A physics-based algorithm to perform predictions in football leagues

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Abstract

In this work, we extended a stochastic model for football leagues based on the team’s potential [R. da Silva et al. Comput. Phys. Commun. \textbf{184} 661–670 (2013)] for making predictions instead of only performing a successful characterization of the statistics on the punctuation of the real leagues. Our adaptation considers the advantage of playing at home when considering the potential of the home and away teams. The algorithm predicts the tournament’s outcome by using the market value or/and the ongoing team’s performance as initial conditions in the context of Monte Carlo simulations. We present and compare our results to the worldwide known SPI predictions performed by the “FiveThirtyEight” project. The results show that the algorithm can deliver good predictions even with a few ingredients and in more complicated seasons like the 2020 editions where the matches were played without fans in the stadiums.

1. Introduction

Football, if on the one hand, seems frivolous for a part of the world population and socially unequal looking at the amounts of money received for some players in major championships compared with those in many country-side cities in Latin America well as other undeveloped countries, on the other hand, it has its attractions. For example, it is professionally performed in more than 200 countries according to FIFA (Federation Internationale de Football Association) and generates many employments.

In addition, we have recently observed the excellent side of the social pressure against the creation of the European Super League (https://www.theguardian.com/football/2021/apr/20/european-super-league-unravelling-as-manchester-city-and-chelsea-withdraw). After all, the monetary discrepancies only reflect an unequal society since football is just a product of that same society.

Called soccer in the USA, this sport, which is the world’s most popular one, has other economic and social importance. For example, many advances in sports science, medicine, and nutrition have maximized the players’ performance. Translating these points into money is difficult, but we have a rough idea of how they can impact people’s lives, especially the poorest ones. Not necessarily, for this reason, physicists have devoted some work and time to describe statistics related to football (see, for example, Refs. [1, 2]) in order to understand the stochastic and deterministic aspects of this exciting research area.
Until the 1980s, the football’s scoring system was 2-1-0, standing for points by the win, draw, and loss, respectively. This system, however, had benefited many “cross-country” teams, i.e., teams that only play to draw, as this becomes very advantageous in this championship modality.

To make the game more attractive and, in a way, more competitive, an evolution has started since then. The scoring system 3-1-0 prevailed over the previous one, as well as other improvements. For example, when some player now kicks back the ball to the goalkeeper, he/she is prohibited from catching it with his/her hands. This simple rule increased the time of the ball in motion in matches. Such rules are constantly updated, and all improvements (or throwbacks), as well as new rules, can always be found in the webpage related to “laws of the game” in the international football association board (IFAB).¹

The magic point of football, which also occurs in many other sport modalities, is that the best team does not always win the match, and in some championships, “dark horses,” a term used to describe a little known candidate or competitor, who unexpectedly wins or succeeds, are ubiquitous.

In Brazil, this “upset victory” is called a ”zebra,” name created by Gentil Cardoso, a Portuguesa iconic football coaching who told a reporter during an interview about the possibility of his team beating Vasco da Gama in a match valid for the regional championship in Rio de Janeiro, Brazil. Cardoso, at the time, used the word ”zebra” to refer to an animal not included in a popular (but prohibited) gambling in Brazil, which we translate as ”animal game” in a free translation.

The so-called ”zebras” are more frequent than one could imagine and many times they break the deterministic characteristics of championships, making the favorite teams not always the champions.² Commonly, the champion has a good cast of players, which, in turn, depends, for instance, on the financial power of the team (although not always). In balanced championships like those in Brazil, where many teams have already become champions, the symbiosis of the team’s players, combined with a good coach, can even change history. Based on these assumptions, in this paper, we intend to answer the following question: Are we able to predict champions and relegations in a football championship using mathematical models?

To shed light on this question, we first consider that each team can beat its opponent, which, in turn, is translated as the probability of victory. This probability depends on amounts defined as the team’s potentials. These potentials consider several factors, for example, the economic power of the team and its history of successes along with the championship. This study also takes into consideration a previous model that successfully described the statistics of team scores in different double round-robin system (DRRS) championships, i.e., the championships in which all teams play each other twice, in turn, and return.

