Dynamical Gauge Bosons and Grand Unified Theory

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1. Introduction
2. Dynamical Gauge Bosons and Compositeness Condition
3. Is Standard Symmetry Asymptotically Free?
4. Infrared Fixed Point Structure in ANF Theories
5. Future Aspects

My talk is based on the papers written with J. Sato and K. Yoshioka, T. Takeuchi, T. Onogi and J. Sato, and Y. Taniguchi and S. Tanimura.

1 Introduction
The idea of dynamically generated gauge bosons has long history since 1960’s. Originally Bjorken argued that gauge bosons are NG bosons responsible for spontaneous breaking of the Lorentz invariance. Since then attempts to generate gauge bosons dynamically using various models especially, NJL-type models were investigated by many authors. However it was very hard to get massless vector bound states and people had to be content with an approximate gauge symmetry generation where the masses of vector bound states are very small compared with the relevant scale. This is because gauge symmetry has generically its profound background and is hardly generated unless it exists from the first in the original system.

The second approach to dynamical gauge bosons is the Berry phase mechanism, which made clear that a gauge symmetry can be induced when the system has two kinds of degrees of freedom, fast and slow ones. During evolution of slow variable, fast one causes a static gauge field acting on slow one and a gauge field is induced by a mechanism of the Berry phase. The Berry phase mechanism made clear that the generation of gauge symmetry is a common phenomena which occurs whenever we reduce the degrees of freedom of a system. However, even in this case gauge symmetry is exact only when the energy levels associated with the fast variables can be completely neglected. This had been always a serious problem of any dynamical gauge boson until the notion of hidden local symmetry came into play. The hidden local symmetry
was first found in the context of supergravity theory, a l’a the non-linear sigma
model\(^{15,16}\) (for a review see Ref.18). Thanks to the hidden gauge symmetry,
gauge invariance is guaranteed exactly and massless vector bosons are possible
to be generated dynamically at quantum level and they can be identified as
corresponding gauge bosons. As such an example, we proposed the possibility
that vector mesons in QCD (\(\rho\) or \(A_1\) mesons) may be identified as Higgsed dy-
namical gauge bosons of hidden local \(SU(2)_V\) symmetry in the non-linear chiral
\(SU(2) \times SU(2)/SU(2)_V\) Lagrangian.\(^{18,19}\) Thanks to the Recent arguments of
duality, the idea becomes now very popular that gauge symmetries might not
be fundamental and they will appear as the consequences of non-perturbative
dynamical effects.\(^{26}\)

With hidden local symmetry at hand, attempts to generate gauge fields
associated with hidden local symmetry were made using \(CP^n-1\) models.\(^{22,24}\) and
it was found that in 2- or 3-dimensional models, gauge fields acquire their own
kinetic terms via quantum effects, and their poles are developed dynamically\(^{20,21}\). However, these attempts have not been successful in four dimensions;
they can be generated only in cutoff theories.\(^{22}\) Here we would like to
propose that cutoff theories are sufficient as low energy effective theories: they
behave just like elementary gauge bosons in the low energy region. The only
difference of dynamical gauge bosons from elementary ones is the existence of
the compositeness condition at some scale \(\Lambda\) below which they behave
as if they are asymptotically non-free (ANF) gauge fields. We shall explain
this in the section 2. Then we argue that ANF gauge theories may provide
another possible scenario of the “standard model” below GUT scale (section
3), where we shall see that infra-red stable points play essential roles (section
4). Final section is devoted to future aspects and related problems.

