Higgs to $\gamma Z$ decay as a probe of flavour changing neutral Yukawa couplings

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With the deeper study of Higgs particle, Higgs precision measurements can be served to probe new physics indirectly. In many new physics models, vector-like quarks $T_L, T_R$ occur naturally. It is important to probe their couplings with SM particles. In this work, we consider the singlet $T_L, T_R$ extended models and show how to constrain the $T$th couplings through the $h \rightarrow \gamma Z$ decay at HL-LHC. Firstly, we derive the perturbative unitarity bounds on $|y_{T_L}^{T}|$ with other couplings set to be zeros simply. In order to optimize the situation, we take $m_{T}=400\text{GeV}$ and $s_L=0.2$ considering the experimental constraints. Under this benchmark point, we find that the future bounds from $h \rightarrow \gamma Z$ decay can limit the real parts of $y_{L,R}^{T}$ in the positive direction to be $\mathcal{O}(1)$ because of the double enhancement. For the real parts of $y_{L,R}^{T}$ in the negative direction, it is always surpassed by the perturbative unitarity. Moreover, we find that top quark CEDM can give more strong bounds (especially the imaginary parts of $y_{L,R}^{T}$) than the perturbative unitarity and $h \rightarrow \gamma Z$ decay in the off-axis regions for some scenarios.

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I. INTRODUCTION

The standard model (SM) of elementary particle physics has been proposed since the 1960s [1], and it is verified to be quite successful up to now. However, there are still many problems beyond the ability of SM. Thus new physics beyond the SM (BSM) are motivated in the high energy physics community. Many of these BSM models predict the existence of heavy fermions, for example, composite Higgs models [2, 3], little Higgs models [4, 5], grand unified theories [6], extra dimension models [7]. In these models, there can be a heavy up-type quark $T$ which interacts with the SM particles through $TbW, TtZ, Tth$ interactions. Analyses on these couplings may tell us some clues on the new physics. $TbW$ coupling can be constrained from single production of $T$ quark, but there are always many assumptions for most of the current constraints. It will be hard for the detection of the flavour changing neutral (FCN) couplings $TtZ, Tth$, because $T$ productions from $tZ, th$ fusion are highly suppressed. If there exists other new decay channels for $T$ quark even the bounded $TbW$ coupling can be saturated. Since the discovery of Higgs boson at the Large Hadron Collider (LHC) [8], it can also be a probe to such new physics.

Currently, all the main production and decay channels of the Higgs boson have been discovered at LHC. Then the next step is to measure the observed channels more accurately. At the same time, attention should be paid to the undiscovered channels. Precision measurements of the Higgs particle may help us decipher the nature of electro-weak symmetry breaking (EWSB) [9] and open the door to new physics [10]. $hVV$ and $hff$ couplings inferred from the observed channels are SM-like now, while there can still be large deviations for the rare decay modes, for example $h \to \gamma Z, \mu^+ \mu^-$. This decay mode has drawn much attention of this community. It can be used to detect CP violation [11–13] and many new physics scenarios [14–16]. Here we will show how to constrain the FCN Yukawa (FCNY) couplings through the $h \to \gamma Z$ decay mode indirectly. The constraints from $h \to \gamma Z$ don’t depend on the total width of $T$ quark, namely in spite of other decay modes.

In this paper, we build the framework of FCN couplings in Sec. II firstly. Sec. III is devoted to the theoretical and experimental constraints on the simplified model. In Sec. IV we compute the new physics contributions to the partial decay width of $h \to \gamma Z$. Then we perform the numerical constraints on the FCNY interactions in Sec. V Finally, we give the summary and conclusions in Sec. VI.

II. FRAMEWORK OF FLAVOUR CHANGING NEUTRAL COUPLINGS

II.1. UV complete model

It is strongly constrained for the mixings between heavy particles and the first two generations because of the bounds from flavour physics [17][19]. What’s more, the third generation is more likely to be concerned with new physics theoretically owing to the mass hierarchy. For convenience and simplicity, we only take into account the mixings between the third generation and heavy quarks.

Usually, we extend the SM fermion sector by introducing vector-like particles to avoid the quantum anamaly. The minimal extension of quark sector is to add one pair of vector-like quarks (VLQ) [20] [21]. For the non-minimally extended models, the scalar sector can also be augmented. Besides the VLQ, we can also plus a real gauge singlet scalar [23], a Higgs doublet [24] and even both the

1 Say $T \to tS$, here $S$ can be a CP even or odd new scalar.
singlet-doublet scalars at the same time \textsuperscript{24} \textsuperscript{25}. In these models, there can exist other decay channels \textsuperscript{26} \textsuperscript{27}. FCN couplings $TtZ$ and $Tth$ show different patterns in different models. For simplicity, we will only consider the case where there is one pair of VLQ $T_L$ and $T_R$. In the following, we will give two specific examples: the minimal extension with a pair of singlet quarks $T_L,T_R$ and the model further enlarged with an extra real singlet scalar.

II.1.1. Minimal vector-like quark model

Let’s start with the model by adding a pair of singlet fermions $T_L,T_R$ to SM, which is dubbed as VLQT model. The Lagrangian can be written as \textsuperscript{21}

\[
\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{Yukawa}^{Yukawa} + \mathcal{L}_{T}^{gauge},
\]

\[
\mathcal{L}_{Yukawa}^{Yukawa} = -\Gamma^i_3 \bar{Q}_L^i \Phi T_R - M_T \bar{T}_L T_R + \text{h.c.}, \quad \mathcal{L}_{T}^{gauge} = \bar{T}_L i \partial \Phi T_L + \bar{T}_R i \partial \Phi T_R,
\]

where $\Phi = i \sigma_2 \Phi^*$ and the covariant derivative is defined as $D_{\mu} = \partial_{\mu} - ig Y T B_\mu$. $Y_T$ and $Q_T$ are the $U_Y(1)$ and electric charge of the $T$ quark, respectively. The Higgs doublet is parametrized as $\Phi^T = [\phi^+, \frac{\phi^+ + \phi^-}{\sqrt{2}}]$. It is easy to obtain the mass terms of $t$ and $T$ quarks \textsuperscript{3}

\[
\mathcal{L}_{mass} \supset - \begin{bmatrix} \bar{t}_L & \bar{T}_L \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \Gamma_3^v & \frac{1}{\sqrt{2}} \Gamma_3^3 v \\ 0 & M_T \end{bmatrix} \begin{bmatrix} t_L \\ T_R \end{bmatrix} + \text{h.c.}.
\]

Here $\Gamma_3^v$ and $\Gamma_3^3$ are the gauge eigenstate Yukawa couplings in front of $\bar{Q}_L^3 \Phi T_R$ and $\bar{Q}_L^3 \Phi t_R$ individually. The $t$ and $T$ quarks can be rotated into mass eigenstates by the following transformations:

\[
\begin{bmatrix} t_L \\ T_L \end{bmatrix} \rightarrow \begin{bmatrix} \cos \theta_L & \sin \theta_L \\ -\sin \theta_L & \cos \theta_L \end{bmatrix} \begin{bmatrix} t_L \\ T_L \end{bmatrix}, \quad \begin{bmatrix} t_R \\ T_R \end{bmatrix} \rightarrow \begin{bmatrix} \cos \theta_R & \sin \theta_R \\ -\sin \theta_R & \cos \theta_R \end{bmatrix} \begin{bmatrix} t_R \\ T_R \end{bmatrix}.
\]

Then we have the following mass eigenstate Yukawa interactions:

\[
\mathcal{L}_{Yukawa} \supset -m_t \bar{t}t - m_T \bar{T}T - \frac{m_t}{v} c_L \bar{t} \hat{h}T - \frac{m_T}{v} s_L \bar{T} \hat{h}T
\]

\[
- \frac{m_T}{v} s_L c_L h (\bar{t}_L T_R + \bar{T}_R t_L) - \frac{m_t}{v} s_L c_L h (\bar{T}_L t_R + \bar{t}_R T_L).
\]

Here $s_L,c_L,s_R,c_R$ are the short-hand marks for $\sin \theta_L, \cos \theta_L, \sin \theta_R, \cos \theta_R$, respectively. Similarly, we take $\sin \theta, \cos \theta$ as $s_\theta, c_\theta$ in the following context. In this model, we have two independent extra parameters $m_T$ and $\theta_L$. There are two relations for the mixing angles and $t$ and $T$ quark masses:

\[
\tan \theta_R = \frac{m_t}{m_T} \tan \theta_L, \quad M_T^2 = m_T^2 c_L^2 + m_t^2 s_L^2.
\]

\textsuperscript{2} Of course you can build one model with more $T_L,T_R$ quarks. But the mass matrix may be equal to and even greater than three dimensions, which are quite complex.

\textsuperscript{3} Although the mass mixing term $\bar{T}_L t_R$ can appear, it will be removed via field redefinition \textsuperscript{24} \textsuperscript{25}. 
For the $T_L$ and $T_R$ quark, we will use $I^T_3$ to denote the third component of weak isospin generally. Then the gauge eigenstate $t, T$ quarks will interact with $Z, W$ boson through the following form:

$$
\mathcal{L}_{\text{gauge}} \supset \frac{g}{c_W} Z_{\mu} \bar{l}_L \gamma^\mu \left( \frac{1}{2} - \frac{2}{3} s_W^2 \right) t_L - \frac{2}{3} s_W^2 \bar{t}_R \gamma^\mu t_R + \bar{T}_L (I^T_3 - Q_T s_W^2) \gamma^\mu T_L \\
+ \tilde{T}_R (I^T_3 - Q_T s_W^2) \gamma^\mu T_R] + \frac{g}{\sqrt{2}} (W^+_{\mu} \bar{l}_L \gamma^\mu b_L + W^-_{\mu} \bar{b}_L \gamma^\mu t_L).
$$

(6)

Here $t$ and $T$ quarks can be rotated into mass eigenstates by the transformations in Eq. (3), thus we have the following mass eigenstate gauge interactions:

$$
\mathcal{L}_{\text{gauge}} \supset \frac{g}{c_W} Z_{\mu} \left[ \left( \frac{1}{2} c_L^2 + I^T_3 s_L^2 - \frac{2}{3} s_W^2 \right) \bar{l}_L \gamma^\mu t_L + \left( \frac{1}{2} s_L^2 + I^T_3 c_L^2 - \frac{2}{3} s_W^2 \right) \bar{T}_L \gamma^\mu T_L \\
+ \left( \frac{1}{2} - I^T_3 \right) s_L c_L \left( \bar{l}_L \gamma^\mu T_L + \bar{T}_L \gamma^\mu t_L \right) + \left( - \frac{2}{3} s_W^2 + I^T_3 s_R^2 \right) \bar{t}_R \gamma^\mu t_R + \left( - \frac{2}{3} s_W^2 + I^T_3 c_R^2 \right) \bar{T}_R \gamma^\mu T_R \\
- I^T_3 s_R c_R \left( \bar{l}_L \gamma^\mu T_R + \bar{T}_R \gamma^\mu t_R \right)] + \frac{g c_L}{\sqrt{2}} (W^+_{\mu} \bar{l}_L \gamma^\mu b_L + W^-_{\mu} \bar{b}_L \gamma^\mu t_L) + \frac{g s_L}{\sqrt{2}} (W^+_{\mu} \bar{T}_L \gamma^\mu b_L + W^-_{\mu} \bar{b}_L \gamma^\mu T_L).
$$

(7)

For the singlet case, i.e., $I^T_3 = 0$, the interactions can be simplified as

$$
\mathcal{L}_{\text{gauge}} \supset \frac{g}{c_W} Z_{\mu} \left[ \left( \frac{1}{2} c_L^2 - \frac{2}{3} s_W^2 \right) \bar{l}_L \gamma^\mu t_L + \left( \frac{1}{2} s_L^2 + \frac{2}{3} s_W^2 \right) \bar{T}_L \gamma^\mu T_L \\
- \frac{2}{3} s_W^2 \bar{t}_R \gamma^\mu t_R - \frac{2}{3} s_W^2 \bar{T}_R \gamma^\mu T_R \right] + \frac{g c_L}{\sqrt{2}} (W^+_{\mu} \bar{l}_L \gamma^\mu b_L + W^-_{\mu} \bar{b}_L \gamma^\mu t_L) + \frac{g s_L}{\sqrt{2}} (W^+_{\mu} \bar{T}_L \gamma^\mu b_L + W^-_{\mu} \bar{b}_L \gamma^\mu T_L).
$$

(8)

\section*{II.1.2. Vector-like quark and one singlet scalar model}

Now let’s consider the model with SM extended by a pair of singlet VLQ $T_L, T_R$ and a real singlet scalar $S$, which is named as VLQT+S. The Lagrangian can be written as \cite{23, 29, 30, 31, 32, 33, 34}.

