\omega\text{-nucleus bound states in the Walecka model}

K. Saito \footnote{ksaito@nucl.phys.tohoku.ac.jp}

Physics Division, Tohoku College of Pharmacy
Sendai 981-8558, Japan

K. Tsushima \footnote{ktsushim@physics.adelaide.edu.au}, D.H. Lu \footnote{dlu@physics.adelaide.edu.au} and A.W. Thomas \footnote{athomas@physics.adelaide.edu.au}

Special Research Center for the Subatomic Structure of Matter
and Department of Physics and Mathematical Physics
The University of Adelaide, SA 5005, Australia

Abstract

Using the Walecka model, we investigate theoretically whether an \omega\ meson is bound to finite nuclei. We study several nuclei from $^6\text{He}$ to $^{208}\text{Pb}$, and compare the results with those in the quark-meson coupling (QMC) model. Our calculation shows that deeper \omega\text{-nucleus bound states are predicted in the Walecka model than in QMC. One can expect to detect such bound states in the proposed experiment involving the $(d,^3\text{He})$ reaction at GSI.

PACS: 24.10.Jv, 21.10.Dr, 21.30.Fe, 14.40.-n
The study of the properties of hadrons in a hot and/or dense nuclear medium is one of the most exciting new directions in nuclear physics [1]–[8]. The recent experimental data observed at the CERN/SPS by the CERES [9] and HELIOS [10] collaborations has been interpreted as evidence for a downward shift of the vector meson mass in dense nuclear matter [11]. To draw a more definite conclusion, measurements of the dilepton spectrum from vector mesons produced in nuclei are planned at TJNAF [12] and GSI [13] (see also Refs.[14, 15]).

Recently a novel approach to the study of meson mass shifts in nuclei was suggested by Hayano et al. [16], using the (d, 3He) reaction at GSI [17] to produce real η and ω mesons with nearly zero recoil. If the meson feels a large enough, attractive (Lorentz scalar) force inside a nucleus, the meson is expected to form meson-nucleus bound states. Hayano et al. [18] have estimated the binding energies for various η- and ω-mesic nuclei.

We have also reported possibility of such bound states [19] using the quark-meson coupling (QMC) model [5], in which the structure of the nucleus can be solved self-consistently, including the explicit quark structure of the nucleons. In this report we study several ω-mesic nuclei (6He, 11B, 26Mg, 16O, 40Ca, 90Zr and 208Pb – the first three are the final nuclei in the proposed experiments at GSI [16, 18]) using an alternative, relativistic nuclear model, namely, the Walecka model or Quantum Hadrodynamics (QHD) [20]. We compare the results with those found in QMC [19].

In Ref.[8] we have already studied the propagation of the ω meson with finite three momentum in infinite, symmetric nuclear matter within QHD-I, using the relativistic Hartree approximation. We also calculated the dispersion relation (in the time-like region) to get the “invariant” mass of the ω within the relativistic, random-phase approximation. The “invariant” mass, $m^*_{\omega}$, is defined by $\sqrt{q_0^2 - q^2}$, where $q_0$ and $q = |\vec{q}|$ are the energy and three momentum of the ω, respectively, and they are chosen so that the real part of the dielectric function in the full propagator vanishes. We do not repeat the details of the calculation here. Instead, we show $m^*_{\omega}$ in Fig. [1] as a function of the nuclear density $\rho_B$. (For more information, see Ref.[8].) The result shown includes the effect of
\(\sigma\)-\(\omega\) mixing in nuclear matter, which is, however, not large below normal nuclear matter density \((\rho_0 = 0.15 \text{ fm}^{-3})\). Furthermore, for low \(q\) the separation between the longitudinal (L) and transverse (T) modes is very small.

Since the proposed experiment at GSI \([16, 18]\) might produce an \(\omega\) meson with nearly zero recoil in a nucleus, it should be sufficient to consider the \(\omega\) with low \(q\) and ignore the separation between the L and T modes. Using the results shown in the figure we shall parametrize the “invariant” mass of the \(\omega\) with \(q = 1\) MeV (the solid curve in Fig. 1) as a function of density. It is approximately given by

\[
m^*_\omega \simeq m_\omega - 312.45x + 199.40x^2 - 59.277x^3 + 8.8427x^4 - 0.52x^5,
\]

where all quoted numbers are in MeV, \(m_\omega(=783\text{ MeV})\) is the mass in free space and \(x = \rho_B/\rho_0\). This reproduces \(m^*_\omega\) well up to three times normal nuclear matter density.

