Action with manifest duality for maximally supersymmetric six-dimensional supergravity

GIANCARLO DE POL\textsuperscript{a}, HARVENDRA SINGH\textsuperscript{b}, MARIO TONIN\textsuperscript{a}

\textsuperscript{a} Dipartimento di Fisica ‘Galileo Galilei’, INFN Sezione di Padova, Università di Padova, Via F. Marzolo 8, 35131 Padova, Italia

\textsuperscript{b} Theory Division, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Calcutta-700 064, India

ABSTRACT

We perform explicitly the toroidal compactification of eleven dimensional supergravity to six dimensions and present its action in a manifestly $SO(5,5)\overline{SO(5)\times SO(5)}$ invariant form using the recently proposed covariant formulation of theories involving chiral fields.

\textsuperscript{1}e-mail: depol@pd.infn.it
\textsuperscript{2}e-mail: hsingh@tnp.saha.ernet.in
\textsuperscript{3}e-mail: tonin@pd.infn.it
1 Introduction

The compactification of $d = 11$ and $d = 10$ supergravities to lower dimensions is an old subject that recently has acquired new interest, due to the important rôle that dualities have reached in modern string theories and in M-theory. Even the simplest compactifications of $d = 11$ supergravity $\mathbb{H}$ on flat tori have received new interest. They give rise to lower dimensional supergravities with maximal supersymmetry and maximal duality groups (U-duality). The presence of these hidden symmetries was recognized from the beginning \cite{2} and they are by now completely classified \cite{3} and widely investigated \cite{4}-\cite{11}. These maximal supergravity models are useful at least as a first step to study more elaborate (and more interesting) models derived from them. An example are the massive and gauged supergravities in different dimensions \cite{12, 8}. Another related example is given by compactifications on spheres \cite{10, 11}, now so popular after the advent of the AdS/CFT duality. A characteristic feature of U-duality is that, in general, the bosonic multiplets of the duality group in $d$ dimensions contain both, let say, $p$-forms and the duals of $(d - p - 2)$-forms. This happens because U-duality contains transformations which correspond to electromagnetic duality and are conjectured to arise at a non-perturbative level in string models. In particular, the duality group of the maximal $d = 6$ supergravity coming from the compactification of $d = 11$ supergravity on a five-torus is $SO(5, 5)$, and one must consider the five 2-form potentials which arise from the reduction of the 3-form potential in $d = 11$, together with their duals potentials, to fill the ten-dimensional vector multiplet of $SO(5, 5)$. The maximal $d = 6$ supergravity is well known and its action has been obtained long time ago in a beautiful paper by Tanii \cite{13}. This action has been derived from its field content and not as an explicit dimensional reduction from $d = 11$ supergravity. Moreover, the $SO(5, 5)$ duality in the sector of the 2-form potentials is manifest in the action of \cite{13} only at the level of field equations. Recently a formulation of covariant actions with manifest duality has been available \cite{14}. In \cite{15} - \cite{17} this approach has been applied to $d = 6$ supergravity models. In the present paper we obtain an action for the maximal $d = 6$ supergravity by performing an explicit toroidal compactification from $d = 11$ and, by using this approach, we present this action in a form that is covariant and manifestly invariant under the $SO(5, 5)$ duality group. Our initial motivation to consider this case, was just our wish to use the approach of ref.\cite{14} at work in a simple case, as a first step to study more interesting situations. However, in this program we met some technical subtleties that convinced us to write this paper. For instance, the one-form potentials in $d = 6$ belong to a Weyl-Majorana spinor of
SO(5, 5), and in order to write the action terms involving their field strengths in a form with manifest SO(5, 5) invariance, one needs a vielbein-like matrix which belongs to the coset \( \text{SO}(5, 5)/\left(\text{SO}(5) \times \text{SO}(5)\right) \). However, the construction of this matrix in terms of the scalars that arise from the reduction is not straightforward.

In the short section 2 we review briefly the \( d = 11 \) supergravity \[1\], writing its action and supersymmetry transformations, in order to set our notations. In section 3 we perform the reduction on \( M_6 \times T_5 \) of the bosonic sector of this action and express the reduced action in a covariant form which is manifestly invariant under \( \text{SO}(5, 5) \) duality. In particular we give the explicit form of vielbein-like matrix \( V \) in terms of the scalars coming from the reduction. In section 4 we describe the reduction of the fermions in order to complete the action and write the supersymmetry transformations of the \( N = 4, d = 6 \) supergravity (however, the four-fermion terms have not been worked out explicitly).

### 2 11-dimensional Supergravity

In this section we set our notations and briefly review some elements of the eleven-dimensional supergravity. We use mostly plus signature for the spacetime metric. The latin alphabet is employed for vector indices while greek letters are used to describe spinor indices, but the last will be often suppressed. In our convention, a \( p \)-form and its Hodge-dual in \( d \)-dimensional space of signature \( t \) are defined by

\[
V(p) = \frac{1}{p!} e^{a_1 \ldots e^{a_p}} V_{a_1 \ldots a_p}
\]

\[
*V(p) = \frac{1}{(d-p)!p!} e^{a_1 \ldots e^{a_{d-p}} \epsilon_{a_1 \ldots a_p} b_1 \ldots b_p} V_{b_1 \ldots b_p}
\]

so that \( **V(p) = (-)^{t+d(d-1)/2} V(p) \). Here, \( e^a \) are one-form vielbeins, \( \gamma^{a_1 \ldots a_p} \) denotes the antisymmetric product of \( p \) Dirac matrices \( \gamma^a \), normalized to 1,

\[
\gamma^{(p)} = \frac{e^{a_1} \ldots e^{a_p}}{p!} \gamma_{a_1 \ldots a_p}
\]

and \( \Gamma^{(p)} = C(d) \gamma^{(p)} \), where \( C(d) \) is the charge conjugation in \( d \) dimension. Of course, \( V(p) V(q) = (-1)^{pq} V(q) V(p) \) is the wedge product of differential forms. Objects in eleven dimension will be marked by a hat.

