Front speed simulation with a hyperbolic model of atmospheric dynamics

M S Yudin

1 Institute of Computational Mathematics and Mathematical Geophysics SB RAS, Novosibirsk, Russia

e-mail m.yudin@ommgp.sscc.ru

Abstract. In this paper the front speed is simulated with a non-hydrostatic finite-element model of atmospheric dynamics. The model is based on the compressible Navier-Stokes equations in two dimensions. Artificial compressibility is introduced into the model in order to make the governing equations hyperbolic. Two tests of the model on different spatial scales are considered: the propagation of a mesoscale atmospheric gravity current (cold front) over flat terrain and the motion of a small scale coastal flow over irregular terrain. In the first test the calculated values of the simulation are compared with an empirical formula first introduced by T. von Karman and later developed for atmospheric fronts by many authors. In the second test, the model is applied to simulating the dynamic pressure effects caused by changes in roughness and interaction of the flow with the non-homogeneous terrain. For this purpose, we consider flow over a low hill at a coastal site. Both model simulation results are compared with available observations and simulations performed by other authors. In general, good agreement between the results of the test calculations and the theory has been obtained.

1. Introduction

In the present paper, atmospheric fronts are simulated with a non-hydrostatic finite-element model of atmospheric dynamics, which is applied to the calculation of the speed of propagation of a cold front in the atmosphere over flat terrain and the motion of a small scale coastal flow over irregular terrain. In the former, the calculated values of the simulation are compared with an empirical formula that has been used for the calculation of the speeds of atmospheric fronts by numerous authors ([3], [11]). In the second test, the model is applied to simulating the dynamic pressure effects caused by changes in roughness and interaction of the flow with the non-homogeneous terrain.

The model used in the present study can be considered, in the terminology of I Peshkov and E I Romenski, as the zeroth order approximation in an asymptotic expansion with respect to the small parameter of relaxation [21]. The model is constructed on some basic principles developed by S K Godunov and E I Romenski. These are: hyperbolicity, fully divergent form of the governing equations, and consistency with the laws of thermodynamics ([4],[21]). Hyperbolic systems have numerous theoretical and technical advantages. With specially-chosen variables, such systems can be transformed to symmetric form. Recently, hyperbolic models have been constructed for viscous Newtonian flows as well [2].

Fronts in the atmosphere are gravity currents that occupy a wide range of length scales from several meters to thousands of kilometres and examples of flows that take place in a variety of forms: breeze...
fronts, storm flows etc. [1]. Atmospheric phenomena of great theoretical and practical importance are cold atmospheric fronts that propagate near the surface with high speeds (see [1] for a review). These currents can be retarded and changed in shape under the influence of the underlying surface and stratification of the atmosphere. The retardation of a frontal system on the windward side of a mountain is a commonly observed phenomenon ([1], [12],[14]). These currents can be subdivided into classes varying from micro- to macro-scales. Atmospheric gravity currents occupy a wide range of length scales from several meters to thousands of kilometers. Mesoscale flows lie in an interval approximately from two to two thousand kilometers [9]. The flows of interest in the present study are mesoscale and microscale currents. These flows are relatively shallow: they belong to the atmospheric boundary layer and range only a few kilometers from the surface in the lower atmosphere.

An important property that any numerical model must contain to realistically simulate these phenomena is a detailed and accurate description of the orography. A popular approach to the incorporation of orography into a model is the use of a terrain-following coordinate system. This is, perhaps, the most widely employed method in the models of atmospheric processes that has so far been used to accurately describe the underlying surface. The domain of simulation is thus transformed into a rectangular one, and many computational problems become simpler to deal with. However, the model equations now contain additional terms which complicate the problem of constructing a numerical method that will conserve the invariants of the original system of equations. In addition, this approach is valid only for rather smooth transformation functions.

