AXION IN LARGE EXTRA DIMENSION

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We examine the axion model in the large extra dimensions scenario with TeV scale gravity. To obtain an intermediate-scale decay constant of the axion, the axion is assumed to live in a sub-spacetime (brane) of the whole bulk. In this model there appear Kaluza-Klein modes of the axion which have stronger interaction than those of the graviton. The axion brane plays a role of absorber of the graviton Kaluza-Klein modes. We discuss various cosmological constraints as well as astrophysical ones and show that the model is viable for certain choices of the dimensionality of the axion brane. The structure of the model proposed here provides a viable realization of the fat brane idea to relax otherwise very severe cosmological constraints.

1 Introduction

It has been suggested that the fundamental scale of nature can be as low as TeV, whereas the largeness of the effective Planck scale or the weakness of the gravity in a long distance can be explained by introducing large extra dimensions. When there exist \( n \) of such extra dimensions, the relation between the gravitational scale in 4 dimensions and the fundamental scale in \((4+n)\) dimensions is given

\[
M_{pl(4)}^2 \sim V_n M_{pl(4+n)}^{n+2},
\]

where \( V_n \) is the volume of the extra dimensional space. For \( M_{pl(4+n)} \approx 1 \) TeV, the size of the extra dimensions \( r_n \) is computed as

\[
r_n \sim V_n^{1/n} \sim 10^{32-17/2} M_{\text{TeV}}^{-2} \text{ cm},
\]

where \( M_{\text{TeV}} \equiv M_{pl(4+n)}/\text{TeV} \). The case \( n = 1 \) is excluded because the gravitational law would change at macroscopic level, but the cases \( n \geq 2 \) are allowed by gravity experiments.

Because there exist a tower of graviton Kaluza-Klein (KK) modes, various astrophysical and cosmological bounds will be applied. We may avoid some of these problems by assuming an extremely low reheating temperature of the early universe. But the reheating temperature cannot be smaller than 1 MeV.

On the other hand, the simplest model of this type would not provide intermediate scales which are necessary to explain phenomenological issues like, small neutrino masses and the strong CP problem. It was pointed out that particles dwelling in the extra dimensions other than graviton can also have similar effective interaction terms in four-dimensional physics. The interaction between bulk matter and normal matters is suppressed as gravity.

In this paper we will propose an axion model in TeV scale gravity for various numbers of the extra dimensions, with special emphasis on cosmological constraints: the addition of the axion in the large-extra-dimension model substantially alters the cosmology.

We will use the convention \( M_{pl} \equiv M_{pl(4)} \) and \( M_* \equiv M_{pl(4+n)} \).
2 PQ scale in extra-dimension

If an axion is a boundary field confined on 4 dimensions, the Peccei-Quinn (PQ) scale $f_{PQ}$ is bounded by $M_* \sim 1$ TeV. To obtain a higher PQ scale, the axion field has to be inevitably a bulk field. If it lives in the whole $(4+n)$ dimensional bulk, the PQ scale will be $f_{PQ} \sim M_{pl}$.

However, the damped coherent oscillation of the axion with $f_{PQ} \sim M_{pl}$ would overclose the universe. A conventional argument gives an upper bound of $F_{PQ} \sim 10^{12}$ GeV. Even when an entropy production takes place after the QCD phase transition (e.g. the reheating temperature is smaller than $\sim 1$ GeV), $f_{PQ}$ cannot be much larger than $10^{15}$ GeV. Provided that the coherent oscillation of the inflaton is followed by the reheating process,

$$f_{PQ} < 10^{15}\text{GeV} \left(\frac{h}{0.7}\right) \left(\frac{\pi/2}{\theta}\right) \left(\frac{\text{MeV}}{T_R}\right)^{1/2},$$

where $h$ is from Hubble constant in units of 100km sec$^{-1}$Mpc$^{-1}$, $\theta$ is the initial value of PQ vacuum angle and $T_R$ is the reheating temperature after inflation. To avoid over-closure problem, we propose a natural way to realize an intermediate scale axion, i.e. axion as an $(4+m)$ dimensional sub-spacetime field ($(3+m)$ brane) ($m < n$). The idea of using sub-spacetime to realize an intermediate scale already appeared in the context of neutrinos.