$$
\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_S,
$$

$$
\mathcal{L}_{\text{Yukawa}} = - \Gamma_3 \bar{Q}_L \Phi T_R - MT_T \bar{T}_L T_R - y_{1s}^S S \bar{T}_L T_R + \text{h.c.}, \quad \mathcal{L}_{\text{gauge}} = \bar{T}_L \Phi^T T_L + \bar{T}_R \Phi^T T_R,
$$

$$
\mathcal{L}_S = \frac{1}{2} \partial_{\mu} S \partial^{\mu} S - V_{\Phi S}, \quad V_{\Phi S} = \mu_{\Phi S} \Phi^\dagger \Phi S + \lambda_{\Phi S} \Phi^\dagger \Phi S^2 + t_S S + m_S^2 S^2 + \mu_S S^3 + \lambda_S S^4.
$$

(9)

Note that the Lagrangian form is invariant after shifting $S$, thus we can set $\langle S \rangle = 0$ through redefinition of the scalar field $S$ \cite{30, 31}. Here $h$ can mix with $S$, so we should transform them into mass eigenstates via following rotations:

$$
\begin{bmatrix}
\begin{array}{c}
h \\
S
\end{array}
\end{bmatrix} \rightarrow \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
\begin{array}{c}
h \\
S
\end{array}
\end{bmatrix}.
$$

(10)

The mass terms of $t$ and $T$ quarks are exactly the same as those in Eq. (2), thus they can be rotated into mass eigenstates by the same transformations of Eq. (3). There is one extra Yukawa
term $-y_3^5 S T L T R$ compared to the model VLQ, so the Yukawa interactions in this model are more complex. Then we have the following mass eigenstate Yukawa interactions:

\[ \mathcal{L}_{\text{Yukawa}} = -m_t \bar{t} t - m_{3T} \bar{3T} T T - \frac{m_t^2}{\sqrt{2}} c_L c_0 - \text{Re}(y_T^s s_L s_H s_0) \bar{h} t - \frac{m_{3T}}{\sqrt{2}} s_L c_0 - \text{Re}(y_T^s) c_L c_R s_0 | h \bar{T} T \]

\[- \frac{m_{3T}}{\sqrt{2}} s_L c_0 + \text{Re}(y_T^s) c_L c_R s_0 \bar{h} (\bar{L} T R + \bar{T} R L) - \frac{m_t^2}{\sqrt{2}} s_L c_L c_0 + \text{Re}(y_T^s) c_L c_R s_0 \bar{h} (\bar{T}_L t_R + \bar{t}_R T_L). \]

+ i \text{Im}(y_T^s) c_L c_R s_0 \bar{h} \gamma^5 T

\[- i \text{Im}(y_T^s) c_L c_R s_0 \bar{h} (\bar{T}_L T R + \bar{T}_R L) - i \text{Im}(y_T^s) c_L c_R s_0 \bar{h} (\bar{T}_L T R - \bar{T}_R L). \]

The gauge interactions for $t$ and $T$ quarks are fully the same as those in Eq. (7) and Eq. (8).

In this model, there are four interesting parameters $\theta_L, m_t, \theta, y_3^5$. The other parameters in scalar potential don’t have relation with the $h \to \gamma Z, \gamma \gamma$ processes.

### II.2. Simplified model

Here we will adopt one more general and model independent framework \[35\]. In the simplified model case, we can write down the related mass eigenstate state interactions generally:

\[ \mathcal{L} \supset -m_t \bar{t} t - m_{3T} \bar{3T} T T - c A_{\mu} \sum_{f=t,T} Q_f \bar{f} \gamma^{\mu} f + c Z_{\mu}(y_L^T \omega_+ + y_R^T \omega_+) t + \bar{T} \gamma^{\mu}(y_L^T \omega_+ + y_R^T \omega_+) t + \bar{t} \gamma^{\mu}(y_L^T \omega_+ + y_R^T \omega_+) t \]

\[ + \bar{t} \gamma^{\mu}(y_L^T \omega_+ + y_R^T \omega_+) t + \bar{T} \gamma^{\mu}(y_L^T \omega_+ + y_R^T \omega_+) t - \frac{m_t^2}{\sqrt{2}} h t (\kappa_t + i \gamma^5 \kappa_t) t + h \bar{T} (y_T + i \gamma^5 y_T) t \]

\[ h \bar{T}(y_L^T \omega_+ + y_R^T \omega_+) t + h \bar{T}(y_L^T \omega_+ + y_R^T \omega_+) t + \frac{g_{CL}}{\sqrt{2}}(W_\mu^+ \bar{T}_L \gamma^\mu b_L + W_{-\mu} \bar{b}_L \gamma^\mu t_L) \]

\[ + \frac{g_{SR}}{\sqrt{2}}(W_\mu^+ \bar{T}_L \gamma^\mu b_L + W_{-\mu} \bar{b}_L \gamma^\mu t_L). \]

Here $\omega_{\pm}$ are the chirality projection operators $(1 \pm \gamma^5)/2$ and the gauge couplings are listed as follows:

\[ g_L^T = \frac{1}{s_{WCW}}(\frac{1}{2} c_L^2 + I_3^T s_L^2 - \frac{2}{3} s_W^2), \quad g_R^T = \frac{1}{s_{WCW}}(\frac{1}{2} c_L^2 + I_3^T c_L^2 - \frac{2}{3} s_W^2), \quad g_{CL}^T = \frac{1}{s_{WCW}}(\frac{1}{2} - I_3^T) s_L c_L, \]

\[ g_s^T = \frac{1}{s_{WCW}}(- \frac{2}{3} s_W^2 + I_3^T s_R^2), \quad g_T^T = \frac{1}{s_{WCW}}(- \frac{2}{3} s_W^2 + I_3^T c_R^2), \quad g_{CL}^T = \frac{-I_3^T s_{BCR}}{s_{WCW}}. \]

For giving $I_3^T$, there are generally nine parameters $m_T, \theta_L, \theta_R, \kappa_t, \kappa_R, \gamma, y_T, y_{3T}, y_{3T}$. Here $m_T, \theta_L, \theta_R, \kappa_t, \kappa_R, \gamma, y_T, y_{3T}$ are all real parameters, while $y_{3T}$ can be complex numbers. If CP conservation is assumed, we will have $\kappa_t = \bar{\kappa}_t = 0, \{y_T, y_{3T}\} \in \mathbb{R}$.

In Tab. [I] we give the expressions of $\kappa_t, \gamma, y_T, y_{3T}$ in three models. In Tab. [III] we give the expressions of $g_{CL}^T(y_{3T}^T)^*, g_T^T(y_{3T}^T)^*, g_{CL}^T(y_{3T}^T)^*, g_T^T(y_{3T}^T)^*$ in the VLQ and VLQ+T models. In this work, we will only study the singlet VLQ case. Here we want to show the feasibility to constrain the FCNY couplings through the $h \to \gamma Z$ decay channel, thus it is best to avoid drowning in elaborate theoretical calculations and collider phenomenology details. There is no doubt that the FCNY couplings in VLQ and VLQ+T are not free parameters. But here we want to make a more general analysis naively, for example, we can extend the SM by one pair of VLQ $T_L, T_R$ and many real singlet scalars.

4 Just as the above two models, $\theta_L, \theta_R$ may be not independent rotation angles in specific models \[24\].
\[ \kappa_t \quad y_T \quad \text{Re}(y_T^T) \quad \text{Im}(y_T^T) \]

\begin{tabular}{|c|c|c|c|c|}
\hline
SM & 1 & \times & \times & \times \\
VLQT & \frac{c_l^2}{s_L^2} - \frac{u_{L,cl}^2}{s_L^2} \text{Re}(y_T^T) & \frac{m_T}{v} s_L^2 c_0 + \text{Re}(y_T^T) c_L c_R s_0 & -\frac{m_T}{v} s_L c_L c_0 - \text{Re}(y_T^T) c_L c_R s_0 & -\frac{m_T}{v} s_L c_L c_0 - \text{Re}(y_T^T) s_L c_R s_0 \\
VLQT + S & \kappa_t \times y_T & \text{Im}(y_T^T) & \text{Im}(y_T^T) \\
SM & 0 & \times & \times & \times \\
VLQT & 0 & 0 & 0 & 0 \\
VLQT + S & \frac{c_l^2}{s_L^2} \text{Im}(y_T^T) s_L c_R s_0 & \text{Im}(y_T^T) c_L c_R s_0 & \text{Im}(y_T^T) c_L c_R s_0 & \text{Im}(y_T^T) s_L c_R s_0 \\
\hline
\end{tabular}

TABLE I. Patterns of Yukawa coefficients in SM, VLQT and VLQT+ S. There is no T quark in SM, so we use the symbol \( \times \) for \( T \) couplings.

\[ \kappa_t \quad y_T \quad \text{Re}(y_T^T) \quad \text{Im}(y_T^T) \]

\begin{tabular}{|c|c|c|c|c|}
\hline
SM & 1 & \times & \times & \times \\
VLQT & \frac{c_l^2}{s_L^2} - \frac{u_{L,cl}^2}{s_L^2} \text{Re}(y_T^T) & \frac{m_T}{v} s_L^2 c_0 + \text{Re}(y_T^T) c_L c_R s_0 & -\frac{m_T}{v} s_L c_L c_0 - \text{Re}(y_T^T) c_L c_R s_0 & -\frac{m_T}{v} s_L c_L c_0 - \text{Re}(y_T^T) s_L c_R s_0 \\
VLQT + S & \kappa_t \times y_T & \text{Im}(y_T^T) c_L c_R s_0 & \text{Im}(y_T^T) c_L c_R s_0 & \text{Im}(y_T^T) c_L c_R s_0 \\
SM & 0 & \times & \times & \times \\
VLQT & 0 & 0 & 0 & 0 \\
VLQT + S & \frac{c_l^2}{s_L^2} \text{Im}(y_T^T) s_L c_R s_0 & \text{Im}(y_T^T) c_L c_R s_0 & \text{Im}(y_T^T) c_L c_R s_0 & \text{Im}(y_T^T) s_L c_R s_0 \\
\hline
\end{tabular}

TABLE II. Patterns of Yukawa coefficients in SM, VLQT and VLQT+ S. There is no T quark in SM, so we use the symbol \( \times \) for \( T \) couplings.

\[ \kappa_t \quad y_T \quad \text{Re}(y_T^T) \quad \text{Im}(y_T^T) \]

\begin{tabular}{|c|c|c|c|c|}
\hline
general & \text{Re}(y_T^T) \quad \text{Im}(y_T^T) \quad \text{Re}(y_T^T) \quad \text{Im}(y_T^T) \quad \text{Re}(y_T^T) \\
& \frac{1}{s_W c_W} \left( \frac{1}{2} - I_3^T \right) s_L c_L (y_T^T)^* & \frac{1}{s_W c_W} \left( \frac{1}{2} - I_3^T \right) c_L c_R (y_T^T)^* & \frac{1}{s_W c_W} \left( \frac{1}{2} - I_3^T \right) s_L c_R (y_T^T)^* & \frac{1}{s_W c_W} \left( \frac{1}{2} - I_3^T \right) c_L c_R (y_T^T)^* \\
VLQT & -\frac{m_T}{v} s_L c_L & 0 & \frac{m_T}{v} s_L c_L & 0 \\
VLQT + S & \frac{-m_T}{v} s_L c_L - \frac{u_{L,cl}^2}{s_L^2} \text{Re}(y_T^T) & 0 & \frac{-m_T}{v} s_L c_L - \frac{u_{L,cl}^2}{s_L^2} \text{Re}(y_T^T) & 0 \\
\hline
\end{tabular}

TABLE III. Patterns of the multiplication of FCN couplings in VLQT and VLQT+ S.

III. CONSTRAINTS ON THE MODEL

III.1. Perturbative unitarity bound

From theoretical point of view, large couplings may cause the problem of perturbative unitarity violation. One famous example is the upper limit of Higgs self-coupling (or the Higgs mass) in the SM [20]. For the scattering amplitude, we can perform the partial wave expansion: \( M = 16\pi \sum_{l=0}^{\infty} (2l + 1) a_l(s) P_l(\cos \theta) \). Then the partial l-wave amplitude is \( a_l(s) = \frac{1}{2\pi} \int_{-1}^{1} d(\cos \theta) P_l(\cos \theta) M \). Espe-
cially, we have \( a_0(s) = \frac{1}{2\pi} \int_{-1}^{1} d(\cos \theta) M \), which should satisfy \( |\text{Re}(a_0)| \leq \frac{1}{2} \). When you consider the 2-to-2 Higgs and longitudinally polarized vector boson scattering processes, S-wave unitarity will lead to the bound.

Similarly, we can bound the Yukawa couplings \( y_{T,L,R}^{T} \) from fermion scattering \(^{37, 38}\). Then we need to consider the 2-to-2 scattering processes with fermions. Obviously, there are two kinds of fermion processes: two-fermion and four-fermion processes. Actually, we only need to consider the neutral initial and final states. In order to simplify the analysis, we keep the \( y_{L}^{T}, y_{R}^{T} \) couplings but turn off the other couplings. After tedious computations, we get the following constraints (more details are given in App. A):

\[
\sqrt{(|y_{L}^{T}|^2 + |y_{R}^{T}|^2)^2 + 12|y_{L}^{T}|^2|y_{R}^{T}|^2} + |y_{L}^{T}|^2 + |y_{R}^{T}|^2 \leq 16\pi.
\]

In Fig. 1, we plot the parameter space region allowed by Eq. (14).

