Microturbulence-mediated route for energetic ion transport and Alfvénic mode intermittency in tokamaks

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Abstract

We report on a theoretical discovery of new regimes of Alfvén eigenmode (AE) induced fast ion transport in tokamak plasmas, where microturbulence plays the role of a mediator of fast ion relaxation. Coulomb collisional scattering alone leads to small AE amplitudes and does not reproduce the steady state regimes observed in experiments. We show that in nonlinear regimes the effective pitch angle scattering due to microturbulence can lead to steady state AE amplitude evolution. This indicates a new route for fast ion losses, which is beyond the scenarios described in “Energetic ion transport by microturbulence is insignificant in tokamaks” [D. C. Pace et al., Phys. Plasmas 20 (2013) 056108]. As a result, microturbulence can significantly increase the amplitude of AEs in predictive simulations of burning plasma experiments such as ITER.
The success of the next generation of fusion devices relies on their ability to confine fusion alpha particle products long enough to transfer a substantial fraction of their energies to the reacting thermal ions. The International Thermonuclear Experimental Reactor (ITER) is predicted to have tight tolerance for fast ion losses in order to sustain burning plasmas [1]. ITER is expected to have a multitude of unstable modes in the toroidal Alfvén eigenmode (TAE) frequency range [2, 3] and, therefore, can likely lead to more global losses via resonance overlapping. It is essential, therefore, to develop efficient and robust capabilities to predict the relaxation of energetic ion component in tokamak experiments. The effect of the pitch angle scattering due to the microturbulence on the saturation amplitudes has previously been ignored, which leads to a significant underestimation of the level of activity of AEs.

This Letter shows that in fusion plasmas, the scattering frequency, \( \nu_{\chi} \), of fast ion pitch angle, \( \chi = v_{\parallel}/v \), due to Coulomb collisions is too small to bring the unstable AEs to steady state regimes observed in experiments [4]. It has been noted earlier that Coulomb scattering is not sufficient to explain the observed AE amplitudes in TFTR when the ion cyclotron resonance heating (ICRH) was applied [5]. In those experiments, strong scattering was required and was shown to be the result of applied ICRH. Subsequent analysis using a cubic amplitude evolution equation [6] helped to describe the nonlinear saturation of \( n = 2 \) TAE at \( \sim 10 \) times higher amplitude, as well as helped to evaluate the growth rate of the unstable mode.

By itself the effect of pitch angle scattering is not new and was reproduced in many publications, including more recent ones, such as [7] and [8]. If multiple unstable AEs are mediated by the microturbulence, the induced scattering can play a profound role in energetic particle (EP) relaxation. Namely, the additional scattering frequency can strongly enhance the AE amplitudes and drive them into steady state regimes that in turn account for substantial fast ion losses. Ref. [9] considered the effects of microturbulence and Alfvén waves on fast ion transport separately and concluded that the effect of the microturbulence with or without AEs is weak and does not lead to a significant experimentally observable EP transport. We show with the example of a single mode that even though the direct effect of the microturbulence is small for fast ion transport, together with AEs it offers a new route for fast ion radial transport and losses by enlarging the resonance extent, and thereby, by boosting the amplitude of each eigenmode. We show that two effects are indissociable since
the turbulence strength is key in setting the amplitudes. The amplitudes of AEs, in turn, set the EP radial transport and losses. This leads to the conclusion that the additional scattering mechanism is required, in concert with the microturbulence scattering considered in the development and validation of a criterion for whether AEs should exhibit a chirping or a quasi-steady frequency response \[10, 11\].

The pitch angle scattering self-consistently enters the quasilinear (QL) methodology via the second order differential scattering operator acting on the EP distribution function and via the broadening of the resonance layer \[12\], as discussed later. As such, the additional scattering is crucial for QL model to work properly. Most of the initial value codes, such as those recently benchmarked in linear regimes in Ref. \[13\] as well as recent ITER projection models \[2, 3\], employ a scattering operator ignoring contributions from microturbulence.

