Deeply bound pionic states and the effective pion mass in nuclear systems

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Abstract

We show that the s-wave pion-nuclear potential which reproduces the deeply bound pionic states in Pb, recently discovered at GSI, is remarkably close to the one constructed directly from low energy theorems based on chiral symmetry. Converting this information into an effective pion mass we find $m_π^*/m_π \simeq 1.13$ in the center of the Pb nucleus, and $m_π^*/m_π \simeq 1.07$ in symmetric nuclear matter.

Introduction. The deeply bound pionic states in Pb, recently discovered at GSI [1] and previously predicted in refs. [2, 3], have stimulated renewed interest in the nature of the s-wave part of the pion-nuclear optical potential [4]. For such states to exist with sufficiently long lifetime, there must be a subtle cancelation between the $\pi^-$ Coulomb and strong interactions in the bulk nucleus. In fact, the effective s-wave $\pi$-nucleon repulsion must roughly compensate the attractive Coulomb force in the interior of the Pb nucleus. If this were not the case, the deeply bound pion wave function would have substantial overlap with the nuclear density distribution, and the pion would then experience strong absorption at the nuclear surface. This would in

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turn prohibit the appearance of the narrow pionic line in the $^{208}$Pb(d,$^3$He) spectrum as it is observed in ref. [1].

The 2p and, even better, the 1s pionic bound states in such heavy systems therefore provide strong quantitative constraints on the s-wave $\pi^-$ interactions with nucleons in nuclei, given that the p-wave interactions are quite well established by data on pionic atoms in higher orbits [4]. In the present note we would like to update this discussion from the point of view of low energy theorems based on the underlying chiral symmetry of QCD. We demonstrate that an s-wave $\pi^-$ optical potential derived from chiral symmetry alone is remarkably consistent with the information provided by ref. [1]. It is useful to convert this information into a statement about the effective pion mass in nuclear matter. The results confirm and sharpen our previous knowledge about the pion-nuclear optical potential.

**Pion self-energy and effective mass in matter.** We start with a reminder of the pion self-energy in nuclear matter. Let $\rho_p$ and $\rho_n$ be the proton and neutron densities, respectively. To leading order in those densities, the self-energy $\Pi$ (or optical potential $U$) of a pion with energy $\omega$ and momentum $\vec{q}$ is given by:

$$
\Pi(\omega, \vec{q}; \rho_p, \rho_n) \equiv 2\omega U(\omega, \vec{q}; \rho_p, \rho_n) = -T^{(+)}(\omega, \vec{q})(\rho_p + \rho_n) - T^{(-)}(\omega, \vec{q})(\rho_p - \rho_n).
$$

Here $T^{(\pm)}$ are the isospin even and odd $\pi$-nucleon (forward) amplitudes. For a $\pi^-$,

$$
T^{(\pm)} = \frac{1}{2} [T_{\pi^-p} \pm T_{\pi^-n}].
$$

Pionic modes of excitation in nuclear matter are determined by solutions $\omega(\vec{q})$ of the equation

$$
\omega^2 - \vec{q}^2 - m_{\pi}^2 - \Pi(\omega, \vec{q}; \rho_p, \rho_n) = 0.
$$

The effective pion mass $m_{\pi}^*(\rho)$ is defined by:

$$
m_{\pi}^{*2} = m_{\pi}^2 + \text{Re} \Pi(\omega = m_{\pi}^*, \vec{q} = 0; \rho).
$$

it is identified with the real part of the energy $\omega(\vec{q} = 0)$ for a pion at rest in matter.
Chiral s-wave potential. Chiral symmetry imposes strong constraints on the near-threshold behavior of the amplitudes (2). According to the low-energy theorem of Tomozawa and Weinberg (TW) [5], the $\pi^-N$ amplitudes at $\vec{q} = 0$ behave as

$$T_{\pi^-p}(\omega, \vec{q} = 0) = -T_{\pi^-n}(\omega, \vec{q} = 0) = \frac{\omega}{2f^2} + O(\omega^2, m^2_\pi),$$

where $f$ is the pion decay constant taken in the chiral limit ($m_\pi \to 0$). Its physical value, $f_\pi = 92.4$ MeV [6], differs from $f$ by terms of order $m^2_\pi$. We use $f = 86$ MeV in the following [7].

