A NEW NONPERTURBATIVE APPROACH TO QCD BY BF THEORY

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Abstract

Yang-Mills theory in the first order formalism appears as the deformation of a topological field theory, the pure BF theory. In this approach new non local observables are inherited from the topological theory and the operators entering the ’t Hooft algebra find an explicit realization. A calculation of the vev’s of these operators is performed in the Abelian Projection gauge.

2 First order formalism

The first order formalism for pure euclidean Yang-Mills theory is described by the action functional

\[ S_{BF-YM} = \int_{\mathbb{R}^4} d^4x \left( \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} B_{\mu\nu}^a F_{\alpha\beta}^a + g^2 \int_{\mathbb{R}^4} d^4x B_{\mu\nu}^a B_{\alpha\beta}^a \right), \]

(2.1)

with the usual definition

\[ F_{\mu\nu}^a = 2 \partial_{[\mu} A_{\nu]}^a + f^{abc} A_{\mu}^b A_{\nu}^c \]

and where \( B \) is a Lie algebra valued 2-form; the usual gauge symmetry takes the form \( \delta A_{\mu}^a = D_{\mu} c, \delta B_{\mu\nu}^a = -[c, B_{\mu\nu}] \) with \( D_{\mu} \equiv \partial_{\mu} - i[A_{\mu}, \cdot] \). The first term in the r.h.s. of (2.1) is the action of a topological quantum field theory, the so-called BF theory; indeed the action (2.1) acquires a “topological” symmetry iff \( g = 0 \). In this case

\[ \delta A_{\mu}^a = 0, \quad \delta B_{\mu\nu}^a = 2 \partial_{[\mu} \psi_{\nu]}^a + 2 f^{abc} A_{[\mu}^b \psi_{\nu]}^c, \]

(2.2)

where \( \psi \) is a 1-form ghost. Note that due to zero modes \( \delta \psi_{\mu} = D_{\mu} \phi \) and \( \delta \phi = 0 \) in the transformations (2.2). This symmetry is then reducible, allowing for a ghosts of ghosts structure. The second term in r.h.s. of (2.1) is an explicit symmetry breaking for the topological symmetry (2.2) and restores a local gauge dynamics. The field equations of (2.1) read

\[ \ast F_{\mu\nu}^a = \frac{ig^2}{2} B_{\mu\nu}^a, \quad \varepsilon^{\mu\nu\alpha\beta} D_{\alpha} B_{\beta}^a = 0, \]

(2.3)

where \( \ast F_{\mu\nu} \equiv \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} \) is the dual field strength. Note that on-shell \( B \) coincides with the dual field strength and satisfies the Bianchi identities. This is no longer true off-shell. In this case the quantum theory may introduce monopoles charges through \( B \) which is not constrained by a Bianchi identity. These topologically non-trivial magnetic configurations should enter the YM vacuum and be in correspondence with a deformation of the instanton sector. By performing a formal perturbation theory in \( g \neq 0 \) around the topological BF theory (the topological embedded sector in YM) it is easily seen that the BF topological vacuum in an axial gauge, say \( B^- = 0 \), is given by the quasi anti-self-dual instanton configurations \( F_{\mu\nu} = o(g) \).
3 A new observable

A new non-local observable associated to an orientable surface $\Sigma \in M^4$ is naturally introduced in the BF formulation of QCD

$$M(\Sigma, C) \equiv \text{Tr} \exp \{ i k \int_C d^2y \, \text{Hol}_y^\Sigma (\gamma) B(y) \text{Hol}_y^\gamma (\gamma') \} ,$$

(3.4)

where $\text{Hol}_y^\Sigma (\gamma)$ denotes the holonomy along the open path $\gamma \equiv \gamma_{\bar{x}y}$ with initial and final point $\bar{x}$ and $y$ respectively,

$$\text{Hol}_y^\Sigma (\gamma) \equiv P \exp \{ i \int_{\bar{x}}^{y} dx^a A_\mu (x) \} .$$

(3.5)

In (3.4) $k$ is an arbitrary expansion parameter, $\bar{x}$ is a fixed point (we do not integrate over $\bar{x}$) and the relation between the assigned paths $\gamma$, $\gamma'$ over $\Sigma$ and the closed contour $C$ is the following: $C$ starts from the fixed point $\bar{x}$, connects a point $y \in \Sigma$ by the open path $\gamma_{\bar{x}y}$ and then returns back to the neighborhood of $\bar{x}$ by $\gamma_{y\bar{x}}$, (which is not restricted to coincide with the inverse $(\gamma_{\bar{x}y})^{-1} = \gamma_{y\bar{x}}$). From the neighborhood of $\bar{x}$ the path starts again to connect another point $y' \in \Sigma$. Then it returns back to the neighborhood of $\bar{x}$ and so on until all point on $\Sigma$ are connected. The path $C$ is generic and no particular ordering prescription is required. Of course the quantity (3.3) is path dependent and our strategy is to regard it as a loop variable once the surface $\Sigma$ is given.