![FIG. 1. The allowed region from perturbative unitarity in the plane of \(|y_{L}^{T}| - |y_{R}^{T}|\).](image)

III.2. Constraints from direct search

In the minimal extensions, the decay final states of \( T \) are \( bW^+, tZ, th \). According to the Goldstone boson equivalence theorem, the partial decay widths satisfy the identity \( \Gamma(T \rightarrow tZ) \approx \Gamma(T \rightarrow th) \approx \frac{1}{2} \Gamma(T \rightarrow bW) \) approximately (or \( \text{Br}(T \rightarrow tZ) \approx \text{Br}(T \rightarrow th) \approx 25\%, \text{Br}(T \rightarrow bW) \approx 50\% \)). For the pair production of VLQ, the cross section is determined by the strong interaction. It will give us the model independent bound on the \( T \) quark mass, but we can’t get the information of \( T \) quark couplings. Assuming \( \text{Br}(T \rightarrow tZ) + \text{Br}(T \rightarrow th) + \text{Br}(T \rightarrow bW) = 1 \), the \( T \) quark mass below 700GeV~TeV is excluded at 95% confidence level (CL) \(^{39, 40}\). The \( T \) quark can also be singly produced through \( TbW \) coupling. In the singlet \( T \) quark case, current constraints are \(|s_{L}| \leq 0.2\)\(^{41}\).

Current experiments give strong constraints on minimal VLQ models, but it will be relaxed in models with additional states. The mass can be light as 400GeV if there exists additional state mediated decay channels \(^{27}\). For the more complicated flavour and scalar sectors, there can be more than one mixing angles. The mixing angle is allowed to be larger.
III.3. Constraints from electro-weak precision measurements

The singlet VLQ $T_L, T_R$ will contribute to the S,T parameters [42, 43]. The oblique corrections are mainly from the modification of SM gauge couplings and new particle loops. Their analytical expressions have been calculated in the previous studies [19, 44, 45]:

$$\Delta S \equiv S - S^{SM}$$
$$= - \frac{N_C s_L^2}{18\pi} [-2 \log r_{IT} + c_L^2 \frac{5 - 22 r_{IT}^2 + 5 r_{IT}^4}{(1 - r_{IT}^2)^2} + c_L^2 \frac{6(1 + r_{IT}^2)(1 - 4 r_{IT}^2 + r_{IT}^4)}{(1 - r_{IT}^2)^3} \log r_{IT}],$$

$$\Delta T \equiv T - T^{SM}$$
$$= \frac{N_C m_t^2 s_L^2}{16\pi s_W^2 m_W^2} (-1 - c_L^2 + \frac{s_L^2}{r_{IT}} - \frac{4c_L^2}{1 - r_{IT}^2} \log r_{IT}),$$

(15)

with $r_{IT} \equiv \frac{m_t}{m_t}$. Now let’s define the $\Delta \chi^2$ as

$$\Delta \chi^2 \equiv \sum_{i,j=1,2} (O_i - O_i^{exp})(\sigma^2)_{ij}^{-1}(O_j - O_j^{exp}),$$

(16)

where $O_i \in \{\Delta S, \Delta T\}, (\sigma^2)_{ij} = \sigma_i \rho_{ij} \sigma_j$. Their values are listed as follows [46]:

$$\Delta S^{exp} = 0.02, \sigma_{\Delta S} = 0.07, \Delta T^{exp} = 0.06, \sigma_{\Delta T} = 0.06,$$

$$\rho = \begin{bmatrix} 1 & 0.92 \\ 0.92 & 1 \end{bmatrix}, \sigma^2 = \begin{bmatrix} \sigma_{\Delta S} & 0 \\ 0 & \sigma_{\Delta T} \end{bmatrix} \rho \begin{bmatrix} \sigma_{\Delta S} & 0 \\ 0 & \sigma_{\Delta T} \end{bmatrix}.$$ 

(17)

In this paper, we choose the parameters to be $m_Z = 91.1876\,\text{GeV}, m_W = 80.387\,\text{GeV}, m_h =$

![Image](image.png)

FIG. 2. The constraints on $m_T, s_L$ from $\chi^2$-fit of the $S,T$ parameters. Here the green and red areas are allowed at 1σ, 2σ CL, respectively.
125.09 GeV, $m_t = 172.74 \text{GeV}$, $G_F = 1.1664 \times 10^{-5} \text{GeV}^{-2}$ and $c_W = m_W/m_Z$. In Fig. 2 we get the constraints from the global fits of $S,T$ parameters. In Fig. 2, we get the constraints from the global fits of $S,T$ parameters.

### III.4. Constraints from top physics

There are also constraints from the $tbW$ anomalous coupling, which gives the bound $V_{tb} \geq 0.92$ at 95% CL assuming $V_{tb} \leq 1$. Then we have $s_L \leq \sqrt{1-V_{tb}} \approx 0.3$.

### III.5. Constraints from Higgs physics

In App. B, we give the exhaustive computations and analyses in both the SM and new physics model. When we take $\kappa_t = c_L^2, \tilde{\kappa}_t = 0, y_T = -m_t^2 s_L^2, \bar{y}_T = 0$ naively, the following expressions are obtained:

$$
\mu_{\gamma\gamma} \equiv \frac{\sigma(gg \to h)\Gamma(h \to \gamma\gamma)}{\sigma^{SM}(gg \to h)\Gamma^{SM}(h \to \gamma\gamma)} = \frac{\Gamma(h \to gg)\Gamma(h \to \gamma\gamma)}{\Gamma^{SM}(h \to gg)\Gamma^{SM}(h \to \gamma\gamma)} = |c_L^2 + s_L^2 F_T(\tau_T)|^2 \frac{|N_L^C Q_L^2 c_T^2 F_T(\tau_T) + s_L^2 F_T(\tau_T)| + F_W(\tau_W)|^2}{|N_L^C Q_L^2 F_T(\tau_T) + F_W(\tau_W)|^2}.
$$

In Fig. 3 we show the contour plot of $(\mu_{\gamma\gamma} - 1)$ in the parameter space of $m_T, s_L$. We find that the typical deviation $(\mu_{\gamma\gamma} - 1)$ is at the level of $-0.5\% \sim -5\%$, which is within the precision of current measurements. As with the results in [48], the constraints from Higgs signal strength are quite loose.

![Contour plot of $\mu_{\gamma\gamma} - 1$](image)

**FIG. 3.** The contour plot of the Higgs signal strength deviation for the $gg \to h \to \gamma\gamma$ channel in the $m_T - s_L$ plane.

5 $t-T$ mixing will also enter into $Zb\bar{b}$ coupling through one-loop correction, but here we won't consider them anymore.
III.6. Constraints from EDM

If there exists CP violation in the FCN interactions, it will contribute to the electron electric dipole moment (EDM). The neutron EDM and chromo EDM (CEDM) will also be affected. Then the imaginary parts of $y^{T}_{L,R}$ can be constrained. Here we also set $\tilde{\kappa}_{t} = 0, \tilde{y}_{T} = 0$ naively, that means CP violation is only from the FCNY interactions.

Firstly, the FCN couplings can alter the electron EDM through Barr-Zee diagrams at two-loop level [49] (see the left and middle diagrams of Fig. 4). Here the contributions originate from the $Z$ boson, because there are no FCN couplings for the photon. Due to the $C$ invariance, only vectorial couplings can contribute [49]. Now we can make a sketchy estimation. Compared to the photon diagram, $Z$ boson mediated Barr-Zee diagrams are suppressed by $\lambda^{2} \equiv g_{tT}^{2} + g_{tL}^{2} \frac{1-4s_{W}^{2}}{4s_{W}c_{W}}$. The CP violated $htt$ coupling has been bounded as $|\tilde{\kappa}_{t}| < 0.01$ from the electron EDM in [50]. From a naive analog, the constraints on FCNY couplings are typically $0.01/\lambda^{2} \sim O(1)$. But this argument is not persuasive, because the two-loop results are unknown for these FCN coupling mediated diagrams. Therefore, we need to resort to other methods.

Secondly, the FCN couplings can be constrained from top quark EDM and CEDM. The top quark EDM can be stringently constrained from $b \to s\gamma$ decay with the upper limit of $|d_{t}^{EDM}| < 1.9 \times 10^{-16} e \cdot cm$ or $|m_{t}d_{t}^{EDM}/e| < 1.7$ at 95% CL [51–53]. The severe constraint on top quark CEDM is inferred from the neutron EDM with the magnitude of $|d_{t}^{CEDM}| < 2.1 \times 10^{-3}$ at 90% CL [53]. In the right diagram of Fig. 4 we also show the Feynman diagram contributing to the top quark EDM. When the photon is replaced by a gluon, we can get the contribution to top CEDM. The related interactions induced at one-loop have the following form:

$$L \supset - \frac{i}{2} d_{t}^{EDM} \sigma^{\mu \nu} \gamma^{5} t F_{\mu \nu} - \frac{ig_{s}}{2} d_{t}^{CEDM} \sigma^{\mu \nu} a_{\mu} \gamma^{5} t G_{\mu \nu}. \quad (19)$$

With the expressions of

$$d_{t}^{EDM} = \frac{eQ_{T}m_{T}[y^{T}_{R}(y^{T}_{L})^{*} - y^{T}_{L}(y^{T}_{R})^{*}]}{16\pi^{2}} C_{1}, \quad d_{t}^{CEDM} = \frac{m_{T}[y^{T}_{R}(y^{T}_{L})^{*} - y^{T}_{L}(y^{T}_{R})^{*}]}{16\pi^{2}} C_{1}, \quad (20)$$

where $C_{1}$ is defined as

$$C_{1} = \frac{1}{4m_{t}^{2}}[B_{0}(m_{t}^{2}, m_{Q}^{2}, m_{W}^{2}) - B_{0}(0, m_{t}^{2}, m_{T}^{2}) + (m_{T}^{2} - m_{t}^{2} - m_{W}^{2})] C_{0}(m_{t}^{2}, 0, m_{t}^{2}, m_{T}^{2}, m_{T}^{2}).$$
\[ |y_R^T(y_L^T)^* - y_L^T(y_R^T)^*| \] can also be rewritten as \(2i(\text{Re}y_R^T \text{Im}y_R^T - \text{Re}y_L^T \text{Im}y_L^T)\), thus \(d_T^{EDM}, d_T^{CEDM}\) will vanish if the imaginary parts of \(y_{LR}^T\) are both turned off. If we take \(m_T = 700\text{GeV}\), top EDM and CEDM set the upper limits of \(|y_R^T(y_L^T)^* - y_L^T(y_R^T)^*|\) to be 6477 and 4.8, respectively. If we take \(m_T = 400\text{GeV}\), the corresponding upper limits of \(|y_R^T(y_L^T)^* - y_L^T(y_R^T)^*|\) are 3635 and 2.7, respectively. Thus top quark CEDM will give much stronger constraints than top EDM.

IV. PARTIAL DECAY WIDTH FORMULA OF \(h \to \gamma Z\)

IV.1. The SM result

\[
\Gamma^{SM}(h \to \gamma Z) = \frac{G_F \alpha^2 m_h^3}{64 \sqrt{2} \pi^3} (1 - \frac{m_Z^2}{m_h^2})^3 \sum_f (2N_f^f Q_f) \frac{f^4}{s_W c_W} A_f(\tau_f, \lambda_f) + A_W(\tau_W, \lambda_W)|^2. \tag{21}
\]

Here \(\tau_i\) and \(\lambda_i\) are defined as
\[
\tau_f = \frac{4m_f^2}{m_h^2}, \lambda_f = \frac{4m_f^2}{m_Z^2}, \tau_W = \frac{4m_W^2}{m_h^2}, \lambda_W = \frac{4m_W^2}{m_Z^2}. \tag{22}
\]

and the \(A_f, A_W\) are defined as
\[
A_f(\tau_f, \lambda_f) \equiv I_1(\tau_f, \lambda_f) - I_2(\tau_f, \lambda_f),
\]
\[
A_W(\tau_W, \lambda_W) \equiv I_1(\tau_W, \lambda_W) + 4(3 - \lambda_W^2)I_2(\tau_W, \lambda_W),
\]
\[
I_1(\tau, \lambda) = \frac{\tau \lambda}{2(\tau - \lambda)} + \frac{\tau^2 \lambda^2}{2(\tau - \lambda)^2}|f(\tau) - f(\lambda)| + \frac{\lambda^2}{(\tau - \lambda)^2}[g(\tau) - g(\lambda)],
\]
\[
I_2(\tau, \lambda) = -\frac{\tau \lambda}{2(\tau - \lambda)}[f(\tau) - f(\lambda)]. \tag{23}
\]

Here \(f(\tau)\) is given in App. E and \(g(\tau)\) is defined as
\[
g(\tau) \equiv \begin{cases} \sqrt{\tau - 1} \arcsin\left(\frac{1}{\sqrt{\tau}}\right), & \text{for } \tau \geq 1 \\ \frac{1}{2} \sqrt{1 - \tau} [\log \left(1 + \frac{1}{\sqrt{\tau}}\right) - i\pi], & \text{for } \tau < 1 \end{cases}. \tag{24}
\]
The fermionic part is dominated by top quark because of the largest Yukawa coupling. Numerically, we can get \((2N_t^CQ_t)\frac{\omega^2_{	ext{Higgs}}}{s_Wc_W}\) \(A_f(\tau_t, \lambda_t) \sim -0.65\), \(A_W(\tau_W, \lambda_W) \sim 12.03\), which means the gauge boson contributions are almost 18.5 times larger than the fermionic ones. It is obvious that the fermionic part and gauge boson part interfere destructively in the SM.

