Relativistic effects of spin and pseudospin symmetries

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Abstract

Dirac Hamiltonian is scaled in the atomic units $\hbar = m = 1$, which allows us to take the non-relativistic limit by setting the Compton wavelength $\lambda \rightarrow 0$. The evolutions of the spin and pseudospin symmetries towards the non-relativistic limit are investigated by solving the Dirac equation with the parameter $\lambda$. With $\lambda$ transformation from the original Compton wavelength to 0, the spin splittings decrease monotonously in all spin doublets, and the pseudospin splittings increase in several pseudospin doublets, no change, or even reduce in several other pseudospin doublets. The various energy splitting behaviors of both the spin and pseudospin doublets with $\lambda$ are well explained by the perturbation calculations of Dirac Hamiltonian in the present units. It indicates that the origin of spin symmetry is entirely due to the relativistic effect, while the origin of pseudospin symmetry cannot be uniquely attributed to the relativistic effect.

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It is well known that the spin and pseudospin symmetries play critical role in the shell structure and its evolution. The introduction of spin-orbit potential made the single-particle shell model can well explain the experimentally observed existence of magic numbers for nuclei close to the valley of $\beta$-stability \cite{1,2}. To understand the near degeneracy observed in heavy nuclei between two single-particle states with the quantum numbers $(n-1, l+2, j = l + 3/2)$ and $(n, l, j = l + 1/2)$, the pseudospin symmetry (PSS) was introduced by defining the pseudospin doublets $(\tilde{n} = n-1, \tilde{l} = l+1, j = \tilde{l} \pm 1/2)$ \cite{3,4}, which has explained numerous phenomena in nuclear structure including deformation \cite{5}, superdeformation \cite{6}, identical bands \cite{7}, and magnetic moment \cite{8}. Because of these successes, there have been comprehensive efforts to understand their origins as well as the breaking mechanisms. For the spin symmetry (SS), the spin-orbit potential can be obtained naturally from the solutions of Dirac equation. Thus, the SS can be regarded as a relativistic symmetry. For the PSS, its origin has not been fully clarified until now. It is worth reviewing some of the major progresses in understanding the underlying mechanism of PSS. In Ref.\cite{9}, a helicity unitary transformation of a non-relativistic single-particle Hamiltonian was introduced to discuss the PSS in the non-relativistic harmonic oscillator. The particular condition between the coefficients of spin-orbit and orbit-orbit terms was indicated in the corresponding non-relativistic single particle Hamiltonian for the requirement of PSS. The same kind of unitary transformation was considered in Ref.\cite{10}, where the application of the helicity operator to the non-relativistic single-particle wavefunction maps the normal state $(l, s)$ onto the pseudo-state $(\tilde{l}, \tilde{s})$, while keeping all other global symmetries. A substantial progress was achieved in Ref.\cite{11}, where the relativistic feature of PSS was recognized. The pseudo-orbital angular momentum $\tilde{l}$ is nothing but the orbital angular momentum of the lower component of Dirac spinor, and the equality in magnitude but difference in sign of the scalar potential $S$ and vector potential $V$ was suggested as the exact PSS limit. Meng et al. showed that exact PSS occurs in the Dirac equation when the sum of the scalar $S$ and vector $V$ potentials is equal to a constant \cite{12}. Unfortunately, the exact PSS cannot be met in real nuclei, much effort has been devoted to the cause of splitting. In Ref.\cite{13-15}, it was pointed out that the observed pseudospin splitting arises from a cancellation of the several energy components, and the PSS in nuclei has a dynamical character. A similar conclusion was reached in Refs.\cite{16,17}. In addition, it was noted that, unlike the spin symmetry, the pseudospin breaking cannot be treated as a perturbation of pseudospin-symmetric Hamiltonian.
The non-perturbation nature of PSS has also been indicated in Ref. [19]. Regardless of these pioneering studies, the origins of the spin and pseudospin symmetries have not been fully understood in the relativistic framework. Recently, we have checked the PSS by use of the similarity renormalization group and shown explicitly the relativistic origin of this symmetry [20]. However, the dependence of the quality of PSS on the relativistic effect has not been checked until now. In this paper, we study the evolution of the spin and pseudospin symmetries from the relativistic to the non-relativistic to explore the relativistic relevance of this symmetries.

