Amplitude ambiguities in pseudoscalar meson photoproduction

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(August 16, 2018)

Abstract

We consider the problem of determining amplitudes from observables for the case of pseudoscalar meson photoproduction. We find a number of surprisingly simple constraints which give necessary conditions for a complete set of measurements. These results contradict one of the selection rules derived previously.

PACS Numbers: 13.60.Le, 13.88.+e
Experiments conducted at CEBAF will soon yield a flood of new and precise data for the photo- and electroproduction of mesons. This has motivated a renewed examination of the observables required for the determination of underlying amplitudes. The electroproduction of pseudoscalar mesons has been studied by Dmitrasinovic, Donnelly and Gross [1] and, more recently, the photoproduction of vector mesons has been studied by Tabakin and co-workers [2]. This type of analysis was applied to the photoproduction of pseudoscalar mesons by a number of groups. The work of Barker, Donnachie and Storrow [3] is generally quoted as the standard reference.

In Ref. [3], the requirements for a complete set of measurements were studied within the transversity representation. This representation is particularly useful as measurements of the cross section and single polarization observables directly determine the magnitudes of the 4 independent amplitudes. It then remains to determine relative phases from double polarization measurements. In Ref. [3] it was claimed that 5 double polarization measurements are required in order to resolve all ambiguities (apart from an overall phase) in the transversity amplitudes. In choosing the 5 double polarization measurements one had only to insure that fewer than 4 were taken from any one set of beam-target (BT), beam-recoil (BR), or target-recoil (TR) observables. Only 3 double polarization measurements (not all from the same set) were found necessary to determine the amplitudes up to "quadrant ambiguities".

As analyses are generally performed using helicity amplitudes, we reexamined the question of complete experiments within this basis. (In order to avoid confusion, we retain the naming scheme of Ref. [3].) The 4 independent helicity amplitudes are denoted $S_1$, $S_2$ (single spin-flip), $N$ (no spin-flip), and $D$ (double spin-flip). Apart from overall factors, the relations between amplitudes and observables are given in Table I. If one examines the set of cross section and single polarization measurements, a number of ambiguities are evident. The first (trivial) ambiguity results from the freedom to alter the overall phase of these amplitudes. It is not hard to find 3 more ambiguity relations associated with the set of cross section and single polarization observables.

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Ambiguity I

\[ S_1 \leftrightarrow S_2 \text{ and } N \leftrightarrow -D \]  \hspace{1cm} (1)

Ambiguity II

\[ S_1 \rightarrow N \]
\[ N \rightarrow -S_1 \]
\[ S_2 \rightarrow -D \]
\[ D \rightarrow S_2 \]  \hspace{1cm} (2)

Ambiguity III

\[ S_1 \rightarrow D \]
\[ D \rightarrow -S_1 \]
\[ N \rightarrow S_2 \]
\[ S_2 \rightarrow -N \]  \hspace{1cm} (3)

If the operations indicated above are carried out, the first four observables listed in Table I are unchanged. The remaining double polarization observables are either unchanged, or changed by at most a sign.

The operations I, II and III are associated with ambiguities in the TR, BR, and BT observables respectively [4]. These discrete symmetries are in fact special cases of three continuous symmetries [5] which correspond to changing the three angles (left unspecified by single polarization measurements) between the four transversity amplitudes \(b_1, b_2, b_3,\) and \(b_4\). In order to determine these angles, up to discrete quadrant ambiguities, we require a set of three double-polarization measurements which is not invariant under operations I, II, or III. Therefore, no more than two measurements can be of the same type (see Table I). This is identical to the requirement stated in Ref. [3].

If we were only able to produce necessary conditions which agreed with the results of Ref. [3], this approach would have limited usefulness. However, the determination of a set of amplitudes which resolves all ambiguities, apart from the
trivial one, is a much more difficult problem. Here the utility of this method becomes clear. In order to explore this problem, and test the conclusion of Ref. [3], we generate one additional ambiguity involving complex conjugation [6].

Ambiguity IV

\[ S_1 \rightarrow -S_1^* \]
\[ S_2 \rightarrow -S_2^* \]
\[ N \rightarrow N^* \]
\[ D \rightarrow D^* \] (4)

If the statements [3] regarding discrete ambiguities were correct, then by choosing no more than 2 observables from each double polarization set, we would resolve this ambiguity as well. However, it is easy to see that certain choices will leave ambiguity IV unresolved. In fact, we can measure the 6 observables \( H, E, O_x, C_z, T_x, \) and \( L_z \) without resolving this ambiguity. Further constraints on observable choices can be generated by composing the transformation given in set IV with sets I, II and III. The resulting additions to Table I are easily found by multiplying elements from the corresponding columns.

