Excitation and emission of electron cyclotron waves in spherical tori

A.K. Ram and A. Bers

Plasma Science & Fusion Center, MIT, Cambridge, MA, 02139, USA
E-mail: abhay@mit.edu

Received 3 December 2002, accepted for publication 15 September 2003
Published 3 November 2003
Online at stacks.iop.org/NF/43/1305

Abstract

The conventional ordinary and extraordinary modes in the electron cyclotron range of frequencies are not suitable for heating of and/or driving currents in spherical tori (ST) plasmas. However, electron Bernstein waves (EBWs) offer an attractive possibility for heating and current drive in this range of frequencies. In this paper, we summarize our theoretical and numerical results that describe the excitation of EBWs in ST plasmas when the extraordinary mode or the ordinary mode are coupled into the plasma from an external source. In our discussion on the conversion of the ordinary mode to EBWs (via the slow extraordinary mode) we illustrate very important physics, relevant to this conversion process, that has been ignored in previous studies. This particular physics has to do with the conversion of the slow extraordinary mode to the fast extraordinary mode that can then propagate out of the plasma and thus reduce the mode conversion to EBWs. This reduction in the mode conversion can occur even when the wave numbers are such that the ordinary mode cutoff and the slow extraordinary mode cutoff are coincident in space. Furthermore, we also consider the emission of EBWs from a thermal plasma. This emission mode converts to extraordinary and ordinary modes in the vicinity of the upper hybrid resonance. We describe the general relationship between the conversion coefficients when exciting EBWs using either the extraordinary mode or the ordinary mode, and the emission coefficients when thermally emitted EBWs convert to the extraordinary and ordinary modes.

PACS numbers: 52.35.-g, 52.35.Hr

1. Introduction

Plasmas in high-$\beta$ spherical tori (ST), e.g. in NSTX [1] and MAST [2], present a special experimental challenge when considering heating and current drive by waves in the electron cyclotron range of frequencies. This is primarily due to such plasmas being overdense, i.e. $\omega_{pe}/\omega_{ce} \gg 1$ where $\omega_{pe}$ and $\omega_{ce}$ are the electron plasma and electron cyclotron frequencies, respectively. For fundamental or second harmonic heating in such plasmas, the conventional extraordinary X mode and the ordinary O mode are cut off near the edge of the plasma and cannot access the core. For higher harmonics the plasma is essentially transparent to the X and O modes.

Mode conversion near the plasma edge allows X or O mode polarized power, incident from free space, to couple to electron Bernstein waves (EBWs). The X mode couples to EBWs in the vicinity of the upper hybrid resonance (UHR). The O mode coupling to EBWs is via the slow X mode whereby power from the externally excited O mode is first mode converted to the slow X mode, which subsequently mode converts to EBWs near the UHR. The propagation of EBWs is not density limited and the waves damp effectively on electrons in the vicinity of the Doppler-shifted electron cyclotron resonance (or its harmonics) [3]. In this paper, we discuss theoretical details of the constraints imposed on the mode conversion of X and O modes to the EBWs in ST. We shall refer to the two conversion process as X–B and O–B, respectively, with an understanding that the O–B process requires an intermediate step whereby the O mode converts to the slow X mode.

From ray tracing, we have previously shown [3] that EBWs damp very effectively on electrons in the vicinity of the Doppler-shifted electron cyclotron resonance (or its harmonics). Consequently, EBWs can also be emitted by a thermal plasma from near these absorption regions. Such EBWs can propagate out to the UHR at the edge of the plasma where they mode convert to the X and O modes. The X and O modes propagate towards the lower density region and the vacuum, at the edge where they become free space propagating modes and are observed in the region outside the plasma. A number of experiments have studied this emission of EBWs in ST [4–8]. We have also shown that the mode conversion coefficients for EBW emission are directly related to the mode conversion coefficients for the excitation of EBWs by sources.
external to the plasma [9, 10]. This will be illustrated through results obtained from numerical solutions of a full-wave kinetic description of mode conversion among EBW and the X and O modes.

It is important to note that when we mention X and O modes in this paper we are really discussing ‘quasi-X’ and ‘quasi-O’ modes. It is only for purely perpendicular propagation across the ambient magnetic field that we have X and O modes—the X mode being elliptically polarized and the O mode being linearly polarized. For oblique propagation the polarizations are mixed. However, the two modes can still be clearly distinguished for oblique propagation.

