Propagation of dust ion acoustic solitary waves in dusty plasma with Boltzmann electrons

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Abstract. In this multispecies plasma model, consisting of negative mobile dusts, non-thermal ions and Boltzmann electrons, dust-ion acoustic solitary waves are studied through reductive perturbative technique by deriving corresponding Korteweg-de Vries (KdV) equation. The number of dust charge contained in a dust particle ($Z_d$) and the streaming speeds of mobile dusts ($u_{d0}$) and ions ($u_{i0}$) are observed to play very important role to form dust-ion acoustic compressive and rarefactive KdV solitons. Remarkably, some initial streaming of dust ($u_{d0}$) are found to be instrumental for both compressive and rarefactive KdV solitons in a short range of $Z_d$ separating both kinds by some asymptotic lines. Also, the presence of low dust charges and lower ion streaming, compressive and rarefactive KdV solitons of either concave or convex characters are shown to reflect. For the higher streaming of mobile dusts, the amplitudes of the rarefactive KdV solitons characteristically changes from higher to lower showing convex character for this plasma model. A rigorous theoretical investigation has been made to show how the number of dust charge contained in a dust particle ($Z_d$) drastically change the amplitudes of the compressive and rarefactive KdV solitons.

1. Introduction
The Ion-Acoustic Soliton (IAS) is one of the beautiful manifestations of nonlinear structures of nature and its study have started since seventy of the last century with very simple stage of two component pasmas. The study of solitary wave was started in 1985 by Korteweg and de Vries [1] and then Washimi and Taniuti [2] in 1966.

In last few decades, IASW have been studied by many workers by taking two and multispecies of the plasmas (not reported here). But gradually the study of multispecies plasmas have been extended to insertion of various characteristics effects viz. magnetized or unmagnetized, plasmas of either cold or warm, relativistic or non-relativistic effects, consideration of pressure variations, quantum effects etc. Dust particles are omnipresent and are invariably immersed in the ambient astrophysical and space plasma. The existence and the effects of dusty plasmas are nicely depicted in planetary rings, earth’s magnetosphere, stars’ neighbourhood, Cometary tails, Asteroid zones, Circumstellar disks, dark molecular clouds, interstellar clouds, nebulae etc. [3-8]. Also, because of the tremendous applications of Dust –Ion Acoustic Waves (DIAW) in the various theoretical and laboratory activities, DIAW has become very popular in the field of plasma physics [9-11].

Properties of solitary waves are greatly affected [12-16] by the presence of dust particles in space. In absence of magnetic field, static dust grains in plasma can generate extremely low frequency dust acoustic (DA) waves [17] and can modify the existing wave spectra [18]. Investigation of DIAWs under
various plasma compositions is studied by many authors [6, 19-21]. In a dusty plasma with inertial charged dust and Boltzmann distributed electrons and ions, it have been studied [17] that DIAWs propagates linearly as a normal mode and nonlinearly as subsonic solitons of either positive or negative electrostatic potentials. Again, DA solitary wave have been studied in a dusty plasma by taking non-thermal parameter [22] in the nonthermal distribution of ions as variable. Creation of spatial inhomogeneity is observed [23, 24] for the distribution of immobile dust particles. It is also shown that an unmagnetized dusty plasma with the effects of vortex-like and nonthermal ion, large amplitude rarefactive as well as compressive DA solitons exists [25]. In an unmagnetized multicomponent plasma, existence of solitary waves are investigated with pressure variations in ions, negative ions and isothermal electrons [26]. For stationery dust, DIA solitary waves are investigated for a plasma consisting ions and electrons with their pressure (or temperature) variations for both lower and higher order situations followed by a comparative study [27]. Formation of DIA compressive and rarefactive solitons are also shown to exist for a relativistic plasma with stationary [28] and mobile [29] dust. Again, DIA waves are studied for non-thermal Cairns like distribution of ions in plasma [30]. Dust acoustic (DA) waves have been investigated foe a dusty plasma showing the implicit role of Cairns distributed cold ions [31]. The existence of both positive and negative solitary structures in some parameter space have been investigated with sufficient non thermality in the ions and negative charges on the dust along with electrons [32] following Cairns distribution [33].

In this paper, the existence of both compressive and rarefactive DIA KdV solitons are shown for the multispecies plasma model consisting of negative mobile dust, non-thermal ions and Boltzmann electrons. Also, the characteristic role (in the formation of DIA solitary waves) of several plasma parameters especially number of dust charges (Zd) and streaming speeds of dusts (ud0) and ions (ui0) are reported in this paper. The paper is organized as ‘Introduction’ in section 1, ‘Basic equations governing the dynamics of motion’ in section 2, ‘derivation of Korteweg-de Vries (KdV) equation’ in section 3, ‘Solution of KdV equation’ in section 4, ‘Results and discussion’ in section 5, followed by the ‘References’.

