Parity-time-symmetry enhanced optomechanically-induced-transparency

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We propose and analyze a scheme to enhance optomechanically-induced-transparency (OMIT) based on parity-time-symmetric optomechanical system. Our results predict that an OMIT window which does not exist originally can appear in weak optomechanical coupling and driving system via coupling an auxiliary active cavity with optical gain. This phenomenon is quite different from those reported in previous works in which the gain is considered just to damage OMIT phenomenon even leads to electromagnetically induced absorption or inverted-OMIT. Such enhanced OMIT effects are ascribed to the additional gain which can increase photon number in cavity without reducing effective decay. We also discuss the scheme feasibility by analyzing recent experiment parameters. Our work provide a promising platform for the coherent manipulation and slow light operation, which has potential applications for quantum information processing and quantum optical device.

In quantum mechanics and quantum optics, it seems almost inevitable that the quantum system will interact with the environment around it. The dynamics of such an open quantum system can be described by a non-Hermitian Hamiltonian ($\hat{H} = \hat{H}^\dagger$) with complex eigenvalues, which will result in different phenomena from ones in closed quantum system1. Recently, a special class of physical system with a so-called parity-time ($PT$) symmetry has attracted great attention2–9. It has been proved that a $PT$-symmetric non-Hermitian Hamiltonian ($[\hat{H}, PT] = 0$) can also have real eigenvalue spectra10,11. This characteristic make open system subject to unitary time evolution like closed system, which causes some environmental damage effects, for example decoherence, can be suppressed effectively. Up to now, $PT$-symmetric system has shown its extensive application prospects in quantum optics and quantum information processing (QIP), including strengthening optics nonlinearity12,13, enhancing photon blockade14, realizing quantum chaos15, soliton active controlling16,17, and so on.

In addition to theoretical research, $PT$-symmetric Hamiltonian has also been realized experimentally in a variety of physical systems18–22. Among them, a simple and intuitive scheme is to link two coupled cavities with optical gain and loss respectively. Especially in refs 18,20, $PT$-symmetric region is obviously observed by measuring the eigenfrequencies of supermodes. These progresses of experiments provide a solid platform for exploring the fundamental $PT$ symmetry and corresponding strange phenomena.

Although ($PT$)-symmetry can enhance many quantum optical effects by balancing the loss with extra gain, there still are opposite phenomena in which environmental loss play a positive role. The most typical two of such phenomena are optical cooling23–25 and electromagnetically induced transparency (EIT)26–28. It is well known that a larger decay rate is necessary whether for EIT in atomic system29 or for optomechanically-induced-transparency (OMIT) in cavity optomechanical system (OMS)30, and a gain is considered just to damage EIT (OMIT) phenomenon even leads to electromagnetically induced absorption (EIA) or inverted-EIT (inverted-OMIT) in previous works30,31. Up until now, enhancing EIT or OMIT still relies on additional coupled dissipation systems32–35. Correspondingly, the extra dissipation needs strong driving and nonlinear coupling to maintain the interference effect in OMIT, which still remains twofold difficulties because strong driving may lead nonlinear system to instability36 and strong nonlinear coupling is hard to realize in OMS37.

In this paper, we propose and analyze an enhanced optomechanically induced transparency in OMS via coupling an auxiliary active cavity with optical gain. A transparency window can appear in weak optomechanical coupling and driving system, however, similar window can not be obtained in dissipation system unless the
driving (or the optomechanical coupling) intensity is magnified roughly 20000 times. This scheme is quite different from ones in existing works because our results show that extra gain can exert positive effect on OMIT phenomenon. Though the conclusion is counter-intuitive, we have analyzed the physical mechanisms of our scheme and explained the reasons of this kind of enhancement. After simulating, we find that absorption and transparency can be of controllable flexibilities by switching the fiber. We believe it can provide a promising platform for the coherent manipulation and slow light operation.

Results

In this part, we present the main results of this work by introducing dynamics analyses and enhanced electromagnetically induced transparency phenomena in parity-time-symmetric optomechanical system. The details of calculation and simulation can be found in METHODS.

Model and dynamics analysis. We consider a typical \( \mathcal{PT} \)-symmetric system for realizing the enhanced OMIT effect (see Fig. 1 for detail) and it is necessary to analyze its dynamics firstly.

