Some detection tests for low complexity data models and unknown background distribution.

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Abstract—We consider several detection situations where, under the alternative hypothesis, the signal admits a low complexity model and, under both the null and the alternative hypotheses, the distribution of the background noise is unknown. We present several detection strategies for such cases, whose design relies on exogenous or on endogenous data. These testing procedures have been inspired by and are applied to two specific problems in Astrophysics, namely the detection of exoplanets from radial velocity curves and of distant galaxies in hyperspectral data cubes.

1 Introduction

Several fields like Genomics or Astrophysics often require to perform a large number of statistical tests simultaneously. During the last two decades, these applications have favored the emergence of theoretical works in multiple testing. Low-complexity- or sparsity-adaptive tests have been designed, including for instance the Benjamini-Hochberg procedure, which controls the false discovery rate (FDR), or several variants of Donoho and Jin’s higher criticism (HC) and of the Berk-Jones (BJ) tests. In practice however, the distribution of the noise background may be not well known or even unknown. This may explain why such multiple testing procedures, which require the knowledge of the distribution under the null hypothesis, are not used in practical applications where they would yet be useful. Users may prefer in practice considering more ad hoc procedures with the hope, sometimes illusory, that the test is performed at a controlled level. A recent example in Astrophysics is the claimed exoplanet detection around \( \alpha \) Centauri Bb, with a P-value \( (i.e., \) the probability, under \( H_0 \), of observing a test statistic at least as extreme as the one observed) evaluated at 0.02\% \cite{8}. This detection was strongly discarded by further analyses \cite{9,10}, stressing out the necessity of considering effects that are difficult to control, such as the noise background created by the star hosting the planet.

This papers presents some testing procedures with correctly controlled error risks, even when the distribution of the test statistics is not well known. We consider two cases: exoplanet detection from radial velocity data \cite{11} and the detection of distant galaxies in the integral field spectrograph MUSE installed at the Very Large Telescope in Chile \cite{12,13}.

2 Exoplanet detection from time series

The presence of a planet orbiting a star induces a reflex motion of the star around the star-planet mass centre. This motion induces a quasi-periodic modulation of the radial velocity of the star with respect to a distant observer and imprints its signature on the stellar spectrum by the Doppler effect \cite{14}. By measuring the Doppler shift in the lines of the stellar spectrum, one obtains the radial velocity of the star at a given time. Data thus consist of a time series containing such measurements.

One way to setup the detection problem considers the following composite (i.e., with unknown parameters) hypotheses:

\[ H_0 : X(t_j) = E(t_j) \]
\[ H_1 : X(t_j) = \sum_{q=1}^{N_s} \alpha_q \sin(2\pi f_q t_j + \varphi_q) + E(t_j) \]

where \( X(t_j = j \Delta t), j = 1, \ldots, N \), are the data points and \( E(t_j) \) is a zero-mean second-order stationary Gaussian noise. The signal under the alternative hypothesis \( H_1 \) has low complexity since it is modeled by a few sine waves, in some unknown number and with unknown parameters. That is, the planetary signature is sparse in the Fourier space. Under both hypotheses, the noise covariance matrix \( \Sigma \) (or equivalently the power spectrum density \( S \)) is unknown, owing to the adver- sal presence of correlated stellar noise. As a consequence, the distribution of the periodogram \( P \) at each frequency \( \nu \), which depends on \( S \), is also unknown. For instance, under \( H_0 \), \( P(\nu) \) is (asymptotically) a \( \chi^2 \) random variable scaled by \( S(\nu) \):

\[ P(\nu) \sim S(\nu) \chi^2/2. \]

To circumvent this difficulty we propose an exogenous approach, \( i.e., \) based on ancillary data under \( H_0 \). These data are simulated time series corresponding to stellar noise, available via astrophysical simulations (see our recent work \cite{15}). The computation load of such simulations is however very demanding, so the number (say, \( L \)) of training time series is small (up to a few tens, typically).

The considered framework is also relevant in other contexts. For instance, in asteroseismology, one wants to detect periodic signals in photometric time series, where the noise is mainly due to atmospheric perturbations. In modern photometers, auxiliary optical channels are often devoted to monitor “empty” regions of the sky and stable calibration stars \cite{16}. Note that radar systems also use training noise samples for detection since ages. However, an important difference is that adaptive test statistics in radar typically rely on the estimation of the noise covariance matrix and therefore require \( L > N \) \cite{17}. This is a very different regime from that considered here, where \( L \ll N \).

The method simply consists in building an averaged periodogram (say, \( \overline{P}(\nu) \)) from the \( L \) training time series. Then, the standardized periodogram is defined as \( Z(\nu) := P(\nu)/\overline{P}(\nu) \). Let \( Z_1, \ldots, Z_N \) denote a subset of its compo-
ponents. For evenly sampled data, the distributions of $Z_j$ admit analytical expressions under both hypotheses [13]. Under standard assumptions [19], one can show that $Z_1, \ldots, Z_N$ are asymptotically independent with $Z_j \sim \mathcal{F}(2,2L)$ (a Fisher $F$-distribution with parameters 2 and 2L) under the null hypothesis and $Z_j \sim F_{\mathcal{N}}(2,2L)$ (non central $F$-distribution), whose non-centrality parameters $\lambda_j$ depend on the planetary signature.