The TW theorem states that

$$T^{(+)}(\omega, \vec{q} = 0) = 0 + O(\omega^2, m^2_\pi),$$
$$T^{(-)}(\omega, \vec{q} = 0) = \frac{\omega}{2f^2} + O(\omega^3, \ldots).$$

At threshold ($\omega = m_\pi$) one has to leading order:

$$T_{\text{thr}}^{(-)} = \frac{m_\pi}{2f^2} = 1.86 \text{ fm}.$$ 

In chiral perturbation theory, non-leading corrections in $T_{\text{thr}}^{(-)}$ have been calculated to fourth order [8]. These calculations use the physical $f_\pi$ in the leading order term and then find that the higher order corrections increase $T_{\text{thr}}^{(-)} = m_\pi/2f^2$ by about 15%, so that a value close to eq. (7) results. For $T^{(+)}$ the second order corrections are altogether small but involve cancelations between large pieces, the $\pi N$ sigma term and other $O(\omega^2)$ terms. In comparison with the empirical threshold amplitudes\[7,8\],

$$T_{\text{thr}}^{(+)} = (-0.22 \pm 0.15) \text{ fm}, \quad \text{(Sigg et al. [9])}$$
$$(-0.16 \pm 0.06) \text{ fm}; \quad \text{(KH, [10])}$$
$$T_{\text{thr}}^{(-)} = (1.96 \pm 0.14) \text{ fm}, \quad \text{(Sigg et al. [9])}$$
$$(1.87 \pm 0.04) \text{ fm}; \quad \text{(KH, [10])}$$

the chiral leading order results (6a,b) are already remarkably close, and we shall use them as reference points.

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*The $T$-amplitudes are related to scattering lengths by $T_{\text{thr}} = 4\pi(1 + \frac{m_\pi}{M_N})a$. The sign convention is such that $a < 0$ implies repulsion, $a > 0$ corresponds to attraction.
Note that at this level, with $T^{(+)} = 0$, the s-wave optical potential is simply

$$U_s = \frac{\rho_n - \rho_p}{4f^2} \simeq 44 \text{ MeV} \left( \frac{\rho_n - \rho_p}{\rho_0} \right).$$

Here we have introduced the density of symmetric nuclear matter, $\rho_0 = 0.17 \text{ fm}^{-3}$, as a convenient scale.

Given that $T^{(+)} \simeq 0$, rescattering corrections are important as already pointed out in ref. [11]. For symmetric nuclear matter, the $T^{(+)}$ in eq. (1) at $\vec{q} = 0$ is to be replaced by

$$T^{(+)}_{\text{eff}}(\omega, \vec{q} = 0) = T^{(+)}(\omega, \vec{q} = 0) - \left[ T^{(+)}^2 + 2T^{(-)}^2 \right] \left( \frac{e^{ikr}}{4\pi f} \right),$$

where the averaged spherical pion wave with $k = \sqrt{\omega^2 - m_\pi^2}$ propagates between pairs of nucleons. For a Fermi gas we have $\langle e^{ikr}/r \rangle \simeq \frac{3}{2} \frac{p_F}{\pi} + ik$, where $p_F = (3\pi^2 \rho/2)^{1/3}$ is the Fermi momentum, and we can drop the small $ik$ term for $k \ll p_F$. Using eqs. (6) in leading chiral order we find

$$T^{(+)}_{\text{eff}}(\omega, \vec{q} = 0) = -3p_F^2 \left( \frac{\omega}{4\pi f} \right)^2 \simeq -0.36 \frac{\omega^2}{m_\pi^2} \left( \frac{\rho}{\rho_0} \right)^{1/3} \text{ fm}.$$

