Very deep spectroscopy of the bright Saturn Nebula NGC 7009 – II. Analysis of the rich optical recombination spectrum

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ABSTRACT
In Paper I, we presented deep, long-slit spectrum of the bright Saturn nebula NGC 7009. Numerous permitted lines emitted by the C\(^+\), N\(^+\), O\(^+\) and Ne\(^+\) ions were detected. Gaussian profile fitting to the spectrum yielded more than 1000 lines, the majority of which are optical recombination lines (ORLs) of heavy element ions. In the current paper, we present a critical analysis of the rich optical recombination spectrum of NGC 7009, in the context of the bi-abundance nebular model proposed by Liu et al. Transitions from individual multiplets are checked carefully for potential blended lines. The observed relative intensities are compared with the theoretical predictions based on high quality effective recombination coefficients, now available for the recombination line spectrum of a number of heavy element ions.

The possibility of plasma diagnostics using the ORLs of various heavy element ions is discussed in detail. The line ratios that can be used to determine electron temperature are presented for each ion, although there is still a lack of adequate atomic data and some of the lines are still not detected in the spectrum of NGC 7009 due to weakness and/or line blending. Plasma diagnostics based on the N\(^{\text{II}}\) and O\(^{\text{II}}\) recombination spectra both yield electron temperatures close to 1000 K, which is lower than those derived from the collisionally excited line (CEL) ratios (e.g., the [O III] and [N II] nebular-to-auroral line ratios; see Paper I for details) by nearly one order of magnitude. The very low temperatures yielded by the O\(^{\text{II}}\) and N\(^{\text{II}}\) ORLs indicate that they originate from very cold regions. The C\(^{2+}/\text{H}^{+}\), N\(^{2+}/\text{H}^{+}\), O\(^{2+}/\text{H}^{+}\) and Ne\(^{2+}/\text{H}^{+}\) ionic abundance ratios derived from ORLs are consistently higher, by about a factor of 5, than the corresponding values derived from CELs. In calculating the ORL ionic abundance ratios, we have used the newly available high quality effective recombination coefficients, and adopted an electron temperature of \(~1000\) K, as given by the ORL diagnostics and as a consequence presumably representing the physical conditions prevailing in the regions where the heavy element ORLs arise. Measurements of the ultraviolet (UV) and infrared (IR) CELs from the literature are used to calculate CEL ionic abundance ratios when optical data are not available for the ionic species. A comparison of results of plasma diagnostics and abundance determinations for NGC 7009 points to the existence of “cold”, metal-rich (i.e., H-deficient) inclusions embedded in the hot, diffuse ionized gas, first postulated by Liu et al.

At electron temperatures yielded by the N\(^{\text{II}}\) and O\(^{\text{II}}\) ORLs, the predicted relative intensities of ORLs agree well with the observed values, indicating that the current quantum calculations of the recombination spectra of those two ionic species well represent the recombination processes under nebular conditions. Deviations from the LS coupling, noticed in an earlier quantitative spectroscopy by Liu et al. for the same object, are again confirmed, especially for recombination lines of the 4f – 3d transition array. For N\(^{\text{II}}\), as well as for O\(^{\text{II}}\), the ionic abundances derived from different J-resolved transitions within a multiplet, or from the transitions belonging to different multiplets, agree with each other. This is another evidence that the new effective recombination coefficients are reliable. New calculations of the effective recombination coefficients for the Ne\(^{\text{II}}\) lines at nebular temperatures and densities are needed.

Key words: atomic data – atomic processes – ISM: abundances – planetary nebulae: individual: NGC 7009
1 INTRODUCTION

The bright Saturn Nebula NGC 7009 is known for its rich and prominent optical recombination lines (ORLs) of heavy element ions, especially those of O II, ever since the spectrophotographic observations of Wyse (1942), who published and analyzed deep spectra of the Orion Nebula and nine planetary nebulae (PNe), including NGC 7009. He identified and measured several dozen O II permitted lines in NGC 7009 in the wavelength range 3700 – 6750 Å, although accurate measurements of many of those O II lines were hampered by line blending. At the end of this paper Wyse (1942) expressed the desire of having more accurate measurements of the O II permitted lines. Aller & Kaler (1964) identified more than 100 O II permitted lines in the spectrum of NGC 7009. Large numbers of permitted lines of other ionic species, such as C II, N II, N III, O III, Ne II, were also detected. The majority of these permitted lines are mainly excited by recombination. Other possible excitation mechanisms, such as the dielectronic recombination, radiative charge transfer, and resonance fluorescence by starlight or by some other prominent nebular emission lines, are all by their nature selective, which means that they tend to excite lines from specific spectral terms of certain parity and multiplicity only (e.g., Grandi 1976; Liu & Danziger 1993; Liu, Danziger & Murdin 1993). With high signal-to-noise ratio, high spectral resolution and wide wavelength-coverage spectra of PNe now available, more and more ORLs of fainter intensities from heavy element ions that arise from many different multiplets are observed and provide an opportunity to study the radiative and dielectronic recombination processes and test the accuracy of the recombination theories for non-hydrogenic ions. The first systematic study of the ORLs in NGC 7009 was carried out by Liu et al. (1995, hereafter LSBC), who analyzed dozens of O II ORLs, using effective recombination coefficients calculated in the intermediate coupling scheme for transitions from the 3d–3p and 4f–3d arrays, and coefficients calculated in the LS coupling scheme for transitions from the 3p–3s array. LSBC found clear deviations from the LS coupling scheme in the 3d–3p and 4f–3d transitions. Luo, Liu & Barlow (2001, hereafter LLB01) presented high-quality observations of several dozens Ne II ORLs in NGC 7009, and derived the Ne\(^{2+}/H^+\) abundance ratios from them.

Along with the advance of observational techniques that have enabled the detections of many faint ORLs of heavy element ions in photoionized gaseous nebulae, the recombination theories of heavy element ions, such as C II, N II, O II, and Ne II, have seen steady improvements since early 1980s (e.g. Storey 1981; Nussbaumer & Storey 1983, 1984, 1986; Escalante & Victor 1990; Péguynot, Petitjean & Boisson 1991; Storey 1994; LSBC; Kisielius et al. 1998; Davey, Storey & Kisielius 2000; Kisielius & Storey 1999, 2002; Fang, Storey & Liu 2011). The high-quality atomic data have been widely used to reveal the physical conditions (electron temperatures and densities) under which the ORLs of heavy element ions arise, and to determine ionic and elemental abundances from them (e.g. Liu et al. 2000).

In nebular astrophysics there has been a long-standing dichotomy whereby the ionic and elemental abundances of C, N, O and Ne relative to hydrogen determined from ORLs (e.g. C II M6 λ4267, N II M396 λ4041, O II M1 λ4649 and M48a λ4089, Ne II M55e λ3432) are systematically higher than those derived from the much brighter collisionally excited lines (CELs, often referred to as forbidden lines). With high-quality optical spectra now available, detailed studies of this problem have been carried out for several archetypal PNe (LSBC and LLB01 for NGC 7009; Liu et al. 2000 for NGC 6153; Liu et al. 2001b for M l-42 and M 2-36; Liu et al. 2006 for Hf 2-2; Garnett & Dinerstein 2001 for NGC 6720). Several deep optical spectroscopic surveys of PNe, which allow for the analyses of nebulae based on ORLs, have been carried out during the past decade (Tsamis et al. 2003, 2004; Liu et al. 2004a,b; Robertson-Tessi & Garnett 2005; Wesson, Liu & Barlow 2005; Wang & Liu 2007). The abundance discrepancy factors (ADFs), defined as the ratio of the abundance derived from ORLs to that derived from CELs, typically lie in the range 1 – 3. But for a significant number of PNe, ADF values exceeding 5, or even 10, are seen. The highest ADF value (~70) of all PNe is found in Hf 2-2 (Liu et al. 2006). Another dichotomy that is closely related to the problem of abundance discrepancy is that nebular electron temperatures derived from the traditional diagnostic [O III] nebular-to-aauroral line ratio are generally higher than those derived from the Balmer jump (BJ) of hydrogen recombination spectrum (e.g. Peimbert 1971; Liu & Danziger 1993b). A number of postulations have been raised to explain these problems (e.g. Peimbert 1967; Rubin 1989; Viegas & Clegg 1994), but all failed to provide a consistent interpretation of all the available observations. Recently, Nicholls, Dopita & Sutherland (2012) explored the possibility that electrons in H II regions and PNe depart from a Maxwell-Boltzmann equilibrium energy distribution and suggested that a “κ-distribution” for the electron energies, which are widely found in solar system plasmas, can explain the temperature and abundance discrepancies in H II regions and PNe. The bi-abundance nebular model proposed by Liu et al. (2000), who postulated that PNe (probably also H II regions) contain H-deficient inclusions, provides a better and natural explanation of the dichotomy. In this model, the faint ORLs of heavy element ions originate mainly from the “cold”, H-deficient inclusions, while the stronger CELs are emitted from the warmer ambient plasma with ‘normal’ chemical composition. Deep spectroscopic surveys and recombination line analysis of individual nebulae in the past decade has yielded strong evidence for the existence of such a “cold” component (see recent reviews by Liu 2003, 2006a, 2011).

This is the second of the two papers devoted to very deep spectroscopy of NGC 7009. In the previous paper (Fang & Liu 2011, hereafter Paper I), we presented high-quality spectra of NGC 7009 and tabulation of all detected lines, including their observed and dereddened intensities, many of which were obtained via careful deblending using the technique of multi-Gaussian profile fitting. We also carried out plasma diagnostics using the CEL ratios, the H I recombination spectrum (including the Balmer and Paschen decrements of the line spectrum, and the Balmer and Paschen jumps of the continuum spectrum), and the He I and He II recombination spectrum (including the He I recombination line ratios, and discontinuities of the He I and He II recombination continua). The average electron temperature yielded by CELs, \(T_e(\text{CELs})\), is higher than that from the H I Balmer jump, \(T_e(\text{H I BJ})\), which in turn is higher than the temperature derived from the He I recombination line ratios, \(T_e(\text{He I})\). The current paper focuses on analyses of the optical recombination spectra of heavy element ions detected in the spectrum of NGC 7009. New effective recombination coefficients, including those for the N II and O II recombination spectrum that were calculated in the intermediate coupling scheme, are now available and are utilized in the analyses. Plasma diagnostics based on the ORLs of heavy element ions are carried out in Section 2, and the electron temperatures derived from the N II and O II ORL ratios agree with each other and are both close to 1200 K. Thus the general pattern of electron temperatures, \(T_e(\text{CELs}) \gtrsim T_e(\text{H I BJ}) \gtrsim T_e(\text{He I}) \gtrsim T_e(\text{N II, O II ORLs})\), which was predicted by the bi-
abundance nebular model (Liu 2003) and has been seen in many PNe, is confirmed in the current analysis of NGC 7009. A comprehensive analysis of individual multiplets of the C II, N II, O II, and Ne II recombination spectra are presented in Section 3. The lines are critically examined for potential blending effects. Comparison is made for the observed and predicted relative intensities of the best observed transitions, using the latest effective recombination coefficients. Ionic and elemental abundances are derived in Section 4, where ADFs for the C, N, O, and Ne ionic abundances are calculated. The results are discussed in Section 5, followed by a summary in Section 6.

2 PLASMA DIAGNOSTICS BASED ON THE ORLS OF HEAVY ELEMENT IONS

2.1 Effective recombination coefficients

Reliable atomic data, most importantly the effective recombination coefficients of abundant heavy element ions such as C II, N II, O II, and Ne II, are key to the spectroscopic analysis of photoionized gaseous nebulae. Most of the ab initio calculations of heavy element ions aimed for astrophysical applications hitherto were carried out in the LS coupling scheme. This approximation tacitly assumes a statistical distribution in the population of the fine-structure levels of the recombining ions (i.e., 1 : 2 for the N II, O II, and Ne II, and 1 : 3 : 5 for the O III, Ne III, and Ne IV levels in the case of O II). The LS coupling may give satisfactory results for some of the low-lying transitions such as those belonging to the 3p – 3s configuration, but not for many of the transitions from the higher 3d – 3p or 4f – 3d configurations. In low-density objects such as H II regions and evolved PNe, the relative populations of the ground-term fine-structure levels of the recombining ion actually have density-dependence and deviate from the statistical distribution, and so do the relative emissivities of resultant recombination lines. A better treatment of the recombination and the following cascading in a proper coupling scheme is vital for probing the physical conditions in gaseous nebulae.

New ab initio calculation of the effective recombination coefficients for the N II recombination spectrum was presented by Fang, Storey & Liu (2011), hereafter FSL11, who took into account the density dependence of effective recombination coefficients arising from the density-dependence of relative populations of the ground-term fine-structure levels of the recombining ion (i.e., N II, O II, and Ne II, and 1 : 3 : 5 for the O III, Ne III, and Ne IV levels in the case of O II). The LS coupling may give satisfactory results for some of the low-lying transitions such as those belonging to the 3p – 3s configuration, but not for many of the transitions from the higher 3d – 3p or 4f – 3d configurations. In low-density objects such as H II regions and evolved PNe, the relative populations of the ground-term fine-structure levels of the recombining ion actually have density-dependence and deviate from the statistical distribution, and so do the relative emissivities of resultant recombination lines. A better treatment of the recombination and the following cascading in a proper coupling scheme is vital for probing the physical conditions in gaseous nebulae.

New ab initio calculation of the effective recombination coefficients for the Ne II recombination spectrum was presented by Kasiselius et al. (1998), who took into account the density dependence of effective recombination coefficients arising from the density-dependence of relative populations of the ground-term fine-structure levels of the recombining ion (i.e., N II, O II, and Ne II, and 1 : 3 : 5 for the O III, Ne III, and Ne IV levels in the case of O II). The LS coupling may give satisfactory results for some of the low-lying transitions such as those belonging to the 3p – 3s configuration, but not for many of the transitions from the higher 3d – 3p or 4f – 3d configurations. In low-density objects such as H II regions and evolved PNe, the relative populations of the ground-term fine-structure levels of the recombining ion actually have density-dependence and deviate from the statistical distribution, and so do the relative emissivities of resultant recombination lines. A better treatment of the recombination and the following cascading in a proper coupling scheme is vital for probing the physical conditions in gaseous nebulae.

Kasiselius et al. (1998) published the Ne II effective recombination coefficients that were calculated in the LS coupling scheme. Only transitions between states with l ≤ 2 were presented. Preliminary effective recombination coefficients for a few selected lines from the 4f – 3d configuration are available (P. J. Storey, private communication), but only for a single temperature and density case. All the previous calculations of the Ne II recombination spectrum assumed that the three ground-term fine-structure levels of the recombining ion Ne II, 3s 3p, and Ne IV, 3s 3p, are thermalized, i.e. they are populated according to the statistical weights. However, the 3s 3p and 3s 3p levels have relatively large critical densities: 2.0 × 10^7 cm^{-3} for 3s 3p and 2.9 × 10^7 cm^{-3} for 3s 3p at 10,000 K, and these values drop to about half when the electron temperature decreases to 10,000 K. At physical conditions lower than the critical densities, the 3s 3p and 3s 3p levels are populated according to the values under thermal equilibrium. Fig. 9 shows the fractional populations of the three Ne II levels as a function of electron den-
The effects of the non-equilibrium level populations of Ne III on the effective recombination coefficients for the 4f – 3d transitions are not clear and may vary from line to line. For the strongest 4f – 3d lines that form exclusively from recombination of target 3P₂ plus cascades, their effective recombination coefficients will be underestimated if a thermal equilibrium of the Ne III ground levels is assumed, and that will cause a corresponding overestimation of the derived Ne²⁺/H⁺.

Many Ne II recombination lines from different multiplets have been observed in deep spectra of PNe and H II regions and ionic abundances derived (e.g. LLB01). However, a proper analysis of those data requires new calculations in an appropriate coupling scheme for the strongest Ne II recombination lines, especially those belonging to the 3d – 3p and 4f – 3d transition arrays.

2.2 Electron temperature from the C II recombination lines

Most C II lines detected in the spectrum of NGC 7009 are mainly excited by radiative recombination, except for a few for which dielectronic recombination dominates. Examples of the latter include the C II M28.01 3d ²F⁰ – 3p ²D λ8797 multiplet, which originates from dielectronic capture of an electron to the 2s2p(3P₂)3d ²F⁰ autoionizing state that lies 0.41 eV (Moore 1993) above the first ionization threshold 2s² ¹S₀ and the subsequent decay to the 2s2p(3P₂)3p ²D bound state that lies about 1.00 eV below the ionization threshold. Figure 10 is a schematic diagram that shows the dielectronic and radiative recombination of C II.

The electron on an autoionizing state either decays to another autoionizing or bound state with the emission of radiation, or autoionizes to a true continuum state leaving an ion and a free electron with no emission of radiation. The latter process usually dominates, and the population of autoionization states is close to that given by Saha and Boltzmann equations as in the case of the local thermodynamic equilibrium (LTE). The emissivity of a dielectronic recombination line is sensitive to electron temperature through the Boltzmann factor \( \exp \left( -\frac{E}{kT} \right) \), where \( E \) is the excitation energy of the upper state relative to the ionization threshold. By comparing the strength of a dielectronic recombination line to that of an ordinary (i.e. radiative recombination dominated) recombination line, whose emissivity has a relatively weak power-law dependence on electron temperature (\( \sim T^{α₀} \), where \( α \sim 1 \)), one can determine the electron temperature. The C II dielectronic lines have been used to determine electron temperatures in stellar winds of PNe (e.g. De Marco et al. 1998). The strongest C II recombination line detected in the spectra of nebulae is the M6 4f ²F⁰ – 3d ²D λ4267 line, which is excited by radiative recombination only. The upper state of the λ4267 line lies about 3.4 eV below the ionization threshold 2s² ¹S₀ (see Fig. 10), and its population is far from LTE, and thus has a very different temperature-dependence from that of the upper state of the M28.01 λ8797 transition (i.e. 3d ²F⁰). We use the intensity ratio of the λ8793.80 (3d ²F⁰/2 – 3p ²D₅/₂) line, the stronger fine-structure component of the C II M28.01 multiplet, and the λ4267 line to determine electron temperature. In NGC 7009, this line ratio yields a temperature of 3000 K, as shown in Fig. 11.

The atomic data used here are the effective dielectronic and radiative recombination coefficients of Nussbaumer & Storey (1984) and Péquignot, Petitjean & Boisson (1991), respectively. Measurements of the C II M28.01 lines are presented in Section 3.1.2.

2.3 Electron temperatures and densities from the N II and O II recombination lines

In the low-density conditions in nebulae, the relative populations of the ground-term fine-structure levels of a recombining ion (e.g.,
Figure 1. Fractional populations of the N II \(^{2+} 2p^0_{3/2}\) and \(^{2}p^0_{3/2}\) fine-structure levels. Four temperature cases, 200, 1000, 5000 and 10 000 K, are shown. This figure is obtained by solving level population equations for a five-level atomic model.

Figure 2. Fractional intensities of the N II M3 \(2p^3_{3D} \rightarrow 2p^3_{3P}\) λ5679 multiplet as a function of electron density. The numbers in the brackets \((J_2 - J_1)\) following the wavelength labels are the total angular momentum quantum numbers of the upper and lower levels, respectively. Transitions from the upper levels with the same angular momentum quantum number \(J_2\) are represented by curves of the same color and line type. Four temperature cases, \(\log T_e [K] = 2.5, 3.0, 3.5,\) and 4.0, are presented. The calculations were based on the effective recombination coefficients of FSL11.
Figure 3. Same as Fig. 2 but for the fractional intensities of the N ii M19 2p3d 3F0 – 2p3p 3D λ5004 multiplet.

Figure 4. Same as Fig. 2 but for the fractional intensities of the N ii M39 2p4f G[7/2,9/2] – 2p3d 3F0 λ4041 multiplet.
Figure 5. Fractional populations of the O$^{2+}$ $^3P_0$, $^3P_1$ and $^3P_2$ fine-structure levels. Four temperature cases, 200, 1000, 5000 and 10 000 K, are shown. This figure is obtained by solving the level population equations for a five-level atomic model.

Figure 6. Same as Fig. 2 but for the fractional intensities of the O II M1 2p$^2$3p $^4$D$^o$ – 2p$^2$3s $^4$P $\lambda$4652 multiplet. The calculations were based on the unpublished effective recombination coefficients of PJS.
Figure 7. Same as Fig. 6 but for the fractional intensities of the O II M10 2p$^2$3d$^4$F – 2p$^2$3p$^4$D$^o$ λ4075 multiplet.

Figure 8. Same as Fig. 6 but for the fractional intensities of the O II M48 4fG(5,4,3)$^o$ – 3d$^4$F λ4089 multiplet.
the λ5666.63 line is free of blending and is amongst the best observed N II ORLs, while the λ5679.56 and λ4041.31 lines are affected by line blending. Accurate measurements of the latter two lines were obtained using multi-Gaussian profile fits (see Section 5.2). The M19 2p3d 3P0 – 2p3p 3D multiplet is the strongest of the 3D – 3p configuration of N II. The density-dependence of the relative emissivities of the two strongest fine-structure components of M19 is noticeable (Fig. 2). The intensity ratio of those components, λ5005.15/(λ5001.14 + λ5001.48), may serve as another density diagnostic. Similarly, the intensity ratio of the λ5005.15 and M39b λ4041.31 lines may be used to determine electron temperature. Figs. [14] and [15] show the λ5005/λ5001 and λ5005/λ4041 ratios of N II as a function of electron density and temperature, respectively. However, accurate measurements of the N II M19 lines are essentially impossible due to the presence of the extraordinarily strong [O III] λ5007 line, which is often strongly saturated in deep spectra.

Some N II states of parentage other than 2P0 have energies even higher than the 2(2P0)4f G[9/2] spectral term, which is the upper state of the M39b λ4041.31 line. The intensity ratio of an N II recombination line that originates from one of those high-energy states to the M3 λ5679.56 line can also be used as a temperature diagnostic. Possible candidates in the optical waveband for such application are, e.g. the M63 3P0 5D0 – 3S0 5P, M66 3d 5F – 3p 5P0 and M72 4f 5G0 – 3d 5P0 multiplets. According to the experimental data given by NIST[2], the upper state of the M63 multiplet is about 1.85 eV below the ionization threshold N II 2P3/2, while the upper states of the M66 and M72 multiplets are 0.53 and 3.67 eV, respectively, above this threshold. The R-matrix calculation of the bound-state energy levels of N II in FSL11 only extends to about 0.45 eV (corresponding to n = 11 in the principal series of N II) below the ionization threshold. Thus only the energy levels of the 2s2p2(2P) 3S and 2s2p2(2P) 3P configurations (i.e., the levels of the 5P, 3P, 3S, 5D, 5P, 3P, 3D, 5S and 5P spectral terms, in the energy order given by NIST) are included in the R-matrix calculation and the N II recombination lines that originate from those levels are precisely calculated. In principle, the intensity ratio of the λ5679.56 and the λ5535.36 lines, the strongest fine-structure components of the M3 3P0 5D0 – 3S0 5P and the M63 3P0 5P0 – 3S0 5P multiplets of N II, respectively, can be used to determine electron temperature. Fig. [16] shows the λ5679/λ5535 ratio as a function of electron temperature, and this relation is quite sensitive to electron density in the logarithmic scale. However, accurate measurements of the λ5535.36 line is difficult due to weakness (about 104 times weaker than Hβ). We have not detected any N II lines of the parentage other than 2P0 in the deep spectrum of NGC 7009.

The λ4649.13 line is the strongest of the O II M1 3p 4D0 – 3S0 3P multiplet, and forms only from recombination of the 2P0 core plus cascades from higher energy levels, while another O II M1 line λ4661.63 can form, in addition, from recombination of the 1P0 and 3P1 cores. For the recombining ion O2+, the population of the fine-structure level 3P2 relative to 2P1 increases with electron density due to collisional excitation, and this results in an increase of the λ5679.56 intensity relative to the λ5666.63 line with density, as shown in Fig. 2. Thus the λ5679/λ5666 ratio can be used as a density diagnostic. The λ4041.31 line belongs to the N II M39b 4f G[9/2] – 3d 3P0 multiplet and is the strongest among the N II 4f – 3d array. It forms from recombination of the 2P3/2 core plus cascades from higher levels. The intensity ratio of the λ5679.56 and λ4041.31 lines has a relatively strong temperature dependence, and thus can serve as a temperature diagnostic. In the spectrum of NGC 7009,
the λ4649.13 lines has a strong temperature dependence, and can be used to determine electron temperature. Figs. 17 and 18 show that the observed O II line ratios λ4649/λ4089 and λ4649/λ4602 in NGC 7009 yield an electron temperature of ~1400 ± 300 K and a density of 2500 – 4000 cm⁻³, respectively. Although the λ4611.63 line is the third strongest in the O II M1 multiplet, it is free from line blending and thus best observed, while the λ4649.13 and λ4089.29 lines both suffer from line blending: the λ4649.13 line is blended with another O II M1 line λ4650.84 and the three C III M1 lines λ4647.42, 4650.25 and 4651.47; the λ4089.29 line is contaminated by the Si IV M1 λ4088.86 (4p²P_,2/3 – 4s²S_1/2) line. Multi-Gaussian fitting was carried out to derive the intensities of the two O II ORLs, and both intensities are accurate to within 20 per cent. Details of spectral fits are given in Section 3.3.

The M10 3d ⁴F – 3p ⁴D⁰ λ4075 multiplet is the strongest transition of the 3d – 3p configuration of O II. Given the opposite trends of the fractional intensities of the λ4075.86 and λ4069.89 lines, the three fine-structure components of M10, as a function of electron density, as shown in Fig. 7, the intensity ratio of the two lines may serve as another density diagnostic. Here the λ4075.86 line is the strongest component of the M10 multiplet. The intensity ratio of the λ4075.86 line and the λ4089.29 line, the strongest fine-structure component of the M48a 4f(G)⁵S – 3d ⁴F multiplet of O II, can be used as another temperature diagnostic. Figs. 19 and 20 show the λ4076/λ4089 and λ4076/λ4070 as a function of electron temperature and density, respectively. Here the intensity of the λ4070 line is a sum of the λ4069.89 (M10 3d ⁴F_5/2 – 3p ⁴D_0/2) and λ4069.62 (M10 3d ⁴F_3/2 – 3p ⁴D_1/2) lines. If we assume a density of about 4300 cm⁻³, as derived from CEL ratios (Paper I), the electron temperature deduced from the O II λ4076/λ4089 ratio is 1150 ± 300 K for NGC 7009. The electron density derived from the O II λ4076/λ4070 ratio is of large uncertainty, due to the relatively large measurement uncertainties of the two lines. The λ4075.86 line is blended with the [S II] λ4067.35 (3p²P_1/2 – ⁴S_1/2) line, while the λ4069.89,62 line is blended with the [S II] λ4068.60 (3p²P_1/2 – ⁴S_1/2) line and the three C III M16 5g ⁵G – 4f ⁴F⁰ lines λ4067.94, 4068.92 and 4070.31. Multi-Gaussian profile fitting was carried out to obtain line fluxes (c.f. Section 3.3).

Several O II recombination lines with parentage other than ³P have been detected in the spectrum of NGC 7009. These lines belong to the M15 3p² ⁴F⁰ – 3s¹ ⁴D, M36 3d² ⁴G – 3p² ⁴F⁰, M101 4f¹ H[5]¹/₂ – 3d² ⁴G, and M105 4f¹ H[1]³/₂ – 3d² ⁴S multiplets of O II. According to the experimental data from NIST, the upper states of the M15 and M36 multiplets are 6.76 and 3.80 eV, respectively, below the ionization threshold O II P, while the upper states of the M101 and M105 multiplets are about 0.89 and 0.87 eV, respectively, below this threshold. In the most recent calculation of PFS for the O II effective recombination coefficients, only the transitions between the levels with principal quantum number n ≤ 6 (i.e. corresponding to 1.50 eV below the ionization threshold ³P) were presented. Thus only the effective recombination coefficients of the M15 and M36 lines are available. The strongest fine-structure components of the M15 and M36 multiplets are λ4590.97 (3p² ⁴F_1/2 – 3s² ⁴D_1/2) and λ4189.79 (3d² ⁴G_0/2 – 3p² ⁴F_1/2), respectively, and both lines are detected in the deep spectrum of NGC 7009, as shown in Figs. 17 and 18. Although the effective recombination coefficients of the M101 and M105 multiplets are not available, the strongest fine-structure components of the M101 multiplet, the λ4253.90 (4f¹ H[5]¹/₂ – 3d² ⁴G_0/2) line, is also detected (Fig. 21). Fig. 21 shows the intensity ratios λ4649/λ4590.97 and λ4649.13/λ4189.79 as a function of electron temperature. The figure also shows that the two line ratio–temperature relations are insensitive to electron density, indicating that they are good temperature diagnostics. Both line ratios detected in the spectrum of NGC 7009 yield electron temperatures close to 3600 K.
2.4 The Ne II recombination lines as potential plasma diagnostics

So far no efforts have been attempted for plasma diagnostics based on the Ne II recombination spectrum, partly due to the lack of suitable effective recombination coefficients. Since all Ne II effective recombination coefficients were calculated under the LS coupling scheme, and relative populations of the $^3P_2$, $^3P_1$ and $^3P_0$ parent levels were assumed to be proportional to the statistical weights, no density diagnostic is possible with the current available atomic data. However, the Ne II recombination line ratios may still serve as temperature diagnostics using the effective recombination coefficients of Kisielius et al. (1998). The M2 $3p^2 D^o - 3s^2 P$ $\lambda3337$ multiplet is the strongest transition of the $3p - 3s$ configuration, and M13 $3d^4 F - 3s^2 D^o \lambda3220$ is the strongest multiplet of the $3d - 3p$ configuration of Ne II. The intensity ratio of the strongest fine-structure components of those two multiplets, $\lambda3334.84/\lambda3218.19$, may be serve as a temperature diagnostic, as shown in the upper panel of Fig. 22. In order to obtain reliable electron temperature, the measurement uncertainty of the $\lambda3334/\lambda3218$ ratio needs to be less than 10 per cent, which is very demanding to achieve.
Recombination of the Ne$^{2+}$ 1D core plus cascades gives rise to another series of Ne II recombination lines, and the strongest multiplet of this series is $\lambda$ 3967. The intensity ratio of the $\lambda$ 33568.50 line, the strongest fine-structure component of the M9 multiplet, and the M2 $\lambda$3334.84 line may be used as another temperature diagnostic, as shown in the upper panel of Fig. 22. The calculation of Kisielius et al. (1998) shows that the $\lambda$3568/$\lambda$3334 ratio is only marginally sensitive to electron temperature. In order to derive reliable temperature, the line ratio (especially the $\lambda$3568 line) needs to be measured to a very high accuracy level. Although the temperature range considered in the calculation of Kisielius et al. (1998) is from 1000 to 20000 K, the current analysis lytic fit to the effective recombination coefficient for the $\lambda$3568 line is only valid for 2000–20000 K. As a consequence, the usage of the diagnostic curve of the $\lambda$3568/$\lambda$3334 ratio in Fig. 22 outside this temperature range is not recommended. The Ne II $\lambda$3334/$\lambda$3218 and $\lambda$3568/$\lambda$3334 ratios observed in NGC 7009 are 1.86 and 0.40, respectively, both falling outside the diagnostic ranges of Fig. 22.

2.5 The C III, N III and O III recombination lines as potential temperature diagnostics

In this Section, we discuss the possibility of using the C III, N III and O III optical recombination line ratios to determine electron temperatures. Although some of those lines are detected in the spectrum of NGC 7009, they are not used for plasma diagnostics in the current paper, due to the lack of adequate atomic data. Unless otherwise specified, the effective recombination coefficients of Nussbaumer & Storey (1984) and Péquignot, Petitjean & Boisson (1991) are used to create the diagnostic curves.

The C III lines are excited by recombination only, and the ratios of the best observed lines can be used as temperature probes. The intensity ratio of the M1 $\lambda$4649 (3p $^3P^o$ – 3s $^3S$) and M16 $\lambda$4069 (5g $^3G$– 4f $^3F^o$) multiplets of C III is sensitive to electron temperature, as shown in the upper panel of Fig. 23. The intensity ratio of the C III triplet M1 $\lambda$4649 and singlet M18 $\lambda$4187 (5g $^3G$ – 4f $^3F^o$) can also be used to determine electron temperature, as shown in the lower panel of Fig. 23. However, those C III lines all suffer from line blending. The $\lambda$4187 line is blended with the O II $\lambda$3218 and $\lambda$3216. The C III $\lambda$4185.45 (3d $^2G_{7/2}$ – 3p $^2P_{5/2}$) line, but its intensity can be measured to a high accuracy using multi-Gaussian profile fitting (Fig. 31). The C III M1 $\lambda$4649 triplets are blended with the O II M1 3p $^3D^o$– 3s $^3P$ lines $\lambda$4649.13 and 4650.84 (Fig. 32), and the C III M16 $\lambda$4069 triplets are blended with three O II M10 3d $^3F$– 3p $^3D^o$ lines and the [S II] $\lambda$4068.60 line (Fig. 33). Intensities of the C III M1 and M16 multiplets are obtained from multi-Gaussian profile fitting. The intensity ratio of the fine-structure components of each multiplet was assumed to be as in LS coupling (Sections 3.3.1 and 3.3.2).
The Optical Recombination Spectrum of NGC 7009

3.3.2. In NGC 7009, the C III $I(M1 \lambda 4649)/I(M16 \lambda 4069)$ and $I(M1 \lambda 4649)/I(M18 \lambda 4187)$ ratios are 1.05 and 3.41, respectively. Both ratios are beyond the diagnostic ranges of Fig. 23.

Fig. 23 shows that the C III $I(M1 \lambda 4649)/I(M16 \lambda 4069)$ and $I(M1 \lambda 4649)/I(M18 \lambda 4187)$ ratios increase with electron temperature below 10,000 K, but both decrease when the temperature goes beyond $\sim 12,600$ K ($\log T_e \sim 4.1$). In order to explain these trends, the effective radiative ($\alpha_{r,\text{eff}}^E$) and dielectronic ($\alpha_{d,\text{eff}}^E$) recombination coefficients as well as the total effective recombination coefficients ($\alpha_{\text{eff}}^E$) of the C III M1 and M16 multiplets are shown in Fig. 24 as a function of electron temperature. Below 10,000 K, the C III M16 multiplet is dominated by radiative recombination. In this temperature regime, the total effective recombination coefficient of the C III M16 multiplet decreases much faster than that of the M1 multiplet as temperature increases. When the temperature goes above 10,000 K, the decreasing rate of the M16 multiplet slows down because its monotonically increasing dielectronic recombination coefficient becomes relatively significant, while that of the M1 multiplet does not change much.

