The Quantum Hall effect, Skyrmions and Anomalies

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Abstract.

We discuss the properties of Skyrmions in the Fractional Quantum Hall effect (FQHE). We begin with a brief description of the Chern-Simons-Landau-Ginzburg description of the FQHE, which provides the framework in which to understand a new derivation of the properties of FQHE Skyrmions (S. Baez, A.P. Balachandran, A. Stern and A. Travesset cond-mat 9712151) from anomaly and edge considerations.

I THE QUANTUM HALL EFFECT

Experiments carried out starting at the end of the 70’s (see [2]) revealed that in some samples, the Hall effect presents fascinating and completely unexpected properties, the most outstanding being the broad plateaus for the transverse conductivity $\sigma_H$ (or for the transverse resistivity $R_H = 1/\sigma_H$) and the vanishing of the longitudinal resistivity at this plateaus, see Fig. 1. There are other important properties, such as the incompressibility of the Hall liquid and the presence of quantum mechanically induced currents if the sample has edges. It is fair to say that, in spite that there are still open questions, theorist have succeeded in giving successful approaches to the problem. We give an introduction to the most common field theory formulation [3], but we will necessarily be very descriptive. We refer, for example, to [4] for a pedagogical review and [5] for a recent and more detailed analysis. The second part of the talk is devoted to the properties of FQHE Skyrmions, a problem which has already been addressed in this MRST meeting [6].
FIGURE 1. The plateaus in the integer Hall effect resistivity plotted versus the filling fraction. The left vertical axis is the inverse transverse resistivity in dimensionless units, and the right one the longitudinal in arbitrary units. With higher resolution settings smaller plateaus at fractional filling fractions may be observed as well.

A The CSLG Field theory of the FQHE

A very intuitive picture to understand the FQHE is to consider the total flux in the sample \( \Phi = BA \), where \( B \) is the external magnetic field and \( A \) the total area, as being carried by \( N_f \) particles, the fluxons, each one carrying an elementary flux unit \( \Phi_0 = \frac{2\pi \hbar c}{e} \). The physical electrons are represented as composite objects, a bare bosonic particle with an odd number of fluxons attached to it restoring its fermionic nature. The filling fraction is defined as

\[
\nu = \frac{N_{el}}{N_f},
\]

where \( N_{el} \) is the number of physical electrons in the sample. If \( \nu = \nu_k \equiv \frac{1}{2k+1}, k \in \mathbb{N}, \) there are exactly \( 2k + 1 \) fluxons per electron. It is, therefore, an “optimal” situation. In any other filling fraction \(^1\) there will be an unbalanced number of fluxons with respect to electrons. How the system responds to this situation is the key for understanding the properties of the QH liquid.

The previous picture may be described in a Landau-Ginzburg formalism, the Chern-Simons-Landau-Ginzburg (CSLG) Lagrangian density being [3]

\[
\mathcal{L} = \phi^\dagger(r)[i\partial_t - ea_0]\phi(r) - \frac{1}{2m^*} \left| \phi^\dagger(r) \left[ -i\nabla - e(A(r) + a(r)) \right] \phi(r) \right|^2 - \\
- \frac{1}{2} \int dr' V(r - r')[|\phi(r')|^2 - \rho_0][|\phi(r')|^2 - \rho_0] - \frac{1}{2} g\mu_B B \phi^\dagger(r)\sigma_z \phi(r) -
\]

\(^1\) unless explicitly stated, \( \hbar = c = 1 \) units are used.

\(^1\) We do not consider generalized filling fractions.
First of all $\phi = (\phi_1, \phi_2)$ is a two-component complex bosonic field (the “bare electron”). The field $a(r)$ is a gauge field that attaches $\theta = (2k + 1)$ fluxons to the boson, which makes the physical electron. $\rho_0$ is the actual density of electrons (which obviously coincides with the density of bosons). The external magnetic field is described by $A$ ($\nabla \times A = -Be_z$). There are two extra terms, $V(r) = \frac{e^2}{2|r|}$ accounts for the Coulomb repulsion between electrons, and the coupling between the spin and the magnetic field, which involves $g^s$, the gyromagnetic ratio, the Bohr magneton $\mu_B$ and the Pauli matrix $\sigma_z$.