2 Dynamical Gauge Bosons and Compositeness Condition

Usually gauge interactions are required to be asymptotically free (AF), oth-
wise the theory will become trivial because of the existence of the Landau
singularity at scale \(\Lambda_L\). We interpret this singularity as an indication of the
so-called compositeness condition. This idea of the compositeness condition
at finite scale was first argued by Bardeen, Hill and Lindner\(^23\), who claimed
that the composite Higgs bosons is characterized by the vanishing of the wave-
function renormalization factor, \(Z = 0\), which is translated into the divergence
of the rescaled Yukawa coupling, \(y_r = \infty\) by rescaling the Higgs fields. Similar
arguments may be applied to dynamical gauge bosons and the divergence of
running gauge couplings at high energy scale \(\Lambda\) can be interpreted as a
compositeness condition for gauge bosons.
Let us consider a simple model in which dynamical gauge bosons are generated associated with the hidden local symmetry in non-linear SU($N_f$) / [SU($N_c$) × SU($N_f - N_c$)] Grassmannian like model. Consider $N_c \times N_f$ complex scalar fields $\phi_{a\bar{a}}$ and auxiliary SU($N_c$) fields $A_{\mu}$ coupled to the index $a = 1, \ldots, N_c$. The hidden local gauge fields $A_{\mu}$ accompanied by the breaking of SU($N_f$)local and redundant degrees of freedom of the model. The hidden local gauge fields $A_{\mu}$ are transformed under SU($N_f$)$_{\text{global}} \times$ SU($N_c$)$_{\text{hidden local}}$ as $\phi(x) \rightarrow h(x)\phi(x)g^\dagger$, where $h(x) \in SU(N_c)_{\text{hidden local}}$ and $g \in SU(N_f)_{\text{global}}$. The Lagrangian is given as

$$\mathcal{L} = (D_{\mu}\phi)_{ia}^\dagger (D^\mu\phi)^{ia} - \lambda_{ab} \left( \phi a_{ia}^\dagger - \frac{N_f}{\omega} \delta_{ab} \right) + \mathcal{L}_{\text{GF+FP}},$$

(1)

with the covariant derivative $D_{\mu}\phi$; $D_{\mu}\phi = \partial_{\mu}\phi - iA_{\mu}\phi$. Here $\omega$ is a dimensionful coupling and $\mathcal{L}_{\text{GF+FP}}$ is the gauge fixing and the FP ghost term for $A_{\mu}$. The hidden local gauge fields $A_{\mu}$ do not have their kinetic terms and represent redundant degrees of freedom of the model. The $N_c \times N_c$ hermitian scalar field $\phi_{ab}$ is the Lagrange multiplier imposing the constraint; $\phi_{ai}^\dagger \phi_{ib} = \frac{N_f}{\omega} \delta_{ab}$. With this constraint $A_{\mu}$ can be eliminated by substituting the equation of motion; $A_{\mu} = -\frac{\omega}{2N_f} (\partial_{\mu}\phi\phi^\dagger - \phi\partial_{\mu}\phi^\dagger)$ and we obtain the following form:

$$\mathcal{L} = \text{Tr} \left[ \partial_{\mu}\phi_1 \partial^\mu \phi + \frac{\omega}{4N_f} (\phi_1^\dagger \partial_{\mu}\phi - \partial_{\mu}\phi^\dagger \phi)^2 \right],$$

(2)

which is the original non-linear sigma model without hidden local symmetry.

There are two alternatives in identifying the massless mode in $A_{\mu}$. We can identify it as (1) Goldstone mode of broken SU($N_c$) local symmetry, (2) Wigner mode of SU($N_c$) gauge symmetry. In the broken phase (1), the breaking of SU($N_c$) local symmetry by the V.E.V. of $\langle \phi \rangle = \sqrt{N_f}v (\delta_{ab}, 0)$ is accompanied by the breaking of SU($N_f$) symmetry. On the other hand, in the symmetric phase (2), $A_{\mu}$ is a massless vector boson and no symmetry should be broken. The phase of the model is determined by the effective potential. Substituting the V.E.V's of $\phi$ and $\lambda$, $\langle \phi \rangle = \sqrt{N_f}v (\delta_{ab}, 0)$, $\langle \lambda \rangle = \lambda \delta_{ab}$, into [3], the effective potential is given in the 1/$N_f$ approximation as

$$V_{\text{eff}} = N_f N_c \left[ \lambda \left( v^2 - \frac{1}{\omega} \right) + \int \frac{d^4k}{i(2\pi)^4} \ln(\lambda - k^2) \right].$$

(3)

This potential coincides with that of the $CP^{N-1}$ model [4] within this approximation, and the ground state is determined by the equations

$$\frac{1}{N_f N_c} \frac{\partial V_{\text{eff}}}{\partial \lambda} = v^2 - \frac{1}{\omega} + \int \frac{d^4k}{i(2\pi)^4} \frac{1}{\lambda - k^2} = 0,$$

(4)

$$\frac{1}{N_f N_c} \frac{\partial V_{\text{eff}}}{\partial v} = 2\lambda v = 0.$$  