Recent experiments have shown the resilience of EP profiles to the neutral beam injection (NBI) power \[4\]. Those experiments showed the transport regulated by stochasticity and therefore the anomalous scattering can be the key in understanding the dynamics. More than ten Alfvénic modes with low amplitudes $\delta B_\theta/B \sim O(10^{-4} - 10^{-2})$ were excited in steady state regimes during the experiments, which produced a noticeable effect on EP confinement setting up “stiff” density profiles. We should note that even though we discuss a single mode saturation behavior our conclusions are important for multiple instabilities which were shown to be relevant for EP relaxation \[14\] although more detailed analysis is beyond this paper goals. Correct diffusion representation due to a single mode in the nonlinear regime is important even for the case when AE modes and their resonances overlap.

Here we apply two codes. The first one is the Resonance Broadened Quasilinear (RBQ) code validated for near-threshold conditions \[8\] expected in experiments \[4\]. And, the second code is the kinetic simulation code BOT which solves the relaxation of a bump-on-tail distribution function in a model formulation \[15, 16\].

**Formulation.** The RBQ code has been built with the goal of efficiently computing the relaxation of EP distribution function in the presence of multiple AEs \[8, 17\]. It implements the QL equations to compute the EP distribution function and relaxes it in time along the canonical toroidal momentum $P_\phi$ according to the equation which sums the diffusion operator of all the modes under consideration:

$$
\frac{\partial f}{\partial t} = \sum_{k,p,m,m'} \frac{\partial}{\partial P_\phi} D_{kp}(P_\phi; t) \frac{\partial}{\partial P_\phi} f + \left\langle \frac{1}{r} \frac{\partial}{\partial r} D_{rh} \frac{\partial(f - f_0)}{\partial r} \right\rangle + \left\langle \frac{\partial}{\partial \chi} \nu_{\chi\chi} \frac{\partial(f - f_0)}{\partial \chi} \right\rangle, \tag{1}
$$
where the diffusion coefficients due to AEs are expressed as $D_{kp}(P_\varphi; t) = \pi C_k^2(t) E^2 R_t G^{*}_{kmp} G_{kmp}$, $R_t = R_t(P_\varphi - P_{\varphi r})$ is the resonance window function analytically formulated in Ref. [12], $G_{kmp}$ are the wave-particle interaction matrix elements of the $k$-th mode of amplitude $C_k$, $m$-th poloidal harmonic, $p$-th resonant side-band, the resonance center is given by the condition $P_\varphi = P_{\varphi r}$, $\chi = v_\parallel / v$ is the pitch angle, and $\nu_{\chi\chi} = \nu_\perp (1 - \chi^2)$ for the case of Coulomb collisions with $\nu_\perp$ being the $90^\circ$ scattering frequency [18], $D_{rh}$ is the radial diffusion coefficient of energetic or hot ions due to the microturbulence [19] and $\langle ... \rangle$ denotes orbit averaging. The second order derivative terms on the RHS of Eq. (1) are the terms which need to be kept near the resonances since they are responsible for the variation of the distribution function $f$ in their vicinity where $f$ deviates the most from the initial equilibrium distribution function, $f_0$. The time scale of resonant particle dynamics near the resonance is very short, on the order of $0.1 - 0.5 \text{ msec}$, which is much shorter than the injection time or slowing down scale, which is $20 - 100 \text{ msec}$. Such time scale separation is sufficient to describe the problems prescribing the EP flux intermittency.

Equation (1) is supplemented by the equation for AE amplitudes $dC_k^2(t)/dt = 2(\gamma_{L,k} + \gamma_{d,k}) C_k^2(t)$, where local growth rates, $\gamma_{L,k} = \gamma_{L,k}(t)$, are computed at each time $t$ using the distribution function $f$ although the damping rate is fixed in time.

