The hydrodynamical modeling of the Supernovae Ia explosion by means adaptive nested meshes on supercomputers

Igor Kulikov, Igor Chernykh and Viktor Protasov
Institute of Computational Mathematics and Mathematical Geophysics SB RAS, Novosibirsk, Russia
E-mail: kulikov@ssd.sscc.ru

Abstract. The results of the study of an explosion point of supernova type Ia (SNIa) with using of mathematical modeling on supercomputers is given in the paper. Hydrodynamical model closed by the stellar equation of state and supplemented by Poisson equation for gravitational potential is used for modeling of a white dwarf. The nuclear combustion of carbon, for which the analytical solution is constructed, is accounted in the model. A multi-level organization of computations on nested grids is used in the solution. The new high-order accuracy numerical method based on the Godunov method, the Rusanov scheme and the piecewise parabolic method on local stencil, adapted for computations on nested grids, is built. The parallel implementation is based on the idea of distributed computations, where the architecture with shared memory is used for modeling of the hydrodynamic evolution of white dwarfs, when the critical values of temperature and density are reached, a new task is launched on the architecture with distributed memory, in which the evolution of hydrodynamic turbulence leading to supersonic nuclear combustion of carbon is simulated.

1. Introduction
A major scenario [1] of supernova explosion is based on the merging of two degenerate white dwarfs with subsequent collapse of the new star when it reaches the Chandrasekhar mass, ignition of the carbon burning process, and type Ia supernova explosion. Another reason for the high temperature of the shell of a degenerate dwarf (a component of a close binary star) may be tidal heating of its shell and mantle. A study of tidal heating of a degenerate component [2] has shown that the maximum temperature is achieved in the dwarf mantle at a point whose mass is about 90% of the total dwarf mass. Dissipation of the main energy of the satellite substance jet in the mantle makes its base the hottest zone. A simple estimation of nuclear fuel burning shows that at a density greater than $\sim 10^5 \text{ g/cm}^3$ the fuel burning time is less than the dynamic time of the dwarf, which is only several seconds. That is, at such densities noncentral ignition of the nuclear fuel at a point is sufficient for the simulation of nuclear fuel burning. The ignition point depth remains an important parameter to be determined in a separate study. The goal of this paper is to determine the role of the ignition point in nuclear fuel burning and in the dynamics of the remnants of a degenerate dwarf explosion.

In section 2, a hydrodynamics model of white dwarfs with a star equation and nuclear carbon burning is presented. In section 3, a numerical method for solving the equations and parallel &
distributed parallel computing is described. Section 4 is devoted to mathematical simulations. Conclusions to the paper are given in section 5.

2. Physics

The conservative form of the equations of gravitational gas dynamics we should use in following form:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0,
\]

\[
\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla p - \rho \nabla \Phi,
\]

\[
\frac{\partial}{\partial t} \left[ E + \rho \frac{\vec{u}^2}{2} \right] + \nabla \cdot \left( \left[ E + \rho \frac{\vec{u}^2}{2} \right] \vec{u} \right) = -\nabla \cdot (p \vec{u}) - (\rho \nabla \Phi, \vec{u}) + Q,
\]

\[
\Delta \Phi = 4\pi G \rho,
\]

where \( \rho \) is the density, \( \vec{u} \) is the velocity, \( p \) is the pressure, \( \Phi \) is the gravitational potential, \( E \) is the internal energy of the gas, \( G \) is the gravitational constant, and \( Q \) is a source of energy due to nuclear reactions. The equation of state for stars consists of the pressure of a nondegenerate hot gas and the pressure due to radiation and a degenerate gas [3].

As nuclear carbon burning we first consider a nuclear reaction responsible for the bombardment of carbon by carbon yielding natrium and proton

\[ 12C \ (12C, p) 23Na, \]

where \( Q = 2.24 \text{ MeV} \) is the energy released during the nuclear reaction. Assume that the nuclear reaction rate \( k_{12C(12C, p)23Na} \) is known from the literature [4]. The change in the carbon concentration \( n_{12C} \) in nuclear burning (2) can be written in the form of an ordinary differential equation:

\[
\frac{dn_{12C}}{dt} = -k_{12C(12C, p)23Na}n_{12C}^2.
\]

The concentrations of natrium \( n_{12C} = n_{12C}(t) \) and protons \( p \) are obtained from the balance of masses. Equation (2) has a simple analytical solution:

\[
n_{12C} (\tau) = \frac{1}{k_{12C(12C, p)23Na} \tau + \frac{n_{12C}^0}{n_{12C}^0}},
\]

where \( n_{12C}^0 = n_{12C}(0) \) is the initial (per time step) concentration of carbon. It is evident that as \( \tau \to \infty \) we have \( n_{12C} \to 0 \), which does not take place in a finite time. Hence, carbon burning is not complete and, therefore, some carbon will be contained in the remnants of a white dwarf explosion.

