COLLECTIVE FIELD REPRESENTATION OF NONRELATIVISTIC FERMIONS IN (1+1) DIMENSIONS

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ABSTRACT

A collective field formalism for nonrelativistic fermions in (1+1) dimensions is presented. The quantum mechanical fermionic problem is bosonized and converted to a second quantized Schrödinger field theory. A formulation in terms of current and density variables gives rise to the collective field representation. Applications to the $D = 1$ hermitian matrix model and the system of one-dimensional fermions in the presence of a weak electromagnetic field are discussed.

* Talk presented at the XXth International Conference on Differential Geometric Methods in Theoretical Physics at Baruch College, CUNY, May 1991.
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1. Introduction

The main motivation for studying one-dimensional nonrelativistic fermions came from the recent revival of interest in the matrix models in the framework of string theory. In particular the singlet sector of the one-dimensional \( N \times N \) hermitian matrix model, which provides the triangulation for the \( D = 1 \) string theory [1,9-14], is equivalent to a system of \( N \) nonrelativistic fermions in an external potential [2].

In this talk we shall present a review of the collective field approach to a quantum mechanical system of one-dimensional nonrelativistic fermions, in terms of which a bosonic field theoretic description is derived.

Beyond the string theory context there is a variety of condensed matter systems which are approximately described by low dimensional nonrelativistic fermions to which our formalism is also applicable.

After presenting the collective field formalism, we shall discuss its application to the \( D = 1 \) hermitian matrix model and the system of one-dimensional fermions in the presence of a weak electromagnetic field. Nontrivial soliton solutions of the collective theory in the absence of an external potential are discussed.

This talk is based on work done in collaboration with Bunji Sakita [3,4,5].
2. Bosonization of Fermions

We start with a system of \( N \) nonrelativistic fermions of mass \( m \) described by the Hamiltonian

\[
H = \frac{1}{2m} \sum_{a=1}^{N} (-i\partial_a - A(x_a))^2 + \sum_{a} V(x_a) + \sum_{a \neq b} v(x_a - x_b) \tag{1}
\]

where \( A \) is a gauge potential, \( V \) is a common potential and \( v \) describes a two-body interaction. The Schrödinger equation is given by

\[
H \Psi(x_1, x_2, \cdots, x_N) = E \Psi(x_1, x_2, \cdots, x_N) \tag{2}
\]

where \( \Psi(x_1, x_2, \cdots, x_N) \) is a totally antisymmetric wave function.

The bosonization of the fermions is achieved by a singular gauge transformation \[3\]

\[
\Psi(x_1, x_2, \cdots, x_N) = e^{i\Theta(x_1, x_2, \cdots, x_N)} \Phi(x_1, x_2, \cdots, x_N) \tag{3}
\]

where \( \Theta \) should be such that under an interchange of a pair of variables:

\[
e^{i\Theta(x_1, x_2, \cdots, x_N)} x_a \leftrightarrow x_b = -e^{i\Theta(x_1, x_2, \cdots, x_N)} \tag{4}
\]

Then \( \Phi \) is a totally symmetric wave function. A realization of \( \Theta \) is given by

\[
\Theta(x_1, x_2, \cdots, x_N) = \sum_{a>b} \text{Im} \ln(x_a - x_b - i\epsilon) \tag{5}
\]

Since (3) is a (singular) gauge transformation, the Hamiltonian for \( \Phi \) is obtained from (1) by adding the following (singular) gauge term to \( A(x_a) \)

\[
a(x_a) = -\pi \left( \sum_{b>a} \delta(x_a - x_b) - \sum_{b<a} \delta(x_a - x_b) \right) \tag{6}
\]