2. Excitation of EBWs—cold plasma model

We can obtain important analytical results by first considering a simplified model in which the plasma is assumed to be cold and the poloidal magnetic field is ignored. The wave fields are assumed to be independent of any poloidal variations. Then the excitation of EBWs is studied in a slab geometry model where the z coordinate is in the direction of the inhomogeneity, y is along the poloidal direction, and z along the toroidal direction. Assuming that the magnetic field is along z, the propagation of waves in an inhomogeneous cold and collisionless plasma is given by

\[ K_{\perp} \frac{d^2 E_y}{dx^2} + \frac{\omega^2}{c^2} \left( K_{\perp}^2 - K_X^2 - \frac{c^2}{\omega^2} K_{\perp}^2 \right) E_y = k_I K_X F \]  

\[ \frac{d}{dx} \left[ \frac{1}{K_{\perp}} \frac{dF}{dx} \right] + \frac{\omega^2}{c^2} \left( 1 - \frac{c^2}{\omega^2} K_{\perp}^2 \right) F = \frac{\omega^2}{c^2} K_X \frac{d}{dx} E_y \]

where \( F = ik_I E_y - dE_z/dx, \) dF/dx = \( (\omega^2/c^2)K_{\perp}E_z, \) c is the speed of light, \( \omega \) is the wave frequency, \( k_I \) is the component of the wave vector \( \vec{k} \) along the magnetic field \( \vec{B} = \hat{z}B_0(y), \)

\[ K_{\perp} = 1 - \frac{\omega_{ce}^2(x)}{\omega^2(x) - \omega_{ce}^2(x)} \]

\[ K_X = -\frac{\omega_{ce}(x)}{\omega} \left( \frac{\omega_{pe}(x)}{\omega^2 - \omega_{pe}^2} \right) \]

\[ K_{\parallel} = 1 - \frac{\omega_{pe}(x)}{\omega^2} \].

The variation of the fields in the z directions is assumed to be of the form exp(ikz) and the time dependence is assumed to be of the form exp(−iot).

For the case when \( k_I = 0, \) (1) and (2) are uncoupled and describe the propagation of the X mode and the O mode, respectively, across the magnetic field. The X mode equation (1) has a resonance at \( K_{\perp} = 0 \) corresponding to the UHR. Equation (1) also has two cutoffs corresponding to the zeros of \( K_{\perp}^2 - K_X^2. \) The resonance absorption of X mode power at the UHR corresponds to the power mode converted to the EBWs [3]. Since the O mode is completely decoupled from the X mode, if an O mode is excited at the edge there will be no resonance absorption. The power will be reflected back out at the O mode cutoff located at the position where \( K_{\perp} = 0. \) In this analysis, we also assume that there is no electron cyclotron resonance between the edge of the plasma and the cutoffs of the X and O modes. This is generally true in NSTX and MAST. Thus, the only absorption is at the UHR.

If we assume that, for sufficiently small \( n_I \approx \omega_{ce}/\omega, \) only the X mode is excited at the plasma boundary, then the equation for X mode propagation is approximately given by

\[ (K_{\perp} - n_I^2) \frac{d^2 E_y}{dx^2} + \frac{\omega^2}{c^2} [(K_{\perp} - n_I^2)^2 - K_X^2] E_y = 0 \]

This equation is obtained from (1) and (2) by assuming that \( E_z \approx 0. \) In this approximation, the X mode resonance is no longer at \( K_{\perp} = 0 \) but at \( K_{\perp} = n_I^2, \) which, for small \( n_I, \) is a small correction. In this case (4) describes the propagation of the X mode into a region of increasing density as it passes through the right-hand cutoff, the UHR, and the left-hand cutoff. We refer to this as the triplet cutoff–resonance–cutoff mode conversion process that has been discussed extensively in [11]. Following the recipe provided in [11], we find that the fraction of the power on the X mode that is converted to EBWs is given by

\[ C_{XB}(\eta, \phi) = 4e^{-\pi \eta}(1 - e^{-\pi \eta}) \cos^2(\phi) \]

where

\[ \phi = \frac{\pi}{2} + \frac{1}{2} \cos^{-1}\left( \frac{N_R^2 - N_k^2}{N_R^2 + N_k^2} \right) \]

\[ N = \ln \left( \frac{\eta}{\eta_1} \right) + \pi \left[ i \coth \left( \frac{\pi}{2} \eta \right) + \cot \left( \frac{\eta}{2} \right) \right] - \frac{1}{\eta_1} - i \eta \]

\[ \psi \left( 1 + \frac{\eta}{2} \right) - \psi \left( 1 + \frac{i \eta}{2} \right) \]

\[ \psi \]

is the Psi function, and

\[ \eta = (1 - n_I^2)^{3/2} \left[ \frac{\omega_{pe}L_n}{\alpha \omega} (\sqrt{1 + \omega^2} - 1) \right]_{\text{UHR}} \]

\[ \eta_1 = (1 - n_I^2)^{3/2} \left[ \frac{\omega_{pe}L_n}{\alpha \omega} (\sqrt{1 + \omega^2} + 1) \right]_{\text{UHR}} \]

