Predictive modelling of fuel shortages during hurricane evacuation: An epidemiological approach

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Abstract
High-volume evacuations, disruptions to the supply chain, and fuel hoarding from non-evacuees have led to localized fuel shortages lasting several days during recent hurricanes. While news reports mention fuel shortages in past hurricanes, the crowdsource platform Gasbuddy has quantified the fuel shortages in the recent hurricanes. The analysis of this fuel shortage data suggested fuel shortages exhibited characteristics of an epidemic. Here, a Susceptible-Infected-Recovered (SIR) epidemic model is developed to study the evolution of fuel shortage during a hurricane evacuation. Additionally, we apply optimal control theory to identify an effective intervention strategy. The study found a linear correlation between traffic demand during the evacuation of Hurricane Irma and the resulting fuel shortage data. This correlation is used in conjunction with the Statewide Regional Evacuation Study Program (SRESP) surveys to estimate the evacuation traffic and fuel shortages for potential hurricanes affecting south Florida. Results indicate that evacuation of Miami-Dade County in the event of a Category-3 hurricane landfall in the region, could lead to fuel shortages in up to 90% of the local refuelling stations. The model indicates that this reduces to 28% by providing relief to 75% of the gas stations during the first two days of the evacuation.

1 INTRODUCTION

Mass evacuations, particularly those at a state wide level, place a significant burden on fuel supplies. The sudden and drastic increase in travel demand, compounded by disruptions to the supply chain and fuel hoarding from non-evacuees has been shown to cause localized fuel shortages during these large, single-event traffic movements [1]. Hurricane evacuees also tend to make longer, intercity trips to stay with friends and family, further increasing fuel demand [2]. During evacuations, fuel shortages can result in stranded cars and exacerbate traffic problems in an emergency [3]. During the 2017 Hurricane Irma evacuation, localized fuel shortages lasted several days and led to a cascade of problems. For example, the fuel shortages gave rise to unpredictable increases in fuel prices, placing additional barriers for evacuees living on low wage incomes. Fuel shortages also led to increased highway congestion near freeway off-ramps and service areas and resulted in evacuees detouring from designated routes, where less traffic was expected or planned for [4]. Even after the hurricane passed, fuel shortages continued to impede the recovery efforts, as utility crews struggled to fuel needed equipment necessary to restore power supply [4]. Understanding the characteristics of fuel shortages during a hurricane evacuation is crucial to the mitigation of these problems and reducing the casualties caused by an imminent hurricane.

While news reports have documented fuel shortages during past hurricanes, crowd-sourced data from the social media platform Gasbuddy [5] has quantified the shortages during recent hurricanes, including Hurricane Irma. The analysis of crowd sourced data performed in this paper suggest characteristics of an epidemic for the evolution of fuel shortages. For example, as one gas station runs out of gas, drivers look to other stations in the vicinity to refuel. This places additional demand on neighbouring gas stations, increasing the likelihood these stations will run out of fuel also. In this fashion, a fuel shortage at one location has “spread” to another. The goal of this paper is to develop a predictive model for estimating fuel shortages based on the Susceptible-Infected-Recovered (SIR) epidemic model.
using data collected from the 2017 Hurricane Irma evacuation. To limit the spread of fuel shortages, this paper identifies an effective “vaccination” strategy based on optimal control theory. To demonstrate the applicability of the proposed approach, we apply the model to a hypothetical storm making landfall in South Florida. The results of the analysis suggest that using the optimal control theory strategy to prioritize the refuelling of gas stations, could lead to a 50% reduction in the number of depleted gas stations.

1.1 Literature review and background

Sociologists and computational scientists have long studied the spread of social phenomena using epidemiological models with different contact and network parameters. Analyzing the connected social and behavioural events of a population as contagion leads to the study of these phenomena from a different perspective. For instance, dynamic equations developed in the context of contagious disease modelling are useful to study social phenomena like, contagious adoption of health related behaviours [6], information spreading [7] and spread of obesity through social ties [8]. The social contagion in rumour spreading in social media [9], and spread of influential and public opinions in a population [10] exhibit similar characteristics. Several studies [11–13] in various fields indicate the effectiveness of epidemiological models originally developed for disease studies in examining and predicting socially contagious phenomenon.

