A combined multifrequency map for point source subtraction

P. Naselsky,1,2⋆ D. Novikov3,4⋆ and Joseph Silk3⋆

1Theoretical Astrophysics Centre, Juliane Maries Vej 30, 2100 Copenhagen, Ø Denmark
2Rostov State University, Zorge 5, 344090 Rostov-Don, Russia
3Astronomy Department, University of Oxford, NAPL, Keble Road, Oxford OX1 3RH
4Astro-Space Centre of Lebedev Physical Institute, Profsoyuznaya 84/32, Moscow 117810, Russia

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ABSTRACT

A method is proposed for combining multifrequency maps in order to produce a catalogue of extragalactic point sources using data from future high-precision satellite experiments. We have found the optimal way of combining maps at different frequencies in order to maximize the signal (point sources) to noise (rest of the signal) ratio. Our approach is a natural multifrequency generalization of the bandpass filter introduced by Tegmark & de Oliveira-Costa. We show that combination of different frequency maps gives us the possibility of creating a more complete catalogue of point sources.

Key words: methods: statistical – cosmic microwave background – cosmology: observations.

1 INTRODUCTION

In the next few years, the new generation of cosmic microwave background (CMB) experiments (MAP and Planck) will provide all-sky maps of the CMB with high resolutions and sensitivities. Theoretical investigations of different kinds of foregrounds, instrumental noise properties, scan strategies and related topics for the future Planck mission are now in progress. The Planck LFI and HFI instruments will be able to measure the CMB anisotropy and polarization using 10 frequency channels that cover the frequency range 30–857 GHz (Mandolesi et al. 2000). The maps provided by Planck will contain contributions from various physical components, including the primary and secondary CMB signals and foregrounds such as free–free and synchrotron emission, the Sunyaev–Zeldovich effect, galactic dust and extragalactic point sources. One of the important goals of this experiment, apart from the separation of the CMB from the remaining parts of the signal, is the creation of a point source catalogue. The aim of this paper is to show how one may optimize the strategy for producing the point source catalogue by using the multifrequency properties of the radio/submillimetre sky observations.

The effect of point sources on satellite observations has been widely discussed in the literature [see the review by Vielva et al. (2001), and references therein]. Many authors have developed various linear and non-linear methods for point source extraction. Hobson et al. (1999) suggested the combination of the maximum-entropy method (MEM) with Mexican Hat wavelet filtering in order to separate all the physical components of the microwave sky including extragalactic point sources, using different frequency channels.

*E-mail: naselsky@tac.dk (PN); novikov@astro.ox.ac.uk (DN); silk@astro.ox.ac.uk (JS)

Tegmark & de Oliveira-Costa (1998, hereafter TO-98) described the linear filtering of point sources in the form of a bandpass filter in order to render point sources on each frequency map more visible relative to the background of other physical components. Cayon et al. (2000), Sanz et al. (1999) and Vielva et al. (2001) applied the Mexican Hat wavelet method to de-noising the CMB maps and for production of the Planck point source catalogue. Naselsky, Novikov & Silk (2002) and Chiang et al. (2001) proposed the approach of amplitude–phase analysis and applied it to simulated maps of the CMB with foregrounds, point sources and pixel noise included for the accurate determination of the locations of bright point sources and separation of these sources from the rest of the signal by an iteration technique. All of the methods mentioned above can complement each other in the framework of the future highly sensitive CMB experiments because of the different sensitivities of each method to different properties of the signal.

In this paper we focus our attention on the combination of maps with different frequencies in order to maximize the signal \((S – \text{point sources})\) to noise \((N – \text{rest of the signal})\) ratio. We show that co-addition of harmonics from different maps with specific weights leads to an increase in the accuracy of the point source subtraction. The aim of our paper is to produce the best possible combination of all frequency channels in order to construct a map in which the presence of point sources of some particular population is most visible. This procedure allows us to obtain a more complete and accurate catalogue of point sources than can be obtained by independently investigating each frequency channel (TO-98).

