Metastable level properties of the excited configuration \(4p^64d^84f\)

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December 22, 2014

Abstract. Metastable levels in rhodium-like ions with the ground configuration \(4p^64d^9\) and the excited configurations \(4p^64d^84f\) and \(4p^54d^{10}\) are investigated. The \textit{ab initio} calculations of the level energies, radiative multipole transition probabilities are performed in a quasirelativistic Hartree-Fock approximation employing an extensive configuration interaction based on quasirelativistic transformed radial orbitals. A systematic trends in behavior of calculated radiative lifetimes of the metastable levels are studied for the ions from \(Z = 60\) to \(Z = 92\). The significance of the radiative transitions of higher multipole order (\(M2\) and \(E3\)) for the calculated radiative lifetimes is demonstrated and discussed.

Keywords: quasirelativistic approach, many-electron ions, metastable levels, radiative lifetimes

PACS: 31.10.+Z, 31.15.ag, 32.70.Cs
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1. Introduction

Our previous investigation of the $4p^54d^{N+1}$ and $4p^54d^{N-1}4f$ configuration metastable levels \cite{1} has revealed that the ions of the rhodium isoelectronic sequence have large number of metastable levels with comparatively large radiative lifetimes. Therefore, there is an evident need for a more detailed study. The relevance and significance of the magnetic quadrupole ($M_2$) and electric octupole ($E_3$) transitions for such metastable levels has been quite comprehensively described in \cite{1}. That investigation of the significance of the $M_2$ and $E_3$ transitions involved large number of atoms of different ionization stages, therefore the calculations were performed without inclusion of correlation corrections.

Metastable levels, which do not have the electric dipole decay channels, are essential in study of both astrophysical and laboratory plasmas \cite{2, 3, 4} or in analysis of experimental spectra of various ions \cite{6, 7}, including very important tungsten ions (see, e.g., \cite{4, 5, 8}). As it was concluded in \cite{2}, the parameters of high multipole order radiative transitions and the radiative lifetimes of metastable levels were “essential building blocks in radiative-collisional models simulating spectra”. Traditionally, the radiative lifetimes for the excited metastable levels of the ground configuration are determined from the calculated electric quadrupole ($E_2$) and magnetic dipole ($M_1$) transition probabilities, see \cite{3, 9, 10, 11}. These types of radiative transitions also can be important when the radiative lifetimes of the levels of excited configurations are calculated \cite{1, 6, 5}. Very recently the multiconfiguration Dirac-Hartree-Fock approach was applied to determine the $E_1$, $M_1$, $E_2$, and $M_2$ transition parameters for the ions up to $Z = 36$ (see \cite{12, 13} and the references therein).

Many of the excited opposite-parity configuration levels are metastable ones due to a limited number of the $E1$-decay channels when the ground configuration has just few levels. In such a situation, the $E3$ and $M2$ radiative transitions become important decay channels - although those transitions traditionally are considered as weak and insignificant for the calculated radiative lifetimes \cite{6, 14, 15}. The relativistic many-body perturbation theory was employed to investigate the Fe$^{9+}$ ion properties in \cite{14}. There the details describing the rise and decay of metastable levels are given and discussed. Transitions originating from the W$^{28+}$ ion metastable levels were observed in EBIT experiment \cite{6}. It was concluded after additional relativistic many-body perturbation theory (RMBPT) calculations that some of the excited configuration $4p^64d^94f$ levels have “extremely long lifetimes”. The radiative transitions in Ni-like ions from $Z = 30$ to $Z = 100$ were studied in \cite{15} using RMBPT which includes the Breit interaction.

In our earlier work \cite{1} we have demonstrated that the metastable levels of the excited configurations also exist for the ions other than Ni-like \cite{15} or Pd-like ions \cite{6} that have all closed shells in the ground configurations. Similar metastable levels can exist in the ions with the ground configurations, which have open 4d shells, e.g., in the Rh-like ions with the ground configuration $4p^64d^9$. Unfortunately, to our best knowledge, there are no theoretical or experimental works investigating the metastable level properties of
The aim of the present work is to demonstrate how the correlation corrections can change calculated radiative lifetimes of the metastable levels. Another issue, which has emerged during our previous investigation, is the location of the $4p^64d^85s$ and other excited configurations. The location in the energy spectrum of the excited configurations is important as the additional decay channels can emerge for some levels of the configuration $4p^64d^84f$.

Traditionally, the levels are considered as the metastable ones when the electric dipole ($E1$) transitions from these levels are forbidden by the selection rules for the total orbital quantum number $J$. The metastable levels have two important channels of the radiative decay. One channel is comprised from the magnetic dipole and electric quadrupole transitions to the levels of the same parity, usually to the levels of the same configuration; nevertheless, the transitions among the levels of different configurations with the same parity are also allowed. The second channel constitutes the magnetic quadrupole and electric octupole transitions to the levels of the different-parity configurations. The strongest radiative transitions are those connecting the excited configuration levels to the ground configuration levels. Nevertheless, some radiative transitions connecting the levels of opposite-parity excited configurations also can be relatively strong and significant in the calculation of radiative lifetimes.

