CONSTITUENT QUARK MODEL DESCRIPTION OF
CHARMONIUM PHENOMENOLOGY

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We review the ability of the quark models to describe the phenomenology of the charm meson sector. The spectroscopy and decays of charmonium and open charm mesons are described in a particular quark model and compared with the data and the results of other models existing in the literature. A quite reasonable global description of the heavy meson spectra is reached. A new assignment of the $\psi(4415)$ resonance as a $3D$ state leaving aside the $4S$ state to the $X(4360)$ is tested through the analysis of the resonance structure in $e^+e^-$ exclusive reactions around the $\psi(4415)$ energy region. We make tentative assignments of some of the XYZ mesons.

To elucidate the structure of the $1^+$ $c\bar{s}$ states, i.e. $D_{s1}(2460)$ and $D_{s1}(2536)$, we study the strong decay properties of the $D_{s1}(2536)$ meson. We also perform a calculation of the branching fractions for the semileptonic decays of $B$ and $B_s$ mesons into final states containing orbitally excited charmed and charmed-strange mesons, which have become a very important source of information about the structure of heavy mesons. Analysis of the nonleptonic $B$ meson decays into $D^{(*)}D_{s,J}$ are also included.

Keywords: potential models; Heavy quarkonia; charmed mesons; bottom mesons; models
1. Introduction

After the November revolution in 1974, the next significant step in the understanding of charmonium physics was the starting of the experimental activity at the dawn of the XXI century of the B-factories and the advent of the LHCb.

These machines have produced a huge amount of data which will allow a better understanding of the quarkonium phenomenology. Reviews of the theoretical importance and experimental status of heavy quarkonium have recently been given, among others, by Quigg, Galik, the CERN Quarkonium Working Group, Seth and Swarnicki.

Although the spectrum of charmonium and their transitions have been the subject of a great number of studies, from the seminal paper of Eichten to the more recent results presented by Radford and Repko, there are only few global studies of all the data produced at the B factories. As most of the new states lie above the open charm threshold, the study of its decay properties through the $D^{(*)} \bar{D}^{(*)}$ channels are very important. This implies a consistent description of both, the parent meson and the $D^{(*)}$ mesons involved in the decay. Furthermore, most of the new resonances are produced through a weak process, so that a wealth of information about the new states can be obtained from the study of the weak decay of B mesons.

All these processes have been partially studied in the literature but the aim of this review is to present a coherent description of as many experimental data as possible in an unique framework. We will provide the comparison with the results of other models where there exist.

With this idea, after a short introduction to the model used, we will describe the spectroscopy of the hidden and open charm mesons as $q\bar{q}$ states, discussing whose of the new XYZ states can be assigned to this structure. The description of these states will be complemented with the study of the electromagnetic and strong decays.

One of the outcomes of our calculation is a new quantum number assignment of the $\psi(4415)$ state motivated by the appearance in the $J^{PC} = 1^{--}$ spectrum of the $X(4360)$ state. A detailed study of the reactions $e^+e^- \rightarrow D^0\bar{D}^0\pi^+$ and $e^+e^- \rightarrow D^0\bar{D}^{*0}\pi^+$ has been performed in order to justify our result.

In the same way, a study of the decay properties of the $D_{s1}(2536)^+$ meson has been performed and compare with the data of the BELLE collaboration to give more insight in the structure of the $1^+ c\bar{s}$ states.

Finally, most of the new states have been discovered from the semileptonic and non-leptonic B decays into open charm states. The theoretical analysis of the data, which includes both weak and strong processes, opens an interesting possibility to study the structure of this type of mesons.
The paper is organized as follows. In Sec. 2 we will introduce our constituent quark model, paying special attention to the terms that determine the spectra of heavy mesons. After that, we will present in Sec. 3 the spectrum of hidden-charm and open charm mesons and its electromagnetic decays. Sec. 4 is devoted to the study of strong decays and reactions, whereas in Sec. 6 we perform the study of the B weak decays into open charm states. We end by summarizing the work and giving some conclusions in Sec. 7.

2. Constituent Quark Model

Constituent quark models have a long history starting from the Isgur seminal work (see, for example Refs. [10] and [11]) in which the potential between two massive quarks (constituents) was modeled by a quadratic confinement potential plus a chromomagnetic interaction. This model was successful in explaining the baryon and meson spectra known at that time. In the eighties it was realized that the constituent mass is a consequence of the chiral symmetry breaking in the light quark sector at a momentum scale $\Lambda_{sb}$ greater than the confinement scale $\Lambda_{conf}$.

In the region between the two scales, due to this breaking, the quark propagator gets modified and quarks acquire a dynamical momentum dependent mass. The Lagrangian describing this scenario must contains chiral fields to compensate the mass term. The Goldstone bosons associated to the chiral fields leads to an additional interaction between light quarks. This fact does not affect to the heavy quark sector but is of paramount importance in the molecular picture because the only remaining interaction between the two molecular components, due to its color singlet nature, is the one driven by the Goldstone boson exchanges between the light quarks.

The simplest Lagrangian which contain chiral fields to compensate the mass term can be expressed as

$$\mathcal{L} = \overline{\psi}(i\partial - M(q^2)U^{\gamma_5})\psi$$

where $U^{\gamma_5} = \exp(i\pi^a\lambda^a\gamma_5/f_\pi)$, $\pi^a$ denotes nine pseudoscalar fields ($\eta_0, \bar{\eta}, K_1, \eta_8$) with $i = 1, \ldots, 4$ and $M(q^2)$ is the constituent mass. This constituent quark mass, which vanishes at large momenta and is frozen at low momenta at a value around 300 MeV, can be explicitly obtained from the theory but its theoretical behavior can be simulated by parametrizing $M(q^2) = m_q F(q^2)$ where $m_q \simeq 300$ MeV, and

$$F(q^2) = \left[\frac{\Lambda^2}{\Lambda^2 + q^2}\right]^{1/2}.$$  

The cut-off $\Lambda$ fixes the chiral symmetry breaking scale.

The Goldstone boson field matrix $U^{\gamma_5}$ can be expanded in terms of boson fields,

$$U^{\gamma_5} = 1 + \frac{i}{f_\pi} \gamma_5 \lambda^a \pi^a - \frac{1}{2f_\pi^2} \bar{\pi}^a \pi^a + ...$$

(3)
The first term of the expansion generates the constituent quark mass, while the second gives rise to a one-boson exchange interaction between quarks. The main contribution of the third term comes from the two-pion exchange which has been simulated by means of a scalar exchange potential.

In the heavy quark sector chiral symmetry is explicitly broken and this type of interaction does not act. However it constrains the model parameters through the light meson phenomenology and provides a natural way to incorporate the pion exchange interaction in the molecular dynamics.

Beyond the chiral symmetry breaking scale one expects the dynamics to be governed by QCD perturbative effects. They are taken into account through the one gluon-exchange interaction.

The one-gluon exchange potential is generated from the vertex Lagrangian
\[
\mathcal{L}_{qqg} = i\sqrt{4\pi\alpha_s} \bar{\psi} \gamma_{\mu} G_{\mu}^{\nu} \Lambda^c \psi,
\]
where \(\Lambda^c\) are the SU(3) color matrices and \(G_{\mu}^{\nu}\) is the gluon field. The resulting potential contains central, tensor and spin-orbit contributions given by

\[
\begin{align*}
V_{\text{OGE}}^c(\vec{r}_{ij}) &= \frac{1}{4} \alpha_s \left( \vec{X}_i \cdot \vec{X}_j \right) \left[ \frac{1}{r_{ij}} - \frac{1}{6m_i m_j} (\vec{\sigma}_i \cdot \vec{\sigma}_j) e^{-r_{ij}/r_0(\mu)} \right], \\
V_{\text{OGE}}^T(\vec{r}_{ij}) &= -\frac{1}{16} \frac{\alpha_s}{m_i m_j} \left( \vec{X}_i \cdot \vec{X}_j \right) \left[ \frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}} \left( \frac{1}{r_{ij}^2} + \frac{3}{r_{ij}^2(\mu)} + \frac{1}{r_{ij} r_g(\mu)} \right) \right] S_{ij}, \\
V_{\text{OGE}}^{SO}(\vec{r}_{ij}) &= -\frac{1}{16} \frac{\alpha_s}{m_i^2 m_j^2} \left( \vec{X}_i \cdot \vec{X}_j \right) \left[ \frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}^3} \left( 1 + \frac{r_{ij}}{r_g(\mu)} \right) \right] \times \left[ ((m_i + m_j)^2 + 2m_i m_j)(\vec{S}_+ \cdot \vec{L}) + (m_j^2 - m_i^2)(\vec{S}_- \cdot \vec{L}) \right],
\end{align*}
\]

where \(r_0(\mu_{ij}) = \hat{r}_0(\mu_{ij})\) and \(r_g(\mu_{ij}) = \hat{r}_g(\mu_{ij})\) are regulators. Note that the contact term of the central part of the one-gluon exchange potential has been regularized as follows

\[
\delta(\vec{r}_{ij}) \sim \frac{1}{4\pi r_0^2} \frac{e^{-r_{ij}/r_0}}{r_{ij}}.
\]

To improve the description of mesons with different flavored quarks we include
one-loop corrections to the OGE potential as derived by Gupta et al.\cite{Gupta}

\begin{equation}
\begin{aligned}
V^{1\text{-loop,}C}_{\text{OGE}}(r_{ij}) &= 0, \\
V^{1\text{-loop,}T}_{\text{OGE}}(r_{ij}) &= \frac{C_F}{4\pi} \frac{\alpha_s^2}{m_im_j} \frac{1}{r_{ij}} S_{ij} \left[ \frac{b_0}{2} \left( \ln(\mu r_{ij}) + \gamma_E - \frac{4}{3} \right) + \frac{5}{12} b_0 - \frac{2}{3} C_A \\
&+ \frac{1}{2} \left( C_A + 2C_F - 2C_A \left( \ln(\sqrt{m_im_j} r_{ij}) + \gamma_E - \frac{4}{3} \right) \right) \right], \\
V^{1\text{-loop,SO}}_{\text{OGE}}(r_{ij}) &= \frac{C_F}{4\pi} \frac{\alpha_s^2}{m_im_j} \frac{1}{r_{ij}} \times \\
&\times \left\{ \left( \vec{S}_+ \cdot \vec{L} \right) \left[ (m_i + m_j)^2 + 2m_im_j \left( C_F + C_A - C_A \left( \ln(\sqrt{m_im_j} r_{ij}) + \gamma_E \right) \right) \\
+ 4m_im_j \left( \frac{b_0}{2} \left( \ln(\mu r_{ij}) + \gamma_E \right) - \frac{1}{12} b_0 - \frac{1}{2} C_F - \frac{7}{6} C_A + \frac{C_A}{2} \left( \ln(\sqrt{m_im_j} r_{ij}) + \gamma_E \right) \right) \\
+ \frac{1}{2} (m_i^2 - m_j^2) C_A \ln \left( \frac{m_j}{m_i} \right) \right] \\
+ \left( \vec{S}_- \cdot \vec{L} \right) \left[ (m_j^2 - m_i^2) \left( C_F + C_A - C_A \left( \ln(\sqrt{m_im_j} r_{ij}) + \gamma_E \right) \right) \\
+ \frac{1}{2} (m_i + m_j)^2 C_A \ln \left( \frac{m_j}{m_i} \right) \right] \right\}, \\
\end{aligned}
\end{equation}

where \( C_F = 4/3, C_A = 3, b_0 = 9, \gamma_E = 0.5772 \) and the scale \( \mu \sim 1 \text{ GeV} \).

Although there is no analytical proof, it is a general belief that confinement emerges from the force between the gluon color charges. When two quarks are separated, due to the non-Abelian character of the theory, the gluon fields self-interact forming color strings which bring the quarks together.

In a pure gluon gauge theory the potential energy of the quark-antiquark \( \bar{q}q \) pair grows linearly with the quark-antiquark distance. However, in full QCD the presence of sea quarks may soften the linear potential, due to the screening of the color charges, and eventually leads to the breaking of the string. This characteristic can be translated into a screened potential in such a way that the potential saturates at the same interquark distance.

\begin{equation}
\begin{aligned}
V^{C}_{\text{CON}}(r_{ij}) &= [-a_c(1 - e^{-\mu r_{ij}}) + \Delta] \left( \vec{X}_i \cdot \vec{X}_j \right), \\
V^{SO}_{\text{CON}}(r_{ij}) &= -(\vec{X}_i \cdot \vec{X}_j) \frac{a_c e^{-\mu r_{ij}}}{4m_i^2m_j^2r_{ij}} \times \\
&\times \left[ (m_i^2 + m_j^2)(1 - 2a_s) + 4m_im_j(1 - a_s) \right] \left( \vec{S}_+ \cdot \vec{L} \right) \\
&+ (m_j^2 - m_i^2)(1 - 2a_s) \left( \vec{S}_- \cdot \vec{L} \right),
\end{aligned}
\end{equation}

where \( a_c \) controls the mixture between the scalar and vector Lorentz structures of the confinement. At short distances this potential presents a linear behavior with
an effective confinement strength, \( \sigma = -a_c \mu_c (\vec{X}_i \cdot \vec{X}_j) \), while it becomes constant at large distances. This type of potential shows a threshold defined by

\[
V_{\text{thr}} = \{-a_c + \Delta\} (\vec{X}_i \cdot \vec{X}_j).
\]

(9)

No \( q \bar{q} \) bound states can be found for energies higher than this threshold. The system suffers a transition from a color string configuration between two static color sources into a pair of static mesons due to the breaking of the color string and the most favored decay into hadrons.

Among the different methods to solve the Schrödinger equation and find the quark-antiquark bound states, we use the Gaussian Expansion Method\(^{15}\) because it provides enough accuracy and it makes the subsequent evaluation of the decay amplitude matrix elements easier.

This procedure provides the radial wave function solution of the Schrödinger equation as an expansion in terms of basis functions

\[
R_\alpha (r) = \sum_{n=1}^{n_{\text{max}}} c^\alpha_n \phi_{n}(r),
\]

(10)

where \( \alpha \) refers to the channel quantum numbers. The coefficients, \( c^\alpha_n \), and the eigenvalue, \( E \), are determined from the Rayleigh-Ritz variational principle

\[
\sum_{n=1}^{n_{\text{max}}} \left[ (T^\alpha_{n,n} - EN^\alpha_{n,n}) c^\alpha_n + \sum_{\alpha'} V^{\alpha \alpha'}_{n,n} c^\alpha_{n} c^{\alpha'}_{n} \right] = 0,
\]

(11)

where \( T^\alpha_{n,n} \), \( N^\alpha_{n,n} \) and \( V^{\alpha \alpha'}_{n,n} \) are the matrix elements of the kinetic energy, the normalization and the potential, respectively. \( T^\alpha_{n,n} \) and \( N^\alpha_{n,n} \) are diagonal whereas the mixing between different channels is given by \( V^{\alpha \alpha'}_{n,n} \).

Following Ref.\(^{15}\), we employ Gaussian trial functions with ranges in geometric progression. This enables the optimization of ranges employing a small number of free parameters. Moreover, the geometric progression is dense at short distances, so that it allows the description of the dynamics mediated by short range potentials. The fast damping of the gaussian tail is not a problem, since we can choose the maximal range much longer than the hadronic size.

