Semileptonic decays of $B_{s1}$, $B_{s2}^*$, $B_{s0}$ and $B'_{s1}$

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Stimulated by recent observations of the excited bottom-strange mesons $B_{s1}$ and $B_{s2}^*$, we calculate the semileptonic decays $B_{s0}$, $B_{s1}$, $B_{s2}^* \rightarrow [D_s(1968), D_s^*(2112), D_{sJ}(2317), D_{sJ}(2460)] \ell \nu$, which is relevant for the exploration of the potential of searching these semileptonic decays in experiment.

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I. INTRODUCTION

Recently, the CDF collaboration announced the observation of two orbitally excited narrow $B_s$ mesons. Their masses are $m_{B_{s1}} = 5829.4 \pm 0.7$ MeV and $m_{B_{s2}^*} = 5839.6 \pm 0.7$ MeV.\textsuperscript{1} Later the D0 collaboration confirmed the $B_{s2}^*$ state with $m_{B_{s2}^*} = 5839.6 \pm 1.1$ (stat.) $\pm 0.7$ (syst.) MeV.\textsuperscript{2} Meanwhile, the D0 collaboration indicated that $B_{s1}$ was not observed with the available data set.\textsuperscript{2} In the heavy quark effective theory (HQET), the charmed or bottom mesons can be categorized into doublets since the angular momentum of the light components is a good quantum number in the $m_Q \rightarrow \infty$ limit. They are $j^P_l = \frac{1}{2}^-$, the $H$ doublet ($0^-, 1^-$) with the orbital angular momentum $L = 0$, $j^P_l = \frac{3}{2}^-$, the $S$ doublet ($0^+, 1^+$) and $j^P_l = \frac{1}{2}^+$, the $T$ doublet ($1^+, 2^+$) with $L = 1$. The recently observed $B_{s1}$ and $B_{s2}^*$ mesons belong to the $T$ doublet.\textsuperscript{1,2}

The observations of $B_{s1}$ and $B_{s2}^*$ not only enrich the mass spectrum of the bottom-strange system\textsuperscript{1} but also inspire our interest in these two unobserved P-wave bottom-strange mesons. Since $B_{s1}$ and $B_{s2}^*$ lie above the thresholds of $B K$ and $B^* K$, their strong decay is dominant. In our recent work,\textsuperscript{4} we calculated the strong decays of $B_{s1}$ and $B_{s2}^*$ with the $P_0$ model. Our result indicates that the two-body strong decay widths of $B_{s1}$ and $B_{s2}^*$ can reach up to 98 keV and 5 MeV, respectively. In contrast, $B_{s0}$ and $B_{s1}^*$ were generally speculated to lie below the threshold of $B K$ and $B^* K$ in Ref.\textsuperscript{3}. In fact, they are expected to be narrow resonances with a width around several tens of keV\textsuperscript{6} since their main decay modes are the isospin-violating strong decays and electromagnetic decays. These two states are still missing in the experiments. Up to now, they have not been observed experimentally.

The semileptonic decays of $B_{s0}$, $B_{s1}$, $B_{s2}^*$, $B_{s0}$ and $B_{s1}^*$ have not been explored. In this work, we will calculate the semileptonic decays $B_{s0}$, $B_{s1}$, $B_{s2}^*$, $B_{s0}$ and $B_{s1}^*$ in future experiments such as CDF, D0 and the forthcoming Large Hadron Collider beauty (LHCb). If these semileptonic decays can be reached by the future experiments, one may compare the bottom-strange mesons with the conventional charmed ones. The observations of the above narrow charm-strange mesons have resulted in an extensive study of their properties in the past five years.\textsuperscript{2}

This work is organized as follows. After the introduction, we briefly introduce the theoretical framework, i.e. the QCD sum rule (QSR) approach, the mass of $D_{sJ}(2460)$ agrees well with the experimental value.\textsuperscript{16} Therefore, $D_{sJ}(2317)$ and $D_{sJ}(2460)$ are very probably conventional $c \bar{s}$ states with $J^P = 0^+$ and $J^P = 1^+$.\textsuperscript{16}

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\textsuperscript{1} Besides $B_{s1}$ and $B_{s2}^*$ observed by CDF and D0, there are only two established bottom-strange states ($B_s$ and $B_s^*$) listed in Particle Data group (PDG) so far.\textsuperscript{3}
\textsuperscript{2} $D_{sJ}(2317)$ and $D_{sJ}(2460)$ with spin parity structures $J^P = 0^+$ and $J^P = 1^+$,\textsuperscript{7,8,9} have inspired heated debates about their structures. A detailed review can be found in Ref.\textsuperscript{10}. Possible interpretations include the $(0^+, 1^+)$ chiral partners of $D_s$ and $D_s^*$,\textsuperscript{11} P-wave excited states of $D_s$ and $D_s^*$,\textsuperscript{12} coupled-channel effects between $c\bar{s}$ states and the $DK$ continuum,\textsuperscript{13} conventional $c\bar{s}$ states\textsuperscript{14,15}, four-quark states\textsuperscript{16,17} etc. Considering the large contribution of the S-wave $DK$ continuum in the QCD sum rule (QSR) approach, the mass of $D_{sJ}(2317)$ agrees well with the experimental value.\textsuperscript{16} Therefore, $D_{sJ}(2317)$ and $D_{sJ}(2460)$ are very probably conventional $c\bar{s}$ states with $J^P = 0^+$ and $J^P = 1^+$.\textsuperscript{16}
II. THE CQM MODEL

We first give a brief review of the CQM model [20, 21]. The effective Lagrangian of the CQM model incorporates both the heavy-quark spin-flavor symmetry and the chiral symmetry [20]

\[ \mathcal{L}_{CQM} = \bar{\chi}[\gamma \cdot (i \partial + V)] \chi + \tilde{\chi} \gamma \cdot A \gamma_5 \chi - m_q \tilde{\chi} \chi + \frac{f_s^2}{8} Tr[\partial^\mu \Sigma \partial_\mu \Sigma^\dagger] + \bar{h}_v (iv \cdot \partial) h_v \\
- \left[ \bar{\chi} (\bar{H} + \bar{S} + i \bar{\mu} \partial_\mu) h_v + h.c. \right] + \frac{1}{2G_3} Tr[(\bar{H} + \bar{S})(H - S)] + \frac{1}{2G_4} Tr[\bar{T}^\mu T_\mu]. \]