By setting \( \hbar = 1 \), the total Hamiltonian of such a system can be written by increasing oscillator free term and optomechanical interaction on the basis of ordinary cavity Hamiltonian:

\[
\hat{H}_c = \hat{h}_c + \omega_m \hat{b}^\dagger \hat{b} - g_e \hat{a}_1^\dagger \hat{a}_1 (\hat{b}^\dagger + \hat{b}).
\]

The Hamiltonian of the cavity can be written as:

\[
\hat{H}_c = \omega_c (\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2) + J (\hat{a}_1^\dagger \hat{a}_2 + H.c.) + \hat{H}_d + \hat{H}_r,
\]

where \( \hat{H}_d = i \Omega_d (\hat{a}_1^\dagger e^{-i\omega_d t} + \hat{a}_2 e^{i\omega_d t}) \) denotes the Hamiltonian of the driving field and correspondingly, the probe field inside the cavity is described by \( \hat{H}_r = i (\hat{a}_1^\dagger \epsilon_p e^{-i\omega_p t} - \hat{a}_2^\dagger \epsilon_p e^{i\omega_p t}) \). Here we make a frame rotating via \( \hat{H} \rightarrow U^\dagger \hat{H} U - i U^\dagger \partial_t U \) and let \( U = \exp \{ -i \omega_d (\hat{a}_1^\dagger \hat{a}_2^\dagger + \hat{a}_2 \hat{a}_1) t \} \), then the total Hamiltonian of the whole system becomes:

\[
\hat{H} = \Delta_c (\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2) + \omega_m \hat{b}^\dagger \hat{b} - g_e \hat{a}_1^\dagger \hat{a}_1 (\hat{b}^\dagger + \hat{b}) + i \Omega_d (\hat{a}_1^\dagger - \hat{a}_1)
\]

\[
+ i (\hat{a}_1^\dagger \epsilon_p e^{-i\omega_p t} - \hat{a}_2^\dagger \epsilon_p e^{i\omega_p t}) - J (\hat{a}_1^\dagger \hat{a}_2 + H.c.).
\]

In this expression, \( \hat{a}_1, \hat{a}_2 \) and \( \hat{b}^\dagger, \hat{b} \) are the annihilation (creation) operators of the cavity mode and mechanical mode, respectively. \( \Omega_d \) and \( \epsilon_p \) are the amplitudes of the driving field and the probe field, which are respectively related to the input powers \( (P_d, P_p) \) and decay rate \( (\gamma) \), i.e., \( |\Omega_d| = \sqrt{2 P_d} / (\hbar \omega_d) \) and \( |\epsilon_p| = \sqrt{2 P_p} / (\hbar \omega_d) \). \( \Delta_c = \omega_c - \omega_m \) is the detuning between the driving field and cavity field. \( \gamma (\gamma_m) \) is the optical (mechanical) decay rate and \( \kappa \) is the gain rate of the active cavity. \( g_e \) is the single-photon coupling intensity of the radiation pressure interaction between the passive field and the oscillator. For the convenience of discussion, we define the dimensionless position operator \( \hat{x} = (1/\sqrt{2}) (\hat{b}^\dagger + \hat{b}) \) and momentum operator \( \hat{p} = (i/\sqrt{2}) (\hat{b}^\dagger - \hat{b}) \) instead of the pho-

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Figure 1. (a) Schematic illustration of our enhanced OMIT. Here the \( \mathcal{PT} \)-symmetric OMS consists of a passive OMS coupled to an active cavity with tunnelling strength \( J \). (b) Level diagram of the OMIT in OMS. \( |m, n\rangle \) denotes the state of \( m \) photons and \( n \) phonons in the displaced frame. (c) Eigenfrequencies of supermodes as a function of coupling \( J/(\kappa + \gamma) \).
non operators in Eq. (2). Then the dynamics of such a system can be determined by quantum Langevin equation
\( \dot{\hat{\omega}} = [\hat{\omega}, \hat{H}] + \hat{N}_{\text{det}}, \) where \( \hat{\omega} \) is an arbitrary optical or mechanical operator and \( \hat{N}_{\text{det}} \) is the dissipation term.

Note that both the stability analyses in \( PT \)-symmetric system and the OMIT phenomena in OMS need to be considered only in the mean value regime\(^{32,36}\). Similarly to previous EIT works in atomic systems, optomechanically induced transparency only deals with the mean response of the system to the probe field without including quantum fluctuation. On the other hand, the non-Hermitian Hamiltonian will be regard as an exact description of an OMS only if the input-bath operators are ignored. Therefore in order to explore the nonlinear dynamics of the system, we employ the semi-classical Langevin equations of motion, that is, \( \delta = (\delta) \) in the corresponding quantum Langevin equations. Then the mean values can be obtained by following dynamics equations\(^{35,36}\):

\[
\begin{align*}
\dot{\hat{x}} &= \omega_m P \\
\dot{\hat{p}} &= -\gamma_m \hat{p} - \omega_m x + \sqrt{\gamma} g_0 \Re[a_1] \\
\dot{\hat{a}_1} &= -\frac{\gamma}{2} \hat{a}_1 + j \hat{a}_2 + i \sqrt{\gamma} g_0 a_1 x + \Omega_d + \varepsilon e^{-i \omega t} \\
\dot{\hat{a}_2} &= -i \Delta + \frac{\kappa}{2} \hat{a}_2 + j \hat{a}_1.
\end{align*}
\]