The standard supergravity \[1\] action in eleven dimensions is (for \( \kappa = 1 \))

\[
S_{(11)} = \int \left[ \frac{\hat{e} \hat{R}}{4} - \frac{1}{2} \hat{G}(4) *\hat{G}(4) - \frac{1}{3} \hat{G}(4) \hat{G}(4) \hat{C}(3) - \frac{1}{2} \hat{\Psi} \hat{\Gamma}^{(8)} \hat{D} \hat{\Psi} \right]
\]
\[
-(\dot{Q}(7) + \dot{Q}(4)\dot{G}(4) - \frac{1}{2}(\dot{Q}(7) + \dot{Q}(4))\dot{Q}(4) + \frac{1}{8}\dot{\Psi}\dot{\Gamma}^{(8)}\dot{Q}(1)\dot{\Psi})
\]

(3)

where \( \dot{e} = det(\dot{e}_\hat{m}) \), \( \dot{R} \) is the scalar curvature, \( \dot{\Psi} = \dot{e}^\alpha \dot{\Psi}_\alpha \) is a one-form Majorana spinor that represents the gravitino and \( \dot{C}(3) \) is a 3-form gauge potential with curvature \( \dot{G}(4) = d\dot{C}(3) \). Moreover

\[
\dot{D}\dot{\Psi} = d\dot{\Psi} - \frac{1}{4}\omega_{\hat{a}\hat{b}}\hat{\gamma}_{\hat{b}\hat{a}}\dot{\Psi}
\]

(4)

\[
\dot{Q}(4) = \frac{1}{4}\dot{\Psi}\dot{\Gamma}^{(2)}\dot{\Psi}
\]

(5)

\[
\dot{Q}(7) = -\frac{1}{4}\dot{\Psi}\dot{\Gamma}^{(5)}\dot{\Psi}
\]

(6)

\[
\dot{Q}(1)\dot{\alpha} \dot{\beta} = -\frac{1}{8}\dot{e}^\hat{c}(\dot{\Psi} f \dot{\Gamma}\delta_{\hat{a}\hat{b}} f \dot{g} \dot{\Psi}) (\dot{\gamma}_{\hat{a}\hat{b}})\dot{\alpha} \dot{\beta}.
\]

(7)

and the supersymmetry transformations are

\[
\delta\dot{e}^\alpha = \dot{\epsilon} \Gamma^\alpha \dot{\Psi}
\]

(8)

\[
\delta\dot{C}(3) = -\frac{1}{2}\dot{\epsilon} \dot{\Gamma}^{(2)}\dot{\Psi}
\]

(9)

\[
\delta\dot{\Psi} = \dot{D}\dot{\epsilon} - \frac{1}{4}\dot{Q}(1)\dot{\epsilon}
\]

(10)

\[
-\frac{1}{3!4!}\left(\dot{\Gamma}_{b_1...b_4} - \dot{Q}_{b_1...b_4}\right) (\dot{e}^\hat{a} \Gamma^{b_4...b_1}_{(1)} + 8\dot{e}^\hat{b_4} \Gamma^{b_3b_2b_1}) \dot{\epsilon}.
\]

(11)

where \( \dot{\epsilon} \) are the local supersymmetry parameters (a real spinor in \( d = 11 \)).

### 3 Reduction on \( M_6 \times T^5 \)

Let us consider the reduction of the \( d = 11 \) spacetime to a six dimensional manifold \( M_6 \) and a 5-torus \( T^5 \). The spacetime coordinates \( \hat{x}^{\hat{m}} \) split up into two sets. For the ones of \( M_6 \) we shall use \( x^m, \ (m,n,...) \), without hat, and for those of the five-torus we shall use \( y^u, \ (u,v,...) \). The corresponding tangent space indices are \( (a,b,...) \) and \( (i,j,...) \) respectively. The ansatz for the vielbeins is

\[
\dot{e}^\hat{a} = \left(\omega^{-1/6}e^a, \ \omega^{1/3}\eta^u e^i_u\right)
\]

(12)

where \( \omega^{-1} = det(e^i_u) \) and the metric on the internal space is \( g_{uv} = e^i_u \delta_{ij} e^j_v \). The one-form \( \eta^u \) is \( \eta^u = dy^u + A^u \) where \( A^u \) are the Kaluza-Klein gauge fields.

It is now straightforward to work out the reduction of the Einstein term in the action \( (3) \). One finds that
\[
\int \frac{1}{4} \hat{e} R = \int \frac{1}{4} \left( e R + \frac{1}{4} d g^{uv} * d g_{uv} + \frac{1}{2} \omega F^u * F^v g_{uv} \right)
\]  
(13)

where \( F^u = dA^u \). One must note that the Einstein term on the right hand side of eq. (13) is independent of \( \omega \) which is due to judicious choice of scaling factors in the ansatz (12).

**Reduction of \( \hat{G}(4) \)**

The 3-form potential reduces as follows:

\[
\hat{C}(3) = C'(3) + dy^u B'_u + \frac{1}{2} dy^u dy^v A'_{(1)uv} + \frac{1}{6} dy^u dy^v dy^w \phi(0)_{uvw},
\]

The forms on the r.h.s. of the above equation do not depend upon the coordinates \( y^u \) of the torus. The reduction of corresponding 4-form field strength can be written as

\[
\hat{G}(4) = G(4) - \eta^u H(3)_{1u} + \frac{1}{2} \eta^u \eta^v (F(2)_{vu} + \phi(0)_{uvw} F(w)) - \frac{1}{6} \eta^u \eta^v \eta^w S(1)_{uvw}
\]

(14)

where (dropping the suffixes \( p \) that mark the form degree)

\[
S_{uvw} = d\phi_{uvw}, \quad F_{uv} = dA_{uv},
\]

\[
H_u = dB_u - \frac{1}{2} (A_{uv} F^v + F_{uv} A^v),
\]

\[
G = dC + B_u F^u - \frac{1}{2} A_{uv} A^v F^u,
\]

and

\[
A_{uv} = A'_{uv} - \phi_{uvw} A^w
\]

\[
B_u = B'_u - \frac{1}{2} A_{uv} A^v - \frac{1}{2} \phi_{uvw} A^w A^v,
\]

\[
C = C' - B_u A^u - \frac{1}{6} \phi_{uvw} A^w A^v A^w.
\]

(15)

Then the kinetic term for the 3-form potential \( \hat{C} \) will reduce to

\[
- \frac{1}{2} \int \hat{G} * \hat{G} = - \frac{1}{2} \int \left[ \omega G * G - H_u * H^u + \frac{\omega^{-1}}{2} (F_{uv} + \phi_{uvw} F^w) *(F^v + \phi^{vw} F_w) \right.
\]

\[
- \frac{\omega^{-2}}{6} S_{uvw} * S_{uvw}\]

(17)

and the Chern-Simon term becomes

\[
- \frac{1}{3} \int \hat{C} \hat{G} \hat{G} = \int \left( G F_{uv} \Phi^{uv} - G F^z \tilde{\phi}_z + H_u H_u \Phi^{uv} \right) + \int \frac{\epsilon_{uvwxy}}{4} F_{yx} F_{uv} (B_u + \frac{1}{2} A_{uz} A^z)
\]

(18)
where we have defined
\[ \Phi_{uv} = \frac{1}{3!} \epsilon^{uvxyz} \phi_{yzw}, \quad \tilde{\phi}_v = -\frac{1}{2} \phi_{vxy} \Phi_{yxw}. \] (19)

Also let us write the interaction term between fermions and \( \hat{G} \) as
\[- \int (\hat{Q}(7) + * \hat{Q}(4)) \hat{G} = \int \left( G * Q(g) + H_u Q^u(h) + (F_{uv} + \phi_{uvw} F^w)Q_{(u)} + S_{uvw} Q^{uvw} \right) \] (20)
where \( Q(g), Q^u(h) \) etc. are forms bilinear in the fermions whose specific form will be determined later.