2. Basic equations governing the dynamics of motion

In this model of multispecies plasma, to investigate the propagation of solitary waves, we consider negative mobile dusts, non-thermal ions and Boltzmann electrons as follows

For dust particles,
\[
\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0
\]
(1)

\[
\left( \frac{\partial}{\partial t} + u_d \frac{\partial}{\partial x} \right) n_d = Z_d \frac{\partial \varphi}{\partial x}
\]
(2)

for ions,
\[
\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0
\]
(3)

\[
\left( \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x} \right) n_i = -\frac{1}{Q} \frac{\partial \varphi}{\partial x}, \text{ where } Q = \frac{n_{do}}{n_{io}}
\]
(4)

for electrons
\[
n_e = e^\theta
\]
(5)

and the Poisson equation
\[
\frac{\partial^2 \varphi}{\partial x^2} = Z_d n_d + n_e - n_i
\]
(6)
where \( u_d, u_i \); \( m_d, m_i \) and \( n_d, n_i \) are velocities, masses and densities the dust and ions respectively and \( Z_d \) is the number of charges contained in a dust grain and \( n_e \) is the density of electrons.

3. Derivation of the Korteweg-de Vries (KdV) equation

In order to derive the KdV equation from the set of equations (1)-(6), we use the stretched variables in terms of the smallness parameter \( \varepsilon \) as

\[
\xi = \varepsilon^2 (x - Vt), \quad \tau = \varepsilon^2 V t
\]

with the phase velocity \( V \) of the waves. We expand the flow variables asymptotically about the equilibrium state in terms of the smallness parameter \( \varepsilon \) as

\[
\begin{align*}
  n_d &= n_{d0} + \varepsilon n_{d1} + \varepsilon^2 n_{d2} + \ldots \\
n_i &= n_{i0} + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \ldots \\
n_e &= n_{e0} + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \ldots \\
u_d &= u_{d0} + \varepsilon u_{d1} + \varepsilon^2 u_{d2} + \ldots \\
u_i &= u_{i0} + \varepsilon u_{i1} + \varepsilon^2 u_{i2} + \ldots \\
\varphi &= \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \ldots
\end{align*}
\]

Using the standard perturbation method, with the use of transformation (7), expansions (8) in the normalized set of equations (1)-(6), we determine the \( \varepsilon \)-order equations. From the coefficients of \( \varepsilon \)-order equations, on integration and using the boundary conditions

\[
n_{i0} = 0, \quad n_{e0} = 0; \quad \nu_{i0} = 0, \quad \nu_{e0} = 0 \quad \text{and} \quad \varphi_{i0} = 0 \quad \text{at} \quad |\xi| \to \infty
\]

the first-order perturbed quantities are obtained as

\[
\begin{align*}
n_{d1} &= -\frac{n_{d0} Z_d}{(u_{d0} - V)^2}, \quad n_{i1} = \frac{Z_d}{u_{d0} - V}, \quad n_{e1} = \frac{n_{e0}}{(u_{e0} - V)^2}, \\
u_{d1} &= -\frac{1}{Q(u_{d0} - V)}, \quad n_{e1} = 1
\end{align*}
\]

Also, the use of stretched variables (8) and the expansions (9) in equation (6) following equations yield

\[
1 - Z_d \sigma = \frac{1}{n_{d0}}, \quad \text{where} \quad \sigma = \frac{n_{d0}}{n_{i0}}
\]

and

\[
Z_d n_{d1} + n_{i1} - n_{i0} = 0
\]

We use the first-order quantities from (10) in (12), to get the phase velocity (V) equation as

\[
1 - Z_d \sigma - \frac{1}{Q(u_{i0} - V)^2} \left( \frac{Z_d}{u_{d0} - V} \right)^2 = 0
\]