The Landau-Ginzburg ground state is the lowest energy solution of Eq. 2. Let us try the ansatz $\phi = \sqrt{\rho_0}(1,0)$. The equations of motion imply

$$\frac{Ne_l}{A} = \rho_0 = \frac{B \pi}{\Phi_0 \vartheta} = \frac{1}{2k + 1} \frac{N_f}{A}.$$  

So, the optimal situation described above, in which each electron captures an odd number fluxons ($2k + 1$ of them) is the ground state. The filling fraction is then $\nu = \nu_k = \frac{1}{2k + 1}$, the longitudinal resistivity is zero and the transverse conductivity $\sigma_H = \frac{\nu e^2}{2\pi}$. The case $g^s \to 0$ is special, since the solution is not unique as $\phi = \sqrt{\rho_0}(1,0)U$, $U \in SU(2)$ is a solution of the equations of motion. However, the true ground state is ferromagnetic, i.e. corresponds to a unique choice of $U$. The $SU(2)$-symmetry is then spontaneously broken to $U(1)$. The ground state remains spin polarized at $g^s = 0$ as well (although spins do not necessarily point to the same direction of the magnetic field). As a consequence, there are Goldstone modes in the system manifesting in the form of neutral dissipationless spin waves (magnons). But, remarkably, the properties of the ground state are essentially independ of $g^s$.

To go further, let us temporally simplify Eq. 2 and neglect the Coulomb repulsion. Let us add the term

$$-\frac{\lambda}{2}(\phi^\dagger(x)\phi(x) - \rho_0)^2.$$  

If $B$ is the external magnetic field at the ground state, we find solutions to the equations of motion in the form

$$\phi_1(r, \theta) = g_V(r)e^{-i\theta}$$
$$\phi_2(r, \theta) = 0$$
$$a_r(r, \theta) = 0$$
$$a_\theta(r, \theta) = \frac{eB}{2}r + n_V(r)$$
$$a_0(r, \theta) = m_V(r)$$

where $\lim_{r \to \infty} g_V(r) = \sqrt{\rho_0}$, $\lim_{r \to 0} g_V(r) = 0$ and $\lim_{r \to \infty} n_V(r) = \frac{N_{\text{vort}}}{r}$. This solution has finite energy and corresponds to a vortex of vorticity $N_{\text{vort}}$ located
at \( r = 0 \) with a finite core \( \xi \sim \sqrt{\lambda} \). The addition of the term Eq. 5 determines the size of the vortex. The vortex may be regarded as a particle on its own, the pseudo-particle, having electric charge and spin. As an example, we compute the total electric charge in the ground state of a QH liquid in the presence of a vortex. The equations of motions imply

\[
J_{em}^0 = \frac{e}{2\theta} e^{\nu_\sigma} \partial_\sigma \alpha_\sigma
\]  

(6)

So, the total electric charge is

\[
Q = \int_\Sigma d^2r J_{em}^0(r) = \frac{e}{2\theta} \oint_{\partial \Sigma} da_l = -e(N_{el} - \nu_k N_{vort})
\]  

(7)

A vortex has charge \( \frac{e}{2k+1} \). It is then a fractionally charged object. Its spin may be also be computed with result \( \frac{2k+1}{2k+1} \). So, in general, FQHE vortices are anyons.

