The Large $N$ Limit of $\mathcal{N} = 1$ Field Theories from F Theory

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Abstract

We study AdS/CFT correspondence in four dimensional $\mathcal{N} = 1$ field theories realized on the worldvolume of D3 branes near the intersections between D7 and D7’ branes in F theory studied by Aharony et al. We consider the compactification of F theory on elliptically fibered Calabi-Yau threefolds corresponding to two sets of parallel D7 branes sharing six spacetime directions. This can be viewed as orbifolds of six torus $T^6$ by $\mathbb{Z}_p \times \mathbb{Z}_q (p, q = 2, 3, 4, 6)$. We find the large $N$ spectrum of chiral primary operators by exploiting the property of AdS/CFT correspondence. Moreover, we discuss supergravity solutions for D3 branes in D7 and D7’ branes background.

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1 Introduction

In [1], the large $N$ limit of superconformal field theories (SCFT) was described by taking the supergravity limit for compactifications on anti-de Sitter (AdS) space. The correlation functions of SCFT can be obtained from the supergravity action dependence on the fields at the boundary of the AdS space [2, 3, 4]. In particular, the $\mathcal{N} = 4$ super $SU(N)$ Yang-Mills theory in 4 dimensions is obtained in the compactification of type IIB string theory on $AdS_5 \times S^5$. The gauge group can be replaced by $SO(N)/Sp(N)$ [5] by taking appropriate orientifold operations (See related works [6, 7, 8]).

There are $\mathcal{N} = 2, 1, 0$ superconformal theories in 4 dimensions for which the corresponding supergravity description on orbifolds of $AdS_5 \times S^5$ has been initiated in [9, 10]. This proposed duality was tested by studying the Kaluza-Klein (KK) states of supergravity theory and by comparing them with the chiral primary operators of the SCFT on the boundary [11]. Furthermore, the field theory/M theory duality gives rise after compactification of on $AdS_4$ or $AdS_7$ to some superconformal theories in 3 and 6 dimensions, respectively. The maximally supersymmetric theories have been studied in [3, 12, 13, 14, 15] and the lower supersymmetric case was also realized on the world-volume of M theory at orbifold singularities [16] (See also [17]). Along the line of [11], the KK states of supergravity theory on the orbifolds of $AdS_4 \times S^7$ were studied and compared with the chiral primary operators of the SCFT on the boundary [18, 19]. The analysis for orbifolds of $AdS_7 \times S^4$ was worked out in [20]. The KK spectrum description on the twisted states of $AdS_5$ orbifolds was discussed in [21].

Recently, it has been found [22] that in $\mathcal{N} = 2, 1$ field theories by worldvolume of D3 branes near D7 branes, their large $N$ spectrum of chiral primary operators is computed by using their string theory duals. (Other related papers are [21, 22, 23, 24, 25, 26].) In this paper, we discuss the following two things. i) First, we will compute the large $N$ spectrum of $\mathcal{N} = 1$ SCFT in section 3 which was not done in [22] explicitly. ii) Second, we discuss the qualitative property of supergravity solution in the nonconformal case in section 4.

In section 2, we review some known results for the moduli space of F theory on elliptically fibered Calabi-Yau threefolds where the coupling remains constant. In section 3, we can read the value for $k$ characteristic for the irreducible representations of $SU(4)$ isometry group from the KK spectrum of original $AdS_5 \times S^5$ from the metric describing the D7 and D7’ brane background. The value for $k$ are determined by the three momenta corresponding to the three angular variables coming from three $S^1$’s and two additional nonnegative integers [22]. We can read off $U(1)_R$ charge from the relation between the dimension of chiral operators and $U(1)_R$ charge in the $\mathcal{N} = 1$ superconformal algebra. We see the representations giving rise to chiral primary operators in the boundary $\mathcal{N} = 1$.
Finally in section 4, we consider the solution of supergravity in nonconformal case in D7 and D7' branes background qualitatively.

# 2 F Theory on Calabi-Yau Threefolds at Constant Coupling

In this section we review the part of [27] which is necessary for our present aim. Let us consider the compactification of F theory on elliptically fibered CY 3-fold over $\mathbb{CP}^1 \times \mathbb{CP}^1$ where the coupling is constant over the base.