Let $\chi$ be a complex scalar field which contains PQ axion in 4+m dimension; $\tilde{a}$.

If the axion field lives only on $4+m$ dimensional sub-spacetime where $m < n$ and the volume of extra-dimension is $V_m$,

$$\mathcal{L}_\chi = \int dx^{4+m} \partial^M \chi^* \partial_M \chi + \int dx^i \tilde{a}(x^A = 0) \langle \chi \rangle \bar{F} F,$$

where $x^A$ means an extra-dimension coordinate. Assuming that the vacuum expectation value of the $\chi$ field does not depend on the extra-dimension coordinates, we obtain

$$f_{PQ} \sim \sqrt{V_m} \langle \chi \rangle \sim r_n^{m/2} M_*^{1+m/2} \sim M_* \left(\frac{M_{pl}}{M_*}\right)^{m/n}$$

$$\sim 10^{3(1+5m/n)} M_{1\text{TeV}}^{1-m/n} \text{GeV}.$$  

Here we have defined the 4-D axion field as $a = \sqrt{V_m} \tilde{a}(x^A = 0)$ and assumed that the size $r_n$ is common for all extra dimensions.

A lower bound of $f_{PQ}$ comes from astrophysical bounds. It is known that $f_{PQ}$ should be larger than $10^9$GeV. In extra dimension physics, KK modes also contribute to supernova cooling if their masses are smaller than the core temperature ($\sim 30$ MeV) as we will show shortly.

To have $10^9$ GeV < $f_{PQ} \leq 10^{15}$ GeV, we need $2/5 < m/n \leq 4/5$. Possible sets of $(m, n)$ with $f_{PQ}$ where $M_* = 1$ TeV and 10 TeV can be found in tables.
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### 3 Laboratory and Astrophysical constraints

The strongest bound from astrophysics is the bound from the Supernova cooling. In SN1987A observation, it was calculated that

\[
\frac{1}{\tilde{f}_{PQ}} < 10^{-18} \text{ GeV}^{-2}, \tag{6}
\]

Since the axion KK modes interact exactly the same way as the conventional axion, the effective interaction of the KK modes at the core temperature \( T \simeq 30 \text{ MeV} \) is

\[
\frac{1}{\tilde{f}_{PQ}^2} \times (T r_n)^m \sim \frac{T^m}{M_*^{m+2}} < 10^{-18} \text{ GeV}^{-2}. \tag{7}
\]

This gives a bound on the fundamental scale

\[
M_* > (10^{18} \times 0.03^m)^{-m/2} \text{ GeV}. \tag{8}
\]
For $m = 1$, $M_\ast > 300$ TeV and $m = 2$, $M_\ast > 5$ TeV. $M_\ast \sim 1$ TeV is allowed only if $m > 2$.

In near future, high energy accelerator experiments may probe the axion KK mode emission. The graviton KK mode signals in collider were discussed in detail recently. The scattering cross section of the graviton KK mode emission from high energy scattering with center of mass frame energy $\sqrt{s}$ is

$$\sigma \propto \frac{1}{M^2_{pl}} (\sqrt{s/r_n})^n \sim \left( \frac{\sqrt{s}}{M} \right)^{n+2} \frac{1}{s},$$

while the KK axion production is

$$\sigma \propto \frac{1}{f_{PQ}^2} (\sqrt{s/r_n})^m \sim \left( \frac{\sqrt{s}}{M} \right)^{m+2} \frac{1}{s}.$$  \hspace{1cm} (9)

Since the energy dependence of the axion KK mode cross-section is different from the graviton KK mode cross-section, it might be possible to detect this difference at TeV scale collider experiments.

4 Thermal production of Axion KK mode

The axion KK mode masses are proportional to $r_n^{-1}$ and they have stronger couplings than the graviton KK modes to the matters in our universe in general cases. If there is no hidden particle which couples to the axion, the main decay channel of rather light KK axion is to two photons.

$$\Gamma_{a_{KK} \to 2\gamma} \sim \frac{C^2_{a\gamma}}{64\pi} \left( \frac{\alpha}{\pi} \right)^2 m_A^3 \frac{1}{f_{PQ}^2} \sim 3 \cdot 10^{-8} C^2_{a\gamma} \frac{m_A^3}{f_{PQ}^2},$$

where $m_A$ is the mass of the axion KK mode and $C_{a\gamma}$ is the axion-photon coupling which is usually within 0.1 to 1. This decay can be cosmologically dangerous. For instance, for $f_{PQ} = 10^{12}$ GeV and $m_A = 1$ MeV, life time of KK mode is $\tau_A \sim 10^{17}$ sec, which is about the age of the universe.

