An investigation of the discriminant power and dimensionality of items used for assessing health condition of elderly people

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Abstract

With reference to the questionnaire adopted within the Italian project “Ulisse” to assess health condition of elderly people, we investigate two important issues: discriminant power and actual number of dimensions measured by the items composing the questionnaire. The adopted statistical approach is based on the joint use of the latent class model and a multidimensional item response theory model based on the 2PL parametrization. The latter allows us to account for the different discriminant power of these items. The analysis is based on the data collected on a sample of 1699 elderly people hosted in 37 nursing homes in Italy. This analysis shows that the selected items indeed measure a different number of dimensions of the health status and that they considerably differ in terms of discriminant power (effectiveness in measuring the actual health status). Implications for the assessment of the performance of nursing homes from a policy-maker prospective are discussed.

Keywords: Latent class model, item response theory models, performance evaluation.
1 Introduction

The progressive ageing of the contemporary society, due to the increasing life expectation, has raised the demand for health assistance, stimulating the debate on the quality of the care provided by nursing homes. According to the OECD, the percentage of elderly people in the population of the industrialized countries will increase from 13% of 2000 to 25% of 2040. As a consequence, the cost of health care, especially for long term care assistance, will rapidly increase. Anderson and Hussey (2000), considering the eight most industrialized OECD countries, found that the cost for care assistance of elderly people increased from 3% to 5% of the GDP between 1993 and 1999 and the cost for long term health assistance increased from 0.9% and 1.6% in the same period. In Italy, public spending for long term assistance in 2002 was, in terms of GDP, about 0.7%, which is expected to strongly increase in the next decade.

A direct consequence of the increasing cost for health assistance is that the Governments of industrialized countries have begun to consider the problem of the rationalization of public interventions, in terms of public spending, regulation, and policies. Public intervention is also crucial to guarantee the accessibility to care facilities of elderly people with low income, without affecting the economic status of their families. Since the access to long term care assistance is conditioned on the possibility to pay for the services provided by nursing homes, there could be an artificial low level of demand for long term care assistance; see Alexxih and Kennell (1994).

The above arguments imply that methodological tools to analyze the performance of facilities for care assistance of elderly people are of central interest for policy-makers. The aim is twofold: (i) to promote the rise of the quality of existing institutions (especially nursing homes) that have to satisfy standard criteria for care assistance and (ii) to implement a strategy to reform the role of pub-
lic institutions in this context. At this regard, a great debate arises about the
construction of indicators able to measure nursing home performance and then to
effectively rank the facilities in a certain geographical region; see Phillips et al.
(2007) and Harrington et al. (2003). In the United States, one of the most import-
ant projects of nursing home facility ranking is “Medicare”, which is supported
by the Department of Health and Human Services. This project has developed
an evaluation system which is based on indicators that are generally linked to
the psycho-physics condition of elderly people which are related, in particular, to
mobility diseases, behavioral disorders, and memory problems. Data are collected
by public institutions through questionnaires administered at regularly repeated
occasions.

One of the main ideas behind the facility ranking is that unidimensional criteria
to classify the health conditions of elderly people are available; see Phillips et al.
(2007) and Mor et al. (2003). This assumption implies that the difference between
two subjects in responding to a set of items on the health condition only depends
on a single latent trait summarizing this condition. This may obviously be a
restrictive assumption. For instance, this does not allow us to classify subjects
who show a degenerative health status in relation to a specific pathology, but
apparently have an overall good health status. Violations of this assumption may
lead to misleading conclusions reached on the basis of a unidimensional ranking
which relies on a single score assigned to each facility.

On the other hand, Kane (1998) identified at list five different groups of res-
idents of nursing homes. These groups include subjects recovering from an acute
episode and are likely to return home, residents who are cognitive impaired, resid-
ents who are cognitively intact but suffer from physical challenges, residents who
are in vegetative state, and residents that are terminally ill. These subjects have
different needs and different levels of the quality of life; see Kane (1981). Con-
sequently, the actions of the nursing homes could be more forceful in relation to
specific areas of intervention, so that some facilities can be specialized in improving
the state of health in relation to some pathologies. Along these lines, dimension-
ality of health condition becomes a central issue for obtaining a consistent and
generalizable ranking index for the performance of the nursing homes.

In the present paper, we simultaneously address the issue of item selection and
of dimensionality on the basis of a formal statistical procedure (Bartolucci, 2007),
which exploits the latent class (LC) model and a class of item response theory (IRT)
models; see Lazarsfeld and Henry (1968), Goodman (1974, 1978), and Hambleton
(1996). Through these methodologies, we study the above issues on the basis of
a dataset coming from the database “Ulisse”, which is collected within a survey
carried out in Italy on the basis of the RAI-MDS questionnaire (Morris, 1997). The
questionnaire covers several aspects of the health status of elderly people hosted in
nursing homes. In particular, we consider 89 among the around 300 available items.
These items characterize: (i) cognitive conditions, (ii) auditory and view fields,
(iii) humor and behavioral disorders, (iv) activities of daily living, (v) incontinence,
(vi) nutritional field, (vii) dental disorders, and (viii) skin conditions. By using
a so large number of items we can fully characterize the health status of elderly
people hosted in the nursing homes, without imposing any \textit{a-priori} restriction on
the relevance of its different components. From the original set of 89 items we then
extract a subset of 35 items on the basis of their discriminant power, that is the
effectiveness in measuring these conditions. The adopted methodology is based
on the joint use of LC and IRT models and represents a useful tool to reduce the
size of the present and similar questionnaires, with the obvious consequence of
reducing the survey costs.