This paper proposes a model that makes predictions of the champions, the four best teams (G4), and the four worst teams (Z4) in a championship. For this purpose, we adopted two different parameters, the market value, and the teams’ performance, as shown in Section 2. In Section 1

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¹https://www.theifab.com/history/ifab
²The reference [3] is related to a report about football organized by I. Zolkernevic highlighting the contribution in this area of some Brazilian authors that instead of us. It includes also comments about the work of Ribeiro et al. (see also [5]). The direct link to this report is: https://revistapesquisa.fapesp.br/en/upset-victories-common-brazilian-soccer-championship
we divide our results into two different parts: the first one corresponds to an exploration of the tuning/calibration of some parameters with special attention to the influence of market value and memory effects on modeling and, in the second part of the results, we take into account the optimal parameters obtained previously to test our model presenting the predictions for the Brazilian Championship A Series and comparing them with some specialized websites. Finally, some conclusions are presented in Section 4.

2. Model and optimization

Our model is based on the agent-based model [4, 6, 7] that considers a system of $N$ teams playing against each other according to DRRS [3]. Here, it is noteworthy that although the goal of this study is to make predictions about the top Brazilian professional league for men’s football clubs called Brazilian Championship A Series (commonly referred as “Brasileirão”), our approach is generic and then, can be applied to other leagues as well.

After a given season starts, if team $i$ plays as host (home team) against team $j$ as visitor (away team) at the $k$-th round, and based on the fact that the number of goals in a match follows a Poisson distribution [2, 8, 9], the probability of the match to result in a draw can be written as [4]:

$$
\begin{align*}
    r_{\text{draw}}^{(i,j)}(k) &= \Pr \left( (g_i = g_j) \mid (\phi^{(i)}_k, \psi^{(j)}_k) \right) \\
    &= \sum_{g=0}^{\infty} \frac{(\phi^{(i)}_k \psi^{(j)}_k)^g}{g!^2} e^{-\left(\phi^{(i)}_k \psi^{(j)}_k\right)} \\
    &= e^{-\left(\phi^{(i)}_k \psi^{(j)}_k\right)} I_0 \left(2 \sqrt{\phi^{(i)}_k \psi^{(j)}_k}\right)
\end{align*}
$$

where $g$ is the number of goals scored by each team, $\phi$ and $\psi$ correspond to the host’s and the visitor’s potentials respectively, and

$$
I_v(z) = \left(\frac{1}{2}\right)^v \sum_{n=0}^{\infty} \frac{(\frac{1}{4}z^2)^n}{n! \Gamma(v + n + 1)}
$$

is the modified Bessel function of the first kind. Our reason to define a home team potential ($\phi$) and an away team potential ($\psi$), resides on the well known fact that teams that are hosting the match have statistical advantage over their visitors [10, 11] for a number of reasons such as travelling distances, crowd size, number of time zones crossed by the visiting team, altitude of the home stadium etc.

The probability that team $i$ has of winning the match against team $j$ at $q$-th round considers the product of the complement of Eq. [1] and a factor that weights the potentials of the teams involved.

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[3] https://en.wikipedia.org/wiki/Round-robin_tournament
such as follows:

\[ \Pr(g_i > g_j, q) = \left[ 1 - r^{(i,j)}_{\text{draw}}(q) \right] \cdot \frac{\phi^{(j)}_q}{\phi^{(i)}_q + \psi^{(j)}_q}. \]  

(2)

The chances of the team \( j \) to win the match is defined by the complement:

\[ \Pr(g_i < g_j, q) = 1 - \Pr(g_i > g_j, q) - r^{(i,j)}_{\text{draw}}(q) \]

\[ = \left[ 1 - r^{(i,j)}_{\text{draw}}(q) \right] \cdot \frac{\psi^{(i)}_q}{\phi^{(i)}_q + \psi^{(j)}_q}. \]  

(3)

The team’s potential is updated after each match. Therefore, we must consider the final result of the match, i.e., whether the team won, lost or ended in a draw. With this in mind, we defined that the winner and the loser teams will have their potentials changed respectively by the quantities \( \Delta \phi_q \) and \( \Delta \psi_q \). By considering that the home team \( i \) plays against the visiting team \( j \), we have

\[
\Delta \phi^{(i)}_q = \begin{cases} 
-\Delta \psi^{(j)}_q = 3 & \text{if } g_i^{(q)} > g_j^{(q)}, \\
\Delta \psi^{(j)}_q = 1 & \text{if } g_i^{(q)} = g_j^{(q)}, \\
-\Delta \psi^{(j)}_q = -3 & \text{if } g_i^{(q)} < g_j^{(q)},
\end{cases}
\]

yielding \( \Delta \psi^{(i)}_q = \Delta \phi^{(j)}_q = 0 \) in all conditions.