In Ref. [3], the resolution of quadrant ambiguities was demonstrated for measurements of \( G, F, E, L_x, \) and one other observable not from the BT set. This set does indeed resolve all of the ambiguities listed in Table I. As the authors point out, the enumeration of all possibilities is “exceedingly tedious” (the ‘no four from any set’ criterion allows 768 possible combinations). It is easy to see how the constraint given by Ambiguity IV could be missed, since it involves measurements from all three double polarization sets.

In summary, the examination of ambiguity relations provides a simple and useful check of proposed complete sets of experiments. We have found that the rules for choosing observables are more complicated than those given in Ref. [3]. Note that some measurements of \( G, H, \) and \( O_x \) exist, and these measurements resolve all the ambiguities listed above. Unfortunately, while necessary conditions are relatively easy to generate, the proof of sufficient conditions is still difficult. We are continuing to work on this aspect of the problem.
This work was supported in part by the U.S. Department of Energy Grants DE-FG05-88ER40454 and DE-FG05-95ER40709A.
REFERENCES

[1] V. Dmitrasinovic, T.W. Donnelly, and F. Gross, "Research Program at CEBAF (III), RPAC III," published by CEBAF (Jan. 1988), edited by F. Gross, p.547.

[2] F. Tabakin, M. Pichowsky, and C. Savkli, Nucl. Phys. A370, 311 (1994).

[3] I.S. Barker, A. Donnachie, and J.K. Storrow, Nucl Phys. B95, 347 (1975).

[4] Note that Ambiguity I can be obtained from the composition of Ambiguities II and III.

[5] See the related work by N.W. Dean and P. Lee, Phys. Rev. D 5, 2741 (1972) in which this method is applied to spin-0−spin-$\frac{1}{2}$ scattering.

[6] In the transversity representation, we can apply the transformation: $(b_1, b_2, b_3, b_4) \rightarrow (b^*_1, b^*_2, b^*_3, b^*_4)$. 
TABLES

TABLE I. Result of ambiguity relations applied to observables. Overall factors have been removed in the relations between amplitudes and observables. The observables are either invariant (+) or change sign (−) under these operations.

| Observable | Type | Helicity Rep. | ( I ) | ( II ) | ( III ) | ( IV ) |
|------------|------|---------------|-------|--------|---------|--------|
| σ(θ)       |      | |N|² + |S₁|² + |S₂|² + |D|² | +  | +  | +  | +  |
| Σ          | S    | 2 Re (S₁*S₂* − ND*) | +   | +  | +  | +  |
| T          |      | 2 Im (S₁N* − S₂D*) | +   | +  | +  | +  |
| P          |      | 2 Im (S₂N* − S₁D*) | +   | +  | +  | +  |
| G          |      | −2 Im (S₁S₂* + ND*) | −   | −  | +  | −  |
| H          | BT   | −2 Im (S₁D* + S₂N*) | −   | −  | +  | +  |
| E          |      | |S₂|² − |S₁|² − |D|² + |N|² | −   | −  | +  | +  |
| F          |      | 2 Re (S₂D* + S₁N*) | −   | −  | +  | −  |
| Oₓ         |      | −2 Im (S₂D* + S₁N*) | −   | +  | −  | +  |
| Oᵧ         | BR   | −2 Im (S₂S₁* + ND*) | −   | +  | −  | −  |
| Cₓ         |      | −2 Re (S₂N* + S₁D*) | −   | +  | −  | −  |
| Cᵧ         |      | |S₂|² − |S₁|² − |N|² − |D|² | −   | +  | −  | +  |
| Tₓ         |      | 2 Re (S₁S₂* + ND*) | +   | −  | −  | +  |
| Tᵧ         | TR   | 2 Re (S₁N* − S₂D*) | +   | −  | −  | −  |
| Lₓ         |      | 2 Re (S₂N* − S₁D*) | +   | −  | −  | −  |
| Lᵧ         |      | |S₁|² + |S₂|² − |N|² − |D|² | +   | −  | −  | +  |