In (8) and (9), the right-hand sides are evaluated at the UHR \((K_{\perp} = 0), \) \( L_n \) is the density scalelength, and \( \alpha = \omega_{pe}/\omega_{ce}. \)

In figure 1, we plot

\[ C_{XB}^{(\text{max})} = 4e^{-\pi \eta}(1 - e^{-\pi \eta}) \]

as a function of \( \eta. \) This gives the envelope of (5), i.e. the maximum possible fractional power of the X mode that can be mode converted to EBWs. Alternatively, from (5),

\[ C_{XB}^{(\text{max})} = C_{XB}(\eta, \pi \eta) \]

where \( n \) is any integer including zero. For \( \eta \approx 0.22, \) \( C_{XB}^{(\text{max})} = 1, \) which implies that all of the X mode power is converted to EBW. From figure 1 we note that \( C_{XB}^{(\text{max})} > 0.5 \) if \( 0.05 \lesssim \eta \lesssim 0.6. \) For \( n_I = 0, \) from (8) we find that the necessary condition for the X-B mode conversion to be better than 50% requires

\[ 0.013 T \text{ cm} \lesssim |BL_n|_{\text{UHR}} \lesssim 0.16 T \text{ cm} \]
This shows that the X–B mode conversion process is efficient for sharp density gradients, i.e. short density scalelengths. As \( n_\parallel \) is increased from zero, the fraction of the X mode power that is coupled to EBWs decreases.

From (2), it is clear that for \( n_\parallel = 0 \) the O mode does not couple power to the EBW. For the O mode power to couple to EBW, it is necessary to have oblique propagation of the O mode, i.e. \( n_\parallel \neq 0 \). Then the O mode power can couple to the X mode, which, in turn, can undergo resonance absorption. The necessary condition for optimum coupling of the O mode power to the X mode is [12]

\[
n_\parallel,c = \frac{c k_\parallel}{\omega} = \left[ 1 + \frac{\omega}{\omega_{p_e}} \right]^{-1/2},
\]

where the right-hand side is to be evaluated at the spatial location where \( \omega = \omega_{p_e} \). However, in order for the O mode power not to get reflected back out on the X mode requires that \( \eta > 1 \) [3,9]. From (8) we find that this condition is satisfied when the density scalelength is longer than that required for optimum mode conversion from the X mode.

From the above discussion, we conclude that the X–B and the O–B mode conversion processes are optimized in different regions of the two-dimensional parameter space spanned by wave frequency and parallel wavelength, and at different locations inside the plasma. The X–B mode conversion process is optimized for \( k_\parallel \approx 0 \) and wave frequencies that place the UHR in the short density scalelength region. The O–B mode conversion process is optimized for \( k_\parallel \) given in (12) and for wave frequencies that place the UHR in a longer density scalelength region of the plasma. The optimum X–B process occurs closer to the edge of the plasma than the optimum O–B conversion.

3. Excitation of EBWs—kinetic plasma model

We extend the analysis of the previous section to include the EBW explicitly into the mode conversion equations. Furthermore, we will explicitly include the poloidal magnetic field as this forms an important component of the mode conversion equations in a ST plasma. The development of the approximate kinetic analysis has been detailed previously [3,9,13]. Here, we just provide the basic equations for completeness.

The equilibrium magnetic field is assumed to be sheared with the form

\[
\vec{B}_0(x) = B_y(x) \hat{y} + B_z(x) \hat{z} = B_0(x) \sin \Psi(x) \hat{y} + B_0(x) \cos \Psi(x) \hat{z}
\]

where \( \Psi \) is the angle between \( \vec{B}_0 \) and the \( z \)-axis. The variation of the fields in the \( y \) and \( z \) directions is assumed to be of the form \( \exp(ik_y y + ik_z z) \) where \( k_y \) and \( k_z \) are the appropriate components of the wave vectors. The approximate full wave description of the propagation of the X mode, O mode, and the EBWs is given by

\[
\frac{d \vec{F}}{d \xi} = i \vec{A}_k \cdot \vec{F}
\]

where \( \xi = \omega x/c \) is the normalized spatial variable,

\[
\vec{F}^T = [E_x E_y E_z (i\chi_1 E_x + B_z( -c B_y))] (15)
\]