Our main result is that the observable $M$ gives an explicit analytic realization of the ’t Hooft loop operator $\{1\}$ (the dual variable to the Wilson loop). This may be realized using $3 + 1$-hamiltonian formalism. When considering the action of $M(C)$ on a physical state $|A >$ we find $\{1\}$, in the fundamental representation,

$$M(C) |A(\bar{x}) > \simeq \text{Tr} \{ e^{i g q C} \mathbb{1} \} |A(\bar{x}) > ,$$

(3.6)

where $\Phi_C \equiv \frac{2\pi k(N - 1)}{g} \text{Link}(C)$. The self-linking $\text{Link}(C)$ is an integer defined as the linking between the curve $C$ and its framing contour $C'$, the latter being a point splitting regularization of the former, i.e. if $C \equiv \{ x'(t) \}$ then $C' \equiv \{ y'(t) = x'(t) + \epsilon n'(t) \}$ with $\epsilon > 0$ and $n'$ a versor orthogonal to $C$. $\text{Link}(C,C') \equiv \text{Link}(C,C')_{\epsilon \to 0}$ equals the number of windings of $C'$ around $C$. Thus with a proper choice of the parameter $k = 4\pi / (N^2 - 1)$ the operator $M(C)$ generates multivalued (i.e. singular) gauge transformations in the center of the group $SU(N)$; this is the defining property of the ’t Hooft magnetic operator. The vacuum expectation values of the operator $M(C)$ and of the Wilson loop $W(C)$ label the phases of the theory; in the confinement phase, $M(C)$ and $W(C)$ are expected to develop a perimeter and an area law respectively.

4 Computation of $< M >$

We compute the vev of $M$ in the abelian projection gauge $\{1\}$, which in our case is better implemented choosing $B$ in the adjoint representation of $SU(N)$. In this approach a monopole dynamics is supposed to arise and dominate $\{1\}$ the YM vacuum, discarding the “charged” $\{2\}$ gauge degrees of freedom. Indeed in this approach we find $\{1\}$ magnetic abelian configurations $\bar{\alpha}$ of the gauge field satisfying a monopole equation

$$\frac{1}{2} F_{\mu\nu} \alpha^\mu \partial_\nu \bar{\alpha}_i = \frac{4\pi g}{N} \omega_{\Sigma,\mu} \cos \left( g \oint_{C'} \bar{\alpha}_i \right) ,$$

(4.7)

$\omega_{\Sigma}$ is a closed form and may be chosen self-dual, i.e. $*\omega_{\Sigma} = \omega_{\Sigma}$. Locally it can be written as $\omega_{\Sigma} \simeq \delta^2(\Sigma)$. The saddle point evaluation for $< M >$ gives (in form language $\omega_{\Sigma} = \omega_{\Sigma,\mu} dx^\mu \otimes dx^\nu$)

$$< M(C) > \simeq N \exp \left\{ - \frac{2\pi n^2}{g^2} \sum_i \int \omega_{\Sigma} \wedge \omega_{\Sigma'} \cos \left( g \oint_{C'} \bar{\alpha}_i \right) \cos \left( g \oint_C \bar{\alpha}_i' \right) \right\} ,$$

(4.8)

where $n \in \mathbb{Z}$. The flux of the magnetic solutions of (4.7) satisfies a proper Dirac quantization $g \oint_C \bar{\alpha}_i = \frac{4\pi n}{N} = 2\pi n$, with $n \in \mathbb{Z}$. In this case, the quantity $\int \omega_{\Sigma} \wedge \omega_{\Sigma'}$ in (4.8), where $\Sigma(\subset C')$ is a
point splitting regularization of \( \Sigma(\geq C) \), equals \( \text{sLink}(C) \). In a lattice regularization it is found that \( \text{sLink}(C) \sim \frac{1}{L(C)} \), where \( L \) is the perimeter of \( C \) measured in lattice spacing \( a \). To summarize, in the continuum limit we find the expected perimeter law for the 't Hooft operator

\[
\frac{< M(C) >}{< 1 >} \sim \exp\{- \frac{8\pi^2 n^2 N}{4g^2} L(C) \} \quad , \tag{4.9}
\]

\( l \) is the magnetic vortex penetration length. It can be shown \( \text{[4]} \) that one-loop quantum effects amount to the replacement of the coupling \( g \) in (4.9) with the renormalized one, i.e. \( 8\pi/g_B^2 = 8\pi/g_0^2 - \beta_1 \ln\left(\frac{\mu}{\nu} \right) \), where \( p \) is the momentum scale, \( \mu \) is the subtraction point and \( \beta_1 = \frac{11}{3}N \).