On the basis of the applied methodology we show that the 35 selected items
indeed measure five different dimensions which may be referred to as: (i) cognitive
conditions, (ii) auditory and view fields, (iii) activities of daily living and incontinence, (iv) humor and behavioral disorders and skin conditions, and (v) nutritional field and dental disorders. These dimensions have a clear interpretation; this seems to confirm the robustness of the proposed analysis.

The reminder of the paper is organized as follows. In Section 2 we describe the dataset on health conditions of elderly people hosted in certain of nursing homes across the Italian regions. In Section 3 we briefly review the statistical methodology based on LC and IRT models. In Section 4 we describe in detail the empirical analysis and in Section 5 we report the main conclusions of the study.

2 The Ulisse database

We consider a dataset collected within the “Ulisse” project, which is carried out by the Italian Ministry of Health jointly with the Italian Society of Gerontology and Geriatrics. The project is based on a longitudinal survey, covering 17 Italian regions, about the assistance level provided to patients hosted in 37 randomly chosen nursing homes. This survey is carried out since 2004 through the repeated administration of a questionnaire (every 6 months) which is filled up by the nursing assistant of each patient and concerns several aspects of the everyday life. For our analysis we consider only the first interview, which covers 1699 patients.

Table 1 reports the geographical distribution, on the Italian territory, of the elderly people and the nursing homes included in the study. We observe that most of the sample is in the north regions: the percentage of patients in these regions is about 85%. In particular, 40% of the full sample is located in Lombardia (17%), Veneto (11%), and Emilia Romagna (12%). In Table 2 we report some descriptive statistics on gender and age of patients of the nursing homes.

We observe that most of the sample is composed by women (71%), with only
Table 1: Regional distribution of the elderly people and nursing homes included in the study.

| Region         | Number of subjects | Percentage | Numbers of nursing homes |
|----------------|--------------------|------------|--------------------------|
| North          | Piemonte           | 105        | 6.18                     | 2                        |
|                | Lombardia          | 292        | 17.19                    | 6                        |
|                | Trentino A. A.     | 74         | 4.36                     | 1                        |
|                | Veneto             | 194        | 11.42                    | 5                        |
|                | Friuli V. G.       | 92         | 5.41                     | 1                        |
|                | Liguria            | 133        | 7.83                     | 2                        |
|                | Emilia R.          | 214        | 12.60                    | 4                        |
| Center         | Toscany            | 83         | 4.89                     | 3                        |
|                | Umbria             | 142        | 8.36                     | 3                        |
|                | Marche             | 48         | 2.83                     | 1                        |
| South          | Abruzzo            | 78         | 4.59                     | 2                        |
|                | Molise             | 46         | 2.71                     | 2                        |
|                | Campania           | 52         | 3.06                     | 1                        |
|                | Puglia             | 51         | 3.00                     | 1                        |
|                | Calabria           | 42         | 2.47                     | 1                        |
| Islands        | Sicilia            | 31         | 1.82                     | 1                        |
|                | Sardegna           | 22         | 1.29                     | 1                        |
| Total          |                    | 1,699      | 100.00                   | 37                       |

Table 2: Nursing homes population by age and gender.

| Age      | Male | Female | Total |
|----------|------|--------|-------|
| > 70     | 4.72 | 3.24   | 7.96  |
| 70 – 75  | 4.78 | 5.37   | 10.14 |
| 75 – 80  | 6.07 | 11.85  | 17.92 |
| 80 – 85  | 5.72 | 15.74  | 21.46 |
| 85 – 90  | 3.36 | 15.68  | 19.04 |
| > 90     | 4.19 | 19.28  | 23.47 |
| Total    | 28.83| 71.17  | 100.00|

29% of men. Moreover, the age distribution differs in relation to gender, with a higher proportion of females with age 85 and over and a relative younger male population. The presence of a so high percentage of old women can obviously condition the analysis about the care facility performance.

From the original questionnaire we single out 89 among the items concerning: (i) cognitive conditions, (ii) auditory and view fields, (iii) humor and behavioral disorders, (iv) activities of daily living, (v) incontinence, (vi) nutritional field, (vii) dental disorders, and (viii) skin conditions. The complete list of items is reported in Appendix 1.
3 The statistical methodology

In the following, we briefly review the LC model (Goodman, 1974) and the IRT model proposed by Bartolucci (2007) for the study of multidimensionality. The aim of the methodology based on these models is: (i) to include in the analysis only the items necessary to identify the number of latent traits; (ii) to identify the latent structure representing the health status of elderly people; (iii) to investigate the different discriminant power of the items. This methodology allows us to choose a convenient partition of the selected items according to the dimension they measure and to analyze its correspondence in terms of facility care.

3.1 The latent class model

The LC model (Lazarsfeld and Henry, 1968; Goodman, 1974, 1978) assumes that the observed sample is drawn from a population which is partitioned into \( k \) latent classes, with \( \pi_c \) being the prior probability (or weight) of class \( c \) (\( c = 1, \ldots, k \)). For each subject \( i \) (\( i = 1, \ldots, n \)), we observe a vector \( y_i = (y_{i1}, \ldots, y_{iJ}) \) of binary response variables corresponding to \( J \) items. Given that the subject is in class \( c \) and with reference to item \( j \), the conditional probability of success is denoted by \( \lambda_{jc} \).