Before any season starts, the market value\(^1\) of each team is supposed to be one of the main factors to reflect its potential performance, which has influence on its chances of becoming the future champion of the league. After the start of a season, the real performance of the teams comes to light as new and important information that must be taken into account in predictive algorithms. Thus, we define the both potentials of a team \( i \) at the \( q \)-th round as:

\[
\phi^{(i)}_q = \alpha M^{(i)} + \beta P^{(i)}_{k_0|\phi} + \gamma \sum_{j=k_0}^{q-1} \Delta \phi_j
\]

\[
\psi^{(i)}_q = \alpha M^{(i)} + \beta P^{(i)}_{k_0|\psi} + \gamma \sum_{j=k_0}^{q-1} \Delta \psi_j
\]

(4)

where \( \phi^{(i)}_{k_0} = \alpha M^{(i)} + \beta P^{(i)}_{k_0|\phi} \) and \( \psi^{(i)}_{k_0} = \alpha M^{(i)} + \beta P^{(i)}_{k_0|\psi} \) are the initial potentials, \( M^{(i)} \) is the market value of team \( i \), and therefore, \( \alpha \) is the market coefficient. Similarly, we also use the scored points to perform predictions, thus \( P^{(i)}_{k_0|\phi} \) is the cumulative scored points in the first \( k_0 \) rounds and \( \beta \) is a coefficient related to the performance of the team. The quantities \( P^{(i)}_{k_0|\phi} \) and \( P^{(i)}_{k_0|\psi} \) are related to the cumulative scored points as the team plays at home and when it plays as a visitor, respectively, and

\(^1\)https://www.transfermarkt.com/
of course, $P_{k_0}^{(i)} = P_{k_0}^{(i)} + P_{k_0}^{(j)}$. Finally, we complete the contribution for the potentials by including the sum of the successive increments/decrements from $k_0$ up to the $q$-th round, i.e., $\sum_{j=k_0}^{q-1} \Delta \phi_j^{(i)}$ and $\sum_{j=k_0}^{q-1} \Delta \psi_j^{(i)}$. Thus, $\gamma$ is a coefficient associated to this quantity.

Now, it comes the problem: how to determine $\alpha$, $\beta$, and $\gamma$? Once these values are estimated, we are able to obtain $\phi_q^{(i)}$ and $\psi_q^{(i)}$ for arbitrary $q$-th round. To reach this goal, we define $k$ such that a memory of $\Delta k = k - k_0$ works as a “learning interval” to determine the optimal values $\alpha_{opt}$, $\beta_{opt}$, and $\gamma_{opt}$, which maximizes relevant parameters that compare the results from simulation and real matches obtained in this interval. Thus, let us explain this process that basically works with three algorithms.

The main algorithm (Algorithm 1) defines the evolution of the potentials described by Eq. (4) in a general way, as well as the evolution of the scores.

**Algorithm 1: Main**

1. if $(\text{rand}[0, 1] < r_{\text{draw}})$ then
2. $p^{(i)} = p^{(i)} + 1$ and $p^{(j)} = p^{(j)} + 1$
3. else
4. if $(\text{rand}[0, 1] < \left(\frac{\phi^{(i)}}{\phi^{(i)} + \psi^{(j)}}\right))$ then
5. $p^{(i)} = p^{(i)} + 3; \phi^{(i)} = \phi^{(i)} + \gamma \ast \Delta \phi^{(i)}; \psi^{(j)} = \psi^{(j)} + \gamma \ast \Delta \psi^{(j)}$;
6. else
7. $p^{(j)} = p^{(j)} + 3; \psi^{(j)} = \psi^{(j)} + \gamma \ast \Delta \psi^{(j)}; \phi^{(i)} = \phi^{(i)} + \gamma \ast \Delta \phi^{(i)}$;
8. end if
9. end if

First, it is necessary to perform the optimization. For that, we have alternatives based on some metrics. We define our first metrics denoted by matching hits:

$$\xi_q = \frac{2}{N} \sum_{l=1}^{N/2} \delta(\phi_q^{(l)}|S, \phi_q^{(l)}|R)$$

(5)

where $\phi_q^{(l)}|S$ and $\phi_q^{(l)}|R$ are the results of the $l$-th match in the $q$-th round of the simulated (S) and the real world (R), respectively, and $\delta(x, y)$ is the well-known Kronecker symbol. Here, one considers only the result and the exact number of goals is not taken into account. Thus, in the tuning algorithm (Algorithm 2), we perform the optimization of our model in order to obtain the values $\alpha_{opt}$, $\beta_{opt}$, and $\gamma_{opt}$ as presented below.
Algorithm 2: Tuning

