The nonlinear anisotropic model of the Universe with tadpole

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Some subclasses of Horndeski theory allow for non-standard behavior of anisotropy in the homogeneous Bianchi I cosmology. For example, the anisotropy are damped near the initial singularity instead of tending to infinity. In this article, we analyze nonlinear anisotropic models with a tadpole term. We have considered an example of such a theory for which the anisotropy is maximal and finite at the initial moment of time and approaches zero at later times. The anisotropy suppression occurs during the inflationary stage. This cosmological model does not contain a singular point.

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I.INTRODUCTION

There is a widespread belief that the modern Universe is homogeneous and isotropic. In his work \cite{1} Mizner C. W speaks about this mysterious fact. The question arises. Has there been an anisotropic phase in the past? The isotropy of the Universe in the past does not follow from any general principles. One of the main arguments for the existence of the anisotropic phase of the Universe is anisotropy at various scales of the microscopic wave background (CMB). The CMB contains information about the past of the Universe. There are anomalies in the observed CMB at the largest scales \cite{2}-\cite{7}. The Bianchi Universe can explain these anomalies of the CMB \cite{8}.

An important criterion for the viability of any anisotropic model is its isotropization time. Observational data indicate that the isotropization of the Universe occurred quite early, no later than the beginning of primary nucleosynthesis, \( t \sim 1 \text{ s} \) \cite{9}. Authors \cite{10} argue that the particle production provides the early isotropization. The anisotropic terms in the Einstein equations become dominant when one goes backwards in time, then they endure an infinite discontinuity. When the universe expands, the anisotropy terms decrease faster than the contribution of other forms of energy subject, and the Universe rapidly approaches a locally isotropic state \cite{11}. The state of affairs changes for modified theories of gravity. Some subclasses of Horndeski gravity (HG) allow for non-standard behavior of anisotropy in the homogeneous Bianchi I cosmology. For example, in works \cite{13}, \cite{14}, the HG theories were considered, in which the anisotropies show a maximum at intermediate times and approach zero at early and late times.

The HG is determined by the following action density \cite{15}:

\[
L_H = \sqrt{-g} \left( L_2 + L_3 + L_4 + L_5 \right),
\]

one has (in the parameterization of ref.\cite{12}):

\[
L_2 = G_2(\phi, X), \quad L_3 = -G_3(\phi, X)\Box \phi,
\]

\[
L_4 = G_4(\phi, X)R + G_{4X}(\phi, X) \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right],
\]

\[
L_5 = G_5(\phi, X)G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{1}{6}G_{5X} \left[ (\Box \phi)^3 - 3 \Box \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3 \right],
\]

respectively, where \( g \) is the determinant of metric tensor \( g_{\mu\nu} \); \( R \) is the Ricci scalar and \( G_{\mu\nu} \) is the Einstein tensor; the factors \( G_i \) (\( i = 2, 3, 4, 5 \)) are arbitrary functions of the scalar field \( \phi \) and the canonical kinetic term, \( X = -\frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi \).

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We consider the definitions \( G_{iX} \equiv \partial G_i/\partial X \), \( (\nabla_{\mu} \nabla_{\nu} \phi)^2 \equiv \nabla_{\mu} \nabla_{\nu} \phi \nabla^\rho \nabla^\sigma \phi \), and \( (\nabla_{\mu} \nabla_{\nu} \phi)^3 \equiv \nabla_{\mu} \nabla_{\nu} \phi \nabla^\rho \nabla^\sigma \phi \nabla^\tau \phi \). The HG occupies a special place among the modified models. The field equations in GR are differential equations of the second order, thus, evading Ostrogradski instabilities arising \([16, 17]\). The HG is the most general variant of the scalar-tensor theory of gravitation with motion equations of the second order. The action density of HG contains several functions that provide a broad phenomenology. This makes it possible to solve important cosmological and astrophysical problems (screening of the cosmological constant, kinetic inflation, late de Sitter stage, hairy black holes, etc.) \([15, 23]\).