The most prominent permitted transitions of N III in optical, the M1 $\lambda 4100$ ($3p^2P^o – 3s^2S$) and M2 $\lambda 4641$ ($3d^2D – 3p^2P^o$) multiplets, are affected by the Bowen fluorescence mechanism (e.g. Bowen 1934, 1935). The N III $\lambda 4379.11$ ($5g^2G – 4f^2F^o$) line is amongst the best observed N III lines in the spectrum of NGC 7009, which are not affected by the fluorescence processes. The intensity ratio of the $\lambda 4379.11$ line and the $\lambda 4195.76$ line, which is the second strongest fine-structure component of the N III M6 multiplet, can be used as a temperature diagnostic. Fig. 25 shows the N III $\lambda 4379/\lambda 4196$ ratio as a function of electron temperature. The dominant excitation mechanism of the N III M18 multiplet is radiative recombination, while the M6 multiplet is mainly excited by dielectronic recombination. The other two fine-structure components of the N III M6 multiplet, $\lambda 4200.10$
and 4215.77 cannot be used: the former one is blended with the He II \( \lambda 4199.83 \) (11g \( ^2G \rightarrow 4f \) \( ^2F^o \)) line, which is more than three times stronger, and the latter one cannot be accurately measured due to weakness (<10^{-4} of the H\( \beta \) intensity). Another N III multiplet, M17 \( 5f \) \( ^2F^o \rightarrow 4d \) \( ^2D \) \( \lambda 4003 \), when used in pair with the N III M6 \( \lambda 4195.76 \) line, may also be a temperature diagnostic, but its radiative recombination coefficients are unknown. The N III M17 lines are detected in the spectrum of NGC 7009 (Fig. 25).

The majority of the O III triplets of the 3d – 3p and 3p – 3s configurations detected in the spectrum of NGC 7009 are mainly excited by the fluorescence or charge-transfer mechanism (e.g. Liu & Danziger 1993a; Liu, Danziger & Murdin 1993). Thus those excited by the fluorescence or charge-transfer mechanism (e.g. Liu et al. 1993), thus those those excited by radiative recombination at temperatures below 5000 K, and the contribution of dielectronic recombination to the total recombination rate catches up with that of the radiative recombination at about 16000 K (Nussbaumer & Storey 1984; Péquignot, Petitjean & Boisson 1991). The O III 5g - 4f lines are dominantly excited by radiative recombination. The upper panel of Fig. 26 shows the intensity ratio of the M46b \( \lambda 4435 \) (5g \( ^2H[11/2] \rightarrow 4f \) \( G[9/2] \)) multiplet, the strongest transition of the 5g – 4f configuration, and the M8 \( \lambda 3265.32 \) line as a function of electron temperature. Péquignot, Petitjean & Boisson (1991) present the radiative recombination coefficients of both the M8 and M46b multiplets of O III, while Nussbaumer & Storey (1984) only give the dielectronic recombination coefficients of M8. The lower panel of Fig. 26 shows the intensity ratio of the \( \lambda 4434.60 \) (5g \( ^2H[11/2] \rightarrow 4f \) \( G[9/2] \)) line, the strongest fine-structure component of the M46b multiplet, and the M8 \( \lambda 3265.32 \) line as a function of electron temperature. The effective recombination coefficients of the \( \lambda 4434.60 \) line are adopted from Kisielius & Storey (1999), whose calculations for the O III 5g - 4f recombination spectrum were carried out in the intermediate coupling scheme and valid from 5000 to 20000 K. Measurement of the \( \lambda 4434.60 \) line is of large uncertainty due to line blending. The other O III 5g - 4f lines are not detected in the spectrum of NGC 7009.

2.6 Summary of the ORL diagnostics

We have discussed about the possibility of using various recombination line ratios of heavy element ions to determine electron temperatures and densities. The line ratios are illustrated as a function of temperature or density. For some cases, there is still a lack of adequate effective recombination coefficients or the expected lines are not detected in the spectrum of NGC 7009 due to weakness and/or line blending. The O III lines of the 5g – 4f configuration are not detected in the spectrum of NGC 7009 due to weakness and/or line blending. The other O III 5g – 4f lines are not detected in the spectrum of NGC 7009.

The applicability of Figs. 14 and 15 is quite small, given that the N II M19 lines are close to the O III \( \lambda 5007 \) line. The N II and O II recombination lines of parentage other than the ground states of the recombining ions are good temperature diagnostics. As shown in Figs. 16 and 21 those line ratios are insensitive to electron density. The electron temperatures derived from the two O II line ratios (\( \lambda 4649/\lambda 4591 \) and \( \lambda 4649/\lambda 4189 \)) in Fig. 21 are consistent with each other, and are close to the temperature value yielded by the C II \( \lambda 8794/\lambda 4267 \) ratio as shown in Fig. 11. Although no N II lines of parentage other than \( ^2P \) are detected in the spectrum of NGC 7009 due to weakness, they are promising diagnostic tools in spectroscopy. The most reliable temperatures derived from the ORLs of heavy element ions are those yielded by the N II and O II lines, as shown in Figs. 12 and 17. Currently only the N II and O II recombination lines can be used to determine electron density, since the density dependence of the population distributions of the energetically lowest fine-structure levels of the recombining ions have been taken into account in the recombination calculations for those two ions (this effect was also considered by Kisielius & Storey 1999). The calculation of the 5g – 4f recombination lines of O III), Figs. 13, 18 and 29 all show large scatter in electron density for a given line ratio observed in NGC 7009. This is reasonable because the ORL ratios only have very weak density dependence.
The Optical Recombination Spectrum of NGC 7009

The effective recombination coefficients of the C III M1 λ4649 (black curves) and M16 λ4069 (red curves) multiplets as a function of electron temperature. Different curves of the same color represent different recombination coefficients: radiative recombination coefficient $\alpha_{R}^{\text{eff}}$ (dotted line), dielectronic recombination coefficient $\alpha_{D}^{\text{eff}}$ (dashed line), and total effective recombination coefficient $\alpha_{\text{Tot}}^{\text{eff}}$ (solid line). Data source of the plot is the same as Fig. 23.

Figure 25. The N III $\lambda$(M18 λ4379)/I(λ4196) ratio as a function of electron temperature. Data source of the figure is the same as Fig. 23.

New treatment of the Ne II recombination in the intermediate coupling scheme is needed.

3 THE OPTICAL RECOMBINATION SPECTRUM OF HEAVY ELEMENTS

In this section, we present a comprehensive analysis of the most significant permitted transitions of C II, N II, O II, and Ne II as well as C III, N III and O III detected in the spectrum of NGC 7009.

Figure 26. Upper: The O III $\lambda$4435/$\lambda$3265 ratio as a function of electron temperature. Here the $\lambda$4435 line is the M46b 5g $H[11/2]$ – 4f $G[9/2]$ multiplet of O III. The figure is based on the dielectronic and radiative recombination coefficients of Nussbaumer & Storey (1984) and P´equignot, Petit-jean & Boisson (1991), respectively. Lower: The O III $\lambda$4434.60/$\lambda$3265 ratio as a function of electron temperature. Here the $\lambda$4434.60 line is the strongest fine-structure component of the O III M46b multiplet, and the effective recombination coefficients of this line are from Kisielius & Storey (1999), whose calculations were carried out in the intermediate coupling scheme and are valid from 5000 to 20000 K.

In Paper I, we derived a mean value of 0.174 for the logarithmic extinction at Hβ, $c(H\beta)$, using the observed H I Balmer line ratios $H\alpha/H\beta$ and $H\gamma/H\beta$. The predicted H I line ratios in the Case B assumption were adopted from Storey & Hummer (1995), with $T_e = 10000$ K and $N_e = 10000$ cm$^{-3}$.

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the N II and O II ORL diagnostics (Figs. [12] and [17]). The wavelengths of atomic transitions are adopted from the compilation of laboratory and theoretical values of Hirata & Horaguchi (1995). The extinction-corrected flux of Hβ, I(Hβ), of NGC 7009 is derived using log I(Hβ) = log F(Hβ) + c(Hβ), where F(Hβ) is the observed Hβ flux (−9.80 in logarithm), which is adopted from Cahn, Kaler & Stanghellini (1992), and c(Hβ) is the logarithmic extinction at Hβ, which was derived from the H I Balmer line ratios Hα/Hβ and Hγ/Hβ (Paper I). The value of c(Hβ) we derived for NGC 7009 is 0.174, which agrees with the value (0.17) given by Cahn, Kaler & Stanghellini (1992), who used the radio/Hβ flux ratio. Thus in NGC 7009 we have I(Hβ) = 10^{−9.63} erg cm\(^{-2}\) s\(^{-1}\). 

3.1 The C II optical recombination spectrum

Several dozen emission lines were identified as the permitted transitions of C II, with 41 being solid identifications (Paper I). The strongest transitions are presented in the current paper. As an example, we give principles of fits for the multiplets M6 and M28.01 in this section. The other multiplets are presented in Appendix A. The effective recombination coefficients of Davey, Storey & Kisielius (2000) are used for ORL analysis.

3.1.1 Multiplet 6, 4f\(^2\)F\(^o\) – 3d\(^2\)D

C II M6 \(λ4267\) is the strongest C II multiplet observed in NGC 7009 (see Fig. [14]). The three fine-structure components of this multiplet have close wavelengths: 4267.00, 4267.26 and 4267.26 Å. Single Gaussian profile fitting to the emission feature gives an intensity of 0.880 (normalized to a scale where Hβ = 100), with an uncertainty of less than 5 per cent. Here the contributions from the O II M53c 4fD[1]\(^3\)3p\(^3\)P\(^o\)/2 – 3d\(^2\)F\(^o\)/2 \(λ4263.27\) and Ne II M57c 4f \(\frac{1}{2}\)\(^3\)F\(^o\)/2 – 3d \(\frac{1}{2}\)\(^3\)F\(^o\)/2 \(λ4267.38\) lines are negligible. This intensity value agrees with LSBC, whose observation yields a value of 0.838. The calculation of Bastin (2006) hereafter B06) that the Case A effective recombination coefficient for the C II M6 \(λ4267\) line differs from that in Case B by 1.5 per cent, and a similar difference is given by Davey, Storey & Kisielius (2000), indicating that this transition is case insensitive.

3.1.2 Multiplet 28.01, 3d\(^2\)F\(^o\) – 3p\(^2\)D

This multiplet is a dielectronic transition, which is a result of cascading from the autoionization state 2s2p\(^3\)P\(^o\) 3d\(^2\)F\(^o\) that lies about 0.41 eV above the first ionization threshold to the 2s2p\(^3\)P\(^o\) 3p\(^2\)D state that lies 1.00 eV below the ionization threshold (Moore [1993]). The features of the two fine-structure components, \(λ8793.80\) (3d\(^2\)F\(^o\)/2 – 3p\(^2\)D\(^0\)/2) and \(λ8799.90\) (3d\(^2\)F\(^o\)/2 – 3p\(^2\)D\(^1\)/2) are very broad (Fig. [27]). The wings of the two lines obviously affect the weaker emission features nearby. Detailed analysis of the complex indicates that at least two more emission lines are blended with the two C II lines: one is the He II 23p\(^3\)P\(^o\) – 6s\(^3\)S \(λ8799.0\) line, while the other is unknown. The results of fitting to the features are shown in Fig. [27].

For each of the two C II M28.01 lines, we used a simulated Lorentz profile with an intrinsic width of 6.80 Å convolved with a Gaussian instrumental profile with a full width at half-maximum (FWHM) of 3.00 Å to fit the observed feature. The convolution of the Lorentz profile and Gaussian gives a Voigt profile with a width of 8.50 Å, which fits the observed features quite well (Fig. [27]). The intensity contribution of the blended He II \(λ8799\) line was estimated from the He II 4f\(^2\)F\(^o\) – 3d\(^2\)D \(λ4686\) line and the hydrogenic theory of Storey & Hummer (1995). an electron temperature of 10,000 K and a density of 10,000 cm\(^{-3}\) were assumed. After correcting for the contribution from the He II line, the intensity of the \(λ8799.90\) line is 0.022±0.003. With the assumption that the relative intensity of the two C II M28.01 lines is as in the LS coupling, i.e. 1.0:0.7, the intensity of the \(λ8793.80\) line is 0.032, which is lower than the total intensity of the broad feature at \(λ8793\) (see Fig. [27]), indicating that there is probably unknown blending. Multiple trial fitting to the profile shows that an emission feature with an observed wavelength of 8793.20 Å best fits the feature, and its intensity is <0.01. The EMIL code (Sharpee et al. 2003) identified this probably blended weak line as a [Cr II] line with a laboratory wavelength of 8795.17 Å. More efforts are needed to verify this identification.

The intensity ratio of the C II \(λ8793.80\) line and the C II M6 \(λ4267\) multiplet is 0.036, and that yields an electron temperature of \(~3000\) K (Section 2.2 and Fig. [11]). As discussed in Section 2.2, this temperature is questionable, due to different excitation mechanisms of the C II M28.01 \(λ8797\) and the M6 \(λ4267\) multiplets. Measurements of the C II M28.01 lines are inaccurate unless detailed modeling of the autoionization levels of C II is carried out. Besides, too many skylines in the near-red of the spectrum of NGC 7009 and the relatively poor sky subtraction in this wavelength region also makes accurate measurements of the C II lines difficult (c.f. Section 2.1 in Paper I).

3.1.3 Comments on the C II recombination spectrum

The current effective recombination coefficients used for analysis of the C II lines are mainly from Davey, Storey & Kisielius (2000) and Bastin (2006). The former calculation takes much care in the low temperature case (\(T_e < 5000\) K), while the latter one mainly covers higher temperatures (5000–50 000 K) and includes the effects of high temperature dielectronic recombination, which rapidly becomes important above an electron temperature of \(15000\) K. Both calculations were carried out in the LS coupling assumption and only for the transitions of the parentage \(^5\)S. In the current analysis, we adopt the calculation of Davey, Storey & Kisielius (2000), assuming a temperature of 1000 K.

The best observed (i.e. the most accurate) multiplets of C II in the spectrum of NGC 7009 are M6 (4f\(^2\)F\(^o\) – 3d\(^2\)D and some transitions that belong to the \(ng – 4f\) (\(n \geq 5\) ) array. For the transitions with parentage other than \(^5\)S, only the M28.01 (3d\(^2\)F\(^o\) – 3p\(^2\)D) multiplet is detected, but measurements of this multiplet could be unreliable, as mentioned in Section 3.1.2. The effective dielectronic recombination coefficients of Nussbaumer & Storey (1984) and radiative recombination coefficients of Péquignot, Petitjean & Boisson (1991) were used for the analysis of the M28.01 multiplet. A full treatment of the C II recombination in an appropriate coupling assumption (i.e. intermediate coupling), with transitions between the autoionization levels taken into account, is needed in the future.

\(^4\) EMIL is developed by Dr. B. Sharpee et al. and is designed to aid in the identification of weak emission lines, particularly the weak recombination lines seen in high dispersion and high signal-to-noise (S/N) spectra. URL: [http://www.pa.msu.edu/astro/software/emil](http://www.pa.msu.edu/astro/software/emil)
for the M3 multiplet.

In this section, spectral fits and discussion of results are only given for the M3 multiplet. An electron temperature of 1000 K is assumed throughout the analysis. The observed relative intensities of the N II λ5679.56 are 0.238 and 0.215, which agree better with the intermediate coupling (Table 1). However, the intensity of this line is 0.020 ± 0.003, which seems to agree better with the LS coupling assumption (column 3) and the intermediate coupling (column 4). The theoretical predictions in intermediate coupling are calculated from the N II effective recombination coefficients of FSL11. Comparisons of the observed and predicted relative intensities are in columns 6 and 7. Results of multi-Gaussian profile fitting to the wavelength range 5650–5760 Å are also presented in Fig. 28.

The strongest M3 line, λ5679.56, is blended with λ5676.02 of the same multiplet (Fig. 28). Two Gaussian profiles with the same width were used to fit them. The intensities of λ5679.56 and 5676.02 are 0.135 ± 0.007 and 0.035 ± 0.004, respectively. Thus the

3.2 The N II optical recombination spectrum

In this section, we present intensities of the N II ORLs detected in the spectrum of NGC 7009, and analyze these lines using the N II effective recombination coefficients of FSL11. Unless otherwise specified, the theoretical relative intensities of the N II lines quoted in this section are all based on that calculation. Comparison of the observed and predicted relative intensities of the N II lines with accurate intensities is made to assess the new atomic data. An electron temperature of 1000 K is assumed throughout the analysis.

In this section, spectral fits and discussion of results are only given for the M3 3p 3D–3s 3P o multiplets, M28 3d 3D o–3p 3P multiplets and the strongest multiplets M39a,b of the 4f – 3d transition array. Discussion of other multiplets of N II are given in Appendix B.

This multiplet is the strongest of N II in optical. The intensities of the N II M3 lines are presented in column 5 of Table 1 in units of $I(\lambda 5679.56) = 1.0$. Also presented are the theoretical relative intensities in the LS coupling assumption (column 3) and the intermediate coupling (column 4). The theoretical predictions in intermediate coupling are calculated from the N II effective recombination coefficients of FSL11. Comparisons of the observed and predicted relative intensities are in columns 6 and 7. Results of multi-Gaussian profile fitting to the wavelength range 5650–5760 Å are also presented in Fig. 28.

The strongest M3 line, λ5679.56, is blended with λ5676.02 of the same multiplet (Fig. 28). Two Gaussian profiles with the same width were used to fit them. The intensities of λ5679.56 and 5676.02 are 0.135 ± 0.007 and 0.035 ± 0.004, respectively. Thus the

Table 1. Comparison of the observed and predicted relative intensities of the N II M3 lines detected in the spectrum of NGC 7009. $I_{LS}$ is the theoretical intensity deduced from the effective recombination coefficients of FSL11, and $I_{IC}$ is the value in the LS coupling assumption. The above two symbols have the same meaning in other tables of the current paper. An electron temperature of 1000 K is assumed for the theoretical predictions $I_{IC}$.

| Line         | $J_2 - J_1$ | $I_{LS}$ | $I_{IC}$ | $I_{obs}$ | $I_{obs}/I_{LS}$ | $I_{obs}/I_{IC}$ |
|--------------|-------------|----------|----------|-----------|------------------|------------------|
| λ5676.63     | 2 – 1       | 0.536    | 0.466    | 0.481     | 0.897            | 1.032            |
| λ5676.02     | 1 – 0       | 0.238    | 0.215    | 0.273     | 1.147            | 1.271            |
| λ5679.56     | 3 – 2       | 1.000    | 1.000    | 1.000     | 1.000            | 1.000            |
| λ5686.21     | 1 – 1       | 0.179    | 0.128    | 0.187     | 1.047            | 1.464            |
| λ5710.77     | 2 – 2       | 0.179    | 0.167    | 0.151     | 0.844            | 0.902            |

Figure 27. Spectrum of NGC 7009 from 8740 to 8840 Å showing the two C II M28.01 autoionization lines $\lambda\lambda$8793.80 and 8799.90. The dashed curve is the sum of Voigt profile fits to the two C II lines, which probably blend with two weaker features, He II $\lambda 8799.00$ and an unknown component, whose profiles are assumed to be Gaussian and are represented by the dotted curve. The solid continuous curve is the sum of all fits. Here a Voigt profile is the convolution of a Gaussian (~3.0 Å FWHM) and a Lorentz profile (~6.86 Å FWHM). Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.

Figure 28. Spectrum of NGC 7009 from 5650 to 5760 Å showing the N II M3 lines and other emission features. The continuous curve is the sum of multi-Gaussian profile fits. Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.

$\lambda 5676.02/\lambda 5679.56$ ratio is 0.273, which agrees with both theoretical ratios within errors (Table 1). Another two lines, $\lambda\lambda$5666.63 and 5710.77, are free of line blending. The intensity of the $\lambda 5710.77$ line is 0.020 ± 0.003, which agrees better with the intermediate coupling (Table 1). The intensity of the $\lambda 5666.63$ line is 0.065 ± 0.007, which also agrees better with intermediate coupling (Table 1). Another M3 line $\lambda 5686.21$ is partially blended with a weaker feature, which was identified as Mn V $\lambda 5692.00$ (Fig. 28). The fitted intensity of $\lambda 5686.21$ is 0.024 ± 0.005, which seems to agree better with LS coupling (Table 1). However, the intensity of this line is questionable due to weakness. The other M3 line, $\lambda 5730.65$, is not observed in our spectrum.

3.2.2 Multiplet 28, 3d 3D o–3p 3P

Gaussian profile fitting to M28 $\lambda\lambda$5941.65 (3d 3D o–3p 3P o) yields an intensity of 0.030 ± 0.004. The intensity contribution from the M28 $\lambda\lambda$5940.24 (3d 3D o–3p 3P o) line is negligible. The intensity ratio of $\lambda 5941.65$ and the N II M3 $\lambda 5679.56$ line is 0.228, which is 42 per cent lower than the theoretical ratio.
Table 2. Same as Table 1 but for a comparison of the observed and predicted relative intensities of the N II M28 lines detected in the spectrum of NGC 7009.

| Line          | $J_2 - J_1$ | $I_{LS}$ | $I_{LC}$ | $I_{obs}$ | $I_{obs} / I_{LS}$ | $I_{obs} / I_{LC}$ |
|--------------|-------------|----------|----------|-----------|-------------------|-------------------|
| λ5941.65     | 3 – 2       | 1.000    | 1.000    | 1.000     | 1.000             | 1.000             |
| λ5952.39     | 2 – 2       | 0.152    | 0.122    | 0.255     | 1.686             | 2.103             |
| λ5951.78     | 2 – 1       | 0.455    | 0.412    | 0.471     | 1.037             | 1.143             |
| λ5960.90d    | 1 – 2       | 0.010    | 0.009    | –         | 0.000             | 0.000             |
| λ5927.81     | 1 – 0       | 0.202    | 0.219    | 0.317     | 1.570             | 1.448             |

a Including the λ5940.24 (3d $^3D_2^o$ – 3p $^3P_1$) line.
b Corrected for the contribution from the He II λ5952.93 (24h $^2H^o$ – 5g $^2G$) line (74 per cent).
c Corrected for the contribution from the He II λ5931.83 (25h $^2H^o$ – 5g $^2G$) line (57 per cent).
d Not detected.

Figure 29. Spectrum of NGC 7009 from 5850 to 6000 Å showing the N II M28 lines and other emission features. The continuous curve is the sum of Gaussian profile fits. Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.

Another M28 line λ5927.81 (3d $^3D_2^o$ – 3p $^3P_0$) is blended with M28 λ5931.78 (3d $^3D_2^o$ – 3p $^3P_1$), which coincides in wavelength with the He II λ25h $^2H^o$ – 5g $^2G$ λ5931.83 line. Two Gaussian profiles were used to fit the complex, and the intensity of the λ5927.81 line is 0.007, with a large uncertainty. The intensity ratio of λ5927.81 to the λ5941.65 line is 0.250, which agrees with the predicted ratio 0.235. Assuming that the He II λ5931.83 line contributes 55 per cent to the total intensity of the blend at λ5932, as estimated from the hydrogenic theory of Storey & Hummer (1995), we obtained an intensity of the λ5931.78 line which is much higher than the theoretical prediction. Another M28 line λ5952.39 (3d $^3D_2^o$ – 3p $^3P_2$) is blended with the He II λ24h $^2H^o$ – 5g $^2G$ λ5952.93 line. The intensity of the λ5952.39 line is much higher than the predicted value. The other M28 line λ5960.90 (3d $^3D_2^o$ – 3p $^3P_0$) is not observed.

3.2.3 4f – 3d transitions

The 4f – 3d transitions of N II are located in the blue side of our spectrum (<4500 Å) and suffer from line blending. Accurate measurements of most of them are difficult. Table 2 presents the observed and predicted relative intensities of those 4f – 3d lines with the most reliable intensities. Figs. 30 and 31 show some of the N II lines of the 4f – 3d transition array detected in the spectrum of NGC 7009. The results of multi-Gaussian profile fitting are overplotted. Only the M39a and M39b multiplets are discussed here. Discussion of other 4f – 3d transitions of N II are presented in Appendix B8.

3.2.3.1 Multiplet 39a, 4f G[7/2] – 3d $^3F_o$: The λ4035.08 (4f G[7/2]$_3$ – 3d $^3F_3$) and λ4043.53 (4f G[7/2]$_4$ – 3d $^3F_4$) lines are shown in Fig. 30. The λ4035.08 line is blended with the O II λ5019 4f F[3]$_{5/2}$ – 3d $^3F_3/2$ λ4035.07 line, which contributes 15 per cent to the total intensity, and the O II M50b 4f F[3]$_{5/2}$ – 3d $^3F_3/2$ λ4035.46 line, which is negligible. The λ4035 line has an intensity 0.037±0.006. Here the contribution from the O II λ4035.07 line has been corrected for. This intensity agrees with the predicted relative intensity. The λ4043.53 line is partially blended with the N II M39b 4f G[9/2]$_3$ – 3d $^3F_3$ λ4043.31 line, which is more than 2 times stronger. The intensity of the λ4043.53 line also agrees well with the predicted value. Reliable measurements of the λ4044.78 (4f G[7/2]$_4$ – 3d $^3F_3$) line are difficult. The other two lines λ4056.90 (4f G[7/2]$_4$ – 3d $^3F_2$) and λ4058.16 (4f G[7/2]$_1$ – 3d $^3F_2$) are too weak.

3.2.3.2 Multiplet 39b, 4f G[9/2] – 3d $^3F_o$: The λ4041.31 (4f G[9/2]$_3$ – 3d $^3F_3$) line is blended with the O II M50c 4f F[2]$_{5/2}$ – 3d $^3F_3/2$ λ4041.28 and O II M50c 4f F[2]$_{5/2}$ – 3d $^3F_5/2$ λ4041.95 lines. The two O II lines contribute only ~7 per cent to the total intensity of the blend at λ4041. The intensity of the λ4041.31 line is 0.083±0.008. The intensity ratio of λ4041.31 to the N II M3 λ5679.56 line is 0.604, which agrees quite well with the predicted ratio 0.598. Another M39b line λ4026.08 (4f G[9/2]$_4$ – 3d $^3F_3$) is blended with the He I M18 5d $^3D$ – 2p $^3P_0$ λ4026.20 line. The other M39b line λ4039.35 (4f G[9/2]$_1$ – 3d $^3F_2$) is not observed.

3.2.4 Comments on the N II recombination spectrum

The effective recombination coefficients used for the analysis of the N II recombination spectrum are from FSL11, which is dedicated to low temperatures ($T_e <$ 10 000 K) and is an improvement over all previous calculations for this ion, as described Section 2.1. The best observed N II lines in our spectrum are M3 (3p $^3P$ – 3s $^3P$), M12 (3p $^3P$ – 3s $^1P$), and the strongest lines of the 4f – 3d array, e.g. λ4041.31 (M39b 4f G[9/2]$_3$ – 3d $^3F_3$), λ4043.53 (M39a 4f G[7/2]$_3$ – 3d $^3F_3$). Those N II lines have been used for plasma diagnostics (Section 2.3). The fine-structure components of the N II multiplets M5 (3p $^3P$ – 3s $^3P$), M20 (3d $^3D$ – 3p $^3D$), M28 (3d $^3D$ – 3p $^3P$), and M29 (3d $^3P$ – 3p $^3P$) observed in our spectrum are incomplete due to line blending. Those that are detected are also blended with other lines. Although multi-Gaussian profile fitting has been carried out and effective recombination coefficients used to correct for the blended line of other ionic species, the derived intensities of those N II lines could still be questionable. Grandi (1976) shows that the strongest components of the M3, M5 and M28 multiplets are affected by the fluorescence mechanism in the Orion nebula. However, such effects is probably insignificant.
in NGC 7009 (C. Morisset, private communication). The 4f – 3d transitions are almost certainly free of fluorescence enhancement because they require such high excitation energy photons that the central star of NGC 7009 cannot afford.

3.3 The O II optical recombination spectrum

LSBC observed eight multiplets of the 3 – 3 transition arrays and a few dozen 4f – 3d lines of O II. The effective radiative recombination coefficients for the O II 3d – 3p and 4f – 3d transitions were calculated under the intermediate coupling scheme, and were used for spectral analysis. For the 3p–3s transitions, the effective recombination coefficients of Storey (1994), whose calculations were carried out in the LS coupling scheme, were utilized. LSBC confirmed the breakdown of LS coupling in the O II transitions, especially those of the 4f – 3d configuration. In Paper I, we presented very deep spectrum of NGC 7009. The data quality is higher than that in LSBC. In the current paper, we present the intensities of the O II ORLs, and analyze the O II recombination spectrum using the new O II effective recombination coefficients of PJS. Unless otherwise specified, the theoretical relative intensities of the O II lines quoted in this section are all based on that calculation. Comparison of the observed and predicted relative intensities of the O II lines with accurate intensities is made to assess the new atomic data. An electron temperature of 1000 K is assumed throughout the analysis. In this section, spectral fits and discussion of the results are only given for the M1 3p 4D o – 3s 4P, M10 3d 4F – 3p 4D o multiplets and the strongest multiplets M39a,b of the 4f – 3d transition array. Discussion of other multiplets of O II are given in Appendix C.
Table 3. Same as Table 1 but for a comparison of the observed and predicted relative intensities of the N II 4f – 3d lines detected in the spectrum of NGC 7009.

| Line     | $J_2 - J_1$ | $I_{HC}$ | $I_{obs}$ | $I_{HC}$/ $I_{obs}$ |
|----------|-------------|----------|-----------|---------------------|
| M39a 4f G[7/2] – 3d $^3$F⁰ | 3–2 | 0.477 | 0.540 | 1.132 |
| λ4035.08 | 3–2 | 0.436 | 0.432 | 0.992 |
| M48b 4f G[9/2] – 3d $^3$F⁰ | 5–4 | 1.000 | 1.000 | 1.000 |
| M43a 4f F[5/2] – 3d $^4$D⁰ | 3–2 | 0.293 | 0.343 | 1.171 |
| M44b 4f F[7/2] – 3d $^4$D⁰ | 3–2 | 0.296 | 0.281 | 0.950 |
| M44a 4f F[5/2] – 3d $^3$D⁰ | 2–1 | 0.539 | 0.669 | 1.241 |
| M55a 4f D[5/2] – 3d $^3$P⁰ | 4–3 | 0.984 | 0.996 | 1.012 |
| λ4422.02 | 2–1 | 0.130 | 0.242 | 1.859 |
| M58a 4f G[7/2] – 3d $^1$F⁰ | 4–3 | 0.201 | 0.395 | 1.965 |
| λ4552.53 | 4–3 | 0.474 | 0.593 | 1.251 |
| M58b 4f G[9/2] – 3d $^3$P⁰ | 4–3 | 0.147 | 0.222 | 1.510 |
| λ4649.64 | 2–1 | 0.447 | 0.623 | 1.400 |

a Including the contribution from the O II M50b 4fF[3]5/2 – 3d $^3$F₅/₂, λ4305.07 line. Neglecting the O II M50b 4fF[3]5/2 – 3d $^3$F₅/₂, λ4035.46 line.

b Neglecting the contributions from the O II M50c 4fF[2]1/2 – 3d $^4$F₅/₂, λ4041.28 and O II M50c 4fF[2]1/2 – 3d $^3$F₅/₂, λ4041.95 lines.

c Including N II M43a 4fF[5/2] – 3d $^3$D₂, λ4175.66.

d Including N II M48b 4fF[7/2] – 3d $^3$D₂, λ4237.05.

e Including N II M48a 4fF[5/2] – 3d $^3$D₂, λ4241.78. Neglecting N II M48a 4fF[5/2] – 3d $^3$D₂, λ4241.24 and O II M52c 4fF[2]1/2 – 3d $^3$D₂, λ4242.04.

f Including Ne II M60b 4fF[1]1/2 – 3d $^3$F₅/₂, λ4422.69. Neglecting O III M49b 5gF[3]3/2 – 3d $^4$D₂, λ4422.02.

g Including Ne II M55d 4fF[2]1/2 – 3d $^3$F₃/₂, λ4553.17 and Si III M2 4p $^3$P₂ – 4s $^3$S₁, λ552.62.

h Including N III M3 3p $^3$D₄/₂ – 3s $^4$P₀/₂, λ4530.86.

i Overestimated.

3.3.1 Multiplet 1, 3p $^4$D⁰ – 3s $^4$P

The M1 multiplet is the strongest amongst all the O II permitted transitions, and is one of the best observed multiplets. Comparisons of the observed and predicted relative intensities of the M1 fine-structure components are presented in Table 4. The results of multi-Gaussian profile fitting are shown in Fig. 32.

The strongest M1 line λ4649.13 is blended with λ4650.84 of the same multiplet; also blended are the three lines of C II M1 3p $^3$P⁰ – 3s $^3$S: λ4647.42, 4650.25 and 4651.47. Five Gaussian profiles of the same FWHM were used to fit the complex, with the laboratory wavelength differences utilized. The relative intensities of the three C II M1 lines were assumed to be as in LS coupling, i.e. 5:3:1, but the relative intensities of the two O II lines were not constrained. The intensity of the λ4649.13 line is 0.667 ± 0.030. The intensity of the λ4650.84 line is 0.169 ± 0.008. The intensity ratio of the λ4650.84 and λ4649.13 lines agrees with the theoretical ratios predicted in the intermediate coupling, but is slightly higher than the LS coupling value (Table 4). Another three M1 lines, λ4661.63, 4673.73 and 4676.24, are free of line blending. The fitted intensities of the three lines agree with the predicted values, except for λ4673.73, whose measurement is obviously higher than both predicted values (Table 4). The measurement uncertainties of the three lines are all less than 10 per cent. Such large difference between the observed and predicted intensity of λ4673.73 cannot be explained explicitly. λ4673.73 coincides in wavelength with C II M5 3p $^3$P₁ – 3d $^3$P₂, λ4673.95. However, as discussed in LSBC, a significant contribution from the C II λ4673.95 line was unlikely because another C III M5 line λ4665.86, which is expected to be much stronger than λ4673.95, is not observed. λ4676.24 is blended with O II M91 4fG[4]3/2 – 3d $^3$D₂, λ4677.07 and N II M61b 4fD[3/2] – 3d $^3$P₀, λ4678.14, but the contributions of these two lines are probably insignificant, as estimated from the new effective recombination coefficients.

