EFFECT OF DISRUPTION RISK ON A SUPPLY CHAIN WITH PRICE-DEPENDENT DEMAND

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ABSTRACT. Supply chain disruption management has been a hot issue for a long time. This paper studies a supply chain consisting of two suppliers and a retailer. The suppliers may have correlated disruption risks and they choose their respective wholesale price. The retailer’s demand is selling price-dependent. In a scenario of ex-ante pricing scheme, i.e., the retailer chooses selling price before the suppliers’ disruption is resolved, we find that the disruption correlation affects the supply chain members’ profits in a nonmonotonic way. In a scenario of responsive pricing scheme, i.e., the retailer chooses selling price after the suppliers’ disruption is resolved, we show that when the suppliers’ disruption correlation becomes stronger, the suppliers’ profit will decrease while the retailer’s profit will increase. Moreover, the retailer always earns a higher profit under the responsive pricing scheme than under the ex-ante pricing scheme. In an extension, we show that the suppliers’ disruption correlations affects the supply chain members’ profits in a similar way if the supply chain has two competitive retailers. However, in this extension, if the suppliers’ disruption correlation is low, the retailers’ profit is higher under the responsive pricing scheme; otherwise, the retailers’ profit is higher under the ex-ante pricing scheme.

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1. Introduction. According to a survey by McKinsey Global Firm Managers in 2006, nearly two-thirds of executives believe that supply chain risks faced by firms have been increasing in the past few years, which has become one focus of the recent round of strategic planning. The sources of supply risk include strike, fire, natural disaster, financial crisis, etc., that prevent suppliers from delivering products successfully. Consequently, downstream firms of supply chains may be confronted with operational challenges. In 1993, a plant of Sumitomo Chemical, a leading Japanese supplier of semiconductor materials, exploded and the supply was interrupted. This seriously threatened many semiconductor manufacturers in the downstream and brought them great losses [24]. For another example, in 2000, Ericsson lost $400 million in sales, and its market share fell from 12% to 9% due to supply disruption [29].

A rich literature has studied many different strategies for supply chain risk management, e.g., the single-supplier strategy and the portfolio-supplier management strategy. The single-supplier strategy refers to the situation that a firm chooses an ideal supplier among all the suppliers to be a partner. Burke et al. [4] indicate that single sourcing is a dominant strategy only when supplier capacities are large relative to the product demand and when the firm does not obtain diversification benefits. The portfolio-supplier management strategy refers to the situation that a firm keeps in touch with many suppliers simultaneously and allocates order quantities among the suppliers. According to an empirical study conducted by Shin et al. [27], dual or multiple sourcing is a common business practice because it may help firms reduce the loss of disruption risks.

From the point of default, supply chain risks ultimately relate to the default of member enterprises, which are related to each other. We also call this phenomenon default correlation [41]. On the one hand, the default probability of different firms is affected by common systemic factors including macro-factors and industrial-factors that lead to indirect default correlation. On the other hand, if there is a direct business relationship among firms, the default of one enterprise may spread and increase the default probability of related firms [7, 11]. For example, two supplier factories in southern Japan were closed at the same time after the earthquake in 2016, and consequently Toyota Motor Co.’s supply was interrupted [15]. Obviously, the default process of these two suppliers were correlated. Similarly, the agricultural product suppliers located in the same geographic area are usually affected by the same weather. Usually, the common way for enterprises to mitigate the adverse consequences of supplier default is to implement multi-sourcing procurement strategy. The premise of this strategy is to assume that the default of one supplier is largely unrelated to that of other suppliers. However, this implicit assumption is not applicable to all cases. Due to factors such as production equipments, suppliers, geographical environments, national politics and economy, etc., the default probability of different firms may be correlated. Lucas [20], DeServigny and Renault [26] and Das et al. [8] discuss the empirical evidence of the correlation among financial defaults. In this paper, we study the impact of default correlation between suppliers in the sourcing context.

Based on the supply disruption problem, our study examines two unreliable suppliers’ models, one for a single retailer and the other for two competing retailers. We are concerned about the effect of default correlation between the two suppliers and pay close attention to pricing, ordering, and revenue. The correlation of supplier defaults may have a certain impact on diversification and price competition.
We take an example illustrate the main idea. If the relationship between the two default processes of two suppliers is perfectly positively correlated, then, for the retailers, the two suppliers will be identical and the benefit of diversification will disappear. Such a situation will lead to loss of the suppliers’ profits and increment of the retailer’s profit. It will enable the retailer to gain all the surplus from the supply chain. With the reduction of default correlation, the benefit of diversification increases while price competition decreases. If the relationship between the default processes of the two suppliers is perfectly negatively correlated, only one of the two suppliers will deliver the products successfully. In this case, price competition will disappear. Therefore, these two suppliers can extract all surplus from the supply chain together.

Suppliers have two types of default: partial breach and complete breach. In this study, we assume that the supplier’s default is a complete breach \[3, 2\]. We associate a price-dependent demand with two different pricing schemes in the model and obtain the explicit equilibrium of the game. Under the ex-ante pricing scheme of the single retailer, profits of the two suppliers and the retailer are nonmonotonic under the effects of competition and diversification. The endogenous selling price brings more profit to the retailer. Under the responsive pricing scheme of the single retailer, the retailer’s profit is always higher than that under the ex-ante pricing scheme.

In addition, we extend our model to incorporate the two retailers’ competition. A new price-dependent demand function is introduced, in which there are two parameters for demand sensitivity. Under the ex-ante pricing scheme of the two retailers, the equilibrium shows that the default correlation has a similar effect on the supply chain. Besides, we fix one of the parameters for the demand sensitivity and change the other. We derive that profits of the two suppliers and two retailers are increasing with the varying parameter. It is interesting that the retailers’ preference over the two pricing schemes changes with the default correlation. The competition between two retailers has played a role in the extension, which brings about a result that is completely different from the result in basic model.

The rest of this paper is arranged as follows. In section 2, we review the related literature on supply chains with supply disruptions. In section 3, we present the basic model and main results. Section 4 compares the supply chain performance under different pricing schemes. Section 5 extends the basic model to competitive retailers. Conclusions are presented in section 6.

2. Literature review. Supply chain disruption management has received wide attention recently \[32, 39, 16, 33, 28\]. The single-supplier scenario has been extensively studied \[36, 23, 6, 14\]. However, the literature on two or multiple unreliable suppliers is limited \[12, 21\]. In our paper, we consider a supply chain consisting of two supplier who may both be subjected to disruption risks. Anupindi and Akella \[1\] assume that the default processes of suppliers are independent and investigate three different delivery contracts for suppliers: all-or-nothing delivery; partial delivery in the current period; partial delivery delivery in the next period. Different from Anupindi and Akella \[1\], we assume that the suppliers’ disruption risks may be correlated. Such a scenario is studied by Babich et al. \[3\]. Babich et al. \[3\] study that a retailer, concerned with random supply disruption, faces a tradeoff between diversification and price competition effects, both of which might depend on the correlation of supplier defaults. In our paper, we employ a supply chain structure similar to Babich et al. \[3\]. However, our paper differs from Babich et al. \[3\] in...
several important aspects. First, Babich et al. [3] assume that the market demand is characterized by a constant or a random variable. We assume that the demand is price-dependent, which is more general. Second, we investigate the effect of default correlation on the profit of supply chain members when the retailer use two different pricing schemes, which is not discussed in Babich et al. [3]. We assume that the retailer may use an ex-ante pricing scheme or responsive pricing scheme. Third, Babich et al. [3] consider the competition of two or multiple suppliers. We not only analyze the competition of two suppliers, but also increase the competition of two retailers. Because the model settings have much difference between Babich et al. [3] and our paper, the management insights in these two studies are much different.