1: for $\alpha = 0$ to $\alpha_{\text{max}}$ step $\Delta \alpha$ do
2:    for $\beta = 0$ to $\beta_{\text{max}}$ step $\Delta \beta$ do
3:        for $\gamma = 0$ to $\gamma_{\text{max}}$ step $\Delta \gamma$ do
4:            for $i_{\text{run}} = 1$ to $N_{\text{run}}$ do
5:                for $i_{\text{round}} = k_0$ to $k$ do
6:                    run Main Algorithm;
7:                end for
8:            end for
9:            if $\xi > \xi_{\text{max}}$ then
10:               $\alpha_{\text{opt}} = \alpha; \beta_{\text{opt}} = \alpha; \gamma_{\text{opt}} = \gamma; \xi_{\text{max}} = \xi$;
11:          end if
12:      end for
13:  end for
14: end for

The last algorithm, the Algorithm 3 shown in the following, is designed to perform the final evolution of the simulation delivering the championship ranking table by considering the values of $\alpha_{\text{opt}}, \beta_{\text{opt}},$ and $\gamma_{\text{opt}}$ obtained previously.

Algorithm 3: Final evolution

1: for $i_{\text{run}} = 1$ to $N_{\text{run}}$ do
2:    for $i_{\text{round}} = k$ to $N_{\text{last}}$ do
3:        run Main Algorithm using $\alpha_{\text{opt}}, \beta_{\text{opt}},$ and $\gamma_{\text{opt}}$;
4:    end for
5: end for

The purpose of each of these algorithms can be summarized as follows

1. Algorithm 2 calls Algorithm 1 $N_{\text{run}}$ times. The simulation considers the current championship, from $k_0$ up to the $k$-th round for the variables $\alpha$, $\beta$, and $\gamma$, using as input $\phi_{k_0}^{(i)}, \psi_{k_0}^{(i)}, p_{k_0|\phi}^{(i)},$ and $p_{k_0|\psi}^{(i)}$ for $i = 1, \ldots, N$, in order to obtain the average value of $\xi_q$ for these $\frac{N}{2} \Delta k$ matches.

2. By performing three external loops, Algorithm 2 increments the values of $\alpha$, $\beta$, and $\gamma$ by the amount $\Delta \alpha$, $\Delta \beta$, and $\Delta \gamma$ and then, we repeat the step 1 for each set $\alpha$, $\beta$, and $\gamma \in [0, 1]$ searching for $\alpha_{\text{opt}}, \beta_{\text{opt}},$ and $\gamma_{\text{opt}}$ that led to a greater average value of $\xi_q$.

3. With $\alpha_{\text{opt}}, \beta_{\text{opt}},$ and $\gamma_{\text{opt}}$ in hand, one runs $N_{\text{run}}$ times once again the championship from the round $k$ until the last round ($N_{\text{last}} = 2(N - 1)$, where $N$ is the number of teams) in order to obtain the average results of each team (how much times it becomes the champion, its participation in $g_4$, and its participation in the $z_4$), according to Algorithm 3 (final evolution).

In addition, we can use alternative metrics to match the hits defined in Eq. (5). One interesting metric is based on two amounts: one based on scored points and one based on variable ranking.
The first one measures the predictive capability of our algorithm to measure the difference of the scored points in simulations (S) and those obtained of the real world (R), which are given by $P_k|S$ and $P_k|R$ per team, respectively, until a $q$-th round defined as

$$\mu_q = \frac{1}{N} \sum_{l=1}^{N} \eta \left( P_q^{(l)}|S, P_q^{(l)}|R \right),$$

where

$$\eta(x, y) = \begin{cases} 1 & \text{if } |x - y| \leq \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

and $\varepsilon$ is an error parameter. If $\varepsilon = 0$, we have what we called the strong predictor. On the other hand, $\varepsilon = 1$ is called medium predictor and $\varepsilon = 2$ means weak predictor. The ranking-based variable takes into account the difference between the team’s ranking position in the simulation and the real league classification table as follows

$$\nu_q = \frac{1}{N} \sum_{l=1}^{N} \eta \left( Q_q^{(l)}|S, Q_q^{(l)}|R \right),$$

where $Q_q^{(l)}$ denotes the ranking position of the team $l$ at the $q$-th round.