(1)

Here \( H, S \) and \( T \) denote the super-fields corresponding to the \((0^-, 1^-), (0^+, 1^+)\) and \((1^+, 2^+)\) doublets respectively, whose explicit matrix representations are [22]

\[ H = \frac{1 + \gamma^5}{2} [P^s \gamma^\mu - P \gamma^5], \]

(2)

\[ S = \frac{1 + \gamma^5}{2} [P^s \gamma^\mu \gamma_5 - P_0], \]

(3)

\[ T^\mu = \frac{1 + \gamma^5}{2} \left\{ P_2^{\mu \nu} \gamma_\nu - \sqrt{\frac{3}{2}} P_1^{\mu \nu} \left[ g^{\mu \nu} - \frac{1}{3} \gamma^\nu (\gamma^\mu - \gamma^5) \right] \right\}. \]

(4)

\( P, P^s \), \( P_0 \) and \( P_1^{\mu \nu} \) correspond to the annihilation operators of the pseudoscalar, vector, scalar and axial vector mesons respectively. They are normalized as

\[ \langle 0 | P | M(0^-) \rangle = \sqrt{M_H}, \quad \langle 0 | P^s | M(1^-) \rangle = \sqrt{M_S}, \quad \langle 0 | P_0 | M(0^+) \rangle = \sqrt{M_S}, \quad \langle 0 | P_1^{\mu \nu} | M(1^+) \rangle = \sqrt{M_T}, \]

\[ \langle 0 | P_2^{\mu \nu} | M(2^+) \rangle = \sqrt{M_T}. \]

In Eq. (1), the fifth term represents the kinetic term of heavy quarks with \( h_v = h_\nu \). \( \chi = \xi q(q = u, d, s) \) denotes the light-quark field with \( \xi = e^{\frac{i q}{2}} \) and \( M \) is the octet pseudoscalar matrix. \( V^\mu \) and \( A^\mu \) are defined as

\[ V^\mu = \frac{1}{2} (\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger), \]

(5)

\[ A^\mu = \frac{-i}{2} (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger). \]

(6)

An important feature of the effective Lagrangian in Eq. (1) is that \( \mathcal{L}_{CQM} \) describes the interaction vertex of the heavy-meson with heavy and light quarks, which makes the study of the phenomenology of heavy-meson physics at the quark level feasible. The CQM model has been applied to study the heavy meson phenomenology [23, 24, 25, 26, 27]. In Ref. [27], the semileptonic decays \( B_s \rightarrow D_{sJ}(2317, 2460) \ell \bar{\nu} \) have been studied in the CQM model. In this work, we extend the same formalism to the semileptonic decays of \( B_{s1} \) and \( B_{s2}^* \). The interested readers may also consult the review paper of the CQM model in [20].

III. THE SEMILEPTONIC DECAYS OF \( B_{s1}, B_{s2}^*, B_{s0} \) AND \( B_{s4}^* \)

In this section, we calculate the semileptonic decays of \( B_{s1}, B_{s2}^*, B_{s0} \) and \( B_{s4}^* \) in the CQM model. We are interested in the semileptonic decay modes of \( B_{s1}, B_{s2}^*, B_{s0} \) and \( B_{s4}^* \) including \( D_{sJ}(1968)\ell \bar{\nu}, D_{sJ}^*(2112)\ell \bar{\nu}, D_{sJ}(2317)\ell \bar{\nu} \) and \( D_{s}(2460)\ell \bar{\nu} \), which are depicted in Fig. (4).

The four-fermion operator describing \( b \rightarrow c + \ell \bar{\nu} \) is

\[ \mathcal{O} = \frac{G_F V_{cb}}{\sqrt{2}} \bar{\ell} \gamma^\mu (1 - \gamma_5) b \gamma_\mu (1 - \gamma_5) \ell. \]

(7)
Thus the general decay amplitudes of \( B_{s1}(B_{s2}^*) \rightarrow D_s(D_s, D_{sJ}(2317, 2460)) \ell \bar{\nu} \) read
\[
M[(b \bar{s}) \rightarrow (c \bar{s}) + \ell \bar{\nu}] = \frac{G_F V_{cb}}{\sqrt{2}} \langle (c \bar{s}) | \bar{c} \gamma^\mu (1 - \gamma_5) b | (b \bar{s}) \rangle \langle \ell \bar{\nu} | \bar{\nu} \gamma_\mu (1 - \gamma_5) \ell | 0 \rangle,
\]
where we use \((c \bar{s})\) and \((b \bar{s})\) to denote the charm-strange and bottom-strange mesons, respectively. The hadronic matrix element \( \langle (c \bar{s}) | \bar{c} \gamma^\mu (1 - \gamma_5) b | (b \bar{s}) \rangle \) has to be calculated using phenomenological models, such as QSR and the CQM model. Here we adopt the CQM model to calculate the hadronic matrix element. Figure 1 illustrates the semileptonic decays \( B_{s1}(B_{s2}^*) \rightarrow D_s \ell \bar{\nu} \) in the CQM model.

