Atom localization in cascade type system

Kalan Mal 1,2,*, Suman Mondal 1, Dipankar Bhattacharyya 3 and Amitava Bandyopadhyay 1, #

1Department of Physics, Visva-Bharati, Santiniketan, PIN 731235, West Bengal, India
2Department of Physics, Suri Vidyasagar College, Suri, PIN 731101, West Bengal, India
3Department of Physics, Santipur College, Santipur, PIN 741404, West Bengal, India

Email: *kmsvc08@gmail.com, #m2amitava@gmail.com

Abstract: A three-level cascade type system is subjected to a standing wave (SW) field acting between the ground energy level and the intermediate energy level of the system and the probe field scans the uppermost energy level from the intermediate energy level. Optical Bloch equations (OBE) for this three-level system are derived from the Liouville equation (Master equation) where the decay terms are added phenomenologically. These OBEs are solved analytically under steady state condition by using weak probe approximation. Under doppler free condition precession of localization was controlled by tuning the SW rabi frequency and relative orientation of the applied fields.

Introduction:

Atom localization [1] is a process in which atoms get confined within a very narrow spatial region. Precession measurement of a single atom has potential application in nanolithography [2], Bose Einstein Condensation [3] and laser cooling [4]. The strong localization of atoms in cold atomic system also modifies the optical properties of the medium and can be used in fabricating optical logic gates, storage of light etc. There are several reports on different techniques to localize atoms within a narrow spatial region. Thomas and his co-workers demonstrated that sub optical wavelength localization could be achieved via a light-shift gradient for atom imaging [5]. Later atom localization was achieved by atoms interacting with a standing wave and this was confirmed by using the phase shift measurement of the optical field [6], homodyne detection [7] and quantum trajectories [8]. Later phase shift of atomic dipole-moment [9] and entanglement between the atomic position and its internal state were used to localize the atom without directly affecting the spatial wave function of the particle [10]. Detection of spontaneously emitted photon due to its interaction with a classical standing wave field and the reservoir modes [11] has also been suggested by several groups but it is not easy to control spontaneous emission experimentally. To overcome this difficulty measurement of upper level population [12], probe absorption [13] and coherent population trapping [14] were used for atom localization study. All these mentioned phenomena [12-14] have experimental realization in pump-probe experiment. B.K. Dutta et al discussed the electromagnetically induced grating [15] phenomenon by using a three level Ξ type system interacting with one dimensional (1D) standing wave field. Ivanov and Rozhdestvensky have proposed a two-dimensional (2D) atom localization scheme using a four-level tripod system via measurement of the population in the upper state or in any ground state [16]. Atom localization via spontaneous emission in a five-level M-type atomic system
[17] and probe absorption in a microwave-driven [18] four-level atomic system were also studied. Knight et al [12] observed 1D subwavelength localization of a moving atom in a three-level Λ type system by measuring the upper level population and recently Rahamatullah et al extended this study [19] for 2D atom localization by probe absorption measurement. In this article a three level Ξ type atomic system interacting with 1D strong standing wave and a weak travelling field has been studied for measurement of atom localization using population of different levels as well as probe absorption. This study is done in Doppler free environment and the parameters of $^{87}$Rb $5S_{1/2} \rightarrow 5P_{3/2} \rightarrow 5D_{3/2}$ transitions are used in simulation. The position and precession of atom localization are controlled by varying Rabi frequency of the standing wave field and relative orientation of the applied fields.