Dirac equation of a particle of mass $m$ in external scalar $S$ and vector $V$ potentials is given by

$$H = c\bar{\alpha} \cdot \vec{p} + \beta (mc^2 + S) + V,$$

where $\bar{\alpha}$ and $\beta$ are the usual Dirac matrices. For a spherical system, the Dirac spinor $\psi$ has the form

$$\psi = \frac{1}{r} \begin{pmatrix} iG_{nk}(r) \phi_{kmj}(\vartheta, \varphi) \\ F_{nk}(r) \vec{\sigma} \cdot \hat{r} \phi_{kmj}(\vartheta, \varphi) \end{pmatrix},$$

where $n$ is the radial quantum number, and $m_j$ is the projection of angular momentum on the third axis. $\kappa = \pm (j + 1/2)$ with $-$ for aligned spin ($s_{1/2}, p_{3/2}$, etc.), and $+$ for unaligned spin ($p_{1/2}, d_{3/2}$, etc.). Splitting off the angular part and leaving the radial functions satisfy the following equation

$$\begin{pmatrix} mc^2 + \Sigma(r) & -c \frac{d}{dr} + \frac{c \kappa}{r} \\ c \frac{d}{dr} + \frac{c \kappa}{r} & -mc^2 + \Delta(r) \end{pmatrix} \begin{pmatrix} G(r) \\ F(r) \end{pmatrix} = \varepsilon \begin{pmatrix} G(r) \\ F(r) \end{pmatrix},$$

where $\Sigma(r) = V(r) + S(r)$ and $\Delta(r) = V(r) - S(r)$. Based on Eq. (3), a lot of work has been carried out to check the origins of the spin and pseudospin symmetries [11, 13–15, 18, 19]. Although they are recognized as the symmetries of Dirac Hamiltonian, it is still not very clear the important role of relativistic effect. In order to explore the relativistic effects of these symmetries, the atomic units $\hbar = m = 1$ are adopted instead of the conventional relativistic units $\hbar = c = 1$ in the present system. For simplicity, the operator $H$ is measured in unit of the rest mass, $mc^2$. Then the equation (3) is presented as

$$\begin{pmatrix} 1 + \lambda^2 \Sigma & \lambda \left( -\frac{d}{dr} + \frac{\kappa}{r} \right) \\ \lambda \left( \frac{d}{dr} + \frac{\kappa}{r} \right) & -1 + \lambda^2 \Delta \end{pmatrix} \begin{pmatrix} G(r) \\ F(r) \end{pmatrix} = \varepsilon \begin{pmatrix} G(r) \\ F(r) \end{pmatrix},$$

where $\kappa = \pm (j + 1/2)$ with $-$ for aligned spin ($s_{1/2}, p_{3/2}$, etc.), and $+$ for unaligned spin ($p_{1/2}, d_{3/2}$, etc.).
where the Compton wavelength $\lambda = \hbar/mc = 1/c$. In this units, the result in the non-relativistic limit can be obtained in a very simple, intuitive, and straightforward manner by taking the speed of light $c \to \infty$ or the Compton wavelength $\lambda \to 0$, which is not possible in the latter units since $c = 1$.

In order to investigate the evolution from the relativistic to the non-relativistic, $\lambda$ is regarded as a parameter and the original Compton wavelength $\lambda = \hbar/mc$ is labelled as $\lambda_0$. The relativistic result corresponds to the solution of Eq.$(4)$ with $\lambda = \lambda_0$. The result in the non-relativistic limit can be obtained from Eq.$(4)$ by setting $\lambda \to 0$. Thus, the evolution from the relativistic to the non-relativistic can be checked by transforming $\lambda$ from $\lambda_0$ to 0. Then, the relativistic effects of the spin and pseudospin symmetries can be investigated by extracting the energy splittings between the spin or pseudospin doublets, and this symmetries develop toward the non-relativistic limit can be checked, and vice versa.

In order to make this clear, we have solved Eq.$(4)$ for a Woods-Saxon type potential for $\Sigma(r)$ and $\Delta(r)$, i.e., $\Sigma(r) = \Sigma_0 f(a_{\Sigma}, r_{\Sigma}, r)$ and $\Delta(r) = \Delta_0 f(a_{\Delta}, r_{\Delta}, r)$ with

$$f(a_0, r_0, r) = \frac{1}{1 + \exp\left(\frac{r-r_0}{a_0}\right)}.$$

The corresponding parameters are determined by fitting the energy spectra from the RMF calculations for $^{208}\text{Pb}$ (to see Ref. [21]). The energy spectra of Eq.$(4)$ are calculated by expansion in harmonic oscillator basis.