| Interaction | Vertex |
|-------------|--------|
| \( b - B_{s1} - \bar{s} \) | \( \langle \bar{s} \rangle \sqrt{\frac{\pi}{2}} \sqrt{m_{B_{s1}} Z_T} \frac{1 + \gamma_5}{2} \{ \frac{\gamma \cdot k}{\pi} - \frac{1}{2}(\gamma \cdot \nu \cdot k) \} \) |
| \( b - B_{s2}^* - \bar{s} \) | \( - \frac{1}{2} \sqrt{m_{B_{s2}^*} Z_T} \frac{1 + \gamma_5}{2} \{ \frac{\gamma \cdot p}{\pi} \} \) |
| \( b - B_{s0} - \bar{s} \) | \( i \sqrt{m_{B_{s0}} Z_S} \frac{1 + \gamma_5}{2} \{ \frac{\gamma \cdot \nu}{\pi} \} \) |
| \( b - B_{s1}' - \bar{s} \) | \( -i \sqrt{m_{B_{s1}'} Z_S} \frac{1 + \gamma_5}{2} \{ \frac{\gamma \cdot \nu}{\pi} \} \) |
| \( c - D_s - \bar{s} \) | \( -i \sqrt{m_{D_s} Z_H} \frac{1 + \gamma_5}{2} \{ \frac{\gamma \cdot \nu}{\pi} \} \) |
| \( c - D_{sJ} - \bar{s} \) | \( -i \sqrt{m_{D_{sJ}} Z_H} \frac{1 + \gamma_5}{2} \{ \frac{\gamma \cdot \nu}{\pi} \} \) |
| \( c - D_{s0} - \bar{s} \) | \( i \sqrt{m_{D_{s0}} Z_S} \frac{1 + \gamma_5}{2} \{ \frac{\gamma \cdot \nu}{\pi} \} \) |
| \( c - D_{s1}' - \bar{s} \) | \( -i \sqrt{m_{D_{s1}'} Z_S} \frac{1 + \gamma_5}{2} \{ \frac{\gamma \cdot \nu}{\pi} \} \) |

TABLE I: The interaction vertex for the semileptonic decays of \( B_{s1}, B_{s2}^*, B_{s0} \) and \( B_{s1}' \).

According to Eq. 11, we give the interaction vertices in Table I which are related to the semileptonic decays of \( B_{s1}, B_{s2}^*, B_{s0} \) and \( B_{s1}' \). Here \( \epsilon, \eta, \epsilon_1 \) and \( \epsilon_2 \) denote the polarization vectors of the heavy mesons. Note that we use \( D_{s0} \) and \( D_{s1}' \) to denote \( D_{sJ}(2317) \) and \( D_{sJ}(2460) \) respectively. The normalization constants \( Z_{H,S,T} \) are given in Ref. 20:

\[
Z_H^{-1} = (\Delta_H + m_s) \frac{\partial I_3(\Delta_H)}{\partial \Delta_H} + I_3(\Delta_H),
\]

\[
Z_S^{-1} = (\Delta_S - m_s) \frac{\partial I_3(\Delta_S)}{\partial \Delta_S} + I_3(\Delta_S),
\]

\[
Z_T^{-1} = \frac{1}{3\Delta^2} \left\{ (\Delta_T^2 - m_s^2) I_3(\Delta_T) + (m_s + \Delta_T) \frac{\partial I_3(\Delta_T)}{\partial \Delta_T} \right\} + (m_s + \Delta_T) I_1 + 2 \Delta_T I_3(\Delta_T) + \frac{\partial I_0(\Delta_T)}{\partial \Delta_T} + I_0(\Delta_T) + \Delta_T I_1
\]

with the definitions of \( I_{0,1,3} \) listed in the appendix.
In terms of the above expressions, we can write the general expression of the hadronic matrix element \((\langle c \bar{s} | \bar{c} \gamma^\mu (1 - \gamma_5) b | (b \bar{s}) \rangle)\) in the CQM model

\[
\langle c \bar{s} | \bar{c} \gamma^\mu (1 - \gamma_5) b | (b \bar{s}) \rangle = -i N_c \frac{16 \pi^4}{T} \int d^4 l \frac{Tr [\gamma^\mu (1 - \gamma_5) \Gamma_a (\gamma \cdot l + m_a) \Gamma_b]}{(l^2 - m_b^2) (v \cdot l + \alpha) (v' \cdot l + \beta)},
\]

(12)

where \(\Gamma_a\) corresponds to the vertex of the interaction of \(B_{s1}(B_{s2}^*, B_{s0}, B_{s1}^*)\) with \(b\) and \(\bar{s}\). \(\Gamma_b\) is the vertex describing the interaction of charm-strange meson with \(c\) and \(\bar{s}\) quarks. \(N_c = 3\). \(\alpha(\beta) = \Delta_{H,S,T}\) denotes the mass difference between the heavy mesons and the heavy quark \(21\).

In the following, by substituting \(\Gamma_a\) and \(\Gamma_b\) with the expression listed in Table I, one obtains the hadron matrix elements relevant to the semileptonic decays of \(B_{s1}\) and \(B_{s2}^*\) with the transitions of \(B_{s1}(B_{s2}^* \rightarrow D_{s0}(D'_{s1})\):

\[
\langle D_{s0}(v') | \gamma^\mu (1 - \gamma_5) | B_{s1}(v, \bar{e}) \rangle = \sqrt{m_{B_{s0}} m_{D_{s0}}} \zeta(\omega) \left[ f_{1 \bar{e}} e_{1 v} + f_{2 \bar{e}} e_{2 v} + h_{1} (\bar{e}_{1} \cdot \bar{e}) e_{1 v} + h_{2} (\bar{e}_{2} \cdot \bar{e}) e_{2 v} + i h_{3} \varepsilon^{\mu \nu \lambda \rho} e_{25} v_{\lambda} \varepsilon_{\rho} + i h_{4} \varepsilon^{\mu \nu \lambda \rho} e_{28} v_{\lambda} \varepsilon_{\rho} \right],
\]

(13)

\[
\langle D'_{s1}(v', \bar{e}_{2}) | \gamma^\mu (1 - \gamma_5) | B_{s1}(v, \bar{e}) \rangle = \sqrt{m_{D_{s1}} m_{D'_{s1}}} \zeta(\omega) \left[ f_{1 \bar{e}} e_{1 v} + f_{2 \bar{e}} e_{2 v} + h_{1} (\bar{e}_{1} \cdot \bar{e}) e_{1 v} + h_{2} (\bar{e}_{2} \cdot \bar{e}) e_{2 v} + i h_{3} \varepsilon^{\mu \nu \lambda \rho} e_{25} v_{\lambda} \varepsilon_{\rho} + i h_{4} \varepsilon^{\mu \nu \lambda \rho} e_{28} v_{\lambda} \varepsilon_{\rho} \right],
\]

