Approximately Efficient Bilateral Trade

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ABSTRACT
We study bilateral trade between two strategic agents. The celebrated result of Myerson and Satterthwaite states that in general, no incentive-compatible, individually rational and weakly budget balanced mechanism can be efficient. I.e., no mechanism with these properties can guarantee a trade whenever buyer value exceeds seller cost. Given this, a natural question is whether there exists a mechanism with these properties that guarantees a constant fraction of the first-best gains-from-trade, namely a constant fraction of the gains-from-trade attainable whenever buyer’s value weakly exceeds seller’s cost. In this work, we positively resolve this long-standing open question on constant-factor approximation, mentioned in several previous works, using a simple mechanism that obtains a $\frac{1}{2.25} \approx 0.121$ fraction of the first-best.

CCS CONCEPTS
• Theory of computation → Approximation algorithms analysis
• Applied computing → Electronic commerce
• Mathematics of computing → Probability and statistics.

KEYWORDS
bilateral trade, mechanism design, approximation algorithms

1 INTRODUCTION
In a bilateral trade, a seller holds an item and is trying to sell it to a buyer. The buyer’s private value $v$ is drawn from a cumulative distribution function (CDF) $F$, and the seller’s private cost $c$ for selling the item is independently drawn from a CDF $G$. If a transaction happens between $v$ and $c$, the society as a whole gains utility of $v - c$. The gains-from-trade (GFT) refer to the expected utility gain from trading. If trading probability between $v$ and $c$ is $x(v, c)$, then

$$GFT = E_{v \sim F, c \sim G} [(v - c) \cdot x(v, c)].$$

Ideally, to maximize GFT, a trade should always happen when $v > c$ and never happen when $v < c$. The resulting optimal GFT is termed the first best (FB). Namely,

$$FB = E_{v \sim F, c \sim G} [(v - c) \cdot 1\{v \geq c\}].$$

The seminal work of Myerson and Satterthwaite [18] shows that if both agents are self-interested, it is impossible to devise a Bayesian incentive-compatible (BIC), individually rational (IR) and weakly budget balanced mechanism that achieves the first-best GFT, as long as the distribution supports of $F$ and $G$ “overlap”. This impossibility result motivates the natural question of whether there exists a mechanism with these properties (BIC, IR, and WBB) that guarantees a constant fraction of the first-best GFT.

Benchmarks and Simple Mechanisms. To better explain related work and our results, we first introduce some benchmarks and simple mechanisms. For simplicity, we assume $F$ and $G$ are both continuous distributions supported on a bounded interval $[0, 1]$ with positive densities. We show in Remark 2 why this is without loss of generality. We slightly overload the notations below and use them to denote both the mechanisms and the gains-from-trade they obtain.

• FB: the first best. It captures the optimal GFT if the agents are not strategic and there is a trade whenever $v \geq c$. Formally,

$$FB = \int_0^1 \int_c^1 \frac{v - c}{v} dF(v) dG(c).$$

• SB: the second best. It captures the optimal GFT achieved by any Bayesian incentive-compatible (BIC), individually rational (IR) and weakly budget balanced (WBB) mechanism.

• FixedP: the fixed-price mechanism. The mechanism sets a fixed price $p$, and there is a trade if buyer’s value is at least $p$ and seller’s cost is at most $p$. Formally,

$$FixedP = \max_p \int_0^p \int_c^1 (v - c) dF(v) dG(c).$$

A mechanism is weakly budget balanced if the total payment of the participants is ex-post non-negative, i.e., the payment from the buyer is always at least the revenue of the seller. Moreover, a mechanism is strongly budget balanced if the total payment of the participants is ex-post exactly 0, i.e., the payment from the buyer is always the revenue of the seller.
• SellerP: the seller-pricing mechanism. The mechanism delegates the pricing power to the seller, who in turn posts a price \( r_c \) to maximize her profit with knowledge of \( c \) and \( F \). The buyer then decides whether to buy depending on whether \( v \geq r_c \). Formally, \( \text{SellerP} = \int_0^1 \int_0^{r_c} (v - c) \, dF(v) \, dG(c) \), where \( r_c \in \text{arg max}_p \left( p - c \right) \cdot (1 - F(p)) \) is the price the seller sets when she has cost \( c \). The mechanism is BIC, IR and SBB.

