Scaling ideas in neutrino scattering reactions: application to the MiniBooNE experiment

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Abstract. An exhaustive analysis of the world data on electron scattering has shown that scaling and superscaling properties are fulfilled with great accuracy. The relativistic impulse approximation (RIA) yields a satisfactory description of experimental data. Not only scaling and superscaling behavior emerge, but also the specific asymmetric shape of the experimental scaling function is reproduced. In this work two relativistic approaches are considered to evaluate the neutrino-nucleus cross sections. One, based on the relativistic impulse approximation, relies on the microscopic description of nuclear dynamics using relativistic mean field theory, and incorporates a description of final state interactions. The second is based on the superscaling behavior exhibited by electron scattering data and its applicability, due to the universal character of the scaling function, to the analysis of neutrino scattering reactions. The role played by the vector meson-exchange currents in the two-particle two-hole sector is also incorporated and the results obtained are compared with the recent data for neutrinos measured by the MiniBooNE Collaboration.

1. Introduction

The data on muon neutrino charged-current quasielastic (CCQE) cross sections recently obtained by the MiniBooNE collaboration [1], and its comparison with several theoretical calculations, have led to an important debate concerning the role played by various ingredients entering in the description of the reaction. Although no definitive conclusions are yet in hand, a detailed study of modeling versus experiment for inclusive quasielastic electron scattering and its extension to neutrino processes can shed light on the different interpretations of the discrepancy between theory and experiment.

When compared with MiniBooNE CCQE data, the Relativistic Fermi Gas (RFG) model underestimates the total cross section unless an axial mass $M_A$ of the order of 1.35 GeV/$c^2$ is employed. This value is considerably larger than the accepted world average value [2]. However, although the RFG incorporates a fully relativistic treatment, its description of the
nuclear dynamics is clearly too crude to draw specific conclusions on the value of the anomalous axial mass. At the level of the impulse approximation, a number of much more sophisticated descriptions of the nuclear dynamics also underpredict the measured CCQE cross section. Among the difficulties that one faces when comparing models, is that the effect of the ingredients in the model, such as interactions in the final state (FSI), may differ greatly from model to model. A systematic analysis of the world inclusive \((e,e')\) data has clearly demonstrated that, for sufficiently large momentum transfers, at energy transfers below the QE peak the property of superscaling works rather well [3, 4, 5, 6]. Moreover, from the longitudinal response a phenomenological scaling function has been extracted that shows a clear asymmetry with a long tail extended to larger energy transfers. Assuming the scaling function to be universal, \(i.e.,\) valid for electromagnetic and weak interactions, in [7, 8] CCQE neutrino-nucleus cross sections were evaluated by using the scaling function extracted from \((e,e')\) data and multiplying it by the corresponding elementary weak cross section. This approach, denoted simply as “SuSA,” provides nuclear-model-independent neutrino-nucleus cross sections, but its reliability rests on a basic assumption: the scaling function (extracted from longitudinal \((e,e')\) data) is appropriate for all of the various weak responses involved in neutrino scattering, and is independent of the vector or axial nature of the nuclear current entering the hadronic tensor.

In a recent paper [9] SuSA predictions have been compared with the MiniBooNE data for the double-differential neutrino cross section showing a systematic discrepancy between theory and experiment. Inclusion of 2p-2h Meson Exchange Current (MEC) contributions yields larger cross sections and accordingly better agreement with the data. However, theory still lies below the data at larger angles where the cross sections are smaller. Before drawing definitive conclusions on the anomalous axial mass, it is important to explore alternative approaches that have been shown to be successful in describing inclusive QE \((e,e')\) processes. This is the case for the Relativistic Mean Field (RMF), where a fully relativistic description of the process is incorporated, and FSI are taken into account by using the same relativistic scalar and vector energy-independent potentials considered in the description of the initial bound states.

2. Analysis of results

In this section we discuss the results obtained with the different approaches considered and compare with the experimental data. Details have been presented in previous works. In particular, the SuSA approach and its extension to CC neutrino reactions can be reviewed in [8], whereas the basic ingredients entering in the RMF model applied to inclusive electron and CCQE neutrino reactions are given in [10, 11, 12, 13].

In Fig. 1 we show the double-differential cross section averaged over the neutrino energy flux as a function of the muon kinetic energy \(T_\mu\). In each panel the results have been averaged over the corresponding angular bin of \(\cos \theta\). In all cases we use the standard value of the nucleon axial mass, \(i.e.,\) \(M_A = 1.03 \text{ GeV}/c^2\). We compare the theoretical results evaluated using the three approaches, SuSA (green line), SuSA+MEC (blue) and RMF (red), with the MiniBooNE data [1]. We show that the 2p-2h MEC increase the cross section, yielding results that are closer to experiment, specifically, for data up to \(\cos \theta \sim 0.6\). At larger angles, the discrepancy with experiment becomes larger while, on the other hand, the role of MEC is seen to be less significant, that is, the difference between SuSA and SuSA+MEC becomes smaller as the scattering angle increases.