(14)

for the semileptonic decays of \(B_{s1}\),

\[
\langle D_{s0}(v') | \gamma^\mu (1 - \gamma_5) | B_{s2}^*(v, \bar{e}) \rangle = \sqrt{m_{B_{s0}} m_{D_{s0}}} \zeta(\omega) \left[ f_{1 \bar{e}} e_{1 v} + f_{2 \bar{e}} e_{2 v} + h_{1} (\bar{e}_{1} \cdot \bar{e}) e_{1 v} + h_{2} (\bar{e}_{2} \cdot \bar{e}) e_{2 v} + i h_{3} \varepsilon^{\mu \nu \lambda \rho} e_{25} v_{\lambda} \varepsilon_{\rho} + i h_{4} \varepsilon^{\mu \nu \lambda \rho} e_{28} v_{\lambda} \varepsilon_{\rho} \right] \eta_{a \beta},
\]

(15)

\[
\langle D'_{s1}(v', \bar{e}_{2}) | \gamma^\mu (1 - \gamma_5) | B_{s2}^*(v, \bar{e}) \rangle = \sqrt{m_{D_{s1}} m_{D'_{s1}}} \zeta(\omega) \left[ f_{1 \bar{e}} e_{1 v} + f_{2 \bar{e}} e_{2 v} + h_{1} (\bar{e}_{1} \cdot \bar{e}) e_{1 v} + h_{2} (\bar{e}_{2} \cdot \bar{e}) e_{2 v} + i h_{3} \varepsilon^{\mu \nu \lambda \rho} e_{25} v_{\lambda} \varepsilon_{\rho} + i h_{4} \varepsilon^{\mu \nu \lambda \rho} e_{28} v_{\lambda} \varepsilon_{\rho} \right] \eta_{a \beta},
\]

(16)

for the semileptonic decays of \(B_{s2}^*\). Here \(\omega = v \cdot v'\).

| \(B_{s1}\) | \(\sqrt{6} f_1\) | \(\sqrt{6} f_2\) | \(\sqrt{6} h_1\) | \(\sqrt{6} h_2\) | \(\sqrt{6} h'_{1}\) | \(\sqrt{6} h'_{2}\) | \(\sqrt{6} h'_{3}\) | \(\sqrt{6} h'_{4}\) | \(\sqrt{6} h'_{5}\)
|---|---|---|---|---|---|---|---|---|
| \(D_s\) | \(- (\omega^2 - 1)\) | - | - | \(- \omega - 2\) | -3 | - | \(- (\omega + 1)\) | - | -
| \(D_{s0}\) | \(- (\omega^2 - 1)\) | - | - | \(- \omega + 2\) | -3 | - | \(- (\omega - 1)\) | - | -
| \(D_{s1}\) | - | \(\omega + 1\) | 2(\(\omega + 1\)) | \(- (\omega + 1)\) | \(\omega + 1\) | -3 | \(\omega + 1\) | \(- (\omega + 1)\) | 3
| \(D'_{s1}\) | - | \(- (\omega - 1)\) | 2(\(\omega - 1\)) | \(\omega - 1\) | \(- \omega - 1\) | -3 | \(- (\omega - 1)\) | \(- (\omega - 1)\) | 3

**Table II:** The coefficients relevant to the transitions of \(B_{s1} \rightarrow D_s, D_{s0}, D_{s1}\).
$B_{s1}(B_{s2}^*) \to D_{s0}(D_{s1}')$, should be replaced by $\xi(\omega)$ for $B_{s1}(B_{s2}^*) \to D_s(D_s^*)$. In the above expressions of the hadron element matrices, there exist two independent form factors $\xi(\omega)$ and $\zeta(\omega)$:

$$\xi(\omega) = \xi(\omega) = \Lambda^{-1} \sqrt{Z_T Z_H} \left[ A_2 + A_3 - B_2 m_s \right]_{\alpha=\Delta^T, \beta=\Delta_H},$$

$$\zeta(\omega) = \zeta(\omega) = \Lambda^{-1} \sqrt{Z_T Z_{\bar{Z}}} \left[ A_2 - A_3 + B_2 m_s \right]_{\alpha=\Delta^T, \beta=\Delta_S}. \tag{17}$$

Equations (13)-(16) are consistent with the heavy-quark spin-flavor symmetry. The expressions of $A_2$, $A_3$, and $B_2$ are given in the appendix. Finally, the differential rates of the semileptonic decays of $B_{s1}$ and $B_{s2}$ are

$$\frac{d\Gamma}{d\omega}(B_{s1} \to D_{sJ} \ell \bar{\nu}) = \frac{G_F^2 |V_{cb}|^2}{(2J + 1)4\pi} m_{D_{sJ}} r^2 (\omega^2 - 1)^{1/2} \Theta[B_{sJ}, D_{sJ'}], \tag{19}$$

with

$$\Theta[B_{sJ}, D_{sJ'}] = (q_{\mu} q_{\nu} - q^2 g_{\mu\nu}) W_{\mu\nu}^{\ell \bar{\nu}}(D_{sJ'}), \tag{20}$$

where

$$W_{\mu\nu}^{\ell \bar{\nu}}(D_{sJ'}) = \sum_{spins} \langle D_{sJ'}| \gamma^\mu (1 - \gamma_5) |B_{sJ} \rangle \langle B_{sJ}| \gamma^\nu (1 - \gamma_5) |D_{sJ'} \rangle^*, \tag{21}$$

and $r = m_{D_{sJ'}}/m_{D_{sJ}}$. The allowed integral range for $\omega$ is

$$0 \leq \omega - 1 \leq \frac{(m_{D_{sJ}} - m_{D_{sJ'}})^2}{2m_{D_{sJ}} m_{D_{sJ'}}}. \tag{22}$$

For the processes of $B_{s1} \to D_s \ell \bar{\nu}$, $B_{s1} \to D_s^* \ell \bar{\nu}$, $B_{s1} \to D_{s0} \ell \bar{\nu}$ and $B_{s1} \to D_{s1}' \ell \bar{\nu}$, the expressions of $\Theta[B_{s1}, D_{sJ'}]$ are respectively

$$\Theta[B_{s1}, D_{s0}(D_s)] = m_{D_{sJ}}^4 r |\xi(\omega)|^2 \left\{ [(r^2 - 1)f + (\omega^2 - 1)(r h_1 + h_2)]^2 + 2(r^2 - 2r + 1)[f^2 + (\omega^2 - 1)h_3^2] \right\}, \tag{23}$$

$$\Theta[B_{s1}, D_{s1}'(D_s^*)] = m_{D_{sJ}}^4 r |\xi(\omega)|^2 \left\{ (\omega^2 - 1)\left\{[(r^2 - 1)f + (r - \omega)f_2 + \omega(r h_1 + h_2) + (\omega^2 - 1)h_3^2] + 2(r h_1 + h_2)^2 \right. \right.