is the transpose of the field vector \( \vec{F} \), \( E'_x = (dE_x/d\xi) \),

\[
\vec{A}_k = \begin{bmatrix} 0 & 0 & -\chi_1^{-1} & 0 & 0 \\ n_y & 0 & 0 & 0 & 1 \\ n_z & 0 & 0 & 0 & 1 \\ K_{x y} & \chi_{x y} & \chi_{x z} & 0 & n_y n_z \\ -\chi_{x y} & K_{y y} - n_z^2 & \chi_{x z} + n_z n_y & 0 & 0 \\ -\chi_{x z} & n_z n_y & K_{z z} - n_y^2 & 0 & 0 \end{bmatrix} (16)
\]

\[
\vec{K} = I + \vec{\chi}
\]

\[
\chi = -\frac{\omega_p^2}{(\omega^2 - \omega_{pe}^2)} \begin{bmatrix} 1 & -i \omega_{ce}/\omega & i \omega_{ce}/\omega \\ -i \omega_{ce}/\omega & 1 - \omega_{ce}^2/\omega^2 & -\omega_{ce} \omega_{ce}/\omega^2 \\ -i \omega_{ce}/\omega & -\omega_{ce} \omega_{ce}/\omega^2 & 1 - \omega_{ce}^2/\omega^2 \end{bmatrix} (18)
\]

where \( \omega_p(x) \) is the electron plasma angular frequency, \( \omega_{ce}(x) = c B_z(x)/m_e \) and \( \omega_{ce}(x) = c B_z(x)/m_e \) are the electron cyclotron angular frequencies for the poloidal and toroidal fields, respectively, \( \omega_{ce} = \sqrt{\omega_{ce}^2 + \omega_{ce}^2} \), \( n_y = c k_y/\omega \), \( n_z = c k_z/\omega \),

\[
\chi_1 = \left( \frac{v_T}{c} \frac{\omega_{pe}}{\omega_k} \right) \left( \frac{\omega^2}{\omega^2 - \omega_{ce}^2} - \frac{\omega_{ce}^2}{\omega^2 - 4 \omega_{ce}^2} \right) (19)
\]

and \( v_T = \sqrt{T(x)/m_e} \) is the electron thermal velocity corresponding to the temperature \( T(x) \).

The only component of the electromagnetic wave field whose evolution is not given by (14) is \( B_z \), which can be obtained from Faraday’s equation [9]

\[
c B_z = n_y E_z - n_z E_y
\]

Thus, the kinetic full-wave dynamics given by (14)–(20) describe the coupling between the X mode, the O mode, and the EBW, i.e. the kinetic mode conversion process. The procedure to solve these equations numerically with the appropriate boundary conditions has been detailed in [9].
The total, electromagnetic and kinetic, time-averaged energy flow density in the $x$ direction is [3, 13]

$$\langle \tilde{s} \rangle_x = \frac{1}{4} \frac{e_0}{\mu_0} \bar{F}^\dagger \cdot \bar{F} \quad (21)$$

where $\bar{F}^\dagger$ is the transpose of the complex conjugate of $\bar{F}$ and

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then, from (14) we find that

$$\frac{d}{d\bar{e}} (\bar{F}^\dagger \cdot \bar{R} \cdot \bar{F}) = 0 \quad (23)$$

This equation is needed to ensure that the numerical scheme solving the mode conversion equations is conserving the total, electromagnetic and kinetic, time-averaged energy flow density. The conservation condition is consistent with the assumption that there is no damping in the mode conversion region; i.e., we assume that the EBW damping at the Doppler-shifted electron cyclotron harmonic occurs away from the mode conversion region inside the plasma.

In the plasma region away from where the mode conversion occurs, the solutions to (14) can be obtained using the WKB technique. In this case, the $X$ mode, the $O$ mode, and the EBWs are distinctly identifiable, and the energy flow density along these waves can be determined using (21).

The various mode conversion coefficients are obtained by the appropriate ratios of these energy flow densities.

The mode conversion equations (14)–(20) are not amenable to analytical solutions. We have to resort to a fully numerical integration of (14)–(20) with the appropriate boundary conditions. To illustrate the physics involved in the mode conversion coupling between the $X$ mode, the $O$ mode, and the EBWs, we will study numerical solutions obtained for NSTX-type parameters [3] corresponding to a high-$\beta$ scenario.