Let $z$ and $w$ be the affine coordinates \(^*\) of the base $\mathbb{CP}^1 \times \mathbb{CP}^1$. Then the elliptically fibered CY 3-fold can be described \([28, 29]\) in the Weirstrass form

$$y^2 = x^3 + f(z, w)x + g(z, w)$$

where $f$ and $g$ are the polynomials of degree 8, 12 respectively in each of the variables. The $j$-invariant of the modular parameter $\tau(z, w)$ of the fiber is given by

$$j(\tau(z, w)) = \frac{4(24f(z, w))^3}{\Delta(z, w)},$$

(2.1)

where the discriminant $\Delta(z, w)$ is

$$\Delta(z, w) = 4(f(z, w))^3 + 27(g(z, w))^2.$$

The fibers are degenerated when $\Delta(z, w) = 0$ and the type IIB 7-branes are located at these points. The $j$-invariant is invariant under $SL(2, \mathbb{Z})$ transformation of $\tau$ and determines the complex structure of the elliptic fiber.

By requiring the modular parameter $\tau(z, w)$ to be constant up to an $SL(2, \mathbb{Z})$ transformation, we obtain $f^3 \sim g^2$. To simplify the problem, we will assume that $f$ and $g$ are factorized. That is, $f(z, w) = \alpha f_1(z)f_2(w)$ and $g(z, w) = g_1(z)g_2(w)$ where $\alpha$ is a constant. Then we have the following possible cases.

Case 1): $\mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

The solution for constant modulus obtained by rescaling $y$ and $x$ and setting the overall coefficient to be 1 has been found in [30] in the form:

$$f_1(z) = \prod_{i=1}^{4}(z - z_i)^2, \quad f_2(w) = \prod_{i=1}^{4}(w - w_i)^2,$$

\(^*\) We define $z \equiv x^8 + ix^9$ and $w \equiv x^6 + ix^7$ in section 3.
$$g_1(z) = \prod_{i=1}^{4}(z - z_i)^3, \quad g_2(w) = \prod_{i=1}^{4}(w - w_i)^3, \quad (2.2)$$

where $z_i$'s and $w_i$'s are constants.

Case 2) : $T^6/Z_3 \times Z_2$

As pointed out in [31], in the limit of $\alpha \to 0$, we get $j(\tau(z, w)) = 0$ from which $\tau(z, w) = e^{\frac{2\pi}{3}}$. The polynomials are given by

$$f_1(z) = 0, \quad g_1(z) = \prod_{i=1}^{3}(z - z_i)^4,$$
$$f_2(w) = \prod_{i=1}^{4}(w - w_i)^2, \quad g_2(w) = \prod_{i=1}^{4}(w - w_i)^3, \quad (2.3)$$

where the 12(12) zeroes of $g_1(z)(g_2(w))$ coalesce into 3(4) identical ones of order 4(2) each.

Case 3) : $T^6/Z_3 \times Z_3$

When the 12 zeroes of $g_2(w)$ have coalesced into 3 identical ones of order 4 and the ones of $g_1(z)$ are given as in eq.(2.3), we have the following:

$$f_1 = 0, \quad f_2 = 0, \quad g_1(z) = \prod_{i=1}^{3}(z - z_i)^4, \quad g_2(w) = \prod_{i=1}^{4}(w - w_i)^4, \quad (2.4)$$

where the discriminant is given by $\Delta(z, w) = 27 \prod_{i=1}^{3}(z - z_i)^8 \prod_{j=1}^{3}(w - w_j)^8$.

Case 4) : $T^6/Z_3 \times Z_6$

If the 12 zeroes of $g_2(w)$ has coalesced into 3 zeroes of order 5, 4, 3 each and those of $g_1(z)$ are the same as before like (2.3), it is easy to see that we have the following:

$$f_1 = 0, \quad f_2 = 0, \quad g_1(z) = \prod_{i=1}^{3}(z - z_i)^4, \quad g_2(w) = (w - w_1)^5(w - w_2)^4(w - w_3)^3. \quad (2.5)$$

Each point $w = w_i$ on the second $\mathbb{CP}^1$ factor has a deficit angle of $5\pi/3, 4\pi/3$ and $\pi$ all together deforming the $\mathbb{CP}^1$ to $T^2/Z_6$.