Since the reduced action is at most quadratic in \( G \), one can dualize \( C \) in standard way: one lets \( G \) to be generic, adds the term \( (G - (B_u - \frac{1}{2} A_{uv} A^v) F^u) dA^\otimes \) (where \( A^\otimes \) is a Lagrange multiplier 1-form) and integrates over \( G \). Then the part of the action involving \( G \) can be replaced by the dual action
\[ S_{dual} = \int \left[ \frac{1}{2} \omega^{-1} (F^\otimes + F_{vu} \Phi_{uv} - F^z \tilde{\phi}_z) * Q(g) \right] (F^\otimes + F_{vu} \Phi_{uv} - F^z \tilde{\phi}_z + * Q(g)) \]
\[ - (B_u - \frac{1}{2} A_{uv} A^v) F^u F^\otimes \] (21)
where \( F^\otimes = dA^\otimes \) is the field strength of the one-form \( A^\otimes \). One can note that the equation of motion for \( A^\otimes \) exactly reproduces the Bianchi identity \( dG = H_u F^u \) for its dual 3-form \( C \).

The gauge transformations of the potentials \( A^u, A_{uv}, A^\otimes \) are
\[ \delta A^u = d\lambda^u \]
\[ \delta A_{uv} = d\lambda_{uv} \]
\[ \delta A^\otimes = d\lambda^\otimes \] (22)
and since \( H_u \) must be gauge invariant it follows from eq. (13) that also \( B_u \) transforms under (22) as
\[ \delta B_u = \frac{1}{2} (\lambda_{uv} F^v + F_{uv} \lambda^v). \] (23)
Moreover \( B_u \) transforms under its own gauge transformation: \( \delta B_u = d\Lambda_u \) where \( \Lambda_u \) is a one-form.

Let us count the total number of 1-form potentials after reduction. All 1-form gauge fields \( A^u, A_{uv}, A^\otimes \) add up to sixteen, which can be arranged into a Majorana-Weyl representation of the duality group \( SO(5, 5) \). Before dealing with it, we must explain our notations for \( SO(5, 5) \) in next subsection.

6
**Action for 1-form gauge fields**

The spinorial representation for $SO(5,5)$ is 32-dimensional. We denote vector indices of $SO(5,5)$ by capital letters $R, S, ...$ and a vector $v^R$ is composed of two five-dimensional parts $v^R = (v_1^R, v_2^R)$; $r = 1, ..., 5$. The metric is off-diagonal $\eta_{RS}v^Rv^S = (v_1^Rv_2^S + v_2^Rv_1^S)$. The $\gamma$-matrices can be conveniently constructed as a tensor product of two $SO(5)$ $\gamma$-matrices and Pauli matrices:

$$\gamma^r \times I_4 \times \sigma_1, \quad I_4 \times \gamma^r \times (i\sigma_2)$$

and in the chosen metric they are

$$\gamma^R = (\gamma_1^r, \gamma_2^r)$$

where

$$\gamma_1^r = \frac{1}{\sqrt{2}} (\gamma_r \times I_4 \times \sigma_1 + I_4 \times \gamma_r \times i\sigma_2)$$

$$\gamma_2^r = \frac{1}{\sqrt{2}} (\gamma_r \times I_4 \times \sigma_1 - I_4 \times \gamma_r \times i\sigma_2).$$

In $SO(5,5)$ there exist Majorana-Weyl spinors with 16 components $\psi_{\mu} \equiv \psi_{\dot{\mu}}; \mu = 1, ..., 16, \mu, \dot{\mu} = 1, ..., 4$ that satisfy the charge conjugation condition

$$\psi^c_{\mu} = (C^{-1})_{\mu}^{\nu} \bar{\psi}_{\nu} = \psi_{\mu}.$$

Now let us come to the reduced action. Keeping from eqs.(13), (17), (21) the terms quadratic in the $F$’s one gets the kinetic action:

$$S_A = \int \left[ \frac{1}{2} \omega^{-1} (F^\circ + F_{uv}\Phi^{uv} - F^z \bar{\phi}_z)^*(F^\circ + F_{uv}\Phi^{uv} - F^z \bar{\phi}_z) 
- \frac{\omega^{-1}}{4}(F_{uv} + \phi_{uvw}F^{vw})^*(F^{vu} + \bar{\phi}^{vwu}F_{w}) + \frac{\omega}{8} F^{uw}F_{uw} \right]$$

As already noted, the sixteen 1-form potentials $(A^\circ, A^u, A_{uv})$ can be arranged into a Weyl-Majorana $SO(5,5)$ spinor of, say, positive chirality; this spinor is

$$A_{\mu} = A^\circ \delta_\mu^\dot{\mu} + \frac{1}{2} A^u (\gamma_u)_{\mu} \dot{\mu} + \frac{1}{2} A_{uv} (\gamma_{uv})_{\mu} \dot{\mu}$$

where $\gamma_u$ with underlined vector indices are constant $SO(5)$ matrices, so that

$$F_{\mu} = dA_{\mu}$$

---

4More precisely $\psi = \left( \begin{array}{c} \psi_\mu \\ 0 \end{array} \right)$.
is also a Majorana-Weyl (W-M) spinor. To write an action invariant under \(SO(5,5)\) one needs a veilbein-like matrix \(V_{\alpha}^{\mu}\) that acts as a bridge between \(SO(5,5)\) and its maximal subgroup \(SO(5) \times SO(5)\). The index \(\alpha\) is \(\alpha \equiv (\alpha^\dagger \alpha)\). \(V_{\alpha}^{\mu}\) transforms as a W-M spinor of negative chirality under \(SO(5,5)\) from the right and as spinors of the two \(SO(5)\) from the left. The spinor indices of the two \(SO(5)\), \(\alpha\) and \(\dot{\alpha}\), can be raised and lowered with the metric given by the \(SO(5)\) charge conjugation. Then the action (27) should be rewritten as

\[
S_A = -\frac{1}{2(16)} \int \mathcal{F}_{\alpha}^* \mathcal{F}_{\alpha} \quad (30)
\]

\[
\mathcal{F}_{\alpha} = V_{\alpha}^{\mu} \mathcal{F}_{\mu}
\]