(5)
with $\Lambda$ being a cutoff to define the integration. Then we have,

$$v^2 - f(\Lambda, \lambda) = \frac{1}{\omega_r(\mu)} - \frac{1}{\omega_{cr}(\Lambda, \mu)},$$

(6)

where

$$\omega_r \equiv \frac{1}{\omega} - \int \frac{d^4k}{i(2\pi)^4} \frac{1}{\mu^2 - k^2},$$

(7)

$$\omega_{cr} \equiv \int \frac{d^4k}{i(2\pi)^4} \frac{1}{\mu^2 - k^2} - \int \frac{d^4k}{i(2\pi)^4} \frac{1}{\lambda - k^2},$$

(8)

$$f(\Lambda, \lambda) \equiv \int \frac{d^4k}{i(2\pi)^4} \frac{1}{\mu^2 - k^2} - \int \frac{d^4k}{i(2\pi)^4} \frac{1}{\lambda - k^2}.$$  

(9)

$\omega_r$ is a redefined coupling at the renormalization point $\mu$. If the dimension is less than four, $\omega_r$, $\omega_{cr}$, and $f$ remain finite in the limit $\Lambda \to \infty$ \cite{4}. However, in four dimensions they become divergent, and so Eq. (6) makes sense only for the finite cutoff $\Lambda$. The function $f$ is understood as a non-negative function of $\lambda$, vanishing at $\lambda = 0$. The critical coupling $\omega_{cr}$ which separates two phases;

1) broken phase : $\lambda = 0$, $v \neq 0$, when $\omega_r < \omega_{cr}$,

2) symmetric phase : $\lambda \neq 0$, $v = 0$, when $\omega_r > \omega_{cr}$. The critical coupling $\omega_{cr}$ is proportional to the reciprocal of the logarithmic divergence: $\omega_{cr}^{-1} \sim \log \Lambda$. Thus $\omega_{cr}$ becomes smaller as the cutoff $\Lambda$ becomes larger and the symmetric phase is always realized by taking $\Lambda$ to be sufficiently large. In this phase the scalar field $\phi$ is massive ($m^2 = \lambda$), whereas the vector field $A_\mu$ becomes massless. Hence this composite vector field is stable. In the broken phase, the $\phi$ field becomes massless (NG bosons associated with the broken symmetries), while the gauge field $A_\mu$, if it exists, becomes massive through the Higgs mechanism. This massive gauge boson is unstable as it can decay into two $\phi$ bosons.

To see the generation of massless gauge bosons explicitly in the symmetric phase with nonzero $\lambda$ we obtain the effective Lagrangian of the composite gauge boson by calculating the one-loop Feynman diagrams which contribute to the leading order terms of expansion with respect to $N_c/N_f$; \cite{5}

$$\mathcal{L} = \text{Tr} \left( |\partial_\mu \phi - i A_\mu \phi|^2 - \lambda |\phi|^2 \right) - \frac{1}{4} Z_0 \left( \partial_\mu A_\nu^A - \partial_\nu A_\mu^A \right)^2$$

$$- \frac{1}{2} Z_1 f^{ABC} \left( \partial_\mu A_\nu^A - \partial_\nu A_\mu^A \right) A_\mu^A A_\nu^B A_\sigma^C - \frac{1}{4} Z_2 f^{EAB} f^{ECD} A_\mu^A A_\nu^B A_\sigma^C A_\tau^D$$

(10)

*We use a dimensional regularization method in order to preserve the hidden gauge invariance. We then replace the regularization parameter $1/\bar{\epsilon}$ with cutoff $\Lambda^2$. 

4
with $Z$ factors, $Z_0 = Z_1 = Z_2 = \frac{1}{16\pi^2} N_f \ln(\frac{\Lambda^2}{\mu^2})$, which are dependent on $\Lambda$. In conventional normalization, $Z(\mu) = 1$ at any scale $\mu$. Thus, if we redefine the field $A_{r\mu}$ so as to normalize the kinetic term in (10), $A_{r\mu} = \sqrt{Z_0} A_{\mu}$, the renormalized coupling is determined as

$$g_r = \frac{1}{\sqrt{Z_0}} = \frac{Z_1}{(\sqrt{Z_0})^3} = \frac{\sqrt{Z_2}}{Z_0},$$  \hspace{1cm} (11)$$

