ASSEMBLY SYSTEM WITH OMNICHANNEL COORDINATION

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Abstract. Assembly system with omnichannel is rarely studied in literature. This paper explores the equilibrium and coordination issues for an omnichannel assembly system. Four different game-theoretical model types applying four operational strategies - a total of sixteen analytical models - are developed and analyzed for both omnichannel and pure channel modes. The numerical analysis of an electronic product assembly system provides a clearer understanding of the solutions and their effects on the profits in the assembly system for different model types and operational strategies. A further sensitivity analysis with focus on an omnichannel with offline channel subsidy (OMS) creates better insights regarding how changes of key parameters affect the assembly system profits. It is found that the omnichannel mode with or without offline channel subsidy can deliver much better operational performance to the assembly system via mutual fusion effect than that of a pure online- or offline-channel mode. Furthermore, the offline channel subsidy can amplify to a very large extent the mutual fusion effect to increase the product demand dramatically and thus improving the operational performance of the assembly system in the omnichannel business scenario. The best operational strategy for the assembly system in the omnichannel business scenario is the coordination strategy with offline channel subsidy.

1. Introduction. In the era of rapid development of mobile commerce and payment, the omnichannel business mode has quickly evolved and transformed the retailing landscape in the last ten years. Omnichannel mode refers to the seamless integration of online- and offline- channels to improve customer experience hoping to capture the customers’ demands, anytime and anywhere [8]. This mode emerged...

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in recent years with the intention to ensure the retailer marketing strategies are geared toward enabling customers to convert on any channel [32].

According to an industry report from Shopify Plus, considering that an estimated 81% of shoppers conduct online research before making big purchases, being able to channel even a small percentage of these customers straight from their online research to the offline stores would represent a massive potential for sales [25, 12]. Owing to the dual advantages of online information acquisition and offline product experience, the omnichannel mode could bring online customers to brick-and-mortar stores and vice versa, which provides a better shopping experience for customers and more business opportunities for retailers and their supply chains. Hence, many traditional retail enterprises, e-commerce enterprises, manufacturers or their product and service suppliers are working together to transform their supply chains to an omnichannel mode.

Many industry leaders now believe that China leads the world in omnichannel development because there is a combination of high adoption of mobile devices and an acceptance of mobile payment. For example, SUNING, a large appliances retail chain store enterprise in China, actively implements the consumer-oriented omnichannel business mode and the supplier-oriented omnichannel platform, which devoted to the integration of its own online electronic-/mobile-commerce platform and its own offline brick-and-mortar stores. The online e- and m-commerce platform provides customers information of the shopping guide staff in the nearest offline brick-and-mortar stores to guide them to offline brick-and-mortar stores for free product experiences and free consulting services; meanwhile, the offline brick-and-mortar store establishes self-pickup points and launches the self-pickup rebates to supplement the online shopping effect, places physical products and posts QR code posters in the store to facilitate the customer’s product experience, and arranges shopping guide staff to provide free product experience consultation and guide customers to online electronic-/mobile-commerce platform for product information acquisition and query. Furthermore, SUNING launches the price unification practice of online-and offline-channel to achieve seamless docking of inter-channel, enhance mutual fusion effect of inter-channel and strengthen consumer experience to make customers feel that each channel has enough value. In the U.S. market, Amazon and Best Buy supported by their joint assembler Toshiba work together to create an omnichannel business mode that will integrate Amazon’s online- and Best Buy’s offline- channels. Best Buy will exclusively sell Amazon Fire-edition smart TVs from Toshiba and Insignia, Best Buy’s in-house brand. The TVs will have Amazon Fire TV built in. They will come with a voice-activated remote with Alexa, Amazon’s voice-enabled assistant, and can be paired with any Echo device [35]. As a typical assembler, DELL acquires PC components and modules from upstream suppliers, assembles PC products, and then sells PC products to end customers through its own online e-commerce platform and offline brick-and-mortar store partners. Obviously, DELL’s channel strategy necessitates an omnichannel solutions that provides the customer a consistent, engaging experience across channels. “Dell EMC believes in an omni channel strategy and leaves the choice to the customer in terms of whom they wish to do business with.” Said by Anil Sethi, the vice president of channels India at Dell EMC [7].

By reshaping the structure and mechanism of the traditional assembly system, the omnichannel mode brings more opportunities but also challenges to the assembly system. From the perspective of mode selection and strategic choice, the
following research questions emerge and urgently need to be studied: what is the value/strength of the omnichannel mode compared with the pure online/offline channel mode regarding their assembly systems? What operational strategies should the omnichannel assembly system take to improve their operational performance? What is the effect of an offline-channel subsidy on the operational performance of an omnichannel assembly system? Under what conditions would an omnichannel assembly system be incentivized economically to implement an offline-channel subsidy?

Based on the comparative modelling and numerical analyses, several game-theoretical approaches toward exploring these issues are developed in this paper, which will help omnichannel assembly systems make appropriate operational decisions & strategies and improve their operational performances. Novel findings of research results reveal that, relative to the pure online and offline modes, the omnichannel mode can improve the operational performance of the assembly system, the coordination strategy outperforms equilibrium strategies regarding operational decisions and outcomes for the omnichannel assembly system, and the omnichannel assembly system would have economic incentives to implement an offline-channel subsidy when incentive condition $\alpha(\kappa) > 1$ holds. This study will supplement the literature shortage and add to the managerial insights for the omnichannel practitioners and their supply chain partners.

In the following sections, the corresponding literature are first reviewed in section 2. The modeling notations and assumptions of a generic assembly system with an omnichannel mode are defined for this study in section 3. Section 4 contains the game-theoretical decision modeling analysis of this study. The numerical and sensitive analyses of an electronic product case for all developed analytical models are conducted and the results and comparisons are synthesized in section 5. Section 6 discusses the managerial insights, research limitations and future research opportunities. The final section concludes the research contributions and foresights of this study.

2. Literature review. The omnichannel mode is fast becoming a new business model replacing the dual-channels. Under an omnichannel mode, the boundary between the online channel and offline channel has been removed, creating a dual advantage from both online information search and offline product experience. Incorporating the omnichannel mode has become an important strategic direction for both the brick-and-mortar and ecommerce retailers to achieve a symbiotic, integrated and mutually reinforced retail supply chain. The omnichannel mode is reshaping the structure and mechanism of the retail competition arena and bringing not only opportunities but also challenges for the theoretical research. The related literature has evolved from the assembly system view to the dual-channel supply chain view, and further to the omnichannel supply chain view.

Assembly system View

Literatures regarding the assembly system management mainly touch upon the issues of competition and cooperation among suppliers, and cooperation and coordination in the assembly system. Carr and Karmarkar [3] studied the competition in multi-echelon supply chains with an assembly network structure and derived closed-form expressions for equilibrium quantities and prices. Zhang [39] investigated the behavior of the two-echelon assembly system under decentralized control, examined the role of horizontal information sharing on the inventory status between the
suppliers, and proposed a demand-independent coordination scheme for managing decentralized assembly systems. Nagarajan et al. [23] modelled the multilateral negotiations between an assembler and its suppliers sequentially using Nash bargaining concept for the assembly supply chain. Nagarajan et al. [24] analyzed alliance/coalition formation between suppliers in a decentralized assembly system using a two-stage approach. Jiang and Wang [15] studied supplier competition in decentralized assembly systems with price-sensitive and uncertain demand. Yin et al. [38] studied the coalition formation among perfectly complementary suppliers in a price-sensitive assembly supply chain. Kalkanci and Erhun [16] analyzed decentralized assembly systems under asymmetric demand information and sequential contracting.

**Dual-channel Supply Chain View**

Over the past decades, the dual-channel supply chains composed of e-commerce and traditional channels has been fully developed. Literatures regarding the dual-channel supply chains mainly touch upon the optimal operational mechanism and performance under the dual-channel competition, cooperation and coordination. Tsay and Agrawal [30] found that increasing electronic channel could increase the profits of both members of the dual-channel supply chain. Dumrongsiri et al. [9] developed conditions under which the manufacturer and the retailer share the market in equilibrium for the dual channel supply chain, and showed that adding a direct channel will increase the overall profit. Cai [2] investigated the influence of channel structures and channel coordination on the supplier, the retailer, and the entire supply chain in the context of two single-channel and two dual-channel supply chains. Xu et al. [36] explored the manufacturer’s inventory coordination problem when it developed its own electronic channel beyond retailer’s traditional channel, focusing on the substitution of the same product in different channels with the manufacturer as the retailer’s competitor. Panda et al. [26] explored pricing and replenishment policies for a high-tech product in a dual-channel supply chain that consists of a brick-and-mortar channel and an internet channel. Kong et al. [18] built a benchmark model to explore the dual-channel price and service competition, extended the model in a reverse supply chain via setting the return rate as the function of the service level, analyzed the optimal pricing and service decision under the centralization and decentralization scenarios, and designed a revenue-sharing contract model to achieve a Pareto optimization for all players. Saha et al. [29] explored optimal pricing policies of a two-level dual-channel supply chain under price- and delivery time-sensitive demand. Modak and Kelle [22] examined the effect of delivery lead time and customers’ channel preference for the dual-channel supply chain combining the traditional retail channel with a direct online channel under price and delivery-time dependent stochastic demand.