The search for the values of $\alpha$, $\beta$, and $\gamma$ that maximizes $\xi_q$ in Eq. (5) is simply changed by

$$\xi_q = \mu_q + \nu_q$$

which is denoted as score-ranking metric. This metric mixes the influence of the rank and score of the teams simultaneously and, for this reason, we performed the predictions using this metric instead of the matching hits. In this paper, we decided to use $\varepsilon = 0$, i.e., the strongest criterion for our approach.

### 3. Results

First, we explore how the Algorithm 2 works and in this preparatory part, we will show how the parameters behave. We consider $N_{\text{run}} = 100$ and $\Delta \alpha = \Delta \beta = \Delta \gamma = 0.05$. In Fig. 1, we present the evolution of these parameters for the 2020 edition of the Brazilian Championship A Series considering the two criteria for the optimization: highest average value of matching hits as shown in Fig. 1(a) and highest average value of the $\mu + \nu$ (score-ranking metric) presented in Fig. 1(b).

In Fig. 1(a), we observe that with the matching hits criterion, the coefficients seem to fluctuate slightly less than when considering the score-ranking metric (Figure 1(b)), mainly when one observes the evolution of $\alpha$ which is related to the market value of the team. As can be seen, we considered $\Delta k = 10$ in this study. An interesting analysis can be performed if we fix $\beta$ and $\gamma$ focusing only in behavior of $\alpha$ for different rounds. In this case, we find by the optimal value of $\alpha$ and consider $\beta = \gamma = 1$. This analysis helps us to understand how much of the market value must be added to perform good predictions.
Figure 1: Evolution of the parameters $\alpha_{opt}$, $\beta_{opt}$, and $\gamma_{opt}$ per round for the 2020 edition of the Brazilian Championship A Series. (a) corresponds to the case where we used the matching hits criterion and (b) corresponds to the case where the criterion was the maximization of the $\mu + \nu$.

Figure 2 shows the behavior of matching heats (mh) parameter as function of $\alpha$ for different rounds and several seasons of the considered Brazilian Championship A Series. We fixed $\beta = \gamma = 1$ to perform these plots.

Particularly for season 2020 shown in Fig. 2, an “almost” monotonic decay is observed only for the largest $k$, $k = 35$. For the other cases with $k \leq 30$, one observes an initial increase and a subsequent decrease of this quantity as function of $\alpha$, showing that $\alpha_{opt}$ corresponds to the peak of these curves. However, for the other seasons, one observes that even for intermediate values of $k$, this “almost” monotonic decay behavior is already observed indicating less influence of the market value on the matching hits.

Differently from the matching hits, the parameters to compose the score-ranking metric $\mu$ and $\nu$ present slow increasing tendency on $\alpha$ as can be observed in Fig. 3 (a) and (b), respectively. Without loss of generality, for saving space, in Fig. 3, we only present the results for 2020 season since it is the most debatable season. Although not presented here, the other seasons have similar behavior. Since we show the way to obtain the $\alpha_{opt}$, it is interesting to present a typical plot of $\alpha_{opt}$ as function of round ($k$). Figure 4 shows our results by using the two considered criteria: matching hits and score-ranking metric. Fig 4 (a) presents our estimates for the matching hits criterion and Fig. 4 (b) shows the results for the score-ranking one.

One can observe that $\alpha_{opt}$ has a tendency to decrease as $k$ increases by using the matching hits criterion. On the other hand, $\alpha_{opt}$ varies strongly with the score-ranking criterion. This scenario must be reflected in our study.

Once we understand how to obtain the optimal parameters, we are able to continue the study of the proposed model in order to make predictions of the championship results. Some questions should be raised in relation to the memory $\Delta k$ and, to answer them we will consider only the matching hits metric. In Fig. 5 we show a plot of $m_{h_{opt}}$, denoting the average matching hits calculated for the $\alpha_{opt}$, as function of $\Delta k$ for three seasons of the Brazilian Championship A series:
Figure 2: Matching hits versus $\alpha$ for different rounds: $k = 10, 15, 20, 25, 30,$ and 35. Plots (a), (b), (c), and (d), shows the results for the seasons 2020, 2019, 2018, and 2017, respectively. In all situations one used $\Delta k = 10, N_{run} = 1000,$ and $\beta = \gamma = 1.$
Figure 3: Parameters of score $\mu_k$ and ranking $v_k$ as function of $\alpha$ for different rounds $k$.