which confirms the gauge invariance. Note that the $Z$ factors vanish at $\mu = \Lambda$, at which the dynamically generated kinetic as well as induced interaction terms disappear. We call this the compositeness condition at the cutoff $\Lambda$. This, in turn, indicates that the running coupling $g_r$ in (11) becomes infinity at scale $\Lambda$, implying that the theory is an ANF theory. The beta function obtained from (11) properly reflects this ANF behavior,

$$\beta(g_r) = \mu \frac{\partial}{\partial \mu} g_r = g_r^3 \frac{1}{16\pi^2} \frac{1}{6} N_f,$$  \hspace{1cm} (12)$$

from which we can see that the beta function (12) of the composite theory coincides with that of the elementary theory (the low energy theory with elementary gauge field),

$$\mathcal{L} = \text{Tr} \left( |D_\mu \phi|^2 - \lambda |\phi|^2 \right) - \frac{1}{4} \text{Tr} (F_{\mu\nu})^2 + \mathcal{L}_{FP+GF},$$  \hspace{1cm} (13)$$

$$\beta(g_r) = g_r^3 \frac{1}{16\pi^2} \left( \frac{1}{6} N_f - \frac{11}{3} N_c \right),$$  \hspace{1cm} (14)$$

in the $N_c/N_f$ expansion. With this approximation, the first term in (14) dominates (the beta function is positive), suggesting ANF character. The singularity at $\Lambda_L$ which appears in ANF gauge theory can be interpreted as $Z = 0$ in the composite theory, and above the cutoff $\Lambda_L$ the gauge field loses its identity as an elementary particle. Thus the ANF gauge theory (13) is an indication of a dynamically generated gauge theory.

One may notice that a positive beta function in the above always appears in the leading order of $N_c/N_f$. In this sense matters always have tendency to generate dynamical gauge bosons while gauge bosons themselves make negative contributions (the second term in (14)) \cite{25}. It should be noted that if the beta function becomes negative due to the next order terms, the theory does not satisfy the compositeness condition but is to be understood as an elementary gauge theory without cutoff. For this case, the $N_c/N_f$ expansion is no more applicable.
3 Is Standard Symmetry Asymptotically Free?

In this section we discuss the possibility that the present standard gauge group may not be AF. The Minimal Supersymmetric Standard Model (MSSM) is very popular because of its success in attaining gauge coupling unification, which is crucial if one wishes to construct a Grand Unified Theory (GUT). Another attractive feature of the MSSM is the unification of the $b$ and $\tau$ Yukawa couplings: If one assumes at the GUT scale

$$R_{b\tau}(M_{GUT}) = Y_b(M_{GUT})/Y_{\tau}(M_{GUT}) = 1,$$

the MSSM can reproduce the experimental value of $R_{b\tau}(M_Z) \approx 1.8$ with an appropriate strength of $Y_{\tau}(M_{GUT})$. It is interesting to investigate an extension of the MSSM having AF property. In extending the MSSM by introducing extra matter superfields, we must keep two things in mind: (1) the matter superfields must be introduced in such a way that gauge coupling unification (and anomaly cancellation) of the MSSM is preserved, and (2) the fermion content must be compatible with the constraints placed by LEP measurements, namely the so called Peskin–Takeuchi constraint. The simplest way to satisfy these requirements is to introduce 2 extra generations which form a generation–mirror generation pair, which we will call the 4th and anti–4th generations with $SU(2)_L \times U(1)_Y$ invariant Dirac masses, which we call Extended Supersymmetric Standard Model (ESSM). The typical behaviors of the running gauge couplings are shown in Fig. They are unified at the same scale in both cases but with different unified couplings.

Then the question arises: Are the above predictions still preserve in ESSM? One immediate consequence of ESSM is that all three gauge couplings of $SU(3)_C, SU(2)_L$ and $U(1)_Y$, are AF: they become larger as they are evolved up to coincide at the unification scale. We extend the analysis of ESSM and study how the existence of the extra generations will affect the running of the Yukawa couplings of the 3rd generation fermions. In addition to the $SU(2)_L \times U(1)_Y$ invariant masses, we also couple the 4th and 4th generation fermions to the two Higgs doublets in the same way as the other generations;

$$Y_U = Y_t, \quad Y_D = Y_b, \quad Y_E = Y_{\tau}, \quad Y_{\bar{D}} = Y_{\bar{U}} = Y_{\bar{E}} = 0,$$

and set all the 1st and 2nd generation Yukawa couplings to zero. As in the MSSM case, we will impose a unification condition on the Yukawa couplings at $M_{GUT}$ and determine the parameter range in which our model can predict the correct top, bottom and $\tau$-lepton masses. At this point one may think that such a program is doomed to failure from the beginning. Since the QCD coupling is AF, the QCD enhancement of $R_{b\tau}$ from $M_{GUT}$ to $M_Z$ will be...
Figure 1: Typical $\mu$ dependence of $\alpha_1(\mu)$, $\alpha_2(\mu)$, $\alpha_3(\mu)$ in the MSSM (dashed lines) and the MSSM + 1 EVF (solid lines).