For asymmetric nuclear matter we can still assume approximately equal inverse correlation lengths $\langle 1/r \rangle$ for protons and neutrons and express them in terms of a common Fermi momentum determined by $\rho = \rho_p + \rho_n$. Additional rescattering corrections proportional to $T^{(+)}T^{(-)}(1/4\pi r)(\rho_n - \rho_p)$ vanish for $T^{(+)} = 0$ and can safely be ignored when using $T^{(+)}$ from eq. (8a). Then the self-energy $\Pi = 2\omega U = -T^{(+)}_{\text{eff}}(\rho_p + \rho_n) - T^{(-)}(\rho_p - \rho_n)$, taken at $\omega = m_\pi$ and $\vec{q} = 0$, gives the threshold s-wave optical potential

$$U_s^{(0)} \simeq 8.5 \text{ MeV} \left( \frac{\rho_p + \rho_n}{\rho_0} \right)^{4/3} + 44 \text{ MeV} \left( \frac{\rho_p - \rho_n}{\rho_0} \right).$$

We refer to eq. (12) in the following as the “chiral” s-wave potential. A non-zero $T^{(+)}_{\text{thr}} \simeq -0.1 \text{ fm}$ would add a correction $\Delta U_s \simeq 2 \text{ MeV} (\rho_p + \rho_n)/\rho_0$ to eq. (12).

**Effective pion mass.** The effective pion mass $m_{\pi}^*(\rho_p, \rho_n)$ resulting from eq. (4) when one uses the “chiral” s-wave potential (12), are shown in Fig.
1 for symmetric matter and for a typical example of asymmetric matter. Clearly the mass shift is small for matter with \( N = Z \), less than 10% of the free mass at \( \rho = \rho_0 \). For systems with a large neutron excess the effect of \( T^{(-)} \) takes over, and this is obviously relevant for nuclei such as Pb.

The detailed analysis of \( \pi \)-nuclear bound states requires of course to go beyond just the “chiral” s-wave potential. Apart from the Coulomb potential, the p-wave term of the optical potential has to be added, and an absorptive potential must be included. This is done in the standard and time-honored way [4, 11]. The Klein-Gordon equation to be solved in \( \vec{r} \)-space is:

\[
\left[ \vec{\nabla}^2 - m_\pi^2 + (\omega - V_c(\vec{r}))^2 - 2\omega U(\omega, \vec{r}) \right] \phi(\vec{r}) = 0 ,
\]  
(13)

where \( V_c(\vec{r}) \) is the Coulomb potential generated by the charge distribution \( \rho_p(\vec{r}) \). For the optical potential \( U = U_s + U_p \) we use the chiral s-wave potential as before but add a phenomenological absorption term:

\[
U_s(\vec{r}) = U_s^{(0)}(\vec{r}) + B\rho^2(\vec{r}) ,
\]  
(14)

where \( U_s^{(0)}(\vec{r}) \) is given by eq. (12) but now with local density distributions \( \rho_{p,n}(\vec{r}) \). The \( B\rho^2 \) term is introduced as usual to parameterize absorption effects, with \( \text{Im} B = -0.27 m_\pi^{-5} \) fitted to the widths of a large amount of pionic atom levels in higher orbits. We choose \( \text{Re} B = 0 \) in our standard set. The nonlocal p-wave term is of the form

\[
U_p(\vec{r}) = \frac{2\pi}{m_\pi} \vec{\nabla} F(\vec{r}) \vec{\nabla} ,
\]  
(15)

with canonical input for the complex function \( F(\vec{r}) \) as specified in ref. [4].

In table 1 we show results for the energies and widths of deeply bound 1s and 2p states of the \((\pi^-^{207}\text{Pb})\) system as calculated with the optical potential (14, 15). The proton and neutron density distributions have been obtained from a realistic Skyrme-Hartree-Fock calculation [12] which reproduces the measured charge distribution of \( ^{208}\text{Pb} \).