• BuyerP: the buyer-pricing mechanism. Symmetric to SellerP, the mechanism delegates the pricing power to the buyer, who sets a price \( r_b' \) to maximize his utility, and then the seller decides whether to sell based on whether \( c \leq r_b' \). Formally, \( \text{BuyerP} = \int_0^1 \int_0^{r_b'} (v - c) \, dG(c) \, dF(v) \), where \( r_b' \in \text{arg max}_p \left( v - p \right) \cdot G(p) \). The mechanism is BIC, IR and SBB.

With these notations, the result of Myerson and Satterthwaite [18] demonstrates that \( SB < FB \) whenever the supports of \( F \) and \( G \) overlap. It has remained an open question ever since, on how far apart \( SB \) and \( FB \) can be. Specifically, is it the case that \( SB \) is always at least a constant fraction of \( FB \)? We answer this question in the positive. In particular, the better of (or a randomization over) SellerP and BuyerP guarantees at least 10% of the best-gains-from-trade.

Theorem 1.1. \( FB \leq 2 \cdot \text{SellerP} + 8 \cdot \text{BuyerP} \leq 10 \cdot SB \).

Remark 1. In Appendix A, we improve the constant to get a 8.23-approximation.

2 RELATED WORK
There has been a large body of work towards answering whether \( SB = \Omega(1) \cdot FB \). In particular, McAfee [17] shows \( \text{FixedP} \geq \frac{1}{2} \cdot FB \) when the median of buyer values (\( F \)) is at least as high as the median of seller cost (\( G \)) — in this case, any price between the two medians gives at least a \( \frac{1}{2} \)-fraction of \( FB \). Blumrosen and Mizrahi [6] show \( \text{SellerP} \geq \frac{1}{2} \cdot FB \) when \( F \) satisfies the monotone-hazard-rate condition. As for the negative direction, Blumrosen and Mizrahi [6] show that for any \( \epsilon > 0 \), \( SB \leq \left( \frac{3}{2} + \epsilon \right) \cdot FB \) in some instance. Blumrosen and Dobzinski [5] show that for any \( \epsilon > 0 \), there is some instance where \( \text{FixedP} < \epsilon \cdot FB \) (and further \( \text{FixedP} < \epsilon \cdot SB \)).

Brustle, Cai, Wu, and Zhao [7] show that \( SB \leq \text{SellerP + BuyerP} \), thus presenting a 2-approximation to \( SB \). Our result is “stronger” in the sense that we show the better of \( \text{SellerP} \) and \( \text{BuyerP} \) is a constant-approximation of \( FB \), not just \( SB \). Given the result of [7] that \( SB \leq \text{SellerP + BuyerP} \), it is natural to ask whether \( FB \leq \text{SellerP + BuyerP} \). It turns out the answer is no: there are distributions where \( FB > (1+\epsilon) \cdot (\text{SellerP + BuyerP}) \) for some \( \epsilon > 0 \), independently shown by Babaioff, Dobzinski, and Kupfer [2] and Cai, Goldner, Ma, and Zhao [8] in their arXiv version.

A closely related question of welfare approximation has also been extensively studied before. The welfare equals GFT plus the term \( E_{\leq,-G}[c] \), i.e., the welfare equals buyer’s value when trade happens and it equals seller’s cost when no trade happens. Maximizing the welfare is equivalent to maximizing the gains-from-trade, but providing a constant approximation to first-best gains-from-trade is much harder than doing so for first-best welfare. For instance, not having any trade at all already gives a non-zero (and often good) approximation to welfare, but it gives a zero approximation to gains-from-trade. Blumrosen and Dobzinski [5] show a \( \left( 1 - \frac{1}{2} \right) \)-approximation to the first-best welfare. Kang, Pernice, and Vondrák [14] improve the approximation ratio to \( 1 - \frac{4}{3} \cdot 10^{-4} \).

