Magnetosphere response to stationary solar wind forcing at various plasma resistivities in 2D MHD simulation

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Abstract. The magnetosphere response to the stationary solar wind forcing at various values of the plasma resistivity is studied. The simulation is performed within two-dimensional (2D) MHD resistive model; the meridional plane is considered. The solar wind and southward interplanetary magnetic field (IMF) are supposed to be stationary. The calculations were carried out for two plasma resistivity values $\eta_p = \{10^9, 10^{10}\} \Omega m$, assumed constant in the whole flow field. At higher plasma resistivity the steady magnetosphere convection (SMC) takes place; the solution is symmetric relative to the $x$-axis. At lower resistivity the sawtooth event is realized; the solution is quasi-periodic and asymmetric. The asymmetry is connected with the formation of the vortex street of Karman type. The sequence of phases of the sawtooth cycle, as well as the evolution of magnetic lines of force and the flow structure in each phase is analyzed. In both solutions the day side magnetic reconnection occurs through the two x-points disposed symmetrically at the top and bottom peaks of the magnetic island.

1. Introduction.

The magnetic reconnection is an essential element of the magnetosphere dynamics causing magnetic substorms – the main factors of space "weather" (see, i.e., books [1], [2]). In MHD approach the solar wind acts as the reconnection driver, different parameters of which (i.e. velocity, density, resistivity, and interplanetary magnetic field (IMF)) can onset reconnection in various ways. According to modern concepts, in a number of physical mechanisms influencing the reconnection process, the leading role belongs to the tearing-instability [3]. This instability develops at the condition $Lu > 1$ fulfilled for the current sheet (the Lundquist number $Lu = \mu \nu / \eta$, where $\eta, \nu$ are characteristic values of the resistivity, Alfvén speed, and sheet thickness, respectively) [1]. Thus, to know local value of the Lundquist number needs to perform global modeling of the magnetosphere dynamics at given solar wind parameters. Although the MHD approach is based on the macroscopic fluid dynamics and cannot describe all the microscopic kinetic details of the magnetospheric response to solar wind forcing, it remains an important tool for studying the dynamics of the magnetosphere as a system as a whole.

There is a number of works devoted to the problem of magnetosphere MHD activity under the solar wind forcing. In [4] the role of the two solar wind parameters, the velocity and the southward magnetic field, was studied with the use of the global MHD simulation. It was concluded that the solar wind velocity is the leading factor controlling the state of the geomagnetic tail under the same other driving conditions (see also [5], [6]). In [7] in the framework of the 3D MHD simulation the onset of
reconnection in the magnetotail at the southward IMF was investigated. The resistivity was varied in calculations only to suppress numerical oscillations. Its value remained close to the order of the Spitzer magnitude (i.e., practically collisionless plasma) and did not affect the physical results.

As argued in [8], many different attempts have been made to incorporate microscopic physics into MHD approach (hybrid and kinetic models - see, for example, [9], [10]), but all of them are inferior in versatility and efficiency to global MHD models. In addition, the use of kinetic approaches does not always give the best result when model is validated on satellite observations ([11], [12]). Therefore, global MHD models continue to be used in magnetospheric studies. Numerous references to relevant works can be found in reviews and books (see cited above [1], [2]).

In this paper, the driving impact of the solar wind resistivity on the magnetosphere behavior is considered. To select basic value of resistivity, it was estimated according to the formula for the Sagdeev’s resistivity [13] and to the measured width of the MHD bow shock [14]. These two estimations give equal order of magnitude, \( \eta \sim 10^5 \, \Omega \cdot m \). In section 1, we describe the setup used for global MHD simulation. Results of global MHD simulations at high resistivity relating to steady magnetospheric convection are briefly presented in section 2. Simulation data on low resistivity describing sawtooth events are analyzed in section 3. These data were given special attention.