The layout of our paper is as follows. In Section 2 we introduce the definition of the ‘best’ map combination, derive the analytical solution for the maximum possible signal-to-noise ratio and make some numerical predictions for the Planck experiment. We also present a possible special combination of the multifrequency maps. We briefly summarize our results in Section 3.
2 OPTIMAL COMBINATION OF DIFFERENT FREQUENCY CHANNELS

In this section, we derive the optimal way of using different frequency channels to produce the best possible map for detection and subsequent subtraction of extragalactic point sources. The idea behind our approach is quite simple and transparent. We define the ‘best’ map as the combination of different frequency maps that maximizes the signal-to-noise ratio. In our case, the signal is the signal from point sources. The noise consists of other physical components including pixel noise, CMB, dust, and synchrotron and free–free emission. All of these components limit our capability to detect point sources.

2.1 Optimization

Suppose that we have N different maps of the same region of the sky made using N different channels with frequencies

\[ v_j, \quad i = 1, \ldots, N. \]

The flux of each point source

\[ S_i = S_i(v_j) \]

is a function of frequency, and therefore differs from one channel to another, while the position of this source is, obviously, the same for all maps. In this subsection, we consider for simplicity one particular population of point sources with the same dependence of fluxes on frequency. The contribution of point sources to the temperature map in the jth frequency channel can be written as a set of Dirac delta functions with coefficients

\[ S_j \]

proportional to the flux of the source at position \( \vec{r}_j \):

\[
\Delta T_{ps}^j = \sum_{i=1}^{N_p} S_j \delta(\vec{r}_j, \vec{r}).
\]  

Here, \( N_p \) is the total number of point sources. The temperature fluctuations on the map convolved with the antenna beam are therefore

\[
\Delta T^j(\vec{r}) = \sum_{i} s_{j} B(\vec{r}_i - \vec{r}) + \sum_{lm} a_{lm} Y_{lm}(\vec{r}) = \sum_{lm} T^j_{lm} Y_{lm}(\vec{r}),
\]  

where \( B(\vec{r}) \) is the beam for the jth channel; the amplitude of the point source \( s_{j} \) is proportional to the flux and depends on the antenna width: \( s_{j} \sim S_{j} / \theta^2 \). The coefficients \( a_{lm} \) have zero mean and variance \( \langle a_{lm} a^*_{lm} \rangle = C_{lm} \theta^2 + C_{pix} \) (pixel noise is not convolved with the beam). The set of harmonics \( T^j_{lm} \), \( j = 1, \ldots, N \), can be considered as the N-dimensional vector \( T^j_{lm} \). The aim of our paper is to combine N given maps with known values of the expected noise to construct the map with the best signal-to-noise ratio. In order to produce such a map, we introduce the vector filter \( \tilde{T}_{lm} \) with components \( \tilde{T}_{lm}^j, \ldots, \tilde{T}_{lm}^j \) and consider the combination of N maps in the following form:

\[
\Delta \tilde{T} = \sum_{lm} \tilde{T}_{lm} Y_{lm}(\vec{r}) = \sum_{lm} \tilde{T}_{lm} Y_{lm}(\vec{r}),
\]  

where \( \tilde{T}_{lm} = \sum_{j=1}^{N} \tilde{T}_{lm} \tilde{T}_{lm}^j \). We consider a symmetric antenna beam and subsequently drop the subscript \( m \) because \( \langle \tilde{T}_{lm} \tilde{T}_{lm}^j \rangle = \langle \tilde{T}_{lm} \tilde{T}_{lm}^j \rangle \). Thus the filter that we use should depend on only \( l \): \( \tilde{T}_l = \tilde{T}_l \). Our approach can readily be generalized to the case of an anisotropic beam, but this is beyond the scope of the present paper. Our problem now is to find vectors \( \tilde{T}_l \) for each \( l \) so that the ratio \( \gamma = \text{signal/noise} \) is maximized:

\[
\gamma = \Delta T_{ps}(\vec{r}) / \sigma_{ps} = \Delta T_{ps}^j(\vec{r}j) / \sigma_{ps}.
\]

\[
\Delta T_{ps}^j(\vec{r}j) = \sum_{i} (2l + 1) b^j_l f_i,
\]

\[
\sigma_{ps}^2 = \langle \Delta T_{ps}^j \rangle = \sum_{i} (2l + 1) f_i^2 M_i f_i.
\]  