**Omnichannel Supply Chain View**

Literature regarding the supply chain management and its optimal operational mechanism under omnichannel mode (or O2O mode) has gained more interests in recent years. Zhang et al. [40] developed the operational models in an O2O supply chain considering the service level competition under three scenarios, and compared the corresponding supply chain performances. In the same year, the same authors [41] also explored an improved revenue-sharing contract model to coordinate a supply chain consisting of one supplier and multiple O2O retailers, considering the impact of disruptions in the supply chain. Chen et al. [4] developed the Supplier-Stackelberg, Retailer-Stackelberg, and Nash game models for the retail
service supply chain with an online-to-offline (O2O) mixed dual-channel, obtained
the optimal prices and maximal profits for both retailer and supplier under dif-
ferent power structures, and discussed the corresponding managerial implications
and pricing strategies. Gao and Su [11] built a stylized model to study the impact
of the buy online and pick up in store (BOPS) initiative on store operations and
customers’ channel-choices. Modak [21] explored pricing, stocking and delivery lead
time for a two-level omni-channel supply chain under price and delivery time sen-
sitive additive stochastic demand. Ji et al. [13] developed three decision models to
investigate initial carbon allowance allocation rules in an O2O retail supply chain
with the cap-and-trade regulation. Gao and Su [10] developed a stylized theoretical
model to study the impact of self-order technologies on customer demand, employ-
ment levels, and restaurant profits in the omnichannel restaurants. Paul et al. [27]
studied how to best share capacity between the routes associated with different sales
channels for an omni-channel grocery retailer in the Netherlands. Chen et al. [5]
studied mutual promotional effects, operational strategies and subsidy policies for
the O2O supply chain. Chen and Su [6] explored the cooperation mechanism for the
consignment supply chain with complementary products under O2O mode. Jiang
et al. [14] studied the optimal pricing decisions for an omni-channel supply chain
with retail service. Yang and Zhang [37] investigated the impact of ship-to-store
(STS) and quick response on fast-fashion operations under omnichannel retailing.

In brief, the existing research are rooted from the dual-supply chain and mainly
focused on the service competition, power structure, closed-loop structure, demand
disruption, cooperative advertising, lateral inventory transshipment and their im-
 pact on the related decisions between the buyer and the supplier in the omnichannel
mode. Nevertheless, the existing literature do not consider the following critical is-
 sues for the assembly system with omnichannel: (1) the impact of an omnichannel
 mode on key decisions and operational performances of assembly system; (2) the im-
 pact of an omnichannel mode with offline subsidy on key decisions and operational
 performances of the assembly system.

This paper intends to explore the role and economic behaviors of the assembly
system with omnichannel and compare the research results with those of the pure
offline- and pure online- channel modes to bridge the literature gap.

3. Modelling notations and assumptions. Traditionally, an assembler can sell
independently through either online- or offline- channel. Nowadays, some assemblers
start to integrate the online and offline channels by setting the same retail prices
for both channels or even subsidizing the physical channels, i.e. offline, from the
virtual channels, i.e. online. The assembly system utilizing an omnichannel mode
include examples such as the laptop, desktop, smart phones, televisions and many
others.

A generic assembly system with omnichannel is conceptualized for this study as
shown in Figure 1. This system includes n module or component suppliers and an
assembler with omnichannel. In this system, each supplier produces and provides a
module/component to the omnichannel assembler, the assembler assembles multiple
modules/components into the final product, and the assembler sells the final product
to the consumers in the market via omnichannel, i.e., an integrated channel of
both offline and online channels. Each supplier could negotiate either jointly (a
centralized strategy), independently (a decentralized strategy) or collaboratively (a
coordination strategy) with the assembler regarding the wholesale price that will
then affect the retail pricing and demands. Through the omnichannel integration, the final product will be sold at a regular retail price in a regular sales season, and the leftover stock will be sold at a salvage price in the following clearance season. The omnichannel assembly system take advantages of both the fast and 24/7 information search of the electronic/mobile commerce online platforms and the convenient physical product experiencing of the offline brick-and-mortar stores.

![A Generic Assembly System with Omnichannel](image)

Table 1 presents the list of all parameter and variable notations and explanations for the modeling use. For tracking purposes, notation $i$ is used to distinguish suppliers (or modules), where $i \in N, N = \{1, 2, \ldots, n\}$.

| Parameter/Variable | Explanations |
|--------------------|--------------|
| $c$                | Unit assembly cost of the final product |
| $c_i$              | Unit cost of the $i^{th}$ module |
| $w_i$              | Wholesale price of the $i^{th}$ module |
| $c_a$              | Operational cost of the online channel |
| $c_s$              | Operational cost of the offline channel |
| $p$                | Retail price of the final product in the online/offline channel |
| $z$                | Stock factor |
| $a$                | Positive constant number |
| $b$                | Price-elasticity index of the expected demand |
| $\theta$           | Mutual fusion coefficient between channels |
| $\eta$             | Clearance discount price factor, and $0 < \eta < 1$ |
| $\lambda_0$        | Market demand share of the online channel, and $0 < \lambda_0 < 1$ |
| $\delta$           | The offline channel subsidy factor, and $0 < \delta < 1$ |
| $\kappa$           | The offline channel discount price factor, and $\kappa = 1 - \delta$ |
| $h$                | The reaction extent of $\lambda(\kappa)$ w.r.t. the change of $\kappa$ |
| $\phi$             | Revenue keeping rate, and $0 < \phi < 1$ |
| $x$                | The random factor defined in the range $[A, B]$ with $B > A > 0$ |
| $\mu$              | Mean value of random factor |
| $\sigma$           | Standard deviation of random factor |

According to the modelling requirements and constraints, several assumptions are made and explained below:

A1: The information acquisition advantage of online channel and the product experience advantage of offline channel are seamlessly docked and perfectly integrated...
in an omnichannel to create a mutual fusion effect between online and offline channels, thus generate more product demands. The mutual fusion effect is positively related to the service quality of both online and offline channels, i.e., the higher the service quality is in an omnichannel, the stronger the mutual promotional effect will be.

A2: Following Chen et al. [5], let \( d_e(p) \) and \( d_s(p) \) denote the online demand function and the offline demand function respectively. They can be defined as

\[
d_e(p) = \lambda_0 y(p)x \quad \text{and} \quad d_s(p) = (1 - \lambda_0) y(p)x.
\]

In both demand functions, \( \lambda_0 \) is the market demand share of the online channel and \( (1 - \lambda_0) \) is the market demand share of the offline channel, \( 0 < \lambda_0 < 1 \). \( y(p) \) is a deterministic and decreasing function of price \( p \). \( y(p) = ap^{-b}p^d = ap^{-(b+\theta)} \), where \( a \) is the positive constant number and \( b \) is the price-elasticity index of the expected demand. \( \theta \in (0, 1) \) measures the mutual fusion coefficient between channels, and \( b > n > 1 > \theta > 0 \). \( x \) is a random factor defined in the range \([A, B]\) with \( B > A > 0 \). The CDF (Cumulative Distribution Function) and PDF (Probability Density Function) of \( x \) are \( F(\cdot) \) and \( f(\cdot) \), and the mean value and standard deviation of \( x \) are \( \mu \) and \( \sigma \). Hence, the total market demand function \( d(p) \), which is composed of the online demand function \( d_e(p) \) and the offline demand function \( d_s(p) \), can be defined as

\[
d(p) = d_e(p) + d_s(p) = y(p)x.
\]

A3: Following Petruzzi and Dada [28], Wang et al. [11], and Wang [34], \( z = \frac{q}{y(p)} \) is defined as the 'stock factor' where \( q \) is the order quantity. Thus, \( q = y(p)z \).

In this study, the 'stock factor' is used to model the equilibrium and coordination conditions of assembly system.

A4: The generalized failure rate is defined as \( g(x) = xh(x) \) because of its relationship with the classical failure rate \( h(x) = \frac{f(x)}{1-F(x)} \), which is the frequency with which the product inventory fails to satisfy the product demand. While the failure rate gives (roughly) the percentage decrease in the probability of a stockout from increasing the stocking quantity by one unit, the generalized failure rate gives (roughly) the percentage decrease in the probability of a stockout from increasing the stocking quantity by 1%[20].

A5: When the distribution of random factor \( x \) in the demand function satisfies the Increasing Generalized Failure Rate (IGFR) condition, i.e., \( \frac{dg(x)}{dx} = h(x) + \frac{xh(x)}{\sigma} > 0 \), the first order conditions of the expect profit function with respect to \( p \) and \( z \) provide a unique solution to the problem of maximizing the expected profit function [33],[20, 19].

A6: Under the centralized decision, module suppliers and the omnichannel assembler jointly determine the optimal price and stock factor to maximize the expected profit of the omnichannel assembly system. The decision sequence of the centralized decision model is as follows: the centralized supply chain first determines the stock factor of the assembled product, i.e., \( z \), and finally determines the retail prices of the product, i.e., \( p \).

A7: Under the decentralized decision, module suppliers and the omnichannel assembler make decisions of the price and stock factor separately to maximize their own profits. The decision sequence of the decentralized decision models is as follows: first, module suppliers decide the wholesale prices of modules simultaneously or sequentially; then, the omnichannel assembler determines the stock factor of the product; finally, the omnichannel assembler determines the retail price of the product.
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A8: Under the coordination decision, module suppliers simultaneously offer a revenue sharing contract to the omnichannel assembler either accepts or rejects the contract. If the omnichannel assembler accepts, he will share a proportion of his revenue to module suppliers. The decision sequence of the coordination decision models is as follows: first, module suppliers will produce and deliver modules at lower wholesale prices, \( w_1, w_2, \ldots, w_n \) to the omnichannel assembler under the premise that the revenue sharing rate is set; then, the omnichannel assembler will place module orders with quantity \( q \) to module suppliers; finally, the omnichannel assembler will sell the final product at a regular retail price \( p \) through omnichannel when the selling season starts and sell the leftover stock at salvage price \( \eta p \) through omnichannel in the clearance season. Afterwards, the omnichannel assembler will share a fraction \( (1 - \phi) \) of his revenue to module suppliers, where \( \phi \) is the revenue keeping rate of the omnichannel assembler, and \( 0 < \phi < 1 \).