Once one knows the density distribution of a nucleus, one can extract an effective potential for the \(\omega\) meson from the “invariant” mass, assuming local density approximation. Because the \(\omega\) consists of the (same-flavor) quark and antiquark, we expect that the \(\omega\) meson does not feel the repulsive, Lorentz vector potential generated by the nuclear environment. The total potential felt by the \(\omega\) is then given by \(m^*_\omega(r) - m_\omega\), where \(m^*_\omega\) now depends on the position from the center of the nucleus. In Fig. 2, we show the potential for an \(\omega\) meson in \(^{40}\text{Ca}\), together with the density distribution. We can see that the potential generated in QHD is rather deeper than that given by QMC. (To get the density distribution in QHD we have used the program of Horowitz \textit{et al.} \[21\].) In a nucleus the (static) \(\omega\)-meson field, \(\phi_\omega\), is then governed by the Klein-Gordon equation:

\[
\left[\nabla^2 + E^2_\omega - m^2_\omega(r)\right] \phi_\omega(\vec{r}) = 0.
\]

An additional complication, which has not been added so far, is the meson absorption in the nucleus. This requires an imaginary part for the potential to describe the effect. At the moment, we have not been able to calculate the in-medium width of the meson, or the imaginary part of the potential in medium, self-consistently within the model. In order to make a more realistic estimate for the meson-nucleus bound states,
we shall include the width of the $\omega$ meson in the nucleus assuming the phenomenological form:

$$\tilde{m}_\omega^*(r) = m_\omega^*(r) - \frac{i}{2} [(m_\omega - m_\omega^*(r))\gamma_\omega + \Gamma_\omega],$$

(3)

$$\equiv m_\omega^*(r) - \frac{i}{2}\Gamma_\omega^*(r),$$

(4)

where $\Gamma_\omega$ (=8.43 MeV) is the width in free space. In Eq. (4), $\gamma_\omega$ is treated as a phenomenological parameter chosen so as to describe the in-medium meson width, $\Gamma_\omega^*$.

According to the estimates in Refs. [6, 7], the width of the $\omega$ at normal nuclear matter density is not large, typically a few tens of MeV: $\Gamma_\omega^* \sim 30 - 40$ MeV. Thus, we calculate the single-particle energies using the values for the parameter appearing in Eq. (4), $\gamma_\omega = 0, 0.2$ and 0.4, which covers the estimated range. Thus we actually solve the following, modified Klein-Gordon equation:

$$\left[\nabla^2 + E_\omega^2 - \tilde{m}_\omega^2(r)\right] \phi_\omega(\vec{r}) = 0.$$  

Equation (5) has been solved in momentum space by the method developed in Ref. [22]. (We should mention that the advantage of solving the Klein-Gordon equation in momentum space is that it can handle quadratic terms arising in the potentials without any trouble, as was demonstrated in Ref. [22].)

Now we are in a position to show our main results. In Tables I and II the calculated single-particle energies for the $\omega$ meson are listed. (In Table II the results of QMC [19] are shown for comparison.) Our results suggest that one should expect to find bound $\omega$-nucleus states, as suggested by Hayano et al. [16, 18] and by our previous work [19]. We have found that much deeper levels are predicted in QHD than in QMC because of the stronger, attractive force in QHD – as shown in Fig.2. Note that the real part of the eigenenergy of the $\omega$ meson is very insensitive to the in-medium width. We may understand this quantitatively, because the correction to the real part of the eigenenergy should be of order $\Gamma_\omega^2/8m_\omega$, which is a few MeV (repulsive) if we choose $\Gamma_\omega^* \sim 100$ MeV. For a more consistent treatment, we need to calculate the in-medium meson width self-consistently within the model.
To summarize, we have calculated the single-particle energies for $\omega$-mesic nuclei using QHD and compared the results with those of QMC. Although the specific form for the width of the meson in medium could not be calculated in this model, our results suggest that one should observe $\omega$-nucleus bound states for a relatively wide range of the in-medium meson width. In particular, even in the light nuclei QHD gives very deep single-particle levels ($\sim 100$ MeV), while QMC predicts much shallower levels. If the $\omega$-nucleus bound states could be observed in the future it would enable us to distinguish between QHD and QMC.

Acknowledgment

We would like to thank R.S Hayano, S. Hirenzaki, H. Toki and W. Weise for useful discussions. This work was supported by the Australian Research Council.
References

[1] Quark Matter ’97, to be published in Nucl. Phys. A (1998).