Another two M1 lines, λ4638.86 and 4641.81, are both blended with two N III M2 lines λ4640.64 and 4641.85, which are excited by the Bowen fluorescence mechanism. Also blended with this feature is N II M5 3p $^3$P₁ – 3s $^3$P₂, λ4643.09. Taking into account another N III M2 line λ4634.14 and N II M5 3p $^3$P₂ – 3s $^3$P₂, λ4630.54, we used six Gaussian profiles (the O II λ4641.81 line coincides in wavelength with N II λ4641.85, thus they were treated as a single component) to fit the complex, assuming that all the six components had the same FWHM. The results of the fitting are plotted in Fig. 32. Here the contribution of N III M2 λ4641.85 to the total intensity of the blend at λ4642 was estimated from N III M2 λ4634.14, which is free of line blending. The intensity ratio of the two N III lines was assumed to be as in pure LS coupling, i.e. 1:5, considering the fact that the two lines decay from the upper level, thus their intensity ratio depends only on the coupling scheme instead of the excitation mechanism. The relative intensity of the two O II M1 lines were not constrained. The intensity of λ4641.81 thus obtained is 0.437 ± 0.085. This measurement agrees well with the predicted value in the intermediate coupling (Table 4). The resultant intensity of the λ4638.86 line is higher than the theoretical ratios (Table 4), but its intensity could be unreliable due to the strength of the N II M2 λ4640.64 line, which is more than 10 times stronger. The C II M12.01 6d $^2$D – 4p $^4$P⁰ lines, λ4637.63, 4638.91 and 4639.07, may be also blended in the λ4638 feature, but taking them into account makes the task of line deblending more difficult.

The other M1 line λ4696.35, which is expected to be the faintest of O II M1, is observed (the inset in Fig. 32). It coincides in wavelength with O II M89a 4fD[3]5/2 – 3d $^3$D₂, λ4696.35, and is partially blended with another weak feature which was identified as N II M61a 4fD[5]5/2 – 3d $^1$P₁, λ4694.64. Accurate measurements of λ4696.35 are difficult due to weakness. Assuming that the O II M89a λ4696.35 line contributes 38 per cent to total flux of the blend at λ4696, as estimated from the new O II effective recombination coefficients, we obtained an intensity of 0.015 for the M1 λ4696.35 line, which agrees well with the newly predicted value (Table 4).

3.3.2 Multiplet 10, 3d $^4$F – 3p $^4$D⁰

The observed and predicted relative intensities of O II M10 lines detected in NGC 7009 are presented in Table 5. The emission features of the M10 lines and the results of Gaussian profile fitting are shown in Fig. 33. λ4069.62 and 4069.89, are blended with [S II] 3p $^3$P₀/₂ – 3p $^3$P₂, λ4068.60 and the three C III M16 5g $^3$G
Figure 32. Spectrum of NGC 7009 from 4625 to 4680 Å showing the O II M1 lines and other emission features. The solid continuous curve is the sum of Gaussian profile fits. The dashed curve is the sum of the Gaussian profiles of the three C III M1 (3p 3P 0 – 3s 3S) lines λλ4647.42, 4650.25 and 4651.47, whose intensity ratio is fixed to be as in LS coupling, i.e. 1 : 3 : 5. The inset shows the very weak O II M1 λ4696.35 (3p 4D 3/2 – 3s 3P 1/2) line, which is located in a different wavelength region. Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.

Table 4. Same as Table 1 but for a comparison of the observed and predicted relative intensities of O II M1 lines detected in the spectrum of NGC 7009.

| Line         | J2 – J1 | I LS | I IC | I obs | I obs / I LS | I obs / I IC |
|--------------|---------|------|------|-------|-------------|-------------|
| λ4638.86     | 3/2–1/2 | 0.208| 0.283| 0.502 | 2.414        | 1.774       |
| λ4641.81     | 5/2–3/2 | 0.525| 0.632| 0.656 | 1.249        | 1.037       |
| λ4649.13     | 7/2–5/2 | 1.000| 1.000| 1.000 | 1.000        | 1.000       |
| λ4650.84     | 1/2–1/2 | 0.208| 0.290| 0.263 | 1.264        | 0.907       |
| λ4661.63     | 3/2–3/2 | 0.267| 0.317| 0.326 | 1.222        | 1.028       |
| λ4673.73     | 1/2–3/2 | 0.042| 0.051| 0.078 | 1.857        | 1.535       |
| λ4676.24     | 5/2–5/2 | 0.225| 0.220| 0.237 | 1.054        | 1.078       |
| λ4696.35     | 3/2–5/2 | 0.025| 0.024| 0.023 | 0.920        | 0.962       |

- 4f 3F 0 lines: λλ4067.94, 4068.91 and 4070.26. The blend feature at λ4070 is also partially resolved from another O II M10 line λ4072.16. Six Gaussian profiles of the same FWHM were used to fit the complex (λλ4069.62 and 4069.89 were treated as a single component, given their close wavelengths), assuming that the differences of the observed wavelengths were the same as those of the laboratory ones. Assumption was also made that the relative intensities of the three C III M16 lines were as in pure LS coupling, i.e. 1.00 : 1.31 : 1.71. The λ4072.16 line blends with N II M38b 4f F[7/2] 3 – 3d 3F 2 λ4073.05 and O II M48a 4f G[3/2] 3 – 3d 3F 2 λ4071.23. The intensity of the N II λ4073.05 line was assumed to be negligible, and the O II λ4071.23 line contributes about 9 per cent to the total intensity of the blend at λ4072. Despite of the assumptions above, no further constraint was set for the relative intensities of the following four features in Gaussian profile fitting: The C III M16 multiplet, [S II] λ4068.60, the two O II M10 lines at λ4070, and O II λ4072.15. The resultant total intensity of λ4069.62, 89 is 0.635 ± 0.060. The intensity of the λ4072.16 line derived from the fits is 0.549 ± 0.029. These intensities agree well with the theoretical values predicted in the intermediate coupling (Table 5).

The total intensity of the C III M16 λ4069 multiplet yielded by Gaussian profile fitting is 0.288 ± 0.058. The intensity ratio of the C III M16 multiplet to the C III M16 λ4650 multiplet (the measurements of C III M1 are described in Section 3.3.1) agrees with the predicted ratio within errors. Here the predicted C III M16/M1 ratio is derived based on the radiative and dielectronic recombination coefficients given by Péquignot, Petitjean & Boisson (1991) and Nussbaumer & Storey (1984), respectively. The C III M16 λ4650 multiplet is mainly excited by dielectronic recombination, while the C III M16 λ4069 multiplet is by radiative recombination (LSBC).

λ4075.86 is expected to be the strongest in O II M10 (Table 5). It is blended with [S II] 3p 3P 1/2 – 3p 3P 3/2, λ4076.35. We used the same technique as LSBC to derive the intensities. The flux contribution of [S II] λ4076.35 to the blend at λ4076 was estimated from the measured intensity of [S II] λ4068.60. A five-level atomic model was constructed to calculate the level population of S +, with an appropriate electron temperature and density assumed. The calculated [S II] λ4068.60/λ4076.35 intensity ratio was 3.04, the same as the ratio given by LSBC. The resultant intensity of the O II λ4075.86 line is 0.688 ± 0.070. The intensity ratio of the λ4075.86 line and the O II M1 λ4661.63 (the measurements of
Table 5. Same as Table 4 but for a comparison of the observed and predicted relative intensities of O II M10 lines detected in the spectrum of NGC 7009. Only the components with the most reliable measurements are presented.

| Line       | $J_2 - J_1$ | $I_{LS}$ | $I_{IC}$ | $I_{obs}$ | $I_{obs}/I_{LS}$ | $I_{obs}/I_{IC}$ |
|------------|-------------|----------|----------|-----------|------------------|------------------|
| $\lambda4069.89^a$ | 5/2–3/2     | 0.730    | 0.956    | 0.923     | 1.264            | 0.965            |
| $\lambda4072.16^b$ | 7/2–5/2     | 0.686    | 0.807    | 0.798     | 1.163            | 0.989            |
| $\lambda4075.86$ | 9/2–7/2     | 1.000    | 1.000    | 1.000     | 1.000            | 1.000            |
| $\lambda4078.84$ | 3/2–3/2     | 0.112    | 0.141    | 0.130     | 1.161            | 0.922            |
| $\lambda4085.11^c$ | 5/2–5/2     | 0.146    | 0.162    | 0.165     | 1.129            | 1.018            |

$^a$ Including the contribution from the O II M10 3d $^4F_{3/2} - 3p^2D_{3/2}^L$ $\lambda4069.62$ line.

$^b$ Corrected for the contribution from the O II M48a 4F G(5)$^0_{1/2}$ – 3d $^2F_{7/2}$ $\lambda4071.23$ line, which is about 9 per cent. Neglecting the N II M38b 4F G(7)$^2_{1/2}$ – 3d $^2F_{9/2}$ $\lambda4073.05$ line (about 2 per cent).

$^c$ Neglecting N II M38b 4F G(7)$^2_{1/2}$ – 3d $^2P_{3/2}$ $\lambda4082.89$ (less than 2 per cent).

3.3.3 4f – 3d transitions

Several dozen transitions of this group were identified and presented in the emission line list of NGC 7009 (Paper I). Table 5 gives the observed and predicted relative intensities of the 4f – 3d lines of O II with the most reliable measurements. For most cases, the measured intensities agree with both calculations. Here we present spectral fits and discussion of the M48a 4F G(5)$^0_{1/2}$ – 3d $^4F_{7/2}$ multiplet of O II. Discussion of other multiplets of the O II 4f – 3d configuration are given in Appendix C. Figs 34 and 35 show some of the detected O II ORLs from the 4f – 3d configuration. The strongest 4f – 3d line of O II observed in NGC 7009, $\lambda4089.29$ (M48a 4F G(5)$^0_{1/2}$ – 3d $^4F_{9/2}$), is shown in Fig 33. A few O II lines of the 4f – 3d configuration, which are blended with the N II lines of the same configuration, are shown in Fig 30.

The $\lambda4089.29$ (M48a 4F G(5)$^0_{1/2}$ – 3d $^4F_{9/2}$) line is blended with the $\lambda4088.27$ (M48a 4F G(5)$^0_{1/2}$ – 3d $^4F_{9/2}$) and the Si IV M1 4p $^3P_1$ – 4s $^2S_{1/2}$ $\lambda4088.86$ lines (Fig 30). The $\lambda4088.27$ line contributes less than 2 per cent to the total flux of the blend at $\lambda4089$. The contribution of the Si IV $\lambda4088.86$ line was estimated from the observed Si IV M1 4p $^3P_1$ – 4s $^2S_{1/2}$ $\lambda4116.10$, assuming that the relative intensities of the two Si IV M1 lines are as in the pure LS coupling, i.e.: 2 : 1. The resultant intensity of the $\lambda4089.29$ line is 0.265±0.013. The intensity ratio of $\lambda4089.29$ and the O II M1 $\lambda4649.13$ line is 0.398, which agrees with the new theoretical prediction (0.387) within measurement errors. The other M48a line $\lambda4071.23$ (4F G(5)$^0_{1/2}$ – 3d $^4F_{7/2}$) is blended with the O II M10 3d $^4F_{7/2}$ – 3p $^3D_{3/2}^L$ $\lambda4072.16$ line (see Section 3.3.2), which is 10 times stronger.

3.3.4 Multiplets with parentage other than $^3P$

The O II ORLs from multiplets with parentage other than $^3P$ are detected in NGC 7009, and they include M15, M36, M101 and M105, which were also observed by LSBC. However, the effective recombination coefficients are only available for two of those multiplets (c.f. discussion in Section 2.3. Table 7 presents the line intensities. The intensities observed by LSBC are also listed. The O II M15 3p $^3P_1$ – 3s $^2S_{1/2}$, $\lambda4590.97$ and 4596.18 are shown in Fig 31. The O II M36 3d $^4G$ – $^3P_2$ 2P lines $\lambda4185.45$ and 4189.79 are shown in Fig 31.

3.3.5 Comments on the O II recombination spectrum

The unpublished effective recombination coefficients of P.J. Storey used in the current analysis of the O II recombination spectrum are accurate at low temperatures ($T_e < 10,000$ K). Appropriate assumptions have been made in the calculation, as described in Sec-
The Optical Recombination Spectrum of NGC 7009

| Line         | J₂ − J₁ | Iₚₑᵈ | Iₚᵟₑᵈ | Iₒᵦᵠ | Iₒᵦᵠₚₑᵈ | Iₒᵦᵠₚᵟₑᵈ |
|--------------|---------|------|-------|-------|-----------|-----------|
| M48a 4f G[5]² – 3d ²F | 11/2–9/2 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| M48b 4f G[4]² – 3d ²F | 7/2–5/2 | 0.285 | 0.316 | 0.326 | 1.141 | 1.032 |
| M48c 4f G[3]² – 3d ²F | 5/2–3/2 | 0.271 | 0.347 | 0.347 | 1.280 | 1.000 |
| M50a 4f F[4]² – 3d ²F | 9/2–9/2 | 0.125 | 0.126 | 0.137 | 1.096 | 1.087 |
| M50b 4f F[3]² – 3d ²F | 7/2–7/2 | 0.063 | 0.068 | 0.076 | 1.206 | 1.120 |
| M53a 4f D[3]² – 3d ²P | 7/2–5/2 | 0.413 | 0.522 | 0.534 | 1.293 | 1.022 |
| M53b 4f D[2]² – 3d ²P | 5/2–3/2 | 0.232 | 0.326 | 0.253 | 1.091 | 0.776 |
| M4294.78² | 3/2–1/2 | 0.105 | 0.118 | 0.108 | 1.031 | 0.919 |
| M53c 4f D[1]² – 3d ²P | 3/2–1/2 | 0.151 | 0.123 | 0.145 | 0.958 | 1.176 |
| M55 4f G[3]² – 3d ²P | 7/2–5/2 | 0.156 | 0.188 | 0.221 | 1.414 | 1.176 |
| M63a 4f D[3]² – 3d ²D | 7/2–5/2 | 0.057 | 0.088 | 0.094 | 1.651 | 1.067 |
| M3357.25² | 5/2–3/2 | 0.154 | 0.168 | 0.185 | 1.200 | 1.011 |
| M4282.96² | 9/2–7/2 | 0.097 | 0.109 | 0.127 | 1.303 | 1.159 |
| M576b 4f G[4]² – 3d ²F | 9/2–7/2 | 0.121 | 0.133 | 0.139 | 1.150 | 1.045 |
| M4371.62² | 5/2–3/2 | 0.189 | 0.264 | 0.229 | 1.208 | 0.869 |
| M4285.69 | 3/2–1/2 | 0.137 | 0.198 | 0.215 | 1.569 | 1.086 |
| M4491.23 | 3/2–1/2 | 0.065 | 0.082 | 0.083 | 1.271 | 1.004 |
| M4894.49 | 5/2–3/2 | 0.086 | 0.109 | 0.113 | 1.316 | 1.108 |
| M4477.96² | 7/2–5/2 | 0.428 | 0.444 | 0.483 | 1.126 | 1.088 |
| M4602.13² | 5/2–3/2 | 0.171 | 0.194 | 0.195 | 1.139 | 1.003 |

² Corrected for the contribution from O II 4f 3p – 3d 3D₇/₂ – 3d ⁴D₉/₂ λ4303.61 (about 12 per cent). Neglecting O II M53a 4f D[3]² – 3d ⁴P₅/₂ λ4304.08 (~3 per cent).
³ Including O II M53b 4f D[2]² – 3d ⁴P₅/₂ λ4294.92 (~12 per cent).
⁴ Including O II M53c 4f D[1]² – 3d ⁴P₅/₂ λ4288.82.
⁵ Including O II M78c 4f F[2]² – 3d ²F₅/₂ λ4292.21. Neglecting O II M55 4f G[3]² – 3d ⁴P₅/₂ λ4291.86 and O II M78c 4f F[2]² – 3d ²F₅/₂ λ4292.95.
⁶ Including O II M63a 4f D[3]² – 3d ²D₂/₂ λ4357.52. Neglecting O II M63a 4f D[3]² – 3d ⁴D₅/₂ λ4357.52.
⁷ Including O II M63c 4f D[3]² – 3d ⁴D₅/₂ λ4283.25. Neglecting O II M78a 4f F[4]² – 3d ²F₅/₂ λ4282.02.
⁸ Neglecting O II M76b 4f G[4]² – 3d ²F₅/₂ λ4371.24 (less than 2 per cent).
⁹ Corrected for the contribution from O II M78a 4f F[4]² – 3d ²F₅/₂ λ4312.44 (less than 2 per cent).
¹⁰ Neglecting O II M45a 5g H[1]² – 4f G[9/2] λ4479.91 (less than 2 per cent).
¹¹ Corrected for the contribution from N II M5 3p ³P₂ – 3s ³P₉/₂ λ4601.48 (26 per cent). Neglecting N II M45a 5g H[1]² – 4f G[9/2] λ4479.91 (less than 2 per cent).

Table 6. Comparison of the observed and predicted relative intensities of the O II 4f – 3d lines detected in the spectrum of NGC 7009. LSBC is the predicted intensities based on the effective recombination coefficients of Liu et al. [1995], and PJS is based on the unpublished effective recombination coefficients of P. J. Storey. An electron temperature of 1000 K is assumed for the theoretical predictions.
Figure 33. Spectrum of NGC 7009 from 4060 to 4115 Å showing the O II M10 lines and some O II lines from the 4f – 3d configuration. The strongest O II 4f – 3d line λ4089.29 (M48a 4f G[5] o 11/2 – 3d F9/2) is observed. The solid continuous curve is the sum of Gaussian profile fits. The dashed curve is the sum of the Gaussian profiles of the three C III M16 (5g 3g – 4f 3Fo) lines λλ4067.94, 4068.92 and 4070.31, whose intensity ratio is fixed to be as in LS coupling, i.e. 1.00 : 1.31 : 1.71. Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.

Table 7. The O II optical recombination lines in NGC 7009 with parentage other than 3P J. Line intensities measured by LSBC are also presented. All intensities are normalized such that \( I(H/β) = 100 \).

| Line (Å) | Mult. | Transition | Current | LSBC |
|----------|-------|------------|---------|------|
|          |       |            | PA=0°   | PA=45° |
| λ4590.97 | M15   | \((^1D)3p \ 2F_2/2 – (^1D)3s \ 2D_5/2\) | 0.087 | 0.0907 | 0.0752 |
| λ4596.18 | M15   | \((^1D)3p \ 2F_2/2 – (^1D)3s \ 3D_5/2\) | 0.062 | 0.0424 | 0.0476 |
| λ4185.45 | M36   | \((^1D)3d \ 2G_7/2 – (^1D)3p \ 2F_2/2\) | 0.070 | 0.0572 | 0.0580 |
| λ4189.79 | M36   | \((^1D)3d \ 2G_9/2 – (^1D)3p \ 2F_2/2\) | 0.083 | 0.0703 | 0.0747 |
| λ4253.89 | M101  | \((^1D)4f \ H[5]_o 11/2 – (^1D)3d \ 2G_9/2\) | 0.058 | 0.0318 | 0.0249 |
| λ4843.37 | M105  | \((^1D)4f \ P[1]_o 3/2 – (^1D)3d \ 2S_1/2\) | 0.021 |       |       |

- a Including O II M15 \((^1D)3p \ 2F_2/2 – (^1D)3s \ 2D_5/2\) λ4595.96.
- b Including O II M36 \((^1D)3d \ 2G_7/2 – (^1D)3p \ 2F_2/2\) λ4189.59.
- c Including O II M101 \((^1D)4f \ H[5]_o 11/2 \ 3D_3 \ 2G_9/2\) λ4253.91 and O II M101 \((^1D)4f \ H[5]_o 11/2 \ 3D_3 \ 2G_7/2\) λ4254.12.
- d Including O II λ4843.37 M105 \((^1D)4f \ P[1]_o 3/2 \ 3D_3 \ 2S_1/2\).
from Nussbaumer & Storey (1984). The calculation of PJS does not reach such high energy.

### 3.4 The Ne II optical recombination spectrum

Several dozen emission features in the spectrum of NGC 7009 were identified as the Ne II permitted lines. In this section, we present spectral fits and discussion of the results only for the M2 3p^2 D^0 3s^3 P, M13 3d^1 F 3p^4 D^0, M9 3p^0 F^0 3s^3 D, and the M5S 3f^2 2F 3d^3 F transitions with the most reliable measurements. The predicted intensities are based on the preliminary effective recombination coefficients calculated by P. J. Storey (private communication), which differs from the laws used. Besides, measurements of λ3334.84 could be overestimated due to the O III line. Another line λ3355.02 (3p^0 D^0 3s^3 P) blends with He I M8 7p^1 S^0 2s 1S 3354.55 (Fig. D1), which contributes 43 per cent to the total intensity, as estimated from the calculations of Benjamin, Skillman & Smith (1999). The resultant intensity of λ3355.02 is 0.223±0.045. The intensity ratio λ3355.02/λ3334.84 agrees well with the predicted value. The intensity of another line λ3360.60 (3p^0 D^0 3s^3 P) is 0.040, which is unreliable (Fig. D1). The other M2 lines are not observed.

#### 3.4.1 Multiplet 2, 3p^2 D^0 3s^3 P

This is the strongest multiplet of Ne II. Fig. 35 shows that λ3334.84 (3p^0 D^0 3s^3 P) is affected by the O III fluorescence line M3 3p^3 S^1 3s^3 P^0, which is more than 10 times stronger. The fitted intensity of λ3334.84 is 0.428, which is higher than LLB01 (0.345). Given that our observational data are the same as LLB01, such difference is probably due to the different redening laws used. Besides, measurements of λ3334.84 could be overestimated due to the O III line.

#### 3.4.2 Multiplet 13, 3d^1 F 3p^4 D^0

Ne II M13 lines locate in the far-blue and they are difficult to observe due to weakness as well as the relatively poor S/N. Only λ3218.19 (3d^1 F^0 3p^4 D^0 3s^3 P) and λ3244.09 (3d^1 F^0 3s^3 P) are observed, and they are shown in Fig. 36. The fitted intensity of λ3218.19 is 0.234, which agrees well with LLB01 (0.229). Here the blended Ne II M16 3d^1 F^0 3s^3 P and λ3217.30 was assumed to be negligible (~3 per cent). The intensity ratio λ33218.19 to the Ne II M2 λ3334.84 line is 0.546, which agrees with the predicted ratio 0.594. The intensity of the λ3244.09 line is 0.074, higher than the measurement of LLB01 (0.0576).

#### 3.4.3 Multiplet 9, 3p^0 F^0 3s^3 D

This is the only Ne II multiplet of an excited-state parentage (2s^2 2p^4 1D) detected in the spectrum of NGC 7009, and shown in Fig. 37. The fitted intensity of λ3568.50 (3p^0 F^0 3s^3 D) is 0.168±0.016. The measured total intensity of the other two lines λ3574.18, 61 is 0.053, with an uncertainty of about 10 per cent. Thus the intensity ratio (λ3574.18+3574.61)/λ3568.50 is 0.32, which differs from the Ls coupling value 0.75. No measurements of this multiplet are given in LLB01.

#### 3.4.4 4f – 3d transitions

Several dozen Ne II ORLs of the 4f – 3d configuration are detected or estimated by deblending techniques (i.e. Gaussian profile fitting). The measured intensities are 10^−2 of Hβ or even lower. Table 8 presents the measured and predicted relative intensities of the 4f – 3d transitions with the most reliable measurements. The predicted intensities are based on the preliminary effective recombination coefficients calculated by P. J. Storey (private communication).
for a few selected Ne II 4f – 3d quartet lines. In this section, we only present spectral fits and results for the M55ε 4f 2[5]o – 3d 4F multiplet of Ne II. Discussion of other multiplets of the 4f – 3d configuration are given in Appendix D. Some of the Ne II lines belonging to the M52a 4f 2[4]o – 3d 4D and M52b 4f 2[3]o – 3d 4D multiplets are shown in Fig. 37. Figure 37 shows many Ne II lines detected in the spectrum of NGC 7009.

The λ4391.99 (M55ε 4f 2[5]o 1/2 – 3d 4F0/2) line observed in NGC 7009 is shown in Fig. 37. Its intensity is 0.077 ± 0.008, which agrees with LLB01: 0.0728 (ESO 1.52 m), 0.0779 (WHT 1997) and 0.0713 (WHT 1996) and 0.0713 (WHT 1997). The contribution from the λ4392.00 (4f 2[5]o 3/2 – 3d 4F1/2) line of the same multiplet is negligible. The other M55ε line λ4409.30 (4f 2[5]o 5/2 – 3d 4F2/2) is blended with the Ne II λ4409.78 (M55b 4f 2[3]o 3/2 – 3d 4F0/2) line, whose intensity is negligible, and the O III λ4408.29 (M46a 5g H[5] – 4f G[4]) line, which contributes about 10 per cent to the total intensity, as estimated from the effective recombination coefficients of Kisielius & Storey (1999). The resultant intensity of the λ4409.30 line is 0.060, which agrees with LLB01: 0.0631 (ESO 1.52 m), 0.0615 (WHT 1996) and 0.0624 (WHT 1997). The intensity ratio λ4409.30/λ4391.99 observed in NGC 7009 is 20 per cent higher than the predicted value (Table 8).

3.4.5 Comments on the Ne II recombination spectrum

The Ne II effective recombination coefficients currently used are mainly from Kisielius et al. (1998). This calculation is carried out for the transitions of l ≲ 2. Although some effective recombination coefficients for the Ne II 4f – 3d transitions are available (P. J. Storey, private communication), only a few selected Ne II lines are calculated, and the results are quite preliminary. The best observed Ne II transitions of the 3 – 3 transitions is M2 (3p 4D – 3s 3P). For the Ne II multiplets, M12 (3d 4D – 3p 4D0), M13 (3d 4F – 3p 4D0), M20 (3d 4F – 3p 4D0), M21 (3d 4D – 3p 4D0), M28 (3d 4P – 3p 4S0) and M34 (3d 4P – 3p 4S0), only the strongest fine-structure components are observed, and the weaker components are either blended with other lines or not detected. Measurements of the M1 (3p 4P0 – 3s 3P), M5 (3p 4D0 – 3s 3P) and M6 (3p 4S0 – 3s 3P) lines are of large uncertainties due to line blending. The M9 (3p 4P0 – 3s 3D) multiplet is the only Ne II transition observed in out spectrum of parentage other than 2P. The effective recombination coefficients for this multiplet are available from Kisielius et al. (1998), which are probably unreliable considering that the calculation is in the LS coupling assumption. The possibility of using the Ne II lines to determine electron temperatures is discussed in Section 5. In NGC 7009, the effects of the fluorescence mechanism on the Ne II M1 and M2 multiplets are probably not important. Thus those lines

Table 8. Comparison of the observed and predicted relative intensities of the Ne II 4f – 3d lines detected in the spectrum of NGC 7009. The predicted intensities Ipred are based on the preliminary effective recombination coefficients for some of the strongest Ne II lines from the 4f – 3d configuration (P. J. Storey, private communication).

| Line | J2 – J1 | Ipred | Iobs | Iobs/Ipred |
|------|---------|-------|------|------------|
| M55ε 4f 2[5]o – 3d 4F | λ4391.99 | 11/2 – 9/2 | 1.000 | 1.000 | 1.000 |
| λ4409.30 | 9/2 – 7/2 | 0.665 | 0.812 | 1.221 |
| M52a 4f 2[4]o – 3d 4D | λ4219.75 | 9/2 – 7/2 | 0.555 | 0.831 | 1.497 |
| λ4233.85 | 7/2 – 5/2 | 0.139 | 0.227 | 1.631 |
| M52b 4f 2[3]o – 3d 4D | λ4231.64 | 7/2 – 5/2 | 0.131 | 0.370 | 2.811 |
| λ4250.65 | 5/2 – 3/2 | 0.088 | 0.216 | 2.461 |
| M57b 4f 1[4]o – 3d 4F | λ4397.99 | 7/2 – 5/2 | 0.346 | 0.325 | 0.941 |
| λ4428.64 | 7/2 – 5/2 | 0.437 | 0.591 | 1.353 |
| M60c 4f 1[3]o – 3s 3P | λ4430.94 | 7/2 – 5/2 | 0.283 | 0.452 | 1.598 |
| λ4457.05 | 5/2 – 3/2 | 0.098 | 0.361 | 3.662 |
| M65 4f 0[3]o – 3d 4P | λ4413.22 | 7/2 – 5/2 | 0.239 | 0.376 | 1.574 |
| M66c 4f 1[3]o – 3d 4P | λ4421.39 | 5/2 – 3/2 | 0.089 | 0.113 | 1.263 |

a Neglecting Ne II M55ε 4f 2[5]o 1/2 – 3d 4F0/2 λ4392.00.
b Corrected for the contribution from the O III M46a 5g H[5] – 4f G[4] λ4408.29 line (∼10 per cent). Neglecting Ne II M55b 4f 2[3]o 3/2 – 3d 4F1/2 λ4409.78.
c Including Ne II M52a 4f 2[4]o 3/2 – 3d 4D5/2 λ4219.37.
d Including Ne II M52b 4f 2[3]o 3/2 – 3d 4D5/2 λ4231.53.
e Including Ne II M61b 4f 2[3]o 3/2 – 3d 4D5/2 λ4428.52.
f Including Ne II M60c 4f 1[3]o 3/2 – 3d 4F3/2 λ4452.52 and Ne II M61b 4f 2[3]o 3/2 – 3d 4D5/2 λ4428.41.
g Neglecting Ne II M57a 4f 1[2]o 1/2 – 3d 4F3/2 λ4430.90 and Ne II M55a 4f 2[4]o 1/2 – 3d 4F7/2 λ4430.06. Neglecting Ne II M57a 4f 1[2]o 1/2 – 3d 4F3/2 λ4431.11.
h Including Ne II M57c 4f 1[3]o 1/2 – 3d 4F3/2 λ4431.11.

Figure 37. Spectrum of NGC 7009 from 3540 to 3595 Å showing the Ne II M9 and M34 lines as well as other emission features. The continuous curve is the sum of Gaussian profile fits. Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.
could be safe for plasma diagnostics and abundance determinations.

3.5 The C III permitted lines

More than 20 lines in NGC 7009 were identified as C III permitted transitions (Paper I); some identifications could be questionable. In this section, we only introduce three multiplets: M1 3p 'D(2 D)–3s'P(2 P)–3s'3S, M16 5g 'G(2 G)–4f'P(2 P) and M18 5g 'G(2 G)–4f'P(2 P). The three C III M1 lines, λλλ 4647.42, 4650.25 and 4651.47, are blended with the O II M1 (3p 'D(3 D)–3s'P(2 P)) lines λλλ 4649.13 and 4650.84, as shown in Fig. 32. Techniques used to obtain the total intensity of the C III M1 multiplet are illustrated in Section 3.3.1. The intensity ratio of the three lines was assumed to be as in the LS coupling, i.e. 1 : 3 : 5. The total intensity is 0.303, which is accurate to 20 per cent. This intensity agrees with the measurements of LSBC: 0.274 (PA = 45°) and 0.438 (PA = 0°).

The C III M16 lines λλλ 4067.94, 4068.91 and 4070.26 are blended with the [S II] λλλ 4068 (3p 'D(3D)–3p'P(2 P))/2 and two O II M10 (3d 'F(3 F)–3p'P(2 P)) lines λλλ 4069.62 and 4069.89, as shown in Fig. 33. Details of deriving the total intensity of the C III M16 multiplet are given in Section 3.5.2. The intensity ratio of the three C III M16 lines was also assumed to be as in the LS coupling, i.e. 1.00 : 1.31 : 1.71. The total intensity is 0.288, with a large uncertainty (~40 per cent). The intensity ratio of the C III M16 λλλ 4069 and the C III M1 λλλ 4650 multiplets is 0.952. The predicted ratio of the two C III multiplets is 0.922, which is calculated from the radiative and dielectronic recombination coefficients of Péquignot, Petitjean & Boisson (1991) and Nussbaumer & Storey (1984), respectively.

The C III M18 λλλ 4186.90 (5g 'G(4 G)–4f'F(4 F)) line is blended with the O II M36 λλλ 4185.44 (3d 'P(3 P)–3p'P(2 P))/2 line, as shown in Fig. 31. Multi-Gaussian fits yield an intensity of 0.089±0.018 for the λλλ 4186.90 line. This intensity agrees with those given by LSBC: 0.0533 (PA = 45°) and 0.102 (PA = 0°). The intensity ratio of the C III M18 λλλ 4187 and the C III M1 λλλ 4650 multiplets is 0.293, which agrees with the predicted ratio (0.331) within errors.

3.6 The N III permitted lines

More than 30 lines were identified as the N III permitted transitions, including multiplets M1 and M2, which are mainly excited by the Bowen fluorescence mechanism. Transitions from the states with excited parentage (i.e. other than '1S) are detected. Most N III lines suffer from line blending. In this section, we only present intensity measurements and discussion for the M3 3p 'D–3s'4P and M18 5g 'G–4f'2P multiplets. Discussion of other multiplets of N III is given in Appendix E.

3.6.1 Multiplet 3. 3p 'D–3s'4P

λλλ 4510.91 (3p 'D(3 D)–3s'4P(2 P)/2 and 3p' 'D(3 D)–3s'4P(1 P)/2) blends with [K IV] λλλ 3p 'S(3 S)–3p'D(2 D)/2 λλλ 4510.92, whose intensity contribution is unknown. Another M3 line λλλ 4514.86 (3p 'D(3 D)–3s'4P(2 P)/2 is partially resolved from Ne II M58a 4f [4] 2P 2(3 P)[4] 2F 2(3 F)/2 λλλ 4517.83 (Fig. 35). Several Ne II ORLs, M58b λλλ 4514.88, M64d λλλ 4516.66 and M58a λλλ 4517.83, are also blended in the complex, which makes line measurements very difficult. Another line λλλ 4518.15 (3p 'D(3 D)–3s'4P(1 P)/2) blends with the Ne II M58a λλλ 4517.83 line, which probably dominates the total intensity.

Another two lines λλλ 4523.58 (3p 'D(3 D)–3s'4P(2 P)/2 and 4547.30 (3p 'D(3 D)–3s'4P(2 P)) decay from the same upper level, thus their intensity ratio depends only on the coupling scheme. The measured line ratio λλλ 4523.58/4547.30 = 0.119, which is slightly higher than the LS coupling value 0.094. Measurements of the λλλ 4547.30 line could be of large error due to weakness (Fig. 35).

Another M3 line λλλ 4454.58 (3p 'D(3 D)–3s'4P(2 P)/2 blends with O III M48 λλλ 4547.30 5g [4] 4f [4] 5S(4 S)–4d [3] 4f 4547.31 and Ne II M55b 4f 2[3] 2P 2(3 P)[4] 4547.64 as well as another three Ne II lines M55b λλλ 4543.52, M55c λλλ 4533.57, and M55c λλλ 4535.77, whose intensity contribution could be negligible. The other line λλλ 3653.86 (3p 'D(3 D)–3s'4P(1 P)/2) blends with N II M58b 4f [4] 2p 41 3d 'F(3 F) λλλ 4530.41, which is probably more than 3 times stronger.

3.6.2 Multiplet 18. 5g 'G–4f'2F

A3437.11 (5g 2G(2 G)–4f 2F(2 F)/2 and 5g 2G(2 G)–4f 2F(2 F)/2) is detected in Fig. 31. Gaussian profile fitting gives an intensity of 0.367, with an uncertainty of less than 10 per cent. Measurements for this line given by LSBC are 0.312 (PA = 45°) and 0.397 (PA = 0°). A3437.11 blends with Ne II M60b 4f [4] 2P 2(3 P)[4] 4537.55, which contributes about 10 per cent to the total intensity, and Ne II M60b 4f [4] 2P 2(3 P)[4] 4537.40 and O III M93c 5g G(2 G)–4f F(2 F)/4 λλλ 4379.58, both of which were assumed to be negligible. This line is used to derive N III/H II abundance ratio.

