Static conformal models for anisotropic charged fluids

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Abstract

We investigate the role played by conformal symmetries in the study of exact solutions in static spherically symmetric spacetimes. We consider the gravitational field associated with anisotropic charged fluids. The existence of a conformal symmetry provides us with a functional relation between the metric functions. This relationship yields a general solution to the Einstein-Maxwell equations. Various conformally symmetric models for fluids that are charged or anisotropic or both are shown to be special cases of the general exact solution generated in this paper. In particular we generate a conformally flat Finch–Skea type model for an isotropic charged fluid.

1. Introduction

The existence of spacetime symmetries in the manifold in a general relativity setting has been the subject of many investigations in recent years. This is particularly true for a conformal symmetry as it preserves the metric up to a conformal factor, and generates a conserved quantity along null geodesics for massless particles. There are several reasons for studying symmetries resulting from Lie derivatives of geometric quantities which determine the change for tensor fields along the flow defined by a vector field. Firstly, symmetries lead to substantial simplifications of the nonlinear field equations. Secondly, symmetries have geometric significance and provide an insight into the spacetime structure. Thirdly, symmetries have physical significance and can be related to astrophysical and cosmological models. In particular spacetimes with spherical symmetry have been widely studied. For a detailed investigation and classification of conformal symmetries in spherically symmetric spacetimes see Tupper et al [1], Maartens et al [2] and Mason and Maartens [3] have considered general relativistic anisotropic fluids with conformal symmetry. Such relativistic fluids with spherical symmetry have been comprehensively analysed in terms of their kinematics and dynamics by Coley and Tupper [4].

Due to the high level of symmetry in spacetimes with spherical symmetry, the corresponding conformal Killing vectors can be identified by integrating the conformal Killing equations. The case of shear–free spacetimes was analysed by Moopanar and Maharaj [5], and the general case of expanding, accelerating and shearing spacetimes was considered in the treatment Moopanar and Maharaj [6]. All the static and nonstatic conformal Killing vectors in the static spherical symmetrical spacetimes were found by Maharaj et al [7] and Maartens et al [8, 9]. Manjonjo et al [10] showed that the components of the Weyl tensor can be used to classify conformal symmetries in the static spherical manifolds. Conformally flat spacetimes and nonconformally flat spacetimes with the static assumption were also studied by Grøn and Johannesen [11] and Herrera et al [12]. In a general treatment Manjonjo et al [13] showed that an explicit functional relation relates the two metric components of the gravitational field. This relationship can be exploited to find new solutions to the Einstein field equations and model relativistic stars. Shee et al [14] found anisotropic compact relativistic stars with nonstatic conformal symmetry. Newton Singh et al [15] discovered electrically charged dense stars in the presence of a static conformal Killing vector. Mafa Takisa et al [16], Kileba Matondo et al [17] and Kileba Matondo et al [18] generated new stellar models with conformal symmetries with radii, masses, densities and redshifts which are consistent with observed astronomical objects. These models have been found by integrating the Einstein-Maxwell field equations for selected forms of the metric functions.
From the above it is clear that there exists a connection between the electric charge, anisotropic pressures, the field equations and the presence of a conformal symmetry. It is desirable to find the precise nature of this connection and identify the line elements that may arise under general conditions. This forms the basis of our study in static spherically symmetric spacetimes. We do not prescribe forms for the gravitational potentials, the existence of a conformal Killing vector is assumed, and we attempt to solve the Einstein-Maxwell equations in general. Many authors have approached the study of conformal symmetries in finding exact solutions to the field equations by imposing particular restrictions on the metric potentials [19]. In the search for a general solution to the Einstein-Maxwell equations for spacetimes with a conformal symmetry we seek to impose minimal restrictions on the metric functions. We only assume the existence of a conformal symmetry. We make no further assumptions on the form of the metric functions other than those arising from the integrability condition for the conformal Killing vector. This leads to a more general form of the metric functions. The work done by various authors can then be shown to be only special cases of the work done here. Manjonjo et al [13] considered the case of the conformal symmetry in the presence of anisotropic pressures. We extend this work to include the electromagnetic field. Note that our results apply to spacetime manifolds of dimension four.