Now, already at this stage, we can explain some of the phenomenology of the FQHE. Let us assume that the QH liquid is at its ground state. Each fermion is a composite boson with an even number of fluxons attached. If the filling fraction is changed from \( \nu_k = \frac{1}{2k+1} \) by tuning the external magnetic field for example, the statistical gauge field \( a \) screens the excess of magnetic field keeping Eq. 3 unaltered. That is, the density of electrons \( \rho_0 \) remains constant under small variations of the magnetic field. The system is therefore incompressible. Furthermore, as the number of electrons does not change, the area of the sample \( A \) remains constant as well, a property that is usually referred to as area-preserving. At some point, however, the magnetic field is enough to create/destroy a fluxon. The QH-liquid responds creating a vortex/anti-vortex. A vortex (or an antivortex) may change dramatically the properties of the liquid. Being a charged object, the FQHE properties may be destroyed because of the motion of those vortices. However, metals with a sufficient amount of disorder may pin down those vortices, and the properties of the ground state like the longitudinal and transverse conductivity are not changed. That explains the broad plateaus observed. Eventually, more vortices cannot be accommodated in the sample, and the longitudinal conductance grows from its zero value. This corresponds to a transition to the next ground state filling fraction. The CSLG description nicely accounts for the experimental situation in Fig. 1.

The case \( g_s \sim 0 \) is experimentally relevant in some cases such as GaAs samples. We may try solutions to the equations of motion in the form

\[
\phi_1(r, \theta) = g_S^{(1)}(r)e^{-ig}
\]
\[
\phi_2(r, \theta) = g_S^{(2)}(r)
\]
\[
a_r(r, \theta) = 0
\]
\[
a_\theta(r, \theta) = \frac{eB}{2}r + n_S(r)
\]
\[
a_0(r, \theta) = m_S(r)
\]  

(8)
where the asymptotics are the same as in Eq. 8, but in addition, we require
\[ \lim_{r \to 0} g_S^{(2)}(r) \neq 0, \lim_{r \to \infty} g_S^{(2)}(r) = 0. \]
This solution minimizes the Coulomb energy; when the spin up component starts to decrease the spin down increases and the electron density deviations from the ground state value \( \rho_0 \) are much smaller than for the vortex. The spin contribution increases, but it is anyhow negligible since the assumed small gyromagnetic ratio. It should not come as a surprise then, that in this regime, fluctuations in spin have a lower energy than vortices, and become the relevant quasi-particles [7]. those solutions are of a topological nature as well, they are Skyrmions, as we will see.

Although the ground state is essentially independent of \( g^S \), the properties of the pseudo-particles are very different. There is a critical value \( g^S_c \) such that for \( g^S < g^S_c \) those are Skyrmions, while for \( g^S > g^S_c \) they are vortices.

We have introduced the pseudo-particles and explained the crucial role they have in explaining the properties of the QH fluid. From now on, we concentrate on the study of its properties, and more specifically of FQHE Skyrmions. In the case of vortices, this may be accomplished by integrating out the bosonic field, i.e. going to the dual picture [4]. The properties of vortices, such as the statistics or its electric charge, are explicit.

In [1] we have generalized this description to the case of Skyrmions. As it is well known, the spin of Skyrmions may be determined from the coefficient of the Hopf term. We include this term and read the spin from its coefficient. To determine the couplings of the different terms in the Lagrangian we consider a sample with edges. In the presence of edges, there are quantum mechanically induced chiral gapless modes [8]. We shall determine all the properties of FQHE Skyrmions by imposing this chirality condition, together with the fact that Skyrmions may be created and destroyed.

## B The properties of FQHE Skyrmions

Let us write the direction to which the spin field is pointing at in terms of a unitary vector \( \vec{n} \). It is possible to everywhere identify an \( SU(2) \) field degree of freedom \( g \), associated with spin fluctuations from

\[ n_i = \frac{1}{2} \text{Tr}(\sigma_i g^\dagger \sigma_3 g) . \]