The bosonic Hamiltonian is now given by \[3\]

\[
H = \frac{1}{2m} \sum_{a=1}^{N} (-i\partial_a - A(x_a))^2 + \sum_{a} V(x_a) + \sum_{a \neq b} v(x_a - x_b)
+ \frac{\pi^2}{m} \left[ \sum_{a>b>c} \delta(x_a - x_b) \delta(x_a - x_c) + \sum_{a>b} \delta^2(x_a - x_b) \right] \tag{7}
\]

The bosonization procedure gave rise to a three-body and a singular two-body interaction.
3. Quantized Schrödinger Bose Field Formalism

It is useful to express this many body system by using the second quantized formalism. As usual we introduce a quantum field operator $\psi(x)$ and its conjugate $\psi^\dagger(x)$, which satisfy the Bose commutation relation at equal times

$$[\psi(x), \psi^\dagger(y)] = \delta(x - y)$$
$$[\psi(x), \psi(y)] = [\psi^\dagger(x), \psi^\dagger(y)] = 0 \quad (8)$$

We construct a symmetric basis vector

$$|x_1, x_2, \ldots, x_N> = \frac{1}{(N!)^{1/2}}\psi^\dagger(x_1)\psi^\dagger(x_2)\cdots\psi^\dagger(x_N)|0> \quad (9)$$

Since

$$\int dx dy \psi^\dagger(x)\psi^\dagger(y)V(x,y)\psi(y)\psi(x)|x_1, x_2, \ldots, x_N> = \sum_{a\neq b} V(x_a, x_b)|x_1, x_2, \ldots, x_N>$$

$$\int dx dy dz \psi^\dagger(x)\psi^\dagger(y)\psi^\dagger(z)V(x,y,z)\psi(z)\psi(y)\psi(x)|x_1, x_2, \ldots, x_N> =$$

$$\sum_{a\neq b\neq c} V(x_a, x_b, x_c)|x_1, x_2, \ldots, x_N> \quad (10)$$

we find that the second quantized expression of eq.(7) is given by

$$H = \int dx \left[\frac{1}{2m}(D\psi(x))^\dagger(D\psi(x)) + \psi^\dagger(x)\psi(x)V(x) + \frac{\pi^2}{6m}(\psi^\dagger(x)\psi(x))^3\right.$$

$$\left. - \frac{\pi^2}{6m}\delta^2(0)\psi^\dagger(x)\psi(x)\right] + \int dx dy \psi^\dagger(x)\psi^\dagger(y)\psi(y)\psi(x)v(x - y) \quad (11)$$

where $D$ is a covariant derivative $D = \frac{\partial}{\partial x} - i A(x)$.

An analogous procedure of bosonization and second quantization can be applied to a $(2+1)$ dimensional system of nonrelativistic fermions. In this case one obtains a Hamiltonian for the bosonic field $\psi$ with an additional coupling to a Chern-Simons gauge field [3].
4. Collective Field Formalism

The Hamiltonian $H$ in eq.(11) can be written as a functional of the charge density $\hat{\rho}(x) = \psi^\dagger(x)\psi(x)$ and the current density $\hat{j}(x) = i\psi^\dagger(x)\partial\psi(x)$

$$H[\hat{j}, \hat{\rho}] = \int dx \left[ \frac{1}{2m} \left( \partial\hat{\rho}(x) + i\hat{j}(x) + i\hat{\rho}(x)A(x) \right) - \frac{1}{\hat{\rho}(x)} \left( -i\hat{j}(x) - i\hat{\rho}(x)A(x) \right) + \hat{\rho}(x)V(x) \right] + \frac{\pi^2}{6m}\hat{\rho}^3(x) - \frac{\pi^2}{6m}\delta^2(0)\hat{\rho}(x) + \int dx dy (\hat{\rho}(x)\hat{\rho}(y) - \delta(x-y)\hat{\rho}(x))v(x-y)$$ (12)