A9: To promote the sale of the products under omnichannel mode, the omnichannel assembly system may subsidize its offline channel to enhance customers’ product using experience or provide customers the convenience of “buy online and pickup in store”. Therefore, it is assumed that \( \delta \) is the offline channel subsidy factor and thus \( \kappa = 1 - \delta \) is the offline channel discount price factor, \( 0 \leq \delta, \kappa \leq 1 \). On this basis, \( \delta p \) is the subsidy of a unit product to the offline channel and the retail price in the offline channel is \( \kappa p \equiv (1 - \delta)p \).

Table 2 provides a framework of the game-theoretical decision models developed and analyzed in section 4. The numerical and sensitivity analyses will be conducted and discussed in section 5.

### Table 2. Framework of Game-Theoretical Decision Models

| Section | Channel Strategy | Game-Theoretical Decision Models | Theories Applied |
|---------|------------------|----------------------------------|------------------|
| 4.1     | Omnichannel mode without offline channel subsidy (OMO mode) | 4.1.1 Centralized Decision Model | OT & BC          |
|         |                   | 4.1.2 Decentralized Decision Model | SG & BC          |
|         |                   | 4.1.2.1 Assembler’s Decision     | SG & BC          |
|         |                   | 4.1.2.2 Suppliers’ Simultaneous Decision | SG & BC |
|         |                   | 4.1.2.3 Suppliers’ Sequential Decision | SG & BC |
|         |                   | 4.1.3 Coordination Decision Model | RSC & BC         |
| 4.2     | Omnichannel mode with offline channel subsidy (OMS mode) | Centralized/Decentralized/Coordination Decision Models under OMS mode | OT+SG+RSC+BC    |
| 4.3     | Pure online/offline channel mode (POC/PFC mode) | Centralized/Decentralized/Coordination Decision Models under POC/PFC mode | OT+SG+RSC+BC    |

Notation: **OT**: Optimization Theory; **BC**: Bertrand Competition; **SG**: Stackelberg Game; **RSC**: Revenue Sharing Contract

4. **Game-theoretical decision models for omnichannel assembly system.**
Based on section 3 modelling notations and assumptions, this section conducts an extensive game-theoretical modeling of the equilibrium and coordination conditions for the assembly system with omnichannel. In the models to follow, note that the superscript or subscript \( c \) represents centralized decision and coordination decision under omnichannel mode without offline channel subsidy (i.e., OMO mode); the superscript or subscript \( d \) : decentralized decision with suppliers simultaneous action under OMO mode; the superscript or subscript \( d' \) : decentralized decision with
suppliers’ sequential action under OMO mode; the superscript or subscript \( sc \) : centralized decision and coordination decision under omnichannel mode with offline channel subsidy (i.e., OMS mode); the superscript or subscript \( sd \) : decentralized decision with suppliers’ simultaneous action under OMS mode; and the superscript or subscript \( sd' \) : decentralized decision with suppliers’ sequential action under OMS mode.

4.1. Game-theoretical decision models under OMO mode.

4.1.1. Centralized decision model. When the assembly system under OMO mode takes a centralized strategy, the optimal profit function of the system can be formulated as follows:

\[
\max_{p,z} \Pi_{SC}(p, z) = py(p) \cdot E[\min\{z, x\}] + \eta py(p) \cdot E[(z-x)^+] \]

\[
- \left[ c + \lambda_0 c_e + (1 - \lambda_0) c_s + \sum_{i=1}^{n} c_i \right] y(p) z
\]  

(1)

In the right-hand-side (RHS) of this equation, the first term \( py(p) \cdot E[\min\{z, x\}] \) represents the expected sales revenue of the assembly system in regular sales season, the second term \( \eta py(p) \cdot E[(z-x)^+] \) represents the expected salvage revenue of the assembly system in clearance season, the third term \( c + \lambda_0 c_e + (1 - \lambda_0) c_s + \sum_{i=1}^{n} c_i \) \( y(p) z \) represents the total cost of the assembly system with omnichannel (including component/modules cost, product assembling cost and operational costs of online- and offline-channel). When the distribution of random variable \( x \) satisfies the IGFR condition, the first order conditions \( (p_c, z_c) \) determine a unique solution to the above optimization problem. Solving the first-order condition of the optimization profit function with respect to the stock factor \( z \) and the retail price \( p \), we can get the optimal retail price and the distribution function of the centralized optimal stock factor as follows:

\[
p_c = \frac{b - \theta}{b - \theta - 1} \frac{c + \lambda_0 c_e + (1 - \lambda_0) c_s + \sum_{i=1}^{n} c_i \ z_c}{z_c - (1 - \eta) \Lambda (z_c)}
\]  

(2)

\[
F(z_c) = \frac{1}{(1 - \eta)(b - \theta)z_c} + \frac{(b - \theta - 1) \Lambda (z_c)}{(b - \theta)z_c}
\]  

(3)

Where \( \Lambda (z_c) = \int_{A}^{z_c} (z_c - x) f(x)dx \)

Then, we can have the centralized optimal order quantity as follows:

\[
q_c = y(p_c) z_c
\]  

(4)

Substituting the optimal stock factor \( z_c \) and the optimal retail price \( p_c \) into the profit function of the assembly system under OMO mode, we can obtain the optimal profit of the assembly system as follows:

\[
\Pi^c_{SC} = \frac{1}{b - \theta - 1} \left[ c + \lambda_0 c_e + (1 - \lambda_0) c_s + \sum_{i=1}^{n} c_i \right] q_c
\]  

(5)

4.1.2. Decentralized decision model.
4.1.2.1 Assembler’s Decision. When the assembler takes a decentralized strategy, the optimal profit function can be formulated as follows:

$$\max_{p, z} \Pi_A(p, z) = py(p) \cdot E[\min\{z, x\}] + \eta(py) \cdot E[(z - x)^+]$$

$$- \left[ c + \lambda_0 c_e + (1 - \lambda_0) c_s + \sum_{i=1}^n w_i \right] y(p)z$$  \hspace{1cm} (6)

In the RHS of this equation, the first term $py(p) \cdot E[\min\{z, x\}]$ represents the expected sales revenue of the assembler in regular sales season, the second term $\eta(py) \cdot E[(z - x)^+]$ represents the expected salvage revenue of the assembler in clearance season, the third term $[c + \lambda_0 c_e + (1 - \lambda_0) c_s + \sum_{i=1}^n w_i] y(p)z$ represents the total cost of the omnichannel assembler (including product assembling cost, operational costs of online- and offline-channel, and procurement costs of components/modules, i.e., wholesale prices of components/modules). Solving the first-order condition of the optimization problem with respect to the stock factor $z$ and the retail price $p$, we can get the reaction function of optimal retail price with respect to (w.r.t.) the wholesale price and the distribution function of the optimal stock factor as follows:

$$p_d(w_1, \ldots, w_i, \ldots, w_n) = \frac{c + \lambda_0 c_e + (1 - \lambda_0) c_s + \sum_{i=1}^n w_i}{c + \lambda_0 c_e + (1 - \lambda_0) c_s + \sum_{i=1}^n c_i} p_c$$  \hspace{1cm} (7)

$$F(z_d) = F(z_c)$$  \hspace{1cm} (8)

Where $\Lambda(z_d) = \int_{\Lambda}^{z_d} (z_d - x) f(x) dx$

Then, we have the reaction function of the decentralized optimal order quantity w.r.t. the wholesale price as follows:

$$q_d(w_1, \ldots, w_i, \ldots, w_n) = \left[ \frac{c + \lambda_0 c_e + (1 - \lambda_0) c_s + \sum_{i=1}^n c_i}{c + \lambda_0 c_e + (1 - \lambda_0) c_s + \sum_{i=1}^n w_i} \right]^{b - \theta} q_c$$  \hspace{1cm} (9)

The suppliers in a decentralized assembly system under OMO mode can make the pricing decisions in a simultaneous or a sequential way.

4.1.2.2 Suppliers’ Simultaneous Decisions. For the case when the module suppliers make a simultaneous decision, substituting the reaction function of the optimal order quantity w.r.t. the wholesale price $q_d(w_1, \ldots, w_i, \ldots, w_n)$ into the module supplier i’s profit function, we can obtain the optimal profit function for the module supplier $i$ as follows:

$$\max_{w_i} \Pi_{S_i}(w_i) = (w_i - c_i) q_d(w_1, \ldots, w_i, \ldots, w_n), i = 1, 2, \ldots, n$$  \hspace{1cm} (10)

In the RHS of this equation, the first term $(w_i - c_i)$ represents the marginal profit of module $i$, the second term $q_d(w_1, \ldots, w_i, \ldots, w_n)$ represents the order quantity of module $i$, thus, the RHS of this equation represents the profit of the module supplier $i$. Solving the first-order condition of the module supplier i’s profit function with respect to the wholesale price $w_i$, and deriving the reaction function of the supplier i’s wholesale price w.r.t. the other suppliers’ wholesale price $\{w_1, \ldots, w_{i-1}, w_{i+1}, \ldots, w_n\}$, when condition $b - \theta > n$ holds, we can obtain the unique Nash equilibrium wholesale price $w_i^d$ of the $i$th supplier as follows:

$$w_i^d = \frac{1}{b - \theta - n} \left[ c + \lambda_0 c_e + (1 - \lambda_0) c_s + \sum_{i=1}^n c_i \right] + c_i, i = 1, 2, \ldots, n$$  \hspace{1cm} (11)

Plugging the supplier i’s equilibrium wholesale price $w_i^d$ into the reaction function of the optimal retail price w.r.t. the wholesale price $p_d(w_1, \ldots, w_i, \ldots, w_n)$, and the reaction function of the optimal order quantity w.r.t. the wholesale price
\( q_d (w_1, \ldots, w_i, \ldots, w_n), \) then we can get the equilibrium retail price, the equilibrium
stock factor and the equilibrium ordering quantity as follows:

\[
p_d = \frac{b - \theta}{b - \theta - n} p_c \tag{12}
\]

\[
F(z_d) = F(z_c) \tag{13}
\]

\[
q_d = \left(\frac{b - \theta - n}{b - \theta}\right)^{b - \theta} q_c \tag{14}
\]