Under $H_0$, the distribution of the standardized periodogram is independent of $S$. This property, together with asymptotic independence, allows one to build global hypothesis tests of level $\alpha$ from values of $Z_j$, e.g., with the test of the maximum: this test rejects the null hypothesis if $\max(Z_j, 1 \leq j \leq N)$ is larger than a threshold $c_\alpha$, calibrated so that

$$1 - \left(1 - \left(\frac{L}{c\alpha + L}\right)^L\right)^N = \alpha.$$ 

Similar results can be obtained for several other tests like that of Fisher [20] or its variants [21][22][23]. This also allows one to use approaches such as HC et BJ in a context where the exact distribution under $H_0$ is not known.

In simulations, we have shown that the asymptotic approximation is accurate when the length of the time series is sufficiently large with respect to the typical correlation time of the noise, and that the tests are conducted at the target false alarm rates, with the predicted probabilities of detection. Moreover, the analytical models allow to study the asymptotic powers of the tests, whose expressions are also accurate for moderate values of $N$. Such results allow for detectability studies combining specific planetary parameters and instrumental configurations [11]. An example is shown in the left panel of Fig. 1 for the test defined by the maximum of the periodogram $Z_j$.

The error criterion considered here is that of the FDR, that is, the expected value of the proportion of false discoveries over the total number of positions found as possibly corresponding to a line [11][13]. In contrast to the previous application, the approach considered here is endogenous, in the sense that it does not rely on auxiliary data. We have shown on simulated data that the distribution of the local maxima under the joint null hypothesis (i.e., there is no line in the whole data cube) can be estimated from the data itself, and that the whole procedure (including nuisance cancellation and matched filtering) controls the FDR (see Fig. 1 right panel). A detailed presentation of astrophysical results obtained with this algorithm can be found in [13].

3 Detection of galaxies in MUSE

The integral field spectrograph MUSE, installed on one of the 8-meter telescopes of the Very Large Telescope (Chile) ac-

quires very rich hyperspectral images (typically $300 \times 300$ pixels in 3 600 optical channels). In such a data "cube", we seek to detect very distant and faint galaxies, whose signatures essentially consist of a weak increase of the light flux corresponding to an emission line, convolved by the spatial point spread function of the instrument. The shape of the lines is approximately known, but not their amplitudes, their positions in the data cube, and nor their number—a few hundreds typically, against several hundreds of millions of voxels. In practice, the lines amplitudes can be very weak with respect to other astrophysical sources and to the background level, caused by the detector noise, residual sky emission, instrumental artifacts, etc.

In this framework, we have considered a detection approach in three steps: the nuisance sources are first eliminated by linear subspace filtering techniques; a specific matched filtering step is applied to the residual; the detection is performed in the filtered data residuals. In order to focus on the sparse and localized nature of the target signatures, the considered test statistics are the local maxima of the resulting filtered data cube. Because of the numerous and complex preprocessing steps applied to the data before the detection step, the distribution of the test statistics under $H_0$ is unknown, with little hope to ever establish a reliable theoretical characterization.

In the considered problem there are several null hypotheses, each one linked to the position of a local maximum [24]. If we denote by $x, y, z$ the position of a local maximum ($x$ and $y$ are spatial coordinates, while $z$ is the wavelength channel), we consider the two hypotheses:

$$\begin{align*}
H_0, x, y, z : & \text{ there is no emission line at position } (x, y, z), \\
H_1, x, y, z : & \text{ there is one such line at this position.}
\end{align*}$$

The error criterion considered here is that of the FDR, that is, the expected value of the proportion of false discoveries over the total number of positions found as possibly corresponding to a line [11][13]. In contrast to the previous application, the approach considered here is endogenous, in the sense that it does not rely on auxiliary data. We have shown on simulated data that the distribution of the local maxima under the joint null hypothesis (i.e., there is no line in the whole data cube) can be estimated from the data itself, and that the whole procedure (including nuisance cancellation and matched filtering) controls the FDR (see Fig. 1 right panel). A detailed presentation of astrophysical results obtained with this algorithm can be found in [13].

4 Conclusions and perspectives

In the signal processing community, there is a large literature on constant false alarm rate tests—see for instance [25][26]. To our knowledge, however, the regime, approach and results described in Section 2 are new. In the statistics community, the validation of testing procedures learning automatically the null distribution traces back to permutation and randomization tests. In the framework of multiple testing and of the FDR considered here, some results have been brought recently, for instance in [27][28][29]. In the studies considered in Section 3 the validation of the approach is merely done by numerical simulations so far. One direction of research is to obtain theoretical guarantees, possibly by modifying or simplifying the procedures and models, inspired by such recent results.
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