The idea of using the density of currents to describe the Schrödinger wave field theory is due to Dashen and Sharp [6]. Our definition of the current density is slightly different from theirs for later convenience, see eq.(19). The Schrödinger equation is given by

$$H |\Phi > = E |\Phi >$$ (13)

where $|\Phi >$ is an $N$ particle Bose state expressed in terms of a symmetric wave function $\Phi(x_1, x_2, \cdots, x_N)$ as

$$|\Phi > = \int \cdots \int dx_1 dx_2 \cdots dx_N \Phi (x_1, x_2, \cdots, x_N) |x_1, x_2, \cdots, x_N >$$ (14)

where $|x_1, x_2, \cdots, x_N >$ is the basis vector defined by eq. (9). Since $\Phi$ is a symmetric function, we may regard it as a functional of the density variable, namely

$$\Phi(x_1, x_2, \cdots, x_N) = \Phi[\rho(x)] = \sum_a \delta(x - x_a)$$ (15)

Thus,

$$|\Phi > = \int \cdots \int dx_1 dx_2 \cdots dx_N \Phi[\hat{\rho}] |x_1, x_2, \cdots, x_N >$$

$$= \Phi[\hat{\rho}] \frac{1}{(N!)^{1/2}} \left( \int dx\psi^\dagger(x) \right)^N |0 >$$ (16)

where we used

$$\hat{\rho}(x) |x_1, x_2, \cdots, x_N > = \sum_a \delta(x - x_a) |x_1, x_2, \cdots, x_N >$$ (17)

Next we use the following commutation relations of charge and current density
in order to obtain the collective field representation of the Schrödinger equation:

\[
[j(x), \dot{\rho}(y)] = i\dot{\rho}(x)\partial_x\delta(x - y), \quad [\dot{\rho}(x), \dot{\rho}(y)] = 0
\]  

(18)

Since

\[
\dot{j}(x) \frac{1}{(N!)^{1/2}} \left( \int dx \psi_\dag(x) \right)^N |0 > = 0
\]  

(19)

we obtain

\[
\dot{j}(x)|\Phi > = [\dot{j}(x), \Phi[\dot{\rho}]] \frac{1}{(N!)^{1/2}} \left( \int dx \psi_\dag(x) \right)^N |0 >
\]

\[
= (i\dot{\rho}(x)\partial_x \frac{\delta}{\delta \rho(x)} \Phi[\dot{\rho}]) \frac{1}{(N!)^{1/2}} \left( \int dx \psi_\dag(x) \right)^N |0 >
\]

(20)

Therefore the collective field representation of the Schrödinger equation is given by

\[
H[ i\rho \partial_x \frac{\delta}{\delta \rho}, \rho ] \Phi[\rho] = E \Phi[\rho]
\]

(21)

This is not however the final form of the Schrödinger equation. The transition to the density variables \(\rho(x)\) introduces a nontrivial Jacobian factor. The inner product of the states in the functional space of \(\rho(x)\) is

\[
< \Phi' | \Phi > = \int D\rho J[\rho] \Phi'[\rho] \Phi[\rho]
\]

(22)

where \(J[\rho]\) is given by [3]

\[
J[\rho] = \int \cdots \int dx_1 dx_2 \cdots dx_N \prod_x \delta(\rho(x) - \sum_a \delta(x - x_a))
\]

(23)

The Jacobian \(J\) can be calculated order by order in \(1/N\) expansion [3,4,7]. We find that

\[
J[\rho] = \delta(\int dx \rho(x) - N) j[\rho]
\]

where

\[
j[\rho] = \exp \left[ -\int dx \rho(x) \ln \rho(x) - \frac{\delta(0)}{2} \int dx \ln \rho(x) - \frac{\delta^2(0)}{12} \int dx \frac{\rho(x)}{\rho(x)} + O(N^{-2}) \right]
\]

(24)