The 2017 evacuation from Hurricane Irma was the largest evacuation in the history of the nation [14]. Approximately 6.5 million Floridians were under either mandatory or voluntary evacuation orders [4]. The overwhelming response to Hurricane Irma was driven by several factors that were unique to the storm: (1) Hurricane Irma had already devastated a number of Caribbean islands, including the U.S. Virgin Islands and Puerto Rico, resulting in several known deaths at the time [15]. (2) Hurricane Irma was the fifth strongest hurricane ever recorded in the Atlantic Ocean. (3) The storm path and “cone-of-uncertainty” threatened nearly the entire state of Florida. (4) Fluctuations in the storm’s path indicated possible devastating storm surge to nearly all of Florida’s coastal areas, where the majority of residents live. The overwhelming response to evacuation orders led to fuel shortages documented by Florida Department of Transportation (FDOT) [4].

Prior research into evacuation traffic as focused primarily on evacuation decision-making [16–19] or estimating traffic volumes and congestion levels [20–25]. Wolshon [21] used ground-based detectors to report traffic volumes in Louisiana to estimate the vehicle response to Hurricane Katrina. Li et al. [22] used traffic count information collected from tollbooths to investigate evacuation response curves for Hurricane Irene from a single county in New Jersey. Later efforts included historical travel time data and weigh-in-motion stations [23]. Li et al studied the spatial patterns for Hurricane Sandy evacuation [24]. Prior attempts to model and predict the fuel shortages during an evacuation have used social media post to identified regions experiencing fuel shortages. The research then predicted where shortages were likely to occur [25]. In earlier work, Islam and co-workers utilized Gasbuddy data to parameterise an SIR model using Unscented Kalman Filter (UKF) [26]. In this paper, we extend the model to correlate with traffic and survey data to predict the fuel shortages and suggest optimal control based remedial intervention strategies.

2 METHODOLOGY

The research methods seeks to pioneer a fuel shortage prediction model for application during evacuations by leveraging existing techniques found in epidemiological modelling. Broadly, this chapter describes the data sources, epidemic model development for fuel shortages, and the predictive analysis for evacuation traffic. The following sections detail these processes.

2.1 Data sources

The Florida Department of Transportation’s (FDOTs) Transportation Data and Analytics Office gathers roadway data from across the State of Florida. Real-time traffic information is available during emergencies such as hurricanes and wildfires. Telemetric monitoring stations located throughout the state collect hourly traffic information including volume, speed, and vehicle classification. There are 255 data collection sites on Florida roads at the time of this study, each providing bidirectional hourly counts and speeds. This data is public record accessible online (https://www.fdot.gov/statistics/trafficdata/default.shtm) and is also available by contacting FDOT. For the analysis of the Hurricane Irma evacuation, we collected and analysed the data corresponding to a 36-day period beginning August 27, 2017 and ending October 1, 2017.

We acquired the data pertaining to percentage of fuel stations out of gas from the Gasbuddy website. Gasbuddy is an online database containing vital roadside information on more than 150,000 fuel stations. The website also provides real-time fuel price information to drivers through a designated mobile app created for both iOS and Android platforms [5]. Along with that, the website also enables existing drivers to report and review various refuelling establishments throughout the United States. Gasbuddy played a crucial role during Hurricanes Irma and subsequent hurricanes by informing evacuees with real time information on fuel availability in different affected areas as they were evacuating. The crowd-sourced mobile app enabled drivers to report on fuel stations that were out of fuel in affected areas, thus contributing to driver awareness about fuel availability during the evacuation [27].

2.2 Epidemic model for fuel shortages

We use a Susceptible-Infected-Recovered (SIR) dynamic epidemic model [28] to model the fuel shortages with a vaccination analogue to represent the intervention efforts to address the fuel shortage. In a conventional disease epidemic, individuals
are divided into the three compartments corresponding to susceptible, infected and recovered based on their infection status, and the dynamic parameters that describe this evolution are assessed by comparisons with empirical data.

A similar approach is used for refuelling stations, wherein the percentage of refuelling stations without gasoline is considered to be “infected (I)”, percentage of refuelling stations with gasoline that are prone to running out of gasoline is “susceptible (S)” and percentage filled with gasoline after running out of fuel as “recovered (R)”. The recovered refuelling stations do not get re-infected (experience fuel shortage) in this case as the model and the on-ground situation represents a short-term outbreak.

In terms of differential equations, the dynamic model for the SIR is:

$$\frac{dS}{dt} = -\beta S(t) I(t) - u_r S(t)$$
$$\frac{dI}{dt} = \beta S(t) I(t) - \gamma I(t)$$

The parameters $\beta$ and $\gamma$ represent the transmission rate per capita and recovery rate, which in the current context represent the rate at which the susceptible refuelling stations are emptying and the empty gas stations are refilling respectively. As in a conventional SIR model the mean infectious period, that is, period in which most fuel stations are without fuel, is $1/\gamma$. The quantity $\beta S(0)/\gamma$ is a threshold quantity known as a basic reproduction number ($R_0$). This is the percentage of refuelling without fuel in a region, because of 1% of stations going out of fuel.

