Evaluation for p-y Method for Offshore Large Diameter Monopile in Sand

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Abstract. Large monopile is widely used in offshore wind farm due to their relatively simple installation. Monopiles are customarily designed by applying the p-y method. That being said, there is no detailed and uniform interpretation on the reasons for limitations in the current p-y method. An elaborate study is performed to determine the main reasons of such limitations. In this study, a FLAC3D model is presented to evaluate the current p-y method for piles in sand of three different relative densities under a static load, in which the non-linear modulus is taken into account. The results show that the initial stiffness of the p-y method is overestimated and even more magnified in the large depth. The diameter effect should be taken into account under diameters exceeding 3m in loose and medium sand, as well as diameters more than 5m in dense sand.

1. Introduction

Monopiles are generally the preferred foundation type for offshore wind turbines. The piles can withstand large or reciprocating horizontal loads and bending moments caused by wind and wave. Large wind turbines need to be supported by hefty dimensions of monopile in deeper water depths. Diameters up to 8m are applied to the offshore wind turbines. The slenderness ratio(L/D) of monopiles lies around 5-6, where D is the monopile’s outer diameter and L is the embedded length. In the design of the large-diameter monopile, the p-y method recommended in the offshore guidelines of the American Petroleum Institute (2010)[1] and Det Norske Veritas (2013)[2] simulates the soil reaction p in dependence on the horizontal displacement y. The commonly used p-y method is assumed to be sufficiently accurate for slender piles. However, it currently fails to accurately address the soil reaction force, when applied to the large diameter monopile. Wiemann K et al. (2006)[3][4] indicated that the p-y method overestimates initial stiffness for small operational loads, and underestimates stiffness for extreme loads. Pender et al. (2007)[4] conducted a series of numerical research to investigate the effect of the pile diameter on the initial stiffness of the p-y curve, meanwhile, the research showed that the impact of the pile diameter was negligible. Sørensen et al. (2009,2010) [5][6]reported that initial stiffness of the soil-pile interaction is affected by the diameter of monopile and the p-y method happens to overestimate initial stiffness. Kallehaved (2012)[7] performed some research about the initial stiffness of the soil-pile interaction based on the Terzaghi of bulb of pressure. The results showed the p-y curve underestimates initial stiffness and that the pile diameter has a significant influence on initial stiffness.

As a matter of fact, the study on the initial stiffness of different sand types is rare and need to be examined properly. In this study, a series of numerical calculations were performed in order to
evaluate the initial stiffness of the $p$-$y$ curve in sand of three different relative densities, and in which the elastic modulus increases non-linearly with depth. This paper analyses the main reasons for the errors of the $p$-$y$ curve calculation results for rigid pile, based on the data obtained.

2. $p$-$y$ method

Monopiles are currently subjected to large horizontal loads. Large horizontal loads may lead to the extensive horizontal displacement of the top monopile and non-linear strain of the soil surrounding the pile. Since the $p$-$y$ method by recommended API can well describe the soil’s nonlinearity, it is therefore applied to analyse the pile’s horizontal displacement. The $p$-$y$ method is as follows.

$$p = A_P \tan(h) \frac{k}{A_H} y$$

Where $k$ represents the initial coefficient of subgrade reaction depending on the angle of internal friction or relative density of the soil, its values can be obtained from Eq (2) or the fig. 1(a), which has been fitted by Augustesen et al. (2009)[8]. $H$ is the depth. $p$ is soil reaction force of the surrounding pile. $y$ is horizontal displacement of the pile or horizontal strain of soil.

![Fig.1](a) $p$-$y$ curves for sand and (b) variation of initial coefficient of subgrade reaction

$$k = \left\{ \begin{array}{ll}
0.008085 \times 10^3 [kPa/m] & \text{for } 29^\circ \leq \phi \leq 45^\circ
\end{array} \right.$$  

Where $\phi$ is the angle of internal friction in degrees.

$P_u$ is the ultimate resistance of the soil and is determined by the following Eq (3).

$$P_u(z) = \min\{C_1z + C_2z^2, C_3z\}$$

Where $C_1$, $C_2$, $C_3$ are functions of the angle of internal friction, that can be determined by the following equations.

$$C_1 = 0.115 \times 10^{0.0405\phi}, \quad C_2 = 0.571 \times 10^{0.022\phi}, \quad C_3 = 0.646 \times 10^{0.0555\phi}$$

The coefficient is determined by the following expressions.

$$A = \left\{ \begin{array}{ll}
3.0 - 0.8z & \text{for static loading} \\
0.9 & \text{for cyclic loading}
\end{array} \right.$$  

The derivative of the equation (1) is as follows. It shows that the initial subgrade stiffness $E_{py}$ is increase linearly as the depth.
\[ E_{py} = \frac{dp}{dy} \bigg|_{y=0} = \frac{kz}{\cosh^2 \left( \frac{kz}{2A_{py}} \right) y=0} = k \]  

(7)

3. Numerical model verification
A three-dimensional numerical model of a monopile foundation is created in the finite difference program Flac3D. The model is applied to analyse model tests piles of Prasad and Chari (1999)[9]. Each pile was 1135 mm long, 5.6 mm in wall thickness and 102 mm in external diameter. Three interfaces around the monopile, the side interfaces and the bottom interface are set between the pile and the soil to simulate the soil-pile interaction. The shear and normal stiffness of the interfaces are set to be equal to \(10^6\) kPa/m and the strength parameters of the interfaces are taken as 0 kPa and the soil internal friction of 0.6 times. The pile was buried to a depth of 612 mm and then lateral load was applied to an eccentricity of 150 mm above the soil surface. Parameters of soil are presented in the table1. The non-linear modulus of soil is considered in numerical model, as follows Eq (8) [10].