which is manifestly invariant under global \(SO(5,5)\) transformations as well as local \(SO(5) \times SO(5)\) rotations. A suitable gauge choice of these local transformations should allow us to express \(V_{\alpha}^{\mu}\) in terms of 25 scalar fields \(g^{uv}\) and \(\Phi^{uv}\), and to make eq.(27) to exactly reproduce (30). A convenient gauge fixed choice for \(V_{\alpha}^{\mu}\) is

\[
V = \exp X \cdot \exp Y \quad (31)
\]

\[
X = \frac{1}{4} F^{uv}[\gamma_u \otimes I_4 \otimes I_2 - I_4 \otimes \gamma_u \otimes I_2 - 2 \gamma_u \otimes \gamma_u \otimes \sigma_3] \quad (32)
\]

\[
Y = \frac{1}{4} F^{uv}[\gamma_u \otimes I_4 \otimes I_2 + I_4 \otimes \gamma_u \otimes I_2] + \frac{h^{uv}}{2}(\gamma_u \otimes \gamma_u \otimes \sigma_3) \quad (33)
\]

where \(X, Y\) belong to \(SO(5,5)\) Lie algebra and \(h^{uv}, f^{uv}\) are related to the scalars \(g^{uv}, \Phi^{uv}\). Given the “vielbeins”

\[
e_{u}^{\underline{u}} = \{exp(h + f)\}_{u}^{\underline{u}}
\]

and its inverse

\[
e_{\underline{u}}^{u} = \{exp - (h + f)\}_{\underline{u}}^{u},
\]

one has

\[
e_{u}^{\underline{u}} h^{uv} e_{v}^{\underline{v}} = g^{uv} \quad (34)
\]

and

\[
e_{u}^{\underline{u}} f^{uv} e_{v}^{\underline{v}} = \Phi^{uv}. \quad (35)
\]

Here, \(\gamma_u = e_{u}^{\underline{u}} \gamma_{\underline{u}}\) and so on. That the gauge fixed matrix \(V_{\alpha}^{\mu}\) defined in (31) allows eq.(27) to coincide with eq.(30) can be easily verified by noting that \(exp[Y]\) applied to \(\mathcal{F}\) dresses the constant \(\gamma\) matrices in \(\mathcal{F}\) as

\[
((expY) \ \mathcal{F})_\sigma = F^\gamma \omega^{-\frac{1}{2}} \delta_\delta^{\dot{\sigma}} + \frac{1}{2} F^u \omega^{\frac{1}{2}} (\gamma_u)_\sigma^{\dot{\sigma}} + \frac{1}{2} F_{uv} \omega^{-\frac{1}{2}} (\gamma^{uv})_\sigma^{\dot{\sigma}} \equiv \mathcal{F}^{(d)}_\alpha \quad (36)
\]
and
\[ ((\exp \mathcal{X}) \mathcal{F}^{(d)})_{\underline{\alpha}} = \omega^{-\frac{1}{2}} \left( F^{\Phi} + F_{uv} \Phi_{vu} - F_{\tilde{u}} \tilde{\phi}_u \right) \delta_{\alpha}^{\tilde{\alpha}} + \frac{1}{2} \omega^{-\frac{1}{2}} F_{\alpha}^{(\gamma_u)} \right) \delta_{\alpha}^{\tilde{\alpha}} + \frac{1}{2} \omega^{-\frac{1}{2}} (F_{uv} + F_{wv} \Phi_{uvw}) (\gamma_{uv})_{\alpha} \right]. \] (37)

The matrix \( V \) given above belongs to the coset \( SO(5,5)/SO(5) \times SO(5) \) and transforms as
\[ V = h(g) V g \] (38)
where \( g \) is a spinorial transformation matrix of \( SO(5,5) \) and \( h(g) \) is the compensating \( SO(5) \times SO(5) \) transformation. The transformation (38) implies the nonlinear transformation of the scalars under \( SO(5,5) \). One can realize a partial gauge-fixing by allowing the coefficients \( f_{wu} \) in eqs. (32) and (33) to be different and maintaining eq. (35) only for the coefficients \( f_{wu} \) in \( Y \) (that is in eq. (33)). In that case the action (30) remains invariant under local transformations of the diagonal \( SO(5) \) which corresponds to the Lorentz group for the compactified space.

**Action for B-fields**

Let us collect from eqs. (17), (18), (21) the terms involving the 2-form potentials together with those quartic in the gauge potentials. The action for the 2-form potentials is
\[ S_B = \int \left[ \frac{1}{2} H_{u}^{*} H_{u} + H_{u} H_{v} \Phi_{vu} + H_{u} Q_{(h)}^{u} + \frac{\epsilon^{uvwx,y} yz}{4} F_{yx} F_{wv} (B_{u} + \frac{1}{2} A_{uw} A^{v}) - F_{u} F^{\Phi} (B_{u} - \frac{1}{2} A_{uw} A^{v}) \right]. \] (39)

The terms quartic in the gauge potentials are present in order to make eq. (39) invariant under the gauge transformations (22) and (23). Let us rewrite \( Q_{(h)}^{u} \) as
\[ Q_{(h)}^{u} = -Q^{(1)u} + Q^{(2)u} - 2 \Phi_{uv} Q^{(2)}_{v} \] (40)
and rename \( B_{u} \) to \( B_{u}^{(2)} \) to conform with our notation for vectors of \( SO(5,5) \). Moreover, let us define its field strength as
\[ H_{u}^{(2)} = H_{u} + Q_{u}^{(2)} = dB_{u}^{(2)} + J_{u}^{(2)} + Q_{u}^{(2)}. \] (41)
where
\[ J_{u}^{(2)} = \frac{1}{2} (F_{w} A_{wu} + A^{w} F_{wu}). \] (42)

