Analytic Estimation of Three-dimensional Variables for SOC in Iron Phosphate Lithium-ion Battery

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Abstract. A three-dimensional variable analysis method for the estimation of state of charge (SOC) in iron phosphate lithium-ion battery is proposed. The 100Ah/3.6V iron phosphate lithium-ion battery is regarded as the research object. The relationship between SOC and single factor was got by using the control variable method, and the specific expression was got by using the way of data fitting. Then the situation when multiple factors work together is analyzed ,and the function between SOC and the three factors—temperature(T),charging voltage(U) and charging current(I) was got by using the way of data fitting. The feasibility of this method is tested with experimental data, and the error analysis has been done. The results indicate that the SOC estimation error of this method is less than 2%. The two-dimensional surface and contour curve is given when charging voltage(U) is 3.3V and under the condition of different charging current and temperature.

Keywords: State of Charge Estimation, The Control Variable Method, Data Fitting, Error Analysis

1 Introduction
Iron phosphate lithium-ion battery [1] has become an ideal power device for replacing petroleum energy devices, widely used in the automotive field and green energy. In order to give full play to the power performance of batteries and prevent over charging or over discharging from affecting the safety and service life of batteries, it is necessary to accurately estimate the state of charge (SOC) of batteries. However, due to the influence of the non-linear characteristics of batteries and many other factors, improving the SOC estimation accuracy has always been the difficulty and key technology of battery management system for electric vehicles [2-3].

At present, the SOC estimation of battery management system of electric vehicle usually adopts the ampere-time method, i.e. current integration method. This method is simple in calculation and occupies little memory, but it is easy to produce large SOC estimation error due to inaccurate current measurement. At the same time, the accumulated effect further increases the error. Open-circuit voltage method [4] can accurately estimate SOC, but because of the long measuring time and only static measurement, it is not real-time and difficult to be practical. Neural network method [5] can accurately estimate SOC, but it needs a lot of reference data to train, and the estimation error is deeply affected by training data and training methods. Kalman filter method [6-9] can accurately estimate
SOC in the case of intense current fluctuation and give the SOC estimation error, but it requires the model parameters to be quite accurate, which reduces its estimation accuracy. Reference [10] synthetically considered the influence of voltage and current, obtained the two-dimensional variable SOC estimation method, and simply revised the temperature, which improved the SOC estimation accuracy. However, because of the complex influence of temperature on SOC, the accuracy in the operating environment of electric vehicle with large temperature fluctuation was significantly reduced.

Firstly, the experiment of SOC control variables is carried out, and the experimental method under the condition of single variable is given, and the experimental data are qualitatively analyzed in this paper. On this basis, the three-dimensional variables of SOC are estimated and analyzed. The relationships between SOC and U, SOC and I, SOC and T are quantitatively analyzed, and the corresponding analytical formulas are obtained. Then a comprehensive analysis is carried out to obtain the three-dimensional variable estimation analytic expressions of SOC and U, I and T, and to test the performance of the analytic expressions. The two-dimensional variable surface and contour distribution curves of SOC with T and I are given when U = 3.3V. The test results show that this method can be applied to the estimation of SOC in practical production.

2 SOC Control Variable Experiments
The SOC estimation of lithium iron phosphate batteries is related to terminal voltage, electrolyte density, discharge current, charging current, battery aging and battery life, and the corresponding relationship is usually non-linear. Under certain conditions, only some factors have the most significant impact on SOC. Through these significant factors, a more accurate SOC estimation formula can be obtained. According to the actual application conditions of lithium iron phosphate battery, the working voltage (U), current (I) and temperature (T) are the three main factors affecting SOC. By using the control variable method, that is, under the condition that the other two factors (such as U, I) are constant, the relationship between SOC and one factor (such as T) can be studied, and the functional expression of SOC and that factor (such as T) can be obtained. Combining the relationship between SOC and single variable (U, I or T), the relationship between SOC and three variables (U, I and T) is obtained.

The SOC estimation experiments were carried out under several charging voltage, current and temperature conditions.

2.1 The relationship between SOC and operating voltage under the Conditions of constant working current and temperature
The experimental methods are as follows: (1) The battery is discharged to the cut-off state, that is, when the battery discharges to the discharge cut-off voltage of 2.3V, it stops discharging. (2) The temperature of the surrounding environment of the tested battery is kept constant by the thermostat, and the temperature is controlled separately at T = 11.5°C, 15°C, 20°C, 25°C. (3) The batteries are charged by constant current power supply with current I = 8A, 5A, 10A, 15A, and the working voltage U at the end of charging is recorded. (4) After charging, the battery is stationary for a period of time, and then the SOC is estimated and recorded in the same environment by the ampere-time method. The recorded data are traced as shown in Fig. 1. Fig. 1 shows that the curve of SOC varying with operating voltage is basically parallel under different operating currents and temperatures, and under the same working current and temperature, the curve shape of SOC varying with working voltage is close to a straight line.

