Effects of friction on modes in collisional multicomponent plasmas

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Abstract. The friction in multi-component plasmas is discussed, and its effects on ion acoustic (IA) and drift waves. In the case of the IA wave it is shown that the friction between the two species has no effect on the mode in a quasi-neutral plasma. Using the Poisson equation instead of the quasi-neutrality reveals the possibility for an instability driven by the collisional energy transfer. However, the different starting temperatures of the two species imply an evolving background. It is shown that the relaxation time of the background electron-ion plasma is, in fact, always shorter than the growth rate time. Therefore the instability is unlikely to develop. The drift wave instability is also reexamined taking into account the ion response in the direction parallel to the magnetic field lines, which appears due to friction with electrons and which can not be omitted in view of the momentum conservation. A modified instability threshold is obtained. In plasmas with dominant electron collisions with neutrals, the instability threshold is shifted towards higher frequencies, compared to the case of dominant electron collisions with ions. The difference between the two cases vanishes when the ion sound response is negligible, i.e., when the instability threshold disappears, and both ions and neutrals react to the electron friction in the same manner. The results obtained here should contribute to the definite clarification of some contradictory results obtained in the past.

1. Introduction
Plasmas both in the laboratory and in space are frequently in the state of partial thermodynamic equilibrium (i.e., with an initial temperature disparity of the plasma constituents). Collisions in such plasmas will after some time eventually result in equal temperatures of the species, implying an evolving plasma. There exists a long standing controversy in the literature, which deals with the effects of this temperature disparity on the perturbations in such an evolving plasma, more precisely on the ion acoustic (IA) waves. In Ref. [1] it is claimed that the corresponding energy transfer may result in the instability of the acoustic mode at large wavelengths (within the quasi-neutrality limit), and that this growth may be described within the fluid theory. The necessary condition for the instability obtained in Ref. [1] for an electron-ion plasma is, in fact, very easily satisfied because it requires only a very small temperature difference between the two species (electrons and ions), viz. $T_e > 4T_i/3$. This instability condition is obtained
by using the energy equations including the source/sink terms originating from the collisional transfer, together with the corresponding friction force terms in the momentum equations. The sufficient instability condition is stronger because of additional dissipative effects, like viscosity and thermal conductivity. However, the current-less instability described in Ref. [1] is based on a model which disregards the same temperature disparity in the description of the spatially homogeneous background, which, due to the same reasons, must be time evolving. In other words, the effects of collisions in the background plasma have been explicitly neglected. Note that because of the time evolution, the term background is used instead of the equilibrium. These effects of collisions have been discussed in Ref. [2], published one year after Ref. [1], and for the same quasi-neutrality case. There, it is claimed that there is no instability for any temperature ratio of the two plasma components, and moreover, that this holds even in a current-carrying plasma, as long as the difference between the electron and ion equilibrium velocity remains below the sound speed. All that was needed to come to that conclusion was to let the background plasma evolve freely in the presence of the given temperature difference. However, we observe that Ref. [2] has apparently remained almost unnoticed by researchers, in contrast to the widely cited Ref. [1]. In the present work, this controversy is revisited for any two-component plasma.

Essential for the problem is the energy equation describing the temperature variation. In the simplified form that we shall use, it contains only the collisional energy transfer source/sink term on the right-hand side. This simplified form is used for clarity only because, in the absence of currents, that term alone is supposed to yield an instability.

Regarding the drift wave, it is known that it can be excited by electron collisions. In this case, the usual relation between the electron perturbed density and the perturbed potential becomes modified due to the presence of the collisional term, so that the potential lags behind the density [3, 4]. The effect appears regardless whether the electrons collide, in their predominant motion along the magnetic field vector, with ions or with neutrals [5, 6]. Compared with the kinetic instability (which is due to the inverse electron Landau damping effect, that appears because the mode frequency is slightly below the diamagnetic frequency), the collisional instability is dominant provided that the electron parallel mean-free path is smaller than the parallel wavelength. Hence the interest in the drift modes with very large parallel wave-lengths and relatively short perpendicular wave-lengths. The modification in the perturbed electron number density is due to the friction, which is typically kept in the electron momentum equation only. However, the momentum conservation requires the presence of the corresponding term in the ion momentum too. Here we perform derivations with such a full friction force term for some simple cases in order to demonstrate the differences introduced by this friction-induced response of the heavier species.