(3)

A necessary condition for obtaining an available OMIT phenomenon is that there should exist an asymptotic steady state and the corresponding system will keep the state for a long evolution time. As we all know, the probe field in OMIT typically has a small amplitude and it would not have an influence on the stability of the system. Therefore, here we firstly ignore the term \( \varepsilon e^{-i \omega t} \) in Eq. (3) for a more simple stability analysis and the stability or stochastic property of the system is characterized by the Jacobian matrix

\[
M = \begin{pmatrix}
0 & \omega_m & 0 & 0 & 0 \\
-\omega_m & -\gamma / 2 & 2 \sqrt{\gamma} g_0 \Re[a_1] & 2 \sqrt{\gamma} g_0 \Im[a_1] & 0 \\
0 & -\gamma & \Delta + \sqrt{\gamma} g_0 x & 0 & -J \\
2 \sqrt{\gamma} g_0 \Re[a_1] & 0 & -\Delta + \sqrt{\gamma} g_0 x & -\gamma / 2 & J \\
0 & 0 & 0 & 0 & -J
\end{pmatrix}
\]

(4)

which can be obtained by linearizing Eq. (3) under the base vector \( \hat{u} = (\delta x, \delta p, \delta \Re[a_1], \delta \Im[a_1], \delta \Re[a_2], \delta \Im[a_2]) \). Here we neglect the optomechanical interaction in Eq. (4) under the condition of weak-coupling regime \( g_0 / \gamma \ll 1 \). In this case, the eigenvalues of this Jacobian matrix can be easily solved as

\[
\begin{align*}
\lambda_{1,2} &= -\frac{\gamma}{4} \pm \sqrt{\left(\frac{\gamma}{4}\right)^2 - \omega_m^2} \\
\lambda_{3,4} &= \frac{\kappa - \gamma}{4} + i \left(\Delta \pm \sqrt{\left(\frac{\gamma + \kappa}{4}\right)^2 - \frac{\gamma}{4}}\right) \\
\lambda_{5,6} &= \frac{\kappa - \gamma}{4} - i \left(\Delta \pm \sqrt{\left(\frac{\gamma + \kappa}{4}\right)^2 - \frac{\gamma}{4}}\right).
\end{align*}
\]

(5)

The real parts of above eigenvalues are also known as the Lyapunov exponents of this nonlinear dynamical system\(^{37,38}\). Whether the system is stable or not can be judged by comparing the largest Lyapunov exponent with zero\(^{38,40}\). Equation (5) implies that if \( J > (\kappa + \gamma)/4 \), the second terms in \( \lambda_{3,4,5,6} \) will be pure imaginary numbers. The real parts of these eigenvalues will not change with the parameter \( J \) and the corresponding Lyapunov exponents are \( \lambda_{1,2} = -\gamma / 4 \) and \( \lambda_{3,4,5,6} = (\kappa - \gamma) / 4 \). If the decay and gain rates are in the regime \( \kappa < \gamma \), all Lyapunov exponents of this system will be negative, which can ensure that the mean value trajectories will tend to a fixed point in phase space. On the contrary, the second terms in \( \lambda_{3,4,5,6} \) will present non-zero real parts if \( J < (\kappa + \gamma)/4 \), which causes positive Lyapunov exponents. In the unstable system, neglected terms \( g_0 \Re[a_1], g_0 \Im[a_1] \) and \( g_0 \Re[a_2], g_0 \Im[a_2] \) in Jacobian matrix will continue to be amplified, and this phenomenon is equivalent to enlarging the nonlinear coupling coefficient and the system will become more unstable in this case.