Case 5) : $T^6/Z_4 \times Z_2$

Another possibility is as follows:

$$f_1(z) = (z - z_1)^3(z - z_2)^3(z - z_3)^2, \quad g_1(z) = 0,$$
\[ f_2(w) = \prod_{i=1}^{4} (w - w_i)^2, \quad g_2(w) = \prod_{i=1}^{4} (w - w_i)^3, \] (2.6)

which corresponds to \( \tau = i \) from \( j(\tau(z, w)) = 13824. \)

Case 6) \( \mathbf{T}^6/\mathbb{Z}_4 \times \mathbb{Z}_4 \)

Finally, we have the case when the 8 zeroes of \( f_1(z)(f_2(w)) \) coalesce into 3 of orders 3, 3 and 2 respectively and \( g_1 = g_2 = 0 \)

\[ f_1(z) = (z - z_1)^3(z - z_2)^3(z - z_3)^2, \quad g_1(z) = 0, \]
\[ f_2(w) = (w - w_1)^3(w - w_2)^3(w - w_3)^2, \quad g_2(w) = 0. \] (2.7)

Note that there are other two cases, the intersection of \( SO(8) \) and \( E_8 \), the intersection of \( E_8 \) and \( E_8 \) which are ruled out because they violate the CY 3-fold condition. The intersection of \( E_6 \) and \( E_7 \) and the intersection of \( E_7 \) and \( E_8 \) do not live in F theory moduli space where the couplings remain constant. In the next section, we will consider how the above six cases can be realized in the context of AdS/CFT correspondence and how to obtain predictions for the full large \( N \) spectrum of chiral primaries.

3 The Surviving Kaluza-Klein Spectrum

Let us take \( N \) D3 branes with the worldvolume along \((x^0, x^1, x^2, x^3)\), the appropriate number of D7 branes with worldvolumes along \((x^0, x^1, x^2, x^3, x^4, x^5, x^6, x^7)\), and D7' branes in \((x^0, x^1, x^2, x^3, x^4, x^5, x^8, x^9)\). By adding D3 branes into D7 and D7' brane system, the worldvolume field theory has \( \mathcal{N} = 1 \) supersymmetry. The position of the D7 brane is given by \( z \), while the D7' brane position is given by \( w \). As usual, O7 plane and O7' plane have their worldvolume parallel with the ones of the D7 and D7' brane respectively.

\[
\text{D3} : (x^0, x^1, x^2, x^3)\\n\text{D7/O7} : (x^0, x^1, x^2, x^3, x^4, x^5, x^6, x^7)\\n\text{D7'/O7'} : (x^0, x^1, x^2, x^3, x^4, x^5, x^8, x^9). \] (3.1)

Let us study the F theory on the vicinity of a CY 3-fold singularity (local F theory geometry) which corresponds to coincident D7 and D7' branes in type IIB theory where the complexified coupling (modular parameter of the fiber) \( \tau(z, w) \) is a constant. The
The metric describing the D7 and D7' brane background looks like $AdS_5 \times S^5$ and can be (up to a constant normalization) summarized by:

\[
SO(8) \times SO(8): \quad ds^2 = |z^{-\frac{1}{2}}dz|^2 + |w^{-\frac{1}{2}}dw|^2, \\
E_6 \times SO(8): \quad ds^2 = |z^{-\frac{2}{3}}dz|^2 + |w^{-\frac{4}{3}}dw|^2, \\
E_6 \times E_6: \quad ds^2 = |z^{-\frac{2}{3}}dz|^2 + |w^{-\frac{4}{3}}dw|^2, \\
E_7 \times SO(8): \quad ds^2 = |z^{-\frac{3}{4}}dz|^2 + |w^{-\frac{1}{2}}dw|^2, \\
E_7 \times E_7: \quad ds^2 = |z^{-\frac{3}{4}}dz|^2 + |w^{-\frac{1}{2}}dw|^2.
\]

The metric can be explained in the same way as in [8]. These spaces are described as the orbifolds $C^2/Z_p \times Z_q$ with $p, q = 2, 3, 4, 6$ that have been already considered in section 2. Thus the covering space is the complex $u, v$ surface, with $z = u^p$ and $w = v^q$. Now by descending the Euclidean metric $ds^2 = |du|^2 + |dv|^2$ on the covering space, we obtain the desired metric which is equivariant under the orientifold action $u \rightarrow \exp(2\pi i/p)u$ and $v \rightarrow \exp(2\pi i/q)v$.

At various types of intersection points there exists an enhancement of gauge symmetries which gives rise to conformal field theories on the D3 branes. For example when one considers Case 1) for which there are $SO(8)$ gauge fields living both on the D7 and D7' branes, there exists an $SO(8) \times SO(8)$ global symmetry on the D3 branes.