2. Friction in ion acoustic mode

In view of the controversy mentioned above, here we give some details following Braginskii [7] where the energy equation for any species $a$ is given in the form:

$$\frac{3}{2} n_a \frac{\partial T_a}{\partial t} + n_a \bar{v}_a \cdot \nabla \bar{v}_a + \frac{3}{2} n_a (\bar{v}_a \cdot \nabla) T_a = Q_a. \tag{1}$$

The corresponding equation for the species $b$ has the same shape, but with a minus sign on the right-hand side. We use the Landau formula for the energy transfer source/sink term [8]

$$Q_a = 3 m_b \nu_{ba} n_b (T_b - T_a) / m_a, \tag{2}$$

where $\nu_{ba}$ is given by $L_{ba} = \log(r_d / b_0)$, $r_d = r_{da} r_{db} / (r_{da}^2 + r_{db}^2)^{1/2}$, $r_{dij} = v_{rij} / \omega_{pj}$, and $b_0 = [|g_a q_b| / (4 \pi e_0)] / [3(T_a + T_b)]$ is the impact parameter.
2.1. Non-evolving background
The two perturbed energy equations without the evolving background effects, corresponding to the model from Ref. [1], are:

\[ \frac{\partial T_{(a,b)}}{\partial t} + 2\frac{2}{3} T_{(a,b)} \nabla \cdot \vec{v}_{(a,b)} = \pm 2 \frac{m_b}{m_a} \nu_{ab} (T_{b1} - T_{a1}) \pm 2 \nu_{ba} \left( \frac{m_b}{m_a} (T_{b0} - T_{a0}) \right) n_{b1}, \]

Here, the minus sign applies to the species b.

The momentum equation which we use throughout the text for the species a is of the form

\[ m_a n_a \frac{\partial \vec{v}_a}{\partial t} = \pm q_a n_a \nabla \phi - \nabla (n_a T_a) - m_a n_a \nu_{ab} (\vec{v}_a - \vec{v}_b), \]

and the continuity equation has its standard form. Similar equations are used for the species b, where the friction term is of the form \( \vec{F}_{fb} = -m_b n_b \nu_{ba} (\vec{v}_b - \vec{v}_a). \)

In the case of quasi-neutral perturbations, the two number densities \( n_{(a,b)1} \) are calculated from the continuity equations and are made equal. The dispersion equation in that case reads:

\[ \left( \omega + \frac{i 4 m_b \nu_{ba}}{m_a} \right) \left( \omega^2 - \frac{5 k^2 T_{a0} + T_{b0}}{3} \frac{1}{m_a + m_b} \right) = 0. \] (3)

Hence, even using the same model as in Ref. [1] there is neither an instability nor damping of the acoustic mode, regardless of the ratio \( T_{a0}/T_{b0}. \)

Observe that the momentum conservation condition \( \nu_{ab} = m_b n_b \nu_{ba}/(m_a n_a) \) is nowhere used in the derivation of Eq. (3). This is because the friction terms vanish in any case. In fact, from the two continuity equations we have the velocities \( \nu_{a1} - \nu_{b1} = 0 \) (i.e., \( \nu_{a1} = \nu_{b1} \)) and \( \nu_{ba} = \nu_{ab} \).

Using this fact, we have obtained in the model from Ref. [1] that the friction completely cancels the semi-neutral conditions. Otherwise, for a varying charge we would have \( \nu_{a1} - \nu_{b1} = \omega n_{a1}/(k n_{a0})[1 - z_a z_b]/(z_b z_a) \), where \( z_j \) denotes the charge number, and therefore the friction effects would remain. Further in the text we assume singly charged species.

2.2. Isothermal perturbations
The derivations are now repeated for isothermal quasi-neutral perturbations. Keeping the full friction force \( \vec{F}_f \) in both momentum equations, and within the same quasi-neutrality limit, yields a real dispersion equation \( \omega^2 = k^2 (T_{a0} + T_{b0})/(m_a + m_b) \). Hence, in the given limit the collisions (through friction) do not affect the isothermal ion acoustic mode.

This fact is usually overlooked in the literature, and in the limit \( m_a \gg m_b \), a typical mistake is that the friction is kept for the lighter species only, in the incomplete simplified form \( \vec{F}_f = -m_b n_b \nu_{ba} \vec{v}_b \). For an electron-ion plasma this gives the incorrect result \( \omega = \pm k (\mu_e^2 + \mu_i^2)^{1/2} - \nu_{ei}/2 \).