Evidently, the chiral s-wave potential (12) works remarkably well when combined with the non-local p-wave potential (15) and the absorptive parts. In table 1 we have also examined the sensitivity to changes of \( T^{(+)}_{\text{eff}} \) by adding a correction \( \delta T^{(+)}_{\text{eff}} \) to eq. (11). The best fit to the deeply bound states in Pb is found for values of \( \delta T^{(+)}_{\text{eff}} \) well within the empirical range of uncertainties.
in eq. (8a). There seems to be no need for a substantial dispersive real part, Re $B$, in the s-wave absorptive potential. The “chiral” effective s-wave potential for $^{207}\text{Pb}$ produces deeply bound $\pi^-$ states with $E_{2p} = -5.39$ MeV and $E_{1s} = -7.27$ MeV. The widths of these states are smaller than 1 MeV.

We have checked the stability of these results with respect to changes of the p-wave potential and the s-wave absorptive part by using several parameterizations available in the literature [13]. The differences are marginal as long as these potentials fit the large amount of pionic atom data for higher orbits.

The effective pion mass profile resulting from the self-consistent solution of

$$m^*_{\pi}^2(r) = m^2_{\pi} + 2\omega \text{Re} U_s(r)$$

for Pb in the absence of the Coulomb potential is shown in Fig. 2. In the nuclear center the effective mass increases by a little less than 20 MeV as compared to its free space value. About half of this shift comes from $T_{\text{eff}}^{(+)}$, the other half results from $T^{(-)}$ and the neutron excess.

We conclude that most of the weakly repulsive s-wave $\pi^-$-nuclear potential can indeed be directly understood in terms of basic theorems derived from chiral symmetry. The new data on deeply bound pionic states have sharpened the quantitative constraints on this potential considerably. In particular, the widths of these states are a sensitive measure of the subtle balance between s-wave repulsion and Coulomb attraction.

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Table 1: Energies $E = \omega - m_\pi$ and widths $\Gamma$ for pionic 1s and 2p states in $^{207}$Pb calculated with the optical potential (14, 15). The “standard” set with $\delta T_{\text{eff}}^{(+)} = 0$ uses the chiral s-wave potential $U_{s}^{(0)}$ of eq. (12) and Re $B = 0$. Results obtained with a correction $\delta T_{\text{eff}}^{(+)}$ to $T_{\text{eff}}^{(+)}$ of eq. (11) are also shown. The experimental data from ref. [1] are displayed at the bottom of the table.

| $\delta T_{\text{eff}}^{(+)}$ [fm] | $E_{1s}$ [MeV] | $\Gamma_{1s}$ [keV] | $E_{2p}$ [MeV] | $\Gamma_{2p}$ [keV] |
|-----------------------------------|-----------------|---------------------|-----------------|---------------------|
| -0.10                             | -7.135          | 787                 | -5.313          | 567                 |
| -0.05                             | -7.200          | 871                 | -5.352          | 630                 |
| 0                                 | -7.266          | 968                 | -5.391          | 702                 |
| +0.05                             | -7.333          | 1077                | -5.430          | 784                 |
| +0.10                             | -7.400          | 1198                | -5.468          | 876                 |
| exp. [1]                          |                |                     | -5.4 ± 0.2      | < 0.8 MeV           |

Figure Captions:

Fig. 1: Effective pion mass in symmetric nuclear matter ($x = 1$, dashed line) and an example of asymmetric matter ($x = \rho_n/\rho_p = 1.6$, solid line) as a function of density $\rho = \rho_p + \rho_n$, calculated with the “chiral” s-wave potential (12).

Fig. 2: Profile of the effective $\pi^-$ mass in $^{207}$Pb deduced from eq. (16).
Figure 1
Figure 2