The graviton KK modes have similar cosmological problems because they can overclose our universe or decay into photons at a late stage of cosmological evolution. It was suggested that a “fat brane” can solve the cosmological problems by absorbing most of the decay products of the KK modes. However massless particles in the higher dimensional brane are not massless in our four-dimensional universe, thus it cannot solve over-closure problem.

If we add an axion as a “brane particle”, the graviton KK mode will decay to the “brane” axion more efficiently, since its decay width is enhanced by factor $(mr_n)^m$. Because the massive axion KK mode can decay into the photon pairs stronger than graviton KK mode, the primordial graviton KK mode will not over-close the universe. Instead, it will contribute to the cosmological background radiation. Axions can be produced thermally during the reheating process, which is also much stronger process than graviton. This can be a severe constraint to the axion model.

In our longer paper the yields $Y$ from four different sources of the axion KK modes were calculated, but here we will present only two relevant sources:
Figure 1. The function $I(T)$ for various $m_A$ in the KK mode yield $Y_I$.

I. the axion KK mode from the pion scattering ($\pi\pi \rightarrow \pi a_{KK}$); this process dominates if $T_R > 10$ MeV

II. the axion KK mode from the two photon inverse decay ($2\gamma \rightarrow a_{KK}$); this gives significant contribution when $m_A \sim T_R$.

\[ Y_I \simeq 6 \cdot 10^{-10} \left( \frac{10^{12}\text{GeV}}{f_{PQ}} \right)^2 \left( \frac{T_R}{100\text{MeV}} \right)^3 A_I, \]  

(12)

\[ A_I = C_{a\pi}^2 \left( \frac{10}{g_*(T_R)} \right)^{3/2} \left( \frac{I(T_R)}{1000} \right), \]  

(13)

\[ Y_{II} \simeq 2 \cdot 10^{-16} \left( \frac{10^{12}\text{GeV}}{f_{PQ}} \right)^2 \left( \frac{T_R}{100\text{MeV}} \right) C_{a\gamma}^2 \left( \frac{10}{g_*(T_R)} \right)^{3/2} \left( \frac{m_A}{T_R} \right)^3. \]  

(14)

For the details of these calculations and the definition of function $I(T)$, see ref. [10].

The lifetime of each KK mode for given $(n, m)$ can be found in tables.

5 Cosmological constraints

In table 1 and 2, $n \geq 5$ in both $M_* = 1$ and 10 TeV cases and $n = 4$ in 1 TeV have minimal KK mode mass greater than 1 MeV. If one takes the minimal reheating temperature of 1 MeV, these cases are cosmologically safe. On the other hand $m \leq 2$ in $M_* = 1$ TeV and $m = 1$ in $M_* = 10$ TeV is forbidden by the astrophysical bound. The cases $n = 4, m = 3$ at $M_* = 1$ TeV and $n = 3, m = 2$ at $M_* = 10$ TeV are not trivially safe or ruled out by the cosmological constraints.
5.1 Big bang nucleosynthesis

At the temperature of the universe around 1 MeV, it is required that there should not be additional particles which contribute to the energy density significantly. Otherwise $^4$He would be produced more than what is observed.

The yield of the axion KK mode at BBN for $T_R > 10$ MeV is

$$\frac{\rho_A}{s} \bigg|_{BBN} \simeq m_1 Y_I (m_1 r_n)^m < 0.1 \text{ MeV}, \quad (15)$$

KK mode may practically have maximal mass $m_1 \equiv \max\{m_π, T_R\}$ which is produced in the thermal bath.

If $T_R \sim m_π$, approximately

$$M_* > 10^{\frac{2m}{m_π}} \times m_π. \quad (16)$$

For $m = 1$, this reads $M_* > 600$ TeV, and for $m = 2$, $M_* > 15$ TeV. But if the reheating temperature is as low as 10 MeV, this bound is not important since $A_I(T_R)$ is suppressed exponentially.