We now briefly summarise the work covered in this paper. In section 2 we examine the static spherically symmetric model admitting conformal symmetry. We use the Weyl tensor to give an equation connecting the metric potentials. In section 3 we solve the Einstein-Maxwell equations for a static spacetime with a conformal symmetry. In section 4 we find as a particular case a metric corresponding to the Finch-Skea model. In section 5 we give a brief summary of some solutions found by other authors which we obtained from our own general analysis in section 3. We follow the notation of and conventions of Manjonjo et al [13]. In section 6 we give a brief conclusion of the work contained in this paper.

2. The model

The line element for static spherically symmetric spacetimes has the form

$$ds^2 = -e^{2\nu}(t)dt^2 + e^{2\lambda}(r)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

in Schwarzschild coordinates [20]. The functions $\nu(r)$ and $\lambda(r)$ are the gravitational potentials. We study this geometry in the situation where the anisotropic pressures and an electric field may be present. In the presence of strong gravity this means we need to analyse the Einstein-Maxwell system of equations. The Einstein-Maxwell field equations are the general relativistic generalisation of Newton’s gravitational laws. The matter distribution is taken to be an anisotropic charged fluid with a fluid 4-vector $u^a = e^{-\nu}\delta^a_0$ which is comoving. The energy momentum tensor can be written in the form

$$T_{ab} = (\rho + p_r)u_a u_b + p_r g_{ab} + (p_\perp - p_r) v_a v_b + E_{ab},$$

where $\rho$, $p_r$ and $p_\perp$ are the energy density, radial pressure and tangential pressure respectively. The vector $\mathbf{v}$ is a unit vector orthogonal to $\mathbf{u}$, and $E_{ab}$ is the electromagnetic field tensor. The Einstein-Maxwell field equations are then given by

$$\frac{1}{r^2}[r(1 - e^{2\lambda})]_r = \rho + \frac{1}{2}E^2,$$

$$e^{-2\lambda}\left[\frac{2\nu}{r} + \frac{1}{r^2}\right] - \frac{1}{r^2} = p_r - \frac{1}{2}E^2,$$

$$e^{-2\lambda}\left[\nu_r + \nu_t^2 + \frac{\nu_t}{r} - \frac{\lambda_t}{r} - \nu_r \lambda_t\right] = p_\perp + \frac{1}{2}E^2,$$

$$\frac{e^{-\lambda}}{r^2}(r^2\nu_t') = \sigma,$$

where $E$ is the electric field intensity and $\sigma$ is the proper charge density. In the absence of charge ($E = 0$) and for isotropic pressure ($\Delta = 0$, $p_r = p_\perp$) we obtain a perfect fluid.

We introduce new transformation due to Durgapal and Bannierji [21] to transform (3a)–(3d) into a new simpler system. We let $x$ be a new variable, and $y$ and $Z$ be new functions defined as follows

$$x = r^2,$$

$$Z(x) = e^{-2\lambda(r)},$$

$$y^2(x) = e^{2\nu(r)}.$$  

Equations (4a)–(4c) transform the system (3a)–(3d) into

$$\frac{1 - Z}{x} = 2Z = \rho + \frac{1}{2}E^2,$$
\[ 4Z \left( \frac{\dot{y}}{y} \right) + \frac{Z - 1}{x} = p_1 - \frac{1}{2} E^2, \]  
(5b)

\[ 4xZ \left( \frac{\dot{y}}{y} \right) + (4Z + 2x\dot{Z}) \frac{\dot{y}}{y} + \dot{Z} = p_1 + \frac{1}{2} E^2, \]  
(5c)

\[ \frac{4Z}{x} (xE + E)^2 = \sigma^2, \]  
(5d)

where \( (\cdot) \) represents differentiation with respect to \( x \). From (5b) and (5c) we obtain
\[ 4xZ \left( \frac{\dot{y}}{y} \right) + 2x\dot{Z} \frac{\dot{y}}{y} + \dot{Z} = \frac{Z - 1}{x} - E^2 = \Delta, \]  
(6)

where \( \Delta = p_1 - p_\perp \) is the degree of pressure anisotropy. Equation (6) is the condition of pressure isotropy generalized to include anisotropy and the electromagnetic field. When \( E = 0 \) we regain the anisotropic case considered by Manjonjo et al \[13\].