2. Problem formulation.

The model problem of the solar wind flow past the Earth’s magnetosphere is studied. An infinitely long cylindrical body carrying magnetic dipole is flowed over by a hypersonic flow of fully ionized hydrogen plasma. The body has a square cross section; the velocity vector of the incoming flow is directed normal to the side face of the body. The computational area occupies the rectangle extending from \(-600 \, R\) to \(+100 \, R\) in the \(x\) direction and from \(-300 \, R\) to \(300 \, R\) in the \(z\) direction. The scale parameter \(R\) equals to \(6 \cdot 10^6 \, m\). The side of the body cross section with the central point at \(x = 0\) and \(z = 0\) is 2.8 \(R\). The magnetic field is created by oppositely directed currents flowing in two infinitely thin parallel wires. Wires are laid along the normal to the body cross section and pass through it symmetrically relative to the center of coordinates at the distances of \(1R\). In each wire the electrical current of \(170 \, MA\) is supplied.

At the sunward boundary the solar wind velocity, pressure, density and the interplanetary magnetic field are given. At other boundaries of the calculation area the “soft” conditions are applied. At the body boundaries the zero velocity normal component is preset.

We accept the following values of solar wind parameters: velocity \(u_0 = -4 \cdot 10^5 \, m/s\), \(w_0 = 0\); interplanetary magnetic field \(B_20 = -1 \, nT\), \(B_x0 = 0\); pressure \(P_0 = 10^{-10} \, Pa\); density \(\rho_0 = 2 \cdot 10^{-20} \, kg/m^3\). The plasma resistivity is assumed to be constant and is evaluated on the base of the satellite measurements of the bow shock width. The data processing performed by [14] indicates that the magnitude order of the shock width amounts to \(\Delta \sim 10^6 \, m\). Estimating the plasma resistivity as \(\eta_p = \mu \eta_0 \Delta\) for the the solar wind velocity \(u_0\) the result turns out \(\eta_p \sim 10^5 \, \Omega \cdot m\). The same value gives Sagdeev’s formula [13] at the conditions of high overheating of ions relative to electrons. For the analysis of the resistivity influence on the reconnection mode the two resistivity values of \(10^6\) and \(10^4 \, \Omega \cdot m\) are accepted. The magnetic Reynolds numbers corresponding to these solutions are 6.4 and 640, respectively. The resistivity of the rod matter equals to \(10^{11} \, \Omega \cdot m\).

The gasdynamic part of the problem is described by the Euler equations with the MHD source terms:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho w) = 0 \tag{1}
\]

\[
\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2 + \rho w^2) + \frac{\partial}{\partial y}(\rho u w) = -\frac{\partial p}{\partial x} + j_x B_z \tag{2}
\]

\[
\frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial x}(\rho u w) + \frac{\partial}{\partial y}(\rho w^2) = -\frac{\partial p}{\partial y} - j_y B_z \tag{3}
\]

\[
\frac{\partial}{\partial t}(\rho (\varepsilon + \frac{u^2 + w^2}{2})) + \frac{\partial}{\partial x}(\rho u (\varepsilon + \frac{u^2 + w^2}{2})) + \frac{\partial}{\partial y}(\rho w (\varepsilon + \frac{u^2 + w^2}{2})) = -\frac{\partial}{\partial x}(pu) - \frac{\partial}{\partial y}(pw) + j_y E_x \tag{4}
\]
\[ \varepsilon = \frac{1}{\gamma - 1} \frac{p}{\rho} \]  
(5)

The medium is considered as an inviscid non-heat-conducting perfect gas consisting of protons and electrons with the specific heat ratio \( \gamma = 5/3 \).

The electrodynamics is described by the Maxwell equations and the Ohm’s law (the Hall effect is neglected):

\[ j_y = -\frac{1}{\mu} \left( \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial z} \right) \]  
(6)

\[ \frac{\partial B_y}{\partial t} = \frac{\partial E_y}{\partial z} \]  
(7)

\[ \frac{\partial B_x}{\partial t} = \frac{\partial E_x}{\partial x} \]  
(8)

\[ E_y = \frac{j_y}{\sigma} - w B_x + u B_z \]  
(9)

All quantities in equations (1)-(9) are given in usual physical notation.