In tokamaks, the effect of microturbulence on the EP relaxation can be projected onto one direction with good accuracy when $n \gg 1$ and $\omega \ll \omega_c$, where $\omega_c$ is the cyclotron frequency. This is often the case in experiments. Since the canonical momentum is linearly proportional to $\chi$ and the poloidal magnetic flux function, $\psi$, $P_\varphi = e\psi / 2\pi mc - \chi vRB_\varphi / B$, one finds that $dP_\varphi|_{\psi, E} = -(vRB_\varphi / B) d\chi$ and $dP_\varphi|_{\chi, E} = \frac{e}{2\pi mc} d\psi$. From Eq. (1) it follows that the last two terms on its RHS contribute to the diffusion in $P_\varphi$ direction. Then combining them reduces Eq. (1) to

$$\frac{df}{dt} \simeq \sum_{k,p,m,m'} \frac{\partial}{\partial P_\varphi} D_{kp}(P_\varphi; t) \frac{\partial}{\partial P_\varphi} f + \left[ R_{Dh} + 1 \right] \frac{\partial P_\varphi}{\partial \chi} \frac{\partial}{\partial P_\varphi} \nu_{\chi\chi} \frac{\partial P_\varphi}{\partial \chi} \frac{\partial (f - f_0)}{\partial P_\varphi}, \quad (2)$$

where the ratio of hot particle diffusion rates in $P_\varphi$ direction due to the turbulence and collisions can be written for the case of weak radial dependence of $\nu_{\chi\chi}$ and $D_{rh}$ considered here as

$$R_{Dh} \simeq D_{rh} \left( \frac{\partial P_\varphi}{\partial \psi} \right)^2_{\chi, \chi} \left( \frac{\partial \psi}{\partial r} \right)^2_{\chi, \chi} \left( \frac{\partial P_\varphi}{\partial \chi} \right)^2_{\psi, \chi} \nu_{\chi\chi} = \frac{D_{rh} \left( \frac{e}{2\pi mc} \frac{\partial \psi}{\partial r} \right)^2_{\psi, \chi}}{\nu_{\chi\chi} \left( \frac{vRB_{\varphi}}{B} \right)^2_{\chi, \chi}} = \frac{D_{rh}}{\nu_{\chi\chi} \left( \frac{\partial \psi}{\partial r} \right)^2_{\chi, \chi}}. \quad (3)$$
Here the expression for $R_{Dh}$ is the same as in Ref.[19] except that we do not rely on large poloidal mode number and define the Larmor radius variable as $\rho_h = v/\omega_c$.

The first term on the RHS of Eqs.(12) has AE driven diffusion coefficient which is coming primarily from the window resonance function $R_l$ dependence on the canonical momentum \cite{12}. The second term in Eq.(2) is responsible for the pitch angle scattering which could be locally dominated by either Coulomb collisions (when $R_{Dh} \ll 1$) or microturbulence (when $R_{Dh} \gg 1$) induced radial diffusion since it has complicated dependencies in the phase space, $D_{rh} = D_{rh}(P_\varphi, \mathcal{E}, \mu)$ \cite{20,21}. Eq.(2) can be rewritten in formal action variables with the resonant frequency $\Omega$ being a function of three constants of the unperturbed motion \cite{22}: $P_\varphi$, magnetic moment, $\mu$, and energy, $\mathcal{E}$, which extended the collisionless QL theory originally developed by Kaufman \cite{23}. We can then rewrite it in the form

$$\frac{\partial f}{\partial t} = \frac{\pi}{2} \sum_{k,p,m,m'} \frac{\partial}{\partial \Omega_{kp}} \left| \omega^2_0 \right|^2 R(\Omega_{kp}) \frac{\partial}{\partial \Omega_{kp}} f + \nu^3_{eff} \left( \frac{\partial^2 f - f}{\partial \Omega_{kp}^2} \right), \quad (4)$$

where

$$\nu^3_{eff} \simeq [R_{Dh} + 1] \left( \frac{\partial \Omega}{\partial \chi} \right)^2 \nu^2_{\chi\chi}, \quad (5)$$

and the window, or resonance, function $R$ replaces the resonance $\delta$-function and automatically satisfies $\int_{-\infty}^{\infty} R(\Omega) d\Omega = 1$ \cite{12}.