The general conformal Killing vector equation for spacetimes admitting a conformal symmetry is given by
\[ \mathcal{L}_X g_{ab} = 2\psi g_{ab}, \]  
(7)

where \( \mathcal{L}_X \) is the Lie derivative operator along the vector \( X \) and \( \psi(x^a) \) is the conformal factor. From the spherical symmetry assumption in (1), we observe that \( X \) is reduced to a function of \( t \) and \( r \) only, and is given by
\[ X = \alpha(t, r) \frac{\partial}{\partial t} + \beta(t, r) \frac{\partial}{\partial r}, \]  
(8a)

\[ \psi = \psi(t, r). \]  
(8b)

The conformal symmetry may be nonstatic even though the metric is static as discussed in \[13\]; the field equations further restrict the forms of vector \( X \) and conformal factor \( \psi \). Equation (7) is not sufficient to find the exact forms of the metric potentials. Following \[13\], we introduce an integrability condition for (7) which is expressed in terms terms of the Weyl tensor. This condition is given by
\[ \mathcal{L}_X C^a_{bcd} = 0, \]  
(9)

where \( C^a_{bcd} \) denotes the nonzero components of the Weyl tensor. This integrability condition is a direct consequence of (7). Then we can show that equation (9) is simplified to yield
\[ \nu_{rr} + \nu_r^2 - \lambda_r \nu_r + r^{-1}(\lambda_r - \nu_r) + \frac{1}{r^2} = (1 + k) \frac{e^{2\lambda}}{r^2}, \]  
(10)

where \( k \) is a constant. Equation (10) is highly nonlinear but it can be integrated in general (see \[13\]). The solution to (10) is given by
\[
e^\nu = \begin{cases} 
Ar \exp \left( \sqrt{1 + k} \int \frac{e^{\lambda}}{r} dr \right) 
+ Br \exp \left( - \sqrt{1 + k} \int \frac{e^{\lambda}}{r} dr \right), & 1 + k > 0 \\
Ar \int e^{\lambda} dr + Br, & 1 + k = 0 \\
Ar \exp \left( \sqrt{- (1 + k)} \int \frac{e^{\lambda}}{r} dr \right) 
+ Br \exp \left( - \sqrt{- (1 + k)} \int \frac{e^{\lambda}}{r} dr \right), & 1 + k < 0,
\end{cases}
\]  
(11)

where \( A \) and \( B \) are constants. This general conformal solution applies to both conformally flat \( (k = 0) \) and non-conformally flat \( (k \neq 0) \) spacetimes.

### 3. Exact solutions

Expressing the conformal conditions (11) in terms of \( x, y \) and \( Z \) and letting \( n = \frac{k + 2}{2} \) we obtain an equivalent condition given by
The conformally flat case is given here by $n = 1$. Substituting (12) into (6) for all values of $n$ and simplifying yields the same equation

$$\dot{Z} = \frac{1}{x}Z + \frac{n}{x} \equiv \frac{1}{2}(\Delta + E^2), \quad \text{for } n \leq \frac{1}{2},$$

(13)

which is a simple linear equation in $Z$. We can integrate (13) to obtain

$$Z = x \int \frac{\Delta + E^2}{2x} dx + mx + n,$$

(14)

where $m$ is an integration constant. The integral in (14) can be evaluated if $\Delta$ and $E$ are specified. Hence the gravitational potentials $y(=e^\nu)$ and $Z(=e^{-2\lambda})$ have been found in general with anisotropy and an electric field when a conformal symmetry exists.

The matter variables $\rho, p_\parallel, p_\perp,$ and $\sigma$ can be written in terms of $\Delta$ and $E$. For the values $n > \frac{1}{2}, n = \frac{1}{2}$ and $n < \frac{1}{2}$ we obtain their explicit forms given below:

**3.1. $n > \frac{1}{2}$**

$$\rho = \frac{1 - n}{x} - \left(3 m + \Delta + \frac{3}{2}E^2\right) - 3 \int \frac{\Delta + E^2}{2x} dx,$$

(15a)

$$p_\parallel = \frac{2 \sqrt{(2n - 1)} \left(\int \frac{\Delta + E^2}{2x} dx + mx + n\right)}{x}$$

$$\times \left\{ \frac{A \exp \left[\int \frac{\sqrt{(2n - 1)} dx}{x \left(\int \frac{\Delta + E^2}{2x} dx + mx + n\right)}\right] - B}{A \exp \left[\int \frac{\sqrt{(2n - 1)} dx}{x \left(\int \frac{\Delta + E^2}{2x} dx + mx + n\right)}\right] + B} \right\} + 3 \int \frac{\Delta + E^2}{2x} dx$$

$$+ \frac{3n - 1}{x} + 3 m + \frac{1}{2} E^2,$$

(15b)

$$p_\perp = p_\parallel + \Delta,$$

(15c)

$$\sigma^2 = \left[4 \int \frac{\Delta + E^2}{2x} dx + \frac{4n}{x} + 4 m\right] (xE + E)^2.$$ 