Dütting, Fusco, Lazos, Leonardi, and Reifenhäuser [13] and Kang, Pernice, and Vondrák [14] consider settings without full knowledge of the distributions. Cesa-Bianchi, Cesari, Colomboni, Fusco, and Leonardi [9] give a regret analysis of the FixedP mechanism in a bilateral trade setting where the seller and buyer repeatedly interact over the course of \( T \) rounds.

In a different direction, McAfee [15] considers a continuous quantities version of the bilateral trade setting, and obtains insights beyond those in the discrete quantity model of Myerson and Satterthwaite [18].

The double auction is a generalized version of bilateral trade, where there are multiple buyers and sellers in the market. There has been a significant amount of work on characterizing and approximating the efficient solutions in related settings; see e.g. [1, 3, 4, 7, 8, 10–12, 16].

3 PROOF OF CONSTANT APPROXIMATION
In the statement of Theorem 1.1, since \( \text{SellerP} \) and \( \text{BuyerP} \) are BIC, IR and SBB, they are at most \( SB \), and hence \( 2 \cdot \text{SellerP} + 8 \cdot \text{BuyerP} \leq 10 \cdot SB \). The main difficulty is in showing that \( FB \leq 2 \cdot \text{SellerP} + 8 \cdot \text{BuyerP} \).

Recall that \( F \) and \( G \) are independent continuous CDFs supported on \([0, 1]\) with positive densities (see Remark 2 for why everything apart from independence is without loss of generality). Let \( \mu(x) = F_\geq x \) (where \( F_\geq x(z) = 0 \) for \( z < x \) and \( F_\geq x(z) = \frac{F(z) - F(x)}{1 - F(x)} \) for \( z \geq x \)), i.e., \( \mu(x) = F^{-(1)} \left( \frac{1 + F(x)}{2} \right) \). Let \( \mu(k)(\cdot) \) be the composition of \( k \) functions of \( \mu(\cdot) \), and \( \mu^{(k)}(\cdot) \) be its inverse \( 0 \) if it does not exist). Our proof aims to relate \( FB \) and \( SB \) by reducing them to similar forms, where we frequently and crucially use the function \( \mu(\cdot) \). We first give immediate bounds for \( FB \), \( \text{SellerP} \) and \( \text{BuyerP} \):

\[
\begin{align*}
FB &= \int_0^1 \int_0^1 (v - c) \, dF(v) \, dG(c) \\
&\leq 2 \int_0^1 \int_0^{\mu(c)(v-c)} (v - c) \, dF(v) \, dG(c),
\end{align*}
\]

where the second step holds since \( \mu(x) \) is the median of \( F_\geq x \), and we preserve the better half.

For \( \text{SellerP} \), we have

\[
\text{SellerP} \geq \int_0^1 \int_0^{\mu(c)(c)} (\mu(c) - c) \, dF(v) \, dG(c).
\]

The right-hand side (RHS) is the seller’s expected payoff when setting the price at \( \mu(c) \) for each \( c \), which is at most her optimal profit and thus at most the gains-from-trade in the profit-optimal seller-pricing mechanism (which is the left-hand side (LHS)).

Similarly, for \( \text{BuyerP} \), we have

\[
\begin{align*}
\text{BuyerP} &\geq \int_0^1 \int_0^{\mu^{(-2)}(v)} (v - \mu^{(-2)}(v)) \, dG(c) \, dF(v) \\
&= \int_0^1 \int_0^{\mu^{(-2)}(c)} (v - \mu^{(-2)}(v)) \, dF(v) \, dG(c).
\end{align*}
\]
Again, the RHS is the buyer’s expected utility when setting the price at $\mu^{-2}(\cdot)$ for each $v$, which is at most the gains-from-trade in the utility-optimal buyer-pricing mechanism (the LHS).