---

5For the moment this splitting is quite arbitrary, but see below.
Then eq. (39) yields the equation of motion
\[ d \left[ \ast H^{(2)u} - 2 \Phi^{uv} H^{(2)v} - \frac{1}{4} \epsilon^{uvwxy} F_{yx} A_{uv} + \frac{1}{2} ( F^u A^\otimes + A^u F^\otimes ) - Q^{(1)u} \right] = 0 \] (43)
while the Bianchi identity for \( B^{(2)u} \) is
\[ d(H^{(2)u} - J^{(2)u} - Q^{(2)u}) = 0. \] (44)
If we define
\[ H^{(1)u} = \ast H^{(2)u} - 2 \Phi^{uv} H^{(2)v} \] (45)
and
\[ J^{(1)u} = \frac{1}{4} \epsilon^{uvwxy} A_{gk} F_{uv} - \frac{1}{2} ( A^u F^\otimes + F^u A^\otimes ) \] (46)
eq (43) becomes
\[ d(H^{(1)u} - J^{(1)u} - Q^{(1)u}) = 0. \] (47)
The closure of this 3-form allows us to define locally a 2-form \( B^{(1)u} \), dual to \( B^{(2)u} \), such that
\[ H^{(1)u} = dB^{(1)u} + J^{(1)u} + Q^{(1)u}. \] (48)
In the notations introduced in eqs. (24), (25), (28), (29), equations (42), (46) become
\[ J^R = -\frac{1}{16} A_\mu (\Gamma^R)^\mu_\nu F_\nu \] (49)
where \( \Gamma^R = C \gamma^R \), \( J^R = \begin{pmatrix} J^{(1)u} \\ J^{(2)u} \end{pmatrix} \) and \( C \) is the \( SO(5,5) \) charge conjugation. Eq. (49) shows that \( J^R \) is an \( SO(5,5) \) vector. We shall also write
\[ B^R = \begin{pmatrix} B^{(1)u} \\ B^{(2)u} \end{pmatrix}, \quad H^R = \begin{pmatrix} H^{(1)u} \\ H^{(2)u} \end{pmatrix}, \quad Q^R = \begin{pmatrix} Q^{(1)u} \\ Q^{(2)u} \end{pmatrix} \] (50)
so that
\[ H^R = dB^R + J^R + Q^R \] (51)
We postulate that \( B^R \) transforms as a vector of \( SO(5,5) \), and therefore \( H^R \) also transforms as an \( SO(5,5) \) vector, since, as we shall show below, \( Q^R \) is an \( SO(5,5) \) vector, if the splitting in eq. (40) is chosen appropriately.

The field equation and Bianchi identity (43), (44) are equivalently described by the definitions (41), (48), (51) together with the duality condition (45). Let us rewrite eq. (45) and its Hodge-dual as
\[ H^{(1)u} + 2\Phi^{uv} H_v^{(2)} = H^{(2)u}, \tag{52} \]
\[ g^{uv} H_v^{(2)} = (H^{(1)u} + 2\Phi^{uv} H_v^{(2)}). \tag{53} \]

Now let us introduce the matrix
\[
L_{IR} = \left( \begin{array}{cc}
L^i_u & L^i_v \\
\tilde{L}^\tilde{i}_u & \tilde{L}^\tilde{i}_v \\
\end{array} \right) = \frac{1}{\sqrt{2}} \left( \begin{array}{cc}
e^i_u & e^i_v (g^{uv} + 2\Phi^{vu}) \\
e^\tilde{i}_u & -e^\tilde{i}_v (g^{uv} - 2\Phi^{vu}) \\
\end{array} \right) \tag{54} \]

\[ I = (i, \tilde{i}), \; i, \tilde{i} = 1, \ldots, 5, \] which can be written in more compact form as
\[ L_{IR} = \frac{1}{\sqrt{2}} \left( \begin{array}{cc}
E & E^{-1} E^T + 2E\Phi \\
E^{-1} E^T - 2\Phi E & E^{-1} - 2\Phi E \end{array} \right). \]

The transposed matrix is
\[ L^T_{RI} = \frac{1}{\sqrt{2}} \left( \begin{array}{cc}
E^T & E^T \\
E^{-1} - 2\Phi E & -E^{-1} - 2\Phi E \end{array} \right), \]
indices \( I, J \) are raised (and lowered) with the metric \( \tau^{IJ} = \left( \begin{array}{cc}
I_5 & 0 \\
0 & -I_5 \end{array} \right) \) (and its inverse).

\( I_p \) stands for \( p \)-dimensional identity matrix, \( E \) is the matrix with elements \( e^i_r \). Some useful formulas are:

\[ (L\eta L^T)^{IJ} \equiv \tau^{IJ} \]

and

\[ (L^T \tau L)_{RS} = \eta_{RS}, \]

where
\[ \eta_{RS} = \left( \begin{array}{cc}
0 & I_5 \\
I_5 & 0 \end{array} \right)_{RS} \]
is the \( SO(5,5) \) metric. If , instead of \( \tau \), we use the metric \( I_{10} \)
\[ M \equiv (L^T I_{10} L) = \left( \begin{array}{cc}
G & 2G\Phi \\
-2\Phi G & G^{-1} - 4\Phi G \Phi \end{array} \right). \tag{55} \]

\( G \) is the matrix with elements \( g_{uv} \) and \( \Phi \) is the matrix with elements \( \Phi^{uv} \).

One can easily verify that
\[ L^T_R = \frac{1}{32} Tr \left( \gamma^I V \gamma_R V^{-1} \right) \tag{56} \]
where, in terms of $SO(5)$ Dirac matrices, and in obvious notation, $\gamma^I = (\gamma^i \times I_4 \times \sigma_1, I_4 \times \gamma^j \times i\sigma_2)$, while $\gamma^R$ is defined in (24). Eq.(5) implies that $L^I_R \ (\bar{L}^I_R)$ transforms as a vector of $SO(5,5)$ from the right and as a vector of the first (the second) $SO(5)$ from the left. To be more precise eq.(56) leads to eq.(54) if one uses in it the partially gauge fixed $V^\mu_\alpha$ that preserves local invariance under the diagonal $SO(5)$ (the internal Lorentz group). To get full local $SO(5) \times SO(5)$ invariance one must introduce in eq.(56) the full ungauged $V^\mu_\alpha$. In that case eq. (56) still reproduces eq. (54) but now the vielbeins $e^i_u$ and $\tilde{e}^i_u$, even if constrained to give the same metric $g_{uv}$, are different.

If we define

$$H^I = L^I_R \ H^R \quad (57)$$

that is $H^i = L^I_R \ H^R$, $\bar{H}^i = \bar{L}^I_R \ H^R$, the sum and difference of eqs.(52) yield

$$H^i = *H^i, \quad \bar{H}^i = - *\bar{H}^i \quad (58)$$

Now it is straightforward to write down the action for the $B$ tensors using the formalism in [14], [15]. First we need to define

$$H^i = H^i - *H^i \ (\text{self-dual})$$
$$\bar{H}^i_+ = \bar{H}^i + *\bar{H}^i \ (\text{antiself-dual}) \quad (59)$$

and then we introduce an auxiliary 1-form $u$

$$u = da, \quad v = \frac{1}{\sqrt{-u^2}} \quad (60)$$

such that $v_m v^m = -1$, where $a(x)$ is an auxiliary scalar field. Using the vector $v^m$ we can define the 2-forms

$$h^i_- = i_v H^i_-, \quad \bar{h}^i_+ = i_v \bar{H}^i_+ \quad (61)$$

One can also write

$$h^i_+ = i_v \left( L^R_i H^i_- + \bar{L}^R_i \bar{H}^i_+ \right)$$
$$= L^R_i h^i_- + \bar{L}^R_i \bar{h}^i_+ \quad (62)$$

