION

The precise study of electroweak interactions has now become a classical domain of $e^+e^-$ physics, with a wealth of useful data available from the LEP and SLD experiments [1]. Nevertheless, with the upstart of data-taking at a high-energy and high-luminosity Linear Collider, a new level of precision will be accessible, setting unprecedented requirements both on the experimental analysis and on the accuracy of theoretical predictions.

The success of the Standard Model (SM) in explaining electroweak data establishes the nonabelian $SU_2 \times U_1$ gauge theory as a firm theoretical basis. Even in the absence of a light Higgs boson, the subset of the SM which incorporates only the known fermions and vector bosons serves as a useful effective theory valid up to the TeV energy range [2]. This sets the framework for the interpretation of precision data: Any deviation from the standard predictions can be parameterized by gauge-invariant operators of higher dimension in the electroweak Lagrangian (anomalous couplings). Only if there are striking signals such as the appearance of new particles, the SM effective-theory description must be abandoned in some sector of the theory and a new model placed there instead. Such scenarios are discussed in the reports on Supersymmetry [3] and on Alternative Theories [4] in these proceedings.
The Task & the Challenge

If models of electroweak symmetry breaking and physics beyond the SM are to be disentangled by their indirect effects on anomalous couplings, a complete coverage of electroweak interactions is essential. All bilinear, trilinear and quartic self-couplings of gauge bosons as well as their interactions with fermions should be measured as precisely as possible.

Fortunately, the contributions which have been presented at this Workshop (and which will be summarized below) show that all of the entries in this list are being addressed, leading to the generic conclusion that such a program is in fact feasible at a Linear Collider.

To get an impression of the expected magnitude of deviations from the SM predictions, one may consider a scenario in which no light Higgs boson exists and the $W$ and $Z$ bosons (their longitudinal modes, to be exact) become strongly interacting at TeV energies. In this case, the SM, lacking the Higgs particle as a regulator, predicts observables only to leading order in an expansion in powers of the energy [2]. At next-to-leading order (NLO) a certain number of new unknown parameters enter the game which in a consistent picture must be at least comparable to the size of one-loop radiative corrections [5]:

$$\frac{1}{16\pi^2} = 0.0063$$

This factor, essentially geometric in origin, is also quite typical for other scenarios which include a light Higgs boson: In this case, many new-physics effects decouple, and their presence can only be felt by finite loop corrections in SM interactions, which are again suppressed by the magic number $1/16\pi^2$.

Even though this conclusion may be pessimistic, without further knowledge of the underlying theory of electroweak symmetry breaking one has to be prepared for measuring all relevant interactions at the percent level or even better. This is a challenge for experiments, theory and simulation tools. One has to:

- Measure the observables with percent accuracy. Clearly, this requires a high luminosity of the machine: For a relative deviation as small as $1/16\pi^2$ to be a one-sigma effect, one needs at least 25,000 signal events in a certain channel. This problem can be partly overcome by a higher c.m. energy since the effects of anomalous couplings on observables typically increase with energy.

- Predict the signal to better than percent accuracy. The SM amplitudes have to be known with radiative corrections incorporated. Even NLO predictions are not always sufficient. In particular for multi-particle final states, this is still a long way to go.

- Understand the background to better than percent accuracy. Although the signal-to-background ratio in the $e^+e^-$ environment is much more favorable
than in hadronic collisions, this still demands the inclusion of complete matrix elements with leading radiative corrections and the exact treatment of multi-particle phase space in Monte Carlo generators. The large number and complexity of the processes to be considered clearly calls for flexible and automatic solutions.

The Big If

While the requirements on the accuracy of the experimental analysis and the theoretical prediction are independent of the scenario of electroweak symmetry breaking realized in Nature, the interpretation of actual experimental results will not:

• If a light Higgs boson exists (a fact that can be checked with confidence at a Linear Collider, if not elsewhere), in practical terms the SM is a complete renormalizable theory.\(^1\) Measuring electroweak interactions probes the structure of the non-abelian symmetry, and deviations would give only indirect hints for extensions or the breakdown of the picture: extra matter, extra gauge interactions, extra dimensions, or effects we do not even think of at present. However, although such new physics seems to be associated with any attempt to reconcile the strong and electroweak interactions with gravity, the theory does not really require it up to energy scales which are probably inaccessible to any collider.

• On the other hand, if no light Higgs boson exists, we are lacking a straightforward explanation for electroweak symmetry breaking. The mechanism responsible for it should manifest itself in the interactions of the particles which are most strongly coupled to the symmetry-breaking sector, namely the massive electroweak gauge bosons and the heavy top and bottom quarks. Precise measurements of their properties and interactions would then play the key role in uncovering the underlying theory which could explain, at least, the presence of gauge boson and fermion masses, and possibly shed light on the origin of flavor physics as a whole.