Here we give the feasibility analysis of our parameters used in above discussions. The dimensionless parameters are in accord with recent microcavity experiments, that is, \( \omega_m = 190 \text{THz} \) and \( \omega_m / 2\pi = 23.4 \text{MHz} \). The Q-factor of passive cavity is \( Q_m \sim 3 \times 10^5 \) corresponding to \( \gamma / 2\pi \approx 6.33 \text{MHz} \) and \( \gamma / 2\pi \approx 1 \text{MHz} \). The optomechanical interaction intensity in this cavity is \( g_0 = 7.4 \times 10^{-5} \gamma \) and corresponding oscillator decay is \( \gamma_m / 2\pi \approx 38 \text{kHz} \) owing to \( Q_m = 2Q / 10^{18-29} \). The driving and probe powers are \( P_d = 8 \text{pW} \) and \( P_p = 0.08 \text{pW} \), respectively. In addition, a small gain rate and fiber coupling \( (J, \kappa < \gamma) \) are also easy to be achieved in experiments\(^{41,42}\). After adopting these parameters, we select \( \omega_m = 1 \) as an unit to nondimensionalize the other parameters, i.e., \( \gamma / 2\pi = 1/23.4, \gamma_m / 2\pi = 0.038 \gamma \) and \( \Omega_d = 10\gamma \). It can been found that the parameters used in our simulations are similar with ones in previous works\(^{43-45}\). In Fig. 2, we verify above discussions by numerically calculating the oscillator evolutions and the largest Lyapunov exponents. Figure 2(a) shows that the critical point of the system stability is at \( J = (\kappa + \gamma) / 4 \). When \( J \) is increased to be greater than the critical point, a significant asymptotic steady state emerges in this system (see
Fig. 2. (a) Analytical (blue) and numerical (red) Lyapunov exponents as a function of coupling $J/(\kappa + \gamma)$. (b,c) Evolutions of dimensionless mechanical displacement $x$ in $\mathcal{PT}$-SP and $\mathcal{PT}$-BP, respectively. The dimensionless parameters in this simulation are: $\Delta_c = \omega_m = 1$, $\gamma = 1/234 = 0.043$, $\gamma_m = 0.038\gamma = 1.652 \times 10^{-3}$, $g_0 = 7.4 \times 10^{-5}\gamma = 3.217 \times 10^{-6}$, $\Omega_\eta = 10\gamma = 0.43$ and $\kappa = 0.8\gamma = 3.479 \times 10^{-2}$. In (b,c) $J/(\gamma + \kappa)$ is taken as 1 and 0.2, respectively.

Enhanced optomechanically induced transparency. The steady-state solution of Eq. (3) can be expanded to contain many Fourier components. In the limit of weak probe field, the high order terms of $\varepsilon_P$ are neglected in our work and each operator will have the following form:\(^{26,43}\):

$$
\sigma = \sigma_0 + \varepsilon_P e^{-i\theta} + \varepsilon_P^* e^{i\theta}
$$

under the condition of $t \to \infty$. Substituting Eq. (7) into Eq. (3) and ignoring these terms containing $\varepsilon_P^2$, $\varepsilon_P^*\varepsilon_P$ and $|\varepsilon_P|^4$, we can respectively gain the zero-order and first-order steady-state equations (see METHODS for detail). The steady-state solutions can be given finally by:

$$
0_x = \sqrt{\frac{2}{\omega_m}}|a_1|^2
$$

$$
0 a_1 = \frac{\Omega_d}{i(\Delta_c - \Delta) + \frac{\gamma}{2} + \frac{\beta^2}{2(\Delta_c - \Delta)} + \frac{\delta - i\delta_\gamma m/2}{2(\Delta_c - \Delta)} + \beta i(2\Delta + L - M)}
$$

and

$$
\dot{a}_1 = \frac{\left\{ (\omega_m^2 - \delta^2 - i\delta\gamma_m/2) - i(\Delta + \delta) + \frac{\gamma}{2} + M \right\} + \beta}{(\omega_m^2 - \delta^2 - i\delta\gamma_m/2) - i(\Delta - \delta) + \frac{\gamma}{2} + \left\{ (\Delta + \delta) + \frac{\gamma}{2} + M + \beta i(2\Delta + L - M) \right\}}.
$$

in which the undefined variables are:
\[ L = \frac{i(\Delta_0 - \delta)}{\gamma}, \quad M = \frac{-i(\Delta_0 + \delta)}{\gamma}, \quad \beta = \frac{i\sqrt{2}g_0\omega_m}{x}, \quad \Delta = \Delta_0 - \Delta', \quad \Delta' = \omega - \omega_d - \sqrt{2}g_0^2 \delta. \] \tag{10}

In order to obtain the OMIT behavior in this model, the output field should be solved to illustrate its response. Based on the input-output relation, the output field can be got by

\[ \epsilon_{out}(t) = \epsilon_p e^{-i\delta t} + \Omega_d a_1. \] \tag{11}

We also extend the output field \( \epsilon_{out}(t) \) in the form of

\[ \epsilon_{out}(t) = \epsilon_{out}^0 e^{+i\delta t} + \epsilon_{out}^0 e^{-i\delta t} + -\epsilon_{out}^0 e^{i\delta t}. \] \tag{12}

and the following relations can be achieved by substituting Eq. (12) into Eq. (11):

\[ \epsilon_{out}^0 = \gamma a_1 - \Omega_d, \quad \epsilon_{out}^+ = \gamma a_1 - 1, \quad \epsilon_{out}^- = \gamma a_1. \] \tag{13}

Observing OMIT phenomenon requires us to specifically focus on the response of the cavity optomechanical system to the probe field in the presence of the coupling field. In Eq. (12), the component of the output field oscillating at the probe frequency corresponds to the term \( \epsilon_{out}^0 \) and one can intuitively find from Eq. (13) that \( \epsilon_{out}^0 \) does not vary with the coupling field. Similarly to previous works, we concentrate on the behavior of \( \epsilon_{out}^0 \) and define

\[ \chi = \epsilon_{out}^0 + 1 = \gamma a_1 \] for the convenience of discussion. The relevant absorption and dispersion theories inspire us that real part and imagery part of \( \chi \) respectively represent the behaviors of absorption and dispersion\(^{26}\).