2.2 The Relation between SOC and Temperature under the Conditions of Constant Operating Voltage and Current
The experimental methods are as follows: (1) The battery is discharged to the cut-off state, that is, when the battery discharges to the discharge cut-off voltage of 2.3V, it stops discharging. (2) The constant temperature of the environment around the tested battery is guaranteed by a thermostat, and a series of experiments under different temperature conditions are carried out. (3) The battery is charged
by a constant voltage power supply with working voltage $U = 3.3V$, and the working current at the end of charging is recorded. (4) After charging, the battery is stationary for a period of time, and then the SOC is estimated and recorded in the same environment by the ampere-time method. The recorded data are traced, as shown in Fig. 2. Fig. 2 shows that the curve of SOC varying with temperature is basically parallel under different charging voltage and current, and under the same charging voltage and current, the trend of SOC varying with temperature is not linear, but nonlinear with larger curvature.

2.3 The Relation between SOC and Working Current under the Conditions of Constant Working Voltage and Temperature

The experimental methods are as follows: (1) The battery is discharged to the cut-off state, that is, when the battery discharges to the discharge cut-off voltage of 2.3V, it stops discharging. (2) The temperature of the surrounding environment of the tested battery is kept constant by the thermostat, and the temperature is controlled separately at $T = 11.5°C, 15°C, 20°C, 25°C$. (3) A constant voltage power supply with working voltage $U = 3.3V$ is used to charge the battery, and the working current $I$ at the end of charging is recorded. (4) After charging, the battery is stationary for a period of time, and then the SOC is estimated and recorded in the same environment by the ampere-time method. The recorded data are traced, as shown in Fig. 3. Fig. 3 shows that the curve of SOC varying with working voltage is basically parallel under different working currents and temperatures, and under the same charging voltage and temperature, the trend of SOC varying with charging current is not a simple linear relationship, but a non-linear relationship with smaller curvature.

![Fig. 1](image1.png)

**Fig. 1** The variation of SOC with charging voltage under constant charging current and constant temperature

![Fig. 2](image2.png)

**Fig. 2** The variation of SOC with temperature under constant charging current and constant charging voltage
3. Estimation and Analysis of Three-dimensional Variables of SOC

3.1 Fitting Analysis of the Relation between SOC and Working Voltage under the Conditions of Constant Working Current and Temperature

According to the rule of Section 2.1, under the condition of constant working current and temperature, the curve of SOC varying with working voltage is close to a straight line, so the relationship between SOC and working voltage can be fitted by least square estimation using first-order polynomial and second-order polynomial. The specific fitting curve expressions are shown in Table 1, and the corresponding error curves are shown in Fig. 4 ~ Fig. 7. From Table 1, it can be seen that for different charging currents and temperatures, the expressions obtained by first-order polynomial fitting have roughly the same slope. That is, the difference of temperature and current does not affect the first term of SOC under voltage action, but only the constant term. The expression obtained by second-order polynomial fitting has no common law, that is, the difference of temperature and current directly affects the effect of voltage on SOC, which is affected by the coupling effect of temperature and voltage, current and voltage. Therefore, considering the principle of simplicity of analysis, the first-order polynomial fitting is better. From the fitting error results of Fig. 4 ~ Fig. 7, we can see that the error produced by first-order polynomial fitting is basically the same as that produced by second-order polynomial fitting. Therefore, considering the principle of simplicity of calculation, the first-order polynomial fitting is better. Based on the above conclusions, the first order polynomial is used to fit the relationship between SOC and charging voltage.

**Table 1** Fitting Estimation of SOC Variation with Charging Voltage under Constant Charging Current and Temperature

| Estimation index | First-order polynomial fitting | Second-order polynomial fitting |
|------------------|--------------------------------|---------------------------------|
| $I = 8.4$, $T = 11.5 \, ^\circ C$ | $SOC = 0.1025 \times U + 0.1544$ | $SOC = 0.0006 \times U^2 + 0.0983 \times U + 0.1612$ |
| $I = 5.4$, $T = 15 \, ^\circ C$ | $SOC = 0.0968 \times U + 0.1575$ | $SOC = -0.0163 \times U^2 + 0.2039 \times U - 0.0183$ |
| $I = 10.4$, $T = 20 \, ^\circ C$ | $SOC = 0.1003 \times U + 0.3066$ | $SOC = -0.0028 \times U^2 + 0.1187 \times U + 0.2763$ |
| $I = 15.4$, $T = 25 \, ^\circ C$ | $SOC = 0.1004 \times U + 0.4710$ | $SOC = -0.0154 \times U^2 + 0.2017 \times U + 0.3048$ |