The object of the present investigation is the configuration $4p^64d^84f$ levels with high values of the total angular momentum $J$ and their possible decay channels. We demonstrate how their relative positions in the energy spectra and their radiative properties change along the rhodium isoelectronic sequence.

The calculations were performed in the quasi-relativistic (QR) Hartree-Fock approximation [16, 17], using several different configuration interaction (CI) expansion sets. As the first step, the energy spectra were calculated for the ions from $Z = 48$ to $Z = 92$ adopting a small CI expansion, which consisted of few configurations for each parity. In this particular approximation (further we call it CI1), all electrons, both from configurations under study and the admixed ones, were described purely by solutions of the quasi-relativistic equations [16, 17]. After the results of the energy spectra were analyzed, the subsequent investigation of the rhodium-like ions was limited to the ions starting from $Z = 60$. Further, at the second step, the correlation corrections were included using the extensive CI wavefunction expansion. In this case, the electrons of the admixed configurations were described by the solutions of the quasi-relativistic equations and by the quasi-relativistic transformed radial orbitals [18].

2. Calculation method

We use the quasi-relativistic Hartree-Fock approximation in our \textit{ab initio} calculation of the ion level energies and radiative transition parameters, such as the line strengths $S$, weighted oscillator strengths $g_f$, radiative transition probabilities $A$. The quasi-relativistic calculations are performed in a way described in our previous studies.
At the start, the QR equations for the ground configuration $4p^64d^9$ are solved. Then the QR equations for the $4f$ and $5l$ (up to $5g$) radial orbitals (RO) are solved in a frozen-core potential. The relativistic corrections are included in the Breit-Pauli approach specially adopted for the quasirelativistic radial orbitals [18]. We employ the same RO basis both for the even and odd configurations.

The correlation corrections are included by applying the CI method. In our basic approach, called CI1, the CI includes only the configurations under research. The even configurations $4p^64d^9$, $4p^64d^85s$ and $4p^64d^75s^2$ and the odd ones $4p^54d^{10}$, $4p^64d^85p$ and $4p^64d^84f$ make the smallest CI expansion sets used in the calculations. No admixed configurations are included in this approach. The results obtained in this approach (CI1) are used for the rough evaluation of the energy spectra of the configurations under investigation and the radiative transitions of different multipole orders.

The subsequent CI expansion is formed from the set of the admixed configurations generated by virtually exciting one or two electrons from the $4l$ and $5l$ shells of the adjusted configurations. The virtually excited electrons up to $5g$ in the admixed configurations are described by the quasirelativistic RO. Although this RO basis is rather limited, the total amount of the admixed configurations, which can interact with the adjusted ground configuration, reaches 72 for $Z = 60$. For the complex of the odd configurations $4p^54d^{10}$ and $4p^64d^84f$, the number of possible admixed configurations adds to 146. The strongly interacting configurations are selected by selection methods [21, 22] applied in all our previously described calculations. The selection criterion [18] is $1 \cdot 10^{-4}$. This means that all configurations with the mean weight larger than $1 \cdot 10^{-4}$ are included into the CI expansion. There are 40 admixed even configurations and 31 odd one after the selection rules are applied for the ion $Z = 60$. The admixed configurations produce very large amount of configuration state functions (CSF). This number is reduced by applying the CSF reduction methods described in [23].

For each ion in the isoelectronic sequence, its own quasirelativistic RO basis is determined, the list of admixed configurations is generated by the same virtual excitations, and the admixed configurations are selected using the same criterion. Therefore the CI expansion differs for each particular ion and the number of admixed configurations decreases with increasing nuclear charge $Z$. For $Z = 92$, the CI expansion is significantly smaller, and only 28 even configurations and 22 odd ones are selected. The results obtained by such a way are labeled as CI2.

In order to perform more accurate and reliable calculations in the multiconfiguration approximation, the basis of determined quasirelativistic RO is supplemented with the transformed radial orbitals (TRO) described by variable parameters [18, 19, 20]. The TRO are determined for the radial orbitals having the principal quantum number $6 \leq n \leq 10$ and for all possible values of the orbital quantum number $l$, i.e. $l \leq 9$. The admixed configurations are generated by virtually exciting one or two electrons from the $4l$ and $5l$ shells of the adjusted configurations. The virtually excited electrons are described by the quasirelativistic RO ($n \leq 5$ and $l \leq 4$) and by the quasirelativistic TRO for the remaining virtually excited electrons.
Table 1. The radiative lifetimes (in ns) of the W$^{29+}$ ion configuration 4d$^5$4f metastable levels.