There is an \( U(1) \) subgroup, which we arbitrarily take to be generated by the third Pauli matrix \( \sigma_3 \), which is gauged via the coupling to a statistical gauge field (the same gauge field mentioned above), so that the gauge invariant observables are defined on \( S^2 \) (parametrized by \( \vec{n} \)). Energy finiteness generally demands that \( g \) goes to the above \( U(1) \) subgroup, i.e. \( g \to \exp i\chi \sigma_3 \), at spatial infinity. This corresponds to \( \phi^\dagger \phi \) going to the ground state value at spatial infinity, and in effect this compactifies \( \mathbb{R}^2 \) to \( S^2 \). This shows the topological nature of the FQHE Skyrmions.
They are naturally associated with $\Pi_2(S^2)$, the elements of $\Pi_2(S^2)$ being labeled by the winding number

$$N_{\text{Sky}} = \int_{R^2} d^2x \ T^0(g) = \frac{i}{4\pi} \int_{R^2} d \ Tr \sigma_3 g^\dagger dg ,$$

where $T^0(g)$ is the time component of the topological current,

$$T^\mu(g) = \frac{i}{4\pi} \epsilon^{\mu\nu\lambda} \ Tr \sigma_3 \partial_\nu g^\dagger \partial_\lambda g .$$

We first show what are the consequences of not including a Hopf term. Our starting point is the standard bulk action of the dual picture [8], including a coupling to the Skyrmion current

$$S_H = \int_{\Sigma \times R^1} d^3x \left( \frac{\sigma_H}{2} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda - e A_\mu T^\mu \right) ,$$

where $\Sigma$ is the two dimensional spatial domain of the sample, $R^1$ accounts for time, and $eT^\mu$ is the Skyrmion current. Additional terms may be added [1], but we just consider the topological sector that determines the charge and the spin. For us, $A_\mu$ is the external electromagnetic field which is not a dynamical variable. Its variations therefore just define the bulk current $J^\mu_{\text{em}}$ by

$$J^\mu_{\text{em}} = -\frac{\delta S_H}{\delta A_\mu} .$$

According to Eq. 12, the bulk electromagnetic current $J^\mu_{\text{em}}$ is

$$J^\mu_{\text{em}} = -\frac{\sigma_H}{2} \epsilon^{\mu\nu\lambda} F_{\nu\lambda} + eT^\mu .$$

For consistency the current $J^\mu_{\text{em}}$, and consequently $T^\mu$, must be conserved. This is the case for $T^\mu$ proportional to the topological current $T^\mu$:

$$T^\mu = \kappa T^\mu .$$

Eq. 13 implies that the electric charge density is $J^0_{\text{em}} = -\sigma_H F_{12} + e\kappa T^0$. Integrating it over the whole sample gives the total electric charge as $-eN_{el} + e\kappa N_{\text{Sky}}$, where $N_{el}$ is the total number of electrons at the corresponding filling fraction $\nu = \frac{1}{2k+1}$. Thus, $\kappa$ times $e$ is the charge of a Skyrmion of unit winding number.

We now examine under what conditions the bulk action $S_H$ is consistent with the existence of chiral edge currents. For the case of filling fraction $\nu = 1$, there is a single edge current on the boundary $\partial\Sigma$ of $\Sigma$, which may be represented by a 2d massless chiral relativistic Dirac fermion [8], while for fractional values of $\nu$ one gets a Luttinger liquid. Chirality implies that the electromagnetic current $J^\mu_{\text{em}}$ of the edge fermions satisfies $J^\mu_{\text{em}} = \frac{1}{\sqrt{2}} (J^0_{\text{em}} + J^1_{\text{em}}) = 0$. In the quantum theory this is known to lead to an anomaly, i.e. $\partial_\mu J^\mu_{\text{em}} \neq 0$.

It is convenient to bosonize the edge theory [9], and for this we shall introduce a scalar field $\phi$ on $\partial\Sigma$. In terms of this field, chirality will mean the following:

$$D^- \phi = f(x^-) ,$$
where \( x^\pm = \frac{1}{\sqrt{2}}(x^0 \mp x^1) \), \( D_\pm = \frac{1}{\sqrt{2}}(D_0 \pm D_1) \) and \( D_\mu \) denotes a covariant derivative. (Usually the more restrictive condition \( f(x^-) = 0 \) is assumed, but Eq. 15 seems enough for us.)