The $R_0$ value determines whether there is an epidemic taking place or not. If $R_0 < 1$, the epidemic dies out, while $R_0 > 1$ results in an epidemic [28, 29]. The term $u_r$ is the per capita rate of refuelling. Keeping congruency with our model parameters, $u_r$ is the rate of refilling the susceptible gas stations to prevent them from emptying out every day. Practically, the level of $u_r$ that is available is constrained by the infrastructure and planning that is in place to address the problem. For example, the gasoline reserves in proximity, the availability of transport vehicles etc. In extreme demand situations arising from large-scale evacuations, the control resources are often limited. We develop an optimal strategy to refuel, such that fuel shortage is at a favourable level throughout the interval under consideration, while constrained by the limited fuel resources.

The refuelling problem considered in this paper is a specific form of the following general optimal control problem:

Determine the optimal control $u_r(t), t_0 \leq t < T$, where $T$ is the unknown final time that minimizes the cost function

$$J = \int_{t_0}^{T} L(x, u) dt$$

subject to the constraints:

$$\dot{x} = f(x, u); \quad x(t_0) = x_0$$
$$g(u) \leq 0$$

Equation (4) states that the state vector $x \in \mathbb{R}^n$ is dependent on the control input $u \in \mathbb{R}^m$ based on the system dynamics, while Equation (5) represents an inequality constraint on the control input. The classical approach is to append the dynamical system constraint (4) to the cost function as follows:

$$J' = \int_{t_0}^{T} \left\{ L(x, u) - \lambda^T \left( f(x, u) - \dot{x} \right) \right\} dt$$
$$= \int_{t_0}^{T} \left\{ H(x, u, \lambda) - \lambda^T \dot{x} \right\} dt$$

where $J'$ represents the augmented cost function, $\lambda \in \mathbb{R}^n$ is a vector of Lagrange multipliers, also known as the co-state vector, and the Hamiltonian $H$ is defined as:

$$H(x, u, \lambda) = L(x, u) + \lambda^T f(x, u)$$

Euler–Lagrange equations provide a classical solution to this optimal control problem for an unconstrained control input [30]:

$$\dot{\lambda} = -\frac{\partial H}{\partial x} (\text{Co - State Equation})$$
$$\dot{x} = \frac{\partial H}{\partial u} = f(x, u) (\text{State Equation})$$
$$\frac{\partial H}{\partial u} = 0 (\text{Stationarity Condition})$$

$$\lambda^T (T) d\dot{x}(T) + H(T) dt = 0 (\text{Transversality Condition})$$

For constrained inputs, the stationarity condition (Equation 5) is generalized using Pontryagin’s Minimum Principle, which states that the optimal control input corresponds to the control that minimizes the Hamiltonian when the state and co-state are fixed at their optimal values [31]:

$$H(x, u, \lambda) \leq H(x, u, \lambda) \forall u \in U$$

where $U$ represents the set of all admissible control inputs.

For the optimal refuelling problem considered in this paper, the input constraint corresponds to the following resource constraint:

$$R = \int_{t_0}^{T} u_r(t) S(t) dt \leq r_{max}$$

The dynamics define a third state variable $r$:

$$r = u_r S$$
By defining the state vector as $\mathbf{x} = [S \ I \ r]^T$, and the co-state vector as $\lambda = [\lambda_S \ \lambda_I \ \lambda_r]^T$ with the control input $u_r$, the cost function takes the form:

$$J = \int_{t_0}^{T} L(\mathbf{x}, u) \ dt = \int_{t_0}^{T} \beta S I \ I (t) \ dt$$

The final time is defined as the time when the number of infected states reaches a selected threshold, $I(T) = I_{\text{min}}$, which would represent the end of the epidemic. The full system dynamics model $\dot{\mathbf{x}} = f(\mathbf{x}, u)$ is:

$$\begin{bmatrix}
\dot{S} \\
\dot{I} \\
\dot{r}
\end{bmatrix} = 
\begin{bmatrix}
-\beta S I - u_r S \\
\beta SI - r I \\
\mu I
\end{bmatrix}$$

This corresponds to a nonlinear, time-invariant system. The control is constrained as follows:

$$0 \leq u_r \leq u_{r,\text{max}}$$

The Hamiltonian takes the form:

$$H(\mathbf{x}, u, \lambda) = \beta SI - \lambda_S (\beta SI + u_r S) + \lambda_I (\beta SI - r I) + \lambda_r u_r S$$