$$+2(1 + r^2 - 2r \omega)(f_2^2 + f_2^2) \right\} + \left. 2[(r^2 - 1)h_3 + (r - \omega) h_4]^2 + 2(1 + r^2 - 2r \omega)\left\{(\omega h_3 + h_4)^2 \right. \right.

$$\left. +[h_3 + \omega h_4 + (\omega^2 - 1)h_2]^2 \right\}. \tag{24}$$

For the semileptonic decays of $B_{s2}^*$, the $\Theta[B_{s2}, D_{sJ'}]$ read

$$\Theta[B_{s2}, D_{s0}(D_s)] = m_{D_{sJ}}^4 r |\xi(\omega)|^2 (\omega^2 - 1) \left\{ \frac{2}{3} \left[(r^2 - 1)f + r(\omega^2 - 1)g \right]^2 + (1 + r^2 - 2r \omega)[f^2 + (\omega^2 - 1)h_2^2] \right\}, \tag{25}$$

| $B_{s2}^* \to$ | $f$ | $f_1$ | $f_2$ | $g$ | $h$ | $h_1$ | $h_2$ |
|----------------|---------|---------|---------|---------|---------|---------|---------|
| $D_{s0}$ | $\omega + 1$ | - | - | -1 | 1 | - | - |
| $D_{s0}$ | $\omega - 1$ | - | - | -1 | 1 | - | - |
| $D_{s1}$ | -1 | 1 | 1 | -1 | - | 1 | 1 |
| $D_{s1}$ | -1 | -1 | 1 | 1 | - | -1 | 1 |

**TABLE III:** The coefficients relevant to the transitions of $B_{s2}^* \to D_s, D_s^*, D_{s0}, D_{s1}$. 
\begin{equation}
\Theta[B_{s0}, D_{s1}^{*}(D_{s}^{*})] = m_{B_{s0}}^{4} r |\zeta(\omega)|^2 (\omega^2 - 1) \left\{ (\omega^2 - 1) \left[ \frac{2}{3} (r - \omega) f + \omega (r f_1 + f_2) + (r \omega - 1) g \right]^2 + (rf_1 + f_2)^2 \right. \\
+ (rh_1 + h_2)^2 + (1 + r^2 - 2r \omega) \left[ \frac{4}{3} f^2 + g^2 + h_1^2 - 2h_2^2 \right] \right\} + \frac{10}{3} (1 + r^2 - 2r \omega)(h_1 + \omega h_2)^2 \right\}.
\end{equation}

| $B_{s0}$ | $f$ | $h_1$ | $h_2$ | $h_3$ |
|---------|-----|------|------|------|
| $D_s$ | $\omega - 1$ | - | -1 | 0 |
| $D_{s0}$ | $\omega + 1$ | - | 1 | 0 |
| $D_{s1}^{*}$ | $\omega - 1$ | -1 | 1 | -1 |
| $D_{s1}'$ | - | -1 | 1 | 0 |

**TABLE IV:** The coefficients relevant to the transitions of $B_{s0} \to D_s, D_{s0}^{*}, D_{s0}, D_{s1}'$.

| $B_{s1}'$ | $f$ | $f_1$ | $f_2$ | $h_1$ | $h_2$ | $h_3$ | $h_4$ |
|---------|-----|------|------|------|------|------|------|
| $D_s$ | $\omega - 1$ | - | -1 | 0 | - | 1 |
| $D_{s0}$ | $\omega + 1$ | - | 1 | 0 | - | 1 |
| $D_{s1}^{*}$ | $\omega - 1$ | -1 | 1 | -1 |
| $D_{s1}'$ | - | -1 | 1 | 0 |

**TABLE V:** The coefficients relevant to the transitions of $B_{s1}' \to D_s, D_{s0}, D_{s0}, D_{s1}'$.

In this work, we also calculate the semileptonic decays of the $B_{s0}$ and $B_{s1}'$ mesons in the $S$ doublet, i.e. $B_{s0}(B_{s1}') \to D_s(D_{s}^{*}, D_{s0}, D_{s1}') \ell \bar{\nu}$. For $B_{s0} \to D_s(D_{s}^{*}, D_{s0}, D_{s1}') \ell \bar{\nu}$, we have

\begin{equation}
\Theta[B_{s0}, D_{s0}] = m_{B_{s0}}^{4} r |\chi(\omega)|^2 (\omega^2 - 1)(rh_1 + h_2)^2,
\end{equation}

\begin{equation}
\Theta[B_{s0}, D_{s1}^{*}] = m_{B_{s0}}^{4} r |\chi(\omega)|^2 \left\{ (\omega^2 - 1) f + (\omega^2 - 1)(rh_1 + h_2)^2 + 2(r^2 - 2r \omega + 1)[f^2 + (\omega^2 - 1)h_3^2] \right\},
\end{equation}

where

\begin{equation}
\chi(\omega) = Z_S [B_1 + B_2 + m_s]_{\alpha=\Delta_S, \beta=\Delta_S}.
\end{equation}

The $\Theta[B_{s1}', D_{s0}]$ and $\Theta[B_{s1}', D_{s1}]$ functions for the $B_{s1}' \to D_{s0}(D_{s}^{*}, D_{s0}, D_{s1}') \ell \bar{\nu}$ decays are similar to Eqs. (23) - (24), where $\zeta(\omega)$ has to be replaced by the new form factor $\chi(\omega)$. The corresponding parameters and coefficients are listed in Tables IV and V.