### IV.2. New physics result

\(h \rightarrow \gamma Z\) decay has already been considered in many models, for example, composite Higgs models \([60, 61]\), minimal supersymmetric standard model (MSSM) \([62, 63]\), next-to-minimal supersymmetric standard model (NMSSM) \([63, 64]\), extended scalar sector models \([14, 65, 66]\) and other new physics models \([67]\). In VLQ models, there are additional fermion contributions: pure new quark interactions. Such off-diagonal contributions are always ingored in most studies \([57, 60]\), because they are small compared to the diagonal terms. As a second thought, this channel can be sensitive to large non-diagonal couplings. Here we don’t enumerate models with more fermions, where the effects of non-diagonal couplings will be diluted or concealed. Besides, we only focus on the cases in which the scalar sector is extended with real gauge singlet scalars. In more complex scalar sector models, the charged Higgs contributions will also attenuate the flavour off-diagonal contributions.

\[
\text{FIG. 6. Possible new fermion contritutions to the } h \rightarrow \gamma Z \text{ decay. For the fermion loops, counter-clockwise diagrams should be included.}
\]

Now let’s consider the partial decay width of \(h \rightarrow \gamma Z\) with the general interactions in Eq. (12). Due to \(U_{EM}(1)\) gauge symmetry, the \(h \rightarrow \gamma Z\) amplitude possesses the following tensor structure:

\[
i\mathcal{M} = i\epsilon_{\mu}(p_1)\epsilon_{\nu}(p_2)\left[(p_2^\nu p_1^\mu - p_1^\nu p_2^\mu)A + \epsilon^{\mu\nu\rho\sigma}p_1^\rho p_2^\sigma\right]
\]

\[
A = \frac{\alpha^2}{8\pi^2v}(A_W + A_t + A_T + A_{\tau T}), \quad B = \frac{\alpha^2}{8\pi^2v}(B_t + B_T + B_{\tau t}).
\]

Where \(A_W, A_t, A_T, A_{\tau T}\) denote the contributions from \(W\) boson, top quark, \(T\) quark and \(t - T\) mixed loops, respectively. Their expressions are given as

\[
A_W = A_W(\tau_W, \lambda_W),
\]

\[
A_t = 2N_t^CQ_t(g_{L}^t + g_{R}^t)\kappa t A_f(\tau_t, \lambda_t) = 2N_t^CQ_t\kappa t \frac{1}{2}c_L^2 + I_3^T(s_L^2 + s_R^2) - \frac{4}{3}s_W^2 A_f(\tau_t, \lambda_t),
\]

\[
A_T = -2N_T^CQ_T \frac{g_{T}^T}{m_T} (g_{L}^T + g_{R}^T)A_f(\tau_T, \lambda_T) = -2N_T^CQ_T \frac{1}{2}c_L^2 + I_3^T(s_L^2 + s_R^2) - \frac{4}{3}s_W^2 A_f(\tau_T, \lambda_T),
\]

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\[ A_T = -4N_C^T Q_T \frac{v}{m_T^2 - m_Z^2} \{m_t \text{Re}(g_L^{T*}(y_L^T) + g_R^{T*}(y_R^T))[(\frac{m_Z^2 - m_h^2}{2}) - m_t^2]C_0(0, m_Z^2, m_h^2, m_t^2, m_t^2) \]
\[ - m_t^2 C_0(0, m_Z^2, m_h^2, m_t^2, m_t^2, m_{t'}^2) - m_Z^2 B_0(m_t^2, m_{t'}^2, m_t^2) - B_0(m_t^2, m_{t'}^2, m_t^2) - 1] \]
\[ + m_T \text{Re}(g_L^{T*}(y_L^T) + g_R^{T*}(y_R^T))[(\frac{m_Z^2 - m_h^2}{2}) - m_t^2]C_0(0, m_Z^2, m_h^2, m_t^2, m_t^2, m_{t'}^2) \]
\[ - m_t^2 C_0(0, m_Z^2, m_h^2, m_t^2, m_t^2, m_{t'}^2) - m_Z^2 B_0(m_t^2, m_{t'}^2, m_t^2) - B_0(m_t^2, m_{t'}^2, m_t^2) - 1]. \] (26)

Similarly, the expressions of \( B_t, B_T, B_{TT} \) are given as

\[ B_t = -2N_C^T Q_t(g_L^{T*} + g_R^{T*}) \bar{C}_t m_t^2 C_0(0, m_Z^2, m_h^2, m_t^2, m_t^2), \]
\[ B_T = 2N_C^T Q_T(g_L^{T*} + g_R^{T*}) \bar{y}_{tT} m_{tT} C_0(0, m_Z^2, m_h^2, m_t^2, m_t^2), \]
\[ B_{TT} = -2N_C^T Q_T \{m_t \text{Im}(g_R^{T*}(y_R^T) + g_L^{T*}(y_L^T))C_0(0, m_Z^2, m_h^2, m_t^2, m_t^2, m_{t'}^2) \]
\[ - m_T \text{Im}(g_R^{T*}(y_R^T) + g_L^{T*}(y_L^T))C_0(0, m_Z^2, m_h^2, m_t^2, m_t^2, m_{t'}^2) \}. \] (27)

Taking the mass of \( t, T \) quarks to be infinity, \( A_t, A_T \) can be expanded as

\[ A_t \approx -2N_C^T Q_t \frac{1}{2} c_L^2 + I_3^L (s_L^2 + s_R^2) - \frac{4}{3} s_W^2 \left[1 + \frac{7m_t^2 + 11m_h^2}{120m_t^2} \right] + \mathcal{O}(\frac{m_t^2}{m_t^2}), \]
\[ A_T \approx -2N_C^T Q_T \left( -\frac{m_T^2}{2} \right) \left( \frac{1}{2} s_L^2 + I_3^L (c_L^2 + c_R^2) - \frac{4}{3} s_W^2 \right) \left[1 + \frac{7m_t^2 + 11m_h^2}{120m_t^2} \right] + \mathcal{O}(\frac{m_t^2}{m_t^2}). \]
\[ A_t + A_T \approx -2N_C^T Q_T \left[ \frac{1}{2} c_L^2 + I_3^L (s_L^2 + s_R^2) - \frac{4}{3} s_W^2 \right] \left[1 + \frac{7m_t^2 + 11m_h^2}{120m_t^2} \right] \]
\[ + \left( \frac{m_T^2}{m_T^2} \right) \left[ \frac{1}{2} s_L^2 + I_3^L (c_L^2 + c_R^2) - \frac{4}{3} s_W^2 \right] \left[1 + \frac{7m_t^2 + 11m_h^2}{120m_t^2} \right]. \] (28)

For the \( \frac{1}{m_t^2} \) suppressed contributions, we can get \( \frac{7m_t^2 + 11m_h^2}{120m_t^2} \approx 5.5\%, \frac{7m_t^2 + 11m_h^2}{120m_t^2} \approx 1\% \) if \( m_T \gg 400GeV \).

The expansion of \( A_{TT} \) is a little bit complicated:

\[ A_{TT} \approx -4N_C^T Q_T \frac{v}{m_T^2 - m_Z^2} \{m_t \text{Re}(g_L^{T*}(y_L^T) + g_R^{T*}(y_R^T))[(\frac{m_Z^2 - m_h^2}{2}) - m_t^2]C_0(0, m_Z^2, m_h^2, m_t^2, m_t^2) \]
\[ + \mathcal{O}(\frac{m_t^2}{m_t^2}, \frac{m_t^2}{m_Z^2}, \frac{m_z^2}{m_T^2}, \frac{m_T^2}{m_T^2}) \}
\[ + m_T \text{Re}(g_L^{T*}(y_L^T) + g_R^{T*}(y_R^T))[(\frac{m_Z^2 - m_h^2}{2}) - m_t^2]C_0(0, m_Z^2, m_h^2, m_t^2, m_t^2, m_{t'}^2) \]
\[ - m_t^2 C_0(0, m_Z^2, m_h^2, m_t^2, m_t^2, m_{t'}^2) - m_Z^2 B_0(m_t^2, m_{t'}^2, m_t^2) - B_0(m_t^2, m_{t'}^2, m_t^2) - 1]. \] (29)
Similarly, we can expand $B_t, B_T, B_{1T}$ as follows:

$$B_t \approx N_C^C Q_s (g_L^4 + g_R^4), \quad B_T \approx -N_C^C Q_T (g_L^T + g_R^T), \quad B_{1T} \approx -N_C^C Q_{1T} (g_L^{1T} + g_R^{1T}).$$

In Tab. [IV] we list the expressions of $A_t + A_T, A_{1T}, B_t + B_T, B_{1T}$ in three models, where we have neglected the $1/m_{t,T}^2$ suppressed terms but keep the $\log r_{1T}^2$ enhanced terms.

| Model | $\tilde{A}_t + \tilde{A}_T$ | $\tilde{A}_{1T}$ |
|-------|-----------------|-----------------|
| SM    | $1 - \frac{5}{3} s_W^2$ | $x$ |
| VLQT  | $1 - \frac{5}{3} s_W^2 - 2 L T c_L^2$ | $\frac{1}{2} s_L c_L (1 - r_{1T}^2 (3 + 2 \log r_{1T}^2))$ |
| VLQT+S | $\phi (1 - \frac{5}{3} s_W^2 - 2 L T c_L^2) + \frac{c_R (g_L^2)^2}{m_T} (\frac{5}{4} s_W^2 - 2 L T c_L^2) L T c_L (\frac{1}{2} s_L c_L (1 - r_{1T}^2 (3 + 2 \log r_{1T}^2)) + \frac{c_R (g_L^2)^2}{m_T} (s_L c_R - r_{1T}^2 (3 + 2 \log r_{1T}^2)) s_R c_L)$ |

TABLE IV. The expressions of $\tilde{A}_t + \tilde{A}_T, \tilde{A}_{1T}, \tilde{B}_t + \tilde{B}_T, \tilde{B}_{1T}$ in the SM, VLQT and VLQT+S. Here we extract the common factor $-\frac{N_C^C Q_T}{3 s_W c_W}$ for convenience, that is redefinition of $A(B)$ with $-\frac{N_C^C Q_T}{3 s_W c_W} \tilde{A}(\tilde{B})$. We take $\tilde{A}_T = \tilde{B}_T = 0$ naively in SM because of the absence of $T$ quark.

The partial decay width formula is computed as

$$\Gamma(h \to \gamma Z) = \frac{G_F m_h^3}{64 \sqrt{2} \pi^3} \left[ (1 - \frac{m_Z^2}{m_h^2})^3 |A_t + A_T + A_{1T} + A_{1T}|^2 + |B_t + B_T + B_{1T}|^2 \right].$$

IV.3. Comments from the viewpoint of low energy theorem

As a matter of fact, we can also understand some behaviors of the $h \to \gamma Z$ amplitude resorting to the low energy theorem [70, 71]. Just as the calculation of $h \to \gamma \gamma$ amplitude from photon self-energy contribution [72], we may get the $h \to \gamma Z$ amplitude through $\gamma - Z$ mixed self-energy contribution [73]. But what confuses us is that there seems no off-diagonal fermion contributions to the $\gamma - Z$ two-point function because photon can only couple to the same flavour particle. The reason is that off-diagonal couplings are proportional to the mixing angle, which is suppressed by the heavy fermion mass. Thus off-diagonal contributions to the $h \to \gamma Z$ amplitude vanish in the limit of $p_h \to 0$, consistent with the corollary of low energy theorem. In other words, this channel will give looser constraints on off-diagonal couplings once one flavour of the loop particles becomes heavier.
V. NUMERICAL RESULTS AND CONSTRAINT PROSPECTS

Just similar to the VLQT model, we take \( \tan \theta_R = \frac{m_T}{m_T} \tan \theta_L, \kappa_t = c_L^2, \bar{\kappa}_t = 0, y_T = -\frac{m_T}{v} s_L^2, \bar{y}_T = 0 \) for simplicity, but let \( \text{Re}(y_L^{(T)}), \text{Re}(y_R^{(T)}), \text{Im}(y_L^{(T)}), \text{Im}(y_R^{(T)}) \) to be free. Then we can choose several benchmark scenarios and estimate the constraints on the magnitude and sign of the FCNY couplings.