Table\(^{1}\) shows the model parameters fitted over all meson spectra and relevant for the heavy quark sectors, which have been taken from Ref.\(^{16}\).
Table 1. Model parameters fitted over all meson spectra and relevant for the heavy quark sectors.

| Quark masses | $m_n$ (MeV) | 313 |
|--------------|-------------|-----|
|              | $m_s$ (MeV) | 555 |
|              | $m_c$ (MeV) | 1763 |
|              | $m_b$ (MeV) | 5110 |
| OGE          | $\alpha_0$ | 2.118 |
|              | $\Lambda_0$ (fm$^{-1}$) | 0.113 |
|              | $\mu_0$ (MeV) | 36.976 |
|              | $\tilde{r}_0$ (fm) | 0.181 |
|              | $\tilde{r}_g$ (fm) | 0.259 |
| Confinement  | $a_c$ (MeV) | 507.4 |
|              | $\mu_c$ (fm$^{-1}$) | 0.576 |
|              | $\Delta$ (MeV) | 184.432 |
|              | $a_s$ | 0.81 |

3. Spectroscopy

3.1. Charmonium

Shortly after the discovery by BELLE of the missing $\eta_c'(21S_0)$, new states containing charm quarks have appeared in great profusion. Some of them have been identified as canonical $c\bar{c}$ states, but others, called collectively as XYZ states, exhibit unexpected properties which hardly fit with those of two quark states.

The charmonium spectrum is given in Table 2. We compare our results with the experimental data and with those predicted by other significant quark models in the literature: S. Godfrey and N. Isgur; and D. Ebert, R.N. Faustov and V.O. Galkin. Some tentative XYZ assignments attending to the masses have been done. The experimental masses are taken from Particle Data Group (PDG) for the well established states and from their respective original works for XYZ mesons.

As one can see in Table 2, we obtain a quite reasonable global description of the charmonium sector. This feature is also reached by other quark models. The spectrum predicted by the different models is quite similar at least for the low lying levels. However, while our confining term is based on a screened linear potential at large interquark distances, the remaining models implement a linear potential for all distances. This can be translated into a different prediction of the masses for the higher excited states, the screened linear potential reduces the masses of higher excited states, see Table 2. This has an important consequence, the new assignment of the $\psi(4415)$. Usually this state has been assigned as a $4S$ state. Our particular choice of the potential includes the new $X(4360)$ as a $4S$ state between the well established $\psi(4160)$ and $\psi(4415)$ which are both predicted as $D$-wave
states. Moreover we can assign as $1^{--}c\bar{c}$ structures the new $X(4630)$ and $X(4660)$ mesons whose nature is still unclear.

It is important to remark that a nonrelativistic treatment of the quark-antiquark system is performed in our approach. However, a relativistic scheme is used for the quark models of Refs. 18 and 19. The relativistic effects should be small due to the large mass of the $c$-quark. Therefore, the differences on the spectrum between both schemes are negligible and can be absorbed in the reparametrization of the model.

The $\eta_c(1S)$ is the lowest state of charmonium. The model predicts a mass of 2990 MeV, in good agreement with the experimental one. The splitting between $1^3S_0$ and $1^3S_1$ is given by the Dirac delta term of the OGE potential. This splitting is measured experimentally to be $116.6 \pm 1.2$ MeV which is in reasonable agreement with our prediction of 106 MeV. Moreover, our predicted mass for the $\eta_c(2S)$ is 3643 MeV, which agrees with the experimental value.

Lattice data show a vanishing long-range component of the spin-spin potential. Thus, this part of the potential appears to be entirely dominated by its short-range, delta-like term, suggesting that the $1^3P_1$ should be close to the center-of-gravity of the $3P_J$ system. The precision measurement of the $h_c(1P)$ mass was reported by CLEO in 2008 $25 \pm 12$ MeV. Later, BES III $27 \pm 13$ MeV. The centroid of the $3P_J$ states is known to be $3525.30 \pm 0.04$ MeV $17$ and then the hyperfine splitting is $+0.02 \pm 0.23$ MeV from CLEO and $-0.10 \pm 0.22$ MeV from BES III. The comparison in our model between the centroid of $3P_J$ states and the corresponding $h_c$ mass shows that our spin-spin interaction is negligible for these channels and it is in perfect agreement with the lattice expectations and the experimental measurements for the ground state.

As shown in Table 2, the long known $1^3P_J$ states are in agreement with the model results. The mean $2P$ multiplet mass is predicted to be near 3.95 GeV. Although no $2P$ $c\bar{c}$ state has been clearly seen experimentally, there are reports from the different Collaborations which claim enhancements in that energy region. Among them one can cite the $X(3872)$, $X(3915)$, $Y(3940)$, $X(3940)$ and $Z(3930)$.

The $X(3872)$ mass is difficult to reproduce by the standard quark models, see Table 2. The $X(3872)$ mass is extremely close to the $D^0D^{*0}$ threshold so it appears as a natural candidate to an even $C$-parity $D^0D^{*0}$ molecule. The molecular interpretation will also explain the large isospin violation, but runs into trouble when it tries to explain the high $\gamma\psi'$ decay rate. This puzzling situation suggests for the $X(3872)$ state a combination of a $2P$ $c\bar{c}$ state and a weakly-bound $D^0D^{*0}$ molecule. In Ref. 28 we have performed a coupled channel calculation of the $1^{++}c\bar{c}$ sector including $qq$ and $q\bar{q}q\bar{q}$ configurations. Two and four quark configurations are coupled nonperturbatively using the $3P_0$ model. The elusive $X(3872)$ meson appears as a new state with a high probability for the $DD^*$ molecular component. The original $c\bar{c}(2^3P_1)$ state acquires a sizable $DD^*$ component and can be identified with the $X(3940)$. 
The $Y(3940) \rightarrow \omega J/\psi$ enhancement was initially found by Belle\textsuperscript{29} in $B^+ \rightarrow K^+ Y(3940)$ decays. It was confirmed by BaBar\textsuperscript{30} with more statistics, albeit with somewhat smaller mass. But Belle\textsuperscript{20} also found a statistically compelling resonant structure $X(3915)$ in $\gamma \gamma$ fusion decaying to $\omega J/\psi$. It shares the same production and decay signature as that of BaBar’s $Y(3940)$, which has mass and width consistent with the $X(3915)$. An interpretation of these two states as being the same appears as a widely accepted idea and the name which is conserved is $X(3915)$. We only know at the moment that this state has an even $C$-parity. If $X(3915)$ was a $c \bar{c}$ state, the most probable quantum numbers would be $0^{++}$. The mass predicted for the $2^3 P_0$ is 3909, in very good agreement with the experimental measurement.

In 2005 Belle\textsuperscript{23} observed an enhancement in the $D \bar{D}$ mass spectrum from $e^+ e^- \rightarrow e^+ e^- D \bar{D}$ events with a statistical significance of 5.3$\sigma$. It was initially dubbed the $Z(3930)$, but since then it has been widely (if not universally) accepted as the $\chi_{c2}(2P)$. There is some Lattice calculations\textsuperscript{31} which suggest that the $\chi_{c2}(2P)$ and the $1^3 F_2$ state could be quite close in mass, so that perhaps the $Z(3930)$ is not the $2^3 P_2$ but rather the $1^3 F_2$. Table\textsuperscript{2} shows that all quark models predict a mass splitting between both states of about tens of MeV, so we do not consider that those states are nearby degenerated and assign the $Z(3930)$ as the $2^3 P_0$ state.

The Belle Collaboration has recently reported measurements of $B \rightarrow \chi_{c1} \gamma K$ and $\chi_{c2} \gamma K$\textsuperscript{24} They found evidence of a new resonance in the $\chi_{c1} \gamma$ final state with a mass of $(3823.1 \pm 1.8 \pm 0.7)$ MeV, a value which is consistent with the $1^3 D_2$ $c \bar{c}$ state according to our model, 3812 MeV. We expect that the $1^1 D_2$ state appears in the same energy range of the $X(3823)$, however, as we will see below, this state should appear in the $h c \gamma$ channel.
Table 2. Masses, in MeV, of charmonium states. Some tentative XYZ assignments attending to the masses have been done. The experimental masses are taken from Particle Data Group (PDG) for the well established states and from their respective original works for XYZ mesons. We compare our results (labeled as The.) with those predicted by other significant quark models in the literature: S. Godfrey and N. Isgur and D. Ebert, R.N. Faustov and V.O. Galkin.

| Ref. | Assignment | $J^{PC}$ | nL | The. | Ref. [18] | Ref. [19] | Exp. |
|------|------------|----------|----|------|-----------|-----------|------|
| 17   | $\eta_c(1S)$ | 0$^+$   | 1S | 2990 | 2970 | 2981 | 2981.0 ± 1.1 |
| 17   | $\eta_c(2S)$ | 2S      | 3643 | 3620 | 3635 | 3638.9 ± 1.3 |
|      |            | 3S      | 4054 | 4060 | 3989 | -          |
| 17   | $\chi_{c0}(1P)$ | 0$^{++}$ | 1P | 3452 | 3440 | 3413 | 3414.75 ± 0.31 |
| 20   | $X(3915)$  | 2P      | 3909 | 3920 | 3870 | 3915 ± 3 ± 2 |
|      |            | 3P      | 4242 | -    | 4301 | -          |
| 17   | $h_c(1P)$  | 1$^{-+}$ | 1P | 3515 | 3520 | 3525 | 3525.41 ± 0.16 |
|      |            | 2P      | 3956 | 3960 | 3926 | -          |
|      |            | 3P      | 4278 | -    | 4337 | -          |
| 17   | $J/\psi$   | 1$^{--}$ | 1S | 3096 | 3100 | 3096 | 3096.916 ± 0.011 |
| 17   | $\psi(2S)$ | 2S      | 3703 | 3680 | 3685 | 3686.108 ± 0.018 |
| 17   | $\psi(3770)$ | 1D    | 3796 | 3820 | 3783 | 3778.1 ± 1.2 |
| 17   | $\psi(4040)$ | 3S    | 4097 | 4100 | 4039 | 4039 ± 1 |
| 17   | $\psi(4160)$ | 2D    | 4153 | 4190 | 4150 | 4153 ± 3 |
| 21   | $X(4360)$  | 4S      | 4389 | 4450 | 4427 | 4361 ± 9 ± 9 |
| 21   | $X(4660)$  | 4D      | 4614 | -    | 4837 | 4634$^{+8}_{-7-8}$ |
| 21   | $Z(3823)$  | 2$^{--}$ | 1D | 3810 | 3840 | 3807 | - |
|      |            | 2D      | 4166 | 4210 | 4196 | - |
|      |            | 3D      | 4437 | -    | 4549 | - |

References:
- PDG
- S. Godfrey and N. Isgur
- D. Ebert, R.N. Faustov and V.O. Galkin.
3.2. Charmed and charmed-strange mesons

A simple analysis about the properties of hadrons containing a single heavy quark \( Q = c, b \) can be carried out in the \( m_Q \to \infty \) limit. In such a limit, the heavy quark acts as a static color source for the rest of the hadron, its spin \( \vec{s}_Q \) is decoupled from the total angular momentum of the light degrees of freedom \( \vec{j} = \vec{s}_q + \vec{l} \), and they are separately conserved. Heavy mesons can be organized in doublets, each one corresponding to a particular value of \( j^P \) and parity. The lowest lying \( Q\bar{q} \) mesons correspond to \( l = 0 \) (S-wave states of the quark model) with \( j^P_q = \frac{1}{2}^- \). This doublet comprises two states with spin-parity \( j^P = (0^-, 1^-) \). For \( l = 1 \) (P-wave states of the quark model), it could be either \( j^P_q = \frac{1}{2}^+ \) or \( j^P_q = \frac{3}{2}^+ \), the two corresponding doublets having \( j^P = (1^+, 2^+) \) and \( j^P = (1^+, 2^+) \).

However, the experimental results show intriguing aspects which contradict this analysis, especially in the charm strange sector. The abnormally light mass of the mesons \( D^*_s(2317) \) and \( D_s(2460) \) below the \( DK \) and \( D*K \) thresholds respectively make these states very narrow since the only allowed decays violate isospin. The unexpected feature of these mesons is that they have masses close (or even lower) than their charmed partners. Moreover, the masses predicted by most of the theoretical approaches are considerably heavier than the experimental ones.

Very recently, new \( D \) and \( D_s \) resonances have been discovered. Thus BaBar collaboration\(^{32}\) reported four new resonances: \( D(2550)^0 \), \( D(2600)^0 \), \( D(2750)^0 \) and \( D^*(2760)^0 \). These resonances have been recently confirmed by LHCb collaboration\(^{33}\) adding two more states \( D(3000)^0 \) and \( D^*(3000)^+ \). The results of both collaboration are compatible except in the case of the width of the \( D(2600)^0 \) measured as \( \Gamma = 93 \pm 6 \pm 13 \) by BaBar collaboration and \( \Gamma = 140 \pm 17 \pm 18 \) by LHCb. Concerning the charmed strange sector, three new states have been reported\(^{34}\) \( D^*_s(2710)^+ \), \( D^*_{sJ}(2860)^+ \) and \( D^*_{sJ}(3040)^+ \). These states have been also confirmed by LHCb collaboration\(^{35}\).

The spectra of \( D \) and \( D_s \) are given in Table 3 and Table 4. We compare our results with those predicted by other significant quark models in the literature: D. Ebert, R.N. Faustov and V.O. Galkin\(^{36}\) and Di Pierro and Eichten\(^{37}\). Assignments for the well established states taken from Particle Data Group (PDG)\(^{17}\) are also given.

The masses predicted for the \( 0^- \) and \( 1^- \) states – the \( j^P_q = \frac{1}{2}^- \) doublet – agree with the experimental measurements in both sectors. The doublet \( j^P_q = \frac{3}{2}^+ \), which corresponds to the \( 2^+ \) state and one of the low lying \( 1^+ \) states, is in reasonable agreement with experiment.