In the functional space of \(\rho(x)\) with measure \(D\rho(x) = \prod_x d\rho(x) \delta(\int dx \rho(x) - N)\), the
wavefunction is defined through the similarity transformation

\[ j^{1/2} \Phi[\rho] = \Psi[\rho] \]  

and the corresponding Hamiltonian is given by

\[ j^{1/2} H[ -i \rho \frac{\delta}{\delta \rho}, \rho ] \cdot j^{-1/2} = H[ \rho \partial \pi - \frac{i}{2} \partial \rho + \delta(0) \frac{i \partial \rho}{4 \rho} - \delta^2(0) \frac{i \partial \rho}{12 \rho^2} + \cdots, \rho ] \]  

where \( \pi(x) = -i \frac{\delta}{\delta \rho(x)} \).

The second quantized form of the Hamiltonian can now be expressed in terms of the canonical fields \( \rho(x) \) and \( \pi(x) \) as

\[ H = \int dx \left[ \frac{1}{2m} \rho(x) \left( \partial \pi(x) + \frac{i}{4} \frac{\delta(0) \partial \rho(x)}{\rho^2(x)} - \frac{i}{12} \frac{\delta^2(0) \partial \rho(x)}{\rho^3(x)} + \cdots - A(x) \right)^2 
+ \frac{1}{8m} \left( \partial \rho(x) \right)^2 + \frac{\pi^2}{6m} \rho^3(x) + V(x) \rho(x) + \frac{1}{4m} \frac{\delta''(0)}{\delta(0)} - \frac{\pi^2}{6m} \delta^2(0) \rho(x) \right] 
+ \int dx dy \rho(x) \rho(y) v(x, y) - Nv(0) \]  

where the density constraint \( \int dx \rho(x) = N \) was used. This expression for the Hamiltonian agrees, up to the subleading in \( N \) singular terms introduced in the calculation of the Jacobian, eq.(24), with the expression derived by applying the collective field approach of [8] on the bosonized version of the many-body Hamiltonian (7).

Equation (27) provides now a bosonic field Hamiltonian amenable to a semiclassical treatment.

5. Applications

5.1. \( D = 1 \) Matrix Model

The relation between one-dimensional nonrelativistic fermions and the \( D = 1 \) hermitian matrix model has been established in the classic paper of Brézin, Itzykson, Parisi and Zuber [2] whose analysis we briefly outline.

The dynamics of the \( D = 1 \) hermitian matrix model is determined by the Lagrangian

\[ L = tr \left[ \frac{1}{2} M^2 - V(M) \right] \]  

where \( M \) is an \( N \times N \) hermitian matrix and \( V(M) \) is a polynomial in \( M \). In particular we consider functions \( V(x) \) which scale with \( N \) like \( V(x) = NV(x/\sqrt{N}) \). One quantizes the system in “cartesian coordinates” defined by \( M_i = tr(M t_i) \), \( i = 1, \cdots, N^2 \), where \( t_i \) is a hermitian basis of the fundamental representation of \( U(N) \) Lie algebra.
The Lagrangian (28) is invariant under a time independent $U(N)$ transformation $M \rightarrow UMU^\dagger$, hence it is natural to consider an analogue of the polar coordinate basis

$$M = U X U^\dagger, \quad X_{ab} = x_a \delta_{ab}$$

(29)

The integration measure is given by

$$\prod_i dM_i = \Delta^2(x) \left( \prod_a dx_a \right) dU$$

(30)

where $\Delta(x) \equiv \prod_{a>b}(x_a - x_b)$ is the Vandermonde determinant. We shall restrict our attention to the singlet sector; the wave function $\Omega$ is a symmetric function of the eigenvalues $x_a, \ a = 1, ..., N$. The Jacobian factor $\Delta^2(x)$, eq.(30), which appears in changing variables from the “cartesian coordinates” to the “polar coordinates” can be absorbed by appropriately redefining the wavefunction and the Hamiltonian. This is done by redefining an antisymmetric wavefunction

$$\Psi(x_1, \cdots, x_N) = \Delta(x) \Omega(x_1, \cdots, x_N)$$

(31)

The corresponding Schrödinger equation is

$$\sum_a \left[ -\frac{1}{2} \frac{\partial^2}{\partial x_a^2} + V(x_a) \right] \Psi(x_1, \cdots, x_N) = E \Psi(x_1, \cdots, x_N)$$

(32)

Therefore the singlet sector of the original matrix model can be equivalently described by a set of $N$ decoupled nonrelativistic fermions in the external potential $V(x_a)$ [2].