Since the branching ratio of \( h \rightarrow \gamma Z \) is about \( 1.5 \times 10^{-3} \), the modification of \( h\gamma Z \) partial decay width will cause negligible effects on the Higgs total width. At the high luminosity LHC (HL-LHC), \( h\gamma Z \) coupling can be measured accurately \([74, 75]\). The expected width will cause negligible effects on the Higgs total width. At the high luminosity LHC (HL-LHC),

\[ \frac{|\Gamma(h \rightarrow \gamma Z)/\Gamma^{SM}(h \rightarrow \gamma Z) - 1|}{19.1\%} \leq 19.1\%. \] (32)

It means \(|(|A|^2 + |B|^2)/|A^{SM}|^2 - 1| \leq 19.1\%\). From now on, we will choose \( m_T = 400 \text{GeV} \) and \( s_L = 0.2 \). As mentioned above, there are four interesting parameters \( \text{Re}(y_L^{(T)}), \text{Re}(y_R^{(T)}), \text{Im}(y_L^{(T)}), \text{Im}(y_R^{(T)}) \). In the following, we will plot the allowed two-dimensional parameter space by setting two of them to be zeros or imposing two conditions.

In Fig.6 we plot the parameter space regions allowed by perturbative unitarity in Eq. (14) and the expected constraints at HL-LHC in Eq. (32) in different scenarios. When evaluating the scalar loop functions, LoopTools is employed \([76]\). In the first plot, we can find that \( h \rightarrow \gamma Z \) decay gives a little stronger constraints than perturbative unitarity in the first and third quadrants in the case of vanishing imaginary parts of \( y_L^{(T)}, y_R^{(T)} \). In the presence of imaginary part, the real parts can be constrained to be less than 3 roughly in the positive direction, while it will give looser bound than the unitary constraints in the negative direction. In the case of vanishing real parts of \( y_L^{(T)}, y_R^{(T)} \), the imaginary parts can only be constrained by unitarity. When the couplings are pure left or pure right, the real parts are also constrained to be less than 3 roughly in the positive direction. For the cases of equal or conjugate \( y_L^{(T)}, y_R^{(T)} \), the real parts can be bounded to be less than 1.5 in the positive direction and greater than -3 in the negative direction.

As a matter of fact, the behaviours in Fig.7 can be explained by the results in Sec. IV.2 qualitatively. In \(|A_t + A_R + A_{TR} + A_{WR}(\tau_W, \lambda_W)|^2, A_{TR} \) can interfere constructively or destructively with \( A_W(\tau_W, \lambda_W) \), while \( |B_{TR}|^2 \) always enhances the partial width. It will give strong constraints for the constructive case because of double enhancement from \( A_{TR}, B_{TR} \). \( A_{TR} \) is proportional to real parts of \( y_{L,R}^{(T)} \), while \( B_{TR} \) receives the contribution from the imaginary parts of \( y_{L,R}^{(T)} \). Thus real parts of \( y_{L,R}^{(T)} \) are more tightly constrained than the imaginary parts because of the interference with the large \( A_W(\tau_W, \lambda_W) \) term. If \( A_{TR} > 0 \) (or \(|m_T \text{Re}(y_R^{(T)}) - (3 + 2 \log r_{TT}^2) \text{Im}(y_L^{(T)})| > 0 \)), it will interfere constructively with \( A_W(\tau_W, \lambda_W) \). The appearance of \( B_{TR} \) will enhance the partial width further, thus this case is more strongly bounded. If \( A_{TR} < 0 \), there will be some cancellation between the destructive interference with \( A_W(\tau_W, \lambda_W) \) and the enhancement from \( B_{TR} \). Thus this case is more loosely bounded.

Although \( m_T g_L^{(T)}(y_L^{(T)})^* \) term is suppressed by the factor \( \frac{m_T}{m_T} \) compared to \( m_T g_R^{(T)}(y_R^{(T)})^* \), it is \( \log r_{TT}^2 \) enhanced. We should take both of them into account. Because of \( A_{TR} \sim |m_T \text{Re}(y_R^{(T)}) - (3 + 2 \log r_{TT}^2) \text{Im}(y_L^{(T)})| \), the regions of \( \text{Re}(y_L^{(T)}), \text{Re}(y_R^{(T)}) \) with same sign are more strongly bounded than those with opposite sign. Because of \( B_{TR} \sim |m_t(1 + \log r_{TT}^2) \text{Im}(y_L^{(T)}) + m_T \text{Im}(y_R^{(T)})| \), the regions of \( \text{Im}(y_L^{(T)}), \text{Im}(y_R^{(T)}) \) with opposite sign are more strongly bounded than those with same sign (compare \( y_L^{(T)} = y_R^{(T)} \) case with the \( y_L^{(T)} = (y_R^{(T)})^* \) case in Fig.7).

Although the attempts show that the constraints are quite loose, it is still worth investigating the FCNY couplings through the \( h \rightarrow \gamma Z \) decay mode. The contributions of FCN couplings are suppressed by both \( s_L \) and \( \frac{m_T}{m_T} \). If \( s_L \) is not very small, it can give considerable constraints on the
FIG. 7. The allowed regions of $y_{LT}^T, y_{RT}^T$ in several scenarios. In the above plots, we take $\text{Im}(y_{LT}^T) = \text{Im}(y_{RT}^T) = 0$ (upper left), $\text{Im}(y_{LT}^T) = \text{Re}(y_{RT}^T) = 0$ (upper central), $\text{Re}(y_{LT}^T) = \text{Im}(y_{RT}^T) = 0$ (upper right), $\text{Re}(y_{LT}^T) = \text{Re}(y_{RT}^T) = 0$ (middle left), $y_{RT}^T = 0$ (middle central), $y_{LT}^T = 0$ (middle right), $y_{LT}^T = y_{RT}^T$ (lower left) and $y_{LT}^T = (y_{RT}^T)^*$ (lower right), respectively.

FCNY couplings. When $s_L$ becomes very small (say $s_L = 0.1$), $h \rightarrow \gamma Z$ decay will lose the power to constrain FCNY couplings (looser than the perturbative unitarity bound). When $m_T$ becomes very heavy (say TeV), it will also lose the power to constrain FCNY couplings.

In Sec. III.6 we have illustrated that the top quark CEDM may give some bounds on the FCNY
Because we have the identity $y^T_R(y^T_L)^* - y^T_L(y^T_R)^* = 2i(\text{Re}y^T_R\text{Im}y^T_L - \text{Re}y^T_L\text{Im}y^T_R)$, the blind directions from top CEDM are $y^T_R = 0, y^T_L = y^*_R, \text{Im}y^T_R = \text{Im}y^T_L = 0, \text{Re}y^T_R = \text{Re}y^T_L = 0$. For the three cases $\text{Im}(y^T_L) = \text{Re}(y^T_R) = 0, \text{Re}(y^T_L) = \text{Im}(y^T_R) = 0, y^T_L = (y^T_R)^*$, the top quark CEDM can give strong constraints. In Fig. 8, we plot the parameter space regions allowed by perturbative unitarity in Eq. (14), the expected constraints at HL-LHC in Eq. (32) and the top quark CEDM in these three scenarios. From these plots, we can find that the off-axis regions are strongly bounded by top CEDM. While it loses the constraining power in the near axis regions.

By the way, $h \rightarrow \gamma\gamma$ depends only on the same flavour Yukawa couplings, while $h \rightarrow \gamma\gamma$ decay is also controlled by the FCN couplings. By combing $h \rightarrow \gamma\gamma, \gamma Z$ together, it is possible to disentangle the FCNY couplings from the same flavour Yukawa couplings. Certainly, the FCN couplings can show up in other processes too. For example, we can search for new physics through the di-Higgs production $e^+e^- \rightarrow hh$, while the $gg \rightarrow hh$ process suffers from the anomalous $hhh$ coupling. The $e^+e^- \rightarrow h\gamma$ production at electron-positron colliders is also an interesting process and it has drawn much attention of the community. It can also be a probe of the anomalous $h\gamma Z$ and $h\gamma\gamma$ couplings. The SM analysis for this process is given in [80] [82]. There are also some works on this process in many new physics models, for example MSSM [83] [86], extended scalar sector models [87] [88], VLQ models [88], EFT framework [89] [91] and simplified scenarios [92]. Besides, we can also probe the FCNY couplings through direct production processes $pp \rightarrow Tth, Tt, ThW, ThJ$. But they suffer from low event rate. Although the FCNY couplings may also be constrained from other processes, the detailed analyses in these channels are beyond the scope of this work.

VI. SUMMARY AND CONCLUSIONS

There can exist FCN interactions between the top quark and new heavy quark. In order to unravel the nature of flavour structure and EWSB, it is of great importance to probe such couplings. Unfortunately, it is difficult to constrain the FCN couplings at both current and future experiments. Here we show how to bound the FCNY couplings in simplified singlet $T_L, T_R$ extended models generally.

In this paper, we have summarized the main constraints from theoretical and experimental view-
points. By turning off other couplings naively, we get the perturbative unitarity bounds on $|y_{L,R}^{T}|$. After considering the constraints from direct search, $S,T$ parameters, top physics and Higgs signal strength, we take $m_T=400\text{GeV}$ and $s_L = 0.2$ as the benchmark point to get the optimal situation. Under this benchmark point, we consider the future bounds from $h \to \gamma Z$ decay at HL-LHC numerically. The real parts of $y_{L,R}^{T}$ in the positive direction can be limited to be less than $1.5 \sim 3$ because of the double enhancement. For the real parts of $y_{L,R}^{T}$ in the negative direction, they are mainly bounded by the perturbative unitarity. Finally, we find that top quark CEDM can give more strong bounds (especially the imaginary parts of $y_{L,R}^{T}$) than the perturbative unitarity and $h \to \gamma Z$ decay in the off-axis regions for some scenarios.

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[1] S. L. Glashow. Partial Symmetries of Weak Interactions. *Nucl. Phys.*, 22:579–588, 1961.
Steven Weinberg. A Model of Leptons. *Phys. Rev. Lett.*, 19:1264–1266, 1967.
Abdus Salam. Weak and Electromagnetic Interactions. *Conf. Proc.*, C680519:367–377, 1968.

[2] Kaustubh Agashe, Roberto Contino, and Alex Pomarol. The Minimal composite Higgs model. *Nucl. Phys.*, B719:165–187, 2005.

[3] Giuliano Panico and Andrea Wulzer. The Composite Nambu-Goldstone Higgs. *Lect. Notes Phys.*, 913:pp.1–316, 2016.

[4] N. Arkani-Hamed, A. G. Cohen, E. Katz, and A. E. Nelson. The Littlest Higgs. *JHEP*, 07:034, 2002.

[5] Martin Schmaltz and David Tucker-Smith. Little Higgs review. *Ann. Rev. Nucl. Part. Sci.*, 55:229–270, 2005.

[6] JoAnne L. Hewett and Thomas G. Rizzo. Low-Energy Phenomenology of Superstring Inspired E(6) Models. *Phys. Rept.*, 183:193, 1989.

[7] Roberto Contino, Thomas Kramer, Minho Son, and Raman Sundrum. Warped/composite phenomenology simplified. *JHEP*, 05:074, 2007.

[8] Georges Aad et al. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. *Phys. Lett.*, B716:1–29, 2012.

[9] F. Englert and R. Brout. Broken Symmetry and the Mass of Gauge Vector Mesons. *Phys. Rev. Lett.*, 13:321–323, 1964.

[10] S. Dittmaier et al. Handbook of LHC Higgs Cross Sections: 1. Inclusive Observables. 2011.
S. Dittmaier et al. Handbook of LHC Higgs Cross Sections: 2. Differential Distributions. 2012.
J R Andersen et al. Handbook of LHC Higgs Cross Sections: 3. Higgs Properties. 2013.
D. de Florian et al. Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector. 2016.

[11] Yi Chen, Adam Falkowski, Ian Low, and Roberto Vega-Morales. New Observables for CP Violation in Higgs Decays. *Phys. Rev.*, D90(11):113006, 2014.
[12] Marco Farina, Yuval Grossman, and Dean J. Robinson. Probing CP violation in $h \to Z\gamma$ with background interference. *Phys. Rev.*, D92(7):073007, 2015.

[13] Xuan Chen, Gang Li, and Xia Wan. Probe CP violation in $H \to \gamma Z$ through forward-backward asymmetry. *Phys. Rev.*, D96(5):055023, 2017.

[14] Chian-Shu Chen, Chao-Qiang Geng, Da Huang, and Lu-Hsing Tsai. New Scalar Contributions to $h \to Z\gamma$. *Phys. Rev.*, D95(7):075019, 2017.

[15] Jose Miguel No and Michael Spannowsky. A Boost to $h \to Z\gamma$: from LHC to Future $e^+e^-$ Colliders. *Phys. Rev.*, D95(7):075027, 2017.

[16] Sally Dawson and Pier Paolo Giardino. Higgs decays to $ZZ$ and $Z\gamma$ in the standard model effective field theory: An NLO analysis. *Phys. Rev.*, D97(9):093003, 2018.

[17] F. del Aguila, M. Perez-Victoria, and Jose Santiago. Observable contributions of new exotic quarks to quark mixing. *JHEP*, 09:011, 2000.

[18] F. del Aguila, M. Perez-Victoria, and Jose Santiago. Effects of mixing with quark singlets. *Phys. Rev.*, D67:035003, 2003. [Erratum: *Phys. Rev.* D69,099901(2004)].