For $B_{s0} \to D_s(D_{s}^{*}) \ell \bar{\nu}$, $\Theta[B_{s0}, D_s]$ and $\Theta[B_{s0}, D_{s}^{*}]$ can be obtained after replacing $\chi(\omega)$ by $\lambda(\omega)$ in Eq. (27) - (28), where

\begin{equation}
\lambda(\omega) = \sqrt{Z_H Z_S} [B_1 - B_2 + m_s]_{\alpha=\Delta_H, \beta=\Delta_H}.
\end{equation}

Similarly, one gets the functions $\Theta[B_{s1}', D_s]$ and $\Theta[B_{s1}', D_{s}^{*}]$ for $B_{s1}' \to D_s(D_{s}^{*}) \ell \bar{\nu}$ with the replacement $\zeta(\omega) \to \lambda(\omega)$ in Eqs. (23) - (24).

**IV. NUMERICAL RESULTS**

We now collect the input parameters: $G_F = 1.1664 \times 10^{-5}$ GeV$^{-2}$, $V_{cb} = 0.043$, $M_{B_{s1}} = 5829.4$ MeV, $M_{B_{s0}} = 5839.6$ MeV, $M_{D_{s}} = 1968.2$ MeV, $M_{D_{s}^{*}} = 2112.0$ MeV, $M_{D_{s0}}(2317) = 2317.3$ MeV, $M_{D_{s1}}(2460) = 2458.9$ MeV, $M_{B_{s0}} = 5718$ MeV and $M_{B_{s1}} = 5765$ MeV, $m_s = 0.5$ GeV, $\Lambda = 1.25$ GeV, the infrared cutoff $\mu = 0.593$ GeV and $\Delta_S = \Delta_H = 335 \pm 35$ MeV.

In Table IV, we give the ranges of $\Delta_{H,S,T}$ for the strange sector, which are given in Ref. [27, 28]. In terms of the definitions of $Z_{H,S,T}$ in Eqs. (29) - (31), we obtain the values of $Z_{H,S,T}$ listed in Table VI.
TABLE VI: The values of $\Delta_{H,S,T}$ and the corresponding $Z_{H,S,T}$. Here $\Delta_{H,S,T}$ and $Z_{H,S,T}$ are in units of GeV and GeV$^{-1}$, respectively.

| Type | $\Delta_H$ | $\Delta_S$ | $\Delta_T$ | $Z_H$ | $Z_S$ | $Z_T$ |
|------|------------|------------|------------|-------|-------|-------|
| (a)  | 0.5        | 0.86       | 0.84       | 4.87  | 2.95  | 3.26  |
| (b)  | 0.6        | 0.91       | 0.94       | 3.45  | 2.28  | 1.91  |
| (c)  | 0.7        | 0.97       | 1.04       | 2.37  | 1.66  | 1.06  |

FIG. 2: (a) The dependence of $\xi(\omega)$ on $\omega$. Here the solid, dashed and dotted lines correspond to the results with parameters $(\Delta_T = 0.84, \Delta_H = 0.5)$, $(\Delta_T = 0.94, \Delta_H = 0.6)$, $(\Delta_T = 1.04, \Delta_H = 0.7)$, respectively. (b) The variation of $\zeta(\omega)$ with $\omega$. Here the solid, dashed and dotted lines correspond to the results with parameters $(\Delta_T = 0.84, \Delta_S = 0.86)$, $(\Delta_T = 0.94, \Delta_S = 0.91)$, $(\Delta_T = 1.04, \Delta_S = 0.97)$, respectively. (c) The dependence of $\chi(\omega)$ on $\omega$. Here the solid, dashed and dotted lines correspond to the results with parameters $\Delta_S = 0.86, \Delta_S = 0.91, \Delta_S = 0.97$, respectively. (d) The variation of $\lambda(\omega)$ with $\omega$. Here the solid, dashed and dotted lines correspond to the results with parameters $(\Delta_S = 0.86, \Delta_H = 0.5)$, $(\Delta_S = 0.91, \Delta_H = 0.6)$, $(\Delta_S = 0.97, \Delta_H = 0.7)$, respectively.

With the above parameters, Fig. 2 illustrates the dependence of the form factors $\xi(\omega)$, $\zeta(\omega)$, $\chi(\omega)$ and $\lambda(\omega)$ on $\omega$. In Tables VII and VIII we give the decay widths of the semileptonic decays of $B_{s1}$, $B_{s2}^*$, $B_{s0}$ and $B_{s1}^*$ with the three different choices of parameters $\Delta_{H,S,T}$ listed in Table VI. Up to now, the CDF and D0 collaborations have not measured the decay widths of $B_{s1}$ and $B_{s2}^*$.

Using the semileptonic decays of $B_{s1}$ as an example, we show the variation of the differential rates of the semileptonic decays of $B_{s1}$ with $\omega$ in Fig. 3. The differential rates of $B_{s1} \to D_s t \bar{\nu}$ and $B_{s1} \to D_s^* t \bar{\nu}$ increase with $\omega$ monotonically.
In contrast, the variation of the differential rates with the three parameter combinations listed in Table VI. The decay widths are in units of GeV.

|       | $B_{s1} \rightarrow D_s \ell \bar{\nu}$ | $B_{s2} \rightarrow D_s' \ell \bar{\nu}$ | $B_{s3} \rightarrow D_s \ell \bar{\nu}$ | $B_{s3}^* \rightarrow D_s' \ell \bar{\nu}$ |
|-------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| $D_s \ell \bar{\nu}$ | $2.1 \times 10^{-15}$ | $1.8 \times 10^{-15}$ | $1.6 \times 10^{-15}$ | $2.1 \times 10^{-15}$ |
| $D_s' \ell \bar{\nu}$ | $4.9 \times 10^{-15}$ | $4.4 \times 10^{-15}$ | $3.9 \times 10^{-15}$ | $5.0 \times 10^{-15}$ |
| $D_{s0} \ell \bar{\nu}$ | $8.7 \times 10^{-20}$ | $4.7 \times 10^{-20}$ | $6.9 \times 10^{-20}$ | $5.6 \times 10^{-20}$ |
| $D_{s1} \ell \bar{\nu}$ | $1.0 \times 10^{-19}$ | $8.7 \times 10^{-20}$ | $1.5 \times 10^{-19}$ | $1.2 \times 10^{-19}$ |

TABLE VII: The decay widths of the semileptonic decays of $B_{s1}$ and $B_{s2}$. Here columns (a), (b), (c) correspond to the results with the three parameter combinations listed in Table VI. The decay widths are in units of GeV.