3.7 The O III permitted lines

In Paper I, about two dozen O III permitted transitions from the 3–3 configuration were identified, including multiplets M2 3p 'D–3s'4P and M3 3p 'S–3s'P, M4 3p 'P–3s'P, M12 3d 'P–3s'P and M15 3d 'P–3p'P, which are mainly excited by the Bowen fluorescence or charge-transfer (Liu & Danziger 1993a). All the O III 5g–4f lines are blended with other emission features. In this section, we only present emission line measurements of the M8
Spectrum of NGC 7009 from 3250 to 3305 Å showing the O III M8 lines λ3260.85 and 3265.32. The continuous curve is the sum of Gaussian profile fits. Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.

3d³P⁰ – 3p³D and M15 3d³P⁰ – 3p³P multiplets. Measurement results of other O III multiplets are given in Appendix F. Discussion of the O III fluorescence lines is in Section 3.7.4.

3.7.1 Multiplet 8, 3d³F⁰ – 3p³D

Only λ3260.85 (3d³P⁰ – 3p³D₁) and λ3265.32 (3d³P⁰ – 3p³D₃) are observed (Fig. 39). Measurements of the two lines could be of relatively large error due to poor S/N in the far blue of the spectrum. However, this multiplet cannot be excited by either the Bowen fluorescence or charge transfer, and the two lines are still used to determine the O³⁺/H⁺ abundance ratio.

3.7.2 Multiplet 15, 3d³P⁰ – 3p³P

Fig. 40 shows the O III M15 lines detected in the spectrum of NGC 7009. Single-Gaussian profile fitting yields an intensity of 11.575 for the λ3444.06 line, with an uncertainty of about 3 per cent. The intensity contribution from the blended He I M7 λ3447.59 (6p¹P¹₀ – 2s¹S₀) line is only 2 per cent. Another M15 line λ3428.62 is blended with the λ3430.57 line of the same multiplet, which is marginally resolved in Fig. 40. Two Gaussian profiles with the same FWHM were used to fit the feature, and the resultant intensities of the λ3428.62 and the λ3430.57 lines are 1.409 and 0.319, respectively, both with uncertainties of less than 10 per cent.

Another M15 line λ3415.26 is partially resolved from the Ne II M21 λ3416.91 (3d⁵D₅/₂ – 3p⁵D₅/₂) line; the other two M15 lines, λ3405.71 and 3408.12 are also detected in the spectrum. The three O IV M2 3d²D – 3p⁵P lines, λ3403.52, 3411.69 and 3413.64, are blended among the above three O III M15 lines, as is shown in Fig. 40. Multi-Gaussian profile fitting was carried out for the complex. The intensity of the λ3415.26 line is 0.415 ± 0.021. The intensities of the λ3405.71 and 3408.12 lines are 0.210 and 0.127, respectively, both with uncertainties of 10 to 15 per cent.

Analysis of the measured intensities of the O IV M2 lines are in Section 3.8.

The observed intensity ratio λ3428.62/λ3444.06 is 0.122, lower than the theoretical prediction (0.336) given by Saraph & Seaton (1980), who assumed that the relative intensities within the O III M15 multiplet to be as in LS coupling, but agrees with the intermediate calculations of Kastner et al. (1983). The observed intensity ratio of the three O III M15 lines λ3405.71, 3415.26 and 3430.57, which have the common upper level, is 1 : 2.102 : 1.615. This ratio differs from that in the pure LS coupling, i.e. 1 : 0.75 : 1.25. Discussion of the intensity ratios of the O III M15 lines is presented in Section 3.7.4.

3.7.3 5g – 4f transitions

The O III permitted transitions of 5g – 4f configuration are in the wavelength range 4300–4600 Å, where numerous ORLs of the singly-ionized ions of C, N, O and Ne are located. The typical intensities of the 5g – 4f lines of O III are ~10⁻⁵–10⁻⁶ of Hβ, and accurate measurements of these lines are difficult due to line blending. The strongest O III line of the 5g – 4f configuration, λ4344.60 (M46b 5gH[6]⁰ – 4fG[5]γ), is marginally detected in the spectrum of NGC 7009, as is shown in Fig. 42. Multi-Gaussian profile fitting yields an intensity of 0.024 ± 0.003, from which we derived an O³⁺/H⁺ abundance ratio that is higher than that deduced from the O III M8 lines (see Section 3.7.1), this much higher O³⁺/H⁺ abundance ratio derived from the λ4344.60 line is questionable. The effective recombination coefficients calculated by Kisielius & Storey (1999)
for the 5g – 4f transitions of O III are utilized to estimate the intensity contributions where necessary.

3.7.4 Fluorescence of the O III permitted lines

In Table 2 we compare the observed and predicted intensity ratios for several pairs of O III Bowen fluorescence lines originating from the same upper level. The observed intensity ratios (column 2 of Table 2) are based on the measurements in NGC 7009 described in Section 3.7.2. The errors are estimated from the measurement errors, including Gaussian profile fit and noise level of the local continuum. The profiles of the strong O III Bowen lines deviate from the exact Gaussian due to charge transfer effect, thus we used Gaussian profiles to fit these emission lines. The error from Gaussian profile fit is 5 – 10 per cent for strong O III Bowen lines. These O III Bowen lines are close to the blue end of the spectrum, the uncertainties due to the poor S/N in this wavelength region are also taken into account. The errors are significant for the two ratios L8791/L3755 and L3774/L3757, because the flux errors are large for the three relatively weak lines L8791.28 (about 21 per cent), 3757 (about 20 per cent) and 3774 (about 32 per cent), and thus the resulting uncertainties estimated from the error propagation formula are comparable to the ratio values. The observations of Liu & Danziger (1993a) for the same object are also presented for comparison.

The radiative transition probabilities of O II have been calculated since late 1960s (Nussbaumer 1969), and later by Saraph & Seaton (1980) and Luo et al. (1989). In these approaches LS coupling was assumed. Calculations of the O II transition probabilities with the scheme of intermediate coupling are presented by Kastner et al. (1983) and Kastner & Bhatia (1990) for some improved values for cascading from the 2p3d 3P2 level. Fischer (1994) carried out a non-relativistic configuration interaction calculation with relativistic correction and showed obvious improvement over the previous work. Simultaneously, a full relativistic configuration interaction calculation of Tong et al. (1994) gave predicted O III Bowen line ratios which were further improvement, especially for the 3133/3444 ratio. Their results are in agreement with observed ratios within the accuracy of observations. A more recent variational Breit-Pauli calculation was done by Tachiev & Fischer (2001) for the carbon-like sequence. The current measurements agree better with the intermediate coupling values.

3.8 The O IV permitted lines

Measurements of the O IV lines, λλ3403.52, 3411.69 and 3413.64 of the M2 3d 5D – 3p 3P0 multiplet are of large uncertainty due to weakness and line blending (Fig. 30). The intensity of the λ3411.69 line derived from multi-Gaussian profile fitting is 0.057±0.014. The intensity ratio of the other two O IV M2 lines λλ3403.52 and 3413.64, which share the same upper level 3D3/2, is 1.318. This intensity ratio is significantly lower than the value in the LS coupling assumption, i.e. 5.0. So far the only effective recombination coefficients available for the O IV lines are the radiative recombination coefficients given by Péquignot, Petitjean & Boisson (1991) and the dielectronic recombination coefficients given by Nussbaumer & Storey (1984). Both calculations were carried out in the LS coupling scheme. The O IV M2 lines are the only lines detected for this ion in the spectrum of NGC 7009, but are not used in abundance determinations.

4 IONIC AND ELEMENTAL ABUNDANCES

In this section, we present the ionic abundances of helium and heavy elements derived from ORLs. For heavy element ions, the electron temperatures deduced from the N II and O II recombination line ratios are assumed (Section 4.1). For He++/H++, a value of 5100 K as deduced from the He λ4713/4686 ratio is adopted. For He++/H++, an electron temperature of 10 000 K, roughly the value deduced from the 5694 Å discontinuity of the He II recombination continuum spectrum, is assumed. The average electron temperature derived from the various optical CEL ratios is ∼ 10 000 K (Paper I), and is used to determine the forbidden line abundances. Given that the electron densities derived from the N II and O II ORLs are close to those from the optical CELs, and that emissivities of the heavy element ORLs are only marginally sensitive to electron density, we have assumed a constant density of 4300 cm⁻³ throughout the abundance determinations.

4.1 Ionic abundances from ORLs

Ionic abundances of He, C, N, O, Ne, Mg and Si are derived in this Section, using the extinction-corrected fluxes of ORLs given in Paper I. A critical analysis of the C II, N II, O II and Ne II permitted lines detected in the spectrum of NGC 7009 is presented in Section 3, and is used to guide the determinations of ORL abundances of heavy element ions. Again, the new effective recombination coefficients for the N II and O II recombination spectrum are utilized.

4.1.1 He++/H++ and He++/H++

Numerous calculations have been dedicated to the recombination spectrum of He I (e.g. Mathis 1957; Burgess & Seaton 1960a, 1960b; Pottasch 1962; Robbins 1968, 1970; Robbins & Robinson 1971; Brocklehurst 1972; Almqvist & Netzer 1989; Smits 1991, 1996; Benjamin, Skillman & Smits 1999, 2002; Porter et al. 2005; Bauman et al. 2005). The recombination-cascade spectrum of He I, with no collision taken into account, was first computed by Brocklehurst (1972). A better treatment of emissivities from He I in nebular environment is that of Smits (1996), who used more accurate calculations of radiative transition rates and photoionization cross-sections, and corrected an error in Brocklehurst (1972). Benjamin, Skillman & Smits (1999) combined the detailed He I recombination model of Smits (1996) with the collisional transitions of Sawey & Berrington (1993) and calculated more accurate He I recombination line emissivities that include the effects of collisional excitation from both the 2s 2S and 2s 2S levels. Benjamin, Skillman & Smits (2002) studied the effects of the optical depth of the 2s 2S metastable level on the He λ line intensities. The availability of these improved atomic data has made it possible to obtain secure measurements of the ionic and elemental abundances of helium, given high-quality spectroscopic data. Zhang et al. (2005a, b) developed the nebular plasma diagnostics based on the He I ORLs, using the theoretical emissivities of He I lines provided by Benjamin, Skillman & Smits (1999). In Paper I, we derived electron temperatures from He I line ratios using the method of Zhang et al. (2005a). In order to keep consistency, we derived ionic and elemental abundances of helium in this Section, using the atomic data of Benjamin, Skillman & Smits (1999) and assuming the temperature to be that yielded by the He I lines (∼ 5000 K).

Ionic and elemental abundances of helium relative to hydrogen derived from the He I and He II ORLs are presented in Table 10.
The adopted $\text{He}^+/\text{H}^+$ ratio (0.999) is an average of the values derived from the He I $\lambda\lambda4471, 5876$ and 6678 lines with weights of 1:3:1, roughly proportional to their intrinsic intensities (Benjamin, Skillman & Smits 1999). Here the effective recombination coefficients were adopted from Benjamin, Skillman & Smits 1999. Case A recombination was assumed for the triplet lines and Case B for the singlet. An electron temperature of 5000 K, as derived from the He I $\lambda7281/\lambda6678$ ratio, and a density of 10,000 cm$^{-3}$ were assumed. Under this physical condition, the effective recombination coefficient for the $\lambda4471$ line given by Benjamin, Skillman & Smits 1999 is 6 per cent higher than that of Brocklehurst 1972. This difference is 7 per cent when the temperature increases to 10,000 K. The difference between the calculations of Benjamin, Skillman & Smits 1999 and Brocklehurst 1972 for the other two He I lines are less than 2 per cent at 5100 K. The $\lambda4471$ line suffers most from line blending among the three He I lines: It is blended with two O II M86c lines $\lambda\lambda4469.46$ and $\lambda4469.48$, three O III lines M49c $\lambda\lambda4471.02, 4475.17$ and M45b $\lambda4476.11$, and one Ne II line M61b $\lambda4468.91$. Both wings of the $\lambda4471$ line are also affected by weak features (Fig. 35). All the blending brings an uncertainty of ~10 per cent to the intensity of the $\lambda4471$ line. The $\lambda4471$ line intensity adopted in the current analysis has been corrected for the contributions from the blended lines listed above, using the effective recombination coefficients available for the O II and O III lines. The intensity of the He I $\lambda6678$ line has also been corrected for the contribution from the blended He II $\lambda6683.20$ (13h $^2$He$^+$ – $^5$g$^-$G) line, which brings an uncertainty of about 3 per cent. The measurement uncertainty of the He I $\lambda5876$ line is less than 2 per cent.

The $\text{He}^+/\text{H}^+$ abundances listed in Table 10 agree with each other reasonably well, except for those yielded by the triplet line $\lambda7065$ ($3s^1\text{S}^3–2p^1\text{P}^0$) and the singlet line $\lambda5016$ ($3p^1\text{P}^–2s^1\text{S}^0$). The former one is more than two times higher than the average abundance, while the latter is nearly half of the value. The abnormally high abundance value is probably due to the enhanced $\lambda7065$ line as a result of self-absorption from the metastable $2s^1\text{S}^0$ level. By comparing the observed intensities of the He I singlet lines of the $\text{ns}^1\text{S}^1–2p^1\text{P}^0$, $\text{np}^1\text{P}^–2s^1\text{S}^0$ and $\text{nd}^1\text{D}^–2p^1\text{P}^0$ series, relative to the $\lambda4922$ ($4d^3\text{D}_2–2p^1\text{P}^0$) line, with the theoretical predictions (c.f. Section 4.5 in Paper I), we concluded that the singlet transitions of He I in NGC 7009 are close to the Case B assumption. Departure from Case B of the He I singlet lines as a result of the He I Lyman photons being destroyed by photoionization of neutral hydrogen and/or absorption by dust grains (Liu et al. 2001) is unlikely to be significant. Thus the low ionic abundance yielded by the $\lambda5016$ line in Table 10 is mainly due to self-absorption from the metastable $2s^1\text{S}^0$ level. The $\lambda5016$ line is blending with the N II M19 3d$^3\text{F}^2$ – 3p$^3\text{D}_2$ $\lambda5016.39$, whose contribution is negligible (<1.0 per cent).

The $\text{He}^{2+}/\text{H}^+$ abundance ratios were derived from two He II lines $\lambda3203$ ($5f^3\text{P}^0$ – $3d^3\text{D}$) and $\lambda4686$ ($4f^3\text{P}^0$ – $3d^3\text{D}$). The effective recombination coefficients of the two lines were adopted from the hydrogenic calculation of Storey & Hummer (1995). Although the electron temperature (~11 000 K) derived from the discontinuity at $\lambda5694$ Å of the He II recombination continuum is of large uncertainty due to weakness of the jump, we assumed an electron temperature of 10,000 K when deriving the $\text{He}^{2+}/\text{H}^+$ abundance ratio. The total elemental abundance of helium relative to hydrogen is 0.112, which is calculated from He/H = $\text{He}^{2+}/\text{H}^+$ + He$^{2+}/\text{H}^+$. This agrees well with the value of 0.109 given by LSBC.

Several He I recombination line series have been observed in our spectrum, and relative intensities of these lines are presented in Table 11. Also presented in the Table are the theoretical predictions given by Benjamin, Skillman & Smits 1999, Brocklehurst 1972, and Smits 1996. Case A was assumed for the triplet lines and Case B for the singlet. The observed intensities of the $\text{nd}^3\text{D}^–2p^3\text{P}^0$ and $\text{nd}^1\text{D}^–2p^1\text{P}^0$ series of He I, relative to the $\lambda4471$ line, agree well with those predicted by recombination theory. The obvious weakness of the $\text{np}^1\text{P}^–2s^1\text{S}^0$ series, compared with theoretical intensities, is caused by self-absorption from the metastable $2s^1\text{S}^0$ level. Such self-absorption should at the same time lead to the enhancement of the $\text{ns}^1\text{S}^–2p^1\text{P}^0$ series. However, what we observed as shown in Table 11 are opposite: The $\lambda7281$ line seems weaker than the recent prediction. The $\lambda3889$ ($3p^3\text{P}^0–2s^1\text{S}^0$) line is affected by self-absorption. Enhancement of the $\text{ns}^1\text{S}^–2p^3\text{P}^0$ series, in particular the $\lambda7065$ line, is clearly observed. Similar patterns in the relative intensities of the He I lines are also observed in NGC 6153 (Liu et al. 2000). M 1-42 and M 2-36 (Liu et al. 2001).

Table 9. Comparison of the observed and predicted Bowen fluorescence line ratios.

| Line Ratio | Current obs. | LD93$^a$ | SS80$^b$ | KBB83$^c$ | FF94$^d$ | TZLL94$^e$ | TFF01$^f$ |
|------------|--------------|----------|----------|----------|----------|----------|----------|
| $\lambda 3133/\lambda 4344$ | 3.261±0.071 | 3.140±0.440 | 3.610 | 4.450 | 3.170 | 3.290 | 3.342 |
| $\lambda 3299/\lambda 3341$ | 0.252±0.117 | 0.285±0.022 | 0.201 | 0.228 | 0.264 | 0.260 | 0.268 |
| $\lambda 3312/\lambda 3341$ | 0.698±0.086 | 0.651±0.048 | 0.606 | 0.656 | 0.728 | 0.717 | 0.736 |
| $\lambda 3428/\lambda 3444$ | 0.122±0.027 | 0.204±0.032 | 0.336 | 0.164 | 0.150 | 0.149 | 0.154 |
| $\lambda 3791/\lambda 3755$ | 0.271±0.213 | 0.232±0.037 | 0.330 | 0.309 | 0.296 | 0.301 | 0.299 |
| $\lambda 3774/\lambda 3757$ | 0.539±0.379 | 0.569±0.127 | 0.750 | 0.715 | 0.708 | 0.701 | 0.704 |

$^a$ Liu & Danziger (1993a);
$^b$ Saraph & Seaton (1980);
$^c$ Kastner et al. (1983);
$^d$ Fischer (1994);
$^e$ Tong et al. (1994);
$^f$ Tachiev & Fischer (2001).

4.1.2 $\text{C}^{2+}/\text{H}^+$ abundances from ORLs

The $\text{C}^{2+}/\text{H}^+$ abundance ratios derived from the $3–3$ and $4f–3d$ transitions as well as from the $\text{ng}–4f$ transition array are presented in Table 12. The effective recombination coefficients of Davey, Storey & Kisielius (2000) were used. Their calculation was carried out from 500 to 20,000 K. As described earlier, an electron temperature of 1000 K deduced from the N II ORLs and probably prevalent in the postulated “cold” component where the ORLs of heavy elements arise (Liu et al. 2000), was assumed in the cal-

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culations. Transitions between doublet states were assumed to be in Case B given the ground term C $^2P^o$. For those doublet transitions whose Case B effective recombination coefficients are not available in Davey, Storey & Kisielius (2000), the more recent calculations of Bastin (2006) were adopted. Bastin (2006) calculated the effective recombination coefficients for both transitions in both Case A and B from 5000 to 50 000 K. Accurately extrapolating the effective recombination coefficients of Bastin (2006) more recent to 1000 K is difficult because the recombination coefficients are not exactly a linear function of electron temperature. The H I Balmer jump temperature 6500 K and a density of 4300 cm$^{-3}$ were assumed when we derived the C$^2P^o$/H$^+$ abundance using the coefficients of Bastin (2006).

The C$^2P^o$/H$^+$ abundance ratio derived from the C$^2$ II $\lambda$4267 line is 5.507 $\times$ 10$^{-5}$. In the calculation of Davey, Storey & Kisielius (2000), the Case A effective recombination coefficient for the C$^2$ II $\lambda$4267 line differs from Case B by 0.8 per cent. If a temperature of 10 000 K is assumed, the C$^2P^o$/H$^+$ ratio derived will increase to 8.432 $\times$ 10$^{-4}$, which then agrees with the abundance given by LSBC. At 10 000 K, the effective recombination coefficient for the C$^2$ II $\lambda$4267 line given by Davey, Storey & Kisielius (2000) differs from that of Bastin (2006) by only 1.4 per cent in Case B, and 1.8 per cent in Case A. Thus we expect that the C$^2P^o$/H$^+$ ratio derived from the coefficients of Bastin (2006) should agree with LSBC.

In Table 12 the C$^2P^o$/H$^+$ abundance ratios derived from the three ng $^3G$ $-$ 4f $^3F^o$ lines (M17.02 $\lambda$9903, M17.04 $\lambda$6462, and M17.06 $\lambda$5342) and the M16.04 $^3F^o$ $-$ 4d $^3D$ $\lambda$6151 line are based on the Case B effective recombination coefficients of Bastin (2006). The abundance ratios all agree with those given by LSBC. The Case B effective recombination coefficients for the C$^2$ II ng $^3G$ $-$ 4f $^3F^o$ ($n = 5, 6$ and 7) transitions calculated by Bastin (2006) are almost identical to the Case A values at 6500 K; for the M16.04 $^3F^o$ $-$ 4d $^3D$ $\lambda$6151 transition, the effective recombination coefficients of the two cases differ by only 2 per cent. The C$^2$ II M4 $^3S^o$ $-$ 3p $^3P^o$ A3920 multiplet was not given by Davey, Storey & Kisielius (2000), and the Case B recombination coefficient of Bastin (2006) was used. The derived C$^2P^o$/H$^+$ abundances

| Table 10. Recombination line helium abundances. |
|-----------------------------------------------|
| He$^+$/H$^+$ | Line (Å) | Abundance |
|-----------------|-----------------|------------|
|                  |                  |            |
| Triplet lines   |                  |            |
| He$^+$/H$^+$    | He I $\lambda$3187.74 | 0.094      |
| He$^+$/H$^+$    | He I $\lambda$3888.64 | 0.089      |
| He$^+$/H$^+$    | He I $\lambda$4026.20 | 0.103      |
| He$^+$/H$^+$    | He I $\lambda$4471.50 | 0.098      |
| He$^+$/H$^+$    | He I $\lambda$5875.60 | 0.103      |
| He$^+$/H$^+$    | He I $\lambda$7065.71 | 0.267      |
| Singlet lines   |                  |            |
| He$^+$/H$^+$    | He I $\lambda$3921.93 | 0.099      |
| He$^+$/H$^+$    | He I $\lambda$5015.68 | 0.065      |
| He$^+$/H$^+$    | He I $\lambda$6678.15 | 0.095      |
| He$^+$/H$^+$    | He I $\lambda$7281.35 | 0.090      |
| He$^+$/H$^+$    | Mean            | 0.099      |
|                  |                  |            |
| He$^2+/H^+$     | He II $\lambda$4685.68 | 0.012      |
| He$^2+/H^+$     | He II $\lambda$3203.17 | 0.012      |
|                  |                  |            |
| He/H             |                  | 0.112      |
The M3 3p$^3$D$^o$ – 3s$^3$P$^o$ λ5680 multiplet is the strongest N $\Pi$ permitted transition. At $T_e = 1000$ K, the LS-coupling effective recombination coefficient of N $\Pi$ M3 λ5680 multiplet calculated by Kisielius & Storey (2002) is case-insensitive, with the Case B value being only 20 per cent higher than that for Case A. In the calculation of FSL11, the Case B effective recombination coefficient for the λ5679.56 line, which is the strongest component of the N $\Pi$ M3 multiplet, is 27 per cent higher than in Case A. The Case B effective recombination coefficients for another two N $\Pi$ M3 lines, λ5666.63 and 5676.02, are higher than their corresponding Case A values by 28 and 24 per cent, respectively. The Case B coefficients of the other N $\Pi$ M3 lines are about 24 to 28 per cent higher than for Case A. In Table 13, the $N^2+/H^+$ abundance ratios derived from the N $\Pi$ M3 λ5680 multiplet lines almost agree with the value derived from the total intensity of this multiplet. The N $\Pi$ singlet line 3p$^3$D$^o$ – 3s$^3$P$^o$ λ3995 is case-insensitive, with its Case B effective recombination coefficient being only 3.6 per cent higher than for Case A. The calculation of FSL11 shows that the Case B effective recombination coefficient for the λ3995 line is higher than the Case A value by about 5 per cent at 1000 K.

The M5 3p$^3$P$^o$ – 3s$^3$P$^o$, M20 3d$^3$D$^o$ – 3p$^3$D, M28 3d$^3$D$^o$ – 3p$^3$P and M29 3d$^3$P$^o$ – 3p$^3$P multiplets of N $\Pi$ are all case-sensitive. The Case B effective recombination coefficient for the M5 3p$^3$P$^o$ – 3s$^3$P$^o$ λ4623 multiplet calculated by Kisielius & Storey (2002) is higher than in Case A by a factor of 9. FSL11 shows that the Case B effective recombination coefficient for the λ4630.54 line, which is the strongest component of the N $\Pi$ M5 multiplet, is higher than for Case A by a factor of 8. The Case B effective recombination coefficients for another two N $\Pi$ M5 lines, λ4640.18 and 4621.39, are 8–9 per cent higher than in Case A. For the weakest lines in the N $\Pi$ M5 multiplet (the λ4607.16, 4613.87 and 4643.09 lines), their Case B effective recombination coefficients do not differ much from the Case A values.

In Table 13, the $N^2+/H^+$ abundance ratios derived from the N $\Pi$ M5 multiplets in the Case B assumption agree with those yielded by the case-insensitive N $\Pi$ M3 λ5680 lines, which signifies Case B is a better assumption for the N $\Pi$ M5 multiplet.

Kisielius & Storey (2002) shows that the Case B effective recombination coefficients for the N $\Pi$ M20 3d$^3$D$^o$ – 3p$^3$D λ4794, M28 3d$^3$D$^o$ – 3p$^3$P λ5939 and M29 3d$^3$P$^o$ – 3p$^3$P λ5479 multiplets are higher than those for Case A by a factor of 52, 51 and 25, respectively. FSL11 reveals that the Case B effective recombination coefficients for the strongest fine-structure components in the above three multiplets are 48.49 and 16 times higher than those for Case A, at a given temperature of 1000 K. Given such large differences between the two cases, the $N^2+/H^+$ abundance ratios derived from the three N $\Pi$ multiplets (Table 13) suggest that Case B is a better assumption for the N $\Pi$ M5 multiplet.

### 4.1.3 $N^2+/H^+$ abundances from ORLs

The $N^2+/H^+$ abundance ratios derived from the N $\Pi$ ORLs detected in the spectrum of NGC 7009, with the most reliable measurements, are presented in Table 13. The M3 multiplet of the 3p$^3$–3s$^3$ configuration is the best observed amongst all the 3–3 transitions. The N $\Pi$ effective recombination coefficients of Kisielius & Storey (2002) are used when we derive the $N^2+/H^+$ abundance ratios from the total intensity of an N $\Pi$ multiplet, which is a sum of line intensities of all the fine-structure components. If some components are missing from a multiplet, i.e., not detected due to weakness or line blending, the total intensity of that multiplet is then calculated by assuming the relative intensities of the fine-structure components are as in LS coupling. The most recent N $\Pi$ effective recombination coefficients calculated by FSL11 are used when we derive the $N^2+/H^+$ abundances from the fine-structure components of each N $\Pi$ multiplet. The Case B recombination was assumed for the triplets and Case A for the singlets. An electron temperature of 1000 K and a density of 4300 cm$^{-3}$ were assumed throughout the abundance determinations. Under such physical condition, the effective recombination coefficient for H$\beta$ adopted is 1.86$\times$10$^{-13}$ cm$^3$ s$^{-1}$, which is calculated from the hydrogenic theory of Storey & Hummer (1995).
We adopt the mean value ($3.45 \times 10^{-4}$) derived by averaging the abundances from the M3 multiplet and the total intensity of the 4f–3d transitions as the recombination line N$^{+}/H^+$ abundance of NGC 7009. This value is about 10 per cent higher than $3.10 \times 10^{-4}$ (slit position angle PA = 45°) given by LSBC, who used the N $\equiv$ M39b $\lambda 4041.31$ and M39a $\lambda 4043.53$ lines to derive the N$^{+}/H^+$ abundance of NGC 7009.

4.1.4 $O^{2+}/H^+$ abundances from ORLs

In the spectrum of NGC 7009, O $\equiv$ has the most abundant optical recombination spectrum amongst all the heavy element ions detected. The most prominent multiplets of the O $\equiv$ transitions are presented in Section 4.1.3. The spectrum analyzed in the current paper covers very broad wavelength range (3040–1100 Å) and is among the deepest CCD spectra ever taken for an emission line nebula, and also has higher quality than that published by LSBC. In the wavelength ranges 3040–4048 Å and 3990–4980 Å, where the most prominent recombination lines of O $\equiv$ are located, our data quality is as good as that of Liu et al. (2000) for PN NGC 6153, which was observed using the same instruments mounted on the ESO 1.5 m telescope. The $O^{2+}/H^+$ abundance ratios derived from the O $\equiv$ ORLs with the most reliable measurements are presented in Tables 14 (the 3d–3p and 3p–3s transitions) and 15 (the 4f–3d transitions). The Case B effective recombination coefficients of O $\equiv$ calculated by PJS are adopted for the abundance determinations. An electron temperature of 100 K is assumed. For purposes of comparison, the effective recombination coefficients of Storey (1994) for the 3p–3s transitions, and LSBC for the 3d–3p and 4f–3d transitions, are also used. Case B is assumed for the quartet transitions, and Case A for the doublets. An electron temperature of 5000 K is assumed when using the data of Storey (1994), whose calculation is valid from 5000 to 20000 K. Since the calculation of Storey (1994) is only spectral term-resolved, we deduce the effective recombination coefficients for each fine-structure component of a multiplet with the assumption that their relative intensities are as in the LS coupling. As in the case of the N $\equiv$ lines (Section 4.1.3), for each multiplet in Table 14 we calculate the abundance value using the co-added intensities from all fine-structure components observed; for the O $\equiv$ 4f–3d transitions in Table 15 we also calculate the abundance after co-adding the intensities of all detected lines.

The O $\equiv$ M1 3p$^4$4D$^0$–3s$^3$3P $\lambda 4650$ multiplet is case-insensitive. At 5000 K, the Case B effective recombination coefficient of the M1 $\lambda 4650$ multiplet given by Storey (1994) is only 3.4 per cent higher than for Case A. Péquignot, Petitjean & Boisson (1991) shows that the difference between the effective radiative recombination coefficients in the two cases for this multiplet is 2.5 per cent. The most recent calculation of PJS reveals that the difference between the effective recombination coefficients in the two cases for the strongest O $\equiv$ M1 line $\lambda 4649.13$ is less than 5 per cent. The coefficients in the two cases for the other O $\equiv$ M1 fine-structure lines are of similar order. The O$^{2+}/H^+$ abundances derived from the individual O M1 lines agree with each other, except for $\lambda 4638.86$ and 4673.73, which yield very high abundance values. The average O$^{2+}/H^+$ from O $\equiv$ M1 is 16.2$ \times 10^{-4}$, which agrees well with the values derived from the recombination coefficients of Péquignot, Petitjean & Boisson (1991) and Storey (1994). The current measurements of the multiplet also agree with LSBC, who gives 15.3$ \times 10^{-4}$ (PA = 0°) and 13.5$ \times 10^{-4}$ (PA = 45°).

Of the seven observed 3d–3p multiplets in Table 14, the intensities of those from the upper terms 2p$^2$3d 4F and 2p$^2$3d 4D are

$\lambda 4553.17$ lines. The intensity of the N $\equiv$ M61a 4F $\lambda 5(5/2)_2$ – 3d$^1$P$^1_1$ $\lambda 4694.64$ line is probably overestimated due to unknown blend, which can be noticed from the profile of the feature. The three lines $\lambda \lambda 4039.35$, 4552.53 and 4694.64 are excluded from calculating the total intensity and the average abundance ratio. The N$^{2+}/H^+$ abundances derived from the $\lambda 4041.31$, 4043.53 lines agree with those from the N $\equiv$ M3 $\lambda 5666.63$, 5679.56 lines. The N $\equiv$ M3 $\lambda 5676.02$ line yields a relatively high abundance. Measurements of this line could be unreliable due to the blended $\lambda 5679.56$ line, which is 4 times stronger (Fig. 28). The relatively low S/N’s at this wavelength region also affect the measurements.

It has been known for decades that the N $\equiv$ permitted lines from the low-lying 3d–3p and 3p–3s triplet arrays, whose upper levels are linked to the ground term 2p$^2$3P by resonance lines, can be enhanced by fluorescence excitation. Grandi (1976) used photoionization models to study the excitation mechanisms of permitted transitions from common heavy element ions observed in the spectra of the Orion nebula and PNe NGC 7027 and NGC 7662, and found that while the N $\equiv$ M28 3d$^3$D$^0$–3p$^3$3P $\lambda 5942$ multiplet is excited by both recombination and continuum fluorescence of the starlight, emission of the N $\equiv$ M3 3p$^3$D$^0$–3s$^3$3P $\lambda 5680$, M5 3p$^3$P$^0$–3s$^3$3P $\lambda 4630$ and M30 4s$^3$P$^0$–3p$^3$3P $\lambda 4383$ multiplets are dominated by fluorescence excitation of the N $\equiv$ M3 4s$^3$P$^0$ level by the He I $\lambda 5876$, $\lambda 6000$ and $\lambda 6093$ resonance lines, which coincides in wavelength with the N $\equiv$ M24 3p$^3$P$^0$–2p$^2$3P$^0$ $\lambda 5808.68$ line. Fluorescence excitation, by line or continuum, however, cannot excite the singlet transitions or transitions from the 3d–4f configuration. Escalante & Morisset (2005) analyzed the N $\equiv$ spectrum of the Orion nebula by using nebular and stellar atmosphere models. Their modeling shows that the intensity of most of the N $\equiv$ permitted lines in Orion could be explained by fluorescence of the starlight continuum. Recombination of N$^{2+}$ contributes a minor part of the observed intensities of lines from the 3p and 3d levels connected to the ground state. They confined the effective temperature of the ionizing star to be lower than 38,000 K in order to reproduce the observed line intensities. Our current analysis shows that the N$^{2+}/H^+$ abundances derived from the 3p–3s transitions agree with those derived from the 4f–3d recombination lines (the values in boldface in Table 13), which are unlikely to be affected by the fluorescence mechanisms. That indicates fluorescence excitation of the N $\equiv$ multiplet and the total intensity of the N $\equiv$ transitions (e.g. M29) that yield abnormally high abundances are excluded from averaging. The N$^{2+}/H^+$ abundance derived by co-adding the line intensities of the 4f–3d transitions is 3.42$ \times 10^{-4}$, which agrees well with the abundance calculated from the total intensity of the M3 multiplet of N $\equiv$ (the values in boldface in Table 13). The abundances derived from total intensities of the 4f–3d transitions are preferred over the average values of abundances from individual lines, since strong lines are better detected with smaller (relative) flux uncertainties.