![Table 1. Soil parameters of model test piles of Prasad and Chari[9]](image)

| Relative density(%) | k' | \(\lambda\) | Effective unit weight (kN/m\(^3\)) | Angle of internal friction(°) |
|---------------------|----|---------|-----------------------------|-------------------------|
| 25                  | 325| 0.65    | 16.5                        | 35.0                    |
| 50                  | 400| 0.60    | 17.3                        | 41.0                    |
| 75                  | 500| 0.55    | 18.3                        | 45.5                    |

![Figure 2 (a) The numerical model of the test and (b) comparison of predictions with model test data[9]](image)

\[ E_0 = k'^*\sigma_{ref} \left( \frac{\sigma_3}{\sigma_{ref}} \right)^{\lambda} \]  

(8)

Where \(E_0\) is the elastic modulus, \(k'\) and \(\lambda\), \(n\) are functions of the relative density. \(\sigma_3\) is the min principle stress. \(\sigma_{ref}\) is the reference stress.

The comparison of the measured load-displacement curves for the piles and the calculation results of the numerical model and p-y method at the three different relative densities is plotted in Fig2(b). The result shows p-y method overestimates the initial stiffness and the initial stiffness of the numerical result is the relatively well agreement with the test data. It proves the validity of calculating the initial stiffness with a numerical model in this study.
4. The comparison study of the p-y method and the numerical model

In this section, the numerical model is applied to analyse the initial stiffness in three different relative density sites 25%, 50% and 75% respectively. The ratio of the buried depth and the diameter was 6 and the diameter varied from 2-8m. In the section, the same and minute lateral displacements of 0.02m were applied to the top monopile.

4.1. The Influence of soil nonlinearity on initial stiffness

A dimensionless analysis is applied to the numerical result. $K_1$ is the initial stiffness at one specific depth, whereas $K_i$ is the initial stiffness at the any depths.

The dimensionless stiffness comparisons are illustrated in figure 3-5, which shows that initial stiffness of the p-y method is directly proportional to the depth and that the initial stiffness of the numerical calculation increases non-linearly with the depth. It indicates the p-y method overestimates the initial stiffness, especially in large depths.

4.2. The Influence of pile diameter on initial stiffness

This section depicts the impact of pile diameter on initial stiffness. $K_1$ is the initial stiffness of 2m pile and $K_D$ is the initial stiffness for piles of any diameters. The comparisons are displayed under the p-y method calculation and numerical model in figure 6-8, which proves that the dimensionless stiffness of p-y method and numerical model is consistent as long as the diameter does not exceed 3m in loose and medium sand. The dimensionless stiffness rises from 35.8%, 26% to 127.5%, 100.2% as the
diameter increases, under the diameters above 3m in loose and medium sand. Nevertheless, when the diameter of the pile is less than 5m in dense sand, the dimensionless stiffness is in agreement, see figure 8. The dimensionless stiffness increases from 16.3% to 62.6% under the diameters over 5m in dense sand. Overall, the dimensionless stiffness is increasing. The variation of the pile diameter will alters the stress-stain relationship of the soil around the pile in the numerical model. The Influence of pile diameter on initial stiffness isn’t considered in the p-y method, and the limit reaction force remains underestimated, so in the calculation the modulus of the p-y method rapidly decreases. Thus a discrepancy is generated between the calculation of p-y method and the numerical model calculations.

![Figure 6](image1.png)

**Figure 6.** The comparison of the different diameter dimensionless stiffness for API and the numerical model in loose sand

![Figure 7](image2.png)

**Figure 7.** The comparison of the different diameter dimensionless stiffness for API and the numerical model in medium sand

![Figure 8](image3.png)

**Figure 8.** The comparison of the different diameter dimensionless stiffness for API and the numerical model in dense sand

### 5. Conclusion

A series of numerical calculations were performed to assess the main factors linked with the limitations of the current p-y method limitations applied to analyse offshore monopiles. The comparison resulting from the numerical model and the p-y method showed the non-linear modulus and the diameter had a significant influence on the initial stiffness. The main conclusions are listed as follows:

1. The non-linear relationship of the initial stiffness with depth is neglected in the p-y method, hence the initial stiffness is overestimated and more overblown in large depths.
(2) The influence of the diameter is insignificant for the initial stiffness under diameters less than 3m in loose and medium sand. Withal, the diameter effect should be considered under pile diameters more than 3m. When the diameter is more than 5m in dense sand, the diameter effect on initial stiffness is not neglected.

According to the results of the numerical calculation, it is suggested that the $p$-$y$ method was carefully applied to large diameter monopiles, which have diameters over 3m.

6. References

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