The single particle energy varying with the parameter $\lambda$ is displayed in Fig.1, where it can be seen that the energy decreases monotonously with $\lambda$ decreasing for all the levels available. The trend of energy with $\lambda$ is towards the direction of non-relativistic limit. With the decreasing of $\lambda$, the calculation is closer to the non-relativistic result. When $\lambda$ is reduced to $\lambda/\lambda_0 = 0.1$, the solution of Eq.$(4)$ is almost same as the non-relativistic result. Furthermore, for the different single-particle states, the sensitivity of energy to $\lambda$ is different. For the spin unaligned states, the decreasing of energy is faster than that for the spin aligned states, which leads to the energy splittings of the spin doublets reduce with $\lambda$ decreasing. When $\lambda$ is reduced to $\lambda/\lambda_0 = 0.1$, the spin-orbit splittings almost disappear for all the spin doublets. These indicate that the spin symmetry becomes better as $\lambda$ decreases, and the spin symmetry breaking is entirely due to the relativistic effect.

To better understand the preceding claim, the energies in several $\lambda$ values are listed in Table I for all single particle levels. For comparison, Table I does also display the data
of the non-relativistic calculations (the last column), which are obtained by solving the Schrödinger equation $H\psi(r) = E\psi(r)$ with $H = -\frac{\hbar^2}{2m} \left( \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} \right) + \Sigma(r)$. From Table I, it can be seen that the relativistic spin-orbit splitting ($\lambda = \lambda_0$) is considerably large. This splitting decreases with the decreasing of $\lambda$. When $\lambda/\lambda_0 = 0.001$, the energy of the spin unaligned state in conjunction with that of the spin aligned state degenerates to the non-relativistic result. These indicate that Eq. (4) reproduces well the process of development from the relativistic to the non-relativistic, and both results of the relativistic and non-relativistic can be obtained well from Eq. (4) with an appropriate value of $\lambda$. Hence, the relativistic effects of the spin and pseudospin symmetries can be checked from the solutions of Dirac equation with the parameter $\lambda$.

In order to recognize clearly the relativistic effect of spin symmetry, the energy splittings of spin doublets varying with $\lambda$ are plotted in Fig.2, where it is shown that the energy splittings decrease monotonously with $\lambda$ reducing for all the spin partners. When $\lambda$ is reduced to $\lambda/\lambda_0 = 0.1$, the energy splittings of all the spin doublets are almost reduced to zero. The detailed observation shows that the energy splittings are more sensitive to $\lambda$ for the states with higher orbital angular momentum in the same radial quantum number. For the states with the same orbital angular momentum, the energy splittings appear crosses in the different radial quantum number. These reflect that the relativistic sensitivity is different for the states with different quantum numbers. When $\lambda$ is reduced to zero, the spin-orbit splittings disappear for all the spin partners, the non-relativistic results are obtained in excellent agreement with those from the solutions of Schrödinger equation. Namely, the spin-orbit splitting arises completely from the relativistic effect, and can be treated as a perturbation of spin-symmetric Hamiltonian as indicated in Ref. [18].

Different from the spin symmetry, the relativistic origin of pseudospin symmetry is more complicated. In Fig.3, we display the energy splittings of pseudospin doublets varying with the parameter $\lambda$. From there, it can be observed that the energy splittings increase significantly with $\lambda$ decreasing for the pseudospin partners $(2g_9/2,1i_{11/2})$, $(2f_7/2,1h_{9/2})$, and $(3p_3/2,2f_{5/2})$. Especially for $(2g_9/2,1i_{11/2})$, the increasing of energy splitting is very obvious. For the doublets $(2d_{5/2},1g_{7/2})$, the increasing of energy splitting with $\lambda$ decreasing is relatively small. When $\lambda/\lambda_0$ decreases below than 0.6, the energy splitting goes toward a stable value. The same phenomenon also appears in the doublet $(3s_1/2,2d_{3/2})$. However for the pseudospin doublets $(2p_3/2,1f_{5/2})$ and $(2s_1/2,1d_{3/2})$, an opposite evolution of energy splitting with $\lambda$ is
disclosed. It shows the origin of pseudospin symmetry is more complicated than that of spin symmetry. The pseudospin splitting cannot be attributed uniquely to the relativistic effect. The quantum number of single-particle states and the shape of potential make important influence on this symmetry.