The Euler–Lagrange equations (Equation 8) for the co-states are:

$$\begin{bmatrix}
\dot{\lambda}_S \\
\dot{\lambda}_I \\
\dot{\lambda}_r
\end{bmatrix} = 
\begin{bmatrix}
-(1 + \lambda_S - \lambda_I) \beta I + (\lambda_S - \lambda_I) u_r \\
(-1 + \lambda_S - \lambda_I) \beta S + \lambda_I \gamma \\
0
\end{bmatrix}$$

The optimal control input $u_{r,\star}$ is determined using Pontryagin’s Minimum Principle:

$$u_{r,\star} (\lambda_{r,\star} - \lambda_{S,\star}) \leq u_r (\lambda_{r,\star} - \lambda_{S,\star}) \forall 0 \leq u_r \leq u_{r,\text{max}}$$

This is equivalent to requiring that the optimal control minimize the left-hand side of the inequality. As a result, the optimal control policy corresponds to a bang-bang control law in which the control takes on either its maximum or minimum constrained value based on the sign of $\lambda_{r,\star} - \lambda_{S,\star}$, which is known as the switching function:

$$u_{r,\star} = \begin{cases} 
0 & \lambda_{r,\star} > \lambda_{S,\star} \\
? & \lambda_{r,\star} = \lambda_{S,\star} \\
u_{r,\text{max}} & \lambda_{r,\star} < \lambda_{S,\star}
\end{cases}$$

Note that the optimal control is undefined when $\lambda_{r,\star} = \lambda_{S,\star}$; however, following a similar argument as presented in Ref. [29], we can show that this condition does not occur for a finite period. Therefore, the optimal control policy is well-defined at all times and is purely bang-bang in nature.

In order to determine the optimal policy resulting from Equation (21), it is necessary to compute the switching time(s) in the control law. First, since $I(T)$ and $r(T)$ are fixed, only $S(T)$ is free. As a result, the transversality boundary condition in Equation (9) reduces to:

$$\lambda_S (T) dS (T) + H (T) \ dT = 0$$

There is no direct relationship between $S(T)$ and $T$; therefore Equation (20) results in $\lambda_S (T) = 0$ and $H (T) = 0$. Since the cost function and dynamics model are time-invariant, the Hamiltonian is also time-invariant. Therefore, $H (t) = 0 \forall t \in [t_0, T]$.

Ref. [30] presents a detailed proof, in the context of a parallel vaccination problem that first demonstrates that there is only a single switching time in the solution. Then, we show that, starting with zero control and switching to the maximum vaccination rate does not correspond to an optimal policy. As a result, the optimal solution corresponds to a policy of applying the maximum vaccination rate from time $t_0$ until a single switching time $t_s$ and then reducing it to zero. Applying this result to the refuelling problem results in the following optimal refuelling policy:

$$u_{r,\star} = \begin{cases} 
0 & t \in [t_0, t_s) \\
u_{r,\text{max}} & t \in [t_s, T]
\end{cases}$$

In this case, the resupply to depleted fuelling stations starts at $t_0$, at the maximum rate until the switching time $t_s$. The switching time $t_s$ is defined as the time when the number of susceptible fuelling stations is $S(t_s) = \frac{\gamma}{\beta}$. The final time $T$ corresponds to the time at which the number of gas stations with fuel shortages (i.e., the number of infected states) corresponds to $I(T) = I_{\text{min}}$.

We parameterize the model using the data for Hurricane Irma from the Gasbuddy crowdsourcing platform. While there are several possible ways to estimate the parameters $\beta$ and $\gamma$ in this paper, we use an Unscented Kalman Filter (UKF) for this purpose. The Kalman Filter [31], developed in the early 1960’s, is an effective method for estimating the parameters from empirical data with a measurement correction. The classical Kalman filter is effective in providing the optimal state and parameter estimation for linear systems subject to a Gaussian noise. However, the model equations for the SIR problem (Equations 1 and 2) are inherently nonlinear. The Unscented Kalman Filter characterizes the estimation error by propagating a set of sigma points through the nonlinear dynamics model; therefore, does not require the assumption of Gaussian white noise [32]. Various applications in engineering and epidemiology use this approach [33, 34]. Ref. [26] provides a detailed description of the parameter estimation using UKF for the SIR model.
3 | PREDICTIVE ANALYSIS OF EVACUATION TRAFFIC AND FUEL SHORTAGE