To proceed we shall pose an action principle for the edge field \( \phi \). The edge action \( S_{\partial \Sigma \times R^1} \) should be such that: i) The total action \( S = S_H + S_{\partial \Sigma \times R^1} \) is gauge invariant. ii) It is consistent with chirality, i.e. Eq. 15. We will show that these two conditions lead to a chiral electromagnetic current \( J_{\text{em}}^\mu = 0 \), which at the boundary is defined by \( J_{\text{em}}^\mu = -\frac{\delta S}{\delta A^\mu} \big|_{\partial \Sigma \times R^1} \). Requirements i) and ii) also lead to the anomaly. For this recall that the one loop effects responsible for the anomaly in the fermionic theory appear at tree level in the bosonized theory. Thus we can expect to recover the anomaly from the classical equation of motion for \( \phi \) [10].

We begin by addressing the issue i) of gauge invariance. If we ignore boundary effects, the bulk action is separately gauge invariant under transformations of the electromagnetic potentials \( A_\mu \),

\[
A_\mu \rightarrow A_\mu + \partial_\mu \Lambda ,
\]

as well as under transformations of the fields \( g \),

\[
g \rightarrow g e^{i\lambda \sigma_3} ,
\]

where both \( \Lambda \) and \( \lambda \) are functions of space-time coordinates. On the other hand, taking into account the boundary \( \partial \Sigma \), one finds instead that Eq. 16 gives the surface terms

\[
\delta S_H = -\frac{\sigma_H}{2} \int_{\partial \Sigma \times R^1} d\Lambda \wedge A + \frac{e\kappa i}{4\pi} \int_{\partial \Sigma \times R^1} d\Lambda \wedge \text{Tr} \sigma_3 g^\dagger dg ,
\]

while gauge invariance under transformations Eq. 17) persists. We now specify that under gauge transformations Eq. 16), the edge field \( \phi \) transforms according to

\[
\phi \rightarrow \phi + e\Lambda .
\]

Then we can cancel both of the above boundary terms in Eq. 18 if we assume the following action for the scalar field \( \phi \):

\[
S_{\partial \Sigma \times R^1} = \frac{R^2}{8\pi} \int_{\partial \Sigma \times R^1} d^2 x (D_\mu \phi)^2 + \frac{\sigma_H}{2e} \int_{\partial \Sigma \times R^1} d\phi \wedge A - \frac{\kappa i}{4\pi} \int_{\partial \Sigma \times R^1} d\phi \wedge \text{Tr} \sigma_3 g^\dagger dg .
\]

In (20) we have added a kinetic energy term for \( \phi \), where the covariant derivative is defined by \( D_\mu \phi = \partial_\mu \phi - eA_\mu \). The coefficient \( R \) is real and is known to correspond to the square root of the filling fraction \( \nu_k \).

Concerning ii), extremizing Eq. 20) with respect to \( \phi \) gives

\[
R^2 \partial_\mu D^\mu \phi = -\frac{2\pi \sigma_H}{e} F_{01} - 4\pi T^r ,
\]
$F_{01}$ being the electric field strength at the boundary and the index $r$ denoting the direction normal to the surface. This equation can be rewritten as

$$2R^2 \partial_+ \mathcal{D}_- \phi = (eR^2 - \frac{2\pi \sigma_H}{e})F_{01} - 4\pi \mathcal{T}^r,$$  

(22)

using $\partial_+ = \frac{1}{\sqrt{2}}(\partial_0 + \partial_1)$ and $\text{diag}(1,-1)$ for the Lorenz metric. But the chirality condition Eq. 15 requires that the right hand side of Eq. 22) vanishes. For this we can set

$$\sigma_H = \frac{\epsilon^2 R^2}{2\pi},$$  

(23)

which is the usual relation for the Hall conductivity (after identifying $R^2$ with the filling fraction $\nu_k$). But we also need

$$\mathcal{T}^r = 0 \text{ at } \partial \Sigma.$$  

(24)