Considering inventory models with price-dependent demands, most studies focus on a product from perfectly reliable supply [34, 22, 17, 13]. Tang [30] analyzes two pricing policies in the presence of supply uncertainty, and show that the retailer would always obtain a higher expected profit under the responsive pricing policy. Our paper differs from their papers in several ways. First, our paper examine the firm’s diversification and ordering decisions with two unreliable suppliers while there is only one supplier in their model. Second, we consider the suppliers’ disruption risks may be correlated. Li et al. [18] study a firm’s sourcing and pricing decisions with unreliable dependent suppliers and a price-dependent demand. However, they assume that the wholesale price is exogenously determined. Our paper assumes that the wholesale price is endogenously determined, which is more general because the primary purpose of having two or multiple suppliers is to increase competition.

In a model extension, we consider the effect of default correlation on order quantities, selling prices, and wholesale prices in the background of retailers’ competition. Retailers’ competition in the background of unreliable supply has been extensively studied [38, 35, 37, 40]. Xiao and Qi [36] focus on the cooperation between a manufacturer and two competing retailers. The impact of cost disruption on the supply chain is discussed based on two different cooperation mechanisms. Based on the work of Xiao and Qi [36], Cao et al. [5] propose a cooperation mechanism between a manufacturer and multiple competing retailers. The changes in order quantities and wholesale price are compared under different costs and demand disruptions. Our paper differs from their papers. We consider two unreliable suppliers while there is only one supplier in their model.

3. Basic model. In the basic model, we study a supply chain model consisting of one retailer and two suppliers. The suppliers produce perfectly substitutable products with identical production lead time. Each supplier may be exposed to disruption risk, and the retailer’s orders to each supplier may not be delivered successfully. Let \( t_i \in \{0, 1\}, \ i = 1, 2 \), where \( t_i = 0 \) means the products are delivered successfully and \( t_i = 1 \) means that the products are lost. \( p_{t_1t_2} \) denotes the joint probability where supplier 1’s delivery state is \( t_1 \) and supplier 2’s delivery state is \( t_2 \). For example, \( p_{00} \) is a probability where supplier 1 delivers the products successfully and supplier 2 also delivers the products successfully. Default correlation is described by \( p_{11} \). Obviously, \( p_{00} + p_{01} + p_{10} + p_{11} = 1 \) [3]. In the basic model, we focus on the effects of the default correlation between the two suppliers on the supply chain and assume that \( \alpha = p_{00} + p_{01} \) and \( \beta = p_{00} + p_{10} \). This assumption indicates that the delivery rates of the two suppliers are fixed. Therefore, as \( p_{00} \) increases, \( p_{01} \) and \( p_{10} \) will decrease (and from subscripts, 01 and 10, only one supplier delivers products successfully), and the the default correlation, \( p_{11} \), increases.
The delivery rates are known to all agents and the supply chain members are assumed to be risk-neutral. In fact, when \( p_{01} = p_{10} = 0 \) and \( \alpha = \beta \), the defaults are perfectly positively correlated. When \( p_{00} = p_{11} = 0 \), the defaults are perfectly negatively correlated. It is necessary to discuss all the ranges from perfect negative correlation to perfect positive correlation. In the following numerical analysis, we set \( \alpha = \beta = 0.5 \) and allow \( p_{00} \) to vary between 0 and 0.5.

We assume that the retailer faces a price-dependent demand \( d = a - s \), where \( a \) is the size of the demand market and \( s \) is the selling price. The linear form is widely adopted in the literature [9, 18]. Without loss of generality, we assume that the production costs of the two suppliers are zero [19]. In addition, we assume that the retailer shoulders the cost of the lost orders, and the penalty cost caused by unsatisfied demand and salvage value of unsold products are zero.

We assume that the delivery risk is taken by the retailer. Because in life, there are many large-scale retailers ordering from small suppliers. The delivery risk may lead to the bankruptcy of the small suppliers and the retailer cannot recoup any fund. Babich et al. [3] use the same assumption.

The sequence of the game is as follows: (1) Under the ex-ante pricing scheme, the suppliers set their respective wholesale price simultaneously. Then, the retailer orders from the suppliers after the delivery uncertainty is resolved. (2) Under the responsive pricing scheme, suppliers set their respective wholesale prices simultaneously. Then, the retailer orders from the suppliers and sets a selling price before the delivery uncertainty is resolved.

In the following, we investigate the retailer’s optimal ordering and pricing decisions. Then we compare the performance of the two pricing schemes and investigate the effect of suppliers’ default correlation on the supply chain.

### 3.1. Ex-ante pricing scheme

The retailer’s profit function is

\[
\pi_R = sE_{p_{11}, p_{12}} \{ \min\{a - s, (1 - t_1)q_1 + (1 - t_2)q_2\} \} - w_1q_1 - w_2q_2, \tag{1}
\]

where \( q_1 \) and \( q_2 \) are the orders that the retailer makes to suppliers 1 and 2, respectively.

**Lemma 3.1.** Given wholesale prices \( w_i \), \( i = 1, 2 \), the retailer’s optimal selling price is

\[
s^* = \begin{cases} 
\tau & \text{if } \tau > \left( \frac{w_{12}}{p_{01}} + \frac{w_{2}}{p_{02}} \right)^+, \\
\frac{w_{11}}{p_{01}} & \text{if } \frac{w_{11}}{p_{01}} > \left( \tau, \frac{w_{2}}{p_{02}} \right)^+, \\
\frac{w_{11}}{p_{02}} & \text{if } \frac{w_{11}}{p_{02}} > \left( \tau, \frac{w_{2}}{p_{01}} \right)^+,
\end{cases}
\]

and the retailer’s optimal ordering quantities are

\[
(q^*_1, q^*_2) = \begin{cases} 
(a - \tau, a - \tau) & \text{if } \tau > \left( \frac{w_{11}}{p_{01}} + \frac{w_{2}}{p_{02}} \right)^+, \\
(a - \frac{w_{2}}{p_{02}}, a - \frac{w_{11}}{p_{01}}) & \text{if } \frac{w_{11}}{p_{01}} > \left( \tau, \frac{w_{2}}{p_{02}} \right)^+, \\
(a - \frac{w_{2}}{p_{02}}, a - \frac{w_{11}}{p_{02}}) & \text{if } \frac{w_{11}}{p_{02}} > \left( \tau, \frac{w_{2}}{p_{01}} \right)^+,
\end{cases}
\]

where \((x, y)^+ = \max\{x, y\}, (x, y)^- = \min\{x, y\}\) and \(\tau = \frac{a}{2} + \frac{\tau_{12} + \tau_{21}}{2(p_{00} + p_{01} + p_{02})} \cdot \frac{1}{\tau_1 + \tau_2} \cdot \frac{1}{\tau_1 + \tau_2} \).
First, the optimal selling price increases in the default correlation for the first three cases in Lemma 3.1. Second, we find that the total optimal order is two times of market demand. This is because in our model, if the default happens at one supplier, the retailer loses all the products from that supplier; to hedge against the default risks, the retailer may order a total quantity that is bigger than the market demand.

Supplier $i$’s profit is

$$\pi_{Si} = w_i q_i,$$

where $i = 1, 2$, we can derive the equilibrium of the game.

We denote $\Phi = \max\{0, \min\left[\frac{3p_{01} - 2p_{10}}{2}, \frac{3p_{10} - 2p_{01}}{2}\right]\}$.