The specific plasma parameters are as follows. The Shafranov-shifted major radius is $R = 1.05\ m$, the minor radius is $a = 0.44\ m$, the peak electron density is $n_0 = 3 \times 10^{19}\ \text{m}^{-3}$, the peak electron temperature is $T_0 = 3\ \text{keV}$, the density profile is $n_e = n_0 + (n_0 - n_E)(1 - x^2/a^2)^{1/2}$, and the temperature profile is $T_e = T_0 + (T_0 - T_E)(1 - x^2/a^2)^2$, where $n_E$ and $T_E$ are the edge density and temperature, respectively, with $n_E/n_0 = 0.02$ and $T_0/T_E = 0.02$. The magnetic field profile is taken to be that shown in figure 2.

4. Mode conversion to EBWs—no poloidal field

Let us consider the coupling of an externally launched $X$ or $O$ mode to EBWs. To test our analytical analysis of the previous section we will assume that, in (13), $\Psi(x) \equiv 0$ and $B_0$ is the same as $B$ (shown in figure 2). This primarily eliminates the effect of the poloidal field on the mode conversion process and simplifies the analysis. The influence of the poloidal field on mode conversion will be addressed in a subsequent section. In what follows in this section we will set $n_y = 0$ so that, consequently, $n_z = n_E$ is the wave number along the magnetic field. Without the poloidal magnetic field the $y$-variation is not needed since one can transform to a frame where the $z$-axis is along the magnetic field and the $x$-axis is perpendicular to it.

4.1. Mode conversion from an $X$ mode

Figure 3 compares the power mode conversion coefficient to EBWs from the $X$ mode for a range of frequencies. In this figure, we have $C_{XB}^{(\text{max})}$ as obtained from (10), $C_{XB}$ as obtained from (5)–(9), and $C_{XB}^{(\text{kin})}$ as obtained from the numerical solution of (14)–(20). For frequencies below about 15.5 GHz, $C_{XB}^{(\text{kin})}$ is bounded by $C_{XB}^{(\text{max})}$ and $C_{XB}^{(\text{kin})}$. This indicates that the theoretical maximum $C_{XB}^{(\text{kin})}$ is also a maximum of the kinetic energy flow density.
mode conversion process. Also, the analytical calculation of the phase in (5) is reasonably good. For frequencies greater than about 16 GHz there are differences between the analytical estimates and the kinetic numerical results. The reason for this difference is simple. For frequencies below 15.5 GHz the UHR is between the first and second harmonics of the electron cyclotron frequencies. For frequencies above 15.5 GHz the UHR is between the second and third harmonics of the electron cyclotron frequency. As the UHR frequency $\omega_{U}$ transits through these domains the character of the kinetic dispersion relation changes [9]. For $\omega_{U} < 2\omega_{ce}$ the slow X mode couples to the EBW at the UHR and does not propagate past the UHR towards the low density region of the plasma. For $\omega_{U} > 2\omega_{ce}$ the slow X mode propagates through the UHR towards the lower density region and couples to the backward propagating EBW closer to the edge of the plasma than the location of the UHR. This change in the character of the slow X mode coupling to the EBW is not part of the cold plasma description. It appears only in the kinetic formulation (see figures 6 and 7 in [9]). For $\omega_{U} < 2\omega_{ce}$ the mode conversion region is described by a cutoff–resonance–cutoff triplet [9, 11], while for $\omega_{U} > 2\omega_{ce}$ the propagation of the slow X mode through the UHR changes the description to a cutoff–cutoff doublet [9]. The oscillations in $C(k)$ are possibly due to the change in wave phase between the cutoffs. It is difficult to develop an analytical picture as the conversion equation, even for perpendicular propagation, is at least of fourth order. Even though the wave properties change for $\omega_{U} > 2\omega_{ce}$, $C(\max)$ still represents, approximately, the maximum power mode conversion coefficient. In plotting $C(\max)$ in figure 3 there is a gap around 15.5 GHz. Around this frequency the second electron cyclotron harmonic transits through the mode conversion region. Since electron cyclotron damping is not included in the formulation of the kinetic mode conversion equations [9], we do not carry out any computations in this region. An important reason for not including damping in the kinetic formulation is that it would not be desirable to have electron cyclotron resonances near the mode conversion region in actual experiments. The presence of such resonances would inhibit the mode conversion process and lead to electron absorption near the edge of the plasma where mode conversion in STs will take place.

Let us consider the mode conversion at a particular frequency and as a function of the toroidal wave number $n_z$. In figure 4, we plot the various power conversion coefficients for an incoming X mode frequency of 16.5 GHz. For $n_z = 0$ there is no coupling to the O mode as the power reflection coefficient $R_{XO}$ is zero. This is consistent with equations (1) and (2), where for $n_z = 0$ the O mode equation (2) is completely decoupled from the X mode equation (1). For small $n_z$ there is essentially no power coupled to the O mode.