In this article, we review the HG within the framework of the Bianchi I cosmological model. In the case \( G_{5X} \neq 0 \), the gravitational equations give consequences containing nonlinear anisotropic terms. Here we study the effects of nonlinear anisotropy for the function \( G_2(X, \phi) = -l \cdot \phi + A(X) \) \([24, 25]\). The dynamics of field functions will be provided by a tadpole term \( l \cdot \phi \). In the work \([14]\), the nonlinear anisotropy was studied for the model

\[
G_2 = X - \Lambda, \quad G_3 = 0, \quad G_4 = \text{const}, \quad G_5 = \text{const} + \xi \sqrt{2X}.
\]

This model contains early and late inflation with suppressed anisotropy. Anisotropy shows a maximum at intermediate times. In the work \([27]\), the nonlinearity leads to the effect of "anisotropization" in the later times, that is, the Universe evolves from an isotropic state to an anisotropic one.

## II. BIANCHI I MODEL

We assume that the space-time is anisotropic, spatially homogeneous and is described by the metric (Bianchi I model):

\[
\mathrm{ds}^2 = -dt^2 + a_1^2(t)dx^2 + a_2^2(t)dy^2 + a_3^2(t)dz^2.
\]

The scale factors \( a_i \) and the scalar field \( \phi(t) \) depend only on \( t \). Then the gravity equations of take the form \([14]\):

\[
G_0^0 \left( \mathcal{G} - 2G_{4X} \dot{\phi}^2 - 2G_{4X} X \dot{\phi}^4 + 2G_{5X} \dot{\phi}^2 + G_{5X} \dot{\phi}^4 \right) = G_2 - G_{2X} \dot{\phi}^2 - 3G_{3X} H \dot{\phi}^3 + 3G_{5X} H \dot{\phi}^2 + G_{5X} \dot{\phi}^3 H - 5G_{5X} H_1 H_2 H_3 \dot{\phi}^3 - G_{5X} H_1 H_2 H_3 \dot{\phi}^5,
\]

\[
\mathcal{G} G_i^j - (H_j + H_k) \frac{dG_i}{dt} = G_2 - l \cdot \phi \frac{dG_3}{dt} + 2 \frac{d}{dt}(G_4 \dot{\phi}) - \frac{d}{dt}(G_5 \dot{\phi}^2 H_j H_k) - 3G_{5X} \dot{\phi}^3 H_j H_k (H_j + H_k).
\]

Here the dot denotes the \( t \)-derivative, one has \( H_i = \dot{a}_i/a_i \), and the average Hubble parameter is \( H = \frac{1}{3} \sum_{i=1}^{3} H_i \equiv \dot{a}/a \) with \( a = (a_1 a_2 a_3)^{1/3} \). The Einstein tensor components are

\[
G_0^0 = -(H_1 H_2 + H_2 H_3 + H_3 H_1),
\]

\[
G_i^j = -\left( \dot{H}_j + \dot{H}_k + H_j^2 + H_k^2 + H_j H_k \right),
\]

where the triples of indices \( \{i, j, k\} \) take values \( \{1, 2, 3\}, \{2, 3, 1\}, \) or \( \{3, 1, 2\} \). In addition, we have defined

\[
\mathcal{G} = 2G_4 - 2G_{4X} \dot{\phi}^2 + G_{5X} \dot{\phi}^2.
\]

Variation of action \([11]\) on \( \phi(t) \) gives the equation:

\[
\frac{1}{a^3} \frac{d}{dt}(a^3 J) = \mathcal{P},
\]

with

\[
J = \dot{\phi} \left[ G_{2X} - 2G_{3X} + 3H \dot{\phi}(G_{3X} - 2G_{4X} \phi) + G_0^0(-2G_{4X} - 2\phi^2 G_{4XX} + 2G_{5X} + G_{5X} \phi^2) + H_1 H_2 H_3 (3G_{5X} \dot{\phi} + G_{5X} \dot{\phi}^3) \right],
\]
The functions \(e^5, e^6\) and \(e^{10}\) we obtain the consequences

\[
\frac{3}{8\pi}(H^2 - \sigma^2) = l\phi - A + \dot{\phi}^2A_X + 3G_{3X}H\ddot{\phi}^3 +
\]

\[+\dot{\phi}^3(5G_{5X} + G_{5XX}\dot{\phi}^2)(H - 2\dot{\beta}_+)[(H + \dot{\beta}_+)^2 - 3\dot{\beta}_+^2],\]  