**Proposition 1.** Under the ex-ante pricing scheme, the equilibrium of the game is given in below.

(a) The optimal wholesale prices are

$$(w_1^*, w_2^*) = \begin{cases} \left(\frac{(p_{00} + p_{01} + p_{10})a}{3}, \frac{(p_{00} + p_{01} + p_{10})a}{3}\right) & \text{if } p_{00} \leq \Phi, \\ \left(\frac{(p_{00} + p_{01} + p_{10})a}{2p_{00} + p_{01} + p_{10}}, \frac{(p_{00} + p_{01} + p_{10})a}{2p_{00} + p_{01} + p_{10}}\right) & \text{otherwise.} \end{cases}$$

(b) The retailer’s optimal selling price is

$$s^* = \begin{cases} \frac{5a}{p} & \text{if } p_{00} \leq \Phi, \\ \frac{(p_{00} + p_{01} + p_{10})a}{2p_{00} + p_{01} + p_{10}} & \text{otherwise,} \end{cases}$$

and optimal order quantities are

$$(q_1^*, q_2^*) = \begin{cases} \left(\frac{a}{p}, \frac{a}{p}\right) & \text{if } p_{00} \leq \Phi, \\ \frac{(p_{00}a}{2p_{00} + p_{01} + p_{10}}, \frac{p_{00}}{2p_{00} + p_{01} + p_{10}}\right) & \text{otherwise.} \end{cases}$$

If the two suppliers’ defaults are perfectly negatively correlated ($p_{01} + p_{10} = 1$), i.e., only one supplier’s products are being delivered successfully, there will be no competition between the suppliers and the suppliers obtain the maximal expected profit. In contrast, if the two suppliers’ defaults are perfectly positively correlated ($p_{00} + p_{11} = 1$), i.e., the suppliers both deliver the products successfully or both do not, then the two suppliers are perfect substitutable by each other for the retailer. Thus the retailer gets all profit of the supply chain and the suppliers get no profit.

From Proposition 1, we also find that wholesale prices decrease with the default correlation, the selling price is nonincreasing with the default correlation, and the order quantity is nondecreasing with the default correlation. The relationship between the profits of all members and the default correlation is as follows.

**Remark 1.** When $p_{00} \leq \Phi$, if the default correlation between the two suppliers increases, the profits of suppliers and the retailer will decrease; otherwise, the retailer’s profit will increase, while the two suppliers’ profits will be non-monotonic.

Remark 1 shows that the the default correlation between the two suppliers affects the supply chain in different ways on the two sides of the threshold. When $p_{00} < \hat{p}_{00}$, as $p_{00}$ increases (i.e., the competition between the two suppliers becomes stronger), the suppliers reduce their respective wholesale price, and the retailer does not change the selling price and order quantity; the profits of suppliers and the retailer are all reduced. The suppliers are harmed due to the more fierce competition between them, while the retailer is harmed as the profit loss from the weaker diversification effect dominates the profit gain from the fierce competition between the suppliers.

\[ \begin{align*}
\pi_{Si} &= w_i q_i, \\
\Phi &= \max\{0, \min\left[\frac{3p_{01} - 2p_{10}}{2}, \frac{3p_{10} - 2p_{01}}{2}\right]\}.
\end{align*} \]
Therefore, the increase of $p_{00}$ hurts all members of the supply chain. When $p_{00} > \hat{p}_{00}$, as $p_{00}$ increases, the retailer gets a bigger profit as it benefits more from the supply diversification effect. Note that now the two suppliers’ profits are non-monotonic in $p_{00}$. Our results is an important complement to Babich et al. [3], who show that the suppliers are always harmed and the retailer is always benefited as the default correlation increases.

3.2. Responsive pricing scheme. In this section, we study the case of responsive pricing scheme, i.e., the retailer chooses its selling price after the disruption risk is resolved. The retailer’s expected profit is

$$\pi_R = E_{p_{1t},p_{2t}}[\Lambda(q_1,q_2)] - w_1q_1 - w_2q_2,$$

where

$$\Lambda(q_1,q_2) = \max_{s \in [0,a]} s \min\{a - s,(1-t_1)q_1 + (1-t_2)q_2\}.$$

Since the retailer always prefers $q_1 \leq \frac{a}{2}$, $q_2 \leq \frac{a}{2}$, and $q_1 + q_2 \geq \frac{a}{2}$, the retailer’s optimization problem will be:

$$\pi_R = p_{01}(a - q_1)q_1 + p_{10}(a - q_2)q_2 + \frac{p_{00}a^2}{4} - w_1q_1 - w_2q_2,$$

s.t. $q_1 + q_2 \geq \frac{a}{2}$,

$$0 \leq q_i \leq \frac{a}{2}, \ i = 1, 2.$$

Lemma 3.2. Given wholesale prices $w_i$, $i = 1, 2$, the retailer’s optimal ordering quantities are

$$(q_1^*,q_2^*) = \left(\left(\frac{a}{2} - \frac{w_1}{2p_{01}}\right)^+,\left(\frac{a}{2} - \frac{w_2}{2p_{10}}\right)^+\right),$$

where $(x)^+ = \max\{x,0\}$.

With the increase of the default correlation, the retailer’s order quantities from the two supplier both decrease. It is interesting to note that the retailer’s order
quantity from one supplier only depends on the supplier’s wholesale price. Based on supplier \( i \)'s profit function (2), we can derive the equilibrium of the game.

**Proposition 2.** Under the responsive pricing scheme, the equilibrium of the game is given below.

(a) The optimal wholesale prices are 
\[
(w^*_1, w^*_2) = \left( \frac{p_{01} \alpha}{2}, \frac{p_{10} \alpha}{2} \right).
\]

(b) The optimal order quantities are 
\[
(q^*_1, q^*_2) = \left( \frac{a}{4}, \frac{a}{4} \right).
\]

Proposition 2 shows that under the responsive pricing scheme, the default correlation only affects the suppliers’ optimal wholesale prices, but does not affect the retailer’s optimal ordering quantities. So the benefit of the diversification effect will disappear. The retailer benefits from the intensified competition between the two suppliers. In the responsive pricing scheme, the total optimal order quantity is exactly the market demand. This is because the retailer has ability to adjust selling price after the yield is realized. For example, if the yield is lower than expected market demand, the retailer can raise selling price to compensate for the loss of insufficient products. Therefore, the retailer does not need to place orders with twice the market demand as he does under the ex-ante pricing scheme.

**Remark 2.** As the default correlation between the two suppliers increases, the retailer’s optimal profit increases, while the two suppliers’ respective optimal profits decrease.

Remark 2 is an important complement to Li et al. [18], who find that the suppliers’ capacity correlation does affect the firm’s optimal order quantities under the exogenous wholesale price. We consider the effect of suppliers’ default correlation on retailer’s profit under endogenous wholesale price. We see that, with the increase in default correlation, the supply chain’s profit and the suppliers’ profit all decrease, but the retailer’s profit increases.

4. Impact of pricing scheme. In this section, we compare the performance of the supply chain under the two pricing schemes. By comparing the market demand and the total order quantity under the two pricing schemes, we can find that the market demand of the ex-ante pricing scheme is smaller than that of the responsive pricing scheme under the same other conditions. Because pricing and ordering at the same stage may result in a loss of profit. However, pricing after the yield is realized can reduce the loss of supply uncertainty under the responsive pricing scheme. As for the difference in the order quantity, it is also caused by the different pricing schemes. The retailer can reduce the loss caused by insufficient orders or the loss caused by excessive orders under the responsive pricing scheme, so the total order quantity is market demand. On the other hand, it is meaningless for the retailer to order part of market demand to suppliers under the ex-ante pricing scheme. The optimal order quantity is two times of market demand.

**Proposition 3.** Suppose that \( \alpha \geq \beta \).

(a) \( \pi^*_{R} \geq \pi^*_{R} \).