From Eq. 23) and Eq. 24, variations of $A_\mu$ give the following result for the edge current

$$J_{\mu \text{em}}^\mu = -\frac{\delta S}{\delta A_\mu} |_{\partial \Sigma \times R^3} = \frac{eR^2}{4\pi} (\mathcal{D}_\mu + \epsilon^{\mu\nu}\mathcal{D}_\nu) \phi,$$  

(25)

and thus it is chiral, i.e. $J_{\text{em}}^\mu = 0$. Here $\epsilon^{01} = -\epsilon^{10} = 1$. By taking its divergence we also recover the anomaly:

$$\partial_\mu J_{\text{em}}^\mu = -\frac{eR^2}{4\pi} \partial_\mu (\mathcal{D}_\mu + \epsilon^{\mu\nu}\mathcal{D}_\nu) \phi = -\frac{e^2 R^2}{2\pi} F_{01},$$  

(26)

where we again used Eq. 23 and Eq. 24.

In order to satisfy chirality in the above discussion, we needed not only to constrain the values of coefficients, but we also found it necessary to impose a boundary condition Eq. 24 on the topological current. As a result, the topological flux, and moreover Skyrmions, cannot penetrate the edge. Thus, provided $g$ is everywhere defined in $\Sigma$, the total Skyrmion number within the bulk $\int_\Sigma d^2x \mathcal{T}^0(g)$ is a conserved quantity, and for example, a nonzero value for the total topological charge cannot be adiabatically generated from the ground state.

Below, we generalize to the situation where the total Skyrmion number in the bulk is not restricted to being a constant. For this we need to drop the boundary condition Eq. 24, and thus allow for a nonzero topological flux into or out of the sample. One may interpret this as Skyrmions being created or destroyed at the edges. For this purpose, we consider an extension of the above description, where the Hopf term

$$S_{WZ} = \frac{\Theta}{24\pi^2} \int_{\Sigma \times R^3} \text{Tr}(g^\dagger dg)^3$$  

(27)
is added to the bulk action $S_H$. [Note that Eq. 27 is a local version of the Hopf term]. This term does not affect the classical equations of motion since it is the integral of a closed three form. That term gives a nontrivial spin to the Skyrmion

$$\frac{\Theta N_{Sky}}{2\pi}.$$  \hspace{1cm} (28)

We thus need the numerical value of $\Theta$ to determine the spin. For this purpose we now reexamine the boundary dynamics taking into account the Hopf term. We once again require i) gauge invariance and ii) chirality.

Concerning i), as before, the bulk action is not invariant under gauge transformations Eq. 16 of the electromagnetic potentials $A_\mu$. In addition, unlike before, it is not invariant under gauge transformations Eq. 17 of the fields $g$. From $S_{WZ}$ we pick up the surface term

$$\delta S_{WZ} = \frac{i\Theta}{8\pi^2} \int_{\partial\Sigma \times R^1} d\lambda \wedge \text{Tr}\sigma_3 g^\dagger dg.$$ \hspace{1cm} (29)

To cancel this variation along with Eq. 18, we once again assume the existence of an edge field $\phi$ which transforms like Eq. 19, simultaneously with the gauge transformations Eq. 16 of the electromagnetic potentials $A_\mu$. We further specify that $\phi$ transforms according to

$$\phi \rightarrow \phi + \lambda,$$ \hspace{1cm} (30)

simultaneously with the gauge transformations Eq. 17 of the fields $g$. Recall that this transformation is the one compatible with the covariant derivative in Eq. 2. Then we can cancel both of the boundary terms Eq. 18 and Eq. 29, making our theory anomaly free, if we assume the following action for the scalar field $\phi$:

$$S_{\partial\Sigma \times R^1} = \frac{R^2}{8\pi} \int_{\partial\Sigma \times R^1} d^2 x \left( D_\mu \phi \right)^2 + \frac{\sigma H}{2e} \int_{\partial\Sigma \times R^1} d\phi \wedge A$$

$$-\frac{i}{4} \int_{\partial\Sigma \times R^1} \left( \Phi \left( \frac{\sigma H}{2e} \right)^2 d\phi + \frac{\sigma H}{e} A \right) \wedge \text{Tr}\sigma_3 g^\dagger dg,$$ \hspace{1cm} (31)

provided we also impose that

$$\kappa = \frac{\pi \sigma H}{e^2} + \frac{\Theta}{2\pi}.$$ \hspace{1cm} (32)