| level                  | CI1   | CI2   | CI3   | CI4   |
|------------------------|-------|-------|-------|-------|
| 4d$^5$(3F)4f 4$I_{15/2}$ | 2.16E+07 | 1.67E+07 | 1.71E+07 | 1.70E+07 |
| 4d$^5$(G)4f 2$K_{15/2}$ | 4.64E+04 | 4.73E+04 | 4.77E+04 | 4.80E+04 |
| 4d$^5$(3F)4f 4$I_{13/2}$ | 5.54E+15 | 2.52E+15 | 3.76E+15 | 4.27E+15 |
| 4d$^5$(G)4f 4$H_{13/2}$ | 6.04E+06 | 6.54E+06 | 6.70E+06 | 6.75E+06 |
| 4d$^5$(3F)4f 2$K_{13/2}$ | 3.18E+04 | 3.13E+04 | 3.28E+04 | 3.30E+04 |
| 4d$^5$(3F)4f 2$I_{13/2}$ | 1.60E+04 | 1.67E+04 | 1.65E+04 | 1.66E+04 |
| 4d$^5$(G)4f 2$I_{13/2}$ | 3.24E+04 | 3.37E+04 | 3.41E+04 | 3.44E+04 |
| 4d$^5$(3F)4f 4$H_{11/2}$ | 1.82E+07 | 1.97E+07 | 2.12E+07 | 2.18E+07 |
| 4d$^5$(3F)4f 2$H_{11/2}$ | 6.03E+05 | 6.27E+05 | 6.54E+05 | 6.56E+05 |
| 4d$^5$(3F)4f 4$G_{11/2}$ | 4.52E+05 | 4.94E+05 | 4.92E+05 | 5.01E+05 |
| 4d$^5$(3F)4f 4$I_{11/2}$ | 1.58E+04 | 1.60E+04 | 1.61E+04 | 1.62E+04 |
| 4d$^5$(D)4f 2$H_{11/2}$ | 8.10E+03 | 8.31E+03 | 8.36E+03 | 8.40E+03 |
| 4d$^5$(F)4f 4$F_{9/2}$  | 5.35E+04 | 4.72E+04 | 4.78E+04 | 4.81E+04 |
| 4d$^5$(F)4f 2$G_{9/2}$  | 3.02E+04 | 2.53E+04 | 2.52E+04 | 2.51E+04 |
| 4d$^5$(D)4f 2$H_{9/2}$  | 1.49E+06 | 1.64E+06 | 1.69E+06 | 1.72E+06 |
| 4d$^5$(D)4f 2$G_{9/2}$  | 3.90E+05 | 4.49E+05 | 4.53E+05 | 4.64E+05 |
| 4d$^5$(F)4f 4$I_{9/2}$  | 1.69E+04 | 1.67E+04 | 1.68E+04 | 1.68E+04 |
| 4d$^5$(P)4f 4$F_{9/2}$  | 2.22E+04 | 2.26E+04 | 2.29E+04 | 2.31E+04 |

Due to the larger basis of the RO, the number of admixed configurations is larger than that in the previous CI2 expansion. 83 even configurations and 73 odd ones are selected for the ion Z = 60 using the same selection criterion 1 \cdot 10^{-4}. The number of admixed configurations in the CI expansion consistently decreases along the isoelectronic sequence, and, for Z = 92, there are just 42 even and 39 odd admixed configurations. The results obtained by such technique are named as CI3. The comparison of the results, determined in the CI2 and CI3 approaches, indicates that the level radiative lifetime values are quite close along the isoelectronic sequence, the difference does not exceed 10%.

The calculations employing even the larger CI expansion have been performed for several selected ions in order to verify applied approach and to prove the convergence of the CI. These results were determined using the same large base of RO as in the CI3 approach, but the selection criterion 1 \cdot 10^{-5} was applied. In Table II they are named as CI4. The number of admixed configurations in the CI4 approach reaches several hundreds for each parity. These calculations are very complex, whereas their results differ from the CI3 data only slightly. The differences among of the lifetime values obtained employing different CI expansions are demonstrated in Table II.

We must underline that we use the LS-coupling scheme for the identification of all investigated levels of the rhodium isoelectronic sequence. Although this identification can vary along the sequence, we use a uniform identification according to that of the tungsten ion (Z = 74) levels.

This table contains the radiative lifetime values of the W$^{29+}$ ion metastable levels determined using different CI expansion sets. The levels discussed in the present work
are included in Table 1. The averaged difference between the values determined in the CI1 approach and those from the CI2 approach is approximately 10%, the averaged difference between the values determined in the CI2 approach and the CI3 approach is approximately 6%. Further, the averaged difference between the results from the CI3 and CI4 approximations decreases to nearly 1%. So it is clear that larger CI expansions are unnecessary as the determined lifetimes of metastable levels change just very slightly. The exception is the level $4d^8(3F)4f^4I_{13/2}$ with very large radiative lifetime. It is obvious that the convergence test fails for this level. Here the radiative transition probabilities are strongly affected by the cancellation effects causing the lifetime value to change significantly.