The computational domain was divided by an orthogonal uniform grid with a step of 0.2R, i.e. the grid dimension is 3500\times1500. The hydrodynamic equations have been integrated with the usage of the explicit second order accurate TVD scheme [15]. The electrodynamic equations have been solved using the described in [16] explicit scheme with second order in space and first order in time resolution.

3. Steady Magnetospheric Convection event. Symmetric solution

At the higher resistivity, \( \eta = 10^6 \ \Omega \cdot m \), the simulation represents magnetosphere steady-state response to the stationary solar wind forcing (i.e. steady magnetospheric convection event is realized). The solution is symmetric relative to the \( x \)-axis. Figure 1 presents magnetic force lines with colored electric current density (left panel) and flow streamlines with colored plasma pressure (right panel). Due to the symmetry of the solution, the figure shows its upper half.

**Figure 1.** Magnetic force lines with background distribution of the electric current density (left panel) and flow streamlines with background distribution of the pressure (right panel). The vector lines and the background contours are symmetric relative to the \( x \)-axis.

The classic structure of magnetosphere in meridian plane is realized with the all standard elements. The bow shock is detached from the body at the distance \( x = 12 \ R \). The velocity ratio at the central part of the bow shock is near to the limiting value \( (\gamma + 1)/(\gamma - 1) \approx 4 \). At the day side and the night side of the body the attached fluid double vortices are formed.

The balanced reconnection occurs with the rate \( E_y = 0.4 \) mV/m both at the day and night sides. The dayside reconnection occurs via magnetic island (not seen in Figure 1, left panel, because of tiny dimensions). The magnetic island O-point is located at \( x = 11 \ R \), the two X-points are shifted vertically relative the O-point. The nightside X-point is located at \( x = -35 \ R \) and implements contact between the dipole and the IMF.
4. Sawtooth event. Asymmetric solution

In this section the lower resistivity, $\eta_p = 10^4 \Omega \cdot m$, is accepted. This leads to arising of the quasi-periodic substorm or the sawtooth event. The sawtooth event cycle contains the all substorm phases. In Figure 2 the temporal variation of the open magnetic flux per cylinder unit length $\Phi_0$ is shown. The open flux $\Phi_0$ is determined as the difference between values of magnetic vector potential component $A_y$ at the dayside and the nightside $x$-points.

![Figure 2](image1.png)

**Figure 2.** The sawtooth cycle series. The temporal variation of the open magnetic flux per cylinder unit length $\Phi_0$ is shown. The growth, expansion and recovery phases are indicated by shaded vertical bands in yellow, light violet and azure, respectively.

The temporal evolution of magnetic field and flow velocity topology is being considered for the third sawtooth cycle of 8.2-12.8 ms interval (the period is approximately 4.6 ks). The cycle starts at the minimum value of the open magnetic flux. The flux $\Phi_0$ is monotone increasing with the reconnection moment of 9.9 ms switching to the expansion phase. Reaching the sharp peak it drops to the next minimum value what terminates the cycle.

The topology of the magnetic field in the dayside of magnetosphere is presented in Figure 3. The structure of magnetic field is practically steady and symmetric and the only its northern part is shown. Just in the front of the bow shock the magnetic island is formed. It is elongated vertically with the $X$-points located at the coordinates $x = 6.5 R, z = \pm 5.6 R$. The formation of the open magnetic force lines and the magnetopause is seen. The positive current generated in the shock layer is balanced by the negative current in the cusp.

![Figure 3](image2.png)

**Figure 3.** The dayside magnetic field force lines and the colored current density (upper part of the symmetrical picture is shown).