The above optimal control algorithm and the epidemic model are used for examining fuel shortage and for determining the optimal intervention control parameters. This model is parameterised with a prediction of fuel shortage data for a potential hurricane impacting south Florida. The step-by-step methodology for the model application is as follows:

1. Analyse the cumulative traffic trends for evacuation during past hurricanes using transportation data. We use the FDOT data from the SunGuide program for Hurricane Irma for this analysis.
2. Analyse the fuel shortage trends for past hurricanes using the crowdsourced data from the Gasbuddy app. We use the data form 2017 Hurricane Irma for this analysis.
3. Obtain the correlation between traffic and fuel shortage. For Hurricane Irma there is a near linear correlation. Note that the parameterization for steps 1–3 can improve significantly by using the data from historical and future hurricanes to develop the relation between evacuation and fuel shortage in specific contexts.
4. The calculation of the total evacuation traffic volume due to a hurricane impacting south Florida is based on the emergency response surveys as explained below. The traffic loading over the evacuation period uses the traffic distribution from Step 1.
5. The relation between traffic and fuel shortages from step 3 and the traffic estimate from step 4 form the basis for predicting the fuel shortage distribution over the evacuation period. The optimal control methodology described in the above section provides the optimal refuelling plan to mitigate the expected fuel shortage.
6. A predictive model is then proposed where the UKF can be utilized to evaluate the SIR dynamic parameters from incoming fuel shortage during the initial stages of the hurricane. Due to the nature of the Ordinary Differential Equations (ODE) of SIR dynamics, we can estimate the infection rate \( \beta \) from the data collection of initial stages of the evacuation.
7. By varying the Basic Reproduction number \( R_0 \), we produce predictive trends and use the optimal refuelling strategy to demonstrate effective countermeasures.

We calculate the total evacuation traffic volume based on the state wide surveys conducted as a part of emergency planning. In response to the active hurricane seasons of 2004 and 2005, the Florida State Legislature authorized the development of regional evacuation studies from across the state. Florida’s Regional Planning Councils developed the Statewide Regional Evacuation Study Program (SRESP) to support and update local government emergency management plans [18]. SRESP includes a series of surveys that aim to improve the understanding of the evacuation behaviour and facilitate better behavioural assumptions in evacuating modelling and shelter planning. The behaviour assumptions collected as part of this survey were: evacuation rate, out-of-county trips, type of refuge, percent of available vehicles, and evacuation timing. 400 residents in each of Florida’s 67 counties took part in the survey [18].

The results of the SRESP surveys are available on the Florida disaster-planning website (http://www.sfrpc.com/sresp.htm). We used the survey results to estimate the fraction of residents that would evacuate by road in response to a Category 1 and Category 3 hurricane landfall in Broward and Miami-Dade county FL. In addition, we utilised the 2017 census data [35] to estimate the number of site-built and mobile homes in both regions. We combine this data with the SRESP survey results to estimate the evacuation participation rate, percent of vehicles used, and the number of evacuating vehicles. Through this approach, we estimate the “maximum probable” number of evacuating vehicles for hypothetical storms [17, 18]. This analysis suggests that 173,914 vehicles would likely evacuate out of Miami Dade County in the event of a Category 1 hurricane landfall in the region. 342,379 evacuating vehicles would evacuate from the area in the event of a Category 3 storm.

4 | RESULT AND DISCUSSIONS

We first present the results linking traffic volume and fuel shortage for various population centres in Florida. We use the hourly counts of vehicles obtained from FDOT’s SunGuide program, to calculate the cumulative outgoing traffic count over the days impacted by Hurricane Irma. Irma made two landfalls, the first in Cujoe Key and the second on to US mainland at Marco Island near Naples on Sep 10 at 3:35 PM [36]. The arrow in Figures 1(a–d) corresponds to the second landfall. The dotted line in Figure 1 shows the cumulative egress from Naples-Fort Myers metropolitan area. The evacuation out of this area started three to four days before the hurricane landfall and reached peak by September 9. The flat peak around the hurricane landfall period indicates the reduced traffic volume during the hours impacted by the hurricane. The solid line in Figure 2(b) shows the fuel shortage reported by Gasbuddy crowdsourced platform in the form of percent refueling stations without fuel. The peak for the fuel shortage lags the peak evacuation traffic as can be expected. When faced with a threat of looming hurricane, people usually fill-up their vehicles in preparation for a potential evacuation, even if they eventually decide not to evacuate. Figure 1(b) shows that up to 60% of the refuelling stations in Naples-Fort Myers area were without gas as the evacuation traffic was peaking.