An approach based on the use of finite-element approximations for the functions to be simulated may be considered as an alternative to the above one. In the present study, to validate such an approach, a finite-element model based on the compressible Navier-Stokes equations in two dimensions is used. The model is thus calculated with two tests on different spatial scales: the propagation of a mesoscale atmospheric gravity current (cold front) over flat terrain and the motion of a small scale coastal flow over irregular terrain.

Artificial compressibility is introduced into the model in order to make the equations hyperbolic. The property of hyperbolicity is used to make the correct formulation of the proper boundary conditions. It also allows an efficient implementation of the computational algorithms and methods used in the simulation. An important step forward was recently made by Peshkov and Romenski [2] in formulating a hyperbolic model for viscous flows.

2. Model equations

The Navier-Stokes equations for a compressible air flow are used here for the calculation of gravity flows in a neutrally stratified atmosphere The exact form of the equations is given in [4]. A more detailed description of the model can be found, for instance, in [17]. In the present study, a two-dimensional finite-element version of the model is employed [16]. The time discretization is similar to that proposed in [13]. It is also described in paper [16]. A general form of the basic equations is as follows:
where \( p' \), \( \theta' \) are deviations from a reference state for pressure \( \bar{p} \) and potential temperature \( \bar{\theta} \), respectively, \( s \) is the specific humidity, \( C_s \) is the sound wave speed, \( u_g \) and \( v_g \) are geostrophic wind components representing the synoptic part of the pressure, \( f_1 \) and \( f_2 \) are the Coriolis parameters, \( g \) is the gravity constant. \( R_u, R_v, R_\omega, R_\theta, \text{ and } R_s \) are terms describing the subgrid-scale processes in the K-theory for turbulent diffusion in meteorology [4].

For the simulations in the present study a small-scale non-hydrostatic model is used, which is essentially a form of the Navier-Stokes equations in the Boussinesq approximation. A version of the equations relevant for a compressible fluid is used here. Since sound waves in the atmosphere have small effects on gravity flows like fronts, a compressible version of the Navier-Stokes equations is taken here to make the equations hyperbolic, and also for some pure technical reasons. The equations are discretized with a splitting method in time, and the compressible form of the equations is suitable for an efficient splitting. Sound waves are then eliminated by the use of an Asselin-type filter [7].

The problem examined in this study has very small physical diffusion, and the governing equations can be considered as almost inviscid and adiabatic. Triangular elements are used in the finite-element model [16]. In this model version a suitable scheme for the advection of a scalar, e.g. temperature, is implemented.

For the fully hyperbolic problems, explicit schemes have been widely used in the past and will probably continue to be used in the future. The problems for which the governing equations are hyperbolic have traditionally been solved by methods of characteristics. Sometimes the conventional methods of characteristics are slower than competitive finite difference methods. However, some finite difference schemes use a form of the governing equations such that knowledge of the characteristic locations can be exploited in an efficient way [10].

In this paper, the advection of a scalar like temperature is treated with an efficient semi-Lagrangian finite difference scheme of third-order [10] that is used as a reasonable compromise between cost and accuracy. A detailed description of the scheme can be found in [17].

3. Test1: cold front propagation speed calculation
The results of a series of calculations are presented here to simulate the front speed in the propagation of an atmospheric gravity current (cold front) over flat terrain under neutral stratification. The model parameters are taken from paper [11]. The calculation domain is 25x25 km. In contrast to [11], where the front is generated by a volume of cold air, in the present study the front is initially given in the form of a step-function of 400 m in height.

Figure 1 shows the initial location of the front in its propagation along flat terrain. The calculated values of the simulation are compared with an empirical formula that was first introduced by T von Karman in 1940 ([3], [12]). Since that time, it has been used for the calculation of the speeds of atmospheric fronts by numerous authors ([11], [3], [11]). This is the well-known empirical formula for the front speed and typical values of the front parameters [3]:

\[ v_F = k (gh_{IFH} \Delta \theta / \theta)^{0.5} \]
\[ k = 0.81 \]
\[ h_{IFH} = 400m \]
\[ \theta = 288K \]
\[ \Delta \theta = 2K \]
\[ v_F = 4.2m/s \]

Table 1 shows the calculated speed of cold front propagation over flat regular terrain under neutral stratification. A reasonably good agreement between the results of the calculations and the theory has been obtained.