(b) When \( \alpha \leq \frac{27 \beta}{25} \), \( \pi^*_{S_1} \leq \pi^*_{S_1} \) if \( p_{00} \leq \frac{5 \alpha - 43}{5} \), and \( \pi^*_{S_1} > \pi^*_{S_1} \) if \( p_{00} > \frac{5 \alpha - 43}{5} \). If \( p_{00} = \Phi \), then \( \pi^*_{S_1} \leq \pi^*_{S_1} \).

(c) If \( p_{00} \leq \max(0, \frac{5 \beta - 4 \alpha}{5}) \), \( \pi^*_{S_2} \leq \pi^*_{S_2} \). Otherwise, \( \pi^*_{S_2} > \pi^*_{S_2} \).
The result of Proposition 3 is illustrated in Figure 2. When the default correlation is weak (i.e., $p_{00} < 0.1$), the suppliers get a higher profit under the responsive pricing scheme; otherwise they get a higher profit under the ex-ante pricing scheme. In contrast, the retailer always get a higher profit under the responsive pricing scheme than under the ex-ante pricing scheme. Specially, when $p_{00}$ is small, the suppliers and the retailer may achieve a win-win result by using the responsive pricing. Proposition 3 is an important complement to Dong et al. [10], who show that the responsive pricing is quite efficient in mitigating the risk. We consider the impact of suppliers’ default correlation on the retailer’s profit under the two pricing schemes with endogenous wholesale price. We find that the retailer prefers the responsive pricing scheme to the ex-ante pricing scheme.

5. The model of two retailers. In this section, we extend our model to incorporate the competition between two retailers. Considering a market that consists of two suppliers and two retailers, the settings of the suppliers are the same as in the basic model. The two retailers set their respective selling prices; that is, $s_i$ for retailer $i$, $i=1,2$. The demand of retailer $i$ is $D_i(s_i, s_j) = a - \theta_1 s_i + \theta_2 s_j$, for $i=1,2$, $j=3-i$, where $\theta_1$ represents the sensitivity of retailer $i$’s demand to its own selling price $s_i$, and $\theta_2$ represents the sensitivity of retailer $i$’s demand to its rival’s selling price $s_j$. This linear function is widely used in the competition between two retailers [36, 2, 25]. We assume that $\theta_1 > \theta_2$, and the two retailers set prices simultaneously.

5.1. Ex-ante pricing scheme. Retailer $i$’s profit function is
\[
\Pi_{R_i} = p_{00} s_i \min \{q_{i1} + q_{i2}, D_i(s_i, s_j)\} + p_{01} s_i \min \{q_{i1}, D_i(s_i, s_j)\}
+ p_{10} s_i \min \{q_{i2}, D_i(s_i, s_j)\} - w_1 q_{i1} - w_2 q_{i2},
\]
where $q_{ik}$ indicates retailer $i$’s order to supplier $k$, for $i=1,2$, $k=1,2$. 

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Figure 2. Ex-ante pricing scheme vs. responsive pricing scheme in the basic model.
We can see that retailer $i$ always prefers $q_{it} \leq D_i(s_i, s_j)$ and $q_{it} + q_{ij} \geq D_i(s_i, s_j)$, $i = 1, 2$. We denote

$$\Phi_1 = \max\{0, \min\{\alpha - \frac{(2\theta_1 - \theta_2)\beta}{3\theta_1 - 2\theta_2}, \beta - \frac{(2\theta_1 - \theta_2)\alpha}{3\theta_1 - 2\theta_2}\}\},$$

$$\Phi_2 = \max\{\frac{(\alpha + \beta)(\theta_1 - \theta_2)}{2\theta_1 - \theta_2}, \Phi_1\}.$$

Let $\Omega = \{(\theta_1, \theta_2) | 0 \leq \Phi_1, \Phi_2 \leq \min\{\alpha, \beta\}\}$. Supplier $k$'s profit is

$$\Pi_{Sk} = w_k(q_{1k} + q_{2k}),$$

where $k = 1, 2$. Based on supplier $k$'s profit function (3), we can derive the equilibrium of the game and the result is given in below.

**Proposition 4.** Under the ex-ante pricing scheme of two retailers, when $(\theta_1, \theta_2) \in \Omega$,

(a) the optimal wholesale prices are

$$(w_1^*, w_2^*) = \begin{cases} 
\left(\frac{(\alpha + \beta - p_{00})\alpha}{3\theta_1 - 2\theta_2}, \frac{(\alpha + \beta - p_{00})\alpha}{3\theta_1 - 2\theta_2}\right) & \text{if } 0 \leq p_{00} \leq \Phi_1, \\
\left(\frac{(\alpha - p_{00})\alpha}{2\theta_1 - \theta_2}, \frac{(\beta - p_{00})\alpha}{2\theta_1 - \theta_2}\right) & \text{if } \Phi_2 \leq p_{00} \leq \min\{\alpha, \beta\}, \\
\left(\frac{(\alpha + \beta - p_{00})\alpha}{(\alpha + \beta)(\theta_1 - \theta_2) + p_{00}\theta_2}, \frac{(\alpha + \beta - p_{00})\alpha}{(\alpha + \beta)(\theta_1 - \theta_2) + p_{00}\theta_2}\right) & \text{otherwise};
\end{cases}$$

(b) the optimal selling prices are

$$(s_1^*, s_2^*) = \begin{cases} 
\left(\frac{5\theta_1 - 3\theta_2}{2(\theta_1 - \theta_2)}, \frac{5\theta_1 - 3\theta_2}{2(\theta_1 - \theta_2)}\right) & \text{if } 0 \leq p_{00} \leq \Phi_1, \\
\left(\frac{a}{2(\theta_1 - \theta_2)}, \frac{a}{2(\theta_1 - \theta_2)}\right) & \text{if } \Phi_2 \leq p_{00} \leq \min\{\alpha, \beta\}, \\
\left(\frac{(\alpha + \beta - p_{00})\alpha}{(\alpha + \beta)(\theta_1 - \theta_2) + p_{00}\theta_2}, \frac{(\alpha + \beta - p_{00})\alpha}{(\alpha + \beta)(\theta_1 - \theta_2) + p_{00}\theta_2}\right) & \text{otherwise},
\end{cases}$$

and optimal order quantities are

$$(D_1^*, D_2^*) = \begin{cases} 
\left(\frac{\theta_1}{3(2\theta_1 - \theta_2)}, \frac{\theta_1}{3(2\theta_1 - \theta_2)}\right) & \text{if } 0 \leq p_{00} \leq \Phi_1, \\
\left(\frac{a}{2}, \frac{a}{2}\right) & \text{if } \Phi_2 \leq p_{00} \leq \min\{\alpha, \beta\}, \\
\left(\frac{p_{00}\theta_1}{(\alpha + \beta)(\theta_1 - \theta_2) + p_{00}\theta_2}, \frac{p_{00}\theta_1}{(\alpha + \beta)(\theta_1 - \theta_2) + p_{00}\theta_2}\right) & \text{otherwise};
\end{cases}$$

The result of Proposition 4 is illustrated in Figure 3. We assume that the condition $(\theta_1, \theta_2) \in \Omega$ holds in Proposition 4, so we can find that the curves of the profits of supply chain members is divided into three parts in Figure 3. If the condition does not hold, the result of Proposition 4 will degenerate. For example, If $\Phi_1 > \min\{\alpha, \beta\}$, the optimal decision of supply chain members will degenerate into the first case; if $\Phi_2 > \min\{\alpha, \beta\}$, the second case will not exist. Therefore, it is more general to limit the $(\theta_1, \theta_2)$ to the interval $\Omega$.