We stress again that the collisions appear in Eq. (3) only from the energy equations, yet they do not affect the IA mode.

Using the Poisson equation instead of quasi-neutrality, for isothermal perturbations we obtain coupled and damped IA and Langmuir waves

\[ \omega^4 + i(\nu_{ab} + \nu_{ba}) \omega^3 - \left[ k^2 \left( \frac{v^2}{T_a} + \frac{v^2}{T_b} \right) + \omega^2 \right] \omega^2 - \omega^2 \left( \nu_{ab} \frac{v^2}{T_b} + \nu_{ba} \frac{v^2}{T_a} \right) = 0. \] (4)

In the low frequency limit \( \omega \ll \omega_{p(a,b)} \) and for an e-i plasma, from Eq. (4) we have

\[ \omega = \pm k v_s \left( 1 - r^2_{de} k^2 \frac{\omega^2 v_s^2}{v_e^2} \right)^{1/2} - i \nu_{ei} r^2_{de} k^2. \] (5)

The IA mode is damped and the damping is \( k \)-dependent. Here, \( v_s^2 = \omega^2 + v^2_{ei} \) and we have used the momentum conservation \( \nu_{ei} = m_e \nu_{ei}/m_i. \)
2.3. Evolving plasma

From Eq. (1) it is seen that the background temperature is also evolving in time as

$$ \frac{\partial T_{(a,b)0}}{\partial t} = \pm 2 \frac{m_b}{m_a} \nu_{ba} (T_b - T_a) \tag{6} $$

Keeping the collision frequencies constant (the approximation discussed below), this gives the temperatures for the two species

$$ T_{(a,b)0}(t) = [T_{(a,b)0}(1 + \exp(-4\nu_{ab}t)) + \hat{T}_{(a,b)0}(1 - \exp(-4\nu_{ab}t))] / 2. $$

It is seen that they evolve towards the common value \((\hat{T}_{a0} + \hat{T}_{b0})/2\).

On the other hand, solving Eq. (6) numerically, with time dependent collision frequencies (2) gives a slightly faster relaxation for the two temperatures. To get a feeling on the relaxation time scale, this is presented in Fig. 1 by taking \(n_0 = 10^{18} \text{ m}^{-3}\) and \(\hat{T}_{a0} = 0.1 \text{ eV}, \hat{T}_{b0} = 3\hat{T}_{a0}\).

![Figure 1](image1.png)  
**Figure 1.** Approximative (full lines), and exact relaxation with time-dependent collision frequencies (dashed lines) of the background plasma temperatures (6).

Eq. (6) is to be used in the linearization of Eq. (1), which in the case \(n_{a0} = n_{b0} = n_0\) yields:

$$ \frac{\partial T_{a1}}{\partial t} + \frac{2}{3} T_{a0} \nabla \cdot \vec{v}_{a1} = +2 \frac{m_b}{m_a} \nu_{ba} (T_{b1} - T_{a1}) - 2 \nu_{ba} \frac{m_b n_{a1} - n_{b1}}{n_0} (T_{b0} - T_{a0}) \tag{7} $$

The corresponding equation for the component \(b\) is

$$ \frac{\partial T_{b1}}{\partial t} + \frac{2}{3} T_{b0} \nabla \cdot \vec{v}_{b1} = -2 \frac{m_b}{m_a} \nu_{ba} (T_{b1} - T_{a1}) . \tag{8} $$

We stress that Eq. (3) is obtained also by using Eqs. (7, 8) in the quasi-neutral limit (implying that the last term in Eq. (7) is omitted). Hence, the IA mode appears unaffected by friction in the quasi-neutral limit even if the energy equations are used, and if the background is described correctly as evolving.