Expression \(l \cdot \phi\) corresponds the tadpole term. In the future, it will provide a dynamic solution to \(\dot{\phi}(t), \dot{\beta}_\pm(t)\). The theory with \(G_5(X)\) gives nontrivial behavior of anisotropy. Taking into account (13), (14) and (15) from equations (5), (6) and (10) we obtain the consequences

\[
\frac{3}{8\pi}(H^2 - \sigma^2) = l\phi - A + \dot{\phi}^2A_X + 3G_{3X}H\ddot{\phi}^3 +
\]

\[+\dot{\phi}^3(5G_{5X} + G_{5XX}\dot{\phi}^2)(H - 2\dot{\beta}_+)[(H + \dot{\beta}_+)^2 - 3\dot{\beta}_+^2],\]  

\[
\frac{1}{8\pi}(2H + 3H^2 + 3\sigma^2) = l\phi - A + G_{3X}\dot{\phi}^2\ddot{\phi} +
\]

\[+\frac{d}{dt}
\left[
G_{5X}\ddot{\phi}^3(H^2 - \sigma^2)
\right] + 2G_{5X}\ddot{\phi}^3 \left(H^3 + \dot{\beta}_+^2 - 3\dot{\beta}_+^2\right),\]  

\[
\frac{\dot{\beta}_+}{8\pi} + G_{5X}\ddot{\phi}^3 \left(\dot{\beta}_+^2 - \dot{\beta}_+^2 - H\dot{\beta}_+\right) = C_+,\]  

\[
\frac{\dot{\beta}_-}{8\pi} + G_{5X}\ddot{\phi}^3 \left(2\dot{\beta}_+\dot{\beta}_- - H\dot{\beta}_-\right) = C_-,\]  

\[
\dot{\phi} \left[A_X + 3HG_{3X}\dot{\phi}^2 + (H - 2\dot{\beta}_+)(H + \dot{\beta}_+)^2 - 3\dot{\beta}_+^2\right] \times
\]

\[
(3G_{5X}\dot{\phi} + G_{5XX}\ddot{\phi}^3)\right] = \frac{C_\phi}{a^3} - \frac{l}{a^3}\int a^3(t)dt,
\]

with \(C_\phi, C_+\) and \(C_-\) being integration constants. Constants \(C_\pm\) correspond to the anisotropic charges. For convenience, introduced

\[
\sigma^2 = \dot{\beta}_+^2 + \dot{\beta}_-^2.
\]

The theory with \(G_5(X) \neq 0\) gives nonlinear equations (18), (19) for \(\dot{\beta}_\pm\). From this point of view, we consider the nonlinear anisotropic model.

Further, we put

\[
C_\phi = C_- = C_+ = 0.
\]

The simplest solution is then the isotropic one,

\[
\dot{\beta}_\pm = 0.
\]
Thus, assumption model (26).

In addition, since the equations are nonlinear in the anisotropies, there are also solutions with \( \dot{\beta}_\pm \neq 0 \):

\[
\dot{\beta}_+ = \frac{1}{2} \left( H - \frac{1}{8\pi \cdot G_{5X} \phi^3} \right), \quad \dot{\beta}_- = \pm \frac{\sqrt{3}}{2} \left( H - \frac{1}{8\pi \cdot G_{5X} \phi^3} \right),
\]

\[
\sigma^2 = \left( H - \frac{1}{8\pi \cdot G_{5X} \phi^3} \right)^2.
\]

In the solution [24], the signs "+" and "-" of \( \dot{\beta}_- \) correspond to

\[(+): H_1 = 3H - 2 \cdot \frac{1}{8\pi \cdot G_{5X} \phi^3}, \quad H_2 = H_3 = \frac{1}{8\pi \cdot G_{5X} \phi^3},\]

\[(-): H_2 = 3H - 2 \cdot \frac{1}{8\pi \cdot G_{5X} \phi^3}, \quad H_1 = H_3 = \frac{1}{8\pi \cdot G_{5X} \phi^3}.\]

Thus, assumption \( C_- = C_+ = 0 \) gives a locally rotationally symmetric (LRS) Bianchi I model. We will consider the model (26).