By comparing Proposition 4 and Proposition 1, we can find that the optimal order quantity of two retailers is still two times of market demand. The main reason for this phenomenon is caused by the ex-ante pricing. In addition, when the condition $(\theta_1, \theta_2) \in \Omega$ holds, the model of two retailers has similar structure to that of single retailer. From the first case of Proposition 4, we can see that the profits of suppliers and retailers decrease with the default correlation. In other words, there is a win-win situation between suppliers and retailers. Proposition 4 is an important complement to Tang and Kouvelis [31], who find that retailers benefit from a lower supplier correlation. We consider that wholesale prices for suppliers are endogenous, and the equilibrium obtained gives firms managerial insights. From the suppliers'
point of view, the default correlation between one supplier and the other competing supplier may be beneficial to their own profits; from the retailers’ point of view, retailers may benefit from a higher default correlation between suppliers.

**Remark 3.** When $p_{00} \leq \Phi_1$, if the default correlation between the two suppliers increases, the profits of the two suppliers and two retailers will decrease; otherwise, the two retailers’ profits will increase, while the two suppliers’ profits will be non-monotonic.

Figure 3 illustrates that under the ex-ante pricing scheme, the default correlation has a similar effect on the supply chain with two competing retailers.

**Remark 4.** All things being equal, the profits of the two suppliers and two retailers are increasing with $\theta_2$.

Remark 4 tells us an interesting result, i.e., when the competition between the retailers becomes stronger ($\theta_2$ is large), the suppliers and the retailers all get a higher profit. This result is illustrated by Figure 4. Moreover, we find that $\Phi_1$ decreases with $\theta_2$.

### 5.2. Responsive pricing scheme.

Under the responsive pricing scheme, the two suppliers set their wholesale prices, and then the two retailers place orders with the two suppliers, respectively. Finally, the two retailers set their own selling prices respectively and simultaneously after the disruption risk is resolved. Let $Q_i = (1-t_1)q_{i1} + (1-t_2)q_{i2}$, $i = 1, 2$. Then, retailer $i$’s profit is

$$\Pi_{Ri} = E_{p_{11.22}}[\Psi(Q_i)] - w_1q_{i1} - w_2q_{i2},$$

where

$$\Psi(Q_i) = \max_{s_i \in [0,a]} \min_{s_j \in [0,a]} \{D_i(s_i, s_j), Q_i\}, \text{ for } i = 1, 2, j = 3 - i.$$

Based on supplier $k$’s profit function (3), we can derive the equilibrium of the game shown in the following proposition.

**Proposition 5.** Under the responsive pricing scheme of two retailers, the equilibrium of the game is given in below.
Figure 4. Impact of the demand sensitivity under the ex-ante pricing scheme of the two retailers

(a) The optimal wholesale prices are
\[
(w^*_1, w^*_2) = \left( \frac{(\alpha - p_{00})a}{2(\theta_1 - \theta_2)}, \frac{(\beta - p_{00})a}{2(\theta_1 - \theta_2)} \right).
\]

(b) The optimal order quantities are
\[
(q^*_i, q^*_i) = \left( \frac{(\theta_1 + \theta_2)a}{4\theta_1 + 2\theta_2}, \frac{(\theta_1 + \theta_2)a}{4\theta_1 + 2\theta_2} \right), \quad i = 1, 2.
\]

Remark 5. As the default correlation between the two suppliers increases, the two retailers’ profits increase, while the two suppliers’ profits decrease.

From Figure 5, we find that the profits of the two suppliers and two retailers are increasing with \(\theta_2\). \(\theta_2\) is independent of with the default correlation.

After that, we compare the performances of the two different pricing schemes under the impact of the retailers’ competition. We denote \(\hat{p}_{00} = \frac{(\alpha + \beta)(\theta_1 + \theta_2)\theta_1}{2(\theta_1 + \theta_2)^2} + 4\theta_1^2 + 5\theta_1\theta_2 - 9\theta_2^2\),
\[
\lambda_1 = \alpha - \frac{8\theta_1^2 + 43\theta_1\theta_2}{10\theta_1^2 + 5\theta_1\theta_2 - 9\theta_2^2}, \quad \text{and} \quad \lambda_2 = \beta - \frac{8\theta_1\theta_2^2 + 4\theta_1^2\theta_2}{10\theta_1^2 + 5\theta_1\theta_2 - 9\theta_2^2}.
\]

Proposition 6. Suppose that \((\theta_1, \theta_2) \in \Omega\).

(a) If \(0 \leq p_{00} \leq \hat{p}_{00}\), \(\Pi^*_R = \Pi^*_S\), \(i = 1, 2\). Otherwise, \(\Pi^*_R > \Pi^*_S\), \(i = 1, 2\).

(b) If \(\lambda_i < \Phi_1\) and \(p_{00} \leq \lambda_i\), then \(\Pi^*_S < \Pi^*_R\); if \(\lambda_i < \Phi_1\) and \(p_{00} > \lambda_i\), then \(\Pi^*_S > \Pi^*_R\); otherwise, \(\Pi^*_S \leq \Pi^*_R\), for \(i = 1, 2\).

The result of Proposition 6 is illustrated in Figure 6. In the model of two competitive retailers, the default correlation affects the suppliers’ profit in the similar way as that in the model of one retailer, which is shown in Proposition 3. In contrast, the retailers get higher profits under the ex-ante pricing scheme if \(p_{00}\) is larger than \(\hat{p}_{00}\), which is different from that in the model of one retailer. Therefore, the retailers’ competition may affect their preference over different pricing schemes. Proposition 6 is an important complement to Tang and Yin [30], who suggest that the retailer would always obtain a higher expected profit under the responsive pricing policy.
Figure 5. Impact of the demand sensitivity under the responsive pricing scheme of the two retailers

Figure 6. Ex-ante pricing scheme vs. responsive pricing scheme in the model of the two retailers

6. Conclusion. This study investigates the effects of suppliers' default correlation on a supply chain with price-dependent demand. In the model of two suppliers and one retailer, we show that the effect of diversification and competition dominates each other in different parameter regions. When suppliers' default correlation is in a lower range, stronger default correlation will reduce the suppliers’ and the retailer’s profit simultaneously; when the default correlation in a higher range, stronger default correlation will increase the retailer’s profit and affect the suppliers’ profit nonmonotonically. On different pricing schemes, we show that the retailer always
get a higher profit using the responsive pricing than using the ex-ante pricing. The suppliers’ preference on different pricing schemes depends on the parameter region.

We also consider the case of two competitive retailers. We show that the monotonicity of the supply chain members’ profits with the default correlation is similar to that in the model with a unique retailer. However, different from that in the model with a unique retailer, the competitive retailers’ preference over the two pricing schemes changes with the default correlation. When the default correlation is low, retailers prefer the responsive pricing scheme; otherwise, they prefer the ex-ante pricing scheme.

The results of our study have the following management implications for firms sourcing from unreliable suppliers: First, when selecting suppliers, firms need to consider the degree of default correlation between suppliers. Second, suppliers need to pay attention to maintaining certain default correlation with other competitors. Third, the two pricing schemes, ex-ante pricing and responsive pricing, may help firms and suppliers achieve win-win situation under certain conditions.

This study provides new insights on the relative strength of diversification and competition, which contributes to the literature on supply chain disruption. In our analysis, we have made several assumptions. For example, we assume that market demand is a specific form, so we can get the impact of default correlation between suppliers on the profits of supply chain members under different pricing schemes. In future research, the analysis in our study can be extended to the case where the yield is multiplicative so that a fraction of the ordered quantity is received. Most managerial insights are valid in these extensions. We can consider whether the main insights of this paper is still valid for stochastic demand. It is also meaningful to include firms’ effort to improve the supply chain reliability into our model.

Appendix.