As one can see, most of the models cannot reproduce the mass splittings between the \( D^*_s(2317) \), \( D_s(2460) \) and \( D_{s1}(2536) \) mesons. This feature is shared by other quark models, but also by other approaches like lattice QCD calculations\(^{38}\). The charmed and charmed-strange \( 0^+ \) states are sensitive to the one-loop corrections of the OGE potential included in our model which bring their masses closer to experiment. This is in agreement with the conclusion of Ref.\(^ {39}\). However, their
Table 3. Masses, in MeV, of charmed mesons predicted by the constituent quark model. We compare our results with those of other significant quark models in the literature from Refs. [36] and [37]. The experimental data are from the PDG.

| Assignment | $J^P$ | The. | Ref. [36] | Ref. [37] | Exp. |
|------------|-------|------|-----------|-----------|------|
| $D$        | $0^-$ | 1896 | 1871      | 1868      | 1867.7 ± 0.3 |
| $D(2550)$  |       | 2695 | 2581      | 2589      | 2539.4 ± 4.5 ± 6.8 |
|            |       | 3154 | 3062      | 3141      | 2896 ± 0.6 |
| $D^*$      | $1^-$ | 2014 | 2010      | 2005      | 2010.25 ± 0.14 |
|            |       | 2754 | 2632      | 2692      | 2812 ± 0.8 |
|            |       | 2905 | 3096      | 3226      | 3042 ± 1.2 |
| $D_0(2400)$| $0^+$ | 2362 | 2406      | 2377      | 2318 ± 29 |
|            |       | 2925 | 2919      | 2949      | 2953 ± 0.6 |
|            |       | 3292 |           |           | 3042 ± 0.8 |
| $D_1(2420)$| $1^+$ | 2499 | 2426      | 2417      | 2421.4 ± 0.6 |
| $D_1(2430)$|       | 2535 | 2469      | 2426      | 2427 ± 26 ± 25 |
|            |       | 3033 | 2932      | 2995      | 3042 ± 1.2 |
| $D_2(2460)$| $2^+$ | 2544 | 2460      | 2460      | 2462.6 ± 0.6 |
|            |       | 3059 | 3012      | 3035      | 3042 ± 1.2 |
|            | $2^-$ | 2822 | 2806      | 2775      | 2812 ± 0.8 |
|            |       | 2962 | 2850      | 2873      | 3042 ± 0.8 |
|            | $3^+$ | 3094 | 2863      | 2799      | 3042 ± 0.8 |
|            |       | 3240 | 3335      |           | 3042 ± 0.8 |
| $3^-$      | $3^-$ | 2863 | 3129      | 3123      | 3042 ± 0.8 |
|            |       | 3260 | 3145      |           | 3042 ± 0.8 |

The contribution are not enough to solve the puzzle in the $1^+$ sector. The importance of the meson-meson continuum in the $1^+ c\bar{s}$ sector will be studied later.
Table 4. Masses, in MeV, of charmed-strange mesons predicted by the constituent quark model. We compare our results with those of other significant quark models in the literature from Refs. [36] and PhysRevD.64.114004 (2001). The experimental data are from the PDG [17].

| Assignment          | J^P | The. | Ref. [36] | Ref. [37] | Exp.     |
|---------------------|-----|------|-----------|-----------|----------|
| D_s                 | 0^- | 1984 | 1969      | 1965      | 1968.5 ± 0.32 |
|                     |     | 2729 | 2688      | 2750      |          |
|                     |     | 3178 | 3259      |           |          |
| D_s^* (2700)        | 1^- | 2104 | 2111      | 2113      | 2112.3 ± 0.5 |
|                     |     | 2794 | 2731      | 2806      | 2709.0 ± 0.4 |
|                     |     | 2890 | 2913      | 2913      |          |
| D_s^* (2317)        | 0^+ | 2383 | 2509      | 2487      | 2317.8 ± 0.6 |
|                     |     | 2934 | 3067      |           |          |
|                     |     | 3310 |           |           |          |
| D_s (2460)          | 1^+ | 2560 | 2536      | 2535      | 2459.6 ± 0.6 |
|                     |     | 2570 | 2574      | 2605      | 2535.12 ± 0.13 |
|                     |     | 3061 | 3114      |           |          |
| D_s (2526)          | 2^+ | 2609 | 2571      | 2581      | 2571.9 ± 0.8 |
|                     |     | 3094 | 3142      | 3157      |           |
|                     |     | 2888 | 2931      | 2900      |          |
|                     |     | 2943 | 2961      | 2953      |          |
|                     |     | 3151 | 3254      | 3203      |          |
|                     |     | 3215 | 3266      | 3247      |          |
|                     |     | 2922 | 2971      | 2925      |          |
|                     |     | 3304 | 3469      |           |          |

We postpone the assignment of the new states to the strong decay section were the strong width of these meson will be present in detail.

3.3. Electromagnetic decays

The knowledge of the leptonic decay width of higher 1^{--} charmonium states is important for several reasons. First of all it allows to test the wave function at very short distances. Moreover it can help to distinguish between conventional c\bar{c} mesons and multiquark structures which have much smaller dielectron widths [19]. The leptonic widths are compared in Table 5, we include the recent data reported by the BES Collaboration in Ref. [41].

As we have mentioned, one striking feature of our model is the new assignment of the ψ(4415). Usually this state has been assigned as a 4S state. Our particular choice of the potential includes the new X (4360) as a 4S state between the well established ψ(4160) and ψ(4415) which are both predicted as D-wave states. Whether or not
Table 5. Leptonic decay widths, in keV, of $\psi$ states.

| $(nL)$ | State | $M_{\text{The.}}$ (MeV) | $\Gamma_{\text{The.}}$ (keV) | $\Gamma_{\text{Exp.}}$ (keV) |
|--------|-------|--------------------------|-----------------------------|-----------------------------|
| $(1S)$ | $J/\psi$ | 3096 | 3.93 | $5.55 \pm 0.14 \pm 0.02$ |
| $(2S)$ | $\psi(2S)$ | 3703 | 1.78 | $2.33 \pm 0.07$ |
| $(1D)$ | $\psi(3770)$ | 3796 | 0.22 | $0.22 \pm 0.05$ |
| $(3S)$ | $\psi(4040)$ | 4097 | 1.11 | $0.83 \pm 0.20$ |
| $(2D)$ | $\psi(4160)$ | 4153 | 0.30 | $0.48 \pm 0.22$ |
| $(4S)$ | $X(4360)$ | 4389 | 0.78 | - |
| $(3D)$ | $\psi(4415)$ | 4426 | 0.33 | $0.35 \pm 0.12$ |
| $(5S)$ | $X(4630)$ | 4614 | 0.57 | - |
| $(4D)$ | $X(4660)$ | 4641 | 0.31 | - |

Table 6. Branching fraction for the decay $\psi(2S) \rightarrow \gamma(\gamma J/\psi)\chi_{cJ}$. Experimental data are from Ref. 43.

| Mode | $\Gamma_{\text{The.}}$ | $\Gamma_{\text{Exp.}}$ |
|------|-------------------|-------------------|
| $\gamma(\gamma J/\psi)\chi_{c0}$ | 0.156 | $0.125 \pm 0.007 \pm 0.013$ |
| $\gamma(\gamma J/\psi)\chi_{c1}$ | 4.423 | $3.56 \pm 0.03 \pm 0.12$ |
| $\gamma(\gamma J/\psi)\chi_{c2}$ | 2.099 | $1.95 \pm 0.02 \pm 0.07$ |

This assignment is correct can be tested with the $e^+e^-$ leptonic widths. From Table 5 one can see that the width of the $4S$ state is 0.78 keV, whereas the experimental value for the $\psi(4415)$ is $\Gamma_{e^+e^-} = 0.35 \pm 0.12$ keV, in excellent agreement with the result for the $3D$ state (0.33 keV). The measurement of the leptonic width for the $X(4360)$ is very important and would clarify the situation.

It is generally assumed that the $1^{--} c \bar{c}$ mesons are a mixture of $^3S_1$ and $^3D_1$ states in order to reproduce the leptonic widths. In our model the mixing is not fitted to the experimental data but driven by the tensor piece of the quark-antiquark interaction. All are almost pure states either $^3S_1$ or $^3D_1$ and we can reasonably reproduce the leptonic widths.

The study of higher multipole contributions to the radiative transitions between spin-triplet states involves an alternative way to disentangle the mixing between $S$- and $D$-waves in $1^{--} c \bar{c}$ mesons. The radiative decay sequences

$$e^+e^- \rightarrow \psi(2S), \quad \psi(2S) \rightarrow \gamma \chi_{c(1,c2)}, \quad \chi_{c(1,c2)} \rightarrow \gamma J/\psi, \quad J/\psi \rightarrow e^+e^- or \mu^+\mu^-,$$

(12)

has been studied experimentally in Ref. 42. The electric dipole amplitudes are dominant but higher multipole contributions are allowed.
Figure 1. Figure from Ref. [42]. Experimental values of the magnetic quadrupole amplitudes obtained by the CLEO Collaboration and their comparison with previous experimental data and theoretical expectations.

For the $\chi_{cJ}$ ($J = 1, 2$) sequences, they search for two multipole amplitudes $b_{2}^{J=1,2}$ and $a_{2}^{J=1,2}$, where $b$ stands for the amplitude where $\chi_{cJ}$ is a reaction product ($\psi' \rightarrow \gamma \chi_{cJ}$) and $a$ stands for the amplitude where $\chi_{cJ}$ is the decay particle ($\chi_{cJ} \rightarrow \gamma J/\psi$).

We show in Fig. 1 the experimental data (solid circles) obtained by the CLEO Collaboration in Ref. [42]. The rest of the data are previous to Ref. [42]. Our theoretical estimations are represented by a vertical solid line. The same theoretical estimations considering a $c$-quark mass ($m_c = 1.5$ GeV) closer to the PDG value are represented by a vertical dashed line as given in Ref. [42]. The last experimental measurements and the theoretical estimations agree well. In some sense it indicates that the mixing between $S$ and $D$-waves in the $1^{-+} c\bar{c}$ states is small, but also in others as the $2^{++}$ channel where the mixing is between the $P$ and $F$-waves.

To end the above discussion, one can calculate the branching fraction of the process $\psi(2S) \rightarrow J/\psi \gamma \gamma$ through $\gamma \chi_{cJ}$. In Table 6 we compare our results with the most recent experimental data. We reproduce not only the tendency of the experimental data but also the absolute value.

Table 7 shows the $E_1$ radiative decay widths for the first two states of $\eta_{c2}$ and $\psi_2$ that may be useful for experimentalists. The recently reported $X(3823)$ state has been assigned to the $1^{3}D_{2}$ $c\bar{c}$ state. An upper limit of the branching...
Table 7. E1 radiative transitions for the first two states of $\eta_c^2$ and $\psi_2$.

| Initial meson | Final meson | $\Gamma_{CQM}$ (keV) |
|---------------|-------------|---------------------|
| $\eta_c^2(1^1D_2)$ | $h_c(1^1P_1)$ | 276.95 |
| $\eta_c^2(2^1D_2)$ | $h_c(1^1P_1)$ | 114.66 |
| | $h_c(2^1P_1)$ | 211.78 |
| $\psi_2(1^3D_2)$ | $\chi_c(1^3P_1)$ | 224.10 |
| | $\chi_c(1^3P_2)$ | 53.74 |
| $\psi_2(2^3D_2)$ | $\chi_c(1^3P_1)$ | 95.44 |
| | $\chi_c(2^3P_2)$ | 19.92 |
| | $\chi_c(2^3P_1)$ | 164.35 |
| | $\chi_c(2^3P_2)$ | 47.92 |
| | $\chi_c(1^3F_2)$ | 3.88 |

The ratio $B(X(3823) \to \chi_c(1P)\gamma)/B(\chi_c(1P)\gamma) < 0.41$ has been also given by experimentalists. Our value, 0.24, is below that limit and assures our assignment. The reason why the $1^3D_2$ $c\bar{c}$ state has been difficult to observe is that open-flavor decay modes are not allowed. The same situation appears for the $1^1D_2$ state being the E1 radiative decays the most plausible decay channels in which this particle can be observed.

4. Strong Decays

Meson strong decay is a complex nonperturbative process that has not yet been described from first principles of QCD. This leads to a rather poorly understood area of hadronic physics which is a problem because decay widths comprise a large portion of our knowledge of the strong interaction.

Several phenomenological models have been developed to deal with this topic. The most popular is the $^3P_0$ model which assumes that a quark-antiquark pair is created with vacuum quantum numbers, $J^{PC} = 0^{++}$. The $^3P_0$ model was first proposed by Micu. Le Yaouanc et al. applied subsequently this model to meson and baryon open-flavor strong decays in a series of publications in the 1970s.

We calculate in this Section the total decay widths of the mesons which belong to charmed and charmed-strange through a modified version of the $^3P_0$ model with a scale dependent strength $\gamma$ of the decay interaction given by

$$\gamma(\mu) = \frac{\gamma_0}{\log \left( \frac{\mu}{m_\gamma} \right)},$$

(13)

where $\mu$ is the reduced mass of the $q\bar{q}$ pair of the decaying meson and $\gamma_0 = 0.81 \pm 0.02$. 
Table 8. Calculated through the $^3P_0$ model, the strong total decay widths of the mesons which belong to charmed, charmed-strange, hidden charm and hidden bottom sectors. The value of the parameter $\gamma$ in every quark sector is given by Eq. (13).

| Meson | J    | P    | C    | Mass (MeV) | $\Gamma_{\text{Exp.}}$ (MeV) | $\Gamma_{\text{The.}}$ (MeV) |
|-------|------|------|------|------------|-------------------------------|-------------------------------|
| $\psi(3770)$ | 1    | −1   | −1   | 3775.2 ± 1.7 | 27.6 ± 1.0                   | 26.5 ± 1.7                   |
| $\psi(4040)$ | 1    | −1   | −1   | 4039 ± 1     | 80 ± 10                      | 111.2 ± 7.0                  |
| $\psi(4160)$ | 1    | −1   | −1   | 4153 ± 3     | 103 ± 8                      | 115.9 ± 7.3                  |
| $X(4360)$ | 1    | −1   | −1   | 4361 ± 9     | 74 ± 18                      | 113.9 ± 7.2                  |
| $\psi(4415)$ | 1    | −1   | −1   | 4421 ± 4     | 119 ± 14                     | 159.0 ± 10.0                 |
| $X(4640)$ | 1    | −1   | −1   | 4634 ± 8     | 92 ± 52                      | 206.3 ± 13.0                 |
| $X(4660)$ | 1    | −1   | −1   | 4664 ± 11    | 48 ± 15                      | 135.0 ± 8.6                  |

$D^*(2010)^+$ | 1    | −1   | −1   | 2010.25 ± 0.14 | 0.096 ± 0.022               | 0.036 ± 0.003               |
$D^*_1(2400)^+$ | 0    | +1   | −1   | 2403 ± 38     | 283 ± 42                    | 212.0 ± 17.1                |
$D^*_1(2420)^+$ | 1    | +1   | −1   | 2423.4 ± 3.1  | 25 ± 6                      | 25.3 ± 2.0                  |
$D^*_1(2430)^0$ | 1    | +1   | −1   | 2427 ± 36     | 384 ± 150                   | 229.2 ± 18.5                |
$D^*_2(2460)^+$ | 2    | +1   | −1   | 2460.1 ± 4.4  | 37 ± 6                      | 64.1 ± 5.2                  |
$D^*(2550)^0$ | 0    | −1   | −1   | 2539.4 ± 8.2  | 130 ± 18                    | 132.1 ± 10.7                |
$D^*(2600)^0$ | 1    | −1   | −1   | 2608.7 ± 3.5  | 93 ± 14                     | 96.9 ± 7.8                  |
$D^*_1(2750)^0$ | 2    | −1   | −1   | 2752.4 ± 3.2  | 71 ± 13                     | 229.9 ± 18.6                |
$D^*_2(2760)^0$ | 3    | −1   | −1   | 2763.3 ± 3.3  | 60.9 ± 6.2                  | 116.4 ± 9.3                 |

$D^*_1(2536)^+$ | 1    | +1   | −1   | 2535.12 ± 0.25 | 1.03 ± 0.15                 | 0.99 ± 0.07                 |
$D^*_2(2575)^+$ | 2    | +1   | −1   | 2572.6 ± 0.9  | 20 ± 5                      | 18.7 ± 1.3                  |
$D^*_1(2710)^+$ | 1    | −1   | −1   | 2710 ± 14     | 149 ± 65                    | 170.8 ± 12.1                |
$D^*_2(2860)^+$ | 1    | −1   | −1   | 2862 ± 6      | 48 ± 7                      | 153.2 ± 10.9                |
$D^*_3(3040)^+$ | 1    | +1   | −1   | 3044 ± 31     | 239 ± 71                    | 301.5 ± 21.5                |

and $\mu_\gamma = 49.84 ± 2.58$ MeV are parameters determined through the fit to some selected the total decay widths. Reference [47] provides a detailed explanation on how the fit was performed and on the convention for the definition of $\gamma$ (see Eq. (2) of Ref. [47]).