Under stable stratification the front moves faster and shows an abrupt pressure jump at the point of observation. The introduction of an inversion layer into the atmosphere increases the pressure further. In contrast to the slow evolution of surface pressure under neutral stratification, there is a considerable pressure jump in a stable atmosphere. This effect is increased by the introduction of an inversion layer. This phenomenon was first explained by Charba [3]. The results of some previous calculations of these phenomena with the present model are in good agreement with Charba’s theory [4].

| Initial front height (m) | Stratification (K / 100 m) | Speed (m/s) |
|-------------------------|---------------------------|-------------|
| 400                     | 0.00                      | 4.5         |

Table 1. Calculated speed of cold front propagation
Neutral stratification.
4. Test2 : small scale coastal flow simulation

The purpose of this section is to describe the use of the model to simulate the dynamic pressure effects caused by changes in roughness in the interaction of a flow with a non-homogeneous terrain. For this purpose, we consider a small-scale water-land flow over a low hill at a coastal site [20].

The height of the hill is about 3 meters, which is achieved gradually at a distance of about 50 meters. The roughness coefficient is 0.001 cm for water and 1.00 cm for land (grass). The observations were performed at three 12-m masts having instruments at 1-, 2-, 3-, 5-, 8-, and 12-m levels. The topography of the domain and the location of the three masts are shown in Fig. 2. The measurements of the horizontal wind speed were taken on 17 October 1974 and described by Peterson et al. [20] and Takle et al. [19].

As reported by Takle et al. [19], notable features of the wind speed profiles are that the low-level flow experiences a decrease in speed due to increased roughness. At the upper levels, there is an increase in the wind speed caused by the increase in terrain elevation. Takle et al. [19] have shown that these effects cannot be realistically simulated with a hydrostatic model. They used a procedure proposed by Song et al. [18] to parameterize the non-hydrostatic terrain effects into their hydrostatic model. Instead of this, in the present study, the non-hydrostatic finite-element model described in Section 1 is used to simulate the flow.

The simulation results for horizontal wind speed are shown in Fig. 3 for comparison with observations performed at mast 2 (located at a distance of about 70 meters from the shore). The initial conditions are a neutral temperature profile and a wind field driven by a 10 m/s wind at the top.

It is not an easy task to compare in detail our results with the results of simulations performed in [19] by using the procedure of Song et al. [18], because some important parameters of the calculations may be different. In general, the agreement is good and, therefore, the above application of the non-hydrostatic model to the simulation of land-water boundary flows may be considered as a useful alternative to the procedure proposed in ([18], [19]).
5. Conclusions
In this paper the front speed was simulated with a non-hydrostatic finite-element model of atmospheric dynamics. The model is based on the compressible Navier-Stokes equations in two dimensions. Artificial compressibility is introduced into the model in order to make the governing equations...
hyperbolic. Two tests of the model on different spatial scales were considered: the propagation of a mesoscale atmospheric gravity current (cold front) over flat terrain and the motion of a small scale coastal flow over irregular terrain.

In the first test, the calculated values of the simulation were compared with an empirical formula first introduced by T. von Karman and later developed for atmospheric fronts by many authors ([3], [11]). In the second test, the model was applied to simulating the dynamic pressure effects caused by changes in roughness and interaction of the flow with a non-homogeneous terrain. For this purpose, we considered the flow over a low hill at a coastal site. Both model simulation results were compared with available observations and simulations performed by other authors. It has been shown that the non-hydrostatic model can be used to simulate realistically the effects of flows that are subject to dynamic influence of low terrain features and roughness changes at water-land boundaries. In general, in both tests, good agreement between the results of the test calculations and the theory has been obtained.