In Fig. 3(a,b), we first exhibit the enhancement effect on the OMIT phenomenon in this \( PT\)-symmetric OMS (blue dashed line). The real part and imagery part of \( \chi \) show an obvious transparent window emerges at the frequency \( \delta = \omega_p - \omega_d = \omega_m \). This OMIT phenomenon can still appear even in the weak driving and weak optomechanical coupling regimes, which is quite different from the OMIT in the normal passive system. When \( \beta = 0 \), the \( PT\)-symmetric OMS returns to a normal passive system and the gain rate \( \kappa \) no longer exist and the system only has decay rates \( \gamma \) and \( \gamma_m \). Figure 3 illustrates that the absorption and dispersion properties perform like a light falling on a normal medium and there are not any transparent windows in this case since the driving and coupling are both too weak. The comparisons between blue line and red line in Fig. 3 show OMIT phenomenon is enhanced by the \( PT\)-symmetric system. The insets in Fig. 3(a,b) show that such an enhancement effect in fact can be carried out in a wide range of parameters. As a criterion of enhanced OMIT, the depth of transparent window is almost unchanged even though the \( \gamma \) and \( \gamma_m \) are both enlarged. What is influenced by dissipation is the width of transparent window and it is widened significantly when \( \gamma \) and \( \gamma_m \) are increased, implying that the enhancement effect discussed in our work is a general conclusion to a certain extent, i.e., it does not require that \( \gamma \) and \( \gamma_m \) are both extremely small. It can be known from Fig. 4, that OMIT phenomenon can not be obtained in normal passive system unless there exists very strong optical driving or optomechanical coupling. In Fig. 4(a), we show that, in order to obtain OMIT, increasing the driving intensity or coupling intensity is equivalent in this system and Fig. 4(b) indicates that \( \Omega_{dg} \) is required to be amplified 15000 times for achieving a similar OMIT phenomenon \( \text{Re}(\chi) \rightarrow 0 \). Based on the above discussions, one can conclude that a \( PT\)-symmetric OMS can indeed enhance the OMIT phenomenon.

Figure 3. Changes of \( \chi \) with probe field frequency. (a,b) Respectively correspond to the real and imaginary parts of \( \chi \). In both figures, the blue dash lines denote the \( \chi \) in \( PT\)-symmetric OMS \((I = 0.2501(\gamma + \kappa)) \) and red solid lines are \( \chi \) in normal passive system \((I = 0) \). In this simulation, the gain rate is set as weak intensity \( \kappa/\gamma = 10^{-4} \) and other parameters are the same with Fig. 2. The insets in (a,b) describe the changes of Re\( \chi \) and Im\( \chi \) with enlarged cavity and oscillator dissipations. Here the blue lines are of the same parameters with ones in main figures and the green lines refer to the case we amplify the cavity dissipation 2 times and oscillator dissipation 20 times \((\gamma = 2/23.4 \text{ and } \gamma_m = 0.38\gamma)\), and the black lines denote the both dissipations are amplified 4 and 40 times, respectively \((\gamma = 4/23.4 \text{ and } \gamma_m = 0.38\gamma)\).
Further more, we introduce an optomechanically induced amplification phenomenon in this enhanced OMIT, that is, the probe field may be amplified by the active cavity if we increase its gain rate $\kappa$. We find that the amplification is selective, meaning that it can be realized in this system only when the probe field satisfies $\omega_p - \omega_d = \omega_m$. To describe this property, we consider the variable $\mu \equiv -\epsilon_{\text{out}}^+$ and plot it in Fig. 5. In Eq. (11), we have defined the output corresponding to the probe field is of the amplitude $\epsilon_p$. Therefore, $\epsilon_{\text{out}}^+$ can be regarded as a magnification since it is an amplitude ratio between the response and probe field. In normal OMIT phenomenon, the maximum response amplitude should be $\mu = 1$ due to the conservation of energy. Physically, this case means all probe fields are exported without any dispersion. However, the existence of the active cavity allows response field to carry more energy than the probe field. In Fig. 5(a), one can observe significant amplification effect with the increasing of $\kappa$. In particular, the probe field will be doubled at $\kappa = 0.1$. It is worth noting that amplified probe (response) field is still a small quantity compared to the driving field. Thus, the dynamics analyses in Eqs (4)–(13) are still accurate. The inset in Fig. 5(a) shows that this amplification phenomenon only exists at the transparent frequency $\delta = \omega_m$.

Correspondingly, the dispersion spectra (see Fig. 5(b) and its inset) illustrate that there are no dispersion effects in this case. The above two properties clarify the reason why we think this amplification is selective. Moreover, the insets in Fig. 5 show the absorption and dispersion spectra are similar with those in normal OMIT phenomena except for negative $\text{Re}(\chi)$. Especially, the widths of the transparency windows almost keep unchanged. Therefore, this enhanced OMIT can be directly used in some schemes of OMIT applications without changing other properties.