Again, from the coefficients of \( \varepsilon^2 \)-order equations, the following equations are obtained

\[
\begin{align*}
B_d \frac{\partial \hat{m}_{d2}}{\partial \xi} + n_{d0} \frac{\partial \hat{u}_{d2}}{\partial \xi} + \frac{\partial}{\partial \tau} \left( n_{d1} u_{d1} \right) + V \frac{\partial n_{d1}}{\partial \tau} &= 0 \\
B_d \frac{\partial \hat{u}_{d2}}{\partial \xi} + n_{d0} \frac{\partial \hat{u}_{d2}}{\partial \xi} + u_{d1} \frac{\partial \hat{u}_{d1}}{\partial \xi} + V \frac{\partial u_{d1}}{\partial \tau} - Z_d \frac{\partial \varphi_2}{\partial \xi} &= 0
\end{align*}
\]
From the equations (14)-(18), we find the values of \( \frac{\partial n_{i2}}{\partial \xi}, \frac{\partial n_{i+}}{\partial \xi}, \frac{\partial n_{i-}}{\partial \xi} \) and using equation (19), the KdV equation is obtained as

\[
\frac{\partial \varphi_1}{\partial \tau} + p \varphi_1 \frac{\partial \varphi_1}{\partial \xi} + q \frac{\partial^3 \varphi_1}{\partial \xi^3} = 0
\]  
(20)

where 

\[
p = \frac{(1 - Z_d \sigma) B_d^3 B_d Q - 3 B_d^4 + 3 \sigma B_d^4 Q^2 Z_d^4}{2VB_d Q(B_d^3 + \sigma Z_d^2 B_d^3)}, \quad q = \frac{(1 - Z_d \sigma) B_d^3 B_d Q - 3 B_d^4 + 3 \sigma B_d^4 Q^2 Z_d^4}{2V((1 - Z_d \sigma) B_d^3 + \sigma Z_d^2 B_d^3)}
\]

\( B_i = u_{i0} - V, \quad B_d = u_{d0} - V \)

4. Solution of Korteweg-de Vries (KdV) equation

To derive the solution of the KdV equation (20), we introduce the transformation \( \eta = \xi - C_1 \tau \) where \( C_1 \) is the soliton speed in the linear \( \eta \)-space and using the boundary conditions \( \varphi_1 = \frac{\partial \varphi_1}{\partial \tau} = \frac{\partial^2 \varphi_1}{\partial \tau^2} = 0 \) as \( |\eta| \to \infty \), the solution is obtained as

\[
\varphi_1 = \varphi_0 \sec h^2 \left( \frac{\eta}{\delta} \right)
\]

where \( \varphi_0 \) and \( \delta \) are the amplitude and width of the solitary wave respectively and are given by \( \varphi_0 = \frac{3C_1}{p} \) and \( \sigma = \sqrt{\frac{4q}{C_1}} \).

5. Results and discussion

In this model of multispecies dusty plasma with negative mobile dusts, non-thermal ions and Boltzmann electrons, dust-ion acoustic (DIA) compressive and rarefactive KdV solitons are shown to exist in the plasma. Throughout the consideration of the paper, \( Q = 0.54 \times 10^{-3} \) is taken to trace the curves.
Figure 1: Amplitudes of compressive and rarefactive KdV solitons versus dust charges ($Z_d$) for fixed $u_{d0} = 4, \sigma = 0.001$ and $u_{i0}=4$ (i), 5 (ii), 6 (iii).

The smaller values of initial streamings of ions and dusts [$u_{d0} = 4, u_{i0} = 4$ (i), 5 (ii), 6 (iii)] are found instrumental for the generation of compressive and rarefactive DIA KdV solitons simultaneously (Fig. 1) with $\sigma = 0.001$. Starting with the linear increase of amplitudes of compressive DIA KdV solitons, suddenly shows a rapid and convex increase within a very small interval of $Z_d$ but their rarefactive counter parts sharply decrease concavely for each ion streaming ($u_{i0}$) in a similar but opposite pattern. After attaining a fixed and certain range of $Z_d$, called critical range (e.g., $Z_{dr} \approx 130 – 150$) of dust charges, both the kinds (compressive and rarefactive) of DIA solitons are separated by disjoint lines like asymptotes. Within this critical range of dust charge ($Z_{dr}$), the initial streamings of ions and mobile dusts become almost equal (Fig. 1). But outside this critical range, for $u_{d0} \leq u_{i0}$ (or $u_{i0} < u_{d0}$) cause to generate compressive (rarefactive) DIA solitons of low and high amplitudes together. Immediate left/right of the critical range of dust charge ($Z_{dr}$) the compressive/rarefactive DIA solitons of much higher amplitudes form within a span of very small interval of $Z_d$. From very far away from the critical range of dust charge ($Z_{dr}$), for higher number of dust charges ($Z_d$) and smaller initial streamings of dust ($u_{d0}$) and ions ($u_{i0}$), the amplitudes of the compressive solitons become very low whereas because of much greater number of dust charges compared to $u_{d0}$ and $u_{i0}$ starting from very near of critical range of dust charge ($Z_{dr}$) to very far, the amplitudes of rarefactive DIA solitons follow similar character.
Figure 2: Amplitudes of compressive KdV solitons versus ion to dust density ratio ($\sigma$) for fixed $u_{i0}=2$, $u_{d0}=0.5$ and $Z_d=20$ (i), 25 (ii), 30 (iii).