In view of (24) and (25), from equations (16) and (20) we obtain

\[
3H \left\{ G_{5X} \phi^3 + \frac{1}{(8\pi)^2 \cdot G_{5X} \phi^3} \left[ 3 + G_{5XX} \cdot \phi^2 \right] \right\} =
\]

\[
= -l \phi + A - \dot{\phi}^2 A_X + \frac{1}{(8\pi)^3 \cdot (G_{5X} \phi^3)^2} \left[ 7 + \frac{2G_{5XX} \cdot \phi^2}{G_{5X}} \right],
\]

\[
3H \left\{ G_{5X} \phi^3 + \frac{1}{(8\pi)^2 \cdot G_{5X} \phi^3} \left[ 3 + G_{5XX} \cdot \phi^2 \right] \right\} =
\]

\[
= -\dot{\phi}^2 A_X - \frac{l \cdot \dot{\phi}}{a^3} \int a^3 dt + \frac{2}{(8\pi)^3 \cdot (G_{5X} \phi^3)^2} \left[ 3 + \frac{G_{5XX} \cdot \phi^2}{G_{5X}} \right].
\]

The equation (17) can be ignored, since it is automatically fulfilled by virtue of the Bianchi identities. The combination of (28) and (29) gives the equation:

\[
\frac{1}{(8\pi)^3 \cdot (G_{5X} \phi^3)^2} - l \phi + A = -\frac{l \cdot \dot{\phi}}{a^3} \int a^3 dt.
\]

We will note an important property of the presented model. If \( l = 0 \) then the system [24, 28, 29] can only have a stationary solution \( \dot{\beta}_\pm \), \( H, \dot{\phi} = const \). A necessary condition for dynamic solution \( \dot{\beta}_\pm(t), \phi(t) \) is the presence of the tadpole term \( l \cdot \dot{\phi} \neq 0 \). In the work [14], the dynamics of the solution was provided by nonzero charges \( C_\phi, C_\pm \).

Let’s make an assumption

\[
\frac{l}{a^3} \int a^3 dt = \mu = const > 0.
\]

In this case, the Universe is accelerating:

\[
H = \frac{l}{3\mu} = const, \quad a(t) = a_* \exp \left( \frac{l \cdot t}{3\mu} \right), \quad l > 0.
\]

In other words, we will consider the isotropization process of the Universe, which is approaching de Sitter’s world. We choose function \( A(X) \) as follows

\[
A(X) = -\mu \dot{\phi} = -\mu \sqrt{2X}, \quad \dot{\phi} \geq 0.
\]
The model with \( G_2 \propto \sqrt{X} \) correspond to \textit{Cuscuton scenarios} [28]. From the equation (30) it follows
\[
\frac{1}{(8\pi)^3 \cdot (G_5 X \phi^3)^2} - l \cdot \phi = 0 \Rightarrow \frac{1}{8\pi \cdot G_5 X \phi^3} = \pm \sqrt{8\pi l} \cdot \phi. \tag{34}
\]
We choose the "+" sign. In this case, the Universe expands in all directions, \( H_i > 0 \) (see (26), (27)).

We want to get a model that is isotropic in later times \((t \to +\infty)\). Therefore, it must be fulfilled
\[
\frac{\dot{\beta}_\pm}{H} \to 0 \text{ as } t \to +\infty, \tag{35}
\]
i.e.
\[
\frac{1}{8\pi \cdot G_5 X \phi^3} \to H \text{ as } t \to +\infty. \tag{36}
\]

The dynamics of \( \dot{\phi}(t) \) allows one to construct solution (24) with isotropization. To satisfy the isotropization condition

\[
\frac{\dot{\beta}_\pm}{H \cdot s'_\pm} \text{, } \frac{1}{8\pi \cdot G_5 X \phi^3} = H + \gamma \phi^{1/3}, \gamma = \text{const.} \tag{37}
\]

Equation (34) is then transformed
\[
H + \gamma \phi^{1/3} = \sqrt{8\pi l} \cdot \phi. \tag{38}
\]
This equation has a nonsingular solution
\[
\phi = \frac{H^2}{8\pi l} \cdot \frac{1 + \frac{8\pi l H}{|\gamma|^3} t}{\left(1 + \sqrt{1 + \frac{8\pi l H}{|\gamma|^3} t}\right)^2}, \gamma < 0, t \geq t_* \equiv -\frac{|\gamma|^3}{8\pi l H}. \tag{39}
\]
Considering (24), (37) and (39) we get
\[
\frac{\dot{\beta}_\pm}{H} = \frac{s_\pm^*}{1 + \sqrt{1 + \frac{t}{|t_*|}}} \to 0 \text{ as } t \to +\infty; s_+^* = 1/2, s_-^* = \sqrt{3}/2;
\]
\[
\frac{\dot{\beta}_\pm}{H} \to s_\pm^* \text{ as } t \to t_*.
\]