**Theory:**

In the theoretical model we have considered a three-level Ξ type system. The strong standing wave field (control or pump field) acts between the ground level $|1\rangle$ and the intermediate level $|2\rangle$ whereas level $|2\rangle$ and level $|3\rangle$ are coupled by a weak probe field. The model is shown in figure 1. The standing wave is formed along the $x$-direction due to counter-propagating components of pump fields. The probe field which is assumed to be spatially uniform along the $x$-direction can pass through the standing wave regime along the orthogonal $z$-direction. The relative orientation (not shown in figure 1.) of propagation vectors of the probe field ($\vec{K}_p$) and SW field ($\vec{K}_c$) is denoted by the angle $\phi$. The components of $\vec{K}_c$ along $z$-axis and $x$-axis are $K_{cz}$ and $K_{cx}$ respectively and $K_p$ is the magnitude of $\vec{K}_p$. The propagation vectors of the probe field and SW field are given below.

$$\vec{K}_p = K_p \hat{z}, \vec{K}_c = K_{cz} \hat{z} \pm K_{cx} \hat{x}$$

![Figure 1: Schematic diagram of three-level Ξ type system](image)

The Hamiltonian for the three-level Ξ type system is given by

$$H = H_0 + H_I$$

$$H_0 = \sum_{i=1}^{3} \hbar \omega_i |i\rangle \langle i|$$

$$H_I = \frac{\hbar \alpha}{2} \sin \left( \frac{\pi x}{L} \right) |2\rangle \langle 1| e^{-i\omega_c t} + c. c.) + \frac{\hbar \alpha}{2} (|3\rangle \langle 2| e^{-i\omega_p t} + c. c.)$$
The time evolution of the density matrix operator $\rho$ of the system is governed by the Liouville equation (or the Master equation) with the phenomenological decay terms added.

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H, \rho] + \Lambda_{\text{relax}}$$

(H$_0$ is the Hamiltonian of the bare atomic system and $H_t$ is interaction Hamiltonian of the system concerned. $\Omega_p$ and $\Omega_c$ stand for the Rabi frequencies of the probe field and control or pump field respectively. $\omega_p$ and $\omega_c$ represent the probe and control frequencies in angular scale. $\Lambda$ denotes the separation between two consecutive nodes and antinodes of the standing wave. $\gamma_{32}$ and $\gamma_{21}$ are the spontaneous decay rates from the level $|3\rangle$ to level $|2\rangle$ and from level $|2\rangle$ to level $|1\rangle$ respectively and these are shown by dashed lines in Fig 1. A set of nine Optical Bloch equations (OBEs) for the three-level system are derived under rotating wave approximation.

$$\frac{\partial \rho_{11}}{\partial t} = \frac{i \Omega_p}{2} \sin\left(\frac{\pi x}{\Lambda}\right) (\rho_{21} - \rho_{12}) + \gamma_{21} \rho_{22}$$

(5)

$$\frac{\partial \rho_{22}}{\partial t} = -\frac{i \Omega_p}{2} \sin\left(\frac{\pi x}{\Lambda}\right) (\rho_{21} - \rho_{12}) + \frac{i \Omega_c}{2} (\rho_{32} - \rho_{23}) - \gamma_{23} \rho_{22} + \gamma_{32} \rho_{33}$$

(6)

$$\frac{\partial \rho_{33}}{\partial t} = -\frac{i \Omega_c}{2} (\rho_{32} - \rho_{23}) - \gamma_{32} \rho_{33}$$

(7)

$$\frac{\partial \rho_{21}}{\partial t} = (-\Gamma_{21} + i \Delta_c) \rho_{21} - \frac{i \Omega_c}{2} \sin\left(\frac{\pi x}{\Lambda}\right) (\rho_{22} - \rho_{11}) + \frac{i \Omega_p}{2} \rho_{31}$$

(8)

$$\frac{\partial \rho_{31}}{\partial t} = (-\Gamma_{31} + i (\Delta_p + \Delta_c)) \rho_{31} - \frac{i \Omega_c}{2} \sin\left(\frac{\pi x}{\Lambda}\right) \rho_{32} + \frac{i \Omega_p}{2} \rho_{21}$$

(9)

$$\frac{\partial \rho_{32}}{\partial t} = (-\Gamma_{32} + i \Delta_p) \rho_{32} - \frac{i \Omega_p}{2} (\rho_{33} - \rho_{22}) - \frac{i \Omega_c}{2} \sin\left(\frac{\pi x}{\Lambda}\right) \rho_{31}$$

(10)