(15d)

**3.2. $n = \frac{1}{2}$**

$$\rho = \frac{1}{2x} - \left(3 m + \Delta + \frac{3}{2}E^2\right) - 3 \int \frac{\Delta + E^2}{2x} dx,$$

(16a)

$$p_\parallel = \frac{2A \sqrt{x} \left(\int \frac{\Delta + E^2}{2x} dx + mx + \frac{1}{2}\right)}{x \left[\int \frac{\Delta + E^2}{2x} dx + mx + \frac{1}{2}\right]} + 3 \int \frac{\Delta + E^2}{2x} dx$$

$$+ \frac{1}{2x} + 3 m + \frac{1}{2} E^2,$$

(16b)

$$p_\perp = p_\parallel + \Delta,$$

(16c)

**3.3. $n < \frac{1}{2}$**

$$\rho = \frac{1}{2x} - \left(3 m + \Delta + \frac{3}{2}E^2\right) - 3 \int \frac{\Delta + E^2}{2x} dx,$$

(16a)

$$p_\parallel = \frac{2A \sqrt{x} \left(\int \frac{\Delta + E^2}{2x} dx + mx + \frac{1}{2}\right)}{x \left[\int \frac{\Delta + E^2}{2x} dx + mx + \frac{1}{2}\right]} + 3 \int \frac{\Delta + E^2}{2x} dx$$

$$+ \frac{1}{2x} + 3 m + \frac{1}{2} E^2,$$

(16b)

$$p_\perp = p_\parallel + \Delta,$$
\[ \sigma^2 = \left[ 4 \int \frac{\Delta + E^2}{2x} dx + \frac{2}{x} + 4m \right] (x\dot{E} + E)^2. \] (16d)

3.3. \( n < \frac{1}{2} \)

\[ \rho = \frac{1 - n}{x} - \left( 3m + \Delta + \frac{3}{2}E^2 \right) - 3 \int \frac{\Delta + E^2}{2x} dx, \] (17a)

\[ p_\parallel = \frac{2}{x} \left[ -(2n - 1) \left( x \int \frac{\Delta + E^2}{2x} dx + mx + n \right) \right. \]

\[ \times \left\{ A \exp \left( \int \frac{\sqrt{(12n - 1)x} dx}{x} \right) - B \right\} \]

\[ + \frac{3n - 1}{x} + 3m + \frac{1}{2} E^2, \] (17b)

\[ p_\perp = p_\parallel + \Delta, \] (17c)

\[ \sigma^2 = \left[ 4 \int \frac{\Delta + E^2}{2x} dx + \frac{4n}{x} + 4m \right] (x\dot{E} + E)^2. \] (17d)

From the above we observe that the matter variables \( \rho, p_\parallel, p_\perp \) and \( \sigma \) have been expressed in terms of integrals containing \( \Delta \) and \( E \). Thus any choice of \( \Delta \) and \( E \) will lead to an explicit solution of the Einstein-Maxwell system of equations. Physical conditions on the matter distribution e.g. causality and regularity, will restrict the choices made for \( \Delta \) and \( E \).

We can express our general result in the form of the following theorem.

**Theorem 1.** For a static charged anisotropic fluid distribution with a spherical conformal symmetry \( \mathbf{X} = \alpha(t, r) \frac{\partial}{\partial t} + \beta(t, r) \frac{\partial}{\partial r} \) with conformal factor \( \psi(t, r) \), the general solution of the Einstein-Maxwell equations is fully determined. The gravitational potentials \( \gamma(= e^\sigma) \) and \( Z(= e^{-2\sigma}) \) are given by (12) and (14) respectively. The matter variables \( \rho, p_\parallel, p_\perp \) and \( \sigma \) depend on the anisotropy \( \Delta \) and electric field \( E \).