Under the assumption of local independence, the probability of the response pattern \( y_i \) is

\[
p(y_i) = \sum_c p(y_i|c) \pi_c,
\]

\[
p(y_i|c) = \prod_j \lambda_{jc}^{y_{ij}} (1 - \lambda_{jc})^{1-y_{ij}}.
\]

The log-likelihood function of the LC model, which is used for parameter estima-
tion, is then

\[ \ell(\theta) = \sum_i \log p(y_i) \]  

(1)

where \( \theta \) is a short-hand notation for all model parameters. This function is maximized by the EM algorithm (Dempster et al., 1977; Goodman, 1978). This is an iterative algorithm which is based on two steps to be repeated until convergence:

- **E-step**: compute the conditional expected value of the log-likelihood given the observed data and the current value of the parameters;

- **M-step**: update the model parameters by maximizing the expected log-likelihood obtained at the E-step.

We initialize the algorithm by both deterministic and random starting values in order to prevent the problem of multimodality of the likelihood. This is a typical problem of latent variable models.

Obviously, when the LC model is applied to analyze a dataset, choosing the number of latent classes is necessary. At this aim, we rely on the Bayesian Information Criterion (BIC) of Schwarz (1978), which is based on the index:

\[ BIC_k = -2\hat{\ell}_k + m_k \log(n), \]

where, for a given number of classes \( k \), \( \hat{\ell}_k \) is the maximum of the log-likelihood given in (1) and \( m_k \) is the corresponding number of parameters. The latter is taken as a measure of complexity of the model on which the above penalization term is based. According to this criterion, the number of classes corresponding to the minimum of \( BIC_k \) has to be selected. This number is indicated by \( \hat{k} \).

Through the EM algorithm we obtain, for each latent class \( c \), the maximum likelihood estimate of the weight, denoted by \( \hat{\pi}_c \), and of the conditional probabilities of success, denoted by \( \hat{\lambda}_{jc}, j = 1, \ldots, J \). On the basis of the latter ones we
can obtain a measure of the discriminant power of the items measuring each dimension. In our study, each dimension corresponds to a group of items measuring a different aspect of the health status of elderly people. In particular, we measure the discriminant power of item $j$ by the following ratio:

$$D_j = \frac{M_j}{\max_h(M_h)},$$

$$M_j = \max_c(\hat{\lambda}_{jc}) - \min_c(\hat{\lambda}_{jc}),$$

where $\max_c(\hat{\lambda}_{jc})$ is the maximum value (across the latent classes) of the probability of success for item $j$ and $\min_c(\hat{\lambda}_{jc})$ is the minimum. Moreover, $\max_h(j)$ at the denominator of (2) stands for the maximum of the difference at the numerator with respect to all the items measuring the same dimension of item $j$. In this way, the above index is always between 0 and 1 and we exclude from the analysis those items which have an index value lower than a given threshold because they show a reduced discriminant power. This approach for reducing the number of items is compared with the one based on the IRT model that we present in the following section.

### 3.2 The multidimensional two-parameter logistic model

This model is a constrained version of the LC model which directly includes, for each item, a parameter measuring its discriminant power. The model is based on the following multidimensional two-parameter logistic (2PL) parametrization of the conditional probabilities of success:

$$\logit(\lambda_{jc}) = \gamma_j \left( \sum_d \delta_{jd} \theta_{cd} - \beta_j \right), \quad j = 1, \ldots, J.$$  

\[9\]
In the above expression, $\delta_{jd}$ is a dummy variable equal to 1 if item $j$ measures dimension $d$ ($d = 1, \ldots, s$) and to 0 otherwise. Moreover, $\theta_{cd}$ is a measure of the latent trait (dimension $d$) for the subjects in latent class $c$ (typically referred to as ability), $\beta_j$ is a measure of the overall tendency to respond 0 to item $j$ (typically referred to as difficulty), and $\gamma_j$ measures the discriminant power of this item (typically referred to as discriminant index).

The log-likelihood function of the model may again be expressed as:

$$\ell(\theta) = \sum_i \log \sum_c p(y_i|c) \pi_c;$$

maximization of $\ell(\theta)$ is again performed by the EM algorithm of Dempster et al. (1977), which has a the same structure outlined in the previous section.

From the EM algorithm we also obtain, for each item $j$, the estimate $\hat{\gamma}_j$ of the discriminant index. In order to select a suitable set of items on the basis of these estimates, we rely on the ratio

$$D_j^* = \frac{\hat{\gamma}_j}{\max_{h(j)}(\hat{\gamma}_h)},$$

This index has a structure similar to that in (2), with the difference $M_j$ between the maximum and the minimum of the estimated response probabilities substituted by the estimated discriminant index. Then, items with a value of $D_j^*$ lower than a certain threshold are dropped.

Another analysis that is allowed by the 2PL model presented above is that of dimensionality. In particular, through this model we can test the hypothesis, indicated in the following by $H_0$, that the items actually measure a reduced number of dimensions. For instance, we can test the hypothesis that the items measure $s - 1$ instead of $s$ dimensions. The $s - 1$ dimensions are specified by collapsing two dimensions into one and then grouping the corresponding items.
In order to test $H_0$, we use the likelihood ratio statistic

$$LR = 2 \sum_y n(y) \log \left[ \frac{\hat{p}(y)}{\hat{p}_0(y)} \right],$$

where the sum is extended to all the possible response configurations $y$, $n(y)$ stands for the observed frequency of configuration $y$, $\hat{p}(y)$ is estimated probability of this configuration under the model with $s$ dimensions, and $\hat{p}_0(y)$ is the corresponding estimate under the reduced model with $s - 1$ dimensions. Under $H_0$, this statistic has a $\chi^2$ asymptotic distribution with a number of degrees of freedom equal to the $\hat{k} - 2$, where $\hat{k}$ is the adopted number of latent classes. Then, the hypothesis is rejected if the observed value of $LR$ is larger than a suitable percentile of this distribution.