[19] J. A. Aguilar-Saavedra. Effects of mixing with quark singlets. *Phys. Rev.*, D86:015021, 2012.

[20] Chien-Yi Chen, S. Dawson, and I. M. Lewis. Exploring resonant di-Higgs boson production in the Higgs singlet model. *Phys. Rev.*, D91(3):035015, 2015.

[21] Shinya Kanemura, Mariko Kikuchi, and Kei Yagyu. Radiative corrections to the Higgs boson couplings in the model with an additional real singlet scalar field. *Nucl. Phys.*, B907:286–322, 2016.

[22] Shi-Ping He and Shou-hua Zhu. One-Loop Radiative Correction to the Triple Higgs Coupling in the Higgs Singlet Model. *Phys. Lett.*, B764:31–37, 2017.

[23] Mathieu Buchkremer, Giacomo Cacciapaglia, Aldo Deandrea, and Luca Panizzi. Model Independent Framework for Searches of Top Partners. *Nucl. Phys.*, B876:376–417, 2013.
Morad Aaboud et al. Combination of the searches for pair-produced vector-like partners of the third-generation quarks at \( \sqrt{s} = 13 \) TeV with the ATLAS detector. *Phys. Rev. Lett.*, 121(21):211801, 2018.

Albert M Sirunyan et al. Search for pair production of vectorlike quarks in the fully hadronic final state. *Phys. Rev. Lett.*, 100(7):072001, 2018.

Morad Aaboud et al. Search for single production of vector-like quarks decaying into \( Wb \) in \( pp \) collisions at \( \sqrt{s} = 13 \) TeV with the ATLAS detector. *JHEP*, 05:164, 2019.

Michael E. Peskin and Tatsu Takeuchi. A New constraint on a strongly interacting Higgs sector. *Phys. Rev. Lett.*, 65:964–967, 1990.

Michael E. Peskin and Tatsu Takeuchi. Estimation of oblique electroweak corrections. *Phys. Rev.*, D46:381–409, 1992.
21

[65] Cheng-Wei Chiang and Kei Yagyu. Higgs boson decays to $\gamma\gamma$ and $Z\gamma$ in models with Higgs extensions. *Phys. Rev.*, D87(3):033003, 2013.

[66] Bogumiła Świeżewska and Maria Krawczyk. Diphoton rate in the inert doublet model with a 125 GeV Higgs boson. *Phys. Rev.*, D88(3):035019, 2013.

[67] Chian-Shu Chen, Chao-Qiang Geng, Da Huang, and Lu-Hsing Tsai. $h \to Z\gamma$ in Type-II seesaw neutrino model. *Phys. Lett.*, B723:156–160, 2013.

[68] R. Mertig, M. Bohm, and Ansgar Denner. FEYN CALC: Computer algebraic calculation of Feynman amplitudes. *Comput. Phys. Commun.*, 64:345–359, 1991.

[69] Vladyslav Shtabovenko, Rolf Mertig, and Frederik Orellana. New Developments in FeynCalc 9.0. *Comput. Phys. Commun.*, 207:432–444, 2016.

[70] John R. Ellis, Mary K. Gaillard, and Dimitri V. Nanopoulos. A Phenomenological Profile of the Higgs Boson. *Nucl. Phys.*, B106:292, 1976.

[71] Mikhail A. Shifman, A. I. Vainshtein, M. B. Voloshin, and Valentin I. Zakharov. Low-Energy Theorems for Higgs Boson Couplings to Photons. *Sov. J. Nucl. Phys.*, 30:711–716, 1979. [Yad. Fiz.30,1368(1979)].

[72] Marcela Carena, Ian Low, and Carlos E. M. Wagner. Implications of a Modified Higgs to Diphoton Decay Width. *JHEP*, 08:060, 2012.

[73] Bernd A. Kniehl and Michael Spira. Low-energy theorems in Higgs physics. *Z. Phys.*, C69:77–88, 1995.

[74] M. Cepeda et al. Higgs Physics at the HL-LHC and HE-LHC. 2019.

[75] Florian Goertz, Eric Madge, Pedro Schwaller, and Valentin Titus Tenorth. Discovering the $h \to Z\gamma$ Decay in $t\bar{t}$ Associated Production. 2019.

[76] T. Hahn and M. Perez-Victoria. Automatized one loop calculations in four-dimensions and D-dimensions. *Comput. Phys. Commun.*, 118:153–165, 1999.

[77] T. Plehn, M. Spira, and P. M. Zerwas. Pair production of neutral Higgs particles in gluon-gluon collisions. *Nucl. Phys.*, B479:46–64, 1996. [Erratum: Nucl. Phys.B531,655(1998)].

[78] Chih-Ting Lu, Jung Chang, Kingman Cheung, and Jae Sik Lee. An exploratory study of Higgs-boson pair production. *JHEP*, 08:133, 2015.

[79] Shinya Kanemura, Kentarou Mawatari, and Kodai Sakurai. Single Higgs production in association with a photon at electron-positron colliders. *Phys. Rev.*, D99(3):035023, 2019.

[80] Daruosh Haji Raissi, Seddigheh Tizchang, and Mojtaba Mohammadi Najafabadi. Loop induced singlet scalar production through the vector like top quark at future lepton colliders. 2018.

[81] G. J. Gounaris, F. M. Renard, and N. D. Vlachos. Tests of anomalous Higgs boson couplings through $e^-e^+\to HZ$ and $H\gamma$. *Nucl. Phys.*, B459:51–74, 1996.

[82] Hong-Yu Ren. New Physics Searches with Higgs-photon associated production at the Higgs Factory. *Chin. Phys.*, C39(11):113101, 2015.
APPENDIX

A. PERTURBATIVE UNITARITY ANALYSIS

A.1. Two-fermion process analysis

For the two-fermion process, we take the $t\bar{t} \rightarrow hh$ process as an example. In Fig. 9, we give the Feynman diagrams for the $t\bar{t} \rightarrow hh$ scattering process.

Feynman diagrams\(^7\) The amplitude with general helicity can be written as

\[
i\mathcal{M}^{\alpha\beta}(t\bar{t} \rightarrow hh) = \frac{1}{p_1 - \vec{k}_1 - m_T} \left[ \frac{1}{p_1 - \vec{k}_2 - m_T} \left[ \frac{1}{y_{L}^{T} \omega_{-} + y_{R}^{T} \omega_{+}} \right] + \frac{1}{y_{L}^{T} (y_{R}^{T})^{*} \omega_{-} + y_{R}^{T} (y_{L}^{T})^{*} \omega_{+}} \right] u^{\alpha}(p_1) \right]
\]

\[
= \frac{m_T (y_{L}^{T} [y_{R}^{T}]^{2} \omega_{-} + |y_{R}^{T}|^{2} \omega_{+}) + m_T (y_{L}^{T} (y_{R}^{T})^{*} \omega_{-} + y_{R}^{T} (y_{L}^{T})^{*} \omega_{+}) - ([y_{L}^{T}]^{2} \omega_{-} + |y_{R}^{T}|^{2} \omega_{+}) \vec{k}_1}{(p_1 - k_1)^2 - m_T^2} \]

\[
+ \frac{m_T (y_{L}^{T} [y_{R}^{T}]^{2} \omega_{-} + |y_{R}^{T}|^{2} \omega_{+}) + m_T (y_{L}^{T} (y_{R}^{T})^{*} \omega_{-} + y_{R}^{T} (y_{L}^{T})^{*} \omega_{+}) - ([y_{L}^{T}]^{2} \omega_{-} + |y_{R}^{T}|^{2} \omega_{+}) \vec{k}_2}{(p_1 - k_2)^2 - m_T^2} u^{\beta}(p_1).
\]

In the high energy limit $p_{1,2}^0 \rightarrow \infty$, it can be approximated as

\[
i\mathcal{M}^{\alpha\beta}(t\bar{t} \rightarrow hh) \approx i\bar{v}^{\sigma} (p_2) \left[ (y_{L}^{T} |y_{R}^{T}|^{2} \omega_{-} + [y_{R}^{T}]^{2} \omega_{+}) \right] \frac{\vec{k}_1}{(p_1 - k_1)^2 - m_T^2} + \frac{\vec{k}_2}{(p_1 - k_2)^2 - m_T^2} u^{\beta}(p_1). \tag{34}
\]

To calculate the above amplitude, we need to choose a reference frame. In the center of mass (COM) frame of initial particles, we can parametrize the momenta $p_1, p_2, k_1, k_2$ and spinors as follows\[^{38, 93}\]:

\[
p_{1}^{\mu} = (E_p, 0, 0, |\vec{p}|), \quad p_{2}^{\mu} = (E_p, 0, 0, -|\vec{p}|),
\]

\[
k_{1}^{\mu} = (E_k, |\vec{k}| \sin \theta, 0, |\vec{k}| \cos \theta), \quad k_{2}^{\mu} = (E_k, -|\vec{k}| \sin \theta, 0, -|\vec{k}| \cos \theta),
\]

\[
s = (p_1 + p_2)^2 = (2E_p)^2, \quad t = (p_1 - k_1)^2, \quad u = (p_1 - k_2)^2.
\]

\[^{7}\] The diagrams mediated by s-channel Higgs propagator vanish in the high energy limit because of the suppression.
Thus we derive the following results:

\[ u^+(p_1) = \left[ \frac{\sqrt{E_p - |\vec{p}|}}{\sqrt{E_p + |\vec{p}|}}, u^-(p_1) = \left[ \frac{\sqrt{E_p + |\vec{p}|}}{\sqrt{E_p - |\vec{p}|}}, \frac{\xi^+}{1}, \frac{\xi^-}{0} \right] \right], \]

\[ v^+(p_2) = \left[ \frac{\sqrt{E_p + |\vec{p}|}}{\sqrt{E_p - |\vec{p}|}}, v^-(p_2) = \left[ \frac{\sqrt{E_p - |\vec{p}|}}{\sqrt{E_p + |\vec{p}|}}, \frac{\eta^+}{0}, \frac{\eta^-}{-1/1} \right] \right]. \quad (35) \]

In the high energy limit, we have:

\[ p^1 \approx (E, 0, 0, E), \quad p^2 \approx (E, 0, 0, -E), \]
\[ k^1 \approx (E, E \sin \theta, 0, E \cos \theta), \quad k^2 \approx (E, -E \sin \theta, 0, -E \cos \theta), \]
\[ s \approx (2E)^2, \quad t \approx -2E^2(1 - \cos \theta), \quad u \approx -2E^2(1 + \cos \theta). \]

\[ u^+(p_1) \approx \sqrt{2} \frac{\mathbf{0}}{\xi^+}, \quad u^-(p_1) \approx \sqrt{2} \mathbf{0}, \quad v^+(p_2) \approx \sqrt{2} \mathbf{0}, \quad v^-(p_2) \approx \sqrt{2} \mathbf{0}. \quad (36) \]

Thus we derive the following results:

\[ iM^{++}(t\bar{t} \to hh) \approx -i |y^T_k|^2 \frac{1}{(m_h^2 + m_t^2 - m_T^2)/(2E^2) - (1 - \cos \theta)} - \frac{1}{(m_h^2 + m_t^2 - m_T^2)/(2E^2) - (1 + \cos \theta)}, \]
\[ iM^{+-}(t\bar{t} \to hh) \approx -i |y^T_k|^2 \sin \theta \frac{1}{(m_h^2 + m_t^2 - m_T^2)/(2E^2) - (1 - \cos \theta)} - \frac{1}{(m_h^2 + m_t^2 - m_T^2)/(2E^2) - (1 + \cos \theta)}, \]
\[ iM^{-+}(t\bar{t} \to hh) \approx -i |y^T_k|^2 \sin \theta \frac{1}{(m_h^2 + m_t^2 - m_T^2)/(2E^2) - (1 - \cos \theta)} - \frac{1}{(m_h^2 + m_t^2 - m_T^2)/(2E^2) - (1 + \cos \theta)}, \]
\[ iM^{--}(t\bar{t} \to hh) \approx -i |y^T_k|^2 \frac{1}{(m_h^2 + m_t^2 - m_T^2)/(2E^2) - (1 - \cos \theta)} - \frac{1}{(m_h^2 + m_t^2 - m_T^2)/(2E^2) - (1 + \cos \theta)}. \quad (37) \]

As we can see, there is no S-wave in this channel, namely,

\[ a^{++}_0(t\bar{t} \to hh) \approx a^{--}_0(t\bar{t} \to hh) \approx a^{++}_0(t\bar{t} \to hh) \approx a^{--}_0(t\bar{t} \to hh) \approx 0. \quad (38) \]

Of course, there are many other two-fermion processes (for example \( t\bar{T} \to hh, T\bar{t} \to hh, T\bar{T} \to hh, t\bar{t} \to W^+W^-, t\bar{t} \to ZZ, t\bar{t} \to Zh \)) depending on the initial and final state particles. Actually, all the two-fermion processes don’t contribute to the S-wave \[37\].