Figure 1(b) shows a similar plot for Tampa-St Petersburg metropolitan area. The plot shows similar trend as that of Naples, with respect to flat traffic peak around landfall and the period impacting this region. Fuel shortages of up to 60% are lagging the peak of evacuation traffic for this region as well. Figure 1(c) shows similar data for Miami-Fort Lauderdale Metropolitan area. This is the largest metropolitan area in Florida with significantly higher population. While the trend is similar to that observed in Figure 1(a,b), there are fluctuations in both traffic peak and fuel peak. This is due to the early uncertainty in the hurricane path. While the hurricane
FIGURE 1  (a) Fuel shortage and cumulative egress for Naples, FL. (b) Fuel shortage and cumulative egress for Tampa, FL. (c) Fuel shortage and cumulative egress for Miami/Ft Lauderdale, FL. (d) Fuel shortage and cumulative egress for Jacksonville, FL.

FIGURE 2  Relationship between fuel demand and cumulative egress for Miami-Ft Lauderdale

eventually made landfall close to Naples, early predictions indicated a possible landfall near Miami [36]. This may have prompted early increase in evacuations, which stabilized, as the hurricane path was more certain. Figure 1(d) shows the similar plot for Jacksonville.

The data in the above Figure 1(a–d) helps obtain a correlation function between the evacuation traffic volume and fuel shortage. Figure 2 shows this correlation for Miami-Ft Lauderdale area. Since there is a delay in the peaks of both fuel demand and cumulative egress, we shift the plot for fuel demand to match
the peak value around the same time as the cumulative egress for all the city/area. We hypothesize that there is a direct correlation between the occurrence of these peak values without taking the uncertainties of evacuation into account. Table 1 shows the time delay ($\Delta T$) between the fuel demand and cumulative egress for different city/areas.

### Predictive modelling and optimal control

Consider a hurricane that would make a direct impact on South Florida. In this section, we analyse the results of a predictive model for fuel shortages due to evacuation from such a hurricane and develop an optimal intervention strategy for such fuel shortages. We estimate the total traffic volume evacuating out of Miami-Dade County based on the State wide Regional Evacuation Study Program (SRESP) surveys [18]. We scale the survey results on the likelihood of evacuation for residents in different surge protection zones, to the population of metropolitan area and average vehicle availability for the households from US census to estimate the total evacuation numbers. Table 2 below provides the details of the calculation for a category 3 hurricane that would make landfall in Miami-Dade County. A similar estimate for Category 1 hurricane generates 173,914 vehicles evacuating Miami metropolitan area.

We distribute the total evacuation traffic across the evacuation period based on the traffic distribution during Hurricane Irma. We use a normalized aggregate traffic distribution, which is essentially a combination of the traffic data from Figure 1(a–d), for this purpose. We then estimate the resulting fuel shortage due to the traffic volume using the regression coefficients in the Table for the Miami region. Figure 3 shows the estimate of fuel shortage. Both the prediction of the loading pattern and the fuel shortage prediction can improve significantly, as data from additional historical and future hurricanes inform the relation between evacuation and fuel shortage for specific contexts.

Once we use the predictive model to parameterize the fuel shortage data, we use the optimal refuelling strategy for further analysis of the mitigation measures. We use an Unscented Kalman Filter (UKF) algorithm [32] to estimate the transmission rate per capita ($\beta$) and the recovery rate ($\gamma$) of the predicted fuel shortage as shown in Table 3. In Figure 4, Equations (1) and (2) are utilized to deterministically produce the time-invariant data of the SIR dynamics without intervention, that is, using $u_v = 0$. The epidemiological parameters corresponding to the transmission per capita rate and the recovery rate for the predicted category 3 and category 1 hurricanes as shown in Table 2. The use of UKF for parameter estimation is necessary as the evolution of infected gas stations ($I(t)$) from the prediction model was time-variant, but the optimal refuelling strategy developed in the methods section is a time-invariant, continuous-time non-linear dynamic system.

Figure 5(a,b) shows the change in percentage of empty fuel stations or $I(t)$ as the optimal refuelling strategy is applied. With $u_v = 0$ (i.e. no intervention), for a category 3 hurricane making a direct hit on Miami, the model predicts that the fuel shortages will be more than 80%. If the resources are used to provide relief to 75% of the gas stations at any given time (i.e. $u_v = 0.75$), the fuel shortages would be reduced to 29%. Figure 6(a) shows that the optimal switching time for the bang-bang controller with