On the basis of the above testing procedure, we can implement a hierarchical algorithm for clustering items into a reduced number of groups. Items in the same group are supposed to measure the same dimension and then each group corresponds to a different dimension. In particular, the clustering algorithm begins by fitting the model in which items are grouped according the structure of the questionnaire. Then, all the possible ways to collapse two groups are considered and the one giving the smallest value of $LR$ with respect to the previous step is selected. This procedure is repeated until the unidimensional IRT model (in which all items are included in the same group) is fitted. Finally, the selected number of groups (and then of dimensions) is the smallest number for which the value of $LR$, computed with respect to the previous classification, is smaller than a suitable percentile of the asymptotic distribution. For a detailed illustration of the algorithm see Bartolucci (2007). We perform this classification once the items with a reduced discriminating power are eliminated as indicated above.
4 Application to the Ulisse dataset

Following the empirical strategy outlined in the previous section, we analyze the Ulisse dataset described in Section 2. We first present the results from the LC model and then those from the IRT model.

4.1 Latent class analysis

As starting point we choose the number of latent classes for the LC model applied to the 89 selected items. At this aim, Table 3 reports the maximum log-likelihood and the corresponding value of the BIC index for a number of latent classes from 1 to 7.

Table 3: Selection of the number of latent classes for the LC model. For each number of classes from 1 to 7, \( \hat{\ell}_k \) is the maximum log-likelihood of the model, \( m_k \) is the number of parameters, and \( BIC_k \) is the corresponding value of the BIC index. In boldface are reported the quantities corresponding to the selected model.

| \( k \) | \( \hat{\ell}_k \) | \( m_k \) | \( BIC_k \) |
|---|---|---|---|
| 1 | -37175.939 | 89 | 75013.842 |
| 2 | -32773.208 | 179 | 66877.781 |
| 3 | -31444.523 | 269 | 64889.813 |
| 4 | -30432.381 | 359 | 63534.930 |
| 5 | -29875.393 | 449 | 63090.356 |
| 6 | **-29423.379** | **539** | **62855.730** |
| 7 | -29159.367 | 629 | 62997.107 |

On the basis of these results we select \( \hat{k} = 6 \) latent classes. These classes correspond to different degrees of impairment of the elderly people health status. In order to interpret these classes, Figure 1 reports the graph of the conditional probabilities of success estimated under the selected LC model for each class. To have a clearer interpretation of these results, we order the six classes according to the probability of success for the first item.
Figure 1: Plot of the estimated conditional probabilities of success $\hat{\lambda}_{jc}$ under the selected LC model. Each panel corresponds to a different latent class $c$, ordered according to the probability of success for the first item.

The estimated conditional probabilities obtained from the LC model are then used to assess the discriminant power of the 89 selected items. At this aim, in Table 4 we report for each item $j$ the value of the index $D_j$, computed according to (2), together with the weighted mean and standard deviation of the estimated success probabilities, computed with weights corresponding to the estimated class probabilities.
Table 4: Weighted mean and standard deviation of the estimated success probabilities $\lambda_{j,c}$ for each item $j$, together with the indices $M_j$ and $D_j$ used to measure the discriminant power. In boldface are the quantities referred to the item that for each dimension $d$ has the highest discriminant power.

| $j$ | item | $d$ | mean | std  | $M_j$ | $D_j$ |
|-----|------|-----|------|------|-------|-------|
| 1   | CC1  | 1   | 0.629| 0.107| 0.753 | 0.760 |
| 2   | CC2  | 1   | 0.577| 0.126| 0.807 | 0.814 |
| 3   | CC3  | 1   | 0.575| 0.126| 0.807 | 0.814 |
| 4   | CC4  | 1   | 0.404| 0.277| 0.991 | 1.000 |
| 5   | CC5  | 1   | 0.442| 0.226| 0.921 | 0.929 |
| 6   | CC6  | 1   | 0.398| 0.264| 0.921 | 0.929 |
| 7   | CC7  | 1   | 0.609| 0.142| 0.865 | 0.873 |
| 8   | CC8  | 1   | 0.642| 0.119| 0.827 | 0.835 |
| 9   | CC9  | 1   | 0.426| 0.159| 0.761 | 0.768 |
| 10  | CC10 | 1   | 0.410| 0.154| 0.711 | 0.717 |
| 11  | CC11 | 1   | 0.401| 0.126| 0.668 | 0.674 |
| 12  | CC12 | 1   | 0.316| 0.173| 0.533 | 0.538 |
| 13  | CC13 | 1   | 0.439| 0.086| 0.593 | 0.598 |

As described in Section 3.1, the index $D_j$ may be used to select a subset of items which provide a similar amount of information than the full set of items. This is obtained by comparing the values of this index with a suitable threshold between 0 and 1. In particular, for different threshold levels we report in Table 5 the number of selected items for each dimension. For instance, with a threshold of 0.5 we retain in the analysis 63 items.
Table 5: Results from the item selection procedure based on the indices $D_j$ in terms of number of items retained for each dimension.