According to the collective field formalism developed earlier the field theoretic Hamiltonian appropriate for describing the singlet sector of the $N$ hermitian $D = 1$ matrix model is given by

$$H = \int dx \left[ \frac{1}{2} \partial\pi(x) \rho(x) \partial\pi(x) + \frac{1}{8} (\partial\rho(x))^2 \rho(x) + \frac{\pi^2}{6} \rho^3(x) + V(x) \rho(x) \right. + \frac{\pi^2}{6} \delta^2(0) \rho(x) + \frac{i \delta(0)}{8} \{ \partial\pi(x), \frac{\partial\rho(x)}{\rho(x)} \} - \delta^2(0) \frac{(\partial\rho(x))^2}{32 \rho^3(x)} \\
- \frac{i \delta^2(0)}{24} \{ \partial\pi(x), \frac{\partial\rho(x)}{\rho^2(x)} \} + \delta^3(0) \frac{(\partial\rho(x))^2}{48 \rho^4(x)} + \cdots \left. \right] - e \left( \int dx \rho(x) - N \right)$$

(33)

where $e$ is a Lagrange multiplier for the density constraint.
Of particular interest is the double scaling limit of the \( D = 1 \) hermitian matrix model. This provides a definition of two-dimensional gravity coupled to a scalar field \[1\]. In the fermionic description the double scaling limit of the theory is taken by keeping the difference between the value of the potential at the local maximum and the fermi energy fixed as \( N \to \infty \), \( V(x_0) - \epsilon_F \to \mu \)[9],[10]. Expanding the potential \( V \) around the local maximum \( x_0 \) and then taking the large \( N \) limit, one concludes that the double scaling limit of the \( D = 1 \) hermitian matrix model is equivalent to a system of fermions in an inverted harmonic oscillator potential [11],[12]. According to the previous discussion the corresponding bosonized Schrödinger wave field Hamiltonian for this system is (up to a constant)

\[
H = \int dx \left[ \frac{1}{2} \partial \psi \dagger \partial \psi + \frac{\pi^2}{6} (\psi \dagger \psi)^3 + (\mu - \frac{x^2}{2})\psi \dagger \psi \right]
\] (34)

The corresponding collective field representation of the Hamiltonian is given by

\[
H = \int dx \left[ \frac{1}{2} \partial \pi \dagger \partial \pi + \frac{1}{8} (\partial \rho(x))^2 + \frac{\pi^2}{6} \rho^3(x) + (\mu - \frac{x^2}{2})\rho(x) + \cdots \right]
\] (35)

where \( \cdots \) contains the singular terms (see eq. 33). We notice that the second term in (35), whose origin lies on the term \( \int \rho \ln \rho \) of the Jacobian (24), is absent from the collective field Hamiltonian of [8] used by Das and Jevicki [13] for the discussion of the double scaling limit.