Definitely, the values obtained employing the large expansion CI4 are more accurate and reliable, but, for a systematic study of the rhodium-like ion properties, the results of the CI3 approach are accurate enough. Furthermore, although the number of the admixed configurations in CI3 is not very large, the total number of CSF, due to the open $4d$ shells, is very large even after the reduction procedure. Consequently, the calculation consumes very large amount of computing resources. This must be considered as an important factor since we investigate an isoelectronic sequence which has large number of ions.

As it is mentioned in the beginning of this section, our calculation method is discussed extensively in [18]. All two-electron interactions are included in the same way as it is done in the conventional Breit-Pauli approximation. For this purpose, the codes [24, 25, 26] are adapted for the quasirelativistic radial orbital peculiarities together with our own computer codes.

The level energies, radiative lifetimes and spectroscopic parameters of the $E1$, $M1$, $E2$, $M2$ and $E3$ transitions calculated using the CI3 approach are published in the database ADAMANT (http://www.adamant.tfai.vu.lt/database) for the ions from $Z = 60$ to $Z = 92$.

3. Results and discussion

The ground configuration of the rhodium-like ions is $4p^64d^9$. It has two energy levels, $^2D_{3/2}$ and $^2D_{5/2}$. The excited configuration $4p^54d^{10}$ is also quite a simple one and has only two levels, $^2P_{1/2}$ and $^2P_{3/2}$. These two excited levels have an usual radiative decay channel through the electric dipole ($E1$) transitions, therefore they are not metastable levels. Further in this work, we do not discuss them. The excited configuration $4p^64d^84f$ has quite a large number of the levels with high values of the total angular momentum $J$. These levels can not decay to the ground configuration by the $E1$ transition. Therefore their radiative lifetimes can be large if the excited levels with the opposite parity, e.g., belonging to the configuration $4p^64d^85s$, are located above them. The location of the excited configurations, including the $4p^64d^85s$, $4p^64d^75s^2$ and $4p^64d^85p$ is discussed further in this section.
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3.1. Configuration locations

At the start of our investigation, calculations in the CI1 approach along the isoelectronic sequence (from $Z = 48$ to $Z = 92$) are performed. The level energies of the even configurations $4p^64d^85s$, $4p^64d^75s^2$ and the odd-configuration complex $4p^54d^84f + 4p^54d^{10}$ relative to the lowest level $4p^64d^92D_{5/2}$ of each ion.

Figure 1. Energy levels (in $10^6$ cm$^{-1}$) of the even configurations $4p^64d^85s$, $4p^64d^75s^2$ and the odd-configuration complex $4p^54d^84f + 4p^54d^{10}$ and the even configurations $4p^64d^9$ of each ion.

3.1. Configuration locations

At the start of our investigation, calculations in the CI1 approach along the isoelectronic sequence (from $Z = 48$ to $Z = 92$) are performed. The level energies of the even configurations $4p^64d^9$, $4p^64d^85s$, and $4p^64d^75s^2$ and the odd ones $4p^54d^{10}$, $4p^64d^85p$, and $4p^64d^84f$ are determined by this rather simple way. As it is described in the previous section, only the interactions between configurations under research (adjusted configurations) are included. The CI1 results show how the configuration position changes with $Z$ increasing. A fraction of these results is presented in Fig. 1. The levels of the odd configurations $4p^54d^{10}$ and $4p^64d^84f$ and the even configurations $4p^64d^9$...
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and $4p^6 4d^7 5s^2$ are plotted relative to the ground level $4p^6 4d^9 2D_{5/2}$.

When $Z = 48$, the levels of the $4p^6 4d^8 4f$ configuration have additional decay channels - not only those to the levels of the ground configuration. The decay channels to the levels of the $4p^6 4d^8 5s$ and $4p^6 4d^7 5s^2$ are open as these configurations are energetically close to the ground configuration and are lower than the $4p^6 4d^8 4f$ configuration levels. As one can see from Fig. 1, the energy difference between the $4p^6 4d^7 5s^2$ configuration and other configurations increases rapidly. Consequently, this configuration does not have any significance for the lifetimes of the $4p^6 4d^8 4f$ levels already at $Z = 52$. The configuration $4p^6 4d^8 5s$ is located lower than the configuration $4p^6 4d^8 4f$ at $Z \leq 60$.

Using a strict definition, the $4p^6 4d^8 4f$ levels with $J \geq 9/2$ are not metastable ones in the case of such location of the excited configuration; only the levels with $J = 15/2$ can be specified as metastable ones. The $E1$ transitions from the levels with $J = 15/2$ to the configurations $4p^6 4d^8 5s$ and $4p^6 4d^7 5s^2$ are forbidden by the selection rules for $J$. These transitions are allowed from the levels with $J = 13/2$, but they appear only for the ion $Z = 48$; moreover, they are weaker than the $M1$ and $E2$ transitions. For other ions, the levels with $J = 13/2$ are metastable ones, because the levels of $4p^6 4d^7 5s^2$ are located higher.