We can choose coordinates so that the angular part of (3.2) becomes \[22\]
\[
ds^2 = d\theta^2 + \cos^2 \theta d\phi_1^2 + \sin^2 \theta \left[ d\psi^2 + \cos^2 \psi d\phi_2^2 + \sin^2 \psi d\phi_3^2 \right],
\]
with $\alpha$ deformed boundary conditions which will appear later and monodromies in the $\phi_1$ and $\phi_2$ variables. We have various types of singularities at $\sin \theta = 0$ which is an $S^3$ in the compact space. This means two intersection of singularity types coming from D7 and D7' branes along two $S^3$ in the compact space which intersect along an $S^1$.

The Laplacian in this metric is
\[
\nabla^2 = \frac{1}{\sin^4 \theta \cos \theta} \frac{d}{d\theta} \sin^3 \theta \cos \theta \frac{d}{d\theta} + \frac{1}{\sin^2 \theta \sin \psi} \sin \psi \cos \psi \frac{d}{d\psi} \\
- \frac{m_1^2}{\cos^2 \theta} - \frac{m_2^2}{\sin^2 \theta \cos^2 \psi} - \frac{m_3^2}{\sin^2 \theta \sin^2 \psi},
\]
where $m_1^2, m_2^2$ and $m_3^2$ are the eigenvalues of the Laplacian on three $S^1$’s respectively. The $SU(4)$ isometry corresponding to the global symmetry in 4 dimensional field theory breaks into $U(1) \times U(1) \times U(1)$ due to the presence of D7 and D7' branes. The $U(1)_R$
charge in the $\mathcal{N} = 1$ superconformal algebra can be written as the sum of the charges for the $SO(2)$'s acting on $\phi_1, \phi_2, \phi_3$, so it is:

$$U(1)_R = 2(m_1 + m_2 + m_3).$$  \hfill (3.5)

The spherical harmonics, denoted by $Y(m_1, m_2, m_3, m, n)$ are classified by the momenta $m_1, m_2$ and $m_3$ in the $\phi$ variables and by two additional non-negative integers $m$ and $n$, such that the eigenvalue of the total Laplacian \[22\] is $k(k + 4)$ where $k = |m_1| + |m_2| + |m_3| + 2m + 2n$. In the $S^5$ case all these numbers are integers. In the present case the periodicity conditions on $\phi_1, \phi_2$ are different, so $m_i = \tilde{m}_i/(1 - \alpha_i/2)$ ($i = 1, 2$), but $m_3, n$ and $m$ are still integers. The values of $\alpha_i$'s are corresponding to each singularity types: $1, 4/3, 3/2, 5/3$ for $SO(8), E_6, E_7$ and $E_8$. Note that only six combinations are possible as discussed in section 2. We have the eigenfunctions $e^{i\tilde{m}_i\phi/(1 - \alpha_i/2)}$ rather than $e^{im\phi}$. Then

$$k = \frac{|\tilde{m}_1|}{(1 - \alpha_1/2)} + \frac{|\tilde{m}_2|}{(1 - \alpha_2/2)} + |m_3| + 2m + 2n. \hfill (3.6)$$

From the allowed values for $\alpha_1$, we see that $k$ has always integer values.

We are now ready to compare the KK spectrum of supergravity and corresponding chiral primary operators living on the boundary of $AdS_5$. Using the relation between the dimensions of the $R$-symmetry representations, we can see that the condition to obtain a chiral primary field in the superconformal algebra is $m = n = 0$. In this case,

$$k = \frac{|\tilde{m}_1|}{(1 - \alpha_1/2)} + \frac{|\tilde{m}_2|}{(1 - \alpha_2/2)} + |m_3| \hfill (3.7)$$

We can now make a discussion over some surviving KK states. This can be done by decomposing $SU(4)$ representations of KK states of four dimensional $\mathcal{N} = 4$ supersymmetry under representations of the smaller global symmetry group of our theory with $\mathcal{N} = 1$ and keeping only states with right $R$ symmetry charges which are determined by scaling dimensions of corresponding chiral primary operators.