...even larger than the MSSM case making it impossible to bring $R_{b\tau}(M_Z)$ down to $\sim 1.8$ even with large Yukawa couplings. However, an $SO(10)$-GUT with an 126-Higgs predicts

$$Y_t(M_{GUT}) = Y_b(M_{GUT}) = \frac{1}{3} Y_\tau(M_{GUT}) \rightarrow R_{b\tau}(M_{GUT}) = \frac{1}{3},$$

which is the unification condition we will adopt here. In this case, the extra enhancement from QCD is actually welcome since $R_{b\tau}$ must be enhanced by a factor of $5 \sim 6$ to reproduce the experimental value of $R_{b\tau}(M_Z)$.

The number of adjustable parameters is four: the unification scale $M_{GUT}$, the unified gauge coupling $\alpha_{GUT}$, the unified Yukawa coupling $Y_{GUT}$, and the mixing angle of the low lying Higgs fields $\tan \beta = v_2/v_1 \left( \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV} \right)$. We restrict $\alpha_{GUT}$ and $Y_{GUT}$ to the region, $\alpha_{GUT} < 1.0, Y_{GUT} < 0.7$, which guarantees that the gauge and Yukawa couplings are kept still within their perturbative regions throughout the evolution from $M_{GUT}$ to $M_Z$. We will use the $\tau$-lepton mass to fix $\tan \beta$ from In view of the relatively large coupling strengths near the unification scale due to the ANFness, we use the fully coupled 2-loop RGE’s to evolve the gauge and Yukawa couplings. Taking the masses at $M_{EVF} = M_{SUSY} = 1 \text{ TeV}$, and by fixing the values of $\alpha_{GUT}$

$\dagger$ An 126-Higgs is necessary to give a direct Majorana mass term to the right-handed neutrino.
and $M_{\text{GUT}}$ in the range allowed by gauge coupling unification, we calculate the evolution of the Yukawa couplings for different values of $Y_{\text{GUT}}$.

The experimentally determined $\overline{\text{MS}}$ running masses of the $\tau$–lepton and the $b$ quark at $\mu = M_Z$ are given by

$$m_{\tau}(M_{\text{SUSY}}) = \frac{\nu}{\sqrt{2}} Y_{\tau}(M_{\text{SUSY}}) \cos \beta$$

$$m_{\tau}(M_Z) = 1.75 \pm 0.01 \text{ GeV}, \quad m_b(M_Z) = 3.1 \pm 0.4 \text{ GeV},$$

from which we conclude $R_{b\tau}(M_Z) = 1.6 \sim 2.0$. The calculated result of $R_{b\tau}(M_Z)$ within reasonable values of $\alpha_{\text{GUT}}$ and $M_{\text{GUT}}$ is shown in Fig. 2. From the figure we see that there is a large set of $(\alpha_{\text{GUT}}, M_{\text{GUT}})$ values which keeps $R_{b\tau}(M_Z)$ below 2.

We would like to stress that the ANFness of the gauge couplings has a strong focusing effect on the top (and also bottom) Yukawa couplings as they evolve down in scale and as a result, the value converges to IR fixed point by the time they reach the SUSY breaking scale $M_{\text{SUSY}} = 1 \text{ TeV}$. In the next section we shall show the explicit picture of the evolutions of the $t$ Yukawa couplings. Also we shall see that for the case of the $\tau$ Yukawa coupling, the
situation is rather different. Near the GUT scale it tends to focus itself due to its larger size at $M_{GUT}$ (Recall $Y_\tau(M_{GUT}) = 3Y_{GUT}$). However, unlike the top and bottom Yukawa couplings, once $Y_\tau$ becomes small at lower scales, it runs slowly and does not quite converge to its IR fixed point ($y_\tau = 0$).