Note that in the RBQ quasilinear methodology used here the resonance function is broadened over a characteristic width of $\Delta \Omega \simeq 2.58 \nu_{eff} \cite{12}$ whereas in BOT \cite{15,16} it is computed using the kinetic equation. Even though the RBQ one-dimensional results in comparison with the experimental data were favorable \cite{8}, they lacked AE amplitude steady state saturation. The pitch-angle scattering in those simulations was due to classical Coulomb collisions and the used diffusion rate was taken at a time of maximum amplitudes.

**Comparison between RBQ and BOT simulations.** Here we compare RBQ simulations in tokamak geometry with fully nonlinear BOT results obtained in a 1D model geometry where the kinetic equation solution scheme resolves the structures near one resonance in the Fourier space. A comparison between the QL methodology in a model geometry \cite{24} and BOT has been performed using a heuristic broadening function has shown that although QL and BOT simulations can exhibit fair qualitative agreement, quantitatively the agreement only occurs in a limited parameter range. BOT and RBQ comparison is done as close as possible by using the same input parameter of growth and damping rates and effective pitch angle scattering.
In RBQ simulations, we consider one reversed-shear Alfvén eigenmode (RSAE) with toroidal number $n = 3$ corresponding to the observed unstable mode at $t = 805 m sec$ of DIII-D discharge #159243 described in Refs. [4], which is one of the modes extensively analyzed recently in several publications [8, 13]. Its mode structure computed by the ideal MHD code NOVA [25] is shown in Fig. 1 as RSAE poloidal harmonics of $\xi \cdot \nabla \psi_\theta$ radial dependence versus the minor radius flux variable.

We have found that if the $\chi$ scattering is given by Coulomb collisions, $\nu_{\chi \chi}$, RBQ simulations (see Fig. 2b) lead to the overshoot (first maximum in time) point with quick amplitude decaying with the damping rate, $\gamma_d < 0$. The growing phase time is determined by the $\gamma_L + \gamma_d > 0$ rate. In the collisional case the effective particle source due to the scattering operator of Eq. (1) is sufficiently weak to replenish the resonant ions near the resonant region and the cycle does not repeat at later times. However if the scattering is set up to a larger value the fast ion population in the resonance region is replenished and a new growing phase emerges. In a later case a classical predator-prey interplay in the AE nonlinear dynamics outlined in Ref. [26] occurs. We should note that even though our model includes EP source through the pitch angle scattering only, because of the time separation energy slowing down contribution is much weaker (or slower) than the scattering in pitch angles.

Within the QL methodology the interplay between background damping and the scattering frequency controls the repetition rate for AE peaks. These results, shown in figure 2b, are consistent with BOT shown in Fig. 2a. Both figures are plotted for the same scattering rate values indicated on the contour map, Fig. 3 as white circles. They correspond to the nominal scattering frequency, $\nu_{eff} = 8.017 \times 10^3 sec^{-1}$ or $\nu_{Col} = [R_{Dh} + 1] \left( \frac{\partial P_e}{\partial x} \right)^2 \nu_{\chi \chi} = 8.9 sec^{-1}$.
computed by the NOVA-K code but with fixed value independent on the minor radius. The sequence of used points (going up vertically) on Fig. 3(a) are indicated. Used value in BOT, \( \nu_{eff} = 0.618 \gamma_L \) (nominal, red curves), correspond to RBQ scattering \( \nu_{Col} \) and the same rate, \( \gamma_L = 1.3 \times 10^4 \, \text{sec}^{-1} \).

In comparison with previous model studies \[24\], we show that the kinetic simulations of BOT agrees much better with RBQ for the oscillatory behavior of the Alfvénic modes. This is illustrated on Fig.2 and is due to the fact that the resonance function used in RBQ is derived self-consistently \[12\].