In order to better understand the relativistic effects of the spin and pseudospin symmetries, we expand perturbatively the Dirac Hamiltonian in Eq.(4) to analyze the effects of each higher order term on the energy splitting behaviors of both spin and pseudospin doublets. Following Ref. [20], for Dirac particle, the expanded Hamiltonian up to the order 1/m^3 is

$$H = \Sigma \left(\frac{p^2}{2m} - \lambda^2 \frac{1}{2m^2} \left(SP^2 - S' \frac{d}{dr}\right) - \lambda^2 \kappa \frac{\Delta'}{r4m^2} + \lambda^4 \frac{S}{2m^4} \left(SP^2 - 2S' \frac{d}{dr}\right) + \lambda^4 \kappa \frac{S\Delta'}{r2m^3}\right) + \lambda^2 \frac{\Sigma''}{8m^2} - \lambda^2 \frac{p^4}{8m^3} - \lambda^4 \frac{\Sigma''^2 - 2S'\Delta' + 4S\Sigma''}{16m^3},$$

where $p^2 = -\frac{d^2}{dr^2} + \frac{\kappa(\kappa+1)}{r^2}$. Based on the same considerations as Ref. [20], $H$ is decomposed into the eight components: $\Sigma \left(\frac{p^2}{2m}\right), -\lambda^2 \frac{1}{2m^2} \left(2SP^2 - S' \frac{d}{dr}\right), -\lambda^2 \kappa \frac{\Delta'}{4m^2}, +\lambda^4 \frac{S}{2m^4} \left(SP^2 - 2S' \frac{d}{dr}\right), +\lambda^4 \kappa \frac{S\Delta'}{2m^3}, +\lambda^2 \frac{\Sigma''}{8m^2}, -\lambda^2 \frac{p^4}{8m^3}, -\lambda^4 \frac{\Sigma''^2 - 2S'\Delta' + 4S\Sigma''}{16m^3}$, which are respectively labelled as $O_1, O_2, \cdots, O_8$. $O_1$ corresponds to the Hamiltonian in the non-relativistic limit, i.e., the Schrödinger part of $H$. $O_2(O_4)$ is the dynamical term relating to the order 1/m^2(1/m^3). $O_3(O_5)$ is the spin-orbit coupling corresponding to the order 1/m^2(1/m^3). The eigenvalues of $H$ are calculated with the fully same $\Sigma(r)$ and $\Delta(r)$ as that in calculating the exact solutions of Eq.(4).

For recognizing the relativistic effect of SS, we analyze the reason why the energies of the spin unaligned states decrease faster than those of the spin aligned states when $\lambda$ decreases. As an illustrated example, we display the energy splittings of every component $O_i(i = 1, 2, \cdots, 5)$ varying with $\lambda$ for the spin doublets $(1p_{1/2}, 1p_{3/2})$ and $(1g_{7/2}, 1g_{9/2})$ in Fig[1] where we neglect the results of $O_6, O_7, and O_8$ because their contributions to the energy splitting are minor and do not influence on the total energy splitting behavior with $\lambda$. From Fig[1] it can be seen that the contributions of all the $O_i(i = 2, 3, 4, 5)$ to the energy splittings between the spin unaligned states and the spin aligned states are positive, and the positive energy splittings decrease with $\lambda$ decreasing. It is for this reason that the energies of the spin unaligned states decrease faster than those of the spin aligned states with $\lambda$ decreasing. Compared with $O_3$ (the spin-orbit coupling corresponding to the order 1/m^2), and the contributions of $O_2, O_4$, and $O_5$ to the spin energy splittings are relatively minor. The total
energy splittings are dominated by the contribution of \( O_3 \) when \( \lambda \) is sufficiently small. This means that, as the relativistic effect becomes weak, the spin splittings are almost entirely due to the spin-orbit coupling. For the different spin partners, the energy splitting behaviors with \( \lambda \) are same except for the extent of splittings, as displayed in Fig.4 for the spin doublets \((1p_{1/2}, 1p_{3/2})\) and \((1g_{7/2}, 1g_{9/2})\). These indicate that the spin symmetry origins completely from the relativistic effect, and possesses the perturbation attribute claimed in Ref.[18]. To understand the relativistic effect of PSS, we analyze the cause of the various energy splitting behaviors of pseudospin doublets. In Fig.5, we show the energy splittings of each component \( O_i (i = 1, 2, \cdots, 5) \) varying with \( \lambda \) for the pseudospin partners \((2s_{1/2}, 1d_{3/2})\) and \((2f_{7/2}, 1h_{9/2})\). From there, it can be seen that the pseudospin energy splittings caused by the Schrödinger part of \( H \) are dominated. This splittings are reduced by the contribution of spin-orbit coupling, and added by the contribution of dynamical terms. For the pseudospin partner \((2s_{1/2}, 1d_{3/2})\), with the decreasing of \( \lambda \), the contribution of the pseudospin breaking (the dynamical terms) declines faster than that of the pseudospin improvement (the spin-orbit coupling), which results in better PSS when \( \lambda \) decreases. However for the \((2f_{7/2}, 1h_{9/2})\), the energy splittings caused by the pseudospin breaking varying with \( \lambda \) are relatively slower than that by the spin-orbit coupling, which leads to the PSS becomes worse with \( \lambda \) decreasing. These cause the different energy splitting behaviors of pseudospin doublets with \( \lambda \). Hence, the pseudospin splitting can not be regarded as a perturbation in agreement with the claim in Ref.[18].