| threshold | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | overall |
|-----------|---|---|---|---|---|---|---|---|---------|
| 0.0       | 13| 5 | 21| 17| 16| 8 | 6 | 3 | 89      |
| 0.1       | 13| 5 | 21| 18| 8 | 7 | 6 | 3 | 81      |
| 0.2       | 13| 5 | 21| 18| 8 | 6 | 4 | 3 | 78      |
| 0.3       | 13| 5 | 21| 17| 6 | 5 | 4 | 2 | 73      |
| 0.4       | 13| 4 | 20| 17| 3 | 4 | 4 | 2 | 67      |
| 0.5       | 13| 4 | 18| 15| 3 | 4 | 4 | 2 | 63      |
| 0.6       | 11| 3 | 15| 11| 2 | 3 | 3 | 2 | 50      |
| 0.7       | 10| 3 | 12| 10| 2 | 3 | 2 | 2 | 44      |
| 0.8       | 7 | 3 | 6 | 8 | 2 | 3 | 1 | 2 | 32      |
| 0.9       | 2 | 2 | 1 | 5 | 1 | 3 | 1 | 2 | 17      |
| 1.0       | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1       |

4.2 Item response theory analysis

To complete the item selection analysis we exploit the alternative approach based on the 2PL model illustrated in Section 3.2. The same model is then used to evaluate the dimensionality of health condition of elderly people. The results of the analysis are finally compared with those presented in the previous section and based on the LC model. Since the adopted IRT model can be seen as a constrained version of the LC model, there are no compatibility problems in comparing the results of the two analyses.

On the basis of the same number of classes selected above, $\hat{k} = 6$, we obtain the estimates of the parameters in (3). In particular, for each item $j$ we report in Table 6 the estimated difficulty level $\hat{\beta}_j$ and discriminant index $\hat{\gamma}_j$, together with the index $D^*_j$ defined in (4). On the basis of the values of this index, we can select a suitable number of items. The results of this selection process are reported in Table 7 for different threshold levels between 0 to 1.
Table 6: Parameter estimates under the 2PL model. For each item $j$, $\hat{\beta}_j$ is the estimated difficulty, $\hat{\gamma}_j$ is the estimated discriminant power, and $D^*_j$ is the relative discriminant power. In boldface are the quantities referred to the item that for each dimension $d$ has the highest discriminant power.

| $d$ | $j$ | item | $\hat{\gamma}_j$ | $\hat{\beta}_j$ | $D^*_j$ |
|-----|-----|------|-----------------|-----------------|--------|
| 1   | 1   | CC1  | 1.000           | 0.000           | 0.706  |
| 1   | 2   | CC2  | 1.161           | 0.492           | 0.820  |
| 1   | 3   | CC3  | 1.416           | 0.875           | 1.000  |
| 1   | 4   | CC4  | 1.178           | 1.742           | 0.832  |
| 1   | 5   | CC5  | 0.989           | 1.347           | 0.699  |
| 1   | 6   | CC6  | 1.098           | 1.666           | 0.775  |
| 1   | 7   | CC7  | 1.280           | 0.283           | 0.904  |
| 1   | 8   | CC8  | 0.938           | 1.134           | 0.663  |
| 1   | 9   | CC9  | 0.928           | 2.693           | 0.655  |
| 1   | 10  | CC10 | 0.805           | 2.515           | 0.586  |
| 1   | 11  | CC11 | 0.750           | 2.561           | 0.538  |
| 1   | 12  | CC12 | 0.657           | 2.355           | 0.527  |
| 1   | 13  | CC13 | 0.463           | 2.355           | 0.527  |
| 2   | 14  | CAS1 | 1.000           | 0.000           | 0.177  |
| 2   | 15  | CAS2 | 4.954           | -0.806          | 0.879  |
| 2   | 16  | CAS3 | 3.673           | -0.883          | 0.652  |
| 2   | 17  | CAS4 | 5.636           | -0.829          | 1.000  |
| 2   | 18  | CAS5 | 1.702           | -0.602          | 0.302  |
| 3   | 19  | HBD1 | 1.000           | 0.000           | 0.070  |
| 3   | 20  | HBD2 | 5.773           | -1.646          | 0.403  |
| 3   | 21  | HBD3 | 3.630           | -1.453          | 0.253  |
| 3   | 22  | HBD4 | 2.722           | -1.459          | 0.190  |
| 3   | 23  | HBD5 | 0.508           | 2.805           | 0.130  |
| 3   | 24  | HBD6 | 2.959           | -1.352          | 0.206  |
| 3   | 25  | HBD7 | 3.225           | -1.110          | 0.225  |
| 3   | 26  | HBD8 | 1.111           | -0.626          | 0.078  |
| 3   | 27  | HBD9 | 1.967           | -1.208          | 0.130  |
| 3   | 28  | HBD10| 3.443           | -1.425          | 0.240  |
| 3   | 29  | HBD11| 3.885           | -1.586          | 0.271  |
| 3   | 30  | HBD12| 3.285           | -1.693          | 0.229  |
| 3   | 31  | HBD13| 2.297           | -1.144          | 0.169  |
| 3   | 32  | HBD14| 12.159          | -1.787          | 0.848  |
| 3   | 33  | HBD15| 5.465           | -1.813          | 0.381  |
| 3   | 34  | HBD16| 6.356           | -1.869          | 0.443  |
| 3   | 35  | HBD17| 14.337          | -1.729          | 1.000  |
| 3   | 36  | HBD18| 3.580           | -1.494          | 0.259  |
| 3   | 37  | HBD19| 8.130           | -1.606          | 0.567  |
| 3   | 38  | HBD20| 9.393           | -1.713          | 0.655  |
| 3   | 39  | HBD21| 4.940           | -1.568          | 0.345  |