Although we have shown only six OBEs here, the remaining three can be obtained easily by taking complex conjugation of equations 8, 9 and 10. Here $\Delta_p$ and $\Delta_c$ are the detunings of probe and control fields respectively with $\Delta_c = \omega_c - \omega_{21}$ and $\Delta_p = \omega_p - \omega_{32}$. The off-diagonal decay rates are defined as $\Gamma_{ij}$ with $i \neq j$. These OBEs are solved analytically under steady state condition ($\frac{\partial \rho_{ij}}{\partial t} = 0$, with $i,j=1,2,3$) in weak probe regime ($\Omega_p \ll \Omega_c$) using population conservation $\sum_{i=1}^{3} \rho_{ii} = 1$ to determine the population ($\rho_{ii}$) of different levels and probe coherence term ($\rho_{32}$). These solutions are given below:

$$\rho_{11} = \frac{\Omega_p^2 \sin^2\left(\frac{\pi x}{\Lambda}\right) L + 2 \gamma_{21}}{2 \Omega_p^2 \sin^2\left(\frac{\pi x}{\Lambda}\right) L + \gamma_{21}}$$

(11)

$$\rho_{22} = \frac{\Omega_c^2 \sin^2\left(\frac{\pi x}{\Lambda}\right) L + 2 \gamma_{31}}{2 \Omega_c^2 \sin^2\left(\frac{\pi x}{\Lambda}\right) L + \gamma_{31}}$$

(12)

$$\rho_{33} = 0$$

(13)

$$\rho_{32} = -\frac{i}{2} \frac{\Omega_p}{\Omega_p^2 + \Omega_c^2} \frac{A C (\rho_{33} - \rho_{22}) + B C \rho_{22} - \rho_{11}}{BC (\rho_{32} - \rho_{22}) + A C \rho_{22} - \rho_{11}}$$

(14)

Here $L = \frac{\Gamma_{21}}{\Gamma_{21} + \Delta_c}$; $A = -\Gamma_{32} + i \Delta_p$; $B = -\Gamma_{21} + i \Delta_c$; $C = -\Gamma_{31} + i (\Delta_p + \Delta_c)$

The expressions of the probe absorption (imaginary part of $\rho_{32}$ ) and $\rho_{ii}$ are used to study atom localization.
Results:

The population of levels $|1>$ and $|2>$ vs different position of the atoms is plotted for different $\Omega_c$ (figure 2). At low $\Omega_c$ almost entire population is trapped in level $|1>$ and shows little undulation along x direction. The strength of control field is position dependent due to the standing wave formation in x direction and this variation becomes prominent for higher values of $\Omega_c$ resulting in redistribution of population between the energy levels. At $\Omega_c = 30 \text{ MHz}$ a periodic array of sharp spikes are observed in the anti-node positions of the standing wave indicating strong atom localization at these positions within sub-wavelength region. The degree of localization increases if level $|2>$ be metastable state but for experimental realization of this theoretical study we have used the parameters of $^{85}\text{Rb} \begin{array}{c} 5S_{1/2} \\ \rightarrow \\ 5P_{3/2} \\ \rightarrow \\ 5D_{3/2} \end{array}$ where $5P_{3/2}$ is not a metastable state. At node positions the effective strength of the control field is maximum and the population is distributed almost equally between the two states $|1>$ and $|2>$ (50% each). The separation between two consecutive node and antinode ($\Lambda$) depends on the wavelength of the control field as well as the relative orientation of the applied fields. This causes alteration in position of atom localization.

![Figure 2](image)

**Figure 2:** Plots of population vs. atomic position under Doppler free condition at different control Rabi frequencies. The control field is kept on-resonant.