There most important difference between two simulations is the recovery time between the peaks is about 30 to 50% larger in RBQ for the same scattering frequency than in BOT. This is because the coefficient \( (\partial P\phi/\partial \chi)^2 \) in Eq. (2) is proportional to \( v^2 \) and with the same \( \nu_{\chi \chi} \) and estimates for the resonance velocity in RBQ (as shown below), we find larger time for the resonant particles and thus the recovery rate. What comes as a surprise is that the experimental point lies near the threshold of the existence and non-existence of steady state regimes in both RBQ and BOT simulations whereas in DIII-D discharge of interest the AE amplitudes are in a steady state regime \[4\]. As discussed in Ref. \[8\] the boundary conditions (BC) should not be physical in RBQ, which are reflective in the plasma center, but they should be the fixed value BC to ensure the expected analytic amplitude scaling \( \delta B_\theta/B \sim \nu_{eff}^2 \) \[26\].

A contour plot obtained with the BOT code is shown in Fig. 3. One notable consistency between Figs. 2 (a) and (b), is the value of the normalized nonlinear bounce frequency, \( \omega_b \), coming out of RBQ and BOT simulations. Both models fairly agree with each other when they are in near-threshold regimes.

**Microturbulence as an origin for anomalous beam ion pitch angle scattering.**

The pitch angle scattering considered recently for the problem of AE frequency chirping \[10, 11\], is proposed here as a mediator for the EP driven AE amplitude saturation. The scattering can be expressed with the help of the canonical momentum if the radial diffusion is known. However, the radial diffusion and EP pitch angle scattering are difficult to evaluate without accurate knowledge of the level of the microturbulence. Here we consider the upper and lower bounds for the scattering frequency.

Let us first compare the Coulomb scattering frequency with the scattering frequency resulting from the microturbulence acting on fast ions. Two expressions for EP diffusion
Figure 2. AE amplitude vs time from RBQ1D and BOT for different degrees of collisionality. Left figure (BOT) has the effective frequency rates (going from the bottom figure up) $0.49 \gamma_L, 0.618 \gamma_L, 0.778 \gamma_L$, and $0.98 \gamma_L$ ($\gamma_L$ is an input parameter of BOT). They correspond to the RBQ scattering rates $\nu_{Col}/2, \nu_{Col}, 2\nu_{Col}$ and $4\nu_{Col}$ of the nominal scattering frequency $\nu_{Col} = 8.9 \text{sec}^{-1}$ computed by NOVA-K (right figure). Figures a and b have the same color coding for the corresponding scattering frequencies, i.e. the red curve is the nominal (collisional) scattering frequency. We also plot a much larger value of the scattering frequency curve, $10\nu_{Col}$ for RBQ simulations as blue dashed like.

Figure 3. Contour map of AE oscillations normalized by the saturation amplitude computed by BOT simulations in units of the bounce frequency at saturation $\omega_{b,\text{sat}}$. The white dots indicate the parameters used in Fig. 2. In the purple region, the solutions have a pulsating amplitude pattern that prevents a steady state to be achieved.
coefficient exist which are projections from thermal ion heat conductivity up to the energetic particle energies. The first one is by Angioni which is the diffusion coefficient averaged over EP distribution \( D^4 \), \[ D^4 \approx \frac{2}{3} (\chi_i + \chi_e) \left[ 0.02 + 4.5 \frac{T_i}{E_{i0}} + 8 \frac{T^2_i}{E^2_{i0}} + 350 \frac{T^3_i}{E^3_{i0}} \right] \] where \( E_{i0} \) is the injection energy. It includes the diffusion transport produced by the electrostatic background plasma microturbulence. It is computed using the quasilinear microturbulence models and fitted with the help of several gyrokinetic codes. The second expression results from the electrostatic GTC simulations \[ D = D_{r,i} = \frac{2}{3} \chi_i \frac{5T_i}{E_{br}} \] where \( E_{br} \) is the fast ion energy at the resonance with the mode. This expression was successfully validated within the chirping criterion to DIII-D \[ 10, 11 \]. Both expressions need to be evaluated using realistic estimates for the mode frequency, which is upshifted due to the finite plasma pressure \[ 28, 29 \]:

\[
\sqrt{\omega_{GAM}^2 + \omega^2_T + \omega^2_{AE}} \approx \left( k_{\parallel} \pm \frac{1}{qR} \right) \chi |v|, \quad (6)
\]

where \( \omega_{GAM} \) is the geodesic acoustic modes (GAM) frequency, \( \omega_T \) is the pressure gradient contribution to the frequency shift, and \( \omega_{AE} = k_{\parallel} v_A \) is the frequency of AE eigenmode ignoring those effects. For classical TAEs \( k_{\parallel} = 1/2qR \), \( \omega_{TAE} = v_A/2qR \approx \sqrt{\omega_{GAM}^2 + \omega_T^2} \), and one can get \( |v_{\parallel}| = |v_A, v_A/3| \) resonances from this equation if the GAM frequency is negligible. In case of DIII-D, the RSAE mode upshift frequency can be small, near the GAM value, \( \omega_{RSAE} \approx \omega_{GAM} + \omega_T^2 \) and \( |k_{\parallel}| \ll \frac{1}{qR} \). So that with good accuracy we estimate for this case \( v/v_A \approx \frac{\sqrt{\omega_{GAM}^2 + \omega_T^2}}{2\omega_{TAE} |\chi|} \approx \frac{1}{2 |\chi|} \). (7)

Alternatively, it can sweep up to the TAE frequency when the resonant ion velocity goes down to \( v/v_A \approx (3 |\chi|)^{-1} \).

Extensive gyrokinetic simulations are required to evaluate \( R_{Dh} \) at each time of the discharge. Instead we infer the electron and ion thermal conductivities from TRANSP simulations for DIII-D shot \#159243 at \( t = 805 msec \) to be \( \chi_e = 2.28 m^2/sec \) and \( \chi_i = 1.27 m^2/sec \). The above projections of the thermal ion conductivity inferred from TRANSP modeling to EP effective scattering rate provide the value of \( R_{Dh} = 0.15 \) in case of Ref. \[ 20 \] expression and \( R_{Dh} = 0.65 \) in case of Ref. \[ 27 \] expression taken at \( E_{br} = E_0 \). These estimates go in Eq. \[ 5 \] and are compared with pure Coulomb collisional scattering \( \nu_{Col} = 8.9 sec^{-1} \) when
\( R_{Dh} = 0 \). More accurate gyrokinetic simulations, such as given in Ref. [30] are required.

We note thought that the above estimates imply that depending on the injection angle, \( E_{br} \) could be as low as \( m_h (v_A^2/9)/2 \) and as high as \( E_0 \).

There are other significant factors which need to be considered for better estimates of \( R_{Dh} \). The most important is the averaging over the mode structure of thermal ion and electron conductivities. TRANS analysis shows that from \( q|_{\bar{\psi}^{1/2} = 0.45} = q_{\min} \) surface towards \( \bar{\psi}^{1/2} = 0.7 \) where the RSAE structure is bounded, electron and ion thermal conductivity approximately doubles. Another factor is that both Angioni and Zhang’s projections were done in the electrostatic limit whereas the electromagnetic turbulence [21] was ignored. In our study we did not quantitatively evaluate those effects. Our results indicate that the anomalous scattering due to the microturbulence should be routinely included in experimental interpretations and can be similar or stronger than the classical Coulomb scattering.