From figure 3 we note that the conversion of the X mode to EBWs is most effective for frequencies around 16 GHz. The conversion of the O mode to the slow X mode has been extensively studied, theoretically, in [12, 15–19]. The coupling of O mode power to slow X mode occurs near a critical parallel wave number (given by equation (12)) when the left-hand X cutoff is near the O mode cutoff [12]. The previous studies have primarily looked at the transmission coefficient from the O mode to the slow X mode through the two respective cutoffs [12, 15, 16, 18, 19]. It is then assumed that the O mode power that tunnels to the slow X mode will couple to the EBW at the UHR. The transmission coefficient is dependent on the density scalelength in the vicinity of the cutoffs, and the range of parallel wavenumbers that can lead to effective transmission of the O mode to the slow X mode increases as the density scalelength decreases [16, 19]. These studies completely ignore the possibility that the slow X mode, in the vicinity of the UHR, can couple power to the fast X mode, which, subsequently, propagates out to the edge of the plasma and reduces the power available to the EBWs. For small scalelengths the right-hand X mode cutoff moves closer to the UHR, reducing $n$ in equation (8). This increases the tunnelling of the slow X mode to the fast X mode. So the condition for coupling effectively from the O mode to the slow X mode requires that the fast X mode cutoff be sufficiently removed from the UHR, i.e. $\eta > 1$, or long scalelengths.

4.2. Mode conversion from an O mode

The conversion of the O mode to the slow X mode has been extensively studied, theoretically, in [12, 15–19]. The coupling of O mode power to slow X mode occurs near a critical parallel wave number (given by equation (12)) when the left-hand X cutoff is near the O mode cutoff [12]. The previous studies have primarily looked at the transmission coefficient from the O mode to the slow X mode through the two respective cutoffs [12, 15, 16, 18, 19]. It is then assumed that the O mode power that tunnels to the slow X mode will couple to the EBW at the UHR. The transmission coefficient is dependent on the density scalelength in the vicinity of the cutoffs, and the range of parallel wavenumbers that can lead to effective transmission of the O mode to the slow X mode increases as the density scalelength decreases [16, 19]. These studies completely ignore the possibility that the slow X mode, in the vicinity of the UHR, can couple power to the fast X mode, which, subsequently, propagates out to the edge of the plasma and reduces the power available to the EBWs. For small scalelengths the right-hand X mode cutoff moves closer to the UHR, reducing $n$ in equation (8). This increases the tunnelling of the slow X mode to the fast X mode. So the condition for coupling effectively from the O mode to the slow X mode requires that the fast X mode cutoff be sufficiently removed from the UHR, i.e. $\eta > 1$, or long scalelengths.
Figure 5 shows the power conversion coefficients, as a function of \( n_z \), when an O mode, with a frequency of 16.5 GHz, is launched into the plasma. From equation (12), the critical value of \( n_z,c \equiv n_{z,c} \) is approximately 0.57. In the proximity of this critical value we note that the reflection on the O mode \( R_{OO} \) is nearly zero and the mode converted power \( C_{OB} \) is a maximum. However, this maximum value is less than 0.5, which implies that less than half of the input O mode power is being coupled to the EBWs even though we are at the optimum \( n_z \). The rest of the power is being reflected back out on the X mode and is indicated by \( R_{OX} \). This important effect has been completely ignored in previous studies of O–B mode conversion and is very important to properly understand the mode conversion of the O mode to EBWs via the slow X mode.

If we increase the O mode frequency to 28 GHz, the corresponding power conversion coefficients are plotted in figure 6. For this frequency the critical value of \( n_z,c \equiv n_{z,c} \) is approximately 0.47. We now note, from \( C_{OB} \), that nearly all of the O mode power is converted to the EBWs. The reflection out on the X mode \( R_{OX} \) is small. When \( C_{OB} \) is a maximum, \( R_{OO} \) is a minimum. From numerical calculations we find that 28 GHz is approximately the lowest frequency where 100% mode conversion is possible in the O–B mode conversion scenario. This corresponds to higher harmonic resonances being inside the plasma than for the case of optimized X–B conversion.

A comparison of figures 5 and 6 provides a physical insight into the conversion of the O mode to EBWs. The range of \( n_z \) for which \( C_{OB} \) is about 50% of its maximum value increases as the frequency decreases. A decrease in the frequency moves the mode conversion region farther towards the edge of the plasma into the region of small density scalelengths. This picture is consistent with previous theories on the conversion of the O mode to EBWs [12, 15, 16, 18, 19]. However, the maximum power mode conversion coefficient decreases with decreasing frequency as power reflected on the X mode, \( R_{OX} \), increases. The reflection coefficient \( R_{OX} \) is proportional to \( \exp(-\pi \eta) \) with \( \eta \) given in equation (8). As the scalelength becomes smaller, \( \eta \) becomes smaller and \( R_{OX} \) increases. Thus, for the efficient conversion of O mode power to EBWs, it is not only necessary to have a critical \( n_{z,c} \) (given by (12)) but, importantly, to also have \( \eta > 1 \).