Compared to the LC model (see Table 5), the IRT model chooses a reduced number of items to keep into the analysis, appearing less conservative in terms of the item selection process. In fact, by using a critical value of 0.5 we select 35 items, instead of 63 chosen by the LC based procedure. The choice of that critical value has been addressed to keep in the analysis a relevant number of items, without loosing too much information in relation to the analyzed phenomenon.
Table 7: Results from the item selection mechanism based on the indices $D_j^*$ in terms of number of items retained for each dimension.

| dimension | threshold | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | overall |
|-----------|-----------|---|---|---|---|---|---|---|---|---------|
|           | 0.0  | 13 | 5 | 21 | 18 | 15 | 8 | 6 | 3 | 89      |
|           | 0.1  | 13 | 5 | 18 | 18 | 1  | 7 | 4 | 3 | 69      |
|           | 0.2  | 13 | 4 | 15 | 17 | 1  | 5 | 4 | 3 | 62      |
|           | 0.3  | 13 | 4 | 8  | 16 | 1  | 5 | 3 | 3 | 53      |
|           | 0.4  | 12 | 3 | 6  | 11 | 1  | 3 | 3 | 2 | 41      |
|           | 0.5  | 11 | 3 | 4  | 10 | 1  | 1 | 3 | 2 | 35      |
|           | 0.6  | 9  | 3 | 3  | 8  | 1  | 1 | 2 | 1 | 28      |
|           | 0.7  | 6  | 2 | 2  | 6  | 1  | 1 | 2 | 1 | 21      |
|           | 0.8  | 4  | 2 | 2  | 5  | 1  | 1 | 1 | 1 | 17      |
|           | 0.9  | 2  | 1 | 1  | 3  | 1  | 1 | 1 | 1 | 11      |
|           | 1.0  | 1  | 1 | 1  | 1  | 1  | 1 | 1 | 1 | 8       |

Once we have selected the number of items, we perform the hierarchical cluster analysis on the dimensionality, starting from the eight dimensions defined by the structure of the questionnaire. The approach that we use at this aim is described at the end of Section 3.2 and the results obtained from its application are reported in Table 8 and represented by the dendrogram in Figure 2. In particular, the table shows the list of the dimensions formed by the collection of the items corresponding to the eight initial dimensions, together with the statistic $LR$ computed with respect to the model chosen at the previous step and the corresponding $p$-value.

Table 8: Output of the hierarchical cluster algorithm based on the 2PL multidimensional model.

| $h$ | $s$ | clusters                          | $LR$  | $p$-value |
|-----|-----|----------------------------------|-------|-----------|
| 1   | 7   | $\{1\}, \{2\}, \{3\}, \{6\}, \{7\}, \{8\}, \{4, 5\}$ | 0.379 | 0.984     |
| 2   | 6   | $\{1\}, \{2\}, \{6\}, \{7\}, \{4, 5\}, \{3, 8\}$ | 3.871 | 0.424     |
| 3   | 5   | $\{1\}, \{2\}, \{4, 5\}, \{3, 8\}, \{6, 7\}$ | 5.235 | 0.264     |
| 4   | 4   | $\{1\}, \{2\}, \{3, 8\}, \{4, 5, 6, 7\}$ | 29.045 | 0.000     |
| 5   | 3   | $\{3, 8\}, \{4, 5, 7, 8\}, \{1, 2\}$ | 60.142 | 0.000     |
| 6   | 2   | $\{1, 2\}, \{3, 4, 5, 6, 7, 8\}$ | 42.107 | 0.000     |
| 7   | 1   | $\{1, 2, 3, 4, 5, 6, 7, 8\}$ | 187.440 | 0.000     |
These results show evidence of five dimensions which have the following structure: \{1\}, \{2\}, \{4, 5\}, \{3, 8\}, \{6, 7\}. These dimensions correspond to: (i) cognitive conditions, (ii) auditory and view fields, (iii) activities of daily living and incontinence, (iv) humor and behavioral disorders and skin conditions, and (v) nutritional field and dental disorders. These dimensions have a clear interpretation and seems to confirm the robustness of the proposed analysis.

Figure 2: *Dendrogram based on the 2PL multidimensional model. The eight initial dimensions characterize:* (i) cognitive conditions, (ii) auditory and view fields, (iii) humor and behavioral disorders, (iv) activities of daily living, (v) continence, (vi) nutritional field, (vii) dental disorders, and (viii) skin conditions.

Finally, in Table 9 we report the estimated abilities $\hat{\theta}_{cd}$ for each latent class, together with estimated class weights $\hat{\pi}_c$. With reference to every dimension $d$, each parameter $\theta_{cd}$ corresponds to the latent trait level for the subjects in class $c$. 

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In the present study, high values of the parameter correspond to high probability to suffer from a certain pathology.