The amplitudes of compressive DIA KdV solitons for very very small ion to dust density ratio ($\sigma$) starting with a reasonable (very small) amplitudes for fixed but smaller initial streamings of ion and dust viz. $u_{i0} = 2, u_{d0} = 0.5$ respectively corresponding to the small number of dust charge $Z_d = 20$ increases slowly (Fig. 2). But with the increase of number of dust charge [$Z_d = 25$ (i), 30 (ii)], the amplitudes of the DIA KdV solitons increases convexly (Fig. 2) but sharply for a smaller range of $\sigma$ for the same values of other parameters.

Figure 3: Amplitudes of rarefactive KdV solitons versus streaming of dust ($u_{d0}$) for fixed $Z_d=300$, $\sigma = 0.001$ and $u_{i0}=3$(i), 5(ii), 7(iii).

On the other hand, the growth of the amplitudes of rarefactive DIA KdV solitons decreases concavely (Fig. 3) relative to the variation of streamings of dusts for fixed values of $Z_d = 300, \sigma = 0.001$ and
\( u_{d0} = 3(i), 5(ii) \) and \( 7(iii) \). Initially, positive amplitudes of the rarefactive DIA KdV solitons are higher for very small initial streamings of dust \( (u_{d0} < 2) \) but with the increase of dust charge \( u_{d0} \) (Fig. 3).

**Figure 4:** Amplitudes of compressive KdV solitons versus streaming of dust \( (u_{d0}) \) for fixed \( Z_d = 200, 0.001 \) and \( u_{i0} = 35 \) (i), 40 (ii), 45 (iii).

Contrary to this, the smaller number of negative dust charges \( Z_d = 200 \) (compared to Fig. 3) into the plasma but greater initial streaming of ions \( u_{i0} = 35 \) (i), 40 (ii), 45 (iii) and \( \sigma = 0.001 \), the amplitudes of compressive DIA KdV solitons slowly increases with the smaller increase of initial streamings of dust i.e., \( u_{d0} < 15 \) for other parametric values of the plasma parameters as above. At some critical initial streamings of dusts \( u_{dc} \approx 15 \), the amplitudes of compressive DIA KdV solitons become same for all the three values of ion streamings. After the critical \( u_{dc} \), the amplitudes of compressive DIA KdV solitons sharply increases (Fig. 4) with the increase of dust initial streaming \( (u_{d0}) \).
Figure 5: Amplitudes of compressive KdV solitons versus ion streaming $u_{i0}$ for fixed $Z_d=10$, $u_{d0}=0.1$ and $\sigma=0.001$ (i), 0.002 (ii), 0.003 (iii).

The amplitudes of compressive DIA KdV solitons increases almost linearly (Fig. 5) as initial streamings of ion ($u_{i0}$) increases for small $Z_d = 10$ very smaller initial streaming of dust $u_{d0} = 0.1$ and $\sigma = 0.001$. Of course, smaller the values of dust to ion density ratio ($\sigma$), smaller is the amplitude of DIA KdV solitons.

Figure 6: Amplitudes of rarefactive KdV solitons versus $Z_d$ for fixed $u_{d0}=50$, $\sigma=0.001$ and $u_{i0}=3$ (i), 5 (ii), 7 (iii).

Smaller number of dust charges ($Z_d$), the amplitudes of compressive DIA KdV solitons decreases sharply from higher to lower in a very short range of $Z_d$ and then convexly tend to almost zero with the other plasma parameters as $u_{d0} = 50, \sigma = 0.001$. With the increase of initial streaming of ions [(i.e., $u_{i0} = 3$ (i) to 5 (ii)] keeping the other parametric values viz. $u_{d0} = 50, \sigma = 0.001$, the rate of decrease
of amplitudes of DIA KdV solitons become more sharp from higher to lower and the interval of \( Z_d \) becomes more contracted (Fig. 6).

Frequent bursts of solar winds and magnetic storms effects the plasma scenario and as a result huge nonlinearity is created in space plasma. These nonlinearity create hazards to space vehicles and probes. Our present study of DIA solitary waves with the plasma parameters (mentioned in this paper with different situation) may be helpful (to some extent) to meet the hazards in dusty space plasma.

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