In a similar way we shall also write

$$J^R = L^R_i J^i + \bar{L}^R_i \bar{J}^i \quad (63)$$

\footnote{$i_v \nabla (p)$ denotes the contraction of the $p$-form $V(p)$ with the vector $v = v^\mu \partial_\mu$.}
and

\[ Q^R = L_i^R Q^i + \tilde{L}_i^R \tilde{Q}^i. \tag{64} \]

Then \cite{14,15} the action for chiral fields is

\[
S_B = \int \eta_{RS} \left( v h^R H^S + \frac{1}{2} dB^R (J^S + Q^S) + \frac{1}{2} J^R Q^S \right)
\]

\[
= \int \left( v h^i H_i + \frac{1}{2} H^i (J_i + Q_i) + \frac{1}{2} J^i Q_i \right) - \left( v \tilde{h}^i \tilde{H}_i + \frac{1}{2} \tilde{H}^i (\tilde{J}_i + \tilde{Q}_i) + \frac{1}{2} \tilde{J}^i \tilde{Q}_i \right). \tag{65} \]

One can see that the action \( S_B \) is invariant under the following transformations

\[ i) \delta B^R = d\Lambda^R, \quad \delta a = 0 \tag{66} \]

\[ ii) \delta B^R = -\frac{2h^R \xi}{\sqrt{-u^2}}, \quad \delta a = \xi \tag{67} \]

\[ iii) \delta B^R = \zeta^R da, \quad \delta a = 0 \tag{68} \]

where the two-form \( \Lambda^R \), the one-form \( \zeta^R \) and \( \xi \) are local parameters. The transformation \( i) \) is the usual \( B \)-gauge transformation, invariance under \( ii) \) implies the auxiliary status of the scalar field \( a(x) \) while the symmetry \( iii) \) allows to eliminate half of the propagating degrees of freedom carried by \( B \). Indeed the field equations coming from the action (65) are

\[
d(v h^i) = 0
\]

\[
d(v \tilde{h}^i) = 0 \tag{69} \]

and a suitable gauge fixing of the symmetry \( iii) \) together with the equations (69) leads to the chirality conditions in (58)

\[ H^i_\perp = 0 = \tilde{H}^i_\perp. \]

Eq.(65) is also invariant under the \( A \)-gauge transformations eqs.(22) which can be written as

\[ \delta A_\mu = d\lambda_\mu. \tag{70} \]

It follows from (70) and the transformation of \( H^R \) that

\[
\delta J^R = -\frac{1}{16} d(\lambda_\mu (\Gamma^R)_{\mu\nu} F^\nu) \\
\delta B^R = \frac{1}{16} \lambda_\mu (\Gamma^R)_{\mu\nu} F^\nu. \tag{71} \]
The invariance of \( S_B \) under (70) and (71) is manifest except for the term \( \frac{1}{2} \int \eta_{RS} dB^R J^S \). For this term it follows from (71) and the cyclic identity of \( \Gamma \) matrices in 10 dimensions. Finally \( S_B \) is manifestly invariant under \( SO(5,5) \) and, trivially, under \( SO(5) \times SO(5) \).

**Action for scalar fields**

If we collect the bosonic terms involving scalars not yet considered, we get

\[
S_{\text{scalar}} = \frac{1}{4} \int \left( \frac{1}{4} d g^{uv} * d g_{vu} + g_{uy} g_{vx} d \Phi^{uv} * d \Phi^{xy} \right) \tag{72}
\]

Now it is straightforward to check that it can be written in a manifestly \( SO(5,5) \) invariant form

\[
S_{\text{scalar}} = \frac{1}{2} \int d M_{RS} * d M^{RS} \tag{73}
\]

where \( M \) is given in (55). This action can also be presented in different equivalent forms. For instance, let us define, in terms of the matrix \( L \),

\[
P_{\tilde{\alpha} i} = L^i_R d \bar{L}^\dagger_i RS \eta - d L^i_R \bar{L}^\dagger_i S \eta^{RS} \tag{74}
\]

then it is straightforward to see that \( S_{\text{scalar}} \) can also be rewritten as

\[
S_{\text{scalar}} = - \frac{1}{16} \int P_{\tilde{\alpha} i} * P^{\tilde{\alpha} i} \tag{75}
\]

## 4 Reduction of Fermions

In the reduction of the eleven dimensional manifold to \( M_6 \times T^5 \), the \( \gamma \)-matrices in \( d = 11 \) reduce as follows

\[
\gamma^{\hat{\alpha}} = (\gamma^a, \tilde{\gamma}^i \gamma)
\]

where \( \gamma^a \) (\( a = 0, 1, \ldots 5 \)) are Dirac matrices in \( d = 6 \) and \( \tilde{\gamma}^i \) (\( i = 1, \ldots 5 \)) are Dirac matrices in \( d = 5 \); \( \gamma = \gamma^0 \gamma^1 \ldots \gamma^5 \) is the chiral matrix in \( d = 6 \). A Majorana spinor in eleven dimensions is a spinor (32 complex components) that satisfies the Majorana condition

\[
\chi^c = C_{(11)} \bar{\chi} = \chi.
\]

Its reduction to six dimensions gives us four symplectic W-M spinors with positive chirality \( \chi^c_+ \) and four W-M spinors with negative chirality \( \chi^c_- \), the symplectic condition being

\[
\chi^c_{\pm a} = (C_{(6)} \bar{\chi}_\pm)_a = (C_{(5)})_{a\beta} \chi^c_\beta.
\]
The fermions in $d = 11$ are described by a 1-form Majorana spinor $\hat{\Psi}$ which represents the gravitino. Its reduction to $d = 6$ consists of four 1-form symplectic Majorana spinors $\psi^\alpha = e^a \psi_a^\alpha$ (the gravitinos) of both chirality and twenty symplectic Majorana spinors of both chirality $\lambda_i^\alpha$ ($i = 1, \ldots, 5$). To be more specific

$$\hat{\Psi} = \omega^{-\frac{1}{2}} (\Psi + \sqrt{\omega} \Lambda)$$ (76)

where

$$\Psi \equiv e^a \psi_a = e^a \psi_a - \frac{1}{4} e^a \gamma_a \tilde{\gamma} \tilde{\gamma} \lambda_i = e^a \psi_a + \frac{1}{6} e^a \gamma_a \tilde{\gamma} \tilde{\gamma} \gamma \lambda_i$$

$$\Lambda \equiv e^i \lambda_i = e^i (\lambda_i - \frac{1}{5} \tilde{\gamma} \gamma \lambda_j) \pm \frac{2}{15} e^i \tilde{\gamma} \gamma \lambda_j$$ (77)