Figure 4: Evolution of the parameter $\alpha_{opt}$ per round keeping fixed $\beta = \gamma = 1$, for the 2020 edition of the Brazilian Championship A Series. (a) corresponds the case where we used matching hits criterion, and (b) is the case where the criteria was the maximization of the $\mu + v$. 
Figure 5: $mh_{opt}$, which denotes the average matching hits obtained with $\alpha_{opt}$, as function of $\Delta k$ for 2018 (a), 2019 (b), and 2020 (c) editions of the championship.

2018, 2019, and 2020, and keeping fixed $\beta = \gamma = 1$.

From this figure, we can observe that $\alpha_{opt}$ presents small fluctuations around a mean value for $\Delta k \geq 10$ and, for this reason, we will use this value until the end of this paper. But the question is whether such optimal values can bring good predictions for the championships. As we will show below, the answer is yes and the optimal parameters used as input in Algorithm 3 are capable to deliver good predictions to the results of the championships.

To study the predictive abilities of our model, we feed Algorithm 3 with data of 2017, 2018, 2019 and 2020 seasons of the Brazilian Championship A Series in order to be able to compare our simulations outcome with the final results of the seasons. However, the 2020 season will deserve more attention because it was a particularly complex season as it was decided only in the last round and held during the Covid-19 pandemic.

In this work, we are not only concerned with the champion of the league. Instead, our goal is to compare the league champion, the top four teams (usually denoted as $g4$) since their positions
in the ranking table qualify them to the stage groups of the continental cup known as Conmebol Libertadores (Spanish and Portuguese for liberators) cup\(^4\) (in the case of South American clubs), and the last four teams (usually denoted as \(z4\)) once they are relegated to a lower league division (Brazilian Championship B Series) during the next season. For that, we define the following variables:

\[
\begin{align*}
    f_w &= \frac{1}{N_{\text{run}}} \sum_{i=1}^{N_{\text{run}}} \theta_i \\
    f_{g4} &= \frac{1}{4N_{\text{run}}} \sum_{i=1}^{N_{\text{run}}} \iota_i \\
    f_{z4} &= \frac{1}{4N_{\text{run}}} \sum_{i=1}^{N_{\text{run}}} \kappa_i
\end{align*}
\] (8)

where \(\theta\) is equal to 1 if the simulation got the champion right and 0 otherwise, \(\iota\) and \(\kappa\) can assume integer values in the range \([0, 4]\) depending on the number of teams that the simulations successfully hit in the \(g4\) and \(z4\) regions, respectively, of the ranking table.

The evolution of predictive parameters \(f_w\), \(f_{g4}\), and \(f_{z4}\) for the 2018, 2019, and 2020 seasons is shown in Fig. 6. The black curve (circle) shows the parameters obtained with score-ranking

\(^4\)http://www.conmebol.com/pt-br/torneos/conmebol-libertadores
metric, dark gray (square) corresponds to those obtained with matching hits criterion. Following, gray (diamond) and light-gray (triangle) correspond to matching hits and score-ranking metrics, respectively, by fixing $\beta = \gamma = 1$, and only the market value influence was optimized in this case.

We can observe that score-ranking metric with three optimized parameters presents a more oscillating evolution. All situations are good in predicting $g_4$, $z_4$, and the champion after a certain reasonable number of rounds. It is important to mention that in the 2020 season, the prediction of the champion was more complicated. Soccer Power Index (SPI) from the project FiveThirtyEight (https://projects.fivethirtyeight.com/soccerpredictions/brasileirao/ (2020)) corroborates this observation, but as we present below, our model was capable of obtaining predictions as good or better as those of SPI. Thus, let us summarize them in numbers!

The results obtained for the champion of 2017, 2018 and 2019 seasons are shown in Table 1. For the sake of simplicity, the results obtained for $g_4$ and $z_4$ ranking table are not shown in this table. It is important to mention that such championships were well behaved when compared to the 2020 season.

Our results also present strong correlations with those of SPI, as can be seen in the last line of Table 1 showing that our predictions are as good as the SPI ones (https://projects.fivethirtyeight.com/soccerpredictions/brasileirao/ (2020)). The predictions are very similar, which is not exactly a surprise.