In Figure 4 the tooth variation of the magnetic field topology with the background current density $j_y$ is shown. The sharp change in the tail dynamics is observed comparing to Figure 1. At first the asymmetric longitudinal oscillations of the magnetic force lines and the current layer appear. Then the formation of plasmoids and their intrainment with plasma flow takes place. The moving downstream large plasmoid, magnetic island, born in the previous tooth is seen in Figure 4a. The plasmoid has been separated from the growing dipole night sector by the IMF. The IMF lines are strongly stretched sunward resulting in the intense current sheet. In the sheet the secondary magnetic island arises (see Figure 4b).
Figure 4. The magnetic field vector lines and the colored current density for the tooth cycle of 8.2–12.9 ks.

In the IMF interval the transverse oscillations of the current sheet appear. On top and below the oscillations extremums the moderate spots of the current density are exposed. They produce the lateral force $\mathbf{j} \times \mathbf{B}$ which generates the transverse oscillations of plasma flow (see below Figure 5).

At the time moment of 9.9 ks the magnetic field reconnection sets in indicating the transition from the growth phase to the expansion phase. Inside of the dipole night sector the magnetic island is growing what increases the sector dimensions. The magnetic island separates from the dipole night sector at the time 11.6 ks starting the recovery phase. The island is bulging and speeding off by the plasma mantle.

The streamlines and the colored vorticity $\mathbf{\omega} = \text{rot} \mathbf{v}$ are shown in the Figure 5. In the growth phase (see Figure 5a,b) the beginning of the tail vortex street is seen. In the first frame the splitting of the upper attached vortex occurs. The splitting originates at the point where the plasma velocity reduces to zero and the length-wise net force ($-\nabla p + \mathbf{j} \times \mathbf{B}$) alternates its sign what stretches the vortex out and tears it in two parts. This mechanism is argued in [17]. Also we see the streamlines oscillations in front of the vortex wake which are consistent with the transverse oscillations of the magnetic force lines in Figure 4a,b. The vortex shedding is continuing in the expansion phase, Figure 5c,d. The vortex street is similar to that of the classic Karman vortex street. The vortex wake is confined by the dipole magnetic field with the implanted magnetic island.
Figure 5. The plasma streamlines and the colored vorticity for the tooth cycle of 8.2-12.9 ks.

When in the recovery phase, Figure 5e,f, the magnetic island separates and the dipole sector shrinks the vortex train is quenched by the magnetic field shear. The only two vortices are attached to the body. Their extension will produce the vortex splitting and shedding in the next tooth. The frequency of vortex formation depends on the phase of the sawtooth event: it is near 185 c in the growth phase, near 165 c in the expansion phase, and near 120 s in the recovery phase.

5. Conclusions
Solar wind resistivity controls magnetospheric tail response modes, the SMC and the sawtooth events, under conditions of the rest solar wind parameters being of the same values. The MHD modeling resistivity might be evaluated from the measured width of the bow shock and from the computed anomalous collision frequency. In computations the two values of resistivity are accepted. At high resistivity of $10^6 \, \Omega \cdot m$ the simulation presents the SMC flow. At low resistivity of $10^4 \, \Omega \cdot m$ the tail sawtooth events are realized.

The SMC evolution of topology of the magnetic field and the flow velocity illustrates the dayside and the night side flow dynamics. In the day side the reconnection is quasistationary. It is formed in the
shock layer through the thin magnetic island with two vertically positioned X-points. Downstream the shock and the magnetic island the double vortex is generated. In the night side the sawtooth cycle sets in when the open magnetic flux arrives at minimum and starts to grow. In the near tail the dipole constituent of magnetic field is expanding. The attached fluid vortices are splitted and begin with shedding. The splitting is provided by the combined tensile impact of the ponderomotive force and the pressure gradient. In the expansion phase the vortex street is formed. The vortex street propagates within the range of the dipole and decays at the magnetic island separation. After the magnetic island separation the open magnetic flux decreases, the magnetic dipole and the attached double vortex are spreading in the near wake. This eventually accomplishes the sawtooth cycle.

6. References

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