We now use the two energy equations (7, 8) with the Poisson equation. The dispersion equation becomes [10]

$$ \omega^2 + i \nu_{ba} \left(1 + \frac{5m_b}{m_a}\right) \omega^2 - \omega^4 \left[\frac{5}{3} k^2 \left(v_{i_a}^2 + v_{i_b}^2\right) + \omega_{pa}^2 + \omega_{pb}^2 + 4 \nu_{ba}^2 \frac{m_b}{m_a} \left(1 + \frac{m_b}{m_a}\right)\right] $$

![Figure 2](image2.png)  
**Figure 2.** The growth rate of the IA mode in electron-proton plasma with \(n_0 = 10^{18} \text{ m}^{-3}\) and for several values of \(T_e/T_i\).
Solving for the IA mode yields approximately
\[ \omega^2 \left[ \frac{25}{9} k^2 \nu_{ba}^2 v_{T_a}^2 + \frac{5 k^2}{3} v_{T_a}^2 + \nu_{pa}^2 + \frac{v_{T_b}^2}{\nu_{pa}} \right] + i \omega^2 \left[ \frac{25}{9} k^2 \nu_{ba}^2 v_{T_a}^2 \nu_{pa}^2 - \frac{5 k^2}{3} v_{T_a}^2 \nu_{pa}^2 + \frac{v_{T_b}^2}{\nu_{pa}} \nu_{pa}^2 \right] + \frac{8 m_b^2}{3 m_a} + \frac{2 v_{T_b}^2}{\nu_{pa}^2} + \frac{v_{T_b}^2}{\nu_{pa}^2} \nu_{pa}^2 \right] = 0. \] (9)

The corresponding imaginary part of the frequency is:
\[ \gamma = \frac{1}{2} \left( \omega_{pa}^2 + \omega_{pb}^2 + (5/3) k^2 (v_{T_a}^2 + \nu_{pa}^2) - 2 \omega_{T_a}^2 \right) \left\{ \nu_{ab} \left( \omega_{T_a}^2 - (5/3) k^2 \nu_{pa}^2 \right) \right\} \left( 1 + (4/3) k^2 v_{T_a}^2 / \omega_{T_a}^2 \right) \]
\[ + \nu_{ab} \left( \omega_{T_a}^2 - (5/3) k^2 \nu_{pa}^2 \right) \left[ 1 + (4/3) \nu_{ab} k^2 v_{T_a}^2 / (\nu_{ba} \omega_{T_a}^2) \right] + 2 \left( 1 - T_{ab}/T_{ba} \right) \nu_{ab} k^2 c_s^2 / \omega_{T_a}^2 \left[ \omega_{T_a}^2 - \nu_{ab} k^2 v_{T_a}^2 \right] \]
\[ - (5/3) k^2 v_{T_b}^2 - (8 \nu_{ab} \nu_{ba} / \omega_{T_a}^2) \left( \omega_{T_a}^2 - k^2 v_{T_a}^2 / \omega_{T_a}^2 \right) + (8 \nu_{ab} / \omega_{T_a}^2) \left( \omega_{T_a}^2 - k^2 v_{T_a}^2 \right) \right\}. \] (11)

In principle, Eq. (11) reveals the possibility for a growing IA mode if the Poisson equation is used instead of the quasi-neutrality, in a time-evolving background plasma. For example, this can be easily demonstrated in the limit of negligible terms originating from the last term in Eq. (7).

This is permissible on condition \(|(T_{a1} - T_{b1})/(n_{a1} - n_{b1})| \gg |T_{a0} - T_{b0}|/n_0, \) or in an alternative form, \(|(T_{a1} - T_{b1})/(T_{a0} - T_{b0})| \gg \nu_{ab}^2 k^2 / q_b \phi_1 / T_{b0}. \) In that limit, the numerical solution of Eq. (9) yields the growth-rate of the IA mode in an electron-ion plasma that is presented in Fig. 2. Here, \( n_0 = 10^{18} \text{ m}^{-3} \) and we take several values of \( T_e/T_i, \) where \( T_i = 0.1 \text{ eV.} \) The growth rate increases with \( T_e/T_i \) but only up to \( T_e/T_i \approx 3. \) For even higher values of the temperature ratio the instability ceases, this is represented by the dashed \( (T_e/T_i \approx 10) \) line.

However, we stress that the system evolves in time. Thus, in order to have a reasonably fast growth of the perturbations, the following condition must be satisfied [cf. Eq. (6)]:
\[ \gamma_r \equiv 2 (m_b/m_a) \nu_{ba} \ll \gamma. \] (12)