Table 8 shows our results for the total strong decay widths of the mesons which belong to hidden charm, charmed and charmed-strange sectors. In the case of mesons containing a single $c$-quark, we have considered the newly observed charmed mesons $D(2550)$, $D^*(2600)$, $D^*_1(2750)$ and $D^*_1(2760)$, and charmed-strange mesons $D^*_2(2710)$, $D^*_2(2860)$ and $D^*_3(3040)$. We get a quite reasonable global description of the total decay widths. A study of the theoretical uncertainties has been performed. It consists on a montecarlo study of the variation of the total decay widths taking into account the uncertainties of the $\gamma$ parameters in Eq. (13).

The detailed analysis of the decay modes of every resonance is beyond the scope of this report. However, let us comment in more detail each sector discussing briefly...
the most significant aspects.

The results predicted by the $^3P_0$ model for the well established charmed mesons are in good agreement with the experimental data except for one case, the total decay width of the $D^*$ meson. The $D^*$ decays only into $D\pi$ channel via strong interaction and it is assumed that the total decay width is given mainly by this decay mode. However, the disagreement may be due to the very small available phase space which enhances possible effects of the final-state interactions.

With respect to the new states reported by Babar \cite{32}, the $J^P = 0^-$ is the most plausible assignment for the $D(2550)$ meson. The total width predicted by the $^3P_0$ model with this assignment is in very good agreement with the experimental data. The helicity-angle distribution of $D^*(2600)$ is found to be consistent with $J^P = 1^-$. Moreover, its mass makes it the perfect candidate to be the spin partner of the $D(2550)$ meson. The predicted mass is about 100 MeV above the experimental value while our prediction of the total decay width as the $^2S_1$ state agrees with the data Babar data but is in clear disagreement with the LHCb data.

There is a strong discussion in the literature about the possible quantum numbers that could have the mesons $D_{J}(2750)$ and $D_{J}^*(2760)$ providing a wide range of assignments. It is important to take into account the experimental observations about these two mesons reported in Ref. \cite{32} before assigning any quantum number. First, despite that the two mesons are close in mass and their total widths are similar, they are considered different particles. Second, the helicity-angle distribution of both mesons is compatible with an angular momentum between quark and antiquark equal to $L = 2$. Third, the $D_{J}(2750)$ and $D_{J}^*(2760)$ mesons have only been seen in the decay mode $D^*\pi$ and $D\pi$, respectively. And finally, the following branching ratio has been measured

$$\frac{B(D_{J}^*(2760)^0 \rightarrow D^+\pi^-)}{B(D_{J}^*(2760)^0 \rightarrow D^{*+}\pi^-)} = 0.42 \pm 0.05 \pm 0.11.$$ \hspace{1cm} (14)

The assignment of the $D_{J}(2750)$ meson as the $nJ^P = 12^-$ state and the $D_{J}^*(2760)$ meson as the $nJ^P = 13^-$ state seems the most plausible in our model. This assignment agrees with those of Ref. \cite{50}.

The predicted masses are of the order of 100 MeV above the experimental data and we obtain a value of 0.68 for the branching ratio written in Eq. (14). However the predicted widths are in clear disagreement with the experimental ones and the problems with the identification of these two states still remains open.

Our theoretical results are in good agreement with the experimental data in the charmed-strange sector. Two new charmed-strange resonances, $D_{s1}(2710)$ and $D_{sJ}(2860)$, have been observed by the BaBar Collaboration in both $DK$ and $D^*K$ channels \cite{34}. In the $D^*K$ channel, the BaBar Collaboration has also found evidence for the $D_{sJ}(3040)$, but there is no signal of $D_{sJ}(3040)$ in the $DK$ channel. It is commonly believed that the $D_{s1}(2710)$ is the first excitation of the $D_{s1}^*$ meson. With this assignment, the prediction of the $^3P_0$ model is in agreement with the experimental data. In Table \hspace{1cm} we show the total strong decay width of the $D_{sJ}^*(2860)$
as the third excitation of the 1$^-$ meson and as the ground state of the 3$^-$ meson. The comparison between experimental data and our results favors the $n J^P = 1^-1$ assignment. The 2$^P$ multiplet mean mass is predicted in our model to be near the mass of the $D_{sJ}(3040)$ resonance. The only decay mode in which the $D_{sJ}(3040)$ has been seen until now is the $D^*K$, and so the most possible assignment is that the $D_{sJ}(3040)$ meson being the next excitation in the $1^+$ channel. Table 8 shows our prediction of the $D_{sJ}(3040)$ decay width as the $nJ^P = 3^-1$ or $4^-1$ state. Both are large but compatible with the experimental data.

One can see that the general trend of the total decay widths is well reproduced in the $1^--c\bar{c}$ sector. There are two particular cases in which the theoretical results exceed the experimental ones. The first case is the $\psi(4415)$ resonance, where we predict a total width of 159 MeV while the PDG average value is $62\pm20$ MeV. However, one should mention that the experimental data are clustered around two values ($\sim100$ MeV and $\sim50$ MeV) corresponding to lower one to very old measurements. If we compare our result with the recent experimental data reported by Seth et al. ($\Gamma = 119\pm16$ MeV), they are more compatible. The second result which disagrees with the experimental data is the corresponding to the pair of states in the vicinity of 4 GeV. The smallest total width of the $X(4660)$ favors the $4D_1$ option for this state although interference between the two states can be the origin of the disagreement.

4.1. Description of the $D_{s1}(2536)^+$ strong decay properties

Recently, new observables of the $D_{s1}(2536)^+$ have been measured. The BaBar Collaboration has performed a high precision measurement of the $D_{s1}(2536)$ decay width obtaining a value of $(1.03\pm0.05\pm0.12)$ MeV. Furthermore, the Belle Collaboration has reported the first observation of the $D_{s1}(2536)^+\rightarrow D^+\pi^-K^+$ decay measuring the branching fraction

$$\frac{D_{s1}(2536)^+\rightarrow D^+\pi^-K^+}{D_{s1}(2536)^+\rightarrow D^{*+}K^0} = (3.27\pm0.18\pm0.37)\%.$$  \hspace{1cm} (15)

They also measured the ratio of the S-wave amplitude in the $D_{s1}(2536)^+\rightarrow D^{*+}K^0$ decay finding a value of $0.72\pm0.05\pm0.01$.

In order to gain insight into the structure of the $1^+$ charm-strange mesons, we study the reaction $D_{s1}(2536)^+\rightarrow D^+\pi^-K^+$ as well as the angular decomposition of the $D_{s1}(2536)^+\rightarrow D^{*+}K^0$ decay.

In the model described in this work, a tetraquark $c\bar{s}n\bar{n}$ state has been predicted by Vijande et al. in Ref. [52] with quantum numbers $IJ^P = 0^+1$ and mass $M = 2841$ MeV. If this state is present it should be coupled to the $J^P = 1^+ c\bar{s}$ states.

Working in the HQS limit, the $c\bar{s}n\bar{n}$ tetraquark has three different spin states, $|01/2\rangle$, $|11/2\rangle$ and $|13/2\rangle$ where the first index denotes the spin of the $n\bar{n}$ pair and the second the coupling with the $\bar{s}$ spin. Although we use the $^3P_0$ model to calculate
the meson decay widths, a description of the coupling between the $D_s$ meson and the tetraquark based on this model is beyond the scope of the present calculation. However, we use it here to select the dominant couplings and parametrize the vertex as a constant $C_S$. The model assumes that the $n\bar{n}$ pair created is in a $J = 0$ state which means that the $D_s$ states will only couple with the first tetraquark component which has spin $1/2$ for the three light quarks. In the HQS limit the heavy quark is an spectator and the angular momentum of the light quarks has to be conserved so that the tetraquark will only couple to the $c\bar{s} j_q = 1/2$ state.

For that reason we couple the tetraquark structure with the $j_q = 1/2 c\bar{s}$ state. This choice differs from the one performed in Ref. 52 where the tetraquark is only coupled to the $1^P_1$ state and not to the $3^P_1$. However, this choice has several advantages: it has the correct heavy quark limit, it may reproduce the narrow width of the $D_s^+(2536)$ state and it is in agreement with the experimental situation which tells us that the prediction of the heavy quark limit is reasonable for the $j_q = 3/2$ state but not for the $j_q = 1/2$ one.

In this scenario we diagonalize the matrix

$$M = \begin{pmatrix} M_{1P_1} & C_{SO} & \sqrt{\frac{2}{3}} C_S \\ C_{SO} & M_{1P_1} & \sqrt{\frac{1}{3}} C_S \\ \sqrt{\frac{1}{3}} C_S & \sqrt{\frac{1}{3}} C_S & M_{c\bar{n}\bar{n}} \end{pmatrix}, \quad (16)$$

where $M_{1P_1} = 2571.5$ MeV, $M_{1P_1} = 2576.0$ MeV and $M_{c\bar{n}\bar{n}} = 2841$ MeV are the masses of the states without couplings, the $C_{SO} = 19.6$ MeV is the coupling induced by the antisymmetric spin-orbit interaction calculated within the model and $C_S$ is the parameter that gives the coupling between the $j_q = 1/2$ component of the $3^P_1$ and $1^P_1$ states and the tetraquark. The value of the parameter $C_S = 224$ MeV is fitted to the mass of the $D_{s1}(2460)$. We get the three eigenstates shown in Table 9. There we also show the probabilities of the three components for each state and the relative phases between different components. One can see that the $D_{s1}(2460)$ meson has a sizable non-$q\bar{q}$ component whereas the $D_{s1}(2536)$ is almost a pure $q\bar{q}$ state. The presence of non-$q\bar{q}$ degrees of freedom in the $1^+ c\bar{s}$ channel enhances the $j_q = 3/2$ component of the $D_{s1}(2536)$ meson. Moreover, a $1^+$ state with an important component of $c\bar{n}\bar{n}$ tetraquark structure is found at 2973 MeV.

We now calculate the different decay widths for the $D_{s1}(2536)^+$ state of Table 9. As expected, the $D^*K$ decay width is narrow $\Gamma = 0.99$ MeV. As the $DK$ decay is suppressed, the total width would be mainly given by the $D^*K$ channel and is in the order of the experimental value $\Gamma_{exp} = (1.03 \pm 0.05 \pm 0.12)$ MeV measured by BaBar. Of course the value strongly depends on the $3^P_0 \gamma$ strength parameter that has been determined before by a global fit of the total decay widths of heavy mesons. It also depends on the fact that we have only coupled the $1/2$ state with the tetraquark making the remaining state a purer $3/2$ which makes it narrower. If we would include an small coupling between the $3/2$ state and the tetraquark our $D_{s1}(2536)$ will be broader.
Table 9. Masses and probability distributions for the three eigenstates obtained from the coupling of the $D_s$ and tetraquark states. The relative sign to the tetraquark component is also shown.

|   | $M$ (MeV) | $S(3P_1)$ | $P(3P_1)$ | $S(1P_1)$ | $P(1P_1)$ | $S(c\bar{s}n\bar{n})$ | $P(c\bar{s}n\bar{n})$ |
|---|-----------|-----------|-----------|-----------|-----------|----------------|----------------|
|   | 2459 | −         | 55.7      | −         | 18.8      | +              | 25.5            |
|   | 2557 | +         | 27.7      | −         | 72.1      | +              | 0.2             |
|   | 2973 | +         | 16.6      | +         | 9.1       | +              | 74.3            |

Table 10. Width and the three branching ratios defined in the text. The first row shows the experimental data and the second shows our results for the $D_{s1}(2536)$ state given in Table 9. For completeness we give in the last two rows the results for the two $1^+$ $cs$ states predicted by the naive CQM.

|   | $M$ (MeV) | $\Gamma$ (MeV) | $R_1$ | $R_2$ | $R_3$ (%) |
|---|-----------|----------------|-------|-------|-----------|
| Exp. | 1.03 ± 0.05 ± 0.12 | 1.27 ± 0.21 | 0.72 ± 0.05 ± 0.01 | 3.27 ± 0.18 ± 0.37 |
| 2557 | 0.99     | 1.31         | 0.66  | 14.07  |
| 2593 | 190.17   | 1.09         | 1.00  | 13.13  |
| 2554 | 11.24    | 1.11         | 0.97  | 13.19  |

There are two other experimental data that do not depend on the $\gamma$ parameter, namely the branching ratio

$$R_1 = \frac{\Gamma(D_{s1}(2536)^+ \to D^{*0}K^+)}{\Gamma(D_{s1}(2536)^+ \to D^{*+}K^0)} = 1.27 \pm 0.21,$$

and the ratio of $S$-wave over the full width for the $D^{*+}K^0$ decay

$$R_2 = \frac{\Gamma_S(D_{s1}(2536)^+ \to D^{*+}K^0)}{\Gamma(D_{s1}(2536)^+ \to D^{*+}K^0)} = 0.72 \pm 0.05 \pm 0.01.$$

The first branching ratio should be 1 if the isospin symmetry was exact. However, the charge symmetry breaking in the phase space makes it different from this value. The effect is sizable since the $D_{s1}(2536)^+$ is close to the $D^+K$ threshold and for this reason it also depends on the details of the $D_{s1}$ wave function. We get for this ratio the value $R_1 = 1.31$, in good agreement with the experimental one.

Notice that in order to get $R_2$ different from one, we need to have a state with high $j_q = 3/2$ component. In our case we get a value of $R_2 = 0.66$, close to the experimental data. The fact that our result is smaller than the experimental one indicates that the probability of the $j_q = 3/2$ state is high which is in agreement with the fact that we get a narrower state.
Finally we calculate the branching
\[
R_3 = \frac{\Gamma(D_{s1}(2536)^+ \to D^+\pi^-K^+)}{\Gamma(D_{s1}(2536)^+ \to D^{*+}K^0)} = (3.27 \pm 0.18 \pm 0.37)\%.
\]
(19)

As the \(D^+\pi^-\) pair in the final state is the only \(D\pi\) combination that cannot come from a \(D^*\) resonance, we describe the reaction through a virtual \(D^*\) meson since \(M_{D^*0} < M_{D^+} + M_{\pi^-}\). We get \(R_3 = 14.1\%\), a factor 3 to 4 greater than the experimental data. This value seems not to depend on the details of the \(D_{s1}\) wave function.

All these results for the width and the ratios \(R_1\), \(R_2\) and \(R_3\) are summarized in Table 10. We also show, for the sake of completeness, the results for the two \(1^+\) states without coupling to the \(c\bar{s}n\bar{n}\) tetraquark. None of these two states agree with the full set of experimental values.