These preliminary results show that the finite-element model can be used for the simulation of atmospheric front propagation on different spatial scales over regular and irregular orographic obstacles. Although the present study is of limited scope, the above simulation results show that the numerical tools considered in this study can be used for the numerical simulation of gravity flows in the atmosphere.

Acknowledgements
This work was supported by the Russian Foundation for Basic Research under grant 17-01-00137 and ICMMG SB RAS under target program 0315-2016-0004.

References
[1] Schultz, D. M., 2005 A Review of Cold Fronts with Prefrontal Troughs and Wind Shifts, Mon. Wea. Rev., 133 pp 2449-2472
[2] Peshkov, I., Romenski, E., 2016 A hyperbolic model for viscous newtonian flows, Continuum Mechanics and Thermodynamics, 28 pp 85-104
[3] Charba, J., 1974 Application of a gravity current model to analysis of squall-line gust fronts, Mon. Wea. Rev., 102 pp 140-156
[4] Yudin M S 2016 A numerical study of gravity waves in the atmosphere: smooth and steep orography effects IOP Conference Series: Earth and Environmental Science 48 DOI http://dx.doi.org/10.1088/1755-1315/48/1/012024
[5] Marchuk G I 1974 Numerical Solution of Atmospheric and Oceanic Problems (Leningrad: Gidgometeoizdat) p 303
[6] Marchuk G I 1982 Mathematical Models in Environmental Problems (Moscow: Nauka) p 319
[7] Penenko V V, Aloyan A E 1985 Models and Methods for Environmental Protection Problems (Novosibirsk: Nauka) p 256
[8] Pielke R A 2002 Mesoscale Meteorological Modeling (San Diego: Academic Press) p 676
[9] Davies H C 1984 On the Orographic Retardation of a Cold Front Beitr. Phys. Atmos. 57 pp 409-418
[10] Ritchie H 1987 Semi-Lagrangian Advection on a Gaussian Grid Mon. Wea. Rev. 115 pp 136-146
[11] Bischoff-Gauss I, Gross G 1989 Numerical Studies on Cold Fronts. Part 1: Gravity Flows in a Neutral and Stratified Atmosphere, Meteorol. Atmos. Phys. 40 pp 150-158
[12] Bischoff-Gauss I, Gross G, and Wippermann F 1989 Numerical Studies on Cold Fronts. Part 2: Orographic Effects on Gravity Flows Meteorol. Atmos. Phys. 40 pp 159-169
[13] Ikawa M 1988 Comparison of Some Schemes for Non-Hydrostatic Models with Orography J. Meteor. Soc. Japan 66 pp 753-776
[14] Schumann U 1987 Influence of Mesoscale Orography on Idealized Cold Fronts J. Atmos. Sci. 44 pp 3423-3441
[15] Yudin M S 2011 Propagation of a Gravity Current in the Atmosphere over a Valley Bull. Novos. Comput. Center 14 pp 65-70
[16] Yudin M S, Wilderotter K 2006 Simulating Atmospheric Flows in the Vicinity of a Water Basin Computational Technologies 11 pp 128-134
[17] Yudin M S 2012 Comparison of FDM and FEM Models for a 2D Gravity Current in the Atmosphere over a Valley Bull. Novos. Comput. Center 13 pp 95-101
[18] Song, J.L., Pielke, R.A., Segal, M., Arritt, R.W. and Kessler, R.C., 1985 A method to determine non-hydrostatic effects within subdomains in a mesoscale model J. Atmos. Sci. 42 pp 2110-2120
[19] Takle E S, Russel R D 1988 Modeling the Atmospheric Boundary Layer Comput.Math.Appl 16 pp 57-68
[20] Peterson, E.W., Taylor, P.A., Hojistrup, J., Jensen, N.O., Kristensen, L. and Petersen, E.L., 1980 Further Investigations into the Effects of Local Terrain Irregularities on Tower-Measure Wind Profiles, Boundary-Layer Meteorology 19 pp 303-313
[21] Dumbser M, Peshkov I, Romenski E, Zanotti O 2016 J. Comput. Phys. 314 pp 824-862