Furthermore, the lifetimes of the levels with $J = 13/2$ are reduced because of the $E2$ and $M1$ transitions to the configuration $4p^6 4d^8 5p$ of the same parity as the $4p^6 4d^8 4f$ one. The position of the $4p^6 4d^8 5p$ configuration in the energy spectra is presented in Fig. 2. One can see from Fig. 2 that the $4p^6 4d^8 5p$ configuration is located in the same area as the $4p^6 4d^8 4f$ and $4p^5 4d^{10}$ configurations for the ions $Z \leq 60$. Hence, for the reliable results, these configurations must be investigated as a single complex. Figures 1 and 2 demonstrate that the levels of the configuration $4p^6 4d^8 4f$ with high values of $J$ can be called the metastable ones only for the ions with $Z \geq 60$.

Figures 1 and 2 present the results of the simple CI1 approach. This approach is accurate enough for the basic calculation and for the initial assessment of the determined results. In order to determine more accurate values of the radiative transition parameters, the approaches described in the Sect. 2 with the larger CI expansions are applied for the ions from $Z = 60$ up to $Z = 92$. The results of the approaches CI2 and CI3 have been compared, and they indicate similar trends in properties of metastable levels. Nevertheless, we use the results of more reliable CI3 approach for the comprehensive study. The results CI4 are the most accurate data, but the CI4 calculations were performed only for few ions, because of their complexity and large use of the computer resources.

3.2. The lifetimes of $J = 15/2$ levels

The configuration $4p^6 4d^8 4f$ has two levels with $J = 15/2$. The lower one is $4d^8 (3F)4f \ ^4I_{15/2}$ and the higher level is $4d^8 (1G)4f \ ^2K_{15/2}$. The radiative lifetimes of these levels are presented in Fig. 3 for the ions from $Z = 60$ to $Z = 92$.

The radiative lifetime of the lower level is very large throughout the entire
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isoelectronic sequence due to its location in the energy spectra. When $Z = 60$ and $Z = 62$, this level does not have radiative transitions of the calculated multipole orders (in our case - up to $E3$) as there are no lower-lying levels with an appropriate $J$. With $Z$ increasing, the energy of the level $4d^8(^3F)4f\ ^4I_{15/2}$ increases. Moreover, it increases faster than the energy of some other levels. The transition probabilities for a decay of the level $4d^8(^3F)4f\ ^4I_{15/2}$ are presented in Fig. 4. The $E2$ transitions are denoted by circles, $M1$ transitions are denoted by triangles in this and in other figures, presenting the transition probabilities.

Another level with $J = 15/2$, namely $4d^8(^1G)4f\ ^2K_{15/2}$, is located quite high in the energy spectra. Here the radiative lifetime is determined by some dozen $E2$ and $M1$
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Figure 3. Radiative lifetimes (in ns) of the levels with \( J = 15/2 \).

Figure 4. Transition probabilities (in s\(^{-1}\)) from the metastable level 4d⁸(3F)4f \( ^4I_{15/2} \). Black triangles and black circles denote transitions to the 4d⁸(3F)4f \( ^4I_{13/2} \), white circles - to the 4d⁸(3F)4f \( ^4H_{11/2} \).
transitions with slowly increasing transition probability values. The strongest one is the $M1$ transition to the $4d^8(^3F)4f^4I_{15/2}$ level. The transition probability value increases from $3.9 \cdot 10^2$ s$^{-1}$ (for $Z = 60$) to $8.4 \cdot 10^5$ s$^{-1}$ (for $Z = 92$). As it is seen from Table 1 for $Z = 74$, the lifetime is $4.8 \cdot 10^4$ s$^{-1}$. In general, the $M1$ transitions determine the radiative lifetimes of this level for all investigated Rh-like ions.

### 3.3. The lifetimes of $J = 13/2$ levels

The configuration $4p^64d^84f$ has five levels with $J = 13/2$. The radiative lifetimes of the lowest level $4d^8(^3F)4f^4I_{13/2}$ are presented in Fig. 5. The lifetimes of this level increase with the $Z$ increasing, because the location of this level in the energy spectrum is going down, i.e. the energy of this level relative to the ground level decreases unlike in the case of the lowest level with $J = 15/2$, where the energy difference increases when the $Z$ increases.

The $4d^8(^3F)4f^4I_{13/2}$ level has three decay channels open when $Z = 60$ and $Z = 62$. When $Z \geq 64$, only one $E2$ transition to the $4d^8(^3F)4f^4I_{9/2}$ level is allowed. This transition determines the lifetimes of the level $4d^8(^3F)4f^4I_{13/2}$ up to $Z = 80$. For the higher $Z$, the decay channel to the $4d^8(^3F)4f^4I_{9/2}$ is also closed. For the ions with $Z \geq 82$, the $4d^8(^3F)4f^4I_{13/2}$ level does not have a radiative decay channel, because the radiative transitions of all calculated multipole orders to the levels, which are lower than the investigated one, are forbidden by the selection rules for $J$. The lifetime of this level calculated for the tungsten ion is $3.8 \cdot 10^{15}$ s$^{-1}$ (see Table 1), this is close to 49 days.