Here \( \vec{b}_j \) is the vector with components \( b^j_l, j = 1, \ldots, N \) (\( h_0 \) is the beam) and \( M_i = [m_{i,j}^l] \) is the covariance matrix: \( m_{i,j}^l = \langle \alpha_{i,l}^m \alpha_{j,m}^l \rangle \). In our calculations, we use

\[
m_{i,j}^l = \langle C_{i}^l C_{j}^l \rangle^{1/2} b^l f_i^2 + C_{pix}^{1/2} \delta_{ij},
\]  

where \( \delta_{ij} \) is the Kronecker symbol. The ratio \( \gamma \) from equation (4) is maximized if we choose the filter to be

\[
\tilde{T}_l = M_l^{-1} \vec{b}_l.
\]  

Consequently, one can conclude that point sources with some particular dependence of flux on frequency are most clearly seen in the map that is the combination of N maps taken with these specific weights of harmonics. If only one channel is under consideration, then this formula becomes especially simple and is exactly the same as the one obtained by TO-98:

\[
\tilde{T}_l = \sum_{j} (2l + 1) b^l_j M_l^{-1} \vec{b}_j.
\]  

Using equations (4) and (6), one can get the final result for the maximum possible signal-to-noise ratio \( \gamma \), which is achieved in the combined map:

\[
\gamma = \left( \sum_{j} (2l + 1) \beta^l_j M_l^{-1} \vec{b}_l \right)^{1/2}.
\]  

2.2 Estimates for the Planck experiment

A multifrequency analysis gives us the possibility of detecting point sources in coincidence with different frequency maps. In our calculations, we assume the frequency dependence of the intensity for some particular point source population in the form of a simple power law: \( \nu^\alpha \). Multiplying this intensity by the conversion factor between surface brightness and temperature (Tegmark & Efstathiou 1996) and taking into account the resolution \( \Theta \), one can obtain the dependence of the point source temperature on frequency:

\[
\Delta T_{ps}^j \sim \nu^\alpha \sin h_j (\nu_j / 56.8 \text{ GHz}) / \Theta \nu_j, \quad j = 1, \ldots, 10
\]  

where \( j \) denotes the Planck channel number. Channels are numbered in order of increasing frequency (see Tables 1 and 2). Two different populations have been chosen: radio and far-infrared sources. Up to about 200 GHz, we can expect mostly radio sources to be detected, while at higher frequencies far-infrared sources dominate. For the radio source population, we take the spectral index \( \alpha \) from Vielva et al. (2001) to be close to zero up to 100–200 GHz (Table 1) and increasing to \( \infty \) at higher frequency. For the far-infrared population, this spectral index has the opposite behaviour and we choose \( \alpha \) to be \( \approx 2.7 \) for 857–352 GHz with practically no contribution from these sources to the signal at lower frequencies. Of course, our estimates are applicable only for these two specific populations. Each particular population of point sources should be investigated separately. This means that we should perform the combination of channels with specific filters \( \tilde{T}_l \) which correspond to the appropriate spectral index \( \alpha \) for this population.