5. Charmonium resonances in \(e^+e^-\) exclusive reactions around the \(\psi(4415)\) region

The Belle Collaboration has recently performed measurements of the exclusive cross section for the processes \(e^+e^- \to D^0D^-\pi^+\) and \(e^+e^- \to D^0D^{*-}\pi^+\) over the center-of-mass energy range 4.0 GeV to 5.0 GeV. In the first reaction they found a prominent peak in the cross section which is interpreted as the \(\psi(4415)\). From the study of the resonant structure in the \(\psi(4415)\) decay, they conclude that the final channel \(D^0D^-\pi^+\) is reached through the \(DD^*_2(2460)\) intermediate state. Using a relativistic Breit-Wigner function parametrization, they obtain the value of the \(B(\psi(4415) \to DD^*_2(2460)) \times B(DD^*_2(2460) \to D\pi^+)\) product of branching fractions and the mass and width of the \(\psi(4415)\). From the measurement of the \(e^+e^- \to D^0D^{*-}\pi^+\) exclusive cross section reported in Ref. 54 they provide upper limits on the peak cross section for the process \(e^+e^- \to X \to D^0D^{*-}\pi^+\) where \(X\) denotes \(X(4260), X(4360), \psi(4415), X(4630)\) and \(X(4660)\). Although only the value concerning the \(\psi(4415)\) is significant.

We have seen that our assignment of the \(\psi(4415)\) as a \(D\)-wave state leaving the \(4S\) state for the \(X(4360)\) agrees with the last measurements of the leptonic and total decay widths. Now we want to perform a study of the two above reactions to test if our result is also compatible with the measurements of Belle.

We assume the reaction \(e^+e^- \to X \to DD^{(*)}\pi\) and parametrize the cross section using a relativistic Breit-Wigner function including Blatt-Weisskopf corrections. The relativistic Breit-Wigner amplitude for the process \(e^+e^- \to \text{resonance} \to \text{hadronic final state } f\) at center-of-mass energy \(\sqrt{S}\) can be written as
\[
\mathcal{T}_f(\sqrt{S}) = \frac{M_r \sqrt{\Gamma^e \Gamma^f}}{S - M_r^2 + iM_r \Gamma_r} e^{i\delta_r},
\]
(20)
where \(r\) indicates the resonance being studied, \(M_r\) is the nominal mass, \(\Gamma_r\) is the full width, \(\Gamma^e\) is the leptonic width, \(\Gamma^f\) is the hadronic width for the decaying channel \(f\) and \(\delta_r\) is a relative phase.
When there are more than one resonance in the same energy range and we measure the same decay channel, the spin-averaged cross section is a coherent sum of the Breit-Wigner amplitudes for each resonance
\[
\sigma(\sqrt{S}) = \frac{(2J + 1)}{(2S_1 + 1)(2S_2 + 1)} \frac{16\pi}{S} \left| \sum_r \frac{M_r \sqrt{\Gamma_r^e \Gamma_r^f}}{S - M_r^2 + iM_r \Gamma_r} e^{i\delta_r} \right|^2 .
\]  
(21)

Now, we introduce the energy dependence of the widths following Ref. [41]. The angular momentum dominant partial width of a resonance decaying into one channel is given by
\[
\Gamma_f^r(\sqrt{S}) = \hat{\Gamma}_r Z_f^{2L+1} \frac{B_L}{B_L},
\]  
(22)
with \( Z_f \) defined as \( Z_f \equiv \rho P_f \), where \( P_f \) is the decay momentum and \( \rho \) is a free parameter whose value is around the range of the interaction, in the order of a few fermis. The energy-dependent partial wave functions \( B_L(Z_f) \) are given in either Ref. [55] or [56]
\[
\begin{align*}
B_0 &= 1, \\
B_1 &= 1 + Z_f^2, \\
B_2 &= 9 + 3Z_f^2 + Z_f^4, \\
B_3 &= 225 + 45Z_f^2 + 6Z_f^4 + Z_f^6,
\end{align*}
\]  
(23)
and \( \hat{\Gamma}_r \) is related with the partial width at the mass of the resonance, \( \Gamma_0 \), as
\[
\hat{\Gamma}_r = \Gamma_0 \frac{B_L(P_0)}{Z_f^{2L+1}(P_0)}. 
\]  
(24)

Then, our final expressions for the partial and total width are given by
\[
\begin{align*}
\Gamma_f^r(\sqrt{S}) &= \Gamma_0 \frac{Z_f^{2L+1}(P_f) B_L(P_0)}{Z_f^{2L+1}(P_0) B_L(P_f)}, \\
\Gamma_r(\sqrt{S}) &= \frac{2M_r}{M_r + \sqrt{S}} \sum_f \Gamma_f^r(\sqrt{S}),
\end{align*}
\]  
(25)
where the term \( \frac{2M_r}{M_r + \sqrt{S}} \) is a relativistic correction factor [55].

5.1. The process \( e^+e^- \rightarrow D^0D^-\pi^+ \)

This process has been studied by Pakhlova et al. in Ref. [53]. They perform a separate study of the \( e^+e^- \rightarrow DD_0^2(2460) \) and \( e^+e^- \rightarrow D(D\pi)_{non-D_0^2(2460)} \) concluding that the \( e^+e^- \rightarrow D^0D^-\pi^+ \) is dominated by \( X \rightarrow DD_0^2(2460) \).

Assuming \( X \equiv \psi(4415) \) and a relativistic Breit-Wigner function to fit the data, the peak cross section for the process \( e^+e^- \rightarrow X \rightarrow DD_0^2(2460) \) is
\[
\sigma(e^+e^- \rightarrow X \rightarrow DD_0^2(2460)) = (0.74 \pm 0.17 \pm 0.08) \text{nb}.
\]
Table 11. Resonance parameters predicted by our constituent quark model for the \( X(4360) \) and \( \psi(4415) \). The experimental data are taken from Ref. 17 for \( X(4360) \) and Ref. 41 for \( \psi(4415) \).

|          | \( X(4360) \)          | \( \psi(4415) \)          |
|----------|--------------------------|--------------------------|
|          | Theory | Experiment    | Theory | Experiment    |
| Mass (MeV) | 4389  | 4361 ± 9 ± 9   | 4426  | 4415.1 ± 7.9  |
| \( \Gamma_{\text{tot}} \) (MeV) | 113.9 | 74 ± 15 ± 10   | 159.0 | 71.5 ± 19.0   |
| \( \Gamma_{\text{ee}} \) (keV) | 0.78  | -              | 0.33  | 0.35 ± 0.12   |

Using

\[
\sigma(e^+e^- \to X) = \frac{12\pi}{m_X^2} \frac{\Gamma_{\text{ee}}}{\Gamma_{\text{tot}}},
\]

the authors of Ref. 53 estimate \( B(\psi(4415) \to D\bar{D}_2^*(2460)) \times B(\bar{D}_2^*(2460) \to D\pi^+) = (10.5 \pm 2.4 \pm 3.8)\% \) or \( (19.5 \pm 4.5 \pm 9.2)\% \) depending on the different parametrization of the \( \psi(4415) \) resonance (Refs. 17 and 41, respectively).

Furthermore, taken from Ref. 17 the branching fraction for \( \bar{D}_2^*(2460) \to D\pi^+ \), one can estimate \( B(\psi(4415) \to DD_2^*) = 0.47 \) using the resonance parameters of Ref. 17 or 0.86 using those of Ref. 41. Note that there are two final charged states in the calculation of \( B(\bar{D}_2^*(2460) \to D\pi^+) \) and we give the branching fraction of the process \( \psi(4415) \to DD_2^* \) in function of the \( DD_2^* \) state and not in function of the \( D\bar{D}_2^* \) one.

The theoretical calculation of the \( e^+e^- \to D^0D^-\pi^+ \) cross section can be divided in three steps. The first one is the resonance production \( e^+e^- \to X \) which can be given in terms of the leptonic width. The second and third steps are the strong decays \( \psi(4415) \to D\bar{D}_2^*(2460) \) and \( \bar{D}_2^*(2460) \to D\pi^+ \) which can be calculated using the \( ^3P_0 \) model. These two partial widths are involved in the calculation of the \( \Gamma_f \) in Eq. (21) because in the case under study we have

\[
\Gamma_f = \Gamma(X \equiv \psi(4415) \to DD_2^*(2460) \to DD\pi^+) = \Gamma(X \equiv \psi(4415) \to DD\bar{D}_2^*(2460)) \times B(D\bar{D}_2^*(2460) \to D\pi^+).
\]

We show the prediction of our model for the mass, the total width and the leptonic width of the resonance \( \psi(4415) \) in Table 11. First, we calculate the branching fractions

\[
B(D_2^{*+} \to D^0\pi^+) = 0.43 \text{ (Exp.: 0.44 \pm 0.09)},
\]

\[
B(D_2^{*0} \to D^-\pi^+) = 0.43 \text{ (Exp.: 0.47 \pm 0.03)},
\]

which agree with the experimental values of Ref. 17. Furthermore the ratios

\[
R_1 = \frac{\Gamma(D_2^{*+} \to D^0\pi^+)}{\Gamma(D_2^{*0} \to D^0\pi^+)} = 1.81 \text{ (Exp.: 1.9 \pm 1.1 \pm 0.3)},
\]

\[
R_2 = \frac{\Gamma(D_2^{*0} \to D^-\pi^+)}{\Gamma(D_2^{*0} \to D^-\pi^+)} = 1.81 \text{ (Exp.: 1.56 \pm 0.16)},
\]

with the experimental values of Ref. 17.
also agree with the experimental data of Ref. [17].

However, when in a similar way we calculate the \( B(\psi(4415) \rightarrow DD^*_2) \), we obtain 0.15 which clearly disagrees with the estimation of Ref. [53].

Our model prediction for the cross section is shown in panel (a) of Fig. 2. One can see that our result is very far from the experimental data. In order to test if this disagreement is due to the 3\(D\) character of our resonance, we repeat the calculation using the parametrization of Ref. [57] where the \( \psi(4415) \) is described as a 4\(S\) state. Although the result approaches the experimental data, see Fig. 2(b), it still does not describe the full cross section. Certainly, the theoretical results have some uncertainties coming either from the wave functions used in the 3\(P_0\) model or the leptonic width. To minimized these uncertainties we have used in Fig. 2(b) the experimental value for the leptonic width [17]. Using the value \( \Gamma_{e^+e^-} \) predicted by the model of Ref. [57] the result would be a factor \( \sim 3 \) smaller.

Taken into account that the energy window around the nominal \( \psi(4415) \) mass in the experiment of Ref. [53] is \( \pm 100 \text{ MeV} \), we introduce in the calculation the resonance \( X(4360) \) which appears as a 4\(S\) 1:\(-\) \( \bar{c}c \) meson in our model. The predicted
mass, total and leptonic widths are shown in Table I. Panel (c) of Fig. 2 shows how this resonance alone cannot reproduce the data but the interference between the X(4360) and ψ(4415), panel (d) of Fig. 2, produces a remarkable agreement with the data.

Using the interference of the two resonances, the theoretical value for the exclusive cross section \( \sigma(e^+e^- \to D\bar{D}^*_0(2460) \to D^0D^*\pi^+) \) at the \( \psi(4415) \) mass is 0.48 nb, within the error bars of the experimental one: \( (0.62^{+0.14}_{-0.13}) \) nb. Our result indicates that the two resonances are needed to explain the experimental data.

5.2. The process \( e^+e^- \to D^0D^{*-}\pi^+ \)

Using the same philosophy we check the \( e^+e^- \to D^0D^{*-}\pi^+ \) exclusive cross section measured by the Belle Collaboration. The experimental analysis estimates from the amplitude of a relativistic Breit-Wigner function fitted to the data an upper limit of 0.76 nb for the peak cross section at \( E_{cm} = M_{\psi(4415)} \).

We calculate the cross section following the same procedure as before. Again the resonance production \( e^+e^- \to X \) has been calculated and is given in Table I. Now, the second and third steps are the strong decays \( \psi(4415) \to D^{*-}D^{**} \) and \( D^{**} \to D^0\pi^+ \).

The theoretical result for the branching fraction \( B(D^{**} \to D^0\pi^+) \) is 0.687, in very good agreement with the experimental value \( 0.677 \pm 0.006 \) of Ref. 17. For the other branching fraction, \( B(\psi(4415) \to D^*D^*) \), there is no experimental data. Our theoretical result is 0.20.

The calculation of the cross section including the \( \psi(3D) \) resonance with \( M = 4426 \text{ MeV} \) alone does not reproduce the full strength of the resonance at \( E_{cm} = M_{\psi(4415)} \) and the result is improved when the \( X(4360) \) is added. See Fig. 3(b).

From the cross section of Fig. 3(b), we calculate the peak cross section for the \( e^+e^- \to D^0D^{*-}\pi^+ \) process at \( M(D^0D^{*-}\pi^+) = 4415 \text{ MeV} \) obtaining 0.45 nb, which
Figure 4. (a): Model prediction of the reaction $e^+e^- \rightarrow D^0D^{*-}\pi^+$ with the resonances $X(4360)$ and $\psi(4415)$ (dashed line) and including $\psi(5S)$ and $\psi(4D)$ (solid line). (b): Model prediction of the reaction $e^+e^- \rightarrow D^0D^{-}\pi^+$ with the resonances $X(4360)$ and $\psi(4415)$ (dashed line) and including $\psi(5S)$ and $\psi(4D)$ (solid line).

is compatible with the experimental upper limit 0.76 nb at 90% C.L. This result is also compatible with the upper limits measured in Ref. [54] for the branchings $B_{ee} \times B(X \rightarrow D^0D^{*-}\pi^+)$ where $X$ denotes the $X(4360)$ and $\psi(4415)$. We obtain the value $0.25 \times 10^{-6}$ for the $X(4360)$ and $0.35 \times 10^{-6}$ for the $\psi(4415)$. They should be compared with the upper limits $< 0.72 \times 10^{-6}$ and $< 0.99 \times 10^{-6}$, respectively.

The $X(4360)$ resonance has been sometimes assigned as an unconventional charmonium state since it was discovered in the $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ decay [21] and its open-charm decays were assumed to be suppressed. Ref. [54] gives the branching ratio $B(X \rightarrow D^0D^{*-}\pi^+)/B(X \rightarrow \pi^+\pi^-\psi(2S)) < 8$. Since the $X \rightarrow \pi^+\pi^-\psi(2S)$ is an Okubo-Zweig-Iizuka (OZI) suppressed decay the value of this upper limit means that the open-charm $D^{*+}D^{*-}$ decay, where $D^{*+}$ decays into $D^0\pi^+$, should be small. This is actually the case in our model. We get $\Gamma(D^0D^{*-}\pi^+) = \Gamma(X(4360) \rightarrow D^{*+}D^{*-})B(D^{*+} \rightarrow D^0\pi^+) = 3.0 \text{MeV}$ and combined with the experimental information we obtain $\Gamma(X(4360) \rightarrow \psi(2S)\pi^+\pi^-) \gtrsim 375 \text{keV}$ which is in the same order of magnitude that other similar decays. The decay of the $\psi(2S)$ meson into $J/\psi\pi\pi$ has a width of 147 keV according to PDG [17].

Finally, data of Ref. [54] show a bump around 4.6 GeV although data of Ref. [53] do not show this bump. Our model predicts two states $\psi(5S)$ and $\psi(4D)$ in this energy region. The inclusion of these two resonances improves the agreement with the cross section in the bump region, as we can see in panel (a) of Fig. 4. This bump should not clearly appear in the $e^+e^- \rightarrow D^0D^{-}\pi^+$, as one can see in panel (b) of Fig. 4 due to the negligible decay width, predicted by the $^3P_0$ model, of the $\psi(5S)$ and $\psi(4D)$ states into $DD^*_2(2460)$ channel.
6. Weak Decays

B-factories have become an important source of data on heavy hadrons. Bottomonium states decay mainly into BB pairs, and these B mesons decay subsequently into charmed and charmless hadrons via the weak interaction.