5.2 Over-closure of Universe

The total energy of the axion KK modes at present must not exceed the critical density:

$$\rho_A < \rho_c = 3 \times 10^{-6} s_0 h^2 \text{ MeV}, \quad (17)$$

where $s_0 \simeq 3000$ cm$^{-3}$ is the entropy of the present universe.

For the case that the KK modes decay into some relativistic particles, we can divide the bound in two parts; decay before the present time and do not decay till now:

$$\frac{\rho_A}{s_0} \sim \int_{m_0}^{m_2} dm_A (m_{Arn})^m Y_A + \int_{m_2}^{m_1} dm_A (m_{Arn})^m \frac{Y_A T_0}{T(m_A)} \quad (18)$$

where the axion KK mode with mass $m_2$ decays at the present time.

5.3 Cosmological Microwave Background Radiation

If the massive KK modes decay after $10^6$ sec but before the recombination era, the produced photons may give a distortion of the cosmological microwave background radiation. The COBE observation gives a bound

$$\frac{\Delta \rho_\gamma}{s} \leq 2.5 \times 10^{-5} T_D \quad (19)$$

where $T_D$ is temperature at KK mode decay.

5.4 Diffuse photon background

Observations of diffuse photon backgrounds at the present universe give upper bounds on additional contributions to photon spectrum. For example, for the
energy range $800 \text{ keV} < E < 30 \text{ MeV}$

$$\frac{dF}{d\Omega} < 78 \left( \frac{E}{1\text{keV}} \right)^{-1.4}.$$  \hspace{1cm} (20)

Constraints on other ranges of the photon energy can be found, e.g. in Ref.\textsuperscript{12}

Theoretical prediction is

$$\frac{dF}{d\Omega} = \frac{n_A c}{4\pi} \times Br$$  \hspace{1cm} (21)

for the life-time of KK mode shorter than the age of the universe, and

$$\frac{dF}{d\Omega} \sim Br \times \frac{n_A c \Gamma_{\alpha KK \rightarrow 2\gamma}}{4\pi Br H_0} \left( \frac{2E}{m_A c^2} \right)^{3/2} (m_A r_n)^{m}$$  \hspace{1cm} (22)

for its life time longer than the age of the universe.\textsuperscript{13} $Br$ is a branching ratio of axion decay into two photons.

Since the axion decay width is highly suppressed by $\left( \frac{\alpha}{\pi} \right)^2$ with $\alpha$ being the fine structure constant it is easy to get a low branching ratio to decay into the photon.

We can assume that there exist another 4D wall in the brane. Or one can just imagine that there are some unknown particles on our wall. Most of the axions decay into the other wall if the coupling constant, the color factor of the other gauge interaction, and/or the number of the fermions with PQ charges in the decay loop diagram in the other wall (or the other particles) are large enough.

5.5 Results

I. $n = 4$, $m = 3$ and $M_* = 1 \text{ TeV}$ case:

1. BBN bound

$$T_R < 80 \text{ MeV}$$ \hspace{1cm} (23)

2. Over-closure bound,

$$T_R < 12 \left( \frac{C_{\alpha \gamma}^2}{A_I^2 Br} \right)^{1/4} \text{ MeV} \simeq 30 \text{ MeV} \text{ (for } Br = 1).$$ \hspace{1cm} (24)

3. CMBR bound, (for $m_A \geq 100 \text{ MeV}$ and $T_R > 10 \text{ MeV}$)

$$T_R < 2 \cdot 10^{-2} \left( \frac{C_{\alpha \gamma}^2}{A_I^2 Br} \right)^{1/5} \text{ MeV}$$ \hspace{1cm} (25)

4. Diffused photon bound,

for $T_R > 10 \text{ MeV},$

$$T_R < 2 \cdot 10^{-2} \left( \frac{Br}{C_{\alpha \gamma}} \right)^{-0.63} \left( \frac{m_A}{10 \text{MeV}} \right)^{-0.53} A_I^{-1/3} \text{ MeV},$$ \hspace{1cm} (26)

for $T_R < 10 \text{ MeV},$

$$T_R < 0.3 Br^{-0.73} \text{ MeV}. $$ \hspace{1cm} (27)
for $Br < \Gamma_{a \rightarrow 2\gamma}/H_0$, and

$$T_R < 5 C_{a\gamma}^{-0.48} \text{ MeV}$$  \hspace{1cm} (28)

where $Br > \Gamma_{a \rightarrow 2\gamma}/H_0$.