**Fig. 3** The variation of SOC with charging current at constant temperature and constant charging voltage
Fig. 4 The error comparison of fitting using first-order polynomial and second-order polynomial when I=8A, T=11.5°C

Fig. 5 The error comparison of fitting using first-order polynomial and second-order polynomial when I=5A, T=15°C
3.2 Fitting Analysis of the Relation between SOC and Temperature under the Conditions of Constant Operating Voltage and Current

According to the rule of Section 2.2, under the condition of constant working voltage and current, the trend of SOC changing with temperature is not linear, but nonlinear with larger curvature. Therefore, the relationship between SOC and temperature can be fitted by least square estimation using second-order polynomial and third-order polynomial. The specific fitting curve expression is shown in Table 2, and the corresponding error curve has been omitted. From Table 2, it can be seen that for different charging currents and voltages, the expressions obtained by second-order polynomial fitting have the same quadratic term and approximately the same primary term. That is, the difference of
charging current and voltage does not affect the quadratic term and primary term of SOC under temperature action, but only the constant term. The third-order polynomial fitting results show that the third-order polynomial approximates zero, which is roughly the same as the corresponding second-order polynomial. Therefore, considering the principle of simplicity of analysis, the second-order polynomial fitting is better. The fitting error results show that the error generated by second-order polynomial fitting is basically the same as that generated by third-order polynomial fitting. Therefore, considering the principle of simplicity of calculation, second-order polynomial fitting is better. Based on the above conclusions, the second-order polynomial is used to fit the relationship between SOC and temperature

Table 2 Fitting Estimation of SOC Variation with Temperature under Constant Charging Voltage and Current

| Estimation index | Condition | Second-order polynomial fitting | Third-order polynomial fitting |
|------------------|-----------|--------------------------------|-----------------------------|
|                  |           | SOC = 0.0008×T²−0.0115×T+0.3779 | SOC = 0.0000×T³+0.0008×T²−0.0112×T+0.3786 |
|                  |           | SOC = 0.0008×T²−0.0112×T+0.4018 | SOC = 0.0000×T³+0.0008×T²−0.0114×T+0.4088 |
|                  |           | SOC = 0.0008×T²−0.0104×T+0.4535 | SOC = 0.0000×T³+0.0006×T²−0.0086×T+0.4818 |
|                  |           | SOC = 0.0008×T²−0.0112×T+0.4984 | SOC = 0.0000×T³−0.0088×T²+0.4876 |
|                  |           | SOC = 0.0008×T²−0.0112×T+0.5415 | SOC = −0.0000×T³+0.0008×T²−0.0115×T+0.5415 |
|                  |           | SOC = 0.0008×T²−0.0112×T+0.5801 | SOC = −0.0000×T³+0.0008×T²−0.0114×T+0.5802 |

3.3 Fitting Analysis of the Relation between SOC and Working Current under the Conditions of Constant Working Voltage and Temperature

According to the rule of section 2.3, under the condition of constant working voltage and temperature, the trend of SOC changing with charging current is not linear, but a non-linear relationship with smaller curvature. Therefore, the relationship between SOC and charging current can be fitted by least square estimation using first-order polynomial and second-order polynomial. The specific fitting curve expression is shown in Table 3, and the corresponding error curve has been omitted. As can be seen from Table 3, for different charging voltages and temperatures, the expressions obtained by second-order polynomial fitting have the same quadratic term and approximately the same primary term. That is, the difference of temperature and charging voltage does not affect the quadratic term and primary term of SOC under current action, but only the constant term. Therefore, considering the principle of simplicity, second-order polynomial fitting is better. The fitting error results show that the error of the expression obtained by the first-order polynomial fitting is obviously greater than that by the second-order polynomial fitting. Therefore, considering the accuracy principle, the second-order polynomial fitting is better. Based on the above conclusions, the second-order polynomial is used to fit the relationship between SOC and charging current

Table 3 Fitting Estimation of SOC Variation with Charging Current under Constant Charging Voltage and Temperature

| Estimation index | Condition | Second-order polynomial fitting | First-order polynomial fitting |
|------------------|-----------|--------------------------------|-------------------------------|
|                  |           | SOC = −0.0008×T²+0.0270×T+0.5405 | SOC = 0.0150×I+0.5722 |
|                  |           | SOC = −0.0007×T²+0.0268×T+0.5013 | SOC = 0.0150×I+0.5328 |
|                  |           | SOC = −0.0007×T²+0.0248×T+0.4154 | SOC = 0.0140×I+0.4479 |
|                  |           | SOC = −0.0008×T²+0.0267×T+0.3307 | SOC = 0.0139×I+0.3707 |
|                  |           | SOC = −0.0007×T²+0.0261×T+0.319 | SOC = 0.0143×I+0.3190 |
|                  |           | SOC = −0.0008×T²+0.0279×T+0.4786 | SOC = 0.0151×I+0.5060 |
3.4 Fitting Analysis of the Relation between SOC and Charging Voltage, Charging Current and Temperature