For each cost $c$, define $FB(c) = 2\int_{\mu(c)}^{1} (v-c) \, dF(v)$, $SellerP(c) = \int_{\mu(c)}^{1} (\mu(c)-c) \, dF(v)$ and $BuyerP(c) = \int_{\mu(c)}^{1} (v-\mu^{-2}(v)) \, dF(v)$. The bounds above simplify to:

$$FB \leq \int_{0}^{1} FB(c) \, dG(c),$$
$$SellerP \geq \int_{0}^{1} SellerP(c) \, dG(c),$$
$$BuyerP \geq \int_{0}^{1} BuyerP(c) \, dG(c).$$

We now show that, for any $c$, $FB(c) \leq 2 \cdot SellerP(c) + 8 \cdot BuyerP(c)$, thus proving our result. The crux of our proof is a partition of value space by quantile. A similar partition can be seen in the work of Colini-Baldeschi, Goldberg, de Keijzer, Leonardi, and Turchetta [12]. We decompose $FB(c)$ into two parts and bound them by $SellerP(c)$ and $BuyerP(c)$ separately.

**Lemma 3.1.** For any $c$, $FB(c) \leq 2 \cdot SellerP(c) + 8 \cdot BuyerP(c)$.

**Proof.** We first have

$$FB(c) = 2\int_{\mu(c)}^{1} (v-c) \, dF(v)$$
$$= 2\int_{\mu(c)}^{1} (\mu(c)-c) \, dF(v) + 2\int_{\mu(c)}^{1} (v-\mu(c)) \, dF(v)$$
$$= 2 \cdot SellerP(c) + 2\int_{\mu(c)}^{1} (v-\mu(c)) \, dF(v).$$

We now proceed to show that $\int_{\mu(c)}^{1} (v-\mu(c)) \, dF(v) \leq 4 \cdot BuyerP(c)$.

Observe that

$$\int_{\mu(c)}^{1} (v-\mu(c)) \, dF(v)$$
$$= \sum_{k=1}^{\infty} \int_{\mu^{(k)}(c)}^{\mu^{(k+1)}(c)} (v-\mu(c)) \, dF(v)$$
$$\leq \sum_{k=1}^{\infty} \int_{\mu^{(k)}(c)}^{\mu^{(k+1)}(c)} (\mu^{(k+1)}(c)-\mu(c)) \, dF(v)$$
$$= \sum_{k=1}^{\infty} \sum_{t=1}^{k} \int_{\mu^{(k)}(c)}^{\mu^{(k+1)}(c)} (\mu^{(t+1)}(c)-\mu^{(t)}(c)) \, dF(v)$$
$$= \sum_{t=1}^{\infty} \sum_{k=t}^{\infty} \int_{\mu^{(k)}(c)}^{\mu^{(k+1)}(c)} (\mu^{(t+1)}(c)-\mu^{(t)}(c)) \, dF(v)$$
$$= 2\int_{\mu(c)}^{1} \int_{c}^{1} (v-c) \, dF(v) \, dG(c).$$

The last step uses the definition of $\mu(\cdot)$: $k = t$ counts for half the probability of all $k \geq t$.