BILINEAR INTERACTIONS

In the Higgs-less scenario, it is customary to express corrections to the bilinear interactions of electroweak gauge bosons in terms of three parameters (e.g., \(S\), \(T\) and \(U\) [7]) which incorporate the leading effects in a low-energy expansion up to dimension four. These parameters can be identified with the coefficients of bilinear gauge-invariant operators [2]. Similarly, in the light-Higgs scenario deviations from

\(^1\) With the current lower limit on the Higgs mass, the vacuum instability bound of the SM is well beyond the reach of colliders [6].
the SM predictions are parameterized to leading order by gauge-invariant operators of dimension six [8].

In any case, these parameters quantify modifications in the way the physical $W$, $Z$ and photon fields are related to the proper $SU_2 \times U_1$ gauge fields. This would be visible in deviations from the tree-level prediction for the $W$ and $Z$ masses in terms of low-energy parameters (the Fermi coupling, the electromagnetic coupling constant and the weak mixing angle):

$$M_W = \frac{e}{\sin \theta_w} (\sqrt{2} G_F)^{-1/2} \quad \text{and} \quad M_W = \cos \theta_w M_Z$$

Such deviations are caused by matter carrying both $SU_2$ and $U_1$ quantum numbers [7] and by violations of the custodial $SU_2^c$ isospin symmetry [9] which in the SM relates the right-handed up- and down-type fermions. Radiative corrections within the SM also affect these relations.

At LEP1 and SLC, the high cross section on the $Z$ resonance allowed for a test of the relations (2) which is precise enough that SM loop corrections have to be taken into account. There is little hope to improve on this by measurements at higher energies unless one encounters a new resonance in $e^+e^-$ scattering. However, by exploiting the high-luminosity capability of the Linear Collider on the $Z$ resonance again, a new level of precision is accessible. This Giga-Z option is reviewed by K. Mönig [10]. (See also [11,12] for experimental issues at Giga-Z. The impact of this option for $b$ physics is further discussed in [13].)

One should note that with the experimental accuracy achievable at Giga-Z, there is need for the inclusion of two-loop (NNLO) corrections in the theoretical prediction [14]. In terms of the effective-theory approach this means that operators of NNLO in the low-energy expansion have to be included as well, and the description in terms of three parameters (like $S$, $T$, $U$) is no longer adequate.

Within the context of a definite model such as the SM or its minimal supersymmetric extension (MSSM), S. Heinemeyer [14] shows how to turn this argument around: Deviations from the relations (2) determine extra unknown parameters of the model which are difficult to access directly. For example, the parameters of the stop sector of the MSSM can be read off the electroweak observables if all other relevant quantities are assumed to be known.

**TRILINEAR INTERACTIONS**

In $e^+e^-$ collisions, trilinear interactions of electroweak gauge bosons affect four-fermion production. Depending on the assumed physical scenario (with or without Higgs) and the assumed underlying symmetry (electromagnetic gauge invariance, CP invariance, custodial symmetry) the number of independent parameters which govern the triple gauge couplings of $W$, $Z$ bosons and photons at NLO vary between two and fourteen [2,15]. Improving on the bounds obtained at the LEP2
experiments, a high-energy Linear Collider will lift the state of knowledge of the trilinear anomalous couplings to the level of the bilinear couplings right now.

In the study by Wolfgang Menges [16] this fact is verified in a refined analysis of \( e^+e^- \to W^+W^- \to 4f \), which takes into account initial-state radiation, beamstrahlung, beam polarization and detector effects, and uses the full spin correlation in the final state to extract the anomalous couplings from simulated event samples. The precision achievable is of the order \( 10^{-4} \) for the \textit{“standard”} couplings and of the order \( 10^{-3} \) for the CP-violating ones, reaching and even surpassing the \textit{magic number} \( 1/16\pi^2 \).

A meaningful measurement of four-fermion production at this level of accuracy requires a theoretical understanding of this process which has not yet been fully achieved. In their respective contributions, W. Placzek [17] and D. Wackeroth [18] review the current status of the four-fermion Monte-Carlo generators \texttt{YFSWW/KORALW} and \texttt{RACOONWW}. They incorporate the resummation of multiple photon radiation in the initial state beyond the leading-logarithmic level. In addition, genuine electroweak loop corrections are taken into account. Since a full one-loop calculation is not yet available, all generators rely on the so-called double-pole approximation for \( e^+e^- \to 4f \), which incorporates all radiative corrections at NLO near the doubly-resonant kinematic configuration. The technical agreement of the two codes is satisfactory, and the LEP2 data are accurately described by the simulation. However, regarding the experimental prospects at a Linear Collider, the level of accuracy is only barely sufficient, and improvements in the theoretical prediction are still needed.