The lifetimes of other four levels with $J = 13/2$ are presented in Fig. 5. One can see from this figure that the radiative lifetimes consistently decrease going up along the
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Figure 6. Radiative lifetimes (in ns) of four levels with $J = 13/2$.

Figure 7. Transition probabilities (in $s^{-1}$) from the level $4d^8(3F)4f\ ^4H_{13/2}$. Black triangles and circles denote transitions to the level $4d^8(3F)4f\ ^4H_{11/2}$, white triangles and circles - to the level $4d^8(3F)4f\ ^4I_{13/2}$, grey triangles and circles - to the $4d^8(3F)4f\ ^4I_{15/2}$ level, black&white circles - to the $4d^8(3F)4f\ ^4F_{9/2}$ level.
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sequence. The lifetimes of three levels are very similar in magnitude, but the lifetimes of the level $4d^8(3F)4f^4H_{13/2}$ are always noticeably (by two orders of magnitude) larger. This particular level is located relatively low in the energy spectra. Nevertheless, it is located higher compared to the level $4d^8(3F)4f^4I_{13/2}$. The level $4d^8(3F)4f^4H_{13/2}$ has seven allowed transitions to the lower levels, three of these being the $M1$ transitions and four the $E2$ ones. Their transition probabilities are presented in Fig. 7.

As one can see from Fig. 7, the dependence of the transition probability $A$ on a nuclear charge $Z$ has a marked minimum for some lines. This is caused by cancellation effects. A radiative transition probability $A$ is proportional to a square of the transition operator matrix element. When this matrix element changes its sign, it’s value becomes very close 0 at a particular value of $Z$. Consequently, the square of the radiative transition operator matrix element has a sharp minimum at this $Z$, causing such a peculiar shape of $A(Z)$. The similar shapes of $A(Z)$ can also occur in other isoelectronic sequences [1]. Very similar behavior of the radiative transition parameters proportional to a square of matrix element one can see in [15].

Another three levels with $J = 13/2$ are located comparatively high in the energy spectra and have much more open decay channels. Furthermore, the strongest transition probabilities from these levels are from 10 to 100 times larger than those of the low-lying level $4d^8(3F)4f^4H_{13/2}$. The $M1$ transitions are dominant, as in Fig. 7 and they determine the radiative lifetimes of the levels with $J = 13/2$. 

\begin{figure}[h]
\centering
\includegraphics[width=\linewidth]{figure8.png}
\caption{Radiative lifetimes (in ns) of five levels with $J = 11/2$.}
\end{figure}
The configuration $4p^64d^84f$ has nine levels with $J = 11/2$. Figure 8 presents the radiative lifetimes of five levels, whereas the lifetimes of other four levels are in the region between the lowest two curves. The level $4d^8(3F)4f\ 4H_{11/2}$ has the largest lifetime. The transition probability values of the radiative transitions from this level are presented in Fig. 9. The $M1$ transition probabilities are significantly larger than the $E2$ transition probabilities. This feature is similar to properties of the levels with $J \geq 13/2$.

Unlike the levels with $J \geq 13/2$, this level (as well as other levels with $J = 11/2$) can decay to the ground configuration in accordance to the selection rules for $J$. This complementary decay channel is an electric octupole transition. The probabilities of the $E3$ transitions are marked by squares in Fig. 9 (and in the following plots, as they appear). The $E3$ transition probability for the $4d^8(3F)4f\ 4H_{11/2}$ level has the largest value at the top of the investigated isoelectronic sequence. That means that the $E3$ transition is the dominant one in determining the radiative lifetime. The inclusion of the $E3$ transition probability into the radiative lifetime calculation decreases the lifetime by factor of 1.8 at the top of the sequence. It is seen from Fig. 9 that the probabilities of two $M1$ transitions and one $E3$ transition determine the calculated radiative lifetime value of the $4d^8(3F)4f\ 4H_{11/2}$ level, because the probabilities of three $E2$ transitions are distinctly smaller. For the calculated lifetime of this level, the $E3$ transition is critical, especially at the top end of investigated sequence, where its value is largest. Other levels with $J = 11/2$ also acquire an open decay channel via the $E3$ transition.
Nonetheless, here the $E3$ transition probability is always somewhat smaller compared to several $M1$ transitions, although it is larger than the $E2$ transition probabilities. For these levels, the number of $M1$ transitions is large, but there is only one $E3$ transition, therefore it does not constitute a significant decay channel as it does in the case of level $4d^8(3F)4f\,^4H_{11/2}$.

Here we must explain that all calculated transition probabilities with the values larger than $1\cdot10^{-18}$ s$^{-1}$ from the levels under consideration are included into determining of the radiative lifetime. It is important because only a limited number of transitions are allowed from the low-lying levels. If the level lies high in the energy spectra, the number of the $M1$ and $E2$ transitions is not so limited since a large number of the levels with appropriate $J$ values are located lower. Since there are only two levels of the ground configuration, the number of the $M2$ and $E3$ transitions always is one or two depending of the $J$ value of the initial level. Usually, the high-lying levels have relatively small radiative lifetimes as they have much more open decay channels to the levels of the same configuration.