**In conclusion**, we show that the classical Coulomb collisions are too small to provide the pitch angle scattering and to replenish the resonant ion population near the AE resonances. This means that the unstable AEs can be in a saturated steady state regime if additional scattering is involved. We show that to see such steady state regimes one needs to include additional microturbulence induced scattering which is expected to be 2-5 times stronger than the classical Coulomb scattering. This conclusion provides an alternate route for fast ion losses with regard to the arguments of Ref. [9]. We observe that microturbulence acts as a mediator of fast ion redistribution by increasing the overall effective pitch angle scattering and thereby increasing the level of saturation of AEs. This, in turn, leads to enhanced Alfvénic transport that would not occur in the absence of turbulence. Essential to our analysis is the conclusion of Ref. [31] that, sufficiently near marginal stability, the effect of scattering collisions on a single resonance dynamics is to erase the system memory so that quasilinear and nonlinear theories give the same governing evolution equation for near-threshold instabilities. In comparison with previous model studies [24], RBQ (which uses a resonance window function derived self-consistently from first principles [12]) has found much better agreement with the kinetic simulations of BOT for the oscillatory behavior of the Alfvénic modes. The route found to enhance EP redistribution is expected to significantly enhance the Alfvénic mode driven effects on fusion alphas in ITER plasmas [2,3], which in-depth consideration is beyond the scope of this paper. However in our earlier evaluations of effective pitch angle scattering [30] where Fig.9 illustrates approximately an
order of magnitude stronger scattering in the presence of micro-turbulence. We stress that the intermittency of AE in the nonlinear regime as described by the QL theory is justified by our comparison of RBQ and BOT simulations. However our analysis does not include the fast ion scattering and resonance overlaps by other AEs, which also could have similar effect as the microturbulence. Nonlinear wave-wave interaction also is not considered in this work but could be important if the amplitudes becomes significant.

ACKNOWLEDGMENTS

The authors appreciate suggestions made by H. L. Berk on the manuscript. This work was supported by the US Department of Energy under contract DE-AC02-09CH11466.

[1] A. Fasoli, C. Gormezano, H. L. Berk, B. Breizman, S. Briguglio, D. S. Darrow, N. Gorelenkov, W. W. Heidbrink, A. Jaun, S. V. Konovalov, R. Nazikian, J.-M. Noterdaeme, S. Sharapov, K. Shinohara, D. Testa, K. Tobita, Y. Todo, G. Vlad, and F. Zonca, Progress in the ITER Physics Basis Chapter 5: Physics of energetic ions 2007 Nucl. Fusion 47, S264.
[2] M. Fitzgerald, S. E. Sharapov, P. Rodrigues, and D. Borba, Nucl. Fusion 56, 112010 (2016).
[3] M. Schneller, P. Lauber, M. Brüdgam, S. D. Pinches, and S. Günter, Plasma Phys. Control Fusion 58, 014019 (2016).
[4] C. S. Collins, W. W. Heidbrink, M. E. Austin, G. J. Kramer, D. C. Pace, C. C. Petty, L. Stagner, M. A. Van Zeeland, R. B. White, Y. B. Zhu, and The DIII-D team, Phys. Rev. Letters 116, 095001 (2016).
[5] K. L. Wong, R. Majeski, M. Petrov, J. H. Rogers, G. Schilling, J. R. Wilson, H. L. Berk, B. N. Breizman, M. Pekker, and H. V. Wong, Phys. Plasmas 4, 393 (1997).
[6] H. L. Berk, B. N. Breizman, and M. S. Pekker, Phys. Rev. Letters 76, 1256 (1996).
[7] C. Slaby, A. Könies, R. Kleiber, and J. M. García-Regaña, Nucl. Fusion 58, 082018 (2018).
[8] N. N. Gorelenkov, V. N. Duarte, C. S. Collins, M. Podestà, and R. B. White, Phys. Plasmas 26, 072507 (2019).
[9] D. C. Pace, M. E. Austin, E. M. Bass, R. V. Budny, W. W. Heidbrink, J. C. Hillesheim, C. T. Holcomb, M. Gorelenkova, B. A. Grierson, D. C. McCune, G. R. McKee, C. M.
Muscatello, J. M. Park, C. C. Petty, T. L. Rhodes, G. M. Staebler, T. Suzuki, M. A. Van Zeeland, R. E. Waltz, G. Wang, A. E. White, Z. Yan, X. Yuan, and Y. B. Zhu, Phys. Plasmas 20, 056108 (2013).

[10] V. N. Duarte, H. L. Berk, N. N. Gorelenkov, W. W. Heidbrink, G. J. Kramer, R. Nazikian, D. C. Pace, M. Podestà, B. J. Tobias, and M. A. Van Zeeland, Nucl. Fusion 57, 054001 (2017).