[2] K. Saito, T. Maruyama and K. Soutome, Phys. Rev. C40, 407 (1989);
    H. Kurasawa and T. Suzuki, Prog. Theor. Phys. 84, 1030 (1990);
    J.C. Caillon and J. Labarsouque, Phys. Lett. B311, 19 (1993);
    H.-C. Jean, J. Piekarewicz and A.G. Williams, Phys. Rev. C49, 1981 (1994);
    T. Hatsuda, H. Shiomi and H. Kuwabara, Prog. Thor. Phys. 95, 1009 (1996).

[3] G.E. Brown and M. Rho, Phys. Rev. Lett. 66, 2720 (1991).

[4] T. Hatsuda and Su H. Lee, Phys. Rev. C46, R34 (1993);
    M. Asakawa and C.M. Ko, Phys. Rev. C48, R526 (1993);
    F. Klingl, N. Kaiser and W. Weise, Nucl. Phys. A624, 527 (1997);
    Su H. Lee, Phys. Rev. C57, 927 (1998).

[5] K. Saito, K. Tsushima and A.W. Thomas, Phys. Rev. C55, 2637 (1997); ibid. C56, 566 (1997) and references therein.

[6] B. Friman, preprint GSI-98-7, nucl-th/9801053.

[7] F. Klingl and W. Weise, talk at the XXXVI Int. Winter Meeting on Nuclear Physics, hep-ph/9802211.

[8] K. Saito, K. Tsushima, A.W. Thomas and A.G. Williams, ADP-98-8/T287, nucl-th/9804015, to appear in Phys. Lett. B.

[9] P. Wurm for the CERES collaboration, Nucl. Phys. A590, 103c (1995).

[10] M. Masera for the HELIOS collaboration, Nucl. Phys. A590, 93c (1995).

[11] G.Q. Li, C.M. Ko and G.E. Brown, Nucl. Phys. A606, 568 (1996);
    G. Chanfray and R. Rapp and J. Wambach, Phys. Rev. Lett. 76, 368 (1996).
[12] M. Kossov et al., TJNAF proposal No. PR-94-002 (1994); See also, P.Y. Bertin and P.A.M. Guichon, Phys. Rev. C42, 1133 (1990).

[13] HADES proposal, see HADES home page: http://piggy.physik.uni-giessen.de/hades/

[14] G.J. Lolos et al., Phys. Rev. Lett. 80, 241 (1998).

[15] M. Mirazita et al., Phys. Lett. B407, 225 (1997).

[16] R.S. Hayano et al., experimental proposal for GSI/SIS, September (1997);
R.S. Hayano, talk given at 2nd international symposium on symmetries in subatomic physics, University of Washington, Seattle, June 25-28 (1997);
R.S. Hayano and S. Hirenzaki, contribution paper (III-E-1), Quark Matter ’97, Tsukuba, December 1-5 (1997).

[17] T. Yamazaki et al., Z. Phys. A355, 219 (1996).

[18] R.S. Hayano, S. Hirenzaki and A. Gillitzer, nucl-th/9806012.

[19] K. Tsushima, D.H. Lu, A.W. Thomas and K. Saito, preprint ADP-98-28/T302, nucl-th/9806043.

[20] B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. 16, 1 (1986).

[21] C.J. Horowitz, D.P. Murdoch and B.D. Serot, in Computational Nuclear Physics 1, edited by K. Langanke, J.A. Maruhn and S.E. Koonin (Springer-Verlag, Berlin, 1991), p.129.

[22] Dinghui H. Lu and Rubin H. Landau, Phys. Rev. C49, 878 (1994);
Y.R. Kwon and F. Tabakin, Phys. Rev. C18, 932 (1978).
Table 1: Calculated $\omega$ meson single-particle energies in QHD, $E = Re(E_\omega - m_\omega)$, and full widths, $\Gamma$, (both in MeV) in various nuclei, where the complex eigenenergies are, $E_\omega = E + m_\omega - i\Gamma/2$. See Eq. (4) for the definition of $\gamma_\omega$. In the light of $\Gamma$ in Refs. [6, 7], the results with $\gamma_\omega = 0.2$ are expected to correspond best with experiment. The first three nuclei are the final nuclei in the proposed experiment using the $(d,^3\text{He})$ reaction at GSI [16, 18].