In addition to the energy splittings associated with the relativistic effects, the wave function splittings between the (pseudo)spin doublets are also associated with the relativistic effects. An illustrated example is displayed in Fig.6 where the upper component of Dirac spinor for the states \( 1g_{7/2,9/2} \) is depicted in several \( \lambda \) values. From Fig.6, it can be seen that the wavefunction splitting of spin doublet is obvious for a relativistic particle \( (\lambda/\lambda_0 = 1) \). With the development towards the non-relativistic direction (to reduce \( \lambda \)), the wavefunction splitting of spin doublet decreases, which is in agreement with the case of level splitting. For the pseudospin symmetry, the lower component of Dirac spinor for the pseudospin doublet \((2g_{9/2}, 1i_{11/2})\) is drawn in Fig.7 in several \( \lambda \) values. The wavefunction splitting of pseudospin doublet is obvious when \( \lambda/\lambda_0 = 1 \). Different from the spin splitting, we can not see that the pseudospin splitting reduces with \( \lambda \) decreasing, which is consistent with the case of level splitting.
In summary, Dirac Hamiltonian is scaled in the atomic units $\hbar = m = 1$, which allows us to take the non-relativistic limit by setting the speed of light $c \to \infty$ or the Compton wavelength $\lambda \to 0$. The evolution towards the non-relativistic limit is investigated from the solutions of Dirac equation by a continuous transformation of the parameter $\lambda$. The solutions of Dirac equation corresponding to $\lambda = \hbar/mc$ and $\lambda = 0$ represent respectively the relativistic result and that in non-relativistic limit. To transform the parameter $\lambda$ from $\hbar/mc$ to 0, the solutions of Dirac equation show the evolution from the relativistic to the non-relativistic limit. The relativistic effects of the spin and pseudospin symmetries are checked from the solutions of Dirac equation with the parameter $\lambda$. It shows the spin splittings decrease monotonously with $\lambda$ reducing for all the spin partners. When $\lambda$ is reduced to zero, the spin-orbit splittings disappear, which is in agreement with the result in the non-relativistic calculations. For the pseudospin symmetry, the energy splittings increase in several partners, no change, or even decrease in another some partners. Compared with the spin symmetry, the origin of pseudospin symmetry is more complicated, and cannot be attributed uniquely to the relativistic effect. The quantum number of single-particle states and the shape of potential make important influence on this symmetry. By the perturbation calculations of Dirac Hamiltonian, the various energy splitting behaviors of both spin and pseudospin doublets with $\lambda$ are explained, which origins from the different contributions of each component to the energy splittings. The result supports the claim in Ref.[18], the spin splitting can be treated as a perturbation, while the pseudospin splitting can not be regarded as a perturbation. The same conclusion can also be obtained from the wavefunction.