Plugging the equilibrium stock factor, the equilibrium retail price, and the equi-
librium ordering quantity into the profit functions, we can obtain the equilibrium
profits of the module supplier \(i\), the assembler and the assembly system as follows:

\[
\Pi_i^d = \frac{b - \theta - 1}{b - \theta} \left(\frac{b - \theta - n}{b - \theta}\right)^{b - \theta - 1} \Pi_i^{SC}, i = 1, 2, \ldots, n \tag{15}
\]

\[
\Pi_A^d = \left(\frac{b - \theta - n}{b - \theta}\right)^{b - \theta - 1} \Pi_A^{SC} \tag{16}
\]

\[
\Pi_{SC}^d = \left[(n + 1) - \frac{n}{b - \theta}\right] \left(\frac{b - \theta - n}{b - \theta}\right)^{b - \theta - 1} \Pi_{SC}^{d} \tag{17}
\]

Based on the analytical results derived above in this section, the following findings
are summarized as:

**Remark 1.** When the suppliers make simultaneous decisions under the OMO mode,
the assembler’s equilibrium profit is \((\frac{(b-\theta)}{(b-\theta-1)})\) times of any supplier’s profit. That is,
\(
\frac{\Pi_A^d}{\Pi_i^d} = \frac{b-\theta}{b-\theta-1}, i = 1, 2, \ldots, n.
\)

**Proposition 1.** When the suppliers make simultaneous decisions under the OMO
mode, (i) all suppliers gain the same profit even though they may incur different
production costs; (ii) the assembly system and its members all decrease their equi-
librium profits as the number of the suppliers increases.

4.1.2.3 Suppliers’ Sequential Decisions. When the module suppliers make sequential
decisions, substituting the reaction function of the optimal order quantity w.r.t. the
wholesale price \(q_d (w_i)\) into the module supplier \(i\) ’s profit function, assuming the
wholesale price of the 1st, 2nd, \ldots, \((n-1)\)th module supplier \(w_1, w_2, \ldots, w_{n-1}\) is
given, then we can obtain the optimal profit function for the module supplier \(n\) as
follows:

\[
\max_{w_n} \Pi_{S_n} (w_n \mid w_1, w_2, \ldots, w_{n-1}) = (w_n - c_n) q_d (w_1, \ldots, w_i, \ldots, w_n) \tag{18}
\]

Solving the first-order condition of the module supplier \(n\) ’s profit function with
respect to the wholesale price \(w_n\), deriving the reaction function of the supplier \(n\)
’s wholesale price \(w_n\) w.r.t. the other suppliers’ wholesale price \(\{w_1, \ldots, w_{n-1}\}\),
plugging \(w_n^d (w_1, \ldots, w_{n-1})\) into the supplier \((n-1)\) ’s profit function, and solving
the first-order condition of the module supplier \((n-1)\) ’s profit function with respect
to the wholesale price \(w_{n-1}\), we can get the reaction function of the supplier \((n-1)\)
’s wholesale price \(w_{n-1}\) w.r.t. the other suppliers’ wholesale price \(\{w_1, \ldots, w_{n-2}\}\)
as \(w_{n-1}^d (w_1, \ldots, w_{n-2})\) likewise, we can get \(w_{n-2}^d (w_1, \ldots, w_{n-3})\), \ldots, \(w_2^d (w_1), w_1^d\),
then we can obtain \(w_2^d, \ldots, w_n^d\) via backward induction. Hence, for \(b - \theta > 1\), the
unique Nash-equilibrium wholesale price \(w_i^d\) of the ith supplier is as follows:
\[ w_i^{d'} = \frac{(b - \theta)^{i-1}}{(b - \theta - 1)^i} \left[ c + \lambda_0 c_e + (1 - \lambda_0) c_s + \sum_{i=1}^{n} c_i \right] + c_i, i = 1, 2, \ldots, n \]  

Plugging the supplier \( i \)'s equilibrium wholesale price \( w_i^{d'} \) into the reaction function of the optimal retail price w.r.t. the wholesale price \( p_d'(w_1, \ldots, w_i, \ldots, w_n) \), and the reaction function of the optimal order quantity w.r.t. the wholesale price \( q_d'(w_1, \ldots, w_i, \ldots, w_n) \), then we can get the equilibrium retail price, the equilibrium stock factor and the equilibrium ordering quantity as follows:

\[ p_d' = \left( \frac{b - \theta}{b - \theta - 1} \right)^n p_c \]  
\[ F(z_d') = F(z_c) \]  
\[ q_d' = \left( \frac{b - \theta - 1}{b - \theta} \right)^{n(b-\theta)} q_c \]

Plugging the equilibrium stock factor, the equilibrium retail price, and the equilibrium ordering quantity into the profit functions, we can obtain the equilibrium profits of the supplier \( i \), the assembler and the assembly system as follows:

\[ \Pi^d_{S_i} = \left( \frac{b - \theta - 1}{b - \theta} \right)^{n(b-\theta)-i+1} \Pi^{SC}_{i}, i = 1, 2, \ldots, n \]  
\[ \Pi^d_A = \left( \frac{b - \theta - 1}{b - \theta} \right)^{n(b-\theta-1)} \Pi^{SC}_A \]  
\[ \Pi^d_{SC} = \left[ (b - \theta) \left( \frac{b - \theta - 1}{b - \theta} \right)^{n(b-\theta-1)} - (b - \theta - 1) \left( \frac{b - \theta - 1}{b - \theta} \right)^{n(b-\theta)} \right] \Pi^{SC}_{SC} \]

Based on the analytical results derived above in this section, the following findings are summarized as:

**Remark 2.** When the suppliers make sequential decisions under the OMO mode, the supplier \( i+1 \) gains \( \frac{(b-\theta)}{(b-\theta-1)} \) times of the supplier \( i \)'s profit. That is, \( \frac{\Pi^d_{i+1}}{\Pi^d_i} = \frac{b-\theta}{b-\theta-1}, i = 1, 2, \ldots, n. \)

**Proposition 2.** When the suppliers make sequential decisions under the OMO mode, (i) the supplier who moves later, gains more profits, i.e., there exists a last-mover advantage when the suppliers make sequential decisions; (ii) the equilibrium profits of the assembly system and its members are all decreased as the number of the suppliers increases.

4.1.3. **Coordination decision model.** Based on both group and individual rationality, a revenue sharing contract approach can be applied to achieve the supply chain coordination through a non-cooperative way [1]. A revenue sharing contract for the coordination decision of assembly system under OMO mode will be modeled and discussed in this section.

The event sequence in a revenue sharing contract can be described as follows. The suppliers simultaneously offer the assembler a revenue sharing contract to the assembler. The assembler either accepts or rejects the contract. If the assembler accepts, he will share a proportion of his revenue to the supplier. The revenue keeping rate will be negotiated between supplier and the assembler. After the revenue sharing rate is set, module suppliers will produce and deliver modules at
lower wholesale prices \(w_1, \ldots, w_i, \ldots, w_n\) to the omnichannel assembler, and the assembler will place module orders with quantity \(q\) to the module suppliers, after the final product is assembled by modules or components, he will sell the final product through omnichannel at regular retail price \(p\) when the selling season starts, and sell the leftover stock through omnichannel at salvage price \(ηp\) in the clearance season. Afterwards, the assembler will share a fraction \(1 - \phi\) of his revenue to the suppliers (supplier \(i\) will get a fraction \(\frac{c_i}{\sum_{i=1}^{n} c_i}(1 - \phi)\) of the assembler’s sharing revenue), where \(\phi\) is the revenue keeping fraction of the assembler, and \(0 \leq \phi \leq 1\). The profit maximization function of the expected profit for the assembler under the revenue sharing contract is as follows:

\[
\max_{p, z} \Pi_A(p, z) = \phi py(p) \cdot E[\min\{z, x\}] + \phi \eta py(p) \cdot E[(z - x)^+] - \left[ \phi c + \phi \lambda_0 c_e + \phi (1 - \lambda_0) c_s + \sum_{i=1}^{n} w_i \right] y(p) z
\]  

(26)

In the RHS of this equation, the first term \(\phi py(p) \cdot E[\min\{z, x\}]\) represents the expected sales revenue of the assembler in regular sales season under revenue sharing contract, the second term \(\phi \eta py(p) \cdot E[(z - x)^+]\) represents the expected salvage revenue of the assembler in clearance season under revenue sharing contract, the third term \([\phi c + \phi \lambda_0 c_e + \phi (1 - \lambda_0) c_s + \sum_{i=1}^{n} w_i] y(p) z\) represents the total cost of the omnichannel assembler under revenue sharing contract. This model has the same structure as that of a centralized assembly system except \(c_i\) is replaced by \(w_i\). Hence, under the condition of IGFR regarding the distribution function \(F(\cdot)\) of the random variable \(x\), solving the first-order condition of the optimization profit function with respect to the stock factor \(z\) and the retail price \(p\), we can get the reaction function of optimal retail price w.r.t. the wholesale price, the distribution function of the optimal stock factor and the reaction function of optimal ordering quantity w.r.t. the wholesale price as follows:

\[
p_r(w_1, \ldots, w_i, \ldots, w_n) = \frac{\phi c + \phi \lambda_0 c_e + \phi (1 - \lambda_0) c_s + \sum_{i=1}^{n} w_i}{\phi c + \phi \lambda_0 c_e + \phi (1 - \lambda_0) c_s + \phi \sum_{i=1}^{n} c_i} p_c
\]  

(27)

\[
F(z_r) = F(z_c)
\]  

(28)

\[
q_c(w_1, \ldots, w_i, \ldots, w_n) = \left[ \frac{\phi c + \phi \lambda_0 c_e + \phi (1 - \lambda_0) c_s + \phi \sum_{i=1}^{n} c_i}{\phi c + \phi \lambda_0 c_e + \phi (1 - \lambda_0) c_s + \phi \sum_{i=1}^{n} c_i} \right]^b
\]  

(29)

Where \(A(z_r) = \int_{z_r}^{\infty} (z_r - x) f(x) dx\).