Using the results of the previous subsection, one can calculate the signal-to-noise ratio for each channel separately \( \gamma(j) \) (if only the jth channel is under consideration (TO-98)) and for the combination of N channels. From equations (7) and (8) one finds
The contribution to the signal-to-noise ratio in each 

\( \gamma \), significant improvement can be achieved by considering both populations and consequently two channels with different frequencies:

Let us for simplicity consider a special case by dealing with only one source in the 10th channel is 1.2. Therefore, we introduce the parameter \( \rho \). This parameter contains information about amplitudes of temperature fluctuations arising from the same point sources in different channels:

\[ \Delta T_{ps} = \rho \sigma_{tot}, \quad \Delta T_{ps}^2 = (1 - \rho) \sigma_{tot}^2. \]

In Fig. 3 we show the signal-to-noise ratio that can be found by applying TO-98’s filters for the 143- and 217-GHz channels separately and for the combination of these two maps with different possible ratios of amplitudes \( \Delta T_{ps}^1 \) and \( \Delta T_{ps}^2 \). We choose these two channels because significant steepening or even a break in the point source spectrum because of synchrotron emission is expected over this range of frequencies, so it is hard to predict the exact value of \( \rho \).

For smaller values of \( \nu \) quite a large number of point sources are expected to be found in coincidence for different channels. It is interesting to investigate what our filter looks like if we combine, for example, the 70- and 100-GHz (LFI) maps.

According to formula (6) one can obtain the filter \( f_{ij} \) with components \( f_{ij}^1, f_{ij}^2 \):

\[ f_{ij}^1 = \frac{\beta_i^j m_{12} - \beta_i^j m_{11}}{\text{det}(M_j)}, \quad f_{ij}^2 = \frac{\beta_i^j m_{11} - \beta_i^j m_{12}}{\text{det}(M_j)}. \]

### Table 1

Results of the combination of 10 Planck channels for the population of radio sources with given spectral behaviour. The frequency of each channel is indicated in column 2 for both LFI and HFI. The third column gives spectral indices as calculated by Vielva et al. (2001). \(-\infty\) denotes a spectral break. The fourth column shows the initial signal-to-noise ratio in units of \( \sigma_{tot}^2 \) under the condition that the point source in the first channel has level \( 1 \sigma_{tot} \). The fifth column gives this ratio after applying the TO-98 filter to each channel separately. In the last column we show this ratio for the best combination of channels from 1 to \( j \). This means that the second row gives the result for the combination of the first and second channels, the third gives the combination of channels 1–3 and so on.

| Channel number \( (j) \) | Frequency (GHz) | Spectral index \( (\alpha) \) | Signal-to-noise ratio (initial) | Signal-to-noise ratio (TO-98) | Signal-to-noise ratio (combination) |
|-------------------------|-----------------|-----------------|-------------------------|-------------------------|-------------------------|
| 1                       | 30              | 0.08            | 1.000                   | 3.083                   | 3.083                   |
| 2                       | 44              | 0.08            | 0.986                   | 3.091                   | 5.894                   |
| 3                       | 70              | –0.10           | 0.935                   | 2.495                   | 8.579                   |
| 4                       | 100 (LFI)       | –0.16           | 0.947                   | 2.490                   | 10.862                  |
| 5                       | 100 (HFI)       | –              | 0.872                   | 3.622                   | 13.026                  |
| 6                       | 143             | –0.55           | 0.766                   | 3.429                   | 14.858                  |
| 7                       | 217             | –0.55           | 0.952                   | 3.357                   | 15.747                  |
| 8                       | 353             | –0.55           | 0.895                   | 1.886                   | 15.834                  |
| 9                       | 545             | –\( \infty \)   | 0.000                   | 0.000                   | 15.847                  |
| 10                      | 857             | –              | 0.000                   | 0.000                   | 15.925                  |

### Table 2

As Table 1 but for the far-infrared source population. The channels are shown in the opposite order. Initial signal-to-noise ratios are given under the assumption that the amplitude of the source in the 10th channel is \( 1 \sigma_{tot} \).