Finally, we would like to look into the numbers of a sui generis season: 2020. This season was held during the pandemic, which led to the absence of fans in the stadium, and the champion

Table 1: Hit estimates of the proposed model in relation to the champion of the Brazilian Championship A Series in 2017, 2018, and 2019 seasons. Here (*) denotes the case where we fixed $\beta = 1.0, \gamma = 1.0$

| Season | 2017 | 2018 | 2019 |
|--------|------|------|------|
| $k$    | 25   | 30   | 35   | 25   | 30   | 35   | 25   | 30   | 35   |
| score-ranking metric | 1.00 | 1.00 | 1.00 | 0.23 | 1.00 | 0.93 | 0.83 | 1.00 | 1.00 |
| score-ranking metric (*) | 0.87 | 0.76 | 1.00 | 0.20 | 0.74 | 0.96 | 0.94 | 0.99 | 1.00 |
| matching hits metric | 0.67 | 0.54 | 1.00 | 0.12 | 0.67 | 0.93 | 0.89 | 0.85 | 1.00 |
| matching hits metric (*) | 0.75 | 0.68 | 1.00 | 0.16 | 0.65 | 0.92 | 0.88 | 0.97 | 1.00 |
| SPI    | 0.78 | 0.74 | 0.99 | 0.26 | 0.77 | 0.97 | 0.94 | 0.97 | 1.00 |

Table 2: Hit estimates of the proposed model in relation to the champion of the Brazilian Championship A Series in 2020 season — during the pandemic. The bold lines stress our best metrics predictions compared with SPI predictions. We did not find predictions for the rounds 29, 31, and 34 made available by SPI. Here (*) denotes the case where we fixed $\beta = 1.0, \gamma = 1.0$

| $k$    | 25   | 26   | 27   | 28   | 29   | 30   | 31   | 32   | 33   | 34   | 35   |
|--------|------|------|------|------|------|------|------|------|------|------|------|
| score-ranking metric | 0.56 | 0.53 | 0.47 | 0.39 | 0.41 | 0.35 | 0.40 | 0.20 | 0.15 | 0.35 | 0.00 |
| score-ranking metric (*) | 0.42 | 0.43 | 0.28 | 0.18 | 0.11 | 0.13 | 0.30 | 0.14 | 0.14 | 0.28 | 0.43 |
| matching hits metric | **0.65** | **0.62** | **0.37** | **0.46** | **0.28** | **0.29** | **0.54** | **0.24** | **0.19** | **0.21** | **0.44** |
| matching hits metric (*) | 0.42 | 0.41 | 0.28 | 0.18 | 0.12 | 0.18 | 0.26 | 0.12 | 0.17 | 0.27 | 0.42 |
| SPI    | **0.34** | **0.43** | **0.43** | **0.28** | – | **0.16** | – | **0.23** | **0.15** | – | **0.25** |
(Flamengo) was known only at the end of the last round, which dramatically affects the prediction. To get an idea, Flamengo finished the championship with just 71 points, one point ahead of Internacional, the vice-champion. Therefore, a single goal against Corinthians in the last round would give the cup to Internacional, changing the championship’s history. As observed in Table 2, one of our versions is always better than the SPI predictions. In order to perform a fair comparison, we choose our best case (matching hits metric) that is better in all comparisons with SPI, except by the prediction of the 27th round where one has a dead heat.

4. Summaries, conclusions, and discussions

Our algorithm was capable of determining essential statistics of the real Brazilian Championship A Series. Our predictions are compatible with an alternative and commercial way to perform predictions (SPI) since, as shown, both predictions are strongly correlated to each other. On the other hand, our method presented more reliable estimates in a more complicated season: 2020. Our additional idea is to take a different path from SPI, making explicit all of our ingredients. The model uses many concepts related to the teams’ scoring process during a championship and the market values of the teams.

In addition, the method can be easily applied in any championship to obtain good probabilities of hitting the champion. Estimates for $g_4$ and $z_4$ show notable agreement between the metrics considered when the predictions are performed from the 26th round on. In fact, from this round on, one obtains something between 80%–100% of probability to point out the exact $g_4$ or $z_4$ (see Fig. 6). We believe that our predictive method can be changed to contemplate other championships based on the double-round robin system scheme. Other authors [12] explored machine learning for the premier league in seasons 2014 and 2015 with good predictions, and the topic seems to be extremely promising for future works by including other sports.

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