Given the Hamiltonians (34), (35) one can apply a semiclassical analysis. Of particular interest are nontrivial classical solutions and their interpretation in the \( D = 1 \) string theory framework.*

The classical equations of motion corresponding to eq.(35) are

\[
\dot{\rho} = -\partial (\rho \partial \pi)
\]

\[
-\dot{\pi} = \frac{1}{2} (\partial \pi)^2 + \frac{1}{8} (\partial \rho(x))^2 - 2 \frac{\partial ^2 \rho}{\rho} + \frac{\pi^2}{2} \rho^2 + (\mu - \frac{x^2}{2})\rho + \cdots
\] (36)

The equivalence of this set of equations of motion and the one derived from (34) is straightforward given that the classical Schrödinger wave field and the collective field are related to each other by

\[
\psi(x) = \rho^{1/2}(x)e^{i\pi(x)}
\] (37)

The presence of the inverted harmonic oscillator potential complicates the search for explicit analytic solutions of (36). It was shown though in [16] that the harmonic

* Classical solutions of the cubic collective theory (without the \( (\partial \rho)^2/\rho \)) have been studied in refs.[14-16].
oscillator potential can be induced through a reparametrization. It is interesting then to study solutions of eq.(36) without the harmonic oscillator potential term [5,17]. In this case we find a nontrivial static solution of the form

\[ \rho(x) = \rho_0 \left[ 1 - \frac{36\rho_0}{\alpha^{-1}e^{2\pi \rho_0 x} + 24\rho_0 + 36\rho_0^2 \alpha e^{-2\pi \rho_0 x}} \right] \] (38)

where \( \rho_0 = \frac{p_F}{\pi} \). Because of translation invariance there is a free parameter \( \alpha \), which controls the position of the minimum of the configuration. The derivative term \((\partial \rho)^2/\rho\) is very crucial for the existence of such a solution.

After some straightforward algebra we find that the energy of this static soliton configuration is

\[ E = H[\rho] - H[\rho_0] = p_F^2 \frac{\sqrt{3}}{2\pi} \log \left( \frac{1}{2 - \sqrt{3}} \right) \] (39)

The time-dependent solution corresponding to the moving soliton is of the form [5]

\[ \rho(x,t) = \rho_0 - \frac{36c}{(\pi^4\rho_0^2 + 3\pi^2 v^2)\alpha^{-1}e^{-2\sqrt{c}(x+vt)} + 24\pi^2 \rho_0 + 36\alpha e^{2\sqrt{c}(x+vt)}} \] (40)

where \( c = \pi^2\rho_0^2 - v^2 \) and \( |v| \leq \pi^2\rho_0^2 \). The corresponding energy is

\[ E = H[\rho] - H[\rho_0] = (p_F^2 - v^2) \frac{\sqrt{3}}{2\pi} \log \frac{\sqrt{p_F^2 + 3v^2}}{2p_F - \sqrt{3(p_F^2 - v^2)}} \] (41)

These soliton solutions, although different from the ones discussed by Jevicki in ref.[16], display similar features.

5.2. One-Dimensional Fermions in External Electromagnetic Field

We start with a system of \( N \) nonrelativistic fermions in the presence of a weak electromagnetic field. The second quantized Hamiltonian for this system in terms of a fermionic field \( \psi \) is given by

\[ H = \int dx \left[ \frac{1}{2m} (D\psi(x))\psi(x) + A_0(x)\psi(x) \right] \] (42)

where \( D \) is the covariant derivative \( D = \frac{\partial}{\partial x} - iA(x) \).
It is known that excitations near the Fermi surface admit a relativistic field theoretic description [18]. The corresponding relativistic Lagrangian is given by

\[ L = \bar{\Psi} \gamma^\mu (i \partial_\mu - A_\mu) \Psi \]  

(43)

where \( \Psi = (\psi_L, \psi_R) \), \( A^\mu = (A^0, v_F A) \), \( x^\mu = (t, \frac{x}{v_F}) \), \( \gamma^\mu = (\gamma^1, -i \gamma^2) \), \( \gamma^5 = \gamma^0 \gamma^1 = \tau^3 \), \( g^{\mu\nu} = \text{diag}(1, -1) \) and \( \epsilon^{01} = 1 \).