- Scalar fields with Dynkin labels $(0, k, 0), \ k = 2, 3, 4, \cdots$

Let us consider the case of $k = 2$ which would correspond to one non-zero $\tilde{m}$ or to $m_3 = 2$. How does the representation $20'$ of $SU(4)$? The corresponding dimension $\Delta = k = 2$ of the chiral primary operator has $R$ charge $4/3$. Obviously there is no solution for nonnegative integers $\tilde{m}_1, \tilde{m}_2$ and $m_3$ satisfying this constraint. There is no any dimension 2 chiral primary operators in the boundary $\mathcal{N} = 1$ CFT. We can go on further to $k = 3$. How does the representation $50$ of $SU(4)$ go? The dimension $\Delta = k = 3$ chiral primary operator has $R$ charge 2. One of the solutions, $m_1 = m_2 = 1, m_3 = -1$ which will give the right number of $k$. Since it does not depend on the values of $\alpha_i$.
and \( \alpha_2 \), all the six possibilities we had in section 2 are valid in this case. So we expect dimension 3 chiral primary operators in the boundary \( \mathcal{N} = 1 \) CFT. That is, there exists a state given by the spherical harmonics, \( Y(1,1,-1,0,0) \). What happens for \( k = 4 \)? That is, 105. The corresponding dimension \( \Delta = k = 4 \) chiral primary operator has \( R \) charge 8/3. There is no solution for \( \tilde{m}_1, \tilde{m}_2 \) and \( m_3 \) satisfying this constraint like as the previous consideration.

- Scalar fields with Dynkin labels \((0,k,2)\), \( k = 0, 1, 2, \ldots \)

Take the representation 45 of \( SU(4) \)? This is the case \( k = 1 \). The dimension \( \Delta = k + 3 = 4 \) chiral primary operators of the boundary theory should have \( U(1)_R \) charge 8/3. Again this does not lead to any chiral primary operators in the boundary.

So far we have studied the chiral primaries of \( \mathcal{N} = 1 \) SCFT from bulk modes. In addition to that, there also exists a contribution from fields living at the singularities. The spectrum of operators is known when we consider the intersection of two \( SO(8) \) singularities which is \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) orientifold of the type IIB string theory [30]. For \( N \) D3 brane probes the theory on D3 branes is \( Sp(2N) \times Sp(2N) \) gauge theory with two chiral superfields in \((2N, 2N)\) representation from the D3 brane coordinates along the \((x^0, x^7, x^8, x^9)\) directions, superfields in the \((N(2N - 1) - 1, 1) \oplus (1, 1) \oplus (1, N(2N - 1) - 1) \oplus (1, 1)\) from the D3 brane coordinates in the \((x^4, x^5)\), eight \((2N, 1)\) which arises from D7 branes located at a particular value of \( z \) and \((1, 2N)\) from the ones at particular value of \( w \), superfields from the D3-D7 strings. However, it is not known how to write down appropriate superpotential which preserve the gauge symmetry and the global symmetry. Unlike the case of single singularity [22], it is not clear how to compare the analysis of KK mode on the \( S^1 \) with the corresponding field theory.

## 4 Supergravity Solution for 3 Branes

So far we considered particular limit of orbifold approach where the metric is known. In this section we will comment on some qualitative features of D3 brane solution in the background of D7 and D7' branes. Let us assume that there exists a supergravity solution:

\[
    ds^2 = dx_{\parallel}^2 + g_{ij}dx^i dx^j, \tag{4.1}
\]

where \( dx_{\parallel}^2 \) is the flat Minkowski metric in the directions \((x^0, x^1, x^2, x^3)\). As before, we realize this geometrically as an elliptically fibered CY 3-fold over a base \( \mathbb{C}P^1 \times \mathbb{C}P^1 \). The metric \( g \) is a Kähler metric and the complexified IIB coupling \( \tau(z, w) = \chi + ie^{-\phi} \) determines the modular invariants of the elliptic fiber of CY 3-fold over the base \( \mathbb{C}P^1 \times \mathbb{C}P^1 \).
\( \text{CP}^1 \). We assume that the metric of the base \( ds^2_{\text{Base}} \) is diagonal
\[
d s^2_{\text{Base}} = g_{zz}(z, \bar{z})dz d\bar{z} + g_{w\bar{w}}(w, \bar{w})dw d\bar{w}.
\] (4.2)