| Channel number \( (j) \) | Frequency (GHz) | Spectral index \( (\alpha) \) | Signal-to-noise ratio (initial) | Signal-to-noise ratio (TO-98) | Signal-to-noise ratio (combination) |
|-------------------------|-----------------|-----------------|-------------------------|-------------------------|-------------------------|
| 10                      | 857             | 2.46            | 1.000                   | 6.281                   | 6.281                   |
| 9                       | 545             | 2.71            | 0.811                   | 2.257                   | 6.575                   |
| 8                       | 353             | 2.71            | 0.551                   | 1.162                   | 6.960                   |
| 7                       | 217             | \( \infty \)    | 0.000                   | 0.000                   | 7.405                   |
| 6                       | 143             | –              | 0.000                   | 0.000                   | 7.460                   |
| 5                       | 100 (HFI)       | –              | 0.000                   | 0.000                   | 7.468                   |
| 4                       | 100 (LFI)       | –              | 0.000                   | 0.000                   | 7.470                   |
| 3                       | 70              | –              | 0.000                   | 0.000                   | 7.470                   |
| 2                       | 44              | –              | 0.000                   | 0.000                   | 7.470                   |
| 1                       | 30              | –              | 0.000                   | 0.000                   | 7.470                   |
Figure 1. Signal-to-noise ratio (in arbitrary units) for each channel shown separately after applying the TO-98 filter (dashed lines) and for the combination of 10 Planck maps (solid line). The calculation is performed for the population of radio sources with spectral parameter $\alpha$ taken from Table 1. The dotted line shows the same ratio, but for a break in the spectrum between 143 and 217 GHz.

Figure 2. As Fig. 1 but for the population of far-infrared sources.

Figure 3. Signal-to-noise ratio for 143- and 217-GHz channels after performing TO-98 filtering and for the combination of channels. Lower straight lines show signal-to-noise ratio for initial maps (in units of $\sigma_{\text{tot}}$) while upper straight lines show this ratio after TO-98 filtering. Dotted lines: 217 GHz; dashed lines: 143 GHz, solid line: combination of maps.

Substituting (13) in (12) we obtain

$$f_1^l \approx \sqrt{C_2^l \det(M)} \left( \beta_1^l \sqrt{C_1^l} - \beta_2^l \sqrt{C_2^l} \right).$$

$$f_2^l \approx \sqrt{C_1^l \det(M)} \left( \beta_2^l \sqrt{C_1^l} - \beta_1^l \sqrt{C_2^l} \right).$$

Therefore harmonics with small $l$ from different channels are taken with the opposite sign:

Figure 4. Components of the filter $f_l^i$ for combining the best possible map from the 70- and 100-GHz (LFI) channels (solid lines) in the comparison with TO-98 filters for each map taken separately (dashed lines). The filter is given in arbitrary units.
Thus the contributions from the first and second maps to the combined map compensate each other. Finally the total signal in the combined map turns to be close to zero at large scales. This means that our filter removes contributions from the CMB and other physical components at scales larger than the typical scale of the point sources.

For \( l \)-values that correspond to scales comparable to the antenna beam, the pixel noise is dominant and

\[
\begin{align*}
\mathcal{M}^{11}_l &\approx C^{1\text{pix}}_l, \\
\mathcal{M}^{12}_l &\approx C^{2\text{pix}}_l, \\
\mathcal{M}^{12}_l &\approx 0.
\end{align*}
\]

Therefore the components of the filter \( \hat{f}_l \) are close to the filters found in TO-98 for each channel separately (see Fig. 4):

\[
\begin{align*}
\hat{f}^{1}_l &\approx \frac{\beta^{1}}{C^{\text{pix}}_l}, \\
\hat{f}^{2}_l &\approx \frac{\beta^{2}}{C^{\text{pix}}_l}.
\end{align*}
\]

These components are both positive and add to form peaks that correspond to point sources in the combined map.

## 3 CONCLUSIONS

We have presented an analysis of microwave sky observations based on the combination of channels with different frequencies in order to separate extragalactic point sources for subsequent production of a point source catalogue. We made some numerical estimates for the future Planck experiment and showed that a multifrequency analysis allows us to increase the signal-to-noise ratio by a factor of 1.2–4.4 to create a more complete and precise catalogue of point sources.

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