A comparison of the X–B and O–B conversion processes shows that the X–B conversion is optimized for lower frequencies leading to the mode conversion region being in the short density scalelength part of the plasma. The optimum O–B conversion requires longer density scalelengths and occurs farther into the plasma. Furthermore, the X–B mode conversion process is efficient over a wider range of parallel wavenumbers than the O–B conversion process.

5. Mode conversion to EBWs—effects of poloidal field

Let us now study the changes in the mode conversion process due to the poloidal magnetic field. This also requires that we include the effect of the poloidal wave numbers \( n_y \). The parameters used for our studies are NSTX-type discussed earlier and the magnetic field is as shown in figure 2. The major effect of including the poloidal field is that the power flow mode conversion coefficients are no longer symmetric in \( n_y \) (or \( n_z \)). In figure 7 we plot the fractional power mode converted to EBWs from an X mode, \( C_{XB} \), as function of \( n_y \) for three different values of \( n_z \). The asymmetry in both \( n_y \) and \( n_z \) is evident. This asymmetry is further amplified for the O–B conversion process as illustrated in figure 8. (In the case of figure 6, corresponding to no poloidal field, the conversion coefficients are found to be completely symmetric around \( n_y = 0 \).) For the O–B conversion process it is important to choose the O mode launch angle appropriately with respect to the poloidal field and the toroidal field. For X–B conversion there is a broad range of \( n_y \) and \( n_z \) for effective conversion to EBWs.

6. Emission of EBWs

From ray tracing analysis we have shown that EBWs are locally and strongly absorbed at the Doppler-shifted electron cyclotron
Excitation and emission of electron cyclotron waves

Figure 7. Fractional power mode converted to EBWs, $C_{XB}$ as a function of $n_y$, for an externally launched X mode for three different values of $n_z$. The wave frequency is 16.5 GHz and the poloidal field is included in the calculations.

Figure 8. Fractional power mode converted to EBWs, $C_{OB}$ as a function of $n_y$, for an externally launched O mode for three different values of $n_z$. The wave frequency is 28 GHz and the poloidal field is included in the calculations.

resonance or its harmonics [3]. The strong and localized absorption implies that thermal emission of EBWs can occur for frequencies corresponding to the local Doppler-shifted electron cyclotron frequency. This emission then converts, at the UHR, to the X and O modes that are then observed in the vacuum region. The excitation and emission of EBWs has been observed experimentally on ST [4–8].

The kinetic model we have developed allows us to study the mode conversion of emitted EBWs to X and O modes. This is not possible in the cold plasma model where the power flow conversion coefficient to the EBW is given by the power resonantly absorbed at the UHR. In order to study the emission we need to have a proper description of the EBWs.

Figure 9 shows the mode conversion emission coefficients for an EBW propagating out to the edge and converting to X and O modes at the UHR. The emission coefficients of the X and O modes are given by $E_X$ and $E_O$, respectively, and $R_B$ is the fraction of EBW emitted power that is reflected back into the plasma.

is the fraction of the emitted EBW that is reflected back into the plasma. The results in figure 9 are for the case of no poloidal field and the parameters are similar to those in section 4.

If we compare figure 9 with figures 4 and 5 it becomes evident that

\[ E_X = C_{XB} \quad \text{and} \quad E_O = C_{OB} \]  
\[ (24) \]

In other words, the fraction of the X mode power flow that is mode converted to EBWs when an X mode is launched from the low density side is the same as the emitted EBW energy flow that is mode converted to an X mode that propagates out into the low density region. Similarly, the fraction of the O mode energy flow that is mode converted to EBWs when an O mode is launched from the low density side is the same as the emitted EBW energy flow that is mode converted to an O mode that propagates out into the low density region. The emitted X and O modes propagate out into the vacuum region where they are observed in experiments.

Also upon comparing figures 4 and 5 we note that

\[ R_{XO} = R_{OX} \]  
\[ (25) \]

This implies that for an X mode launched from the outside, the fraction of the X mode power that is reflected back out on the O mode is the same as the fraction of the O mode power that is reflected back out on the X mode for an O mode launched from the outside.

The symmetry relations in (24) and (25) are, in fact, quite general and can be derived, from first principles, using considerations of energy flow conservation and energy flow under time reversal [9,10]. Thus, the symmetry relations hold even for the case when the effect of the poloidal field is included in our studies of mode conversion.