Here we denote the functional dependence of \( g_{zz} \) and \( g_{w\bar{w}} \) explicitly. So far a general solution for this metric is not known. However there is a partial attempt to get a particular solution. For example, by solving the Einstein equations, Asano [33] provided a metric \( g_{zz} = \text{Im} \tau \mid \eta^2 \prod_{i=1}^n (z - z_i)^{-1/12} \mid^2 \), \( g_{w\bar{w}} = | \prod_{i=1}^n (w - w_i)^{-1/2} \mid^2 \); where the Dedekind \( \eta \) function is given by \( \eta(\tau) = q^{1/24} \prod_{n=1}^\infty (1 - q^n) \), \( q = \exp(2\pi i \tau) \). The positions of D7 and D7’ branes are given by \( z_i \) and \( w_i \) and the modular invariant \( \tau(z, w) \) is a function of \( z \) only for this solution. Since the fibration itself is not symmetric w.r.t. \( z \) and \( w \), it is not strange to have an asymmetric metric of the base. One can see that both \( g_{zz}dz d\bar{z} \) and \( g_{w\bar{w}}dw d\bar{w} \) describe the metric on \( \text{CP}^1 \) globally because \( g_{zz} \to (z \bar{z})^{-2} \) as \( z \to \infty \) and \( g_{w\bar{w}} \to (w \bar{w})^{-2} \) as \( w \to \infty \).

Let us introduce D3 branes into this problem: this can be done in such a way as to respect the identification under the orientifold group alternatively. The solution for D3 branes reads [34]
\[
d s^2 = f^{-1/2}dx^2_\parallel + f^{1/2}g_{ij}dx^i dx^j,
\] (4.3)

and the five form field \( F_{0123i} \) is given by \( F_{0123i} = -\frac{1}{4}\partial_i f^{-1} \) and \( f(x^i) \) is a function of the coordinates transverse to D3 branes. Then we have to solve
\[
\frac{1}{\sqrt{g}} \partial_i (\sqrt{g} g^{ij} \partial_j f) \sim -\sqrt{g} N \delta^6(x - x^0),
\] (4.4)

where the right hand side indicates a source term at the position of \( N \) D3 branes. The equation for this leads to
\[
\left[ g_{zz} g_{w\bar{w}} \nabla_y^2 + 2 (g_{w\bar{w}} \partial_z \bar{z} + g_{zz} \partial_w \partial_{\bar{w}}) \right] f \sim -N \delta^2(\bar{z} - z_0) \delta^2(w - w_0) \delta^2(y).
\] (4.5)

It is difficult to find an exact solution for this so we will solve it perturbatively. Let us consider the case of regular metric at \( z_0 \) and \( w_0 \) and in the new variables, \( \bar{z} \) and \( \bar{w} \) they satisfy \( \bar{z}_0 = \bar{w}_0 = 0 \). Then the solution for (4.3) can be obtained by expanding \( g \) and \( f \) like as \( g = 1 + c(|\bar{z}|^2 + |\bar{w}|^2) + \cdots \) and \( f = f_0 + f_1 + \cdots \), iteratively. One can find
\[
f_0 \sim \frac{N}{(y^2 + 2|\bar{z}|^2 + 2|\bar{w}|^2)^2}
\] (4.6)

which behaves as \( N/y^4 \) for small \( \bar{z} \) and \( \bar{w} \) and
\[
\left[ \nabla_y^2 + 2 \partial_{\bar{z}} \partial_{\bar{z}} + 2 \partial_{\bar{w}} \partial_{\bar{w}} \right] f_1 = -c \left(|\bar{z}|^2 + |\bar{w}|^2\right) \nabla_y^2 f_0.
\] (4.7)

From this the expression for \( f_1 \) is given by
\[
f_1 \sim -\frac{4c}{3} \left( \frac{|\bar{z}|^4 + |\bar{w}|^4 + 2|\bar{z}|^2 |\bar{w}|^2}{(y^2 + 2|\bar{z}|^2 + 2|\bar{w}|^2)^2} \right) f_0.
\] (4.8)

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which is small when both $\bar{z}$ and $\bar{w}$ are near zero value. Plugging these values back all other terms in $f$ can be obtained.

One can also study D3 branes moving on an ALE space or near conifold singularity. A theory away from the conformal point can be obtained by moving D3 branes away. Again we need to solve (4.4) in this background. In principle we can generalize our present method to three types of parallel D7 branes with any two sharing 6 dimensions and all of them sharing 4 dimensions keeping 4 supercharges (this can be realized by F theory on Calabi-Yau fourfold). In this case the supersymmetry can be preserved when we add extra D3 branes in appropriate directions. It would be interesting to elaborate this further.

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