On the other hand, recall $BuyerP(c) = \int_{\mu^{(2)}(c)}^{1} (v-\mu^{-2}(v)) \, dF(v)$ and we have

$$\int_{\mu^{(2)}(c)}^{1} (v-\mu^{-2}(v)) \, dF(v)$$
$$= \sum_{t=1}^{\infty} \int_{\mu^{(t+1)}(c)}^{\mu^{(t)}(c)} (v-\mu^{-2}(v)) \, dF(v)$$
$$\geq 2\sum_{t=1}^{\infty} \int_{\mu^{(t+1)}(c)}^{\mu^{(t)}(c)} (\mu^{(t+1)}(c)-\mu^{(t)}(c)) \, dF(v)$$
$$= \frac{1}{2} \sum_{t=1}^{\infty} \int_{\mu^{(t+1)}(c)}^{\mu^{(t)}(c)} (\mu^{(t+1)}(c)-\mu^{(t)}(c)) \, dF(v),$$

where the last step again applies the definition of $\mu(\cdot)$. Therefore, we conclude our proof that $FB(c) \leq 2 \cdot SellerP(c) + 8 \cdot BuyerP(c)$. □

**Remark 2.** It is without loss of generality to consider bounded continuous distributions with positive densities. First we show that for bounded distributions, the continuity assumption is without loss of generality. For any general distributions $F$ and $G$ on $[0, 1]$, there is a sequence of continuous distributions on $[0, 1]$ with positive densities converging to it in the Lévy metric. All of $FB$, seller’s profit in $SellerP$ and buyer’s utility in $BuyerP$ are continuous in $F$ and $G$ (in the Lévy metric), thus yielding our result. Next we show that the bounded-support condition is also without loss of generality. Consider the case where $FB$ is finite. Here, almost all of the contribution comes from $(v, c) \in [-\ell, \ell]^2$ for some $\ell$, and the better of seller’s profit in $SellerP$ and buyer’s utility in $BuyerP$ gives a 10-approximation for the first best there. Therefore, having bounded supports is also without loss of generality.

**Remark 3.** By symmetry, we also have $FB \leq 8 \cdot SellerP + 2 \cdot BuyerP$. Thus, using $SellerP$ with probability $\alpha$ and $BuyerP$ with probability $1 - \alpha$ for any $\alpha \in [0.2, 0.8]$ gives a 10-approximation to $FB$.

**Remark 4.** We can use a parameter to control the quantile of $\mu(x)$ (instead of using the median) in $\tilde{F}_{\geq x}$. This improves the approximation constant to 8.23. The details are deferred to Appendix A.

### A IMPROVING THE CONSTANT

To improve the approximation constant of Theorem 1.1, instead of setting $\mu(x)$ to be the median of $\tilde{F}_{\geq x}$, we introduce a parameter $\lambda$ to control the quantile of $\mu(x)$ in $\tilde{F}_{\geq x}:

$$\mu(x) := \tilde{F}^{-1}\left(\lambda + (1 - \lambda)x \right).$$

Similar to the proof of Theorem 1.1, we have

$$FB = \int_{0}^{1} \int_{c}^{1} (v-c) \, dF(v) \, dG(c)$$
$$\leq \frac{1}{1-\lambda} \int_{0}^{1} \int_{\mu(c)}^{1} (v-c) \, dF(v) \, dG(c),$$

\[\text{In the case where } FB \text{ is infinite, we can similarly show } SB \text{ is also infinite: For any } M, \text{ there is an interval } [-\ell, \ell] \text{ so that the first best when } (v, c) \in [-\ell, \ell]^2 \text{ is at least } 10M. \text{ Our result in the paper, invoked for the bounded support case, implies that the second best in this interval is at least } M. \text{ Thus } SB \text{ is infinite whenever } FB \text{ is.}\]
and
\[ \text{SellerP} \geq \int_0^1 \int_{\mu(c)}^1 \left( v - \mu^{-2}(v) \right) dF(v) \, dG(c), \]
and
\[ \text{BuyerP} \geq \int_0^1 \int_{\mu^{-2}(v)}^1 \left( v - \mu^{-2}(v) \right) dG(c) \, dF(v) \]
\[ = \int_0^1 \int_{\mu^{-2}(v)}^1 \left( v - \mu^{-2}(v) \right) dG(c) \, dF(v). \]