### A.2. Four-fermion process analysis

For the four-fermion processes, we take the \( t\bar{t} \to T\bar{T} \) process as an example. In Fig. 10, we give
In the COM frame of initial particles, the representations of spinors are listed as follows:

\[
iM^{\rho\sigma\alpha\beta}(t\bar{t} \to TT) = -\frac{i}{(p_1 - k_1)^2 - m_h^2} \bar{u}^\alpha(k_1)((y_R^T)^* \omega_- + (y_L^T)^* \omega_+)u^\rho(p_1)\bar{v}^\sigma(p_2)(y_L^T \omega_- + y_R^T \omega_+)v^\beta(k_2).
\]  

(39)

In general, the initial and final states both can be \( t, p_1, \rho \) and \( t, p_2, \sigma \). Thus the coupled channel matrix is \( 16 \times 16 \) (four states plus four helicity cases) even if we don’t consider the color degrees of freedom.

In the Feynman diagram. The amplitude with general helicity can be written as

\[
\approx \left(\begin{array}{c} 0 \\ \xi^+ \end{array}\right), \quad \approx \left(\begin{array}{c} \xi^- \\ 0 \end{array}\right), \quad v^+(p_2) \approx \sqrt{2E} \left(\begin{array}{c} \eta^+ \\ 0 \end{array}\right), \quad v^-(p_2) \approx \sqrt{2E} \left(\begin{array}{c} -\bar{\eta}^- \\ 0 \end{array}\right),
\]

(40)

Then we can get the polarized amplitudes:

\[
iM^{++++}(t\bar{t} \to TT) \approx iM^{++--}(t\bar{t} \to TT) \approx iM^{++--}(t\bar{t} \to TT) \approx 0,
\]

\[
iM^{++--}(t\bar{t} \to TT) \approx \frac{is|y_L^T|^2 \sin^2 \frac{\theta}{2}}{(p_1 - k_1)^2 - m_h^2},
\]

\[
iM^{++--}(t\bar{t} \to TT) \approx iM^{++--}(t\bar{t} \to TT) \approx 0,
\]

\[
iM^{++--}(t\bar{t} \to TT) \approx -\frac{is|y_L^T|^2 \sin^2 \frac{\theta}{2}}{(p_1 - k_1)^2 - m_h^2},
\]

(41)
To make the problem as simple as possible, we only turn on the $y^T_L, y^T_R$ couplings. Under this consideration, the non-zero coupled channel amplitudes are

$$
\mathcal{M}^{++-} (t\bar{t} \to T\bar{T}) \approx \frac{s y^T_R (y^T_L)^* \sin^2 \frac{\theta}{2}}{(p_1 - k_1)^2 - m_h^2}, \quad \mathcal{M}^{+-+} (t\bar{t} \to T\bar{T}) \approx -\frac{s |y^T_L|^2 \sin^2 \frac{\theta}{2}}{(p_1 - k_1)^2 - m_h^2},
$$

$$
\mathcal{M}^{-++} (t\bar{t} \to T\bar{T}) \approx -\frac{s |y^T_R|^2 \sin^2 \frac{\theta}{2}}{(p_1 - k_1)^2 - m_h^2}, \quad \mathcal{M}^{---} (t\bar{t} \to T\bar{T}) \approx \frac{s y^T_L (y^T_R)^* \sin^2 \frac{\theta}{2}}{(p_1 - k_1)^2 - m_h^2}. \quad (42)
$$

The corresponding S-wave amplitudes are calculated to be

$$
a^{++-}_0 (t\bar{t} \to T\bar{T}) \approx -\frac{y^T_R (y^T_L)^*}{16\pi}, \quad a^{+-+}_0 (t\bar{t} \to T\bar{T}) \approx \frac{|y^T_L|^2}{16\pi},
$$

$$
a^{-++}_0 (t\bar{t} \to T\bar{T}) \approx \frac{|y^T_R|^2}{16\pi}, \quad a^{---}_0 (t\bar{t} \to T\bar{T}) \approx -\frac{y^T_L (y^T_R)^*}{16\pi}. \quad (43)
$$

In the basis of $++, +-, --, --$, we can get the following coupled channel matrix for this process:

$$
a_0 (t\bar{t} \to T\bar{T}) = \frac{1}{16\pi} \begin{bmatrix}
0 & 0 & 0 & -y^T_R (y^T_L)^* \\
0 & 0 & |y^T_L|^2 & 0 \\
0 & |y^T_R|^2 & 0 & 0 \\
-y^T_L (y^T_R)^* & 0 & 0 & 0
\end{bmatrix}. \quad (44)
$$

Similarly, we can get the coupled channel matrices for the other processes in the basis of $++, +-, --, --$:

$$
a_0 (T\bar{T} \to t\bar{t}) = \frac{1}{16\pi} \begin{bmatrix}
0 & 0 & 0 & -y^T_R (y^T_L)^* \\
0 & 0 & |y^T_L|^2 & 0 \\
0 & |y^T_R|^2 & 0 & 0 \\
-y^T_L (y^T_R)^* & 0 & 0 & 0
\end{bmatrix}. \quad (45)
$$

$$
a_0 (t\bar{T} \to t\bar{t}) = \frac{1}{16\pi} \begin{bmatrix}
-y^T_L (y^T_R)^* & 0 & 0 & y^T_T (y^T_L)^* \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -|y^T_R|^2
\end{bmatrix}. \quad (46)
$$

$$
a_0 (t\bar{T} \to T\bar{t}) = \frac{1}{16\pi} \begin{bmatrix}
-(y^T_L)^* (y^T_R)^* & 0 & 0 & 0 \\
0 & 0 & (y^T_L)^* (y^T_R)^* & 0 \\
0 & (y^T_L)^* (y^T_R)^* & 0 & 0 \\
0 & 0 & 0 & -(y^T_L)^* (y^T_R)^*
\end{bmatrix}. \quad (47)
$$

$$
a_0 (T\bar{t} \to t\bar{T}) = \frac{1}{16\pi} \begin{bmatrix}
-y^T_T y^T_R & 0 & 0 & 0 \\
0 & y^T_T y^T_R & 0 & 0 \\
0 & y^T_T y^T_R & 0 & 0 \\
0 & 0 & 0 & -y^T_T y^T_R
\end{bmatrix}. \quad (48)
$$
Note that all the eigenvalues must satisfy

$$\lambda = \frac{1}{16\pi} \begin{bmatrix} -|y_L^{T}|^2 & 0 & y_L^{T}(y_L^{T})^* \\ 0 & 0 & 0 \\ y_L^{T}(y_L^{T})^* & 0 & -|y_R^{T}|^2 \end{bmatrix}.$$  \hspace{1cm} (49)

In the basis of $t\bar{t}, T\bar{T}, t\bar{T}, T\bar{t}$, we can get the following coupled channel matrix for all the four-fermion processes without regard to the quark color:

$$a_0 = \begin{bmatrix} 0_{4\times4} & a_0(t\bar{t} \rightarrow T\bar{T}) & 0_{4\times4} & 0_{4\times4} \\ a_0(T\bar{T} \rightarrow t\bar{t}) & 0_{4\times4} & 0_{4\times4} & 0_{4\times4} \\ 0_{4\times4} & 0_{4\times4} & 0_{4\times4} & a_0(T\bar{t} \rightarrow T\bar{t}) \\ 0_{4\times4} & 0_{4\times4} & a_0(T\bar{t} \rightarrow T\bar{t}) & 0_{4\times4} \end{bmatrix}. \hspace{1cm} (50)$$

In the above, we write the coupled channel matrix in the block form. Obviously, this square matrix is $16 \times 16$. Then we can get the eigenvalues of $a_0$ as follows\(^8\)

\[
\begin{align*}
\lambda_1^+ &= \frac{1}{16\pi} |y_L^{T}|^2, & \lambda_1^- &= -\frac{1}{16\pi} |y_L^{T}|^2, & \lambda_2^+ &= \frac{1}{16\pi} |y_R^{T}|^2, & \lambda_2^- &= -\frac{1}{16\pi} |y_R^{T}|^2, \\
\lambda_3^+ &= \frac{1}{32\pi} (|y_L^{T}|^2 + |y_R^{T}|^2), & \lambda_3^- &= -\frac{1}{16\pi} |y_L^{T}| |y_R^{T}|, \\
\lambda_4^+ &= \frac{1}{32\pi} (\sqrt{(|y_L^{T}|^2 + |y_R^{T}|^2)^2 + 12|y_L^{T}|^2|y_R^{T}|^2} - |y_L^{T}|^2 - |y_R^{T}|^2), \\
\lambda_4^- &= \frac{1}{32\pi} (-\sqrt{(|y_L^{T}|^2 + |y_R^{T}|^2)^2 + 12|y_L^{T}|^2|y_R^{T}|^2} - |y_L^{T}|^2 - |y_R^{T}|^2).
\end{align*}
\]  \hspace{1cm} (51)

Where $\lambda_1^-, \lambda_2^-$ are doubly degenerate and $\lambda_3^+, \lambda_3^-$ are four-fold degenerate. S-wave unitarity requires that all the eigenvalues must satisfy $|\text{Re}(\lambda_i)| \leq \frac{1}{2}$. It will lead to the following constraints\(^9\)

$$\sqrt{(|y_L^{T}|^2 + |y_R^{T}|^2)^2 + 12|y_L^{T}|^2|y_R^{T}|^2} + |y_L^{T}|^2 + |y_R^{T}|^2 \leq 16\pi. \hspace{1cm} (52)$$

Note that $|y_L^{T}| \leq \sqrt{8\pi}, |y_R^{T}| \leq \sqrt{8\pi}, \sqrt{|y_L^{T}| |y_R^{T}|} \leq \sqrt{8\pi}$ hold automatically in the above bound.

**B. $h \rightarrow \gamma\gamma$ CHANNEL ANALYSIS**

**B.1. The SM result**

The partial decay width of $h \rightarrow \gamma\gamma$ for SM is given in Ref.\([56][58]\)

$$\Gamma^{SM}(h \rightarrow \gamma\gamma) = \frac{G_F a^2 m_h^3}{128 \sqrt{2} \pi^3} \sum_f N_f^2 Q_f^2 F_f(\tau_f) + F_W(\tau_W)^2 \left( \tau_f = \frac{4m_f^2}{m_h^2}, \tau_W = \frac{4m_W^2}{m_h^2} \right). \hspace{1cm} (53)$$

---

\(^8\) When we take the quark color into account further, the matrix will become $48 \times 48$. Although the unitarity bounds may be improved, the matrix will be quite large and complex to deal with.

\(^9\) Remember that the bounds are just a rough estimation. If we turn on the other couplings (say $y_f, \tilde{y}_f, \kappa_\ell, \bar{\kappa}_\ell$), these constraints may be altered.
With the $F_f, F_W$ defined by

$$F_f(\tau_f) \equiv -2\tau_f[1 + (1 - \tau_f)f(\tau_f)], \quad F_W(\tau_W) \equiv 2 + 3\tau_W + 3\tau_W(2 - \tau_W)f(\tau_W),$$

$$f(\tau) \equiv \begin{cases} \arcsin^2\left(\frac{1}{\sqrt{\tau}}\right), & \text{for } \tau \geq 1 \\ -\frac{1}{3}\log\frac{\tau}{\tau + 1 - \sqrt{\tau}} - i\pi^2, & \text{for } \tau < 1 \end{cases} \quad (54)$$

For the fermionic part, top quark is dominated because of the largest Yukawa coupling. Numerically, we can get $N_t^CQ_t^2F_f(\tau_t) \sim -1.84, F_W(\tau_W) \sim 8.32$. This means the gauge boson contributions are almost 4.5 times larger than the fermionic ones.