To describe theoretically the properties of the mentioned c-quark mesons (conventional or unexpected), one must deal with weak interaction observables which are generally concerned with the semileptonic and nonleptonic decays of b-hadrons. We perform in this Section a study of the semileptonic and nonleptonic B decays into orbitally excited charmed and charmed-strange mesons in order to gain insight on the structure of the charmed mesons.

6.1. Semileptonic B (B_s) decays into D** (D_s**) mesons

Different Collaborations have recently reported semileptonic B decays into orbitally excited charmed mesons providing detailed results of branching fractions. The theoretical analysis of these data, which include both weak and strong decays, offers the possibility for a stringent test of meson models.

The Belle Collaboration reported data\cite{Belle} on the product of branching fractions $\mathcal{B}(B^+ \to D^{**} l^+ \nu_l) \mathcal{B}(D^{**} \to D^{(*)} \pi)$, where, in the usual notation, $l$ stands for a light $e$ or $\mu$ lepton. The $D_0^*(2400)$, $D_1(2430)$, $D_4(2420)$ and $D_2(2460)$ mesons are denoted generically as $D^{**}$, and the $D^*$ and $D$ mesons as $D^{(*)}$.

$D^{**}$ decays are reconstructed in the decay chains $D^{**} \to D^*\pi^\pm$ and $D^{**} \to D\pi^\pm$. In particular, the $D_0^*(2400)$ meson decays only through the $D\pi$ channel, while the $D_1(2430)$ and $D_4(2420)$ mesons only via $D^*\pi$. Both $D\pi$ and $D^*\pi$ channels are opened for $D_2^*(2460)$.

In the case of BaBar data\cite{BaBar}, the branching fractions $\mathcal{B}(D_2^*(2460) \to D^{(*)}\pi)$ include both the $D^*$ and $D$ contributions. As they also provide the ratio $\mathcal{B}_{B_{1/2}^{(*)}}$, we estimate separately the $D^*$ and $D$ contributions.

A similar analysis can be done in the charmed strange sector for the $B_s$ meson semileptonic decays. Here the intermediate states are the orbitally charmed-strange mesons, $D_{s**}$, and the available final channels are $DK$ and $D^*K$. The PDG only reports the value of the following product of branching fractions $\mathcal{B}(B_s^0 \to D_{s1}(2536)^- \mu^+\nu_\mu)\mathcal{B}(D_{s1}(2536)^- \to D^{***} K^0) = 2.4 \pm 0.5\%$ based on their best value for $\mathcal{B}(\bar{b} \to B_s^0)\mathcal{B}(B_s^0 \to D_{s1}(2536)^- \mu^+\nu_\mu)\mathcal{B}(D_{s1}(2536)^- \to D^{***} K^0)$ measured by the D0 Collaboration\cite{D0}.

All these magnitudes can be consistently calculated in the framework of constituent quark models because they can simultaneously account for the hadronic part of the weak process and the strong meson decays. In this context, meson strong decays will be described through the $^3P_0$ model presented before. As for the weak process the matrix elements factorize into a leptonic and a hadronic part. It is the hadronic part that contains the nonperturbative strong interaction effects and we will evaluate it within our constituent quark model. Further details on the semileptonic decay calculation can be found on Refs.\cite{Segovia, Entem, Fernandez}. 

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**References:**

\cite{Belle}, \cite{BaBar}, \cite{D0}, \cite{Segovia}, \cite{Entem}, \cite{Fernandez}
Table 12. Probability distributions and their relative phases for the four states predicted by CQM. In the $1^+$ strange sector the effects of non-$q\bar{q}$ components are included; see text for details.

| State     | $D_0^*(2400)$ | $D_1(2420)$ | $D_1(2430)$ | $D_2^*(2460)$ |
|-----------|----------------|-------------|-------------|---------------|
| $^3P_0$   | +, 1.0000      | -           | -           | -             |
| $^1P_1$   | -              | -           | 0.5903      | -             |
| $^3P_1$   | -              | +           | 0.4097      | -             |
| $^3P_2$   | -              | -           | -           | +, 0.99993    |

| State     | $D_{s0}^*(2317)$ | $D_{s1}(2536)$ | $D_{s1}(2460)$ | $D_{s2}^*(2573)$ |
|-----------|------------------|----------------|----------------|------------------|
| $^3P_0$   | +, 1.0000        | -              | -              | -                |
| $^1P_1$   | -                | -              | 0.7210         | -                |
| $^3P_1$   | -                | +              | 0.2770         | -                |
| $^3P_2$   | -                | -              | -              | +, 0.99991       |

The mesons involved in the reactions have been discussed in previous sections of this work. The most relevant features to take into account here are: we have reached a good description of the singlet- and triplet-spin $S$-wave charmed mesons, $D$ and $D^*$, and charmed-strange mesons, $D_s$ and $D_{s}^*$. We have seen that the interpretation of the $D_{s0}^*(2317)$ as a canonical $c\bar{s}$ state is plausible since its mass goes down to the experimental value when the one-loop QCD corrections to the OGE potential are taken into account. The $D_{s1}(2460)$ meson has an important non-$q\bar{q}$ contribution. The presence of non-$q\bar{q}$ degrees of freedom in the $J^P = 1^+$ charmed-strange meson sector enhances the $j_q = 3/2$ component of the $D_{s1}(2536)$ meson, which is almost a pure $q\bar{q}$ state. Table 12 shows only the $q\bar{q}$ probabilities of the orbitally excited charmed and charmed strange mesons.

Table 13 shows the final results and their comparison with the experimental data in the case of the $B$ semileptonic decays into orbitally excited charmed mesons.

The meson $D_0^*(2400)$ has $J^P = 0^+$ quantum numbers and, therefore, due to parity conservation, it decays only into $D\pi$, so that we have $\mathcal{B}(D_0^*(2400)^0 \rightarrow D\pi) - \mathcal{B}(D_0^*(2400)^+ \rightarrow D\pi)$...
Table 13. Most recent experimental measurements reported by the Belle and BaBar Collaborations and their comparison with our results. $l$ stands for a light $e$ or $\mu$ lepton. The symbol (*) indicates the estimated results from the original data using $B_{D/D^*}$.

| D*$_1$(2400) | | |
|---|---|---|
| $B(B^+ \rightarrow D^*_1(2400)^0\bar{\nu}_\tau)B(D^*_1(2400)^0 \rightarrow D^-\pi^+)$ | $2.4 \pm 0.4 \pm 0.6$ | $2.6 \pm 0.5 \pm 0.4$ |
| $B(B^0 \rightarrow D^*_1(2400)^-\tau^+\nu)B(D^*_1(2400)^+ \rightarrow D^0\pi^-)$ | $2.0 \pm 0.7 \pm 0.5$ | $4.4 \pm 0.8 \pm 0.6$ |

| D$_1$(2430) | | |
|---|---|---|
| $B(B^+ \rightarrow \bar{D}_1(2430)^0\bar{\nu}_\tau)B(\bar{D}_1(2430)^0 \rightarrow D^-\pi^+)$ | < 0.7 | $2.7 \pm 0.4 \pm 0.5$ |
| $B(B^0 \rightarrow D_1(2430)^-\tau^+\nu)B(D_1(2430)^+ \rightarrow D^0\pi^-)$ | < 5 | $3.1 \pm 0.7 \pm 0.5$ |

| D$_1$(2420) | | |
|---|---|---|
| $B(B^+ \rightarrow D_1(2420)^0\bar{\nu}_\tau)B(D_1(2420)^0 \rightarrow D^-\pi^+)$ | $4.2 \pm 0.7 \pm 0.7$ | $2.97 \pm 0.17 \pm 0.17$ |
| $B(B^0 \rightarrow D_1(2420)^-\tau^+\nu)B(D_1(2420)^+ \rightarrow D^0\pi^-)$ | $5.4 \pm 1.9 \pm 0.9$ | $2.78 \pm 0.24 \pm 0.25$ |

| D$_2$(2460) | | |
|---|---|---|
| $B(B^+ \rightarrow \bar{D}_2(2460)^0\bar{\nu}_\tau)B(\bar{D}_2(2460)^0 \rightarrow D^-\pi^+)$ | $2.2 \pm 0.3 \pm 0.4$ | $1.4 \pm 0.2 \pm 0.2^{(*)}$ |
| $B(B^+ \rightarrow D_2^+(2460)^0\bar{\nu}_\tau)B(D_2^+(2460)^0 \rightarrow D^-\pi^+)$ | $1.8 \pm 0.6 \pm 0.3$ | $0.9 \pm 0.2 \pm 0.2^{(*)}$ |
| $B(B^+ \rightarrow D_2^+(2460)^0\bar{\nu}_\tau)B(D_2^+(2460)^0 \rightarrow D^+\pi^-)$ | $4.0 \pm 0.7 \pm 0.5$ | $2.3 \pm 0.2 \pm 0.2$ |
| $B(B^0 \rightarrow D_2^+(2460)^0\tau^+\nu)B(D_2^+(2460)^+ \rightarrow D^0\pi^-)$ | $2.2 \pm 0.4 \pm 0.4$ | $1.1 \pm 0.2 \pm 0.1^{(*)}$ |
| $B(B^0 \rightarrow D_2^+(2460)^0\tau^+\nu)B(D_2^+(2460)^+ \rightarrow D^0\pi^-)$ | < 3 | $0.7 \pm 0.2 \pm 0.1^{(*)}$ |
| $B(B^0 \rightarrow D_2^+(2460)^0\tau^+\nu)B(D_2^+(2460)^+ \rightarrow D^0\pi^-)$ | < 5.2 | $1.8 \pm 0.3 \pm 0.1$ |

$B_{D/D^*}$ | $0.55 \pm 0.03$ | $0.62 \pm 0.03 \pm 0.02$ |

$D^-\pi^+ = B(D_0^-(2400)^- \rightarrow D^0\pi^-) = 2/3$ coming from isospin symmetry. One can see in Table 13 that the theoretical product of branching fractions agrees well with the latest BaBar data. The difference between the semileptonic width of the charged and neutral $B$ mesons is due to the large mass difference between the $D_0^*(2400)^0$ and $D_0^*(2400)^\pm$ mesons for which we take the masses reported in Ref. [17].

The only OZI-allowed decay channel for the $D_1(2430)$ meson is the $D_1(2430) \rightarrow D^*\pi$ so that isospin symmetry predicts a branching fraction $B(D_1(2430) \rightarrow D^*\pi^\pm) = 2/3$. We have in this case for the product of branching fractions shown in Table 13 that our value is roughly a factor of 2 smaller than the results from the BaBar Collaboration. Note however the disagreement between BaBar and Belle data for the product of branching fractions in which the $\bar{D}_1(2430)^0$ meson is involved.

As in the previous case, the branching fraction $B(D_1(2420) \rightarrow D^*\pi^\pm)$ is again 2/3 in our model because $D_1(2420) \rightarrow D^*\pi$ is the only OZI-allowed decay channel. The products of branching fractions compare very well with the latest BaBar data.
as seen in Table 13. The strong decays which appear in the decay chains that involve the $D_s^*(2460)$ meson are $D_s^*(2460) \rightarrow D\pi$ and $D_s^*(2460) \rightarrow D^*\pi$. In Table 14 we show the strong decay branching ratios obtained with the $^3P_0$ model. They are in good agreement with experimental data. Considering that the total width of the $D_s^*(2460)$ meson is the sum of the partial widths of $D\pi$ and $D^*\pi$ channels, since these are the only OZI-allowed processes, we are able to predict the products of branching fractions in Table 13 which are in very good agreement with BaBar data.

The semileptonic decays of $B_s$ meson into orbitally excited charmed-strange mesons ($D_s^*$) provides an extra opportunity to get more insight into the structure of these latter mesons.

We have calculated the semileptonic $B_s$ decays assuming that the $D_s^*$ mesons are pure $q\bar{q}$ systems. For the $D_{s0}^*(2317)$ and $D_{s1}(2460)$, which are below the corresponding $DK$ and $D^*K$ thresholds, we only quote the weak decay branching fractions. Concerning the $D_{s1}(2460)$, the $^1P_1$ and $^3P_1$ probabilities change with the coupling to non-$q\bar{q}$ degrees of freedom. What we do here is to vary these probabilities (including the phase) in order to obtain the limits of the decay width in the case of the $D_{s1}(2460)$ being a pure $q\bar{q}$ state, see Fig. 5. Assuming that non-$q\bar{q}$ components will give a small contribution to the weak decay, experimental results lower than these limits will be an indication of a more complex structure for this meson.

For the decay into $D_{s1}(2536)$, our model predicts the weak decay branching fraction $\mathcal{B}(B_s^0 \rightarrow D_{s1}(2536)\mu^+\nu_\mu) = 4.77 \times 10^{-3}$ and the strong branching fraction $\mathcal{B}(D_{s1}(2536)^- \rightarrow D^{*-}K^0) = 0.43$. The final result appears in Table 15. It is compatible with the existing experimental data which to us is a confirmation of our result about the $q\bar{q}$ nature of this state.

In the case of the $D_{s2}^*(2573)$ meson the open strong decays are $DK$ and $D^*K$, so the experimental measurements must be referred to $\mathcal{B}(B_s^0 \rightarrow D_{s2}^*(2573)^-\mu^+\nu_\mu)$ $\mathcal{B}(D_{s2}^*(2573)^- \rightarrow D^*K^0)$ and $\mathcal{B}(B_s^0 \rightarrow D_{s2}^*(2573)^-\mu^+\nu_\mu)\mathcal{B}(D_{s2}^*(2573)^- \rightarrow D^*K^0)$. For the weak branching fraction we get in this case $\mathcal{B}(B_s^0 \rightarrow D_{s2}^*(2573)^-\mu^+\nu_\mu) = 3.76 \times 10^{-3}$. For the strong decay part of the reaction, we

| Branching ratio | Theory | Experiment |
|-----------------|--------|------------|
| $\Gamma(D_s^{*+} \rightarrow D^0\pi^+)/\Gamma(D_s^{*+} \rightarrow D^{*0}\pi^+)$ | 1.80 | $1.9 \pm 1.1 \pm 0.3$ |
| $\Gamma(D_s^{*0} \rightarrow D^+\pi^-)/\Gamma(D_s^{*0} \rightarrow D^{*+}\pi^-)$ | 1.82 | $1.56 \pm 0.16$ |
| $\Gamma(D_s^{*} \rightarrow D\pi)/\Gamma(D_s^{*} \rightarrow D^{(*)}\pi)$ | 0.65 | $0.62 \pm 0.03 \pm 0.02$ |
Figure 5. Decay width for the $B_0^s \to D_s(2460)^- \mu^+ \nu_\mu$ decay as a function of the $^1P_1$ component probability. The sign reflects the relative phase between $^1P_1$ and $^3P_1$ components: $-1$ opposite phase and $+1$ same phase.