To coordinate the omnichannel in the assembly system, the following conditions need to be satisfied: \(p_r(w_1, \ldots, w_i, \ldots, w_n) = p_c\), and \(F(z_r) = F(z_c)\). Then we have the reaction function of coordinated wholesale prices w.r.t. the revenue keeping rate: \(w_i^c(\phi) = \phi c_i\).

Plugging the optimal stock factor, the optimal retail price, the optimal ordering quantity and the coordinated wholesale prices into the profit functions, we can obtain the reaction functions of coordinated profits of the module supplier \(i\) and the assembler w.r.t. the revenue keeping rate as follows:

\[
\Pi_{S_i}^c(\phi) = \frac{c_i}{\sum_{i=1}^{n} c_i}(1 - \phi)\Pi_{SC}, i = 1, 2, \ldots, n
\]

\[
\Pi_A^c(\phi) = \phi\Pi_{SC}
\]

Only when the following two conditions hold: \(\Pi_{S_i}^c(\phi) \geq \max \left\{ \Pi_{A}, \Pi_A \right\}, \Pi_{S_i}^d(\phi) \geq \max \left\{ \Pi_{A}, \Pi_A \right\}\), the members of assembly system would have the economic incentive to coordinate the omnichannel and achieve Pareto improvement of operational
performance. Thus, the reasonable interval of revenue keeping rate can be derived as follows: \( \phi^* \in [\underline{\phi}, \bar{\phi}] \). Hereinto, \( \bar{\phi} = \max \left\{ \left( \frac{b-\theta-n}{b-\theta} \right)^{b-\theta-1}, \left( \frac{b-\theta-1}{b-\theta} \right)^{n(b-\theta-1)} \right\} \), \( \underline{\phi} = \min_{i \in N} \left\{ 1 - \sum_{i=1}^{n} c_i \max \left( \frac{b-\theta-1}{b-\theta}, \left( \frac{b-\theta-1}{b-\theta} \right)^{n(b-\theta-1)} \right) \right\} \).

On this basis, the coordinated wholesale prices, the coordinated profits of the module supplier \( i \) and the assembler can be obtained as follows:

\[
w_i^c = \phi^* c_i, \quad i = 1, 2, \ldots, n
\]

\[
\Pi_{S_i}^c = \frac{c_i}{\sum_{i=1}^{n} c_i} (1 - \phi^*) \Pi_{SC}^c, \quad i = 1, 2, \ldots, n
\]

\[
\Pi_A^c = \phi^* \Pi_{SC}^c
\]

**Proposition 3.** In the coordination decision of assembly system under the OMO mode, the supplier who incurs more costs gains more coordinated profits under the OMO mode.

The analytical results of section 4.1 are summarized in Table 2. The centralized strategy neglects the roles of the suppliers in making crucial pricing and order quantity decisions and therefore is inferior to the coordination strategy regarding the derived solutions. Thus, the centralized decision results are not shown in Table 3 and will be ruled out in the coming discussions.

4.2. **Game-theoretical decision models under OMS mode.** Under the Omnichannel mode with offline channel subsidy (OMS mode), the market demand share of the offline channel \( (1 - \lambda) \) increases as the offline subsidy factor \( \delta \) increases, i.e., the market demand share of the online channel \( \lambda \) increases as \( \kappa \) increases. Then, we can define the reaction function of market demand share of the online channel \( \lambda \) w.r.t. \( \kappa \) as \( \lambda(\kappa) = \lambda_0 - h \delta(\kappa) = \lambda_0 - h(1 - \kappa) \), and \( 0 < h < 1 \). On this basis, the offline demand function and online demand function can be defined as follows:

\[
d_o(p) = \lambda(\kappa) a p^{-b} \kappa p^b x = \lambda(\kappa) \kappa^b y(p)x
\]

\[
d_s(p) = [1 - \lambda(\kappa)] a (kp)^{-b} p^b x = [1 - \lambda(\kappa)] \kappa^{-b} y(p)x
\]

Hence, the total market demand function \( d(p) \) can be defined as:

\[
d(p) = d_o(p) + d_s(p) = g(\kappa)y(p)x
\]

where \( g(\kappa) \equiv \lambda(\kappa) \kappa^b + [1 - \lambda(\kappa)] \kappa^{-b} \).

Likewise, we can define \( z = \frac{q}{g(\kappa)y(p)} \) as the stock factor, where \( q \) is the order quantity of the assembled product.

On this basis, the profit functions of the module supplier \( i \), the assembler and the assembly system under OMS mode can be expressed as follows:

\[
\Pi_{S_i}(w_i) = \left( w_i - c_i \right) g(\kappa)y(p)x, \quad i = 1, 2, \ldots, n
\]

\[
\Pi_A(p, z) = g(\kappa) \left\{ p y(p) \cdot E[\min\{z, x\}] + \eta p y(p) \cdot E[(z - x)^+] \right\} - \left\{ c + \lambda(\kappa)c_e + [1 - \lambda(\kappa)]c_s + \sum_{i=1}^{n} w_i \right\} y(p)x
\]

\[
\Pi_{SC}(p, z) = g(\kappa) \left\{ p y(p) \cdot E[\min\{z, x\}] + \eta p y(p) \cdot E[(z - x)^+] \right\} - \left\{ c + \lambda(\kappa)c_e + [1 - \lambda(\kappa)]c_s + \sum_{i=1}^{n} c_i \right\} y(p)x
\]

Conducting the same analysis as the previous modeling sub-sections, the analytical results of section 4.2 are summarized in Table 4.

Comparing the analytical results under OMS mode (Table 4) with those under OMO mode (Table 3), the key findings can be summarized in the following remarks:
Remark 3. Under the OMS mode, (i) the optimal stock factor equals to that under the OMO mode; (ii) the optimal retail price of the final product is $l(k)$ times of that under the OMO mode; (iii) the optimal ordering quantity of the final product is $o(k)$ times of that under the OMO mode; (iv) the optimal profit of the assembly system is $\alpha(k)$ times of that under the OMO mode. That is,

$$\frac{z_{sc}}{z_c} = 1, \frac{P_{sc}}{P_c} = l(k), \frac{q_{sc}}{q_c} = o(k), \frac{\Pi_{sc}}{\Pi_{SC}} = \alpha(k)$$

Hereinto, $l(k) \equiv \frac{c + \lambda(k)c_a + \frac{1 - \lambda(k)}{e + e_0c_a + (1 - \lambda_0)c_a + \sum_{i=1}^{n} c_i}}{c + \lambda(k)c_a}$, $o(k) = g(k)[l(k)]^{-(b - \theta)}$, $\alpha(k) = o(k)l(k)$

Remark 4. Under the scenario of suppliers’ simultaneous decision, (i) the equilibrium stock factor under the OMS mode equals to that under the OMO mode; (ii) the module supplier $i$’s profit margin under the OMS mode is $l(k)$ times of that under the OMO mode; (iii) the equilibrium retail price of the final product under the OMS mode is $l(k)$ times of that under the OMO mode; (iv) the equilibrium ordering quantity of the final product under the OMS mode is $o(k)$ times of that under the OMO mode; (v) the equilibrium profits of the module supplier $i$, the assembler and the assembly system under the OMS mode are $\alpha(k)$ times of those under the OMO mode, respectively. That is,

$$\frac{z_{sd}}{z_d} = \frac{z_{sc}}{z_c} = 1, \frac{P_{sd}}{P_d} = l(k), \frac{q_{sd}}{q_d} = o(k), \frac{\Pi_{sd}}{\Pi_{SD}} = \frac{\Pi_{sc}}{\Pi_{SC}} = \frac{\Pi_{sd}}{\Pi_{SD}} = \alpha(k), i = 1, 2, \ldots, n$$

Remark 5. Under the scenario of suppliers’ sequential decision, (i) the equilibrium stock factor under the OMS mode equals to that under the OMO mode; (ii) the module supplier $i$’s profit margin under the OMS mode is $l(k)$ times of that under the OMO mode; (iii) the equilibrium retail price of the final product under the OMS mode is $l(k)$ times of that under the OMO mode; (iv) the equilibrium ordering quantity of the final product under the OMS mode is $o(k)$ times of that under the OMO mode; (v) the equilibrium profits of the module supplier $i$, the assembler and the assembly system under the OMS mode are $\alpha(k)$ times of those under the OMO mode, respectively. That is,

$$\frac{z_{sd}}{z_d} = \frac{z_{sc}}{z_c} = 1, \frac{P_{sd}}{P_d} = l(k), \frac{q_{sd}}{q_d} = o(k), \frac{\Pi_{sd}}{\Pi_{SD}} = \frac{\Pi_{sc}}{\Pi_{SC}} = \frac{\Pi_{sd}}{\Pi_{SD}} = \alpha(k), i = 1, 2, \ldots, n$$

The coefficients of $l(k), o(k)$ and $\alpha(k)$ appear in the above three remarks, thus, it is necessary to discuss these coefficient values.

(1) Coefficient $l(k)$ is jointly affected by the market demand share of the online channel, offline channel subsidy factor, price-elasticity index of the expected demand, mutual fusion coefficient between channels, assembly cost, module costs, and online and offline channel costs, thus, $l(k)$ may be higher than/equal to/lower than 1. Especially, when $c_s = c_a, l(k) = 1$, the module supplier $i$’s equilibrium profit margin under the OMS mode is equal to that under the OMO mode, besides, the assembler’s retail price under the OMS mode is equal to that under the OMO mode.