Table 9: Estimated ability parameters $\hat{\theta}_{cd}$ for each latent class $c$ and dimension $d$, together with the estimated weights $\hat{\pi}_c$.

| latent dimension | 1     | 2     | 3     | 4     | 5     | weight |
|------------------|-------|-------|-------|-------|-------|--------|
| class 1          | -2.516| -2.690| -5.179| -5.137| -4.815| 0.213  |
| class 2          | 1.142 | -1.171| -3.402| -2.224| -3.927| 0.153  |
| class 3          | -1.253| -2.711| -2.227| -3.203| 1.050 | 0.131  |
| class 4          | 3.996 | 0.702 | -1.960| 0.525 | -2.025| 0.102  |
| class 5          | 2.068 | -1.667| -1.946| 1.050 | 1.333 | 0.160  |
| class 6          | 4.386 | -0.451| -0.727| 2.117 | 1.787 | 0.238  |

For each pair of dimensions $(d_1, d_2)$, it may be interesting to compute the correlation $\rho_{d_1,d_2}$ between the estimated ability levels. Taking into account the class weights, we compute these correlation indices as

$$
\rho_{d_1,d_2} = \frac{\sum_c (\hat{\theta}_{cd_1} - \hat{\theta}_{d_1}) (\hat{\theta}_{cd_2} - \hat{\theta}_{d_2}) \hat{\pi}_c}{\sqrt{\sum_c (\hat{\theta}_{cd_1} - \hat{\theta}_{d_1}) \hat{\pi}_c \sqrt{\sum_c (\hat{\theta}_{cd_2} - \hat{\theta}_{d_2}) \hat{\pi}_c}},
$$

where $\hat{\theta}_d = \sum_c \hat{\theta}_{cd} \hat{\pi}_c$ is the average ability level for dimension $d$. The results are reported in Table 10.

Table 10: Correlation between the estimated abilities for each pair of dimensions $(d_1, d_2)$.

| 2nd dimension | 1     | 2     | 3     | 4     | 5     |
|---------------|-------|-------|-------|-------|-------|
| 1             | 1.000 |       |       |       |       |
| 2             | 0.912 | 1.000 |       |       |       |
| 3             | 0.866 | 0.648 | 1.000 |       |       |
| 4             | 0.964 | 0.863 | 0.898 | 1.000 |       |
| 5             | 0.611 | 0.287 | 0.905 | 0.661 | 1.000 |
Note that, apart from three pairs of dimensions for which these correlations are particularly high (1st and 2nd, 1st and 4th, and 3rd and 5th), the other correlations are smaller than 0.9. In particular, the correlation between the 2nd and the 5th dimensions is smaller than 0.3; moreover, the correlation is smaller than 0.7 for three other pairs of dimensions (1st and 5th, 2nd and 3rd, and 4th and 5th). This confirms that the dimensions found by the clustering algorithm are actually distinct and then measuring the health condition of elderly people necessarily requires a multidimensional scale.

5 Conclusions

In the present paper we simultaneously study the issue of item selection and of dimensionality of health status of elderly people hosted in nursing homes. Our statistical approach is based on the joint use of latent class (LC) and item response theory (IRT) models.

The study is based on a dataset collected in Italy within the “Ulisse” project, which relies on a sample of 1699 elderly people hosted in 37 nursing homes. The health status of these patients is assessed by a set of items which are administered at repeated occasions. From the original dataset, we extract 89 items, which characterize different areas of the health status at the first visit. In particular, we consider eight groups of items (each corresponding to a different dimension), which measure: (i) cognitive conditions, (ii) auditory and view fields, (iii) humor and behavioral disorders, (iv) activities of daily living, (v) incontinence, (vi) nutritional field, (vii) dental disorders, and (viii) skin conditions.

The analysis initially exploits the LC model for selecting a subset of items which provides an amount of information close to that of the full set of items. In particular, through the Bayesian Information Criterion we find evidence of the
presence of six latent classes. Then, through the estimated conditional probabilities of the LC model we start ranking the items according to their discriminant power. This is measured by the standardized difference between the estimated conditional probabilities across latent classes. However, this selection process appears too conservative.

The items selection analysis is then completed by using an IRT model based on a multidimensional 2PL parametrization. In particular, the applied strategy first selects a benchmark model which has a number of dimensions equal to the eight initial dimensions defined by the questionnaire and then, by applying the 2PL model, selects the items depending on their discriminatory power in measuring the latent trait. The subset of selected items is then used to study the dimensionality of the health condition of elderly people. A this aim, we apply a hierarchical clustering algorithm which, starting from the multidimensional model with eight dimensions, ends with the unidimensional model. Within these two extremes we find all the possible numbers of dimensions of the analyzed phenomenon and select the most suitable by a series of likelihood ratio tests. On the basis of this procedure we find evidence of five dimensions obtained by collapsing the initial eight dimensions. These groups have the following structure: \{1\}, \{2\}, \{4,5\}, \{3,8\}, \{6,7\}. These five dimensions may be referred to as: (i) cognitive conditions, (ii) auditory and view fields, (iii) activities of daily living and incontinence, (iv) humor and behavioral disorders and skin conditions, and (v) nutritional field and dental disorders.