**Discussion**

**Transformation between inverted-OMIT and OMIT.** In the above section, we have already shown a transparent window ($\text{Re}(\chi) \to 0$) and $\text{Im}(\chi) \to 0$ will appear after we add an extra gain to keep the system in $\mathcal{PT}$ SP. A natural question is what this transparent window corresponds to, an ordinary OMIT or an inverted-OMIT. The achieved answer is that $\mathcal{PT}$ symmetry can enhance both OMIT and inverted-OMIT. The inverted-OMIT in $\mathcal{PT}$-symmetric OMS has been reported in ref. 30 recently. The OMIT effect, never being observed in gain systems, can also be enhanced under certain parameters. In order to show our conclusions intuitively, we calculate the transmission rate $T(\delta) = |\epsilon_{\text{out}}^+|^2$, i.e., the square of amplitude ratio between the output field and input field amplitudes, to distinguish OMIT or inverted-OMIT30. Using the input-output relationship in Eq. (13), the transmission rate can be simplified as

\begin{equation}
\eta(\delta) = \frac{|\epsilon_{\text{out}}^+|^2}{|\epsilon_p|^2},
\end{equation}
We find that the gain can increase the photon number. Under some certain parameters, it is
possible to increase the gain by adjusting the coupling intensity. The underlying physics of OM IT
is formally similar to that of ordinary EIT in atomic systems, Huang and Agarwal in ref. 47, discussed
this relation and sketched three conditions for engendering OM IT.

\[ \eta(\delta) = \left| \frac{e^{i\Delta}\alpha e^{-\delta t}}{e^{i\Delta}\alpha e^{-i\delta t}} \right|^2 = \left| \gamma^\eta a_1 - 1 \right|^2 \]  

(14)
due to narrow bandwidth driving and detecting fields. Generally speaking, a typical OM IT-like transmission rate

corresponds to such a one with a transparency window and two sideband dips in frequency domain. It takes on, however, a significant difference with inverted OM IT phenomenon in which corresponding transmission rate

exhibits a transmission dip and two sideband peaks. It can be known from Fig. 6(a) that under some certain parameters, one can make the system switch between transparency and inverted transparency by only adjusting the coupling intensity any fiber and the gain \( \kappa \) on the auxiliary cavity. The transmission rate \( \eta \) takes on a significant decline at \( \delta = \omega_m \) when \( f = 0 \). Physically, this means that the optomechanical system corresponds to an absorption process on the probe field. When the fiber is connected (\( f = 0 \)) and its coupling is adjusted to the appropriate interaction strength, a transmission window with \( \eta \sim 1 \) appears at \( \delta = \omega_m \) with two sideband dips. By setting different \( f \) and \( \kappa \), an inverted OM IT phenomenon can also be observed in our system, meaning that we can achieve both slow light and fast light operations in such a system and the absorption and transparency can be of controllable flexibilities by switching the fiber. The inset in Fig. 6(a) shows that the optomechanically induced amplification phenomenon exhibits similar transmission rate curve with the OM IT phenomenon. The only difference between optomechanically induced amplification and OM IT is that \( \eta(\omega) \) of the former will be greater than 1 because the probe field is amplified by the auxiliary cavity.

In Fig. 6(b), we show that the OM IT-like transmission rate can also appear even if the auxiliary cavity.

Physical mechanism of enhancing OM IT. Now we discuss the physical mechanism why parity-time-
symmetric system can enhance OM IT by an extra gain. The underlying physics of OM IT is formally similar
to that of ordinary EIT in atomic system and we know that emerging traditional EIT in \( \Lambda \)-type atom system
requires three necessary conditions. Analogizing optomechanically induced transparency to EIT in atomic systems, Huang and Agarwal in ref. 47, discussed this relation and sketched three conditions for engendering OM IT, that is,

i. Driving frequency is set in the red sideband, that is, \( \Delta_\omega = \omega_\omega - \omega_\phi = \omega_\omega m \).

ii. Optical field loss needs to be much greater than oscillator dissipation, i.e., \( \gamma_\omega \ll \gamma \).

iii. Steady state of oscillator displacement is not zero, i.e., \( x(t \rightarrow \infty) \neq 0 \).

Intuitively, conditions 2 and 3 are contradictory in a normal dissipative optomechanical system because the condition \( \gamma_\omega \ll \gamma \) requires a large optical field decay, which will result in a decrease of the photon number in the cavity. When the photon number is reduced, the oscillator displacement will approach to equilibrium position and the condition iii is violated in this case. In order to solve this contradiction, strong driving is needed in previous works to ensure more photons preserved in the cavity and strong optomechanical interaction is demanded for a nonvanishing oscillator displacement \( x \). That is why Fig. 4 shows the OM IT effect appears only with strong driving and optomechanical interaction.