The average N$^{2+}/H^+$ abundance from the 3–3 transitions is 3.70$ \times 10^{-4}$, which agrees with the average value ($3.71 \times 10^{-4}$) from the 4f–3d transitions. Here the N $\equiv$ transitions (e.g. M29) that yield abnormally high abundances are excluded from averaging. The N$^{2+}/H^+$ abundance derived by co-adding the line intensities of the 4f–3d transitions is 3.42$ \times 10^{-4}$, which agrees well with the abundance calculated from the total intensity of the M3 multiplet of N $\equiv$ (the values in boldface in Table 13). The abundances derived from total intensities of the 4f–3d transitions are preferred over the average values of abundances from individual lines, since strong lines are better detected with smaller (relative) flux uncertainties.
Table 13. Recombination line N2+/H+ abundances. Intensities are normalized such that H$eta$ = 100. The N II effective recombination coefficients of FSL11 and Kisielius & Storey (2002) KS02) are both used for purpose of comparison.

| Line | Mult. | $I_{\text{obs}}$ (Å) | N2+/H+ $\times 10^{-4}$ (FSL11) | N2+/H+ $\times 10^{-4}$ (KS02) |
|------|-------|----------------------|-------------------------------|-------------------------------|
|      |       |                      | FSL11                        | KS02                          |
| 3 – 3 transitions                  |       |                      |                               |                               |
| A5666.63 | M3 | 0.064               | 3.442                        |                               |
| A5676.02 | M3 | 0.036               | 4.075                        |                               |
| A5679.56 | M3 | 0.130               | 3.319                        |                               |
| A5862.21 | M3 | 0.024               | 4.695                        |                               |
| A5710.77 | M3 | 0.020               | 2.886                        |                               |
| **M3 3p – 3d** – 3s**P**          | 0.280 | 3.482               | 3.358                        |                               |
| A4601.48 | M5 | 0.016               | 3.087                        |                               |
| A4621.39 | M5 | 0.020               | 4.423                        |                               |
| A4630.54 | M5 | 0.067               | 3.549                        |                               |
| **M5 3p – 3s**            | 0.102 | 3.598               | 2.242                        |                               |
| A3994.99 | M12 | 0.033               | 6.655                        | 6.776                         |
| **M12 3p – 3s**            | 0.033 | 6.655               | 6.776                        |                               |
| A4803.29 | M20 | 0.032               | 2.723                        |                               |
| **M20 3d** – 3p**D**          | 0.078 | 4.412               | 3.464                        |                               |
| A5941.65 | M28 | 0.030               | 1.795                        |                               |
| **M28 3d** – 3p**P**          | 0.063 | 3.038               | 2.119                        |                               |
| A5480.06 | M29 | 0.012               | 9.409                        |                               |
| **M29 3d** – 3p**P**          | 0.061 | 14.949              | 7.590                        |                               |
| Average                        |       |                      |                               | 3.695                         |
| 4f – 3d transitions             |       |                      |                               |                               |
| A4035.08 | M39a | 0.035               | 2.926                        |                               |
| A4041.31 | M39b | 0.081               | 3.174                        |                               |
| A4043.53 | M39a | 0.035               | 3.168                        |                               |
| A4171.61 | M43b | 0.023               | 3.033                        |                               |
| A4176.16 | M43a | 0.021               | 3.739                        |                               |
| A4236.91 | M48a | 0.036               | 3.727                        |                               |
| A4241.78 | M48a | 0.093               | 3.689                        |                               |
| A4179.67 | M50a | 0.010               | 3.308                        |                               |
| A4432.74 | M55a | 0.036               | 3.089                        |                               |
| A4442.02 | M55a | 0.011               | 3.204                        |                               |
| A4552.53 | M58a | 0.032               | 3.681                        |                               |
| A4530.41 | M58b | 0.046               | 3.792                        |                               |
| A4678.14 | M61b | 0.012               | 3.049                        |                               |
| A4694.64 | M61a | 0.020               | 5.393                        |                               |
| Sum                            |       |                      | 0.466                        | 3.419                         |
| Average                        |       |                      | 3.705                        |                               |

* Including O II M52c lines 4fF[2]3/2 – 3d2D5/2 λ4621.27. Neglecting O II M52c 4fF[2]3/2 – 3d2D5/2 λ4622.14.

* Including the Ne II M52a 3d3P2 – 3p3P1 λ5940.24 line, which contributes less than 1 per cent to the total intensity.

* Including the Ne II M52a 3d3P2 – 3p3P1 λ5940.24 line, which contributes less than 1 per cent to the total intensity.

* Corrected for the contribution from the O II M50c 4fF[2]3/2 – 3d4F7/2 λ4041.28 line (7 per cent).

* Corrected for the contribution from the O II M48b 4fF[7/2]3/2 – 3d3D2 λ4237.05 line (∼30 per cent).

* Including the Ne II M52a 4fF[7/2]3/2 – 3d3D2 λ4237.05 line (∼30 per cent).

* Including the contribution from the Ne II M52a 4fF[7/2]3/2 – 3d3D2 λ4241.78. Neglecting Ne II M52a 4fF[7/2]3/2 – 3d4F7/2 λ4553.40.
The Optical Recombination Spectrum of NGC 7009

4.1.5 Ne$^{2+}$/H$^+$ abundances from ORLs

Tables 16 and 17 present the recombination line Ne$^{2+}$/H$^+$ abundances derived from the 3–3 and 4f–3d transitions, respectively. For the 3d–3p and 3p–3s transitions, the effective recombination coefficients calculated in the LS coupling assumption by Kisielius et al. (1998) are adopted. Case A is assumed for the quartet transitions and Case B for the doublets. The calculation of Kisielius et al. (1998) is valid from 1000 to 20000 K, and four density cases, 10$^2$, 10$^5$, 10$^7$, and 10$^9$ cm$^{-3}$, were calculated. We assumed an electron density of 10$^5$ cm$^{-3}$ and a temperature of 10000 K in the abundance determinations. For purpose of comparison, also presented in Tables 16 and 17 are the recombination line Ne$^{2+}$/H$^+$ abundances derived by LLB01, who used the same CCD spectrum as analyzed in the current paper but assumed an electron temperature of 7100 K, as derived from the Balmer discontinuity. In general, the Ne$^{2+}$/H$^+$ abundance ratios derived from the 3d–3p and 3p–3s transitions in the current work are lower than those given by LLB01. This is mainly due to the different temperatures adopted. For the 4f–3d transitions, the abundances derived by us are systematically lower than those given by LLB01 by about 13 per cent, except the strongest lines of this transition array, e.g. λ4391.99 (M55e 4f$^2$[5]$^{1/2}$ - 3d$^3$F$_{9/2}$) and λ4409.30 (M55e 4f$^2$[5]$^{1/2}$ - 3d$^3$F$_{7/2}$), which yield close Ne$^{2+}$/H$^+$ abundance ratios from the two analyses. The difference between the abundances given by the two analyses are partially contributed by the different extinctions used: The logarithmic extinction at H$\beta$, $c$(H$\beta$), derived by LLB01 is 0.07, while ours is 0.174 (Paper I).
The Ne II M21 3d $^2D_{3/2} - 3p^2 D_{5/2}$ λ3416.91 line yields relatively higher abundance, probably due to a blend of the Ne II M19 3d $^2F_{5/2} - 3p^2 D_{5/2}$ λ3417.69 line. The nearby O II Bowen fluorescence line M15 3d $^3P_{1/2} - 3p^3 P_1$ λ3415.26 also affects the measurement of the λ3416.91 line (Fig. 30). The Ne II lines of the M9 3p $^2P_0 - 3s^2 S^0$ multiplet, λ3568.50 and 3574.61 are detected in the spectrum of NGC 7009 (Fig. 37). The Ne$^2+$/H$^+$ abundance derived from this multiplet is much higher than yielded by other multiplets. Similar results are also observed by LLB01 and in another two PNNe, M 1-42 and M 2-36 (Liu et al. 2001b). LLB01 pointed out that the high abundance yielded by M9 is possibly due to the underestimated effective recombination coefficients for this multiplet. The 3p $^3D_{5/2} - 3s^3 P_1/2$ λ3355.02 line of multiplet M2 is blended with the He I M8 7p $^1P_1 - 2s^1 S_0$ λ3354.55 line, and is also partially blended with the [Cl III] 3p $^3P_0 - ^3S_1/2 - 3p^3 S_1/2$ λ3353.17 line, as shown in Fig. D2. The intensity of the [Cl III] line was obtained from line fitting with two Gaussian profiles. The intensity contribution from the He I line was corrected for, as LLB01 did, using the observed intensity of the He I M6 5p $^1P_1 - 2s^1 S_0$ λ3613.64 line, assuming the line ratio $I(3354.55)/I(3613.64) = 0.35$ (in Case B assumption), as predicted by Brocklehurst (1972). Here an electron temperature of 5000 K, as derived from the He I line ratios (Paper I), and a density of 10000 cm$^{-3}$ were assumed. The correction for the He I line amounts to 26 per cent, close to the result of LLB01 (30 per cent). The reason that we use the He I λ3613.64 line, instead of the He I M8 4d $^1D_2 - 2p^1 P_1$ λ4921.93 line, to correct for the intensity contribution from the He I λ3354.55 line is that given the small wavelength span between the λ3354.55, 3613.64 lines, measurements of their intensity ratio are much less sensitive to any uncertainties in reddening corrections and flux calibration.

The Ne II 4f – 3d recombination lines in Table D17 are those whose preliminary effective recombination coefficients are available (P. J. Storey, private communication). The Ne$^2+$/H$^+$ abundances derived from the 4f – 3d transitions are systematically higher than those derived from the 3 – 3 transitions by about 50...
per cent. The difference between the abundances derived from the 3 – 3 and 4f – 3d transitions is mainly due to the inadequacy of the Ne II effective recombination coefficients. The average Ne II/H+ abundance from the 4f – 3d transitions is 8.5 × 10^{-4}, about 0.15 dex higher than the average value deduced from the individual lines of the 3 – 3 transition array. Here the two abnormally high abundances yielded by the Ne II M52b λ4250.65 (4f2[3]P2/2 − 3d3D3/2) and M61d λ4457.05 (4f2[3]P2/2 − 3d3D3/2) lines are excluded from averaging. We adopt the abundance value 8.42 × 10^{-4}, which is calculated by co-adding the intensities of the 4f – 3d transitions, as the recombination line Ne II/H+ abundance of NGC 7009. The two Ne II lines λλ4250.65 and 4457.05 that yield abnormally high abundances are excluded from the abundance calculation.

4.1.6 ORL abundances of other ions

Table 18 presents the recombination line C IV/H+, C IV/H+, N III/H+, O III/H+, and O IV/H+ abundances. The C IV/H+ abundance ratios are derived from the M1 λ4650, M16 λ4069 and M18 λ4187 multiplets. The abundances from the three multiplets agree with each other, and the adopted C IV/H+ ratio in NGC 7009 is an average from them. The C IV/H+ abundance ratio 2.20 × 10^{-5} is derived from the C IV M8 λ4658 line, which is slightly contaminated by [Fe II] λ4658. This abundance agrees with those given by LSBC: 0.239 × 10^{-4} (PA = 45°) and 0.182 × 10^{-4} (PA = 0°).

The N III/H+ abundance ratio is derived from N III M18 λ4379. The N III M17 lines, λλ3998.63 and 4003.58,72 are observed, but only the effective dielectronic recombination coefficients for this multiplet are available (Nussbaumer & Storey 1984). If we adopt the dielectronic data, the derived N III/H+ abundance from the M17 multiplet will be more than one order of magnitude higher than that from the N III M18 λ4379 line (Table 15). This indicates that the excitation of the N III M17 lines is probably dominated by radiative recombination. We adopt the N III/H+ ratio (1.31 × 10^{-4}) derived from the N III M18 λ4379 line as the abundance in NGC 7009. This N III/H+ abundance agrees with those given by LSBC who also used the λ4379 line: 1.34 × 10^{-4} (PA = 45°) and 1.71 × 10^{-4} (PA = 0°). We observed the N III M1 and M2 lines, which are excited by the secondary Bowen fluorescence mechanism. Also detected in the spectra of NGC 7009 are the N III λλ3574.61 and 3568.50 lines that yield abnormally high abundances derived from the 3 – 3 transitions. Intensities are normalized such that I(H β) = 100. The abundances of LLBO1 are presented for purpose of comparison.

| Line | Mult. | I_{obs} | Ne II/H+ (×10^{-4}) |
|------|-------|---------|---------------------|
|      |       |         | LLBO1               |
| λ3694.21 | M1   | 0.254   | 7.932 | 8.97 |
| λ3709.62 | M1   | 0.105   | 8.209 | 8.40 |
| λ3777.14 | M1   | 0.048   | 3.859 | 3.54 |
| M1 3p 3P^o − 3s 4P |       | 0.686   | 7.544 | 7.45 |
| M3 3p 3D^o − 3s 3P |       | 0.275   | 7.140 | 7.45 |
| M1 3d 3D^o − 3p 3P^o |       | 0.571   | 7.692 | 7.67 |
| M3 3d 4D − 3p 3P^o |       | 0.194   | 8.051 | 7.67 |
| M3 3d 4F − 3p 3D^o |       | 0.636   | 4.840 | 3.68 |
| M3 3d 4D − 3p 3D^o |       | 0.179   | 2.832 | 3.38 |
| M3 3d 4F − 3p 3D^o |       | 0.179   | 2.832 | 3.38 |
| λ3416.91 | M21  | 0.075   | 11.818 | 13.00 |
| λ3453.07 | M21  | 0.018   | 4.448 | 4.25 |
| λ3549.16 | M34  | 0.111   | 5.043 | 5.04 |
| M3 3d 4F − 3p 3S^o |       | 0.065   | 3.006 | 3.05 |
| M3 3p 3P − 3s 3P |       | 0.221   | 36.125 | 36.125 |
| Average |       | 6.040   |         |       |

a Corrected for the contribution from the He I M8 7p 1P^o 7/2 − 2s 3P^o 1/2 line λ3354.55 line.

b Probably overestimated due to Ne II M19 3d 4F_{7/2} − 3p 3D^o_{5/2} λ3417.69, Excluded from calculating the average Ne II/H+ abundance ratio.

c Including the Ne II M9 3p 3P_{3/2} − 3s 3P^o_{3/2} λ3574.18 line.
The ORL analysis is summarized in Table 18. They are mostly averaged from the abundance ratios that are calculated by co-adding the line intensities of individual multiplets (or transition arrays). Atomic data references used for the ORL analysis are listed in Table 21.

Several permitted lines emitted by silicon ions were observed or deblended (Paper I). As the first and the second ionization potentials of atomic silicon are 8.15 and 16.35 eV, respectively, we expect that the main ionization stages of silicon are doubly and triply ionized, while the amount of Si IV should exist but is of much lower abundance compared to Si III and Si IV.

The Si III/He I abundance ratios derived from Si III M2 and M5 lines are presented in Table 20. The Si IV line at 4116.10 Å is also observed (Fig. 25), which means that λ4088.86 of the same multiplet should also exist. Since only the effective dielectronic recombination coefficients for a few selected Si III transitions are available combination lines are summarized in Table 19. They are mostly averaged from the abundance ratios that are calculated by co-adding the line intensities of individual multiplets (or transition arrays). Atomic data references used for the ORL analysis are listed in Table 21.

Several permitted lines emitted by silicon ions were observed or deblended (Paper I). As the first and the second ionization potentials of atomic silicon are 8.15 and 16.35 eV, respectively, we expect that the main ionization stages of silicon are doubly and triply ionized, while the amount of Si IV should exist but is of much lower abundance compared to Si III and Si IV.

The Si III/He I abundance ratios derived from Si III M2 and M5 lines are presented in Table 20. The Si IV line at 4116.10 Å is also observed (Fig. 25), which means that λ4088.86 of the same multiplet should also exist. Since only the effective dielectronic recombination coefficients for a few selected Si III transitions are available.
The Optical Recombination Spectrum of NGC 7009

Table 19. Adopted recombination line abundances for the C, N, O, and Ne ions.

| Ion       | Abundances ($\times 10^{-4}$) | log[X$^+$/H$^+$]+12 |
|-----------|-------------------------------|---------------------|
| C$^+/H^+$ | 6.865                         | 8.796               |
| C$^+/H^+$ | 1.480                         | 8.170               |
| C$^+/H^+$ | 0.220                         | 7.342               |
| N$^+/H^+$ | 3.450                         | 8.538               |
| N$^+/H^+$ | 1.313                         | 8.118               |
| O$^+/H^+$ | 14.176                        | 9.152               |
| O$^+/H^+$ | 0.659                         | 7.819               |
| O$^+/H^+$ | 0.158                         | 7.198               |
| Ne$^+/H^+$| 8.425                         | 8.926               |

Table 20. Recombination line Si$^{3+}$/H$^+$ and Mg$^{2+}$/H$^+$ abundances. Intensities are normalized such that $I$($H$β) = 100.

| Line (Å)  | Mult. | $I_{obs}$ | Si$^{3+}$/H$^+$ ($\times 10^{-5}$) |
|-----------|-------|-----------|----------------------------------|
| λ4552.62  | M2    | 0.0175    | 0.220                             |
| λ4567.82  | M2    | 0.0114    | 0.0352                            |
| λ4574.76  | M2    | 0.0041    | 0.0326                            |
| M2 4p$^1$P$^1$ – 4s $^3$S | M5 | 0.022 | 0.0105 |
| M5 4d $^3$D – 4p $^3$P$^o$ | M5 | 0.0396 | 0.438 |

| Wavelength (Å) | Mult. | $I_{obs}$ | Mg$^{2+}$/H$^+$ ($\times 10^{-5}$) |
|----------------|-------|-----------|----------------------------------|
| λ4481.20     | M4    | 0.0303    | 3.179                             |
| M4 4f$^1$F$^1$ – 3d $^2$D | M4 | 0.0309 | 3.179 |

We assume that the Mg II 4f–3d λ4481 line has an effective recombination coefficient equal to that of the C II 4f–3d λ4267 line, given the similarity between the atomic structure of Mg II and C II.

from Nussbaumer & Storey (1986), we only present the Si$^{3+}$/H$^+$ abundance ratios. The averaged Si$^{3+}$/H$^+$ ratio is 6.04×10$^{-6}$.

The Mg II M4 λ4481 line is observed (Fig. 15), and the Mg$^{2+}$/H$^+$ abundance derived is presented in Table 20. Since the ionization potentials of neutral Mg$^0$ and Mg$^{2+}$ are 7.65 and 80.14 eV, respectively, we assume that magnesium in NGC 7009 is mainly doubly ionized. Unfortunately no effective recombination coefficients for Mg II lines are available. Given the similarity between the atomic structure of Mg II and C II, we assumed that the effective recombination coefficient of the Mg II M4 4f$^1$F$^1$ – 3d $^2$D λ4481 line is equal to, or at least close to, that of the C II M6 4f$^1$F$^1$ – 3d $^2$D λ4267 transition. The effective recombination coefficient (in Case B) for the C II λ4267 line is adopted from Bastin (2000), with the assumption of $T_e = 10000$ K and $N_e = 10000$ cm$^{-3}$. The calculation of Davey, Storey & Kiskielius (2000) differs from that of Bastin (2000) by 1.5 per cent for the C II λ4267 line. The Mg$^{2+}$/H$^+$ abundance derived from the λ4481 line is 3.18×10$^{-5}$.

4.2 Ionic abundances from CELs

4.2.1 Ionic abundances from the optical CELs

The ionic abundances derived from optical CELs detected in the spectrum of NGC 7009 are presented in Table 22. An electron temperature of 10 000 K, which is an average from different CEL diagnostic ratios (Paper I), and a constant density of 4300 cm$^{-3}$, an average derived from a variety of optical CEL ratios, are assumed throughout the abundance determinations. In addition to the ionic abundances of N, O and Ne, abundances are also derived for the ions of F, Mg, Si, S, Cl, Ar from the CELs detected in the spectrum of NGC 7009. The atomic data references used for CEL analysis are listed in Table 22.

4.2.2 Ionic abundances from the IR and UV CELs

NGC 7009 has been observed in wavelength range other than optical: the IUE Short Wavelength Prime (SWP) and Long Wavelength Redundant (LWR) observations by Perinotto & Benvenuti (1981), the IRAS Low Resolution Spectrometer (LRS) observations by Pottasch et al. (1985), the ISO Short Wavelength Spectrometer (SWS) and Long Wavelength Spectrometer (LWS) observations by Liu et al. (2001a), and the Kuiper Airborne Observatory (KAO) observations by Rubin et al. (1997). Ionic abundances derived from ten near- to far-infrared lines and seven ultraviolet lines are presented in Table 22.

The dereddened and normalized intensities of three infrared lines, the [Ni II] 12.8µm and the [Ne III] 15.5 and 36.0µm, are adopted from LLB01. The Ne$^{2+}$/H$^+$ abundance ratio derived from the [Ne III] 12.8µm line, assuming a temperature of 10 020 K and a density of 4300 cm$^{-3}$. The derived Ne$^{2+}$/H$^+$ ratio is 1.32×10$^{-5}$, which agrees with 1.38×10$^{-5}$ given by LLB01, as is expected. The critical densities of the [Ne III] $^3$P$_1$ and $^3$P$_0$ levels are $2.1×10^7$ and $3.1×10^7$ cm$^{-3}$ (at $T_e = 10000$ K; Osterbrock & Ferland 2006), respectively, much larger than the average electron density. The Ne$^{2+}$/H$^+$ abundance ratios deduced from the [Ne III] 15.5 and 36µm lines are $1.67×10^{-4}$ and $1.48×10^{-4}$, respectively. The two ratio values agree with each other within errors. We adopt a value of $1.65×10^{-4}$, which is derived from the sum of the intensities of the two [Ne III] infrared (IR) fine-structure lines, as the Ne$^{2+}$/H$^+$ ratio in NGC 7009. It agrees with the value of $1.63×10^{-4}$ given by LLB01. An electron temperature of 9980 K and a density of 3930 cm$^{-3}$ were assumed in LLB01.

The observed line fluxes, in units of erg cm$^{-2}$ s$^{-1}$, of the [N III] 57µm and the [O III] 52 and 88µm fine-structure lines are adopted from Liu et al. (2001a). These fluxes were normalized using the observed total Hβ flux, $10^{-15}$ erg cm$^{-2}$ s$^{-1}$. The extinction of the three IR lines, as pointed out by Liu et al. (2001a), should be negligible. Since the critical density of the [N III] $^3$P$_1$/2 level is $1.5×10^3$ cm$^{-3}$, comparable to the density of NGC 7009, we have assumed a density of 1260 cm$^{-3}$, deduced from the [O III] 52µm/88µm line ratio, in deriving the N$^{2+}$/H$^+$ abundance ratio from the [N III] 57µm line. Here an electron temperature of 10 020 K is again assumed. The derived N$^{2+}$/H$^+$ ratio is $4.97×10^{-5}$, in close agreement with $4.91×10^{-5}$ given by Liu et al. (2001a).

The O$^{2+}$/H$^+$ abundance ratios derived from the [O III] 52 and 88µm lines are $2.79×10^{-4}$ and $2.76×10^{-4}$, respectively. Here an electron temperature of 9800 K derived from the [O III] λ4959/λ3463 ratio, and a density of 1260 cm$^{-3}$ derived from the [O III] 52µm/88µm ratio, were assumed. Given that the critical densities of the [O III] $^3$P$_1$ and $^3$P$_2$ fine-structure levels are $5.1×10^2$ and $3.6×10^3$ cm$^{-3}$, respectively (Osterbrock & Ferland 2006), the density value of 1260 cm$^{-3}$ for the O III IR-line abundance is appropriate. We adopt a value of $2.99×10^{-4}$, which is derived from the sum of the two [O III] IR lines, as the O$^{2+}$/H$^+$ abundance.
The flux of the [O IV] 25.9μm line were estimated from the \( F([\text{O IV}] 25.9\mu m)/F([\text{S III}] 18.7\mu m) \) ratio given by Rubin et al. (1997), who obtained far-IR observations of the PNe NGC 7009, NGC 7027 and NGC 6210 with the Kuiper Airborne Observatory (KAO). The ISO SWS observations by X.-W. Liu (unpublished) during the ISO Orbit #344 in 1996 gives the \( F([\text{O IV}] 25.9\mu m)/F([\text{S III}] 18.7\mu m) \) ratio that differs from that of Rubin et al. (1997) by more than 30 per cent. The \( \text{O}^{+}/\text{H}^{+} \) abundance ratio derived from the above two observations are given in Table 24. Here the flux of the [S III] 18.7μm line was adopted from Pottasch et al. (1986).

The IRAS fluxes (in units of erg cm\(^{-2}\) s\(^{-1}\)) of the [Ne v] 14.3μm, the [S III] 18.7μm and the [S IV] 10.52μm lines, as well as the total flux of Hβ, are adopted from Pottasch et al. (1986). The Ne\(^{+}/\text{H}^{+} \) abundance ratio derived from the [Ne v] 14.3μm line is 6.11×10\(^{-7}\). Here an electron temperature of 10 020 K is assumed. The critical densities for the [Ne v] \(^3\)P\(_1\) and \(^3\)P\(_2\) fine-structure levels are 6.2×10\(^3\) and 3.5×10\(^4\) cm\(^{-3}\) (Osterbrock & Ferland 2006), respectively. Thus the electron density of 1260 cm\(^{-3}\) from the [O III] 52μm/88μm ratio is again assumed. If we adopt a density value of 4300 cm\(^{-3}\), the derived Ne\(^{+}/\text{H}^{+} \) abundance ratio slightly increases to 6.87×10\(^{-7}\).

The S\(^{2+}/\text{H}^{+} \) abundance ratio derived from the [S III] 18.7μm line is 8.36×10\(^{-7}\). Here an electron temperature of 10 020 K and a density of 1260 cm\(^{-3}\) are assumed. Since the critical densities of the [S III] \(^3\)P\(_1\) and \(^3\)P\(_2\) fine-structure levels are 1.98×10\(^4\) and 1.54×10\(^5\) cm\(^{-3}\), respectively, a density of 1260 cm\(^{-3}\) is reasonable. If we increase the density value to 4300 cm\(^{-3}\), the S\(^{2+}/\text{H}^{+} \) ratio derived then increases to 9.93×10\(^{-7}\). The S\(^{2+}/\text{H}^{+} \) abundance ratio derived from the [S IV] 10.52μm line is 7.34×10\(^{-6}\). The same temperature is assumed.

The observed fluxes (in unit of erg cm\(^{-2}\) s\(^{-1}\)) for the seven ultraviolet (UV) lines in Table 24 are adopted from Perinotto & Bonvenuti (1981). The fluxes are normalized using their Hβ flux, which should be multiplied by a factor 0.48, the fraction of the Hβ flux entering into the IUE slot in position of SWP and LWR images. Using the logarithmic reddening constant \( c(H/\beta) = 0.174 \) derived in Paper I and the extinction curve of Howarth (1983), we derived the dereddened intensities of those UV lines. The ionic abundances are presented in Table 24. Here an electron temperature of 10 020 K and a density of 4300 cm\(^{-3}\) are assumed in the abundance calculations.

### 4.3 Comparison of the ORL and CEL abundances

#### 4.3.1 Ionic abundances

In Fig 41 the ionic abundances of C, N, O and Ne derived from ORLs are compared with the corresponding values derived from the optical, UV and far-IR CELs. Here the ionic abundances of C, N, O and Ne derived from ORLs are from Table 19 and the ionic abundances derived from CELs are from Tables 23 (optical) and 24 (UV and IR). The IR fine-structure line fluxes are adopted from the recent ISO observations (Liu et al. 2001a). We also make use

### Table 21. References for the ORL atomic data.

| ion   | Effec.recomb. coefficients | ORLs                        | Comments          |
|-------|-----------------------------|-----------------------------|-------------------|
| H I   | Storey & Hummer (1995)      |                             | Case B            |
| He I  | Benjamin, Skillman & Smits (1999) |                          | Case B; singlets  |
|       | Brocklehurst (1972)         |                             | Case A; triplets  |
| He II | Storey & Hummer (1995)      |                             | Case B            |
| C I   | Escalante & Victor (1990)   |                             | Case A; singlets  |
|       | Escalante & Victor (1990)   |                             | Case B; triplets  |
| C II  | Davey, Storey & Kisielius (2000) |                         | Case B            |
| C III | Péquignot, Petitjean & Boisson (1991) |                     | Case A            |
|       | Nussbaumer & Storey (1984)  |                             | Dielectronic recombination |
| C IV  | Péquignot, Petitjean & Boisson (1991) |                     | Case A            |
| N I   | Péquignot, Petitjean & Boisson (1991) |                     | Case A; doublets  |
| N II  | Nussbaumer & Storey (1984)  |                             | Case B; quartets  |
| N III | Péquignot, Petitjean & Boisson (1991) |                     | Case A            |
| O I   | Péquignot, Petitjean & Boisson (1991) |                     | Case A            |
| O II  | P. J. Storey (PJS, private communication) |             | Case B            |
| O III | Péquignot, Petitjean & Boisson (1991) |                     | Case A            |
| O IV  | Péquignot, Petitjean & Boisson (1991) |                     | Case A            |
| Ne II | Kisielius et al. (1996)     |                             | Case B; doublets  |
|       | Storey (unpublished)        |                             | Case A; quartets  |
| Nussbaumer & Storey (1984) |                             | Dielectronic recombination |
| Mg II | Davey, Storey & Kisielius (2000) |                          | Case B            |
| Si II | Nussbaumer & Storey (1986)  |                             | Dielectronic recombination |
| Si III| Nussbaumer & Storey (1986)  |                             | Dielectronic recombination |

\(^{a}\) Given the similarity between the atomic structure of Mg II and C II, we have assumed that the Mg II M4 4f \(^2\)F\(_0\) – 3d \(^2\)D \(\lambda 4481 \) line has an effective recombination coefficient equal to that of the C II M6 4f \(^2\)F\(_0\) – 3d \(^2\)D \(\lambda 4267 \) line (Zhang et al. 2005b).
of the IR line fluxes from observations of IRAS in the literature. The UV line fluxes from the IUE observations are dereddened using the extinction derived in Paper I, and they are used to derive ionic abundances for highly ionized heavy elemental ions. The recombination line C and N derived from CELs were adopted from the earlier observations of the Galactic disc and bulge PNe from literature. The C/H, N/H, abundance from Asplund et al. (2009) and average abundances for highly ionized heavy elements can be lower than the abundance ratios derived from CELs by nearly a factor of 5 (i.e. ADF = 5), in agreement with what was observed by LSBC (for C, N and O) and LLB01 (for Ne). However, the ADF differs from 5 when the abundances derived from UV CELs are used: The N3+/H+ abundance derived from ORL is higher than the value derived from the N IV \( \lambda 486 \) UV line by a factor of 2. The recombination line O3+/H+ abundance is higher than the abundance derived from the O IV \( \lambda 403 \) UV line by only 65% per cent. This is probably mainly due to systematic difference in flux calculations, given that the UV data are adopted from the early IUE observations of Perinotto & Benvenuti (1981). The ADF value of O3+ is close to 5 when the abundance derived from the [O IV] 25.9\( \mu m \) IR line is used.

### Table 22. Ionic abundances derived from optical CELs. Line intensities are normalized such that \( I(H\beta) = 100 \).

| Ions | Lines | \( I_{\text{obs}} \) | \( X^{i+}/H^+ \) | Abundances |
|------|-------|--------------------|-----------------|------------|
| C I  | λλ8024.13,9850.26 | 0.035 | C0/H+ | \( 8.483 \times 10^{-9} \) |
| N I  | λ5568.90,5200.26 | 0.091 | N0/H+ | \( 8.435 \times 10^{-8} \) |
| O I  | λλ5197,5090.81,7751.10 | 0.740 | O1/H+ | \( 8.395 \times 10^{-7} \) |
| N II | λ5754.64 | 0.390 | N2+/H+ | \( 2.876 \times 10^{-6} \) |
| N II | λ6548.04,6583.46 | 0.206 | N2+/H+ | \( 2.725 \times 10^{-6} \) |
| O II | λλ3726.03,3728.81 | 0.1997 | O2+/H+ | \( 8.779 \times 10^{-6} \) |
| O II | λ6371.99,7330.73 | 2.300 | O3+/H+ | \( 1.996 \times 10^{-5} \) |
| O III | λ4363.21 | 7.300 | O3+/H+ | \( 2.435 \times 10^{-4} \) |
| O III | λ4031.23 | 0.120 | O3+/H+ | \( 2.455 \times 10^{-4} \) |
| O III | λλ4958.91,5006.84 | 1550.753 | O2+/H+ | \( 3.231 \times 10^{-4} \) |
| F II | λ4799.45 | 0.017 | F+/F+ | \( 2.579 \times 10^{-8} \) |
| F IV | λ4959.90 | 0.012 | F4+ | \( 4.387 \times 10^{-9} \) |
| Ne III | λ3342.50 | 0.755 | Ne2+/H+ | \( 3.079 \times 10^{-4} \) |
| Ne III | λλ3868.76 | 118.837 | Ne2+/H+ | \( 1.296 \times 10^{-4} \) |
| Ne III | λ4012.01 | 0.014 | Ne2+/H+ | \( 2.161 \times 10^{-4} \) |
| Ne IV | λλ4734.17,4725.67 | 0.042 | Ne3+/H+ | \( 1.501 \times 10^{-5} \) |
| Ne IV | λλ4714.17,4715.66 | 0.067 | Ne3+/H+ | \( 3.845 \times 10^{-5} \) |
| S II | λλ4066.60,4076.35 | 0.960 | S+/H+ | \( 1.085 \times 10^{-7} \) |
| S II | λ6716.44,6730.82 | 3.700 | S+/H+ | \( 1.192 \times 10^{-7} \) |
| S III | λ3727.81 | 1.045 | S2+/H+ | \( 2.781 \times 10^{-6} \) |
| S III | λ6352.10 | 1.400 | S2+/H+ | \( 2.265 \times 10^{-6} \) |
| S III | λλ6964.60,9543.60 | 64.000 | S2+/H+ | \( 1.965 \times 10^{-6} \) |
| Cl II | λ6161.84 | 0.008 | Cl+/H+ | \( 3.343 \times 10^{-8} \) |
| Cl II | λ8578.69,9123.60 | 0.078 | Cl+/H+ | \( 4.994 \times 10^{-10} \) |
| Cl III | λ3355.17 | 0.076 | Cl2+/H+ | \( 1.388 \times 10^{-7} \) |
| Cl III | λλ5517.72,5537.89 | 1.000 | Cl2+/H+ | \( 5.513 \times 10^{-8} \) |
| Cl III | λ4848.85 | 0.020 | Cl2+/H+ | \( 8.319 \times 10^{-8} \) |
| Cl IV | λ5323.28 | 0.012 | Cl3+/H+ | \( 2.525 \times 10^{-8} \) |
| Cl IV | λλ7530.80,8054.63 | 0.990 | Cl3+/H+ | \( 5.475 \times 10^{-8} \) |
| Ar III | λ3109.17 | 0.175 | Ar2+/H+ | \( 5.956 \times 10^{-7} \) |
| Ar III | λ5191.82 | 0.100 | Ar2+/H+ | \( 8.584 \times 10^{-7} \) |
| Ar III | λλ7135.80,7751.10 | 18.300 | Ar3+/H+ | \( 1.027 \times 10^{-6} \) |
| Ar IV | λλ4711.37,4740.17 | 7.600 | Ar3+/H+ | \( 5.592 \times 10^{-7} \) |
| Ar IV | λλ7237.40,7262.76 | 0.372 | Ar3+/H+ | \( 2.713 \times 10^{-6} \) |
| Ar V | λ6463.10,7005.67 | 0.065 | Ar4+/H+ | \( 6.654 \times 10^{-9} \) |
| K IV | λ6101.83,6795.10 | 0.196 | K2+/H+ | \( 1.146 \times 10^{-8} \) |

#### 4.3.2 Total elemental abundances

Elemental abundances derived from ORLs and CELs are compared in Table 25. Abundance errors (numbers in brackets) are estimated from measurement uncertainties, which is calculated by quadratically adding line flux errors from Gaussian profile fitting and the systematic uncertainties in line measurements e.g. subtraction of the continuum. Uncertainties introduced by ionization correction methods, i.e. the ionization correction factors (icf’s), are not taken into account in error estimate. Also given in this table are the solar abundances from Asplund et al. (2009) and average abundances of the Galactic disc and bulge PN e from literature. The C/H, N/H, O/H and Ne/H elemental abundances derived from ORLs are higher than the corresponding values derived from CELs by a factor of 5.4, 6.9, 4.7 and 5.3, respectively. This result is similar to LSBC, who derived ADF values of 6.1, 4.1 and 4.7 for C, N and O in NGC 7009. The Ne abundance discrepancy derived by LLB01 is about 4. In the analyses of LSBC, the elemental abundances of C and N derived from CELs were adopted from the earlier observations of Barker (1983), who first discussed the large discrepancy between the C2+/H+ abundances derived from the C II \( \lambda 4267 \) recombination line and from the UV CEL C III \( \lambda 4919, 7097 \).
Whenever available, the ionization correction factors (icf’s) given by Kingsburg & Barlow (1994) were used. In Section 4.2.2 we have derived 16 ionic abundances from UV and IR data available from the literature (Table 24). These ionic abundances can be used as an aid to derive total elemental abundances. When they are not necessary, we can use them to check the elemental abundance values derived from icf’s, by adding these UV and IR abundances with the optical abundance ratios in Table 22 and compare with the ionization corrected total abundances.