(40)

The Universe becomes isotropic in later times. The function \(\dot{\beta}_\pm\) are finite at the beginning (\(t_\ast\)) of the Universe. This is the non-standard behavior of the Universe anisotropy. The profiles of \(\dot{\beta}_\pm\) are shown in Fig[1]

The Universe is expanding on the \(x, y\) and \(z\) axes:
\[
H_1 = H \cdot \frac{3 + \sqrt{1 + \frac{t}{|t_*|}}}{1 + \sqrt{1 + \frac{t}{|t_*|}}} > 0, \quad H_2 = H_3 = H \cdot \frac{\sqrt{1 + \frac{t}{|t_*|}}}{1 + \sqrt{1 + \frac{t}{|t_*|}}} > 0.
\]

(41)

The profiles of \(H_i\) are shown in Fig[2] On integration (41), we get the scale factors

\[
a_1(t) = a_1^* \exp \left\{ H|t_*| \left[ 1 + \frac{t}{|t_*|} + 4 \sqrt{1 + \frac{t}{|t_*|}} - 4 \ln \left( 1 + \sqrt{1 + \frac{t}{|t_*|}} \right) \right] \right\},
\]
\[
a_{2,3}(t) = a_{2,3}^* \exp \left\{ H|t_*| \left[ 1 + \frac{t}{|t_*|} - 2 \sqrt{1 + \frac{t}{|t_*|}} + 2 \ln \left( 1 + \sqrt{1 + \frac{t}{|t_*|}} \right) \right] \right\}, \quad t \geq t_*.
\]

(42)

FIG. 2: Hubble parameter profiles \(H_i\).
This cosmological model does not contain a singular point. The profiles of $a_i$ are shown in Fig. 3.

Using (32) and (33) from the equation (28), we restore the function $G_3(X)$:

$$G_3X = \frac{1}{(8\pi)^2 \cdot G_5X \phi^6} \left(3 + \frac{G_{5XX} \cdot \phi^2}{G_5X} \right) \left(-1 + \frac{1}{12H \pi \cdot G_5X \phi^3} \right),$$ (43)

then

$$G_3 = \frac{\gamma}{48 \sqrt{2} \pi} \left[-X^{-1/3} + \frac{4\sqrt{2} \gamma}{HX^{1/6}} \right] + \text{const.}.$$ (44)

From assumption (37), we restore the function $G_5(X)$:

$$G_5 = -\frac{3|\gamma|^2}{8 \cdot 2^{1/6} \pi H^3 X^{1/6}} - \frac{3|\gamma|}{16 \cdot 2^{1/3} \pi H^2 X^{1/3}} - \frac{1}{8 \cdot 2^{1/2} \pi H X^{1/2}} +$$

$$+ \frac{3|\gamma|^3}{8\pi H^4} \ln \left[\frac{2^{1/6} |\gamma| X^{1/6}}{2^{1/6} |\gamma| X^{1/6} - H} \right] + \text{const.}.$$ (45)

Thus, we presented a theory that allows the process of isotropization of the Universe with finite anisotropy.

**III. CONCLUSION**

We have studied one homogeneous and anisotropic cosmological model of Bianchi I within the Horndeski gravity with the tadpole. Our aim was to see what nonlinear anisotropy with tadpole has to offer. First, the anisotropic terms
are maximal and finite at the initial moment of time; they approaches zero at later times. Secondly, the anisotropy suppression occurs during the inflationary stage. Third, this cosmological model does not contain a singular point.

Setting the integration constants $C_\phi = C_\pm = 0$ to zero, we studied the isolated influence of the tadpole on the dynamics of field functions. Zero values of anisotropic charges affect the symmetry of space-time. Assumption $C_\pm = 0$ gives a locally rotationally symmetric Bianchi I model: $a_1 \neq a_2 = a_3$.

In this article, we introduced the Lagrangian functions $G_2$, $G_4$ by hand. We obtained the rest of the functions $G_3$, $G_5$ using the reconstruction method. A priori, the de Sitter world ($H = \text{const}$) was set. The isotropization process is also modeled. Such ansatzes allow you to restore the desired functions $G_3$, $G_5$. The reconstruction method is often used in modified theories of gravity [23], [24], [25], [26].

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