Table 15. Our predictions and their comparison with the available experimental data for semileptonic $B_s$ decays into orbitally excited charmed-strange mesons.

| Decay | Experiment $(\times 10^{-3})$ | Theory $(\times 10^{-3})$ |
|-------|-------------------------------|-----------------------------|
| $D^{*0}_{s0}(2317)$ | $\mathcal{B}(B_0^s \to D^{*0}_{s0}(2317)^- \mu^+ \nu_\mu)$ | $4.43$ |
| $D_{s1}(2460)$ | $\mathcal{B}(B_0^s \to D_{s1}(2460)^- \mu^+ \nu_\mu)$ | $1.74 - 5.70$ |
| $D_{s1}(2536)$ | $\mathcal{B}(B_0^s \to D_{s1}(2536)^- \mu^+ \nu_\mu) \mathcal{B}(D_{s1}(2536)^- \to D^{*-} \bar{K}^0)$ | $2.4 \pm 0.4^{[15]}$ |
| $D^{*+}_{s2}(2573)$ | $\mathcal{B}(B_0^s \to D^{*+}_{s2}(2573)^- \mu^+ \nu_\mu) \mathcal{B}(D^{*+}_{s2}(2573)^- \to D^{*-} \bar{K}^0)$ | $1.70$ |
| | $\mathcal{B}(B_0^s \to D^{*+}_{s2}(2573)^- \mu^+ \nu_\mu) \mathcal{B}(D^{*+}_{s2}(2573)^- \to D^{(*)-} \bar{K}^0)$ | $0.18$ |
| | $\mathcal{B}(B_0^s \to D^{*+}_{s2}(2573)^- \mu^+ \nu_\mu) \mathcal{B}(D^{*+}_{s2}(2573)^- \to D^{(*)-} \bar{K}^0)$ | $1.88$ |

obtain

$$\mathcal{B}(D^{*-}_{s2} \to D^- \bar{K}^0) = 0.45$$
$$\mathcal{B}(D^{*+}_{s2} \to D^{*-} \bar{K}^0) = 0.047$$

(29)
using the $^3P_0$ model. Besides we predict the ratio
\[
\frac{\Gamma(D_{s2}^- \to DK)}{\Gamma(D_{s2}^- \to DK) + \Gamma(D_{s2}^- \to D^*K)} = 0.91.
\] (30)

Our final results can be seen in Table 15.

6.2. Nonleptonic $B$ decays into $D^{(*)}D_sJ$ final states

The nonleptonic decays of $B$ mesons have been used to search for new charmonium and charmed-strange mesons and to study their properties in detail. For instance, the properties of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons were not well known until the Belle Collaboration observed the $B \to \bar{D}D_{s0}^*(2317)$ and $B \to \bar{D}D_{s1}(2460)$ decays.

First observations of the $B \to \bar{D}D_{s1}(2536)$ decay modes have been reported by BaBar and an upper limit on the decay $B^0 \to D^{*-}D_{s1}(2536)^+$ was also obtained by Belle. The most recent analysis of the production of $D_{s1}(2536)^+$ in double charmed $B$ meson decays has been reported by the Belle Collaboration in Ref. 69. Using the latest measurements of the $B \to D^{(*)}D_{sJ}$ branching fractions, they calculated the ratios
\[
R_{D_{s0}} = \frac{\mathcal{B}(B \to DD_{s0}^*(2317))}{\mathcal{B}(B \to DD_s)} = 0.10 \pm 0.03,
\]
\[
R_{D_{s0}^*} = \frac{\mathcal{B}(B \to D^{*}D_{s0}^*(2317))}{\mathcal{B}(B \to D^{*}D_s)} = 0.15 \pm 0.06,
\]
\[
R_{D_{s1}} = \frac{\mathcal{B}(B \to DD_{s1}(2460))}{\mathcal{B}(B \to DD_s^*)} = 0.44 \pm 0.11,
\]
\[
R_{D_{s1}^*} = \frac{\mathcal{B}(B \to D^{*}D_{s1}(2460))}{\mathcal{B}(B \to D^{*}D_s^*)} = 0.58 \pm 0.12.
\] (31)

In addition, the same ratios were calculated for $B \to D^{(*)}D_{s1}(2536)^+$ decays using combined results by the BaBar and Belle Collaborations
\[
R_{D_{s1}V} = \frac{\mathcal{B}(B \to DD_{s1}(2536))}{\mathcal{B}(B \to DD_s^*)} = 0.049 \pm 0.010,
\]
\[
R_{D_{s1}V^*} = \frac{\mathcal{B}(B \to D^{*}D_{s1}(2536))}{\mathcal{B}(B \to D^{*}D_s^*)} = 0.044 \pm 0.010.
\] (32)

From a theoretical point of view, this kind of decays can be described using the factorization approximation. This amounts to evaluate the matrix element which describes the $B \to D^{(*)}D_{sJ}$ weak decay process as a product of two matrix elements, the first one to describe the $B$ weak transition into the $D^{(*)}$ meson and the second one for the weak creation of the $c \bar{s}$ pair which makes the $D_{sJ}$ meson. The latter matrix element is proportional to the corresponding $D_{sJ}$ meson decay constant.

The $D_{sJ}$ meson decay constants are not known experimentally except for the ground state, $D_s$, which has been measured by different Collaborations. Another way to study $D_{sJ}$ mesons, that does not rely on the knowledge of their decay
constants, is through the decays \( B_s \rightarrow D_{sJ}M \), where \( M \) is a meson with a well known decay constant. However, the experimental study of these processes is currently difficult for several reasons. First, \( B \)-factories would need to collect data at the \( \Upsilon(5S) \) resonance. Second, the kinematically clean environment of \( B \) meson decays does not hold in \( B_s \) decays. And finally, the fraction of events with a pair of \( B_s \) mesons over the total number of events with a pair of \( b \)-flavored hadrons has been measured to be relatively small, \( f_s[\Upsilon(5S)] = 0.193 \pm 0.029 \). These difficulties leave, for the time being, the \( B \rightarrow D(\ast)D_{sJ} \) decay processes as our best option to study \( D_{sJ} \) meson properties.

According to Refs. 70 and 71, within the factorization approximation and in the heavy quark limit, the ratios in Eqs. (31) and (32) can be written as

\[
R_{D0} = R_{D^*0} = \frac{|f_{D_{s0}^{*}(2317)}|}{|f_{D_s}|}^2,
\]

\[
R_{D1} = R_{D^*1} = \frac{|f_{D_{s1}^{*}(2460)}|}{|f_{D_s^*}|}^2,
\]

\[
R_{D1'} = R_{D^*1'} = \frac{|f_{D_{s1}^{*}(2536)}|}{|f_{D_s^*}|}^2,
\]

where the phase space effects are neglected because they are subleading in the heavy quark expansion. Now, in the heavy quark limit one has \( f_{D_{s0}^{*}(2317)}^{\ast} = f_{D_{s1}^{*}(2460)} \), \( f_{D_s} = f_{D_s^*} \) and \( f_{D_{s1}^{*}(2536)} = 0 \). Moreover, there are several estimates of the decay constants, always in the heavy quark limit, \( f_{D_{s0}^{*}(2317)} \) that predict for \( P \)-wave, \( j_q = 1/2 \) states similar decay constants as for the ground state mesons (i.e. \( f_{D_{s0}^{*}(2317)} = f_{D_s} \) and \( f_{D_{s1}^{*}(2460)} = f_{D_s^*} \)), and very small decay constants for \( P \)-wave, \( j_q = 3/2 \) states. Thus, in the heavy quark limit one would expect \( R_{D0} \sim 1 \), \( R_{D1} \sim 1 \) and \( R_{D1'} \ll 1 \). While the decay into \( D_{s1}(2536) \) follows the expectations, this is not the case for the \( D_{s0}^{*}(2317) \) and \( D_{s1}(2460) \) mesons. This fact has motivated to argue that either those two states are not canonical \( c\bar{s} \) mesons or that the factorization approximation does not hold for decays involving those particles.

Leaving aside that the factorization approximation has been recently analyzed in Refs. 75, 76, 77 finding that it works well in this kind of processes, we will concentrate in the influence of the effect of the finite \( c \)-quark mass in the theoretical predictions. As found in Ref. 78, \( 1/m_Q \) contributions give large corrections to various quantities describing \( B \rightarrow D^{**} \) transitions and we expect they also play an important role in this case. It is possible that taking into account the finite mass of the charmed quark one can distinguish better between \( q\bar{q} \) and non-\( q\bar{q} \) structures for the \( D_{sJ} \) mesons.

The nonleptonic decay width for \( B \rightarrow D(\ast)D_{sJ} \) processes in the factorization approximation and using helicity formalism \( 62, 63 \) is given in Ref. 79. Using...
experimental masses we obtain the ratios

\[
R_{D^0} = 0.90 \times \left| \frac{f_{D^0_{s(2317)}}}{f_{D^0}} \right|^2,
\]

\[
R_{D^{*0}} = 0.72 \times \left| \frac{f_{D^0_{s(2317)}}}{f_{D^0}} \right|^2.
\]

(34)

The double ratio \(R_{D^*0}/R_{D0}\) does not depend on decay constants, and in our model we obtain \(R_{D^*0}/R_{D0} = 0.80\). The experimental value is given by \(R_{D^*0}/R_{D0} = 1.50 \pm 0.75\). Our result is small compared to the central experimental value but we are compatible within 1\(\sigma\). In the case of the meson \(D_{s1}(2460)\) we obtain

\[
R_{D^1} = 0.70 \times \left| \frac{f_{D_{s1}(2460)}}{f_{D^1}} \right|^2,
\]

\[
R_{D^{*1}} = 1.00 \times \left| \frac{f_{D_{s1}(2460)}}{f_{D^1}} \right|^2.
\]

(35)

and for the double ratio \(R_{D^{*1}}/R_{D1}\) we get 1.43, which agrees well with the experimental result \(R_{D^{*1}}/R_{D1} = 1.32 \pm 0.43\). Finally, for the meson \(D_{s1}(2536)\) we obtain

\[
R_{D^{1'}} = 0.64 \times \left| \frac{f_{D_{s1}(2536)}}{f_{D^1}} \right|^2,
\]

\[
R_{D^{*1'}} = 0.99 \times \left| \frac{f_{D_{s1}(2536)}}{f_{D^1}} \right|^2.
\]

(36)

and for the double ratio \(R_{D^{*1'}}/R_{D^{1'}}\), our value is 1.56 which in this case is 2\(\sigma\) above the experimental one, 0.90 \(\pm\) 0.27.

The quality of the experimental numbers does not allow to be very conclusive as to the goodness of the factorization approximation. But one can conclude from Eqs. (34), (35) and (36) that the phase-space and weak matrix element corrections cannot be ignored, as done when using the infinite heavy quark mass limit.

The decay constants of pseudoscalar and vector mesons in charmed and charmed-strange sectors are given in Table 16. We compare our results with the experimental data and those predicted by different approaches and collected in Refs. 17, 80. Our original values are those with the symbol (†). The decay constants of vector mesons agree with other approaches. In the case of the pseudoscalar mesons, the decay constants are simply too large. The reason for that is the following: Our CQM presents an OGE potential which has a spin-spin contact hyperfine interaction that is proportional to a Dirac delta function, conveniently regularized, at the origin. The corresponding regularization parameter was fitted to determine the hyperfine splittings between the \(^nS_0\) and \(^nS_1\) states in the different flavor sectors, achieving a good agreement in all of them. While most of the physical observables are insensitive to the regularization of this delta term, those related with annihilation processes are affected. The effect is very small in the \(^3S_1\) channel as
the delta term is repulsive in this case. It is negligible for higher partial waves due to the centrifugal barrier. However, it is sizable in the $^1S_0$ channel for which the delta term is attractive.

One expects that the wave functions of the $^1S_0$ and $^3S_1$ states are very similar. In fact, they are equal if the Dirac delta term is ignored. The values with the symbol (‡) in Table 16 are referred to the pseudoscalar decay constants which have been calculated using the wave function of the corresponding $^3S_1$ state. We recover the agreement with experiment and also with the predictions of different theoretical approaches. The $f_{D^*}/f_D$ and $f_{D^*/f_D}$ ratios are also shown in the last column of Table 16. They are not very sensitive to the delta term and our values agree nicely with experiment and the values obtained in other approaches.

Table 17 summarizes the remaining decay constants needed for the calculation we are interested in. There, we show the results from the constituent quark model in which the 1-loop QCD corrections to the OGE potential and the presence of
Table 17. Decay constants calculated within the CQM including one-loop QCD corrections to the OGE potential and a non-$q\bar{q}$ structure in channel $1^+$. 

| Meson          | $f_D$ (MeV) | $\sqrt{M_D f_D}$ (GeV$^{3/2}$) |
|----------------|-------------|--------------------------------|
| $D_{s0}^*(2317)$ | 118.706     | 0.181                          |
| $D_{s1}(2460)$   | 165.097     | 0.259                          |
| $D_{s1}(2536)$   | 59.176      | 0.094                          |

Table 18. Ratios of branching fractions for nonleptonic decays $B \rightarrow D^{(*)} D_{sJ}$. The symbol $(*)$ indicates that the ratios have been calculated using the experimental pseudoscalar decay constant in Table 16. For the $D_{s1}(2460)$ and $D_{s1}(2536)$ mesons, the ratios have been calculated without (1) and with (2) taking into account the non-$q\bar{q}$ degrees of freedom in the $J^P = 1^+$ channel.

| $X \equiv D_{s0}^{(*)}(2317)$ | $X \equiv D_{s1}(2460)$ | $X \equiv D_{s1}(2536)$ |
|--------------------------------|-------------------------|-------------------------|
| The. | Exp. | The. | Exp. | The. | Exp. |
| $B(B \rightarrow DX)/B(B \rightarrow D D_{s0})$ | 0.19$^{(*)}$ | 0.10$^{(*)}$ ± 0.03 | - | - | - |
| $B(B \rightarrow D^*X)/B(B \rightarrow D^* D_{s1})$ | 0.15$^{(*)}$ | 0.15$^{(*)}$ ± 0.06 | - | - | - |
| $B(B \rightarrow DX)/B(B \rightarrow D D_{s1})$ | 0.15$^{(*)}$ | 0.15$^{(*)}$ ± 0.06 | - | - | - |
| $B(B \rightarrow D^*X)/B(B \rightarrow D^* D_{s1})$ | 0.15$^{(*)}$ | 0.15$^{(*)}$ ± 0.06 | - | - | - |
| $B(B \rightarrow D X)/B(B \rightarrow D D_{s1})$ | 0.15$^{(*)}$ | 0.15$^{(*)}$ ± 0.06 | - | - | - |
| $B(B \rightarrow D^* X)/B(B \rightarrow D^* D_{s1})$ | 0.15$^{(*)}$ | 0.15$^{(*)}$ ± 0.06 | - | - | - |

non-$q\bar{q}$ degrees of freedom in $J^P = 1^+$ charmed-strange meson sector are included. If one compares $f_{D_s}$ ($f_{D_s}$) to $f_{D_{s0}^*(2317)}$ ($f_{D_{s1}(2460)}$), one finds that the latter is suppressed.