Probe absorption ($\text{Im}[\rho_{32}]$) vs $\phi$ (angle between probe field and control field) is plotted in figure 3. Both the fields are kept on-resonant i.e. $\Delta_c = 0$ and $\Delta_p = 0$. The probe absorption depends on two separate phenomena. It is a superposition of atom localization and Electromagnetically induced transparency (EIT) [20]. As mentioned in the previous section the node and antinode positions depend on the relative orientation of the applied fields ($\phi$). Varying $\phi$ means controlling node and anti-node positions. At node positions (sharp dips at $\phi = 0, \pm \frac{\pi}{6}$ and $\pm \frac{\pi}{2}$ in figure 3) the transparency is due to the strong localization of atom in the ground level. In this condition the probe field finds no population in the intermediate level to transfer in the uppermost level and the probe absorption is almost zero. The transparency in probe field absorption also occurs at anti-node position although 50% population exists in the intermediate level. The reason for this transparency is the destructive quantum interference resulting the formation of EIT (dips at $\phi = \pm \frac{\pi}{12}$ and $\pm \frac{\pi}{4}$ in figure 3). The
widths of these transparency windows depend differently on \( \Omega_c \), it decreases for atom localization and increases for EIT with increases in \( \Omega_c \).

Figure 3: Plots of probe absorption in arb. unit vs. relative orientation of the applied fields under Doppler free condition at different control Rabi frequencies with \( \Delta_p = \Delta_c = 0 \) and \( \Omega_p = 1 \) MHz.

Conclusion:

In this work the 1D atom localization in a three-level \( \Xi \) type system, where unlike the conventional pump-probe study we have applied a strong standing wave field between the ground level and intermediate level, has been studied using population measurement of different energy levels as well as probe absorption. The degree of atom localization can be controlled by Rabi frequency of control field. Under this atom localization almost 100% reduction in probe absorption has been made possible by tuning the relative orientation of the applied fields. Atom localization within a narrow region can be used in nanolithography and periodic transparencies in probe absorption profile can have potential application in optical devices fabrication.

Acknowledgement

KM acknowledges UGC (ERO) for granting a minor research project (F. No. PSW: 050(2015-16), dated-16/11/2016). DB acknowledges SERB for granting a project under Teaching Associateship for Research Excellence (TARE) scheme (sanction no.TAR/2018/000710).

References

[1] Mitsunga M and Imotto N 1999 Phys. Rev. A 59 4773
[2] Johnson K S, Thywissen J H, Dekker N H, Berggren K K, Chu A P, Younkin R and Prentiss M 1998 Science 280 1583
[3] Collins G P 1996 Phys. Today 49(8) 18
[4] Phillips W D 1998 Rev. Mod. Phys. 70 72
[5] Thomas J E 1990 Phys. Rev. A 42 5652
[6] Storey P, Collet M and Walls D 1992 Phys. Rev. Lett. 68 472
[7] Quadt R, Collet M and Walls D F 1995 Phys. Rev. Lett. 74 351
[8] Carmichael H J 1993 An Open System Approach to Quantum Optics: Lecture Notes in Physics (Berlin Springer-Verlag)
[9] Kunze S, Rempe G and Wilkens M 1994 Euro Phys. Lett. 27, 115
[10] Kunze S, Dieckmann K and Rempe G 1997 Phys. Rev. Lett. 78 2038
[11] Ghafoor F, Qamar S and Zubairy M S 2002 Phys. Rev. A 65 043819
[12] Paspalakis E and Knight P L 2001 Phys. Rev. A 63 065802
[13] Kapale K T and Zubairy M S 2006 Phys. Rev. A 73 023813
[14] Agarwal G S and Kapale K T 2006 J. Phys. B At. Mol. Opt. Phys. 39 3437
[15] Dutta B K and Mahapatra P K 2006 J. Phys. B At. Mol. Opt. Phys. 39 1145
[16] Ivanov V and Rozdestvensky Y 2010 Phys. Rev. A 81 033809
[17] Ding C, Li J and Yang X 2011 Phys. Rev. A 83 063834
[18] Ding C, Li J, Yang X and Xiong H 2011 Phys. Rev. A 84 043840
[19] Rahamatullah and Qamar S 2013 Phys. Rev. A 88 013846
[20] Harris S E, Field J E and Imamoglu A 1990 Phys. Rev. Lett. 64 1107