### 4 Infrared Fixed Point Structure in ANF Theories

We have shown a possible scenario of the standard gauge symmetry with ANF character and mentioned that due to the ANF gauge couplings the top Yukawa coupling is determined almost insensitive to their initial values fixed at GUT scale $M_G$. We would like to stress that such strong convergence of Yukawa couplings to their infrared fixed points is a common feature appearing in ANF theories. This section is devoted to present how strongly the couplings are focused into their infrared points in ANF theories and demonstrate the structure of the renormalization group flow. As illustrations we take the supersymmetric standard models with AF and ANF gauge couplings and compare them by concentrating on their infrared structure.

Before this let us consider a simple gauged Yukawa system with one gauge coupling $g$ and one Yukawa coupling $y$, whose 1-loop $\beta$-functions are as follows:

\[
\frac{d\alpha}{dt} = -\frac{b}{2\pi} \alpha^2, \quad \frac{d\alpha_y}{dt} = \frac{\alpha_y}{2\pi} \left( a\alpha_y - c \alpha \right),
\]

where,

\[
\alpha \equiv \frac{g^2}{4\pi}, \quad \alpha_y \equiv \frac{y^2}{4\pi}, \quad t = \ln \left( \frac{\mu}{\mu_0} \right).
\]

The system is AF (ANF) for $b > 0$ ($b < 0$) and always $a > 0$, $c \geq 0$. Then,

\[
\frac{dR}{dt} = \frac{a}{2\pi} \alpha R (R - R^*), \quad \left( R \equiv \frac{\alpha_y}{\alpha}, \quad R^* = \frac{c - b}{a} \right).
\]

If we take $\mu_0 = M_G$ and $\alpha(0) = \alpha(M_G)$, we get from (21)

\[
\frac{R(t) - R^*}{R(t)} = \left( \frac{\alpha(t)}{\alpha(M_G)} \right)^B \left( \frac{R(M_G) - R^*}{R(M_G)} \right), \quad B \equiv 1 - \frac{c}{b}.
\]

where $R^*$ is an infrared fixed point if $R^*>0$ and we see that the suppression factor $\xi = \left( \frac{\alpha(t)}{\alpha(M_G)} \right)^B$, gives the criterion on how fast $R$ approaches to $R^*$, depends only on the gauge coupling $g$. The $b$-dependence on the suppression factor $\xi$ is shown in Fig. 3, from which we find a big difference between AF
(b > 0) and ANF (b < 0) cases. In the AF case the point b = c above which B becomes negative and ξ becomes larger than 1 and R*(< 0) is no more an infrared fixed point. On the other hand, in the ANF case there is always a nontrivial infrared fixed point R*(> 0) and the convergence to R* becomes much better. Let us compare the case of MSSM(AF) with that of the ESSM(ANF).

Let us compare the typical values corresponding to AF and ANF cases, respectively, by taking the realistic values of α = α₃, α_y = α_t; 1) MSSM: b = 3, c = 16 \Rightarrow ξ \sim 0.423 and 2) ESSM: b = -1, c = 16 \Rightarrow ξ \sim 10^{-6}.

The suppression factors can be read off from Fig.3 (indicated by arrows). We can see more clearly the situation by comparing the µ-dependence of α_t/α₃ in AF and ANF cases. In the ESSM case the convergence to the infrared fixed point is much better than that in the MSSM (Fig.3) and its fixed point value depends very weakly on the initial value at M_GUT (Fig.4). This is because of the effect from the gauge couplings which becomes very large at M_G.

The RG flow in (α₃, α_t) plane of those 2 cases behaves quite differently each other, where the ANF case is in remarkable contrast to the AF case,
Figure 4: $R_t$ in the MSSM ($M_G = 1.6 \times 10^{16}$ GeV, $\alpha_{GUT} = 0.04$)