In PT-symmetric OMS, we find that the gain can increase the photon number. Under some certain parameters, however, this gain almost does not reduce the effective dissipation of the system, which ensures the condition
iii is also well satisfied. In other words, the contradictory OMIT conditions can be well met simultaneously in PT-symmetric OMS even with weak optomechanical coupling and driving, and it is the reason for the positive effect of PT-symmetric OMS on OMIT.

Now we verify the above analyses in mathematics and let us re-examine the steady state solution in Eq. (8). The oscillator displacement depends on the photon number in the cavity, which jointly affect the OMIT phenomenon via parameter β (see Eq. (9)). For the quantitative interpretation, we simplify β as

$$\beta = \frac{\sqrt{2}(g_0(\omega_p)^2}{\omega_m[\Delta - \Delta_{c} + \Delta_{c} + \gamma/4]} + \frac{\gamma}{2 - \kappa/4} \right)^{2} \times \frac{(g_0(\omega_p)^2}{\Delta_{c}^2 + \Gamma_{c}^2/4} \right),$$

(15)

by substituting Eq. (10) into Eq. (8) and neglecting Δ'. In Eq. (15), we find that β depends on the product rather than independent driving and coupling intensities, and it is the reason why increasing driving intensity or coupling intensity is equivalent and hyperbolic contour appears in Fig. 4. More importantly, it can be known from Eq. (15) that the extra auxiliary cavity can provide frequency and decay corrections in the expression of β, that is, Δ_{eff} = Δ_{c}(1 - c) and Γ_{eff} = γ - κ, where c = f^2/(κ^2 + γ^2/4). If κ is positive, i.e., a gain effect on the auxiliary cavity, both Δ_{eff} and Γ_{eff} will be reduced and correspondingly, β will be amplified because Δ_{eff} and Γ_{eff} appear in the denominator of Eq. (15).

Here we emphasize that the corrections of frequency (Δ_{eff}) and decay (Γ_{eff}) described above are only applicable to the discussion of β. In addition to the impact of β, the extra auxiliary cavity can also change dynamics characteristics of the system. As shown in Fig. 1(c), the cavity mode will take place a model splitting Δω = √(16/2 - (γ + κ)^2)/2 in the PT-SP regime. If we set J to be very close to the EP (J - 0.2501(κ + γ)), two splitting models will degenerate again with an effective decay γ_{eff} = γ - κ (see effective energy level diagram in Fig. 1(b)). A small κ allows us to ignore it, which causes the effective energy level decay is not affected by the auxiliary cavity. Therefore, the condition γ_{m} ≪ γ - κ - γ can still be well satisfied.

However, if the auxiliary cavity is also a dissipative cavity without gain, the auxiliary cavity (or other coupled quantum systems) cannot always guarantee to increase β because Γ_{eff} will increase when κ is negative. Though γ_{eff} = γ + |κ| is increased in this case, it cannot always guarantee for a positive effect on OMIT. This is the reason why our scheme is superior to other schemes which use passive quantum system to enhance OMIT.

In summary, we have proposed a theoretical scheme to enhance the OMIT phenomenon in PT-symmetric OMS. By calculating absorption (dispersion) spectrum and transmission rate, we have achieved an obvious OMIT window with experimentally accessible parameter values. In contrast to the single passive nonlinear OMS or coupled passive systems, our results illustrate that Re(χ) - 0 and Im(χ) - 0 are easy to be satisfied even though both driving and non-linear coupling are extremely weak. This enhancement effect is due to the additional gain. Specifically, a special physical mechanism ensures that the photon number in cavity is increased without reducing effective decay, and this is the key difference between our scheme and previous works in which the gain is considered just to damage OMIT phenomenon. We have found that this scheme allows us to flexibly control absorption and transparency by switching the fiber. Moreover, the probe field can also exhibit an electromagnetically induced amplification effect with the increasing gain rate. We thus believe the scheme proposed here may provide a promising choice for the unachievable strong nonlinear coupling in quantum optical devices, which is of potential applications for coherent manipulation, slow light operation and other utilizations in QIP.