The number of the transitions to the ground configuration is determined only by the selection rules. This number does not depend on the level location in the energy spectra. The $E3$ transition probability values are similar in magnitude as the $M1$ probability values for a particular level. Nevertheless, the $E3$ transitions are rather insignificant decay channels for the high-lying levels as there is a large number of the $M1$ transitions allowed from these levels. Furthermore, the dominant $M1$ transition probabilities are larger for the high-lying levels, as in the case of levels with $J = 13/2$.

3.5. The lifetimes of $J = 9/2$ levels

The configuration $4p^64d^84f$ has thirteen levels with $J = 9/2$. Figure 10 presents the radiative lifetimes only for six levels. The lifetimes of other seven levels are close to two curves, which present the lifetimes of levels $4d^8(3F)4f\,^4I_{9/2}$ and $4d^8(3P)4f\,^4F_{9/2}$ (white and gray triangles). The excluded levels are located higher than the presented ones, and their radiative lifetime dependencies on $Z$ are very similar to those of the latter two levels, both in a magnitude and in a shape.

The radiative lifetimes of the lowest level $4d^8(3F)4f\,^4F_{9/2}$ (white circles) are not the largest ones, unlike to the previous ($J > 9/2$) cases. These lifetimes are significantly smaller as there is an additional decay channel allowed by the selection rules. The transition probabilities for this level are presented in Fig. 11. One can notice from Fig. 11 that the $M2$ transition (black diamonds) to the ground configuration is the strongest one. On top of that, two $E3$ transitions are allowed, and their transition probability values are larger than those of the $E2$ transitions. These three radiative transitions ($M2$ and $E3$) to the ground configuration are very significant. At the lower end of the investigated sequence, their inclusion decreases the calculated radiative lifetime almost by 2000 times. Further, the influence of these transitions rapidly decreases, and the inclusion of them into the radiative lifetime calculations decreases the lifetime value.
Figure 10. Radiative lifetimes (in ns) of six levels with $J = 9/2$.

Figure 11. Transition probabilities (in s$^{-1}$) from the level $4d^9(3F)4f\ ^4F_{9/2}$. Black triangles and circles denote transitions to the level $4d^9(3F)4f\ ^4F_{7/2}$, white circles - to the level $4d^9(3F)4f\ ^4F_{9/2}$, black diamonds and squares - to the level $4d^9\ ^2D_{5/2}$, white squares - to the level $4d^9\ ^2D_{3/2}$.
approximately 13 times at \( Z = 92 \). The reasons for a rapid decrease of the \( E3 \) transition values in Fig. 11 are explained in Sect. 3.3. According to the selection rules for \( J \), one \( M2 \) transition and two \( E3 \) transitions are allowed from this and all other levels with \( J = 9/2 \). The number of the \( M1 \) and \( E2 \) transitions depends on the location of level examined.

Next level with \( J = 9/2 \) is the level \( 4d^8(3F)4f \, ^2G_{9/2} \). Black squares denote the radiative lifetime of this level in Fig. 11. Here the \( M2 \) transition is the strongest one. But the \( M2 \) transition probability value starts to decrease slowly from \( Z = 84 \). Several strong \( M1 \) transitions behave in a similar way. Such a drop of the transition probability values causes the radiative lifetime for this level to increase when \( Z \geq 84 \). The \( M2 \) and \( E3 \) transitions are the most significant ones at \( Z = 64 \) and \( Z = 66 \). The inclusion of their probabilities into the the radiative lifetime calculations decrease the lifetime value by more than 450 times. At the top end of the sequence, this decrease drops to 3 times.

At the beginning of the sequence, the level \( 4d^8(1D)4f \, ^2H_{9/2} \) has the largest radiative lifetime value (see Fig. 11 black circles). The quite sharp change of the calculated lifetime for this level between \( Z = 80 \) and \( Z = 86 \) is caused by the rapid increase of the strongest \( M2 \) transition probability value together with that of several \( M1 \) transitions. Like for other previously discussed levels with \( J = 9/2 \), there is one \( M2 \) transition, two \( E3 \) transitions and few \( M1 \) and \( E2 \) transitions. The \( M2 \) transition is the strongest one for \( Z > 70 \), the \( E3 \) transitions are not so strong, but their transition probability values are larger than those of the \( E2 \) transitions. The \( M2 \) and \( E3 \) transitions affect the lifetime of \( 4d^8(1D)4f \, ^2H_{9/2} \), but no so significantly as the lifetimes of previously discussed levels due to the amount of the \( M1 \) and \( E2 \) transitions allowed from this level.

The radiative lifetimes of other levels with \( J = 9/2 \), located higher in the energy spectra than the discussed ones, are determined mainly by the numerous \( M1 \) transitions. The \( E2 \) transitions are also allowed, but they are much weaker than the \( M1 \) transitions. So here the influence of the \( E3 \) and \( M2 \) transitions is not so outstanding. Although these transitions cannot be discarded at all, when the nuclear charge \( Z \) is between 60 and 70, these transitions decrease the calculated lifetime value up to 4 times. Usually, the decrease drops very quickly. For heavy ions \( (Z > 80) \), the lifetime values change only by few percent.