Let \( FB(c) = \frac{1}{1 - \lambda} \int_{\mu(c)}^1 (v - c) \, dF(v) \), \( \text{SellerP}(c) = \int_{\mu(c)}^1 (\mu(c) - c) \, dF(v) \) and \( \text{BuyerP}(c) = \int_{\mu(c)}^1 (c - \mu^{-2}(v)) \, dF(v) \). This gives
\[ FB \leq \int_0^1 FB(c) \, dG(c), \]
\[ \text{SellerP} \geq \int_0^1 \text{SellerP}(c) \, dG(c), \]
\[ \text{BuyerP} \geq \int_0^1 \text{BuyerP}(c) \, dG(c). \]

Fix any \( c \) from now on. We have
\[ (1 - \lambda) \cdot FB(c) \]
\[ = \int_{\mu(c)}^1 (v - c) \, dF(v) \]
\[ = \int_{\mu(c)}^1 (\mu(c) - c) \, dF(v) + \int_{\mu(c)}^1 (v - \mu(c)) \, dF(v) \]
\[ = \text{SellerP}(c) + \sum_{k=1}^{\infty} \int_{\mu(c)}^1 1 \left( \mu^{(k+1)}(c) - \mu(c) \right) \, dF(v) \]
\[ \leq \text{SellerP}(c) + \sum_{k=1}^{\infty} \int_{\mu(c)}^1 1 \left( \mu^{(k+1)}(c) - \mu(c) \right) \, dF(v) \]
\[ = \text{SellerP}(c) + \sum_{k=1}^{\infty} \sum_{i=1}^{k} \int_{\mu(c)}^1 1 \left( \mu^{(i+1)}(c) - \mu^{(i)}(c) \right) \, dF(v) \]
\[ = \text{SellerP}(c) + \sum_{k=1}^{\infty} \sum_{i=1}^{k} \int_{\mu(c)}^1 1 \left( \mu^{(i+1)}(c) - \mu^{(i)}(c) \right) \, dF(v) \]
\[ = \text{SellerP}(c) + \sum_{k=1}^{\infty} \sum_{i=1}^{k} \int_{\mu(c)}^1 1 \left( \mu^{(i+1)}(c) - \mu^{(i)}(c) \right) \, dF(v). \]

The last step uses the definition of \( \mu(\cdot) \): \( k = t \) counts for \( \lambda \)-fraction of the probability of all \( k \geq t \).

Additionally, we have
\[ \text{BuyerP}(c) = \int_{\mu(c)}^1 1 \left( v - \mu^{-2}(v) \right) \, dF(v) \]
\[ = \sum_{i=1}^{\infty} \int_{\mu^{-2}(v)}^1 1 \left( \mu^{-2}(v) \right) \, dF(v) \]
\[ \geq \sum_{i=1}^{\infty} \int_{\mu^{-2}(v)}^1 1 \left( \mu^{(i+1)}(c) - \mu^{(i)}(c) \right) \, dF(v) \]
\[ = (1 - \lambda) \sum_{i=1}^{\infty} \int_{\mu^{-2}(v)}^1 1 \left( \mu^{(i+1)}(c) - \mu^{(i)}(c) \right) \, dF(v). \]

Therefore, \( FB(c) \leq \frac{1}{1 - \lambda} \sum_{i=1}^{\infty} \int_{\mu^{-2}(v)}^1 1 \left( \mu^{(i+1)}(c) - \mu^{(i)}(c) \right) \, dF(v). \)

Setting \( \lambda = 0.311 \) gives \( FB \leq 8.23 \cdot \text{max}(\text{SellerP}, \text{BuyerP}). \)

REFERENCES

[1] Moshe Babaioff, Yang Cai, Yannai A. Gonczarowski, and Mengfei Zhao. 2018. The Best of Both Worlds: Asymptotically Efficient Mechanisms with a Guarantee on the Expected Gains-From-Trade. In EC 373. https://doi.org/10.1145/3219166.3219203