(2) Coefficient $o(k)$ is jointly affected by the market demand share of the online channel, offline channel subsidy factor, price-elasticity index of the expected demand, mutual fusion coefficient between channels, assembly cost, module costs, and online and offline channel costs, thus, $o(k)$ may be higher than/equal to/lower than 1. (i) if $o(k) > 1$, the optimal (or equilibrium) ordering quantity of the final
product under the OMS mode is higher than that under the OMO mode; (ii) if \( o(\kappa) = 1 \), the optimal (or equilibrium) ordering quantity of the final product under the OMS mode equals to that under the OMO mode; (iii) if \( o(k) < 1 \), the optimal (or equilibrium) ordering quantity of the final product under the OMS mode is lower than that under the OMO mode.

(3) Incentive coefficient \( \alpha(\kappa) \) is jointly affected by the market demand share of the online channel, offline channel subsidy factor, price-elasticity index of the expected demand, mutual fusion coefficient between channels, assembly cost, module costs, and online and offline channel costs, thus, incentive coefficient \( \alpha(\kappa) \) may be higher than/equal to/lower than 1. (i) if incentive coefficient \( \alpha(\kappa) > 1 \), the optimal (or equilibrium) profits of the assembly system and its members under the OMS mode are higher than those under the OMO mode; (ii) if incentive coefficient \( \alpha(\kappa) = 1 \), the optimal (or equilibrium) profits of the assembly system and its members under the OMS mode equal to those under the OMO mode; (iii) if incentive coefficient \( \alpha(\kappa) < 1 \), the optimal (or equilibrium) profits of the assembly system and its members under the OMS mode are lower than those under the OMO mode.

In brief, the optimal stock factor will remain the same, regardless of whether it is OMO mode or OMS mode; however, the retail price, ordering quantities and thus the related profits in the assembly system will be jointly affected by multiple parameters discussed above.

4.3. **Game-theoretical decision models under POC/PFC mode** (\( \lambda(k) = 1 \) or \( 0, \theta = 0 \)). For the purpose of comparison, the assembly system under pure online/offline channel (POC/PFC) mode will be modeled by the same analytical approaches used in section 4.1.

In the POC mode, since the demand proportion of the online channel \( \lambda_0 = 1 \), the offline subsidy factor \( \delta = 0 \), i.e., \( \kappa = 1 \), thus we have \( \lambda(\kappa) = 1 \). Besides, the mutual fusion coefficient between channels \( \theta = 0 \). Conducting the same analysis as the previous modeling sub-sections, the analytical results are summarized in Table 5.

In the PFC mode, since the demand proportion of the online channel \( \lambda_0 = 0 \), the offline subsidy factor \( \delta = 0 \), i.e., \( \kappa = 1 \), thus we have \( \lambda(\kappa) = 0 \). Besides, the mutual fusion coefficient between channels \( \theta = 0 \). Conducting the same analysis as the previous modeling sub-sections, the analytical results are summarized in Table 6.

4.4. **Analytical results discussions.** Based on the remarks derived in the above section, the following findings are identified and discussed:

(1) Under the OMO mode and OMS mode, as the price-elasticity index of the expected demand decreases, the ordering quantities of the final product for both the online channel and offline channel will increase, and the profit of the assembly system will also increase. Furthermore, as the mutual fusion coefficient between channels increases, the ordering quantities of the final product for both the online channel and offline channel and the profit of the assembly system will increase.

(2) In all game-theoretical decision models listed in Table 2, comparing the analytical results shown from Table 3 to Table 6, it is found that as the module costs, the assembly cost, the operational cost of the online- or offline- channel decreases, the optimal retail price will be reduced, and the optimal ordering quantity and the optimal profits of the assembly system and its members will be increased. The findings imply that the assembly system and its members have the motivation to pursue
cost reduction strategies that may allow the members in the assembly system to lower price to stimulate more sales and boost their profits.

(3) Whether it is OMO mode (Table 3), OMS mode (Table 4), POC mode (Table 5) or PFC mode (Table 6), when the decentralized strategy is taken, the retail price of the decentralized strategy with suppliers’ sequential actions is less than that of the decentralized strategy with suppliers’ simultaneous actions, the ordering quantity of the decentralized strategy with suppliers’ sequential actions is higher than that of the decentralized decision with suppliers’ simultaneous actions, and the profits of the decentralized strategy with suppliers’ sequential actions are higher than those of the decentralized strategy with suppliers’ simultaneous actions.

In brief, the suppliers’ sequential actions can deliver better results than those with the suppliers’ simultaneous actions. Furthermore, comparing the analytical results of the coordination strategy with those of the decentralized strategy, the retail price of the coordination strategy is less than that of the decentralized strategy with suppliers’ sequential actions, the ordering quantity of the coordination strategy is higher than that of the decentralized strategy with suppliers’ sequential actions, and the profits of coordination strategy are higher than those of the decentralized strategy with suppliers’ sequential actions. In brief, the coordination strategy can deliver better results than those with the decentralized strategies. These findings imply that the coordination strategy actually outperforms the decentralized strategy no matter what actions suppliers take.

(4) Whether it is OMO mode, OMS mode, POC mode or PFC mode, the revenue sharing contract mechanism can effectively coordinate the members of the assembly system with omnichannel to make the best pricing and quantity decisions that create the best profits to all members.

The numerical and sensitivity analyses in the next section validate and reveal the key analytical findings of this section by a real example, thus, provide a more powerful explanation to the theoretical findings and comparisons in this section.

Table 3. Analytical Results under the OMO Mode

| Results/Strategy | Decentralized (Equilibrium) strategy | Coordination strategy |
|------------------|------------------------------------|----------------------|
| $F_{w}$         | $F_{w} = F_{s}$                    | $F_{w} = F_{s}$      |
| $p_{s}$         | $p_{s} = \frac{\lambda c_{s}}{1 - \alpha}$ | $p_{s} = \frac{\lambda c_{s}}{1 - \alpha}$ |
| $\theta$        | $\theta_{l} = \frac{\lambda c_{l}}{1 - \alpha}$ | $\theta = \theta_{l}$ |
| $\Pi_{l}$       | $\Pi_{l} = \frac{\lambda c_{l}}{1 - \alpha} \Pi_{l}$ | $\Pi_{l} = \frac{\lambda c_{l}}{1 - \alpha} \Pi_{l}$ |
| $\Pi_{c}$       | $\Pi_{c} = \frac{\lambda c_{c}}{1 - \alpha} \Pi_{c}$ | $\Pi_{c} = \frac{\lambda c_{c}}{1 - \alpha} \Pi_{c}$ |
| $g_{s}$         | $g_{s} = \frac{\lambda c_{s}}{1 - \alpha}$ | $g_{s} = \frac{\lambda c_{s}}{1 - \alpha}$ |

**Note:** $\lambda = \frac{\sum_{i=1}^{n} \eta_{i}}{n}$, $\theta = \frac{\sum_{i=1}^{n} \theta_{i}}{n}$

5. **Numerical and sensitivity analyses.** An electronic product is selected from the China market for the numerical and sensitivity analyses purpose [17]. The parameters and their values are listed in Table 7. They will serve as the inputs to the analytical models developed in sections 4.1, 4.2, 4.3 and 4.4.
Table 4. Analytical Results under the OMS Mode

| Results | Suppliers’ Simultaneous Actions | Suppliers’ Sequential Actions | Coordination Strategy |
|---------|---------------------------------|-------------------------------|-----------------------|
| $F(z_o)$ | $F(z_o) = F(x_o)$ | $F(z_o) = F(x_o)$ | $F(z_o) = \frac{1}{2} \left( 1 - (1 - \theta) \pi_o \right)$ |
| $p_o$ | $p_o = \frac{1}{2} \theta p_o$ | $p_o = \frac{1}{2} \theta p_o$ | $p_o = \frac{1}{2} \theta p_o$ |
| $q_o$ | $q_{od} = \left( \frac{1}{2} \lambda \right) q_{od}$ | $q_{od} = \left( \frac{1}{2} \lambda \right) q_{od}$ | $q_{od} = \left( \frac{1}{2} \lambda \right) q_{od}$ |
| $s^*_i$ | $w^*_i = \frac{1}{2} \left( \frac{1}{\lambda} \right) \left( c + g + \sum c_i \right) + 1$ | $w^*_i = \frac{1}{2} \left( \frac{1}{\lambda} \right) \left( c + g + \sum c_i \right) + 1$ | $w^*_i = \frac{1}{2} \left( \frac{1}{\lambda} \right) \left( c + g + \sum c_i \right) + 1$ |
| $N_b$ | $N^b = \left( \frac{1}{2} \lambda \right) N^b$ | $N^b = \left( \frac{1}{2} \lambda \right) N^b$ | $N^b = \left( \frac{1}{2} \lambda \right) N^b$ |
| $N_o$ | $N^o = \left( \frac{1}{2} \lambda \right) N^o$ | $N^o = \left( \frac{1}{2} \lambda \right) N^o$ | $N^o = \left( \frac{1}{2} \lambda \right) N^o$ |
| $\theta^*$ | $\theta^* = \left( \frac{1}{2} \lambda \right) \theta^*$ | $\theta^* = \left( \frac{1}{2} \lambda \right) \theta^*$ | $\theta^* = \left( \frac{1}{2} \lambda \right) \theta^*$ |

Note: $z = \max \left( \left( \frac{1}{2} \lambda \right), \left( \frac{1}{2} \lambda \right) \right)$, $\theta = \min \left( \theta, \frac{1}{2} \lambda \right)$, $\max \left( \left( \frac{1}{2} \lambda \right), \left( \frac{1}{2} \lambda \right) \right)$