### 4. Summary

The configurations \( 4p^64d^9 \), \( 4p^54d^{10} \), \( 4p^64d^85s \), \( 4p^64d^75s^2 \), \( 4p^64d^85p \) and \( 4p^64d^84f \) of the Rh-like ions are investigated using the quasirelativistic approach with a small CI1 expansion for the range from \( Z = 48 \) up to \( Z = 92 \). Calculation results demonstrate that the ions with \( Z < 60 \) do not have metastable levels in the excited configurations, except for two levels of the the configuration \( 4p^64d^84f \) with the largest total angular momentum, \( J = 15/2 \).

The levels with \( J = 9/2 \) of the \( 4p^64d^85s \) and \( 4p^64d^75s^2 \) configurations are
located relatively low in the energy spectra, and the $E1$ transitions from the $4p^64d^84f$ configuration levels with $J < 15/2$ are possible to the levels of these configurations. Furthermore, the decay channels through the $M2$ and $E3$ transitions to the $4p^64d^85s$ and $4p^64d^75s^2$ are open.

On the other hand, the decay channel through the $M1$ and $E2$ transitions is open for the $4p^64d^85p$ configuration. For the ions with $Z < 60$, the configuration $4p^64d^85p$ is close in energy to the $4p^64d^84f$ and $4p^54d^{10}$. These energetically close and strongly interacting excited configurations must be investigated as a single configuration complex. The present investigation was devoted to the detailed investigation of the configuration $4p^64d^84f$ metastable levels. The $4p^64d^84f$ configuration levels with $9/2 \leq J \leq 15/2$ are metastable ones for the highly-charged ions with $Z \geq 60$. These ions are investigated thoroughly, using a large CI3 expansion within the basis of the quasirelativistic transformed radial orbitals.

The radiative lifetimes of the levels with $J = 15/2$ and $J = 13/2$ are determined by transitions to the levels with the same parity, i.e., the $M1$ and $E2$ transitions. It should be noted, that the $M1$ transition probability values for all investigated ions are much larger (more than 100 times) than the ones of the $E2$ transition. The radiative lifetimes of the levels $4d^8(^3F)4f \, ^4I_{15/2}$ and $4d^8(^3F)4f \, ^4I_{13/2}$ are extremely large (more than 10 seconds) due to their position in the energy spectra. Although these two levels are relatively very low in the spectra, the level $4d^8(^3F)4f \, ^4I_{15/2}$ has three open radiative decay channels through rather weak transitions (one $M1$ and two $E2$). The level $4d^8(^3F)4f \, ^4I_{13/2}$ has only one decay channel through the $E2$ transition.

Since the total number of possible $M1$ and $E2$ transitions depends on a particular level location, for the high-lying levels with $J = 13/2$, as well as for other high-lying metastable levels, the $M1$ transitions primarily determine the calculated level radiative lifetime as their probability values are significantly larger compared to those of the $E2$ transitions.

The lifetimes of the levels with $J = 11/2$ and $J = 9/2$ are also determined by the transitions to the levels of the same parity, but in this case, the $M2$ and $E3$ transitions to the ground configuration of the opposite parity are allowed. Since there are only two levels in the ground configuration, the number of $M2$ and $E3$ transitions is strictly defined. Only one $E3$ transition is possible from each level with $J = 11/2$; two $E3$ transitions and one $M2$ transition is allowed from each level with $J = 9/2$. The number of possible $E2$ and $M1$ transitions can vary depending on the location of the particular level in the energy spectra - when a particular level is located higher, the number of allowed $M1$ and $E2$ transitions becomes larger.

The $E3$ transition probability value is similar in magnitude with that of the $M1$ transitions and is larger than the transition probabilities of $E2$ for the levels with $J = 11/2$. For the levels with $J = 9/2$, the $M2$ transition probability has the largest value for most ions. The $E3$ transition probability values are smaller than those of the $M1$ transitions, but are larger than the $E2$ transition probabilities for these levels.

The inclusion of $M2$ and $E3$ transitions into the radiative lifetime calculation can
Metastable level properties of the excited configuration \(4p^64d^84f\) significantly change determined total radiative lifetime value. If the level position is comparatively high in the energy spectra, the \(M2\) and the \(E3\) transitions are not so significant, although they can not be neglected. For the Rh-like ions, the decay of the metastable levels through the \(M1\) and the \(M2\) transitions is more significant compared to the decay through the \(E3\) ones. The \(E2\) transition probability values are the smallest ones for all metastable levels, except for the level \(4d^8(3F)4f^{4}I_{13/2}\), which can decay only through the \(E2\) transition.

Acknowledgment

Current research is funded by the European Social Fund under the Global Grant measure, project VP1-3.1-ŠMM-07-K-02-013.

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