| City/Area      | $\Delta T$ (Days) | $l_{\text{MAX}}$ | Cum. Egress$_{\text{MAX}}$ | No. of Fuel Stations |
|----------------|-------------------|------------------|-----------------------------|---------------------|
| Ft Myers-Naples| 3.25              | 0.61             | 123356                      | 76                  |
| Tampa-St Petersburg| 1.25             | 0.6              | 114312                      | 922                 |
| Miami-Ft Lauderdale| 1                | 0.66             | 234924                      | 1545                |
| West Palm Beach   | 0                 | 0.56             | 36968                       | 34                  |
| Jacksonville      | 0                 | 0.56             | 185861                      | 453                 |

| Parameter          | Category 1 | Category 3 |
|--------------------|------------|------------|
| Transmission rate per capita ($\beta$) | 0.0028/day | 0.0043/day |
| Recovery rate ($\gamma$)                 | 0.0453/day | 0.0105/day |

### Table 2 Calculation of total evacuating vehicles for Miami-Dade County for a Category-3 hurricane. The sources for data are Baker et al [18], Downs et al [19] and US census [35] |

| EVAC ZONE | HOUSE [38] | EVAC RATE [17] | VEH USE RATE [17] | VEH/ HH [18] | EVAC VEH |
|-----------|------------|----------------|------------------|--------------|----------|
|           | BUILT      | MOBILE        | BUILT            | MOBILE      |          |
| Cat 1     | 56,046     | 871           | 65%              | 85%          | 80%      | 85%      | 1.5  | 43,716 | 944 |
| Cat 2     | 80,323     | 1144          | 60%              | 80%          | 70%      | 75%      | 1.9  | 64,098 | 1,305 |
| Cat 3     | 94,206     | 2346          | 60%              | 80%          | 70%      | 75%      | 1.9  | 75,177 | 2,675 |
| Cat 4     | 175,122    | 1704          | 30%              | 75%          | 65%      | 70%      | 2.1  | 71,713 | 1,879 |
| Cat 5     | 188,004    | 3084          | 15%              | 75%          | 65%      | 70%      | 2.1  | 38,494 | 3,401 |
| Inland    | 422,067    | 7,039         | 5%               | 65%          | 75%      | 80%      | 2    | 31,656 | 7,321 |

Total Vehicles = 342,379
FIGURE 3  Prediction of cumulative egress and fuel shortage for: (a) Category 3 hurricane directly impacting Miami, FL, and (b) Category 1 hurricane directly impacting Miami, FL.

FIGURE 4  Fuel shortage prediction model and the Time-invariant continuous SIR data computed using UKF parameter estimation for (a) category 3 hurricane and (b) category 1 hurricane.

FIGURE 5  Change in the percentage of empty fuel stations due to varying levels of intervention ($\sigma_{\text{max}}$) for (a) category 3 hurricane and (b) category 1 hurricane.
\( u_v = 0.75 \) is 3 days. This implies that we can mitigate the infectious behaviour of fuel shortage, by providing relief to 75\% of operating refuelling stations for the first 3 days after the start of evacuation. This essentially means that the R0 parameter described earlier is less than 1 under these conditions, that is, 1\% gas stations going out of fuel does not cascade into fuel shortage beyond this 1\%. When R0 is higher than 1, like with \( (u_v = 0) \), 1\% gas stations going out of fuel leads to an additional R0 percent refuelling stations going out of fuel due to the infectious behaviour. Figures 5 and 6 show how the switching function \( (t_s) \) and the level of intervention \( (u_v) \) variation can optimally control the problem and mitigate the infectious behaviour.

The initial data gathered during early stages of a hurricane evacuation can be an input as measurement update to the UKF to evaluate the \( \beta \) parameter of the SIR dynamics model for fuel shortage. The \( \gamma \) parameter relates to the recovery rate and we can estimate it using the \( \beta \) value and an approximate recovery period based on historical data or by varying the R0. Figure 7 shows the predictive model for such case for Fort Myers-Naples area during Hurricane Irma. The R0 is valued to determine the recovery rate \( (\gamma) \) to produce the mechanistic data for the predictive model. For this analysis, we assume that Day 1 fuel shortage data for Fort Myers-Naples area is available, and we use the parameter estimation to determine the possible scenarios. The best fit R0 is the one that was determined as described in the methods section.

Figures 8–11 show the optimal refuelling strategy for different R0s corresponding to Figure 7 for Naples-Fort Myers. Figure 8(a) shows the evolution of susceptible gas stations with different levels of intervention for R0 = 5. Similarly, Figure 8(b,c) show the corresponding infected (or empty) gas stations and the switching time for various levels of intervention. Figures 9–11 are similar plots for different R0 values. This approach is useful during the early stages of an ongoing hurricane to establish upper and lower bounds for the possible levels of fuel shortage and the levels of intervention needed. The on-the-fly prediction will improve as more data becomes available.