We also have \( 1/l \to \Lambda_{QCD} \) with \( \Lambda_{QCD} \) some typical physical mass scale in QCD.

5 Computation of \( < W > \) and conclusions

We consider now the computation of the vev of the Wilson loop, which in the confinement phase of QCD is expected to develop an area law. We start by rewriting it in terms of the non abelian Stokes theorem \( \text{[7]} \)

\[
W(\Sigma) \equiv W(\Sigma, C) = Tr P_{\Sigma} \exp\{ i \int_{\Sigma} \text{Hol}^\gamma_F(x) \} \quad , \tag{5.10}
\]

where \( \mathcal{C} = \partial \Sigma \) and \( P_{\Sigma} \) means surface path ordering. Using the functional identity \( F_{\mu\nu}^a(x) \exp\{ - \frac{i}{4} \int C_{\mu\nu} F_{\mu\nu}^a \} = 4i\delta/\partial \ast B_{\mu\nu}^a(x) \exp\{ - \frac{i}{4} \int B_{\mu\nu}^a C_{\mu\nu} \} \) we obtain \( \text{[4]} \) the "duality relation"

\[
< W(\Sigma) > \equiv < M^* (\Sigma, C) > \equiv < Tr P_{\Sigma} \exp\{ - \frac{g^2}{2} \int_{\Sigma} \text{Hol}^\gamma_F(x) \ast B(x) \text{Hol}^\gamma_F(x) \} > . \tag{5.11}
\]

To calculate (5.11) we expand \( M^* \) in \( g \); the first relevant contraction encountered at lower level is given in terms of \( < A \ast B > \), involving the off diagonal propagator \( < AB > \) present both in the BF-YM theory \( \text{[4]} \) as well as in the pure BF theory. Therefore we find

\[
\frac{< M^* (\Sigma) >}{< 1 >} = e^{- \frac{g^2}{2} c_2(t) \int \text{Link}(C, \Sigma) \} \quad , \tag{5.12}
\]

where \( c_2(t) \) is the 2th Casimir of the group representation and \( \Delta(\Sigma) \) depends on higher order integrations over \( \Sigma \). It may be shown \( \text{[7]} \) that

\[
\int \text{Link}(C, \Sigma) \sim \int_{\Sigma} \text{Link}(C, \Sigma^\ast) \quad , \tag{5.13}
\]

where \( \text{Link}(C, \Sigma^\ast) \) is the linking number between the curve \( C \) and the dual plane \( \Sigma^\ast \) in \( x \) to \( \Sigma \). Using again a lattice regularization, (13) is found to be proportional to the area of \( \Sigma \). We then obtain

\[
\frac{< W(\Sigma) >}{< 1 >} \sim \exp\{ - \sigma(l) A(\Sigma) \} \quad , \tag{5.14}
\]

where \( \sigma(l) \) is the string tension defined at tree level by

\[
\sigma(l) \equiv g^2 \left( \frac{N^2 - 1}{16N} \right) \frac{1}{l^2} \quad . \tag{5.15}
\]

Again quantum effect should amount to the replacement of the bare coupling with the running one and also \( \sigma \) with the renormalization group invariant quantity \( \tilde{\sigma}(\Lambda_{QCD}) \). A rough estimate of \( \tilde{\sigma} \) for energy scales between 10 and 100(Gev)$^2$ and \( \Lambda_{QCD} \sim 0.5 \text{ Gev} \) gives \( \tilde{\sigma} \sim 1.2 \times 0.25(\text{Gev})^2 \), which is of the same order of magnitude of the experimental value.

In conclusion, in the framework of the first order (BF) formalism for YM theory we have introduced an explicit analytic representation for the 't Hooft algebra, using non local operators whose vev are naturally given in terms of geometrical quantities like linking numbers between curves and surfaces. The underlying dynamics is understood in terms of monopole vortex lines which should enter the theory by means of the auxiliary field \( B \), thus supporting the picture of the dual superconductor vacuum. The explicit calculation produces perimeter and area laws for the operators \( M(C) \) and \( W(\Sigma) \), corresponding to the expected confining phase for the theory.

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