Proof of Proposition 1. When \((q_1^*, q_2^*) = (a - s, a - s)\) with \(s^* = \frac{a}{2} + \frac{w_{11} + w_{22}}{2(p_{00} + p_{01} + p_{10})}\), we have

\[
\begin{align*}
\pi_{S_1} &= w_1(a - (\frac{a}{2} + \frac{w_{11} + w_{22}}{2(p_{00} + p_{01} + p_{10})})), \\
\pi_{S_2} &= w_2(a - (\frac{a}{2} + \frac{w_{11} + w_{22}}{2(p_{00} + p_{01} + p_{10})})),
\end{align*}
\]

where \(\frac{a}{2} + \frac{w_{11} + w_{22}}{2(p_{00} + p_{01} + p_{10})} \geq \max\{\frac{w_{12}}{p_{01}}, \frac{w_{21}}{p_{10}}\}\).

We can easily solve this problem by K-K-T conditions, and then find the optimal wholesale prices. When \(p_{00} \leq \Phi\), we can get \((w_{11}^*, w_{22}^*) = \left(\frac{(p_{00} + p_{01} + p_{10})a}{3}, \frac{(p_{00} + p_{01} + p_{10})a}{3}\right)\). Otherwise, we have the optimal wholesale prices when the conditions \(\frac{a}{2} + \frac{w_{11} + w_{22}}{2(p_{00} + p_{01} + p_{10})} = \frac{w_{12}}{p_{01}}\) and \(\frac{a}{2} + \frac{w_{11} + w_{22}}{2(p_{00} + p_{01} + p_{10})} = \frac{w_{21}}{p_{10}}\) hold.

\[
(w_{11}^*, w_{22}^*) = \begin{cases}
\left(\frac{(p_{00} + p_{01} + p_{10})a}{3}, \frac{(p_{00} + p_{01} + p_{10})a}{3}\right) & p_{00} \leq \Phi, \\
\left(\frac{(p_{00} + p_{01} + p_{10})a}{2p_{00} + p_{01} + p_{10}}, \frac{(p_{00} + p_{01} + p_{10})a}{2p_{00} + p_{01} + p_{10}}\right) & \text{otherwise}.
\end{cases}
\]

Proof of Remark 1. According to Proposition 1, we can derive the optimal profits of the retailer \((\pi_{R}^*)\), suppliers \((\pi_{S_1}^* \text{ and } \pi_{S_2}^*)\), and system \((U^*)\) are
Proof of Remark 2. We use backward induction to solve the model. It is easy to derive that the optimal wholesale prices are \((w_1^*, w_2^*) = \left(\frac{p_0a}{2}, \frac{p_1a}{10}\right)\). Therefore, the optimal profits of the retailer \((\pi_R^*)\), suppliers \((\pi_{S_1}^*, \pi_{S_2}^*)\), and system \((U^*)\) are

\[
\pi_R^* = \frac{(p_0 + p_{10} + 3p_0)a^2}{16},
\]

\[
\pi_{S_1}^* = \frac{p_0a^2}{8},
\]

\[
\pi_{S_2}^* = \frac{p_{10}a^2}{8},
\]

\[
U^{**} = \frac{(3p_0 + 3p_{10} + 4p_00)a^2}{16}.
\]

Proof of Proposition 3. When \(p_{00} \leq \Phi\) and \(\alpha \geq \beta\), that is, \(\Phi = \max\{0, 3\beta - 2\alpha\}\),

\[
\pi_R^* - \pi_R^{**} = -\frac{(32p_00 + 5p_0 + 5p_{10})a^2}{144} < 0,
\]

so \(\pi_R^* < \pi_R^{**}\).

\[
\pi_{S_1}^* - \pi_{S_1}^{**} = \frac{(4p_{10} - 5p_0 + 4p_010)a^2}{72} = \frac{(5p_{10} + 4\beta - 5\alpha)a^2}{72},
\]

\[
\pi_{S_2}^* - \pi_{S_2}^{**} = \frac{(4p_0 + 4p_0 - 5p_{10})a^2}{72} = \frac{(5p_0 + 4\alpha - 5\beta)a^2}{72}.
\]

Let \(\mu_1 = \frac{5\alpha - 4\beta}{5}\). If \(\alpha \leq \frac{27\beta}{25}\), that is \(\mu_1 \leq \Phi\), when \(p_{00} \leq \mu_1, \pi_{S_1}^* \leq \pi_{S_1}^{**}\); when \(\mu_1 < p_{00} \leq \Phi\), \(\pi_{S_1}^* > \pi_{S_1}^{**}\). If \(\alpha > \frac{27\beta}{25}\), that is \(\mu_1 > \Phi\), then \(\pi_{S_1}^* < \pi_{S_1}^{**}\).

Let \(\mu_2 = \frac{5\beta - 4\alpha}{5}\). When \(p_{00} \leq \max\{0, \mu_2\}\), \(\pi_{S_2}^* \leq \pi_{S_2}^{**}\); when \(\mu_2 < p_{00} \leq \Phi\), \(\pi_{S_2}^* > \pi_{S_2}^{**}\).

When \(p_{00} > \Phi\),

\[
\pi_R^* - \pi_R^{**} = -\frac{(p_0 + p_{10})(4p_00 + 8p_00p_0 + 8p_00p_{10} + p_0^2 + 2p_01p_{10} + p_{10}^2)a^2}{16(2p_0 + p_010)^2} < 0,
\]

\[
\pi_{S_1}^* - \pi_{S_1}^{**} = \frac{(4p_00 + 4p_00p_0 + 4p_00p_{10} - p_0^2 - 2p_01p_{10} - p_{10}^2)p_01a^2}{8(2p_0 + p_010)^2} > 0,
\]

\[
\pi_{S_2}^* - \pi_{S_2}^{**} = \frac{(4p_00 + 4p_00p_0 + 4p_00p_{10} - p_0^2 - 2p_01p_{10} - p_{10}^2)p_01a^2}{8(2p_0 + p_010)^2} > 0.
\]

Therefore, \(\pi_R^* < \pi_R^{**}, \pi_{S_1}^* > \pi_{S_1}^{**}\), and \(\pi_{S_2}^* > \pi_{S_2}^{**}\).
We have its to the game is unique and the optimal profits of the retailers $\Pi_s = p_{s1}q_1 + p_{s2}q_2 - w_1q_1 - w_2q_2$, s.t. $0 \leq q_1 \leq D_1(s_1, s_2)$.

If $p_{s1} > w_2$ and $p_{s0} > w_1$, the optimal order of retailer 1 is $(D_1(s_1, s_2), D_1(s_1, s_2))$. Similarly, the optimal order of retailer 2 is $(D_2(s_2, s_1), D_2(s_2, s_1))$. We are easy to get the optimal selling prices,

$$(s_1^*, s_2^*) = \left( \frac{ap_{s0} + ap_{s1} + ap_{s10} + \theta_1w_1 + \theta_2w_2}{2(\theta_1 - \theta_2)(p_{s0} + p_{s0} + p_{s0})}, \frac{ap_{s0} + ap_{s01} + ap_{s10} + \theta_1w_1 + \theta_2w_2}{2(\theta_1 - \theta_2)(p_{s0} + p_{s1} + p_{s0})} \right).$$

By K-K-T conditions, we derive the optimal wholesale prices. When $s_i^* \geq \max \left\{ \frac{w_1}{p_{s0}}, \frac{w_2}{p_{s0}} \right\}$, $i = 1, 2$, $(w_1^*, w_2^*) = \left( \frac{(p_{s0} + p_{s1} + p_{s0})a}{3(\theta_1 - \theta_2)}, \frac{(p_{s0} + p_{s01} + p_{s10})a}{3(\theta_1 - \theta_2)} \right)$, where $p_00 > \frac{\alpha_1 - \alpha a_2 + \beta_1 - \beta a_2}{2a_1 - a_2}$. Otherwise, $s_i^* = \frac{w_i}{p_{s0}}$, $i = 1, 2$,

$$\frac{ap_{s0} + ap_{s01} + ap_{s10} + \theta_1w_1 + \theta_2w_2}{2(\theta_1 - \theta_2)(p_{s0} + p_{s0} + p_{s0})} = \frac{w_1}{p_{s0}} = \frac{w_2}{p_{s0}}.$$