With increasing collider energy a new scale discrepancy of \( \sqrt{s} \) vs. \( M_W, M_Z \) complicates the calculation of radiative corrections. At ultra-high energies Sudakov-type logarithms of such scale ratios pile up, invalidating finite-order predictions and calling for new methods of resummation. Fortunately, as shown by M. Melles [19], these contributions are under control: they factorize and exponentiate and can thus be absorbed into universal correction factors.

If no light Higgs boson exists, a conceivable side-effect of electroweak symmetry breaking is a heavy vector resonance in \( WW \) scattering [20,21]. This would mix with the \( Z \) boson, leading to an effective form factor in the \( ZWW \) coupling. As pointed out by T. Barklow [22], if anomalous triple gauge couplings are interpreted in this way, the presence of such a vector resonance with a mass as high as 2.5 TeV could easily be detected in \( e^+e^- \to 4f \). Here, due to the \( s \)-channel nature of the process, high luminosity at lower energy (500 fb\(^{-1}\) at 800 GeV) is more promising than lower luminosity at higher energy (200 fb\(^{-1}\) at 1.5 TeV).

The measurement of \( W \) pair production and the disentangling of the various couplings is greatly simplified by charm tagging, which removes ambiguities in processes with \( W \) decaying into hadrons. This possibility is being investigated in the present context by W. Walkowiak [23].
QUARTIC INTERACTIONS

The study of quartic vector boson interactions has not been possible at any existing collider, and the Linear Collider in conjunction with the LHC will play a pioneer role [24,25]. These interactions are particularly interesting since in the absence of a scalar resonance (the Higgs boson) the scattering amplitudes for the processes $W W \to W W$ and $WW \to ZZ$ become strong in the TeV range, violating tree-level unitarity [26] and thus calling for new physical effects which regulate the high-energy behavior.

Conversely, if the Higgs exists, there would be a strong cancellation in this class of processes which would be interesting to observe directly: the Higgs mechanism at work.

The processes $WW \to WW$ and $WW \to ZZ$ are accessible at a Linear Collider as subprocesses of $e^+e^- \to \bar{\nu}\nu + 4f$ (and $e^-e^- \to \nu\nu + 4f$), where the “initial” $W$ bosons are radiated off the incoming electron/positron. While at ultra-high energies this effect can be described by an effective structure-function approach [27], at Linear Collider energies of the order $0.5\ldots1$ TeV this is not sufficient, and complete matrix elements should be used for a reliable calculation. Therefore, the analysis presented by R. Chierici [28] uses the new generic Monte-Carlo package WHIZARD [29] to simulate the complete six-fermion signal without such approximations.

The difficulty here is threefold: First, $WW$ and $ZZ$ states must be clearly separated from each other using their hadronic decays. Second, a large background from the subprocesses $\gamma\gamma \to WW$ and $\gamma W \to ZW$ where the electron radiating the photon vanishes in the beampipe must be reduced. Finally, anomalous quartic couplings primarily affect the longitudinal degrees of freedom of the vector bosons, which should be extracted from appropriate angular correlations.

The analysis shows that for the second-stage TESLA parameters $\sqrt{s} = 800$ GeV and $\int L = 1$ ab$^{-1}$ a meaningful measurement is in fact possible. Assuming for simplicity CP invariance and exact custodial symmetry, which reduces the dimensionality of the NLO parameter space to two, the remaining anomalous couplings $\alpha_4$ and $\alpha_5$ can be measured with an accuracy of the order $10^{-2}$. This already comes near the magic number $1/16\pi^2$. Beam polarization and the inclusion of further observables in the analysis further improve this result.

Obviously, going to even higher energies is another option for increasing the impact of new effects in $WW$ scattering on observables. A. de Roeck [30] demonstrates the power of a CLIC design with $\sqrt{s} = 3$ TeV to disentangle various possible scenarios for the high-energy behavior of this process.

ELECTROWEAK TOP QUARK INTERACTIONS

Recent theoretical developments have shown that the heaviness of the top quark (or, equivalently, the lightness of all other fermions) may indicate its direct involvement in the mechanism of electroweak symmetry breaking [31]. It is therefore
important to look also into processes like $W^+W^- \rightarrow t\bar{t}$, another interaction that becomes strong at high energies if not regulated by a Higgs-like resonance.

Although the larger top mass makes it more difficult, $WW \rightarrow tt$ (and $WZ \rightarrow tb$) scattering can be accessed by methods similar to elastic vector boson scattering. T. Han [32] and J. Alcaraz [33] present studies which investigate the possibility to observe resonances in these channels. As a result, the detection of resonances with a mass up to $2/3$ of the collider energy seems feasible.