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TABLE I: The relativistic bound energies \((E = \varepsilon - m, \text{in MeV})\) of a Dirac particle for Woods-Saxon potential with \(\lambda/\lambda_0 = 1, 0.5, 0.1, 0.01, 0.001\). The last column represents the non-relativistic results, which are obtained from the solutions of the Schrödinger equation \(H\psi(r) = E\psi(r)\) with \(H = -\frac{\hbar^2}{2m} \left( \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} \right) + \Sigma(r)\).

| \(\lambda/\lambda_0\) | 1       | 0.5     | 0.1     | 0.01    | 0.001   | non     |
|----------------------|---------|---------|---------|---------|---------|---------|
| 1s\(1/2\)            | -59.206 | -60.756 | -61.109 | -61.123 | -61.123 |
| 2s\(1/2\)            | -41.592 | -47.056 | -48.305 | -48.353 | -48.353 |
| 3s\(1/2\)            | -18.358 | -27.968 | -30.287 | -30.376 | -30.377 |
| 1p\(3/2\)            | -52.763 | -55.631 | -56.306 | -56.332 | -56.332 |
| 1p\(1/2\)            | -52.263 | -55.578 | -56.304 | -56.332 | -56.332 |
| 2p\(3/2\)            | -31.401 | -38.680 | -40.396 | -40.462 | -40.463 |
| 2p\(1/2\)            | -30.611 | -38.576 | -40.393 | -40.462 | -40.463 |
| 3p\(3/2\)            | -7.694  | -17.957 | -20.633 | -20.737 | -20.738 |
| 3p\(1/2\)            | -6.999  | -17.822 | -20.628 | -20.737 | -20.738 |
| 1d\(5/2\)            | -45.234 | -49.459 | -50.493 | -50.533 | -50.534 |
| 1d\(3/2\)            | -44.055 | -49.329 | -50.489 | -50.533 | -50.534 |
| 2d\(5/2\)            | -20.999 | -29.752 | -31.897 | -31.980 | -31.981 |
| 2d\(3/2\)            | -19.573 | -29.543 | -31.890 | -31.980 | -31.981 |
| 1f\(7/2\)            | -36.882 | -42.381 | -43.786 | -43.841 | -43.842 |
| 1f\(5/2\)            | -34.775 | -42.137 | -43.779 | -43.841 | -43.842 |
| 2f\(7/2\)            | -10.759 | -20.436 | -22.933 | -23.031 | -23.032 |
| 2f\(5/2\)            | -8.777  | -20.102 | -22.922 | -23.031 | -23.032 |
| 1g\(9/2\)            | -27.921 | -34.508 | -36.276 | -36.346 | -36.347 |
| 1g\(7/2\)            | -24.701 | -34.114 | -36.264 | -36.346 | -36.347 |
| 1h\(11/2\)           | -18.545 | -25.944 | -28.044 | -28.127 | -28.128 |
| 1h\(9/2\)            | -14.117 | -25.366 | -28.025 | -28.127 | -28.128 |
| 1i\(13/2\)           | -8.942  | -16.792 | -19.167 | -19.262 | -19.263 |
| 1i\(11/2\)           | -3.361  | -16.000 | -19.141 | -19.262 | -19.263 |

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FIG. 1: (Color online) Variation of single particle energy with $\lambda/\lambda_0$.

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FIG. 2: (Color online) The energy splittings of spin doublets $\Delta E = E_{n,l-1/2} - E_{n,l+1/2}$ varying with $\lambda/\lambda_0$.

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FIG. 3: (Color online) The energy splittings of pseudospin doublets $\Delta E = E_{n,\tilde{l}-1/2} - E_{n-1,\tilde{l}+1/2}$ varying with $\lambda/\lambda_0$. 
FIG. 4: (Color online) The spin energy splittings of each component $O_i$ ($i = 1, 2, \ldots, 5$) varying with $\lambda$, where the splittings caused by the $O_1$, $O_2$, $\ldots$, $O_5$ are respectively labelled as nonrela, dynam1, spin-orb1, dynam2, spin-orb2, and the total energy splitting is labelled as total.
FIG. 5: (Color online) The same as Fig. 4 but for the pseudospin energy splittings.

FIG. 6: (Color online) The upper component of Dirac spinor $G(r)/r$ for the spin doublet $(1g_{7/2}, 1g_{9/2})$ with $\lambda/\lambda_0 = 1.0, 0.8, 0.6, 0.4$. 

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FIG. 7: (Color online) The lower component of Dirac spinor $F(r)/r$ for the pseudospin doublet $(2g_9/2,1i_{11}/2)$ with $\lambda/\lambda_0 = 1.0, 0.8, 0.6, 0.4$. 