Table 5. Analytical Results under the POC Mode

| Results | Suppliers’ Simultaneous Actions | Suppliers’ Sequential Actions | Coordination Strategy |
|---------|---------------------------------|-------------------------------|-----------------------|
| $F(z_c)$ | $F(z_c) = F(x_c)$ | $F(z_c) = F(x_c)$ | $F(z_c) = \frac{1}{2} \left( 1 - (1 - \theta) \pi_c \right)$ |
| $p_c$ | $p_{cd} = \frac{1}{2} \theta p_{cd}$ | $p_{cd} = \frac{1}{2} \theta p_{cd}$ | $p_{cd} = \frac{1}{2} \theta p_{cd}$ |
| $q_c$ | $q_{cd} = \left( \frac{1}{2} \lambda \right) q_{cd}$ | $q_{cd} = \left( \frac{1}{2} \lambda \right) q_{cd}$ | $q_{cd} = \left( \frac{1}{2} \lambda \right) q_{cd}$ |
| $s^*_i$ | $w^*_i = \frac{1}{2} \left( \frac{1}{\lambda} \right) \left( c + g + \sum c_i \right) + 1$ | $w^*_i = \frac{1}{2} \left( \frac{1}{\lambda} \right) \left( c + g + \sum c_i \right) + 1$ | $w^*_i = \frac{1}{2} \left( \frac{1}{\lambda} \right) \left( c + g + \sum c_i \right) + 1$ |
| $N_b$ | $N^b = \left( \frac{1}{2} \lambda \right) N^b$ | $N^b = \left( \frac{1}{2} \lambda \right) N^b$ | $N^b = \left( \frac{1}{2} \lambda \right) N^b$ |
| $N_o$ | $N^o = \left( \frac{1}{2} \lambda \right) N^o$ | $N^o = \left( \frac{1}{2} \lambda \right) N^o$ | $N^o = \left( \frac{1}{2} \lambda \right) N^o$ |
| $\theta^*$ | $\theta^* = \left( \frac{1}{2} \lambda \right) \theta^*$ | $\theta^* = \left( \frac{1}{2} \lambda \right) \theta^*$ | $\theta^* = \left( \frac{1}{2} \lambda \right) \theta^*$ |

Note: $z = \max \left( \left( \frac{1}{2} \lambda \right), \left( \frac{1}{2} \lambda \right) \right)$, $\theta = \min \left( \theta, \frac{1}{2} \lambda \right)$, $\max \left( \left( \frac{1}{2} \lambda \right), \left( \frac{1}{2} \lambda \right) \right)$

Table 6. Analytical Results under the PFC Mode

| Results | Suppliers’ Simultaneous Actions | Suppliers’ Sequential Actions | Coordination Strategy |
|---------|---------------------------------|-------------------------------|-----------------------|
| $F(z_c)$ | $F(z_c) = F(x_c)$ | $F(z_c) = F(x_c)$ | $F(z_c) = \frac{1}{2} \left( 1 - (1 - \theta) \pi_c \right)$ |
| $p_c$ | $p_{cd} = \frac{1}{2} \theta p_{cd}$ | $p_{cd} = \frac{1}{2} \theta p_{cd}$ | $p_{cd} = \frac{1}{2} \theta p_{cd}$ |
| $q_c$ | $q_{cd} = \left( \frac{1}{2} \lambda \right) q_{cd}$ | $q_{cd} = \left( \frac{1}{2} \lambda \right) q_{cd}$ | $q_{cd} = \left( \frac{1}{2} \lambda \right) q_{cd}$ |
| $s^*_i$ | $w^*_i = \frac{1}{2} \left( \frac{1}{\lambda} \right) \left( c + g + \sum c_i \right) + 1$ | $w^*_i = \frac{1}{2} \left( \frac{1}{\lambda} \right) \left( c + g + \sum c_i \right) + 1$ | $w^*_i = \frac{1}{2} \left( \frac{1}{\lambda} \right) \left( c + g + \sum c_i \right) + 1$ |
| $N_b$ | $N^b = \left( \frac{1}{2} \lambda \right) N^b$ | $N^b = \left( \frac{1}{2} \lambda \right) N^b$ | $N^b = \left( \frac{1}{2} \lambda \right) N^b$ |
| $N_o$ | $N^o = \left( \frac{1}{2} \lambda \right) N^o$ | $N^o = \left( \frac{1}{2} \lambda \right) N^o$ | $N^o = \left( \frac{1}{2} \lambda \right) N^o$ |
| $\theta^*$ | $\theta^* = \left( \frac{1}{2} \lambda \right) \theta^*$ | $\theta^* = \left( \frac{1}{2} \lambda \right) \theta^*$ | $\theta^* = \left( \frac{1}{2} \lambda \right) \theta^*$ |

Note: $z = \max \left( \left( \frac{1}{2} \lambda \right), \left( \frac{1}{2} \lambda \right) \right)$, $\theta = \min \left( \theta, \frac{1}{2} \lambda \right)$, $\max \left( \left( \frac{1}{2} \lambda \right), \left( \frac{1}{2} \lambda \right) \right)$

The assembly system with omnichannel is composed of four key module suppliers and one assembler with omnichannel, i.e., $n = 4$, and $i = 1, 2, 3, 4$. The unit cost of the modules are represented by $c_1, c_2, c_3,$ and $c_4$ valued at 149.53, 80, and 60 USD/unit respectively. The unit assembly cost of the final product for the assembler $c$ is 50 USD/unit. The operational cost of the online channel $c_o$ is 26 USD/unit and
Table 7. Parameter Values for Numerical Analysis

| Parameters | Value |
|------------|-------|
| $c$ | Assembly cost (USD/unit) | 50 |
| $c_1$ | 1\textsuperscript{st} module cost (USD/unit) | 149 |
| $c_2$ | 2\textsuperscript{nd} module cost (USD/unit) | 53 |
| $c_3$ | 3\textsuperscript{rd} module cost (USD/unit) | 80 |
| $c_4$ | 4\textsuperscript{th} module cost (USD/unit) | 60 |
| $c_e$ | Operational cost of the online channel (USD/unit) | 26 |
| $c_s$ | Operational cost of offline channel (USD/unit) | 39 |
| $a$ | Positive constant number | $1E+18$ |
| $b$ | Price-elasticity index of the expected demand | 5.0 |
| $\theta$ | Mutual fusion coefficient between channels | 0.5 |
| $\eta$ | Clearance discount price factor | 50\% |
| $\lambda_0$ | Market demand share of the online channel | 0.6 |
| $\delta$ | The offline channel subsidy factor, and $0 < \delta < 1$ | 0.1 |
| $\kappa$ | The offline channel discount price factor, and $\kappa = 1 - \delta$ | 0.9 |
| $h$ | the reaction extent of $\lambda(\kappa)$ w.r.t. the change of $\kappa$ | 0.5 |
| $\phi$ | Revenue keeping rate | 0.7 |
| $\mu$ | Mean value of random factor | 100 |
| $\sigma$ | Standard deviation of random factor | 10 |

the operational cost of the offline channel $c_s$ is 39 USD/unit. The clearance discount price factor $\eta$ is set at 50\%; the market demand share of the online channel $\lambda_0$ is set at 0.6; the offline channel subsidy factor for unit product $\delta$ is set at 0.1; the positive constant number $a$ is set at $1E+18$; the price-elasticity index of expected demand $b$ is set at 5.0; and the mutual fusion coefficient between channels $\theta$ is set at 0.5. The random factor $x$ obeys normal distribution, i.e. $x \sim N(\mu, \sigma^2)$. $A$ is set at 0.1 and $B$ is set at 1000.

5.1. Numerical analysis. The numerical analysis results of all models are shown in Table 8 (OMO mode vs OMS mode) and Table 9 (POC mode vs PFC mode) for further comparison purpose. It is noted that the incentive condition $\alpha(k) > 1$ holds in this case.

Selective pairwise comparisons between models are conducted to further unravel the key differences between the models and identify the best strategy. The centralized strategy is not included in the following analysis since it cannot compete with the coordination strategy as we have already discussed in the previous section. The findings are summarized and discussed below:

**OMO vs POC**
Comparing the assembly system under OMO mode (Table 8) with that under the pure online channel (POC) mode (Table 9), it is found that: (i) the stock factor and the retail price under OMO mode are higher than those under the POC mode; (ii) the ordering quantity under OMO mode is higher than that under the POC mode; and (iii) the profits of the assembly system and its members under OMO mode are higher than those under the POC mode.

**OMO vs PFC**
Comparing the assembly system under OMO mode (Table 8) with that under pure offline channel (PFC) mode (Table 9), it is found that: (i) the stock factor and
the retail price under OMO mode are higher than those under PFC mode; (ii) the ordering quantity under OMO mode is higher than that under PFC mode; and (iii) the profits of the assembly system and its members under OMO mode are higher than those under PFC mode.

**OMO vs OMS**

Comparing the assembly system under OMO mode (without offline channel subsidy) with that under OMS mode (with offline channel subsidy) in Table 8, it is found that: (i) the stock factor and the retail price under OMS mode equal to those under OMO mode; (ii) the ordering quantity under OMS mode is higher than that under OMO mode; and (iii) the profits of the assembly system and its members under OMS mode are higher than those under OMO mode.

**Simultaneous (SI) vs Sequential (SE) vs Coordination (CO) Decisions**

For all model types, it is found that: (i) the stock factor: SI=SE=CO; (ii) the retail price: CO < SE < SI; (iii) the ordering quantity: CO > SE > SI; and (iv) the profits of the assembly system and its members: CO > SE > SI.

**Coordination Mechanism based on Revenue Sharing Contract**

Whether it is the assembly system under OMO mode, OMS mode, POC mode, or PFC mode, a revenue sharing contract can effectively coordinate the members in the assembly system and achieve better operational performances for its members.