The equality between the emission coefficients and the excitation coefficients also points to the fact that optimized mode conversion heating and current drive experiments can be designed on the basis of the optimized emission results. The...
7. Conclusions

From our studies of the mode conversion process in which power from an externally launched X or O mode can be coupled to EBWs in a ST plasma, we note that two processes, X–B and O–B, are optimized in different regions of parameter space spanned by the parallel wave numbers and the wave frequency. The X–B conversion is optimized for a broad range of parallel wave numbers with a peak at, or near, perpendicular (to the magnetic field) propagation. The O–B conversion process is optimized over a narrower range of parallel wave numbers and peaks around a critical parallel wave number for which the O mode cutoff and the left-hand X mode cutoff coincide. Furthermore, the O–B conversion process is a maximum at a higher wave frequency compared to the X–B conversion process. Consequently, the O–B conversion takes place farther into the plasma than the X–B conversion. This implies that the density gradient scalelength within the mode conversion region is longer for the O–B process than for the X–B process. If the O–B process is moved into the region of short density scalelengths, part of the O mode power is reflected back out on the X mode. This effect has been ignored in previous studies. These studies showed that the short density scalelengths in the mode conversion region increased the range of parallel wave numbers for which the O–B conversion process was efficient. They completely ignored the coupling of the slow X mode to the fast X mode that neutralizes this proposed advantage.

We have shown that the mode conversion emission coefficients are related to the mode conversion excitation coefficients. This symmetry is very useful from an experimental point of view. Optimized mode conversion heating and current drive experiments can be designed on the basis of the emission results. The conditions for which the X mode (or the O mode) emission is most pronounced will be the conditions for which the X–B (or the O–B) mode conversion is maximized.

Acknowledgments

This work is supported by US Department of Energy Contracts DE-FG02-91ER-54109 and DE-FG02-99ER-54521.

References

[1] Ono M. et al 1998 Proc. 17th International Conf. on Fusion Energy 1998 (Yokohama, 1998) (Vienna: IAEA) CD-ROM file ICP/01 and http://www.iaea.org/programmes/ripc/physics/fec1998/pdf/icp_01.pdf
[2] Sykes A. 1998 Proc. 17th International Conf. on Fusion Energy 1998 (Yokohama, 1998) (Vienna: IAEA) CD-ROM file OV2/5 and http://www.iaea.org/programmes/ripc/physics/fec1998/pdf/ov2_5.pdf
[3] Ram A.K. and Schultz S.D. 2000 Phys. Plasmas 7 4084
[4] Taylor G. et al 2002 Phys. Plasmas 9 167
[5] Chattopadhyay P.K. et al 2002 Phys. Plasmas 9 752
[6] Shevchenko V., Cunningham G. and Field A. 2001 Abstracts of Invited and Contributed Papers: 28th European Physical Society (EPS) Conf. on Controlled Fusion and Plasma Physics ed C. Silva et al (European Physical Society) p 381 and http://www.cfn.ist.utl.pt/EPS2001/fin/pdf/p3.101.pdf
[7] Jones B. et al 2003 Phys. Rev. Lett. 90 165001
[8] Taylor G. et al 2003 Phys. Plasmas 10 1395
[9] Ram A.K., Bers A. and Lashmore-Davies C.N. 2002 Phys. Plasmas 9 409
[10] Bers A. and Ram A.K. 2002 Phys. Lett. A 301 442
[11] Ram A.K., Bers A., Schultz S.D. and Fuchs V. 1996 Phys. Plasmas 3 1976
[12] Preinhaelter J. and Kopecky V. 1973 J. Plasma Phys. 10 1
[13] Bers A., Ram A.K. and Schultz S.D. 1994 Proc. 2nd Europhysics Topical Conf. on RF Heating and Current Drive of Fusion Devices 1998 (Brussels, Belgium, 1998), vol 22A ed J. Jacquinet et al (Petit-Lancy: EPS) pp 237–40
[14] Ram A.K. and Bers A. 1991 Phys. Fluids B 3 1059
[15] Weitzner H. and Batchelor D.B. 1979 Phys. Fluids 22 1355
[16] Mjoljus E. 1984 J. Plasma Phys. 31 7
[17] Tokman M.D. 1985 Sov. J. Plasma Phys. 11 689
[18] Hansen F.R., Lynov J.P., Mardi C. and Petrillo V. 1988 J. Plasma Phys. 39 319
[19] Cairns R.A. and Lashmore-Davies C.N. 2000 Phys. Plasmas 7 4126