We have

$$w_1^* = \frac{a(\alpha - p_{s0})(\alpha + \beta - p_{s0})}{\alpha_1 - \alpha a_2 + \beta_1 - \beta a_2 + p_{s0}a_2},$$
$$w_2^* = \frac{a(\beta - p_{s0})(\alpha + \beta - p_{s0})}{\alpha_1 - \alpha a_2 + \beta_1 - \beta a_2 + p_{s0}a_2},$$

where $p_{s0} \in \left[ \min\{\alpha - \frac{(2a_1 - a_2)\beta}{3a_1 - 2a_2}, \beta - \frac{(2a_1 - a_2)\alpha}{3a_1 - 2a_2}, \frac{\alpha - \alpha a_2 + \beta_1 - \beta a_2}{2a_1 - a_2}, \frac{\beta - \beta a_2 - \alpha a_1 + \alpha a_2}{2a_1 - a_2} \right]$. 

**Proof of Proposition 4.**

$$\Pi_{R1} = p_{s0}q_1D_1(s_1, s_2) + p_{s1}q_1D_1(s_1, s_2) + p_{s1}q_1D_1(s_1, s_2) - w_1q_1 - w_2q_2,$$

s.t. $0 \leq q_1 \leq D_1(s_1, s_2)$.

We are easy to get the optimal selling prices,

$$(s_1^*, s_2^*) = \left( \frac{ap_{s0} + ap_{s1} + ap_{s10} + \theta_1w_1 + \theta_2w_2}{2(\theta_1 - \theta_2)(p_{s0} + p_{s0} + p_{s0})}, \frac{ap_{s0} + ap_{s01} + ap_{s10} + \theta_1w_1 + \theta_2w_2}{2(\theta_1 - \theta_2)(p_{s0} + p_{s1} + p_{s0})} \right).$$

By K-K-T conditions, we derive the optimal wholesale prices. When $s_i^* \geq \max \left\{ \frac{w_1}{p_{s0}}, \frac{w_2}{p_{s0}} \right\}$, $i = 1, 2$, $(w_1^*, w_2^*) = \left( \frac{(p_{s0} + p_{s1} + p_{s0})a}{3(\theta_1 - \theta_2)}, \frac{(p_{s0} + p_{s01} + p_{s10})a}{3(\theta_1 - \theta_2)} \right)$, where $p_00 > \frac{\alpha_1 - \alpha a_2 + \beta_1 - \beta a_2}{2a_1 - a_2}$. Otherwise, $s_i^* = \frac{w_i}{p_{s0}}$, $i = 1, 2$,

$$\frac{ap_{s0} + ap_{s01} + ap_{s10} + \theta_1w_1 + \theta_2w_2}{2(\theta_1 - \theta_2)(p_{s0} + p_{s0} + p_{s0})} = \frac{w_1}{p_{s0}} = \frac{w_2}{p_{s0}}.$$

We have

$$w_1^* = \frac{a(\alpha - p_{s0})(\alpha + \beta - p_{s0})}{\alpha_1 - \alpha a_2 + \beta_1 - \beta a_2 + p_{s0}a_2},$$
$$w_2^* = \frac{a(\beta - p_{s0})(\alpha + \beta - p_{s0})}{\alpha_1 - \alpha a_2 + \beta_1 - \beta a_2 + p_{s0}a_2},$$

where $p_{s0} \in \left[ \min\{\alpha - \frac{(2a_1 - a_2)\beta}{3a_1 - 2a_2}, \beta - \frac{(2a_1 - a_2)\alpha}{3a_1 - 2a_2}, \frac{\alpha - \alpha a_2 + \beta_1 - \beta a_2}{2a_1 - a_2}, \frac{\beta - \beta a_2 - \alpha a_1 + \alpha a_2}{2a_1 - a_2} \right]$. 

**Proof of Remark 3.** According to Proposition 4, we can derive the equilibrium profits to the game is unique and the optimal profits of the retailers $(\Pi_{s1}^{r*})$, suppliers $(\Pi_{s1}^{s*})$, and system $(V_{r*})$ are

$$\Pi^{s*}_{s1} = \left\{ \begin{array}{ll} \frac{2a(\alpha + \beta - p_{s0})a_12a_1}{2(a_1)\theta_1(\theta_1 - \theta_2)2a_1} & \text{if } 0 \leq p_{s0} \leq \Phi_1, \\
\frac{a(\alpha - p_{s0})a_12a_1}{2(a_1)\theta_1(\theta_1 - \theta_2)2a_1} & \text{if } \Phi_2 \leq p_{s0} \leq \min\{\alpha, \beta\}, \\
\frac{2a(\alpha + \beta - p_{s0})a_12a_1}{(a_1)\theta_1(\theta_1 - \theta_2)2a_1} & \text{otherwise,} \end{array} \right.$$
\[
V^{e^a} = \begin{cases}
\frac{2(\alpha+\beta-p_{00})(\beta_1-\beta_2)\theta_1a^2}{9(\beta_1-\beta_2)^2(\theta_1-\theta_2)} & \text{if } 0 \leq p_{00} \leq \Phi_1, \\
\frac{(\alpha+\beta-p_{00})a^2}{2(\theta_1-\theta_2)} & \text{if } \Phi_2 \leq p_{00} \leq \min\{\alpha, \beta\}, \\
\frac{2(\alpha+\beta-p_{00})(\alpha+\beta-p_{00})p_{00}\theta_1a^2}{(\alpha+\beta)(\theta_1-\theta_2) + p_{00}\theta_2} & \text{otherwise}.
\end{cases}
\]

Proof of Proposition 5. We use backward induction to derive the retailers’ optimal pricing decision. After the disruption risk is resolved, the pricing decisions of the two retailers are as follows.

(1) If \(Q_i \geq \bar{q}, \ i = 1, 2\),

\[
\begin{align*}
 s_1 &= \frac{a}{2\theta_1 - \theta_2}, \\
 s_2 &= \frac{a}{2\theta_1 - \theta_2}.
\end{align*}
\]

(2) If \(Q_i < \bar{q}, \ i = 1, 2\),

\[
\begin{align*}
 s_1 &= \frac{\theta_1a + \theta_2a - \theta_1Q_1 - \theta_2Q_2}{\theta_1^2 - \theta_2^2}, \\
 s_2 &= \frac{\theta_1a + \theta_2a - \theta_2Q_1 - \theta_1Q_2}{\theta_1^2 - \theta_2^2}.
\end{align*}
\]

(3) If \(Q_1 \geq \bar{q}\) and \(Q_2 < \bar{q}\),

\[
\begin{align*}
 s_1 &= \frac{\theta_1a + \theta_2a - \theta_2Q_2}{2\theta_1^2 - 2\theta_2^2}, \\
 s_2 &= \frac{2\theta_1a - \theta_2Q_2 - 2\theta_1^2Q_2 + \theta_2Q_1 + \theta_1\theta_2a}{2\theta_1(\theta_1^2 - \theta_2^2)}.
\end{align*}
\]

(4) If \(Q_1 < \bar{q}\) and \(Q_2 \geq \bar{q}\),

\[
\begin{align*}
 s_1 &= \frac{\theta_1a + \theta_2a - \theta_2Q_1}{2\theta_1^2 - 2\theta_2^2}, \\
 s_2 &= \frac{2\theta_1a - \theta_2Q_2 - 2\theta_1^2Q_1 + \theta_2Q_1 + \theta_1\theta_2a}{2\theta_1(\theta_1^2 - \theta_2^2)},
\end{align*}
\]

where \(\bar{q} = \frac{\theta_1a}{2\theta_1 - \theta_2}\).