|       | $\gamma_\omega=0$ | $\gamma_\omega=0.2$ | $\gamma_\omega=0.4$ |
|-------|------------------|------------------|------------------|
| $^6\text{He}$ | $E$ | $\Gamma$ | $E$ | $\Gamma$ | $E$ | $\Gamma$ |
| 1s     | -97.4 | 7.9   | -97.4 | 33.5 | -97.2 | 59.1   |
| $^{11}\text{B}$ | 1s | -129.0 | 8.0   | -129.0 | 38.5 | -128.9 | 69.0   |
| $^{20}\text{Mg}$ | 1s | -143.6 | 8.2   | -143.6 | 39.8 | -143.6 | 71.5   |
| 1p     | -120.9 | 7.9   | -120.9 | 37.8 | -120.9 | 67.7   |
| 2s     | -80.7  | 7.7   | -80.7  | 33.2 | -80.6  | 58.8   |
| $^{16}\text{O}$ | 1s | -134.1 | 8.1   | -134.1 | 38.7 | -134.0 | 69.3   |
| 1p     | -103.4 | 7.9   | -103.4 | 35.5 | -103.4 | 63.3   |
| $^{40}\text{Ca}$ | 1s | -147.6 | 8.2   | -147.6 | 40.1 | -147.6 | 72.0   |
| 1p     | -128.7 | 8.0   | -128.6 | 38.3 | -128.6 | 68.6   |
| 2s     | -99.8  | 7.8   | -99.8  | 35.6 | -99.8  | 63.5   |
| $^{90}\text{Zr}$ | 1s | -154.3 | 8.3   | -154.3 | 40.6 | -154.3 | 73.0   |
| 1p     | -143.3 | 8.2   | -143.3 | 39.8 | -143.3 | 71.4   |
| 2s     | -123.4 | 8.0   | -123.4 | 38.0 | -123.4 | 68.0   |
| $^{208}\text{Pb}$ | 1s | -157.4 | 8.4   | -157.4 | 40.8 | -157.4 | 73.3   |
| 1p     | -151.3 | 8.3   | -151.3 | 40.5 | -151.3 | 72.7   |
| 2s     | -139.4 | 8.1   | -139.4 | 39.5 | -139.4 | 70.8   |
Table 2: As in Fig.1, but for QMC.

|       | $\gamma=0$ |       |       | $\gamma=0.2$ |       |       | $\gamma=0.4$ |       |
|-------|------------|-------|-------|--------------|-------|-------|--------------|-------|
|       | $E$   | $\Gamma$ | $E$   | $\Gamma$ | $E$   | $\Gamma$ | $E$   | $\Gamma$ |
| $^6$He | 1s    | -55.7 | 8.1   | -55.6 | 24.7 | -55.4 | 41.3 |
| $^{11}$B | 1s    | -80.8 | 8.1   | -80.8 | 28.8 | -80.6 | 49.5 |
| $^{20}$Mg | 1s    | -99.7 | 8.2   | -99.7 | 31.1 | -99.7 | 54.0 |
|       | 1p    | -78.5 | 8.0   | -78.5 | 29.4 | -78.4 | 50.8 |
|       | 2s    | -42.9 | 7.9   | -42.8 | 24.8 | -42.5 | 41.9 |
| $^{16}$O | 1s    | -93.5 | 8.1   | -93.4 | 30.6 | -93.4 | 53.1 |
|       | 1p    | -64.8 | 7.9   | -64.7 | 27.8 | -64.6 | 47.7 |
| $^{40}$Ca | 1s    | -111.3 | 8.2   | -111.3 | 33.1 | -111.3 | 58.1 |
|       | 1p    | -90.8 | 8.1   | -90.8 | 31.0 | -90.7 | 54.0 |
|       | 2s    | -65.6 | 7.9   | -65.5 | 28.9 | -65.4 | 49.9 |
| $^{90}$Zr | 1s    | -117.3 | 8.3   | -117.3 | 33.4 | -117.3 | 58.6 |
|       | 1p    | -104.8 | 8.2   | -104.8 | 32.3 | -104.8 | 56.5 |
|       | 2s    | -86.4 | 8.0   | -86.4 | 30.7 | -86.4 | 53.4 |
| $^{208}$Pb | 1s    | -118.5 | 8.4   | -118.4 | 33.1 | -118.4 | 57.8 |
|       | 1p    | -111.3 | 8.3   | -111.3 | 32.5 | -111.3 | 56.8 |
|       | 2s    | -100.2 | 8.2   | -100.2 | 31.7 | -100.2 | 55.3 |
Figure 1: The “invariant” mass of the $\omega$ meson in matter, including $\sigma$-$\omega$ mixing. The solid curve is for $q = 1$ MeV, where the L and T modes are almost degenerate. The dashed curves are for $q = 500$ MeV, in which case the L and T modes are well separated.
Figure 2: Potentials for the $\omega$ meson and the density distributions in $^{40}$Ca. The results for QHD are shown by dashed curves, while those for QMC are shown by solid curves.