Finally, the applied methodology suggests that the dimensionality of health status of elderly people is a relevant aspect to be considered in order to obtain a clear classification of the nursing home facilities in the Italian context. Moreover, the IRT analysis shows that the identified dimensions have not the same discriminating power in determining the health status.
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### Appendix 1: Description of the selected Items

| n | item | Our Ulisse item description |
|---|------|-------------------------------|
| 01 | CC1 | recalls what recently happened (5 minutes) |
| 02 | CC2 | keeps some past memories green |
| 03 | CC3 | recall the actual season |
| 04 | CC4 | recalls where him room |
| 05 | CC5 | recalls the names and faces of the staff |
| 06 | CC6 | recalls where he is |
| 07 | CC7 | e4 decides about his daily activities |
| 08 | e5a | gets easily sidetracked |
| 09 | CC8 | shows episodes of altered perception or awareness of surrounding |
| 10 | CC9 | e5c shows episodes of disorganized speech |
| 11 | CC10 | e5d has periods of restlessness movements |
| 12 | e5e | shows lethargic spans |
| 13 | CC11 | does its cognitive conditions change during the day |
| 14 | CAS1 | e6 shows hearing deficiency |
| 15 | CAS2 | makes self-understood |
| 16 | CAS3 | has a clear language |
| 17 | CAS4 | is capable of understand others |
| 18 | CAS5 | is able to see in conditions of adequate lighting |
| 19 | HBD1 | made negative statements |
| 20 | HBD2 | made repetitive questions |
| 21 | HBD3 | made repetitive verbalizations |
| 22 | HBD4 | shows persistent anger with self or others |
| 23 | HBD5 | shows self depreciation disesteem |
| 24 | HBD6 | expresses fears are not real |
| 25 | HBD7 | believes itself to be dying |
| 26 | HBD8 | complains about his health |
| 27 | HBD9 | i1 has problems with sleep |
| 28 | HBD10 | i1 has problems with sleep |
| 29 | HBD11 | i1 has problems with sleep |
| 30 | HBD12 | i1 shows expressions of sad-faced |
| 31 | HBD13 | i1 easily tears |
| 32 | HBD14 | i1 shows repetitive movements |
| 33 | HBD15 | i1 abstains from activities of interest |
| 34 | HBD16 | i1 shows reduced local interactions |
| 35 | HBD17 | i1 shows repetitive questions |
| 36 | HBD18 | i1 shows offensive language |
| 37 | HBD19 | i1 is physically aggressive |
| 38 | HBD20 | i1 has a socially inappropriate behavior |
| 39 | HBD21 | i1 refuses assistance |
| 40 | ADL1 | j1a needs support in moving to/from lying position |
| 41 | ADL2 | j1b needs support in moving to/from bed, chair, wheelchair |
| 42 | ADL3 | j1c walks between different points within the room |
| 43 | ADL4 | j1d walks in the corridor |
| 44 | ADL5 | j1e walks into the nursing home ward |
| 45 | ADL6 | j1f walks outside the nursing home ward |
| 46 | ADL7 | j1g needs support for dressing |
| 47 | ADL8 | j1h needs support for eating |
| 48 | ADL9 | j1i needs support using the toilet room |
| 49 | ADL10 | j1j needs support for personal hygiene |
| 50 | ADL11 | j1k needs support for taking full-body bath/shower |
| 51 | ADL12 | j1l shows balance problems |
| 52 | ADL13 | j1m shows loss of mobility in the neck |
| 53 | ADL14 | j1n shows loss of mobility in the arm including shoulder or elbow |
| 54 | ADL15 | j1o shows limitations in the movements of the hand including wrist or finger |
| 55 | ADL16 | j1p shows loss of mobility of the foot and ankle |
| 56 | ADL17 | j1q shows limitations in other movements |
| 57 | ADL18 | j1r sets a goal to achieve |
| 58 | I1 | i1a fecal incontinence |
| 59 | I2 | i1b urinary incontinence |
| 60 | I3 | i2a elimination of faces (constipation) |
| 61 | I4 | i2b incontinence |
| 62 | I5 | i2c diarrhea |
| 63 | I6 | i2d fecoloma |
| 64 | I7 | k1b need aids (external catheter) |
| 65 | I8 | k1c need aids (indwelling catheter) |
| 66 | I9 | k1d need aids (intermittent catheter) |
| 67 | I10 | k1e need aids (urological) |
| 68 | I11 | k1f failed to use toilet room/commode/urina |
| 69 | I12 | k1g pads/briefs used |
| 70 | I13 | k1h耘 cleanings/irritations used |
| 71 | I14 | k1i stump present |
| 72 | I15 | k1j need aids (others) |
| 73 | N1 | n1a chewing problem |
| 74 | N2 | n1b swallowing problem |
| 75 | N3 | n1c mouth pain |
| 76 | N4 | n1d soreness of mouth/teeth |
| 77 | N5 | n1e gain weight |
| 78 | N6 | n1f complain about the taste of many foods |
| 79 | N7 | n1g complaints of being hungry |
| 80 | N8 | n1h leave the food on his plate |
| 81 | D1 | d1a debris (soft, easily movable substances) present in mouth prior to going to bed at night |
| 82 | D2 | d1b has dentures or removable bridge |
| 83 | D3 | d2c some/all natural teeth lost does not have or does not use dentures (or partial plates) |
| 84 | D4 | d2d broken, loose, or curved teeth |
| 85 | D5 | d3a inflamed gums (gingiva); swollen or bleeding gums; oral abscesses; ulcers or rashes |
| 86 | D6 | d4a daily cleaning of teeth/dentures or daily mouth care by resident or staff |
| 87 | SK1 | g2a pressure ulcer |
| 88 | SK2 | g2b stasis ulcers |
| 89 | SK3 | p3 a ulcer that was resolved or cured |