|       | $B_{s0} \rightarrow D_s \ell \bar{\nu}$ | $B_{s1} \rightarrow D_s' \ell \bar{\nu}$ | $B_{s2} \rightarrow D_s' \ell \bar{\nu}$ |
|-------|----------------------------------------|----------------------------------------|----------------------------------------|
| $D_s \ell \bar{\nu}$ | $2.5 \times 10^{-15}$ | $1.3 \times 10^{-14}$ | $5.9 \times 10^{-15}$ |
| $D_s' \ell \bar{\nu}$ | $2.5 \times 10^{-15}$ | $1.3 \times 10^{-14}$ | $5.9 \times 10^{-15}$ |
| $D_{s0} \ell \bar{\nu}$ | $1.6 \times 10^{-15}$ | $3.9 \times 10^{-16}$ | $2.8 \times 10^{-15}$ |
| $D_{s1} \ell \bar{\nu}$ | $1.5 \times 10^{-15}$ | $3.3 \times 10^{-15}$ | $1.3 \times 10^{-14}$ |

TABLE VIII: The decay widths of the semileptonic decays of $B_{s0}$ and $B_{s1}$. Here columns (a), (b), (c) correspond to the results with the three parameter combinations listed in Table VI. The decay widths are in units of GeV.

In contrast, the variation of the differential rates of $B_{s1} \rightarrow D_{s0} \ell \bar{\nu}$ and $B_{s1} \rightarrow D_{s1} \ell \bar{\nu}$ with $\omega$ are not monotonic. The different line shapes of the differential rates of the semileptonic decays of $B_{s1}$ account for the results of the decay widths in Table VII. i.e. it tells us why the decay rates for type (c) are not always larger than the results for the other types. The same observation holds for the semileptonic decays of $B_{s2}$, $B_{s0}$, $B_{s1}$.

![Graphs](https://example.com/graphs.png)

FIG. 3: (a), (b), (c) and (d) illustrate the dependence of the differential rates of the semileptonic decays of $B_{s1}$ on $\omega$. Here the solid, dashed and dotted lines correspond to type (a), (b) and (c) in Table VI.
V. DISCUSSION

The semileptonic decay of the bottom-strange meson is an interesting topic. Due to the lack of experimental information of the excited bottom-strange mesons, theorists mainly focused on the semileptonic decay of $B_s$ using theoretical approaches such as the QCD sum rule approach \[ 29 \] and the quark model \[ 20 \]. The observations of $B_s0$, $B'_s1$, $B_s1$, $B'_s2 \rightarrow [D_s(1968), D'_s(2112), D_{sJ}(2317), D_{sJ}(2460)]\bar{\nu}$ in the framework of the CQM model. Our numerical results indicate that (1) the decay width of $B_s1(B'_s2) \rightarrow D_s(D'_s)\bar{\nu}$ is around $10^{-15}$ GeV, which is 4 $\sim$ 5 orders of magnitude larger than that of $B_s1(B'_s2) \rightarrow D_{sJ}(2317, 2460)\bar{\nu}$; (2) the decay width of $B_s0(B'_s1) \rightarrow D_s(D'_s, D_{sJ}(2317, 2460))\bar{\nu}$ is $10^{-14} \sim 10^{-15}$ GeV.

Although the CDF and D0 experiments did not measure the decay widths of $B'_s1$ and $B'_s2$ and the experiments did not observe $B'_s2$, $B'_s3$, $B'_s4$, and $B'_s5$. In Ref. \[ 4 \], one obtains the two-body strong decay widths of $B_s1$ and $B'_s2$ as 98 keV and 5 MeV respectively. $B_s0$ and $B'_s1$ are expected to be narrow resonances with a width around several tens of keV\[ 3 \], since their main decay modes are the isospin violating strong decays and electromagnetic decays. Thus it is reasonable to take the strong decay width as the total width approximately for $B_s1$, $B'_s2$, $B'_s3$, and $B'_s4$. We further show the order of magnitude of the semileptonic decay of $B_s1$, $B'_s2$, $B'_s3$, and $B'_s4$ in Table IX which is convenient for the experimentalist to conclude whether the current and future experiments can reach these semileptonic decays.

|          | $B_s0 \rightarrow$ | $B'_s1 \rightarrow$ | $B_s1 \rightarrow$ | $B'_s2 \rightarrow$ |
|----------|-------------------|-------------------|-------------------|-------------------|
| $D_s\bar{\nu}$ | $10^{-9} \sim 10^{-10}$ | $10^{-9} \sim 10^{-10}$ | $\sim 10^{-11}$ | $\sim 10^{-13}$ |
| $D'_s\bar{\nu}$ | $10^{-9} \sim 10^{-10}$ | $10^{-9} \sim 10^{-10}$ | $\sim 10^{-11}$ | $\sim 10^{-13}$ |
| $D_{s0}\bar{\nu}$ | $10^{-10} \sim 10^{-11}$ | $10^{-10} \sim 10^{-11}$ | $\sim 10^{-16}$ | $\sim 10^{-18}$ |
| $D'_{s0}\bar{\nu}$ | $10^{-9} \sim 10^{-10}$ | $10^{-9} \sim 10^{-10}$ | $\sim 10^{-16}$ | $\sim 10^{-17}$ |

TABLE IX: The estimation of the branching fractions of the semileptonic decays of $B_s0$, $B'_s1$, $B_s1$, and $B'_s2$ according to our numerical result shown in Table VII and VIII.