Our results for the decay constants clearly deviate from the ones obtained in the infinite heavy quark mass limit. In that limit one gets $f_{D_{s0}^*(2317)} = f_{D_s}$, $f_{D_{s1}(2460)} = f_{D_s}$ and $f_{D_{s1}(2536)} = 0$, results that lead to a strong disagreement with experiment for the decay width ratios in Eqs. (31) and (32). That was already noticed in Ref. [71] where the authors, using the experimental ratios, estimated that $f_{D_{s0}^*(2317)} \sim \frac{1}{3} f_{D_s}$ and $f_{D_{s0}^*(2317)} \sim f_{D_{s1}(2460)}$ instead. We obtain $f_{D_{s0}^*(2317)}/f_{D_s} = 0.36$, $f_{D_{s0}^*(2317)}/f_{D_{s1}(2460)} = 0.72 f_{D_{s1}(2460)}$ and $f_{D_{s1}(2536)} = 59.176$ MeV, the latter being small compared to the others but certainly not zero.

Finally, we show in Table 18 our results for the ratios written in Eqs. (31) and (32). The symbol $(*)$ indicates that the ratios have been calculated using the experimental pseudoscalar decay constant in Table 16. We get results close to or within the experimental error bars for the $D_{s0}^*(2317)$ meson, which to us is an indication that this meson could be a canonical $c\bar{s}$ state. The incorporation of the non-$q\bar{q}$ degrees of freedom in the $J^P = 1^+$ channel, enhances the $f_q = 3/2$ component of the $D_{s1}(2536)$ meson and it gives rise to ratios in better agreement.
with experiment. Note that the $D_{s1}(2536)$ meson is an almost pure $q\bar{q}$ state in our description.

The situation is more complicated for the $D_{s1}(2460)$ meson. The probability distributions of its $^1P_1$ and $^3P_1$ components are corrected by the inclusion of non-$q\bar{q}$ degrees of freedom, the latter making a $\sim 25\%$ of the wave function. In our calculation, only the pure $q\bar{q}$ component of the $D_{s1}(2460)$ meson has been used to evaluate the $\Gamma(B \to D^{(*)}D_{s1}(2460))$ decay width. The values we get for the corresponding ratios in Eqs. (31) are lower than the experimental data.

7. Summary

A study of heavy meson properties within a nonrelativistic constituent quark model, which successfully describes hadron phenomenology and hadronic reactions, has been presented in this review. Within the heavy quark sector, we have focused on the spectroscopy and on the electromagnetic, strong and weak processes.

An exhaustive study of heavy meson spectra in terms of $q\bar{q}$ components has been performed. The model can be used as a template against which to compare the new mesons, whose nature is still unknown and some of them are in conflict with naive quark model expectations. Some electromagnetic decays have been included. The study of these processes could provide valuable information on the meson structure since the operator of electromagnetic transitions is very well known.

A quite reasonable global description of the charmonium spectra and decay widths has been reached. One striking feature of our model is the new assignment of the $\psi(4415)$ as a $D$-wave state leaving the $4S$ state for the $X(4360)$. This agrees with the last measurements of its leptonic and total decay widths. We have also tested that our result is compatible with the measured exclusive cross section for the processes $e^+e^- \to D^0D^-\pi^+$ and $e^+e^- \to D^0D^{*-}\pi^+$. Tentative assignments of some $XYZ$ mesons as $c\bar{c}$ states have been done.

The situation is more complicated in the open charm and charmed-strange sectors. We describe the charmed and charmed-strange mesons $D^{(*)}$ and $D_{s}^{(*)}$ and the $j_q^P = \frac{1}{2}^+$ doublet of the orbitally excited states. For the $j_q^P = \frac{3}{2}^+$ doublet the introduction of one loop correction to the OGE potential brings the mass of the $0^+$ state closer to experiment but it is not enough to solve the puzzle of these $P$-wave states.

We have assumed the presence of non-$q\bar{q}$ degrees of freedom in the $J^P = 1^+$ charmed-strange meson sector to enhance the $j_q = 3/2$ component of the $D_{s1}(2536)$ meson. Independently of the mechanism that produces this effect, it has become clear that the description of the $D_{s1}(2536)$ meson as a $j_q = 3/2 c\bar{s}$ state is necessary to simultaneously explain its decay properties. The $J^P = 1^+$ $D_{s1}(2460)$ has an important non-$q\bar{q}$ contribution in our framework.

In the last years several new resonances on the open charm sector have been observed. We have discussed their possible quantum numbers attending to their masses and strong decays. We can only partially describe these states, although the
experimental situation is still not clear.

We have also performed a calculation of the branching fractions for the semileptonic decays of $B$ and $B_s$ mesons into final states containing orbitally excited charmed and charmed-strange mesons, respectively. Our results for $B$ semileptonic decays into $D_0^*(2400), D_1(2420)$ and $D_2^*(2460)$ are in good agreement with the latest experimental measurements. In the case of the $D_1(2430)$ meson, the prediction lies a factor of 2 below BaBar. In the case of $B_s$ semileptonic decays, our prediction for the $\mathcal{B}(B_s^0 \to D_{s1}(2536)^- \mu^+\nu_\mu)\mathcal{B}(D_{s1}(2536)^- \to D^{*-}\bar{K}^0)$ product of branching fractions is in agreement with the experimental data. This, together with the strong decay properties studied for the $D_{s1}(2536)$ meson, is to us evidence of a dominant $q\bar{q}$ structure for this state. We have given also predictions for decays into other $D_s^{**}$ mesons which can be useful to test the $q\bar{q}$ nature of these states.

An analysis of the nonleptonic $B$ meson decays into $D^{(*)}D_{sJ}$ has been also included since it provides valuable information about the structure of the $D_{s0}^*(2317), D_{s1}(2460)$ and $D_{s1}(2536)$ mesons. The strong disagreement found between the heavy quark limit predictions and the experimental data is an indication of the finite $c$-quark mass effects, which are included in the context of the constituent quark model. We have got results close to or within the experimental error bars for the $D_{s0}^*(2317)$ meson, which is an indication that this meson could be a canonical $c\bar{s}$ state. The description of the $D_{s1}(2536)$ meson as an almost $1^+, j_q = 3/2$ $c\bar{s}$ state provides theoretical ratios in better agreement with experiment.

To conclude, we have tried to show in this review that many aspects of the charmonium physics can be understood within the framework of the constituent quark model. However there remains interesting open questions which need further theoretical and experimental effort to be clarified.

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Bibliography

1. C. Quigg, Quarkonium: New developments, in The XVIII Rencontres de la Vallee d’Aoste Conference Proceedings, (2004). FERMILAB-conf-04/033-T, arXiv:hep-ph/0403187v2.
2. R. Galik, Quarkonium production and decay, in The XXIV Physics in Collisions Conference Proceedings, (2004). arXiv:hep-ph/0408190.
3. N. Brambilla et al., The European Physical Journal C - Particles and Fields 71 (2011) 1.
4. K. K. Seth, New results from cleo and bes, in Journal of Physics: Conference Series, (2005), p. 32.
5. K. K. Seth, Heavy Quarkonia Results from CLEO, in Nuclear Physics B: Proceedings Supplements, (2005), p. 344.
6. K. Seth et al., Heavy Quarkonia – A review of the experimental status, in Nuclear Physics B: Proceedings Supplements, (1) (2006), p. 207.
7. T. Skwarnicki, Cleo results on transitions in heavy quarkonia, in The 40th Rencontres De Moriond On QCD And High Energy Hadronic Interactions Conference Proceedings, (2005). arXiv:hep-ex/0505050.
8. E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D 17 (1978) 3090.
9. S. F. Radford and W. W. Repko, Phys. Rev. D 75 (2007) 074031.
10. N. Isgur and G. Karl, Phys. Rev. D 19 (1979) 2653.
11. N. Isgur, International Journal of Modern Physics E 01 (1992) 465.
12. A. Manohar and H. Georgi, Nuclear Physics B 234 (1984) 189.
13. D. Dyakonov and V. Petrov, Nuclear Physics B 272 (1986) 457.
14. S. N. Gupta and S. F. Radford, Phys. Rev. D 24 (1981) 2309.
15. E. Hiyama, Y. Kino and M. Kamimura, Prog. Part. Nucl. Phys. 51 (2003) 223.
16. J. Segovia, A. M. Yasser, D. R. Entem and F. Fernandez, Phys. Rev. D 78 (2008) 114033.
17. J. Beringer et al., Phys. Rev. D 86 (2012) 010001.
18. S. Godfrey and N. Isgur, Phys. Rev. D 32 (Jul 1985) 189.
19. D. Ebert, R. Faustov and V. Galkin, Eur. Phys. J. C71 (2011) 1825.
20. Belle Collaboration (S. Uchara et al.), Phys. Rev. Lett. 104 (2010) 092001.
21. Belle Collaboration (X. L. Wang et al.), Phys. Rev. Lett. 99 (2007) 142002.
22. Belle Collaboration (G. Pakhlova et al.), Phys. Rev. Lett. 101 (2008) 172001.
23. Belle Collaboration (S. Uchara et al.), Phys. Rev. Lett. 96 (2006) 082003.
24. Belle Collaboration (V. Bhardwaj et al.), arXiv:hep-ex/1304.3975 (2013).
25. Belle Collaboration (S.-K. Choi et al.), Phys. Rev. Lett. 89 (2002) 102001.
26. CLEO Collaboration (S. Dobbs et al.), Phys. Rev. Lett. 101 (2008) 182003.
27. BESIII Collaboration (M. Ablikim et al.), Phys. Rev. Lett. 104 (2010) 132002.
28. P. G. Ortega, J. Segovia, D. R. Entem and F. Fernandez, Phys. Rev. D 81 (2010) 054023.
29. Belle Collaboration (S.-K. Choi et al.), Phys. Rev. Lett. 94 (2005) 182002.
30. BaBar Collaboration (B. Aubert et al.), Phys. Rev. Lett. 101 (2008) 082001.
31. J. J. Dudek, R. G. Edwards and C. E. Thomas, Phys. Rev. D 79 (May 2009) 094504.
32. BaBar Collaboration (P. del Amo Sanchez et al.), Phys. Rev. D 82 (2010) 111101.
33. LHCb Collaboration (R. Aaij et al.) (2013) arXiv:1307.4556 [hep-ex].
34. BaBar Collaboration (B. Aubert et al.), Phys. Rev. D 80 (2009) 092003.
35. LHCb Collaboration Collaboration (R. Aaij et al.), JHEP 1210 (2012) 151, arXiv:1207.6016 [hep-ex].
36. D. Ebert, R. Faustov and V. Galkin, Eur. Phys. J. C66 (2010) 197.
37. M. Di Pierro and E. Eichten, Phys. Rev. D 64 (Oct 2001) 114004.
38. G. S. Bali, Phys. Rev. D 68 (2003) 071501.
39. O. Lakhina and E. S. Swanson, Physics Letters B 650 (2007) 159.
40. A. Badalian, B. Ioffe and A. V. Smilga, Nucl. Phys. B281 (1987) 85.
41. M. Ablikim et al., Physics Letters B 660 (2008) 315.
42. BaBar Collaboration (M. Artuso et al.), Phys. Rev. D 80 (2009) 112003.
43. CLEO Collaboration (H. Mendez et al.), Phys. Rev. D 78 (2008) 011102.
44. L. Micu, *Nucl. Phys.* B10 (1969) 521.
45. A. Le Yaouanc, L. Oliver, O. Pène and J. C. Raynal, *Phys. Rev. D* 8 (1973) 2223.
46. A. Le Yaouanc, L. Oliver, O. Pène and J.-C. Raynal, *Phys. Rev. D* 9 (1974) 1415.
47. J. Segovia, D. Entem and F. Fernandez, *Phys.Lett.* B715 (2012) 322.
48. K. K. Seth, *Phys. Rev. D* 72 (2005) 017501.
49. B. Aubert, A precision measurement of the d_s 1 (2536) meson mass and decay width, in 33rd International Conference on High-Energy Physics, (2006). arXiv:hep-ex/0607084.
50. P. Colangelo, F. De Fazio, F. Gianmuzzi and S. Nicotri, *Phys. Rev. D* 86 (2012) 054024.
51. Belle Collaboration (V. Balagura et al.), *Phys. Rev. D* 77 (2008) 032001.
52. J. Vijande, F. Fernández and A. Valcarce, *Phys. Rev. D* 73 (2006) 034002.
53. Belle Collaboration (G. Pakhlova et al.), *Phys. Rev. Lett.* 100 (2008) 062001.
54. Belle Collaboration (G. Pakhlova et al.), *Phys. Rev. D* 80 (2009) 091101.
55. J. M. Blatt and V. F. Weisskopf, *Theoretical nuclear physics* (Dover Pubns, 1991).
56. T. Barnes, S. Godfrey and E. S. Swanson, *Phys. Rev. D* 72 (2005) 054026.
57. Belle Collaboration (D. Liventsev et al.), *Phys. Rev. D* 77 (2008) 091503.
58. BaBar Collaboration (B. Aubert et al.), *Phys. Rev. Lett.* 101 (2008) 261802.
59. BaBar Collaboration (B. Aubert et al.), *Phys. Rev. Lett.* 103 (2009) 051803.
60. D0 Collaboration (V. M. Abazov et al.), *Phys. Rev. Lett.* 102 (2009) 051801.
61. M. A. Ivanov, J. G. Körner and P. Santorelli, *Phys. Rev. D* 73 (2006) 054024.
62. E. Hernández, J. Nieves and J. M. Verde-Velasco, *Phys. Rev. D* 74 (2006) 074008.
63. J. Segovia, C. Albertus, D. R. Entem, F. Fernández, E. Hernández and M. A. Perez-García, *Phys. Rev. D* 84 (2011) 094029.
64. Belle Collaboration (P. Krokovny et al.), *Phys. Rev. Lett.* 91 (2003) 262002.
65. BaBar Collaboration (D. Aubert et al.), *Phys. Rev. D* 74 (2006) 091101.
66. BaBar Collaboration (D. Aubert et al.), *Phys. Rev. D* 77 (2008) 011102.
67. Belle Collaboration (J. Dalseno et al.), *Phys. Rev. D* 76 (2007) 072004.
68. Belle Collaboration (T. Asheev et al.), *Phys. Rev. D* 83 (2011) 051102.
69. A. L. Yaouanc, L. Oliver, O. Pène and J.-C. Raynal, *Physics Letters B* 387 (1996) 582.
70. A. Datta and P. O’Donnell, *Physics Letters B* 572 (2003) 164.
71. A. L. Yaouanc, L. Oliver, O. Pène and J.-C. Raynal and V. Moréñas, *Physics Letters B* 520 (2001) 59.
72. P. Colangelo and F. D. Fazio, *Physics Letters B* 532 (2002) 193.
73. P. Colangelo, F. De Fazio, G. Nardulli, N. Paver and Riazuddin, *Phys. Rev. D* 60 (1999) 033002.
74. Z. Luo and J. L. Rosner, *Phys. Rev. D* 64 (2001) 094001.
75. A. Abd El-Hady, A. Datta and J. P. Vary, *Phys. Rev. D* 58 (1998) 014007.
76. A. Abd El-Hady, A. Datta, K. S. Gupta and J. P. Vary, *Phys. Rev. D* 55 (1997) 6780.
77. F. Jugeau, A. Le Yaouanc, L. Oliver and J.-C. Raynal, *Phys. Rev. D* 72 (2005) 094010.
78. J. Segovia, C. Albertus, E. Hernández, F. Fernández and D. R. Entem, *Phys. Rev. D* 86 (2012) 014010.
79. Guo-Li and Wang, *Physics Letters B* 633 (2006) 492.
80. M.B. and Voloshin, *Progress in Particle and Nuclear Physics* 61 (2008) 455.