Therefore any solution to the field equations for a static charged anisotropic fluid distribution, in the presence of spherical conformal symmetry, is fully specified by means of the two quantities \( \Delta \) and \( E \). Any solution found using a particular conformal symmetry vector \( \mathbf{X} \) will be contained in our general class. When \( E = 0 \) we regain the anisotropic uncharged fluids and the results of Manjonjo et al [13]. It is possible to find all perfect fluid solutions with conformal symmetry as shown by [13]; the line elements can be written explicitly. We note that Lake [22] established an algorithm to obtain static perfect fluid solutions to the Einstein field equations with a single generating function. Herrera et al [23] extended this work to anisotropic fluids requiring two generating functions. In contrast our treatment requires the presence of conformal vector \( \mathbf{X} \); \( \Delta \) and \( E \) play the role of generating functions.

4. Finch-Skea geometry

The general class of exact solutions with conformal symmetry will contain models of general relativistic compact stars which are of astrophysical significance. We illustrate this by generating a model with Finch-Skea geometry. The Finch and Skea metric [24] has received considerable attention in modeling compact objects as it is a regular solution and it satisfies all criteria for physical acceptability. Hansraj and Maharaj [25] generated charged stars with this geometry describing realistic stellar bodies in terms of elementary functions, Bessel functions and modified Bessel functions. Maharaj et al [26] produced anisotropic charged stellar objects with the Finch-Skea ansatz with a barotropic equation of state. Pandya et al [27] found models with geometry consistent with observed radii and masses for dense stars. The Finch-Skea geometry has also been studied for higher dimensional gravitational geometries [28–31] and trace-free gravity [32].
We take \( \Delta = 0, \ m = 0 \) and \( n = 1 \) in (14) and select an electric field
\[
E^2 = \frac{2x}{(1 + x)^2}.
\] (18)
This gives the potential
\[
Z = \frac{1}{1 + x},
\] (19)
which is in the Finch-Skea model form. Then with \( n = 1 \), equation (12) becomes
\[
y = Ax^2 \exp\left(\frac{1}{2} \int \frac{\sqrt{1 + x}}{x} \, dx\right) + Bx^2 \exp\left(-\frac{1}{2} \int \frac{\sqrt{1 + x}}{x} \, dx\right)
\]
\[
= A(\sqrt{1 + x} - 1)\exp(\sqrt{1 + x}) + B(\sqrt{1 + x} + 1)\exp(-\sqrt{1 + x}).
\] (20)
Then we can find the forms of the matter variables in (5a)–(5d). These are given by
\[
\rho = \frac{3}{(1 + x)^2},
\] (21a)
\[
\rho = (p_\parallel = p_\perp)
\]
\[
= \frac{2}{1 + x} \times \frac{\tanh(\sqrt{1 + x}) + \beta_1}{(\beta_1\sqrt{1 + x} - 1)\tanh(\sqrt{1 + x}) + (\sqrt{1 + x} - \beta_1)}
\]
\[
- \frac{1}{1 + x},
\] (21b)
\[
\sigma^2 = \frac{2(3 + x)^2}{(1 + x)^5}.
\] (21c)

where \( \beta_1 = \frac{A - \beta}{A + \beta} \). Equations (18)–(21c) then comprise an exact solution to the Einstein-Maxwell system with a conformal Killing vector with Finch-Skea geometry. This exact solution is regular and well-behaved with isotropic pressure and an electric field. This model is a special case of the model covered in section 3.3 of the paper by Hansraj and Maharaj [25]. The solution (18)–(21c) is a special case of (43)–(48) in [25] with \( \psi = 1 \) (this \( \psi \) is not to be confused with the conformal factor in this paper). In addition, we observe that \( n = 1 \) implies the spacetime is conformally flat. Therefore the Finch-Skea model of this form is necessarily conformally flat.

5. Known solutions

The main result of this paper is summarized in theorem 1 which states that all solutions of the Einstein-Maxwell equations with conformal symmetry are fully determined. We showed in section 4 that a particular exact solution with a Finch-Skea geometry can be generated from this paper. Also many authors have approached the study of exact models admitting a conformal symmetry by imposing special restrictions on the metric potentials. These particular cases can all be regained from our results; these more restricted models are just special cases of the general solution obtained here. We have collected, in table 1, some of the recent exact models with anisotropy and charge that have been found. We first give the values of the parameters \( n, A \) and \( B \) which are relevant. The metric functions \( y \) and \( Z \) are written in terms of new constants which simplifies comparison with these earlier published papers. We regain the isotropic charged exact solutions of Usmani et al [33] and Mak and Harko [34]. The anisotropic distributions, where charge is absent, correspond to the results of Rahaman et al [35], Rahaman et al [36], Shee et al [14], Mafa Takisa et al [16], Esculpi and Aloma [37]. Models with both charge and anisotropy, include particular exact solutions of Esculpi and Aloma [37] and Newton Singh et al [15]. It is clear that other exact solutions can be generated by considering other functional forms of the anisotropy \( \Delta \) and charge \( E \). Our general conformal solution provides a unifying basis for particular solutions that exist.