**CONCLUSIONS**

A Linear Collider with an energy in the $0.5 \ldots 1$ TeV range will provide an appropriate environment to measure electroweak interactions with such a precision that one can not only check the overall consistency of the gauge theory, but be sensitive to the physics that lies at the origin of electroweak symmetry breaking. However, to reach the necessary high level of accuracy, strong requirements on the machine, the detector, the analysis, and on theory must be fulfilled. Many details have yet to be clarified and work still needs to be done, but as the contributions presented at this Workshop have shown, this program is realistic.

**ACKNOWLEDGMENTS**

I would like to thank the participants of the Electroweak Working Group for stimulating discussions, and the organizers for their invitation and for the pleasant atmosphere at this meeting.

**REFERENCES**

1. A. Gurtu, to appear in: Proc. XXXth International Conference on High Energy Physics (ICHEP 2000), Osaka, Japan, World Scientific, Singapore 2001.
2. T. Appelquist and C. Bernard, *Phys. Rev.* **D22**, 200 (1980); A. Longhitano, *Phys. Rev. D22*, 1166 (1980); *Nucl. Phys. B188*, 118 (1981); T. Appelquist and G.-H. Wu, *Phys. Rev. D48*, 3235 (1993).
3. H.-U. Martyn, in these proceedings.
4. C. S. Kim, in these proceedings.
5. S. Weinberg, *Physica 96A*, 327 (1979).
6. T. Hambye and K. Riesselmann, DESY-97-152, hep-ph/9708416.
7. M.E. Peskin and T. Takeuchi, *Phys. Rev. Lett.* **65**, 964 (1990); *Phys. Rev. D46*, 381 (1992).
8. W. Buchmüller and D. Wyler, *Nucl. Phys. B268*, 621 (1986).
9. M. Veltman, *Act. Phys. Pol.* **B8**, 475 (1977); *Nucl. Phys. B123*, 89 (1977); P. Sikivie, L. Susskind, M. Voloshin, and V. Zakharov, *Nucl. Phys. B173*, 189 (1980).
10. K. Mönig, in these proceedings.
11. P. C. Rowson and M. Woods, in these proceedings.
12. K. Fuji and T. Omori, in these proceedings.
13. B. Schumm, in these proceedings; A. Ali, D. Benson, I. Bigi, R. Hawkings, and T. Mannel, DESY 00–183, hep-ph/0012218.
14. S. Heinemeyer, in these proceedings.
15. K.J.F. Gaemers, G.J. Gounaris, Z. Phys. C1, 259 (1979); K. Hagiwara, K. Hikasa, R.D. Peccei, and D. Zeppenfeld, Nucl. Phys. B282, 253 (1987); G.J. Gounaris, J.L. Kneur, D. Zeppenfeld et al., Triple Gauge Boson Couplings, in CERN 96-01.
16. W. Menges, in these proceedings.
17. W. Placzek, in these proceedings.
18. D. Wackeroth, in these proceedings.
19. M. Melles, in these proceedings.
20. S. Weinberg, Phys. Rev. D19, 1277 (1979); L. Susskind, Phys. Rev. D20, 2619 (1979).
21. R. Casalbuoni et al, in: Proc. e+e− Collisions at 500 GeV: The Physics Potential, Munich–Annecy–Hamburg, 1991-93, DESY 92-123A+B, 93-123C.
22. T. Barklow, in these proceedings.
23. W. Walkowiak, in these proceedings.
24. V. Barger et al., Phys. Rev. D52, 3815 (1995); E. Boos et al., Phys. Rev. D57, 1553 (1998); ibid. D61, 077901 (2000).
25. J. Bagger et al., Phys. Rev. D49, 1246 (1994); ibid. D52, 3878 (1995).
26. B. Lee, C. Quigg, and H. Thacker, Phys. Rev. Lett. 38, 883 (1977).
27. M. S. Chanowitz and M. K. Gaillard, Phys. Lett. B142, 85 (1984); G. L. Kane, W. W. Repko, and W. R. Rolnick, Phys. Lett. B148, 367 (1984); S. Dawson, Nucl. Phys. B249, 42 (1985).
28. R. Chierici, S. Rosati, and M. Kobel, in these proceedings.
29. W. Kilian, Comput. Phys. Commun., in preparation; T. Ohl, in these proceedings.
30. A. De Roeck, in these proceedings.
31. B. Dobrescu, in these proceedings.
32. T. Han, in these proceedings.
33. J. Alcaraz and E. Ruiz Morales, in these proceedings.