From Table IX, we can exclude the possibility of finding the semileptonic decay of $B'_s2 \rightarrow [D_s, D'_s, D_{s0}, D'_{s0}]\bar{\nu}$ and $B_s1 \rightarrow [D_{s0}, D'_{s0}]\bar{\nu}$ in experiments. However, for $B_s1 \rightarrow [D_s, D'_s]\bar{\nu}$ and the semileptonic decays of $B_s0$ and $B'_s1$, the upper limit of the branching ratio can reach to $10^{-9}$. The present precision of the experimental measurement of the branching fraction of the $B$ mesons has reached up to $10^{-7} \sim 10^{-8}$ \[ 3 \]. The decays $B_s1 \rightarrow [D_s, D'_s]\bar{\nu}$ and the semileptonic decays of $B_s0$ and $B'_s1$ may be observed in future experiments. Especially, the forthcoming LHCb experiments will produce an enormous amount of data of heavy-flavor hadrons, which is one of the potential experiments in which to search the $B_s1 \rightarrow [D_s, D'_s]\bar{\nu}$ and the semileptonic decays of $B_s0$ and $B'_s1$. Then these semileptonic decays will be helpful to further test the structure of $D_{sJ}(2317)$ and $D_{sJ}(2460)$. In our calculation, we have assumed $D_{sJ}(2317)$ and $D_{sJ}(2460)$ as the charm-strange mesons with $J^P = 0^+$ and $1^+$. If the experimental measurement of these semileptonic decays are consistent with our prediction, it will provide strong support of the $c\bar{s}$ structure for $D_{sJ}(2317)$ and $D_{sJ}(2460)$.

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3 In Ref. \[ 6 \], authors calculated the isospin-violating strong decay widths of $B_s0$ and $B'_s1$, which are 35 keV and 38 keV, respectively.
Appendix

The definitions of $I_0(\alpha), I_1(\alpha, \beta, \omega), I_5(\alpha, \beta, \omega), I_6(\alpha, \beta, \omega), I_3(\alpha), A_2(\alpha, \beta, \omega), A_3(\alpha, \beta, \omega), B_1(\alpha, \beta, \omega)$ and $B_2(\alpha, \beta, \omega)$ are

$$A_2(\alpha, \beta, \omega) = \frac{[(\omega^2 - 1)T(\alpha, \beta, \omega) - 6\omega U(\alpha, \beta, \omega) + (2\omega^2 + 1)S(\alpha, \beta, \omega)]/[2(\omega^2 - 1)]}{2^{1/2}},$$

$$A_3(\alpha, \beta, \omega) = \frac{[-\omega(\omega^2 - 1)T(\alpha, \beta, \omega) + 2(2\omega^2 + 1)U(\alpha, \beta, \omega) - 3\omega(S(\alpha, \beta, \omega) + S(\beta, \alpha, \omega))] /[2(\omega^2 - 1)]}{2^{1/2}},$$

$$B_1(\alpha, \beta, \omega) = \frac{\omega X(\beta, \alpha, \omega) - X(\alpha, \beta, \omega)}{\omega^2 - 1},$$

$$B_2(\alpha, \beta, \omega) = \frac{\omega X(\alpha, \beta, \omega) - X(\beta, \alpha, \omega)}{\omega^2 - 1},$$

with

$$T(\alpha, \beta, \omega) = I_6(\alpha, \beta, \omega) + m_s^2 I_5(\alpha, \beta, \omega),$$

$$U(\alpha, \beta, \omega) = I_1 + \alpha I_3(\alpha) + \beta I_3(\beta) + \alpha\beta I_5(\alpha, \beta, \omega),$$

$$S(\alpha, \beta, \omega) = \omega[I_1 + \beta I_3(\beta)] + \alpha I_3(\beta) + \alpha^2 I_5(\alpha, \beta, \omega),$$

$$X(\alpha, \beta, \omega) = -I_3(\beta) - \alpha I_5(\alpha, \beta, \omega),$$

$$I_0(\alpha) = \frac{N_c}{16\pi^{3/2}} \int_{1/\Lambda^2}^{1/\mu^2} \frac{dy}{y^{3/2}} e^{-y(m_s^2 - \alpha^2)} \left[ \frac{3}{2y} + m_s^2 + \alpha^2 \right],$$

$$\times [1 + \text{erf}(\alpha\sqrt{y})] - \frac{N_c m_s^2}{16\pi^2} \Gamma\left(-1, \frac{m_s^2}{\Lambda^2}, \frac{m_s^2}{\mu^2}\right),$$

$$I_1 = \frac{N_c m_s^2}{16\pi^2} \Gamma\left(-1, \frac{m_s^2}{\Lambda^2}, \frac{m_s^2}{\mu^2}\right),$$

$$I_3(\alpha) = \frac{N_c}{16\pi^{3/2}} \int_{1/\Lambda^2}^{1/\mu^2} \frac{dy}{y^{3/2}} \exp[-y(m_s^2 - \alpha^2)] (1 + \text{erf}(\alpha\sqrt{y})), $$
\[ \mathcal{I}_5(\alpha, \beta, \omega) = \int_0^1 dx \frac{4}{7 + 2x^2(1 - \omega) + 2x(\omega - 1)} \left\{ \frac{2N_c}{16\pi^3} \int_{1/\Lambda^2}^{1/\mu^2} dy \sigma e^{-(m^2_s - \sigma^2)y^{-1}} \left[ (1 + \text{erf}(\sqrt{y})) \right] + \frac{2N_c}{16\pi^2} \int_{1/\Lambda^2}^{1/\mu^2} dy e^{-y m_s^2} y^{-1} \right\}, \]

\[ \mathcal{I}_6(\alpha, \beta, \omega) = \mathcal{I}_1 \int_0^1 dx \frac{\sigma}{1 + 2x^2(1 - \omega) + 2x(\omega - 1)} - \frac{N_c}{16\pi^3/2} \int_0^1 dx \frac{1}{1 + 2x^2(1 - \omega) + 2x(\omega - 1)} \]

\[ \times \int_{1/\Lambda^2}^{1/\mu^2} dy \sigma^{-3/2} e^{-\nu(m^2_s - \sigma^2)} \left\{ \sigma \left[ 1 + \text{erf}(\sqrt{y}) \right] [1 + 2y(m^2_s - \sigma^2)] + 2 \sqrt{\frac{y}{\pi}} \left[ \frac{3}{2y} + (m^2_s - \sigma^2) \right] \right\}, \]

where

\[ \sigma(\alpha, \beta, \omega) = \frac{(1 - x)\alpha + x\beta}{\sqrt{1 + 2x^2(1 - \omega) + 2x(\omega - 1)}}. \]
[29] M.Q. Huang, Phys. Rev. D 69, 114015 (2004); T.M. Aliev and M. Savci, Phys. Rev. D 73 114010 (2006); T.M. Aliev, K. Azizi and A. Ozpineci, Eur. Phys. J. C 51, 593-599 (2007), arXiv: hep-ph/0608264