Here, \( \gamma_r^{-1} \) determines the relaxation time for the background. Taking the electron-ion case like in Ref. [1] and the corresponding self-evident conditions \( m_b \ll m_a, T_{a0} < T_{b0}, k^2 v_{T_a}^2 < \omega_{T_a}^2 < \omega_{pa}^2 < \omega_{pb}^2, \) from Eq. (11) to the leading order terms we obtain
\[ \gamma - \gamma_r \approx - \nu_{ba} \right\} \left\{ \nu_{ba} \omega_{T_a}^2 - k^2 v_{T_a}^2 \omega_{T_a}^2 \right\} + 2 k^2 c_s^2 \omega_{T_a} \right\} \left( \omega_{T_a}^2 - k^2 v_{T_a}^2 \right) + \frac{8 \nu_{ab}^2}{\omega_{T_a}^2} \right\}. \] (13)

Hence, because always \( \omega_{pa} \geq \omega_{T_a}, \) here we have
\[ \gamma < \gamma_r, \] (14)
i.e., the system relaxes on a time scale that is (much) shorter than the eventual growth time of the perturbations. Consequently the assumed instability actually can not develop.
3. Drift wave and friction

In the presence of electron collisions the drift wave can be growing. The friction force term in the electron parallel momentum equation is usually written in the form \(-m_en_e\nu_{ej}\vec{v}_e\), where \(j = i, n\). As a result one obtains a standard phase shift in the electron Boltzmann distribution \(n_1/n_0 = (e\phi_1/\kappa T_e)(1 - i\delta)\), that is responsible for the mode growth. Here and further in the text the temperature is in K.

However, the conservation of momentum implies that the friction term in the electron momentum equation should read \(-m_en_e\nu_{ej}(\vec{v}_e - \vec{v}_j)\) even when conventional criteria for a negligible parallel dynamics of heavier particles are fulfilled, so that the corresponding momentum component of the heavier species includes the friction term \(-m_jn_j\nu_{je}(\vec{v}_j - \vec{v}_e)\).

Below, we perform derivations with such a full friction force term for some simple cases \([11]\) in order to demonstrate the differences introduced by this friction-induced response of the heavier species.

3.1. Plasma with dominant collisions between charged particles

We stress that this case may also include a rather weakly ionized plasma with \(n_0 \ll n_i\) (the index 0 here and below denotes the electron or ion equilibrium quantities). This is because of the much larger cross section for collisions between charged particles. To have dominant collisions with protons in a plasma containing electrons, protons and neutral atoms, the electron number density should satisfy the condition

\[
n_0 > \frac{3\sigma_{en}(4\pi\epsilon_v\kappa T_e)^2}{(8\pi)^{1/2}e^4L_{ei}}.
\]  

(15)

Here we used the standard notation, and \(L_{ei}\) is the Coulomb logarithm. Taking as an example \(T_e = 10^4\) K, which gives \([12, 13]\) \(\sigma_{en} = 2.5 \cdot 10^{-19}\) m\(^2\) (here \(L_{ei} = 6\) for \(n_0 = 10^{18}\) m\(^{-3}\)), it turns out that the electron-ion collisions are more frequent than the electron-neutral collisions provided that \(n_0/n_i > 0.009\). For the given temperature this is close to well known estimate \([14]\) showing that, in terms of electron collisions, an ion is equivalent to \(3.4 \cdot 10^5(300/T_e)^2 \approx 300\) neutral atoms.

We use the continuity equation for electrons and ions placed in an external magnetic field \(\vec{B}_0 = B_0\vec{e}_z\)

\[
\frac{\partial n_j}{\partial t} + \nabla_\perp(n_j\vec{v}_{\perp j}) + \nabla_\parallel(n_j\vec{v}_{\parallel j}) = 0, \quad j = e, i,
\]  

(16)

The linearized perpendicular velocities of electrons and ions are given by

\[
\vec{v}_{\perp e1} = \frac{1}{B_0}\vec{e}_z \times \nabla_\perp \phi_1 - \frac{\nu_T^2 e}{\Omega_e} \vec{e}_z \times \nabla_\perp n_{e1}/n_{e0}, \quad \nu_T^2 = \kappa T_e/m_e,
\]  

(17)

\[
\vec{v}_{\perp i1} = \frac{1}{B_0}\vec{e}_z \times \nabla_\perp \phi_1 - \frac{1}{\Omega_i B_0} \frac{\partial}{\partial t} \nabla_\perp \phi_1.
\]  

(18)

The electron parallel velocity is determined from

\[
0 = e\sigma_0 \frac{\partial \phi_1}{\partial z} - \kappa T_e \frac{\partial n_1}{\partial z} - m_en_0\nu_{ei}(v_{ez1} - v_{iz1}),
\]  

(19)

and the ion velocity from

\[
\frac{\partial v_{iz1}}{\partial t} = -\frac{e}{m_i} \frac{\partial \phi_1}{\partial z} - \nu_{ie}(v_{iz1} - v_{ez1}).
\]  

(20)
In 'standard' derivations the ion parallel velocity is only due to the parallel component of the perturbed electric field, while the friction induced term in (20) is omitted with the usual excuse of the huge difference in mass between the two species. Similarly, the corresponding ion term is neglected in Eq. (19).