The forbidden line O/H abundance ratio was calculated from the O\(^+\)/H\(^+\) ratio derived from the [O III] \(\lambda\lambda 3726,3729\) line\(^6\) and the O\(^{2+}\)/H\(^+\) ratio derived from the [O III] \(\lambda\lambda 4959,5007\) lines, correcting for the unseen O\(^{3+}\) in optical using

\[
\frac{O}{H} = icf(O) \times \left(\frac{O^+}{H^+}\right) \left(\frac{O^{2+}}{H^+}\right) = \left[\frac{He^+ + He^{2+}}{He^{3+}}\right]^{2/3} \times \left(\frac{O^+}{H^+}\right) \left(\frac{O^{2+}}{H^+}\right).
\]

From the He\(^+\) and He\(^{2+}\) abundances given in Table 10 we have icf(O) = 1.086. Thus derived O/H ratio is 3.603\(\times 10^{-4}\), close to the value 3.716\(\times 10^{-4}\), which is derived by combining the O ions from UV, IR and optical lines, O/H = O\(^+/H^+\) + O\(^{2+}/H^+\) + O\(^{3+}/H^+\). Here the O\(^{3+}/H^+\) is derived from the O IV \(\lambda\lambda 1403\) UV line (Section 4.2.2).

The recombination line abundance O\(^+/H^+\) is not available, thus in order to make use of the above equation, we assume that the recombination line O\(^+/O^{2+}\) ratio is the same as that derived from the CELs. Given the small ionic concentration of O\(^+\) (less than 10 per cent in NGC 7009), the errors introduced should be negligible. The O\(^{2+}/H^+\), O\(^{3+}/H^+\) and O\(^{4+}/H^+\) abundance ratios derived from ORLs are available from Table 19. The recombination line O/H abundance ratio thus derived is 1.667\(\times 10^{-3}\).

Both C\(^{2+}/H^+\) and C\(^{3+}/H^+\) abundance ratios have been derived from ORLs and are presented in Table 19. The C\(^{4+}/H^+\) abundance ratio has also been derived from the C IV \(\lambda\lambda 1465,8,30\) line, but probably is not acceptable because it blends with [Fe III] \(\lambda\lambda 4658,05\). The unseen C\(^+/H^+\) are corrected for using the equation of LSBC.

\[
\frac{C}{H} = \frac{C^2 + C^3}{H^+} \times icf(C)
\]
Table 24. Ionic abundances derived from the UV and far-IR fine-structure CELs. Intensities are normalized such that $I(H/\beta) = 100$.

| Ions | Lines | $I_{obs}$ | $X^{+}/H^{+}$ | log($X^{+}/H^{+}$)+12 | Ref. |
|------|-------|-----------|---------------|-----------------|-----|
| IR Lines |
| [N III] | 57 µm | 24.315 | N$^{2+}/H^{+}$ | 4.974×10^{-5} | 8.065 | (1) |
| [O III] | 52 µm | 165.940 | O$^{2+}/H^{+}$ | 2.792×10^{-4} | 8.446 | (1) |
| [O III] | 88 µm | 55.029 | O$^{2+}/H^{+}$ | 2.762×10^{-4} | 8.441 | (1) |
| [O IV] | 25.9 µm | 20.29±1.0 | O$^{3+}/H^{+}$ | 1.275×10^{-5} | 7.105 | (2) |
| [O IV] | 25.9 µm | 14.16±0.76 | O$^{3+}/H^{+}$ | 8.893×10^{-6} | 6.950 | (2) |
| [Ne III] | 12.8 µm | 9.9±3.0 | Ne$^{2+}/H^{+}$ | 1.322×10^{-5} | 7.121 | (3) |
| [Ne III] | 15.5 µm | 250±12 | Ne$^{2+}/H^{+}$ | 1.667×10^{-4} | 8.222 | (3) |
| [Ne III] | 36.0 µm | 18.0±1.8 | Ne$^{2+}/H^{+}$ | 1.477×10^{-4} | 8.160 | (3) |
| [Ne V] | 14.3 µm | 8.696 | Ne$^{4+}/H^{+}$ | 6.107×10^{-7} | 5.786 | (4) |
| [S III] | 18.7 µm | 7.826 | S$^{2+}/H^{+}$ | 8.356×10^{-7} | 5.922 | (4) |
| [S IV] | 10.52 µm | 230.434 | S$^{3+}/H^{+}$ | 7.342×10^{-6} | 6.866 | (4) |

UV Lines (Å)

| C II | λ2326 | 3.821 | C$^{+}/H^{+}$ | 4.555×10^{-6} | 5.629 | (5) |
| C III | λ1908 | 44.832 | C$^{2+}/H^{+}$ | 1.308×10^{-4} | 8.117 | (5) |
| N III | λ1751 | 6.959 | N$^{2+}/H^{+}$ | 5.535×10^{-5} | 7.743 | (5) |
| N IV | λ486 | 6.197 | N$^{3+}/H^{+}$ | 6.151×10^{-5} | 7.789 | (5) |
| O III | λ1663 | 4.313 | O$^{2+}/H^{+}$ | 2.387×10^{-4} | 8.378 | (5) |
| O IV | λ1403 | 0.705 | O$^{3+}/H^{+}$ | 3.977×10^{-4} | 7.600 | (5) |
| [Ne IV] | λ2424 | 7.789 | Ne$^{3+}/H^{+}$ | 1.895×10^{-5} | 7.278 | (5) |

(1) The observed flux is from Liu et al. (2001a).
(2) The observed flux is estimated from the flux ratio $F([O IV]) / F([S III])$ adopted from the ISO/LWS observations during ISO #344 orbit in 1996 (Liu et al., unpublished), using the [S III] 18.7 µm flux adopted from Pottasch et al. (1986).
(3) The observed flux is from LLB01.
(4) The observed flux is from Pottasch et al. (1986).
(5) The observed flux is from Perinotto & Benvenuti (1981).

\[ icf(C) = \left( \frac{He^{+} + He^{2+}}{He} \right)^{1/3} \times \left( \frac{O^{+} + O^{2+}}{O^{2+}} \right). \] (3)

The ionic abundances of O$^{+}$ and O$^{2+}$ in the above equation are assumed to be those derived from the forbidden line measurements. In NGC 7009, most of the carbon should exist in the form of C$^{2+}$ and C$^{3+}$, and the correction required for unobserved C$^{+}$ and C$^{4+}$ is quite small. The Equation [3] gives $icf(C) = 1.07$. Thus derived recombination line C/H abundance ratio is 8.932×10^{-4}, which is only 3 per cent lower than the value given by LSBC.

For the collisionally excited lines, C$^{+}/H^{+}$ and C$^{2+}/H^{+}$ are derived from UV lines (Table 24). We assume C$^{3+}/C^{2+} = 0.216$, as given by the ORL abundance ratios. Thus the CEL ratio of carbon is C/H = C$^{+}/H^{+}$ + C$^{2+}/H^{+}$ + C$^{3+}/H^{+}$, which is 1.636×10^{-3}.

Recombination line abundances are available for the N$^{2+}/H^{+}$ and N$^{3+}/H^{+}$ ratios (Table 19) but not for N$^{+}/H^{+}$. The latter is available from the collisionally excited [N II] $\lambda\lambda$6548 and 6584 lines. The N$^{2+}/H^{+}$ ratio derived from the UV collisionally excited N III] $\lambda$1751 line is about 10 per cent higher than that deduced from the [N III] 57 µm far-IR fine-structure line (Table 24). Given the weakness of the $\lambda$1751 and the relative new observations of the 75 µm line, we adopt the N$^{2+}/H^{+}$ abundance ratio derived from the far-IR line. N$^{2+}/H^{+}$ from the 57 µm and N$^{+}/H^{+}$ from the $\lambda\lambda$6548 and 6584 lines yield N$^{+}/N^{2+} = 0.0548$. We assume that this is also valid for corresponding abundances derived from ORLs. The N IV

Figure 41. Comparison of the ionic abundances derived from ORLs and the optical, UV and IR CELs. Error bars on the ORL abundances are from the propagation calculations of the ORL measurement errors.
optical recombination lines are not clearly detected in the spectrum of NGC 7009, but we detected the O IV M2 3d 2D – 3p 2P λ3412 line, and the O 3+/H 1 abundance ratio is derived from it (Table 18). Given the ionization potential of N 3+ (77.42 eV) is the same to that of O 3+ (77.4 eV), we assume N 3+/N = O 3+/O. Thus the total recombination line N/H abundance is given by

\[
\frac{N}{H} = (1.0548 \times \frac{N^2}{H^2} + \frac{N^3}{H^3})/(1 - \frac{O^2}{O^2}).
\]

To obtain the forbidden-line N/H abundance, we correct for the unseen N 3+/H + assuming N 3+/N 2+ = 0.380, as given by ORLs, so that

\[
\frac{N}{H} = \frac{N^+}{H^+} + 1.380 \times \frac{N^{2+}}{H^{2+}}.
\]

The total CEL N/H abundance thus derived is 7.01 \times 10^{-5}. Here the N 4+/H + is neglected, given its very low abundance, if we assume that the N 3+/N 2+ ratio from CELs is the same as that given by the ORL values. If we take into account the N 3+/H + abundance ratio derived from the UV N IV λ1486 line (Table 2), the total CEL abundance ratio is then N/H = N^+ + N^2+/H^+ + N^3+/H^3 = 1.140 \times 10^{-4}, which is 64 per cent higher than the value derived by the ionization correction Equation 5. Since the recent IR data should be more reliable, we adopt the ratio derived from Equation 5.

Ne^+/H^+, Ne^2+/H^+ and Ne^3+/H^+ ionic abundances are available from IR (Table 24) and optical CELs (Table 23). For the Ne 3+/H + ratio, we adopt the abundance derived from the [Ne iii] λ3867 optical line, which is only about 21 per cent lower than that derived from the ISO SWS observation of the [Ne iii] λ15.54+36 μm IR fine-structure lines. For the Ne 3+/H + ratio, we adopt the value derived from the [Ne iv] λλ4724.17 and 4725.67 optical lines. From the IUE observation by Perinotto & Benvenuti (1981), we also derived the Ne 3+/H + abundance ratio from their observed Ne iv λλ2422 and 2424 lines (Table 24), which is 26 per cent higher than that from optical. Thus the CEL Ne/H ratio obtained from the equation

\[
\frac{Ne}{H} = \frac{Ne^+}{H^+} + \frac{Ne^{2+}}{H^{2+}} + \frac{Ne^{3+}}{H^{3+}}
\]

is 1.578 \times 10^{-4} (Table 25). This is 9 per cent lower than the Ne/H ratio derived by LLB01.

Only the Ne 2+/H + ratio is available from ORLs. The Ne 3+/Ne 2+ ratio from CELs is 0.102, with and Ne 3+/Ne 2+ from optical CELs is 0.116. Assuming these ionization ratios from CELs are the same as given by ORLs, we obtain a recombination line Ne/H abundance ratio value of 8.398 \times 10^{-5} by using

\[
\frac{Ne}{H} = (0.102 + 1.0 + 0.116) \times \frac{Ne^{2+}}{H^{2+}}.
\]

This ratio value is 21 per cent higher than that given by LLB01.

CELs emitted by [F ii] and [F iv] ions are detected, with the F 2+H + and F 3+/H + abundance ratios derived from the [F ii] λ4789 and [F iv] λ4060 lines respectively (Table 22). O has ionization potential comparable to F and Ne 3+ has that comparable to F 3+. Zhang & Liu (2005) suggested that F/O = F 3+/O 2+ for low-excitation PN, and F/O = (F 3+/Ne 3+)/(Ne/O) for high-excitation PN. Here we adopt the assumption of Zhang & Liu (2005). Given that NGC 7009 is a medium excitation PN, we derive the Ne 3+/H + abundance ratio from the equation

\[
\frac{F}{H} = \frac{F^+ + F^{3+}}{H^+ + H^{3+}}/(1 - \frac{O^2}{O})
\]

The derived F/H abundance ratio is 2.923 \times 10^{-7}. The F 3+/H + should be negligible because the ionization potential of F 3+ is 87 eV, which is too high. No recombination line from the fluorescent ions are detected.

For the elements heavier than Ne, we detected CELs emitted by S, Cl, Ar and K ions (Table 27), and ORLs by Si and Mg ions (Table 20). Considering that the first ionization potential of silicon atom is only 8.2 eV, and the ionization potential of Si 3+ is 166.8 eV, which is a huge jump from that of Si 2+, 45 eV, we assume that the Si 3+/H + abundance is negligible, and the main ionization stages of Si in NGC 7009 are Si 2+, Si 3+ and Si 3+. Several Si iii multiplets and the Si iv multiplet V1 are detected from NGC 7009. Only effective dielectronic recombination coefficients for Si iii multiplets are available from Nussbaumer & Storey (1984), thus only the Si 3+/H + ratio is obtained from the Si iii multiplets M2 4p 3P0 – 4s 1S0 and M5 4d 3D – 4p 3P0 (Table 20). Since the ionization potential of Si 4+ (16.3 eV) is close to that of F 3+ (17.4 eV), and the ionization potential of Si 3+ (45 eV) is comparable to that of N 3+ (47.4 eV), we assume the relation: Si 3+/Si = F 3+/O = O 3+/O, and Si 3+/Si = N 3+/1. Thus the total Si/H abundance ratio from ORLs can be derived from the equation

\[
\frac{Si}{H} = \frac{Si^{3+}}{H^+}/(1 - \frac{O^2}{O} - \frac{N^{3+}}{N}) = \frac{Si^{3+}}{H^+}/(1 - \frac{F^+}{F} - \frac{N^{3+}}{N}).
\]

The Si/H abundance ratio thus derived is 8.278 \times 10^{-5}. In Equation 9 the O/H ratio is from optical CELs, and the N 3+/H ratio is from ORLs.

The Mg 2+/H + abundance ratio is derived from the Mg ii M4 4f 2F0 – 3d 3D λ4481 line. The ionization potentials of Mg 2+ and Mg 3+ are 7.65 eV and 80.1 eV, respectively, thus we assume that magnesium in NGC 7009 are mainly doubly ionized, and Mg 2+ and Mg 3+ are negligible. No effective recombination coefficients for Mg ii lines are available. Given the similarity between the atomic structure of Mg ii and C ii, we assume that the Mg ii λ4481 line has an effective recombination coefficient equal to that of C ii M6 λ4267. Thus we have Mg/H = Mg 2+/H +.

For S, we have S 3+/H + derived from the [S ii] λ6716 and 6731 optical lines, S 3+/H + derived from the [S iii] λ6312 optical line. The S 3+/H + is also available from the [S iii] λ18.7 μm IR line. S 3+/H + is derived from the [S iv] 10.5 μm far-IR fine-structure line, which was adopted from the IRAS observations by Pottasch et al. (1986). We use the S 3+/H + ratio derived from the λ6312 line. S 4+ is not observed. Since S 3+ has an ionization potential of 47.3 eV, very close to the value of 47.4 eV for N 3+, we assume that S 3+/S = N 3+/N. The N 3+/H ratio is from ORLs. Thus the S/H ratio is obtained from

\[
\frac{S}{H} = (\frac{S^+}{H^+} + \frac{S^3+}{H^{3+}})/(1 - \frac{N^{3+}}{N}).
\]

The derived S/H ratio is 1.299 \times 10^{-5}. If we adopt the S 3+/H + ratio derived from the [S iii] λ18.7 μm IR line, then the S/H ratio is

\[
7 \text{ Another [Ne iii] nebular line λ3967 is saturated in the higher resolution spectrum, and is not saturated but blended with the H I λ3970 line in the low-resolution spectrum.}
The Cl/H abundance ratio can be obtained from the equation
\[
\frac{Cl}{H} = \frac{(Cl^{+} + Cl^{2+} + Cl^{3+})}{(1 - \frac{He^{2+}}{He} - \frac{Ne^{5+}}{Ne})}.
\]
(11)

The derived total Cl/H abundance is 1.927 \times 10^{-7}.

Ar^{2+}/H^{+} is derived from the [Ar \(II\)] \(\lambda\lambda 7136\) and 7751 lines, Ar^{2+}/H^{+} is from the [Ar \(I\)] \(\lambda 4711\) and 4740, and Ar^{4+}/H^{+} is from the [Ar \(III\)] \(\lambda 6435\) and 7006 lines (Table 2). The uncorrected Ar^{4+} is for assuming Ar^{4+}/Ar = N^{+}/N, where the N^{+}/N is derived from optical CELs. The total Ar/H ratio can be obtained using
\[
\frac{Ar}{H} = \frac{(Ar^{2+} + Ar^{3+} + Ar^{4+})}{(1 - \frac{N^{+}}{N})}.
\]
(12)

The finally derived Ar/H abundance ratio is 2.570 \times 10^{-6}.

We have derived the K^{+}/H^{+} abundance ratio from the [K \(I\)] \(\lambda\lambda 6102\) and 6795 lines. A very faint feature, which might be the [K \(V\)] \(\lambda 4163\) line is detected in the spectrum of NGC 7009, but we could not confirm that. K^{+} is probably negligible in NGC 7009, judging from the very low ionization potential of K. Since the ionization potential of K^{+} (31.6 eV) is comparable to that of N^{+} (29.6 eV), and ionization potential of K^{2+} (60.9 eV) is comparable to that of F^{2+} (62.7 eV), we assume K^{2+}/K = N^{2+}/N and K^{4+}/K = F^{3+}/F. Here the N^{2+}/N ratio is from the ORLs, and the F^{3+}/F ratio is from CELs. We use the following equation to correct for the unseen K^{2+} and K^{4+} ions,
\[
\frac{K}{H} = \frac{K^{3+}}{H^{+}}/(1 - \frac{N^{2+}}{N} - \frac{F^{3+}}{F}).
\]
(13)

The derived total K/H abundance is 4.242 \times 10^{-8}.

### 5 DISCUSSION

Average elemental abundances for the Galactic disc and bulge PNe taken from Kingsburg & Barlow (1994) and Exter, Barlow & Walton (2004) are presented in Table 25 for purpose of comparison. Also presented in this table are the average abundances for 23 Galactic bulge PNe of Wang & Liu (2007) plus two bulge PNe M 1-42 and M 2-36 studied by Liu et al. (2001b), and 58 Galactic disc PNe selected from Tsamis et al. (2003, 2004), Liu et al. (2004a, b) and Wesson et al. (2005). The helium abundance derived from the current analyses agrees well with the average value of the 58 disc sample, but 0.12 dex higher than the most recent solar value (Asplund et al. 2009). The recombination line C/H abundance 8.95 derived for NGC 7009 also agrees with the average value (9.03) of the disc sample, but 0.14 dex lower than the 23 bulge sample of Wang & Liu (2007). The forbidden-line C/H abundance for NGC 7009 is lower than all the values quoted from literature. The elemental N/H abundance derived from CELs for NGC 7009 agrees well with the the Sun, but lower than all other average values. Our recombination line N/H abundance is lower than the average values of the bulge and disc samples compiled by Wang & Liu (2007), but is higher than the average value of the sample by Kingsburg & Barlow (1994) plus Exter, Barlow & Walton (2004). The forbidden-line O/H abundance for NGC 7009 is 0.13 dex lower than the solar value, and lower than the average value of the disc sample of Wang & Liu (2007) by the same amount. Our C/O abundance ratio derived from CELs is 0.96, slightly lower than all other ratios from literature, indicating that NGC 7009 might be enriched in oxygen, which is consistent with the fact that the spectrum of NGC 7009 is obviously rich in oxygen emission lines. The forbidden-line neon abundance of NGC 7009 is 0.27 dex (\(\sim\)) higher than the solar value, and agrees with the average value (8.13) of the disc sample of Wang & Liu (2007). The forbidden-line Ne/O ratio observed in NGC 7009 is higher than the solar ratio by a factor of 2.5. However, Wang & Liu (2008) suggests that the solar Ne/O ratio of Asplund, Grevesse & Sauval (2005) be revised upwards by 0.22 dex, i.e. increased to 0.37. Similarly high forbidden-line Ne/O ratio (0.34–0.36) is also found in NGC 6153 by Liu et al. (2000). The sulfur and argon abundances of NGC 7009 both agree with the solar values. The Mg/H abundance of NGC 7009 agrees with the average abundance of the bulge PNe sample of Wang & Liu (2007), and is 0.1 dex lower than the solar value.

Elemental abundances derived from ORLs and CELs observed in NGC 7009 are presented in Table 25. By using the IR and UV observed line fluxes from the literature and correcting for extinction, we are able to derive elemental abundances of C, N, O, and Ne relative to hydrogen from both ORLs and CELs. The resultant ORL abundances of the above four elements are all higher than the abundance ratios derived from CELs, by a factor of 5–7. After deep spectroscopic observations of the ORLs detected in the spectrum of NGC 7009 by LSBC (for C, N and O) and LLB01 (for Ne), those heavy-element abundance discrepancy problems are again studied quantitatively, using the deepest CCD spectrum ever taken for a gaseous nebula and the new effective recombination coefficients calculated for the N \(\text{II}\) and O \(\text{II}\) recombination spectra under the nebular conditions in the intermediate coupling scheme. The analyses of the spectrum are carried out in a very consistent manner, i.e. an electron temperature of 1000 K as derived from the N \(\text{II}\) and O \(\text{II}\) ORL ratios is assumed throughout analyses of the heavy-element ORLs, while a temperature of about 10000 K as yielded by CELs is assumed in calculating the CEL abundances. In the previous deep spectroscopic of NGC 7009 by LSBC, the electron temperature (7100 K) derived from the H \(\text{I}\) Balmer jump was used to calculate the recombination line neon abundance. Liu et al. (2000) assumed a temperature of 9100 K, as derived from the [O \(\text{III}\)] nebular-to-auroral line ratio was adopted in calculating the ORL abundances. In the analyses of the neon abundance in NGC 7009 by LLB01, the electron temperature (7100 K) derived from the H \(\text{I}\) Balmer jump was used to calculate the recombination line neon abundance. Liu et al. (2000) assumed a temperature of 9100 K, as derived from the [O \(\text{III}\)] CEL ratio, in calculating the recombination line abundances for the C, N, O and Ne ions in NGC 6153. In the analyses of Galactic bulge PNe M 1-42 and M 2-36, Liu et al. (2010) used the H \(\alpha\) Balmer jump temperature (3560 K for M 1-42 and 5900 K for M 2-36) to derived the recombination line abundances. However, the recombination line abundances derived in the studies mentioned above are questionable, given that the temperatures assumed are not derived from the heavy-element ORLs. The effective recombination coefficients of the permitted transitions of heavy element ions, which are mainly excited by radiative recombination, usually decrease as the electron temperature increases, except for the dielectronic transitions (with high-lying parent), whose effective recombination coefficients increases as the temperature increases. When we use the recombination lines that are mainly excited by radiative recombina-
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Table 25. Total elemental abundances derived from ORLs and CELs, in units such that \( \log N(\text{H}) = 12 \).

| Element | ORLs | X/H | CELs | \( \log[\text{X/H}]+12 \) | TLW\(^a\) | TLW\(^b\) | KB94\(^c\) | Solar\(^d\) |
|---------|------|-----|------|------------------------|----------|----------|----------|---------|
| He      | 0.112 | 11.049 |     | 11.02 | 11.06 | 11.06 | 11.06 | 10.93 |
| C       | 8.93(±0.15)×10\(^{-4}\) | 1.64(±0.33)×10\(^{-4}\) | 8.95 | 8.21 | 9.09 | 8.52 | 9.03 | 8.56 | 8.74 | 8.43 |
| N       | 4.92(±0.20)×10\(^{-4}\) | 7.14(±0.35)×10\(^{-5}\) | 8.69 | 7.85 | 8.94 | 8.17 | 9.14 | 8.34 | 8.38 | 7.83 |
| O       | 1.67(±0.06)×10\(^{-3}\) | 3.60(±0.07)×10\(^{-4}\) | 9.22 | 8.56 | 9.22 | 8.60 | 9.32 | 8.70 | 8.66 | 8.69 |
| F       | 2.92(±0.58)×10\(^{-7}\) | 5.47 | 5.47 | 4.56 |
| Ne      | 8.40(±0.42)×10\(^{-4}\) | 1.58(±0.08)×10\(^{-4}\) | 8.92 | 8.20 | 9.06 | 7.99 | 9.07 | 8.13 | 8.06 | 7.93 |
| Mg\(^e\) | 3.18(±0.16)×10\(^{-5}\) | 7.50 | 7.56 | 7.71 | 7.60 |
| Si\(^f\) | 8.28(±0.80)×10\(^{-6}\) | 6.92 | 7.51 |
| S       | 1.30(±0.11)×10\(^{-5}\) | 7.11 | 6.84 | 7.05 | 6.99 | 7.12 |
| Cl      | 1.93(±0.21)×10\(^{-7}\) | 5.28 | 5.35 | 5.29 | 5.50 |
| Ar      | 2.57(±0.38)×10\(^{-6}\) | 6.41 | 6.20 | 6.34 | 6.51 | 6.40 |
| Kr\(^g\) | 4.24(±0.84)×10\(^{-8}\) | 4.63 | 5.03 |

\(^a\) Average abundances for 23 Galactic bulge PNe of Wang & Liu (2007) plus Galactic bulge PNe M 1-42 and M 2-36 analyzed by Liu et al. (2001b).

\(^b\) Average abundances given by Wang & Liu (2007) for 58 Galactic disc PNe which were selected from Tsamis et al. (2003, 2004), Liu et al. (2004a b) and Wesson, Liu & Barlow (2005).

\(^c\) Average abundances of Galactic disc and bulge PNe (Kingsburg & Barlow 1994; Exter, Barlow & Walton 2004), all based on CEL analyses except for helium for which ORLs were used.

\(^d\) Solar values from Asplund et al. (2009).

\(^e\) Mg\(^+\)/H\(^+\) and Mg\(^{2+}\)/H\(^+\) are neglected in calculating the total ORL abundance.

\(^f\) Si\(^+\)/H\(^+\) is neglected in calculating the total ORL abundance.

\(^g\) K\(^+\)/H\(^+\) and K\(^{2+}\)/H\(^+\) are neglected in calculating the total CEL abundance.

NGC 6153 by Liu et al. (2000) yielded an ADF of about 10 for that object and, based on the empirical composite models, Liu et al. (2000) proposed a bi-abundance nebular model as a possible explanation to such high abundance discrepancy.

Ever since the detection of a Balmer jump temperature as low as 3560 K for M 1-42, 5660 K lower than the [O III] forbidden line temperature for the same nebula (Liu et al. 2001b), it has become increasingly clear that PNe, at least those exhibiting large ADFs, must contain another component of previously unknown ionized gas. This component of gas mainly emits ORLs yet is essentially invisible in strong CELs because the electron temperature prevailing in the component is too low to excite any UV or optical CELs. The low temperature condition in that component is probably due to the much enhanced cooling by the IR fine-structure lines of the heavy element ions, which is a consequence of one of a very high metallicity (i.e. H-deficient). This physical idea is supported by detailed photoionization modeling of Péquignot et al. (2003) and Tylenda (2003) and also by direct measurements of the average electron temperatures under which various types of emission lines are emitted (c.f. Liu 2003, 2006a, b and 2011 for reviews; Zhang et al. 2004 for the plasma diagnostics based on the H I recombination spectrum and Zhang et al. 2005a for the plasma diagnostics based on the He I recombination spectrum). The discovery of a dramatically high ADF value of \(~\sim 70\) and the remarkably low Balmer jump temperature (\(~\sim 900\) K) in PN Hf 2-2 by Liu et al. (2006a) strengthens the validity of the bi-abundance nebular model. In order to reproduce the multi-waveband spectroscopic and imaging observations of NGC 6153 and investigate the nature and origin of the H-deficient inclusions, Yuan et al. (2011) constructed three-dimensional photoionization models, using the Monte Carlo photoionization code MOCASSIN developed by Ercolano et al. (2003). Modeling of NGC 6153 showed that chemically homogeneous models yielded small electron temperature fluctuations and failed to reproduce the strengths of the ORLs of heavy ele-
ment ions. In contrast, bi-abundance models incorporating a small amount of metal-rich inclusions (∼1.3 per cent of the total nebular mass) are able to match all the observations within measurement uncertainties. The metal-rich inclusions, cooled down in a very low temperature (∼800 K) by ionized IR fine-structure lines, dominate the emission of heavy-element ORLs, but contribute almost nil to the emission of most CELs. The current analyses of the optical recombination spectrum of NGC 7009 are carried out under the context of the bi-abundance nebular model, and the results of plasma diagnostics based on various types of emission lines and abundance determinations are consistent with that context: the temperature sequence $T_e(\text{[O III]}) \gtrsim T_e(\text{H i BJ}) \gtrsim T_e(\text{He i}) \gtrsim T_e(\text{N II} \& \text{O II ORLs})$ is consistent with predictions from the bi-abundance model; the $\text{C}^{2+}/\text{H}^{+}$, $\text{N}^{2+}/\text{H}^{+}$, $\text{O}^{2+}/\text{H}^{+}$ and $\text{Ne}^{2+}/\text{H}^{+}$ ionic abundances derived from ORLs, using the new effective recombination coefficients and the electron temperature yielded by the N II and O II ORLs, are systematically higher, by about a factor of 5, than the corresponding abundances derived from CELs. It has been shown from optical observations that the ADF varies with position in several high-ADF PNe and is highest close to the central star. The “cold” abundances should be cooled via the IR fine-structure lines of heavy element ions. Thus it is interesting to see if the IR fine-structure line fluxes relative to optical/UV CELs peak where the ADF peaks in PNe with large ADFs. Recently, Herschel and Hubble observations of NGC 7009 have been carried out and the results show that within the first ∼5 arcsec from the central star of NGC 7009, the [O III] 88.9 µm/λ5007 flux ratio seems to increase towards center (R. H. Rubin, private communication). With the very deep spectrum and the high-quality atomic data now available, we can derive more precise physical properties e.g. total mass, spatial distribution of the “cold”, metal-rich inclusions in NGC 7009 through three-dimensional photoionization modeling.

Given that the ADFs found in PNe are all larger than unity, the metal-rich (H-deficient) inclusions are probably a real feature of PNe. However, the presence of those “cold” inclusions is not predicted by the current theories of stellar evolution. Iben, Kaler & Truran (1983) proposed that an evolved star undergoing a very late helium flash (the so-called ‘born-again’ PNe) may harbour H-deficient material, such as the H-deficient knots detected in the two ‘born-again’ PNe Abell 30 and Abell 58. Wesson, Liu & Barlow (2003) and Wesson et al. (2008) found that the H-deficient knots in Abell 30 and Abell 58 are O-rich, in contradiction with the expectation of the ‘born-again’ scenario. The ADFs are found to be high in the PNe with Wolf-Rayet central stars, and that can be explained by the scenario of a single post-asymptotic giant branch (post-AGB) star experiencing a list helium shell flash, but not all PNe with large ADFs have an H-deficient central star, such as the case of NGC 7009 studied in the current paper. García-Rojas, Peña & Peimbert (2009) observed the faint ORLs in Galactic PNe with [WC] nucleus and found the results that argue against the presence of H-deficient knots coming from a late thermal pulse event. De Marco (2008) suggested a binary scenario to explain the observations that are in contradiction with the theory of the single post-AGB evolution. In this respect, it is interesting to note that Abell 36 have experienced a nova-like outburst (Clayton & De Marco 1997). Lutz et al. (1998) has also found the central star of H 2-2, a PN with the largest ADF value (∼70), Liu et al. (2006a) ever found for an emission line nebulae, to be a close binary system. An alternative scenario of the origin of the H-deficient inclusions is that they evolve from metal-rich planetary material, such as icy planetesimals left over from the debris of planetary system of the progenitor star of the PN (Liu 2003 2006a). Both high spectral- and spatial-resolution observations in the future, in combination with detailed three-dimensional photoionization modeling as has been carried out for NGC 6153 (Yuan et al. 2011), will help to reveal the possible astrophysical origins of the H-deficient inclusions in NGC 7009.