If \(q_{ij} + q_{ij} < \bar{q}\), \(\Pi_{R_i}\) is an increasing function with \(q_{ij}\), for \(i = 1, 2\), \(j = 3 - i\), otherwise, the optimal order is zero, retailer \(i\) has no positive profit.

If each order exceeds \(\bar{q}\), then retailer \(i\)'s profit is

\[
\Pi_{R_i} = \frac{(p_{00} + p_{01} + p_{10})(\theta_1 + \theta_2)\theta_1a^2}{(2\theta_1 + \theta_2)^2(\theta_1 - \theta_2)} - w_1q_{i1} - w_2q_{i2},
\]

s.t. \(q_{ik} \geq \bar{q}, \ i = 1, 2, \ k = 1, 2\).

It is clear that the profit function of retailer \(i\) is a decreasing function of the order, so the optimal order is \(\bar{q}\). As \(\bar{q}\) does not affect the wholesale prices, the two suppliers can raise the wholesale prices arbitrarily and eventually take all the profits from the retailers. Therefore, retailer \(i\) will not choose such a way of ordering when they know that their own profits are zero finally.

The following focus is on the case that each order does not exceed the threshold. Retailer \(i\)'s profit function is
Through the backward induction, the optimal wholesale prices are 
\[ (w^*_1, w^*_2) = \left( \frac{(\alpha - p_{00})a}{2(\theta_1 - \theta_2)}, \frac{(\beta - p_{00})a}{2(\theta_1 - \theta_2)} \right). \]

The optimal orders are 
\[ (q^*_1, q^*_2) = \left( \frac{(\theta_1 + \theta_2)a}{4\theta_1 + 2\theta_2}, \frac{(\theta_1 + \theta_2)a}{4\theta_1 + 2\theta_2} \right), i = 1, 2. \]

**Proof of Remark 5.** According to Proposition 5, we can derive the optimal profits of the retailers (\( \Pi_{R_i}^* \) and \( \Pi_{R_i}^* \)), suppliers (\( \Pi_{S_i}^* \) and \( \Pi_{S_i}^* \)), and system (\( V^{**} \)) are 
\[
\begin{align*}
\Pi_{R_i}^* &= \frac{(\theta_1 + \theta_2)(2p_{00} + \alpha + \beta)\theta_1a^2}{4(\theta_1 - \theta_2)(2\theta_1 + \theta_2)^2}, i = 1, 2, \\
\Pi_{S_i}^* &= \frac{(\alpha - p_{00})(\theta_1 + \theta_2)a^2}{4\theta_1^2 - 2\theta_1\theta_2 - 2\theta_2^2}, \\
\Pi_{S_2}^* &= \frac{2\theta_1 + \theta_2 - 2p_{00}\theta_2\theta_2}{4(\theta_1 - \theta_2)(2\theta_1 + \theta_2)^2}.
\end{align*}
\]

**Proof of Proposition 6.** Let \( f(p_{00}) = \Pi_{R_i}^* - \Pi_{R_i}^* \), and \((\theta_1, \theta_2) \in \Omega.\)

**Case 1.** \( 0 \leq p_{00} \leq \min\{\alpha - \frac{(2\theta_1 - \theta_2)\beta}{3\theta_1 - 2\theta_2}, \beta - \frac{(2\theta_1 - \theta_2)\alpha}{3\theta_1 - 2\theta_2}\} \),
\[\Pi_{S_i}^* - \Pi_{S_i}^* = 0,\]
\[\Rightarrow p_{00}^{(1)} = \alpha - \frac{8\beta\theta_1^2 + 4\beta\theta_1\theta_2}{10\theta_1^2 + 5\theta_1\theta_2 - 9\theta_2^2}.\]

When \( p_{00}^{(1)} < \Phi_1, \Pi_{S_i}^* \leq \Pi_{S_i}^* \) if \( p_{00} \leq p_{00}^{(1)} \); \( \Pi_{S_i}^* > \Pi_{S_i}^* \) if \( p_{00}^{(1)} < p_{00} < \Phi_1. \) When \( p_{00}^{(1)} \geq \Phi_1, \Pi_{S_i}^* \leq \Pi_{S_i}^*.\)
\[\Pi_{S_2}^* - \Pi_{S_2}^* = 0,\]
\[\Rightarrow p_{00}^{(2)} = \beta - \frac{8\alpha\theta_1^2 + 4\alpha\theta_1\theta_2}{10\theta_1^2 + 5\theta_1\theta_2 - 9\theta_2^2}.\]

When \( p_{00}^{(2)} < \Phi_1, \Pi_{S_i}^* \leq \Pi_{S_i}^* \) if \( p_{00} \leq p_{00}^{(2)} \); \( \Pi_{S_i}^* > \Pi_{S_i}^* \) if \( p_{00}^{(2)} < p_{00} < \Phi_1. \) When \( p_{00}^{(2)} \geq \Phi_1, \Pi_{S_i}^* \leq \Pi_{S_i}^*.\)
\[\frac{\partial(\Pi_{R_i}^* - \Pi_{R_i}^*)}{\partial p_{00}} > 0,\]
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\[ f(0) = \frac{a^2 \theta_1 (\alpha + \beta)(20\theta_1^1 - 15\theta_1\theta_2^2 + 13\theta_2^3)}{36(\theta_1 - \theta_2)(4\theta_1^2 - \theta_2^2)^2} > 0. \]

**Case 2.** \( p_{00} \geq \frac{\alpha_1 - \alpha_2 + \beta_1 - \beta_2}{2\theta_1 - \theta_2}, \)

\[ \Pi_{S_1}^e - \Pi_{S_1}^r = 0, \]

\[ \Rightarrow p_{00}^{(3)} = \alpha, \]

so \( \Pi_{S_1}^e > \Pi_{S_1}^r. \)

\[ \Pi_{S_2}^e - \Pi_{S_2}^r = 0, \]

\[ \Rightarrow p_{00}^{(4)} = \beta, \]

so \( \Pi_{S_2}^e > \Pi_{S_2}^r. \)

\[ \frac{\partial(\Pi_{R}^e - \Pi_{R}^r)}{dp_{00}} < 0, \]

\[ f(p_{00}) = 0 \Rightarrow p_{00}^{*} = \frac{(\alpha + \beta)(\theta_1 + \theta_2)\theta_1}{2\theta_1^2 + 2\theta_1\theta_2 + \theta_2^2}. \]

**Case 3.** otherwise,

\[ \Pi_{S_1}^e - \Pi_{S_1}^r = 0, \]

\[ \Rightarrow p_{00}^{(5)} = \alpha, \]

so \( \Pi_{S_1}^e > \Pi_{S_1}^r. \)

\[ \Pi_{S_2}^e - \Pi_{S_2}^r = 0, \]

\[ \Rightarrow p_{00}^{(6)} = \beta, \]

so \( \Pi_{S_2}^e > \Pi_{S_2}^r. \)

\[ \frac{\partial(\Pi_{R}^e - \Pi_{R}^r)}{dp_{00}} > 0. \]

Therefore, when \((\theta_1, \theta_2) \in \Omega \) and \( p_{00} \leq p_{00}^{*}, \) \( \Pi_{R}^e > \Pi_{R}^r; \) otherwise, \( \Pi_{R}^e \leq \Pi_{R}^r. \)

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