3. Numerical method

As noted in the introduction, the computational model is implemented using the idea of distributed calculations: a shared memory architecture is used to simulate the hydrodynamic evolution of white dwarf. When the temperature and density reach some critical values, a new task is started on a distributed memory architecture to simulate the development of hydrodynamic turbulence leading to supersonic nuclear carbon burning (see figure 1). Adaptive nested meshes are used to discrete the calculation domain, since the supernova explosion process at the stage of basic calculations takes place on different scales. For this, a regular root mesh is constructed in the calculation domain, where each cell is a nested mesh.
The hydrodynamical numerical method to solve the equations of hydrodynamics is based on a combination of Godunov’s method for conservation laws by calculating fluxes through the boundaries [5], an operator splitting method to construct a scheme that is invariant with respect to rotation to approximate the advection terms [6, 7, 8], and Rusanov’s method to solve Riemann problems [9] for determining the fluxes with vectorization of the calculations [10]. A compact scheme for a piecewise-parabolic representation of the solution in each of the directions is used to solve the Riemann problems [11, 12, 13]. To solve the Poisson equation on the root mesh for basic calculations and on the mesh for satellite calculations, an algorithm based on the fast Fourier transform is used. The method of successive over-relaxation is used for the nested meshes at the stage of basic calculations [14].

4. Numerical simulation

Let us simulate a white dwarf with one solar mass and temperature $T = 10^9$ K. The goal of this paper is to determine the role of the ignition point in nuclear fuel burning and in the dynamics of the remnants of a degenerate dwarf explosion. Figures 2–5 shows the simulation results density dynamics from the onset of the explosion to its passage through the bulk of the star by variation of position of the ignition point.

According to modeling result, see figures 2–5, degeneration of “O”-like structure of the SNIa residuals explosion occurs and the “C”-like structure is formed, when the detonation point is shifted on 20 percent. It means, that in most cases of SNIa explosions the mass of the residuals is concentrated mostly in flat one-way wave. It leads to propagation of the new “life”-elements in particular direction. Apparently, irregularity of the complex elements and compounds distribution is connected with different regimes of the SNIa explosions. Direction and velocity of the SNIa residuals distribution and its further interaction with molecular hydrogen clouds allows to predict the area of planetary systems formation and, possibly, exoplanets and moons. Such planetary systems are rich in metal and its compounds. Therefore, despite the catastrophic scenario of the SNIa explosion with the formation of an accentuated shock wave, this mechanism is one of the contenders for the scenario of formation of the planetary systems.
Figure 2. Relative density distribution from the onset of the explosion to its passage through the bulk of the star for position of ignition point is equal to zero.

Figure 3. Relative density distribution from the onset of the explosion to its passage through the bulk of the star for position of ignition point is equal to ten percent of radius.
Figure 4. Relative density distribution from the onset of the explosion to its passage through the bulk of the star for position of ignition point is equal to 30 percent of radius.

Figure 5. Relative density distribution from the onset of the explosion to its passage through the bulk of the star for position of ignition point is equal to 90 percent of radius.
with a rich chemical composition.

5. Conclusion
The results of the study of an explosion point of supernova type Ia (SNIa) with using of mathematical modeling on supercomputers is given in the paper. Hydrodynamical model closed by the stellar equation of state and supplemented by Poisson equation for gravitational potential is used for modeling of a white dwarf. The nuclear combustion of carbon, for which the analytical solution is constructed, is accounted in the model. A multi-level organization of computations on nested grids is used in the solution. The new high-order accuracy numerical method based on the Godunov method, the Rusanov scheme and the piecewise parabolic method on local stencil, adapted for computations on nested grids, is built. The parameters of the detonation point, when the degeneration of the “O”-like structure of the SNIa residuals explosion occurs and the “C”-like structure is formed.

Acknowledgments
This work was supported by Russian Science Foundation (project No. 18-11-00044).

References
[1] Iben I and Tutukov A 1999 The Astrophysical Journal 511 324-34
[2] Iben I, Tutukov A and Fedorova A 1998 The Astrophysical Journal 503 344-49.
[3] Timmes F and Arnett D 1999 The Astrophysical Journal Supplement Series 125 277-94
[4] Spillane T 2007 Physical Review Letters 98 122501
[5] Godunov S and Kulikov I 2014 Computational Mathematics and Mathematical Physics 54 1012-24
[6] Kulikov I, Chernykh I, Snytnikov A, Protasov V, Tutukov A and Glinsky B 2014 Parallel Programming: Practical Aspects, Models and Current Limitations 71-116
[7] Vshivkov V, Lazareva G, Snytnikov A, Kulikov I and Tutukov A 2011 Journal of Inverse and Ill-posed Problems 19 151-66
[8] Kulikov I, Lazareva G, Snytnikov A and Vshivkov V 2009 Lecture Notes of Computer Science 5698 414-22
[9] Rusanov V 1961 Computational Mathematics and Mathematical Physics 1 267-79
[10] Kulikov I, Chernykh I and Tutukov A 2018 Lobachevskii Journal of Mathematics 39 1207-16
[11] Popov M and Ustyugov S 2007 Computational Mathematics and Mathematical Physics 47 1970-89
[12] Popov M and Ustyugov S 2008 Computational Mathematics and Mathematical Physics 48 477-99
[13] Kulikov I and Vorobyov E 2016 Journal of Computational Physics 317 318-46
[14] Kulikov I 2018 Journal of Physics: Conference Series 1103 012011