6. Conclusion

In this study we have examined static spherically spacetimes which admit a conformal symmetry. We have used this conformal symmetry to obtain a relationship between the metric potentials \( \nu(r) \) and \( \lambda(r) \). This relation enabled us to generate the general exact solution to the Einstein-Maxwell system for a static charged anisotropic fluid distribution. This solution is fully determined by the degree of pressure anisotropy \( \Delta \) and electric field intensity \( E \). Any choices of \( \Delta \) and \( E \) will yield a particular solution to the Einstein-Maxwell system. There is no guarantee that an arbitrary choice will lead to a physically acceptable model. Physical assumptions, such as upper
Table 1. A summary of some of the exact solutions admitting a conformal symmetry found by various authors with anisotropy ($\Delta$) or/and charge ($E$).

| Parameters | Metric functions | $\Delta$ | $E$ | Model |
|------------|------------------|---------|-----|-------|
| $A = 0,$  $n = \frac{1}{2}$ | $y = Bx^\tau$  \( Z = \psi_0 x \) | N       | Y   | Usmani et al [33] |
| $A = 0,$  $n = \frac{1}{2}$ | $y = Bx^\tau$  \( Z = \frac{1 - \tilde{B}x}{3} \) | N       | Y   | Mak and Harko [34] |
| $A = 0,$  $n = \frac{1}{2}$ | $y = Bx^\tau$  \( Z = (1 - a) - bx + \frac{C}{C_f^2 x^2} \) | Y       | N   | Rahaman et al [35] |
| $A = 0,$  $n = \frac{1}{2}$ | $y = Bx^\tau$  \( Z = \frac{1}{C_f^3} \left( \frac{x^2}{\tau^2} \right)^{\alpha - 1} + \frac{C}{3C - 1} \) | Y       | N   | Rahaman et al [36] |
| $A = 0,$  $n = \frac{1}{2}$ | $y = Bx^\tau$  \( Z = -(1 + a_1) + c_1 \ln x + 2K_1 x \) \( 2C_f^2 \) | Y       | N   | Shue et al [14] |
| $A = 0,$  $n = \frac{1}{2}$ | $y = Bx^\tau$  \( Z = \frac{k}{2} \int \frac{dx}{x^{2/\tau}} \) | Y       | N   | Mafa Takisa et al [16] |
| $A = 0,$  $n = \frac{1}{2}$ | $y = Bx^\tau$  \( Z = \frac{1 + \tilde{n A} - \tilde{b} x - \tilde{C}}{2(1 + 2n - C) \left[ 1 + \tilde{n A} - 3B^2 \right]} \) | Y       | Y   | Esculpi and Aloma [37] |
| $A = 0,$  $n = \frac{1}{2}$ | $y = Bx^\tau$  \( Z = \frac{1 + \tilde{n A}}{3n + 1} \left[ 1 + b x - \frac{C}{3R^2 n} \right] \left[ 1 + \frac{C}{3R^2 n} \right] \left[ 1 + \frac{C}{3R^2 n} \right] \) | Y       | Y   | Esculpi and Aloma [37] |
| $A = 0,$  $n = \frac{1}{2}$ | $y = Bx^\tau$  \( Z = \frac{1 + \tilde{n A} - \tilde{b} x - \tilde{C}}{2(1 + 2n - C) \left[ 1 + \tilde{n A} - 3B^2 \right]} \) | Y       | Y   | Newton Singh et al [15] |

Bound on the speed of sound, will place restrictions on the matter variables and metric functions. The results are encapsulated in theorem 1. The work covered by various authors are shown to be special cases of this general solution. These solutions are recovered by setting specific values on some of the parameters in our results. We presented these particular cases in table 1. Finally we examined the Finch-Skea model admitting a conformal symmetry. We have shown that there exists an isotropic charged model with the Finch-Skea type geometry which is conformally flat.
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