Cross sections evaluated with the RMF model also yield reasonable agreement with data for smaller angles, the discrepancy becoming larger as \(\theta\) increases. However, some differences emerge from the comparison between the RMF and SuSA predictions. As observed, RMF cross sections are in general larger than the SuSA ones. In particular, in the region close to the peak in the cross section, the RMF result becomes larger than the one obtained with SuSA+MEC. This holds especially for large scattering angles. On the contrary, SuSA and SuSA+MEC get more
In Fig. 2 RMF cross sections are the lowest for the smallest values of \( \cos \theta \), the data at large muon scattering angles, particularly for small forward scattering angles. With respect to comparison with data, the RMF also underestimates the 2p-2h MEC yield an enhancement of the cross section whose magnitude increases for more T data, improving the agreement as \( \cos \theta \) increases. As we move to more positive \( \cos \theta \), the RMF cross section grows faster, lying above the results corresponding to SuSA+MEC in the intermediate region. Finally, for smaller values of the scattering angle, namely \( \cos \theta \) approaching 0.9, while RMF inverts its behavior and decreases very rapidly, SuSA and SuSA+MEC approaches to \( \cos \theta = 0.9 \) show a much softer slope. In fact, this is the region where the discrepancy between RMF and SuSA-based models can be better appreciated. It is very illustrative to point out that the general shape presented by the RMF cross section as a function of \( \cos \theta \) fits perfectly well the shape shown by data, although RMF predictions fall below the data for small muon momenta. The different behaviour of the models is partly due to the fact that the RMF is better describing the low-energy excitation region whereas, as already

**Figure 1.** (color online) Flux-integrated double-differential cross section per target nucleon for the \( \nu_\mu \) CCQE process on \( ^{12}\text{C} \) evaluated in the SuSA (green line), SuSA+MEC (blue) and RMF (red) models and displayed versus the muon kinetic energy \( T_\mu \) for various bins of \( \cos \theta \). The data are from MiniBooNE [1].

strength in the region of high muon kinetic energies. This can be attributed to the breakdown of zeroth-kind scaling in the RMF, i.e., the longitudinal and transverse scaling functions to be not equal.

In Fig. 2 we plot the neutrino-flux-averaged cross section versus the scattering angle at fixed \( T_\mu \) (averaged over each bin). For low muon momenta the three models tend to underestimate the data, improving the agreement as \( T_\mu \) increases. As observed, when added to the SuSA results, the 2p-2h MEC yield an enhancement of the cross section whose magnitude increases for more forward scattering angles. With respect to comparison with data, the RMF also underestimates the data at large muon scattering angles, particularly for small \( T_\mu \). However, some important differences between RMF and SuSA-based models also emerge. In the six panels presented in Fig. 2 RMF cross sections are the lowest for the smallest values of \( \cos \theta \). As we move to more positive \( \cos \theta \), the RMF cross section grows faster, lying above the results corresponding to SuSA+MEC in the intermediate region. Finally, for smaller values of the scattering angle, namely \( \cos \theta \) approaching 0.9, while RMF inverts its behavior and decreases very rapidly, SuSA and SuSA+MEC approaches to \( \cos \theta = 0.9 \) show a much softer slope. In fact, this is the region where the discrepancy between RMF and SuSA-based models can be better appreciated. It is very illustrative to point out that the general shape presented by the RMF cross section as a function of \( \cos \theta \) fits perfectly well the shape shown by data, although RMF predictions fall below the data for small muon momenta. The different behaviour of the models is partly due to the fact that the RMF is better describing the low-energy excitation region whereas, as already
pointed out, the SuSA model has no predictive power at very low angles, where the cross section is dominated by low excitation energies and the superscaling ideas are not supposed to apply.

Figure 2. (color online) Double-differential $\nu_\mu$ CCQE cross section for $^{12}$C integrated over neutrino flux versus the outgoing muon scattering angle for various bins of the muon kinetic energy $T_\mu$. Results are given for RMF (red lines), SuSA (green) and SuSA+MEC (blue).

Figure 3. (color online) Total CCQE cross section per neutron versus the neutrino energy. The curves corresponding to different nuclear models are compared with the flux unfolded MiniBooNE data [1].

To conclude, in Fig. 3 we display the total QE cross section per neutron as a function of the neutrino energy and compared with the experimental data. As observed, the discrepancies between the various models tend to be washed out by the integration, yielding very similar results for the models that include FSI (SuSA, RMF and rROP), all of them giving a lower total cross section than the models without FSI (RFG and RPWIA). On the other hand the SuSA+MEC curve, while being closer to the data at high neutrino energies, has a somewhat different shape with respect to the other models, in qualitative agreement with the relativistic calculation of [14].
3. Concluding remarks

In this work we apply the RMF model to CCQE neutrino reactions on $^{12}$C corresponding to the kinematics of the MiniBooNE experiment. Results for the flux-averaged double-differential cross sections are compared with data and the predictions given by SuSA and SuSA+MEC models. Generally speaking, the RMF model underestimates the data especially at large muon scattering angles and low muon energies. This was already observed with SuSA and to a somewhat lesser extent with SuSA+MEC. However, the specific behavior shown by RMF clearly differs from that of SuSA and SuSA+MEC; the maximum in $d^2\sigma/d\cos\theta dT_\mu$ as a function of $T_\mu$ for various bins of $\cos\theta$ gets higher for RMF, whereas the tail at high $T_\mu$ is more pronounced for the SuSA-based models. Also, the general trend shown by the curve corresponding to the double-differential cross section as a function of the scattering angle for bins of $T_\mu$, clearly differs for RMF and SuSA (SuSA+MEC) approaches. Here, it is very interesting to point out that the specific shape followed by RMF predictions fits perfectly well the slope shown by data.

In spite of the discrepancies introduced by the models in the double-differential cross sections, RMF and SuSA approaches provide almost identical results for the single-differential cross section, this being found to lie below the data. Although the inclusion of 2p-2h MEC contributions increases the differential cross section being closer to data, its shape is best reproduced by the RMF and does not improve with the inclusion of the 2p-2h MEC contributions in SuSA. Finally, the impact of the 2p-2h contribution on the total cross section increases with the neutrino energy, suggesting that the data can be explained without the need for a large nucleon axial mass. However more refined calculations taking care of correlation currents, MEC effects in the axial-vector channel, etc., should be performed before definitive conclusions can be drawn.

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