**Methods**

**Derivation of the absorption and dispersion spectra.** As we have discussed in RESULTS, the steady-state solution of Eq. (3) can be expanded to the following form:

$$\dot{\phi} = \phi_0 + \phi_0 e^{-i\omega t} + \phi_0 e^{i\omega t},$$

(16)

under the condition of t → ∞, and the relation

$$\dot{\phi} = 0 - i\beta \phi_0 e^{-i\omega t} + i\beta \phi_0 e^{i\omega t},$$

(17)

can be intuitively obtained. Substituting Eq. (17) into Eq. (3), the dynamics equations of systemic mean values become:

$$0 = \omega_m P_0$$

$$0 = \frac{-\omega_m P_0 - \omega_m x_0 + \sqrt{g_0(\omega_p)^2}}{2}$$

$$0 = \left[-i\Delta_c + \frac{\gamma}{2}\right] a_1 + i\gamma a_2 + i\sqrt{g_0(\omega_p)^2} a_0 x + \Omega_d$$

$$0 = \left[-i\Delta_c + \frac{\kappa}{2}\right] a_2 + i\gamma a_1$$

$$0 = \left[i\Delta_c + \frac{\gamma}{2}\right] a_1 - i\gamma a_2$$

$$0 = \left[i\Delta_c + \frac{\kappa}{2}\right] a_2 - i\gamma a_1,$$

(18)
and

\[-i\delta^+ x e^{-i\delta t} = \omega_m\frac{p}{p} e^{-i\delta t}\]
\[-i\delta^+ p e^{-i\delta t} = -\frac{\gamma_m}{2} p e^{-i\delta t} - \omega_m x e^{-i\delta t} + \sqrt{2} g_0 (a_{1}^+ a_{1} + a_{1}^+ a_{1})\]
\[-i\delta^+ a_{1} e^{-i\delta t} = \left(-i\Delta_c - \frac{\gamma}{2}\right) a_{1} e^{-i\delta t} + if^+ a_{2} e^{-i\delta t} + i\sqrt{2} g_0 (a_{1}^+ x + x^+ a_{1}) + 1\]
\[-i\delta^+ a_{2} e^{-i\delta t} = \left(-i\Delta_c + \frac{\gamma}{2}\right) a_{2} e^{-i\delta t} + if^+ a_{1} e^{-i\delta t}\]
\[-i\delta^+ a_{1}^+ e^{-i\delta t} = \left(i\Delta_c - \frac{\gamma}{2}\right) a_{1}^+ e^{-i\delta t} - if^+ a_{2}^+ e^{-i\delta t} + i\sqrt{2} g_0 (a_{1}^+ x + x^+ a_{1})\]
\[-i\delta^+ a_{2}^+ e^{-i\delta t} = \left(i\Delta_c + \frac{\gamma}{2}\right) a_{2}^+ e^{-i\delta t} - if^+ a_{1}^+ e^{-i\delta t}\]

After ignoring these terms which contain \(e_p^2, e_p^2\) and \(|e_p|^4\). Here Eqs 18 and 19 correspond respectively to first-order and second-order steady states. Therefore, Eq. 19 can be further simplified as:

\[-i\delta^+ x = \omega_m\frac{p}{p}\]
\[-i\delta^+ p = -\frac{\gamma_m}{2} p - \omega_m x + \sqrt{2} g_0 (a_{1}^+ a_{1} + a_{1}^+ a_{1})\]
\[-i\delta^+ a_{1} = \left(-i\Delta_c - \frac{\gamma}{2}\right) a_{1} + if^+ a_{2} + i\sqrt{2} g_0 (a_{1}^+ x + x^+ a_{1}) + 1\]
\[-i\delta^+ a_{2} = \left(-i\Delta_c + \frac{\gamma}{2}\right) a_{2} + if^+ a_{1}\]
\[-i\delta^+ a_{1}^+ = \left(i\Delta_c - \frac{\gamma}{2}\right) a_{1}^+ - if^+ a_{2}^+ - i\sqrt{2} g_0 (a_{1}^+ x + x^+ a_{1})\]
\[-i\delta^+ a_{2}^+ = \left(i\Delta_c + \frac{\gamma}{2}\right) a_{2}^+ - if^+ a_{1}^+\]

Based on Eqs 18 and 20, the steady-state solutions can be given finally by:

\[0 x = \frac{\sqrt{2} g_0}{\omega_m} |a_{1}|\]
\[0 a_{1} = \frac{\Omega_d}{i(\Delta_c - \Delta')} + \frac{\gamma}{2} + \frac{\beta}{i(\Delta_c - \Delta')}\]

and

\[+ a_{1} = \frac{\left\{\omega_m^2 - \delta^2 - i\delta\gamma_m/2\right\} - i(\Delta - \delta) + \frac{\gamma}{2} + L - i(\Delta + \delta) + \frac{\gamma}{2} + M + \beta}{(\omega_m^2 - \delta^2 - i\delta\gamma_m/2)(i(\Delta - \delta) + \frac{\gamma}{2} + L - i(\Delta + \delta) + \frac{\gamma}{2} + M + \beta)(12\Delta + L - M)}\]

which are exactly the same with Eqs 8 and 9 in Results.

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Author Contributions
W.L. and H.S. proposed the basic idea of enhanced optomechanically induced transparency in parity-time-symmetric optomechanical system; Y.J. and C.L. analyzed the system stability corresponding to Figure 2; W.L. and H.S. wrote the main manuscript text and all authors have given approval to the final version of the manuscript.

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