Considering the conclusion of section 3.1, the relationship between SOC and charging voltage under different temperature and charging current can be obtained as follow:

\[ SOC = K \times U + f(T, I) \]  

(1)

Where \( K \) is a constant, and taking the average slope of the first order polynomial in Table 1 as follow:

\[ K = \frac{0.1025 + 0.0968 + 0.1003 + 0.1004}{4} = 0.1000 \]  

(2)

Considering the conclusion of section 3.2, we can get the relationship between SOC and temperature under different charging voltage and current as follow:

\[ SOC = a \times T^2 + b \times T + g(U, I) \]  

(3)

Where \( a \) and \( b \) are constants. The average values of the coefficients of the quadratic term and the first term of the second order polynomial in Table 2 are taken respectively as follow:

\[
\begin{align*}
    a &= \frac{0.0008 + 0.0008 + 0.0008 + 0.0008 + 0.0008 + 0.0008}{6} = 0.0008 \\
    b &= \frac{-0.0115 - 0.0115 - 0.0014 - 0.0112 - 0.0112 - 0.0112}{6} = -0.0111
\end{align*}
\]  

(4)

Considering the conclusion of section 3.3, the relationship between SOC and charging current under different charging voltage and temperature can be obtained as follow:

\[ SOC = c \times I^2 + d \times I + w(U, T) \]  

(5)

Where \( c \) and \( d \) are constants. The average values of the coefficients of the quadratic term and the first term of the second-order polynomial in Table 3 are taken respectively as follow:

\[
\begin{align*}
    c &= \frac{-0.0008 - 0.0007 - 0.0007 - 0.0008 - 0.0007 - 0.0008}{6} = -0.0007 \\
    d &= \frac{0.0270 + 0.0268 + 0.0248 + 0.0267 + 0.0261 + 0.0279}{6} = 0.0266
\end{align*}
\]  

(6)

The relationship between SOC and charging current, charging voltage and temperature can be obtained by using formula (1), (3) and (5) as follow:

\[ SOC = K \times U + a \times T^2 + b \times T + c \times I^2 + d \times I + e \]  

(7)

Among them, \( K, a, b, c,d \) and \( e \) are all constants, and \( K, a, b, c \) and \( d \) are formulated as (2), (4), (6) respectively. \( e \) can be obtained by formula (8) ~ (11). Making

\[ SOC_{-1} = K \times U + a \times T^2 + b \times T + c \times I^2 + d \times I \]  

(8)

So according to formula (7) and (8), there are

\[ SOC = SOC_{-1} + e \]  

(9)

Considering the interference caused by the actual measurement, the expectations on both sides of equation (9) are obtained as follow:

\[ e_0 = E(e) = E(SOC) - E(SOC_{-1}) \]  

(10)

That is
From this we can get $e_0 = 0.0145$. The variance of $e$ is calculated as $3.7428 \times 10^{-5}$, so the estimation is reliable. That is $e \approx e_0 = 0.0145$.

### 3.5 The Three-Dimensional Variable Estimation Error Analysis of SOC

The measured $SOC_1$ is compared with the estimated $SOC_2$ by fitting formula (7). The relative error obtained by the relative error formula (12) is plotted as a curve, as shown in Fig. 8. The figure shows that the estimated error accuracy of this method is less than 2%.

$$\eta = \frac{|SOC_2 - SOC_1|}{SOC_1} \times 100\%$$

### 3.6 Two-Dimensional Variable Surface and Contour Distribution of SOC at a Certain Charging Voltage

Considering that the effect of charging voltage on SOC is linear, the corresponding SOC value can be obtained by simple surface translation. Fig. 9 shows the two-dimensional variable surface of SOC with charge current and temperature obtained from equation (7) when charging voltage $U = 3.3V$. Fig. 10 shows the contour distribution of SOC under different charging currents and temperatures obtained from equation (7) when charging voltage $U = 3.3V$. 

$$e_i = \frac{\sum (SOC_i - SOC_{i-1})}{n}$$

(11)
4 Conclusion
A large number of charging experiments have been carried out at different temperatures, terminal voltages and currents. The experimental data are analyzed in depth. By using the method of control variables, the estimating three-dimensional variables analytical formulas between SOC of lithium iron phosphate batteries and temperature, terminal voltage and terminal current were found. The test results show that this method can make SOC estimation error less than 2%. Because of its simplicity and high accuracy, it is feasible to apply this method to SOC estimation in practical production.

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