In this case, the perturbed ion number density is described by [3, 4]

\[
\frac{n_1}{n_0} = \left( \frac{\omega_s}{\omega} + \frac{k_y^2 \rho_s^2}{\omega^2} - k_z^2 \rho_s^2 \right) \frac{e\phi_1}{\kappa T_e}. \tag{21}
\]

The electron number density is calculated from Eqs. (16), (19) yielding the dispersion equation:

\[
\omega^2 (1 + k_y^2 \rho_s^2) - \omega_s \omega - k_z^2 \nu_s^2 + \frac{i\omega}{k_z^2 D_z} \left[ \omega (\omega - \omega_s) - k_z^2 \nu_s^2 \right] = 0, \tag{22}
\]

\[
D_z = \frac{v_{Te}^2}{\nu_{ei}}, \quad \omega_s = -\frac{k_y \kappa T_e}{eB_0} \frac{n_0'}{n_0}, \quad \rho_s = \frac{c_s}{\Omega_i}.
\]

Here, we have taken \( \nabla n_0 = \vec{e}_z d n_0 / dx \), the perturbations are assumed to be of the form \( \sim f(x) \exp(-i\omega t + ik_y y + ik_z z) \), and we work in the frame of a local approximation. The imaginary part of the frequency is

\[
\omega_i \approx \frac{\omega^2}{k_z^2 D_z} \frac{\omega_s^2 k_y^2 \rho_s^2 - k_y^2 \nu_s^2}{\omega^2 (1 + k_y^2 \rho_s^2) + k_y^2 \nu_s^2}. \tag{23}
\]

Hence, there appears to be a threshold for the instability, that appears due to the ion sound response in the parallel direction.

In deriving (23) we have used the limit

\[
|\omega| \ll k_z^2 D_z, \tag{24}
\]

and assume that \( \omega \) and \( \omega_s \) are of the same order. Used for convenience, the condition (24) is in fact not always easily satisfied. Physically it describes the condition of isothermal electrons along the field lines that has been assumed. It can be rewritten as \( (\omega/k_z)/v_{Te} \ll k_z v_{Te}/\nu_{ei} \). The right-hand side gives the ratio of the electron mean free path \( v_{Te}/\nu_{ei} \) and the parallel wavelength, that in fact must be much less than unity in order to remain within a proper fluid theory (i.e., for collisions being able to maintain Maxwellian distribution).

However, a self-consistent analysis should include the full friction force effect in both, i.e., the ion and electron, parallel equations. This is simply due to the fact that the two forces are necessarily equal by magnitude. Hence, we keep Eqs. (19, 20) as they are, with the complete friction terms.

Combining the two parallel momentum equations, and using the conservation of momentum \( m_i \nu_{ie} = m_e \nu_{ei} \), yields \( v_{i+1} = k_z c_s^2 n_1/(\omega n_0) \). Instead of Eq. (21), we now have

\[
\frac{n_1}{n_0} \left( 1 - \frac{k_z^2 c_s^2}{\omega^2} \right) = \left( \frac{\omega_s}{\omega} - k_y^2 \rho_s^2 \right) \frac{e\phi_1}{\kappa T_e}. \tag{25}
\]

A procedure similar as earlier, now yields the electron number density

\[
\frac{n_1}{n_0} = \frac{\omega_s + ik_z^2 D_z}{\omega - k_z^2 c_s^2 / \omega + ik_z^2 D_z} \frac{e\phi_1}{\kappa T_e}. \tag{26}
\]
Within the same approximations as earlier (i.e., $\omega^2, \omega, k_z^2 c_s^2 \ll k_y^2 D_z^2$), the dispersion equation that we now have is:

$$\omega^2 (1 + k_y^2 \rho_s^2) - \omega^2 - k_z^2 c_s^2 + \frac{i(\omega^2 - k_z^2 c_s^2)(\omega^2 - \omega_s^2 c_s^2)}{\omega k_y^2 D_z} = 0. \quad (27)$$