The $\omega$-dependent coefficients in (76) are fixed by the requirement that the reduced kinetic action will be $\omega$-independent and the coefficients in (77) are chosen to get the reduced kinetic action in the standard diagonal form, that is to have

$$\hat{\Psi} \hat{\Gamma}^{(8)} D \hat{\Psi} = \psi \Gamma^{(3)} \tilde{\gamma} D \psi + \lambda^i \Gamma^a D_a \lambda_i.$$ (78)

The spinors $\psi_a, \lambda_i$ must be split in their chiral and antichiral components $\psi_{\pm a}, \lambda_{\pm i}$. All these spinors are invariant under $SO(5, 5)$. Inserting eqs. (76), (77) in eq. (8) one can compute the coupling between bosons and fermions and the four-fermion interaction terms. In particular the interaction terms of the fermions with the scalars $\Phi^{uv}$, the gauge fields $A^\alpha$ and $A_{uv}$ and the 2-form potential can be obtained from eq. (20). In this respect it is useful to notice that the Hodge dual of the current $Q_\gamma + * Q_4$ of eq. (20) has the simple expression

$$* Q_7 + Q_4 = \frac{1}{4} \hat{\Psi} \hat{\Gamma}^{[b_1} \Gamma^{(4)} \gamma_{b_2]} \hat{\Psi}.$$ (79)

As before the resulting action terms turn out to be invariant under global transformations of $SO(5, 5)$ but only local transformations of the diagonal $SO(5)$ which corresponds to the Lorentz group of the internal space. However, invariance under both $SO(5)$ can be recovered if one assumes that the components of positive and negative chirality of $\lambda_i$ are vectors of the first and the second $SO(5)$ respectively and that $\psi^\alpha_a$ and $\lambda^\alpha_i$ are spinors of the first $SO(5)$, whereas $\psi_{\alpha a}$ and $\lambda_{\alpha i}$ are spinors of the second $SO(5)$ (and of course one uses the non gauge-fixed forms of $V_\mu$ and $L_{LM}$).

After a long calculation we find that the action that describes the interaction between fermions and gauge fields and scalars can be written as

$$S_{(I)} = \frac{1}{16} \int (Q^{\alpha \dot{\alpha}} V_{\alpha \dot{\alpha}} \mathcal{F}_{\mu} + Q^{\dot{\mu}} P_{\dot{\mu}})$$ (80)
where
\[ ^*Q^{\alpha\dot{\alpha}} = \frac{1}{2} \psi^\alpha_+ \Gamma^c \Gamma^{(2) \Gamma_4} \psi^-_d + \frac{1}{4} \psi^\beta_+ \Gamma^{(2) \Gamma_4} \chi^\dot{\alpha}_+ (\gamma^i)_{\beta} + \frac{1}{4} \psi^\dot{\beta}_- \Gamma^{(2) \Gamma_4} \chi^-_i (\gamma^j)_{\dot{\alpha}} \]
\[ + \frac{1}{4} (\Lambda^i_+ \gamma^i)^{\dot{\alpha}} \Gamma^{(2)} (\Lambda^-_i \gamma^j)^{\alpha} \]
\[ ^*Q^{\dot{\alpha}} = \psi^{\alpha}_+ \Gamma^{(1) \Gamma_4} (\gamma^i_+ \chi^-_i)_{\alpha} + \psi^\dot{\alpha}_- \Gamma^{(1) \Gamma_4} (\gamma^j_+ \chi^-_j)_{\dot{\alpha}}. \]

Action (81) is manifestly invariant under global $SO(5, 5)$ and local $SO(5) \times SO(5)$ transformations. Moreover, we find that the splitting in eq.(64) can be performed so that fermionic 3-forms $Q^I$ which appear in eq.(64) are
\[ Q^i = -\frac{1}{4} \psi^\alpha_+ \Gamma^{(1) \Gamma_4} (\gamma^i)^{\beta} \psi_{+\beta} + \frac{1}{2} \psi^\dot{\alpha}_- \Gamma^{(2) \Gamma_4} \lambda^{+\dot{\alpha}} - \frac{1}{8} \lambda^{-i\alpha} \Gamma^{(3) \Gamma_4} \lambda_{-i\dot{\beta}} \]
\[ Q^j = -\frac{1}{4} \psi^\alpha_+ \Gamma^{(1) \Gamma_4} (\gamma^j)^{\dot{\beta}} \psi_{-\dot{\beta}} + \frac{1}{2} \psi^\dot{\alpha}_- \Gamma^{(2) \Gamma_4} \lambda^{i-\dot{\alpha}} - \frac{1}{8} \lambda^{i\dot{\dot{\alpha}}} \Gamma^{(3) \Gamma_4} \lambda_{i\dot{\beta}} \]
where $Q^i = L^i_R Q^R$ and $Q^\dot{i} = L^\dot{i}_R Q^R$. Then $Q^i$, $Q^\dot{i}$ are vectors of the two $SO(5)$s. It follows that $Q^R$, defined in eq.(64) is an $SO(5, 5)$ vector, as anticipated in the previous section.

In conclusion the $d = 6$, $N = 4$ supergravity action is
\[ S = \int \left[ \frac{1}{4} e R - \frac{1}{2(16)} F_{\mu} \ast F_{\mu} + \eta_{RS} \left( v \, R^R \, H^S + \frac{1}{2} dB^R (J^S + Q^S) + \frac{1}{2} J^R Q^S \right) \right. \]
\[ - \frac{1}{(16)} P_{\dot{i}i} \ast P^{\dot{i}i} - \frac{1}{2} (\psi^\alpha_+ \Gamma^{(3) \Gamma_4} \Gamma \psi^-_i + \psi^\dot{\alpha}_- \Gamma^{(3) \Gamma_4} \Gamma \psi^-_i) \]
\[ - \frac{1}{2} \left( \lambda^{ia} \Gamma_4 D_a \Lambda_{+i\dot{a}} + \lambda^{i\dot{a}} \Gamma_4 D_a \Lambda_{-i\dot{a}} \right) \]
\[ + \frac{1}{16} (Q^{\alpha\dot{\alpha}} V_{\alpha\dot{\alpha}} F_F + Q^{\dot{\alpha}} P_{\dot{i}i}) + S_4 \]