Figure 5: $R_t$ in the MSSM + 1 EVF ($M_G = 7.0 \times 10^{16}$ GeV, $\alpha_{GUT} = 1.0$)
the details of which is left to the reference\cite{1}. In the case of ANF, the ratio $\frac{\alpha}{\alpha_3}$ evidently has infrared fixed point in all the region in the $(\alpha_3, \alpha_t)$ plane and one loop approximation becomes more and more available in the infrared region. This is in remarkable contrast to the AF case. This kind of infrared stability is very attractive and may give us a feasibility of determining low-energy Yukawa coupling constants. Finally we make a comment on the infrared structure of $R_\tau$. $R_\tau$ does not reach to the non-trivial infrared fixed point but to zero in the low energy limit. On the other hand, the top Yukawa coupling constant is quite remarkable (Fig.\ref{fig:fig3}); we find that it indeed reaches to their fixed point, whose value hardly be affected by the initial values at $M_G$ because of the ANF gauge couplings. We are sure that the infrared fixed points obtained from these solutions are physically significant and provide us with reliable low-energy parameters. In the case at hand, by using these fixed point solutions and the experimental value of $\alpha_3(1\text{TeV}) \sim 0.093$, we get for example\cite{12}:

$$m_t(M_Z) \sim 178 \text{GeV}, \quad m_b(M_Z) \sim 3.2 \text{GeV} \quad (\tan \beta \sim 58)$$

These values are certainly consistent with the experimental values\cite{38}:

$$m_t(M_Z) \sim 180 \pm 10 \text{GeV}, \quad m_b(M_Z) \sim 3.1 \pm 0.4 \text{GeV} \quad .$$

\section{Future Aspects}

The points in the parameter-space of Lagrangian, to which the renormalization flow of the parameters in the Lagrangian converge in the high (low energy) limit, are called ultraviolet (infrared) fixed points and they play important roles in characterizing the physics. The theory is called “asymptotically free” (AF) when its ultraviolet fixed points of the parameters are zero (trivial ultraviolet fixed points). If we trace back to the low energy region of AF theory, on the other hand, naive continuation leads larger and larger couplings at lower energy, showing infrared divergence. In contrast to AF theories, we have seen that ANF theories have nontrivial infrared fixed points which have close relations to low-energy physics. Usually ANF theories have been considered to be unphysical and people thought that couplings should never diverge in high energy limit. However we have demonstrated that ANF theories may provide possible candidates for models to unify all existing particles and fields. This may be guaranteed so far as the ultraviolet cutoff $\Lambda$ has a well defined

\footnote{The value $\tan \beta \sim 58$ is determined from the experimental value $m_{\tau}(M_Z)$\cite{12}.}
physical meaning and if (at least) one of the couplings becomes infinity at a certain scale $\Lambda$, which is just an indication of some new physics above $\Lambda$. Recently in some theories having nontrivial fixed points, it is found that there exist some relations between different kinds of coupling constants, and the symmetry structure of the system is enlarged in this limit. In this context we may hope that infrared structure appearing in ANF theories may provides important information on the low energy physics.

Then we can proceed further to consider the scenario of GUT with ANF standard gauge groups. As an example, we have seen that a vector-like family pair added to MSSM attains gauge unification at 2-loop level and reproduces experimental bottom tau ratio quite easily, and further it connects the top quark mass to an infrared fixed point. The existence of extra fermions have been discussed from various points of view; in deriving CP violation, dynamical SUSY breaking, hierarchical mass matrix, especially we are aware that any GUT model motivated by string models or supergravity models, predicts additional fermions quite naturally. At present it is very difficult to predict their masses, but we must not exclude the cases in which they are even below 1 TeV.\(^{42, 43}\) If it is so, we may find their consequences in near future. ANF model predicts that the strong coupling constant decreases very slowly or even increases at higher scale. This will be tested by measuring the excess of jet production cross section around 1 TeV or so in the future supercollider experiments.

We have argued that any non-linear sigma model has its own hidden local symmetry and there may be a possibility that the gauge bosons associated with this hidden local symmetry are generated dynamically. However this is not enough for those vector fields to be real gauge bosons. If one introduces matter fields, for example, there is no reason that those matters couple to those gauge bosons with universal strength and they couple to hidden gauge bosons with arbitrary strength. The only case for the hidden gauge bosons to have universal couplings, seems to assign all the existing matters as NG bosons living in the coset space $G/H$. Then the universality of gauge couplings is always guaranteed. This possibility is very attractive and once investigated elaborately in 1986\(^{16}\) within the framework of $N = 8$ supergravity theories, where the hidden local symmetry has played an important role. In this scenario it is assumed that all the existing matters and fields except graviton are bound states composed of $N = 8$ gravity multiplets.\(^{17}\) Of course there are many problems; anomaly, chirality, the cosmological constant, etc. I do hope that we shall be able to overcome for all those problem in the near future.
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