**B.2. The new physics result**

Due to $U_{EM}(1)$ gauge symmetry, the $h \to \gamma\gamma$ amplitude possesses the following tensor structure:

$$i\mathcal{M} = i\epsilon_\mu(p_1)\epsilon_\nu(p_2)\left[\left(p_2^\mu p_1^\nu - p_1^\mu p_2^\nu\right)A + \epsilon^{\mu\nu\rho\sigma}p_2^\rho B\right], \quad A \equiv \frac{e^2}{8\pi\nu^2}(-F_W(\tau_W) + A_T + A_T), \quad B \equiv \frac{e^2}{8\pi\nu^2}(B_t + B_T). \quad (55)$$

The expressions of $A_t, A_T, B_t, B_T$ are given as

$$A_t = -N_t^CQ_t^2\kappa_t F_f(\tau_t), \quad A_T = N_t^CQ_t^2\frac{y_{\tau\nu}}{m_T}F_f(\tau_T),$$

$$B_t = -2N_t^CQ_t^2\tilde{\kappa}_t \cdot 2\tau_tf(\tau_t), \quad B_T = N_t^CQ_t^2\frac{y_{\tau\nu}}{m_T} \cdot 2\tau_Tf(\tau_T). \quad (56)$$

Taking the mass of $t, T$ quarks to be infinity, they can be expanded as

$$A_t \approx -N_t^CQ_t^2\kappa_t\left(-\frac{4}{3} - \frac{7m_h^2}{90m_T^2}\right), \quad A_T \approx N_t^CQ_t^2\frac{y_{\tau\nu}}{m_T}\left(-\frac{4}{3} - \frac{7m_h^2}{90m_T^2}\right),$$

$$A_t + A_T \approx \frac{4}{3}N_t^CQ_t^2\left[\left(\kappa_t - \frac{y_{\tau\nu}}{m_T}\right) + \kappa_t \cdot \frac{7m_h^2}{120m_T^2} - \frac{y_{\tau\nu}}{m_T} \cdot \frac{7m_h^2}{120m_T^2}\right],$$

$$B_t \approx -2N_t^CQ_t^2\tilde{\kappa}_t(1 + \frac{m_h^2}{12m_T^2}), \quad B_T \approx 2N_t^CQ_t^2\frac{y_{\tau\nu}}{m_T}(1 + \frac{m_h^2}{12m_T^2}),$$

$$B_t + B_T \approx -2N_t^CQ_t^2\left[\left(\tilde{\kappa}_t - \frac{y_{\tau\nu}}{m_T}\right) + \tilde{\kappa}_t \cdot \frac{m_h^2}{12m_T^2} - \frac{y_{\tau\nu}}{m_T} \cdot \frac{m_h^2}{12m_T^2}\right]. \quad (57)$$

The partial decay width formula is computed as

$$\Gamma(h \to \gamma\gamma) = \frac{G_F^2\alpha^2m_h^3}{128\sqrt{2}\pi^3}||A_t + A_T - F_W(\tau_W)||^2 + |B_t + B_T|^2$$

$$= \frac{G_F^2\alpha^2m_h^3}{128\sqrt{2}\pi^3} \left[|N_t^CQ_t^2\kappa_t F_f(\tau_t) - \frac{y_{\tau\nu}}{m_T}F_f(\tau_T)| + F_W(\tau_W)|^2 + 4(N_t^CQ_t^2)^2|\kappa_t \cdot \tau_f(\tau_t) - \frac{y_{\tau\nu}}{m_T} \cdot \tau_Tf(\tau_T)|^2\right]. \quad (58)$$

---

10 The expression of $\Gamma(h \to gg)$ has similar form of the fermionic part of the $\gamma\gamma$ decay.
In Tab. V we list the expressions of $A_t + A_T$, $B_t + B_T$ in several models, where we have neglected the $\frac{1}{m_{T}^2}$ suppressed terms. We can see that the $A_t + A_T$, $B_t + B_T$ in VLQ and VLQ+S models are close to those in SM. In fact, it is difficult to detect VLQ in the $h\gamma\gamma$ decay channel [21].

|       | $\tilde{A}_t + \tilde{A}_T$ | $\tilde{B}_t + \tilde{B}_T$ |
|-------|-----------------------------|-----------------------------|
| **general** | $\frac{4}{3}[(\kappa_t - \frac{y_t}{m_T}) + \kappa_t - \frac{7m_T^2}{12m_{T}^2} - \frac{y_{t^2}}{m_T}], \frac{7m_T^2}{12m_{T}^2}$ | $-2[(\kappa_t - \frac{y_t}{m_T}) + \kappa_t + \frac{m_T^2}{12m_{T}^2} - \frac{y_{t^2}}{m_T}, \frac{m_T^2}{12m_{T}^2}]$ |
| **SM** | $\frac{4}{3}(1 + \frac{7m_T^2}{12m_{T}^2})$ | 0 |
| **VLQ** | $\frac{4}{3}(1 + c_{L}^2), \frac{7m_T^2}{12m_{T}^2} + s_{L}^2, \frac{7m_T^2}{12m_{T}^2}$ | 0 |
| **VLQ+S** | $\frac{4}{3}[c_{B} - \frac{y_{B}}{m_{T}}Re(y_{B}^2)s_{B}c_{B} + (c_{L}^2c_{B} - \frac{y_{B}}{m_{T}}Re(y_{B}^2)s_{B}c_{L}c_{B}) - \frac{7m_T^2}{12m_{T}^2}]$ | $\frac{2y_{B}}{m_{T}^2}Im(y_{B}^2)s_{B}c_{B} + \frac{2y_{B}}{m_{T}^2}Im(y_{B}^2)s_{B}c_{B} + \frac{7m_T^2}{12m_{T}^2}[1 + s_{L}^2, \frac{7m_T^2}{12m_{T}^2} + c_{L}^2, \frac{7m_T^2}{12m_{T}^2}]$ |

**Table V.** The expressions of $\tilde{A}_t + \tilde{A}_T$, $\tilde{B}_t + \tilde{B}_T$ in the SM, VLQ and VLQ+S under the heavy quark limit. Here we extract the common factor $N_{u}^2 Q_{T}^2$ for convenience, that is redefinition of $A(B)$ with $N_{u}^2 Q_{T}^2 \tilde{A}(\tilde{B})$. We take $\tilde{A}_T = \tilde{B}_T = 0$ naively in SM because of the absence of $T$ quark.

### C. Asymptotic Behaviors of the Loop Functions

$B_0$ function is defined as

$$B_0(k^2, m_0^2, m_1^2) = \frac{1}{(2\pi)^{4-D} i\pi^2} \int d^Dq \frac{1}{(q^2 - m_0^2)[(q + k)^2 - m_1^2]}$$

$$= \Delta_\epsilon - \int_0^1 dx \log \frac{xm_0^2 + (1-x)m_1^2 - x(1-x)k^2}{\mu^2} \quad (\Delta_\epsilon = \frac{1}{\epsilon} - \gamma_E + \log 4\pi). \quad (59)$$

In the limit of $k^2 \ll m_0^2, m_1^2$, $B_0$ function can be expanded as

$$B_0(k^2, m_0^2, m_1^2) = B_0(0, m_0^2, m_1^2) + \frac{\partial B_0(k^2, m_0^2, m_1^2)}{\partial k^2} \bigg|_{k^2 = 0} k^2 + O\left(\frac{k^4}{m_0^2}, \frac{k^4}{m_0^2 m_1^2}, \frac{k^4}{m_1^4}\right)$$

$$= \Delta_\epsilon + 1 - \frac{m_0^2 \log \frac{m_0^2}{m_1^2} - m_1^2 \log \frac{m_1^2}{m_0^2}}{m_0^2 - m_1^2} + \frac{m_0^4 - m_1^4 + 2m_0^2 m_1^2 \log \frac{m_0^2}{m_1^2}}{2(m_0^2 - m_1^2)^3} k^2 + O\left(\frac{k^4}{m_0^2}, \frac{k^4}{m_0^2 m_1^2}, \frac{k^4}{m_1^4}\right). \quad (60)$$

$C_0$ function is defined as

$$C_0(k_1^2, k_2^2, k_3^2, m_0^2, m_1^2, m_2^2) \quad (k_12 \equiv k_1 - k_2)$$

$$= \frac{1}{(2\pi)^{4-D} i\pi^2} \int d^Dq \frac{1}{(q^2 - m_0^2)[(q + k_1)^2 - m_1^2][q + k_2)^2 - m_2^2]}$$

$$= - \int_0^1 \int_0^1 \int_0^1 dx dy dz \frac{\delta(x + y + z - 1)}{(y k_1 + z k_2)^2 + x m_0^2 + y m_1^2 + z m_2^2 - y k_1^2 - z k_2^2}. \quad (61)$$
Then we have:
\[
C_0(0, m_Z^2, m_h^2, m_t^2, m_T^2) = C_0(0, m_Z^2, m_h^2, m_T^2, m_t^2)
\]
\[
= -\int_0^1 \int_0^1 \int_0^1 dx dy dz \, \delta(x + y + z - 1)
\]
\[
\times [yp_1 + z(p_1 + p_2)]^2 + (x + y)m_T^2 + zm_T^2 - yp_1^2 - z(p_1 + p_2)^2
\]
\[
= -\int_0^1 \int_0^1 \int_0^1 dx dy dz \, \delta(x + y + z - 1)
\]
\[
\times yz(m_h^2 - m_Z^2) + z^2m_h^2 + (x + y)m_T^2 + zm_T^2 - zm_h^2.
\] (62)

In the limit of \(m_h, m_Z \ll m_f\), \(C_0(0, m_Z^2, m_h^2, m_t^2, m_T^2)\) function can be expanded as
\[
C_0(0, m_Z^2, m_h^2, m_t^2, m_T^2) = -\frac{1}{2m_f^2} \left[ 1 + \frac{m_h^2 + m_Z^2}{12m_f^2} + \mathcal{O}\left(\frac{m_h^4, m_h^2 m_Z^2, m_Z^4}{m_f^4}\right) \right].
\] (63)

In the limit of \(m_h, m_Z \ll m_t, m_T\), \(C_0(0, m_Z^2, m_h^2, m_t^2, m_T^2)\) function can be expanded as
\[
C_0(0, m_Z^2, m_h^2, m_t^2, m_T^2) = C_0(0, 0, 0, m_t^2, m_T^2) + \frac{\partial C_0(0, m_Z^2, m_h^2, m_t^2, m_T^2)}{\partial m_h^2}|_{(m_h, 0, m_Z = 0)} m_h^2
\]
\[
+ \frac{\partial C_0(0, m_Z^2, m_h^2, m_t^2, m_T^2)}{\partial m_T^2}|_{(m_h, 0, m_Z = 0)} m_T^2
\]
\[
+ \mathcal{O}\left(\frac{m_h^4, m_h^2 m_Z^2, m_Z^4}{m_f^4}\right).
\] (64)

With
\[
\frac{\partial C_0(0, 0, 0, m_t^2, m_T^2)}{\partial m_T^2}|_{(m_h, 0, m_Z = 0)} = \frac{\partial C_0(0, m_Z^2, m_h^2, m_t^2, m_T^2)}{\partial m_Z^2}|_{(m_h, 0, m_Z = 0)}
\]
\[
= \frac{1}{m_T^4} \left( 2(1 + 2r_T^2) \log r_T^2 + 5 - 4r_T^2 - r_T^4 \right).
\] (65)

Thus we have:
\[
C_0(0, m_Z^2, m_h^2, m_t^2, m_T^2) \approx \frac{1}{m_T^4} \left[ 1 + \log r_T^2 + \frac{m_h^2 + m_Z^2}{m_T^4} \frac{5 + 2(1 + 2r_T^2) \log r_T^2}{4} \right].
\] (66)

For the case of \(C_0(0, m_Z^2, m_h^2, m_T^2, m_t^2)\), we can get the corresponding results via the replacement \(m_t \leftrightarrow m_T\). For example:
\[
C_0(0, m_Z^2, m_h^2, m_T^2, m_t^2) \approx -\frac{1}{m_T^4} \left[ 1 + r_T^2 \log r_T^2 + \frac{m_h^2 + m_Z^2}{m_T^4} \frac{1 + 4r_T^2 \log r_T^2}{4} \right].
\] (67)

Here we also give the heavy \(m_f\) expansion of the following functions:
\[
f(\tau_f) = \frac{m_h^2}{4m_f^2} + \frac{m_h^4}{48m_f^4} + \mathcal{O}\left(\frac{m_h^6}{m_f^6}\right), \quad F_f(\tau_f) = -\frac{4}{3} - \frac{7m_h^2}{90m_f^2} + \mathcal{O}\left(\frac{m_h^4}{m_f^4}\right).
\] (68)
and
\[ A_f(\tau_f, \lambda_f) = \frac{m_h^2}{m_h^2 - m_Z^2} \left[ (m_h^2 - m_Z^2 - 4m_f^2)C_0(0, m_Z^2, m_h^2, m_f^2, m_f^2) - 2m_Z^2 \frac{B_0(m_h^2, m_f^2, m_f^2) - B_0(m_Z^2, m_f^2, m_f^2)}{m_h^2 - m_Z^2} - 2 \right] \]
\[ = - \frac{1}{3} - \frac{7m_h^2 + 11m_Z^2}{360m_f^2} + O\left(\frac{m_h^4, m_h^2m_Z^2, m_Z^2}{m_f^4}\right). \] (69)

In the following, we list some special limits of the loop integrals:

\[ \lim_{\tau_f \to 0} F_f(\tau_f) = 0, \quad \lim_{\tau_f \to \infty} F_f(\tau_f) = -\frac{4}{3}, \]
\[ \lim_{\tau_W \to 0} F_W(\tau_W) = 2, \quad \lim_{\tau_W \to \infty} F_W(\tau_W) = 7, \]
\[ \lim_{\lambda \to \infty} I_1(\tau, \lambda) = \frac{\tau^2}{2} f(\tau) - \frac{\tau}{2}, \quad \lim_{\lambda \to \infty} I_2(\tau, \lambda) = \frac{\tau}{2} f(\tau), \]
\[ \lim_{\lambda_f \to \infty} A_f(\tau_f, \lambda_f) = \frac{\tau_f}{2} [(\tau_f - 1)f(\tau_f) - 1] = \frac{1}{4} F_f(\tau_f), \]
\[ \lim_{\lambda_W \to \infty} A_W(\tau_W, \lambda_W) = \frac{1}{2t_W} [(5 - t_W^2)\tau_W(2 - \tau_W)f(\tau_W) + 2 + 5\tau_W - t_W^2(2 + \tau_W)]. \] (70)