[11] V. N. Duarte, H. L. Berk, N. N. Gorelenkov, W. W. Heidbrink, G. J. Kramer, R. Nazikian, D. C. Pace, M. Podestà, and M. A. Van Zeeland, Phys. Plasmas 24, 122508 (2017).

[12] V. N. Duarte, N. N. Gorelenkov, R. B. White, and H. L. Berk, Phys. Plasmas 26, 120701 (2019).

[13] S. Taimourzadeh, E. M. Bass, Y. Chen, C. Collins, N. N. Gorelenkov, A. Könies, Z. X. Lu, D. A. Spong, Y. Todo, M. E. Austin, J. Bao, M. Borchardt, A. Bottino, W. W. Heidbrink, Z. Lin, R. Kleiber, A. Mishchenko, L. Shi, J. Varela, R. E. Waltz, G. Yu, W. L. Zhang, and Y. Zhu, Nucl. Fusion 59, 066006 (2019).

[14] H. L. Berk, B. N. Breizman, J. Fitzpatrick, and H. V. Wong, Nucl. Fusion 35, 1661 (1995).

[15] M. K. Lilley, B. N. Breizman, and S. E. SharaPov, Phys. Plasmas 17, 092305 (2010), http://dx.doi.org/10.1063/1.3486535.

[16] M. K. Lilley, BOT manual, see https://code.google.com/archive/p/bump-on-tail or https://github.com/mklilley/BOT.

[17] V. N. Duarte, Quasilinear and nonlinear dynamics of energetic-ion-driven Alfvén eigen-modes, http://www.teses.usp.br/teses/disponiveis/43/43134/tde-01082017-195849/, Ph.D. thesis, University of São Paulo, Brazil (2017).

[18] R. J. Goldston, D. C. McCune, H. H. Towner, S. L. Davis, R. J. Hawryluk, and G. L. Schmidt, J. Comput. Phys. 43, 61 (1981).

[19] J. Lang and G.-Y. Fu, Phys. Plasmas 18, 055902 (2011).

[20] W. Zhang, Z. Lin, and L. Chen, Phys. Rev. Lett. 101 (2008).

[21] T. Hauff, M. J. Pueschel, T. Dannert, and F. Jenko, Phys. Rev. Lett. 102, 075004 (2009).

[22] H. L. Berk, B. N. Breizman, and M. S. Pekker, Plasma Phys. Reports 23, 778 (1997).

[23] A. N. Kaufman, Phys. Fluids 15, 1063 (1972).

[24] K. Ghantous, H. L. Berk, and N. N. Gorelenkov, Phys. Plasmas 21, 032119 (2014).

[25] C. Z. Cheng and M. S. Chance, Phys. Fluids 29, 3695 (1986).

[26] H. L. Berk, B. N. Breizman, and H. Ye, Phys. Rev. Lett. 68, 3563 (1992).

[27] C. Angioni, A. G. Peters, G. V. Pereverzev, A. Botino, J. Candy, R. Dux, E. Fable, E. Hein,
and R. E. Waltz, Nucl. Fusion 49, 055013 (2009).

[28] M. A. Van Zeeland, W. W. Heidbrink, S. E. Sharapov, D. Spong, A. Cappa, X. Chen, C. Collins, M. García Muñoz, N. N. Gorelenkov, G. J. Kramer, P. Lauber, Z. Lin, and C. Petty, Nucl. Fusion 56, 112007 (2016).

[29] N. Gorelenkov, G. Kramer, and R. Nazikian, Plasma Phys. Control. Fusion 48, 1255 (2006).

[30] V. N. Duarte, N. N. Gorelenkov, M. Schneller, E. D. Fredrickson, M. Podestà, and H. L. Berk, Nucl. Fusion 58, 082013 (2018).

[31] V. N. Duarte and N. N. Gorelenkov, Nucl. Fusion 59, 044003 (2019)