**II. $n = 3$, $m = 2$ and $M_\ast = 10 \text{ TeV}$ case:**

1. BBN bound

$$T_R < 90 \text{ MeV}. \hspace{1cm} (29)$$

2. Over-closure bound,

$$T_R < 28 \left( \frac{C_{a\gamma}^2}{A_f^2 Br} \right)^\frac{1}{2} \text{ MeV} \simeq 40 \text{ MeV} \text{ (for } Br = 1). \hspace{1cm} (30)$$

3. CMBR bound,

$$T_R < 4 \cdot 10^{-2} \left( \frac{C_{a\gamma}^2}{A_f^2 Br^2} \right)^\frac{1}{2} \text{ MeV}. \hspace{1cm} (31)$$

4. Diffused photon bound, for $T_R > 10 \text{ MeV},$

$$T_R < 3 \cdot 10^{-2} \left( \frac{Br}{C_{a\gamma}} \right)^{-0.63} \left( \frac{m_A}{10 \text{ MeV}} \right)^{-0.2} A_f^{-1/3} \text{ MeV}, \hspace{1cm} (32)$$

for $T_R < 10 \text{ MeV},$

$$T_R < 0.1 Br^{-1.2} \text{ MeV}, \hspace{1cm} (33)$$

for $Br < \Gamma_{a \rightarrow 2\gamma}/H_0$, and

$$T_R < 3 \text{ MeV} \hspace{1cm} (34)$$

where $Br > \Gamma_{a \rightarrow 2\gamma}/H_0$.

We also performed computer calculations on both $n = 4$, $m = 3$ and $M_\ast = 1 \text{ TeV}$ and $n = 3$, $m = 2$ and $M_\ast = 10 \text{ TeV}$ cases. Fig. 2 and 3 show that the DPBR is the stringent bound if the branching ratio is small. But in an extremely small branching ratio case, CMBR is dominant. This is because the life time of the axion KK modes becomes shorter than $10^{13} \text{ sec}$, and so the produced photons will disturb the CMBR spectrum.

We also performed full computer calculations for all possible $(m, n)$ sets in tables in our longer paper.

### 6 conclusions

In this paper, we discussed the axion model in the extra dimensions whose PQ scale lies in an intermediate scale $f_{PQ} \leq 10^{15} \text{ GeV}$. This intermediate scale can be obtained by introducing a $3 + m$ dimensional brane in the $4 + n$ dimension bulk.

If we include the axion as a brane particle, it will change the extra dimension physics, especially in the cosmological aspect. Since the graviton KK mode will decay into the axion KK mode, the over-closure problem is not as serious as
the original model of Arkani-Hamed et.al. On the other hand, the argument from stars and supernova cooling will give a more strict bound on the axion production. Among other things, the most severe cosmological bound comes from photon emission through the decays of the KK modes of the axion. We found that the astrophysical argument restrict the number of the dimensionality of the sub-spacetime where the axion lives: $m > 2$ for $M_\ast = 1$ TeV and $m > 1$ for $M_\ast = 10$ TeV. The latter case requires quite a low reheating temperature after the inflation.

To lift this bound, we can introduce the hidden matter/gauge fields to another four-dimensional wall (or to our wall itself) which has much stronger coupling to axion and/or much more generations of particles, (or maybe much lower QCD phase
transition scale, etc). This can significantly lower the branch ratio of the axion KK mode decay into photons.

This idea can be used to improve the original fat brane model. The intermediate scale brane can absorb most of graviton KK modes, and the particle in this brane can decay into relativistic particles in the four dimensional wall(s). This mechanism can solve the problem of the overclosure of the universe by the KK modes. If we introduce, in this brane, a massless particle (which is not an axion) which does not decay into photons or any dangerous particles in 4-D, we can avoid other cosmological problems such as the ones related to the cosmic photon backgrounds.

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