Since $\phi$ admits gauge transformations Eq. 30, as well as Eq. 19, we must redefine the covariant derivative appearing in Eq. 31 according to

$$D_\mu \phi = \partial_\mu \phi - \beta_\mu,$$

$$\beta_\mu = e A_\mu - \frac{i}{2} \text{Tr}\sigma_3 g^\dagger \partial_\mu g.$$ \hspace{1cm} (33)

With regard to ii), the equation of motion for $\phi$ is
\[ R^2 \partial_\mu D^\mu \phi = -\frac{2\pi \sigma_H}{e} F_{01} - 2\Theta T^r, \]  

(34)

which can be rewritten as

\[ 2R^2 \partial_\mu D^\mu \phi = (eR^2 - \frac{2\pi \sigma_H}{e}) F_{01} + 2(\pi R^2 - \Theta) T^r. \]  

(35)

We recover the chirality condition Eq. 15 upon setting

\[ \Theta = \pi R^2, \]  

(36)

as well as (23). From Eq. 23) and Eq. 36), variations of \( A_\mu \) again give the edge current in the form of Eq. 25 (although the covariant derivative is now defined differently) and thus \( J^e_\mu \) = 0. By taking its divergence we get the anomaly equation:

\[ \partial_\mu J^e_\mu = \frac{eR^2}{2\pi} \varepsilon_{\mu\nu} \partial_\nu \beta_\nu. \]  

(37)

Thus now we can satisfy the criterion of chirality without imposing any boundary conditions on the topological current. Substituting Eq. 36) into Eq. 32 (and using \( R^2 = \nu_k \)) also fixes \( \kappa \) to be the filling fraction. It follows that the Skyrmion charge is \( e\nu_k N_{Sky} \). Eqs. 28, 36 then give the value for the spin to be \( \frac{N_{Sky} \nu_k}{2} \). Therefore, within the above assumptions, a winding number one Skyrmion is a fermion when the filling fraction is one.

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REFERENCES

1. S. Baez, A.P. Balachandran, A. Stern and A. Travesset, \textit{cond-mat} 9712151.
2. R.E. Prange and S.M. Girvin, \textit{The Quantum Hall effect}, Springer-Verlag, New York (1990) and references therein;
3. S. C. Zhang, H. Hansson and S. Kivelson, Phys. Rev. Lett. \textbf{62}, 82 (1989); \textbf{62}, 980 (1989); D. H. Lee and S. C. Zhang, Phys. Rev. Lett. \textbf{66}, 1220 (1991).
4. S. C. Zhang, Int. J. of Mod. Phys. \textbf{B} 6, 25 (1992) and references therein.
5. J.M. Leinaas and S. Viefers, \textit{cond-mat} 9712009.
6. R. Ray, This proceedings.
7. S.L. Sondhi, A. Karlhede, S.A. Kivelson and E.H. Rezayi, Phys. Rev. \textbf{B} 47, 16419 (1993).
8. X.Wen, Adv. In Phys. 44 (1995) 405-473 and references therein.
9. A.P. Balachandran, L. Chandar and B. Sathiapalan, Nucl. Phys. \textbf{B} 443, 465 (1996); Int. J. of Mod. Phys. \textbf{A} 11, 3587 (1996).
10. C.G. Callan Jr. and J.A. Harvey, Nucl. Phys. B 250 (1985) 427; S.G. Naculich, Nucl. Phys. B 296 (1988) 837.