6 SUMMARY AND CONCLUSIONS

Nearly two decades after the first analysis of the O II optical recombination spectrum of the bright PN NGC 7009 (LSBC), once again we focus on the rich ORLs of heavy element ions detected in very deep CCD spectrum of the same object. Thanks to much advance in observational techniques, which enables accurate detection of the weak ORLs of heavy element ions, and the steady improvements in atomic data, especially the recombination theories of heavy element ions in the physical conditions of photoionized nebulae, we are now clear that the long-standing dichotomy between nebular plasma diagnostics and abundance determinations using CELs on the one hand and ORLs on the other, are real rather than cause by, e.g., observational uncertainties or errors in atomic data. Unremmiting efforts in nebular research of the past 40 years gradually lead to new understanding of the problems in nebular astrophysics. Various mechanisms (e.g. temperature fluctuations and/or density inhomogeneities, abundance inhomogeneities, non-Maxwell-Boltzmann equilibrium electrons e.g. the $\kappa$-distribution of electron energies) have been proposed to explain the discrepancies in plasma diagnostics and abundance determinations, and debate over these mechanisms are still going on.

In the context of bi-abundance nebular model postulated by Liu et al. (2000), we present a comprehensive and critical analysis of the rich optical recombination spectrum of NGC 7009. Transitions from individual multiplets of heavy element ions, e.g. C II, N II, O II and Ne II, are checked carefully for line blending, and accurate dereddened line fluxes of the most prominent transitions of those ions are obtained through multi-Gaussian profile fitting. In addition to the accurate observations of ORLs, we have finished new calculations of the effective recombination coefficients for the N II recombination spectrum. The new effective recombination coefficients for the nebular O II lines calculated by P. J. Storey (unpublished) help to enlarge our current atomic dataset for nebular recombination line study. Both calculations were carried out in the intermediate coupling scheme, and have taken into account the density-dependence of the relative populations of the ground fine-structure levels of recombining ions (i.e. $\text{N}^{2+}$, $\text{O}^{2+}$) and $\text{O}^{2+}$, $\text{He}^{2+}$, $\text{O}^{3+}$, $\text{Ne}^{2+}$, and $\text{He}^{3+}$, $\text{O}^{2+}$, $\text{Ne}^{2+}$, $\text{Fe}^{2+}$, and $\text{Ni}^{2+}$ for O II). The new effective recombination coefficients of N II and O II are of high quality and make nebular density diagnostics using the ORLs of heavy element ions possible for the first time.

The observed relative intensities of ORLs are compared with theoretical predictions that are based on the new effective recombination coefficients. At a given electron temperature ($T_e = 1000$ K) as yielded by the ORL ratios of N II and O II, the predicted relative intensities of ORLs agree with the observed values. Plasma diagnostics based on the best observed N II and O II ORLs (i.e. the $I(\text{M}3 \lambda 5679)/I(\text{M}39 \lambda 4041)$ ratio of N II and $I(\text{I}1 \lambda 4649)/I(\text{M}48a \lambda 4089)$ ratio of O II) both yield electron temperatures close to 1000 K, which is lower than those derived from the CEL ratios by nearly one order of magnitude. The low temperatures yielded by the N II and O II ORLs indicate that the recombination lines of heavy element ions originate from very cold regions. The electron temperatures derived from
the intensity ratios of the O II high-excitation recombination lines M15 λ4591 and M36 λ4189, which are formed from recombination of excited-state parent (i.e. O^{2+} 2p^2 3D), relative to the O II M1 λ4649 line agree with each other (~3600 K), and is consistent with the fact that very cold (~1000 K) inclusions probably exist in the nebula. The electron temperature (~3000 K) yielded by the C II I(M28.01 λ8794)/(I(M6 λ4267) dielectronic-to-radiative recombination line ratio also agrees with the conjecture of very cold inclusions. The C^{2+}/H^+, N^{2+}/H^+, O^{2+}/H^+ and Ne^{2+}/H^+ ionic abundance ratios derived from ORLs, using the new effective recombination coefficients of N II and O II, are consistently higher than the corresponding values derived from CELs, by a factor of 5. An electron temperature of 1000 K is yielded by the best observed N II and O II recombination line ratios and as a consequence presumably represents the physical condition prevailing in the regions where the heavy element ORLs arise, has been assumed throughout the recombination-line abundance determinations. The results of plasma diagnostics and abundance determinations for NGC 7009 points to the existence of “cold”, metal-rich inclusions in NGC 7009, and is thus consistent with the context of the current spectral analyses, i.e. the bi-abundance nebular model.

Recombination line analysis for NGC 7009 helps to assess the new atomic data. The agreement between the observed and predicted relative intensities of the N II and O II ORLs indicates that the current calculations of the recombination spectra of those two ionic species well represent the physical processes, i.e. radiative and dielectronic recombination, under nebular conditions. Our nebular analysis also shows that the recombination lines of different multiplets, or different J-resolved fine-structure components of a multiplet yield consistent ionic abundances (e.g. N^{2+}/H^+ and O^{2+}/H^+). This is another evidence that the new effective recombination coefficients are reliable. However, the Ne^{2+}/H^+ abundance ratio derived from the total intensity of the 4f–3d transitions is higher, by nearly 0.2 dex, than the average value derived from the multiplets of the 3–3 configuration. That indicates new calculations of the effective recombination coefficients for the Ne II lines are needed.

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APPENDIX A: THE C II OPTICAL RECOMBINATION SPECTRUM

A1 Multiplet 6, 4f 2F0 – 3d 2D
See Section 3.1.1

A2 Multiplet 3, 3d 2D – 3p 2P0
Only the \( \lambda 7231.32 \) (3d 2D 3/2 – 3p 2P0 1/2) line is detected; the other two components (\( \lambda 7236.42 \) (3d 2D 1/2 – 3p 2P0 3/2) and \( \lambda 7237.17 \) (3d 2D 3/2 – 3p 2P0 1/2)) are blended with the [Ar iv] 3p 2P0 1/2 – 3p 2P0 3/2, \( \lambda 7237.40 \) line. The intensity of \( \lambda 7231.32 \) is 0.130, with an uncertainty of less than 5 per cent. Assuming that the intensity ratios of the C II M3 lines are as in the LS coupling, i.e. 1:5:9, we obtain an total intensity ratio of M3 \( \lambda 7235 \) to C II M6 \( \lambda 4267 \) of 0.442, which is lower than the predicted ratio 1.204 in Case B but much higher than the Case A value (0.017). Those theoretical ratios are based on the C II effective recombination coefficients of B06. Since the calculation of B06 for the C II recombination spectrum is valid from 5000 K, here an electron temperature of 10 000 K and a density of 10 000 cm\(^{-3}\) were assumed. In this section, an electron temperature of 5000 or 10 000 K is assumed when we use the effective recombination coefficients of B06.

The calculation of Davey, Storey & Kisielius (2000) is valid from 500 to 20 000 K. Using their C II effective recombination coefficients, we derived a theoretical C II \( \lambda 7235 \) (M3 \( \lambda 7235 \))/I(M6 \( \lambda 4267 \)) ratio of 0.015 in Case A and 0.907 in Case B. An electron temperature of 10 000 K and a density of 10 000 cm\(^{-3}\) are assumed when we use the atomic data of Davey, Storey & Kisielius (2000) in the current paper. Given that the effective recombination coefficients are insensitive to electron density at low temperature, we expect that the intensity ratio of those two C II multiplets at 10 000 cm\(^{-3}\) does not differ much from that at 2000 – 4000 cm\(^{-3}\), a density range yielded by the N II and O II recombination line diagnostics (see Figs 12 and 17). Thus the C II \( \lambda 7235 \)/I(M6 \( \lambda 4267 \)) ratio observed in NGC 7009 (0.442) lies between the two cases.

A3 Multiplet 4, 4s 2S – 3p 2P0
The \( \lambda 3918.98 \) (4s 2S1/2 – 3p 2P0 1/2) component is blending with the O II M17 3p 2P0 1/2 – 3s 2D3/2 \( \lambda 3919.29 \) and N II M17 3d 1P1 – 3p 1P1 \( \lambda 3919.00 \) lines, and is partially blended with the other M4 line \( \lambda 3920.69 \) (4s 2S1/2 – 3p 2P0 3/2). Two Gaussian profiles were used to fit the complex. Using the O II effective recombination coefficients of PJS, we estimated that the O II line contributes 42 per cent to the total intensity of the blend at \( \lambda 3919 \). The contribution of the N II line is negligible according to the calculation of FSL11. The intensities of the \( \lambda 3918.98 \) and \( \lambda 3920.69 \) lines have 0.016 ± 0.003 and 0.034±0.004, respectively. Thus the intensity ratio of C II M4 \( \lambda 3920 \) to the C II M6 \( \lambda 4267 \) multiplet is 0.057, which agrees with the predicted ratio in Case B derived from the coefficients of B06, but higher than the Case A value 0.018. An electron temperature of 10 000 K was assumed. At 5000 K, the lowest temperature of the calculation by B06, this C II ratio is 0.011 in Case A and 0.035 in Case B. The C II M4 multiplet is not presented in the calculation of Davey, Storey & Kisielius (2000).

A4 Multiplet 16.04, 6f 2F0 – 4d 2D
All fine-structure components of this multiplet have an identical laboratory wavelength 6151.43 Å. Single Gaussian profile fitting to this feature gives an intensity of 0.028±0.004. The contribution from the blended N II M3 4p 2D2 – 3s 2P2 \( \lambda 6150.75 \) line is negligible. Thus the intensity ratio of this multiplet to C II M6 \( \lambda 4267 \) is 0.031, agrees with the predicted ratio 0.027 deduced from the coefficients of Davey, Storey & Kisielius (2000). The predicted ratio of the two C II multiplets, based on the coefficients of B06, is 0.037. Both Davey, Storey & Kisielius (2000) and B06 show that the Case A effective recombination coefficient for the \( \lambda 6151 \) multiplet differs from its Case B value by only 2 per cent. For the C II transitions of large \( l (l > 4) \), only Case A coefficients are presented by Davey, Storey & Kisielius (2000). Thus in this section, the theoretical intensity ratios of those C II transitions relative to the M6 \( \lambda 4267 \) multiplet are all presented in Case A.

A5 Multiplet 17.02, 5g 2G – 4f 2F0
This multiplet (\( \lambda 9903 \)) is free of blending, and has an intensity 0.213±0.020. The intensity ratio of M17.02 \( \lambda 9903 \) to the C II M6 \( \lambda 4267 \) multiplet is 0.234, which is slightly lower than the predicted ratio 0.302 deduced from the coefficients of Davey, Storey & Kisielius (2000). At 10 000 K, the effective recombination coefficient for this multiplet given by Davey, Storey & Kisielius (2000) differs from that of B06 by only 3 per cent.

A6 Multiplet 17.04, 6g 2G – 4f 2F0
This multiplet (\( \lambda 6462 \)) has an intensity 0.078±0.004. The intensity ratio of \( \lambda 6462 \) to the C II M6 \( \lambda 4267 \) multiplet is 0.088, which agrees with the predicted value 0.076 (deduced from the atomic data of Davey, Storey & Kisielius (2000)) within errors.

A7 Multiplet 17.06, 7g 2G – 4f 2F0
The C II M17.06 \( \lambda 5342 \) multiplet is partially blending with a very weak feature, which was identified as [Kr iv] \( \lambda 5346.02 \) (Paper I). Two Gaussian profiles were used to fit the complex, and that gives an intensity of 0.037±0.002 for \( \lambda 5342 \). The intensity ratio of \( \lambda 5342 \) to the C II M6 \( \lambda 4267 \) multiplet is 0.043, which agrees well with the predicted ratio 0.045 deduced from the data of Davey, Storey & Kisielius (2000).

A8 Multiplet 28.01, 3d 2F0 – 3p 2D
See Section 3.1.2

APPENDIX B: THE N II OPTICAL RECOMBINATION SPECTRUM

B1 Multiplet 3, 3p 2D – 3s 3P0
See Section 3.2.1

B2 Multiplet 5, 3p 3P – 3s 3P0
The strongest component of the N II M5 multiplet, \( \lambda 4630.54 \) (3p 2P0 – 3s 3P2) is partially blended with the N II M2 3d 2D3/2 – 3p 2P0 3/2 \( \lambda 4634.14 \) fluorescence line (Fig 32). Gaussian profile fitting yields an intensity of 0.067±0.013 for \( \lambda 4630.54 \). The intensity ratio of the \( \lambda 4630.54 \) line and the N II M3 \( \lambda 5679.56 \) line is 0.512, which agrees well with the predicted ratio 0.491. Another M5 line \( \lambda 4601.48 \) (3p 2P2 – 3s 3P0) is blended with the O II M92b
I. The other four M5 lines all suffer from serious line blending. The λ4607.16 (3p\(^3\)P\(_1\) – 3s\(^3\)P\(_0\)) line is blended with the [Fe III] λ4607.03 line, whose contribution to the total intensity is uncertain. The blend of λ4607 is again partially blended with a much stronger feature centered at λ4609, which is a blend of O II M92a 4fF[3]\(^5/2\) – 3d\(^2\)D\(_{3/2}\) λ4609.44 and O II M92c 4fF[2]\(^5/2\) – 3d\(^2\)D\(_{3/2}\) λ4610.20 (Fig. B1). (2) The λ4613.87 (3p\(^3\)P\(_1\) – 3s\(^3\)P\(_1\)) line is blended with at least three lines, O II M92b 4fF[3]\(^5/2\) – 3d\(^2\)D\(_{3/2}\) λ4613.14, O II M92b 4fF[3]\(^5/2\) – 3d\(^2\)D\(_{5/2}\) λ4613.68, and Ne II M64b 4fF[2]\(^3/2\) – 3d\(^4\)P\(_{1/2}\) λ4612.93. (3) The λ4621.39 (3p\(^3\)P\(_0\) – 3s\(^3\)P\(_0\)) line is blended with at least two lines, O II M92c 4fF[2]\(^5/2\) – 3d\(^2\)D\(_{5/2}\) λ4621.27 and O II M92c 4fF[2]\(^3/2\) – 3d\(^2\)D\(_{5/2}\) λ4622.14. (4) The λ4643.09 (3p\(^3\)P\(_2\) – 3s\(^3\)P\(_2\)) line is blended with the N III M2 λ4640.64 and 4641.85 fluorescence lines (Fig. C3). Accurate measurements of these four N II M5 lines are difficult.

B3 Multiplet 12, 3p\(^1\)D – 3s\(^1\)P\(^0\)

This singlet (λ3994.99) is shown in Fig. C3. Gaussian profile fitting yields an intensity of 0.033±0.004. The intensity ratio of the λ3994.99 line to the N III M5 λ5679.56 line is 0.257, which is higher than the theoretical prediction 0.123.

B4 Multiplet 19, 3d\(^3\)F\(^\circ\) – 3p\(^3\)D

The M19 λ5004 lines are blended with or seriously affected by the saturated [O III] λ5007 line.

B5 Multiplet 20, 3d\(^3\)D\(^\circ\) – 3p\(^3\)D

The M20 λ4803.29 (3d\(^3\)D\(_{5/2}\) – 3p\(^3\)D\(_{3/2}\)) line is blended with the C II M17.08 8g\(^2\)G – 4f\(^2\)F\(_{2}\) λ4802.70 line, which contributes ~50 per cent to the total intensity of the blend at λ4803, as estimated from the C II effective recombination coefficients of Davey, Storey & Kisielius (2000). The intensity of the λ4803.29 line is 0.032±0.016. The intensity ratio of λ4803.29 and the N II M3 λ5679.56 line is 0.248, which agrees with the predicted value 0.282 within errors. The other M20 lines are not detected.

B6 Multiplet 28, 3d\(^3\)D\(^\circ\) – 3p\(^3\)P

See Section 3.2.3.2.

B7 Multiplet 29, 3d\(^3\)P\(^\circ\) – 3p\(^3\)P

Accurate measurements of this multiplet are difficult. Only the λ5454.22 (3d\(^3\)P\(_2\) – 3p\(^3\)P\(_2\)) and λ5480.06 (3d\(^3\)P\(_2\) – 3p\(^3\)P\(_1\)) lines are detected. The former line is blended with the S II M6 4p\(^4\)D\(_{5/2}\) – 4s\(^4\)P\(_{1/2}\) λ5453.83 line and the latter one is blended with the λ5478.10 (3d\(^3\)P\(_2\) – 3p\(^3\)P\(_1\)) line of the same multiplet.

B8 4f – 3d transitions

B8.1 Multiplet 39a, 4fG[7/2] – 3d\(^3\)F\(^\circ\):

See Section 3.2.3.1.

B8.2 Multiplet 39b, 4fG[9/2] – 3d\(^3\)F\(^\circ\):

See Section 3.2.3.3.

B8.3 Multiplet 38a, 4fF[5/2] – 3d\(^3\)F\(^\circ\):

Accurate measurements of this multiplet are difficult. The λ4076.91 (4fF[5/2] – 3d\(^3\)F\(_2\)) and λ4077.40 (4fF[5/2] – 3d\(^3\)F\(_2\)) lines are blended with the [S II] λ4076.35 (3p\(^3\)2P\(_{1/2}\) – 3p\(^3\)2S\(_{1/2}\)) line. The λ4086.83 (4fF[5/2] – 3d\(^3\)F\(_2\)) and λ4087.30 (4fF[5/2] – 3d\(^3\)F\(_2\)) lines are blended with the O II M48c 4fG[3]\(^5/2\) – 3d\(^3\)F\(_3/2\) λ4087.15 line. The other M38a line λ4100.97 (4fF[5/2] – 3d\(^3\)F\(_2\)) line is blended with the H I λ4101 line.

B8.4 Multiplet 38b, 4fF[7/2] – 3d\(^3\)F\(^\circ\):

Accurate measurements of the M38b lines are difficult. The λ4073.04 (4fF[7/2] – 3d\(^3\)F\(_2\)) line is blended with the much stronger O II M10 λ4072.16 line. The λ4095.90 (4fF[7/2] – 3d\(^3\)F\(_2\)) line and λ4096.58 (4fF[7/2] – 3d\(^3\)F\(_2\)) lines are blended with the N III M1 3p\(^3\)P\(_{3/2}\) – 3s\(^3\)S\(_{1/2}\) λ4097.33 fluorescence line. The other lines are not observed.

B8.5 Multiplet 43a, 4fF[5/2] – 3d\(^3\)D\(^\circ\):

The intensity of the λ4176.16 (4fF[5/2] – 3d\(^3\)D\(_{5/2}\)) line is 0.028±0.004, which includes the contribution from the N II M43a 4fF[5/2] – 3d\(^3\)D\(_{5/2}\) λ4175.66 line (about 20 per cent).
B8.6 Multiplet 43b, 4f F[7/2] – 3d D°:

The λ4171.61 (4f F[7/2]) line is partially blended with the O II λ4169.22 line. The line at 4171.61 has an intensity of 0.023 ± 0.003, which agrees well with the predicted value (Table 3).

B8.7 Multiplet 48a, 4f F[5/2] – 3d D°:

The λ4241.78 (4f F[5/2]) line coincides in wavelength with the N II λ4241.78 line. The line at 4241.78 (4f F[5/2]) line is blended with the N II λ4241.78 line (Fig. 31). Gaussian profile fitting yields an intensity of 0.055 ± 0.011 for the λ4241.78 line, which includes the contribution from the N II λ4241.78 line (~30 per cent). The λ4241.79 line intensity is 24 per cent higher than what is predicted (Table 2). Measurements of the other two M48a lines, λ4246.71 (4f F[5/2]) and λ4247.20 (4f F[5/2]) are difficult.

B8.8 Multiplet 48b, 4f F[7/2] – 3d D°:

The λ4241.78 (4f F[7/2]) line coincides in wavelength with the N II M48b 4f F[7/2] line. The contribution from the Ne II M48b 4f F[7/2] line is blended with the N II M48b 4f F[7/2] line (Fig. 31). Gaussian profile fitting yields an intensity of 0.003, which agrees well with the predicted value (Table 3). The contribution from the Ne II M48b 4f F[7/2] line is blended with the N II M48b 4f F[7/2] line (Fig. 31). The other two M48b lines are not observed.

B8.9 Multiplet 49a, 4f G[7/2] – 3d D°:

The λ4195.97 (4f G[7/2]) line is blended with the N II M6 3p 2 3d D° line, which contributes 8 per cent to the total intensity. The other two M49a lines, λ4199.98 (4f G[7/2]) and λ4201.35 (4f G[7/2]) are blended with the much stronger He II λ4199.83 line.

B8.10 Multiplet 49b, 4f G[9/2] – 3d D°:

The λ4181.10 (4f G[9/2]) line is blended with the N II M50a λ4178.86 (4f G[9/2]) and M50a λ4179.67 (4f G[9/2]) lines, whose intensity contribution is less than 40 per cent. [Fe v] λ4180.60 is probably also blended (Fig. 31), but its intensity is uncertain.

B8.11 Multiplet 50a, 4f D[5/2] – 3d D°:

The λ4169.38 (4f D[5/2]) line is blended with the O II M19 λ4169.22 line, which contributes ~70 per cent to the total intensity. The line at 4169.38 (4f D[5/2]) and λ4179.67 (4f D[5/2]) lines are blended with the N II M49b 4f 2 3d D° line, which contributes more than 60 per cent to total intensity. The other two M50a lines λ4173.57 (4f D[5/2]) and λ4174.38 (4f D[5/2]) are not observed.

B8.12 Multiplet 50b, 4f D[3/2] – 3d D°:

The λ4156.39 (4f D[3/2]) and λ4157.01 (4f D[3/2]) lines are blended with the O II M19 λ4156.53 (3d 4p 3f/2 – 3p 3P) line, which contributes ~75 per cent to the total intensity of the blend at λ4157. The other M50b lines are not observed.

B8.13 Multiplet 55a, 4f D[5/2] – 3d P°:

The λ4442.02 (4f D[5/2]) line is blended with the Ne II M60b 4f 1 3d 2P° line. The line at 4442.02 (4f D[5/2]) line is partially blended with the Ne II M60b 4f 1 3d 2P° line (Table 3). The other two M55a lines are not observed.

B8.14 Multiplet 58a, 4f G[7/2] – 3d F°:

The λ4552.53 (4f G[7/2]) line is blended with the N II M55d 4f 2 3d 2F° line, which contributes ~12 per cent to the total intensity. The intensity of the λ4552.53 line is 0.049 ± 0.007, which agrees with the theoretical prediction (Table 3).

B8.15 Multiplet 58b, 4f G[9/2] – 3d F°:

The λ4694.64 (4f G[9/2]) line is partially blended with the weak O II M1 λ4694.64 (3p 4d 2D° → 3s 2P°) line. The λ4694.64 line has an intensity that is 50 per cent higher than what is predicted (Table 3). The intensity of this line is unreliable due to the weakness.

APPENDIX C: THE O II OPTICAL RECOMBINATION SPECTRUM

C1 Multiplet 1, 3p 3D° – 3s 4P

See Section 3.3.1

C2 Multiplet 2, 3p 3P° – 3s 4P

The observed and predicted relative intensities of O II M2 lines with most reliable measurements are presented in Table C1. The features of these lines as well as results of Gaussian profile fitting are shown in Fig. C1. A single Gaussian profile fit to the feature of λ4349.43 gives an intensity of 0.195 ± 0.029. The intensity ratio of the λ4349.43 line to the O II M1 λ4349.13 line is 0.292, slightly higher than the predicted value 0.274. The intensity of the λ4349.43 line could be overestimated due to the saturated H I λ4340 line (Fig. C1). The O II
Table C1. Comparison of the observed and predicted relative intensities of the O II M2 lines detected in the spectrum of NGC 7009. Only lines with the most reliable measurements are presented. Here an electron temperature of 1000 K and a density of 4300 cm$^{-3}$ are assumed for the theoretical intensities $I_{\text{IC}}$.

| Line               | $J_2 - J_1$ | $I_{\text{LS}}$ | $I_{\text{IC}}$ | $I_{\text{obs}}$ | $I_{\text{obs}}/I_{\text{LS}}$ | $I_{\text{obs}}/I_{\text{IC}}$ |
|--------------------|-------------|-----------------|-----------------|-----------------|-------------------------------|-------------------------------|
| $\lambda 3417.14a$ | 3/2–1/2     | 0.397           | 0.553           | 0.551           | 1.388                         | 0.996                         |
| $\lambda 3419.63$  | 5/2–3/2     | 0.429           | 0.365           | 0.322           | 0.750                         | 0.882                         |
| $\lambda 3425.76$  | 1/2–1/2     | 0.079           | 0.119           | 0.147           | 1.861                         | 1.235                         |
| $\lambda 3439.43$  | 3/2–5/2     | 1.000           | 1.000           | 1.000           | 1.000                         |                               |
| $\lambda 3466.89$  | 3/2–5/2     | 0.429           | 0.578           | 0.640           | 1.492                         | 1.107                         |


$^a$ Corrected for the contribution from O II M53a 4f D[3]$_{5/2}^0 – 3d$ $^4P_{3/2}^0$ $\lambda 4317.70$, which is about 15 per cent.

M16 3p$^2$ D$^0_{3/2} – 3s^2$ D$^0_{3/2}$ $\lambda 4351.26$ and 3p$^2$ D$^0_{5/2} – 3s^2$ D$^0_{5/2}$ $\lambda 4351.45$ lines are also blended with the A4349.43 line, but their contributions are probably insignificant.

Another three M2 lines, $\lambda\lambda 3417.14, 4319.63$ and $4325.76$, are less affected by the saturated H I $\lambda 4340$ line. $\lambda 4317.14$ blends with O II M53a 4f D[3]$_{5/2}^0 – 3d$ $^4P_{3/2}^0$ $\lambda 4317.70$, and this blended feature is partially resolved from $\lambda 4319.63$ (Fig. C1). Three O II M63c and two O II M78b lines are blended with the left wing of $\lambda 4317.14$, and we treated these five O II lines as a single component since they are all weak and have close wavelengths. Three Gaussian profiles were used to fit the complex. The result intensities of $\lambda\lambda 4317.14$ and $\lambda 4319.63$ both agree better with the newly predicted values (Table C1). The intensity contribution of the blended O II M53a $\lambda 4317.70$ line was estimated from the most recent effective recombination coefficients and was subtracted. $\lambda 4325.76$ is free of blend, and its fitted intensity (with an uncertainty of 10–20 per cent) also agrees better with the intermediate coupling prediction. The main source of uncertainties in the $\lambda 4325.76$ intensity is due to the saturated H I $\lambda 4340$ line, while those in $\lambda\lambda 4317.14$ and 4319.63 are mainly due to blending.

Measurements of the remaining three M2 lines, 4336.86, 4345.56 and 4366.89, are difficult: the former two are blended with the saturated H I $\lambda 4340$ line, and the latter one is affected by [O III] $\lambda 4363$ (Fig. C1). The measurement of $\lambda 4366.89$ seems to agree with the predicted intensity (Table C1). The fitting error of the $\lambda 4363$ intensity is 12 per cent, but the actual uncertainty could be even larger. The contributions of O III M39a 5g G[4]$_{3/2}^1 – 4f$ F[3]$_1$, A4366.99 and O II M75a 4f D[3]$_{7/2}^0 – 3d$ $^2F_{5/2}^0$ $\lambda 4366.53$ were assumed to be negligible.

C3 Multiplet 5, 3p$^2$ D$^0 – 3s^2$ D$^0$

Accurate measurements of the three M5 lines are difficult. $\lambda 4414.90$ (3p$^2$ D$^0_{3/2} – 3s^2$ D$^0_{3/2}$) is partially resolved from another M5 line $\lambda 4416.97$ (3p$^2$ D$^0_{5/2} – 3s^2$ D$^0_{5/2}$), as is shown in Fig. C3. $\lambda 4414.90$ coincides in wavelength O III M46a 5g H[5]$_5^0 – 4f$ G[4]$_4^0$ (Fig. C2). $\lambda 4414.85$, whose contribution to the total flux of the blend at $\lambda 4415$ was estimated from the effective recombination coefficients of Kisielius & Storey (1999), $\lambda 4414.90$ is also affected by at least three Ne II lines from the 4f–3d configuration, and for most these Ne II lines, the effective recombination coefficients are not available. The $\lambda 4416.97$ line is blended with N II M55b 4f D[3]$_{3/2}^0 – 3d$ $^2P_{3/2}^0$ $\lambda 4417.07$, N II M55b 4f D[3]$_{1/2}^0 – 3d$ $^3P_{3/2}^0$ $\lambda 4417.82$, Ne II M61d 4f $^2[2]_{7/2}^0 – 3d$ $^2D_{5/2}^0$ $\lambda 4416.76$ and Ne II M61d 4f $^2[2]_{9/2}^0 – 3d$ $^2D_{5/2}^0$ $\lambda 4416.77$. The contributions of the two Ne II lines to the total flux of the blend at $\lambda 4417$ were estimated from the new Ne II effective recombination coefficients, while those of the two Ne II lines could be negligible. The other line $\lambda 4452.37$ (3p$^2$ D$^0_{3/2} – 3s^2$ D$^0_{3/2}$) is blended with O III M49a 5g F[4]$_4^1 – 4f$ D[3]$_1^0$. $\lambda 4454.03$ and [Fe II] $\lambda 4452.10$ (Fig. C1). The contribution from the O III $\lambda 4454.03$ line was estimated from the effective recombination coefficients of Kisielius & Storey (1999), but that of the [Fe II] line is uncertain.

C4 Multiplet 6, 3p$^2$ P$^0 – 3s^2$ P

The strongest M6 line $\lambda 3973.26$ (3p$^2$ P$^0_{1/2} – 3s^2$ P$^0_{1/2}$) is blended with the saturated H I $\lambda 3970$ line (Fig. C1). $\lambda 3945.04$ (3p$^2$ P$^0_{1/2} – 3s^2$ P$^0_{1/2}$) and 3954.36 (3p$^2$ P$^0_{3/2} – 3s^2$ P$^0_{3/2}$) are free of blend, but are affected by the saturated [Ne III] $\lambda 3967$ line (Fig. C2). The measured intensities of $\lambda 3945.04$ and 3954.36 are higher than the predicted values (in IC) by 45 and 83 per cent, respectively. Their measurements are of large uncertainties ($\gtrsim 30$ per cent).

Accurate measurements of the other M6 line $\lambda 3982.71$ (3p$^2$ P$^0_{1/2} – 3s^2$ P$^0_{1/2}$) are difficult due to the saturated H I $\lambda 3970$ line. It is also partially resolved from a weak feature that was identified as O IV M43 3d$^2$ D$^1_1 – 4d$ $^3P^0_1$ $\lambda 3985.55$. If this identification is correct, $\lambda 3982.71$ should blend with another O IV M43 line $\lambda 3983.73$. Unfortunately no effective recombination coefficients for O IV M43 are available for us to confirm the conjecture.

C5 Multiplet 10, 3d$^2$ F$^0 – 3p^2$ D$^0$

See Section 3.3.2.
Figure C2. Spectrum of NGC 7009 from 4380 to 4445 Å showing the O II M5 lines and other emission features. Many Ne II lines of the 4f – 3d configuration are also detected. The inset shows the spectral region 4449–4460 Å, where the O II M5 λ4452.37 line is located. The continuous curve is the sum of Gaussian profile fits. Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.

Figure C3. Spectrum of NGC 7009 from 3940 to 4006 Å showing the O II M6 lines and other emission features. The continuous curve is the sum of Gaussian profile fits. The profile of the O II M17 λ3961.57 line, whose accurate measurement is impossible due to the saturated [Ne III] λ3967 line, is only an estimate. The inset shows the profiles of the [Ne III] λ3967 and H I λ3970 lines, both of which are saturated but assumed to be Gaussian. Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.

C6 Multiplet 12, 3d 4D – 3p 4Do

The features of O II M12 lines are shown in Fig. C4. λ3882.19 (3d 4D(7/2) – 3p 4Do(7/2)) is close to H I λ3889, which may affect its measurement. A single-Gaussian profile fit yields an intensity value of 0.084 ± 0.005 for the λ3882.19 line. The intensity of the λ3882 line, which is more than 50 per cent higher than the predicted value, is overestimated due to the saturated [Ne III] λ3868. The other M12 lines are either much affected by the saturated [Ne III] line or are not observed due to weakness.

C7 Multiplet 19, 3d 4P – 3p 4Po

The O II M19 lines are observed in Fig. C5 and the observed and predicted relative intensities are presented in Table C2. λ4169.22 blends with He I M52 6s 1S0 – 2p 1P0 λ4168.97 and N II M50a 4f 2[3/2] – 3d 3Do λ4169.38. The He I λ4168.97 line contributes 30 per cent to the total intensity, which was estimated from the theoretical He I line intensities of Benjamin, Skillman & Smits [1999] while the N II line was assumed to be negligible. The resultant intensity of the λ4169.22 line is 0.082±0.008. The predicted intensity of this line is 0.090, which is derived from the most recent O II effective recombination coefficients (the measured intensity of O II M1 λ4649.13 was used). The measured intensity ratio of λ4153.30 to λ4169.22 is much higher than the predicted value. No explanation can be given for this large discrepancy except there is an unknown blend.

Although the measured intensity of λ4156.53 is also higher than theory, its actual intensity should agree well with the predicted value, given the fact that it is blended with N II M50b 4f D[3/2] – 3d 3Do λ4156.39 and N II M50b 4f D[3/2] – 3d 3Do λ4157.01, which contribute about 25 per cent to the total flux of the blend at λ4156. The measurements of another two M19 lines λλ4129.32 and 4132.80 both agree with the newly predicted intensities. These
two lines blend with three Si II intensity.

I because it blends with He λ = 4121.46 lines are treated as a single component, and the two O II λ4120 lines are treated as another one) were used to fit the feature. The intensity ratio of the three O II M20 lines ($\lambda$λ4120.28 + $\lambda$λ4120.54)/$\lambda$λ4119.22 were assumed to be the predicted value (0.278), and the intensity of the He I λ4120.99 line was estimated from the atomic data of Benjain, Skillman & Smits (1999). The fitting results agree well with the observed spectrum, as is shown in Fig. C5. Accurate measurements of $\lambda$λ4110.79 and λ4120.99 line was estimated from the atomic data of Benjain, Skillman & Smits (1999). The fitting results agree well with the observed spectrum, as is shown in Fig. C5. Accurate measurements of $\lambda$λ4110.79 and λ4120.99 is 0.779 : 1, and differs from the newly predicted ratio in the intermediate coupling scheme is 3.49 : 2.09 : 1.0.