The real frequency is the same as in the earlier, obtained from the real part of Eq. (27). The imaginary part of the frequency now becomes

$$\omega_i \approx \frac{\omega_y^2 k_y^2 \rho_s^2}{k_y^2 D_z} \frac{\omega_r^2 - k_z^2 c_s^2}{\omega_r^2 (1 + k_y^2 \rho_s^2) + k_z^2 c_s^2} = \frac{\omega_y^2 k_y^2 \rho_s^2}{k_z^2 D_z} \frac{\omega_s^2 \omega - k_z^2 \rho_s^2}{\omega_r^2 (1 + k_y^2 \rho_s^2) + k_z^2 c_s^2}. \quad (28)$$

We remark the obvious difference in the instability threshold in the expression (23), and the correct expression (28).

3.2. On electron-ion momentum transfer

The huge difference in mass is a typical excuse for neglecting the ion response to the friction. However, this difference in mass may be compensated by the frequent electron collisions with ions, so that sooner or later the ions start to move in the parallel direction due to the electron drag. To get a feeling on the effects of collisions and the corresponding time scales, we may discuss the following two separate cases.

a) Assuming that the following condition is satisfied $v_{Te} \gg \omega/k_z \gg c_s$, $c_s = (\kappa T_e/m_i)^{1/2}$, the ions respond in the parallel direction only through the friction. The electron velocity $V_0$ is assumed nearly constant due to the parallel electric field of the wave. From (20), assuming that the ions are initially at rest, the ion velocity, normalized to $V_0$, becomes $1 - \exp(-\nu_{ie} t)$. Taking $n_0 = 10^{12}$ m$^{-3}$, $T_e = 10^4$ K, $T_i = 2 \cdot 10^3$ K, we have $\nu_{ie} = 7 \cdot 10^3$ Hz. A simple plot of the ion velocity reveals that it becomes close to 1 already after about 0.0007 seconds.

b) Taking another extreme case where electrons initially, due to any external reason acquire a velocity $v_{e0} = V_0$, without any additional force, and where $v_{i0} = 0$. In this case the electron velocity is not kept constant, the interaction of the two fluids yields the evolution of the two velocities:

$$\vec{v}_e = \frac{\nu_{ie} V_0}{\nu_{ei} + \nu_{ie}} + \frac{\nu_{ei} V_0}{\nu_{ei} + \nu_{ie}} \cdot \exp[-(\nu_{ei} + \nu_{ie})t], \quad (29)$$

$$\vec{v}_i = \frac{\nu_{ie} V_0}{\nu_{ei} + \nu_{ie}} - \frac{\nu_{ei} V_0}{\nu_{ei} + \nu_{ie}} \cdot \exp[-(\nu_{ei} + \nu_{ie})t]. \quad (30)$$

Here, the electron and ion velocities monotonously change in time towards the common velocity (the first term on the right-hand side) $v_e \simeq V_0 m_e/m_i \ll V_0$ which, for the same parameters as above, is achieved within the time interval shorter than $10^{-6}$ sec. The characteristic time for the velocity relaxation is $\sim 1/(\nu_{ei} + \nu_{ie})$.

A real physical situation, as in the case of the drift wave discussed earlier, is expected to be somewhere in between the two extremes presented above. Hence, in spite of a huge mass difference, the collisions (friction) will force ions to move along the magnetic field lines, and, due to the same reason, the electron velocity amplitude associated with the drift wave is expected to be considerably smaller.
3.3. Dominant collisions with neutrals

The electron parallel momentum Eq. (19) now reads
\[ 0 = e n_0 \frac{\partial \phi_1}{\partial z} - \kappa T_e \frac{\partial n_1}{\partial z} - m_e n_0 \nu_{en} (v_{e\perp 1} - v_{n\perp 1}). \]  
(31)

The ion dynamics is the same as above, so we use Eq. (21). The dynamics of neutrals is completely described by
\[ \frac{\partial v_{n\perp 1}}{\partial t} = -\nu_{ne} (v_{n\perp 1} - v_{e\perp 1}). \]  
(32)