The collective Hamiltonian is

\[ H = \int dx \left[ \frac{1}{2m} \rho(x) \left( \partial_\pi(x) - A(x) \right)^2 + \frac{1}{8m} \frac{(\partial \rho(x))^2}{\rho(x)} + A_0(x) \rho(x) + \frac{\pi^2}{6m} \rho^3(x) \right] - e \left( \int dx \rho(x) - N \right) + \cdots \]  

(45)

We shall now employ a semiclassical treatment by expanding around the time independent classical solutions \( \rho(x) = \rho_0(x) + \delta \rho(x) \) where \( \rho_0(x) \) satisfies

\[ -\frac{1}{8} \left( \frac{\partial \rho_0(x)}{\rho_0(x)} \right)^2 - \frac{2 \partial^2 \rho_0(x)}{\rho_0(x)} + \frac{\pi^2}{2} \rho_0^2(x) - e m = 0 \]  

\[ \int \rho_0(x) dx = N \]  

(46)

The most obvious solution is the constant solution \( \rho_0 = \frac{N}{L} = \frac{2e}{\pi} \).

The Hamiltonian for the excitations \( \delta \rho \) is up to a constant (assuming that the external electromagnetic field is weak and the fields are slowly varying)

\[ H = \int dx \left[ \frac{\rho_0}{2m} \partial_\pi(x) \partial_\pi(x) - \frac{\rho_0}{m} \partial_\pi(x) A(x) + A_0(x) \delta \rho(x) + \frac{\pi^2}{2m} \rho_0^2(\delta \rho(x))^2 \right] \]  

(47)

where \( \delta \rho \), \( \pi \) are conjugate field variables with a subsidiary condition \( \int \delta \rho(x) dx = 0 \).
This Hamiltonian can be derived from a local Lagrangian of the form

$$\mathcal{L} = \frac{1}{8\pi} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2\pi \sqrt{v_F}} \epsilon^{\mu\nu} A_\mu \partial_\nu \chi$$

(48)

where $\delta \rho = \frac{1}{2} \sqrt{\frac{m}{m_\pi}} \partial \chi$. The relativistic notation we used has been indicated earlier. The above Lagrangian is essentially the bosonized form of its fermionic counterpart in eq.(43) [20].

The anomaly equation (44) is now expressed in terms of the equation of motion for the field $\chi$. We can define the vector current $j_\mu$ as

$$j_\mu(x) \equiv -\frac{\partial \mathcal{L}}{\partial A_\mu(x)} = \frac{1}{2\pi \sqrt{v_F}} \epsilon^{\mu\nu} \partial_\nu \chi(x)$$

(49)

Then using the equations of motion for $\chi$ we find that

$$-\epsilon^{\mu\nu} \partial_\mu j_\nu = \frac{1}{2\pi \sqrt{v_F}} \partial_\mu \partial^\mu \chi = \frac{1}{2\pi v_F} \epsilon^{\mu\nu} F_{\mu\nu}$$

(50)

which is the anomaly equation.

In the above analysis we considered only the constant solution of the static classical equations (46) and the fluctuations around it. But eq. (46) admits also a soliton solution, eq.(38). A semiclassical analysis around this nontrivial configuration has to be done and appropriately interpreted in the fermionic picture.

6. Discussion

In the previous section we talked about soliton solutions of the classical equations of motion of the system of one-dimensional nonrelativistic fermions in the collective field representation in the absence of an external potential. It would be very interesting to find if soliton-like solutions persist in the presence of an external potential, particularly an inverted harmonic oscillator potential, which is the case in the double scaling limit of the $D = 1$ matrix model. The existence of new classical solutions of the equations of motion of the collective Hamiltonian describing the double scaling limit of the $D = 1$ matrix model might make clearer the connection between the field theory of the $D = 1$ matrix model and the $D = 1$ string theory.
7. Acknowledgements

I would like to thank Prof. B. Sakita for a fruitful collaboration on which the results presented in this talk are based. I would also like to thank Antal Jevicki for discussions. This research is supported by NSF grant PHY90-20495.

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