**Summary**

Across the game-theoretical decision models, we have observed several phenomena. First, the worst strategy is the decentralized assembly system with suppliers’ simultaneous actions which has set an exceptionally high retail price causing an extremely low demand for the assembled product. Second, the decentralized strategy with suppliers’ sequential actions provides much better solutions and profits than those of the decentralized strategy with suppliers’ simultaneous actions. Third, the centralized and the coordinated strategies provide the same results in exception that each member in the assembly system applying the centralized strategy does not need to make individual pricing and order quantity decisions. Fourth, the coordinated strategy outperforms the decentralized strategy regarding the profits in the assembly system and for its members.

In sum, through the above pairwise model comparisons, the best operational strategy for the assembly system sold through omnichannel is identified as the coordination strategy that subsidizes the offline channel to build stronger mutual fusion effect. This has allowed the omnichannel assembler to set a lower retail price to the market and create the higher product demand and profits that could be later shared with the suppliers to compensate the low wholesale prices of the modules and eventually bring out the highest profits to the suppliers. This finding provides a profound practical implication to the omnichannel practitioners implementing an offline subsidy strategy.

5.2. **Sensitivity analysis.** Since, in section 5.1 analysis, OMS mode is found to be the most attractive business mode to the omnichannel practitioners, the sensitivity analysis will focus on how the changes of three key parameters of the models under OMS mode impact the profits of the members in the assembly system. Three key parameters are: *offline channel subsidy factor* ($\delta$), *price-elasticity index of the expected demand* ($b$), and *mutual fusion coefficient between channels* ($\theta$). The increment scale and range of the change of each parameter in the sensitivity analysis are listed in Table 10.
### Table 8. Numerical Analysis Results for the Omnichannel mode

| Strategy          | Decentralized strategy | Coordination strategy | Decentralized strategy | Coordination strategy |
|-------------------|------------------------|-----------------------|------------------------|-----------------------|
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Coordination      | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
|                   | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
| Decentralized     | Simultaneous actions   | Sequential actions    | Simultaneous actions   | Sequential actions    |
For the suppliers, Figure 2 shows that the higher module cost supplier can gain more profits than those with lower module costs, however, the relationship is not linear rather exponential. In other words, the impact of offline subsidy factor on profits is much more dramatic.

5.2.2. Price-elasticity index of the expected demand \((b)\). The sensitivity analysis results of the price-elasticity index of the expected demand \((b)\) are shown in Figure 3. It is clear there is a reverse exponential relationship between profits and \(b\). In this case, it implies that products with a lower price-elasticity index (less than 5, and closer to 4.5) are better choices to the assembly system with omnichannel than those with a higher index.

Similar to Figure 2, for the suppliers, Figure 3 shows that the higher module cost supplier can gain more profits than those with lower module costs, however, the relationship is not a positive exponential rather a reverse exponential one. In other words, the impact of the price-elasticity index of the expected demand on profits is very dramatic but in a reverse direction, i.e., the higher the price-elasticity, the lower the profits the assembly system and its members can get.

5.2.3. Mutual fusion coefficient between channels \((\theta)\). The sensitivity analysis results of the mutual fusion coefficient between channels \((\theta)\) against profits are shown in Figure 4. There is a positive exponential relationship between profits and \(\theta\). It implies that the stronger the mutual fusion effects between online- and offline-channels, the higher the profits to suppliers and assembler (thus the assembly system) can be achieved and the impact can be especially high when the mutual fusion coefficient increases beyond 0.5.

Similar to Figure 2, for the suppliers, Figure 4 shows that the higher module cost supplier can gain more profits than those with lower module costs. In other words, the impact of the mutual fusion coefficient on profits is very dramatic, i.e., the higher the mutual fusion coefficient, the higher the profits the assembly system and its members can get; in particular, when \(\theta\) increases beyond 0.7.
4.5 4.6 4.7 4.8 4.9 5 5.1 5.2 5.3 5.4 5.5
price-elasticity index of the expected demand

Figure 3. Impact of Price Elasticity Index of the Expected Demand (b) Change on Profits

0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
mutual fusion coefficient between channels

Figure 4. Impact of Mutual fusion Coefficient (θ) Change on Profits

In summary, the sensitivity analysis of three key parameters on OMS mode provides valuable findings that have added to both theoretical and practical understanding of the research questions.

6. Theoretical and managerial insights. Based on the analytical and numerical results and discussions, this section first discusses the theoretical insights, followed by the managerial insights gained from this study.
6.1. **Theoretical insights.** The theoretical insights for the optimal operations management of the assembly system with omnichannel are discussed and summarized in this section.

(1) Regardless of whether it is OMO mode, OMS mode, POC mode or PFC mode, when the decentralized strategy is taken, compared with simultaneous actions, adopting the sequential actions among the suppliers can effectively improve the operational performance for all members of the assembly system. This is a typical 'late-mover advantage'.

(2) Regardless of whether it is OMO mode, OMS mode, POC mode or PFC mode, compared with decentralized strategy, the coordination strategy via the revenue sharing contract can effectively improve operational performance for all members of assembly system.

(3) Compared with the pure channel mode, adopting the omnichannel mode can effectively improve the operational performance of the assembly system. By giving full play to the advantages of information-accessing in online channel and product-experience in offline channels, the omnichannel mode can bring about mutual fusion effect between the online- and offline- channels for the assembly system, and thus increasing the demand and profit in both the online- and offline- channels.

(4) Compared with the OMO mode, adopting offline channel subsidy under OMS mode (omnichannel mode with offline channel subsidy) can effectively improve the operational performance of the assembly system, when the incentive condition $\alpha(\kappa) > 1$ holds. By attracting more customers to involve product experiences through offline channel subsidy, the omnichannel mode with offline channel subsidy can strengthen mutual fusion effect for omnichannel for the assembly system and enhance value added and creation for customers, and thus increasing the demand and profit in both the online- and offline- channels.

(5) Regardless of whether it is OMO mode, OMS mode, POC mode or PFC mode, reducing the module costs, the assembly cost and the operational costs of online- or offline- channel, can effectively improve the operational performance of the assembly system.

6.2. **Managerial insights.** Despite its tremendous growth over the last few years globally, ecommerce sales still only represent 8.3% of total retail sales in the U.S. [31]. Even though more consumers are getting used to buying things like books, shoes and electronics online, the majority of spending still takes place in brick and mortar outlets. In fact, apart from Amazon, all of the top ten retailers in the U.S. are old-school, brick-and-mortar stores. Majority of shoppers conduct online research before making big purchases. Being able to channel even a small percentage of these customers straight from their online research to offline stores would represent a massive potential for brick-and-mortar stores. This potential has driven major retailers, either e-commerce or traditional retailers, to jump into an omnichannel commerce wagon.

This study and its key theoretical findings provide new and useful managerial insights to the omnichannel practitioners. First, it is shown that an omnichannel business mode can deliver better financial performance than that of a pure online- or offline- channel mode to the assembly system. Furthermore, OMS mode (omnichannel mode with the offline channel subsidy) can bring more demand and create more profits for the assembly system than those of OMO mode. Therefore, adopting OMS mode can effectively improve the operational performance for the assembly
Second, the coordination strategy via a revenue sharing contract mechanism is shown theoretically as the best operational strategy to increase demands and profits for the assembly system with omnichannel. Furthermore, if the coordination strategy is ruled out due to non-economic reasons, the equilibrium strategy with suppliers’ sequential actions would be the second-best choice for the assembly system with omnichannel.

Finally, this study shows that a higher offline subsidy factor, a lower price-elasticity index of the expected demand and a higher mutual fusion coefficient between channels will create more profits for the assembly system under OMS mode. Therefore, setting a higher offline channel subsidy to attract more demand to the offline brick-and-mortar channel, assembling and selling a lower price elasticity product, and enhancing the communication and integration between the ecommerce and physical channels, would be good marketing and operational strategies for assembly system in omnichannel business scenario.

7. Conclusion. Unlike existing omnichannel literature with focus on the study of more visible segments in a supply chain, i.e., seller and buyer relationship, this paper explores beyond seller and buyer relationship in the omnichannel commerce and investigates extensively how an assembly system - less visible segment in an omnichannel business mode - interacts with an omnichannel market regarding key business decisions. It has built a new theoretical foundation for the future supply chain-oriented studies in the omnichannel commerce.

There are three key findings in this study: (1) compared with the pure online- or offline-channel mode, the omnichannel business mode with or without offline channel subsidy can deliver much better operational performance to the assembly system via mutual fusion effect. (2) the offline channel subsidy can effectively enhance the mutual fusion effect to increase the demand dramatically and create better operational performance for the assembly system under an omnichannel business mode. (3) the coordination strategy outperforms the decentralized strategy regarding the operational performance of the assembly system, regardless of whether it is omnichannel mode (with/without offline channel subsidy) or pure channel mode (pure online- or offline-channel mode).

Judging from the fast convergence of the e-commerce and brick-and-mortar commerce, the omnichannel commerce is moving forward to becoming a major business model for many retailers and even assemblers or manufacturers. Academic research is currently lacking behind the business development in the omnichannel commerce. More efforts should be devoted into the omnichannel commerce study.

Due to the constraints of the relevant literature shortage, research fund limitation and empirical data collection difficulty, this study focuses mainly on the theoretical exploration of the assembly system in an omnichannel business mode. Even though insightful findings are discovered, there are still many important research issues worthy of further exploration in the future. First, the service quality variable may be considered in the assembly system with omnichannel. Second, the assembly system may be extended to be three-echelon supply chain composed of multiple complementary module suppliers, an assembler and omnichannel retailer in the future research. Third, different market power and related decision structure may be taken into account in the assembly system with omnichannel. Fourth, other
types of coordination contracts can also be considered in the assembly system with omnichannel. Finally, fairness concern or overconfidence of decision-makers in the assembly system can be considered in the future research.

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