[2] Moshe Babaioff, Shahar Dobzinski, and Ron Kupfer. 2021. A Note on the Gains from Trade of the Random-Offerer Mechanism. CoRR abs/2111.07799 (2021). arXiv:2111.07799

[3] Moshe Babaioff, Kira Goldberg, and Yannai A. Gonczarowski. 2020. Buwow-Klemerer-Style Results for Welfare Maximization in Two-Sided Markets. In SODA 2452–2471. https://doi.org/10.1137/1.9781611975994.150

[4] Santiago R. Balseiro, Vahab S. Mirrokni, Renato Pare Eles, and Song Zuo. 2019. Dynamic Double Auctions: towards First Best. In SODA 157–172. https://doi.org/10.1137/1.9781611975482.11

[5] Liad Blumrosen and Shahar Dobzinski. 2016. (Almost) Efficient Mechanisms for Bilateral Trading. CoRR abs/1604.04876 (2016). arXiv:1604.04876

[6] Liad Blumrosen and Yehonatan Mizrahi. 2016. Approximating Gains-From-Trade in Bilateral Trading. In WINE 408–413. https://doi.org/10.1007/978-3-662-54110-4_28

[7] Johannes Brustle, Yang Cai, Fa Wu, and Mengfei Zhao. 2017. Approximating Gains from Trade in Two-sided Markets via Simple Mechanisms. In EC 589–590. https://doi.org/10.1137/1.9781611973274.158

[8] Yang Cai, Kira Goldberg, Steven Ma, and Mengfei Zhao. 2021. On Multi-Dimensional Gains from Trade Maximization. In SODA: 1079–1098. https://doi.org/10.1137/1.9781611975465.67

[9] Nicolò Cesa-Bianchi, Tommaso R. Cesari, Roberto Colomboni, Federico Fusco, and Stefano Leonardi. 2021. A Regret Analysis of Bilateral Trade. In EC 289–309. https://doi.org/10.1137/1.978364563476645

[10] Riccardo Colini-Baldeschi, Bart de Keijzer, Stefano Leonardi, and Stefano Turchetta. 2016. Approximately Efficient Double Auctions with Strong Budget Balance. In SODA 1424–1443. https://doi.org/10.1137/1.9781611974351.ch98

[11] Riccardo Colini-Baldeschi, Paul W. Goldberg, Bart de Keijzer, Stefano Leonardi, Tim Roughgarden, and Stefano Turchetta. 2017. Approximately Efficient Two-Sided Combinatorial Auctions. In EC 591–606. https://doi.org/10.1137/1.9781611973274.28

[12] Riccardo Colini-Baldeschi, Paul W. Goldberg, Bart de Keijzer, Stefano Leonardi, and Stefano Turchetta. 2017. Fixed Price Approximability of the Optimal Gain from Trade In WINE 146–160. https://doi.org/10.1007/978-3-319-79294-5_11

[13] Paul Dütting, Federico Fusco, Philip Lazos, Stefano Leonardi, and Rebecca Reiffenhäuser. 2021. Efficient two-sided markets with limited information. In SOSTOC 1452–1465. https://doi.org/10.1145/3406325.3451076

[14] Zi Yi Kang, Francisco Pernice, and Jan Vondrák. 2022. Fixed-Price Approximations in Bilateral Trade. In SODA 2964–2985. https://doi.org/10.1137/1.9781611977073.115

[15] R.Preston McAfee. 1991. Efficient allocation with continuous quantities. Journal of Economic Theory 50, 1 (1991), 51–74.

[16] R.Preston McAfee. 1992. A dominant strategy double auction. Journal of Economic Theory 56, 2 (1992), 434–450.

[17] R Preston McAfee. 2008. The gains from trade under fixed price mechanisms. Applied economics research bulletin 1, 1 (2008), 1–10.

[18] Roger B. Myerson and Mark A. Satterthwaite. 1983. Efficient mechanisms for bilateral trading. Journal of economic theory 29, 2 (1983), 265–281.