Table C2. Same as Table C1 but for the O II M1 lines detected in the spectrum of NGC 7009.

| Line       | $J_2 - J_1$ | $I_{LS}$  | $I_{IC}$ | $I_{Obs}$  | $I_{LS}/I_{IC}$ | $I_{Obs}/I_{IC}$ |
|------------|-------------|-----------|----------|------------|-----------------|-----------------|
| $\lambda$4129.32 | 1/2–3/2      | 0.397     | 0.335    | 0.389      | 0.981           | 1.162           |
| $\lambda$4132.80$^b$ | 3/2–1/2      | 0.397     | 0.768    | 0.918      | 2.311           | 1.195           |
| $\lambda$4140.70$^b$ | 3/2–3/2      | 0.127     | 0.028    | 0.060      | 0.316           | 1.415           |
| $\lambda$4153.30 | 5/2–3/2      | 0.429     | 0.909    | 1.336      | 3.180           | 1.500           |
| $\lambda$4165.53$^c$ | 3/2–5/2      | 0.429     | 0.474    | 0.688      | 2.229           | 2.017           |
| $\lambda$4169.22 | 5/2–5/2      | 1.000     | 1.000    | 1.000      | 1.000           | 1.000           |

$^a$ Probably including unknown blend.

$^b$ Overestimated due to He I M53 He I M53 6d $^1D_2$ – $^2P_1^o$ $\lambda$4143.76, which is stronger by early two orders of magnitude.

$^c$ Neglecting N II M50b 4f D[3/2]$_2$ – 3d $^3D_1^o$ $\lambda$4156.39 and N II M50b 4f D[2]$_1$ – 3d $^3D_1^o$ $\lambda$4157.01, which contribute about 4 per cent to the total intensity.

The three M28 lines are shown in Fig. C5. The other M20 lines are not observed.

C9 Multiplet 25, 3d $^2F$ – 3p $^2D^o$

Fig. C5 shows two O II M25 lines $\lambda$4699.22 (5/2 – 3/2) and 4705.35 (7/2 – 5/2). $\lambda$4699.22 is partially resolved from [Fe III] $\lambda$4701.53, and it also blends with O II M40 3d $^2F_{7/2}$ – 3p $^2D_{1/2}^o$ $\lambda$4699.00 which contributes about 30 per cent to the total flux of the blend at $\lambda$4699. $\lambda$4705.35 is affected by the much stronger [Ar IV] $\lambda$4711. The measured intensity ratio $\lambda$4699.22 : $\lambda$4705.35 is 0.779 : 1.00, which agrees with the newly predicted ratio 0.765 : 1, and differs from the value in pure LS coupling, i.e. 0.7 : 1.0. Here the contribution of the blended O II M40 $\lambda$4699.00 line has been subtracted.

C10 Multiplet 28, 3d $^3P$ – 3p $^4S^o$

The three M28 lines are shown in Fig. C7 $\lambda$4924.53 (5/2 – 3/2) is partially resolved from He I M48 4d $^1D_2$ – 2p $^1P_1^o$ $\lambda$4921.93, and the other two $\lambda$4890.86 (1/2 – 3/2) and $\lambda$4906.83 (3/2 – 3/2) are free of blend. The observed relative intensity of the three lines is 1.0 : 3.52 : 5.73, which deviates from the pure LS coupling ratio 1 : 2 : 3, and also differs from the newly predicted ratio in the intermediate coupling scheme is 3.49 : 2.09 : 1.0.
C11 4f – 3d transitions

C11.1 Multiplet 48a, 4f G[5] – 3d 4F

See Section 3.3.3

C11.2 Multiplet 48b, 4f G[4f] – 3d 4F

\(\lambda 4083.90\) (4f G[4f]2/1 – 3d 4F5/2) blends with O II M10 3d 4F5/2 – 3p 4D5/2 λ4085.11, which contributes more than 50 per cent to the total intensity. The other lines are not observed.

C11.3 Multiplet 48c, 4f G[3f] – 3d 4F

\(\lambda 4087.15\) (4f G[3f]5/1 – 3d 4F3/2) lies between the features of O II M48a λ4089.29 and O II M10 λ4085.11. Its intensity given by the fit agrees with the predicted value (Table 6), while the measurement uncertainty is large due to weakness. Here the contributions of the blended N II M38a 4f F[5/2] – 3d 4F0 4086.83 and N II M38a 4f F[5/2] – 3d 4F2 4087.30 lines were assumed to be negligible. The other M48c lines are not observed.

C11.4 Multiplet 50a, 4f F[4f] – 3d 4F

Only \(\lambda 4062.94\) (4f F[4f]3/2 – 3d 4F4/2) is observed, but its measurements are quite uncertain due to weakness.

C11.5 Multiplet 50b, 4f F[3f] – 3d 4F

Only \(\lambda 4048.21\) (4f F[3f]5/2 – 3d 4F7/2) is observed. \(\lambda 4048.21\) blends with \(\lambda 4047.80\) (4f F[3f]1/2 – 3d 4F7/2), whose intensity was assumed to be negligible. The measured intensity of \(\lambda 4048.21\) is slightly higher than the current prediction, but its uncertainty could be large due to weakness. \(\lambda 4035.07\) (4f F[3f]5/2 – 3d 4F5/2) and \(\lambda 4035.46\) (4f F[3f]1/2 – 3d 4F5/2) blend with N II M39a 4f G[7/2] \(\lambda 4035.08\) (Section 3.2.3.1), which contributes more than 10 per cent to the total intensity of the blend at \(\lambda 4035\).

C11.6 Multiplet 50c, 4f F[2f] – 3d 4F

\(\lambda 4032.48\) (4f F[2f]5/2 – 3d 4F3/2) and \(\lambda 4033.16\) (4f F[2f]3/2 – 3d 4F5/2) blend with O II M50a 4f F[4f]2/1 – 3d 4F3/2 \(\lambda 4032.25\), and measurements are difficult. \(\lambda 4041.28\) (4f F[2f]5/2 – 3d 4F3/2) and \(\lambda 4041.95\) (4f F[2f]3/2 – 3d 4F3/2) blended with N II M39b 4f G[9/2]5/2 – 3d 4F2 4041.31, which contributes more than 90 per cent to the total intensity of the blend at \(\lambda 4041\). The other line \(\lambda 4054.08\) (4f F[2f]7/2 – 3d 4F7/2) is not observed.

C11.7 Multiplet 53a, 4f D[3f] – 3d 4P

Only \(\lambda 4303.82\) (4f D[3f]5/2 – 3d 4P5/2) is observed, which is shown in Fig. 54. It blends with another M33a line \(\lambda 4304.08\) (4f D[3f]5/2 – 3d 4P5/2) and O II M65a 4f G[5/2] – 3d 4D7/2 \(\lambda 4303.61\), whose total intensity contribution to the blend amounts to about 15 per cent. The resultant measured intensity of \(\lambda 4303.83\) agrees well with the most recent predicted value (Table 6). The other line \(\lambda 4317.70\) (4f D[3f]5/2 – 3d 4P3/2) is blended with O II M2 3p 4P3/2 – 3s 4P1/2 \(\lambda 4317.14\) which is 5 – 6 times stronger.

C11.8 Multiplet 53b, 4f D[2f] – 3d 4P

\(\lambda 4294.78\) (4f D[2f]5/2 – 3d 4P3/2) coincides in wavelength with \(\lambda 4294.92\) (4f D[2f]5/2 – 3d 4P3/2) which contributes about 12 per cent to the total intensity of the feature at \(\lambda 4295\). The total intensity of the two \(\lambda 4295\) lines agrees with the predicted new value. Another line \(\lambda 4307.23\) (4f D[2f]5/2 – 3d 4P1/2) is too weak to perform accurate measurements although it is observed (Fig. 54). The other M33b lines are not observed.

C11.9 Multiplet 53c, 4f D[1f] – 3d 4P

\(\lambda 4288.82\) is a blend of two M53c lines (4f D[1f]5/2 – 3d 4P1/2) and 4f D[1f]3/2 – 3d 4P1/2. The measured intensity of \(\lambda 4288.82\) agrees with the newly predicted value, but is much lower than the one predicted based on the previous calculations (Table 6). The other lines are not observed.

C11.10 Multiplet 55, 4f G[3f] – 3d 4P

\(\lambda 4291.25\) (4f G[3f]5/2 – 3d 4P5/2) blends with O II M78c 4f F[2f]2/1 – 3d 4P5/2 \(\lambda 4292.21\), which contributes about 18 per cent to the total intensity. The contribution from O II \(\lambda 4291.86\) (4f G[3f]5/2 – 3d 4P5/2) was assumed to be negligible. The other line \(\lambda 4305.39\) is not observed.

C11.11 Multiplet 63a, 4f D[3f] – 3d 4D

Measurements of \(\lambda 4357.25\) (4f D[3f]5/2 – 3d 4D3/2) and 4f D[3f]3/2 – 3d 4D3/2 are affected by O II H \(\lambda 4363\), as well as the saturated H I \(\lambda 4340\) line (Fig. 57). The other M63a lines are not observed.
C11.12 **Multiplet 65a, 4f G[5] → 3d 4D**

Λ4303.61 (4f G[5] 7/2 → 3d 4D7/2) blends with O II M53a 4f D[3] 7/2 → 3d 4F5/2 λ4303.82, which is about 7–8 times stronger, and O II M53a 4f D[3] 7/2 → 3d 4F5/2 λ4304.08, which contributes about 3 per cent to the total intensity of the blend at Λ4304.

C11.13 **Multiplet 65b, 4f G[4] → 3d 4D**

Measurements of this multiplet are difficult due to the saturated H I Λ4340 line.

C11.14 **Multiplet 65c, 4f G[3] → 3d 4D**

Measurements of this multiplet are difficult due to the saturated H I Λ4340 line.

C11.15 **Multiplet 67a, 4f F[4] → 3d 4D**

In Fig. 34, the three lines of this multiplet blend with several O II lines. Using the effective recombination coefficients of PJS, we estimated that the Λ4275.55 contribution about 35 per cent to the total intensity of the feature at Λ4275, which is formed by more than 10 O II ORLs from the 4f → 3d configuration. The other two M67a lines contribute very little.

C11.16 **Multiplet 67b, 4f F[3] → 3d 4D**

Measurements of this multiplet are difficult due to the reason given in Section C11.15. Using the new O II effective recombination coefficients, we estimated that the Λ4276.75 (4f F[3] 5/2 → 3d 4D5/2) line, which is the strongest in O II M67a, contributes about 17 per cent to the total intensity of the feature at Λ4275.

C11.17 **Multiplet 67c, 4f F[2] → 3d 4D**

Measurements of this multiplet are difficult due to blending.

C11.18 **Multiplet 76b, 4f G[4] → 3d 4F**

Λ4371.62 (4f G[4] 5/2 → 3d 4F5/2) blends with Λ4371.24 (4f G[4] 3/2 → 3d 4F3/2) whose intensity contribution was assumed to be negligible. The measurement of Λ4371.62 agrees with the predicted intensity, but this intensity could be of large uncertainty due to weakness as well as the strength of [O III] Λ4363.

C11.19 **Multiplet 76c, 4f G[3] → 3d 4F**

Only Λ4353.59 (4f G[3] 5/2 → 3d 4F5/2) is observed, which is shown in Fig. C1. Its fitted intensity is higher than the newly predicted value. This measurement is overestimated due to the saturated Hγ. The intensity contribution from Λ4354.18 (4f G[3] 3/2 → 3d 4F3/2) was assumed to be negligible (only about 6 per cent). The other two M76c lines Λ4384.70 and 4385.32 are blended with He I M51 5d 1D2 → 2p 1P0 Λ4387.93 (Fig. C2).

C11.20 **Multiplet 78a, 4f F[4] → 3d 4F**

The measured intensity of Λ4313.44 (4f F[4] 7/2 → 3d 4F7/2) agrees with the most recent prediction, but is probably of large uncertainty due to weakness. Here the contribution from Λ4312.11 (4f F[4] 5/2 → 3d 4F5/2), which is about 32 per cent, has been corrected for. The other line Λ4282.01 (4f F[4] 3/2 → 3d 4F5/2) is not observed.

C11.21 **Multiplet 78b, 4f F[3] → 3d 4F**

Λ4285.69 (4f F[3] 5/2 → 3d 4F5/2) blends with Λ4285.21 (4f F[3] 3/2 → 3d 4F3/2) which contributes only about 1 per cent to the total intensity. It is also partially resolved from a feature at Λ4283, which is a blend of O II M67a 4f F[2] 7/2 → 3d 4D5/2 Λ4282.96 and Ne II M57c 4f F[1] 3/2 → 3d 4F7/2 Λ4283.73. The fitted intensity of Λ4285.69 agrees with predicted value. The other lines are not observed.

C11.22 **Multiplet 80a, 4f D[3] → 3d 4P**

In Fig. 35, Λ4491.23 (4f D[3] 7/2 → 3d 4P3/2) is partially resolved from a feature at Λ4488 which is a blend of the O III M104 4f D[2] 7/2 → 3d 4P1/2 lines (Λ4487.72 3/2 → 1/2, and Λ4488.20 5/2 → 3/2 and 2/2 → 3/2) and O II M86b 4f D[2] 5/2 → 3d 4P1/2 Λ4489.49. Four Gaussian profiles were used to fit the complex. Since the effective recombination coefficients for the O III M104 lines are not available, we assumed that their relative intensities are as in pure LS coupling, i.e. 5 : 9 : 1. The resultant intensity of Λ4491.23 agrees well with the most recent theoretical value.

C11.23 **Multiplet 80b, 4f D[2] → 3d 4P**

Λ4489.49 (4f D[2] 5/2 → 3d 4P1/2) blends with O III M104 Λ4488.20 and O II M86a Λ4491.23. The fitted intensity of Λ4489.49 agrees with the newly predicted value quite well. Details of line fits are given in Section C11.22. Accurate measurements of Λ4466.41, 59 are difficult due to He I Λ4471, which is stronger by more than two orders of magnitude.

C11.24 **Multiplet 88, 4f G[3] → 3d 4P**

Λ4477.90 (4f G[3] 5/2 → 3d 4P3/2) is observed in Fig. 35. It is blended with O III M45a 5g H[11/2] 5/2 → 4f G[9/2] 3/2 Λ4477.91, which contributes about 1–2 per cent to the total intensity and thus is negligible (an estimate based on the effective recombination coefficients for the O III 5g → 4f recombination spectrum given by Kisielius & Storey [1999]). The measured intensity of Λ4477.90 agrees well with the newly predicted value (Table 6).

C11.25 **Multiplet 92a, 4f F[4] → 3d 4D**

Λ4609.44 (4f F[4] 7/2 → 3d 4D3/2) is observed in Fig. 35. It blends with O III M92c 4f F[2] 3/2 → 3d 4D3/2 Λ4610.20 and O II M92c 4f F[2] 3/2 → 3d 4D3/2 Λ4611.07, which contribute about 20 per cent to the total intensity. After subtracting the blend, the resultant intensity of Λ4609.44 agrees well with the new prediction (Table 6).
C11.26 Multiplet 92b, 4f F[3]o – 3d 2D

λ4602.13 (4f F[3]o 2/2 – 3d 2D3/2) is observed in Fig. B1. It blends with N II M5 3p 3P2 – 3s 3P1 λ4601.48, which contributes about 26 per cent to the total intensity. Subtraction of the blend gives an intensity of λ4602.13 that agrees well with the most recent prediction. The contribution from Ne II M64d 4f 2[2]3/2 – 3d 3P5/2 λ4600.16 was assumed to be negligible. Fig. B1 also shows the weak feature of λ4613.14 (4f F[3]o 5/2 – 3d 2D5/2) which blends with λ4613.68 (4f F[3]o 7/2 – 3d 3D7/2). The total measured intensity of the two lines agrees with both of the predicted values (Table 6). Correction has been made for the contribution from Ne II M5 3p 3P1 – 3s 3P0 λ4613.87, which is about 7 per cent, and the contribution from Ne II M64b 4f 2[3]7/2 – 3d 3P5/2 λ4612.93 was assumed to be negligible.

APPENDIX D: THE NE II OPTICAL RECOMBINATION SPECTRUM

D1 Multiplet 1, 3p 4Po – 3s 4P

Fig. F5 shows that λ3694.21 (3p 4P5/2 – 3s 4P5/2) lies between the two HII lines H18 λ3691.55 and H17 λ3697.15. The fitted intensity of λ3694.21 is 0.254±0.026. This measurement agrees with the values of LLB01: 0.294 (ESO 1.52 m) and 0.224 (WHT). Another M1 line λ3777.14 (3p 4P5/2 – 3s 4P1/2) is observed in Fig. F1. Its measured intensity is 0.050, with an uncertainty of about 20 to 25 per cent, and also agrees with LLB01: 0.045 (ESO 1.52 m) and 0.059 (WHT). The intensity ratio λ3777.14/λ3694.21 is 0.195, lower than the LS coupling ratio 0.397. The other lines are not observed.

D2 Multiplet 2, 3p 4Do – 3s 4P

See Section 3.4.4.

D3 Multiplet 5, 3p 2Do – 3s 2P

Fig. F5 shows that λ3713.08 (3p 2D5/2 – 3s 2P3/2) is blended with H I H15 λ3711.97, which is more than 5 times stronger. Also blended here is O III M3 3d 3D5/2 – 3s 3P2 λ3715.08. Three Gaussian profiles were used to fit the complex, and the fitted intensity of λ3713.08 is 0.306, which could be of large uncertainty (over 50 per cent). The intensity ratio of λ3713.08 to Ne II M2 λ3334.84 agrees with the LS coupling ratio. The other M2 lines are not observed.

D4 Multiplet 6, 3p 2So – 3s 2P

Λ3481.93 (3p 2S1/2 – 3s 2P3/2) is partially resolved from He I M43 15d 3D – 2p 3Po λ3478.97, which is about 2 times stronger. The fitted intensity of λ3481.93 is 0.043, which is slightly lower than LLB01 (0.051). The other line λ3557.81 is not observed.

D5 Multiplet 8, 3d 4D – 3p 4Po

The wavelengths of the Ne II M8 lines are in the range 3017–3054 Å. The blue cutoff of our spectrum is around 3040 Å, and only three lines have the wavelengths longer than this cutoff. Measurements of these lines are difficult: The λλ3045.56 and 3047.56 lines are blended with the much stronger O III M4 λ3047.12 (3p 3P2 – 3s 3P2) line, which is affected by the Bowen fluorescence mechanism; the λ3054.67 line is not observed.

D6 Multiplet 12, 3d 4D – 3p 4Do

The intensity of λ3329.16 is 0.053, which is slightly higher than that of LLB01 (0.0464). This measurement is of large uncertainty due to weakness of the line, as is shown in Fig. F2. Another M2 line λ3366.98 blends with Ne II M20 3d 2F7/2 – 3p 3D5/2 λ3367.22 (Fig. F1), which contributes more than 90 per cent to the total intensity. The other M12 lines are not observed.

D7 Multiplet 13, 3d 4F – 3p 4Do

See Section 3.4.2.

D8 Multiplet 20, 3d 4F – 3p 4Do

Λ3367.22 (3d 2F7/2 – 3p 2D3/2) blends with Ne II M12 3d 4D7/2 – 3p 3D5/2 λ3366.98, whose intensity contribution is negligible (Section D6). The fitted intensity of λ3367.22 is 0.106±0.016, which agrees with that of LLB01 (0.110). The other M20 lines are not observed.

D9 Multiplet 21, 3d 4D – 3p 4Do

Fig. F0 shows that λ3441.91 (3d 2D5/2 – 3p 2D3/2) is partially resolved from O III M15 3d 3P3 – 3p 3P1 λ3445.26, which is excited by the Bowen fluorescence mechanism. It also blends with Ne II M19 3d 2F7/2 – 3p 3D5/2 λ3417.69, whose intensity contribution is unknown due to the lack of atomic data. The total intensity of the feature at λ3441 given by fit is 0.114, higher than the measurement of LLB01 (0.0825). The other M21 lines are not observed.
D10  Multiplet 28, 3d 2P – 3p 2S

The fitted intensity of λ3456.61 (3d 2P3/2 – 3p 2S0/2) is 0.031, which is slightly lower than the intensity value 0.0334 given by LLB01. Here the contribution from He I 19d 3D – 2p 3P0 λ3456.86 was assumed to be negligible. The other line λ3503.58 is not observed.

D11  Multiplet 34, 3d 4P – 3p 4S

The measured intensity of λ3542.85 (3d 4P5/2 – 3p 4S0/2) and is 0.031, which agrees with the measurement given by LLB01 (0.0325). Fig. 37 shows that λ3565.82 (3d 4P3/2 – 3p 4S0/2) is partially resolved from Na II M9 3p 2F7/2 – 3s 2D5/2 λ3568.50, which is about 7 times stronger. The fitted intensity of λ3565.82 is 0.025, with a relatively large uncertainty due to the weakness. This measurement also agrees with that of LLB01 (0.0243). The intensity ratio of these two M34 lines is thus 1.24 : 1.0, which differs from the LS coupling value 1.5 : 1. The other line λ3594.16 is not observed.

D12  Multiplet 9, 3p 2F0 – 3s 2D

See Section 3.4.3

D13  4f – 3d transitions

D13.1  Multiplet 55c, 4f2[5]f0 – 3d 4F

See Section 3.4.4

D13.2  Multiplet 52a, 4f2[4]f0 – 3d 4D

λ4219.75 (4f 2[4]f0/2 – 3d 4Df/2) blends with Ne II M52a 4f 2[4]f0/2 – 3d 4Df/2 λ4219.37 and Ne II M52d 4f 2[4]f0/2 – 3d 4Df/2 λ4220.89 (Fig. D1). The intensity contribution of λ4219.37 (a few per cent) could be negligible. The fitted intensity of λ4219.75 is 0.064, which agrees with the measurements given by LLB01: 0.0674 (ESO 1.52 m), 0.0699 (WHT 1996) and 0.0616 (WHT 1997). This measurement is higher than the predicted value (Table 5). The measured intensity of the other line λ4233.85 agrees well with LLB01, but is higher than the predicted value. This measurement is probably unreliable due to weakness of the λ4233.85 line.

D13.3  Multiplet 52b, 4f2[3]f0 – 3d 4D

The measured intensity of λ4231.64 (4f 2[3]f0/2 – 3d 4Df/2) is 0.027, which agrees with the measurements of LLB01: 0.0244 (ESO 1.52m), 0.0328 (WHT 1996) and 0.0237 (WHT 1997). Here the contribution from Ne II M52b line λ4250.65 is 0.016, and it also agrees with those of LLB01: 0.0169 (ESO 1.52 m) and 0.0154 (WHT 1996), and 0.0121 (WHT 1997). Both measurements are higher than the predicted values (Table 5), thus λ4231.64 and 4250.65 are not used for abundance determinations. The other two lines λ4217.17 and 4217.15 are too faint to measure accurately.

D13.4  Multiplet 57b, 4f 1[4]f0 – 3d 4F

Only λ4397.99 (4f 1[4]f0/2 – 3d 4Ff/2) is observed (Fig. C2). Its fitted intensity is 0.024±0.004. This intensity value is slightly higher than those given by LLB01: 0.0219 (ESO 1.52 m), 0.0184 (WHT 1996) and 0.0185 (WHT 1997), but agrees with the predicted value within errors (Table 8). The other M57b lines are not observed.

D13.5  Multiplet 60c, 4f 1[3]f0 – 3d 2F

Only λ4428.64 (4f 1[3]f0/2 – 3d 2Ff/2) is observed, and its fitted intensity is 0.044, which agrees with the measurements given by LLB01: 0.0483 (ESO 1.52 m), 0.0451 (WHT 1996) and 0.0479 (WHT 1997). Here the contribution from Ne II M61b 4f 2[3]f0/2 – 3d 2Ff/2 λ4428.52 is included, but the other two blended lines Ne II M60c 4f 1[3]f0/2 – 3d 2Ff/2 λ4428.52 and Ne II M61b 4f 2[3]f0/2 – 3d 2Ff/2 λ4428.41 were assumed to be negligible.

D13.6  Multiplet 61a, 4f 2[4]f0 – 3d 2D

The measured intensity of λ4430.94 (4f 2[4]f0/2 – 3d 2Df/2) is 0.040, which is higher than the observations of LLB01: 0.0331 (ESO 1.52 m), 0.0340 (WHT 1996) and 0.0317 (WHT 1997). This measurement is of large uncertainty due to blending. The contribution from Ne II M57a 4f 1[2]f0/2 – 3d 2Ff/2 λ4430.90 and Ne II M55a 4f 2[4]f0/2 – 3d 2Ff/2 λ4430.06 are included, but Ne II M57a 4f 1[2]f0/2 – 3d 2Ff/2 λ4431.11 was assumed to be negligible.

D13.7  Multiplet 61d, 4f 2[2]f0 – 3d 2D

λ4457.05 (4f 2[2]f0/2 – 3d 2Df/2) is partially resolved from O III M49a 5g F[4]0/2 – 4f D[3]1 λ4458.55 (Fig. 35). The fitted intensity of λ4457.05 is 0.026, with an uncertainty of about 20 per cent. Here the contribution from Ne II M66c 4f 1[3]f0/2 – 3d 2Ff/2 λ4457.36 is included, but the other two blended lines Ne II M61d 4f 2[2]f0/2 – 3d 2Df/2 λ4457.24 and Ne II M66c 4f 1[3]f0/2 – 3d 2Ff/2 λ4457.24 were assumed to be negligible. This measurement agrees with those of LLB01: 0.0250 (ESO 1.52 m), 0.0247 (WHT 1996) and 0.0211 (WHT 1997), but is much higher than the predicted value (Table 5). The other M61d lines are not observed.

D13.8  Multiplet 65, 4f 0[3]f0 – 3d 4P

λ4413.22 (4f 0[3]f0/2 – 3d 2Pf/2) blends with O III M5 3p 3Df/2 – 3s 2Pf/2 λ4414.90 (Fig. C2). The fitted intensity of λ4413.22 is 0.028, which agrees with the measurements of LLB01: 0.0296 (ESO 1.52 m), 0.0243 (WHT 1996) and 0.0271 (WHT 1997), but is higher than the predicted value (Table 3). Here the contribution from Ne II M65 4f 0[3]f0/2 – 3d 2Pf/2 λ4413.11 was assumed to be negligible, but Ne II M57c 4f 1[3]f0/2 – 3d 4Ff/2 λ4413.11 is included. The other line λ3777.98 blends with N III M18 λ4379.

D13.9  Multiplet 66c, 4f 1[3]f0 – 3d 4P

λ4421.39 (4f 1[3]f0/2 – 3d 2Pf/2) is observed in Fig. C2. Its fitted intensity is 0.0084. The uncertainty of this measurement could be large due to weakness of this line. The other two M66c lines λ4457.24 and 4457.36 blend with Ne II M61d 4f 2[2]f0/2 – 3d 2Df/2 λ4457.05.
**APPENDIX E: THE N III PERMITTED LINES**

**E1 Multiplet 1, 3p2P0 – 3s3S**

λ4097.33 is partially resolved from H I λ4101 (Fig. [3]). Its fitted intensity is 2.246±0.112. Several O II and N II ORLs are blended with λ4097.33, and they contribute about 15–20 per cent to the total intensity. The other line λ4103.39 blends with H I λ4101, and contributes less than 1 per cent to the total intensity.

**E2 Multiplet 2, 3d2D – 3p2P0**

Measurements of this multiplet are presented in Section [3.1]. Here the ratio of the two lines λ4634.14 and 4641.85 were assumed to be as in LS coupling. The resultant intensities of the λ4634.14, 4640.64 and 4641.85 lines are 1.133, 2.262 and 0.226, respectively, and the uncertainties are all less than 10 per cent. Thus the intensity ratio of λ4640.64 to λ4641.81 is 10 : 1, which differs from the pure LS coupling ratio, i.e. 9 : 1.

**E3 Multiplet 3, 3p'4D – 3s'4P0**

See Section [3.6.1].

**E4 Multiplet 6, 3p'2D – 3s'2P0**

λ4195.76 (3p'2D3/2 – 3s'2P3/2) is observed in Fig. [31]. Its measured intensity is 0.046±0.008. Here the intensity contribution from N II M49a 4f [24]3 – 3d 1D0 λ4195.97, which is about 8 per cent, has been corrected. Another line λ4215.77 (3p'2D3/2 – 3s'2P3/2) is partially resolved from Ne II M52b 4f 2[3]f7/2 – 3d 1D2 λ4217.17. Two Gaussian profiles were used to fit the complex, and the fitted intensity of λ4215.77 is 0.0086, with an uncertainty of more than 20 per cent. Thus the ratio λ4195.76 / λ4215.77 is 0.185, slightly higher than the LS coupling value (0.20). This is interesting because these two lines decay from the same upper level, thus their intensity ratio depends only on the coupling scheme. The other line λ4200.10 (3p'2D5/2 – 3s'2P3/2) blends with He II 11g G – 4f 2F4 λ4199.83 which is probably about 4 times stronger.

**E5 Multiplet 9, 3d'4F0 – 3p'4D**

Only λ4881.81 (3d' 4F9/2 – 3p' 4D5/2) and λ4884.13 (3d' 4F7/2 – 3p' 4D7/2) are observed (Fig. [37]). λ4881.81 blends with [Fe II] λ4881.00, whose intensity contribution is unknown. Measurements of λ4884.13 are of large uncertainty due to weakness of the line.

**E6 Multiplet 12, 5s3S – 4p2P0**

Measurements of λ4544.85 (5s2S1/2 – 4p2P3/2) could be overestimated due to the strength of He II 9g 2G – 4f 2F4 λ4541.59 (Fig. [38]), which is more than 10 times stronger. The other line λ4539.71 is blended with He II λ4541.59.

**E7 Multiplet 17, 5f2F0 – 4d2D**

λ3998.63 (5f2F7/2 – 4d2D5/2) is detected in Fig. [33]. Its fitted intensity is 0.025±0.005. The other two lines λ4003.58 and 4003.72 are blending together. The measured intensity ratio (λ4003.58 / λ4003.58) / λ3998.63 is 1.54, which agrees with the value in LS coupling, i.e. 1.50.

**APPENDIX F: THE O III PERMITTED LINES**

**F1 Multiplet 2, 3p3P0 – 3s3P**

Table [F1] presents the observed and predicted relative intensities of the O III M2 lines detected in NGC 7009. Fig. [F1] shows the observed M2 lines. λ3757.24 is partially resolved from λ3754.69 and λ3759.87. Three Gaussian profile fits were used to fit the complex, and resultant relative intensities deviate from LS coupling. The uncertainty of the λ3754.69 intensity is the largest among the three (over 30 per cent), because it also blends with He I M66 14d 1D2 – 2p 1P1 λ3756.10, whose contribution is unknown, and He II 23g 2G – 4f 2F4 λ3758.15, which contributes less than 10 per cent to the total intensity (an estimate based on the hydrogenic theory of Storey & Hummer [1995]). The uncertainties of λ3754.69 and 3759.87 are both less than 5 per cent. Measurements of λ3774.02 is probably of large error due to the strength of H I H11 λ3770.63, which is more than 20 times stronger. Another line λ3791.27 is free of blend, but its fitted intensity is 45 per cent lower than the value in pure LS coupling. The other line λ3810.99 is not observed. Both of the line ratios λ3791.27/λ3754.69 and λ3774.02/λ3757.24, where the two lines of each of the ratios decay from the same upper levels, differ from the LS coupling assumption, and agree with the measurements given by Liu et al. (1995a) for NGC 7009. Discussion of these line ratios is in Section [3.7.4].

**F2 Multiplet 3, 3p3S – 3s3P0**

Fig. [F2] shows the observed O III M3 lines in NGC 7009. λ3340.76 blends with the [Ne III] auroral line 2p1 1S0 – 2p3 1D2 λ3342.50 and Ne II M2 3p 1D2 3/2 – 3s 1P1/2 λ3344.40. The intensity of [Ne III] λ3342.50 was estimated from the theoretical nebular-to-auroral line ratio (λ3368.76 + 3967.47)/λ3342.50 which was calculated by solving the population equations for a five-level atomic model at a particular N_e and T_e. We estimated that [Ne III] λ3342.50 contributes about 12 per cent to the total intensity of the blend at λ3341. The contribution from Ne II M2 λ3344.40 is negligible. The resultant λ3299.40/λ3340.76 and λ3312.32/λ3340.76 ratios are discussed in Section [3.7.4].

**F3 Multiplet 4, 3p3P – 3s3P0**

Only λ3047.12 and λ3059.30 are observed in the near-UV, but their measurements are quite unreliable due to poor S/N.
Figure F1. Spectrum of NGC 7009 from 3740 to 3825 Å showing the O III M2 lines and other emission features. The continuous curve is the sum of Gaussian profile fits. Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.

Figure F2. Spectrum of NGC 7009 from 3290 to 3348 Å showing the O III M3 lines and other emission features. The continuous curve is the sum of Gaussian profile fits. Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.

Figure F3. Spectrum of NGC 7009 from 5500 to 5650 Å showing the O III M5 λ5592.24 line. The [Cl III] λ5517, 5537 doublet lines are also detected. Note the residual of [O I] λ5577 due to poor sky subtraction. The continuous curve is the sum of Gaussian profile fits. Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.

F4 Multiplet 5, 3p 1P – 3s 1P o

λ5592.24 is observed in Fig. [4]. Its fitted intensity is 0.047, which is slightly lower than the observation of Liu & Danziger (1993a). 0.0510±0.0031. The uncertainty of the current measurement is about 10 per cent.

F5 Multiplet 8, 3d 3F o – 3p 3D

See Section 3.7.1.

F6 Multiplet 12, 3d 3P o – 3p 3S

The O II M12 lines are shown in Fig. [F4]. The fitted intensity of λ3132.79 is 37.81, with an uncertainty of about 7 per cent. Our measurement is lower than that of Liu & Danziger (1993a): 43.4±4.3. The measured intensity ratio of the three lines λ3115.68 : λ3121.64 : λ3132.79 is 1 : 13 : 343, which differs significantly from the ratio in the pure LS coupling assumption, i.e. 1 : 3 : 5. That is because this multiplet is mainly excited by the Bowen fluorescence mechanism.

F7 Multiplet 14, 3d 3D o – 3p 3P

λ3715.08 (3d 3D o 3 – 3p 3P 1) is partially blended with H I H15 λ3711.97 (Fig. [F5]). Also blended here are λ3714.03 (3d 3D o 1 – 3p 3P 1), Ne II M5 3p 2D 5/2 – 3s 2P 3/2 λ3713.08 and He II 29g 2G – 4f 2F o λ3715.16. Reliable measurements of the two M14 lines are difficult. Accurate measurements of another line λ3707.25 are also difficult due to weakness. The other lines are not observed.

F8 Multiplet 15, 3d 3P o – 3p 3P

See Section 3.7.2.
Figure F4. Spectrum of NGC 7009 from 3100 to 3175 Å showing the O III M12 lines. The continuous curve is the sum of Gaussian profile fit. Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.

Figure F5. Spectrum of NGC 7009 from 3680 to 3745 Å showing the O III M14 lines and other emission lines. The H I Balmer series are marked on top of the figure. The Ne II M1 line λ3694.21 (3p 4P5/2 – 3s 4P5/2) is also detected. The continuous curve is the sum of Gaussian profile fits. Continuum has been subtracted and the spectrum has been normalized such that Hβ has an integrated flux of 100. Extinction has not been corrected for.