The last term $S_4$ represents the four fermion interactions. It can be obtained straightforwardly from the reduction of the four fermion terms in (3), but we have not computed it explicitly here. In addition to be manifestly invariant under $SO(5, 5)$ duality the action (83) is also invariant under local supersymmetry. The supersymmetry transformations can be obtained easily from the reduction of eqs.(8) to (11) and using invariance under duality. However, a remark is in order: the field equations for the 2-form potentials from the action (64) coincide with the standard ones only after a (suitable) gauge fixing of the symmetry $\tilde{i}i$) in eq. (63). Therefore, as discussed in [15] - [19] the standard supersymmetric transformations of the fermions that involve the field strength $H_u$ must be modified. The correct recipe is to replace in them the 3-forms $H^R$ with the 3-forms
\[ K^R = H^R + h^R \nu. \]
Concluding supersymmetry transformations are

\[
\delta e^a = \psi^\beta _+ \Gamma^a c_{\beta \alpha} e^\alpha_+ + \psi^\delta _- \Gamma^a c_{\beta \alpha} e^\alpha_- \\
\delta V_{\alpha \dot{\alpha}} = \frac{1}{2} V_{\beta \bar{\alpha}} \left( \bar{\gamma}^i \right)_\alpha \beta \left( \bar{\gamma}^i \right)_{\dot{\alpha}} \left( \lambda_{+ i \bar{\gamma}} \right)_\delta \epsilon_{- \delta} + \left( \lambda_{- \bar{\gamma} i} \right)_\delta \epsilon_{- \delta} \\
\delta A^{\mu}_{(1)} = \frac{1}{2} V_{\alpha \dot{\alpha}} \left( \psi^\alpha_+ \epsilon^\alpha_+ + \epsilon^\dot{\alpha} \psi^\alpha_- + \frac{1}{2} \lambda^\dot{\alpha} \Gamma^{(1)} (\epsilon^\alpha_+ \bar{\gamma}^i \epsilon^\alpha_-) + \frac{1}{2} (\epsilon^\alpha_+ \bar{\gamma}^i \epsilon^\alpha_-) \Gamma^{(1)} \lambda^\alpha_- \right) \\
\delta B^{R}_{(2)} = \frac{1}{2} L^i_R \left( \psi^\alpha_+ \Gamma^{(1)} (\bar{\gamma}^i \epsilon^\alpha_+) \alpha + \lambda^\dot{\alpha} \Gamma^{(2)} (\epsilon_{- \dot{\alpha}}) \right) + \frac{1}{2} \tilde{L}^i_R \left( \psi^\dot{\alpha} \Gamma^{(1)} (\bar{\gamma}^i \epsilon_{- \dot{\alpha}}) \alpha + \lambda^\alpha \Gamma^{(2)} (\epsilon_{+ \alpha}) \right) \\
\delta A^{\mu}_{(2)} = \frac{1}{2} \delta A^{\mu}(\Gamma^{R})_{\mu \nu} A^{\nu} \\
\delta \psi_{+ \alpha} = D \epsilon_{+ \alpha} + e^a V_{\alpha \dot{\alpha}} \left( \frac{1}{8} \gamma^i \epsilon^\alpha_+ \gamma^j \epsilon^\alpha_- \right) + \frac{3}{4} \gamma^{ab} \epsilon^\alpha_+ F^{ab}_{\mu} \\
\delta \lambda_{+ i \alpha} = \frac{1}{4} P^i_{a \dot{a}} \gamma^a (\bar{\gamma}^i \epsilon_{- \dot{\alpha}}) \alpha + \frac{1}{4} V_{\alpha \dot{\alpha}} \gamma^{ab} (\bar{\gamma}^i \epsilon^\alpha_+) \alpha + \frac{1}{12} K_{- iabc} \gamma^{abc} \epsilon_{- \dot{\alpha}}, \quad (84)
\]

Transformations for the \( \psi_{- \dot{\alpha}} \) and \( \lambda_{- i \alpha} \) can be obtained from the above by exchanging \(+ \leftrightarrow -\), \(+ \leftrightarrow \dot{+}\), \( \alpha \leftrightarrow \dot{\alpha} \) in the positive chirality fields respectively.

**Acknowledgements**

This work was supported by the European Commission TMR programme ERBFMRX-CT96-0045. H.S. was also supported in part by INFN fellowship.
References

[1] E.Cremmer, B.Julia and J.Scherk, Phys.Lett. B76 (1978) 409.

[2] A. Salam and E. Sezgin “Supergravities in Diverse Dimensions”, North-Holland Physics, Amsterdam, 1989.

[3] B. Julia “Group disintegration” Nuffield Gravity Workshop, Cambridge 1980 ; E.Cremmer “Supergravities in 5 dimensions”, ibidem.

[4] for recent discussions see E. Cremmer, B. Julia, H. Lu and C.N. Pope, Nucl. Phys. B523 (1998), 73; N.A. Obers and B. Pioline, Phys. Rept.318:113-225,1999, hep-th/9809039.

[5] B. de Wit and H. Nicolai, Nucl. Phys. B243 (1984), 91; Nucl. Phys. B274 (1986) 363; Phys. Lett. B187 (1987).

[6] B. de Wit and H. Nicolai, Nucl. Phys. B281 (1987), 211.

[7] M.J. Duff, B.E. Nilsson, C.N. Pope and N.P. Warner, Phys. Lett. B149 (1984) 90.

[8] A. Chamseddine and M.S. Volkov, Phys. Rev. D57 (1998) 6242; M. Volkov, hep-th/9910116.

[9] N. Kaloper and R.C. Myers JHEP 9905:010,1999, hep-th/9901045.

[10] M. Cvetic, J.T. Liu, H. Lu and C.N. Pope, Nucl. Phys. B 560:230-256,1999, hep-th/9905096; M. Cvetic, H. Lu and C.N. Pope, Phys. Rev. Lett.83:5226-5229,1999, hep-th/9906221; CTP-TAMU-42-99, hep-th/9910025.

[11] H. Nastase, D. Vaman and P. van Nieuwenhuizen, Phys. Lett. B469:96-102,1999, hep-th/9905075; ITP-SB-99-56, hep-th/9911238; H. Nastase and D. Vaman, hep-th/0002028.

[12] see for example: H. Singh, hep-th/9801038, hep-th/9808181; P. Cowdall and P. Townsend, hep-th/9801165.

[13] Y. Tanii, Phys. Lett. B145 (1984) 197.

[14] P. Pasti, D. Sorokin and M. Tonin, Phys. Rev. D52 (1995) 4277; Phys. Rev. D55 (1997) 6292.
[15] G. Dall’Agata, K. Lechner and M. Tonin, Nucl. Phys. B512 (1998) 179.

[16] F. Riccioni and A. Sagnotti, hep-th/9812042.

[17] K. Van Hoof, hep-th/9910175.

[18] G. Dall’Agata, K. Lechner and D. Sorokin, Class. Quant. Grav 14 (1997) L195; G. Dall’Agata, K. Lechner and M. Tonin, J.H.E.P. 9807 (1998) 017.

[19] I. Bandos, N. Berkovits and D. Sorokin, Nucl.Phys.B522:214-233,1998, hep-th/9711055.