Supercattering Channels of Nonspherical structurers

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Abstract. A superscattering structure is an efficient energy-mapping device that of particular importance for various electromagnetic experiment methods, with potential sensing and energy harvesting applications. We study in this work the scattering cross-section of outgoing channels in the irreducible and singular basis for an arbitrary shape scatterer. The superscattering status is shown to occur within a single outgoing channel of an optimized cluster of cylinders, a forbidden mechanism in spherically symmetric Mie resonators.

1. Introduction
The expanded and outgoing investigation of the optical responses of high-index low-losses materials has resulted in elaborated design methods to enhance scattering significantly [1]–[8]. This is particularly important to increase efficiency of optical devices and technologies for various applications[5], [9]–[13]. For a given scattering problem, the far-field scattering is fully described by a set of incoming and outgoing spherical waves mapped through the S-matrix. It has been shown that the contribution of a single scattering channel is limited to $2(2n + 1)\pi/k^2$ for scatterers with spherical symmetry, where $n$ is the multipole order; and every Mie mode contributes to one multipolar channel [6], [14]. It is therefore established that if scattering exceeds (by far) the dipolar channel limitation ($6\pi/k^2$) it is regarded as superscattering[6]. However, any degradation of the special spherical symmetry allows for, along with the Mie modes, the excitation of the Fabry-Perot modes that oscillate transversely to the scatterer plane and scatter at various channels imposed by the new reduced symmetry. In this scenario, the limitation of the maximum power that can be carried out by a scattering channel (ex. spherical basis) can be determined by a collective of degenerate modes. We attempt in this work to obtain general symmetry-related limitation on these channels. Nevertheless, a structure with limited rotational or reflection symmetries allows for different approaches to enhance the overall scattering [15]. Considering the realization simplicity, we optimize a cylinder as a resonating building block in a symmetric cluster with a discrete point group symmetry ($D_{4h}$), achieving unrepresented values of superscattering.

2. Theory
To define a new basis for the scattering channels that is a property of the scatterer symmetry, we incorporate two methods acting on the S-matrix as defined in the spherical basis [16]: the singular value decomposition (SVD) and the irreducible representation. Starting from the SVD of the S-matrix, energy conservation requires a limitation on its singular values ($\lambda_w(S) \leq 1$) [17]. Using the triangle inequality and submultiplicative properties of the SVD method, and the
Figure 1. T-matrix block-diagonal form after being projected on the irreducible basis of $D_{4h}$ symmetry with each ($T_\mu$) represents a subspace. On the right side, the schematic illustrates that the differential scattering of each scattering channel in the irreducible basis is a combination of a set of spherical scattering channels.

relation of the S-matrix $S = 2T + I$, we obtain the following limitation on the T-matrix singular values

$$\lambda_w(T) = \frac{1}{2} \lambda_w(S - I) \leq \frac{1}{2} (1 + \lambda_w(S)),$$

which implies also that the T-matrix singular values are equal or less than unity. We then take a step further to exploit the symmetry properties of the scatterer with the projection of the T-matrix on the irreducible basis; which describes the transformation properties of a set of eigenfunctions and corresponds to a distinct energy eigenvalue. Transforming the reducible representation into the irreducible one breaks up the energy levels into fewer degenerate levels. For particles with discrete geometrical symmetries, the T-matrix can be made block diagonal $T_{\text{irr}} = P^{-1}TP = \text{diag}(T_1, \ldots, T_C)$ using group theory, where $P$ is an orthogonal projector matrix [16], $C$ is the number of the irreducible representations and it is identical to the number of the classes of the scatterer point group symmetry (see fig.1). Every irreducible representation is a regular matrix with $(w_\mu \times w_\mu)$, where $\mu = 1, \ldots, C$. Now we reapply the SVD for the T matrix in the irreducible basis $W_{\text{irr}}^\dagger T_{\text{irr}} N_{\text{irr}} = \Lambda_{\text{irr}}$, where $\Lambda_{\text{irr}} = \text{diag} (\lambda_{11}, \ldots, \lambda_{\mu w_\mu}) \in \mathbb{R}^{w \times w}$ since the projector matrix performs a similarity transformation, the singular values $\lambda_{\mu w_\mu}$ are identical to the singular values of the T-matrix in the reducible basis and thus subject to the aforementioned inequality. The left and right singular matrices $W_{\text{irr}} \in \mathbb{C}^{w \times w}$ and $N_{\text{irr}} \in \mathbb{C}^{w \times w}$ are unitary, orthonormal, and take the same block diagonal form of the T-matrix in the irreducible representation. Therefore, every submatrix of the degenerate subspace is, in fact, being decomposed separately from the other subspaces. We exploit this feature to derive the scattering cross section in the following form

$$\sigma_{\text{sca}} = \frac{2\pi}{k^2} ||Ta||^2 = \frac{2\pi}{k^2} ||P^{-1}W_{\text{irr}}^\dagger \Lambda_{\text{irr}} N_{\text{irr}}^{\dagger} Pa||^2 = \frac{2\pi}{k^2} \sum_{\mu} \sum_{w_\mu} \lambda_{\mu w_\mu}^2 \left| \mathbf{n}_{w_\mu}^{(\dagger)} \cdot \mathbf{a} \right|^2,$$

where the vector $a$ represents the spherical expansion coefficients of the incident wave, the sub-vector $\mathbf{a} = Pa \in \mathbb{C}^{w_\mu \times w_\mu}$ refers to the projected input vector that is transferred by the
Figure 2. (a) Scattering cross section decomposed according to the power mapped by subspaces ($\mu$) in the irreducible representation of the T-matrix [16]; and normalized to the dipolar-resonance maximum scattering in a spherical object $N_s = \sigma_{sca}/(6\pi/k^2)$. The optimized structure as shown in the inset is a D4h cluster of identical cylinders positioned in the xy-plane around the coordinate origin. The cylinders made from dielectric ceramic materials with height 9.82(mm), radius 3.778(mm), and permittivity 24.5. (b) Spherical multipole decomposition of scattering cross section. For clarity purposes the decomposition here is presented for only the first 4 multipoles orders, however to fully recover scattering 9 orders were considered [18]. The legend designate multipole order with either electric (E) or Magnetic polarity (M), and the red line is the total normalized scattering cross section. (c) Norm $|E_x|$ of the total electric field at wavelength maxima 21.95(mm). Notice the incident plane wave is x-polarized and propagating in the z-direction.

3. Results
In Fig. 2, we present an optimized result of scattering by a dielectric cluster (see caption for details), these result has nearly tripled what has been shown in literature [6], [14]. Every element in the cluster is a resonating Huygens source with a highly asymmetric scattering pattern, which has reduced the destructive interference in the whole structure. As can be seen from Fig. 2(a), most of the degenerate levels transfer energy that is higher than the spherical dipolar level. However, the total scattering power is more evenly distributed between them if compared to the conventional spherical decomposition (Fig. 2(b)). We also note that the magnetic octupole channel carries four times the power of the spherical dipolar limit. Note that the octupolar channel remains in superscattering status if a normalization standard is considered relative to its maxima in an uncoupled spherical system $(14\pi/k^2)$. The near-filed profile in Fig. 2(c) illustrates the strong and directive concentrations of the input wave in the forward direction.

4. Conclusion
In conclusion, the incident wave and the symmetry of a scatterer have been formed in a rigorous way to determine a limit on a single channel scattering power for arbitrarily-shape scatterer. We
also carried out a numerical optimization of a cluster of cylinders and obtain large superscattering values amount up to 18 times that of the maximum dipolar channel in a spherical scatterer.

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