Fuel supply is a critical component towards a resilient transportation system. During an evacuation, vehicles without fuel require roadside services, which can reduce traffic flow. A worst-case scenario would be that these vehicles are abandoned; forming roadside hazards or even obstructing the roadway. Therefore, vulnerabilities in the fuel supply chain have the potential to cascade into the transportation network, upon which evacuees are dependent. This research represents an opportunity to better predict where and when fuel shortages may occur and how they might spread. The paper also provides insights into planning techniques to prevent widespread shortages.

The predictive analysis for potentially affected regions combined with the optimal refuelling methodology discussed above can help assess the levels of fuel supply required to mitigate the fuel shortage crisis in the affected regions, and thereby assist decision makers in allocating limited resources in a dynamically evolving emergency. In response to the high fuel shortages during the Hurricane Irma evacuation, FDOT has five recommendations for limiting the impact of fuel shortages for future mass evacuation events [4]. These include: (1) Provide law enforcement escorts for refuelling vehicles, (2) communicate the fuel availability during the event, (3) provide regional waivers for the transportation of fuel across state lines, (4) identify critical gas stations along state evacuation routes and (5) provide traffic management plans for critical stations. The current study provides a data analytic approach that can help inform the implementation of some of these recommendations. While the
FIGURE 8 Evolution of (a) susceptible (operational), (b) infected (empty) gas stations, (c) the optimal application and switching time, $t_s$, for different refuelling rates and the effect of refuelling for Fort-Myers-Naples during Hurricane Irma for the predictive model with $R_0 = 5$.

FIGURE 9 Evolution of (a) susceptible (operational), (b) infected (empty) gas stations, (c) the optimal application and switching time, $t_s$, for different refuelling rates and the effect of refuelling for Fort-Myers-Naples during Hurricane Irma for the predictive model with $R_0 = 7$.

Current study is at a coarser scale than identifying individual gas stations, our model can assess (a) the level of fuel needed in critical regions (using $u_v$), and (b) the duration to keep-up the refuelling supply for optimally addressing the emergency (using $t_s$). These can be crucial inputs in planning and implementation of the mitigation procedures. In the case of a hurricane impacting a large geographical area (entire state or multiple states), the duration to keep-up the refuelling varies by location (city), and would help focus the resources for the aforementioned recommendations to different cities as needed. The level of fuel supply needed for each region would be another crucial input for the planning. While we parameterize the current study for

FIGURE 10 Evolution of (a) susceptible (operational), (b) infected (empty) gas stations, (c) The optimal application and switching time, $t_s$, for different refuelling rates and the effect of refuelling for Fort-Myers-Naples during Hurricane Irma for the predictive model with $R_0 = 3.5$. 

hurricane evacuations in Florida, the modelling approach is generally applicable to similar resource shortages due to evacuations in any location. Future directions in extending the current work can focus on identifying critical fuelling stations within a region using fine-scale individual based stochastic epidemic models.

5 CONCLUSIONS

In this paper, we present a predictive model formulation for the evolution of fuel shortages during hurricanes and an optimal control strategy to mitigate such shortages. The latent demand due to the depletion of feeling stations spreads to neighbouring stations and throughout the community, similar to an epidemiological outbreak. This realization allows the application of well-established epidemiological research and models to the problem of fuel shortages and mitigation strategies. In addition, we develop an epidemiological analogue for resource allocation based on vaccination models and optimal control theory.

The data analysis of the evacuation traffic and crowd sourced fuel shortage data suggests that there is a direct correlation between the two. The analysis suggests the evacuation from Hurricane Irma and related activities depleted over 60% of the fuelling stations in Tampa, Miami/Fort Lauderdale, and Naples, while Jacksonville saw depletion rates as high as 56%. Using epidemiological analogy, the fuel shortage epidemic is under control when the basic reproduction number ($R_0$) is less than 1. The predictive model suggests that there can be a fuel shortage in up to 90% of the refuelling stations in Miami-Dade County, due to an evacuation from a Category 3 hurricane impacting Miami. The optimal control algorithm suggests the level and duration of intervention required to keep these fuel shortages from becoming an epidemic. While the application focused on hurricanes impacting Florida, the model is generally applicable to similar resource shortages due to evacuations in any location. We utilize the model to predict the level of fuel shortage and the effect of intervention, when limited data is available during the early stages of a hurricane evacuation. This approach is demonstrated using early data from Hurricane Irma for Naples-Fort Myers region.

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