This is used in Eq. (31), with the momentum conservation condition that now reads \( m_n n_0 \nu_{ne} = m_e n_0 \nu_{en} \), yielding
\[ v_{e\perp 1} = \frac{i a k_z v_e^2}{\nu_{en}} \left( \frac{\epsilon \phi_1}{\kappa T_e} - \frac{n_1}{n_0} \right), \quad a = 1 + \frac{i \epsilon \nu_{en}}{\omega}. \]  
(33)

Here, \( \epsilon = m_e n_0 / (m_n n_a) \), and this is a small quantity for any plasma. For instance, for an electron-proton plasma in a hydrogen gas \( m_n = m_i = m_p \), it is of the order \( 10^{-6} \) or less. Equation (33) is used in the electron continuity yielding
\[ \frac{n_1}{n_0} = \frac{\omega_e + i a k_z^2 D}{\omega + i a k_z^2 D}, \quad D = \frac{v_e^2}{\nu_{en}}. \]  
(34)

In the case \( \omega, \omega_e \ll k_z^2 D \), Eq. (34) can be written as
\[ \frac{n_1}{n_0} = \left[ 1 + \frac{\epsilon^2 v_{en}^2}{\omega^2} - \frac{\epsilon^2 v_{en}^2}{\omega} \frac{\omega_e + \omega_s}{k_z^2 D} + \frac{i (\omega - \omega_s)}{k_z^2 D} \right] \left( 1 + \frac{\epsilon^2 v_{en}^2}{\omega^2} - \frac{2 \epsilon v_{en}}{k_z^2 D} \right)^{-1}. \]  
(35)

We have \( 1 \gg 2 \epsilon v_{en}/k_z^2 D \gg \epsilon^2 v_{en}^2/\omega^2 \), the real part is therefore very close to unity, while the imaginary part is simply \( i (\omega - \omega_s)/k_z^2 D \). Consequently, combining Eq. (35) with Eq. (21), the real part of the frequency appears described as earlier, while the imaginary part becomes
\[ \omega_i \simeq \frac{\omega_e^2}{k_z^2 D} \frac{\omega_s^2 k_g^2 - k_z^2 c_s^2}{\omega_e^2 (1 + k_g^2 \rho_s^2) + k_z^2 c_s^2}. \]  
(36)

Compared to the previously discussed \( e - i \) collision case (28), it is seen that i) in both cases the threshold is caused by the ion sound response, however, ii) the instability threshold in Eq. (36) is shifted towards higher frequencies (because \( k_g \rho_s \) is usually less than unity).

Although the two cases describe two physically different plasma environments, the explanation for case ii) should be as follows. The lower threshold frequency in the \( e - i \) case implies that electrons experience a larger amount of collisions with ions within a wave period, which is in fact necessary to compensate for the ion movement in the parallel direction (due to the parallel electric field). This is because moving ions (in the same direction as electrons) represent a less efficient barrier for electron parallel motion and, in order to have the necessary phase shift between the density and potential, the electrons should have more collisions for the instability to take place. On the other hand, in the \( e - n \) case, a higher frequency (equivalent to a smaller amount of collisions) for the instability to develop is possible because neutrals are less movable in the parallel direction (they do not react to the parallel electric field), and therefore they represent a more effective barrier.
4. Summary
The results obtained for the IA case can be summarized as follows. i) The friction does not affect the IA mode in the limit of quasi-neutral perturbations. ii) Even using the non-evolving model equivalent to Ref. [1], there is no instability of the IA mode, contrary to claims from Ref. [1]. iii) When the background plasma is properly described as evolving in time, and as long as the quasi-neutrality is used, collisions do not produce a growth of the ion acoustic mode. iv) When the Poisson equation is used instead of quasi-neutrality, in principle there is a possibility for a positive growth-rate of the IA mode. It appears as a combined effect of the breakdown of the charge neutrality from one side (introduced by the Poisson equation), and the heat transfer (the compressibility and advection in energy equation) from the other side, all within the background of a time-evolving plasma. However, as the equilibrium plasma evolves in time, with the relaxation time \( t \), the obtained growth time must be (much) shorter than the relaxation time. Yet, this shows to be impossible and we conclude that there is no instability in the electron-ion plasma with an initial temperature disparity if the plasma evolves freely.

In the case of the drift wave, the self-consistent inclusion of the momentum conservation in the ion and electron equations, which originates from the collisions between the two fluids, yields a different instability threshold that, to the best of our knowledge, has not been discussed in the literature so far. The correct expressions (28) and (36) should be used for estimates of the growth rate and the instability threshold for the collisional drift mode.

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